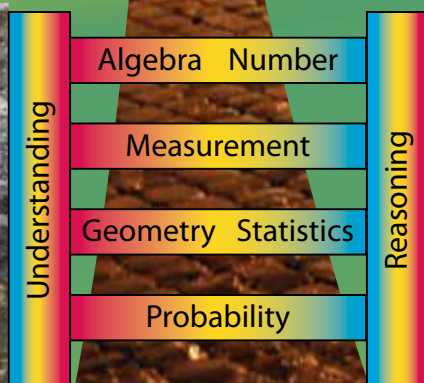


Mathematics

7



Mathematics

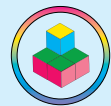
National Curriculum



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The cover: Maths is the pathway to success in our technological economy and society.

“In today’s world, economic access and full citizenship depend crucially on math and science literacy.” - Robert P. Moses.

“Mathematics makes a significant contribution to our modern society; its basic skills are vital for the life opportunities of the youth. Let us all embrace and encourage the study of this indispensable subject.”

- Alfred L. Teye.

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Preface

This text has been written for Year 7 students. The aim of the text is to assist students in investigating and understanding the exciting and very important world of Mathematics and to implement the intent of the Australian Mathematics Curriculum.

A literature review of learning from school textbooks was used to enhance the format of this textbook.

Each chapter, apart from Review, contains:

- ★ Numerous worked examples
- ★ Numerous sets of graded exercises
- ★ An open-ended rich task
- ★ Mental computation
- ★ Technology in mathematics
- ★ Investigations
- ★ Puzzles
- ★ NAPLAN questions
- ★ Maths competition preparation
- ★ A mathematics game
- ★ A mathematics trick
- ★ A bit of mathematics history
- ★ Careers using mathematics
- ★ Chapter review

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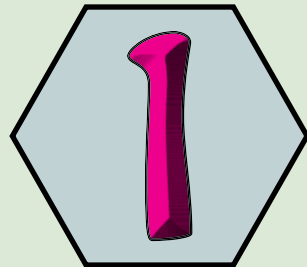
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- ★ **Workprogram**
- ★ **Study guides** for each term.
- ★ Detailed **lesson plans** for each term.
- ★ Sample **assessment items**.
- ★ **Ebooks** of this textbook.

Number 1



- ★ Express one quantity as a fraction of another.
- ★ Connect fractions, decimals and percentages and carry out simple conversions.
- ★ Find percentages of quantities and express one quantity as a percentage of another.
- ★ Justify choice of written, mental or calculator strategies for solving specific problems including those involving large numbers.

A TASK

The Babylonian base 60 system used only two symbols.

∟ to count units.

∟ to count tens.

Mesopotamia, the Tigris and Euphrates river system around 3000 BC, is considered the cradle of civilisation.

The fertility and wealth of Mesopotamia was due to the regular flooding and deposit of rich silts onto the vast fields. Because this happened about every 360 days, 60 was chosen as a number base. Have you noticed how many things have six as a factor?

Debate the statement: “Changing to base 10 was a dumb act. We would have been better staying with base 6”.

A LITTLE BIT OF HISTORY

In 1542, Robert Recorde published a method of multiplying up to 10×10 by knowing only the 5×5 times table.

Example: 9×7

9 7 Write 9 and 7 in the top row.
1 3 The bottom row is the difference
 \times between each number and 10.

The tens is the difference in the diagonals: $9-3 = 7-1 = 6$

The unit is the product of the numbers in the bottom row: $1 \times 3 = 3$

Thus: $9 \times 7 = 63$



60 secs = 1 minute.
60 mins = 1 hour.
 360° = 1 circle.
12 months = 1 year.

Fractions

$$\frac{a}{b}$$

← Numerator

← Denominator

Who invented fractions?
Henry the Eighth.



Proper fraction a less than b

Examples: $\frac{3}{5}$, $\frac{15}{17}$, $\frac{165}{219}$

Improper fraction a greater than b

Examples: $\frac{3}{2}$, $\frac{9}{2}$, $\frac{538}{47}$

Mixed number whole number **and** fraction.

Examples: $2\frac{3}{4}$, $3\frac{2}{5}$, $1\frac{19}{24}$

Exercise 1.1

Write the following words as fractions:

One-half = $\frac{1}{2}$	One-fifth = $\frac{1}{5}$	Two and one-half = $2\frac{1}{2}$
Three-quarters = $\frac{3}{4}$	Three-fifths = $\frac{3}{5}$	Four and three-quarters = $4\frac{3}{4}$

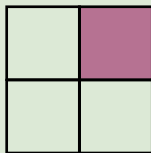
- | | |
|--------------------------|-----------------------------|
| 1 One-half | 2 Two-thirds |
| 3 One-quarter | 4 Four-fifths |
| 5 Five and one-quarter | 6 Two and seven-eighths |
| 7 Six and five-ninths | 8 Three and seven-elevenths |
| 9 Two and three-sevenths | 10 Three-fifths |

Write the following fractions as words:

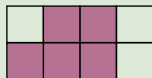
- 11 $\frac{1}{5}$ 12 $\frac{2}{3}$ 13 $4\frac{3}{10}$ 14 $5\frac{4}{9}$ 15 $2\frac{8}{17}$

What fraction is the shaded part of the whole:

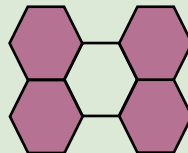
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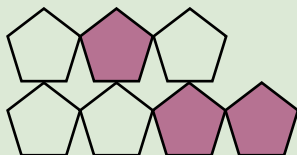
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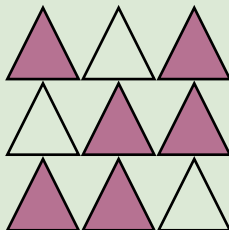
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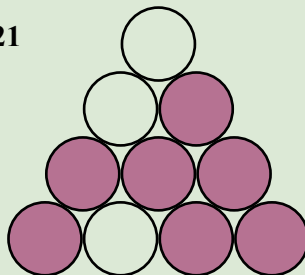
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20



21



Mixed Numbers

What fraction is 10 g of 25 g?

$$= \frac{10}{25}$$

$$= \frac{2}{5} \quad \{\text{Divide by 5}\}$$

What fraction is 75c of \$1?

$$= \frac{75c}{100c} \quad \{\text{Change the \$1 to cents}\}$$

$$= \frac{3}{4} \quad \{\text{Divide by 25}\}$$

22 What fraction is 50c of \$1?

23 What fraction is 25c of \$1?

24 What fraction is 4 m of 10 m?

25 What fraction is 20 m of 40 m?

26 What fraction is 30 mins of 1 hr?

27 What fraction is 45 mins of 1 hr?

27 What fraction is \$60 of \$100?

28 What fraction is \$30 of \$100?

29 What fraction is 500 mL of 1 L?

30 What fraction is 250 mL of 1 L?

31 What fraction is 90° of 180° ?

32 What fraction is 135° of 180° ?

$$1000\text{mL} = 1\text{L}$$

Exercise 1.2

1 Change each of the following mixed numbers to an improper fraction:

$$\begin{aligned} 2\frac{3}{5} &= 2 + \frac{3}{5} \\ &= \frac{10}{5} + \frac{3}{5} \\ &= \frac{13}{5} \end{aligned}$$

$$\begin{aligned} 2\frac{3}{5} &= 2 \text{ wholes and } 3 \text{ fifths.} \\ &= 10 \text{ fifths and } 3 \text{ fifths} \\ &= 13 \text{ fifths} \\ &= \frac{13}{5} \end{aligned}$$

This might be easier:

$$2 \times 5 + 3 = 13$$

$$\text{Thus: } \frac{13}{5}$$

a) $2\frac{1}{2}$

b) $1\frac{3}{5}$

c) $1\frac{2}{5}$

d) $2\frac{2}{3}$

e) $1\frac{1}{8}$

f) $3\frac{3}{4}$

g) $2\frac{1}{4}$

h) $3\frac{7}{4}$

2 Change each of the following improper fractions to a mixed number:

$$\begin{aligned} \frac{13}{5} \quad 13 \div 5 &= 2 \text{ remainder } 3 \\ &= 2\frac{3}{5} \end{aligned}$$

a) $\frac{5}{2}$

b) $\frac{7}{3}$

c) $\frac{9}{4}$

d) $\frac{9}{5}$

e) $\frac{8}{5}$

f) $\frac{10}{3}$



g) $\frac{19}{7}$

h) $\frac{37}{11}$

Fraction of a Quantity

Exercise 1.3

Find:

$\frac{2}{5}$ of 10 $\frac{2}{5}$ of  2 lots of  {a fifth of 10 is 2} <u>= 4</u>	OR $\frac{2}{5}$ of 10 = 2 lots of $\frac{1}{5}$ of 10 $= 2 \times (10 \div 5)$ $= 2 \times 2$ <u>= 4</u>
---	---

1 $\frac{1}{2}$ of 10

2 $\frac{3}{5}$ of 10

3 $\frac{4}{5}$ of 10

4 $\frac{2}{3}$ of 9

5 $\frac{3}{4}$ of 8

6 $\frac{1}{4}$ of 8

7 $\frac{2}{5}$ of 15

8 $\frac{3}{5}$ of 15

9 $\frac{1}{4}$ of 20

10 $\frac{3}{4}$ of 20

11 $\frac{1}{2}$ of 16

12 $\frac{5}{8}$ of 16

13 $\frac{5}{9}$ of 27

14 $\frac{2}{3}$ of 12

15 $\frac{1}{12}$ of 36

16 Three-fifths of the class of 25 students are female. How many of the class are female? How many are male?

17 Jessica plans to bank one-quarter of her weekly wages. If Jessica's weekly wage is \$800, how much will Mary bank each week?

18 How much is one-quarter of a kilogram?

19 How much is one-quarter of 4 kilograms?

20 How long is three-quarters of 4 kilometres?

21 How much is four-fifths of 1 ha (1 ha = 10 000 m²)?

22 A project returned \$10 000. The profit is expected to be two-fifths of the return. How much profit?

23 How much sand is needed to make 30 tonnes of concrete, if three-tenths of the concrete mix is sand?

24 On an average weekday three-quarters of the train seats are occupied. If the train has 400 seats, how many seats would be occupied on an average weekday?

25 Mark covered five-eighths of his journey of 800 km on the first day. How far did Mark travel on his first day?

Fractions & Percentages

Fractions and percentages are used hundreds of thousands of times every day for comparison and analysis.

Fractions and percentages are different representations of the same number.

$$\div 100$$

To change a percent to a fraction.

Exercise 1.4

Write each of the following percentages as a fraction:

$30\% = \frac{30}{100}$ $= \frac{3}{10}$	$37\% = \frac{37}{100}$	$425\% = \frac{425}{100}$ $= 4\frac{25}{100}$ $= 4\frac{1}{4}$
---	-------------------------	--



Use a calculator to simplify complicated fractions.
See Technology 1.1

- | | | | |
|---------|---------|---------|---------|
| 1 10% | 2 20% | 3 30% | 4 40% |
| 5 50% | 6 60% | 7 70% | 8 80% |
| 9 90% | 10 100% | 11 1% | 12 5% |
| 13 7% | 14 12% | 15 120% | 16 150% |
| 17 200% | 18 500% | 19 156% | 20 217% |

Write each of the following fractions as a percentage
(if necessary round to 2 decimal places):

$$\frac{3}{5} = \frac{300}{500}$$

$$= 60\%$$

$$\times 100\%$$

To change a fraction to a percent.

$$\left\{ \times 100\% = \times \frac{100}{100} = \times 1 \right\}.$$

- | | | | |
|-------------------|--------------------|-------------------|-------------------|
| 21 $\frac{1}{10}$ | 22 $\frac{2}{10}$ | 23 $\frac{3}{10}$ | 24 $\frac{4}{10}$ |
| 25 $\frac{5}{10}$ | 26 $\frac{6}{10}$ | 27 $\frac{7}{10}$ | 28 $\frac{8}{10}$ |
| 29 $\frac{9}{10}$ | 30 $\frac{10}{10}$ | 31 $\frac{1}{2}$ | 32 $\frac{1}{4}$ |
| 33 $\frac{2}{4}$ | 34 $\frac{3}{4}$ | 35 $\frac{1}{5}$ | 36 $\frac{2}{5}$ |
| 37 $\frac{3}{5}$ | 38 $\frac{4}{5}$ | 39 $\frac{5}{5}$ | 40 $\frac{1}{3}$ |

Percentage of a Quantity

Exercise 1.5

Calculate each of the following:

Find 20% of 80 $= \frac{20}{100} \times \frac{80}{1}$ $= \frac{1600}{100}$ $= \underline{16}$	Find 70% of 126 $= \frac{70}{100} \times \frac{126}{1}$ $= \frac{8820}{100}$ $= \underline{88.2}$	Find 31% of 65 $= \frac{31}{100} \times \frac{65}{1}$ $= \frac{2015}{100}$ $= \underline{20.15}$
--	--	---

- 1 25% of 60
- 2 10% of 65
- 3 75% of 24
- 4 30% of 70
- 5 50% of 80
- 6 40% of 30
- 7 60% of 250
- 8 80% of 25
- 9 90% of 80
- 10 15% of 80
- 11 25% of 80
- 12 75% of 80
- 13 About 70% of a 50 kg person's weight is water. How much of the 50 kg is water?
- 14 40% of a \$650 wage is spent on rent. How much is the rent?
- 15 If 22% of a sale is profit, how much profit in a sale of \$8200?
- 16 15% of the class of 24 students are left-handed. How many left-handed students?
- 17 60% of the class of 23 students have brown eyes. How many brown-eyed students?
- 18 Mia pays 37% tax on an income of \$56 000 pa. How much tax does Mia pay?
- 19 The management fund returned 9% on an investment of \$35 000. How much was returned?
- 20 The bank deposit of \$5000 returned 6% interest. How much interest was earned?
- 21 A fortnightly wage of \$860 was increased by 4%. What is the new wage?
- 22 A monthly rent return of \$1500 is reduced by a 7% management fee. How much is paid to the landlord?
- 23 What is the hotel room bill after GST of 10% is added to \$235?
- 24 What is the electricity bill after GST of 10% is added to \$350?

Divide by 100?
Move decimal point
two places left.

GST

The Goods and Services Tax is a broad sales tax of 10% on most goods and services.

Fractions & Percentages

Exercise 1.6

Express each of the following as a fraction and as a percentage:

<p>Ethan scored 8 out of 10.</p> <p>Fraction: $= \frac{8}{10} = \frac{4}{5}$</p> <p>Percentage: $= \frac{8}{10} \times \frac{100\%}{1} = 80\%$</p>	<p>School occupies 6 hours of a day.</p> <p>Fraction: $= \frac{6}{24} = \frac{1}{4}$</p> <p>Percentage: $= \frac{6}{24} \times \frac{100\%}{1} = 25\%$</p>
--	--

- Chloe scored 7 out of 10 for the mental computation.
- Lenny scored 9 out of 10 for the test.
- Lara scored 8 out of 10 for the test.
- Ven scored 46 out of 50 for the test.
- Wal scored 64 out of 100 for the test.
- Aaron scored 63 out of 100 for the test.
- Mony scored 16 out of 25 for the test.
- Megan scored 17 out of 20 for the test.
- Derin scored 18 out of 20 for the test.
- Sunny scored 21 out of 50 for the test.
- GST is a tax of 10c in the dollar.
- The poll showed that around 7 in 10 politicians favoured a policy change.
- 94 out of 100 students passed the Mathematics test.
- Jo worked 6 hours today.
- 5800 of the 58 000 football crowd were under 12 years of age.
- There are 610 boys in a school of 1 000 students.



Estimate the fraction of the Australian flag that is blue?

Park Rangers control, supervise, and manage national parks, nature reserves, and other important ecological, historical, and recreational areas.

- School subjects are one or more of English, Mathematics, Biology, Chemistry, Geography.
- Courses range from traineeships to University degrees.

Estimation

It is often convenient to use an estimate rather than an exact value.

The estimate is made by rounding numbers.

☺ The radius of the Earth (mean radius = 6371 km) may be rounded to 6400 km.

☺ The height of the book (height = 24.13 cm) may be rounded to 24 cm.

☺ The temperature ($t = 24.79^\circ\text{C}$) may be rounded to 25°C .

Exercise 1.7

Use rounding to the first digit to estimate the following answers:

1	$4.36 + 3.78 = 4 + 4$ <u>Estimate = 8</u>	Round 4.36 to 4 Round 3.78 to 4
2	$38.7 - 24.6 = 40 - 20$ <u>Estimate = 20</u>	Round 38.7 to 40 Round 24.6 to 20
3	$62.1 \times 3.72 = 60 \times 4$ <u>Estimate = 240</u>	Round 62.1 to 60 Round 3.72 to 4
4	$\frac{896}{3.12} = \frac{900}{3}$ <u>Estimate = 300</u>	Round 896 to 900 Round 3.12 to 3

Leading digit estimation
Rounding to the first digit gives a fairly rough estimate, but is close enough for many purposes.

Estimation is also useful for checking the *reasonableness* of an answer.

- | | | | | | |
|----|-----------------------|----|----------------------|----|----------------------|
| 1 | $7.27 + 6.93$ | 2 | $23.12 + 25.83$ | 3 | $24.2 - 5.96$ |
| 4 | 34.38×5.12 | 5 | $19.051 + 7.26$ | 6 | $504.2 - 38.39$ |
| 7 | 72.4×1.695 | 8 | $20.01 - 6.92$ | 9 | 58.03×0.813 |
| 10 | $\frac{58}{27}$ | 11 | $\frac{16}{3.7}$ | 12 | $\frac{252}{19.34}$ |
| 13 | $682.1 + 43.18$ | 14 | $86.7 - 17.68$ | 15 | 1.32×621.76 |
| 16 | $4287.1 - 862$ | 17 | $147.23 + 65.70$ | 18 | 7.89×2.606 |
| 19 | $31.09 + 23.46$ | 20 | 70.76×2.934 | 21 | $896.92 - 349.2$ |
| 22 | $\frac{235.6}{36.71}$ | 23 | $\frac{0.058}{3.4}$ | 24 | $\frac{47}{213}$ |
| 25 | $707.2 - 475.6$ | 26 | 56.02×31.94 | 27 | $938.2 + 382.1$ |

Exercise 1.8

Which of the following answers is correct (Estimate the correct answer by using rounding to the first digit)?

- | | (a) | (b) | (c) | (d) |
|---|-------------------|-------|--------|--------|
| 1 | 3.4×19.1 | 6.494 | 48.24 | 64.94 |
| 2 | $1.32 + 17.82$ | 19.14 | 1.914 | 191.4 |
| 3 | $532.3 - 64.32$ | 46.98 | 517.98 | 467.98 |
| 4 | $\frac{208}{24}$ | 0.087 | 0.87 | 8.67 |

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 1.9

- 1 Spell fraction.
- 2 Change $2\frac{1}{4}$ to an improper fraction.
- 3 Change $\frac{5}{3}$ to a mixed number.
- 4 Change $\frac{3}{10}$ to a percentage.
- 5 Change 50% to a fraction.
- 6 24×5
- 7 10% of 64
- 8 I buy a loaf of bread for \$3.45 and a biro for 40c, what is the total?
- 9 I buy a \$1.50 newspaper with a \$5 note, how much change?
- 10 If I paid \$20 deposit and 10 payments of \$5. How much did I pay?

To multiply by 5:
Multiply by 10 then divide by 2.

$$24 \times 5 = 24 \times 10 \div 2 = 240 \div 2 = 120.$$

Exercise 1.10

- 1 Spell percentage.
- 2 Change $3\frac{2}{3}$ to an improper fraction.
- 3 Change $\frac{5}{2}$ to a mixed number.
- 4 Change $\frac{9}{10}$ to a percentage.
- 5 Change 40% to a fraction.
- 6 46×5
- 7 10% of 91
- 8 I buy a carton of milk for \$3.15 and a paper for \$1.50, what is the total?
- 9 I buy a \$4.20 loaf of bread with a \$5 note, how much change?
- 10 If I paid \$50 deposit and 10 payments of \$5. How much did I pay?

$$10\% \text{ of } 64 = 6.4$$

Exercise 1.11

- 1 Spell fraction.
- 2 Change $2\frac{2}{5}$ to an improper fraction.
- 3 Change $\frac{7}{2}$ to a mixed number.
- 4 Change $\frac{7}{10}$ to a percentage.
- 5 Change 30% to a fraction.
- 6 16×5
- 7 10% of 37
- 8 I buy a paper for \$2.50 and a chocolate for \$1.60, what is the total?
- 9 I buy a \$3.55 carton of milk with a \$5 note, how much change?
- 10 If I paid \$50 deposit and 5 payments of \$20. How much did I pay?



What did one maths book say to the other maths book?

I have a lot of problems.

NAPLAN Questions

Exercise 1.12

- 1 What is exactly halfway between $\frac{1}{3}$ and $\frac{2}{3}$?
- 2 What is exactly halfway between $\frac{1}{4}$ and $\frac{3}{4}$?
- 3 One-quarter of the class were involved in the school play. How many students are in the class if 5 students were involved in the school play?
- 4 75% of the 800 tickets cost less than \$50. How many tickets cost less than \$50?
- 5 Two-thirds of 4863 people at the festival wore a hat of some kind. How many people wore a hat?
- 6 The binoculars, priced at \$50, are to be reduced by 20%.
What is then the price of the binoculars?
- 7 Put the following fractions in order from smallest to largest:
 $\frac{1}{2}, \frac{3}{5}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}$



- 8 What is \$20 as a fraction of \$50?
- 9 15 of the 45 students have brown eyes. What fraction of the students have brown eyes?
- 10 Which is the best estimate of the total: $\$4521 + \$5289 + \$2867$?
a) \$10 000 b) \$11 000 c) \$12 000
- 11 The basket contains 12 hen eggs and 18 bantam eggs. What fraction of the eggs are hen eggs.
- 12 $\frac{3}{5}$ of the concrete is gravel, $\frac{1}{5}$ of the concrete mix is sand, and the rest is cement. What fraction of the concrete is cement?
- 13 Jack contributed half, Olivia contributed one-tenth, and Ella contributed the rest. How much did Ella contribute?
- 14 Write $\frac{23}{25}$ as a percentage.
- 15 The pizza is cut into 12 equal slices. How many slices are left after two-thirds of the pizza is eaten?
- 16 The painting contract returned \$20 000. Labour and materials cost 50% and 30% of the remainder was paid in tax. How much was left?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.



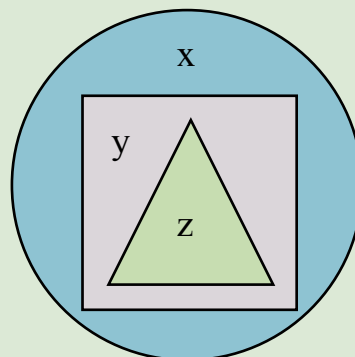
Exercise 1.13

- Which of the following fractions is the largest?
 - $\frac{11}{22}$
 - $\frac{22}{33}$
 - $\frac{33}{44}$
 - $\frac{44}{55}$
- Find the value of $\frac{0.16}{4}$
- Find the value of $\frac{1}{2}(1.5 + 4.5)$
- What is the value of $3 + \frac{5}{10} + \frac{7}{100}$?
- Find the value of x in each of the following:
 - $\frac{1}{2} = \frac{5}{x}$
 - $\frac{x}{7} = \frac{6}{21}$
 - $\frac{3}{4} \times \frac{x}{3} = \frac{1}{2}$
- How many minutes in three-quarters of an hour?
- A bus journey takes two and one-quarter hours. If the bus leaves at 7:15 am, when would the bus be expected to arrive?
- Two-thirds of a number is two. What is the number?
- Adam received $\frac{1}{2}$ of the inheritance. Matthew received $\frac{1}{3}$ of the inheritance. If there was \$300 000 left, what was the value of the inheritance?
- The salad dressing suggested that 20 mL of oil be mixed with 40 mL of vinegar. Tim decided to use $\frac{3}{4}$ of the vinegar instead. What fraction of Tim's dressing is oil?
- The recipe for potting mix suggested that 8 buckets of compost be mixed with 3 parts of soil. Sienna decided to use $\frac{3}{4}$ of the amount of compost and the same amount of soil. What fraction of the potting mix will be compost?
- Find the value of x , y , and z .

The numbers in the circle total 11.

The numbers in the square total 6.

The largest number is 5.



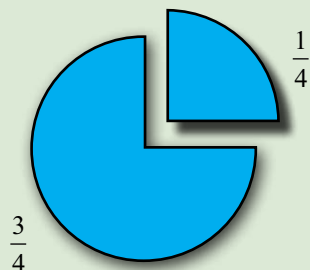
Investigations

Investigation 1.1 Limits?

Use a calculator to investigate: $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \dots$

A good starting point would be to complete the following:

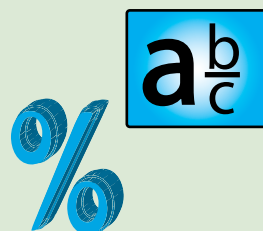
$$\begin{aligned} \frac{1}{2} &= 0.5 \\ \frac{1}{2} + \frac{1}{3} &= 0.833 \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} &= 1.083 \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} &= \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} &= \\ \dots\dots\dots &= \end{aligned}$$



Investigation 1.2 Fraction Percentage Collage

A collage is an art form that may include cuttings from newspapers, brochures, magazines, coloured papers, photographs and other objects, glued to a board of paper or other material.

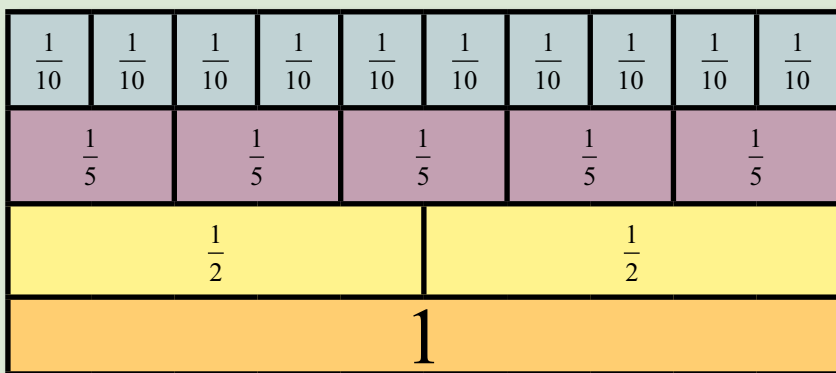
Make a collage of fractions and percentages.



Investigation 1.3 Equivalent Fractions

How many equivalent fractions can you find in the figure below?

For example: One whole = two halves ie., $1 = \frac{2}{2}$
 One-half = five tenths ie., $\frac{1}{2} = \frac{5}{10}$



A Couple of Puzzles

Exercise 1.14

1 Find the missing number in each of the following:

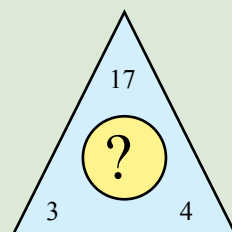
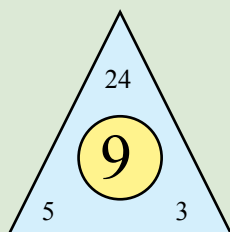
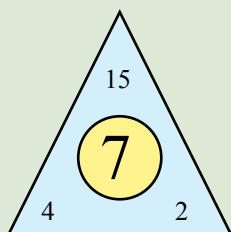
a) $\frac{3}{4} = \frac{*}{16}$

b) $\frac{2}{5} = \frac{*}{35}$

c) $\frac{4}{5} = \frac{24}{*}$

d) $2\frac{1}{2} = \frac{10}{*}$

2 Use the numbers in the corners to make the central number the same way in all three cases. What is the unknown number?



A Game: Cut Five

Write 5 random digits (from 0 to 9)

5 4 9 5 1

Each of two players take turns to either

- Erase a zero and all digits to the right
- Reduce a number (not less than 0)

The loser is the person who removes the last digit.

Sample Game

Start

5 4 9 5 1

Player 1 (reduce the 2nd 5 to 0)

5 4 9 0 1

Player 2 (erase from the 0)

5 4 9

Player 1 (reduce the 5 to 1)

1 4 9

Player 2 (reduce the 4 to 0)

1 0 9

Player 1 (erase from the 0)

1

Player 2 (reduce the 1 to 0)

0

Player 1 (erase from the 0)

Lose (last move)

A Sweet Trick

1 Ask your audience to enter their favourite number from 1 to 9 in a calculator.

5

2 Ask them to multiply by 3

$5 \times 3 = 15$

3 Ask them to multiply by 37

$15 \times 37 = 555$



This works for every number from 1 to 9.
Why? $3 \times 37 = ?$

Wow. Lots more of
their favourite number.

Technology

Technology 1.1 Calculators and fractions

Use a calculator to simplify $\frac{75}{100}$

Enter $\boxed{75}$ $\boxed{a \frac{b}{c}}$ $\boxed{100}$ in your calculator.

$\boxed{\text{Enter}}$ will then give $\frac{3}{4}$



How does a cow add?
By using a cow-culator.

Use your calculator to simplify the following fractions:

a) $\frac{30}{100}$

b) $\frac{70}{100}$

c) $\frac{15}{100}$

d) $\frac{25}{100}$

e) $\frac{65}{100}$

f) $\frac{64}{100}$

g) $\frac{36}{42}$

h) $\frac{24}{36}$

Technology 1.2 Calculators and fractions

Calculators, with $\boxed{a \frac{b}{c}}$, are very good at handling fractions.

Experiment with the fraction button to learn how to do each of the following:

1 Show that $\frac{2}{5} \times \frac{100}{1} = 40$

2 Show that $\frac{3}{4} \times \frac{100}{1} = 75$

3 Show that $\frac{6}{24} \times \frac{100}{1} = 25$

Use the fraction button on your calculator to check your answers to the previous exercises.

Technology 1.3 Fraction games and applets

Search the Internet for some of the many fraction games and applets.

Give special attention to games and applets that demonstrate fractions, mixed numbers, and percentages.

What is the difference between a Java applet and a Flash applet?

Chapter Review 1

Exercise 1.15

1 Write the following words as fractions:

a) One-half

b) Two-thirds

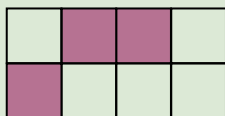
2 Write the following fractions as words:

a) $\frac{1}{5}$

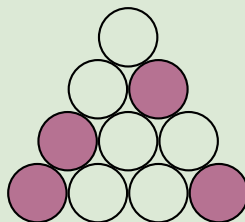
b) $4\frac{3}{10}$

3 What fraction is the shaded part of the whole:

a)



b)



4 Calculate each of the following:

a) What fraction is 50c of \$1?

b) What fraction is 25c of \$1?

5 Change each of the following mixed numbers to an improper fraction:

a) $1\frac{3}{5}$

b) $2\frac{1}{2}$

c) $1\frac{2}{5}$

d) $2\frac{2}{3}$

6 Change each of the following improper fractions to a mixed number:

a) $\frac{5}{2}$

b) $\frac{7}{3}$

c) $\frac{9}{4}$

d) $\frac{9}{5}$

7 Calculate each of the following:

a) $\frac{5}{9}$ of 27

b) $\frac{2}{3}$ of 12

c) 25% of 60

d) 10% of 65

e) $\frac{1}{2}$ of 10

f) $\frac{3}{5}$ of 25

8 Two-thirds of the class of 24 students are male. How many of the class are male? How many are female?

9 Express each of the following as a fraction and as a percentage:

a) Sebastian scored 8 out of 10 for the mental computation.

b) Eun-Young scored 43 out of 50 for the test.

10 Which of the following answers is correct (Estimate the correct answer by using rounding to the first digit)?

	(a)	(b)	(c)	(d)
a) 52×18	66	616	936	1546
b) $186 + 320$	406	506	606	706
c) $892 - 287$	455	505	555	605
d) $\frac{329}{59}$	1.58	5.58	15.58	55.58

Chapter Review 2

Exercise 1.16

1 Write the following words as fractions:

a) One-half

b) Three-quarters

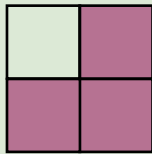
2 Write the following fractions as words:

a) $\frac{1}{3}$

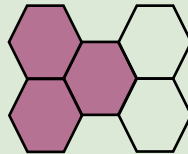
b) $2\frac{2}{5}$

3 What fraction is the shaded part of the whole:

a)



b)



4 Calculate each of the following:

a) What fraction is 25c of \$1?

b) What fraction is 75c of \$1?

5 Change each of the following mixed numbers to an improper fraction:

a) $1\frac{1}{2}$

b) $1\frac{3}{4}$

c) $2\frac{2}{5}$

d) $4\frac{3}{7}$

6 Change each of the following improper fractions to a mixed number:

a) $\frac{5}{2}$

b) $\frac{5}{3}$

c) $\frac{7}{4}$

d) $\frac{9}{5}$

7 Calculate each of the following:

a) $\frac{1}{3}$ of 27

b) $\frac{2}{5}$ of 10

c) 25% of 40

d) 10% of 45

e) $\frac{3}{5}$ of 20

f) $\frac{3}{4}$ of 24

8 Two-thirds of the class of 27 students are female. How many of the class are female? How many are male?

9 Express each of the following as a fraction and as a percentage:

a) Isabella scored 9 out of 10 for the mental computation.

b) Lachlan scored 47 out of 50 for the test.

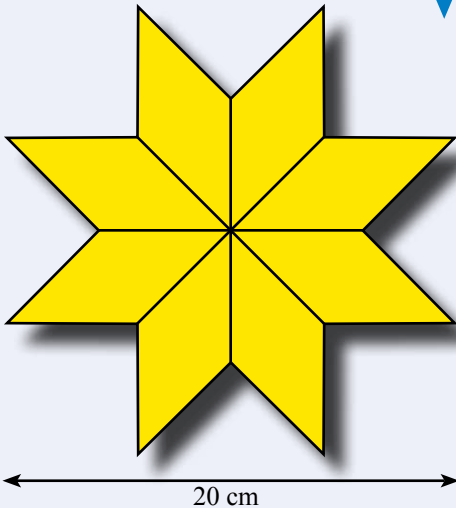
10 Which of the following answers is correct (Estimate the correct answer by using rounding to the first digit)?

	(a)	(b)	(c)	(d)
a) 42×38	1296	1396	1496	1596
b) $292 + 710$	802	902	1002	1102
c) $789 - 397$	362	392	462	492
d) $\frac{791}{42}$	12.83	15.83	18.83	24.83

Area



- ★ Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.
- ★ Use area formulas for rectangles and triangles to solve problems involving areas of surfaces.



A TASK

The shape on the left is to be covered with 23 karat gold leaf? How much will the gold leaf cost?

- How much for a leaf of 23 karat gold?
- What is the size of a leaf of 23 karat gold?
- What is the area of the shape?
- How is gold leaf applied?
- How many gold leaves are needed to cover the shape?

A LITTLE BIT OF HISTORY

There are just five Platonic solids (tetrahedron, cube, octahedron, dodecahedron, icosahedron).

Platonic solids, named after Plato (427BC-347BC), are solids with every face the same size and shape. Platonic solids were thought to be the basic shapes of the Universe.



Tetrahedron (4 faces)



Cube (6 faces)



Octahedron (8 faces)

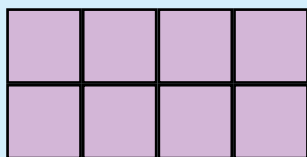
Area

Area n. 1. the amount of surface;
the international metric unit of
area is square metre (m^2).

Exercise 2.1

Calculate the area of each of the following rectangles by:

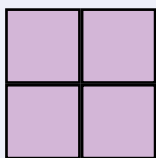
- counting the number of squares (assuming each square is 1cm^2),
- multiplying the length by the breadth.



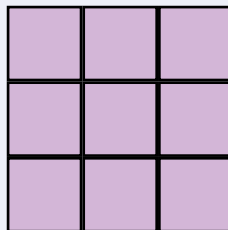
a) Area = 8 squares
= 8 cm^2

b) Area = $4\text{ cm} \times 2\text{ cm}$
= 8 cm^2

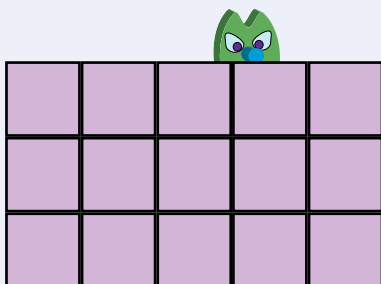
1



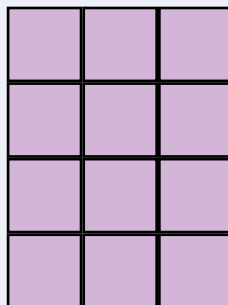
2



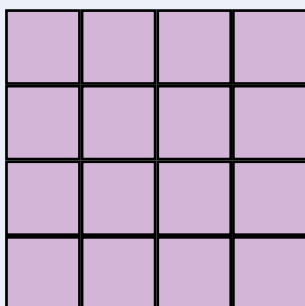
3



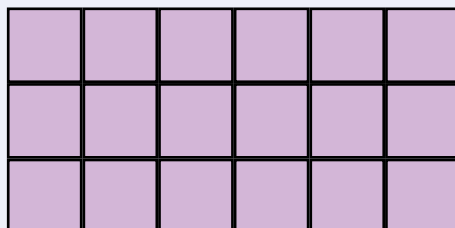
4



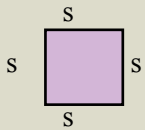
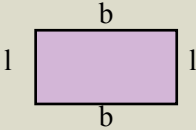
5



6





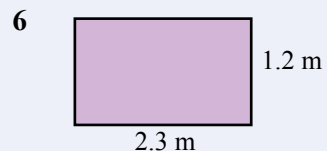
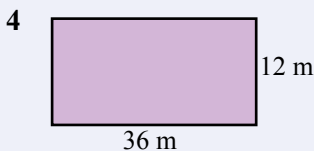
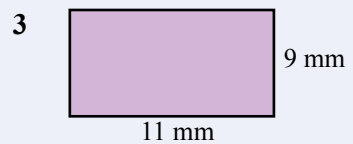
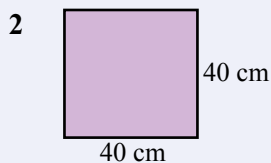
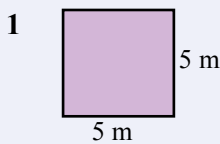
Area

Square  $\text{Area} = s \times s = s^2$	Rectangle  $\text{Area} = l \times b$
--	--

Exercise 2.2

Calculate the area of each of the following shapes:

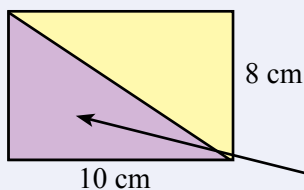
 $\text{Area} = s^2$ $= 8^2 \text{ m}^2$ $= \underline{64 \text{ m}^2}$	 $\text{Area} = l \times b$ $= 22 \text{ mm} \times 13 \text{ mm}$ $= \underline{286 \text{ mm}^2}$
---	--



- 7 A bedroom is 3 m by 3 m. How many square metres of carpet is needed to cover the floor of the room?
- 8 A bedroom is 2.6 m by 2.8 m. How many square metres of carpet is needed to cover the floor of the room?
- 9 What is the area of a bedroom wall that is 2.6 m wide by 1.2 m high? Roughly how much paint is needed for two coatings of the wall (A litre of paint will cover 15 m^2 on average)?
- 10 A kitchen bench top is 1.1 m by 2.3 m. How many square metres of laminate is needed to cover the top of the bench?
- 11 A rectangular paddock is 124 m by 110 m. What is the area of the paddock in square metres and hectares ($1 \text{ hectare} = 10\,000 \text{ m}^2$)?
- 12 The builder wants to put a 2 m wide concrete path around the outside of a 12 m square building. What is the area of the path?

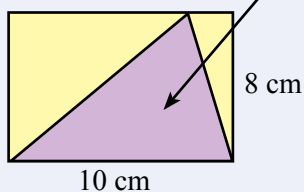
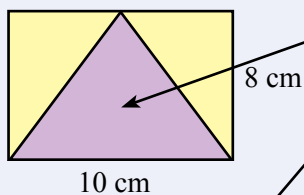
A hectare is the area of a square 100 m by 100 m.

Area of a Triangle



Area of the rectangle = 80 cm^2

Area of the triangle = 40 cm^2

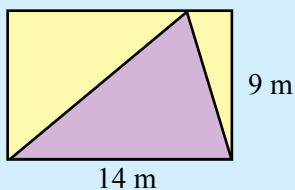


Use a pair of scissors to test whether the purple areas and the yellow areas are the same size.

The area of the triangle is half the area of the rectangle it fits in.

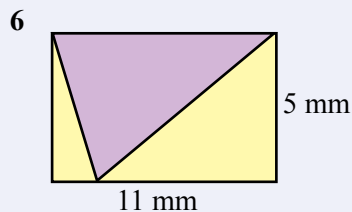
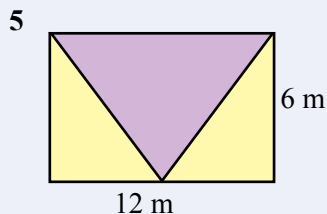
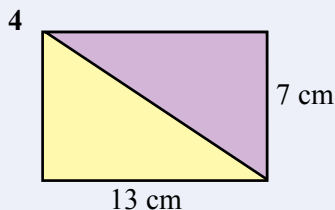
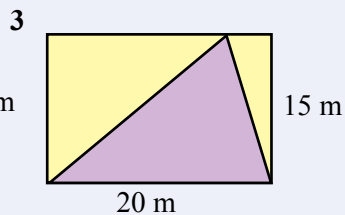
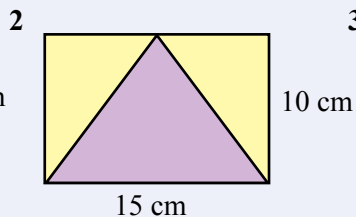
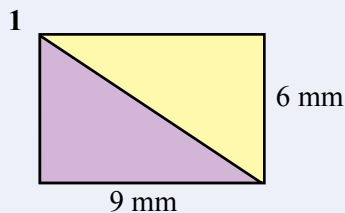
Exercise 2.3

Find the area of the purple triangle inside each rectangle:



$$\begin{aligned}\text{Area of rectangle} &= l \times b \\ &= 14 \text{ m} \times 9 \text{ m} \\ &= \underline{126 \text{ m}^2}\end{aligned}$$

$$\begin{aligned}\text{Area of triangle} &= \frac{1}{2} \text{ Area of rectangle} \\ &= \frac{1}{2} \times 126 \text{ m}^2 \\ &= \underline{63 \text{ m}^2}\end{aligned}$$

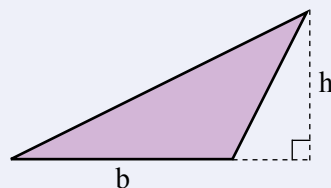
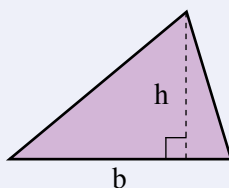
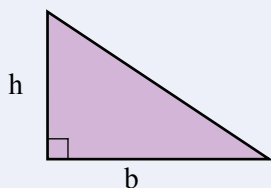


Area of a Triangle

$$\text{Area of } \triangle = \frac{1}{2}bh$$

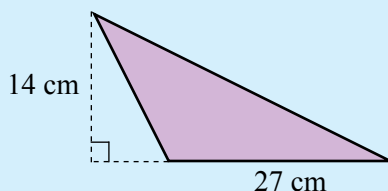
$$\text{or Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

b for base
h for height



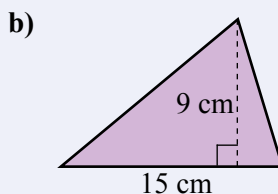
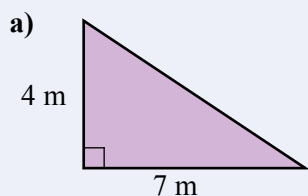
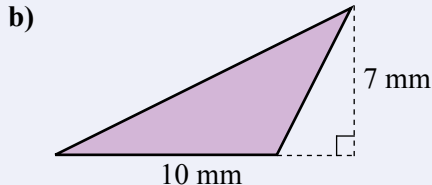
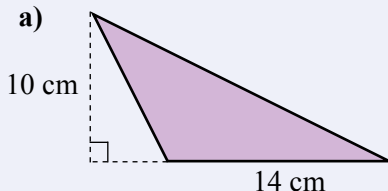
Exercise 2.4

Calculate the area of the following triangle:



$$\begin{aligned} \text{Area of } \triangle &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 27 \text{ cm} \times 14 \text{ cm} \\ &= \underline{189 \text{ cm}^2} \end{aligned}$$

1 Calculate the area of each of the following triangles:



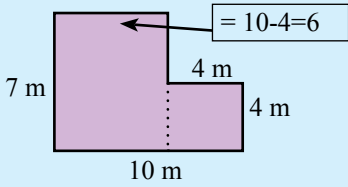
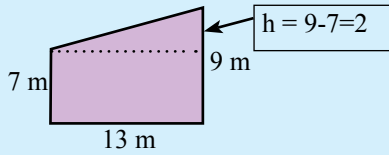
- A triangular road sign has a base of 90 cm and a perpendicular height of 120 cm. What is the area of the road sign?
- A scarf has a triangular shape with a base of 120 cm and a perpendicular height of 50 cm. What area of material was used to make the scarf?
- The front gable of a house has a triangular shape with a base of 4 m and a perpendicular height of 2 m. What is the area of the gable?
- A paddock, in the shape of a triangle, has a base of 200 m and a perpendicular height of 50 m. What is the area of the paddock in square metres and hectares (1 hectare = 10 000 m²)?
- A paddock, in the shape of a triangle, has a base of 650 m and a perpendicular height of 450 m. What is the area of the paddock in square metres and hectares?

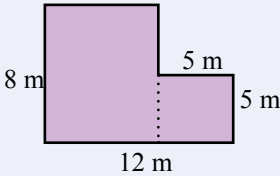
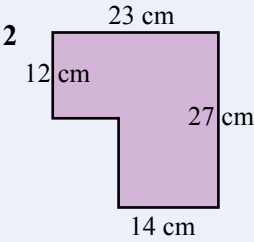
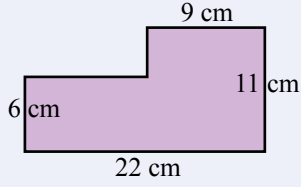
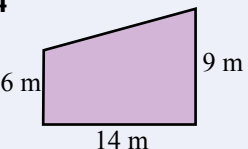
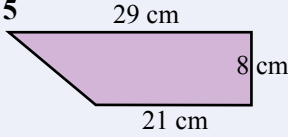
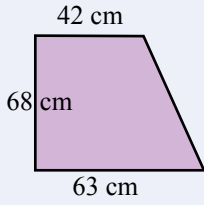
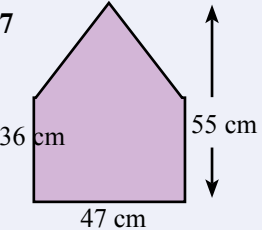
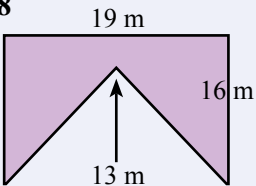
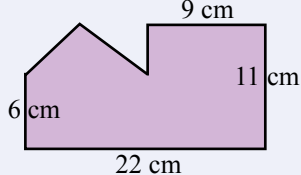
Composite Shapes

Composite shapes can be squares, rectangles, and triangles composed together.

Exercise 2.5

Calculate the area of each of the following composite shapes:

 <p>Area = rectangle + square $= lb + s^2$ $= 7 \times 6 + 4 \times 4 \text{ m}^2$ $= 42 + 16 \text{ m}^2$ $= \underline{58 \text{ m}^2}$</p>	 <p>Area = triangle + rectangle $= \frac{1}{2}bh + lb$ $= 0.5 \times 13 \times 2 + 13 \times 7 \text{ m}^2$ $= 13 + 91 \text{ m}^2$ $= \underline{104 \text{ m}^2}$</p>
--	---

<p>1</p> 	<p>2</p> 	<p>3</p> 
<p>4</p> 	<p>5</p> 	<p>6</p> 
<p>7</p> 	<p>8</p> 	<p>9</p> 

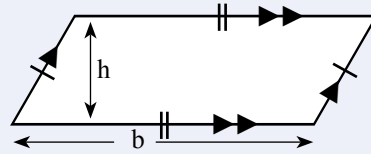
Navy Officers manage, lead, and train non-commissioned sailors. They may also be involved in specialised responsibilities such as weapons, dental, medical, engineering, etc.

- Relevant school subjects are Mathematics and English.
- Courses usually involve a University Bachelor degree.

Area of a Parallelogram

A parallelogram is a quadrilateral with opposite sides parallel.

Area of a parallelogram = bh



Why is the area = bh ?
See Investigation 2.1

Exercise 2.6

Calculate the area of each of the following shapes:

Area

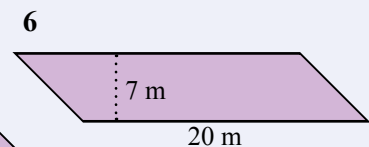
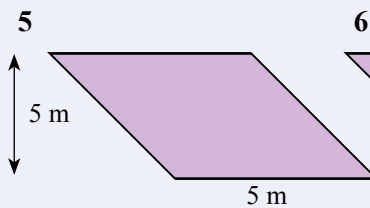
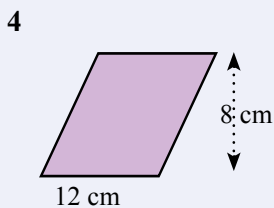
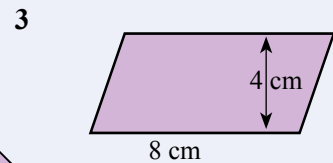
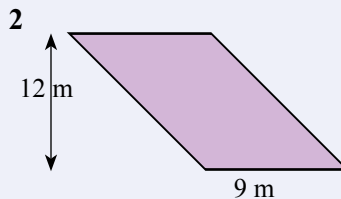
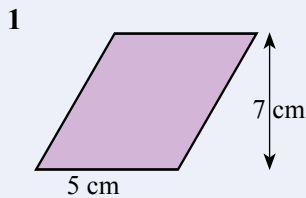
= bh

= $6\text{ cm} \times 9\text{ cm}$

= 54 cm^2

6 cm

9 cm



- 7 The base of a parallelogram is 10 cm and the perpendicular height is 7 cm. What is the area of the parallelogram?
- 8 The city council wishes to paint, two coatings, ten angled-parking car parking spaces. Each car space is in the shape of a parallelogram 2 m base with 3 m perpendicular height.
If a litre of road paint covers 10 m^2 , how much paint is needed?
- 9 In words only, describe a parallelogram.
- 10 If you were to rename the parallelogram shape, what would you name it?

All Together

Area is the amount of surface enclosed within a shape.

Exercise 2.7

1 Which unit of area would you use to measure each of the following:

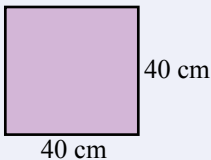
- The area of your classroom ceiling?
- The area of the desk top?
- The area of the schoolground?
- The area of Victoria?
- The area of a leaf?
- The area of your monitor screen?
- The area of your mobile display?
- The area of your TV screen?
- The area of your watchface?

Units of area:

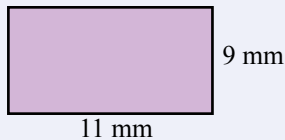
square centimetre	cm ²
square metre	m ²
hectare	ha
square kilometre	km ²

2 Calculate the area of each of the following shapes:

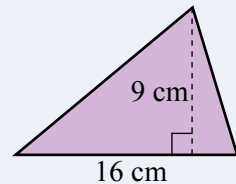
a)



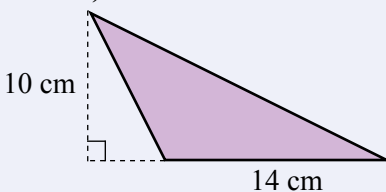
b)



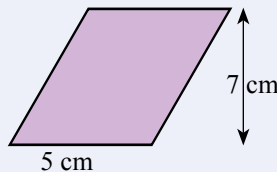
c)



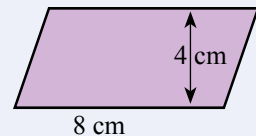
d)



e)



f)



3 A rectangular paddock has been roughly measured as $300 \text{ m} \pm 10 \text{ m}$ by $450 \text{ m} \pm 10 \text{ m}$.

- What is the largest possible area of the paddock?
- What is the smallest possible area of the paddock?
- If the value of land is \$18 000 per hectare, what is the largest value of the paddock?

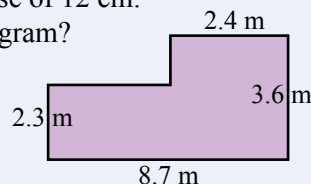
$$1 \text{ ha} = 10\,000 \text{ m}^2$$

4 The cliff sign has a base of 0.8 m and a perpendicular height of 0.9 m. How much sheet metal is needed to make 100 cliff warning signs?



5 A parallelogram has an area of 150 cm^2 and a base of 12 cm. What is the perpendicular height of the parallelogram?

6 A company has an offer to supply and install a laminate floor for \$54.50 per square metre. How much will it cost to laminate the floor?



Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 2.8

- 1 Spell triangle.
- 2 21×4
- 3 What is 10% of 54?
- 4 What is the formula for the area of a triangle?
- 5 What is the area of a parallelogram: base = 5 m, height = 8 m?
- 6 What is the area of a rectangle 4 m by 8 m?
- 7 How many m^2 in a hectare?
- 8 Change 25% to a fraction.
- 9 Change $\frac{7}{3}$ to a mixed number.
- 10 I buy a USB stick for \$7.30 with a \$20 note. How much change?

Exercise 2.9

- 1 Spell quadrilateral.
- 2 23×4
- 3 What is 10% of 79?
- 4 What is the formula for the area of a rectangle?
- 5 What is the area of a triangle: base = 7 m, height = 4 m?
- 6 What is the area of a parallelogram: base = 8 cm, height = 7 cm?
- 7 How many m^2 in a hectare?
- 8 Change 50% to a fraction.
- 9 Change $\frac{11}{4}$ to a mixed number.
- 10 I buy a music voucher for \$15.80 with a \$20 note. How much change?



What English word has three consecutive double letters?

Bookkeeper

Exercise 2.10

- 1 Spell parallelogram.
- 2 24×4
- 3 What is 10% of 63?
- 4 What is the formula for the area of a parallelogram?
- 5 What is the area of a triangle: base = 10 m, height = 5 m?
- 6 What is the area of a rectangle 4 mm by 23 mm?
- 7 How many m^2 in a hectare?
- 8 Change 75% to a fraction.
- 9 Change $\frac{15}{7}$ to a mixed number.
- 10 I buy a pie for \$5.60 with a \$10 note. How much change?

Opportunity is missed by most people because it is dressed in overalls and looks like work - Thomas Edison.



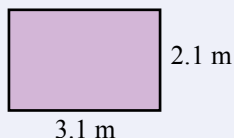
NAPLAN Questions



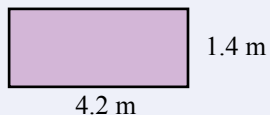
Exercise 2.11

1 Which room has the largest area?

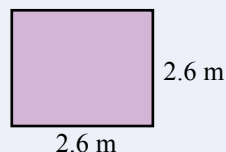
a)



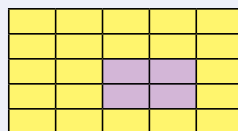
b)



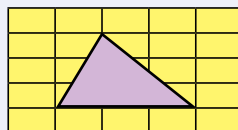
c)



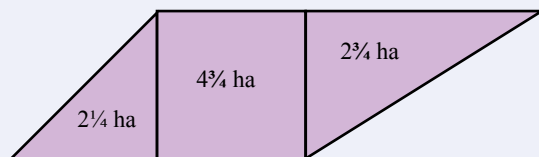
2 The total area of the purple rectangles in the middle is 16 m^2 . What is the total area of the yellow rectangles?



3 The yellow rectangle is 5 cm by 5 cm. What is the area of the triangle?

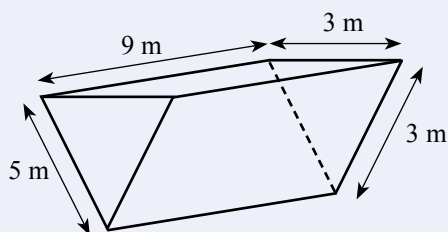


4 A farm is made up of 3 paddocks. What is the total area of the farm?



5 The area of a square is $28\,900 \text{ m}^2$. What is the length of one side?

6 What is the total surface area of the triangular prism - each end is 5 m^2 ?



7 The area of the rectangle is 72 m^2 . What is the length of the rectangle?



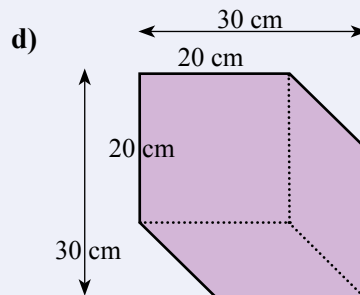
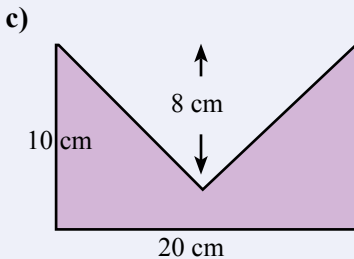
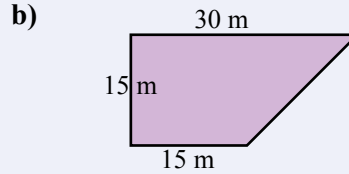
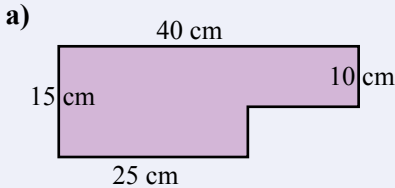
Competition Questions



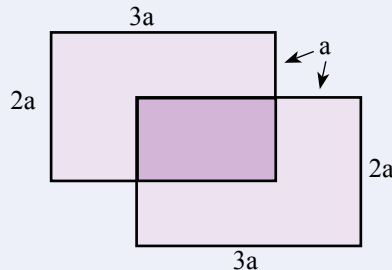
Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 2.12

- The area of a square is 169 cm^2 . What is the length of each side?
- Each floor of an office block has rectangular floors 28 m by 15 m. If the total office space is 4200 m^2 , how many floors in the building?
- Find the area of each of the following shapes:



- If the base of a parallelogram has been tripled, what has happened to the area of the parallelogram?
- If the perpendicular height of a triangle has been tripled, what has happened to the area of the triangle?
- If the length of a rectangle has been doubled and the width tripled, what has happened to the area of the rectangle?
- The two rectangles overlap. What is the area of the overlap?



- A 3 cm border is cut from a rectangular sheet of paper so that the remaining rectangle has an area one-quarter of the original rectangular sheet of paper. If the length was 12 cm what was the breadth?



? cm^2	150 cm^2
300 cm^2	450 cm^2

- What is the area of the greenish rectangle?

Investigations

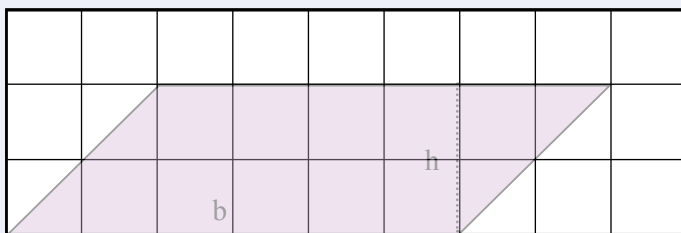
Investigation 2.1 The area of a parallelogram

A parallelogram is a quadrilateral with opposite sides parallel (and with equal length).



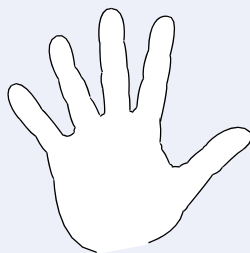
- 1 Make a copy of the above parallelogram - include the letters.
- 2 Cut along the dotted line and rearrange the pieces to make a rectangle?
- 3 What is the area of the rectangle? Check that this is the formula for the area of a parallelogram.
- 4 Paste your results into your maths pad.

Investigation 2.2 The area of a parallelogram



- 1 For the above diagram, each square is 1 cm^2 .
- 2 What is the area of the parallelogram - count the squares?
- 3 The base, $b=6 \text{ cm}$ and the height, $h=2 \text{ cm}$. Thus $bh = 6 \text{ cm} \times 2 \text{ cm} = 12 \text{ cm}^2$.
- 4 Is the area of a parallelogram $= bh$?
- 5 Check this result by drawing your own parallelograms on cm-squared paper.

Investigation 2.3 The area of your hand



- 1 Trace your hand on cm-squared graph paper.
- 2 Count the number of squares.
- 3 Can you guess the area of someone else's hand?
- 4 How close were you to their answer?

Investigations

Investigation 2.4 How big is 1 square centimetre (cm²)?

To become familiar with the area of a square centimetre, place a 1 cm by 1 cm marker on the whiteboard or classroom wall.

Use the 1 square centimetre to estimate areas in your classroom:

- The area of this page.
- The area of your calculator, etc.

$$100\text{cm} \times 100\text{cm} = 10\,000\text{cm}^2 = 1\text{m}^2$$

How close were your estimates to the actual areas?

Investigation 2.5 How big is 1 square metre (m²)?

To become familiar with the area of a square metre, place a 1 m by 1 m marker on the whiteboard or classroom wall.

Use the 1 square metre to estimate areas in your classroom:

- The area of the whiteboard.
- The area of the walls of the classroom, etc.

$$100\text{m} \times 100\text{m} = 10\,000\text{m}^2 = 1\text{ha}$$

How close were your estimates to the actual areas?

Investigation 2.6 Triangle Collage

A collage is an art form that may include cuttings from newspapers, brochures, magazines, coloured papers, photographs and other objects, glued to a board of paper or other material.

Make a collage of triangles.



Triangles are everywhere!

Technology

Technology 2.1 Area Spreadsheets

Make a spreadsheet to check your answers to the earlier exercises.

Shape	Base	Height	Area
Rectangle	26	17	442
Triangle	30	15	225
Parallelogram	9	12	108

Enter the formula:
=b2*c2

Enter the formula:
=0.5*b3*c3

Enter the formula:
=b4*c4

Technology 2.2 Area games and applets

Search the Internet for some of the many area applets.

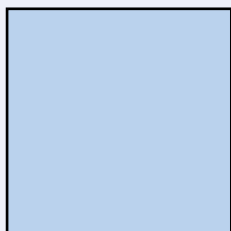
Give special attention to applets that demonstrate how the formulas for rectangle, triangle, and parallelogram are derived.

A Couple of Puzzles

Exercise 2.13

- Find a path from the top left corner to the bottom right corner that totals 15.
- Divide the square into six squares.

1	2	1	9
4	7	3	2
5	4	5	3
2	6	8	3



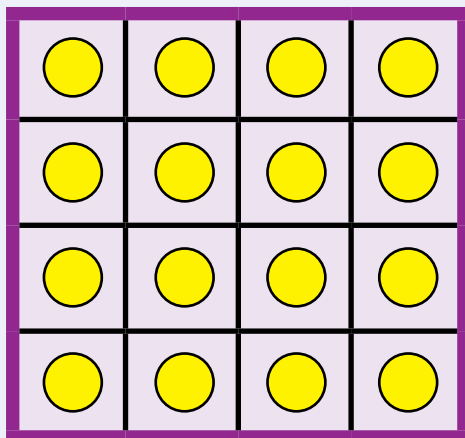
The six squares don't have to be the same size.

- 6 has exactly four factors (1,2,3,6). What is another number that has exactly four factors.

A Game

Squeeze is a two-player board game.

- Each player, in turn, must remove either 1, 2, 3 or 4 markers from one row or column.
- The person who takes the last marker is the loser.



A Sweet Trick

- You race your audience. They have a calculator, you don't.
- Your audience volunteers a two digit number: **73**
- When someone says go**
They multiply by 3, by 7, by 13, by 37
- You write up the answer faster than they can use a calculator.

Why so?

What is $3 \times 7 \times 13 \times 37$?

737373



Chapter Review 1

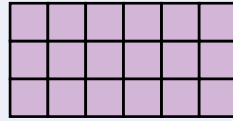
Exercise 2.14

- 1 Calculate the area of each of the following rectangles by:
- counting the number of squares (assuming each square is 1 cm^2),
 - multiplying the length by the breadth.

a)

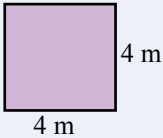


b)

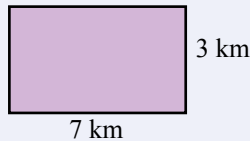


- 2 Calculate the area of each of the following shapes:

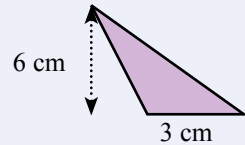
a)



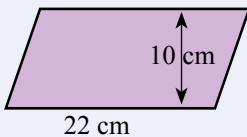
b)



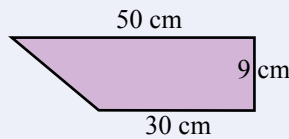
c)



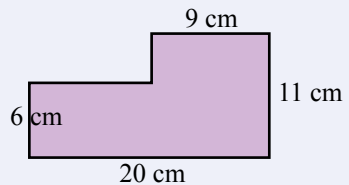
d)



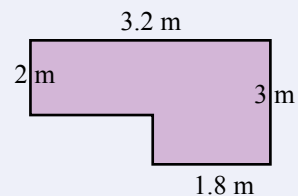
e)



f)



- 3 What is the area of a bedroom wall that is 3.4 m wide by 2.2 m high? Roughly how much paint is needed for two coatings of the wall (A litre of paint will cover 15 m^2 on average)?
- 4 A rectangular paddock is 250 m by 130 m. What is the area of the paddock in square metres and hectares ($1\text{ hectare} = 10\,000\text{ m}^2$)?
- 5 The slippery sign has a base of 0.6 m and a perpendicular height of 0.8 m. How much sheet metal is needed to make 50 slippery warning signs?
- 6 A company has an offer to supply and install carpet at \$68.90 per square metre. How much will it cost to carpet the lounge?
- 7 A rectangular paddock has been roughly measured as $400\text{ m} \pm 10\text{ m}$ by $450\text{ m} \pm 10\text{ m}$.
- What is the largest possible area of the paddock?
 - What is the smallest possible area of the paddock?
 - If the value of land is \$28 000 per hectare, what is the largest value of the paddock?



Chapter Review 2

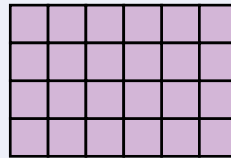
Exercise 2.15

- 1 Calculate the area of each of the following rectangles by:
- counting the number of squares (assuming each square is 1 cm^2),
 - multiplying the length by the breadth.

a)

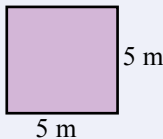


b)

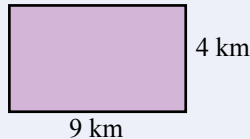


- 2 Calculate the area of each of the following shapes:

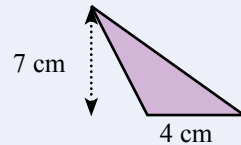
a)



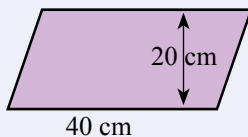
b)



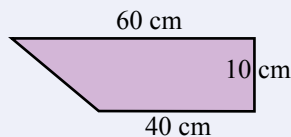
c)



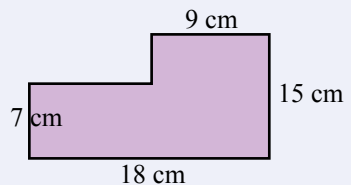
d)



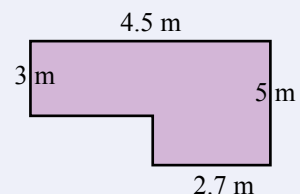
e)



f)



- 3 What is the area of a bedroom wall that is 2.8 m wide by 2.4 m high? Roughly how much paint is needed for two coatings of the wall (A litre of paint will cover 15 m^2 on average)?
- 4 A rectangular paddock is 350 m by 270 m. What is the area of the paddock in square metres and hectares ($1\text{ hectare} = 10\,000\text{ m}^2$)?
- 5 The burn hazard sign has a base of 0.9 m and a perpendicular height of 1.2 m. How much sheet metal is needed to make 150 burn hazard signs?
- 6 A company has an offer to supply and install carpet at \$94.90 per square metre. How much will it cost to carpet the lounge?
- 7 A rectangular paddock has been roughly measured as $500\text{ m} \pm 20\text{ m}$ by $420\text{ m} \pm 20\text{ m}$.
- What is the largest possible area of the paddock?
 - What is the smallest possible area of the paddock?
 - If the value of land is \$14 000 per hectare, what is the largest value of the paddock?



Chance

3

- ★ Discuss the meaning of probability terminology (for example probability, sample space, favourable outcomes, trial, chance events and experiments).
- ★ Construct sample spaces for single-step experiments with equally likely outcomes
- ★ Express probabilities in common and decimal fractional and percentage forms.
- ★ Understand the advantages and limitations of calculating theoretical probabilities.

Two-up is a fair game.



A TASK

To decide who wins the free concert ticket the following game was played by repeatedly tossing two coins.

REPEAT

- Each person in the class either put both hands on their head (two heads), or put both hands on their tail (two tails), or put one hand on their head and one hand on their tail (head and tail).
- The people who match the coin toss stay in the game.

UNTIL one person is left.

- ★ Is this game fair?

A LITTLE BIT OF HISTORY

Two-up is a traditional Australian gambling game that is now legal to play on ANZAC day.

The game, using two coins, was played in Australia as early as 1798.

The game appears to have developed from a game called pitch and toss which used one coin. Many other gambling games have similar outcomes such as Heads, Evens, or Tails. Most gambling games have odds that are unfair.

In two-up a head and a tail is called odds and means toss again.

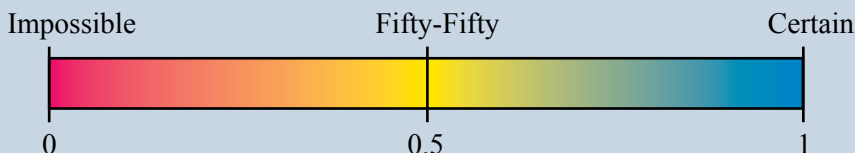


Probability

Probability is the chance of something happening.

So many things involve probability.

- Insurance is about probability.
- Weather prediction is about probability.
- Gambling is about probability.
- Even atomic theory is about probability.
- etc., etc., etc.



Exercise 3.1

1 Copy the above probability scale and add each of the following to the scale.

- The day after Monday will be Tuesday.
- Next year will have 390 days.
- We will have a new Maths teacher next week.
- I will watch TV tonight.
- I will become the Prime Minister.
- I will drink some milk today.
- It will rain tomorrow.
- I will get my driver's licence before I am 21.
- When a coin is thrown it will land tails up.
- I will send a text message today.
- The next movie I watch will be an action movie.
- The next person to walk through the door will be a girl.

Recent survey:

1 out of 2 people make up 50% of the population.



2 Brainstorm each of the following:

- List 3 events in your everyday life that will be certain.
- List 3 events in your everyday life that will be impossible.
- List 3 events in your everyday life that will be approximately fifty-fifty.
- The probability of tossing a coin and getting a head is fifty-fifty, or 0.5 or $\frac{1}{2}$. If a coin is tossed 100 times, will the number of heads be 50?
- The probability of rolling a die and getting a 6 is one-sixth, or 0.17, or $\frac{1}{6}$. If a die is rolled 60 times, will the number of 6s be 10?
- Almost every gambling game involving money is unfair.
- We have Buckley's chance of winning the match.

Next time you get a chance, find out about Buckley's chance.



Sample Space

A Sample Space is a complete list of all possible outcomes.

Tossing a coin:

There are two possibilities: head or Tail.

The Sample Space is: $S = \{H, T\}$

Exercise 3.2

- a) Write the Sample Space for spinning the spinner.



$S = \{\text{Green, Blue, Red, Yellow}\}$

- b) Calculate the probability of getting blue.

$$\text{Probability of Blue} = \frac{1}{4} = 0.25$$

- c) Spin the spinner 40 times. What fraction of the spins are blue?

G	B	R	Y
13	8	9	10

$$\text{Fraction Blue} = \frac{8}{40} = 0.20$$

- d) Is your answer to b) and c) different? Which answer is more reliable?

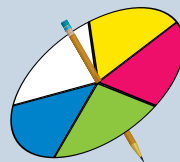
The answers are different. The Sample Space gives the theoretical answer. The spinner is experimental - the more spins, the more likely the answer will be 0.25.

- a) Write the Sample Space for spinning the spinner.

b) Calculate the probability of getting blue.

c) Spin the spinner 50 times. What fraction of the spins are blue?

d) Is your answer to b) and c) different? Which answer is more reliable?



- a) Write the Sample Space for tossing a coin.

b) Calculate the probability of getting a tail.

c) Toss a coin 50 times. What fraction of the tosses were tails?

d) Is your answer to b) and c) different? Which answer is more reliable?

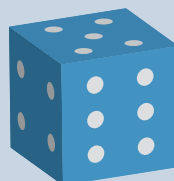


- a) Write the Sample Space for throwing a die.

b) Calculate the probability of getting a 6.

c) Toss a die 60 times. What fraction of the tosses were 6?

d) Is your answer to b) and c) different? Which answer is more reliable?



Theoretical Probability

The probability of an event is the number of favourable outcomes divided by the number of possible outcomes.

$$\text{Theoretical Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

The Sample Space is the complete list of all possible outcomes.

Exercise 3.3

Four balls are numbered 1 to 4. What is the probability of selecting a ball that shows:

- a) 2? b) even? c) >1 ?

Sample space = {1,2,3,4}

a) $P(2) = \frac{1}{4} = 0.25$ {there is one 2}

b) $P(\text{even}) = \frac{2}{4} = 0.5$ {2,4 are even}

c) $P(>1) = \frac{3}{4} = 0.75$ {2,3,4 are >1 }

A standard 6-sided die is thrown. What is the probability of each of the following happening?

- a) 3 b) odd c) >4

Sample space = {1,2,3,4,5,6}

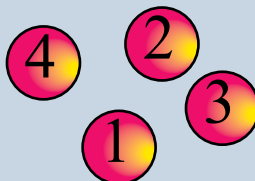
a) $P(3) = \frac{1}{6} = 0.17$ {there is one 3}

b) $P(\text{odd}) = \frac{3}{6} = 0.5$ {1,3,5 are odd}

c) $P(<4) = \frac{3}{6} = 0.5$ {1,2,3 are <4 }

- 1 Four balls are numbered 1 to 4. What is the probability of selecting a ball that shows:

- a) 4? b) odd?
c) >2 ? d) <2 ?

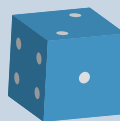


- 2 Five balls are numbered 1 to 5. What is the probability of selecting a ball that shows:

- a) 4? b) even? c) >3 ?

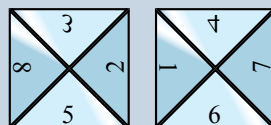
- 3 A six-sided die is thrown. What is the probability of each of the following:

- a) 3? b) even?
c) odd? d) 5?
e) <3 ? f) >4 ?
g) >6 ? h) a number divisible by 3?



- 4 An eight-sided die is thrown. What is the probability of each of the following:

- a) 5? b) even?
c) odd? d) 7?
e) <5 ? f) >7 ?
g) >5 ? h) a number divisible by 3?



- 5 The numbers 1 to 4 are each written on a piece of paper and placed in a bag. If a piece of paper is drawn from the bag, what is the probability that the number on the paper is:

a) 3? b) even?
c) >3 ? d) 5?

- 6 A class raffle is conducted by writing the numbers from 1 to 26 on a piece of paper and placed in a bag. What is the probability that the drawn number is:

a) 7? b) 20?
c) odd? d) even?
e) >20 ? f) <15 ?
g) divisible by 3? h) divisible by 4?
i) divisible by 5? j) divisible by 6?
k) a single digit number?
l) a number whose digits sum to 7?

- 7 A lottery is conducted by printing and selling tickets with the numbers from 1 to 1 000 000.

a) What is the probability that the drawn number is 321 718?
b) If you purchase one ticket, what is your chance of winning?
c) If you purchase 5 tickets, what is your chance of winning?

- 8 A letter is randomly chosen from the word CHANCE.

What is the probability that the letter is:

a) a vowel?
b) a consonant?
c) the letter A?

Vowels are a, e, i, o, u.
A vowel is a sound made with little restriction of the oral cavity.

- 9 A letter is randomly chosen from the word PROBABILITY.

What is the probability that the letter is:

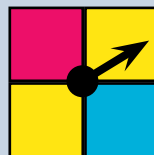
a) a vowel?
b) a consonant?
c) the letter A?

Consonants are non-vowels.
A consonant is a sound made with complete or partial closure of the oral cavity.

- 10 The spinner is one-quarter red, one-quarter blue, and one-half yellow.

What is the probability of each of the following:

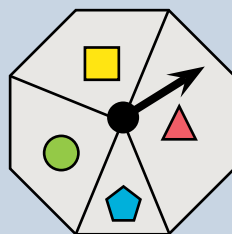
a) Red? b) blue?
c) Yellow? d) Green?



- 11 The spinner is marked as shown:

What is the probability of each of the following:

a) Red? b) blue?
c) Yellow? d) Green?



Experimental Probability

If the experiment is fair then with a large number of trials, the experimental probability and the theoretical probability should be similar.

$$\text{Experimental probability} = \frac{\text{Number of successful trials}}{\text{Total number of trials}}$$

Experimental probability changes from trial to trial.

Exercise 3.4

A coin was tossed 20 times:

H	T
HHH	HHH
HTT	HTT

Number of heads = 9

Number of tails = 11

$$\begin{aligned} P(\text{head}) &= \frac{\text{Number of heads}}{\text{Total no of tosses}} \\ &= \frac{9}{20} = 0.45 \end{aligned}$$

$$\begin{aligned} P(\text{tail}) &= \frac{\text{Number of tails}}{\text{Total no of tosses}} \\ &= \frac{11}{20} = 0.55 \end{aligned}$$

- Investigate the experimental probability of heads (and tails) by tossing a coin 50 times. Use a procedure similar to the above example.
 - Increase the number of trials from 50 to 500, using a table similar to this table:



	50 throws	500 throws
P(head)	$= \frac{\quad}{50} = 0.$	$= \frac{\quad}{500} = 0.$
P(tail)	$= \frac{\quad}{50} = 0.$	$= \frac{\quad}{500} = 0.$

- 3** Combine your results with other members of your class. As the number of trials increase, does the experimental probability get closer to the theoretical probability?

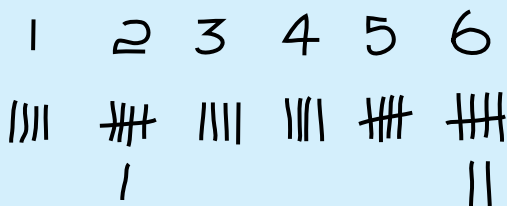
Theoretical probability of tossing a coin.

$$S = \{\text{head, tail}\}$$

$$P(\text{head}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(head) = \frac{1}{2} = 0.5$$

A die was tossed 30 times:



No. of 1s = 4
 No. of 2s = 6
 No. of 3s = 4
 No. of 4s = 4
 No. of 5s = 5
 No. of 6s = 7
 TOTAL = 30

$$P(1) = \frac{\text{No of 1s}}{\text{Total tosses}} = \frac{4}{30} = 0.13$$

$$P(2) = \frac{\text{No of 2s}}{\text{Total tosses}} = \frac{6}{30} = 0.20$$

$$P(3) = \frac{\text{No of 3s}}{\text{Total tosses}} = \frac{4}{30} = 0.13$$

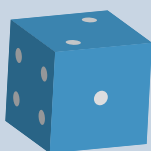
$$P(4) = \frac{\text{No of 4s}}{\text{Total tosses}} = \frac{4}{30} = 0.13$$

$$P(5) = \frac{\text{No of 5s}}{\text{Total tosses}} = \frac{5}{30} = 0.17$$

$$P(6) = \frac{\text{No of 6s}}{\text{Total tosses}} = \frac{7}{30} = 0.23$$

- 4 Investigate the experimental probability of tossing a die.
 Toss a die 60 times. Use a procedure similar to the above example.

- 5 Increase the number of trials from 60 to 600, using a table similar to this table:



	60 throws	600 throws
P(1)	$= \frac{\quad}{60} = 0.$	$= \frac{\quad}{600} = 0.$
P(2)	$= \frac{\quad}{60} = 0.$	$= \frac{\quad}{600} = 0.$
P(3)	$= \frac{\quad}{60} = 0.$	$= \frac{\quad}{600} = 0.$
P(4)	$= \frac{\quad}{60} = 0.$	$= \frac{\quad}{600} = 0.$
P(5)	$= \frac{\quad}{60} = 0.$	$= \frac{\quad}{600} = 0.$
P(6)	$= \frac{\quad}{60} = 0.$	$= \frac{\quad}{600} = 0.$

- 6 Combine your results with other members of your class.
 As the number of trials increase, does the experimental probability get closer to the theoretical probability?

Theoretical probability of tossing a die.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(1) = \frac{1}{6} = 0.17 \quad P(2) = \frac{1}{6} = 0.17$$

$$P(3) = \frac{1}{6} = 0.17 \quad P(4) = \frac{1}{6} = 0.17$$

$$P(5) = \frac{1}{6} = 0.17 \quad P(6) = \frac{1}{6} = 0.17$$

Games of Chance

A game of chance is a game which uses an instrument to give an uncertain outcome.

Game of chance instruments may be: coins, dice, cards, numbered wheels, computers etc.

If the game of chance involves money then it is gambling.
Most gambling games are unfair.

Exercise 3.5

<p>a) Play the following game with a die:</p> <ul style="list-style-type: none"> You score 1 point if you throw a number greater than 4. Your partner scores 1 point if they throw an even number. The first to 10 wins. 	<p>b) Comment on the fairness of the game.</p> <p>$S = \{1, 2, 3, 4, 5, 6\}$</p> <p>$P(> 4) = \frac{2}{6} = 0.33$ $P(\text{even}) = \frac{3}{6} = 0.50$</p> <p>The game is not fair because your partner has a 0.5 chance of scoring a point. You have a 0.33 chance of scoring a point. Your partner will win most of the games.</p>
---	--

- 1
 - a) Play the following game with a die:
 - You score 5 points if you throw a 6.
 - Your partner scores 1 point if the number is not a 6.
 - The first to 10 wins.
 - b) Comment on the fairness of the game.
- 2
 - a) Play the following game with a die:
 - You score 5 points if you throw a 6.
 - You lose 1 point if you throw a number that is not a 6.
 - You win if your score is positive after 10 throws
 - b) Comment on the fairness of the game.
- 3
 - a) Play the following game with a die:

A **Low** is a 1 or a 2

A **High** is a 5 or a 6

Choose either Low or High before the die is thrown.

 - If you are correct you score 1 point.
 - If you are wrong you lose 1 point.
 - You win if your score is positive after 10 throws
 - b) Comment on the fairness of the game.

Foresters establish, manage, and harvest forests to ensure a continued supply of timber for society's needs.

- Relevant school subjects are Science, English, Mathematics.
- Courses normally involve a Bachelor degree with a major in forestry.

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 3.6

- 1 Spell probability.
- 2 24×11
- 3 A coin is tossed, what is the chance of a tail?
- 4 A 6-sided die is tossed, what is $P(>4)$?
- 5 What is the formula for the area of a triangle?
- 6 What is the area of a rectangle 3 cm by 6 cm?
- 7 What is the area of a parallelogram: base = 8 m, height = 5 m?
- 8 Change 25% to a fraction.
- 9 Change $\frac{5}{2}$ to a mixed number.
- 10 I buy 1 kg of tomatoes for \$5.70 with a \$10 note, how much change?

Can you spot the method?

$21 \times 11 = 231$	$32 \times 11 = 352$
$22 \times 11 = 242$	$33 \times 11 = 363$
$23 \times 11 = 253$	$34 \times 11 = 373$
$24 \times 11 = 264$	
$25 \times 11 = 275$	

Exercise 3.7

- 1 Spell chance.
- 2 72×11
- 3 A coin is tossed, what is the chance of a head?
- 4 A 6-sided die is tossed, what is $P(<4)$?
- 5 What is the area of a triangle: base = 6 m, height = 5 m?
- 6 What is the formula for the area of a rectangle?
- 7 What is the area of a parallelogram: base = 9 m, height = 3 m?
- 8 Change 50% to a fraction.
- 9 Change $\frac{7}{4}$ to a mixed number.
- 10 I buy 1 kg of dried tomatoes for \$17.80 with a \$20 note, how much change?

The shin bone is a device for finding furniture in a dark room.

Exercise 3.8

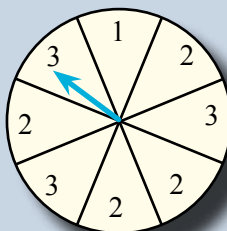
- 1 Spell theoretical.
- 2 31×11
- 3 A coin is tossed, what is the chance of a tail?
- 4 A 6-sided die is tossed, what is $P(>5)$?
- 5 What is the area of a triangle: base = 8 m, height = 7 m?
- 6 What is the area of a rectangle 7 cm by 5 cm?
- 7 What is the formula for the area of a parallelogram?
- 8 Change 75% to a fraction.
- 9 Change $\frac{11}{3}$ to a mixed number.
- 10 I buy a cooking pot for \$32.50 with a \$50 note, how much change?

Lottery: A tax on the statistically-challenged.



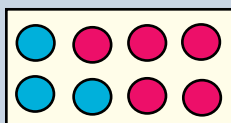
Exercise 3.9

- 1 If the spinner is spun, on which number is the arrow **least likely** to stop?

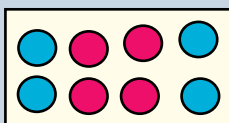


- 2 From which box does Lin have a 75% chance of selecting a red ball?

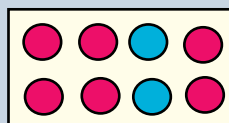
a)



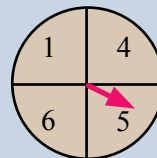
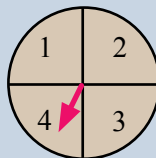
b)



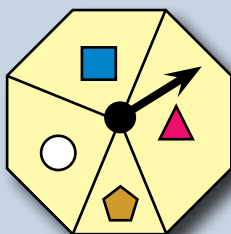
c)



- 3 The two spinners show a total of 9.
If the spinners are spun again, in how many different ways can a total of 7 be obtained?



- 4 A traffic light is amber and red for 30 seconds. The traffic light is green for 20 seconds. What is the probability that the traffic light is green?
- 5 There are 50 Australian stamps and 30 New Zealand stamps. What is the probability of choosing an Australian stamp?
- 6 A jar of jelly beans contains jelly beans with the following colours: 30 black, 25 white, 25 red and 20 green. What is the chance of picking a white jelly bean.
- 7 A bag contains 50 marbles of which 20 are red, the others are yellow. What is the chance of picking a red marble from the bag?
- 8 Jo spins the arrow 100 times. Which is the most likely table of results?



a)

Shape	Number
	25
	30
	25
	20

b)

Shape	Number
	25
	35
	15
	25

Competition Questions

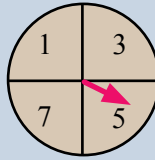
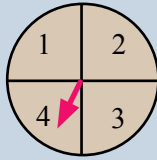


Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 3.10

- 1 The two spinners show a total of 9. Which total is impossible?

- a) 1
b) 3
c) 5
d) 7



How many two digit numbers, divisible by 2, can be made with the following cards?



All possible two digit numbers:

52 53

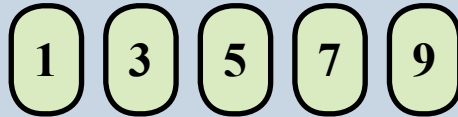
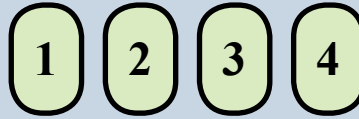
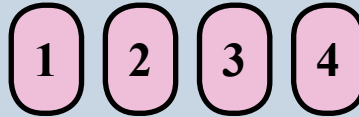
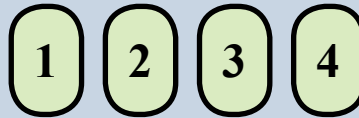
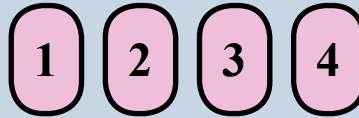
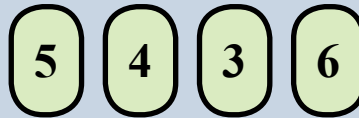
25 23

32 35

Divisible by 2 must be even

2 numbers are divisible by 2

- 2 How many two digit numbers, divisible by 2, can be made with the following cards?
- 3 How many two digit numbers, divisible by 3, can be made with the following cards?
- 4 How many three digit numbers, divisible by 2, can be made with the following cards?
- 5 How many three digit numbers, divisible by 4, can be made with the following cards?
- 6 How many three digit numbers, divisible by 3, can be made with the following cards?
- 7 How many three digit numbers, divisible by 3, can be made with the following cards?
- 8 How many three digit numbers, divisible by 3, can be made with the following cards?



