Excel

YEARS 9 and 10

ADVANCED MATHEMATICS STUDY GUIDE

Excel

Essential Skills

J. Compton & A. Jones

MARR

Excel

YEARS 9 & 10

ADVANCED MATHS

J. Compton and A. Jones

PASCAL
PRESS

2^{nd} revised edition 1997
Reprinted 1999, 2001, 2002, 2004

ISBN 1 8753 1295 1

Pascal Press
PO Box 250
Glebe NSW 2037
(02) 8585 4044
www.pascalpress.com.au

Cover: DiZign
Typeset by JMV Computer Services P/L
Printed in Singapore by Green Giant Press

Preface

Excel Advanced Mathematics is an invaluable preparation for Year 11 Maths or Extension 1 Maths. Students will find that, together with *Excel Intermediate Maths*, it provides a solid foundation for starting Year 11.

Extension students will find *Excel Advanced Mathematics* a solid revision of the basic mathematics taken for granted in Year 11.

Each chapter has many worked examples. You should cover the solutions and attempt the examples. Continue this process until you have mastered the concepts. Only then should you attempt the examples at the end of the chapter.

Most questions have worked solutions. The solutions should only be used as a last resort or to check your working after you have completed the questions.

You should be completely familiar with the buttons necessary to perform all mathematical functions on YOUR calculator. Read the manual. Most calculator sequences shown in this book refer to the CASIO fx82 series.

Contents

		Page
Chapter 1	Consumer Arithmetic, Rates and Variation	1
Chapter 2	Algebra and Quadratics	19
Chapter 3	Real numbers, Surds and Indices	38
Chapter 4	Equations and Inequalities	48
Chapter 5	Graphs	60
Chapter 6	Coordinate Geometry	75
Chapter 7	Chance and Data	93
Chapter 8	Measurement including Area, Surface Area and Volume	118
Chapter 9	Similarity	134
Chapter 10	Trigonometry	143
Chapter 11	Basic Geometry	162
Chapter 12	Option — Further Geometry — The Circle	182
Chapter 13	Option — Logarithms and Functions	203
Chapter 14	Option — Polynomials and Curve Sketching	222
Chapter 15	Practice Paper	235
Chapter 16	Worked Solutions to Exercises	243

Chapter 1

CONSUMER ARITHMETIC, RATES AND VARIATION

1.1 Earning money

1.1.1 Salary and wages

Examples

(a) Find Carol's wage if she cleans for a period of 32 hours per week and is paid $7.82 per hour.

SOLUTION

Wage = $7.82 × 32

= $250.24

Carol's wage is $250.24 per week.

(b) Two jobs as mechanic are advertised. The first pays $401.45 for a 35-hour week, while the second pays $433.20 for a 38-hour week. Which job pays the higher hourly rate?

SOLUTION

First job: Hourly rate

= $401.45 ÷ 35

= $11.47

Second job: Hourly rate

= $433.20 ÷ 38

= $11.40

First job pays a higher hourly rate.

(c) Find the yearly salary of a person whose fortnightly income is $1620.

SOLUTION

Salary = $1620 × 26

= $42 120

The salary is $42 120 per year.

(d) Sue works for Hannah's Realty and her commission on sales is calculated as follows:

3% on the first $12 000

2% on the next $30 000

$1\frac{1}{2}$% on the remainder.

Calculate Sue's commission if she sells a block of land for $71 000.

SOLUTION

$71 000 = $12 000 + $30 000 + $29 000

Commission

= 3% of $12 000 + 2% of $30 000

+ $1\frac{1}{2}$% of $29 000

= 0.03 × $12 000 + 0.02 × $30 000

+ 0.015 × $29 000

= $360 + $600 + $435

= $1395

Sue's commission is $1395.

(e) Fiona works in a clothing factory and is paid $2.70 for each shirt completed. Find her pay if 290 shirts are completed.

SOLUTION: Pay = $2.70 × 290

= $783

Fiona is paid $783.

(f) Daniel works at Costless Supermarket when required. If he is called on to work for six-and-a-half hours at $11.97 per hour, find his total pay.

SOLUTION

Total pay = $11.97 × 6.5

= $77.805

= $77.81 (rounded to the nearest cent)

Daniel is paid $77.81.

1.1.2 Overtime rates

A person may work more than a normal week and be paid at an hourly rate more than the normal rate. This may be $1\frac{1}{2}$ (time-and-a-half) or 2 (double) times his normal rate.

Examples

(a) Adam's employment contract states that he is paid normal pay for the first eight hours he works. He is paid time-and-a-half for the next two hours and double time for any time after that. Find Adam's pay if he works a thirteen-hour day and his hourly pay rate is $14.75.

SOLUTION

13 hours = 8 hours + 2 hours + 3 hours

$$\begin{aligned}\text{Pay} &= \$14.75 \times 8 + \$14.75 \times 1.5 \times 2 \\ &\quad + \$14.75 \times 2 \times 3 \\ &= \$118 + \$44.25 + \$88.50 \\ &= \$250.75\end{aligned}$$

Adam is paid $250.75.

(b) Laura is paid normal rates when she works during the week, time-and-a-half on Saturdays and double time on Sundays. Her normal pay rate is $13.70 per hour. Find her weekly pay if she works the following hours:

Day	Start	Finish	Hours
Wed–Fri	9:00	5:30	$8\frac{1}{2} \times 3 = 25\frac{1}{2}$
Sat	9:00	4:00	7
Sun	10:00	3:00	5

SOLUTION

Weekly pay

$$\begin{aligned}&= \$13.70 \times 25\tfrac{1}{2} + \$13.70 \times 1.5 \times 7 \\ &\qquad + \$13.70 \times 2 \times 5 \\ &= \$349.35 + \$143.85 + \$137 \\ &= \$630.20\end{aligned}$$

Laura's weekly pay is $630.20.

1.1.3 Other payments: bonus, holiday loading, etc.

Examples

(a) Mitchell's boss, Mr Skrooge, was so impressed by his productivity improvement that he offered him a Christmas bonus equal to 40% of his normal weekly pay. If Mitchell received $280 extra in his pay that week, find his usual pay.

SOLUTION

$$\begin{aligned}\text{Bonus : } 40\% \text{ of pay} &= \$280 \\ 1\% \text{ of pay} &= \$7 \\ 100\% \text{ of pay} &= \$700\end{aligned}$$

Mitchell's usual pay is $700 per week.

(b) Sarah's employment award includes an annual $17\frac{1}{2}\%$ holiday loading on four week's normal wages. Her weekly wage is presently $960. Her employer has offered a weekly pay rise of $15 to replace this holiday loading. Would Sarah be better off with the pay rise and by how much?

SOLUTION

Sarah's holiday loading

$$\begin{aligned}&= 17\tfrac{1}{2}\% \times \$960 \times 4 \text{ weeks} \\ &= 0.175 \times 960 \times 4 \\ &= \$672\end{aligned}$$

$$\begin{aligned}\text{Sarah's pay rise} &= \$15 \times 52 \Leftarrow \boxed{\text{52 weeks per year}} \\ &= \$780\end{aligned}$$

Sarah is better off by $780 – $672, that is, Sarah is better off by $108.

(c) Craig works outdoors and is paid an additional allowance for rain or extreme hot weather (when the thermometer rises to over 35°). For rainy days he is paid an extra $15 per day, while for hot days he receives an additional $19 per day. How much will Craig be paid in allowances if in one week there are two hot days and one wet day?

SOLUTION

$$\begin{aligned}\text{Allowances} &= \$19 \times 2 + \$15 \\ &= \$38 + \$15 = \$43\end{aligned}$$

Craig receives an extra $43.

1.2 Deductions from pay

Most employees have deductions taken from their pay. These include taxation, superannuation payments, loan repayments and health fund payments.

1.2.1 Taxation

Income tax is paid by employees and is usually of the form called PAYE (pay as you earn). The amount is calculated by the employer and is paid to the government before the employee gets paid. An annual tax return is completed by the employee and any deductions claimed may lead to a refund.

Example

Using the table below, calculate the tax to be paid on taxable income of $41 760.

These are called tax brackets.

Taxable income $	Tax
$1 — $5 400	Nil
$5 401 — $20 700	Nil plus 20 cents for each $1 over $5 400
$20 701 — $36 000	$3 060 plus 38 cents for each $1 over $20 700
* $36 001 — $50 000	$8 874 plus 46 cents for each $1 over $36 000
$50 001 and over	$15 314 plus 47 cents for each $1 over $50 000

SOLUTION

Taxable income of $41 760, that is, in the bracket marked with *.

$$\begin{aligned}\text{Tax} &= \$8874 + (\$41\,760 - \$36\,001) \times 0.46 \leftarrow \boxed{\text{46 cents}} \\ &= \$8874 + \$5759 \times 0.46 \\ &= \$8874 + \$2649.14 \\ &= \$11\,523.14.\end{aligned}$$

1.2.2 Other deductions

Examples

(a) Jason's gross pay for a fortnight was $1232. His deductions were as follows: taxation $372.61; superannuation $32.46; miscellaneous $176.50. Find Jason's net pay.

SOLUTION:

$$\boxed{\text{Net} = \text{gross} - \text{deductions}}$$

$$\begin{aligned}\text{Net pay} &= \$1232 - (\$372.61 \\ &\qquad + \$32.46 + \$176.50) \\ &= \$1232 - \$581.57 \\ &= \$650.43.\end{aligned}$$

Jason's net pay was $650.43.

(b) Cliff pays $42 per month to his health fund to provide private health insurance. He also pays the Medicare levy which is calculated using the table at right

Taxable income	Medicare levy
Less than $11 746	Nil
$11 746 – $12 528	20 c for every dollar above $11 746
More than $12 528	1.25% of taxable income

If Cliff's yearly salary is $39 760, find his annual health insurance cost by adding his private health insurance to his Medicare levy.

SOLUTION: Cost of private insurance

$$= \$42 \times 12 = \$504$$

$$\begin{aligned}\text{Cost of Medicare levy} &= \$39\,760 \times 1.25\% \\ &= \$39\,760 \times 0.0125 = \$497\end{aligned}$$

$$\begin{aligned}\text{Total insurance cost} &= \$504 + \$497 \\ &= \$1001\end{aligned}$$

Cliff's cost for health insurance is $1001 per annum.

1.3 Spending money

1.3.1 Budgeting

A budget is a plan for the use of expected income.

Example

Anne's monthly budget is outlined below:

Monthly income		**Fixed expenses**		**Variable expenses**	
Job	$1740	Rent	$660	Petrol	$100
Son's board	$220	Electricity	$70	Food	$480
Investments	$65	Water	$56	Entertainment	$140
Total	$2025	Rates	$52	Telephone	$75
		Other	$40	Other	$40
		Total	$878	Total	$835

What is Anne's expected monthly balance?

Balance = income – expenses
= $2025 – ($878 + $835)
= $2025 – $1713
= $312

Anne's expected monthly balance is $312.

1.3.2 Shopping

Examples

(a) Determine the best buy:

300 mL of orange juice for 72 c
600 mL of orange juice for $1.35
2 L of orange juice for $3.82
5 L of orange juice for $6.90.

SOLUTION: There are a few ways to attack this question — one is to find the cost per unit (here 1 mL).

300 mL, 72 c,
∴ 72 ÷ 300 = 0.24 c/mL.
600 mL, $1.35,
∴ 135 ÷ 600 = 0.225 c/mL.
2 L, $3.82,
∴ 382 ÷ 2000 = 0.191 c/mL.
5 L, $6.90,
∴ 690 ÷ 5000 = 0.138 c/mL.

Note: Could have found the cost per 100 mL.

The cheapest per mL is the 5 L container.

Best buy is 5 L for $6.90.

(b) Michael lives in Consumerland, a country about to introduce a goods and services tax (GST) on most articles that are purchased. The GST will be set at 12%. Some products already have a sales tax as given in the examples. Find the price without sales tax and then the new price with the GST added.

(i) A car tyre is priced at $108 (including a 20% sales tax).

SOLUTION

120% of price before tax = $108

1% of price before tax = $\frac{\$108}{120}$
= $0.90

100% of price before tax = $0.90 × 100
= $90

Now add on 12% GST:

112% of price before tax = $0.90 × 112
= $100.80.

Price without sales tax is $90 and with GST is $100.80.

(ii) A freezer is priced at $829 (including a $7\frac{1}{2}$% sales tax).

SOLUTION

$107\frac{1}{2}$% of price before tax = $829

1% of price before tax = $\frac{\$829}{107.5}$
= $7.71
(to the nearest cent)

100% of price before tax = $7.71 × 100
= $771.16
(to the nearest cent)

Now add on 12% GST

112% of price before tax = $7.71 × 112
= $863.70
(to the nearest cent)

Price without sales tax is $771.16 and with GST is $863.70.

(c) Jenny decided to purchase a stereo on time payment using the following terms.

Stereo : $1249
Deposit : 15%
Interest on balance : 17% per annum

36 monthly repayments (or instalments)

Find the size of each monthly instalment.

SOLUTION

Deposit ← Paid before Jenny takes the stereo out of the shop
= 15% of $1249
= 0.15 × $1249 = $187.35

Balance owing
= $1249 – $187.35 = $1061.65

Interest ← This is known as 'simple interest'
= $1061.65 × 17% over 3 years
= $1061.65 × 0.17 × 3
= $541.44 (to the nearest cent)

Total repayments
= $1061.65 + $541.44
= $1603.09 (to the nearest cent)

Monthly repayments
= $1603.09 ÷ 36
= $44.53 (to the nearest cent)

Jenny needs to repay $44.53 per month.

Note: Jenny will be paying $541.44 extra by buying the stereo on time payment. [This is the amount of the interest she has to pay.]

(d) Amy bought a video recorder with a marked price of $720. She paid a deposit of 15% in cash, and borrowed the remainder to be paid back in 12 monthly instalments of $62.22. Find the interest rate that Amy was charged for the money she borrowed.

SOLUTION: Deposit = 15% of $720
= 0.15 × 720
= $108

Amount borrowed = $720 – $108
= $612

Total instalments = $62.22 × 12
= $746.64

Total amount paid = $108 + $746.64
= $854.64

Total interest paid = $854.64 – $720
= $134.64

$$\boxed{\text{Interest rate} = \frac{\text{interest}}{\text{amount borrowed}} \times 100\%}$$

$$\text{Interest rate} = \frac{\$134.64}{\$612} \times 100$$
= 22%.

Interest rate is 22% per annum.

1.4 Saving and borrowing

1.4.1 Simple interest

$SI = PRt$, where SI = simple interest, P = principal, R = annual interest rate expressed as a decimal, t = number of years.

Note: $R = \frac{r}{100}$ when r is expressed as a percentage.

Examples

Find the simple interest gained on the following:

(a) $\$1600$ at $12\frac{1}{2}\%$ p.a. for 3 months.

SOLUTION

3 months is $\frac{3}{12}$ of a year

$$SI = PRt$$
$$= \$1600 \times 0.125 \times \tfrac{3}{12}$$
$$= \$50$$

Interest gained is $50.

(b) $\$500$ at 4.75% p.a. for 18 months.

SOLUTION

$\frac{18}{12} = 1\frac{1}{2}$
$= 1.5$

$$SI = PRt$$
$$= \$500 \times 0.0475 \times 1.5$$
$$= \$35.625$$
$$= \$35.63 \text{ [to the nearest cent]}$$

Interest gained is $35.63.

(c) $\$760$ at 9% p.a. for 1 day.

SOLUTION

$365\frac{1}{4}$ days in each year,
$\therefore \dfrac{1}{365\frac{1}{4}}$ of a year

$$SI = PRt$$
$$= \$760 \times 0.09 \times \frac{1}{365\frac{1}{4}}$$
$$= \$0.187\,268\,993$$
$$= \$0.19 \text{ [to the nearest cent]}$$

Interest gained is $0.19.

1.4.2 Compound interest

This is more realistic than simple interest mentioned above and assumes depositors will gain interest on their interest as their money lies untouched in their account, rather than withdrawing interest on the day it is earned.

For example, $1000 gaining compound interest of 12% per annum over three years can be calculated the following way:

$\$1000 \times 1.12 \times 1.12 \times 1.12$
deposit + interest for first year
Year 1's total + interest for second year
Year 2's total + interest for third year,

$$\text{that is, total after 3 years} = \$1000 \times 1.12 \times 1.12 \times 1.12$$
$$= \$1000 \times 1.12^3$$
$$= \$1404.93 \text{ (to the nearest cent)}$$
$$\text{Compound interest} = \$1404.93 - \$1000 \leftarrow$$
$$= \$404.93.$$

$1000 is the original deposit

Remember: To increase $1000 by 12% we could find 12% of $1000 and add to $1000 — it is easier to simply multiply: $\$1000 \times 1.12$

We can use the formula:

$$A = P(1+R)^n$$

where A = accumulated Amount
P = principal
R = annual interest rate as a decimal.
n = number of years (or time periods).

Note: $R = \dfrac{r}{100}$ when r is expressed as a percentage.

Examples

(a) Terry invested $42 000 in a savings account which attracted an 8.5% interest rate compounded annually. Find out how much money Terry has in the account after 4 years.

Continued

SOLUTION

$$A = P(1+R)^n$$
$$= \$42\,000(1+0.085)^4$$
$$= \$42\,000 \times 1.085^4$$
$$= \$58\,206.07 \text{ (to the nearest cent)}$$

Terry has $58 206.07 in his savings account.

(b) Mercia works as a manager of a clothing factory. Her pay conditions involve an increase of 6% every year. If this year she is paid a salary of $48 320, what will her salary be in seven years time?

SOLUTION

$$A = P(1+R)^n$$
$$= \$48\,320(1+0.06)^7$$
$$= \$48\,320 \times 1.06^7$$
$$= \$72\,655.41 \text{ (to the nearest cent)}$$

Mercia will be paid a salary of $72 655.41.

(c) Susan can choose between two accounts: the first offering simple interest at a rate of 12% and the second compound interest at a rate of 8%. If she had $400 to invest for three years, which account should she use to gain the greater amount of interest?

SOLUTION

$$\text{Simple interest} = \$400 \times 0.12 \times 3$$
$$= \$144$$

For compound interest,

$$A = 400(1.08)^3$$
$$= \$503.88$$
$$\text{(to the nearest cent)}$$
$$\text{Compound interest} = \$503.88 - \$400$$
$$= \$103.88.$$

Susan would gain more interest by using the simple interest account.

(d) Noel deposits $1000 in an account which offers interest at 6% per annum *compounded monthly*. He invests the money for two years. Find his balance after two years.

SOLUTION: As the interest is determined monthly, there will be 24 payments over the two years. Also, the rate is 6% per annum, that is $\frac{6}{12} = 0.5\%$ per month.

$$A = P(1+R)^n$$

Note: 0.5% = 0.005

$$= \$1000(1+0.005)^{24}$$
$$= \$1000(1.005)^{24}$$
$$= \$1127.16.$$

Noel will have $1127.16 in his account after two years.

1.5 Depreciation

Whereas compound interest is calculating values that are increasing, depreciation is the opposite — that is, calculating values that are decreasing.

We can use the formula: $A = P(1-R)^n$

where A = final value
P = initial value
R = annual depreciation rate expressed as a decimal.
n = number of years (or time periods).

Note: $R = \dfrac{r}{100}$ when r is expressed as a percentage.

Examples

(a) Lloyd buys a second-hand car for $14 400. If it depreciates in value at 13% per annum, find its value after four years.

SOLUTION

$$A = P(1-R)^n$$
$$= \$14\,400(1-0.13)^4$$
$$= \$14\,400(0.87)^4$$
$$= \$8249.73 \text{ (to the nearest cent)}$$

Lloyd's car is valued at $8249.73.

(b) Christine purchases a computer valued at $2100. If it depreciates at 20% per annum, how much will it depreciate in its third year?

SOLUTION: $A = P(1-R)^n$

Value after two years:

$$\$2100(1-0.2)^2 = \$2100(0.8)^2 = \$1344$$

Value after three years:

$$\$2100(1-0.2)^3 = \$2100(0.8)^3 = \$1075.20.$$

Amount of depreciation

in third year: $= \$1344 - \$1075.20$
$= \$268.80$

The computer depreciates $268.80 in its third year.

(c) Helen purchases a television set for $1000. If it depreciates at 20% per annum, how many years will elapse until it is valued at $512?

SOLUTION:

$$A = P(1-R)^n$$
$$512 = 1000(1-0.2)^n$$
$$512 = 1000(0.8)^n$$
$$0.8^n = \frac{512}{1000}$$
$$0.8^n = 0.512$$

By trial and error, using the $\boxed{x^y}$ key:

$n = 3$.

You could also take logs of both sides (See Chapter 13.)

After three years the television set is valued at $512.

1.6 Loans

1.6.1 Loans involving flat-rate interest

Loans involving flat-rate interest involve simple interest which is calculated on the original amount borrowed, regardless of the amount of money owing.

Examples

(a) Peter borrowed $4000 from the bank and was charged flat-rate interest of 11% per annum. He was allowed to repay the loan in 36 monthly instalments. Find the amount of each instalment.

SOLUTION

Note: This is similar to Example (c) in Section 1.3.2.

Interest $= \$4000 \times 0.11 \times 3$
$= \$1320$

Monthly instalment
$= (\$4000 + \$1320) \div 36$
$= \$147.78$ (to the nearest cent)

Peter repays $147.78 in each instalment.

(b) Mary borrowed money for a trip to England. Her loan was for a period of two years and each monthly repayment was $300. If she was charged flat-rate interest of 10% per annum, find her original loan.

SOLUTION

Total repayments $= \$300 \times 24$
$= \$7200$

Let original loan $= L$

Interest $= L \times 10\% \times 2$

$$L + L \times 10\% \times 2 = 7200$$
$$L + L \times 0.1 \times 2 = 7200$$
$$L + 0.2L = 7200$$
$$L(1+0.2) = 7200$$
$$L(1.2) = 7200$$
$$1.2L = 7200$$
$$\frac{1.2L}{1.2} = \frac{7200}{1.2}$$
$$L = 6000$$

Original loan was $6000.

1.6.2 Loans involving reducible interest

The vast majority of loans involve reducible interest, that is, interest is calculated on the amount owing at specified times throughout the loan period.

Examples

(a) Brad borrows $9000 from a bank and agrees to meet the annual repayments of $2100. He is charged interest at 14% per annum reducible. How much will he owe the bank after two repayments?

SOLUTION: Each year Brad is charged interest on the balance owing and then makes his annual repayment.

Amount owing after one year
$= \$9000 \times 1.14 - \$2100$
$= \$8160.$

Amount owing after two years
$= \$8160 \times 1.14 - \$2100$
$= \$7202.40.$

Brad still owes $7202.40 at the end of two years.

(b) Carolyn decides to borrow $72 000 for a home loan. At the end of each month, interest is calculated before the monthly repayment is made. The interest rate is 15% per annum, monthly reducible. She decides to repay the loan at $950 per month. How much will she owe after her first monthly repayment?

SOLUTION: 15% per annum means $\frac{15}{12}\%$ per month, that is, 1.25% per month.

Note: $1.25\% = 0.0125$

Amount owing after one month
$= \$72\,000 \times 1.0125 - \$950$
$= \$71\,950.$

[Note that after repaying $950, only $50 has come off the borrowed amount.]

1.7 Sales

1.7.1 Sales discount

Examples

(a) At a sale offering discounts of 15% for cash, Ross purchased a CD player selling for $220. Find the cash price paid by Ross.

SOLUTION

$100\% - 15\% = 85\%$

Cash price $= 85\%$ of $\$220$
$= 0.85 \times 220$
$= \$187$

Ross will pay $187.

Note: We could have found 15% of $220, and subtracted this from $220.

(b) Paul, a plumber, purchases supplies from a hardware store and gains a trade discount of 12%. His purchases total $145 and he is allowed a further discount of 8% for paying cash. Find his final bill.

SOLUTION

Cost after trade discount
$= 88\%$ of $\$145$
$= \$127.60$

Cost after further discount
$= 92\%$ of $\$127.60$
$= \$117.39$ (to the nearest cent)

Paul has to pay $117.39.

[*Note:* This is different to simply adding the two discounts.]

(c)

20% off

STOREWIDE SALE

Elene paid $38.40 for a cutlery set at a sale which offered 20% discount on all goods. How much did Elene save on the original price?

SOLUTION: Elene's $38.40 represents 80% of the original price.

80% of original price = $38.40
20% of original price = $38.40 ÷ 4
= $9.60
100% of original price = $9.60 × 5
= $48

The original price was $48.

Note: We could have brought it down to 10%, or 1%, etc., rather than 20%.

1.7.2 Sales profit and loss

Examples

(a) A dealer buys a car for $7200 and resells it for $9600. Find the profit expressed as a percentage of the cost price.

SOLUTION: Profit = $9600 − $7200
= $2400

$$\text{Profit as \% (cost)} = \frac{2400}{7200} \times 100 = 33.\dot{3}\%$$

Profit is $33\frac{1}{3}$%.

Note:

- Profit as % (cost) $= \frac{\text{profit}}{\text{cost}} \times 100\%$
- Profit as % (selling) $= \frac{\text{profit}}{\text{selling}} \times 100\%$

(b) Jordie purchased a case of tomatoes for $4.60 and sold them, making a profit of 24% on the cost price. Find the price at which Jordie sold the case.

SOLUTION: Price = 1.24 × $4.60
= $5.70 (to the nearest cent)

Jordie sold the case for $5.70.

(c) A shoe shop marks up the price of shoes by 50%. If Lyndy buys a pair of shoes from the shop for $60, what was the shop's profit?

SOLUTION

As the shop's cost price = 100%
shop's selling price = 150%.
150% of cost price = $60
1% of cost price = $60 ÷ 150
= $0.40 (to the nearest cent)
100% of cost price = $0.40 × 100
= $40 (to the nearest cent)
Store profit = $60 – $40
= $20

Store profit is $20.

(d) Alan bought a car for $18 700 and sold it, making a 60% loss on the original purchase. Find the amount of money that Alan received when he sold the car.

SOLUTION

Alan's selling price = 40% of $18 700
= 0.4 × $18 700
= $7480

Alan sold the car for $7480.

(e) Jill sold for $77 a vacuum cleaner which she bought three years previously. If this amounted to a loss of 45% on the original purchase, find the cost of the cleaner when new.

SOLUTION

55% of the original price = $77
1% of the original price = $77 ÷ 55
= $1.40 (to the nearest cent)
100% of the original price = $1.40 × 100
= $140.00 (to the nearest cent)

Jill originally paid $140.00 for the vacuum cleaner.

1.8 Rates

Rates are a comparison of quantities expressed in different units.

Examples

(a) While doing my homework I notice that I am breathing once every four seconds. Express this as breaths per minute and breaths per hour.

SOLUTION

Breaths: 1/4 seconds
∴ 15/60 seconds
(by multiplying by 15)
∴ 15/minute
∴ 900/hour
(by multiplying by 60)

(b) Nurse Fiona records Cliff's pulse rate as 17 beats per 15 seconds. What is the pulse rate per minute?

SOLUTION

Rate = 17 beats/15 seconds
∴ 68 beats /minute
(by multiplying by 4)

(c) At Murray's Fruit Shop, bananas sell for $1.69/kg and tomatoes at $2.99/kg.

(i) How much will 1259 g of bananas cost?

SOLUTION

Now 1250 g = 1.25 kg
∴ Cost = 1.25 × $1.69
= $2.11
(to 2 decimal places)

(ii) What weight of tomatoes can I buy for $4? (Leave answer correct to 1 decimal place.)

SOLUTION

Weight = $4 ÷ $2.99
= 1.337 792 642
that is, the weight is 1.3 kg.

(d) It is found that a truck uses fuel at 22 litres per 100 km. If the truck travels a distance of 380 km, how much fuel does it use?

SOLUTION

380 ÷ 100 = 3.8
∴ fuel usage = 22 × 3.8
= 83.6,
∴ fuel usage is 83.6 litres.

(e) A rectangular prism with sides 4 m by 3 m by 12 m is to be painted. If a tin of paint which covers 24 m^2 is purchased for $21, find the cost of painting the prism.

SOLUTION

Surface area of a prism
$= 2lb + 2lh + 2bh$
$= 2 \times 4 \times 3 + 2 \times 4 \times 12 + 2 \times 3 \times 12$
$= 24 + 96 + 72$
$= 192\ \text{m}^2$
Now, number of tins
= 192 ÷ 24
= 8,
cost = $21 × 8
= $168,
∴ the cost is $168.

(f) A farmer purchases 60 tonnes of superphosphate to spread over her 400 hectare property. Calculate the rate of application in kg per ha.

SOLUTION

Rate of application

$$= \frac{60\,000}{400} = 150$$

∴ the rate is 150 kg/ha.

1.8.1 Conversion of rates

Examples

(a) Convert 10 m/sec to km/h.

SOLUTION

10 m/sec

∴ 600 m/min (by multiplying by 60)

∴ 36 000 m/h (by multiplying by 60)

∴ 36 km/h (÷ by 1000)

∴ 10 m/s = 36 km/h

(b) A long distance runner completes a marathon of 42.2 kilometres in 2 hours 15 minutes. Calculate his average speed in km/h and m/s. (Correct to 3 decimal places.)

SOLUTION

42.2 km/2 hours 15 minutes

∴ 42.2 km/2.25 h

∴ $18.7\dot{5}$ km/h (by dividing by 2.25)

∴ average speed is 18.756 km/h

Now, 18.756 km/h = 18 756 m/h

312.6 m/min (by dividing by 60)

5.21 m/s (by dividing by 60)

∴ average speed is 5.21 m/s

(c) When Mitchell won the Under 7 100 m race, he averaged 19 km/h. What was his time for the event, to two decimal places?

SOLUTION

19 km/h

∴ 19 000 m/h

∴ 19 000 m/60 min

∴ 19 000 m/3600 s

100 m/18.947 368 42 s

(by dividing by 190)

that is, 18.95 seconds to complete 100 m

1.8.2 Financial Exchange Rates

Example

$1 Australian buys:

Country	Currency	Rate
US	dollar	0.79
UK	pound	0.49
Japan	yen (¥)	96.50
NZ	dollar	1.13

Use the table above to answer the following questions:

(a) Convert $70 Australian t0 $US

SOLUTION

$70 \times 0.79 = 55.3$

That is, $70 Aus = $55.30 US

(b) Convert $100 US to $A.

SOLUTION

$100 \div 0.79 = 126.582\,278\,5$

That is, $100 US = $126.58Australian

(c) Convert $30 US to Japanes yen.

SOLUTION

Firstly change $30 US to $A.

∴ $30 \div 0.79 = 37.974\,683\,54$

that is, $30 US = $37.97 Aus

Now, change $37.97 Australian to Japanese yen (¥).

∴ $37.97 \times 96.5 = 3664.105$

that is, $30 US = $37.97 Aus

= 3664 ¥

1.9 Variation

There are two two types of variation questions. Say, we have two quantities x and y.

- If $x\uparrow$ and $y\uparrow$, we say that x is **directly proportional** to y. (*Note*: Similarly if $x\downarrow, y\downarrow$.)
- If $x\uparrow$ nad $y\downarrow$, we say that y is **indirectly proportional** to x.

1.9.1 Direct variation

- If x and y are directly proportional then

$y \propto x$ $\propto$ means 'is proportional to'

that is $y = kx$ where k is some constant.

Examples

(a) A car is travelling at a constant speed. If it travells 400 km in 5 hours, how far will it travel in 9 hours?

SOLUTION

- Look to see what we have to eventually find, that is distance

∴ $D \propto T$

that is, $D = kT$

- Now use the information given to us to find k,

i.e. $D = kT$

$400 = k \times 5$

i.e. $5k = 400$

$k = 80$

- Now, put $k = 80$ back into equation

∴ $D = 80T$

Substitute in $T = 9$

∴ $D = 80 \times 9$

$= 720$

In 9 hours the car will travel 720 km.

(b) We know that P varies directly with Q. If $P = 12$, $Q = 4$, find P when $Q = 6$.

SOLUTION

Set $P = ?$, because we must eventually find P

∴ $P \propto Q$

$P = kQ$

Substitute $P = 12, Q = 4$

∴ $12 = k.4$

∴ $4k = 12$

$k = 3$

∴ $P = 3Q$

Substitute in $Q = 6$

∴ $P = 3 \times 6$

$= 18$

(c) If y varies as the square root of x, and $y = 3$ when $x = 16$, find y when $x = 49$.

SOLUTION

$y \propto \sqrt{x}$

∴ $y = k\sqrt{x}$

Substitute $y = 3, x = 16$

∴ $3 = k.\sqrt{16}$

∴ $4k = 3$

$k = \frac{3}{4}$

∴ $y = \frac{3}{4}\sqrt{x}$

$y = \frac{3}{4}\sqrt{49}$

$= \frac{3}{4} \times 7$

∴ $y = \frac{21}{4}$

$= 5\frac{1}{4}$

1.9.2 Inverse variation

- If x and y are inversely proportional then

$$y \propto \frac{1}{x}$$

that is, $y = \frac{k}{x}$

Examples

(a) It is known that p is inversely proportional to q. If $p = 8$ and $q = 4$, find p when $q = 16$.

SOLUTION

$$\therefore \quad p \propto \frac{1}{q}$$

that is, $p = \dfrac{k}{q}$

Substitute in $p = 8, q = 4$

$$\therefore \quad 8 = \frac{k}{4}$$

$$\therefore \quad k = 32$$

$$\therefore \quad p = \frac{32}{q}$$

Now, substitute in $q = 16$

$$\therefore \quad p = \frac{32}{16} = 2$$

(b) The rate of vibration of a string varies inversely as its length. A string 15 cm long vibrates at 5000 hertz. What length of string will vibrate at 4000 hertz?

SOLUTION

Let L = length, V = vibration in hertz.

$$\therefore \quad L \propto \frac{1}{V}, \quad \text{that is,} \quad L = \frac{k}{V}$$

Substitute in $L = 15, V = 5000$

$$\therefore \quad 15 = \frac{k}{5000}$$

$$\therefore \quad k = 5000 \times 15 = 75\,000$$

Hence, $L = \dfrac{75\,000}{V}$

Now, substitute in $V = 4000$

$$\therefore \quad l = \frac{75\,000}{4000} = 18.75$$

$\therefore$ String is 18.75 cm long.

(c) **The cost per passenger of hiring a bus is inversely proportional to the number of passengers on the bus. If there are twenty passengers, the cost per passenger is $2. What is the cost per passenger when there are thirty passengers?**

Let P = number of passengers, C = cost/passenger

$$C \propto \frac{1}{P}$$

$$\therefore \quad C = \frac{k}{P}$$

$$2 = \frac{k}{20}$$

$$\therefore \quad k = 40$$

$$\therefore \quad C = \frac{40}{P}$$

Now, $P = 30$

$$\therefore \quad C = \frac{30}{40} = 1.\dot{3}$$

that is, cost is $1.33/passenger

1.10 Exercises

1. Leo is a console operator at a service station and works for 16 hours per week. If he is paid $14.25 per hour, find his weekly pay.

2. Ken is a builder's labourer and is paid a wage of $647.50 for a 35-hour week. Find Ken's hourly pay rate.

3. Marge sells vacuum cleaners door-to-door, is paid $215 per week and receives 12% of all her sales. Find Marge's total pay for a week when she sells $2550 worth of vacuum cleaners.

4. Toni is a mathematics teacher and is paid $39 000 per year. If she receives an increase in her salary of $7\frac{1}{2}\%$, find her new salary.

5. Jennifer's hourly rate is increased from $8.40 to $9.00. Express this increase as a percentage of the original rate (correct to one decimal place).

6. A wall tiler charges $28 per m^2. Find the cost of tiling an area measuring 4 metres by 3.2 metres.

7.

Day	Hours
Monday	8
Tuesday	8
Wednesday	8
Thursday	8
Friday	-
Saturday	-
Sunday	6

In one week Janelle worked the hours detailed in the table. She is paid time-and-a-half rates on Saturday, double time on Sundays and normal rate on any other day. Find Janelle's pay for this period if she is paid $11.47 per hour.

8. Mr Johnston received a $66 bonus from his employer. If this represented 12% of his normal weekly pay, find his usual pay.

9. Maria's holiday loading is set at $17\frac{1}{2}\%$ of four weeks' normal pay. If her weekly wage is set at $620, find her holiday loading.

10. Sarah receives a wet-weather allowance of 7% of her pay for any wet day. In one fortnight when she worked ten days it rained on three days and she received her allowance of $42. Find her total pay for the fortnight.

11.

Taxable income $	Tax
$1 — $5 400	Nil
$5 401 — $20 700	Nil plus 20 cents for each $1 over $5 400
$20 701 — $36 000	$3 060 plus 38 cents for each $1 over $20 700
$36 001 — $50 000	$8 874 plus 46 cents for each $1 over $36 000
$50 001 and over	$15 314 plus 47 cents for each $1 over $50 000

Find the tax paid on a taxable income of
(a) $22 376 (b) $47 389

12. Using the table in Question 11, what would be the taxable income if a person paid tax of:
(a) 20 c? (b) $3475? (c) $19 625?

13. To fund government assistance for hospital and medical expenses, a Medicare levy is imposed on people whose income exceeds $11 745.

Taxable income	Medicare levy
Less than $11 745	Nil
$11 745 – $12 528	20 c for every dollar above $11 745
More than $12 528	1.25% of taxable income

Using the above table, find the Medicare levy for a person whose taxable income is:

(a) $11 314 (b) $12 004 (c) $32 476

14. Determine the best buy:

(a) A. 300 mL of milk for 35 c
B. 600 mL of milk for 72 c
C. 1 L of milk for $1.10
D. 2 L of milk for $2.15

(b) 170 g of Spreadmite for $1.45
235 g of Spreadmite for $1.90
340 g of Spreadmite for $2.65
500 g of Spreadmite for $3.95

15. A sales tax of 20% is imposed on all electrical 'whitegoods'. Find the new price of the following articles if the pre-tax prices are:
(a) freezer: $518
(b) washing machine: $815.

16. The Government decides to reduce the tax imposed on cosmetics from $27\frac{1}{2}$% to 15%. Find the savings on a bottle of perfume priced at $51 before the reduction in tax.

17. The Government imposed an increase in tax on cigarettes of 3%. Find the new price of a packet which already attracts a tax of 22% and is presently priced at $4.80.

18. Georgio plans to buy a new television set marked at $940. He agrees to the following terms:

Deposit: 15%
Interest: 22% per annum
24 monthly instalments.

(a) Find the size of each monthly instalment.

(b) How much extra does Georgio have to pay compared to paying cash?

19. Chan purchases a wall unit by paying $75 as a deposit and twenty-four monthly instalments of $68.

(a) Find the total cost of Chan's wall unit.

(b) If Chan's deposit was 5% of the original cash price, how much extra did he pay as interest in buying the wall unit by instalments.

(c) What was the *annual* interest rate Chan was charged, correct to one decimal place?

20. Find the simple interest on:

(a) $1240 at 9.25% per annum for 6 months.

(b) $65 at 6% per annum for 1 month.

(c) $207 at 9% per annum for 2 days.

(d) $400 at 1.25% per month for 2 months.

21. Find the compound interest on:

(a) $215 at 8% per annum for 3 years.

(b) $604 at 11% per annum for 6 years.

22. The town of Singleton had a population in 1980 of 11 420 and it increases at 3% per year. Find the population in the year 2000.

23. A certain country has an inflation rate of 8% per annum. If a resident of that country has an annual salary of $41 000 which increases each year by an amount equal to the inflation rate, find the salary in 15 years' time.

24. Sam's building society has an account that offers 5% p.a. interest, compounded monthly. If he invests $4000 in this account and withdraws the interest when it is paid twice annually, at six and twelve months, find his total interest.

25. A lounge suite depreciates annually at a rate of 17%. If it was purchased originally for $1470, find its value after 4 years.

26. A car is purchased for $24 000. If it is depreciated in value by 23% per annum, how much will it depreciate in value in its third year?

27. Sureloan's credit union structures its loans on a flat-rate interest of 9% per annum. If Yazu borrows $2500 and agrees to thirty monthly payments, find the size of each payment.

28. Timothy gained a loan from a financial centre and was charged a flat-rate interest of 18% per annum. If the term of the loan was three years and each monthly instalment was $616, find the amount that Timothy borrowed. (To the nearest dollar.)

29. Matthew borrows $40 000 from a bank to buy a block of land. His monthly repayments are $420. If he is charged an interest rate of 12% per annum reducible, how much will he owe the bank after:

(a) one month?
(b) two months?

30. If a store reduces prices by 15%, how much will a watch cost if it was originally marked at $79?

31. Konrad's purchases at Fred's Wreckers totalled $375. As Konrad was a vehicle rebuilder, he gained a 12% trade discount. Find the cost of Konrad's purchases. If he gained a further 5% reduction for paying the bill within 30 days, find the final cost of his purchases.

32. 'This year was 35% wetter than last year.' Assuming that this statement is based on total rainfall, find last year's total if this year's rainfall is 810 mm.

33. A car dealer increases the price of a car by 18%. If the car sells for $28 910, find the mark-up on the cost of the car. If 96% of this mark-up is for warranty, wages, rent, etc., how much profit is made by the car dealer on this sale?

34. Karen's car travels 100 km on 8 litres of petrol. Petrol costs 74 cents per litre.

(a) How much does Karen pay for the petrol to travel 100 km?

(b) How far will Karen travel on $20 worth of petrol? (To one decimal place.)

35.Annette's motorbike can travel 100 km on 5.2 litres of petrol. She purchases petrol at the price of 76.9 cents/L.

(a) How much will it cost Annette to travel 100 km?

(b) Find the distance that Annette will travel using $15 worth of petrol.

36. A car driven by Ronnie is moving at a steady speed. When his speed is 80 km/h, the car consumes 8 litres of petrol for every 100 km travelled.

(a) Ronnie's petrol tank holds 64 litres. How many kilometres can the car travel on full tank of petrol when its speed is 80 km/h?

(b) When the speed is 110 km/h, the car consumes 30% more petrol. Calculate the number of litres of petrol per 100 km that the car consumes at 110 km/h.

37.Convert the following:

(a) 100 km/h to m/s (to one decimal place).
(b) 40 km/h to m/s (to one decimal place)
(c) 20 m/s to km/h
(d) 250 m/min to km/h.

38. At the 1896 Olympic Games, Australia's Edwin Flack won a gold medal in the 800 m in a time of 2 minutes 11 seconds.

(a) Find the average speed in m/s, to one decimal place.
(b) Express this speed in km/h.

39. At the 1992 Olympic Games Australia's Kieren Perkins won a gold medal in the 1500 m swimming race in a time of 14 minutes 43.48 seconds.

(a) Find the average speed in m/s, correct to 3 significant figures.
(b) Express his speed in km/h.

40. The population of Bridgetown five years ago was 7420 and now it is 8280. Find the average annual rate of population increase.

41. On a property sold for $60 000, a real estate agent receives a commission of $1200. At what rate in the dollar is the commission calculated?

42. For a particular trip John averaged 90 km/h for 3 hours, stopped for 30 minutes, then averaged 80 km/h for the next 2 hours, rested again for 30 minutes and then drove for another hour covering 60 km. Find

(a) the total distance covered
(b) his average speed for the entire trip in km/h.

43. Premiums are paid on an insurance policy at a rate of \$3.25 per \$100 of value of the goods to be insured. What is the premium payable on electrical equipment valued at \$3250?

44. A council charges ratepayers 2.1 cents in the dollar on the unimproved capital value (UCV) of their properties.
    (a) What will be paid on a property valued at \$47 000?
    (b) If the property is revalued at \$53 000, what additional amount will the ratepayer be charged?

45. It is known that $y \propto x$. If $y = 6$ and $x = 2$, find y when $x = 7$.

46. At a constant speed, distance travelled varies directly as time. If a man walks 32 km in 5 hours, how far would he have walked in 8 hours at the same constant speed?

47. The distance travelled by bicycle is directly proportional to the number of revolutions of the front wheel. If it travels 48 m in 20 revolutions of the front wheel, find:
    (a) the distance travelled for 12 revolutions,
    (b) the number of revolutions required to travel a distance of 36 m.

48. The time t taken for a pendulum to swing varies as the square root of its length ℓ. If one swing of a pendulum 81 cm long takes two seconds, find the time taken for one swing of a pendulum 16 cm long.

49. The distance of the horizon is proportional to the square root of the height above sea level. If Jacki is at the top of a building 125 m high, she can see 40 km to the horizon. How far could she see if she was at the top of a 20 m tower?

50. x varies inversely as y. If $x = 8$ and $y = 9$, find x when $y = 18$.

51. a varies inversely as $\sqrt{b}$. If $a = 9$ when $b = 16$, find a when $b = 64$.

52. The air pressure available from a bicycle pump varies inversely as the square of its radius. If a pump of radius 2 units can supply a pressure of 12 units, find the pressure that can be supplied by a pump of radius 3 units.

53. The intensity of light varies inversely as the square of the distance from the light source. If the intensity is 10 units 5 m away from the source, find:
    (a) the intensity when observed 15 metres away
    (b) the distance, the observer must be from the light source for intensity to be 14 units. (Correct to one decimal place.

Chapter 2
ALGEBRA AND QUADRATICS

2.1 Generalised arithmetic

Examples

(a) The sum of x and y is: $x + y$.

Sum: add
Product: multiply
Difference: subtract
Quotient: divide

(b) The average of

a, b and c is: $\frac{a+b+c}{3}$

$$\text{Average} = \text{mean} = \frac{\text{sum of scores}}{\text{no. of scores}}$$

(c) The number 4 more than c is: $c + 4$.

(d) The next three consecutive numbers after x are $x + 1, x + 2$ and $x + 3$.

Note: In generalised arithmetic, it can be helpful to substitute numbers for the pronumerals.

(e) If y is odd, find the next three consecutive odd numbers.

SOLUTION

$y + 2, y + 4, y + 6$

All odd, and even, numbers are separated by two.

(f) Convert:

(i) \$$y$ to cents.

SOLUTION
$100 \times y = 100y$
\$$y = 100y$ cents.

Try \$7
$\therefore 7 \times 100$
i.e. \$7 = 700 ¢

(ii) p litres to mL.

SOLUTION

$p \times 1000 = 1000p$
p litres = $1000p$ mL.

Try 8 litres.
$\therefore 8 \times 1000$
$= 8000$ mL

(iii) y minutes to hours.

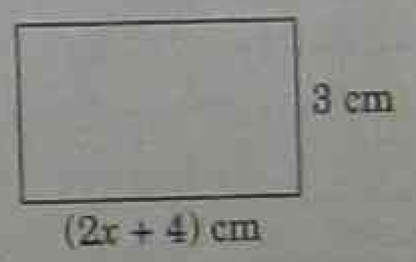
SOLUTION

$$y \div 60 = \frac{y}{60}$$

y minutes $= \frac{y}{60}$ hours

Try 120 minutes.
$\therefore \frac{120}{60} = 2$

(g) Find the area of the rectangle:



SOLUTION

$$A = (2x + 4) \times 3$$
$$= 3(2x + 4)$$

Area is $3(2x + 4)$ cm^2.

(h) Find the change from \$5 if y cakes are purchased at k cents each.

SOLUTION

Cost $= y \times k$ or $k \times y$
$= yk$ cents or ky cents.

Change $= 500 - ky$

Change is $(500 - ky)$ cents.

2.2 Substitution into algebraic expressions

Examples

(a) If $a = 3, b = 4$ and $c = -5$, evaluate:

(i) $$\begin{aligned} ab + c &= 3 \times 4 + (-5) \\ &= 12 - 5 \\ &= 7 \end{aligned}$$

(ii) $$\begin{aligned} b - c &= 4 - (-5) \\ &= 4 + 5 \\ &= 9 \end{aligned}$$

(iii) $$\begin{aligned} \frac{bc-1}{a} &= \frac{4\times -5-1}{3} \\ &= \frac{-20-1}{3} \\ &= \frac{-21}{3} \\ &= -7 \end{aligned}$$

(iv) $$\begin{aligned} c(a-b) &= -5(3-4) \\ &= -5(-1) \\ &= 5 \end{aligned}$$

(v) $$\begin{aligned} b^2 + c^2 &= (4)^2 + (-5)^2 \\ &= 16 + 25 \\ &= 41 \end{aligned}$$

(vi) $$\begin{aligned} 2c^2 - (2c)^2 &= 2(-5)^2 - (2\times -5)^2 \\ &= 2(25) - (-10)^2 \\ &= 50 - 100 \\ &= -50 \end{aligned}$$

2.3 Simplifying algebraic expressions

Examples: Expressions are simplified as follows:

(a) $3x + 5x + 12x = 20x$

(b) $4x - 2y + 3x + 4y = 7x + 2y$

(c) $$\begin{aligned} 12xy - 3yx &= 12xy - 3xy \\ &= 9xy \end{aligned}$$

Remember : $ab = ba$

(d) $4a \times (-2b) = -8ab$

(e) $$\begin{aligned} (-6y)^2 &= -6y \times -6y \\ &= 36y^2 \end{aligned}$$

(f) $$\begin{aligned} cd \div c &= \frac{cd}{c} \\ &= \frac{{}^1\cancel{c}\times d}{\cancel{c}_1} \\ &= \frac{d}{1} \\ &= d \end{aligned}$$

(g) $$\begin{aligned} 12ab \div 3a &= \frac{12ab}{3a} \\ &= \frac{{}^4\cancel{12}\times \cancel{a}^1 \times b}{{}_1\cancel{3}\times \cancel{a}_1} \\ &= \frac{4\times b}{1} \\ &= 4b \end{aligned}$$

(h) $$\begin{aligned} 5pq \div p^2q &= \frac{5pq}{p^2q} \\ &= \frac{5\times \cancel{p}^1 \times \cancel{q}^1}{{}_1\cancel{p}\times p \times \cancel{q}_1} \\ &= \frac{5\times 1}{1\times p} \\ &= \frac{5}{p} \end{aligned}$$

(i) $$\begin{aligned} \frac{12a+3a}{5} &= \frac{15a}{5} \\ &= \frac{\cancel{15}^3\times a}{\cancel{5}_1} \\ &= 3a \end{aligned}$$

2.4 Simple algebraic fractions

2.4.1 Addition and subtraction

Find a common denominator and then add or subtract the numerators.

Examples: Fractions are added or subtracted as follows:

(a) $$\begin{aligned} \frac{5y}{2} + \frac{y}{3} &= \frac{15y}{6} + \frac{2y}{6} \\ &= \frac{17y}{6} \end{aligned}$$

- Lowest common denominator of 2 and 3 is 6
- 2 times 3 is 6

∴ $5y$ times $3 = 15y$
and so on ...

(b) $$\begin{aligned} \frac{3x}{4} - \frac{x}{2} &= \frac{3x}{4} - \frac{2x}{4} \\ &= \frac{x}{4} \end{aligned}$$

(c) $$\begin{aligned} \frac{4}{x} - \frac{3}{2x} &= \frac{8}{2x} - \frac{3}{2x} \\ &= \frac{5}{2x} \end{aligned}$$

(d) $$\begin{aligned} \frac{5}{2y} + \frac{3}{5y} &= \frac{25}{10y} + \frac{6}{10y} \\ &= \frac{31}{10y} \end{aligned}$$

2.4.2 Multiplication and division

To multiply: Cancel and then multiply numerators and denominators.

To divide: Find the **reciprocal** of the second fraction (that is, turn it upside down) and then multiply.

Examples: Fractions are multiplied or divided as follows:

(a) $$\begin{aligned} \frac{x}{3}\times\frac{9}{2x} &= \frac{\cancel{x}^1}{\cancel{3}_1}\times\frac{\cancel{9}^3}{2\cancel{x}_1} \\ &= \frac{3}{2} \\ &= 1\tfrac{1}{2} \end{aligned}$$

(b) $$\begin{aligned} \frac{4y}{7}\times\frac{21}{6y} &= \frac{{}^2\cancel{4}\cancel{y}^1}{\cancel{7}_1}\times\frac{\cancel{21}^{\cancel{3}^1}}{{}_1\cancel{3}\,\cancel{6}\cancel{y}_1} \\ &= \frac{2}{1} \\ &= 2 \end{aligned}$$

(c) $$\begin{aligned} \frac{x+2}{6}\times\frac{18}{x+2} &= \frac{\cancel{x+2}^1}{\cancel{6}_1}\times\frac{\cancel{18}^3}{\cancel{x+2}_1} \\ &= 3 \end{aligned}$$

(d) $$\begin{aligned} \frac{ab}{4}\div\frac{a}{6} &= \frac{{}^1\cancel{a}b}{\cancel{4}_2}\times\frac{\cancel{6}^3}{\cancel{a}_1} \\ &= \frac{3b}{2} \end{aligned}$$

(e) $$\begin{aligned} \frac{4x}{5ab}\div\frac{12}{10b} &= \frac{{}^1\cancel{4}x}{{}_1\cancel{5}a\cancel{b}_1}\times\frac{{}^2\cancel{10}\cancel{b}^1}{\cancel{12}_3} \\ &= \frac{2x}{3a} \end{aligned}$$

2.5 Removing grouping symbols

The term outside the grouping symbols multiplies the contents of the grouping symbols.

Examples

(a) Expanding and simplifying:

(i) $$\begin{aligned} 3(2x+5y) &= 3\times 2x + 3\times 5y \\ &= 6x + 15y \end{aligned}$$

(ii) $$\begin{aligned} a(2a-7) &= a\times 2a + a\times -7 \\ &= 2a^2 - 7a \end{aligned}$$

(iii) $$\begin{aligned} -(3-4y) &= -1(3-4y) \\ &= -1\times 3 - 1\times(-4y) \\ &= -3 + 4y \end{aligned}$$

Note: – sign before the grouping symbols has the effect of negating the contents of grouping symbols.

(iv) $$\begin{aligned} &5(2x-4) - 3(5-x) \\ &= 10x - 20 - 15 + 3x \\ &= 13x - 35 \end{aligned}$$

(v) $$\begin{aligned} &x(x+3) - 2(x+3) \\ &= x^2 + 3x - 2x - 6 \\ &= x^2 + x - 6 \end{aligned}$$

(b) Simplifying fractions:

(i) $\frac{x+2}{3}+\frac{x-4}{4}=\frac{4(x+2)}{12}+\frac{3(x-4)}{12}$

Good to include this step with grouping symbols

$$=\frac{4(x+2)+3(x-4)}{12}$$
$$=\frac{4x+8+3x-12}{12}$$
$$=\frac{7x-4}{12}$$

(ii) $\frac{x}{5}-\frac{x+1}{2}=\frac{2x}{10}-\frac{5(x+1)}{10}$

$$=\frac{2x-5(x+1)}{10}$$
$$=\frac{2x-5x-5}{10}$$
$$=\frac{-3x-5}{10}$$

(c) Find the difference between $7x^2 - 4x$ and $3x + 5x^2$.

SOLUTION

$7x^2-4x-\left(3x+5x^2\right)$
$=7x^2-4x-3x-5x^2$
$=2x^2-7x$

2.6 Binomial products

A binomial expression has two terms, for example $2x + 1$, so a binomial product is the result of multiplying two binomial expressions.

Each term in the first binomial expression multiplies each term in the second expression.

Examples:
Expand and simplify:

(a) $(x+2)(x+4)$
$=x(x+4)+2(x+4)$
$=x^2+4x+2x+8$
$=x^2+6x+8$

(b) $(x-2y)(x-3y)$
$=x(x-3y)-2y(x-3y)$
$=x^2-3xy-2yx+6y^2$
$=x^2-5xy+6y^2$

(c) $(a-b)(a+b)$
$=a(a+b)-b(a+b)$
$=a^2+\cancel{ab}-\cancel{ba}-b^2$
$=a^2-b^2$

This method can be shortened of course, or other methods used, such as:

- The Robin Hood method (with arrows …).

Example: Expand and simplify
$(x + 4)(x - 3)$

$(x+4)(x-3)=x^2-3x+4x-12$
$= x^2+x-12$

- The FOIL method (**F**irst, **O**utside, **I**nside, **L**ast)

Example: Expand and simplify
$(3x + 1)(2x + 5)$

$(3x + 1)\ (2x + 5)$

$= \underset{\substack{\uparrow\\ \text{first}}}{(3x)\ (2x)} + \underset{\substack{\uparrow\\ \text{outside}}}{(3x)\ (5)} + \underset{\substack{\uparrow\\ \text{inside}}}{(1)\ (2x)} + \underset{\substack{\uparrow\\ \text{last}}}{(1)\ (5)}$

$= 6x^2 + 15x + 2x + 5$

$= 6x^2 + 17x + 5.$

2.6.1 Special products

The following results are very important and must be known for success in Year 10:

$$(a+b)^2=a^2+2ab+b^2$$
$$(a-b)^2=a^2-2ab+b^2$$
$$(a-b)(a+b)=a^2-b^2$$

Examples: Expanding and simplifying:

(a) $(x+5)^2=(x)^2+2(x)(5)+(5)^2$
$=x^2+10x+25$

(b) $(a-2)^2=(a)^2+2(a)(2)+(2)^2$
$=a^2-4a+4$

(c) $(3x-4)^2=(3x)^2+2(3x)(4)+(4)^2$
$=9x^2-24x+16$

(d) $(d-2)(d+2)$
$=(d)^2-(2)^2$
$=d^2-4$

(e) $(5y+3)(5y-3)$
$=(5y)^2-(3)^2$
$=25y^2-9$

(f) $\left(x^2-y^2\right)\left(x^2+y^2\right)$
$=\left(x^2\right)^2-\left(y^2\right)^2$
$=x^4-y^4$

2.6.2 Using these special products

Examples: Using the special products to evaluate

(a) $101^2=(100+1)^2$
$=100^2+2(100)(1)+1^2$ $\left[(a+b)^2=a^2+2ab+b^2\right]$
$=10\,000+200+1$
$=10201$

(b) $98^2=(100-2)^2$
$=100^2-2(100)(2)+2^2$ $\left[(a-b)^2=a^2-2ab+b^2\right]$
$=10\,000-400+4$
$=9604$

2.6.3 More-difficult expansions

Examples: Expanding and simplifying:

(a) $(x-3)(x+3)-(x-4)^2$
$=x^2-9-\left(x^2-8x+16\right)$
$=x^2-9-x^2+8x-16$
$=8x-25$

(b) $2(x+6)^2-(4-x)(x+4)$
$=2\left(x^2+12x+36\right)-(4-x)(4+x)$
$=2x^2+24x+72-\left(16-x^2\right)$
$=2x^2+24x+72-16+x^2$
$=3x^2+24x+56$

2.7 Factorisations

2.7.1 Common factors

We look for the highest, or largest, factor common to the terms in the expression — this is the opposite to expanding.

Examples: Factorising

(a) $4x-6=2(2x-3)$

Check by expanding.

(b) $xy-3x=x(y-3)$

(c) $12ab-14a=2a(6b-7)$

You must take both 2 and a as common factors.

(d) $3x^2-6x-3=3(x^2-2x-1)$

(e) $-12x-4=-4(3x+1)$

(f) $9a^2+12a^2b=3a^2(3+4b)$

(g) $4x(x+y)+3(x+y)$

$$=(x+y)[4x+3]$$
$$=(x+y)(4x+3)$$

2.7.2 Factorising by grouping in pairs

We can factorise four-term expressions by grouping in pairs.

Examples: Factorising:

(a) $ax+bx\ +ay+by$

$$=x(a+b)+y(a+b)$$
$$=(a+b)[x+y]$$
$$=(a+b)(x+y)$$

(c) $xy+y^2-x-y$

$$=y(x+y)-1(x+y)$$
$$=(x+y)(y-1)$$

(b) $p^2+mq+pq+mp$

$$=\underbrace{p^2+pq}+\underbrace{mq+mp}$$
$$=p(p+q)+m(p+q)$$
$$=(p+q)(p+m)$$

2.7.3 Difference of two squares

We can reverse an earlier rule: $a^2-b^2=(a-b)(a+b)$

Examples: Factorising:

(a) $c^2-9=(c)^2-(3)^2$

$$=(c-3)(c+3)$$

Note: It does not matter if your answer has this order:
$(c\ +3)(c\ -3)$.
The order of factors is not important, that is, $2\times3=3\times2$.

(b) $49-4a^2=(7)^2-(2a)^2$

$$=(7-2a)(7+2a)$$

(c) $x^4-1=(x^2)^2-(1)^2$

$$=(x^2-1)(x^2+1)$$
$$=(x-1)(x+1)(x^2+1)$$

2.7.4 Completing the square

Numbers such as 1, 4, 9, 16, … are known as square numbers while expressions such as $(x-4)^2$, $(x+5)^2$, etc. are known as perfect squares.

Examples: What must be added to the following expressions to give perfect squares?

(a) x^2-4x

↑ We halve the coefficient of x and square it

that is, x^2-4x+4 $\boxed{\text{as } \left(\frac{-4}{2}\right)^2=4}$

$\therefore$ we add 4

that is, $x^2-4x+4=(x-2)^2$

$\boxed{a^2-2ab+b^2=(a-b)^2}$

(b) x^2+6x

$\therefore\ x^2+6x+9=(x+3)^2$

$\boxed{a^2+2ab+b^2=(a+b)^2}$

that is, we add 9

(c) x^2-5x

$\therefore\ x^2-5x+\left(\frac{-5}{2}\right)^2$

that is, $x^2-5x+\frac{25}{4}$

that is, $x^2-5x+6\frac{1}{4}=\left(x-\frac{5}{2}\right)^2$

$\therefore$ we add $6\frac{1}{4}$

2.7.5 The monic quadratic trinomial

- An expression with three terms is called a *trinomial*.
- A trinomial with a highest power of 2 is called a *quadratic*.
- If the coefficient (number in front of) the term with the power of 2 is 1, the quadratic trinomial is *monic*, $\therefore$ x^2+5x+6 is a monic quadratic trinomial.

As
$$(x+a)(x+b)=x^2+bx+ax+ab$$
$$=x^2+(a+b)x+ab$$

then, factorising, $x^2+(a+b)x+ab=(x+a)(x+b)$.

Hence, to factorise x^2+5x+6 we are looking for two numbers that add together to give 5, (that is, $a+b=5$) and multiply together to give 6 (that is, $ab=6$).

Examples: Factorising:

(a) x^2+5x+4
(that is, $a+b=5,\ ab=4$)
$=(x+4)(x+1)$

(b) x^2-5x+6
(that is, $a+b=-5,\ ab=6$)
$=(x-3)(x-2)$

(c) x^2-3x-4
(that is, $a+b=-3,\ ab=-4$)
$=(x-4)(x+1)$

(d) $x^2+5x-14$
(that is, $a+b=5,\ ab=-14$)
$=(x+7)(x-2)$

Another helpful rule is:

- For $x^2 + 5x + 4$
  ↑↑ positive here, we say 'both'
  'this sign'; that is, **both positive**.
- For $x^2 - 3x - 4$
  ↑ ↑ negative here, we say 'the bigger one is'
  'this sign'; that is, **bigger one is negative**.

Examples: Factorising:

(e) $x^2+7x+12=(x+4)(x+3)$
↑
both positive

(f) $x^2-8x+15=(x-5)(x-3)$
↑
both negative

(g) $x^2-4x-12=(x-6)(x+2)$

$\boxed{\text{The 'bigger one' is negative.}}$

(h) $x^2+x-20=(x+5)(x-4)$

$\boxed{\text{The 'bigger one' is positive.}}$

2.7.6 Non-monic quadratic trinomial

Most of the hints given above no longer apply, although:

- the two numbers a and b still multiply together (ab) to give our constant term;
- if this constant term is positive, we can still say 'both positive/negative'. (See above.)

$\boxed{\text{In } ax^2 + bx + c, \text{ the } c \text{ is the constant term and is said to be independent of } x.}$

Examples:

(a) Factorise $2x^2 + 13x + 21$.

SOLUTION

$2x^2 + 13x + 21 = (2x \qquad)(x \qquad)$.

We can see that they are both positive,

$\therefore\ 2x^2 + 13x + 21 = (2x + \qquad)(x + \qquad)$.

Now, it's simply trial and error, and checking by expanding the answer to get back to the expression.

$\therefore\ 2x^2 + 13x + 21 = (2x + 7)(x + 3)$.

(b) Factorise $3x^2 - 2x - 8$.

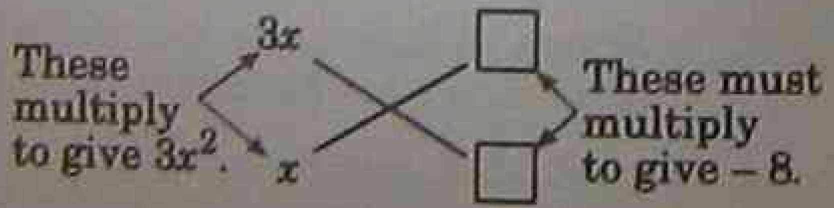
SOLUTION

Start with $(3x \qquad)(x \qquad)$ and trial and error, knowing that we are looking for two numbers that multiply together to give -8.

$\therefore\ 3x^2 - 2x - 8 = (3x + 4)(x - 2)$.

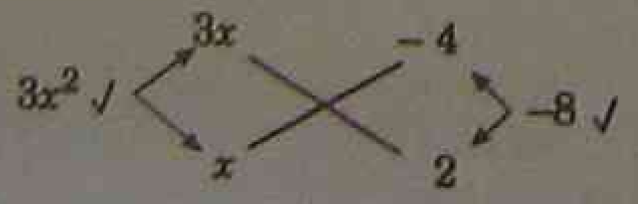
Now try a second method: This is called the cross method, where the guess and verify stage occurs around a cross.

(c) Factorise $3x^2 + 10x - 8$.



We then multiply across the diagonals and add terms.

Thus:

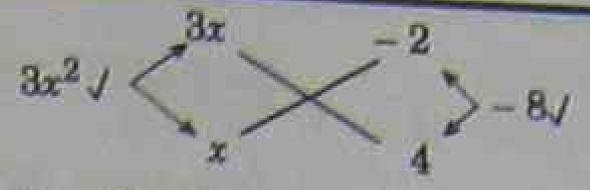


$$\begin{aligned}\therefore\quad & 3x \times 2 + x \times -4\\ &= 6x - 4x\\ &= 2x.\end{aligned}$$

$\boxed{\text{No! It must be } 10x.}$

Try again:

Continued



$$\begin{aligned}\therefore\quad & 3x \times 4 + x \times -2\\ &= 12x - 2x\\ &= 10x.\end{aligned}$$

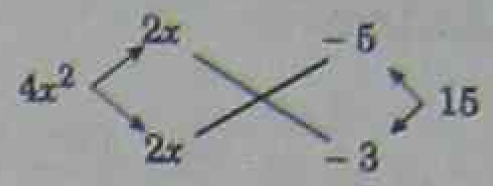
$\boxed{\text{Yes,}}$

$\therefore\ (3x - 2)(x + 4)$.

(d) Factorise $4x^2 - 16x + 15$.

SOLUTION: Note that the $4x^2$ means we could choose $4x$ and x or $2x$ and $2x$.

Once again, by trial and error:



$$\begin{aligned} & 2x \times -3 + 2x \times -5\\ &= -6x - 10x\\ &= -16x,\end{aligned}$$

$\therefore\ 4x^2 - 16x + 15 = (2x - 5)(2x - 3)$.

Another method of factorising non-monic quadratic trinomial is as follows:

If the trinomial is of the form $ax^2 + bx + c$, we rewrite bx as two separate terms, which allows us to factorise pairs of terms.

Examples

(e) Factorise $2x^2 + 5x - 3$.

- We multiply $2x^2$ by -3, that is, $-6x^2$.
- We now look for two factors which add together to give 5 (from the $5x$) and multiply together to give -6 (from the $-6x^2$).

Thus the factors are 6 and -1.

That is,

$$\begin{aligned}2x^2+5x-3 &= 2x^2+6x-x-3\\ &= 2x(x+3)-1(x+3)\\ &= (x+3)(2x-1)\\ &= (2x-1)(x+3).\end{aligned}$$

(f) Factorise $8a^2 + 10a - 3$

SOLUTION

$8a^2 \times -3 = -24a^2$,

therefore look for factors which add to give 10 and multiply to give -24.

That is, 12 and -2.

Therefore

$$\begin{aligned}8a^2+10a-3 &= 8a^2-2a+12a-3\\ &= 2a(4a-1)+3(4a-1)\\ &= (4a-1)(2a+3).\end{aligned}$$

A further method is as follows:

Examples

(g) Factorise $4a^2 - 13a + 9$.

SOLUTION

- The coefficient of a^2 and the constant term are multiplied … here $4 \times 9 = 36$.
- The coefficient of a^2 (here it is 4) is put in both brackets and as a denominator.

Thus,

㊱

$$4a^2-13a+9=\frac{(4a\qquad)(4a\qquad)}{4}$$

- We now look for two numbers that add together to get -13 (from $-13a$), and multiply together to get 36.

That is, -9 and -4.

Therefore:

$$4a^2-13a+9=\frac{(4a-9)(4a-4)}{4}.$$

- We then cancel … here $4a - 4$ is divided by 4 to get $a - 1$.

Thus $4a^2 - 13a + 9 = (4a - 9)(a - 1)$.

(h) Factorise $6x^2 + 11x - 10$.

SOLUTION

$\boxed{-60}$

$$6x^2+11x-10=\frac{(6x\quad)(6x\quad)}{6}$$

Now $6 \times x - 10 = -60$,
$\therefore$ add to get 11 and multiply to get -60, that is 15, -4.

$$=\frac{(6x+15)(6x-4)}{6}$$
$$=\frac{{}^{1}\cancel{3}(2x+5)\times{}^{1}\cancel{2}(3x-2)}{\cancel{6}\,\cancel{2}_{1}}$$
$$=(2x+5)(3x-2)$$

Thus $6x^2 + 11x - 10 = (2x + 5)(3x - 2)$.

2.7.7 Further examples

Examples: Factorising:

(a) $x^3 - x = x(x^2-1)$
$\quad = x(x-1)(x+1)$

(b) $y^8 - 1$
$= (y^4-1)(y^4+1)$ Now factor $y^4 - 1$
$= (y^2-1)(y^2+1)(y^4+1)$ Now factor $y^2 - 1$
$= (y-1)(y+1)(y^2+1)(y^4+1)$

(c) $6x^3 + 13x^2 - 5x$
$\quad = x(6x^2+13x-5)$
$\quad = x(3x-1)(2x+5)$

Always look first for common factors.

(d) $(x-3)^2-(y+2)^2$
$= [(x-3)+(y+2)][(x-3)-(y+2)]$
$= (x-3+y+2)(x-3-y-2)$
$= (x+y-1)(x-y-5)$.

(e) $a^4 + 2a^2 + 1$
$\quad = (a^2+1)(a^2+1)$
$\quad = (a^2+1)^2$

(f) $c^6 + 3c^3 - 4$
$\quad = (c^3+4)(c^3-1)$

Note: In Year 11 you might learn to factorise $x^3 - 1$: a difference of two cubes.

2.8 More algebraic fractions

2.8.1 Factorising numerators and denominators

Numerators and denominators of fractions should be factorised if possible.

Examples: Simplifying:

(a) $$\frac{3x+12}{3}=\frac{{}^{1}\cancel{3}(x+4)}{\cancel{3}_{1}}$$
$$=x+4$$

(b) $$\frac{5x+15y}{2x+6y}=\frac{5\overset{1}{\cancel{(x+3y)}}}{2\underset{1}{\cancel{(x+3y)}}}$$
$$=\frac{5}{2}$$
$$=2\tfrac{1}{2}$$

(c) $$\frac{4}{4x+8}=\frac{\cancel{4}^{1}}{{}_{1}\cancel{4}(x+2)}$$
$$=\frac{1}{x+2}$$

Don't forget the 1 as the numerator

(d) $$\frac{2a-2b}{b-a}=\frac{2(a-b)}{-1(-b+a)}$$
$$=\frac{2\cancel{(a-b)}1}{-1\cancel{(a-b)}1}$$
$$=\frac{2}{-1}$$
$$=-2$$

Remember:
$x - y = -(y - x)$

(e) $$\frac{x^2+5x-6}{x^2-36}=\frac{\overset{1}{\cancel{(x+6)}}(x-1)}{(x-6)\underset{1}{\cancel{(x+6)}}}$$
$$=\frac{x-1}{x-6}$$

(f) $$\frac{ax+ay+2x+2y}{x^2-y^2}$$
$$=\frac{a(x+y)+2(x+y)}{(x+y)(x-y)}$$
$$=\frac{\overset{1}{\cancel{(x+y)}}(a+2)}{\underset{1}{\cancel{(x+y)}}(x-y)}$$
$$=\frac{a+2}{x-y}$$

2.8.2 Algebraic fractions involving multiplication and division

Examples: Simplifying:

(a) $$\frac{a^2-9}{a-3}\times\frac{5}{a+3}$$
$$=\frac{\overset{1}{\cancel{(a-3)}}\overset{1}{\cancel{(a+3)}}}{\underset{1}{\cancel{a-3}}}\times\frac{5}{\underset{1}{\cancel{a+3}}}$$
$$=5$$

(b) $$\frac{y^2+7y+10}{y^2-8y+12}\times\frac{y-6}{y+5}$$
$$=\frac{\overset{1}{\cancel{(y+5)}}(y+2)}{\underset{1}{\cancel{(y-6)}}(y-2)}\times\frac{\overset{1}{\cancel{y-6}}}{\underset{1}{\cancel{y+5}}}$$
$$=\frac{y+2}{y-2}$$

(c) $$\frac{3b^2-27}{ab^2-16a}\div\frac{b+3}{2b+8}$$

We multiply by the reciprocal... ↵

$$=\frac{3(b-3)\overset{1}{\cancel{(b+3)}}}{a(b-4)\underset{1}{\cancel{(b+4)}}}\times\frac{2\overset{1}{\cancel{(b+4)}}}{\underset{1}{\cancel{b+3}}}$$
$$=\frac{6(b-3)}{a(b-4)}$$

(d) $$\frac{x^2-4x+3}{x^2-x-6}\div\frac{x^2+3x-4}{x^2+x-12}$$
$$=\frac{\overset{1}{\cancel{(x-3)}}\cancel{(x-1)}}{\underset{1}{\cancel{(x-3)}}(x+2)}\times\frac{\overset{1}{\cancel{(x+4)}}(x-3)}{\underset{1}{\cancel{(x+4)}}\underset{1}{\cancel{(x-1)}}}$$
$$=\frac{x-3}{x+2}$$

2.8.3 Algebraic fractions involving addition and subtraction

Examples: Simplifying:

(a) $$\frac{3}{x+1}-\frac{4}{x+3}$$
$$=\frac{3(x+3)-4(x+1)}{(x+1)(x+3)}$$
$$=\frac{3x+9-4x-4}{(x+1)(x+3)}$$
$$=\frac{-x+5}{(x+1)(x+3)}$$

Remember:
- Common denominator by multiplying the two denominators,
- then:
  $x + 1$ times ? $= (x + 1)(x + 3)$
  $\therefore\ x + 3$,
- then:
  $x + 3$ times the numerator (3).

(b)
$$\begin{aligned}&\frac{2}{x-1}+\frac{3}{x^2-1}\\&=\frac{2}{x-1}+\frac{3}{(x+1)(x-1)}\\&=\frac{2(x+1)+3}{(x+1)(x-1)}\\&=\frac{2x+2+3}{(x+1)(x-1)}\\&=\frac{2x+5}{(x+1)(x-1)}\end{aligned}$$

(c)
$$\begin{aligned}&\frac{4}{a^2+a}-\frac{3}{a^2-1}\\&=\frac{4}{a(a+1)}-\frac{3}{(a+1)(a-1)}\\&=\frac{4(a-1)-3a}{a(a+1)(a-1)}\\&=\frac{4a-4-3a}{a(a+1)(a-1)}\\&=\frac{a-4}{a(a+1)(a-1)}\end{aligned}$$

2.9 Solving quadratic equations

A quadratic equation is in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Using the result that if $ab = 0$, then either $a = 0$ or $b = 0$ (or $a = b = 0$), we attempt to factorise our quadratic expression and then solve it.

2.9.1 Factorising and solving quadratic equations

Note: Students should be confident with Chapter 2 work on factorising quadratic trinomials.

Examples: Solve:

(a)
$$\begin{aligned}x^2-5x-6&=0\\(x-6)(x+1)&=0\\x&=6,\ -1\end{aligned}$$

We could check by substitution.

This means that $x = 6$ or -1

(b)
$$\begin{aligned}x^2-4x&=-4\\x^2-4x+4&=0\\(x-2)(x-2)&=0\\x&=2\end{aligned}$$

We must have the quadratic equalling zero.

(c)
$$\begin{aligned}x^2&=5x\\x^2-5x&=0\\x(x-5)&=0\\x&=0,\ 5\end{aligned}$$

(d)
$$6+x-x^2=0$$

Multiply through by – 1.

$$\begin{aligned}x^2-x-6&=0\\(x-3)(x+2)&=0\\x&=3,\ -2\end{aligned}$$

(e)
$$\begin{aligned}&2x^2+5x-3=0\\&(2x-1)(x+3)=0\\&\ \Downarrow\qquad\qquad\ \Downarrow\ \leftarrow\\&2x-1=0\quad x+3=0\\&2x=1\qquad\quad x=-3\\&x=\tfrac{1}{2}\end{aligned}$$

You should be able to do this step in your head.

Therefore, $x=\frac{1}{2},\ -3$.

(f)
$$\begin{aligned}&3x^2-8x+5=0\\&(3x-5)(x-1)=0\\&\ \Downarrow\qquad\qquad \Downarrow\\&3x=5\qquad x=1\\&x=\tfrac{5}{3}\end{aligned}$$

Therefore, $x=1\frac{2}{3},\ 1$.

(g)
$$\begin{aligned}x^2-5x+6&=2\\x^2-5x+6-2&=0\\x^2-5x+4&=0\\(x-4)(x-1)&=0\\x&=4,\ 1\end{aligned}$$

We must always make one side equal to zero.

(h)
$$\begin{aligned}2(x-3)(x-2)&=x^2-3x+2\\2(x^2-5x+6)&=x^2-3x+2\\2x^2-10x+12&=x^2-3x+2\\2x^2-10x+12-x^2+3x-2&=0\\x^2-7x+10&=0\\(x-5)(x-2)&=0\\x&=5,\ 2\end{aligned}$$

(i)
$$\begin{aligned}x&=\frac{x+12}{x}\\x^2&=x+12\\x^2-x-12&=0\\(x-4)(x+3)&=0\\x&=4,\ -3\end{aligned}$$

Multiply through by x.

(j)
$$\begin{aligned}(x-5)^2&=9\\x-5&=\pm\sqrt{9}\\x-5&=\pm3\end{aligned}$$

Therefore, $x = 8,\ 2$.

We could expand the LHS and treat it like above examples.

Take the square root of both sides (SRBS).

2.9.2 The quadratic formula

When we cannot factorise the quadratic, we can use the quadratic formula:

For $ax^2 + bx + c = 0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Note: The algebraic expression 'housed under' the square root has a great bearing on the final solution — it determines whether the solutions are surds, or there is only one solution, or there is *no real solution*.

Examples: Solve the equation, leaving answers as simplified surds where possible.

(a) $x^2 - 5x + 2 = 0$.

SOLUTION

Here $a = 1$, $b = –5$, $c = 2$.

Note: It is a good idea to write down the values for a, b and c before substituting in the formula.

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(2)}}{2(1)}\\&=\frac{5\pm\sqrt{25-8}}{2}\\&=\frac{5\pm\sqrt{17}}{2}\end{aligned}$$

This is often called the *exact form*.

(b) $3x^2 - 4x - 5 = 0$ $(a = 3, b = –4, c = –5)$

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(-5)}}{2(3)}\\&=\frac{4\pm\sqrt{16+60}}{6}\\&=\frac{4\pm\sqrt{76}}{6}\\&=\frac{4\pm2\sqrt{19}}{6}\\&=\frac{{}^{1}\cancel{2}(2\pm\sqrt{19})}{\cancel{6}_{3}}\\&=\frac{2\pm\sqrt{19}}{3}\end{aligned}$$

Solve, correct to two decimal places if necessary:

(c) $2x^2 + 3x + 8 = 0 \quad (a = 2, b = 3, c = 8)$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4(2)(8)}}{2(2)} \\ &= \frac{-3 \pm \sqrt{9 - 64}}{4} \\ &= \frac{-3 \pm \sqrt{-55}}{4} \end{aligned}$$

No real solution.

Cannot find $\sqrt{-55}$ in 'real' number system.

(d) $1 - 4x - 5x^2 = 0 \quad (a = -5, b = -4, c = 1)$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-5)(1)}}{2(-5)} \\ &= \frac{4 \pm \sqrt{16 + 20}}{-10} \\ &= \frac{4 \pm \sqrt{36}}{-10} \\ &= \frac{4 \pm 6}{-10} \\ &= \frac{10}{-10} \text{ or } \frac{-2}{-10} \\ &= -1, \tfrac{1}{5} \end{aligned}$$

Note: This could have been factorised as the roots are rational.

(e) $2x^2 - 5x - 1 = 0$
$(a = 2, b = -5, c = -1)$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{5 \pm \sqrt{25 + 8}}{4} \\ &= \frac{5 \pm \sqrt{33}}{4} \\ &= \frac{5 + \sqrt{33}}{4} \text{ or } \frac{5 - \sqrt{33}}{4} \\ &= 2.69 \text{ or } -0.19. \end{aligned}$$

Calculator steps:

$\boxed{(}\ 5\ \boxed{+}\ 33\ \boxed{\sqrt{\ }}\ \boxed{)}\ \boxed{\div}\ 4\ \boxed{=}$

or we can use memory:

$33\ \boxed{\sqrt{\ }}\ \boxed{\text{Min}}$

$\boxed{(}\ 5\ \boxed{+}\ \boxed{\text{MR}}\ \boxed{)}\ \boxed{\div}\ 4\ \boxed{=}$

and similarly with the other case.

2.9.3 Simultaneous equations leading to quadratic equations

Example

Solve $2x + y = 9 \quad \ldots (1)$
$xy = 4 \quad \ldots (2)$

SOLUTION

From (1), $y = 9 - 2x$

Substituting in (2):

$$\begin{aligned} x(9 - 2x) &= 4 \\ 9x - 2x^2 &= 4 \\ 2x^2 - 9x + 4 &= 0 \\ (2x - 1)(x - 4) &= 0 \end{aligned}$$

Therefore, $x = \frac{1}{2}, 4.$

Now, there are two values of x to substitute in (1).

Substitute $x = \frac{1}{2}$:

$$\begin{aligned} 2\left(\tfrac{1}{2}\right) + y &= 9 \\ 1 + y &= 9 \\ y &= 9 - 1 \\ &= 8 \end{aligned}$$

Therefore, $x = \frac{1}{2}, \ y = 8.$

Also, $x = 4$:

$$\begin{aligned} 2(4) + y &= 9 \\ 8 + y &= 9 \\ y &= 9 - 8 \\ y &= 1 \end{aligned}$$

Therefore, $x = 4, \ y = 1.$

Therefore, $x = 4, y = 1.$

The answer is:

$x = \frac{1}{2}, \ y = 8,$ or $x = 4, \ y = 1,$

or: $\left(\frac{1}{2}, 8\right)$ or $(4, 1)$.

2.9.4 Use of equations to solve problems

Examples

(a) The sum of two consecutive even numbers is 22. Find the numbers.

SOLUTION

Let the numbers be x and $x + 2$.

Then
$$\begin{aligned} x + x + 2 &= 22 \\ 2x + 2 &= 22 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

Therefore, $x = 10, x + 2 = 12$, and so the numbers are 10 and 12.

(b) Find two numbers such that their sum is 20, while half their difference is 1.

SOLUTION

Let the numbers be x and y.

Then $x + y = 20 \quad \ldots (1)$

$\frac{1}{2}(x - y) = 1 \quad \ldots (2)$

$2 \times (2) \quad x - y = 2 \quad \ldots (3)$

$[(1) - (3)] \quad 2y = 18$

$y = 9$

Substituting $y = 9$ in (1):

$$\begin{aligned} x + 9 &= 20 \\ x &= 20 - 9 \\ x &= 11 \end{aligned}$$

Therefore, $x = 11, y = 9$, and so the numbers are 11 and 9.

(c) If the sides of an isosceles triangle are $(x + 20)$ cm, $(3x - 16)$ cm and $(x + 2)$ cm, what are the possible values of x?

SOLUTION:

Two of the three sides are equal. We form equations and solve them to find value(s) of x, but there is no point in setting $x + 20 = x + 2$ as this is nonsense.

$$\begin{aligned} 3x - 16 &= x + 20 \\ 3x - x &= 20 + 16 \\ 2x &= 32 \\ x &= 16 \end{aligned} \quad \textit{or} \quad \begin{aligned} 3x - 16 &= x + 2 \\ 3x - x &= 2 + 16 \\ 2x &= 18 \\ x &= 9 \end{aligned}$$

Therefore, the values of x could be 16 or 9.

(d) John is ten years older than Allyn, but twenty-five years ago, John was twice Allyn's age. Find Allyn's present age.

SOLUTION

Let Allyn's age $= x$

John's age $= x + 10$

Twenty-five years ago:

Allyn $= x - 25$

John $= x + 10 - 25 = x - 15$

Therefore twenty-five years ago:

$$\begin{aligned} 2(x - 25) &= x - 15 \\ 2x - 50 &= x - 15 \\ 2x - x &= 50 - 15 \\ x &= 35. \end{aligned}$$

Therefore Allyn's present age is 35 years.

(e) Daniel has 22 coins, each being either a one-dollar coin or a two-dollar coin. He has a total of $35. How many of each coin does he have?

SOLUTION: Let x be the number of $1 coins, and y be the number of $2 coins.

The equations are:

$x + y = 22$ [he has 22 coins] … (1)

$x + 2y = 35$ [as $1 and $2] … (2)

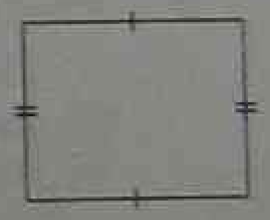
$[(2) - (1)] \quad y = 13$

Substituting $y = 13$ in (1):

$$\begin{aligned} x + 13 &= 22 \\ x &= 9 \end{aligned}$$

Therefore he has nine $1 coins and thirteen $2 coins. [Check this.]

(f) The perimeter of the rectangle is 40 cm and the area is 96 cm². Find its dimensions.



SOLUTION

Let the dimensions be x cm by y cm.

$2x + 2y = 40$
… (1)

$xy = 96 \quad \ldots (2)$

From (2), make y the subject:

$y = \frac{96}{x} \quad \ldots (3)$

We have to use the substitution method — not the elimination method.

Substituting (3) in (1):

$$2x + 2\left[\frac{96}{x}\right] = 40$$

Continued

$$2x+\frac{192}{x}=40$$

$$2x^2+192=40x$$

Multiply by x.

$$2x^2-40x+192=0$$

$$x^2-20x+96=0$$

$$(x-12)(x-8)=0$$

$$x=12,\ 8$$

Divide through by 2.

In some questions, one of the solutions might be negative, so just disregard it. That is, we cannot have negative length.

Now substituting in (3):

If $x=12$, $y=\frac{96}{12}$.
Therefore $y=8$.
If $x=8$, $y=12$.

Therefore the dimensions are 12 cm and 8 cm.

(g) Use algebra to find the points of intersection of the straight line $y=x-14$ and the parabola $y=x^2+6x-8$.

SOLUTION

The simultaneous equations are:

$$y=x-14 \qquad \ldots(1)$$

$$y=x^2+6x-8 \qquad \ldots(2)$$

Either eliminate y by subtracting, *or*, both equations equal y, so they are equal to each other.

$$x^2+6x-8=x-14$$

$$x^2+6x-8-x+14=0$$

$$x^2+5x+6=0$$

$$(x+3)(x+2)=0$$

Therefore $x=-3,-2$.

Substituting in (1):
If $x=-3$, $y=-3-14$
$=-17$
and if $x=-2$, $y=-2-14$
$=-16$.

The points of intersection are:
$x=-3, y=-17$, or $(-3,-17)$
$x=-2, y=-16$, or $(-2,-16)$.

2.10 Exercises

1. The maximum speed of a car is x kilometres per hour, but when it has a trailer, it is slowed up by y kilometres per hour. What would be its maximum speed when towing the trailer?

2. A boy is now 14 years old.
   (a) How old will he be in x years time?
   (b) How old was he y years ago?
   (c) If his mother was p years old when he was born, how old is she now?
   (d) How old will his mother be in q years time?

3. A brick wall x metres long and y metres high has r windows, each m metres high and q metres wide. Find the:
   (a) overall area of the wall;
   (b) area of one window;
   (c) area of r windows;
   (d) area of the wall which is completely brick.

4. Convert:
   (a) k cm to mm.
   (b) p days to minutes.
   (c) k kilolitres to litres.
   (d) r cents to dollars.
   (e) v grams to kilograms.

5. A barrel of sugar contains 16.4 kg. If x kilograms is removed but then y grams is put back, find the amount of sugar now in the barrel.

6. Find the number of cents in x dollars and y cents.

7. If 20 books cost \$$c$, find the cost of 3 books.

8. In a triangle, two angles are found to be $p°$ and $q°$. Find the size of the third angle.

9. (a) How far will Sharon walk at s km/h in t hours?
   (b) What is Bob's average speed if his car travels k km in p hours?
   (c) How long will it take Pat to cycle b km if she travels at c km/h?

10. Evaluate the following expressions, if $a=-2, b=3, c=-5$:
    (a) abc (b) $bc-a$
    (c) $\frac{1}{a}-\frac{1}{b}$ (d) $\frac{ab}{c-1}$
    (e) a^2-b^2 (f) c^2-2a^2

11. Simplify:
    (a) $12y-3x-6y+7x$
    (b) $5y^2-2y+3y^2-4y$
    (c) $12xy-4xy+xy$
    (d) $3a\times(-2b)$ (e) $(-5y)^2$
    (f) $7a\times a\times a$ (g) $12xy\times-2x$
    (h) $14ab\div7a$ (i) $3xy\div-x$
    (j) $(4x+6x)\div5x$

12. Simplify:
    (a) $\frac{2x}{3}+\frac{x}{2}$ (b) $\frac{5a}{4}-\frac{a}{2}$
    (c) $\frac{12x}{13}-\frac{x}{2}$ (d) $\frac{3}{x}-\frac{1}{x}$
    (e) $\frac{5}{2a}-\frac{3}{a}$ (f) $\frac{a}{b}-\frac{c}{d}$
    (g) $\frac{2a}{3b}-\frac{4a}{5b}$ (h) $\frac{6}{y}\times\frac{2y}{3}$
    (i) $\frac{6m}{5}\times\frac{10}{18m^2}$ (j) $\frac{3}{2m}\div\frac{6}{4m}$
    (k) $\frac{6a}{5}\div\frac{4a}{25}$ (l) $\frac{3a}{8bc}\times\frac{b}{9a}\times4c$
    (m) $\frac{4}{x}\div\frac{3}{x}\times1\frac{1}{3}$

13. Expand and simplify:
    (a) $4(2x-7)$ (b) $-3(5a-12)$
    (c) $2+3(4y-1)$ (d) $7-5(x-1)$
    (e) $-(3x-4)$ (f) $4(x-1)+2(x-5)$
    (g) $5(2a-5)-6(4-a)$
    (h) $x(x-7)-2(x-7)$

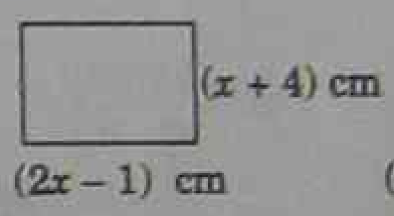
14. Simplify:
    (a) $\frac{3x-4}{2}+\frac{x-1}{4}$ (b) $\frac{5x-4}{2}-\frac{3-x}{3}$
    (c) $\frac{4a-2}{2x}-\frac{3a-4}{3x}$

15. Find the difference between:
    (a) $4x-2$ and $x+5$
    (b) $12y-4$ and $5-2y$
    (c) $3x-x^2$ and $7x^2-5x+2$

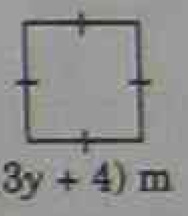
16. Expand and simplify:
    (a) $(a-2)(a+4)$ (b) $(2x-1)(x+3)$
    (c) $(4x-7)(x+8)$ (d) $(4-x)(3x+1)$
    (e) $(x-4)^2$ (f) $(4x+1)^2$
    (g) $(2y-1)(2y+1)$ (h) $(5y-7)(5y+7)$

17. Find the area of:

(a)



(b)



18. Expand and simplify:
    (a) $x^2-(x-4)^2$
    (b) $(x-1)^2-(x+1)^2$
    (c) $\left(x+\frac{1}{x}\right)^2$
    (d) $(a-b)(b-a)-a^2-b^2$
    (e) $3(x-4)^2$
    (f) $2(a-1)^2-2(a-1)(a+1)$

19. Factorise:
    (a) $5x-10y$ (b) $8a-6$
    (c) $12xy-8y$ (d) $15y^2-10y$
    (e) $-10a-18$ (f) $4y^2z-6yz$
    (g) $3x^2-9x-48$ (h) $\pi r^2-2\pi r$
    (i) $2l+2b$
    (j) $3(x+y)+x(x+y)$
    (k) $a(a-7)-b(a-7)$

20. Factorise:

(a) $2x+2y+ax+ay$
(b) $-2a+2b-ca+cb$
(c) $6a-ct+ac-6t$
(d) g^3-3g^2-2g+6
(e) $xy-y^2-7x+7y$

21. Factorise:

(a) u^2-36 (b) $1-25p^2$
(c) $4a^2-9b^2$ (d) $25k^2-144m^2$
(e) $16x^4-1$ (f) $1-x^4y^4$
(g) $2a^2-2$ (h) $5a^2-125$

22. Factorise:

(a) x^2+2x-8 (b) $k^2-12k+11$
(c) $h^2+13h-48$ (d) $b^2-3b-88$
(e) $x^2-6xy+8y^2$ (f) $x^2+3xy-4y^2$

23. Factorise:

(a) $2x^2-x-1$ (b) $3p^2+11p+8$
(c) $3c^2+5c-12$ (d) $9x^2+17x+8$
(e) $8v^2-v-7$ (f) $3b^2+16bc+5c^2$

24. Simplify:

(a) $\frac{8}{2a-6}$ (b) $\frac{3y-4}{9y-12}$

(c) $\frac{a^2-4a}{6a}$ (d) $\frac{a^2-6a+9}{a^2-9}$

(e) $\frac{x^2+7x+12}{x^2-16}$

(f) $\frac{p^2+4p+pq+4q}{p^2-q^2}$

25. Simplify:

(a) $\frac{y-6}{y^2-36}\times\frac{y+6}{2}$

(b) $\frac{x^2-1}{x-1}\div\frac{x+1}{5}$

(c) $\frac{a^2-6a-7}{a^2-1}\times\frac{a^2+a}{a^2}$

26. Simplify:

(a) $\frac{4}{x+3}+\frac{5}{x+7}$ (b) $\frac{2}{x-3}+\frac{4}{x-5}$

(c) $\frac{x-3}{x^2-5x+6}-\frac{3x}{x^2+4x}$

(d) $\frac{2}{c^2-49}+\frac{4}{c^2-4c-21}$

(e) $\frac{y+2}{y^2+2y-3}-\frac{y}{y^2+7y+12}$

27. Solve the following quadratic equations:

(a) $(3x-1)(x+2)=0$
(b) $x^2-5x=0$
(c) $3x=x^2$
(d) $x^2-4x+4=0$
(e) $5x-6-x^2=0$
(f) $x^2=49$
(g) $4x^2=36$
(h) $5x^2-2x-3=0$
(i) $9x^2=2(9x+8)$
(j) $4(x^2-3)+13x=0$
(k) $3x^2-8x+6=x^2-3x+4$
(l) $(x-3)^2=16$
(m) $\frac{3x+4}{x}=x$

28. Solve the following equations, leaving your answer as a surd:

(a) $x^2-4x-1=0$
(b) $x^2-8x-5=0$

29. Solve the following equations, writing your answer correct to three decimal places.

(a) $3x^2+9x+5=0$
(b) $5x^2=2x+4$

30. (a) If 5 is subtracted from twice a number, the result is the same as when 4 is added to the number. Find the number.

(b) The product of two consecutive numbers is 30. By solving a quadratic equation, find the numbers.

(c) The sum of two numbers is 10 and the sum of their squares is 52. Find the numbers.

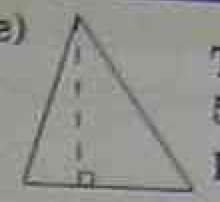
(d)

For the above rectangle, the length is known to be 8 cm longer than the width.

(i) If the width is x cm, find the length of the rectangle in terms of x.

(ii) If its area is 240 cm², find the dimensions of the rectangle.

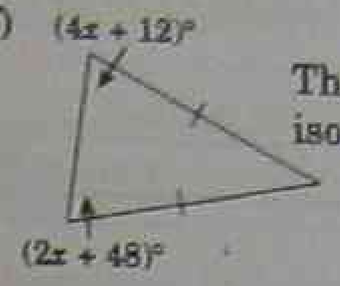
(e)



The base of a triangle is 5 cm longer than the perpendicular height.

If the area of the triangle is 18 cm², find the length of the base.

(f)



The base angles of an isosceles triangle are given.

Find the size of *all* the angles of the triangle.

(g) Find the points of intersection of

$y=x^2+6x-2$ and $y=2x+3$.

(h) What positive integer, when increased by 20, will be equal to its square.

31. Complete the square in the following:

(a) x^2-6x
(b) x^2+2x
(c) a^2-7a
(d) x^2+3x

Chapter 3

REAL NUMBERS, SURDS AND INDICES

3.1 Real numbers

3.1.1 Rational numbers

A number is rational if it can be expressed in terms of $\frac{a}{b}$ where a and b are integers, and $b \neq 0$.

Examples:

Examples of rational numbers are:

$$\frac{4}{7},\ -\frac{2}{3},\ 1\tfrac{2}{3}\left(=\frac{5}{3}\right),\ 27\%\left(=\frac{27}{100}\right),\ 2\left(=\frac{2}{1}\right),\ \sqrt{9}\left(=3=\frac{3}{1}\right),\ 4:5\left(=\frac{4}{5}\right)\text{ etc.}$$

3.1.2 Converting fractions to decimals

Examples:

(a) $\frac{3}{8}$ (b) $\frac{4}{9}$ (c) $\frac{8}{11}$

SOLUTION

(a) $\frac{3}{8}$

$$8\overline{)3.000}\quad 0.375$$

$\therefore\ \frac{3}{8}=0.375$

(this is a **terminating** decimal)

(b) $\frac{4}{9}$

$$9\overline{)4.000\ldots}\quad 0.444\ldots$$

$\therefore\ \frac{4}{9}=0.\dot{4}$ (this is a **repeating** or **recurring** decimal)

(c) $\frac{8}{11}$

$$11\overline{)8.000\ldots}\quad 0.7272\ldots$$

$\therefore\ \frac{8}{11}=0.\dot{7}\dot{2}$

3.1.3 Converting simple recurring decimals to fractions

Look at this pattern: $\frac{1}{9}=0.\dot{1}$, $\frac{2}{9}=0.\dot{2}$, $\frac{3}{9}=0.\dot{3}$, and so on.

Examples: Express as rationals, that is in form $\frac{a}{b}$:

(a) $0.\dot{7}$ (b) $0.\dot{9}$ (c) $3.\dot{6}$

SOLUTION

(a) $0.\dot{7}=\frac{7}{9}$

(b) $0.\dot{9}=\frac{9}{9}=1$ (we take $0.\dot{9}=0.9999\ldots$ as equalling 1)

(c) $3.\dot{6}=3\tfrac{6}{9}$

$=3\tfrac{2}{3}$.

3.1.4 Real numbers

A real number can be represented on a number line and combines rational numbers with **irrational** numbers (such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, etc.).

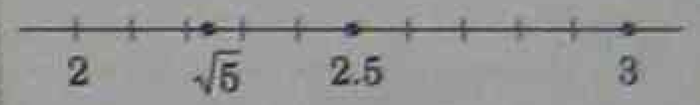
Examples: Using your calculator for assistance,position the following real numbers on a number line

(a) 3, $\sqrt{5}$, 2.5

(b) 118%, $\sqrt{2}$, 7 : 4

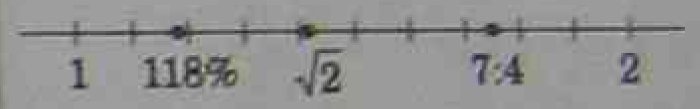
SOLUTION

(a) In ascending order: $\sqrt{5}\approx 2.23$, 2.5, 3



(b) In ascending order:

118% = 1.18, $\sqrt{2}\approx 1.41$, 7 : 4 = 1.75



3.2 Surds

Surds are numerical expressions which involve irrational numbers.

3.2.1 Approximation of surds

The calculator can be used to approximate surds.

Example

Arrange the following numbers in order, from smallest to largest:

3, $\sqrt{5}$, $\sqrt{7}$, 2, 4

SOLUTION:

$3=\sqrt{9}$, $\sqrt{5}$, $\sqrt{7}$, $2=\sqrt{4}$, $4=\sqrt{16}$

In order : $\sqrt{4}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{9}$, $\sqrt{16}$

That is, 2, $\sqrt{5}$, $\sqrt{7}$, 3, 4.

3.2.2 Rules for surds

- $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$
- $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$
- $\left(\sqrt{a}\right)^2=a$

Examples

Simplify the following:

(a) $\sqrt{48}$ (b) $3\sqrt{12}$

(c) $\sqrt{\frac{49}{64}}$ (d) $\left(\sqrt{16}\right)^2$

Note: We look for two factors, the first of which is a perfect square.

SOLUTIONS

(a) $\sqrt{48}=\sqrt{16}\times\sqrt{3}$

$=4\times\sqrt{3}$

$=4\sqrt{3}$

(b) $2\sqrt{12} = 3\times\sqrt{4}\times\sqrt{3}$

$= 3\times 2\times\sqrt{3}$

$= 6\sqrt{3}$

(c) $\sqrt{\frac{49}{64}} = \frac{7}{8}$

(d) $\left(\sqrt{16}\right)^2 = 4^2 = 16$

3.2.3 Addition and subtraction of surds

We can add or subtract only like terms.

In algebra, like terms are $3a$, $7a$, $2a$, etc. — like surds are $\sqrt{3},\ 3\sqrt{3},\ -7\sqrt{3}$, etc.

Examples: Simplify:

(a) $4\sqrt{3} + 2\sqrt{3} - \sqrt{3}$

(b) $2\sqrt{5} + \sqrt{20}$

(c) $5\sqrt{7} - \sqrt{63} + 2\sqrt{28}$

(d) $\sqrt{a^3} + 3a\sqrt{a}$

SOLUTIONS

(a) $4\sqrt{3} + 2\sqrt{3} - \sqrt{3} = 5\sqrt{3}$

(b) $2\sqrt{5} + \sqrt{20} = 2\sqrt{5} + \sqrt{4}\times\sqrt{5}$

$= 2\sqrt{5} + 2\sqrt{5}$

$= 4\sqrt{5}$

We may need to simplify surds before adding or subtracting.

(c) $5\sqrt{7} - \sqrt{63} + 2\sqrt{28}$

$= 5\sqrt{7} - \sqrt{9}\times\sqrt{7} + 2\times\sqrt{4}\times\sqrt{7}$

$= 5\sqrt{7} - 3\sqrt{7} + 4\sqrt{7}$

$= 6\sqrt{7}$

(d) $\sqrt{a^3} + 3a\sqrt{a}$

$= \sqrt{a^2}\times\sqrt{a} + 3a\sqrt{a}$

$= a\sqrt{a} + 3a\sqrt{a}$

$= 4a\sqrt{a}$

3.2.4 Multiplication and division of surds

We reverse our surd rules: $\sqrt{a}\times\sqrt{b} = \sqrt{ab}$ $\qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Examples

Simplify:

(a) $\sqrt{5}\times\sqrt{5}$ (b) $\sqrt{7}\times\sqrt{3}$

(c) $2\sqrt{3}\times\sqrt{3}$ (d) $3\sqrt{5}\times 2\sqrt{7}$

(e) $\left(3\sqrt{2}\right)^2$ (f) $\sqrt{4a+4}$

(g) $\sqrt{54}\div\sqrt{18}$ (h) $\frac{3\sqrt{2}\times\sqrt{6}}{\sqrt{3}}$

Expand and simplify:

(i) $\sqrt{3}\left(2-\sqrt{3}\right)$ (j) $4\sqrt{2}\left(\sqrt{2}-1\right)$

SOLUTIONS

(a) $\sqrt{5}\times\sqrt{5} = \sqrt{25}$

$= 5$

(b) $\sqrt{7}\times\sqrt{3} = \sqrt{21}$

(c) $2\sqrt{3}\times\sqrt{3} = 2\times 3 = 6$

(d) $3\sqrt{5}\times 2\sqrt{7} = 6\sqrt{35}$

(e) $\left(3\sqrt{2}\right)^2 = 3\sqrt{2}\times 3\sqrt{2}$

$= 9\sqrt{4}$

$= 9\times 2$

$= 18$

(f) $\sqrt{4a+4} = \sqrt{4(a+1)}$

$= \sqrt{4}\times\sqrt{a+1}$

$= 2\sqrt{a+1}$

(g) $\sqrt{54}\div\sqrt{18} = \sqrt{\frac{54}{18}}$

$= \sqrt{3}$

(h) $\frac{3\sqrt{2}\times\sqrt{6}}{\sqrt{3}} = \frac{3\sqrt{12}}{\sqrt{3}}$

$= \frac{3\times\sqrt{4}\times\cancel{\sqrt{3}}^{1}}{\cancel{\sqrt{3}}_{1}}$

$= 3\times 2$

$= 6$

(i) $\sqrt{3}\left(2-\sqrt{3}\right) = \sqrt{3}\times 2 - \sqrt{3}\times\sqrt{3}$

$= 2\sqrt{3} - \sqrt{9}$

$= 2\sqrt{3} - 3$

(j) $4\sqrt{2}\left(\sqrt{2}-1\right) = 4\sqrt{2}\times\sqrt{2} - 4\sqrt{2}\times 1$

$= 4\times\sqrt{4} - 4\sqrt{2}$

$= 4\times 2 - 4\sqrt{2}$

$= 8 - 4\sqrt{2}$

3.2.5 Binomial products

We can use the following three rules:

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a-b)(a+b) = a^2 - b^2$

Examples

Expand and simplify:

(a) $\left(\sqrt{2}+1\right)^2$ (b) $\left(\sqrt{3}-\sqrt{5}\right)^2$

(c) $\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)$

(d) $\left(2\sqrt{3}-1\right)\left(\sqrt{5}-\sqrt{2}\right)$

SOLUTIONS

(a) $\left(\sqrt{2}+1\right)^2 = \left(\sqrt{2}\right)^2 + 2\left(\sqrt{2}\right)(1) + 1^2$

$= 2 + 2\sqrt{2} + 1$

$= 3 + 2\sqrt{2}$

(b) $\left(\sqrt{3}-\sqrt{5}\right)^2$

$= \left(\sqrt{3}\right)^2 - 2\left(\sqrt{3}\right)\left(\sqrt{5}\right) + \left(\sqrt{5}\right)^2$

$= 3 - 2\sqrt{15} + 5$

$= 8 - 2\sqrt{15}$

(c) $\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right) = \left(\sqrt{2}\right)^2 - 1^2$

$= 2 - 1$

$= 1$

(d) $\left(2\sqrt{3}-1\right)\left(\sqrt{5}-\sqrt{2}\right)$

$= 2\sqrt{3}\left(\sqrt{5}-\sqrt{2}\right) - 1\left(\sqrt{5}-\sqrt{2}\right)$

$= 2\sqrt{15} - 2\sqrt{6} - \sqrt{5} + \sqrt{2}$

3.2.6 Rationalising the denominator

If the denominator of a fraction is irrational, we can multiply 'top and bottom' by the same surd to *rationalise* the denominator.

Examples: Rationalise the denominator:

(a) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (b) $\frac{\sqrt{2}-1}{5\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}-1}$ (d) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

SOLUTIONS

(a) $\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{\sqrt{3}+1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}\left(\sqrt{3}+1\right)}{\sqrt{4}}$

$= \frac{\sqrt{6}+\sqrt{2}}{2}$

((b) $\frac{\sqrt{2}-1}{5\sqrt{3}} = \frac{\sqrt{2}-1}{5\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}\left(\sqrt{2}-1\right)}{5\sqrt{9}}$

$= \frac{\sqrt{6}-\sqrt{3}}{15}$

Note: We don't need to multiply by $5\sqrt{3}$; $\sqrt{3}$ will do.

(c)
$$\frac{2}{\sqrt{3}-1}=\frac{2}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$=\frac{2\sqrt{3}+2}{\sqrt{9}-1}$$
$$=\frac{2\sqrt{3}+2}{3-1}$$
$$=\frac{2\sqrt{3}+2}{2}$$
$$=\frac{{}^{1}\cancel{2}(\sqrt{3}+1)}{\cancel{2}_{1}}$$
$$=\sqrt{3}+1$$

Note: We multiply by the *conjugate* of the denominator — the same expression but with the opposite sign between the terms.

(d)
$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$
$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$$
$$=\frac{\left(\sqrt{3}\right)^2-2\left(\sqrt{3}\right)\left(\sqrt{2}\right)+\left(\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$
$$=\frac{3-2\sqrt{6}+2}{3-2}$$
$$=\frac{5-2\sqrt{6}}{1}$$
$$=5-2\sqrt{6}$$

3.3 Indices

For x^n, x is the base, and n is the index.

Example:

Use your calculator to evaluate 3^7.

SOLUTION: We use our x^y button.

$3^7 = 2187$, that is, $3\ \boxed{x^y}\ 7\ \boxed{=}$

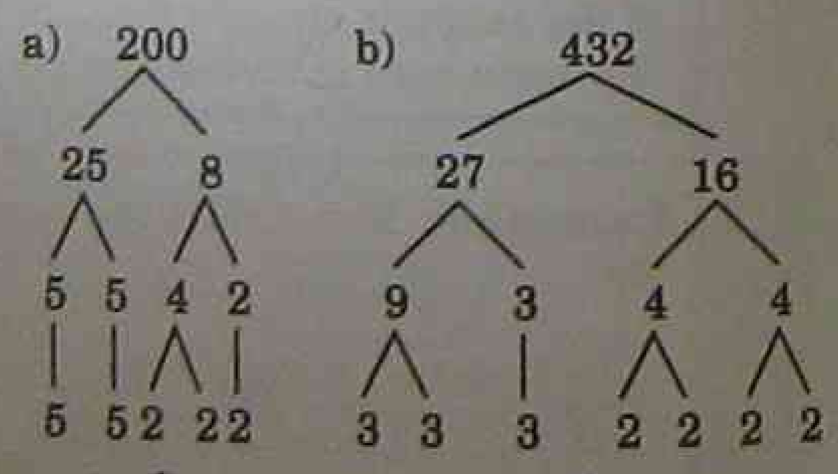
3.3.1 Factorising integers

Examples: Express as a product of their prime factors in index form:

(a) 200

(b) 432

SOLUTIONS



$\therefore = 5^2 \times 2^3$

$= 3^3 \times 2^4$

3.3.2 Index rules

- $x^a \times x^b = x^{a+b}$
- $x^a \div x^b = x^{a-b}$
- $\left(x^a\right)^b = x^{ab}$
- $x^0 = 1$ (*Note*: $x \neq 0$)

Examples: Simplify:

(a) $p^4 \times p^2$ (b) $p^6 \div p$

(c) $4^5 \div 4^3$ (d) $12a^4b \div 3a^3$

(e) $\left(x^2\right)^4$ (f) $\left(a^{\frac{1}{2}}\right)^3$

(g) $\left(4y^2\right)^3$ (h) $4x^0$

(i) $(3a)^0 + 3a^0$

SOLUTIONS

(a)
$$p^4 \times p^2 = p^{(4+2)}$$
$$= p^6$$

(b)
$$p^6 \div p = p^6 \div p^1$$
$$= p^{6-1}$$
$$= p^5$$

(c)
$$4^5 \div 4^3 = 4^{5-3}$$
$$= 4^2$$
$$= 16$$

(d)
$$12a^4b \div 3a^3 = 4ab$$

(e)
$$\left(x^2\right)^4 = x^{2\times 4}$$
$$= x^8$$

(f)
$$\left(a^{\frac{1}{2}}\right)^3 = a^{\frac{3}{2}}$$

(g)
$$\left(4y^2\right)^3 = 4^3 \times \left(y^2\right)^3$$
$$= 64y^6$$

(h)
$$4x^0 = 4 \times x^0$$
$$= 4 \times 1$$
$$= 4$$

(i)
$$(3a)^0 + 3a^0 = 1 + 3 \times 1$$
$$= 1 + 3$$
$$= 4$$

3.3.3 Negative powers

- $x^{-a} = \dfrac{1}{x^a}$

Examples: Simplify:

(a) 4^{-2} (b) $\left(\frac{1}{2}\right)^{-3}$

(c) $3x^{-2}$ (d) $\left(5x^3\right)^{-2}$

(e) $\dfrac{x^2}{x^5}$

SOLUTIONS

(a)
$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

(b)
$$\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}}$$
$$= 1 \div \frac{1}{8}$$
$$= 1 \times \frac{8}{1}$$
$$= 8$$

Note: The negative power is the reciprocal.

(c)
$$3x^{-2} = 3 \times x^{-2}$$
$$= 3 \times \frac{1}{x^2}$$
$$= \frac{3}{x^2}$$

(d)
$$\left(5x^3\right)^{-2} = \frac{1}{\left(5x^3\right)^2}$$
$$= \frac{1}{25x^6}$$

(e)
$$\frac{x^2}{x^5} = x^2 \div x^5$$
$$= x^{-3}$$

3.3.4 Fractional powers

- $x^{\frac{1}{a}} = \sqrt[a]{x}$

That is, $x^{\frac{1}{2}} = \sqrt{x}$, $x^{\frac{1}{3}} = \sqrt[3]{x}$,

$x^{\frac{1}{n}} = n^{\text{th}}$ power of x

- $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

Examples: Simplify:

(a) $4^{\frac{1}{2}}$ (b) $8^{\frac{1}{3}}$

(c) $4a^{\frac{1}{3}}$ (d) $(16y)^{\frac{1}{2}}$

(e) $16^{\frac{3}{2}}$ (f) $16^{-\frac{3}{4}}$

(g) $x\sqrt{x}$ (h) $\dfrac{x}{\sqrt{x}}$

SOLUTIONS

(a) $4^{\frac{1}{2}} = \sqrt{4} = 2$

(b) $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

(c) $4a^{\frac{1}{3}} = 4\sqrt[3]{a}$

(d) $(16y)^{\frac{1}{2}} = 16^{\frac{1}{2}} \times y^{\frac{1}{2}} = \sqrt{16} \times \sqrt{y} = 4\sqrt{y}$

(e) $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$

To keep numbers small it is best to take the root first and then the power.

(f)

$$16^{-\frac{3}{4}} = \left(\left((16)^{-1}\right)^{\frac{1}{4}}\right)^3 = \left(\left(\frac{1}{16}\right)^{\frac{1}{4}}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

It's getting complicated — let's try the calculator!

(g) $x\sqrt{x} = x \times x^{\frac{1}{2}} = x^1 \times x^{\frac{1}{2}} = x^{1\frac{1}{2}} = x^{\frac{3}{2}} = \sqrt{x^3}$

(h) $\dfrac{x}{\sqrt{x}} = x \div x^{\frac{1}{2}} = x^1 \div x^{\frac{1}{2}} = x^{\frac{1}{2}} = \sqrt{x}$

3.3.5 Using calculators to evaluate difficult indices

Calculators can be used to evaluate difficult indices.

Examples: Evaluate:

(a) $256^{\frac{1}{8}}$

(b) $\left(\dfrac{1}{16}\right)^{-\frac{3}{2}}$

SOLUTIONS

(a) $256^{\frac{1}{8}} = 2$

(b) $\left(\dfrac{1}{16}\right)^{-\frac{3}{2}} = 64$

Either use your $x^{\frac{1}{y}}$ key, or use your x^y key and enter the power as a fraction.

3.3.6 Solving algebraic problems involving indices

Example: Solve for x:

(i) $2^x = 64$ (ii) $3^{x+2} = 81$

(iii) $2^{2x-1} = 3125$

① Express both sides as a power of the same base number.

② Then the indices must be equal.

SOLUTION

(i) $2^x = 2^6$ Both as powers of 2.

$\therefore x = 6$ Indices are equal

(ii) $3^{x+2} = 3^4$ Both as powers of 3.

$\therefore x + 2 = 4$ Indices are equal

$x = 2$

(iii) $25^{2x-1} = 3125$

$\left(5^2\right)^{2x-1} = 5^5$

$5^{4x-2} = 5^5$ Powers of 5

$\therefore 4x - 2 = 5$

$4x = 7$

$x = \frac{7}{4}$

3.4 Standard, or scientific, notation

3.4.1 Definition

Very large, or very small, numbers are written as a product of a number between 1 and 10, and a power of ten.

Examples: Express in scientific notation:

(a) 181.4 (b) 2 000 000

(c) 0.0215 (d) 0.0001

Express in ordinary, or decimal, notation:

(e) 4.271×10^2 (f) 3.08×10^4

(g) 1.56×10^{-2} (h) 5×10^{-3}

SOLUTIONS

(a) $181.4 = 1.\overset{\frown}{81}.4$, therefore 2 places

$= 1.814 \times 100$

$= 1.814 \times 10^2$

Note: The number of decimal places moved by the decimal point equals the power of ten.

(b) $2\,000\,000 = 2.\overset{\frown}{000\,000}.$, therefore 6 places

$= 2 \times 1\,000\,000$

$= 2 \times 10^6$

(c) $0.0215 = 0.\overset{\frown}{02}.15$, therefore 2 places.

$= 2.15 \div 100$

$= 2.15 \times \dfrac{1}{100}$

$= 2.15 \times 10^{-2}$

(d) $0.0001 = 0.\overset{\frown}{0001}.$, therefore 4 places.

$= 1 \div 10\,000$

$= 1 \times \dfrac{1}{10\,000}$

$= 1 \times 10^{-4}$

(e) $4.271 \times 10^2 = 4.271 \times 100$, therefore 2 places.

$= 4.\overset{\frown}{27}.1$

$= 427.1$

(f) $3.08 \times 10^4 = 3.08 \times 10\,000$, therefore 4 places.

$= 3.\overset{\frown}{0800}$

$= 30\,800$

(g) $1.56 \times 10^{-2} = 1.56 \times \dfrac{1}{100}$

$= 1.56 \div 100$, therefore 2 places.

$= 0.\overset{\frown}{01}.56$

$= 0.0156$

(h) $5 \times 10^{-3} = 5 \times \dfrac{1}{1000}$

$= 5 \div 1000$, therefore 3 places.

$= 0.\overset{\frown}{005}.$

$= 0.005$

3.4.2 Standard notation and the calculator

A scientific calculator can leave very large, or very small, numbers in scientific (or standard) notation.

Examples: Calculate, leaving your answer in scientific notation:

(a) 13 364 × 176 000 000
(b) $(0.2)^4$
(c) 43 ÷ 14 767

SOLUTIONS

(a) 13 364 × 176 000 000
= 2.352 064 12 (calculator display)
= $2.352\ 064 \times 10^{12}$

(b) $(0.2)^4$ = 1.6 – 03 (calculator display)
= 1.6×10^{-3}

(c) 43 ÷ 14 767
= 2.911 898 151 – 03 (calc. display)
= $2.911\ 898\ 151 \times 10^{-3}$

Often we can leave the answer correct to significant figures.

Examples: Express in scientific notation, correct to two significant figures:

(a) 76 294 320 (b) 0.004 768 7

SOLUTIONS

(a) $7.629\ 432 \times 10^7 \approx 7.6 \times 10^7$
(b) $0.004\ 768\ 7 = 4.7687 \times 10^{-3}$
$\approx 4.8 \times 10^{-3}$

Note: Standard notation can be used to correct numbers to a given significance. It allows the zero problem to be avoided.

For example:

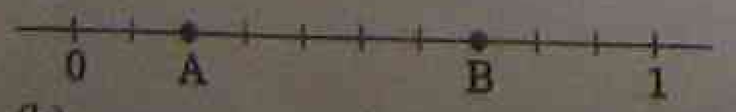
(a) Write 60.432 correct to three significant figures.
$60.432 = \underline{6.04}32 \times 10^1$
$= 6.04 \times 10^1$
= 60.4 (three significant figures)

(b) Write 0.004 27 correct to two significant figures.
$0.004\ 27 = 4.27 \times 10^{-3}$
$= 4.3 \times 10^{-3}$
=0.0043 (two significant figures)

3.5 Exercises

1. What fractions are represented by the points A and B on the following number lines?

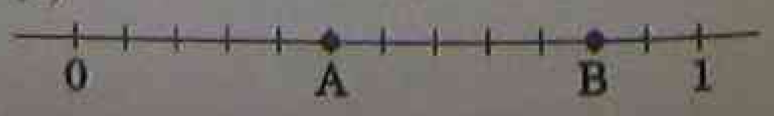
(a)



(b)

(c)

(d)



2. Express as decimals:

(a) $\frac{5}{8}$ (b) $\frac{3}{7}$ (c) $\frac{4}{11}$ (d) $\frac{7}{13}$

3. Rewrite in the form $\frac{a}{b}$:

(a) 17% (b) $1\frac{1}{2}$ (c) 0.2 (d) $\sqrt{81}$
(e) 4 : 5 (f) 12 : 7 (g) $0.\dot{5}$

4. Arrange in ascending order (that is, from smallest to largest):
$\sqrt{11}$, $\sqrt{7}$, $\sqrt{12}$, 3, 5

5. Simplify:

(a) $\sqrt{27}$ (b) $5\sqrt{8}$ (c) $\sqrt{\frac{25}{81}}$ (d) $\left(\sqrt{3}\right)^2$

6. Simplify:

(a) $12\sqrt{5}+\sqrt{5}-7\sqrt{5}$ (b) $\sqrt{28}-\sqrt{63}$
(c) $2\sqrt{2}+\sqrt{128}$
(d) $x\sqrt{x}+\sqrt{x^3}-\sqrt{4x}$

7. Simplify:

(a) $\sqrt{3}\times\sqrt{5}\times\sqrt{3}$ (b) $6\sqrt{3}\times -2\sqrt{3}$
(c) $\left(4\sqrt{3}\right)^2$ (d) $\sqrt{9y-18}$
(e) $\sqrt{27}\div\sqrt{3}$ (f) $\frac{4\sqrt{3}\times\sqrt{6}}{3\sqrt{2}}$

8. Expand and simplify:

(a) $\sqrt{5}\left(\sqrt{5}-2\right)$ (b) $2\sqrt{3}\left(\sqrt{3}-2\right)$

9. Expand and simplify:

(a) $\left(2\sqrt{2}-1\right)^2$ (b) $\left(3\sqrt{2}+\sqrt{3}\right)^2$
(c) $\left(\sqrt{3}-2\right)\left(\sqrt{3}+2\right)$ (d) $\left(\sqrt{5}-1\right)\left(2\sqrt{3}+1\right)$

10. Rationalise the denominator:

(a) $\frac{6}{\sqrt{3}}$ (b) $\frac{4\sqrt{2}}{3\sqrt{5}}$
(c) $\frac{12}{\sqrt{7}+\sqrt{3}}$ (d) $\frac{5+\sqrt{2}}{5-\sqrt{2}}$

11. Calculate:

(a) 4.2^4 (b) 18^3

12. Express as a product of their prime factors in index form:

(a) 216 (b) 1152

13. Simplify:

(a) $k^4\times k^3\div k^2$ (b) $3^5\times 3^2\div 3^6$
(c) $-4x^2\times 3xy$ (d) $\left(y^3\right)^2$
(e) $\left(2y^4\right)^3$ (f) $3a^0$
(g) $\left(4^3\times 4^2\right)^0$

14. Simplify:

(a) 5^{-2} (b) $\left(\frac{1}{3}\right)^{-2}$ (c) $\left(\frac{2}{5}\right)^{-1}$
(d) $4a^{-3}$ (e) $(5y)^{-3}$ (f) $x^{-4}\times x^{-2}$

15. Simplify:

(a) $64^{\frac{1}{2}}$ (b) $27^{\frac{1}{3}}$ (c) $5x^{\frac{1}{2}}$
(d) $\left(a^6\right)^{\frac{1}{2}}$ (e) $\left(9k^4\right)^{\frac{1}{2}}$ (f) $25^{\frac{3}{2}}$
(g) $4^{-\frac{5}{2}}$

16. Evaluate, using your calculator:

(a) $243^{\frac{2}{5}}$ (b) $(0.008)^{-\frac{1}{3}}$

17. Express in scientific notation:

(a) 684.31 (b) 4798
(c) 0.4307 (d) 0.000 08

18. Express in decimal form:

(a) 3.68×10^3 (b) 4.076×10^{-2}
(c) 4×10^{-4}

19. Calculate, leaving your answer in decimal form:

(a) $(0.0005)^2$ (b) 4 ÷ 17 680

20. Solve for a:

(a) $3^a = 2187$ (b) $a^2 = 256$
(c) $5^{4a} = 625$ (d) $10^{2a-1} = 10\ 000$
(e) $2^a = \frac{1}{16}$ (f) $5^{a+1} = \frac{1}{25}$
(g) $\left(\sqrt{2}\right)^a = 32$ (h) $\left(\frac{1}{\sqrt{5}}\right)^a = 25^{2a-5}$

Chapter 4

EQUATIONS AND INEQUALITIES

4.1 Solution of simple equations and inequalities

4.1.1 Solutions of equations

The main rules to remember are:

- Pronumeral on one side and number on the other side.
- To *change sides* means we have to *change signs*.

Examples: Solve:

(a)
$$3x - 1 = x + 2$$
this is +3x
this is −1
this is +x
this is +2

$$3x - x = 2 + 1$$
$$2x = 3$$
$$\frac{^1\cancel{2}x}{\cancel{2}_1} = \frac{3}{2} \leftarrow$$
$$x = 1\tfrac{1}{2}$$

We divide by the coefficient (no. in front) of x.

(b)
$$5y - 7 = 3(2y + 4) \leftarrow$$
$$5y - 7 = 6y + 12$$
$$5y - 6y = 12 + 7$$
$$-y = 19$$
$$\frac{-y}{-1} = \frac{19}{-1} \leftarrow$$
$$y = -19$$

First we expand the bracket.

The coefficient of y is −1.

(c)
$$\frac{3a - 4}{2} = 8$$
$$^1\cancel{2}\left[\frac{3a - 4}{\cancel{2}_1}\right] = 8 \times 2$$
$$3a - 4 = 16$$
$$3a = 16 + 4$$
$$3a = 20$$
$$\frac{^1\cancel{3}a}{\cancel{3}_1} = \frac{20}{3}$$
$$a = 6\tfrac{2}{3}$$

We have to get rid of denominators, so we multiply both sides by 2.

We can leave this as a mixed numeral or $6.\dot{6}$

(d)
$$\frac{5a - 1}{2} = \frac{4a + 8}{3}$$
$$^3\cancel{6}\left[\frac{5a - 1}{\cancel{2}_1}\right] = {}^2\cancel{6}\left[\frac{4a + 8}{\cancel{3}_1}\right]$$
$$3(5a - 1) = 2(4a + 8)$$
$$15a - 3 = 8a + 16$$
$$15a - 8a = 16 + 3$$
$$7a = 19$$
$$\frac{^1\cancel{7}a}{_1\cancel{7}} = \frac{19}{7}$$
$$a = 2\tfrac{5}{7}$$

Best to multiply by lowest common denominator

Note that we could have jumped to this step by cross - multiplying : ☒

(e)
$$\frac{5}{2a} - \frac{3}{4a} = \frac{1}{2}$$
$$\overset{2}{\cancel{4a}}\left[\frac{5}{\underset{1}{\cancel{2a}}}\right] - \overset{1}{\cancel{4a}}\left[\frac{3}{\underset{1}{\cancel{4a}}}\right] = \overset{2}{\cancel{4}}a\left[\frac{1}{\underset{1}{\cancel{2}}}\right]$$
$$2(5) - 1(3) = 2a(1)$$
$$10 - 3 = 2a$$
$$7 = 2a$$
$$2a = 7$$
$$\frac{^1\cancel{2}a}{\cancel{2}_1} = \frac{7}{2}$$
$$a = 3\tfrac{1}{2}$$

The lowest common denominator is $4a$.

Simply swap sides around.

(f)
$$\frac{5}{y} - \frac{2}{y - 1} = 0$$
$$\frac{5}{y} = \frac{2}{y - 1}$$
$$5(y - 1) = 2y$$

By cross - multiplying. ☒

$$5y - 5 = 2y$$
$$5y - 2y = 5$$
$$3y = 5$$
$$\frac{^1\cancel{3}y}{_1\cancel{3}} = \frac{5}{3}$$
$$y = 1\tfrac{2}{3}$$

4.1.2 Solution of inequalities

Algebraic expressions with an inequality symbol have a range of solutions.
An important rule is:

- When multiplying or dividing an inequality by a negative we must reverse the inequality symbol.

Examples: Solve:

(a)
$$3x - 4 > x + 7$$
$$3x - x > 7 + 4$$
$$2x > 11$$
$$\frac{2x}{2} > \frac{11}{2}$$
$$x > 5\tfrac{1}{2}$$

(b)
$$x + 7 \geq 3x - 4$$
$$x - 3x \geq -4 - 7$$
$$-2x \geq -11$$
$$\frac{-2x}{-2} \leq \frac{-11}{-2}$$
$$x \leq 5\tfrac{1}{2}$$

When dividing by a negative we reverse the inequality.

(c)
$$7 - 5x > 12$$
$$-5x > 12 - 7$$
$$-5x > 5$$
$$\frac{-5x}{-5} < \frac{5}{-5}$$
$$x < -1$$

(d)
$$4(3 - 2x) \geq \frac{2x - 5}{5}$$
$$5[4(3 - 2x)] \geq {}^1\cancel{5}\left[\frac{2x - 5}{\cancel{5}_1}\right]$$
$$20(3 - 2x) \geq 2x - 5$$
$$60 - 40x \geq 2x - 5$$
$$-40x - 2x \geq -5 - 60$$
$$-42x \geq -65$$
$$\frac{^1\cancel{-42}x}{\cancel{-42}_1} \leq \frac{-65}{-42}$$
$$x \leq 1\tfrac{23}{42}$$

4.1.3 Graphing on the number line

Examples

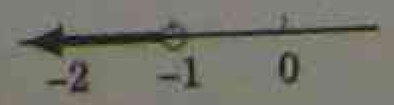
Graph on separate number lines

(a) $x \geq 2$

We use a closed circle, to denote the inclusion of 2.

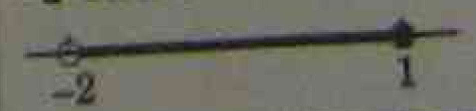
Hint: The arrow-head looks like the inequality symbol.

(b) $x < -1$



We use an open circle to denote the *exclusion* of −1.

(c) $-2 < x \leq 1$



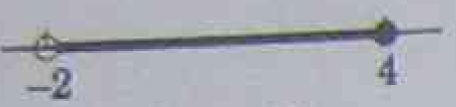
We want all values of x greater than −2 but less than or equal to 1.

Write down the inequality that corresponds to the graph

(d)

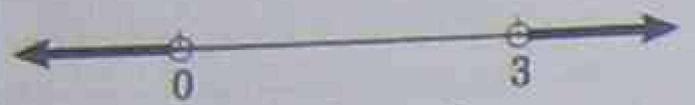
$x < 2$

(e)



$-2 < x \le 4$

(f)



$x < 0, x > 3$

Solve, and graph your solution on a number line:

(g) $2x - 4 > x - 1$

$2x - x > -1 + 4$

$x > 3$

(h)

$$\begin{aligned} 4 &\le 3(4-5x) \\ 4 &\le 12 - 15x \\ 15x &\le 12 - 4 \\ 15x &\le 11 \\ \frac{\cancel{15}x}{\cancel{15}} &\le \frac{11}{15} \\ x &\le \frac{11}{15} \end{aligned}$$

Note: $\frac{10}{15} = \frac{2}{3}$ and $\frac{12}{15} = \frac{4}{5}$

4.1.4 Restrictions on inequality solutions

Sometimes restrictions are imposed which influence the solution.

Examples: Solve, or find the solution set of:

(a)

$$\begin{aligned} 3x - 4 &< x + 7, \quad \text{if } x > 3. \\ 3x - x &< 7 + 4 \\ 2x &< 11 \\ \frac{2x}{2} &< \frac{11}{2} \\ x &< \frac{11}{2} \\ x &< 5\tfrac{1}{2} \end{aligned}$$

But, $x > 3$,

therefore $3 < x < 5\frac{1}{2}$.

(b)

$$\begin{aligned} 7(4-2x) &< x, \quad \text{if } x < 2. \\ 28 - 14x &< x \\ -14x - x &< -28 \\ -15x &< -28 \\ \frac{-15x}{-15} &> \frac{-28}{-15} \\ x &> 1\tfrac{13}{15}. \end{aligned}$$

But $x < 2$,

therefore $1\frac{13}{15} < x < 2$.

(c)

$$\begin{aligned} 5x + 4 &> 6x, \quad x \in N \\ 5x - 6x &> -4 \\ -x &> -4 \\ \frac{-x}{-1} &< \frac{-4}{-1} \\ x &< 4 \end{aligned}$$

But $x \in N$,

therefore $x = 1, 2, 3$.

$x \in$ N means x is an element of, or belongs to, the set of positive integers.

4.2 Formulae

Formulae are widely used and usually represent physical quantities. Unlike equations, formulae involve more than one pronumeral. The subject of a formula is the pronumeral on the left-hand side of the equals sign (the pronumeral by itself).

4.2.1 Substitution into a formula

Examples

(a) If $S = n(n+1)$, find S when $n = 16$.

SOLUTION

$$\begin{aligned} S &= n(n+1) \\ &= 16(16+1) \\ &= 16(17) \\ &= 272 \end{aligned}$$

(b) If $A = P\left(1 + \frac{r}{100}\right)^n$, find A when:

$P = 10\,000, r = 20, n = 2$.

SOLUTION

$$\begin{aligned} A &= P\left(1 + \frac{r}{100}\right)^n \\ &= 10\,000\left(1 + \frac{20}{100}\right)^2 \\ &= 10\,000(1.2)^2 \\ &= 14\,400 \end{aligned}$$

(c) If $T = \frac{n}{2}[2a + (n-1)d]$, find T if: $a = 6$, $d = 3, n = 10$.

SOLUTION

$$\begin{aligned} T &= \frac{n}{2}[2a + (n-1)d] \\ &= \frac{10}{2}[2(6) + (10-1) \times 3] \\ &= 5[12 + 9 \times 3] \\ &= 5[12 + 27] \\ &= 5[39] \\ &= 195 \end{aligned}$$

(d) If $M = \frac{1}{M_1} + \frac{1}{M_2}$, find M when $M_1 = 1.6$ and $M_2 = 0.5$.

SOLUTION

$$\begin{aligned} M &= \frac{1}{M_1} + \frac{1}{M_2} \\ &= \frac{1}{1.6} + \frac{1}{0.5} \end{aligned}$$

CALCULATOR

$1.6\ \boxed{1/x}\ \boxed{+}\ 0.5\ \boxed{1/x}\ \boxed{=}$

Note: Your calculator may require $\boxed{\text{INV}}\ \overset{1/x}{\boxed{}}$

$$\begin{aligned} &= 0.625 + 2 \\ &= 2.625 \end{aligned}$$

4.2.2 Constructing formulae

Examples

(a) Write down a formula for the speed, in kilometres/hour if a vehicle has travelled D kilometres in T hours.

SOLUTION

$$S = \frac{D}{T}$$

(b) The table shows the relation between speed S km/h and the stopping distance D metres, of a passenger van.

S	40	60	80	100
D	16	36	64	100

Continued

(i) Find a formula linking D with S, that is, expressing D in terms of S.

SOLUTION

We can see that D is found by dividing S by 10 and then squaring,

that is, $D = \left(\frac{S}{10}\right)^2$,

therefore $D = \frac{S^2}{100}$

(ii) Find D when $S = 70$.

SOLUTION

$$\begin{aligned} D &= \frac{70^2}{100} \\ &= \frac{490\cancel{0}}{10\cancel{0}} \\ &= 49 \end{aligned}$$

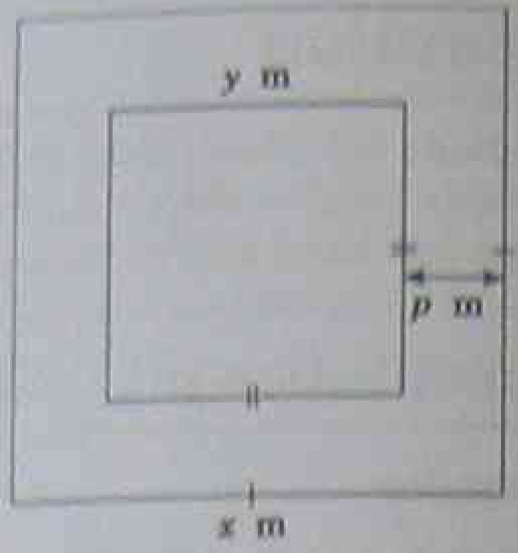
(iii) What would be the stopping distance of a passenger van travelling at 120 km/h?

SOLUTION

$$\begin{aligned} D &= \frac{S^2}{100} \\ &= \frac{(120)^2}{100} \\ &= \frac{14\,40\cancel{0}}{10\cancel{0}} \\ &= 144 \end{aligned}$$

The distance is 144 metres.

(e) Consider a square garden of side x metres. A path p metres wide surrounds a square area of lawn with side y metres, as shown in the diagram.



Write down a formula for:

(i) y in terms of x and p.

SOLUTION

$$\begin{aligned} 2p &= x - y \\ y &= x - 2p \end{aligned}$$

Path on both sides, ∴ $2p$.

(ii) the area A of the path in terms of x and y.

SOLUTION

$$A = x^2 - y^2$$

Area of larger square minus area of smaller square.

(iii) the area A of the path in terms of x and p.

SOLUTION

$$\begin{aligned} A &= x^2 - y^2 \\ \text{But}\quad y &= x - 2p \qquad \text{[from (i)]} \\ \text{therefore}\quad A &= x^2 - (x - 2p)^2 \\ &= x^2 - \left(x^2 - 4xp + 4p^2\right) \\ &= \cancel{x}^2 - \cancel{x}^2 + 4xp - 4p^2 \\ &= 4xp - 4p^2 \\ &= 4p(x - p) \end{aligned}$$

4.2.3 Changing the subject of a formula

A formula such as $v^2 = u^2 - 2as$ has v^2 as the subject and we can easily find v, if we know u, a and s, by simple substitution (and taking the square root). However to find u, a or s it is best to change the subject to the unknown pronumeral.

Examples: Make y the subject:

(a)
$$\begin{aligned} 2x + y &= 7 \\ y &= 7 - 2x \end{aligned}$$

(b)
$$\begin{aligned} 3x - y &= 5 \\ 3x - 5 &= y \\ y &= 3x - 5 \end{aligned}$$

Taking y to the RHS changes its sign. Then we just swap sides.

Note: We could have left the y on the LHS as $-y$ and then divide through by -1.

(c)
$$\begin{aligned} 4x - 3y &= 12 \\ 4x - 12 &= 3y \\ 3y &= 4x - 12 \\ \frac{\cancel{3}y}{\cancel{3}} &= \frac{4x - 12}{3} \\ y &= \frac{4x - 12}{3} \end{aligned}$$

(d)
$$\begin{aligned} xy - 7 &= 3a + 2b \\ xy &= 3a + 2b + 7 \\ \frac{\cancel{x}y}{\cancel{x}} &= \frac{3a + 2b + 7}{x} \\ y &= \frac{3a + 2b + 7}{x} \end{aligned}$$

Divide both sides by x.

(e)
$$\begin{aligned} 3(4x - 2y) &= 13x + 1 \\ 12x - 6y &= 13x + 1 \\ 12x - 13x - 1 &= 6y \\ 6y &= -x - 1 \\ \frac{\cancel{6}y}{\cancel{6}} &= \frac{-x - 1}{6} \\ y &= \frac{-x - 1}{6} \end{aligned}$$

Expand the bracket first.

(f)
$$\begin{aligned} A &= \frac{3xy}{7} \\ 7[A] &= \left[\frac{3xy}{\cancel{7}_1}\right]\cancel{7}_1 \\ 7A &= 3xy \\ 3xy &= 7A \\ \frac{\cancel{3x}y}{\cancel{3x}} &= \frac{7A}{3x} \\ y &= \frac{7A}{3x} \end{aligned}$$

Multiply both sides by 7.

(g)
$$\begin{aligned} P &= \frac{3x - 4}{y} \\ y[P] &= \left[\frac{3x - 4}{\cancel{y}_1}\right]\cancel{y}^1 \\ Py &= 3x - 4 \\ \frac{\cancel{P}y}{\cancel{P}} &= \frac{3x - 4}{P} \\ y &= \frac{3x - 4}{P} \end{aligned}$$

(h)
$$\begin{aligned} x &= \sqrt{\frac{A}{y}} \\ x^2 &= \frac{A}{y} \\ x^2 y &= A \\ y &= \frac{A}{x^2} \end{aligned}$$

Squaring both sides.

Multiplying both sides by y.

Dividing both sides by x^2.

(i)
$$\begin{aligned} 3xy &= 7x - y \\ 3xy + y &= 7x \\ y(3x + 1) &= 7x \\ \frac{y\cancel{(3x+1)}}{\cancel{3x+1}} &= \frac{7x}{3x + 1} \\ y &= \frac{7x}{3x + 1} \end{aligned}$$

Take y as a common factor.

(j)
$$\begin{aligned} x^2 &= y^2 - 4x \\ y^2 &= x^2 + 4x \\ y &= \pm\sqrt{x^2 + 4x} \end{aligned}$$

Don't forget both cases.

(k)
$$\begin{aligned} x &= \frac{y}{y + 3} \\ x(y + 3) &= y \\ xy + 3x &= y \\ xy - y &= -3x \\ y(x - 1) &= -3x \\ \frac{y\cancel{(x-1)}}{\cancel{x-1}} &= \frac{-3x}{x - 1} \\ y &= \frac{-3x}{x - 1} \end{aligned}$$

Cross-multiplying

This could be written as

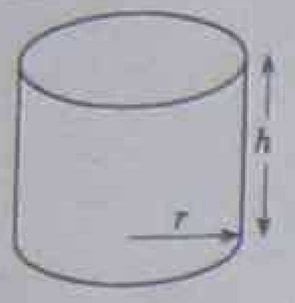
$$\frac{\cancel{-}3x}{\cancel{-}(1-x)} \text{ or } \frac{3x}{1-x}.$$

4.2.4 Solving problems

Examples

(a) Write down the formula for the curved surface area A of a cylinder:

$$A = 2\pi rh$$



(i) Find the area in terms of π if the radius is 12 cm and the height is 10 cm.

SOLUTION

$$\begin{aligned} A &= 2\pi rh \\ A &= 2\times\pi\times 12\times 10 \\ &= 240\pi \end{aligned}$$

The area is 240π cm².

(ii) Find the height h if the area A of the curved surface is 132π cm² and the radius is 6 cm.

SOLUTION: $A = 2\pi rh$

Making h the subject:

$$\begin{aligned} 2\pi rh &= A \\ \frac{\cancel{2\pi r}h}{\cancel{2\pi r}} &= \frac{A}{2\pi r} \\ h &= \frac{A}{2\pi r} \\ \text{therefore } h &= \frac{132\pi}{2\times\pi\times 6} \\ &= \frac{132\pi}{12\pi} \\ &= 11 \text{ (by cancelling)} \end{aligned}$$

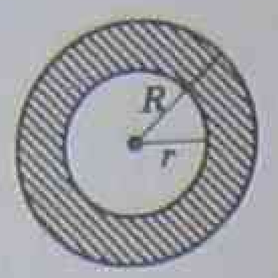
Or, we can substitute early:

$$\begin{aligned} A &= 2\pi rh \\ 132\pi &= 2\times\pi\times 6\times h \\ 132\pi &= 12\pi h \\ 12\pi h &= 132\pi \\ \frac{12\pi h}{12\pi} &= \frac{132\pi}{12\pi} \end{aligned}$$

therefore $h = 11$.

The radius is 11 cm.

(b) Write down the formula for the area A of the annulus (next column) if the radius of the larger circle is R while the radius of the smaller circle is r.



That is,

$$\begin{aligned} A &= \pi R^2 - \pi r^2 \\ &= \pi\left(R^2 - r^2\right). \end{aligned}$$

(i) Find the area of an annulus where $R = 6.2$ cm and $r = 4.7$ cm, correct to three significant figures.

SOLUTION

$$\begin{aligned} A &= \pi\left(R^2 - r^2\right) \\ &= \pi\left(6.2^2 - 4.7^2\right) \\ &= \pi(38.44 - 22.09) \\ &= \pi(16.35) \\ &= 51.365\,039\,89 \\ &= 51.4, \text{ to three significant figures.} \end{aligned}$$

The area is 51.4 cm² (three significant figures)

(ii) Find the radius of the smaller circle if the radius of the larger circle is 10 cm and the area of the annulus is 36π cm².

SOLUTION

$$A = \pi\left(R^2 - r^2\right)$$

Make r the subject:

$$\begin{aligned} A &= \pi R^2 - \pi r^2 \\ \pi r^2 &= \pi R^2 - A \\ r^2 &= \frac{\pi R^2 - A}{\pi} \end{aligned}$$

Substituting $R = 10$, $A = 36\pi$,

$$\begin{aligned} r^2 &= \frac{\pi(10)^2 - 36\pi}{\pi} \\ &= \frac{100\pi - 36\pi}{\pi} \\ &= \frac{64\cancel{\pi}}{\cancel{\pi}} \\ r^2 &= 64 \\ r &= 8 \quad \text{(Cannot have a radius of } -8 \text{ cm)} \end{aligned}$$

Alternatively, we can substitute early:

$$\begin{aligned} A &= \pi\left(R^2 - r^2\right) \\ 36\pi &= \pi\left(10^2 - r^2\right) \\ 36\pi &= \pi\left(100 - r^2\right). \end{aligned}$$

Divide by π:

$$\begin{aligned} 36 &= 100 - r^2 \\ r^2 &= 100 - 36 \\ r^2 &= 64 \\ r &= 8 \end{aligned}$$

The radius of the smaller circle is 8 cm.

4.3 Simultaneous equations

If an equation has two unknowns, for example $x + y = 4$, there is an infinite number of solutions. If there is a pair of equations with two unknowns we may be able to find a solution to satisfy both equations at the same time, that is, simultaneously.

There are three methods of solution: graphically, algebraically through *substitution*, and algebraically through *elimination*. The latter two are treated below:

4.3.1 The substitution method

Examples: Solve the simultaneous equations:

(a)

$2x + y = 5$ ... (1)
$5x - 3y = 7$ ... (2)

We number our equations to help.

SOLUTION

From (1), make y the subject:

$y = 5 - 2x$ ... (3)

It is best here to choose y as it is a monic term (coefficient of 1).

Now substitute into (2):

$$\begin{aligned} 5x - 3(5 - 2x) &= 7 \\ 5x - 15 + 6x &= 7 \\ 11x - 15 &= 7 \\ 11x &= 7 + 15 \\ 11x &= 22 \\ x &= 2 \end{aligned}$$

Now we substitute $x = 2$ in (1).

Equation (1) is 'simpler'.

$$\begin{aligned} 2(2) + y &= 5 \\ 4 + y &= 5 \\ y &= 5 - 4 \\ y &= 1 \end{aligned}$$

Therefore $x = 2, y = 1$
[*Or* (2,1) as an ordered pair.]

It is good to write the complete solution here.

Note: We could check this answer by substituting into both equations, (1) and (2).

(b)

$3x - 4y = 11$
$2x + 3y = -4$

SOLUTION

$3x - 4y = 11$ ... (1)
$2x + 3y = -4$ ... (2)

From (2),

$$\begin{aligned} 2x &= -4 - 3y \\ x &= \frac{-4 - 3y}{2} \end{aligned}$$

Not as easy as Example (a); we choose the term with smallest coefficient.

Substituting in (1):

$$\begin{aligned} 3\left(\frac{-4 - 3y}{2}\right) - 4y &= 11 \\ 3(-4 - 3y) - 8y &= 22 \\ -12 - 9y - 8y &= 22 \\ -12 - 17y &= 22 \\ -17y &= 22 + 12 \\ -17y &= 34 \\ y &= -2 \end{aligned}$$

By multiplying through by 2.

By dividing through by −17.

Substituting $y = -2$ in (2):

$$\begin{aligned} 2x + 3(-2) &= -4 \\ 2x - 6 &= -4 \\ 2x &= -4 + 6 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

Therefore, $x = 1, y = -2$.

4.3.2 The elimination method

The second example above could have been solved in a simpler way by using the elimination method. One of the pronumerals is eliminated.

Examples: Solve:

(a) $2x - 3y = 3$ ... (1)
$4x + 3y = 15$ ... (2)

Add equations (1) and (2):

$6x = 18$
$x = 3$

The pronumeral y has been eliminated by adding.

From here the method is the same as the substitution method.

Substituting in (2):

Remember that we can substitute into either equation — choose the simpler.

$4(3) + 3y = 15$
$12 + 3y = 15$
$3y = 15 - 12$
$3y = 3$
$y = 1$

Therefore, $x = 3$ and $y = 1$.

(b) $3x - 4y = 11$ ... (1)
$2x + 3y = -4$ ... (2)

SOLUTION

$2 \times (1)$ $6x - 8y = 22$ ... (3)
$3 \times (2)$ $6x + 9y = -12$ ... (4)

We chose to eliminate the pronumeral x — lowest common multiple is 6.

Subtracting (4) from (3):

$-17y = 34$
$y = -2$

Substituting in (2):

$2x + 3(-2) = -4$
$2x - 6 = -4$
$2x = -4 + 6$
$2x = 2$
$x = 1$

Therefore $x = 1, y = -2$.

4.4 Use of equations to solve problems

Examples

(a) The sum of two consecutive even numbers is 22. Find the numbers.

SOLUTION
Let the numbers be x and $x + 2$.

Then $x + x + 2 = 22$
$2x + 2 = 22$
$2x = 20$
$x = 10$

Therefore, $x = 10, x + 2 = 12$, and so the numbers are 10 and 12.

(b) Find two numbers such that their sum is 20, while half their difference is 1.

SOLUTION
Let the numbers be x and y.

Then $x + y = 20$ ... (1)
$\frac{1}{2}(x - y) = 1$ ... (2)
$2 \times (2)$ $x - y = 2$ ... (3)
$[(1) - (3)]$ $2y = 18$
$y = 9$

Substituting $y = 9$ in (1):

$x + 9 = 20$
$x = 20 - 9$
$x = 11$

Therefore, $x = 11, y = 9$, and so the numbers are 11 and 9.

(c) If the sides of an isosceles triangle are $(x + 20)$ cm, $(3x - 16)$ cm and $(x + 2)$ cm, what are the possible values of x?

SOLUTION: Two of the three sides are equal. We form equations and solve them to find value(s) of x, but there is no point in setting $x + 20 = x + 2$ as this is nonsense.

$3x - 16 = x + 20$ $\quad$ $3x - 16 = x + 2$
$3x - x = 20 + 16$ *or* $3x - x = 2 + 16$
$2x = 32$ $\quad$ $2x = 18$
$x = 16$ $\quad$ $x = 9$

Therefore, the values of x could be 16 or 9.

(d) John is ten years older than Allyn, but twenty-five years ago, John was twice Allyn's age. Find Allyn's present age.

SOLUTION
Let Allyn's age $= x$
John's age $= x + 10$
Twenty-five years ago:
Allyn $= x - 25$
John $= x + 10 - 25 = x - 15$
Therefore twenty-five years ago:

$2(x - 25) = x - 15$
$2x - 50 = x - 15$
$2x - x = 50 - 15$
$x = 35.$

Therefore Allyn's present age is 35 years.

(e) Daniel has 22 coins, each being either a one-dollar coin or a two-dollar coin. He has a total of $35. How many of each coin does he have?

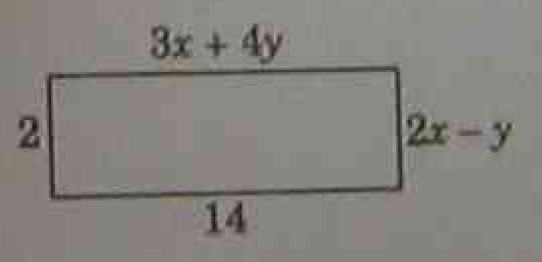
SOLUTION: Let x be the number of $1 coins, and y be the number of $2 coins.
The equations are:
$x + y = 22$ [he has 22 coins] ... (1)
$x + 2y = 35$ [as $1 and $2] ... (2)

$[(2) - (1)]$ $y = 13$
Substituting $y = 13$ in (1):

$x + 13 = 22$
$x = 9$

Therefore he has nine $1 coins and thirteen $2 coins. [Check this.]

(f) Find the value of x and y for the rectangle



SOLUTION

$3x + 4y = 14$ (1)
$2x - y = 2$ (2)
$4 \times (2) \therefore$ $8x - 4y = 8$ (3)
$[(1) - (3)]$ $11x = 22$
$x = 2$
Substituting $x = 2$ in (2)
$4 - y = 2$
$-y = 2$
$-y = -2$
$y = 2$
Therefore $x = 2$ and $y = 2$

(g) Three peaches and four plums cost $2.75 while five peaches and two plums cost$2.95.Find the price of each.

SOLUTION

Let a = price of a peach, b = price of a plum

$\therefore$ $3a + 4b = 275$ (1)
$5a + 2b = 295$ (2)
$2 \times (2)$ $10a + 4b = 590$ (3)
$\therefore [(3) - (1)]$ $7a = 315$
$\therefore$ $a = 45$
Substituting $a = 45$ in (1)
$\therefore$ $135 + 4b = 275$
$\therefore$ $4b = 275 - 135$
$4b = 140$
$b = 35$
$\therefore$ Peach costs 45c, plum costs 35c.

4.5 Exercises

1. Solve for the given variable:
   (a) $4x - 12 = 3x + 6$
   (b) $5y + 2 = 3y + 18$
   (c) $7(2y - 4) = 3(2y - 11)$
   (d) $\frac{4x - 2}{3} = 5$
   (e) $\frac{4 - 2y}{4} = 3y + 1$
   (f) $\frac{7a - 12}{2} - \frac{a + 1}{3} = 5$
   (g) $\frac{12x - 4}{3} = \frac{4x + 11}{2}$

(h) $\frac{5}{2x}=\frac{3}{4}$

(i) $\frac{6}{3y+1}=\frac{2}{2y-1}$

(j) $\frac{3}{2y-1}+4=0$

(k) $\frac{5a-3}{4}-\frac{a-1}{2}=\frac{3}{5}$

2. Solve the following inequations:

(a) $5a-3>2a+12$

(b) $7y-4\le 9y+12$

(c) $4-2y\ge 11$

(d) $3\le 4a+6$

(e) $4-2(x-4)\ge 12$

(f) $\frac{3-4x}{2}\le 3$

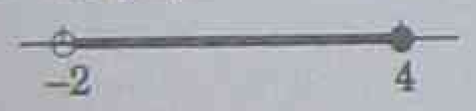
3. Graph on separate number lines:

(a) $-2<x\le 3$

(b) $3\le x<4$

4. Write down the inequality that corresponds to the graph:

(a)



(b)

5. Solve the following, and graph the solution on separate number lines:

(a) $3x-5\ge 2x-10$

(b) $\frac{4-5x}{2}<1$

6. Find the solution set of:

(a) $2x-4>7$, if $x<15$.

(b) $3x-2<x+5$, if x is positive.

(c) $2x+5<9$, if x is a positive integer.

7. If $P=m(v-u)$, find P when $m=16$, $v=11, u=2$.

8. If $S=\frac{a\left(r^n-1\right)}{r-1}$, find S when $a=100, r=2, n=4$.

9. If $I=\frac{PRN}{100}$, find I, if $P=160$, $R=11, N=9$.

10. Examine each pattern of numbers in each table, and find a formula for y in terms of x.

(a)

x	0	1	2	3
y	2	4	6	8

(b)

x	1	2	3	4
y	5	8	13	20

11. The following table shows the cost ($\$C$) of catching a taxi over a variety of distances (t km):

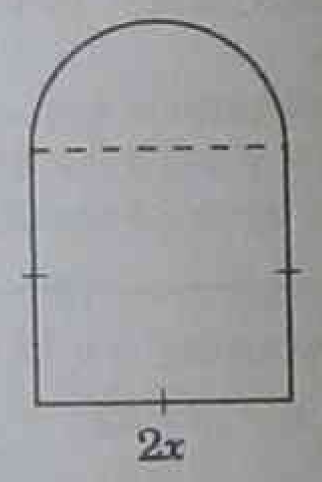
t	0	1	2	3	4
C	2	3.5	5	6.5	8

(a) Find a formula for C in terms of t.

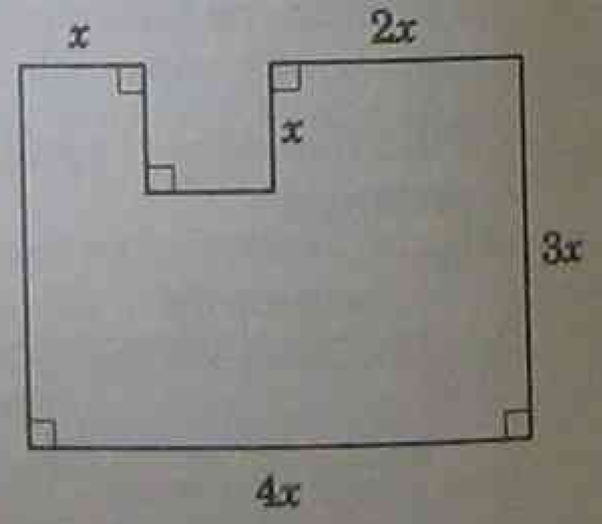
(b) Find the cost of catching a taxi and travelling 6 kilometres.

12. Find a formula for A, the area of the following:

(a)



(b)



13. A 20 cm × 8 cm print is to be surrounded with a t cm-wide frame. Find a formula for A, the area of the entire picture (print + frame) in terms of t.

14. Make y the subject of:

(a) $4x+2y=16$

(b) $5x-3y=11$

(c) $2xy=11-x$

(d) $5y-2xy=7$

(e) $A=\frac{4x}{y}$

(f) $P=\frac{\sqrt{xy}}{2}$

(g) $k=\frac{y-1}{y}$

(h) $x^2=\frac{y}{y-2}$

15. (a) If $V=\pi r^2h$, find h if $V=770, r=7$, $\pi=\frac{22}{7}$.

(b) If $s=ut+\frac{1}{2}at^2$, find u if $s=345.5$, $t=5, a=10$.

16. An object dropped from rest is acted upon by gravity. It will have a velocity of v km/h after falling d metres (ignoring air resistance) according to the formula $v=8.4\sqrt{d}$.

(a) Make d the subject of the formula.

(b) Hence, or otherwise, find how far the object will have to fall before its velocity is 42 km/h?

17. Solve the following pairs of simultaneous equations:

(a) $$\begin{aligned}x+2y&=4\\3x+14y&=36\end{aligned}$$

(b) $$\begin{aligned}3x-8y&=2\\4x+10y&=44\end{aligned}$$

(c) $$\begin{aligned}3x&=5-2y\\4x&=1+3y\end{aligned}$$

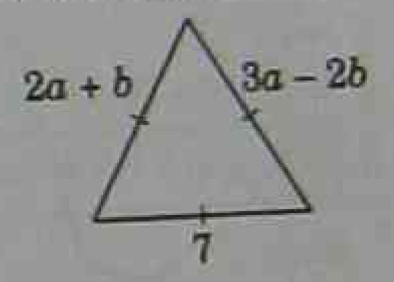
(d) $$\begin{aligned}x-y&=8\\\frac{x}{2}+\frac{y}{3}&=-1\end{aligned}$$

18. (a) If five is subtracted from twice a number, the result is the same as when four is added to the number. Find the number.

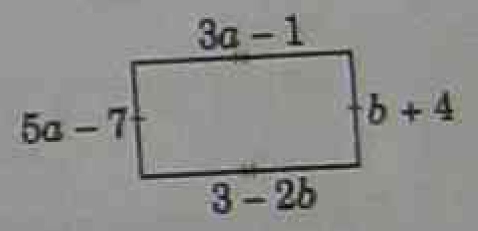
(b) The sum of two numbers is tvelwe and their difference is two. Find the numbers.

(c) Find the values of a and b.

(i)



(ii)



(d) Six pens and five pencils cost $2.50 while three pens and two pencils cost $1.15. What will be the cost of a pen and four pencils?

Chapter 5

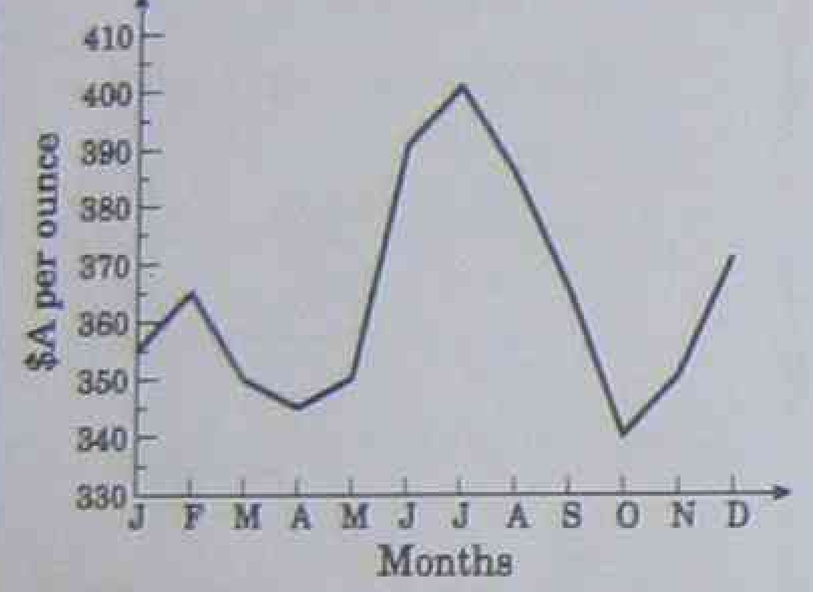
GRAPHS

5.1 Line graphs

A line graph shows the relationship between two variables.

Examples

(a)



The above graph shows the variations in the price of gold as recorded at the beginning of each month from 1 January to 1 December. Use the graph to answer the following questions:

(i) What does the smallest marked unit on the vertical axis represent?

ANSWER: $5 (Australian) per ounce.

(ii) What was the price of gold on 1 March?

ANSWER: $350

(iii) When was the price of gold at its highest point, and what was the price?

ANSWER: 1 July; $400.

(iv) During which month did the greatest price rise occur and how much did it rise?

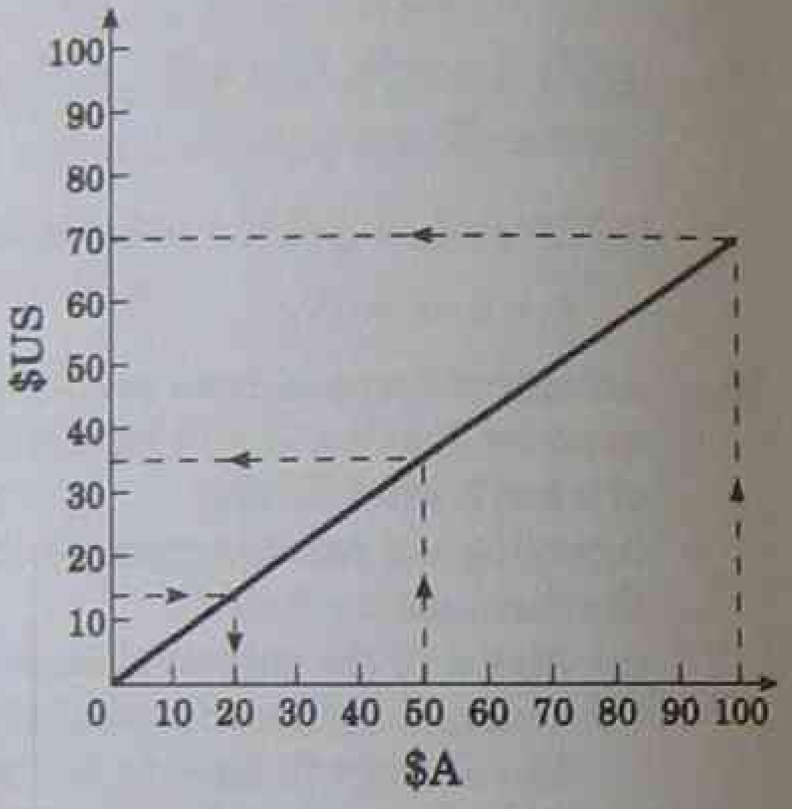
ANSWER: May; $40

Steepest part of graph

(v) When was the price of gold $350 per ounce?

ANSWER: March, May, November.

(b)



The above graph is used to convert Australian dollars to US dollars. From the conversion graph we can see that $100 (Australian) converts to $70 (United States), that is, $A100 = $US70. Use the graph to convert:

(i) $A50 to $US

ANSWER: $A50 = $US35.

(ii) $US14 to $A

ANSWER: $US14 = $A20

(c) Find a formula for $US ($S$) in terms of $A ($A$), by using the information on the graph.

SOLUTION: Take points from the graph: (0, 0) and (100, 70)

Use coordinate geometry formulae, that is,

Continued

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

Thus, $$\frac{S-0}{A-0}=\frac{70-0}{100-0},$$

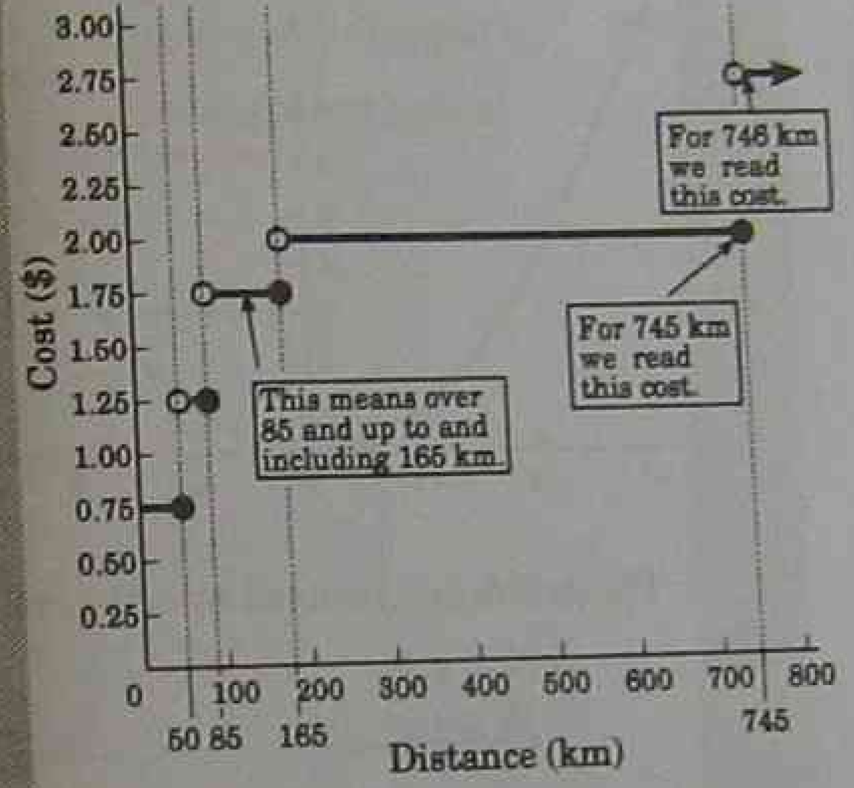
therefore, $$\frac{S}{A}=\frac{7\cancel{0}}{10\cancel{0}}=\frac{7}{10}$$

and $$S=\frac{7A}{10}.$$

5.2 Other types of line graphs

5.2.1 Step graphs

Example: The graph below indicates telephone charges for STD calls lasting five minutes (or part thereof).



Use the graph to calculate the cost of the following calls:

(i) Five minutes over a distance of 100 km.

ANSWER: $1.75.

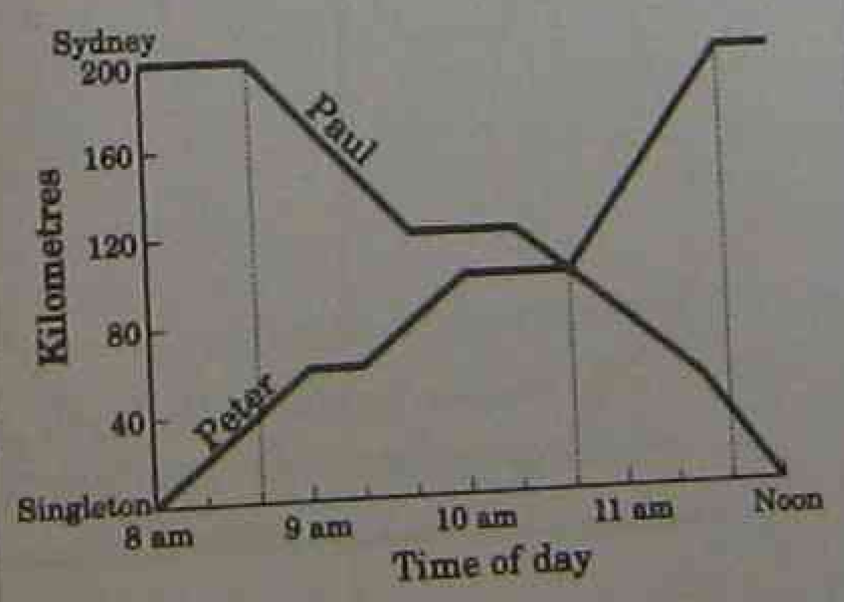
(ii) Ten minutes over a distance of 85 km.

ANSWER: 2 × $1.25 = $2.50.

10 minutes is 2 × 5 minutes

5.2.2 Travel graphs

Example



The above graph shows the travel patterns of Peter who travelled from Singleton to Sydney and Paul who travelled from Sydney to Singleton.

(i) How long did Peter travel for before he stopped?

ANSWER: 1 hour

(ii) What was his average speed before he stopped?

ANSWER: 60 km/h

(iii) At what time did Peter arrive in Sydney?

ANSWER: 11:40 a.m.

(iv) At what time did Paul leave Sydney?

ANSWER: 8:40 a.m.

(v) At what time did Peter meet Paul on the road?

ANSWER: 10:40 a.m.

Continued

(vi) What was Paul's average speed for the entire trip?

SOLUTION: From 8:40 a.m. to 12 noon $= 3$ h 20 min $= 3\frac{1}{3}$ hours.

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{200}{3\frac{1}{3}} = 60 \text{ km/h}$$

5.3 The parabola

5.3.1 The graph of $y = x^2$

To graph any equation which results in a curve we must plot points to make a clear shape.

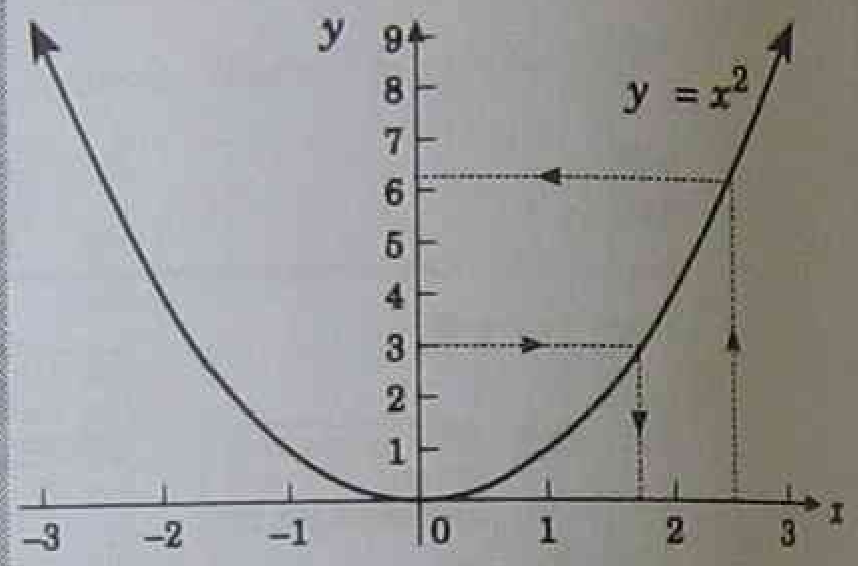
Example: For $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

Our points are $(-3, 9)$, $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$, $(3, 9)$

The y-axis is the axis of symmetry. The curve is symmetrical about the y-axis.

The curve is always above the x--axis because both positives and negatives squared give positive answers.

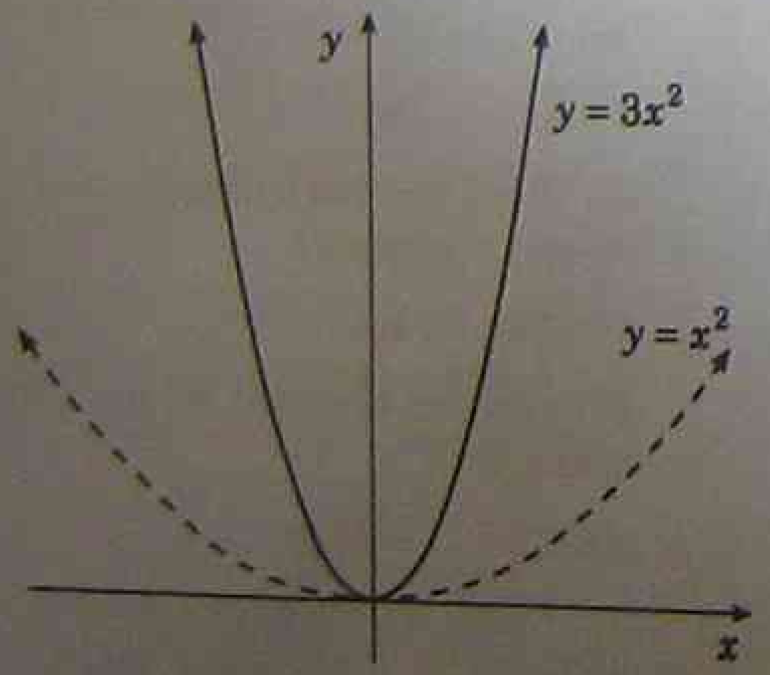


The graph can be used to estimate:

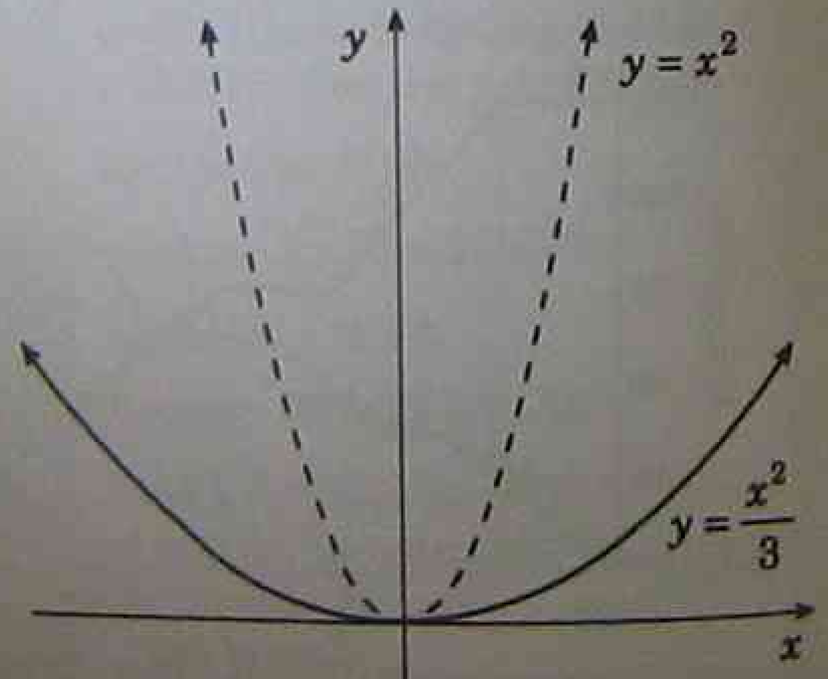
$2.5^2 \approx 6.2$,

$\sqrt{3} \approx 1.7$.

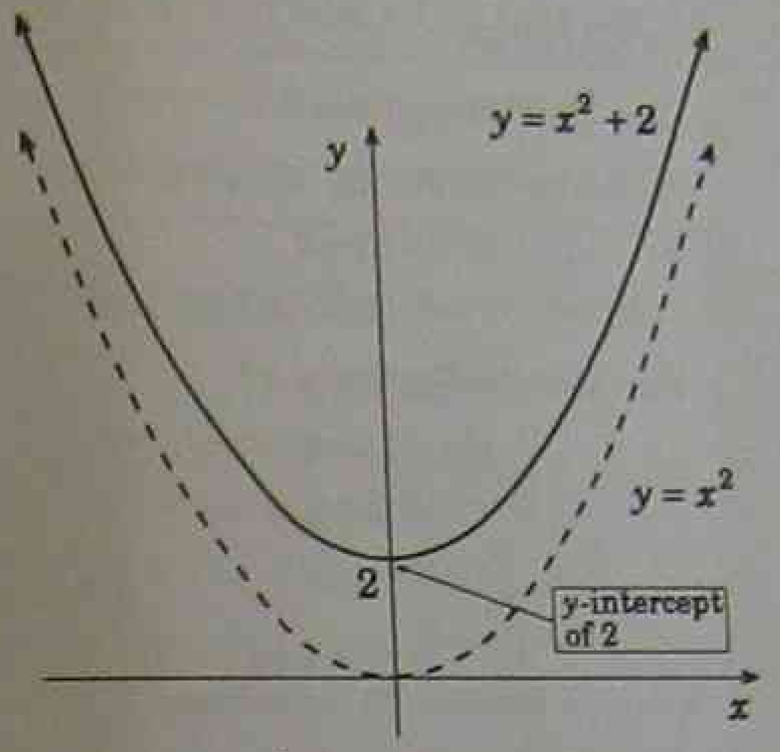
5.3.2 Graphing $y = ax^2 + c$, $y = (x - k)^2$, etc



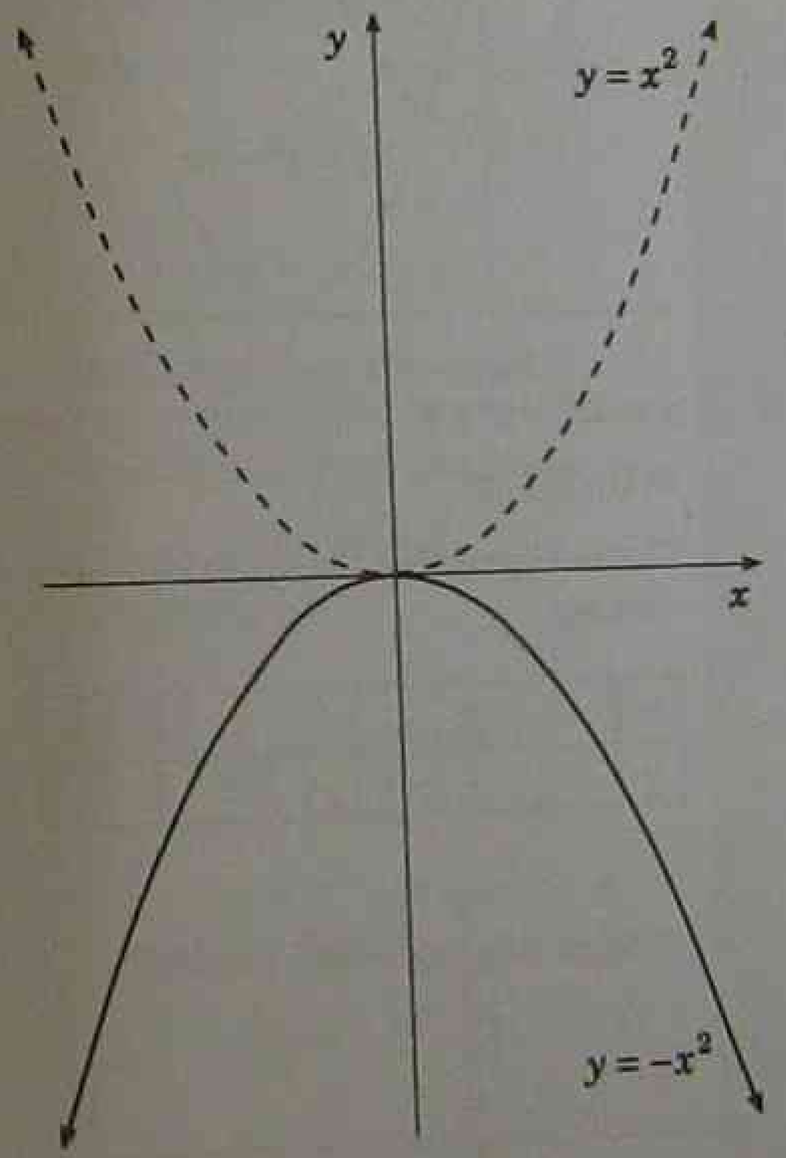
$y = 3x^2$ is 'inside' $y = x^2$



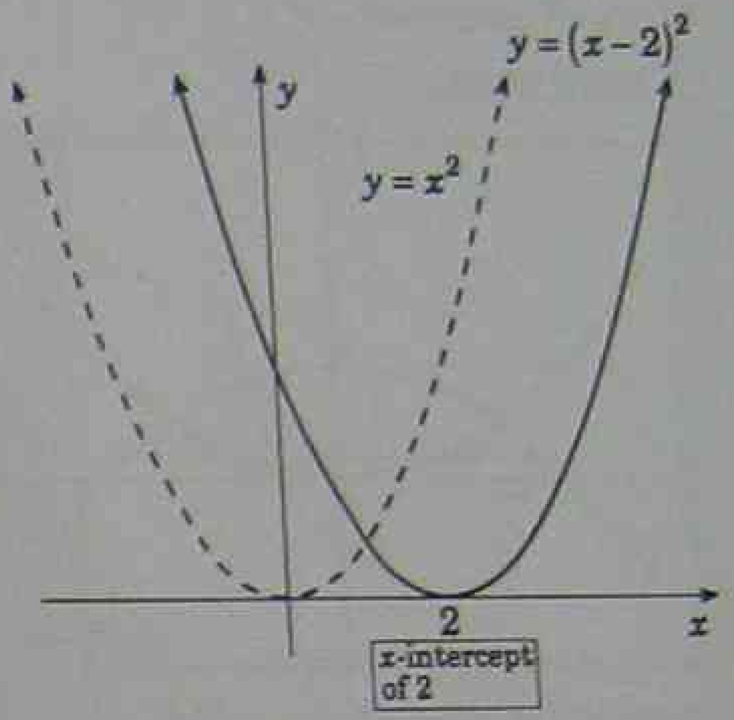
$y = \frac{x^2}{3}$ (or $y = \frac{1}{3}x^2$) is 'outside' $y = x^2$



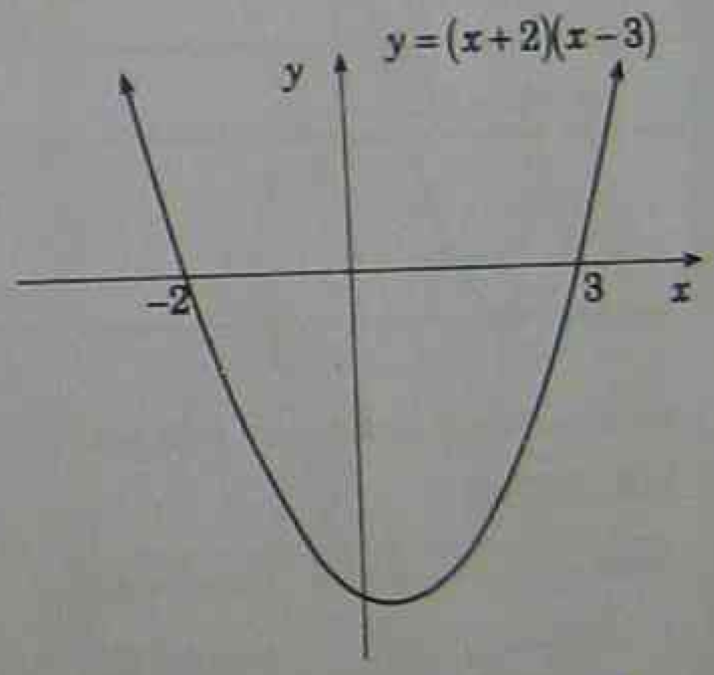
$y = x^2 + 2$ is 'shifted up' two units from $y = x^2$



$y = -x^2$ is 'upside down' view of $y = x^2$, that is coefficient of x^2 is negative, therefore concave down, not concave up.



$y = (x - 2)^2$ is shifted to right two units from $y = x^2$

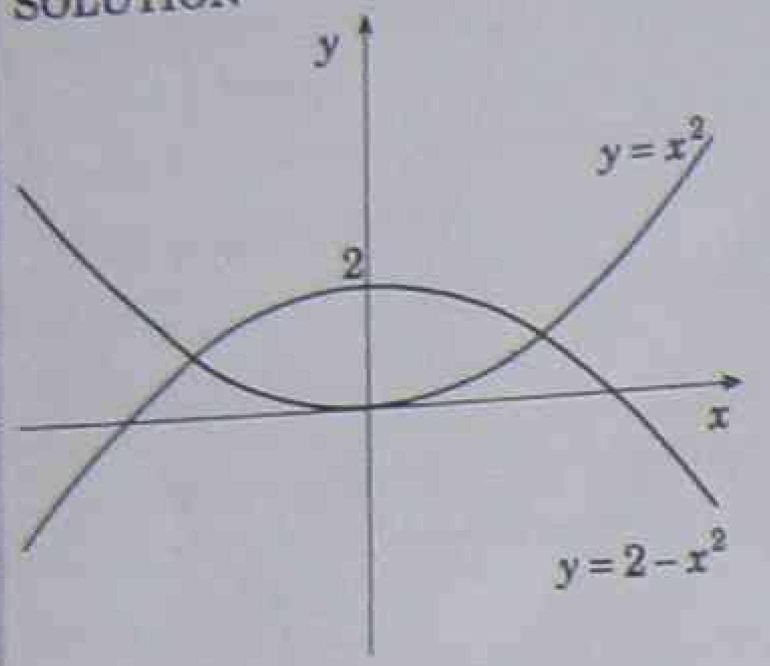


When $x = 0$, $x = -2$ and $x = 3$ give us the x-intercepts.

Examples: Graph the following parabolas on same number plane:

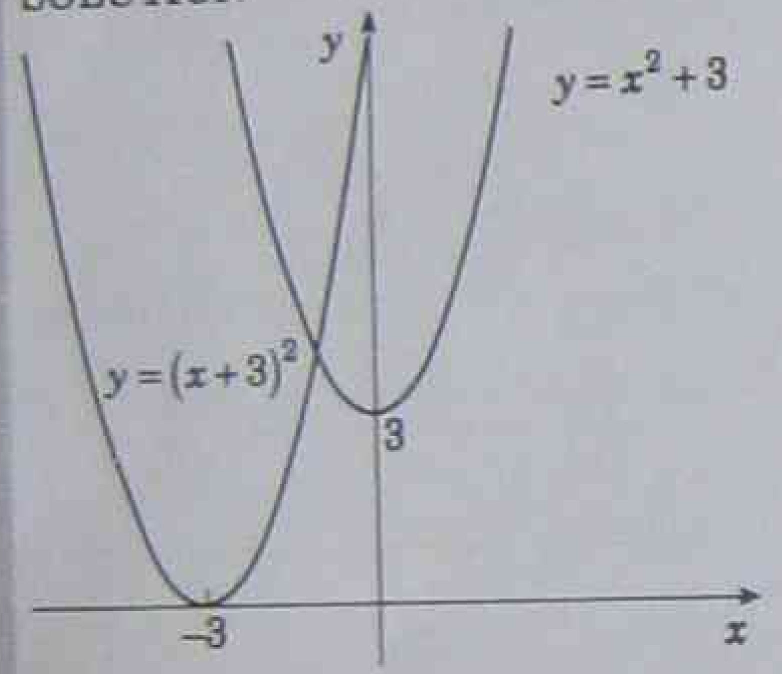
(a) $y = x^2$ and $y = 2 - x^2$

SOLUTION



(b) $y = x^2 + 3$ and $y = (x + 3)^2$

SOLUTION

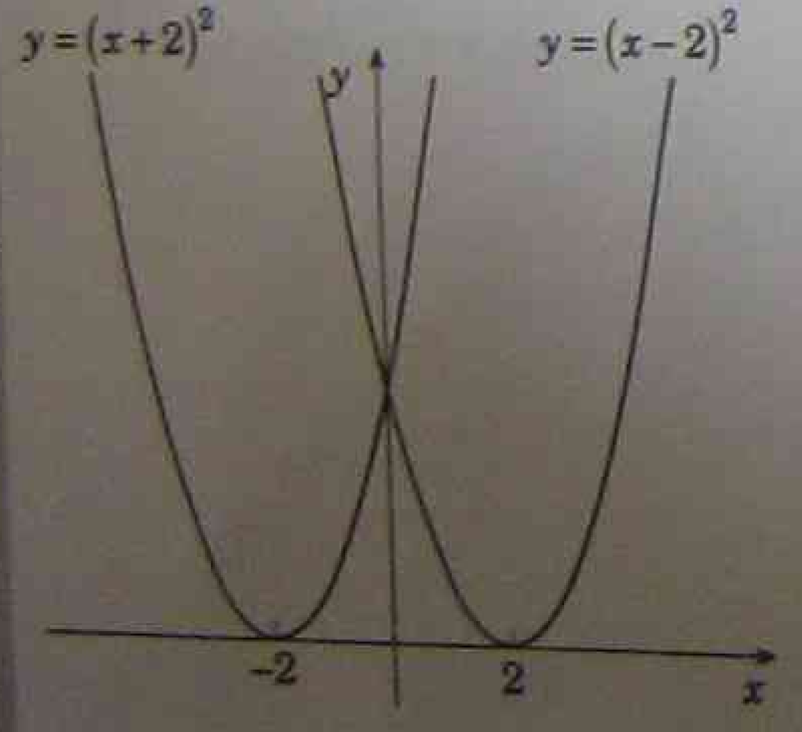


(c) $y = x^2 - 4x + 4$ and $y = x^2 + 4x + 4$

SOLUTION

Now, $y=x^2-4x+4$ $\quad y=x^2+4x+4$

$\therefore \quad y=(x-2)^2 \qquad y=(x+2)^2$



(d) $y = (x + 2)(x - 3)$ and $y = x^2 - 7x$

SOLUTION

For $y=(x+2)(x-3)$

to find x-intercept, let $y=0$

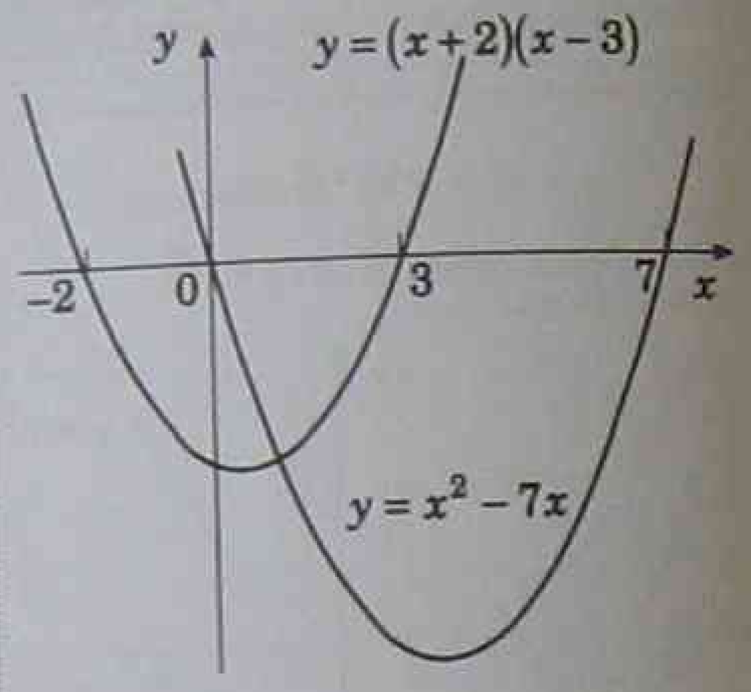
$\therefore \quad (x+2)(x-3)=0$

$x=-2, 3$

For $y=x^2-7x$, let $y=0$

$\therefore \quad x(x-7)=0$

$x=0, 7$

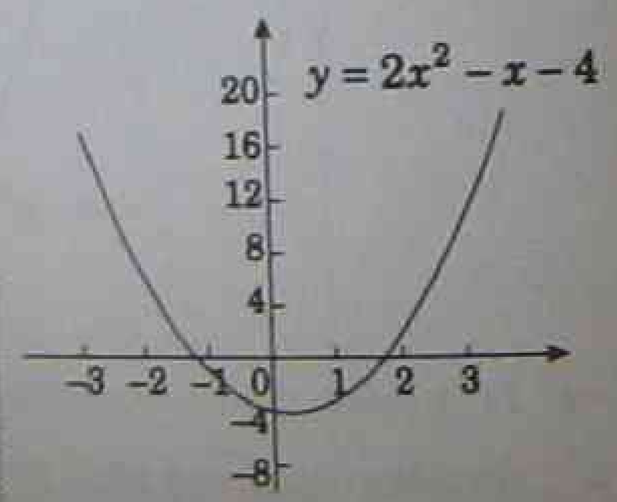


(e) $y = 2x^2 - x - 4$

SOLUTION

Cannot factorise, so must use table of values:

x	−3	−2	−1	0	1	2	3
y	17	6	−1	−4	−3	2	11



5.3.3 The vertex of a parabola

Three methods can be used:

(a) Midpoint of x-intercepts

Example: Find the vertex of

$y = (x + 3)(x - 1)$

SOLUTION

x-intercept when $y=0$

$\therefore \quad (x+3)(x-1)=0$

$x=-3, 1$

$\therefore$ midpoint of -3 and 1

$$= \frac{-3+1}{2}$$

$$=-1$$

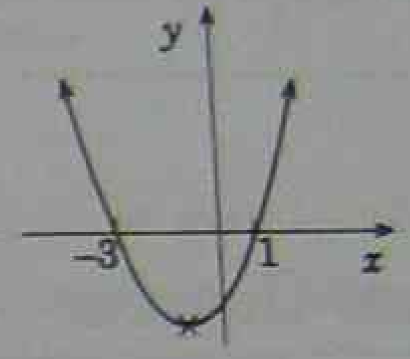
Now substitute in y,

that is, $\quad y=(-1+3)(-1-1)$

$=(2)(-2)$

$=-4$

$\therefore$ vertex is $(-1, -4)$



The parabola has minimum vaue (as $a > 0$) of −4.

(b) Completing the square

Example: Find the vertex of

$y = x^2 - 4x + 3$

SOLUTION

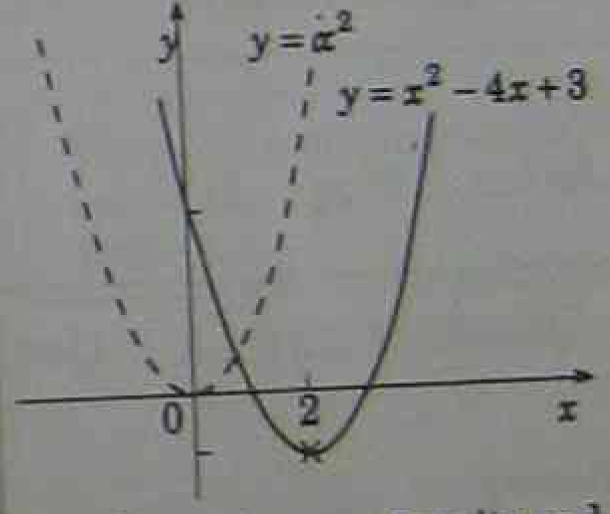
$y=x^2-4x+3$

$\therefore \quad y=x^2-4x \quad +3$

$\therefore \quad y=x^2-4x+4+3-4$

(We have added a 4, so subtract 4.)

$\therefore \quad y=(x-2)^2-1$



$y=x^2$ moved across 2 units and down 1 unit. Vertex is (2, −1). The parabola has minimum value of −1.

(c) Using $x=\dfrac{-b}{2a}$ (axis of symmetry)

Example: Find the vertex of

$y = -x^2 - 3x + 7$

SOLUTION

As $y=-x^2-3x+7$ is in form of

$y=ax^2+bx+c \; \therefore \quad a=-1, b=-3$

$$\therefore \quad x=\frac{-b}{2a}$$

$$=\frac{3}{-2}$$

$$=-1\tfrac{1}{2}$$

[$\therefore$ axis of symmetry is $x=-1\frac{1}{2}$]

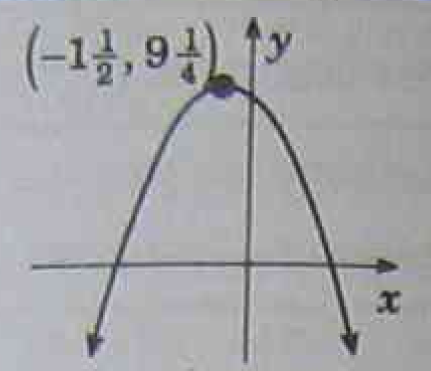
Substitute $x = -1\frac{1}{2}$ in

$$y = -x^2 - 3x + 7$$

$$\therefore \quad y = -\left(1\tfrac{1}{2}\right)^2 - 3\left(-1\tfrac{1}{2}\right) + 7$$

$$= -2\tfrac{1}{4} + 4\tfrac{1}{2} + 7$$

$$= 9\tfrac{1}{4}$$



$\therefore$ vertex is $\left(-1\frac{1}{2}, 9\frac{1}{4}\right)$

[the parabola has a maximum value (as $a < 0$) of $9\frac{1}{4}$]

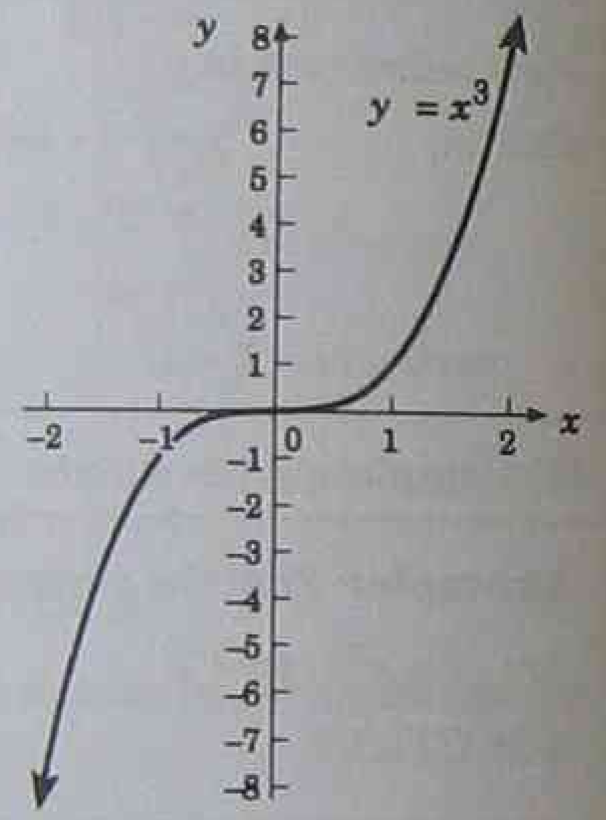
5.4 Other Graphs

5.4.1 The cubic (for example $y = x^3$)

Example: For $y = x^3$ (See at right)

x	-2	-2	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	-8	-1	$-\frac{1}{8}$	0	$\frac{1}{8}$	1	8

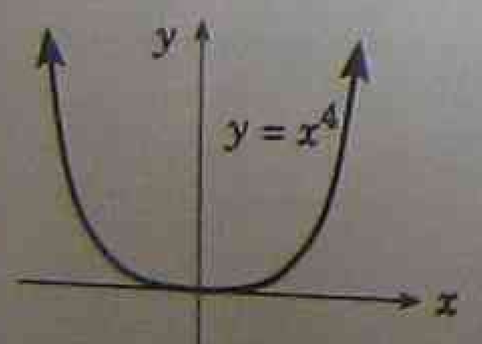
Remember: A positive number cubed gives a positive. A negative number cubed gives a negative.



5.4.2 Quartics and Quintics

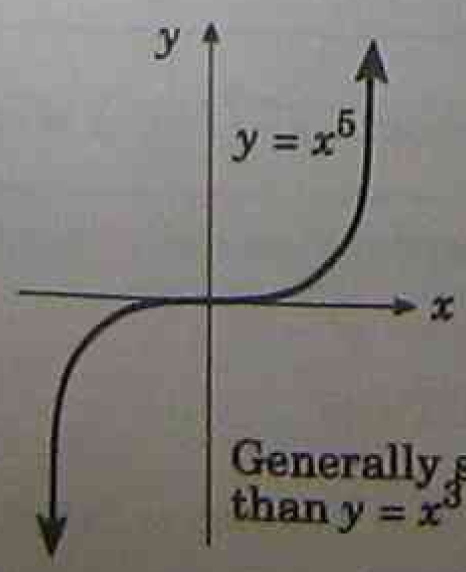
Examples

(a) **Quartic**: $y = x^4$



Generally steeper than $y = x^2$

(b) **Quintic**: $y = x^5$



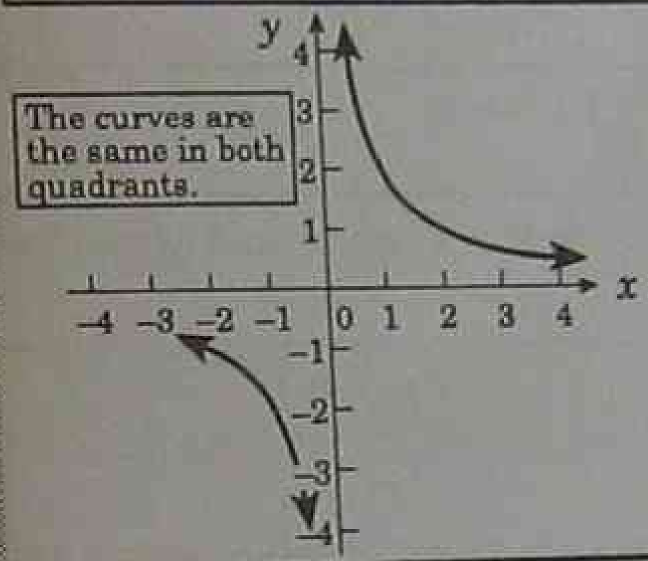
Generally steeper than $y = x^3$

5.4.3 The hyperbola $\left(y = \frac{a}{x}, \text{ or } xy = a\right)$

Example: Graph $y = \frac{2}{x}$

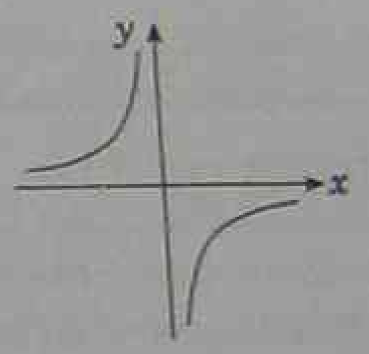
x	-4	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	4
y	$-\frac{1}{2}$	-1	-2	-4	—	4	2	1	$\frac{1}{2}$

Note that there is a problem for x = 0 as $y = \frac{2}{0}$ is indeterminate. Therefore the graph has a discontinuity, that is, it has two parts.



The curve gets closer and closer to the axes without touching. The axes are called the *asymptotes* of the curve.

If $a < 0$, for example, $y = -\frac{2}{x}$, we would graph it as follows:



5.4.4 The exponential (for example, $y = 2^x$)

Example: Complete the table for $y = 2^x$ and graph $y = 2^x$ on a number plane.

x	-3	-2	-1	0	1	2	3
y	0.125	0.25	0.5	1	2	4	8

SOLUTION

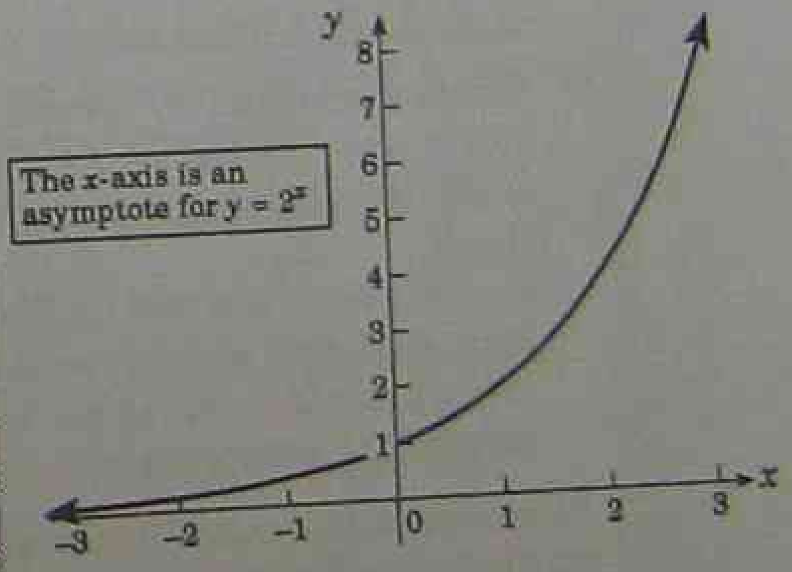
To find 2^{-3} by calculator we use:

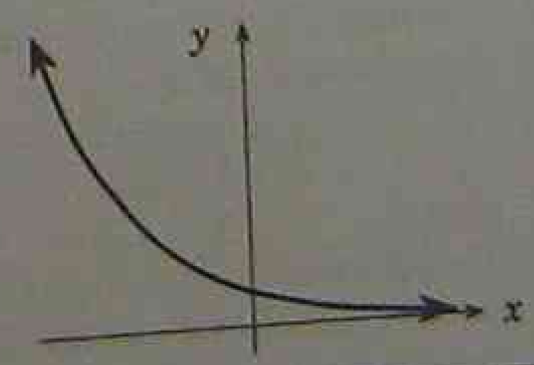
2 $\boxed{x^y}$ 3 $\boxed{+/-}$ $\boxed{=}$

We can also graph $y = 2^{-x}$, that is:

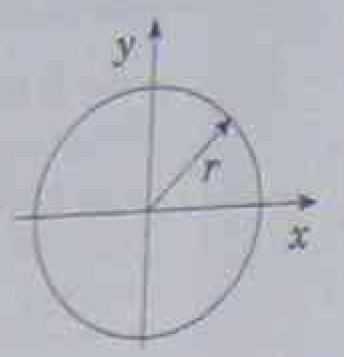
$y = \frac{1}{2^x}$, the reciprocal of $y = 2^x$. →

The curve of an exponential always passes through (0, 1) and is always positive.





5.4.5 The circle $(x^2 + y^2 = r^2)$

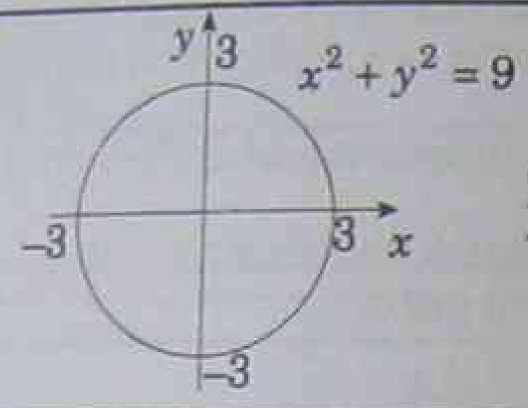


Centre: (0, 0)
Radius = r units

Example:

Graph $x^2 + y^2 = 9$

SOLUTION



Centre: (0, 0)
Radius = 3 units

5.5 Regions on the number plane

When we graph inequalities on a number plane the result is a *region* and covers half the number plane — hence the term *half-plane*.

Examples

(a) Graph $2x + y < 1$ on a number plane.

SOLUTION

- First rewrite it as $y < 1 - 2x$.
- To find the boundary of the region, let $y = 1 - 2x$ and complete a table of values:

x	0	1	2
y	1	−1	−3

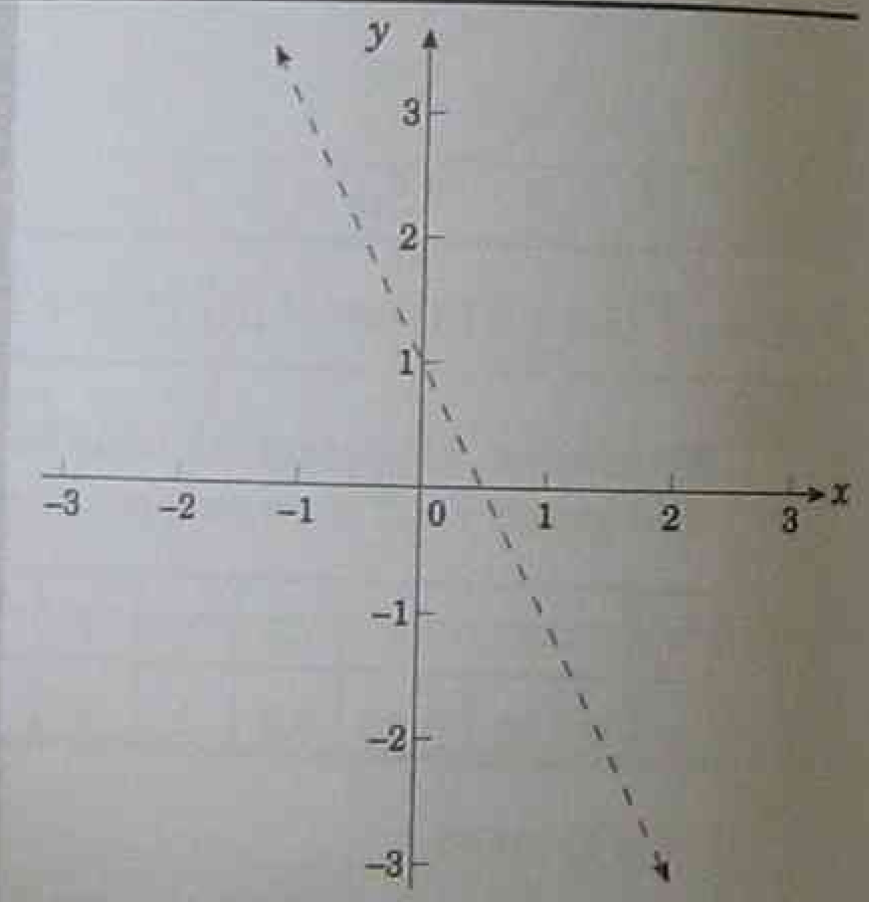
- Check the inequality symbol:
  If it is > or < we have a broken line.
  If it is ≥ or ≤ we have an unbroken line.
  In this case it is a broken line.
- Now we decide which side of this line we shade to represent the points that satisfy the inequality.

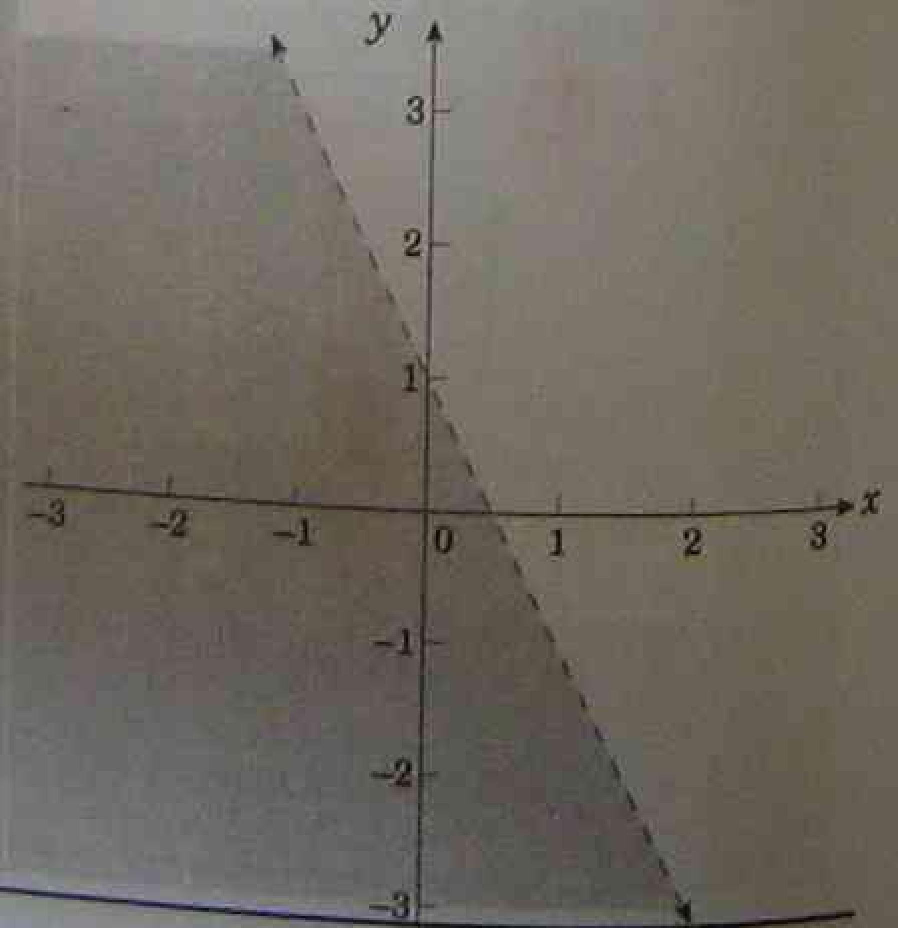
Substitute a representative point. The point (0, 0) is always easy to substitute and it is **not** on the broken line. Substitute (0, 0) in

$$y < 1 - 2x$$
$$0 < 1 - 2(0),$$

that is, $0 < 1$? ... Yes.

Therefore all points on the side of the broken line that (0, 0) was on **must** satisfy, so we shade that side.





(b) Graph the half-plane $y \le x^2 - 2$ on a number plane.

SOLUTION

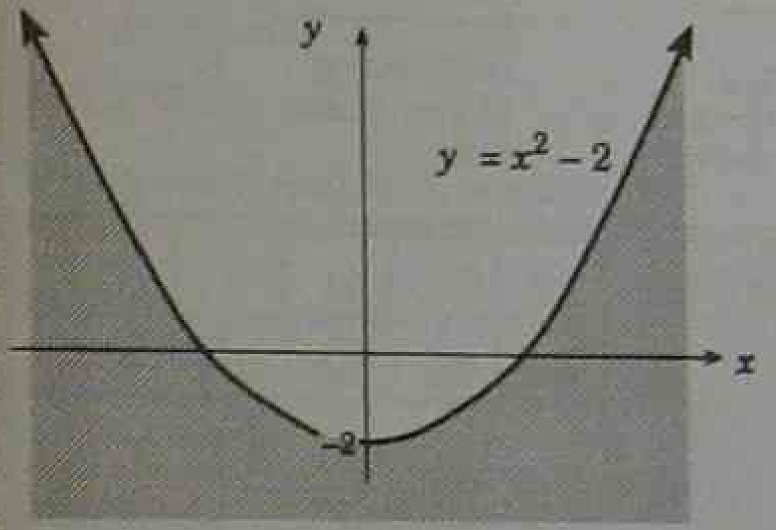
Thus graph $y = x^2 - 2$, with an unbroken line (because of ≤).

Choose (0, 0) and substitute in $y \le x^2 - 2$,

that is, $0 \le 0^2 - 2$

$0 \le -2$? ... No!

Therefore we shade the side that (0, 0) is not on.



(c) Determine whether the points (−1, 2) or (3, -1) lie in the shaded region given as $y \le 2x - 3$

SOLUTION

Substitute each point in inequality:

(−1, 2) ∴ $x = -1, y = 2$ in

$y \le 2x - 3$

∴ $2 \le 2(-1) - 3$

$2 \le -2 - 3$

that is, $2 \le -5$? No!

∴ (−1, 2) is not in $y \le 2x - 3$

(3, - 1) ∴ $x = 3, y = -1$ in

$y \le 2x - 3$

∴ $-1 \le 2(3) - 3$

$-1 \le 6 - 3$

that is, $-1 \le 3$? Yes!

∴ (3, - 1) lies in region $y \le 2x - 3$.

Note: The above examples could have been expressed in a slightly different way using $\{(x, y): ......\}$ which means 'the set of points such that ...',

for example: graph $\{(x, y): x - y < 1\}$ means graph the set of points such that $x - y < 1$.

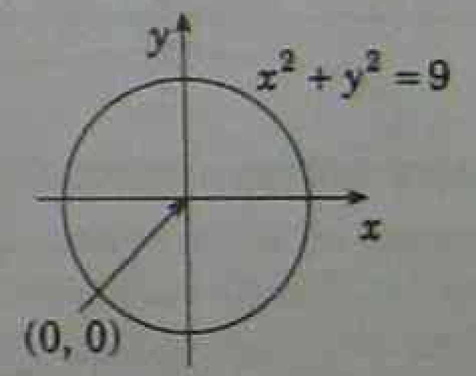
(d) Does the inequality $x^2 + y^2 < 9$ represent the inside or the outside of the circle $x^2 + y^2 = 9$?

SOLUTION

Consider a typical point inside the circle — the centre (0, 0) is an obvious choice.

Test $x = 0, y = 0$ in $x^2 + y^2 < 9$, that is, $0 + 0 < 9$.

As this is a true statement (0, 0) lies in the region $x^2 + y^2 < 9$, that is, $x^2 + y^2 < 9$ represents the inside of the circle.



If a point obviously outside was chosen, for example (4, 0), it will NOT satisfy inequation.

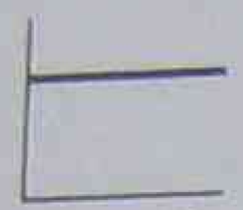
For example, $x = 4, y = 0$ in $x^2 + y^2 < 9$

$16 + 0 \nless 9$

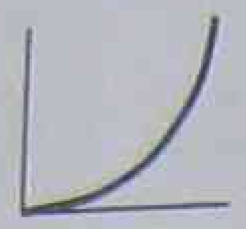
(4, 0) does not lie in the region.

5.6 Matching graphs to physical phenomena

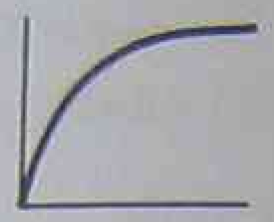
A line graph can represent physical phenomena as follows:



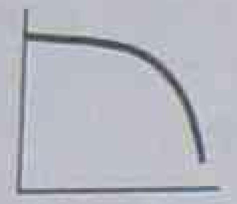
Staying the same



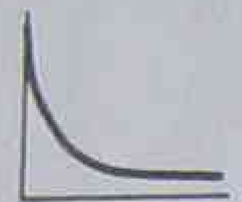
Increasing slowly then speeding up



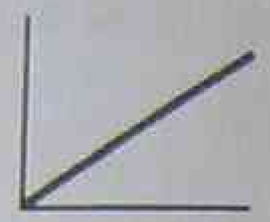
Increasing quickly then slowing down



Decreasing slowly then speeding up

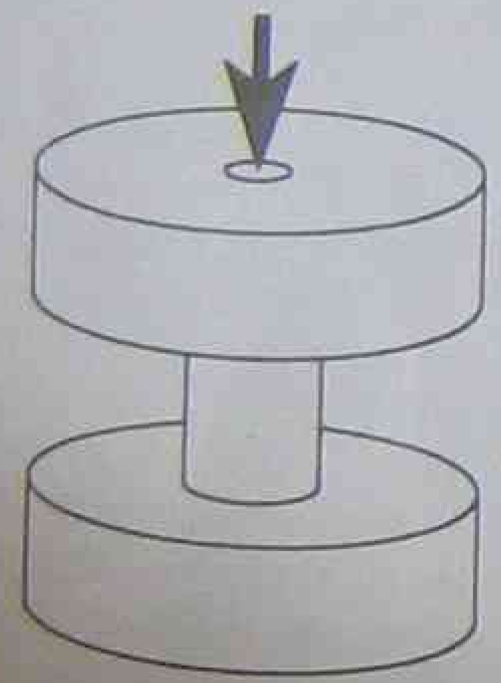


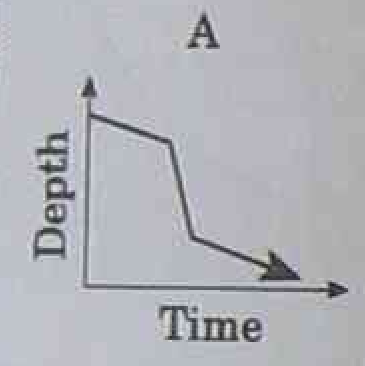
Decreasing quickly then slowing down

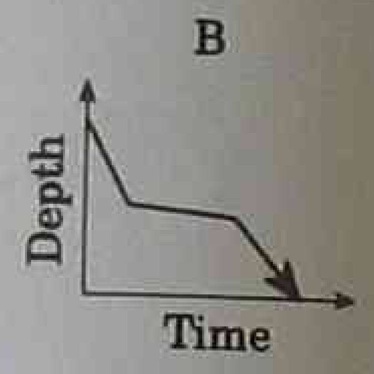


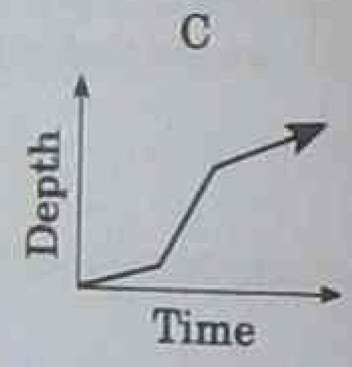
Increasing at the same rate

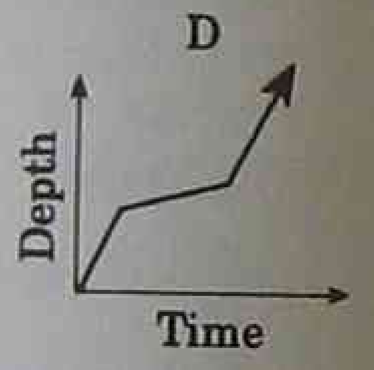
Example: Water is poured into an empty tank at a steady rate and the rise of the water level is graphed. Which graph best represents the change in water level?









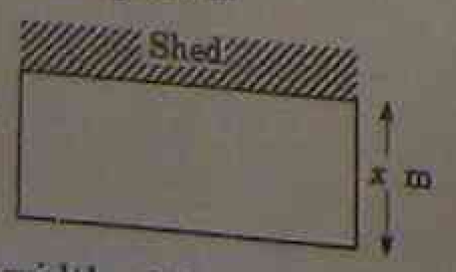


The water level would increase slowly, then quickly, and then slowly again, therefore the gradient is *gentle*, then *steep* and then *gentle*. The answer is C.

5.7 Using graphs to solve problems

Examples

(a) A farmer wishes to use a 60-metre length of wire mesh to make an enclosure which will have as its fourth side an existing shed.



The width of the enclosure is x metres.

(i) Find the length of the enclosure.

SOLUTION: For the rectangle, $x + \text{length} + x = 60$, therefore, $\text{length} = 60 - 2x$.

(ii) Write down an expression for the area A enclosed.

SOLUTION

$$A = \text{length} \times \text{width}$$
$$= (60 - 2x)x$$

Therefore $A = x(60 - 2x)$.

(iii) Find the area enclosed when $x = 15$.

SOLUTION: Substitute $x = 15$ in

$$\begin{aligned} A &= x(60 - 2x) \\ &= 15(60 - 2(15)) \\ &= 15(60 - 30) \\ &= 15(30) \\ &= 450 \end{aligned}$$

The area is 450 m^2.

(iv) Draw a graph to illustrate the possible values of the area in terms of x.

SOLUTION

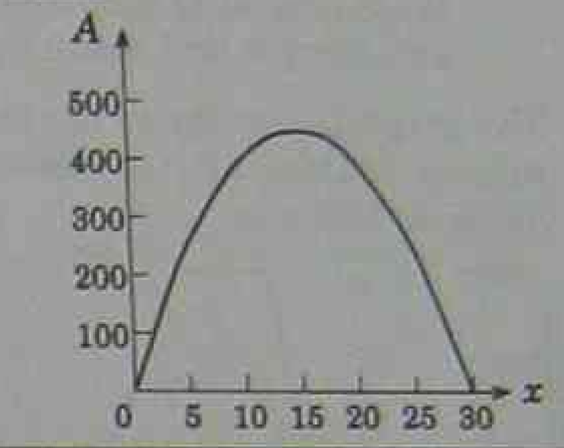
Using the equation $A = x(60 - 2x)$:

x	0	5	10	15	20	25	30
A	0	250	400	450	400	250	0

We can use a table of values or axis of symmetry, intercepts, etc.

Note: x will range between 0 and 30.

Remember that a parabola is symmetrical.



We only need the positive quadrant because width (x) and area (A) can only be positive.

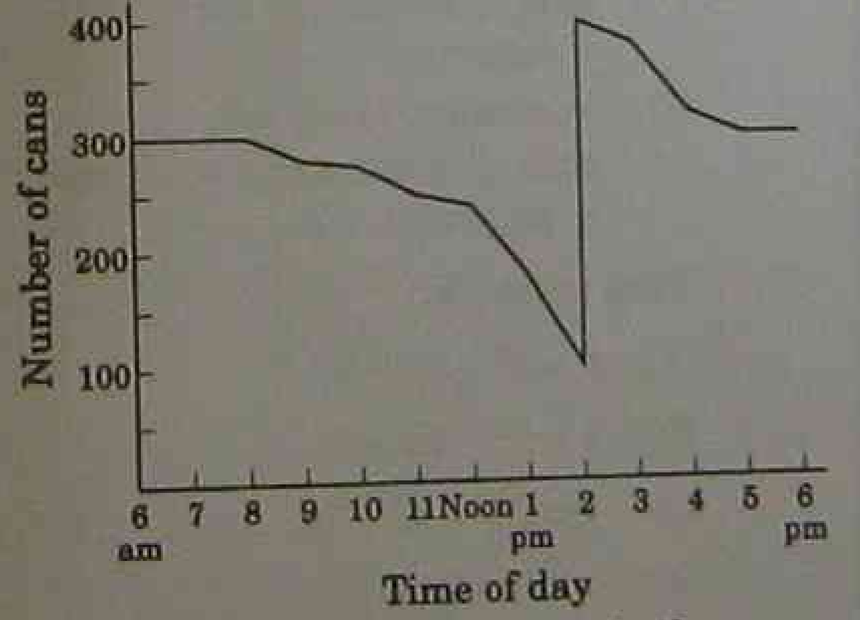
(v) What is the largest area that can be enclosed?

SOLUTION: What we are asked here is: What is the *maximum value* of A?

This occurs at the maximum point on the parabola, that is, 450, therefore the largest area is 450 m^2.

5.8 Exercises

1. The graph represents the number of cans in a vending machine in the main foyer of a large hospital.



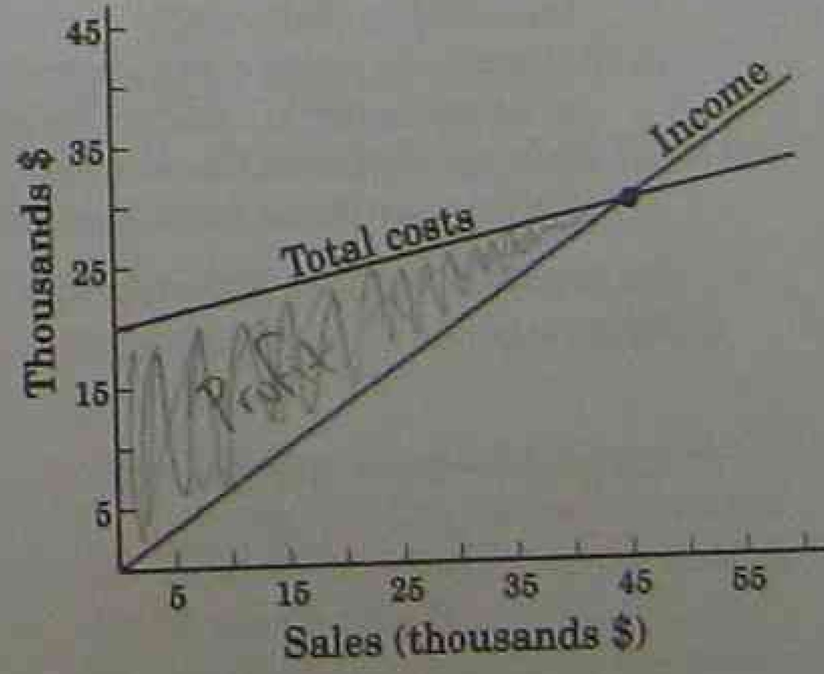
(a) How many cans were in the machine at 6 a.m.?

(b) When was the vending machine refilled and how many cans were added?

(c) How many cans were sold in the 12-hour period?

(d) During which hour were most cans sold?

2. A company graphs its income and total costs. Profit is found by subtracting total costs from income.



(a) What are the total costs of production before any sales are made?

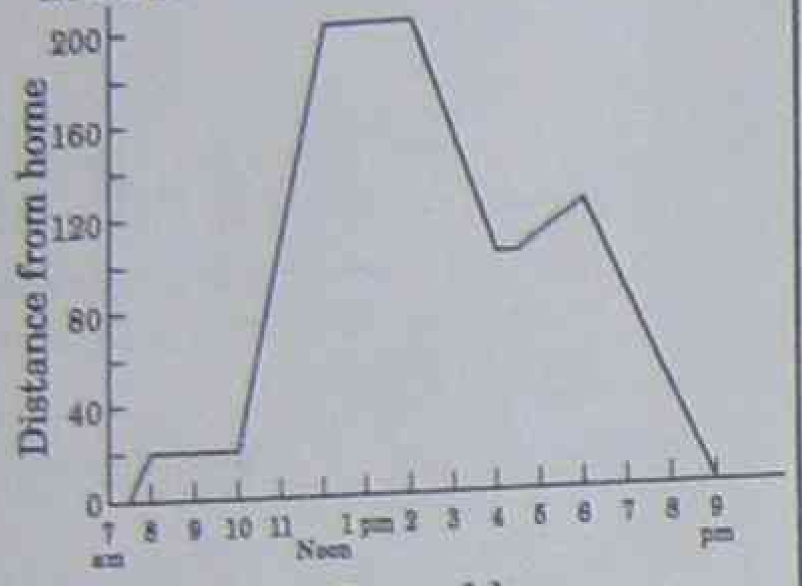
(b) What amount represents the break-even figure — that is, when income = total costs?

Continued

(c) Write a formula for total costs T in terms of sales S and a formula for income I in terms of sales S.

(d) If sales amount to \$81 000, use the formulae to find total costs and income and hence the profit.

3. The graph below shows the travel pattern of Deborah, who leaves her home at 7:30 a.m.



Time of day

(a) At what time did Deborah arrive back at home?

(b) How long was her car stationary during the morning?

(c) When was she travelling at the greatest speed? What was the average speed during this time?

(d) What was the farthest distance Deborah was from home?

(e) What was the total distance travelled by Deborah?

(f) Deborah is given a car allowance of 24 cents per kilometre. Calculate her claim for the allowance for this particular day's travelling.

4. On separate number planes, graph the following:

(a) $y = x^2 - 2$ (b) $y = -2x^2$
(c) $y = 3 - x^2$ (d) $y = x^3 + 1$
(e) $y = x^4 + 2$ (f) $y = x^5 - 1$
(g) $y = \frac{2}{x}$ (h) $xy = 1$
(i) $y = -\frac{3}{x}$ (j) $y = 3^x$
(k) $y = \left(\frac{1}{2}\right)^x$

5. Determine whether the following parabolas are concave up or concave down when graphed:

(a) $y = x^2 - 5x + 2$ (b) $y = x - x^2$
(c) $y = 3 + x^2 - x$ (d) $y = 4x^2 - x + 1$

6. Without graphing them, find the x-intercept and y-intercept for the following parabolas:

(a) $y = x^2 + x - 2$ (b) $y = x^2 - x$
(c) $y = x^2 - 4$ (d) $y = 9 - x^2$

7. Find the equations of the axis of symmetry for each of the following:

(a) $y = 3x^2 - 6x + 2$
(b) $y = 4 - 5x - x^2$
(c) $y = 4x^2 - 8x$
(d) $y = 4 - x^2$

8. Find the maximum value of y if:
(a) $y = 4 - x - x^2$
(b) $y = -x^2 + 4x + 2$

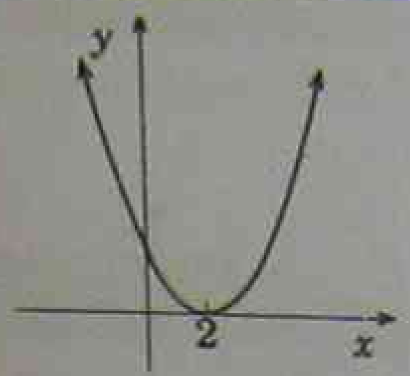
9. Find the minimum value of y if:
(a) $y = x^2 + 6x + 12$
(b) $y = x^2 - 2x + 3$

10. Sketch the parabola with:
(a) maximum value = 3, axis of symmetry $x = 2$, and y- intercept = –1.
(b) minimum value = –4, axis of symmetry $x = 3$, and y- intercept = –2.
(c) minimum value = 0, axis of symmetry $x = -2$, and y- intercept = 3.

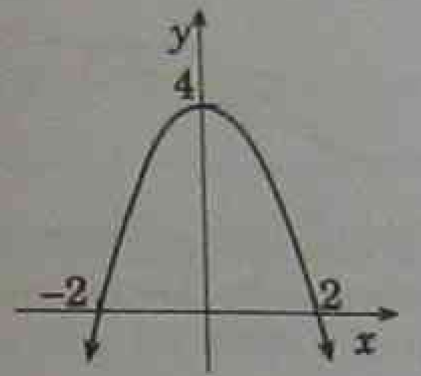
11. Find vertex of:
(a) $y = x^2 - 5x - 6$
(b $y = x^2 + 2x - 8$
(c) $y = 2x^2 - 3x + 1$
(d) $y = 5 - 2x - x^2$

12. Match the following graphs with equations:
1. $y = 2^x$ 2. $x^2 + y^2 = 4$
3. $y = (x + 2)^2$ 4. $y = 4 - x^2$
5. $y = -2x + 2$ 6. $y = x^3 + 2$
7. $y = 2x - 1$ 8. $xy = 2$
9. $y = (x - 2)^2$ 10. $y = -2 - x^3$

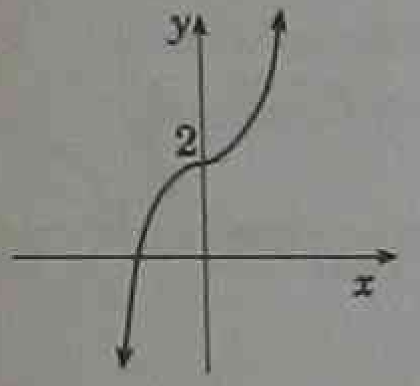
A.



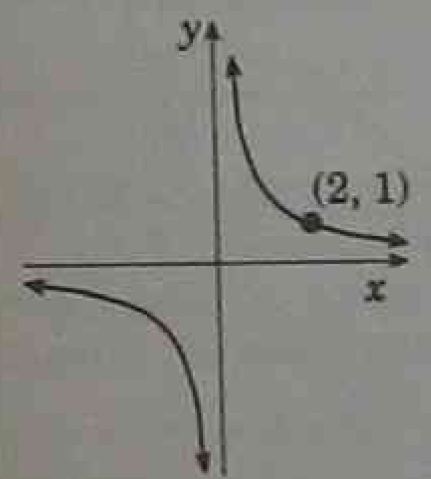
B.



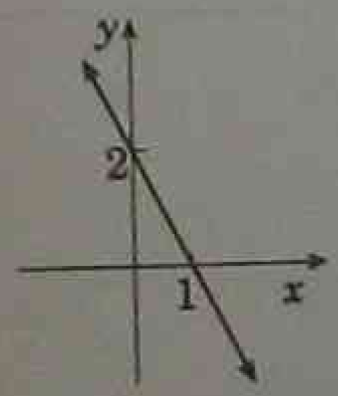
C.



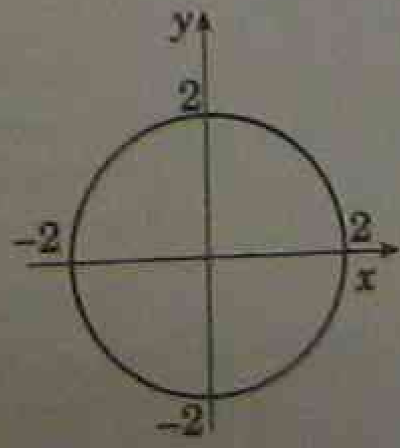
D.



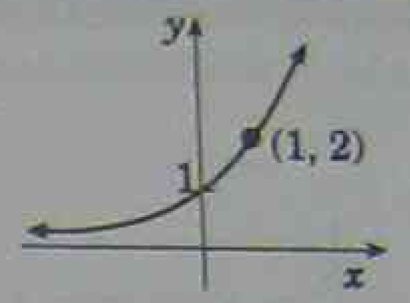
E.



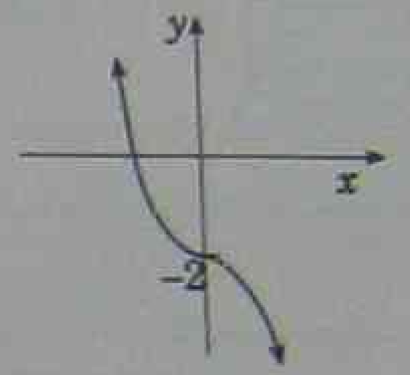
F.



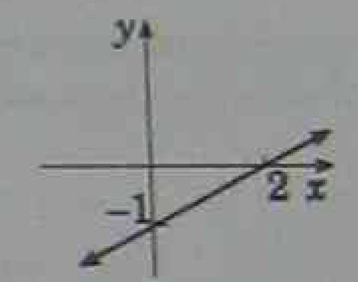
G.



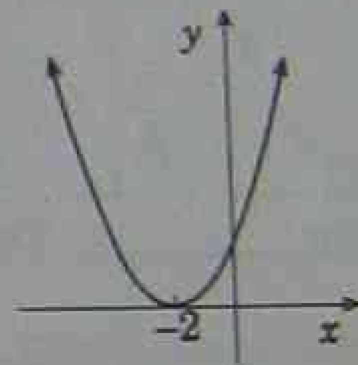
H.



I.



J.



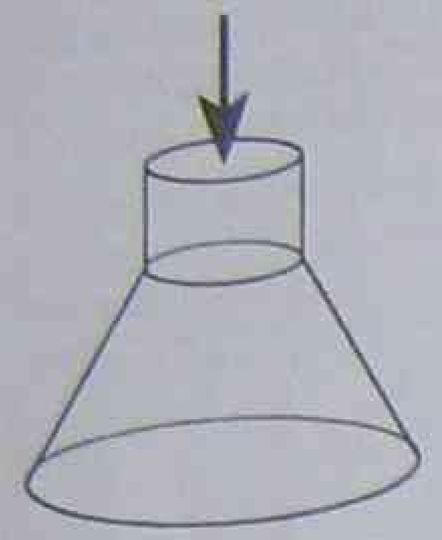
13. Graph the following on separate number planes:
(a) $\{(x, y): y > x + 1\}$
(b) $\{(x, y): y \leq 3 - x\}$
(c) $\{(x, y): y > x^2 + 1\}$
(d) $\{(x, y): y > x\}$

14. Which of the following points lie in the region $y > x + 7$:
(2, 1), (3, –1), (–1, 12)?

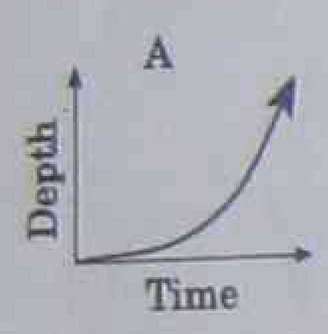
15. Does $y \leq x^2 - x - 1$ contain the point (3, –1)?

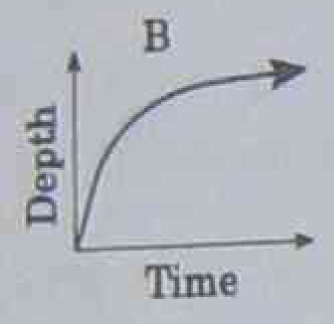
16. A piece of wire 30 cm long is bent to form a rectangle. Let the width be x cm.
(a) Find an expression for the length of the rectangle.
(b) Write an expression for the area A of the rectangle.
(c) Find the area enclosed if the width is 6 cm.
(d) Draw a graph to illustrate the possible values of the area in terms of x.
(e) What is the largest area that can be enclosed?

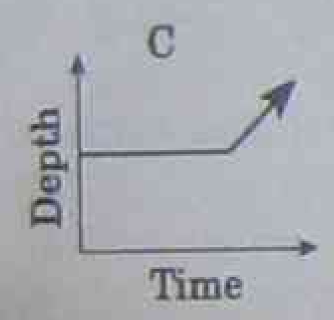
17. Water is poured into an empty container at a constant rate.

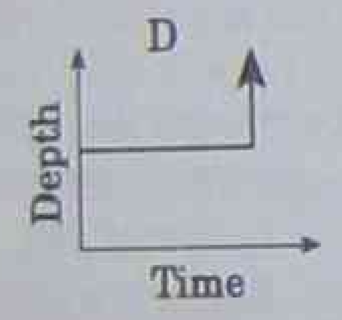


Which graph best represents the depth of water in the container as it fills?

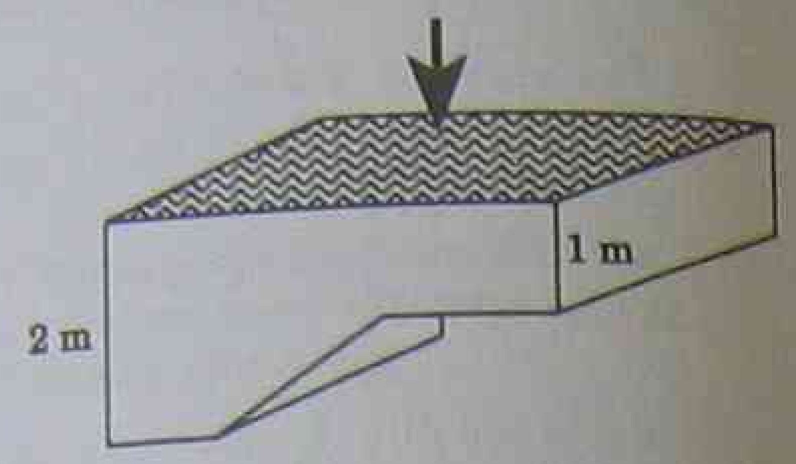




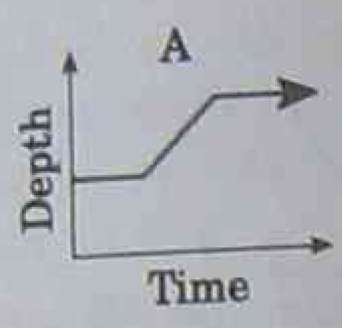


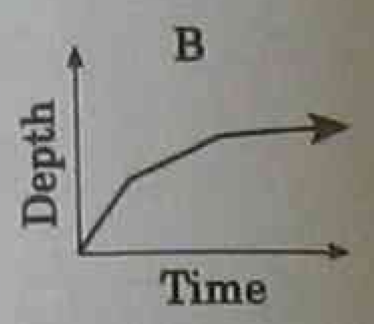


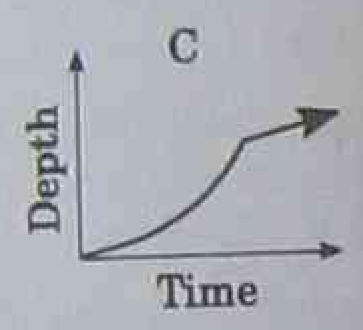
18. A swimming pool is being filled at a constant rate.

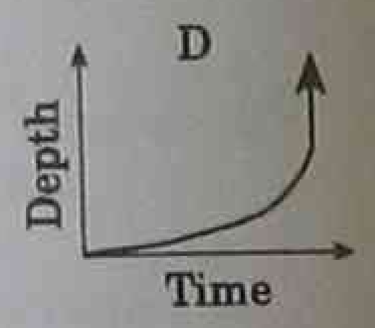


Which graph best represents the depth of water in the pool as it fills?









Chapter 6

COORDINATE GEOMETRY

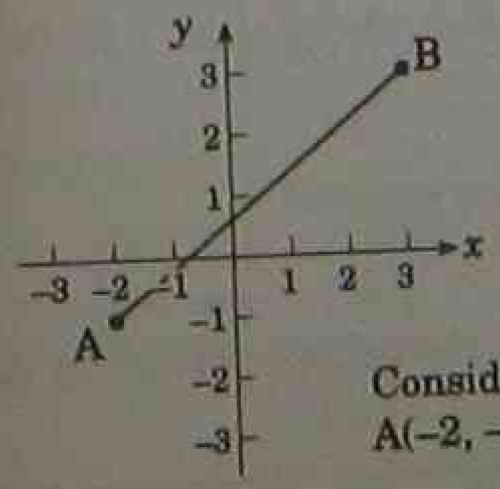
6.1 Important formulae

Three very important formulae used in coordinate geometry are:

1. The distance formula: $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$
   (Based on Pythagoras' Theorem)
2. The midpoint formula: $MP=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
3. The gradient formula: $m=\frac{y_2-y_1}{x_2-x_1}$

The distance formula is used to calculate the straight-line distance between points (x_1, y_1) and (x_2, y_2). The midpoint formula is used to find the coordinates of the midpoint of the interval joining (x_1, y_1) and (x_2, y_2) while the gradient formula is used to find the gradient (or slope) of the line containing the points (x_1, y_1) and (x_2, y_2).

Examples



Consider the points A(–2, –1) and B(3, 3).

Calculate:

(a) Distance AB
(b) Midpoint of AB
(c) Gradient of AB.

SOLUTION

Put $(x_1, y_1) = (-2, -1)$; $(x_2, y_2) = (3, 3)$.

$$\boxed{\begin{array}{l} x_1=-2,\quad x_2=3\\ y_1=-1,\quad y_2=3\end{array}}$$

Note: This is simply an application of Pythagoras' Theorem.

(a)
$$\begin{aligned} d&=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\\ &=\sqrt{(-2-3)^2+(-1-3)^2}\\ &=\sqrt{(-5)^2+(-4)^2}\\ &=\sqrt{25+16}\\ &=\sqrt{41}\end{aligned}$$

AB is $\sqrt{41}$ units long.

Note: This is the exact distance AB.

(b)
$$\begin{aligned} MP&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\ &=\left(\frac{-2+3}{2},\frac{-1+3}{2}\right)\\ &=\left(\tfrac{1}{2},1\right)\end{aligned}$$

Average of x-coordinates, average of y-coordinates.

Midpoint is $\left(\frac{1}{2},1\right)$.

Continued

(c) $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\boxed{\frac{\text{Difference in } y}{\text{Difference in } x}}$

$= \frac{3-(-1)}{3-(-2)}$

$= \frac{4}{5}$

Gradient is $\frac{4}{5}$.

Note: The gradient is always left as a simple fraction. Do not convert either to a decimal or to a mixed number.

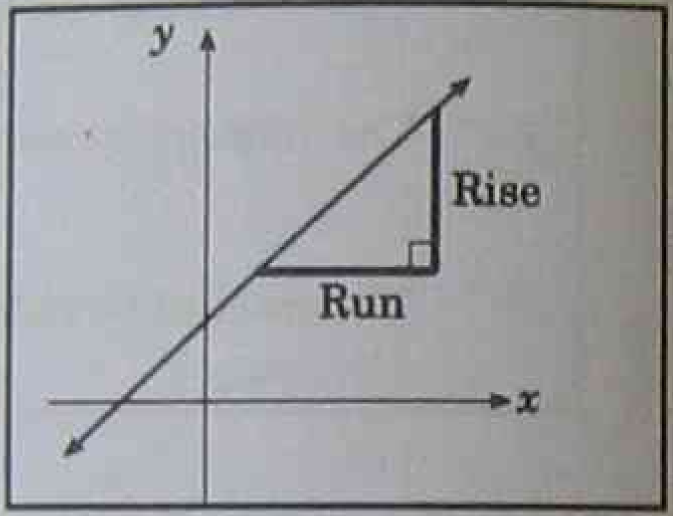
Remember when putting negative numbers on the calculator that it is the $\boxed{+/\_}$ button you use. For example, 3 – (–1) would involve

3 $\boxed{-}$ 1 $\boxed{+/\_}$ $\boxed{=}$

$(-3-2)^2$ is calculated:

3 $\boxed{+/\_}$ $\boxed{-}$ 2 $\boxed{=}$ $\boxed{\text{INV}}$ $\overset{x^2}{\boxed{\sqrt{\ }}}$

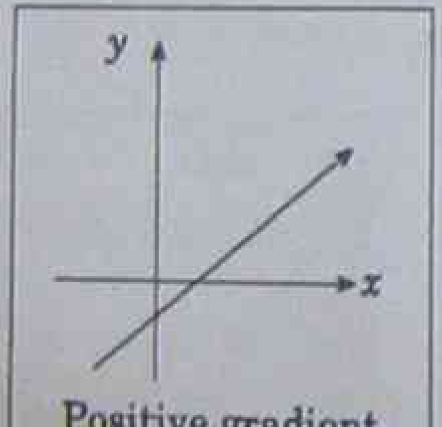
6.1.1 A note on gradient



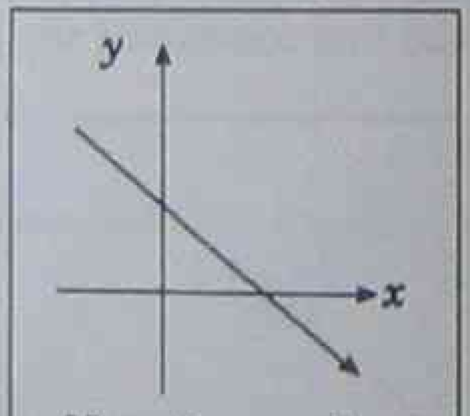
The gradient or slope of a line can be thought of as $\frac{\text{vertical rise}}{\text{horizontal run}}$ or $\frac{\text{rise}}{\text{run}}$.

This is the reason why the y-values are on top in the formula.

The gradient of a line can be either positive or negative. A line sloping upward to the right has a positive gradient; one sloping downward to the right has a negative gradient.



Positive gradient

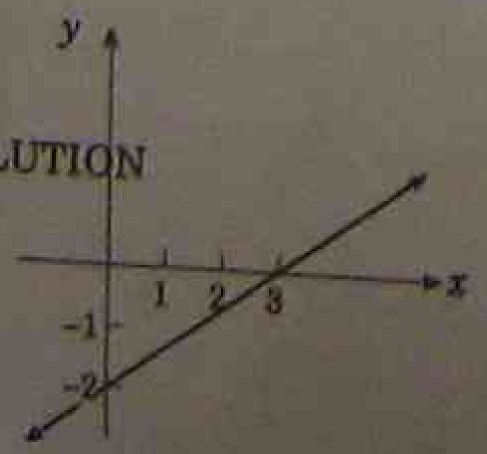


Negative gradient

The gradient can be found from the diagram by looking at the triangle formed by the line and the coordinate axes.

Examples

(a)



SOLUTION

Rise = 2 (↑)

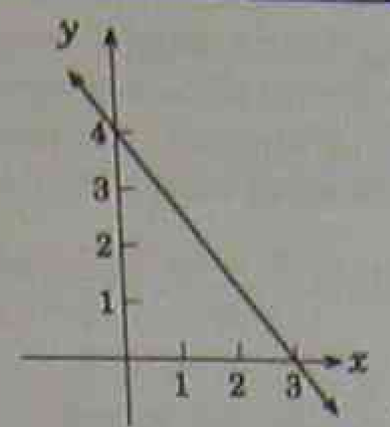
Run = 3 (→)

Direction is positive (↗)

$\text{Gradient} = \frac{\text{rise}}{\text{run}}$

$= \frac{2}{3}$.

(b)



SOLUTION

Rise = 4 (↑)

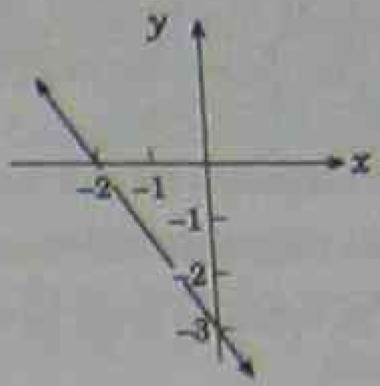
Run = 3 (→)

Direction is negative (↘)

$\text{Gradient} = \frac{\text{rise}}{\text{run}}$

$= -\frac{4}{3}$.

(c)



SOLUTION

Rise = 3

Run = 2

Direction is negative (↘)

$\text{Gradient} = \frac{\text{rise}}{\text{run}}$

$= -\frac{3}{2}$.

Remember: Take rise and run as positive lengths and then consider the sign of the gradient by observing the direction of the line.

6.1.2 Use of the formulae

Examples

(a) Plot the points A(–2, – 2), B(2, –2) and C(0, 3) on a number plane. Prove that:

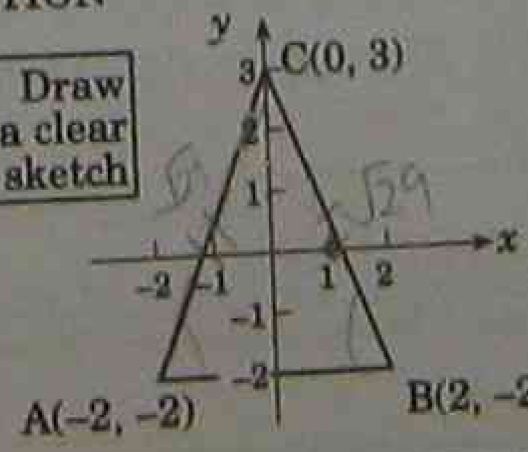
(i) Length AC is $\sqrt{29}$ units.

(ii) ΔABC is isosceles.

(iii) The midpoint of AB lies on the y-axis.

SOLUTION

Draw a clear sketch



(i) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(-2-0)^2 + (-2-3)^2}$

$= \sqrt{4+25}$

$= \sqrt{29}$ units

AC is $\sqrt{29}$ units.

(ii) For ΔABC to be isosceles, two sides must have the same length. Check the length of BC.

$BC = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2-0)^2 + (-2-3)^2}$

$= \sqrt{4+25}$

$= \sqrt{29}$ units,

$\therefore$ AC = BC $\left(\text{both } \sqrt{29}\right)$.

that is, ΔABC is isosceles.

(iii)

$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-2+2}{2}, \frac{-2+-2}{2}\right)$

$= (0, -2)$.

The point (0, –2) lies on the y-axis as the x-coordinate is 0.

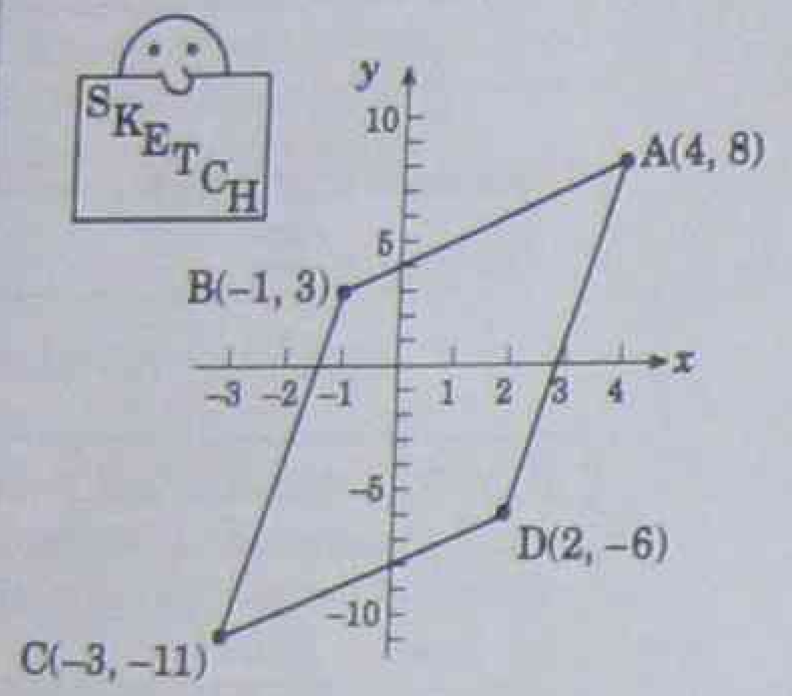
(b) The points A(4, 8), B(–1, 3), C(–3, –11) and D(2, –6) are the vertices of a parallelogram. Show that:

(i) The opposite sides are equal.

(ii) The diagonals bisect each other.

(iii) The opposite sides have equal gradients.

SOLUTION



(i) Using the distance formula:

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ in each case.

$$AB = \sqrt{(-1-4)^2 + (3-8)^2} = \sqrt{50}$$
$$CD = \sqrt{(-3-2)^2 + (-11+6)^2} = \sqrt{50}$$
$$\therefore\ AB = CD$$
$$AD = \sqrt{(4-2)^2 + (8+6)^2} = \sqrt{200}$$
$$BC = \sqrt{(-1+3)^2 + (3+11)^2} = \sqrt{200}$$
$$\therefore\ AD = BC.$$

The opposite sides are equal.

(ii) This example requires the midpoint of each diagonal, AC and BD. If they are the same, the diagonals must bisect each other. Using

$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ in each case

$$MP_{AC} = \left(\frac{-3+4}{2}, \frac{-11+8}{2}\right)$$
$$= \left(\frac{1}{2}, -\frac{3}{2}\right)$$
$$MP_{BD} = \left(\frac{-1+2}{2}, \frac{3-6}{2}\right)$$
$$= \left(\frac{1}{2}, -\frac{3}{2}\right),$$

therefore the diagonals intersect at $\left(\frac{1}{2}, -\frac{3}{2}\right)$, the midpoint of each diagonal.

The diagonals bisect each other.

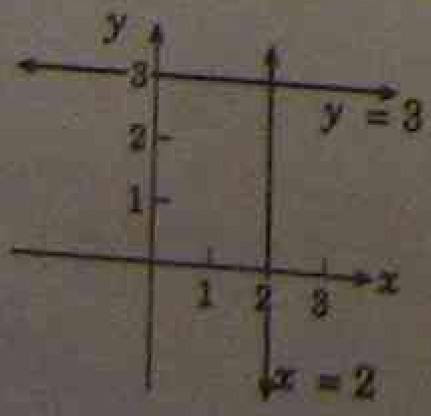
(iii) Using in each case $m = \frac{y_2 - y_1}{x_2 - x_1}$;

$$m_{AB} = \frac{8-3}{4-(-1)} \quad = \frac{5}{5} = 1$$
$$m_{CD} = \frac{-11-(-6)}{-3-2} = \frac{-5}{-5} = 1$$
$$m_{AD} = \frac{8-(-6)}{4-2} \quad = \frac{14}{2} = 7$$
$$m_{BC} = \frac{3-(-11)}{-1-(-3)} \quad = \frac{14}{2} = 7.$$

The opposite sides have equal gradients.

6.2 Forms of the straight-line equation

6.2.1 Lines parallel to the coordinate axes



Line parallel to x-axis: $y = b$

Line parallel to y-axis: $x = a$

The diagram shows the lines $y = 3$, parallel to the x-axis and $x = 2$, parallel to the y-axis.

---

16. Find the equation of the line which is:

(a) parallel to $y = 4x - 7$ and passing through (0, –4).

(b) parallel to $2x - 3y = 5$ and passing through (–1, –1).

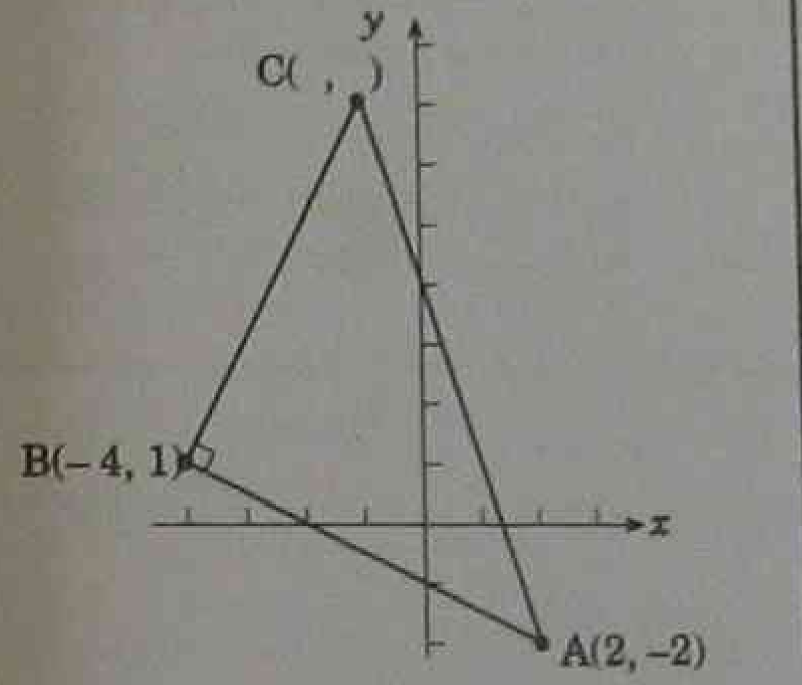
(c) perpendicular to $x + 3y - 6 = 0$ and passing through (4, 1).

(d) perpendicular to $2y = 5 - x$ and passing through (–2, 3).

17. If $2x - ky = 5$ is parallel to $3x + 4y - 7 = 0$, find the value of k.

18. Find the equation of the line which is the perpendicular bisector of the interval AB, given that A is (–3, 4) and B is (1, –2).

19.



A(2, –2), B(– 4, 1) and C are vertices of a right-angled triangle, with the right angle at B.

(a) Find the equation of the line AB.

(b) Find the gradient of the line BC.

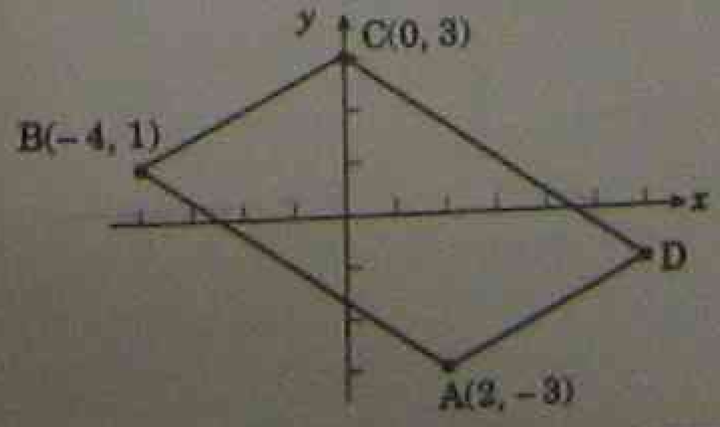
(c) Find the equation of the line BC.

(d) If C has coordinates (–2, 5), calculate the length of BC.

(e) Find the area of ΔABC.

(f) Find the equation of the median drawn from the midpoint of AB to the vertex C.

20.



A(2, –3), B(– 4, 1), C(0, 3) and D are the vertices of the parallelogram ABCD with AC as a diagonal.

(a) Calculate the gradient of AB and hence find its equation.

(b) Find the equation of CD.

(c) Write down the coordinates of D, the fourth vertex.

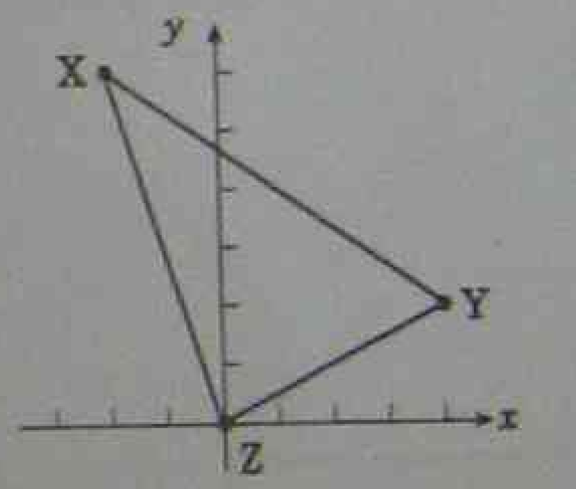
(d) Show that the interval joining the midpoints of AB and CD is parallel to BC.

21. Find the equation of the circle centre (0, –3) and radius 4 units.

22. Find the equation of the circle with diameter AB, where A and B have coordinates (–3, 3) and (1, –5) respectively.

23. Show that the point X(–2, 0) lies on the line passing through P(7, –3) and Q(–5, 1).

24. The vertices of the triangle XYZ are X(–2, 6), Y(4, 2) and Z(0, 0). Let M, N and P be the midpoints of XY, YZ and ZX respectively.



(a) Find the coordinates of M, N and P.

(b) Find the equations of the medians MZ, NX and PY.

(c) Show that the medians are concurrent.

(d) Prove that the product of the gradients of XY, YZ and ZX is 1.

(e) Find the equation of the *altitude* of the triangle through Z.

The *altitude* of a triangle is the line perpendicular to one side and passing through the opposite vertex.

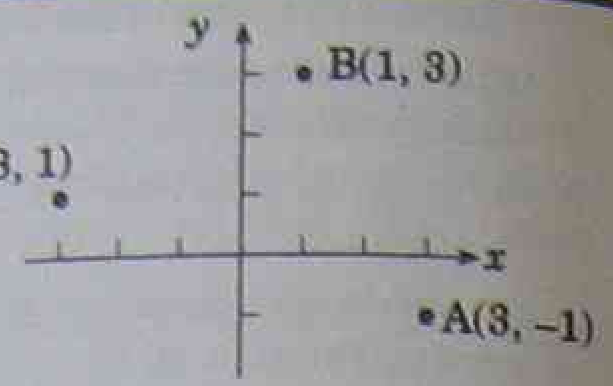
25. The point (2d, 5) lies on the line $2x - y - 8 = 0$. Find d.

26. Show that the quadrilateral with vertices P(−2, 3), Q(1, 5), R(5, −1) and S(2, −3) is a rectangle.

27. Find the equation of the line perpendicular to AB, where A is (−3, 5), B is (2, 4) and passing through the origin.

28. By first plotting the points A(2, 2), B(4, 8) and C(−2, 5), find:
    (a) The distance BC.
    (b) The equation of the line through B and C.
    (c) The equation of the line parallel to BC through A.
    (d) The equation of the line perpendicular to BC through A.
    (e) The midpoint of AB.
    (f) The equation of the median CL, where L is the midpoint of AB.

29. A, B and C are the points (3, −1), (1, 3) and (−3, 1) respectively.



    (a) Calculate the length BC.
    (b) Find the midpoint of BC.
    (c) Find the equation of the line BC.
    (d) Find the equation of the line through A parallel to BC. (Call it *l*).
    (e) Find the coordinates of the fourth vertex D of the parallelogram ABCD (AC being a diagonal).
    (f) Show that D lies on *l*.
    (g) Find the equation of the circle with BC as diameter.

Chapter 7

CHANCE AND DATA

Statistics involves the collection and organisation of information (data) so that:

(a) large masses of information can be easily analysed, and

(b) predictions can be made, based on the collected information.

Tables and graphs allow information to be presented in a clear, concise form. The information can also be readily analysed from the table or graph.

7.1 Simple distributions

7.1.1 Some definitions and formulae

Range = highest score − lowest score

Mode = most common score or score with the highest frequency

Median = middle score when scores are arranged in *ascending* order

$$\text{Mean} = \text{average} = \frac{\text{total of the scores}}{\text{number of scores}} = \frac{\sum xf}{\sum f}$$

7.1.2 A note on notation

x	score
f	frequency
c.f.	cumulative frequency
xf	(score)×(frequency)
$\sum$	sum
$\sum f$	sum of frequency column
$\sum xf$	sum of xf column
$\bar{x}$	mean

Examples

(a) The results of 25 Year One students in a weekly spelling test are:

				4	7	6
7	8	9	10	5	6	7
4	5	3	8	6	5	7
9	9	8	7			
8	6	4	9			

(i) Construct a frequency distribution table to enable you to answer the following questions:

(ii) How many students scored:
  (A) 8 marks.
  (B) 8 or less marks.
  (C) less than 8 marks.
  (D) more than 8 marks.

(iii) What percentage of students scored:
  (A) 8 marks.
  (B) 8 or less marks.
  (C) less than 8 marks.
  (D) more than 8 marks.

(iv) Find the range and the mode.

(v) Calculate the mean.

(vi) Find the median score.

(vii) Construct:
  (A) a frequency histogram.
  (B) a cumulative frequency histogram.

SOLUTION

(i)

Frequency distribution table

Score (x)	Tally	Frequency (f)	Cumulative frequency (c.f.)	Score × frequency xf
3	|	1	1	3 ⇐ (3 × 1)
4	[illegible]	3	4	12 ⇐ (4 × 3)
5	[illegible]	3	7	15 ⇐ (5 × 3)
6	[illegible]	4	11	24
7	[illegible]	5	16	35
8	[illegible]	4	20	32
9	[illegible]	4	24	36
10	|	1	25	10
	$\sum f$	25	$\sum xf$	167

(ii) (A) 4 (from frequency column)
  (B) 20 (from c.f. column)
  (C) 16 [same as 7 or less]
  (D) 25 − 20 = 5 [total less (8 or less)]

(iii) (A) $\text{Percentage} = \frac{4}{25} \times 100\%$
$= 16\%$

(B) $\text{Percentage} = \frac{20}{25} \times 100\%$
$= 80\%$

(C) $\text{Percentage} = \frac{16}{25} \times 100\% = 64\%$

(D) $\text{Percentage} = \frac{5}{25} \times 100\% = 20\%$

(iv) Range
$= \text{highest score} - \text{lowest score}$
$= 10 - 3 = 7$

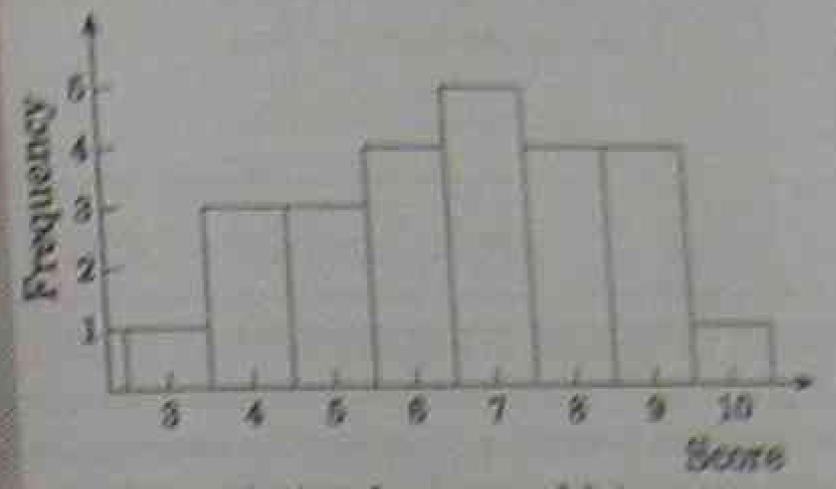
Mode
$= \text{score with highest frequency}$
$= 7$ (frequency is 5)

(v) $\text{Mean} = \frac{\sum xf}{\sum f}$
$= \frac{167}{25}$
$= 6.68$

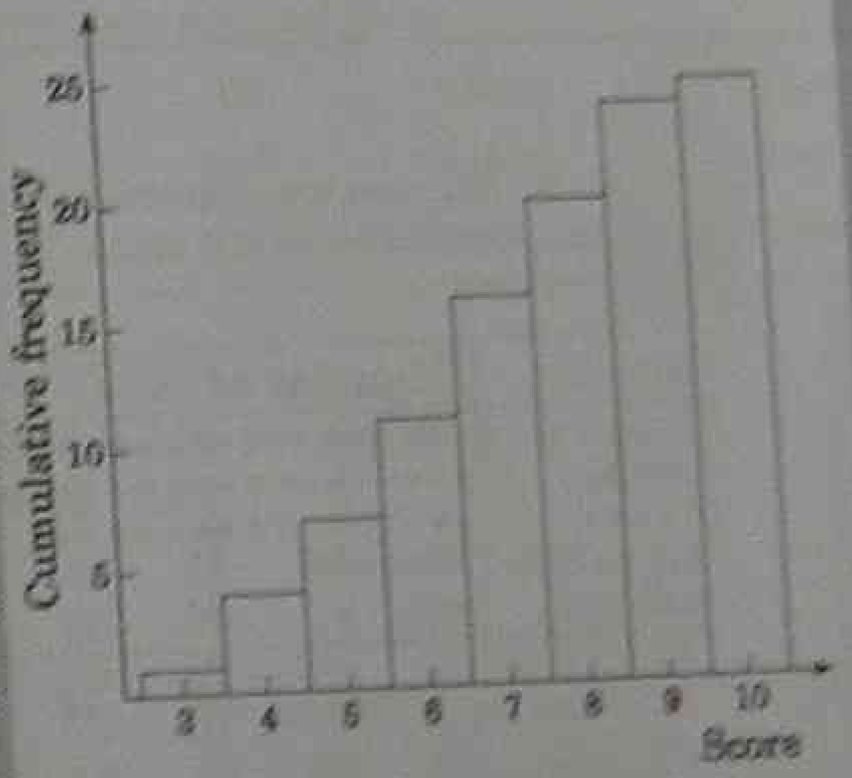
(vi) To find the median, we need the middle score. This is the 13th score. From the c.f. column, the 13th score is a 7. (The 12th, 13th, 14th, 15th, 16th scores are all 7.)

When we have 25 scores the 13th is the middle score, that is, 12 before it and 12 after it.

(vii) (A) Histogram (Column graph)

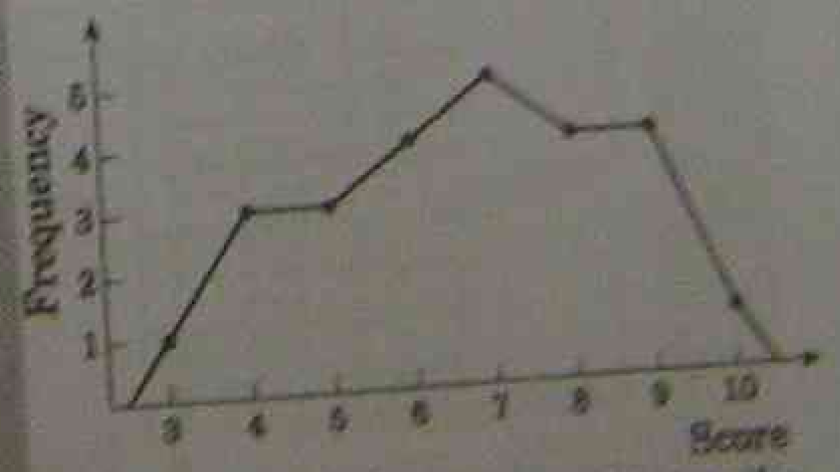


(B) Cumulative frequency histogram



Note that if a frequency polygon had been required, the graph would be as follows:

Frequency polygon



This can also be achieved by joining the midpoints of the tops of the columns in the histogram in (A).

(b) Sometimes, when there are only a few scores, a table is not necessary.

Remember your calculator will calculate the mean. For example: Gregory, over his first six innings, managed scores of 18, 32, 12, 12, 20 and 12.

(i) Find the range of these scores.

(ii) Write down the modal score.

(iii) Calculate the mean and the median.

(iv) If in his next three innings he scores 17, 15 and 20, find the new range, mode, mean and median.

(v) How many runs must he score in his tenth innings to have an overall average of 18?

SOLUTION

(i) Range
$= \text{highest score} - \text{lowest score}$
$= 32 - 12$
$= 20$

(ii) Mode = most common score
$= 12$

(iii) $\text{Mean} = \frac{\text{total of scores}}{\text{number of scores}}$
$= \frac{18 + 32 + 12 + 12 + 20 + 12}{6}$
$= \frac{106}{6}$
$= 17.7$ (one decimal place)

Median = middle score when scores are in ascending order

The scores in ascending order are:
$12, 12, \boxed{12, 18,}\ 20, 32$
↑
Middle scores

$\text{Median} = \frac{12 + 18}{2}$
$= 15$

When there is an even number of scores, the median will be the average of the two middle scores, that is, half-way between the two middle scores.

(iv) The scores are now 18, 32, 12, 12, 20, 12, 17, 15 and 20.

Range = 32 − 12 = 20 [unchanged]
Mode = 12 [unchanged]
Mean

$$= \frac{18+32+12+12+20+12+17+15+20}{9}$$

$$= \frac{158}{9} = 17.6 \text{ (one decimal place)}$$

The scores in ascending order are:
12, 12, 12, 15, 17, 18, 20, 20 and 32.

↑
Middle score

When there is an odd number of scores there is a single middle score.

The mean has dropped marginally while the median has risen.

(v) For Gregory to average 18 over ten innings, his total for ten innings would be (10 × 18) runs, that is, 180 runs. Over nine innings, he has scored 158 runs, thus he would have to score (180 – 158) runs, that is, 22 runs in his tenth innings

7.1.3 Dot plots

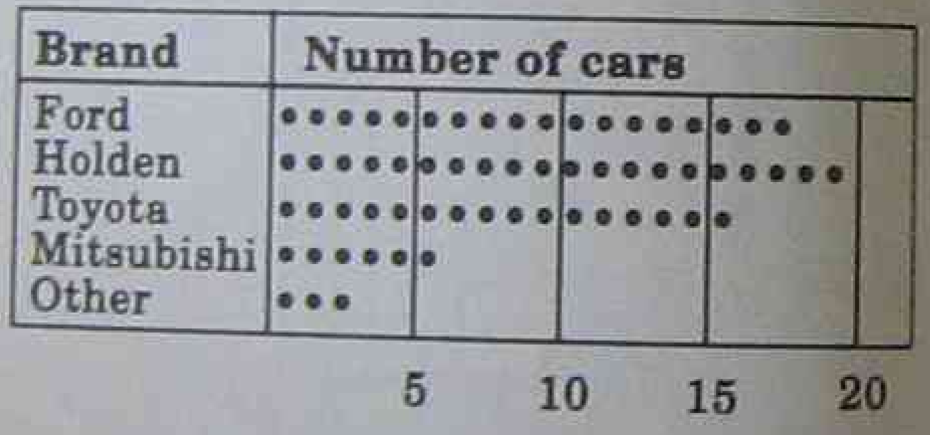
A dot plot is an alternative method of displaying information. It is simpler to draw and simpler to read than a histogram.

Example:

The type of cars passing along Macquarie Street over a half an hour period was noted and the results recorded in the following table:

Brand	Frequency
Ford	18
Holden	20
Toyota	16
Mitsubishi	6
Other	3
Total	63

A dot diagram was constructed from the information in the table.



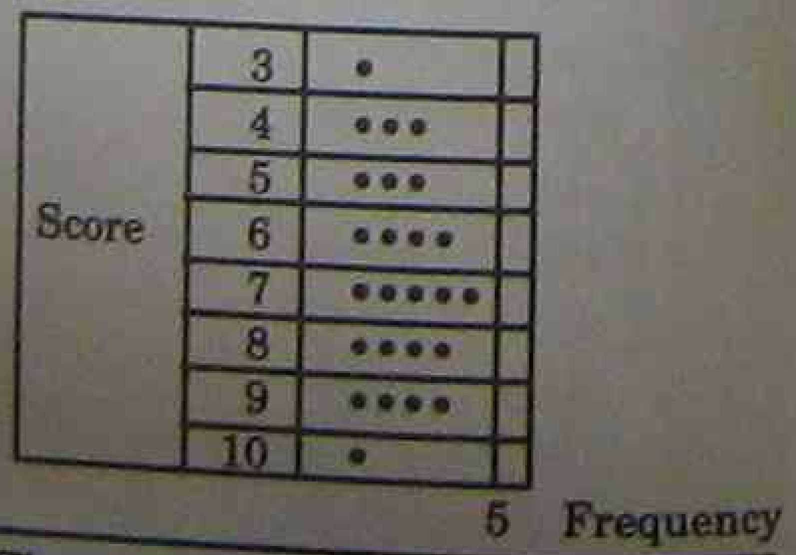
A dot diagram can also be used in the same way as a tally to organise raw data.

Example:

Consider the results of 25 Year One students in a weekly spelling tests:

7	8	9	10	4
7	6	4	5	3
8	5	6	7	9
9	8	7	6	5
7	8	6	4	9

Firstly, a rough dot plot is done



The numbers are transferred to the diagram as dots in the order that they occur.

This is then transferred to a frequency distribution table

Score	Frequency
3	1
4	3
5	3
6	4
7	5
8	4
9	4
10	1
	25

The rough dot plot serves as a tally as well as giving an indication of the size of the distribution and other features such as outliers.

7.1.4 Stem and leaf plots

This is another method of displaying numerical data.The leaf is the FINAL digit of a number where the stem is the FIRST digit or digits. The leaf is always a single digit. The stem can contain any number of digits.

For example consider these numbers: 68, 647, 24317. If these should be organised into stem and leaf then:

68	6 stem	8 leaf
647	64 stem	7 leaf
24 317	2431 stem	7 leaf

Example:

The results in a mathematics test out of 50 were:

19	48	36	40	31	22
18	27	18	20	18	36
49	50	45	13	9	17
22	31	39	26	28	30
44	8	23	19	46	33

Draw a stem and leaf plot for this data.

Initial plot

Stem is tens digit ⇓

Leaf is units digit ⇓

Stem	Leaf
0	9 8
1	9 8 8 8 3 7 9
2	2 7 0 2 6 8 3
3	6 1 6 1 9 0 3
4	8 0 9 5 4 6
5	0

← this row represents the scores 48, 40, 49, 45, 44, 46

Note that in the initial plot the numbers are entered as they occur.

Refined plot

Stem	Leaf
0	8 9
1	3 7 8 8 8 9 9
2	0 2 2 3 6 7 8
3	0 1 1 3 6 6 9
4	0 4 5 6 8 9
5	0

The digits in the leaf are arranged in increasing order in order to enable greater understanding of the scores.

A stem and leaf plot allows the smallest and largest numbers to be seen, as well as the size of the distribution and any clustering of data. It gives an indication of the groupings to be used in a grouped table and makes the tallying easier.

An application of the stem and leaf plot is to find range, mode, mean and median using the plot.

Example

Consider the stem and leaf plot drawn to illustrate the results of 30 students in a mathematics test. Find the range, mode, mean and median.

SOLUTION

Stem	Leaf	Number of leaves f	$f \times$ stem	Sum of leaves
0	89	2	0	17
1	3788899	7	70	52
2	0223678	7	140	28
3	0113669	7	210	26
4	045689	6	240	32
5	0	1	50	0
	$\sum$	30	710	155

- Range = Highest score – Lowest score = 50 – 8 = 42
- Mode = score(s) with highest frequency = 18   Look for digit occuring the most against each stem
- $$\text{Mean} = \frac{\text{Sum of } ([f \times \text{stem}] \text{ and } [\text{sum of leaves}])}{\text{Total } f}$$

$$= \frac{710+155}{30}$$

$$\bar{x} = \frac{\sum (f \times \text{stem}) + \sum \text{Sum of leaves}}{\sum f}$$

$$= \frac{865}{30}$$

$$\approx 28.8 \quad \text{(to 1 decimal place)}$$

- Median is the middle score — cross numbers off in the leaf column one from start and one from end until only a single number (odd number) or two numbers (even number). Median is the single number or average of the two remaining numbers.

and then to here

⇓

Stem	Leaves
0	$\cancel{8}\cancel{9}$
1	$\cancel{3}\cancel{7}\cancel{8}\cancel{8}\cancel{8}\cancel{9}\cancel{9}$
2	$\cancel{0}\cancel{2}\cancel{2}\cancel{3}\cancel{6}78$
3	$\cancel{0}\cancel{1}\cancel{1}\cancel{3}\cancel{6}\cancel{6}\cancel{9}$
4	$\cancel{0}\cancel{4}\cancel{5}\cancel{6}\cancel{8}\cancel{9}$
5	$\cancel{0}$

⇑

Start here

27 and 28 are left
∴ the median is

$$\frac{27+28}{2} = 27.5$$

Median $= 27.5$

Range = 42   Mode = 18   Mean = 28.8   Merdian = 27.5

7.1.5 Interquartile range

When scores are arranged in ascending order:

The **median** is the middle score of the distribution.

The **upper quartile** is the middle score between the median and the highest score.

The **lower quartile** is the middle score between the median and the lowest score.

The **interquartile range** = upper quartile – lower quartile

Note: 50% of the scores are represented by the interquartile range.

Example:

For the scores:

(i) 3, 5, 4, 6, 4, 4, 8, 7, 6, 8

(ii) 18, 20, 16, 16, 18, 14, 18, 21, 19

Find

(a) the median

(b) the upper quartile

(c) the lower quartile

(d) the interquartile range.

SOLUTION

(i) Arrange in ascending order

⇓

3, 4, 4, 4, 5, 6, 6, 7, 8, 8

⇑

(a) Median = Average of 5th and 6th scores

$$= \frac{5+6}{2} = 5.5$$

Even number of scores
∴ 2 middle numbers

(b) For upper quartile use only the scores above the median, that is,

6, 6, 7, 8, 8

⇑

Upper quartile
= middle number of scores above median
= 7

Odd number of scores
∴ one middle number

(c) For lower quartile use only the scores below the median, that is,

3, 4, 4, 4, 5

⇑

Lower quartile
= middle number of scores below median
= 4

(d) Interquartile range
= Upper quartile – Lower quartile
= 7 – 4
= 3

(ii) Arrange in ascending order

⇓

14, 16, 16, 18,18, 18, 19, 20, 21

⇑

(a) Median = middle score = 18

One midddle score

(b) For upper quartile use only the scores above the median, that is,

18, 19, 20, 21

⇑

Upper quartile
= Average of 2nd and 3rd scores

$$= \frac{19+20}{2} = 19.5$$

(b) For lower quartile use only the scores below the median, that is,

14, 16, 16, 18

⇑

Lower quartile
= Average of 2nd and 3rd scores
= 16

(d) Interquartile range
= Upper quartile – Lower quartile
= 19.5 – 16
= 3.5

A stem and leaf plot can be used to find both the median and the quartiles.

Example:

Consider the stem and leaf plot from before. The results of 30 students in a mathematics test out of 50 have been tabulated.

Stem	Leaf
0	~~8~~ ~~9~~
1	~~3~~ ~~7~~ ~~8~~ ~~8~~ ~~8~~ ~~9~~ ~~9~~
2	~~0~~ ~~2~~ ~~2~~ ~~3~~ ~~6~~ 7 8 ←
3	0 1 1 3 6 6 9
4	0 4 5 6 8 9
5	0

There are 30 scores so that the median is the average of the 15th and 16th scores,

that is, median $= \frac{27+28}{2}$

$= 27.5$

For the upper quartile consider only numbers above the median (28, 30, ...). There are 15 of these so upper quartile will be the 8th number above the median.

Cross off 7 numbers

⇓

2	~~8~~
3	~~0~~ ~~1~~ ~~1~~ ~~3~~ ~~6~~ ~~6~~ 9
4	0 4 5 6 8 9
5	0

15 numbers

Upper quartile = 39.

For the lower quartile consider only numbers below the median (8, 9, ..., 27). There are 15 of these so upper quartile will be the 8th number above the smallest score. Cross off the first 7 numbers

⇓

Stem	Leaf
0	~~8~~ ~~9~~
1	~~3~~ ~~7~~ ~~8~~ ~~8~~ ~~8~~ 9 9
2	0 2 2 3 6 7

15 numbers

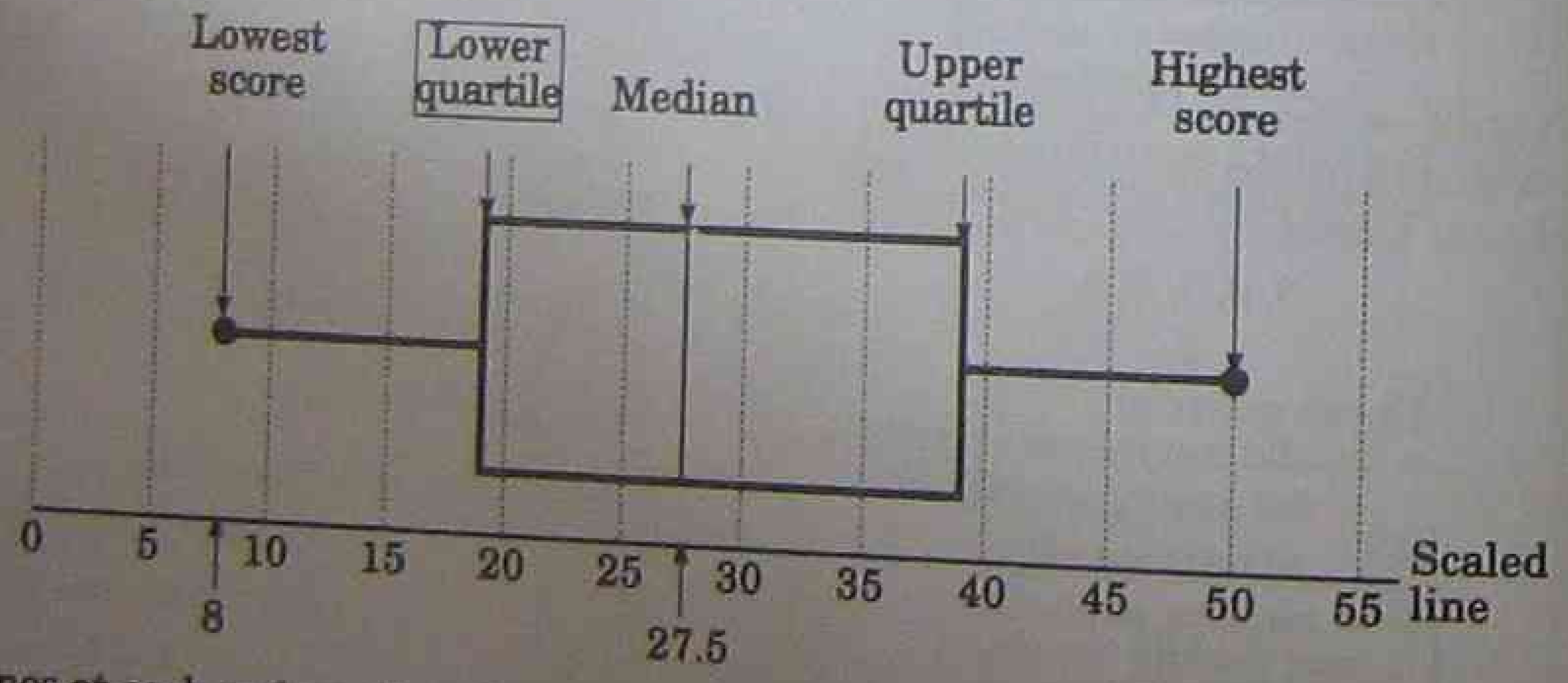
The lower quartile = 19

Interquartile range = 39 − 19 = 20

7.1.6 Box-and-whisker plots

A box-and-whisker plot uses five selected pieces of information — lowest score, highest score, median, upper and lower quartiles — to give an alternate display for a set of scores.

Example: Consider a set of data with lowest score 8, highest score 50, median 27.5, upper quartile 39 and lower quartile 19.



The lines at each end are the whiskers, while the centre rectangle is the box.The box contains 50% of the scores. The lower whisker represents 25% of the scores as does the upper whisker. A box-and-whisker plot can be drawn to illustrate any numerical data regardless of how the quartiles and the median are calculated.

Examples:

(a) For the scores 3, 5, 4, 6, 4, 4, 8, 7, 6, 8 calculate the median, upper and lower quartiles and draw a box-and-whisker plot.

SOLUTION

Rearrange scores:

3, 4, 4, 4, 5, 6, 6, 7, 8, 8

⇑

Median $= \frac{5+6}{2} = 5.5$

For lower quartile: 3, 4, 4, 4, 5

⇑

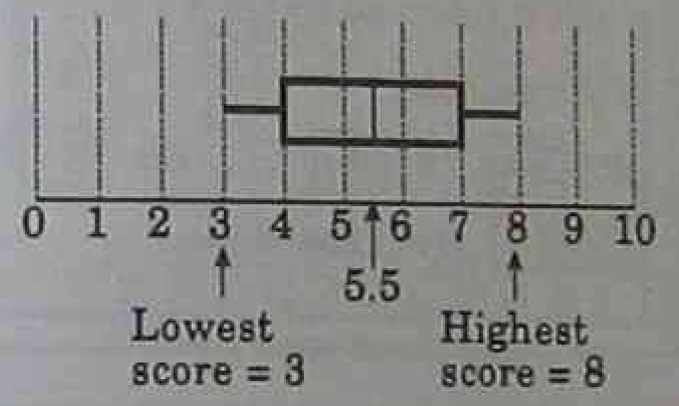
Lower quartile = 4

For upper quartile: 6, 6, 7, 8, 8

⇑

Upper quartile = 7

Interquartile range = 7 − 4 = 3



(b) A stem-and leaf plot has been constructed to represent the scores of 25 students in a spelling test out of 30. Construct a box-and-whisker plot from the information.

Stem	Leaf
0	7 7 8 9 9
1	0 1 1 4 4 6 6 9 — Median
2	1 1 2 2 5 8 9 9 9
3	0 0 0

Middle of lower scores

Middle of upper scores

SOLUTION

There are 25 scores.

∴ median is the 13th score

Median = 19

For lower quartile consider only lower 12 scores. The two middle scores (6th and 7th) are 10 and 11

Lower quartile $= \frac{10+11}{2} = 10.5$

For upper quartile consider only upper 12 scores. The two middle scores are 28 and 29.

Upper quartile $= \frac{28+29}{2} = 28.5$

Summary

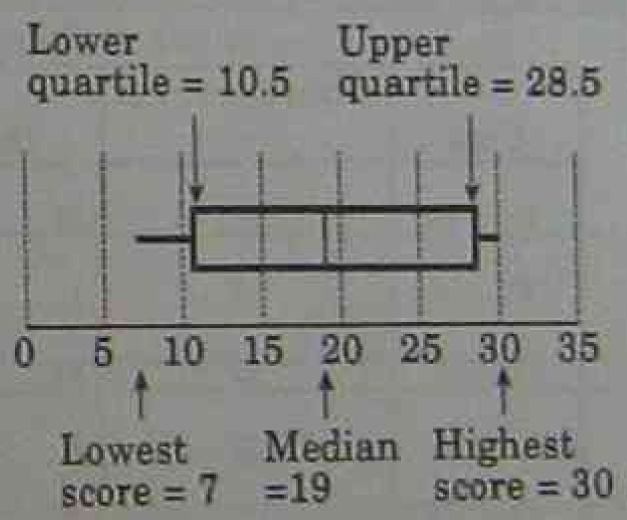
Lower score = 7 Highest score = 30

Lower quartile = 10.5

Median = 19

Upper quartile = 28.5

Box-and-whisker plot



7.2 Grouped data

7.2.1 Grouped frequency distribution

Sometimes the range of a set of data is too large to use a simple frequency distribution table. A grouped frequency distribution table is then used. Instead of considering individual scores, the scores are grouped into classes. The class centre (middle score of the group) is then taken as representative of the group. Consider the following example:

Example

(a) Alison notes the time she devotes to homework each night for 30 consecutive school days. The results (in minutes) are:

(25) (72) 64, 38, 29, 36, 42, 63, 49, 50, 39, 55, 61, 59, 27, 39, 54, 27, 60, 32, 42, 46, 54, 71, 27, 52, 68, 63, 69, 60.

(i) By taking the group 25– 29 as the first class, construct a grouped frequency distribution table. What is the range?

(ii) Find the modal class.

(iii) Calculate the mean.

(iv) Within which group is the median?

(v) Draw a histogram and cumulative frequency polygon.

SOLUTION

(i) See the table on page 87.

For the range, circle the smallest and largest numbers from the original data.

Range = 72 – 25 = 47. This indicates the spread of the distribution.

(ii) Modal class = class with highest frequency

= 60 – 64

(iii) $$\text{Mean} = \frac{\sum x \times f}{\sum f} = \frac{1465}{30} = 48.833\,33 \approx 48.8 \text{ (one decimal place)}$$

This could be done on your calculator using SD mode, that is,

$\boxed{\text{MODE}}\boxed{\bullet}$

Scores are entered individually and each is followed by $\boxed{\text{M+}}$.

For example,

25 $\boxed{\text{M+}}$ 72 $\boxed{\text{M+}}$ . . . 60 $\boxed{\text{M+}}$.

$\boxed{\text{INV}}\underset{\bar{x}}{\boxed{7}}$ gives the mean.

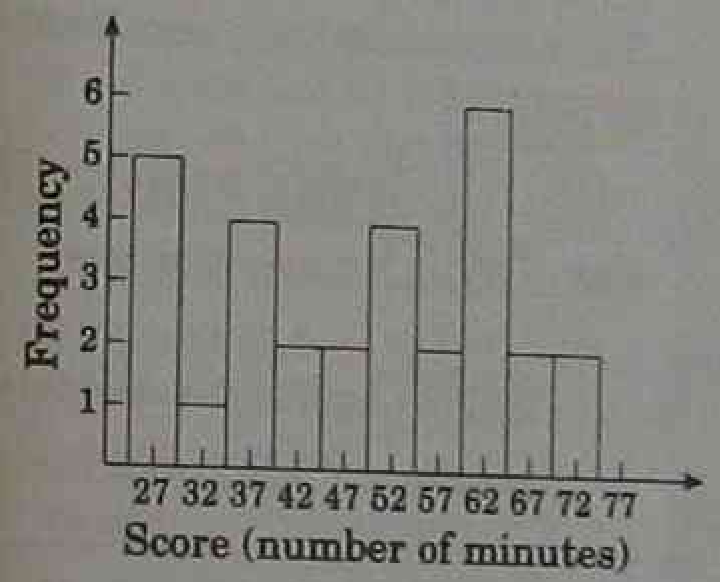
Class	Class centre (x)	Tally	Frequency (f)	Cumulative frequency (c.f.)	$x \times f$
25 – 29	27	卌	5	5	135
30 – 34	32	|	1	6	32
35 – 39	37	||||	4	10	148
40 – 44	42	|	2	12	84
45 – 49	47	|	2	14	94
50 – 54	52	||||	4	18	208
55 – 59	57	|	2	20	114
60 – 64	62	卌 |	6	26	372
65 – 69	67	|	2	28	134
70 – 74	72	|	2	30	144
		Σ	30	Σ	1465

↑ Middle of the group — or average of the end points

When using this method, always note near your answer that a calculator was used. You should not use this method if the question implies that the mean is to be calculated from the table.

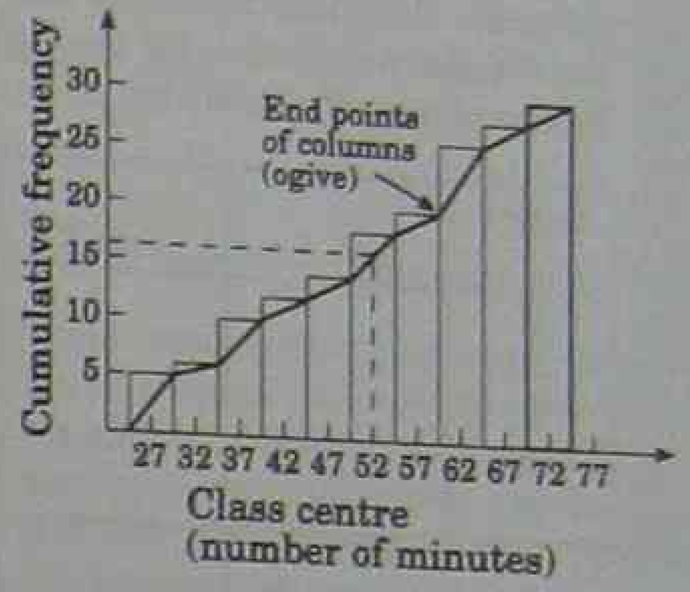
(iv) As there are 30 numbers, the median will lie between the 15th and 16th number after they are arranged in ascending order. From the c.f. column, both the 15th and 16th scores lie in the class 50 – 54. The median lies in the class 50 – 54.

(v) Histogram — a column graph using class centres and frequency.



Note: Class centres are along the horizontal axis.

Cumulative frequency polygon — use cumulative frequencies and class centres



First, construct a c.f. histogram, and then join the ends of the tops of columns. This is also called the *ogive*.

The median can be determined from this graph by drawing a line across from the c.f. axis at the level of the 15th/16th score until it meets the ogive. Drop another line straight down at that point to meet the horizontal axis. The median class can then be read from the axis.

7.3 Measures of dispersion

7.3.1 Measures of central tendency

The mean, median and mode are known as measures of central tendency as they tend to indicate the centre of a distribution.

The extent to which a set of scores tends to spread about an average value is the dispersion or variation of the data. Two common measures of this are the range and the standard deviation. (Range has been discussed in Section 7.1.)

7.3.2 Standard deviation

$$\sigma_n = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

σ_n = standard deviation
$\bar{x}$ = mean
n = number of scores (or $\sum f$)

Note: Look for the σ_n button on your calculator.

Example

(a) Jenny noted the time taken to travel to work each day for a week. The results were: 18, 20, 16, 15 and 21 minutes. Calculate the mean and then the standard deviation for these times by completing the following table:

Score x	$x-\bar{x}$	$(x-\bar{x})^2$
15		
16		
18		
20		
21		
	Σ	

SOLUTION

Score x	$x-\bar{x}$	$(x-\bar{x})^2$
15	−3	9
16	−2	4
18	0	0
20	2	4
21	3	9
	Σ	26

$$\text{Mean} = \frac{15+16+18+20+21}{5}$$
$$= \frac{90}{5}$$
$$= 18$$

Standard deviation

$$= \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$
$$= \sqrt{\frac{26}{5}}$$
$$= 2.280\,3509$$
$$\approx 2.3 \text{ (one decimal place)}$$

Complete the process again using the S.D. functions on your calculator:

$\boxed{\text{MODE}}\,\boxed{\cdot}$

15$\boxed{\text{M+}}$ 16$\boxed{\text{M+}}$ 18$\boxed{\text{M+}}$ 20$\boxed{\text{M+}}$ 21$\boxed{\text{M+}}$

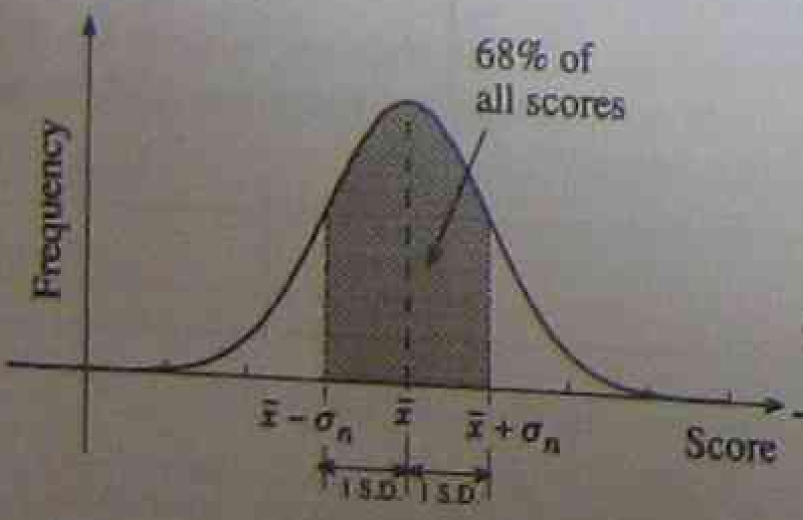
Now $\boxed{\text{INV}}\,\boxed{\bar{x}}$ gives the mean: $\bar{x} = 18$

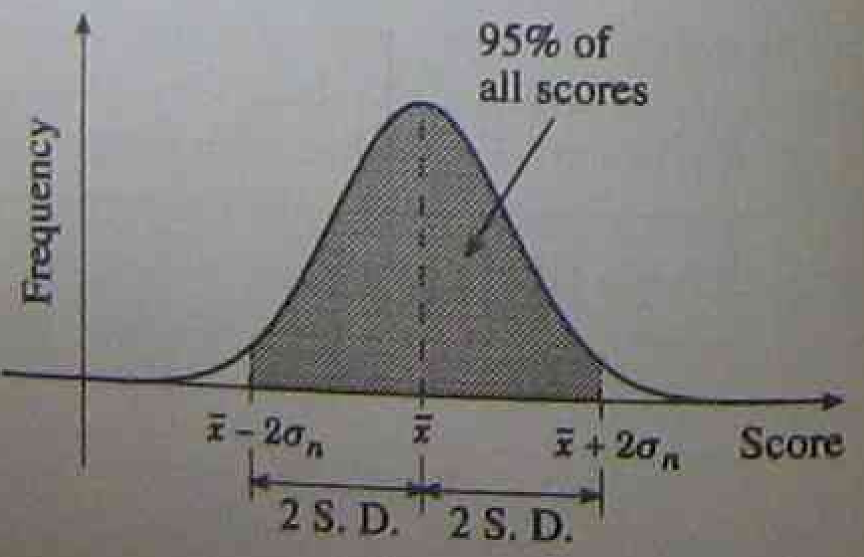
$\boxed{\text{INV}}\,\boxed{\sigma_n}$ gives the standard deviation: $\sigma = 2.3$ (one decimal place)

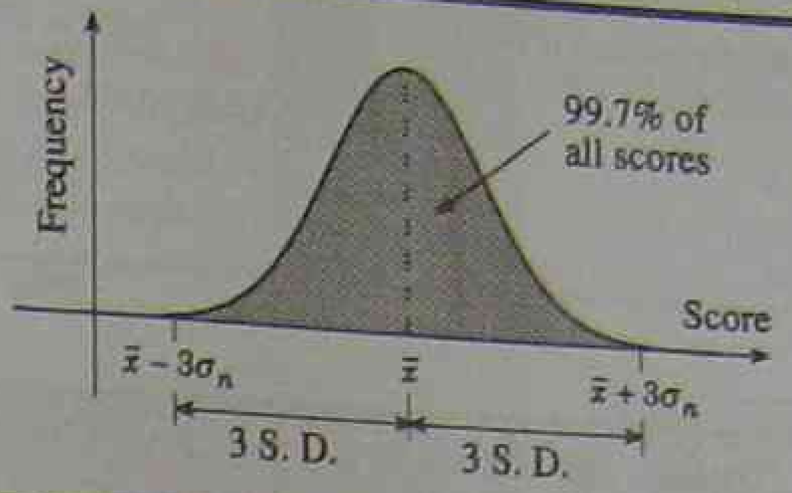
7.3.3 Normal distribution

In a normal distribution (the graph of the distribution approximates a bell-shaped curve)

1. 68% of the scores will lie within one standard deviation (S.D.) of the mean;
2. 95% will lie within two S.D. of the mean; and
3. 99.7% will lie within three S.D. of the mean.







Examples

(a) Shoe laces for a particular new line of Larynx joggers are manufactured 60 cm long with a standard deviation of 0.5 cm.

EXPLANATION

Then 68% of all laces will lie between 60 – 0.5 cm and 60 + 0.5 cm, that is, between 59.5 cm and 60.5 cm.

95% will lie between 60 – 2(0.5) cm and 60 + 2(0.5) cm, that is, 59 cm and 61 cm.

99.7% will lie between 60 – 3(0.5) cm and 60 + 3(0.5) cm, that is, 58.5 cm and 61.5 cm.

(b) Wood-turned wine glasses are produced with an average diameter of 8.8 cm and standard deviation of 0.05 cm. Any glass not within two standard deviations is rejected for sale. Find the largest and smallest diameter a wine glass can have without being rejected.

SOLUTION

$$\text{Mean} + 2\text{S.D.} = 8.8 + 2(0.05) \text{ cm}$$
$$= 8.9 \text{ cm}$$
$$\text{Mean} - 2\text{S.D.} = 8.8 - 2(0.05) \text{ cm}$$
$$= 8.7 \text{ cm}$$

The limits within which wine glasses are not rejected are 8.7 cm to 8.9 cm.

7.4 Relative frequency

$$\text{Relative frequency of a particular score} = \frac{\text{frequency of that score}}{\text{total frequency}}$$

$$\text{r.f.} = \frac{f}{\sum f}$$

Example

(a) The Blondie matchbox factory produces boxes of 50 matches. One hundred boxes are chosen randomly and the number of matches in each box counted. The table shows the results of this survey, including the relative frequency for each different count.

From the table it is a simple matter to answer questions such as:

(i) What percentage of boxes contained 50 matches?

(ii) What percentage of boxes contained 48 matches?

Number of matches	Frequency	Relative frequency
48	3	0.03*
49	8	0.08
50	78	0.78
51	7	0.07
52	2	0.02
53	2	0.02
Σ	100	1.00

$$^{*}\ \text{r.f.} = \frac{3}{100} = 0.03$$

(iii) What percentage contained 50 matches or more?

SOLUTION

(i) $0.78 = 78\%$

(ii) $0.03 = 3\%$

(iii) $0.78 + 0.07 + 0.02 + 0.02$

$= 0.89 = 89\%$

7.5 Probability

A coin is tossed once. There is one chance in two that it will land heads face up. We say that the *probability* of throwing a head is one out of two, or:

$$P(\text{head}) = \tfrac{1}{2}.$$

As each outcome is equally likely, the probability of throwing a tail is also $\frac{1}{2}$,

that is, $$P(\text{tail}) = \tfrac{1}{2}.$$

When throwing a die, there are six faces and each is equally likely to turn up. It cannot be predicted which number will be face up. The probability of throwing a six is one chance out of six, that is $P(6) = \frac{1}{6}$.