Investigations

When tossing a coin, a head or tail is 'equally likely'.

Investigation 3.1 Equally likely outcomes?

Toss a bottle top 50 times:

	Tally
Face up	
Face down	

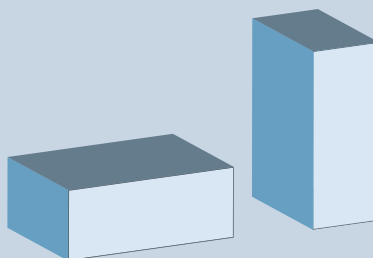


Is face up or face down equally likely?

Investigation 3.2 Equally likely outcomes?



Toss a matchbox 90 times:



	Prediction	Tally
Face		
Side		
End		

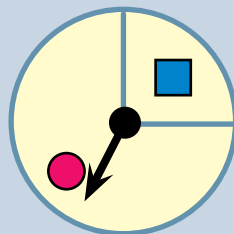


Investigation 3.3

Make a spinner similar to the one shown.

The  covers 90° and the  covers 270° .

- What is the probability of a ?
- What is the probability of a ?
- Spin the spinner 80 times and record the results.
Does the experimental probability agree with the theoretical probability?
- Can you simulate the spinner on a spreadsheet (See Technology 3.3)?



Investigation 3.4

Play the following game with a die:

You score 2 points if you throw a number greater than 4.

Your partner scores 1 point if they throw an even number.

The person with the highest score, after 30 throws, wins.

Comment on the fairness of the game.

Use theoretical probability to support your point of view.



Technology

Technology 3.1

Use the Rand (random) on a calculator to simulate the throwing of a coin.

A throw = $\boxed{2\text{ndF}}$ $\boxed{\text{Rand}}$ $\boxed{\times}$ $\boxed{2}$ $\boxed{+}$ $\boxed{0.5}$ $\boxed{=}$ {Round the answer}

The random function gives a number between 0 and 1.
This is scaled to give a number from 1 to 6.

Technology 3.2

Use a spreadsheet to simulate the tossing of a coin. This is a great way to estimate the experimental probability of a head or tail. Use the Help to find out about the Round function and the Rand function.

Press F9 to get a new set of random numbers.

	1
	1
	2
	2
	1
	1
	1
	2
	2
	1
No of heads	6

Enter the formula
=Randbetween(1,2)

If tails=1, heads=2
To count heads enter:
=Countif(b1:b10,2)

Technology 3.3

Use a spreadsheet to simulate the throwing of a die. This is a great way to estimate the experimental probability of the numbers 1, 2, 3, 4, 5, 6.

Enter the formula
=Randbetween(1,6)

5		
4		
5	No 1s	86
6	No 2s	89
1	No 3s	73
3	No 4s	77
3	No 5s	91
5	No 6s	84
2		
1		

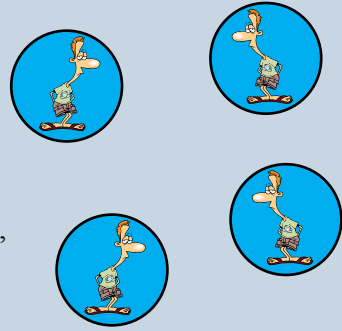
If using 500 rows enter:
=CountIF(a1:a500,1)

If using 500 rows enter:
=CountIF(a1:a500,6)

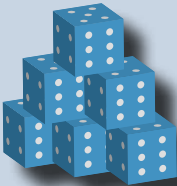
A Couple of Puzzles

Exercise 3.11

- 1 If you had \$1 000 000 and you spent \$500 every day, how long would it take to spend the million dollars?
- 2 Four people meet each other for the first time. If each person shakes hands with each other person, how many handshakes in total?
- 3 Ten people meet each other for the first time. If each person shakes hands with each other person, how many handshakes in total?



4



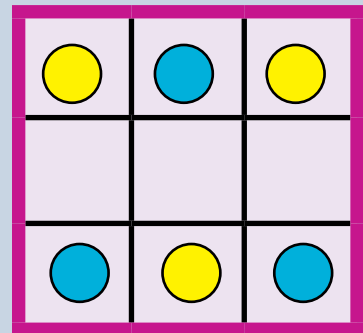
6 dice were used to build this 3 storey structure. How many dice would be needed to build a 10 storey structure?

A Game

3x3 Chaos

- 1 Draw up a 3 by 3.
- 2 Two players place each of their markers, either blue or red, as shown.
- 3 Taking turns, move one marker one place either vertically, horizontally, or diagonally.

The winner is the player who gets their three markers in a vertical, horizontal, or diagonal row.



A Sweet Trick

- 1 You race your audience. They have a calculator, you don't.
- 2 Your audience volunteers a three digit number: 491
- 3 **When someone says go**
They multiply by 7, by 11, by 13
- 4 You write up the answer faster than they can use a calculator.

Why so?

What is $7 \times 11 \times 13$?

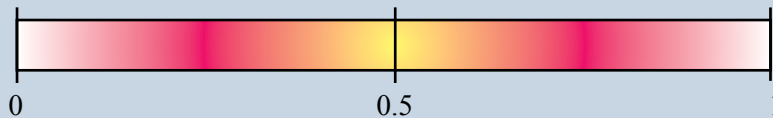
491491



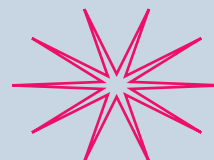
Chapter Review 1

Exercise 3.12

- 1 Write a definition of probability.



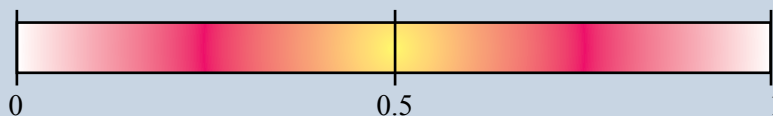
- 2 Determine the probability of each of the following events and mark the result on the above probability scale.
- Roll a 2 with a six-sided die.
 - I will watch TV today.
 - Win first prize in a lottery with one ticket (Assume 1 000 000 tickets in the lottery).
- 3 When tossing a coin the theoretical probabilities are:
Probability of Head = $\frac{1}{2} = 0.5$ Probability of Tail = $\frac{1}{2} = 0.5$
- If a coin is tossed 50 times, how many heads would be expected?
 - If a coin is tossed 50 times, how many tails would be expected?
 - If a coin is tossed 500 times, how many heads would be expected?
- 4 A six-sided die is thrown. What is the probability of each of the following?
- | | | |
|---------|----------------------------|---------|
| a) 3 | b) even | c) odd |
| d) 5 | e) <3 | f) >3 |
| g) >7 | h) a number divisible by 2 | |
- 5 A class raffle is conducted by writing the numbers from 1 to 20 on a piece of paper and placed in a bag. What is the probability that the drawn number is:
- | | |
|---------------------------|--------------------|
| a) 18? | b) 2? |
| c) odd? | d) even? |
| e) >15 ? | f) <10 ? |
| g) divisible by 3? | h) divisible by 4? |
| i) divisible by 5? | j) divisible by 6? |
| k) a single digit number? | |
| l) a square (eg 1, 4, 9)? | |
- 6 A lottery is conducted by printing and selling tickets with the numbers from 1 to 100 000.
- What is the probability that the drawn number is 65 213?
 - If you purchase one ticket, what is your chance of winning?
 - If you purchase 10 tickets, what is your chance of winning?
- 7 A letter is randomly chosen from the word LIKELY.
What is the probability that the letter is:
- a vowel?
 - a consonant?
 - the letter L?



Chapter Review 2

Exercise 3.13

- 1 Write a definition of probability.



- 2 Determine the probability of each of the following events and mark the result on the above probability scale.

- a) The letter A will be randomly selected from the letters of the word PROBABILITY
- b) I will send a text message today.
- c) I will spend more than \$10 today.
- d) I will lose a biro today.

- 3 When tossing a coin the theoretical probabilities are:

Probability of Head = $\frac{1}{2} = 0.5$ Probability of Tail = $\frac{1}{2} = 0.5$

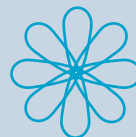
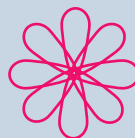
- a) If a coin is tossed 60 times, how many heads would be expected?
- b) If a coin is tossed 60 times, how many tails would be expected?
- c) If a coin is tossed 600 times, how many heads would be expected?

- 4 A six-sided die is thrown. What is the probability of each of the following?

- a) 6 b) even c) odd
- d) 4 e) <4 f) >4
- g) <1 h) a number divisible by 3

- 5 A class raffle is conducted by writing the numbers from 1 to 30 on a piece of paper and placed in a bag. What is the probability that the drawn number is:

- a) 28? b) 2?
- c) odd? d) even?
- e) >25 ? f) <10 ?
- g) divisible by 3? h) divisible by 4?
- i) divisible by 5? j) divisible by 6?
- k) a single digit number?
- l) a square (eg 1, 4, 9)?



- 6 A lottery is conducted by printing and selling tickets with the numbers from 1 to 1 000 000.

- a) What is the probability that the drawn number is 142 871?
- b) If you purchase one ticket, what is your chance of winning?
- c) If you purchase 20 tickets, what is your chance of winning?

- 7 A letter is randomly chosen from the word SPINNER.

What is the probability that the letter is:

- a) a vowel?
- b) a consonant?
- c) the letter S?



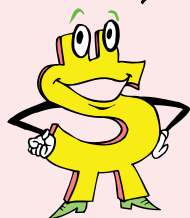
Money

4

- ★ Find percentages of quantities and express one quantity as a percentage of another.
- ★ Use authentic problems to express quantities as percentages of other amounts.
- ★ Apply the unitary method to identify 'best buys' situations, such as comparing the cost per 100g.

A TASK

Use Internet stores as well?



Conduct 'best value research'.

- Make a list of 5 necessary everyday grocery items.
- Visit a couple of grocery stores and collect data such as price and quantity.
- Decide which items are 'best buy'.
- If possible also make decisions about quality.
- Publish your findings - classroom wall?

A LITTLE BIT OF HISTORY

1910 The pound, £, is Australia's currency.

1965 A number of names are suggested for Australia's new currency such as the emu, the digger, the roo etc.

Our Prime Minister at the time, Sir Robert Menzies, preferred the name the royal.

1966 The dollar, \$, is adopted as the name for Australia's new currency.

Australia's first five pound note.



Estimating

An estimate of how much we have to pay is useful.

Leading digit estimation

Rounding to the first digit gives a fairly rough estimate, but is close enough for many purposes.

Exercise 4.1

Use rounding to the first digit to estimate the following shopping bills:

	Estimate
4 items @ \$2.85 each	$4 \times \$3 = \12
5 items @ \$47.99 each	$5 \times \$50 = \250
2 items @ \$209.99 each	$2 \times \$200 = \$400 +$
	<u>Total = \$662</u>

The actual answer is \$671.33

- 1 4 items @ \$5.25 each
5 items @ \$8.12 each

- 2 3 items @ \$2.95 each
6 items @ \$3.10 each

- 3 2 items @ \$2.45 each
7 items @ \$9.99 each
3 items @ \$3.85 each

- 4 1 items @ \$4.95 each
6 items @ \$2.87 each
5 items @ \$7.15 each

- 5 5 items @ \$15.90 each
2 items @ \$7.65 each
1 items @ \$14.99 each
3 items @ \$21.05 each

- 6 4 items @ \$42.00 each
8 items @ \$61.63 each
3 items @ \$53.90 each
1 items @ \$77.14 each

- 7 2 items @ \$43.20 each
3 items @ \$68.00 each
1 items @ \$28.65 each
1 items @ \$97.45 each
5 items @ \$51.75 each

- 8 3 items @ \$25.00 each
3 items @ \$27.50 each
2 items @ \$32.10 each
5 items @ \$17.80 each
6 items @ \$33.40 each



Rounding example:

456 ► 500
443 ► 400

- 9 4 items @ \$668.25 each
1 items @ \$152.00 each
5 items @ \$928.92 each
6 items @ \$88.90 each
1 items @ \$439.70 each
5 items @ \$501.00 each

- 10 5 items @ \$142.05 each
5 items @ \$423.10 each
9 items @ \$207.60 each
2 items @ \$389.80 each
4 items @ \$219.99 each
1 items @ \$155.00 each

GST & Discounts

Exercise 4.2

Add GST (10%) to an item priced at \$37

$$\begin{aligned} &= 37 + 10\% \text{ of } 37 \\ &= 37 + 0.1 \times 37 \\ &= \underline{\$40.70} \end{aligned}$$

The GST is a tax of 10% on most supplies of goods and services in Australia from 1 July 2000.

- 1 Add GST (10%) to the price of each of the following items:
- | | |
|--|--|
| <p>a) A sandwich @ \$2.40</p> <p>c) A watch @ \$48.50</p> <p>e) A hotel room @ \$145</p> <p>g) A telephone bill @ \$125.60</p> | <p>b) Cable subscription @ \$55</p> <p>d) A filing cabinet @ \$240</p> <p>f) Haircut @ \$33</p> <p>h) Electricity account @ \$171.60</p> |
|--|--|

A discount of 10% is offered on a pair of joggers priced at \$180.	a) Discount = 10% of 180 = 0.10×180 = <u>\$18</u>
a) How much is the discount?	
b) What is the new price?	b) New price = $180 - 18$ = <u>\$162</u>

10%
Discount

- 2 A discount of 10% is offered on each of the following items. Find the discount, and the new price:

- | | |
|--|--|
| <p>a) A single bed @ \$230</p> <p>c) A set of saucepans @ \$120</p> <p>e) A chair @ \$35.50</p> <p>g) A calculator @ \$22.50</p> | <p>b) An MP4 player @ \$48</p> <p>d) A vase @ \$25</p> <p>f) A lamp @ \$8.50</p> <p>h) A DVD @ \$12.60</p> |
|--|--|

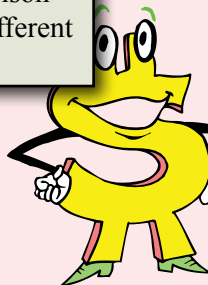
A discount of 20% is offered on a laptop priced at \$650.	a) Discount = 20% of 650 = 0.20×650 = <u>\$130</u>
a) How much is the discount?	
b) What is the new price?	b) New price = $650 - 130$ = <u>\$520</u>

20%
Discount

- 3 A discount of 20% is offered on each of the following items. Find the discount, and the new price:
- | | |
|--|--|
| <p>a) An alarm clock @ \$18</p> <p>c) A necklace @ \$35</p> <p>e) An Angus bull @ \$3200</p> <p>g) A cattle crush @ \$4700</p> | <p>b) A bicycle @ \$390</p> <p>d) A concert ticket @ \$120</p> <p>f) A shovel @ \$24.90</p> <p>h) A 20-year old ute @ \$3700</p> |
|--|--|
- 4 An insurance premium of \$455.50 is given a 10% discount. What is then the new cost of the premium?

Rates

A rate is a comparison of quantities of different kinds.



Examples of Rates

Rent	Dollars per week
Salary	Dollars per fortnight
Pay rate	Dollars per hour
Potato price	Dollars per kilogram
Petrol price	Dollars per litre
Cloth price	Dollars per metre
Call costs	Dollars per minute
Taxi charges	Dollars per kilometre

Exercise 4.3

1 In words, what is meant by each of the following rates?

Petrol price of \$1.95/L	Bean price of \$4.98/kg
<u>Price of \$1.95 per 1 Litre of petrol</u>	<u>Price of \$4.98 for 1 kilogram of beans</u>

- | | |
|----------------------------------|------------------------------|
| a) Petrol price of \$1.79/L | b) Wool price of 960c/kg |
| c) Cheese price of \$13.40/kg | d) Timber price of \$15.90/m |
| e) Land price of \$24 000/ha | f) Call cost of 98c/min |
| g) Barbed wire price of \$320/km | h) Service fee of \$85/hour |

2 Simplify each of the following rates:

<p>\$40 for 5 kilograms</p> <p>$= \\$ \frac{40}{5}$ for $\frac{5}{5}$ kilograms</p> <p>$= \\$8$ for 1 kilogram</p> <p><u>$= \\$8/\text{kg}$</u></p>	<p>\$16 for 10 minutes</p> <p>$= \\$ \frac{16}{10}$ for $\frac{10}{10}$ minutes</p> <p>$= \\$1.60$ for 1 minute</p> <p><u>$= \\$1.60/\text{min}$</u></p>
---	--

- | | |
|-----------------------------|-------------------------|
| a) \$20 for 5 hours | b) \$25 per 5 kg |
| c) \$36 for 6 metres | d) \$18 per 9 minutes |
| e) \$240 for 10 bags | f) \$200 per 4 weeks |
| g) \$565 for 5 hectares | h) \$120 per 5 buckets |
| i) \$210 for 7 weeks | j) \$32 per 8 seconds |
| k) \$75 for 15 pegs | l) \$28 per 8 pens |
| m) \$100 for 20 grams | n) \$320 per 4 holes |
| o) \$39 for 13 buttons | p) \$22 per 11 seconds |
| q) \$45 for 9 kilograms | r) \$456 per 10 litres |
| s) \$9 for 10 grams | t) \$120 per 10 seconds |
| u) \$88 for 10 metres | v) \$65 per 100 minutes |
| w) \$56 for 100 millilitres | x) \$9 per 100 grams |

Rates

Rate problems are very common in everyday life.

Exercise 4.4

A house is let for \$350 per week. What is the rent for 4 weeks?

$$1 \text{ week} = \$350$$

$$4 \text{ weeks} = 4 \times \$350$$

$$= \underline{\$1400}$$

If the price of diesel is \$1.85 per litre, what is the cost of 36 L of diesel?

$$1 \text{ litre} = \$1.85$$

$$36 \text{ litres} = 36 \times \$1.85$$

$$= \underline{\$66.60}$$

- A two bedroom unit is being let for \$465 pw (per week). What is the rent for:
 - 4 weeks?
 - 8 weeks
 - 6 months (26 weeks)?
 - 1 year (52 weeks)?
- Hao earns a salary of \$1250 per week, how much is earned in:
 - 2 weeks?
 - 3 weeks?
 - a month (4 weeks)?
 - 6 months (26 weeks)?
 - 1 year (52 weeks)?
 - 1 decade (10 years)?
- A prepaid mobile phone deal offers 28c per text message, what would be the charge for:
 - 10 text messages?
 - 15 text messages?
 - 37 text messages?
 - 196 text messages?
- The Internet connection costs \$57.50 per month, how much for:
 - 3 months?
 - 6 months?
 - 1 year?
 - 1 decade (10 years)?
- The mobile phone company offers one rate of 48 cents per 30 secs to any mobile or landline within the country. What would it cost for:
 - a 3 min call?
 - a 4 min call?
 - a 7 min call?
 - a 8 min 30 sec call?
- If the price of unleaded petrol is \$1.86 per litre, what is the cost of:
 - 5 L of petrol?
 - 10 L of petrol?
 - 35 L of petrol?
 - 57 L of petrol?
- If 75 mm x 38 mm roof batten timber is \$1.85 per metre. How much will it cost for:
 - 2 m?
 - 4 m?
 - 6850 mm?
 - 87 630 mm?



1000mm=1m
6850mm=6.85m

Unitary Method

The unitary method first finds the rate for one quantity.

The unitary method is a useful method for comparing one rate with another rate.

Exercise 4.5

1 Find the rate for one in each of the following:

$\$40$ for 2 kilograms $= \$ \frac{40}{2}$ for $\frac{2}{2}$ kilograms $= \underline{\$20 \text{ for } 1 \text{ kg}}$	$\$5.34$ for 3 litres $= \$ \frac{5.34}{3}$ for $\frac{3}{3}$ litre $= \underline{\$1.78 \text{ for } 1 \text{ L}}$
---	---

- a) \$14 for 2 kg b) \$32 for 4 litres
 c) \$2.80 for 7 metres d) \$4.50 for 9 minutes
 e) \$8.50 for 50 g f) \$6.40 for 80 grams

2 Which is the best buy?

$\$4.80$ for 2kg of potatoes or $\$10.20$ for 5kg of potatoes? $= \frac{4.80}{2}$ for $\frac{2}{2}$ $= \$2.40$ for 1 kg <u>The 5 kg is the best buy.</u>	$= \frac{10.20}{5}$ for $\frac{5}{5}$ $= \$2.04$ for 1 kg
--	--

- a) \$6.40 for 2 kg of tomatoes or \$9.90 for 3 kg?
 b) \$20.30 for 7 m of timber or \$15.50 for 5 m?
 c) \$560 for 14 ingots of lead or \$663 for 17 ingots?

3 Find the rate for 100 of each of the following:

$\$6$ for 300 grams $= \$ \frac{6}{3}$ for $\frac{300}{3}$ grams $= \underline{\$2 \text{ for } 100 \text{ g}}$	$\$37.80$ for 700 grams $= \$ \frac{37.80}{7}$ for $\frac{700}{7}$ grams $= \underline{\$5.40 \text{ for } 100 \text{ g}}$	The unitary method can also find the rate for 100 (if the second quantity is larger than 100).
---	--	--

- a) \$8 for 200 g b) \$16 for 800 mL
 c) \$36 for 900 g d) \$4.20 for 200 g
 e) \$8.16 for 800 mL f) \$6.30 for 700 mins

4 Which is the best buy?

$\$4.20$ for 400 mL of vegetable oil or $\$10.20$ for 600 mL? $= \frac{4.20}{4}$ for $\frac{400}{4}$ $= \$1.05$ for 100 mL <u>The 400 mL is the best buy.</u>	$= \frac{10.20}{6}$ for $\frac{600}{6}$ $= \$1.70$ for 100 mL
---	--

- a) \$3.42 for 300 mL of vegetable oil or \$2.64 for 200 mL?
 b) \$5.05 for 500 g of breakfast cereal or \$4.10 for 400 g?
 c) \$3.20 for 200 g of white chocolate biscuits or \$4.29 for 300 g?

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 4.6

- 1 Spell estimate.
- 2 Add GST (10%) to \$20.
- 3 \$2.50 per litre. How much for 3 litres?
- 4 A coin is tossed, what is the chance of a tail?
- 5 A 6-sided die is tossed, what is $P(>4)$?
- 6 What is the formula for the area of a triangle?
- 7 What is the area of a rectangle 4 cm by 7 cm?
- 8 Change 20% to a fraction.
- 9 Which is the best buy: \$6 for 2 L or \$10 for 5 L?
- 10 If the price of petrol is \$2 per litre, how much for 45 L?

What insect is good at maths?

An accountAnt.

Exercise 4.7

- 1 Spell discount.
- 2 Add GST (10%) to \$25.
- 3 \$3.50 per kg. How much for 4 kg?
- 4 A coin is tossed, what is the chance of a head?
- 5 A 6-sided die is tossed, what is $P(<4)$?
- 6 What is the formula for the area of a parallelogram?
- 7 What is the area of a triangle base = 6 cm and height = 5 cm?
- 8 Change 30% to a fraction.
- 9 Which is the best buy: \$10 for 2 m or \$20 for 5 m?
- 10 If the price of petrol is \$2 per litre, how much for 54 L?

Tracer works both ways.

Murphy's law of combat.

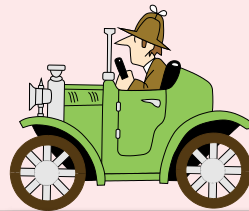
Exercise 4.8

- 1 Spell unitary.
- 2 Add GST (10%) to \$30.
- 3 \$1.50 per glass. How much for 5 glasses?
- 4 A coin is tossed, what is the chance of a tail?
- 5 A 6-sided die is tossed, what is $P(<3)$?
- 6 What is the formula for the area of a rectangle?
- 7 What is the area of a parallelogram base = 5 cm and height = 7 cm?
- 8 Change 40% to a fraction.
- 9 Which is the best buy: \$15 for 3 mins or \$28 for 7 m?
- 10 If the price of petrol is \$2 per litre, how much for 62 L?



Exercise 4.9

- 1 Liam is paid \$18.50 per hour. How much will Liam be paid for 8 hours work?
- 2 Ella was paid \$196 for 8 hours work. What is the pay rate per hour?
- 3 Isabella bought a box of 100 stamps for \$65. What is the cost of 1 stamp?
- 4 A vehicle uses 6 litres of petrol per 100 km. How far would the vehicle travel with 36 litres?
- 5 A vehicle uses 9 litres of petrol per 100 km. How many litres of petrol would the vehicle need to travel 450 km?
- 6 If a cheese is advertised at \$12 per kg, how much cheese can be bought with \$3?
- 7 If a watermelon is \$4.50 per kg, how much will 600 g of watermelon cost?
- 8 If 3 bottles of juice and 4 \$1 ice creams cost \$11.50, what would 4 bottles of juice and 2 \$1 ice creams cost?
- 9 A \$550 laptop is to be reduced by 20%. What is the reduced price?



Don't do too much in your head.
Pen and paper work will get better results.



- 10 Use leading digit estimate to estimate the total cost of each of the following bills

a)

3 items @ \$2.99 each
4 items @ \$2.42 each
4 items @ \$4.39 each
2 items @ \$6.95 each

b)

2 items @ \$73.99 each
3 items @ \$49.65 each
4 items @ \$31.70 each
2 items @ \$64.05 each

- 11 If 1 Australian dollar buys 1.10 US dollars, how many US dollars could be bought with 10 Australian dollars using this exchange rate?
- 12 If 1 Australian dollar buys 0.90 US dollars, how many US dollars could be bought with 20 Australian dollars using this exchange rate?
- 13 A shortcut for adding GST (10%) is to multiply by 1.1. What would be the shortcut for adding 20%? Use the shortcut to increase \$4 by 20%
- 14 The car uses 5 L of petrol per 100 km. How far would the car travel on \$30 of petrol at \$1.50 per litre?

Competition Questions



Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 4.10

- 1 If 16 litres of fertiliser costs \$24, how much will 50 litres of fertiliser cost?
- 2 If the pay rate is \$22.50/h, what would be the pay for 420 minutes?
- 3 How long will a 250 mL bottle of medicine last if a teaspoon (5 mL) of the medicine is used each day?
- 4 How long will a 250 mL bottle of medicine last if 2.5 mL of the medicine is used each day?
- 5 Assume that a tap turned full on releases 10 L per 30 seconds. How long will it take to fill a 50 L drum?
- 6 Assume that a tap turned full on releases 8 L per 30 seconds. If the flow is halved, how long will it take to fill a 50 L drum?
- 7 A resting heart pumps about 5 litres of blood per minute. Roughly how much blood is pumped in a day?
- 8 The heart of an active athlete can pump around 30 litres of blood per minute. Roughly how much blood is pumped in a 2.5 hour marathon?

If 6 people are able to pack the fruit in 10 hours,
how long will it take 4 people to pack the fruit?

6 people pack in 10 hours
= 1 person pack in 60 hours

Thus 4 people in $60\text{h} \div 4$
4 people in 15 hours

- 9 If 8 people are able to pack the fruit in 10 hours, how long will it take 5 people to pack the fruit?
- 10 There is enough poultry food to feed 8 hens for 20 days. For how many days would this feed 10 hens?

The length and breadth of a rectangle is increased by 20%.
What is the increase in the area of the rectangle?

Area = $(l \times 1.2) \times (b \times 1.2)$ $\{\times 1.2 = \text{increase of } 20\%\}$
= $lb \times 1.44$ $\{1.2 \times 1.2 = 1.44\}$
= an increase of 44%

- 11 The length and breadth of a rectangle is increased by 10%. What is the increase in the area of the rectangle?
- 12 The length and breadth of a rectangle is increased by 30%. What is the increase in the area of the rectangle?

Investigations

Investigation 4.1 Real Life

Applications of percentages and rates are everywhere.

- 1 Can you make a list of 20 applications of percentages?
- 2 Can you make a list of 20 applications of rates?
- 3 Make a collage of your results.



Investigation 4.2 The Official Interest Rate

In Australia, the official interest rate is decided by the Reserve Bank of Australia on the first Tuesday of each month.

- 1 Find the official interest rates for the last five years on the Internet.
- 2 Draw a graph of the official interest rates.
- 3 Predict how the interest rates will move. Extend the graph for the rest of the year.
- 4 Put the graph on the classroom wall.
- 5 Each month mark in the official interest rate.

Interest rates are a powerful instrument for keeping the Australian economy under control.

Investigation 4.3 Advertising

Don't miss out Desperate sale - owners bankrupt

Large block of land close to city and beaches in a natural setting.

Expected to sell quickly at \$35 000.

Contact Tom Tool
0417 310 089

- 1 Write down five reasons why you would be tempted to buy the above block of land.
- 2 Write down 5 things that you need to know about this block of land and is not included in the above advertisement.
- 3 What is probably misleading about the above advertisement?

Investigation 4.4 Compound Interest

Sue Saver placed \$200 000 in an e-saver account at the beginning of 2005.

The account is increased by 6% at the end of each year.
How much will be in the account at the end of 2020?



Sports Scientists apply knowledge from a variety of disciplines such as medicine, biomechanics, psychology, etc to maximise sporting performance.

- Relevant school subjects are English, Mathematics, and Science.
- Courses usually involve a University sporting degree.

Technology

Technology 4.1 Using the % button

Calculators can be useful for percentage calculations:



- 1 14 out of 20 as a percent $\boxed{14} \boxed{\div} \boxed{20} \boxed{\%}$ to give the answer 70%

Without the $\boxed{\%}$ key $\boxed{14} \boxed{\div} \boxed{20} \boxed{\times} \boxed{100} \boxed{=}$

- 2 20% of 80 $\boxed{80} \boxed{\times} \boxed{20} \boxed{\%}$ to give the answer 16

Without the $\boxed{\%}$ key $\boxed{80} \boxed{\times} \boxed{20} \boxed{\div} \boxed{100} \boxed{=}$

- 3 Increase 95 by 30% $\boxed{95} \boxed{+} \boxed{30} \boxed{\%}$ to give the answer 123.5

Without the $\boxed{\%}$ key $\boxed{95} \boxed{+} \boxed{95} \boxed{\times} \boxed{30} \boxed{\div} \boxed{100} \boxed{=}$

- 4 Decrease 115 by 25% $\boxed{115} \boxed{-} \boxed{25} \boxed{\%}$ to give the answer 86.25

Without the $\boxed{\%}$ key $\boxed{115} \boxed{-} \boxed{115} \boxed{\times} \boxed{25} \boxed{\div} \boxed{100} \boxed{=}$

Technology 4.2 Unitary Method

- Search the Internet for videos and powerpoints on the 'Unitary Method'
- Select the best two demonstrations of the 'Unitary Method'
- Show them to the rest of the class.

\$	mL
4.85	750
?	100

Technology 4.3 Running Rate

The National Council of Teachers of Mathematics (NCTM) has a well known speed, distance, time simulation - a Java applet.

- Find the applet by using 'nctm distance simulation' as a search phrase.
- Start experimenting with the applet.
- Can you complete the tasks and questions?

This is a wonderful introduction to understanding graphs.

A Couple of Puzzles

Exercise 4.11

- 1 Can you copy and complete the pattern?
- 2 4 has exactly three factors (1,2,4).
What is another number that has exactly three factors.
- 3 I have 3 factors and I am between 20 and 30.
Who am I?
- 4 What is the next number in the sequence:
1, 4, 16, 64, ...?
- 5 A rectangle has an area of 24 cm^2 .
The length and the width are whole numbers.
Find the length and the width that gives the smallest perimeter.

$$\begin{aligned} 99 \times 11 &= 1089 \\ 99 \times 22 &= 2178 \\ 99 \times 33 &= 3267 \\ 99 \times 44 &= 4356 \\ 99 \times 55 &= 5445 \\ 99 \times 66 &= 6534 \\ 99 \times 77 &= 7623 \\ 99 \times 88 &= 8712 \\ 99 \times 99 &= 9801 \end{aligned}$$

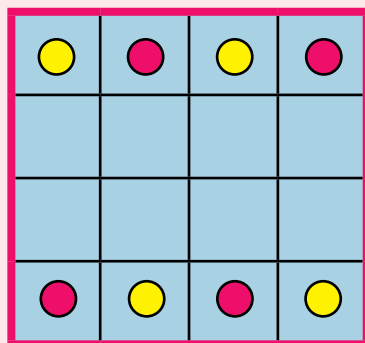
$$\text{Area} = 24 \text{ cm}^2$$

A Game

4x4 Chaos

- 1 Draw up a 4 by 4.
- 2 Two players place each of their markers, either red or yellow, as shown.
- 3 Taking turns, move one marker one place either vertically, horizontally, or diagonally.

The winner is the player who gets their four markers in a vertical, horizontal, or diagonal row.



A Sweet Trick

- 1 You race your audience. They have a calculator, you don't.
- 2 Your audience volunteers a five digit number: **71956**
- 3 **When someone says go**
They multiply by 11, by 9091
- 4 You write up the answer faster than they can use a calculator.



7195671956

Why so?

What is 11×9091 ?

Chapter Review 1

Exercise 4.12

- 1 Use rounding to the first digit to estimate the following shopping bills:
 - a) 2 items @ \$4.75 each
3 items @ \$2.35 each
5 items @ \$8.12 each
3 items @ \$5.95 each
 - b) 4 items @ \$21.00 each
2 items @ \$38.65 each
6 items @ \$43.90 each
3 items @ \$59.84 each
- 2 Add GST (10%) to the price of each of the following items:
 - a) A burger @ \$6
 - b) A book @ \$36.50
- 3 A discount of 10% is offered on each of the following items. Find the discount, and the new price:
 - a) A calculator @ \$23
 - b) An external hard drive @ \$72.50
- 4 A discount of 20% is offered on each of the following items. Find the discount, and the new price:
 - a) A pillow @ \$78
 - b) A bicycle @ \$290
- 5 An insurance premium of \$855.00 is given a 10% discount. What is then the new cost of the premium?
- 6 A prepaid mobile phone deal offers 28c per text message, what would be the charge for 173 text messages?
- 7 The Internet connection costs \$59.95 per month, how much for 1 year?
- 8 If the price of unleaded petrol is \$1.92 per litre, what is the cost of 64 L of petrol?
- 9 Which is the best buy?
 - a) \$4.80 for 2 kg of potatoes or \$10.20 for 5 kg?
 - b) \$64.75 for 7 m of cloth or \$48.50 for 5 m?
 - c) \$5500 for 10 yearlings or \$9200 for 16 yearlings?
- 10 Find the rate for 100 of each of the following:
 - a) \$9 for 300 g
 - b) \$21 for 700 mL
 - c) \$36 for 800 g
 - d) \$4.20 for 300 g
 - e) \$8.19 for 900 mL
 - f) \$6.40 for 500 mins
- 11 Which is the best buy?
 - a) \$6.50 for 500 mL of malt vinegar or \$4.64 for 400 mL?
 - b) \$16.20 for 300 g of coffee or \$9.10 for 200 g?
 - c) \$18.55 for 700 g of fruit and nut mix or \$25.65 for 900 g?

Mary left on a horse on Sunday, was gone for four days, and came back on Sunday. How did that happen?

The horse's name was Sunday!

Chapter Review 2

Exercise 4.13

- 1 Use rounding to the first digit to estimate the following shopping bills:
 - a) 4 items @ \$6.25 each
 - b) 5 items @ \$68.70 each
 - 2 items @ \$3.75 each
 - 3 items @ \$53.15 each
 - 3 items @ \$9.17 each
 - 6 items @ \$79.99 each
 - 3 items @ \$4.05 each
 - 2 items @ \$82.40 each
- 2 Add GST (10%) to the price of each of the following items:
 - a) A prepaid phone @ \$140
 - b) A CD @ \$14.00
- 3 A discount of 10% is offered on each of the following items. Find the discount, and the new price:
 - a) A backpack @ \$26
 - b) An chair @ \$65.50
- 4 A discount of 20% is offered on each of the following items. Find the discount, and the new price:
 - a) A container @ \$2800
 - b) A set of earphones @ \$18.80
- 5 An insurance premium of \$1365.00 is given a 10% discount. What is then the new cost of the premium?
- 6 A prepaid mobile phone deal offers 26c per text message, what would be the charge for 123 text messages?
- 7 The Internet connection costs \$79.95 per month, how much for 1 year?
- 8 If the price of unleaded petrol is \$1.87 per litre, what is the cost of 62 L of petrol?
- 9 Which is the best buy?
 - a) \$9.50 for 5 kg of carrots or \$14.80 for 8 kg?
 - b) \$21 for 6 rolls of yarn or \$23.80 for 7 rolls?
 - c) \$103.20 for 20 solar cells or \$127.75 for 25 solar cells?
- 10 Find the rate for 100 of each of the following:
 - a) \$6 for 200 g
 - b) \$18 for 300 mL
 - c) \$44 for 800 g
 - d) \$7.20 for 500 mm
 - e) \$17.71 for 700 mL
 - f) \$6.60 for 600 mins
- 11 Which is the best buy?
 - a) \$5.50 for 500 mL of yogurt or \$3.60 for 300 mL?
 - b) \$5.25 for 500 g of parrot seed or \$7.42 for 700 g?
 - c) \$8.60 for 400 g of sultanas or \$16.20 for 900 g?

If the enemy is in range,
so are you - Murphy's
laws of combat.

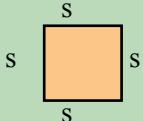
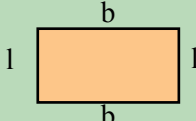
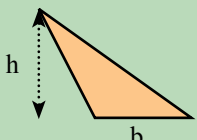
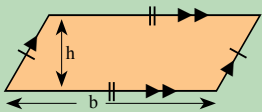
Review 1



Chapter 1 Number 1

Change an improper fraction to a mixed number $\frac{8}{3} = 2\frac{2}{3}$	Fraction of a quantity $\frac{3}{5}$ of 10 = $\frac{3 \times 10}{5} = 6$
Percentage of a quantity $25\% \text{ of } 60 = \frac{25 \times 60}{100} = 15$	

Chapter 2 Area

Square  $\text{Area} = s \times s = s^2$	Rectangle  $\text{Area} = l \times b$
Triangle  $\text{Area} = \frac{1}{2}bh$	Parallelogram  $\text{Area} = bh$

Chapter 3 Chance

The probability of an event is the number of favourable outcomes divided by the number of possible outcomes.	$\text{Theoretical Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$
A Sample Space is a complete list of all possible outcomes.	Tossing a coin: There are two possibilities: head or Tail. The Sample Space is: $S = \{H, T\}$

Chapter 4 Money

Unitary method - best buy $\$4.20$ for 400 mL of vegetable oil or $\$10.20$ for 600 mL? $= \frac{4.20}{4}$ for $\frac{400}{4}$ $= \frac{10.20}{6}$ for $\frac{600}{6}$ $= \$1.05$ for 100 mL $= \$1.70$ for 100 mL <u>The 400 mL is the best buy.</u>	
---	--

Review 1

Exercise 5.1 Mental computation

- 1 Spell percentage.
- 2 Add GST (10%) to \$30.
- 3 \$1.50 per litre. How much for 3 litres?
- 4 A coin is tossed, what is the chance of a tail?
- 5 A 6-sided die is tossed, what is $P(>3)$?
- 6 What is the formula for the area of a triangle?
- 7 What is the area of a rectangle 6 cm by 7 cm?
- 8 Change 20% to a fraction.
- 9 Which is the best buy: \$6 for 3 L or \$15 for 5 L?
- 10 If the price of petrol is \$2 per litre, how much for 42 L?

Exercise 5.2

- 1 Write the following words as fractions:

a) One-half

b) One-quarter

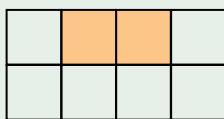
- 2 Write the following fractions as words:

a) $\frac{1}{5}$

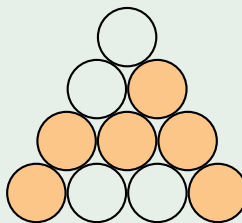
b) $3\frac{1}{2}$

- 3 What fraction is the shaded part of the whole:

a)



b)



- 4 Calculate each of the following:

a) What fraction is 50c of \$1?

b) What fraction is 25c of \$1?

- 5 Change each of the following mixed numbers to an improper fraction:

a) $2\frac{1}{2}$

b) $2\frac{1}{3}$

c) $3\frac{1}{4}$

d) $2\frac{3}{5}$

- 6 Change each of the following improper fractions to a mixed number:

a) $\frac{3}{2}$

b) $\frac{9}{2}$

c) $\frac{11}{3}$

d) $\frac{15}{7}$

- 7 Calculate each of the following:

a) $\frac{1}{2}$ of 24

b) $\frac{2}{3}$ of 12

c) 25% of 40

d) 10% of 45

e) $\frac{4}{5}$ of 20

f) $\frac{3}{4}$ of 12

- 8 Two-thirds of the class of 24 students are male. How many of the class are male? How many are female?

9 Express each of the following as a fraction and as a percentage:

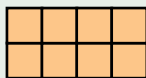
a) Chloe scored 8 out of 10 for the mental computation.

b) Tori scored 44 out of 50 for the test.

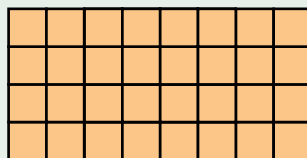
10 Calculate the area of each of the following rectangles by:

- counting the number of squares (assuming each square is 1 cm^2),
- multiplying the length by the breadth.

a)

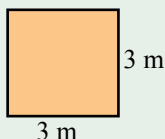


b)

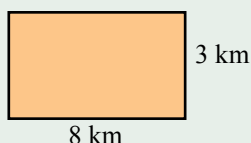


11 Calculate the area of each of the following shapes:

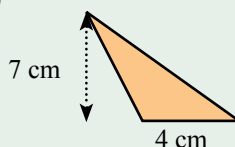
a)



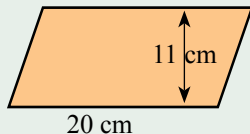
b)



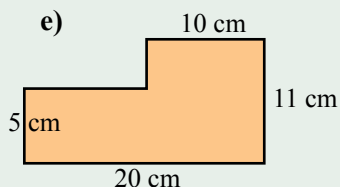
c)



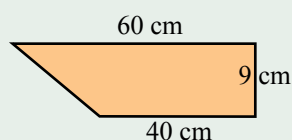
d)



e)



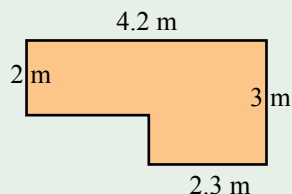
f)



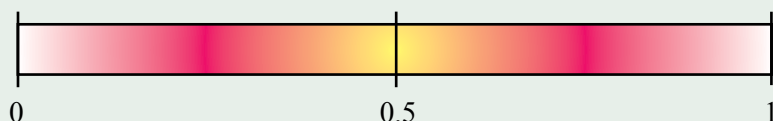
12 What is the area of a bedroom wall that is 3.2 m wide by 2.3 m high? Roughly how much paint is needed for two coatings of the wall (A litre of paint will cover 15 m^2 on average)?

13 A rectangular paddock is 450 m by 120 m. What is the area of the paddock in square metres and hectares (1 hectare = $10\,000\text{ m}^2$)?

14 A company has an offer to supply and install carpet at \$92.90 per square metre. How much will it cost to carpet the lounge?



15 Write a definition of probability.



- 16** Determine the probability of each of the following events and mark the result on the above probability scale.
- Roll a 3 with a six-sided die.
 - I will go to school today.
 - Win first prize in a lottery with one ticket (Assume 1 000 000 tickets in the lottery).
- 17** When tossing a coin the theoretical probabilities are:
 Probability of Head = $\frac{1}{2} = 0.5$ Probability of Tail = $\frac{1}{2} = 0.5$
- If a coin is tossed 70 times, how many heads would be expected?
 - If a coin is tossed 70 times, how many tails would be expected?
 - If a coin is tossed 700 times, how many heads would be expected?
- 18** A six-sided die is thrown. What is the probability of each of the following?
- | | | |
|--------------|-----------------------------------|---------------|
| a) 2 | b) even | c) odd |
| d) 6 | e) <4 | f) >4 |
| g) >6 | h) a number divisible by 2 | |
- 19** A class raffle is conducted by writing the numbers from 1 to 20 on a piece of paper and placed in a bag. What is the probability that the drawn number is:
- | | |
|----------------------------------|---------------------------|
| a) 10? | b) 11? |
| c) odd? | d) even? |
| e) >10? | f) <10? |
| g) divisible by 2? | h) divisible by 3? |
| i) divisible by 4? | j) divisible by 5? |
| k) a square (eg 1, 4, 9)? | |
| l) a cube (eg 1, 8, 27)? | |
- What do all ships weigh, regardless of size?
 Anchor.
- 20** A lottery is conducted by printing and selling tickets with the numbers from 1 to 100 000.
- What is the probability that the drawn number is 55 698?
 - If you purchase one ticket, what is your chance of winning?
 - If you purchase 10 tickets, what is your chance of winning?
- 21** A letter is randomly chosen from the word CHANCE.
 What is the probability that the letter is:
- a vowel?
 - a consonant?
 - the letter C?
- 22** Use rounding to the first digit to estimate the following shopping bills:
- | | |
|---------------------------------|----------------------------------|
| a) 2 items @ \$8.75 each | b) 3 items @ \$92.00 each |
| 4 items @ \$6.72 each | 5 items @ \$85.65 each |
| 3 items @ \$9.43 each | 2 items @ \$73.85 each |
| 6 items @ \$5.95 each | 3 items @ \$69.04 each |
- 23** Add GST (10%) to the price of each of the following items:
- A drink @ \$2
 - A ticket @ \$130.00

- 24** A discount of 20% is offered on each of the following items.
Find the discount, and the new price:
- a) A speaker @ \$42 b) A bicycle @ \$320
- 25** An insurance premium of \$1460.00 is given a 10% discount. What is then the new cost of the premium?
- 26** A prepaid mobile phone deal offers 29c per text message, what would be the charge for 128 text messages?
- 27** The Internet connection costs \$49.95 per month, how much for 1 year?
- 28** If the price of unleaded petrol is \$1.84 per litre, what is the cost of 57 L of petrol?
- 29** Which is the best buy?
a) \$4.60 for 2 kg of potatoes or \$10.00 for 5 kg?
b) \$64.75 for 5 m of fibre or \$111.15 for 9 m?
c) \$895 for 10 rugs or \$1445 for 17 rugs?
- 30** Find the rate for 100 of each of the following:
a) \$10 for 200 g b) \$20 for 400 mL
c) \$32 for 800 g d) \$5.22 for 300 g
e) \$8.12 for 700 mL f) \$6.20 for 500 mins
- 31** Which is the best buy?
a) \$3.90 for 500 mL of spice sauce or \$3.28 for 400 mL?
b) \$1.20 for 300 g of beans or \$0.70 for 200 g?
c) \$2.40 for 600 g of pumpkin or \$3.42 for 900 g?
- Talent is what you possess;
genius is what possesses you
- Malcolm Cowley.

Exercise 5.4

1 Write the following words as fractions:

a) One-half

b) Three-quarters

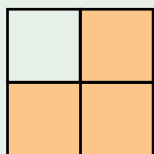
2 Write the following fractions as words:

a) $\frac{3}{5}$

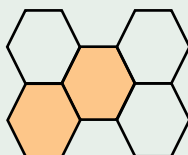
b) $6\frac{3}{4}$

3 What fraction is the shaded part of the whole:

a)



b)



4 Calculate each of the following:

a) What fraction is 50c of \$1?

b) What fraction is 75c of \$1?

5 Change each of the following mixed numbers to an improper fraction:

a) $1\frac{1}{3}$

b) $2\frac{3}{4}$

c) $3\frac{2}{5}$

d) $4\frac{6}{7}$

6 Change each of the following improper fractions to a mixed number:

a) $\frac{5}{2}$

b) $\frac{8}{3}$

c) $\frac{9}{4}$

d) $\frac{13}{5}$

7 Calculate each of the following:

a) $\frac{1}{5}$ of 20

b) $\frac{3}{4}$ of 12

c) 25% of 20

d) 10% of 50

e) $\frac{2}{3}$ of 12

f) $\frac{3}{5}$ of 20

8 Two-thirds of the class of 21 students are female. How many of the class are female? How many are male?

9 Express each of the following as a fraction and as a percentage:

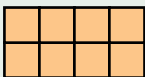
a) Lisa scored 8 out of 10 for the mental computation.

b) Jack scored 41 out of 50 for the test.

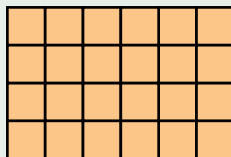
10 Calculate the area of each of the following rectangles by:

- counting the number of squares (assuming each square is 1 cm^2),
- multiplying the length by the breadth.

a)

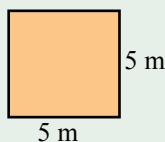


b)

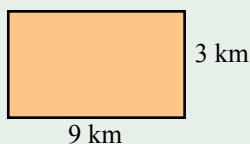


11 Calculate the area of each of the following shapes:

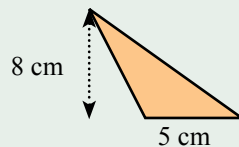
a)



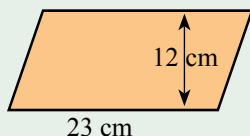
b)



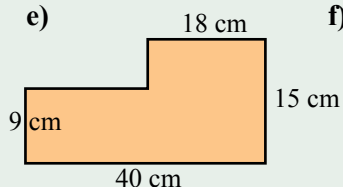
c)



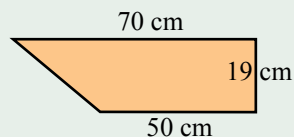
d)



e)



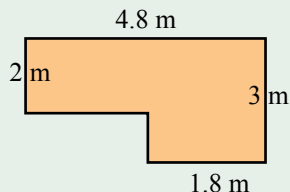
f)



12 What is the area of a bedroom wall that is 3.2 m wide by 2.4 m high? Roughly how much paint is needed for two coatings of the wall (A litre of paint will cover 16 m² on average)?

13 A rectangular paddock is 650 m by 490 m. What is the area of the paddock in square metres and hectares (1 hectare = 10 000m²)?

14 A company has an offer to supply and install carpet at \$106.90 per square metre. How much will it cost to carpet the lounge?



15 Write a definition of probability.



16 Determine the probability of each of the following events and mark the result on the above probability scale.

a) Roll a 6 with a six-sided die.

b) I will watch TV today.

c) Win first prize in a lottery with one ticket (Assume 1 000 000 tickets in the lottery).

17 When tossing a coin the theoretical probabilities are:

Probability of Head = $\frac{1}{2}$ = 0.5 Probability of Tail = $\frac{1}{2}$ = 0.5

a) If a coin is tossed 80 times, how many heads would be expected?

b) If a coin is tossed 80 times, how many tails would be expected?

c) If a coin is tossed 800 times, how many heads would be expected?

- 18 A six-sided die is thrown. What is the probability of each of the following?
- a) 1 b) even c) odd
 - d) 6 e) <5 f) >2
 - g) >7 h) a number divisible by 2
- 19 A class raffle is conducted by writing the numbers from 1 to 20 on a piece of paper and placed in a bag. What is the probability that the drawn number is:
- a) 11? b) 10?
 - c) odd? d) even?
 - e) >18 ? f) <8 ?
 - g) divisible by 3? h) divisible by 4?
 - i) divisible by 5? j) divisible by 6?
 - k) a square (eg 1, 4, 9)?
 - l) an odd number divisible by 3?
- 20 A lottery is conducted by printing and selling tickets with the numbers from 1 to 1 000 000.
- a) What is the probability that the drawn number is 372 968?
 - b) If you purchase one ticket, what is your chance of winning?
 - c) If you purchase 50 tickets, what is your chance of winning?
- 21 A letter is randomly chosen from the word RAFFLE.
What is the probability that the letter is:
- a) a vowel?
 - b) a consonant?
 - c) the letter L?
- When you come to a roadblock, take a detour
- Mary Kay Ash.
- 22 Add GST (10%) to the price of each of the following items:
- a) A clipper @ \$25 b) A keyboard @ \$24.50
- 23 A discount of 20% is offered on each of the following items.
Find the discount, and the new price:
- a) A digital radio receiver @ \$88 b) A TV @ \$690
- 24 An insurance premium of \$945 is given a 10% discount. What is then the new cost of the premium?
- 25 Which is the best buy?
- a) \$7.20 for 2 kg of tomatoes or \$17 for 5 kg?
 - b) \$54 for 12 m of cornice molding or \$69 for 15 m?
 - c) \$240 for 20 buckets or \$396 for 36 buckets?
- 26 Find the rate for 100 of each of the following:
- a) \$8 for 200 g b) \$25 for 500 mL
 - c) \$32 for 800 g d) \$7.20 for 300 g
 - e) \$11.20 for 700 mL f) \$22.52 for 400 mins
- 27 Which is the best buy?
- a) \$6.20 for 500 mL of soy sauce or \$9.60 for 800 mL?
 - b) \$2.24 for 400 g of milk powder or \$3.72 for 600 g?
 - c) \$16.45 for 700 g of fruit and nut mix or \$21.33 for 900 g?

Number 2



- ★ Compare fractions using equivalence (by using a fraction wall or a number line).
- ★ Solve problems involving addition and subtraction of fractions.
- ★ Multiply and divide fractions.

I'm a fraction of myself.



A TASK

- a) How long did Diophantus live?
'My boyhood lasted $\frac{1}{6}$ of my life. My beard grew after $\frac{1}{12}$ more. I married after $\frac{1}{7}$ more. Five years later my son was born. My son lived to $\frac{1}{2}$ of my age. I died four years after my son.'

$$\frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4 = x$$

- b) Make a riddle for your age.

A LITTLE BIT OF HISTORY

Diophantus of Alexandria (circa 250 BC) is known as the father of algebra and wrote a series of books which summarised mathematical knowledge at the time.

Diophantus was the first Greek mathematician to recognise fractions as numbers.

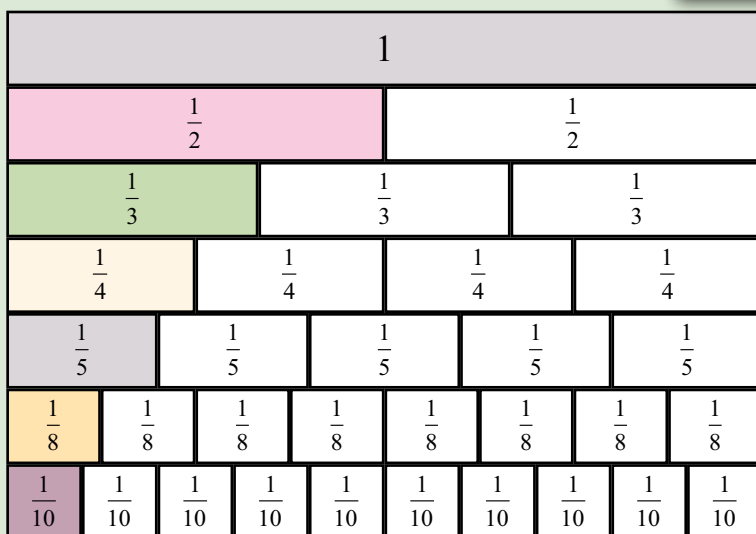
His collection of 13 books, Arithmetica, contained about 150 problems and their solutions. Only 6 books survive.



1621 edition of Arithmetica.

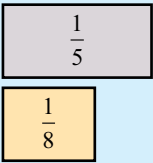
Equivalent Fractions

Fraction walls are useful for comparing fractions.



Exercise 6.1

1 Which fraction is larger?

$\frac{1}{5}$ or $\frac{1}{8}$


$\frac{1}{5}$ is larger than $\frac{1}{8}$

Can you think of another way to compare fractions?



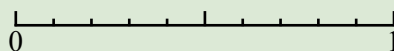
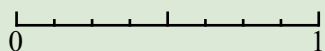
- | | | |
|------------------------------------|-----------------------------------|------------------------------------|
| a) $\frac{1}{2}$ or $\frac{1}{3}$ | b) $\frac{1}{3}$ or $\frac{1}{4}$ | c) $\frac{1}{4}$ or $\frac{1}{5}$ |
| d) $\frac{1}{5}$ or $\frac{1}{8}$ | e) $\frac{1}{3}$ or $\frac{1}{5}$ | f) $\frac{1}{8}$ or $\frac{1}{10}$ |
| g) $\frac{1}{4}$ or $\frac{1}{10}$ | h) $\frac{1}{5}$ or $\frac{1}{3}$ | i) $\frac{1}{4}$ or $\frac{1}{2}$ |

2 Put the following fractions in order from smallest to largest.

- | | |
|--|--|
| a) $\frac{1}{5}, \frac{1}{3}, \frac{1}{8}, \frac{1}{4}$ | b) $\frac{1}{10}, \frac{1}{2}, \frac{1}{5}, \frac{1}{8}$ |
| c) $\frac{1}{5}, \frac{1}{4}, \frac{1}{8}, \frac{1}{10}$ | d) $\frac{1}{5}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ |

3 Put the following fractions on a copy of the number line:

- | | |
|--|--|
| a) $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ | b) $\frac{1}{10}, \frac{1}{5}, \frac{1}{20}$ |
|--|--|



4 Which fraction is larger?

$\frac{3}{5}$ or $\frac{4}{8}$ <u>$\frac{3}{5}$ is larger than $\frac{4}{8}$</u>	<table border="1"> <tr> <td>$\frac{1}{5}$</td> <td>$\frac{1}{5}$</td> <td>$\frac{1}{5}$</td> </tr> <tr> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> </tr> </table>	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$					

a) $\frac{1}{2}$ or $\frac{2}{3}$

b) $\frac{1}{3}$ or $\frac{2}{5}$

c) $\frac{3}{4}$ or $\frac{2}{3}$

d) $\frac{2}{3}$ or $\frac{5}{8}$

e) $\frac{4}{5}$ or $\frac{5}{8}$

f) $\frac{2}{3}$ or $\frac{3}{5}$

5 Put the following fractions in order from smallest to largest.

a) $\frac{2}{5}, \frac{3}{4}, \frac{5}{8}$

b) $\frac{5}{10}, \frac{5}{8}, \frac{3}{5}$

c) $\frac{3}{4}, \frac{2}{3}, \frac{7}{8}$

d) $\frac{9}{10}, \frac{4}{5}, \frac{3}{4}$

6 Use the fraction wall to complete the following:

$\frac{3}{4} = \frac{?}{8}$ <u>$\frac{3}{4} = \frac{6}{8}$</u>	<table border="1"> <tr> <td>$\frac{1}{4}$</td> <td>$\frac{1}{4}$</td> <td>$\frac{1}{4}$</td> </tr> <tr> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> <td>$\frac{1}{8}$</td> </tr> </table>	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$								
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$					

a) $\frac{1}{2} = \frac{?}{4}$

b) $\frac{1}{2} = \frac{?}{8}$

c) $\frac{1}{2} = \frac{?}{10}$

d) $\frac{1}{4} = \frac{?}{8}$

e) $\frac{3}{4} = \frac{?}{8}$

f) $\frac{1}{5} = \frac{?}{10}$

g) $\frac{2}{5} = \frac{?}{10}$

h) $\frac{3}{5} = \frac{?}{10}$

i) $\frac{4}{5} = \frac{?}{10}$

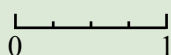
j) $\frac{2}{3} = \frac{?}{6} = \frac{?}{9}$

k) $2 = \frac{8}{4} = \frac{?}{8} = \frac{?}{10}$

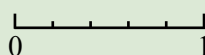
l) $3 = \frac{9}{3} = \frac{?}{5} = \frac{?}{10}$

7 Draw an appropriate number line and place the fractions on the number line (label the number line).

a) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$



b) $\frac{1}{5}, \frac{2}{5}, \frac{4}{5}, 1$



c) $\frac{1}{10}, \frac{2}{5}, \frac{4}{5}, \frac{7}{10}$

d) $\frac{1}{2}, \frac{3}{8}, \frac{3}{4}, \frac{5}{8}, \frac{6}{8}$

Fractions

$$\frac{a}{b}$$

← Numerator
← Denominator

Q Who invented fractions?
A Henry the 1/8th.

Proper fraction $a < b$

Examples: $\frac{3}{5}, \frac{15}{17}, \frac{165}{219}$

Improper fraction $a > b$

Examples: $\frac{3}{2}, \frac{9}{2}, \frac{518}{37}$

Mixed number whole number and fraction.

Examples: $2\frac{3}{4}, 3\frac{2}{5}, 1\frac{19}{24}$

Exercise 6.2

1 Change each of the following mixed numbers to an improper fraction:

$$\begin{aligned} 2\frac{3}{5} &= 2 + \frac{3}{5} \\ &= \frac{10}{5} + \frac{3}{5} \\ &= \frac{13}{5} \end{aligned}$$

This might be easier:

$$\begin{aligned} 2 \times 5 + 3 &= 13 \\ \text{Thus: } &\frac{13}{5} \end{aligned}$$

a) $2\frac{1}{2}$

b) $1\frac{3}{5}$

c) $1\frac{2}{5}$

d) $2\frac{2}{3}$

e) $1\frac{1}{8}$

f) $3\frac{3}{4}$

g) $2\frac{1}{4}$

h) $3\frac{7}{10}$

2 Change each of the following improper fractions to a mixed number:

$$\begin{aligned} \frac{13}{5} \quad 13 \div 5 &= 2 \text{ remainder } 3 \\ &= 2\frac{3}{5} \end{aligned}$$

a) $\frac{5}{2}$

b) $\frac{7}{3}$

c) $\frac{9}{4}$

d) $\frac{9}{5}$

e) $\frac{8}{5}$

f) $\frac{10}{3}$

g) $\frac{19}{7}$

h) $\frac{37}{11}$

Food Scientists use knowledge from biology, chemical engineering, and biochemistry to improve and develop new products for the general public.

- Relevant school subjects are English, Mathematics, Science.
- Courses normally involve a University degree.

Adding Fractions

Adding fractions when the denominators are the same.

Exercise 6.3

$\frac{1}{4} + \frac{2}{4}$

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

1 $\frac{1}{4} + \frac{2}{4}$

2 $\frac{1}{5} + \frac{2}{5}$

3 $\frac{3}{7} + \frac{2}{7}$

4 $\frac{1}{3} + \frac{1}{3}$

5 $\frac{3}{10} + \frac{2}{10}$

6 $\frac{1}{8} + \frac{3}{8}$

7 $\frac{2}{3} + \frac{1}{3}$

8 $\frac{5}{10} + \frac{2}{10}$

9 $\frac{1}{6} + \frac{3}{6}$

10 $\frac{2}{12} + \frac{5}{12}$

11 $\frac{7}{16} + \frac{2}{16}$

12 $\frac{13}{20} + \frac{5}{20}$

Adding fractions when the denominators are different.

The trick is to make the denominators the same.

$\frac{1}{4} + \frac{2}{3}$
 Draw a 4 x 3 grid:

$$= \frac{1 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4}$$

$$= \frac{3}{12} + \frac{8}{12}$$

$$= \frac{11}{12}$$

$$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$$

13 $\frac{1}{2} + \frac{1}{3}$

14 $\frac{1}{2} + \frac{1}{4}$

15 $\frac{1}{2} + \frac{1}{5}$

16 $\frac{1}{2} + \frac{1}{6}$

17 $\frac{1}{3} + \frac{1}{4}$

18 $\frac{1}{3} + \frac{1}{5}$

19 $\frac{1}{3} + \frac{1}{6}$

20 $\frac{1}{3} + \frac{1}{7}$

21 $\frac{1}{2} + \frac{2}{5}$

22 $\frac{1}{2} + \frac{3}{5}$

23 $\frac{1}{2} + \frac{3}{7}$

24 $\frac{1}{2} + \frac{3}{8}$

25 $\frac{2}{5} + \frac{1}{6}$

26 $\frac{2}{3} + \frac{1}{5}$

27 $\frac{3}{5} + \frac{1}{2}$

28 $\frac{2}{3} + \frac{5}{8}$

29 $\frac{5}{8} + \frac{1}{3}$

30 $\frac{3}{7} + \frac{1}{2}$

31 $\frac{3}{4} + \frac{1}{2}$

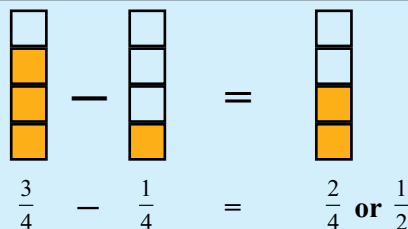
32 $\frac{3}{4} + \frac{2}{3}$

Subtracting Fractions

Subtracting fractions when the denominators are the same.

Exercise 6.4

$$\frac{3}{4} - \frac{1}{4}$$



$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} \text{ or } \frac{1}{2}$$

1 $\frac{3}{4} - \frac{1}{4}$

2 $\frac{2}{3} - \frac{1}{3}$

3 $\frac{5}{9} - \frac{1}{9}$

4 $\frac{4}{5} - \frac{1}{5}$

5 $\frac{3}{5} - \frac{1}{5}$

6 $\frac{3}{5} - \frac{2}{5}$

7 $\frac{7}{8} - \frac{3}{8}$

8 $\frac{4}{9} - \frac{2}{9}$

9 $\frac{7}{10} - \frac{3}{10}$

10 $\frac{8}{10} - \frac{5}{10}$

11 $\frac{13}{20} - \frac{9}{20}$

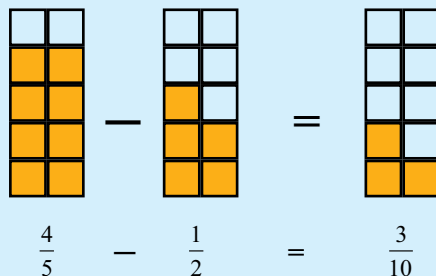
12 $\frac{67}{100} - \frac{35}{100}$

Subtracting fractions when the denominators are different.

The trick is to make the denominators the same.

$$\frac{4}{5} - \frac{1}{2}$$

Draw a 5 x 2 grid:



$$= \frac{4 \times 2}{5 \times 2} - \frac{1 \times 5}{2 \times 5}$$

$$= \frac{8}{10} - \frac{5}{10}$$

$$= \frac{3}{10}$$

13 $\frac{1}{2} - \frac{1}{3}$

14 $\frac{1}{2} - \frac{1}{4}$

15 $\frac{1}{2} - \frac{1}{5}$

16 $\frac{1}{2} - \frac{1}{6}$

17 $\frac{1}{3} - \frac{1}{4}$

18 $\frac{1}{3} - \frac{1}{5}$

19 $\frac{1}{3} - \frac{1}{6}$

20 $\frac{1}{3} - \frac{1}{7}$

21 $\frac{1}{2} - \frac{2}{5}$

22 $\frac{3}{5} - \frac{1}{2}$

23 $\frac{1}{2} - \frac{3}{7}$

24 $\frac{1}{2} - \frac{3}{8}$

25 $\frac{2}{5} - \frac{1}{6}$

26 $\frac{2}{3} - \frac{2}{5}$

27 $\frac{4}{5} - \frac{2}{3}$

28 $\frac{2}{3} - \frac{5}{8}$

29 $\frac{2}{3} - \frac{3}{8}$

30 $\frac{4}{7} - \frac{3}{8}$

31 $\frac{3}{4} - \frac{5}{8}$

32 $\frac{17}{20} - \frac{29}{100}$

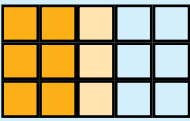
Multiplying Fractions

While calculators handle fractions very well, the skills learned by manually dealing with fractions is very helpful when dealing with algebra and mental computations.

Exercise 6.5

Calculate each of the following:

Multiply the numerators.
Multiply the denominators.

Method 1	Method 2
$\frac{2}{3} \times \frac{3}{5}$ Draw a 3 x 5 grid.  <p>Shade $\frac{3}{5}$ of grid.</p> <p>Shade $\frac{2}{3}$ of the $\frac{3}{5}$</p> $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$	$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5}$ $= \frac{6}{15} = \frac{2}{5}$

1 $\frac{4}{5} \times \frac{3}{5}$

2 $\frac{3}{4} \times \frac{3}{5}$

3 $\frac{2}{5} \times \frac{4}{3}$

4 $\frac{2}{7} \times \frac{3}{5}$

5 $\frac{1}{2} \times \frac{4}{5}$

6 $\frac{2}{5} \times \frac{3}{4}$

7 $\frac{2}{3} \times \frac{3}{4}$

8 $\frac{1}{4} \times \frac{2}{5}$

$\frac{1}{3} \times 1\frac{1}{2} = \frac{1}{3} \times \frac{3}{2}$ $= \frac{3}{6}$ $= \frac{1}{2}$	$8 \times 1\frac{5}{6} = \frac{8}{1} \times \frac{11}{6}$ $= \frac{88}{6}$ $= \frac{44}{3}$ $= 14\frac{2}{3}$
--	---

Change a mixed number to an improper fraction before multiplying.



9 $1\frac{1}{2} \times \frac{3}{4}$

10 $\frac{1}{2} \times 1\frac{3}{4}$

11 $\frac{2}{3} \times 2\frac{1}{2}$

12 $\frac{1}{5} \times 2\frac{1}{2}$

13 $2 \times \frac{4}{5}$

14 $3 \times \frac{3}{4}$

15 $\frac{1}{2} \times 5$

16 $\frac{2}{3} \times 9$

17 $12 \times \frac{2}{3}$

18 $\frac{3}{8} \times \frac{4}{5}$

19 $\frac{3}{4} \times \frac{3}{4}$

20 $(\frac{2}{3})^2$

21 $\frac{2}{5}$ of 20

22 $\frac{2}{10}$ of 400

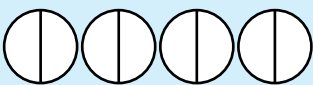
23 $\frac{2}{10}$ of 3.6

24 $1\frac{3}{4}$ of 10

Dividing Fractions

Exercise 6.6

Calculate each of the following:

Method 1	Method 2
$4 \div \frac{1}{2}$ 	$4 \div \frac{1}{2} = \frac{4}{1} \times \frac{2}{1}$ $= \frac{8}{1}$ $= \underline{8}$
1 How many halves in 4?	
2 There are 4 x 2 halves	
3 Answer = $\frac{4}{1} \times \frac{2}{1} = \underline{8}$	

To divide by a fraction, turn the fraction upside down and multiply.

1 $3 \div \frac{1}{2}$

2 $5 \div \frac{1}{2}$

3 $2 \div \frac{1}{2}$

4 $6 \div \frac{1}{2}$

5 $2 \div \frac{1}{3}$

6 $2 \div \frac{2}{3}$

7 $3 \div \frac{1}{4}$

8 $3 \div \frac{3}{4}$

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$ $= \frac{6}{4}$ $= \frac{3}{2}$ $= \underline{1\frac{1}{2}}$	$1\frac{2}{3} \div 1\frac{1}{3} = \frac{5}{3} \div \frac{4}{3}$ $= \frac{5}{3} \times \frac{3}{4}$ $= \frac{15}{12}$ $= \frac{5}{4}$ $= \underline{1\frac{1}{4}}$
---	---

First, change the mixed numeral to an improper fraction.

Calculators are very good at multiplying and dividing fractions.
See Technology 6.3

9 $\frac{3}{4} \div \frac{1}{2}$

10 $\frac{2}{3} \div \frac{3}{4}$

11 $\frac{1}{3} \div \frac{3}{5}$

12 $\frac{2}{3} \div \frac{4}{5}$

13 $\frac{3}{4} \div \frac{2}{5}$

14 $\frac{3}{5} \div \frac{7}{20}$

15 $\frac{6}{7} \div \frac{2}{3}$

16 $2\frac{1}{2} \div \frac{1}{2}$

17 $1\frac{2}{3} \div \frac{1}{3}$

18 $1\frac{1}{2} \div \frac{1}{2}$

19 $\frac{4}{5} \div 1\frac{1}{3}$

20 $1\frac{2}{3} \div 1\frac{1}{2}$

21 $1\frac{2}{3} \div 2\frac{1}{5}$

22 $1\frac{2}{3} \div 2\frac{1}{3}$

23 $1\frac{1}{4} \div 2\frac{1}{3}$

24 $2\frac{1}{2} \div 1\frac{1}{3}$

25 $1\frac{1}{4} \div 3\frac{1}{3}$

26 $2\frac{3}{8} \div 1\frac{1}{2}$

27 $4 \div \frac{1}{2}$

28 $15 \div 1\frac{1}{3}$

Mental Computation

Exercise 6.7

- 1 Spell improper.
- 2 Change 0.5 to a fraction.
- 3 Change $\frac{3}{5}$ to a decimal.
- 4 Change $3\frac{2}{5}$ to an improper fraction.
- 5 What is 20% of 21?
- 6 Calculate: $\frac{2}{7} + \frac{3}{7}$
- 7 Calculate: $\frac{1}{2} \times \frac{3}{5}$
- 8 A 6-sided die is tossed, what is $P(>3)$?
- 9 What is the area of a rectangle 5 cm by 8 cm?
- 10 I buy 40 L of petrol @ \$2/L with \$100. How much change?

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 6.8

- 1 Spell multiply.
- 2 Change 0.4 to a fraction.
- 3 Change $\frac{3}{4}$ to a decimal.
- 4 Change $2\frac{3}{4}$ to an improper fraction.
- 5 What is 20% of 32?
- 6 Calculate: $\frac{3}{4} - \frac{1}{4}$
- 7 Calculate: $\frac{2}{3} \times \frac{1}{5}$
- 8 A 6-sided die is tossed, what is $P(<3)$?
- 9 What is the area of a triangle 3 cm by 8 cm?
- 10 I buy 35 L of petrol @ \$2/L with \$100. How much change?



A horse tied to a rope. The rope is 3 m long. A bale of hay is 5 m in front of the horse. The horse is able to eat the hay, yet does not break the rope. How is that possible?

The rope isn't tied to anything!

Exercise 6.9

- 1 Spell numeral.
- 2 Change 0.3 to a fraction.
- 3 Change $\frac{2}{5}$ to a decimal.
- 4 Change $2\frac{1}{4}$ to an improper fraction.
- 5 What is 20% of 34?
- 6 Calculate: $\frac{1}{4} + \frac{1}{3}$
- 7 Calculate: $\frac{1}{4} \times \frac{3}{5}$
- 8 A 6-sided die is tossed, what is $P(\text{even})$?
- 9 What is the area of a rectangle 5 cm by 7 cm?
- 10 I buy 32 L of petrol @ \$2/L with \$100. How much change?

How is it one careless match can start a bush fire, but it takes a whole box to start a campfire?



Exercise 6.10

- 1 What is exactly halfway between $\frac{1}{4}$ and $\frac{3}{4}$?
- 2 What is exactly halfway between $1\frac{1}{4}$ and $2\frac{3}{4}$?
- 3 What is exactly halfway between $1\frac{3}{4}$ and $3\frac{1}{4}$?
- 4 One-third of the load is 2 tonnes. How heavy was the load?
- 5 One-quarter of the payout is \$50. How much was the payout?
- 6 What is the value of ♥, if ♥ \times 20 = 8?
- 7 The bucket of potting mix contained three-quarters of a bucket of compost. How much compost is needed to make 3 buckets of potting mix?

Halfway between 2 and 5:

$$= \frac{(2+5)}{2} = 3\frac{1}{2}$$



Don't do too much in your head. Pen and paper will give better results.

- 8 Find the value of:
 - a) $\frac{3}{10} + \frac{2}{10}$
 - b) $\frac{1}{2} + \frac{1}{3}$
 - c) $\frac{1}{2} - \frac{1}{3}$
- 9 Write $3\frac{2}{3}$ as a vulgar fraction.
- 10 Write $\frac{15}{4}$ as a mixed numeral.
- 11 Write $5\frac{3}{4}$ as a decimal fraction.
- 12 Write $\frac{3}{4}$ as a percentage.
- 13 A bag has 15 red marbles and 20 blue marbles. What fraction of the marbles are blue?
- 14 The pizza is cut into 12 equal slices. How many slices are left after two-thirds of the pizza is eaten?
- 15 The painting contract returned \$24 000. $\frac{1}{2}$ of the \$24 000 went to labour and materials and $\frac{1}{3}$ of the \$24 000 was paid in tax. How much was left?
- 16 Chloe completed $\frac{2}{5}$ of the task on the first day and $\frac{2}{5}$ of the remaining task on the second day. What percentage of the whole task did Chloe complete on the second day?
- 17 The concrete is made up of two-thirds sand, one-quarter gravel and the remainder is cement. What fraction of the concrete is cement?

Vulgar and improper is the same thing.

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.



Exercise 6.11



- 1 Approximately what fraction of the above rectangle is blue?
- 2 Approximately what fraction of the above rectangle is red?
- 3 Which of the following fractions is the largest?

a) $\frac{11}{22}$

b) $\frac{22}{33}$

c) $\frac{33}{44}$

d) $\frac{44}{55}$

4 Find the value of $\frac{0.5}{2}$

5 Find the value of $\frac{1}{2} \times \left(\frac{1}{2} + \frac{1}{2} \right)$

6 What is the value of $10 + 1 + \frac{1}{10} + \frac{1}{100}$?

7 Find the value of each of the following:

a) $\frac{1}{2} + \frac{1}{3}$

b) $\frac{1}{2} - \frac{1}{4}$

c) $\frac{2}{3} \times \frac{3}{2}$

- 8 How many minutes in three-quarters of an hour?
- 9 A bus journey takes two and one-quarter hours. If the bus leaves at 9:10 am, when would the bus be expected to arrive?
- 10 Two-thirds of a number is twelve. What is the number?
- 11 Jo is $\frac{2}{5}$ the height of her father. Jo's brother, Bill, is $\frac{3}{5}$ the height of his father (Jo's father). What fraction of Bill's height is Jo?
- 12 The recipe for potting mix suggested that 8 buckets of compost be mixed with 3 buckets of soil. Sophie decided to use $\frac{3}{4}$ of the amount of compost and the same amount of soil. What fraction of the potting mix will be compost?
- 13 Amelia received $\frac{1}{2}$ of the inheritance. Arthur received $\frac{1}{3}$ of the inheritance. If there was \$300 000 left, what was the value of the inheritance?
- 14 What is the value of unknown?

a)

$$\frac{3}{?} \times \frac{4}{5} = \frac{2}{5} + \frac{4}{5}$$

b)

$$\frac{3}{4} + \frac{2}{5} = 1\frac{1}{2} - \frac{?}{20}$$

A Diophantus problem
Find two integers such that when one forms their product and adds the square of either integer to it, the result will be a square.

Investigations

Investigation 6.1

Magic Fractions

4	9	2
3	5	7
8	1	6

Magic squares were thought to bring good luck and good health to the wearer.

In this magic square, every row, column, and diagonal sums to 15.

Great - If you are 15!

Make magic squares for 10, 11, 12, 13, and 14 year-old people.

5	10	3
4	6	8
9	2	7

Hint: Adding 1 to every number makes a magic square for an 18 year-old.

What would happen if you subtracted $\frac{1}{3}$ from each number?

Investigation 6.2

Fraction Patterns 1

Copy and complete the following table by finding the fraction of the number:

	1	10	100	1000	10 000	100 000	1 000 000
$\frac{1}{4}$	0.25						
$\frac{1}{2}$			50				
$\frac{3}{4}$		7.5					

Investigation 6.3

Fraction Patterns 2

Use a calculator to investigate each of the following:

Investigate

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$$



Investigate

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$$

Investigate

Cyclic numbers:

$$\frac{1}{7} =$$

$$\frac{4}{7} =$$

$$\frac{2}{7} =$$

$$\frac{5}{7} =$$

$$\frac{3}{7} =$$

$$\frac{6}{7} =$$

A Couple of Puzzles

Exercise 6.12

1 What is $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of a pie?

3

17		1		15
	5	7	14	16
4	6	13		
	12			
11	18	25		9

2

2			7
	8	1	
	10	15	4
			9

Use the numbers 1 to 16, once each, to complete this magic square.

Use the numbers 1 to 25, once each, to complete this magic square.

A Game

Combinations by 3

- 1 Make nine cards with the numbers 1 to 9 written on them.
- 2 Turn the cards face down and mix them up.
- 3 When it is their turn, Each player/team turns up 3 cards and then uses $+$, $-$, \times , \div , and $()$ to make a whole number with an **upper limit of 20**.
- 4 Player/team with largest total score after 5 turns wins.

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

6	3	1
---	---	---

$$3 \times (6 - 1) = 15$$

A Sweet Trick

You race your audience. They have a calculator, you don't.

Your audience volunteers a five digit number:

You write a five digit number:

Your audience volunteers a five digit number:

You write a five digit number:

71956

28043

36170

63829

You write the 9 complement.

When someone says go

They add them up.

You write up the answer faster than they can use a calculator.

199 998



Can you work out why the answer will always be 199 998?

Technology

Technology 6.1 Calculators and fractions 1

Calculators, with $\left[a \frac{b}{c}\right]$, are very good at simplifying fractions:

1 $\frac{6}{9}$ $\left[6\right] \left[a \frac{b}{c}\right] \left[9\right] [=] \left[2r3\right]$ meaning $\frac{2}{3}$

2 $\frac{36}{60}$ $\left[36\right] \left[a \frac{b}{c}\right] \left[60\right] [=] \left[3r5\right]$ meaning $\frac{3}{5}$



Technology 6.2 Calculators and fractions 2

Calculators, with $\left[a \frac{b}{c}\right]$, are very good with improper fractions and mixed numerals:

1 $\frac{15}{7}$ $\left[15\right] \left[a \frac{b}{c}\right] \left[7\right] [=] \left[2r1r7\right]$ meaning $2\frac{1}{7}$

2 Then $\left[2ndF\right] \left[a \frac{b}{c}\right]$ will give $\left[15r7\right]$ meaning $\frac{15}{7}$



Technology 6.3 Calculators and fractions 3

Calculators, with $\left[a \frac{b}{c}\right]$, are very good at fraction operations:

1 $\frac{3}{5} + \frac{2}{7}$ $\left[3\right] \left[a \frac{b}{c}\right] \left[5\right] [+] \left[2\right] \left[a \frac{b}{c}\right] \left[7\right] [=] \left[31r35\right]$ meaning $\frac{31}{35}$

2 $\frac{4}{5} - \frac{2}{3}$ $\left[4\right] \left[a \frac{b}{c}\right] \left[5\right] [-] \left[2\right] \left[a \frac{b}{c}\right] \left[3\right] [=] \left[2r15\right]$ meaning $\frac{2}{15}$

3 $\frac{2}{7} \times \frac{5}{8}$ $\left[2\right] \left[a \frac{b}{c}\right] \left[7\right] [\times] \left[5\right] \left[a \frac{b}{c}\right] \left[8\right] [=] \left[5r28\right]$ meaning $\frac{5}{28}$

4 $\frac{3}{4} \div \frac{5}{8}$ $\left[3\right] \left[a \frac{b}{c}\right] \left[4\right] [\div] \left[5\right] \left[a \frac{b}{c}\right] \left[8\right] [=] \left[1r1r5\right]$ meaning $1\frac{1}{5}$

5 $2\frac{1}{2} + 1\frac{2}{3}$ $\left[2\right] \left[a \frac{b}{c}\right] \left[1\right] \left[a \frac{b}{c}\right] \left[2\right] [+] \left[1\right] \left[a \frac{b}{c}\right] \left[2\right] \left[a \frac{b}{c}\right] \left[3\right] [=]$

$\left[4r1r6\right]$ meaning $4\frac{1}{6}$



Technology 6.4 Fraction games and applets

Search the internet for some of the many fraction games and applets.

Give special attention to games and applets that involve:

- ★ Fraction wall games.
- ★ Mixed numbers.
- ★ Adding and subtracting fractions.
- ★ Multiplying fractions and dividing fractions.

Chapter Review 1

Exercise 6.13

1 Put the following fractions in order from smallest to largest.

a) $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}$

b) $\frac{1}{5}, \frac{1}{9}, \frac{1}{2}, \frac{1}{8}$

c) $\frac{5}{8}, \frac{2}{5}, \frac{3}{4}$

d) $\frac{7}{10}, \frac{5}{8}, \frac{4}{5}$

2 Complete the following:

a) $\frac{2}{5} = \frac{?}{10}$

b) $\frac{3}{4} = \frac{?}{12}$

c) $\frac{2}{3} = \frac{?}{6} = \frac{?}{9}$

d) $2 = \frac{8}{4} = \frac{?}{8} = \frac{?}{10}$

3 Change each of the following improper fractions to a mixed number:

a) $\frac{5}{2}$

b) $\frac{7}{3}$

c) $\frac{9}{4}$

d) $\frac{9}{5}$

e) $\frac{8}{5}$

f) $\frac{10}{3}$

g) $\frac{19}{7}$

h) $\frac{37}{11}$

4 Change each of the following mixed numbers to an improper fraction:

a) $2\frac{1}{2}$

b) $1\frac{3}{5}$

c) $1\frac{2}{5}$

d) $2\frac{2}{3}$

e) $1\frac{1}{8}$

f) $3\frac{3}{4}$

g) $2\frac{1}{4}$

h) $3\frac{7}{10}$

5 Calculate each of the following:

a) $\frac{1}{4} + \frac{1}{4}$

b) $\frac{1}{5} + \frac{2}{5}$

c) $\frac{3}{7} + \frac{2}{7}$

d) $\frac{1}{2} + \frac{1}{3}$

e) $\frac{1}{2} + \frac{1}{4}$

f) $\frac{2}{5} + \frac{1}{4}$

g) $\frac{1}{2} - \frac{1}{5}$

h) $\frac{1}{2} - \frac{1}{3}$

i) $\frac{2}{3} - \frac{1}{3}$

j) $\frac{2}{3} - \frac{1}{5}$

k) $\frac{5}{9} - \frac{3}{9}$

l) $\frac{3}{7} - \frac{2}{5}$

m) $\frac{1}{3} \times \frac{1}{2}$

n) $\frac{1}{3} \times \frac{2}{5}$

o) $\frac{2}{5} \times \frac{3}{4}$

p) $\frac{4}{7} \times \frac{1}{2}$

q) $1\frac{2}{3} \times \frac{3}{4}$

r) $2\frac{2}{5} \times 1\frac{1}{2}$

s) $\frac{1}{3} \div \frac{1}{2}$

t) $\frac{1}{4} \div \frac{1}{2}$

u) $\frac{2}{3} \div \frac{1}{2}$

v) $\frac{3}{4} \div \frac{2}{5}$

w) $2\frac{1}{2} \div \frac{2}{3}$

x) $3\frac{2}{3} \div 1\frac{1}{3}$

Chapter Review 2

Exercise 6.14

1 Put the following fractions in order from smallest to largest.

a) $\frac{1}{3}, \frac{1}{5}, \frac{1}{2}, \frac{1}{7},$

b) $\frac{1}{7}, \frac{1}{12}, \frac{1}{9}, \frac{1}{11},$

c) $\frac{3}{4}, \frac{4}{5}, \frac{2}{3}$

d) $\frac{2}{5}, \frac{3}{10}, \frac{1}{3}$

2 Complete the following:

a) $\frac{3}{5} = \frac{?}{10}$

b) $\frac{2}{3} = \frac{?}{6}$

c) $\frac{1}{4} = \frac{?}{8} = \frac{?}{12}$

d) $3 = \frac{?}{2} = \frac{?}{5} = \frac{?}{7}$

3 Change each of the following improper fractions to a mixed number:

a) $\frac{3}{2}$

b) $\frac{9}{2}$

c) $\frac{9}{5}$

d) $\frac{6}{5}$

e) $\frac{7}{4}$

f) $\frac{17}{4}$

g) $\frac{33}{8}$

h) $\frac{37}{13}$

4 Change each of the following mixed numbers to an improper fraction:

a) $2\frac{1}{4}$

b) $3\frac{3}{4}$

c) $3\frac{7}{10}$

d) $5\frac{4}{5}$

e) $3\frac{3}{8}$

f) $1\frac{3}{5}$

g) $2\frac{1}{2}$

h) $1\frac{2}{5}$

5 Calculate each of the following:

a) $\frac{1}{5} + \frac{1}{5}$

b) $\frac{1}{6} + \frac{4}{6}$

c) $\frac{2}{9} + \frac{5}{9}$

d) $\frac{1}{4} + \frac{1}{4}$

e) $\frac{2}{3} + \frac{1}{2}$

f) $\frac{3}{7} + \frac{2}{3}$

g) $\frac{1}{2} - \frac{1}{5}$

h) $\frac{1}{4} - \frac{1}{5}$

i) $\frac{3}{4} - \frac{1}{3}$

j) $\frac{3}{4} - \frac{1}{2}$

k) $\frac{2}{3} - \frac{3}{5}$

l) $1\frac{1}{3} - \frac{4}{5}$

m) $\frac{1}{3} \times \frac{1}{4}$

n) $\frac{1}{3} \times \frac{2}{5}$

o) $\frac{1}{5} \times \frac{2}{3}$

p) $\frac{2}{5} \times \frac{3}{4}$

q) $2\frac{2}{3} \times \frac{1}{4}$

r) $3\frac{1}{4} \times 1\frac{2}{5}$

s) $\frac{1}{4} \div \frac{1}{5}$

t) $\frac{1}{2} \div \frac{1}{4}$

u) $\frac{3}{5} \div \frac{1}{3}$

v) $\frac{5}{6} \div \frac{2}{3}$

w) $1\frac{3}{4} \div \frac{1}{2}$

x) $2\frac{3}{5} \div 1\frac{1}{6}$

Algebra



- ★ Introduce the concept of variables.
- ★ Move fluently between algebraic and word representations.
- ★ Create algebraic expressions and perform substitutions.
- ★ Identify order of operations.
- ★ Apply the commutative and associative laws to algebraic terms and expressions.

A TASK

7 girls.
Each girl has
7 backpacks.
Each backpack
has 7 cats.
How many legs?



You, a member of the Chamber of Commerce, are presenting a short persuasive speech to a group of teachers. The Chamber of Commerce wants you to convince the teachers that mental computation should be a part of every mathematics lesson.

- Mental computation in the day of a student?
- Mental computation in the day of a business?
- What strategies are used to do mental computation?
- Mental estimation versus exact mental computation?

A LITTLE BIT OF HISTORY

Alexander Aitken (1895-1967), from New Zealand, was recognised as the greatest mathematician of his era.

Aitken was also known for his awesome mental computations:

- He could recite Pi to 707 decimal places.
- Mentally multiply two nine digit numbers in 30 seconds.
- Change fractions to 26 decimal places in five seconds.

For some years from about 15, without telling anyone, I practised mental calculation gradually until what had been difficult at first became easier and easier.



Order of Operations

Order of Operations:

- 1 Work the brackets first.
- 2 Work from left to right and do all \times and \div as you come across them.
- 3 Work from left to right and do all $+$ and $-$ as you come across them.

Everyone in the World
calculates in this order.
Imagine the chaos if they
didn't.



You owe me:
 $8 - 2 \times 3 = 18?$

I owe you:
 $8 - 2 \times 3 = 2?$



Exercise 7.1

Find the value of each of the following:

$(8 - 5) \times 3$ $= 3 \times 3$ {brackets} $= 9$ { \times and \div }	$5 \times 6 - (4 + 2) \div 3 + 2$ $= 5 \times 6 - 6 \div 3 + 2$ {brackets} $= 30 - 2 + 2$ { \times and \div } $= 30$ {+ and -}
--	---

1 $8 - 2 \times 3$

2 $6 \times 5 - 4$

3 $5 + 5 \times 2$

4 $18 - 6 \div 2$

5 $20 - 10 \div 2$

6 $25 + 6 \times 2$

7 $8 \div 2 - 3$

8 $9 \div 3 \times 2 + 1$

9 $3 \times 2 - 2 \times 2$

10 $12 \div 4 + 2 \times 3$

11 $30 - 6 \times 4 \div 8$

12 $(12 + 2) \times 2 + 3$

13 $16 \div (3 + 1) - 2$

14 $2 \times 3 + 8 \div 2 + 1$

15 $18 \div 3 \times 2 + 5$

16 $5 \times 6 - (4 + 2)$

17 $4 \times 3 \div (2 + 1)$

18 $7 + 12 \div (2 + 4)$

19 $5 \times (5 - 3) + 4$

20 $(6 + 2) \div 4 \times 3$

Order of Operations:

- 1 () brackets first.
- 2 \times and \div from left to right.
- 3 $+$ and $-$ from left to right.

Exercise 7.2

Find the value of each of the following:

$ \begin{aligned} &9 - (5 - 3) \times 3 \\ &= 9 - 2 \times 3 \\ &= 9 - 6 \\ &= \underline{3} \end{aligned} $	$ \begin{aligned} &35 - (6 \times 2 - 5) + 8 \times 2 \\ &= 35 - (12 - 5) + 8 \times 2 \\ &= 35 - 7 + 8 \times 2 \\ &= 35 - 7 + 16 \\ &= 28 + 16 \\ &= \underline{44} \end{aligned} $
---	--

1 $15 - 2 \times 5$

3 $6 + (5 \times 2)$

5 $(20 - 10) \div 2$

7 $8 \div 4 - 1$

9 $15 \div 3 - 2 \times 2$

11 $11 - 2 \times (6 \div 2)$

13 $9 \div 3 - (4 - 2)$

15 $35 - (6 \times 2 - 5) + 8 \times 2$

17 $20 \div (3 \times 2 - 1) + 5 \times (7 - 4)$

19 $13 \times 2 - 15 \div (8 - 5) + 4 \times 2$

21 $(12 - 4 \times 2) \times 5 - 6 \times 3 \div 9$

2 $10 \div 2 - 4$

4 $(18 \div 6) - 2$

6 $5 - (1 + 2)$

8 $16 - 4 \times (2 + 1)$

10 $12 \div (4 + 2) \times 2$

12 $(3 + 2) \times 3 + 1$

14 $4 \times (3 + 2) \div 2 + 1$

16 $(3 + 2) \times 4 + 10 \div (8 - 3)$

18 $(8 - 4 \times 1) \div (8 \div (3 + 1))$

20 $12 + 3 \times 2 - 15 \div 3 - 8$

22 $3 + 6 \div (5 - 2) \times 2 + 10$

Calculators calculate in this order.



Use a calculator to check your answers
(Enter the whole problem in one go.)

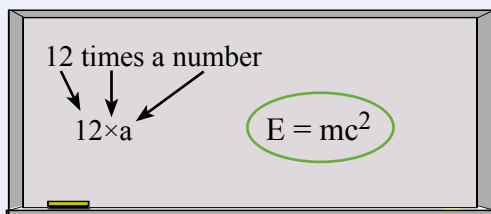
In mathematics, brackets and parentheses are the same thing.

Examples of brackets are:
 $(5 + 2)$, $[5 + 2]$, $\{5 + 2\}$

Algebraic Expressions

Algebraic Expressions:

When letters of the alphabet are used to represent numbers.



Algebra is used to solve millions and millions of problems every day.

$12a$ and $12 \times a$ are the same thing.
 $x \div 3$ and $\frac{x}{3}$ are the same thing.

Use any letter of the alphabet to represent the number. x is a popular letter to use.

Exercise 7.3

Write an algebraic expression for each of the following:

A number plus 5 <u>$a + 5$</u>	Two less than a number <u>$b - 2$</u>	The next consecutive number <u>$x + 1$</u>
--	---	--

- | | |
|---------------------------------|------------------------------|
| 1 A number plus 4 | 2 A number minus 9 |
| 3 6 is added to a number | 4 A number is decreased by 7 |
| 5 The number is increased by 13 | 6 Subtract 12 from a number |
| 7 The sum of a number and 23 | 8 Five less than the number |
| 9 The next consecutive number | 10 Decrease a number by 85 |

A number times 3 <u>$3 \times x$</u>	A number divided by 12 <u>$b \div 12$</u>	Three-fifths of a number <u>$\frac{3}{5} \times x$</u>
--	---	--

- | | |
|-----------------------------------|-------------------------------|
| 11 Double a number | 12 A number divided by 11 |
| 13 Triple a number | 14 Half of a number |
| 15 Multiply a number by 25 | 16 A quarter of a number |
| 17 The product of a number and 87 | 18 Two-thirds of a number |
| 19 Seven times a number | 20 Three-quarters of a number |

The sum of a and b <u>$a + b$</u>	The product of a and b <u>$a \times b$</u>	Double x minus three times y <u>$2 \times x - 3 \times y$</u>
---	--	---

- | | |
|----------------------------|-----------------------------------|
| 21 The sum of b and a | 22 The product of b and a |
| 23 a plus b | 24 b plus a |
| 25 a times b | 26 b times a |
| 27 a is increased by b | 28 a is multiplied by b |
| 29 Triple a and add c | 30 Quadruple x and subtract m |

Substitution

When using substitution in algebra, a variable such as x or y is replaced with its value.

Exercise 7.4

Find the value of $x + 5$ if $x = 4$

$$\begin{aligned}x + 5 &= 4 + 5 \\ &= \underline{9}\end{aligned}$$

Find the value of $2b - 1$ if $b = 3$

$$\begin{aligned}2b - 1 &= 2 \times 3 - 1 \\ &= 6 - 1 \\ &= \underline{5}\end{aligned}$$

- 1 Find the value of each of the following algebraic expressions given that $x = 4$ and $y = 9$.

a) $3x$

b) $2y$

c) $x + 5$

d) $y - 5$

e) $x \div 2$

f) $\frac{y}{3}$

g) $x + y$

h) $y - x$

i) xy

j) $3x + 5$

k) $4x - y$

l) $2y + 10$

- 2 If $x = 10$, what is the value of $7x + 3$?

- 3 $y = 8 - 3x$, what is the value of y when $x = 1.5$?

The area A , of a rectangle of length l , and breadth w , is given by the algebraic formula: $A = lw$.

Find the area of the rectangle if length = 12 m and breadth = 5 m

$$\begin{aligned}A &= lw \\ &= 12 \text{ m} \times 5 \text{ m} \\ &= \underline{60 \text{ m}^2}\end{aligned}$$

- 4 The area, A , of a rectangle of length, l , and breadth, b , is given by the algebraic formula: $A = lb$. Find the area of each of the following rectangles:

a) length = 6 cm and breadth = 5 cm.

b) length = 15 m and breadth = 6 m.

- 5 The weight that a pack mule can carry may be estimated from the formula: $P = 0.2W$, where W is the weight of the mule in kg. Estimate the pack weight that the following mules can carry:

a) Weight = 400 kg.

b) Weight = 450 kg.



- 6 The weight of a pig can be estimated from the formula: $w = 69g^2L$, where g is the girth measurement and L is the length of the pig. Estimate the weight of the following pigs:

a) Girth 1.3 m, length = 1.0 m.

b) Girth 1.1 m, length = 0.9 m.

Commutative Laws

Commutative

An operation is commutative if changing the order of the operands does not change the result.

Addition is commutative because:

$$a + b = b + a$$

Example: $3 + 5 = 5 + 3$

$$a \times b = b \times a \quad a \div b \neq b \div a$$

$$2 \times 5 = 5 \times 2 \quad 8 \div 2 \neq 2 \div 8$$

Commutative means swap?



Exercise 7.5

Calculate each of the following and decide if the operation is commutative:

$7 + 3$ and $3 + 7$ $7 + 3 = 10$ $3 + 7 = 10$ <u>$+$ is commutative</u>	$6 - 4$ and $4 - 6$ $6 - 4 = 2$ $4 - 6 = -2$ <u>$-$ is not commutative</u>	8×2 and 2×8 $8 \times 2 = 16$ $2 \times 8 = 16$ <u>\times is commutative</u>
---	--	--

1 $5 + 4$ and $4 + 5$

2 $8 + 6$ and $6 + 8$

3 $15 + 23$ and $23 + 15$

4 $0 + 2$ and $2 + 0$

5 $3 + 1$ and $1 + 3$

6 $89 + 61$ and $61 + 89$

7 $5 - 4$ and $4 - 5$

8 $8 - 6$ and $6 - 8$

9 $15 - 23$ and $23 - 15$

10 $0 - 2$ and $2 - 0$

11 $3 - 1$ and $1 - 3$

12 $89 - 61$ and $61 - 89$

13 5×4 and 4×5

14 8×6 and 6×8

15 15×23 and 23×15

16 0×2 and 2×0

17 3×1 and 1×3

18 89×61 and 61×89

19 $5 \div 4$ and $4 \div 5$

20 $8 \div 6$ and $6 \div 8$

21 $15 \div 23$ and $23 \div 15$

22 $0 \div 2$ and $2 \div 0$

23 $3 \div 1$ and $1 \div 3$

24 $89 \div 61$ and $61 \div 89$

Addition is commutative

$$a + b = b + a$$

Subtraction is **not** commutative

$$a - b \neq b - a$$

Multiplication is commutative

$$a \times b = b \times a$$

Division is **not** commutative

$$a \div b \neq b \div a$$

Associative Laws

Associative

An operation is associative if changing the grouping of the operands does not change the result.

Addition is associative because:

$$(a + b) + c = a + (b + c)$$

Example: $(3 + 2) + 5 = 3 + (2 + 5)$

$$(a \times b) \times c = a \times (b \times c)$$

$$(4 \times 3) \times 1 = 4 \times (3 \times 1)$$

Associative
means group?



Exercise 7.6

Calculate each of the following and decide if the operation is associative:

$(5 + 3) + 4$ and $5 + (3 + 4)$

$$(5 + 3) + 4 = 8 + 4 = 12$$

$$5 + (3 + 4) = 5 + 7 = 12$$

$+$ is associative

$(7 - 6) - 4$ and $7 - (6 - 4)$

$$(7 - 6) - 4 = 1 - 4 = -3$$

$$7 - (6 - 4) = 7 - 2 = 5$$

$-$ is not associative

$$0 \div a = 0$$

$$a \div 0 = \text{undefined}$$

$$0 \times a = 0$$

$$a \times 0 = 0$$

1 $(2 + 7) + 5$ and $2 + (7 + 5)$

3 $(9 + 0) + 1$ and $9 + (0 + 1)$

5 $(2 - 7) - 5$ and $2 - (7 - 5)$

7 $(9 - 0) - 1$ and $9 - (0 - 1)$

9 $(2 \times 7) \times 5$ and $2 \times (7 \times 5)$

11 $(9 \times 0) \times 1$ and $9 \times (0 \times 1)$

13 $(2 \div 7) \div 5$ and $2 \div (7 \div 5)$

16 $(9 \div 0) \div 1$ and $9 \div (0 \div 1)$

2 $(4 + 6) + 13$ and $4 + (6 + 13)$

4 $(26 + 2) + 98$ and $26 + (2 + 98)$

6 $(2 - 7) - 5$ and $2 - (7 - 5)$

8 $(26 - 2) - 98$ and $26 - (2 - 98)$

10 $(4 \times 6) \times 13$ and $4 \times (6 \times 13)$

12 $(26 \times 2) \times 98$ and $26 \times (2 \times 98)$

14 $(4 \div 6) \div 13$ and $4 \div (6 \div 13)$

16 $(26 \div 2) \div 98$ and $26 \div (2 \div 98)$

Addition is associative

$$(a + b) + c = a + (b + c)$$

Multiplication is associative

$$(a \times b) \times c = a \times (b \times c)$$

Subtraction is **not** associative

$$(a - b) - c \neq a - (b - c)$$

Division is **not** associative

$$(a \div b) \div c \neq a \div (b \div c)$$

Algebraic Expressions

Exercise 7.7

- a) Write an algebraic expression for each of the following pairs:
- b) Find the value of each algebraic expression using $x = 2$, $y = 5$.
- c) Comment on the values obtained.

x plus y and y plus x $x + y$ $y + x$ $2 + 5$ $5 + 2$ $= 7$ $= 7$ <u>$x + y$ and $y + x$ is commutative.</u>	triple x minus y and y minus triple x $3x - y$ $y - 2x$ $3 \times 2 - 5$ $5 - 3 \times 2$ $= 1$ $= -1$ <u>$3x - y$ and $y - 3x$ is not commutative.</u>
---	--

- 1 y plus seven **and** seven plus y .
- 2 The sum of x and 13 **and** the sum of 13 and x .
- 3 Nine plus triple x **and** triple x plus nine.
- 4 Two more than y **and** y more than two.
- 5 The difference between x and y **and** the difference between y and x .
- 6 Double y minus five **and** five minus double y .
- 7 The product of nine and x **and** the product of x and y .
- 8 Triple x times twelve **and** twelve times triple x .
- 9 y divided by two **and** two divided by y .
- 10 Quadruple y divided by two **and** two divided by quadruple y .

Exercise 7.8

- a) Find the value of each algebraic expression using $x = 6$, $y = 3$, $z = 2$.
- b) Comment on the values obtained.

$5 + (x + 2y)$ $5 + (6 + 2 \times 3)$ $= 5 + (6 + 6)$ $= 5 + 12$ $= 17$ <u>$5 + (x + 2y)$ and $(5 + x) + 2y$ is associative.</u>	and	$(5 + x) + 2y$ $(5 + 6) + 2 \times 3$ $= 11 + 6$ $= 17$
---	------------	--

- 1 $(x + y) + z$ **and** $x + (y + z)$
- 2 $(3 + x) + 4$ **and** $3 + (x + 4)$
- 3 $2x + (y + 8)$ **and** $(2x + y) + 8$
- 4 $(z + 10y) + 15$ **and** $z + (10y + 15)$
- 5 $(x - y) - z$ **and** $x - (y - z)$
- 6 $4x - (3 - 2z)$ **and** $(4x - 3) - 2z$
- 7 $9 \times (x \times y)$ **and** $(9 \times x) \times y$
- 8 $3x \times (2y \times z)$ **and** $(3x \times 2y) \times z$
- 9 $x \div (y \div 1)$ **and** $x \div (y \div 1)$
- 10 $(5x \div 2y) \div 10$ **and** $5x \div (2y \div 10)$



What would a 1 km line of \$2 coins be worth?

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 7.9

- 1 Spell substitution.
- 2 Find the value of: $20 - 10 \div 2$
- 3 Find the value of the expression: $5x - 2$ when $x = 3$
- 4 Find the value of the expression: $2(a + 5)$ when $a = 7$
- 5 Is $2a + 5c = 5c + 2a$ true or false?
- 6 Change 0.25 to a fraction.
- 7 Change $\frac{8}{5}$ to a mixed number.
- 8 Calculate: $\frac{2}{9} + \frac{3}{9}$
- 9 Calculate: $\frac{1}{2} \times \frac{1}{3}$
- 10 I buy a \$6.40 magazine with a \$10 note, how much change?

To sleep: perchance to dream: ay,
there's the rub - William Shakespeare.

Exercise 7.10

- 1 Spell commutative.
- 2 Find the value of: $24 + 6 \times 2$
- 3 Find the value of the expression: $2x + 4$ when $x = 2$
- 4 Find the value of the expression: $3b + 2b$ when $b = 3$
- 5 Is $a + (b + c) = (a + b) + c$ true or false?
- 6 Change 0.5 to a fraction.
- 7 Change $\frac{9}{4}$ to a mixed number.
- 8 Calculate: $\frac{1}{3} + \frac{1}{2}$
- 9 Calculate: $\frac{1}{4} \times \frac{1}{3}$
- 10 I buy a \$8.15 magazine with a \$20 note, how much change?

Exercise 7.11

- 1 Spell associative.
- 2 Find the value of: $5 \times 4 - 3$
- 3 Find the value of the expression: $5x \div 2$ when $x = 4$
- 4 Find the value of the expression: $4(3 + x)$ when $x = 2$
- 5 Is $a - d = d - a$ true or false?
- 6 Change 0.75 to a fraction.
- 7 Change $\frac{10}{3}$ to a mixed number.
- 8 Calculate: $\frac{1}{3} + \frac{1}{4}$
- 9 Calculate: $\frac{2}{3} \times \frac{1}{2}$
- 10 I buy a \$16.30 magazine with a \$20 note, how much change?



Why shouldn't you tell secrets
when there's a clock in the room?


Time will tell.




Double check on the calculator.
Check the answer by substituting
back into the question.

Exercise 7.12

- 1 If $a = 3$, what is the value of $8a$?
- 2 If $b = 4$, what is the value of $3b + 5$?
- 3 $y = 8 - 3x$, what is the value of y when $x = 2.5$?

- 4 what is the value of ?




If  = 2,  = 3, and  +  =  + 

- 5 what is the value of ?

If  = 5,  = 2, and  +  -  =  +  + 

- 6 What is the value of $4x - 2x^2 + 7$ when $x = 1$?
- 7 $25 \times \Delta = 35$ What is the value of Δ ?
- 8 The dividend yield, in percent, of shares is given by the formula:
Dividend Yield = $100 \times \text{dividend} \div \text{share price}$.
Calculate the dividend yield of a share with a price of \$25 and a dividend of \$0.80
- 9 A rule for a pattern is multiply by three and then add two. The first three numbers of this pattern are: 5, 8, 11, ... What is the fifth number in this pattern?
- 10 A rule for a pattern is to add two and then multiply by four. The first three numbers of this pattern are: 16, 20, 24, ... What is the tenth number in this pattern?



- 11 What is the missing number?
a) $x \times \text{} = y \times x$
b) $x + (\text{} + \text{}) = (x + y) + z$
- 12 A number is multiplied by itself and then 5 is added. The answer is 14. What is the number?
- 13 Two numbers added together equal 7. The two numbers multiplied together equal 12. What are the two numbers?
- 14 Two numbers added together equal 40. The two numbers multiplied together equal 175. What are the two numbers?

Competition Questions



Build maths muscle and prepare for mathematics competitions at the same time.

Order of Operations:

- 1 () brackets first.
- 2 \times and \div from left to right.
- 3 $+$ and $-$ from left to right.

Exercise 7.13

1 Evaluate each of the following:

- a) $5 + 3 \times 4$
- b) $20 - 12 \div 4$
- c) $2 \times 6 - 4 \div 2$
- d) $(12 + 2) \times 10 - 5$
- e) $6 - (5 - (4 - (3 - (2 - 1))))$

2 Find the unknown in each of the following:

a)

1	2	3	4	
7	10	13	16	?

b)

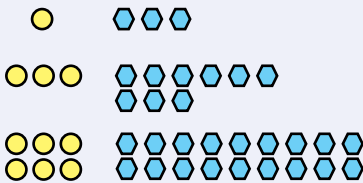
1	3	5	7	
1	9	25	49	?

3 Which rule applies?

x	1	2	3	4	5
y	5	8	11	14	17

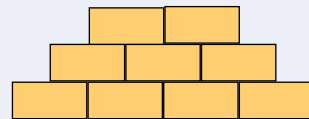
- a) $y = 4x + 1$
- b) $y = 3x + 4$
- c) $y = 3x + 2$
- d) $y = 2x + 3$

4 Which rule applies?



- a) $\text{blue circle} = 4 \times \text{yellow circle} - 1$
- b) $\text{blue circle} = 2 \times \text{yellow circle} + 1$
- c) $\text{blue circle} = \text{yellow circle} + 2$
- d) $\text{blue circle} = 3 \times \text{yellow circle}$

5 Bricks are placed in layers so that a layer has one less brick than the layer below. If there are six layers and the top layer has twelve bricks, how many bricks altogether?



6 When one-sixth of a number is subtracted from itself, the result is 45. What is the number?

7 If x and y are positive numbers, which of the following is the largest:

- a) $a \times b$
- b) $(a + b)^2$
- c) $a^2 + b^2$

Agronomists research and apply knowledge of agricultural crops and grasses to improve agricultural production.

- Relevant school subjects are Mathematics and English.
- Courses usually involve a University Bachelor degree.

Investigations

Investigation 7.1 Commutative

Commutative

An operation is commutative if changing the order of the operands does not change the result.

Addition is commutative because:

$$\mathbf{a + b = b + a}$$

Example: $3 + 5 = 5 + 3$

- 1 Brainstorm five examples of normal-life activities that are not commutative.

Putting on your shoes first and then your socks does not give the same result as putting on your socks first and then your shoes.

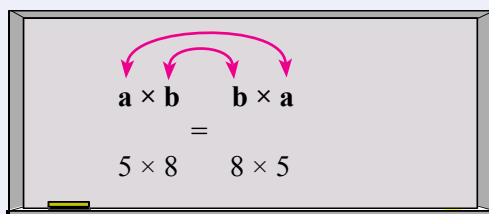
Putting on socks and shoes is not commutative.

- 2 Brainstorm five examples of normal-life activities that are commutative.

Doing your homework first and then calling a friend gives the same result as calling your friend first and then doing your homework.

Investigation 7.2 Online Commutative Activities

- 1 Use a search phrase such as 'Commutative property of addition' to find hundreds of activities about the commutative property.
- 2 Try some of the activities and games.
- 3 Report back to your class about activities that are useful.
- 4 Similarly, use a search phrase such as 'Commutative property of multiplication' to find hundreds of activities about the commutative property.



Investigation 7.3 Online Associative Activities

- 1 Use a search phrase such as 'Associative property of addition' to find hundreds of activities about the associative property.
- 2 Try some of the activities and games.
- 3 Report back to your class about activities that are useful.
- 4 Similarly, use a search phrase such as 'Associative property of multiplication' to find hundreds of activities about the associative property.

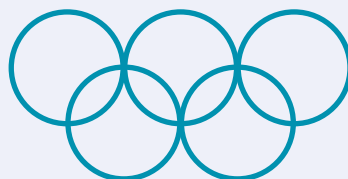
A Couple of Puzzles

Exercise 7.14

- I am a number between 10 and 20.
I am a prime number.
The sum of my digits is 8. Who am I?
- I am a two digit number. If you reverse my digits and add us together, the result is 99. How many of us are there?
- Place each of the numbers 1 to 9 in each space so that the total of the numbers in each ring is 11.

Prime numbers have only two factors: 1 and itself.

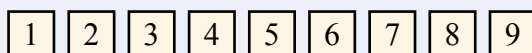
Examples: 2, 3, 5, 7, 11, 13, 17, 19



A Game

Combinations by 4

- Make nine cards with the numbers 1 to 9 written on them.
- Turn the cards face down and mix them up.
- When it is their turn, Each player/team turns up 4 cards and then uses $+$, $-$, \times , \div , and $()$ to make a whole number with an **upper limit of 15**. All 4 cards must be used.
- Player/team with largest total score after 5 turns wins.



$$5 \times (8 \div 4 + 1) = 15$$

A Sweet Trick

You race your audience. They have a calculator, you don't.

Your audience volunteers a five digit number:

You write a five digit number:

Your audience volunteers a five digit number:

You write a five digit number:

Your audience volunteers a five digit number:

You write a five digit number:

71956

28043

36170

63829

83715

16284

You write the 9 complement.

When someone says go

They add them up.

You write up the answer faster than they can use a calculator.



??? ??

Use the trick in Chapter 6 to help you know what the answer should be.

Technology

Technology 7.1 Calculators and Order of Operations

Order of Operations:

- 1 () brackets first.
- 2 \times and \div from left to right.
- 3 $+$ and $-$ from left to right.

Calculators calculate in this order.



Use a calculator to simplify: $4 + 15 \div (3 + 2)$

Enter 4 + 15 \div (3 + 2) =

To give the answer: 7



Enter the expression exactly as it is written.

Use a calculator to check the answers to the earlier exercises.

Technology 7.2 Substitution

Use a spreadsheet to check that the following expressions are equivalent:

	A	B
1	Substituting value	3
2	$2x + 6$	12
3	$2 \times (x + 3)$	12

Use any substituting value other than 0

Enter the first expression
 $=2*B1 + 6$

Enter the second expression
 $=2*(B1 + 3)$

Thus the expressions $2x + 6$ and $2(x + 3)$ are equivalent.

Are the following statements correct?

1 $2x + 6 = 2 \times (x + 3)$

3 $7a + 28 = 28 + 7a$

5 $10x - 80 = 10(x - 8)$

7 $14y - 102 = 14(y - 8)$

2 $9x + 17 = 17 + 9x$

4 $12b + 36 = 6 \times (2b + 6)$

6 $5a + 4a = 2a \times (5 + 2)$

8 $48y + 6y = 6y(8 + 1)$

Chapter Review 1

Exercise 7.15

1 Find the value of each of the following:

a) $12 - 3 \times 2$

b) $3 \times 5 - 2$

c) $5 \times 6 \div (2 + 1)$

d) $4 + 15 \div (3 + 2)$

e) $18 \div (3 \times 2 - 3) + 5 \times 2$

f) $(4 - 2 \times 1) \div (8 \div (3 + 1))$

2 Write an algebraic expression for each of the following:

a) A number plus 5

b) A number minus 3

c) Triple a number

d) Half of a number

e) b is increased by d

f) x is multiplied by y

g) Triple s and then add w

h) Quadruple a and then subtract b

Use any letter of the alphabet to represent the number. x is a popular letter to use.

3 Find the value of each of the following algebraic expressions given that $x = 5$ and $y = 6$.

a) $2x$

b) $3y$

c) $x + 9$

d) $y - 2$

e) $y \div 2$

f) $\frac{y}{3}$

g) $x + y$

h) $y - x$

i) xy

j) $3x + 1$

k) $4x - y$

l) $2y + 10$

4 Write an algebraic expression for each of the following pairs:

Find the value of each algebraic expression using $x = 7$, $y = 4$.
Comment on the values obtained.

a) x plus three **and** three plus x.

b) Nine plus triple x **and** triple x plus nine.

c) Five times y minus eight **and** eight minus five times y.

d) y times ten **and** ten times y.

e) x divided by four **and** four divided by x.

5 Find the value of each algebraic expression using $x = 10$, $y = 8$, $z = 3$.
Comment on the values obtained.

a) $(x + y) + z$ **and** $x + (y + z)$

b) $3x + (y + 1)$ **and** $(3x + y) + 1$

c) $(x - y) - z$ **and** $x - (y - z)$

d) $4 \times (x \times y)$ **and** $(4 \times x) \times y$

e) $x \times (5y \times z)$ **and** $(x \times 5y) \times z$

f) $x \div (y \div z)$ **and** $(x \div y) \div z$

The easy way is always mined - Murphy's Laws of Combat.

6 The power of a wind turbine, in watts, is given by the formula:

Power = $0.62avv^3$, where a is the circular area of the turbine blades, and v is the wind speed. What power is generated by a wind turbine with $a = 600 \text{ m}^2$, and $v = 10 \text{ m/s}$?

Chapter Review 2

Exercise 7.16

1 Find the value of each of the following:

a) $20 - 2 \times 2$

b) $7 \times 4 - 3$

c) $2 \times 8 \div (2 + 2)$

d) $9 + 12 \div (4 + 2)$

e) $15 \div (3 \times 3 - 4) + 1 \times 6$

f) $(7 - 3 \times 1) \div (6 \div (2 + 1))$

2 Write an algebraic expression for each of the following:

a) A number plus 6

b) A number minus 8

c) Double a number

d) Third of a number

e) h is decreased by m

f) a is multiplied by b

g) Triple g and then minus f

h) Quadruple a and then increase by p

Use any letter of the alphabet to represent the number. x is a popular letter to use.

3 Find the value of each of the following algebraic expressions given that $x = 8$ and $y = 3$.

a) $3x$

b) $4y$

c) $x + 6$

d) $y - 1$

e) $x \div 2$

f) $\frac{y}{3}$

g) $x + y$

h) $x - y$

i) xy

j) $2x + 5$

k) $x - 2y$

l) $3y + 9$

4 Write an algebraic expression for each of the following pairs:

Find the value of each algebraic expression using $x = 4$, $y = 1$.

Comment on the values obtained.

a) x plus two **and** two plus x.

b) Seven plus triple x **and** triple x plus seven.

c) Nine times y minus five **and** five minus nine times y.

d) x times ten **and** ten times x.

e) x divided by four **and** four divided by x.

5 Find the value of each algebraic expression using $x = 5$, $y = 6$, $z = 7$.

Comment on the values obtained.

a) $(x + y) + z$ **and** $x + (y + z)$

b) $2x + (y + 3)$ **and** $(2x + y) + 3$

c) $(x - y) - z$ **and** $x - (y - z)$

d) $5 \times (x \times y)$ **and** $(5 \times x) \times y$

e) $x \times (2y \times z)$ **and** $(x \times 2y) \times z$

f) $x \div (y \div z)$ **and** $(x \div y) \div z$

Everything has beauty
but not everyone sees it
- Confucius.

6 The power of a 240v electrical motor, in watts, is given by the formula:

Power = Vie , where V is the voltage, i is the current, and e is the

efficiency of the motor. What is the power output of an electrical motor

with $V = 240$ volts, $i = 3$ amps and $e = 0.75$?

Volume

8

- ★ Calculate volumes of rectangular prisms.
- ★ Investigate volumes of cubes and rectangular prisms.
- ★ Establish and use the formula $V = l \times b \times h$.
- ★ Understand and use cubic units when finding volumes of cubes and rectangular prisms.

Volume is used in everyday life.



A TASK

Volume is used in everyday life much more than most people would suspect.

- Over the next couple of weeks collect examples of volume that appears in all kinds of print material - newspapers, advertisements, brochures etc.
- Categorise the examples into applications and note the units of volume e.g., dams (Sydney Harbours & gigalitres), timber (super feet), engines (cubic centimetres), milk (litres), swimming pools (megalitres & kilolitres) etc.
- Prepare a wall chart for the classroom.

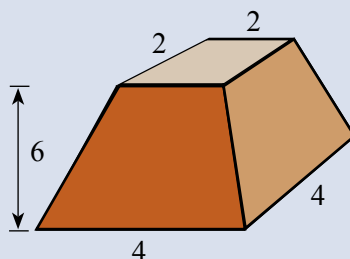
A LITTLE BIT OF HISTORY

An ancient papyrus known as the Moscow Mathematical Papyrus, 1700 BC, shows that the Egyptians knew the formula for the volume of a truncated square based pyramid - a frustum.

$$V = \frac{1}{3}h(a^2 + ab + b^2)$$

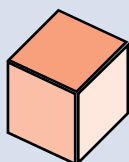
$$V = \frac{1}{3} \times 6 \times (2^2 + 2 \times 4 + 4^2)$$

$$V = 56 \text{ cubic units}$$



Volume

Volume n. 1. the amount of space occupied in three dimensions.
Abbrev. V, the international metric unit of volume is cubic metre (m^3).

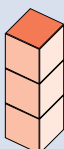


This is 1 cubic centimetre.
It is a cube $1\text{cm} \times 1\text{cm} \times 1\text{cm} = 1\text{cm}^3$

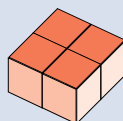
Exercise 8.1

If each cube in each figure below is one cubic centimetre, 1cm^3 , find the volume of each figure:

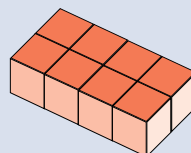
1



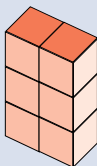
2



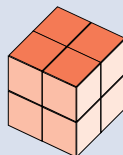
3



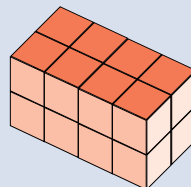
4



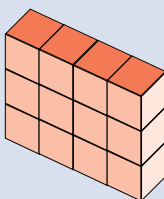
5



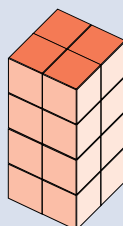
6



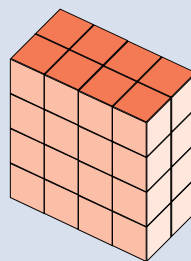
7



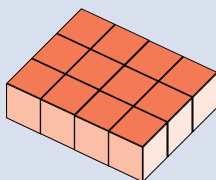
8



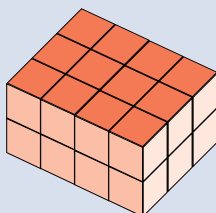
9



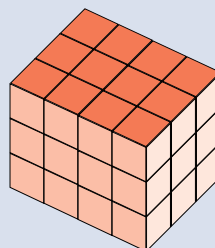
10



11



12

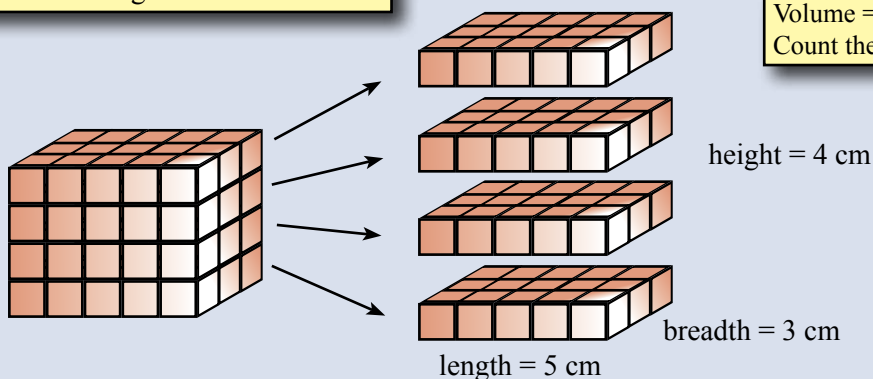


Rectangular Prisms

A **rectangular prism** is a column with a rectangular base.

The volume is the space occupied by the prism.

Volume = 60?
Count them.

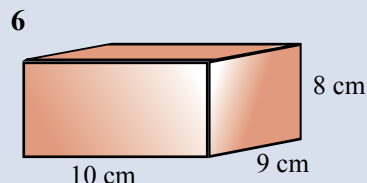
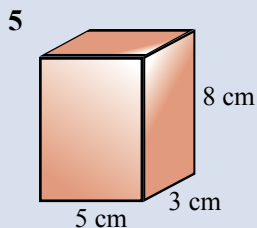
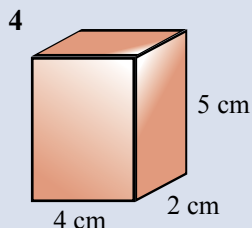
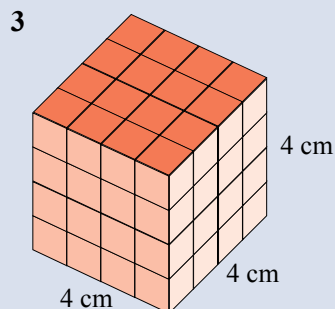
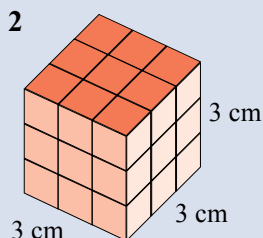
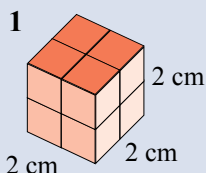
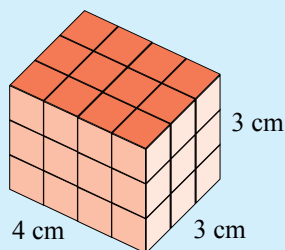


$$\begin{aligned}\text{Volume of prism} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 5 \text{ cm} \times 3 \text{ cm} \times 4 \text{ cm} \\ &= \underline{60 \text{ cm}^3}\end{aligned}$$

Exercise 8.2

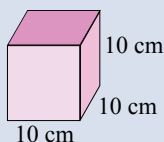
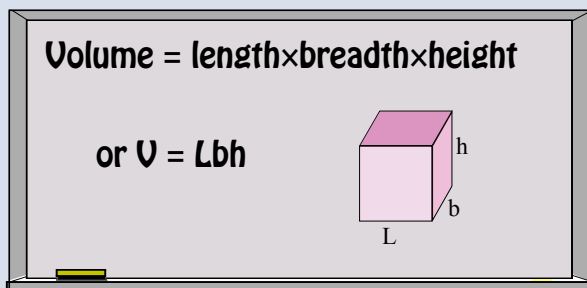
Find the volume of each of the following prisms:

$$\begin{aligned}\text{Volume} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 4 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm} \\ &= \underline{36 \text{ cm}^3}\end{aligned}$$



Rectangular Prisms

A rectangular prism is a column with a rectangular base.



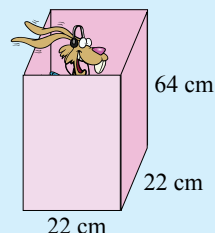
$$\begin{aligned}\text{Volume} &= Lbh \\ &= 10\text{cm} \times 10\text{cm} \times 10\text{cm} \\ &= 1000\text{ cm}^3 \text{ \{cubic centimetres\}}\end{aligned}$$

$$\begin{aligned}1000\text{ cubic centimetres holds } 1\text{ litre.} \\ \mathbf{1000\text{ cm}^3 = 1\text{ L}} \\ \mathbf{1\text{ cm}^3 = 1\text{ mL}}\end{aligned}$$

Exercise 8.3

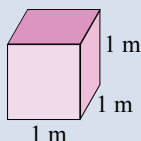
A bin has a base of 22 cm by 22 cm and height 64 cm.
How many litres of water would the bin hold ($1000\text{ cm}^3 = 1\text{ L}$)?

$$\begin{aligned}\text{Volume} &= Lbh \\ &= 22\text{ cm} \times 22\text{ cm} \times 64\text{ cm} \\ &= 30\,976\text{ cm}^3 \\ &= 30\,976 \div 1000\text{ Litres} \quad \{1000\text{ cm}^3 = 1\text{ L}\} \\ &= \underline{30.976\text{ L or } 31\text{ L}}\end{aligned}$$



- 1 A milk carton has a base of 10 cm by 10 cm and a height of 20 cm.
How many litres of milk will the carton hold ($1000\text{ cm}^3 = 1\text{ L}$)?
- 2 A milk carton has a base of 15 cm by 10 cm and a height of 20 cm.
How many litres of milk will the carton hold ($1000\text{ cm}^3 = 1\text{ L}$)?
- 3 A bin has a base of 20 cm by 20 cm and height 40 cm.
How many litres of water would the bin hold ($1000\text{ cm}^3 = 1\text{ L}$)?
- 4 A bin has a base of 25 cm by 25 cm and height 30 cm.
How many litres of water would the bin hold ($1000\text{ cm}^3 = 1\text{ L}$)?
- 5 What is the capacity, in litres, of a refrigerator of length 45 cm, width 45 cm and height 125 cm ($1000\text{ cm}^3 = 1\text{ L}$)?
- 6 Given that 1 L of water weighs 1 kg, what is the weight of water in a bin length 35 cm, breadth 28 cm, and depth 20 cm ($1000\text{ cm}^3 = 1\text{ L}$)?

Rectangular Prisms



$$\begin{aligned}\text{Volume} &= Lbh \\ &= 1\text{ m} \times 1\text{ m} \times 1\text{ m} \\ &= 1\text{ m}^3 \text{ \{1 cubic metre\}}\end{aligned}$$

$$\begin{aligned}\text{Volume} &= Lbh \\ &= 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} \\ &= 1\,000\,000\text{ cm}^3 \text{ \{cubic centimetre\}}\end{aligned}$$

1 cubic metre holds 1000 litres.
 $1\text{ m}^3 = 1000\text{ L}$

$$\begin{aligned}\text{Volume} &= Lbh \\ &= 1000\text{ mm} \times 1000\text{ mm} \times 1000\text{ mm} \\ &= 1\,000\,000\,000\text{ mm}^3 \text{ \{cubic millimetre\}}\end{aligned}$$

Exercise 8.4

How many litres of water is needed to fill a swimming pool 15 m long, 5 m wide, and 1.5 m deep?



$$\begin{aligned}\text{Volume} &= Lbh \\ &= 15\text{ m} \times 5\text{ m} \times 1.5\text{ m} \\ &= 112.5\text{ m}^3\end{aligned}$$

$$\begin{aligned}&= 112.5 \times 1000\text{ Litres} \quad \{1\text{ m}^3 = 1000\text{ L}\} \\ &= \underline{112\,500\text{ Litres}}\end{aligned}$$

- How many litres of water is needed to fill a swimming pool 10 m long, 3 m wide, and 1.4 m deep ($1\text{ m}^3 = 1000\text{ L}$)?
- How many litres of water is needed to fill a swimming pool 20 m long, 10 m wide, and 1.6 m deep ($1\text{ m}^3 = 1000\text{ L}$)?
- How many litres of water is needed to fill a swimming pool 50 m long, 10 m wide, and 1.6 m deep ($1\text{ m}^3 = 1000\text{ L}$)?
- What is the capacity of each of the following containers, in m^3 and litres?



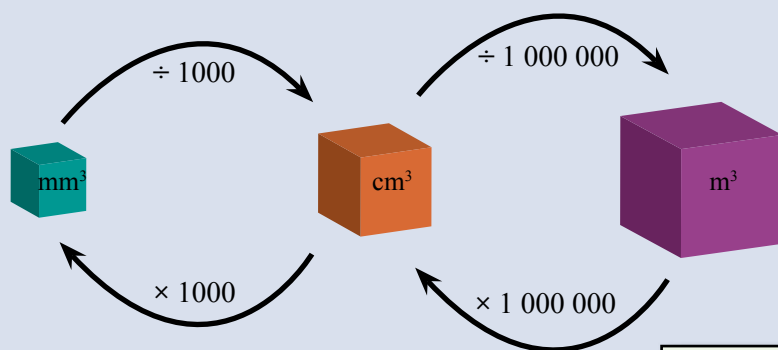
a)

b)

20 foot container	40 foot container
length = 5.89 m	length = 12.01 m
width = 2.31 m	width = 2.33 m
height = 2.33 m	height = 2.33 m

- The tank on a water truck is 4000 mm long, 2480 mm wide, and 1500 mm deep. What is the capacity of the water truck in litres ($1\text{ m}^3 = 1000\text{ L}$)?
- How many cubic metres of concrete is needed for the floor of a shed 12 m long, 6 m wide and 0.3 m deep?

Units of Volume



$$1\text{cm}^3 = 1000\text{mm}^3$$

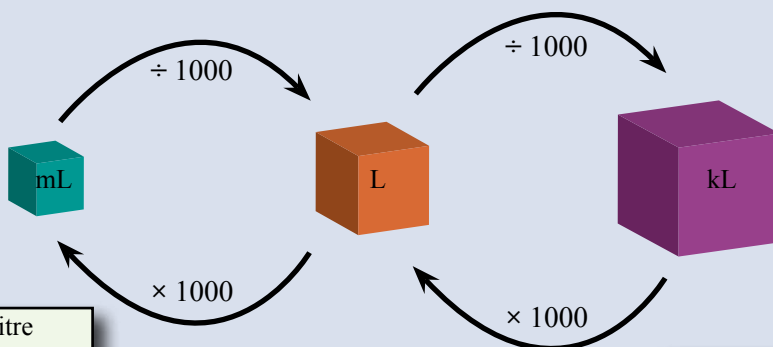
$$1\text{m}^3 = 1\,000\,000\text{cm}^3$$

Exercise 8.5

1 Copy and complete the following unit conversions:

8500 mm^3 to cm^3 $8500\text{ mm}^3 = 8500 \div 1000\text{ cm}^3$ $= \underline{8.5\text{ cm}^3}$	2.3 m^3 to cm^3 $2.3\text{ m}^3 = 2.3 \times 1\,000\,000\text{ cm}^3$ $= \underline{2\,300\,000\text{ cm}^3}$
---	---

- | | |
|---|--|
| a) 6700 mm^3 to cm^3 | b) 900 mm^3 to cm^3 |
| c) $35\,000\,000\text{ cm}^3$ to m^3 | d) $2\,400\,000\text{ cm}^3$ to m^3 |
| e) 4.9 m^3 to cm^3 | f) 1.9 m^3 to cm^3 |
| g) 8.3 cm^3 to mm^3 | h) 0.6 cm^3 to mm^3 |



$$1\text{kL} = 1\text{ kiloLitre}$$

$$1\text{L} = 1000\text{ milliLitres}$$

$$1\text{kL} = 1000\text{L}$$

$$1\text{L} = 1000\text{mL}$$

2 Copy and complete the following unit conversions:

6500 mL to L $6500\text{ mL} = 6500 \div 1000\text{ L}$ $= \underline{6.5\text{ L}}$	1.8 kL to L $1.8\text{ kL} = 1.8 \times 1000\text{ L}$ $= \underline{1800\text{ L}}$
--	--

- | | |
|-----------------------------------|---------------------------------------|
| a) 4200 mL to L | b) $71\,300\text{ mL}$ to L |
| c) 5800 L to kL | d) $960\,000\text{ L}$ to kL |
| e) 7.3 kL to L | f) 0.61 kL to L |
| g) 2.5 L to mL | h) 0.75 L to mL |

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 8.6

- 1 Spell volume.
- 2 What is the formula for the volume of a rectangular prism?
- 3 What is the volume of a 2m by 4m by 5m rectangular prism?
- 4 How many millilitres in a Litre?
- 5 How many litres in a kilolitre?
- 6 Find the value of the expression: $6x - 4$ when $x = 5$
- 7 Find the value of the expression: $3(b + 4)$ when $b = 2$
- 8 Change 0.2 to a fraction.
- 9 Change $\frac{10}{3}$ to a mixed number.
- 10 I buy a \$5.90 magazine with a \$10 note, how much change?

A comma is a shorter pause than a coma yet it is one letter longer ???

Exercise 8.7

- 1 Spell rectangular.
- 2 What is the formula for the volume of a rectangular prism?
- 3 What is the volume of a 3m by 5m by 4m rectangular prism?
- 4 How many millilitres in a Litre?
- 5 How many litres in a kilolitre?
- 6 Find the value of the expression: $10 - 2x$ when $x = 3$
- 7 Find the value of the expression: $3(c \div 4)$ when $c = 8$
- 8 Change 0.3 to a fraction.
- 9 Change $\frac{11}{4}$ to a mixed number.
- 10 I buy a \$9.10 magazine with a \$20 note, how much change?

Your imagination is a preview to life's coming attractions - Albert Einstein.

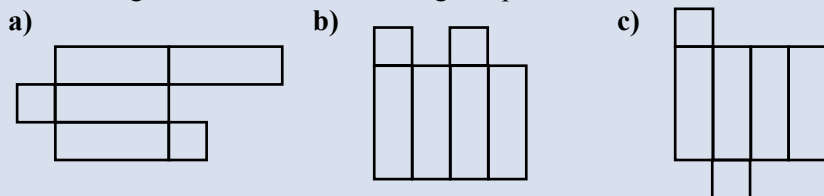
Exercise 8.8

- 1 Spell rectangular prism.
- 2 What is the formula for the volume of a rectangular prism?
- 3 What is the volume of a 6m by 2m by 5m rectangular prism?
- 4 How many millilitres in a Litre?
- 5 How many litres in a kilolitre?
- 6 Find the value of the expression: $5x + 4$ when $x = 10$
- 7 Find the value of the expression: $4(d + 4)$ when $d = 2$
- 8 Change 0.4 to a fraction.
- 9 Change $\frac{14}{5}$ to a mixed number.
- 10 I buy a \$15.30 magazine with a \$20 note, how much change?

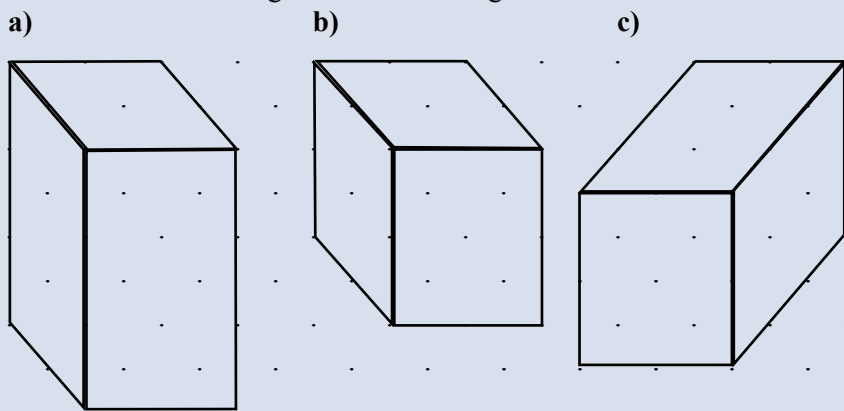
NAPLAN Questions

Exercise 8.9

- 1 Change 4.23 kilolitres to litres.
- 2 What is the volume of a rectangular prism with a base 4 m by 3 m and height 5 m?
- 3 Which diagram is the net of a rectangular prism?



- 4 Which of the following isometric drawings is the odd one out?

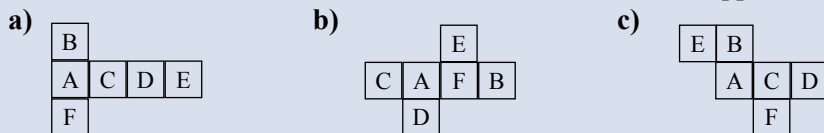


- 5 Find the volume of a cone with radius 5 cm and height 7 cm.

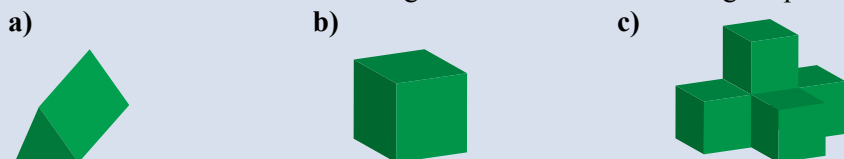
The volume of a cone is: $V = \frac{\pi r^2 h}{3}$



- 6 When the net is folded to make a cube, which letter will be opposite letter A?



- 7 Count the number of faces and edges on each of the following shapes:



Competition Questions



Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 8.10

- 1 Which of the following has 6 faces?

a)



b)



c)

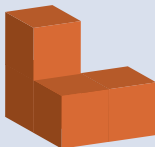


- 2 Which of the following has 22 edges?

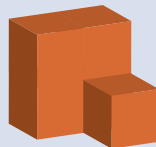
a)



b)



c)



- 3 The smaller cube has a mass of 15 g.
The larger cube has length, breadth, and height four times that of the smaller cube.
What is the mass of the larger cube?

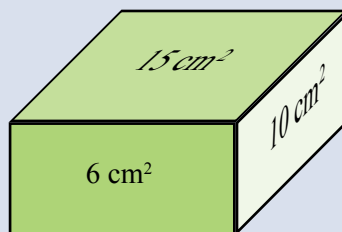


- 4 A box measures 20 cm by 30 cm by 40 cm.
What is the volume, in litres, of the box?

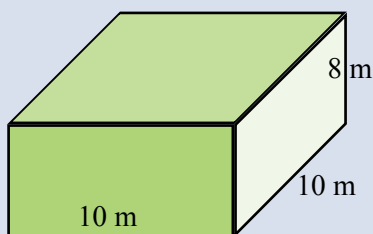
- 5 A box of baking soda measures 3 cm by 5 cm by 7 cm.
How many teaspoons of baking soda are expected to be in the box?
(1 teaspoon = 5 cm³).

- 6 60 centicubes are glued together to form a rectangular based prism.
If the area of the base is 20 cm², what is the height of the prism?

- 7 The areas of three faces of a rectangular based prism are as shown.
What is the volume of the prism?



8



The tank, in the shape of a rectangular based prism has water to a depth of 6 m.
If a solid cube with length of 5 m is put on the bottom of the tank, what will be the new depth of the water?

Technology

Technology 8.1

Use a spreadsheet to calculate volume:

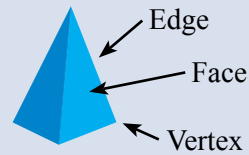
Length	Breadth	Depth	Volume
25.6	17	2	870.4

Enter the formula:
=a2*b2*c2

Technology 8.2

Investigate the number of faces, edges, and vertices on solids.

Use a spreadsheet to summarise and examine a relationship between the number of faces, edges, and vertices on a solid.



Solid	Faces	Edges	Vertices
triangle based pyramid	4	6	4
square based pyramid			
pentagon based pyramid			
square based prism	6	12	8

Technology 8.3

Use search phrases such as 'faces, edges, vertices' and '3D solids' to view video and other information about solid shapes.

Research information about 3D solids.

Also use search phrases such as 'maths shape games' to play some of the many online interactive shape games.

Dieticians apply their knowledge of human nutrition to prevent illness and disease, and to maintain and promote health.

- Relevant school subjects are Mathematics, English, and Chemistry.
- Courses usually involve a University Bachelor degree.

Investigations

Investigation 8.1 How big is 1 cubic centimetre (cm^3)?

To become familiar with the volume of a cubic centimetre, make a 1 cm by 1 cm by 1 cm box or use centicubes.

Use the cubic centimetre to estimate volumes in your classroom:

- The volume of a calculator.
- the volume of a pencil case, etc.

How close were your estimates to the actual volumes?

Investigation 8.2 Getting a feel for volume

- 1 Collect a variety of containers.
- 2 Estimate the capacity of each container.
- 3 Measure the capacity of each container (Fill with water and measure the water).
- 4 Complete a table similar to this one

1 cm^3 of water = 1 mL

1000 mL = 1 L

Container	Estimate	Actual
tablespoon	10 mL	5 mL
small cup		
large cup		
sardine tin		
vase		
sugar bowl		

Investigation 8.3 How big is 1 cubic metre (m^3)?

To become familiar with the space of a cubic metre, make a 1 m by 1 m by 1 m frame

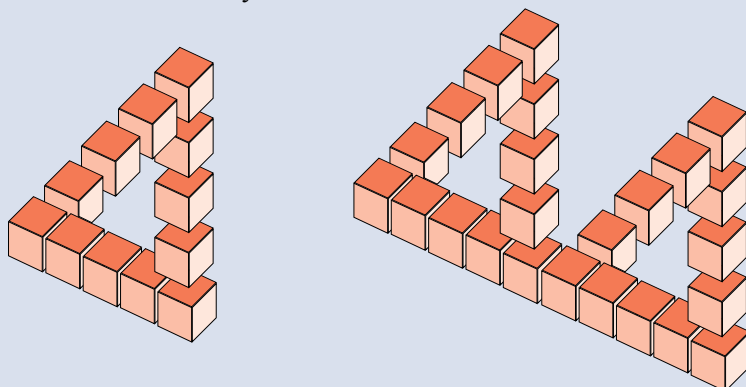
Use the cubic metre to estimate volumes in your classroom:

- The volume of a desk.
- The volume of the classroom.

How close were your estimates to the actual volumes?

Investigation 8.4 Illusion?

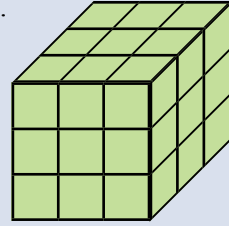
- 1 Can you use centicubes to make this figure?
- 2 Draw and colour one of your own



A Couple of Puzzles

Exercise 8.11

- This cube has been made from a number of smaller cubes.
How many smaller cubes?
- The entire outside of the cube is painted green.
 - How many smaller cubes have 3 faces painted?
 - How many smaller cubes have 2 faces painted?
 - How many smaller cubes have 1 faces painted?
 - How many smaller cubes have 0 faces painted?



A Game

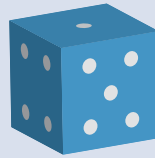
Pig is a dice game for two or more players.

- Take it in turns to throw a die.
The player may throw the die as many times as they wish.
They total the number thrown each time.
When they stop they add their total to their previous total.

If a 1 is thrown then

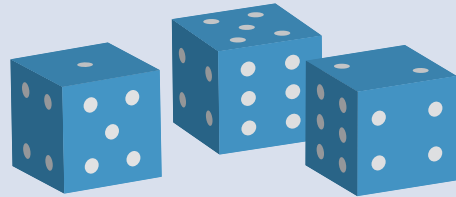
- **their turn stops.**
- **their total for their turn is zero.**

- The first to total 50 or more is the winner.



A Sweet Trick

- Ask your audience to throw three dice.
- Ask your audience to:
 - Double the first number.
 - Add 3.
 - Multiply by 5.
 - Add the second number.
 - Multiply by 10.
 - Add the third number.
- They tell you the answer.
- You tell them the numbers on each die.



$$2 \times 1 = 2$$

$$2 + 3 = 5$$

$$5 \times 5 = 25$$

$$25 + 5 = 30$$

$$30 \times 10 = 300$$

$$300 + 2 = 302$$

302



Subtract 150:

$$302 - 150 = 152$$

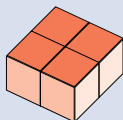
The numbers are 1, 5, and 2

Chapter Review 1

Exercise 8.12

- 1 If each cube in each figure below is one cubic centimetre, 1 cm^3 , find the volume of each figure:

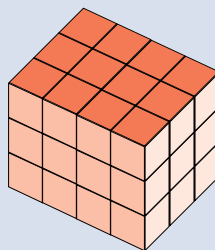
a)



b)

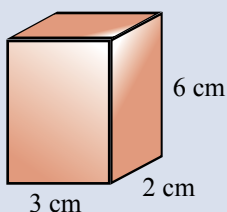


c)

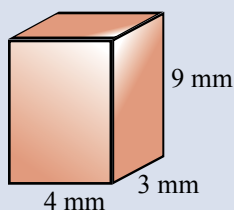


- 2 Find the volume of each of the following prisms:

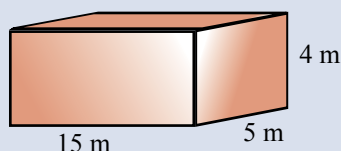
a)



b)



c)



- 3 A milk carton has a base of 10 cm by 10 cm and a height of 20 cm. How many litres of milk will the carton hold ($1000\text{ cm}^3 = 1\text{ L}$)?
- 4 What is the capacity, in litres, of a refrigerator of length 50 cm, width 45 cm and height 140 cm ($1000\text{ cm}^3 = 1\text{ L}$)?
- 5 How many litres of water is needed to fill a swimming pool 30 m long, 10 m wide, and 1.8 m deep ($1\text{ m}^3 = 1000\text{ L}$)?
- 6 What is the capacity, in m^3 and litres, of a 20 foot shipping container with length of 5.89 m, width of 2.31 m, and height of 2.33 m?



- 7 Make the following unit conversions:

a) 6200 mm^3 to cm^3

b) $53\,000\,000\text{ cm}^3$ to m^3

c) 2.4 m^3 to cm^3

d) 0.9 cm^3 to mm^3

e) 7800 mL to L

f) 910 000 L to kL

g) 1.3 kL to L

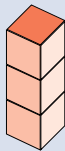
h) 0.25 L to mL

Chapter Review 2

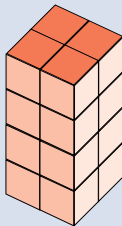
Exercise 8.13

- 1 If each cube in each figure below is one cubic centimetre, 1 cm^3 , find the volume of each figure:

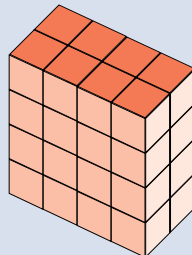
a)



b)

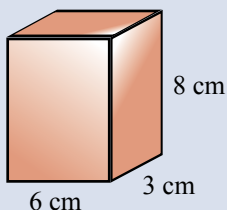


c)

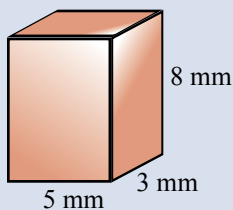


- 2 Find the volume of each of the following prisms:

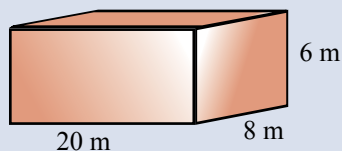
a)



b)



c)



- 3 A milk carton has a base of 15 cm by 10 cm and a height of 20 cm. How many litres of milk will the carton hold ($1000\text{ cm}^3 = 1\text{ L}$)?
- 4 What is the capacity, in litres, of a refrigerator of length 55 cm, width 45 cm and height 140 cm ($1000\text{ cm}^3 = 1\text{ L}$)?
- 5 How many litres of water is needed to fill a swimming pool 50 m long, 10 m wide, and 1.6 m deep ($1\text{ m}^3 = 1000\text{ L}$)?
- 6 What is the capacity, in m^3 and litres, of a 40 foot shipping container with length of 12.01 m, width of 2.33 m, and height of 2.33 m?



- 7 Make the following unit conversions:

a) 1500 mm^3 to cm^3

b) $73\,000\,000\text{ cm}^3$ to m^3

c) 9.2 m^3 to cm^3

d) 0.6 cm^3 to mm^3

e) 9900 mL to L

f) 840 000 L to kL

g) 2.6 kL to L

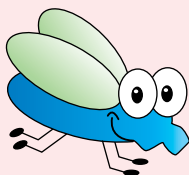
h) 0.75 L to mL

Coordinates



- ★ Compare, order, add and subtract integers.
- ★ Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point.
- ★ Plot points from a table of integer values.
- ★ Recognise simple patterns, such as points that lie on a straight line.

The bartender asked Descartes if he would like another drink. He replied, "I think not." And he vanished.



A TASK

At 110 km/h your car can use up to 25% more fuel than it would cruising at 90 km/h.

What is the most economical speed?

- Research fuel consumption.
- Research speed and fuel consumption.
- Present your findings as a poster/graph.

A LITTLE BIT OF HISTORY

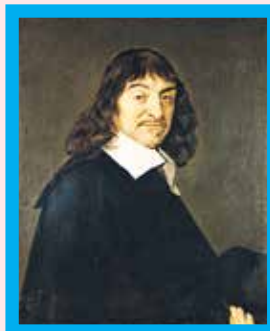
Rene Descartes (1596 - 1650) was a very influential philosopher, mathematician, and physicist. He has been recognised as the 'Father of Modern Philosophy'.

Descartes founded analytic geometry, the bridge between algebra and geometry. Cartesian coordinates were vital for the development of calculus and thus our technological age (No Cartesian coordinates no TV, mobile phones etc etc).

The story is that Descartes was watching a fly and worked out how to fix the fly's position by its distance from the walls and floor of the room - thus Cartesian Coordinates.

Descartes is said to be one of the most influential thinkers of all time:

'I think, therefore I am.'



Integers

Integers are:

Positive whole numbers: 1, 2, 3, 4, 5, 6, ...
and Zero: 0
and Negative whole numbers: -1, -2, -3, -4, -5, -6, ...

Exercise 9.1

Change the following words to an integer:

I lost \$3. <u>-3</u>	A rate drop of 25 points. <u>-25</u>	8 steps up. <u>8</u>
--------------------------	---	-------------------------

8 and +8 are the same thing.

- | | |
|--------------------------|-----------------------|
| 1 A profit of \$65 | 2 A gain of 7 cm. |
| 3 A loss of \$3 million. | 4 No movement. |
| 5 4 m under the sea. | 6 A rise of 4 m. |
| 7 A drop of 3°C. | 8 A increase of 12%. |
| 9 I lost 4 golf balls. | 10 A reduction of 15% |
| 11 A profit of \$220. | 12 \$6000 decrease. |
| 13 \$70 overvalued. | 14 \$31 under value. |
| 15 Drop of 13 floors. | 16 Zero change. |

Assume that up is positive and down is negative:

- | | |
|-------------------------|------------------------|
| 17 50 m down. | 18 20 units down. |
| 19 Move 4 places down. | 20 Up 3 levels. |
| 21 Down 2 amps. | 22 Up 50 basis points. |
| 23 Neither up nor down. | 24 48 steps down. |

Assume that to the right is positive and that to the left is negative:

- | | |
|---------------------------------|-------------------------------|
| 25 Move 8 places to the right. | 26 Don't move. |
| 27 4 steps to the left. | 28 7 steps to the right. |
| 29 Move the biro 3 to the left. | 30 Move 6 places to the left. |

Z is the set of Integers. $Z = \{ \dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots \}$

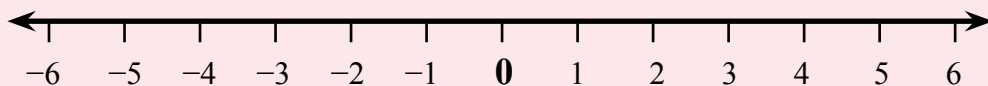
Numbers such as 3.5, -721.2, $2\frac{1}{2}$ are not integers.

Z comes from the German Zahl, meaning number.

Ordering Integers

Integers can be placed on a number line.

Integers on the right are larger than integers on the left.



'less than'
 $-2 < 1$
-2 is to the left of 1



'greater than'
 $3 > -2$
3 is to the right of -2

Exercise 9.2

Copy each pair of numbers and place the correct $<$ or $>$ between them.

3 -1 3 is to the right of -1 3 is greater than -1 <u>$3 > -1$</u>	-4 2 -4 is to the left of 2 -4 is less than 2 <u>$-4 < 2$</u>	-3 -6 -3 is to the right of -6 -3 is greater than -6 <u>$-3 > -6$</u>
--	--	--

1 5 -2

2 -3 1

3 -5 2

4 -4 0

5 0 -1

6 -3 -2

7 2 6

8 4 -2

9 -3 6

10 -5 -6

11 2 0

12 -4 -1

13 6 -1

14 -4 -3

15 5 -2

16 13 5

17 -11 -1

18 -15 -11

Exercise 9.3

Arrange the integers in ascending order (smallest to largest):

$6, -1, 2, -5, -4$ <u>$-5, -4, -1, 2, 6$</u>	From left to right on the number line.
--	--

1 $3, 0, 1, 5$

2 $1, -1, 0, -2$

3 $6, -3, 1, -2$

4 $-5, 3, 2, -3, 0$

5 $-6, 2, -1, -4$

6 $-5, 5, 1, -3, 2$

7 $1, -2, -1, -3, 2$

8 $3, 1, -4, 6, -6$

9 $-4, 2, 4, -3, 6$

10 $2, 6, -6, 0, 4$

11 $5, -3, 6, -4, 1$

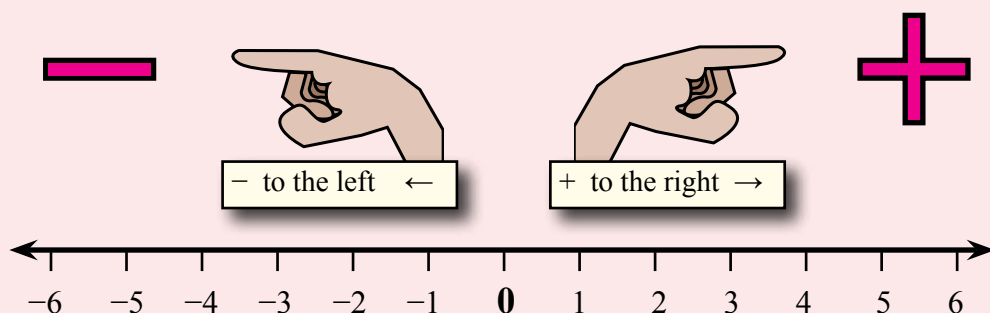
12 $-7, 8, 3, -3, 4$

13 $11, -3, -4, -9, 2$

14 $-2, -3, -1, -12, 9$

15 $-13, 15, 13, -12, -7$

Adding Integers



Exercise 9.4

Add the following integers:

$3 + ^{-}4$ Put your biro on 0. Move biro 3 to right. Move biro 4 to the left. Your biro is at $^{-}1$. <u>$3 + ^{-}4 = ^{-}1$</u>	$^{-}5 + 3$ Put your biro on 0. Move biro 5 to left. Move biro 3 to the right. Your biro is at $^{-}2$. <u>$^{-}5 + 3 = ^{-}2$</u>
---	---

1 $4 + 3$

4 $^{-}1 + ^{-}4$

7 $^{-}7 + 1$

10 $4 + ^{-}6$

13 $7 + 8$

16 $13 + 8$

19 $^{-}2 + ^{-}7$

2 $5 + ^{-}3$

5 $7 + 6$

8 $^{-}4 + ^{-}2$

11 $^{-}3 + 5$

14 $8 + ^{-}3$

17 $12 + ^{-}5$

20 $13 + 17$

3 $^{-}2 + 4$

6 $6 + ^{-}2$

9 $12 + 13$

12 $^{-}2 + ^{-}3$

15 $^{-}5 + 8$

18 $^{-}9 + 6$

21 $^{-}12 + 8$

Exercise 9.5

Copy and complete the following addition tables:

1

+	$^{-}2$	$^{-}1$	0	1
$^{-}2$				
$^{-}1$				
0				
1				

$1 + ^{-}2 = ^{-}1$

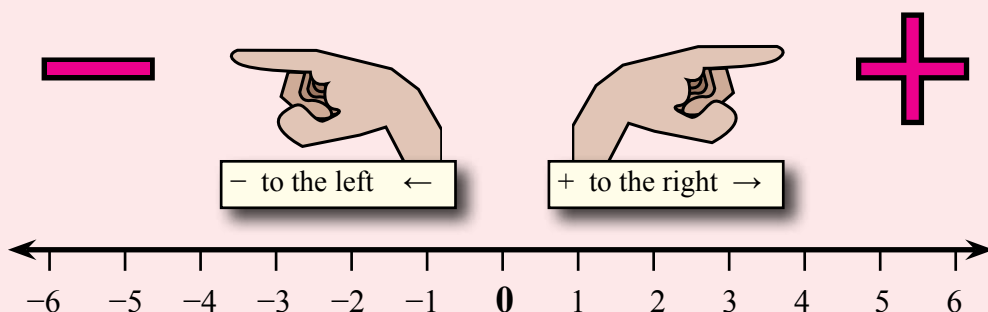
$0 + 1 = 1$

2

+	$^{-}10$	$^{-}5$	0	5
$^{-}10$				
$^{-}5$				
0				
5				

$^{-}5 + ^{-}5 = ^{-}10$

Subtracting Integers



Exercise 9.6

Calculate the following:

$$2 - 4$$

Put your biro on 0.
Move biro 2 to right.
Move biro 4 to the left.
Your biro is at -2 .

$$\underline{2 - 4 = -2}$$

$$^{-}3 - 2$$

Put your biro on 0.
Move biro 3 to left.
Move biro 2 to the left.
Your biro is at -5 .

$$\underline{^{-}3 - 2 = -5}$$

1 $5 - 3$

2 $3 - 4$

3 $^{-}3 + 1$

4 $^{-}4 - 2$

5 $6 - 5$

6 $2 - 5$

7 $^{-}6 + 3$

8 $^{-}3 - 4$

9 $4 - 1$

10 $2 - 6$

11 $^{-}3 + 5$

12 $^{-}1 - 3$

13 $7 - 2$

14 $6 - 9$

15 $^{-}8 + 7$

16 $^{-}5 - 1$

17 $9 - 15$

18 $^{-}12 + 10$

19 $10 - 9$

20 $^{-}15 + 10$

$-$ is the opposite direction of $+$
 $-(-)$ is opposite of $-$ thus $+$

$$\begin{array}{ll} -^{-}3 = 3 & -^{-}1 = 1 \\ -^{-}8 = 8 & -^{-}5 = 5 \end{array}$$

$$3 - ^{-}2$$

Put your biro on 0.
Move biro 3 to right.
 $^{-}2 = 2$
Move biro 2 to the right.
Your biro is at 5.

$$\underline{3 - ^{-}2 = 5}$$

$$^{-}5 - ^{-}1$$

Put your biro on 0.
Move biro 5 to left.
 $^{-}1 = 1$
Move biro 1 to the right.
Your biro is at -4 .

$$\underline{^{-}5 - ^{-}1 = -4}$$

21 $1 - ^{-}4$

22 $5 - ^{-}1$

23 $^{-}3 - ^{-}2$

24 $^{-}2 - ^{-}4$

25 $5 - ^{-}3$

26 $4 - ^{-}3$

27 $^{-}1 - ^{-}7$

28 $^{-}6 - ^{-}2$

29 $3 - ^{-}1$

30 $5 - ^{-}2$

31 $^{-}3 - ^{-}1$

32 $^{-}4 - ^{-}2$

33 $2 - ^{-}6$

34 $8 - ^{-}4$

35 $^{-}8 - ^{-}3$

36 $^{-}4 - ^{-}6$

37 $9 - ^{-}12$

38 $2 - ^{-}7$

39 $^{-}5 - ^{-}4$

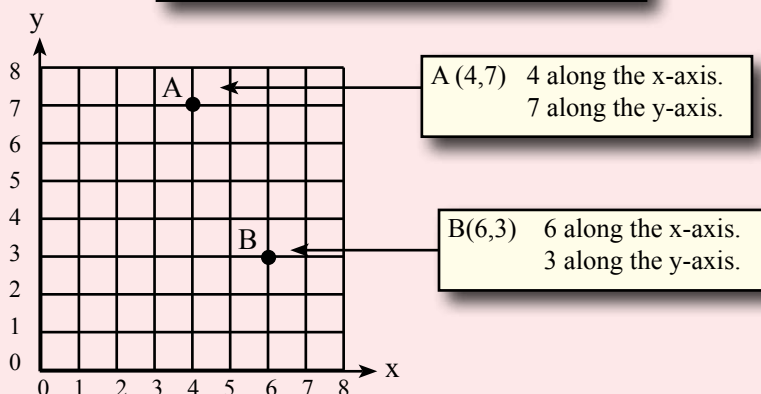
40 $^{-}3 - ^{-}12$

Cartesian Coordinates

Cartesian coordinates are used to fix a point on a two-dimensional plane. The position of the point is referenced by x on the x -axis and y on the y -axis.

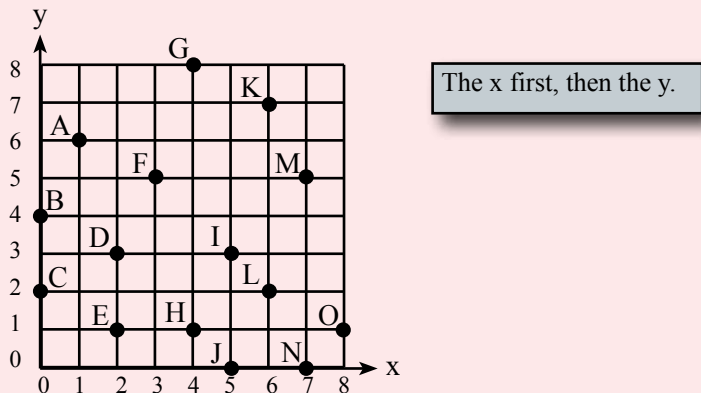
To plot points

- Put your biro on the Origin $(0,0)$.
- Move your biro horizontally the x value.
- Move your biro vertically the y value.



Exercise 9.7

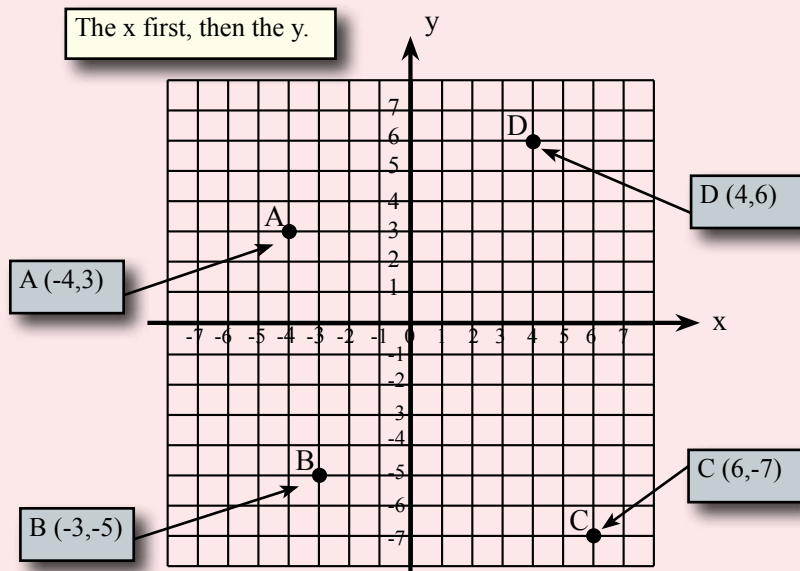
- 1 Write the coordinates of each of the points in the graph below.



- 2 Plot the following points on a set of Cartesian axes:
- | | | |
|-----------|-----------|-----------|
| a) A(2,4) | b) B(4,4) | c) C(5,0) |
| d) D(0,0) | e) E(2,7) | f) F(5,3) |
| g) G(0,4) | h) H(6,6) | i) I(2,3) |
- 3 What is it? Plot the following points on a sheet of grid paper, joining each point to the previous point with a continuous line.
 $(1,3)$ $(3,3)$ $(3,1)$ $(5,1)$ $(5,3)$ $(7,3)$ $(4,7)$ $(1,3)$
- 4 Make your own "What is it?"

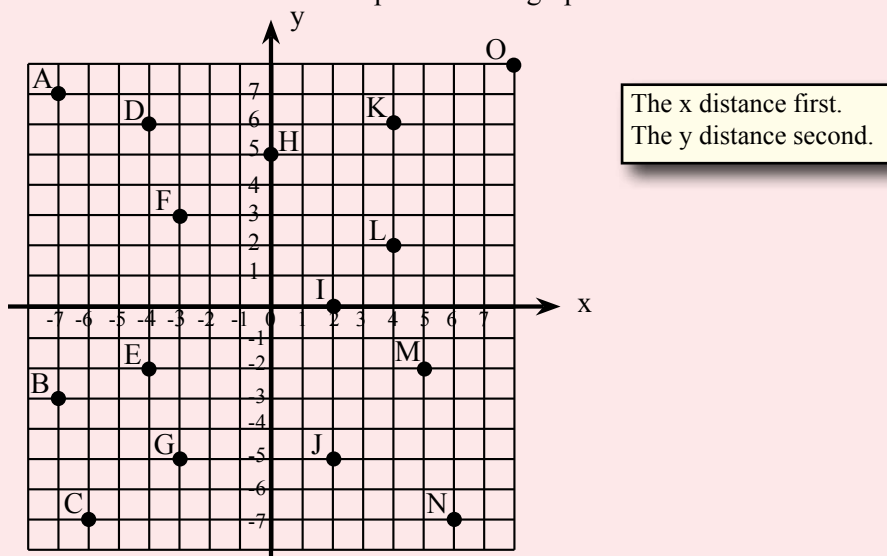
The Four Quadrants

The use of ordered pairs to fix a point in two-dimensional space can be extended to integers (negative numbers as well as positive numbers). The cartesian space is then divided into four quadrants. The origin (0,0) is at the centre.



Exercise 9.8

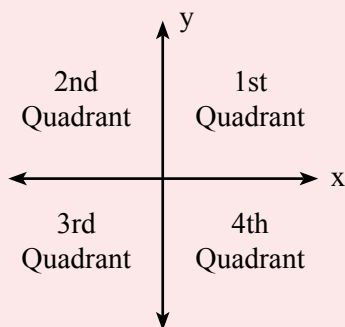
- 1 Write the coordinates of each of the points in the graph below:



- 2 Plot the following points on a set of Cartesian axes:

- | | | | |
|--------------|--------------|-------------|--------------|
| a) A(1, -5) | b) B(-2, 5) | c) C(0, 0) | d) D(1, -6) |
| e) E(3, 4) | f) F(0, -2) | g) G(-5, 0) | h) H(-7, -1) |
| i) I(-1, -1) | j) J(-3, -4) | k) K(0, -4) | l) L(-5, 4) |

The Four Quadrants

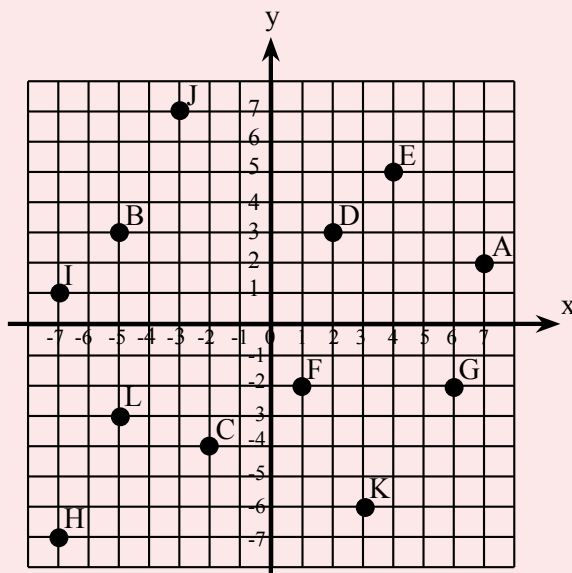


The number plane is divided into four quadrants as shown.

Exercise 9.9

In which quadrant do each of the points belong?

- | | |
|------|------|
| 1 A | 2 B |
| 3 C | 4 D |
| 5 E | 6 F |
| 7 G | 8 H |
| 9 I | 10 J |
| 11 K | 12 L |



In which quadrant do each of the following points belong?

- | | | | |
|-------------|-------------|------------|-------------|
| 13 A(1,2) | 14 B(-1,3) | 15 C(3,-2) | 16 D(1, -4) |
| 17 E(3,-4) | 18 F(-2,-2) | 19 G(-4,3) | 20 H(-4,-2) |
| 21 I(-1,-4) | 22 J(-3,-2) | 23 K(5,-4) | 24 L(-1,2) |

25 Answer True or False to each of the following:

- a) The point (2,5) is 2 to the right on the x-axis and 5 up the y-axis.
- b) The point (4,-3) is 4 to the right on the x-axis and 3 down the y-axis.
- c) (2, -3) is in the third quadrant.
- d) (-1,5) is in the second quadrant.
- e) (-3, -4) is in the third quadrant.
- f) (5, 5) is in the third quadrant.
- g) (2, 0) is a point on the x-axis.
- h) (0, -2) is a point on the y-axis.

Environmental Scientists observe, measure, analyse, and control environmental effects.

- Relevant school subjects are Mathematics and Science.
- Courses usually involve an environmental science degree.

What is it?

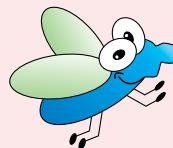
Exercise 9.10

Plot the following points on a sheet of grid paper

- ★ Choose a scale that will include all points in a set.
- ★ Join the points in a set together (Don't join the first to the last).
- ★ Do the same for each set. Eventually a shape will appear.

- 1 A $(-15,3)$ $(-10,8)$ $(-10,3)$ $(-8,3)$ $(-5,7)$
 B $(-8,0)$ $(-6,-6)$ $(-8,-8)$ $(-5,-8)$
 C $(4,-6)$ $(3,-8)$ $(6,-8)$ $(10,10)$
 D $(-8,0)$ $(-12,-2)$ $(-16,2)$ $(-15,3)$ $(-15,1)$
 E $(-5,-8)$ $(-3,-4)$ $(4,-6)$
 F $(10,10)$ $(8,4)$ $(-10,8)$
 G Draw a circle with radius 0.5 at centre $(-12,4)$

Well. What is it?



- 2 A $(8,-8)$ $(3,-6)$ $(2,-3)$ $(10,3)$ $(11,7)$ $(6,15)$
 B $(0,-3)$ $(1,-6)$ $(5,-8)$ $(-2,-8)$ $(0,-6)$ $(-1,-3)$
 C $(7,20)$ $(5,22)$ $(1,22)$ $(-1,20)$ $(-1,16)$ $(3,14)$
 D $(6,15)$ $(11,13.5)$ $(13,15.5)$ $(12,17)$ $(7,17.5)$ $(7,20)$
 E $(3,14)$ $(4,10)$ $(-5,4)$ $(-6,7)$ $(-7,0)$ $(-1,-3)$
 F $(2,-6)$ $(1,-3)$
 G $(5,17)$ $(5,18)$ $(4,18)$ $(4,17)$ $(5,17)$

- 3 A $(10,-9)$ $(11,11)$ $(14,9)$ $(14,16)$ $(9,22)$
 B $(0,-2)$ $(-4,-4)$ $(-1,-10)$ $(-3,-12)$ $(2.7,-12)$
 C $(0,14)$ $(0,16)$ $(1,15)$ $(2,13)$ $(4,12)$ $(4,10)$ $(3,8)$
 D $(8,-4)$ $(3,-2)$ $(1,-7)$ $(4,-8)$ $(2,-14)$ $(10,-9)$
 E $(-3,6)$ $(-6,7)$ $(-7,9)$ $(-6,11)$
 F $(9,22)$ $(4,18)$ $(8,0)$ $(3,8)$
 G $(0,14)$ $(-4.7,12.5)$ $(-6,14)$ $(-7,13)$ $(-6,11)$
 H $(-2,-3)$ $(-3,6)$
 I Draw a circle with radius 0.5 at centre $(1,10)$
 J Draw a circle with radius 1 at centre $(-1,7)$
 K Draw a circle with radius 0.5 at centre $(-4,8)$

- 4 Make your own 'What is it?'.

- ★ Draw or copy an image on a Cartesian plane
- ★ Write the ordered pairs of key points
- ★ Test your 'What is it?' on a classmate.

If you were to spell out numbers, how far would you have to go until you find the letter "a"?

One thousand.

Linear Patterns

A linear pattern increases, or decreases, by the same amount each step.

Linear adj. Relating to a line.

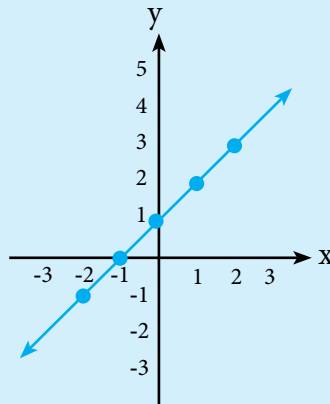
Exercise 9.11

- Plot the following points on the Cartesian plane:
- Are the points linear?
- If the pattern is linear, what is the next point?

$(-2,-1), (-1,0), (0,1), (1,2), (2,3)$.

The points are linear.