7.5.1 Definition of probability

The probability of an event occurring is defined as follows:

$$\text{Probability (event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Consider the die. The number of favourable outcomes was one (throwing a six), while the number of possible outcomes was six (each face).

$$\therefore\ P(6) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$$

Example

(a) A fair die is thrown. Find the probability of rolling:

(i) a 3.

(ii) a 5.

(iii) an odd number.

(iv) an even number.

(v) a number larger than 4.

SOLUTION

(i) $P(3) = \frac{1}{6}$ (ii) $P(5) = \frac{1}{6}$

(iii) There are three odd numbers (1, 3, 5),

$\therefore\ P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$

(iv) $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

(v) There are two numbers larger than 4 — 5 and 6.

$P(> 4) = \frac{2}{6} = \frac{1}{3}$

7.5.2 Range of probability

A certainty has a probability of 1. An impossibility has a probability of 0.

The probability of any specific event occurring has a value between 0 and 1. A value outside this range is meaningless. Thus, if there are two possibilities, the probability of the event (say A) not occurring is $1 - P(A)$. They are complementary events.

Example

(a) Michelle's school-bag contains two black socks and three grey socks. If one sock is drawn out at random, what is the probability that the sock is:

(i) black (ii) grey

SOLUTION

(i) $P(\text{black}) = \dfrac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \dfrac{2}{5}$ ← $\boxed{\begin{array}{l}\text{Total possible outcomes}\\ = \text{total number of socks}\end{array}}$

(ii) $P(\text{grey}) = 1 - P(\text{black})$

$= 1 - \dfrac{2}{5}$

$= \dfrac{3}{5}$

7.5.3 Relative frequency and probability

Nicholas throws an unbiased die 180 times. The results are below:

The probability of throwing a one is theoretically $\frac{1}{6}$, or 0.167 (three decimal places). Nicholas has thrown 28 ones out of 180 throws.

Score	Frequency	Relative frequency
1	28	0.156
2	30	0.167
3	31	0.172
4	29	0.161
5	32	0.178
6	30	0.167
Σ	180	1.000

Experimental probability

$= \dfrac{28}{180} = 0.156$ (three decimal places)

← $\boxed{\text{Remember r.f.} = \dfrac{f}{\text{total } f}}$

Over a large number of trials the experimental probability will eventually reflect the theoretical probability.

7.6 Diagrammatic approach to probability

A diagram is a very useful method of showing both the total number of possibilities and the favourable outcomes.

7.6.1 Table form (Also called dot diagram)

The table form is generally used for questions involving two dice or sets of two numbered cards.

Examples

(a) Mitchell throws two dice. Find the probability that the sum of the numbers face up will be:

(i) 7

(ii) 8

(iii) 12

(iv) greater than 7

(v) 7 or more

(vi) 1

(vii) even or less than 7

(viii) even *and* less than 7.

SOLUTION

Draw up a table to show all possibilities.

First Die

Second Die

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

From the table, count the total number of possible outcomes — 36.

So, all answers will be out of 36.

(i) $P(7) = \frac{6}{36} = \frac{1}{6}$ (Highlighted in table)

(ii) $P(8) = \frac{5}{36}$

(iii) $P(12) = \frac{1}{36}$

(iv) $P(>7) = \frac{15}{36} = \frac{5}{12}$

(v) $P(\geq 7) = \frac{21}{36} = \frac{7}{12}$

(vi) $P(1) = 0$ (impossible with two dice)

(vii) $P(\text{even or less than } 7) = \frac{24}{36} = \frac{2}{3}$

(viii) $P(\text{even and less than } 7) = \frac{9}{36} = \frac{1}{4}$

(b) Scott throws two dice and then calculates the product of the two numbers face up. Find the probability that the product is:

(i) 6 (ii) 36

(iii) less than 24

(iv) even

(v) even or less than 24

(vi) even and less than 24

(vii) is not 36.

SOLUTION

The first step is to draw up the table.

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Now count the total number of possible outcomes — again it is 36. This then becomes the denominator in all the probabilities.

(i) $P(6) = \frac{4}{36} = \frac{1}{9}$ (Highlighted in table)

(ii) $P(36) = \frac{1}{36}$

(iii) $P(< 24) = \frac{30}{36} = \frac{5}{6}$

(iv) $P(\text{even}) = \frac{27}{36} = \frac{3}{4}$

(v) $P(\text{even and less than } 24)$

$$= \frac{22}{36} = \frac{11}{18}$$

(vi) $P(\text{even or less than } 24) = \frac{35}{36}$

(vii) $P(\text{not } 36) = 1 - P(36)$

$$= 1 - \frac{1}{36}$$

$$= \frac{35}{36}$$

(b) How many bears found:
 (i) 4 lumps?
 (ii) 4 or less lumps?
 (iii) less than 4 lumps?

(c) What percentage of bears had:
 (i) 4 lumps?
 (ii) at least 4 lumps?

(d) If a bear is chosen at random, what is the probability that his porridge contained:
 (i) 4 lumps?
 (ii) less than 4 lumps?

(e) Calculate the mean number of lumps and the median.

(f) Draw a histogram.

3. Gai has been playing soccer for six years and has recorded her goal-scoring record. Over these years Gai scored 8, 7, 4, 8, 6, 3 goals.

(a)(i) Calculate Gai's mean number of goals.
 (ii) Find the range and the mode.
 (iii) Find the median.

(b) In her seventh year of soccer Gai struck brilliant form to score 13 goals. How will this affect Gai's mean and median?

(c) After 8 years, Gai's mean number of goals slumps to 6.5. How many goals did she score in her eighth year?

4. For the following sets of scores, find the mean and median. (To one decimal place if necessary.)

(a) 7, 4, 7, 3, 5, 6, 4, 8, 4

(b) 2, 4, 6, 3, 5, 7, 9, 11, 7, 4

5. Zoltan needs to average 75 in his yearly examinations to be allowed to drive the family car. His average for his first four exams is 70. What mark must he score in his fifth and final exam to average 75?

6. 25 students were surveyed. They were asked the question, 'What colour is your favorite cap?' The results of the survey were: Green 8, Red 7, Blue 3, Black 2, Yellow 1 and Other colour 4. Draw a dot plot to illustrate this information.

7. The shoe sizes of 22 students from a Year 7 class were collected.

$6\frac{1}{2}$ 5 8 $7\frac{1}{2}$ 7 5
5 $5\frac{1}{2}$ 7 6 $4\frac{1}{2}$ 4
7 6 $5\frac{1}{2}$ 6 5 4
5 7 7 $6\frac{1}{2}$

By first making a dot plot draw up a frequency distribution table to illustrate this information.

8. The height of 20 Year 7 boys was measured and recorded. The heights in cm were:

148 150 151 155 149 154 163
159 152 162 165 167 147 158
168 154 149 150 160 152

Construct a stem-and-leaf plot to illustrate this information.

9. The results of students in an English test out of 70 were as follows:

26 47 39 64 48 50 52 33 61 55
47 62 46 52 60 39 28 54 39 62
49 53 57 44 64 43 40 33 47 50

(a) Construct a stem-and-leaf plot to illustrate this information.

(b) Write down the range and mode.

(c) Calculate the mean and find the median.

10. For the following sets of scores:

(a) Find the median, the upper and lower quartiles and the interqurtile range.

(b) Construct a box-and-whisker plot.

(i) 7, 9, 8, 6, 6, 5, 4, 8

(ii) 9, 8, 9, 6, 7, 6, 6, 5, 9, 8, 7

(iii) 18, 14, 17, 12, 19, 16, 16, 14, 18, 15

(iv) 11, 7, 9. 12, 12, 7, 8, 11, 12, 8, 7, 14, 13, 10, 9

11. From the following stem-and-leaf plots find the median, the upper and lower quartiles and construct a box-and-whisker plot.

(a)

Stem	Leaf
1	0 7 8
2	1 2 4 6 9
3	7 7 7 8 8 9
4	0 0 6 7 8 8 9
5	1 4

(b)

Stem	Leaf
10	5 8
11	3 3 6 6 9
12	4 4 7
13	5 5 6 8 8
14	7

12. The heights of 30 Year 8 boys were measured and tabulated. The results were as follows:

161 149 150 161 167 169
172 180 182 150 174 167
168 170 175 182 181 168
154 166 162 160 159 169
166 158 164 174 172 170

(a) Construct a stem-and-leaf plot from this information.

(b) From the stem-and-leaf plot find the mean, the upper and lower quartiles and the interquartile range.

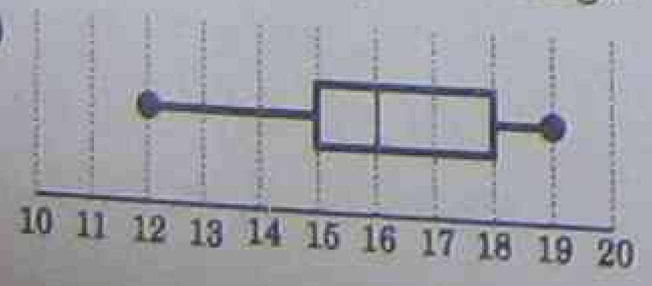
(c) Construct a box-and-whisker plot to illustrate this information.

13. From the following box-and-whisker plots:

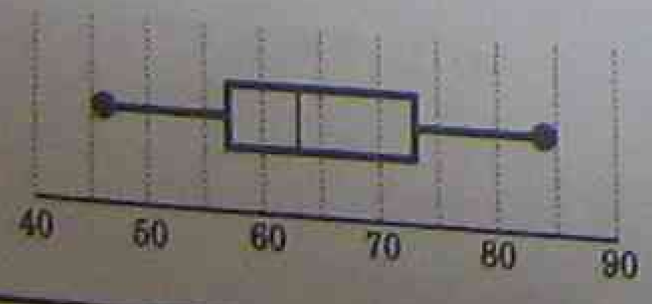
(a) write down the median

(b) calculate the interquartile range.

(i0



(ii)



14. Big Al maintained a record of the barrels of illicit apple cider sold daily from his disused warehouse. Over 30 days his figures were:

18, 24, 27, 19, 20, 14, 31, 20, 26, 18, 11, 23, 26, 32, 15, 20, 26, 29, 17, 27, 30, 24, 32, 19, 30, 26, 24, 27, 23, 36

Commencing with the grouping 11 – 15, construct a frequency distribution table for this information and answer the following questions:

(a) Find the range.

(b) What group is the modal class?

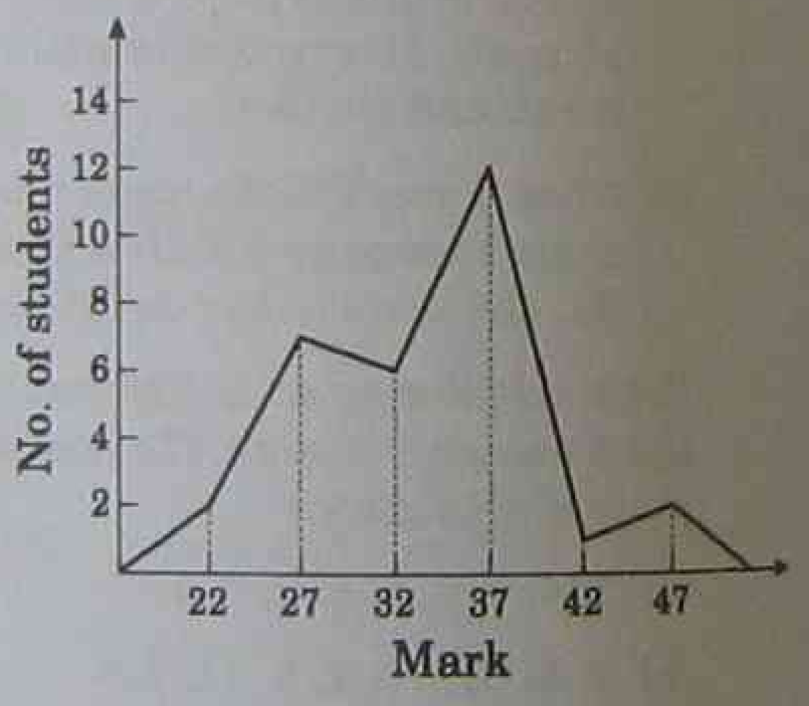
(c) By constructing an *xf* column, calculate the mean.

(d) Find the median class.

(e) Construct a histogram.

(f) Draw an ogive and mark the median on the diagram.

15. Professor Kowalski drew up a frequency polygon to represent his class's marks out of 50. From the diagram answer the following questions:



(a) The midpoint of the first group is 22 and has a size of 5. What is the first grouping?

(b) Draw up a frequency distribution table.

(c) How many students were in the professor's class?

(d) Find the modal class.

(e) Calculate the mean.

(f) Draw a cumulative frequency diagram and mark in the median.

16. Use a calculator to find the mean and standard deviation (correct to one decimal place) for the following sets of scores:

(a) 16, 9, 18, 20, 14, 17, 19

(b) 22, 35, 14, 9, 38, 26, 35, 45, 17

(c) 2.8, 3.6, 4.4, 7.1, 8.2, 6.5, 7.3, 9.1, 8.8, 4.5

17. A country town installs 2000 new electric lights in a new housing estate. These lamps have an average life of 1000 hours with a standard deviation of 200 hours.

(a) What percentage of bulbs would be expected to fail between 800 hours and 1200 hours? How many bulbs (nearest whole number) would this be?

(b) How many bulbs would be expected to last longer than 1600 hours?

18. In a kindergarten class of 40 children the average foot length was calculated to be 10.5 cm with a standard deviation of 0.5 cm.

(a) What percentage of lengths are:

(i) between 10 cm and 11 cm?

(ii) less than 10 cm?

(iii) more than 11.5 cm?

(iv) between 9 cm and 12 cm?

(b) How many children (nearest whole number) have foot lengths:

(i) between 10 cm and 11 cm?

(ii) less than 10 cm?

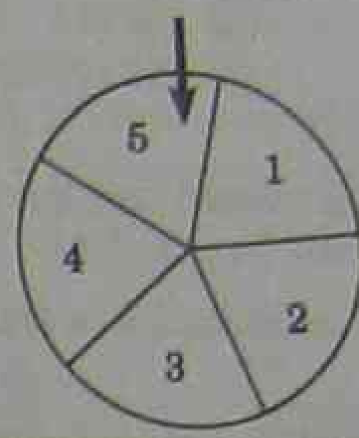
19. A manufacturer of zinc-plated nails manufactures a certain type of nail of mean length 3.8 cm, with standard deviation 0.02 cm. Gauges reject all nails longer than 3.84 cm or shorter than 3.76 cm. What percentage of nails are rejected? In a run of 10 000 nails, how many (nearest whole number) would you expect to be rejected? If one nail is selected at random, what is the probability that it would *not* be rejected?

20. Consider the set of five numbers 8, 3, 7, 4, 3.

(a) Add five to each number in the group. How does this change the mean and standard deviation?

(b) Multiply each of the numbers by five. How does this alter the mean and standard deviation?

21.



A five-sector wheel is spun 100 times and the number of times each sector is uppermost is noted as follows:

Number	Frequency uppermost	Relative frequency
1	17	
2	22	
3	21	
4	19	
5	21	
Σ	100	

(a) Complete the relative frequency column.

(b) From the table, calculate the probability that a 2 was uppermost.

22. A pack has 52 cards divided into four suits: diamonds, hearts, clubs and spades. Each suit contains a 2, 3, 4, 5, … , 10, J(ack), Q(ueen), K(ing), A(ce). If one card is drawn at random from the pack, find the probability that it is:

(a) an A.
(b) a K.
(c) a heart.
(d) a spade.
(e) a black suit.
(f) a picture card (J, Q, K).
(g) not a picture card.
(h) a *number* less than 7.
(i) the six of hearts.
(j) red or a 6.
(k) a red six.

23. The probability of rain at the weekend is assessed as $^7/_{10}$. What is the probability that it will not rain at the weekend?

24. A jar contains seven red, five black and three white jelly babies. If one jelly baby is drawn at random from the jar, find the probability that it is:

(a) red. (b) not red.
(c) black. (d) red or white.
(e) green.

25. It has been calculated that the probability of a male birth is 0.48.

(a) Find the probability of a female birth.

(b) At a large Sydney hospital, 1400 babies were born over a year. How many male babies would you expect?

26. Two dice are thrown (faces numbered 1 to 6). The sum of the two uppermost faces is noted. Find the probability that the sum is:

(a) 9. (b) 5.
(c) odd. (d) greater than 9.
(e) 9 or less. (f) not 9.
(g) odd or less than 5.
(h) odd and less than 5.

27. If, instead of adding the numbers on the dice, the product is calculated, find the probability that this product is:

(a) 24. (b) odd.
(c) less than 12. (d) 12 or less.
(e) divisible by 3.
(f) odd and divisible by 3.
(g) odd or divisible by 3.

28. Two dice (faces numbered 1 to 6) are thrown and the highest number from the two faces is noted. For example, if a 3 and 5 are thrown, the 5 only is noted. Draw up a table to illustrate all possible outcomes. Find the probability that the number noted is:

(a) 6. (b) 1.
(c) not 6. (d) not 6 or 1.
(e) even. (f) a prime.
(g) an even prime.
(h) a prime or even.
(i) divisible by 3.
(j) greater than 6.

29. A pair of four-sided dice, faces numbered 1 to 4 are rolled and the numbers on the bottommost faces noted.

(a) Find the probability that the sum of these numbers is:
(i) 8.
(ii) 1.
(iii) 4.
(iv greater than 4.
(v) at least 4.

(b) Find the probability that the product of these numbers is:
(i) 16.
(ii) even.
(iii) less than 6.
(iv) odd and less than 6.

30. In a supermarket promotion, 10 gold tokens numbered 1 to 10 are hidden. Melville found one of the tokens. Find the probability that Melville found the token with number:

(a) 7. (b) even.
(c) less than 7. (d) more than 7.

31. Robyn keeps loose change in her car ashtray. Last Saturday she noted that there were four 50 cent coins and six 20 cent coins. She selected one coin from the ashtray to pay for a 45c iceblock. What is the probability that the coin selected is sufficient?

32. Three coins are thrown and the uppermost faces noted. Draw a tree diagram to illustrate the possible outcomes. Find the probability that the throw results in:

(a) three heads.
(b) two heads.
(c) one head.
(d) no heads.
(e) at least one head.

33. A bag of Smarties contains three red and two blue ones. Two are drawn at random from the bag, the colour noted and then eaten. Find the probability that they are:

(a) both red.
(b) both blue.
(c) different colours.
(d) the same colour.

---

34. Six cards numbered one to six are placed face down in random order on the table.

(a) One card is drawn at random. Find the probability that it is:
(i) 6.
(ii) even.
(iii) less than 4.

(b) Two cards are turned up in succession, the first digit making up the tens digit of a number and the second being the units (that is, 34). Find the probability that the number formed is:
(i) 34.
(ii) 43.
(iii) odd.
(iv) divisible by 5.
(v) less than 40.
(vi) divisible by 5 and less than 40.

(c) Find the probability that the two digits turned up have a sum:
(i) of 7. (ii) less than 7.

35. In how many ways can seven books be placed on a shelf? How many ways can they be placed if two books must always be together? Find the probability that when seven science books are placed on a shelf the two chemistry books are together.

36. At a twenty-first birthday party there are eight guests. Each guest shakes hands with each other guest once only. How many handshakes are there altogether?

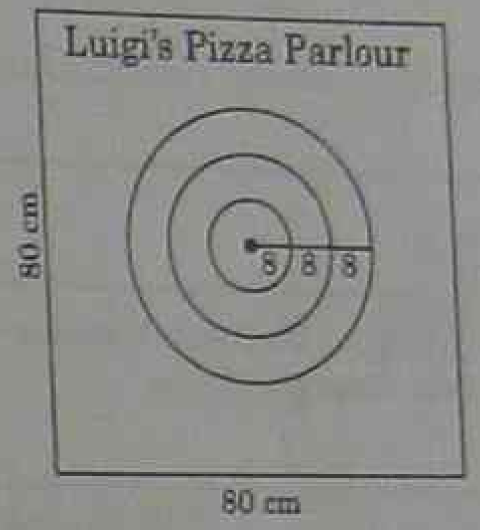
37. A biscuit barrel contains five choc-chip and four walnut biscuits. Two biscuits are drawn in succession and consumed. Find the probability that of the two biscuits:

(a) both are choc-chip.
(b) one is a choc-chip.
(c) only the first is a choc-chip.
(d) neither is a choc-chip.
(e) at least one is a choc-chip.

38. One hundred tickets are sold in a Chook Raffle. Gregory John purchases two tickets, hoping to win both first and second prizes. Find the probability that he wins:

(a) first prize.
(b) both prizes.
(c) only second prize.
(d) a prize.
(e) no prize.
(f) at least one prize.

39.



Luigi has a special incentive scheme at his pizza shop. On the side wall is a specially designed target on an 80 × 80 cm board. The target has three concentric circles with radii 8 cm, 16 cm and 24 cm. Each customer is given one dart to throw at the board. They win a free super-supreme if they hit the inner circle, a medium pizza if they hit the middle area and a thick shake for the area in the outer circle. If they miss the circles, they pay full price for their order. (Anyone who misses the board is allowed a rethrow.)

(a) Find the probability that:
(i) Jill wins a super supreme.
(ii) Peter wins a thick shake.
(iii) Guiseppe wins a prize.
(iv) Alison pays full price.

(b) If each customer is allowed two throws (and possibly can win two prizes) find the probability that:
(i) Pietro wins two prizes.
(ii) Janus wins one prize.
(iii) Leigh fails to win at all.
(iv) Scotty wins at least one prize.

Chapter 8

MEASUREMENT:

TIME, PERIMETER, AREA, SURFACE AREA AND VOLUME

8.1 Time

The $\boxed{\text{DMS}}$, or Degrees–Minutes–Seconds button (also identified as $\boxed{.\;,\;,,}$, is useful to calculate time problems.

Examples:

(a) Find the sum of 2 h 14 m 52 s and 3 h 59 m 18 s.

SOLUTION

[Enter 2 $\boxed{\text{DMS}}$ 14 $\boxed{\text{DMS}}$ 52 $\boxed{\text{DMS}}$ + 3 $\boxed{\text{DMS}}$ 59 $\boxed{\text{DMS}}$ 18 $\boxed{\text{DMS}}$ = 2nd Function $\boxed{\text{DMS}}$]

∴ 6 h 14 m 10 s

(b) Find the average of 2 h 14 m and 3 h 42 s (*Note:* 3 h 0 m 42 s)

SOLUTION

[Enter 2 $\boxed{\text{DMS}}$ 14 $\boxed{\text{DMS}}$ + 3 $\boxed{\text{DMS}}$ 0 $\boxed{\text{DMS}}$ 42 $\boxed{\text{DMS}}$ = ÷ 2 $\boxed{\text{DMS}}$ = 2nd Function $\boxed{\text{DMS}}$]

∴ 2 h 37 m 21 s

(c) Rewrite as hours, minutes and seconds 4.2 h

SOLUTION

[Enter 4.2 2nd Function $\boxed{\text{DMS}}$]

∴ 4.2 h = 4 h 12 m

(d) How much time has elapsed between

(i) 4:27 a.m. and 11:38 a.m.

(ii) 6:58 and 17:21

SOLUTION

(i) [Enter 11 $\boxed{\text{DMS}}$ 38 $\boxed{\text{DMS}}$ − 4 $\boxed{\text{DMS}}$ 27 $\boxed{\text{DMS}}$ = 2nd Function $\boxed{\text{DMS}}$]

∴ 7 h 11 m

(ii) 10 h 23 m

(e) A car travels 25 km at an average speed of 65 km/h. What time will the journey take, to the nearest second?

SOLUTION

As Time = Distance ÷ Speed

= 25 ÷ 65

= 0.384 615 384

= 0 h 23 m 4.62 s

[by 2nd Function $\boxed{\text{DMS}}$]

∴ 23 m 5 s.

8.2 Commonly used units in the metric system

Length

1000 mm = 1 m
100 cm = 1 m
10 mm = 1 cm
1000 m = 1 km

Mass

1000 mg = 1 g
1000 g = 1 kg
1000 kg = 1 t

Area

$1\ \text{m}^2 = 1000\ \text{mm} \times 1000\ \text{mm}$
$= 1\,000\,000\ \text{mm}^2$
$1\ \text{m}^2 = 100\ \text{cm} \times 100\ \text{cm}$
$= 10\,000\ \text{cm}^2$
$1\ \text{ha} = 100\ \text{m} \times 100\ \text{m}$
$= 10\,000\ \text{m}^2$

Capacity

1000 mL = 1 L
1000 L = 1 kL
1000 kL = 1 ML

Volume and capacity

$1\ \text{cm}^3 = 1\ \text{mL}$
$1000\ \text{cm}^3 = 1\ \text{L}$
$1\ \text{m}^3 = 1000\ \text{L} = 1\ \text{kL}$
Also $1\ \text{cm}^3 = 10\ \text{mm} \times 10\ \text{mm} \times 10\ \text{mm}$
$= 1000\ \text{mm}^3$
$1\ \text{m}^3 = 100\ \text{cm} \times 100\ \text{cm} \times 100\ \text{cm}$
$= 1\,000\,000\ \text{cm}^3$

8.3 Less common units of measurement

Prefix	Symbol	Meaning
nano	n	$\frac{1}{1\,000\,000\,000}$
micro	µ	$\frac{1}{1\,000\,000}$
milli	m	$\frac{1}{1000}$
kilo	k	1000
mega	M	1 000 000
giga	G	1 000 000 000

Examples:

Convert:

(a) 1.2 gigabytes to kilobytes.

SOLUTION

∴ 1.2 × 1 000 000 = 1 200 000

∴ 1.2 Gbytes = 1 200 000 kbytes

(b) 3000 nanometres to micrometres.

∴ $3000 \div 1000 = 3$

that is 3000nm = 3 μm.

(c) 4.76 millimetres to nanometres

∴ $4.76 \times 1\,000\,000 = 4\,760\,000$

∴ 4.76 mm = 4 760 000 nm

8.4 Measurements based on logarithmic scales

8.4.1 Sound

Noise	Relative intensity	Decibels (dB)
Minimum of audible sound	1	0
Soft wind on leaves	10	10
Whisper at 1 metre	10^2	20
Heavy traffic	10^8	80
Rock group	10^{11}	110

The table shows various noises and the measurements on a scale of decibels. A whisper at $10^2 = 100$ is ten times as intense as a soft wind on leaves at $10^1 = 10$.

Examples:

(a) A lawn mower has relative intensity of 10^{12}. Find its decibel rating.

Number of decibels = $12 \times 10 = 120$

(b) By how many times is the relative intensity of the average office (50 dB) greater than that of a whisper at 1 metre?

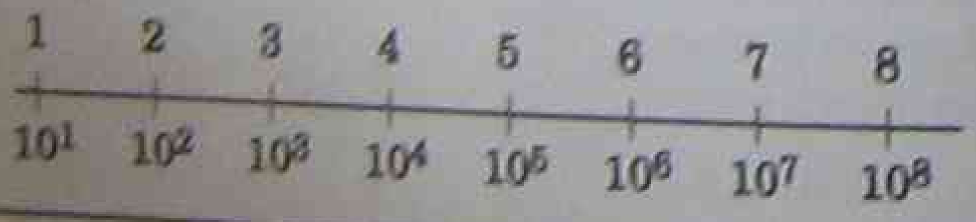
10^2 compared to 10^5

$10^5 \div 10^2 = 10^3 = 1000$

∴ 1000 times greater.

8.4.2 Richter scale

The Richter scale measures earthquakes and each unit is ten times greater than the previous unit.



Examples:

(a) An earthquake measures 6 on Richter scale. How much more powerful is it than one with a reading of 5?

SOLUTION

As $6 \to 10^6$, $5 \to 10^5$

∴ $10^6 \div 10^5 = 10$

that is 10 times greater.

(b) Make a similar comparison between earthquakes measuring 4 and 7.

∴ $4 \to 10^4$ and $7 \to 10^7$

∴ $10^7 \div 10^4 = 10^3$

$= 1000$

∴ thousand times more powerful.

6. I left Laguna at 11:06 and travelled to Cessnock 47 km away, averaging 67 km/h. What time did I arrive at my destination, to nearest minute?

7. Laura walked for 18 minutes at a speed of 9 km/h. How far did she walk?

8. Convert:

   (a) 7 kilometres to micrometres

   (b) 25 megalitres to gigalitres

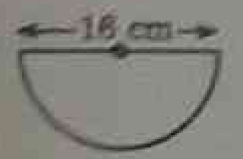
   (c) 10 ms to μs

   (d) 3200 μL to mL.

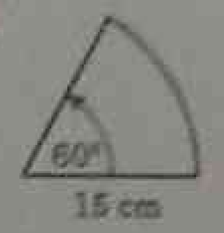
9. An equilateral triangle has a side length of 4.76 mm. Find the perimeter of the triangle.

10. Find the perimeter, correct to three decimal places, of the following figures:

(a)

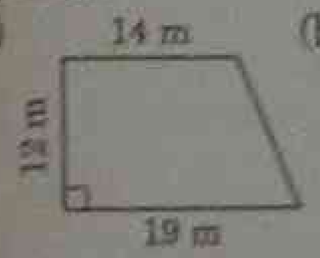


(b)

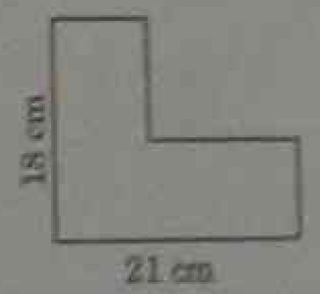


11. Find the perimeter of the following figures:

(a)



(b)

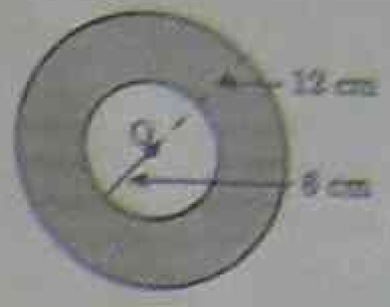


12. A rumpus room measuring 4.4 m by 3.6 m has its floor partly covered by a rug so that a border of 50 cm is left between the wall and the edge of the rug on all four sides. Find the perimeter of the rug.

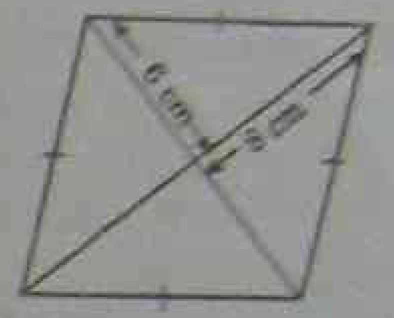
13. Syl decides to fit a length of weather stripping completely around his side door which is in the shape of a semi-circle on a rectangle. Calculate the length of weather stripping required, to the nearest cm.

14. The lengths of the diagonals of a rhombus are 6 cm and 10 cm. Calculate the area of the rhombus.

15. (a) Find the shaded area of the annulus. Leave your answer in terms of π:



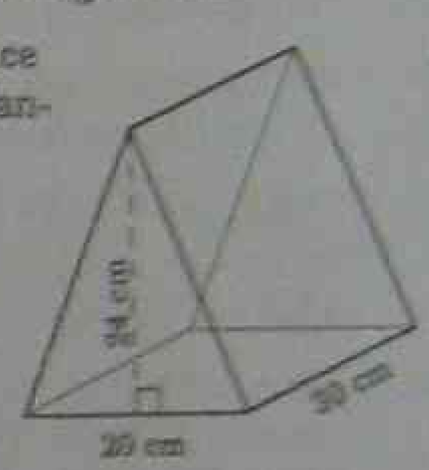
(b) Find the area and perimeter of the rhombus:



16. What is the surface area (correct to two decimal places) of a basketball which has a diameter of 32 cm?

17. Find the area of the curved surface of a cylinder with a radius of 7 cm and a height of 12 cm. (Give your answer to three significant figures.)

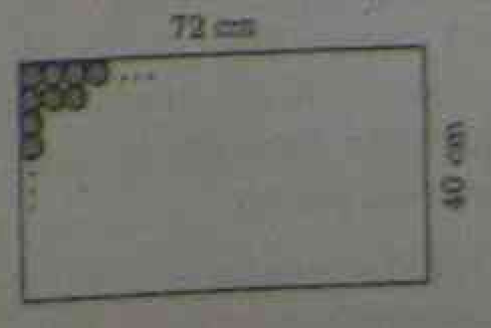
18. Find the surface area of the triangular prism:



19. A rectangular sheet of metal measures 72 cm by 40 cm. From this sheet, circular pieces with diameters of 4 cm are to be stamped out.

   (a) Find the maximum number of stampings.

   (b) Find the amount of scrap metal, to the nearest cm^2.



(b) 3000 nanometres to micrometres.

$\therefore$ $3000 \div 1000 = 3$

that is 3000nm = 3 μm.

(c) 4.76 millimetrs to nanometres

$\therefore$ $4.76 \times 1\,000\,000 = 4\,760\,000$

$\therefore$ 4.76 mm = 4 760 000 nm

8.4 Measurements based on logarithmic scales

8.4.1 Sound

Noise	Relative intensity	Decibels (dB)
Minimum of audible sound	1	0
Soft wind on leaves	10	10
Whisper at 1 metre	10^2	20
Heavy traffic	10^8	80
Rock group	10^{11}	110

The table shows various noises and the measurements on a scale of decibels. A whisper at $10^2 = 100$ is ten times as intense as a soft wind on leaves at $10^1 = 10$.

Examples:

(a) A lawn mower has relative intensity of 10^{12}. Find its decibel rating.

Number of decibels = $12 \times 10 = 120$

(b) By how many times is the relative intensity of the average office (50 dB) greater than that of a whisper at 1 metre?

10^2 compared to 10^5

$10^5 \div 10^2 = 10^3 = 1000$

$\therefore$ 1000 times greater.

8.4.2 Richter scale

The Richter scale measures earthquakes and each unit is ten times greater than the previous unit.

1	2	3	4	5	6	7	8
10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8

Examples:

(a) An earthquake measures 6 on Richter scale. How much more powerful is it than one with a reading of 5?

SOLUTION

As $6 \rightarrow 10^6$, $5 \rightarrow 10^5$

$\therefore$ $10^6 \div 10^5 = 10$

that is 10 times greater.

(b) Make a similar comparison between earthquakes measuring 4 and 7.

$\therefore$ $4 \rightarrow 10^4$ and $7 \rightarrow 10^7$

$\therefore$ $10^7 \div 10^4 = 10^3$

$= 1000$

$\therefore$ thousand times more powerful.

---

6. I left Laguna at 11:06 and travelled to Cessnock 47 km away, averaging 67 km/h. What time did I arrive at my destination, to nearest minute?

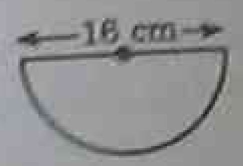
7. Laura walked for 18 minutes at a speed of 9 km/h. How far did she walk?

8. Convert:
   (a) 7 kilometres to micrometres
   (b) 25 megalitres to gigalitres
   (c) 10 ms to μs
   (d) 3200 μL to mL.

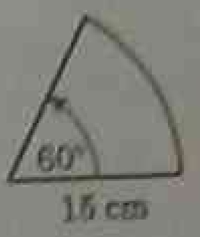
9. An equilateral triangle has a side length of 4.76 mm. Find the perimeter of the triangle.

10. Find the perimeter, correct to three decimal places, of the following figures:

(a)

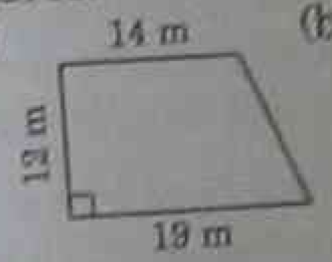


(b)

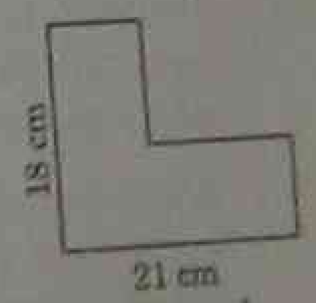


11. Find the perimeter of the following figures:

(a)

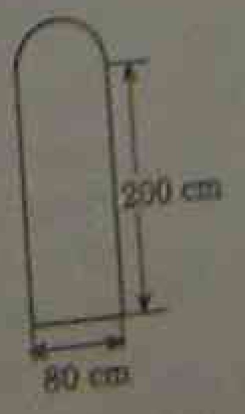


(b)



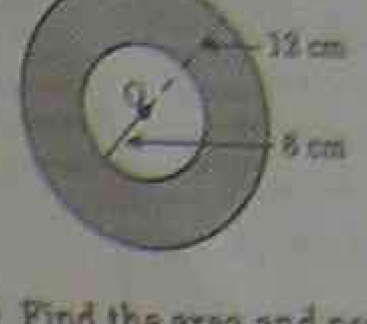
12. A rumpus room measuring 4.4 m by 3.6 m has its floor partly covered by a rug so that a border of 50 cm is left between the wall and the edge of the rug on all four sides. Find the perimeter of the rug.

13. Syl decides to fit a length of weather stripping completely around his side door which is in the shape of a semi-circle on a rectangle. Calculate the length of weather stripping required, to the nearest cm.

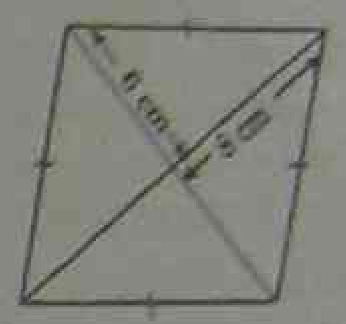


14. The lengths of the diagonals of a rhombus are 6 cm and 10 cm. Calculate the area of the rhombus.

15. (a) Find the shaded area of the annulus. Leave your answer in terms of π:



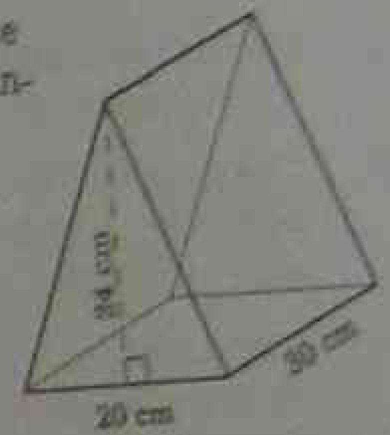
(b) Find the area and perimeter of the rhombus:



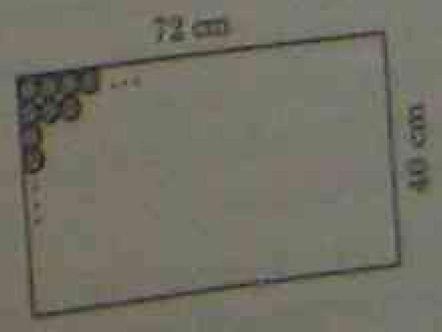
16. What is the surface area (correct to two decimal places) of a basketball which has a diameter of 32 cm?

17. Find the area of the curved surface of a cylinder with a radius of 7 cm and a height of 12 cm. (Give your answer to three significant figures.)

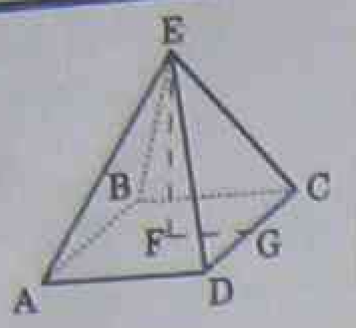
18. Find the surface area of the triangular prism:



19. A rectangular sheet of metal measures 72 cm by 40 cm. From this sheet, circular pieces with diameters of 4 cm are to be stamped out.
   (a) Find the maximum number of stampings.
   (b) Find the amount of scrap metal, to the nearest cm^2.



20. A square-based pyramid has a base length of 12 cm and a perpendicular height of 8 cm. Find, leaving answers as surds if necessary, the:



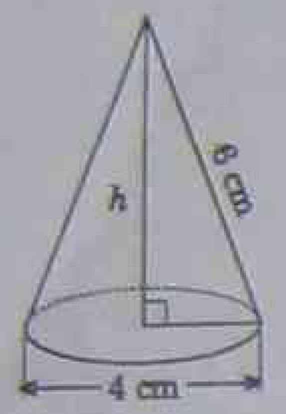
(a) length of FG.

(b) slant height EG.

(c) area of ΔCED.

(d) surface area of the pyramid.

21. A cone has a diameter of 4 cm and a slant height of 6 cm. Find:



(a) the length of its perpendicular height, as a surd.

(b) the area of the curved surface of the cone, to two decimal places.

(c) the total surface area of the cone, to two decimal places.

22. The surface area of a cube is known to be 54 cm^2. Find the length of each edge of the cube.

23. A tin of fruit salad has a diameter of 7 cm and a height of 10 cm. Find the area of its label, to the nearest cm^2.

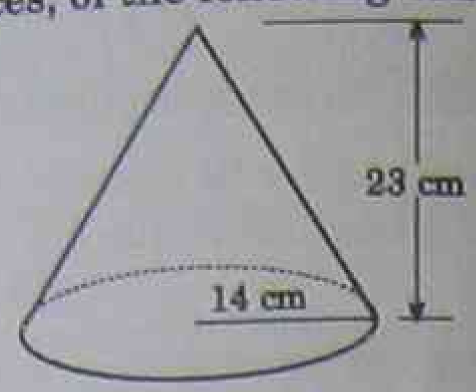
24. A wooden bowl is in the shape of a hemisphere. Its outer surface is to be painted. Find the area to be painted, if the diameter of the bowl is 26 cm. (Correct to one decimal place.)

25. A conical tent has a base diameter of 2.5 metres, and a perpendicular height of 2 metres. The tent is complete with a floor. Find the area of canvas required to make the tent, correct to two decimal places.

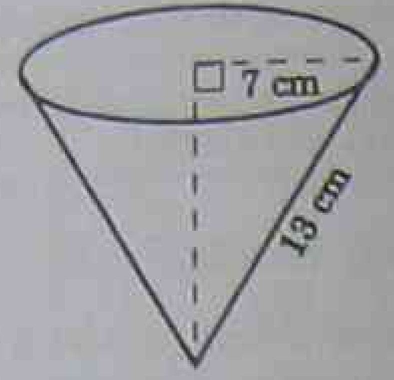
26. Find the volume of a cylinder which has a radius and a height of 7.6 cm. Express your answer correct to three decimal places.

27. Find the volume, correct to two decimal places, of the following solids:

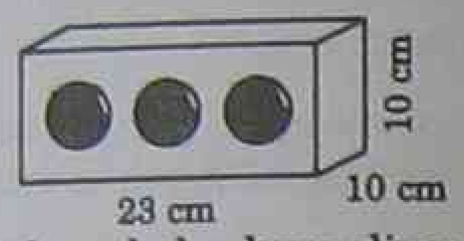
(a)



(b)



(c)



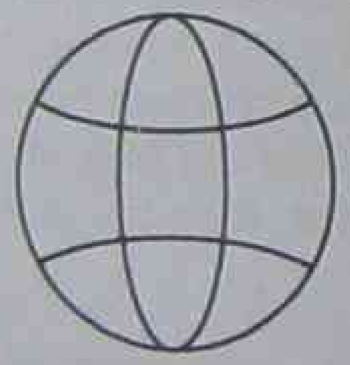
All three holes have diameters of 5 cm.

(d)

28. Find the volume of a square-based pyramid with a side length of 16 cm and a perpendicular height of 15 cm.

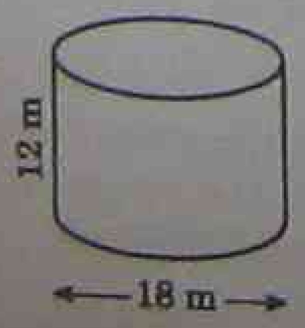
29. A cylindrical water tank has a radius of 1.8 metres and a height of 2.3 metres. Find its capacity to the nearest litre.

30. Calculate the capacity, in litres, of a basketball, whose diameter is 25 cm.

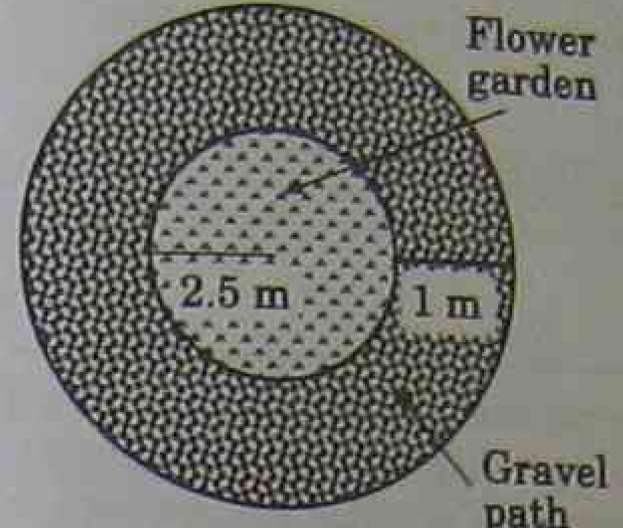


31. Ken decided to preserve some of his peaches. The instructions were to fill a cylindrical preserving jar to two-thirds of its total volume. If the radius of the jar was 20 cm and its height was 30 cm, how many litres were required?

32. A cylindrical oil-storage tank has a height of 12 metres and diameter of 18 metres. Find its capacity to the nearest kilolitre.



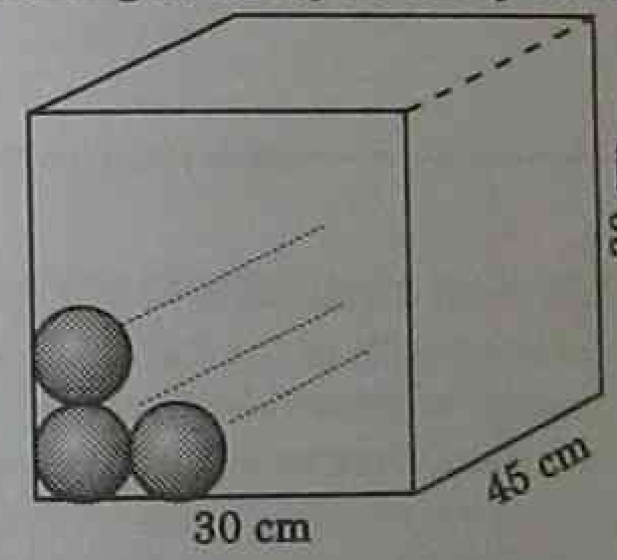
33. Jenny decided to incorporate a Japanese theme into her front yard. She planned to have a 1 m wide circular gravel path around an existing flower garden of 2.5 m radius.



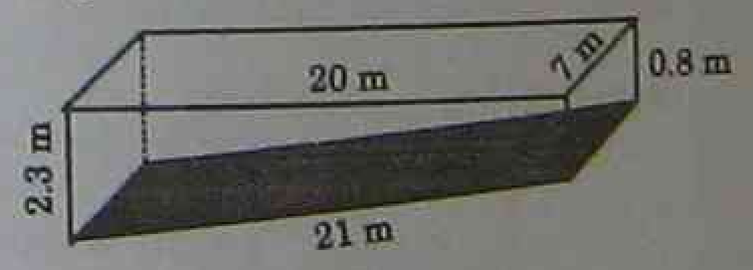
(a) Find the area of the path, in square metres.

(b) The path was to have a depth of 10 cm. Find the volume of gravel required.

34. How many tennis balls, each 7.5 cm in diameter, can be packaged in a box measuring 30 cm by 30 cm by 45 cm.



35. A swimming pool has the shape of a trapezoidal prism, as shown in the diagram.



Find:

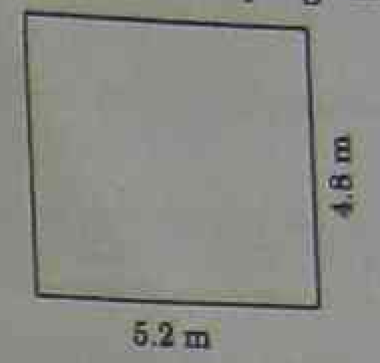
(a) the cost of tiling the base of the pool, if the tiles cost $55/$m^2$.

(b) the cost of tiling the four walls of the pool, if those tiles cost $42/$m^2$.

(c) the volume of the pool in cubic metres.

(d) the volume of water (in kilolitres) required to fill the pool.

36. A paving brick (paver) is 22.5 cm long and 10 cm wide. Ken purchases a pallet of 500 pavers to begin to pave his rectangular-shaped pergola area.

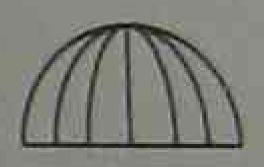


(a) Find the area of the rectangle to be paved.

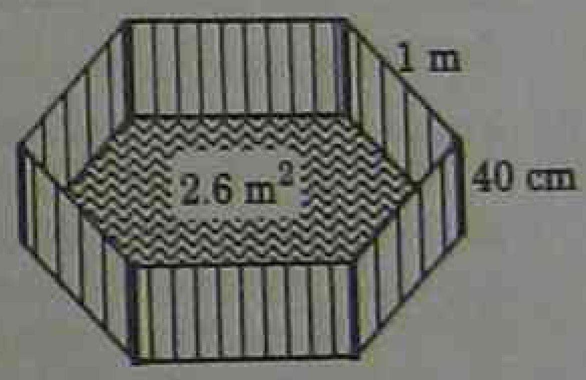
(b) What is the area in cm^2.

(c) How many more pavers will Ken need to complete the job?

37. If 1 cm^3 of steel has a mass of 6.8 g, find the total mass of a solid steel hemispherical paper- weight of 10 cm diameter.



38. A children's swimming pool has a hexagonal shape.



It is constructed of a blue vinyl sheet draped over six pipes supported by six poles. The area of the base is 2.6 m^2.

(a) Find the total surface area of the vinyl sheet.

(b) Find the capacity of the pool.

(c) The pool is to be filled to three-quarters of its capacity. How many kilolitres will it contain?

Chapter 9

SIMILARITY

9.1 Similar figures

Similar figures have the same shape. Similar figures are common in daily life. Building plans, surveyor's plans, road maps, town plans and photographic enlargements are all examples of similar figures. They are all cases where the original shapes have been reduced or enlarged in a given ratio. This is called the **scale** for plans and designs or the **enlargement factor** for other cases.

The scale can be calculated by finding the ratio of any length in the plan to the corresponding length in the original. For example, a block of land 120 m long is represented by a length 12 cm on the plan.

$$\begin{aligned}\text{Scale} &= 12\text{ cm} : 120\text{ m}\\ &= 120\text{ mm} : 120\,000\text{ mm}\\ &= 12\cancel{0} : 120\,00\cancel{0}\\ &= 1 : 1000\end{aligned}$$

Examples

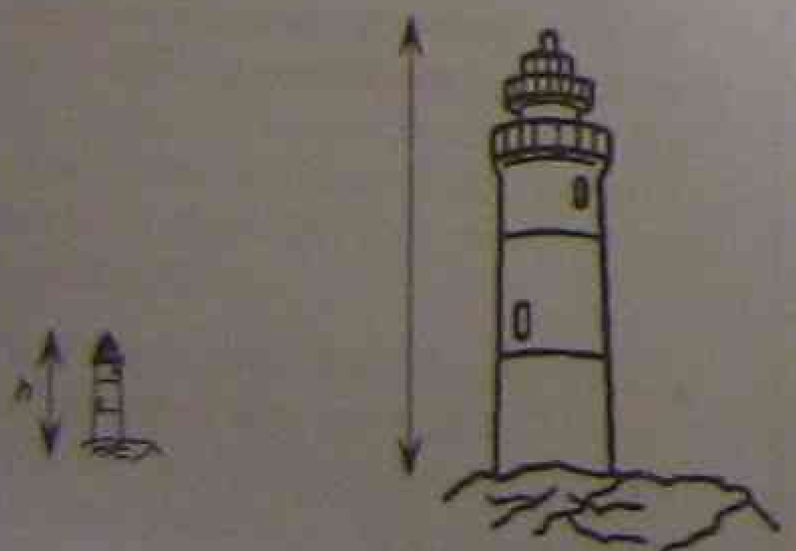
(a) A scale model is made of a 40 m long yacht. If the scale model has a length of 20 cm, find the scale used.

SOLUTION

$$\begin{aligned}\text{Scale} &= 20\text{ cm} : 40\text{ m}\\ &= 20\text{ cm} : 4000\text{ cm}\\ &= 2\cancel{0} : 400\cancel{0}\\ &= 1 : 200\end{aligned}$$

The scale used is 1 : 200.

(b) Beverley wishes to make a scale drawing of the Minmi Lighthouse.



She used a scale of 1 : 1000. If the actual height of the lighthouse is 58 metres, what will be the height in cm of the lighthouse in her scale drawing?

SOLUTION

Call the height of the drawing h cm.

Then

$$\begin{aligned}h\text{ cm} : 58\text{ m} &= 1:1000\\ h:5800 &= 1:1000\\ \frac{h}{5800} &= \frac{1}{1000}\\ 1000h &= 5800\\ h &= 5.8.\end{aligned}$$

Rewrite ratios as equivalent fractions, then cross-multiply

The height on the scale drawing is 5.8 cm.

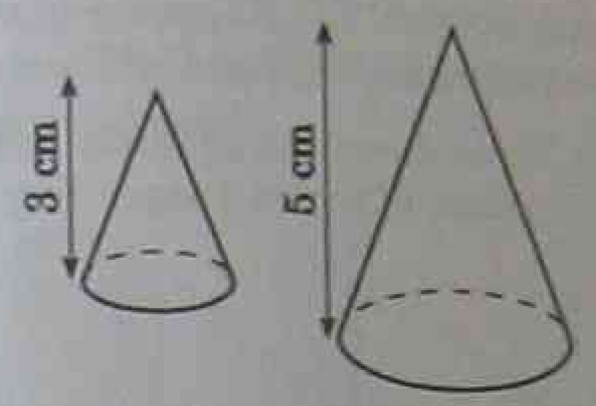
9.5 Volumes of similar solids

Similar solids have the same shape, with the lengths of corresponding dimensions in the same ratio. Similar solids have:

- their surface areas proportional to the squares of the lengths of corresponding sides;
- their volumes proportional to the cubes of the lengths of corresponding sides.

Examples

(a)



For these similar cones, find the ratio of:

(i) their surface areas

(ii) their volumes.

SOLUTION

$$\text{Ratio of heights} = \frac{3}{5}$$

$$\text{(i) Ratio of surface areas} = \frac{3^2}{5^2} = \frac{9}{25}$$

$$\text{(ii) Ratio of volumes} = \frac{3^3}{5^3} = \frac{27}{125}$$

(b) If the volume of the larger cone in (a) is 500 cm^3, calculate the volume of the smaller cone.

SOLUTION

Let the volume be V cm^3.

$$\begin{aligned}\frac{V}{500} &= \frac{27}{125}\\ 125V &= 27\times500\\ V &= \frac{27\times500}{125}\\ &= 108\end{aligned}$$

The volume of the smaller cone is 108 cm^3.

(c) A scale model of a yacht displaces 800 cm^3 of water. If the scale model has a length of 10 cm, compared to the actual length of 20 m, find the volume of water displaced by the real yacht.

SOLUTION

$$\begin{aligned}\text{Ratio of lengths} &= 10\text{ cm} : 20\text{ m}\\ &= 10 : 2000\\ &= 1 : 200\end{aligned}$$

$$\text{Ratio of volumes} = 1:(200)^3$$

Let the volume of the actual yacht be V.

Then

$$\frac{800}{V} = \frac{1}{(200)^3} \qquad \boxed{\frac{\text{plan}}{\text{actual}}}$$

$$\begin{aligned}V &= 800\times200^3\text{ cm}^3\\ &= \frac{800\times8\cancel{000\,000}}{1\cancel{000\,000}}\text{ m}^3\\ &= 6400\text{ m}^3\end{aligned} \qquad \boxed{\begin{aligned}1\text{ m}^3 &= 100\times100\times100\text{ cm}^3\\ &= 1\,000\,000\text{ cm}^3\end{aligned}}$$

Actual displacement is 6400 m^3.

(d) A $\frac{1}{50}$ scale model of the Boganbrey Town Hall requires 80 mL of paint to cover its outside walls. Calculate the quantity of paint needed for the outside walls of the actual Town Hall.

SOLUTION

Let the required area to be covered be A.

$$\begin{aligned}\text{Ratio of lengths} &= 1:50\\ \text{Ratio of surface areas} &= 1:50^2\\ &= 1:2500\end{aligned}$$

$$\boxed{\frac{\text{Area of scale model}}{\text{Area of actual hall}}}$$

$$\text{Then } \frac{0.08}{A} = \frac{1}{2500} \qquad \boxed{80\text{ mL} = 0.08\text{ L}}$$

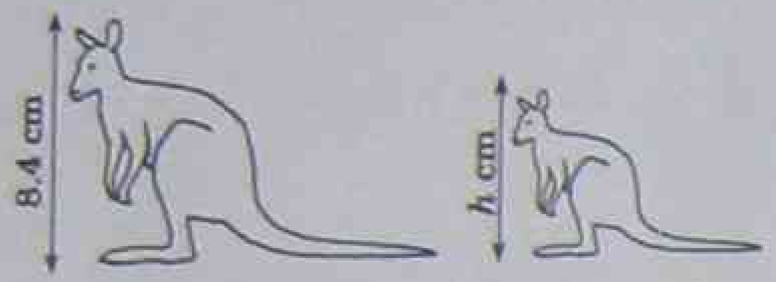
$$\begin{aligned}A &= 0.08\times2500\\ &= 200\end{aligned}$$

The Town Hall surface will require 200 L of paint.

9.6 Exercises

1. A photograph of Vanessa's pet aardvark is enlarged so that the original 7.5 cm picture becomes 0.6 m. Find the enlargement ratio.

2. Two similar kangaroo logos are produced for a supermarket promotion.

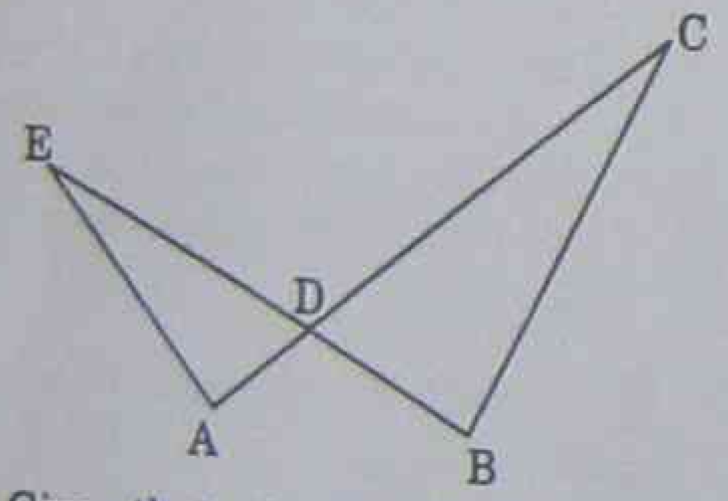


The dimensions of the logos are to be in the ratio 2 : 3. Find the height of the smaller logo if the larger one is 8.4 cm high.

3. A rectangle 5 cm wide by 8 cm long is enlarged by a scale factor of $\frac{5}{4}$. Find the perimeter and area of the enlarged rectangle.

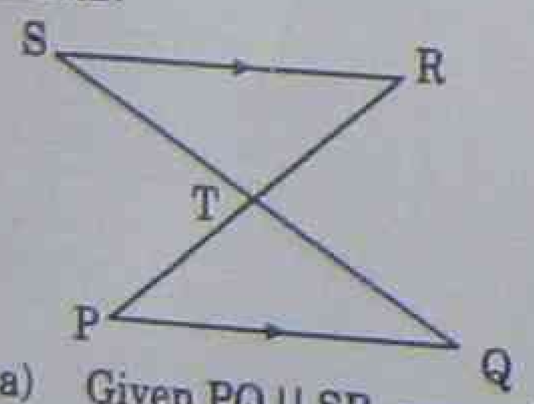
4. A floor plan of a regular pentagonal room is drawn to the scale of 1 : 100. The actual floor has a perimeter of 125 m. Find the length of each side on the plan.

5.



Given that ∠AED = ∠BCD, prove that the triangles ADE and BDC are similar.

6.

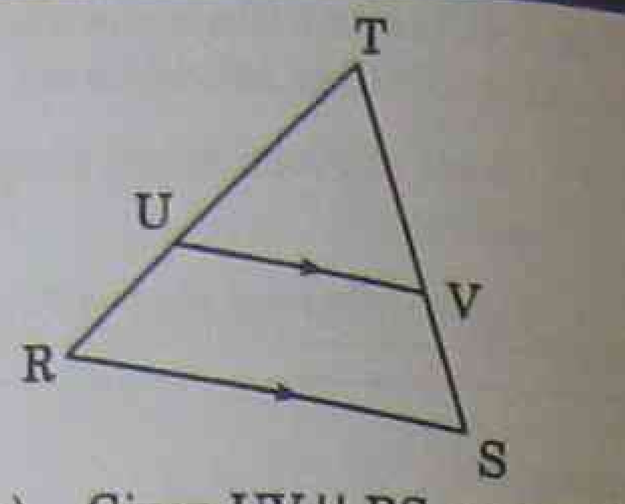


(a) Given PQ || SR, prove that ΔPTQ ||| ΔRTS.

(b) Complete the ratios.

$$\frac{PQ}{\square}=\frac{QT}{\square}=\frac{TP}{\square}$$

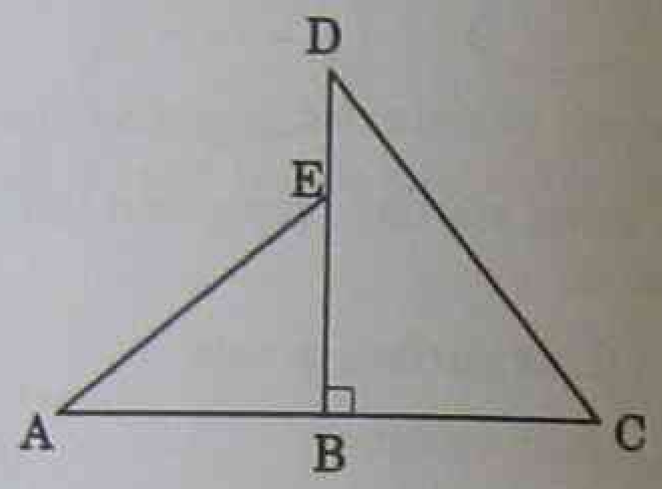
7.



(a) Given UV || RS, prove that the triangles UVT and RST are similar.

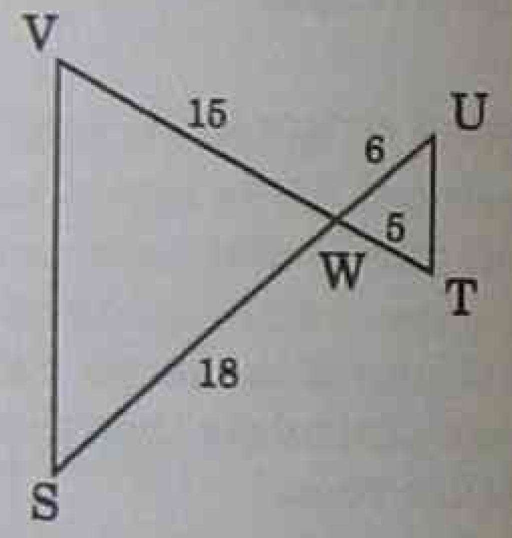
(b) Write down the equality linking the ratios of the corresponding sides.

8.



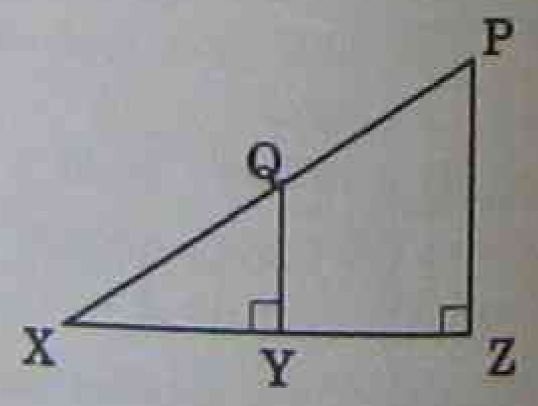
Given DB ⊥ AC, and ∠AEB = ∠BCD, prove that ΔABE ||| ΔDBC.

9.



If VW = 15, SW = 18, UW = 6, WT = 5, prove that the triangles are similar.

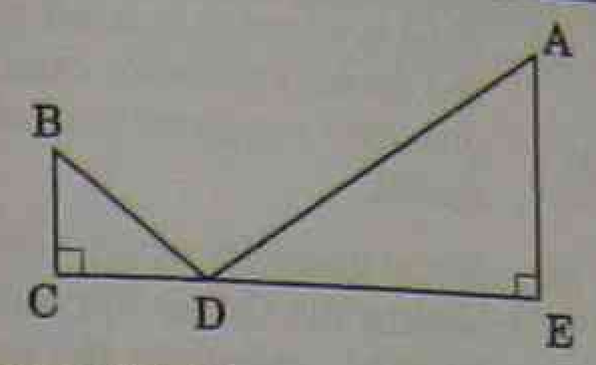
10.



(a) If ∠QYX and ∠PZY are both 90°, prove that ΔXYQ ||| ΔXZP.

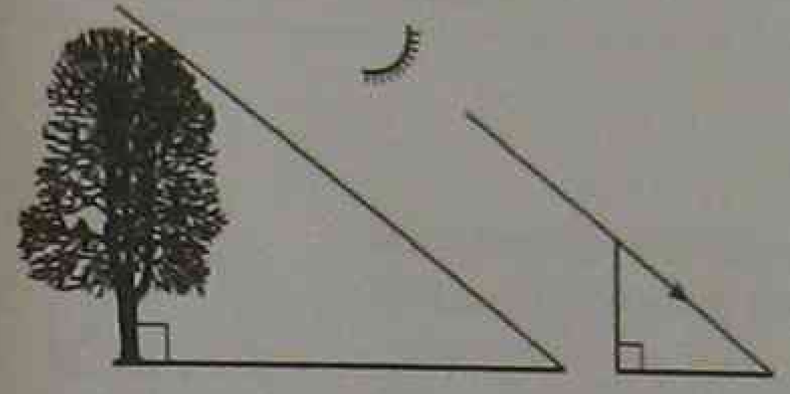
(b) Given that QY = 2.4 m, and that XY = 3.6 m and YZ = 4.8 m, find the length PZ.

11.



A ray of light from source A is reflected in a horizontal mirror at D, and strikes a target B. If B is 1.5 m above ground level, E and C are 18.9 m apart and CD is 4.5 m, find the height of the light source above ground level.

12.



A metre rule casts a shadow 1.8 m long. At the same instant a tree casts a shadow 7.2 m long. Calculate the height of the tree.

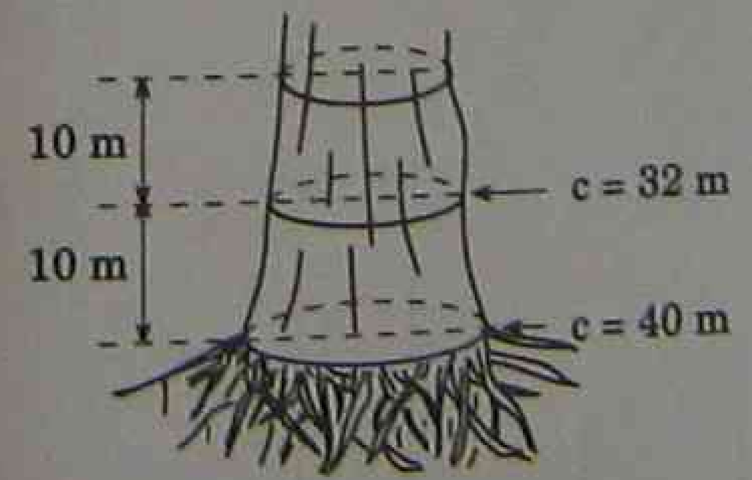
13. Two semi-circles have radii of 3 cm and 6 cm. Find the ratio of:

(a) their perimeters

(b) their areas.

14. Two regular octagons have sides of length 3 cm and 5 cm. If the area of the larger octagon is 45 cm², calculate the area of the smaller one.

15. A giant Deadwood tree has a base circumference of 40 m. Scientists have theorised that this circumference reduces in a fixed ratio for every 10 m of height. The circumference 10 m from the base is measured to be 32 metres.

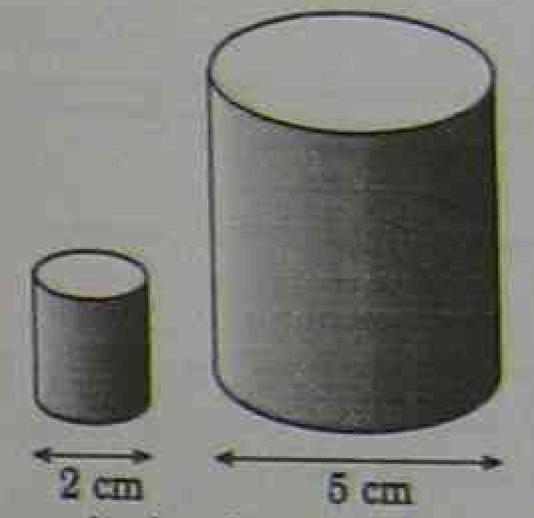


(a) Find the reduction ratio.

(b) Assuming, as have the scientists, that this ratio is fixed, calculate the circumference 20 m from the base of the tree.

(c) Calculate the area of cross-section of the tree, both at the 10 m height and at the 20 m height, given that the area of the base is estimated to be 750 m².

16.



Similar cylinders have base diameters of 2 cm and 5 cm.

(a) Find the ratio of:

(i) their surface areas

(ii) their volumes.

(b) If the larger cylinder has a volume of 60 cm³, find the volume of the smaller one.

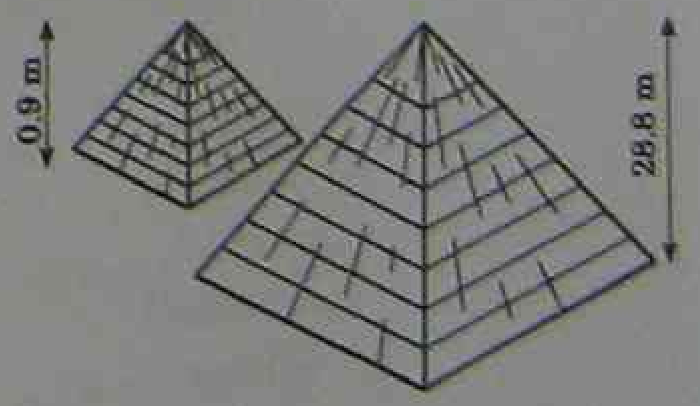
(c) If the smaller cylinder has a surface area of 18 cm², calculate the surface area of the larger one.

17. Two similar cubes with volumes 32 cm³ and 108 cm³ contain spherical glass balls which fit snugly in each one.

(a) Calculate the ratio of the diameters of the glass balls.

(b) Find the ratio of the surface areas of the glass balls.

18.



A scale model of the Great Pyramid of Kotara is to be constructed. The height

Continued

of the Great Pyramid is 28.8 m, while the height of the scale model is 0.9 m.

(a) Calculate the scale used.

(b) Calculate the ratio of the surface areas of the pyramids.

(c) If 48 L of paint is used to coat the Great Pyramid, calculate the quantity required to coat the scale model.

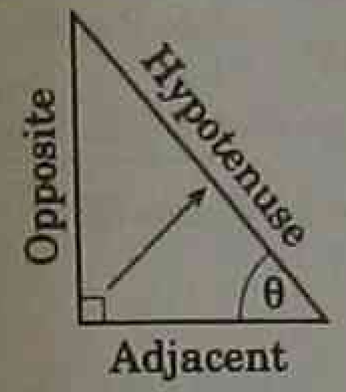
(d) If the quantity of air contained within the Great Pyramid is 8192 kL, calculate the amount contained within the scale model.

Chapter 10

TRIGONOMETRY

10.1 Right-angle triangle trigonometry

10.1.1 Sides and ratios



We name each of the sides. The hypotenuse is the side opposite the right angle. ⇐ The hypotenuse is also the longest side.

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \text{(SOH)}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \text{(CAH)}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} \quad \text{(TOA)}$$

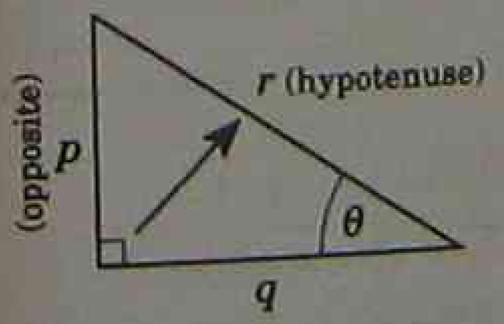
⇐ sin is the abbreviation for sine, cos for cosine and tan for tangent.

A way of remembering these definitions:

S	O	H	C	A	H	T	O	A
O	R	A	U	P	O	O	T	U
M	A	I	T	P	R		H	S
E	N	R	S	E	R		E	S
	G			A	I		R	I
	E			R	B			E
					L			S
					E			

Examples

(a) Find expressions for sin *θ*, cos *θ* and tan *θ*.



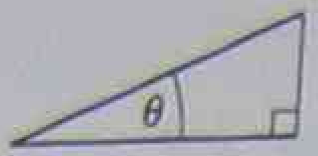
SOLUTION

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{p}{r}$$

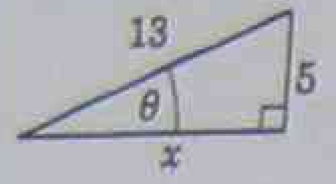
$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{q}{r}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{p}{q}$$

(b) Given theat $\sin\theta = \frac{5}{13}$, find an expression for $\tan\theta$ as a simple fraction.



SOLUTION



$$\sin\theta = \tfrac{5}{13} = \frac{\text{opposite}}{\text{hypotenuse}}$$

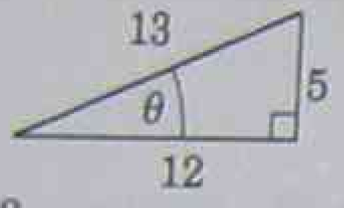
∴ opposite = 5, hypotenuse = 13

By Pythagoras' Theorem

$$x^2 + 5^2 = 13^2$$

$$\therefore \quad x^2 = 169 - 25$$

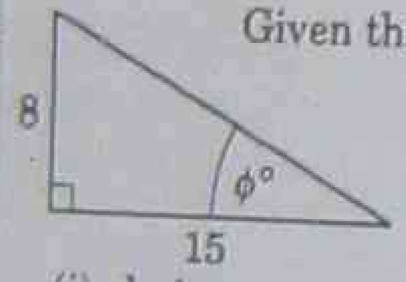
$$= 144$$



$$\therefore \quad x = \sqrt{144} = 12$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}$$

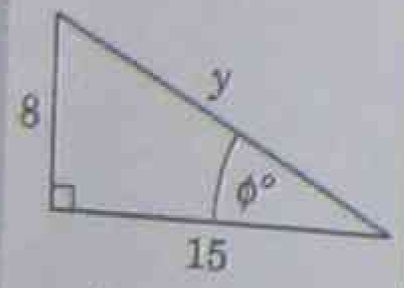
(c) Given that $\tan\phi = \frac{5}{12}$



(i) derive expressions for $\sin\phi$ and $\cos\phi$

(ii) Show that $\tan\phi = \frac{\sin\phi}{\cos\phi}$

SOLUTION



$$\tan\phi = \tfrac{8}{15}$$

$$= \frac{\text{opposite}}{\text{adjacent}}$$

opposite = 8,

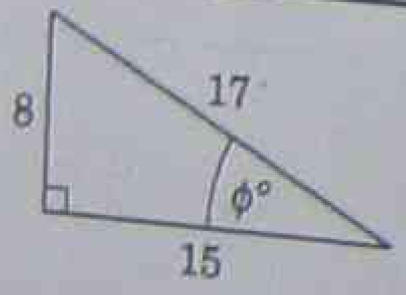
adjacent = 15

$$y^2 = 8^2 + 15^2$$

$$= 289$$

$$\therefore \ y = \sqrt{289}$$

$$= 17$$



(i)

$$\sin\phi = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}$$

$$\cos\phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$$

(ii)

$$\frac{\sin\phi}{\cos\phi} = \sin\phi + \cos\phi = \frac{8}{17} + \frac{15}{17}$$

$$= \frac{8}{17_{\,1}} \times \frac{17^{\,1}}{15} = \frac{8}{15}$$

$$\tan\phi = \frac{8}{15}$$

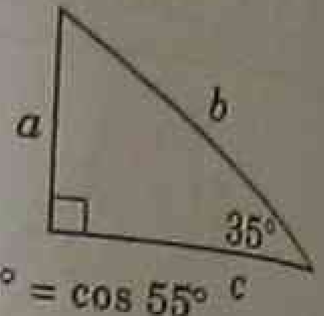
$$\therefore \quad \tan\phi = \frac{\sin\phi}{\cos\phi}$$

(d) Find expressions for

(i) sin 35°, cos 35°

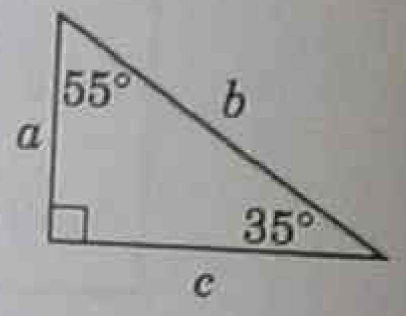
(ii) sin 55°, cos 55°

(iii) Show that sin 35° = cos 55°



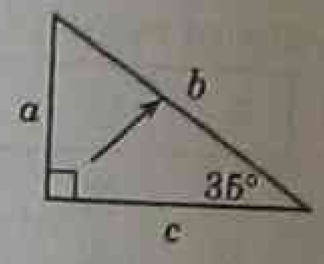
SOLUTION

(i)



$$\sin 35^\circ = \frac{a}{b}$$

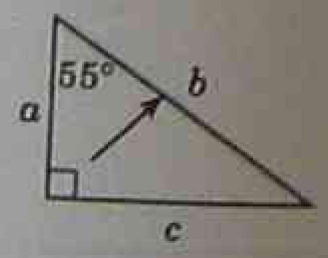
$$\cos 35^\circ = \frac{c}{b}$$



(ii)

$$\sin 55^\circ = \frac{c}{b}$$

$$\cos 55^\circ = \frac{a}{b}$$



(iii) $\sin 35^\circ = \frac{a}{b}$, $\cos 55^\circ = \frac{a}{b}$

$$\therefore \quad \sin 35^\circ = \cos 55^\circ$$

In general

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, for all θ

$\sin(90 - \theta) = \cos\theta$, for all θ $(0 < \theta < 90)$

$\cos(90 - \theta) = \sin\theta$, for all θ $(0 < \theta < 90)$

Examples

(a) sin 60° = cos x°, find x

(b) cos 10° = sin x°, find x

(c) Simplify the expressions:

(i) $\dfrac{\sin 28^\circ}{\cos 62^\circ}$

(ii) $\dfrac{\sin 28^\circ}{\sin 62^\circ}$

SOLUTIONS

(a) sin 60° = cos 30° ∴ x = 30

(b) cos 10° = sin 80° ∴ x = 80

(c) (i) $\dfrac{\sin 28^\circ}{\cos 62^\circ} = \dfrac{\sin 28^\circ}{\sin 28^\circ} = 1$

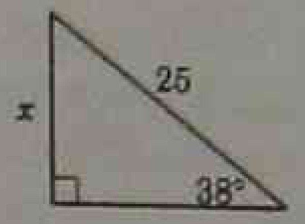
(ii) $\dfrac{\sin 28^\circ}{\sin 62^\circ} = \dfrac{\sin 28^\circ}{\cos 28^\circ} = \tan 28^\circ$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

10.1.2 Finding the sides of right-angled triangles

Examples

(a) Find the side x.



SOLUTION

$$\overset{\text{Opp}}{\underset{\text{Hyp}}{}}\frac{x}{25} = \sin 38^\circ,$$

$$\therefore \quad x = 25 \times \sin 38^\circ$$

$$= 15.391\,537$$

$$\approx 15.4 \text{ (one decimal place)}$$

Always try to put the unknown side (the letter) on the top of the ratio. This is **not** always possible but should be tried first.

Calculator

25 [×] 38 [SIN] [=]

(b) Find the side y.

SOLUTION

$$\overset{\text{Opp}}{\underset{\text{Adj}}{}}\frac{y}{48} = \tan 24^\circ$$

$$y = 48 \times \tan 24^\circ$$

$$= 21.370\,977$$

$$\approx 21.4 \text{ (one decimal place)}$$

Calculator

48 [×] 24 [TAN] [=]

(c) When the altitude of the sun is 63° 14′, a tree casts a shadow 8.6 m long. Calculate the height of the tree correct to three significant figures.

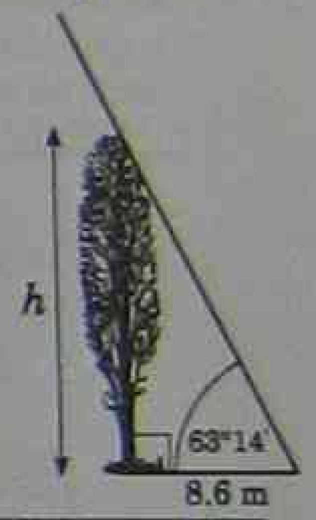
SOLUTION

$$\frac{h}{8.6} = \tan 63^\circ 14'$$

$$h = 8.6 \times \tan 63^\circ 14'$$

$$= 17.049\,746$$

$$\approx 17.0 \text{ (three significant figures)}$$



Calculator

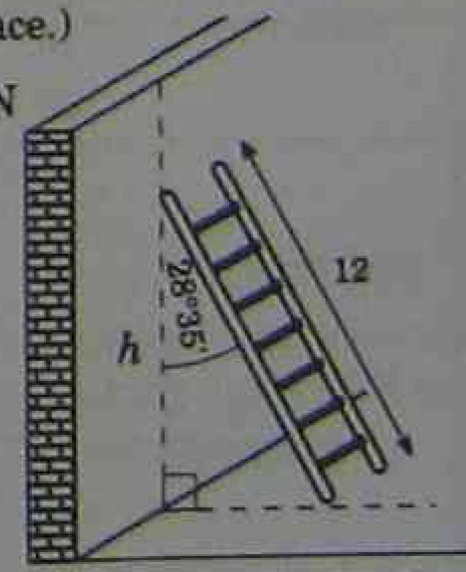
8.6 [×] 63 [° ′ ″] 14 [° ′ ″] [TAN] [=]

The height of the tree is 17.0 metres.

Note: the correct angle from the diagram is the angle between the sun's rays and ground level.

(d) A 12 m ladder leans against a brick wall and is inclined at 28° 35′ to the wall. How far up the wall will the ladder reach? (Answer correct to one decimal place.)

SOLUTION



Continued

$$\frac{h}{12} = \cos 28^{\circ}\,35'$$
$$h = 12 \times \cos 28^{\circ}\,35'$$
$$= 10.537\,466$$
$$\approx 10.5 \text{ (one decimal place)}$$

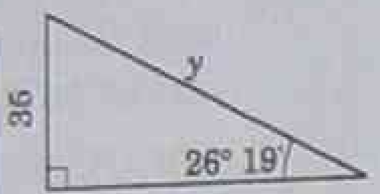
Calculator
$12\ \boxed{\times}\ 28\ \boxed{{}^{\circ}\,{}'\,{}''}\ 35\ \boxed{{}^{\circ}\,{}'\,{}''}\ \boxed{\text{COS}}\ \boxed{=}$

The ladder reaches 10.5 metres up the wall.

Note: The FIX mode can be used to correct an answer to a given number of decimal places.

(e) A special case: Find the hypotenuse.

SOLUTION



We cannot put the letter on top this time as $\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$.

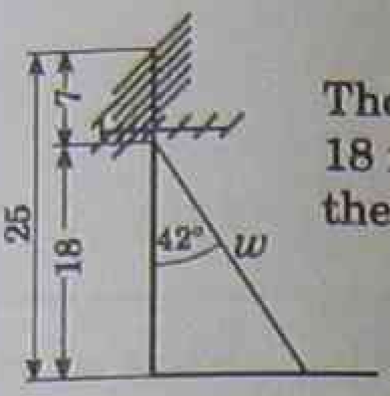
$$\frac{^{\text{Opp}}\ 35}{_{\text{Hyp}}\ y} = \sin 26^{\circ}\,19'$$
$$\frac{1}{y} = \frac{\sin 26^{\circ}\,19'}{35}$$
$$y = \frac{35}{\sin 26^{\circ}\,19'}$$
$$= 78.947\,615$$
$$\approx 78.9 \text{ (one decimal place)}$$

Take reciprocal of both sides.

Calculator
$35\ \boxed{\div}\ 26\ \boxed{{}^{\circ}\,{}'\,{}''}\ 19\ \boxed{{}^{\circ}\,{}'\,{}''}\ \boxed{\text{SIN}}\ \boxed{=}$

(f) A 25 metre high antenna is held in position by four wires fixed 7 m from the top of the antenna. If the wires are attached so as to make an angle of 42° with the antenna, how much wire is used (to the nearest cm).

SOLUTION



The wire is attached 18 m from the base of the antenna.

Do not round off until the end.

$$\frac{18}{w} = \cos 42^{\circ}$$
$$\frac{1}{w} = \frac{\cos 42^{\circ}}{18}$$
$$w = \frac{18}{\cos 42^{\circ}}$$
$$= 24.221\,389$$

Length of wire needed
$$= 4 \times 24.221\,389$$
$$= 96.885\,557$$
$$\approx 96.89 \text{ (to the nearest cm)}$$

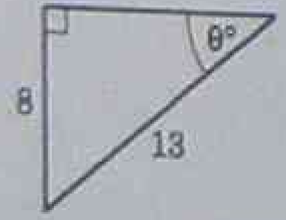
The length of wire needed is 96.89 metres.

Nearest cm is the same as correct to two decimal places. Why?

10.1.3 Finding the angles of right-angled triangles

Examples

(a) Find θ.



SOLUTION

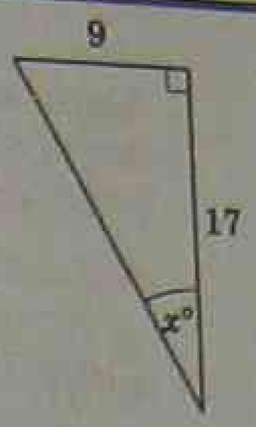
$$\sin\theta = \frac{8}{13}$$
$$\therefore\ \theta = 37^{\circ}\,59'$$

$\frac{\text{Opp}}{\text{Hyp}}$

Calculator
$8\ \boxed{\div}\ 13\ \boxed{=}\ \boxed{\text{INV}}\ \boxed{\text{SIN}}\ \boxed{\text{INV}}\ \boxed{{}^{\circ}\,{}'\,{}''}$

Note: The answer is $37^{\circ}\ 58^{\circ}\ \underline{47}$ this means 37° 58' 47''. To correct this to the nearest minute, consider the third (underlined) number. When this number ≥ 30, the number of minutes rises by one. If the number is < 30, the number of minutes is unchanged.

(b) Find x.



SOLUTION

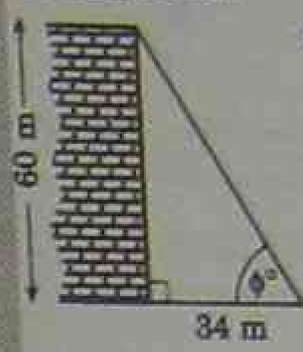
$$\tan x = \frac{9}{17}$$
$$\therefore\ x^{\circ} = 27^{\circ}\,54'$$

$\frac{\text{Opp}}{\text{Adj}}$

Calculator
$9\ \boxed{\div}\ 17\ \boxed{=}\ \boxed{\text{INV}}\ \boxed{\text{TAN}}\ \boxed{\text{INV}}\ \boxed{{}^{\circ}\,{}'\,{}''}$

(c) Bill stands 34 m from a 60 m high building. Find the angle of elevation of the top of the building (to the nearest minute).

SOLUTION



Angle of elevation is ϕ°.

$$\tan\phi = \frac{60}{34}$$
$$\therefore\ \phi = 60^{\circ}\,28'$$

$\frac{\text{Opp}}{\text{Adj}}$

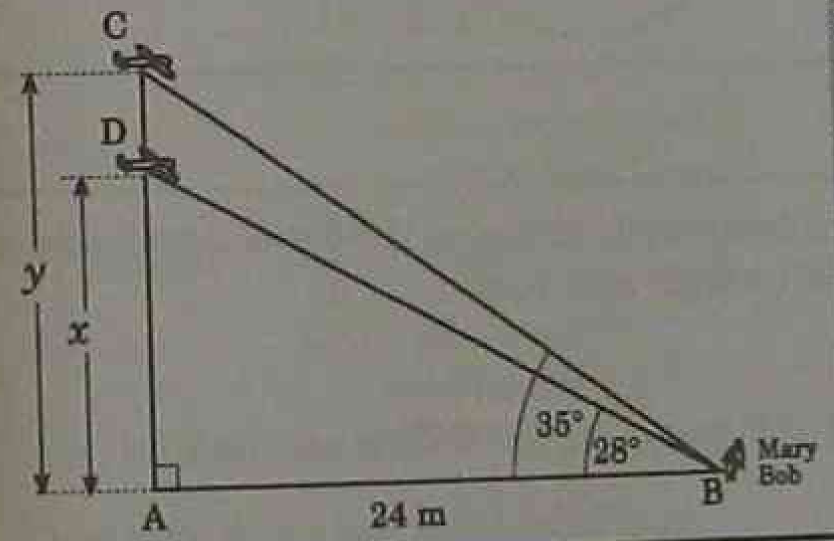
The angle of elevation is 60° 28'.

10.1.4 Two-triangle problems

Example

Mary and Bob are flying model planes so that Mary's plane is directly above Bob's. The planes, held by wires, are flying about the couple in a circle of 24 m radius. The angles that the wires make with the horizontal are 28° and 35° for Bob and Mary's planes respectively. Mary's plane is at C, Bob's at D.

Calculate the vertical distance between the two planes.



SOLUTION

The required distance is CD. This is the difference between AC and AD.

Let AD = x m and AC = y m.

In ΔABC, $\frac{y}{24} = \tan 35^{\circ}$
$\therefore\quad y = 24 \times \tan 35^{\circ}$.

In ΔABD, $\frac{x}{24} = \tan 28^{\circ}$
$\therefore\quad x = 24 \times \tan 28^{\circ}$.

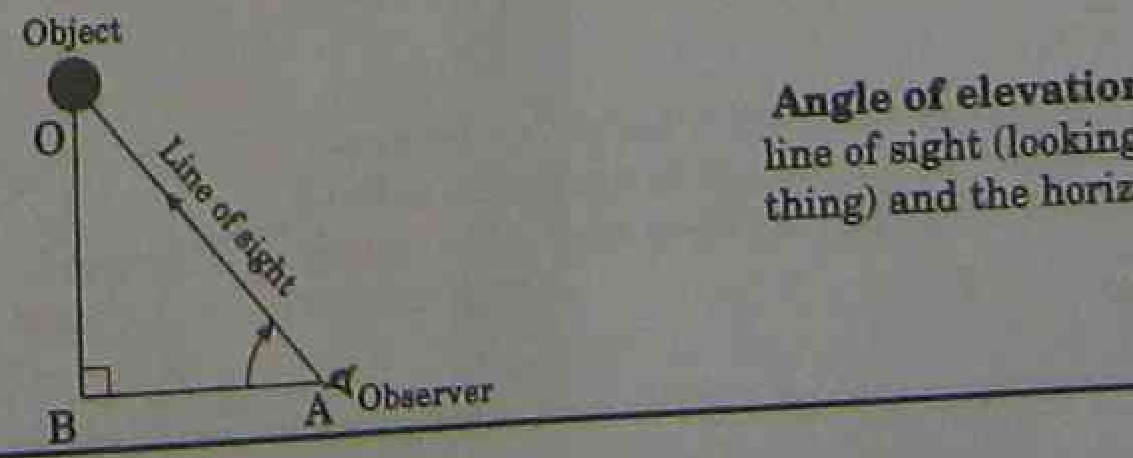
$$\text{Now } CD = y - x$$
$$= 24\tan 35^{\circ} - 24\tan 28^{\circ}$$
$$= 4.043\,9546$$
$$= 4 \text{ (nearest metre)}$$

Mary's plane is four metres above Bob's plane.

10.2 Some notes regarding angles

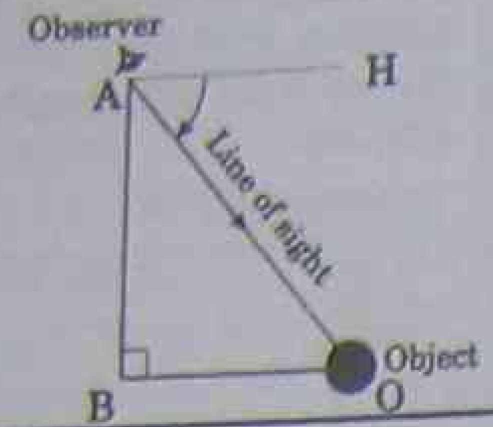
10.2.1 Angles of elevation and depression

Figure 1



Angle of elevation: angle between line of sight (looking upward at something) and the horizontal.

Figure 2

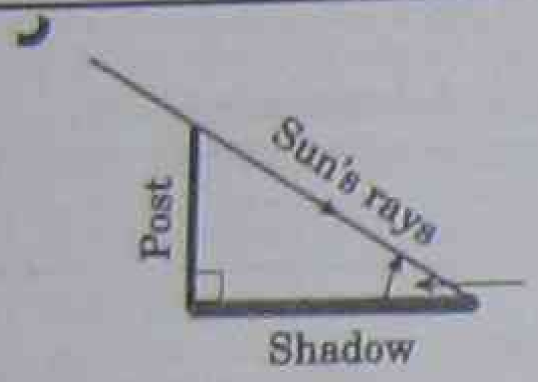


Angle of depression: angle between line of sight (looking downward at something) and the horizontal.

Angle of elevation = Angle of depression

In Figure 2, $\angle HAO = \angle BOA$ (Why?)

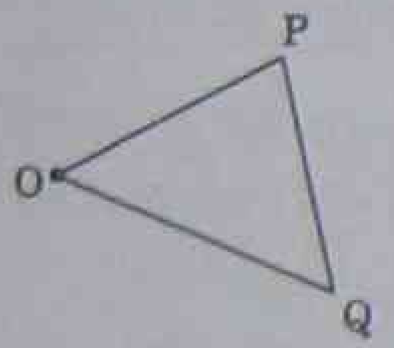
10.2.2 Angle of elevation of the sun



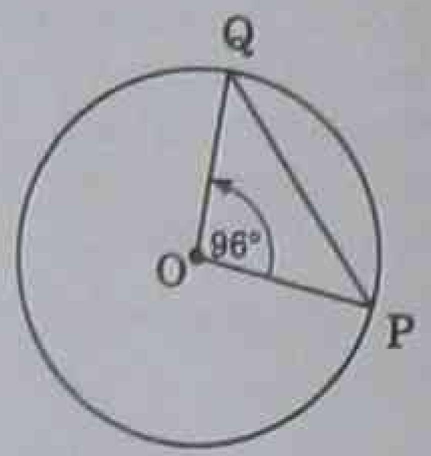
The angle concerned is that between the sun's rays and the horizontal.

Angle of elevation of the sun.

10.2.3 Angles subtended by an interval

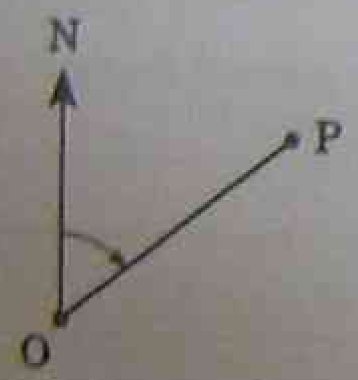


PQ subtends the angle POQ at the point O.



The chord PQ subtends an angle of 96° at the centre, O, of the circle. (This terminology is also used in geometry of the circle.)

10.2.4 Bearings

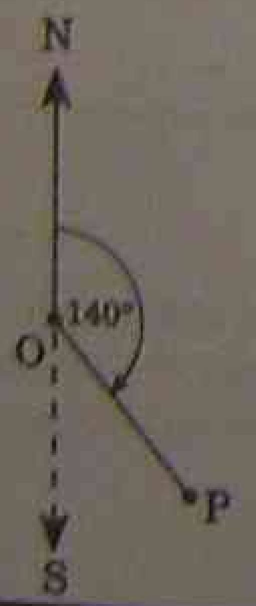


Bearings are measured clockwise from due north and are written as three-digit numbers.

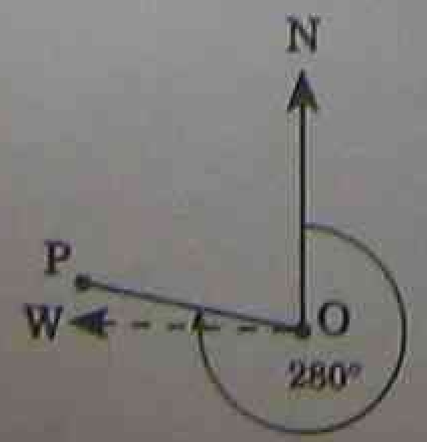
The bearing of P from O is 047°. This means that $\angle NOP = 47°$.

Examples: Bearings of P from O

(a) 140°



(b) 280°



Special cases

NE has bearing 045°.

Due E has bearing 090°.

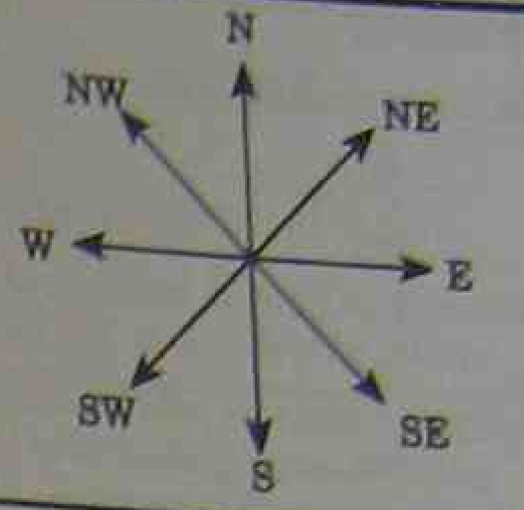
SE has bearing 135°.

Due S has bearing 180°.

The angle between NE and SE is 90°.

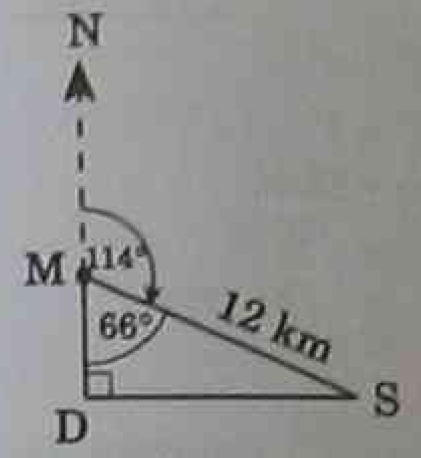
The angle between SW and NW is 90°.

The angle between W and NW is 45°.



10.2.5 Examples involving trigonometry

(a) A ferry leaves Minmi Breakwater and steams 12 km on a bearing of 114° to a point S. Find the distance that the ferry is south of Minmi.



SOLUTION

This asks us to find the distance MD.

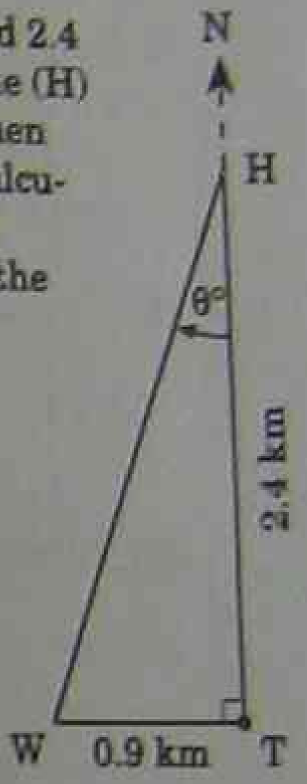
$$\angle NMS = 114°$$
$$\angle DMS = 180° - 114° = 66° \quad \text{(angle sum of straight lines)}$$

In ΔDSM, $\frac{MD}{12} = \cos 66°$

$$\therefore \quad MD = 12 \times \cos 66° = 4.880\,8397 \approx 4.9.$$

The ferry is 4.9 km south of Minmi.

(b) Red Riding Hood walked 2.4 km due south from home (H) to an old tree (T) and then due west for 0.9 km. Calculate Red Riding Hood's bearing from home (to the nearest degree).



SOLUTION

The actual angle required is $\angle NHW$. This is the bearing of W from H.

We must first find $\angle WHT$. Call it $\theta°$.

$$\tan\theta = \frac{0.9}{2.4}$$
$$\theta = 21°$$

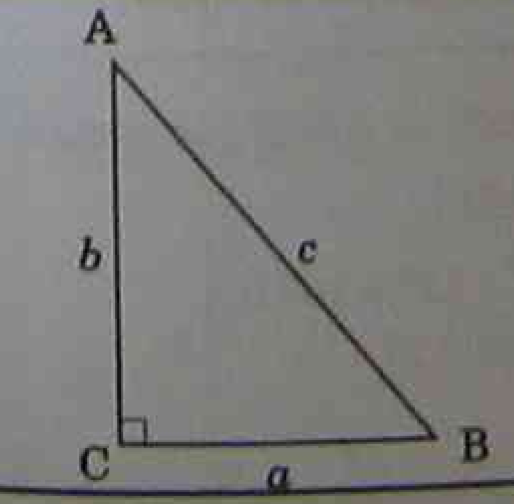
(to the nearest degree)

$$\therefore \angle NHW = 180° + 21° = 201°$$

The bearing of Red Riding Hood is 201°.

10.3 The Theorem of Pythagoras revisited

10.3.1 Pythagoras' Theorem



In any right-angled triangle ABC,

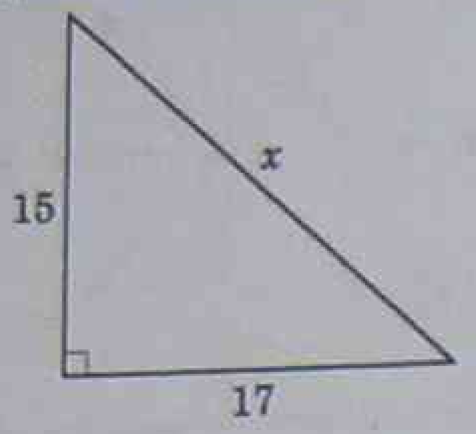
$$c^2 = a^2 + b^2.$$

There are two types of questions involving Pythagoras' Theorem: finding the length of the hypotenuse (longest side — the one opposite the right angle) or finding the length of one of the shorter sides.

Examples

(a) Find x in each diagram.

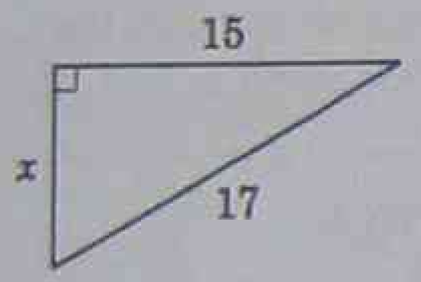
(i)



SOLUTION

$$x^2 = 15^2 + 17^2$$
$$= 514$$
$$\therefore x = \sqrt{514}$$
$$= 22.671568$$
$$\approx 22.7 \text{ (one decimal place)}$$

(ii)

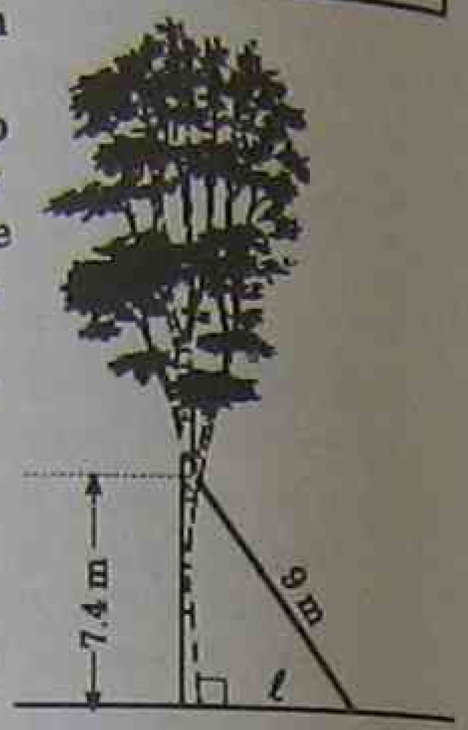


SOLUTION

$$x^2 + 15^2 = 17^2$$
$$x^2 = 17^2 - 15^2$$
$$= 64$$
$$x = \sqrt{64}$$
$$= 8$$

The hypotenuse squared is always written by itself. Then solve the equation for the letter used.

(b) A 9 metre length of timber reaches 7.4 m up a gum tree. How far from the base of the tree is the base of the length of timber (one decimal place).

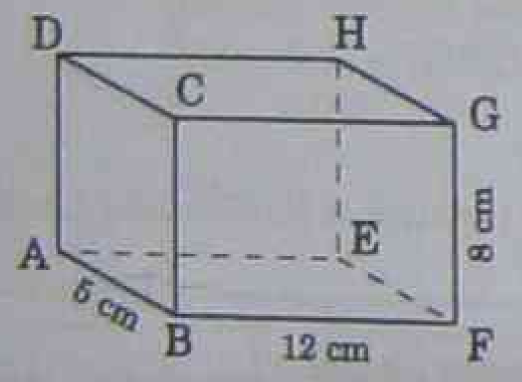


SOLUTION

$$l^2 + 7.4^2 = 9^2$$
$$\therefore \quad l^2 = 9^2 - 7.4^2$$
$$= 26.24$$
$$l = \sqrt{26.24}$$
$$= 5.122\,4994$$
$$\approx 5.1$$

The timber is 5.1 m from the base of the tree.

10.3.2 Three-dimensional problems

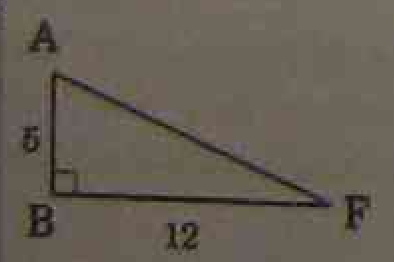


Trigonometry can be used with 3-D figures to calculate lengths and angles. Sometimes it is used in conjunction with Pythagoras' Theorem.

In this problem we have the rectangular prism sides of length 5 cm, 12 cm and 8 cm (as in the diagram). The problem is to find ∠AFD. This could be phrased in many ways but consider this one:

'Louie the fly sits at F and wishes to fly up to see his girl-friend fly, Maisie at D. At what angle to the base plane ABFE must Louie take off to fly the most direct route?'

SOLUTION



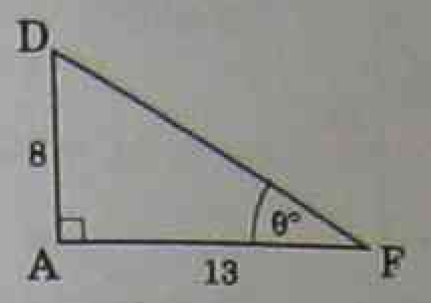
We can break the question into two parts. The first step is to find the length of the base diagonal AF.

∠ABF = 90°, as ABFE is a rectangle.

Using ΔABF,

$$AF^2 = 5^2 + 12^2 = 169$$
$$AF = \sqrt{169} = 13$$

Step two is to use ΔAFD to calculate ∠AFD.



∠DAF = 90°, because AD is perpendicular to the base plane.

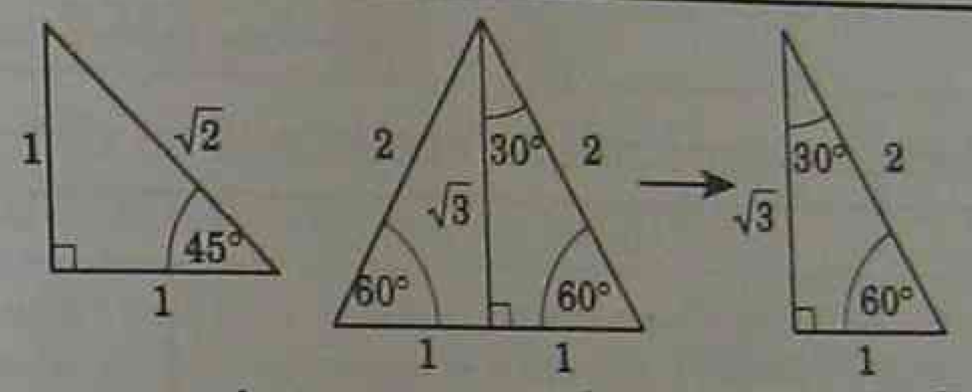
$$\tan\theta = \frac{8}{13}$$
$$\therefore \quad \theta = 31°\,36',$$

that is, $\angle AFD = 31°\,36'$.

Louie must take off at an angle of 31° 36' to fly direct to Maisie.

Also, note: It would be an easy task to find the length DF using Pythagoras' Theorem.

10.3.3 Exact values for ratios involving angles of 30°, 45°, and 60°



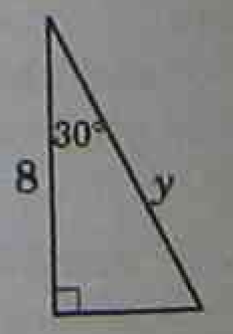
$$\sin 45° = \tfrac{1}{\sqrt{2}} \qquad \sin 30° = \tfrac{1}{2} \qquad \sin 60° = \tfrac{\sqrt{3}}{2}$$
$$\cos 45° = \tfrac{1}{\sqrt{2}} \qquad \cos 30° = \tfrac{\sqrt{3}}{2} \qquad \cos 60° = \tfrac{1}{2}$$
$$\tan 45° = 1 \qquad \tan 30° = \tfrac{1}{\sqrt{3}} \qquad \tan 60° = \sqrt{3}$$

Ratio	Angle		
	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

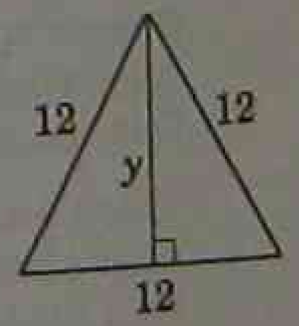
Examples using the exact ratios

(a) Find the exact value of y in these diagrams

(i)

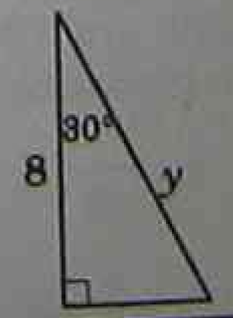


(ii)



SOLUTION

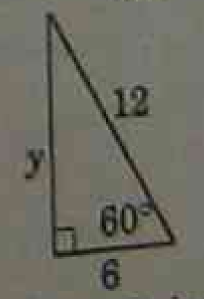
(i)



$$\frac{8}{y} = \cos 30°$$
$$\therefore \quad \frac{8}{y} = \frac{\sqrt{3}}{2}$$
$$\therefore \quad \sqrt{3}y = 16$$
$$y = \frac{16}{\sqrt{3}}$$

Cross multiply

(ii)



Triangle is equilateral.
Each angle is 60°.

Continued

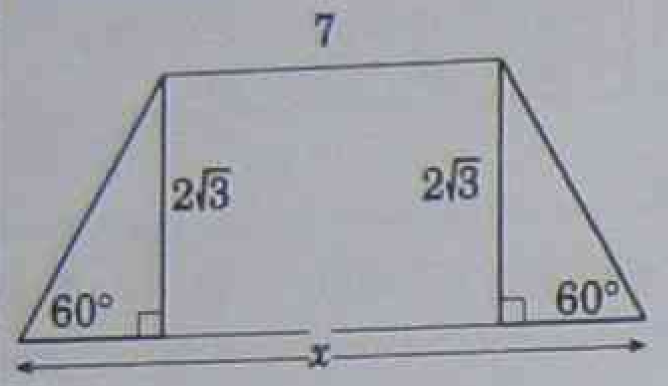
$$\therefore \quad \frac{y}{12} = \sin 60^\circ$$

$$\therefore \quad y = 12 \sin 60^\circ$$

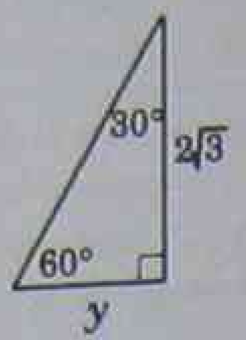
$$= 12 \times \frac{\sqrt{3}}{2}$$

$$= 6\sqrt{3}$$

(b) By first calculating the length x, find the area of the trapezium



SOLUTION



$$\frac{y}{2\sqrt{3}} = \tan 30^\circ$$

$$\therefore \quad y = 2\sqrt{3} \tan 30^\circ$$

$$= 2\sqrt{3} \times \frac{1}{\sqrt{3}}$$

$$= 2$$

Then $x = 2 + 7 + 2 = 11$

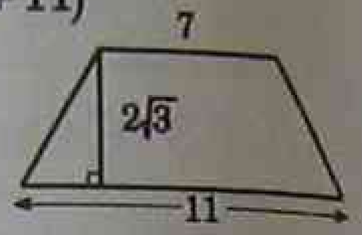
$$\text{Area} = \tfrac{1}{2} h(a+b)$$

$$= \tfrac{1}{2} \times 2\sqrt{3} \times (7+11)$$

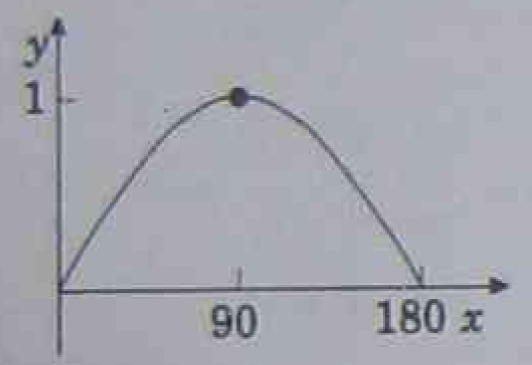
$$= \sqrt{3}(18)$$

$$= 18\sqrt{3}$$

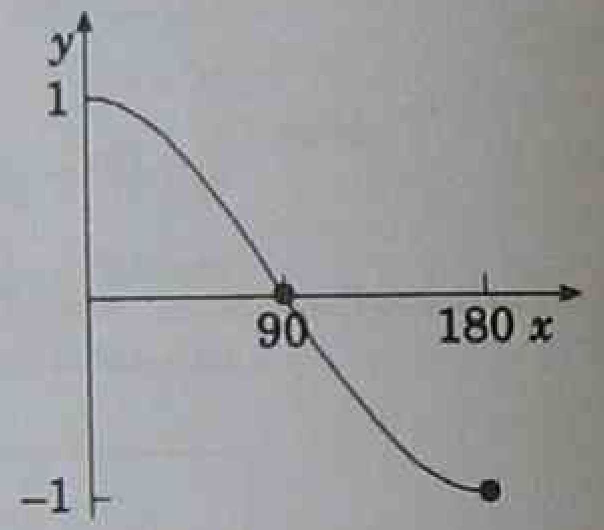
Area is $18\sqrt{3}$ units^2.



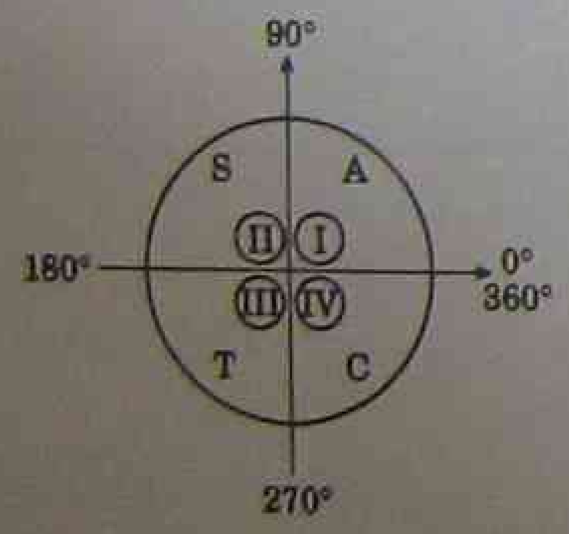
10.3.4 Graphs of sine and cosine functions $(0^\circ \le x^\circ \le 180^\circ)$



$y = \sin x \ (0^\circ \le x^\circ \le 180^\circ)$



10.4 Trigonometric ratios for angles of any magnitude



Remember

All
Stations
To
Central

Trigonometric ratios can be found for angles that cannot exist in right-angled triangles.

Consider:

First quadrant
Angles less than 90°.
All ratios positive.

Second quadrant
Angles between 90° and 180°.
Only the sin ratio is positive.

Third quadrant
Angles between 180° and 270°.
Only the tan ratio is positive.

Fourth quadrant
Angles between 270° and 360°.
Only the cos ratio is positive.

Examples

(a) $\tan 140^\circ = -\tan(180^\circ - 140^\circ)$ $\boxed{\text{Second quadrant}}$
$= -\tan 40^\circ$

(b) $\cos 128^\circ\, 18' = -\cos(180^\circ - 128^\circ\, 18')$
$= -\cos 51^\circ\, 42'$ $\boxed{\text{Second quadrant}}$

> *Note*: Your calculator will give you the correct values for tan 140° and cos 128° 18' directly. Try it.
>
> 140 $\boxed{\text{TAN}}$
>
> 128 $\boxed{\circ\,'\,''}$ 18 $\boxed{\circ\,'\,''}$ $\boxed{\text{COS}}$

(c) Find the size of the obtuse angle θ, given that cos θ = −0.025.

SOLUTION

$$\cos\theta = -0.025$$

$$\theta = 91^\circ\, 26'$$

> Calculator
>
> 0.025 $\boxed{+/-}$ $\boxed{\text{INV}}$ $\boxed{\text{COS}}$ $\boxed{\text{INV}}$ $\boxed{\circ\,'\,''}$

Your calculator automatically gives the obtuse angle if cos is negative.

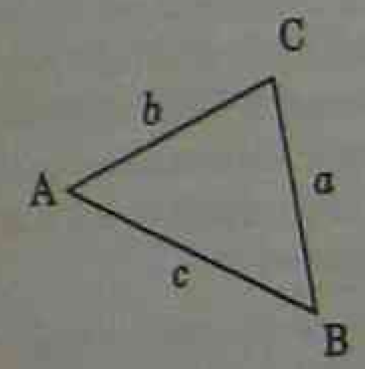
10.5 Triangles without right angles

Trigonometry can be used in all triangles regardless of whether they contain a right angle. There are two methods and each requires a formula. **THESE FORMULAE MUST BE KNOWN.**

10.5.1 The Sine Rule (Rule of opposites)

Use the Sine Rule when you know two angles and one side, or when you know two sides and an angle.

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

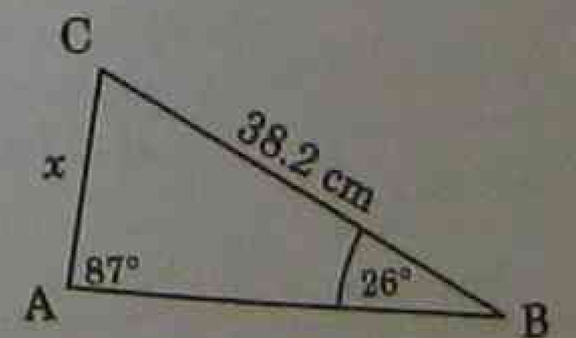


Note the opposites in the formula:

$$\frac{\text{side}}{\sin\,[\text{opposite angle}]} = \frac{\text{side}}{\sin\,[\text{opposite angle}]}$$

Examples

(a) Find x, correct to three significant figures.



SOLUTION

$$\frac{a^{\surd}}{\sin A^{\surd}} = \frac{b^{\surd}}{\sin B^{\surd}} = \frac{c}{\sin C}$$

We have $\hat{A} = 87^\circ$, $\hat{B} = 26^\circ$ and $a = 38.2$.
We want b.

$$\frac{38.2}{\sin 87^\circ} = \frac{x}{\sin 26^\circ}$$

$$\text{Therefore} \quad x = \frac{38.2 \sin 26^\circ}{\sin 87^\circ}$$

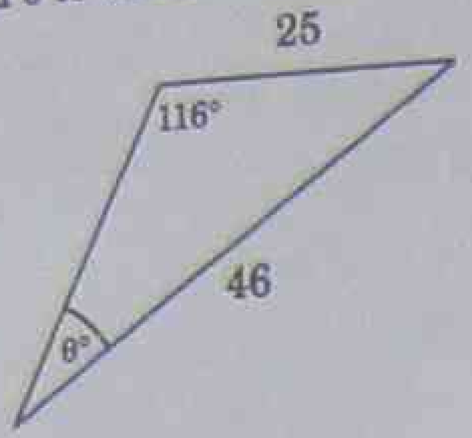
$$= 16.768\,759$$

$$= 16.8$$

(three significant figures)

> Calculator
>
> 38.2 $\boxed{\times}$ 26 $\boxed{\text{SIN}}$ $\boxed{+}$ 87 $\boxed{\text{SIN}}$ $\boxed{=}$

(b) Find θ to the nearest minute.



SOLUTION

$$\frac{25}{\sin\theta} = \frac{46}{\sin 116^\circ} \qquad \frac{\text{Side}}{\sin(\text{Opp})}$$

$$\therefore 46\sin\theta = 25\sin 116^\circ$$

$$\sin\theta = \frac{25\sin 116^\circ}{46}$$

$$\therefore \quad \theta = 29^\circ\, 14'$$

Calculator

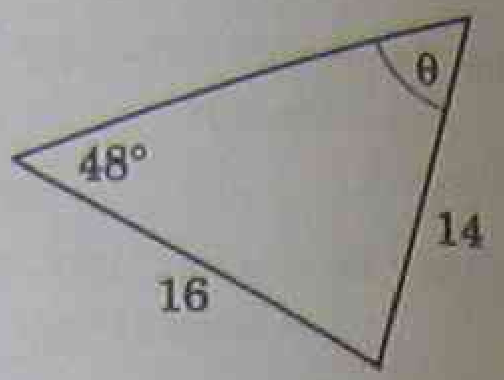
25 $\boxed{\times}$ 116 $\boxed{\text{SIN}}$ $\boxed{\div}$ 46 $\boxed{=}$

$\boxed{\text{INV}}$ $\boxed{\text{SIN}}$ $\boxed{\text{INV}}$ $\boxed{\circ\,'\,''}$

The ambiguous case

Note: The required angle could also be (180° – 29° 14′), that is, 150° 46′, but this would mean the angle sum of the triangle is greater than 180°. As both acute and obtuse angles have sine positive, an obtuse answer is always possible. When a decision cannot be made between the acute result and the obtuse result it is known as the ambiguous case.

(c) Find θ to the nearest minute.



SOLUTION

$$\frac{16}{\sin\theta} = \frac{14}{\sin 48^\circ}$$

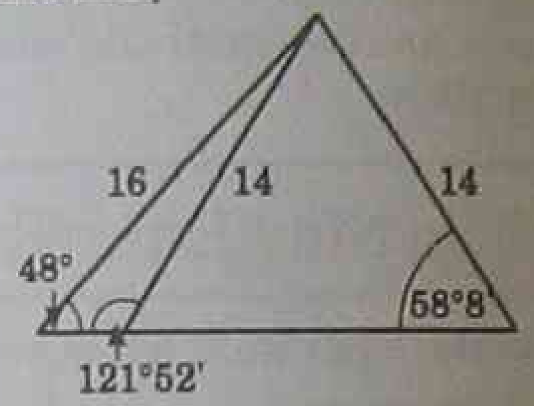
$$14\sin\theta = 16\sin 48^\circ$$

$$\sin\theta = \frac{16\sin 48^\circ}{14}$$

$$\theta = 58^\circ\, 8'$$

$$\text{But also, } \theta = (180^\circ - 58^\circ\, 8') = 121^\circ\, 52'$$

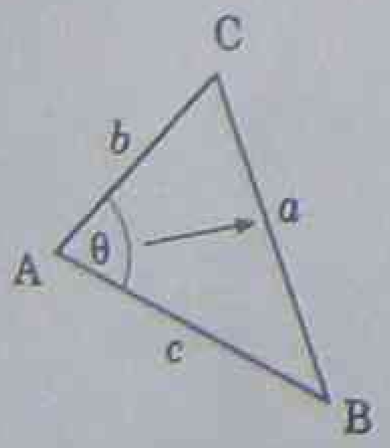
Either answer is possible. From this information,



This may happen when using the Sine Rule to find an angle. Check with the original information — the obtuse value may not be possible. Remember that the angle sum of the triangle is 180°.

10.5.2 The Cosine Rule

Finding a side



The Cosine Rule is used when the question involves two sides and the included angle.

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

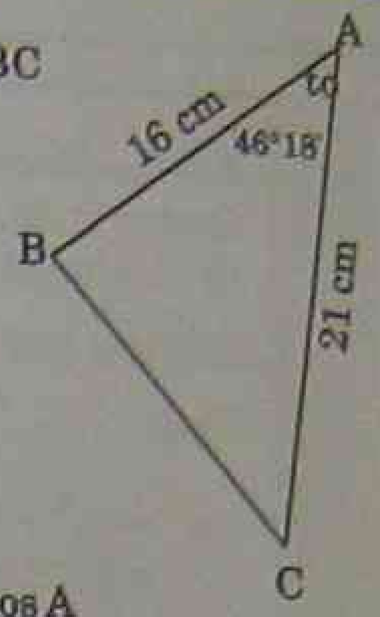
Sides around angle A

Angle opposite the side being found

Similar formulae can be written for b^2 and c^2.

Example

(a) Find the length of BC the nearest cm.



SOLUTION

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 21^2 + 16^2 - 2 \times 21 \times 16 \times \cos 46^\circ\, 18'$$

$$= 232.727\,02 \quad \leftarrow \quad \boxed{\text{Keep this on your screen.}}$$

$$a = \sqrt{232.727\,02}$$

$$= 15.255\,393$$

$$\approx 15$$

BC is 15 cm.

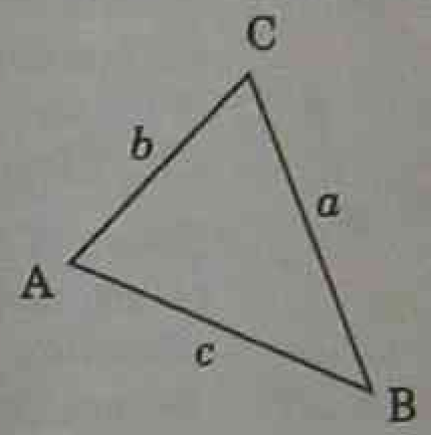
The complete working can be done on your calculator.

21 $\boxed{\text{INV}}$ $\overset{x^2}{\boxed{\sqrt{\ }}}$ $\boxed{+}$ 16 $\boxed{\text{INV}}$ $\overset{x^2}{\boxed{\sqrt{\ }}}$ $\boxed{-}$
2 $\boxed{\times}$ 21 $\boxed{\times}$ 16 $\boxed{\times}$ 46 $\boxed{\circ\,'\,''}$ 18 $\boxed{\circ\,'\,''}$
$\boxed{\text{COS}}$ $\boxed{=}$

If your calculator has a separate $\boxed{x^2}$ button, ignore the two $\boxed{\text{INV}}$.

This calculation gives a^2, so we need to push $\boxed{\sqrt{\ }}$.

To find an angle



The question must contain values for all three sides to be able to use the angle form of the Cosine Rule.

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

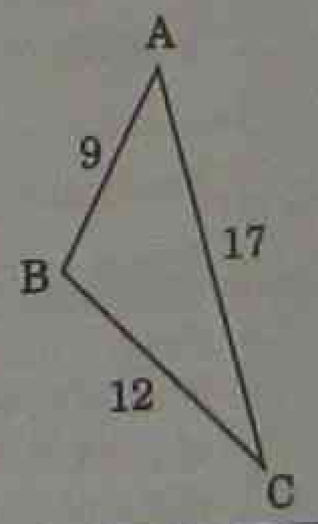
Similar forms can be written for cos B and cos C. Note that the side subtracted is the one opposite the required angle.

Example

(b) In the ΔABC, $a = 12$ cm, $b = 17$ cm and $c = 9$ cm. Find the size of the largest angle.

SOLUTION

The largest angle is always opposite the longest side. Therefore the angle required is B.



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{12^2 + 9^2 - 17^2}{2 \times 12 \times 9}$$

$$= \frac{(12^2 + 9^2 - 17^2)}{(2 \times 12 \times 9)} \qquad \boxed{\text{Use brackets around top and bottom.}}$$

$$= -0.296\,2963 \quad \leftarrow \quad \boxed{\text{Leave this on the screen.}}$$

$$\therefore \ B = 107^\circ\, 14'.$$

Calculator

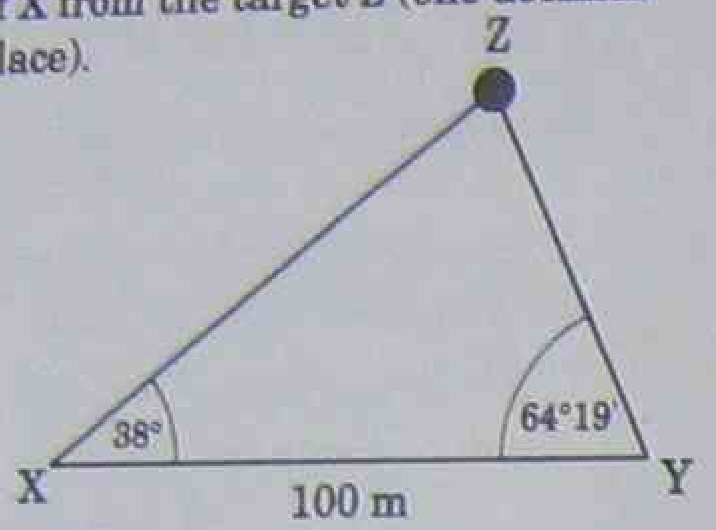
$\boxed{(}$ 12 $\boxed{\text{INV}}$ $\overset{x^2}{\boxed{\sqrt{\ }}}$ $\boxed{+}$ 9 $\boxed{\text{INV}}$ $\overset{x^2}{\boxed{\sqrt{\ }}}$ $\boxed{-}$
17 $\boxed{\text{INV}}$ $\overset{x^2}{\boxed{\sqrt{\ }}}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 2 $\boxed{\times}$ 12 $\boxed{\times}$
9 $\boxed{)}$ $\boxed{=}$

This gives cos B, then use

$\boxed{\text{INV}}$ $\boxed{\text{COS}}$ $\boxed{\text{INV}}$ $\boxed{\circ\,'\,''}$

10.5.3 Miscellaneous worked examples

(a) Two cannons X and Y, 100 metres apart, fire at a target Z. If ∠ZXY = 38° and ∠ZYX = 64° 19′, find the distance of X from the target Z (one decimal place).



SOLUTION

The question asks us to find the length of XZ. We have two angles and a side — use the Sine Rule. *But*, the Sine Rule needs the angles opposite the sides. We must calculate the size of $\hat{Z}$.

$$\hat{Z} = 180° - (64°\,19' + 38°) = 77°\,41'.$$

angle sum of a Δ

Then

$$\frac{\text{side}}{\sin[\text{opp. angle}]}$$

$$\frac{XZ}{\sin 64°\,19'} = \frac{100}{\sin 77°\,41'}$$

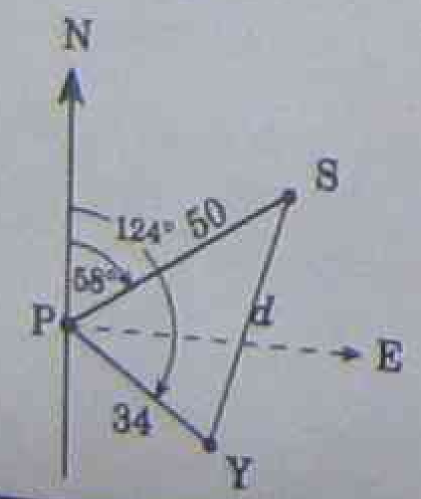
$$\therefore \quad XZ = \frac{100 \sin 64°\,19'}{\sin 77°\,41'} = 92.243\,428 \approx 92.2.$$

The distance of X from the target is 92.2 m.

(b) A ship S leaves port P and travels 50 km on a bearing of 58°, while a yacht Y also leaves P and travels 34 km on a bearing of 124°. Find: (i) the distance between the ship and the yacht; and (ii) the bearing of the yacht as measured from the ship.

SOLUTIONS

(i)



Let the distance SY be d km. (Use the Cosine Rule, as we know two sides and the included angle.)

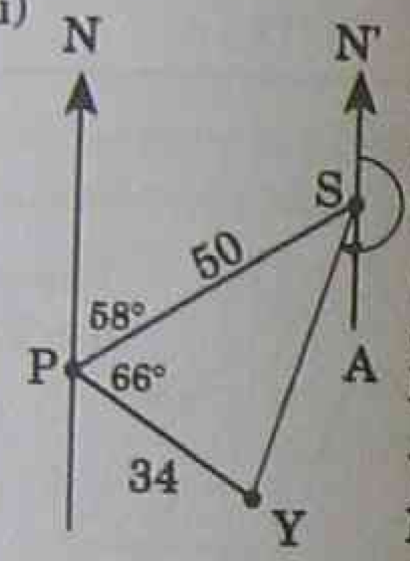
$\angle SPY = 124° - 58° = 66°$.

Then

$$d^2 = 50^2 + 34^2 - 2(50)(34)\cos 66°$$
$$= 2273.0954 \leftarrow \boxed{\text{Leave on screen}}$$
$$\therefore \; d = \sqrt{2273.0954} = 47.676\,99 \approx 47.7 \text{ (one decimal place)}$$

The distance between the boats is 47.7 km.

(ii)



The bearing of Y from S is ∠N′SY. This can be calculated by subtracting ∠ N′SP and ∠PSY from 360°.

Note ∠N′SY is reflex; clockwise direction.

Now ∠N′SP = 180° − 58° = 122° (co-interior angles, NP || N′A).

Use the Cosine Rule to calculate ∠PSY as we have three sides of the triangle.

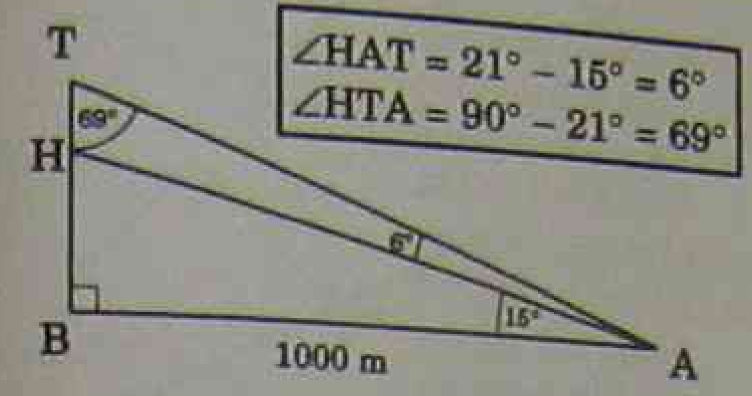
$$\cos\angle PSY = \frac{(50^2 + 47.7^2 - 34^2)}{(2 \times 50 \times 47.7)}$$
$$\angle PSY = 40°\,39'$$
$$\text{Bearing of Y} = 360° - (122° + 40°\,39') = 197°\,21'$$

The bearing of Y is 197° 21′.

(c) **A two-triangle problem**

From a point, A, level with the base of a hill, the angle of elevation of the top of the hill, H, is measured as 15°. The angle of elevation from A to the top of the tower, T, built on the hill is 21°. If A is 1000 m in a direct line from the base (B) of the hill, calculate the length AH and use this information to calculate the height of the tower (TH) correct to one decimal place.

SOLUTION



∠HAT = 21° − 15° = 6°
∠HTA = 90° − 21° = 69°

In ΔABH,

$$\frac{AB}{AH} = \cos 15°$$
$$\therefore \; AH = \frac{1000}{\cos 15°} = 1035.2762$$

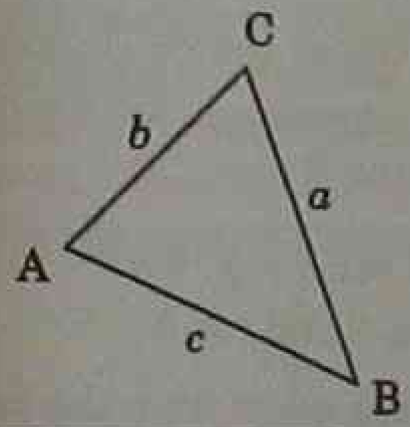
This length is then used as a link to ΔAHT.

In ΔAHT,

$$\frac{TH}{\sin 6°} = \frac{AH}{\sin 69°}$$
$$TH = \frac{AH \sin 6°}{\sin 69°} = \frac{1035.2762 \times \sin 6°}{\sin 69°} = 115.914\,84 \approx 115.9$$

The height of the tower is 115.9 metres.

10.5.4 The area of a triangle



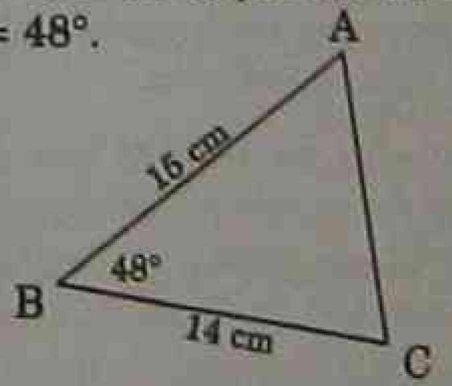
$$\text{Area} = \tfrac{1}{2}ab\sin C$$

Two sides and included angle

A similar formula can be written for any combination of two sides and the angle between them.

Example

(a) Find the area of the triangle ABC where a = 14 cm, c = 15 cm and ∠B = 48°.



$$\text{Area} = \tfrac{1}{2}ac\sin B = \tfrac{1}{2} \times 14 \times 15 \times \sin 48° = 78.030\,207 \approx 78 \text{ (nearest whole number)}$$

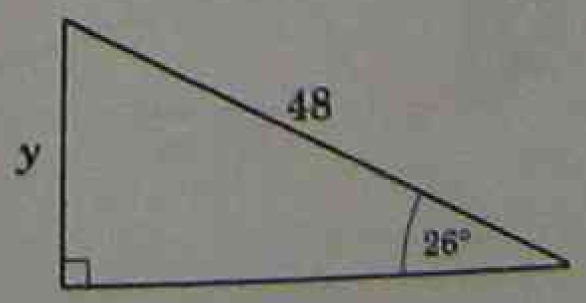
Equivalent formula using angle B

The area is 78 cm².

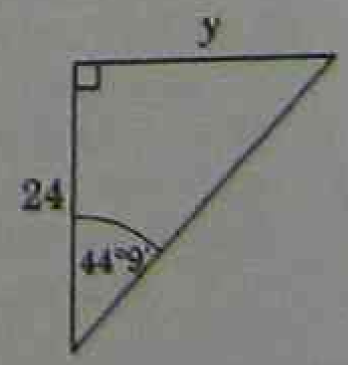
10.6 Exercises

1. Find y from the following figures (correct to one decimal place).

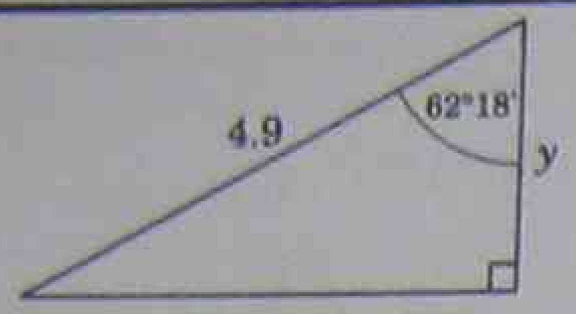
(a)



(b)

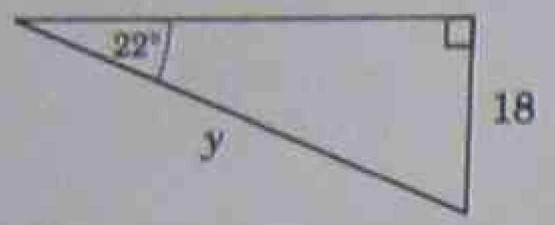


(c)

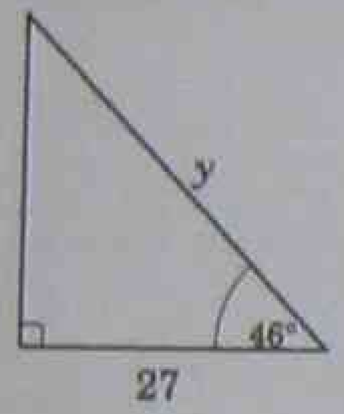


(d)

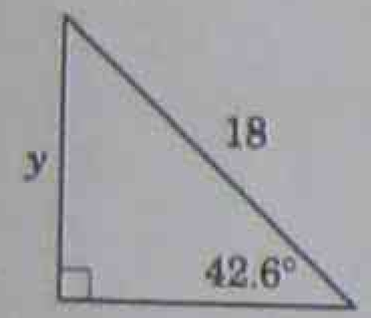
(e)



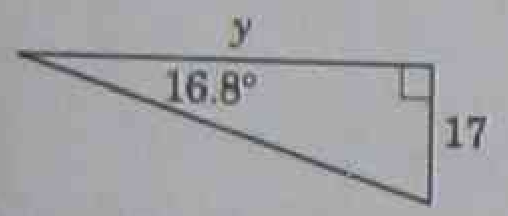
(f)



(g)

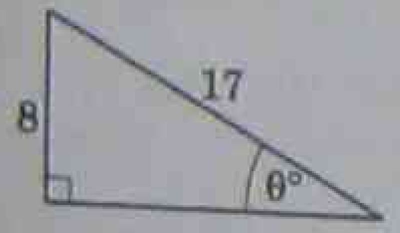


(h)

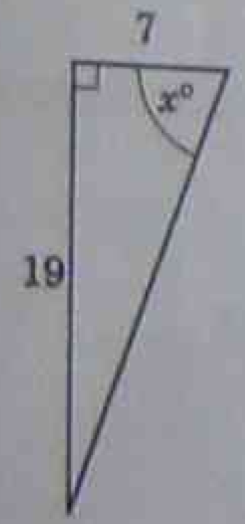


2. Find the size of the marked angle (correct to the nearest minute).

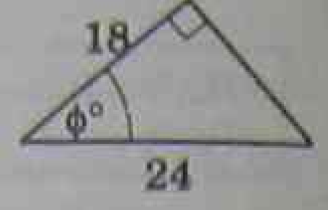
(a)



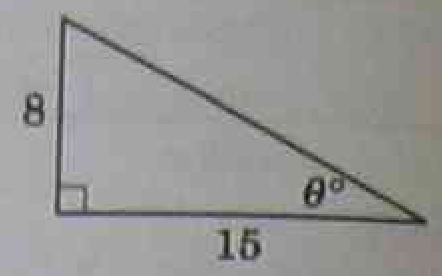
(b)



(c)



(d)



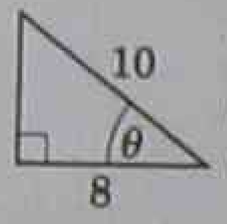
(Answer correct to 1 decimal place.)

3. (a) If $\tan\theta = \frac{4}{7}$, find θ to the nearest degree.

(b) If $\cos\theta = \frac{4}{7}$, find θ to the nearest minute.

(c) If $\sin \frac{4}{7}$, find f correct to the nearest tenth of a degree.

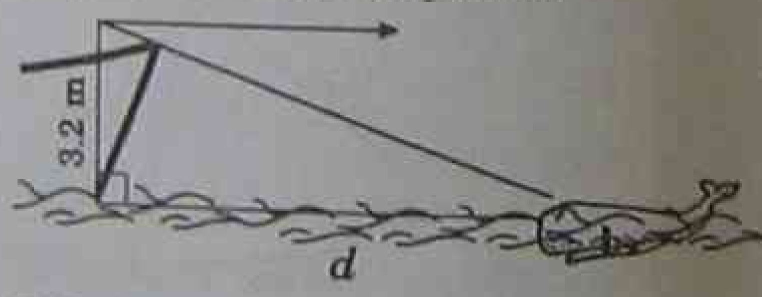
4.



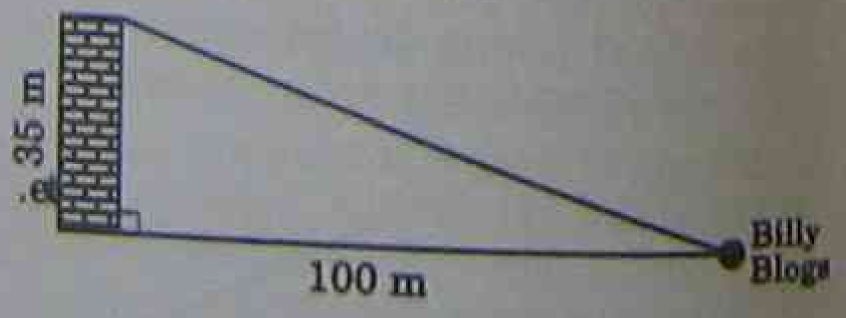
Given $\cos\theta = \frac{8}{10}$, derive expressions for $\sin\theta$ and $\tan\theta$.

5. Given that $\tan\theta = \frac{3}{4}$, derive expressions for $\sin\theta$ and $\cos\theta$.

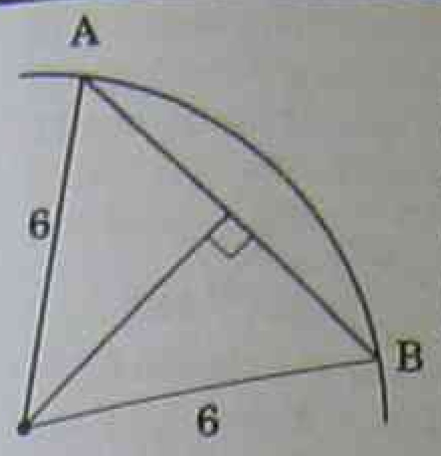
6. Captain Hook notes from the bow of his ship that the angle of depression of a whale is 18°. If the bow of the ship is 3.2 m above the water level, calculate the distance of the whale from the ship (to one decimal place).



7. Billy Blogs measures the angle of elevation of a 35 m building from a position 100 m from the base of the building. Find the angle of elevation, correct to the nearest

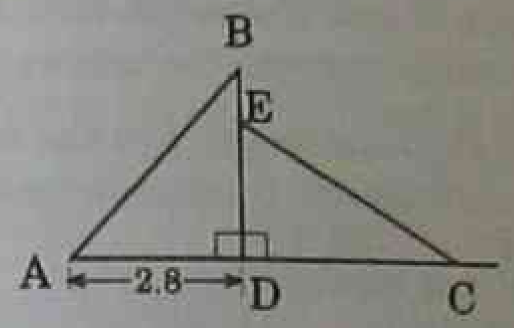


8.

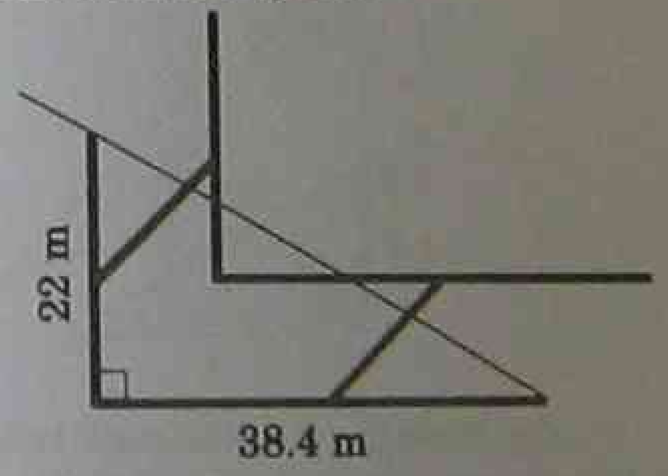


A chord of a circle of radius 6 m, subtends an angle of 84° at the centre. Find the length of the chord correct to one decimal place.

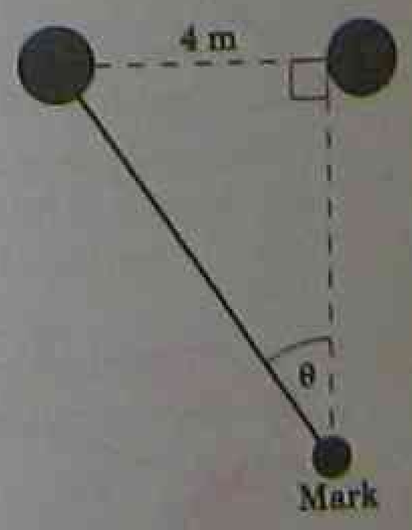
9. Two sails on a yacht are cut so that AD = 2.8 m, ∠DAB = 48°, BE = 0.9 m, and DC = 3.7 m. Calculate the length BD from ΔADB and hence calculate ∠DCE to the nearest minute.



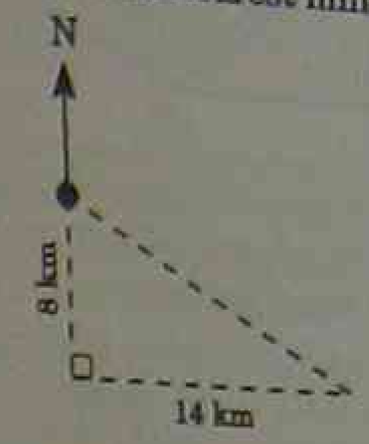
10. The goalposts at Guam stadium are 22 m high. Peter observed that the shadow cast by the posts was 38.4 m long. Calculate the altitude of the sun (to the nearest degree).



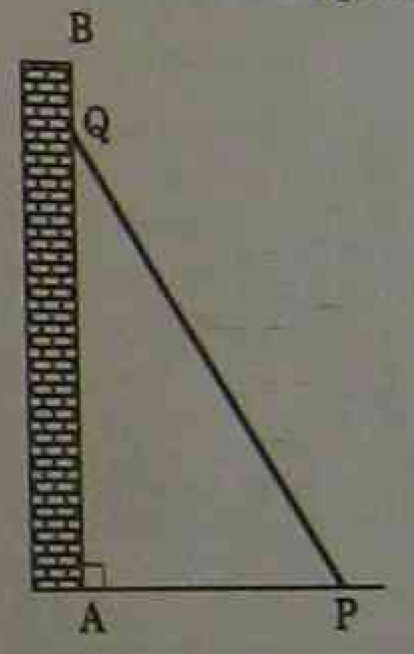
11. Mark and Marika set up garbage cans 4 m apart for soccer posts. If Mark is directly in line with one post, for what range of angles will the soccer ball pass between the posts from a distance of (a) 10 m, (b) 15 m?



12. A large black bear ambles 8 km south and then 14 km east. She then calculates the bearing of her starting point from her new position. Find this bearing to the nearest minute.



13. A metal support PQ holds up a brick wall AB. If the foot of the support is 5.2 m from the wall and reaches 7.8 m up the wall, calculate (a) the length of the support, and (b) the angle the support makes with the ground.

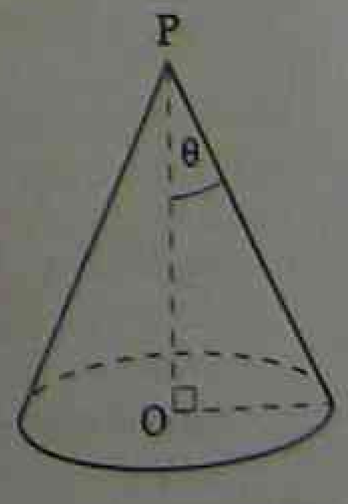


14. Given that $\tan\theta = \frac{5}{12}$, find the value of $\sin\theta$ without calculating θ.

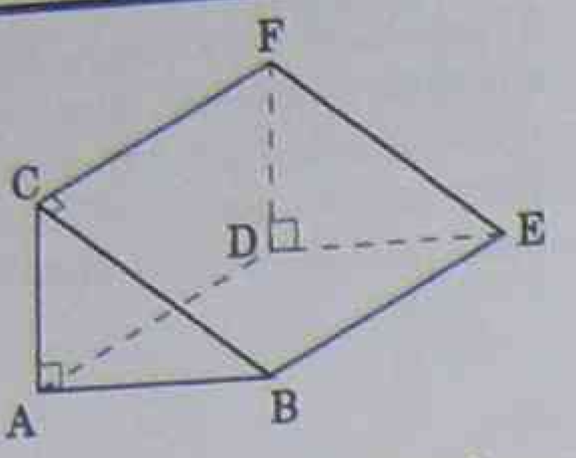
15. If $\cos\phi = \frac{1}{2}$, find the *exact* values of $\sin\phi$ and $\tan\phi$. From any of these ratios, find ϕ.

16. If $\tan\alpha = 1$, find *exact* values for $\sin\alpha$ and $\cos\alpha$. From one of these ratios, calculate the value of α.

17. A right cone of base radius 2 cm has a slant height of 2.8 cm. Find the perpendicular height of the cone and the size of the semivertical angle (marked θ).



18.

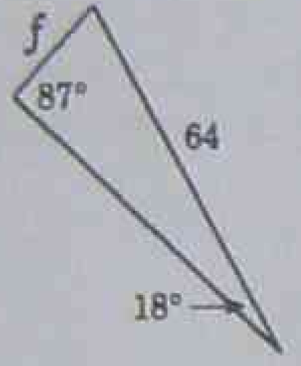


A right-triangular prism has dimensions: AB = 8 cm, AC = 6 cm, and BE = 12 cm. Calculate:

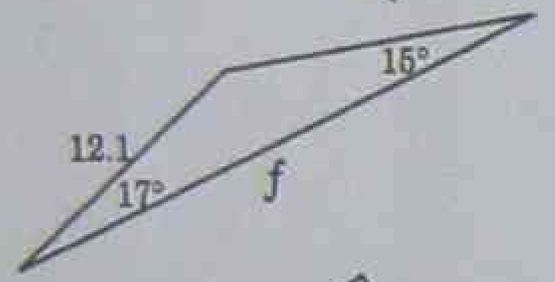
(a) Length BC (b) Length BF
(c) ∠ABC (d) ∠CBF
(e) ∠ACD

19. Find the length f in the following diagrams (to one decimal place).

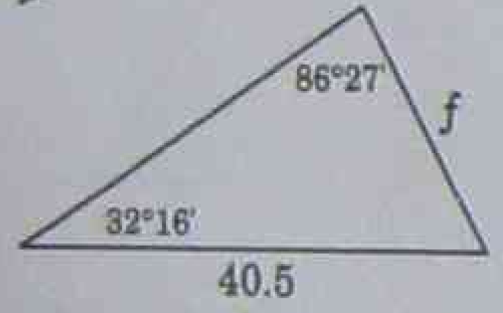
(a)



(b)

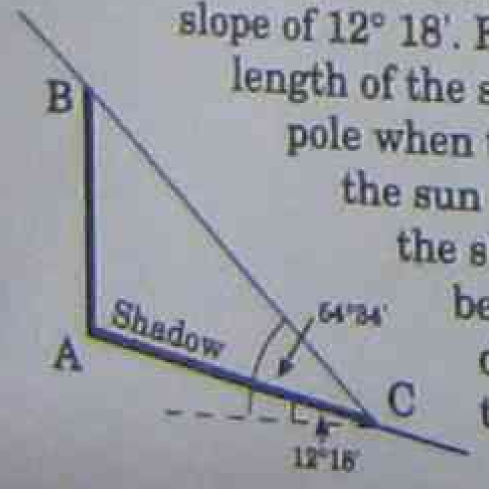


(c)



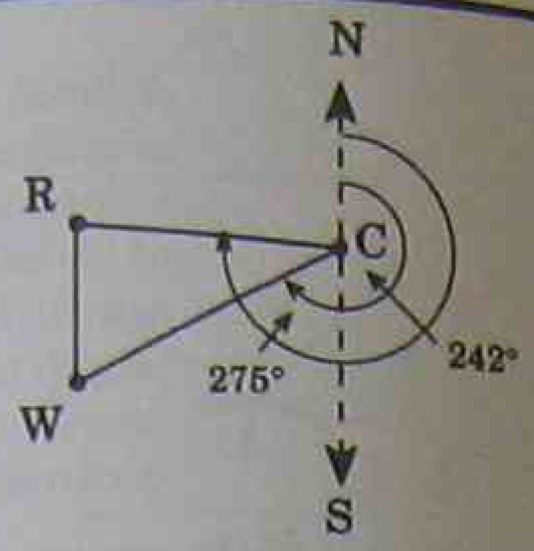
20. A parallelogram has a long diagonal of 12 cm and a side of 7 cm. The obtuse angle between the sides is 114°. Find the size of the angle between the 7 cm side and the diagonal, using the Sine Rule, and then find the area of the parallelogram.