The next point would be $(3,4)$
 $\{ y = x + 1 \}$

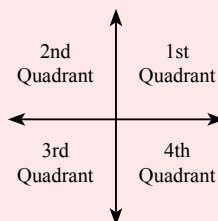


- $(-2,0), (-1,1), (0,2), (1,3), (2,4)$.
- $(-2,1), (-1,2), (0,3), (1,4), (2,5)$.
- $(-2,2), (-1,3), (0,4), (1,5), (2,6)$.
- $(-2,3), (-1,3), (0,4), (1,4), (2,5)$.
- $(-2,3), (-1,4), (0,5), (1,6), (2,7)$.
- $(-2,4), (-1,1), (0,0), (1,1), (2,4)$.
- $(-2,-3), (-1,-2), (0,-1), (1,0), (2,1)$.
- $(-2,-4), (-1,-3), (0,-2), (1,-1), (2,0)$.
- $(-2,-4), (-1,-2), (0,-1), (1,-3), (2,-3)$.
- $(-2,-5), (-1,-4), (0,-3), (1,-2), (2,-1)$.
- $(-2,-6), (-1,-5), (0,-4), (1,-3), (2,-2)$.
- $(-5,1), (-4,1), (-3,1), (-2,1), (-1,1)$.
- $(-5,-3), (-3,-1), (-1,1), (1,3), (3,5)$.
- $(-6,-8), (-3,-5), (0,-2), (3,1), (6,4)$.
- $(-6,-4), (-3,-2), (0,0), (3,2), (6,4)$.



Lines are everywhere.

Mental Computation



Exercise 9.12

- 1 Spell Integer.
- 2 $5 - 7$
- 3 $-2 + -3$
- 4 In which quadrant is $(3, 5)$
- 5 What is the volume of a 3m by 4m by 5m rectangular prism?
- 6 How many litres in a m^3 ?
- 7 Find the value of the expression: $5(b + 4)$ when $b = 2$
- 8 Change 0.5 to a fraction.
- 9 $\frac{2}{3} \times \frac{1}{4}$
- 10 While playing darts I threw a 20, a 5, and a 14. What is my score?

Many everyday problems
are solved mentally.

Exercise 9.13

- 1 Spell Coordinate.
- 2 $4 - 8$
- 3 $-5 + 2$
- 4 In which quadrant is $(-2, 3)$
- 5 What is the volume of a 6m by 3m by 5m rectangular prism?
- 6 How many litres in a m^3 ?
- 7 Find the value of the expression: $5(c - 4)$ when $c = 7$
- 8 Change 0.6 to a fraction.
- 9 $\frac{1}{3} \times \frac{3}{4}$
- 10 While playing darts I threw a 12, a 7, and a 13. What is my score?

As the new model is larger
and roomier, the minivan
will be called a van.

Exercise 9.14

- 1 Spell Linear.
- 2 $9 - 12$
- 3 $-3 + -4$
- 4 In which quadrant is $(-6, -1)$
- 5 What is the volume of a 5m by 3m by 3m rectangular prism?
- 6 How many litres in a m^3 ?
- 7 Find the value of the expression: $6(d + 2)$ when $d = 4$
- 8 Change 0.7 to a fraction.
- 9 $\frac{2}{3} \times \frac{3}{4}$
- 10 While playing darts I threw a 11, a 8, and a 18. What is my score?

NAPLAN Questions



Exercise 9.15

1 Calculate:

a) $5 - 2$

b) $6 - 9$

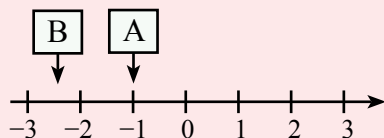
c) $4 + ^{-}2$

d) $^{-}5 + 3$

e) $^{-}3 + ^{-}2$

f) $^{-}4 - ^{-}2$

2 Estimate the value of A and B:



3 Given: $y = 2x - 15$ find the value of y when $x = 3$



Mathematics is useful in almost every career that one can think of.

4 Arrange the following integers in ascending order:

a) $^{-}2, ^{-}3, 1, 0, 2$

b) $^{-}1.5, ^{-}2.5, 1.5, 0.5$

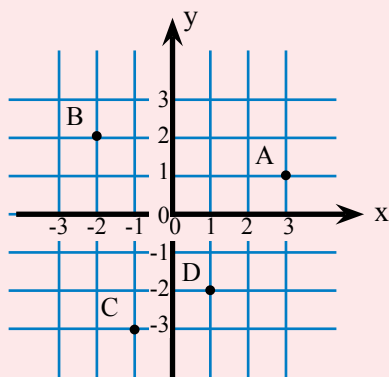
5 What are the coordinates of:

a) A?

b) B?

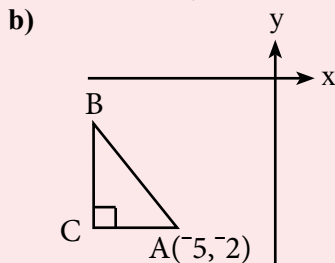
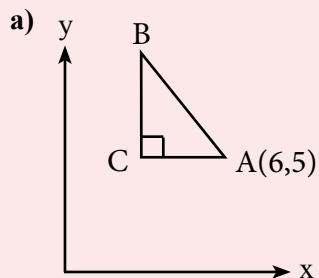
c) C?

d) D?



x coordinate first,
y coordinate second.

6 What are the coordinates of B ($AB = 5$, $BC = 4$, $CA = 3$)?

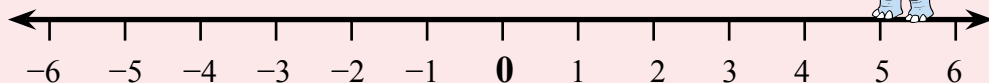


Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.



Exercise 9.16



- 1 Ethan is standing on -4 and facing towards the zero when he is given the following instructions:

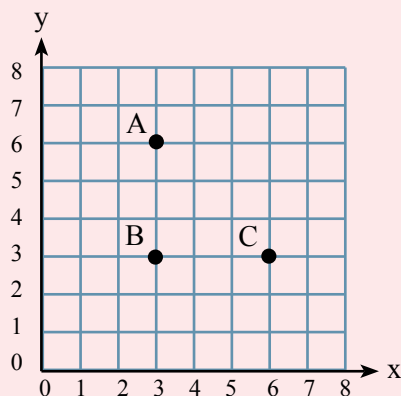
- a) Move 6 units forward. b) Move 3 units backward.
c) Move 5 units forward. d) Move 7 units backward.

Where is Ethan now standing?

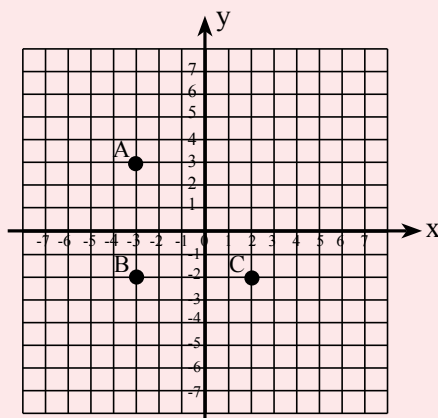
- 2 Calculate:

- a) $5 - 4 + 3 - 2 + 1$ b) $1 - 2 + 3 - 4 + 5$
c) $10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1$
d) $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10$
e) $100 - 99 + 98 - 97 + \dots + 4 - 3 + 2 - 1$
f) $1 - 2 + 3 - 4 + \dots + 97 - 98 + 99 - 100$

- 3 a) What are the coordinates of D, so that ABCD forms a square?
b) What are the coordinates of D, so that ABCD forms a parallelogram?
c) What are the coordinates of D, so that D is an equal distance from A, B, and C?
d) What are the coordinates of D, to form a different parallelogram?



- 4 a) What are the coordinates of D, so that ABCD forms a square?
b) What are the coordinates of D, so that ABCD forms a parallelogram?
c) What are the coordinates of D, so that D is an equal distance from A, B, and C?
d) What are the coordinates of D, to form a different parallelogram?



Investigations

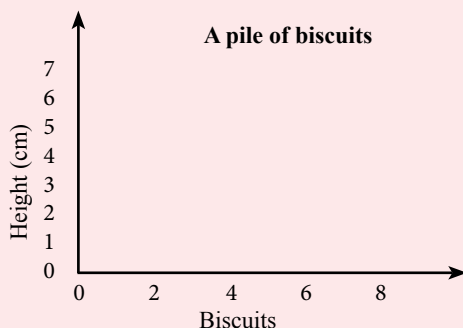
Investigation 9.1 A Linear Relationship?

- 1 Obtain a packet of biscuits.
- 2 Measure the height of a pile of biscuits.
- 3 Make a pile of 2 biscuits and measure its height.
Make a pile of 4 biscuits and measure its height.
Make a pile of 6 biscuits and measure its height.
Make a pile of 8 biscuits and measure its height.

You need:

- biscuits of same size.
- a ruler.

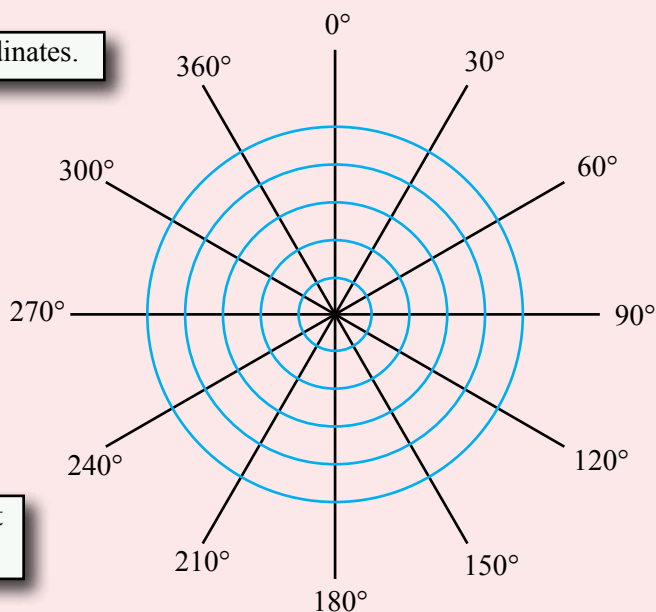
- 4 Draw a graph.



- 5
 - a) Can you use the graph to find the height of 15 biscuits?
 - b) How will the graph change if the biscuits were thicker?
 - c) How will the graph change if the biscuits were thinner?

Investigation 9.2 Polar Coordinates?

Investigate polar coordinates.



Where do you think that
(4, 300°) would be?

A Couple of Puzzles

Exercise 9.17

- 1 When 7 is subtracted from a number, the answer is the same as subtracting the number from 7. What is the number?
- 2 What four different odd numbers add to 20?
- 3 What is the smallest number that is exactly divisible by 2, 3, 4, and 5?
- 4 How many cubes are needed to completely cover a cube of the same size?

Try Guess and Check?

$$10 - 7 = 3, 7 - 10 = -3$$

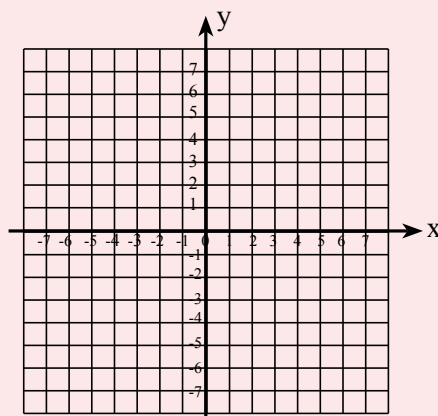


A Game

Coordinate Noughts and Crosses.

Two groups play each other.
Cartesian coordinates are drawn on the board.
Four points in a line is a point (vertically, horizontally, or diagonally).

Each group takes it in turn to call out an ordered pair.
A plotter plots the point.
Keep playing until there is no room left.
The group with the most points is the winner



A Sweet Trick

Hide a piece of paper somewhere with the number 1089 written on it.

- 1 Ask your audience for a 3 digit number. 815
- 2 Arrange the number in descending order. 851
- 3 Reverse and subtract.
$$\begin{array}{r} 851 \\ -158 \\ \hline 693 \end{array}$$
- 4 Reverse and add
$$\begin{array}{r} 693 \\ +396 \\ \hline 1089 \end{array}$$

Dramatically reveal the piece of paper.



The answer will always be 1089.

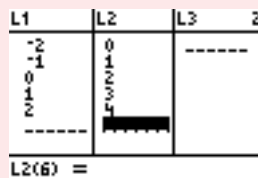
Technology

Technology 9.1

Use a Graphics Calculator to plot the points in Exercise 9.11

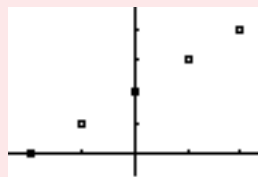
$(-2,0)$, $(-1,1)$, $(0,2)$, $(1,3)$, $(2,4)$

Press **STAT** **EDIT** and enter the points



Press **Y=** and highlight **PLOT 1**

Press **GRAPH** to see a plot



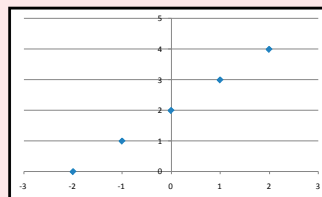
Press **ZOOM** **ZooMStat** to zoom in

Technology 9.2

Use a Spreadsheet to plot the point in Exercise 9.11

$(-2,0)$, $(-1,1)$, $(0,2)$, $(1,3)$, $(2,4)$

-2	-1	0	1	2
0	1	2	3	4



Use the Chart (Scatter) to plot the points:

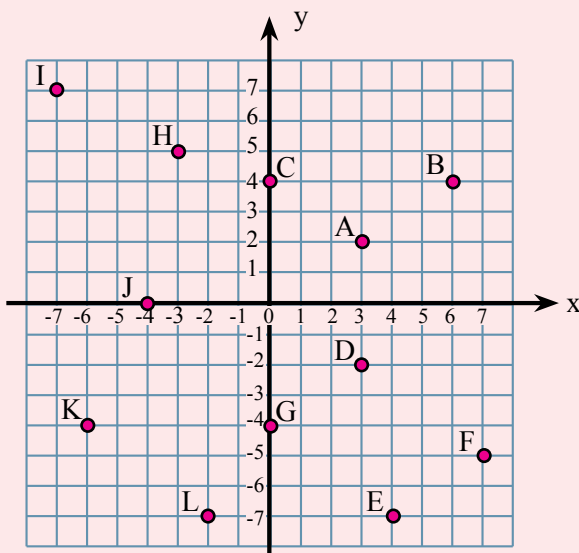
Technology 9.3

- 1 Play some of the integer games on the Internet. Search phrase 'integer game'.
- 2 Play a coordinate game on the Internet. Search phrase 'coordinate game'.
- 3 View some of the coordinate lessons on the Internet. Search phrase 'cartesian coordinates'.
- 4 Try to find an applet that allows you to plot points.

Chapter Review 1

Exercise 9.18

- Change the following words to an integer:
 - A drop of 8°C .
 - A loss of \$75.
 - A increase of 25%.
 - Shortened by 15 cm.
- Copy each pair of numbers and place the correct $<$ or $>$ between them.
 - 6 -3
 - -2 3
 - -1 -4
- Arrange the integers in ascending order (smallest to largest):
 - $-3, 0, 1, -2$
 - $-2, 1, 0, -1$
 - $-2, 3, -1, -3$
- Calculate the following:
 - $7 - 2$
 - $-3 + 2$
 - $-4 - 1$
 - $6 - -1$
 - $-5 - -4$
 - $6 + 3 - -5$
- Write the coordinates of each of the points in the graph below:



The x distance first.
The y distance second.

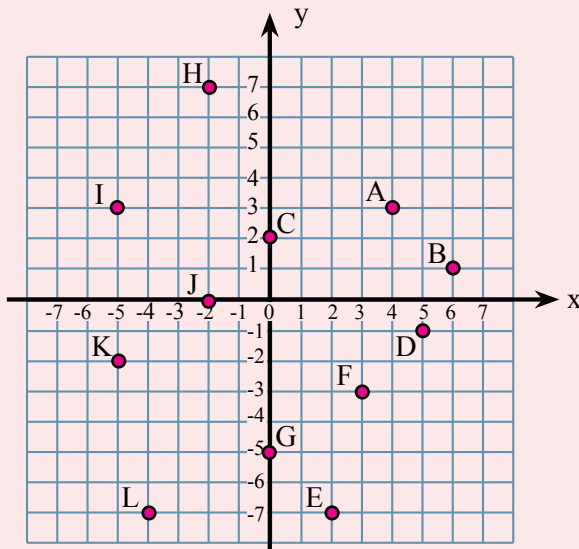
- Plot the following points on a set of Cartesian axes:
 - A(1,3)
 - B(2,1)
 - C(-4,3)
 - D(5,-1)
 - E(-3,-5)
 - F(0,0)
 - G(0,2)
 - H(-5,0)
- Plot the following points on the Cartesian plane and determine if the pattern is linear:
 - $(-2,2), (-1,3), (0,4), (1,5), (2,6)$.
 - $(-2,1), (-1,2), (0,3), (1,4), (2,5)$.
 - $(-2,1), (-1,4), (0,3), (1,2), (2,2)$.
 - $(-2,4), (-1,2), (0,0), (1,-2), (2,-4)$.

Why are there flotation devices under plane seats instead of parachutes?

Chapter Review 2

Exercise 9.19

- Change the following words to an integer:
 - A loss of \$200.
 - A rise of 4m.
 - An increase of 7%.
 - No movement.
- Copy each pair of numbers and place the correct $<$ or $>$ between them.
 - 2 -1
 - -4 2
 - -3 -2
- Arrange the integers in ascending order (smallest to largest):
 - $3, 1, 0, -1$
 - $-4, 0, 4, -2$
 - $-4, 1, -2, -1$
- Calculate the following:
 - $3 - 1$
 - $-3 + 4$
 - $-3 - 2$
 - $1 - -2$
 - $-3 - -6$
 - $5 + -3 - -2$
- Write the coordinates of each of the points in the graph below:

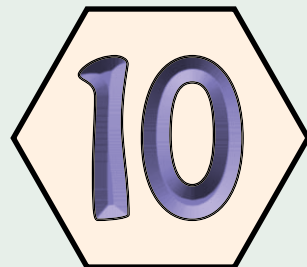


The x distance first.
The y distance second.

- Plot the following points on a set of Cartesian axes:
 - $A(2,1)$
 - $B(4,2)$
 - $C(-1,3)$
 - $D(4,5)$
 - $E(-1,-3)$
 - $F(0,0)$
 - $G(0,3)$
 - $H(-2,0)$
- Plot the following points on the Cartesian plane and determine if the pattern is linear:
 - $(-2,1), (-1,2), (0,3), (1,4), (2,5)$.
 - $(-2,3), (-1,4), (0,5), (1,6), (2,7)$.
 - $(-2,2), (-1,3), (0,3), (1,2), (2,1)$.
 - $(-2,6), (-1,3), (0,0), (1,-3), (2,-6)$.

T Why are you late?
S Because of the sign down the road.
T How come?
S 'School Ahead, Go Slow'

Review 2



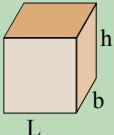
Chapter 6 Number 2

Adding fractions $\frac{2}{3} + \frac{1}{4} = \frac{2 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$	Subtracting fractions $\frac{3}{5} - \frac{1}{2} = \frac{3 \times 2}{5 \times 2} - \frac{1 \times 5}{2 \times 5} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$
Multiplying fractions $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$	Dividing fractions $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$

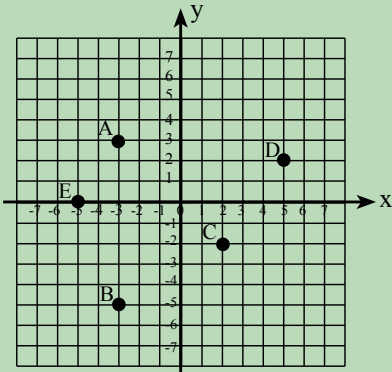
Chapter 7 Algebra

Order of Operations: 1 () brackets first. 2 \times and \div from left to right. 3 $+$ and $-$ from left to right.	Write an algebraic expression for: double a number and add 3 $= 2x + 3$ Find the value of $5x - 3$ when $x = 4$ $= 5 \times 4 - 3 = 20 - 3 = 17$
Commutative law $x + y = y + x$ $xy = yx$	Associative law $(x + y) + z = x + (y + z)$ $(xy)z = x(yz)$

Chapter 8 Volume

A rectangular prism is a column with a rectangular base. Volume = Lbh		$1\text{cm}^3 = 1000\text{mm}^3$ $1\text{m}^3 = 1\,000\,000\text{cm}^3$ $1\text{L} = 1000\text{mL}$ $1\text{kL} = 1000\text{L}$
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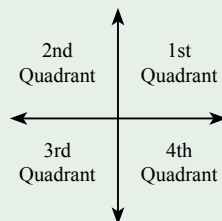
Chapter 9 Coordinates

Integers are: Positive whole numbers: 1, 2, 3, 4, 5, 6, .. and Zero: 0 and Negative whole numbers: -1, -2, -3, -4, -5, -6,..	Adding and subtracting integers: $5 + -7 = -2$ $-2 - -5 = -2 + 5 = 3$
	Coordinates: The x first The y second A(-3,3) B(-3,-5) C(2,-2) D(5,2) E(-5,0)

Review 1

Exercise 10.1 Mental computation

- 1 Spell Commutative.
- 2 $3 - 6$
- 3 $-1 + -4$
- 4 In which quadrant is $(3, 5)$
- 5 What is the volume of a 2m by 4m by 5m rectangular prism?
- 6 How many litres in a m^3 ?
- 7 Find the value of the expression: $4(d + 3)$ when $d = 2$
- 8 Change 0.5 to a fraction.
- 9 $\frac{1}{2} + \frac{1}{3}$
- 10 While playing darts I threw a 10, a 5, and a double 7. What is my score?



Exercise 10.2

- 1 Put the following fractions in order from smallest to largest.

a) $\frac{1}{2}, \frac{1}{7}, \frac{1}{3}, \frac{1}{5}$

b) $\frac{1}{9}, \frac{1}{4}, \frac{1}{8}, \frac{1}{6}$

c) $\frac{2}{5}, \frac{1}{4}, \frac{3}{8}$

d) $\frac{4}{5}, \frac{7}{10}, \frac{2}{3}$

- 2 Complete the following:

a) $\frac{4}{5} = \frac{?}{10}$

b) $\frac{2}{3} = \frac{?}{9}$

c) $\frac{1}{4} = \frac{?}{8} = \frac{?}{12}$

d) $2 = \frac{6}{3} = \frac{?}{5} = \frac{?}{7}$

- 3 Change each of the following improper fractions to a mixed number:

a) $\frac{7}{3}$

b) $\frac{9}{2}$

c) $\frac{7}{5}$

d) $\frac{11}{3}$

e) $\frac{12}{5}$

f) $\frac{13}{6}$

g) $\frac{19}{7}$

h) $\frac{25}{9}$

- 4 Change each of the following mixed numbers to an improper fraction:

a) $1\frac{1}{3}$

b) $2\frac{1}{2}$

c) $2\frac{2}{3}$

d) $3\frac{1}{5}$

e) $2\frac{4}{5}$

f) $3\frac{3}{4}$

g) $3\frac{3}{8}$

h) $5\frac{9}{10}$

- 5 Calculate each of the following:

a) $\frac{1}{5} + \frac{2}{5}$

b) $\frac{1}{2} + \frac{2}{3}$

c) $\frac{3}{5} - \frac{1}{5}$

d) $\frac{3}{4} - \frac{1}{3}$

e) $\frac{1}{2} \times \frac{1}{3}$

f) $\frac{2}{3} \times \frac{1}{4}$

g) $\frac{1}{3} \div \frac{1}{4}$

h) $\frac{2}{3} \div \frac{1}{4}$

() brackets first.
 \times and \div from left to right.
 $+$ and $-$ from left to right.

6 Find the value of each of the following:

- | | |
|--|--|
| a) $10 - 2 \times 3$ | b) $2 \times 5 - 1$ |
| c) $2 \times 6 \div (2 + 1)$ | d) $3 + 14 \div (7 - 5)$ |
| e) $18 \div (3 \times 2 - 3) + 2 \times 3$ | f) $(10 - 2 \times 1) \div (8 \div (3 + 1))$ |

7 Write an algebraic expression for each of the following:

- | | |
|--------------------|------------------------|
| a) A number plus 3 | b) A number minus 4 |
| c) Triple a number | d) Quarter of a number |

8 Find the value of each of the following algebraic expressions given that $x = 3$ and $y = 6$.

- | | | |
|-------------|-------------|-------------|
| a) $3x$ | b) $2y$ | c) $x + 4$ |
| d) $x + y$ | e) $y - x$ | f) $x - y$ |
| g) $2x + 3$ | h) $3x - y$ | i) $2y + 5$ |

9 Write an algebraic expression for each of the following pairs:
 Find the value of each algebraic expression using $x = 5$, $y = 4$.
 Comment on the values obtained.

- a) x plus four **and** four plus x .
 b) y times three **and** three times y .
 c) x minus four **and** four minus by x .

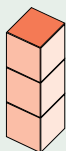
10 Find the value of each algebraic expression using $x = 7$, $y = 6$, $z = 5$.
 Comment on the values obtained.

- a) $(x + y) + z$ **and** $x + (y + z)$
 b) $5x + (y + 1)$ **and** $(5x + y) + 1$
 c) $(x - y) - z$ **and** $x - (y - z)$
 d) $2 \times (x \times y)$ **and** $(2 \times x) \times y$

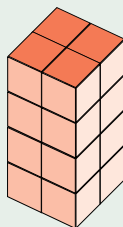
11 The power output of a car speaker, in watts, is given by the formula:
 $\text{Power} = v \times v \div R$, where v is the car's voltage and R is the impedance
 of the speaker being used. What power is generated by a speaker
 with $v = 12$ volts, and $R = 4$ ohms?

12 If each cube in each figure below is one cubic centimetre,
 1cm^3 , find the volume of each figure:

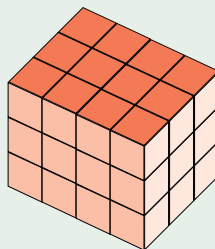
a)



b)

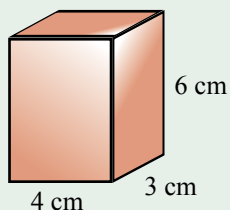


c)

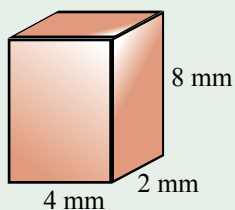


13 Find the volume of each of the following prisms:

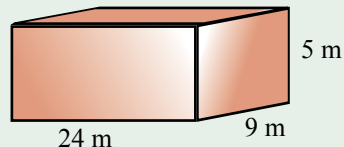
a)



b)



c)



14 A milk carton has a base of 10 cm by 10 cm and a height of 20 cm.
How many litres of milk will the carton hold ($1000 \text{ cm}^3 = 1 \text{ L}$)?

15 How many litres of water is needed to fill a swimming pool 50 m long,
10 m wide, and 1.8 m deep ($1 \text{ m}^3 = 1000 \text{ L}$)?

16 Make the following unit conversions:

a) 3800 mm^3 to cm^3

b) $47\,000\,000 \text{ cm}^3$ to m^3

c) 1.6 m^3 to cm^3

d) 0.3 cm^3 to mm^3

e) 7800 mL to L

f) 880 000 L to kL

g) 5.2 kL to L

h) 0.75 L to mL

17 Change the following words to an integer:

a) A loss of \$600.

b) 7 paces to the right.

c) A decrease of 12%.

d) No movement.

18 Copy each pair of numbers and place the correct $<$ or $>$ between them.

a) 4 -2

b) -1 2

c) -2 -3

19 Arrange the integers in ascending order (smallest to largest):

a) -2 , 1, 0, -1

b) -3 , 0, 2, -2

c) -1 , 2, -2 , 1

20 Calculate the following:

a) $9 - 6$

b) $-5 + 2$

c) $-2 - 1$

d) $4 - -2$

e) $-3 - -1$

f) $6 + -2 - -4$

21 Plot the following points on a set of Cartesian axes:

a) A(1,3)

b) B(2,5)

c) C(-4 ,2)

d) D(3,5)

e) E(-4 , -2)

f) F(0,0)

g) G(0,1)

h) H(-3 ,0)

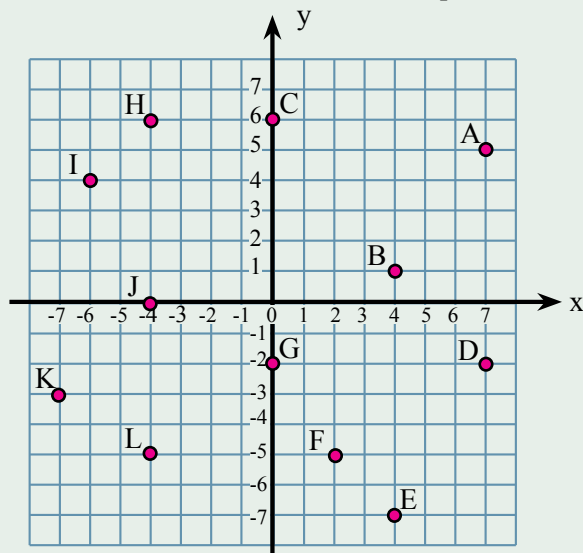
22 Plot the following points on the Cartesian plane and determine
if the pattern is linear:

a) $(-2,1)$, $(-1,2)$, $(0,3)$, $(1,4)$, $(2,5)$.

b) $(-2,0)$, $(-1,2)$, $(0,4)$, $(1,6)$, $(2,8)$.

The x distance first.
The y distance second.

- 23 Write the coordinates of each of the points in the graph below:

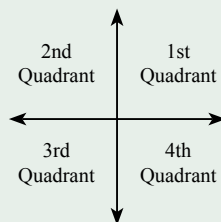


The x distance first.
The y distance second.

Review 2

Exercise 10.3 Mental computation

- 1 Spell Associative.
- 2 $3 - 5$
- 3 $-1 - 4$
- 4 In which quadrant is $(-2, 3)$
- 5 What is the volume of a 3m by 5m by 6m rectangular prism?
- 6 How many millilitres in a litre?
- 7 Find the value of the expression: $4g - 3$ when $g = 3$
- 8 Change 0.8 to a fraction.
- 9 $\frac{1}{2} \div \frac{1}{4}$
- 10 While playing darts I threw a 20, a 1, and a double 18. What is my score?



Exercise 10.4

- 1 Put the following fractions in order from smallest to largest.

a) $\frac{1}{4}, \frac{1}{5}, \frac{1}{2}, \frac{1}{3}$

b) $\frac{1}{2}, \frac{1}{7}, \frac{1}{5}, \frac{1}{9}$

c) $\frac{3}{4}, \frac{2}{3}, \frac{3}{5}$

d) $\frac{3}{8}, \frac{1}{3}, \frac{2}{5}$

- 2 Complete the following:

a) $\frac{3}{5} = \frac{?}{10}$

b) $\frac{3}{4} = \frac{?}{8}$

c) $\frac{2}{3} = \frac{?}{6} = \frac{?}{9}$

d) $3 = \frac{9}{3} = \frac{?}{5} = \frac{?}{9}$

3 Change each of the following improper fractions to a mixed number:

a) $\frac{9}{2}$

b) $\frac{9}{4}$

c) $\frac{8}{5}$

d) $\frac{14}{5}$

e) $\frac{17}{7}$

f) $\frac{13}{9}$

g) $\frac{24}{7}$

h) $\frac{29}{3}$

4 Change each of the following mixed numbers to an improper fraction:

a) $2\frac{1}{3}$

b) $2\frac{1}{2}$

c) $1\frac{3}{4}$

d) $3\frac{2}{5}$

e) $3\frac{5}{6}$

f) $4\frac{2}{7}$

g) $5\frac{4}{5}$

h) $6\frac{9}{10}$

5 Calculate each of the following:

a) $\frac{2}{7} + \frac{3}{7}$

b) $\frac{1}{2} + \frac{3}{5}$

c) $\frac{4}{5} - \frac{1}{5}$

d) $\frac{4}{5} - \frac{1}{3}$

e) $\frac{1}{4} \times \frac{1}{3}$

f) $\frac{3}{4} \times \frac{2}{3}$

g) $\frac{1}{4} \div \frac{1}{3}$

h) $\frac{3}{4} \div \frac{2}{3}$

6 Find the value of each of the following:

a) $12 - 2 \times 3$

b) $2 \times 4 - 1$

c) $2 \times 12 \div (3 + 1)$

d) $3 + 10 \div (5 - 3)$

e) $20 \div (2 \times 5 - 5) + 2 \times 5$

f) $(16 - 2 \times 1) \div (4 \div (1 + 1))$

7 Write an algebraic expression for each of the following:

a) A number plus 7

b) A number decreased by 7

c) Multiply a number by 10

d) A third of a number

8 Find the value of each of the following algebraic expressions given that $x = 5$ and $y = 6$.

a) $2x$

b) $2y$

c) $x + 2$

d) $x + y$

e) $x - y$

f) $y - x$

g) $2x + 3$

h) $x - 2y$

i) $2y - 5$

9 Write an algebraic expression for each of the following pairs:

Find the value of each algebraic expression using $x = 7$, $y = 2$.

Comment on the values obtained.

a) x plus five **and** five plus x .

b) y times x **and** x times y .

c) x minus three **and** three minus x .

10 Find the value of each algebraic expression using $x = 4$, $y = 3$, $z = 1$.

Comment on the values obtained.

a) $(x + y) + z$ **and** $x + (y + z)$

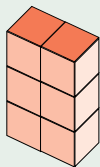
b) $3x + (y + 2)$ **and** $(3x + y) + 2$

c) $(x - y) - z$ **and** $x - (y - z)$

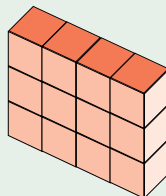
- 11 The power output of a car speaker, in watts, is given by the formula: $\text{Power} = v \times v \div R$, where v is the car's voltage and R is the impedance of the speaker being used. What power is generated by a speaker with $v = 12$ volts, and $R = 2$ ohms?

- 12 If each cube in each figure below is one cubic centimetre, 1 cm^3 , find the volume of each figure:

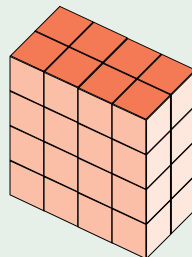
a)



b)

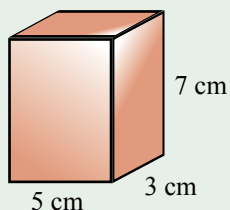


c)

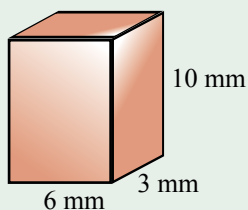


- 13 Find the volume of each of the following prisms:

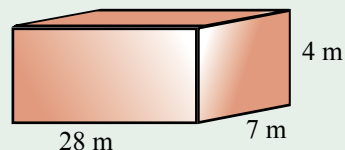
a)



b)



c)



- 14 A milk carton has a base of 10 cm by 15 cm and a height of 20 cm. How many litres of milk will the carton hold ($1000 \text{ cm}^3 = 1 \text{ L}$)?

- 15 How many litres of water is needed to fill a swimming pool 50 m long, 8 m wide, and 1.6 m deep ($1 \text{ m}^3 = 1000 \text{ L}$)?

- 16 Make the following unit conversions:

a) 7900 mm^3 to cm^3

b) $64\,000\,000 \text{ cm}^3$ to m^3

c) 3.2 m^3 to cm^3

d) 0.4 cm^3 to mm^3

e) 5000 mL to L

f) 310 000 L to kL

g) 9.5 kL to L

h) 0.25 L to mL

- 17 Change the following words to an integer:

a) A loss of \$700.

b) 9 paces to the left.

c) An increase of 37%.

d) No movement.

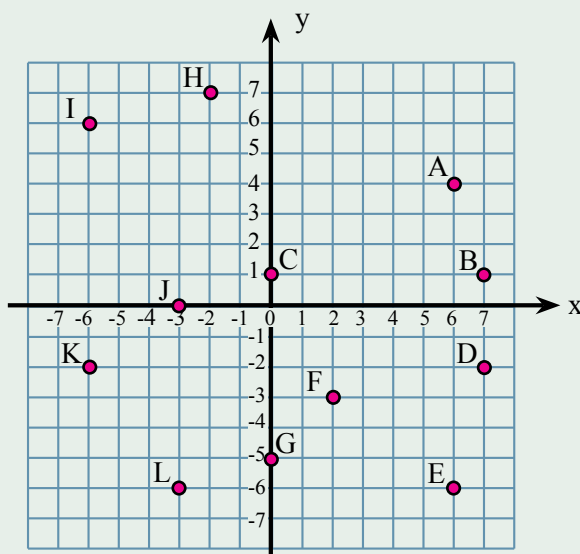
- 18 Copy each pair of numbers and place the correct $<$ or $>$ between them.

a) 1 $^{-}3$

b) $^{-}2$ 1

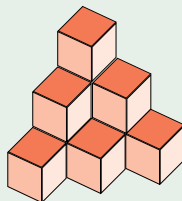
c) $^{-}5$ $^{-}2$

- 19 Arrange the integers in ascending order (smallest to largest):
- a) $-3, 2, 0, -1$ b) $-3, 0, 1, -2$ c) $-1, -3, -2, 1$
- 20 Calculate the following:
- a) $3 - 6$ b) $-5 + 4$ c) $-2 - 3$
d) $5 - -2$ e) $-2 - -1$ f) $7 + -2 - -2$
- 21 Plot the following points on a set of Cartesian axes:
- a) A(2,1) b) B(3,2) c) C(-5,1) d) D(6,-3)
e) E(-3,-4) f) F(0,0) g) G(0,5) h) H(-1,0)
- 22 Plot the following points on the Cartesian plane and determine if the pattern is linear:
- a) $(-2,2), (-1,3), (0,4), (1,5), (2,6)$.
b) $(-2,5), (-1,-3), (0,-1), (1,1), (2,3)$.
- 23 Write the coordinates of each of the points in the graph below:

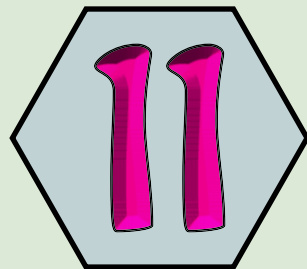


The x distance first.
The y distance second.

- 24 How many more cubes are needed to make a larger cube $3\text{cubes} \times 3\text{cubes} \times 3\text{cubes}$?



Number 3



- ★ Investigate index notation.
- ★ Represent whole numbers as products of powers of prime numbers.
- ★ Define and compare prime and composite numbers.
- ★ Apply knowledge of factors to expressing whole numbers as products of powers of prime factors (repeated division by prime factors, creating factor trees).
- ★ Solve problems involving lowest common multiples and greatest common divisors (highest common factors) for pairs of whole numbers by comparing their prime factorisation.
- ★ Investigate square numbers such as 25 and 36 and developing square-root notation.
- ★ Investigate between which two whole numbers a square root lies.

A TASK

- Become famous and rich by either proving or disproving The Goldbach Conjecture.
- Christian Goldbach (1690-1764), an amateur mathematician, conjectured that every even number greater than 4 can be expressed as the sum of two different prime numbers.
- To prove The Goldbach Conjecture, show that it works for every even number.
- To disprove The Goldbach Conjecture, find just one even number that can't be expressed as the sum of two different prime numbers.

$$\begin{aligned}6 &= 1 + 5 \\16 &= 13 + 3 \\16 &= 11 + 5 \\76 &= 73 + 3 \\76 &= 23 + 53 \\76 &= 71 + 5\end{aligned}$$

A LITTLE BIT OF HISTORY

Perfect numbers.

A number is perfect if it is the sum of its smaller factors.

$$6 = 1 + 2 + 3 \quad \text{Thus 6 is perfect.}$$

$$28 = 1 + 2 + 4 + 7 + 14 \quad \text{Thus 28 is perfect.}$$

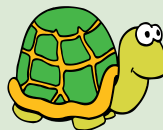
100 AD The known perfect numbers are: 6, 28, 496, 8128

1460 AD The fifth perfect number is known.

1870 AD Eight perfect numbers are known.

1970 AD Twenty-three perfect numbers are known.

All known perfect numbers end in 6 or 28.



Indices

Exercise 11.1

Calculate each of the following:

- | | | | |
|--|--|--|--|
| 1 1×1 | 2 2×2 | 3 3×3 | 4 4×4 |
| 5 5×5 | 6 $1 \times 1 \times 1$ | 7 $2 \times 2 \times 2$ | 8 $3 \times 3 \times 3$ |
| 9 $4 \times 4 \times 4$ | 10 $5 \times 5 \times 5$ | 11 $1 \times 1 \times 1 \times 1$ | 12 $2 \times 2 \times 2 \times 2$ |
| 13 $3 \times 3 \times 3 \times 3$ | 14 $4 \times 4 \times 4 \times 4$ | 15 $5 \times 5 \times 5 \times 5$ | |

16 Copy and complete the following table:

	Square	Cube	Fourth	Fifth
1				
2	$2 \times 2 = 4$		$2 \times 2 \times 2 \times 2 = 16$	
3				
4		$4 \times 4 \times 4 = 64$		
5				

- | | |
|--|--|
| 17 10×10 | 18 $10 \times 10 \times 10$ |
| 19 $10 \times 10 \times 10 \times 10$ | 20 $10 \times 10 \times 10 \times 10 \times 10$ |
| 21 $10 \times 10 \times 10 \times 10 \times 10 \times 10$ | 22 $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ |

$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

On a calculator: 4 ^ 7 = 16384

The larger the numbers, the more useful a calculator will become.



- | | |
|--|--|
| 23 $2 \times 2 \times 2 \times 2 \times 2 \times 2$ | 24 $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ |
| 25 $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$ | 26 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ |

How many 10s are multiplied together to equal:

- 27** 1 million?
- 28** 1 billion (10^9 is generally known as 1 billion)?
- 29** 1 terabyte (A 1 terabyte hard disk drive (HDD) will hold 10^{12} bytes.)?
- 30** 8 gigabytes (1 gigabyte is 10^9 bytes)?
- 31** Can your calculator handle 1 googol (1 googol is known as 10^{100}).

A convenient way of writing $2 \times 2 \times 2$ is

$$2^3 \quad \begin{array}{l} \leftarrow \text{Index} \\ \leftarrow \text{Base} \end{array}$$

Exercise 11.2

Write the following in index form:

$2 \times 2 \times 2 \times 2 \times 2$ $= 2^5$	$10 \times 10 \times 10 \times 10$ $= 10^4$
--	--

- | | | |
|--|---|--|
| 1 $4 \times 4 \times 4$ | 2 $2 \times 2 \times 2 \times 2 \times 2$ | 3 10×10 |
| 4 $10 \times 10 \times 10$ | 5 $10 \times 10 \times 10 \times 10$ | 6 10 |
| 7 $10 \times 10 \times 10 \times 10 \times 10$ | 8 $9 \times 9 \times 9 \times 9$ | 9 $3 \times 3 \times 3 \times 3 \times 3 \times 3$ |

Exercise 11.3

Write the following in factor form:

3^4 $= 3 \times 3 \times 3 \times 3$	10^3 $= 10 \times 10 \times 10$
---	--

- | | | |
|----------|----------|----------|
| 1 4^3 | 2 10^4 | 3 5^2 |
| 4 2^7 | 5 6^2 | 6 10^5 |
| 7 10^1 | 8 10^6 | 9 1^4 |

Exercise 11.4

Calculate each of the following:

2×10^3 $= 2 \times 1000$ $= 2000$	$3 \times 10^2 + 6 \times 10^1 + 8$ $= 3 \times 100 + 6 \times 10 + 8$ $= 300 + 60 + 8$ $= 368$
--	--

- | | | |
|---------------------------------------|---------------------------------------|---------------------------------------|
| 1 3×10^2 | 2 6×10^3 | 3 2×10^1 |
| 4 $4 \times 10^1 + 7$ | 5 $9 \times 10^1 + 4$ | 6 $6 \times 10^2 + 2 \times 10^1 + 5$ |
| 7 $7 \times 10^2 + 8 \times 10^1 + 3$ | 8 $4 \times 10^2 + 3 \times 10^1 + 7$ | 9 $2 \times 10^3 + 1 \times 10^2 + 1$ |

Accountants examine and advise on the finances, records and compliance of clients. Accountancy normally involves a lot of public contact.

- Relevant school subjects are English, Mathematics, IT.
- Courses normally involve a Degree with a major in accounting.

Factors

The factors of a number are the numbers that divide exactly into the number.

The factors of 9 = 1, 3, and 9.

The factors of 12 = 1, 2, 3, 4, 6, and 12.

The factors of 7 = 1 and 7.

Exercise 11.5

Find all the factors of each of the following numbers:

21

$$1 \times 21 = 21$$

$$3 \times 7 = 21$$

The factors of 21 = 1, 3, 7, 21

1 and the number will always be factors.

1 2

2 3

3 4

4 5

5 6

6 7

7 8

8 9

9 10

10 11

11 15

12 16

13 20

14 24

15 27

16 28

17 30

18 31

19 32

20 33

Exercise 11.6

Find the highest common factor of each of the following pairs of numbers

12 and 18

Factors of 12 = 1, 2, 3, 4, 6, 12

Factors of 18 = 1, 2, 3, 6, 9, 18

The highest common factor is 6

1 12 and 15

2 12 and 16

3 8 and 20

4 10 and 25

5 16 and 24

6 9 and 21

7 20 and 30

8 8 and 30

9 14 and 21

10 24 and 30

11 28 and 35

12 16 and 30

Mental computation tip:

$$\begin{aligned} 12 \times 15 &= 6 \times 2 \times 15 \\ &= 6 \times 30 \\ &= 180 \end{aligned}$$

Mental computation tip:

$$\begin{aligned} 20 \times 42 &= 10 \times 2 \times 42 \\ &= 10 \times 84 \\ &= 840 \end{aligned}$$

Prime Numbers

Prime numbers have exactly two factors, one and itself.

The factors of 2 = 1 and 2.

The factors of 3 = 1 and 3.

The factors of 7 = 1 and 7. 2, 3, and 7 are prime numbers.

Composite numbers have more than two factors.

1 only has one factor.
1 is neither prime nor composite.

Exercise 11.7

Use the sieve of Eratosthenes to find the prime numbers between 1 and 100.

Eratosthenes (275 BC to 194 BC) suggested a method for finding the prime numbers by sieving out the composite numbers so that only prime numbers are left.

- 1 Make a table of all of the numbers from 1 to 100.
- 2 Cross out all the numbers that are divisible by two, except two itself.
- 3 Cross out all the numbers that are divisible by three, except three itself.
- 4 Do the same for five.
- 5 Do the same for seven.
- 6 Are the numbers left all the prime numbers from 1 to 100?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
91	92	93	94	95	96	97	98	99	100

Cross out 1.
1 is not a prime number. Why?

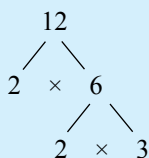
Exercise 11.8

Comment on each of the following:

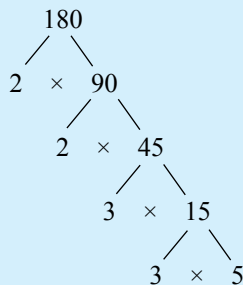
- 1 2 is the only prime number that is even.
- 2 There are more prime numbers between 1 and 100 than between 100 and 200.
- 3 There are 5 prime numbers less than 10.
- 4 The sum of two prime numbers is even.
- 5 The product of prime numbers is odd.
- 6 All numbers can be written as the sum of prime numbers. eg. $9 = 7 + 2$.

Prime Factors

Every composite number can be written as a product of prime factors.

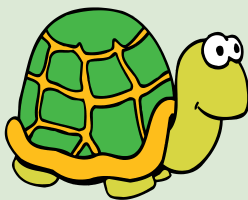


Prime factors of 12 = $2 \times 2 \times 3$
or = $2^2 \times 3$



Prime factors of 180 = $2 \times 2 \times 3 \times 3 \times 5$
or = $2^2 \times 3^2 \times 5$

These are called factor trees.



Keep dividing by prime numbers such as 2, 3, 5, 7, until the result is prime.

Exercise 11.9

Use factor trees to find the prime factors of each of the following numbers:

1 18

2 20

3 24

4 15

5 30

6 32

7 100

8 120

9 180

10 144

11 250

12 1000

13 5000

14 10 000

15 12 000

Exercise 11.10

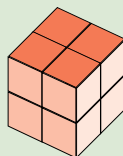
Comment on each of the following:

- 1 12 and 6 have the same prime factors of 2 and 3.
- 2 120 and 60 have the same prime factors 2, 3, and 5.
- 3 360 has the prime factors of 2, 3, and 5.
- 4 There is no number less than 100 with prime factors of 2, 3, 5, and 7.
- 5 There is no number less than 10 000 with prime factors of 2, 3, 5, 7, and 11.
- 6 888 888 has prime factors of just 2 and 3.
- 7 282 475 249 has just one prime factor 7.

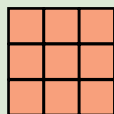
Squares and Cubes



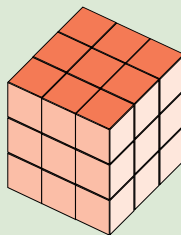
$$\begin{aligned} 2 \times 2 &= 4 \\ 2^2 &= 4 \\ 2 \text{ squared} &= 4 \end{aligned}$$



$$\begin{aligned} 2 \times 2 \times 2 &= 8 \\ 2^3 &= 8 \\ 2 \text{ cubed} &= 8 \end{aligned}$$



$$\begin{aligned} 3 \times 3 &= 9 \\ 3^2 &= 9 \\ 3 \text{ squared} &= 9 \end{aligned}$$



$$\begin{aligned} 3 \times 3 \times 3 &= 27 \\ 3^3 &= 27 \\ 3 \text{ cubed} &= 27 \end{aligned}$$

Exercise 11.11

- | | |
|--------------------------|------------------------|
| 1 Find the square of 4 | 2 Find the cube of 4 |
| 3 Find the square of 5 | 4 Find the cube of 5 |
| 5 Find the square of 6 | 6 Find the cube of 6 |
| 7 Find the square of 7 | 8 Find the cube of 7 |
| 9 Find the square of 8 | 10 Find the cube of 8 |
| 11 Find the square of 9 | 12 Find the cube of 9 |
| 13 Find the square of 10 | 14 Find the cube of 10 |

Mental Computation Trick

To square any two digit number ending in 5:

First digit \times (first digit + 1) then attach 25 to the answer.

Example: 35^2 $\{3 \times 4 = 12\}$ answer = 1225

Example: 55^2 $\{5 \times 6 = 30\}$ answer = 3025

In your head,
what is 45^2 ?



3.2^2 is between which two squares?

3.2 is between 3 and 4

3.2^2 is between 3^2 and 4^2

3.2^2 is between 9 and 16

In your head,
what is 4.5^2 ?

- | | |
|--|--|
| 15 3.7^2 is between which two squares? | 16 4.5^2 is between which two squares? |
| 17 6.5^2 is between which two squares? | 18 7.4^2 is between which two squares? |

Square Roots & Cube Roots



is the symbol for square root.

The square root is the opposite of squaring.



is the symbol for cube root.

The cube root is the opposite of cubing.

Exercise 11.12

What is the square root of 36?

$$\begin{aligned}\sqrt{36} \\ &= \sqrt{6 \times 6} \\ &= \underline{6}\end{aligned}$$

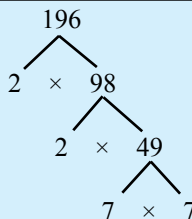
What is the cube root of 64?

$$\begin{aligned}\sqrt[3]{64} \\ &= \sqrt[3]{4 \times 4 \times 4} \\ &= \underline{4}\end{aligned}$$

- | | |
|----------------------------------|---------------------------------|
| 1 What is the square root of 9? | 2 What is the cube root of 8? |
| 3 What is the square root of 16? | 4 What is the cube root of 27? |
| 5 What is the square root of 25? | 6 What is the cube root of 125? |
| 7 What is the square root of 64? | 8 What is the cube root of 216? |

Use a factor tree to find $\sqrt{196}$

$$\begin{aligned}\sqrt{196} \\ &= \sqrt{2 \times 2 \times 7 \times 7} \\ &= 2 \times 7 \\ &= \underline{14}\end{aligned}$$



- | | |
|--|---|
| 9 Use a factor tree to find $\sqrt{400}$ | 10 Use a factor tree to find $\sqrt{144}$ |
| 11 Use a factor tree to find $\sqrt{225}$ | 12 Use a factor tree to find $\sqrt{324}$ |
| 13 Use a factor tree to find $\sqrt[3]{125}$ | 14 Use a factor tree to find $\sqrt[3]{1728}$ |

Between what two numbers does $\sqrt{43}$ lie?

43 is between 36 and 49

$\sqrt{43}$ is between $\sqrt{36}$ and $\sqrt{49}$

$\sqrt{43}$ is between 6 and 7

- | |
|--|
| 15 Between what two numbers does $\sqrt{32}$ lie? |
| 16 Between what two numbers does $\sqrt{74}$ lie? |
| 17 Between what two numbers does $\sqrt[3]{19}$ lie? |



Check your answers with a calculator?

Mental Computation

Mental computation helps people prepare for problems in everyday life.

Exercise 11.13

- 1 Spell index.
- 2 Write $3 \times 3 \times 3$ in index form.
- 3 Calculate 2×10^2 .
- 4 Calculate $5 \times 10^2 + 9 \times 10^1 + 7$.
- 5 What is the square of 5?
- 6 What is the cube root of 8?
- 7 Change $\frac{7}{4}$ to a mixed number.
- 8 Change $\frac{7}{10}$ to a percentage.
- 9 Calculate: 14×15 .
- 10 I buy a loaf of bread for \$3.60 with a \$5 note, how much change?

$$\begin{aligned} 14 \times 15 &= 7 \times 2 \times 15 \\ &= 7 \times 30 \\ &= 210 \end{aligned}$$

Exercise 11.14

- 1 Spell factor.
- 2 Write $2 \times 2 \times 2 \times 2$ in index form.
- 3 Calculate 6×10^1 .
- 4 Calculate $3 \times 10^2 + 8 \times 10^1 + 5$.
- 5 What is the cube of 3?
- 6 What is the square root of 36?
- 7 Change $\frac{5}{2}$ to a mixed number.
- 8 Change $\frac{4}{10}$ to a percentage.
- 9 Calculate: 16×15 .
- 10 I buy a carton of milk for \$3.80 with a \$5 note, how much change?

If at first you don't succeed,
you're like everyone else
who went on to greatness -
Patrick Combs.

Exercise 11.15

- 1 Spell composite.
- 2 Write $5 \times 5 \times 5$ in index form.
- 3 Calculate 4×10^2 .
- 4 Calculate $6 \times 10^2 + 7 \times 10^1 + 1$.
- 5 What is the square of 7?
- 6 What is the cube root of 27?
- 7 Change $\frac{7}{2}$ to a mixed number.
- 8 Change $\frac{2}{5}$ to a percentage.
- 9 Calculate: 18×15 .
- 10 I buy a loaf of bread for \$3.20 with a \$10 note, how much change?

If at first you don't succeed,
call in the artillery
- Murphy's Laws of Combat.





Exercise 11.16

1 Calculate each of the following:

- | | |
|--|--------------------------|
| a) $4 \times 4 \times 4$ | b) $2 \times 2 \times 2$ |
| c) 2^3 | d) 2^4 |
| e) $2^2 + 2^3$ | f) $2^2 + 3^2$ |
| g) $2^3 - 2^2$ | h) $3^3 - 3^2$ |
| i) 10^2 | j) 10^3 |
| k) $5 \times 10^2 + 9 \times 10^1 + 2$ | |
| l) $3 \times 10^3 + 4 \times 10^2 + 7 \times 10^1 + 6$ | |
| m) $7 \times 10^4 + 2 \times 10^3 + 0 \times 10^2 + 8 \times 10^1 + 1$ | |

$$\begin{aligned}
 &7 \times 10^2 + 4 \times 10^1 + 1 \\
 &= 7 \times 100 + 4 \times 10 + 1 \\
 &= 700 + 40 + 1 \\
 &= \underline{741}
 \end{aligned}$$

2 Which of the following numbers is a multiple of both 3 and 7?

- a) 37 b) 43 c) 77 d) 777

3 Which of the following numbers has 4, 6, and 8 as factors?

- a) 12 b) 18 c) 24 d) 30



Who am I?

I am the only prime number between 90 and 100.

4 Which is not a factor of 24?

- a) 3 b) 6 c) 8 d) 14

5 a) $\boxed{3} \times \boxed{?} = \boxed{9} \times \boxed{2}$ b) $\boxed{4} \times \boxed{?} = \boxed{8} \times \boxed{3}$

c) $\boxed{2} \times \boxed{?} = \boxed{8} \times \boxed{9}$ d) $\boxed{3} \times \boxed{?} = \boxed{9} \times \boxed{9}$

6 5^3 is the same as:

- a) 5×3 b) $5 \times 5 \times 5$ c) $5 + 5 + 5$ d) $3 \times 3 \times 3 \times 3$

7 Which is the same as $2^2 \times 2^3$

- a) $2 \times 2 \times 2$ b) $2 \times 2 \times 2 \times 2 \times 2$ c) $2 \times 2 \times 2 \times 2 \times 2$

8 Which is the same as $3^2 \times 3^2$

- a) $3 \times 3 \times 3$ b) $3 \times 3 \times 3 \times 3$ c) $3 \times 3 \times 3 \times 3 \times 3$

Instrument Fitters assemble and install instruments that measure, indicate, transmit, record, and control.

- Relevant school subjects are Mathematics and English.
- Courses usually involve an instrumentation apprenticeship.

Competition Questions

Exercise 11.17

1 Find the value of each of the following:

a) $(0.1)^2$

b) $(0.2)^2$

c) $(0.3)^3$

d) $(0.03)^2$

e) $10 + 10^2 + 10^3$

f) $10 + 10^2 + 10^3 + 10^4 + 10^5$

g) $2 + 2^2 + 2^3 + 2^4$

$$\begin{aligned}(0.05)^2 &= 0.05 \times 0.05 \\ &= \underline{0.0025} \text{ \{4 decimal places\}}\end{aligned}$$



Build maths muscle and prepare for mathematics competitions at the same time.

2 In how many ways can the following numbers be written as the sum of two primes?

18

Make a list {Prime numbers less than 18 are: 2,3,5,7,11,13,17}

$18 = 5 + 13$

$18 = 7 + 11$ {There don't seem to be anymore}

18 can be written as the sum of two primes in two ways.

a) 24

b) 25

3 In how many ways can 18 be written as the sum of three primes?

4 What is the sum of the primes between 10 and 20?

5 The years 1991 and 2002 are both palindromes.

How many years from 2002 to the next palindrome year?

6 How many two digit numbers are divisible by 2?

7 How many two-digit numbers are divisible by 4?

8 How many two-digit numbers are divisible by 8?

9 How many two-digit numbers are divisible by 3?

10 How many two-digit numbers are divisible by 6?

11 If n is a positive whole number, which of the following are odd for every value of n ?

a) $3n$

b) $n + 4$

c) $2n + 1$

d) n^2

e) $2n^2 + 1$

f) $n^2 - 1$

A palindrome is the same either forwards or backwards eg., 1881

Q. If you were running a race, and you passed the person in 2nd place, what place would you then be in?

A. You would be in 2nd place.

Investigations

Investigation 11.1

With the use of a calculator, devise a strategy to decide whether the following numbers are prime numbers:

- a) 321 b) 4321
c) 54 321 d) 654 321



An initial strategy may be to check if the number is divisible by 2, 3, 5, 7, 11,

Investigation 11.2 A shortcut for squaring numbers?

Complete the pattern:

$2^2 = 3 \times 1 + 1 = 4$	$10^2 = ? \times ? + 1 = ?$
$3^2 = 4 \times 2 + 1 = 9$	$14^2 = ? \times ? + ? = ?$
$4^2 = 5 \times 3 + 1 = 16$	$19^2 = ? \times ? + ? = ?$
$5^2 = ? \times ? + 1 = 25$	$50^2 = ? \times ? + ? = ?$
$6^2 = ? \times ? + 1 = ?$	
$7^2 = ? \times ? + ? = ?$	
$8^2 = ? \times ? + ? = ?$	
$9^2 = ? \times ? + ? = ?$	

What larger numbers would be easy to square using this method?

Investigation 11.3 A shortcut for squaring numbers ending in 5?

Complete the pattern:

$$\begin{aligned}
 15^2 &= 10 \times 20 + 25 = 225 \\
 25^2 &= 20 \times 30 + 25 = 625 \\
 35^2 &= ?0 \times ?0 + 25 = ? \\
 65^2 &= ?0 \times ?0 + ?? = ? \\
 85^2 &= ?? \times ?? + ?? = ? \\
 105^2 &=
 \end{aligned}$$

Investigation 11.4 Other squared number patterns

Complete the pattern:

$1^2 = 1$	$101^2 = 10201$
$11^2 = 121$	$1001^2 = 1002001$
$111^2 = 12321$	$10001^2 = ?$
$1111^2 = ?$	$100001^2 = ?$
$11111^2 = ?$	$1000001^2 = ?$
$111111^2 = ?$	$10000001^2 = ?$
$1111111^2 = ?$	$100000001^2 = ?$

A Couple of Puzzles

Exercise 11.18

- What is the largest two digit prime number in which each digit is a prime number?
- a)

Who am I?

- I am a two-digit number.
- I am a prime number
- Each of my digits is prime.
- The sum of my digits is prime.

b)

Who am I?

- My numerator is a prime number between 20 and 28.
- My denominator is a perfect number between 20 and 30.

A Game

Prime Cross

- Two players take turns to use each of the numbers 0,1,2,3,4,5,6,7,8,9.
- A number cannot be used twice.
- The winner is the first player to get a line that totals to a **prime number**.

5	7	0
	4	
6		2

The line total is 11, a prime number.



A Sweet Trick

- Ask your audience to enter their favourite number from 1 to 9 in a calculator.
- Ask them to multiply by 3
- Ask them to multiply by 7
- Ask them to multiply by 11
- Ask them to multiply by 13
- Ask them to multiply by 37

$$\begin{aligned}
 &7 \\
 7 \times 3 &= 21 \\
 21 \times 7 &= 147 \\
 147 \times 11 &= 1617 \\
 1617 \times 13 &= 21021 \\
 21021 \times 37 &= 777777
 \end{aligned}$$

Wow. Lots more of their favourite number.



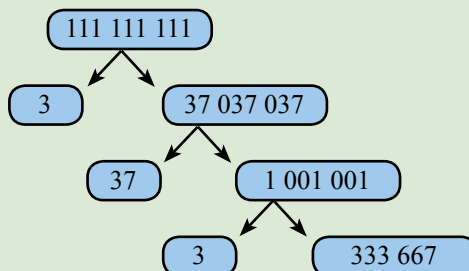
This works for every number from 1 to 9.
Why? $3 \times 7 \times 11 \times 13 \times 37 = ?$

Can you make a trick that returns 9 of their favourite number? Use an Applet to find the factors of 111111111.

Technology

Technology 11.1 Prime Factors

- 1 Use the phrase "prime factor applet" to search the Internet for one of the many prime factor applets.
- 2 Use the applet to find the answers to Exercise 11.9.
- 3 Use the applet to find the prime factors of 111 111 111 as suggested in the Sweet Trick.
- 4 Can you find the factors of 111 111 111 111?



Technology 11.2 Factor Game

- 1 Use the phrase "factor game" to find one of the many factor games on the Internet.
- 2 Play the Factor Game.
- 3 Some sites will allow you to play other players from around the world.

The Factor Game					
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

Technology 11.3 Highest Common Factor

- 1 Use the phrase "highest common factor" to find one of the factor applets on the Internet.
- 2 Use the applet to find the answers to Exercise 11.6.

Highest common factor is also known as the greatest common factor.

Technology 11.4

Use a spreadsheet to calculate powers of the numbers 1 to 10. You may wish to make a graph.

	Square	Cube	Fourth	Fifth
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	
5	25	125		
6				
7				
8				
9				
10				

= a2^5

The ^ is the power

Chapter Review 1

Exercise 11.19

1 Write the following in index form:

- | | | |
|--|--|---|
| a) $3 \times 3 \times 3$ | b) $2 \times 2 \times 2 \times 2 \times 2$ | c) 10×10 |
| d) $10 \times 10 \times 10$ | e) $10 \times 10 \times 10 \times 10$ | f) $10 \times 10 \times 10 \times 10 \times 10$ |
| g) $5 \times 5 \times 5 \times 5 \times 5$ | h) $7 \times 7 \times 7 \times 7$ | i) $3 \times 3 \times 3 \times 3 \times 3 \times 3$ |

2 Write the following in factor form:

- | | | |
|-----------|-----------|-----------|
| a) 5^3 | b) 3^4 | c) 3^2 |
| d) 10^1 | e) 10^2 | f) 10^3 |
| g) 10^4 | h) 10^5 | i) 1^4 |

3 Calculate each of the following:

- | | | |
|--|--|--|
| a) 2×10^2 | b) 5×10^3 | c) 4×10^1 |
| d) $6 \times 10^1 + 8$ | e) $9 \times 10^1 + 7$ | f) $6 \times 10^2 + 1 \times 10^1 + 5$ |
| g) $7 \times 10^2 + 9 \times 10^1 + 2$ | h) $6 \times 10^2 + 6 \times 10^1 + 9$ | i) $8 \times 10^3 + 1 \times 10^2 + 0$ |

$$\begin{aligned} 7 \times 10^2 + 4 \times 10^1 + 1 &= 7 \times 100 + 4 \times 10 + 1 \\ &= 700 + 40 + 1 = 741 \end{aligned}$$

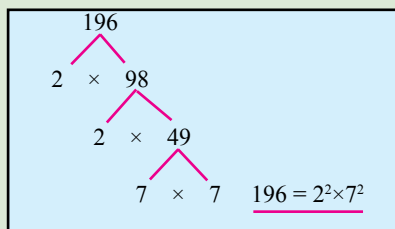
4 Find all the factors of each of the following numbers:

- | | | |
|-------|-------|-------|
| a) 2 | b) 6 | c) 8 |
| d) 10 | e) 15 | f) 20 |
| g) 24 | h) 25 | i) 30 |

5 Find the highest common factor of each of the following pairs of numbers

- | | | |
|--------------|--------------|--------------|
| a) 8 and 10 | b) 14 and 16 | c) 8 and 20 |
| d) 15 and 25 | e) 16 and 24 | f) 12 and 21 |
| g) 24 and 32 | h) 28 and 35 | i) 18 and 30 |

Factors of 12 = 1, 2, 3, 4, 6, 12
 Factors of 18 = 1, 2, 3, 6, 9, 18
 The highest common factor is 6



6 Use factor trees to find the prime factors of each of the following numbers:

- | | | |
|---------|---------|---------|
| a) 12 | b) 15 | c) 16 |
| d) 18 | e) 20 | f) 28 |
| g) 1000 | h) 2000 | i) 6000 |

7 Find the square of 2

8 Find the cube of 2

9 Find the square of 3

10 Find the cube of 3

11 What is the square root of 9?

12 What is the cube root of 8?

13 What is the square root of 16?

14 What is the cube root of 27?

15 Use a factor tree to find $\sqrt{400}$

16 Use a factor tree to find $\sqrt{144}$

Chapter Review 2

Exercise 11.20

1 Write the following in index form:

a) 2×2

b) $3 \times 3 \times 3$

c) 7×7

d) 10×10

e) $10 \times 10 \times 10$

f) $10 \times 10 \times 10 \times 10 \times 10$

g) $8 \times 8 \times 8 \times 8$

h) $6 \times 6 \times 6$

i) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

2 Write the following in factor form:

a) 2^3

b) 3^2

c) 4^5

d) 10^1

e) 10^2

f) 10^3

g) 10^4

h) 10^5

i) 1^7

3 Calculate each of the following:

a) 5×10^1

b) 3×10^2

c) 7×10^3

d) $2 \times 10^1 + 4$

e) $5 \times 10^1 + 3$

f) $9 \times 10^2 + 1 \times 10^1 + 0$

g) $8 \times 10^2 + 1 \times 10^1 + 3$

h) $3 \times 10^2 + 0 \times 10^1 + 4$

i) $4 \times 10^3 + 4 \times 10^2 + 4$

$$\begin{aligned} 7 \times 10^2 + 4 \times 10^1 + 1 &= 7 \times 100 + 4 \times 10 + 1 \\ &= 700 + 40 + 1 = \underline{741} \end{aligned}$$

4 Find all the factors of each of the following numbers:

a) 3

b) 4

c) 5

d) 6

e) 10

f) 12

g) 18

h) 27

i) 40

5 Find the highest common factor of each of the following pairs of numbers

a) 6 and 8

b) 12 and 16

c) 10 and 30

d) 20 and 24

e) 15 and 30

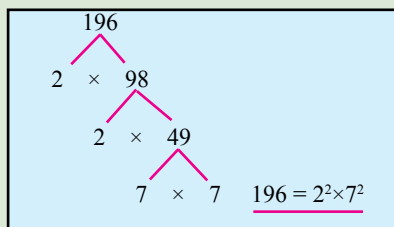
f) 16 and 24

g) 25 and 30

h) 24 and 36

i) 40 and 64

Factors of 12 = 1, 2, 3, 4, **6**, 12
 Factors of 18 = 1, 2, 3, **6**, 9, 18
 The highest common factor is 6



6 Use factor trees to find the prime factors of each of the following numbers:

a) 10

b) 12

c) 18

d) 20

e) 24

f) 30

g) 500

h) 1000

i) 5000

7 Find the square of 2

8 Find the cube of 2

9 Find the square of 5

10 Find the cube of 5

11 What is the square root of 25?

12 What is the cube root of 8?

13 What is the square root of 36?

14 What is the cube root of 125?

15 Use a factor tree to find $\sqrt{225}$

16 Use a factor tree to find $\sqrt{324}$

Linear Equations

12

- ★ Solve equations (use the balance model and explain the need to do the same thing to each side of the equation).
- ★ Use strategies such as backtracking and guess, check and improve to solve equations.
- ★ Use substitution to check solutions.
- ★ Solve real life problems.
- ★ Create linear relationships to represent realistic situations.



Conceptions

Advertising promotions with creative and artistic solutions.

A TASK

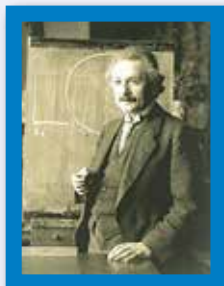
Your advertising company, Conceptions, has just been awarded a contract to conduct a two week advertising campaign promoting Algebra in the local high school.

Brainstorm ideas and put them in action.

A LITTLE BIT OF HISTORY

Albert Einstein (1879-1955) graduated as a maths and physics teacher in 1900.

- In 1905 he wrote three papers: the theory of relativity; mass and energy equivalence; statistical mechanics.
- He earned a PhD in 1905.
- He won the Nobel prize in 1921.



$E = mc^2$ is one of Albert Einstein's famous equations.

E is the energy in joules.

m is the mass in kilograms.

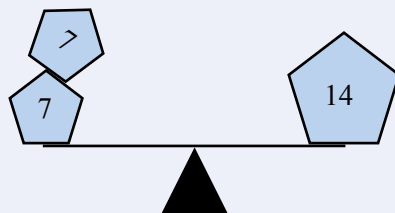
c is the speed of light.

If it were possible to convert the hydrogen (30 g) in a kilogram of water into energy, the amount of energy released would be:

$$\begin{aligned}\text{Energy} &= 0.03 \times 300\,000\,000 \times 300\,000\,000 \text{ joules} \\ &= 2\,700\,000\,000\,000\,000 \text{ joules} \\ &= \underline{2\,700\,000\,000\,000 \text{ kilojoules}}\end{aligned}$$

Word Equations

An equation is a mathematical sentence in which two expressions are equal.



Exercise 12.1

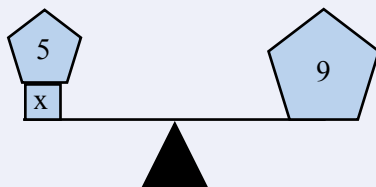
Use any method you wish to find the value of the number in each of the following word equations:

When a number is added to five the answer is nine.

Guess that the number is 4.

Check: $4 + 5 = 9$

The number is 4



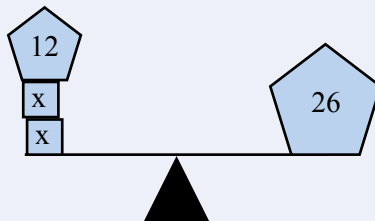
- 1 When a number is added to seven the answer is eleven.
- 2 When a number is multiplied by five the answer is ten.
- 3 When a number is divided by three the answer is eight.
- 4 When four is subtracted from a number the answer is seventeen.
- 5 When twenty-two is increased by a number the answer is thirty-six.
- 6 Eleven more than a number is fifty-nine.
- 7 When a number is tripled the answer is thirty.
- 8 When a number is decreased by fifteen the result is sixteen.
- 9 When a number is halved, twenty-three is the result.
- 10 Twenty-nine less than a number is ten.

When a number is doubled and then twelve added the answer is twenty-six.

Guess that the number is 6.

Check: $6 \times 2 + 12 = 24$

The number is 7



- 11 Nine more than twice the number is twenty-one.
- 12 A number is halved and then subtracted from fifteen to give a result of six.
- 13 When a number is added to thirteen and the result multiplied by four the answer is one hundred and thirty-six.
- 14 A number is added to the sum of forty and thirty-six and the result divided by three to give the answer of thirty-five.

Symbolic Equations

When equations are written with symbols the equations become easier to solve, especially complicated equations.

Hundreds of thousands of problems are solved every day throughout Australasia by a problem being written as a symbolic equation and then by solving the equation.

Exercise 12.2

Rewrite each of the following word equations as symbolic equations:

Word equation	Symbolic equation
When a number is added to five the answer is nine.	$x + 5 = 9$
When a number is divided by six the result is four.	$x \div 6 = 4$
When a number is multiplied by two the result is eight.	$2x = 8$
A number decreased by six is equal to fifteen.	$x - 6 = 15$
A quarter of a number is seven.	$x \div 4 = 7$

- When a number is added to ten the result is fifteen.
- When seven is subtracted from a number the answer is nineteen.
- When a number is divided by eight the answer is nine.
- When a number is multiplied by five the answer is twenty-five.
- Twelve less than a number is twenty-one.
- When a number is decreased by sixteen the answer is thirty-seven.
- Seven more than a number is eleven.
- When a number is divided by four the answer is seven.
- Increase a number by fourteen and the result is forty-nine.
- When a number is tripled the result is twenty-seven.
- When a number is halved the answer is seventeen.
- When a number is subtracted from twenty the answer is two.
- When forty-five is divided by a number the result is five.
- When a number is added to seventy-three the answer is ninety-nine.
- Divide a number by six and the result is seven.

Balancing Equations

Solve equations:
Find the value of the unknown.

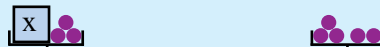


Keep the balance:
Do the same to both sides.

Exercise 12.3

Use the balance method to solve the following equations:

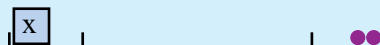
$$x + 3 = 5$$



Subtract 3 from each side

$$x + 3 - 3 = 5 - 3$$

$$\underline{x = 2}$$



1 $x + 2 = 5$

2 $x + 4 = 7$

3 $x + 3 = 8$

4 $a + 5 = 9$

5 $a + 1 = 4$

6 $b + 7 = 10$

7 $y + 4 = 6$

8 $b + 2 = 13$

9 $x + 9 = 13$

10 $x + 17 = 26$

11 $x + 21 = 30$

12 $x + 56 = 73$

13 $x + 7 = 9$

14 $x + 4 = 2$

15 $x + 8 = 3$

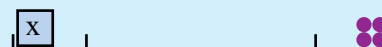
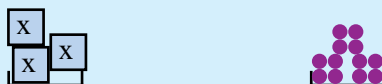
$$3x = 12$$

As your skills develop so will
your ability to solve much
more difficult problems.

Divide both sides by 3

$$3x \div 3 = 12 \div 3$$

$$\underline{x = 4}$$



16 $3x = 9$

17 $2x = 6$

18 $4x = 12$

19 $6x = 18$

20 $4x = 24$

21 $9x = 36$

22 $5x = 14$

23 $8x = 20$

24 $3x = 15$

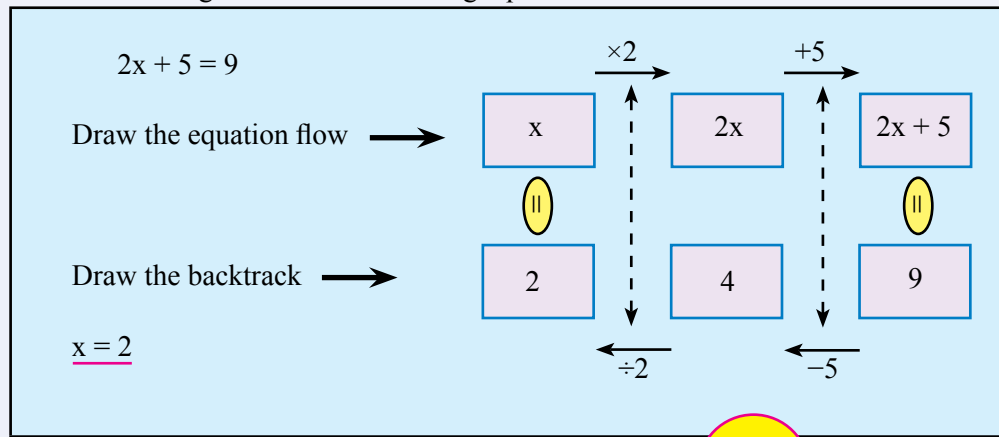
Backtracking Equations

Solve equations:
Find the value of the unknown.

- 1 Write the flow of the equation.
- 2 Backtrack (inverse operations).

Exercise 12.4

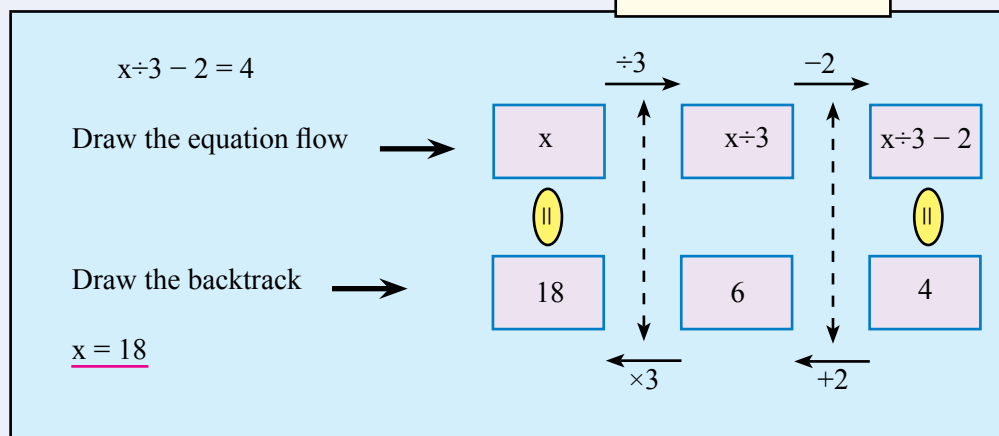
Use backtracking to solve the following equations:



- | | |
|------------------|-----------------|
| 1 $2x + 5 = 7$ | 2 $2x + 7 = 11$ |
| 3 $3x + 4 = 13$ | 4 $3x + 1 = 10$ |
| 5 $4a + 2 = 30$ | 6 $5b + 7 = 22$ |
| 7 $2x + 24 = 28$ | 8 $3x + 8 = 5$ |

Inverse of

+	is	-
-	is	+
\times	is	\div
\div	is	\times



- | | | |
|-----------------------|-----------------------|------------------------|
| 9 $x \div 2 - 5 = 3$ | 10 $x \div 5 - 2 = 8$ | 11 $x \div 4 + 3 = 19$ |
| 12 $5x - 2 = 18$ | 13 $x \div 2 - 7 = 9$ | 14 $6x + 3 = 21$ |
| 15 $x \div 7 - 5 = 2$ | 16 $x \div 5 + 3 = 8$ | 17 $6x + 43 = 79$ |

Problem Solving

Solve equations:
Find the value of the unknown.

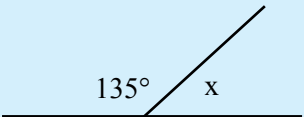
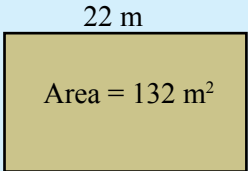
The inverse of $+$ is $-$
The inverse of $-$ is $+$
The inverse of \times is \div
The inverse of \div is \times

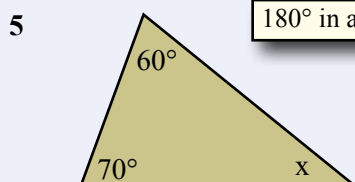
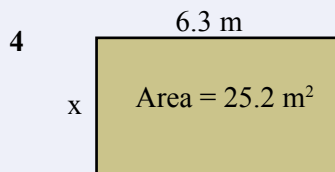
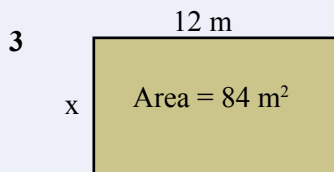
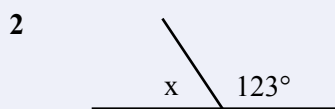
Exercise 12.5

For each of the following problems:

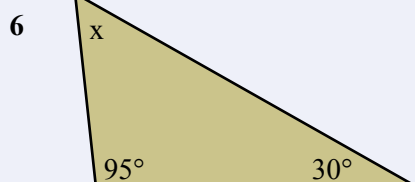
- Write an equation.
- Solve the equation.
- Check the answer.

Substitute your answer into the equation.

	
<p>a) Sum angles straight line = 180° $x + 135 = 180$</p> <p>b) $x + 135 = 180$ $x = 180 - 135$ {Inverse of $+$} $x = 45^\circ$</p> <p>c) Substitute $x = 45$ into $x + 135 = 180$ $x + 135$ $= 45 + 135$ $= 180$ ✓</p>	<p>a) Area = lb $132 = 22x$</p> <p>b) $22x = 132$ $x = 132 \div 22$ {Inverse of \times} $x = 6$ m</p> <p>c) Substitute $x = 6$ into $22x = 132$ $22x$ $= 22 \times 6$ $= 132$ ✓</p>



180° in a triangle.



Problem Solving

Solve equations:
Find the value of the unknown.

The inverse of $+$ is $-$
The inverse of $-$ is $+$
The inverse of \times is \div
The inverse of \div is \times

Exercise 12.6

For each of the following problems:

- a) Write an equation.
- b) Solve the equation.
- c) Check the answer.

Substitute your answer into the equation.

Noah is paid \$110 for the day and \$5 for each box that he makes. How many boxes will Noah have to make so that he earns a total of \$700 for the day.

- a) \$5 per box $\blacktriangleright 5x$
plus \$110 $\blacktriangleright 5x+110$
 $5x + 110 = 700$
- b) $5x + 110 = 700$
 $5x = 700 - 110$ {Inverse of $+$ }
 $5x = 590$
 $x = 590 \div 5$ {Inverse of \times }
 $x = 118$ boxes

- c) Substitute $x = 118$ into $5x+110=700$
 $5 \times 118 + 110$
 $= 590 + 110$
 $= 700$ ✓

Emily will buy a milk carton for \$3.25 and spend the rest of her \$10 on chocolate bars at \$0.75 each. How many chocolate bars can she buy?

- a) \$0.75 per bar $\blacktriangleright 0.75x$
plus \$3.25 $\blacktriangleright 0.75x+3.25$
 $0.75x + 3.25 = 10$
- b) $0.75x + 3.25 = 10$
 $0.75x = 10 - 3.25$ {Inverse of $+$ }
 $0.75x = 6.75$
 $x = 6.75 \div 0.75$ {Inverse of \times }
 $x = 9$ chocolate bars

- c) Substitute $x = 9$ into $0.75x+3.25=10$
 $0.75 \times 9 + 3.25$
 $= 6.75 + 3.25$
 $= 10$ ✓

- Chloe is paid \$100 for the day and \$15 for each box that she makes. How many boxes will Chloe have to make so that she earns a total of \$400 for the day.
- William is paid \$150 for the week and \$45 for each sale that he makes. How many sales will William have to make so that he earns a total of \$510 for the week.
- Charlotte will buy a milk carton for \$3.60 and spend the rest of her \$20 on tomatoes at \$4.20 per kilogram. How many kilograms of tomatoes will she buy?
- Riley will buy a cubic metre of gravel for \$72 and spend the rest of his \$100 on cement at \$8.50 per bag. How many bags of cement will he buy?
- The total mass of the truck must not exceed 6.8 tonnes. If the truck has a mass of 3.4 tonnes, how many cartons, each of mass 0.2 tonnes, can be placed on the truck?

Solving Word Problems

Solve equations:
Find the value of the unknown.

The inverse of $+$ is $-$
 The inverse of $-$ is $+$
 The inverse of \times is \div
 The inverse of \div is \times

Exercise 12.7

For each of the following problems:

- a) Write an equation.
- b) Solve the equation.
- c) Check the answer.

Substitute your answer into the equation.

<p>If a printer charges an upfront \$45 and then \$0.15 per flyer, how many flyers can be printed for \$500?</p> <p>a) \$0.15 per flyer \blacktriangleright $0.15x$ plus \$45 \blacktriangleright $0.15x + 45$ <u>$0.15x + 45 = 500$</u></p>	<p>b) $0.15x + 45 = 500$ $0.15x = 500 - 45$ {Inverse of $+$} $0.15x = 455$ $x = 455 \div 0.15$ {Inverse of \times} <u>$x = 3033$ flyers</u></p> <p>c) Substitute $x = 3033$ into $0.15x + 45 = 500$ $0.15 \times 3033 + 45$ $= 454.95 + 45$ <u>$= \\$499.95$</u> ✓</p>
---	---

- If a printer charges an upfront \$60 and then \$0.11 per flyer, how many flyers can be printed for \$500?
- If a printer charges an upfront \$3500 and then \$6.85 per book, how many books can be printed for \$20 000?
- The plumber charges \$85 callout and then \$75 per hour. How many hours of work will the plumber put in for \$500?
- The electrician charges \$90 callout and then \$70 per hour. How many hours of work will the electrician put in for \$500?
- The taxi charges \$3.00 flagfall and then \$3 per kilometre. How far will the taxi travel for \$50?
- The taxi charges \$3.50 flagfall and then \$2.80 per kilometre. How far will the taxi travel for \$50?
- The Internet phone company charges \$0.10 connection fee and then \$0.15 per minute. How long will a \$5 call last?
- The mobile phone company charges \$0.39 flagfall and then \$0.98 per minute. How long will a \$5 call last?
- It appears that crickets chirp faster during warmer weather. Suppose that the frequency of the chirping, chirps per minute, is seven times the temperature, $^{\circ}\text{C}$, minus 30. What is the temperature if a cricket is chirping at 150 chirps per minute?

Mental Computation

Mental computation can make problems easier and quicker.

Exercise 12.8

- 1 Spell equation.
- 2 Solve: $5x = 15$
- 3 Write in index form: $3 \times 3 \times 3 \times 3$
- 4 Solve: $3x + 2 = 11$
- 5 Simplify: $m^2 \times m^3$
- 6 Simplify: $2^8 \div 2^6$
- 7 Simplify: 5^0
- 8 A taxi charges \$3 flagfall and \$2.50 per km. What is the function?
- 9 Change $3\frac{1}{2}$ to a vulgar fraction.
- 10 $9 + ^{-}12$

Exercise 12.9

- 1 Spell linear.
- 2 Solve: $x \div 4 = 3$
- 3 Write in index form: $2 \times 2 \times 2 \times 2 \times 2$
- 4 Solve: $2x - 5 = 3$
- 5 Simplify: $b^2 \times b^4$
- 6 Simplify: $5^5 \div 5^3$
- 7 Simplify: 4^0
- 8 A call costs 25 cents flagfall and 28 cents per minute. What is the function?
- 9 Change $2\frac{2}{3}$ to a vulgar fraction.
- 10 $^{-}3 + ^{-}4$

Exercise 12.10

- 1 Spell function.
- 2 Solve: $x - 3 = 6$
- 3 Write in index form: $5 \times 5 \times 5 \times 5 \times 5$
- 4 Solve: $4x - 2 = 6$
- 5 Simplify: $d^2 \times d^3$
- 6 Simplify: $3^6 \div 3^3$
- 7 Simplify: 7^0
- 8 The plumber charges \$70 callout and \$80 per hour. What is the function?
- 9 Change $2\frac{3}{4}$ to a vulgar fraction.
- 10 $^{-}5 - ^{-}3$

Success seems to be largely a matter of hanging on after others have let go - William Feather.

What do you plant to grow a seedless watermelon?

Customs Brokers help importers and exporters clear their goods through customs, quarantine and administer freight movement.

- Relevant school subjects are English and Mathematics.
- Courses usually involve a customs brokers course.

NAPLAN Questions

Exercise 12.11



- 1 If $x = 3$, what is the value of $2x$?
- 2 If $x = 4$, what is the value of $5x - 3$?
- 3 $y = 15 - 3x$. What is the value of y when $x = 4.74$?
- 4 $y = 9 - 4x$. What is the value of y when $x = 3.5$?
- 5 If $g = 5$, what is the value of $\frac{2g}{(3g-10)}$?

- 6 Which rule applies to the table?

- a) $y = x$
- b) $y = 2x$
- c) $y = x^2$

x	1	2	3	4	5
y	1	4	9	16	25

- 7 What is the next number in the following pattern?

$\times 5 - 3$

→

$7, 12, 17, ?$

- 8 The time, in minutes, for medium roasting of beef in a conventional oven at 200°C is given by the function $T = 50m + 25$, where m is the mass of the roast in kilograms. What is the mass of a roast that needs 175 minutes to cook?

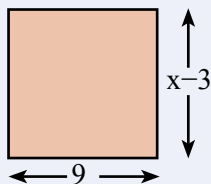


- 9 Solve each of the following:

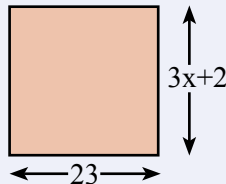
- a) When a number is doubled and then 13 is added the answer is 21.
- b) When a number is tripled and 7 subtracted, the answer is 20.

- 10 What is the value of x in each of the following squares?

a)

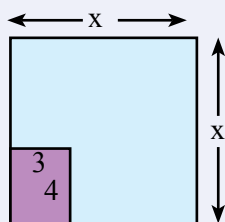


b)

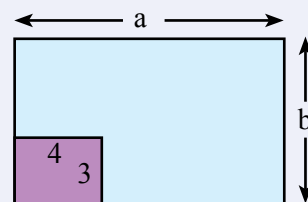


- 11 What is the area of the section shaded blue?

a)



b)



Competition Questions

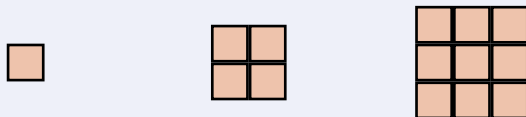
Prepare for mathematics competitions and build maths muscle at the same time.



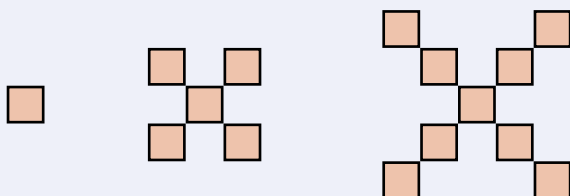
Exercise 12.12

1 How many tiles in the next step of each pattern?


a)

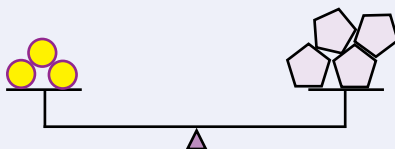


b)



2 $\frac{3}{4} \times ? - 4 = 5$

3 What is the mass of 1 .



4 For each of the following, write an equation and then solve the equation:

The perimeter of a triangle is 50 m. If each side is $2x$, $3x$, and $5x$, what is the value of x ?

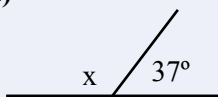
$$2x + 3x + 5x = 50$$

$$10x = 50$$

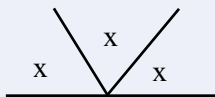
$$x = 50 \div 10$$

$$x = 5 \text{ m}$$

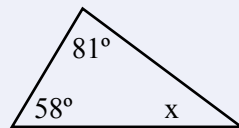
a)



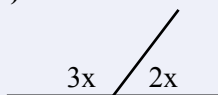
b)



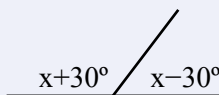
c)



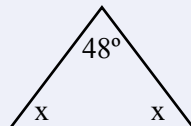
d)



e)



f)



5 The perimeter of a triangle is 50 m. If each side is $2x$, $3x$, and $5x$, what is the value of x ?

6 The three sides of an equilateral triangle are $x + 2$, $2x - 1$, and $4y$. What are the values of x and y ?

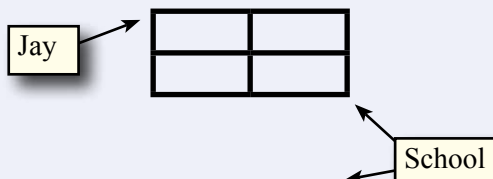
A Couple of Puzzles

Exercise 12.13

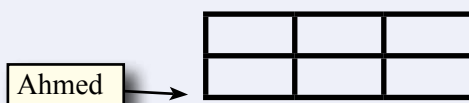
- How many dots in the fourth figure?



- In how many ways can Jay walk to school if each walk is 4 blocks?



- In how many ways can Ahmed walk to school if each walk is 5 blocks?



A Game

What's My Rule?

- The rule maker enters a rule on a calculator without the other team being able to see.
- The other team may now use the calculator but must keep it in constant mode by only pressing a number and =.
- A trial means entering a number and pressing equal:
- The other team then tries to decide 'What's my rule?'
 - The team loses a point for each trial.
 - The team wins 10 points when they 'get the rule'.
 - First team to 50 points wins.

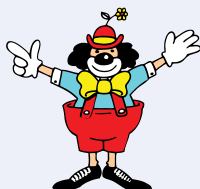
$$6 \div 4 = 1.5$$

$$12 = 3$$

The rule here is:
 $y = x \div 4$

A Sweet Trick

- Ask your audience to write any 3-digit number. 716
- Ask them to rearrange the digits anyway they like. 671
- Ask them to find the difference. $716 - 671 = 45$
- Ask them to add their age. $45 + 17 = 62$
- You tell them their age.



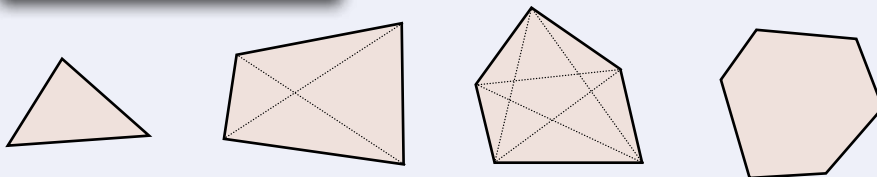
Sum the digits: $6+2 = 8$

Then keep adding 9 until you get nearest their age:
8 or 17 or 26 or 35 or 44 etc.

Investigations

Investigation 12.1 Polygon diagonals

Investigate the number of diagonals in polygons.



Polygon	Number of sides	Number of diagonals
Triangle	3	0
Quadrilateral	4	2
Pentagon	5	
Hexagon		
Heptagon		
Octagon		
Nonagon		
Decagon		

There is an equation/ formula for the number of diagonals in a polygon. Can you find it?

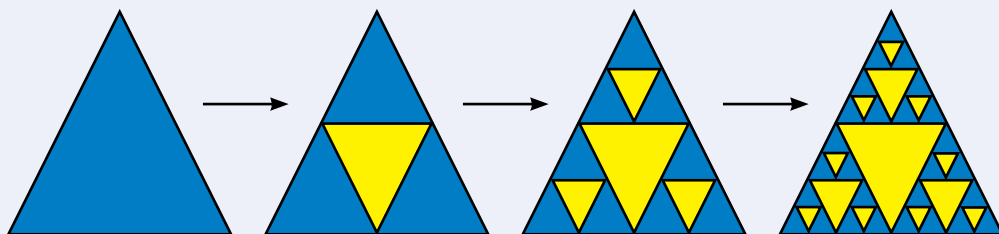
Investigation 12.2 Sierpinski Pattern

Investigate the Sierpinski Pattern.



Investigation 12.2 Sierpinski Triangle

Investigate the Sierpinski Triangle.



Technology

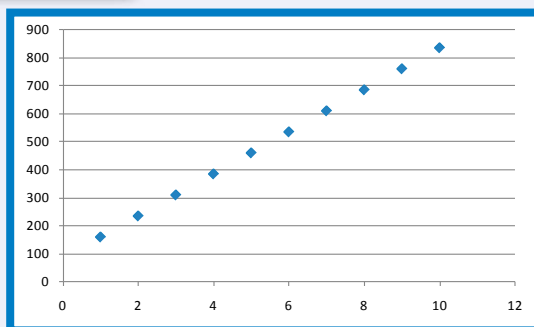
Technology 12.1 Linear equations and the spreadsheet

The plumber charges \$85 callout and charges \$75 per hour.

$$C = 75h + 85$$

Hours	Charge
1	160
2	235
3	310
4	385
5	460
6	535
7	610
8	685
9	760
10	835

Enter the formula:
 $=75*a2+85$



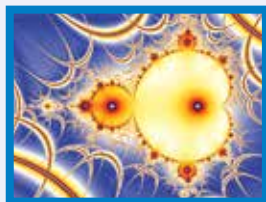
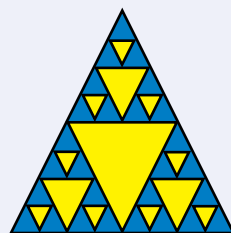
- What is the charge for 5 hours?
- If the charge is \$535, how many hours?

Technology 12.2 Online Equation Solvers

- Use a search phrase similar to 'online equation solver' to try out one of the many equation solvers.
- Enter: $2x + 7 = 15$ as a test.
- Use the equation solver to check your answers.

Technology 12.3 Sierpinski Triangle

- Use a search phrase similar to 'Sierpinski triangle fractal' to use an applet to experiment with the Sierpinski triangle.
- Use a search phrase similar to 'fractal video' to get an introduction to the wonderful world of fractals.
- Can you use a 'fractal generator' to make your own fractals?



Chapter Review 1

I have always admired the ability to bite off more than one can chew and then chew it
- William DeMille.

Exercise 12.14

- 1 Rewrite each of the following word equations as symbolic equations:
- When a number is added to thirteen the result is twenty-three.
 - When nine is subtracted from a number the answer is eleven.
 - When a number is divided by forty-two the answer is six.
 - When a number is multiplied by twenty-four the answer is seventy-two.

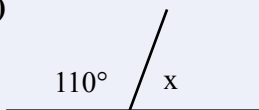
- 2 Solve each of the following equations:

- | | | |
|-------------------|------------------|------------------|
| a) $x + 2 = 5$ | b) $x - 4 = 7$ | c) $5x = 15$ |
| d) $x \div 3 = 4$ | e) $a + 6 = 4$ | f) $4b = 10$ |
| g) $2c + 4 = 6$ | h) $3d - 2 = 13$ | i) $5x + 9 = 13$ |

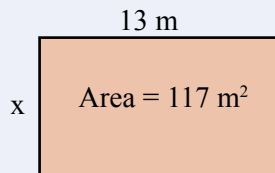
- 3 For each of the following problems:

- Write an equation.
- Solve the equation.
- Check the answer.

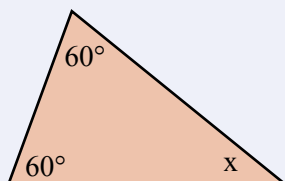
a)



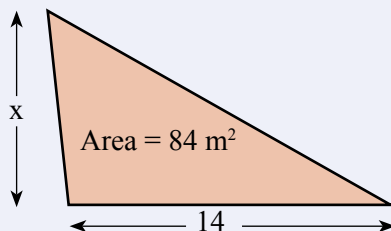
b)



c)



d)



- Tom is paid \$170 for the week and \$60 for each sale that he makes. How many sales will Tom have to make so that he earns a total of \$530 for the week.
- Britney will buy a milk carton for \$3.20 and spend the rest of her \$40 on tomatoes at \$6.50 per kilogram. How many kilograms of tomatoes will she buy?
- Harvey will buy a cubic metre of gravel for \$75 and spend the rest of his \$150 on cement at \$9.50 per bag. How many bags of cement will he buy?
- The total mass of the truck must not exceed 6.2 tonnes. If the truck has a mass of 3.8 tonnes, how many pallets, each of mass 0.3 tonnes, can be placed on the truck?
- The plumber charges \$85 callout and then \$75 per hour. How many hours of work will the plumber put in for \$700?

Chapter Review 2

A smile is a curve that sets everything straight - Phyllis Diller.

Exercise 12.15

1 Rewrite each of the following word equations as symbolic equations:

- a) When a number is added to twelve the result is seventeen.
- b) When fifteen is subtracted from a number the answer is eight.
- c) When a number is divided by sixty-one the answer is three.
- d) When a number is multiplied by seven the answer is ninety-one.

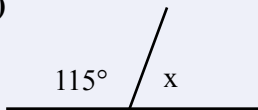
2 Solve each of the following equations:

- a) $x + 5 = 9$
- b) $x - 3 = 4$
- c) $4x = 20$
- d) $x \div 2 = 6$
- e) $a + 5 = 2$
- f) $5b = 11$
- g) $2m + 5 = 11$
- h) $3p - 4 = 14$
- i) $4x + 7 = 13$

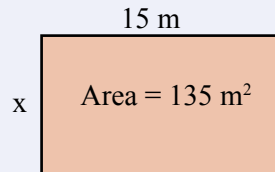
3 For each of the following problems:

- Write an equation.
- Solve the equation.
- Check the answer.

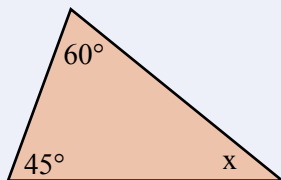
a)



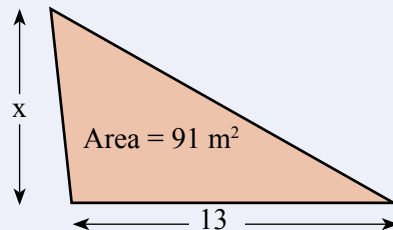
b)



c)



d)



- e) The printer charges an upfront \$35 and then \$0.15 per flyer. How many flyers can be printed for \$1800?
- f) The Internet phone company charges \$0.15 connection fee and then \$0.12 per minute. How long will a \$5 call last?
- g) Albert will buy a cubic metre of mushroom compost for \$55 and spend the rest of his \$150 on potting mix at \$5.20 per bag. How many bags of potting mix will he buy?
- h) The total mass of the truck must not exceed 7.4 tonnes. If the truck has a mass of 4.1 tonnes, how many pallets, each of mass 0.4 tonnes, can be placed on the truck?
- i) The electrician charges \$95 callout and then \$85 per hour. How many hours of work will the electrician put in for \$750?

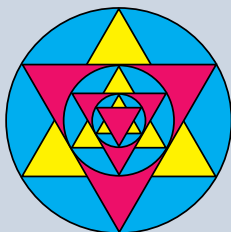
Geometry

13

- ★ Define and classifying angles such as acute, right, obtuse, straight, reflex and revolution, and pairs of angles such as complementary, supplementary, adjacent and vertically opposite.
- ★ Construct parallel and perpendicular lines using their properties, a pair of compasses and a ruler, and dynamic geometry software.
- ★ Define and identify alternate, corresponding and allied angles and the relationships between them for a pair of parallel lines cut by a transversal.
- ★ Identify side and angle properties of scalene, isosceles, right-angled and obtuse-angled triangles.
- ★ Describe squares, rectangles, rhombuses, parallelograms, kites and trapeziums.
- ★ Use concrete materials and digital technologies to investigate the angle sum of a triangle and quadrilateral.

Q How do farmers make crop circles?

A With a pro-tractor.



A TASK

Protractor Designs

- Research crop circles.
- Using just a protractor, a compass, and a ruler, create a design.

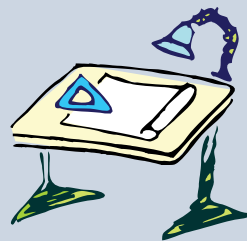


A LITTLE BIT OF HISTORY

Engineers and Designers believe that drafting is the oldest profession - designs have been on the walls of caves for thousands of years.

A design is a way to showing how to turn an idea into reality. Drafting developed as the Egyptians, 3000 BC, drew designs on papyrus and the Chinese developed a process for drawing on rice paper around 800 AD.

Computer-aided design software (CAD) has significantly enhanced the art of drafting.

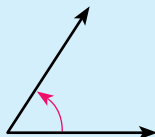


Types of Angles

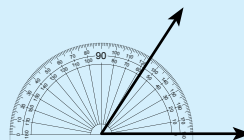
Type	Size	Diagram	Comment
acute	less than 90°		
right	90°		A right angle is a quarter turn.
obtuse	between 90° and 180°		
straight	180°		A straight angle is a half turn.
reflex	between 180° and 360°		
revolution	360°		A revolution is one complete turn.

Exercise 13.1

- a) Estimate and draw an angle of 60° .
b) Measure and name the angle with a protractor.



Place the centre over the vertex.
Match a zero line over one arm.
Measure the angle from the 0° .



The angle is 57° . It is an acute angle.

- a) Estimate and draw an angle of 60° .
b) Measure and name the angle with a protractor.
- a) Estimate and draw an angle of 90° .
b) Measure and name the angle with a protractor.
- a) Estimate and draw an angle of 45° .
b) Measure and name the angle with a protractor.
- a) Estimate and draw an angle of 120° .
b) Measure and name the angle with a protractor.
- a) Estimate and draw an angle of 200° .
b) Measure and name the angle with a protractor.
- a) Estimate and draw an angle of 300° .
b) Measure and name the angle with a protractor.
- a) Estimate and draw an angle of 30° .
b) Measure and name the angle with a protractor.

A vertex is a corner point.

Drawing Angles

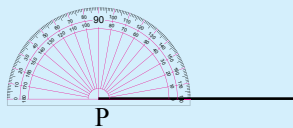
Exercise 13.2

Use a protractor to draw an obtuse angle of 135° .

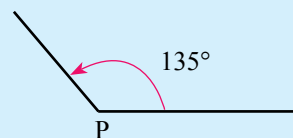
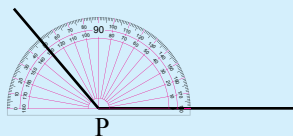
Draw an arm and mark the vertex P.



Place the centre over the vertex.
Match a zero line over the arm



Go around the protractor from 0 to 135
Mark a point and draw the second arm.



- 1 Use a protractor to draw an obtuse angle of 120° .
- 2 Use a protractor to draw an obtuse angle of 160° .
- 3 Use a protractor to draw an acute angle of 45° .
- 4 Use a protractor to draw an acute angle of 60° .
- 5 Use a protractor to draw a reflex angle of 135° .
- 6 Use a protractor to draw a reflex angle of 270° .

Exercise 13.3

Name the type of each of the following angles:

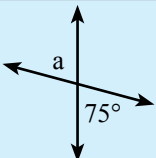
- | | | |
|----------------|----------------|----------------|
| 1 22° | 2 189° | 3 300° |
| 4 105° | 5 90° | 6 360° |
| 7 215° | 8 294° | 9 45° |
| 10 180° | 11 5° | 12 100° |
| 13 96° | 14 320° | 15 218° |

Describing Angles

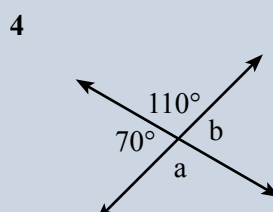
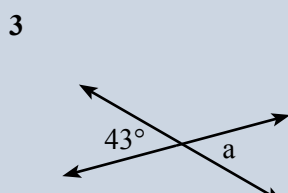
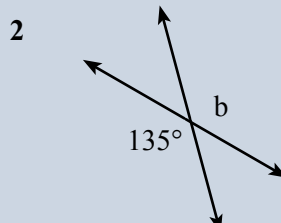
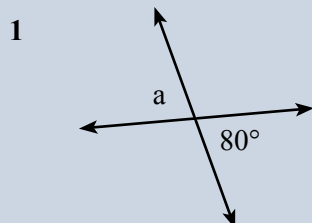
Types of angles	Comment	Diagram	
Complementary	Sum to 90° $a + b = 90^\circ$		40° and 50° are complementary angles.
Supplementary	Sum to 180° $a + b = 180^\circ$		110° and 70° are supplementary angles.
Adjacent	Next to each other. a and b are adjacent angles.		Have the same vertex. Have a common arm.
Vertically opposite	When two lines cross. $a = b$		Vertically opposite angles are equal. There are two pairs.
Angles at a point	Sum to 360° $a + b + c = 360^\circ$		The angles at a point add up to 360° .

Exercise 13.4

Find the size of the unknown angle:



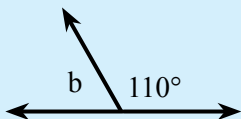
$\underline{a = 75^\circ}$ {vertically opposite angles}



Angle Problems

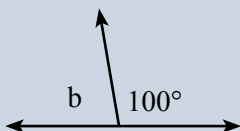
Exercise 13.5

Find the size of the unknown angle:

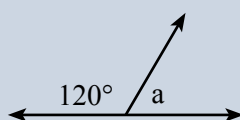


$$\begin{aligned} b + 110^\circ &= 180^\circ && \{\text{complementary angles}\} \\ b &= 180^\circ - 110^\circ \\ \text{Thus } \underline{b} &= 70^\circ \end{aligned}$$

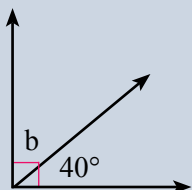
1



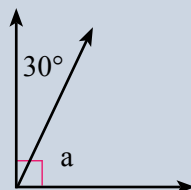
2

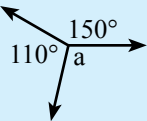


3



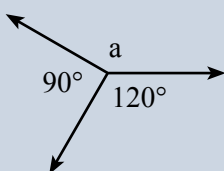
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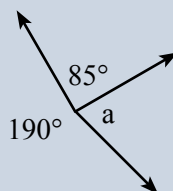


$$\begin{aligned} a + 110^\circ + 135^\circ &= 360^\circ && \{\text{Angles at a point}\} \\ a + 245^\circ &= 360^\circ \\ a &= 360^\circ - 245^\circ \\ \text{Thus } \underline{a} &= 115^\circ \end{aligned}$$

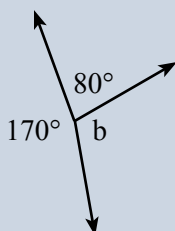
5



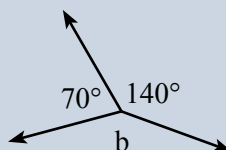
6



7



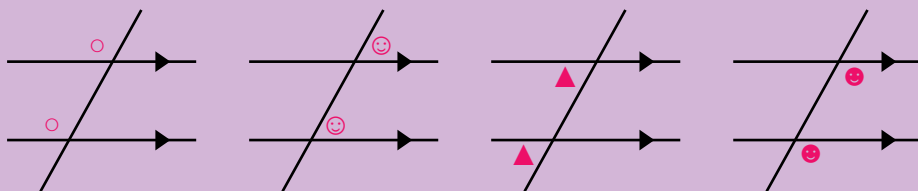
8



Corresponding Angles

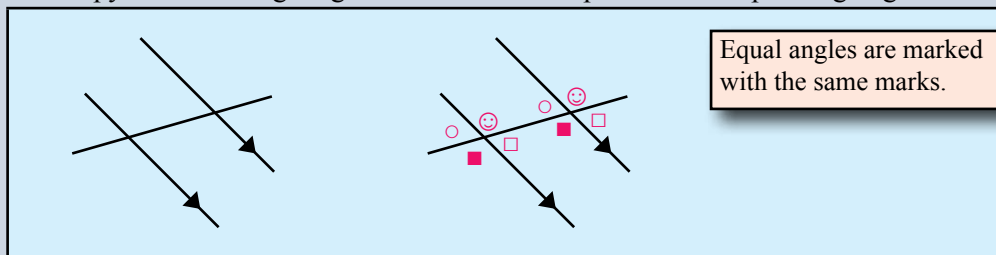
When a transversal cuts two parallel lines four pairs of corresponding angles are formed.

The corresponding angles are equal.

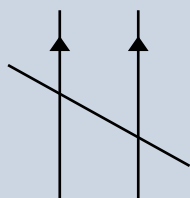


Exercise 13.6

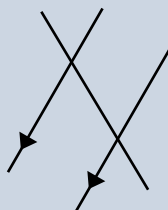
1 Copy the following diagrams and mark four pairs of corresponding angles:



a)

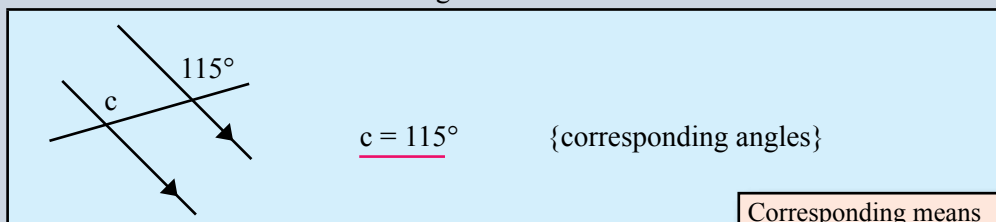


b)

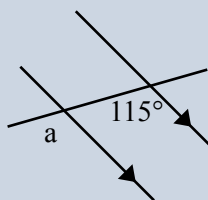


A transversal is a line that cuts other lines.

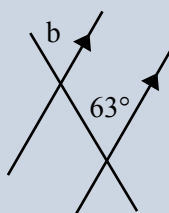
2 Find the size of the unknown angle:



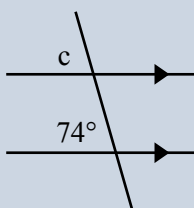
a)



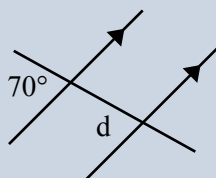
b)



c)



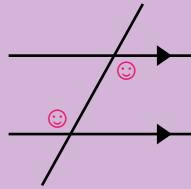
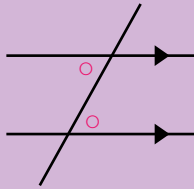
d)



Alternate Angles

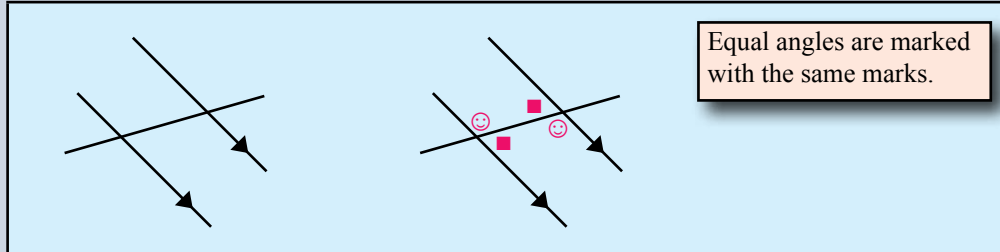
When a transversal cuts two parallel lines two pairs of alternate angles are formed.

The alternate angles are equal.

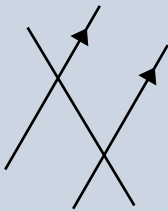


Exercise 13.7

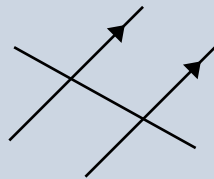
1 Copy the following diagrams and mark two pairs of alternate angles:



a)

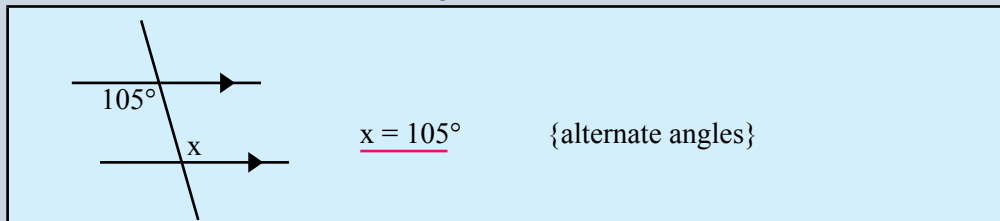


b)

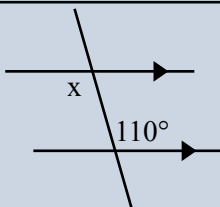


Alternate - on the other side.

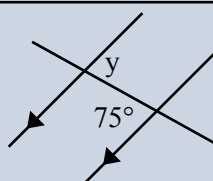
2 Find the size of the unknown angle:



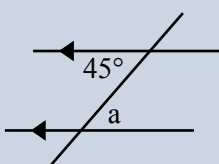
a)



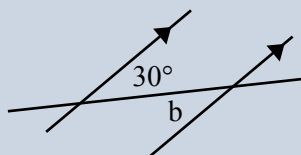
b)



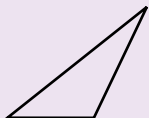

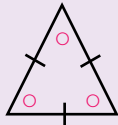
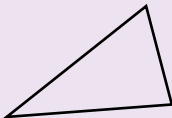
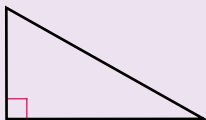
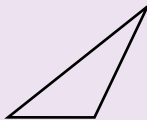
c)



d)

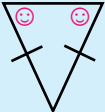
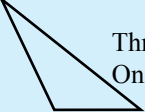



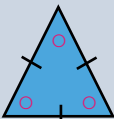
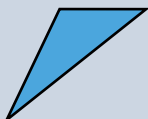
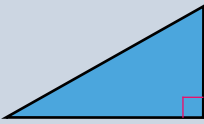
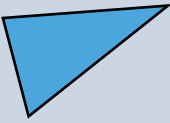
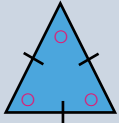
Triangles

Triangles have three sides and three angles			
Sides	Scalene	Isosceles	Equilateral
	No sides equal No angles equal 	Two sides equal Two angles equal 	Three sides equal Three angles equal 
Angles	Acute-angled	Right-angled	Obtuse-angled
	Three angles acute. Each angle less than 90° 	One angle 90° 	One angle greater 90° 

Exercise 13.8


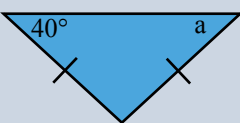
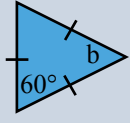
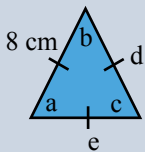
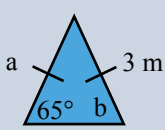
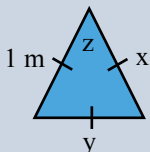
1 Classify the following triangles using the sides and the angles:

 <p>Two equal sides = isosceles Three acute angles = acute-angled</p>	 <p>Three unequal sides = scalene One obtuse angle = obtuse-angled</p>
--	---

- a) 
- b) 
- c) 
- d) 
- e) 
- f) 

2 Find the size of the unknown:

Equal sides are marked with the same marks.

- a) 
- b) 
- c) 
- d) 
- e) 
- f) 

Quadrilaterals

Quadrilaterals have four sides and four angles		
Square	Rectangle	Parallelogram
Four sides equal Opposite sides parallel All angles are right angles	Opposite sides equal Opposite sides parallel All angles are right angles	Opposite sides equal Opposite sides parallel Opposite angles equal
Rhombus	Trapezium	Kite
Four sides equal Opposite sides parallel Opposite angles equal	One pair of parallel sides	Two pairs of adjacent sides equal

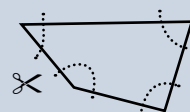
Exercise 13.9

- Draw a square, a rectangle, a parallelogram, a rhombus, a trapezium, and a kite (Instructions and/or templates on the Internet can be found with a search phrase such as 'geometry quadrilaterals').
- Copy the following table and use a ruler and a protractor to complete the table.

	Square	Rectangle	Parallelogram	Rhombus	Trapezium	Kite
Opposite sides equal	✓					
Opposite angles equal						
Opposite sides parallel						
Diagonals equal						
90° angles						

- Which quadrilateral am I?
 - My opposite sides are equal.
 - My opposite angles are equal.
 - My diagonals are equal.
 - My four angles are 90°.
 - My four sides are equal.
 - My diagonals meet at right angles.
 - At least one pair of my opposite sides are parallel.

What is the angle sum of a quadrilateral?
Cut out the corners and put them together.



A Couple of Puzzles

Exercise 13.10

- Who am I? If you multiply me by 25 and add 33 the result is 158.
- Who am I? I'm a two digit number. If you reverse our digits and add us together, the result is 99.
- Place each of the digits 0, 1, 2, 3, 4, 5 in the boxes to give the smallest two-digit answer.

—		
<hr/>		

A Game

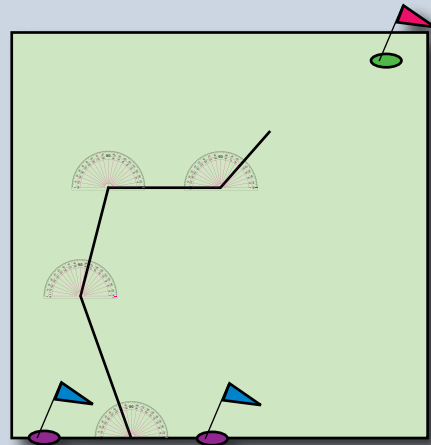
Angle Golf

- Draw a 'hole with flagpole' and a 'tee'.
- Take turns to hit the ball.
- At each turn, a player chooses an angle from the list and throws a die for the distance (5 = 5 cm).

An angle cannot be used twice.

0°	10°	20°	30°
40°	50°	60°	70°
80°	90°	100°	110°

- The winner is the person closest to the hole after 6 hits?



A Sweet Trick

- Ask your audience to enter their favourite number from 1 to 9 in a calculator.
- Ask them to multiply by 271
- Ask them to multiply by 41

$$\begin{aligned}
 &8 \\
 &8 \times 271 = 2168 \\
 &2168 \times 41 = 88888
 \end{aligned}$$

This works for every number from 1 to 9.
Why? $271 \times 41 = ?$



Wow. Lots more of their favourite number.

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 13.11

- 1 Spell obtuse.
- 2 Is a 60° angle acute, obtuse, or reflex?
- 3 How many sides are equal in an equilateral triangle?
- 4 Sketch and label a rhombus.
- 5 Solve: $2x + 5 = 11$
- 6 Simplify: $m^3 \times m^2$
- 7 $3 + ^{-}5$
- 8 A taxi charges \$4 flagfall and \$3 per km. What is the function?
- 9 16×4
- 10 Change $\frac{5}{3}$ to a mixed number.

To multiply by 4 double twice.
 $16 \times 4 = 32 \times 2 = 64$

Exercise 13.12

- 1 Spell alternate.
- 2 Is a 100° angle acute, obtuse, or reflex?
- 3 How many sides are equal in an isosceles triangle?
- 4 Sketch and label a kite.
- 5 Solve: $3x + 5 = 11$
- 6 Simplify: $x^3 \times x^4$
- 7 $4 - 7$
- 8 A taxi charges \$3 flagfall and \$2 per km. What is the function?
- 9 15×4
- 10 Change $\frac{3}{2}$ to a mixed number.

Q Why did he keep a ruler on his newspaper?

A To get the story straight.

Exercise 13.13

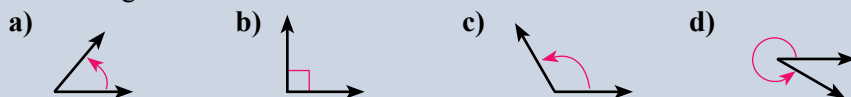
- 1 Spell acute.
- 2 Is a 200° angle acute, obtuse, or reflex?
- 3 How many sides are equal in an equilateral triangle?
- 4 Sketch and label a rhombus.
- 5 Solve: $4x + 5 = 11$
- 6 Simplify: $b^2 \times b^3$
- 7 $^{-}3 + ^{-}2$
- 8 A taxi charges \$4 flagfall and \$2 per km. What is the function?
- 9 21×4
- 10 Change $\frac{5}{2}$ to a mixed number.

It's always too soon to quit.
- Norman Vincent Peale

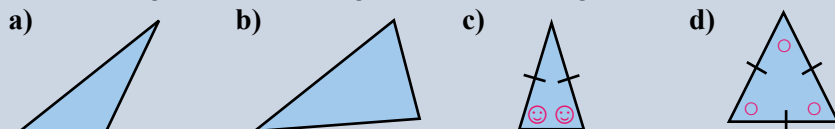


Exercise 13.14

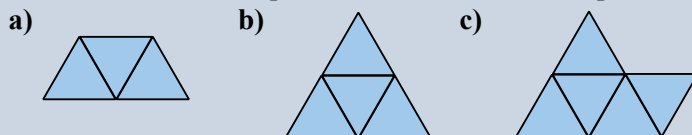
1 Which angle is closest to 120° .



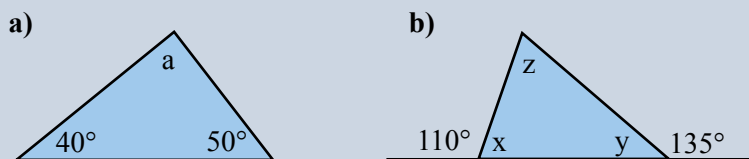
2 Which triangle is an acute-angled isosceles triangle.



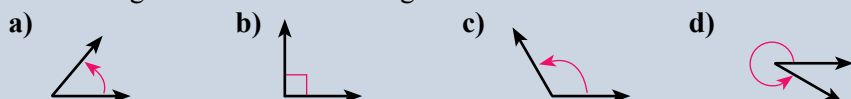
3 The following shapes are made up of identical equilateral triangles. Each side is 10 cm. What is the perimeter of each of the shapes?



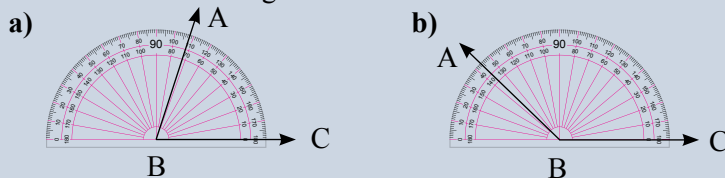
4 Find the value of the unknowns.



5 Which diagram shows a reflex angle?



6 What is the size of angle ABC?



Architectural Drafters prepare drawings and plans for architects and work with builders.

- Relevant school subjects are English and Mathematics.
- Courses generally involve Certificate IV in building design or a Diploma course.

Competition Questions

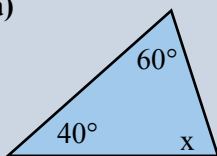


Build maths muscle and prepare for mathematics competitions at the same time.

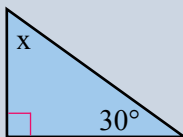
Exercise 13.15

1 Find the value of unknown in each of the following diagrams:

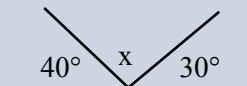
a)



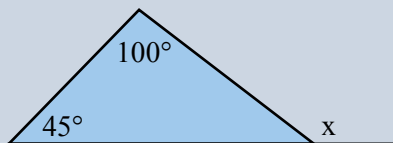
b)



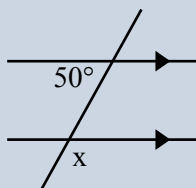
c)



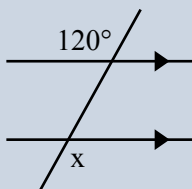
d)



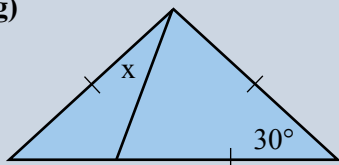
e)



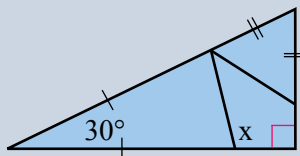
f)



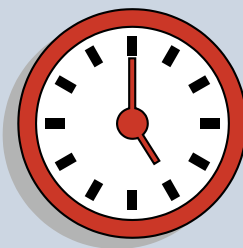
g)



h)



i) The clock shows 5 o'clock. What is the size of the smaller angle between the minute and hour hands?



Two engineers are painting a flag pole and they need to know how tall it is so they can purchase the right amount of paint. One climbs up the pole with a tape measure and falls after getting about half way. A Designer comes along, pulls the pole out of the ground and lays it down and says "Now you can measure it". The two engineers looked at each other and one said "But we want to know how tall it is, not how wide".

Investigations

Investigation 13.1 Collage

Make a collage of angles, lines and triangles by using cuttings from newspapers, brochures, magazines, and photos.

Display the collage in your classroom.

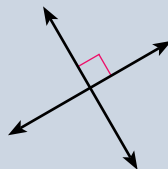


How many parallel lines can you find?
How many perpendicular lines?
How many isosceles triangles?

Investigation 13.2 Perpendicular lines

Lines that meet at right-angles (90°) are called perpendicular lines.

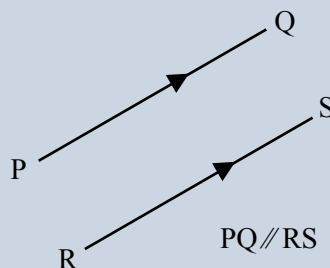
Using just a pen, ruler, and protractor, can you draw perpendicular lines?



Investigation 13.3 Parallel lines

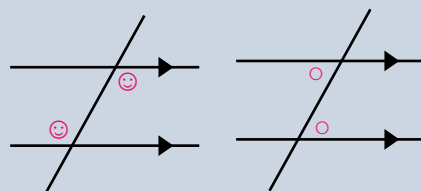
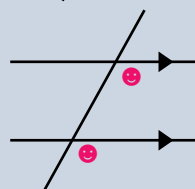
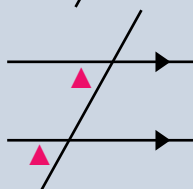
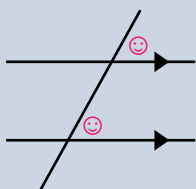
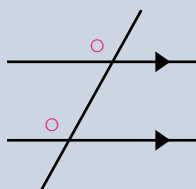
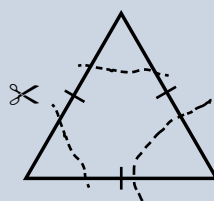
Lines that never meet are called parallel lines. Parallel lines are always the same distance apart.

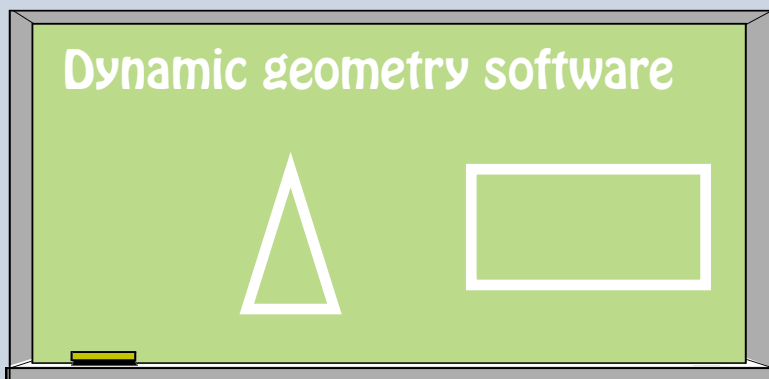
- Using just a pen, ruler, and a pair of compasses, can you draw parallel lines?
- Using just a pen, ruler, and protractor, can you draw parallel lines?



Investigation 13.4 Equal angles

- Draw an equilateral triangle using a ruler and either a protractor or a pair of compasses.
- Cut the three angles out and match them. They should be the same.
- Do the same for:
 - isosceles triangles.
 - corresponding angles.
 - alternate angles.





Technology 13.1 Drawing angles and triangles

- ▶ Interactive geometry software or dynamic geometry environments are about making and manipulating geometric objects.
- ▶ Use a search phrase such as "interactive geometry software" to find a list of such software.
- ▶ Experiment with one or two of them.

Technology 13.2 Applets

- ☞ Use a search phrase such as 'triangle applet' or 'quadrilateral applet' to find one of the many applets on the Internet. Experiment with them.
- ☞ Use a search phrase such as "parallel angle applet" to find an applet that lets you experiment with corresponding, alternate and vertically opposite angles.
- ☞ Use your own search phrases to find some of the hundreds of applets that let you experiment with the geometry in this chapter.
- ☞ Experiment with the applets.

Technology 13.3 Drawing with LOGO

LOGO is a computer programming language created in 1967 for educational use.

- ☞ LOGO is essentially a turtle with a pen on the monitor.
- ☞ The turtle will draw geometric shapes when given movement commands.
- ☞ The LOGO programming language is a powerful programming language.
- ☞ LOGO is considered an ideal introduction to computer programming.

To draw an equilateral triangle:

```
PENDOWN
FORWARD 100
LEFT 120
FORWARD 100
LEFT 120
FORWARD 100
PENUP
```

What would this code produce?

```
TO TRIANGLE REPEAT 3 [FD 100 LEFT 120]
REPEAT 10 [TRIANGLE FD 10 LEFT 36]
```

There are a number of free LOGO applications on the Internet, even LOGO applets, and many tutorials on how to draw objects with LOGO.

Chapter Review 1

Exercise 13.16

1 Name the type of each of the following angles:

a) 42°

b) 259°

c) 320°

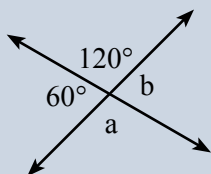
d) 123°

e) 90°

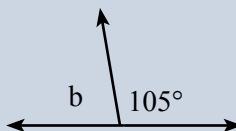
f) 360°

2 Find the size of the unknown angle:

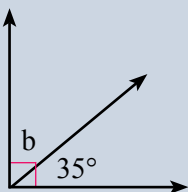
a)



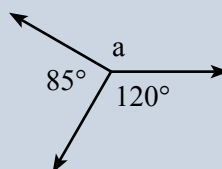
b)



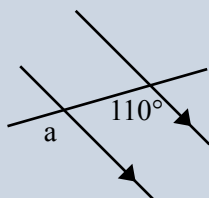
c)



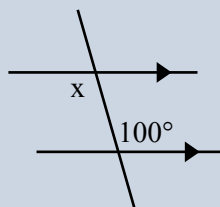
d)



e)



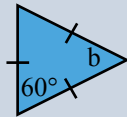
f)



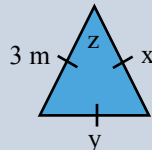
g)



h)



i)



3 Which quadrilateral am I?

a) My opposite sides are equal.

b) My opposite angles are equal.

c) My diagonals are equal.

d) My four angles are 90° .

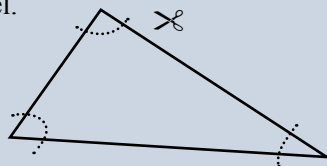
e) My four sides are equal.

f) My diagonals meet at right angles.

g) At least one pair of my opposite sides are parallel.

What is the angle sum of a triangle?

Cut out the corners and put them together.



Data 1



- ★ Calculate mean, median, mode and range for sets of data.
- ★ Use ordered stem-and-leaf plots to record and display numerical data.
- ★ Use mean and median to compare data sets and explain how outliers may affect the comparison.
- ★ Locate mean, median and range on graphs and connect them to real life.

A TASK

Conduct comparative research:

- Brainstorm a research question (eg., Is this winter colder than last winter? Was last summer wetter than this summer? Did the algebra advertising campaign change students' opinion about algebra? Did the Year 7s appreciate the concert more than the Year 10s? Use a 1 to 10 rating?).
- Collect data.
- Draw compound stem and leaf plots.
- Make a conclusion and present your research (Poster? Powerpoint?).

What's a
brainstorm?



A LITTLE BIT OF HISTORY

Pythagoras formed a brotherhood called the Pythagoreans. The Pythagoreans worshipped whole numbers.

The even numbers were thought to be feminine and the odd numbers masculine. 1 was the creator of all numbers and the number 5 was the symbol for marriage because it was the union of 2, the first feminine number, and 3, the first masculine number.

That makes me
rooster number 3?



Data Measures

Calculating data measures can help make sense of the data.

The Range

describes the **spread** of the data.

Range = largest – smallest

Exercise 14.1

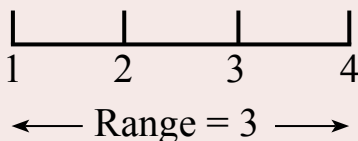
Find the range of each of the following set of scores:

1, 2, 3, 4, 4

Range = largest – smallest

Range = 4 – 1

Range = 3



1 4, 7, 15, 23, 31, 43

3 2, 7, 2, 6, 3, 2, 6, 7

5 232, 235, 236, 232, 231

7 25.2, 26.1, 25.8, 23.2, 24.6

2 4, 15, 32, 83, 112, 145, 231

4 1, 3, 3, 5, 1, 2, 7, 4

6 425, 424, 421, 423, 424, 427

8 -2, -3, -5, -4, -1, -2, -6

The Mode

describes the **middle** of the data.

The mode is the score that occurs most often.

Exercise 14.2

Find the mode of each of the following set of scores:

5, 3, 3, 4, 3, 5, 3

Mode = 3 {Occurs 4 times}

2, 5, 8, 5, 4, 2, 1, 3

Mode = 2 and 5 {Twice each}

This set of scores is bimodal

1 4, 6, 6, 3, 2, 3, 6

3 2, 5, 2, 1, 6, 1, 1, 5

5 5, 5, 5, 3, 4, 3, 3, 5

7 32, 35, 36, 32, 31, 33

9 -1, -3, -4, -2, -1, -2, -1

2 4, 5, 2, 3, 4, 2, 4, 5

4 1, 4, 3, 3, 4, 5, 6, 2

6 1, 2, 3, 2, 1, 2, 4, 1

8 7.1, 7.3, 7.1, 7.2, 7.1

10 a, b, a, c, b, a, d, b, a

Data Measures

The Median

describes the **middle** of the data.

The median is in the middle

Exercise 14.3

Find the median of each of the following set of scores:

5, 7, 3, 4, 3, 5, 3

Put the scores in ascending order

3, 3, 3, 4, 5, 5, 7

Median = 4 {4 is in the middle }

2, 5, 7, 5, 4, 2, 1, 3

Put the scores in ascending order

1, 2, 2, 3, 4, 5, 5, 7

Median = 3.5 {Average of 3 & 4}

1 4, 6, 6, 3, 2, 3, 6

3 2, 5, 2, 1, 6, 1, 1

5 32, 98, 36, 32, 31

7 5.2, 6.1, 5.8, 3.2, 4.6

2 4, 5, 2, 3, 4, 2, 4, 5

4 1, 2, 3, 4, 1, 2, 4, 3

6 21, 24, 23, 23, 56

8 -1, -3, -4, -2, -1, -2, -1, -2vv

The Mean

describes the **middle** of the data.

$$\text{mean} = \frac{\text{Sum of scores}}{\text{Number of scores}}$$

Mean and average are the same. Except that I'm mean and not average.



Exercise 14.4

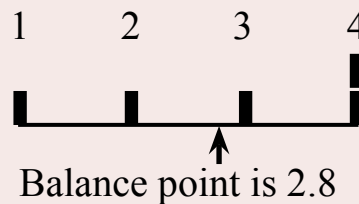
Find the mean of each of the following set of scores:

1, 2, 3, 4, 4

$$\text{mean} = \frac{\text{Sum of scores}}{\text{Number of scores}}$$

$$\text{mean} = \frac{1+2+3+4+4}{5}$$

$$\text{mean} = \underline{2.8}$$



1 1, 2, 3, 4, 4, 4

3 1, 1, 2, 2, 3, 4, 4, 4, 4

5 31, 36, 36, 32, 32, 33

7 101, 106, 106, 102, 102, 103

9 5.1, 5.6, 5.6, 5.2, 5.2, 5.3

2 1, 2, 3, 4, 4, 80

4 1, 6, 6, 2, 2, 3

6 81, 86, 86, 82, 82, 83

8 721, 726, 726, 722, 722, 723

10 -1, -6, -6, -2, -2, -3

Dot Plots

Plots are simple to make and show the shape of the data.

Dot Plots

show stacked dots on a line.

Exercise 14.5

Use a Dot Plot to show the shape of the following data. Also find the range, mode, median, and mean.

The following list shows the ages of the students in the group

12 12 13 12 12 11 12
11 12 12 11 11 12 13
12 11 12 12 12 12

Range = $13 - 11 = 2$

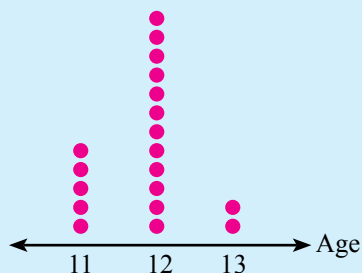
Mode = 12 {13 times}

Median = 12

$$\text{mean} = \frac{\text{Sum of scores}}{\text{Number of scores}}$$

$$\text{mean} = \frac{237}{20}$$

$$\text{mean} = 11.85$$



- 1 Use a part of the number line.
- 2 Label the number line (axis).
- 3 Add the dots.

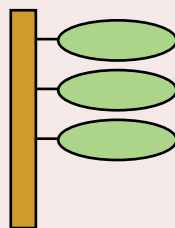
- 1 The ages of students in the group: 12, 13, 13, 12, 13
13, 12, 14, 13, 13
13, 13
- 2 The test marks (out of 20): 16, 16, 18, 19, 16
17, 18, 19, 18, 17,
16, 18, 18, 17,
- 3 Each person was asked how many brothers and sisters they have at this school:
0, 0, 2, 1, 3,
1, 0, 1, 2, 1,
0, 1, 0, 1, 2,
1, 0, 0, 1, 0,
0, 1, 0
- 4 The minimum temperature each day for the last fortnight:
2, -3, -4, 1, -3, 0, -2, -1, -3, 0, 2, 0, -2, 1



Stem and Leaf Plots

Stem and Leaf Plots

show the shape of the data.



Title		
Stem	Leaf	Key 3 4 means 34
2	4 6	
3	0 2 4 5 5 8	
4	1	

This number is 35

Exercise 14.6

Use a stem and leaf plot to represent the following data. Also find the range, mean, mode, and median.

Use a stem and leaf plot to represent the ages of 23 employees of a shire council. Also find the range, mode, median, and mean of the ages.

19 27 34 37 32 49 42 34 25 28 35 56 43 39 23 51 28 32 49 36 34 42 34

First, put the ages in order:

19 23 25 27 28 28 32 32 34 34 34 34 35 36 37 39 42 43 43 49 49 51 56

Then, use the **first digit as the stem** and the **trailing digit as the leaf**:

Employee Age (1|9 means 19)

1	9
2	3 5 7 8 8
3	2 2 4 4 4 4 5 6 7 9
4	2 3 3 9 9
5	1 6

$$\text{Range} = 56 - 19 = 37$$

$$\text{Mode} = 34 \quad \{34 \text{ occurs 4 times}\}$$

$$\text{Median} = 34 \quad \{\text{the middle number is 34}\}$$

$$\text{Mean} = \frac{\text{Sum}}{\text{Number}} = \frac{830}{23}$$

$$\text{Mean} = 36.09$$

- Scores on the test (maximum of 20): 14, 16, 16, 13, 12, 13, 16
- Number of birds visiting the feeder each day: 34, 35, 42, 43, 34, 32, 44, 35
- Ages of people at the birthday party: 12, 35, 22, 21, 16, 21, 21, 34
- The weights (kg) of the forward pack: 121, 132, 103, 124, 131, 122, 134, 123
- The weekly holiday rents (\$): 519, 522, 517, 516, 522, 522, 518, 522, 528
- The weekly house rents (\$): 348, 349, 352, 349, 351, 352, 350, 351, 349, 351, 348, 350, 356, 345, 349
- The daily solar cell voltage readings: 5.2, 6.1, 5.8, 7.2, 4.6, 4.9, 5.8, 7.3, 5.9, 6.8, 6.2, 5.3, 5.2, 5.2

Compound Plots

Compound Plots are useful for comparing data in a single display.

Compound.

Combine to make a whole. Combine two plots side by side.

Form 8A believe that they are a better mathematics class than Form 8B.

- Draw a compound Stem and Leaf plot.
- Calculate the range, mode, median, and mean.
- Make a comment.

8A							8B						
100	94	60	56	85	98	43	25	58	60	73	72	55	64
68	79	97	48	53	65	72	74	72	59	57	58	93	82
32	57	47	79	59	44	63	79	69	67	60	80	73	59
74	37	63	96	52	67	84	65	76	804	67	78	68	78

First, put the data in order:

8A 32, 43, 44, 47, 48, 52, 53, 56, 57, 57, 59, 60, 63, 63, 65, 67, 68, 72, 74, 79, 79, 84, 85, 94, 96, 97, 98, 100

8B 25, 55, 57, 58, 58, 59, 59, 60, 60, 64, 65, 67, 67, 68, 69, 72, 72, 73, 73, 74, 76, 78, 78, 79, 80, 82, 93

8A	8B (2 5 is 25)
	2 5
2	3
8743	4
977632	5 578899
875330	6 00457789
9942	7 223346889
54	8 02
8764	9 3
0	10

The 804 is an outlier.

- It is probably a mistake.
- It was meant to be 84? or 80?
- The 804 has been deleted.

Comment. The stem and leaf plot shows that 8A has five students who have better results than any of the students in 8B. However, of the eight lowest results, seven of the results are from 8A. 8A's results are more spread out than 8Bs. The mean appears to be the most useful central measure. 8A's mean is 67.6 and 8B's mean is 67.4. The median would also be of some use and 8A's median is 64 while 8B's median is 68. There is very little evidence to support 8A's claim that they are a better mathematics class.

Exercise 14.7

- 1 Curious about whether people with glasses are smarter, the following samples of mathematics results were obtained.

- Draw a compound Stem and Leaf plot.
- Calculate the range, mode, median, and mean.
- Make a comment.

Glasses (Maths results)	No Glasses (Maths results)
61 57 99 77 68 67 43 60 71 50	45 53 62 93 75 60 68 57 86 70
31 98 83 69 53 64 79 66 67 50	68 52 75 53 94 49 73 88 60 69
87 63 85 78 73	55 84 58 79 65

- 2 The following data shows the number of hours of power provided by a sample of Brand A batteries and a sample of Brand B batteries.

- Draw a compound Stem and Leaf plot.
- Calculate the range, mode, median, and mean.
- Make a comment.

Brand A	Brand B
32 30 14 46 54 37 26 35 31 40	33 30 29 32 30 41 36 31 42 37
35 51 43 21 30 25 33 24 18	31 30 32 40 28 31 34 40 39

- 3 The following table lists the IQs of a class of Maths students and a class of Biology students.

- Draw a compound Stem and Leaf plot.
- Calculate the range, mode, median, and mean.
- Make a comment.

Maths students	Biology students
100 107 80 122 102 92 94 103	102 82 124 113 106 105 113 117
108 113 105 97 108 93 119 129	97 90 88 91 85 122 125 89
100 87 96 110 109 125 111 92	85 94 124 116 114 98 102 99
115 102 101 119	121 110 109 108

- 4 Believing that Sydney has a generally warmer climate than Melbourne, the average monthly minimums, °C, for two years were recorded.

- Draw a compound Stem and Leaf plot.
- Calculate the range, mode, median, and mean.
- Make a comment.

Melbourne	Sydney
18.3 16.2 14.8 12.1 10.3 9.4 10.2	17.3 15.7 13.7 10.9 9.4 8.3 9.4
12.4 14.9 18.1 19.6 19.3 17.9 16.6	11.4 14.7 17.4 18.4 18.5 17.7 16.3
14.5 12.1 10.9 9.3 10.1 12.7 15.5	14.5 10.7 9.2 8.5 9.2 11.4 14.7
18.3 19.9 19.5	17.6 18.3 18.4

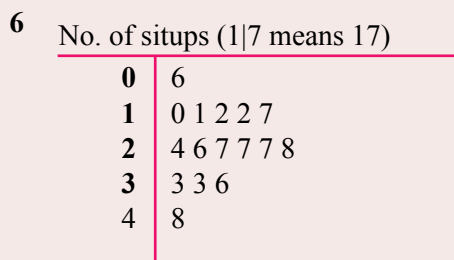
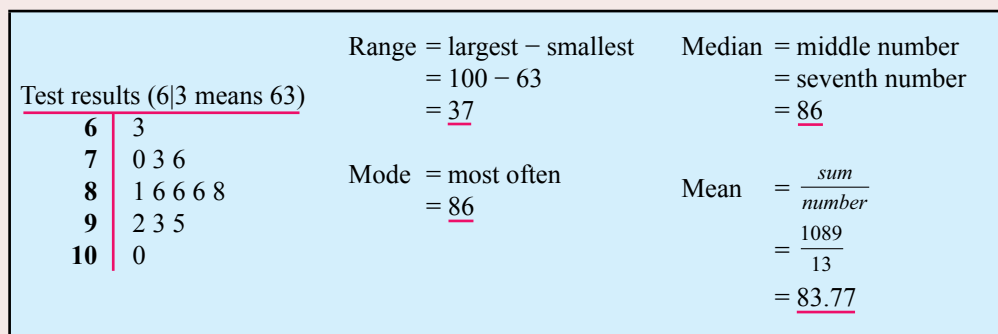
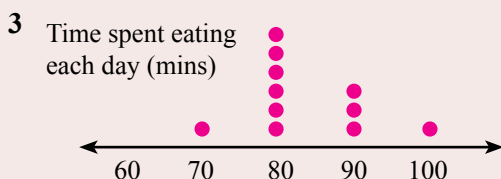
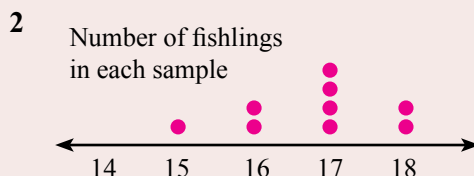
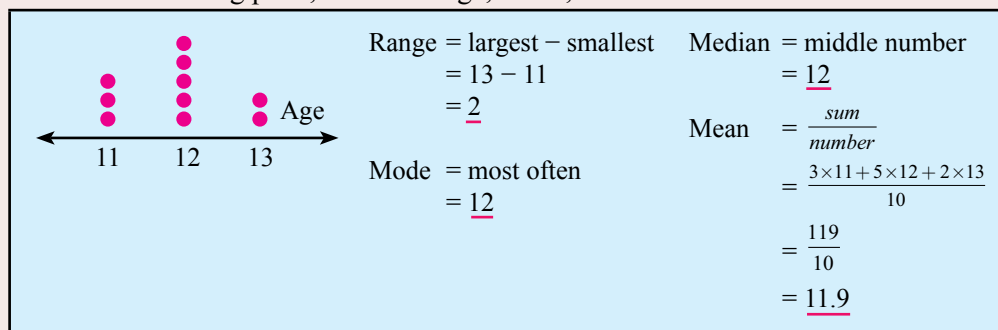
Data Measures from a Plot

Outliers are unusual values.

- Are they a mistake?
- Should you delete them?

Exercise 14.8

Given the following plots, find the range, mode, median and mean.



Mental Computation

Exercise 14.9

1 Spell median.

Given the data set: 2,2,2,4,5

2 What is the range?

3 What is the mode?

4 What is the median?

5 What is the mean?

6 Sketch and label a rhombus.

7 Solve: $2x + 5 = 9$

8 Simplify: $m^4 \times m^2$

9 $3 + \sqrt{-6}$

10 Change $\frac{2}{5}$ to a decimal.

Mental computation helps
you learn how to think.



Exercise 14.10

1 Spell range.

Given the data set: 2,3,3,3,5

2 What is the range?

3 What is the mode?

4 What is the median?

5 What is the mean?

6 Sketch and label a parallelogram.

7 Solve: $3x - 5 = 4$

8 Simplify: $a^3 \times a^2$

9 $\sqrt{-3} + \sqrt{-6}$

10 Change $\frac{1}{4}$ to a decimal.

The most powerful weapon on
earth is the human soul on fire -
Ferdinand Foch.

Exercise 14.11

1 Spell average.

Given the data set: 1,2,2,2,5

2 What is the range?

3 What is the mode?

4 What is the median?

5 What is the mean?

6 Sketch and label a kite.

7 Solve: $5x + 2 = 27$

8 Simplify: $b^5 \times b^3$

9 $\sqrt{-3} + 2$

10 Change $\frac{3}{5}$ to a decimal.

Q What can be measured but has
no length, width, or thickness?

A The temperature/time.



Exercise 14.12

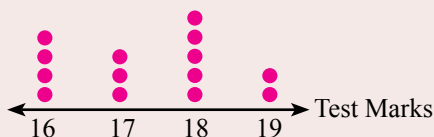
- What is the average of 3 and 7?
- What is halfway between $1\frac{1}{2}$ and $2\frac{1}{2}$?
- Find the mean (average) exercise time:

Monday	Tuesday	Thursday	Saturday	Sunday
30 minutes	50 minutes	20 minutes	30 minutes	30 minutes

- Given the scores: 4, 4, 5, 5, 5, 5, 7, 85
 - What is the mean, median, and mode?
 - What is the mean, median, and mode after 85 is removed?
 - Which is affected the most by removing 85, the mean, median, or mode?
- Find the mean, median, and mode of the following: 1, 2, 2, 2, 3, 5
 - Is Mean > Median = Mode true?

After four innings, the average number of runs per innings was 15. After five innings, the average number of runs per innings was 16. How many runs were scored in the fifth innings?	
After 4 innings: $\text{average} = \frac{\text{total}}{\text{no innings}}$ $15 = \frac{\text{total}}{4}$ Total for 4 innings = 15×4 = 60	After 5 innings: $\text{average} = \frac{\text{total}}{\text{no innings}}$ $16 = \frac{\text{total}}{5}$ Total for 5 innings = 16×5 = 80 No runs in fifth innings = $80 - 60$ = <u>20</u>

- After 4 innings, the average number of runs was 20.
After 5 innings, the average number of runs was 25.
How many runs were scored in the fifth innings?



- What is the median of:
- The basketball team of fifteen players need to score at least 75 points to be competitive. What is the average score per player needed to be competitive?
- The basketball team has twelve players with an average player score of 4.5. What total would the team be expected to score?
- Given the scores: 6, 6, 6, 7, 7, 7, 50. Which central measure is affected the most by removing the 50: the mean, median, or mode?

Competition Questions



Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 14.13

- What is the average of the numbers 3, 0.3, 0.03?
- What is the average of the numbers 33, 3.3, 0.33?
- What is the average of the numbers 333, 33.3, 3.33?
- If 25 million people have total savings of \$10 billion, what is the average savings per person?
- If 5 million people have total savings of \$4 billion, what is the average savings per person?
- Australia has roughly 24 million people inhabiting roughly 8 000 000 km², what is the average number of people per km²?
- Indonesia has roughly 240 million people inhabiting roughly 2 000 000 km², what is the average number of people per km²?



Indonesia has over 400 active volcanoes and more than 3 earthquakes per day.

- The truck is carrying 45 bags of potatoes with an average weight of 100 kg per bag. What is the weight of the 45 bags?

A person has an average of 48 after three tests. What mark must the person get on the fourth test so that the average of the four tests is 50?

$$48 = \frac{\text{total}}{3}$$

Total after 3 tests = 144

$$50 = \frac{\text{total}}{4}$$

Total after 4 tests = 200

Thus need $200 - 144 = \underline{56}$ on the 4th test.

- A person has an average of 81 after three tests. What mark must the person get on the fourth test so that the average of the four tests is 85?
- A person has an average of 65 after four tests. What mark must the person get on the fifth test so that the average of the five tests is 70?
- The mean of 8 test marks was 15. Two people sat the test a day later and scored 13 and 17. What is the mean of the 10 test marks/
- The first truckload of 40 bags potatoes had an average weight of 101 kg per bag. The second truckload of 50 bags of potatoes had an average weight of 110 kg per bag. What was the average weight of the 90 bags of potatoes?

Investigations

Investigation 14.1 A Day in the Life of an Average Year 7 Student

- 1 General classroom discussion about what an average Year 7 student does in 24 hours.
- 2 Decide what normal activities should be included in a survey. Design your survey. It may be similar to the one below:

	School	Sleeping	TV	-	-
Time					

- 3 Have members of your class complete the survey form.
- 4 Analyse the data
 - Look for errors and decide what to do with the errors.
 - Calculate the mean or median time for each activity.
 - Present your results on a pie chart?



- 5 What would a pie chart for your parents/guardians look like?

Investigation 14.2 Length of first names?

- 1 Write a list of the first name of people in your class.
- 2 Count the number of letters.
- 3 What is the average/mean length?
- 4 Write a list of the first name of 25 people from another country.
- 5 Count the number of letters.
- 6 What is the average/mean length?

Name	Letters
Olivia	6
William	7
Ella	4
Jack	4

An Internet search?

Investigation 14.3 Online statistical activities

There are a large number of national and international online statistical activities for Year 7 students. These activities generally involve the collection of data from your class to form a large national data set or international data set. This then forms the basis of informed research on many relevant topics.

Become involved in one of these activities.

Statisticians design, collect, and analyse data using a variety of statistical techniques to provide information for a large range of clients.

- Relevant school subjects are Mathematics.
- Courses range from Bachelor degrees to Master degrees.

Technology

Technology 14.1 The Calculator

Scientific calculators will calculate descriptive statistics such as mean and sum.

- 1 Change the calculator mode to **Stat** or **SD**
- 2 Enter a number then press **M+**
- 3 Repeat entering a number and then pressing **M+**
- 4 Find the \bar{x} button, this is the mean.
- 5 Find the $\sum x$ button, this is the sum of the numbers.

Check your calculator's manual for Statistical calculations.

Technology 14.2 The Spreadsheet

Most spreadsheets will calculate a massive number of descriptive statistics **and** draw a frequency polygon:

- 1 Enter a set of data into the spreadsheet.
- 2 Enter formulas for the mean, median, and mode.

	A	B	C
1	2	mean =	2.6
2	2	median =	2
3	2	mode =	2
4	3		
5	4		

=mean(A1:A5)

=median(A1:A5)

=mode(A1:A5)

Calculate range as:
=max(a1:a5) - min(a1:a5)

Technology 14.3 The Graphics calculator

A graphics calculator will automatically calculate a large number of descriptive statistics **and** draw a frequency column graph:

- 1 Select the STAT menu, EDIT, and enter data into one of the lists.
- 2 Return to the main screen.
- 3 Select the STAT menu, Calc, 1_Stats and enter L1.

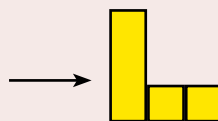
L1	L2	L3
2		
2		
2		
3		
4		

1_Stats
 \bar{x} = 2.6
 median = 2
 n = 5
 $\sum x$ = 13
 + heaps more.

To draw a frequency column graph:

- 1 Enter the numbers in L1 and the frequency in L2.
- 2 Use STATPLOT to set up the graph Xlist= L1 and Frequency = L2.
- 3 Use Zoom and ZooMSTAT to fit the graph if necessary.
- 4 TRACE.

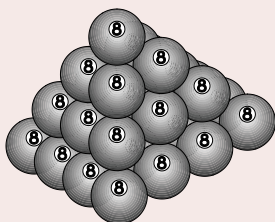
L1	L2	L3
2	3	
3	1	
4	1	



A Couple of Puzzles

Exercise 14.14

- 1 What is the smallest number that is exactly divisible by each of the numbers 2, 3, 4, and 5?
- 2 There are 32 cars and bicycles in the staff car park. If the total number of wheels is 110, how many cars are in the staff car park?
- 3 How many billiard balls?
- 4 Remove 6 rooks to leave an odd number in each row and column.



A Game

Calculator Cricket - Reduce a six digit number to 0 in exactly six balls.

A six digit number is entered into your calculator and you are to reduce it to 0 in six calculations before the other player reduces their six digit number to 0.

- 1 The six digit number must not include 0 and no digit must be repeated.
- 2 A bowl is +, -, ×, or ÷ by any two-digit number.

	217345
+55	217400
÷200	1087
+13	1100
÷100	11
÷11	1
-1	0

A Sweet Trick

- 1 Ask your audience to choose a number from 1 to 6.
- 2 Multiply the number with 9.
- 3 Multiply the result with 111.
- 4 Multiply the result by 1001.
- 5 Divide the answer by 7.

$$\begin{aligned}
 &3 \\
 &3 \times 9 = 27 \\
 &27 \times 111 = 2997 \\
 &2997 \times 1001 = 2999997 \\
 &2999997 \div 7 = 428571
 \end{aligned}$$



All of the numbers
1, 2, 4, 5, 7, 8
will be in the answer.

Chapter Review 1

Exercise 14.15

1 Find the range, mode, median, and mean of each of the following set of scores:

- | | |
|----------------------------|---|
| a) 1, 2, 3, 3, 3, 6 | b) 2, 7, 2, 5, 3, 2, 2, 7 |
| c) 6, 8, 7, 7, 6, 7, 8, 7 | d) 10, 12, 12, 13, 12, 15, 10 |
| e) 2.1, 2.2, 2.3, 2.3, 2.6 | f) -2, -3, -5, -7, -4, -1, -2, -3, -1, -1 |

2 Use a Dot Plot to show the shape of the following data.

Also find the range, mode, median, and mean.

- a) The ages of students in the group: 12, 13, 13, 12, 13
13, 12, 14, 13, 13
13, 12, 14, 13, 12
- b) The test marks (out of 10): 8, 7, 8, 9, 6, 8, 7
8, 8, 9, 8, 7, 9, 7
7, 6, 8, 7, 8

3 Use a stem and leaf plot to represent the following data.

Also find the range, mean, mode, and median.

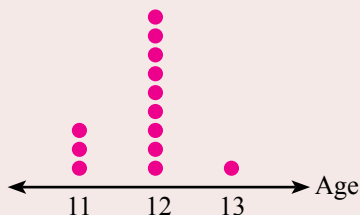
- a) Traffic speed radar readings: 58, 55, 62, 58, 59, 68, 60, 57
- b) Pulse: No. heartbeats/minute: 64, 82, 75, 75, 66, 87, 72, 71, 68

4 In deciding whether to continue with an advertising campaign, data about the number of customers per day just before the campaign started and the number of customers per day after the campaign started was collected. Analyse the data and suggest whether the advertising campaign is making a difference.

Before Advertising Campaign	After Advertising Campaign
180 191 173 224 201 213 197 190	184 192 189 200 193 225 197 206
189 216 194 207 175 192 200 181	202 217 218 206 215 194 192 222
220 195 199 184 215 201 190 205	196 209 213 208 208 197 191 211
	204

5 Given the following plots, find the range, mode, median and mean.

a)



b)

Voltage (5|8 means 5.8)

4	2 7
5	0 1 2 3 3 3 9
6	1 2 8 8
7	0 2

Chapter Review 2

Exercise 14.16

1 Find the range, mode, median, and mean of each of the following set of scores:

- a) 2, 2, 4, 4, 4 b) 1, 6, 4, 1, 3, 1, 2, 5
 c) 5, 8, 6, 7, 6, 6, 8, 5, 6 d) 30, 32, 32, 33, 32, 35, 30
 e) 1.2, 1.4, 1.1, 1.3, 1.3, f) -5, -1, -2, -5, -2, -4, -2, -3, -2, -1

2 Use a Dot Plot to show the shape of the following data.

Also find the range, mode, median, and mean.

- a) The test marks (out of 10):
 6, 7, 9, 9, 6, 7, 7
 7, 8, 9, 8, 7, 9, 9
 7, 6, 7
- b) Each person was asked how many brothers and sisters they have at this school:
 0, 3, 2, 1, 3, 1, 0, 1, 2, 1,
 3, 1, 4, 1, 2, 2, 3, 4, 1, 0,

3 Use a stem and leaf plot to represent the following data.

Also find the range, mean, mode, and median.

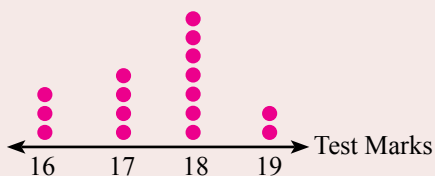
- a) Ages of people at the birthday party: 11, 12, 26, 12, 36, 21, 12, 12
 b) The daily solar cell voltage readings: 4.2, 4.1, 4.0, 4.2, 3.6, 3.9, 3.8

4 A traffic inspector suspects that drivers are more careless with their car speed on public holidays. An initial investigation involved collecting radar gun readings, in a 50 km/h zone, on a public holiday and a normal workday just previous to the public holiday. Analyse the data and make a comment.

Public Holiday	Workday
50 55 52 51 49 63 51 54 57 53 38	52 42 48 44 46 54 51 41 38 35 43
50 55 49 38 65 70 50 58 52 60 51	39 48 50 54 57 72 55 49 37 41 53
54 48 49 56 43 53 57 54	44 45 50 58 70 55 51 47

5 Given the following plots, find the range, mode, median and mean.

a)

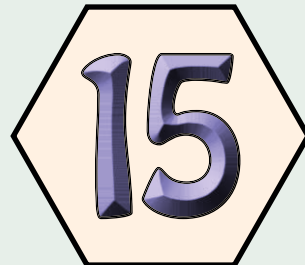


b)

Weight (5|3 means 53)

4	3
5	0 1 1 3 6 9
6	1 2 4
7	3

Review 3



Chapter 11 Number 3

$2 \times 2 \times 2 = 2^3$ <p>Index \swarrow Base \nwarrow</p>	<p>Prime numbers have exactly two factors. The factors of 2 = 1 and 2. The factors of 3 = 1 and 3. 2, 3, and 7 are prime numbers. The factors of 7 = 1 and 7.</p>
<p>Composite numbers have more than two factors. 1 only has one factor. 1 is neither prime nor composite.</p>	<p>Prime factors of 12 = $2 \times 2 \times 3$ or = $2^2 \times 3$</p>

Chapter 12 Linear Equations

<p>Word equation: When a number is added to five the answer is nine.</p> <p>Symbolic equation: $x + 5 = 9$</p>	<p>Solve equations:</p> <p>Find the value of the unknown.</p>												
<p>$2x + 5 = 9$</p> <p>$2x = 9 - 5$ {inverse of + is -}</p> <p>$2x = 4$</p> <p>$x = 4 \div 2$ {inverse of \times is \div}</p> <p>$x = 2$</p>	<p>Inverse of</p> <table><tr><td>+</td><td>is</td><td>-</td></tr><tr><td>-</td><td>is</td><td>+</td></tr><tr><td>\times</td><td>is</td><td>\div</td></tr><tr><td>\div</td><td>is</td><td>\times</td></tr></table>	+	is	-	-	is	+	\times	is	\div	\div	is	\times
+	is	-											
-	is	+											
\times	is	\div											
\div	is	\times											

Chapter 13 Geometry

<p>acute = less than 90° right = 90° obtuse = between 90° and 180° reflex = between 180° and 360° straight = 180° revolution = 360°</p>	<p>Equilateral triangle:</p> <p>all 3 sides equal all 3 angles = 60°</p>
<p>Isosceles triangle</p> <p>2 sides equal angles opposite equal sides equal</p>	<p>Parallelogram:</p> <p>Opposite sides equal Opposite sides parallel Opposite angles equal</p>

Chapter 14 Data 1

<p>Range = largest – smallest Mode = most often Median = in the middle Mean = total/number</p>	<p>Dot plot</p>																				
<p>Stem and leaf plot</p> <table> <tr> <th>Age (1/2 means 12)</th> <th></th> </tr> <tr> <td>1</td> <td>2 2 2 2 2 2</td> </tr> <tr> <td>2</td> <td>3</td> </tr> <tr> <td>3</td> <td>2 6</td> </tr> </table>	Age (1/2 means 12)		1	2 2 2 2 2 2	2	3	3	2 6	<p>Compound plot</p> <table> <tr> <th>Business A</th> <th>Business B (4/6 is 46)</th> </tr> <tr> <td>977</td> <td>1 77889</td> </tr> <tr> <td>8775422</td> <td>2 0013467789</td> </tr> <tr> <td>984331</td> <td>3 02245689</td> </tr> <tr> <td>97643</td> <td>4 6</td> </tr> <tr> <td>8620</td> <td>5 7</td> </tr> </table>	Business A	Business B (4/6 is 46)	977	1 77889	8775422	2 0013467789	984331	3 02245689	97643	4 6	8620	5 7
Age (1/2 means 12)																					
1	2 2 2 2 2 2																				
2	3																				
3	2 6																				
Business A	Business B (4/6 is 46)																				
977	1 77889																				
8775422	2 0013467789																				
984331	3 02245689																				
97643	4 6																				
8620	5 7																				

Review 1

Exercise 15.1 Mental computation

1 Spell equation.

Given the data set: 2,2,2,4,5

2 What is the range?

3 What is the mode?

4 What is the median?

5 What is the mean?

6 Sketch and label a parallelogram.

7 Solve: $2x + 3 = 9$

8 Simplify: $a^3 \times a^2$

9 $2 + \sqrt{5}$

10 Change $2\frac{2}{3}$ to a vulgar fraction.

Genius is one percent inspiration,
ninety-nine percent perspiration -
Thomas Edison.

Q Is an old \$50 note better
than a new one?

A I'd rather have an old \$50
note than a new \$1 note.

Exercise 15.2

1 Write the following in index form:

a) $2 \times 2 \times 2$

b) $3 \times 3 \times 3 \times 3 \times 3$

c) 10×10

d) $10 \times 10 \times 10$

e) $5 \times 5 \times 5 \times 5$

f) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

2 Write the following in factor form:

a) 2^3

b) 3^4

c) 4^2

d) 10^1

e) 10^4

f) 1^5

3 Calculate each of the following:

a) 3×10^2

b) 4×10^3

c) 8×10^1

d) $4 \times 10^1 + 9$

e) $2 \times 10^2 + 3 \times 10^1 + 7$

f) $6 \times 10^3 + 3 \times 10^2 + 0$

4 Find all the factors of each of the following numbers:

a) 2

b) 6

c) 8

d) 10

e) 12

f) 25

5 Find the highest common factor of each of the following pairs of numbers

a) 8 and 12

b) 15 and 20

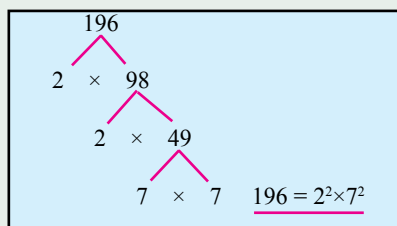
c) 12 and 16

d) 15 and 25

e) 28 and 35

f) 18 and 32

Factors of 12 = 1, 2, 3, 4, 6, 12
Factors of 18 = 1, 2, 3, 6, 9, 18
The highest common factor is 6



6 Use factor trees to find the prime factors of each of the following numbers:

a) 14

b) 12

c) 18

d) 24

e) 1000

f) 4000

7 Find the square of 2

8 Find the cube of 2

9 What is the square root of 9?

10 What is the cube root of 8?

11 Rewrite each of the following word equations as symbolic equations:

- a) When a number is added to eleven the result is thirty-five.
- b) When seven is subtracted from a number the answer is thirteen.
- c) When a number is divided by fifty-two the answer is three.
- d) When a number is multiplied by twelve the answer is eighty-four.

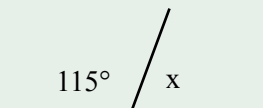
12 Solve each of the following equations:

- | | | |
|-------------------|------------------|------------------|
| a) $x + 3 = 5$ | b) $x - 2 = 8$ | c) $3x = 15$ |
| d) $a \div 5 = 4$ | e) $x + 4 = 9$ | f) $6b = 30$ |
| g) $8c + 2 = 10$ | h) $3x - 3 = 15$ | i) $5m + 7 = 22$ |

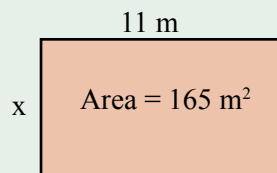
13 For each of the following problems:

- Write an equation.
- Solve the equation.
- Check the answer.

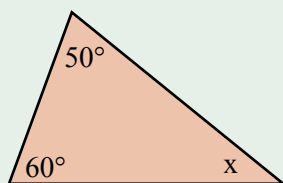
a)



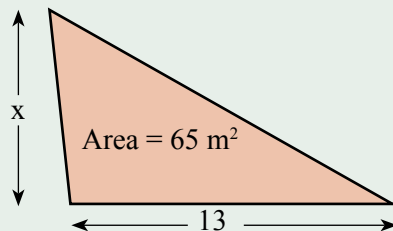
b)



c)



d)

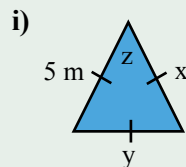
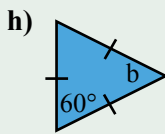
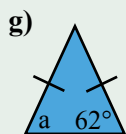
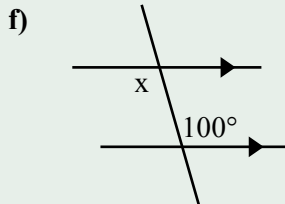
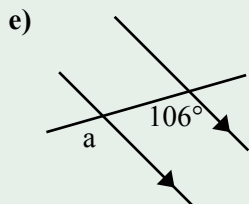
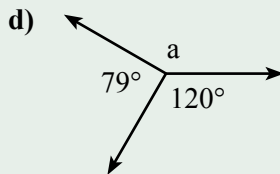
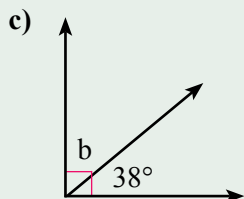
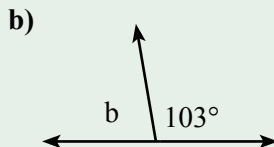
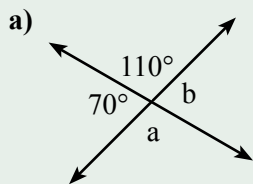


- e) Chloe is paid \$220 for the week and \$50 for each sale that she makes. How many sales will Chloe have to make so that she earns a total of \$570 for the week.
- f) Ethan will buy a milk carton for \$2.80 and spend the rest of his \$45 on tomatoes at \$5.20 per kilogram. How many kilograms of tomatoes will he buy?
- g) The total mass of the truck must not exceed 6.4 tonnes. If the truck has a mass of 3.9 tonnes, how many pallets, each of mass 0.2 tonnes, can be placed on the truck?
- h) The plumber charges \$75 callout and then \$95 per hour. How many hours of work will the plumber put in for \$850?

14 Name the type of each of the following angles:

- | | | |
|----------------|----------------|----------------|
| a) 54° | b) 195° | c) 310° |
| d) 146° | e) 90° | f) 360° |

15 Find the size of the unknown angle:



16 Which quadrilateral am I?

- a) My opposite sides are equal.
- b) My opposite angles are equal.
- c) My diagonals are equal.
- d) My four angles are 90° .
- e) My four sides are equal.
- f) My diagonals meet at right angles.
- g) At least one pair of my opposite sides are parallel.

17 Find the range, mode, median, and mean of each of the following set of scores:

- | | |
|----------------------------|---|
| a) 1, 2, 3, 3, 3, 6 | b) 2, 5, 2, 4, 3, 2, 2, 6 |
| c) 6, 8, 7, 8, 6, 7, 8, 8 | d) 20, 22, 22, 23, 22, 24, 20 |
| e) 6.1, 6.2, 6.3, 6.3, 6.4 | f) -3, -1, -4, -7, -4, -1, -4, -3, -4, -2 |

18 Use a stem and leaf plot to represent the following data.

Also find the range, mean, mode, and median.

- a) Traffic speed radar readings: 78, 84, 79, 78, 89, 78, 80, 77
- b) Pulse: No. heartbeats/minute: 62, 80, 76, 76, 66, 77, 75, 76, 68

- 19 Use a Dot Plot to show the shape of the following data.

Also find the range, mode, median, and mean.

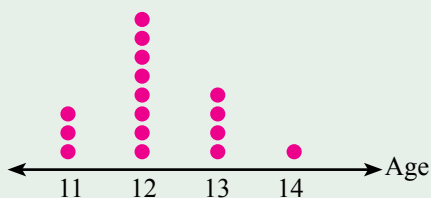
- a) The ages of students in the group: 11, 13, 13, 12, 13
13, 11, 13, 13, 12
12, 12, 11, 13, 12
- b) The test marks (out of 10): 7, 7, 8, 9, 6, 8, 7
9, 8, 6, 8, 7, 9, 7
6, 6, 7, 7, 7, 8

- 20 Suspecting age discrimination, samples of employee ages were taken from two comparable businesses. Analyse the data and make a comment.

Business A	Business B
33 25 19 52 38 33 22 27 22 46 49	21 17 35 38 28 29 32 19 46 18 24
17 27 39 47 17 28 58 56 31 50 24	34 57 20 27 18 36 27 30 39 17 20
34 43 44	32 26 23

- 21 Given the following plots, find the range, mode, median and mean.

a)



b)

Voltage (8|2 means 8.2)

6	1
7	0 1 1 2 9
8	2 2 2 5 7
9	0

Review 2

Exercise 15.3 Mental computation

- 1 Spell quadrilateral.

Given the data set: 2,3,3,3,4

- 2 What is the range?

- 3 What is the mode?

- 4 What is the median?

- 5 What is the mean?

- 6 Sketch and label a rhombus.

- 7 Solve: $3x + 4 = 13$

- 8 Simplify: $x^4 \times x^2$

- 9 $3 + \sqrt{5}$

- 10 Change $\frac{4}{5}$ to a decimal.

Talent is what you possess;
genius is what possesses you
- Malcolm Cowley.