21. A post 12 m high stands on a uniform slope of 12° 18'. Find the length of the shadow of the pole when the altitude of the sun is 54° 34' and the shadow is being cast directly down the hill.



(*Hint*: Use the slope of the rise to find ∠BAC)

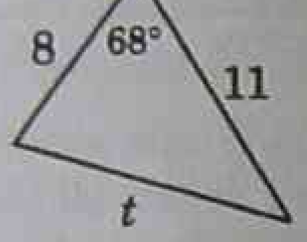
22.



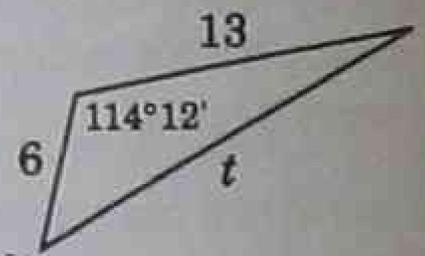
A cabin cruiser in difficulties observes that two ports Redhead (R) and Wallsend (W) have bearings 275° and 242° respectively. Redhead is 25 km due north of Wallsend. How far must the ship travel to the nearest port?

23. (a) Find the length t in the following diagrams (to one decimal place).

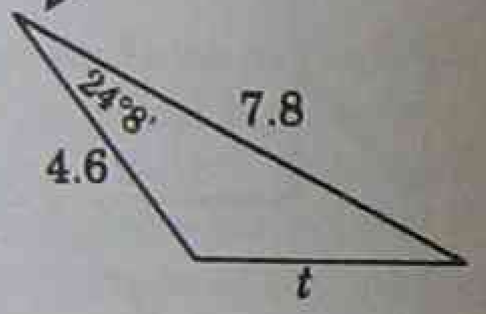
(i)



(ii)



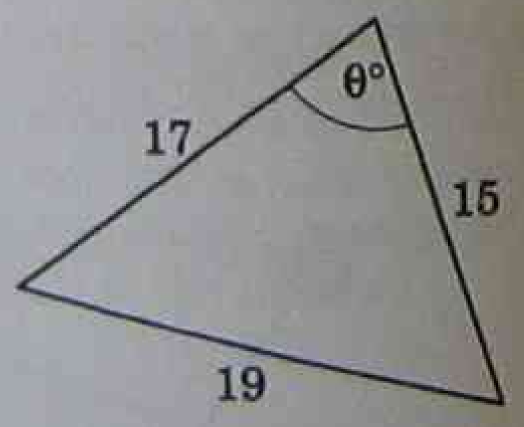
(iii)



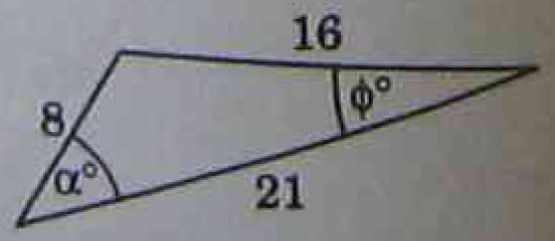
(b) Find the area of each triangle, correct to one decimal place.

24. Calculate the size of the following marked angles to the nearest minute.

(a)



(b)

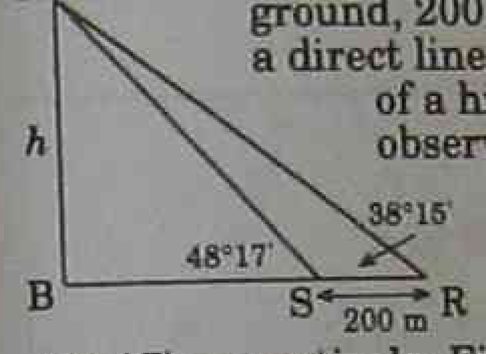


25. Maria cycles 20 km due south from O and then 13 km on a bearing of 035°. How far is she from her starting point?

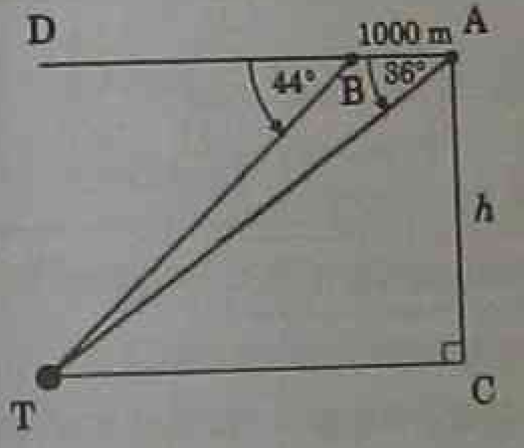
26. Find the size of the largest and smallest angles in a triangle with sides 18 cm, 23 cm and 8 cm. Hence find the area of this triangle (to the nearest 0.1 cm²).

27. Find the angles of a parallelogram and the length of the other diagonal given that it has adjacent sides 30 cm and 40 cm with one diagonal 35 cm.

28. From two points R and S on level ground, 200 m apart and in a direct line with the base of a hill, Pierre observes the angles of elevation of the top of the hill to be 38° 15' and 48° 17' respectively. Find the height h of the hill.

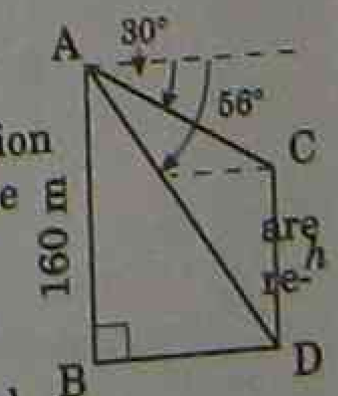


29.

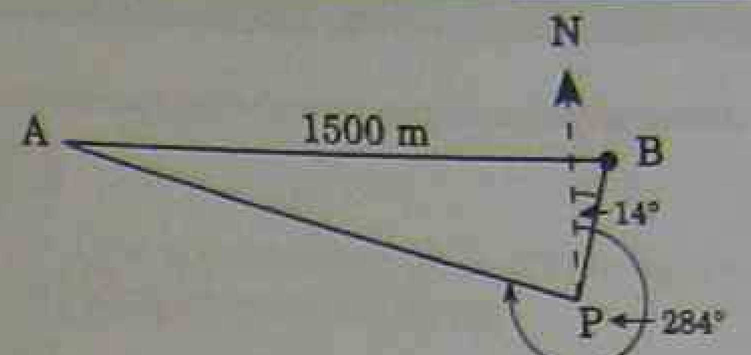


Biggles flying at a constant height on a bombing run observes the angle of depression of his target, T, to be 36°. After flying a further 1 km directly toward the target and at the same altitude, the angle of depression is noted as 44°. Find the altitude h of the plane in km, correct to two significant figures.

30. From the top of a tall building 160 m high, the angles of depression of the top and the base of another building 30° and 56° are respectively. Find the height h of the second building.



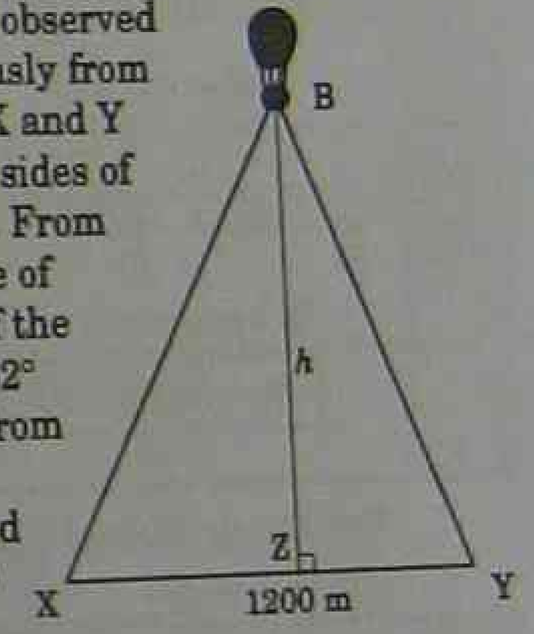
31.



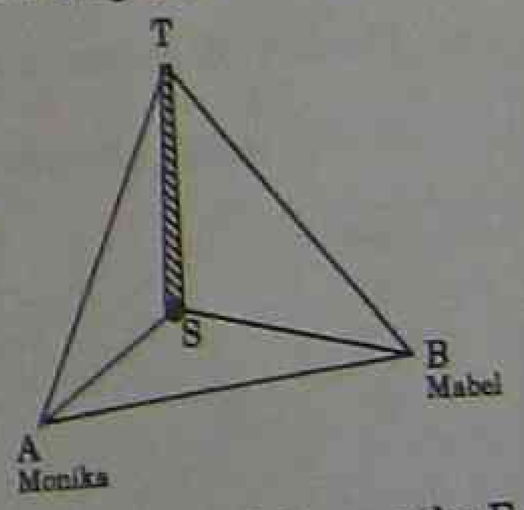
Minmi Breakwater runs east–west for 1500 m. Observers from a prawn trawler note that the bearing of one end of the breakwater is 014°, while that of the other is 284°. Calculate the distance from the prawn trawler to both ends of the breakwater.

32. Two straight roads PQ and PR are inclined to each other at 58° 30'. Two bike riders start simultaneously from P and travel along the roads at 18 and 24 km/h respectively. How long is it before they are 80 km apart? (Answer to the nearest minute.)

33. A balloon is observed simultaneously from two points X and Y on opposite sides of the balloon. From X, the angle of elevation of the balloon is 62° 18', while from Y, it is 68° 12'. If X and Y are 1200 m apart, find the height of the balloon.



34.



Monika and Mabel observe the Eyefull Tower. Monica is due south and Mabel due east of the tower. They note that the angles of elevation of the tower are 48° and 36° respectively. If the tower is 120 m high, calculate the distance between Monika and Mabel (to the nearest metre).

Chapter 11

BASIC GEOMETRY

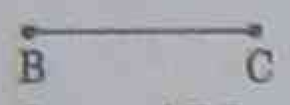
11.1 Drawing geometric sketches

11.1.1 Terminology

Produced: When a line is produced it is extended. The order of the end points indicates the direction that the line is to be extended.

Example

BC is an interval. Produce BC to D.



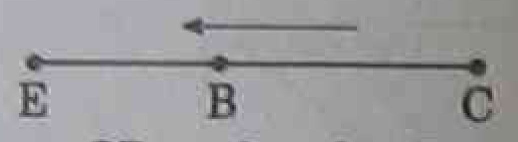
Interval BC

BC produced to D

Note order BC

Produce CB to E.

Interval BC



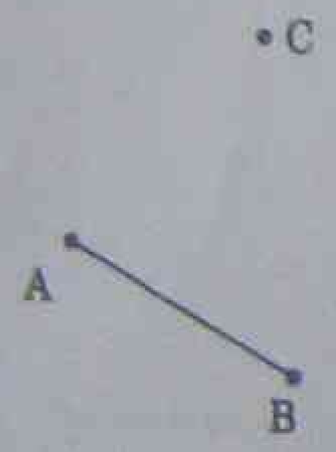
CB produced to E

Note order CB

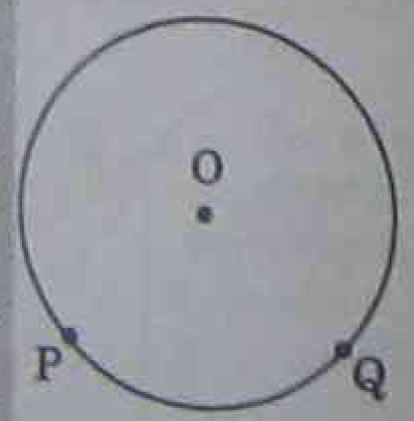
Subtend: when a line or arc subtends an angle then the angle stands on the end points of the line or arc.

Examples

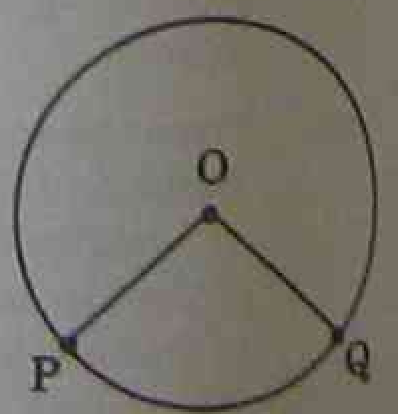
(i) The line AB subtends an angle at C.



(ii) The arc PQ subtends an angle at the centre, O, of the circle



Arc PQ

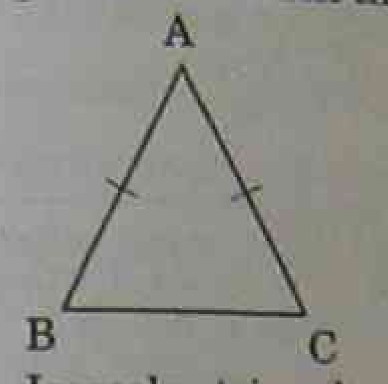


∠ POQ is the angle subtended by PQ at centre O.

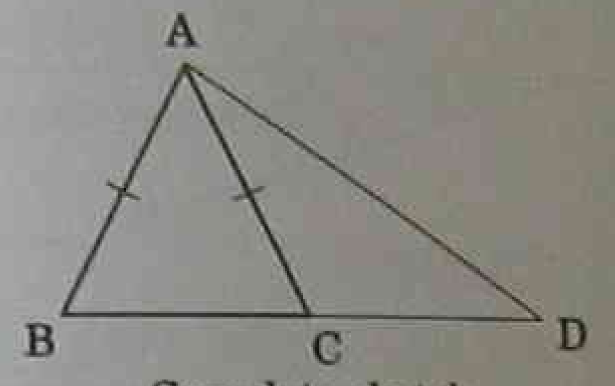
11.1.2 Geometric sketches

Examples

(a) An isosceles triangle ABC has side AB equal to AC. The side BC is produced toD. D is joined A. Sketch this figure.

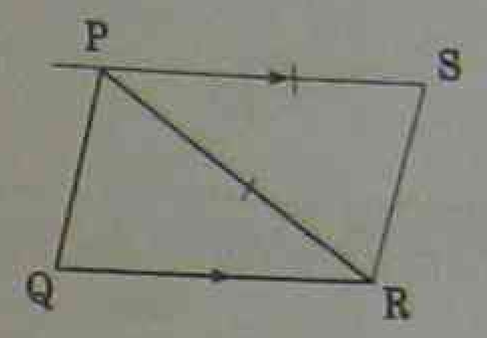


Isosceles triangle with AB = AC



Complete sketch

(b) Give concise description of this figure using geometrical terms.



Description: Through the vertex P of the triangle PQR a line PS is drawn such that PS is parallel to base QR and PS is equal in length to PR. S is then joined to R.

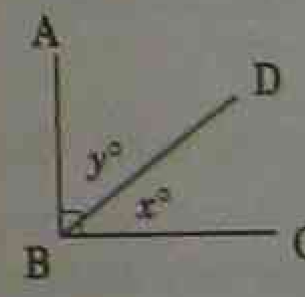
Note: In the construction of triangles the lengths of two sides of any triangle must together be longer than the third.

11.2 Basic angle properties (review of Year 7 and 8 Geometry

The basic angle properties are as follows:

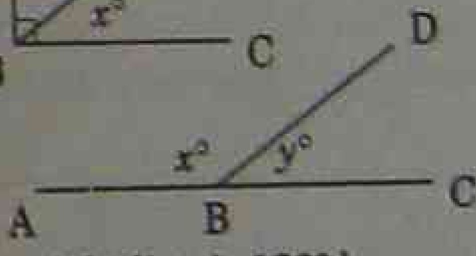
- **Complementary angles have a sum of 90°.**

  $\angle ABC = 90°$ $x + y = 90$



- **Supplementary angles have a sum of 180°.**

  AC is a straight line. $x + y = 180$

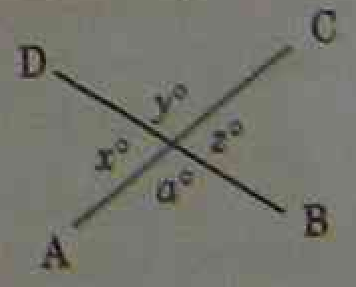


Note: This could also be phrased: 'The angle sum of a straight line is 180°.'

- **Vertically opposite angles are equal.**

  AC and DB are straight lines. $x = z$

  $y = a$

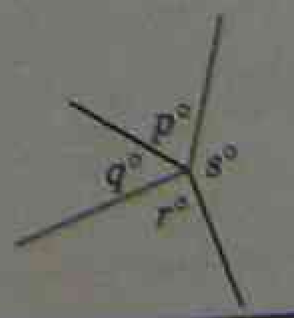


Note also:
$y + z = 180$
$x + a = 180$
$x + y = 180$
$a + z = 180$
(angle sum of a straight line.)

- **The angle sum of a revolution is 360°.**

  $p + q + r + s = 360$

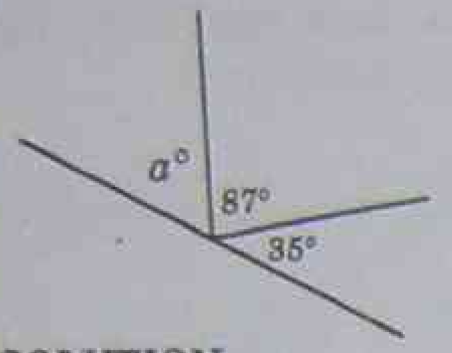
  This property is often referred to as 'angles at a point'.



Examples

Find the value of the pronumerals in the following figures, giving adequate reasons:

(a)



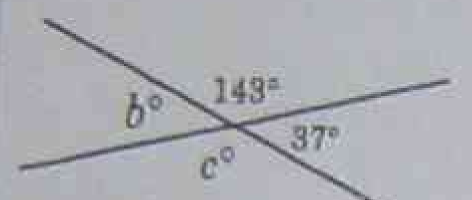
SOLUTION

$a + 87 + 35 = 180$ (∠ sum of straight line)

∴ $a = 180 - 122$

$= 58$

(b)

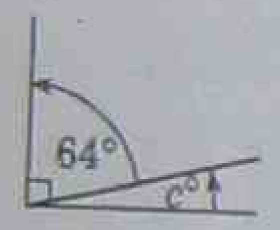


SOLUTION

$b = 37$ (vert. opp. angles)

$c = 143$ (vert. opp. angles)

(c)



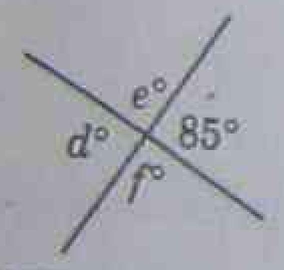
SOLUTION

$c + 64 = 90$ (complementary angles)

∴ $c = 90 - 64$

$= 26$

(d)



SOLUTION

$d = 85$ (vert. opp. angles)

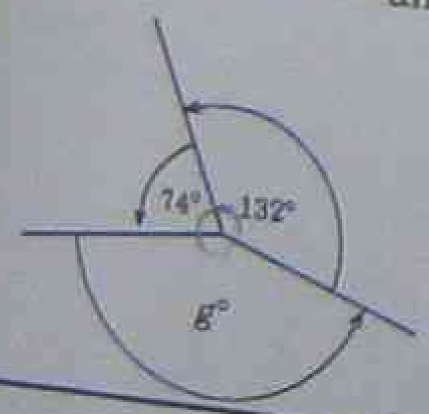
$e + 85 = 180$ (∠ sum of straight line)

∴ $e = 180 - 85$

$= 95$,

also $f = 95$ (vert. opposite angles)

(e)



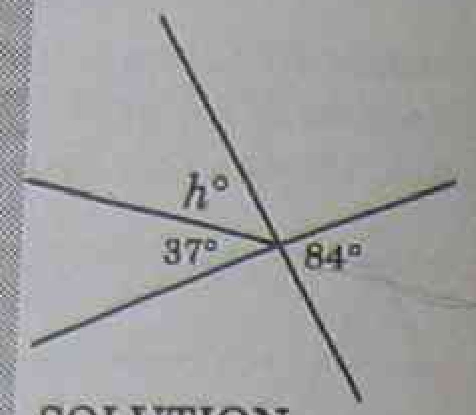
SOLUTION

$g + 74 + 132 = 360$ (angles at a point)

∴ $g = 360 - 206$

$= 154$

(f)



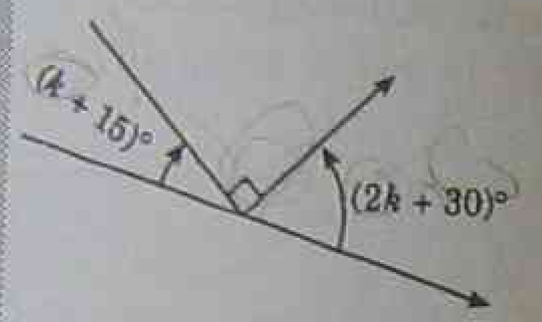
SOLUTION

$h + 37 = 84$ (vert. opp. angles)

∴ $h = 84 - 37$

$= 47$

(g)



SOLUTION

$(k + 15) + 90 + (2k + 30) = 180$ (angle sum of a straight line)

∴ $3k + 135 = 180$

$3k = 45$

∴ $k = 15$

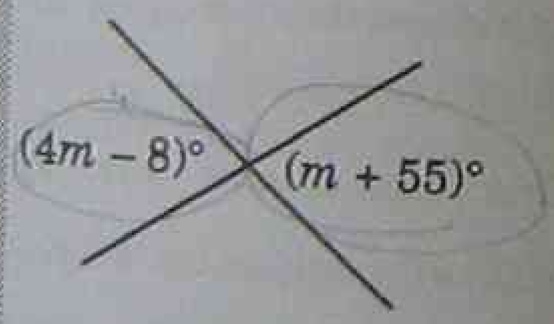
If the question had asked for the size of each unknown angle, then:

$k + 15 = 15 + 15 = 30$

$2k + 30 = 2 \times 15 + 30 = 60$,

∴ the other angles are 30° and 60°. (Check that they sum to 180°.)

(h)



SOLUTION

$4m - 8 = m + 55$ (vert. opp. angles)

∴ $3m = 63$

$m = 21$

If you need to find the size of each angle:

$4m - 8 = 4 \times 21 - 8 = 76$

∴ each angle is 76°.

11.3 Parallel lines

11.3.1 Parallel lines cut by a transversal

When a system of parallel lines is cut by a transversal:

(a) alternate angles so formed are equal;

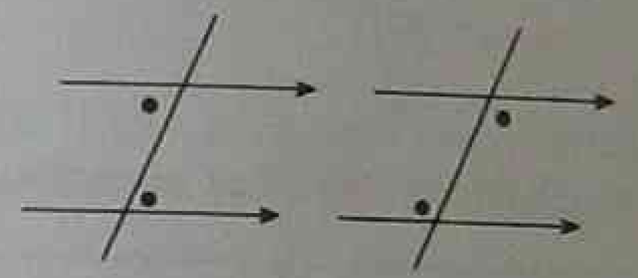
(b) corresponding angles so formed are equal;

(c) co-interior angles so formed are supplementary.

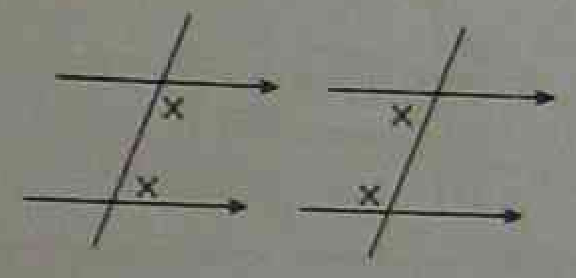
These angles types appear in pairs and can be remembered by associating them with the relevant letter:

- Alternate Z
- Corresponding F
- Co-interior C (Actually [ )

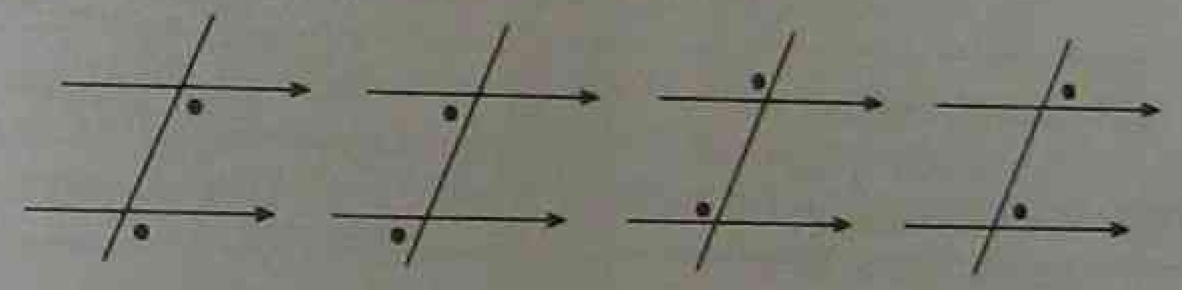
Alternate angles (Z)



Co-interior angles ([ )



Corresponding angles (F)



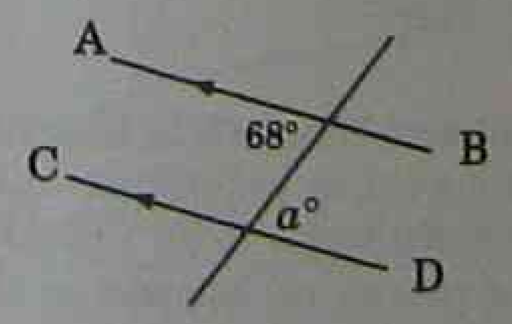
Note: **These angle properties are true only when the lines are parallel.**

This means that alternate angles are only equal when they are between parallel lines. Similarly, corresponding angles are only equal and co-interior angles are only supplementary when the lines are parallel.

Examples

Find the value of the pronumeral in the following figures, giving adequate geometric reasons:

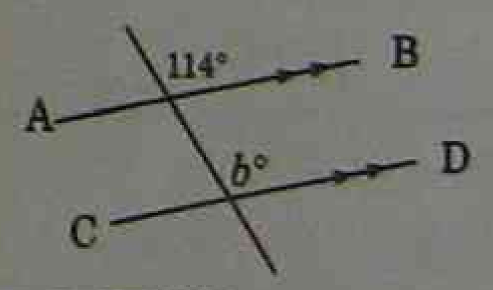
(a)



SOLUTION

$a = 68$ (alternate angles, AB || CD)

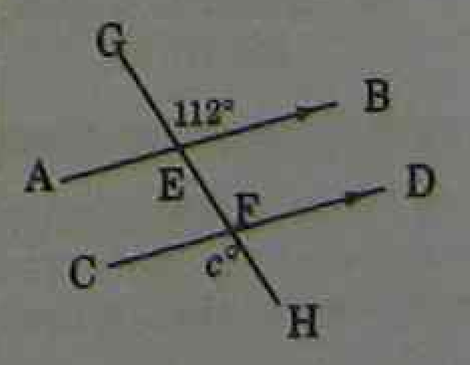
(b)



SOLUTION

$b = 114$ (corresponding angles, AB || CD)

(c)



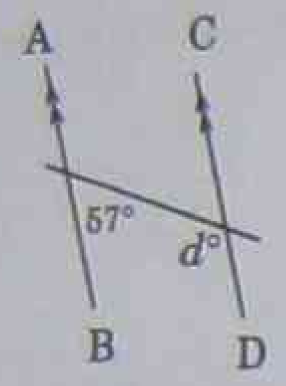
Continued

SOLUTION

$\angle AEF = 112°$ (vert. opp. angles)

$\therefore \quad c = 112$ (corresponding angles, AB || CD)

(d)

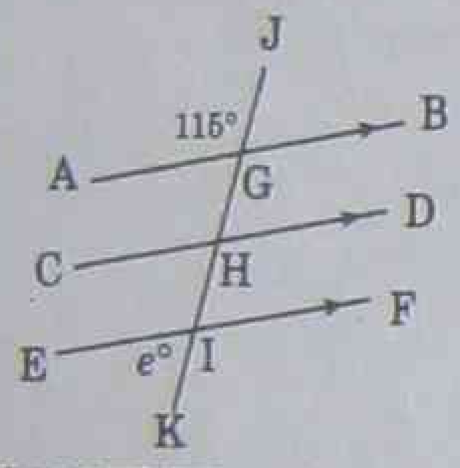


SOLUTION

$d = 180 - 57$ (co-interior angles, AB || CD)

$= 123$

(e)



SOLUTION

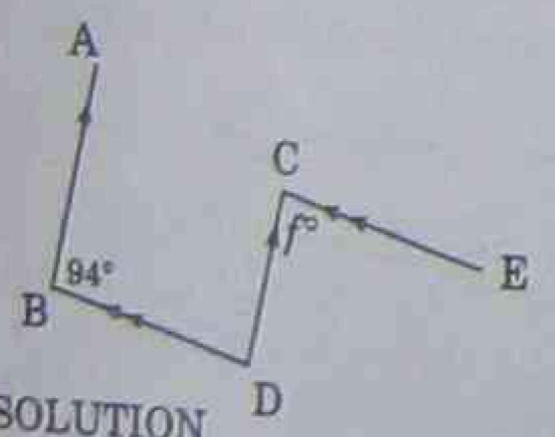
$\angle CHG = 115°$ (corresponding angles, AB || CD)

$\angle EIH = 115°$ (corresponding angles, CD || EF)

$\therefore \quad e = 180 - 115$ (angle sum of straight line)

$e = 65$

(f)



SOLUTION

$\angle BDC = 180° - 94°$ (co-interior angles, AB || CD)

$= 86°$

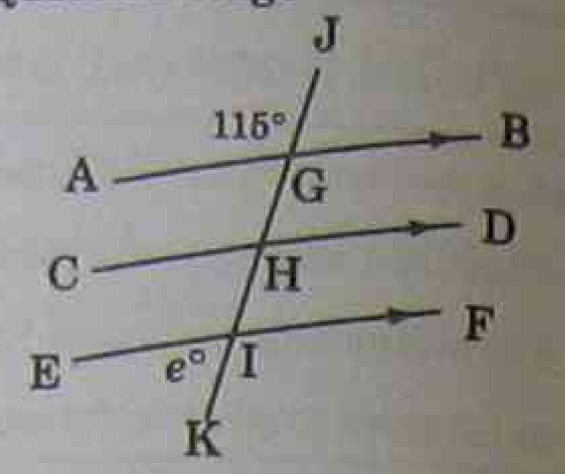
$\therefore \quad f = 86$ (alternate angles, BD || CE)

Note: **Always draw a clear diagram.**

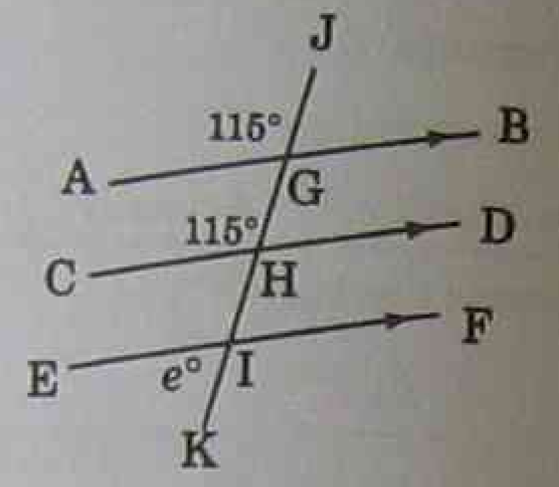
When more than one step is required, fill in the angles as you move around the diagram towards the required angle.

Consider Example (e) again. The stages of your solution would be:

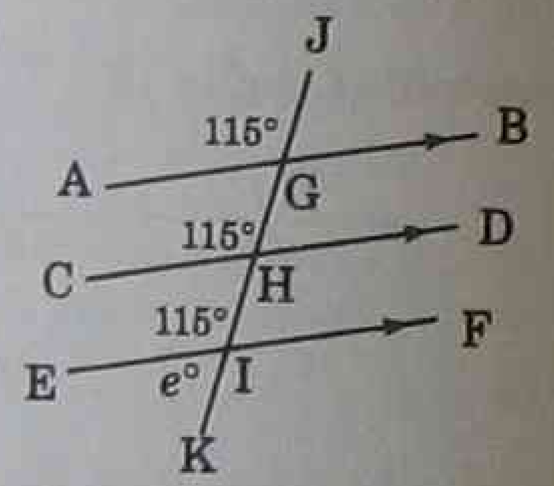
1. Question stage



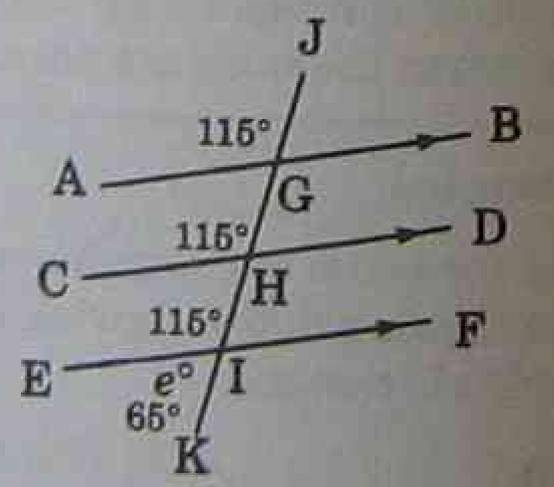
2. $\angle CHG = 115°$, filled in on diagram.



3. $\angle EIH = 115°$, filled in on diagram.



4. $e = 65$, filled in on diagram.



This sequence is drawn to illustrate the procedure used on a diagram in moving from a known angle to an unknown. [It would, of course, all happen on the one diagram.]

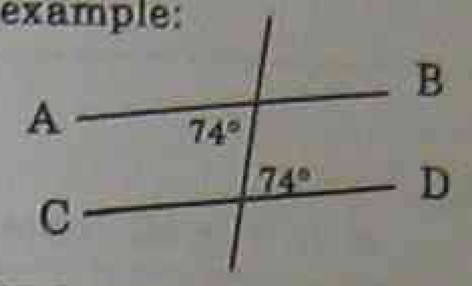
Also remember that in many figures there is more than one way of proceeding around a diagram.

11.3.2 Tests for parallel lines

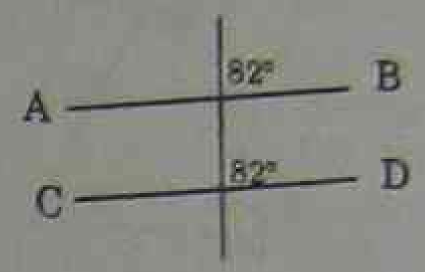
Two lines are parallel if:

- a pair of alternate angles are equal; *or*
- a pair of corresponding angles are equal; *or*
- a pair of co-interior angles are supplementary; *or*
- they are both parallel to a third line.

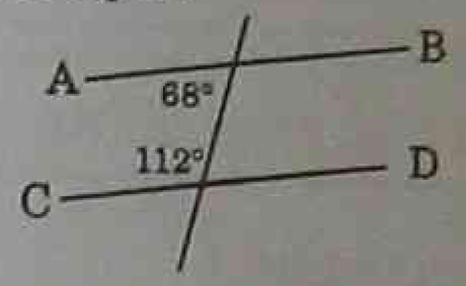
For example:



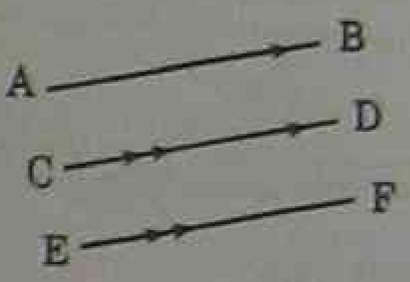
AB || CD because a pair of alternate angles are equal.



AB || CD because a pair of corresponding angles are equal.



AB || CD because a pair of co-interior angles are supplementary.

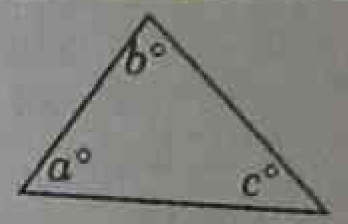


Given AB || CD, and CD || EF, then AB || EF, because both AB and EF are parallel to CD.

11.4 Angle properties of triangles

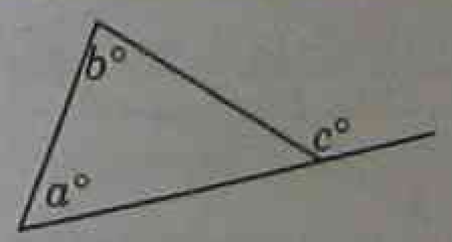
Some definitions: A scalene triangle is a triangle in which no two sides are equal in length. An isosceles triangle is a triangle in which there are two sides equal in length. An equilateral triangle is a triangle in which all side are equal in length.

11.4.1 The angle sum of a triangle



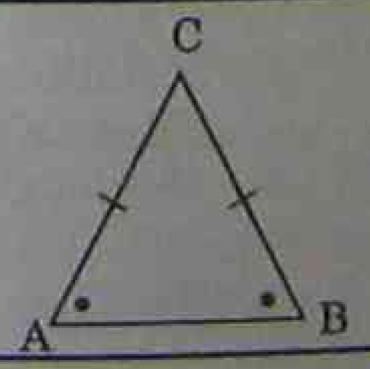
The angle sum of a triangle is 180°, that is, $a + b + c = 180$.

11.4.2 The exterior angle of a triangle



The exterior angle of a triangle equals the sum of the two interior opposite angles, that is, $a + b = c$.

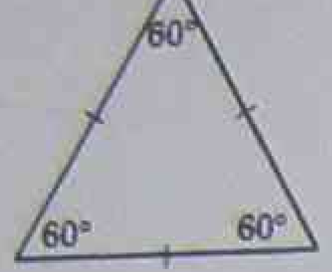
11.4.3 Isosceles triangles



The base angles of an isosceles triangle are equal.

Given that AC = BC, then $\angle A = \angle B$.

11.3.4 Equilateral triangles

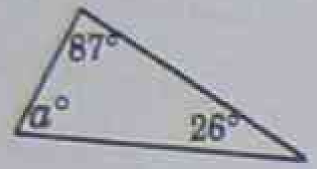


All angles of an equilateral triangle are equal to 60°.

Examples

Find the values of the pronumerals in the following figures:

(a)



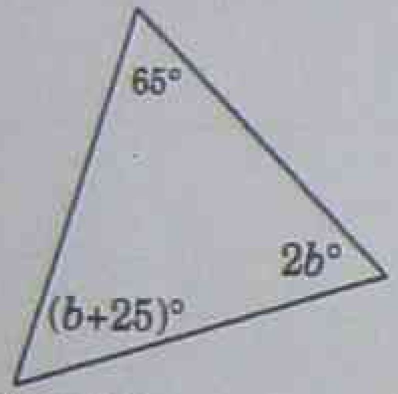
SOLUTION

$a + 26 + 87 = 180$ (angle sum of a Δ)

$\therefore\ a = 180 - 113$

$= 67$

(b)



SOLUTION

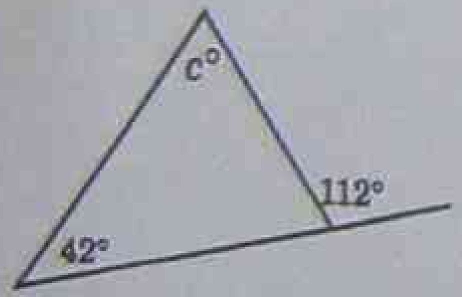
$(b + 25) + 2b + 65 = 180$ (angle sum of a Δ)

$\therefore\ 3b + 90 = 180$

$3b = 90$

$b = 30$

(c)



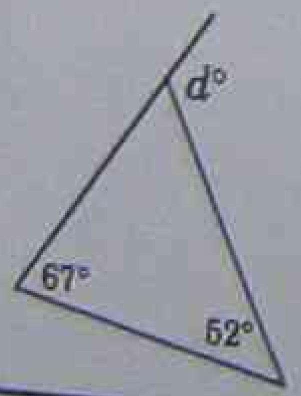
SOLUTION

$c + 42 = 112$ (exterior angle of a triangle)

$\therefore\ c = 112 - 42$

$= 70$

(d)

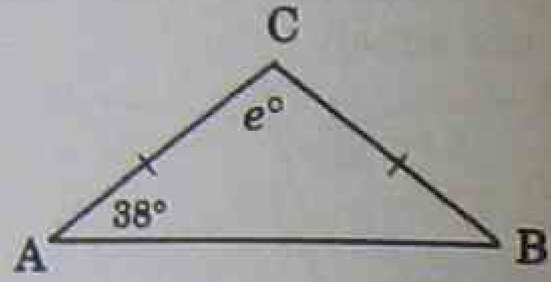


SOLUTION

$d = 67 + 52$ (exterior angle of a Δ)

$\therefore\ d = 119$

(e)



SOLUTION

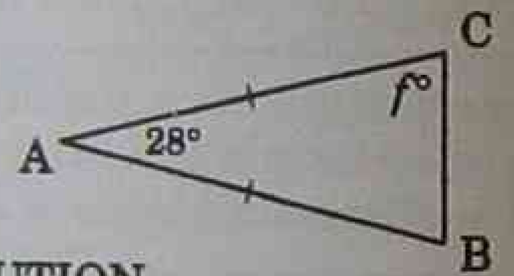
$\angle ABC = 38°$ (base angles of isosc. Δ)

$e + 38 + 38 = 180$ (angle sum of a Δ)

$\therefore\ e = 180 - 76$

$= 104$

(f)



SOLUTION

$\angle ABC = f°$ (base angles of isosc. Δ)

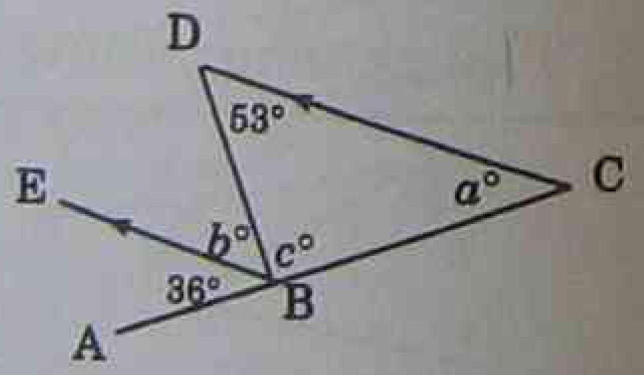
$\therefore\ f + f + 28 = 180$ (angle sum of a Δ)

$2f = 180 - 28$

$2f = 152$

$f = 76$

(g)



SOLUTION

$a = 36$ (corresponding angles, EB ∥ DC)

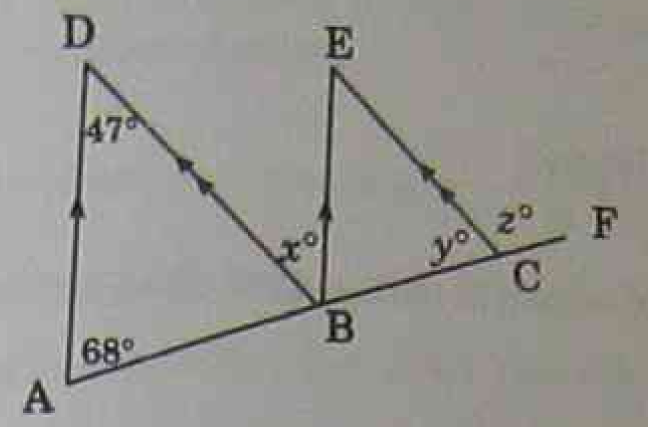
$b = 53$ (alternate angles, EB ∥ DC)

$\therefore\ 36 + 53 + c = 180$ (angle sum of a Δ)

$c + 89 = 180$

$c = 91$

(h)



SOLUTION

$x = 47$ (alternate angles, AD ∥ BE)

$\angle EBC = 68°$ (corresponding angles, AD ∥ BE)

$\angle BEC = 47°$ (alternate angles, DB ∥ EC)

$\therefore\ y + 68 + 47 = 180$ (angle sum of a Δ)

$y = 180 - 115$

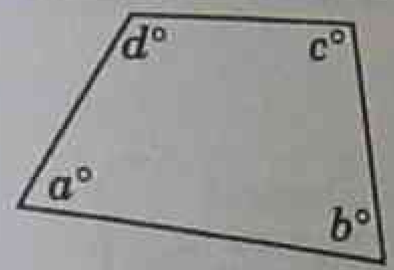
$= 65$

Also, $z = 68 + 47$ (exterior angle of a Δ)

$= 115$

11.5 Quadrilaterals

11.5.1 The angle sum of a quadrilateral

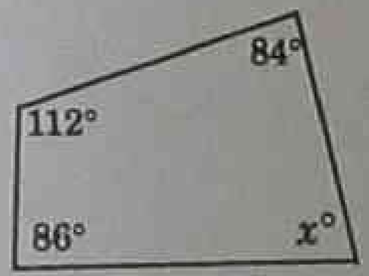


The angle sum of a quadrilateral is 360°, that is,

$a + b + c + d = 360$

Examples

(a)



SOLUTION

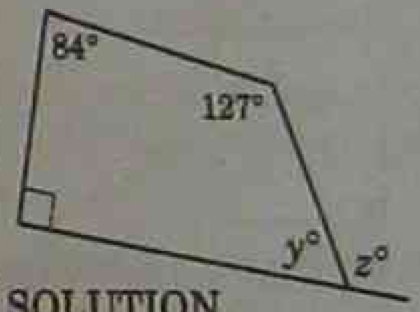
$x + 86 + 112 + 84 = 360$ (angle sum of a quadrilateral)

$\therefore\ x + 282 = 360$

$x = 360 - 282$

$x = 78$

(b)



SOLUTION

$y + 90 + 84 + 127 = 360$ (angle sum of a quadrilateral)

$\therefore\ y + 301 = 360$

$y = 59$

Also, $z + 59 = 180$ (angle sum of a straight line)

$\therefore\ z = 180 - 59$

$z = 121$

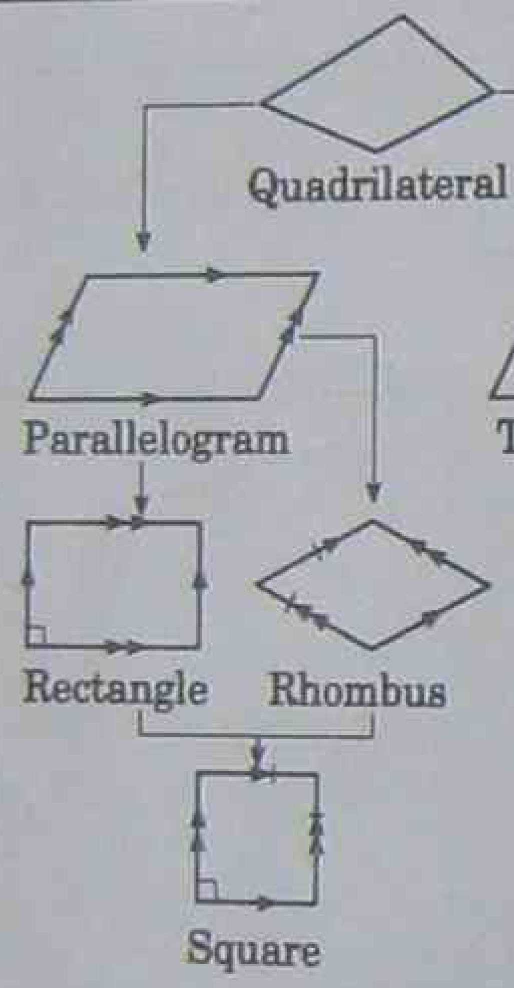
11.5.2 Special quadrilaterals

- **A trapezium** is a quadrilateral in which one pair of opposite sides is parallel.
- **A parallelogram** is a quadrilateral with both pairs of opposite sides parallel. Its special properties are:
  - (a) The opposite sides are equal.
  - (b) The opposite angles are equal.
  - (c) The diagonals bisect each other.
- **A rectangle** is a parallelogram with one angle a right angle. Its special properties are:
  - (a) All the properties of the parallelogram.
  - (b) All angles are right angles.
  - (c) The diagonals are equal.

- **A square** is a rectangle with one pair of adjacent sides equal. Its special properties are:
  - (a) All the properties of the rectangle.
  - (b) All sides are equal.
  - (c) The diagonals bisect each other at right angles.
  - (d) The diagonals bisect the angles of the square.
- **A rhombus** is a parallelogram with one pair of adjacent sides equal. Its special properties are:
  - (a) All the properties of a parallelogram.
  - (b) The diagonals bisect each other at right angles.
  - (c) The diagonals bisect the angles of the rhombus.

The **square** can also be defined in terms of the **rhombus**. A square is a rhombus with one angle a right angle. A square is a quadrilateral which is both a rhombus and a rectangle.

11.5.3 The quadrilateral family tree



Each quadrileteral in the tree has all the properties of those above it, plus its own distinct properties.

11.6 Regular polygons

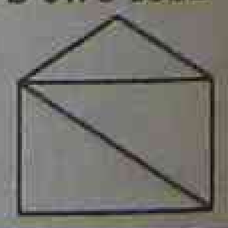
A regular polygon has all angles and sides equal.

11.6.1 Angle sum of an n-sided polygon

Break the polygon into triangles and you will see that there are always two less triangles than sides of the polygon, for example, 5 sides, 3 triangles.

Each triangle has an angle sum of 180°. This leads to the formula:

Angle sum $= (n - 2) \times 180°$, where n = the number of sides.



Think of it this way: (Number of sides less 2) × angle sum of Δ (180°)

In a regular n-sided polygon, each angle can be calculated by dividing the angle sum by the number of sides.

$$\text{Therefore, each angle} = \frac{(n-2)\times 180}{n}.$$

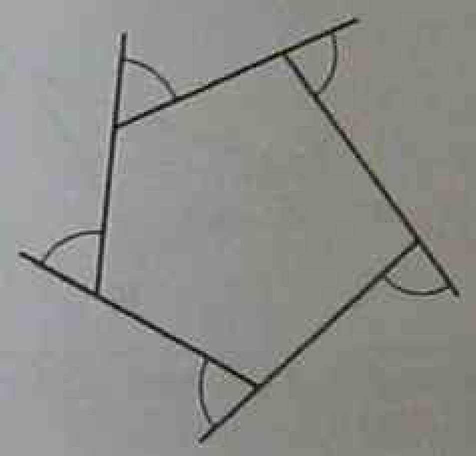
For example: Consider a regular octagon.

$$\text{Angle sum} = (8-2)\times 180° = 6\times 180°$$
$$= 1080°$$

$$\text{Each angle} = \frac{1080°}{8}$$
$$= 135°$$

Number of sides = 8

11.6.2 Exterior angles of polygons



An n-sided polygon has n exterior angles. They are formed by producing (extending) each side in order around the polygon.

In the diagram, there are five sides so there are 5 exterior angles.

The sum of the *exterior* angles of any polygon is 360°. Each exterior angle of a *regular* polygon with n sides is $\frac{360°}{n}$. For example, consider a regular octagon:

$$\text{Each exterior angle} = \frac{360°}{8}$$
$$= 45°$$

8 sides

Note that the exterior angle and the interior angle sum to 180° — they must do this, as they form a straight line.

A consequence of this is that the four *exterior* angles of any *quadrilateral* must sum to 360°.

11.7 Congruent triangles

Congruent triangles are equal in all respects.

11.7.1 Tests for congruent triangles

The tests for congruent triangles are:

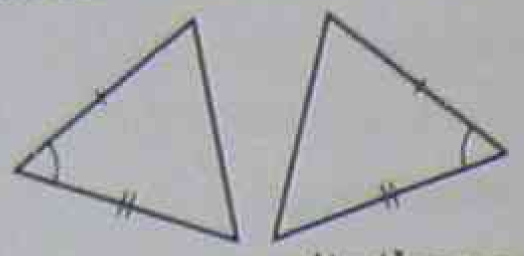
(a) Two triangles are congruent if the three sides of one triangle are respectively equal to the three sides of the other. (SSS)

(b) Two triangles are congruent if two sides of one and the included angle are respectively equal to two sides and the included angle of the other. (SAS)

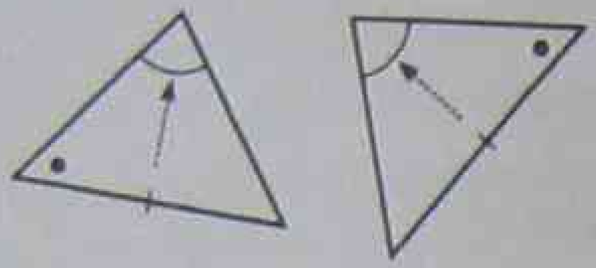
(c) Two triangles are congruent if two angles and one side of one triangle are respectively equal to two angles and the corresponding side of the other. (ASA)

Notes:

1. The included angles are between the equal sides as follows:



2. The corresponding sides are opposite the **same** equal angle in each triangle, as follows:

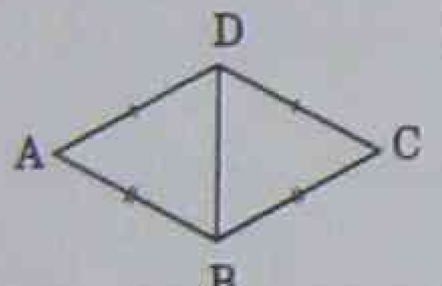


(d) Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other. (RHS)

11.7.2 Using the congruence tests

Examples

(a)

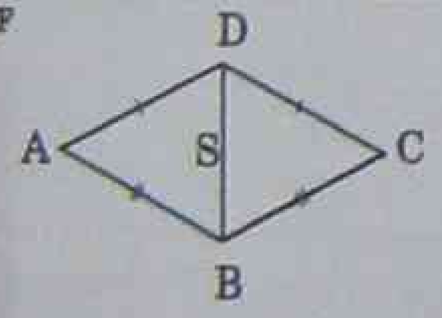


DB is common
DC = AD (given)
AB = BC
(S

Given that AD = DC and AB = BC, prove that ΔABD ≡ ΔDBC.

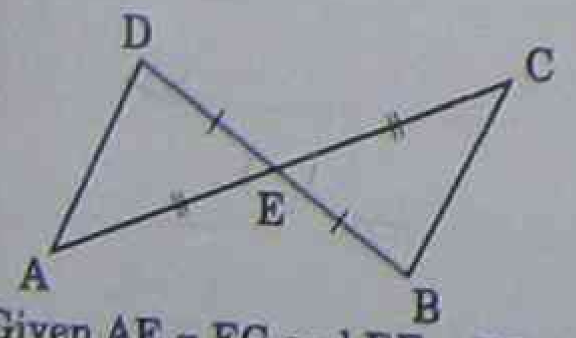
Note: ≡ means 'is congruent to'.

PROOF



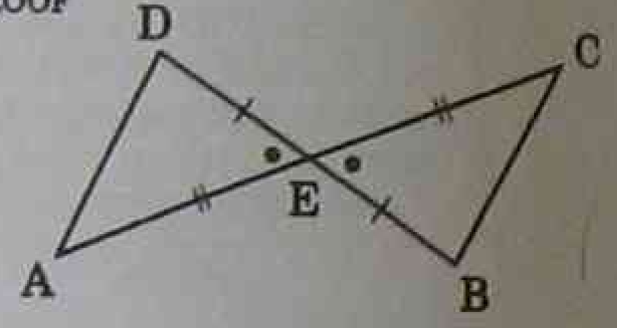
In the triangles ABD and DBC:
AB = BC (data)
AD = DC (data)
BD = BD (common side)
∴ ΔABD ≡ ΔDBC (SSS)

(b)



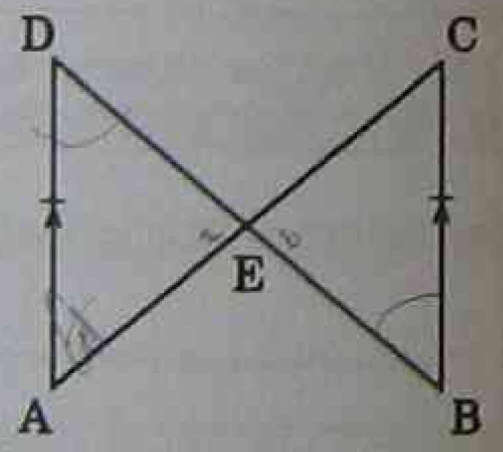
Given AE = EC and DE = EB, prove that ΔAED ≡ ΔEBC.

PROOF



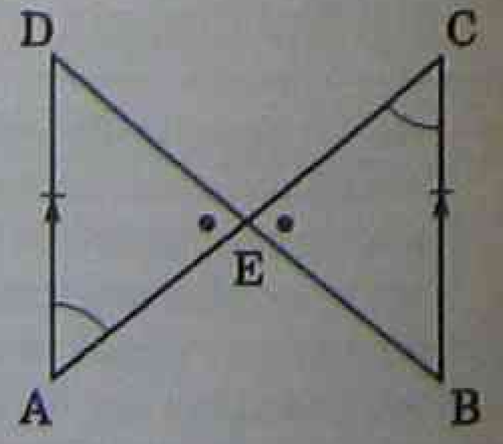
In the triangles ADE and EBC:
AE = EC (data)
DE = EB (data)
∠AED = ∠BEC (vertically opp. angles)
∴ ΔAED ≡ ΔEBC (SAS)

(c)



Given AD = BC and AD || BC, prove that ΔAED ≡ ΔBCE.

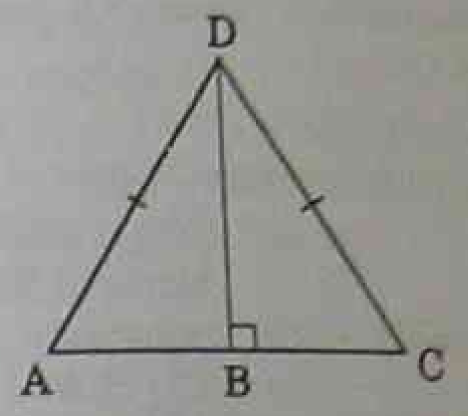
PROOF



Continued

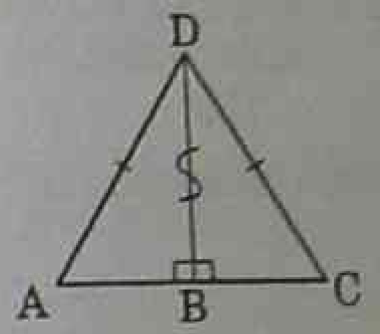
In the triangles AED and BCE:
AD = BC (data)
∠DAE = ∠ECB (alternate angles, AD || BC)
∠AED = ∠CEB (vertically opposite angles)
∴ ΔAED ≡ ΔBCE (ASA)

(d)



Given BD ⊥ AC and AD = DC, prove that ΔABD ≡ ΔBCD.

PROOF



In the triangles ABD and BCD:
AD = DC (data)
BD = BD (common side)
∠ABD = ∠DBC = 90° (BD ⊥ AC)
∴ ΔABD ≡ ΔBCD (RHS)

Important

- Mark the data on the diagram and then mark any other angles and sides that you can show are equal.
- A congruence proof consists of **three** (3) equality statements.
- All congruence assumptions may be applied to right-angled triangles — RHS is just an additional assumption for right-angled triangles.

11.7.3 Beyond the congruence assumptions

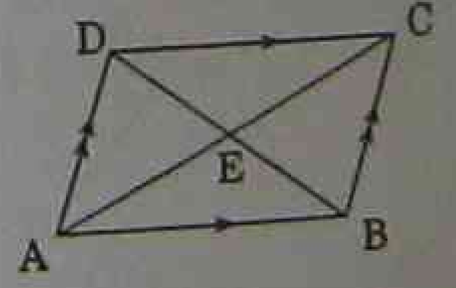
Many important properties can be proved using congruent triangles. Once the triangles have been shown to be congruent, all other corresponding sides and angles of the triangles are then known to be equal.

Example

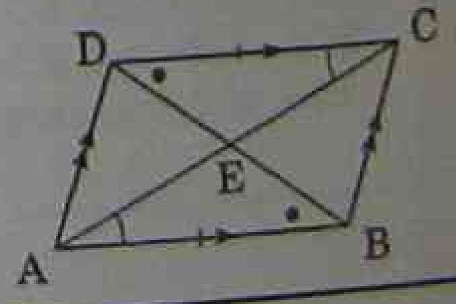
(a) ABCD is a parallelogram with diagonals AC and DB meeting at E. Prove that:

(i) ΔAEB ≡ ΔDEC

(ii) AE = EC and EB = DE, (that is, that the diagonals bisect each other).



PROOF



(i) In triangles AEB and DEC:
AB = DC (opposite sides of a parallelogram)
∠EAB = ∠ECD (alternate angles, AB || CD)
∠ABE = ∠EDC (alternate angles, AB || CD)
∴ ΔAEB ≡ ΔDEC (ASA)

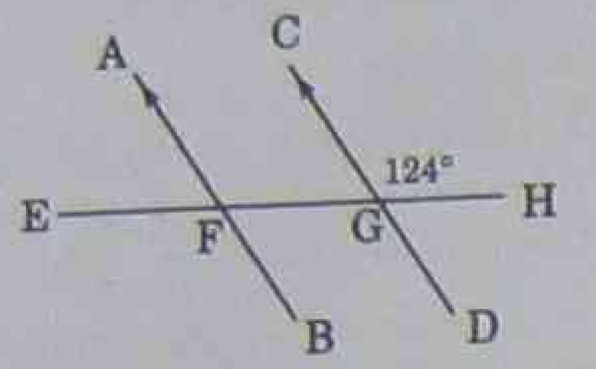
(ii) As triangles AEB and DEC are congruent,
AE = EC (corresp. sides of congruent triangles)
and EB = ED (corresp. sides of congruent triangles)

AE and EC are both opposite equal angles, marked with •.

11.8 Deductive geometry

The following examples use all basic geometry treated in this chapter.

(a)



Given AB || CD and $\angle CGH = 124°$, find $\angle AFE$, giving reasons.

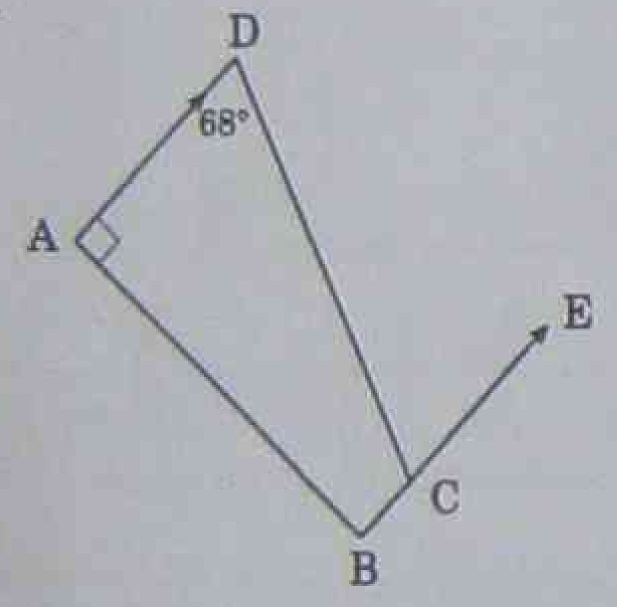
SOLUTION

$\angle AFG = 124°$ (corresponding angles, AB || CD)

$\therefore \angle AFE = 180° - 124°$ (angle sum of a straight line)

$\angle AFE = 56°$

(b)



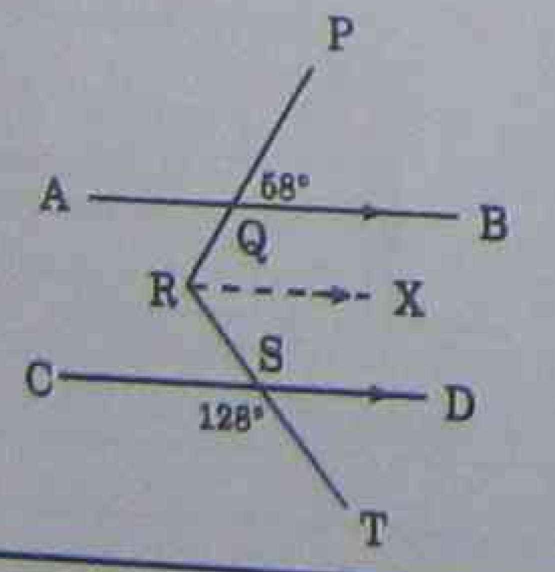
Given $\angle BAD = 90°$, AD || BE and $\angle ADC = 68°$, find $\angle DCE$ giving reasons.

> Remember to solve the problem on your diagram first.

SOLUTION

$\angle DCE = 68°$ (alternate angles AD || BE)

(c)



Given $\angle PQB = 58°$, $\angle CST = 128°$, and AB || CD, find $\angle QRS$.

Construction: Draw RX parallel to AB and CD.

SOLUTION

$\angle CSR = 180° - 128°$
$= 52°$ (angle sum of a straight line)

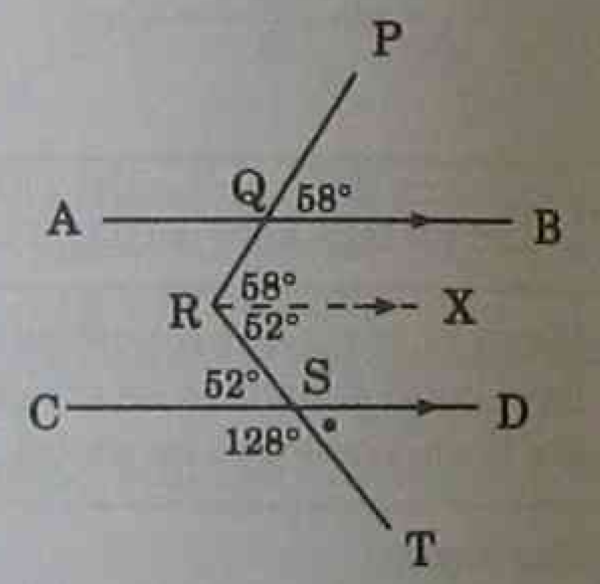
$\angle SRX = 52°$ (alternate angles, CD || RX)

$\angle QRX = 58°$ (corresponding angles, AB || RX)

Now

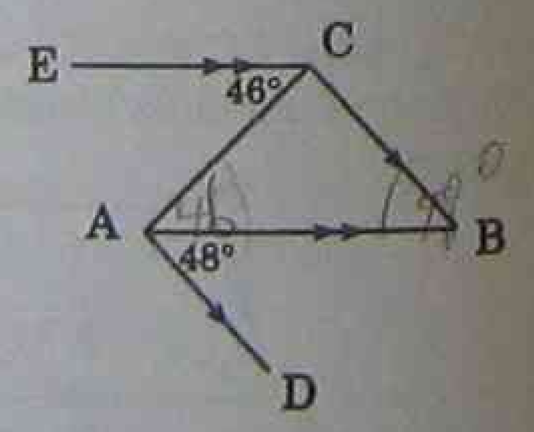
$\angle QRS = \angle QRX + \angle SRX$
$= 58° + 52°$
$= 110°$

Now look at the steps on the diagram:



By working around the diagram, known to unknown, the logical steps for your proof become obvious.

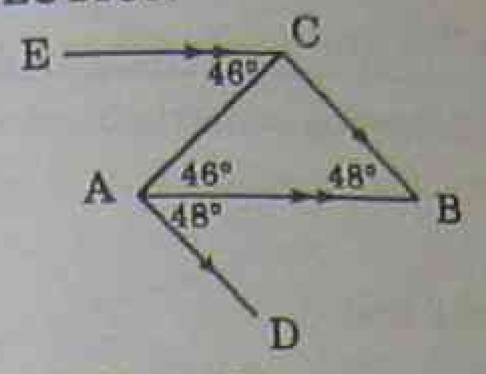
(d)



Given AD || CB, EC || AB, $\angle ECA = 46°$ and $\angle DAB = 48°$, find $\angle ACB$, giving reasons.

Continued

SOLUTION



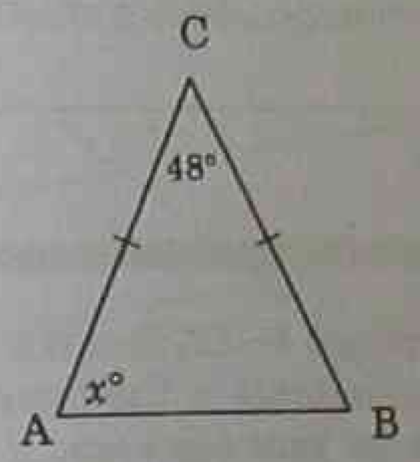
$\angle ABC = 48°$ (alt. angles, AD || CB)

$\angle CAB = 46°$ (alt. angles, EC || AB)

$\therefore \angle ACB = 180° - 48° - 46°$ (angle sum of a triangle)

$\angle ACB = 86°$

(e)



Given AC = BC and $\angle ACB = 48°$, find $\angle CAB$, giving reasons.

SOLUTION

Let $\angle CAB = x°$

$\therefore \angle ABC = x°$ (base angles of an isosceles triangle)

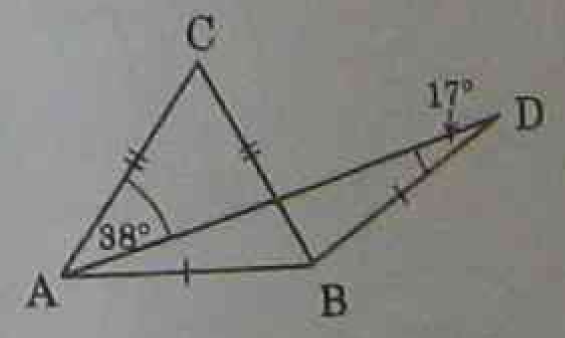
$\therefore 2x + 48 = 180$ (angle sum of a triangle)

$\therefore 2x = 132$

$x = 66$

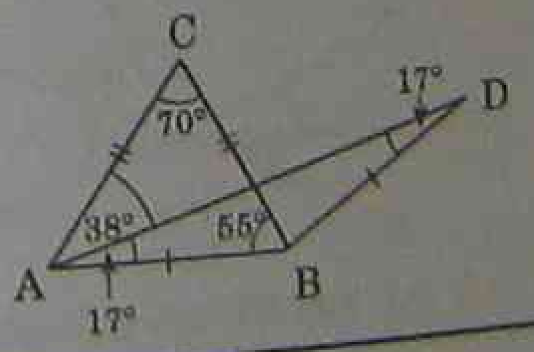
$\therefore \angle CAB = 66°$

(f)



Given AC = CB, AB = BD, $\angle ADB = 17°$ and $\angle CAD = 38°$, find $\angle ACB$, giving reasons.

SOLUTION



$\angle DAB = 17°$ (base angles of isosceles ΔABD)

$\therefore \angle CAB = 17° + 38°$ $(\angle CAD + \angle DAB)$
$= 55°$

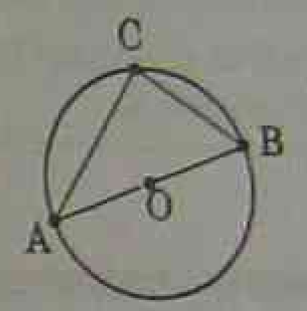
$\therefore \angle ABC = 55°$ (base angles of isosceles ΔABC)

Hence

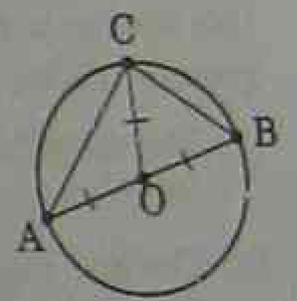
$\angle ACB = 180° - 55° - 55°$ (angle sum of a triangle)

$\angle ACB = 70°$

(g) AB is a diameter of a circle with centre O. $\angle ACB$ is subtended by AB at C. Prove that $\angle ACB = 90°$.



SOLUTION



Join OC. Let $\angle CAO = x°$, $\angle CBO = y°$

In ΔAOC,

AO = OC (equal radii)

$\therefore \angle ACO = x°$ ($\angle$s opposite equal sides)

In ΔCOB,

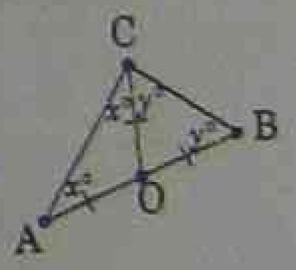
CO = OB (equal radii)

$\therefore \angle OCB = y°$ ($\angle$s opposite equal sides)

Then $\angle ACB = x° + y°$

In ΔABC,

$x° + y° + x° + y° = 180°$ ($\angle$ sum of Δ)



$\therefore 2x + 2y = 180$

$\therefore x + y = 90$

$\therefore \angle ACB = 90°$

11.9 Constructions (See Year 7 Excel Study Guide)

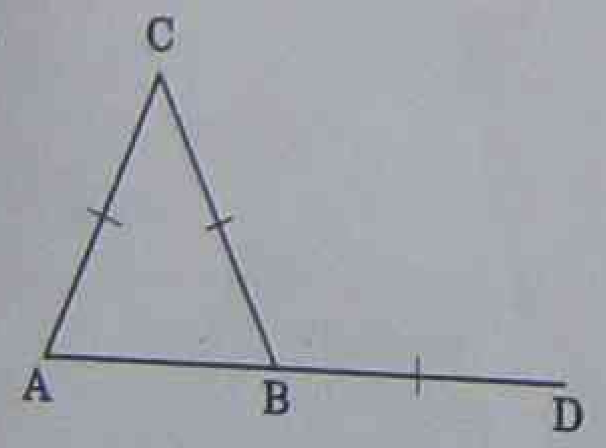
Advanced Level students should be able to complete the following constructions:

- Bisection of angles and intervals.
- Construction of right angles:
  - — perpendicular to a line from a fixed point not on the line.
  - — perpendicular to a line *at* a fixed point *on* the line.
- Construction of triangles from a given set of data, including given three sides, two sides and included angle, two angles and one side.
- Construction of angles of 60°, 30° and 120°
- Construction of quadrilaterals from a given set of data.
- Construction of a line parallel to a fixed line.
- Construction of regular figures, including equilateral triangles, hexagons, squares and octagons in circles.

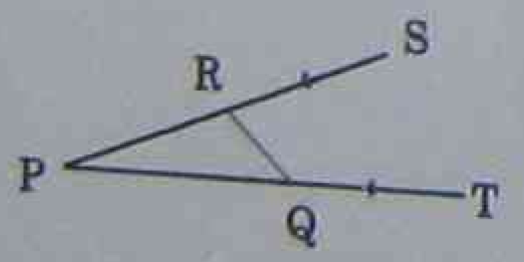
11.9 Exercises

1. Use the description to sketch the geometric shape.
   (a) The diagonals AC and BD of the parallelogram ABCD intersect at E.
   (b) The line PQ is drawn parallel to RS. The interval AB intersects PQ at C and RS at D. An isosceles triangle CDE is constructed such that CD = DE.
   (c) The interval AB is produced to X, while BA is produced to Y such that BX is equal in length to AY. Triangles ABD and YXD are drawn such that AD = BD and YD = XD.
2. Write a clear concise description of the figures shown.

(a)

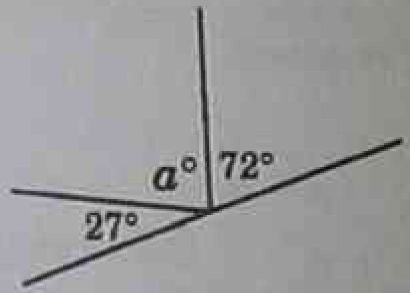


(b)

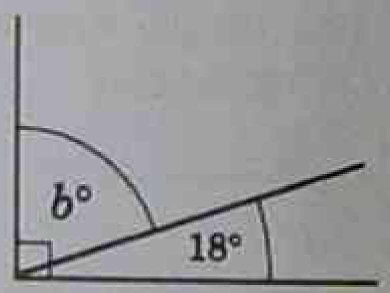


In Questions 3– 32, find the values of the pronumerals in the figures, giving reasons for your answers.

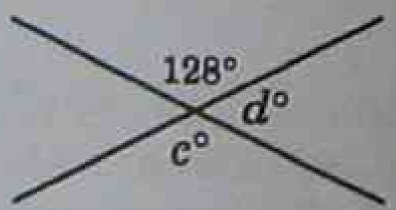
3.



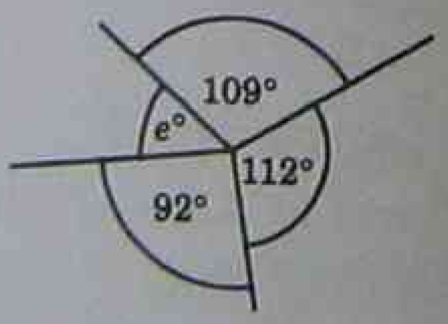
4.



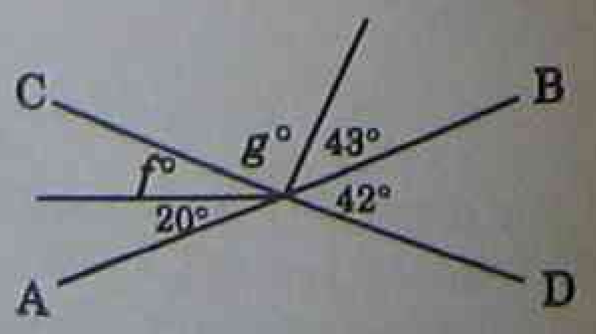
5.



6.

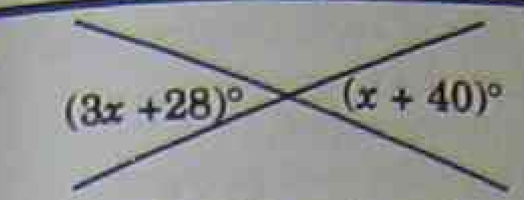


7.



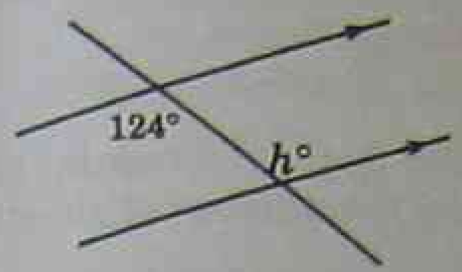
AB and CD are straight lines.

8.

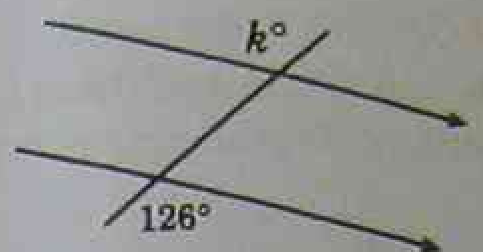


Also find the size of each angle.

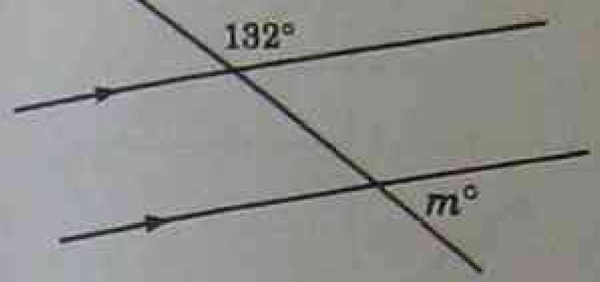
9.



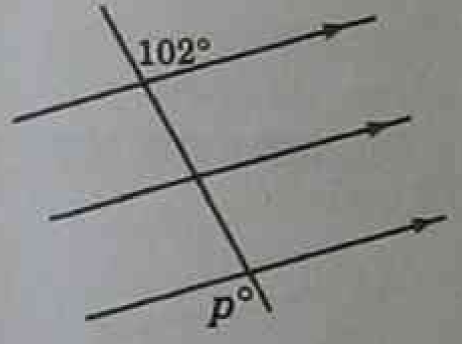
10.



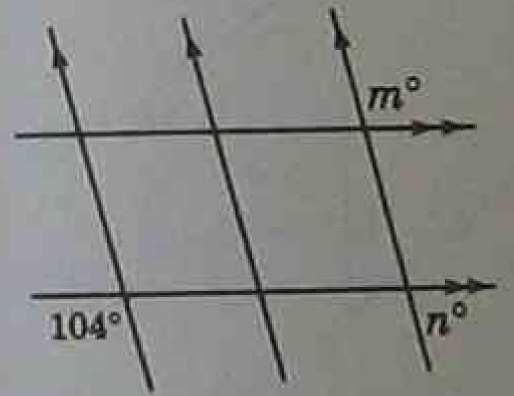
11.



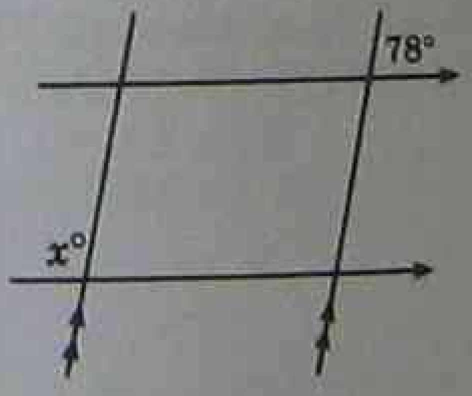
12.



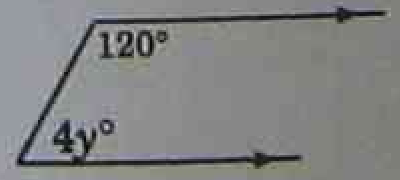
13.



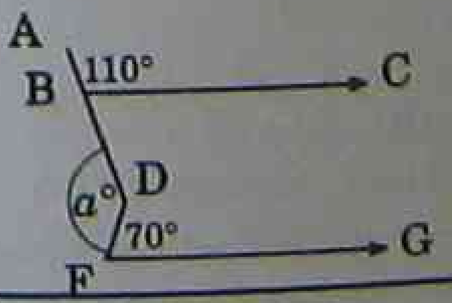
14.



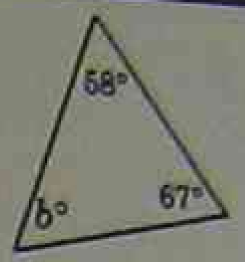
15.



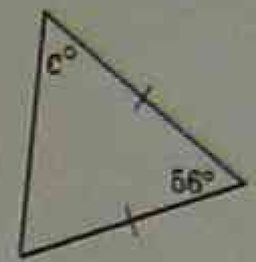
16.



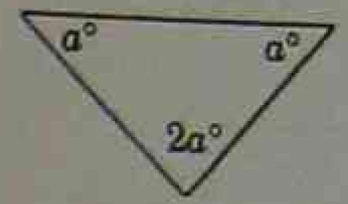
17.



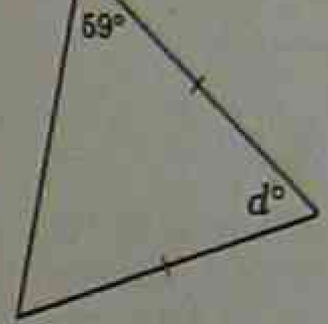
18.



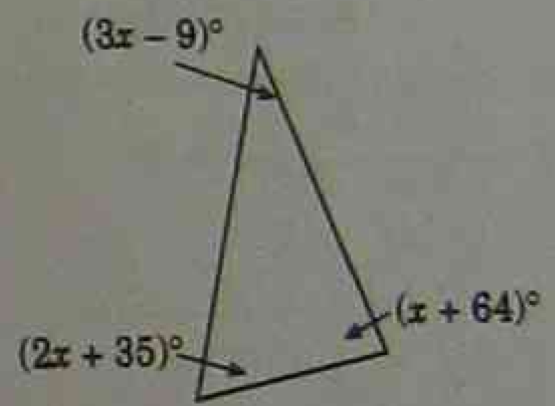
19.



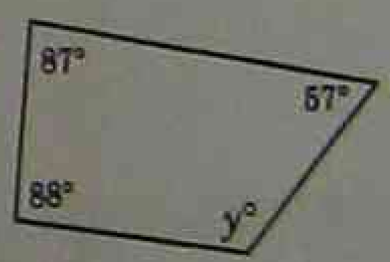
20.



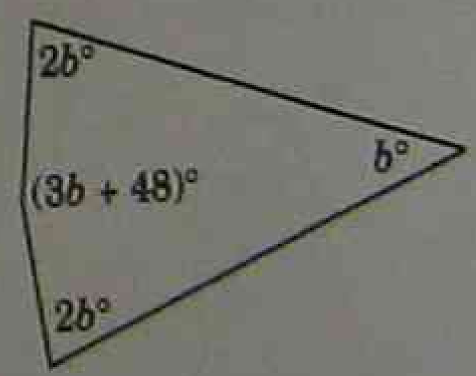
21.



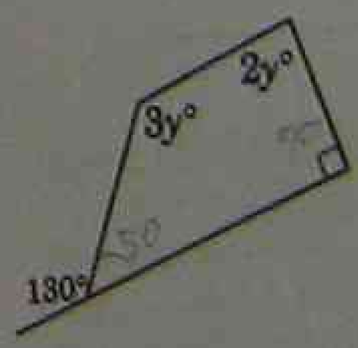
22.



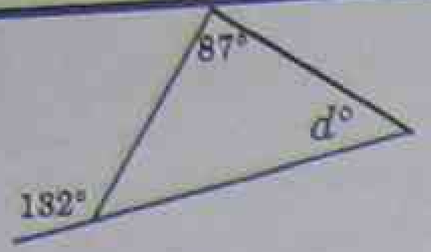
23.



24.

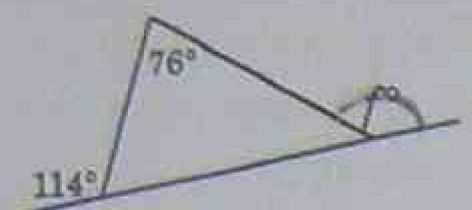


25.

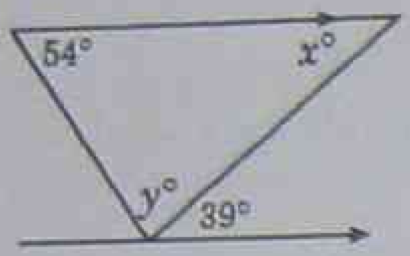


26.

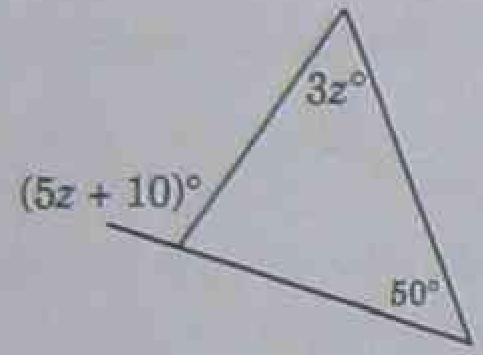
27.



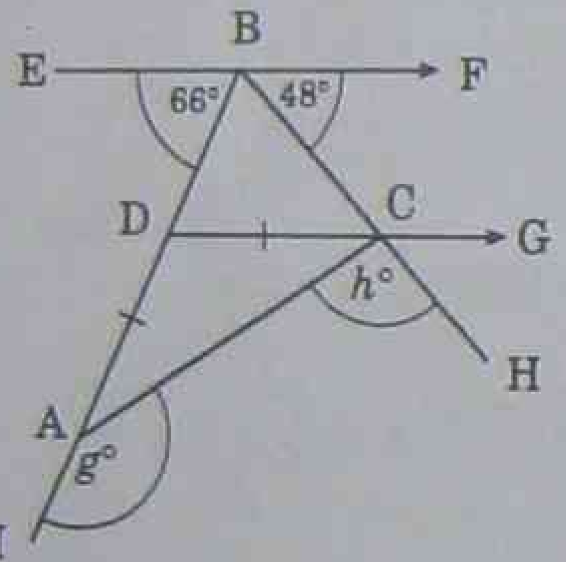
28.



29.

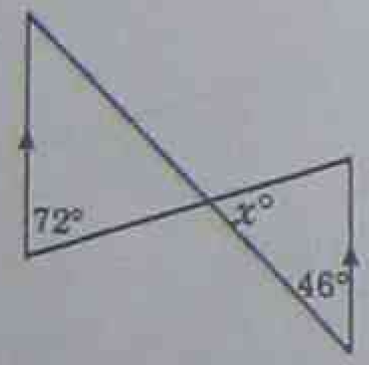


30.

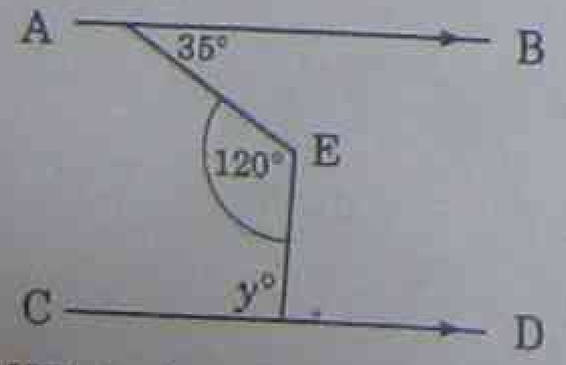


IB and BH are straight lines, AD = DC and EF || DG

31.



32.



[*Hint*: A construction is needed.]

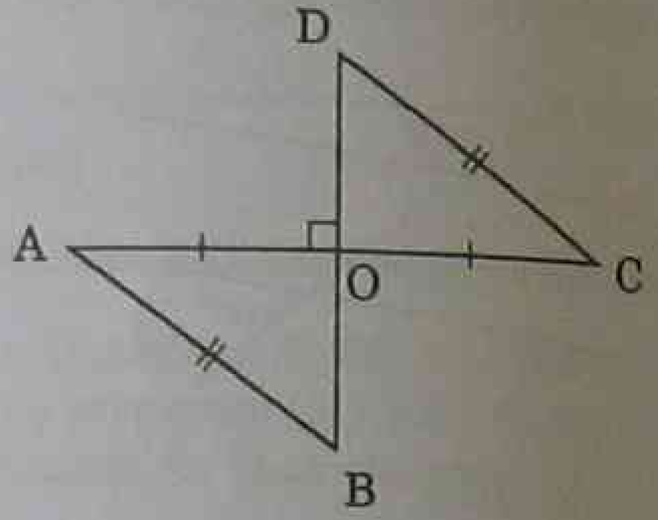
33. Find the size of each interior angle of:
(a) a regular hexagon,
(b) a regular decagon,
(c) a regular 52-sided polygon.

34. Find the size of each exterior angle of:
(a) a regular pentagon,
(b) a regular 12-sided polygon.

35. Each interior angle of a regular polygon is 140°. Find the number of sides of the polygon.

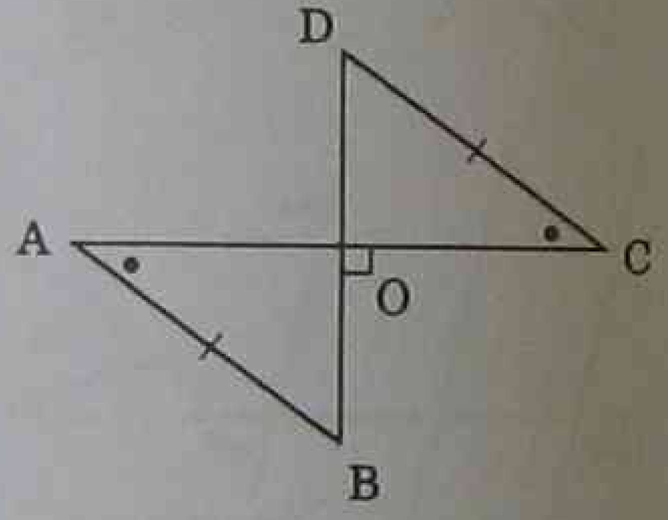
In Questions 36 – 46 prove that these triangles are congruent, giving adequate reasons.

36.



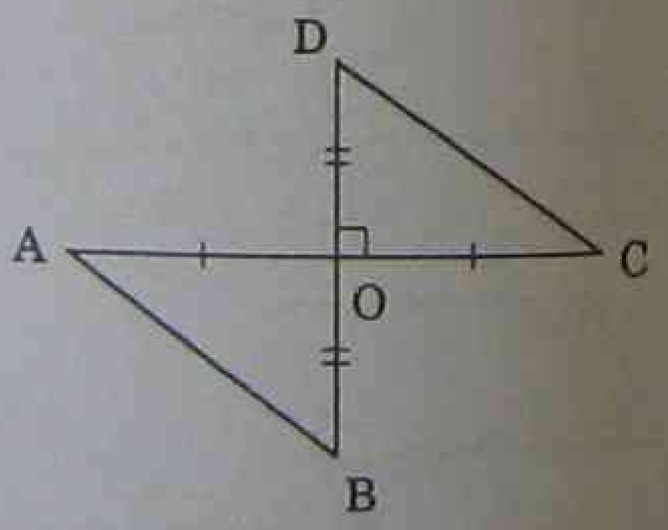
Given AO = OC, AB = CD, AC ⊥ DB, prove ΔAOB ≡ ΔDOC.

37.



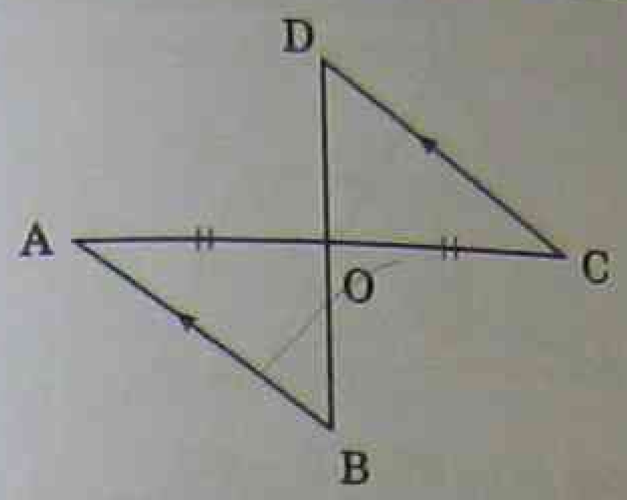
Given AB = DC, ∠OAB = ∠DCO, AC ⊥ DB, prove ΔAOB ≡ ΔDOC.

38.



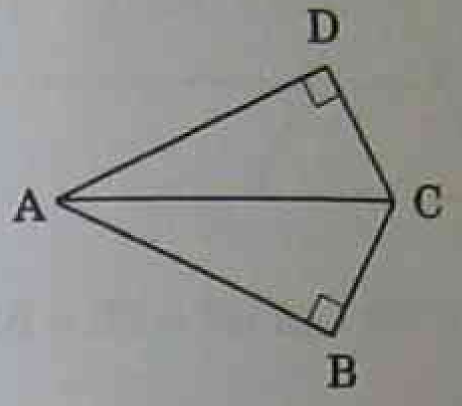
Given AO = OC, BO = OD, AC ⊥ DB, prove ΔAOB ≡ ΔDOC.

39.



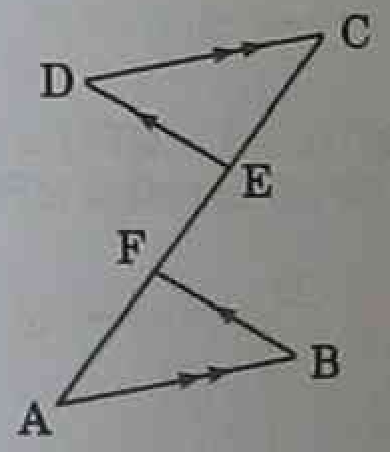
Given AB || DC, AO = OC, prove ΔAOB ≡ ΔDOC.

40.



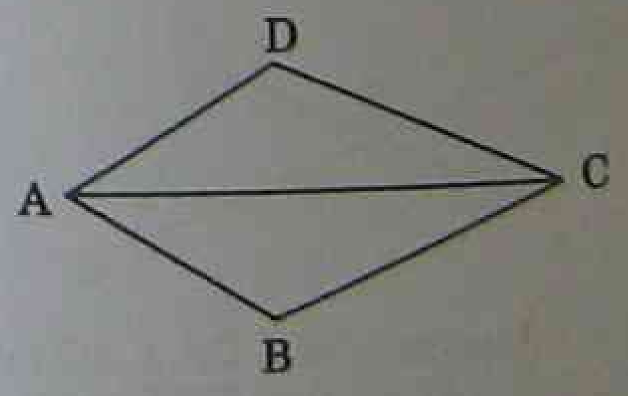
Given that CA bisects ∠DAB, CB ⊥ AB and CD ⊥ AD, prove that ΔABC ≡ ΔADC.

41.



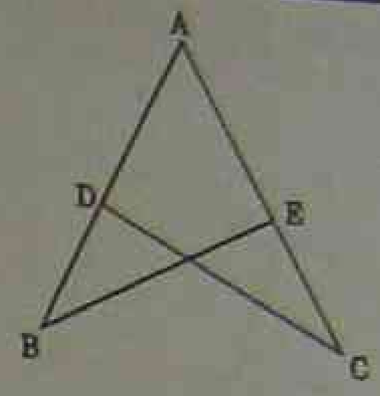
Given CD || AB, DE || FB, AC is a straight line, prove that ΔABF ≡ ΔCDE and hence that
(i) AF = CE
(ii) AE = CF

42.



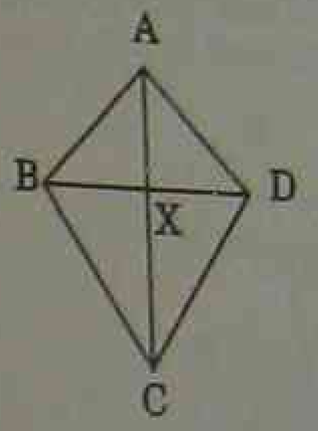
Given AD = AB, DC = BC, prove that ΔADC ≡ ΔABC and hence that ∠DAC = ∠BAC.

43.



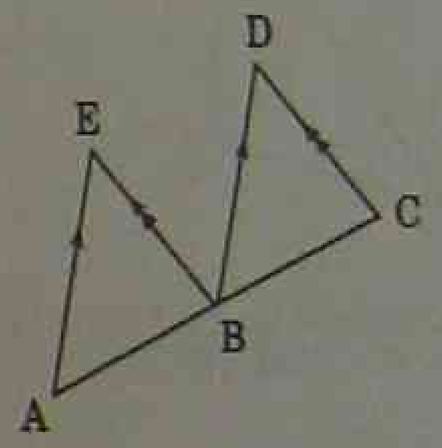
Given AD = AE, BD = EC, prove that ΔABE ≡ ΔADC and hence that ∠ABE = ∠ACD.

44.



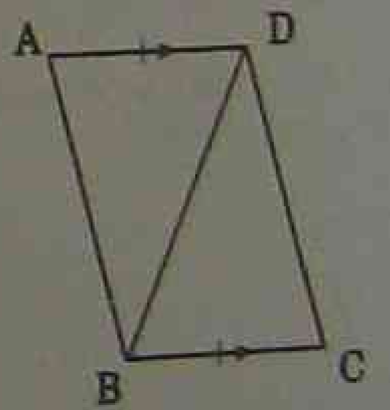
ABCD is a quadrilateral with AB = AD and BC = DC. Prove that
(i) ΔABC ≡ ΔADC
(ii) ΔABX ≡ ΔADX
(iii) AC bisects BD at right angles.

45.



Given B is the midpoint of AC, and that AE || BD, EB || DC, prove that Δ's ABE and BDC are congruent.

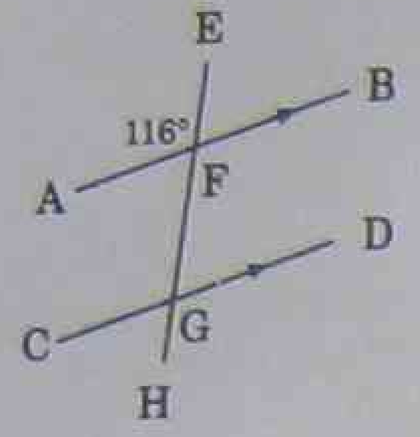
46.



Given AD = BC, AD || BC, prove that Δ's ABD and BCD are congruent.
Hence prove that
(i) AB = DC
(ii) ∠ABD = ∠BDC
(iii) AB || DC

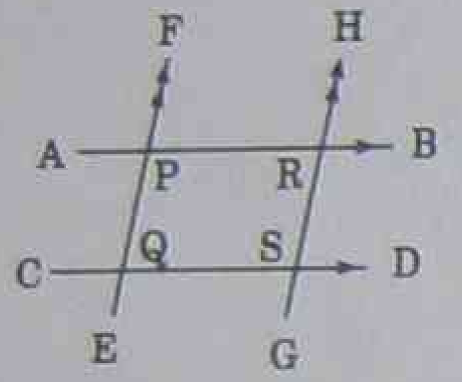
For Questions 47 – 58, give full reasons in each answer.

47.



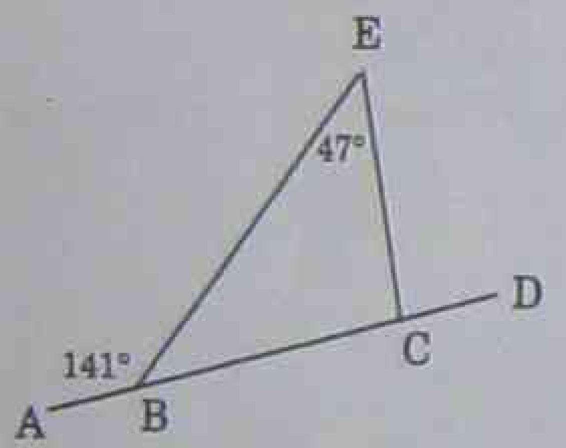
Given AB || CD and ∠AFE = 116°, find ∠CGH.

48.



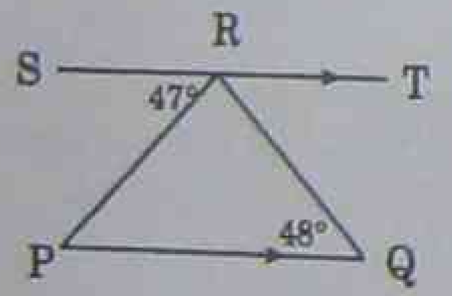
Given AB || CD, EF || GH and ∠CQE = 78°, find ∠BRS.

49.



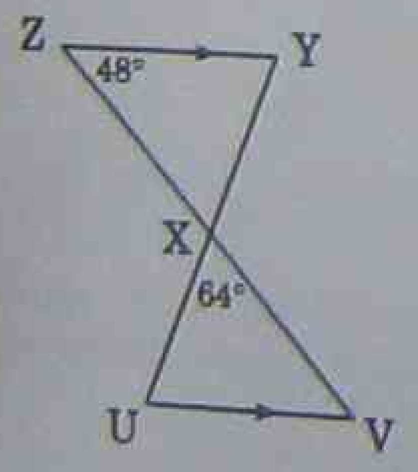
AD is a straight line, ∠ABE = 141° and ∠BEC = 47°. Find ∠ECD.

50.



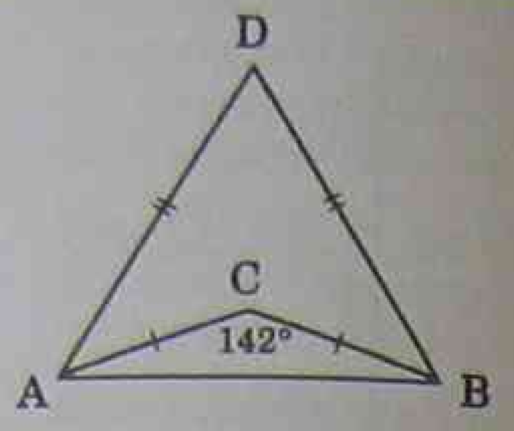
Given PQ || ST, ∠SRP = 47° and ∠PQR = 48°, find ∠PRQ.

51.



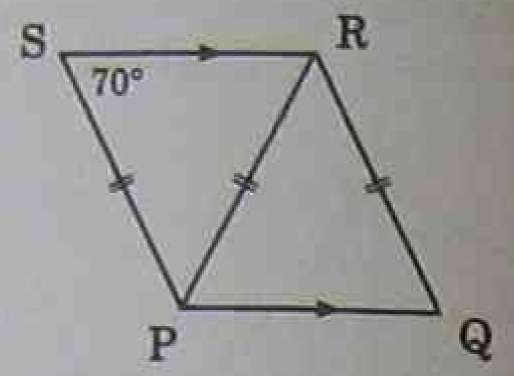
Given UV || ZY, ∠YZX = 48° and ∠UXV = 64°, find ∠XUV.

52.



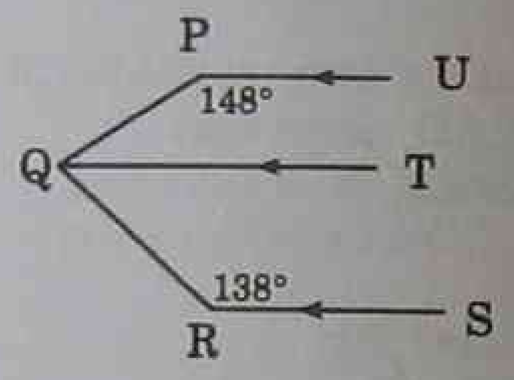
Given AC = BC, AD = BD, ∠ACB = 142° and that AC bisects ∠DAB, find ∠ADB.

53.



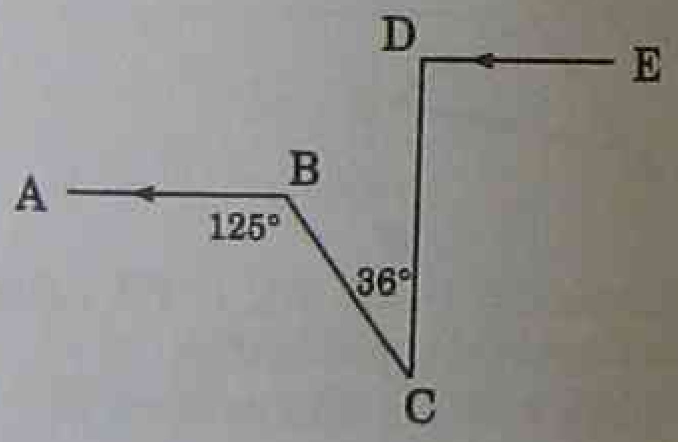
Given PQ || SR and SP = PR = RQ, find ∠PQR.

54.



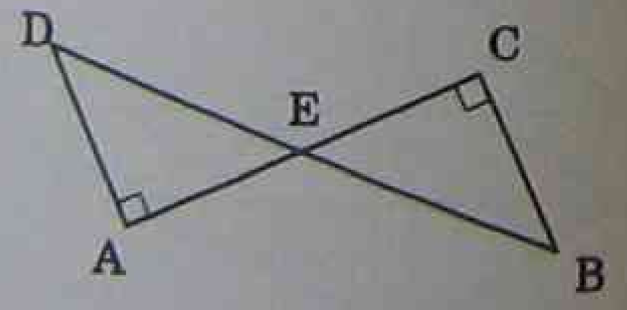
Given PU || QT || RS, ∠QPU = 148° and ∠QRS = 138°, find reflex ∠PQR.

55.



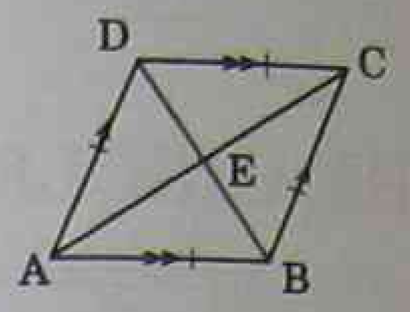
Given AB || DE, ∠ABC = 125° and ∠BCD = 36°, find ∠CDE.

56.



E is midpoint of BD, DA ⊥ AC and BC ⊥ AC. Prove that DB bisects AC.

57. **Prove that the dagonals of any rhombus bisect each other at right angles.**



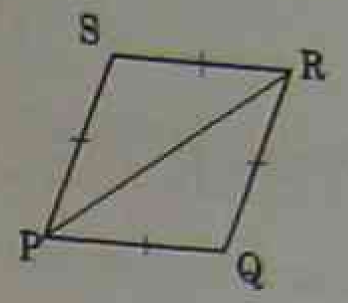
Given AD || BC, AB = BC = DA = CD

Hint: Prove that

(i) ΔDEC ≡ ΔAEB

(ii) ΔDEC = ΔBEC

58. **Prove that, if all sides of a quadrilateral are equal then it is a rhombus.**



Given PQ = QR = RS = SP.

Draw diagonal PR

Hint: Prove that ΔPSR ≡ ΔPQR

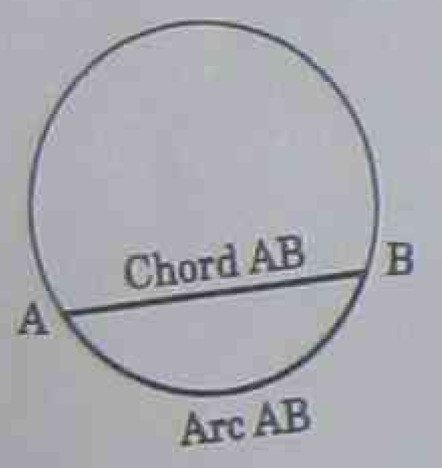
Chapter 12

FURTHER GEOMETRY — THE CIRCLE

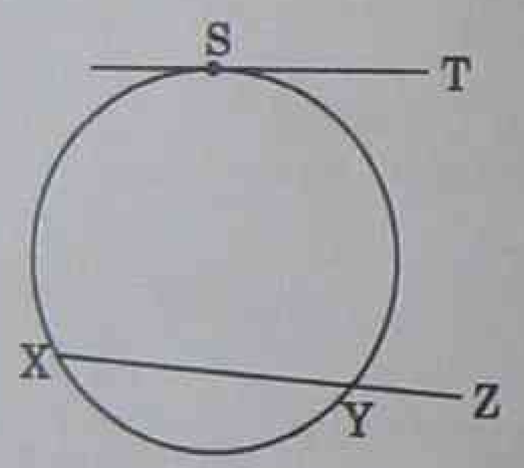
Some important points

- Always draw a clear diagram and mark on the diagram all given information.
- Work from the known towards the desired result.
- If an angle is required, mark sizes of angles on the diagram as you find them. If it is a deductive question it is generally useful to label one of the angles with a pronumeral (one of the angles in the result).
- If sides are involved, consider isosceles triangles or congruent triangles.
- If you become lost, check that all the given information has been used.
- Use any *hints* given in the question.

12.1 Some parts of the circle



Chord AB cuts off arc AB.

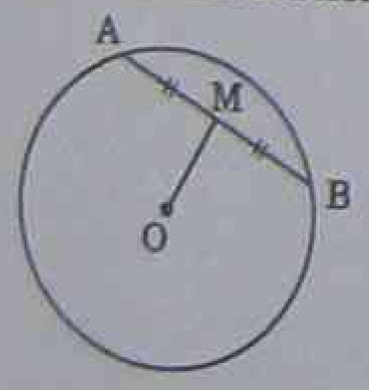


Tangent TS touches the circle at S.

Secant XZ cuts the circle at X and Y.

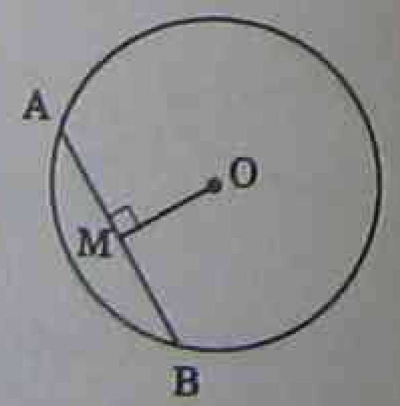
12.2 Properties of chords

All properties in this section rely on the use of congruent triangles.



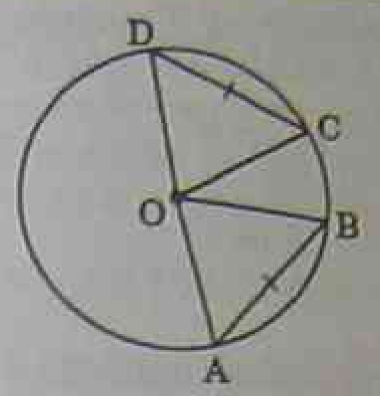
1. A line from the centre of a circle to the midpoint of a chord meets the chord at right angles.

   Given AM = MB, then OM ⊥ AB.



2. A perpendicular drawn to a chord from the centre of a circle bisects the chord.

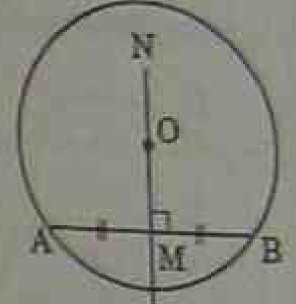
   Given OM ⊥ AB, then AM = MB.



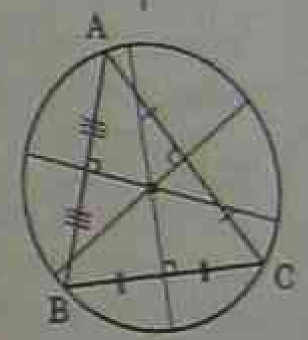
3. Equal chords of a circle *subtend* equal angles at the centre. Given AB = DC, then ∠AOB = ∠DOC.

Other interesting properties arising from these first three properties are:

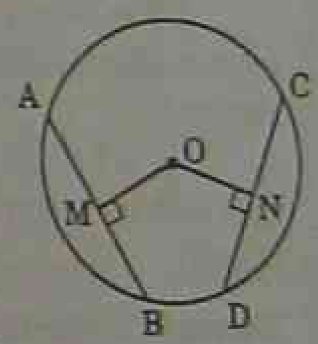
(a) The perpendicular bisector of a chord of a circle must pass through the centre of the circle. If AM = MB and NM ⊥ AB, then NB passes through O.



Note: This property is used to locate the centre of a circle passing through any three non-collinear points.
The centre is the point of intersection of the perpendicular bisectors of lines joining AB, BC and AC.



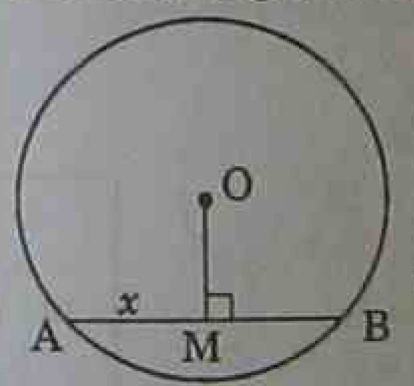
(b) Equal chords of a circle are equidistant from the centre of the circle. Given AB = CD, then OM = ON.
(See Exercises, Question 1)



Simple examples

Find the value of the pronumeral in the following questions:

(a)



Given OM ⊥ AB, O is the centre, AB = 12 cm; find x.

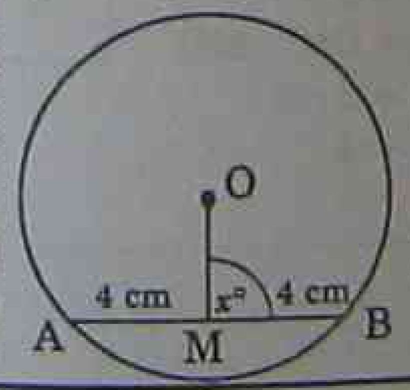
SOLUTION

AM = MB (Line from centre ⊥ chord)

$\therefore$ AM = 6 cm

$\therefore$ $x = 6$

(b)



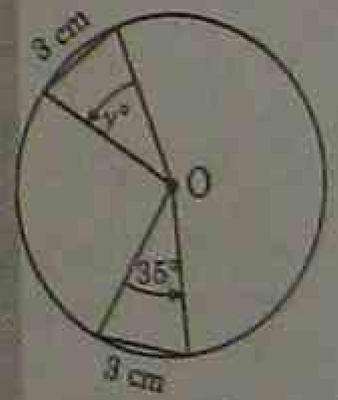
Given O is the centre, AM = MB = 4 cm; find x.

SOLUTION

OM ⊥ AB

$\therefore$ $x = 90$

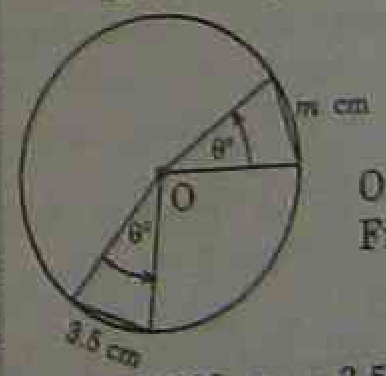
(c)



O is the centre. Find y.

SOLUTION: $y = 35$ (Equal chords subtend equal angles at the centre)

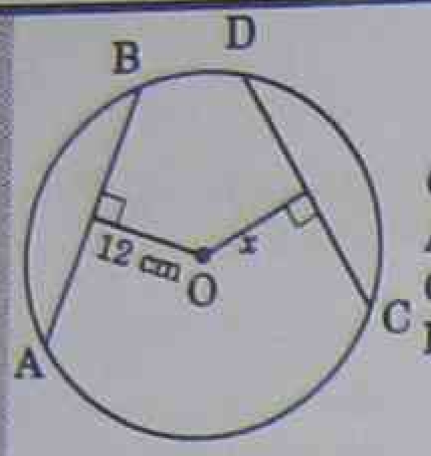
(d)



O is the centre. Find m.

SOLUTION: $m = 3.5$ (Equal chords subtend equal angles at the centre)

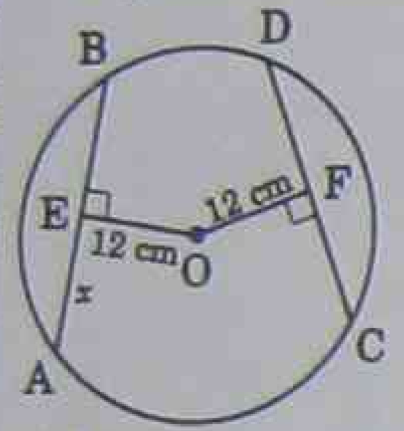
(e)



O is the centre,
AB = 14 cm,
CD = 14 cm.
Find x.

SOLUTION: $x = 12$ (equal chords are equidistant from the centre)

(f)



O is the centre,
EO = 12 cm,
OF = 12 cm,
FC = 5 cm,
AE = x cm.
Find x.

SOLUTION

$DC = 10$ cm

(perpendicular from centre to a chord bisects the chord)

$\therefore \quad AB = 10$ cm

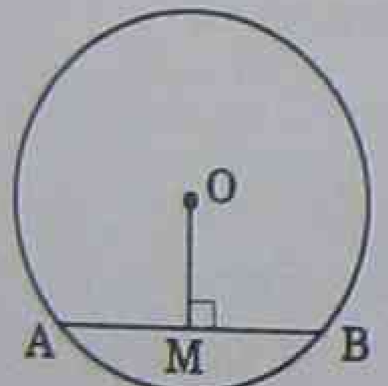
(equal chords are equidistant from the centre)

$\therefore \quad x = 5$

(perp. from centre to chord bisects the chord)

Examples involving Pythagoras' Theorem

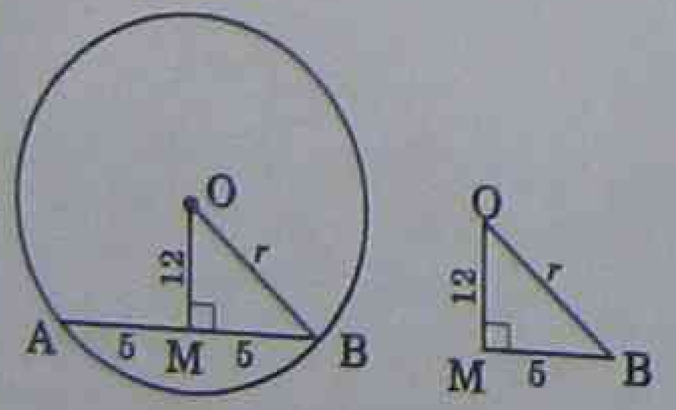
(g)



Given that a chord AB, 10 cm long, is 12 cm from the centre O, find the radius of the circle.

SOLUTION

Join OB and mark the information on the diagram.

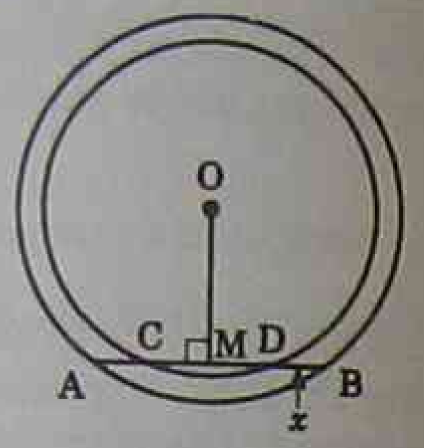


Using Pythagoras' Theorem:

$$\begin{aligned} r^2 &= 12^2 + 5^2 \\ &= 169 \\ \therefore\ r &= \sqrt{169} \\ &= 13 \end{aligned}$$

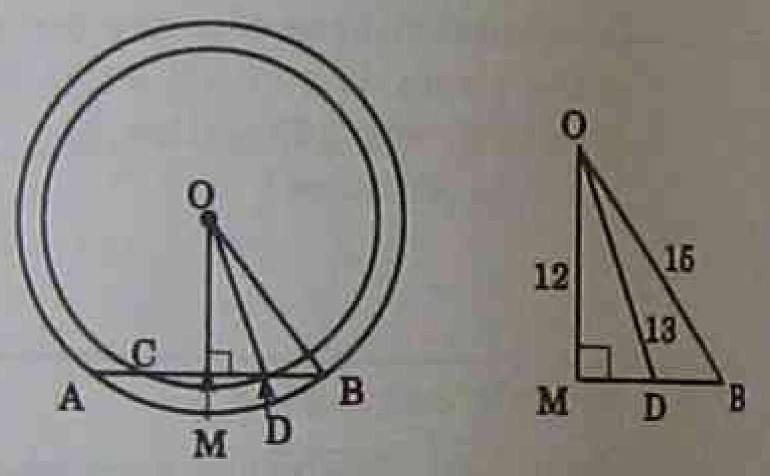
The radius of the circle is 13 cm.

(h) A chord is drawn 12 cm from the centre of concentric circles with radii 13 cm and 15 cm. Find the length of DB in the diagram.

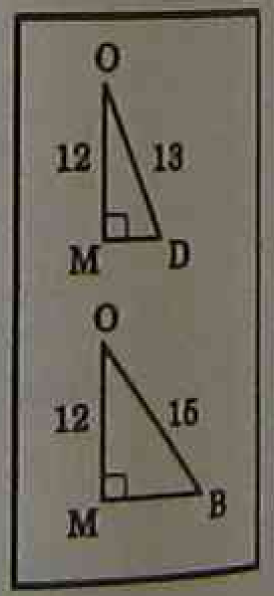


SOLUTION

Join OD, OB and mark the information on the diagram.



Using Pythagoras' Theorem:



From ΔOMD,

$$\begin{aligned} MD^2 + 12^2 &= 13^2 \\ \therefore\quad MD^2 &= 13^2 - 12^2 \\ &= 25 \\ \therefore\quad MD &= 5 \end{aligned}$$

From ΔOMB,

$$\begin{aligned} MB^2 + 12^2 &= 15^2 \\ \therefore\quad MB^2 &= 15^2 - 12^2 \\ &= 81 \\ \therefore\quad MB &= 9 \end{aligned}$$

$$\begin{aligned} \text{Now}\quad DB &= BM - MD \\ &= 9 - 5 \\ &= 4 \end{aligned}$$

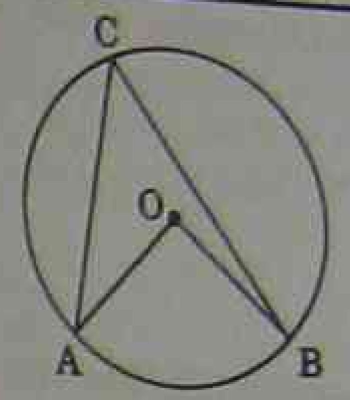
DB is 4 cm.

12.3 Properties of angles

1. The angle at the centre of a circle is double the angle at the circumference standing on the same arc.

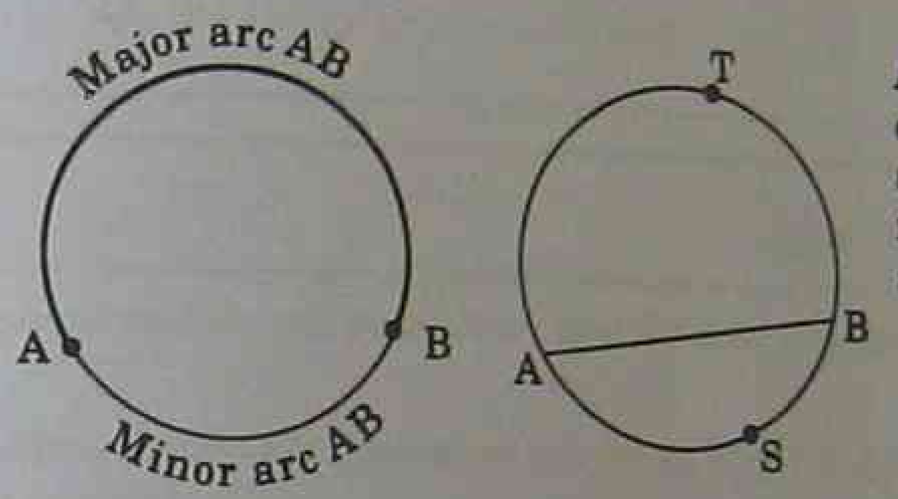
Given O is the centre, then:

$$\angle AOB = 2 \times \angle ACB, \text{ or}$$
$$\angle ACB = \tfrac{1}{2} \times \angle AOB$$



Some important notes

(a) Arcs and chords

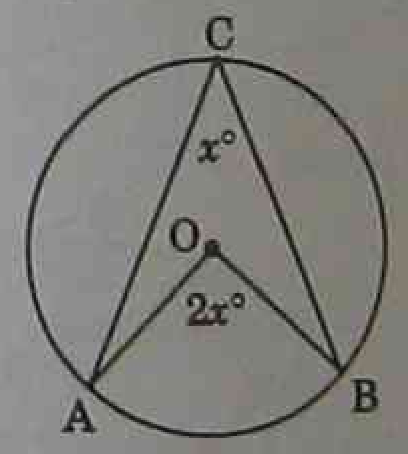


Any chord (AB in this diagram) divides the circumference into two sections. In this diagram the minor arc is ASB and the major arc is ATB.

(b) Reflex angle at the centre

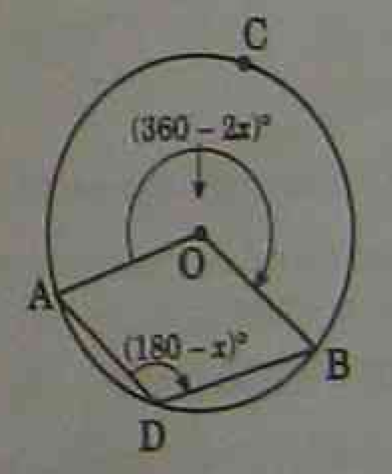
There are always two angles at the centre; one stands on the major arc — the reflex angle, while the other stands on the minor arc — the acute or obtuse angle. This second type is the most common and the easiest to see.

For example:



Normal case

$$\angle AOB = 2 \times \angle ACB, \text{ or}$$
$$\angle ACB = \tfrac{1}{2} \times \angle AOB$$



Reflex angle at the centre case

Reflex $\angle AOB = 2 \times \angle ADB$.

(Both stand on the major arc ACB.)

If $\angle AOB = 2x°$, then

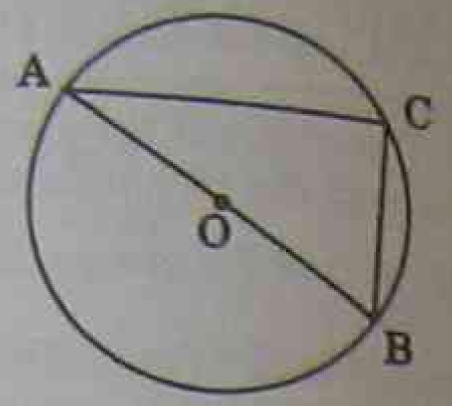
reflex $\angle AOB = (360° - 2x)°$

and $\quad \angle ADB = \tfrac{1}{2}(360 - 2x)°$
$\qquad\qquad = (180 - x)°$

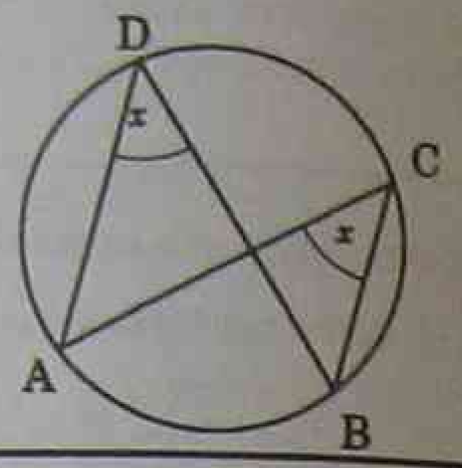
Continued

Some important notes (continued)

2. The angle in a semicircle is a right angle. Given AB is a diameter, ∠ACB = 90°.



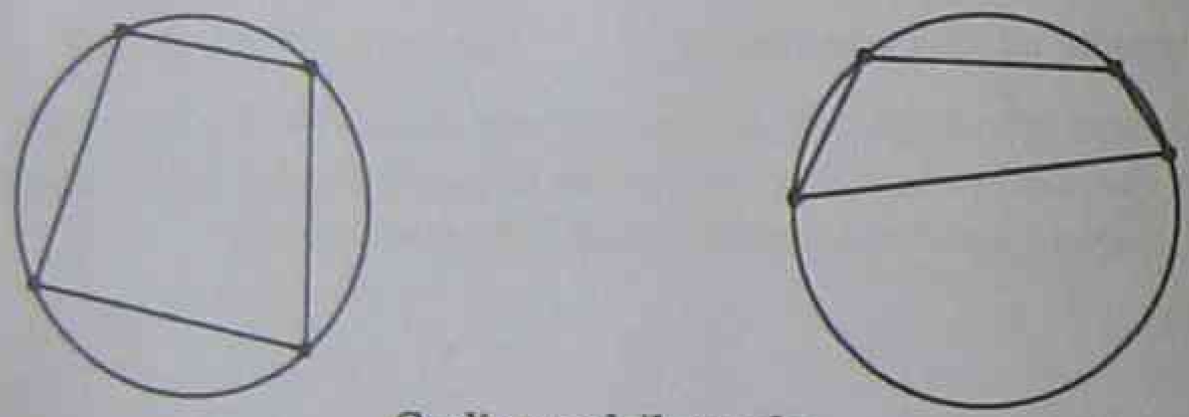
3. Angles in the same segment are equal, [that is, angles at the circumference standing on the same arc]. Given ∠ADB, ∠ACB standing on arc AB, then ∠ADB = ∠ACB.



12.4 Cyclic quadrilaterals

1. A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle.

   Concyclic points are points through which a circle can be drawn, that is, all points lie on the circumference of a circle.



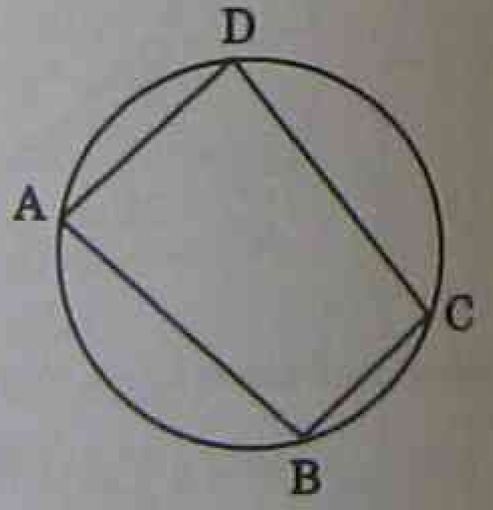
Cyclic quadrilaterals

The four vertices of a cyclic quadrilateral are concyclic points.

2. Opposite angles in a cyclic quadrilateral are supplementary.

   ∠DAB + ∠BCD = 180°, ∠ABC + ∠ADC = 180°.

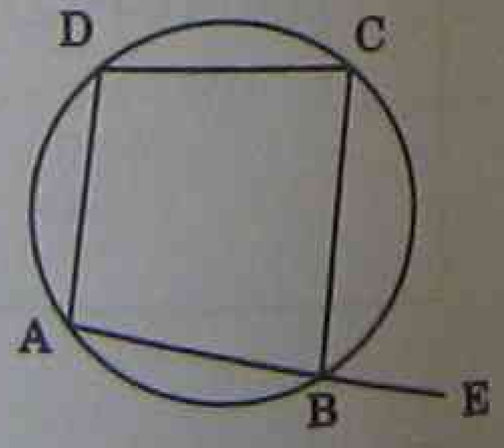
   Conversely, if the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.



3. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. Given that ABCD is a cyclic quadrilateral:

   ∠CBE = ∠ADC.

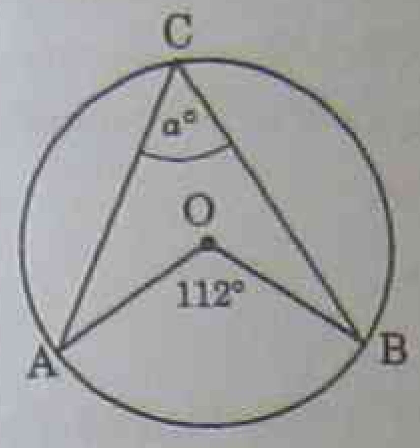
   (Proof of this theorem is Question 5 in Exercises.)



Examples

Find the value of the pronumeral in each diagram, giving adequate reasons. (O is the centre in each diagram.)

(a)



∠AOB = 112°

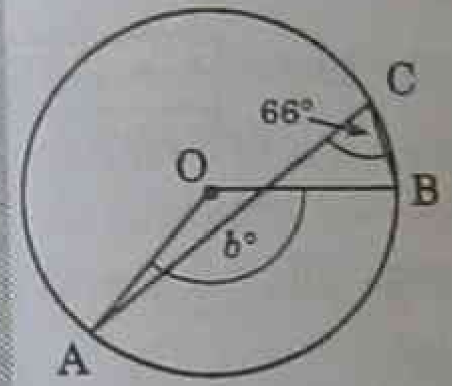
∠ACB = a°

SOLUTION

$a = \frac{1}{2} \times 112 = 56$ (∠ at the circum.

$= \frac{1}{2}$ ∠ at the centre on the same arc)

(b)



∠ACB = 66°

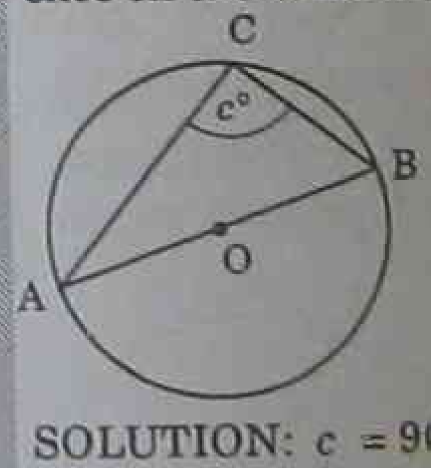
∠AOB = b°

SOLUTION

$b = 2 \times 66 = 132$

(∠ at centre = 2 × ∠ at the circumference on the same arc.)

(c)

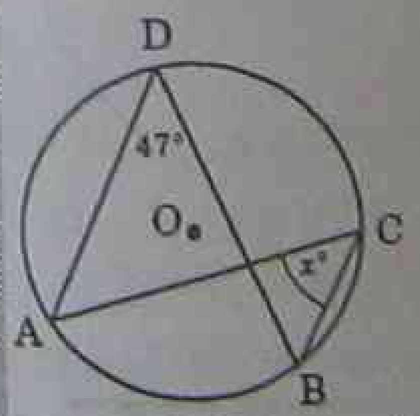


AB is diameter

SOLUTION: $c = 90$

(Angle in a semicircle)

(d)



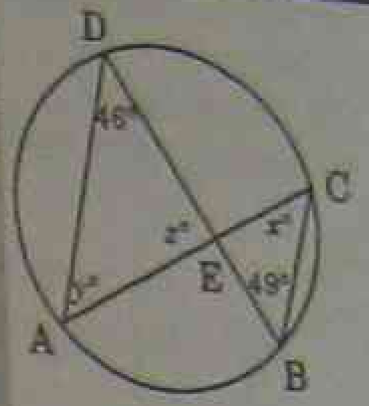
∠ADB = 47°

∠ACB = x°

SOLUTION: $x = 47$

(Angles in the same segment)

(e)



∠ADB = 46°

∠DBC = 49°

∠DAC = y°

∠ACB = x°

∠AED = z°

SOLUTION

$x = 46$

(angles in the same segment standing on arc AB)

$y = 49$

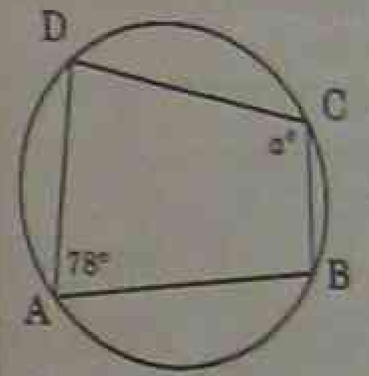
(angles in the same segment, on arc DC)

$z = 180 - (46 + 49)$

(angle sum of ΔADE)

$= 85$

(f)



∠DAB = 78°

∠DCB = a°

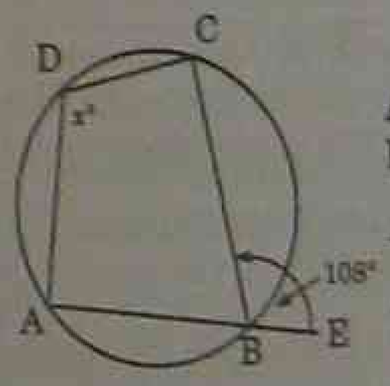
SOLUTION

$a = 180 - 78$

(opposite angles of a cyclic quad'l)

$= 102$

(g)



AE is a straight line.

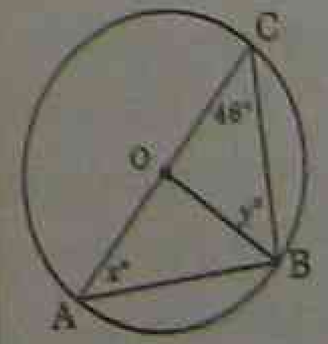
∠CBE = 108°

∠ADC = x°

SOLUTION: $x = 108$

(exterior angle of a cyclic quad'l)

(h)



AC is a diameter.

∠ACB = 46°

∠CAB = x°

∠CBO = y°

SOLUTION

OC = OB (equal radii)

∴ $y = 46$ (base angles, isosceles Δ)

Continued

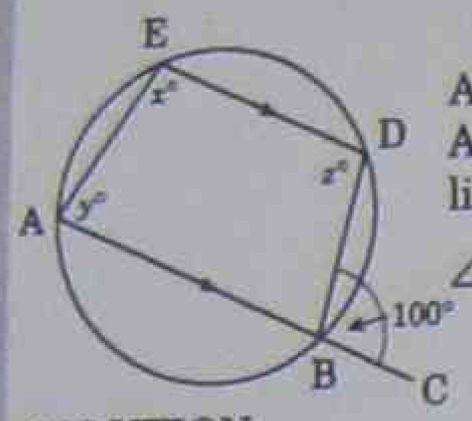
But $\angle ABC = 90°$ ($\angle$ in a semicircle)

$\therefore \quad \angle ABO = 90° - 46° = 44°$

Also AO = OB (equal radii)

$\therefore \quad x = 44$ (base angles, isosceles Δ)

(i)



AB ∥ ED

AC is a straight line.

$\angle DBC = 100°$

SOLUTION

$x = 100$

(exterior angle of a cyclic quad'l)

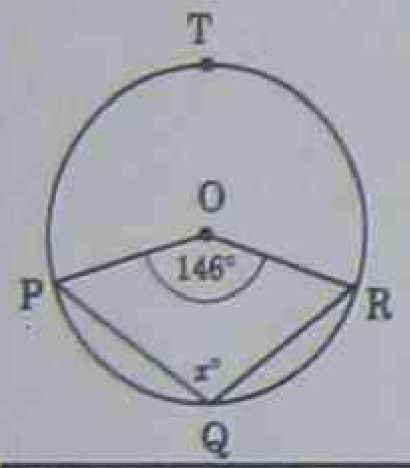
$z = 100$ (alternate angles, AB ∥ ED)

$\therefore \quad y = 180 - 100$

(opposite angles of a cyclic quad'l)

$= 80$

(j)



$\angle POR = 146°$

$\angle PQR = x°$

Note 1. This is the reflex angle at the centre case.

Note 2. PQRO is **not** a cyclic quadrilateral. Only three vertices lie on the circle. The other vertex is at the centre.

SOLUTION

Reflex $\angle POR$ stands on major arc PTR.

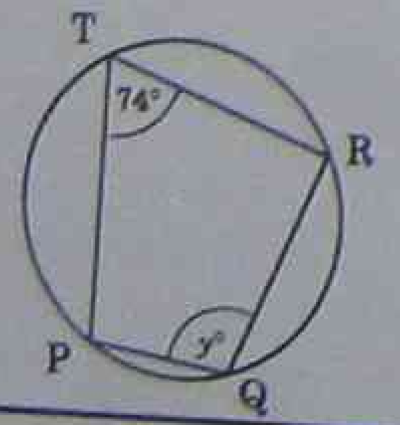
$\angle PQR$ is the angle at the circumference on major arc PTR.

$\angle POR = 360° - 146°$ (angles at a point)

$= 214°$

$\therefore \quad x = 107$ ($\angle$ at circumference $= \frac{1}{2}\angle$ at centre on major arc PTR)

Compare this with the following case:



This is a cyclic quadrilateral as all four vertices lie on the circle.

$\angle PTR = 74°$, $\angle PQR = y°$,

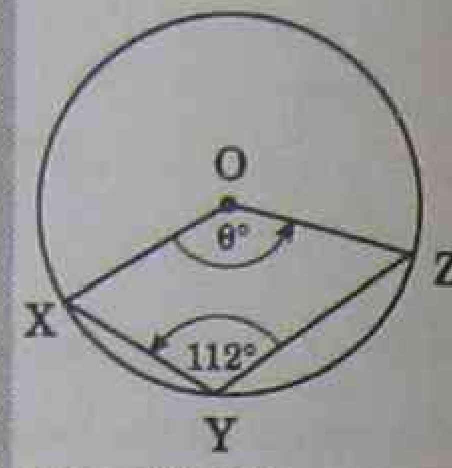
SOLUTION

$y = 180 - 74$

(opposite angles of a cyclic quad'l)

$\therefore \quad y = 106$

(k)



$\angle XYZ = 112°$,

$\angle XOZ = \theta°$

SOLUTION

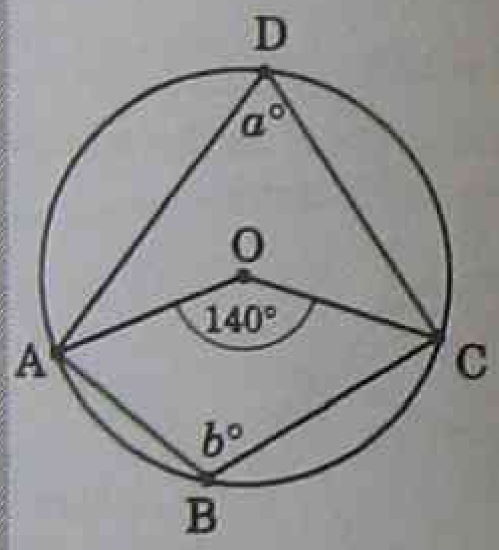
Reflex $\angle XOZ = 224°$ $\boxed{2 \times 112°}$

($\angle$ at centre $= 2 \times \angle$ at circumference on major arc XZ)

$\therefore \quad \theta = 360 - 224$ (angles at a point)

$= 136$

(l)



$\angle AOC = 140°$,

$\angle ADC = a°$,

$\angle ABC = b°$.

SOLUTION

$a = 70$

($\angle$ at circumference $= \frac{1}{2}\angle$ at centre on arc AC)

$\therefore \quad b = 180 - 70$

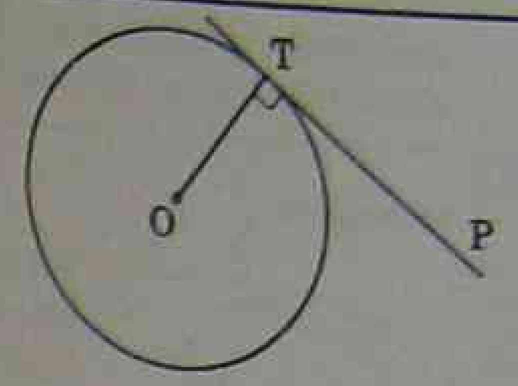
(opposite angles of cyclic quad'l ABCD)

$= 110$

12.5 Properties of tangents

1. The angle between a tangent and a radius drawn to the point of contact is a right angle.

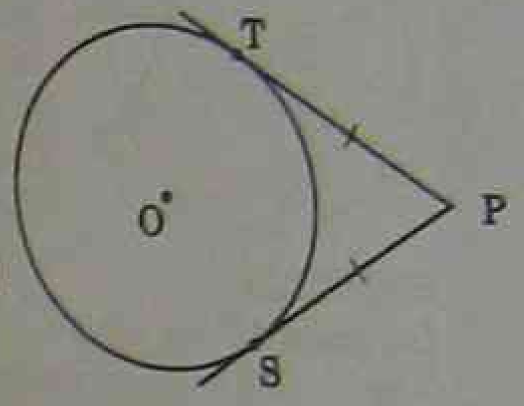
PT is a tangent at T, O is the centre; then $\angle OTP = 90°$.



2. The lengths of tangents drawn from an external point are equal.

PT and PS are tangents; then PT = PS.

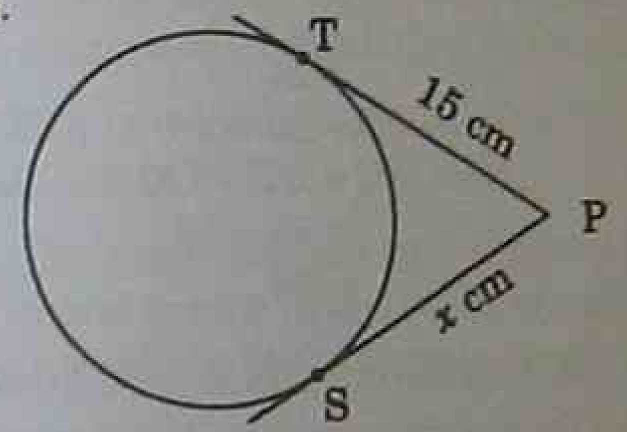
[Congruent triangles are used after joining OT and OS.]



Examples

In Examples (a) – (c), find the value of x.

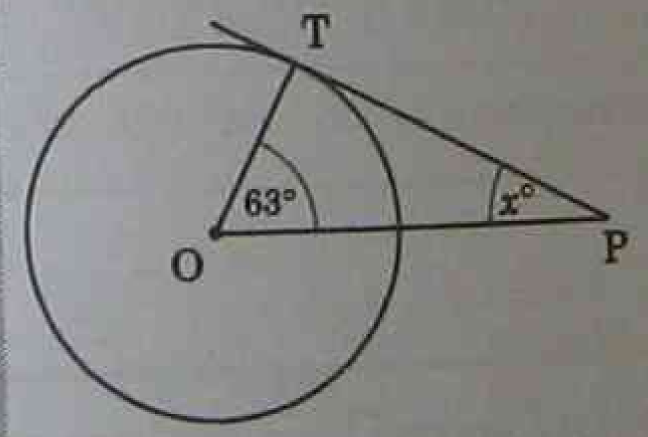
(a)



PT, PS are tangents, PT = 15 cm.

SOLUTION: $x = 15$ cm (tangents drawn from external point)

(b)



O is centre. PT is a tangent touching at T. $\angle TOP = 63°$, $\angle OPT = x°$

SOLUTION

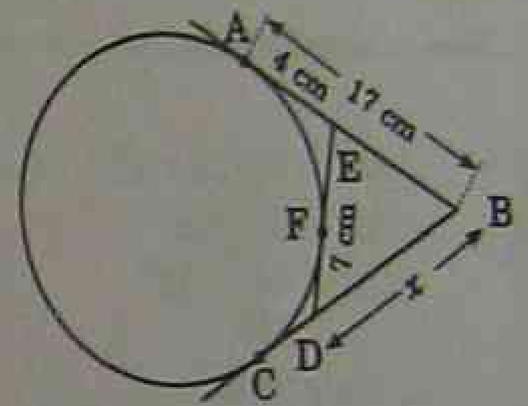
$\angle OTP = 90°$

(radius drawn to a point of contact)

$\therefore \quad x = 90 - 63$ ($\angle$ sum of Δ)

$x = 27$

(c)



AB, BC and ED are tangents touching at A, C and F respectively.

AB = 17 cm, AE = 4 cm, ED = 7 cm, BD = x cm. Find x.

Draw clear diagrams. Mark all known information on the diagram. Use all the given data.

SOLUTION

Look for:

AB = BC (tangents from point B)

$\therefore \quad$ BC = 17 cm

Also AE = EF (tangents from point E)

$\therefore \quad$ EF = 4 cm

Then FD = 3 cm (ED = 7 cm)

But FD = DC (tangents from point D)

$\therefore \quad$ DC = 3 cm

Then BD = 14 cm (BC = 17 cm)

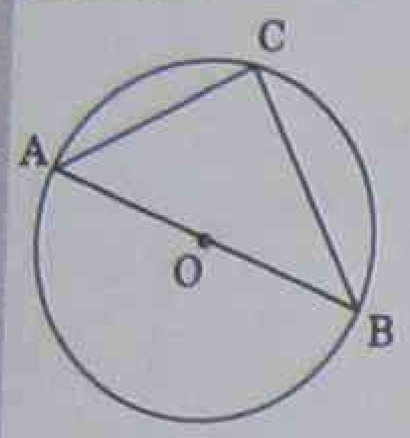
$\therefore \quad x = 14$

12.6 More applications of Pythagoras' Theorem

Remember: Whenever a right angle occurs in a triangle, there are opportunities to use either Pythagoras' Theorem or trigonometry. Here Pythagoras' Theorem is considered.

Examples

(a)



AB is a diameter of a circle, centre O, and radius 20 cm. Given BC = 32 cm, find the length of chord AC.

SOLUTION

Let chord AC = x cm.

Now AB = 40 cm (radius = 20 cm)

Also ∠ACB = 90° (angle in a semicircle)

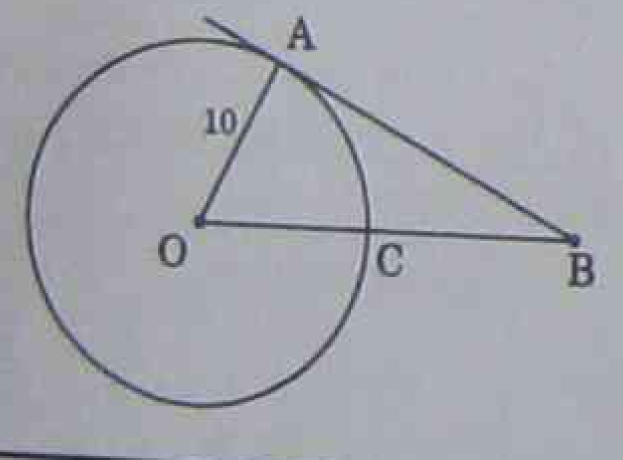
Now, using Pythagoras' Theorem:

$$\begin{aligned} AB^2 &= BC^2 + AC^2 \\ 40^2 &= 32^2 + x^2 \\ \therefore\ x^2 &= 40^2 - 32^2 \\ &= 576 \\ \therefore\ x &= \sqrt{576} \\ &= 24 \end{aligned}$$

∴ AC is 24 cm long.

∴ AC is 24 cm long.

(b)



AB is a tangent to a circle, centre O, radius 10 cm. If AB is 24 cm, find the length of BC.

SOLUTION

To find BC, we must first find BO.

∠OAB = 90°

(angle between the radius and the tangent to the point of contact)

Let OB = y cm.

Using Pythagoras' Theorem in ΔOAB:

$$\begin{aligned} OB^2 &= OA^2 + AB^2 \\ y^2 &= 10^2 + 24^2 \\ &= 676 \\ \therefore\ y &= \sqrt{676} \\ &= 26 \\ OB &= 26 \text{ cm} \\ \text{Now } BC &= OB - OC \\ &= 26 - 10 \\ &= 16 \end{aligned}$$

(OC is a radius and is thus 10 cm)

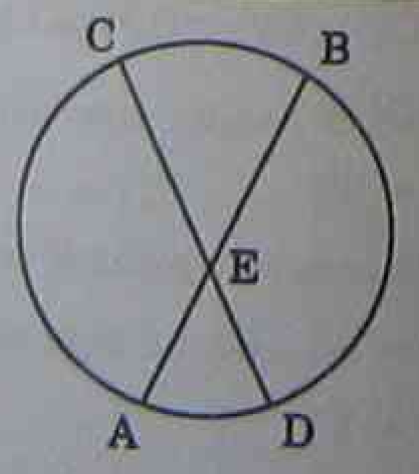
∴ BC is 16 cm long.

12.7 Ratio theorems (Proved using similar triangles)

1. The products of intercepts of intersecting chords are equal, that is:

AE . EB = DE . EC

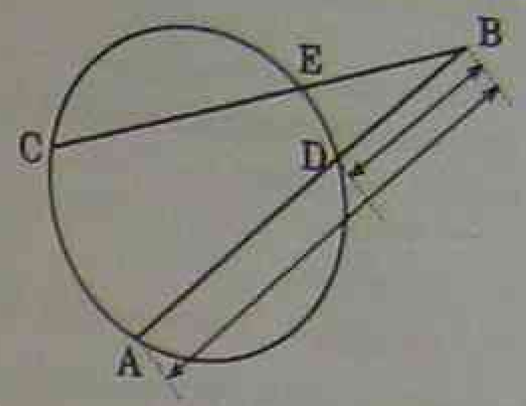
$$\boxed{AE \times EB = DE \times EC}$$



2. An extension of (1) concerning secants from an external point B is illustrated in the diagram.

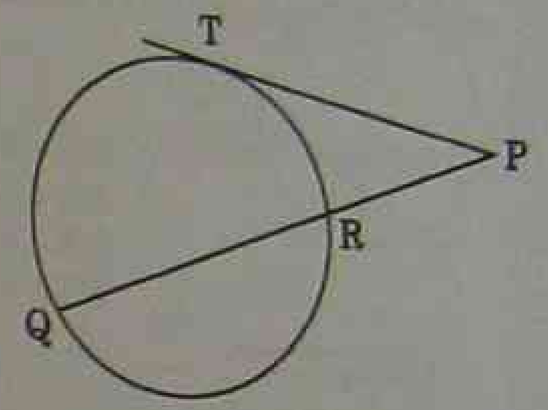
This is a case of chords produced intersecting at an external point. Chords AD and CE are extended to meet at B. Then AB × BD = CB × BE

(End-point A to intersection B × intersection B to other end point D)



3. The square of the length of the tangent is equal to the product of the intercepts of a secant drawn from an external point.

$$PT^2 = QP \times PR$$

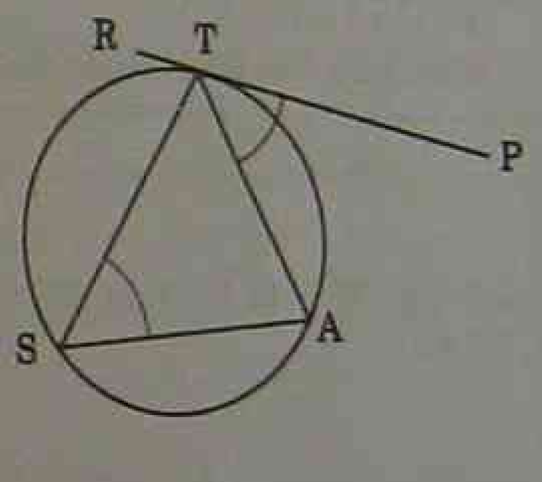


12.8 Angle in the alternate segment

1. PT is a tangent at T. AT is a chord drawn to the point of contact. ∠TSA is the angle at the circumference subtended by chord AT.

∠TSA is the angle in the alternate segment relative to ∠PTA.

2. An angle formed by a tangent to a circle with a chord drawn to the point of contact is equal to any angle in the alternate segment.



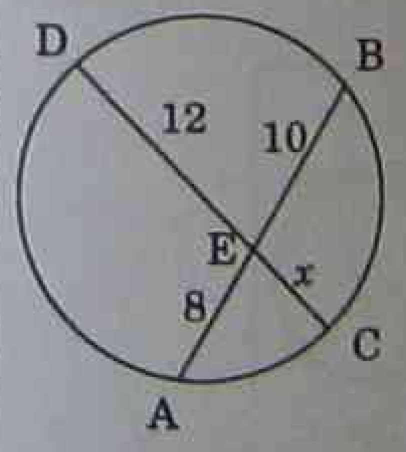
From the diagram, ∠PTA = ∠TSA.

Also note that ∠RTS = ∠TAS for the same reason (tangent RT, chord TS).

12.9 Worked examples

Find the value of the pronumeral in each diagram. All lengths are in cm.

(a)



AE = 8
EB = 10
DE = 12
EC = x

SOLUTION

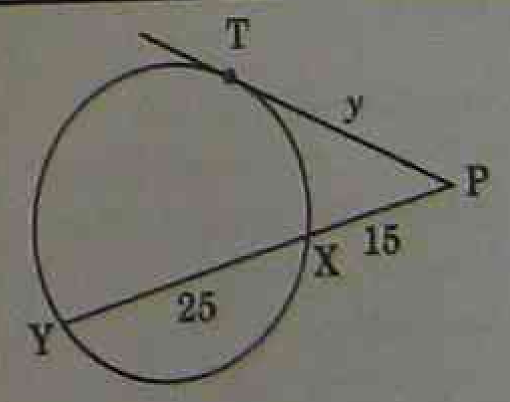
$12 \times x = 8 \times 10$

(intercepts of intersecting chords)

$12x = 80$

$x = \frac{80}{12} = 6\frac{2}{3}$

(b)



XY = 25, XP = 15, PT = y.

Find the exact value.

$$\boxed{\textit{Note}:\ YP = 40}$$

Continued

SOLUTION

$y^2 = 40 \times 15$

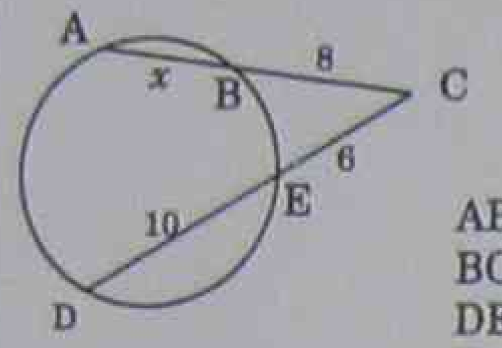
(product of intercepts of secant = square of tangent.)

$= 600$

$\therefore \quad y = \sqrt{600}$

$= 10\sqrt{6}$

(c)



AB = x
BC = 8
DE = 10
EC = 6

Note:
AC = $x + 8$
DC = 16

SOLUTION

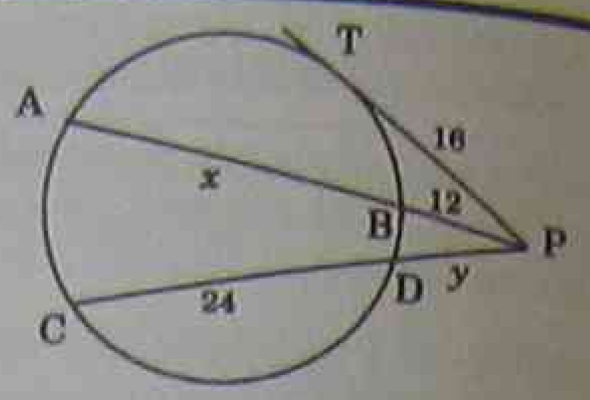
$(x+8) \times 8 = 16 \times 6$

(intercepts of intersecting chords)

$\therefore \quad x + 8 = 12$

$x = 4$

(d) AB = x, BP = 12, CD = 24, DP = y, and TP = 16.



SOLUTION

$16^2 = (x+12)12$

(product of intercepts of secant = square of the tangent)

$\frac{256}{12} = x + 12$

$21\frac{1}{3} = x + 12$

$\therefore \quad x = 9\frac{1}{3}$

$16^2 = (y+24)y$

(product of intercepts of secant = square of the tangent)

$\therefore \quad 256 = y^2 + 24y,$

that is, $y^2 + 24y - 256 = 0$

$(y+32)(y-8) = 0$

$\therefore \quad y = -32 \text{ or } 8,$

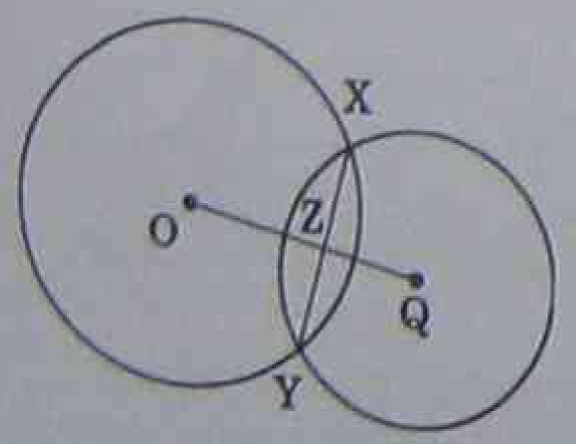
that is, $y = 8$

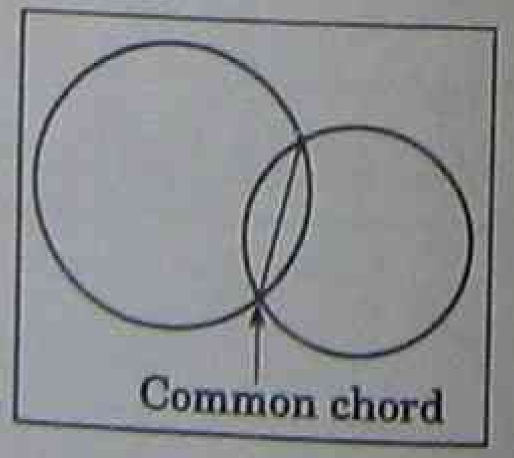
[Length must be positive.]

12.10 Circles that touch

12.10.1 Intersection of two circles

When two circles intersect, the line joining their centres bisects their common chord at right angles.



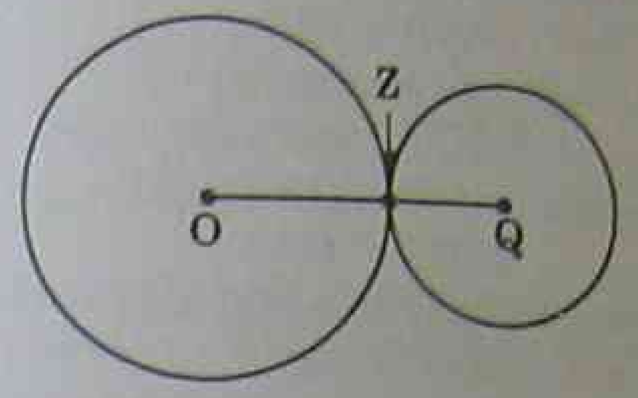


Two circles, centres O and Q intersect at X and Y. The line joining the centres OQ intersect the common chord XY at Z.

Then XZ = ZY and XY ⊥ OQ. (This is proven using congruent triangles.)

12.10.2 Point of contact of two circles

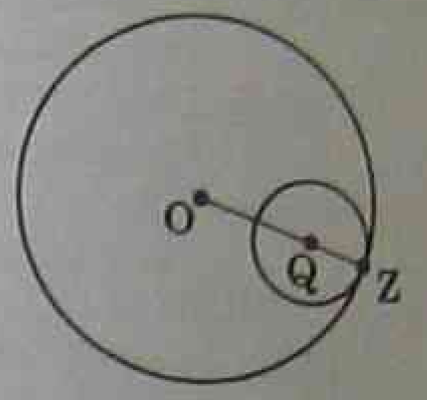
When two circles touch, their centres and the point of contact are collinear.



Two circles with centres O and Q touch at Z.

OQ is a straight line and Z lies on OQ (or OQ produced), that is, O, Q and Z are collinear.

(This is proved by drawing the common tangent at Z.)



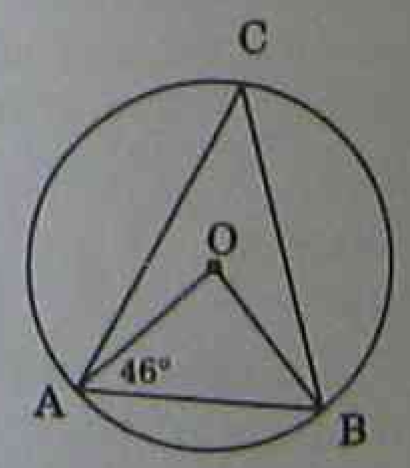
12.11 Worked examples using deductive reasoning

- Always draw a clear diagram.
- Mark on the diagram all the given information.
- Work from the known towards the unknown.
- If you become lost, check that you have used all the data.
- Any known facts can be used — you cannot use the fact that you are asked to prove.

In all diagrams, O is the centre of the circle.

Examples

(a) ∠OAB = 46°. Find ∠ACB.



SOLUTION

AO = OB (equal radii)

$\therefore \angle ABO = 46°$

(base angles of isosceles Δ)

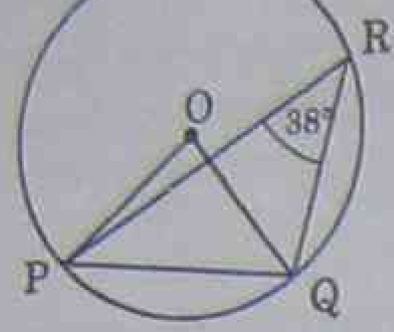
$\therefore \angle AOB = 180° - (46° + 46°)$

$= 88°$ (angle sum of Δ)

$\therefore \angle ACB = 44°$

(angle at circumference = $\frac{1}{2}$ ∠ at centre on arc AB)

(b)

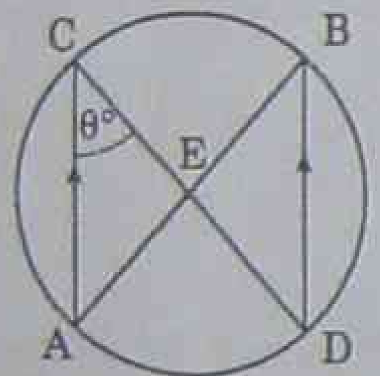


$\angle PRQ = 38°$,
$\angle OQR = 47°$.
Find $\angle OPR$.

SOLUTION

$\angle POQ = 76°$
($\angle$ at centre $= 2 \times \angle$ at circumference on arc PQ)
But $OP = OQ$ (equal radii)
$\therefore \angle OPQ = \angle OQP$
(base angles, isosc. Δ)
$\angle OQP = \angle OPQ$
$= \frac{1}{2}(180° - 76°)$
$= 52°$
Then $\angle PQR = \angle OQP + \angle OQR$
$= 52° + 47°$
$= 99°$
Then $\angle RPQ = 43°$ ($\angle$ sum of ΔPRQ)
But $\angle OPR = \angle OPQ - \angle RPQ$
$= 52° - 43°$
$= 9°$

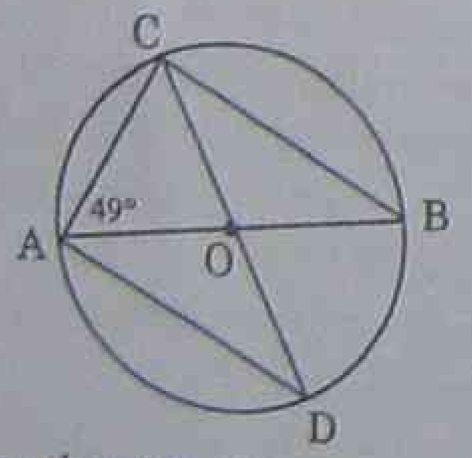
(c)



AC || BD. Prove that $\angle ACE = \angle CAE$.

SOLUTION
Let $\angle ACE = \theta°$
$\therefore \angle EDB = \theta°$
(alternate angles, AC || BD),
and $\angle CAB = \theta°$
(angles in the same segment on arc CB)
Then $\angle ACE = \angle CAE$ (both $\theta°$)

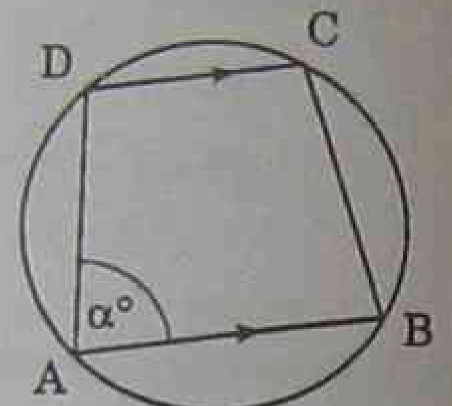
(d)



Given that AB, CD are diameters, $\angle CAB = 49°$. Find $\angle ADC$.

SOLUTION
$AO = OC$ (equal radii)
$\angle ACO = 49°$
(base angles of isosceles Δ)
But $\angle ACB = 90°$
(angle in a semicircle)
$\therefore \angle OCB = 41°$ $(90° - 49°)$
Also, $OC = OB$ (equal radii)
$\therefore \angle OBC = 41°$
(base angles of isosceles Δ),
and $\angle ADC = 41°$
(angles in the same segment)

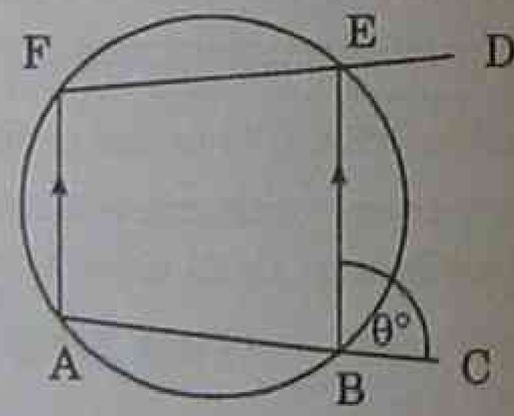
(e)



Given that AB || DC and ABCD is a cyclic quadrilateral, prove that $\angle ADC = \angle BCD$.

SOLUTION
Let $\angle DAB = \alpha°$
$\therefore \angle ADC = (180 - \alpha)°$
(co-interior angles, AB || CD),
and $\angle DCB = (180 - \alpha)°$
(opposite angles of a cyclic quad'l)
$\therefore \angle ADC = \angle BCD$ [both $(180 - \alpha)°$]

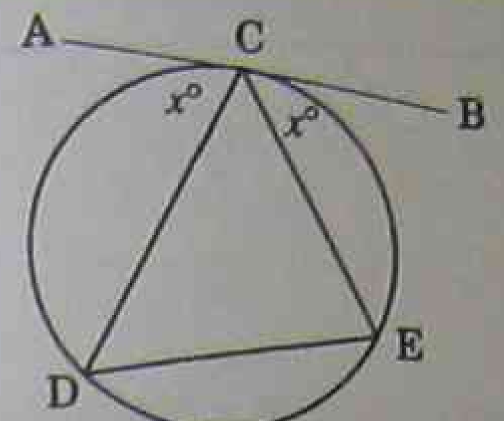
(f)



Given that AF || BE, and ABEF is a cyclic quad'l, prove that $\angle CBE = \angle DEB$.

SOLUTION
Let $\angle EBC = \theta°$
$\therefore \angle FAB = \theta°$
(corresponding angles, AF || BE),
and $\angle DEB = \theta°$
(External $\angle$ of a cyclic quad'l)
$\therefore \angle CBE = \angle DEB$ (both $\theta°$)

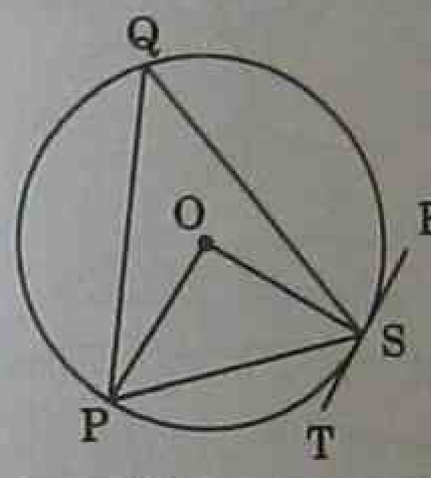
(g)



Given that AB is tangent at C and $\angle ACD = \angle BCE$, prove that DE || AB.

SOLUTION
Let $\angle ACD = x°$
Then $\angle BCE = x°$ (data)
But $\angle BCE = \angle CDE$
(angle in the alternate segment)
$\therefore \angle CDE = x°$
Then $\angle ACD = \angle CDE$ (both $x°$)
$\therefore$ AB || DE
(a pair of alternate angles are equal).

(h)

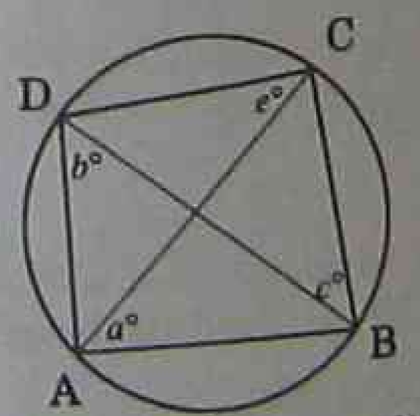


Given that TR is a tangent at S, prove that $\angle POS = 2 \times \angle PST$.

SOLUTION

Let $\angle PST = \theta°$
Now $\angle PQS = \angle PST$
(angle in alternate segment)
$\therefore \angle PQS = \theta°$
But $\angle POS = 2 \times \angle PQS = 2\theta°$
(angle at centre $= 2 \times$ angle at circum. on arc PS)
$\therefore \angle POS = 2 \times \angle PST$

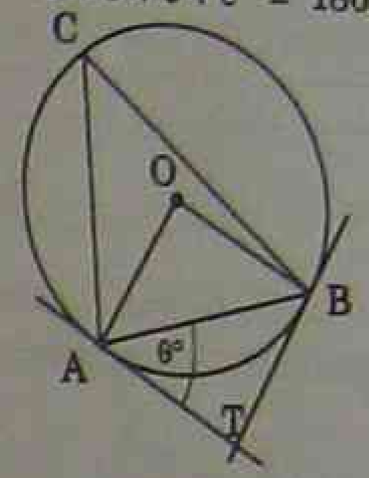
(i)



Given that ABCD is a cyclic quadrilateral and $\angle CAB = a°$, $\angle ADB = b°$, $\angle DBC = c°$, $\angle ACD = e°$, prove that $a + b + c + e = 180°$.

SOLUTION
$\angle BDC = a°$
(angles in same segment)
and $\angle ABD = e°$
(angles in same segment).
But $\angle ADC + \angle ABC = 180°$
(opposite angles of a cyclic quadrilateral)
$\therefore a + b + c + e = 180$

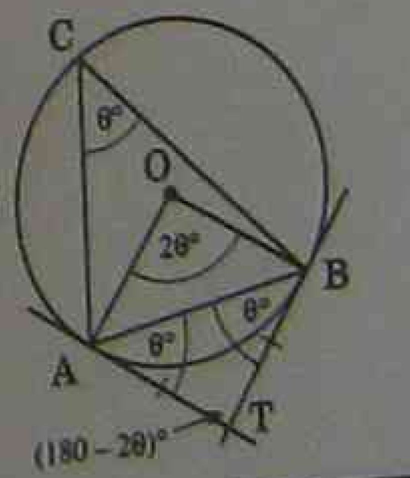
(j)



Given that TA and TB are tangents to a circle centre O and $\angle TAB = \theta°$, prove that ATBO is a cyclic quadrilateral.

SOLUTION
$TA = TB$
(tangents from an external point)
$\therefore \angle TBA = \theta°$
(base angles of isosceles Δ)
Then $\angle ATB = 180° - 2\theta°$
(angle sum of Δ)
But $\angle TAB = \angle ACB$
(angle in alternate segment)
$\therefore \angle ACB = \theta°$
and $\angle AOB = 2\theta°$ (angle at centre $= 2 \times$ angle at circum. on arc AB)
Now
$\angle AOB + \angle ATB = 2\theta° + 180° - 2\theta° = 180°$
$\therefore$ ATBO is cyclic quadrilateral
(opp. angles supplementary).

This is what your diagram should look like before you attempt to write down any steps of your working:



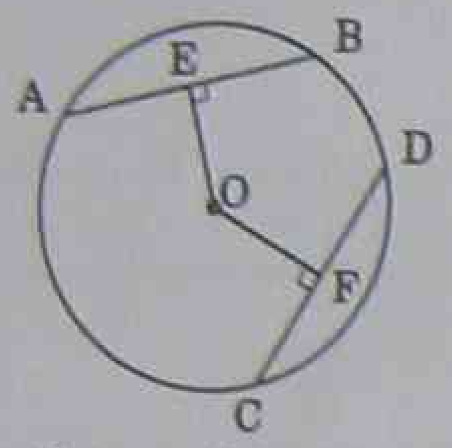
Remember:
- Make sure all data are used.
- Mark information on the diagram.
- Mark angles on the diagram as you find them.
- Work from the known towards the unknown.

To do all this you must have a clearly drawn diagram.

12.12 Exercises

(*Note:* When O is used it always indicates the centre of the circle.)

1.



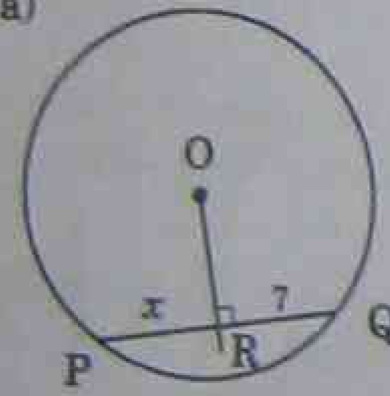
Prove that equal chords are equidistant from the centre, that is, given AB = CD, prove that OE = OF, where OE ⊥ AB and OF ⊥ CD.

Construction: Join OB and OD.

Hint: Use congruent triangles.

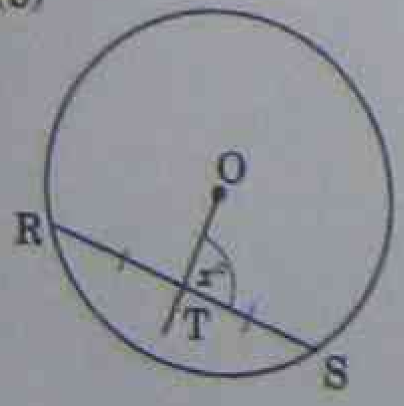
2. Find the value of the pronumeral in the following questions. (All lengths are in cm.) Give adequate reasons.

(a)



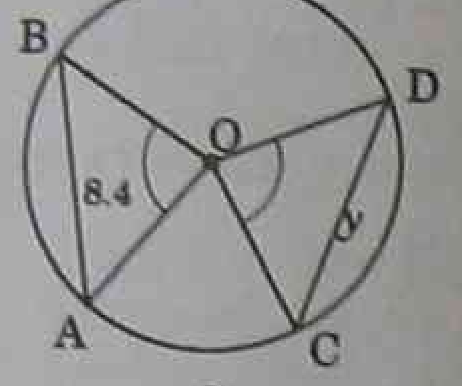
Given: RQ = 7 cm.

(b)



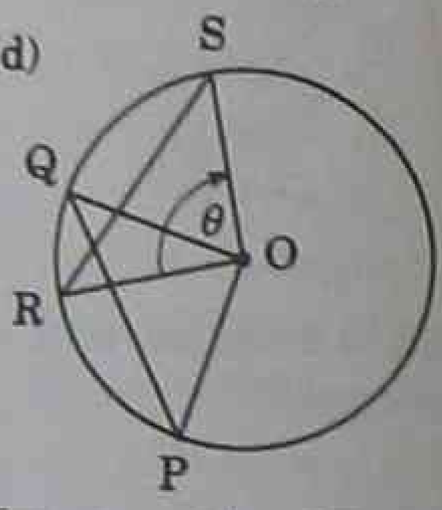
Given: RT = TS.

(c)



Given: ∠AOB = ∠COD, AB = 8.4 cm.

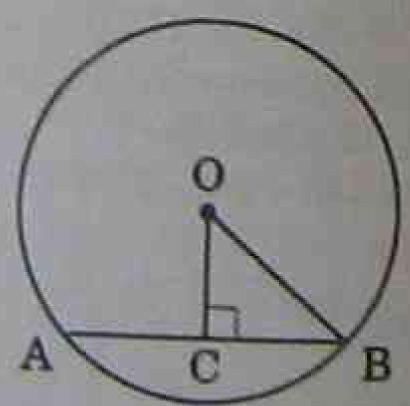
(d)



Given: PQ = RS, ∠POQ = 110°, ∠ROS = θ°.

3. Use properties of the circle and Pythagoras' Theorem in the following questions:

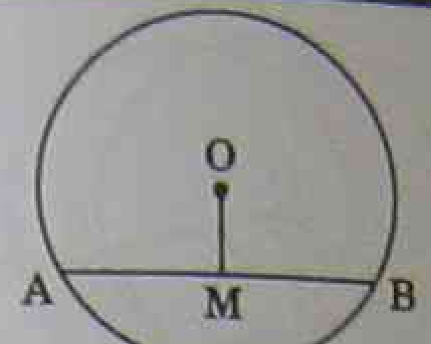
(a)



AB is a 16 cm chord of a circle with radius 10 cm. Find the length OC.

Note: OC ⊥ AB.

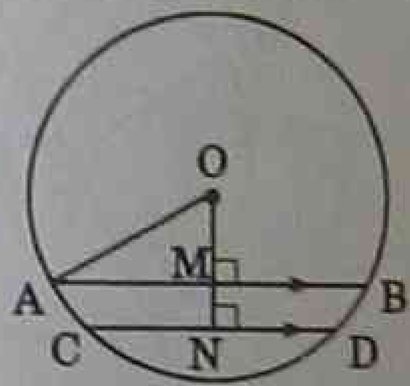
(b)



Given that AM = MB, OM = 10 cm, and the circle has radius 26 cm, find the length of chord AB.

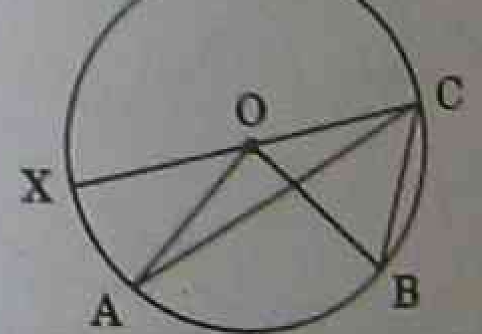
(c) A chord 15 cm long is 4 cm from the centre of a circle. Calculate the radius of the circle correct to one decimal place.

(d) Two parallel chords 32 cm long and 24 cm long are drawn in a circle of radius 20 cm as shown in the figure. Find the distance between the two chords if they are both on the same side of the centre.



Given AB ∥ CD, AB = 32 cm, CD = 24 cm, OA = 20 cm. Find the length MN.

4.

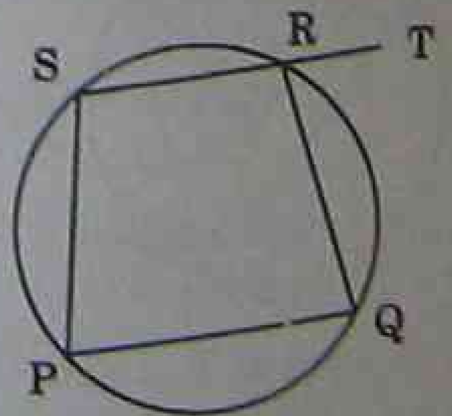


Given that O is the centre of the circle, prove that ∠AOB = 2 × ∠ACB.

Construction: Join CO and produce it to X.

Let ∠OAC = $x°$, ∠OBC = $y°$ and remember that radii are equal.

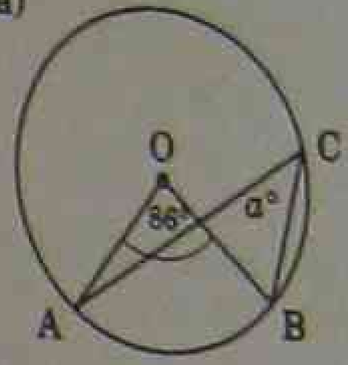
5.



Given that PQRS is a cyclic quadrilateral, prove that ∠SPQ = ∠QRT.

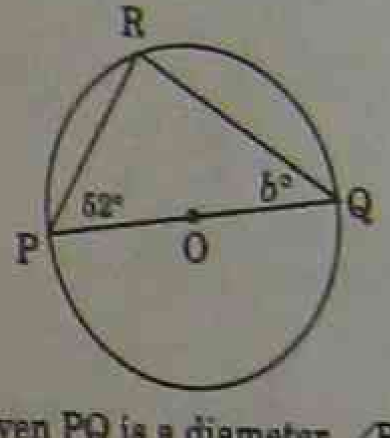
6. Find the value of the pronumeral(s) in each of the following diagrams, giving adequate reasons in each case.

(a)



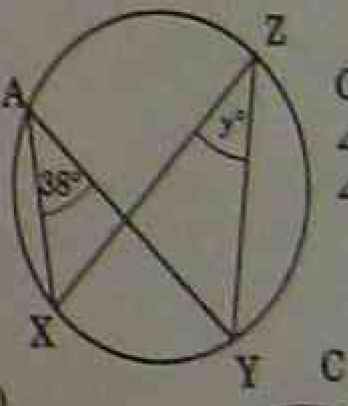
Given ∠AOB = 86°, ∠ACB = $a°$.

(b)



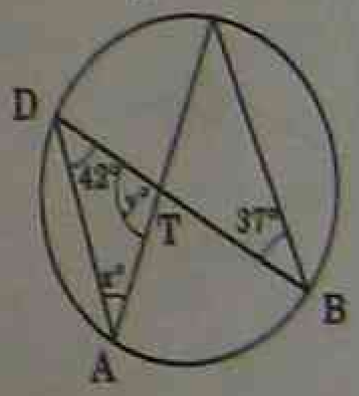
Given PQ is a diameter, ∠RPQ = 52°, ∠RQP = $b°$.

(c)



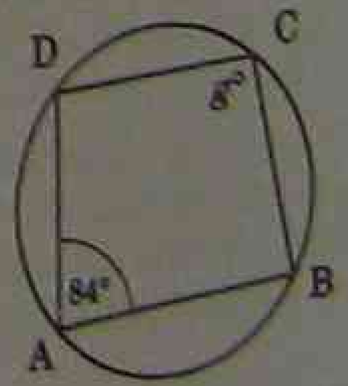
Given ∠XAY = 38°, ∠XZY = $y°$.

(d)



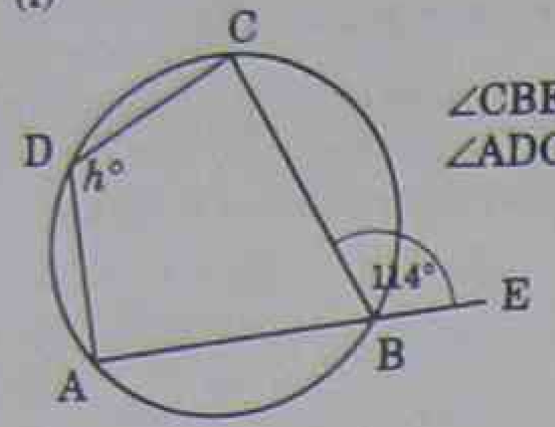
Given ∠ADB = 42°, ∠DBC = 37°, ∠DAC = $x°$, ∠DTA = $y°$.

(e)



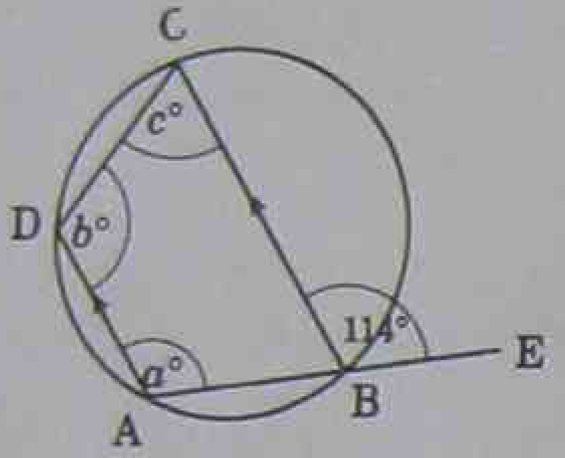
Given ∠DAB = 84°, ∠DCB = $g°$.

(f)



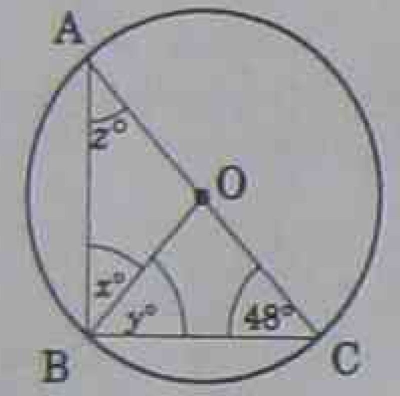
∠CBE = 114°, ∠ADC = h°.

(g)



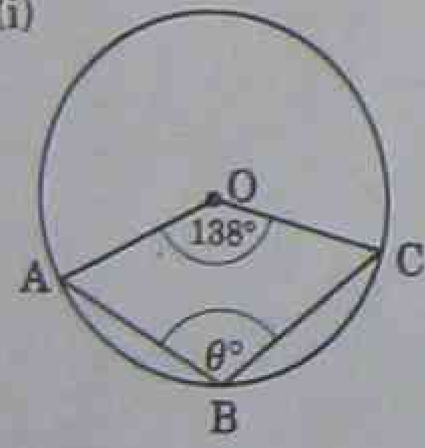
AD ∥ BC, ∠CBE = 114°, ∠BAD = a°, ∠ADC = b°, ∠DCB = c°.

(h)



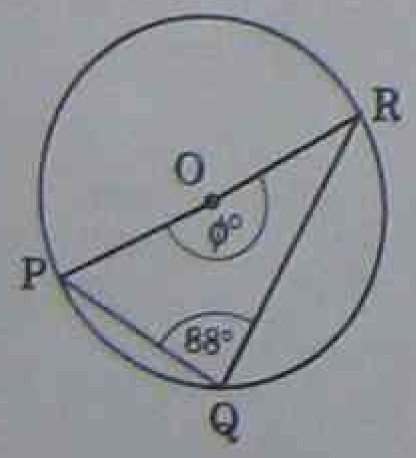
AC is a diameter, ∠BCO = 48°, ∠OBC = y°, ∠ABO = x°, ∠BAC = z°.

(i)



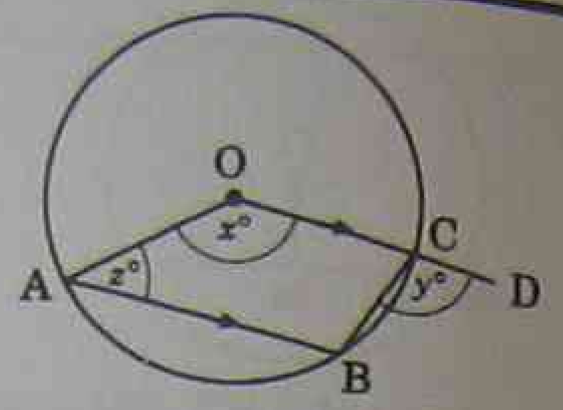
∠AOC = 138°, ∠ABC = θ°.

(j)



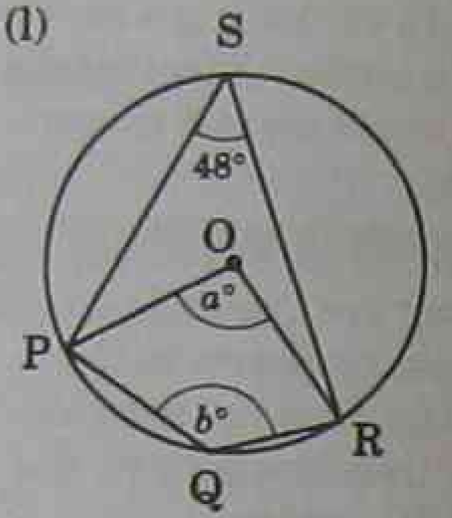
∠PQR = 88°, ∠POR = φ°.

(k)



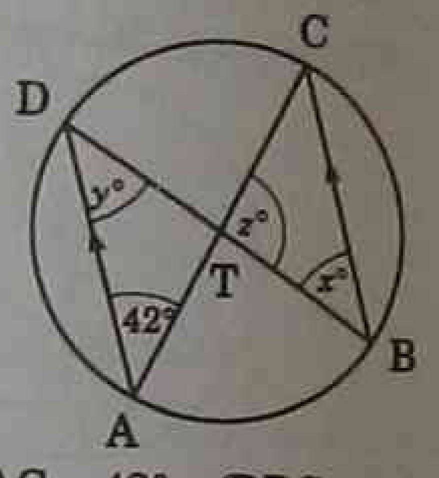
AB ∥ OD, ∠ABC = 124°, ∠AOC = x°, ∠BCD = y°, ∠BAO = z°.

(l)



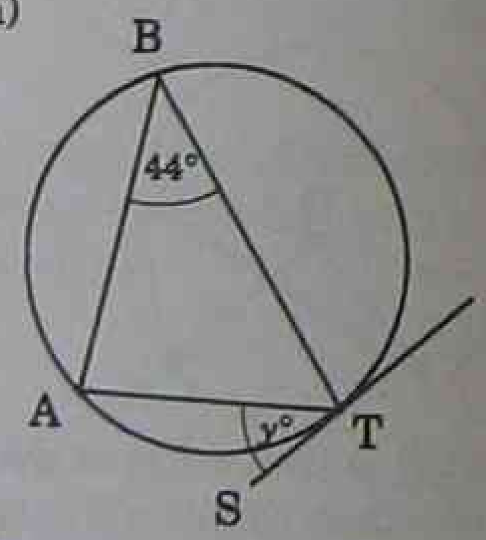
∠PSR = 48°, ∠POR = a°, ∠PQR = b°.

(m)



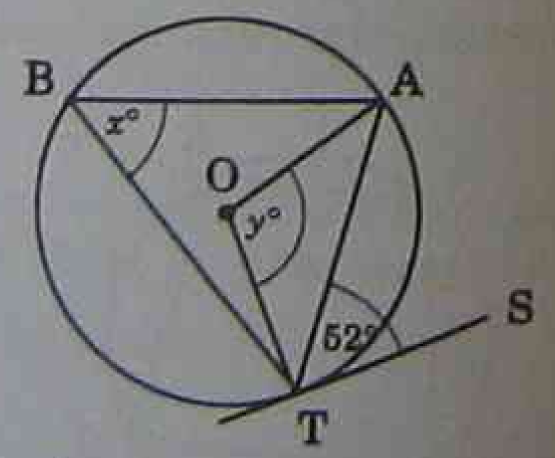
AB ∥ BC, ∠DAC = 42°, ∠DBC = x°, ∠ADB = y°, ∠CTB = z°.

(n)



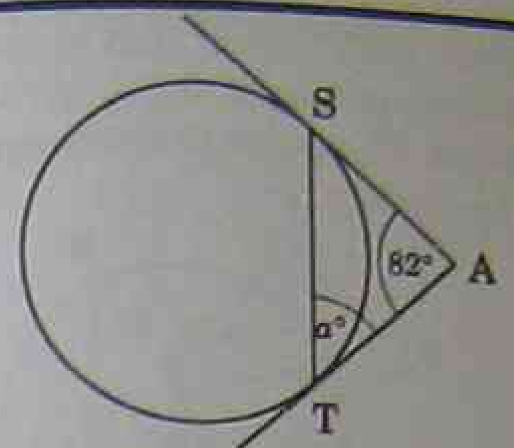
ST is a tangent at T, ∠ABT = 44°, ∠ATS = y°.

(o)



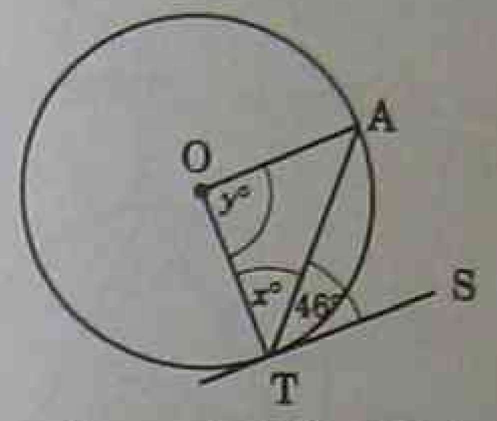
ST is a tangent at T, ∠ATS = 52°, ∠TBA = x°, ∠TOA = y°.

(p)



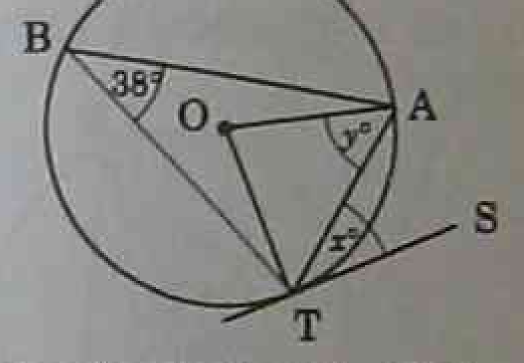
AS and AT are tangents at S and T, ∠ SAT = 82°, ∠STA = a°.

(q)



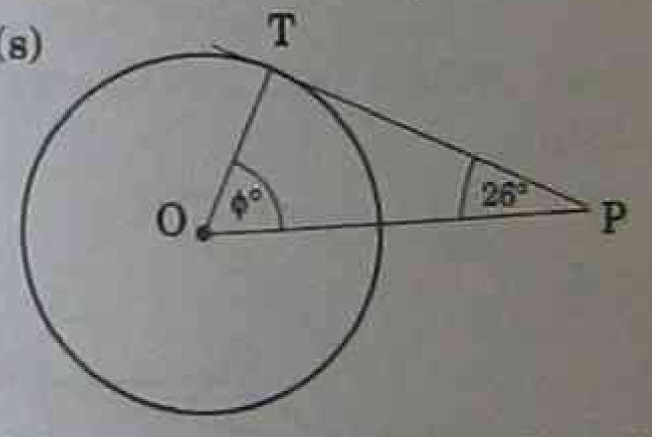
ST is a tangent at T, ∠ATS = 46°, ∠OTA = x°, ∠TOA = y°.

(r)



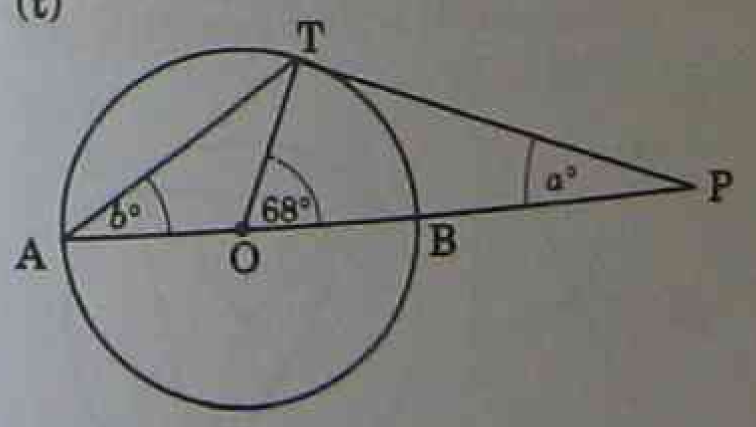
ST is a tangent at T, ∠TBA = 38°, ∠ATS = x°, ∠OAT = y°.

(s)



PT is a tangent at T, ∠OPT = 26°, ∠TOP = φ°.

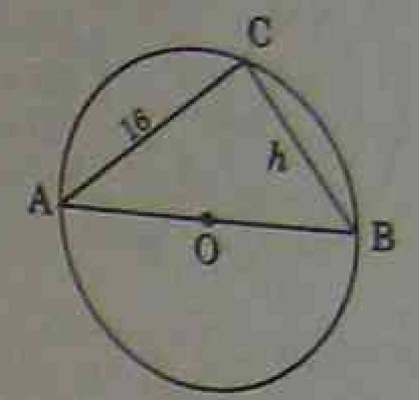
(t)



PT is a tangent at T, AB is a diameter, ∠TOB = 68°, ∠TPB = a°, ∠TAO = b°.

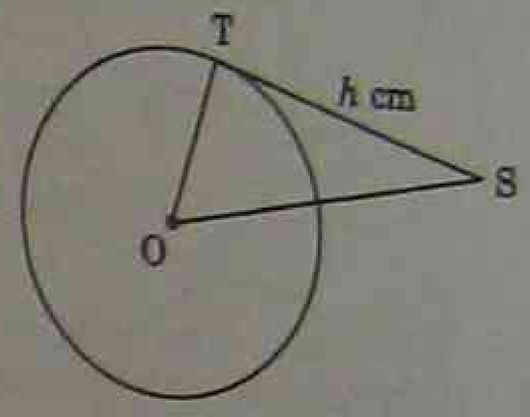
7. Find the value of h in each diagram. All measurements are in cm.

(a)



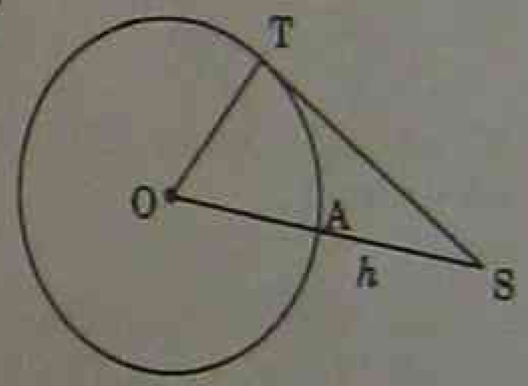
AB is the diameter of a circle of radius 10 cm, while AC = 16 cm, and BC = h cm.

(b)



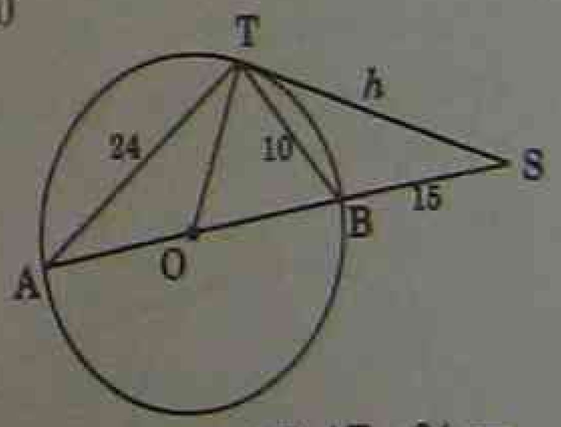
ST is a tangent at T to the circle of radius 15 cm. If OS is 25 cm, calculate h.

(c)



ST is a tangent at T to the circle of radius 2.7 cm. Given that ST = 3.6 cm and AS = h cm, calculate h.

(d)

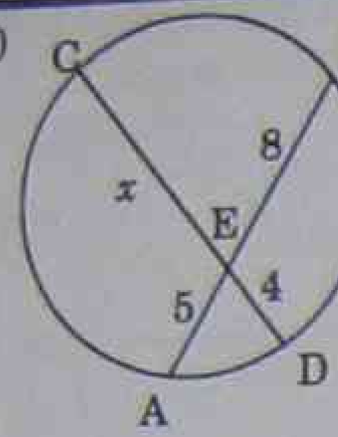


ST is a tangent at T. AT = 24 cm, BT = 10 cm, BS = 15 cm and ST = h cm. Calculate h correct to one decimal place.

8. Calculate the value of the pronumeral in each diagram. All lengths are in cm. Answer correct to one decimal place where necessary.

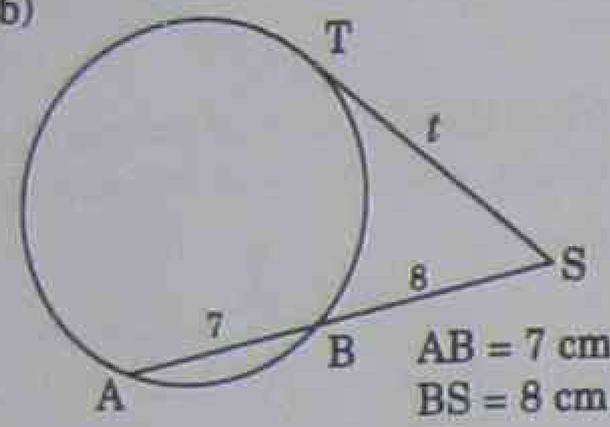
Continued

(a)



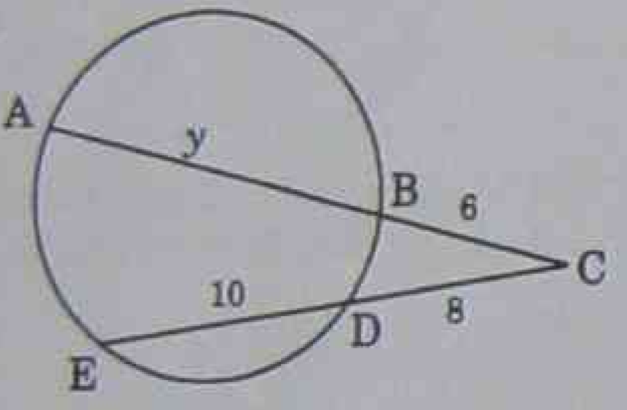
AE = 5 cm,
EB = 8 cm,
ED = 4 cm,
EC = x cm.

(b)



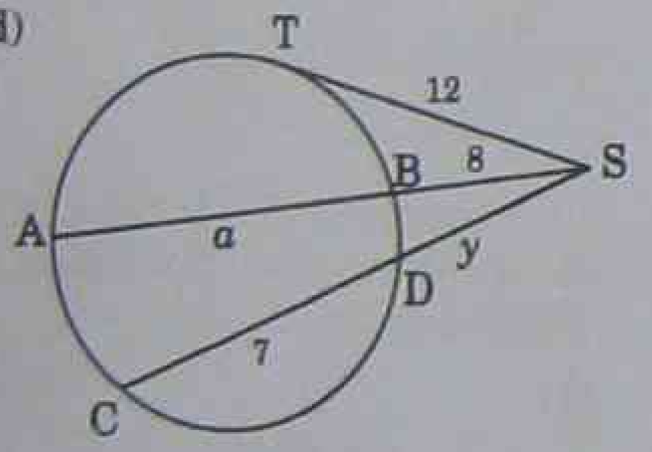
AB = 7 cm,
BS = 8 cm,
TS = t cm.

(c)



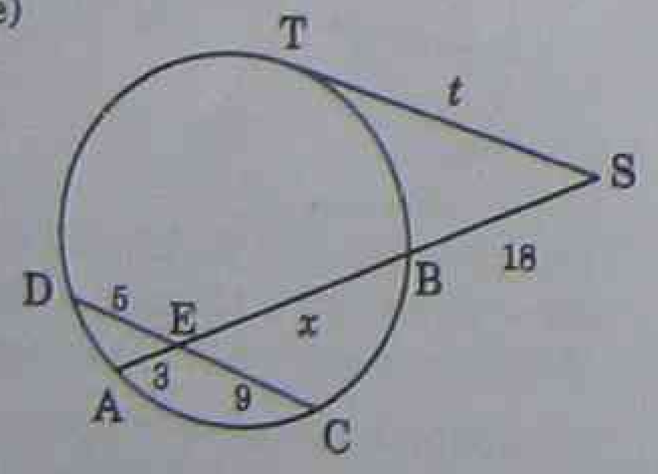
BC = 6 cm, DC = 8 cm, ED = 10 cm, AB = y cm.

(d)



ST = 12 cm, SB = 8 cm, AB = a cm, CD = 7 cm, DS = y cm.

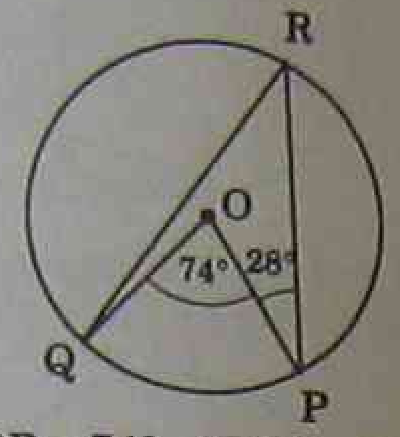
(e)



ST is a tangent at T. ED = 5 cm, EA = 3 cm, EC = 9 cm, EB = x cm, BS = 18 cm, TS = t cm. (Leave t in exact form.)

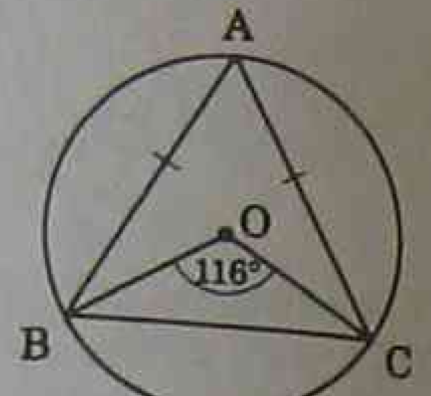
9. Answer each part of this question with a logical sequence of steps, giving adequate reasons. The point O is always the centre of the circle.

(a)



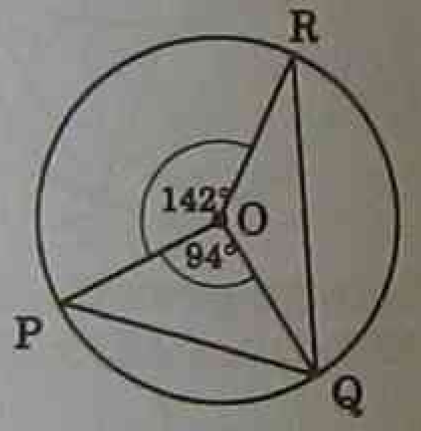
Given $\angle QOP = 74^\circ$, $\angle OPR = 28^\circ$, find $\angle RQO$.

(b)



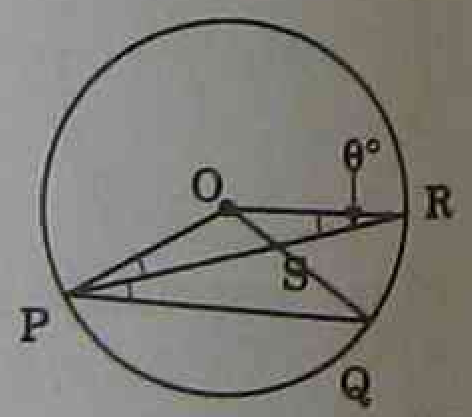
Given AB = AC, $\angle BOC = 116^\circ$, find $\angle ABO$.

(c)



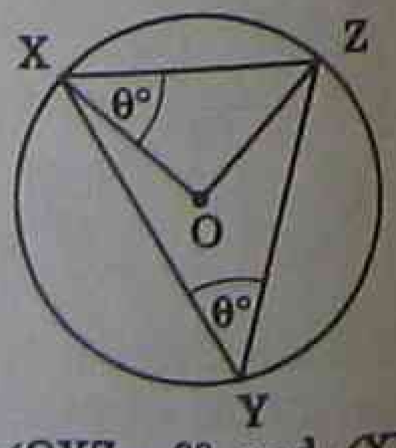
Given $\angle POR = 142^\circ$, $\angle POQ = 94^\circ$, find $\angle ORQ$.

(d)



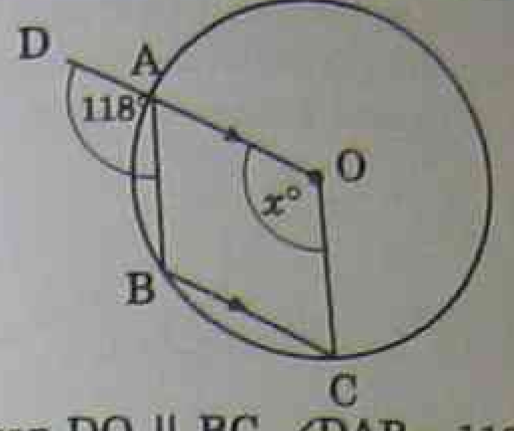
Given $\angle OPR = \angle QPR$, $\angle SRO = \theta^\circ$, prove $\angle ROS = 2\theta^\circ$.

(e)



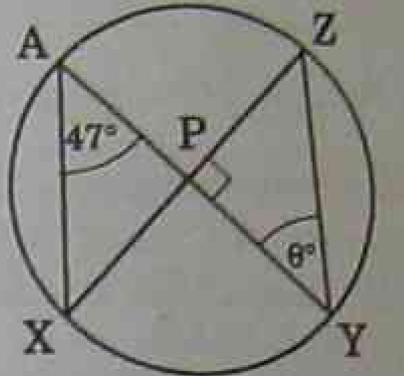
Given that $\angle OXZ = \theta^\circ$, and $\angle XYZ = \theta^\circ$, prove that $\angle XOZ = 90^\circ$.

(f)



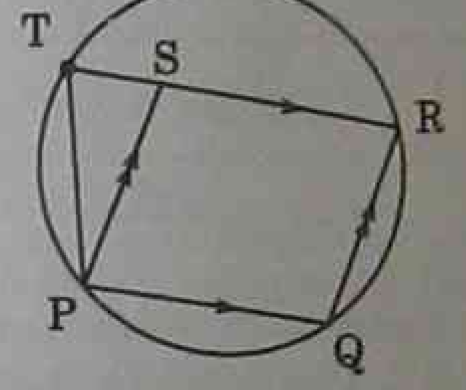
Given DO ∥ BC, $\angle DAB = 118^\circ$, find x.

(g)



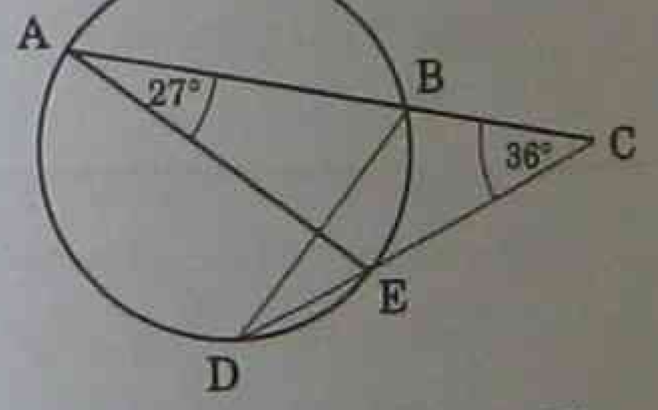
Given $\angle XAY = 47^\circ$, $\angle YPZ = 90^\circ$, find θ.

(h)



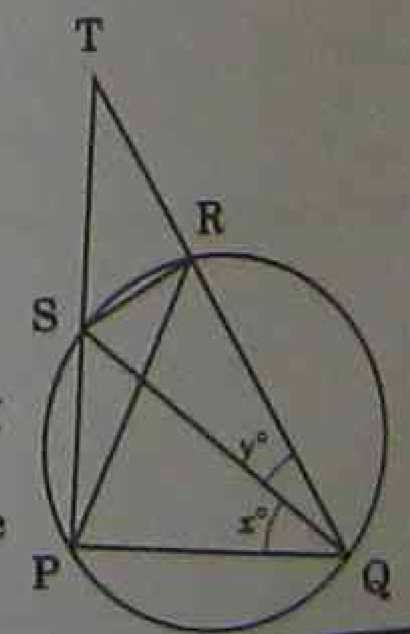
PQRS is a parallelogram, and PQRT is a cyclic quadrilateral. Prove that PT = PS. (Call $\angle PQR = \alpha^\circ$.)

(i)



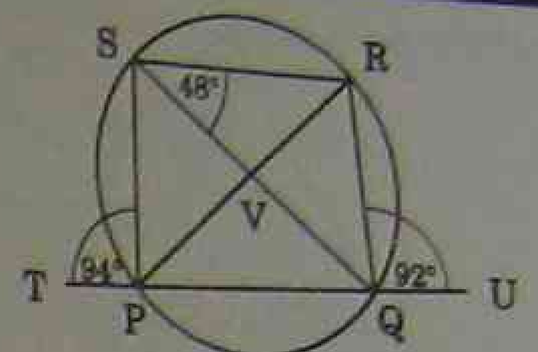
Given $\angle BAE = 27^\circ$, $\angle DCA = 36^\circ$, find $\angle DBC$.

(j)



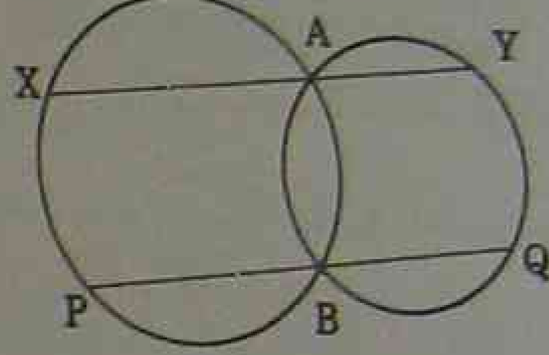
Given PR = PQ, $\angle PQS = x^\circ$, $\angle SQR = y^\circ$, prove that $\angle STR = x^\circ$.

(k)



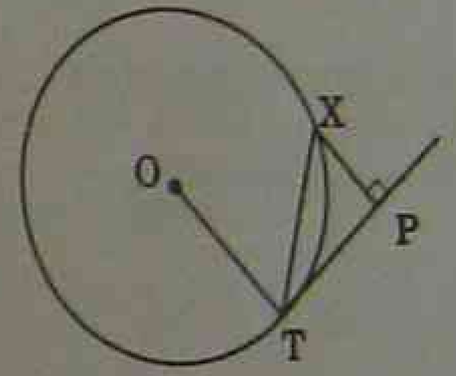
Given $\angle TPS = 94^\circ$, $\angle RQU = 92^\circ$, $\angle QSR = 48^\circ$, find $\angle SVR$.

(l)



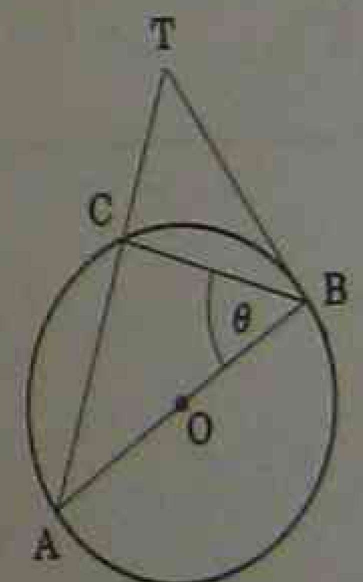
Circles intersect at A and B. XY and PQ are straight lines. Prove that PX ∥ QY.

(m)



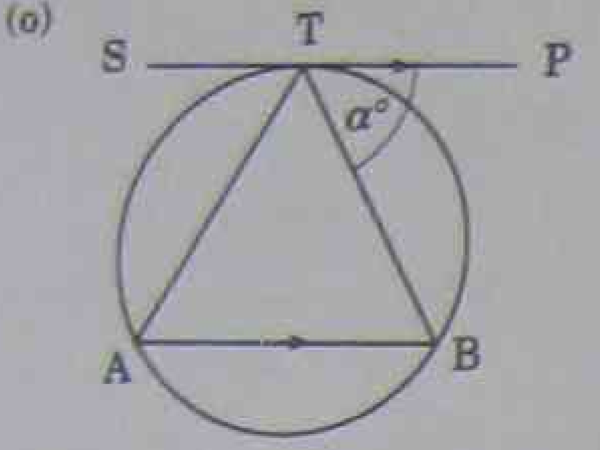
Given that PT is a tangent at T and XP ⊥ PT, prove that $\angle OTX = \angle TXP$.

(n)



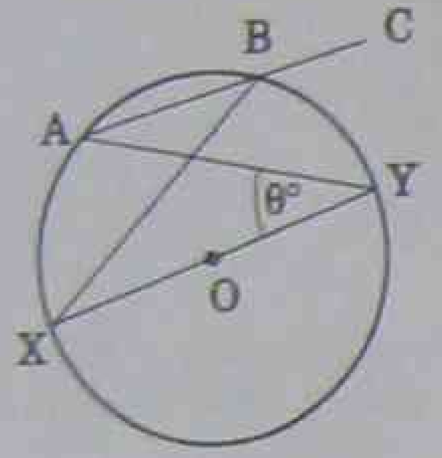
Given that TB is a tangent at B and $\angle ABC = \theta^\circ$. AT is a straight line. Prove that $\angle ATB = \theta^\circ$.

(o)



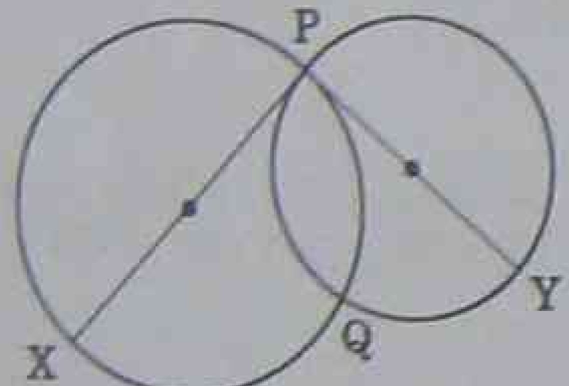
Given that SP ∥ AB, PT is a tangent at T and ∠PTB = α°, prove that ΔABT is isosceles.

(p)



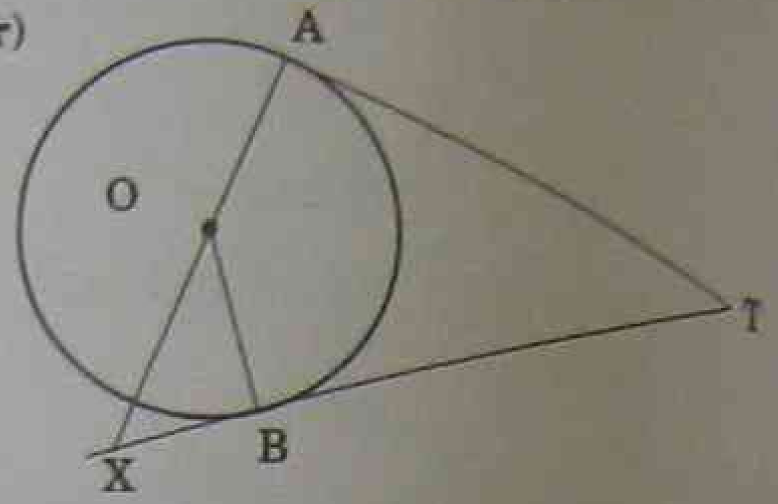
Given that AC is a straight line, XY is a diameter and ∠XYA = θ°, prove that ∠XYA + ∠YBC = 90°.

(q)



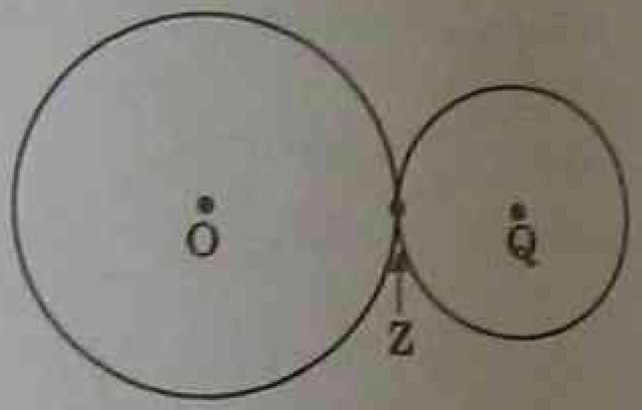
Two circles intersect at P and Q. PX and PY are the respective diameters. Prove that X, Q and Y are collinear.

(r)



TA and TB are tangents at A and B respectively. AX is a straight line. Prove that ∠BOX = ∠ATB and that ∠ATB = 2 × ∠OAB.

(s)



Two circles with centres O and Q intersect at Z. Prove that O, Q and Z are collinear.

(*Hint*: Draw the common tangent through Z. Join OZ and QZ.)

Chapter 13.

LOGARITHMS AND FUNCTIONS

13.1 Indices (Refer to Chapter 3 for greater detail)

13.1.1 Index laws

- $x^a \times x^b = x^{a+b}$
- $x^a \div x^b = x^{a-b}$
- $\left(x^a\right)^b = x^{ab}$

13.1.2 Special indices

(a) **Zero index**

Any non-zero number when raised to zero power will give an answer of 1.

$x^0 = 1,\ (x \neq 0),$

$2^0 = 1,\ 5^0 = 1,\ \left(\frac{3}{4}\right)^0 = 1$

(b) **Fractional index**

Index $\left(\frac{1}{n}\right)$ means the n^th root.

- $x^{\frac{1}{2}} = \sqrt{x}, \quad 4^{\frac{1}{2}} = \sqrt{4} = 2$
- $x^{\frac{1}{3}} = \sqrt[3]{x}, \quad 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
- $x^{\frac{2}{3}} = \sqrt[3]{x^2}$, or $x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$ $\boxed{x^{\frac{2}{3}} = \left(x^2\right)^{\frac{1}{3}}, \text{ or } = \left(x^{\frac{1}{3}}\right)^2}$
- $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

[It is usually easier to take the root, then calculate the power.]

(c) **Negative indices**

The minus sign in the index indicates that the reciprocal should be taken.

- $x^{-n} = \frac{1}{x^n}$
- $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
- $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
- $a^{-\frac{1}{n}} = \frac{1}{a^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{a}}$
- $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$
- $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{27}\right)^2} = \frac{1}{3^2} = \frac{1}{9}$

Note that the number part of the index is unchanged.

13.1.3 Tables of powers

In this chapter it will be useful to have drawn up the powers of the following numbers. Use your calculator to check these tables.

Powers of 2	
2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1 024
2^{11}	2 048
2^{12}	4 096

Powers of 3	
3^0	1
3^1	3
3^2	9
3^3	27
3^4	81
3^5	243
3^6	729
3^7	2 187
3^8	6 561

Powers of 5	
5^0	1
5^1	5
5^2	25
5^3	125
5^4	625
5^5	3 125
5^6	15 625

Powers of 10	
10^0	1
10^1	10
10^2	100
10^3	1 000
10^4	10 000
10^5	100 000
10^6	1 000 000

13.1.4 Using the index laws

Examples

(a) Simplify:

(i) $\frac{10^{-2} \times 10^5}{10^2}$ (ii) $16^{\frac{3}{4}}$

(iii) $125^{-\frac{1}{3}}$

SOLUTION

(i) $\frac{10^{-2} \times 10^5}{10^2} = 10^{-2+5-2} = 10^1 = 10$

(ii) $16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = 2^3 = 8$

(iii) $125^{-\frac{1}{3}} = \frac{1}{125^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$

(b) Solve for x:

(i) $2^x = 64$ (ii) $3^{x+2} = 81$

(iii) $2^{2x-1} = 3125$

① Express both sides as a power of the same base number.

② Then the indices must be equal.

SOLUTION

(i) $2^x = 2^6$ Both as powers of two

$\therefore \; x = 6$ Indices are equal

(ii) $3^{x+2} = 3^4$ Both as powers of 3.

$\therefore \; x + 2 = 4$ Indices are equal

$x = 2$

(iii) $25^{2x-1} = 3125$

$\left(5^2\right)^{2x-1} = 5^5$

$5^{4x-2} = 5^5$ Powers of 5

$\therefore \; 4x - 2 = 5$

$4x = 7$

$x = \frac{7}{4}$

(c) By using the tables of powers, simplify the following calculations:

(i) 27×243 (ii) $\frac{125 \times 3125}{625}$

(iii) $\sqrt{16 \times 32 \times 128}$ (iv) $(32)^2$

SOLUTION

(i) $3^3 \times 3^5 = 3^8 = 6561$

(ii) $\frac{5^3 \times 5^5}{5^4} = \frac{5^8}{5^4} = 5^4 = 625$

(iii) $\sqrt{2^4 \times 2^5 \times 2^7} = \sqrt{2^{16}}$

$= \left(2^{16}\right)^{\frac{1}{2}}$

$= 2^8$

$= 256$

(iv) $\left(2^5\right)^2 = 2^{10} = 1024$

13.2 Definition of a logarithm

The logarithm (log) of a number is the index or the power to which the base must be raised to give that number.

If $N = a^x$, ⇐ where a is the base, x is the index

then $\log_a N = x$ ⇐ where a is the base of the logarithm

Note:

1. Base of index becomes base of log.
2. Index becomes subject of log equation.
3. Number N goes into the logarithm.

or

If $\log_a N = x$
then $N = a^x$

You must be able to change readily from index form to logarithm form and vice versa.

The following statements are identical:

- If $2^5 = 32$, then $\log_2 32 = 5$.
- If $5^4 = 625$, then $\log_5 625 = 4$.
- If $10^3 = 1000$, then $\log_{10} 1000 = 3$.

Examples

(a) Write in logarithm form:

(i) $3^6 = 729$ (ii) $2^{-2} = \frac{1}{4}$

SOLUTION

(i) If $3^6 = 729$, then $\log_3 729 = 6$.

(ii) If $2^{-2} = \frac{1}{4}$, then $\log_2\left(\frac{1}{4}\right) = -2$.

(b) Write in index form:

(i) $\log_{10} 10\,000 = 4$ (ii) $\log_3\left(\frac{1}{27}\right) = -3$

SOLUTION

(i) If $\log_{10} 10\,000 = 4$, then $10^4 = 10\,000$.

(ii) If $\log_3\left(\frac{1}{27}\right) = -3$, then $3^{-3} = \frac{1}{27}$.

13.2.1 Logarithmic equations

Logarithmic equations are solved by changing to equivalent index expressions and solving the index equation.

Examples

(a) Solve the logarithmic equations:

(i) $\log_2 256 = x$ (ii) $\log_3 x = 2$

(iii) $\log_x\left(\frac{1}{8}\right) = -3$

SOLUTION

(i) If $\log_2 256 = x$,
then $2^x = 256$,
that is, $2^x = 2^8$
$x = 8$

Remember:
1. Express in index form with the same base.
2. Equate indices.

(ii) If $\log_3 x = 2$,
then $3^2 = x$,
$x = 9$

(iii) If $\log_x\left(\frac{1}{8}\right) = -3$,
then $x^{-3} = \frac{1}{8}$
$x^{-3} = 2^{-3}$
$x = 2$

13.3 Exponential and logarithm graphs

13.3.1 Using your calculator

Your calculator can be used to evaluate powers, both integral and fractional. The button to be used will have:

$\boxed{x^y}$ or $\overset{x^y}{\boxed{}}$

This relates to Casio calculators.

and $\boxed{x^{\frac{1}{y}}}$ or $\overset{x^{\frac{1}{y}}}{\boxed{}}$

To calculate 4^3:

PRESS 4 $\boxed{x^y}$ 3 $\boxed{=}$

or 4 $\boxed{\text{INV}}$ $\overset{x^y}{\boxed{}}$ 3 $\boxed{=}$

To calculate $2^{\frac{1}{3}}$:

PRESS 2 $\boxed{x^{\frac{1}{y}}}$ 3 $\boxed{=}$

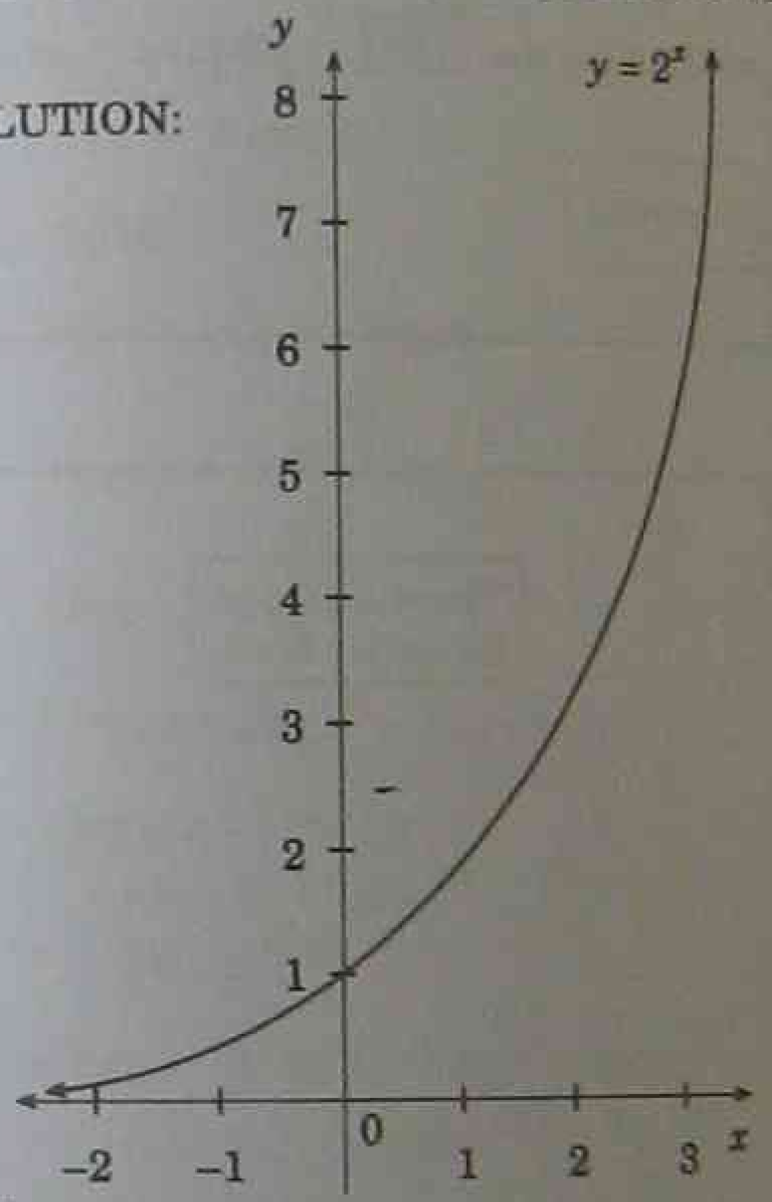
or 2 $\boxed{\text{INV}}$ $\overset{x^{\frac{1}{y}}}{\boxed{}}$ $\boxed{=}$

13.3.2 Exponential graphs

Graph $y = 2^x$ by first completing a table of values:

x	−2	−1	0	1	2	3
y						

SOLUTION:



x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Using $y = 2^x$

$x = -2,\ y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$x = 0,\ y = 2^0 = 1$

$2^0 = 1$, but so is 3^0, 4^0, etc.

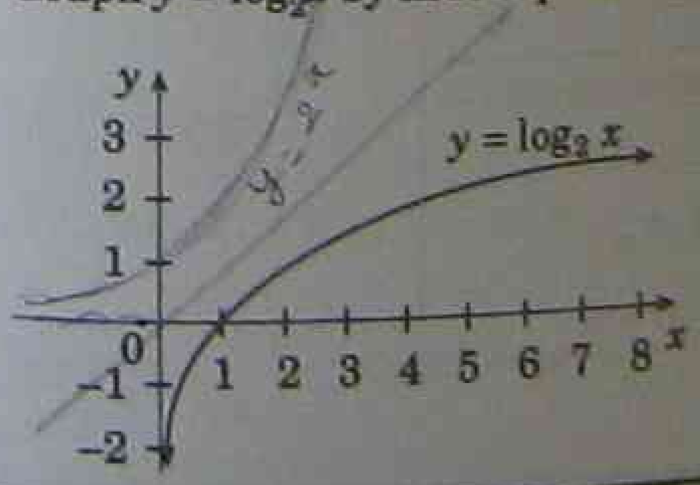
Notes:

1. The graph cuts the y-axis at 1.
2. All y-values are greater than 0.
3. The graph runs along the x-axis but does not cut the x-axis.

These are properties of all exponential graphs of the form: $y = a^x$ $(a > 1)$

13.3.3 Logarithmic graphs

Graph $y = \log_2 x$ by first expressing this as an index equation.



If $y = \log_2 x$, then $x = 2^y$.

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	−2	−1	0	1	2	3

This time we give y-values and calculate the x-values.

Using $x = 2^y$,

when $y = -2$, $x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$y = 0$, $x = 2^0 = 1$

$\log_2 1 = 0$ because $2^0 = 1$.
Similarly:
$\log_3 1 = 0$, $\log_8 1 = 0$, etc.

Notes

1. The graph cuts x-axis at 1.
2. All x-values are greater than zero.
3. The graph runs along the y-axis but does not cut the y-axis.

These are properties of all logarithmic graphs of the form: $y = \log_a x \quad (a > 0)$

$y = 2^x$ and $y = \log_2 x$ are inverse relationships.

The graph of $y = \log_{10} x$ is readily obtained by using the LOG button on your calculator.

To calculate $\log_{10} 0.8$:

Press 0.8 LOG

The answer is now generated.

= not needed

13.4 Laws for logarithms

The laws for logarithms are as follows:

1. $\log_a MN = \log_a M + \log_a N$

These relate to the index laws.

For example,

$\log_2 3 \times 5 = \log_2 3 + \log_2 5$, or

$\log_{10} 7 + \log_{10} 3 = \log_{10} 21$

2. $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

For example,

$\log_3\left(\frac{20}{4}\right) = \log_3 20 - \log_3 4$, or

$\log_{10} 63 - \log_{10} 9 = \log_{10}\left(\frac{63}{9}\right) = \log_{10} 7$

3. $\log_a M^r = r\log_a M$

For example,

$\log_3 4^2 = 2\log_3 4$, or

$\frac{1}{2}\log_5 16 = \log_5 16^{\frac{1}{2}} = \log_5 4$

$16^{\frac{1}{2}} = \sqrt{16}$

There are also the following important cases:

4. $\log_a 1 = 0$

For example,

$\log_5 1 = 0$

$\log_{10} 1 = 0$

$\log_x 1 = 0$

This corresponds to
$5^0 = 1$
$10^0 = 1$
$x^0 = 1$

5. $\log_a a = 1$

For example,

$\log_2 2 = 1$

$\log_{10} 10 = 1$

$\log_x x = 1$

This corresponds to
$2^1 = 2$
$10^1 = 10$
$x^1 = x$

Examples

(a) Simplify (contract) the following:

(i) $\log_2 x + \log_2 y$ (ii) $\log_a S - \log_a T$
(iii) $4\log_a x$ (iv) $\frac{1}{2}\log_2 x$
(v) $3\log_{10} x + 4\log_{10} y$
(vi) $\frac{1}{2}\log_a x - 2\log_a y$

SOLUTION

(i) $\log_2 x + \log_2 y = \log_2 xy$

(ii) $\log_a S - \log_a T = \log_a\left(\frac{S}{T}\right)$

(iii) $4\log_a x = \log_a x^4$

(iv) $\frac{1}{2}\log_2 x = \log_2 x^{\frac{1}{2}} = \log_2 \sqrt{x}$

(v) $3\log_{10} x + 4\log_{10} y$
$= \log_{10} x^3 + \log_{10} y^4$
$= \log_{10} x^3 y^4$

(vi) $\frac{1}{2}\log_a x - 2\log_a y = \log_a x^{\frac{1}{2}} - \log_a y^2$
$= \log_a \sqrt{x} - \log_a y^2$
$= \log_a\left(\frac{\sqrt{x}}{y^2}\right)$

(b) Expand the following expressions:

(i) $\log_2 ab$ (ii) $\log_x\left(\frac{p}{q}\right)$
(iii) $\log_3 a^4$ (iv) $\log_{10} \sqrt[3]{t}$
(v) $\log_n\left(\frac{a^4}{b^2}\right)$ (vi) $\log_e\left[\frac{\sqrt{xy^3}}{z^2}\right]$

SOLUTION

(i) $\log_2 ab = \log_2 a + \log_2 b$

(ii) $\log_x\left(\frac{p}{q}\right) = \log_x p - \log_x q$

(iii) $\log_3 a^4 = 4\log_3 a$

(iv) $\log_{10} \sqrt[3]{t} = \log_{10} t^{\frac{1}{3}} = \frac{1}{3}\log_{10} t$

(v) $\log_n\left(\frac{a^4}{b^2}\right) = \log_n a^4 - \log_n b^2$
$= 4\log_n a - 2\log_n b$

(vi) $\log_e\left[\frac{\sqrt{xy^3}}{z^2}\right]$
$= \log_e\left[\frac{\left(xy^3\right)^{\frac{1}{2}}}{z^2}\right]$
$= \log_e\left[\frac{\left(x^{\frac{1}{2}}y^{\frac{3}{2}}\right)}{z^2}\right]$
$= \log_e x^{\frac{1}{2}} + \log_e y^{\frac{3}{2}} - \log_e z^2$
$= \frac{1}{2}\log_e x + \frac{3}{2}\log_e y - 2\log_e z$

(c) Evaluate the following expressions:

(i) $\log_3 27$ (ii) $\log_a a^2$ (iii) $\log_2 \sqrt{2}$
(iv) $\log_n \sqrt[3]{n}$ (v) $\log_3\left(\frac{1}{3}\right)$ (vi) $\log_3\left(\frac{1}{81}\right)$
(vii) $\log_2 4\sqrt{2}$
(viii) $\log_a\left(a^2 + a\right) - \log_a(a + 1)$
(ix) $\log_{10} b + \log_{10}\left(\frac{1}{b}\right)$
(x) $\log_a[\log_e e]$
(xi) $\log_2 18 - \log_2 9$
(xii) $\log_{10} 250 - \log_{10} 4 + \log_{10} 16$

SOLUTION

Remember: $\log_a a = 1$, $\log_2 2 = 1$, etc.

(i) $\log_3 27 = \log_3 3^3 = 3\log_3 3 = 3$

(ii) $\log_a a^2 = 2\log_a a = 2$

(iii) $\log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}\log_2 2 = \frac{1}{2}$

(iv) $\log_n \sqrt[3]{n} = \log_n n^{\frac{1}{3}} = \frac{1}{3}\log_n n = \frac{1}{3}$

(v) $\log_3\left(\frac{1}{3}\right) = \log_3 3^{-1} = -1\log_3 3 = -1$

(vi) $\log_3\left(\frac{1}{81}\right) = \log_3\left(\frac{1}{3^4}\right) = \log_3 3^{-4}$
$= -4\log_3 3 = -4$

(vii) $\log_2 4\sqrt{2} = \log_2 2^2 \cdot 2^{\frac{1}{2}} = \log_2 2^{2.5}$
$= 2.5\log_2 2 = 2.5$

(viii) $\log_a\left(a^2 + a\right) - \log_a(a+1)$
$= \log_a\left(\frac{a^2+a}{a+1}\right) = \log_a \frac{a\cancel{(a+1)}}{\cancel{(a+1)}}$
$= \log_a a = 1$

(ix) $\log_{10} b + \log_{10}\left(\frac{1}{b}\right) = \log_{10} b + \log_{10} b^{-1}$
$= \log_{10} b - \log_{10} b$
$= 0$

(x) $\log_a[\log_e e] = \log_a 1 = 0$

(xi) $\log_2 18 - \log_2 9 = \log_2\left(\frac{18}{9}\right) = \log_2 2 = 1$

(xii) $\log_{10} 250 - \log_{10} 4 + \log_{10} 16$
$= \log_{10}\left[\frac{250 \times \cancel{16}}{\cancel{4}}\right]$
$= \log_{10} 1000$
$= \log_{10} 10^3$
$= 3\log_{10} 10$
$= 3$

(d) If $p = \log_a 2$ and $q = \log_a 5$, then write in terms of p and q:

(i) $\log_a 8$ (ii) $\log_a \sqrt{5}$ (iii) $\log_a 10$
(iv) $\log_a 0.4$ (v) $\log_a 0.2$ (vi) $\log_a 50$
(vii) $\log_a \sqrt{50}$ (viii) $\log_a 40$
(ix) $\log_a\left(\frac{8}{5}\right)$ (x) $\log_a 0.625$

SOLUTION

(i) $\log_a 8 = \log_a 2^3 = 3\log_a 2 = 3p$

(ii) $\log_a \sqrt{5} = \log_a 5^{\frac{1}{2}} = \frac{1}{2}\log_a 5 = \frac{1}{2}q$

(iii) $\log_a 10 = \log_a(2 \times 5)$
$= \log_a 2 + \log_a 5 = p + q$

(iv) $\log_a 0.4 = \log_a\left(\frac{2}{5}\right)$
$= \log_a 2 - \log_a 5 = p - q$

(v) $\log_a 0.2 = \log_a\left(\frac{1}{5}\right) = \log_a 5^{-1}$
$= -1\log_a 5 = -q$

(vi) $\log_a 50 = \log_a(2 \times 25) = \log_a\left(2 \times 5^2\right)$
$= \log_a 2 + \log_a 5^2$
$= \log_a 2 + 2\log_a 5$
$= p + 2q$

(vii) $\log_a \sqrt{50} = \log_a 50^{\frac{1}{2}} = \frac{1}{2}\log_a 50$
$= \frac{1}{2}(p + 2q) = \frac{1}{2}p + q$

(viii) $\log_a 40 = \log_a(8 \times 5)$
$= \log_a 8 + \log_a 5$
$= \log_a 2^3 + \log_a 5$
$= 3\log_a 2 + \log_a 5$
$= 3p + q$

(ix) $\log_a\left(\frac{8}{5}\right) = \log_a 8 - \log_a 5$
$= \log_a 2^3 - \log_a 5$
$= 3\log_a 2 - \log_a 5$
$= 3p - q$

(x) $\log_a 0.625 = \log_a\left(\frac{5}{8}\right) = \log_a 5 - \log_a 8$
$= \log_a 5 - \log_a 2^3$
$= \log_a 5 - 3\log_a 2$
$= q - 3p$

(e) If $a = \log_{10} 3$ and $b = \log_{10} 4$, write simple log expressions for:

(i) $a + b$ (ii) $a - b$ (iii) $\frac{1}{2}b$
(iv) $3a$ (v) $3a + 2b$ (vi) $2a - b$
(vii) Show that $2a + b - 1 = \log_{10} 3.6$

SOLUTION

(i) $a + b = \log_{10} 3 + \log_{10} 4 = \log_{10}(3 \times 4)$
$= \log_{10} 12$

(ii) $a - b = \log_{10} 3 - \log_{10} 4 = \log_{10}\left(\frac{3}{4}\right)$
$= \log_{10} 0.75$

(iii) $\frac{1}{2}b = \frac{1}{2}\log_{10} 4 = \log_{10}\sqrt{4} = \log_{10} 2$

(iv) $3a = 3\log_{10} 3 = \log_{10} 3^3 = \log_{10} 27$

(v) $3a + 2b = 3\log_{10} 3 + 2\log_{10} 4$
$= \log_{10} 3^3 + \log_{10} 4^2$
$= \log_{10} 27 + \log_{10} 16$
$= \log_{10}(27 \times 16)$
$= \log_{10} 432$

(vi) $2a - b = 2\log_{10} 3 - \log_{10} 4$
$= \log_{10} 3^2 - \log_{10} 4$
$= \log_{10} 9 - \log_{10} 4$
$= \log_{10}\left(\frac{9}{4}\right)$
$= \log_{10} 2.25$

$\boxed{\text{Note this step}}$

(vii) $2a + b - 1 = 2\log_{10} 3 + \log_{10} 4 - \log_{10} 10$
$= \log_{10} 3^2 + \log_{10} 4 - \log_{10} 10$
$= \log_{10} 9 + \log_{10} 4 - \log_{10} 10$
$= \log_{10}\left(\frac{9 \times 4}{10}\right) = \log_{10}\left(\frac{36}{10}\right)$
$= \log_{10} 3.6$

(f) Write an expression without logarithms for y, given:

(i) $\log_a y = \frac{1}{2}\log_a x$
(ii) $\log_a y = \log_a x - \log_a z$
(iii) $\log_a y = \log_a a - 2\log_a x$
(iv) $\log_a y = \frac{1}{2}\log_a x - 2\log_a z$
(v) $\log_a y = \log_a x - 1$

SOLUTION

(i) $\log_a y = \frac{1}{2}\log_a x$
$= \log_a \sqrt{x}$
$\therefore \quad y = \sqrt{x}$

(ii) $\log_a y = \log_a x - \log_a z$
$= \log_a\left(\frac{x}{z}\right)$
$\therefore \quad y = \frac{x}{z}$

(iii) $\log_a y = \log_a a - 2\log_a x$
$= \log_a a - \log_a x^2$
$= \log_a\left(\frac{a}{x^2}\right)$
$\therefore \quad y = \frac{a}{x^2}$

(iv) $\log_a y = \frac{1}{2}\log_a x - 2\log_a z$
$= \log_a x^{\frac{1}{2}} - \log_a z^2$
$= \log_a \sqrt{x} - \log_a z^2$
$= \log_a\left(\frac{\sqrt{x}}{z^2}\right)$
$\therefore \quad y = \frac{\sqrt{x}}{z^2}$

(v) $\log_a y = \log_a x - 1$
$= \log_a x - \log_a a \leftarrow$ $\boxed{\text{Note again}}$
$= \log_a\left(\frac{x}{a}\right)$
$\therefore \quad y = \frac{x}{a}$

(g) Solve for x:

(i) $\log_2 x = \log_2 5 + \log_2 3$
(ii) $\log_2 24 - \log_2 8 = \log_2 x$
(iii) $\log_2 3x + \log_2 7 = \log_2 42$
(iv) $2\log_2 3 + \log_2 x = 3\log_2 3$
(v) $\log_2(x+1) + \log_2(x+3) = 3$
(vi) $\log_2(x+4) - \log_2(x-2) = 1$

SOLUTION

(i) $\log_2 x = \log_2 5 + \log_2 3$
$= \log_2(5 \times 3)$
$= \log_2 15$
$\therefore \quad x = 15$

(ii) $\log_2 24 - \log_2 8 = \log_2 x$
$\therefore \quad \log_2\left(\frac{24}{8}\right) = \log_2 x$
$\therefore \quad \log_2 3 = \log_2 x$
$\therefore \quad x = 3$

(iii) $\log_2 3x + \log_2 7 = \log_2 42$
$\log_2(3x \times 7) = \log_2 42$
$\log_2 21x = \log_2 42$
$\therefore \quad 21x = 42$
$\therefore \quad x = 2$

(iv) $2\log_2 3 + \log_2 x = 3\log_2 3$
$\log_2 3^2 + \log_2 x = \log_2 3^3$
$\therefore \quad \log_2 9 + \log_2 x = \log_2 27$
$\therefore \quad \log_2 9x = \log_2 27$
$9x = 27$
$\therefore \quad x = 3$

(v) $\log_2(x+1) + \log_2(x+3) = 3$
$\therefore \quad \log_2(x+1)(x+3) = 3\log_2 2$
$\log_2\left(x^2 + 4x + 3\right) = \log_2 2^3$
$= \log_2 8$
$\therefore \quad x^2 + 4x + 3 = 8$
$x^2 + 4x - 5 = 0$
$(x+5)(x-1) = 0$
$\therefore \quad x = 1 \text{ or } -5.$

Now $x = -5$ is inadmissible as $\log_2 -4$ or $\log_2 -2$ do not exist,
$\therefore \quad x = 1$

(vi) $\log_2(x+4) - \log_2(x-2) = 1$
$\therefore \quad \log_2\left(\frac{x+4}{x-2}\right) = \log_2 2$
$\therefore \quad \frac{x+4}{x-2} = 2$
$x + 4 = 2x - 4$
$8 = x$
$\therefore \quad x = 8$

13.5 Change of base

The rule for change of base is:

$$\boxed{\log_a N = \frac{\log_{10} N}{\log_{10} a}}$$

This rule allows the base of the logarithm to be changed to allow simplification of the expression or calculation of the logarithm. Remember that the calculator only has the capacity to calculate $\boxed{\log_{10}}$ written as $\boxed{\log}$, or $\boxed{\log_e}$, natural logarithms, sometimes written $\boxed{\ln}$.

Much more will be said about $\boxed{\log_e}$ in Years 11 and 12.

For example, $\log_2 3 = \dfrac{\log_{10} 3}{\log_{10} 2}$

$$\log_5 10 = \frac{\log_{10} 10}{\log_{10} 5} = \frac{1}{\log_{10} 5}$$

We are using $\boxed{\log_{10}}$ on the calculator.

Examples

(a) Calculate correct to four decimal places

(i) $\log_2 3$ (ii) $\log_5 10$

SOLUTION

(i) $\log_2 3 = \dfrac{\log_{10} 3}{\log_{10} 2} = 1.584\,9625$

≈ 1.5850

(correct to four decimal places)

Calculator:

3 $\boxed{\text{LOG}}$ $\boxed{+}$ 2 $\boxed{\text{LOG}}$ $\boxed{=}$

(ii) $\log_5 10 = \dfrac{\log_{10} 10}{\log_{10} 5} = \dfrac{1}{\log_{10} 5}$

$= 1.430\,6766$

≈ 1.4307

(correct to four decimal places)

Calculator:

5 $\boxed{\text{LOG}}$ $\boxed{\text{INV}}$ $\boxed{\frac{1}{x}}$

13.5.1 Further exponential equations

Examples

Solve for x, giving the answer correct to three decimal places:

(i) $4^x = 10$

(ii) $4^x = 5^{2x-1}$

SOLUTION

(i) **Method 1**

Change to log expression

$$4^x = 10$$
$$\Rightarrow x = \log_4 10$$
$$= \frac{\log_{10} 10}{\log_{10} 4}$$
$$= 1.660\,964$$
$$\approx 1.661$$

Method 2

Take logs of both sides. Here logs to base 10 are taken.

$$4^x = 10$$
$$\log_{10} 4^x = \log_{10} 10$$
$$x \log_{10} 4 = 1$$
$$\therefore \quad x = \frac{1}{\log_{10} 4}$$
$$\approx 1.661$$

(ii) For this type of expression we must use Method 2.

$4^x = 5^{2x-1}$ (take logs of both sides)

Continued

$$\log_{10} 4^x = \log_{10} 5^{2x-1}$$
$$\therefore \quad x \log_{10} 4 = (2x-1)\log_{10} 5$$
$$x \log_{10} 4 = 2x \log_{10} 5 - \log_{10} 5$$
$$x \log_{10} 4 - 2x \log_{10} 5 = -\log_{10} 5$$
$$x(\log_{10} 4 - 2\log_{10} 5) = -\log_{10} 5$$
$$\therefore \quad x = \frac{-\log_{10} 5}{\log_{10} 4 - 2\log_{10} 5}$$
$$= 0.878\,2354$$
$$\approx 0.878$$

Calculator:

5 $\boxed{\text{LOG}}$ $\boxed{+/-}$ ÷ $\boxed{(}$

4 $\boxed{\text{LOG}}$ − 2 $\boxed{\times}$ 5

$\boxed{\text{LOG}}$ $\boxed{)}$ $\boxed{=}$

13.6 Functions

13.6.1 Function definition

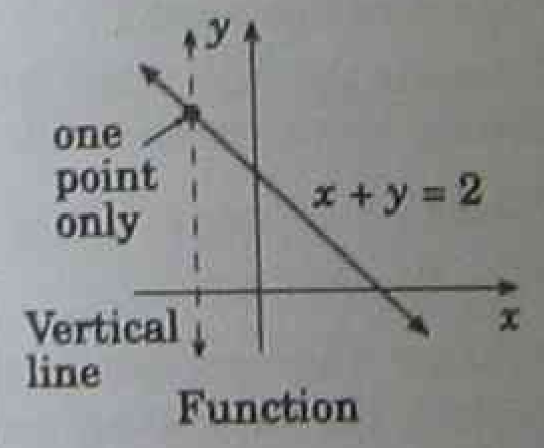
A **function** is a rule or relationship where for each input there is only one output, that is, for each x value there is only one y value

13.6.2 Vertical line test

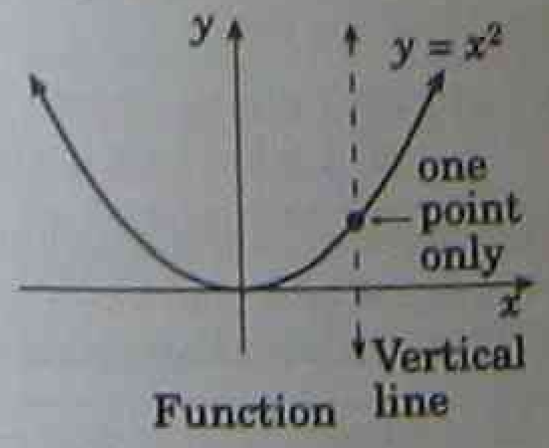
If a vertical line can cut the graph of a relationship at more than one point then it is **not** a function.

Examples

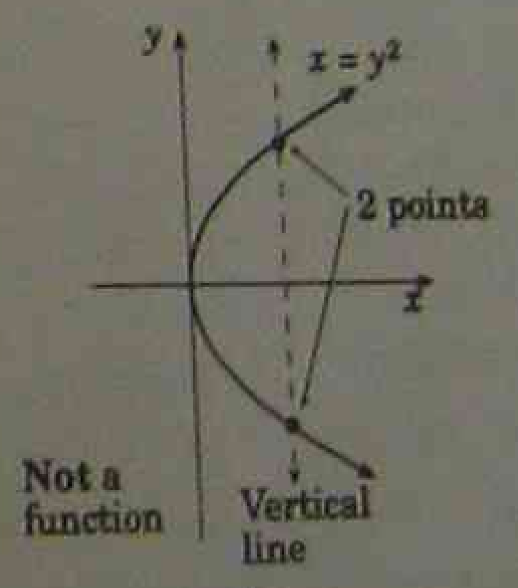
(a)



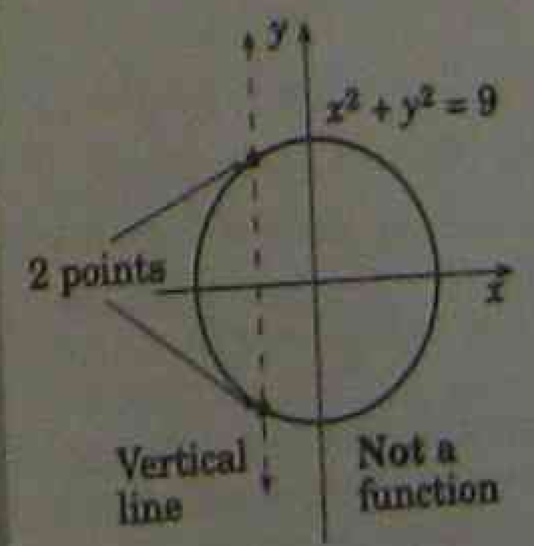
(b)



(c)



(d)



13.6.3 Function notation

The notation $f(x)$ is often used to indicate a function. For example,

$f(x) = x^2 + x$ | Means only that $y = x^2 + x$ is as function |

$f(x) = 7 - 2x$

The notation used does not always have to be $f(x)$ (although it usually is) — $F(x)$, $\phi(x)$, $G(x)$, $H(x)$, $C(t)$ and many others can be used. For example.

$$C(t) = t^2 - 4$$

defines a function of t is $C(t) = t^2 - 4$.

The notation implies substitution into a function. For example, if $f(x) = x^2 - 4$ evaluate $f(2)$. This is the same question as 'given $y = x^2 - 4$ calculate the value of y when $x = 2$'. The notation simplifies the expression of the question.

$$f(x) = x^2 - 4$$

$$\therefore \quad f(2) = (2)^2 - 4 = 4 - 4 = 0$$

Examples

(a) For $f(x) = 3x - 5$ find the following:

(i) $f(2)$
(ii) $f(-2)$
(iii) $f(a)$
(iv) $f(a + 2)$
(v) $f(4a)$
(vi) $f(2 - a^2)$
(vii) $f(x - 1)$

SOLUTION

(i) $f(2) = 3(2) - 5 = 6 - 5 = 1$

(ii) $f(-2) = 3(-2) - 5 = -6 - 5 = -11$

(iii) $f(a) = 3(a) - 5 = 3a - 5$

(iv) $f(a + 2) = 3(a + 2) - 5 = 3a + 6 - 5$
$= 3a + 1$

(v) $f(4a) = 3(4a) - 5 = 12a - 5$

(vi) $f(2 - a^2) = 3(2 - a^2) - 5$
$= 6 - 3a^2 - 5 = 1 - 3a^2$

(vii) $f(x - 1) = 3(x - 1) - 5 = 3x - 3 - 5$
$= 3x - 8$

(b) A stone is thrown from the top of a tower such that the distance of the stone above ground level is a function of the elapsed time. The function is given by $f(t) = 60 + 5t - 5t^2$, where t is the time in seconds and $f(t)$ the distance in metres.

(i) What does the value $f(0)$ represent?

(ii) What does $f(0.5)$ represent? What does this value mean?

(iii) Find the height of the stone after 1 second, 2 seconds and 3 seconds. What does this mean?

(iv) Calculate the time taken for the stone to reach ground level.

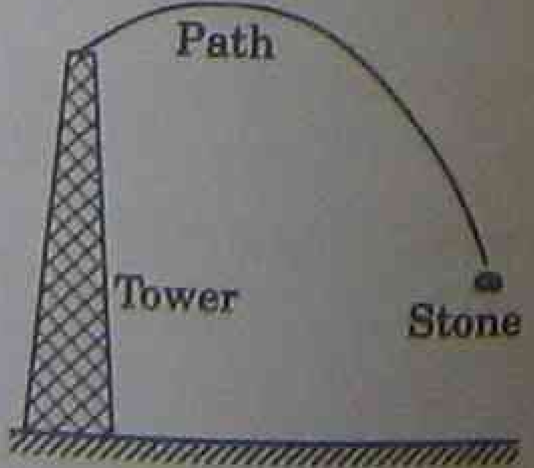
SOLUTION

$$f(t) = 60 + 5t - 5t^2$$

(i) $f(0) = 60$. This represents the height of the stone above ground level when $t = 0$, that is, the initial height, the starting height, the height of the tower.

(ii) $f(0.5) = 60 + 5(0.5) -(0.5)^2$
$= 60 + 2.5 - 1.25 = 61.25$

This represents the heightof the stone $^1/_2$ second after the stone was thrown. The stone has been thrown upwards.



(iii) $f(1) = 60 + 5(1) - 5(1)^2 = 60 + 5 - 5$
$= 60$

$f(2) = 60 + 5(2) - 5(2)^2$
$= 60 + 10 - 20 = 50$

$f(3) = 60 + 5(3) - 5(3)^2$
$= 60 + 15 - 45 = 30$

The stone rose to a maximum height of 61.25 metres and has now begun to fall. When it has reached the height, $f(t) = 0$ then the stone has reached ground level.

(iv) $f(4) = 60 + 5(4) - 5(4)^2$
$= 60 + 20 - 80 = 0$

The particle reaches ground level after 4 seconds.
or we want

$$f(t) = 0$$
$$\therefore \ 60 + 5t - 5t^2 = 0$$
$$\text{i.e.} \ t^2 - t - 12 = 0 \quad \text{(divide by } -5)$$
$$(t - 4)(t + 3) = 0$$
$$\therefore \ t = 4 \ \text{ or } \ -3$$

Negative time has no meaning, thus $t = 4$ seconds.

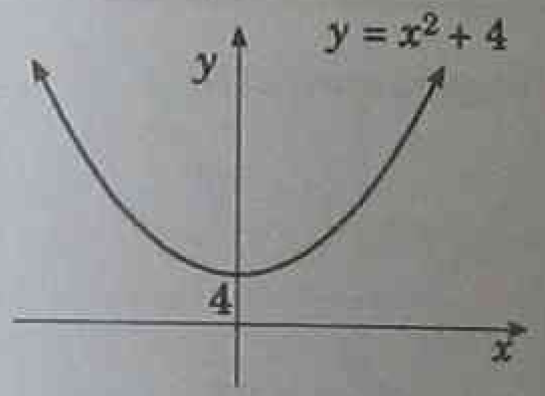
13.6.4 Domain and range

The **domain** of a function is the set of all possible x values (inputs).

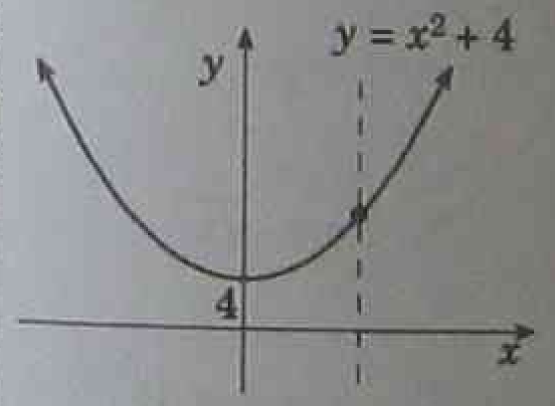
The **range** of a function is the set of all possible y values (outputs).

Examples: For the given sketches state whether or not they are functions and find the domain and range.

(a)



SOLUTION



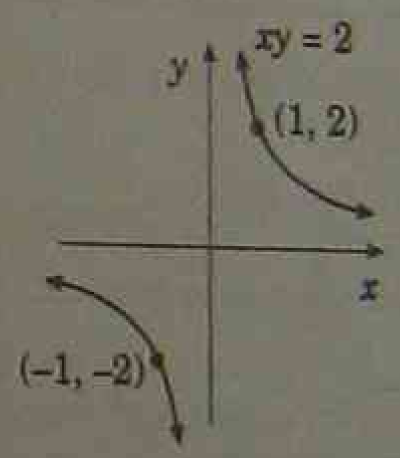
By vertical line test this represents a function.

Domain: All values of x. (No restriction)

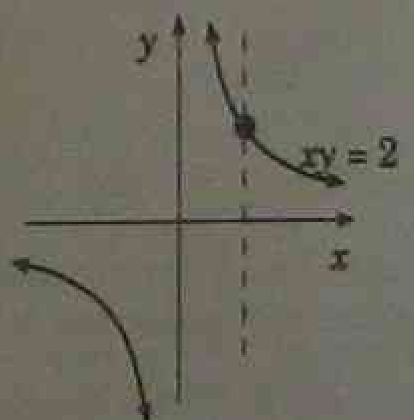
| Any x value can be used as an input. |

Range: Only y values greater or equal to 4 are outputs, that is, $y \geq 4$.

(b)



SOLUTION



By vertical line test this represents a function.
Domain: All x values except 0 can be used, that is $x \neq 0$.
Range: All y values except 0 can be outputs, that is $y \neq 0$.

13.6.5 Inverse functions

If in the equation of a function the x and y are interchanged the inverse function is generated, that is, the x replaces the y and the y replaces the x. For example:

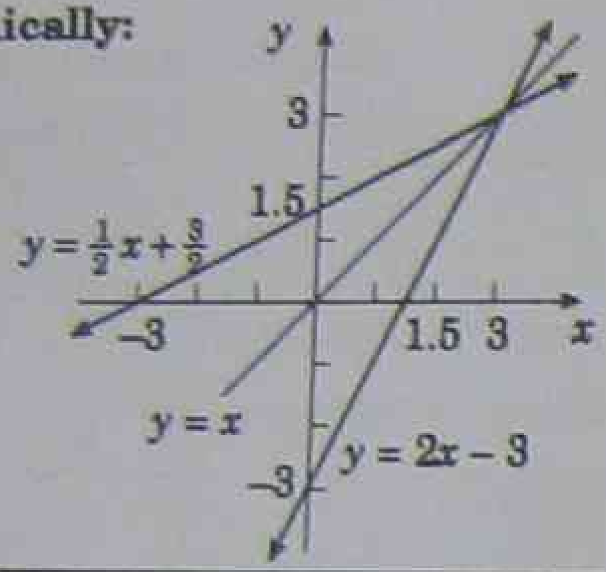
Original function $y = 2x - 3 \rightarrow$ interchange x and y $\rightarrow x = 2y - 3$.

Make y the subject $\rightarrow$ $2y = x + 3$

$$y = \tfrac{1}{2}x + \tfrac{3}{2}$$

$\therefore$ inverse function: $y = \frac{1}{2}x + \frac{3}{2}$

Graphically:



By drawing graphs of $y = 2x - 3$ and $y = \frac{1}{2}x + \frac{3}{2}$ together with $y = x$ it can be seen that the two functions are mirror images through $y = x$.

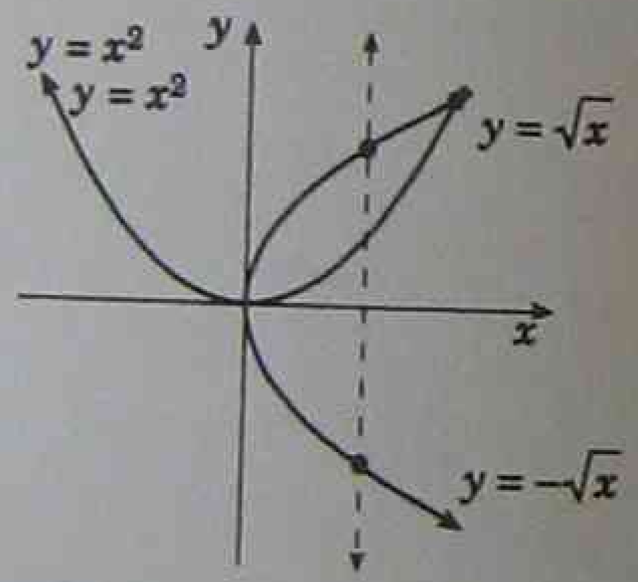
$$\boxed{\text{The graphs of inverse functions are mirror images through } y = x.}$$

Some functions can only have an inverse function if the range is restricted. For example,

Function: $y = x^2$ $\rightarrow$ Interchange x and y $\rightarrow x = y^2$ $\therefore$ $y = \pm\sqrt{x}$.

Now when $y = \pm\sqrt{x}$ is graphed:

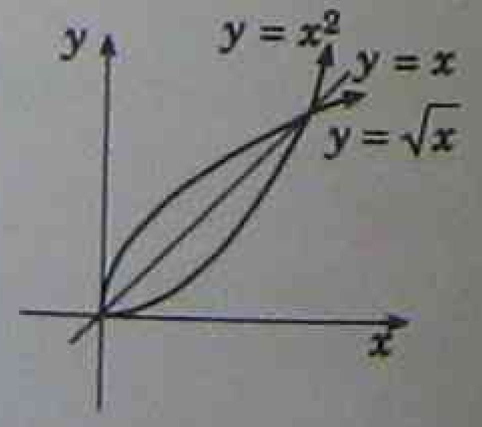
Clearly $y = \pm\sqrt{x}$ (or $x = y^2$) does not represent a function. (Vertical line test)



If the range of $y = x^2$ is restricted such that $y > 0$, then the inverse function will have a domain such that $x > 0$, that is $y = \sqrt{x}$ and the function has an inverse.

Function: $y = x^2,\ y > 0$

Inverse: $y = \sqrt{x},\ x > 0$



$$\boxed{\text{The domain of the function becomes the range of the inverse function and vice versa.}}$$

13.6.6 Using graphs to draw other graphs

Given the graph of $y = f(x)$,

then $y = f(x) + a$ represents a shift of the graph up by 'a' units

$y = f(x) - a$ represents a shift of the graph down by 'a' units

$y = f(x + a)$ represents a shift of the graph to the left by 'a' units

$y = f(x - a)$ represents a shift of the graph to the right by 'a' units

Example: By first sketching $y = x^2$, draw sketches of

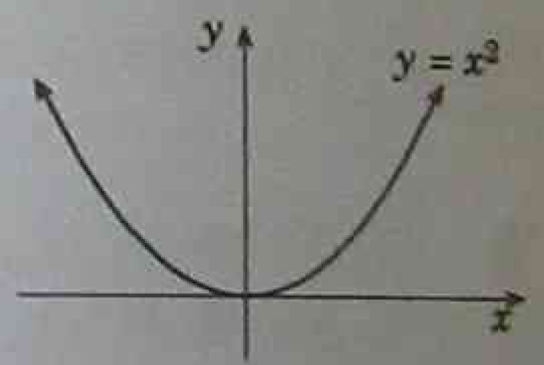
(i) $y = x^2 + 2$

(ii) $y = x^2 - 2$

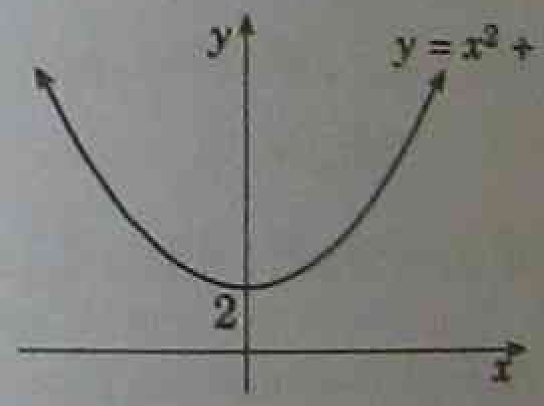
(iii) $y = (x + 2)^2$

(iv) $y = (x - 2)^2$

SOLUTION

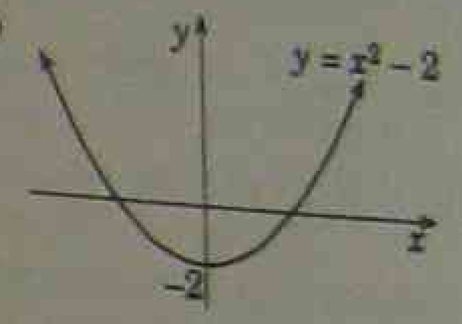


(i)



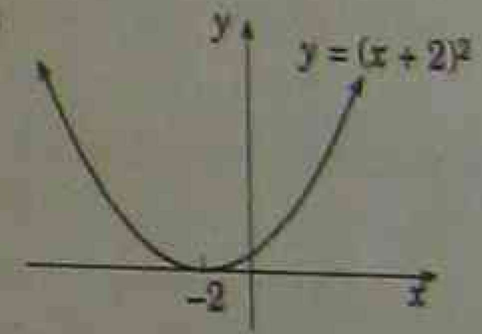
Graph of $y = x^2$ moved up by 2.

(ii)



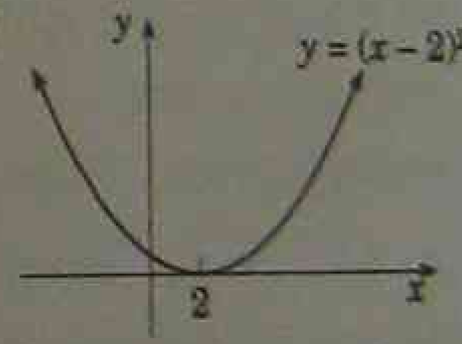
Graph of $y = x^2$ moved down by 2.

(iii)



Graph of $y = x^2$ moved left by 2 units.

(iv)



Graph of $y = x^2$ moved right by 2 units.

13.7 Exercises

1. Simplify

(a) $\dfrac{a^4 \times a^7}{a^9}$

(b) $\dfrac{5a^{12} \times 3a^3}{15a^5}$

(c) $\left(2a^5\right)^4$

(d) $\dfrac{\left(3a^2\right)^4}{\left(2a^3\right)^2}$

(e) $\dfrac{a^{\frac{3}{4}} \times a^{-\frac{1}{2}}}{a^{\frac{1}{4}}}$

(f) $\dfrac{\left(a^{\frac{1}{3}}\right)^4 \times \left(a^{\frac{3}{4}}\right)}{\left(a^{\frac{1}{3}}\right)^3 \times a^{\frac{1}{4}}}$

2. Evaluate:

(a) $81^{\frac{1}{2}}$ (b) $81^{\frac{1}{4}}$ (c) $2^3 \times 64^{\frac{1}{3}}$

(d) $256^{\frac{1}{2}} \times 16^{-\frac{1}{4}}$ (e) $2187^{\frac{4}{7}}$

(f) $125^{-\frac{2}{3}}$ (g) $1000^{\frac{1}{3}} \times 16^{-\frac{3}{4}}$

3. Solve for a:

(a) $3^a = 81$ (b) $a^2 = 1024$

(c) $5^{2a} = 625$ (d) $10^{3x-2} = 10\,000$

(e) $2^x = \frac{1}{4}$ (f) $5^{x+1} = \frac{1}{125}$

(g) $\left(\sqrt{3}\right)^x = 27$ (h) $\left(\frac{1}{\sqrt{5}}\right)^{4x} = 625^{2x-15}$

4. Simplify the following expressions using the tables of powers. Express your answer without an index.

(a) $9 \times 27 \times 27$ (b) $\dfrac{2 \times 16 \times 64 \times 128}{256}$

(c) $\dfrac{\sqrt{729}}{9}$ (d) $\sqrt{\dfrac{3125 \times 125}{625}}$

(e) $\dfrac{(81)^2 \times (9)^3}{(27)^4}$

5. Write in logarithm form:

(a) $5^3 = 125$ (b) $3^{-3} = \frac{1}{27}$ (c) $\sqrt{8} = 2^{\frac{3}{2}}$

6. Write in index form:

(a) $\log_{10} 1000 = 3$ (b) $\log_2 16 = 4$

(c) $\log_2 4\sqrt{2} = 2.5$ (d) $\log_5\left(\frac{1}{25}\right) = -2$

7. Solve the following logarithmic equations:

(a) $\log_2 64 = x$ (b) $\log_5 125 = x$

(c) $x = \log_3 \sqrt{27}$ (d) $\log_{10} 1\,000\,000 = x$

(e) $\log_2\left(\frac{1}{4}\right) = x$ (f) $\log_2 x = 3$

(g) $\log_5 x = 2$ (h) $\log_3 x = \frac{1}{2}$

(i) $\log_{10} x = -1$ (j) $\log_2 x = -3$

(k) $\log_8 x = -\frac{1}{3}$ (l) $\log_x 256 = 8$

(m) $\log_x\left(\frac{1}{9}\right) = -2$ (n) $\log_x 4 = \frac{2}{3}$

8. By first completing the following table of values:

x	-2	-1	0	0.5	1	1.5	2
y							

sketch the graph of $y = 3^x$.

9. Complete the following table for $y = 10^x$ and then sketch the curve.

x	-1	-0.5	0	0.2	0.5	0.8	1
y							

10. Rewrite $y = \log_3 x$ in index form and complete the following table:

x							
y	-2	-1	0	0.5	1	1.5	2

Sketch the curve $y = \log_3 x$.

11. Rewrite $y = \log_{10} x$ in index form and complete the following table:

x							
y	-1	-0.5	0	0.2	0.5	0.8	1

Sketch the curve $y = \log_{10} x$.

12. Simplify the following logarithmic expressions:

(a) $\log_a N + \log_a M$

(b) $\log_a N - \log_a M$

(c) $i \log_a N$

(d) $i \log_a N + j \log_a M$

(e) $\frac{1}{2} \log_a N$

(f) $\frac{1}{2} \log_a N - 2 \log_a M$

13. Expand the following logarithmic expressions:

(a) $\log_a xy$ (b) $\log_a\left(\dfrac{x}{y}\right)$

(c) $\log_a\left(\dfrac{1}{y}\right)$ (d) $\log_a \sqrt{x}$

(e) $\log_a\left(\dfrac{x^2}{y}\right)$ (f) $\log_a\left[\dfrac{x^2\sqrt{y}}{z^4}\right]$

14. Use the laws of logarithms to evaluate the following expressions:

(a) $\log_2 64$ (b) $\log_4 64$

(c) $\log_3 \sqrt{3}$ (d) $\log_5 5\sqrt{5}$

(e) $\log_5\left(\dfrac{1}{\sqrt{5}}\right)$ (f) $\log_a a^3$

(g) $\log_a \sqrt{a}$ (h) $\log_a a + \log_a \sqrt{a}$

(i) $\log_a(\log_2 2)$

(j) $\log_x\left(x^2 - 2x\right) - \log_x(x-2)$

(k) $\log_5 40 - \log_5 8$

(l) $\log_3 54 - \log_3 2$

15. Given $\log_{10} 3 = 0.477$ and $\log_{10} 5 = 0.699$, evaluate the following correct to three decimal places:

(a) $\log_{10} 15$ (b) $\log_{10}\left(\frac{5}{3}\right)$

(c) $\log_{10} \sqrt{3}$ (d) $\log_{10} 3\sqrt{3}$

(e) $\log_{10} 5\sqrt{3}$ (f) $\log_{10}\left(\frac{1}{3}\right)$

(g) $\log_{10} 0.2$ (h) $\log_{10} 45$

(i) $\log_{10} 75$ (j) $\log_{10} 5.4$

16. Given $\log_a 7 = 2.8$ and $\log_a 5 = 2.3$, evaluate the following correct to one decimal place:

(a) $\log_a 35$ (b) $\log_a\left(\frac{7}{5}\right)$

(c) $\log_a 1.4$ (d) $\log_a \sqrt{7}$

(e) $\log_a\left(\frac{1}{7}\right)$ (f) $\log_a\left(\frac{1}{25}\right)$

(g) $\log_a\left(\dfrac{1}{\sqrt{7}}\right)$ (h) $\log_a 175$

(i) $\log_a \sqrt{35}$ (j) $\log_a 9.8$

17. If $x = \log_n 2$ and $y = \log_n 3$, write the following in terms of x and y:

(a) $\log_n 6$ (b) $\log_n\left(\frac{2}{3}\right)$

(c) $\log_n 1.5$ (d) $\log_n \sqrt{2}$

(e) $\log_n \sqrt{6}$ (f) $\log_n 24$

(g) $\log_n 6\sqrt{2}$ (h) $\log_n 4.5$

(i) $\log_n\left(\frac{16}{3}\right)$ (j) $\log_n 0.5$

18. If $N = \log_a 10$ and $M = \log_a 3$, write simple logarithmic expressions for:

(a) $N + M$ (b) $N - M$

(c) $\frac{1}{2}N$ (d) $\frac{1}{2}M - \frac{1}{2}N$

(e) $2N + 3M$ (f) $4M - 2N$

19. Express y in terms of the other variables.

(a) $\log_2 y = \log_2 a + \log_2 b$

(b) $\log_2 y = \frac{1}{3} \log_2 a$

(c) $3 \log_2 y = -\log_2 x$

(d) $\log_2 y = 1 - \frac{1}{2} \log_2 x$

20. Solve the following logarithmic equations for a:

(a) $\log_{10} a = \log_{10} 3 + \log_{10} 8$

(b) $\log_{10} a = \frac{1}{2} \log_{10} 81$

(c) $\log_{10} a = \log_{10} 72 - \log_{10} 9$

(d) $\log_{10} a = 3 \log_{10} 2 - 2 \log_{10} 5$

(e) $\log_{10} a = 1 - 2 \log_{10} 5$

(f) $\log_{10} 6a - \log_{10}(a+4)$

(g) $\log_2 a - \log_2(a+2) = 3$

21. Simplify the following to a single logarithm:

(a) $\dfrac{\log_{10} 5}{\log_{10} 3}$ (b) $\dfrac{\log_a N}{\log_a b}$

22. Calculate the following to four significant figures:

(a) $\log_2 5$ (b) $\log_2 10$

(c) $\log_2 \sqrt{5}$ (d) $\log_2\left(\frac{1}{5}\right)$

(e) $\log_2 0.1$

23. Solve the following for y, giving the answer correct to two decimal places:

(a) $3^x = 5$ (b) $3^x = 10$

(c) $3^{2x} = 5$ (d) $3^x = \sqrt{5}$

(e) $3^x = \left(\frac{1}{5}\right)$ (f) $3^x = 2^{3x-5}$

(g) $2^{3-x} = 5^{2x+1}$

24. Evaluate the following (all to the base 10):

(a) $\dfrac{\log 16}{\log 2}$ (b) $\dfrac{\log 81}{\log 27}$

(c) $\dfrac{\log 8}{\log\left(\frac{1}{4}\right)}$ (d) $\dfrac{\log 2}{\log(0.25)}$

(e) $\log 125 + \log 32 - \log\left(\frac{2}{5}\right)$

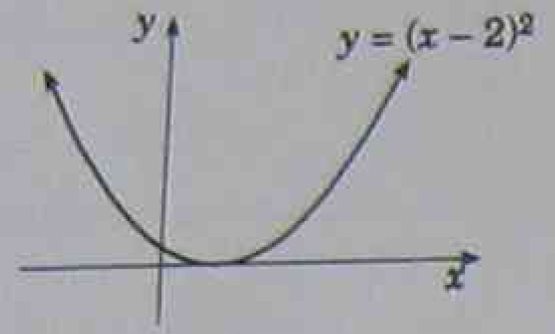
25. Evaluate:

(a) $\log_4(\log_2 16)$ (b) $\log_{10}\left(\log_{10} 10^{10}\right)$

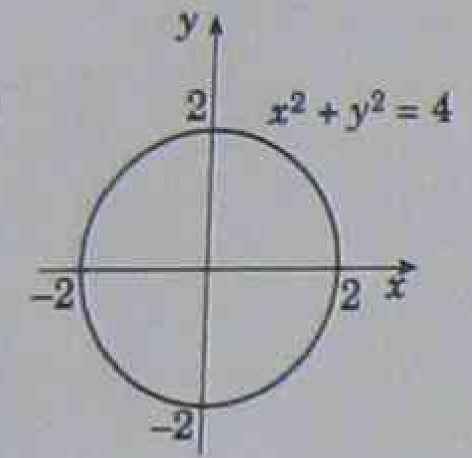
(c) $2 \log_{10}\left(\frac{16}{15}\right) + 3 \log_{10}\left(\frac{5}{3}\right) + \log_{10}\left(\frac{9}{16}\right)$

26. By using a vertical line test decide whether these graphs represent functions.

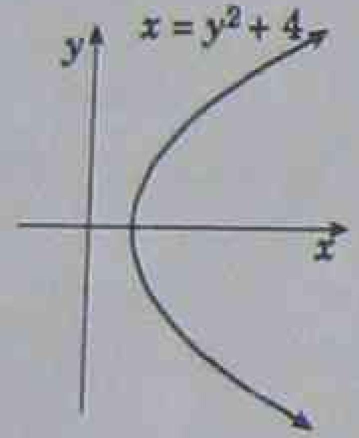
(a)



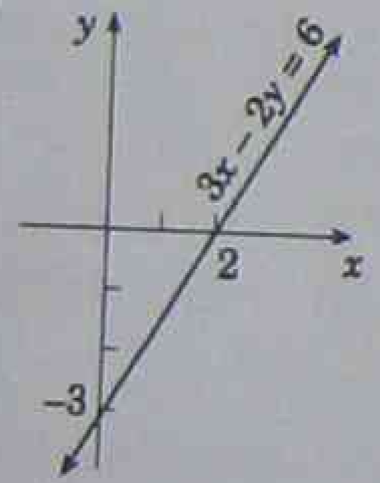
(b)



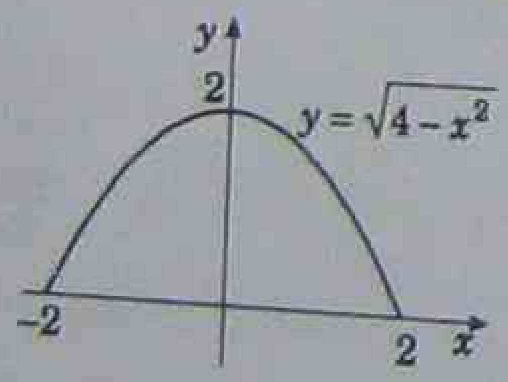
(c)



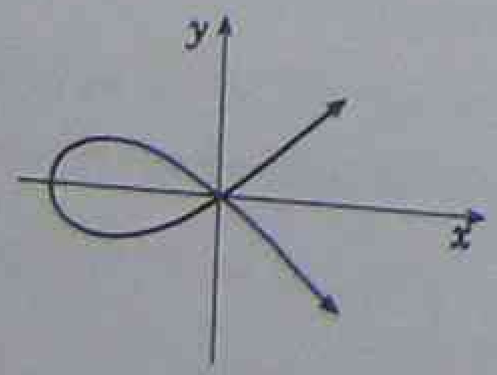
(d)



(e)



(f)



27. Given that $f(x) = 6x - 11$, evaluate

(a) $f(0)$ (b) $f(2)$
(c) $f(-2)$ (d) $f(5)$

28. If $f(x) = x^2 - 4x$, evaluate

(a) $f(0)$ (b) $f(1)$
(c) $f(-1)$ (d) $f(3)$

29. For the following functions of t, evaluate $f(0)$, $f(2)$ and $f(-2)$.

(a) $f(t) = 1 - t^2$

(b) $f(t) = \frac{1+t}{1-t}$

(c) $f(t) = t^2 - t^3$

(d) $f(t) = (t - 3)^2$

(e) $f(t) = (3 - t)^2$

(f) $f(t) = 3^t$

30. When $\phi(x) = x^2 + x$, find expressions (in the simplest form) for

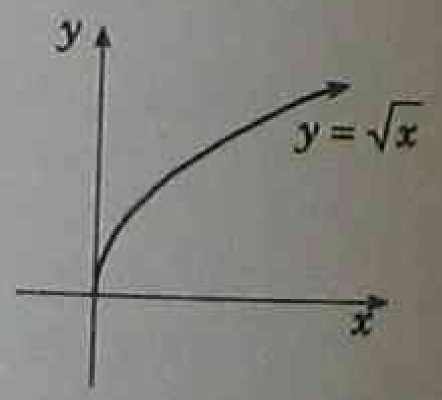
(a) $\phi(h)$ (b) $\phi(x + 2)$
(c) $\phi(x + 2) - \phi(2)$ (d) $\phi(-x)$

31. (a) For $g(x) = x^4$, show that $g(x) = g(-x)$.

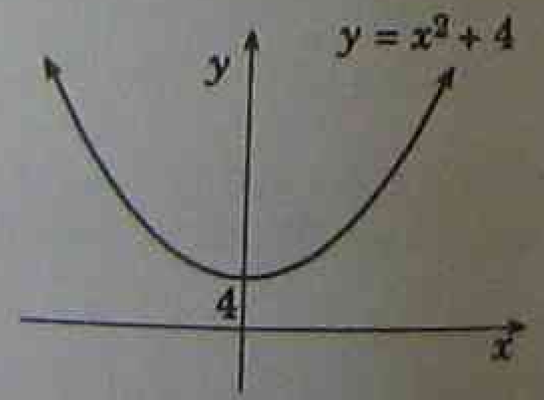
(b) For $H(x) = x^3$, show that $H(-x) = -H(x)$

32. From the given sketches state the domain and range for the functions shown.

(a)

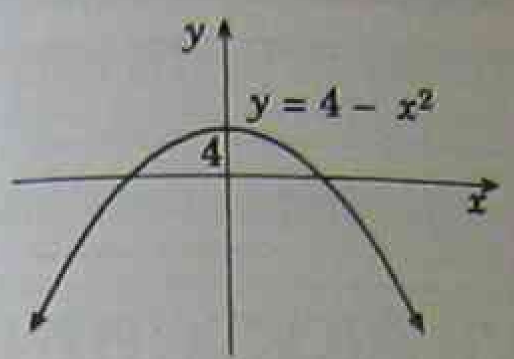


(b)

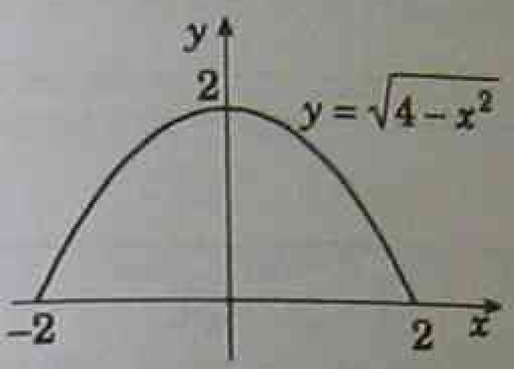


Continued

(c)



(d)



33. For the equation $y = 2x - 3$,

(a) sketch the line representing $y = 2x - 3$.

(b) Derive the equation for the inverse function of $y = 2x - 3$, and sketch this inverse function on the same number plane.

(c) By drawing the line $y = x$, verify that $y = 2x - 3$ and its inverse are mirror images through the line $y = x$.

34. Repeat Question 8. for:

(a) $x - 2y = 4$

(b) $y = -x + 2$

(c) $y = x + 2$

35. By first sketching $y = x^2$, make sketches of:

(a) $y = x^2 - 4$

(b) $y = (x - 4)^2$

36. Sketch the following functions:

(a) $y = -x^2$

(b) $y = 4 - x^2$

(c) $y = -(x - 4)^2$

37. By first sketching $y = x^3$, sketch the following:

(a) $y = x^3 + 2$

(b) $y = x^3 - 2$

(c) $y = (x - 2)^3$

(d) $y = (x + 2)^3$

Chapter 14

POLYNOMIALS AND CURVE SKETCHING

14.1 Definition of a polynomial

$P(x)$ is a polynomial expression in x if:

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$, where $a_n, a_{n-1}, \dots, a_0$ are real numbers and n is an integer, and $n \geq 0$.

Note: n cannot be a fraction or negative.

Therefore, expressions such as $\frac{1}{5}x^4$, 3, $x^3 - x$, are all polynomials in x, but

2^x, $\sqrt{x}$, $\frac{1}{x}$, x^{-2} are **not** polynomials in x.

From the definition of $P(x)$ above:

n = degree of the polynomial

a_n = leading coefficient

$a_n x^n$ = leading term

a_0 = constant term.

Note: If the leading coefficient is 1, then $P(x)$ is a *monic* polynomial, for example, $x^3 - 4x + 1$.

Examples

(a) For the polynomial $P(x) = 3x^4 - 7x - 2$:

(i) the degree of $P(x) = 4$.

(ii) the leading coefficient = 3.

(iii) the leading term: $3x^4$.

(iv) the constant term: -2.

(b) The degree of the following polynomials is as shown:

(i) $4 - 2x^3 + 3x$, degree 3.

(ii) $(4x - 7)(3x^3 - 4)$, degree 4,

[found by multiplying $4x$ and $3x^3$, that is, $12x^4$].

Note: We could have expanded completely to get the same result.

(c) Determine whether the following are polynomials in x:

(i) $\frac{x^2 - x}{x}$

SOLUTION

$$\frac{x^2 - x}{x} = \frac{\cancel{x}^{1}(x-1)}{\cancel{x}_{1}} = x - 1,$$

which is a polynomial in x,

$\therefore \frac{x^2 - x}{x}$ is a polynomial.

(ii) 3^{-1}

SOLUTION

$3^{-1} = \frac{1}{3} = \frac{1}{3}x^0$ which is a polynomial in x.

14.2 Operations with polynomials

14.2.1 Addition of polynomials

Examples

(a) If $P(x) = 3x^2 - 4x + 2$ and $Q(x) = 5x^2 - 3x - 6$, find $P(x) + Q(x)$.

SOLUTION

$$\begin{aligned} P(x) + Q(x) &= 3x^2 - 4x + 2 \\ &\quad + (5x^2 - 3x - 6) \\ &= 3x^2 - 4x + 2 \\ &\quad + 5x^2 - 3x - 6 \\ &= 8x^2 - 7x - 4 \end{aligned}$$

Adding like terms

or

$$\begin{array}{r} 3x^2 - 4x + 2 \quad + \\ \underline{5x^2 - 3x - 6} \\ \underline{8x^2 - 7x - 4} \end{array}$$

We can work left to right across this addition.

$\therefore P(x) + Q(x) = 8x^2 - 7x - 4$.

(b) If $P(x) = x^4 - 5x^2 + 3x + 1$, and $Q(x) = 4x^3 + 2x^2 - 5x - 3$, find $P(x) + Q(x)$.

SOLUTION

$$\begin{array}{r} x^4 + 0x^3 - 5x^2 + 3x + 1 \quad + \\ \underline{4x^3 + 2x^2 - 5x - 3} \\ \underline{x^4 + 4x^3 - 3x^2 - 2x - 2} \end{array}$$

We could have left a gap instead of $0x^3$ but less mistakes will be made this way.

$\therefore P(x) + Q(x) = x^4 + 4x^3 - 3x^2 - 2x - 2$.

14.2.2 Subtraction of polynomials

Examples

(a) Find the difference between $P(x)$ and $Q(x)$ if $P(x) = 5x^3 - 3x + 2$ and

$Q(x) = 2x^2 - 5x + 1$.

SOLUTION

$$\begin{aligned} P(x) - Q(x) &= 5x^3 - 3x + 2 - (2x^2 - 5x + 1) \\ &= 5x^3 - 3x + 2 - 2x^2 + 5x - 1 \\ &= 5x^3 - 2x^2 + 2x + 1 \end{aligned}$$

Remember a negative in front of the bracket means –1 multiplies contents, that is, changes signs.

or

$$\begin{array}{r} 5x^3 + 0x^2 - 3x + 2 \quad - \\ \underline{2x^2 - 5x + 1} \\ \underline{5x^3 - 2x^2 + 2x + 1} \end{array}$$

$\therefore P(x) - Q(x) = 5x^3 - 2x^2 - 2x + 1$.

(b) If $P(x) = 5x^4 - 3x^2 - 5x$ and

$Q(x) = 3x^3 - x^2 + 2$, find $P(x) - Q(x)$.

SOLUTION

$$\begin{array}{r} 5x^4 + 0x^3 - 3x^2 - 5x + 0 \quad - \\ \underline{3x^3 - x^2 + 0x + 2} \\ \underline{5x^4 - 3x^3 - 2x^2 - 5x - 2} \end{array}$$

$\therefore P(x) - Q(x)$

$= 5x^4 - 3x^3 - 2x^2 - 5x - 2$.

14.2.3 Multiplication of polynomials

We have done this many times using quadratics (degree 2), but we now extend the procedure to involve cubics (degree 3), quartics (degree 4), and so on.

Examples

(a) Find $P(x) \cdot Q(x)$ and determine its degree if $P(x) = x^4 - 3x$ and $Q(x) = x - 7$.

SOLUTION

Remember $P(x) \cdot Q(x)$ means $P(x) \times Q(x)$.

$$\begin{aligned} P(x) \cdot Q(x) &= (x^4 - 3x)(x - 7) \\ &= x^4(x - 7) - 3x(x - 7) \\ &= x^5 - 7x^4 - 3x^2 + 21x \end{aligned}$$

Therefore, the degree = 5.

(b) Find the constant term of $P(x) \cdot Q(x)$ if $P(x) = 4x^2 - 3 - x$ and $Q(x) = 5x - 7$.

SOLUTION

$$\begin{aligned} P(x) \cdot Q(x) &= (4x^2 - 3 - x)(5x - 7) \\ &= 4x^2(5x - 7) - 3(5x - 7) \\ &\quad - x(5x - 7) \\ &= 20x^3 - 28x^2 - 15x + 21 - 5x^2 + 7x \\ &= 20x^3 - 33x^2 - 8x + 21 \end{aligned}$$

Therefore, the constant term is 21.

We should have realised that the constant term was to be the product of −3 and −7.

14.2.4 Division of polynomials

When we divide polynomials, the procedure is similar to the long division method using rational numbers, for example: $5764 + 43$,

or $43\overline{)5764}$

Here we go:

$$\begin{array}{lrl} & 134 & \\ & 43\overline{)5764} & \\ \boxed{1 \times 43 = 43} \rightarrow & \underline{43} & \\ \boxed{\text{Subtraction to give 14}} \rightarrow & 146 & \leftarrow \boxed{\text{This 6 has been 'brought down'.}} \\ \boxed{3 \times 43 = 129} \rightarrow & \underline{129} & \\ \boxed{\text{Subtraction to give 17}} \rightarrow & 174 & \leftarrow \boxed{\text{This 4 has been 'brought down'.}} \\ \boxed{4 \times 43 = 172} \rightarrow & \underline{172} & \\ \boxed{\text{Subtraction to give 2}} \rightarrow & 2 & \leftarrow \boxed{\text{This is the remainder.}} \end{array}$$

$$\begin{array}{ccccccccc} \therefore\ 5764 & = & 43 & \times & 134 & + & 2. \\ \Uparrow & & \Uparrow & & \Uparrow & & \Uparrow \\ \boxed{\text{dividend}} & = & \text{divisor} & \times & \text{quotient} & + & \text{remainder} \end{array}$$

Let's now try with some polynomials in x:

Examples

(a) Complete the division:

$x + 1\overline{)2x^3 - 4x^2 + x - 1}$

SOLUTION

Look at the leading terms of x and $2x^3$: How many x in $2x^3$? or x times what is $2x^3$? Answer: $2x^2$

$$\begin{array}{r} 2x^2 - 6x + 7 \\ x + 1\overline{)2x^3 - 4x^2 + x - 1} \\ \underline{2x^3 + 2x^2} \qquad\qquad \\ -6x^2 + x \qquad \\ \underline{-6x^2 - 6x} \qquad \\ 7x - 1 \\ \underline{7x + 7} \\ -8 \end{array}$$

Subtraction always eliminates the leading term.

This x has been brought down.

How many x in $-6x^2$? Answer: $-6x$

The remainder can be positive or negative.

$$\begin{aligned} \therefore\ & 2x^3 - 4x^2 + x - 1 \\ &= (x + 1)(2x^2 - 6x + 7) - 8. \end{aligned}$$

(b) $(x^3 + 6x^2 - 2) + (x + 3)$

SOLUTION

$$\begin{array}{r} x^2 + 3x - 9 \\ x + 3\overline{)x^3 + 6x^2 + 0x - 2} \\ \underline{x^3 + 3x^2} \qquad\qquad\quad \\ 3x^2 + 0x \qquad \\ \underline{3x^2 + 9x} \qquad \\ -9x - 2 \\ \underline{-9x - 27} \\ 25 \end{array}$$

In the absence of a term we can put in a 0 or we can leave a gap

$\therefore\ x^3 + 6x^2 - 2 = (x + 3)(x^2 + 3x - 9) + 25.$

(c) $(-x^3 + 2x^2 + x - 1) + (x + 2)$

SOLUTION

$$\begin{array}{r} -x^2 + 4x - 7 \\ x + 2\overline{)-x^3 + 2x^2 + x - 1} \\ \underline{-x^3 - 2x^2} \qquad\qquad \\ 4x^2 + x \qquad \\ \underline{4x^2 + 8x} \qquad \\ -7x - 1 \\ \underline{-7x - 14} \\ 13 \end{array}$$

$$\begin{aligned} \therefore\ & -x^3 + 2x^2 + x - 1 \\ &= (x + 2)(-x^2 + 4x - 7) + 13. \end{aligned}$$

(d) $(2x^3 - x^2 - x + 1) + (x^2 - 1)$

SOLUTION

$$\begin{array}{r} 2x - 1 \\ x^2 - 1\overline{)2x^3 - x^2 - x + 1} \\ \underline{2x^3 \qquad - 2x} \quad\; \\ -x^2 + x + 1 \\ \underline{-x^2 \qquad + 1} \\ x \end{array}$$

Here we can leave gaps.

Note : The remainder is not always a constant.

$$\begin{aligned} \therefore\ & 2x^3 - x^2 - x + 1 \\ &= (x^2 - 1)(2x - 1) + x \end{aligned}$$

14.3 The Remainder Theorem

The Remainder Theorem can be used when the question is solely about remainders. It states:

If a given polynomial $P(x)$ is divided by $(x - a)$, then the remainder is $P(a)$.

Examples

(a) Find the remainder when $2x^3 - 4x^2 + x + 7$ is divided by $(x - 2)$.

SOLUTION

Let $P(x) = 2x^3 - 4x^2 + x + 7$

Now, as the divisor is $(x - 2)$, the remainder $= P(2)$

$$\begin{aligned} \therefore\ P(2) &= 2(2)^3 - 4(2)^2 + (2) + 7 \\ &= 16 - 16 + 2 + 7 \\ &= 9 \end{aligned}$$

This is a lot shorter than the division method used previously.

$\therefore$ the remainder is 9.

(b) If $x^3 - x^2 + mx + 4$ has a remainder of 4 when divided by $(x + 2)$, find the value of m.

SOLUTION

Let $P(x) = x^3 - x^2 + mx + 4$

Now, as the divisor is $(x + 2)$, that is, $[x - (-2)]$, the remainder $= P(-2) = 4$,

$$\begin{aligned} \therefore\ P(-2) = (-2)^3 - (-2)^2 + m(-2) + 4 &= 4 \\ -8 - 4 - 2m + 4 &= 4 \\ -2m - 8 &= 4 \\ -2m &= 4 + 8 \\ -2m &= 12 \\ m &= -6 \end{aligned}$$

14.4 The Factor Theorem

$(x - a)$ divides $P(x)$, if and only if, $P(a) = 0$.

Thus $(x - a)$ is a factor of $P(x)$, if and only if, $P(a) = 0$, (as there is no remainder following the division).

Examples

(a) Show that $(x+2)$ is a factor of $P(x)=x^3+x^2-4x-4$. Express $P(x)$ as a product of its factors and hence solve $x^3+x^2-4x-4=0$.

SOLUTION

For $(x+2)$ to be a factor, $P(-2)$ must equal zero:

$$\therefore\ P(-2)=(-2)^3+(-2)^2-4(-2)-4$$
$$=-8+4+8-4$$
$$=0$$

$\therefore$ $(x+2)$ is a factor.

Now, to find other factors, we need to divide!

$$\begin{array}{r}
x^2-x-2\\
x+2\overline{)x^3+x^2-4x-4}\\
\underline{x^3+2x^2}\\
-x^2-4x\\
\underline{-x^2-2x}\\
-2x-4\\
\underline{-2x-4}\\
0
\end{array}$$

We knew that the remainder was zero.

$$\therefore\ x^3+x^2-4x-4=(x+2)(x^2-x-2)$$
$$=(x+2)(x-2)(x+1)$$

Factorising (x^2-x-2) gives $(x-2)(x+1)$.

Now, as $x^3+x^2-4x-4=0$

$\therefore$ $(x+2)(x-2)(x+1)=0$

$\therefore$ $x=-2, 2, -1$

Remember that in quadratics, if $(x-a)(x-b)=0$, then either $x-a=0$ or $x-b=0$, $\therefore$ $x=a$ or b.

(b) If x^3+ax+b is divisible by both $x+2$ and $x-3$, find the values of a and b.

SOLUTION

Let $P(x)=x^3+ax+b$

- Divisible by $x+2$

$\therefore$ $P(-2)=0$

$$\therefore\ (-2)^3+a(-2)+b=0$$
$$-8-2a+b=0$$
$$-2a+b=8$$
$$\therefore\ 2a-b=-8 \qquad —(1)$$

- Divisible by $x-3$

$$\therefore\ P(3)=0$$
$$\therefore\ 3^3+a(3)+b=0$$
$$27+3a+b=0$$
$$\therefore\ 3a+b=-27 \qquad —(2)$$

- Solving (1) and (2) simultaneously:

$$2a-b=-8$$
$$3a+b=-27$$
$$(1)+(2):\quad 5a=-35$$
$$a=-7$$

Substitute $a=-7$ in (1):

$$\therefore\ 2(-7)-b=-8$$
$$-14-b=-8$$
$$-b=-8+14$$
$$-b=6$$
$$b=-6$$

$\therefore$ $a=-7$, $b=-6$.

(c) Solve $x^3+2x^2-5x-6=0$.

SOLUTION

We look for a factor, and the clue is the constant term of -6. Therefore try the factors of -6, that is, $\pm1, \pm2, \pm3, \pm6$.

Let $P(x)=x^3+2x^2-5x-6$.

Start with the simplest, i.e. $P(1)$, then the next simplest, etc.

Now we divide $P(x)$ by $(x+1)$.

$$\therefore\ P(1)=1^3+2(1)^2-5(1)-6$$
$$=1+2-5-6$$
$$=-8\neq0$$
$$\text{Try, } P(-1)=(-1)^3+2(-1)^2-5(-1)-6$$
$$=-1+2+5-6$$
$$=0$$
$$\therefore\ P(-1)=0$$

$\therefore$ $x+1$ is a factor.

Continued

$$\begin{array}{r}
x^2+x-6\\
x+1\overline{)x^3+2x^2-5x-6}\\
\underline{x^3+x^2}\\
x^2-5x\\
\underline{x^2+x}\\
-6x-6\\
\underline{-6x-6}\\
0
\end{array}$$

$$\therefore\ x^3+2x^2-5x-6=(x+1)(x^2+x-6)$$
$$=(x+1)(x+3)(x-2)$$

Now, as $x^3+2x^2-5x-6=0$,

then $(x+1)(x+3)(x-2)=0$,

$\therefore$ $x=-1, -3, 2$

14.5 Roots and zeros

In Example (c) above, we solved the equation and the solution was $x=-1, -3, 2$. We say that the roots of $x^3+2x^2-5x-6=0$ are $x=-1, -3, 2$.

Also, these values: -1, -3 and 2 are zeros of the polynomial $x^3+2x^2-5x-6=0$,

that is, *if* a is a zero of $P(x)=0$, then $(x-a)$ is a factor of $P(x)$ and $P(a)=0$.

14.6 Graphs of polynomial functions

When we have to sketch a polynomial function, the zeros tell us where the function cuts the x-axis, so we factorise to find the zeros.

Examples

(a) Sketch $y=(x+2)(x-3)(4-x)$

SOLUTION

Put $y=0$

We put $y=0$ because this is the x-axis.

- By solving $y=0$,

$(x+2)(x-3)(4-x)=0$

$\therefore$ $x=-2, 3, 4$

Therefore there are zeros at $-2, 3, 4$.