Exercise 15.4

1 Write the following in index form:

a) $5 \times 5 \times 5$

b) $2 \times 2 \times 2 \times 2 \times 2$

c) $10 \times 10 \times 10$

d) $10 \times 10 \times 10 \times 10$

e) $4 \times 4 \times 4 \times 4$

f) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

2 Write the following in factor form:

a) 3^2

b) 3^5

c) 2^3

d) 10^1

e) 10^3

f) 1^4

3 Calculate each of the following:

a) 2×10^2

b) 7×10^3

c) 9×10^1

d) $4 \times 10^1 + 6$

e) $5 \times 10^2 + 4 \times 10^1 + 8$

f) $4 \times 10^3 + 2 \times 10^1 + 7$

4 Find all the factors of each of the following numbers:

a) 3

b) 8

c) 10

d) 15

e) 20

f) 24

5 Find the highest common factor of each of the following pairs of numbers

a) 8 and 12

b) 8 and 24

c) 15 and 25

d) 12 and 18

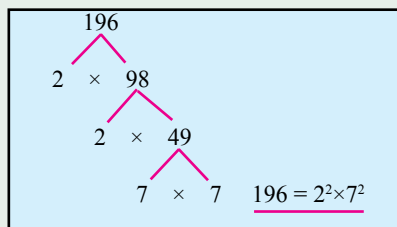
e) 21 and 28

f) 32 and 48

Factors of 12 = 1, 2, 3, 4, **6**, 12

Factors of 18 = 1, 2, 3, **6**, 9, 18

The highest common factor is 6



6 Use factor trees to find the prime factors of each of the following numbers:

a) 8

b) 15

c) 20

d) 24

e) 400

f) 4000

7 Find the square of 3

8 Find the cube of 3

9 What is the square root of 16?

10 What is the cube root of 27?

11 Rewrite each of the following word equations as symbolic equations:

a) When a number is added to eight the result is fifty-one.

b) When thirteen is subtracted from a number the answer is eleven.

c) When a number is divided by twenty-one the answer is five.

d) When a number is multiplied by nine the answer is eighty-one.

12 Solve each of the following equations:

a) $x + 6 = 13$

b) $x - 3 = 7$

c) $5x = 10$

d) $b \div 4 = 6$

e) $x + 7 = 10$

f) $5a = 25$

g) $6c + 3 = 21$

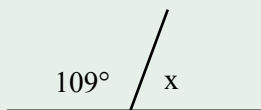
h) $5x - 2 = 18$

i) $4m + 2 = 22$

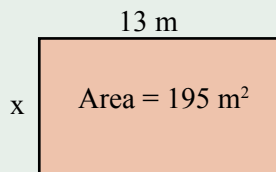
13 For each of the following problems:

- Write an equation.
- Solve the equation.
- Check the answer.

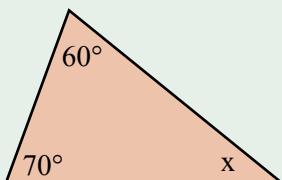
a)



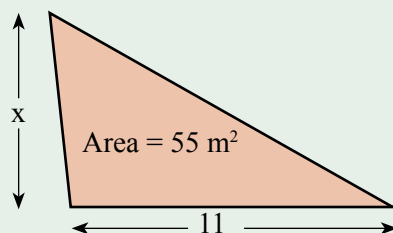
b)



c)



d)



- e)** The printer charges an upfront \$125 and then \$0.15 per flyer. How many flyers can be printed for \$1500?
- f)** The Internet phone company charges \$0.12 connection fee and then \$0.25 per minute. How long will a \$5 call last?
- g)** The total mass of the truck must not exceed 5.6 tonnes. If the truck has a mass of 3.5 tonnes, how many pallets, each of mass 0.4 tonnes, can be placed on the truck?
- h)** The electrician charges \$85 callout and then \$80 per hour. How many hours of work will the plumber put in for \$650?

14 Name the type of each of the following angles:

a) 38°

b) 215°

c) 325°

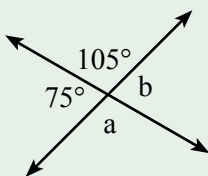
d) 163°

e) 90°

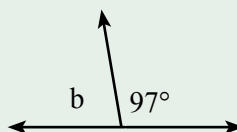
f) 360°

15 Find the size of the unknown angle:

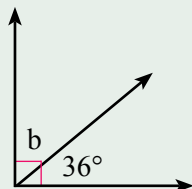
a)



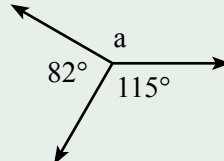
b)

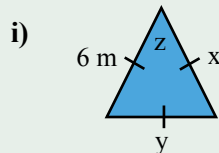
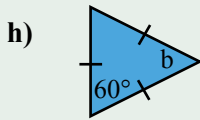
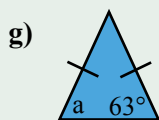
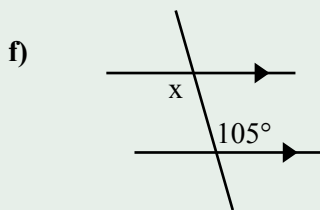
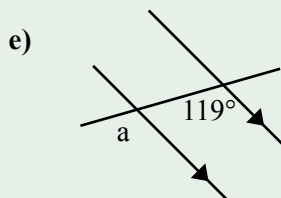


c)



d)





- 16** Which quadrilateral am I?
- My opposite sides are equal.
 - My opposite angles are equal.
 - My diagonals are equal.
 - My four angles are 90° .
 - My four sides are equal.
 - My diagonals meet at right angles.
 - At least one pair of my opposite sides are parallel.
- 17** Find the range, mode, median, and mean of each of the following set of scores:
- 1, 1, 1, 2, 5
 - 2, 3, 3, 4, 3, 2, 3, 4
 - 15, 18, 17, 18, 16
 - 70, 72, 72, 73, 72, 74, 70
 - 8.1, 8.2, 8.3, 8.3, 8.4
 - 2, -1, -3, -5, -4, -3, -4, -3, -3, -2
- 18** Use a stem and leaf plot to represent the following data.
Also find the range, mean, mode, and median.
- Ages of people at the birthday party: 12, 12, 32, 12, 36, 23, 12, 12, 12
 - The daily solar cell voltage readings: 5.2, 5.1, 5.0, 5.1, 4.7, 4.9, 4.8
- 19** Use a Dot Plot to show the shape of the following data.
Also find the range, mode, median, and mean.
- The ages of students in the group: 11, 12, 12, 11, 13, 11, 11, 12, 11, 12, 12, 13, 11, 13, 11
 - The test marks (out of 10): 6, 7, 8, 9, 6, 8, 7, 8, 7, 6, 8, 9, 7, 7, 7, 6, 7, 7, 7, 8, 7
- 20** The manager of a golf course has supplied data about the playing of a golf course when the greens have been watered five days before competition, and when the greens have been watered two days before competition.
Analyse the data and make a comment.

Watered 5 days before	Watered 2 days before
90 94 73 106 83 87 97 102	92 66 125 105 96 94 83 88
82 123 109 68 88 94 114 96	75 89 82 109 85 73 89 81
75 82 126 107 99 91 85 80	95 78 99 113 89 87 96 69
78 70 118 88 115 96 104 104	75 85 81 104 90 79 72 92

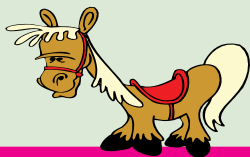
Ratio & Rate

16

- ★ Recognise and solve problems involving simple ratios.
- ★ Understand that rate and ratio problems can be solved using fractions or percentages and choosing the most efficient form to solve a particular problem.
- ★ Find percentages of quantities and express one quantity as a percentage of another.
- ★ Use authentic problems to express quantities as percentages of other amounts.

A TASK

The Golden Ratio is the secret to my good looks.



“The Golden ratio”.

- What is the ‘Golden Ratio’?
- Construct some Golden Rectangles.
- Conduct a survey to answer the question: “Are Golden Rectangles more pleasing to the eye than other rectangles?”
- Report your findings (Powerpoint, poster, classroom presentation, article in the local paper?).

A LITTLE BIT OF HISTORY

Many architects and artists proportion their work using the Golden Ratio in the belief that this ratio is pleasing to the eye.

- c. 450 BC Parthenon dimensions supposedly based on the Golden Ratio proportions.
- c. 300BC Euclid mathematically defined the Golden Ratio.
- c. 1485 Leonardo da Vinci creates the Vitruvian Man in golden ratio proportions.
- 1955 Salvador Dali creates ‘The Sacrament of the Last Supper’ with many Golden Ratio aspects.



The pentagram helps illustrate the connection between man and the golden ratio

Ratio Warm Up

A Ratio is a comparison of two quantities of the same type in the same units.

A ratio can be written as 3: 10, a fraction $\frac{3}{10}$, a decimal 0.3, and as a percentage 30%.

Exercise 16.1

Write the following comparisons as ratios:

18 people passed the test and 5 failed.	
a) What is the ratio of pass to fail?	18 : 5
b) What is the ratio of fail to pass?	5 : 18
c) What is the ratio of pass to the total?	18 : 23
d) What is the ratio of fail to the total	5 : 23

The mathematical symbol for ratio is



- 1 The Maths class has 9 girls and 14 boys.
 - a) What is the ratio of girls to boys?
 - b) What is the ratio of boys to girls?
 - c) What is the ratio of girls to the total number in the class?
 - d) What is the ratio of boys to the total number in the class?
- 2 Last month there were 24 sunny days and 7 cloudy days.
 - a) What is the ratio of sunny to cloudy days?
 - b) What is the ratio of cloudy to sunny days?
 - c) What is the ratio of sunny to the total number of days in the month?
 - d) What is the ratio of cloudy to the total number of days in the month?
- 3 Write each of the following ratios as a fraction, a decimal and a percentage:

	Fraction	Decimal	Percentage
1 : 4	$\frac{1}{4}$	0.25	25%
5 : 2	$\frac{5}{2}$	2.5	250%

Make a percentage by multiplying by 100.

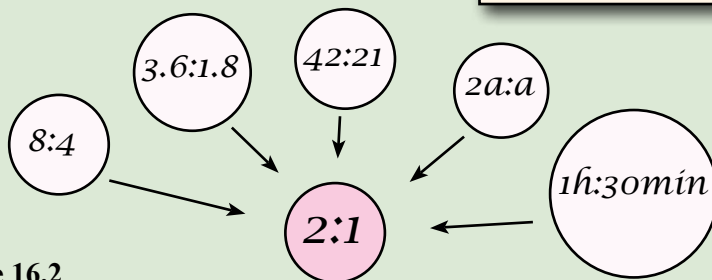
- | | | | |
|-----------|-----------|-----------|-----------|
| a) 1 : 2 | b) 1 : 4 | c) 3 : 4 | d) 1 : 5 |
| e) 2 : 5 | f) 3 : 5 | g) 4 : 5 | h) 1 : 10 |
| i) 2 : 10 | j) 3 : 10 | k) 4 : 10 | l) 5 : 10 |
| m) 6 : 10 | n) 5 : 2 | o) 3 : 2 | p) 7 : 2 |

- 4 Write each of the following fractions as a ratio, a decimal and a percentage:

- | | | | |
|-------------------|-------------------|------------------|-------------------|
| a) $\frac{1}{2}$ | b) $\frac{3}{10}$ | c) $\frac{3}{5}$ | d) $\frac{4}{5}$ |
| e) $\frac{5}{10}$ | f) $\frac{1}{4}$ | g) $\frac{3}{4}$ | h) $2\frac{1}{2}$ |

Ratio

When ratios are large or awkward it sometimes helps to simplify them.



Exercise 16.2

Simplify the following ratios:

$15 : 9$ $= \frac{15}{9}$ $= \frac{5 \times 3}{3 \times 3}$ $= \frac{5}{3} = \underline{5 : 3}$	$18 : 24$ $= \frac{18}{24}$ $= \frac{3 \times 6}{4 \times 6}$ $= \frac{3}{4} = \underline{3 : 4}$	$8x : 4x$ $= \frac{8x}{4x}$ $= \frac{2 \times 4 \times x}{1 \times 4 \times x}$ $= \frac{2}{1} = \underline{2 : 1}$
---	---	---

Calculators are good at simplifying ratios (see Technology 16.1)

- | | | |
|------------|-------------|------------|
| 1 8 : 4 | 2 3 : 12 | 3 12 : 16 |
| 4 3 : 9 | 5 6 : 9 | 6 15 : 5 |
| 7 25 : 15 | 8 24 : 18 | 9 5 : 15 |
| 10 28 : 21 | 11 40 : 12 | 12 20 : 35 |
| 13 2x : 8x | 14 9x : 15x | 15 6x : 9x |

Exercise 16.3

The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a total mixture of 4 L?	<p>Rose food is $\frac{1}{5}$ of the mixture</p> $= \frac{1}{5} \times 4 \text{ L}$ $= \underline{0.8 \text{ L}} \text{ or } 800 \text{ mL}$
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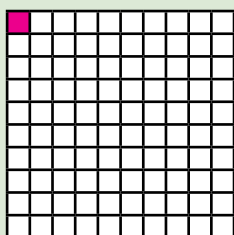
1 part to 4 parts
Total of 5 parts

- The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a mixture of 2 L?
- The fruit juice is made by mixing juice concentrate and water in the ratio of 1 : 3. How much juice concentrate is needed to make 2 L of fruit juice?
- The lunch bill for the 7 guests came to \$154. If Jessica paid for 3 of the guests and Aaron paid for the other 4 guests, how much did each pay?
- Eun-Young paid \$7 towards the Lottery ticket and Mark paid \$3. If the Lottery ticket returned \$30, how much should each receive?
- The concrete is to be made of cement, sand, and gravel in the ratio of 1 : 2 : 3. If the mixer takes 30 shovelfulls, how many shovelfulls of cement is needed?

Percentages

1% means 1 per 100.

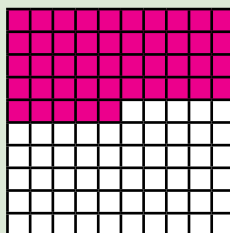
$$1\% = \frac{1}{100}$$



Percentages are used hundreds of thousands of times every day for comparison and analysis.

45% means 45 per 100

$$45\% = \frac{45}{100}$$



Exercise 16.4

Change the following decimals to a percentage:

$\times 100$ Move the decimal point 2 places left.

$$\begin{aligned} 0.2 &= 0.2 \times 100\% \\ &= \underline{20\%} \end{aligned}$$

$$\begin{aligned} 1.72 &= 1.72 \times 100\% \\ &= \underline{172\%} \end{aligned}$$

$$\begin{aligned} 0.0735 &= 0.0735 \times 100\% \\ &= \underline{7.35\%} \end{aligned}$$

- | | | | |
|----------|----------|---------|-----------|
| a) 0.1 | b) 0.2 | c) 0.3 | d) 0.4 |
| e) 0.5 | f) 0.6 | g) 0.7 | h) 0.8 |
| i) 0.9 | j) 1 | k) 1.1 | l) 1.3 |
| m) 0.25 | n) 0.75 | o) 0.42 | p) 0.92 |
| q) 2.3 | r) 0.63 | s) 3.45 | t) 6.2 |
| u) 0.235 | v) 2.306 | w) 0.04 | x) 0.0625 |

Exercise 16.5

Change the following fractions to a percentage:

Change to a percent:
 $\times 100$

$$\begin{aligned} \frac{7}{10} &= \frac{7}{10} \times \frac{100}{1} \% \\ &= \underline{70\%} \end{aligned}$$

$$\begin{aligned} \frac{2}{5} &= \frac{2}{5} \times \frac{100}{1} \% \\ &= \underline{40\%} \end{aligned}$$

$$\begin{aligned} \frac{3}{4} &= \frac{3}{4} \times \frac{100}{1} \% \\ &= \underline{75\%} \end{aligned}$$

- | | | | |
|-------------------|--------------------|-------------------|-------------------|
| a) $\frac{1}{10}$ | b) $\frac{2}{10}$ | c) $\frac{3}{10}$ | d) $\frac{4}{10}$ |
| e) $\frac{5}{10}$ | f) $\frac{6}{10}$ | g) $\frac{7}{10}$ | h) $\frac{8}{10}$ |
| i) $\frac{9}{10}$ | j) $\frac{10}{10}$ | k) $\frac{1}{2}$ | l) $\frac{1}{4}$ |
| m) $\frac{3}{4}$ | n) $\frac{1}{5}$ | o) $\frac{2}{5}$ | p) $\frac{4}{5}$ |

Percentages

It is useful to express one quantity as a percentage of a second quantity.

Exercise 16.6

What percentage is:

30 marks out of 50 marks $= \frac{30}{50} \times \frac{100}{1} \%$ $= \underline{60\%}$	13 L out of 20 L $= \frac{13}{20} \times \frac{100}{1} \%$ $= \underline{65\%}$
--	--



- | | |
|------------------------------|-----------------------------|
| a) 3 marks out of 10 marks? | b) 5 marks out of 10 marks? |
| c) 7 marks out of 10 marks? | d) 8 marks out of 10 marks? |
| e) 10 km out of 50 km? | f) 20 m out of 50 m? |
| g) 30 mins out of 50 mins? | h) \$40 out of \$50? |
| i) 45 m out of 50 m? | j) 50 km out of 50 km? |
| k) 14 books out of 20 books? | l) 15 cm out of 20 cm? |
| m) 16 weeks out of 20 weeks? | n) 17 kg out of 20 kg? |
| o) 46 mm out of 94 mm? | p) 3.15 g out of 9.87 g? |

Calculations often involve finding a percentage of an amount.



rearranged

Exercise 16.7

Calculate each of the following:

Find 10% of 80 $= \frac{10}{100} \times \frac{80}{1}$ $= \underline{8}$	Find 25% of 176 $= \frac{25}{100} \times \frac{176}{1}$ $= \underline{44}$	Find 6% of 540 $= \frac{6}{100} \times \frac{540}{1}$ $= \underline{32.4}$
--	---	---

- | | | | |
|---------------|---------------|--------------|--------------|
| a) 10% of 60 | b) 20% of 65 | c) 30% of 24 | d) 70% of 70 |
| e) 80% of 80 | f) 50% of 80 | g) 25% of 80 | h) 75% of 80 |
| i) 50% of 420 | j) 25% of 400 | k) 5% of 40 | l) 7% of 350 |
- m) About 70% of a 50 kg person's weight is water. How much of the 50 kg is water?
- n) 40% of a \$600 wage is spent on rent. How much is the rent?
- o) If 30% of a sale is profit, how much profit in a sale of \$9500?
- p) The management fund returned 10% on an investment of \$74 500. How much was returned?
- q) Business tax is 30%, What is the tax on a taxable income of \$75 000?
- r) Business tax is 30%, What is the tax on a taxable income of \$48 250?

Rates

A rate is a comparison of quantities of different kinds.

Example: Rent of \$380 per week compares the quantity \$ with the quantity week.

Exercise 16.8

1 Copy and complete the following table:

A few of the thousands of examples of rates		
Rate	Units	Example
a) Pay rates	Dollars per hour	\$23.50/h
b) Speed	Kilometres per hour	75 km/h
c) Cricket run rate	Runs per over	4.5 runs/over
d) Petrol price	Dollars per litre	
e)		
f)		
g)		
h)		
i)		
j)		

2 Write each of the following as a rate:

\$40 for 20 L of petrol $= \frac{40}{20}$ \$ per L $= \underline{\$2/L}$	250 mm of rain in 4 months. $= \frac{250}{4}$ mm per month $= \underline{62.5 \text{ mm per month}}$
--	--

- a) 100 metres in 10 seconds b) 80 points in 5 matches
 c) 120 litres pumped in 60 seconds d) 240 beats in 4 minutes
 e) 7000 people in 200 buses f) 275 marks in 5 tests

A house is let for \$350 per week. What is the rent for 4 weeks? $= 350 \times 4$ $= \underline{\$1400}$	If the price of diesel is \$2.10 per litre, what is the cost of 50 L of diesel? $= 2.10 \times 50$ $= \underline{\$105}$
---	---

- 3 A two bedroom unit is being let for \$465 pw (per week). What is the rent for:
 a) 4 weeks? b) 6 months (26 weeks)? c) 1 year (52 weeks)?
- 4 Megan earns a salary of \$1851.30 per fortnight, how much is earned in:
 a) a month (4 weeks)?
 b) 6 months (26 weeks)?
 c) 1 year (52 weeks)?

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally

Exercise 16.9

- 1 Spell percentage.
 - 2 Rent is \$350 per week. What is the rent for 4 weeks?
 - 3 What is 10% of 28?
 - 4 Simplify the ratio 10 : 8
- Given the data set: 2,2,2,4,5
- 5 What is the mode?
 - 6 What is the mean?
 - 7 Solve: $2x + 3 = 7$
 - 8 Simplify: $x^3 \times x^2$
 - 9 $2 + ^{-}4$
 - 10 If the price of unleaded petrol was \$1.50 per litre, what would it cost to fill a 40 L tank?

$$\begin{aligned} 10\% \text{ of } 28 \\ &= 1/10 \text{ of } 28 \\ &= 2.8 \end{aligned}$$

Exercise 16.10

- 1 Spell unitary.
 - 2 Rent is \$450 per week. What is the rent for 4 weeks?
 - 3 What is 10% of 31?
 - 4 Simplify the ratio 10 : 16
- Given the data set: 1,2,2,2,5
- 5 What is the median?
 - 6 What is the mean?
 - 7 Solve: $3x + 2 = 14$
 - 8 Simplify: $b^4 \times b^2$
 - 9 $^{-}2 + 5$
 - 10 If the price of unleaded petrol was \$1.60 per litre, what would it cost to fill a 50 L tank?

$$\begin{aligned} 10:16 \\ &= \frac{5 \times 2}{8 \times 2} \\ &= \frac{5}{8} \\ &= 5:8 \end{aligned}$$

Exercise 16.11

- 1 Spell discount.
 - 2 Rent is \$550 per week. What is the rent for 4 weeks?
 - 3 What is 10% of 74?
 - 4 Simplify the ratio 10 : 14
- Given the data set: 1,1,1,3,4
- 5 What is the mode?
 - 6 What is the mean?
 - 7 Solve: $5x + 3 = 18$
 - 8 Simplify: $c^5 \times c^3$
 - 9 $3 + ^{-}7$
 - 10 If the price of unleaded petrol was \$1.70 per litre, what would it cost to fill a 40 L tank?



The only place you'll find success before work is in the dictionary - May B Smith.

NAPLAN Questions

Exercise 16.12



- 1 Bucksalot the horse walked for 10 minutes at a rate of 120 metres per minute. How far did Bucksalot walk?
- 2 Josh worked for 6 hours and was paid \$25 per hour. How much did Josh earn?
- 3 The hire car uses 6 litres of petrol for every 100 km. How much petrol is needed to travel 450 km?
- 4 If light travels at 300 000 km per second. How far will light travel in 1 minute?
- 5 How many grams of chocolate can be bought for \$9?
- 6 A shower nozzle with a rate of 20 litres per minute is replaced with a nozzle with a rate of 16 litres of water per minute. How much water is saved in a $3\frac{1}{2}$ minute shower?



Chocolate
\$12 per 100 g



- 7 How much pumpkin is needed to make enough pumpkin soup to serve 48 people?
- 8 There is 45 kg of mortar. The ratio of cement to sand in the mortar mix is 1 to 4. How many kg of cement is in the mortar?
- 9 If 1 Australian dollar buys 1.05 US dollars, how many US dollars could be bought with 200 Australian dollars using this exchange rate?
- 10 The car uses 5 L of petrol per 100 km. How far would the car travel on \$30 of petrol at \$1.50 per litre?



Pumpkin Soup

Serves 6

750 g pumpkin
250 g potatoes
2 medium onions
3 chicken stock cubes
 $3\frac{1}{2}$ cups water
salt and pepper to taste



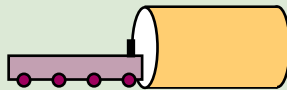
Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

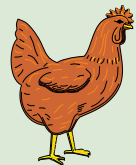


Exercise 16.13

- 1 The straight line distance from Melbourne to Cairns is roughly 2400 km. What would be the average speed of an aeroplane in order to get from Melbourne to Cairns in 4 hours?
- 2 The road distance from Melbourne to Cairns is roughly 3000 km. How long would it take a truck to get from Melbourne to Cairns at an average speed of 90 km/h non-stop?
- 3 A truck is travelling at a speed of 90 km/h. How far does the truck travel in 5 seconds?



- 4 A coal train 1 km long is travelling at a constant speed of 30 km/h. How long does it take the train to pass through a tunnel 1 km long?
- 5 Australia's birth rate is roughly 13 per 1000 people and Australia's death rate is roughly 7 per 1000 people. If Australia's population is 22 million, what is Australia's expected yearly increase in population (neglecting immigration increases)?
- 6 An athlete runs 80 m in 9.6 seconds. At this speed, how long would it take the athlete to run 100 m?
- 7 A poultry hopper has enough grain to feed 8 hens for 6 days. For how many days would the hopper feed 12 hens?
- 8 A poultry hopper has enough grain to feed 8 hens for 6 days or 16 chickens for 6 days. For how many days would the hopper feed 8 hens and 8 chickens?
- 9 Pump A can fill a tank in 30 minutes and pump B can fill the same tank in 20 minutes. How long will it take both pumps, working together, to fill the tank?
- 10 Simplify the ratio $24 : 36 : 15 : 27$
- 11 A, B, C, D share \$500 in the ratio $1 : 2 : 3 : 4$. How much does B get?
- 12 A, B, C, D share in the ratio $1 : 2 : 3 : 4$. If B gets \$500, what was the original total?

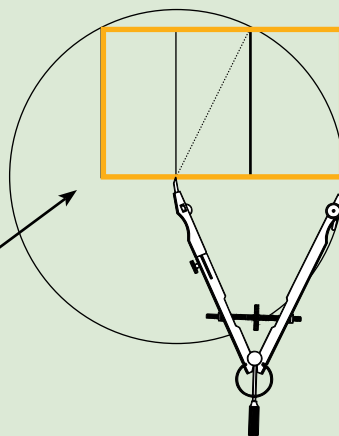


Investigations

The Golden Rectangle is said to be the most visually pleasing of all geometric shapes.

Investigation 16.1 Draw a Golden Rectangle

- 1 Draw a square.
- 2 Divide the square in two.
- 3 Draw a diagonal on one half.
- 4 With a pair of compasses, radius equal to the diagonal, extend the base.
- 5 Complete the Golden Rectangle.



$$\text{Golden Ratio} = \frac{\text{width}}{\text{depth}}$$

$$= ?$$

Investigation 16.2 Golden ratio

Investigate Leonardo Da Vinci's belief about some body measurements and the Golden Ratio.

Leonardo Da Vinci believed that some body measurements matched the Golden Ratio.

- 1 Measure height.
- 2 Measure height of navel.
- 3 Calculate the ratio = $\frac{\text{height}}{\text{navel height}}$

Research the 'Vitruvian Man'.

- 1 Stretch arm and measure from armpit to tip of finger.
- 2 Measure elbow to tip of finger.
- 3 Calculate the ratio = $\frac{\text{arm length}}{\text{elbow to finger}}$

The Golden Ratio, to three decimal places, is 1.618.

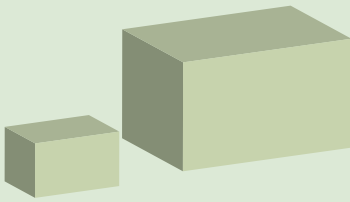
Mining Surveyors survey underground and open-cut mines.

- Relevant school subjects are English and Mathematics.
- Courses usually involve a University Bachelor degree.

A Couple of Puzzles

Exercise 16.14

- 1 Adrian bought a car for \$5000. Sold it for \$4500. Bought it back again for \$3000 and sold it for \$4000. How much profit or loss did Adrian make?



- 2 The smaller block has a mass of 5 kg. The larger block is twice as wide, twice as long, and twice as deep. What is the mass of the larger block?

- 3 Using only the numbers 1, 2, 3, 4, complete the grid so that every column, row, and diagonal sums to 10.

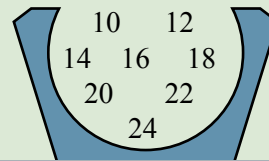
1			
	2		
		3	
			4

A Game

Cross

Each of two players take turns to choose two numbers from the pot.

- Multiply the two numbers.
- Put your mark on your answer on the board.
No answer - no mark.
- When all numbers have been marked, the player with the largest number of marks wins.

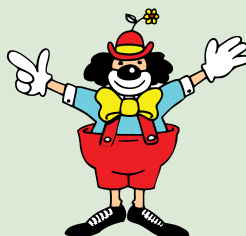


264	352	216	192	336
432	240	280	120	200
440	224	252	528	220
240	320	308	360	140
396	180	480	288	384

A Sweet Trick

Ask your audience to add the last two digits of their birth year to the age they will turn on their birthday this year.

Why?



All answers will be the same.

In 2012 it will be 112.
In 2013 it will be 113.
In 2014 it will be 114.
In 2015 it will be 115.

Technology

Technology 16.1 Simplifying ratios

Scientific calculators are excellent in working with fractions:

1 Simplify $\frac{15}{35}$ $\boxed{15} \boxed{a \frac{b}{c}} \boxed{35} \boxed{=} \boxed{3r7}$ meaning $\frac{3}{7}$

2 Simplify $\frac{18}{4}$ $\boxed{18} \boxed{a \frac{b}{c}} \boxed{4} \boxed{=} \boxed{4r1r2}$ meaning $4\frac{1}{2}$

To change to a vulgar fraction: $\boxed{2ndF} \boxed{a \frac{b}{c}}$ to give $\boxed{9r2}$ ie $\frac{9}{2}$

3 Use a scientific calculator to simplify the following ratios:

a) $3 : 9$

b) $9 : 12$

c) $16 : 24$

d) $2.1 : 3.5$

e) $14.4 : 12.6$

f) $256 : 1024$

Q What are the next two terms: O T T F F S S E _ _
A N T - Nine Ten.

Technology 16.2 Calculator Challenge

1 Enter the number 6294 on your calculator.

In one operation, wipe out the 4.

Example:

$6294 - 4 = 6290$

In one operation,
wipe out the 9.

In one operation,
wipe out the 2.

In one operation,
wipe out the 6.

Show 10 different ways to wipe
ou the 9 in one operation.

2 Assume that the keys 4 and 7 are broken:

Get the calculator to show 47.

Example:

$32 + 15 = 47$

Get the calculator
to show 777

Get the calculator
to show 444

Get the calculator
to show 77.44

Get the calculator to
show 7474.7474

Chapter Review 1

Exercise 16.15

- 1 The Maths class has 11 girls and 13 boys.
 - a) What is the ratio of girls to boys?
 - b) What is the ratio of boys to girls?
 - c) What is the ratio of girls to the total number in the class?
 - d) What is the ratio of boys to the total number in the class?
- 2 Write each of the following ratios as a fraction, a decimal and a percentage:
 - a) 1 : 10 b) 3 : 10 c) 2 : 5 d) 3 : 5
 - e) 1 : 2 f) 1 : 4 g) 3 : 4 h) 3 : 20
- 3 Write each of the following fractions as a ratio, a decimal and a percentage:
 - a) $\frac{1}{2}$ b) $\frac{3}{10}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$
- 4 Simplify the following ratios:
 - a) 8 : 4 b) 5 : 10 c) 12 : 8
 - d) 15 : 10 e) 6x : 15x f) 6x : 9x
- 5 The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a mixture of 4 L?
- 6 The fruit juice is made by mixing juice concentrate and water in the ratio of 1 : 3. How much juice concentrate is needed to make 2 L of fruit juice?
- 7 What percentage is:
 - a) 5 marks out of 10 marks? b) 7 marks out of 10 marks?
 - c) 30 m out of 100 m? d) \$30 out of \$50?
 - e) 16 km out of 20 km? f) 19 mL out of 20 mL?
- 8 Calculate each of the following:
 - a) 10% of 40 b) 50% of 40 c) 25% of 40 d) 75% of 40
 - e) 30% of a \$600 wage is spent on rent. How much is the rent?
- 9 Write each of the following as a rate:
 - a) 100 metres in 10 seconds b) 60 points in 5 matches
 - c) 100 litres pumped in 50 seconds d) 320 beats in 4 minutes
 - e) 45 trees in 5 hours f) \$650 in 5 sales
- 10 Sebastian earns a salary of \$2087.40 per fortnight, how much is earned in:
 - a) a month (4 weeks)?
 - b) 6 months (26 weeks)?
 - c) 1 year (52 weeks)?

Chapter Review 2

Exercise 16.16

- 1 Last month there were 22 sunny days and 9 cloudy days.
 - a) What is the ratio of sunny to cloudy days?
 - b) What is the ratio of cloudy to sunny days?
 - c) What is the ratio of sunny to the total number of days in the month?
 - d) What is the ratio of cloudy to the total number of days in the month?
- 2 Write each of the following ratios as a fraction, a decimal and a percentage:
 - a) 1 : 10 b) 2 : 10 c) 1 : 5 d) 2 : 5
 - e) 1 : 2 f) 1 : 4 g) 3 : 4 h) 5 : 20
- 3 Write each of the following fractions as a ratio, a decimal and a percentage:
 - a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{2}{5}$
- 4 Simplify the following ratios:
 - a) 6 : 3 b) 4 : 10 c) 6 : 8
 - d) 10 : 15 e) 4x : 12x f) 8x : 10x
- 5 The fungicide is mixed with water in the ratio of 1 : 3. How much rose food is needed to make a mixture of 3 L?
- 6 The mortar is made by mixing cement and sand in the ratio of 1 : 4. How much cement is needed to make 40 shovelfulls of mortar?
- 7 What percentage is:
 - a) 7 marks out of 10 marks? b) 9 marks out of 10 marks?
 - c) 60 km out of 100 km? d) \$40 out of \$50?
 - e) 15 tanks out of 20 tanks? f) 17 sheep out of 20 sheep?
- 8 Calculate each of the following:
 - a) 10% of 20 b) 50% of 20 c) 25% of 20 d) 75% of 20
 - e) 40% of a \$700 wage is spent on rent. How much is the rent?
- 9 Write each of the following as a rate:
 - a) 80 metres in 10 seconds b) 30 trucks in 5 days
 - c) 90 stitches 10 wounds d) 360 beats in 5 minutes
 - e) 70 rows in 5 hours f) \$4850 for 20 accounts
- 10 Tim earns a salary of \$2867.40 per fortnight, how much is earned in:
 - a) a month (4 weeks)?
 - b) 6 months (26 weeks)?
 - c) 1 year (52 weeks)?

Linear Equations

17

- ★ Solve simple linear equations.
- ★ Solve real-life problems by using pronumerals to represent unknowns.
- ★ Use travel graphs to investigate and compare the distance travelled to and from school
- ★ Interpret features of travel graphs such as the slope of lines and the meaning of horizontal lines.
- ★ Use graphs of evaporation rates to explore water storage.

A TASK

I meet up with lions most days.



How much water is lost in evaporation?

- Fill a bucket with water and mark the level.
- Measure the changes in level each day.
- Graph your results.

Discuss your results (is the rate of evaporation linear?, can you estimate the volume of water lost to evaporation each week from your suburb's water supply? What can be done to reduce the loss?)

A LITTLE BIT OF HISTORY

Ahmes (c. 1550 BC), an Egyptian scribe and mathematician, wrote linear equation problems on papyrus.

Diophantus (c. 250 AD) is considered to be the 'Father of algebra'.

Al-Khwarizmi (c. 825 AD) wrote the Arabic work *Hisāb al-jabr w'al-muqābala*. The work appears to be the first to mention the term 'al jabr' and involved algebra in the reduction of equations.

Francois Viète, in the sixteenth century, appears to be the first to use the algebra notation that is now used throughout the World.

The World's oldest equation?

Hau (heap) its whole, its seventh, it makes 19 (Ahmes, 1550 BC).



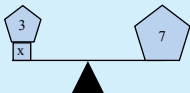
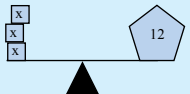
Symbolic Equations

When equations are written with symbols the equations become easier to solve, especially complicated equations.

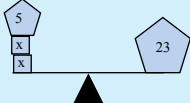
Hundreds of thousands of problems are solved every day throughout Australasia by a problem being written as a symbolic equation and then by solving the equation.

Exercise 17.1

Rewrite each of the following word equations as symbolic equations:

Word equation	Symbolic equation
When a number is added to three the answer is seven.	$\underline{x + 3 = 7}$ 
A number decreased by five is equal to eight.	$\underline{x - 5 = 8}$
When a number is multiplied by three the result is twelve.	$\underline{3x = 12}$ 
When a number is divided by four the result is two.	$\underline{x \div 4 = 2}$

- 1 When a number is added to five the result is nine.
- 2 When six is subtracted from a number the answer is ten.
- 3 When a number is multiplied by five the answer is eighteen.
- 4 When a number is divided by three the answer is seven.

Word equation	Symbolic equation
When a number is doubled and then five added, the result is twenty-three.	$\underline{2x + 5 = 23}$ 
Fourteen less than half of a number is thirty-six.	$\underline{x \div 2 - 14 = 36}$

- 5 When a number is tripled and then six added, the result is thirty-three.
- 6 Twelve less than half of a number is twenty-one.
- 7 Multiply a number by seven, then add four and the result is twenty-eight.
- 8 Divide a number by nine and subtract three to give an answer of twenty-one.
- 9 When a number is trebled and then eight is subtracted, the result is seventy-two.
- 10 Divide the number by six and then subtract one to get eight as the answer.

Solve Equations

Solve equations:
Find the value of the unknown.

- 1 Write the flow of the equation.
- 2 Backtrack (inverse operations).

Exercise 17.2

Use backtracking to solve the following equations:

$x + 5 = 9$

Draw the equation flow \rightarrow x $\xrightarrow{+5}$ x + 5

||

Draw the backtrack \rightarrow 4 $\xleftarrow{-5}$ 9

$x = 4$

- | | |
|-----------------|----------------|
| 1 $x + 5 = 7$ | 2 $x + 7 = 11$ |
| 3 $a + 2 = 14$ | 4 $b + 7 = 22$ |
| 5 $x + 24 = 28$ | 6 $x + 8 = 5$ |

Inverse of	+	is	-
	-	is	+
	\times	is	\div
	\div	is	\times

$x + 7 = 9$ $x = 9 - 7$ {inverse of + is -} <u>$x = 2$</u>	$x - 4 = 7$ $x = 7 + 4$ {inverse of - is +} <u>$x = 11$</u>
$2x = 9$ $x = 9 \div 2$ {inverse of \times is \div } <u>$x = 4.5$</u>	$x \div 5 = 6$ $x = 6 \times 5$ {inverse of \div is \times } <u>$x = 30$</u>

- | | |
|-------------------|--------------------|
| 7 $x + 3 = 8$ | 8 $x + 2 = 13$ |
| 9 $y - 7 = 12$ | 10 $a - 5 = 16$ |
| 11 $6x = 18$ | 12 $5b = 20$ |
| 13 $x \div 2 = 5$ | 14 $c \div 6 = 7$ |
| 15 $x + 8 = 13$ | 16 $b - 5 = 12$ |
| 17 $3m = 45$ | 18 $x \div 3 = 9$ |
| 19 $d + 3 = 36$ | 20 $p - 17 = 20$ |
| 21 $7h = 35$ | 22 $a \div 7 = 11$ |
| 23 $x + 8 = 3$ | 24 $4x - 9 = 4$ |

Solving Equations

Solve equations:
Find the value of the unknown.

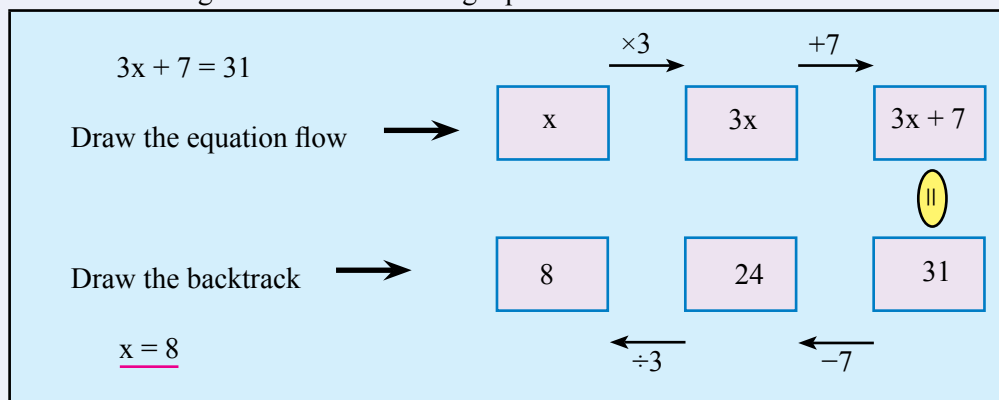
- 1 Write the flow of the equation.
- 2 Backtrack (inverse operations).

Inverse of

+	is	-
-	is	+
×	is	÷
÷	is	×

Exercise 17.3

Use backtracking to solve the following equations:



1 $2x + 5 = 17$

2 $3x + 4 = 13$

3 $4a + 6 = 14$

4 $5b + 7 = 22$

$2x + 7 = 15$

$2x = 15 - 7$ {inverse of + is -}

$2x = 8$

$x = 8 \div 2$ {inverse of \times is \div }

$x = 4$

$(x - 4) \div 3 = 4$

$x - 4 = 4 \times 3$ {inverse of \div is \times }

$x - 4 = 12$

$x = 12 + 4$ {inverse of \div is \times }

$x = 16$

Inverse of

+	is	-
-	is	+
×	is	÷
÷	is	×

5 $2x + 3 = 7$

6 $3x + 2 = 8$

7 $5y + 7 = 17$

8 $4x + 3 = 15$

9 $4r - 8 = 4$

11 $3v - 2 = 7$

13 $(a - 5) \div 2 = 3$

15 $(w - 3) \div 4 = 5$

17 $m \div 4 + 3 = 6$

19 $h \div 3 + 4 = 9$

21 $2x + 3 = 8$

10 $5b - 7 = 8$

12 $2g - 6 = 8$

14 $(t - 6) \div 3 = 6$

16 $(s - 2) \div 3 = 6$

18 $n \div 2 + 1 = 4$

20 $b \div 7 + 3 = 5$

22 $4r - 9 = 4$

Linear Equations

Exercise 17.4

Given the function, $y = 2x + 1$,

- complete the table.
- draw a graph of the function.
- Is the function linear or non-linear?

A linear pattern increases by the same amount each step.

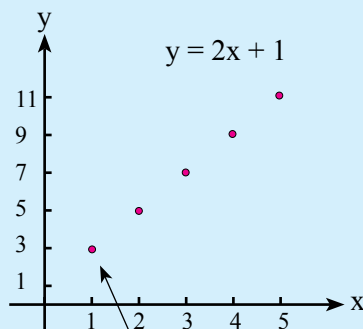
A non-linear pattern does not increase by the same amount each step.

x	1	2	3	4	5
$y=2x+1$					

When $x=1$, $y=2 \times 1 + 1 = 3$
 $x=2$, $y=2 \times 2 + 1 = 5$
 $x=3$, $y=2 \times 3 + 1 = 7$
 $x=4$, $y=2 \times 4 + 1 = 9$
 $x=5$, $y=2 \times 5 + 1 = 11$

x	1	2	3	4	5
$y=2x+1$	3	5	7	9	11

The function is linear because it increases by the same amount, 2, each step, and the graph of the function is linear.



To plot: $x = 1, y = 3$
 Start at the origin
 Go right 1
 Go up 3

1 $y = 2x + 2$

3 $y = 2x + 4$

5 $y = 2x - 2$

7 $y = 3x + 1$

9 $y = 3x + 3$

11 $y = 3x - 2$

13 $y = 3x + 0.5$

15 $y = 0.5x + 1$

17 $y = 0.5x + 3$

19 $y = 1.5x + 1$

21 $y = 1.5x - 1$

23 $y = 2(x + 1)$

25 $y = x(x + 1)$

27 $y = (x + 5) \div x$

2 $y = 2x + 3$

4 $y = 2x - 1$

6 $y = 2x$

8 $y = 3x + 2$

10 $y = 3x - 1$

12 $y = 3x - 3$

14 $y = 3x$

16 $y = 0.5x + 2$

18 $y = 0.5x$

20 $y = 1.5x + 2$

22 $y = 1.5x$

24 $y = x \div 2 + 3$

26 $y = (x + 3) \div 2$

28 $y = 2.5x + 1.5$



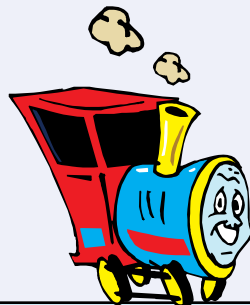
Plot the:
 x first
 y second.

Travel Graphs

Exercise 17.5

A train is travelling at a constant speed of 30 km/h.
The distance travelled is given by the function: $D = 30t$
where D is the distance in km, and t is the time in hours.

- Complete the table.
- Draw a graph of the function.
- Is the function linear or non-linear?

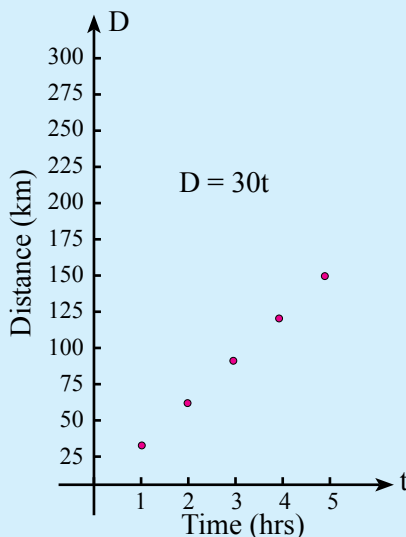


t	1	2	3	4	5
$D = 30t$					

When $x=1$, $y=30 \times 1=30$
 $x=2$, $y=30 \times 2=60$
 $x=3$, $y=30 \times 3=90$
 $x=4$, $y=30 \times 4=120$
 $x=5$, $y=30 \times 5=150$

t	1	2	3	4	5
$D=30t$	30	60	90	120	150

The function is linear because it increases by the same amount, 30, each step, and the graph of the function is linear.



- A train is travelling at a constant speed of 20 km/h.
The distance travelled is given by the function: $D = 20t$
where D is the distance in km, and t is the time in hours.
 - complete the table.
 - draw a graph of the function.
 - Is the function linear or non-linear?
- A train is travelling at a constant speed of 40 km/h.
The distance travelled is given by the function: $D = 40t$
where D is the distance in km, and t is the time in hours.
 - complete the table.
 - draw a graph of the function.
 - Is the function linear or non-linear?
- A train is travelling at a constant speed of 50 km/h.
The distance travelled is given by the function: $D = 50t$
where D is the distance in km, and t is the time in hours.
 - complete the table.
 - draw a graph of the function.
 - Is the function linear or non-linear?

Did you notice that the faster the speed the steeper the slope of the line?

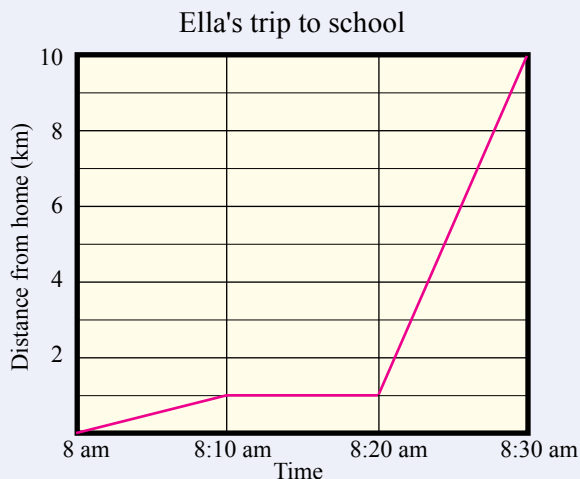


Travel Graphs

Exercise 17.6

Ella lives 10 km from school.
Ella walks to the bus station, 1 km,
and then catches a bus to the
school.

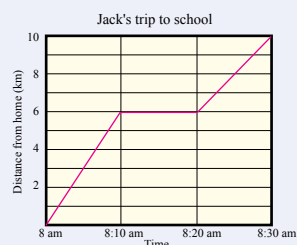
- How long did it take Ella to walk to the bus station?
- How long did Ella wait for the bus?
- How long did it take the bus to get from the bus station to school?
- Can you calculate the speed of the bus?
- Can you calculate Ella's walking speed?



- | | |
|---|--|
| <ol style="list-style-type: none"> 10 mins 10 mins 10 mins | <ol style="list-style-type: none"> The bus travelled 10–1 km in 10 minutes
 $= 9 \times 6 \text{ km in 60 minutes}$
 $= \underline{54 \text{ km/h}}$ {60mins= 1 hour} Ella walked 1 km in 10 minutes
 $= 1 \times 6 \text{ km in 60 minutes}$
 $= \underline{6 \text{ km/h}}$ {60mins= 1 hour} |
|---|--|

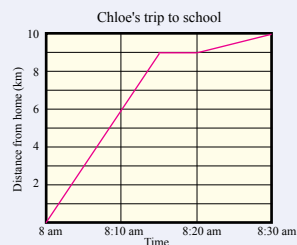
- Jack lives 10 km from school and uses his bike to travel to school. The graph shows Jack's trip to school. Jack first rode his bike to his friend's place and they then travelled together to school.

- How long did it take Jack to get to his friend's place?
- How long did Jack wait at his friend's place?
- Which cycling speed was greatest?
- Calculate Jack's speed in getting to his friend's place.
- Calculate Jack's speed from his friend's place to school.



- Chloe lives 10 km from school. Chloe's mum drops her off at her friend's place. Chloe and her friend then walk to school together.

- How long did it take Chloe to get to her friend's place?
- How long did Chloe wait at her friend's place?
- Calculate Chloe's speed in getting to her friend's place.
- Calculate Chloe's walking speed.



Solving Equations

Solve equations:
Find the value of the unknown.

- 1 Write the flow of the equation.
- 2 Backtrack (inverse operations).

Inverse of	+	is	-
	-	is	+
	\times	is	\div
	\div	is	\times

Exercise 17.7

A train is travelling at a constant speed of 40 km/h. The distance travelled is given by the function: $D = 40t$, where D is the distance in km, and t is the time in hours.

- a)** How far does the train travel in 3 hours?

$$D = 40 \times 3$$

$$\underline{D = 120 \text{ km}}$$

- b)** How long does it take the train to travel 100 km?

$$100 = 40t$$

$$100 \div 40 = t \quad \{\text{inverse of } \times \text{ is } \div\}$$

$$\underline{2.5 \text{ hours}} = t \quad \{\text{or } 2 \text{ h } 30 \text{ mins}\}$$

- 1 A train is travelling at a constant speed of 50 km/h. The distance travelled is given by the function: $D = 50t$, where D is the distance in km, and t is the time in hours.

- a)** How far does the train travel in 3 hours?

- b)** How long does it take the train to travel 110 km?

0.2 hours $= 0.2 \times 60 \text{ mins}$ $= 12 \text{ mins}$
--

- 2 A train is travelling at a constant speed of 30 km/h. The distance travelled is given by the function: $D = 30t$, where D is the distance in km, and t is the time in hours.

- a)** How far does the train travel in 3 hours?

- b)** How long does it take the train to travel 120 km?

- 3 Jack is riding his bike at a constant speed of 25 km/h. The distance travelled is given by the function: $D = 25t$, where D is the distance in km, and t is the time in hours.

- a)** How far does Jack travel in 30 mins (ie., 0.5 h)?

- b)** How long does it Jack to travel 10 km?

- 4 The taxi charge, in \$, is given by the function $C = 3k + 2$, where k is the number of kilometres travelled.

- a)** What is the taxi charge for a journey of 10 km?

- b)** Find the distance travelled if the taxi charge was \$21.50.

- 5 The plumber's charge, in \$, is given by the function $C = 70t + 50$, where t is the time in hours.

- a)** What is the charge for a plumbing job that takes 5 hours?

- b)** Find the time of a job if the charge was \$295.

Mental Computation

Exercise 17.8

- 1 Spell linear.
- 2 Solve: $5x = 15$
- 3 Solve: $3x + 2 = 11$
- 4 What is 10% of 62?
- 5 $2 + ^{-}5$
- 6 What is the median of: 1,3,4,6,8?
- 7 Simplify: 5^0
- 8 Write in index form: $3 \times 3 \times 3 \times 3$
- 9 Simplify: $m^2 \times m^3$
- 10 Simplify: $2^8 \div 2^6$

Mental computation can make problems easier and quicker.

You cannot plough a field by turning it over in your mind
- Anonymous.

Exercise 17.9

- 1 Spell equation.
- 2 Solve: $x \div 4 = 3$
- 3 Solve: $2x - 5 = 3$
- 4 What is 10% of 45?
- 5 $3 + ^{-}4$
- 6 What is the mean of: 1,1,1,2,3?
- 7 Simplify: 4^0
- 8 Write in index form: $2 \times 2 \times 2 \times 2 \times 2$
- 9 Simplify: $b^2 \times b^4$
- 10 Simplify: $5^5 \div 5^3$

Imagine you are in a sinking rowboat surrounded by sharks. How would you survive?

Quit imagining!

Exercise 17.10

- 1 Spell function.
- 2 Solve: $x - 3 = 6$
- 3 Solve: $4x - 2 = 6$
- 4 What is 10% of 76?
- 5 $5 + ^{-}9$
- 6 What is the mode of: 1,1,1,2,3?
- 7 Simplify: 7^0
- 8 Write in index form: $5 \times 5 \times 5 \times 5 \times 5$
- 9 Simplify: $d^2 \times d^3$
- 10 Simplify: $3^6 \div 3^3$

Real Estate Agents use good communication and negotiation skills to arrange the sale of houses, businesses, etc.

- Relevant school subjects are English and Mathematics.
- Courses range from Traineeships to Certificate and Diploma courses.

NAPLAN Questions

Exercise 17.11



- 1 If $x = 2$, what is the value of $5x$?
- 2 If $x = 3$, what is the value of $4x + 5$?
- 3 If $x = 4$, what is the value of $3x - 15$?
- 4 $y = 15 - 2x$. What is the value of y when $x = 3.75$?
- 5 Complete each of the following tables:

a)

t	1	2	3	4	5
$D = 30t$	30	60			

b)

x	1	2	3	4	5
$y = 4x + 3$	7	11			

- 6 What is the next number in the following pattern?

$\times 4 - 3$

→

$1, 5, 9, ?$



- 7 Solve each of the following:
 - a) When a number is doubled and then 12 is added the answer is 23.
 - b) When a number is tripled and 5 subtracted, the answer is 16.
 - c) When a number is halved and then 7 is added the answer is 10.
 - d) When a number decreased by five and then halved, the answer is 19.

- 8 Which rule applies to the table?

a) $y = x + 6$

b) $y = 4x$

c) $y = 3x - 2$

x	1	2	3	4	5
y	4	8	12	16	20

- 9 Which rule applies to the table?

a) $y = 5x$

b) $y = 3x + 2$

c) $y = 2x + 3$

x	1	2	3	4	5
y	5	7	9	11	13

- 10 Which rule applies to the table?

a) $y = 4x - 3$

b) $y = 3x - 4$

c) $y = 5x - 4$

x	1	2	3	4	5
y	1	5	9	13	17

Competition Questions

Exercise 17.12



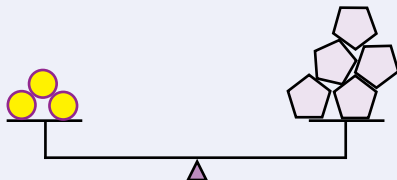
Prepare for mathematics competitions and build maths muscle at the same time.

1

$$\frac{2}{3} \times ? - 2 = 6$$

2

What is the mass of 1 ?



3

Two numbers sum to 50. When five is added to both numbers, the larger is then twice the size of the smaller. What are the original two numbers?

4

Two containers hold a total of 50 balls. When 6 balls are shifted from the first container to the second container, the second container has four as many balls as the first. How many balls are now in the second container?

5

Two containers hold a total of 100 balls. When 26 balls are shifted from the first container to the second container, the second container has four as many balls as the first. How many balls are now in the second container?

6

Grandmother received a lump sum. After giving one-quarter of the money to Alf and one-third of the money to Fay, she had \$1750 left. What was the value of the original lump sum?

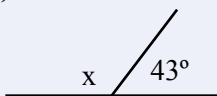
7

$$\frac{3}{4} \times ? + \frac{1}{3} = \frac{19}{3}$$

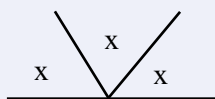
8

Find the value of x :

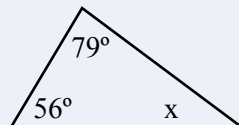
a)



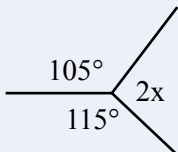
b)



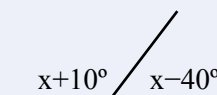
c)



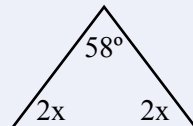
d)



e)



f)



Technology

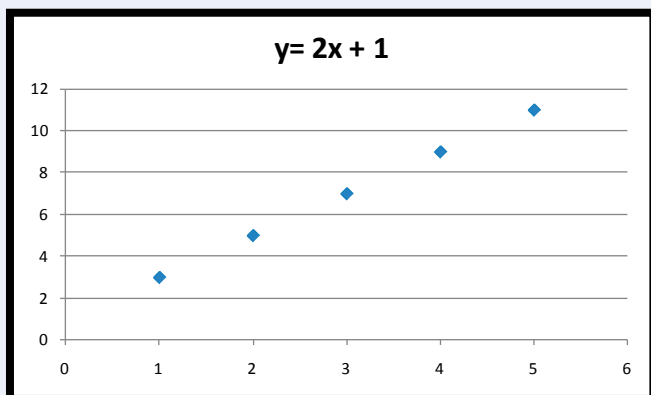
Technology 17.1 Linear equations and the Spreadsheet

Use a spreadsheet to graph the functions in Exercise 17.4 and 17.5.

$$y = 2x + 1$$

	A	B	C	D	E	F
1	x	1	2	3	4	5
2	y	3	5	7	9	11

Enter the formula:
=2*b1+1



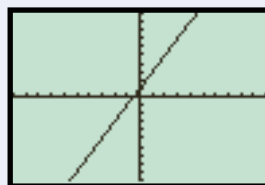
Technology 17.2 Linear equations and the Graphics Calculator

Use a graphics calculator to graph the functions in Exercise 17.4 and 17.5.

$$y = 2x + 1$$

Press **Y=** and enter the function $2x + 1$.

Press **GRAPH** to see a plot of the function.



Press **TABLE** to see a table of the values.

X	Y1
0	1
1	3
2	5
3	7
4	9
5	11

A Couple of Puzzles

Exercise 17.13

- 1 Who am I? If you subtract me from 29 and then double the answer, the result is 38.
- 2 Who am I? I'm a two-digit number. If you reverse my digits and add us together, the result is 55.
- 3 It is now 8 am. What will be the time in 123 hours?

4 The average of 3 numbers 5, 6, x is 7. The value of x?

A Game

Leap Frog. Leap frog is a game played by one.

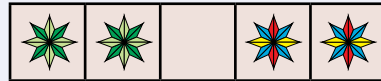
The aim of the game is to swap the counters from each end of the board to the other end of the board.

- ☺ A counter can move one space at a time towards the other end of the board.
- ☺ A counter cannot move backwards.
- ☺ A counter can jump over an opposite counter.
- ☺ A counter cannot jump over a similar counter.

Beginning of the game.



End of the game.



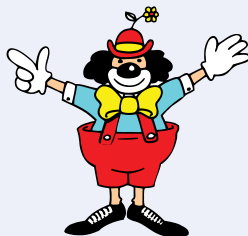
A Sweet Trick

Ask your audience to number themselves 1, 2, 3, etc.

Someone hides a coin in their left or right hand.

With a calculator they do the following:

- | | |
|---|----------------------|
| 1 Enter the number of the person with the coin. | 5 |
| 2 Multiply by 4 | $5 \times 4 = 20$ |
| 3 Add 13 | $20 + 13 = 33$ |
| 4 Multiply by 15 | $33 \times 15 = 495$ |
| 5 Add 9 | $495 + 9 = 504$ |
| 6 Divide by 6 | $504 \div 6 = 84$ |
| 7 If in the left hand, add 1. Add 2 for the right hand. | $84 + 2 = 86$ |
| 8 Add 37 | $86 + 37 = 123$ |



Subtract 71 from their answer ($123 - 71 = 52$)
 5 tells you person 5.
 2 tells you right hand.

Investigations

Investigations 17.1 Falling Stone

The equation for a falling stone.

$$t = 0.45\sqrt{h}$$

t is the time in seconds.
h is the height in metres.

A compass and a variable jump off a cliff at the same time. Which will land first?

The compass. The variable has to ask for directions.

Investigate

Drop stones from various heights and time them.

Graph your results.

Is it a linear function?

Does the above equation work?

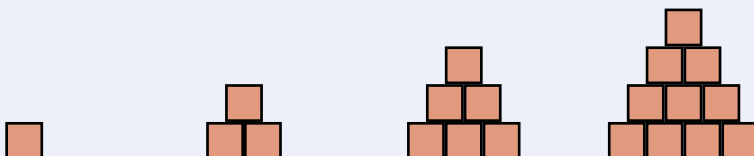
$$h = 4.9t^2$$

Does this equation also work?

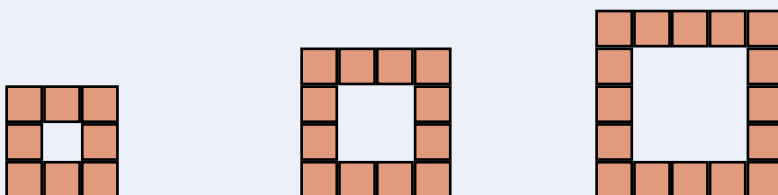
Investigations 17.2 Patterns

Investigate each of the following patterns:

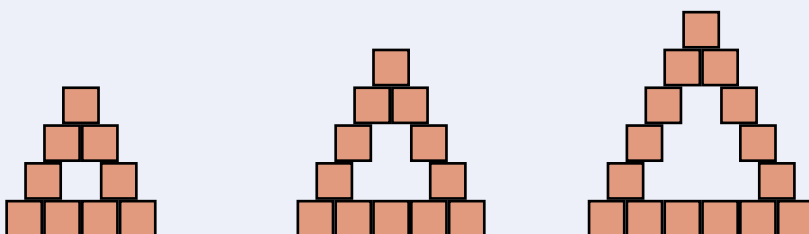
a)



b)



c)



Chapter Review 1

Exercise 17.14

- Rewrite each of the following word equations as symbolic equations:
 - When a number is added to six the result is fourteen.
 - When three is subtracted from a number the answer is seven.
 - When a number is multiplied by seven the answer is thirty-five.
 - When a number is divided by two the answer is fifteen.
 - When a number is tripled and then four added, the result is twenty-two.
 - Eight less than half of a number is thirty-six.

- Solve each of the following equations:

- | | |
|-------------------------|-------------------------|
| a) $x + 2 = 9$ | b) $y - 4 = 17$ |
| c) $5x = 20$ | d) $x \div 3 = 8$ |
| e) $2x + 7 = 15$ | f) $3r - 8 = 4$ |
| g) $(x - 2) \div 2 = 6$ | h) $n \div 3 + 13 = 24$ |

- For each of the following functions:

- Complete the table.
- Draw a graph of the function.
- Is the function linear or non-linear?

a)

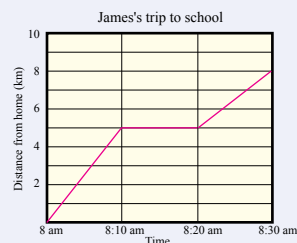
x	1	2	3	4	5
$y = 2x + 3$					

b)

t	1	2	3	4	5
$D = 30t$					

- James lives 8 km from school and uses his bike to travel to school. The graph shows James's trip to school. James first rode his bike to his friend's place and they then travelled together to school.

- How long did it take James to get to his friend's place?
- How long did James wait at his friend's place?
- Which cycling speed was greatest?
- Calculate James's speed in getting to his friend's place.
- Calculate James's speed from his friend's place to school.



- A train is travelling at a constant speed of 50 km/h. The distance travelled is given by the function: $D = 50t$, where D is the distance in km, and t is the time in hours.
 - How far does the train travel in 2 hours?
 - How long does it take the train to travel 120 km?

Chapter Review 2

Exercise 17.15

- Rewrite each of the following word equations as symbolic equations:
 - When a number is added to three the result is fifteen.
 - When seven is subtracted from a number the answer is nine.
 - When a number is multiplied by four the answer is twenty-five.
 - When a number is divided by three the answer is sixty-two.
 - When a number is doubled and then one added, the result is thirty-six.
 - Eight less than one-third of a number is fifty-two.

- Solve each of the following equations:

- | | |
|-------------------------|------------------------|
| a) $x + 3 = 7$ | b) $y - 6 = 32$ |
| c) $4x = 28$ | d) $x \div 4 = 9$ |
| e) $3x + 4 = 13$ | f) $2b - 8 = 5$ |
| g) $(x - 1) \div 4 = 5$ | h) $b \div 3 + 6 = 21$ |

- For each of the following functions:

- Complete the table.
- Draw a graph of the function.
- Is the function linear or non-linear?

a)

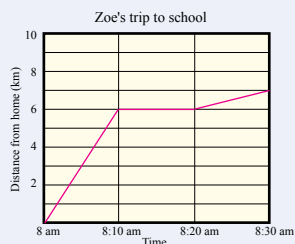
x	1	2	3	4	5
$y = 3x + 1$					

b)

t	1	2	3	4	5
$D = 40t$					

- Zoe lives 7 km from school. Zoe's mum drops her off at her friend's place. Zoe and her friend then walk to school together.

- How long did it take Zoe to get to her friend's place?
- How long did Zoe wait at her friend's place?
- Calculate Zoe's speed in getting to her friend's place.
- Calculate Zoe's walking speed.



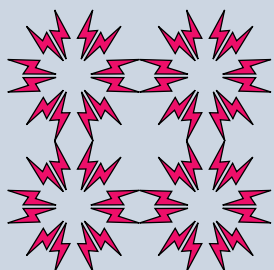
- A train is travelling at a constant speed of 60 km/h. The distance travelled is given by the function: $D = 60t$, where D is the distance in km, and t is the time in hours.
 - How far does the train travel in 2 hours?
 - How long does it take the train to travel 150 km?

Transformations

18

- ★ Describe translations, reflections, and rotations.
- ★ Identify line and rotational symmetries.
- ★ Identify combinations of transformations that produce the same result.
- ★ Using digital technologies to experiment with patterns.

A TASK



Design a wallpaper pattern.

- There are only 17 ways in which a pattern can repeat?
- Research wallpaper patterns.
- Choose a fundamental shape.
- Choose a wallpaper pattern.
- Construct your wallpaper pattern (use software?).
- Present your wallpaper pattern.

A LITTLE BIT OF HISTORY

Tribal art tattoos have clear strong black arcing lines and mirror-image patterns.

Polynesian Tribal art tattoos were strictly banned when the missionaries arrived around 1800 AD.

The Polynesian designs were almost lost apart from detailed sketches by one of the missionaries.

Polynesia, including New Zealand and Tonga, is a group of over a thousand islands in the south Pacific Ocean.

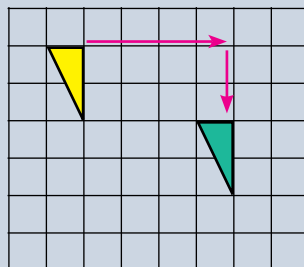


Translation

A **translation** slides a shape across a page.

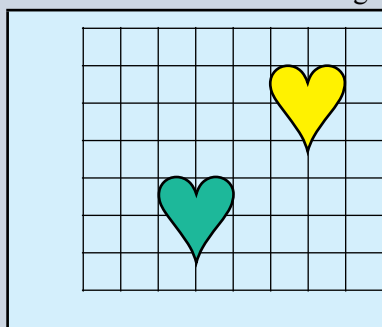
The shape is moved without being rotated, reflected, or changed in size.

The triangle has been translated:
4 units to the right and
2 units down.



Exercise 18.1

Describe each of the following translations (from yellow to green):



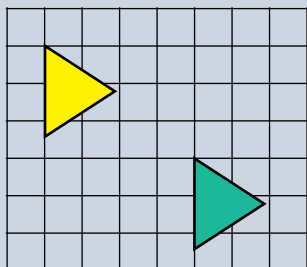
The heart shape has been translated:
3 units left and
3 units down.

or

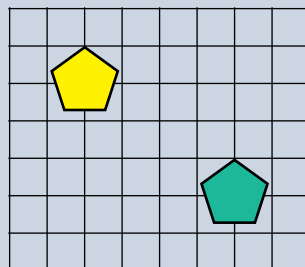
3 units down and
3 units left.

Translation is commutative.

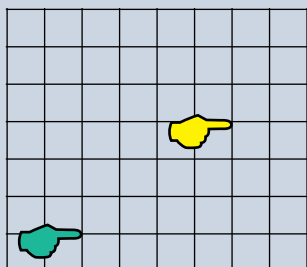
1



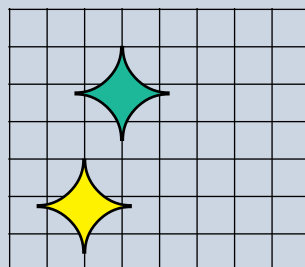
2



3



4



Translation

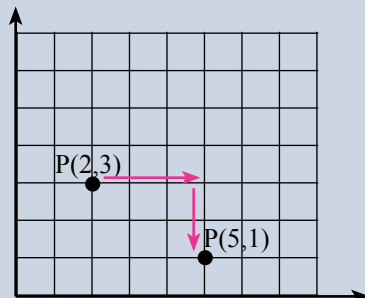
Exercise 18.2

What are the new coordinates after:

The point $P(2,3)$ is translated 3 units right and 2 units down?

$$= P(2+3, 3-2)$$

$$= \underline{P(5,1)}$$

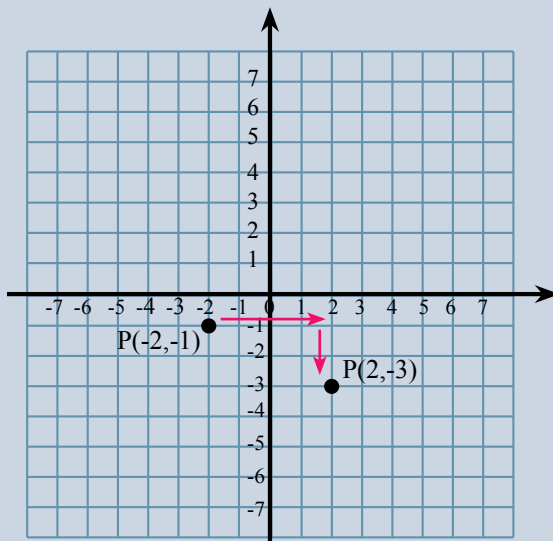


- 1 The point $P(1,3)$ is translated 2 units right and 2 units down?
- 2 The point $P(2,3)$ is translated 3 units left and 1 unit down?
- 3 The point $P(5,4)$ is translated 3 units left and 2 units up?
- 4 The point $P(1,1)$ is translated 6 units right and 3 units up?

The point $P(-2,-1)$ is translated 4 units right and 2 units down?

$$= P(-2+4, -1-2)$$

$$= \underline{P(2,-3)}$$



- 5 The point $P(-1,3)$ is translated 3 units right and 2 units down?
- 6 The point $P(4,-3)$ is translated 3 units left and 1 unit up?
- 7 The point $P(-2,2)$ is translated 3 units left and 2 units down?
- 8 The point $P(-1,-1)$ is translated 5 units right and 1 unit up?

Geometry transformations

- ✓ Translations
- ✓ Rotations
- ✓ Reflections

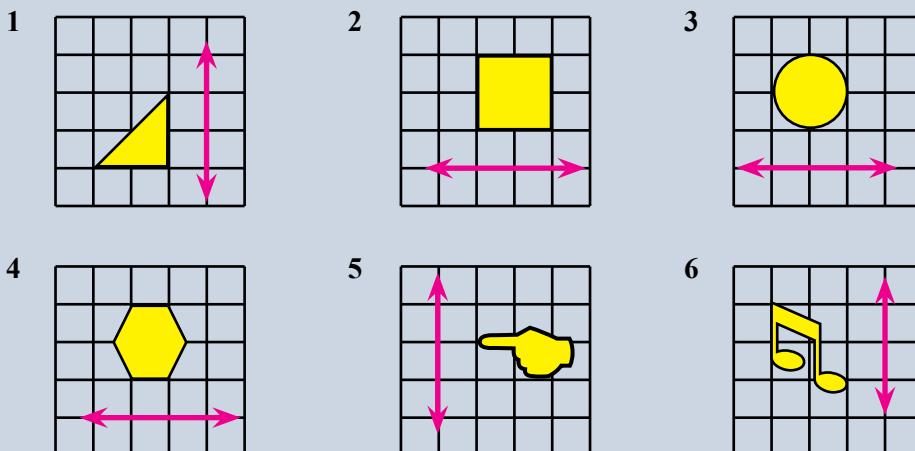
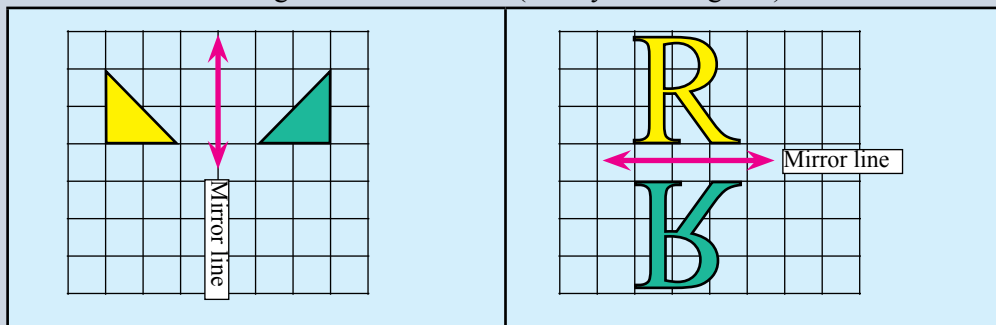
Reflection

A **reflection** is a mirror image.

The shape is reflected without being moved, rotated, or changed in size.

Exercise 18.3

Draw the reflected image in the mirror line (from yellow to green):



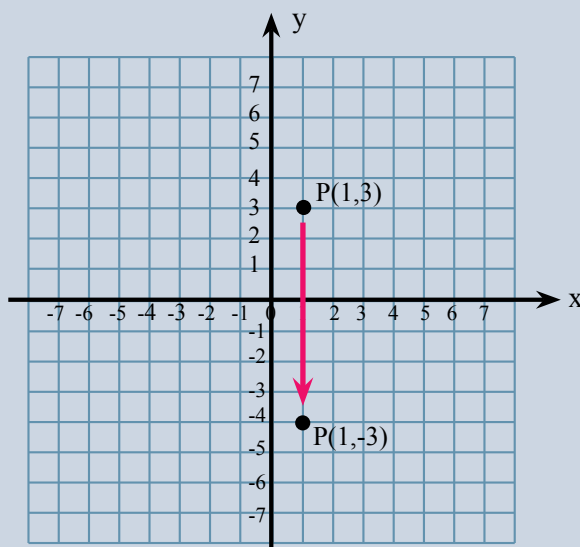
Exercise 18.4

What are the new coordinates after:

The point $P(1,3)$ is reflected in the x-axis?

$$= \underline{P(1,-3)}$$

- 1 The point $P(3,4)$ is reflected in the x-axis?
- 2 The point $P(-2,2)$ is reflected in the x-axis?
- 3 The point $P(-3,-3)$ is reflected in the x-axis?



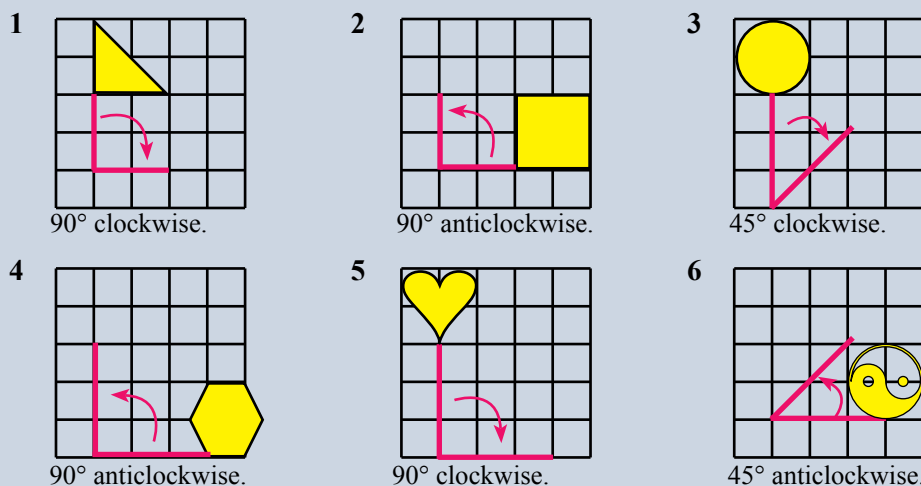
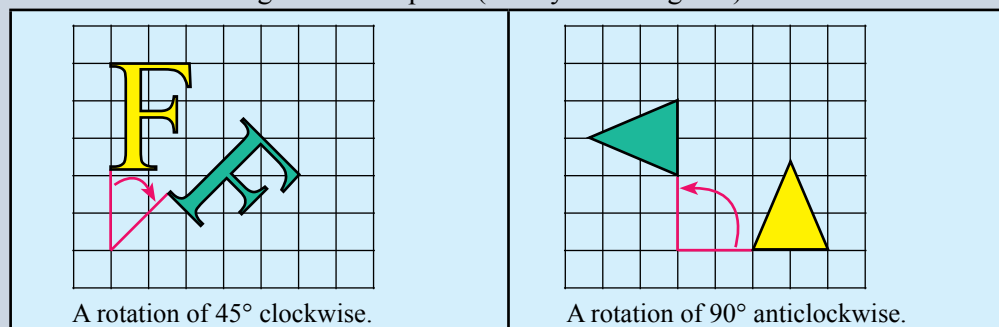
Rotation

A **rotation** turns a shape about a point.

The shape is rotated without being moved, reflected, or changed in size.

Exercise 18.5

Draw the rotated image about the point (from yellow to green):



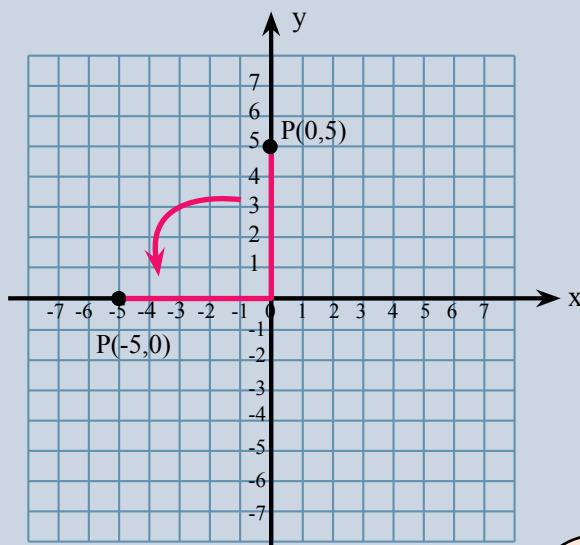
Exercise 18.6

What are the new coordinates after:

The point $P(0,5)$ is rotated 90° anticlockwise about the origin?

$$= \underline{P(-5,0)}$$

- 1 The point $P(0,6)$ is rotated 90° anticlockwise about the origin?
- 2 The point $P(-6,0)$ is rotated 90° clockwise about the origin?
- 3 The point $P(0,-4)$ is rotated 90° anticlockwise about the origin?



Combined Transformations

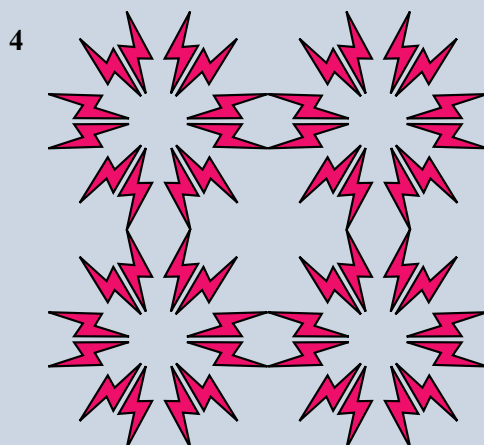
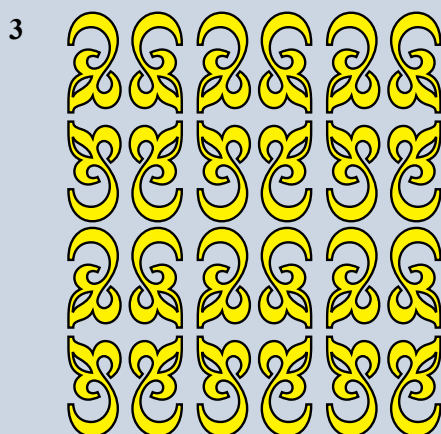
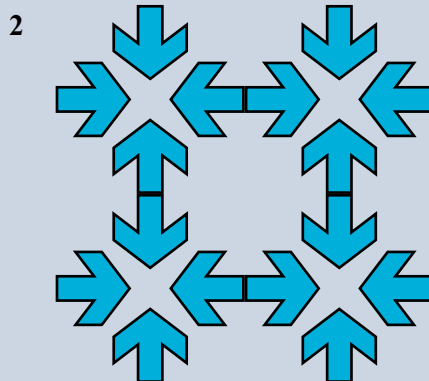
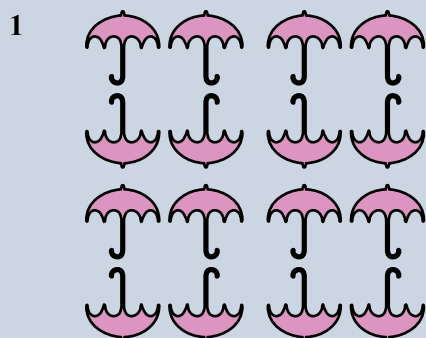
Wallpaper patterns.

A shape may be repeatedly translated, rotated, and reflected to produce a pattern..

Exercise 18.7

- Describe transformations of the original shape to produce the following pattern (there is often more than one answer).
- Replace the shape with your own shape and redraw the pattern.

	<p>The umbrella has been reflected through a vertical axis to produce:</p> <p>The pair of umbrellas have then been reflected through a horizontal axis to produce:</p> <p>The four umbrellas are then translated many times to produce the pattern.</p>
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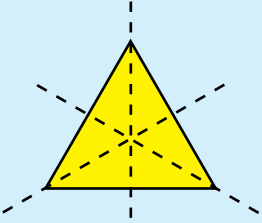
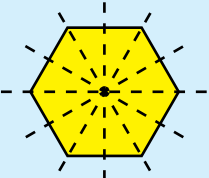

Symmetry

A line of symmetry divides a shape into two equal halves. The halves are a mirror image of each other.

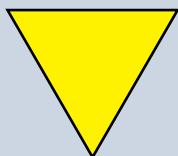
The shape can be folded into two halves that fit exactly on top of each other.

Exercise 18.8

- Copy each shape and draw the line(s) of symmetry if they exist:
- Verify each line of symmetry by making the shape and folding along the line of symmetry (each half should match).

<p>Equilateral triangle</p>  <p>3 lines of symmetry</p>	<p>Regular hexagon</p>  <p>6 lines of symmetry</p>	<p>Parallelogram</p>  <p>No line of symmetry.</p>
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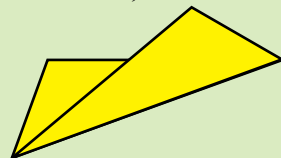
1 Equilateral triangle



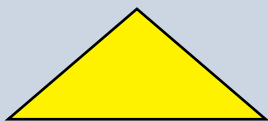
2 Square



If the parallelogram is folded over a diagonal, the halves don't fit on top of each other, no mirror image.



3 Isosceles triangle



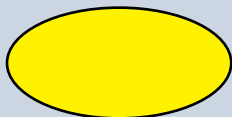
4 Rectangle



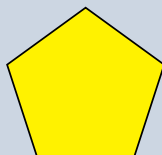
5 Parallelogram



6 Ellipse



7 Pentagon



8 'Charlie'




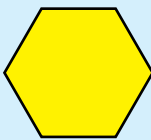

Symmetry

A shape has rotational symmetry when the shape looks the same after turned around a centre point a certain number of degrees.

Rotational symmetry order is the number of matches in a 360° rotation.

Exercise 18.9

- What is the rotational symmetry order of each of the following:
- Verify the rotational symmetry order by copying the shape and counting the number of matches as you rotate the shape.

<p>Equilateral triangle</p>  <p>Order = 3</p>	<p>Regular hexagon</p>  <p>Order = 6</p>	<p>Parallelogram</p>  <p>Order = 1</p>
--	---	--

1 Star

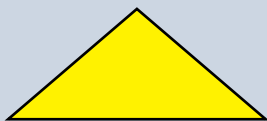


2 Square

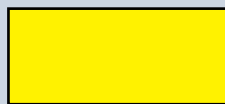


The parallelogram has to be rotated a full 360° before it matches itself again. Many say that an order of just 1 is not really rotational symmetry.

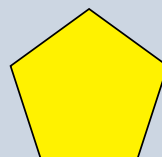
3 Equilateral triangle



4 Rectangle



5 Parallelogram



6 Ellipse



7 Flower



8 Star Fish



Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 18.10

- 1 Spell transformation.
- 2 Translate $P(2,3)$ 2 units to the right and 4 units up.
- 3 How many lines of symmetry in a square?
- 4 25×12
- 5 What is 20% of 31.
- 6 Solve: $3x + 2 = 11$
- 7 $3 + ^{-}5$
- 8 Write in index form: $2 \times 2 \times 2 \times 2 \times 2$
- 9 Simplify: $a^4 \times a^3$
- 10 Simplify: $3^7 \div 3^4$

$$\begin{aligned}25 \times 12 \\&= 25 \times 4 \times 3 \\&= 100 \times 3 \\&= 300\end{aligned}$$

Never forget that your weapon is made by the lowest bidder. -
Murphy's Laws of Combat.

Exercise 18.11

- 1 Spell translation.
- 2 Translate $P(1,3)$ 3 units to the right and 2 units down.
- 3 How many lines of symmetry in an equilateral triangle?
- 4 25×16
- 5 What is 20% of 24.
- 6 Solve: $2x - 3 = 15$
- 7 $^{-}2 + ^{-}3$
- 8 Write in index form: $3 \times 3 \times 3$
- 9 Simplify: $b^3 \times b^3$
- 10 Simplify: $2^6 \div 2^3$

$$\begin{aligned}20\% \text{ of } 24 \\&= 2 \times 10\% \text{ of } 24 \\&= 2 \times 2.4 \\&= 4.8\end{aligned}$$

Great works are performed, not by strength, but by perseverance - Samuel Johnson.

Exercise 18.12

- 1 Spell reflection.
- 2 Translate $P(5,3)$ 3 units to the left and 1 unit down.
- 3 How many lines of symmetry in a parallelogram?
- 4 25×20
- 5 What is 20% of 12.
- 6 Solve: $4x + 5 = 11$
- 7 $^{-}6 + 2$
- 8 Write in index form: $5 \times 5 \times 5 \times 5$
- 9 Simplify: $x^2 \times x^3$
- 10 Simplify: $4^5 \div 4^2$

NAPLAN Questions



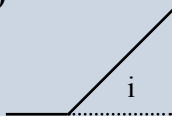
Exercise 18.13

1 Which of the following shows an incline of 60° ?

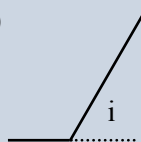
a)



b)

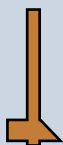


c)



2 Which image shows a 180° rotation of the shape?

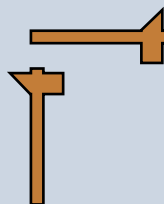
a)



b)



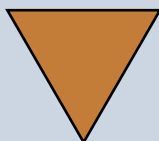
c)



A line of symmetry divides a shape into two equal halves. The halves are a mirror image of each other.

3 Draw the lines of symmetry of each of the following shapes:

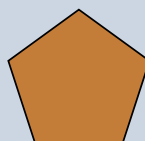
a)



b)



c)



4 Which shape does not have a line of symmetry?

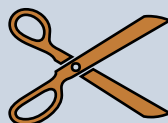
a)



b)



c)



5 Which shape does not have rotational symmetry?

a)



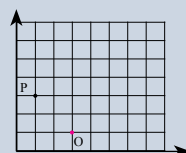
b)



c)



6 The point $P(1,3)$ is rotated 90° clockwise about O . What are the new coordinates of point P ?



Competition Questions




Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 18.14

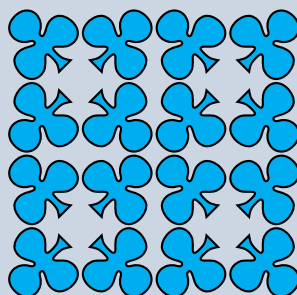
1 The second photo of 'Charlie' is a:

- a) translation?
- b) reflection?
- c) rotation?