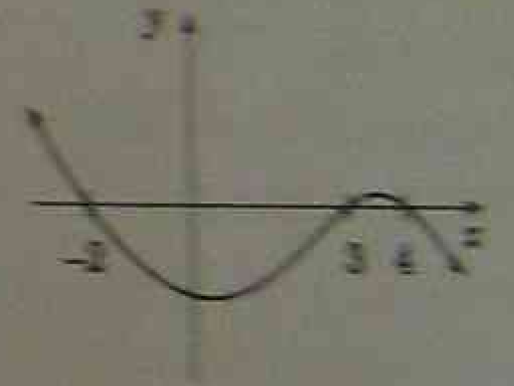
The curve cuts the x-axis at these points.

- Now choose a point other than these zeros and determine the sketch.

Note: $y(5)$ means substitute $x=5$ in y.

For example: $y(5)=(5+2)(5-3)(4-5)$ is negative.

This means that at $x=5$, the curve is below the x-axis and so for all $x>5$ it must be below the axis — it cuts x-axis only at $x=4, 3, -2$



(b) Sketch $y=(x+2)^2(3-x)$

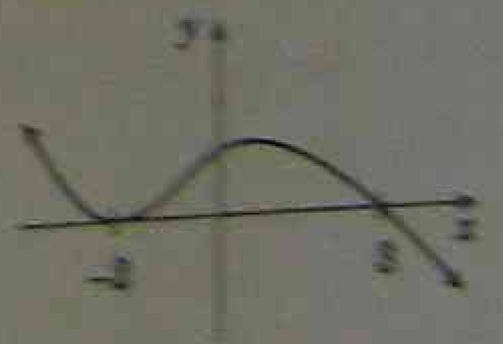
SOLUTION

By solving $(x+2)^2(3-x)=0$ we get a double root at $x=-2$ and another root at $x=3$.

- At the double root the curve just touches the x-axis.

- Substitute point $x=4$.

Then $y(4)=(4+2)^2(3-4)$ which is negative.

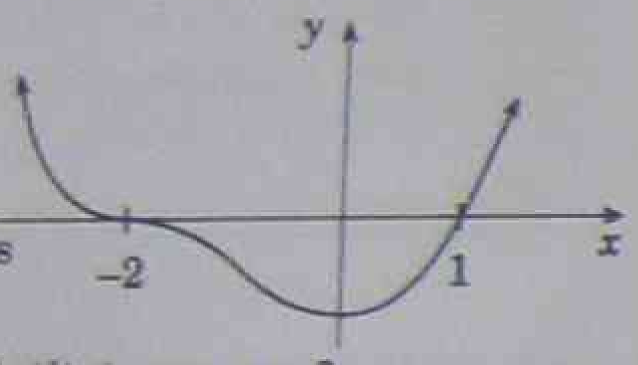


(c) Sketch $y = (x-1)(x+2)^3$.

SOLUTION

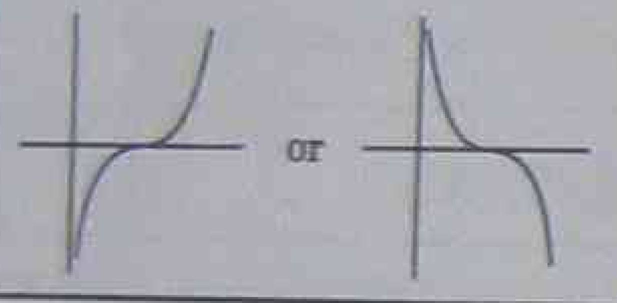
Solving $(x-1)(x+2)^3 = 0$, $x = 1$, and $x = -2$ is a *triple root*.

- At a *triple root* the curve has a *kink*.



- We substitute, say, $x = 2$,
  $\therefore\ y(2) = (2-1)(2+2)^3 > 0$.

The 'kink' can look like



(d) Solve the equation $x^3 + 8x^2 + 17x + 10 = 0$ and sketch the graph of $y = x^3 + 8x^2 + 17x + 10$.

SOLUTION

Let $P(x) = x^3 + 8x^2 + 17x + 10$.

Try a few values of x:

$$\begin{aligned} P(-1) &= (-1)^3 + 8(-1)^2 + 17(-1) + 10 \\ &= -1 + 8 - 17 + 10 \\ &= 0 \end{aligned}$$

The equation had all positive terms so the roots must be negative.

$\therefore\ x = -1$ is a root, and $(x+1)$ is a factor.

Found it on our first try. Fluke!

Now divide $P(x)$ by $(x+1)$.

$$\begin{array}{r} x^2 + 7x + 10 \\ x+1\overline{)\,x^3 + 8x^2 + 17x + 10} \\ \underline{x^3 + x^2} \\ 7x^2 + 17x \\ \underline{7x^2 + 7x} \\ 10x + 10 \\ \underline{10x + 10} \\ 0 \end{array}$$

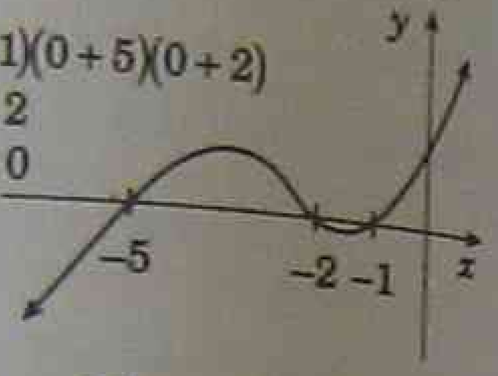
$$\begin{aligned} \therefore\ P(x) &= x^3 + 8x^2 + 17x + 10 \\ &= (x+1)(x^2 + 7x + 10) \\ &= (x+1)(x+5)(x+2) = 0 \end{aligned}$$

$\therefore\ x = -1, -5, -2.$

Therefore the zeros are $-1, -5, -2$.

Now, for $y = x^3 + 8x^2 + 17x + 10$ choose $x = 0$,

$$\begin{aligned} \therefore\ y(0) &= (0+1)(0+5)(0+2) \\ &= 1 \cdot 5 \cdot 2 \\ &= 10 > 0 \end{aligned}$$



(e) Give values of a and b such that the graph of $y = (ax-5)(x-b)^2$ cuts the x-axis at $x = 2.5$ and touches the x-axis at $x = 3$.

SOLUTION

- Cuts at
$$\begin{aligned} x &= 2.5 \\ x &= \tfrac{5}{2} \\ 2x &= 5 \\ 2x - 5 &= 0 \end{aligned}$$

Refer back to $(ax-5)(x-b)^2$

$\therefore\ (2x-5)$ is a factor,

$\therefore\quad 2x - 5 = ax - 5$,

therefore $a = 2$.

- Touches at $x = 3$
$$x - 3 = 0$$

Therefore a double root exists,

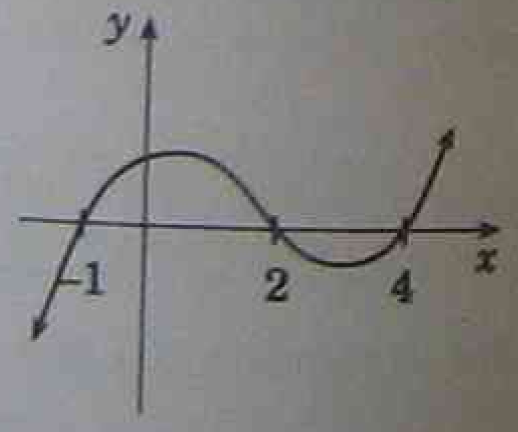
$\therefore\ (x-3)^2$ is a factor,

that is, $x - 3 = x - b$, therefore $b = 3$.

Thus $a = 2$, $b = 3$.

(f) Write down the equations of the polynomials with the following graphs:

(i)



SOLUTION

Either $y = (x+1)(x-2)(x-4)$, (1)

or $y = -(x+1)(x-2)(x-4)$. (2)

But by substituting a value, we can be certain, for example, that:

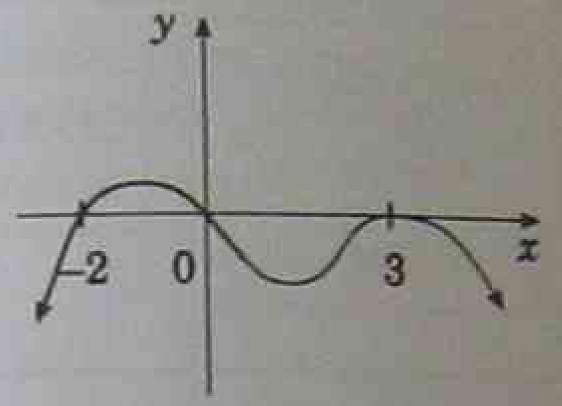
$y(5)$ has to be positive, from the graph.

From (1),
$y(5) = (5+1)(5-2)(5-4) > 0$ ✓

From (2),
$y(5) = -(5+1)(5-2)(5-4) < 0$ ×

$\therefore\ y = (x+1)(x-2)(x-4)$.

(ii)



SOLUTION

$$\begin{aligned} \text{Either}\quad y &= (x+2) \cdot x \cdot (x-3)^2 \\ &= x(x+2)(x-3)^2, &(1) \\ \text{or}\quad y &= -x(x+2)(x-3)^2 &(2) \end{aligned}$$

Try $y(4)$:

In (1): $y(4) = 4(4+2)(4-3)^2 > 0$ *No!*

In (2): $y(4) = -4(4+2)(4-3)^2 < 0$ *Yes*, as $y(4)$ in the graph is negative.

Therefore, $y = -x(x+2)(x-3)^2$.

14.7 Curve sketching

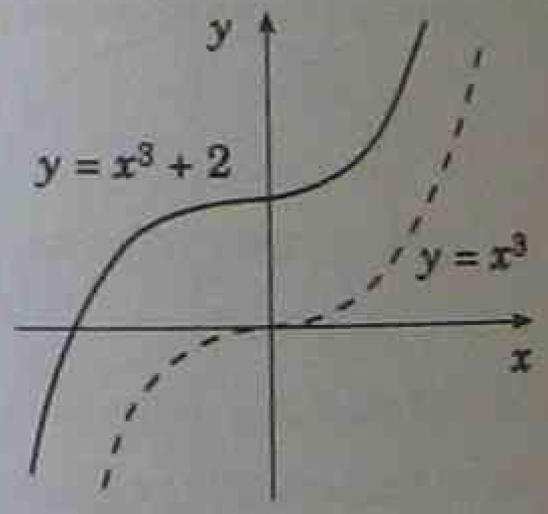
14.7.1 Effects of constants on family graphs

In the chapter on quadratics we investigated the effect constants had on the graphs of parabolas. We now extend this to other graphs.

Examples:
On the same number plane graph:

(a) $y = x^3$ and $y = x^3 + 2$

SOLUTION



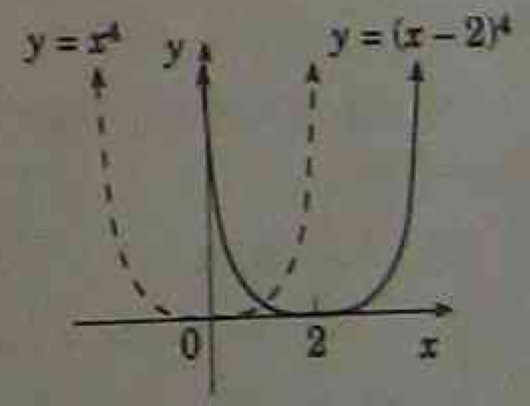
Curve moved up by 2 units.

In general

$y = f(x) + b$ moves curve $f(x)$ up by b units. Similarly $y = f(x) - b$ drops $f(x)$ by b units.

(b) $y = x^4$ and $y = (x-2)^4$

SOLUTION



Curve is moved across x-axis by 2 units.

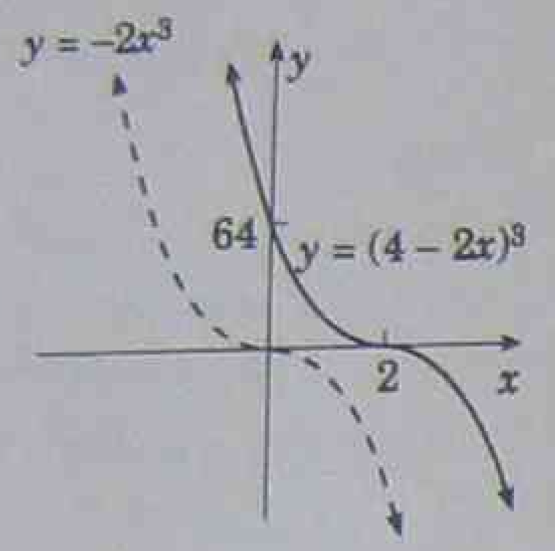
In general

$y = f(x-a)$ is a shift to the right of $f(x)$ by a units.

$y = f(x+a)$ moves $f(x)$ to the left by a units.

(c) $y = -2x^3$ and $y = (4 - 2x)^3$

SOLUTION



$y = (4 - 2x)^3$

Let $x = 0$

$\therefore$ $y = 64$

Also, let $y = 0$

$\therefore$ $(4 - 2x)^3 = 0$

$4 - 2x = 0$

$x = 2$

$$\boxed{\begin{aligned} y &= (4-2x)^3 \\ &= [-(2x-4)]^3 \\ &= -(2x-4)^3 \end{aligned}}$$

Shift of $y = -2x^3$ to right by 4

14.7.2 General equation of circles

$$(x-a)^2 + (y-b)^2 = r^2 \quad \text{where } (a, b) = \text{coordinates of centre}$$
$$r = \text{radius.}$$

Examples:

(a) Find the coordinates of centre and radius of:

(i) $x^2 + y^2 = 49$

(ii) $x^2 + (y-2)^2 = 7$

(iii) $(x-4)^2 + (y-3)^2 = 21$

(iv) $x^2 - 4x + y^2 - 8y = 11$

SOLUTIONS

(i) $x^2 + y^2 = 49$

$\therefore$ C(0, 0), radius = 7 units

(ii) $x^2 + (y-2)^2 = 7$

$\therefore$ C(0, 2), radius = $\sqrt{7}$ units.

(iii) $(x-4)^2 + (y+3)^2 = 21$

$\therefore$ C(4, −3). radius = $\sqrt{21}$ units.

(iv)

$x^2 - 4x + y^2 - 8y = 11$

Now, complete the squares:

$x^2 - 4x \quad + y^2 - 8y \quad = 11$

$\therefore x^2 - 4x + 4 + y^2 - 8y + 16 = 11 + 4 + 16$

$\therefore \quad (x-2)^2 + (y-4)^2 = 31$

$\therefore$ C(2, 4), radius = $\sqrt{31}$ units.

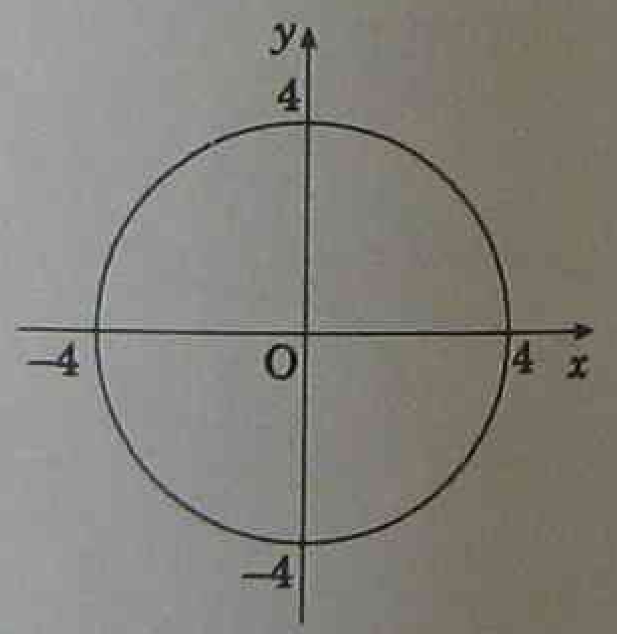
(b) Sketch the circles, labelling intercepts:

(i) $x^2 + y^2 = 16$

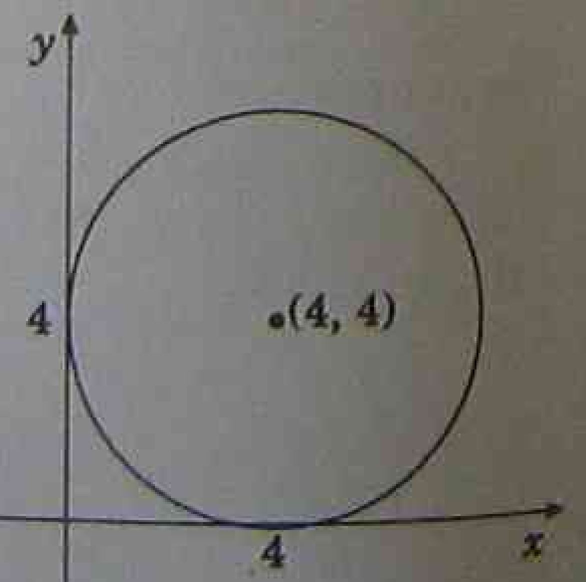
(ii) $(x-4)^2 + (y-4)^2 = 16$

SOLUTION

(i)



(ii)



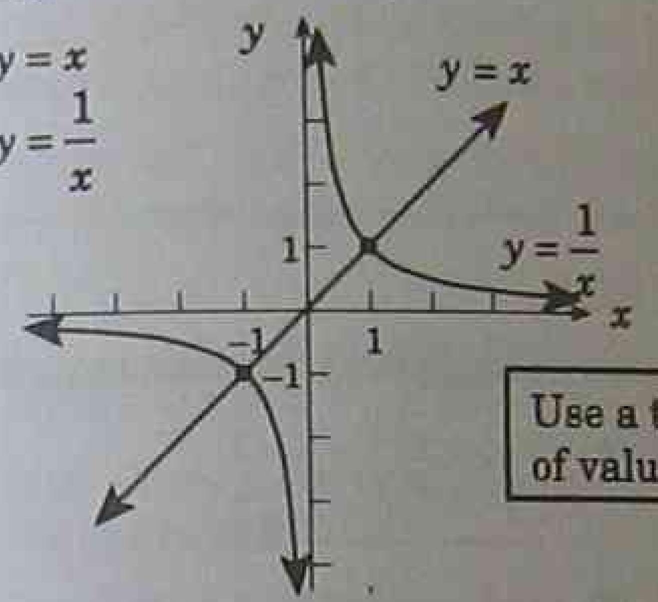
14.8 Intersection of graphs

14.8.1 Solving simultaneous equations graphically

Remember that in Chapter 4 (Section 4.3) we looked at solving simultaneous equations to find the point(s) of intersection of the two graphs. We had the choice of the elimination method and the substitution method. An alternative to the two algebraic methods is the graphical method. Its usefulness depends on the accuracy of the graphs.

Examples: Solve graphically the following simultaneous equations:

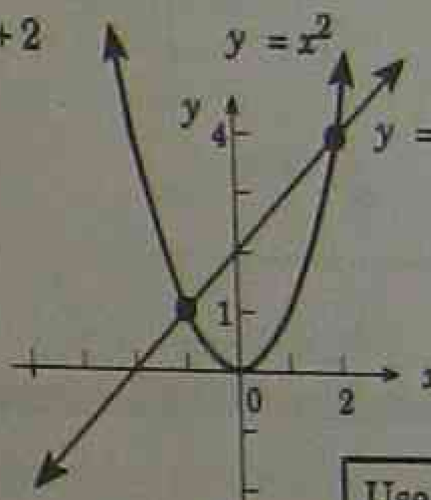
(a) $y = x$
$y = \frac{1}{x}$



Use a table of values.

SOLUTION: (1, 1) or (−1, −1).

(b) $y = x^2$
$y = x + 2$



Use a table of values.

SOLUTION: (2, 4) or (−1, 1).

14.8.2 Graphical solutions of other equations

As we know, to solve the simultaneous equations:

$y = x^2$

$y = x + 6,$

most students would choose to substitute one equation into the other, that is:

$y = x^2 \quad \ldots (1)$

$y = x + 6 \quad \ldots (2)$

Substitute (1) into (2):

$x^2 = x + 6,$

and then solve and substitute back!

Reversing this, if we had to solve $x^2 = x + 6$, we could choose to do so by setting both x^2 and $x + 6$ equal to y; that is, $y = x^2$ and $y = x + 6$ could now be solved using any one of the methods available to us, say, the graphical method. This is very handy if we cannot solve the equation by factorisation.

Examples

(a) Solve $x^3 = x + 2$.

SOLUTION

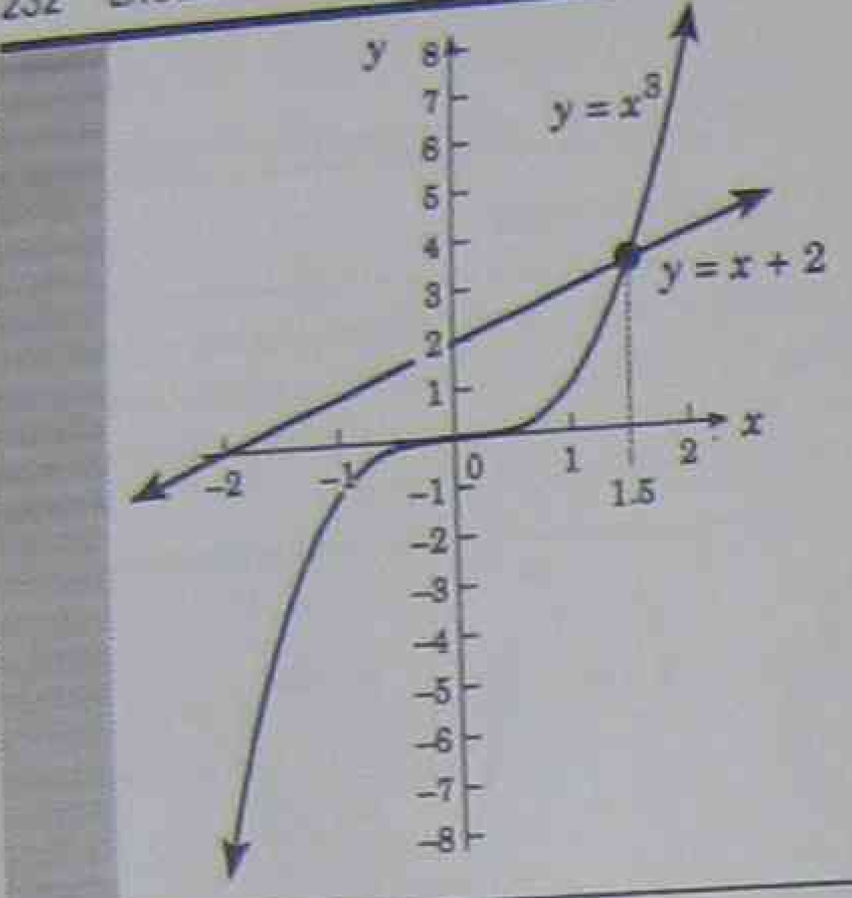
Let $y = x^3$
$y = x + 2$

Now, solving graphically:

(See the sketch on page 232.)

Therefore the solution of $x^3 = x + 2$ is
$x = 1.5$

Continued



(b) The equation $2^x - x + 4 = 0$ could be solved by drawing a line on the graph of $y = 2^x$. Find the equation of that line.

SOLUTION

$2^x - x + 4 = 0$

$2^x = x - 4$

Hence $y = 2^x$ and $y = x - 4$.

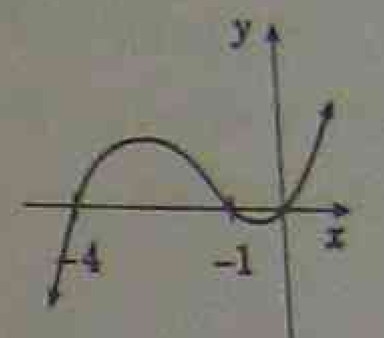
The equation of the line is $y = x - 4$.

14.9 Exercises

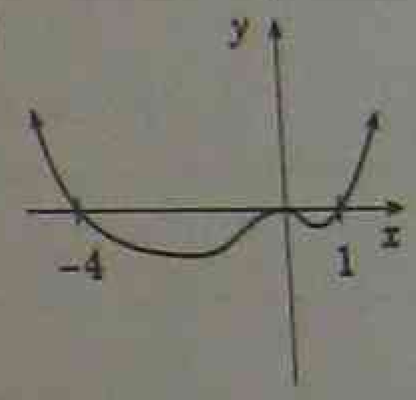
1. For the following polynomials, find the degree, the leading term, leading coefficient and constant term:
   (a) $P(x) = 3x^4 - 7x + 2$
   (b) $P(x) = 4 - 2x^5$
   (c) $P(x) = 3x^5 - 2x^3 + 4x$
   (d) $P(x) = 7$
2. Determine whether the following are polynomials in x:
   (a) $\frac{4}{x} - 3x + 2$
   (b) $\frac{5x^4 - x}{x}$
   (c) $\frac{3x}{4} + 2$
   (d) $4x^4 + \sqrt{x} - 2$
   (e) $(x+4)^2 - 2x + 3$
3. Complete the following:
   (a) $\begin{array}{r} 4x^4 - 2x^2 + 3 \\ \underline{3x^4 - 5x^2 - 7} \end{array}$ +
   (b) $\begin{array}{r} 3x^2 - 4x + 1 \\ \underline{5x^2 + 2x - 5} \end{array}$ +
   (c) $\begin{array}{r} 4x^3 + 12x^2 + 3x + 7 \\ \underline{5x^3 \qquad\quad + x + 3} \end{array}$ +
   (d) $\begin{array}{r} 4x^5 - 21x + 2 \\ \underline{3x^5 + \ 2x - 1} \end{array}$ +
   (e) $\begin{array}{r} 5x^3 + 2x^2 - x - 1 \\ \underline{3x^3 - \ x^2 - x - 5} \end{array}$ −
   (f) $\begin{array}{r} 4x^4 - 2x^2 + 1 \\ \underline{x^4 - 5x^2 - 7} \end{array}$ −
   (g) $\begin{array}{r} 4x^3 \qquad - x + 2 \\ \underline{3x^3 + 2x^2 \qquad - 1} \end{array}$ −
   (h) $\begin{array}{r} 5x^4 \qquad - 2x^2 \qquad + 1 \\ \underline{3x^3 + \ x^2 - 5x + 2} \end{array}$ −
4. Find the degree of $P(x).Q(x)$ if:
   (a) $P(x) = 3x^2 - 4x + 1$ and $Q(x) = 4 - x + x^2$
   (b) $P(x) = 5x^4 - 2x - 1$ and $Q(x) = 3x - 5x^2 + x^3$
5. Find the product of the following polynomials:
   (a) $2x^3 - 4$ and $5x^2 - x + 2$
   (b) $6x^2 - 4x$ and $x^3 + x - 1$
6. Complete the following divisions, leaving your answer in the form: dividend = divisor × quotient + remainder.
   (a) $(x^3 - 3x^2 + 7x - 5) \div (x - 3)$
   (b) $(-x^3 + 2x^2 - 5x - 1) \div (x + 2)$
   (c) $(x^5 + x^3 - x) \div (x^2 + x)$
   (d) $(x^4 - 1) \div (x^2 - 3)$

7. If $P(x) = x^3 + 2x^2 - x + 1$ and $Q(x) = x - 1$, find
   (a) $P(x) + Q(x)$ (b) $P(x) - Q(x)$
   (c) $P(x) \cdot Q(x)$ (d) $\frac{P(x)}{Q(x)}$
8. Find the remainder after the following divisions:
   (a) $(x^3 + 4x^2 + x - 1) \div (x + 2)$
   (b) $(x^3 - 4x + 2) \div (x - 1)$
9. If $x^3 + 3x^2 - 7x + k$ is divided by $(x + 2)$ the remainder is 5. Find the value of k.
10. When $3x^4 - 2x^3 + 2k$ is divided by $(x + 1)$, the remainder is 3. Find the value of k.
11. If $x^2 - 7x + 9$ is divided by $(x - k)$, the remainder is 3. Find the value of k.
12. If $(x - 5)$ is a factor of $x^3 + kx - 10$, find the value of k.
13. Show that $(x - 1)$ is a factor of $x^3 - 2x^2 - 5x + 6$, and hence solve the equation $x^3 - 2x^2 - 5x + 6 = 0$.
14. Factorise $2x^3 - 5x^2 - 4x + 3$.
15. Solve for x, the equation $x^3 + 2x^2 - 9x - 18 = 0$.
16. If $x = -3$ and $x = 1$ are roots of the equation $x^3 - 2x^2 - ax + b$, find the values of a and b, and hence find the third root of the equation.
17. Sketch the graphs of the following polynomials:
   (a) $y = (x - 4)(x - 1)(x + 2)$
   (b) $y = x(x - 2)^2(4 - x)$
   (c) $y = (3 - x)(x + 1)^3(4 - x)$
   (d) $y = x^2(1 - x)^2$
   (e) $y = (1 + x)^3$
18. Factorise the following and then sketch their graphs, showing clearly the zeros of each function:
   (a) $y = x^3 - 4x^2 + x + 6$
   (b) $y = x^3 - x^2 - x + 1$
   (c) $y = x^3 - 3x^2 + 3x - 1$
   (d) $y = -x^3 + 12x + 16$
19. Match the polynomial function to the correct sketch:
   (a) $y = x(x - 1)^2(4 - x)$
   (b) $y = x^2(x - 1)(x + 4)$
   (c) $y = x(x - 1)^3(4 - x)$
   (d) $y = x(x + 1)(4 + x)$

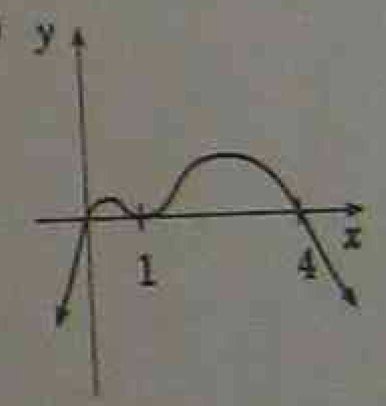
(i)



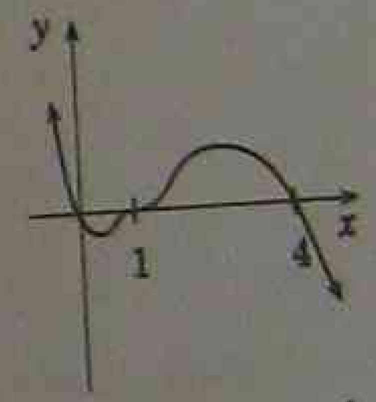
(ii)



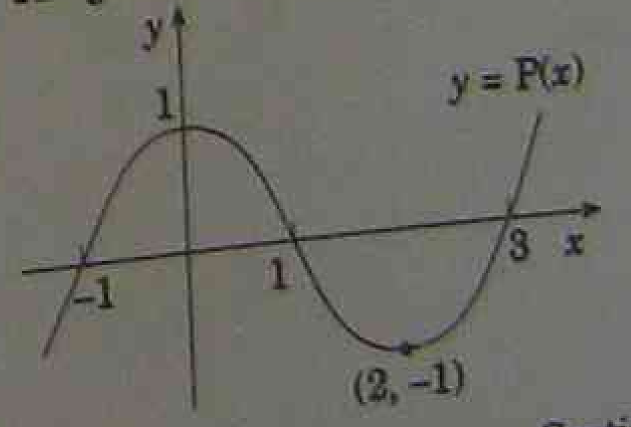
(iii)



(iv)



20. On the same number plane, sketch the following:
   (a) $y = x^3$ and $y = x^3 - 2$
   (b) $y = x^4$ and $y = 2 - x^4$
   (c) $y = x^2 - 2x + 1$ and $y = -x^2 + 2x - 1$
21. The graph of $y = P(x)$ appears below.



Continued

On the same number plane graph:

(a) $y = P(x)$ and $y = P(x) + 2$

(b) $y = P(x)$ and $y = -P(x)$

22. For the equations of circles, find the centre and radius:

(a) $x^2 + (y + 2)^2 = 9$

(b) $x^2 - 4x + y^2 + 12y = 16$

(c) $x^2 - 3x + y^2 = 6$

23. Solve graphically the following simultaneous equations:

(a) $y = 2x + 3$
$y = x^2$

(b) $y = 1$
$y = x^2$

24. The solution of $x^3 = x^2 + 2x - 1$ could be found by discovering the points of intersection of $y = x^3$ and parabola when graphed on the same number plane. Find the equation of the parabola.

25. The equation $x^2 - x - 7 = 0$ could be solved by finding the points of intersection of $y = x^2$, and which straight line?

Chapter 15

PRACTICE PAPER

Time Allowed: 90 minutes

- Part A: 30 Multiple-choice Questions: Allow 35 minutes
- Part B: Single and free response Questions: Allow 55 minutes

Part A Multiple-choice Questions (1 mark each)

Select the answer that best fits the question.

1. If $(x - 2)(3x + 1) = 0$, then x equals:

(A) 2 or –1 (B) 2 or $-\frac{1}{3}$ (C) –2 or –1 (D) –2 or $\frac{1}{3}$

2. The gradient of the line $3x + y = 2$ is:

(A) –3 (B) 3 (C) 2 (D) $\frac{2}{3}$

3. Express $\dfrac{1}{\sqrt{11} - 3}$ with a rational denominator:

(A) $\dfrac{\sqrt{11} + 3}{2}$ (B) $\dfrac{\sqrt{11} + 3}{20}$ (C) $\dfrac{\sqrt{11} - 3}{2}$ (D) $\dfrac{\sqrt{11} - 3}{20}$

4. It is known that an ant travels 10 cm in 20 seconds. Its speed in kilometres per hour is:

(A) 0.018 km/h (B) 0.18 km/h (C) 1.8 km/h (D) 18 km/h

5. A packet contains four balloons: two blue and two red. What is the probability that Fiona selects two balloons at random which are the same colour?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

6. Expand and simplify $4 - 2(5 - 3x)$:

(A) $10 - 6x$ (B) $-6 - 3x$ (C) $-6 + 3x$ (D) $-6 + 6x$

7. If $y = \dfrac{4x + 3}{2x}$, then:

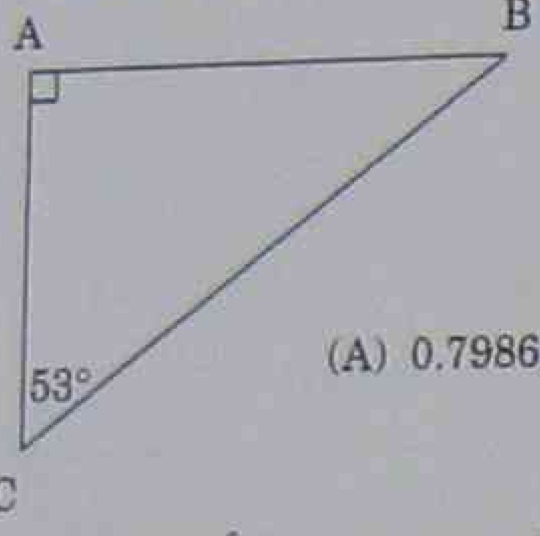
(A) $x = \dfrac{3}{2y - 4}$ (B) $x = \dfrac{y - 3}{2}$ (C) $x = \dfrac{4y + 3}{2y}$ (D) $x = \dfrac{2y - 4}{3}$

8. An income tax scale is as follows:

Taxable income $	Tax
\$20 701 — \$36 000	\$3 060 plus 38 cents for each \$1 over \$20 700
\$36 001 — \$50 000	\$8 874 plus 46 cents for each \$1 over \$36 000
\$50 001 and over	\$15 314 plus 47 cents for each \$1 over \$50 000

Maryanne's taxable income, if she pays \$11 657 tax, is:

(A) \$38 450 (B) \$42 050(C) \$44 010(D) \$44 620

9. 

The ratio of AB to BC is closest to:

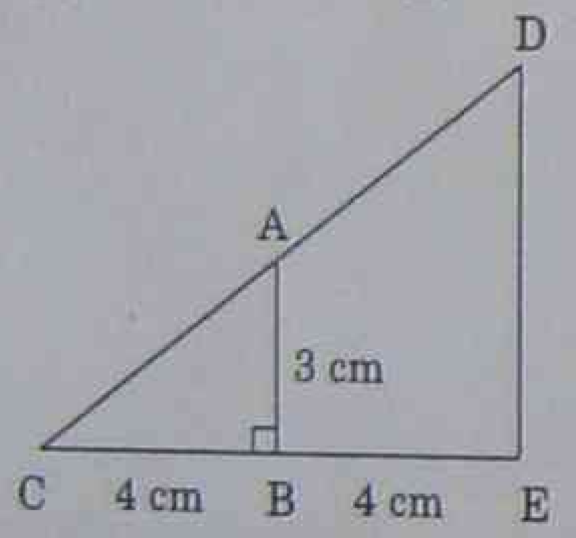
(A) 0.7986 (B) 0.6018 (C) 1.327 (D) 1.2522

10. The term $a^{-\frac{2}{3}}$ can be rewritten as:

(A) $\frac{-2a}{3}$ (B) $\frac{\sqrt[3]{a}}{2}$ (C) $\frac{1}{\sqrt{a^3}}$ (D) $\frac{1}{\sqrt[3]{a^2}}$

11. The value of $\sqrt{(4.2)^2+(3.4)^2-2(4.2)(3.4)(-0.4768)}$ is closest to:

(A) 0.16 (B) 0.4 (C) 6.54 (D) 42.82

12. 

AB is parallel to DE. The length of CD is:

(A) 10 cm (B) 11 cm

(C) 15 cm (D) Not enough information

13. The table shows the amount of energy used per kilogram of body weight per hour of a number of common activities:

Type of activity	*Male* (kJ/kg)	*Female* (kJ/kg)
Sleeping	3.9	3.6
Sitting	6.0	5.6
Jogging	20.2	18.8
Skateboarding	24.8	21.9
Cycling	26.8	24.4
Swimming	30.0	28.0
Sprinting	34.6	31.8

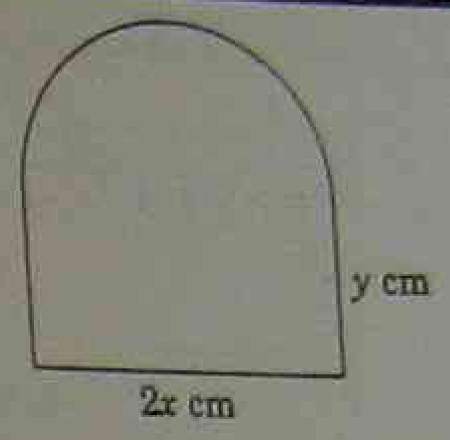
Ken weighs 64 kg and completes an exercise routine which involves a thirty minute jog to the swimming pool, a twenty minute swim and a ten minute ride home in his mother's car. The energy (in kJ) that Ken has expended is approximately:

(A) 60 kJ (B) 1260 kJ (C) 1350 kJ (D) 3600 kJ

14. A shape consists of a semicircle surmounted on a rectangle. The perimeter of the shape would equal:

(A) $(4x + 2y)$ cm (B) $(4\pi x + 2y)$ cm

(C) $(2x + \pi x + 2y)$ cm (D) $(2x + 2\pi x + 2y)$ cm



15. The following statistics were obtained from Year 10 Mathematics and History tests:

Subject	*Mean*	*Standard deviation*
Mathematics	62	7
History	60	8

What mark in History would be equivalent to a mark of 76 in Mathematics?

(A) 70 (B) 74 (C) 76 (D) 78

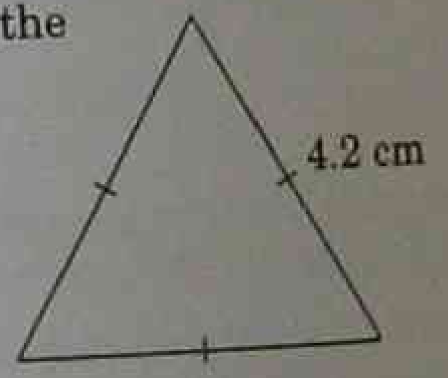
16. Simplify $\frac{3x}{5}-\frac{x}{2}$.

(A) $\frac{x}{10}$ (B) $\frac{2x}{3}$ (C) 1 (D) $\frac{x}{3}$

17. Brett received a 30% discount on a trailer which amounted to a saving of \$27. The price he paid for the trailer was:

(A) \$63 (B) \$57 (C) \$97 (D) \$117

18. The area of the triangle is closest to:

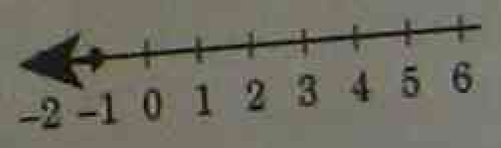


(A) 4.41 cm^2 (B) 7.64 cm^2

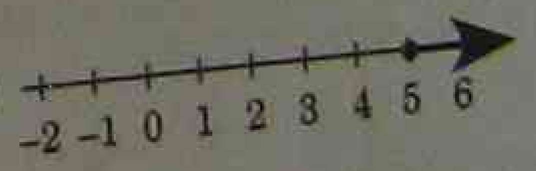
(C) 8.82 cm^2 (D) 15.28 cm^2

19. The solution of $4 - 2x \geq 6$ is represented on the number line as:

(A) 

(B) 

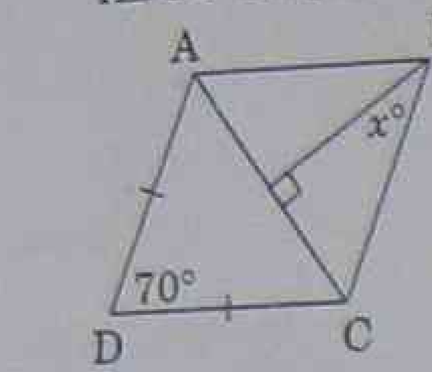
(C) 

(D) 

20. $\left.\begin{array}{r}3x-2y=0\\ x+y=5\end{array}\right\}$ When these simultaneous equations are solved for x:

(A) $x = 10$ (B) $x = 3$ (C) $x = -2$ (D) $x = 2$

21\. ABCD is a parallelogram. The value of x is:



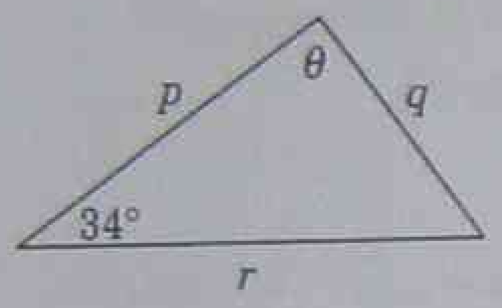
(A) 20 (B) 35

(C) 40 (D) 45

22\. Helen received 8% commission on the first \$400 worth of sales, 6% on the next \$800, and 5% on the remainder. The number of dollars worth of encyclopaedias she sold if she earned \$150 commission was:

(A) \$2300 (B) \$2600 (C) \$3400 (D) \$3940

23\.



The value of $\sin\theta$ is:

(A) $\frac{p \sin 34°}{r}$ (B) $\frac{q \sin 34°}{r}$

(C) $\frac{r \sin 34°}{q}$ (D) $\frac{r \sin 34°}{p}$

24\. In a one hour episode of Rugby League Replay, highlights of a game are shown. During the episode, the ratio of commercials to game was found to be 1 : 4. The fraction of the original game of eighty minutes actually shown during the program is:

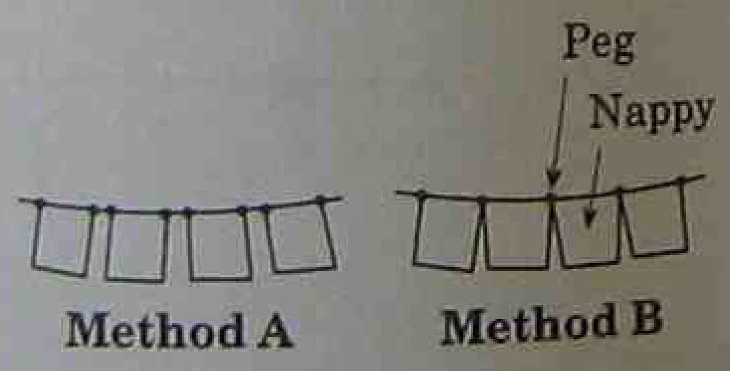
(A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$

25\. Simplify $\frac{a^3b^2}{\left(a^2b\right)^2}$.

(A) ab (B) a (C) $\frac{1}{a}$ (D) ab

26\. There are two ways to hang nappies on a clothes line using pegs.

Method B involves overlapping one nappy onto another and then pegging them on the line. If n nappies are to be hung on a clothes line, how many more pegs would be required using Method A than using Method B.

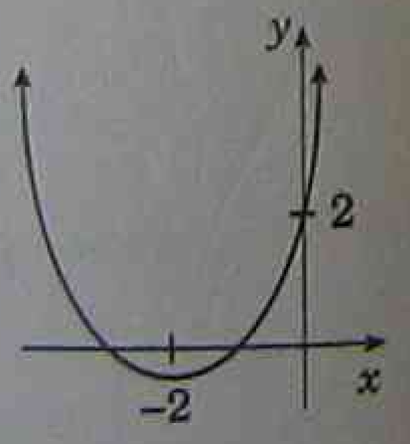


(A) $n - 1$ (B) $n + 1$ (C) $2n - 1$ (D) $2n + 1$

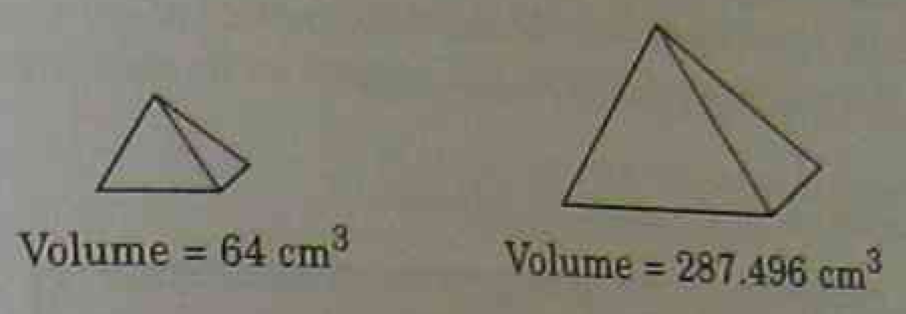
27\. Which of the following is most likely to represent the graph?

(A) $y = x^2 - 4x - 2$ (B) $y = x^2 + 4x - 2$

(C) $y = x^2 - 4x + 2$ (D) $y = x^2 + 4x + 2$



28\. The following solids are known to be similar. If the surface area of the smaller solid is 16 cm^2, the surface area of the larger solid is:



(A) 17.9685 cm^2 (B) 43.56 cm^2 (C) 71.874 cm^2 (D) 95.832 cm^2

29\. Sue purchased a microwave oven by paying \$60 deposit and 18 monthly instalments of \$32.60. How much interest did she pay if the cash price was \$529?

(A) \$57.80 (B) \$117.80 (C) \$177.80 (D) \$192.20

30\. If $x^2 + 8x + p = (x + q)^2$, what is the value of p and q?

(A) $p = 4$ and $q = 16$ (B) $p = 16$ and $q = 16$

(C) $p = 16$ and $q = 4$ (D) $p = 8$ and $q = 4$

Part B Short-answer Questions

31\. (1 mark for each part)

(a) Calculate the distance between (2, −1) and (−3 ,4).

(b) A car leaves Dubbo at 9:30 a.m. and travels 270 km to Wandin's Gap at an average speed of 75 km/h. At what time did the car arrive at Wandin's Gap?

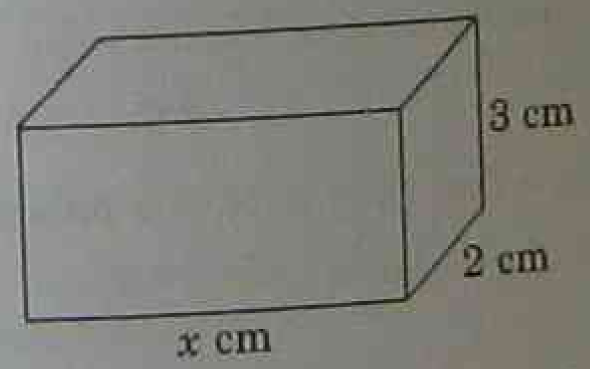
(c) Solve $3x = \frac{x}{2} + 5$.

(d) A four-litre can of white paint costs \$28.20 and has a normal coverage rate of 16 m^2 per litre. Find the cost of painting an area of 96 m^2 with two coats of paint.

(e) Paul's normal hourly rate of pay is \$15.62. What are his weekly earnings if he works 38 hours at normal rate, 6 hours at time-and-a-half and 3 hours at double time?

(f) Given $v = u + at$, find t when $v = 60$, $u = 24$, $a = 6$.

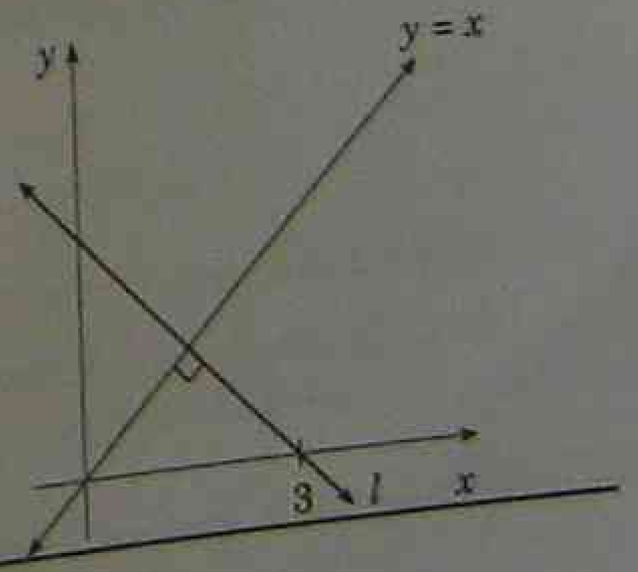
(g)



The surface area of this rectangular prism is 62 cm^2. Find the value of x.

(h) A town B is between towns A and C. If B is 4 times as far from C as from A and the distance from A to C is 65 kilometres, how far is it from A to B?

(i) Find the equation of line ℓ.



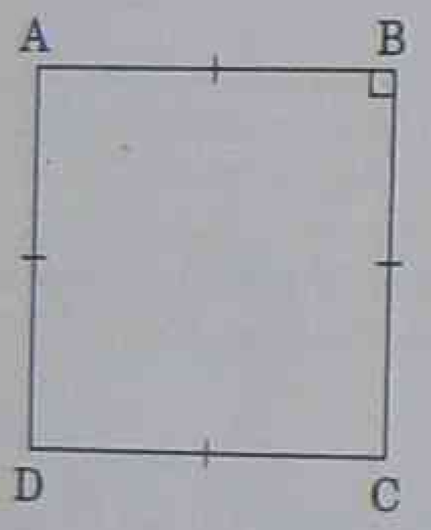
(k) In September, Ken, a real estate salesman sold six properties at an average price of \$204 000. The following month he sold three properties at an average price of \$179 500. Find the average price of all the properties he sold over the two-month period.

(l) The label on a medicine bottle reads as follows:

> VENTOSYRUP
>
> Each 5 mL contains SALBUTAMOL 2 mg.
>
> *Dosage*: To be taken three times a day
>
> *Adults*: 10 mL
>
> *Children*: 0.375 mL per kg body weight
> up to maximum of 10 mL

How many millilitres should a 4 year old boy be given in a single dose if he weighs 19 kilograms?

(m)



Use your geometrical instruments to bisect ∠BCD.

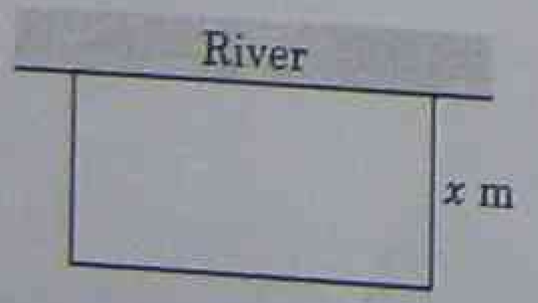
(n) One digit is missing from the stem-and-leaf plot below. The mean of all scores is 13.

0	2 4 4 5
1	0 6 7
2	1 ?
3	0

Find the missing digit.

(o) The sum of two numbers is six but their product is less than 5. Give a possible pair of numbers.

32. (3 marks)



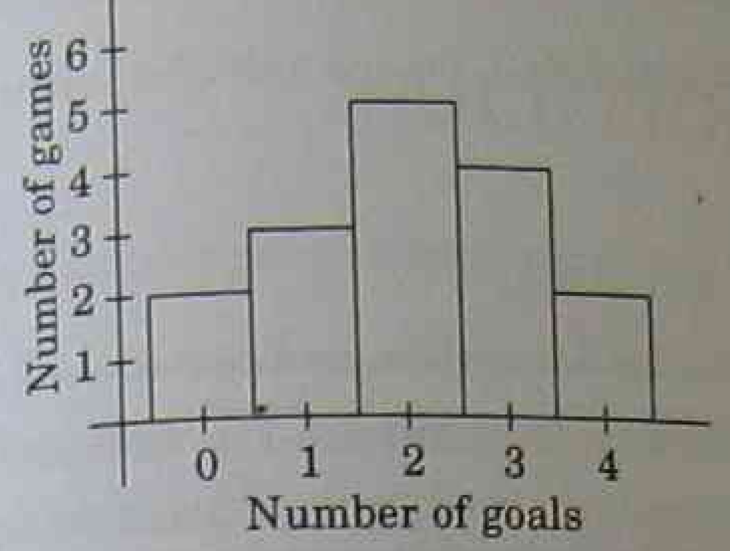
A farmer wishes to fence a rectangular enclosure using an existing river as the fourth side. He has eighty metres of fencing material to use.

Let the width of the enclosure be x metres.

(a) Find an expression for the length of the rectangle in terms of x.

(b) If the length of the enclosure is chosen to be 50 metres, find the width, and hence the area, of the enclosure.

33. (3 marks)



This graph represents the goals scored by a soccer team in its games throughout the season.

(a) How many games were played during the season?

(b) What was the average number of goals scored during the season?

(c) Half the games in the season were played at the team's home ground where two-thirds of the goals were scored. How many goals were scored at these home games?

34. (3 marks)

The height of a cylindrical can varies inversely as the square of its radius;

(a) Write an expression showing the relationship between the height (h) and the radius (r) of the can.

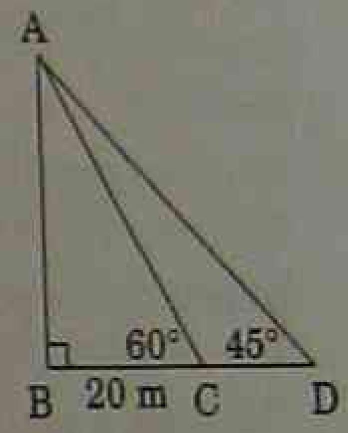
(b) A can with height 14 cm has a radius of 4 cm. Calculate the height of a can with radius 5 cm.

35. (3 marks)

From a point D, the angle of elevation of a tower AB is 45°. Moving to a point C, 20 metres from the base of the tower, the angle of elevation is 60°.

(a) Find the height AB of the tower.

(b) Find the distance between C and D.

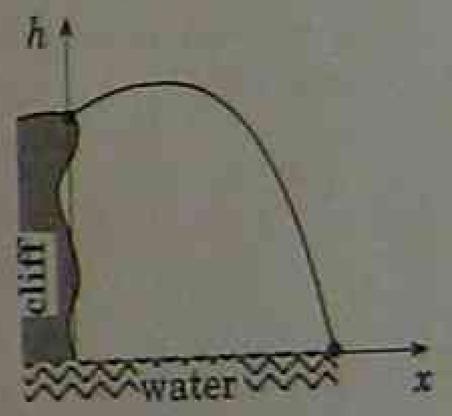


36. (3 marks)

A stone is thrown from a cliff and lands in water at B. For the axes on the diagram, the fligtht path of the ball is the parabola $h = 32 + 4x - x^2$, where x and h are in metres.

(a) Find the distance the stone travels before it lands at B.

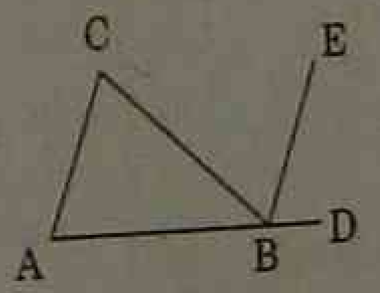
(b) What was the maximum height (above the water) that the stone attained?



37. (3 marks)

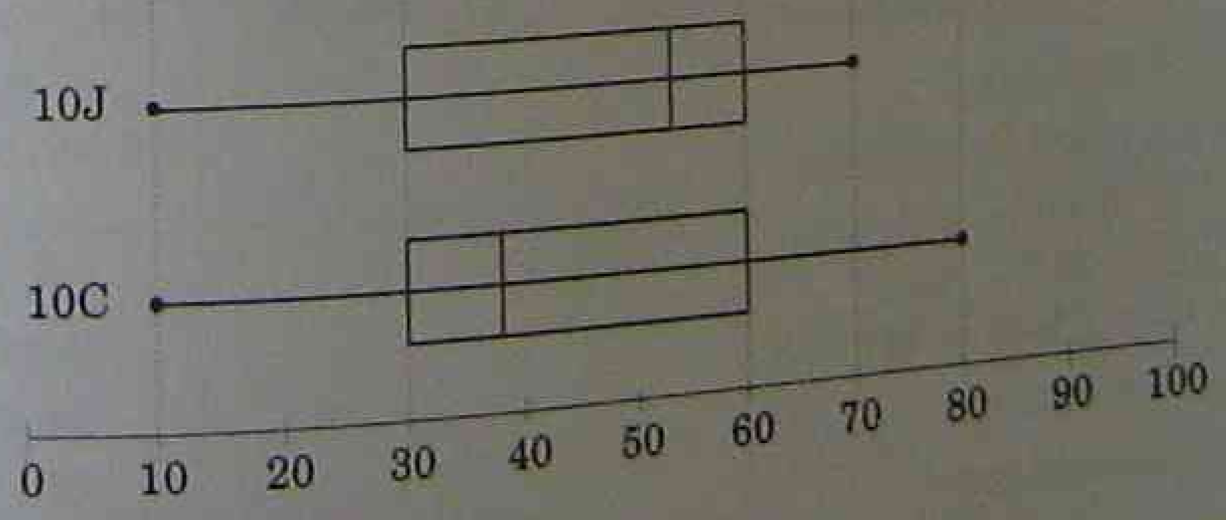
Given AB = BC, BE bisects ∠CBD.

Prove AC || BE



38. (3 marks)

The box-and-whisker plots below show the results for two equal-sized classes on the same mathematics test.



Continued

(a) What was the range of marks for 10C?

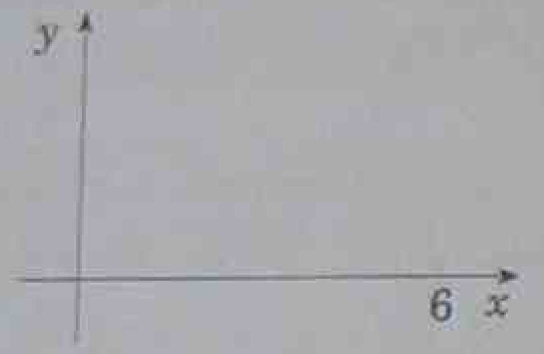
(b) What was the interquartile range for 10J?

(c) The top half of the students in 10J performed better than the top half of students in 10C. Explain how the graph shows this.

39. (3 marks)

It is known that a certain parabola cuts the x-axis at 0 and 6.

(a) Write down equations of two parabolas that fit this description, and sketch them on the number plane.



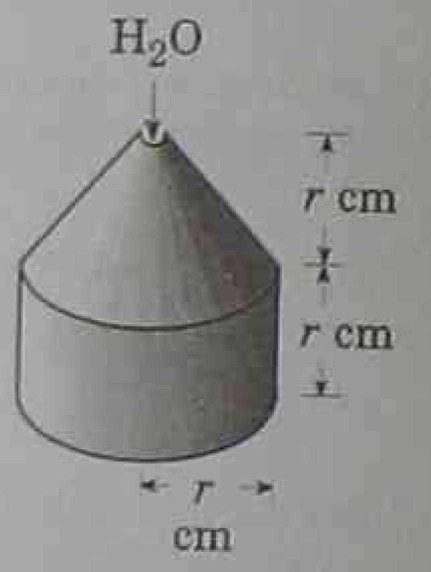
(b) On further investigation, it is discovered that the parabola has a minimum value of −18. Find the equation of the parabola.

40. (3 marks)

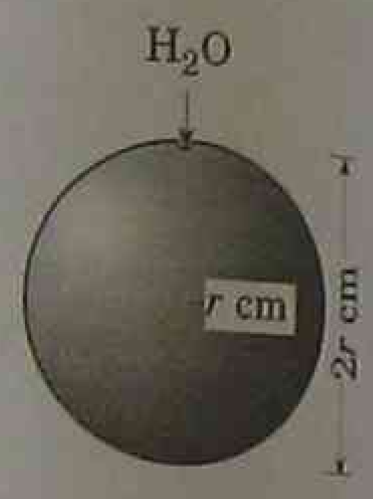
Solid A represents a cone surmounted on a cylinder, solid B represents a sphere.

Water (H_2O) is poured into the solids at a constant identical rate.

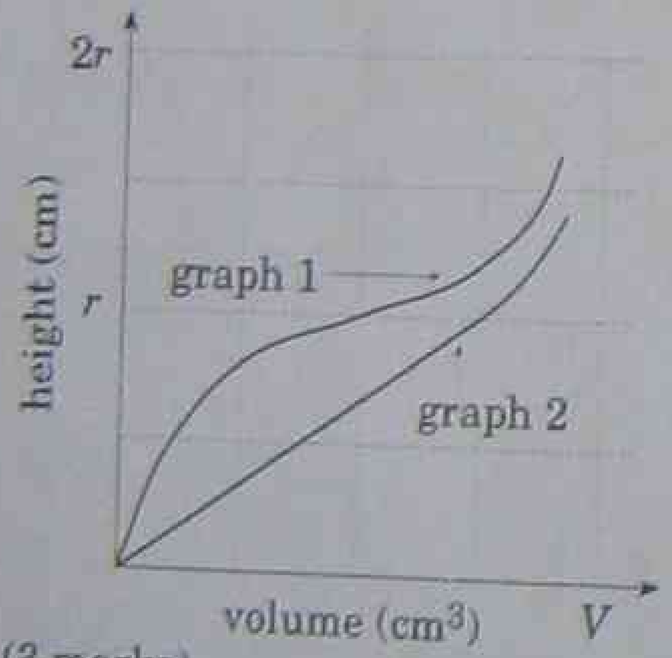
The graph shows the relationship between the volume and the height of the water in each solid.



Solid A



Solid B



(a) Which graph represents the volume/height relationship for solid A?

(b) Find the value of V, in terms of π and r, for solid A.

(c) Give a possible value for r, representing the radius of both solids, if they have the same volume.

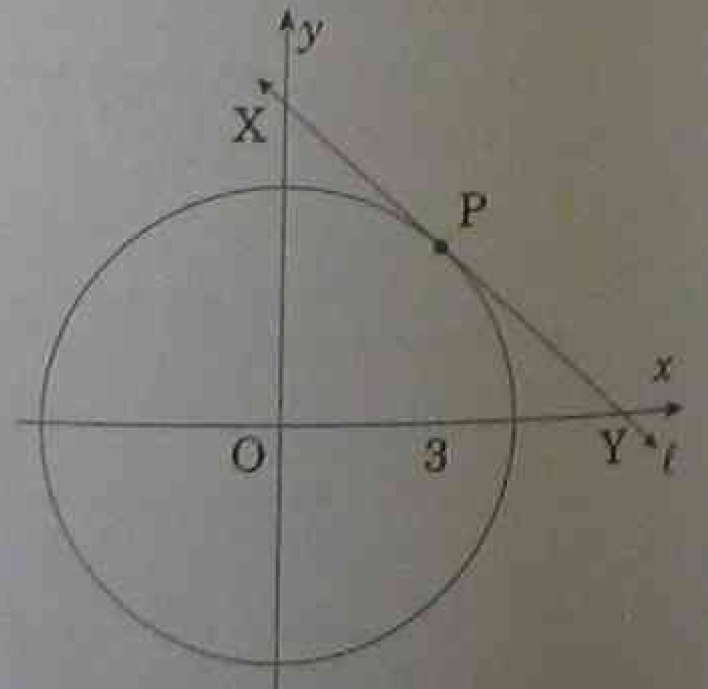
41. (3 marks)

The line l touches the circle $x^2 + y^2 = 25$ at P. It is know that the point P has co-ordinates $(3, a)$.

(a) Find the value of a.

(b) The line l is written in the form
$4x + 3y + k = 0$.
Find the value of k.

(v) Find the area of the triangle OXY.



SOLUTIONS TO EXERCISES

Chapter 1 Consumer arithmetic, rates and variation (page 1)

1. $228
2. $18.50
3. $521
4. $41 925
5. 7.1%
6. $358.40
7. $504.68
8. $550
9. $434
10. $242
11. (a) $3696.88 (b) $14 112.94
12. (a) $5401 (b) $21 792.11
(c) $59 172.34
13. (a) Nothing (b) $51.80
(c) $405.95
14. (a) D (b) C
15. (a) $621.60 (b) $978
16. $5 cheaper
17. $4.92
18. (a) $47.94 (b) $351.56
19. (a) $1707 (b) $207
(c) 7.3%
20. (a) $57.35 (b) 33 cents
(c) 10 cents (d) $10
21. (a) $55.84 (b) $525.73
22. 20 626
23. $130 058.93
24. $202.10
25. $697.64
26. $3272.81
27. $102.08
28. $14 400
29. (a) $39 980 (b) $39 959.80
30. $67.15
31. $330, $313.50
32. 600 mm
33. $176.40
34. (a) $5.92 (b) 337.8 km
35. (a) $4 (b) 375 km
36. (a) 800 km (b) 10.4 L/100 km
37. (a) 27.8 m/s (b) 11.1 m/s
(c) 72 km/h (d) 15 km/h
38. (a) 6.1 m/s (b) 21.96 km/h
39. (a) 1.70 m/s (b) 6.12 km/h
40. 172 people/year
41. $2 in $100
42. (a) 490 km (b) 70 km/h
43. $105.63
44. (a) $987 (b) $136.50

45. $y \propto x$
$y = kx$
When $y = 6, x = 2$
$6 = 2k$
$k = 3$
$\therefore \quad y = 3x$
$\therefore$ when $x = 7, y = 21$

46. Let $d =$ distance, $t =$ time.
$\therefore \quad d \propto t$
$\therefore \quad d = kt$
Substitute $d = 32, t = 5$
$\therefore \quad 32 = k.5$
$\therefore \quad 5k = 32$
$k = 6.4$
$\therefore \quad d = 6.4t$
Now, substitute $t = 8$
$\therefore \quad d = 6.4 \times 8$
$= 51.2$
$\therefore$ man travelled 51.2 km.

47. Let d = distance, r = revolutions

(a) $\therefore\ d \propto r$

$\therefore\ d = kr$

Substitute $d = 48, r = 20$

$\therefore\ 48 = k.20$

$\therefore\ 20k = 48$

$k = 2.4$

$\therefore\ d = 2.4r$

Substitute $r = 12$

$\therefore\ d = 2.4 \times 12$

$= 28.8$

$\therefore$ distance is 28.8 km.

(b) As $d = 2.4r$

substitute $d = 36$

$\therefore\ 36 = 2.4r$

$\therefore\ r = \frac{36}{2.4}$

$= 15$

$\therefore$ 15 revolutions to travel 36 m.

48. $t \propto \sqrt{\ell}$

that is, $t = k\sqrt{\ell}$

Substitute $\ell = 81$ and $t = 2$

that is, $2 = k\sqrt{81}$

that is, $2 = 9k$

$9k = 2$

$k = \frac{2}{9}$

$\therefore\ t = \frac{2}{9}\sqrt{\ell}$

Now, substitute $\ell = 16$

$\therefore\ t = \frac{2}{9}\sqrt{16}$

$t = \frac{2}{9} \times 4$

$= \frac{8}{9}$

$\therefore$ will take $\frac{8}{9}$ second.

49. Let d = distance, h = height

$\therefore\ d \propto \sqrt{h}$

$d = k\sqrt{h}$

Substitute $h = 125, d = 40$

$\therefore\ 40 = k\sqrt{125}$

that is, $40 = k.5\sqrt{5}$

$\therefore\ k = \frac{40^{8}}{{}_{1}5\sqrt{5}}$

that is, $k = \frac{8}{\sqrt{5}}$

$\therefore\ d = \frac{8}{\sqrt{5}}.\sqrt{h}$

Now, substitute $h = 20$

$\therefore\ d = \frac{8}{\sqrt{5}}.\sqrt{20}$

$= \frac{8}{\sqrt{5}}.2\sqrt{5}$

$= 16$

$\therefore$ she can see 16 km.

50. $x \propto \frac{1}{y}$

that is, $x = \frac{k}{y}$

Substitute in $x = 8, y = 9$

$\therefore\ 8 = \frac{k}{9}$

$\therefore\ k = 8 \times 9$

that is, $k = 72$

$\therefore\ x = \frac{72}{y}$

Now, substitute $y = 18$

$\therefore\ x = \frac{72}{18}$

$= 4$

$\therefore\ x = 4$

51. $a \propto \frac{1}{\sqrt{b}}$

that is, $a = \frac{k}{\sqrt{b}}$

Substitute in $a = 9, b = 16$

$\therefore\ 9 = \frac{k}{\sqrt{16}}$

$\therefore\ 9 = \frac{k}{4}$

$\therefore\ k = 36$

$\therefore\ a = \frac{36}{\sqrt{b}}$

Now, substitute in $b = 64$

$\therefore\ a = \frac{36}{\sqrt{64}}$

$= \frac{36}{8}$

$= 4.5$

$\therefore\ a = 4.5.$

52. Let p = pressure, r = radius.

$p \propto \frac{1}{r^2}$

$\therefore\ p = \frac{k}{r^2}$

Substitute $p = 12, r = 2$

$\therefore\ 12 = \frac{k}{2^2}$

$12 = \frac{k}{4}$

$k = 48$

$\therefore\ p = \frac{48}{r^2}$

Now, substitute $r = 3$

$\therefore\ p = \frac{48}{3^2}$

$= \frac{48}{9}$

$= 5\frac{1}{3}$

$\therefore$ pressure of $5\frac{1}{3}$ units.

53. (a) Let light intensity $= i$, distance $= d$

$\therefore\ i \propto \frac{1}{d^2}$

that is, $i = \frac{k}{d^2}$

Substitute in $i = 10, d = 5$

$\therefore\ 10 = \frac{k}{5^2}$

$\therefore\ 10 = \frac{k}{25}$

$\therefore\ k = 250$

$\therefore\ i = \frac{250}{d^2}$

Now, substitute $d = 15$

$\therefore\ i = \frac{250}{15^2}$

$= \frac{250}{225}$

$i = 1.\dot{1}$

$= 1\frac{1}{9}$

$\therefore$ intensity is $1\frac{1}{9}$ units when 15 metres away.

(b) Substitute $i = 14$ in $i = \frac{250}{d^2}$

that is, $14 = \frac{250}{d^2}$

$14d^2 = 250$

$d^2 = \frac{250}{14}$

$\therefore\ d = \sqrt{\frac{250}{14}}$

(Only positive square root required.)

$= 4.225\,7713$

that is, the distance is 4.2 m (to one decimal place).

Chapter 2 Algebra and quadratics (page 19)

1. $(x - y)$ km/hour
2. (a) $(14 + x)$ years old
   (b) $(14 - y)$ years old
   (c) $(p + 14)$ years old
   (d) $(p + 14 + q)$ years old
3. (a) xy m^2 (b) mq m^2
   (c) rmq m^2 (d) $(xy - rmq)$ m^2
4. (a) $10k$ (b) $1440p$
   (c) $1000k$ (d) $\frac{r}{100}$
   (e) $\frac{v}{1000}$
5. $(16\,400 - 1000x + y)$ grams
6. $(100x + y)$ cents
7. $\frac{\$3c}{20}$
8. $180° - (p° + q°)$

9. (a) st km (b) $\frac{k}{p}$ km/h
(c) $\frac{b}{c}$ hours

10. (a) 30 (b) −13 (c) $-\frac{5}{6}$
(d) 1 (e) −5 (f) 17

11. (a) $6y + 4x$ (b) $8y^2 - 6y$
(c) $9xy$ (d) $-6ab$
(e) $25y^2$ (f) $7a^3$
(g) $-24x^2y$ (h) $2b$
(i) $-3y$ (j) 2

12. (a) $\frac{7x}{6}$ (b) $\frac{3a}{4}$
(c) $\frac{11x}{26}$ (d) $\frac{2}{x}$
(e) $-\frac{1}{2a}$ (f) $\frac{ad - bc}{bd}$
(g) $-\frac{2a}{15b}$ (h) 4
(i) $\frac{2}{3m}$ (j) 1
(k) $7\frac{1}{2}$ (l) $\frac{1}{6}$
(m) $1\frac{7}{9}$

13. (a) $8x - 28$ (b) $-15a + 36$
(c) $12y - 1$ (d) $12 - 5x$
(e) $4 - 3x$ (f) $6x - 14$
(g) $16a - 49$ (h) $x^2 - 9x + 14$

14. (a) $\frac{7x - 9}{4}$ (b) $\frac{17x - 18}{6}$
(c) $\frac{3a + 1}{3x}$

15. (a) $3x - 7$ (b) $14y - 9$
(c) $8x - 8x^2 - 2$

16. (a) $a^2 + 2a - 8$ (b) $2x^2 + 5x - 3$
(c) $4x^2 + 25x - 56$ (d) $11x + 4 - 3x^2$
(e) $x^2 - 8x + 16$ (f) $16x^2 + 8x + 1$
(g) $4y^2 - 1$ (h) $25y^2 - 49$

17. (a) $(2x^2 + 7x - 4)$ cm^2
(b) $(9y^2 + 24y + 16)$ m^2

18. (a) $8x - 16$ (b) $-4x$
(c) $x^2 + 2 + \frac{1}{x^2}$ (d) $2ab - 2a^2 - 2b^2$
(e) $3x^2 - 24x + 48$ (f) $-4a + 4$

19. (a) $5(x - 2y)$ (b) $2(4a - 3)$
(c) $4y(3x - 2)$ (d) $5y(3y - 2)$
(e) $-2(5a + 9)$ (f) $2yz(2y - 3)$
(g) $3(x^2 - 3x - 16)$ (h) $\pi r(r - 2)$
(i) $2(l + b)$ (j) $(x + y)(3 + x)$
(k) $(a - 7)(a - b)$

20. (a) $(x + y)(2 + a)$ (b) $(b - a)(2 + c)$
(c) $(c + 6)(a - t)$ (d) $(g - 3)(g^2 - 2)$
(e) $(x - y)(y - 7)$

21. (a) $(u - 6)(u + 6)$
(b) $(1 - 5p)(1 + 5p)$
(c) $(2a - 3b)(2a + 3b)$
(d) $(5k - 12m)(5k + 12m)$
(e) $(4x^2 + 1)(2x + 1)(2x - 1)$
(f) $(1 + x^2y^2)(1 - xy)(1 + xy)$
(g) $2(a - 1)(a + 1)$
(h) $5(a - 5)(a + 5)$

22. (a) $(x + 4)(x - 2)$ (b) $(k - 11)(k - 1)$
(c) $(h + 16)(h - 3)$ (d) $(b - 11)(b + 8)$
(e) $(x - 2y)(x - 4y)$ (f) $(x - y)(x + 4y)$

23. (a) $(2x + 1)(x - 1)$ (b) $(3p + 8)(p + 1)$
(c) $(3c - 4)(c + 3)$ (d) $(9x + 8)(x + 1)$
(e) $(8v + 7)(v - 1)$ (f) $(3b + c)(b + 5c)$

24. (a) $\frac{4}{a - 3}$ (b) $\frac{1}{3}$
(c) $\frac{a - 4}{6}$ (d) $\frac{a - 3}{a + 3}$
(e) $\frac{x + 3}{x - 4}$ (f) $\frac{p + 4}{p - q}$

25. (a) $\frac{1}{2}$ (b) 5 (c) $\frac{(a - 7)(a + 1)}{a(a - 1)}$

26. (a) $\frac{9x + 43}{(x + 3)(x + 7)}$ (b) $\frac{6x - 22}{(x - 3)(x - 5)}$
(c) $\frac{-2x + 10}{(x - 2)(x + 4)}$ (d) $\frac{6c + 34}{(c - 7)(c + 7)(c + 3)}$
(e) $\frac{7y + 8}{(y + 3)(y - 1)(y + 4)}$

27. (a) $\frac{1}{3}$ or −2 (b) 0 or 5
(c) 0 or 3 (d) 2
(e) 3 or 2 (f) ±7
(g) 3 or −3 (h) $-\frac{3}{5}$ or 1
(i) $-\frac{2}{3}$ or $2\frac{2}{3}$ (j) $\frac{3}{4}$ or −4
(k) $\frac{1}{2}$ or 2 (l) 7 or −1
(u) −1 or 4

28. (a) $2 \pm \sqrt{5}$ (b) $4 \pm \sqrt{21}$

29. (a) −0.736 or −2.264
(b) 1.117 or −0.717

30. (a) 9
(b) −6 and −5 or 5 and 6
(c) 6 and 4
(d) (i) Length = $(x + 8)$ cm,
(ii) 20 cm and 12 cm
(e) 9 cm (f) 84°, 84°, 12°
(g) (−5,−7) and (1,5)
(h) Integer is 5

31. (a) $x^2 - 6x + \underline{9} = (x - 3)^2$
(b) $x^2 + 2x + \underline{1} = (x + 1)^2$
(c) $x^2 - 7x + \underline{\left(\frac{7}{2}\right)^2} = \left(x - \frac{7}{2}\right)^2$ (Add $\frac{49}{4}$)
(d) $x^2 + 3x + \underline{\left(\frac{3}{2}\right)^2} = \left(x + \frac{3}{2}\right)^2$ (Add $\frac{9}{4}$)

Chapter 3 Real numbers surds and indices (page 38)

1. (a) A : $\frac{1}{5}$ B : $\frac{7}{10}$
(b) A : $\frac{1}{4}$ B : $\frac{5}{8}$
(c) A : $\frac{1}{3}$ B : $\frac{5}{6}$
(d) A : $\frac{5}{12}$ B : $\frac{5}{6}$

2. (a) 0.625 (b) $0.\dot{4}2857\dot{1}$
(c) $0.\dot{3}\dot{6}$ (d) $0.\dot{5}3846\dot{1}$

3. (a) $\frac{17}{100}$ (b) $\frac{3}{2}$
(c) $\frac{1}{5}$ (d) $\frac{9}{1}$
(e) $\frac{4}{5}$ (f) $\frac{12}{7}$
(g) $\frac{5}{9}$

4. $\sqrt{7}$, 3, $\sqrt{11}$, $\sqrt{12}$, 5

5. (a) $3\sqrt{3}$ (b) $10\sqrt{2}$
(c) $\frac{5}{9}$ (d) 3

6. (a) $6\sqrt{5}$ (b) $-\sqrt{7}$
(c) $10\sqrt{2}$ (d) $(2x - 2)\sqrt{x}$

7. (a) $3\sqrt{5}$ (b) −36
(c) 48 (d) $3\sqrt{y - 2}$
(e) 3 (f) 4

8. (a) $5 - 2\sqrt{5}$ (b) $6 - 4\sqrt{3}$

9. (a) $9 - 4\sqrt{2}$ (b) $21 + 6\sqrt{6}$
(c) −1 (d) $2\sqrt{15} + \sqrt{5} - 2\sqrt{3} - 1$

10. (a) $2\sqrt{3}$ (b) $\frac{4\sqrt{10}}{15}$
(c) $3\left(\sqrt{7} - \sqrt{3}\right)$ (d) $\frac{27 + 10\sqrt{2}}{23}$

11. (a) 311.1696 (b) 5832

12. (a) $3^3 \times 2^3$ (b) $3^2 \times 2^7$

13. (a) k^5 (b) 3
(c) $-12x^3y$ (d) y^6
(e) $8y^{12}$ (f) 3
(g) 1

14. (a) $\frac{1}{25}$ (b) 9 (c) $2\frac{1}{2}$
(d) $\frac{4}{a^3}$ (e) $\frac{1}{125y^3}$ (f) $\frac{1}{x^6}$

15. (a) 8 (b) 3
(c) $5\sqrt[3]{x}$ (d) a^3
(e) $3k^2$ (f) 125
(g) $\frac{1}{32}$

16. (a) 9 (b) 5

17. (a) 6.8431×10^2 (b) 4.798×10^3
(c) 4.307×10^{-1} (d) 8×10^{-5}

18. (a) 3680 (b) 0.040 76
(c) 0.0004

19. (a) 0.000 000 25
(b) 0.000 226 244 3439

20. (a) $3^a = 3^7$ (b) $a^2 = 16^2$
$\therefore\ a = 7$ $\therefore\ a = 16$

(c) $5^{4a} = 5^4$ (d) $10^{2a-1} = 10^4$

$\therefore 4a = 4$ $\therefore 2a - 1 = 4$

$a = 1$ $2a = 5$

$a = 2\frac{1}{2}$

(e) $2^a = 2^{-4}$

$\therefore a = -4$

(f) $5^{a+1} = 5^{-2}$

$\therefore a + 1 = -2$

$\therefore a = -3$

(g) $\left(2^{\frac{1}{2}}\right)^x = 2^5$

$2^{\frac{1}{2}x} = 2^5$

$\therefore \frac{1}{2}x = 5$

$x = 10$

(h) $\left(5^{-\frac{1}{2}}\right)^a = \left(5^2\right)^{2a-5}$

$5^{-\frac{1}{2}a} = 5^{4a-10}$

$\therefore -\frac{1}{2}a = 4a - 10$

$a = -8a + 20$ $(\times -2)$

$9a = 20$ $(+8a)$

$a = \frac{20}{9}$

Chapter 4 Equations and inequalities (page 48)

1. (a) $x = 18$ (b) $y = 8$
(c) $y = -\frac{5}{8}$ (d) $x = 4\frac{1}{4}$
(e) $y = 0$ (f) $a = 3\frac{11}{19}$
(g) $x = 3\frac{5}{12}$ (h) $x = 3\frac{1}{3}$
(i) $y = 1\frac{1}{3}$ (j) $y = \frac{1}{8}$
(k) $a = 1\frac{2}{15}$

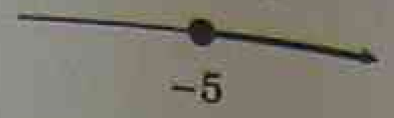
2. (a) $a > 5$ (b) $y \geq -8$
(c) $y \leq -3\frac{1}{2}$ (d) $a \geq -\frac{3}{4}$
(e) $x \leq 0$ (f) $x \geq -\frac{3}{4}$

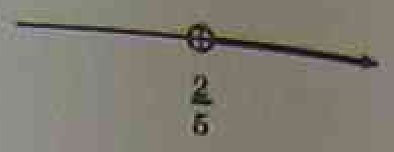
3. (a)

(b)

4. (a) $-2 < x \leq 4$ (b) $-4 \leq x \leq 2$

5. (a) $x \geq -5$





(b) $x > \frac{2}{5}$

6. (a) $5\frac{1}{2} < x < 15$ (b) $0 < x < 3\frac{1}{2}$
(c) $x = 1$

7. 144

8. 1500

9. 158.4

10. (a) $y = 2x + 2$ (b) $y = x^2 + 4$

11. (a) $c = \dfrac{3t}{2} + 2$ (b) \$11

12. (a) $A = 4x^2 + \dfrac{\pi x^2}{2}$
(b) $A = 11x^2$

13. $A = (20 + 2t)(8 + 2t)$

14. (a) $y = 8 - 2x$ (b) $y = \dfrac{5x - 11}{3}$
(c) $y = \dfrac{11 - x}{2x}$ (d) $y = \dfrac{7}{5 - 2x}$
(e) $y = \dfrac{4x}{A}$ (f) $y = \dfrac{4P^2}{x}$
(g) $y = \dfrac{1}{1 - k}$ (h) $y = \dfrac{2x^2}{x^2 - 1}$

15. (a) $h = 5$ (b) $u = 44.1$

16. (a) $d = \dfrac{v^2}{70.56}$ (b) 25 m

17. (a) $x = -2, y = 3$ (b) $x = 6, y = 2$
(c) $x = 1, y = 1$ (d) $x = 2, y = -6$

18. (a) Let number be x.

$\therefore 2x - 5 = x + 4$

$2x - x = 4 + 5$

$x = 9$

$\therefore$ the number is 9.

(b) Let the numbers be x and y.

$\therefore$ $x + y = 12$ (1)

$x - y = 2$ (2)

(1) − (2)

$\therefore$ $2y = 10$

$y = 5$

Substitute in (1)

$\therefore$ $x + 5 = 12$

$x = 7$

$\therefore$ numbers are 7 and 5.

(c) (i) Equilateral triangle

$\therefore$ $3a - 2b = 7$ (1)

$2a + b = 7$ (2)

$2 \times (2)$ $4a + 2b = 14$ (3)

(1) + (3) $7a = 21$

$a = 3$

Substitute in (2)

$\therefore$ $6 + b = 7$

$b = 7 - 6$

$b = 1$

$\therefore$ $a = 3, b = 1.$

(ii)

Opposite sides of rectangle equal.

$\therefore$ $3a - 1 = 3 - 2b$

$5a - 7 = b + 4$

Rewriting these equations:

$\therefore$ $3a + 2b = 4$ (1)

$5a - b = 11$ (2)

$2 \times (2)$ $10a - 2b = 22$ (3)

(1) + (3)

$\therefore$ $13a = 26$

$a = 2$

Substituting in (2):

$10 - b = 11$

$-b = 11 - 10$

$-b = 1$

$\therefore$ $b = -1$

$\therefore$ $a = 2$ and $b = -1.$

(d) Let cost of pens = x cents
cost of pencils = y cents.

$\therefore$ $6x + 5y = 250$ (1)

$3x + 2y = 115$ (2)

$2 \times (2)$ $6x + 4y = 230$ (3)

(1) − (2)

$\therefore$ $y = 20$

Substitute in (2)

$\therefore$ $3x + 40 = 115$

$3x = 115 - 40$

$3x = 75$

$x = 25$

$\therefore$ pens cost 25 c while pencils cost 20 c.

Chapter 5 Graphs (page 60)

1. (a) 300 cans
(b) Refilled at 2 p.m.
$400 - 100 = 300$
$\therefore$ 300 cans.
(c) $(300 - 100) + (400 - 300)$
$= 200 + 100$
$= 300$
$\therefore$ 300 cans
(d) 1 p.m. – 2 p.m. (steepest)

2. (a) \$20 000
(b) \$45 000
(c) Using coordinate geometry formulae:
- **For total costs — two points**

$\therefore$ (0, 20 000) and (45 000, 30 000)

Now using $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$,

i.e. $\dfrac{T - 20\,000}{S - 0} = \dfrac{30\,000 - 20\,000}{45\,000 - 0}$

$\therefore$ $\dfrac{T - 20\,000}{S} = \dfrac{\cancel{10\,000}}{\cancel{45\,000}} = \dfrac{2}{9}$

By cross-multiplying:

$9T - 180\,000 = 2S$

$9T = 2S + 180\,000$

$T = \dfrac{2S + 180\,000}{9}$

$\therefore$ $T = \dfrac{2(S + 90\,000)}{9}$

- **For income** — two points

$\therefore$ (0,0) and (45 000, 30 000)

$$\therefore \quad \frac{I-0}{S-0} = \frac{30\,000-0}{45\,000-0}$$

$$\therefore \quad \frac{I}{S} = \frac{\cancel{30\,000}}{\cancel{45\,000}} = \frac{2}{3}$$

$$\therefore \quad 3I = 2S$$

$$\therefore \quad I = \frac{2S}{3}$$

Hence, $T = \dfrac{2(S+90\,000)}{9}$; $I = \dfrac{2S}{3}$

(d) $S = \$81\,000$

$$\therefore \quad T = \frac{2(81\,000+90\,000)}{9} = \frac{2(171\,000)}{9} = 38\,000$$

$\therefore$ total cost is \$38 000.

Also, $I = \dfrac{2S}{3} = \dfrac{2(81\,000)}{3} = \$54\,000$

$\therefore$ income is \$54 000.

$\therefore$ profit = income − total cost
= \$54 000 − \$38 000
= \$16 000

$\therefore$ profit is \$16 000.

3. (a) 9 p.m. (b) 2 hours
(c) From 10 a.m. to noon

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{200-20}{2} = \frac{180}{2} = 90$$

$\therefore$ speed is 90 km/h.

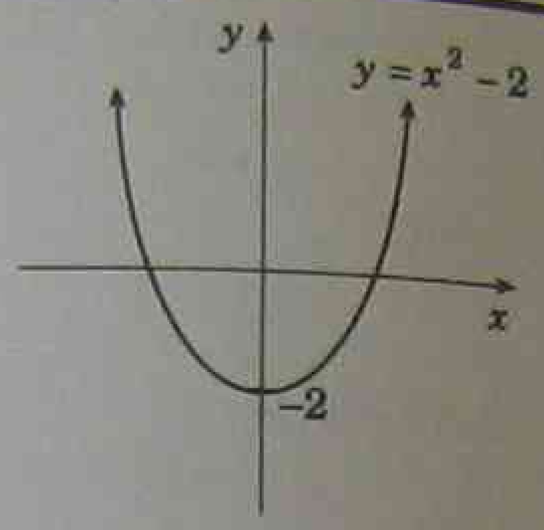
(d) 200 km

(e) Dist. = 200 + 100 + 20 + 20 + 100
= 440

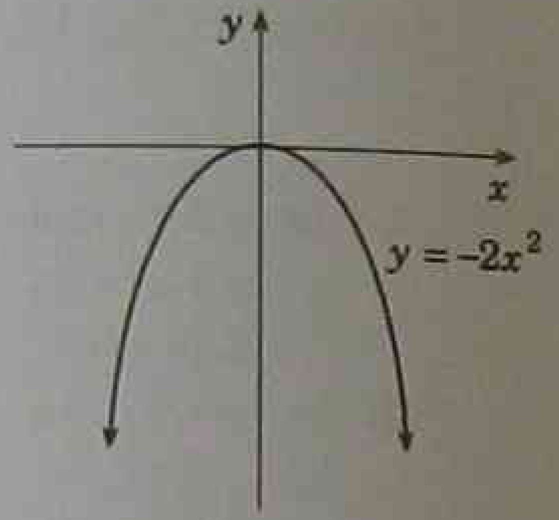
$\therefore$ distance was 440 km.

(f) $A = 440 \times \$0.24$
= \$105.60

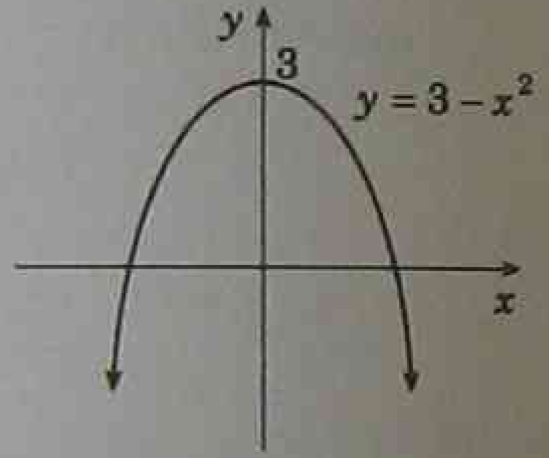
4. (a)



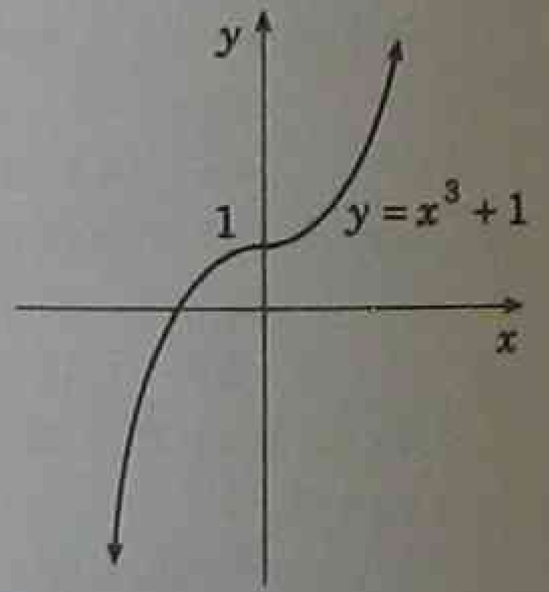
(b)



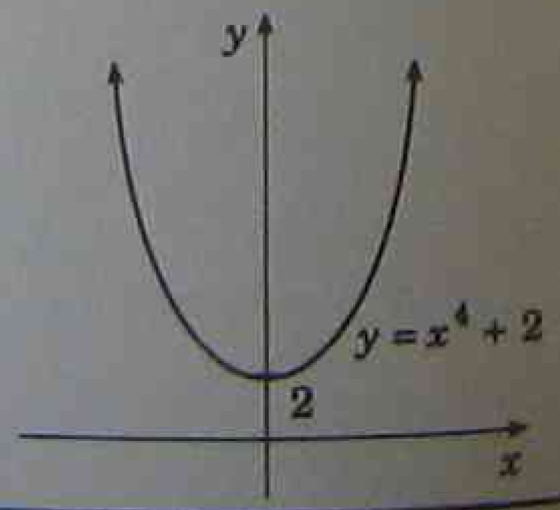
(c)



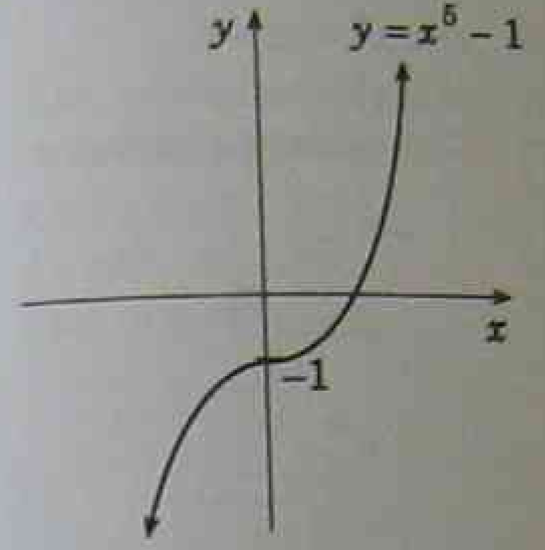
(d)



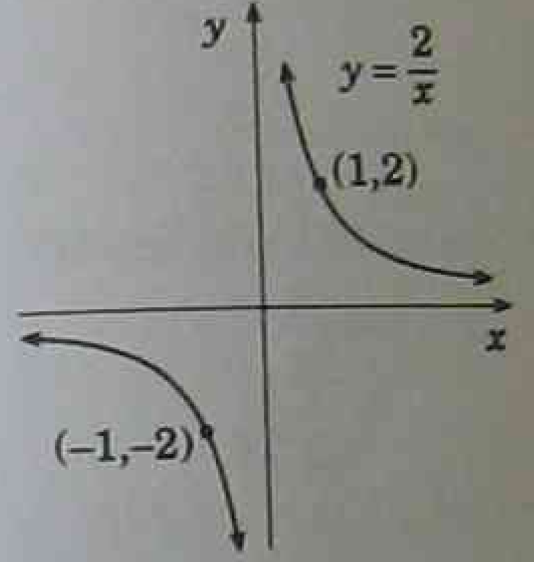
(e)



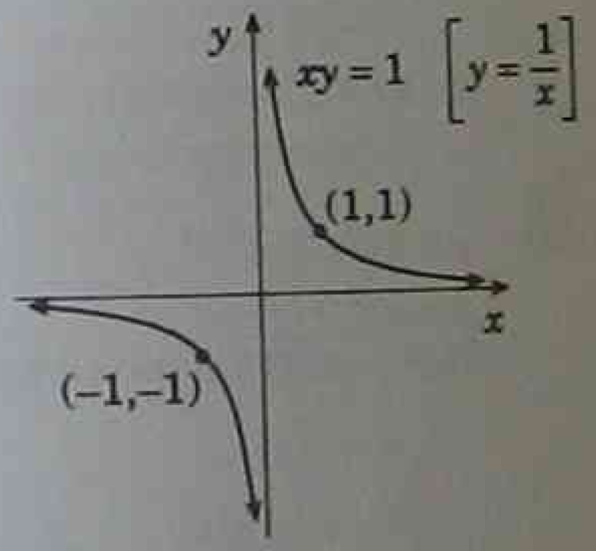
(f)



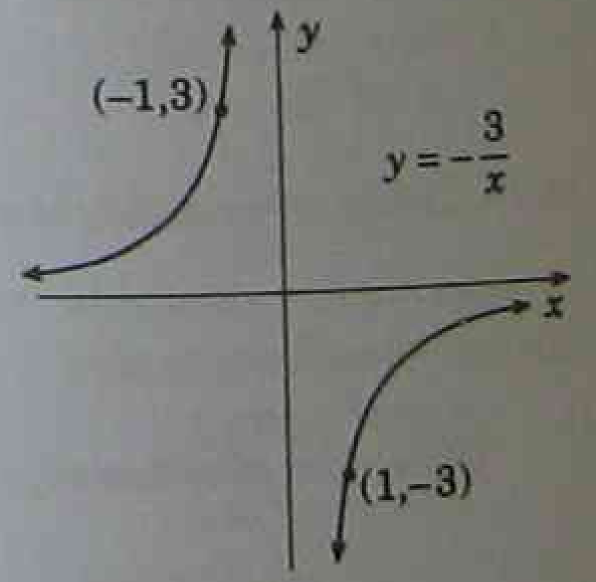
(g)



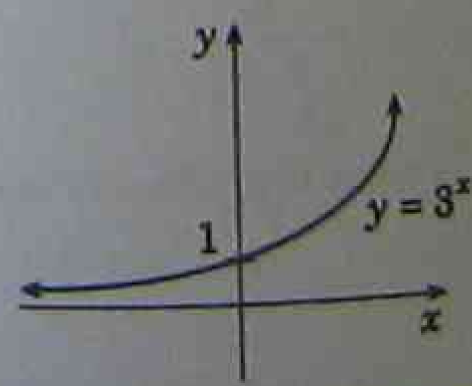
(h)



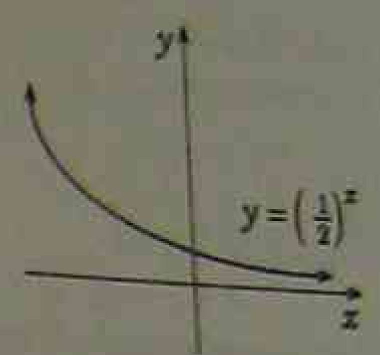
(i)



(j)



(k)

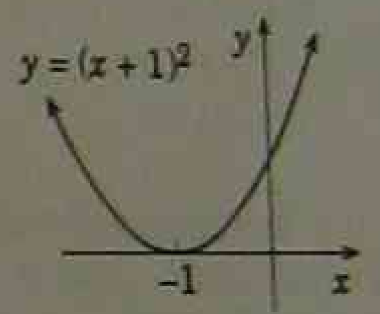


Note: $y=\left(\frac{1}{2}\right)^x$

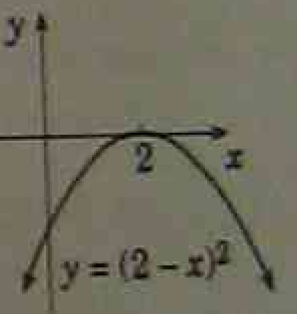
$\therefore\ y=\left(2^{-1}\right)^x$
$=2^{-x}$

$\therefore\ y=\left(\frac{1}{2}\right)^x$ is the same as $y=2^{-x}$.

(l)



(m)



5. (a) $y=x^2-5x+2$
$y=ax^2+bx+c$
$\therefore\ a=1>0$
$\therefore$ concave up.

(b) $y=x-x^2$
$y=ax^2+bx+c$
$\therefore\ a=-1<0$
$\therefore$ concave down.

(c) $y=3+x^2-x$
$y=ax^2+bx+c$
$\therefore\ a=1>0$
$\therefore$ concave up.

(d) $y=4x^2-x+1$
$y=ax^2+bx+c$
$\therefore\ a=4>0$
$\therefore$ concave up.

6\. (a) $y=x^2+x-2$

***x*-intercept** $\therefore\ y=0$

$\therefore\ 0=x^2+x-2$

i.e. $x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2,\ 1$

$\therefore$ x-intercepts of -2 and 1.

***y*-intercept** $\therefore\ x=0$

$\therefore\ y=0^2+0-2$

$y=-2$

$\therefore$ y-intercept of -2.

(b) $y=x^2-x$

***x*-intercept** $\therefore\ y=0$

$\therefore\ 0=x^2-x$

i.e. $x^2-x=0$

$x(x-1)=0$

$x=0,\ 1$

$\therefore$ x-intercepts of 0 and 1.

***y*-intercept** $\therefore\ x=0$

$\therefore\ y=0^2-0$

$y=0$

$\therefore$ y-intercept of 0.

(c) $y=x^2-4$

***x*-intercept** $\therefore\ y=0$

$\therefore\ 0=x^2-4$

i.e. $x^2-4=0$

$x^2=4$

$x=\pm 2$

$\therefore$ x-intercepts of 2 and -2.

***y*-intercept** $\therefore\ x=0$

$\therefore\ y=0^2-4$

$y=0-4$

$=-4$

$\therefore$ y-intercept of -4.

(d) $y=9-x^2$

***x*-intercept** $\therefore\ y=0$

$\therefore\ 0=9-x^2$

i.e. $9-x^2=0$

$x^2=9$

$x=\pm 3$

$\therefore$ x-intercepts of 3 and -3.

***y*-intercept** $\therefore\ x=0$

$\therefore\ y=9-0^2$

$y=9$

$\therefore$ y-intercept of 9.

7\. (a) $y=3x^2-6x+9$

$y=ax^2+bx+c$ $\boxed{\begin{array}{l}a=3\\ b=-6\\ c=9\end{array}}$

Now, axis of symmetry:

$$x=\frac{-b}{2a}=\frac{-(-6)}{2(3)}=\frac{6}{6}=1$$

$\therefore$ axis of symmetry is $x=1$.

(b) $y=4-5x-x^2$

$y=ax^2+bx+c$ $\boxed{\begin{array}{l}a=-1\\ b=-5\\ c=4\end{array}}$

Now, axis of symmetry:

$$x=\frac{-b}{2a}=\frac{-(-5)}{2(-1)}=\frac{5}{-2}=-2\tfrac{1}{2}$$

$\therefore$ axis of symmetry is $x=-2\frac{1}{2}$.

(c) $y=4x^2-8x$

$y=ax^2+bx+c$ $\boxed{\begin{array}{l}a=4\\ b=-8\\ c=0\end{array}}$

Now, axis of symmetry:

$$x=\frac{-b}{2a}=\frac{-(-8)}{2(4)}=\frac{8}{8}=1$$

$\therefore$ axis of symmetry is $x=1$.

(d) $y=4-x^2$

$y=ax^2+bx+c$ $\boxed{\begin{array}{l}a=-1\\ b=0\\ c=4\end{array}}$

Now, axis of symmetry:

$$x=\frac{-b}{2a}=\frac{0}{2(-1)}=0$$

$\therefore$ axis of symmetry is $x=0$.

8\. (a)

Max. value occurs on axis of symmetry, and $a<0$.

$y=4-x-x^2$ ...(1) $\boxed{\begin{array}{l}a=-1<0\\ b=-1\\ c=4\end{array}}$

$y=ax^2+bx+c$

Axis of symmetry:

$$x=\frac{-b}{2a}=\frac{-(-1)}{2(-1)}=\frac{1}{-2}=-\tfrac{1}{2}$$

$\therefore\ x=-\frac{1}{2}$

Now, substitute $x=-\frac{1}{2}$ in equation (1).

$$\therefore\ y(-\tfrac{1}{2})=4-(-\tfrac{1}{2})-(-\tfrac{1}{2})^2=4+\tfrac{1}{2}-\tfrac{1}{4}=4\tfrac{1}{4}$$

$\therefore$ max. value of $4\frac{1}{4}$ (at $x=-\frac{1}{2}$).

(b)

$y=-x^2+4x+2$ ...(1) $\boxed{\begin{array}{l}a=-1<0\\ b=4\\ c=2\end{array}}$

$y=ax^2+bx+c$

Axis of symmetry:

$$x=\frac{-b}{2a}=\frac{-4}{2(-1)}=\frac{-4}{-2}$$

$\therefore\ x=2$

Now, substitute $x=2$ in equation (1).

$$\therefore\ y(2)=-2^2+4(2)+2=-4+8+2=6$$

$\therefore$ max. value of 6 (at $x=2$).

9\. (a) Minimum value occurs on axis of symmetry, and $a>0$.

$y=x^2+6x+12$ ...(1) $\boxed{\begin{array}{l}a=1\\ b=6\\ c=12\end{array}}$

$y=ax^2+bx+c$

Axis of symmetry:

$$x=\frac{-b}{2a}=\frac{-6}{2(1)}=\frac{-6}{2}=-3$$

Now, substitute $x=-3$ in equation (1).

$$\therefore\ y(-3)=(-3)^2+6(-3)+12=9-18+12=3$$

$\therefore$ minimum value of 3 (at $x=-3$).

(b) $y=x^2-2x+3$ ...(1) $\boxed{\begin{array}{l}a=1\\ b=-2\\ c=3\end{array}}$

$y=ax^2+bx+c$

Axis of symmetry:

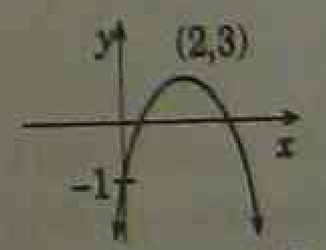
$$x=\frac{-b}{2a}=\frac{-(-2)}{2(1)}=\frac{2}{2}=1$$

Now, substitute $x=1$ in equation (1)

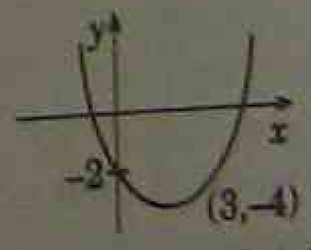
$$\therefore\ y(1)=1^2-2(1)+3=1-2+3=2$$

$\therefore$ min. value of 2 (at $x=1$).

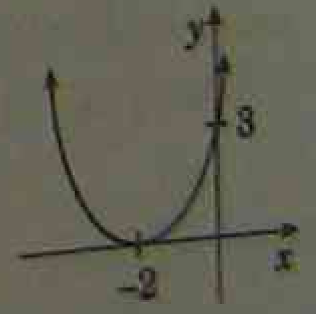
10\. (a) Passes through (2, 3), and y-intercept of -1.



(b) Passes through (3,−4), and y-intercept of -2.



(c) Passes through (−2,0), and y-intercept of 3.



11. (a) $y = x^2 - 5x - 6$

$\therefore \quad y = (x-6)(x+1)$

As $y = 0$

$\therefore (x-6)(x+1) = 0$

$x = 6, -1$

Middle of 6 and -1

$= \frac{6 + {}^-1}{2}$

$= \frac{5}{2} = 2\frac{1}{2}$

Substitute x in $y = (x-6)(x+1)$

$= \left(2\frac{1}{2} - 6\right)\left(2\frac{1}{2} + 1\right)$

$= \left(-3\frac{1}{2}\right)\left(3\frac{1}{2}\right)$

$= -12\frac{1}{4}$

$\therefore$ vertex $\left(2\frac{1}{2}, -12\frac{1}{4}\right)$

(b) $y = x^2 + 2x - 8$

$\therefore \quad y = x^2 + 2x \quad -8$

that is, $y = x^2 + 2x + 1 - 8 - 1$

$y = (x+1)^2 - 9$

$\therefore$ vertex at$(-1, -9)$.

(c) $y = 2x^2 - 3x + 1$

Find equation of axis of symmetry:

$x = \frac{-b}{2a}$

$= \frac{3}{2}$

that is, $x = 1\frac{1}{2}$ gives axis of symmetry.

Substitute in y

$\therefore \quad y = 2\left(1\frac{1}{2}\right)^2 - 3\left(1\frac{1}{2}\right) + 1$

$= 4\frac{1}{2} - 4\frac{1}{2} + 1$

$= 1$

$\therefore$ vertex $\left(1\frac{1}{2}, 1\right)$.

(d) $y = 5 - 2x - x^2$

$\therefore \quad a = -1, b = -2$

$\therefore \quad x = -\frac{b}{2a}$

$= \frac{2}{-2}$

$= -1$

$\therefore$ $x = -1$ is axis of symmetry

Substitute in y

$\therefore \quad y = 5 - 2(-1) - (-1)^2$

$= 5 + 2 - 1$

$= 6$

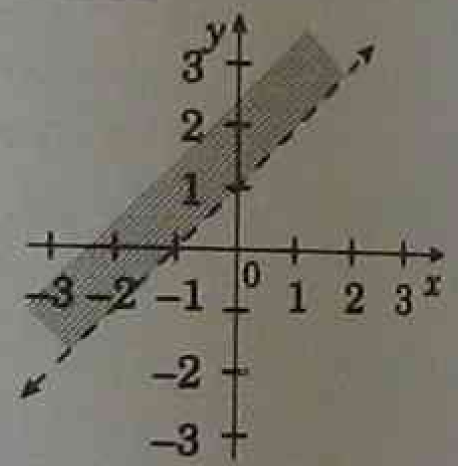
$\therefore$ vertex is $(-1, 6)$.

12. A—9, B—4, C—6, D—8, E—5, F—2, G—1, H—10, I—7, J—3.

13. (a) $\{(x, y): y > x + 1\}$

$y = x + 1$

x	0	1	2
y	1	2	3



[Choose (0,0)

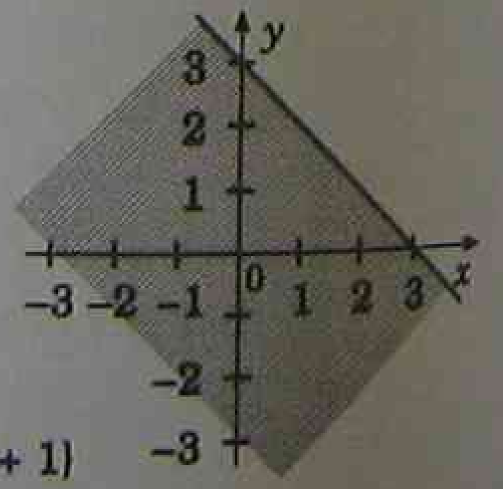
$\therefore 0 > 0 + 1$

No!]

(b) $\{(x, y): y \le 3 - x\}$

$y = 3 - x$

x	0	1	2
y	3	2	1



[Choose (0,0)

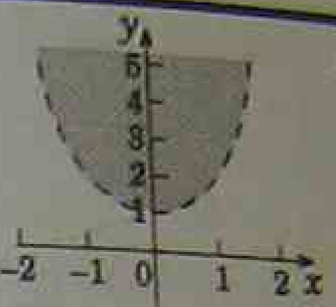
$\therefore 0 \le 3 - 0$

Yes!]

(c) $\{(x, y): y > x^2 + 1\}$

$y = x^2 + 1$

x	-2	-1	0	1	2
y	5	2	1	2	5



[Choose (0,0)

$\therefore 0 > 0^2 + 1$

No!]

14. $y > x + 7$

Substitute coordinates of each point into inequality:

- $(2, 1) \therefore \quad x = 2, y = 1$
  that is, $1 > 2 + 7$? No.
- $(3, -1) \therefore \quad x = 3, y = -1$
  that is, $-1 > 3 + 7$? No.
- $(-1, 12) \therefore \quad x = -1, y = 12$
  that is, $12 > -1 + 7$? Yes.

$\therefore$ $(-1, 12)$ is only point in the region $y > x + 7$.

15. For $y \le x^2 - x - 1$, substitute in $(3, -1)$, that is,

$x = 3, y = -1$

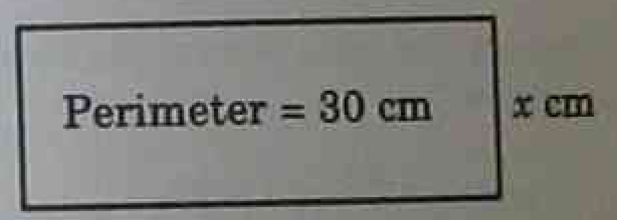
$\therefore \quad -1 \le 3^2 - (3) - 1$

$-1 \le 9 - 3 - 1$

$-1 \le 5$? Yes.

$\therefore$ the region contains $(3, -1)$.

16.



(a) 2 length + $2x = 30$

i.e. $2l + 2x = 30$

$2l = 30 - 2x$

$l = \frac{\cancel{2}(15 - x)}{\cancel{2}}$

$\therefore \quad l = 15 - x$

$\therefore$ length $= (15 - x)$ cm.

(b) $A = (15 - x) \cdot x$

$= x(15 - x)$

$\therefore$ area is $x(15 - x)$ cm^2.

(c) Width $= x = 6$

$\therefore$ area $= x(15 - x)$

$= 6(15 - 6)$

$= 6(9)$

$= 54$

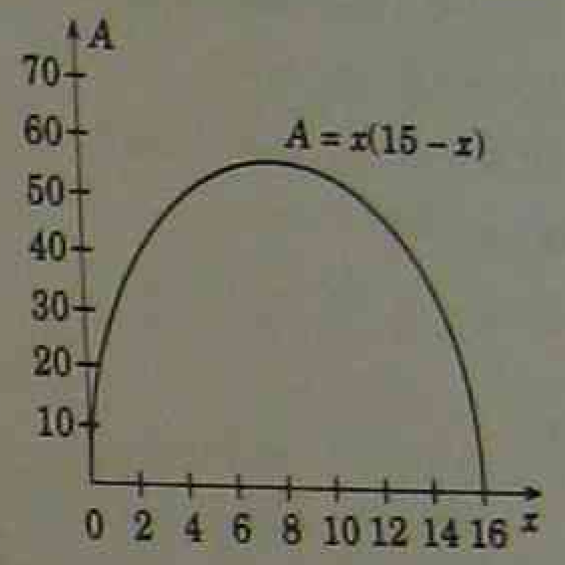
$\therefore$ area is 54 cm^2.

(d) $A = x(15 - x)$

x	0	1	2	3	4	5	6	7	8
A	0	14	26	36	44	50	54	56	56

Continued

9	10	11	12	13	14	15
54	50	44	36	26	14	0



(e) $A = x(15 - x)$

$= 15x - x^2$

$\therefore \quad A = 15x - x^2$

$\therefore$ axis of symmetry:

$x = \frac{-b}{2a}$

$= \frac{-15}{2(-1)}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

Substitute $x = 7\frac{1}{2}$ in A

$A = x(15 - x)$

$= 7\frac{1}{2}\left(15 - 7\frac{1}{2}\right)$

$= 7\frac{1}{2} \times 7\frac{1}{2}$

$= 56.25$

$\therefore$ maximum value is 56.25,

i.e. largest area is 56.25 cm^2.

17. A. [Slowly increases and then rapidly increases.]

18. B. [Quickly increases, then not as quick, then not as quick again.]

Chapter 6 Coordinate geometry (page 75)

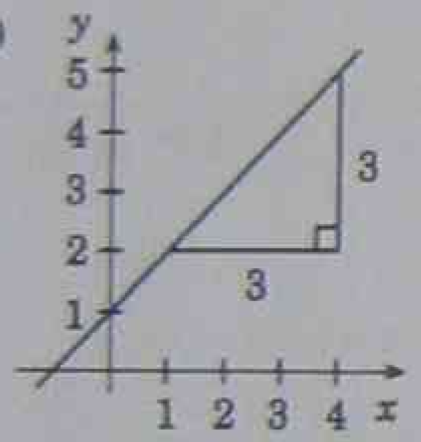
1. (a) $2\sqrt{17}$ (b) 2 (c) 5 (d) $3\sqrt{2}$

2. (a) (3,–2) (b) $\left(-1\frac{1}{2},-1\right)$
(c) (1,4) (d) (2,–2)

3. (a) $m=\frac{1}{4}$ (b) $m=\frac{3}{2}$
(c) $m=-1$ (d) $m=0$ [Line ∥ x-axis.]

4. (a) $m=-\frac{3}{2}$ (b) $m=-2$
(c) $m=-\frac{5}{3}$ (d) $m=\frac{3}{4}$

(e)



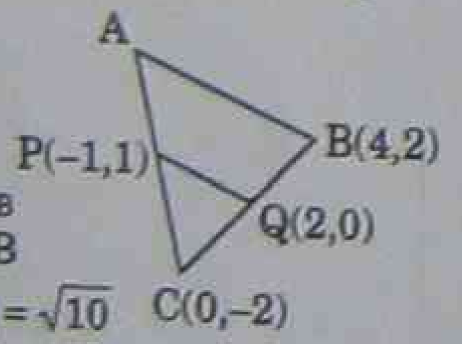
$$\text{Gradient}=\frac{\text{rise}}{\text{run}}=\frac{3}{3}=1,$$
i.e. $m=1$.

5. (a) $AB=\sqrt{40}=2\sqrt{10}$
$AC=\sqrt{40}=2\sqrt{10}$
$\therefore\ AB=AC$
ΔABC is isosceles.

(b) $m_{AB}=-\frac{1}{3}$

(c) Midpoint AC = (–1,1) (Call it P.)

(d) Midpoint BC = (2,0) (Call it Q.)

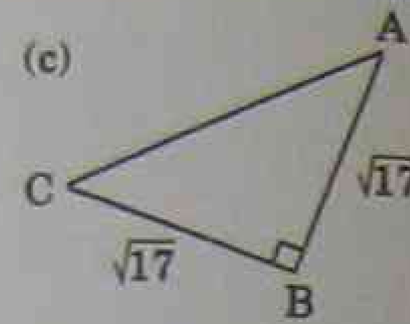


$m_{PQ}=-\frac{1}{3}$
$\therefore\ m_{PQ}=m_{AB}$
i.e. PQ ∥ AB
Distance $PQ=\sqrt{10}$
$=\frac{1}{2}AB$
PQ ∥ AB and equal to half AB.

6. (a) $m_{AB}=4,\ m_{BC}=-\frac{1}{4}$
As $m_{AB}\times m_{BC}=4\times-\frac{1}{4}=-1$
then $AB\perp BC$.
i.e. ΔABC is right angled at B.

(b) $AC=\sqrt{34},\ AB=\sqrt{17},\ BC=\sqrt{17}$
$\therefore\ AC^2=34,\ AB^2=17,\ BC^2=17$
i.e. $AC^2=AB^2+BC^2$.

(c)



$\text{Area}=\frac{1}{2}bh$
$=\frac{1}{2}\times\sqrt{17}\times\sqrt{17}$
$=\frac{17}{2}$
$=8.5\text{ units}^2$.

(d) As AB = BC, ΔABC is isosceles.

7. Using $y=mx+b$
(a) $m=-3$ (b) $m=\frac{1}{4}$
(c) $m=3$ (d) $m=-\frac{1}{5}$

8. $x-2y-4=0$

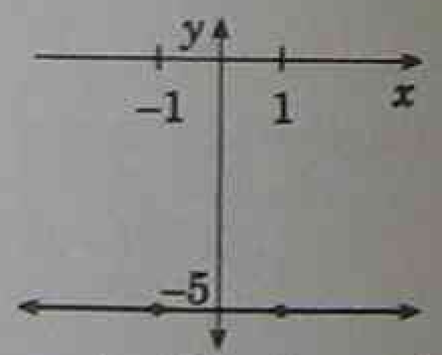
9. $y=-3x+4$ or $3x+y-4=0$

10. Using the point–gradient formula
$y-y_1=m(x-x_1)$:
(a) $2x+y+1=0$
(b) $x-2y=0$
(c) $3x-4y+13=0$

11. $2x-3y-19=0$

12. Using the two-point formula:
(a) $5x-y-6=0$
(b) $x-y+3=0$
(c) $x-6y+8=0$
(d) $\frac{y+5}{x-1}=\frac{-5+5}{-1-1}=\frac{0}{-2}=0$

Consider the points now that you have seen $m=0$.



The required line is parallel to the x-axis and passes through points with y-coordinate -5. The equation is $y=-5$.

13. $2x+y=1$
Substitute each point.
A(–3, 7), LHS = –6 + 7 = 1 = RHS
(–3, 7) lies on $2x+y=1$.
B(3, –7), LHS = 6 – 7 = –1
(3, –7) does not lie on the line.
C(–11, 23), LHS = –22 + 23 = 1
= RHS
(–11, 23) lies on $2x+y=1$.
A and C lie on $2x+y=1$.

14. The equation joining (–1, 3) and (1, 1) is of the form:

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1},$$

i.e. $\frac{y-3}{x+1}=\frac{1-3}{1+1}=\frac{-2}{2}=-1$

$\therefore\ y-3=-1(x+1)$ ☒
$=-x-1$

$\therefore\ x+y-2=0$ is the equation.

Now, for points to be collinear, (4, –2) must lie on this line.

Check (4,–2). $x+y-2=0$
LHS = 4 – 2 – 2 = 0 = RHS
$\therefore$ (4, –2) lies on $x+y-2=0$, i.e. all points are collinear.

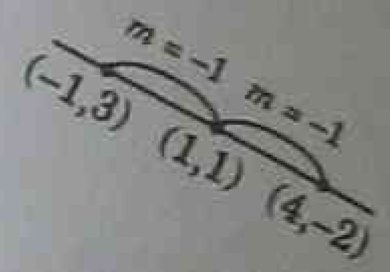
Alternative method using gradients

The gradient of the line joining (–1,3) and (1,1) is:

$$m=\frac{3-1}{-1-1}=\frac{2}{-2}=-1$$

The gradient of the line joining (1,1) and (4,–2) is:

$$m=\frac{1+2}{1-4}=\frac{3}{-3}=-1$$



The line must contain all three points.

15. (a) $2x-3y=7$
$\therefore\ 3y=2x-7$
$y=\frac{2}{3}x-\frac{7}{3}$
Gradient $=\frac{2}{3}$
$6x-12y-5=0$
$12y=6x-5$
$y=\frac{6}{12}x-\frac{5}{12}$
Gradient $=\frac{1}{2}$

Lines are neither parallel nor perpendicular.

(b) $x+2y-3=0$
$\therefore\ 2y=-x+3$
$\therefore\ y=-\frac{1}{2}x+\frac{3}{2}$
Gradient $=-\frac{1}{2}$
$y=2x+1$
Gradient = 2
Now $2\times-\frac{1}{2}=-1$

$\therefore$ lines are perpendicular.

$2x+3y=6$
$3y=-2x+6$
$y=-\frac{2}{3}x+2$
Gradient $=-\frac{2}{3}$
$4y=6x-5$
(c) $y=\frac{3}{2}x-\frac{5}{4}$
Gradient $=\frac{3}{2}$
Now $-\frac{2}{3}\times\frac{3}{2}=-1$

$\therefore$ lines are perpendicular.

16. (a) $y=4x-7,\ m=4$
Line parallel has gradient = 4.
Required line has equation:

$y-y_1=m(x-x_1)$
$y-3=2(x+2)$
$\therefore\ y-3=2x+4$

$\therefore\ 4x-y-4=0$ is the equation.

(b) $2x-3y=5$
$3y=2x-5$
$y=\frac{2}{3}x-\frac{5}{3}$
Gradient $=\frac{2}{3}$
Line parallel will have gradient $=\frac{2}{3}$
The required equation will be:
$y-y_1=m(x-x_1)$
$y+1=\frac{2}{3}(x+1)$
$\therefore\ 3(y+1)=2(x+1)$
$\therefore\ 3y+3=2x+2$
$2x-3y-1=0$ is the equation.

(c) $x+3y-6=0$

$\therefore \quad 3y=-x+6$

$y=-\frac{1}{3}x+2$

Gradient $=-\frac{1}{3}$

The perpendicular line will have gradient 3.

$\boxed{3\times-\frac{1}{3}=-1}$

Equation is of the form:

$$y-y_1=m(x-x_1)$$
$$y-1=3(x-4)$$
$$\therefore \quad y-1=3x-12$$

$\therefore$ $3x-y-11=0$ is the required equation.

(d) $2y=5-x$

$y=-\frac{1}{2}x+\frac{5}{2}$

Gradient $=-\frac{1}{2}$

The line perpendicular will have gradient 2.

$\boxed{-\frac{1}{2}\times2=-1}$

The equation will have the form:

$$y-y_1=m(x-x_1)$$
$$\therefore \quad y+4=4(x-0)$$
$$\therefore \quad y+4=4x$$

$\therefore$ $2x-y+7=0$ is the required equation.

17. Consider $2x-ky=5$ (1)

$\therefore \quad ky=2x-5$

$y=\frac{2}{k}x-\frac{5}{k}$

The gradient of line (1) $=\frac{2}{k}$

Consider $3x+4y-7=0$ (2)

$\therefore \quad 4y=-3x+7$

$y=-\frac{3}{4}x+\frac{7}{4}$

The gradient of line (2) $=-\frac{3}{4}$

As the lines are parallel, their gradients are equal

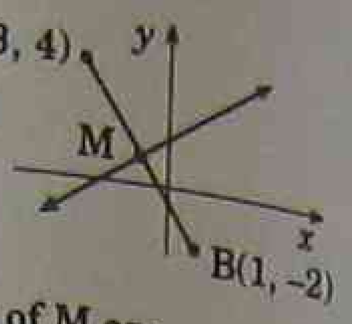
$\therefore \quad \frac{2}{k}=-\frac{3}{4}$ ☒

$-3k=8$

$k=-\frac{8}{3}$

$=-2\frac{2}{3}$

18. Call the midpoint of AB, M.



Then the coordinates of M are

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-3+1}{2},\frac{4+-2}{2}\right)$$
$$=(-1,1)$$

The gradient of AB $=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{4+2}{-3-1}$

$=\frac{6}{-4}$

$=-\frac{3}{2}$

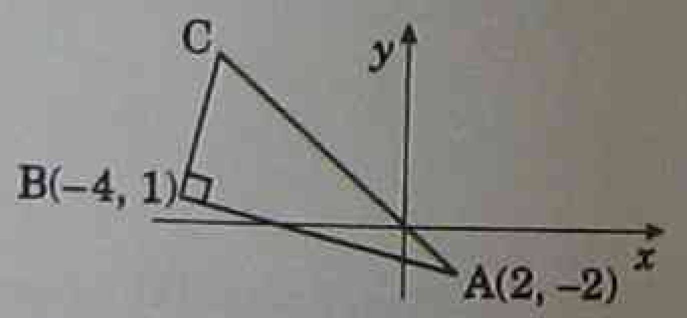
The gradient of the perpendicular $=\frac{2}{3}$ $\boxed{-\frac{3}{2}\times\frac{2}{3}=-1}$

Now the equation of the line ⊥ AB through M is of the form:

$$y-y_1=m(x-x_1)$$
$$y-1=\frac{2}{3}(x+1)$$
$$3(y-1)=2(x+1)$$
$$\therefore \quad 3y-3=2x+2$$

$2x-3y+5=0$ is the equation of the perpendicular bisector.

19.



(a) The equation of AB is of the form:

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$
$$\therefore \quad \frac{y-1}{x+4}=\frac{-2-1}{2+4}=\frac{-3}{6}$$
$$\therefore \quad \frac{y-1}{x+4}=\frac{-1}{2} \text{ [see part (b)]}$$
$$\therefore \quad 2y-2=-x-4$$

☒

$x+2y+2=0$ is the equation of AB.

(b) BC ⊥ AB, hence the gradient of BC will be the negative reciprocal of the gradient of AB.

$m_{AB}=-\frac{1}{2}$ [from part (a)],

$\therefore \quad m_{BC}=2$ $\boxed{-\frac{1}{2}\times2=-1}$

(c) The equation of BC has the form

$$y-y_1=m(x-x_1),$$

that is, $y-1=2(x+4)$

$=2x+8$

$\therefore$ $2x-y+9=0$ is the equation of BC.

(d) Distance BC

$$=\sqrt{(-4+2)^2+(1-5)^2}$$
$$=\sqrt{4+16}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

(e) Distance AB

$$=\sqrt{(2+4)^2+(-2-1)^2}$$
$$=\sqrt{36+9}$$
$$=\sqrt{45}$$
$$=3\sqrt{5}$$

Area of $\Delta ABC=\frac{1}{2}bh$

$=\frac{1}{2}\times AB\times BC$

$=\frac{1}{2}\times3\sqrt{5}\times2\sqrt{5}$

$=15$ square units

$\boxed{\sqrt{5}\times\sqrt{5}=5}$

(f) Midpoint of AB $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$=\left(\frac{-4+2}{2},\frac{1-2}{2}\right)$

$=\left(-1,-\frac{1}{2}\right)$

The points are $\left(-1,-\frac{1}{2}\right)$ and $C(-2,5)$.

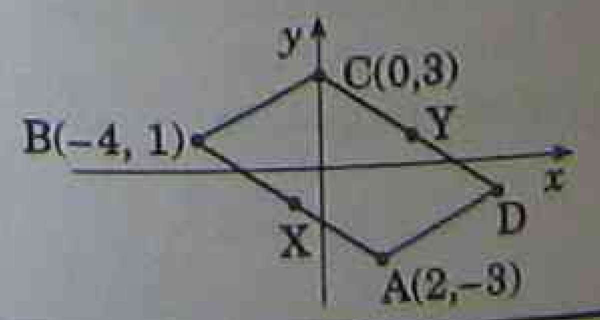
The equation is of the form:

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$
$$\therefore \quad \frac{y-5}{x+2}=\frac{-\frac{1}{2}-5}{-1+2}=\frac{-\frac{11}{2}}{1}$$
$$\therefore \quad \frac{y-5}{x+2}=-\frac{11}{2}$$
$$\therefore \quad 2y-10=-11x-22$$

☒

$11x+2y+12=0$ is the equation of the median.

20.



(a) $m_{AB}=\frac{1+3}{-4-2}$ $\boxed{\frac{y_2-y_1}{x_2-x_1}=m}$

$=\frac{4}{-3}$

$=-\frac{2}{3}$

The equation of AB is of the form:

$y-y_1=m(x-x_1)$

$y+3=-\frac{2}{3}(x-2)$

$3y+9=-2x+4$

Could use either A or B in the formula.

$\therefore$ $2x+3y+5=0$ is the equation of AB.

(b) CD ∥ AB, gradients are equal.

$m_{CD}=-\frac{2}{3}$ also. Using C(0,3), the equation is of the form:

$$y-3=-\frac{2}{3}(x-0)$$
$$\therefore \quad 3y-9=-2x$$

$2x+3y-9=0$ is the equation of CD.

(c) D will be in the same position relative to A as point C is relative to B, that is, right across 4, up 2.

$D=(2+4,-3+2)$

$=(6,-1)$

(d) Midpoint of AB $=\left(\frac{-4+2}{2},\frac{1-3}{2}\right)$

$=(-1,-1)$

Call it X.

The midpoint of CD $=\left(\frac{0+6}{2},\frac{3-1}{2}\right)$

$=(3,1)$

Call it Y.

Using points X and Y:

the gradient of XY $=\frac{-1-1}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$

the gradient of BC $=\frac{3-1}{0+4}=\frac{2}{4}=\frac{1}{2}$.

☒

As the gradients are equal, XY ∥ BC.

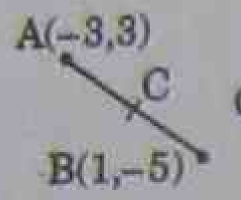
21. The equation of a circle centre (a,b), radius r units, is $(x-a)^2+(y-b)^2=r^2$.

Centre (0,−3), radius 4.

The equation of the circle is:

$$(x-0)^2+(y+3)^2=4^2$$
$$x^2+(y+3)^2=16.$$

22. Let C be the midpoint of AB. C is the centre of the circle, while $\frac{1}{2}$AB will be the radius.



$$C=\left(\frac{-3+1}{2},\frac{3-5}{2}\right)$$
$$=(-1,-1)$$

$$\text{Distance AB}=\sqrt{(-3-1)^2+(3+5)^2}$$
$$=\sqrt{16+64}$$
$$=\sqrt{80}$$
$$=4\sqrt{5}$$

Thus the radius is $\frac{1}{2}\times4\sqrt{5}=2\sqrt{5}$.

The equation of the circle is:

$$(x+1)^2+(y+1)^2=\left(2\sqrt{5}\right)^2$$
$$(x+1)^2+(y+1)^2=20.$$

23. Line PQ will have an equation of the form:

$$\frac{y+3}{x-7}=\frac{1+3}{-5-7}=\frac{4}{-12}$$
$$\therefore\ \frac{y+3}{x-7}=-\frac{1}{3}$$ ☒
$$\therefore\ 3y+9=-x+7$$

$x+3y+2=0$ is the equation of PQ.

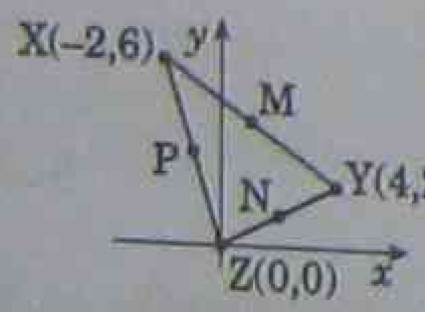
For X to lie on PQ, the coordinates of X must satisfy the equation.

$(-2,0)$ $\quad x+3y+2=0$

$$\text{LHS}=-2+3(0)+2=0$$
$$=\text{RHS}$$

$\therefore$ (–2,0) lies on line PQ.

24.



(a) Using the midpoint formula: $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

The midpoint of XY is:

$$M=\left(\frac{-2+4}{2},\frac{6+2}{2}\right)$$
$$=(1,4)$$

The midpoint of YZ is:

$$N=\left(\frac{4+0}{2},\frac{2+0}{2}\right)$$
$$=(2,1)$$

The midpoint of ZX is:

$$P=\left(\frac{-2+0}{2},\frac{6+0}{2}\right)$$
$$=(-1,3)$$

(b) For the median MZ, use (1,4) and (0,0). The equation is:

Using the two-point form: $\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$ in each case.

$$\frac{y-0}{x-0}=\frac{4-0}{1-0}$$
$$\frac{y}{x}=\frac{4}{1}$$
$$\therefore\ y=4x \text{ or } 4x-y=0.$$

For the median NX, use (2,1) and (–2,6). The equation is:

$$\frac{y-1}{x-2}=\frac{6-1}{-2-2}=\frac{5}{-4}$$
$$\therefore\ \frac{y-1}{x-2}=-\frac{5}{4}$$ ☒
$$4y-4=-5x+10$$

$\therefore$ $5x+4y-14=0$ is the equation of NX.

For the median PY, use (–1,3) and (4,2). The equation is

$$\frac{y-3}{x+1}=\frac{2-3}{4+1}=\frac{-1}{5}$$
$$\therefore\ 5y-15=-x-1$$ ☒

$x+5y-14=0$ is the equation of PY.

(c) For medians to be concurrent they must all pass through the same point. (Call it K.)

Solve MZ and PY to find K.

MZ: $y=4x$ (1)

PY: $x+5y=14$ (2)

Substitute (1) into (2)

$$x+5(4x)=14$$
$$21x=14$$
$$x=\frac{14}{21}$$
$$=\frac{2}{3}$$

Substitute in (1):

$$y=4\left(\tfrac{2}{3}\right)$$
$$=\tfrac{8}{3}$$

K is $\left(\frac{2}{3},\frac{8}{3}\right)$.

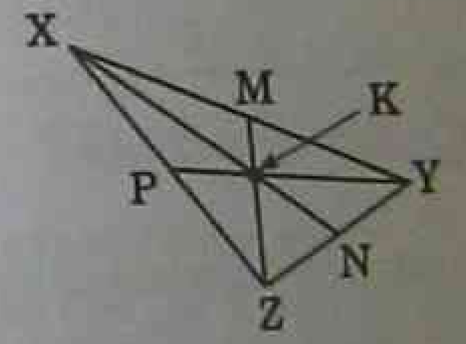
If NX passes through K, all lines pass through K. Test K with NX, $5x+4y-14=0$.

$\left(\frac{2}{3},\frac{8}{3}\right)$. $\quad\text{LHS}=5x+4y-14$

$$=\tfrac{10}{3}+\tfrac{32}{3}-14$$
$$=\tfrac{42}{3}-14$$
$$=14-14$$
$$=0=\text{RHS},$$

that is, NX passes through K, the point of intersection of MZ and PY

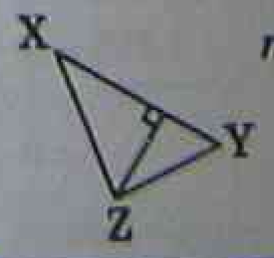
$\therefore$ the medians are concurrent.



(d) $m_{XY}=\frac{6-2}{-2-4}=\frac{4}{-6}$ $\quad$ Using $m=\frac{y_2-y_1}{x_2-x_1}$

$$=-\tfrac{2}{3}$$
$$m_{YZ}=\frac{2-0}{4-0}=\frac{1}{2}$$
$$m_{ZX}=\frac{6-0}{-2-0}=-\frac{3}{1}$$
$$\therefore\ m_{XY}\times m_{YZ}\times m_{ZX}$$
$$=\frac{\cancel{-2}^1}{\cancel{3}_1}\times\frac{1}{\cancel{2}_1}\times\frac{\cancel{-3}^1}{1}=1$$

(e) The altitude through Z will be ⊥ XY and pass through Z.



$m_{XY}=-\frac{2}{3}$, $m_{\text{altitude}}=\frac{3}{2}$.

Using the point–gradient formula with Z(0, 0):

$$y-0=\tfrac{3}{2}(x-0)$$ $\boxed{-\tfrac{2}{3}\times\tfrac{3}{2}=-1}$
$$\therefore\ 2y=3x$$

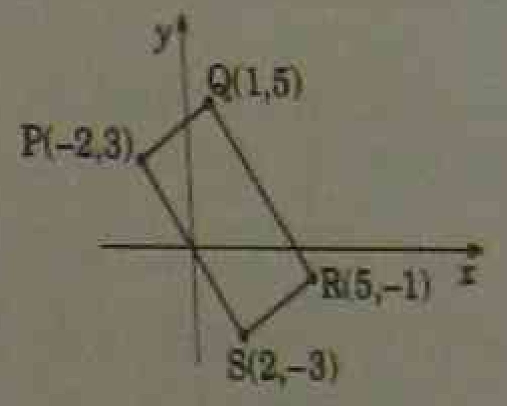
$3x-2y=0$ is the equation of the altitude through Z.

25. The point $(2d,5)$ lies on $2x-y-8=0$, so the coordinates must satisfy the equation.

$(2d,5)$ $\quad 2x-y-8=0$

$$2(2d)-5-8=0$$
$$4d=13$$
$$d=\tfrac{13}{4}=3\tfrac{1}{4}$$

26.



$$m_{PS}=\frac{3+3}{-2-2}=\frac{6}{-4}=-\tfrac{3}{2}$$
$$m_{QR}=\frac{5+1}{1-5}=\frac{6}{-4}=-\tfrac{3}{2}$$

$\therefore$ PS ∥ QR [equal gradients]

$$m_{PQ}=\frac{5-3}{1+2}=\tfrac{2}{3}$$
$$m_{RS}=\frac{-1+3}{5-2}=\tfrac{2}{3}$$

$\therefore$ PQ ∥ RS [equal gradients]

$\therefore$ PQRS is a parallelogram.

But PQ ⊥ PS, as:

$$m_{PQ}\times m_{PS}=\tfrac{2}{3}\times-\tfrac{3}{2}=-1,$$

$\therefore$ PQRS is a rectangle (a parallelogram with one angle a right angle).

27. $m_{AB}=\frac{5-4}{-3-2}=\frac{1}{-5}=-\frac{1}{5}$

A line ⊥ AB will have a gradient of 5.

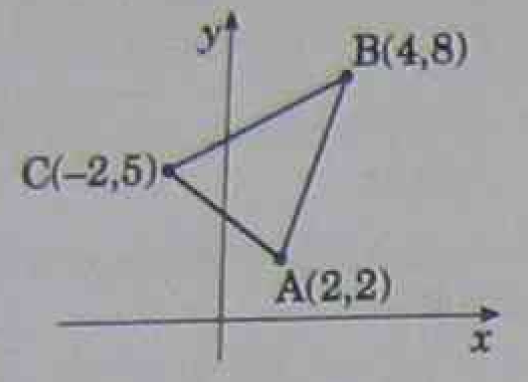
$$\boxed{-\tfrac{1}{5}\times5=-1}$$

The line passes through (0,0) with a gradient of 5.

The equation is $y-0=5(x-0)$

$\therefore$ $y=5x$ is the required equation.

28.



(a) $BC = \sqrt{(4+2)^2 + (8-5)^2}$
$= \sqrt{36+9}$
$= \sqrt{45}$
$= 3\sqrt{5}$

(b) The equation is of the form:

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$
$$\therefore \quad \frac{y-8}{x-4} = \frac{5-8}{-2-4} = \frac{-3}{-6}$$
$$\therefore \quad \frac{y-8}{x-4} = \frac{1}{2}$$
$$\therefore \quad 2y - 16 = x - 4$$

☒

$x - 2y + 12 = 0$ is the required equation.

(c) A line ∥ BC will have the same gradient as BC, that is, $m = \frac{1}{2}$.

Then the equation will be:

$y - 2 = \frac{1}{2}(x-2)$ using A
$\therefore \quad 2y - 4 = x - 2$

$x - 2y + 2 = 0$ is the equation.

(d) A line ⊥ BC will have a gradient of –2. [Negative reciprocal]

The equation will be:

$y - 2 = -2(x-2)$
$y - 2 = -2x + 4$

$\therefore$ $2x + y - 6 = 0$ is the equation.

(e) The midpoint of $AB = \left(\frac{4+2}{2}, \frac{8+2}{2}\right)$
$= (3,5)$

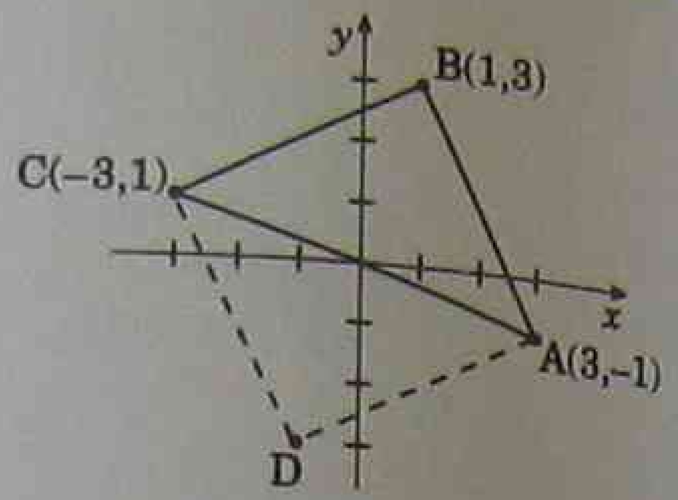
(f) L(3,5), C(–2,5).

The equation is of the form:

$y = b$ as CL is parallel to the x-axis. (Look at the coordinates.)

$\therefore$ $y = 5$ is the equation of the median.

29.



(a) $BC = \sqrt{(-3-1)^2 + (1-3)^2}$
$= \sqrt{16+4}$
$= \sqrt{20}$
$= 2\sqrt{5}$

(b) The midpoint of $BC = \left(\frac{-3+1}{2}, \frac{1+3}{2}\right)$
$= (-1,2)$

(c) The equation of BC is of the form:

$$\frac{y-3}{x-1} = \frac{1-3}{-3-1} = \frac{-2}{-4}$$
$$\therefore \quad \frac{y-3}{x-1} = \frac{1}{2}$$
$$\therefore \quad 2y - 6 = x - 1$$

$x - 2y + 5 = 0$ is the equation of BC.

(d) A line ∥ BC will have the same gradient as BC, that is, m = 1/2 [see (a)]

The equation is of the form:

$y + 1 = \frac{1}{2}(x-3)$
$\therefore \quad 2y + 2 = x - 3$

$x - 2y - 5 = 0$ is the equation of l.

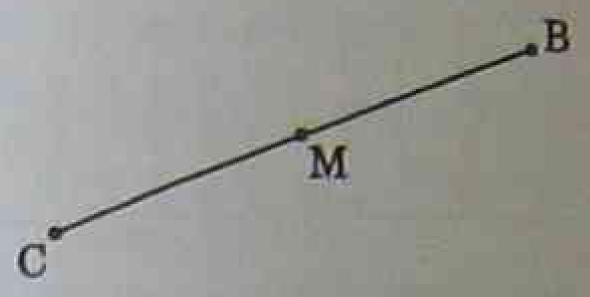
(e) For the coordinates of D, the relationship between C and D is the same as that between B and A, that is: right by 2, down by 4.

Then $D = (-3+2, 1-4)$
$= (-1,-3)$

(f) $(-1,-3)$, $\quad x - 2y - 5 = 0$
LHS $= x - 2y - 5$
$= -1 + 6 - 5$
$= 0 \quad$ = RHS

The point (–1,–3) lies on l.

(g) Call the mid-point of BC, M. Then M is the centre of the circle.



$M = \left(\frac{-3+1}{2}, \frac{1+3}{2}\right)$
$= (-1,2)$ [from (b)]

The radius $= \frac{1}{2}$ of BC
$= \frac{1}{2} \times 2\sqrt{5}$
$= \sqrt{5}$

The equation of the circle is:

$(x-2)^2 + (y+1)^2 = (\sqrt{5})^2$
$(x-2)^2 + (y+1)^2 = 5$

is the equation of the circle with diameter BC.

Chapter 7 Chance and data
(page 93)

1. (a) (i) 5 (from f) (ii) 13 (from c.f.)
(iii) 7

(b) Range = 7 – 2 = 5, Mode = 4 and 5

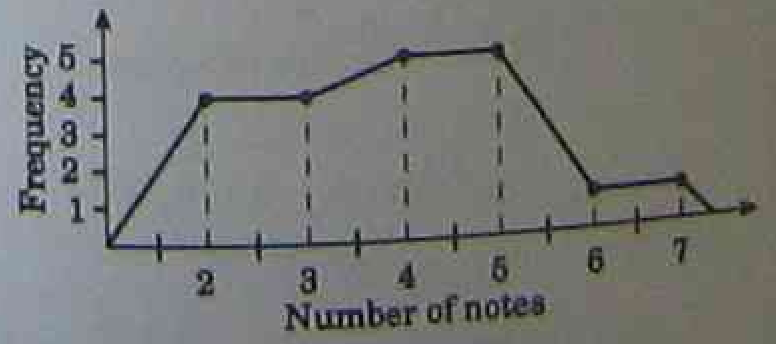
(c) (i) Percentage $= \frac{5}{20} \times 100 = 25\%$

(ii) Percentage $= \frac{13}{20} \times 100 = 65\%$

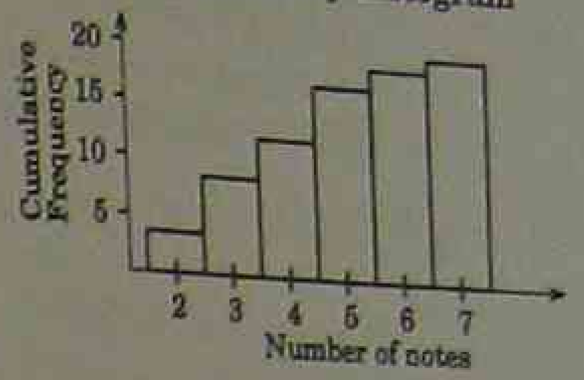
(d) Mean $(\bar{x}) = \frac{\sum xf}{\sum f} = \frac{78}{20} = 3.9$

Median = 4 (from c.f., look for 10th and 11th score. They are both 4.).

(e) Frequency polygon



Cumulative frequency histogram



(f) Probability
$= \frac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}}$
$= \frac{5}{20} = \frac{1}{4}$

2. (a) Range = 7 – 0 = 7, Mode = 4 and 6

(b) (i) 5 (ii) 13 (iii) 8 [3 or less]

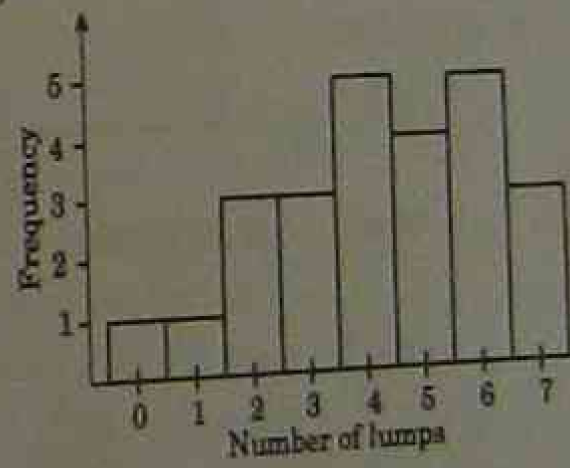
(c) (i) $\frac{5}{25} \times 100 = 20\%$ (ii) $\frac{17}{25} \times 100 = 68\%$

(d) (i) Probability $= \frac{5}{25} = \frac{1}{5}$ (ii) $\frac{8}{25}$

(e) Mean $= \frac{\sum xf}{\sum f} = \frac{107}{25} = 4.28$

Median = 4 [13th score from c.f.]

(f)



3. (a) (i) Mean $= \frac{8+7+4+8+6+3}{6} = 6$

(ii) Range = 8 – 3 = 5, Mode = 8

(iii) 3, 4, $\boxed{6, 7}$, 8, 8

Median $= \frac{6+7}{2} = 6.5$

(b) Scores are now 3, 4, 6, ⑦, 8, 8, 13

Mean $= \frac{49}{7} = 7$ [36 + 13]

Mean has increased by one.

3, 4, 6, 7, 8, 8, 13

Median = 7

Median has increased by 0.5.

(c) Total no. of goals in 8 seasons
$= 6.5 \times 8 = 52$.
Gai scored (52 – 49) goals in the 8th season, that is, 3 goals.

4. (a) Mean $= \frac{48}{9} = 5.3$ (one decimal place)

3, 4, 4, 4 (5) 6, 7, 7, 8

Median = 5

(b) Mean $= \frac{58}{10} = 5.8$

2, 3, 4, 4 (5, 6) 7, 7, 9, 11

Median = 5.5

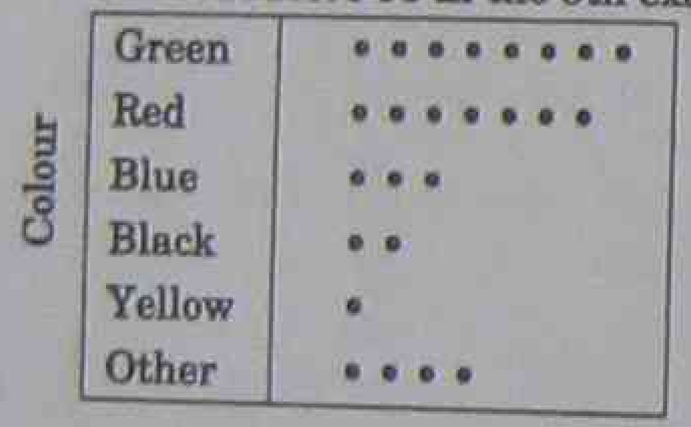
5. Total over 4 exams $= 4 \times 70 = 280$.

Needs to average 75 over 5 exams, that is, total must be $(5 \times 75) = 375$ after 5 exams.

Mark needed $= 375 - 280 = 95$

Zoltan must score 95 in the 5th exam.

6.



7.

→

Shoe size	f
4	2
$4^1/_2$	1
5	5
$5^1/_2$	2
6	3
$6^1/_2$	2
7	5
$7^1/_2$	1
8	1
Total	22

8. Initial plot

Stem	Leaf
14	8 9 7 9
15	0 1 5 4 9 2 8 4 0 2
16	3 2 5 7 8 0

Final plot

Stem	Leaf
14	7 8 9 9
15	0 0 1 2 2 4 4 5 8 9
16	0 2 3 5 7 8

9. Initial plot

Stem	Leaf
2	6 8
3	9 9 3 3 9
4	7 9 7 6 4 8 3 0 7
5	3 7 2 0 2 4 5 0
6	2 4 0 4 1 2

(a) Final plot

Stem	Leaf
2	6 8
3	3 3 9 9 9
4	0 3 4 6 7 7 7 (8 9)
5	0 0 2 2 3 4 5 7
6	0 1 2 2 4 4

⇩

	f	$f \times$ stem	sum of leaves
	2	40	14
	5	150	33
	9	360	51
	8	400	23
	6	360	13
Total	30	1310	134

(b) Range $= 64 - 26 = 38$

Mode = 39, 47

(c) Median between 15th and 16th scores

$$\text{Median} = \frac{48 + 49}{2} = 48.5$$

$$\text{Mean} = \frac{\Sigma(f \times \text{stem}) + \Sigma(\text{sum of leaves})}{\Sigma f}$$

$$= \frac{1310 + 134}{30}$$

$$= 48.1 \text{ (1 decimal place)}$$

10. (i)(a) Rearranged numbers

4, 5, 6, 6, 7, 8, 8, 9

⇑

$$\text{Median} = \frac{6 + 7}{2} = 6.5$$

(Even number of scores)

Consider 4, 5, 6, 6

⇑

$$\text{Lower quartile} = \frac{5 + 6}{2} = 5.5$$

Consider 7, 8, 8, 9

⇑

Upper quartile = 8

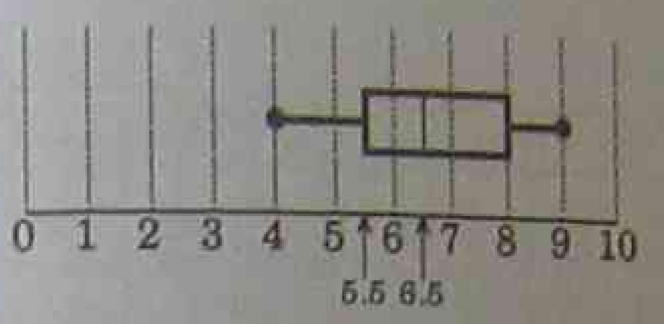
Interquartile range $= 8 - 5.5 = 2.5$

(b) Lowest = 4 highest = 9

Lower quartile = 5.5

Median = 6.5

Upper quartile = 8



(ii)(a) 5, 6, 6, 6, 7, 7, 8, 8, 9, 9, 9

⇑

Median = 7

(Odd number of scores)

Consider 5, 6, 6, 6, 7

⇑

Lower quartile = 6

Consider 8, 8, 9, 9, 9

⇑

Upper quartile = 9

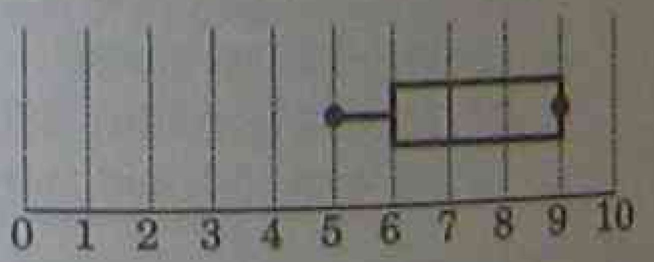
Interquartile range $= 9 - 6 = 3$

(b) Highest = 9 lowest = 5

Lower quartile = 6

Median = 7

Upper quartile = 9



(iii)(a) 12, 14, 14, 15, 16, 16, 17, 18, 18, 19

⇑

Median = 16

Consider 12, 14, 14, 15, 16

⇑

Lower quartile = 14

Consider 16, 17, 18, 18, 19

⇑

Upper quartile = 18

Interquartile range $= 18 - 14 = 4$

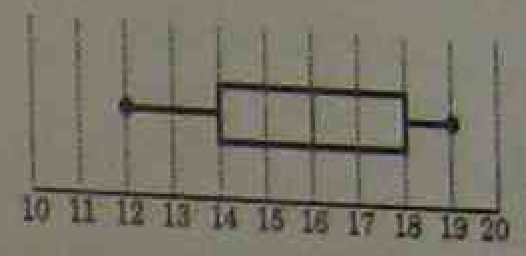
(b) Lowest score = 12

Highest score = 19

Median = 16

Lower quartile = 14

Upper quartile = 18



(iv)(a) 7, 7, 7, 8, 8, 9, 9, 10, 11, 11, 12, 12, 12, 13, 14

⇑

Median = 10

Consider 7, 7, 7, 8, 8, 9, 9

⇑

Lower quartile = 8

Consider 11, 11, 12, 12, 12, 13, 14

⇑

Upper quartile = 12

Interquartile range $= 12 - 8 = 4$

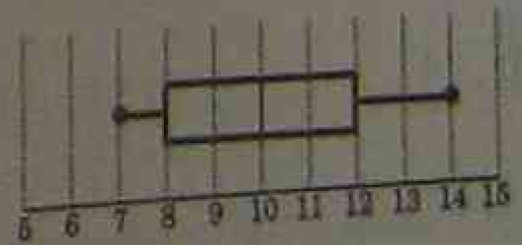
(b) Lowest score = 7

Highest score = 14

Median =10

Lower quartile = 8

Upper quartile = 12



11. (a) 23 scores — middle is 12th score.