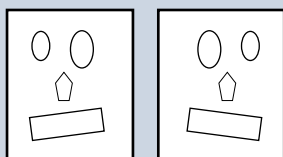
2 The transformations of the original  to form the pattern could not have been a:

- a) rotation of 45° , reflection in x-axis, reflection in y-axis, repeated translations?
- b) rotation of 225° , reflection in x-axis, reflection in y-axis, repeated translations?
- c) rotation of 270° , reflection in x-axis, reflection in y-axis, repeated translations?

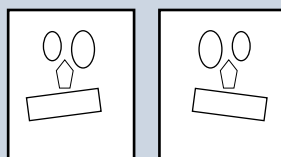


3 Which pair is not a true mirror-image?

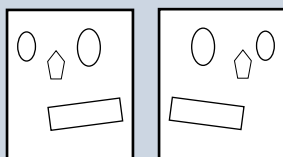
a)



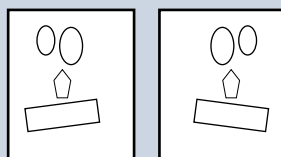
b)



c)



d)

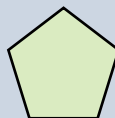


4 Which regular polygon has a rotational symmetry of 135° ?

a)

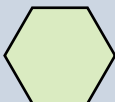


b)

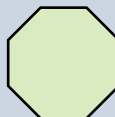


Regular means each side is the same length.

c)



d)



Technology

Technology 18.1 Internet Games

There are numerous games on the Internet that involve reflection, rotation, and translation.

Play some of these games.

Technology 18.2 Internet Puzzles

There are numerous puzzles on the Internet that involve reflection, rotation, and translation.

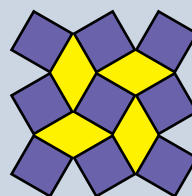
Solve some of these puzzles.

Technology 18.3 Wallpaper Software

Wallpaper patterns involve translations, reflections, and rotations of initial shapes to produce a distinctive pattern.

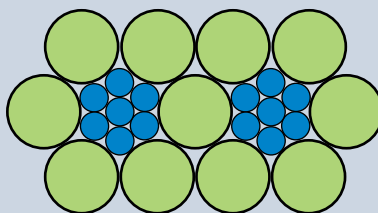
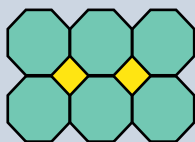


A wallpaper pattern found with an Egyptian mummy.



There are many Wallpaper software applications that help you create two-dimensional patterns using wallpaper symmetry groups.

Essentially the software helps you construct the initial tile and automatically makes transformations of the initial tile.



Experiment with Wallpaper software.

Illustrators design and produce drawings and layouts for advertising, magazines, newspapers, film, television etc.

- Relevant school subjects are English and Mathematics.
- Courses range from Diploma, to Advanced Diplomas, and University Degrees.

Investigations

Investigation 18.1 Reflection

- 1 Write your name on a sheet of paper and use a mirror to see what the reflected image looks like.
- 2 Now, can you write your name on a sheet of paper so that it reads correctly in the mirror?



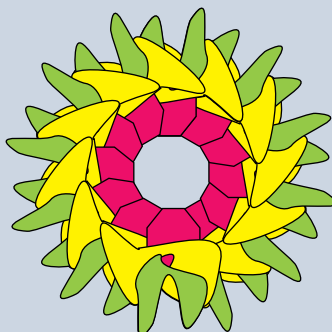
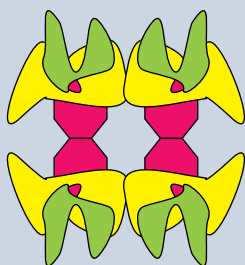
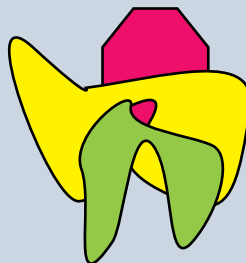
Investigation 18.2 Face Symmetry

- 1 Use a digital camera to get a photo of your face.
- 2 Use graphic software to crop one-half of your face.
- 3 Use reflection tools to make composites of your left side and your right side.
- 4 How symmetrical is your face?



Investigation 18.3 Making Shapes

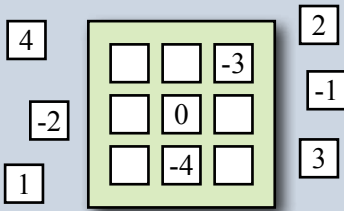
- 1 Use graphic software to make a shape.
- 2 Use the reflection, rotation tools, and translations (copy/paste/move) to make patterns.



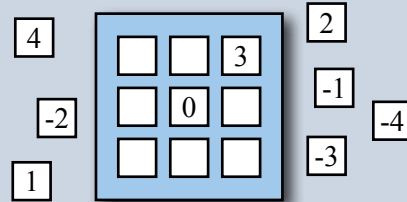
A Couple of Puzzles

Exercise 18.15

- 1 What are the next two numbers in the sequence: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \dots$,



- 2 Place the tiles so that every row, column, and diagonal sum to 0.

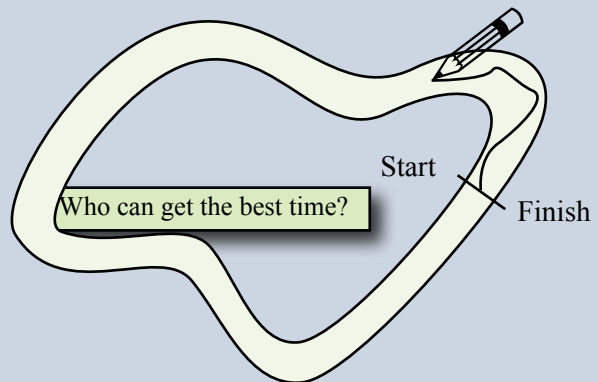


- 3 Place the tiles so that every row, column, and diagonal sum to 0.

A Game

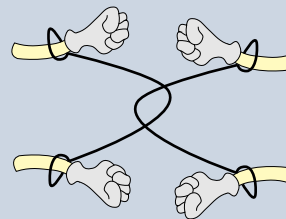
Reflection Race

- 1 Draw a racetrack.
- 2 Place the racetrack in front of a mirror.
- 3 Race a biro around the racetrack by **looking in the mirror only**. If your biro smashes into a wall your race has ended and you start again.

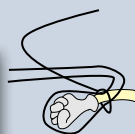


A Sweet Trick

- 1 Have two members of your audience tied up as shown in the diagram.
- 2 Challenge them to free themselves without cutting or untying the rope.
- 3 You free them because they can't.



Loop one rope through the wrist band of the other, over the hand and back through the wrist.



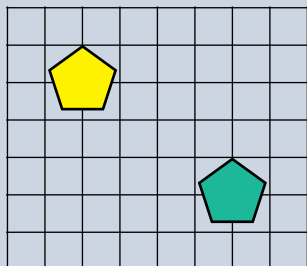
With a bit of practice, you will be amazed at how simple this is.

Chapter Review 1

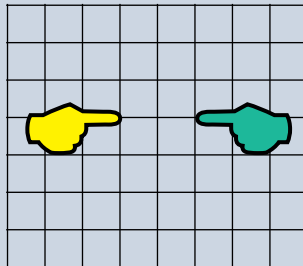
Exercise 18.16

1 Describe each of the following transformations from yellow to green:

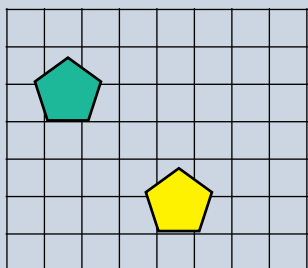
a)



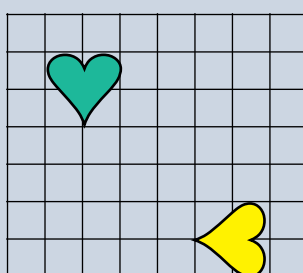
b)



a)



b)



2 What are the new coordinates after:

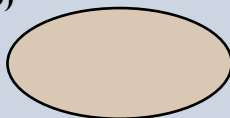
- The point $P(1,3)$ is translated 2 units right and 2 units down?
- The point $P(-2,2)$ is translated 3 units left and 2 units down?
- The point $P(3,4)$ is reflected in the x-axis?
- The point $P(-2,2)$ is reflected in the x-axis?
- The point $P(0,6)$ is rotated 90° anticlockwise about the origin?
- The point $P(-6,0)$ is rotated 90° clockwise about the origin?

3 Draw the lines of symmetry of each of the following shapes:

a)



b)



c)



4 What is the rotational symmetry order of each of the following:

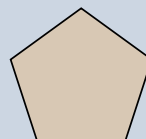
a)



b)



c)

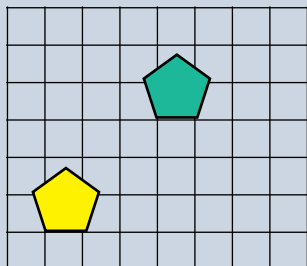


Chapter Review 2

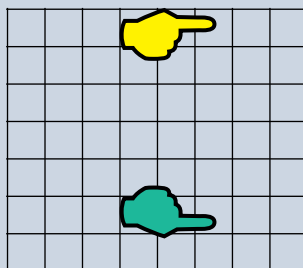
Exercise 18.17

1 Describe each of the following transformations from yellow to green:

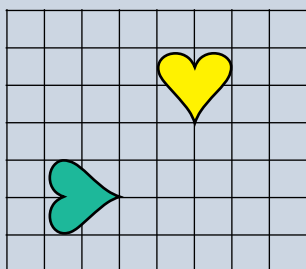
a)



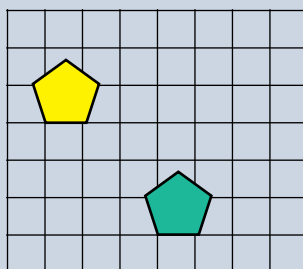
b)



a)



b)



2 What are the new coordinates after:

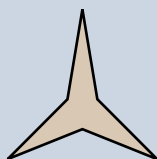
- The point $P(2,3)$ is translated 2 units right and 3 units down?
- The point $P(2,-2)$ is translated 2 units left and 4 units down?
- The point $P(2,5)$ is reflected in the x-axis?
- The point $P(-1,2)$ is reflected in the x-axis?
- The point $P(0,3)$ is rotated 90° clockwise about the origin?
- The point $P(-3,0)$ is rotated 90° anticlockwise about the origin?

3 Draw the lines of symmetry of each of the following shapes:

a)



b)



c)

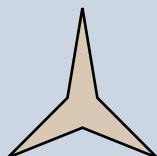


4 What is the rotational symmetry order of each of the following:

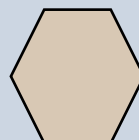
a)



b)



c)



Data 2

19

- ★ Identify and investigate issues collected from primary and secondary sources.
- ★ Investigate secondary data sets to answer comparative questions.
- ★ Construct and compare a range of data displays.
- ★ Use ordered stem-and-leaf plots to record and display numerical data collected in a class investigation.
- ★ Investigate relationships between data.



Research helps us understand the world around us.
Research saves lives.

A TASK

Conduct relationship research:

- Brainstorm a research question.
- Collect data.
- Represent the data (Use one of the methods in this chapter to display the data).
- Interpret the data and draw a conclusion related to the original research question.
- Publish your research.

A LITTLE BIT OF HISTORY

- 1583 Galileo Galilei's research with a pendulum led to the discovery of **the relationship between the length of a pendulum and its swinging time** (the period).

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- 1656 Christian Huygen made the first pendulum clock with an error of less than 1 min per day.

- 1721 George Graham improved the accuracy of the pendulum clock to less than 1 sec per day.

Huygen's design



Comparative Research

Research Question:

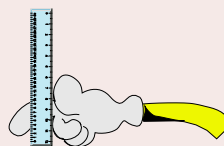
Are the reaction times of girls and boys the same?

Univariate Data

One measurement only
(Eg., reaction time).

Collect Data

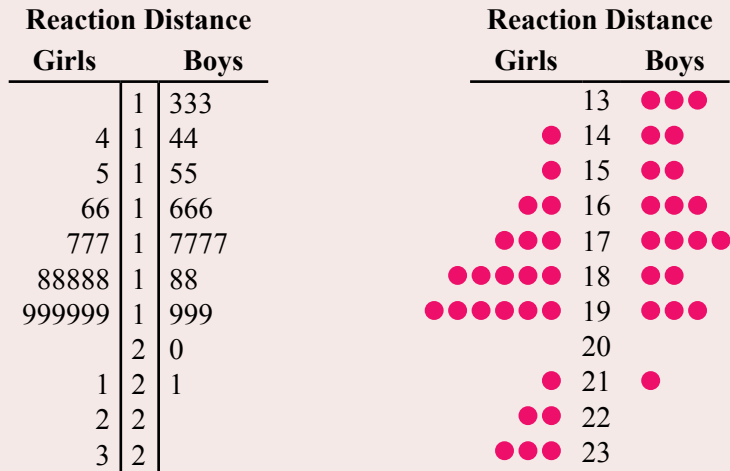
A centimetre ruler is dangled between the fingers of the catcher. The ruler is dropped and where the catcher grabs the ruler with the fingers is the reaction 'distance'.



Reaction Distance (girls)	Reaction Distance (boys)
18 23 19 17 19 15 18	16 19 17 13 18 19 20
14 17 19 16 22 19 17	13 18 15 19 14 17 16
19 18 21 18 19 18 16	17 13 16 17 14 21 15

Represent Data

Use back-to-back stem plots or parallel dot plots to compare univariate data.



Interpret Data and Draw Conclusion

Visually, the girl's reaction distance is grouped around 17, 18, and 19 cm with 6 students around 21, 22, and 23 cm. The boy's reaction distance is evenly distributed from 13 cm to 19 cm. There is little doubt that the girl's reaction distance is longer than the boy's reaction distance.

For this sample, the boys have a faster reaction time than the girls.

Exercise 19.1

1 Conduct the following univariate comparative research:

- Research question: Does movement increase heart rate?
- Collect data (Either collect your own data or use the following data).

Sitting at desk (beats per min)	Walking to class (beats per min)
72 81 54 68 76 91 51	84 78 73 98 83 91 88
64 72 61 58 47 59 67	67 98 75 61 99 89 112
84 73 51 69 56 73 66	97 79 58 79 85 93 107
84 52 50 64	67 95 86 70

- Represent the data (Use back-to-back stem plots or parallel dot plots to compare the univariate data).
- Interpret the data and draw a conclusion related to the original research question.

2 Conduct the following univariate comparative research:

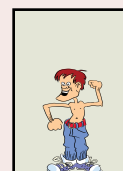
- Research question: Did the tourist promotion campaign increase customers?
- Collect data: A Chamber of Commerce collected the following data before and after a tourist promotion campaign. The data involved the daily number of customers at various outlets.

Before	After
76 56 74 58 82 67 79	48 88 78 55 44 81 127
55 81 39 65 81 24 45	67 67 48 31 76 92 105
54 42 38 23 93 42 81	74 64 80 77 48 59 99
75 86 12	57 70 54

- Represent the data (Use back-to-back stem plots or parallel dot plots to compare the univariate data).
- Interpret the data and draw a conclusion related to the original research question.

3 Conduct research to answer the following research questions:

- Are the results of the term 1 test the same as the results of the term 2 test?
- Will the results on the second diagnostic test be better than the results on the first diagnostic test.
- Will skinfold measurements before lunch be smaller than skinfold measurements after lunch?
- Are we taller in the morning than we are in the afternoon?
- Do girls send more text messages than boys?



Relationship Research

Research Question:

Are height and weight related?

Bivariate Data

Two measurements
(Eg., weight and height).

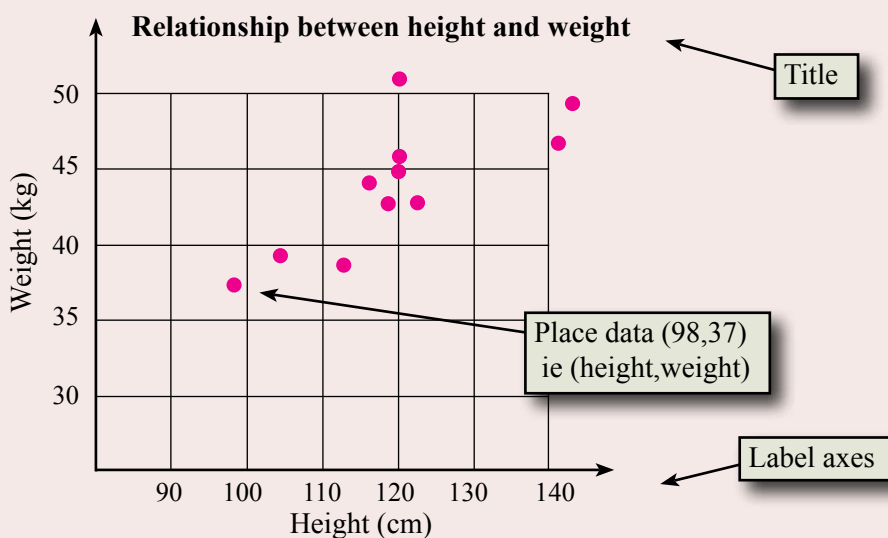
Collect Data

The height and weight of each person, willing to participate, in the class is collected.

Height (cm)	98	104	120	142	116	120	141	112	122	119	120
Weight (kg)	37	39	46	49	44	51	47	38	43	43	45

Represent Data

Use a scatter plot to display and identify patterns in bivariate data.



Interpret Data and Draw Conclusion

Visually, as the height increases the weight generally also increases. This is just a general tendency and is not always the case - three people with a height of 120 cm have different weights.

For this sample, there is a general relationship between height and weight. As the height increases there is a general increase in weight.

Exercise 19.2

1 Conduct the following relationship research:

- Research question: Is income and education related?
- Collect data (Either collect your own data or use the following data).

Education		Education	1	2	3	4	5	6	7	8
1	Less than Year 12	Salary (\$k)	24	33	36	40	57	68	94	98
2	Year 12									
3	Post Year 12									
4	Diploma									
5	Bachelor's degree									
6	Masters degree									
7	Professional degree									
8	Doctoral degree									

\$36 000 per year
(median)

- Represent the data (Use a scatter plot to display the bivariate data).
- Interpret the data and draw a conclusion related to the original research question.

2 Conduct the following relationship research:

- Research question: Are height and pace length related?
- Collect data:

Measure the height of each person in the class.

Calculate the pace length for each person in the class.

(Example: 100 m in 92 paces thus length of pace = $100/92 = 1.09$ m)



- Represent the data (Use a scatter plot to display the bivariate data).
- Interpret the data and draw a conclusion related to the original research question.

3 Conduct the following relationship research:

- Research question: Is it true that the longer you have an exercise machine, the less you use it?
- Collect data:

Hours of exercise per week	6	6	2	10	4	8	2	6
Months Treadmill owned	7	4	9	3	7	2	11	6



- Represent the data (Use a scatter plot to display the bivariate data).
- Interpret the data and draw a conclusion related to the original research question.

4 Conduct research to answer the following research questions:

- Is there a relationship between the month of the year and the average monthly temperature?
- Is there a relationship between the width and length of the leaves of a plant?

Relationship Research

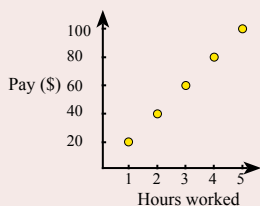
Correlation:

Is a change in one measurement matched by a change in another measurement?

Bivariate Data

Two measurements
(Eg., weight and height).

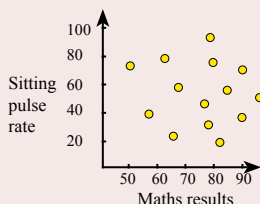
A linear correlation



The points are in a straight line.

As the hours increase, the pay increases the same amount (\$20 per hour).

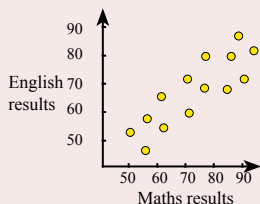
No correlation



There is no obvious pattern.

There appears to be no relationship between pulse rate and maths results.

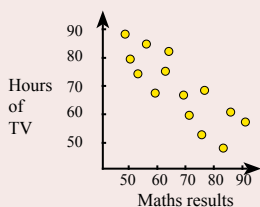
A positive correlation



The points are close to a line.

The English results tend to increase as the Maths results increase.

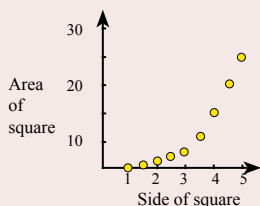
A negative correlation



The points are close to a line

The maths results tend to decrease as the TV hours increase

A non-linear correlation



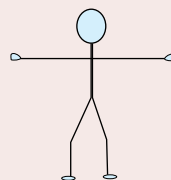
The points are not on a line

There appears to be a relationship between the area and the side of the square.

Exercise 19.3

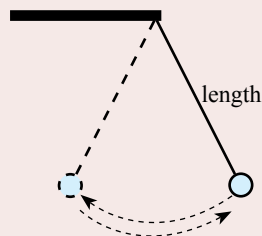
- 1 Conduct the following relationship research:

- Research question: Are height and arm span related?
- Collect data:
Measure the height of each person in the class.
Calculate the arm span for each person in the class.
- Represent the data (Use a scatter plot to display the bivariate data).
- Describe the correlation if any.
- Interpret the data and draw a conclusion related to the original research question.



- 2 Conduct the following relationship research:

- Research question: Is the time of the swing related to the length of the pendulum?
- Collect data (Either collect your own data or use the following data).

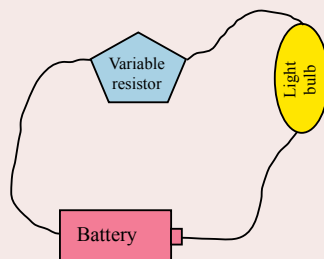


Length (cm)	20	30	40	50	60	70	80	90	100
Time (sec)	37	39	46	49	44	51	47	38	43

- Represent the data (Use a scatter plot to display the bivariate data).
- Describe the correlation if any.
- Draw a conclusion related to the original research question.

- 3 Conduct the following relationship research:

- Research question: What is the relationship between the resistance and the current in the shown circuit?
- Collect data (Either collect your own data or use the following data).



Resistance (ohms)	100	200	300	400	500	700	900
Current (amps)	0.1	0.05	0.04	0.02	0.02	0.01	0.01

- Represent the data (Use a scatter plot to display the bivariate data).
- Describe the correlation if any.
- Interpret the data and draw a conclusion related to the original research question.

Column Graphs

A column graph uses columns to illustrate data.

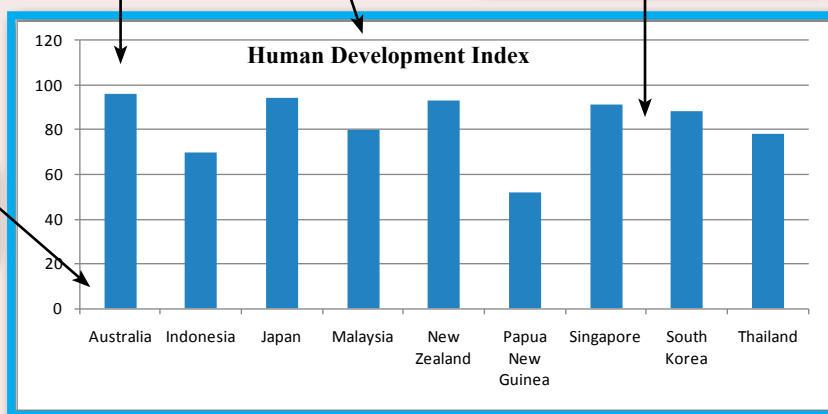
Country	HDI
Australia	96
Indonesia	70
Japan	94
Malaysia	80
New Zealand	93
Papua New Guinea	52
Singapore	91
South Korea	88
Thailand	78

Title

Columns same width

Even gap between columns

Gap before first column



Exercise 19.4

- The above example data represents the Human Development Index, HDI, for some countries in Oceania and Asia. The HDI is an index of education, income, and health. Search the Internet for more recent data and use a column graph to illustrate the data. You may also wish to include other countries such as China, Vietnam, the Philippines etc.

- Use a column graph to illustrate the following data

a)

Country	Healthindex
Australia	7
Indonesia	8
Malaysia	5
New Zealand	6
Singapore	9

b)

Country	GDPindex
Australia	10
Indonesia	5
Malaysia	2
New Zealand	1
Thailand	3

Gross Domestic Product, GDP, is the value of all goods and services produced by a country.

Mental Computation

Mental computation helps you learn how to think.

Exercise 19.5

1 Spell relationship.

Given the data set: 2,2,3,4,5

2 What is the median?

3 What is the mode?

4 Translate P(2,3) 2 units to the right and 4 units up.

5 25×14

6 What is 20% of 31.

7 Solve: $3x + 2 = 17$

8 $4 + ^{-}5$

9 Write in index form: $4 \times 4 \times 4 \times 4$

10 Simplify: $a^5 \times a^3$

$$\begin{aligned} 25 \times 14 \\ &= 25 \times 4 \times 3.5 \\ &= 100 \times 3.5 \\ &= 350 \end{aligned}$$

Exercise 19.6

1 Spell research.

Given the data set: 2,3,3,3,5

2 What is the mode?

3 What is the mean?

4 Translate P(1,3) 3 units to the right and 2 units down.

5 25×18

6 What is 20% of 42.

7 Solve: $2x - 5 = 11$

8 $^{-}3 + ^{-}4$

9 Write in index form: $2 \times 2 \times 2 \times 2 \times 2$

10 Simplify: $b^4 \times b^5$

I never will understand all the good that a simple smile can accomplish - Mother Teresa.

A psychotic thinks that $2 + 2 = 5$.

A neurotic knows that $2 + 2 = 4$... but it worries them.

Exercise 19.7

1 Spell correlation.

Given the data set: 1,2,2,2,5

2 What is the mean?

3 What is the median?

4 Translate P(5,4) 2 units to the left and 4 units down.

5 25×22

6 What is 20% of 63.

7 Solve: $5x + 1 = 16$

8 $^{-}3 + 5$

9 Write in index form: $3 \times 3 \times 3 \times 3$

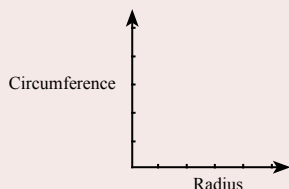
10 Simplify: $c^2 \times c^3$

Investigations

Investigation 19.1 Relationship Research.

Conduct the following relationship research:

- Research question:
Is there a relationship between the radius and the circumference of a circle?
- Collect data.
Measure the radius and the circumference of about 20 circular objects.
- Represent the data (Use a scatter plot to display the bivariate data).



- Describe the correlation if any.
- Draw a conclusion related to the original research question.

Investigation 19.2

Collect the number of daily text messages from a Year 12 or Year 10 form class or maths class and the number of daily text messages from your own form class or maths class. Construct compound stem-and-leaf plots.

- Are the shapes for the other class and your class similar?
Would you have expected the shapes to be similar or different?
- Is the data symmetrical?
Does the mean of your class have a value close to the value of the median of your class?
Does the mean of the other year's class have a value close to the value of the median of the other class?

Investigation 19.3 Online statistical activities

There are a large number of national and international online statistical activities for Year 7 students. These activities generally involve the collection of data from your class to form a large national data set or international data set. This then forms the basis of informed research on many relevant topics.

Become involved in one of these activities.



Research Officers design and undertake research for a large range of clients.

- Relevant school subjects are Mathematics.
- Courses usually involve a Bachelor degree.

Technology

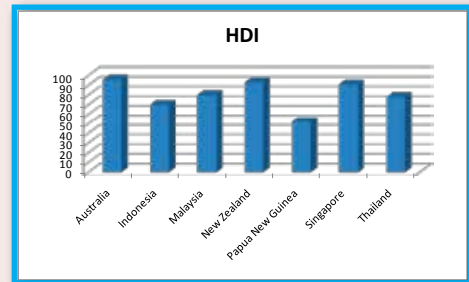
Technology 19.1 The Spreadsheet

Most spreadsheets will draw a considerable number of data graphs:

- 1 Enter a set of data into the spreadsheet.

	Country	HDI
1	Australia	96
2	Indonesia	70
3	Malaysia	80
4	New Zealand	93
5	Papua New Guinea	52
6	Thailand	78

The HDI, Human Development Index, is a measure of a country's education, income, and health.



- 2 Choose: INSERT CHART

The Internet has tutorials for all kinds of spreadsheet graphs.

Technology 19.2 The Graphics calculator

A graphics calculator will automatically calculate a large number of descriptive statistics **and** draw a frequency column graph:

- 1 Select the STAT menu, EDIT, and enter data into one of the lists.
- 2 Return to the main screen.
- 3 Select the STAT menu, Calc, 1_Stats and enter L1.

L1	L2	L3
2		
2		
2		
3		
4		

1_Stats	
\bar{x}	= 2.6
median	= 2
n	= 5
$\sum x$	= 13
+ heaps more.	

To draw a frequency column graph:

- 1 Enter the numbers in L1 and the frequency in L2.
- 2 Use STATPLOT to set up the graph Xlist= L1 and Frequency = L2.
- 3 Use Zoom and ZooMSTAT to fit the graph if necessary.
- 4 TRACE.

L1	L2	L3
2	3	
3	1	
4	1	



A Couple of Puzzles

Exercise 19.8

- Find three consecutive numbers that sum to 15.
- Find three consecutive numbers that sum to 75.
- Find three consecutive numbers that sum to 135.

The three consecutive numbers 3, 4, 5 sum to 12.

♠	♠	♠	6
♣	♣	♥	11
♣	♥	♥	10
10	?	8	

- 4 Each symbol represents a number.
Most row and column totals are shown.
What is the sum of the middle column?

A Game

One player chooses **Even**, the other player is **Odd**

The first player chooses a number from 1 to 9. 7

Repeat

The next player chooses +, −, or ×
and one of the numbers left.

Until no numbers are left.

If the answer is Even **then** even wins
else odd wins.

$$7 \times 3 = 21$$

$$21 + 4 = 25$$

$$25 - 6 = 19$$

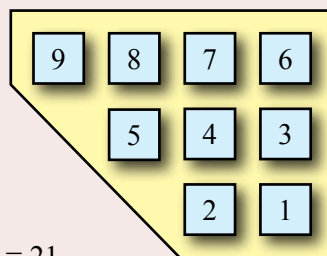
$$19 + 9 = 28$$

$$28 \times 1 = 28$$

$$28 + 8 = 36$$

$$36 \times 2 = 72$$

$$72 + 5 = 77$$



The final answer is odd.
Odds is the winner.

A Sweet Trick

- Ask your audience for a 3 digit number.
- Ask your audience for as many 2 digit combinations of their number as possible
- Ask them to sum the combinations
- Ask them to divide the answer by the sum of the digits in the original number

623

23

26

62

63

32

36 +
242

$242 \div (6+2+3)$

The answer is always 22.



Embellish the trick?

Previously hide a note with 22 written on it?
Previously know a sentence on page 22?

Chapter Review 1

Exercise 19.9

- 1 Research question: Do Victorian cows produce more milk per year than New South Wales cows?

A (hundred litres per year)	B (hundred litres per year)
30 39 50 50 54 49 49	29 36 48 47 50 50 50
51 52 53 54 58 56	49 50 52 50 54 53

58 = 5800 litres of milk per year.

- a) Represent the data (Use back-to-back stem plots or parallel dot plots to compare the data).
- b) Interpret the data and draw a conclusion related to the original research question.
- 2 Research question: Is there a relationship between the diameter of a circle and the circumference of the same circle?

Diameter (cm)	2	3	4	5	6	7
Circumference (cm)	6.3	9.4	12.6	15.7	18.8	22

- a) Represent the data (Use a scatter plot to display the data).
- b) Describe the correlation, if any.
- c) Interpret the data and draw a conclusion related to the original research question.
- 3 Use a column graph to illustrate the following data

a)

Country	Life expectancy
Australia	80
Indonesia	69
Malaysia	72
New Zealand	78
Singapore	82

b)

Country	Median age
Australia	36
Indonesia	26
Malaysia	24
New Zealand	33
Singapore	36

The median age is a measure of the middle of the people's age. The higher the number the less young people there are.

Chapter Review 2

Exercise 19.10

- 1 Research question: Did the two week campaign to reduce playground rubbish actually reduce playground rubbish?

Before Campaign							After Campaign						
22	9	11	10	25	19	18	18	6	10	2	21	2	1
13	22	13	17	24	15		19	5	9	5	15	13	

24 = 24 pieces of rubbish per 100 m².

- a) Represent the data (Use back-to-back stem plots or parallel dot plots to compare the data).
- b) Interpret the data and draw a conclusion related to the original research question.
- 2 Research question: Is there a relationship between the length a pendulum and the time for one complete swing?

Pendulum length (cm)	50	70	90	110	130	150
Period (seconds)	1.4	1.7	1.9	2.1	2.3	2.5

- a) Represent the data (Use a scatter plot to display the data).
- b) Describe the correlation, if any.
- c) Interpret the data and draw a conclusion related to the original research question.
- 3 Use a column graph to illustrate the following data

a)

Country	Population growth rate
Australia	0.9%
Indonesia	1.5%
Malaysia	1.8%
New Zealand	1.1%
Singapore	1.7%

b)

Country	Military spending
Australia	2.8%
Indonesia	1.3%
Malaysia	2.0%
New Zealand	1.0%
Singapore	4.9%

Military spending as a percentage of Gross Domestic Product (GDP).

Review 4

20

Chapter 16 Number 4

A **Ratio** is a comparison of two quantities of the same type in the same units.

Examples: 3: 10, $\frac{3}{10}$, 0.3, 30%.

Percentages. 53% means 53 per 100

$$0.2 = 0.2 \times 100 = \underline{20\%}$$

$$\frac{3}{10} = \frac{3}{10} \times 100 = \underline{30\%}$$

A **Rate** is a comparison of quantities of different kinds.

Example: Rent of \$380 per week compares the quantity \$ with the quantity week.

Find 25% of 176

$$= \frac{25}{100} \times \frac{176}{1}$$

$$= \underline{44}$$

Chapter 17 Linear Equations

Rewrite as a **symbolic equations**:

When a number is tripled and then six added, the result is thirty-three.

$$3x + 6 = 33$$

$$2x + 7 = 15$$

$$2x = 15 - 7 \quad \{\text{inverse of } + \text{ is } -\}$$

$$2x = 8$$

$$x = 8 \div 2 \quad \{\text{inverse of } \times \text{ is } \div\}$$

$$\underline{x = 4}$$

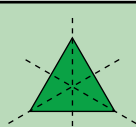
x	1	2	3	4	5
y=2x+1	3	5	7	9	11

Chapter 18 Transformations

A **translation** slides a shape across a page.

A **reflection** is a mirror image.

A **rotation** turns a shape about a point.



No line of symmetry

A **line of symmetry** divides a shape into two equal halves. The halves are a mirror image of each other.

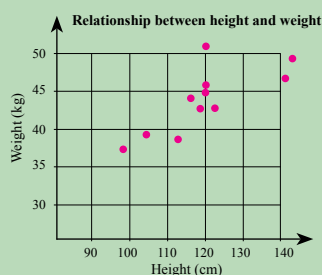
A shape has **rotational symmetry** when the shape looks the same after turned around a centre point a certain number of degrees.

Chapter 19 Data 2

Back-to-back **stem plots** or **parallel dot plots** to compare data.

Reaction Distance		Reaction Distance		
Girls	Boys	Girls	Boys	
	1 333		13	•••
4	1 44		14	•••
5	1 55		15	•••
66	1 666		16	•••
777	1 7777		17	•••
88888	1 88	•••••	18	•••
999999	1 999	•••••	19	•••
	2 0		20	
1	2 1		21	•
2	2		22	
3	2		23	

Scatter plot to display and identify relationships in data



Review 1

Exercise 20.1 Mental computation

1 Spell transformation.

Given the data set: 1,2,2,2,3

2 What is the median?

3 What is the mode?

4 Translate $P(3,5)$ 2 units to the right and 4 units down.

5 25×22

6 What is 20% of 43.

7 Solve: $5x + 2 = 17$

8 $1 + \sqrt{-4}$

9 Write in index form: $4 \times 4 \times 4 \times 4 \times 4$

10 Simplify: $x^4 \times x^2$

Exercise 20.2

1 The Maths class has 12 girls and 13 boys.

a) What is the ratio of girls to boys?

b) What is the ratio of boys to girls?

c) What is the ratio of girls to the total number in the class?

d) What is the ratio of boys to the total number in the class?

2 Write each of the following ratios as a fraction, a decimal and a percentage:

a) 1 : 10

b) 3 : 10

c) 1 : 4

d) 3 : 5

3 Simplify the following ratios:

a) 4 : 8

b) 5 : 10

c) 12 : 9

d) 15 : 10

e) $5x : 10x$

f) $6x : 18x$

4 The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a mixture of 2 L?

5 What percentage is:

a) 3 marks out of 10 marks?

b) 7 marks out of 10 marks?

c) 20 km out of 100 km?

d) \$30 out of \$50?

6 Calculate each of the following:

a) 10% of 50

b) 50% of 60

c) 25% of 40

d) 75% of 40

e) 30% of a \$800 wage is spent on rent. How much is the rent?

7 Write each of the following as a rate:

a) 100 metres in 10 seconds

b) 30 steps in 5 seconds

c) 100 litres pumped in 50 seconds

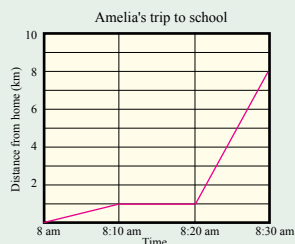
d) \$32 in 4 sessions

- 8 Maddie earns a salary of \$1724.10 per fortnight, how much is earned in:
- a month (4 weeks)?
 - 6 months (26 weeks)?
 - 1 year (52 weeks)?
- 9 Rewrite each of the following word equations as symbolic equations:
- When a number is added to five the result is twelve.
 - When a number is doubled and then three added, the result is twenty-one.
- 10 Solve each of the following equations:
- $x + 5 = 7$
 - $y - 7 = 18$
 - $2x + 3 = 17$
 - $3b - 7 = 5$
- 11 For each of the following functions:
- Complete the table.
 - Draw a graph of the function.
 - Is the function linear or non-linear?

x	1	2	3	4	5
$y=2x+5$					

- 12 Amelia lives 8 km from school. The graph shows Amelia's trip to school. James first walked to her friend's place and then they were both driven to school.

- How long did it take Amelia to get to her friend's place?
- How long did Amelia wait at her friend's place?
- Calculate Amelia's walking speed in getting to her friend's place.
- Calculate Amelia's speed from in getting from her friend's place to school.

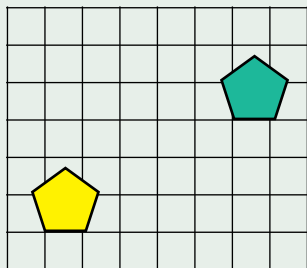


- 13 A train is travelling at a constant speed of 40 km/h. The distance travelled is given by the function: $D = 40t$, where D is the distance in km, and t is the time in hours.
- How far does the train travel in 3 hours?
 - How long does it take the train to travel 100 km?

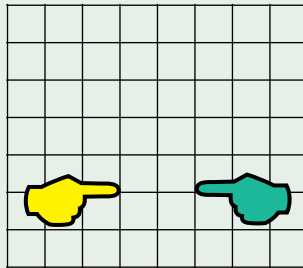
$$3^3 + 4^3 + 5^3 = 6^3$$

14 Describe each of the following transformations from yellow to green:

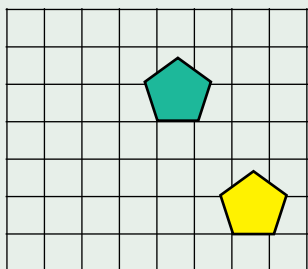
a)



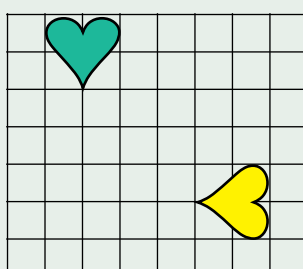
b)



a)



b)

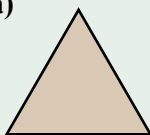


15 What are the new coordinates after:

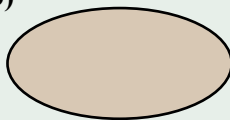
- The point $P(2,5)$ is translated 2 units right and 2 units down?
- The point $P(-1,3)$ is translated 3 units left and 2 units down?
- The point $P(3,1)$ is reflected in the x-axis?
- The point $P(-4,5)$ is reflected in the x-axis?
- The point $P(0,3)$ is rotated 90° anticlockwise about the origin?
- The point $P(-2,0)$ is rotated 90° clockwise about the origin?

16 Draw the lines of symmetry of each of the following shapes:

a)



b)

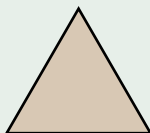


c)

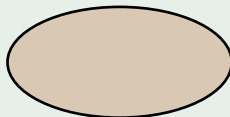


17 What is the rotational symmetry order of each of the following:

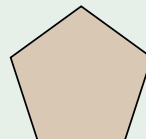
a)



b)



c)



- 18 Research question: Do Year 10 students text more than Year 7 students?

Year 7 (Weekly text messages)	Year 10 (Weekly text messages)
28 23 36 47 12 39 21	27 42 38 35 45 12 39
18 37 41 25 24 19	41 34 23 37 43 41

24 = 24 text messages in a week.

- a) Represent the data (Use back-to-back stem plots or parallel dot plots to compare the data).
- b) Interpret the data and draw a conclusion related to the original research question.
- 19 Research question: Is there a relationship between the voltage and the current in a circuit?

Current (amps)	0.2	0.3	0.4	0.5	0.6	0.7
Voltage (volts)	30	45	60	75	90	105

- a) Represent the data (Use a scatter plot to display the data).
- b) Describe the correlation, if any.
- c) Interpret the data and draw a conclusion related to the original research question.

Review 2

Exercise 20.3 Mental computation

- 1 Spell equation.

Given the data set: 1,2,2,2,3

- 2 What is the median?

- 3 What is the mean?

- 4 Translate P(4,2) 3 units to the left and 3 units up.

- 5 25×18

- 6 What is 20% of 32.

- 7 Solve: $4x + 3 = 15$

- 8 $4 + ^{-}5$

- 9 Write in index form: $2 \times 2 \times 2 \times 2$

- 10 Simplify: $x^4 \times x^3$

Exercise 20.4

- 1 The Maths class has 13 girls and 10 boys.
 - a) What is the ratio of girls to boys?
 - b) What is the ratio of boys to girls?
 - c) What is the ratio of girls to the total number in the class?
 - d) What is the ratio of boys to the total number in the class?
- 2 Write each of the following ratios as a fraction, a decimal and a percentage:
 - a) 1 : 10
 - b) 7 : 10
 - c) 3 : 4
 - d) 2 : 5
- 3 Simplify the following ratios:
 - a) 2 : 6
 - b) 3 : 9
 - c) 10 : 5
 - d) 12 : 4
 - e) $4x : 8x$
 - f) $9x : 24x$
- 4 The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a mixture of 4 L?
- 5 What percentage is:
 - a) 9 marks out of 10 marks?
 - b) 14 marks out of 20 marks?
 - c) 60 km out of 100 km?
 - d) \$40 out of \$50?
- 6 Calculate each of the following:
 - a) 10% of 60
 - b) 50% of 80
 - c) 25% of 80
 - d) 75% of 80
 - e) 40% of a \$900 wage is spent on rent. How much is the rent?
- 7 Write each of the following as a rate:
 - a) 10 metres in 5 seconds
 - b) 15 houses in 5 rows
 - c) 20 litres pumped in 10 seconds
 - d) \$40 for 10 trees
- 8 Aaron earns a salary of \$2763.90 per fortnight, how much is earned in:
 - a) a month (4 weeks)?
 - b) 6 months (26 weeks)?
 - c) 1 year (52 weeks)?
- 9 Rewrite each of the following word equations as symbolic equations:
 - a) When a number is added to seventeen the result is twenty-three.
 - b) When a number is tripled and then six added, the result is sixty-five.
- 10 Solve each of the following equations:
 - a) $x + 8 = 12$
 - b) $y - 9 = 13$
 - c) $5x + 7 = 42$
 - d) $3a - 4 = 32$

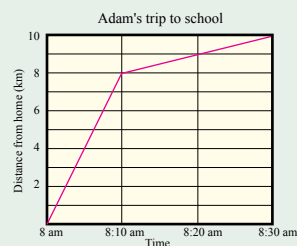
11 For each of the following functions:

- Complete the table.
- Draw a graph of the function.
- Is the function linear or non-linear?

x	1	2	3	4	5
$y=3x+2$					

12 Adam lives 10 km from school. The graph shows Adam's trip to school. Adam was first driven to his friend's place and then they both walked to school.

- How long did it take Adam to get to his friend's place?
- How long did Adam wait at his friend's place?
- Calculate Adam's speed in getting to his friend's place.
- Calculate Adam's walking speed in getting from his friend's place to school.

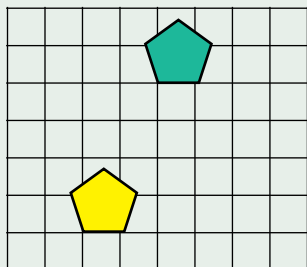


13 A train is travelling at a constant speed of 40 km/h. The distance travelled is given by the function: $D = 40t$, where D is the distance in km, and t is the time in hours.

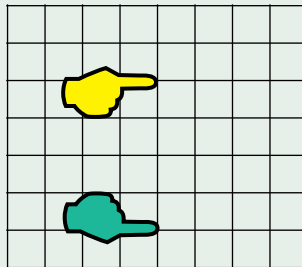
- How far does the train travel in 2 hours?
- How long does it take the train to travel 60 km?

14 Describe each of the following transformations from yellow to green:

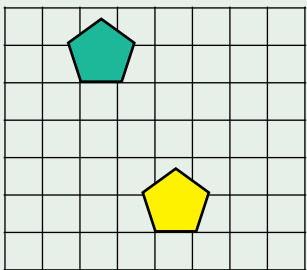
a)



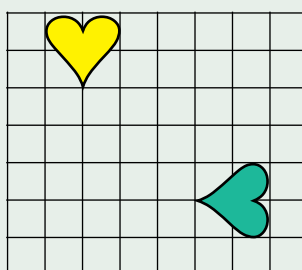
b)



a)



b)

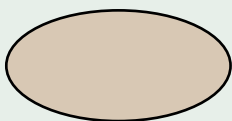


15 What are the new coordinates after:

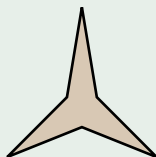
- The point $P(4,2)$ is translated 2 units right and 2 units down?
- The point $P(4,-3)$ is translated 3 units left and 2 units down?
- The point $P(5,1)$ is reflected in the x-axis?
- The point $P(-6,3)$ is reflected in the x-axis?
- The point $P(0,4)$ is rotated 90° anticlockwise about the origin?
- The point $P(-7,0)$ is rotated 90° clockwise about the origin?

16 Draw the lines of symmetry of each of the following shapes:

a)



b)

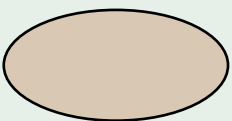


c)

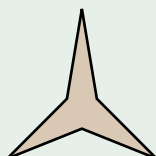


17 What is the rotational symmetry order of each of the following:

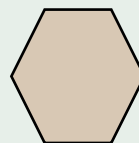
a)



b)



c)



18 Research question: Do Year 12 students text more than Year 10 students?

Year 12 (Weekly text messages)	Year 10 (Weekly text messages)
26 35 53 47 46 39 58	27 42 38 35 45 12 39
11 49 54 32 26 32	41 34 23 37 43 41

- Represent the data (Use back-to-back stem plots or parallel dot plots to compare the data).
- Interpret the data and draw a conclusion related to the original research question.

19 Research question: Is there a relationship between the time it takes a rock to drop and the height it was dropped from?

Time (seconds)	1	2	3	4	5
Height (metres)	5	20	45	80	125

- Represent the data (Use a scatter plot to display the data).
- Describe the correlation, if any.
- Interpret the data and draw a conclusion related to the original research question.

Glossary

Acute – An acute angle is a sharp angle between 0° and 90° .

Angle – An angle is the measure of turn between two lines. Angles are measured in degrees from 0° to 360° , eg. 147° . In later studies other measures such as radians will be introduced.

Angle sum of a polygon –

The sum of the interior angles of a triangle is 180° .

The sum of the interior angles of a quadrilateral is 360° .

The sum of the interior angles of a pentagon is 540° .

The general rule: Sum interior angles = $(\text{no sides} - 2) \times 180^\circ$

Ascending order of numbers is an order from smallest to largest.

Example: 2, 3, 5, 10 is in ascending order.

Average – An average is a central measure. Average and mean are the same.

(The mode, and median, although different, are also central measures).

Area – The area is the amount of surface.

Area of rectangle = length \times breadth

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

Area of circle = $\pi \times \text{radius}^2 = \pi r^2$

Area of parallelogram = base \times height

Bearing – The bearing is the angle measured clockwise from North.

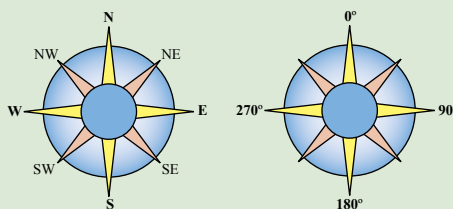
The bearing of North is 0° .

The bearing of East is 90° .

The bearing of South is 180° .

The bearing of South West is 225° .

The bearing of West is 270° .



Bias – Unfair sampling. The sample does not represent the population.

Census – Collection of data from the entire population.

Centimetre – A centimetre is one-hundredth of a metre. $100 \text{ cm} = 1 \text{ m}$.

Circumference – The circumference is the distance around the outside of a circle.

$C = 2\pi r$ or $C = \pi d$

Complementary angles are angles that sum to 90° .

Example: 40° and 50° are complementary angles.

Complementary events - The complement of any event (A) is the event (not A). The probabilities of complementary events add to 1.

Composite number has more than two factors.

Example: 8 has factors of 1, 2, 4, 8. 8 is a composite number.

Compound stem-and-leaf plot – A compound stem-and-leaf plot has two stem-and-leaf plots joined together.

Congruent objects have the same shape and the same size.

The symbol for congruence is \equiv or \cong

The tests of congruent triangles are:

SSS (side, side, side).

SAS (side, angle, side).

ASA (angle, side, angle).

RHS (right-angle, hypotenuse, side).

Consecutive numbers are numbers that follow one another.

Example: 3, 4, 5 are consecutive numbers.

Continuous numbers are numbers that can have any value. Weight is continuous because the weight of an object can be any number on the number line.

Discrete numbers can have only certain values - the number of people in the class must be a whole number. There can't be 4.62 people in the class.

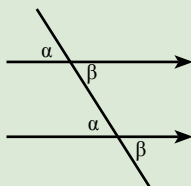
Compound Interest - Compound interest arises when interest is added to the principal. The interest that has been added also earns interest. This addition of interest to the principal is called compounding. Eg A bank account may have its interest compounded every year: in this case, an account with \$1000 initial principal and 20% interest per year would have a balance of \$1200 at the end of the first year, \$1440 at the end of the second year, and so on.

Coordinates – An ordered pair on numbers that fix a point in the cartesian plane.

Example: P(2,5). The point P is 2 units to the right and 5 units up from the origin (0,0).

Corresponding angles – matching angles when a line cuts a pair of lines.

If the lines are parallel, the corresponding angles are equal.



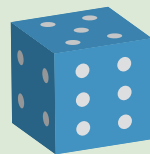
α is one pair of corresponding angles.

β is another pair of corresponding angles.

Cube – A cube is a three-dimensional object with all six faces congruent and each face having the shape of a square. A cube is one of the five platonic solids.

Cubed – A cubed number is the number multiplied by itself three times.

Example: Two cubed = $2^3 = 8$.



Cubic centimetre is the amount of space occupied by a cube with each side of length 1 cm. The unit is 1 cm^3 ($1 \text{ L} = 1\,000 \text{ cm}^3$).

Cubic metre is the amount of space occupied by a cube with each side of length 1 m. The unit is 1 m^3 ($1 \text{ m}^3 = 1\,000 \text{ L}$).

Data - Information collected for analysis or reference.

Decagon is a polygon with 10 sides and 10 angles.

Decimal place – The number of places after the decimal point.

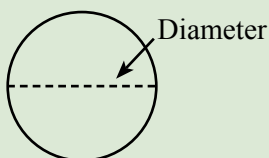
Example: 5.281 has three decimal places.

Denominator in a fraction is the number at the bottom.

Descending order of numbers is an order from largest to smallest.

Example: 10, 5, 3, 2 is in descending order.

Diameter – The diameter of a circle is the length of the line joining two points on the circle and that passes through the centre of the circle.

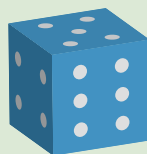


Die – A die is a cube with each of the numbers

1, 2, 3, 4, 5, 6 on each of the six faces.

The opposite sides of a die sum to seven.

Die is singular, dice is plural.



Digit – A digit is a single number.

Example: The number 435 has the digits 4, 3, and 5.

Discrete numbers are numbers that can only have certain values, normally whole numbers.

Example: The number of people in the class is discrete (Can't be 4.62 people).

Distributive law – Each term in the brackets is multiplied by the term outside the brackets. Example: $3(a + 5) = 3a + 15$

Dividend – The dividend is the number being divided. In $45 \div 7$, 45 is the dividend.

Divisor – The divisor is the number dividing. In $45 \div 7$, 7 is the divisor.

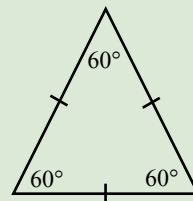
Dodecahedron – A dodecahedron is a three-dimensional object with all twelve faces congruent and each face having the shape of a regular pentagon (5 sides of equal length). A dodecahedron is one of the five platonic solids.

Equation – An equation is a mathematical sentence with an equals sign.

Example: $2x + 5 = 9$ is an equation.

An **equilateral triangle** is a triangle with three equal sides.

Each of the three angles in an equilateral triangle are 60° .



Estimate – To make an approximate guess of the answer.

Example: An estimate of 43×26 is $40 \times 30 = 1200$

Evaluate – To evaluate an expression is to find the value of the expression.

Example: Evaluate $2x(3-1)$

$$2x(3-1) = 2x2 = 4$$

Even numbers are numbers that are exactly divisible by 2.

Example: 2, 4, 6, 8, 10 are even numbers.

Expand – Each term in the brackets is multiplied by the term outside the brackets.

Example: $3(a + 5) = 3a + 15$

Factors – The factors of a number are the numbers which divide exactly into the number.

Example: The factors of 6 are 1, 2, 3, 6.

Factorise – To make into a product.

Example: $3a + 15 = 3(a + 3)$

Finite – A definite number.

Example: $\{2, 3, 1, 6\}$ has a finite number of elements. It has 4 elements.

The opposite of finite is infinite.

Formula – A formula is an equation.

The formula for the perimeter of a circle is: $C = 2\pi r$

Frequency – The number of times a number occurs.

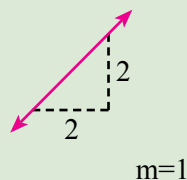
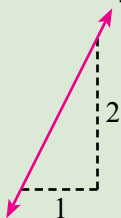
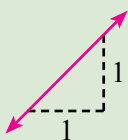
Example: 2, 3, 2, 3, 2, 2 The frequency of 2 is 4 {2 occurs 4 times}

Gram – A gram is a measure of mass and is one thousandth of a kilogram.

$$1\,000\text{ g} = 1\text{ kg}$$

Gradient – A measure of the slope.

Example: The gradient of the line $y = 2x - 1$ is 2. $m = 2$



Greater than – The symbol for greater than is $>$

Example: 6 is greater than 4, $6 > 4$

Heptagon – A polygon with seven sides and seven angles.

Hexagon – A hexagon is a polygon with six sides and six angles.

Highest common factor – The largest factor that is common.

Example: The highest common factor of 12 $\{1, 2, 3, 4, 6, 12\}$ and 8 $\{1, 2, 4, 8\}$ is 4.

Hypotenuse – The longest side in a right-angled triangle.

The hypotenuse is opposite the right-angle.

Icosahedron – An icosahedron is a three-dimensional object with all twenty faces congruent and each face having the shape of an equilateral triangle.

A icosahedron is one of the five platonic solids.

Improper fraction – An improper fraction or vulgar fraction is a fraction with the numerator larger than the denominator.

Example: $\frac{5}{3}$ is an improper fraction.

Index – The power when a number is written in index form.

Example: $9 = 3^2$. The index is 2.

Infinite – Too large/many to be counted. Not finite.

Integers are whole positive numbers and whole negative numbers.

Example: $-2, 3, 4, -7$ are integers. 3.4 is not an integer.

Intersection – The point where two lines cross each other. The common numbers in a set of two numbers.

Irrational number – An irrational number is a number that cannot be written as a common fraction or as a decimal fraction that terminates or recurs.

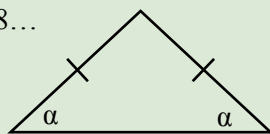
Example: π is irrational because it cannot be written as a decimal

that terminates or recurs. $\pi = 3.14159265358\dots$

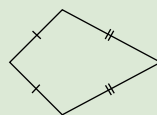
$\sqrt{2}$ is irrational = 1.41421356....

Isosceles – A triangle with two sides of equal lengths.

The angles opposite the equal sides are equal.



Kite – A quadrilateral with two pairs of adjacent sides equal.



Kilogram – A kilogram is a measure of mass. 1 kg = 1 000 g. 1 tonne = 1 000 kg.

Kilometre – A kilometre is a measure of length. 1 km = 1 000 m.

Latitude – The latitude of a position on Earth is the angle North or South of the Equator.

Adelaide's latitude is 34.55°S, Adelaide's longitude is 138.35°E.

Brisbane's latitude is 27.28°S, Brisbane's longitude is 153.01°E.

Canberra's latitude is 35.27°S, Canberra's longitude is 149.12°E.

Darwin's latitude is 12.28°S, Darwin's longitude is 130.50°E.

Hobart's latitude is 42.53°S, Hobart's longitude is 147.19°E.

Melbourne's latitude is 37.82°S, Melbourne's longitude is 144.95°E.

Perth's latitude is 31.95°S, Brisbane's longitude is 115.83°E.

Sydney's latitude is 33.52°S, Sydney's longitude is 151.13°E.

Litre – A litre is a measure of volume. 1 kg = 1 000 mL.

Longitude – The longitude of a position on Earth is the angle East or West of the line of meridian through Greenwich. See Latitude for examples.

Mean – The mean of a set of numbers is the sum of the numbers divided by the number of numbers. The mean of 2, 4, 5, 7 =

Median – The median of a set of numbers is the middle number when the numbers have been put in order.

Example: Find the median of: 4, 5, 2, 3, 6, 7, 2

In order: 2, 2, 3, 4, 5, 6, 7

The median is 4

Find the median of: 1, 3, 1, 0, 4, 3

In order: 0, 1, 1, 3, 3, 4

The median is the mean of 1 and 3 = 2.

Metre – The metre, m, is the standard measure of length.

Millimetre – A millimetre is one thousandth of a metre. $1\text{ m} = 1\,000\text{ mm}$.

Mixed number – A mixed number consists of a whole number and a fraction.

Example: $2\frac{3}{5}$

Mode – The mode of a set of numbers is the number that occurs the most.

Example: 2, 4, 3, 3, 5, 3. The mode is 3 (3 occurs three times).

1, 5, 4, 1, 5, 3. The mode is 1 and 5 (bimodal).

Net – The net of a solid is the shape that can be folded to make the solid.

Numerator – The numerator is the top number in a fraction.

Obtuse angle – An obtuse angle is an angle between 90° and 180° .

Octagon – An octagon is a polygon with eight sides and eight angles.

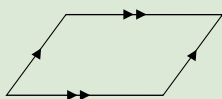
An **octahedron** is a three-dimensional object with all eight faces congruent and each face having the shape of an equilateral triangle. A octahedron is one of the five platonic solids.

Odd numbers are numbers that are not exactly divisible by 2.

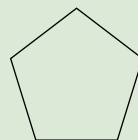
Example: 1, 3, 5, 7, 9 are odd numbers.

Obtuse angle – An angle greater than 90° and less than 180° .

Parallelogram – A parallelogram is a quadrilateral, four sided figure, in which the opposite sides are parallel.



Pentagon – A pentagon is a polygon with five sides and five angles.



Per annum – Per year.

Percentage – A percentage is a fraction of 100. $43\% = \frac{43}{100}$

Perimeter – The perimeter is the distance around the outside edge of a figure.

Perpendicular lines are lines that are at 90° to each other.

Pi, π , is the ratio of the circumference of a circle to the diameter.

$$\pi = 3.14159265358\dots$$

Polygons are shapes made up of straight lines. Triangles (3 sides), quadrilaterals (4 sides), pentagons (5 sides), hexagons (6 sides) etc are polygons.

Polyhedron – A solid shape with flat sides. Cube, dodecahedron, icosahedron, etc

Power – The power of a number is the number of times the number is multiplied by itself.

Example: $3 \times 3 \times 3 \times 3 \times 3 = 3^5$. {3 to the fifth power

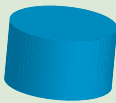
Probability – The chance of an event happening.

Probability ranges from a low of 0 (no chance) to a high of 1 (certain).

$$P(\text{event}) = \frac{\text{No of favourable outcomes}}{\text{Total no of outcomes}}$$

Prime number – A prime number is a number with just two factors, 1 and itself. 2, 3, 5, 7, 11, 13 are prime numbers. 1 is not a prime number.

Prism – A prism is a three-dimensional shape in which the base shape is repeated from the bottom to the top.



Cylinder.

A circular based prism.

Probability is the chance of an event happening.

If a die is thrown, the chance of a 3 showing is

Quadratic – An equation in which the highest power of x is 2

Example: $y = 2x^2 - 5x + 3$

Quadrilateral – A quadrilateral is a figure with four straight lines.

Quartile – A value that divides the data in quarters.

Upper quartile, median, lower quartile.

Quotient – The quotient is the result of a division.

Example: The quotient of $10 \div 5$ is 2.

Radius – The radius of a circle is the distance from the centre of a circle to a point on the circle.

Range – The difference between the highest and the lowest value.

Ratio – A ratio is a comparison of two quantities. A certain two stroke petrol is made by mixing one part of two stroke oil to 32 parts of unleaded petrol (1: 32).

Rational number – A rational number is a number that can be written as a common fraction or as a decimal fraction that either terminates or recurs.

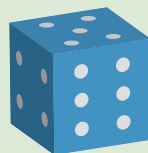
Example: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{-2}{3}$

Example: π is irrational because it cannot be written as a decimal fraction that terminates or recurs. $\pi = 3.14159265358\dots$

Rectangle – A rectangle is a four sided figure in which the opposite sides are parallel and the internal angles are 90° (right-angles).



Rectangular prism – A rectangular prism is a prism in which the base is a rectangle.



Right-angle – A right-angle is 90° .

Rounding - Giving an approximation of a number using the nearest more convenient number is called rounding. When rounding to the nearest ten, 11, 12, 13 and 14, round to 10, whereas 15, 16, 17, 18 and 19 round to 20.

Similar figures have the same shape. Congruent figures have a the same shape and the same size.

Square – A square is figure with four equal sides and each internal angle of size 90° .

Square centimetre, cm^2 , is the area occupied by a square with each side of length 1 cm.

Square metre, m^2 , is the area occupied by a square with each side of length 1 m ($1 \text{ m}^2 = 10\,000 \text{ cm}^2$).

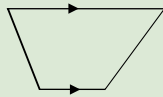
Symmetry - Property of regularity in shape by, for example, reflection or rotation. The letter T is symmetrical by reflection, the letter Z is symmetrical by rotation, the letter H is symmetrical by both reflection and rotation, the letter R is not symmetrical.

Surface area - The surface area of an object is the sum of the area of the various faces that make up the object.

Tonne – A tonne, t, is a measure of mass ($1\text{ t} = 1\,000\text{ kg}$).

Transformation – A movement of figures and objects. The transformations translation (slide), rotation (turn) and reflection (flip) do not change the size or shape of the figure or object.

Trapezium – A trapezium is a four-sided figure with one pair of opposite sides parallel.



Triangle – A triangle is a figure with three sides.

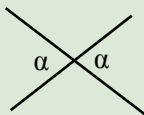
Two-way table – A table that shows the sample space of a two-stage experiment.

Variables are letters used in equations, formulas, and expressions.

Example: x is a variable in the equation: $3x + 4 = 12$.

Vertex – The corner point of an angle.

Vertically opposite angles – A pair of non-adjacent angles formed when two lines intersect. Vertically opposite angles are equal.



Vinculum – The horizontal line separating the numerator from the denominator.

Volume of a figure is a measure of the amount of space occupied by the figure.

Example: The volume of a prism = area of base \times height

Whole numbers are the positive counting numbers.

Example: 0, 1, 2, 3, 4, 5, etc.

x-intercept – The point where the graph cuts the x-axis.

y-intercept – The point where the graph cuts the y-axis.

Answers

Exercise 1.1 1 $1/2$ 2 $2/3$ 3 $1/4$ 4 $4/5$ 5 $5/14$ 6 $2/7$ 7 $6/5$ 8 $3/7$ 9 $2/3$ 10 $3/5$ 11 one-fifth
12 two-thirds 13 four and three-tenths 14 five and four-ninths 15 two and eight-seventeenths 16 $1/4$ 17 $5/8$
18 $4/5$ 19 $3/7$ 20 $6/9=2/3$ 21 $7/10$ 22 $50/100=1/2$ 23 $25/100=1/4$ 24 $4/10=2/5$ 25 $20/40=1/2$ 26 $30/60=1/2$
27 $45/60=9/12=3/4$ 28 $30/100=3/10$ 29 $500/1000=1/2$ 30 $250/1000=1/4$ 31 $90/180=1/2$ 32 $135/180=3/4$

Exercise 1.2 1a $5/2$ 1b $8/5$ 1c $7/5$ 1d $8/3$ 1e $9/8$ 1f $15/4$ 1g $9/4$ 1h $19/4$ 2a $2/1/2$ 2b $2/1/3$ 2c $2/1/4$ 2d $1/4/5$
2e $1/3/5$ 2f $3/1/3$ 2g $2/5/7$ 2h $3/4/11$

Exercise 1.3 1 5 2 6 3 8 4 6 5 6 6 2 7 6 8 9 9 5 10 15 11 8 12 10 13 15 14 8 15 3 16 15 females, 10 males
17 \$200 18 250g 19 1kg 20 3kg 21 8000m² 22 \$4000 23 9tonnes 24 300seats 25 500km

Exercise 1.4 1 $10/100=1/10$ 2 $20/100=1/5$ 3 $30/100=3/10$ 4 $40/100=2/5$ 5 $50/100=1/2$ 6 $60/100=3/5$
7 $70/100=7/10$ 8 $80/100=4/5$ 9 $90/100=9/10$ 10 $100/100=1$ 11 $1/100$ 12 $5/100=1/20$ 13 $7/100$
14 $12/100=3/25$ 15 $120/100=12/10=6/5$ 16 $150/100=15/10=3/2$ 17 $200/100=2$ 18 $500/100=5$
19 $156/100=156/100=156/100$ 20 $217/100=217/100$ 21 10% 22 20% 23 30% 24 40% 25 50% 26 60%
27 70% 28 80% 29 90% 30 100% 31 50% 32 25% 33 50% 34 75% 35 20% 36 40% 37 60% 38 80%
39 100% 40 33%

Exercise 1.5 1 15 2 6.5 3 18 4 21 5 40 6 12 7 150 8 20 9 72 10 12 11 20 12 60 13 35kg 14 \$260 15 \$1804
16 3.6 round to 4 17 13.8 round to 14 18 \$20 720 19 \$3150 20 \$300 21 \$894.40 22 \$1395 23 \$258.50
24 \$385

Exercise 1.6 1 $7/10, 70\%$ 2 $9/10, 90\%$ 3 $8/10=4/5, 80\%$ 4 $46/50=23/25, 92\%$ 5 $64/100=16/25, 64\%$
6 $63/100, 63\%$ 7 $16/25, 64\%$ 8 $17/20, 85\%$ 9 $18/20=9/10, 90\%$ 10 $21/50, 42\%$ 11 $10/100=1/10, 10\%$
12 $7/10, 70\%$ 13 $94/100=47/50, 94\%$ 14 $6/24=1/4, 25\%$ 15 $5800/58000=1/10, 10\%$ 16 $610/1000=61/100, 61\%$

Exercise 1.7 1 14 2 50 3 14 4 150 5 27 6 460 7 140 8 13 9 54 or 60 10 2 11 4 12 15 13 740 14 70 15 600
16 3100 17 170 18 24 19 50 20 210 21 600 22 5 23 0.02 24 $1/4$ or 0.25 25 200 26 1800 27 1300

Exercise 1.8 1 c) 2 a) 3 c) 4 c)

Exercise 1.9 2 9/4 3 $1/2/3$ 4 30% 5 $50/100=1/2$ 6 120 7 6.4 8 \$3.85 9 \$3.50 10 \$70

Exercise 1.10 2 11/3 3 $2/1/2$ 4 90% 5 $40/100=2/5$ 6 230 7 9.1 8 \$4.65 9 \$0.80 10 \$100

Exercise 1.11 2 12/5 3 $3/1/2$ 4 70% 5 $30/100=3/10$ 6 80 7 3.7 8 \$4.10 9 \$1.45 10 \$150

Exercise 1.12 1 $1/2$ 2 $1/2/3$ 20 4 600 5 3242 6 \$40 7 $1/3, 1/2, 3/5, 2/3, 3/4$ 8 $20/50=2/5$ 9 $15/45=1/3$

10 \$12 000 11 $12/30=2/5$ 12 $1/5$ 13 $4/10=2/5$ 14 92% 15 4 16 \$4000

Exercise 1.13 1 $44/55=2/5$ 2 0.04 3 4 3.57 5a $x=10$ 5b $x=2$ 5c $x=2$ 6 45mins 7 9:30am 8 3 9 \$1 800 000

10 $20/50=2/5$ 11 $6/9=2/3$ 12 $x=5, y=4, z=2$

Exercise 1.14 1a 12 1b 14 1c 30 1d 4 2 5 (top numbers - product of bottom 2 numbers)

Exercise 1.15 1a $1/2$ 1b $2/3$ 2a one-fifth 2b four and three-tenths 3a $3/8$ 3b $4/10=2/5$ 4a $50/100=1/2$
4b $25/100=1/4$ 5a $8/5$ 5b $5/2$ 5c $7/5$ 5d $8/3$ 6a $2/1/2$ 6b $2/1/3$ 6c $2/1/4$ 6d $1/4/5$ 7a 15 7b 8 7c 15 7d 6.5
7e 5 7f 15 8 16 male, 8 female 9a $8/10=4/5, 80\%$ 9b 43/50, 86% 10a c) 10b b) 10c d) 10d b)

Exercise 1.16 1a $1/2$ 1b $3/4$ 2a one-third 2b two and two-fifths 3a $3/4$ 3b $3/5$ 4a $25/100=1/4$ 4b $75/100=3/4$
5a $3/2$ 5b $7/4$ 5c $12/5$ 5d $31/7$ 6a $2/1/2$ 6b $1/2/3$ 6c $1/3/4$ 6d $1/4/5$ 7a 9 7b 4 7c 10 7d 4.5 7e 12 7f 18

8 18 female, 9 male 9a $9/10, 90\%$ 9b $47/50, 94\%$ 10a d) 10b c) 10c b) 10d c)

Exercise 2.1 1 4cm² 2 9cm² 3 15cm² 4 12cm² 5 16cm² 6 18cm²

Exercise 2.2 1 25m² 2 1600cm² 3 99mm² 4 432m² 5 21km² 6 2.76m² 7 9m² 8 7.28m²

9 3.12m², about half a litre 10 2.53m² 11 13 640m², 13.64ha 12 112m²

Exercise 2.3 1 27mm² 2 75cm² 3 150m² 4 45.5cm² 5 36m² 6 27.5mm²

Exercise 2.4 1a 70cm² 1b 35mm² 1c 14m² 1d 67.5cm² 2 5400cm² 3 3000cm² 4 4m² 5 5000m², 0.5ha
6 146 250m², 14.625ha

Exercise 2.5 1 81m² 2 486cm² 3 177cm² 4 105m² 5 200cm² 6 3570cm² 7 2138.5cm² 8 180.5m² 9 209.5cm²

Exercise 2.6 1 35cm² 2 108m² 3 32cm² 4 96cm² 5 25m² 6 140m² 7 70cm² 8 12L

9 A quadrilateral with opposite sides parallel 10 Sheared rectangle?

Exercise 2.7 1a m² 1b m² or cm² 1c ha 1d km² 1e cm² 1f cm² 1g cm² 1h m² or cm² 1i cm² 2a 1600cm²

2b 99m² 2c 72cm² 2d 70cm² 2e 35cm² 2f 32cm² 3a 310x460=142 600m² or 14.26ha

3b 290x440=127 600m² or 12.76ha 3c \$256 680 4 36m² 5 12.5cm 6 \$1260.59

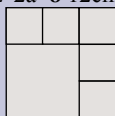
Exercise 2.8 2 84 3 5.4 4 $1/2$ 5 40m² 6 32m² 7 10 000 8 $25/100=1/4$ 9 $2/1/3$ 10 \$12.70

Exercise 2.9 2 92 3 7.9 4 bh 5 14m² 6 56cm² 7 10 000 8 50/100=1/2 9 2/3/4 10 \$4.20

Exercise 2.10 2 96 3 6.3 4 bh 5 25m² 6 92mm² 7 10 000 8 75/100=3/4 9 2/1/7 10 \$4.40

Exercise 2.11 1 c) 2 84m² 3 4.5cm² 4 9/3/4ha 5 170m 6 109m² 7 12m

Exercise 2.12 1 13cm 2 10floors 3a 525cm² 3b 337.5m² 3c 120cm² 3d 800cm² 4 Area is tripled
5 Area is tripled 6 Area is six times larger 7 2a² 8 12cm 9 100cm²



Exercise 2.13 1 1▶2▶1▶3▶2▶3▶3▶2 3 8 (1,2,4,8), 21 (1,3,7,21) and others

Exercise 2.14 1a 4cm² 1b 18cm² 2a 16m² 2b 21km² 2c 9cm² 2d 220cm² 2e 360cm² 2f 165cm² 3 1L

4 32 500m², 3.25ha 5 12m² 6 \$564.98 7a 188 600m² or 18.86ha 7b 171 600m² or 17.16ha 7c \$528 080

Exercise 2.15 1a 8cm² 1b 24cm² 2a 25m² 2b 36km² 2c 14cm² 2d 800cm² 2e 500cm² 2f 198cm² 3 1L

4 94 500m², 9.45ha 5 81m² 6 \$1793.61 7a 228 800m² or 22.88ha 7b 192 000m² or 19.2ha 7c \$320 320

Exercise 3.2 1a S = {Red, Green, Blue, White, Yellow} 1b P(blue)=1/5=0.2 1d The answers are probably different. The Sample Space gives the theoretical answer. The spinner is experimental - the more spins, the more likely the answer will be 0.2 2a S = {Head, Tail} 2b P(tail)=1/2=0.5 2d The answers are probably different. The Sample Space gives the theoretical answer. The tossing is experimental - the more tosses, the more likely the answer will be 0.5. 3a S = {1, 2, 3, 4, 5, 6} 3b P(6) = 1/6=0.17 3d The answers are different. The Sample Space gives the theoretical answer. The throwing of the die is experimental - the more throws, the more likely the answer will be 0.17.

Exercise 3.3 1a P(4) = 1/4 = 0.25 1b P(odd) = 2/4 = 0.5 1c P(>2) = 2/4 = 0.5 1d P(<2) = 1/4 = 0.25

2a P(4) = 1/5 = 0.2 2b P(even) = 2/5 = 0.4 2c P(>3) = 2/5 = 0.4 3a P(3) = 1/6 = 0.17 3b P(even) = 3/6 = 0.5

3c P(odd) = 3/6 = 0.5 3d P(5) = 1/6 = 0.17 3e P(<3) = 2/6 = 0.33 3f P(>4) = 2/6 = 0.33 3g P(>6) = 0/6 = 0

3h P(divisible by 3) = 2/6 = 0.33 4a P(5) = 1/8 = 0.125 4b P(even) = 4/8 = 0.5 4c P(odd) = 4/8 = 0.5

4d P(7) = 1/8 = 0.125 4e P(<5) = 4/8 = 0.5 4f P(>7) = 1/8 = 0.125 4g P(>5) = 3/8 = 0.375

4h P(divisible by 3) = 2/8 = 0.25 5a P(3) = 1/4 = 0.25 5b P(even) = 2/4 = 0.5 5c P(>3) = 1/4 = 0.25

5d P(5) = 0/4 = 0 6a P(7) = 1/26 = 0.04 6b P(20) = 1/26 = 0.04 6c P(odd) = 13/26 = 0.5

6d P(even) = 13/26 = 0.5 6e P(>20) = 6/26 = 0.23 6f P(<15) = 14/26 = 0.54

6g P(divisible by 3) = 8/26 = 0.31 6h P(divisible by 4) = 6/26 = 0.23 6i P(divisible by 5) = 5/26 = 0.19

6j P(divisible by 6) = 4/26 = 0.15 6k P(single digit) = 9/26 = 0.35 6l P(digits sum to 7) = 2/26 = 0.08

7a 1/1 000 000 = 0.000 001 7b 1/1 000 000 = 0.000 001 7c 5/1 000 000 = 0.000 005 8a 2/6 = 0.33

8b 4/6 = 0.67 8c 1/6 = 0.17 9a 4/11 = 0.36 9b 7/11 = 0.64 9c 1/11 = 0.09 10a 1/4 = 0.25 10b 1/4 = 0.25

10c 2/4 = 0.5 10d 0/4 = 0 11a 3/8 = 0.375 11b 1/8 = 0.125

11c 2/8 = 0.25 11d 2/8 = 0.25

Exercise 3.5 1b The game is fair because your partner has a 5/6 chance of scoring a point. You have a 1/6 chance of scoring 5 points. If a die is tossed 6 times, your partner is likely to score 5 points and you are likely to score 5 points. 2b The game is fair because you have a 5/6 chance of losing a point. You have a 1/6 chance of scoring 5 points. If a die is tossed 6 times, you are likely to score 5 points and you are likely to lose 5 points. 2c The game is unfair. You have a 2/6 chance of winning and a 4/6 chance of losing.