Stem	Leaf
1	0 7 8
2	1 2 $\boxed{4}$ 6 9
3	7 7 7 ⑧ 8 9
4	0 0 6 $\boxed{7}$ 8 8 9
5	1 4

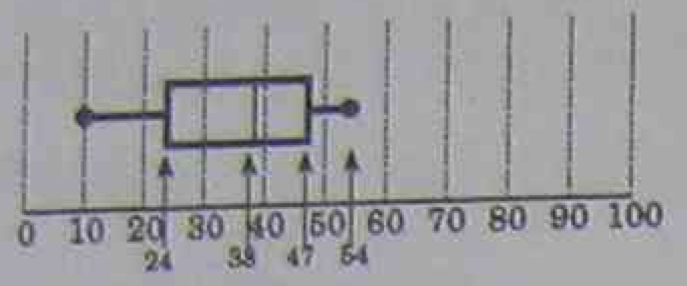
○ Median = 38

☐ Lower quartile = 24

☐ Upper quartile = 47

Lowest score = 10

Highest score = 54



(b) 16 scores — middle is between 8th and 9th scores

Stem	Leaf
10	5 8
11	3 $\boxed{3\ 6}$ 6 9
12	4 4 7
13	5 $\boxed{5\ 6}$ 8 8
14	7

○ Median = 124

☐ Lower quartile

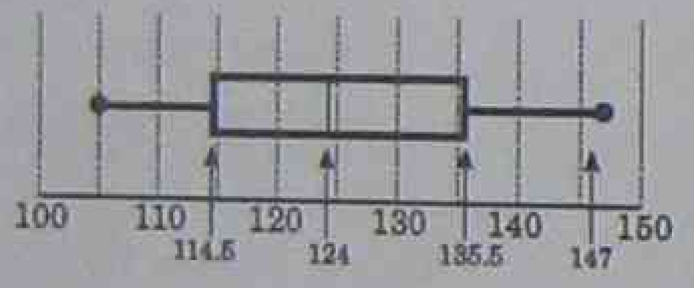
$= \frac{113+116}{2} = 114.5$

☐ Upper quartile

$= \frac{135+136}{2} = 135.5$

Lowest score = 105

Highest score = 147



12. (a) Initial plot

Stem	Leaf
14	9
15	0 0 4 9 8
16	1 1 7 9 7 8 8 6 2 0 9 6 4
17	2 4 0 5 4 2 0
18	0 2 2 1

Final plot

Stem	Leaf
14	9
15	0 0 4 8 9
16	0 $\boxed{1}$ 1 2 4 6 6 7 7 8 8 9 9
17	0 0 2 2 $\boxed{4}$ 4 5
18	0 1 2 2

(b) 30 scores — middle is between 15th and 16th scores.

$$\text{Median } = \frac{167+168}{2} = 167.5$$

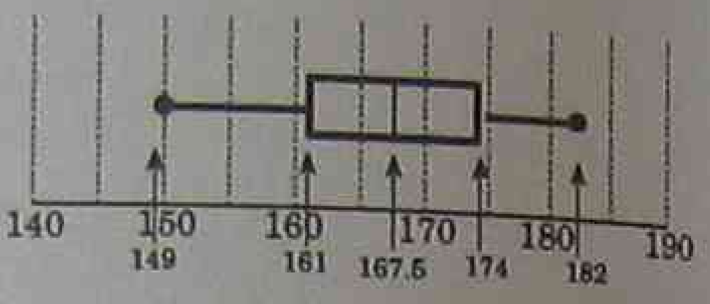
Lower quartile = 161

Upper quartile =174

Interquartile range = 174 – 161 = 13

(c) Lowest score = 149

Highest score = 182



13. (i)

(a) Median = 16

(b) Interquartile range = 18 – 15 = 3

(ii)

(a) Median = 63

(b) Interquartile range = 73 – 57 = 16

14.

11 – 15	13	III	3	3	39
16 – 20	18	~~IIII~~ III	8	11	144
21 – 25	23	~~IIII~~	5	16	115
26 – 30	28	~~IIII~~ ~~IIII~~	10	26	280
31 – 35	33	III	3	29	99
36 – 40	38	I	1	30	38
		∑	30	∑	715

(a) Range = 36 – 11 = 25

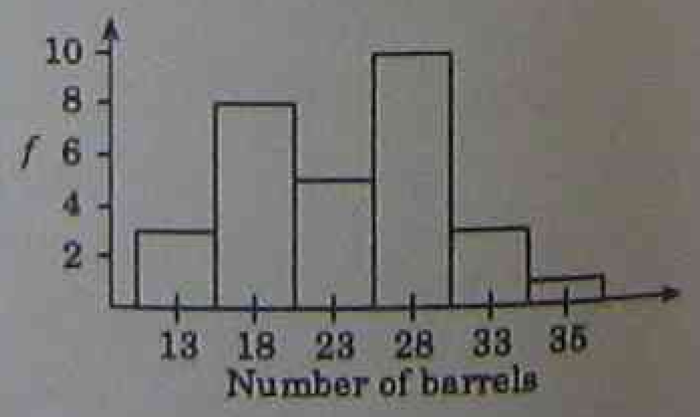
(b) 26 – 30

(c) $\text{Mean} = \frac{\sum xf}{\sum f} = \frac{715}{30} = 23.8$

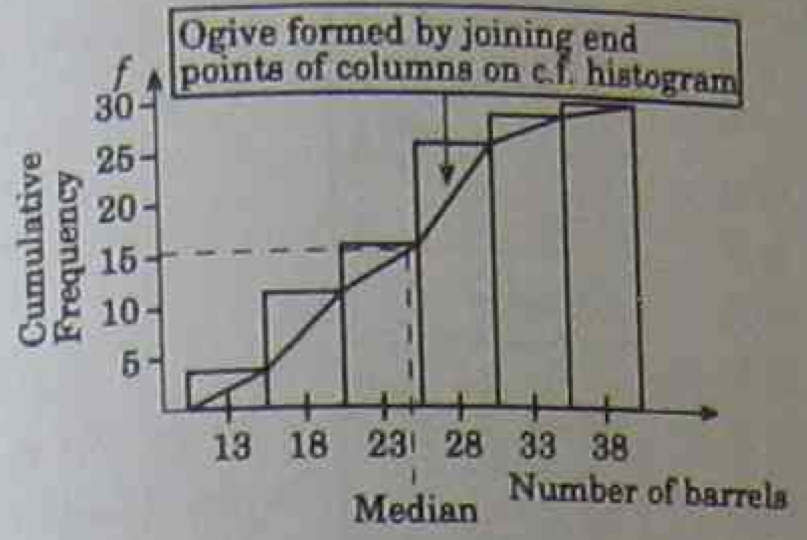
(one decimal place)

(d) Median class = 21 – 25 [Class that contains 15th and 16th scores]

(e)



(f)



15. (a) 20 - 24

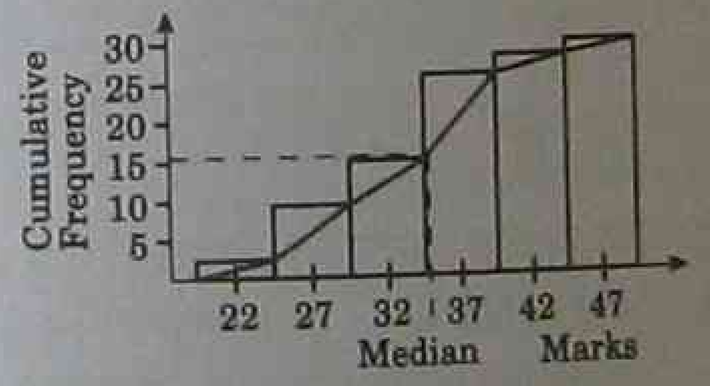
(b)

Class	Centre (x)	Frequency	c.f.	xf
20 – 24	22	2	2	44
25 – 29	27	7	9	189
30 – 34	32	6	15	192
35 – 39	37	12	27	444
40 – 44	42	1	28	42
45 – 49	47	2	30	94
		30		1005

(c) 30 (d) Modal class = 35 – 39

(e) $\text{Mean} = \frac{1005}{30} = 33.5$

(f) Median between 15th and 16th scores



16. (a) $\bar{x} = 16.1$ (one dec. place)
S.D. = 3.4 (one dec. place)
(b) $\bar{x} = 26.8$ (one dec. place)
S.D. = 11.5 (one dec. place)
(c) $\bar{x} = 6.2$ (one dec. place)
S.D. = 2.1 (one dec. place)

17. (a) 800 – 1200 is 1 S.D.

68% fall within 1 S.D. of the mean.

68% of 2000 = 1360

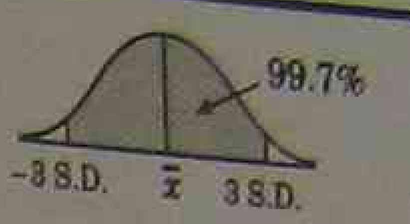
Number of bulbs is 1360.

(b) 3 standard deviations from the mean

99.7% lie within 3 S.D. [0.3% lie outside]

0.3% of 2000 = 6

But 3 lie below 3 S.D., while 3 lie above 3 S.D.



3 bulbs would be expected to last more than 1600 hours.

18. (a) (i) 1 S.D. about the mean 68%

(ii) $\frac{1}{2} \times 32\%$ (100 – 68%) = 16%

16% are less than 10 cm.

(iii) 2 S.D. above mean

95% lie within 2 S.D.

∴ 5% lie outside 2 S.D.

∴ 2.5% are greater than 11.5 cm.

(iv) 99.7% lie within 3 S.D.

(b) (i) 68% × 40 = 27.2 ≈ 27 children

(ii) 16% × 40 = 6.4 ≈ 6 children

19. $\bar{x} = 3.8$, S.D. = 0.02

$\bar{x} + 1$ S.D. = 3.82, $\bar{x} + 2$ S.D. = 3.84,
$\bar{x} + 3$ S.D. = 3.86

$\bar{x} - 1$ S.D. = 3.78, $\bar{x} - 2$ S.D. = 3.76,
$\bar{x} - 3$ S.D. = 3.74

The gauge rejects nails outside 3 S.D. of the mean.

5% of the nails lie outside 2 S.D.

5% of 10 000 = 500

The gauge rejects 500 nails.

$$\text{P(rejected)} = \frac{500}{10\,000} = 0.05$$

∴ P(not rejected) = 1 – 0.05 = 0.95.

20. (a) $\bar{x} = 5$, S.D. = 2.1
New numbers 13, 8, 12, 9, 8
$\bar{x} = 10$, S.D. = 2.1

The S.D. is unchanged; the mean increases by 5.

(b) New numbers 40, 15, 35, 20, 15
$\bar{x} = 25$, S.D. = 10.5

Both the mean and the S.D. have been multiplied by 5.

21.(a)

Number	Frequency	Relative frequency
1	17	0.17
2	22	0.22
3	21	0.21
4	19	0.19
5	21	0.21
Σ	100	1.00

(b) P(2) = 0.22

22. (a) $\frac{4}{52}=\frac{1}{13}$ (b) $\frac{4}{52}=\frac{1}{13}$ (c) $\frac{13}{52}=\frac{1}{4}$
(d) $\frac{13}{52}=\frac{1}{4}$ (e) $\frac{26}{52}=\frac{1}{2}$ (f) $\frac{12}{52}=\frac{3}{13}$
(g) $1-\frac{3}{13}=\frac{10}{13}$
(h) Includes 2, 3, 4, 5, 6 [Ace may be considered as 1]
$P=\frac{5}{13}$ (No Ace) or $P=\frac{6}{13}$ (Ace is 1)
(i) $\frac{1}{52}$ (j) 26 red cards + 2 black 6's,
$P=\frac{28}{52}=\frac{7}{13}$ (k) $\frac{2}{52}=\frac{1}{26}$

23. P(no rain) $=1-\frac{7}{10}=\frac{3}{10}$

24. 7R, 5B, 3W. Total no. = 15

(a) $\frac{7}{15}$ (b) $1-\frac{7}{15}=\frac{8}{15}$ (c) $\frac{5}{15}=\frac{1}{3}$
(d) $\frac{10}{15}=\frac{2}{3}$ (e) 0 [There are no greens.]

25. (a) P(female) = 1 − 0.48 = 0.52
(b) No. of males = 0.48 × 1400 = 672
We would expect 672 males.

26. Refer to the diagram in Chapter 7 (page 108).

(a) $P(9)=\frac{4}{36}=\frac{1}{9}$
(b) $P(5)=\frac{4}{36}=\frac{1}{9}$
(c) $P(\text{odd})=\frac{18}{36}=\frac{1}{2}$
(d) $P(>9)=\frac{6}{36}=\frac{1}{6}$
(e) $P(\leq 9)=1-\frac{1}{6}=\frac{5}{6}$
(f) $P(\neq 9)=1-\frac{1}{9}=\frac{8}{9}$
(g) 6 less than 5 + 16 odd, $P=\frac{22}{36}=\frac{11}{18}$
(h) $P=\frac{2}{36}=\frac{1}{18}$ [only the threes]

27. Refer to the diagram in Chapter 7 (page 108).

(a) $\frac{2}{36}=\frac{1}{18}$ (b) $\frac{9}{36}=\frac{1}{4}$ (c) $\frac{19}{36}$
(d) $\frac{23}{36}$ (e) $\frac{20}{36}=\frac{5}{9}$ (f) $\frac{5}{36}$
(g) $\frac{2}{3}$

28.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

(a) $\frac{11}{36}$ (b) $\frac{1}{36}$
(c) $1-\frac{11}{36}=\frac{25}{36}$
(d) $\frac{24}{36}=\frac{2}{3}$
(e) $\frac{21}{36}=\frac{7}{12}$ (f) $\frac{17}{36}$
(g) $\frac{3}{36}=\frac{1}{12}$ (h) $\frac{35}{36}$
(i) $\frac{16}{36}=\frac{4}{9}$ (j) 0

29. (a)

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Total = 16

(i) $\frac{1}{16}$ (ii) 0
(iii) $\frac{3}{16}$ (iv) $\frac{10}{16}=\frac{5}{8}$
(v) $\frac{13}{16}$

(b)

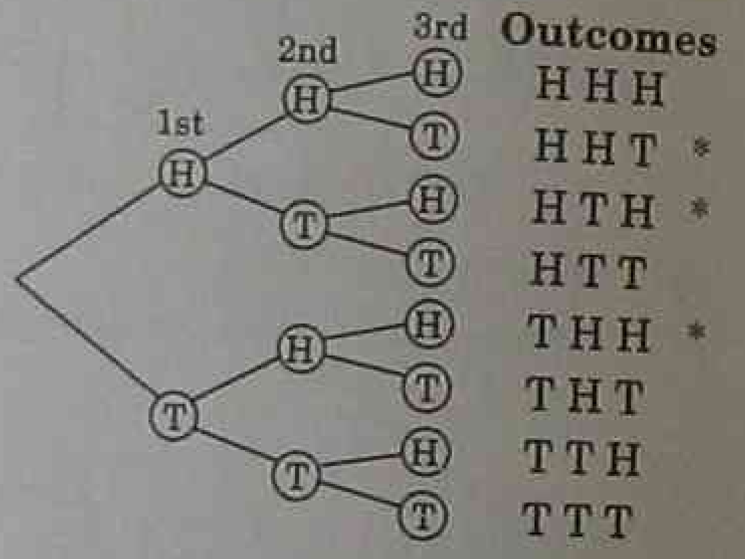
×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

(i) $\frac{1}{16}$ (ii) $\frac{12}{16}=\frac{3}{4}$
(iii) $\frac{8}{16}=\frac{1}{2}$ (iv) $\frac{3}{16}$

30. (a) $\frac{1}{10}$ (b) $\frac{5}{10}=\frac{1}{2}$
(c) $\frac{6}{10}=\frac{3}{5}$ (d) $\frac{3}{10}$

31. Betty needs a 50c coin. $P(50c)=\frac{4}{10}=\frac{2}{5}$

32.



No. of outcomes = 8

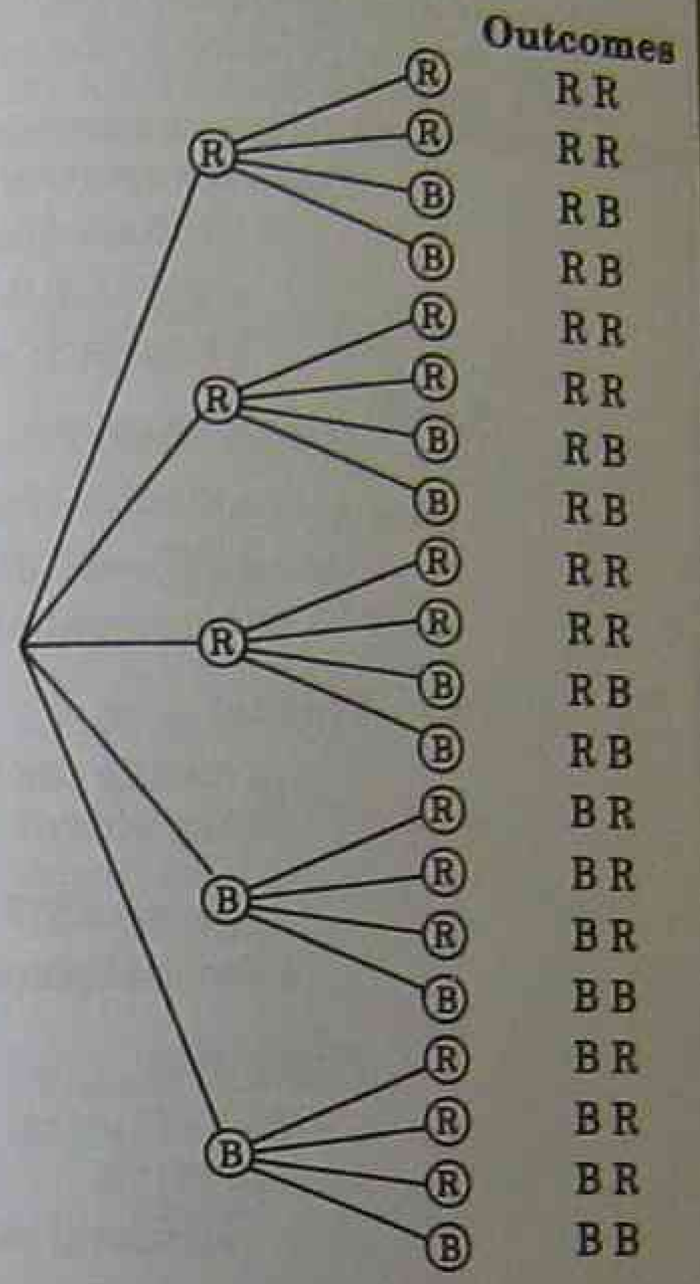
(a) $P(HHH)=\frac{1}{8}$ (b) $P(2H)=\frac{3}{8}$ (marked *)
(c) $P(1H)=\frac{3}{8}$ (d) $P(TTT)=\frac{1}{8}$
(e) $P(\text{at least }1H)=1-P(TTT)=\frac{7}{8}$

33. See the tree diagram at top of the next page.

Total no. of outcomes = 20

Remember: Once one is chosen it is out of the bag.

(a) $P(RR)=\frac{6}{20}=\frac{3}{10}$ (b) $P(BB)=\frac{2}{20}=\frac{1}{10}$
(c) P(different colours)
$=1-P(\text{same colours})=1-\frac{4}{10}=\frac{3}{5}$
(d) $P(\text{same colours})=\frac{2}{5}$



34. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$

(b)

▨	12	13	14	15	16
21	▨	23	24	25	26
31	32	▨	34	35	36
41	42	43	▨	45	46
51	52	53	54	▨	56
61	62	63	64	65	▨

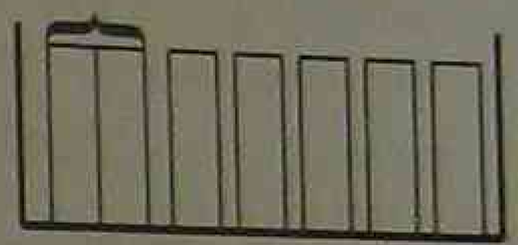
Total number of outcomes = 30
(i) $P(34)=\frac{1}{30}$
(ii) $P(43)=\frac{1}{30}$
(iii) $P(\text{odd})=\frac{15}{30}=\frac{1}{2}$
[Must end in 1, 3, 5]

(iv) $P(\text{divisible by }5)=\frac{1}{6}$
[Must end in 5]
(v) $P(<40)=\frac{15}{30}=\frac{1}{2}$
[Must start with 1, 2, 3]
(vi) $\frac{3}{30}=\frac{1}{10}$
[Start with 1, 2, 3, end with 5]

(c) (i) $P(\text{sum }7)=\frac{6}{30}=\frac{1}{5}$
(ii) $P(<7)=\frac{12}{30}=\frac{2}{5}$

35. There are 7 spaces. Any of 7 books can fill the first space; that leaves any of 6 books to fill the next space, 5 books for the next, etc.

The number of ways
$=7\times6\times5\times4\times3\times2\times1=5040$
7 books can be arranged in 5040 ways.



Treat the books as a single item, and consider how many ways 6 books can be arranged on the shelf.

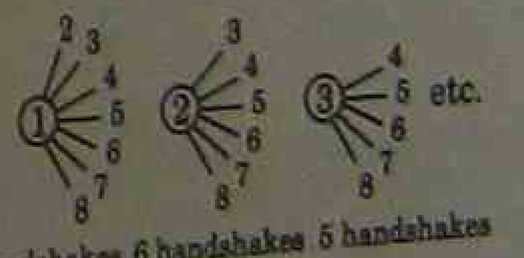
The number of ways 6 books can be arranged $=6\times5\times4\times3\times2\times1=720$.

But the 2 books can be internally arranged in 2 ways: $\boxed{A|B}$ or $\boxed{B|A}$, that is, 2 ways.

∴ the total number of ways 7 books can be arranged with two always together $=2\times720=1440$.

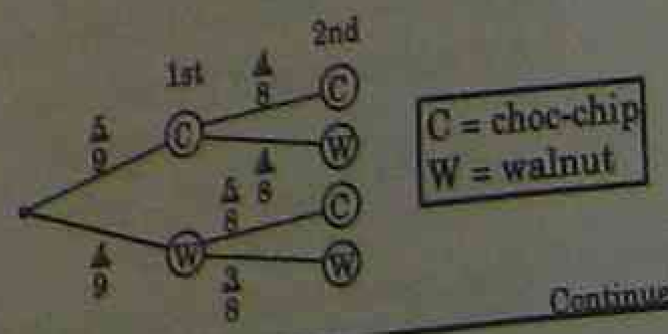
P(chemistry together)
$=\frac{\text{no. of ways with 2 together}}{\text{total no. of arrangements}}$
$=\frac{1440}{5040}$
$=\frac{2}{7}$

36.



No. of handshakes = 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28.

37.



Continued

Outcome	Probability
CC	$P(CC)=\frac{5}{9}\times\frac{4}{8}=\frac{5}{18}$
CW	$P(CW)=\frac{5}{9}\times\frac{4}{8}=\frac{5}{18}$
WC	$P(WC)=\frac{4}{9}\times\frac{5}{8}=\frac{5}{18}$
WW	$P(WW)=\frac{4}{9}\times\frac{3}{18}=\frac{1}{6}$

From the table:

(a) $P(CC)=\frac{5}{18}$

(b) $P(\text{one C})=P(WC)+P(CW)$

$$=\frac{5}{18}+\frac{5}{18}$$

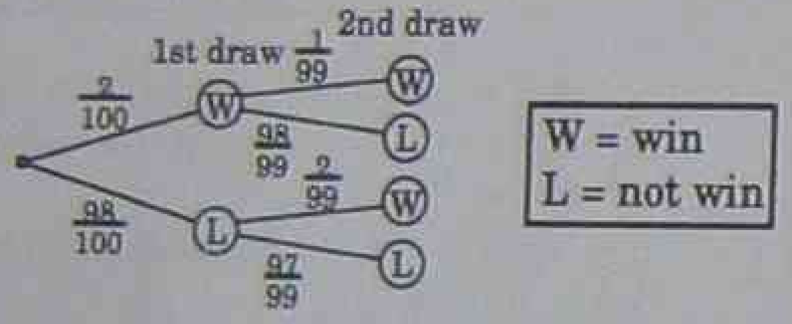
$$=\frac{5}{9}$$

(c) $P(CW)=\frac{5}{18}$ (d) $P(WW)=\frac{1}{6}$

(e) $P(\text{at least one C})=1-P(WW)$

$$=1-\frac{1}{6}=\frac{5}{6}$$

38.



Outcome	Probability
WW	$P(WW)=\frac{2}{100}\times\frac{1}{99}=\frac{1}{4950}$
WL	$P(WL)=\frac{2}{100}\times\frac{98}{99}=\frac{98}{4950}$
LW	$P(LW)=\frac{98}{100}\times\frac{2}{99}=\frac{98}{4950}$
LL	$P(LL)=\frac{98}{100}\times\frac{97}{99}=\frac{4753}{4950}$

From the diagram:

(a) $P(\text{first prize})=\frac{2}{100}=\frac{1}{50}$

(b) $P(WW)=\frac{1}{4950}$

(c) $P(LW)=\frac{98}{4950}=\frac{49}{2475}$

(d) $P(\text{a prize})=P(WW)+P(WL)+P(LW)$

or $1-P(LL)$

$$=1-\frac{4753}{4950}=\frac{197}{4950}$$

(e) $P(LL)=\frac{4753}{4950}$

(f) $\frac{197}{4950}$ [same as (d)]

39. We need to know the relative areas of each region.

A of inner circle $=\pi r^2=\pi(8^2)=64\pi\ \text{cm}^2$

A of middle section $=\pi(16^2)-64\pi$

$$=192\pi\ \text{cm}^2$$

A of outer ring $=\pi(24^2)-192\pi$

$$=384\pi\ \text{cm}^2$$

Also, area of board $=80\times 80=6400\ \text{cm}^2$.

(a) The probability of hitting any section will be the ratio of the area of each section relative to the area of the board.

(i) P(super supreme)

$$=P(\text{hitting centre})$$

$$=\frac{64\pi}{6400}=\frac{\pi}{100}=0.0314$$

(4 decimal places)

(ii) P(thick shake)

$$=P(\text{outer ring})$$

$$=\frac{384\pi}{6400}=0.1885$$

(4 decimal places)

(iii) P(prize)

$$=P\begin{pmatrix}\text{hitting the large circle}\\ \text{anywhere}\end{pmatrix}$$

$$=\frac{\pi(24)^2}{6400}=0.2827$$

(4 decimal places)

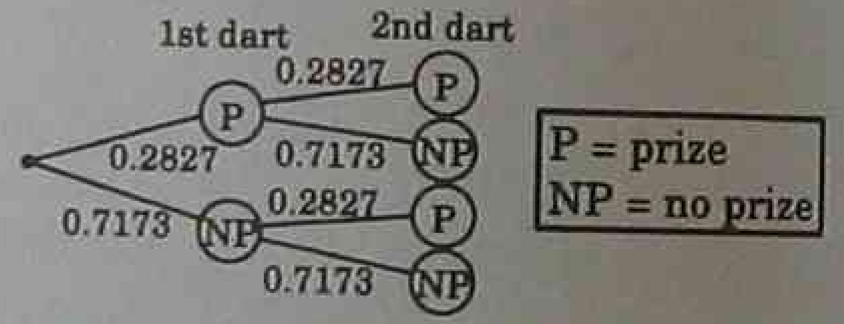
(iv) P(no prize)

$$=1-P(\text{prize})$$

$$=0.7173$$

(4 decimal places)

(b) P(prize) = 0.2827



Outcomes	Probability
P, P	$P(PP) = (0.2827)^2$
P, NP	$P(P,NP) = 0.2827 \times 0.7173$
NP, P	$P(NP,P) = 0.7173 \times 0.2827$
NP, NP	$P(NP,NP) = 0.7173 \times 0.7173$

From the diagram:

(i) $P(P,P) = (0.2827)^2 = 0.0799$
(4 decimal places)

(ii) $P(\text{one prize}) = P(P,NP) + P(NP,P)$
$= 0.4056$ (4 decimal places)

(iii) $P(\text{no prizes}) = (0.7173)^2 = 0.5145$
(4 decimal places)

(iv) P(at least one prize)

$$=1-P(\text{no prize})$$

$$=1-0.5145$$

$$=0.4855$$

(4 decimal places)

Chapter 8 Measurement: time, perimeter, area, surface area and volume (page 118)

1. (a) 6 h 6 m 9 s
   (b) 8 h 54 m 49 s
   (c) 6 h 1 m 3 s
   (d) 1 h 2 m 52 s

2. (a) $\text{Average}=\frac{6\text{ h }41\text{ m }24\text{ s}}{3}$

$$=2\text{ h }13\text{ m }48\text{ s}$$

3. (a) 3.4 h = 3 h 24 m
   (b) 0.76 h = 45 m 36 s
   (c) 2.05 h = 2 h 3 m

4. (a) 2 h 32 m
   (b) 4 h 48 m
   (c) 5:14 to 13:06 = 7 h 52 m
   (d) 6 h 46 m
   (e) 11 h 53 m
   (f) 18 h 2 m

5. (a) $\text{Time}=\frac{\text{Distance}}{\text{Speed}}$

$$=\frac{70}{45}$$

$$=1.\dot{5}\text{ h}$$

$$=1\text{ h }33\text{ m }20\text{ s}$$

(b) $\text{Time}=\frac{80}{54}$

$$=1.\dot{4}8\dot{1}\text{ h}$$

$$=1\text{ h }28\text{ m }53.\dot{3}\text{ s}$$

$$=1\text{ h }28\text{ m }53\text{ s}$$

(to nearest second)

6. $\text{Time}=\frac{47}{67}$

$$=42\text{ m }53.7\text{ s}$$

$$=43\text{ m}$$

(to nearest minute)

∴ From 11:06 add 43 minutes

∴ 11:49

7. Distance = Speed × Time

Now, 18 minutes = 0.3 h

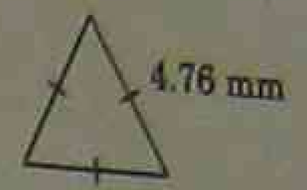
∴ Distance = 9 × 0.3

= 2.7

∴ Laura walked 2.7 km.

8. (a) 7 000 000 000 μm
   (b) 0.025GL
   (c) 10 000 μs
   (d) 3.2 mL

9.

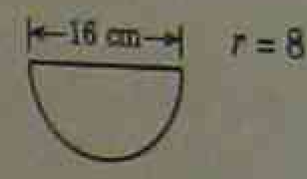


Perimeter = 3(4.76)

= 14.28

∴ perimeter is 14.28 mm.

10. (a)



$$\text{Perimeter}=16+\tfrac{1}{2}\times 2\times\pi\times 8$$

$$=16+25.132\,741\,23$$

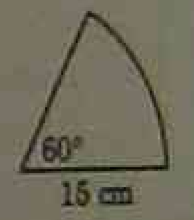
$$=41.132\,741\,23$$

$$=41.133$$

(to three dec. places)

∴ perimeter is 41.133 cm.

(b)



$$\text{Perimeter}=15+15+\tfrac{1}{6}\times 2\times\pi\times 15$$

$$=30+15.707\,963$$

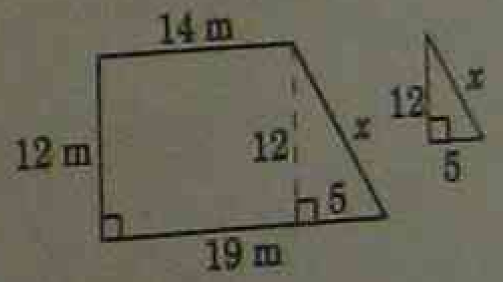
$$=45.707\,963$$

$$=45.708$$

(to three dec. places)

∴ perimeter is 45.708 cm.

11. (a)



Using Pythagoras' Theorem to find x:

$$x^2=5^2+12^2=169$$

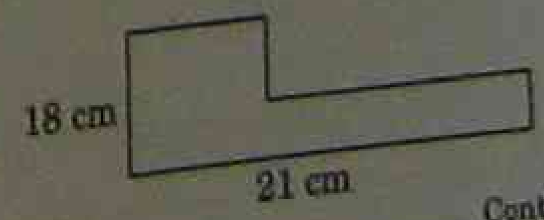
$$x=\sqrt{169}=13.$$

∴ Perimeter = 14 + 12 + 19 + 13

= 58

Perimeter is 58 m.

(b)

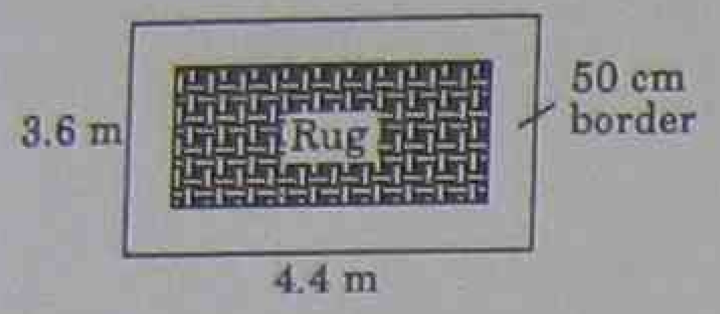


Continued

$$\begin{aligned} \text{Perimeter} &= 2(18+21) \\ &= 2(39) \\ &= 78 \end{aligned}$$

∴ perimeter is 78 cm.

12.

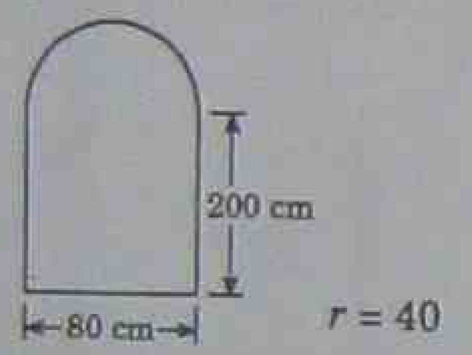


$$\begin{aligned} \text{Rug length} &= 4.4 - 2(0.5) \\ &= 4.4 - 1 \\ &= 3.4 \text{ m} \\ \text{Rug width} &= 3.6 - 2(0.5) \\ &= 3.6 - 1 \\ &= 2.6 \text{ m} \end{aligned}$$

$$\begin{aligned} \therefore \text{ rug perimeter} &= 2(3.4 + 2.6) \\ &= 2(6) \\ &= 12 \end{aligned}$$

∴ rug's perimeter is 12 m.

13.

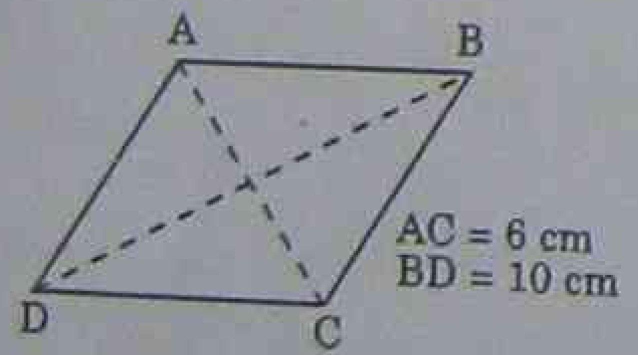


$$\begin{aligned} P &= 200 + 80 + 200 + \tfrac{1}{2} \times 2 \times \pi \times 40 \\ &= 480 + 125.663\,706 \\ &= 605.663\,706 \\ &= 606 \end{aligned}$$

(to the nearest whole number)

∴ perimeter is 606 cm.

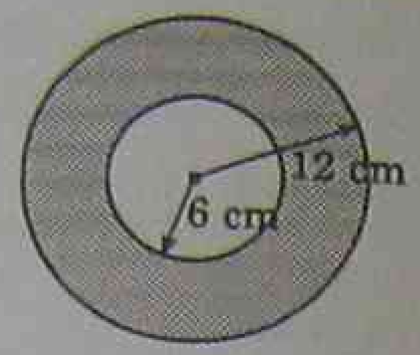
14.



$$\begin{aligned} A &= \tfrac{1}{2}(\text{product of diagonals}) \\ &= \tfrac{1}{2}(6 \times 10) \\ &= \tfrac{1}{2}(60) \\ &= 30 \end{aligned}$$

∴ area is 30 cm^2.

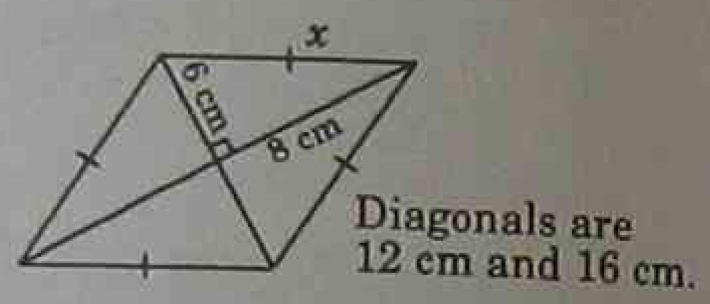
15. (a)



$$\begin{aligned} A &= \pi R^2 - \pi r^2 \\ &= \pi \times 12^2 - \pi \times 6^2 \\ &= \pi\left[12^2 - 6^2\right] \\ &= \pi[144 - 36] \\ &= \pi[108] \\ &= 108\pi \end{aligned}$$

∴ area is 108π cm^2.

(b)



Diagonals are 12 cm and 16 cm.

$$\begin{aligned} \therefore\ A &= \tfrac{1}{2}[\text{product of diagonals}] \\ &= \tfrac{1}{2}(16 \times 12) \\ &= \tfrac{1}{2}(192) \\ &= 96 \end{aligned}$$

∴ area is 96 cm^2.

Now, diagonals meet at right angles

∴ find x, using Pythagoras' Theorem.

$$\begin{aligned} x^2 &= 6^2 + 8^2 \\ &= 36 + 64 \\ &= 100 \\ x &= \sqrt{100} \\ &= 10 \end{aligned}$$

∴ length of one side = 10 cm

∴ perimeter = 4 × 10 = 40

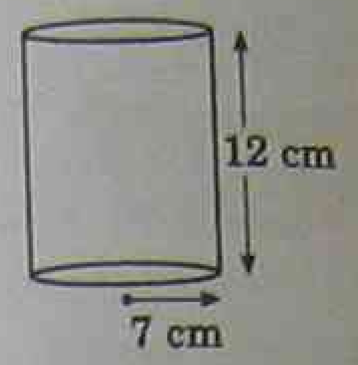
∴ perimeter is 40 cm.

16. $r = 16$

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4 \times \pi \times 16^2 \\ &= 3216.990\,87 \\ &= 3216.99 \text{ [to two dec. places]} \end{aligned}$$

∴ surface area is 3216.99 cm^2.

17.



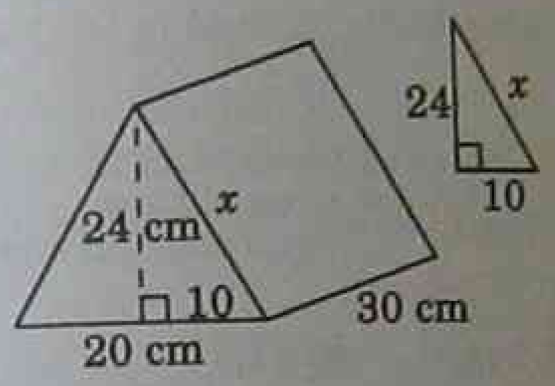
Curved surface area

$$\begin{aligned} &= 2\pi rh \\ &= 2 \times \pi \times 7 \times 12 \\ &= 527.787\,57 \\ &= 528 \end{aligned}$$

[correct to three significant figures]

∴ curved surface area is 528 cm^2.

18.



First find x.

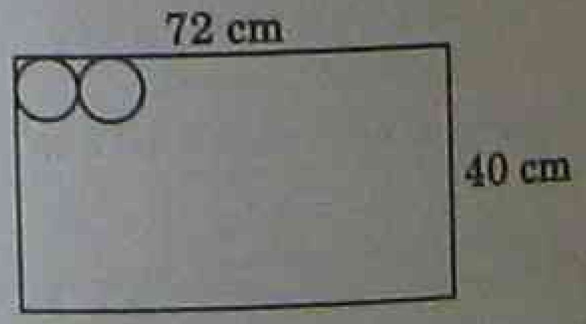
$$\begin{aligned} \therefore x^2 &= 24^2 + 10^2 \\ &= 576 + 100 \\ &= 676 \\ x &= \sqrt{676} \\ &= 26, \end{aligned}$$

∴ surface area

$$\begin{aligned} &= 2 \times \tfrac{1}{2} \times 20 \times 24 \quad \Leftarrow \boxed{\text{Ends}} \\ &\quad + 2 \times 26 \times 30 \quad \Leftarrow \boxed{\text{Sides}} \\ &\quad + 20 \times 30 \quad \Leftarrow \boxed{\text{Bottom}} \\ &= 480 + 1560 + 600 \\ &= 2640, \end{aligned}$$

∴ surface area is 2640 cm^2.

19.



(a) Circles:

$$\begin{aligned} \text{Lengthways} &= 72 \div 4 \\ &= 18 \\ \text{Sideways} &= 40 \div 4 \\ &= 10, \\ \therefore \quad \text{no. of circles} &= 18 \times 10 \\ &= 180, \end{aligned}$$

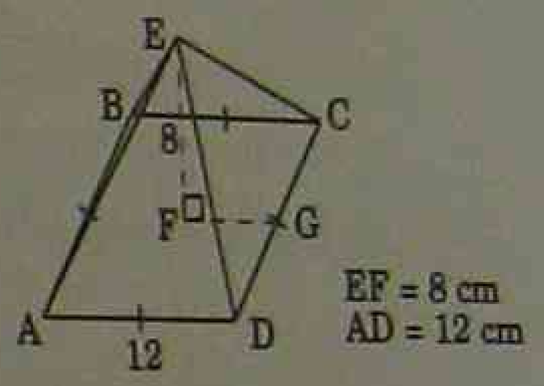
∴ maximum number is 180.

(b) Amount of scrap metal

$$\begin{aligned} &= (\text{area of sheet}) \\ &\quad - (\text{area of all circles}) \\ &= 72 \times 40 - 180\left(\pi \times 2^2\right) \\ &= 2880 - 2261.946\,711 \\ &= 618.053\,2895 \\ &= 618 \text{ [to the nearest whole number]} \end{aligned}$$

∴ the amount is 618 cm^2 of scrap metal.

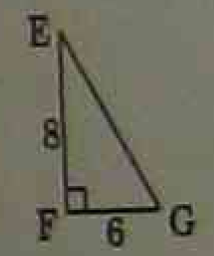
20.



EF = 8 cm
AD = 12 cm

(a) As AD = 12 cm,
FG = 6 cm.

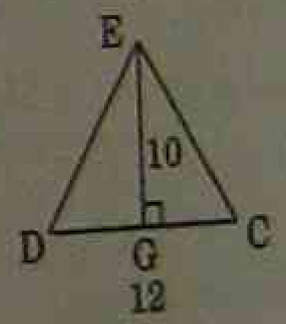
(b) Using Pythagoras' Theorem:



$$\begin{aligned} EG^2 &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \\ EG &= \sqrt{100} \\ &= 10 \end{aligned}$$

∴ the length of EG is 10 cm.

(c)



$$\begin{aligned} \text{Area of } \Delta CED &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times 12 \times 10 \\ &= 60, \end{aligned}$$

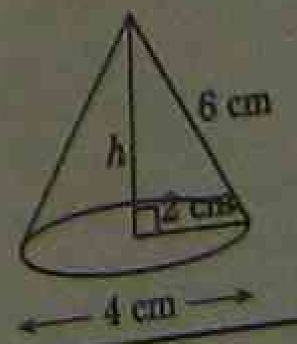
∴ the area is 60 cm^2.

(d)

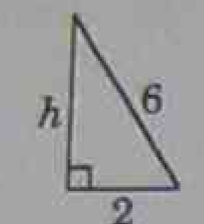
$$\begin{aligned} \text{Surface area} &= 4(60) + 12^2 \\ &= (240 + 144), \end{aligned}$$

∴ the surface area is 384 cm^2.

21.



(a) $h^2 = 6^2 - 2^2$
$= 36 - 4$
$= 32$
$h = \sqrt{32}$
$= \sqrt{16} \times \sqrt{2}$
$= 4\sqrt{2}$,



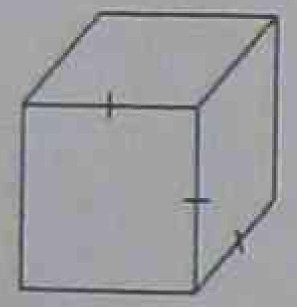
[using Pythagoras' Theorem]

$\therefore$ perpendicular height is $4\sqrt{2}$ cm.

(b) Curved surface area $= \pi rl$
$= \pi \times 2 \times 6$
$= 12\pi$
$= 37.699\,111\,84$
$= 37.70$
[to two decimal places],
$\therefore$ curved surface area is 37.70 cm^2.

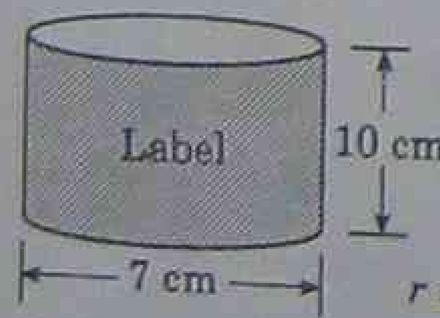
(c) Total surface area $= \pi r^2 + \pi rl$
$= \pi \times 2^2$
$+ \pi \times 2 \times 6$
$= \pi[4 + 12]$
$= \pi[16]$
$= 50.265\,482\,46$
$= 50.27$
[to two decimal places],
$\therefore$ total surface area is 50.27 cm^2.

22.



Surface area $= 6s^2 = 54$
$\therefore$ $s^2 = 9$
$s = 3$
[cannot be $s = -3$]
$\therefore$ each side is 3 cm.

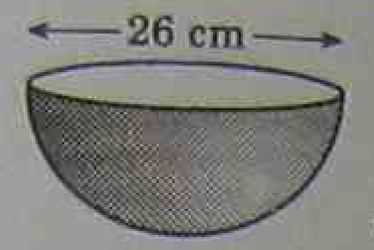
23.



$r = 3.5$ cm

Area of label = curved surface area of cylinder,
$\therefore$ curved surface area
$= 2\pi rh$
$= 2 \times \pi \times 3.5 \times 10$
$= 219.911\,4858$
$= 220$
[to the nearest whole number]
$\therefore$ area of label is 220 cm^2.

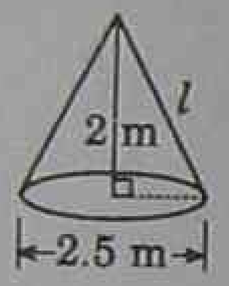
24.



$r = 13$ cm

Surface area of sphere
$= 4\pi r^2$,
$\therefore$ surface area of hemisphere
$= 2\pi r^2$,
$\therefore$ surface area $= 2\pi r^2$
$= 2 \times \pi \times 13^2$
$= 1061.858\,317$
$= 1061.9$
[to one decimal place],
$\therefore$ area to be painted is 1061.9 cm^2.

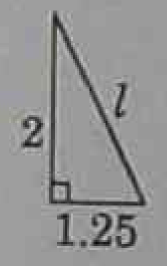
25.



Required area
= area of base
+ curved surface area

$r = 1.25$

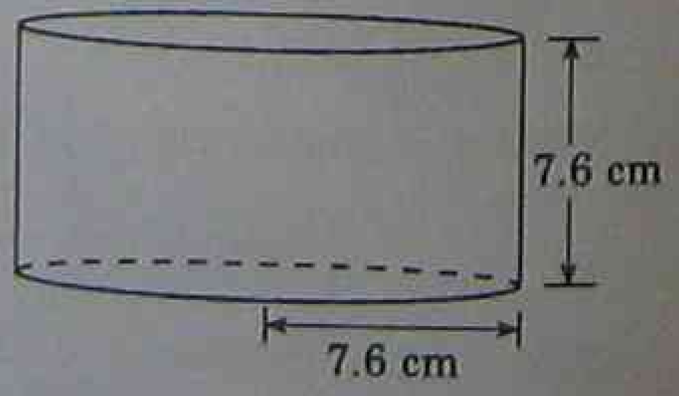
Surface area $= \pi r^2 + \pi rl$,
$\therefore$ to find l, use Pythagoras' Theorem:



$l^2 = 2^2 + 1.25^2$
$= 4 + 1.5625$
$= 5.5625$,
$\therefore$ $l = \sqrt{5.5625}$
$= 2.358\,495\,283$,

$\therefore$ surface area
$= \pi r^2 + \pi rl$
$= \pi(1.25)^2 + \pi(1.25)(2.358\,495\,283)$
$= 4.908\,738\,521 + 9.261\,789\,318$
$= 14.170\,527\,84$
$= 14.17$ [correct to two decimal places],
$\therefore$ area of canvas is 14.17 m^2.

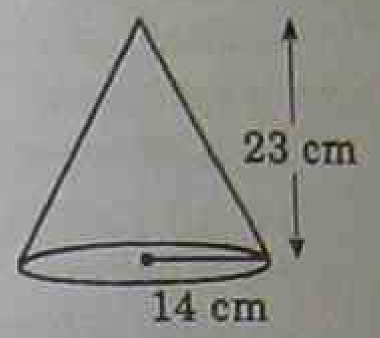
26.



Continued

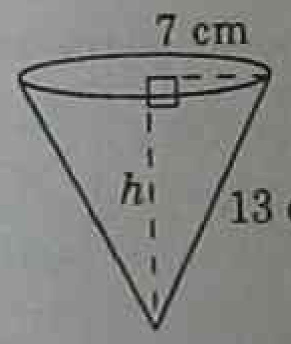
$v = \pi r^2 h$
$= \pi \times 7.6^2 \times 7.6$
$= 1379.083\,777$
$= 1379.084$ [to three dec. places]
$\therefore$ the volume is 1379.084 cm^3.

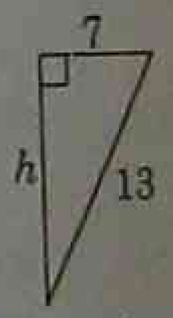
27. (a)



$v = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi \times 14^2 \times 23$
$= 4720.766\,56$
$= 4720.77$ [to two dec. places],
$\therefore$ the volume is 4720.77 cm^3.

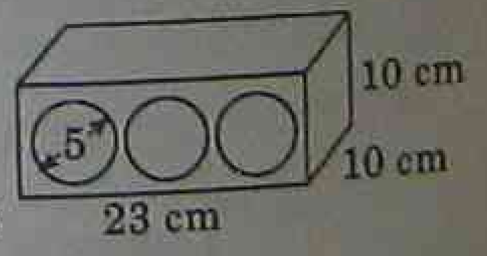
(b)





$h^2 = 13^2 - 7^2$
$= 169 - 49$
$= 120$
$h = \sqrt{120}$
[By Pythagoras' Theorem]
$\therefore$ $v = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi \times 7^2 \times \sqrt{120}$
$= 562.102\,2464$
$= 562.10$ [to two dec. places],
$\therefore$ the volume is 562.10 cm^3.

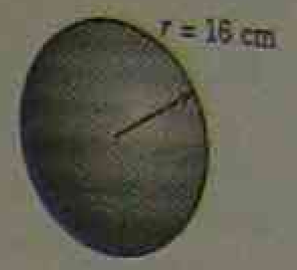
(c)



$r = 2.5$

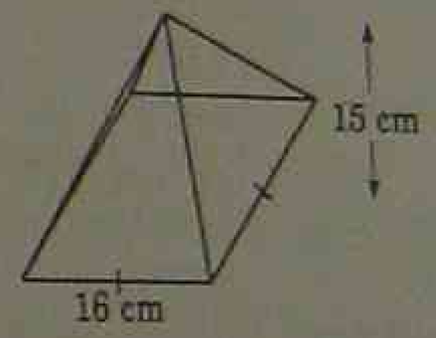
Volume $= 23 \times 10 \times 10$
$- 3 \times \pi \times 2.5 \times 2.5 \times 10$
$= 2300 - 589.048\,6226$
$= 1710.951\,377$
$= 1710.95$ [to two dec. places],
$\therefore$ the volume is 1710.95 cm^3.

(d)



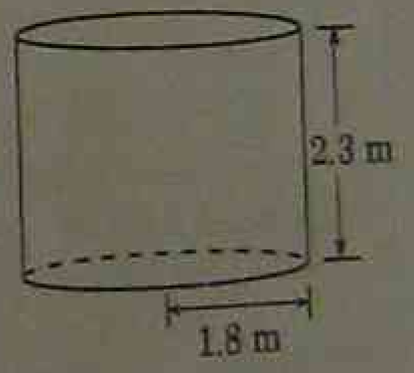
$v = \frac{4}{3}\pi r^3$
$= \frac{4}{3} \times \pi \times 16^3$
$= 17\,157.284\,68$
$= 17\,157.28$ [to two dec. places],
$\therefore$ volume is 17 157.28 cm^3.

28.



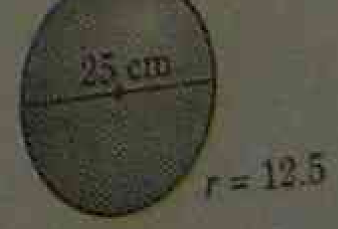
$v = \frac{1}{3}Ah$
$= \frac{1}{3} \times 16 \times 16 \times 15$
$= 1280$
$\therefore$ volume is 1280 cm^3.

29.



$v = \pi r^2 h$
$= \pi \times 1.8^2 \times 2.3$
$= 23.411\,148\,45$
$\therefore$ volume is 23.411 148 45 m^3.
$\therefore$ capacity $= 23.411\,148\,45 \times 1000$
$= 23\,411.148\,45$
$= 23\,411$
[to the nearest litre],
$\therefore$ capacity is 23 411 litres.

30.



Continued

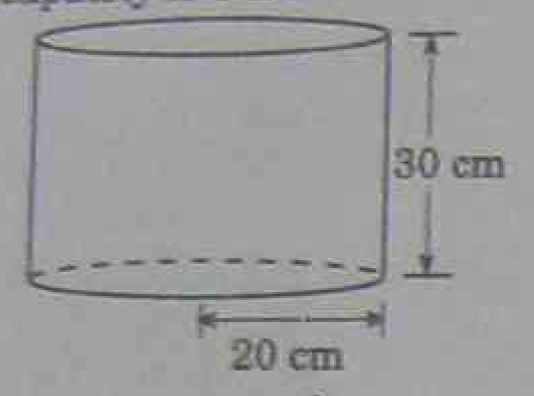
$$
\begin{aligned}
\text{Volume} &= \tfrac{4}{3}\pi r^3\\
&= \tfrac{4}{3}\times\pi\times 12.5^3\\
&= 8181.230\,868
\end{aligned}
$$

$\therefore$ the volume is 8181.230 868 cm^3.

$$
\begin{aligned}
\therefore\ \text{capacity} &= 8181.230\,868 \div 1000\\
&= 8.181\,230\,868\\
&= 8 \text{ [to the nearest litre]},
\end{aligned}
$$

$\therefore$ capacity is 8 litres.

31.



$$
\begin{aligned}
v &= \pi r^2 h\\
&= \pi\times 20^2\times 30\\
&= 37\,699.111\,84 \text{ cm}^3
\end{aligned}
$$

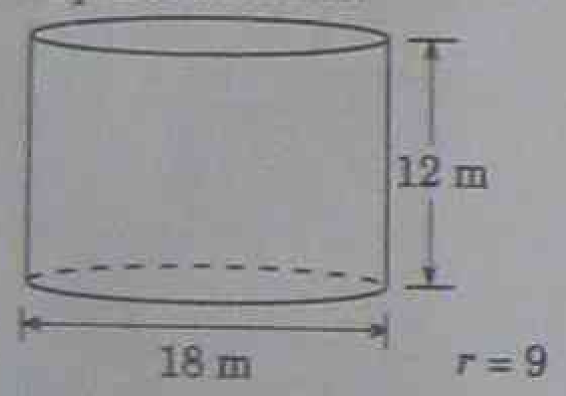
$$
\begin{aligned}
\therefore\ \text{the capacity} &= 37\,699.111\,84 \div 1000\\
&= 37.699\,111\,84 \text{ litres}
\end{aligned}
$$

But if only $\frac{2}{3}$ full,

$$
\begin{aligned}
\text{jar } \tfrac{2}{3} \text{ full} &= \tfrac{2}{3}\times 37.699\,111\,84\\
&= 25.132\,741\,23\\
&= 25 \text{ [to the nearest whole]},
\end{aligned}
$$

$\therefore$ it requires 25 litres.

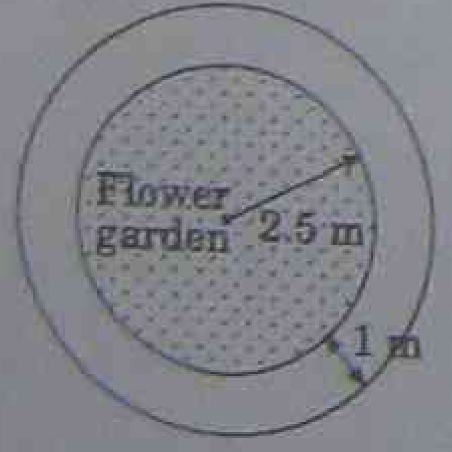
32.



$$
\begin{aligned}
\text{Volume} &= \pi r^2 h\\
&= \pi\times 9^2\times 12\\
&= 3053.628\,059\\
\text{Capacity} &= 3053.628\,059\times 1000\\
&= 3053\,628.059 \text{ litres}\\
&= 3053.628\,059 \text{ kilolitres}\\
&= 3054 \text{ [to the nearest kilolitre]},
\end{aligned}
$$

$\therefore$ the capacity is 3054 kL.

33.



(a) Area of large circle

$$
\begin{aligned}
&= \pi R^2\\
&= \pi(3.5)^2
\end{aligned}
\qquad
\boxed{\begin{aligned}R &= 2.5+1\\ &= 3.5\end{aligned}}
$$

Area of small circle

$$
\begin{aligned}
&= \pi r^2\\
&= \pi(2.5)^2
\end{aligned}
$$

Area of path

$$
\begin{aligned}
&= \pi(3.5)^2 - \pi(2.5)^2\\
&= \pi(3.5+2.5)(3.5-2.5)\\
&= 6\pi\\
&= 18.849\,555\,92\\
&\approx 18.8 \text{ m}^2
\end{aligned}
$$

[to one decimal place]

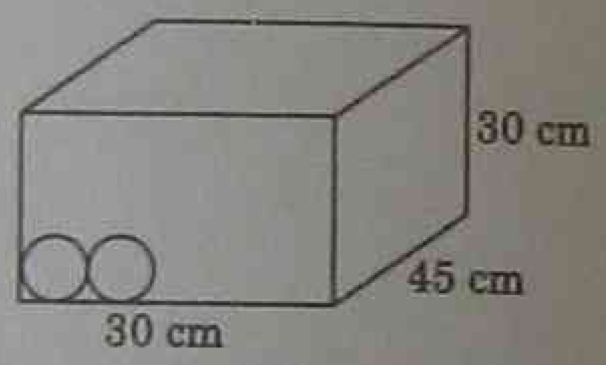
(b) Volume of gravel

$$
\begin{aligned}
&= \text{area of path}\times\text{depth}\\
&= 18.8\times 0.1 \qquad \boxed{10 \text{ cm} = 0.1 \text{ m}}\\
&= 1.88 \approx 2
\end{aligned}
$$

[to the nearest whole number].

Volume of gravel required is 2 m^3.

34.

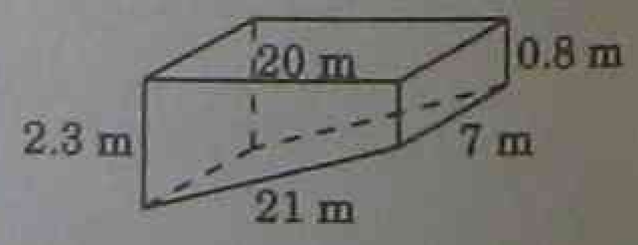


No. of balls in :

$$
\begin{aligned}
\text{length} &= 30 \div 7.5\\
&= 4\\
\text{width} &= 45 \div 7.5\\
&= 6\\
\text{height} &= 30 \div 7.5\\
&= 4,\\
\therefore\ \text{no. of balls} &= 4\times 6\times 4\\
&= 96
\end{aligned}
$$

$\therefore$ 96 balls will fit the container.

35. (a)



$$
\begin{aligned}
\text{Area of base} &= 21\times 7\\
&= 147 \text{ m}^2\\
\therefore\ \text{cost} &= \$55\times 147\\
&= \$8085.00
\end{aligned}
$$

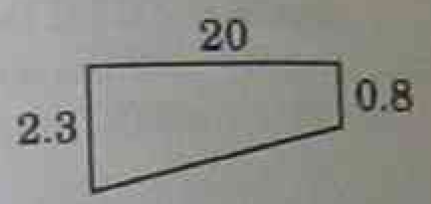
$\therefore$ cost is \$8085.

(b) Area of walls

$$
\begin{aligned}
&= 2 \text{ rectangles} + 2 \text{ trapezia}\\
\therefore\ \text{area} &= 2.3\times 7 + 0.8\times 7 \leftarrow \boxed{\text{Ends}}\\
&\quad + 2\times\tfrac{1}{2}\times 20(0.8+2.3)\\
&= 16.1 + 5.6 + 62 \quad \uparrow\\
&= 83.7 \text{ m}^2, \qquad \boxed{\text{Sides}}\\
\therefore\ \text{cost} &= \$42\times 83.7\\
&= \$3515.40,
\end{aligned}
$$

$\therefore$ the cost is \$3515.40.

(c)



$$
\begin{aligned}
v &= Ah\\
&= \text{area of trapezium}\times\text{height}\\
&= \left[\tfrac{1}{2}\times 20(2.3+0.8)\right]\times 7\\
&= 31\times 7\\
&= 217,
\end{aligned}
$$

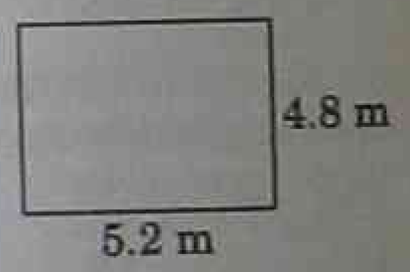
$\therefore$ the volume is 217 m^3.

(d)

$$
\begin{aligned}
\text{Volume} &= 217 \text{ m}^3,\\
\therefore\ \text{capacity} &= 217\times 1000\\
&= 217\,000 \text{ litres}\\
&= 217 \text{ kilolitres},
\end{aligned}
$$

$\therefore$ the capacity is 217 kilolitres.

36. (a)



$$
\begin{aligned}
\text{Area} &= 5.2\times 4.8\\
&= 24.96,
\end{aligned}
$$

$\therefore$ area is 24.96 m^2

(b)

$$
\boxed{\begin{aligned}1 \text{ m}^2 &= 100\times 100\\ &= 10\,000 \text{ cm}^2\end{aligned}}
$$

$$
\begin{aligned}
\text{Area} &= 24.96\times 10\,000\\
&= 249\,600 \text{ cm}^2
\end{aligned}
$$

$\therefore$ the area is 249 600 cm^2.

(c)

$$
\begin{aligned}
\text{Area of paver} &= 22.5\times 10\\
&= 225 \text{ cm}^2.
\end{aligned}
$$

Number of pavers

$$
\begin{aligned}
&= 249\,600 \div 225\\
&= 1109.333\\
&= 1110
\end{aligned}
$$

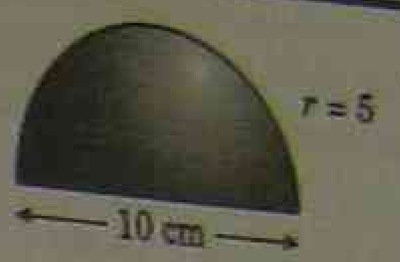
[to the next whole number]

Pavers still needed

$$
\begin{aligned}
&= 1110 - 500\\
&= 610.
\end{aligned}
$$

Still needs 610 pavers.

37.



Vol. of sphere $= \frac{4}{3}\pi r^3$,

$\therefore$ volume of hemisphere

$$
\begin{aligned}
&= \tfrac{1}{2}\times\tfrac{4}{3}\pi r^3\\
&= \tfrac{1}{2}\times\tfrac{4}{3}\times\pi\times 5^3\\
&\approx 261.799\,39,
\end{aligned}
$$

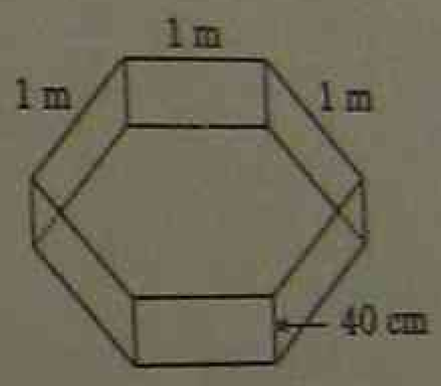
$\therefore$ the volume is 261.799 39 cm^3,

$$
\begin{aligned}
\therefore\ \text{the mass} &= 261.799\,39\times 6.8\\
&= 1780.2358 \text{ g}\\
&= 1780 \text{ g}
\end{aligned}
$$

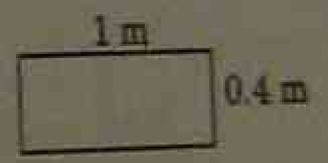
[to the nearest gram],

$\therefore$ the mass is 1780 grams.

38.



(a)



Surface area of vinyl sheet

$$
\begin{aligned}
&= \text{base} + 6 \text{ rectangles}\\
&= 2.6 + 6\times 1\times 0.4\\
&= 2.6 + 2.4\\
&= 5,
\end{aligned}
$$

$\therefore$ total surface area is 5 m^2.

(b)

$$
\begin{aligned}
\text{Volume} &= \text{area of base}\times\text{height}\\
&= 2.6\times 0.4\\
&= 1.04
\end{aligned}
$$

$\therefore$ the volume is 1.04 m^3,

$$
\begin{aligned}
&= 1.04\times 1000 \text{ litres}\\
&= 1040 \text{ litres},
\end{aligned}
$$

$\therefore$ capacity is 1040 litres.

(c) Threequarters capacity

$$
\begin{aligned}
&= \tfrac{3}{4}\times 1040\\
&= 780 \text{ litres}\\
&= 0.780 \text{ kilolitres},
\end{aligned}
$$

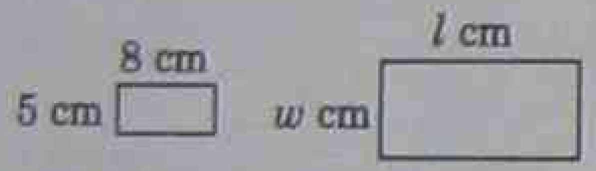
$\therefore$ volume when $\frac{3}{4}$ full is 0.78 kilolitres.

Chapter 9 Similarity (page 134)

1. Scale $= 0.6\text{ m} : 7.5\text{ cm}$
$= 600\text{ mm} : 75\text{ mm}$
$= 8 : 1$
$\therefore$ enlargement ratio is 8 : 1.

2. Ratio $= 2 : 3$
$h : 8.4 = 2 : 3$
[*Note*: Ratio is smaller : larger]
$$\therefore \quad \frac{h}{8.4} = \frac{2}{3}$$
$$3h = 16.8$$
$$\therefore \quad h = \frac{16.8}{3} = 5.6.$$
The smaller logo is 5.6 cm.

3.



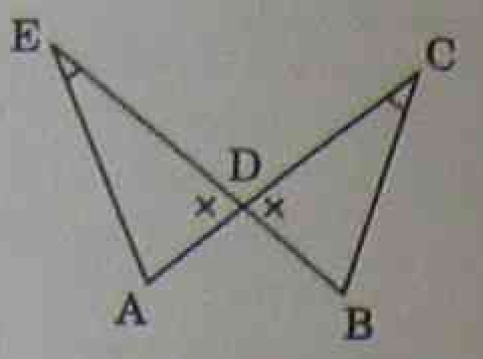
$w : 5 = 5 : 4$
$$\therefore \quad \frac{w}{5} = \frac{5}{4}$$
$$4w = 25$$
$$w = \frac{25}{4} = 6.25.$$
The width becomes 6.25 cm.
$l : 8 = 5 : 4$
$$\therefore \quad \frac{l}{8} = \frac{5}{4}$$
$$\therefore \quad 4l = 40$$
$$l = 10,$$
The length becomes 10 cm,
$\therefore$ the perimeter
$= 2 \times (10 + 6.25) = 32.5$ cm.
The area $= 10 \times 6.25 = 62.5\text{ cm}^2$.

4.

Perimeter $= p$ Perimeter $= 125$ m

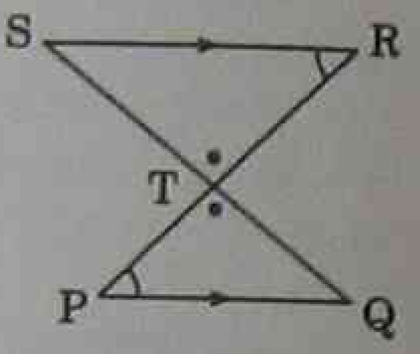
$p : 125 = 1 : 100$
$$\therefore \quad \frac{p}{125} = \frac{1}{100}$$
$$100p = 125$$
$$\therefore \quad p = \frac{125}{100} = 1.25.$$
A regular pentagon has 5 equal sides,
$\therefore$ each side $= 1.25 \div 5$ m
$= 0.25$ m
$= 25$ cm.

5.



In Δ's ADE and BDC,
$\angle AED = \angle BCD$ [data]
$\angle ADE = \angle BDC$ [vertically opposite angles]
$\therefore \triangle ADE \,|||\, \triangle BDC$ [equiangular].

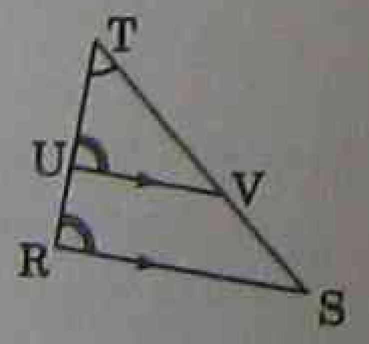
6.



(a) In Δ's PTQ and RTS,
$\angle QPT = \angle SRT$ [alternate angles, SR ∥ PQ]
$\angle PTQ = \angle STR$ [vertically opposite angles]
$\therefore \triangle PTQ \,|||\, \triangle RTS$ [equiangular].

(b) As Δ's are similar, ratios of corresponding sides are equal.
$$\frac{\boxed{PQ}}{\boxed{SR}} = \frac{\boxed{QT}}{ST} = \frac{\boxed{TP}}{TR}$$
Corresponding sides — the sides in each triangle opposite the same equal angle.

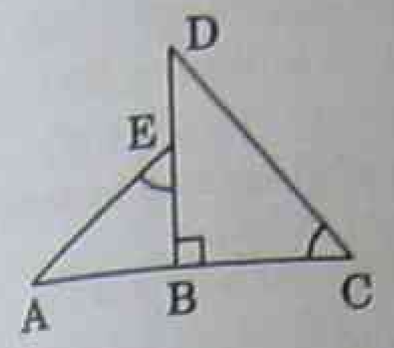
7.



In Δ's TUV and TRS,
$\angle UTV = \angle RTS$ [common angles]
(a) $\angle TUV = \angle TRS$ [corresponding angles, UV ∥ RS],
$\therefore \triangle TUV \,|||\, \triangle TRS$ [equiangular].

(b) As Δ's are similar, ratios of corresponding sides are equal
$$\frac{TU}{TR} = \frac{UV}{RS} = \frac{TV}{TS}.$$

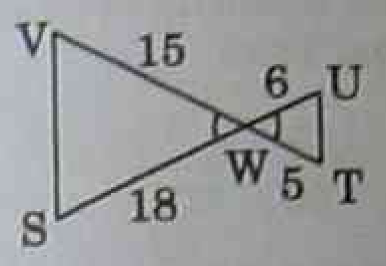
8.



$\angle DBC = 90°$
$\therefore \angle ABE = 90°$
[angle sum of a straight line]

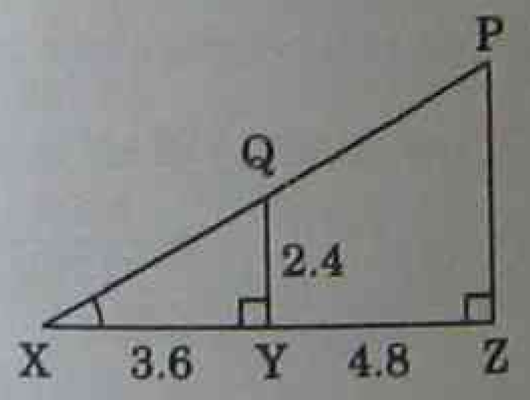
In Δ's ABE and DBC:
$\angle AEB = \angle BCD$ [data]
$\angle ABE = \angle DBC$ [both 90°]
$\therefore \triangle ABE \,|||\, \triangle DBC$ [equiangular]

9.



$$\frac{VW}{WT} = \frac{15}{5} = \frac{3}{1}$$
$$\frac{SW}{WU} = \frac{18}{6} = \frac{3}{1},$$
$$\therefore \quad \frac{VW}{WT} = \frac{SW}{WU}.$$
Also $\angle SWV = \angle TWU$ [vertically opposite angles],
$\therefore \triangle SVW \,|||\, \triangle UTW$ [ratios of two sides in proportion; included angles equal.

10.

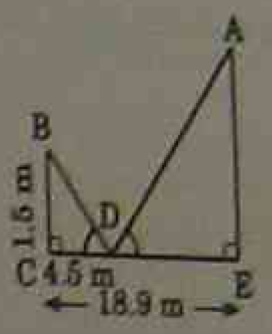


(a) In Δ's XYQ and XZP:
$\angle QYX = \angle PZX$ [both 90°]
$\angle QXY = \angle PXZ$ [common angle],
$\therefore \triangle XYQ \,|||\, \triangle XZP$ [equiangular]

(b) Ratios of corresponding sides are equal,
$$\therefore \quad \frac{XY}{XZ} = \frac{XQ}{XP} = \frac{QY}{PZ}$$
Now XY = 3.6, XZ = 3.6 + 4.8, QY = 2.4.

Now using $\frac{XY}{XZ} = \frac{QY}{PZ}$
$$\frac{3.6}{8.4} = \frac{2.4}{PZ}$$
$$\therefore \quad 3.6\,PZ = 2.4 \times 8.4$$
$$\therefore \quad PZ = \frac{2.4 \times 8.4}{3.6} = 5.6$$
PZ is 5.6 m.

11.



$\angle CDB = \angle EDA$.
Hence
$\triangle BCD \,|||\, \triangle AED$
[equiangular]

Then $\frac{CD}{DE} = \frac{CB}{EA}$
$$\therefore \quad \frac{4.5}{14.4} = \frac{1.5}{EA} \qquad \left[\begin{aligned} DE &= 18.9 - 4.5 \\ &= 14.4 \end{aligned}\right]$$
$$\therefore \quad 4.5\,EA = 1.5 \times 14.4$$
$$EA = \frac{1.5 \times 14.4}{4.5} = 4.8.$$

12. Let height be h m.
$$\frac{h}{1} = \frac{7.2}{1.8}$$
$$\therefore \quad h = 4.$$
The tree is 4 m high.

13. (a) $P_1 : P_2 = 3 : 6 = 1 : 2 \quad \left(\frac{P_1}{P_2} = \frac{1}{2}\right)$

(b) $A_1 : A_2 = r_1^2 : r_2^2$
$= 3^2 : 6^2$
$= 9 : 36$
$= 1 : 4$.

14. $$\frac{A_1}{A_2} = \frac{3^2}{5^2} = \frac{9}{25}.$$
Now $A_2 = 45$,
$$\therefore \quad \frac{A_1}{45} = \frac{9}{25}$$
$$\therefore \quad A_1 = \frac{9 \times 45}{25} = 16.2.$$
The smaller area is 16.2 cm^2.

15. (a) Ratio $= \frac{32}{40} = \frac{4}{5}$.
Reduction ratio is 4 : 5.

(b) Call the new circumference C_2.
Then:

$$\frac{C_2}{32}=\frac{4}{5},$$
$$\therefore\ 5\,C_2=4\times 32$$
$$C_2=\frac{4\times 32}{5}$$
$$=25.6.$$

The circumference 20 m from the base is 25.6 m.

(c) Let the area at 10 m level be A_2.
Then:

$$\frac{A_2}{750}=\frac{4^2}{5^2}=\frac{16}{25},$$
$$\therefore\ 25\,A_2=16\times 750$$
$$A_2=\frac{16\times 750}{25}$$
$$=480.$$

(d) Let the area at 20 m level be A_3.
Then:

$$\frac{A_3}{480}=\frac{16}{25}$$
$$\therefore\ 25\,A_3=16\times 480$$
$$A_3=\frac{16\times 480}{25}$$
$$=307.2,$$

therefore the cross-sectional area is 307.2 m^2.

16. The ratio of diameters is 2 : 5.
(a) (i) Ratio of surface areas = 4 : 25.
(ii) Ratio of volumes $=2^3 : 5^3$
$=8 : 125.$
(b) Let the smaller volume be V. Then

$$\frac{V}{60}=\frac{8}{125},$$
$$\therefore\ 125V=8\times 60$$
$$V=\frac{8\times 60}{125}$$
$$=3.84,$$

therefore the smaller volume is 3.84 cm^3.

(c) Let the larger surface area be A.
Then

$$\frac{18}{A}=\frac{4}{25}$$
$$\therefore\quad 4A=18\times 25$$
$$A=\frac{18\times 25}{4}$$
$$=112.5,$$

therefore the larger area is 112.5 cm^2.

17. The ratio of volumes $=32 : 108$
$=8 : 27$
$=2^3 : 3^3.$
(a) Diameters of glass balls correspond to widths of cubes,
$\therefore$ ratio of diameters = 2 : 3.
(b) The ratio of surface areas $=2^2 : 3^2$
$=4 : 9.$

18. (a) Scale = 0.9 : 28.8
= 1 : 32.
(b) Ratio of surface areas $=1^2 : 32^2$
$=1 : 1024.$
(c) Let the quantity of paint required be x litres. Then

$$\frac{x}{48}=\frac{1}{1024}$$
$$\therefore\quad x=\frac{48}{1024}\text{ L.}$$

The quantity of paint

$$=\frac{48}{1024}\times 1000\text{ mL}$$
$$=46.875\text{ mL}$$
$$\approx 47\text{ mL}$$

The paint required for the model is approximately 47 mL (or $46\frac{7}{8}$ mL).

(d) Let the quantity of air be V kL.

$$\frac{V}{8192}=\frac{1^3}{32^3},$$
$$\therefore\quad V=\frac{8192}{32\,768}\text{ kL.}$$

The quantity of air in model

$$=\frac{8192}{32\,768}\times 1000\text{ L}$$
$$=250\text{ L.}$$

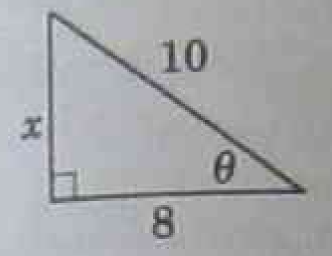
Chapter 10 Trigonometry
(page 143)

1. (a) $y=21.0$ (sine)
(b) $y=23.3$ (tan)
(c) $y=2.3$ (cos)
(d) $y=48.1$ (sine)
(e) $y=125.9$ (tan)
(f) $y=38.9$ (cos)
(g) $y=12.2$ (sine)
(h) $y=56.3$ (tan)
[All correct to one decimal place.]

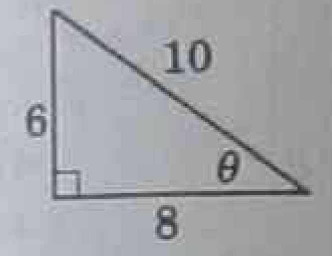
2. (a) $\theta=28°\,04'$ (sine)
(b) $x=69°\,47'$ (tan)
(c) $\phi=41°\,25'$ (cos)
(d) $\theta=28.1°$

3. (a) $\theta=30°$ (tan) (nearest degree)
(b) $\theta=55°\,09'$ (cos)
(c) $\phi=34.8°$

4.

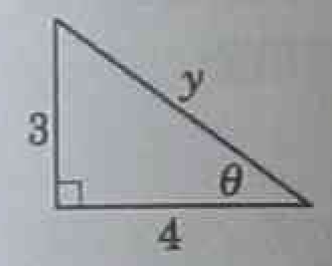


$$x^2+8^2=10^2$$
$$\therefore\quad x^2=100-64$$
$$=36$$
$$x=\sqrt{36}=6$$

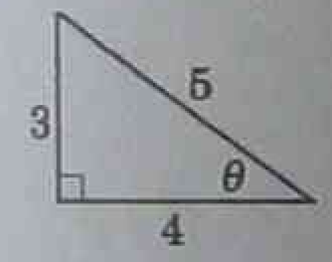


$$\sin\theta=\frac{6}{10}=\frac{3}{5}$$
$$\tan\theta=\frac{6}{8}=\frac{3}{4}$$

5.

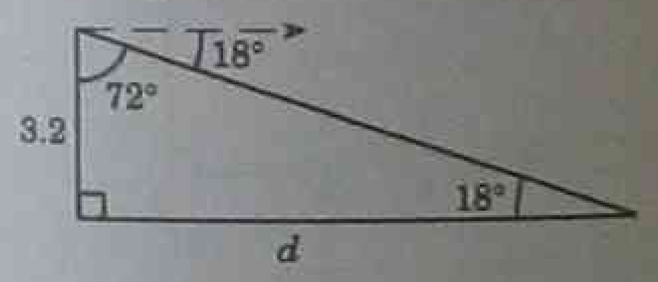


$$y^2=3^2+4^2$$
$$=9+16=25$$
$$y=\sqrt{25}=5$$



$$\sin\theta=\frac{3}{5}$$
$$\cos\theta=\frac{4}{5}$$

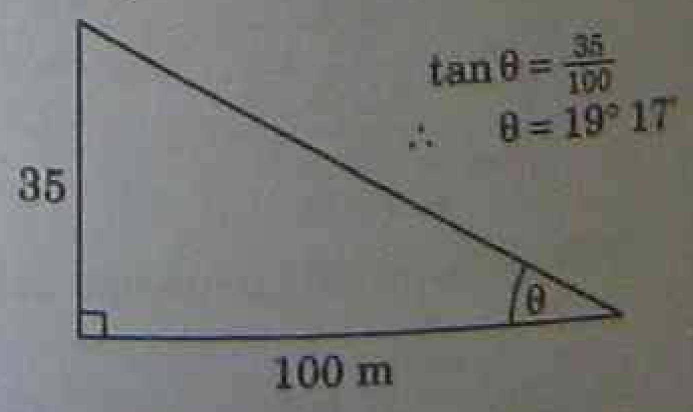
6. Let distance be d metres.
∠ of depression = ∠ of elevation



$$\frac{d}{3.2}=\tan 72°$$
$$\therefore\ d=3.2\times\tan 72°$$
$$=9.848\,5873$$
$$\approx 9.8\text{ [one dec. place]}$$

The whale is 9.8 m from the ship.

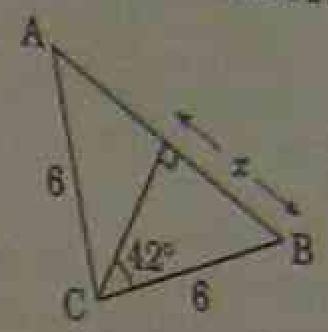
7. Let the angle be θ.



$$\tan\theta=\frac{35}{100}$$
$$\therefore\quad \theta=19°\,17'$$

The angle of elevation is 19° 17′.

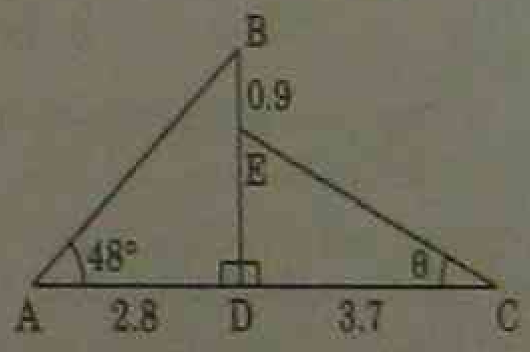
8. Perpendicular from centre to chord bisects the chord and also the vertical angle. Call the half-chord x metres.



$$\text{Then }\frac{x}{6}=\sin 42°$$
$$x=6\times\sin 42°$$
$$=4.014\,7836,$$
$$\therefore\ \text{length of chord}=2\times 4.014\,7836$$
$$=8.029\,5673$$
$$\approx 8.0\text{ [one dec. place].}$$

The chord is 8.0 m long.

9.



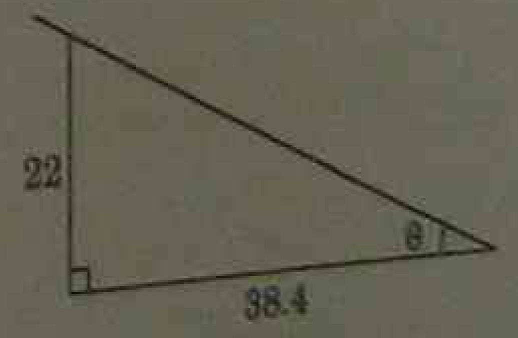
$$\text{From }\Delta ADB,\ \frac{BD}{2.8}=\tan 48°$$
$$\therefore\quad BD=2.8\times\tan 48°$$
$$=3.109\,715,$$
$$\therefore\ DE=3.109\,715-0.9=2.209\,715$$

Let ∠DCE be θ.
From ΔDCE,

$$\tan\theta=\frac{DE}{3.7}=\frac{2.209\,715}{3.7}$$
$$\theta=30°\,51'$$

$\therefore$
DB = 3.1 m [one decimal place]
∠DCE = 30° 51′.

10.

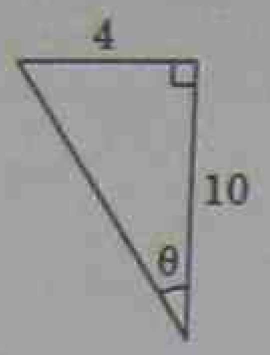


Let the altitude of the sun be θ.

$$\tan\theta=\frac{22}{38.4}$$
$$\therefore\quad \theta=30°\text{ [nearest degree]}$$

The altitude of the sun is 30°.

11. (a)



Let θ be the angle that the ball must be kicked to strike the outer can.

$$\tan\theta = \frac{4}{10}$$
$$\theta = 21^\circ\,48'.$$

The ball can be kicked at any angle between 0° and 21° 48′ relative to the nearest can.

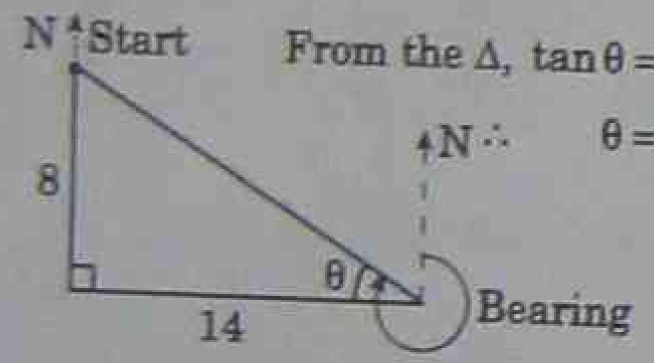
(b) As for (a), but $\tan\theta = \frac{4}{15}$

$$\theta = 14^\circ\,56'.$$

The range of angles is now within 0° and 14° 56′.

12. From the Δ, $\tan\theta = \frac{8}{14}$

$\therefore \quad \theta = 29^\circ\,45'.$

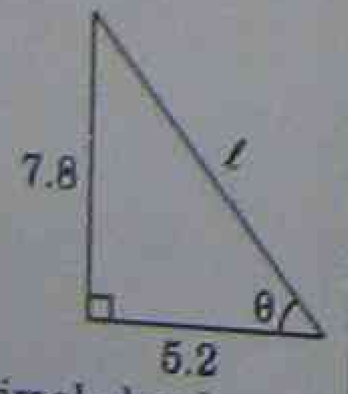


The bearing is measured clockwise from N to the direction of the starting point. This is

$$\begin{aligned} &270 + \theta^\circ \\ &= 270^\circ + 29^\circ\,45' \\ &= 299^\circ 45'. \end{aligned}$$

The bearing of the starting point is 299° 45′.

13. (a) Using Pythagoras' Theorem, let the length of the support be ℓ metres.



$$\begin{aligned} 1^2 &= 7.8^2 + 5.2^2 \\ &= 87.88 \\ \therefore\ 1 &= \sqrt{87.88} \\ &= 9.374\,4333 \\ &\approx 9.4 \text{ [one decimal place].} \end{aligned}$$

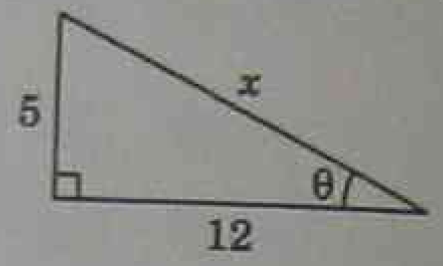
The length of the support is 9.4 m.

(b) Let the angle be θ.

$$\tan\theta = \frac{7.8}{5.2}$$
$$\therefore \quad \theta = 56^\circ\,19'.$$

The support makes an angle of 56° 19′ with the ground.

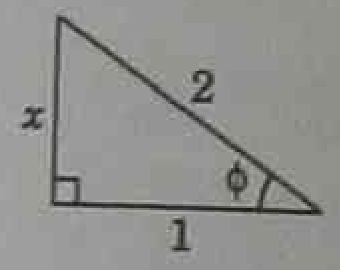
14.



$$\begin{aligned} x^2 &= 5^2 + 12^2 \\ &= 169 \\ \therefore\ x &= \sqrt{169} \\ &= 13 \end{aligned}$$

From the diagram, $\sin\theta = \frac{5}{x} = \frac{5}{13}$.

15.

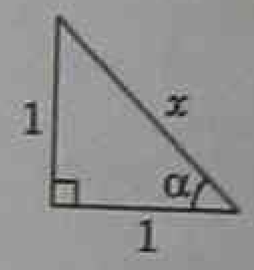


$$\begin{aligned} 1^2 + x^2 &= 2^2 \\ x^2 &= 4 - 1 \\ &= 3 \\ \therefore \quad x &= \sqrt{3}. \end{aligned}$$

From the diagram,

$$\begin{aligned} \sin\phi &= \frac{x}{2} = \frac{\sqrt{3}}{2} \\ \tan\phi &= \frac{x}{1} = \sqrt{3} \\ \cos\phi &= \tfrac{1}{2} \\ \therefore \quad \phi &= 60^\circ. \end{aligned}$$

16.



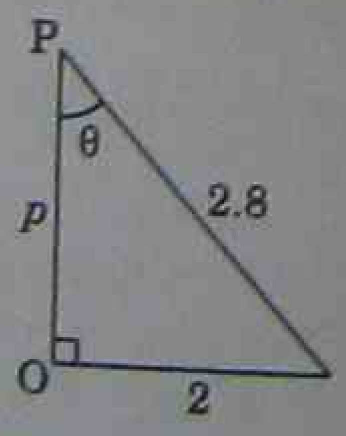
$$\begin{aligned} x^2 &= 1^2 + 1^2 \\ &= 2 \\ \therefore\ x &= \sqrt{2}. \end{aligned}$$

$$\begin{aligned} \sin\alpha &= \frac{1}{x} = \frac{1}{\sqrt{2}} \\ \cos\alpha &= \frac{1}{x} = \frac{1}{\sqrt{2}}. \end{aligned}$$

Now $\tan\alpha = 1$,

$\therefore \quad \alpha = 45^\circ.$

17. Let the perpendicular height be p.



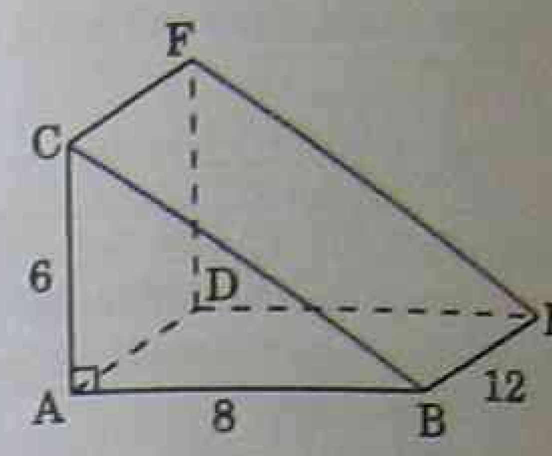
$$\begin{aligned} 2.8^2 &= p^2 + 2^2 \\ \therefore\ p^2 &= 2.8^2 - 2^2 \\ &= 3.84 \\ \therefore\ p &= \sqrt{3.84} \\ &= 1.959\,5918. \end{aligned}$$

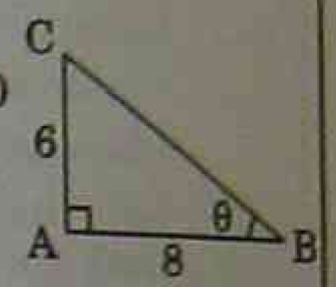
The perpendicular height is 2.0 cm.

$$\sin\theta = \frac{2}{2.8}$$
$$\theta = 45^\circ\,35'.$$

The semi-vertical angle is 45° 35′.

18.



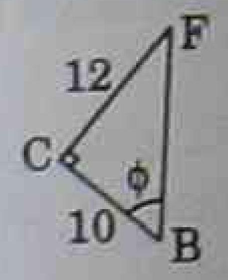


(a) $BC^2 = 6^2 + 8^2 = 100$

$\therefore BC = \sqrt{100} = 10$

BC is 10 cm.

(b)



$$\begin{aligned} BF^2 &= 12^2 + 10^2 \\ &= 244 \\ BF &= \sqrt{244} \\ &= 2\sqrt{61}. \end{aligned}$$

(c) Let ABC = θ [see diagram

$\therefore \quad \tan\theta = \frac{6}{8}$ from (a)]

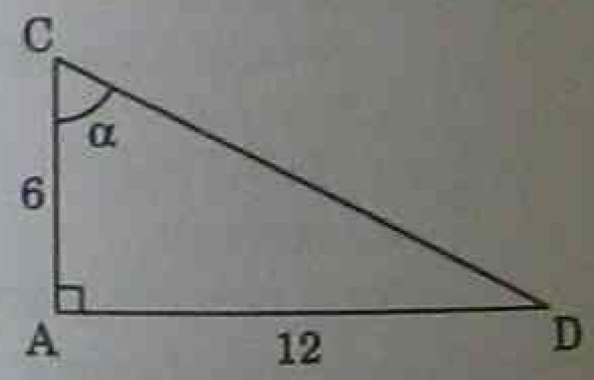
$\theta = 36^\circ\,52'.$

(d) Let ∠CBF = ϕ [see diagram

$\therefore \quad \tan\phi = \frac{12}{10}$ from (b)]

$\phi = 50^\circ\,12'.$

(e) Let ∠ACD = α°



$$\tan\alpha = \frac{12}{6} = 2$$
$$\alpha = 63^\circ\,26'.$$

19. Using the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(a)

$$\begin{aligned} \frac{f}{\sin 18^\circ} &= \frac{64}{\sin 87^\circ} \\ \therefore \quad f &= \frac{64 \sin 18^\circ}{\sin 87^\circ} \\ &= 19.804\,229 \\ \therefore \quad f &= 19.8 \text{ [one dec. place].} \end{aligned}$$

(b) The third angle

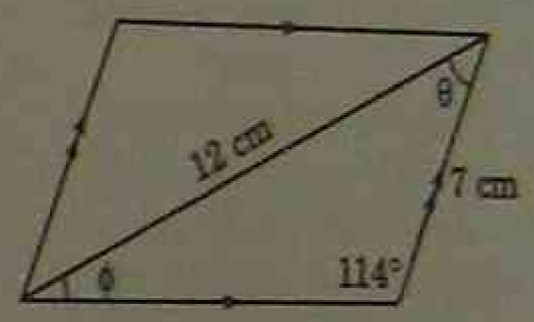
$$\begin{aligned} &= 180^\circ - (17^\circ + 15^\circ) \\ &= 148^\circ \end{aligned}$$

$$\begin{aligned} \frac{f}{\sin 148^\circ} &= \frac{12.1}{\sin 15^\circ} \\ \therefore \quad f &= \frac{12.1 \sin 148^\circ}{\sin 15^\circ} \\ &= 24.774\,155 \\ &\approx 24.8 \text{ [one dec. place].} \end{aligned}$$

(c)

$$\begin{aligned} \frac{f}{\sin 32^\circ\,16'} &= \frac{40.5}{\sin 86^\circ\,27'} \\ \therefore \quad f &= \frac{40.5 \sin 32^\circ\,16'}{\sin 86^\circ\,27'} \\ &= 21.662\,919 \\ &\approx 21.7 \text{ [one dec. place].} \end{aligned}$$

20.



We are required to find the angle θ, but as the sine rule requires opposites, we can only find the third angle. Call this angle ϕ.

$$\begin{aligned} \frac{7}{\sin\phi} &= \frac{12}{\sin 114^\circ} \\ \therefore \frac{\sin\phi}{7} &= \frac{\sin 114^\circ}{12} \\ \therefore\ \sin\phi &= \frac{7 \sin 114^\circ}{12} \\ &= 0.532\,9015 \\ \therefore \quad \phi &= 32^\circ\,12'. \end{aligned}$$

$$\begin{aligned} \text{Then } \theta &= 180^\circ - (114^\circ + 32^\circ\,12') \\ &= 180^\circ - (146^\circ\,12') \\ &= 33^\circ\,48'. \end{aligned}$$

The area of a triangle

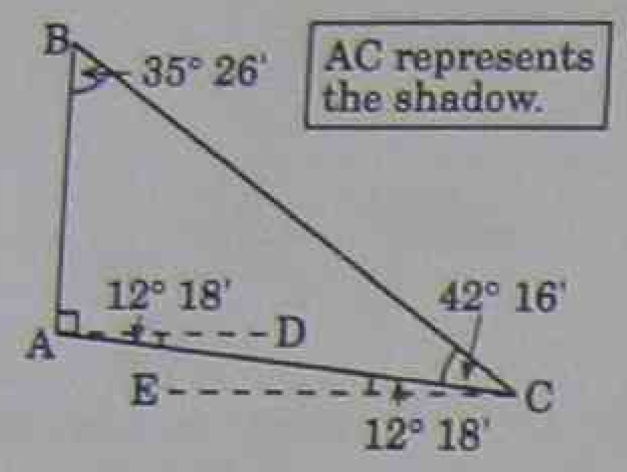
$$\begin{aligned} &= \tfrac{1}{2}ab\sin C \\ &= \tfrac{1}{2} \times 7 \times 12 \times \sin 33^\circ\,48' \\ &= 23.364\,416, \end{aligned}$$

$\therefore$ the area of the parallelogram

$$\begin{aligned} &= 2 \times \text{area of } \Delta \\ &= 46.728\,832 \\ &= 46.7 \text{ cm}^2 \end{aligned}$$

[one decimal place].

21.



AC represents the shadow.

$$\angle BCA = 54°34' - 12°18' = 42°16'$$
$$\angle DAC = 12^\circ 18'$$
as $\angle DAC = \angle ACE$ [alt. angles AD $\parallel$ EC]

$\therefore$ $\angle BAC = 90^\circ + 12^\circ 18' = 102^\circ 18'$,

$\therefore$ $\angle ABC = 180^\circ - (102^\circ 18' + 42^\circ 16')$

$= 180^\circ - 144^\circ 34'$

$= 35^\circ 26'$.

Using the sine rule:

$$\frac{AC}{\sin 35^\circ 26'} = \frac{AB}{\sin 42^\circ 16'}$$

$\therefore$ $AC = \dfrac{12 \sin 35^\circ 26'}{\sin 42^\circ 16'}$

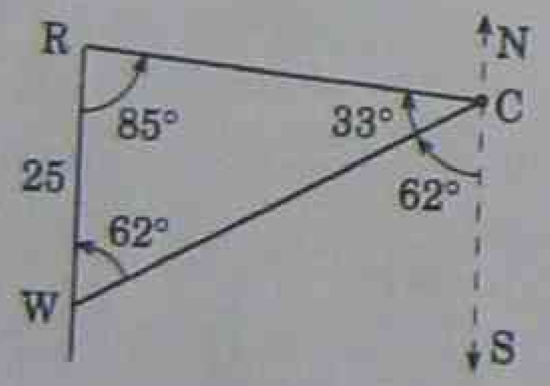
$= 10.343\,813$

≈ 10.3

[one dec. place].

The length of the shadow is 10.3 m.

22.



The bearing of W = 242°

$\therefore$ $\angle SCW = 62^\circ (242^\circ - 180^\circ)$

$\angle WCR = 275^\circ - 242^\circ = 33^\circ$,

and $\angle RWC = \angle WCS$

[alt. angles, RW $\parallel$ NS]

$\therefore$ $\angle RWC = 62^\circ$

Then $\angle WRC = 180^\circ - (33^\circ + 62^\circ)$

$= 180^\circ - 95^\circ$

$= 85^\circ$.

The shortest distance to a port will be RC, as the shortest side of a triangle is opposite the smallest angle ($\angle RWC < \angle WRC$).

Then, using the sine rule:

$$\frac{RC}{\sin 62^\circ} = \frac{25}{\sin 33^\circ}$$

$\therefore$ $RC = \dfrac{25 \sin 62^\circ}{\sin 33^\circ}$

$= 40.529\,026$

≈ 40.5 [one dec. place].

The distance to the nearest port (Redhead) is 40.5 km.

23. Using the cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A,$$

where the angle is the included angle. Note that for the cosine rule we need two sides and an included angle to find the third side.

(a) (i) $t^2 = 8^2 + 11^2 - 2(8)(11)\cos 68^\circ$

$= 119.069\,24$

$\therefore t = \sqrt{119.069\,24}$

$= 10.911\,885$

≈ 10.9 [one dec. place].

(ii) $t^2 = 6^2 + 13^2$

$- 2(6)(13)\cos 114^\circ 12'$

$= 268.947\,99$

$\therefore t = \sqrt{268.947\,99}$

$= 16.399\,634$

≈ 16.4 [one dec. place].

(iii) $t^2 = 4.6^2 + 7.8^2$

$- 2(4.6)(7.8)\cos 24^\circ 8'$

$= 16.512\,078$,

$\therefore t = \sqrt{16.512\,078}$

$= 4.063\,5056$

≈ 4.1 [one dec. place].

(b) Using the area formula:

$$\text{area} = \tfrac{1}{2} ab \sin C.$$

(i) $\text{Area} = \tfrac{1}{2}(8)(11)\sin 68^\circ$

$= 40.796\,09$

≈ 40.8 [one dec. place].

The area is 40.8 units^2.

(ii) $\text{Area} = \tfrac{1}{2}(6)(13)\sin 114^\circ 12'$

$= 35.572\,685$

≈ 35.6 [one dec. place].

The area is 35.6 units^2.

(iii) $\text{Area} = \tfrac{1}{2}(4.6)(7.8)\sin 24^\circ 08'$

$= 7.334\,9745$

≈ 7.3 [one dec. place].

The area is 7.3 units^2.

---

24. Using the angle form of the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

To find any angle of a triangle using the cosine rule we need the length of all three sides.

(a) $\cos\theta = \dfrac{17^2 + 15^2 - 19^2}{2(17)(15)}$

$= \dfrac{153}{510}$

$= 0.3$

$\theta = 72^\circ 33'$.

INV COS

(b) $\cos\alpha = \dfrac{8^2 + 21^2 - 16^2}{2(8)(21)}$

$= \dfrac{249}{336}$

$= 0.741\,0714$,

$\therefore$ $\alpha = 42^\circ 11'$.

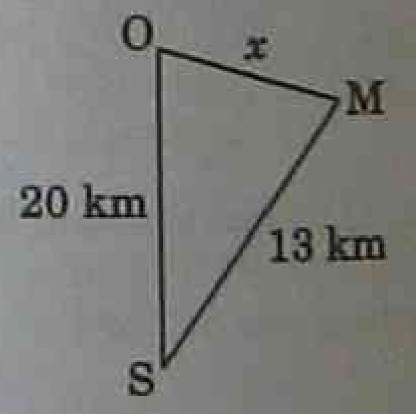
$\cos\phi = \dfrac{16^2 + 21^2 - 8^2}{2(16)(21)}$

$= \dfrac{633}{672}$

$= 0.941\,9642$

$\therefore$ $\phi = 19^\circ 37'$.

25.



$x^2 = 20^2 + 13^2 - 2(20)(13)\cos 35^\circ$

$= 143.040\,94$

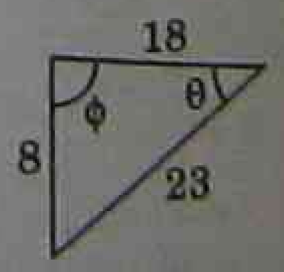
$\therefore x = \sqrt{143.040\,94}$

$= 11.959\,972$

≈ 12.0 [one dec. place].

Maria is 12.0 km from the start.

26.



The smallest ∠ is opposite the shortest side. The largest ∠ is opposite the longest side. Let the smallest angle be θ.

$\cos\theta = \dfrac{18^2 + 23^2 - 8^2}{2(18)(23)}$

$= \dfrac{789}{828}$

$= 0.952\,899$

$\therefore$ $\theta = 17^\circ 39'$.

Let the largest angle be ϕ.

$\cos\phi = \dfrac{8^2 + 18^2 - 23^2}{2(8)(18)}$

$= \dfrac{-141}{288}$

$= -0.489\,5833$

$\phi = 119^\circ 18'$.

The smallest angle is 17° 39' and the largest angle is 119° 18'.

$\text{Area} = \tfrac{1}{2} \times 8 \times 18 \times \sin\phi$

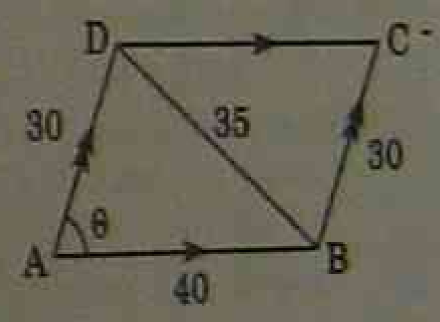
$= 4 \times 18 \times \sin 119^\circ 18'$

$= 62.780\,869$

≈ 62.8 [one dec. place].

Area is 62.8 cm^2.

27.



Let one angle of the parallelogram be θ. [As in the diagram.]

$$\cos\theta = \frac{30^2 + 40^2 - 35^2}{2(30)(40)}$$

$= \dfrac{1275}{2400}$

$= 0.531\,25$

$\therefore$ $\theta = 57^\circ 55'$.

Opposite ∠s are equal,

$\therefore$ $\hat{C} = 57^\circ 55'$.

Also $\angle ABC = \angle ADC$

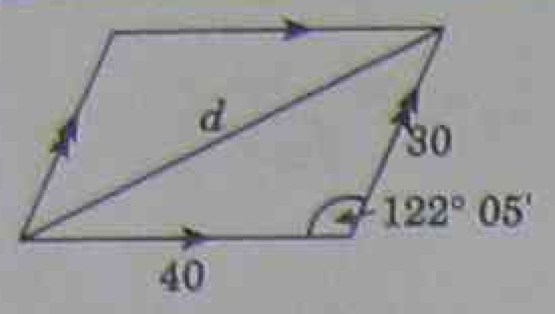
$= 180^\circ - 57^\circ 55'$

$= 122^\circ 05'$,

as there are co-interior ∠s and AD $\parallel$ BC.

Therefore the angles of the parallelogram are 57° 55' and 122° 05'.

Continued

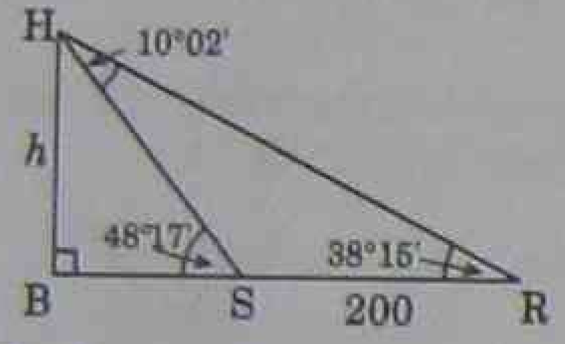


Let the diagonal be d cm.

$$d^2 = 40^2 + 30^2 - 2(40)(30)\cos 122^\circ 05'$$
$$= 3774.7651,$$
$$\therefore\ d = \sqrt{3774.7651}$$
$$= 61.439\,117$$
$$= 61.4 \text{ [one dec. place].}$$

The other diagonal is 61.4 cm.

28.



Link method question.
Find the length of HS by using the sine rule in ΔHSR. We need ∠SHR for the sine rule.
In ΔHSR.

$$\angle SHR = 48^\circ 17' - 38^\circ 15'$$
[∠HSB is the exterior ∠ of ΔHSR]
$$= 10^\circ 02'.$$

Then $\dfrac{HS}{\sin 38^\circ 15'} = \dfrac{200}{\sin 10^\circ 2'}$

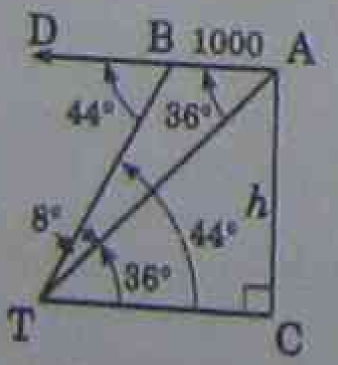
$$\therefore\ HS = \frac{200 \sin 38^\circ 15'}{\sin 10^\circ 2'}$$
$$= 710.699\,22.$$

In ΔHBS,

$$\frac{h}{HS} = \sin 48^\circ 17',$$
$$\therefore\ h = HS \times \sin 48^\circ 17'$$
$$= 710.699\,22 \times \sin 48^\circ 17'$$
$$= 530.497\,62$$
$$\approx 530 \text{ [to the nearest metre].}$$

The hill is 530 m high.

29.



$\angle ATC = 36^\circ$
$\angle BTC = 44^\circ$
[Alt. ∠'s, AB || TC]
$\therefore\ \angle BTA = 8^\circ$
Also $\angle TBA = 136^\circ$
[∠ sum of Δ]

Let the altitude AC be h metres.
[Another link method]
Find AT from ΔBAT and use it in ΔTCA.

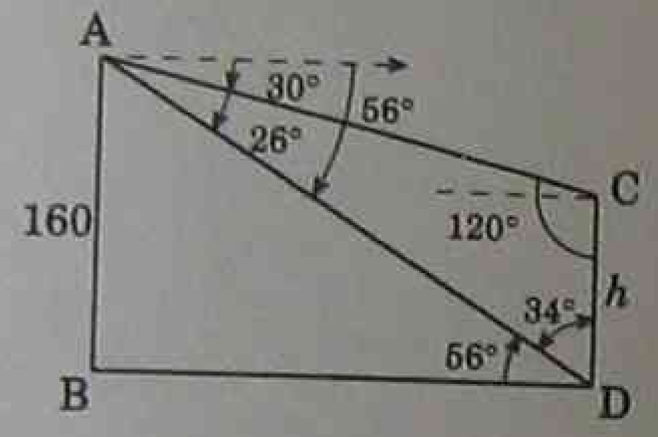
$$\frac{AT}{\sin 136^\circ} = \frac{1000}{\sin 8^\circ}$$
$$\therefore\ AT = \frac{1000 \sin 136^\circ}{\sin 8^\circ}$$
$$= 4991.3264$$

In ΔTCA,

$$\frac{h}{AT} = \sin 36^\circ$$
$$\therefore\ h = AT \times \sin 36^\circ$$
$$= 4991.3264 \times \sin 36^\circ$$
$$= 2933.828 \text{ metres}$$
$$= 2.933\,828 \text{ km}$$
$$\approx 2.9 \text{ km}$$
[two significant figures]

The altitude of the plane is 2.9 km.

30.



[Another link method]

$$\angle ADB = 56^\circ \text{ [equal to } \angle \text{ of depression]}$$
$$\angle CAD = 26^\circ$$
Also $\angle CDA = 90^\circ - 56^\circ = 34^\circ$
$$\therefore\ \angle ACD = 180^\circ - (26^\circ + 34^\circ)$$
$$= 120^\circ.$$

Use ΔABD to find AD and then use AD in ΔADC.
In ΔABD,

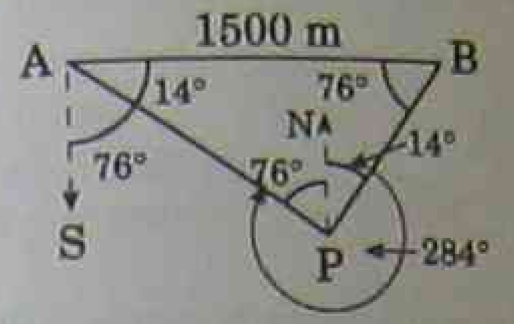
$$\frac{160}{AD} = \sin 56^\circ$$
$$\frac{AD}{160} = \frac{1}{\sin 56^\circ}$$
$$\therefore\ AD = \frac{160}{\sin 56^\circ}$$
$$= 192.994\,87.$$

In ΔADC, using the sine rule:

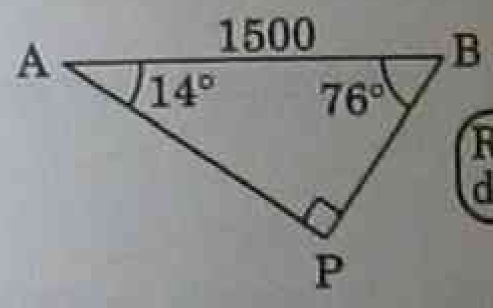
$$\frac{h}{\sin 26^\circ} = \frac{AD}{\sin 120^\circ}$$
$$\therefore\ h = \frac{192.994\,87 \times \sin 26^\circ}{\sin 120^\circ}$$
$$= 97.691\,571$$
$$\approx 98 \text{ [to the nearest metre].}$$

The height of the building is 98 m.

31.



$$\angle NPA = 360^\circ - 284^\circ$$
$$= 76^\circ$$
$$\therefore\ \angle SAP = 76^\circ$$
[Alt. ∠'s, AS || NP].
Then $\angle PAB = 90^\circ - 76^\circ$
$$= 14^\circ$$
[angle between south and east is 90°]
Also $\angle APB = 76^\circ + 14^\circ = 90^\circ$
and $\angle ABP = 76^\circ$.



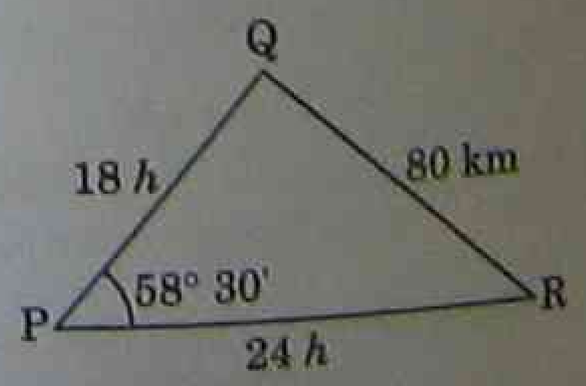
Redraw diagram

We have a right-angled Δ.

$$\frac{AP}{1500} = \sin 76^\circ$$
$$AP = 1500 \times \sin 76^\circ$$
$$= 1455.4436$$
$$\approx 1455 \text{ [to the nearest metre].}$$
$$\frac{BP}{1500} = \sin 14^\circ$$
$$BP = 1500 \times \sin 14^\circ$$
$$= 362.882\,84$$
$$\approx 363 \text{ [to the nearest metre].}$$

The distances of the trawler from the ends of the breakwater are 1455 m and 363 m.

32.



Let the time to be 80 km apart be h hours.

Then $PQ = 18h$
$PR = 24h$

$\boxed{D = S \times T}$

Now using the cosine rule:

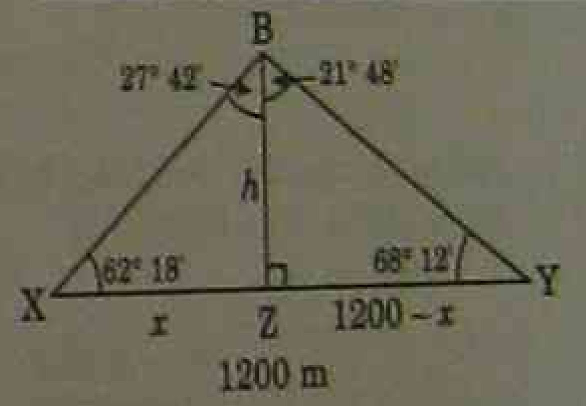
$$80^2 = (18h)^2 + (24h)^2 - 2(18h)(24h)\cos 58^\circ 30'$$
$$= h^2\left[18^2 + 24^2 - 2(18)(24)\cos 58^\circ 30'\right]$$
$$\therefore\ h^2 = \frac{80^2}{18^2 + 24^2 - 2(18)(24)\cos 58^\circ 30'}$$
$$= 14.267\,84,$$
$$\therefore\ h = \sqrt{14.267\,84}$$
$$= 3.777,$$

Use $\boxed{\text{INV}}$ $\boxed{\circ\,'\,''}$ buttons

that is, the time is 3.777 hours
= 3 hours 47 minutes.

Use $\boxed{\text{MEMORY IN}}$ on calculator — calculate denominator first

33.



Let XZ be x metres, then
ZY = $(1200 - x)$ metres.
In ΔBXZ,

$$\frac{x}{h} = \tan 27^\circ 42'$$
$$\therefore\ x = h \tan 27^\circ 42'. \qquad (1)$$

In ΔBZY,

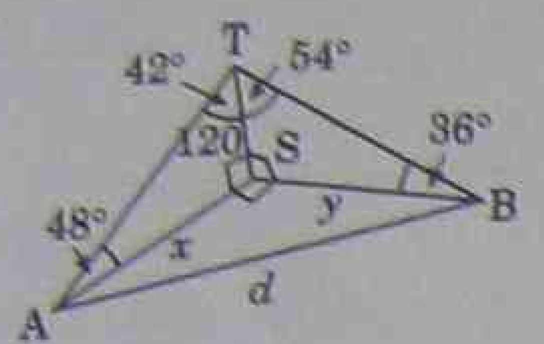
$$\frac{1200 - x}{h} = \tan 21^\circ 48',$$
$$\therefore\ 1200 - x = h \tan 21^\circ 48'. \qquad (2)$$

Substitute (1) into (2).

$$1200 - h\tan 27^\circ 42' = h \tan 21^\circ 48'$$
$$\therefore\ 1200 = h\tan 27^\circ 42' + h\tan 21^\circ 48',$$
that is, $h(\tan 27^\circ 42' + \tan 21^\circ 48')$
$$= 1200,$$
$$\therefore\ h = \frac{1200}{\tan 27^\circ 42' + \tan 21^\circ 48'}$$
$$= 1297.3209$$
$$\approx 1297$$
[to the nearest metre]
$$\approx 1300$$
[to the nearest 100 m]

The height of the balloon is approximately 1300 m.

34.



Monika is at A, Mabel is at B.
∠ASB = 90° [angle between south and east]
Let the distance required, AB, be d metres.
Let AS = x m, SB = y m.

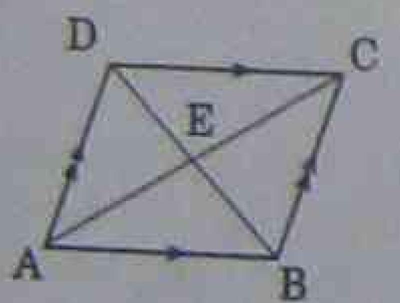
Then from ΔAST, $\frac{x}{120} = \tan 42°$
∴ $x = 120\tan 42°$.
From ΔTSB, $\frac{y}{120} = \tan 54°$
∴ $y = 120\tan 54°$.
Now in ΔABS, $d^2 = x^2 + y^2$ [Pythagoras' Theorem]

$$\therefore\ d^2 = (120\tan 42°)^2 + (120\tan 54°)^2$$
$$= 11\,674.475 + 27\,279.752$$
$$= 38\,954.227$$
$$\therefore\ d = \sqrt{38\,954.227}$$
$$= 197.368\,25$$
$$\approx 197$$
[to the nearest metre].

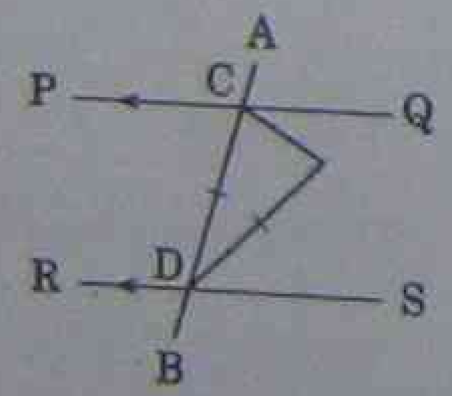
Therefore the distance between Monika and Mabel is 197 metres.

Chapter 11 Basic geometry (page 162)

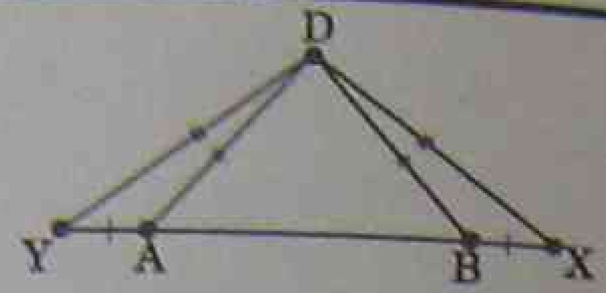
1. (a)



(b)



(c)



2. (a) An isosceles triangle ABC is drawn with sides AC and BC of equal length. The base AB is produced to D such that BD is of equal length to BC.

(b) The sides PR and PQ of the triangle PQR are produced to S and T respectively such that RS is of equal length to QT.

3. $a = 180 - (27 + 72)$
$= 180 - 99$
$= 81$ [∠ sum of a straight line].

4. $b + 18 = 90$ [complementary angles]
∴ $b = 90 - 18$
∴ $b = 72$.

5. $c = 128$ [vertically opposite angles]
also $d = 180 - 128$ [angle sum of a straight line]
∴ $d = 52$.

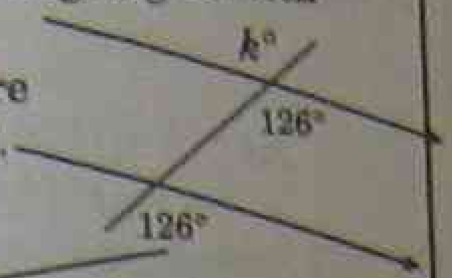
6. $e + 109 + 112 + 92 = 360$ [angles at a point]
∴ $e + 313 = 360$
∴ $e = 360 - 313$
$= 47$.

7. $g + 43 + 42 = 180$ [CD is a str. line]
∴ $g = 180 - 85$
$= 95$.
Then
$f + 20 + 95 + 43 = 180$ [AB is a str. line]
That is $f + 158 = 180$
∴ $f = 180 - 158$
$= 22$.

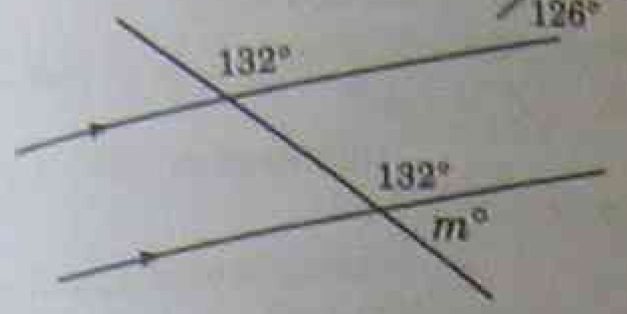
8. $3x + 28 = x + 40$ [vert. opposite angles]
∴ $2x = 12$
$x = 6$.
Then $3x + 28 = 3(6) + 28$
$= 46$
Check $x + 40 = 6 + 40$
$= 46$.
The equal angles are 46°

9. $h = 124$ [alternate angles, parallel lines].

10. $k = 126$ [corresponding angles with parallel lines, then k° and 126° are vertically opposite].

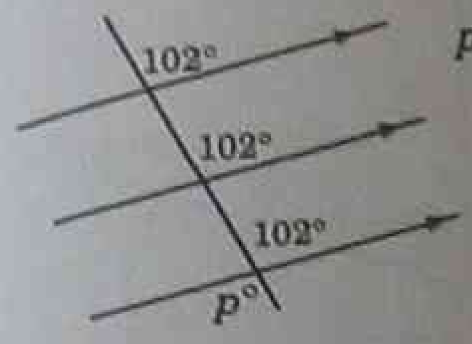


11.



$m + 132 = 180$
$m = 180 - 132$
$= 48$.
[corresponding ∠'s in parallel lines, then m° and 132° are supplementary]

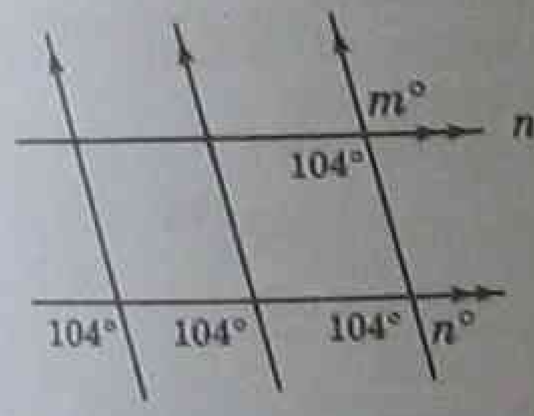
12.



$p = 102$

Corresponding ∠'s in parallel lines, then p° and 102° are vertically opposite.

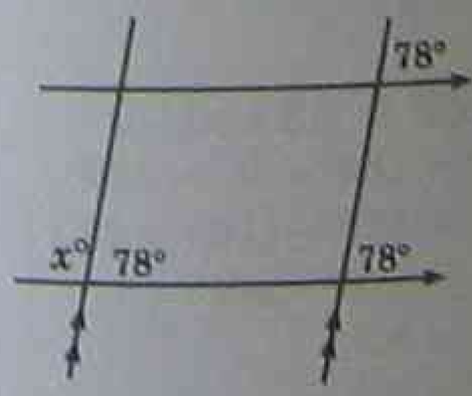
13.



$m = 104$
$n + 104 = 180$
$n = 180 - 104$
$= 76$

For m, corresponding ∠'s with parallel lines, and m° and 104° are vertically opposite. For n, as for m, but $n + 104 = 180$ [straight line].

14.

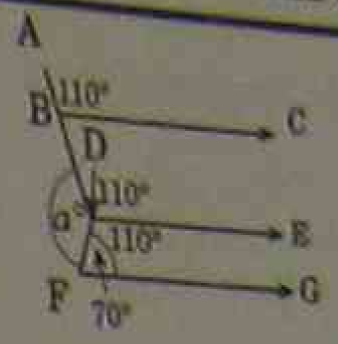


$x + 78 = 180$
$x = 180 - 78$
$= 102$.

[corresponding ∠'s with parallel lines, x° and 78° are supplementary]

15. $4y + 120 = 180$ [cointerior angles, parallel lines]
∴ $4y = 180 - 120$
$4y = 60$
∴ $y = 15$.

16.



Draw a third line parallel to other two at D [as in the diagram].

∠BDE = 110° [corr. angles, BC ∥ DE]
∠FDE = 110° [coint. angles, DE ∥ FG]
∴ ∠ADF = 360° − (110° + 110°)
= 360° − 220°
= 140°
∴ $a = 140$.

17. $b + 58 + 67 = 180$ [∠ sum of Δ]
∴ $b = 180 - 125$
$= 55$.

18. $c + c + 56 = 180$ [base angles of isosceles triangle, angle sum of Δ]
$2c = 180 - 56$
∴ $2c = 124$
∴ $c = 62$.

19. $a + a + 2a = 180$ [∠ sum of a Δ]
$4a = 180$
$a = 45$.

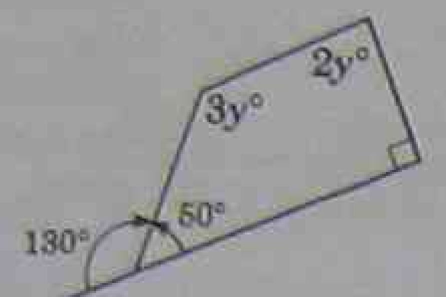
20. $d + 59 + 59 = 180$ [base angles of an isosceles Δ, ∠ sum of a Δ]
∴ $d = 180 - 118$
$= 62$.

21. $(3x - 9) + (x + 64) + (2x + 35) = 180$ [∠ sum of a Δ]
∴ $6x + 90 = 180$
∴ $6x = 90$
∴ $x = \frac{90}{6}$,
that is, $x = 15$.

22. $y + 88 + 87 + 57 = 360$ [∠ sum of a quadrilateral]
∴ $y + 232 = 360$
∴ $y = 360 - 232$
that is, $y = 128$.

23. $(3b + 48) + 2b + b + 2b = 360$ [∠ sum of a quadrilateral]
∴ $8b + 48 = 360$
∴ $8b = 360 - 48$
$= 312$
∴ $b = \frac{312}{8}$,
that is, $b = 39$.

24.



The other angle in the quadrilateral is 50°, [supplementary ∠'s],

then $3y+2y+90+50=360$
[∠ sum of a quadrilateral]

$\therefore\ 5y+140=360$

$\therefore\ 5y=360-140$

$=220$

$\therefore\ y=\dfrac{220}{5}$,

that is, $y=44$.

25. $d+87=132$ [exterior ∠ of a Δ]

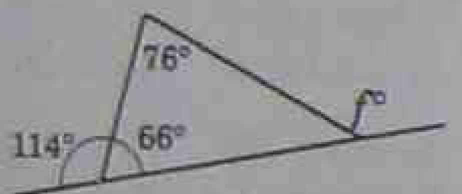
$\therefore\ d=132-87$

$=45$.

26. $e=74+47$ [exterior ∠ of a Δ]

$=121$.

27.



114° and 66° are supplementary.

$f=76+66$ [exterior ∠ of a Δ]

$=142$.

28. $x=39$ [alternate angles, parallel lines]

$y+39+54=180$ [∠ sum of a Δ]

$\therefore\ y+93=180$

$\therefore\ y=180-93$

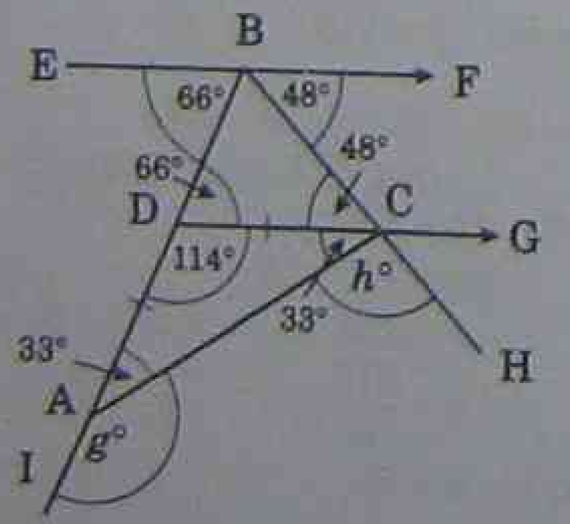
$=87$.

29. $5z+10=3z+50$ [exterior ∠ of a Δ]

$\therefore\ 2z=40$

$z=20$.

30.



$\angle BDC=66°$
[alt. angles, EF ∥ DG]

$\therefore\ \angle ADC=114°$
[AB is a straight line]

As $AD=DC$,

$\angle DAC=\angle DCA=33°$
[base angles of isosceles Δ, ∠ sum of a Δ]

Also, $\angle DCB=48°$
[alt.angles, EF ∥ DG]

Then, $g+33=180$
[IB is a straight line]

$\therefore\ g=180-33$

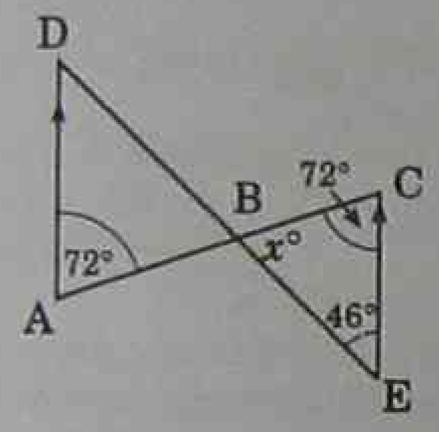
$=147$,

and $h+33+48=180$
[BH is a straight line]

$\therefore\ h=180-81$

$=99$.

31.



$\angle BCE=72°$
[alternate angles. AD ∥ CE]

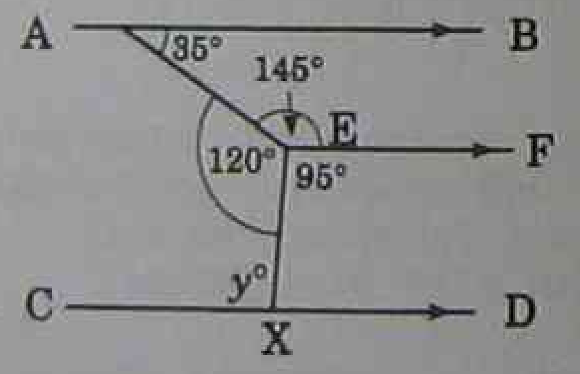
$\therefore\ x+72+46=180$

$\therefore\ x+118=180$

$\therefore\ x=180-118$

$=62$.

32.



Construct EF ∥ AB and CD.

$\angle AEF=180°-35°=145°$

$\therefore\ \angle FEX=360°-(145°+120°)$

$=360°-265°$

$=95°$,

$\therefore\ \angle CXE=95°$
[alternate angles, EF ∥CD]

$\therefore\ y=95$.

33. (a) Angle sum $=(n-2)\times 180°$ $\boxed{n=6}$

$=(4)\times 180°$

$=720°$

Each angle $=720\div 6$

$=120°$.

(b) Angle sum $=(n-2)\times 180°$ $\boxed{n=10}$

$=8\times 180°$

$=1440°$

Each angle $=1440\div 10$

$=144°$.

(c) Angle sum $=(n-2)\times 180°$ $\boxed{n=52}$

$=50\times 180°$

$=9000°$

Each angle $=\dfrac{9000}{52}$

$\approx 173.1°$ [One dec. place]

34. (a) Exterior angle $=\dfrac{360°}{n}$

$=\dfrac{360°}{5}$

$=72°$.

(b) Exterior angle $=\dfrac{360°}{n}$

$=\dfrac{360°}{12}$

$=30°$.

35. Interior angle $=\dfrac{(n-2)\times 180}{n}$

$\therefore\ 140=\dfrac{180n-360}{n}$

$140n=180n-360$

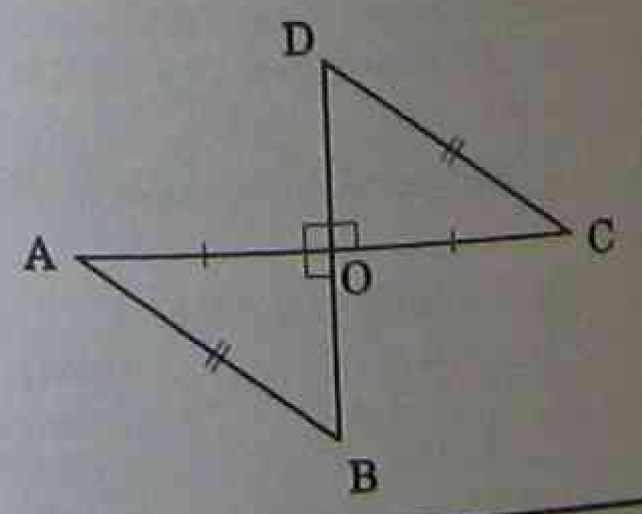
$40n=360$

$n=\dfrac{360}{40}$

$=9$.

The polygon has 9 sides.

36.



Note: **Always mark data on the diagram** and then solve the question on the diagram before attempting to write a solution.

In the ΔAOB and ΔOCD,

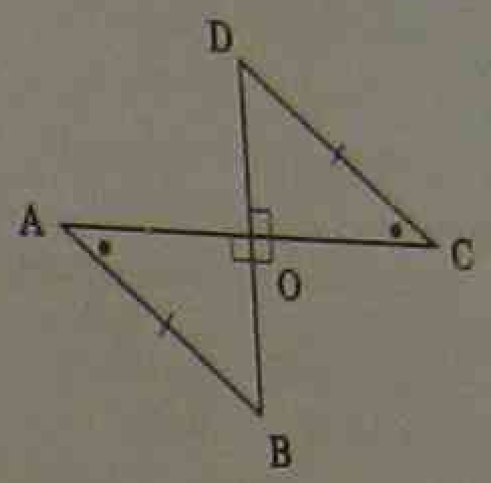
$AB=DC$ [data]

$AO=OC$ [data]

$\angle AOB=\angle DOC=90°$ [AC⊥DB]

$\therefore\ \Delta AOB\equiv\Delta OCD$ [RHS].

37.



In Δ's ABO and CDO

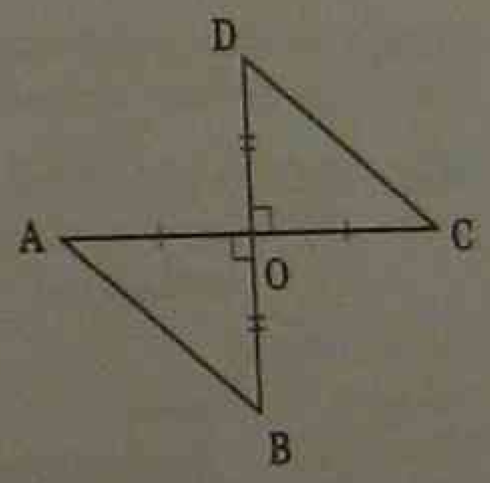
$AB=DC$ [data]

$\angle OAB=\angle OCD$ [data]

$\angle AOB=\angle DOC$ [both 90°, AC⊥DB]

$\therefore\ \Delta ABO\equiv\Delta CDO$ [AAS].

38.



In Δ's ABO and CDO

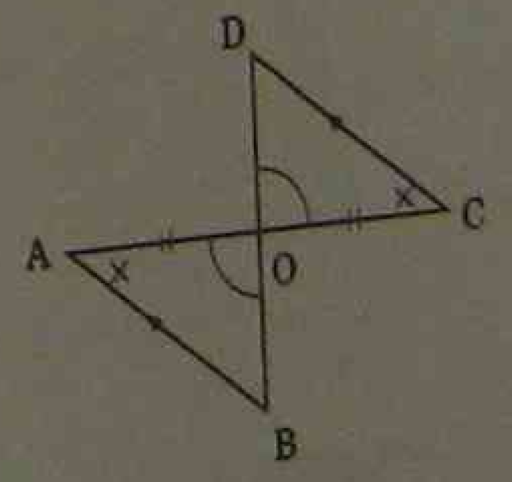
$AO=OC$ [data]

$BO=OD$ [data]

$\angle AOB=\angle DOC$ [both 90°, AC⊥DB]

$\therefore\ \Delta ABO\equiv\Delta CDO$ [SAS].

39.



In Δ's ABO and CDO

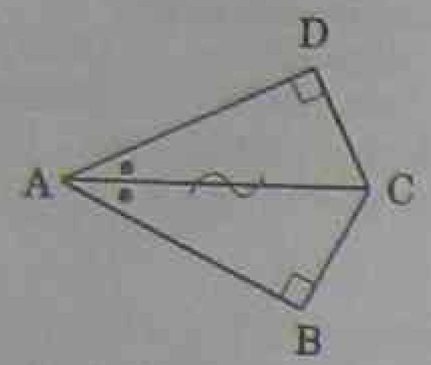
$AO=OC$ [data]

$\angle BAO=\angle DCO$ [alt. ∠'s, AB ∥ CD]

$\angle AOB=\angle COD$ [vert opp. angles]

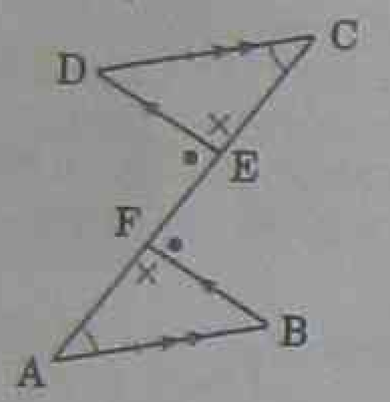
$\therefore\ \Delta ABO\equiv\Delta CDO$ [AAS].

40.



In Δ's ABC and ADC
AC = AC [common side]
$\angle BAC = \angle DAC$ [CA bisects $\angle DAB$]
$\angle ABC = \angle CDA$ [both 90°]
$\therefore \triangle ABC \equiv \triangle ADC$ [AAS].

41.



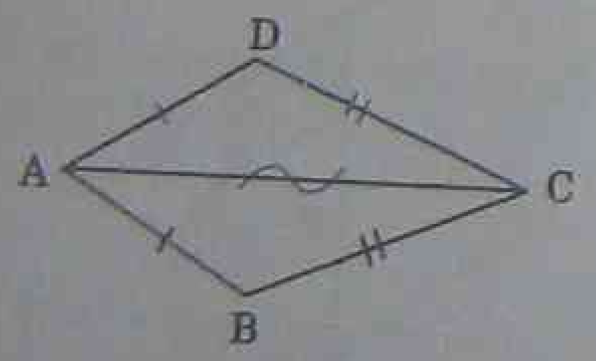
$\angle BFE = \angle FED$ [alt. ∠'s, FB ∥ DE]
$\therefore \angle AFB = \angle DEC$ [supplements of equal angles]

In Δ's AFB and DEC
AB = DC [data]
$\angle AFB = \angle DEC$ [proved above]
$\angle FAB = \angle DCE$ [alt. ∠'s, AB ∥ DC]
$\therefore \triangle AFB \equiv \triangle DEC$ [AAS].

(i) Then AF = CE [corresponding sides in congruent Δ's].

(ii) Also AF + EF = CE + EF [adding EF to both].
$\therefore$ AE = CF.

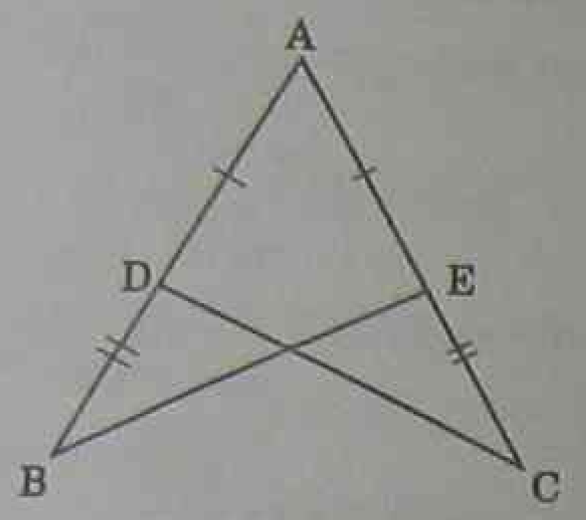
42.



In Δ's ABC and ACD
AB = AD [data]
BC = DC [data]
AC = AC [common side]
$\therefore \triangle ABC \equiv \triangle ACD$ [SSS],

$\therefore \angle BAC = \angle DAC$ [angles in corresponding positions in congruent Δ's].

43.

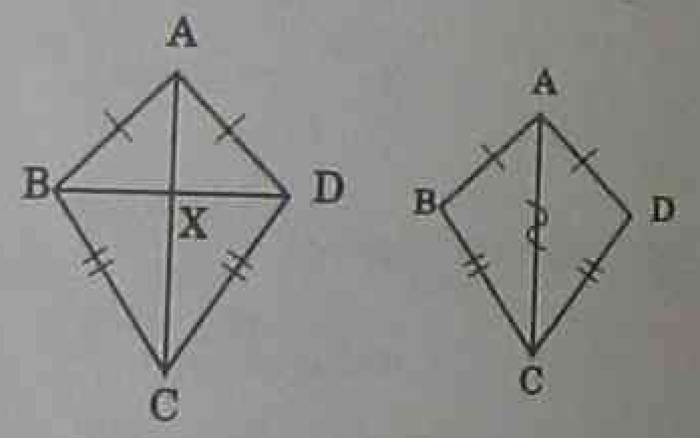


AD + DB = AE + EC [addition of equal sides
$\therefore$ AB = AC.

In Δ's ABE and ADC
AB = AC [proved above]
AE = AD [data]
$\angle BAE = \angle CAD$ [common ∠]
$\therefore \triangle ABE \equiv \triangle ADC$ [SAS],

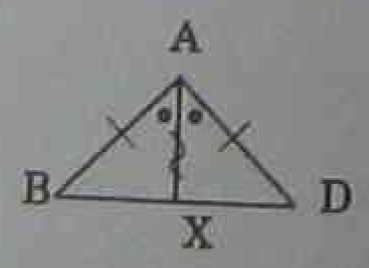
$\therefore \angle ABE = \angle ACD$ [∠'s in corresponding positions in congruent Δ's].

44.



(i) In Δ's ABC and ADC:
AB = AD [data]
BC = DC [data]
AC = AC [common side]
$\therefore \triangle ABC \equiv \triangle ADC$ [SSS].

(ii) Now $\angle BAX = \angle DAX$ [∠'s in corresponding positions in congruent Δ's].



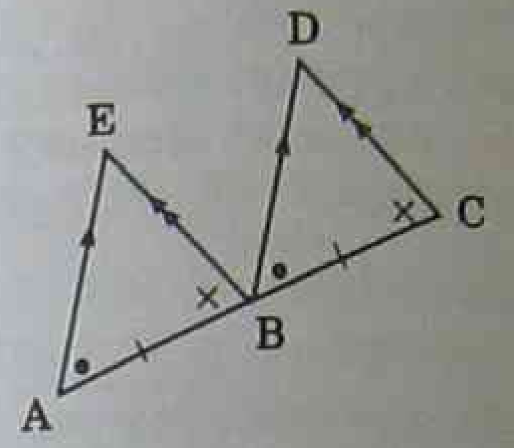
In Δ's ABX and ADX:
AB = AD [data]
AX = AX [common side]
$\angle BAX = \angle DAX$ [proved above]
$\therefore \triangle ABX \equiv \triangle ADX$ [SAS].

(iii) BX = XD [corresponding sides in congruent Δ's].
$\therefore$ AC bisects BD.

Continued

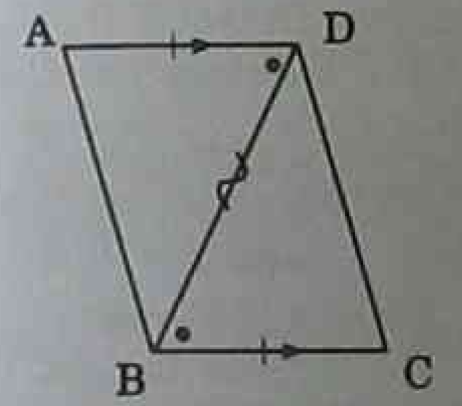
Also $\angle BXA = \angle AXD$ [∠'s in corresponding positions in congruent Δ's].
But BD is a straight line,
$\therefore \angle BXA = \angle AXD = 90°$,
that is, $AC \perp BD$.
AC bisects BD at right angles.

45.



In Δ's ABE and BCD
AB = BC [B is midpoint of AC]
$\angle BAE = \angle CBD$ [corr. ∠'s, AE ∥ BD]
$\angle ABE = \angle BCD$ [corr. ∠'s, BE ∥ CD]
$\therefore \triangle ABE \equiv \triangle BCD$ [AAS].

46.



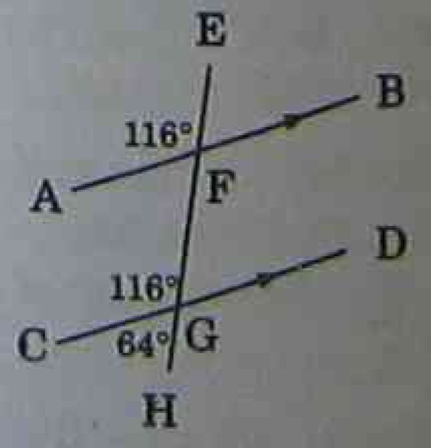
In Δ's ABD and BCD
AD = BC [data]
BD = BD [common side]
$\angle ADB = \angle DBC$ [alt. ∠'s, AD ∥ BC]
$\therefore \triangle ABD \equiv \triangle BCD$ [SAS].

(i) AB = DC [corresponding sides in congruent ∠'s].

(ii) $\angle ABD = \angle BDC$ [∠'s in corresponding positions in congruent Δ's].

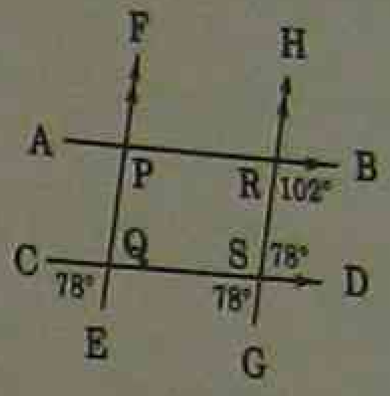
(iii) As $\angle ABD = \angle BDC$, AB ∥ DC [a pair of alternate ∠'s equal].

47.



$\angle CGF = 116°$ [corresp. ∠'s, AB ∥ CD]
$\therefore \angle CGH = 180° - 116°$ [∠ sum of a straight line]
$= 64°$.

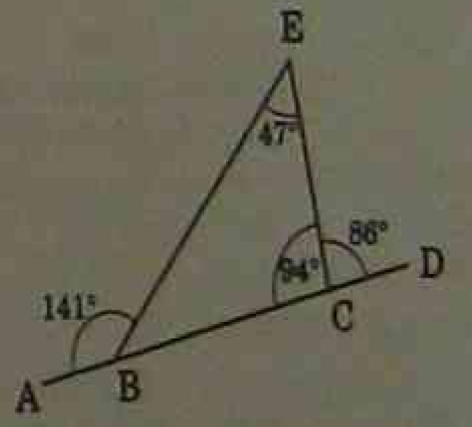
48.



$\angle CQE = 78°$
$\angle GSQ = 78°$ [corr. ∠'s, EF ∥ GH]
and $\angle RSD = 78°$ [vert. opposite ∠'s]
$\therefore \angle BRS = 180° - 78°$ [co-interior ∠'s, AB ∥ CD]
$= 102°$.

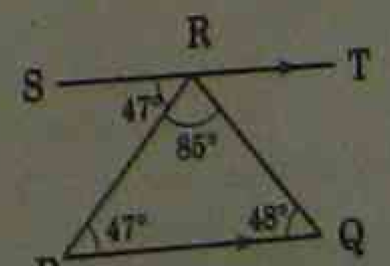
This is one of a number of equally correct solutions for this diagram.

49.



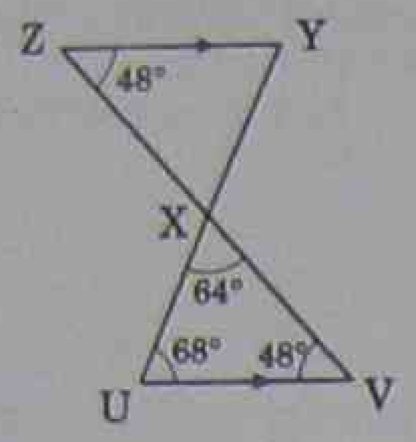
$\angle BCE + 47° = 141°$ [ext. ∠ of a Δ]
$\therefore \angle BCE = 141° - 47°$
$= 94°$.
Then $\angle ECD = 180° - 94°$
$= 86°$ [AD is a straight line].

50.



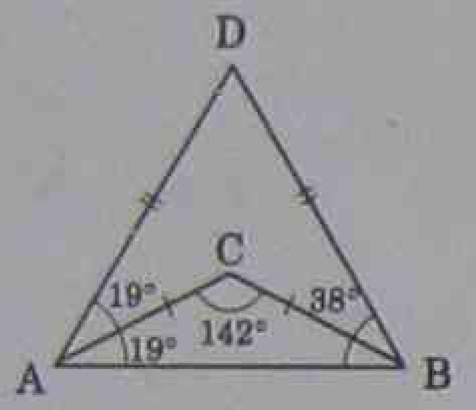
$\angle QPR = 47°$ [alt. ∠'s, ST ∥ PQ]
$\therefore \angle PRQ = 180° - (47° + 48°)$ [∠ sum of a Δ]
$= 180° - 95°$
$= 85°$.

51.