Exercise 3.6 2 264 3 1/2 or 0.5 4 2/6=1/3 5 A=1/2bh 6 18cm² 7 40m² 8 25/100=1/4 9 2/1/2 10 \$4.30

Exercise 3.7 2 792 3 1/2 or 0.5 4 3/6=1/2 5 15m² 6 A=bh 7 27m² 8 50/100=1/2 9 1/3/4 10 \$2.20

Exercise 3.8 2 341 3 1/2 or 0.5 4 1/6 5 28m² 6 35cm² 7 A=bh 8 75/100=3/4 9 3/2/3 10 \$17.50

Exercise 3.9 1 1 2 c 3 3(1+6, 2+5, 3+4) 4 20/50=0.4 5 50/80=0.625 6 25/100=0.25 7 20/50=0.4 8 b

Exercise 3.10 1 1 2 6 3 4 4 12 5 6 6 12 7 24 8 24

Exercise 3.11 1 About 5.5years 2 6 3 45 4 55

Exercise 3.12 1 The probability of an event is the number of favourable outcomes divided by the number of possible outcomes 3a 25 3b 25 3c 250 4a 1/6=0.17 4b 3/6=0.5 4c 3/6=0.5 4d 1/6=0.17 4e 2/6=0.33 4f 3/6=0.5 4g 0 4h 3/6=0.5 5a 1/20=0.05 5b 1/20=0.05 5c 10/20=0.5 5d 10/20=0.5 5e 5/20=0.25

5f 9/20=0.45 5g 6/20=0.3 5h 5/20=0.25 5i 4/20=0.2 5j 3/20=0.15 5k 9/20=0.45 5l 4/20=0.2

6a 1/100 000 = 0.000 01 6b 1/100 000 = 0.000 01 6c 10/100 000 = 0.0001 7a 2/6=0.33 7b 4/6=0.67

7c 2/6=0.33

Exercise 3.13 1 The probability of an event is the number of favourable outcomes divided by the number of possible outcomes 3a 30 3b 30 3c 300 4a 1/6=0.17 4b 3/6=0.5 4c 3/6=0.5 4d 1/6=0.17 4e 3/6=0.5 4f 2/6=0.33 4g 0 4h 2/6=0.33 5a 1/30=0.03 5b 1/30=0.03 5c 15/30=0.5 5d 15/30=0.5 5e 5/30=0.17 5f 9/30=0.3 5g 10/30=0.33 5h 7/30=0.23 5i 6/30=0.2 5j 5/30=0.17 5k 9/30=0.3 5l 5/30=0.17

6a 1/1 000 000 = 0.000 001 6b 1/1 000 000 = 0.000 001 6c 20/100 000 = 0.000 02 7a 2/7=0.29 7b 5/7=0.71

Exercise 4.1 1 \$60 2 \$27 3 \$86 4 \$58 5 \$186 6 \$870 7 \$670 8 \$520 9 \$11 440 10 \$6100

Exercise 4.2 1a \$2.64 1b \$60.50 1c \$53.35 1d \$264 1e \$159.50 1f \$36.30 1g \$138.16 1h \$188.76 2a \$207 2b \$43.20 2c \$108 2d \$22.50 2e \$31.95 2f \$7.65 2g \$20.25 2h \$11.34 3a \$14.40 3b \$312 3c \$28 3d \$96 3e \$2560 3f \$19.92 3g \$3760 3h \$2960 4 \$409.95

Exercise 4.3 1a price of \$1.79 for 1 litre of petrol 1b price of 960cents for 1 kg of wool

1c price of \$13.40 for 1kg of cheese 1d price of \$15.90 for 1m of timber 1e price of \$24 000 for 1ha of land 1f price of 98cents for 1min call 1g price of \$320 for 1km of barbed wire 1h price of \$85 for 1hour 2a \$4/h 2b \$5/kg 2c \$6/m 2d \$2/min 2e \$24/bag 2f \$50/week 2g \$113/ha 2h \$24/bucket 2i \$30/week 2j \$4/s 2k \$5/peg 2l \$3.50/pen 2m \$5/g 2n \$80/hole 2o \$3/button 2p \$2/s 2q \$5/kg 2r \$45.60/L 2s \$0.90/g 2t \$12/s 2u \$8.80/m 2v \$0.65/min 2w \$0.56/mL 2x \$0.09/g

Exercise 4.4 1a \$1860 1b \$3720 1c \$12 090 1d \$24 180 2a \$2500 2b \$3750 2c \$5000 2d \$32 500 2e \$65 000 2f \$650 000 3a \$2.80 3b \$4.20 3c \$10.36 3d \$54.88 4a \$172.50 4b \$345 4c \$690 4d \$6900 5a \$2.88 5b \$3.84 5c \$6.72 5d \$8.16 6a \$9.30 6b \$18.60 6c \$65.10 6d \$106.02 7a \$3.70 7b \$7.40 7c \$12.67 7d \$162.12

Exercise 4.5 1a \$2 for 1kg 1b \$8 for 1L 1c \$0.40 for 1m 1d \$0.50 for 1min 1e \$0.17 for 1g 1f \$0.08 for 1g 2a the 2kg is the best buy 2b the 7m is the best buy 2c the 17 ingots is the best buy 3a \$4 for 100g 3b \$2 for 100mL 3c \$4 for 100g 3d \$2.10 for 100g 3e \$1.02 for 100mL 3f \$0.90 for 100min 4a the 300mL is the best buy 4b the 500g is the best buy 4c the 300g is the best buy

Exercise 4.6 2 \$22 3 \$7.50 4 $1/2=0.5$ 5 $2/6=0.33$ 6 $A=\frac{1}{2}bh$ 7 $28cm^2$ 8 $20/100=1/5$ 9 5L 10 \$90

Exercise 4.7 2 \$27.50 3 \$14 4 $1/2=0.5$ 5 $3/6=0.5$ 6 $A=bh$ 7 $15cm^2$ 8 $30/100=3/10$ 9 5m 10 \$108

Exercise 4.8 2 \$33 3 \$7.50 4 $1/2=0.5$ 5 $2/6=0.33$ 6 $A=bh$ or lb 7 $35cm^2$ 8 $40/100=2/5$ 9 7m 10 \$124

Exercise 4.9 1 \$148 2 \$24.50 3 \$0.65 4 600km 5 40.5L 6 250g 7 \$2.70 8 \$12 9 \$440 10a \$47 10b \$530 11 \$US11 12 \$US18 13 $x1.2$, \$4.80 14 400km

Exercise 4.10 1 \$75 2 \$157.50 3 50days 4 100days 5 2.5mins or 2 mins 30s 6 $187.5s=3min$ 7.5s 7 7200L 8 4500L 9 16h 10 16days 11 21% 12 69%

$99 \times 11 = 1089$
 $99 \times 22 = 2178$
 $99 \times 33 = 3267$
 $99 \times 44 = 4356$
 $99 \times 55 = 5445$
 $99 \times 66 = 6534$
 $99 \times 77 = 7623$
 $99 \times 88 = 8712$
 $99 \times 99 = 9801$

Exercise 4.11 1 2 9(1,3,9) there are many others 3 25(1,5,25) 4 256 5 6cm by 4cm

Exercise 4.12 1a \$74 1b \$580 2a \$6.60 2b \$40.15 3a \$20.70 3b \$65.25 4a \$62.40 4b \$232 5 \$769.50 6 \$48.44 7 \$719.40 8 \$122.88 9a 5kg 9b 7m 9c 10yearlings 10a \$3 for 100g 10b \$3 for 100mL 10c \$4.50 for 100g 10d \$1.40 for 100g 10e \$0.91 for 100mL 10f \$1.28 for 100min 11a 400mL 11b 200g 11c 700g

Exercise 4.13 1a \$71 1b \$1140 2a \$154 2b \$15.40 3a \$23.40 3b \$58.95 4a \$2240 4b \$15.04 5 \$1228.50 6 \$31.98 7 \$959.40 8 \$115.94 9a 8kg 9b 7 rolls 9c 25 solar cells 10a \$3 for 100g 10b \$6 for 100mL 10c \$5.50 for 100g 10d \$1.44 for 100mm 10e \$2.53 for 100mL 10f \$1.10 for 100min 11a 500mL 11b 500g 11c 900g

Exercise 5.1 2 \$33 3 \$4.50 4 $1/2=0.5$ 5 $3/6=0.5$ 6 $A=\frac{1}{2}bh$ 7 $42cm^2$ 8 $20/100=1/5$ 9 3L 10 \$84

Exercise 5.2 1a $1/2$ 1b $1/4$ 2a one-fifth 2b three and one-half 3a $2/8=1/4$ 3b $6/10=3/5$ 4a $50/100=1/2$

4b $25/100=1/4$ 5a $5/2$ 5b $7/3$ 5c $13/4$ 5d $13/5$ 6a $1/1/2$ 6b $4/1/2$ 6c $3/2/3$ 6d $2/1/7$ 7a 12 7b 8 7c 10 7d 4.5

7e 16 7f 9 8 16 male, 8 female 9a $8/10=4/5=80\%$ 9b $44/50=22/25=88\%$ 10a $8cm^2$ 10b $32cm^2$ 11a $9m^2$

11b $24km^2$ 11c $14cm^2$ 11d $220cm^2$ 11e $160cm^2$ 11f $450cm^2$ 12 $14.72m^2$, 1L 13 54 000m², 5.4ha 14 \$994.03

15 The probability of an event is the number of favourable outcomes divided by the number of possible outcomes 17a 35 17b 35 17c 350 18a $1/6=0.17$ 18b $3/6=0.5$ 18c $3/6=0.5$ 18d $1/6=0.17$ 18e $3/6=0.5$

18f $2/6=0.17$ 18g 0 18h $3/6=0.5$ 19a $1/20=0.05$ 19b $1/20=0.05$ 19c $10/20=0.5$ 19d $10/20=0.5$ 19e $10/20=0.5$

19f $9/20=0.45$ 19g $10/20=0.5$ 19h $6/20=0.3$ 19i $5/20=0.25$ 19j $4/20=0.2$ 19k $4/20=0.2$ 19l $2/10=0.1$

20a $1/100\ 000 = 0.000\ 01$ 20b $1/100\ 000 = 0.000\ 01$ 20c $10/100\ 000 = 1/10\ 000 = 0.000\ 1$ 21a $2/6=0.33$

21b $4/6=0.67$ 21c $2/6=0.33$ 22a \$109 22b \$1130 23a \$2.20 23b \$143 24a \$33.60 24b \$256 25 \$1314

26 \$37.12 27 \$599.40 28 \$104.88 29a 5kg 29b 9m 29c 17rugs 30a \$5 for 100g 30b \$5 for 100mL

30c \$4 for 100g 30d \$1.74 for 100g 30e \$1.16 for 100mL 30f \$1.24 for 100min 31a 500mL 31b 200g

31c 900g

Exercise 5.3 2 \$66 3 \$12.50 4 $1/2=0.5$ 5 $4/6=0.67$ 6 A=bh 7 14cm^2 8 $60/100=3/5$ 9 5L 10 \$104

Exercise 5.4 1a $1/2$ 1b $3/4$ 2a three-fifths 2b six and three-quarters 3a $3/4$ 3b $2/5$ 4a $50/100=1/2$

4b $75/100=3/4$ 5a $4/3$ 5b $11/4$ 5c $17/5$ 5d $34/7$ 6a $2/1/2$ 6b $2/2/3$ 6c $2/1/4$ 6d $2/3/5$ 7a 4 7b 9 7c 5 7d 5

7e 8 7f 12 8 14 male, 7 female 9a $8/10=4/5=80\%$ 9b $41/50=82\%$ 10a 8cm^2 10b 24cm^2 11a 25m^2 11b 27km^2

11c 20cm^2 11d 276cm^2 11e 468cm^2 11f 1140cm^2 12 1L 13 318 500m², 31.85ha 14 11.4m^2 , \$1218.66

15 The probability of an event is the number of favourable outcomes divided by the number of possible

outcomes 17a 40 17b 40 17c 400 18a $1/6=0.17$ 18b $3/6=0.5$ 18c $3/6=0.5$ 18d $1/6=0.17$ 18e $4/6=0.67$

18f $4/6=0.67$ 18g 0 18h $3/6=0.5$ 19a $1/20=0.05$ 19b $1/20=0.05$ 19c $10/20=0.5$ 19d $10/20=0.5$ 19e $2/20=0.1$

19f $7/20=0.35$ 19g $6/20=0.3$ 19h $5/20=0.25$ 19i $4/20=0.2$ 19j $3/20=0.15$ 19k $4/20=0.2$ 19l $3/20=0.15$

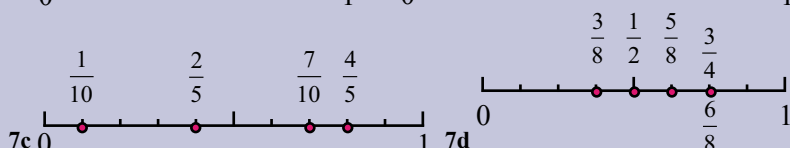
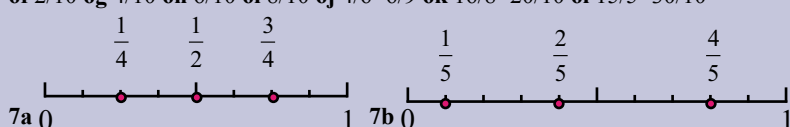
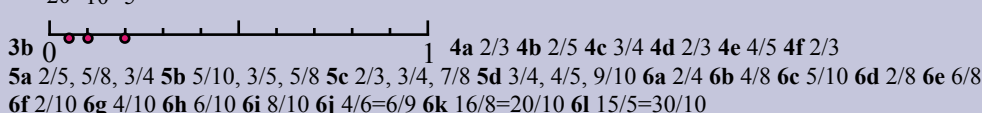
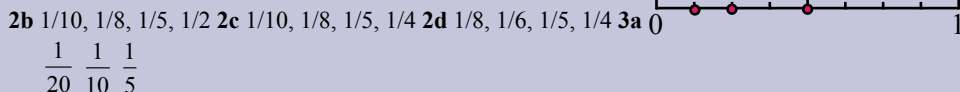
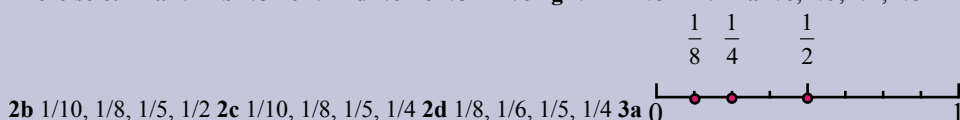
20a $1/1\ 000\ 000 = 0.000\ 001$ 20b $1/1\ 000\ 000 = 0.000\ 001$ 20c $10/1\ 000\ 000 = 0.000\ 05$ 21a $2/6=0.33$

21b $4/6=0.67$ 21c $1/6=0.17$ 22a \$27.50 22b \$26.95 23a \$70.40 23b \$552 24 \$850.50 25a 5kg 25b 12m

25c 36buckets 26a \$4 for 100g 26b \$5 for 100mL 26c \$4 for 100g 26d \$2.40 for 100g 26e \$1.60 for 100mL

26f \$5.63 for 100min 31a 800mL 31b 400g 31c 700g

Exercise 6.1 1a $1/2$ 1b $1/3$ 1c $1/4$ 1d $1/5$ 1e $1/3$ 1f $1/8$ 1g $1/4$ 1h $1/3$ 1i $1/2$ 2a $1/8$, $1/5$, $1/4$, $1/3$



Exercise 6.2 1a $5/2$ 1b $8/5$ 1c $7/5$ 1d $8/3$ 1e $9/8$ 1f $15/4$ 1g $9/4$ 1h $37/10$ 2a $2/1/2$ 2b $2/1/3$ 2c $2/1/4$ 2d $1/4/5$

2e $1/3/5$ 2f $3/1/3$ 2g $2/5/7$ 2h $3/4/11$

Exercise 6.3 1 $3/4$ 2 $3/5$ 3 $5/7$ 4 $2/3$ 5 $5/10$ or $1/2$ 6 $4/8$ or $1/2$ 7 $3/3$ or 1 8 $7/10$ 9 $4/6$ or $2/3$ 10 $7/12$ 11 $9/16$

12 $18/20$ or $9/10$ 13 $5/6$ 14 $3/4$ 15 $7/10$ 16 $8/12$ or $2/3$ 17 $7/12$ 18 $8/15$ 19 $9/18$ or $1/2$ 20 $10/21$

21 $9/10$ 22 $11/10$ or $1/1/10$ 23 $13/14$ 24 $14/16$ or $7/8$ 25 $17/30$ 26 $13/15$ 27 $11/10$ or $1/1/10$

28 $31/24$ or $1/7/24$ 29 $23/24$ 30 $13/14$ 31 $10/8$ or $5/4$ or $1/1/4$ 32 $17/12$ or $1/5/12$

Exercise 6.4 1 $2/4$ or $1/2$ 2 $1/3$ 3 $4/9$ 4 $3/5$ 5 $2/5$ 6 $1/5$ 7 $4/8$ or $1/2$ 8 $2/9$ 9 $4/10$ or $2/5$ 10 $3/10$ 11 $4/20$ or $1/5$

12 $32/100$ or $8/25$ 13 $1/6$ 14 $2/8$ or $1/4$ 15 $3/10$ 16 $4/12$ or $1/3$ 17 $1/12$ 18 $2/15$ 19 $3/18$ or $1/6$ 20 $4/21$

21 $1/10$ 22 $1/10$ 23 $1/14$ 24 $2/16$ or $1/8$ 25 $7/30$ 26 $4/15$ 27 $2/15$ 28 $1/24$ 29 $7/24$ 30 $11/56$ 31 $4/32$ or $1/8$

32 $1120/2000$ or $56/100$ or $14/25$

Exercise 6.5 1 $12/25$ 2 $9/20$ 3 $8/15$ 4 $6/35$ 5 $4/10$ or $2/5$ 6 $6/20$ or $3/10$ 7 $6/12$ or $1/2$ 8 $2/20$ or $1/10$

9 $9/8=1/1/8$ 10 $7/8$ 11 $10/6$ or $5/3=1/2/3$ 12 $5/10$ or $1/2$ 13 $8/5=1/3/5$ 14 $9/4=2/1/4$ 15 $5/2=2/1/2$

16 $18/3=6$ 17 $24/3=8$ 18 $12/40$ or $3/10$ 19 $9/16$ 20 $4/9$ 21 $40/5=8$ 22 $800/10=80$ 23 $7.2/10$ or $3.6/5$ or 0.72

24 $70/4$ or $35/2=17/1/2$

Exercise 6.6 1 6 2 10 3 4 4 12 5 6 6 $6/2=3$ 7 12 8 $12/3=4$ 9 $6/4$ or $3/2=1/1/2$ 10 $8/9$ 11 $5/9$ 12 $10/12=5/6$

13 $15/8=1/7/8$ 14 $60/35$ or $12/7=1/5/7$ 15 $18/14$ or $9/7=1/2/7$ 16 $10/2=5$ 17 $15/3=5$ 18 $6/2=3$

19 $12/20$ or $3/5$ 20 $10/9=1/1/9$ 21 $25/33$ 22 $15/21$ or $5/7$ 23 $15/28$ 24 $15/8=1/7/8$ 25 $15/40$ or $3/8$

26 $38/24$ or $19/12=1/7/12$ 27 8 28 $45/4=11/1/4$

Exercise 6.7 2 $1/2$ 3 0.6 4 $17/5$ 5 4.2 6 $5/7$ 7 $3/10$ 8 $3/6=1/2$ 9 40cm^2 10 \$20

Exercise 6.8 2 $4/10=2/5$ 3 0.75 4 $11/4$ 5 6.2 6 $2/4=1/2$ 7 $2/15$ 8 $2/6=1/3$ 9 12cm^2 10 \$30

Exercise 6.9 2 $3/10$ 3 0.4 4 $9/4$ 5 6.8 6 $7/12$ 7 $3/20$ 8 $3/6=1/2$ 9 35cm^2 10 \$36

Exercise 6.10 1 $1\frac{1}{2}$ 2 2 3 $2\frac{1}{2}$ 4 6 tonnes 5 \$200 6 $\frac{8}{20}$ or $\frac{2}{5}$ or 0.4 7 $\frac{9}{4}=2\frac{1}{4}$ buckets 8 $\frac{5}{10}=1/2$
8b $\frac{5}{6}$ **8c** $\frac{1}{6}$ **9** $11/3$ **10** $3/3/4$ **11** 5.75 **12** 75% **13** $20/35$ or $4/7$ **14** 4 **15** \$4000 **16** 24% **17** $1/12$
Exercise 6.11 1 $1/2$ 2 $2/10$ or $1/5$ 3 $44/55$ 4 $1/4$ or 0.25 5 $1/2$ 6 11.11 **7a** $5/6$ **7b** $1/4$ **7c** 1 **8** 45mins
9 11:25am **10** 18 **11** $2/3$ **12** $2/3$ **13** \$1 800 000 **14a** 2 **14b** 7

2	13	12	7
11	8	1	14
5	10	15	4
16	3	6	9

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Exercise 6.12 1 $1/8$ 2

Exercise 6.13 **1a** $1/6$, $1/5$, $1/4$, $1/3$ **1b** $1/9$, $1/8$, $1/5$, $1/2$ **1c** $2/5$, $5/8$, $3/4$ **1d** $5/8$, $7/10$, $4/5$ **2a** 4 **2b** 9 **2c** 4, 6
2d 16, 20 **3a** $2/12$ **3b** $2/13$ **3c** $2/14$ **3d** $1/4/5$ **3e** $1/3/5$ **3f** $3/1/3$ **3g** $2/5/7$ **3h** $3/4/11$ **4a** $5/2$ **4b** $8/5$ **4c** $7/5$
4d $8/3$ **4e** $9/8$ **4f** $15/4$ **4g** $9/4$ **4h** $37/10$ **5a** $2/4=1/2$ **5b** $3/5$ **5c** $5/7$ **5d** $5/6$ **5e** $3/4$ **5f** $13/20$ **5g** $3/10$ **5h** $1/6$ **5i** $1/3$
5j $7/15$ **5k** $2/9$ **5l** $1/35$ **5m** $1/6$ **5n** $2/15$ **5o** $6/20=3/10$ **5p** $4/14=2/7$ **5q** $15/12=5/4=1\frac{1}{4}$ **5r** $36/10=18/5=3\frac{3}{5}$
5s $2/3$ **5t** $2/4=1/2$ **5u** $4/3=1\frac{1}{3}$ **5v** $15/8=1\frac{7}{8}$ **5w** $15/4=3\frac{3}{4}$ **5x** $33/12=11/4=2\frac{3}{4}$
Exercise 6.14 **1a** $1/7$, $1/5$, $1/3$, $1/2$ **1b** $1/12$, $1/11$, $1/9$, $1/7$ **1c** $2/3$, $3/4$, $4/5$ **1d** $3/10$, $1/3$, $2/5$ **2a** 6 **2b** 4
2c 2, 3 **2d** 6, 15, 21 **3a** $1/12$ **3b** $4/12$ **3c** $1/4/5$ **3d** $1/1/5$ **3e** $1/3/4$ **3f** $4/1/4$ **3g** $4/1/8$ **3h** $2/11/13$ **4a** $9/4$ **4b** $15/4$
4c $37/10$ **4d** $29/5$ **4e** $27/8$ **4f** $8/5$ **4g** $5/2$ **4h** $7/5$ **5a** $2/5$ **5b** $5/6$ **5c** $7/9$ **5d** $2/4=1/2$ **5e** $7/6=1\frac{1}{6}$ **5f** $23/21=1\frac{2}{21}$
5g $3/10$ **5h** $1/20$ **5i** $5/12$ **5j** $2/8=1/4$ **5k** $1/15$ **5l** $8/15$ **5m** $1/12$ **5n** $2/15$ **5o** $6/20=3/10$ **5q** $8/12=2/3$
5r $91/20=4\frac{11}{20}$ **5s** $5/4=1\frac{1}{4}$ **5t** $4/2=2$ **5u** $9/5=1\frac{4}{5}$ **5v** $15/12=5/4=1\frac{1}{4}$ **5w** $14/4=7/2=3\frac{1}{2}$
5x $78/35=2\frac{8}{35}$

Exercise 7.1 1 2 2 26 3 15 4 15 5 15 6 37 7 1 8 7 9 2 10 9 11 27 12 31 13 2 14 11 15 17 16 24 17 4 18 9
 19 14 20 6

Exercise 7.2 1 5 2 1 3 16 4 1 5 5 6 2 7 1 8 4 9 1 10 4 11 5 12 16 13 1 14 11 15 44 16 22 17 19 18 2 19 29
 20 5 21 18 22 17

Exercise 7.3 1 $x+4$ 2 $x-9$ 3 $6+x$ or $x+6$ 4 $x-7$ 5 $x+13$ 6 $x-12$ 7 $x+23$ or $23+x$ 8 $x-5$ 9 $x+1$ 10 $x-85$ 11 $2x$
 12 $x/11$ 13 $3x$ 14 $x/2$ 15 $25x$ or $x \times 25$ 16 $x/4$ 17 $87x$ or $x \times 87$ 18 $2x/3$ 19 $7x$ 20 $3x/4$ 21 $a+b$ 22 ba or ab
 23 $a+b$ 24 $b+a$ 25 ab 26 ba 27 $a+b$ 28 ab 29 $3a+c$ 30 $4x-m$

Exercise 7.4 **1a** 12 **1b** 18 **1c** 9 **1d** 4 **1e** 2 **1f** 3 **1g** 13 **1h** 5 **1i** 36 **1j** 17 **1k** 7 **1l** 28 **2** 73 **3** 3.5 **4a** 30cm^2
4b 90m^2 **5a** 80kg **5b** 90kg **6a** 117kg **6b** 75kg

Exercise 7.5 1 + is commutative 2 + is commutative 3 + is commutative 4 + is commutative
 5 + is commutative 6 + is commutative 7 - is not commutative 8 - is not commutative
 9 - is not commutative 10 - is not commutative 11 - is not commutative 12 - is not commutative
 13 \times is commutative 14 \times is commutative 15 \times is commutative 16 \times is commutative 17 \times is commutative
 18 \times is commutative 19 \div is not commutative 20 \div is not commutative 21 \div is not commutative
 22 \div is not commutative 24 \div is not commutative 24 \div is not commutative

Exercise 7.6 1 + is associative 2 + is associative 3 + is associative 4 + is associative 5 - is not associative
 6 - is not associative 7 - is not associative 8 - is not associative 9 \times is associative 10 \times is associative
 11 \times is associative 12 \times is associative 13 \div is not associative 14 \div is not associative 15 \div is not associative
 16 \div is not associative

Exercise 7.7 1 $y+7$ and $7+y$ is commutative 2 $x+13$ and $13+x$ is commutative
 3 $9+3x$ and $3x+9$ is commutative 4 $2+y$ and $y+2$ is commutative 5 $x-y$ and $y-x$ is not commutative
 6 $2y-5$ and $5-2y$ is not commutative 7 $9x$ and $x9$ is commutative 8 $3x \times 12$ and $12 \times 3x$ is commutative
 9 $y \div 2$ and $2 \div y$ is not commutative 10 $4y \div 2$ and $2 \div 4y$ is not commutative

Exercise 7.8 1 $(x+y)+z$ and $x+(y+z)$ is associative 2 $(3+x)+4$ and $3+(x+4)$ is associative
 3 $2x+(y+8)$ and $(2x+y)+8$ is associative 4 $(z+10y)+15$ and $z+(10y+15)$ is associative
 5 $(x-y)-z$ and $x-(y-z)$ is not associative 6 $4x-(3-2z)$ and $(4x-3)-2z$ is not associative
 7 $9 \times (x \times y)$ and $(9 \times x) \times y$ is associative 8 $3x \times (2y \times z)$ and $(3x \times 2y) \times z$ is associative
 9 $x \div (y \div 1)$ and $x \div (y \div 1)$ is not associative 10 $(5x \div 2y) \div 10$ and $5x \div (2y \div 10)$ is not associative

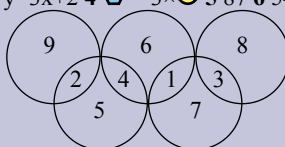
Exercise 7.9 2 15 3 13 4 24 5 true 6 $1/4$ 7 $1/3/5$ 8 $5/9$ 9 $1/6$ 10 \$3.60

Exercise 7.10 2 36 3 8 4 15 5 true 6 $1/2$ 7 $2/1/4$ 8 $5/6$ 9 $1/12$ 10 \$11.85

Exercise 7.11 2 17 3 10 4 20 5 false 6 $3/4$ 7 $3/1/3$ 8 $7/12$ 9 $2/6=1/3$ 10 \$3.70

Exercise 7.12 1 24 2 17 3 0.5 4 4 5 4 6 9 7 $35/25=7/5=1\frac{2}{5}$ 8 3.2% 9 17 10 52 **11a** y **11b** $x+(y+z)=(x+y)+z$
 12 3 13 3 and 4 14 5 and 35

Exercise 7.13 1a 17 1b 17 1c 10 1d 135 1e 3 2a 19 2b 81 3 $y=3x+2$ 4 $\text{⬢} = 3 \times \text{⬢} = 5$ 5 87 6 54 7 $(a+b)^2$



Exercise 7.14 1 17 2 9 (18, 27, 36, 45, 54, 63, 72, 81, 90) 3

Exercise 7.15 1a 6 1b 13 1c 10 1d 7 1e 16 1f 1 2a $x+5$ 2b $x-3$ 2c 3x 2d $x/2$ 2e $b+d$ 2f xy 2g $3s+w$ 2h $4a-b$

3a 10 3b 18 3c 14 3d 4 3e 3 3f 2 3g 11 3h 1 3i 30 3j 16 3k 14 3l 22 4a $x+3$ and $3+x$ is commutative

4b $9+3x$ and $3x+9$ is commutative 4c $5y-8$ and $8-5y$ is not commutative 4d $y \times 10$ and $10 \times y$ is commutative

4e $x/4$ and $4/x$ is not commutative 5a $(x+y)+z$ and $x+(y+z)$ is associative

5b $3x+(y+1)$ and $(3x+y)+1$ is associative 5c $(x-y)-z$ and $x-(y-z)$ is not associative

5d $4 \times (x \times y)$ and $(4 \times x) \times y$ is associative 5e $x \times (5y \times z)$ and $(x \times 5y) \times z$ is associative

5f $x \div (y \div z)$ and $x \div (y \div z)$ is not associative 6 P=372 000 watts or 372kw

Exercise 7.16 1a 16 1b 25 1c 4 1d 11 1e 9 1f 2 2a $x+6$ 2b $x-8$ 2c $2x$ 2d $x/3$ 2e $h-m$ 2f ab 2g $3g-f$ 2h $4a+p$

3a 24 3b 12 3c 14 3d 2 3e 4 3f 1 3g 11 3h 5 3i 24 3j 21 3k 2 3l 18 4a $x+2$ and $2+x$ is commutative

4b $7+3x$ and $3x+7$ is commutative 4c $9y-5$ and $5-9y$ is not commutative 4d $x \times 10$ and $10 \times x$ is commutative

4e $x/4$ and $4/x$ is not commutative 5a $(x+y)+z$ and $x+(y+z)$ is associative

5b $2x+(y+3)$ and $(2x+y)+3$ is associative 5c $(x-y)-z$ and $x-(y-z)$ is not associative

5d $5 \times (x \times y)$ and $(5 \times x) \times y$ is associative 5e $x \times (2y \times z)$ and $(x \times 2y) \times z$ is associative

5f $x \div (y \div z)$ and $(x \div y) \div z$ is not associative 6 P=504 watts

Exercise 8.1 1 3cm^3 2 4cm^3 3 8cm^3 4 6cm^3 5 8cm^3 6 16cm^3 7 12cm^3 8 16cm^3 9 32cm^3 10 12cm^3 11 24cm^3 12 36cm^3

Exercise 8.2 1 8cm^3 2 27cm^3 3 64cm^3 4 40cm^3 5 120cm^3 6 720cm^3

Exercise 8.3 1 2L 2 3L 3 16L 4 18.75L 5 253L 6 19.6kg

Exercise 8.4 1 42 000L 2 320 000L 3 800 000L 4a 31.7m^3 , 31 700L 4b 65.2m^3 , 65 200L 5 14 880L 6 21.6m^3

Exercise 8.5 1a 6.7cm^3 1b 0.9cm^3 1c 35m^3 1d 2.4m^3 1e 4 900 000 cm^3 1f 1 900 000 cm^3 1g 8300 mm^3

1h 600 mm^3 2a 4.2L 2b 71.3L 2c 5.8kL 2d 960kL 2e 7300L 2f 610L 2g 2500mL 2h 750mL

Exercise 8.6 2 V=lbh 3 40m^3 4 1000 5 1000 6 26 7 18 8 $2/10=1/5$ 9 $3/1/3$ 10 \$4.10

Exercise 8.7 2 V=lbh 3 60m^3 4 1000 5 1000 6 4 7 6 8 $3/10$ 9 $2/3/4$ 10 \$10.90

Exercise 8.8 2 V=lbh 3 60m^3 4 1000 5 1000 6 54 7 24 8 $4/10=2/5$ 9 $2/4/5$ 10 \$4.70

Exercise 8.9 1 4230L 2 60m^3 3 c 4 b 5 183.26 cm^3 6a E 6b B 6c D 7a f=5, e=9 7b f=6, e=12 7c f=16, e=36

Exercise 8.10 1 b 2 c 3 960g 4 24L 5 21 6 3cm 7 30 cm^3 8 7.25m

Exercise 8.11 1 27 2a 8 2b 12 2c 6 2d 1

Exercise 8.12 1a 4cm^3 1b 6cm^3 1c 36cm^3 2a 36cm^3 2b 108 mm^3 2c 300 m^3 3 2L 4 315L 5 540 000L

6 31.7m^3 , 31 700L 7a 6.2cm^3 7b 53m^3 7c 2 400 000 cm^3 7d 900 mm^3 7e 7.8L 7f 910kL 7g 1300L 7h 250mL

Exercise 8.13 1a 3cm^3 1b 16cm^3 1c 32cm^3 2a 144cm^3 2b 120 mm^3 2c 960 m^3 3 3L 4 346.5L 5 800 000L

6 65.2m^3 , 65 200L 7a 1.5cm^3 7b 73m^3 7c 9 200 000 cm^3 7d 600 mm^3 7e 9.9L 7f 840kL 7g 2600L 7h 750mL

Exercise 9.1 1 65 2 7 3 3million or 3 000 000 4 0 5 4 6 4 7 3 8 12 9 4 10 15 11 220 12 6000 13 70

14 31 15 13 16 0 17 50 18 20 19 4 20 3 21 2 22 50 23 0 24 48 25 8 26 0 27 4 28 7 29 3 30 6

Exercise 9.2 1 $5 > 2$ 2 $3 < 1$ 3 $5 < 2$ 4 $4 < 0$ 5 $0 > 1$ 6 $3 < 2$ 7 $2 < 6$ 8 $4 > 2$ 9 $3 < 6$ 10 $5 > 6$

11 $2 > 0$ 12 $4 < 1$ 13 $6 > 1$ 14 $4 < 3$ 15 $5 > 2$ 16 $13 > 5$ 17 $11 < 1$ 18 $15 < 11$

Exercise 9.3 1 0,1,3,5 2 2,1,0,1 3 3,2,1,6 4 5,3,0,2,3 5 6,4,1,2 6 5,3,1,2,5 7 3,2,1,1,2

8 6,4,1,3,6 9 4,3,2,4,6 10 6,0,2,4,6 11 4,3,1,5,6 12 7,3,3,4,8 13 9,4,3,2,11 14 12,3,2,1,9

15 13,12,7,13,15

Exercise 9.4 1 7 2 2 3 2 4 5 5 13 6 4 7 7 6 8 6 9 25 10 2 11 2 12 5 13 15 14 5 15 3 16 21 17 7 18 3

19 9 20 30 21 4

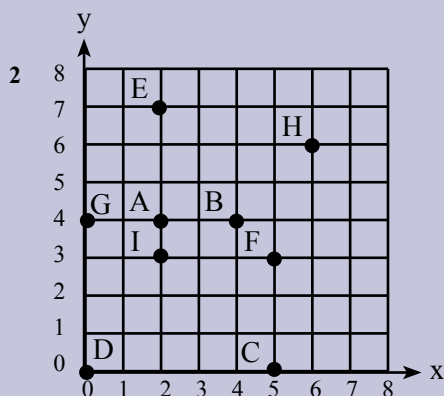
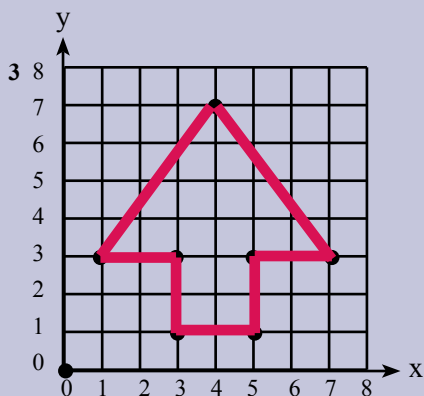
+	-2	-1	0	1
-2	-4	-3	-2	-1
-1	-3	-2	-1	0
0	-2	-1	0	1
1	-1	0	1	2

+	-10	-5	0	5
-10	-20	-15	-10	-5
-5	-15	-10	-5	0
0	-10	-5	0	5
5	-5	0	5	10

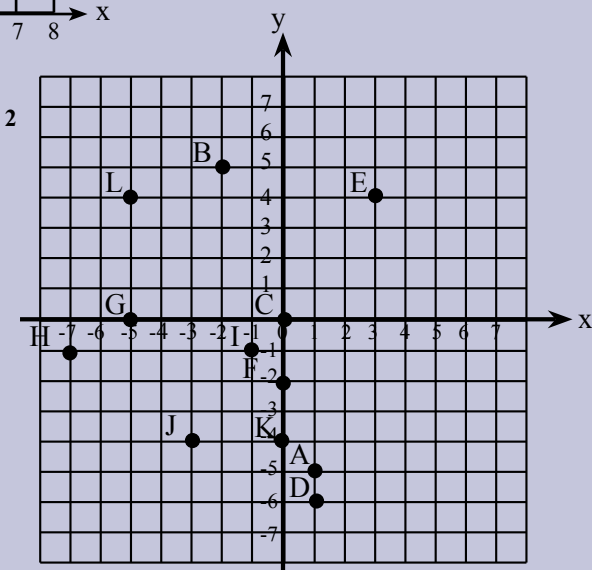
Exercise 9.5 1

Exercise 9.6 1 2 2 -1 3 -2 4 -6 5 1 6 -3 7 3 8 -7 9 3 10 -4 11 2 12 -4 13 5 14 -3 15 -1 16 -6 17 -6 18 -2 19 1 20 -5 21 5 22 6 23 -1 24 2 25 8 26 7 27 6 28 -4 29 4 30 7 31 -2 32 -2 33 8 34 12 35 -5 36 2 37 21 38 9 39 -1 40 9

Exercise 9.7 1 A(1,6), B(0,4), C(0,2), D(2,3), E(2,1), F(3,5), G(4,8) H(4,1), I(5,3), J(6,7), K(6,7), L(6,2), M(7,5), N(7,0), O(8,1)



Exercise 9.8 1 A(-7,7), B(-7,-3), C(-6,-7), D(-4,6), E(-4,-2), F(-3,3), G(-3,-5) H(0,5), I(2,0), J(2,-5), K(4,6), L(4,2), M(5,-2), N(6,-7), O(8,8)



Exercise 9.9 1 1st 2 2nd 3 3rd 4 1st 5 1st 6 4th 7 4th 8 3rd 9 2nd 10 2nd 11 4th 12 3rd 13 1st 14 2nd 15 4th 16 4th 17 4th 18 3rd 19 2nd 20 3rd 21 3rd 22 3rd 23 4th 24 2nd 25a T 25b T 25c F 25d T 25e T 25f F 25g T 25h T

Exercise 9.11 1 linear (3,5) 2 linear (3,6) 3 linear (3,7) 4 non-linear 5 linear (3,8) 6 non-linear 7 linear (3,2) 8 linear (3,1) 9 non-linear 10 linear (3,0) 11 linear (3,-1) 12 linear (0,1) 13 linear (5,7) 14 linear (9,7) 15 linear (9,6)

Exercise 9.12 2 -2 3 -5 4 1st 5 60m³ 6 1000L 7 30 8 5/10=1/2 9 2/12=1/6 10 39

Exercise 9.13 2 -4 3 -3 4 2nd 5 90m³ 6 1000L 7 15 8 6/10=3/5 9 3/12=1/4 10 32

Exercise 9.14 $2^{-3} 3^{-7} 4^3$ 3rd $5 45\text{m}^3$ 6 1000L 7 36 8 $7/10$ 9 $6/12=1/2$ 10 37

Exercise 9.15 1a $3^{-1} b^{-3} 1c^{-2} 1d^{-2} 1e^{-5} 1f^{-2} 2A=-1$, $B=-2.5$ 3 $^{-9} 4a^{-3}$, $^{-2}$, 0, 1, 2 4b $^{-2.5}$, $^{-1.5}$, 0.5, 1.5
5 A(3,1), B($^{-2}$,2), C($^{-1}$,3), D(1, $^{-2}$) 6a B(3,9) 6b B($^{-8}$,2)

Exercise 9.16 $1^{-3} 2a^3 2b^3 2c^5 2d^{-5} 2e^{50} 2f^{-50}$ 3a (6,6) 3b (6,6) or (0,6) or (6,0)

3c (6,6) or (0,6) or (6,0) 3d (6,6) or (0,6) or (6,0) 4a (2,3) 4b (2,3) or ($^{-8}$,3) or (2, $^{-7}$)

4c (2,3) or ($^{-8}$,3) or (2, $^{-7}$) 4d (2,3) or ($^{-8}$,3) or (2, $^{-7}$)

Exercise 9.17 1 7 2 1,3,5,11 is one solution 3 60 4 6

Exercise 9.18 1a $^{-8} 1b^{25} 1c^{-75} 1d^{-15} 2a^{6>3} 2b^{-2<3} 2c^{-1>^{-4}} 3a^{-3}$, $^{-2}$, 0, 1 3b $^{-2}$, $^{-1}$, 0, 1

3c $^{-3}$, $^{-2}$, $^{-1}$, 3 4a 5 4b $^{-1} 4c^{-5} 4d^7 4e^{-1} 4f^{14}$ 5 A(3,2), B(6,4), C(0,4), D(3, $^{-2}$), E(4, $^{-7}$), F(7, $^{-5}$), G(0, $^{-4}$),
H($^{-3}$,5), I($^{-7}$,7), J($^{-4}$,0), K($^{-6}$, $^{-4}$), L($^{-2}$, $^{-7}$) 7a linear 7b linear 7c non-linear 7d linear

Exercise 9.19 1a $^{-200} 1b^4 1c^7 1d^0 2a^{2>^{-1}} 2b^{-4<2} 2c^{-3<2} 3a^{-1}$, 0, 1, 3 3b $^{-4}$, $^{-2}$, 0, 4 3c $^{-4}$, $^{-2}$, $^{-1}$, 1
4a 2 4b 1 4c $^{-5} 4d^3 4e^3 4f^4$ 5 A(4,3), B(6,1), C(0,2), D(5, $^{-1}$), E(2, $^{-7}$), F(3, $^{-3}$), G($^{-5}$,0), H($^{-2}$,7), I($^{-5}$,3),
J($^{-2}$,0), K($^{-5}$,2), L($^{-4}$,7) 7a linear 7b linear 7c non-linear 7d linear

Exercise 10.1 $2^{-3} 3^{-5} 4^1$ 1st $5 40\text{m}^3$ 6 1000L 7 20 8 $5/10=1/2$ 9 $5/6$ 10 29

Exercise 10.2 1a $1/7$, $1/5$, $1/3$, $1/2$ 1b $1/9$, $1/8$, $1/6$, $1/4$ 1c $1/4$, $3/8$, $2/5$ 1d $2/3$, $7/10$, $4/5$ 2a $4/5=8/10$

2b $2/3=6/9$ 2c $1/4=2/8=3/12$ 2d $2=6/3=10/5=14/7$ 3a $2/1/3$ 3b $4/1/2$ 3c $1/2/5$ 3d $3/2/3$ 3e $2/2/5$ 3f $2/1/6$

3g $2/5/7$ 3h $2/7/9$ 4a $4/3$ 4b $5/2$ 4c $8/3$ 4d $16/5$ 4e $14/5$ 4f $15/4$ 4g $27/8$ 4h $59/10$ 5a $3/5$ 5b $7/6=1/1/6$

5c $2/5$ 5d $5/12$ 5e $1/6$ 5f $2/12=1/6$ 5g $4/3=1/1/3$ 5h $8/3=2/2/3$ 6a 4 6b 9 6c 4 6d 10 6e 12 6f 4 7a $x+3$

7b $x-4$ 7c $3x$ 7d $x/4$ 8a 9 8b 12 8c 7 8d 9 8e 3 8f $^{-3} 8g^9 8h^3 8i^{17}$ 9a $x+4$ and $4+x$ are commutative

9b $yx+3$ and $3y$ are commutative 9c $x-4$ and $4-x$ are not commutative

10a $(x+y)+z$ and $x+(y+z)$ are associative 10b $5x+(y+1)$ and $(5x+y)+1$ are associative

10c $(x-y)-z$ and $x-(y-z)$ are not associative 10d $2(xy)$ and $(2x)y$ are associative 11 36watts 12a 3cm^3

12b 16cm^3 12c 36cm^3 13a 72cm^3 13b 64mm^3 13c 1080m^3 14 2L 15 900 000L 16a 3.8cm^3 16b 47m^3

16c 1 600 000 cm^3 16d 300 mm^3 16e 7.8L 16f 880kL 16g 5200L 16h 750mL 17a $^{-600}$ 17b 7 17c $^{-12}$ 17d 0

18a $4>^{-2}$ 18b $^{-1}<2$ 18c $^{-2}>3$ 19a $^{-2}$, $^{-1}$, 0, 1 19b $^{-3}$, $^{-2}$, 0, 2 19c $^{-2}$, $^{-1}$, 1, 2 20a 3 20b $^{-3}$ 20c $^{-3}$ 20d 6

20e $^{-2}$ 20f 8 22a linear 22b linear 23 A(7,5), B(4,1), C(0,6), D(7, $^{-2}$), E(4, $^{-7}$), F(2, $^{-5}$), G(0, $^{-2}$), H(4,6),

I($^{-6}$,4), J($^{-4}$,0), K($^{-7}$,3), L($^{-4}$,5)

Exercise 10.3 $2^{-2} 3^{-5} 4^2$ 2nd $5 90\text{m}^3$ 6 1000 7 9 8 $8/10=4/5$ 9 $4/2=2$ 10 57

Exercise 10.4 1a $1/5$, $1/4$, $1/3$, $1/2$ 1b $1/9$, $1/7$, $1/5$, $1/2$ 1c $3/5$, $2/3$, $3/4$ 1d $1/3$, $3/8$, $2/5$ 2a $3/5=6/10$

2b $3/4=6/8$ 2c $2/3=4/6=6/9$ 2d $3=9/3=15/5=27/9$ 3a $4/1/2$ 3b $2/1/4$ 3c $1/3/5$ 3d $2/4/5$ 3e $2/3/7$ 3f $1/4/9$

3g $3/3/7$ 3h $9/2/3$ 4a $7/3$ 4b $5/2$ 4c $7/4$ 4d $17/5$ 4e $23/6$ 4f $30/7$ 4g $29/5$ 4h $69/10$ 5a $5/7$ 5b $11/10=1/1/10$

5c $3/5$ 5d $7/15$ 5e $1/12$ 5f $6/12=1/2$ 5g $3/4$ 5h $9/8=1/1/8$ 6a 6 6b 7 6c 6 6d 8 6e 14 6f 7 7a $x+7$ 7b $x-7$

7c $x \times 10$ or $10x$ 7d $x/3$ 8a 10 8b 12 8c 7 8d 11 8e $^{-1}$ 8f 1 8g 13 8h $^{-7}$ 8i $7 9a^{x+5}$ and $5+x$ are commutative

9b yx and xy are commutative 9c $x-3$ and $3-x$ are not commutative

10a $(x+y)+z$ and $x+(y+z)$ are associative 10b $3x+(y+2)$ and $(3x+y)+2$ are associative

10c $(x-y)-z$ and $x-(y-z)$ are not associative 11 72watts 12a 6cm^3 12b 12cm^3 12c 32cm^3 13a 105cm^3

13b 180mm^3 13c 784m^3 14 3L 15 640 000L 16a 7.9cm^3 16b 64m^3 16c 3 200 000 cm^3 16d 400 mm^3 16e 5L

16f 310kL 16g 9500L 16h 250mL 17a $^{-700}$ 17b $^{-9}$ 17c 37 17d 0 18a $1>^{-3}$ 18b $^{-2}<1$ 18c $^{-5}<2$

19a $^{-3}$, $^{-1}$, 0, 2 19b $^{-3}$, $^{-2}$, 0, 1 19c $^{-3}$, $^{-2}$, $^{-1}$, 1 20a $^{-3}$ 20b $^{-1}$ 20c $^{-5}$ 20d 7 20e $^{-1}$ 20f 7 22a linear 22b linear

23 A(6,4), B(7,1), C(0,1), D(7,2), E(6, $^{-6}$), F(2,3), G(0, $^{-5}$), H($^{-2}$,7), I($^{-6}$,6), J($^{-3}$,0), K($^{-6}$,2), L($^{-3}$, $^{-6}$)

24 17 more cubes

Exercise 11.1 1 1 2 4 3 9 4 16 5 25 6 1 7 8 8 27 9 64 10 125 11 1 12 16 13 81 14 256 15 625 17 100

18 1000 19 10 000 20 100 000 21 1 000 000 22 10 000 000 23 64 24 78 125 25 1 26 256 27 6 28 9 29 12

30 9 31 normally no

Exercise 11.2 $1^4 2^5 3^{10^2} 4^{10^3} 5^{10^4} 6^{10^1} 7^{10^5} 8^9 9^3$

Exercise 11.3 $1^4 \times 4 \times 4^2 10 \times 10 \times 10 \times 10 \times 3^5 \times 5^4 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5^6 \times 6^6 10 \times 10 \times 10 \times 10 \times 7^6 10$

$8^{10} \times 10 \times 10 \times 10 \times 10 \times 10^9 1 \times 1 \times 1 \times 1$

Exercise 11.4 1 300 2 6000 3 20 4 47 5 94 6 625 7 783 8 437 9 211

Exercise 11.5 1 1,2 2 1,3 3 1,2,4 4 1,5 5 1,2,3,6 6 1,7 7 1,2,4,8 8 1,3,9 9 1,2,5,10 10 1,11 11 1,3,5,15

12 1,2,4,8,16 13 1,2,4,5,10,20 14 1,2,3,4,6,8,12,24 15 1,3,9,27 16 1,2,4,7,14,28 17 1,2,3,5,6,10,15,30

18 1,31 19 1,2,4,8,16,32 20 1,3,11,33

Exercise 11.6 1 3 2 4 3 4 4 5 5 8 6 3 7 10 8 2 9 7 10 6 11 7 12 2

Exercise 11.7 Sieve as shown

Exercise 11.8 1 Yes. All other even numbers have at least 1, 2, and itself as factors. Thus not prime.

2 Yes. The larger the number, the higher the chance that they have a factor other than 1 and itself.

3 There are 4 prime numbers less than 10 - 2, 3, 5, 7

4 With the exception of $2+3=5$ it would appear to be true although not proved.

5 With the exception of $2 \times 3 = 6$ it would appear to be true although not proved.

6 With the exception of 2 and 3.

Exercise 11.9 1 2×3^2 2 $2^2 \times 5$ 3 $2^3 \times 3$ 4 3×5 5 $2 \times 3 \times 5$ 6 2^5 7 $2^2 \times 5^2$ 8 $2^3 \times 3 \times 5$ 9 $2^2 \times 3^2 \times 5$ 10 $2^4 \times 3^2$ 11 2×5^3 12 $2^3 \times 5^3$ 13 $2^3 \times 5^4$ 14 $2^4 \times 5^4$ 15 $2^5 \times 3 \times 5^3$

Exercise 11.10 1 True $12=2^2 \times 3$, $6=2 \times 3$ 2 True $120=2^3 \times 3 \times 5$, $60=2^2 \times 3 \times 5$ 3 $360=2^3 \times 3^2 \times 5$

4 True $2 \times 3 \times 5 \times 7 = 210$ 5 True $2 \times 3 \times 5 \times 7 \times 11 = 2310$ 6 False $888=2^3 \times 3 \times 7 \times 11 \times 13 \times 37$

7 True $282 \ 475 \ 249 = 7^{10}$

Exercise 11.11 1 16 2 64 3 25 4 125 5 36 6 216 7 49 8 343 9 64 10 512 11 81 12 729 13 100 14 1000 15 9 and 16 16 16 and 25 17 36 and 49 18 49 and 64

Exercise 11.12 1 3 2 2 3 4 4 3 5 5 6 5 7 8 8 6 9 20 10 12 11 15 12 18 13 5 14 12 15 5 and 6 16 8 and 9 17 2 and 3

Exercise 11.13 2 3^3 3 200 4 597 5 25 6 2 7 $1/3$ 8 70% 9 210 10 \$1.40

Exercise 11.14 2 2^4 3 60 4 385 5 27 6 6 7 $2/1$ 8 40% 9 240 10 \$1.20

Exercise 11.15 2 5^3 3 400 4 671 5 49 6 3 7 $3/1$ 8 40% 9 270 10 \$6.80

Exercise 11.16 1a 64 1b 8 1c 8 1d 16 1e 12 1f 13 1g 4 1h 18 1i 100 1j 1000 1k 592 1l 3476 1m 72 081 2 d 3 c 4 d 5a 6 5b 6 5c 36 5d 27 6 b 7 b 8 b

Exercise 11.17 1a 0.01 1b 0.04 1c 0.09 1d 0.000 9 1e 1110 1f 111 110 1g 30 2a 19+5, 17+7, 13+11 2b 23+2 3 2+13+3, 2+11+5, 2+9+7, 2+3+11 4 60 5 110 years to 2112 6 45 7 22 8 11 9 30 10 15 11 a,c,e

Exercise 11.18 1 97 2a 29 2b 23/28

Exercise 11.19 1a 3^3 1b 2^5 1c 10^2 1d 10^3 1e 10^4 1f 10^5 1g 5^5 1h 7^4 1i 3^6 2a $5 \times 5 \times 5$ 2b $3 \times 3 \times 3 \times 3$ 2c 3×3 2d 10 2e 10×10 2f $10 \times 10 \times 10$ 2g $10 \times 10 \times 10 \times 10$ 2h $10 \times 10 \times 10 \times 10 \times 10$ 2i $1 \times 1 \times 1 \times 1 \times 1$ 3a 200 3b 5000

3c 40 3d 68 3e 97 3f 615 3g 792 3h 669 3i 8100 4a 1, 2 4b 1, 2, 3, 6 4c 1, 2, 4, 8 4d 1, 2, 5, 10 4e 1, 3, 5, 15 4f 1, 2, 4, 5, 10, 20 4g 1, 2, 3, 4, 6, 8, 12, 24 4h 1, 5, 25 4i 1, 2, 3, 5, 6, 10, 15, 30 5a 2 5b 2 5c 4 5d 5 5e 8 5f 3 5g 8 5h 7 5i 6 6a $2^2 \times 3$ 6b 3×5 6c 2^4 6d 2×3^2 6e $2^2 \times 5$ 6f $2^2 \times 7$ 6g $2^3 \times 5^3$ 6h $2^4 \times 5^3$ 6i $2^4 \times 3 \times 5^3$ 7 4 8 8 9 9 10 27 11 3 12 2 13 4 14 3 15 20 16 12

Exercise 11.20 1a 2^2 1b 3^3 1c 7^2 1d 10^2 1e 10^3 1f 10^5 1g 8^4 1h 6^3 1i 4^6 2a $2 \times 2 \times 2$ 2b 3×3 2c $4 \times 4 \times 4 \times 4$ 2d 10 2e 10×10 2f $10 \times 10 \times 10$ 2g $10 \times 10 \times 10 \times 10$ 2h $10 \times 10 \times 10 \times 10 \times 10$ 2i $1 \times 1 \times 1 \times 1 \times 1 \times 1$ 3a 50 3b 300 3c 7000 3d 24 3e 53 3f 910 3g 813 3h 304 3i 4404 4a 1, 3 4b 1, 2, 4 4c 1, 5 4d 1, 2, 3, 6 4e 1, 2, 5, 10 4f 1, 2, 3, 4, 6, 12 4g 1, 2, 3, 6, 9, 18 4h 1, 3, 9, 27 4i 1, 2, 4, 5, 8, 10, 20, 40 5a 2 5b 4 5c 10 5d 4 5e 15 5f 8 5g 5 5h 12 5i 8 6a 2×5 6b $2^3 \times 3$ 6c 2×3^2 6d $2^2 \times 5$ 6e $2^3 \times 3$ 6f $2 \times 3 \times 5$ 6g $2^2 \times 5^3$ 6h $2^3 \times 5^3$ 6i $2^3 \times 5^4$ 7 4 8 8 9 25 10 125 11 5 12 2 13 6 14 5 15 15 16 18

Exercise 12.1 1 4 2 2 3 24 4 21 5 14 6 48 7 10 8 31 9 46 10 39 11 6 12 18 13 21 14 29

Exercise 12.2 1 $x+10=15$ 2 $x-7=19$ 3 $x \div 8=9$ 4 $5x=25$ 5 $x-12=21$ 6 $x-16=37$ 7 $x+7=11$ 8 $x \div 4=7$ 9 $x+14=49$ 10 $3x=27$ 11 $x \div 2=17$ 12 $20-x=2$ 13 $45 \div x=5$ 14 $x+73=99$ 15 $x \div 6=7$

Exercise 12.3 1 $x=3$ 2 $x=3$ 3 $x=5$ 4 $a=4$ 5 $a=3$ 6 $b=3$ 7 $y=2$ 8 $b=11$ 9 $x=4$ 10 $x=9$ 11 $x=9$ 12 $x=17$ 13 $x=2$ 14 $x=-2$ 15 $x=-5$ 16 $x=3$ 17 $x=3$ 18 $x=3$ 19 $x=3$ 20 $x=6$ 21 $x=4$ 22 $x=2.8$ 23 $x=2.5$ 24 $x=-5$

Exercise 12.4 1 $x=1$ 2 $x=2$ 3 $x=3$ 4 $x=3$ 5 $a=7$ 6 $b=3$ 7 $x=2$ 8 $x=-1$ 9 $x=16$ 10 $x=50$ 11 $x=64$ 12 $x=100$ 13 $x=32$ 14 $x=3$ 15 $x=49$ 16 $x=25$ 17 $x=6$

Exercise 12.5 1 $x=68^\circ$ 2 $x=57^\circ$ 3 $x=7m$ 4 $x=4m$ 5 $x=50^\circ$ 6 $x=55^\circ$

Exercise 12.6 1 20boxes 2 8sales 3 3.9kg 4 3bags 5 17cartons

Exercise 12.7 1 4000flyers 2 2408books 3 5.5hrs or 5hrs 30mins 4 5.8hrs or 5hrs 48 mins 5 15.6km 6 16.6km 7 32.6mins or 32mins 36secs 8 4.7mins or 4mins 42secs 9 $25.7^\circ C$

Exercise 12.8 2 $x=3$ 3 4^4 4 $x=3$ 5 m^5 6 2^2 7 1 8 charge= $2.5 \times km + 3$ 9 $7/2$ 10 7^3

Exercise 12.9 2 $x=12$ 3 2^5 4 $x=4$ 5 b^6 6 5^2 7 1 8 cost= $28 \times \text{time} + 25$ 9 $8/3$ 10 7^7

Exercise 12.10 2 $x=9$ 3 5^6 4 $x=2$ 5 d^5 6 3^3 7 1 8 charge= $80 \times \text{hours} + 70$ 9 $11/4$ 10 7^2

	2	3		5		7			
11		13				17		19	
		23						29	
31						37			
41		43				47			
		53						59	
61						67			
71		73						79	
						97			

Exercise 12.11 1 6 2 17 3 0.78 4 $5 \leq 2 < 6$ $y=x^2$ 7 22 8 3kg 9a 4 9b 9 10a $x=12$ 10b $x=7$ 11a x^2-12
11b $ab-12$

Exercise 12.12 1a 16 1b 13 2 12 3 0.6kg 4a $x=143^\circ$ 4b $x=60^\circ$ 4c $x=41^\circ$ 4d $x=36^\circ$ 4e $x=90^\circ$ 4f $x=66^\circ$
5 $x=5m$ 6 $x=3$, $y=5/4$

Exercise 12.13 1 19 2 6 3 10

Exercise 12.14 1a $x+13=23$ 1b $x-9=11$ 1c $x+42=6$ 1d $24x=72$ 2a $x=3$ 2b $x=11$ 2c $x=3$ 2d $x=12$ 2e $a=-2$
2f $b=2/1/2$ or $b=2.5$ 2g $c=1$ 2h $d=5$ 2i $x=4/5$ or $x=0.8$ 3a $x=70^\circ$ 3b $x=9m$ 3c $x=60^\circ$ 3d $x=12m$ 3e 6sales
3f 5.6kg 3g 7bags 3h 8pallets 3i 8.2hrs or 8hrs 12mins

Exercise 12.15 1a $x+12=17$ 1b $x-15=8$ 1c $x+61=3$ 1d $7x=91$ 2a $x=4$ 2b $x=7$ 2c $x=5$ 2d $x=12$ 2e $a=-3$
2f $b=2/1/5$ or $b=2.2$ 2g $m=3$ 2h $p=6$ 2i $x=1/1/2$ or $x=1.5$ 3a $x=65^\circ$ 3b $x=9m$ 3c $x=75^\circ$ 3d $x=14m$
3e 11 766flyers 3f 40.4mins or 40mins 24secs 3g 18bags 3h 8pallets 3i 7.7hrs or 7hrs 42mins

Exercise 13.3 1 acute 2 reflex 3 reflex 4 obtuse 5 right 6 revolution 7 reflex 8 reflex 9 acute 10 straight
11 acute 12 obtuse 13 obtuse 14 reflex 15 reflex

Exercise 13.4 1 $a=80^\circ$ 2 $b=135^\circ$ 3 $a=43^\circ$ 4 $a=110^\circ$, $b=70^\circ$

Exercise 13.5 1 $b=80^\circ$ 2 $a=60^\circ$ 3 $b=50^\circ$ 4 $a=60^\circ$ 5 $a=150^\circ$ 6 $a=85^\circ$ 7 $b=110^\circ$ 8 $b=150^\circ$

Exercise 13.6 2a $a=115^\circ$ 2b $b=63^\circ$ 2c $c=74^\circ$ 2d $d=70^\circ$

Exercise 13.7 2a $x=110^\circ$ 2b $y=75^\circ$ 2c $a=45^\circ$ 2d $b=30^\circ$

Exercise 13.8 1a isosceles, acute-angled 1b equilateral, acute-angled 1c scalene, obtuse-angled
1d right-angled 1e scalene, acute-angled 1f equilateral, acute-angled 2a $a=70^\circ$ 2b $a=40^\circ$ 2c $b=60^\circ$
2d $a=b=c=60^\circ$, $d=e=8cm$ 2e $b=65^\circ$, $a=3m$ 2f $z=60^\circ$, $x=y=1m$

Exercise 13.9 2

	Square	Rectangle	Parallelogram	Rhombus	Trapezium	Kite
Opposite sides equal	✓	✓	✓	✓		
Opposite angles equal	✓	✓	✓	✓		✓ one pair
Opposite sides parallel	✓	✓	✓	✓	✓ one pair	
Diagonals equal	✓	✓				
90° angles	✓	✓				

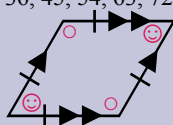
3a square, rectangle, parallelogram, rhombus 3b square, rectangle, parallelogram, rhombus

3c square, rectangle 3d square, rectangle 3e square, rhombus 3f square, rhombus, kite

3g square, rectangle, parallelogram, rhombus, trapezium

Exercise 13.10 1 5 2 18, 27, 36, 45, 54, 63, 72, or 81 3 304–253 or 403–352

Exercise 13.11 2 acute 3 3 4



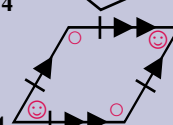
5 $x=3$ 6 m^5 7 $^{-2}$ 8 $\text{charge}=3d+4$ 9 64 10 $1/2/3$

Exercise 13.12 2 obtuse 3 2 4



5 $x=2$ 6 x^7 7 $^{-3}$ 8 $\text{charge}=2d+3$ 9 60 10 $1/1/2$

Exercise 13.13 2 reflex 3 3 4



5 $x=1/1/2$ or $x=1.5$ 6 b^5 7 $^{-5}$ 8 $\text{charge}=2d+4$ 9 84 10 $2/1/2$

Exercise 13.14 1 c 2 c 3a 50cm 3b 60cm 3c 70cm 4a 90° 4b $x=70^\circ$, $y=45^\circ$, $z=65^\circ$ 5 d 6a 72° 6b 136°

Exercise 13.15 1a 80° 1b 60° 1c 110° 1d 145° 1e 130° 1f 120° 1g 45° 1h 105° 1i 150°


Exercise 13.16 1a acute 1b reflex 1c reflex 1d obtuse 1e right 1f revolution 2a $a=120^\circ$, $b=60^\circ$ 2b $b=75^\circ$
 2c $b=55^\circ$ 2d $a=155^\circ$ 2e $a=110^\circ$ 2f $x=100^\circ$ 2g $a=65^\circ$ 2h $b=60^\circ$ 2i $x=3m$, $y=3m$, $z=60^\circ$
 3a square, rectangle, parallelogram, rhombus 3b square, rectangle, parallelogram, rhombus
 3c square, rectangle 3d square, rectangle 3e square, rhombus 3f square, rhombus, kite
 3g square, rectangle, parallelogram, rhombus, trapezium

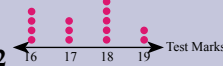
Exercise 14.1 1 39 2 227 3 5 4 6 5 5 6 6 7 2 9 8 5

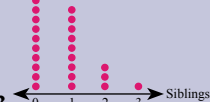
Exercise 14.2 1 6 {3 times} 2 4 {3 times} 3 1 {3 times} 4 3 and 4 {twice each} 5 5 {4 times}
 6 1 and 2 {3 times each} 7 32 {twice} 8 7.1 {3 times} 9 1 {3 times} 10 a {4 times}


Exercise 14.3 1 4 2 4 3 2 4 2.5 5 32 6 23 7 5.2 8 2

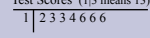
Exercise 14.4 1 3 2 15.67 3 2.9 4 3.33 5 33.33 6 83.33 7 103.33 8 723.33 9 5.33 10 3.33

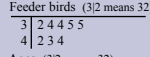
Exercise 14.5 1  range=2, mode=13 {8 times}, median=13, mean=12.83

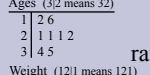
2  range=3, mode=18 {5 times}, median=17.5, mean=17.36

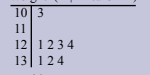
3  range=3, mode=0 {10 times}, median=1, mean=0.78

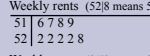
4  range=6, mode=-3 and 0 {3 times}, median=-0.5, mean=-0.86

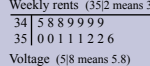
Exercise 14.6 1  range=4, mode=16 {3 times}, median=14, mean=14.29

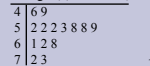
2  range=12, mode=34 and 35 {2 times}, median=35, mean=37.38

3  range=23, mode=21 {3 times}, median=21, mean=22.75

4  range=31, mode=no mode, median=123.5, mean=123.75

5  range=12, mode=522 {4 times}, median=522, mean=520.67

6  range=11, mode=349 {4 times}, median=350, mean=350

7  range=2.7, mode=5.2 {3 times}, median=5.8, mean=5.82

Glasses	No glasses (415 is 45)
1	3
3	4
7300	5
987764310	6
98731	7
753	8
98	9

Exercise 14.7 1

Glasses: range=68, mode= 67 and 50 {2 times each}, median=67, mean=67.96

No glasses: range=49, mode=no mode, median=68, mean=67.64

Brand A	Brand B (410 is 40)
81	1
6541	2
75532100	3
630	4
41	5

Brand A: range=40, mode=30 and 55 {2 times}, median=32, mean=32.89

Brand B: range=14, mode= 30 and 31 {3 times}, median=32, mean=34

Maths	Biology (11/6 is 116)
70	8 25589
764322	9 014789
988753200	10 225689
99532110	11 033467
952	12 12445

3

Maths: range=49, mode=no mode, median=104, mean=104.96

Biology: range=43, mode=no mode, median=105.5, mean=104.61

Melbourne	Sydney (13/7 is 13.7)
43	8 35
9321	9 2244
	10 79
7411	11 44
	12 7
985	13 577
5	14 7
62	15 3
9	16 3467
331	17 3445
9653	18 19

4

Melbourne: range=10.6, mode=12.1 and 18.2 {2 times}, median=14.85, mean=14.70

Sydney: range=10.2, mode=no mode, median=14.6, mean=13.82

Exercise 14.8 1 range=4, mode=4 {5 times}, median=4, mean=3.875

2 range=3, mode=17 {4 times}, median=17, mean=16.78

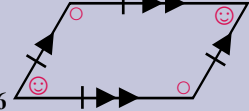
3 range=30, mode=80 {6 times}, median=80, mean=83.64

4 range=300, mode=300 {6 times}, median=300, mean=263.64

5 range=54, mode=142 {3 times}, median=145, mean=149.15

6 range=42, mode=27 {3 times}, median=26.5, mean=23.56

Exercise 14.9 2 3 3 2 4 2 5 3 6  $7x=2$ 8 m^6 9 $^{-3}$ 10 0.4

Exercise 14.10 2 3 3 3 4 3 5 3.2 6  $7x=3$ 8 a^5 9 $^{-9}$ 10 0.25

Exercise 14.11 2 4 3 2 4 2 5 2.4 6  $7x=5$ 8 b^8 9 $^{-1}$ 10 0.6

Exercise 14.12 1 5 2 2 3 32mins 4a mean=15, median=5, mode=5 4b mean=5, median=5, mode=5

4c the mean 5a mean=2.5, median=2, mode=2 5b true 6 45 7 17.5 8 5 9 54 10 the mean

Exercise 14.13 1 1.11 2 12.21 3 123.21 4 \$400 5 \$800 6 3 7 120 8 4500kg or 4.5tonnes 9 97 10 90 11 15 12 106kg



Exercise 14.14 1 60 2 23 cars 3 30 4

Exercise 14.15 1a range=5, mode=3 {3 times}, median=3, mean=3

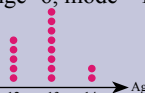
1b range=5, mode=2 {4 times}, median=2.5, mean=3.75

1c range=2, mode=7 {4 times}, median=7, mean=7

1d range=5, mode=12 {3 times}, median=12, mean=12

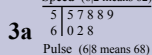
1e range=0.5, mode=2.3 {2 times}, median=2.3, mean=2.3

1f range=6, mode=-1 {3 times}, median=-2.5, mean=-2.9

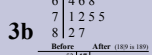
2a  range=2, mode=13 {8 times}, median=13, mean=12.8



range=3, mode=8 {8 times}, median=8, mean=7.63



range=13, mode=58 {2 times}, median=58.5, mean=59.625



range=23, mode=75 {2 times}, median=72, mean=73.33

4 Before: range=51, mode=no mode, median=196, mean=197.17

After: range=41, mode=no mode, median=203, mean=203.38

5a range=2, mode=12 {9 times}, median=12, mean=11.85

5b range=30, mode=53 {3 times}, median=53, mean=57.4

Exercise 14.16 1a range=2, mode=4 {3 times}, median=4, mean=3.2

1b range=5, mode=1 {3 times}, median=2.5, mean=2.875

1c range=3, mode=6 {4 times}, median=6, mean=6.33

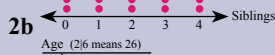
1d range=5, mode=32 {3 times}, median=32, mean=32

1e range=0.3, mode=1.3 {2 times}, median=1.3, mean=1.26

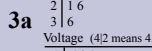
1f range=4, mode=-2 {4 times}, median=-2, mean=-2.7



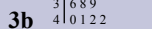
range=3, mode=7 {7 times}, median=7, mean=7.53



range=4, mode=1 {7 times}, median=1.5, mean=1.75



range=25, mode=12 {4 times}, median=12, mean=17.75



range=0.6, mode=4.2 {2 times}, median=4, mean=3.97

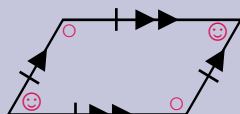
4

Holiday: range=32, mode=no mode, median=52.5, mean=52.83

Workday: range=37, mode=no mode, median=48.5, mean=48.97

5a range=3, mode=18 {7 times}, median=18, mean=17.5

5b range=30, mode=51 {2 times}, median=56, mean=56.64



Exercise 15.1 2 3 3 2 {3 times} 4 2 5 3 6 $7x=3$ 8 a^5 9 $^{-3}$ 10 $\frac{8}{3}$

Exercise 15.2 1a 2^3 **1b** 3^6 **1c** 10^2 **1d** 10^3 **1e** 5^4 **1f** 4^6 **2a** $2 \times 2 \times 2$ **2b** $3 \times 3 \times 3 \times 3$ **2c** 4×4 **2d** 10 **2e** $10 \times 10 \times 10 \times 10$

2f $1 \times 1 \times 1 \times 1 \times 1 = 1$ **3a** 300 **3b** 4000 **3c** 80 **3d** 49 **3e** 237 **3f** 6300 **4a** 1,2 **4b** 1,2,3,6 **4c** 1,2,4,8 **4d** 1,2,5,10

4e 1,2,3,4,6,12 **4f** 1,5,25 **5a** 4 **5b** 5 **5c** 4 **5d** 5 **5e** 7 **5f** 2 **6a** 2×7 **6b** $2^2 \times 3$ **6c** 2×3^2 **6d** $2^3 \times 3$ **6e** $2^3 \times 5^3$ **6f** $2^5 \times 5^3$

7 4 **8** 8 **9** 3 **10** 2 **11a** $x+11=35$ **11b** $x-7=13$ **11c** $x \div 52=3$ **1d** $12x=84$ **12a** $x=2$ **12b** $x=10$ **12c** $x=3$ **12d** $a=20$

12e $x=5$ **12f** $b=5$ **12g** $c=1$ **12h** $x=6$ **12i** $m=3$ **13a** $x=65^\circ$ **13b** $x=15m$ **13c** $x=70^\circ$ **13d** $x=10m$ **13e** 7 sales

13f 8kg **3g** 12pallets **3h** 8.15hrs or 8hrs 9mins **14a** acute **14b** reflex **14c** reflex **14d** obtuse **14e** right

14f revolution **15a** $a=110^\circ$, $b=70^\circ$ **15b** $b=77^\circ$ **15c** $b=52^\circ$ **15d** $a=161^\circ$ **15e** $a=106^\circ$ **15f** $x=100^\circ$ **15g** $a=62^\circ$

15h $b=60^\circ$ **15i** $x=5m$, $y=5m$, $z=60^\circ$ **16a** square, rectangle, parallelogram, rhombus

16b square, rectangle, parallelogram, rhombus **16c** square, rectangle **16d** square, rectangle

16e square, rhombus **16f** square, rhombus, kite **16g** square, rectangle, parallelogram, rhombus, trapezium

17a range=5, mode=3 {3 times}, median=3, mean=3

17b range=4, mode=2 {4 times}, median=2.5, mean=3.25

306

5 days before		2 days before (4/6 is 46)
88530	7	23556899
88753220	8	11235578999
97664410	9	02245669
97644	10	459
854	11	3
632	12	5

20

5 days before: range=58, mode=no mode, median=94, mean=94.5
 2 days before: range=59, mode=89 {3 times}, median=88.5, mean=88.63

Exercise 16.1 1a 9:14 1b 14:9 1c 9:23 1d 14:23 2a 24:7 2b 7:24 2c 24:31 2d 7:31 3a 1/2, 0.5, 50%
 3b 1/4, 0.25, 25% 3c 3/4, 0.75, 75% 3d 1/5, 0.2, 20% 3e 2/5, 0.4, 40% 3f 3/5, 0.6, 60% 3g 4/5, 0.8, 80%
 3h 1/10, 0.1, 10% 3i 2/10, 0.2, 20% 3j 3/10, 0.3, 30% 3k 4/10, 0.4, 40% 3l 5/10, 0.5, 50%
 3m 6/10, 0.6, 60% 3n 5/2 or 2 1/2, 2.5, 250% 3o 3/2 or 1 1/2, 1.5, 150% 3p 7/2 or 3 1/2, 3.5, 350%
 4a 1:2, 0.5, 50% 4b 3:10, 0.3, 30% 4c 3:5, 0.6, 60% 4d 4:5, 0.8, 80% 4e 5:10, 0.5, 50% 4f 1:4, 0.25, 25%
 4g 3:4, 0.75, 75% 4h 5:2, 2.5, 250%

Exercise 16.2 1 2:1 2 1:4 3 3:4 4 1:3 5 2:3 6 3:1 7 5:3 8 4:3 9 1:3 10 4:3 11 10:3 12 4:7 13 1:4 14 3:5 15 2:3

Exercise 16.3 1 0.4L or 400mL 2 0.5L or 500mL 3 Amelia \$66, Jack \$88 4 Sophie \$21, Noah \$9
 5 5 shovelfulls

Exercise 16.4 a) 10% b) 20% c) 30% d) 40% e) 50% f) 60% g) 70% h) 80% i) 90% j) 100% k) 110%
 l) 130% m) 25% n) 75% o) 42% p) 92% q) 230% r) 63% s) 345% t) 620% u) 23.5% v) 230.6% w) 4%
 x) 6.25%

Exercise 16.5 a) 10% b) 20% c) 30% d) 40% e) 50% f) 60% g) 70% h) 80% i) 90% j) 100% k) 50%
 l) 25% m) 75% n) 20% o) 40% p) 80%

Exercise 16.6 a) 30% b) 50% c) 70% d) 80% e) 20% f) 40% g) 60% h) 80% i) 90% j) 100% k) 70%
 l) 75% m) 80% n) 85% o) 48.94% p) 31.91%

Exercise 16.7 a) 6 b) 13 c) 7.2 d) 49 e) 64 f) 40 g) 20 h) 60 i) 210 j) 100 k) 2 l) 24.5 m) 35 kg n) \$240
 o) \$2850 p) \$7450 q) \$22 500 r) \$14 475

Exercise 16.8 2a 10 m/s 2b 16 points/match 2c 2 L/s 2d 60 beats/min 2e 35 people/bus 2f 55 marks/test
 3a \$1860 3b \$12 090 3c \$24 180 4a \$3702.60 4b \$24 066.90 4c \$48 133.80

Exercise 16.9 2 \$1400 3 2.8 4 5:4 5 2 {3 times} 6 3 7 $x=2$ 8 x^5 9 $^{-2}$ 10 \$60

Exercise 16.10 2 \$1800 3 3.1 4 5:8 5 2 6 2.4 7 $x=4$ 8 b^6 9 3 10 \$80

Exercise 16.11 2 \$2200 3 7.4 4 5:7 5 1 {3 times} 6 2 7 $x=3$ 8 c^8 9 $^{-4}$ 10 \$68

Exercise 16.12 1 1200 m 2 \$150 3 27 L 4 18 000 000 km 5 75 g 6 14 L 7 6 kg 8 9 kg 9 \$210 10 400 km

Exercise 16.13 1 600 km/h 2 33 h 20 min 3 125 m 4 4 min 5 132 000 6 12 s 7 3 days 8 3 days 9 12 mins
 10 8:12:59 11 \$100 12 \$2500

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

Exercise 16.14 1 \$500 profit 2 40 kg 3

Exercise 16.15 1a 11:13 1b 13:11 1c 11:24 1d 13:24 2a 1/10, 0.1, 10% 2b 3/10, 0.3, 30% 2c 2/5, 0.4, 40%
 2d 3/5, 0.6, 60% 2e 1/2, 0.5, 50% 2f 1/4, 0.25, 25% 2g 3/4, 0.75, 75% 2h 3/20, 0.15, 15% 3a 1:2, 0.5, 50%
 3b 3:10, 0.3, 30% 3c 3:5, 0.6, 60% 3d 4:5, 0.8, 80% 4a 2:1, 4b 1:2 4c 3:2 4d 3:2 4e 2:5 4f 2:3
 5 0.8 L or 800 mL 6 0.5 L or 500 mL 7a 50% 7b 70% 7c 30% 7d 60% 7e 80% 7f 95% 8a 4 8b 20 8c 10
 8d 30 8e \$180 9a 10 m/s 9b 12 points/match 9c 2 L/s 9d 80 beats/min 9e 9 trees/h 9f \$130/sale
 10a \$4174.80 10b \$27 136.20 10c \$54 272.40

Exercise 16.16 1a 22:9 1b 9:22 1c 22:31 1d 9:31 2a 1/10, 0.1, 10% 2b 2/10, 0.2, 20% 2c 1/5, 0.2, 20%
 2d 2/5, 0.4, 40% 2e 1/2, 0.5, 50% 2f 1/4, 0.25, 25% 2g 3/4, 0.75, 75% 2h 5/20, 0.25, 25% 3a 1:2, 0.5, 50%
 3b 1:4, 0.25, 25% 3c 3:4, 0.75, 75% 3d 2:5, 0.4, 40% 4a 2:1, 4b 2:5 4c 3:4 4d 2:3 4e 1:3 4f 4:5
 5 0.75 L or 750 mL 6 8 shovelfulls 7a 70% 7b 90% 7c 60% 7d 80% 7e 75% 7f 85% 8a 2 8b 10 8c 5
 8d 15 8e \$280 9a 8 m/s 9b 6 trucks/day 9c 9 stitches/wound 9d 72 beats/min 9e 14 rows/h
 9f \$242.50 /account 10a \$5734.80 10b \$37 276.20 10c \$74 552.40

Exercise 17.1 1 $x+5=9$ 2 $x-6=10$ 3 $5x=18$ 4 $x+3=7$ 5 $3x+6=33$ 6 $x+2-12=21$ 7 $7x+4=28$ 8 $x+9-3=21$
 9 $3x-8=72$ 10 $x+6-1=8$

Exercise 17.2 1 $x=2$ 2 $x=4$ 3 $a=12$ 4 $b=15$ 5 $x=4$ 6 $x=3$ 7 $x=5$ 8 $x=11$ 9 $y=19$ 10 $a=21$ 11 $x=3$ 12 $b=4$
 13 $x=10$ 14 $c=42$ 15 $x=5$ 16 $b=17$ 17 $m=15$ 18 $x=27$ 19 $d=33$ 20 $p=37$ 21 $h=5$ 22 $a=77$ 23 $x=5$ 24 $x=3.25$

Exercise 17.3 1 $x=6$ 2 $x=3$ 3 $a=2$ 4 $b=3$ 5 $x=2$ 6 $x=2$ 7 $y=2$ 8 $x=3$ 9 $r=3$ 10 $b=3$ 11 $v=3$ 12 $g=7$ 13 $a=11$
14 $t=24$ 15 $w=23$ 16 $s=20$ 17 $m=12$ 18 $n=6$ 19 $h=15$ 20 $b=14$ 21 $x=2.5$ 22 $r=3.25$

Exercise 17.4 1 Plot (1,4), (2,6), (3,8), (4,10), (5,12)

3 Plot (1,6), (2,8), (3,10), (4,12), (5,14)

5 Plot (1,0), (2,2), (3,4), (4,6), (5,8)

7 Plot (1,4), (2,7), (3,10), (4,13), (5,16)

9 Plot (1,6), (2,9), (3,12), (4,15), (5,18)

11 Plot (1,1), (2,4), (3,7), (4,10), (5,13)

13 Plot (1,3.5), (2,6.5), (3,9.5), (4,12.5), (5,15.5)

15 Plot (1,1.5), (2,2), (3,2.5), (4,3), (5,3.5)

17 Plot (1,3.5), (2,4), (3,4.5), (4,5), (5,5.5)

19 Plot (1,2.5), (2,4), (3,5.5), (4,7), (5,8.5)

21 Plot (1,0.5), (2,2), (3,3.5), (4,5), (5,6.5)

23 Plot (1,4), (2,6), (3,8), (4,10), (5,12)

25 Plot (1,2), (2,6), (3,12), (4,20), (5,30) non-linear

27 Plot (1,6), (2,3.5), (3,2.67), (4,2.25), (5,2) non-linear

2 Plot (1,5), (2,7), (3,9), (4,11), (5,13)

4 Plot (1,1), (2,3), (3,5), (4,7), (5,9)

6 Plot (1,2), (2,4), (3,6), (4,8), (5,10)

8 Plot (1,5), (2,8), (3,11), (4,14), (5,17)

10 Plot (1,2), (2,5), (3,8), (4,11), (5,14)

12 Plot (1,0), (2,3), (3,6), (4,9), (5,12)

14 Plot (1,3), (2,6), (3,9), (4,12), (5,15)

16 Plot (1,2.5), (2,3), (3,3.5), (4,4), (5,4.5)

18 Plot (1,0.5), (2,1), (3,1.5), (4,2), (5,2.5)

20 Plot (1,3.5), (2,5), (3,6.5), (4,8), (5,9.5)

22 Plot (1,1.5), (2,3), (3,4.5), (4,6), (5,7.5)

24 Plot (1,3.5), (2,4), (3,4.5), (4,5), (5,5.5)

26 Plot (1,2), (2,2.5), (3,3), (4,3.5), (5,4)

28 Plot (1,4), (2,6.5), (3,9), (4,11.5), (5,14)

Exercise 17.5 1 Plot (1,20), (2,40), (3,60), (4,80), (5,100) 2 Plot (1,40), (2,80), (3,120), (4,160), (5,200)

3 Plot (1,50), (2,100), (3,150), (4,200), (5,250)

Exercise 17.6 1a 10mins 1b 10mins 1c Jack's home to friend's place 1d 36km/h 1e 24km/h 2a 15mins

2b 5mins 2c 36km/h 2d 6km/h

Exercise 17.7 1a 150km 1b 2.2h or 2h 12 mins 2a 90km 2b 4h 3a 12.5km 3b 0.4h or 24mins 4a \$32

4b 6.5km 5a \$400 5b 3.5h or 3h 30mins

Exercise 17.8 2 $x=3$ 3 $x=3$ 4 6.2 5 7^{-3} 6 4 7 1 8 3^4 9 m^5 10 2^2

Exercise 17.9 2 $x=12$ 3 $x=4$ 4 4.5 5 1 6 1.6 7 1 8 2^5 9 b^6 10 5^2

Exercise 17.10 2 $x=9$ 3 $x=2$ 4 7.6 5 4 6 1 {3 times} 7 1 8 5^6 9 d^5 10 3^3

Exercise 17.11 1 10 2 17 3 7.3 4 7.5 5a (1,30), (2,60), (3,90), (4,120), (5,150)

5b (1,7), (2,11), (3,15), (4,19), (5,23) 6 13 7a $x=5.5$ 7b $x=7$ 7c $x=6$ 7d $x=43$ 8 $y=4x$ 9 $y=2x+3$ 10 $y=4x-3$

Exercise 17.12 1 12 2 0.8kg 3 15,35 4 16 5 46 6 \$4200 7 8 8a 47° 8b 60° 8c 45° 8d 70° 8e 105° 8f 30.5°

Exercise 17.13 1 10 2 23 or 32 or 41 or 14 3 11am 4 $x=10$

Exercise 17.14 1a $x+6=14$ 1b $x-3=7$ 1c $7x=35$ 1d $x \div 2=15$ 1e $3x+4=22$ 1f $x \div 2-8=36$ 2a $x=7$ 2b $y=21$

2c $x=4$ 2d $x=24$ 2e $x=4$ 2f $r=4$ 2g $x=14$ 2h $n=33$ 3a Plot (1,5), (2,7), (3,9), (4,11), (5,13)

3b Plot (1,30), (2,60), (3,90), (4,120), (5,150) 4a 10mins 4b 10mins 4c Home to friend's place 4d 30km/h

4e 18km/h 5a 100km 5b 2.4h or 2h 24mins

Exercise 17.15 1a $x+3=15$ 1b $x-7=9$ 1c $4x=25$ 1d $x \div 3=62$ 1e $2x+1=36$ 1f $x \div 3-8=52$ 2a $x=4$ 2b $y=38$

2c $x=7$ 2d $x=36$ 2e $x=3$ 2f $b=6.5$ 2g $x=21$ 2h $b=45$ 3a Plot (1,4), (2,7), (3,10), (4,13), (5,16)

3b Plot (1,40), (2,80), (3,120), (4,160), (5,200) 4a 10mins 4b 10mins 4c 36km/h 4d 6km/h 5a 120km

5b 2.5h or 2h 30mins

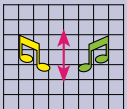
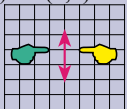
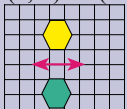
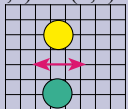
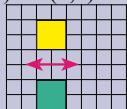
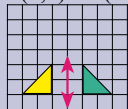
Exercise 18.1 1 4 units right and 3 units down or 3 units down and 4 units right

2 3 units down and 4 units right or 4 units right and 3 units down

3 3 units down and 4 units left or 4 units left and 3 units down

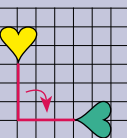
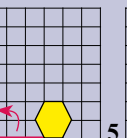
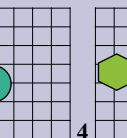
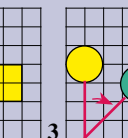
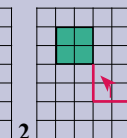
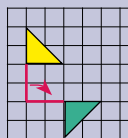
4 1 unit right and 3 units up or 3 units up and 1 unit right

Exercise 18.2 1 P(3,1) 2 P(-1,2) 3 P(2,6) 4 P(7,4) 5 P(2,1) 6 P(1,-2) 7 P(-5,0) 8 P(4,0)

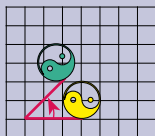


Exercise 18.3 1

Exercise 18.4 1 P(3,-4) 2 P(-2,-2) 3 P(-3,3)



Exercise 18.5 1



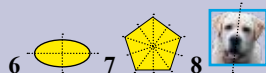
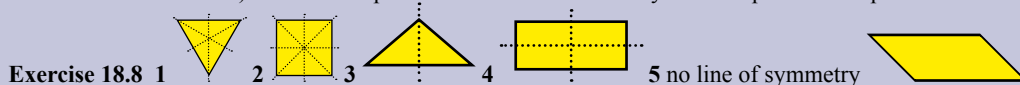
Exercise 18.6 1 P(-6,0) 2 P(0,6) 3 P(4,0)

Exercise 18.7 1 a) The shape has been reflected through a vertical axis. b) The pair of shapes have then been reflected through a horizontal axis. c) The four shapes have then translated many times to produce the pattern.

2 a) The shape has been reflected through a vertical axis. b) The pair of shapes have then been reflected through a horizontal axis. c) The four shapes have then translated many times to produce the pattern.

3 a) The shape has been reflected through a vertical axis. b) The pair of shapes have then been rotated 45° . c) The four shapes have then translated many times to produce the pattern.

4 a) The shape has been reflected through a horizontal axis. b) The pair of shapes have then been repeatedly rotated 60° five times. c) The four shapes have then translated many times to produce the pattern.

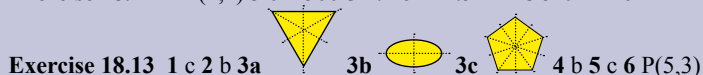


Exercise 18.9 1 order=3 2 order=4 3 order=1 4 order=1 5 order=5 6 order=1 7 order=3 8 order=5

Exercise 18.10 2 P(4,7) 3 4 4 300 5 6.2 6 $x=3$ 7 2 8 2^5 9 a^7 10 3^3

Exercise 18.11 2 P(4,1) 3 3 4 400 5 4.8 6 $x=9$ 7 5 8 3^3 9 b^6 10 2^3

Exercise 18.12 2 P(2,2) 3 0 4 500 5 2.4 6 $x=1.5$ 7 4 8 5^4 9 x^5 10 4^3



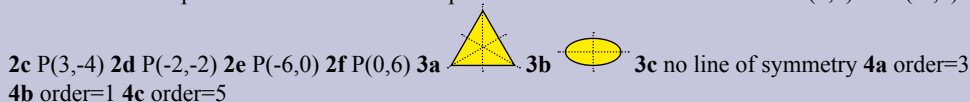
Exercise 18.14 1 reflection 2 c 3 c 4 d

1	4	3
2	0	2
3	4	1

Exercise 18.15 1 $1/16$, $1/32$ 2 3 4

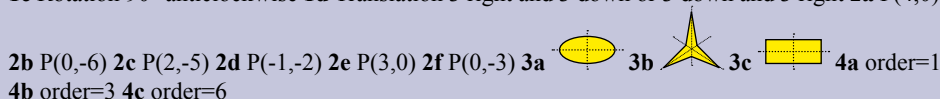
Exercise 18.16 1a Translation 4 right and 3 down or 3 down and 4 right 1b Reflection in a vertical axis

1c Translation 3 up and 3 left or 3 left and 3 up 1d Rotation 90° anticlockwise 2a P(3,1) 2b P(-5,0)



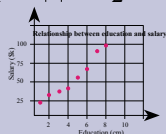
Exercise 18.17 1a Translation 3 right and 3 up or 3 up and 3 right 1b Reflection in a horizontal axis

1c Rotation 90° anticlockwise 1d Translation 3 right and 3 down or 3 down and 3 right 2a P(4,0)

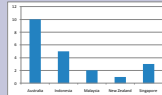
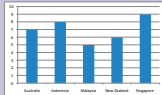


		Customers	
		Before	After
Heart Rate	Walking	47	2
		98	3
		522	4
		8654	5
		75	6
		9654	7
		62111	8
		3	9
		109	10
		11	11
		12	12
Sitting		4	7
		771	6
		998538	7
		986543	8
		9887531	9
		2	10
		11	11
		12	12
		13	13
		14	14

Exercise 19.1 1 2



Exercise 19.2 1



Exercise 19.4 2a

2b

Exercise 19.5 2 3 3 2 {2 times} 4 P(4,7) 5 350 6 6.2 7 $x=5$ 8 $^{-1}$ 9 4^4 10 a^8

Exercise 19.6 2 3 {3 times} 3 3.2 4 P(4,1) 5 450 6 8.4 7 $x=8$ 8 $^{-7}$ 9 2^6 10 b^9

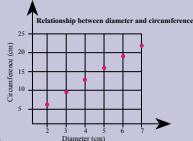
Exercise 19.7 2 2.4 3 2 4 P(3,0) 5 550 6 12.6 7 $x=3$ 8 2 9 3^5 10 c^5

Exercise 19.8 1 4,5,6 2 24,25,26 3 44,45,46 4 9

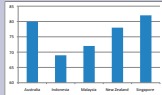
Exercise 19.9 1

Milk Production	
Victoria	NSW
2000	86
2001	88
2002	90
2003	92
2004	94

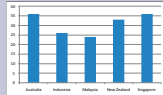
2



3a



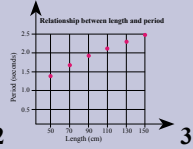
3b



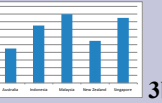
Exercise 19.10 1

Rabbit Campaigns	
Before	After
9	1222569
10	101000
11	101000
12	1

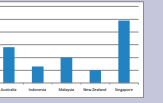
2



3a



3b



Exercise 20.1 2 2 3 2 {3times} 4 P(5,1) 5 550 6 8.6 7 $x=3$ 8 $^{-3}$ 9 4^5 10 x^6

Exercise 20.2 1a 12:13 1b 13:12 1c 12:25 1d 13:25 2a 1/10, 0.1, 10% 2b 3/10, 0.3, 30% 2c 1/4, 0.25, 25%

2d 3/5, 0.6, 60% 3a 1:2 3b 1:2 3c 4:3 3d 3:2 3e 1:2 3f 1:3 4 0.4L or 400mL 5a 30% 5b 70% 5c 20%

5d 60% 6a 5 6b 30 6c 10 6d 30 6e \$240 7a 10m/s 7b 6steps/s 7c 2L/s 7d \$8/session 8a \$3448.20

8b \$22 413.30 8c \$44 826.60 9a $x+5=12$ 9b $2x+3=21$ 10a $x=2$ 10b $y=25$ 10c $x=7$ 10d $b=4$

11 Plot (1,7), (2,9), (3,11), (4,13), (5,15) 12a 10mins 12b 10mins 12c 6km/h 12d 42km/h 13a 120km

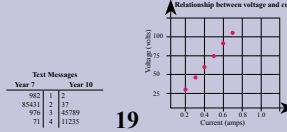
13b 2.5h or 2h 30min 14a translation 5 right and 3 up or 3 up and 5 right 14b reflection in a vertical axis

14c translation 2 left and 3 up or 3 up and 2 left 14d rotation 90° anticlockwise 15a P(4,3) 15b P(4,1)

15c P(3,1) 15d P(4,5) 15e P(3,0) 15f P(0,2) 16a  16b  16c no line of symmetry

17a order=3 17b order=1 17c order=5 18

19



Exercise 20.3 2 2 3 2 4 P(1,5) 5 450 6 6.2 7 $x=3$ 8 $^{-1}$ 9 2^4 10 x^7

Exercise 20.4 1a 13:10 1b 10:13 1c 13:23 1d 10:23 2a 1/10, 0.1, 10% 2b 7/10, 0.7, 70%

2c 3/4, 0.75, 75% 2d 2/5, 0.4, 40% 3a 1:3 3b 1:3 3c 2:1 3d 3:1 3e 1:2 3f 3:8 4 0.8L or 800mL 5a 90%




5b 70% 5c 60% 5d 80% 6a 6 6b 40 6c 20 6d 60 6e \$360 7a 2m/s 7b 3 houses/row 7c 2L/s 7d \$/tree

8a \$5527.80 8b \$35 930.70 8c \$71 861.40 9a $x+17=23$ 9b $3x+6=65$ 10a $x=4$ 10b $y=22$ 10c $x=7$ 10d $a=12$

11 Plot (1,5), (2,8), (3,11), (4,14), (5,17) 12a 10 mins 12b 0 mins 12c 48 km/h 12d 6 km/h 13a 80km

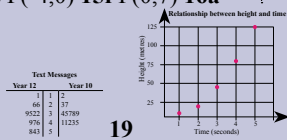
13b 1.5h or 1h 30min 14a translation 2 right and 4 up or 4 up and 2 right 14b reflection in a horizontal axis

14c translation 2 left and 4 up or 4 up and 2 left 14d rotation 90° clockwise 15a P(6,0) 15b P(1,5)

15c P(5,1) 15d P(6,3) 15e P(4,0) 15f P(0,7) 16a  16b  16c  17a order=1

17b order=3 17c order=6 18

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