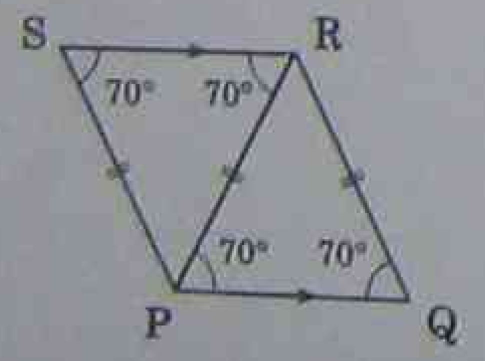
$\angle XVU = 48^\circ$ [alt. $\angle$'s, ZY $\parallel$ UV]
$\therefore \angle XUV = 180^\circ - (64^\circ + 48^\circ)$
[$\angle$ sum of a Δ]
$= 180^\circ - 112^\circ$
$= 68^\circ$.

52.



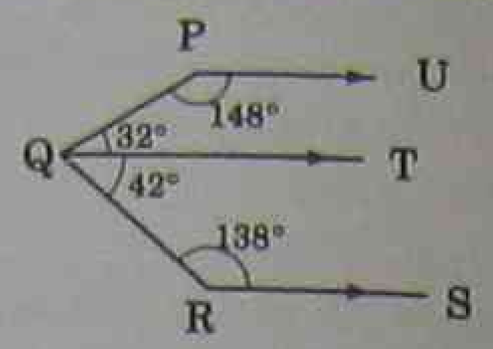
$\angle CAB = \angle CBA$
[base $\angle$'s of an isosceles Δ]
$\angle CAB + \angle CBA = 180^\circ - 142^\circ$
[$\angle$ sum of a Δ]
$= 38^\circ$,
$\therefore$ $\angle CAB = 19^\circ$.
Then $\angle DAC = 19^\circ$
[AC bisects $\angle DAB$].
Then $\angle DAB = 38^\circ$ [$\angle CAB + \angle DAC$]
$\therefore$ $\angle ABD = 38^\circ$
[base $\angle$'s of an isosceles Δ].
Then $\angle ADB = 180^\circ - (38^\circ + 38^\circ)$
$= 104^\circ$ [$\angle$ sum of a Δ].

53.



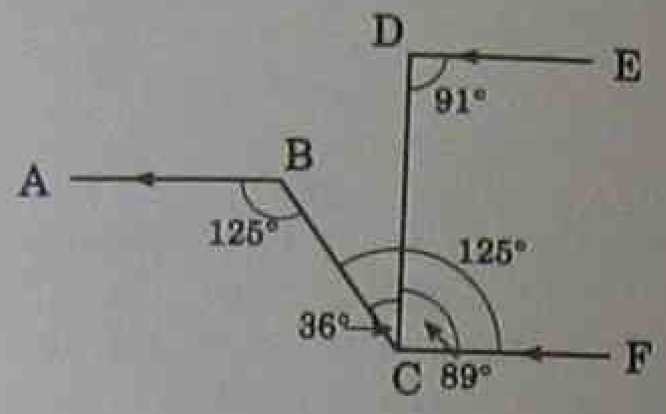
$\angle SRP = 70^\circ$ [base $\angle$'s of an isosceles Δ]
$\therefore$ $\angle RPQ = 70^\circ$ [alt. $\angle$'s, SR $\parallel$ PQ]
$\therefore$ $\angle PQR = 70^\circ$ [base $\angle$'s of an isosceles ΔPQR].

54.



$\angle PQT = 32^\circ$ [co - int. $\angle$'s, PU $\parallel$ QT]
also $\angle RQT = 42^\circ$ [co - int. $\angle$'s, QT $\parallel$ RS]
$\therefore$ $\angle PQR = 74^\circ$ [$32^\circ + 42^\circ$]
Hence reflex $\angle PQR = 360^\circ - 74^\circ$
$= 286^\circ$.

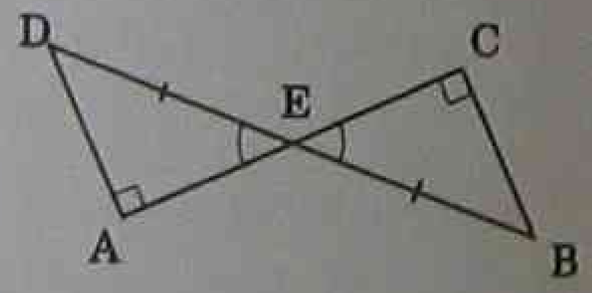
55.



Construction: Draw CF $\parallel$ AB.
Hence CF $\parallel$ DE [both parallel to AB].

Then $\angle BCF = 125^\circ$ [alt.$\angle$'s, AB $\parallel$ CF]
$\therefore$ $\angle DCF = 125^\circ - 36^\circ$ [$\angle BCD = 36^\circ$]
$= 89^\circ$.
Then $\angle CDE = 180^\circ - 89^\circ$
[co - int $\angle$'s, CF $\parallel$ DE]
$= 91^\circ$.

56.



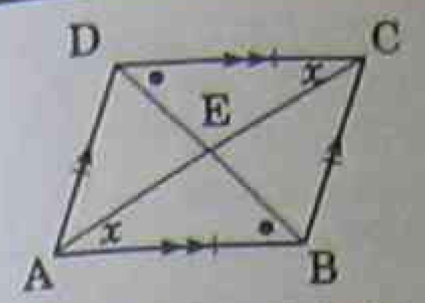
The aim is to prove that AE = EC. Congruent triangles appears to be the method.

In Δ's ADE and CBE
DE = EB [E is the midpoint of DB]
$\angle DEA = \angle BEC$ [vert. opp.$\angle$'s]
$\angle DAE = \angle BCE$ [both 90°]
$\therefore \Delta ADE \equiv \Delta CBE$ [AAS].

Then AE = EC [corresponding sides in congruent Δ's].

That is, E is the midpoint of AC, or DB bisects AC.

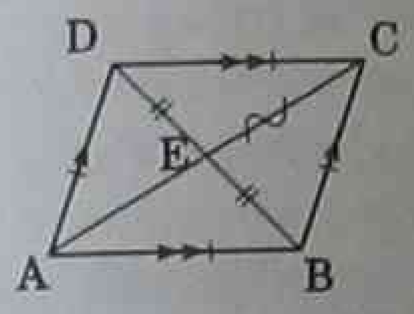
57.



In the Δ's DEC and AED
DC = AB (ABCD is rhombus)
$\angle CDE = \angle EBA$ (Alternate $\angle$'s, CD $\parallel$ AB)
$\angle DCE = \angle EAB$ (Alternate $\angle$'s, CD $\parallel$ AB)
$\therefore$ $\Delta DEC \equiv \Delta AEB$ (AAS)

$$\text{Then } \left.\begin{array}{l} DE = EB \\ CE = EA \end{array}\right\} \left(\begin{array}{c} \text{corresponding sides equal} \\ \text{in congruent triangles} \end{array}\right)$$

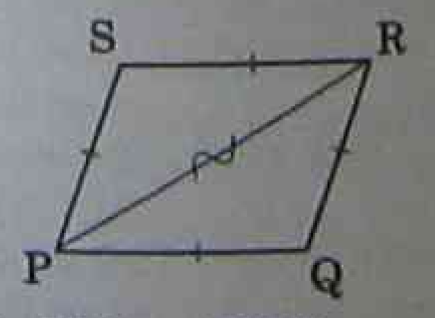
$\therefore$ The diagonals bisect each other.
Also in Δ's DEC and BEC



DC = BC (ABCD is rhombus)
EC = EC (Common side)
DE = EB (Proved above)
$\therefore$ $\Delta DEC \equiv \Delta BEC$ (SSS)

Then $\angle DEC = \angle BEC$
(Angles in corresponding positions in congruent triangles)
But $\angle DEC + \angle BEC = 180^\circ$
(DB is a straight line)
$\therefore$ $\angle DEC = \angle BEC = 90^\circ$
(Similarly $\angle DEB = \angle BEA = 90^\circ$)
$\therefore$ The diagonals bisect each other at right angles.

58.

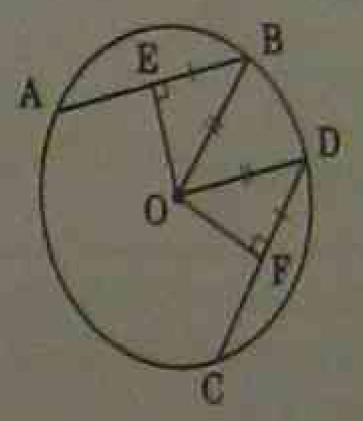


In Δ's PRS and PQR
PS = QR (Given)
SR = PQ (Given)
PR = PR (Common side)
$\therefore$ $\Delta PRS \equiv \Delta PQR$ (SSS)
$\therefore$ $\angle SRP = \angle RPQ$ (Angles in corresponding positions in congruent Δ's)

$\therefore$ SR $\parallel$ PQ
(A pair of alternate angles equal.)
Also $\angle SPR = \angle PRQ$ (Angles in corresponding positions in congruent Δ's.)
$\therefore$ SP $\parallel$ RQ
(A pair of alternate angles equal.)
$\therefore$ PQRS is a rhombus (parallelogram with all sides equal).

Chapter 12 Further geometry — the circle (page 182)

1.



Data: AB = CD,
OE $\perp$ AB,
OF $\perp$ CD, O is the centre of the circle.

Aim: To prove that OE = OF.
Construction: Join OB, OD.
Proof: AE = EB [perpendicular from centre to chord].
CF = FD [perpendicular from centre to chord].
$\therefore$ EB = FD [halves of equal chords AB and CD].

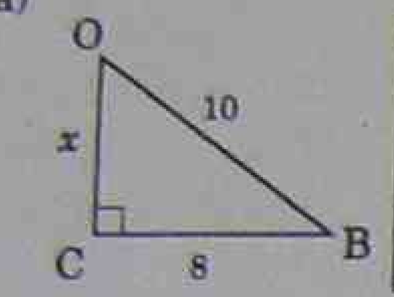
In Δ's EBO and FDO
EB = FD [proved above]
OB = OD [equal radii]
$\angle OEB = \angle OFD$
$= 90^\circ$ [OE $\perp$ AB, OF $\perp$ CD]
$\therefore \Delta EBO \equiv \Delta FDO$ [RHS].

Then EO = FO [corresponding sides in congruent Δ's].

Hence, equal chords are equidistant from the centre.

2. (a) $x = 7$ [perpendicular from centre to chord].
(b) $x = 90$ [line from centre to midpoint of chord].
(c) $y = 8.4$ [equal chords subtend equal $\angle$'s at the centre].
(d) $\theta = 110$ [equal chords subtend equal $\angle$'s at centre].

3. (a)



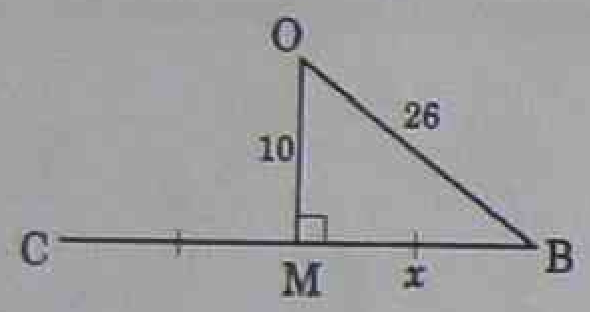
CB = 8 as the perpendicular from the centre bisects the chord.

Let OC = x cm

$$10^2 = x^2 + 8^2$$
$$\therefore \quad x^2 = 100 - 64$$
$$= 36$$
$$x = \sqrt{36}$$
$$= 6$$

OC = 6 cm.

(b)



Line from centre to midpoint of chord is perpendicular to chord,
$\therefore$ $\angle$OMB = 90°.

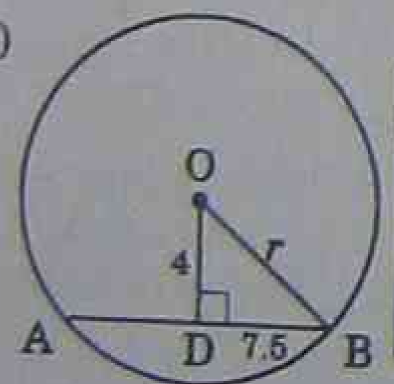
Join the radius OB.
Let MB = x cm.

$$26^2 = 10^2 + x^2$$
$$\therefore \quad x^2 = 26^2 - 10^2$$
$$= 576$$
$$x = \sqrt{576}$$
$$= 24.$$

MB = 24 cm

$$\therefore \quad AB = 2 \times 24 \text{ cm}$$
$$= 48 \text{ cm}.$$

(c)



DB = 7.5 cm. Perpendicular from the centre bisects the chord.

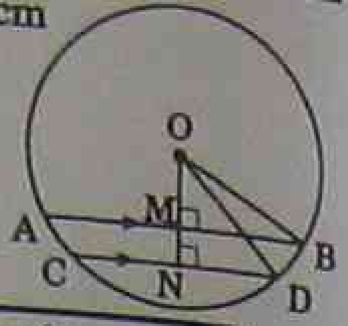
AB = 15 cm, OD = 4 cm
Join the radius OB. Let OB = r cm.

$$r^2 = 4^2 + 7.5^2$$
$$= 72.25$$
$$\therefore \quad r = \sqrt{72.25}$$
$$= 8.5.$$

The radius is 8.5 cm.

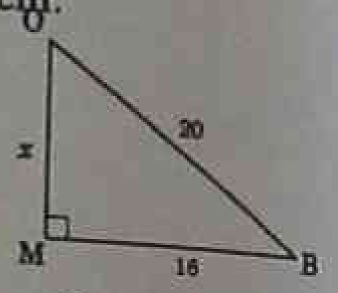
(d)

$$OB = OD = 20 \text{ cm}$$
$$CD = 24 \text{ cm}$$
$$\therefore \quad ND = 12 \text{ cm}$$
$$AB = 32 \text{ cm}$$
$$\therefore \quad MB = 16 \text{ cm}.$$
$$\Uparrow$$



Perpendicular from the centre bisects the chord.

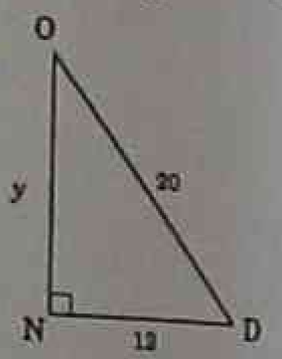
In ΔOMB, let OM = x cm.



$$\text{Then} \quad 20^2 = x^2 + 16^2$$
$$\therefore \quad x^2 = 20^2 - 16^2$$
$$= 144$$
$$\therefore \quad x = \sqrt{144} = 12.$$

OM = 12 cm.

In ΔOND, let ON = y cm.



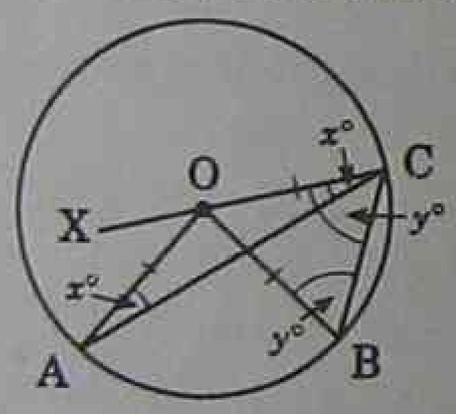
$$20^2 = y^2 + 12^2$$
$$\therefore \quad y^2 = 20^2 - 12^2$$
$$= 256$$
$$\therefore \quad y = \sqrt{256} = 16.$$

ON = 16 cm.

$$\text{Now} \quad MN = ON - OM$$
$$= 16 - 12 \text{ cm}$$
$$= 4 \text{ cm}.$$

The distance between the chords is 4 cm.

4.



Data: O is centre.
Aim: To prove that $\angle AOB = 2 \times \angle ACB$.

Construction: Join CO and produce to X.

Proof : Let $\angle OAC = x°$, $\angle OBC = y°$.

$\angle OCA = x°$ [OA = OC, base $\angle$s of an isosceles Δ]

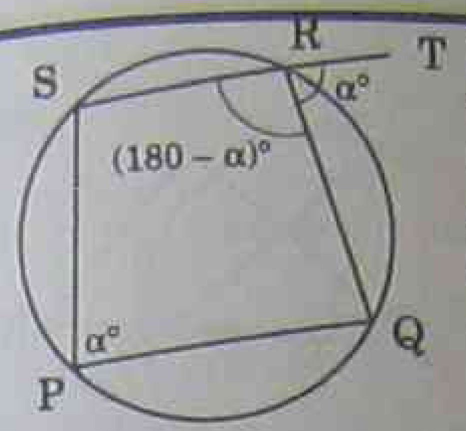
$\therefore$ $\angle XOA = 2x°$ [exterior $\angle$ of ΔOCA].

Also $\angle OCB = y°$ [OB = OC, base $\angle$s of an isosceles Δ],

then $\angle XOB = 2y°$ [exterior $\angle$ of ΔBOC].

$$\text{Then} \quad \angle AOB = \angle XOB - \angle XOA$$
$$= 2y° - 2x°$$
$$= 2(y° - x°)$$
$$\text{and} \quad \angle ACB = \angle OCB - \angle OCA$$
$$= y° - x°.$$
$$\text{Then} \quad \angle AOB = 2 \times \angle ACB$$

5.



Data: PQRS is a cyclic quadrilateral.
Aim: To prove that $\angle SPQ = \angle QRT$.

Proof : Let $\angle SPQ = \alpha°$.
then $\angle SRQ = (180 - \alpha)°$
[opposite $\angle$s of a cyclic quadrilateral].
then $\angle SRQ + \angle TRQ = 180°$
[ST is a straight line],
$\therefore$ $\angle TRQ = \alpha°$
[$\angle TRQ + 180° - \alpha° = 180°$]
that is, $\angle SPQ = \angle TRQ$ [both $\alpha°$].

6. (a) $a = \dfrac{1}{2} \times 86 = 43$
[$\angle$ at the circumference $= \frac{1}{2}\angle$at the centre on the same arc].

(b) $\angle PRQ = 90°$ [$\angle$ in semi-circle]
$\therefore$ $b + 52 = 90$ [$\angle$ sum of a Δ]
$b = 38.$

(c) y = 38 [$\angle$'s in the same segment].

(d) $x = 37$ [$\angle$'s in the same segment]
$\therefore$ $y = 180 - (42 + 37)$
[$\angle$ sum of a Δ]
$\therefore$ $y = 101.$

(e) $g = 180 - 84$
[opp. $\angle$'s of a cyclic quadrilateral]
$= 96.$

(f) $h = 114$ [exterior $\angle$ of a cyclic quadrilateral]

(g) $a = 114$
[corresponding $\angle$'s, AD ∥ BC]
$b = 114$
[ext. $\angle$ of a cyclic quadrilateral]
$c + 114 = 180$
[opp. $\angle$'s of a cyclic quadrilateral],
$\therefore$ $c = 180 - 114$
$= 66.$

(h) OB = OC [equal radii]
$\therefore$ $y = 48$
[base $\angle$'s of an isosceles Δ].
But $\angle ABC = 90°$
[angle in a semi-circle],
$\therefore$ $x = 42$ [90 – 48]
Also OB = OA [equal radii],
$\therefore$ $z = 42$
[base $\angle$'s of an isosceles Δ].

(i) Reflex $\angle AOC = 360° - 138°$
$= 222°$ [$\angle$s at a point],
$\therefore$ $\theta = \dfrac{1}{2}(222)$
[$\angle$ at circumference $= \frac{1}{2}\angle$ at the centre on the same arc],
that is, $\theta = 111.$

(j) Reflex $\angle POR = 2 \times 88°$
$= 176°$
[$\angle$ at the centre = 2× $\angle$ at the circumference on the same arc],
$\therefore$ $\phi = 360 - 176$
$= 184$ [$\angle$s at a point].

(k) $y = 124$
[alternate angles, AB ∥ OD]
Reflex $\angle AOC = 248°$
[$\angle$ at the centre = 2× $\angle$ at the circumference on the same arc]
$\therefore$ $x = 360 - 248$
$= 112$ [$\angle$s at a point],
then $z = 180 - 112$
[co-interior angles, AB ∥ OD]
$= 68.$

Note: ABCO is not a cyclic quadrilateral.

(l) $a = 2 \times 48$
[$\angle$ at the centre = 2× $\angle$ at the circumference on the same arc]
$= 96.$
Then $b = 180 - 48$
[opp. $\angle$'s of a cyclic quadrilateral PQRS]
$\therefore$ $b = 132.$

(m) $x = 42$
[$\angle$s in the same segment]
$\therefore$ $y = 42$
[alternate $\angle$s, AD ∥ BC],
then $\angle ATD = 180° - (42° + 42°)$
$= 96°$ [$\angle$ sum of a Δ],
$\therefore$ $z = 96$
[vertically opposite $\angle$'s].

(n) y = 44 [$\angle$ in alternate segment].

(o) $x = 52$
[$\angle$ in alternate segment]
$\therefore$ $y = 2 \times 52$
[$\angle$ at the centre = 2× $\angle$ at the circumference on the same arc]
$= 104.$

(p) $AT = AS$
[tangents from external point],
then $\angle TSA = a°$
[base ∠'s of an isosceles Δ],
$\therefore\ a + a + 82 = 180$ [∠ sum of a Δ]
$2a = 98$
$a = 49.$

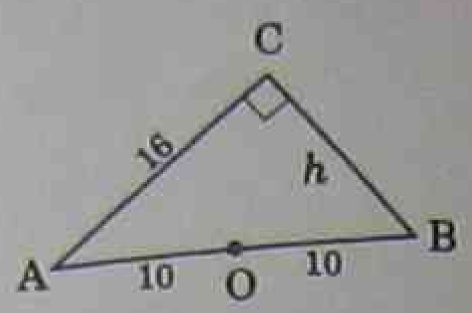
(q) $x + 46 = 90$
[radius drawn to the point of contact of a tangent],
$\therefore\ x = 90 - 46$
$= 44.$
Also $OT = OA$ [equal radii],
$\therefore\ \angle OAT = 44°$
[base ∠'s of an isosceles Δ],
$\therefore\ y + 44 + 44 = 180$ [∠ sum of a Δ],
that is, $y = 180 - 88$
$= 92.$

(r) $x = 38$
[∠ in the alternate segment]
$\angle TOA = 2 \times 38°$
[∠ at the centre = 2×∠ at the circumference on the same arc]
$= 76°.$
$OA = OT$ [equal radii],
$\therefore\ \angle OTA = y°$
[base ∠'s of an isosceles Δ],
then $y + y + 76 = 180$
[∠ sum of a Δ]
$2y = 180 - 76$
$= 104$
$\therefore\ y = 52.$

(s) $\angle OTP = 90°$ [radius drawn to the point of contact],
$\therefore\ \phi + 26 = 90$ [∠ sum of a Δ]
$\phi = 90 - 26$
$= 64.$

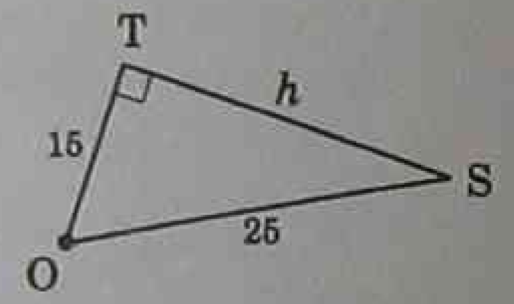
(t) $\angle OTP = 90°$ [radius drawn to the point of contact]
$\therefore\ a + 68 = 90$ [∠ sum of a Δ]
$a = 90 - 68$
$= 22.$
$OA = OT$ [equal radii],
then $\angle ATO = b°$
[base ∠'s of an isosceles Δ],
$\therefore\ b + b = 68$
[external ∠ of ΔAOT],
$\therefore\ 2b = 68$
$\therefore\ b = 34.$

7. (a)



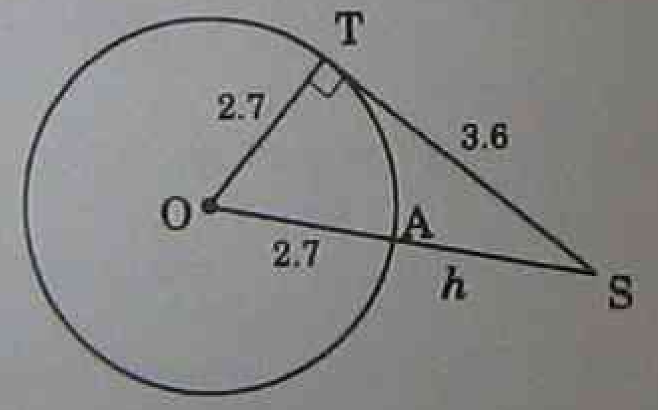
$\angle ACB = 90°$
[∠ in a semicircle]
$AB = 20$ [radius = 10 cm],
$\therefore\ h^2 + 16^2 = 20^2$
[Pythagoras' Theorem]
$\therefore\ h^2 = 20^2 - 16^2$
$= 144$
$h = \sqrt{144}$
$= 12,$
that is, h is 12.

(b)



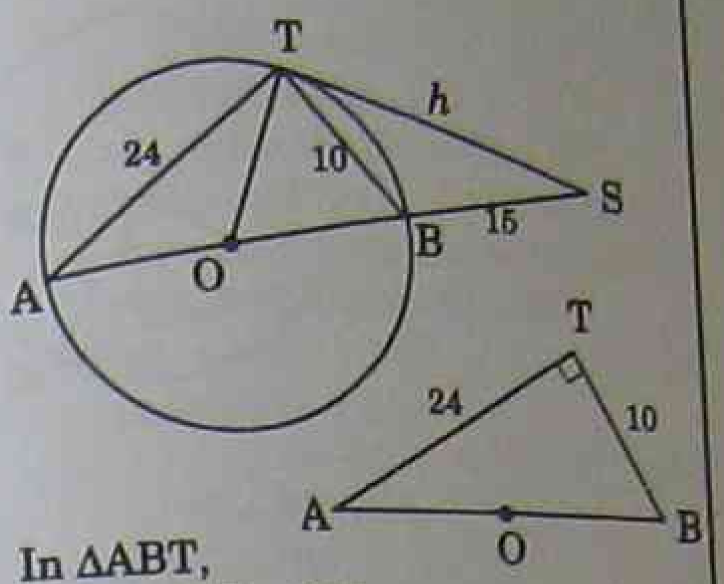
$\angle OTS = 90°$ [radius drawn to the point of contact]
$\therefore\ h^2 + 15^2 = 25^2$
[Pythagoras' Theorem],
$\therefore\ h^2 = 25^2 - 15^2$
$= 400$
$h = \sqrt{400}$
$= 20.$

(c)

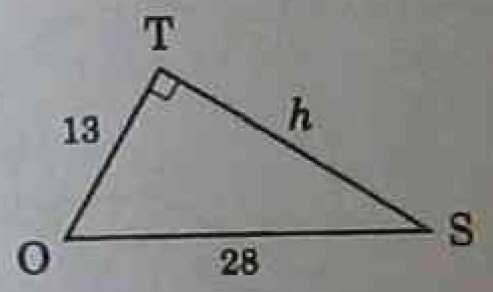


Let $OS = x$ cm
$\angle OTS = 90°$ [radius drawn to the point of contact]
Then $x^2 = 2.7^2 + 3.6^2$
$= 20.25$
$\therefore\ x = \sqrt{20.25}$
$= 4.5.$
Now $h = OS - OA$
$= 4.5 - 2.7$
$= 1.8.$

(d)



In ΔABT,
$\angle ATB = 90°$
[angle in a semi-circle].
Let $AB = x$ cm
$x^2 = 24^2 + 10^2$
$= 676$
$\therefore\ x = \sqrt{676} = 26.$
Then $OB = \frac{1}{2} \times 26 = 13$
[OB is a radius].



In ΔOTS,
$OT = 13$ [radius]
$OS = 13 + 15 = 28.$
Then $h^2 + 13^2 = 28^2$
$h^2 = 28^2 - 13^2$
$= 615$
$\therefore\ h = \sqrt{615}$
$= 24.799\ 194$
$= 24.8$ [one dec. place].

8. (a) $4x = 5 \times 8$ [$DE \times EC = AE \times EB$]
$= 40$
$\therefore\ x = 10.$

(b) $t^2 = 15 \times 8$ [$ST^2 = AS \times SB$]
$\therefore\ t^2 = 120$
$t = \sqrt{120}$
$= 10.954\ 451$
≈ 11.0 [one dec. place].

(c)
$6(y + 6) = 18 \times 8$ [$AC \times BC = EC \times CD$]
$y + 6 = 3 \times 8$
$= 24$
$\therefore\ y = 18.$

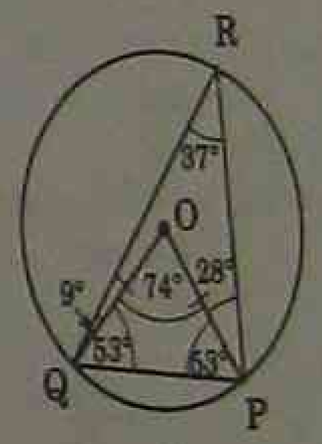
(d) $8(a + 8) = 12^2$ [$AS \times SB = ST^2$]
$8(a + 8) = 144$
$\therefore\ a + 8 = 18$
$\therefore\ a = 10.$
Then $y(7 + y) = 12^2$ [$CS \times SD = TS^2$]
$7y + y^2 = 144,$
that is,
$y^2 + 7y - 144 = 0$
$(y + 16)(y - 9) = 0$
$\therefore\ y = 9$ or $-16.$

$$\boxed{\begin{matrix} & 144 & \\ / & & \backslash \\ 16 & & 9 \end{matrix}}$$

Ignore the negative value as we are finding a length, that is, $y = 9$.

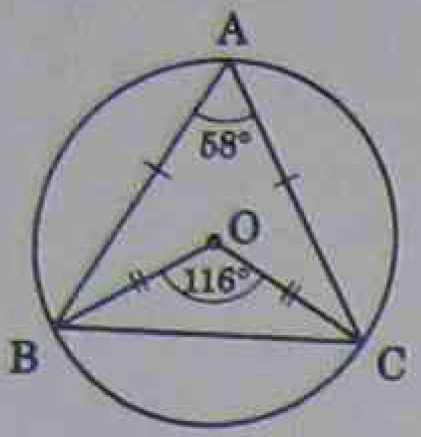
(e) $3x = 5 \times 9$ [$AE \times EB = DE \times EC$]
45
$x = 15.$
Then $AB = 18.$
Also $t^2 = 36 \times 18$ [$TS^2 = AS \times SB$]
$\therefore\ t = \sqrt{648}$ [$\sqrt{18^2 \times 2}$]
$= 18\sqrt{2}.$

9. (a)



$\angle QRP = 37°$
[∠ at the circumference = $\frac{1}{2}$∠ at the centre on the same arc].
Also $OQ = OP$ [equal radii],
then $\angle OQP = \angle OPQ$
[base ∠'s of an isosceles Δ].
But $\angle OQP + \angle OPQ + 74° = 180°$
[∠ sum of a Δ],
then $\angle OQP = \angle OPQ$
$= \frac{180° - 74°}{2}$
$= 53°.$
In ΔRQP, $\angle RQP + 37° + 81° = 180°$
[∠ sum of a Δ, $\angle RPQ = 81°$],
$\therefore\ \angle RQP = 180° - 118°$
$62°.$
Then $\angle RQO = 62° - 53°$
$= 9°.$

(b)



$\angle BAC = 58°$

[$\angle$ at the circumference $= \frac{1}{2}\angle$ at the centre on the same arc]

Now $\angle ABC = \angle ACB$

[base $\angle$'s of an isosceles Δ].

But $\angle ABC + \angle ACB + 58° = 180°$

$\therefore\ 2\angle ABC = 180° - 58°$

$= 122°$

$\therefore\ \angle ABC = 61°$.

Also $BO = OC$ [equal radii],

$\therefore\ \angle OBC = \angle OCB$

[base $\angle$'s of an isosceles Δ].

But $\angle OBC + \angle OCB + 116° = 180°$

[$\angle$ sum of a Δ],

$\therefore\ 2\angle OBC = 180° - 116°$

$= 64°$,

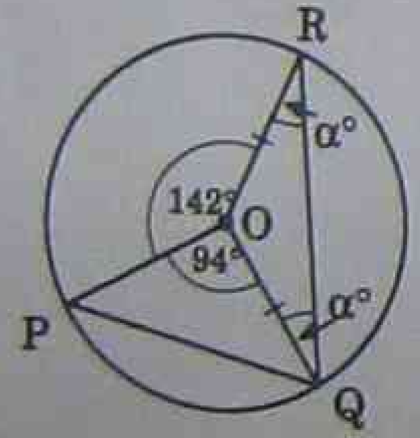
$\therefore\ \angle OBC = 32°$.

But $\angle ABO = \angle ABC - \angle OBC$

$= 61° - 32°$

$= 29°$.

(c)



Let $\angle ORQ = \alpha°$.

But $OR = OQ$ [equal radii],

$\therefore\ \angle OQR = \alpha°$.

$\angle ROQ + 142° + 94° = 360°$

[angles at a point],

$\therefore\ \angle ROQ = 360° - 236°$

$= 124°$.

In ΔROQ, $\alpha + \alpha + 124 = 180$

[$\angle$ sum of a Δ],

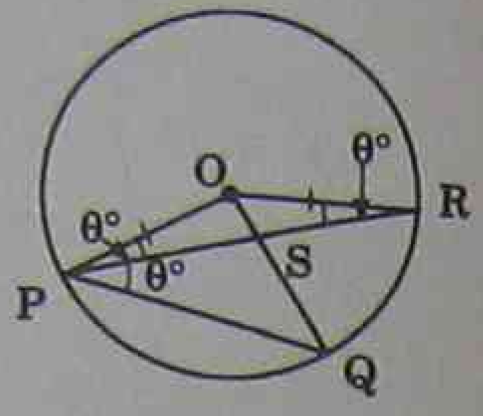
$\therefore\ 2\alpha = 180 - 124$

$= 56$

$\therefore\ \alpha = 28$,

that is, $\angle ORQ = 28°$.

(d)



$\angle ORS = \theta°$

$OP = OR$ [equal radii],

$\therefore\ \angle OPR = \theta°$

[base $\angle$'s of an isosceles Δ],

$\therefore\ \angle RPQ = \theta°$

[given $\angle OPR = \angle QPR$].

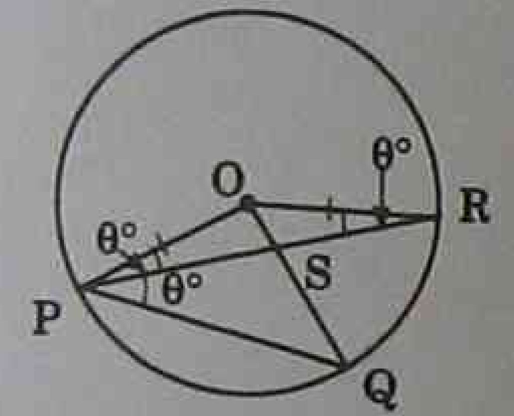
Then $\angle ROQ = 2 \times \angle RPQ$

[$\angle$ at the centre $= 2 \times \angle$ at the circumference on the same arc],

that is, $\angle ROS = 2\theta°$

[$\angle ROS$ is the same as $\angle ROQ$].

(e)



$OX = OZ$ [equal radii]

$\therefore\ \angle OZX = \theta°$

[base $\angle$'s of an isosceles Δ].

Also, $\angle XOZ = 2\theta°$

[$\angle$ at the centre $= 2 \times \angle$ at the circumference on the same arc],

$\therefore\ \theta + \theta + 2\theta = 180$

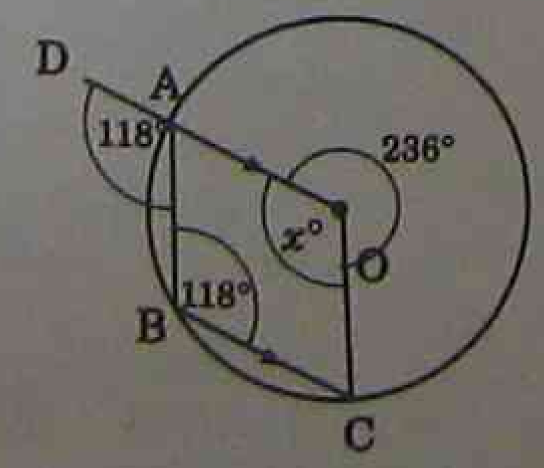
[$\angle$ sum of ΔXOZ]

$4\theta = 180$

$\therefore\ \theta = 45$

$\angle XOZ = 2\theta° = 90°$.

(f)



$\angle ABC = 118°$

[alternate $\angle$'s, $DO \parallel BC$].

Then reflex $\angle AOC = 2 \times 118°$

[$\angle$ at the centre $= 2 \times \angle$ at the circumference on the same arc],

that is, reflex $\angle AOC = 236°$.

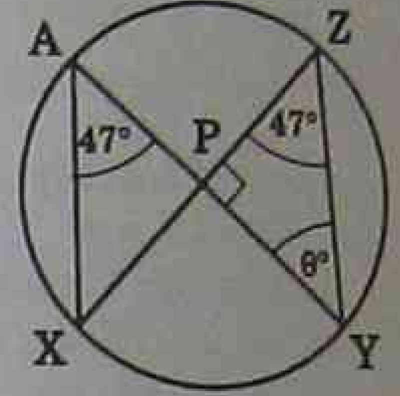
Then $x + 236 = 360$

[angles at a point].

$\therefore\ x = 360 - 236$

$= 124$.

(g)



$\angle XZY = 47°$

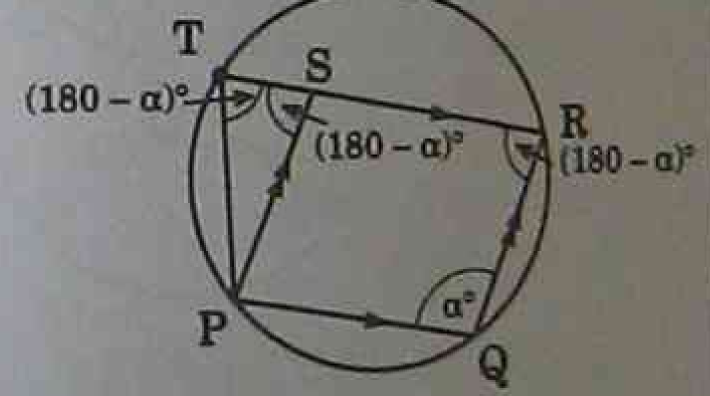
[$\angle$'s in the same segment]

$\therefore\ \theta + 47 + 90 = 180$ [$\angle$ sum of a Δ]

$\therefore\ \theta = 180 - 137$

$= 43$.

(h)



Let $\angle PQR = \alpha°$.

$\angle PTR = (180 - \alpha)°$

[opposite angles of a cyclic quadrilateral]

$\angle SRQ = (180 - \alpha)°$

[co-interior angles, $PQ \parallel SR$]

$\therefore\ \angle TSP = (180 - \alpha)°$

[corresponding angle, $SP \parallel RQ$].

In ΔPST,

$\angle PTS = \angle PST$

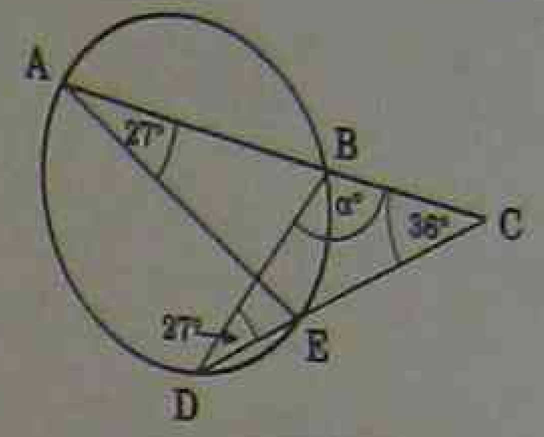
[both $(180 - \alpha)°$],

then ΔPST is isosceles

[base angles equal],

$\therefore\ PT = PS$.

(i)



Let $\angle DBC = \alpha°$.

Now $\angle BDE = 27°$

[angles in the same segment].

In ΔBDC,

$\alpha + 27 + 36 = 180$

[angle sum of a Δ],

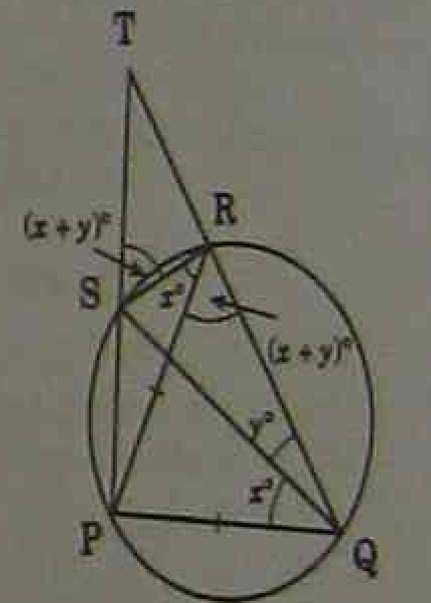
$\therefore\ \alpha + 63 = 180$

$\alpha = 180 - 63$

$= 117$.

$\angle DBC = 117°$.

(j)



$\angle PRQ = (x° + y°)$

[base angles of isosceles ΔRPQ].

Also $\angle SRP = x°$

[angles in the same segment]

and $\angle TSR = (x° + y°)$

[exterior angles of a cyclic quadrilateral PQRS].

Now $\angle SRQ = x° + (x + y)°$

$= (2x + y)°$.

From ΔSRT,

$\angle STR + \angle TSR = \angle SRQ$

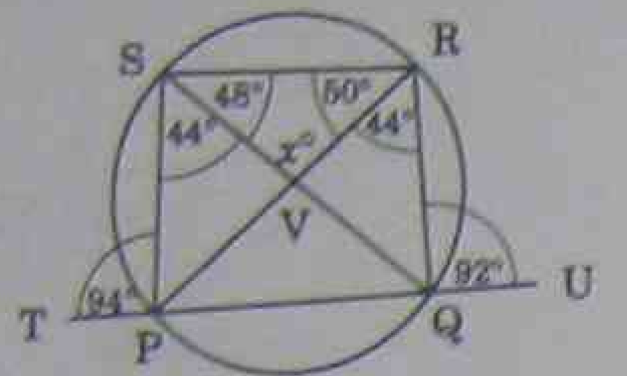
[exterior $\angle$ = sum of interior opposite angles],

$\therefore\ \angle STR + (x + y)° = (2x + y)°$,

that is, $\angle STR = (2x + y)° - (x + y)°$

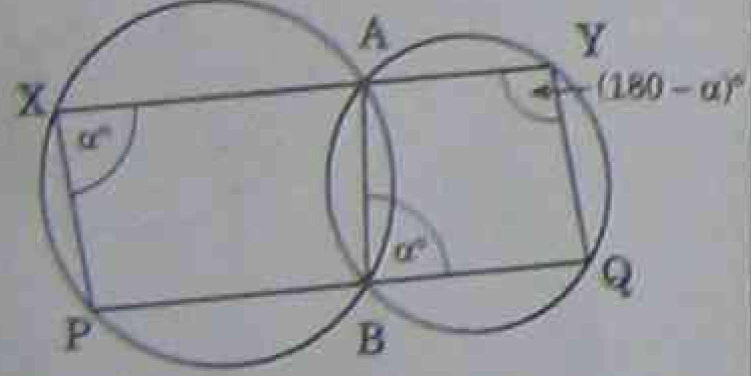
$= x°$.

(k)



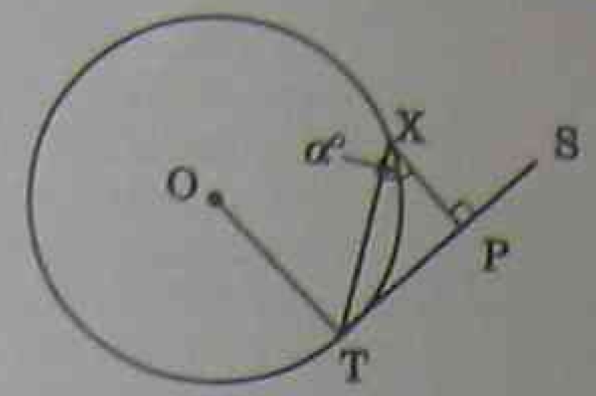
Let $\angle SVR = x°$.
Now $\angle PSR = 92°$
[exterior $\angle$ of a cyclic quadrilateral].
But $\angle PSV = \angle PSR - \angle QSR$
$= 92° - 48°$
$= 44°$,
$\therefore$ $\angle PRQ = 44°$
[angles in the same segment].
But $\angle SRQ = 94°$
[exterior angle of a cyclic quadrilateral].
Now $\angle SRV = \angle SRQ - \angle PRQ$
$= 94° - 44°$
$= 50°$.
In ΔSVR,
$x + 50 + 48 = 180$
[angle sum of a triangle],
$\therefore$ $x = 180 - 98$
$= 82$
$\therefore$ $\angle SVR = 82°$.

(l)



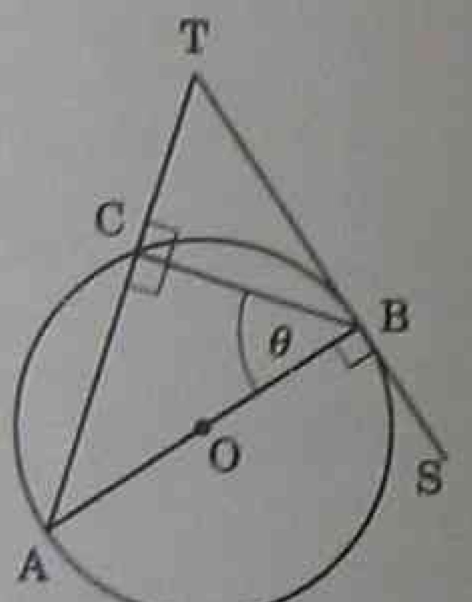
Join PX, BA and QY.
Call $\angle PXA = \alpha°$
$\therefore$ $\angle ABQ = \alpha°$
[exterior angle of a cyclic quadrilateral PBAX].
Also, $\angle AYQ = 180° - \alpha°$
[opposite angles of a cyclic quadrilateral BQYA],
then $\angle PXA + \angle AYQ = \alpha° + 180° - \alpha°$
$= 180°$,
$\therefore$ PX ‖ YQ
[a pair of co-interior angles are supplementary].

(m)



Let $\angle TXP = \alpha°$.
Now $\angle OTP = 90°$
[angle from the centre to the point of contact].
Then $\angle OTP = \angle XPS$ [both 90°]
$\therefore$ OT ‖ XP
[pair of corresponding $\angle$'s equal],
$\therefore$ $\angle OTX = \alpha°$
[alternate angles, OT ‖ XP],
that is, $\angle OTX = \angle TXP$.

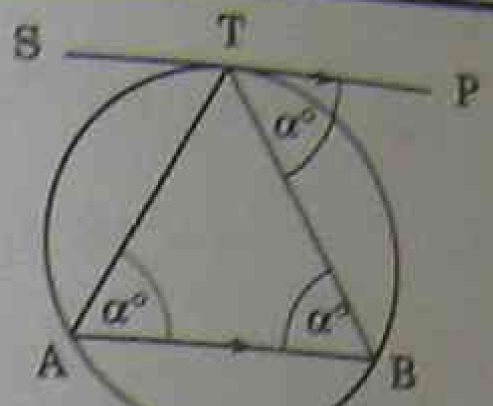
(n)



$\angle ACB = 90°$
[angle in a semi-circle],
$\therefore$ $\angle TCB = 90°$
[AT is a straight line].
$\angle ABS = 90°$
[radius to a point of contact],
$\therefore$ $\angle CBS = \angle CBA + \angle ABS$
$= 90° + \theta°$.

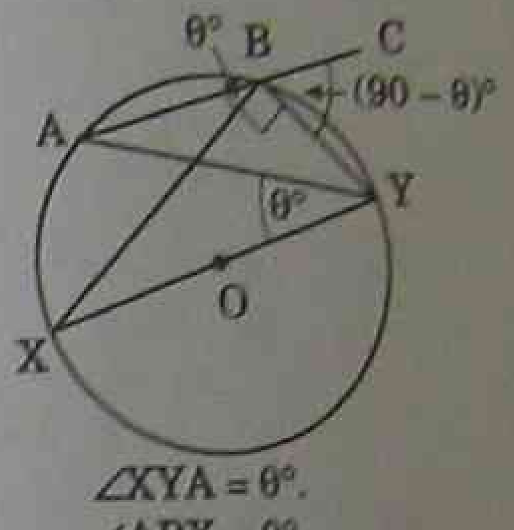
Then in ΔTCB,
$\angle ATB + \angle TCB = \angle CBS$
[exterior angle of a triangle],
that is, $\angle ATB + 90° = 90° + \theta°$
$\therefore$ $\angle ATB = \theta°$.

(o)



$\angle ABT = \alpha°$
[alternate angles, SP ‖ AB],
also $\angle TAB = \alpha°$
[angle in the alternate segment],
$\therefore$ $\angle TAB = \angle TBA$ [both α°],
that is, ΔABT is isosceles
[base angles equal].

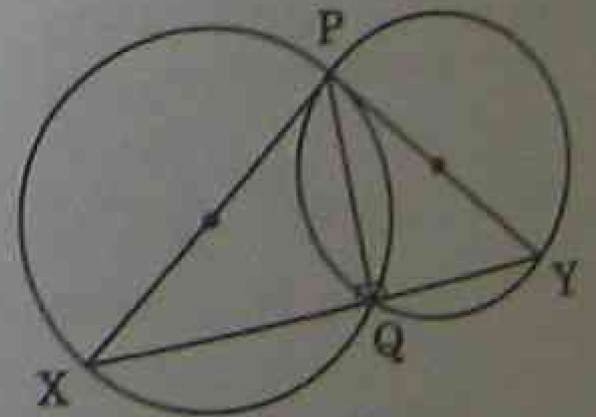
(p)



$\angle XYA = \theta°$.
$\angle ABX = \theta°$
[angles in the same segment],
also $\angle XBY = 90°$
[angle in a semi-circle].
Then $\angle YBC + 90° + \theta° = 180°$
[AC is a straight line],
$\therefore$ $\angle YBC = 180° - 90° - \theta°$
$= 90° - \theta°$.

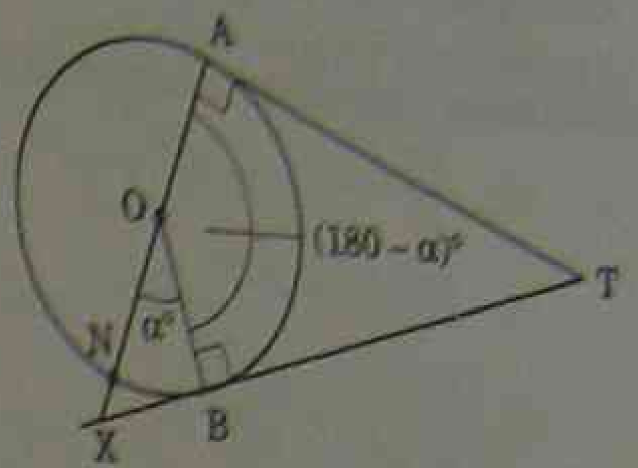
Then
$\angle XYA + \angle YBC = \theta° + 90° - \theta°$
$= 90°$.

(q)



Join XQ and QY.
Now $\angle PQY = 90°$
[angle in a semi-circle].
Also $\angle XQP = 90°$
[angle in a semi-circle].
Then $\angle PQY + \angle XQP = 180°$
$\therefore$ XY is a straight line,
that is, X, Q, Y lie on a straight line.
X, Q, Y are collinear.

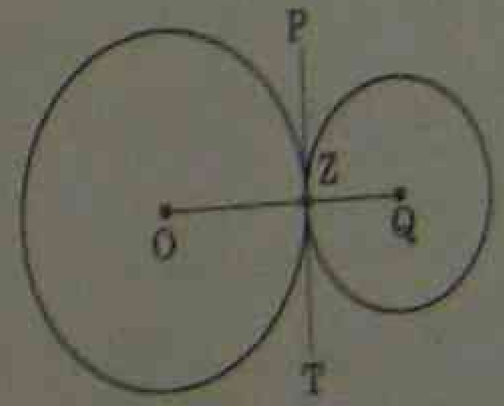
(r)



(i) Let $\angle BOX = \alpha°$
$\therefore$ $\angle BOA = 180° - \alpha°$
[AX is a straight line].
Also $\angle OBT = 90°$
[radius to a point of contact],
and $\angle OAT = 90°$
[radius to a point of contact].
From quadrilateral OBTA,
$180° - \alpha° + 90° + 90° + \angle ATB$
$= 360°$
[angle sum of a quadrilateral],
$\therefore$ $\angle ATB = 360° - 360° + \alpha°$
$= \alpha°$,
$\therefore$ $\angle BOX = \angle ATB$ [both α°].

(ii) $\angle OAB = \frac{1}{2}\alpha°$
[$\angle$ at the circumference $= \frac{1}{2}\angle$ at the centre on the same arc NB].
Then $\angle ATB = 2 \times \angle OAB$
[$\alpha° = 2 \times \frac{1}{2}\alpha°$].

(s)



Draw the tangent PT touching each circle at Z. Join OZ and QZ. Now, PT is a tangent to the circle centre O.
$\therefore$ $\angle OZP = 90°$ ($\angle$ between the radius and the tangent)
Also in cxircle centre Q
$\angle PZQ = 90°$ ($\angle$ between the radius and the tangent)

$\therefore$ $\angle OZQ = 180°$, that is OQ is a straight line and Z lies on OQ.
$\therefore$ O, Q and Z are collinear.

Chapter 13 Logarithms and functions (page 203)

1\. (a) $\frac{a^{11}}{a^9}=a^2$ (b) $\frac{15a^{15}}{15a^5}=a^{10}$

(c) $\left(2^4\right)a^{5\times4}=16a^{20}$

(d) $\frac{\left(3^4\right)a^{2\times4}}{\left(2^2\right)a^{3\times2}}=\frac{81a^8}{4a^6}=\frac{81}{4}a^2$

(e) $\frac{a^{\frac{3}{4}-\frac{1}{2}}}{a^{\frac{1}{4}}}=\frac{a^{\frac{1}{4}}}{a^{\frac{1}{4}}}=1$

(f)
$$\begin{aligned}\frac{a^{\frac{2}{3}\times4}\times a^{\frac{3}{4}}}{a^{\frac{3}{2}\times3}\times a^{\frac{1}{4}}}&=\frac{a^{\frac{8}{3}}\times a^{\frac{3}{4}}}{a^{\frac{9}{2}}\times a^{\frac{1}{4}}}\\&=\frac{a^{\frac{8}{3}+\frac{3}{4}}}{a^{\frac{9}{2}+\frac{1}{4}}}\\&=\frac{a^{\frac{41}{12}}}{a^{\frac{19}{4}}}\\&=a^{\frac{41}{12}-\frac{19}{4}}\\&=a^{-\frac{4}{3}}\end{aligned}$$

2\. (a) $\sqrt{81}=9$ $\boxed{\text{Using table of powers}}$

(b) $\sqrt[4]{81}=3$ (c) $8\times\sqrt[6]{64}=8\times2$
$=16$

(d) $\sqrt{256}\times\frac{1}{\sqrt{16}}=16\times\frac{1}{4}$
$=4$

(e)
$$\begin{aligned}\left(2187^{\frac{1}{7}}\right)^4&=\left(\sqrt[7]{2187}\right)^4\\&=3^4\\&=81\end{aligned}$$

(f)
$$\begin{aligned}125^{-\frac{2}{3}}&=\frac{1}{125^{\frac{2}{3}}}\\&=\frac{1}{\left(125^{\frac{1}{3}}\right)^2}\\&=\frac{1}{\left(\sqrt[3]{125}\right)^2}\\&=\frac{1}{5^2}\\&=\frac{1}{25}\end{aligned}$$

(g)
$$\begin{aligned}\sqrt[3]{1000}\times\left(16^{\frac{1}{4}}\right)^{-3}&=10\times\frac{1}{\left(\sqrt[4]{16}\right)^3}\\&=10\times\frac{1}{2^3}\\&=10\times\frac{1}{8}\\&=\frac{5}{4}\text{ or }1\tfrac{1}{4}\end{aligned}$$

3\. (a) $3^a=3^4$ $\boxed{\text{Using table of powers}}$
$\therefore\ a=4.$

(b) $a^2=1024$
$a=\sqrt{1024}$
$=32.$

(c) $5^{2a}=5^4$
$\therefore\ 2a=4$
$a=2.$

(d) $10^{3x-2}=10^4$
$\therefore\ 3x-2=4$
$\therefore\ 3x=6$
$x=2.$

(e) $2^x=2^{-2}$
$\therefore\ x=-2.$

$$\boxed{\begin{aligned}\tfrac{1}{4}&=\tfrac{1}{2^2}\\&=2^{-2}\end{aligned}}$$

(f) $5^{x+1}=5^{-3}$
$\therefore\ x+1=-3$
$\therefore\ x=-4.$

$$\boxed{\begin{aligned}\tfrac{1}{125}&=\tfrac{1}{5^3}\\&=5^{-3}\end{aligned}}$$

(g) $\left(3^{\frac{1}{2}}\right)^x=3^3$
$\therefore\ 3^{\frac{1}{2}x}=3^3$
$\therefore\ \frac{1}{2}x=3,$
that is, $x=6.$

(h) $\left(5^{-\frac{1}{2}}\right)^{4x}=\left(5^4\right)^{2x-15}$
$\therefore\ 5^{-2x}=5^{4(2x-15)}$
$\therefore\ -2x=8x-60$
that is, $10x=60$
$\therefore\ x=6.$

4\. (a) $3^2\times3^3\times3^3=3^8$
$=6561.$

(b)
$$\begin{aligned}\frac{2\times2^4\times2^6\times2^7}{2^8}&=\frac{2^{18}}{2^8}\\&=2^{10}\\&=1024.\end{aligned}$$

(c) $\frac{\sqrt{3^6}}{3^2}=\frac{3^3}{3^2}=3$

(d) $\sqrt{\frac{3^6}{3^2}}=\sqrt{3^4}=3^2=9$

(e)
$$\begin{aligned}\sqrt{\frac{5^5\times5^3}{5^4}}&=\sqrt{\frac{5^8}{5^4}}\\&=\sqrt{5^4}\\&=5^2\\&=25.\end{aligned}$$

(f)
$$\begin{aligned}\frac{\left(3^4\right)^2\times\left(3^2\right)^3}{\left(3^3\right)^4}&=\frac{3^8\times3^6}{3^{12}}\\&=\frac{3^{14}}{3^{12}}\\&=3^2\\&=9.\end{aligned}$$

5\. (a) $\log_5 125=3$
(b) $\log_3\left(\frac{1}{27}\right)=-3$
(c) $\log_2\sqrt{8}=\frac{3}{2}$

6\. (a) $10^3=1000$
(b) $2^4=16$
(c) $2^{2.5}=4\sqrt{2}$
(d) $5^{-2}=\frac{1}{25}$

7\. Change to index form and solve the index equations as in Question 3.

(a) $2^x=64$ $\boxed{\text{Using table of powers}}$
$\therefore\ 2^x=2^6$
$\therefore\ x=6.$

(b) $5^x=125$
$\therefore\ 5^x=5^3$
$\therefore\ x=3.$

(c) $3^x=\sqrt{27}$
$\therefore\ 3^x=\left(3^3\right)^{\frac{1}{2}}$
$=3^{\frac{3}{2}}$
$\therefore\ x=\frac{3}{2}.$

(d) $10^x=1\,000\,000$
that is, $10^x=10^6$
$\therefore\ x=6.$

(e) $2^x=\frac{1}{4}$
$\therefore\ 2^x=2^{-2}$
$\therefore\ x=-2.$

(f) $2^3=x,$
that is, $x=8.$

(g) $5^2=x$
$\therefore\ x=25.$

(h) $3^{\frac{1}{2}}=x$
$\therefore\ x=\sqrt{3}.$

(i) $10^{-1}=x$
$\therefore\ x=\frac{1}{10}.$

(j) $2^{-3}=x$
$\therefore\ x=\frac{1}{2^3}$
$=\frac{1}{8}.$

(k) $8^{-\frac{1}{3}}=x$
$\therefore\ x=\frac{1}{8^{\frac{1}{3}}}$
$=\frac{1}{\sqrt[3]{8}}$
$=\frac{1}{2}.$

(l) $x^8=256$
$\therefore\ x^8=2^8$
that is, $x=2.$

$$\boxed{\begin{aligned}&\text{or}\\x&=(256)^{\frac{1}{8}}\\&=2\end{aligned}}$$

(m) $x^{-2}=\frac{1}{9}$
that is, $x^{-2}=3^{-2}$
$\therefore\ x=3.$

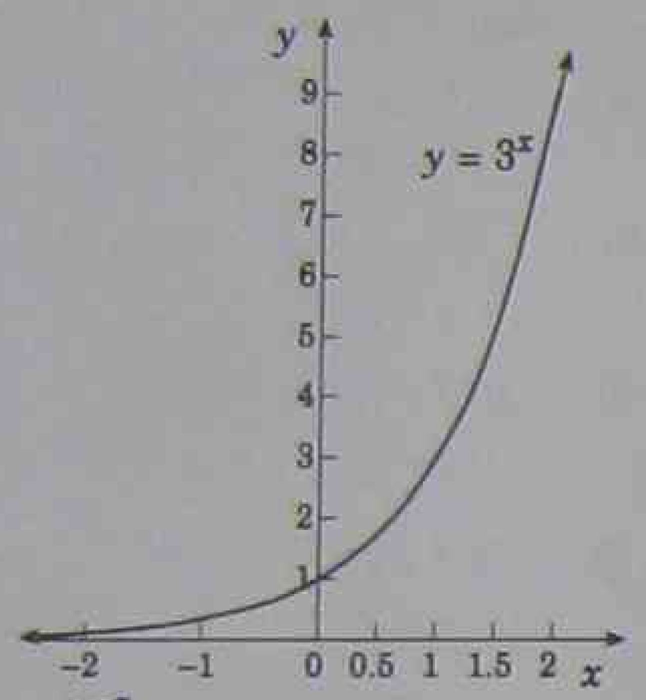
$$\boxed{\begin{aligned}\text{or }x^{-2}&=\tfrac{1}{9}\\\therefore\ x^2&=9\\\therefore\ x&=\sqrt{9}\\&=3\end{aligned}}$$

(n) $x^{\frac{2}{3}}=4$
$\therefore\ x^{\frac{2}{3}}=(8)^{\frac{2}{3}}$
$\therefore\ x=8.$

$$\boxed{\begin{aligned}\text{or }x^{\frac{2}{3}}&=4\\\therefore\ x&=(4)^{\frac{3}{2}}\\&=\left(4^{\frac{1}{2}}\right)^3\\&=2^3\\&=8\end{aligned}}$$

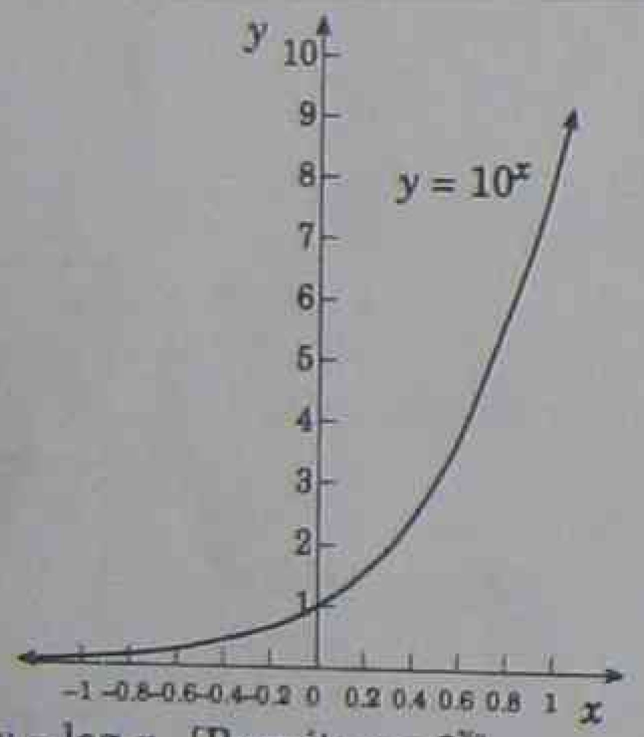
8. $y = 3^x$

x	–2	–1	0	0.5	1	1.5	2
y	$^1/_9$	$^1/_3$	1	≈1.7	3	≈5.2	9



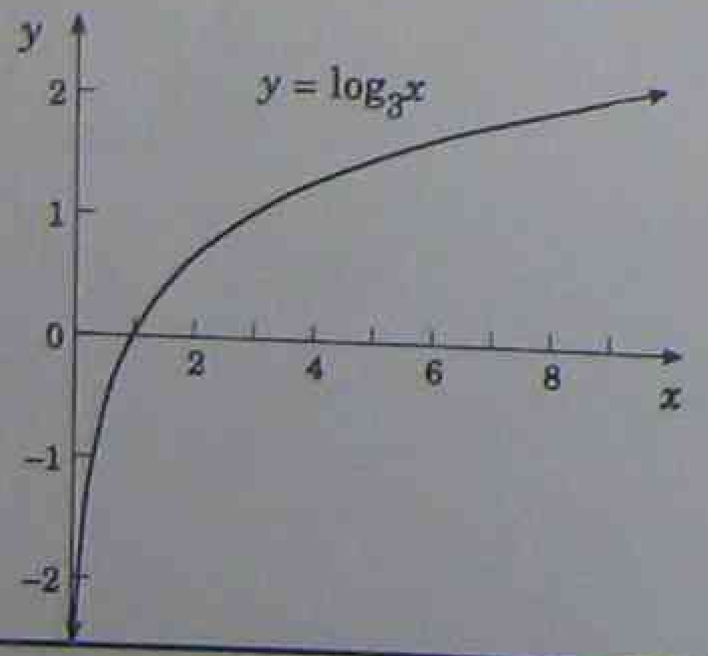
9. $y = 10^x$

x	–1	–0.5	0	0.2	0.5	0.8	1
y	0.1	≈0.3	1	≈1.6	≈3.2	≈6.3	10



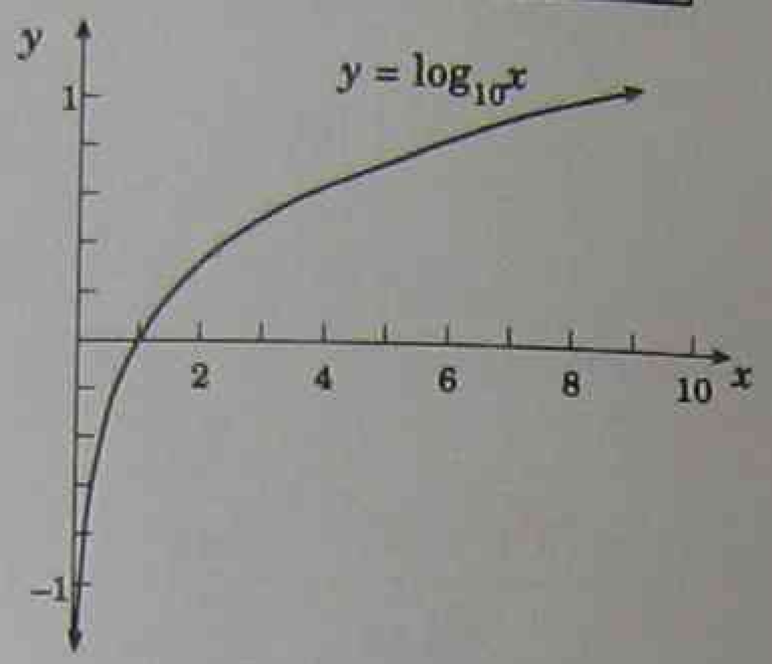
10. $y = \log_3 x$ [Rewrite $x = 3^y$]

x	$^1/_9$	$^1/_3$	1	≈1.7	3	≈5.2	9
y	–2	–1	0	0.5	1	1.5	2



11. $y = \log_{10} x$

x	0.1	≈0.3	1	≈1.6	≈3.2	≈6.3	10
y	–1	–0.5	0	0.2	0.5	0.8	1



12. (a) $\log_a NM$

(b) $\log_a\left(\frac{N}{M}\right)$

(c) $\log_a N^i$

(d) $\log_a N^i M^j$

(e) $\log_a \sqrt{N}$

(f) $\log_a\left(\frac{\sqrt{N}}{m^2}\right)$

13. (a) $\log_a x + \log_a y$

(b) $\log_a x - \log_a y$

(c) $\log_a y^{-1} = -\log_a y$

(d) $\log_a x^{\frac{1}{2}} = \frac{1}{2}\log_a x$

(e) $\log_a x^2 - \log_a y = 2\log_a x - \log_a y$

(f) $\log_a x^2 + \log_a \sqrt{y} - \log_a z^4$

$$= 2\log_a x + \log_a y^{\frac{1}{2}} - 4\log_a z$$
$$= 2\log_a x + \tfrac{1}{2}\log_a y - 4\log_a z$$

14. (a) $\log_2 2^6 = 6\log_2 2$

$$= 6$$

Using $\log_a a = 1$

(b) $\log_4 64 = \log_4 4^3$

$$= 3\log_4 4$$
$$= 3$$

(c) $\log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}}$

$$= \tfrac{1}{2}\log_3 3$$
$$= \tfrac{1}{2}$$

(d) $\log_5 5\sqrt{5} = \log_5 5^{1.5}$

$$= 1.5\log_5 5$$
$$= 1.5$$

(e) $\log_5\left(\frac{1}{5^{\frac{1}{2}}}\right) = \log_5 5^{-\frac{1}{2}}$

$$= -\tfrac{1}{2}\log_5 5$$
$$= -\tfrac{1}{2}$$

(f) $\log_a a^3 = 3\log_a a$

$$= 3$$

(g) $\log_a a^{\frac{1}{2}} = \frac{1}{2}\log_a a$

$$= \tfrac{1}{2}$$

(h) $\log_a a\sqrt{a} = \log_a a^{1.5}$

$$= 1.5\log_a a$$
$$= 1.5$$

(i) $\log_a 1$ [$\log_2 2 = 1$]

$$= 0$$

(j) $\log_x\left(\frac{x^2 - 2x}{x - 2}\right) = \log_x\left(\frac{x\cancel{(x-2)}}{\cancel{(x-2)}}\right)$

$$= \log_x x$$
$$= 1$$

(k) $\log_5\left(\frac{40}{8}\right) = \log_5 5$

$$= 1$$

(l) $\log_3\left(\frac{54}{2}\right) = \log_3 27$

$$= \log_3 3^3$$
$$= 3\log_3 3$$
$$= 3$$

15. (a) $\log_{10} 3 + \log_{10} 5 = 0.477 + 0.699$

$$= 1.176$$

(b) $\log_{10} 5 - \log_{10} 3 = 0.699 - 0.477$

$$= 0.222$$

(c) $\log_{10} 3^{\frac{1}{2}} = \frac{1}{2}\log_{10} 3$

$$= \tfrac{1}{2}(0.477)$$
$$= 0.239$$

(d) $\log_{10} 3\sqrt{3} = \log_{10} 3^{1.5}$

$$= 1.5\log_{10} 3$$
$$= 1.5(0.477)$$
$$= 0.716$$

(e) $\log_{10} 5 + \log_{10} \sqrt{3} = 0.699 + \frac{1}{2}(0.477)$

$$= 0.938$$

(f) $\log_{10} 3^{-1} = -\log_{10} 3$

$$= -0.477$$

(g) $\log_{10} \frac{1}{5} = -\log_{10} 5^{-1}$

$$= -\log_{10} 5$$
$$= -0.699$$

(h) $\boxed{9 \times 5 = 45}$

$$\log_{10} 9 + \log_{10} 5 = \log_{10} 3^2 + \log_{10} 5$$
$$= 2\log_{10} 3 + \log_{10} 5$$
$$= 2(0.477) + 0.699$$
$$= 1.653$$

(i) $\log_{10} 25 + \log_{10} 3 = \log_{10} 5^2 + \log_{10} 3$

$$= 2\log_{10} 5 + \log_{10} 3$$
$$= 2(0.699) + 0.477$$
$$= 1.875$$

(j) $\log_{10} 5.4 = \log_{10}\left(\frac{27}{5}\right)$

$$= \log_{10} 27 - \log_{10} 5$$
$$= \log_{10} 3^3 - \log_{10} 5$$
$$= 3\log_{10} 3 - \log_{10} 5$$
$$= 3(0.477) - 0.699$$
$$= 0.732$$

16. (a) $\log_a 7 + \log_a 5 = 2.8 + 2.3$

$$= 5.1$$

(b) $\log_a 7 - \log_a 5 = 2.8 - 2.3$

$$= 0.5$$

(c) $\log_a 1.4 = \log_a\left(\frac{7}{5}\right)$

$$= 0.5 \text{ [from (b)]}$$

(d) $\log_a \sqrt{7} = \log_a 7^{\frac{1}{2}}$

$$= \tfrac{1}{2}\log_a 7$$
$$= \tfrac{1}{2}(2.8)$$
$$= 1.4$$

(e) $\log_a\left(\frac{1}{7}\right) = \log_a 7^{-1}$

$$= -\log_a 7$$
$$= -2.8$$

(f) $$\begin{aligned}\log_a\left(\tfrac{1}{5^2}\right)&=\log_a 5^{-2}\\&=-2\log_a 5\\&=-2(2.3)\\&=-4.6\end{aligned}$$

(g) $$\begin{aligned}\log_a\left(\tfrac{1}{7^{\frac{1}{2}}}\right)&=\log_a 7^{-\frac{1}{2}}\\&=-\tfrac{1}{2}\log_a 7\\&=-\tfrac{1}{2}(2.8)\\&=-1.4\end{aligned}$$

(h) $$\begin{aligned}\log_a 175&=\log_a 25+\log_a 7\\&=\log_a 5^2+\log_a 7\\&=2\log_a 5+\log_a 7\\&=2(2.3)+2.8\\&=7.4\end{aligned}$$

$\boxed{175 = 25\times 7}$

(i) $$\begin{aligned}\log_a 35^{\frac{1}{2}}&=\tfrac{1}{2}\log_a 35\\&=\tfrac{1}{2}\log_a(5\times 7)\\&=\tfrac{1}{2}[\log_a 5+\log_a 7]\\&=\tfrac{1}{2}[2.8+2.3]\\&=2.6\ \text{[one dec. place]}\end{aligned}$$

(j) $$\begin{aligned}\log_a 9.8&=\log_a\left[9\tfrac{4}{5}\right]\\&=\log_a\left[\tfrac{49}{5}\right]\\&=\log_a 49-\log_a 5\\&=\log_a 7^2-\log_a 5\\&=2\log_a 7-\log_a 5\\&=2(2.8)-2.3\\&=3.3\end{aligned}$$

17\. (a) $$\begin{aligned}\log_n(3\times 2)&=\log_n 3+\log_n 2\\&=y+x.\\&\ \ (\text{or } x+y)\end{aligned}$$

(b) $$\begin{aligned}\log_n\left(\tfrac{2}{3}\right)&=\log_n 2-\log_n 3\\&=x-y.\end{aligned}$$

(c) $$\begin{aligned}\log_n\left(\tfrac{3}{2}\right)&=\log_n 3-\log_n 2\\&=y-x.\end{aligned}$$

(d) $$\begin{aligned}\log_n 2^{\frac{1}{2}}&=\tfrac{1}{2}\log_n 2\\&=\tfrac{1}{2}x.\end{aligned}$$

(e) $$\begin{aligned}\log_n 6^{\frac{1}{2}}&=\tfrac{1}{2}\log_n 6\\&=\tfrac{1}{2}(x+y)\ \text{[from (a)].}\end{aligned}$$

(f) $$\begin{aligned}\log_n(6\times 4)&=\log_n 6+\log_n 4\\&=\log_n 6+\log_n 2^2\\&=\log_n 6+2\log_n 2\\&=x+y+2x\\&=3x+y.\end{aligned}$$

(g) $$\begin{aligned}\log_n\left(6\times\sqrt{2}\right)&=\log_n 6+\log_n 2^{\frac{1}{2}}\\&=\log_n 6+\tfrac{1}{2}\log_n 2\\&=x+y+\tfrac{1}{2}x\\&=\tfrac{3}{2}x+y.\end{aligned}$$

(h) $$\begin{aligned}\log_n\left(4\tfrac{1}{2}\right)&=\log_n\left(\tfrac{9}{2}\right)\\&=\log_n 9-\log_n 2\\&=\log_n 3^2-\log_n 2\\&=2\log_n 3-\log_n 2\\&=2y-x.\end{aligned}$$

(i) $$\begin{aligned}\log_n\left(\tfrac{16}{3}\right)&=\log_n 16-\log_n 3\\&=\log_n 2^4-\log_n 3\\&=4\log_n 2-\log_n 3\\&=4x-y.\end{aligned}$$

(j) $$\begin{aligned}\log_n\left(\tfrac{1}{2}\right)&=\log_n 2^{-1}\\&=-\log_n 2\\&=-x.\end{aligned}$$

18\. (a) $$\begin{aligned}N+M&=\log_a 10+\log_a 3\\&=\log_a(10\times 3)\\&=\log_a 30.\end{aligned}$$

(b) $$\begin{aligned}N-M&=\log_a 10-\log_a 3\\&=\log_a\left(\tfrac{10}{3}\right).\end{aligned}$$

(c) $$\begin{aligned}\tfrac{1}{2}N&=\tfrac{1}{2}\log_a 10\\&=\log_a 10^{\frac{1}{2}}\\&=\log_a\sqrt{10}.\end{aligned}$$

(d) $$\begin{aligned}\tfrac{1}{2}M-\tfrac{1}{2}N&=\tfrac{1}{2}[\log_a 3-\log_a 10]\\&=\tfrac{1}{2}\log_a\tfrac{3}{10}\\&=\log_a\sqrt{0.3}.\end{aligned}$$

(e) $$\begin{aligned}2N+3M&=2\log_a 10+3\log_a 3\\&=\log_a 10^2+\log_a 3^3\\&=\log_a 100+\log_a 27\\&=\log_a(100\times 27)\\&=\log_a 2700.\end{aligned}$$

(f) $$\begin{aligned}4M-2N&=4\log_a 3-2\log_a 10\\&=\log_a 3^4-\log_a 10^2\\&=\log_a 81-\log_a 100\\&=\log_a\left(\tfrac{81}{100}\right)\\&=\log_a 0.81.\end{aligned}$$

19\. (a) $$\begin{aligned}\log_2 y&=\log_2 ab\\ \therefore\quad y&=ab.\end{aligned}$$

(b) $$\begin{aligned}\log_2 y&=\log_2 a^{\frac{1}{3}}\\&=\log_2\sqrt[3]{a}\\ \therefore\quad y&=\sqrt[3]{a}.\end{aligned}$$

(c) $$\begin{aligned}\log_2 y^3&=\log_2 x^{-1}\\ \therefore\quad y^3&=x^{-1}\\&=\tfrac{1}{x}\\ \therefore\quad y&=\sqrt[3]{\tfrac{1}{x}}\ \text{or}\ \frac{1}{\sqrt[3]{x}}.\end{aligned}$$

(d) $$\begin{aligned}\log_2 y&=\log_2 2-\tfrac{1}{2}\log_2 x\\&=\log_2 2-\log_2 x^{\frac{1}{2}}\\&=\log_2 2-\log_2\sqrt{x}\\&=\log_2\left(\frac{2}{\sqrt{x}}\right)\\ \therefore\quad y&=\frac{2}{\sqrt{x}}.\end{aligned}$$

Note use of $1=\log_2 2$ →

20\. (a) $$\begin{aligned}\log_{10}a&=\log_{10}(3\times 8)\\ \therefore\quad a&=3\times 8\\&=24\end{aligned}$$

(b) $$\begin{aligned}\log_{10}a&=\log_{10}81^{\frac{1}{2}}\\&=\log_{10}\sqrt{81}\\ \therefore\quad a&=\sqrt{81}\\&=9\end{aligned}$$

(c) $$\begin{aligned}\log_{10}a&=\log_{10}\left(\tfrac{72}{9}\right)\\&=\log_{10}8\\ \therefore\quad a&=8\end{aligned}$$

(d) $$\begin{aligned}\log_{10}a&=\log_{10}2^3-\log_{10}5^2\\&=\log_{10}8-\log_{10}25\\&=\log_{10}\left(\tfrac{8}{25}\right)\\ \therefore\quad a&=\tfrac{8}{25}\end{aligned}$$

(e) $$\begin{aligned}\log_{10}a&=\log_{10}10-\log_{10}5^2\\&=\log_{10}10-\log_{10}25\\&=\log_{10}\left(\tfrac{10}{25}\right)\\ \therefore\quad a&=\tfrac{10}{25}\\&=\tfrac{2}{5}.\end{aligned}$$

(f) $$\begin{aligned}\log_{10}\left(\tfrac{6a}{a+4}\right)&=\log_{10}10\\ \therefore\quad \tfrac{6a}{a+4}&=10\\ \therefore\quad 6a&=10(a+4)\\&=10a+40\\ \therefore\quad 4a&=-40\\ a&=-10.\end{aligned}$$

(g) $$\begin{aligned}\log_2\left(\tfrac{a}{a+2}\right)&=3\times 1\\&=3\log_2 2\\&=\log_2 2^3\\&=\log_2 8\\ \therefore\quad \tfrac{a}{a+2}&=8\\ \therefore\quad a&=8(a+2)\\&=8a+16\\ \therefore\quad 7a&=-16\\ a&=-\tfrac{16}{7}\end{aligned}$$

Note the use of $1=\log_{10}10$ and $1=\log_2 2$.

21\. (a) $\log_3 5$
(b) $\log_5 N$

22\. (a) $$\begin{aligned}\frac{\log_{10}5}{\log_{10}2}&=\frac{0.698\,97}{0.301\,03}\\&=2.321\,9281\\&=2.322\end{aligned}$$

[to four significant figures]

(b) $$\begin{aligned}\frac{\log_{10}10}{\log_{10}2}&=\frac{1}{\log_{10}2}\\&=3.321\,9281\\&=3.322\end{aligned}$$

[to four significant figures]

(c) $$\begin{aligned}\log_2 5^{\frac{1}{2}}&=\tfrac{1}{2}\log_2 5\\&=\tfrac{1}{2}(2.321\,9281)\quad\text{[from (a)]}\\&=1.160\,964\\&=1.161\end{aligned}$$

[to four significant figures]

(d)
$$\begin{aligned}\log_2\left(\tfrac{1}{5}\right) &= \log_2 5^{-1}\\ &= -\log_2 5\\ &= -2.322 \qquad \text{[from (a)]}\end{aligned}$$

(e)
$$\begin{aligned}\log_2 0.1 &= \log_2\left(\tfrac{1}{10}\right)\\ &= \log_2 10^{-1}\\ &= -\log_2 10\\ &= -3.322 \qquad \text{[from (b)]}\end{aligned}$$

23. (a)
$$\begin{aligned}3^x &= 5\\ \log_{10} 3^x &= \log_{10} 5\\ \therefore\ x\log_{10} 3 &= \log_{10} 5\\ \therefore\ x &= \frac{\log_{10} 5}{\log_{10} 3}\\ &= 1.464\,9735,\\ &\approx 1.46\end{aligned}$$
[to 2 dec. places]

Taking $\log_{10}$ of each side

On calculator

$5\ \boxed{\text{LOG}}\ \boxed{\div}\ 3\ \boxed{\text{LOG}}\ \boxed{=}$

or express $3^x = 5$ in log form.

$$\begin{aligned}x &= \log_3 5\\ &= \frac{\log_{10} 5}{\log_{10} 3}\\ &\approx 1.46\end{aligned}$$
[to two decimal places]

(b)
$$\begin{aligned}3^x &= 10\\ \log_{10} 3^x &= \log_{10} 10\\ \therefore\ x\log_{10} 3 &= 1\\ x &= \frac{1}{\log_{10} 3}\\ &= 2.095\,9033\\ &\approx 2.10\end{aligned}$$
[to two decimal places]

Take logs of both sides.

(c)
$$\begin{aligned}3^{2x} &= 5\\ \log_{10} 3^{2x} &= \log_{10} 5\\ \therefore\ 2x\log_{10} 3 &= \log_{10} 5\\ 2x &= \frac{\log_{10} 5}{\log_{10} 3}\\ x &= \frac{\log_{10} 5}{2\log_{10} 3}\\ &= 0.732\,4867\\ &\approx 0.73\end{aligned}$$
[to two decimal places]

(d)
$$\begin{aligned}3^x &= \sqrt{5}\\ \log_{10} 3^x &= \log_{10}\sqrt{5} = \log_{10} 5^{\frac{1}{2}}\\ \therefore\ x\log_{10} 3 &= \tfrac{1}{2}\log_{10} 5\\ \therefore\ x &= \frac{\frac{1}{2}\log_{10} 5}{\log_{10} 3}\\ &\approx 0.73\end{aligned}$$
[to two decimal places]

(e)
$$\begin{aligned}3^x &= 5^{-1}\\ \log_{10} 3^x &= \log_{10} 5^{-1}\\ \therefore\ x\log_{10} 3 &= -\log_{10} 5\\ \therefore\ x &= \frac{-\log_{10} 5}{\log_{10} 3}\\ &= -1.46 \quad \text{[from (a)]}\end{aligned}$$

(f)
$$\begin{aligned}\log_{10} 3^x &= \log_{10} 2^{3x-5}\\ \therefore\ x\log_{10} 3 &= (3x-5)\log_{10} 2\\ &= 3x\log_{10} 2 - 5\log_{10} 2\\ \therefore\ 3x\log_{10} 2 - x\log_{10} 3 &= 5\log_{10} 2\\ \therefore\ x(3\log_{10} 2 - \log_{10} 3) &= 5\log_{10} 2\\ \therefore\ x &= \frac{5\log_{10} 2}{3\log_{10} 2 - \log_{10} 3}\\ &= 3.533\,4753\\ &\approx 3.53.\end{aligned}$$
[to two decimal places]

Calculator

$2\ \boxed{\text{LOG}}\ \boxed{\times}\ 5\ \boxed{\div}\ \boxed{[}\ 2\ \boxed{\text{LOG}}$
$\boxed{\times}\ 3\ \boxed{-}\ 3\ \boxed{\text{LOG}}\ \boxed{]}\ \boxed{=}$

(g)
$$\begin{aligned}2^{3-x} &= 5^{2x+1}\\ \therefore\ \log_{10} 2^{3-x} &= \log_{10} 5^{2x+1}\\ \therefore\ (3-x)\log_{10} 2 &= (2x+1)\log_{10} 5\\ \therefore\ 3\log_{10} 2 - x\log_{10} 2 &\\ &= 2x\log_{10} 5 + \log_{10} 5\\ \therefore\ 3\log_{10} 2 - \log_{10} 5 &\\ &= 2x\log_{10} 5 + x\log_{10} 2\\ &= x(2\log_{10} 5 + \log_{10} 2)\\ \therefore\ x &= \frac{3\log_{10} 2 - \log_{10} 5}{2\log_{10} 5 + \log_{10} 2}\\ &= 0.120\,1433\\ &\approx 0.12\end{aligned}$$
[to two decimal places]

Continued

Calculator

$\boxed{[}\ 2\ \boxed{\text{LOG}}\ \boxed{\times}\ 3\ \boxed{-}\ 5\ \boxed{\text{LOG}}\ \boxed{]}\ \boxed{\div}$
$\boxed{[}\ 5\ \boxed{\text{LOG}}\ \boxed{\times}\ 2\ \boxed{-}\ 2\ \boxed{\text{LOG}}\ \boxed{]}\ \boxed{=}$

24. (a)
$$\begin{aligned}\frac{\log 2^4}{\log 2} &= \frac{4\log 2}{\log 2}\\ &= 4\end{aligned}$$

(b)
$$\begin{aligned}\frac{\log 3^4}{\log 3^3} &= \frac{4\log 3}{3\log 3}\\ &= \tfrac{4}{3}\end{aligned}$$

(c)
$$\begin{aligned}\frac{\log 2^3}{\log 2^{-2}} &= \frac{3\log 2}{-2\log 2}\\ &= -\tfrac{3}{2}\end{aligned}$$

(d)
$$\begin{aligned}\frac{\log 2}{\log\left(\frac{1}{4}\right)} &= \frac{\log 2}{\log 2^{-2}}\\ &= \frac{\log 2}{-2\log 2}\\ &= \tfrac{1}{-2}\\ &= -\tfrac{1}{2}\end{aligned}$$

(e)
$$\begin{aligned}&\log_{10}\left(125\times 32 \div \tfrac{2}{5}\right)\\ &= \log_{10}\left(125\times \cancel{32}^{16}\times\tfrac{5}{\cancel{2}_1}\right)\\ &= \log_{10} 10\,000\\ &= \log_{10} 10^4\\ &= 4\log_{10} 10 = 4\end{aligned}$$

25. (a)
$$\begin{aligned}\log_4\left(\log_2 2^4\right) &= \log_4(4\log_2 2)\\ &= \log_4 4\\ &= 1\end{aligned}$$

Remember
$\log_a a = 1$

(b)
$$\begin{aligned}\log_{10}(10\log_{10} 10) &= \log_{10} 10\\ &= 1\end{aligned}$$

(c)
$$\begin{aligned}&\log_{10}\left(\tfrac{16}{15}\right)^2 + \log_{10}\left(\tfrac{5}{2}\right)^3 + \log_{10}\left(\tfrac{9}{16}\right)\\ &= \log_{10}\left[\left(\tfrac{16}{15}\right)^2\times\left(\tfrac{5}{2}\right)^3\times\left(\tfrac{9}{16}\right)\right]\\ &= \log_{10} 10\\ &= 1\end{aligned}$$

$\tfrac{16}{15}\times\tfrac{16}{15}\times\tfrac{5}{2}\times\tfrac{5}{2}\times\tfrac{5}{2}\times\tfrac{9}{16}$
$= 10$ after cancelling.

26. (a) Function
(b) Not function
(c) Not function
(d) Function
(e) Function
(f) Not function

27. $f(x) = 6x - 11$
(a) $f(0) = 6(0) - 11 = -11$
(b) $f(2) = 6(2) - 11 = 1$
(c) $f(-2) = 6(-2) - 11 = -23$
(d) $f(5) = 6(5) - 11 = 19$

28. $f(x) = x^2 - 4x$
(a) $f(0) = 0^2 - 4(0) = 0$

(b)
$$\begin{aligned}f(1) &= 1^2 - 4(1)\\ &= 1 - 4 = -3\end{aligned}$$

(c)
$$\begin{aligned}f(-1) &= (-1)^2 - 4(-1)\\ &= 1 + 4 = 5\end{aligned}$$

(d)
$$\begin{aligned}f(3) &= 3^2 - 4(3)\\ &= 9 - 12 = -3\end{aligned}$$

29. (a)
$$\begin{aligned}f(t) &= 1 - t^2\\ f(0) &= 1 - 0 = 1\\ f(2) &= 1 - (2)^2 = -3\\ f(-2) &= 1 - (-2)^2 = -3\end{aligned}$$

(b)
$$\begin{aligned}f(t) &= \frac{1+t}{1-t}\\ f(0) &= 1\\ f(2) &= \frac{1+2}{1-2} = -3\\ f(-2) &= \frac{1+(-2)}{1-(-2)} = \frac{-1}{3}\end{aligned}$$

(c)
$$\begin{aligned}f(t) &= t^2 - t^3\\ f(0) &= 0\\ f(2) &= 2^2 - 2^3 = -4\\ f(-2) &= (-2)^2 - (-2)^3\\ &= 4 - (-8) = 12\end{aligned}$$

(d) $f(t)=(t-3)^2$

$f(0)=(-3)^2=9$

$f(2)=(2-3)^2$

$=(-1)^2=1$

$f(-2)=(-2-3)^2$

$=(-5)^2=25$

(e) $f(t)=(3-t)^2$

$f(0)=(3)^2=9$

$f(2)=(3-2)^2=1$

$f(-2)=(3-{}^{-}2)^2$

$=(5)^2=25$

(f) $f(t)=3^t$

$f(0)=3^0=1$

$f(2)=3^2=9$

$f(-2)=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

30. $\phi(x)=x^2+x$

(a) $\phi(h)=h^2+h$

(b) $\phi(x+2)=(x+2)^2+(x+2)$

$=x^2+4x+4+x+2$

$=x^2+5x+6$

(c) $\phi(x+2)-\phi(2)=x^2+5x+6-\left(2^2+2\right)$

$=x^2+5x+6-(6)$

$=x^2+5x$

(d) $\phi(-x)=(-x)^2+(-x)$ $\boxed{-x\times -x=x^2}$

$=x^2-x$

31. (a) $g(x)=x^4$

$g(-x)=(-x)^4=x^4$ $\boxed{\begin{array}{r}(-x)\times(-x)\times(-x)\\ \times(-x)=x^4\end{array}}$

$\therefore\ g(x)=g(-x)$

A function is called an *even* function when $f(x)=f(-x)$.

(b) $H(x)=x^3$

$H(-x)=(-x)^3$ $\boxed{\begin{array}{l}(-x)\times(-x)\times(-x)\\ =-x^3\end{array}}$

$=-x^3$

$\therefore\ H(-x)=-H(x)$ $\boxed{H(x)=x^3}$

A function is called an *odd* function when $f(-x)=-f(x)$.

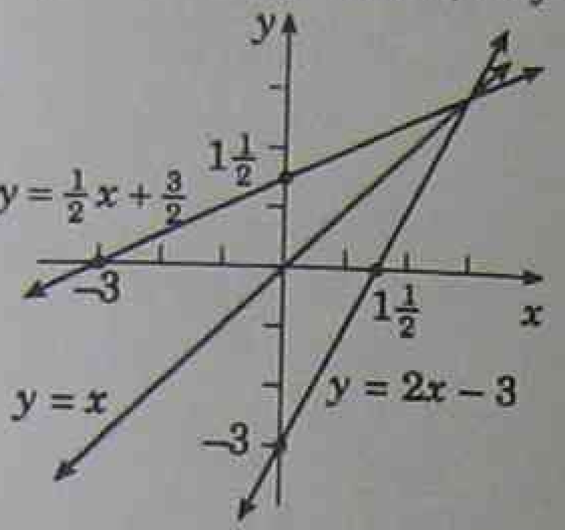
32. (a) Domain: $x\ge 0$ Range: $y\ge 0$

(b) Domain: all values of x can be inputs
Range: $y\ge 4$

(c) Domain: all values of x can be inputs
Range: $y\le 4$

(d) Domain: all values of x between -2 and 2 are inputs, i.e. $-2\le x\le 2$.
Range: all values between 0 and 2 may be outputs, that is, $0\le y\le 2$.

33. (a)



(b) $y=2x-3$

$x=2y-3 \quad \Leftarrow \boxed{\text{Interchange } x \text{ and } y}$

$\therefore\ 2y=x+3$

$y=\frac{1}{2}x+\frac{3}{2} \quad \Leftarrow \boxed{\text{Make } y \text{ the subject}}$

Inverse function is $y=\frac{1}{2}x+\frac{3}{2}$.

(c) From the diagram it can be seen that the two lines are mirror images through $y=x$.

34. (a) $x-2y=4$

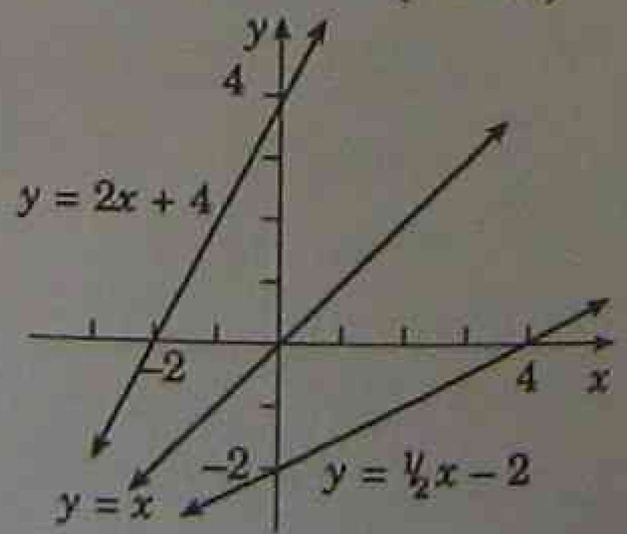
$\therefore\ 2y=x-4$

$y=\frac{1}{2}x-2\ \left(\begin{array}{c}\text{original}\\ \text{function}\end{array}\right)$

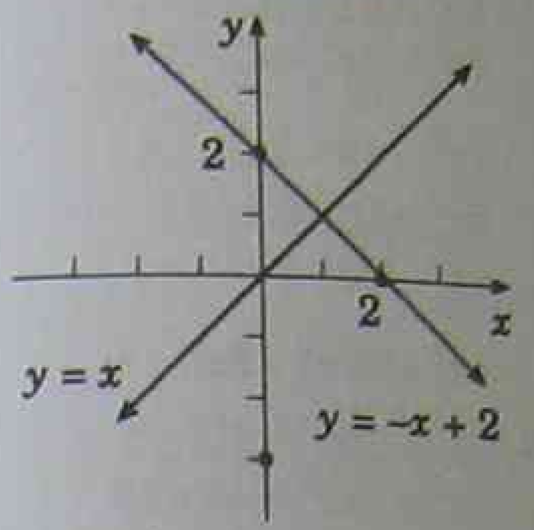
Now, $x=\frac{1}{2}y-2\ \left(\begin{array}{c}\text{interchange}\\ x \text{ and } y\end{array}\right)$

$\therefore\ 2x=y-4$

$y=2x+4\ \left(\begin{array}{c}\text{Inverse}\\ \text{function}\end{array}\right)$



(b) $y=-x+2$ (original)



$x=-y+2$

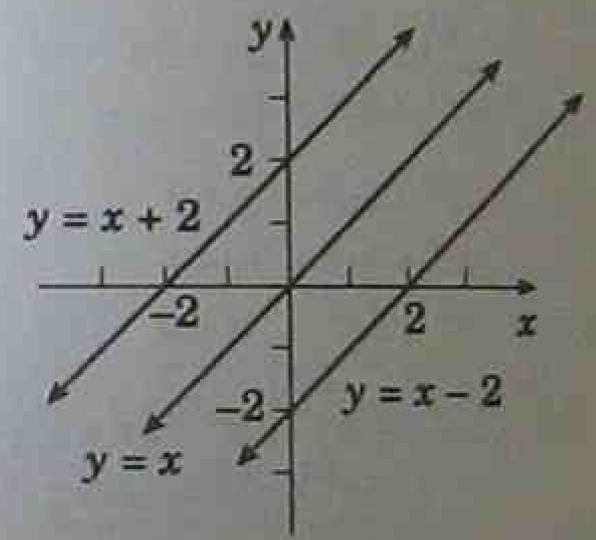
$\therefore\ y=-x+2$ (inverse)

Both original function and inverse function are the same functions, that is, $y=-x+2$.

$y=-x+2\ \perp\ y=x$

and hence is its own reflection.

(c) $y=x+2$ (original)

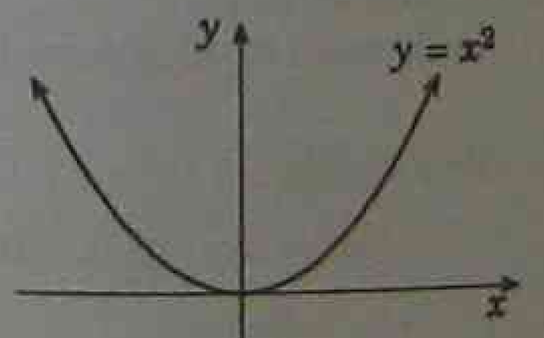


$x=y+2$

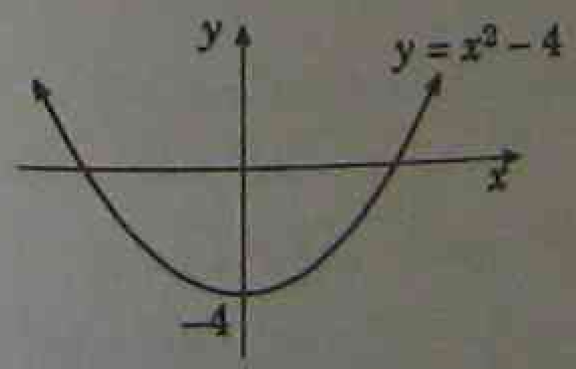
$\therefore\ y=x-2$ (inverse)

Function and inverse function are parallel to and equidistant from $y=x$.

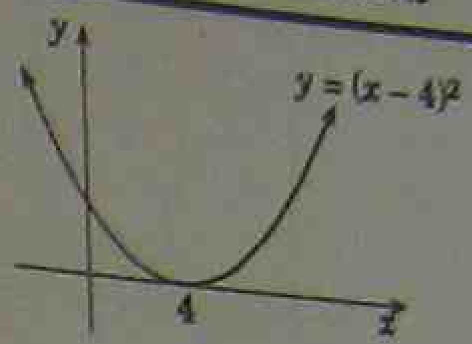
35.



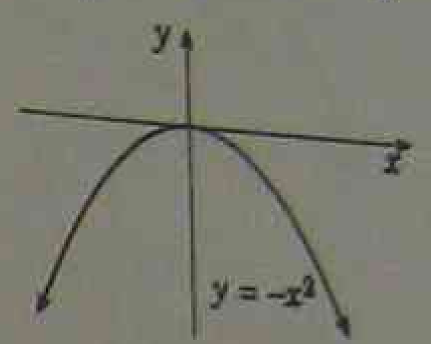
(a)



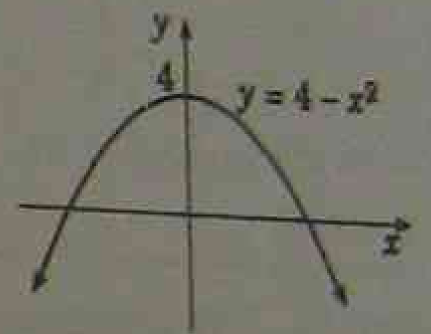
(b)



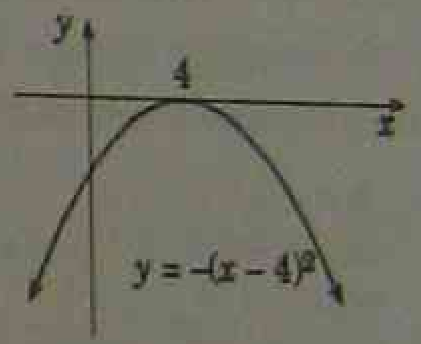
36. (a)



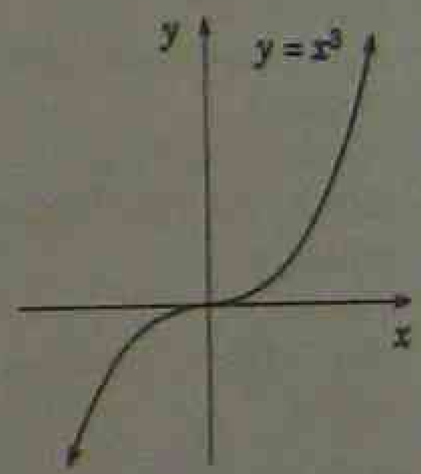
(b)



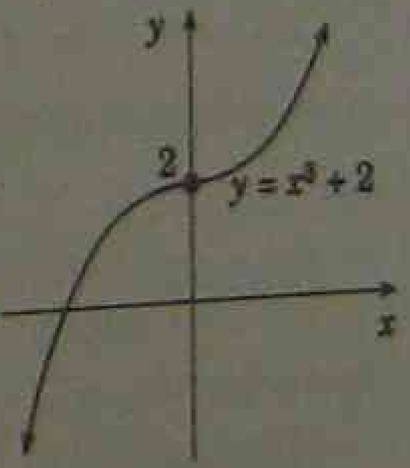
(c)



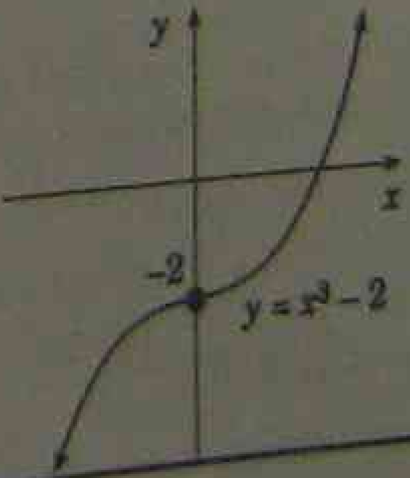
37.



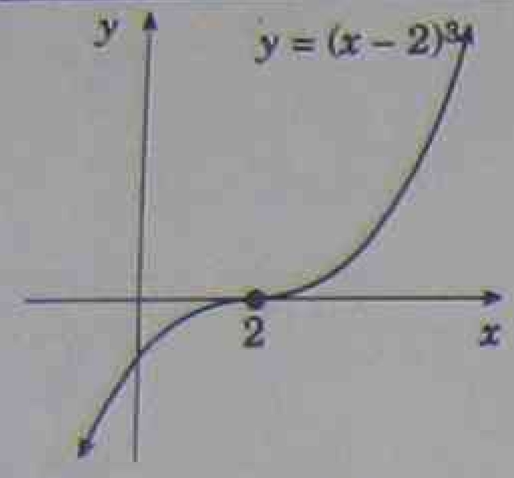
(a)



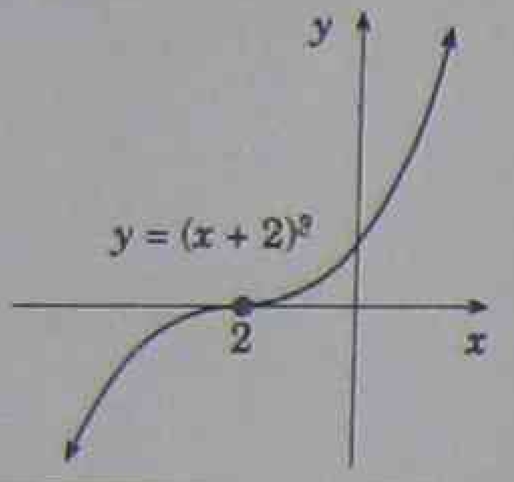
(b)



(c)



(d)



Chapter 14 Polynomials and curve sketching (page 222)

1\. (a) $P(x) = 3x^4 - 7x + 2$
deg: 4, lead t: $3x^4$, lead c = 3
const t: 2

(b) $P(x) = 4 - 2x^5$
deg: 5, lead t: $-2x^5$, lead c = −2
const t: 4

(c) $P(x) = 3x^5 - 2x^3 + 4x$
deg: 5, lead t: $3x^5$, lead c = 3
const t: 0

(d) $P(x) = 7$
deg: 0, lead t: 7, lead c = 7
const t: 7

2\. (a) $\frac{4}{x} - 3x + 2 = 4x^{-1} - 3x + 2$
∴ not polynomial [power of − 1]

(b) $\frac{5x^4 - x}{x} = 5x^3 - 1$
∴ polynomial.

(c) $\frac{3x}{4} + 2 = \frac{3}{4}x + 2$
∴ polynomial.

(d) $4x^4 + \sqrt{x} - 2 = 4x^4 + x^{\frac{1}{2}} - 2$
∴ not polynomial [power of $\frac{1}{2}$]

(e) $(x+4)^2 - 2x + 3$
$= x^2 + 8x + 16 - 2x + 3$
$= x^2 + 6x + 19$
∴ polynomial.

3\. (a)
$$\begin{array}{r} 4x^4-2x^2+3 \\ \underline{3x^4-5x^2-7}\,+ \\ 7x^4-7x^2-4 \end{array}$$

(b)
$$\begin{array}{r} 3x^2-4x+1 \\ \underline{5x^2+2x-5}\,+ \\ 8x^2-2x-4 \end{array}$$

(c)
$$\begin{array}{r} 4x^3+12x^2+3x+\ 7 \\ \underline{5x^3+\ 0x^2+\ \ x+\ 3}\,+ \\ 9x^3+12x^2+4x+10 \end{array}$$

(d)
$$\begin{array}{r} 4x^5-21x+2 \\ \underline{3x^5+\ 2x-1}\,+ \\ 7x^5-19x+1 \end{array}$$

(e)
$$\begin{array}{r} 5x^3+2x^2-x-1 \\ \underline{3x^3-\ x^2-x-5}\,- \\ 2x^3+3x^2\qquad+4 \end{array}$$

(f)
$$\begin{array}{r} 4x^4-2x^2+1 \\ \underline{x^4-5x^2-7}\,- \\ 3x^4+3x^2+8 \end{array}$$

(g)
$$\begin{array}{r} 4x^3+0x^2-\ x+2 \\ \underline{3x^3+2x^2+0x-1}\,- \\ x^3-2x^2-\ x+3 \end{array}$$

(h)
$$\begin{array}{r} 5x^4+0x^3-2x^2+0x+1 \\ \underline{3x^3+\ x^2-5x+2}\,- \\ 5x^4-3x^3-3x^2+5x-1 \end{array}$$

4\. (a) $P(x) \cdot Q(x)$
$= \left(3x^2 - 4x + 1\right)\left(4 - x + x^2\right)$
$= \ldots + 3x^4 + \ldots$
∴ degree is 4
[only need leading term]

(b) $P(x) \cdot Q(x)$
$= \left(5x^4 - 2x - 1\right)\left(3x - 5x^2 + x^3\right)$
$= \ldots + 5x^7 + \ldots$
∴ degree is 7.

5\. (a) $\left(2x^3 - 4\right)\left(5x^2 - x + 2\right)$
$= 10x^5 - 2x^4 + 4x^3 - 20x^2 + 4x - 8$

(b) $\left(6x^2 - 4x\right)\left(x^3 + x - 1\right)$
$= 6x^5 + 6x^3 - 6x^2 - 4x^4 - 4x^2 + 4x$
$= 6x^5 - 4x^4 + 6x^3 - 10x^2 + 4x$

6\. (a)
$$\begin{array}{r} x^2+7 \\ x-3\,\overline{\smash{)}\,x^3-3x^2+\underset{\times}{7x}-\underset{\times}{5}} \\ \underline{x^3-3x^2} \\ 0\ +7x-\ 5 \\ \underline{7x-21} \\ 16 \end{array}$$

∴ $x^3 - 3x^2 + 7x - 5$
$= (x-3)\left(x^2+7\right) + 16$

(b)
$$\begin{array}{r} -x^2+4x-13 \\ x+2\,\overline{\smash{)}\,-x^3+2x^2-\underset{\times}{5x}-\underset{\times}{1}} \\ \underline{-x^3-2x^2} \\ 4x^2-5x \\ \underline{4x^2+8x} \\ -13x-\ 1 \\ \underline{-13x-26} \\ 25 \end{array}$$

∴ $-x^3 + 2x^2 - 5x - 1$
$= (x+2)\left(-x^2 + 4x - 13\right) + 25$

(c)
$$\begin{array}{r} x^3-x^2+2x-2 \\ x^2+x\,\overline{\smash{)}\,x^5+0x^4+\underset{\times}{x^3}+\underset{\times}{0x^2}-\underset{\times}{x}} \\ \underline{x^5+\ x^4} \\ -x^4+x^3 \\ \underline{-x^4-x^3} \\ 2x^3+0x^2 \\ \underline{2x^3+2x^2} \\ -2x^2-\ x \\ \underline{-2x^2-2x} \\ x \end{array}$$

∴ $x^5 + x^3 - x$
$= \left(x^2+x\right)\left(x^3 - x^2 + 2x - 2\right) + x$

(d) $\left(x^4 - 1\right) \div \left(x^2 - 3\right)$

$$\begin{array}{r} x^2+3 \\ x^2-3\,\overline{\smash{)}\,x^4+0x^3+0x^2+0x-1} \\ \underline{x^4\qquad\ -3x^2} \\ 3x^2-1 \\ \underline{3x^2-9} \\ 8 \end{array}$$

∴ $x^4 - 1 = \left(x^2 - 3\right)\left(x^2 + 3\right) + 8$

7\. $p(x) = x^3 + 2x^2 - x + 1$
$q(x) = x - 1$

(a) $p(x) + q(x)$
$$\therefore \begin{array}{r} x^3+2x^2-x+1 \\ \underline{x-1}\,+ \\ x^3+2x^2 \end{array}$$
∴ $p(x) + q(x) = x^3 + 2x^2$

(b) $p(x) - q(x)$
$$\therefore \begin{array}{r} x^3+2x^2-\ x+1 \\ \underline{x-1}\,- \\ x^3+2x^2-2x+2 \end{array}$$
∴ $p(x) - q(x) = x^3 + 2x^2 - 2x + 2$

(c) $p(x) \cdot q(x)$
$= \left(x^3 + 2x^2 - x + 1\right)(x-1)$
$= x^4 - x^3 + 2x^3 - 2x^2 - x^2$
$+ x + x - 1$
$= x^4 + x^3 - 3x^2 + 2x - 1$

(d) $\frac{p(x)}{q(x)}$

$$\therefore \begin{array}{r} x^2+3x+2 \\ x-1\,\overline{\smash{)}\,x^3+2x^2-x+1} \\ \underline{x^3-\ x^2} \\ 3x^2-\ x \\ \underline{3x^2-3x} \\ 2x+1 \\ \underline{2x-2} \\ 3 \end{array}$$

∴ $p(x) + q(x) = \left(x^2 + 3x + 2\right) + 3$

8\. (a) Let $P(x) = x^3 + 4x^2 + x - 1$
Dividing by $(x+2)$, ∴ find $P(-2)$
∴ $P(-2) = (-2)^3 + 4(-2)^2 + (-2) - 1$
$= -8 + 16 - 2 - 1 = 5$
∴ remainder is 5.

(b) Let $P(x) = x^3 - 4x + 2$
Dividing by $(x-1)$, ∴ find $P(1)$
∴ $P(1) = 1^3 - 4(1) + 2$
$= 1 - 4 + 2 = -1$
∴ remainder is − 1.

9. Let $P(x) = x^3 + 3x^2 - 7x + k$

Dividing by $(x+2)$, $\therefore P(-2) = 5$

$$\therefore P(-2) = (-2)^3 + 3(-2)^2 - 7(-2) + k = 5$$
$$-8 + 12 + 14 + k = 5$$
$$18 + k = 5$$
$$k = 5 - 18$$
$$k = -13$$

$\therefore$ value of k is -13.

10. Let $P(x) = 3x^4 - 2x^3 - 2k$

Dividing by $(x+1)$, $\therefore P(-1) = 3$

$$\therefore P(-1) = 3(-1)^4 - 2(-1)^3 + 2k = 3$$
$$3 + 2 + 2k = 3$$
$$5 + 2k = 3$$
$$2k = 3 - 5$$
$$2k = -2$$
$$k = -1$$

$\therefore$ value of k is -1.

11. Let $P(x) = x^2 - 7x + 9$

Dividing by $(x-k)$, $\therefore P(k) = 3$

$\therefore P(k) = k^2 - 7k + 9 = 3$,

that is, $k^2 - 7k + 6 = 0$

$$(k-6)(k-1) = 0$$
$$k = 6 \text{ or } 1$$

$\therefore$ k is 6 or 1.

12. Let $P(x) = x^3 + kx - 10$

Dividing by $(x-5)$, $\therefore P(5) = 0$

$$\therefore P(5) = 5^3 + 5k - 10 = 0$$
$$125 + 5k - 10 = 0$$
$$5k + 115 = 0$$
$$5k = -115$$
$$\frac{5k}{5} = -\frac{115}{5}$$
$$k = -23$$

$\therefore$ value of k is -23.

13. Let $P(x) = x^3 - 2x^2 - 5x + 6$

If $(x-1)$ is a factor, then $P(1) = 0$

$$\therefore P(1) = 1^3 - 2(1)^2 - 5(1) + 6$$
$$= 1 - 2 - 5 + 6$$
$$= 0$$

$\therefore$ as $P(1) = 0$, then $(x-1)$ is a factor of $P(x)$.

$$\begin{array}{r} x^2 - x - 6 \\ x-1\overline{)x^3 - 2x^2 - 5x + 6} \\ \underline{x^3 - x^2} \\ -x^2 - 5x \\ \underline{-x^2 + x} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

$$\therefore \text{ as } \quad x^3 - 2x^2 - 5x + 6 = 0$$
$$\therefore \quad (x-1)(x^2 - x - 6) = 0$$
$$(x-1)(x-3)(x+2) = 0$$

$\therefore$ $x = 1,\ 3,\ -2$,

$\therefore$ the roots of the equation are $x = 1$, 3 and -2.

14. Let $P(x) = 2x^3 - 5x^2 - 4x + 3$

Now find a, where $P(a) = 0$, that is, try

$$P(1) = 2(1)^3 - 5(1)^2 - 4(1) + 3$$
$$= 2 - 5 - 4 + 3$$
$$\neq 0$$

Try

$$P(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$$
$$= -2 - 5 + 4 + 3$$
$$= 0$$

$\therefore$ $P(-1) = 0$, $\therefore$ $(x+1)$ is a factor.

$$\begin{array}{r} 2x^2 - 7x + 3 \\ x+1\overline{)2x^3 - 5x^2 - 4x + 3} \\ \underline{2x^3 + 2x^2} \\ -7x^2 - 4x \\ \underline{-7x^2 - 7x} \\ 3x + 3 \\ \underline{3x + 3} \\ 0 \end{array}$$

$$\therefore 2x^3 - 5x^2 - 4x + 3$$
$$= (x+1)(2x^2 - 7x + 3)$$
$$= (x+1)(2x-1)(x-3).$$

15. Let $P(x) = x^3 + 2x^2 - 9x - 18$

Try $P(1) = 1^3 + 2(1)^2 - 9(1) - 18 \neq 0$

$$P(-1) = (-1)^3 + 2(-1)^2 - 9(-1) - 18 \neq 0$$
$$P(2) = (2)^3 + 2(2)^2 - 9(2) - 18$$
$$= 8 + 8 - 18 - 18 \neq 0$$
$$P(-2) = (-2)^3 + 2(-2)^2 - 9(-2) - 18$$
$$= -8 + 8 + 18 - 18 = 0,$$

$\therefore$ as $P(-2) = 0$, $(x+2)$ is a factor,

$$\begin{array}{r} x^2 - 9 \\ \therefore\ x+2\overline{)x^3 + 2x^2 - 9x - 18} \\ \underline{x^3 + 2x^2} \\ -9x - 18 \\ \underline{-9x - 18} \\ 0 \end{array}$$

As $x^3 + 2x^2 - 9x - 18 = 0$

$$(x+2)(x^2 - 9) = 0$$
$$(x+2)(x-3)(x+3) = 0$$

$\therefore$ $x = -2,\ 3,\ -3$.

16. Let $P(x) = x^3 - 2x^2 - ax + b$

As $x = -3$ is a root, $\quad P(-3) = 0$

$$\therefore \quad P(-3) = (-3)^3 - 2(-3)^2 - a(-3) + b$$
$$= 0$$
$$-27 - 18 + 3a + b = 0$$
$$-45 + 3a + b = 0$$
$$\therefore \quad 3a + b = 45.$$

Also, as $x = 1$ is a root, $\quad P(1) = 0$.

$$\therefore \quad P(1) = (1)^3 - 2(1)^2 - a(1) + b = 0$$
$$1 - 2 - a + b = 0$$
$$-1 - a + b = 0$$
$$\therefore \quad a - b = -1$$

$$\therefore \quad 3a + b = 45 \qquad (1)$$
$$a - b = -1 \qquad (2)$$

$(1) + (2)$

$$4a = 44$$
$$a = 11$$

Substitute $a = 11$ in (2)

$$11 - b = -1$$
$$-b = -1 - 11$$
$$-b = -12$$
$$\therefore \quad b = 12,$$

$\therefore$ $a = 11$, $b = 12$,

$\therefore$ $P(x) = x^3 - 2x^2 - 11x + 12$.

Now as $x = -3$, $x = 1$ are roots,

$\therefore$ $(x+3)$, $(x-1)$ are factors,

$\therefore$ $(x+3)(x-1)$ is also a factor,

$\therefore$ $(x^2 + 2x - 3)$ is a factor,

$$\begin{array}{r} x - 4 \\ \therefore\ x^2 + 2x - 3\overline{)x^3 - 2x^2 - 11x + 12} \\ \underline{x^3 + 2x^2 - 3x} \\ -4x^2 - 8x + 12 \\ \underline{-4x^2 - 8x + 12} \\ 0 \end{array}$$

$$\therefore \quad x^3 - 2x^2 - 11x + 12$$
$$= (x^2 + 2x - 3)(x - 4)$$
$$= (x+3)(x-1)(x-4),$$

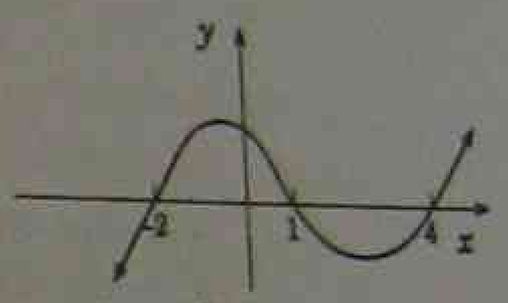
$\therefore$ as $P(x) = 0$

$(x+3)(x-1)(x-4) = 0$,

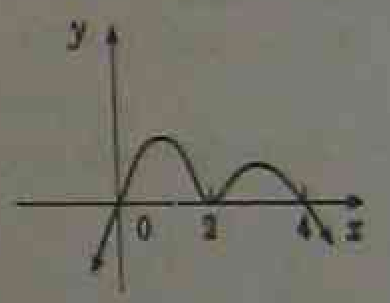
$\therefore$ $x = -3,\ 1,\ 4$

$\therefore$ the other root is $x = 4$.

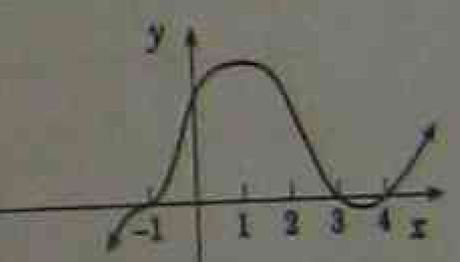
17. (a) $y = (x-4)(x-1)(x+2)$



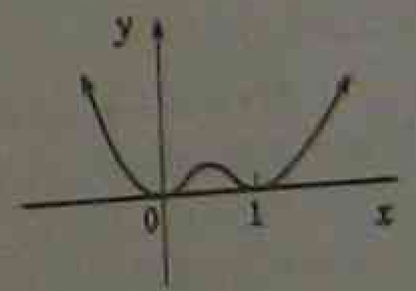
(b) $y = x(x-2)^2(4-x)$



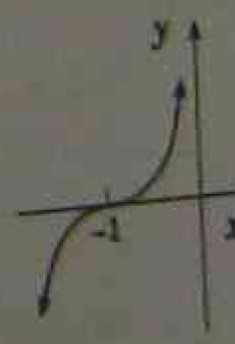
(c) $y = (3-x)(x+1)^3(4-x)$



(d) $y = x^2(1-x)^2$



(e) $y = (1+x)^3$



18. (a) $y = x^3 - 4x^2 + x + 6$

Let $y = P(x)$

$\therefore$ $P(x) = x^3 - 4x^2 + x + 6$.

Try $P(1) = 1^3 - 4(1)^2 + 1 + 6$

$$= 1 - 4 + 1 + 6 \neq 0$$

Continued

$$\begin{aligned} P(-1) &= (-1)^3 - 4(-1)^2 - 1 + 6 \\ &= -1 - 4 - 1 + 6 = 0 \end{aligned}$$

$\therefore$ as $P(-1) = 0$, $x + 1$ is a factor,

$$\therefore \begin{array}{r} x^2 - 5x + 6 \\ x+1\overline{\big)\,x^3 - 4x^2 + x + 6} \\ \underline{x^3 + x^2} \\ -5x^2 + x \\ \underline{-5x^2 - 5x} \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$

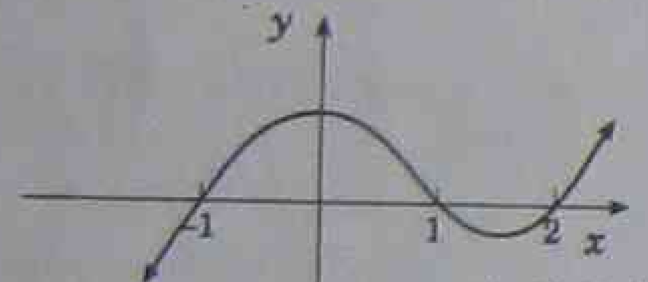
$$\begin{aligned} \therefore \quad P(x) &= x^3 - 4x^2 + x + 6 \\ &= (x+1)(x^2 - 5x + 6) \\ &= (x+1)(x-3)(x-2) \\ \therefore \quad y &= (x+1)(x-3)(x-2) \end{aligned}$$

Substitute $x = 4$ in y,

that is, $y(4) = (4+1)(4-3)(4-1)$

$= (5)(1)(3) > 0$

$\therefore$ above the x-axis at $x = 4$.



zeros at −1, 1, 2

(b) $y = x^3 - x^2 - x + 1.$

Let $y = P(x)$

$\therefore \quad P(x) = x^3 - x^2 - x + 1.$

Try $P(1) = 1^3 - 1^2 - 1 + 1$

$= 0$

$\therefore$ as $P(1) = 0$, $x - 1$ is a factor,

$$\therefore \begin{array}{r} x^2 - 1 \\ x-1\overline{\big)\,x^3 - x^2 - x + 1} \\ \underline{x^3 - x^2} \\ -x + 1 \\ \underline{-x + 1} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \quad P(x) &= x^3 - x^2 - x + 1 \\ &= (x-1)(x^2 - 1) \\ &= (x-1)(x-1)(x+1) \\ &= (x-1)^2(x+1) \end{aligned}$$

$\therefore \quad y = (x-1)^2(x+1).$

Substitute $x = 2$ in y,

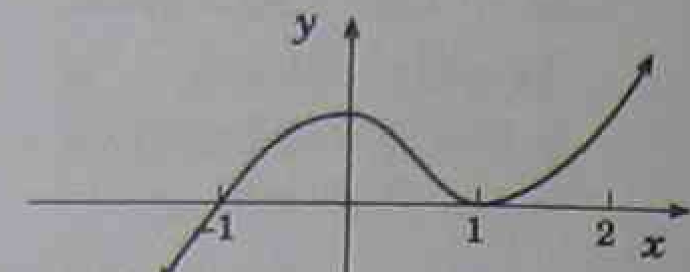
that is, $y(2) = (2-1)^2(2+1)$

$= (1)^2(3) > 0,$

$\therefore$ above x-axis at $x = 4$,

that is, $y > 0$ when $x = 4$,

(that is, $y > 0$ for $x > 1$).



zeros at ±1

(c) $y = x^3 - 3x^2 + 3x - 1$

Let $y = P(x)$

$\therefore \quad P(x) = x^3 - 3x^2 + 3x - 1.$

Try $P(1) = 1^3 - 3(1)^2 + 3(1) - 1$

$= 1 - 3 + 3 - 1$

$= 0$

$\therefore$ as $P(1) = 0$, $(x - 1)$ is a factor,

$$\therefore \begin{array}{r} x^2 - 2x + 1 \\ x-1\overline{\big)\,x^3 - 3x^2 + 3x - 1} \\ \underline{x^3 - x^2} \\ -2x^2 + 3x \\ \underline{-2x^2 + 2x} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \text{ as } P(x) &= x^3 - 3x^2 + 3x - 1 \\ &= (x-1)(x^2 - 2x + 1) \\ &= (x-1)(x-1)(x-1) \\ &= (x-1)^3 \end{aligned}$$

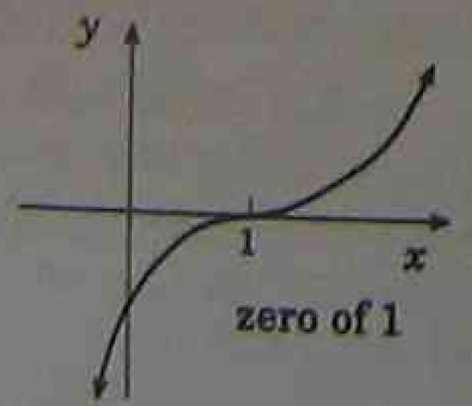
$\therefore \quad y = (x-1)^3$

Substitute $x = 2$ in y, that is,

$y(2) = (2-1)^3 = 1^3 > 0,$

$\therefore$ above x-axis at $x = 2$

$\therefore$ $y > 0$ when $x > 1$.



zero of 1

(d) $y = -x^3 + 12x + 16$

Let $y = P(x)$

$\therefore \quad P(x) = -x^3 + 12x + 16.$

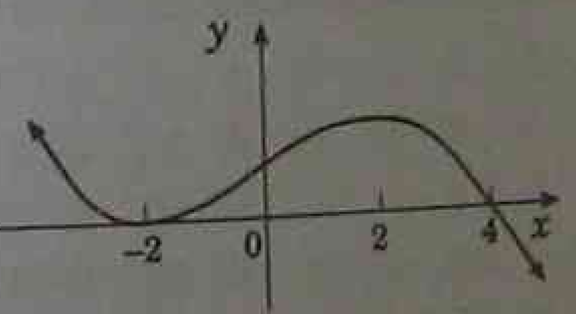
$$\begin{aligned} \text{Try } P(1) &= -1^3 + 12(1) + 16 \\ &\neq 0 \\ P(-1) &= -(-1)^3 + 12(-1) + 16 \\ &\neq 0 \\ P(2) &= -2^3 + 12(2) + 16 \\ &= -8 + 24 + 16 \\ &\neq 0 \\ P(-2) &= -(-2)^3 + 12(-2) + 16 \\ &= 8 - 24 + 16 \\ &= 0, \end{aligned}$$

$\therefore$ as $P(-2) = 0$, $(x + 2)$ is a factor,

$$\therefore \begin{array}{r} -x^2 + 2x + 8 \\ x+2\overline{\big)\,-x^3 + 0x^2 + 12x + 16} \\ \underline{-x^3 - 2x^2} \\ 2x^2 + 12x \\ \underline{2x^2 + 4x} \\ 8x + 16 \\ \underline{8x + 16} \\ 0 \end{array}$$

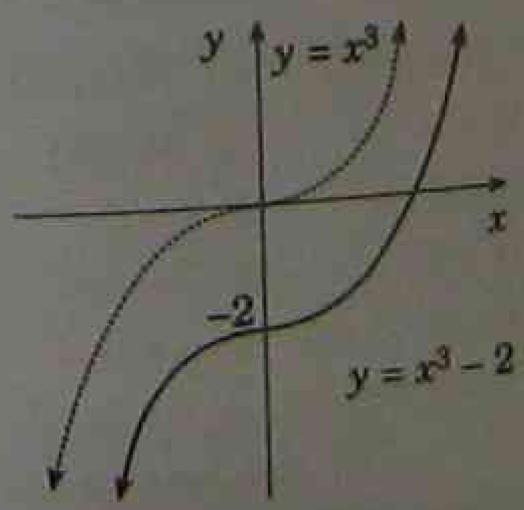
$$\begin{aligned} \therefore \text{ as } P(x) &= -x^3 + 12x + 16 \\ &= (x+2)(-x^2 + 2x + 8) \\ &= -(x+2)(x^2 - 2x - 8) \\ \therefore \quad y &= -(x+2)(x+2)(x-4) \\ &= -(x+2)^2(x-4) \end{aligned}$$

$\therefore$ zeros at -2, 4

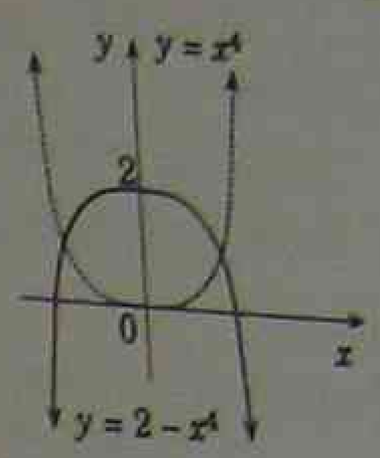


19. (a) (iii) (b) (ii)
(c) (iv) (d) (i)

20. (a)

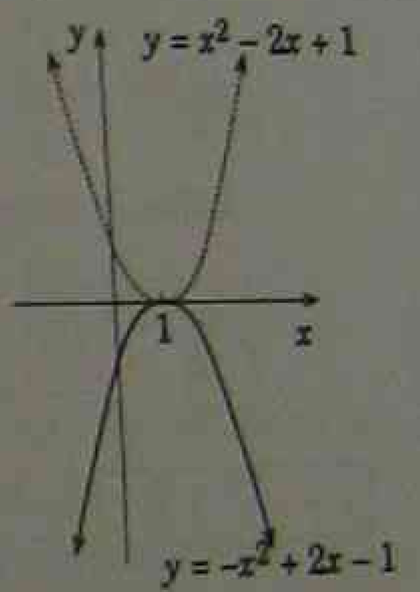


(b)

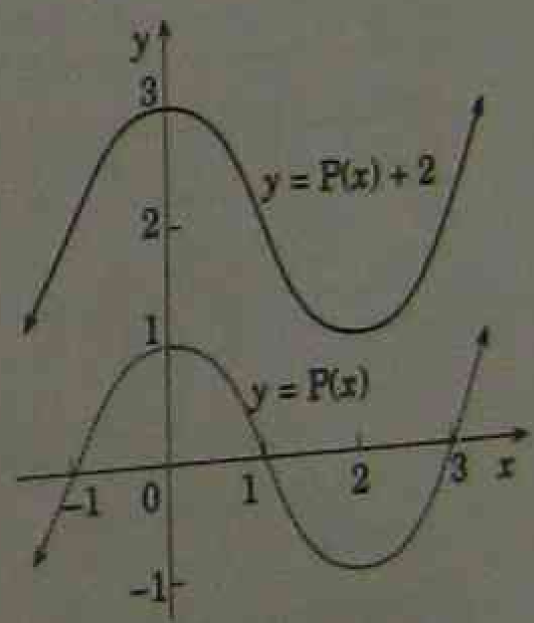


(c)

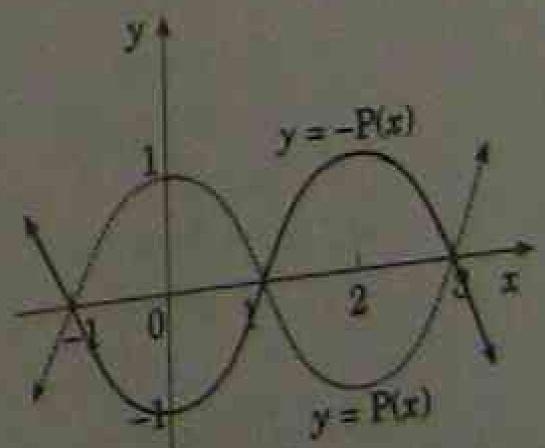
$$\begin{aligned} & y = x^2 - 2x + 1 \\ \therefore \quad & y = (x-1)^2 \\ & y = -x^2 + 2x - 1 \\ \therefore \quad & y = -(x^2 - 2x + 1) \\ & = -(x-1)^2 \end{aligned}$$



21 (a)



(b)



22. (a) $x^2+(y+2)^2=9$

$\therefore$ In form

$(x-h)^2+(y-k)^2=r^2$

$\therefore$ centre $(0,-2)$, radius $=3$ units.

(b) $x^2-4x+y^2+12y=16$

$x^2-4x+4+y^2+12y+36$

$=16+4+36$

$(x-2)^2+(y+6)^2=56$

$\therefore$ centre $(2,-6)$,

radius $=\sqrt{56}$ units.

(c) $x^2-3x+y^2=6$

$\therefore\quad x^2-3x+\left(\frac{3}{2}\right)^2+y^2=6+\left(\frac{3}{2}\right)^2$

$\therefore\quad \left(x-\frac{3}{2}\right)^2+y^2=6+\frac{9}{4}$

that is, $\left(x-\frac{3}{2}\right)^2+y^2=8\frac{1}{4}$

$\therefore$ centre $\left(\frac{3}{2},0\right)$,

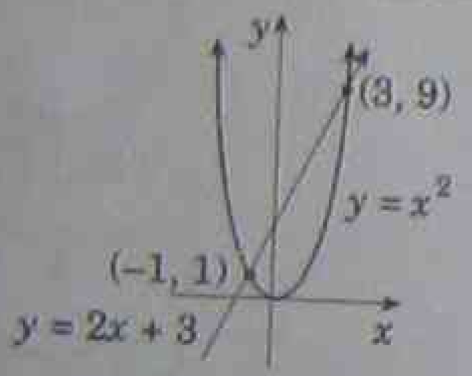
radius $=\sqrt{\frac{33}{4}}=\frac{\sqrt{33}}{2}$ units.

23. (a) $y=2x+3$

x	0	1	2
y	3	5	7

$y=x^2$

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



[Remember, as $y=2x+3$ ...(1)
and $y=x^2$ ...(2)

$\therefore\quad (1)=(2)$

$\therefore\quad x^2=2x+3$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

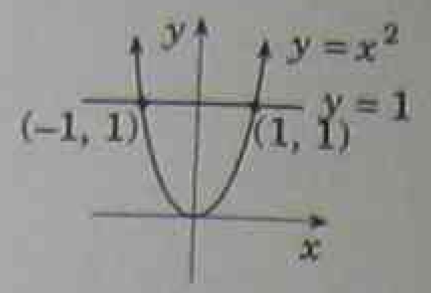
$x=3,\ -1$

and away we go!]

$\therefore$ (−1, 1) and (3, 9)

(b) $y=1$

$y=x^2$



$\therefore$ (−1,1) and (1,1).

24. From $x^3=x^2+2x-1$

$\therefore\quad y=x^3$

$y=x^2+2x-1$

$\therefore$ parabola is $y=x^2+2x-1$.

25. From $x^2-x-7=0$

$\therefore\quad x^2=x+7$

$\therefore\quad y=x^2$

$y=x+7$

$\therefore$ the straight line is $y=x+7$.

Chapter 15 Practice Paper
(page 235)

Part A

1. B 2. A 3. A 4. A 5. B 6. D
7. A 8. B 9. A 10. D 11. C 12. A
13. C 14. C 15. C 16. A 17. A 18. B
19. B 20. D 21. B 22. B 23. C 24. A
25. C 26. A 27. C 28. B 29. B 30. C

Part B

31. (a) $\sqrt{50}$ units (b) 1:06 p.m.
(c) $x=2$ (d) \$84.60
(e) \$827.86 (f) $t=6$
(g) $x=5$ (h) 13 km
(i) $x+y-3=0$ (j) $x=0,2$
(k) \$195 833.33 (l) 7.125 mL
(m) 1 (n) Ask teacher (45°)
(o) 7 and −1, 8 and −2, etc.

32. (a)

Let length $=y$

$\therefore\quad y+2x=80$

$y=80-2x$

$\therefore$ length is $(80-2x)$ m.

(b) $80-2x=50$

$\therefore\quad 2x=30$

$x=15$

$\therefore$ width is 15 m.

Area $=50\times15$

$=750$

$\therefore$ area is 750 m^2.

33. (a) 16 games

(b) $\frac{2\times0+3\times1+5\times2+4\times3+2\times4}{16}$

$=\frac{33}{16}=2\frac{1}{16}$

(c) $\frac{2}{3}\times33=22$

$\therefore$ 22 goals

34. (a) $h\propto\frac{k}{r^2}$

$\therefore\quad h=\frac{k}{r^2}$

(b) $h=14,\ r=4$

$\therefore\quad 14=\frac{k}{4^2}$

$14=\frac{k}{16}$

$\therefore\quad k=16\times14$

$=224$

$\therefore\quad h=\frac{224}{r^2}$

Now, substitute $r=5$

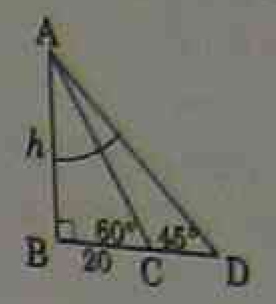
$\therefore\quad h=\frac{224}{5^2}$

$=\frac{224}{25}$

$=8.96$

$\therefore$ height of 8.96 cm.

35.



(a) $\frac{h}{20}=\tan 60$

$\therefore\quad h=20\tan 60$

$=34.64$

$\therefore$ height of 34.64 m.

(b) As ΔABD is isosceles

$\therefore$ BD $=34.64$

$\therefore$ CD $=34.64-20$

$=14.64$

that is, CD is 14.64 m.

36. (a) Let $h=0$

$\therefore\quad 32+4x-x^2=0$

That is, $x^2-4x-32=0$

$(x-8)(x+4)=0$

$\therefore\quad x=8,-4$

$\therefore$ B is 8 metres from the cliff.

(b) Using axis of symmetry formula:

$x=\frac{-b}{2a}$

$=\frac{-4}{2(-1)}$

$=2$

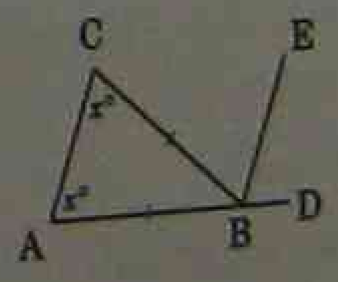
Substitute $x=2$ in h

$\therefore\quad h=32+4(2)-2^2$

$=36$

$\therefore$ maximum height is 36 m.

37.



Let $\angle CAB = x^\circ$

$\therefore$ $\angle ACB = x^\circ$

[base $\angle$'s of isosceles Δ equal]

$\therefore$ $\angle CBD = 2x^\circ$ [ext $\angle$ of Δ equals sum of 2 int. $\angle$'s.]

But $\angle CBE = \angle EBD$

[BE bisects $\angle CBD$]

$\therefore$ $\angle CBE = \angle EBD = x^\circ$

$\therefore$ $\angle CAB = \angle EBD$

$\therefore$ $AC \parallel BE$

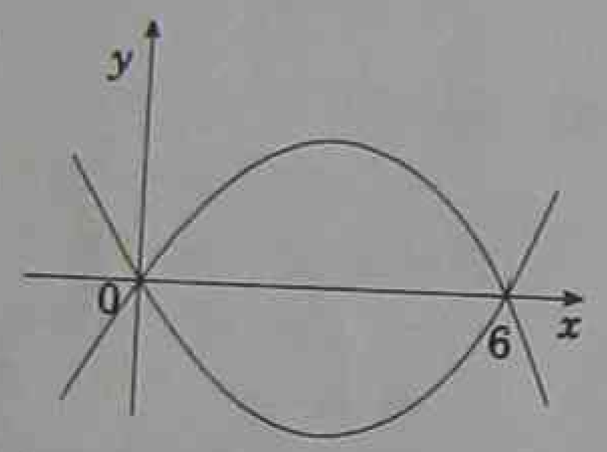
(corresponding angles equal)

38. (a) 10 to 80 $\therefore$ range = 70

(b) 30

(c) Both classes have identical lower and upper quartiles but 10J has higher median. Hence top half of students did better.

39. (a)



Any parabola with equation of form $y = ax(x-6)$, for example, $y = x(x-6)$, $y = -x(x-6)$.

(b) minimum value occurs at $x = 3$

$\therefore$ $(3, -18)$ lies on parabola.

$\therefore$ substitute in $y = ax(x-6)$

$\therefore$ $-18 = 3a(3-6)$

$\therefore$ $-18 = -9a$

$a = 2$

$\therefore$ $y = 2x(x-6)$

40. (a) Graph 2

(b)

$$V = \pi r^2 h + \tfrac{1}{3}\pi r^2 h$$

$$= \pi \times r^2 \times r + \tfrac{1}{3} \times \pi \times r^2 \times r$$

$$= \pi r^3 + \tfrac{1}{3}\pi r^3$$

$$= \tfrac{4}{3}\pi r^3$$

$\therefore$ volume is $\frac{4}{3}\pi r^3$ cm^3.

(c) Now, volume of sphere

$$= \tfrac{4}{3}\pi r^3$$

$\therefore$ volume is $\frac{4}{3}\pi r^3$ cm^3.

As both volumes can be found using the formula, r can take any positive value (radius cannot be negative), that is $r > 0$. $r = 0$ makes no sense.

41. (a) Substitute $x = 3$ in

$x^2 + y^2 = 25$

$\therefore$ $9 + y^2 = 25$

$\therefore$ $y^2 = 16$

$y = \pm 4$

$\therefore$ obviously $y = 4$

$\therefore$ $a = 4$ $\therefore$ $(3, 4)$

(b) Substitute $(3, 4)$ in

$4x + 3y + k = 0$

that is, $4(3) + 3(4) + k = 0$

$12 + 12 + k = 0$

$k = -24$

$\therefore$ $4x + 3y - 24 = 0$ is the equation of line.

(c) x: substitute $y = 0$

$\therefore$ $4x - 24 = 0$

$x = 6$

y: substitute $x = 0$

$\therefore$ $3y - 24 = 0$

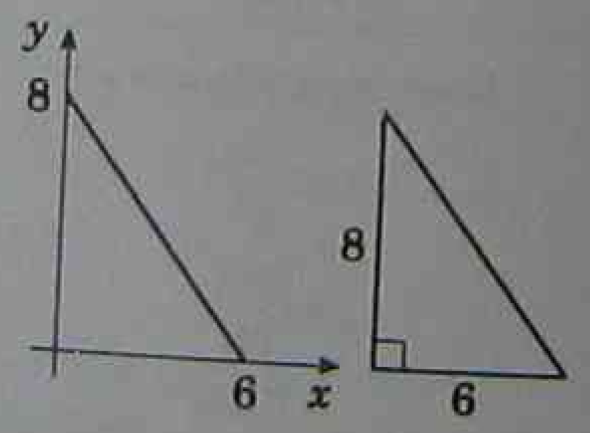
$y = 8$

$\therefore$ Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 8$

$= 24$

$\therefore$ area is 24 units^2.



Essential Skills

Excel
Essential Skills

Years 9 & 10 Advanced Mathematics

In this book you will find:

- ✓ Over 200 pages of practice exercises
- ✓ Comprehensive summaries of core topics
- ✓ Worked examples of each type of question
- ✓ Fully worked solutions for most problems
- ✓ A practice exam paper

25-

This guide is essential for all students who want to Excel in Years 9 and 10 Advanced Maths. The topic summaries include important formulae and principles, and the examples and worked solutions show the different methods used to answer each type of question. This book will help students get a more secure grasp of Advanced Maths.

About the authors

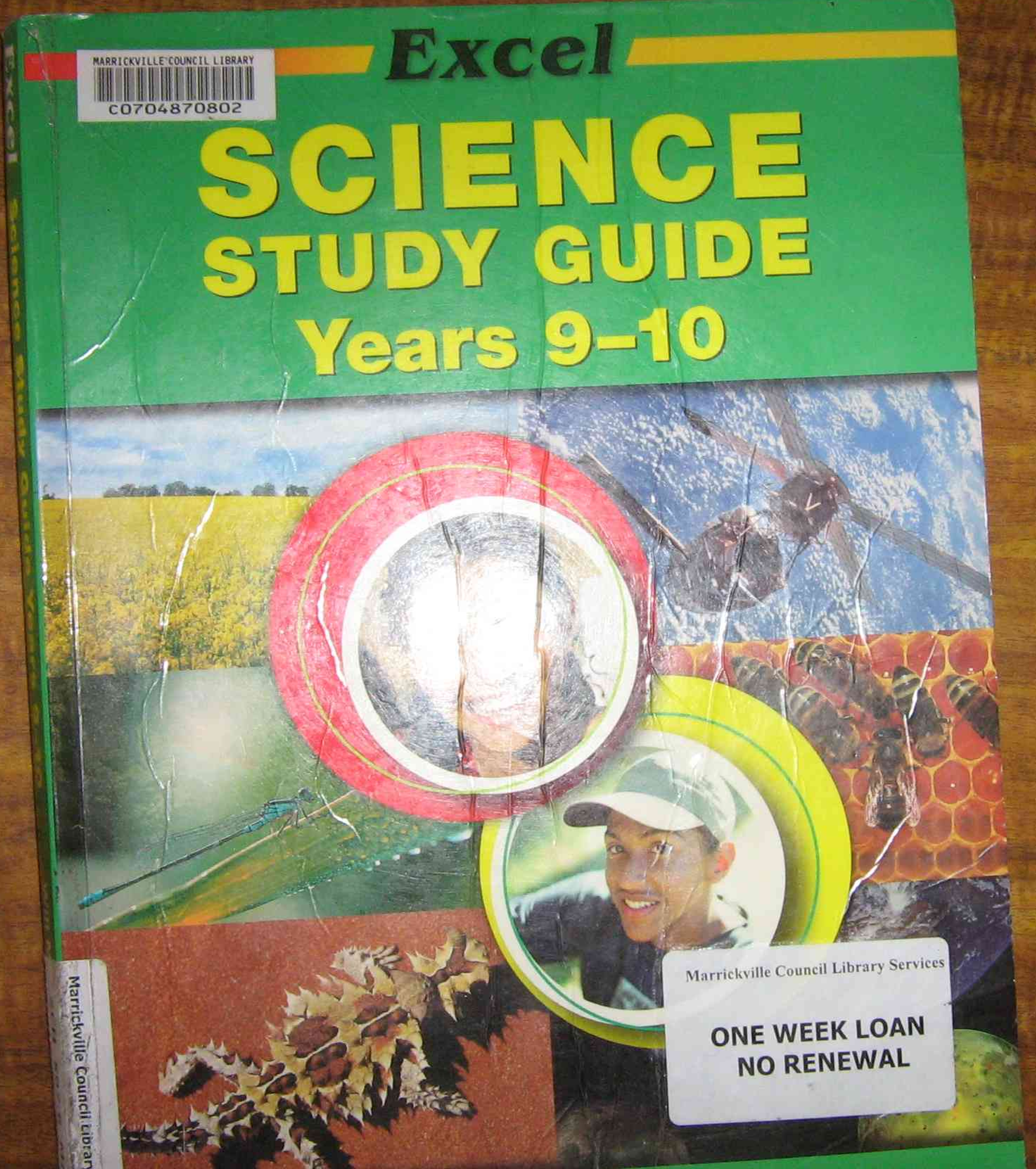
Both authors are experienced classroom teachers, and have written various successful mathematics books, including the Excel Mathematics Study Guides for Years 7 – 10.

Your own **checklist** for books in the Excel series for **Years 9 & 10 students**

Bookseller reference	BOOKS	LEVEL	✓
	English books:		
1-74020-038-1	Excel Essential Skills English Workbook	Year 9	
1-74020-039-X	Excel Essential Skills English Workbook	Year 10	
1-74125-001-3	Excel Essential Skills All You Need to Know About Grammar	Years 7–10	
1-74020-312-7	Excel Essential Skills Essay Writing Step-by-Step	Years 7–10	
	Mathematics books:		
1-74020-034-9	Excel Essential Skills Advanced Mathematics Revision & Exam Workbook	Year 9	
1-74020-035-7	Excel Essential Skills Advanced Mathematics Revision & Exam Workbook	Year 10	
1-86441-299-2	Excel Intermediate Mathematics Study Guide	Years 9-10	
1-74125-061-7	Excel Revise in a Month School Certificate Mathematics	Year 10	
	Science books:		
1-74020-080-2	Excel Essential Skills Science Revision Workbook	Year 9	
1-74020-081-0	Excel Essential Skills Science Revision Workbook	Year 10	
1-74125-063-3	Excel Revise in a Month School Certificate Science	Year 10	
	Other titles:		
1-877085-16-2	Excel School Certificate Australian History, Civics and Citizenship	Year 10	
1-74125-008-0	Excel School Certificate Australian Geography, Civics and Citizenship	Year 10	
1-74125-062-5	Excel Revise in a Month School Certificate Australian History, Geography, Civics & Citizenship	Year 10	

ISBN 1-87531-295-1
9 781875 312955

- Students: visit our website for a complete list of books at **www.pascalpress.com.au**
- Our address is **Pascal Press, PO Box 250, Glebe NSW 2037, ph (02) 8585 4044**



Excel

SCIENCE STUDY GUIDE

Years 9–10

Excel

SCIENCE STUDY GUIDE

YEARS 9–10

Will Marchment
BSc DipEd

PASCAL
PRESS

ISBN 1 74125 005 6

Pascal Press
PO Box 250
Glebe NSW 2037
(02) 8585 4044
www.pascalpress.com.au

Publisher: Vivienne Petris Joannou
Edited by Ken Tate and Christine Eslick
Page design by Larissa Petryca, id360
Typesetting and diagrams by Replika Press Pvt Ltd. (India)
Cover by DiZign
Printed in Singapore by Green Giant Press

Contents

Using this book vi

Chapter 1
Energy, Force and Motion 1

The wave model 1
Glossary 1
Mechanical waves 1
Transverse waves 1
Compression waves 3
Electromagnetic waves 4
Test yourself 5

Newton's laws of motion 7
Glossary 7
Force, mass and acceleration 8
Distance, speed and time 9
Acceleration 10
Analysing motion 11
Test yourself 12

Electrical energy 16
Glossary 16
Electrical circuits 16
Current, voltage and resistance 17
Series and parallel circuits 20
Test yourself 23

Mid-chapter test 26

Light energy 28
Glossary 28
Absorption and reflection 29
Refraction 30
Test yourself 31

Nuclear energy 34
Glossary 34
Energy release from the nucleus 34
Social and environmental issues 36
Test yourself 36

Gravitational force 38
Glossary 38
Mass and weight 38
Gravitational acceleration 38
Test yourself 39

End-chapter test 41
Summary 42

Chapter 2
Atoms, Elements and Compounds 44

Atomic theory 44
Glossary 44
Structure of the atom 44
Elements 45
Modelling subatomic structure 46
Test yourself 48

Elements 49
Glossary 49
Atoms and molecules 50
Periodic Table 50
Test yourself 51

Mid-chapter test 53

Compounds and reactions 54
Glossary 54
Compounds 55
Classification of compounds 56
Names and formulae of compounds 57
Reactions, observations and equations 59
Combustion reactions 61
Corrosion reactions 61
Precipitation reactions 62
Acids on metals and carbonates 62
Neutralisation reactions 64

The role of indicators 65
Decomposition reactions 67
Test yourself 67
End-chapter test 71
Summary 73

Chapter 3
Structure and Function of Living Things 75

Cell theory 75
Glossary 75
Systems serve the needs of cells 75
Cell division 76
Test yourself 77

The Watson–Crick model of DNA 79
Glossary 80
DNA—structure and replication 80
DNA mutations 81
Chromosomes and genes 81
Features of an organism 82
Test yourself 83
Mid-chapter test 85

The theory of evolution and natural selection 87
Glossary 87
Evidence for evolution 87
Natural selection 90
Test yourself 91

Humans 94
Glossary 94
Coordination systems in humans 95
Responses of body systems to disease 98
Human reproduction 101
Test yourself 104
End-chapter test 106
Summary 108

Chapter 4
Earth and Space 111

The Big Bang theory and components of the universe 111
Glossary 111
Origin of the universe 111
Electromagnetic spectrum and astronomy 113
Problems in obtaining information in astronomy 114
Major features of the universe 115
The life of stars 116
Test yourself 118
Mid-chapter test 121

Natural events 123
Glossary 123
Geological history 123
Fossilisation 126
The fossil record and geological time 127
Plate tectonics and continental drift 130
Earthquakes and vulcanism 132
Impact of natural events on the spheres of Earth 134
Test yourself 135
End-chapter test 138
Summary 140

Chapter 5 142
Ecosystems, Resources and Technology 142

Ecosystems 142
Glossary 142
Biotic and abiotic features of the local environment 142
Cycles of materials in ecosystems 144
Impact of human activities in ecosystems 146
Conservation of the environment 147
Test yourself 148

Energy resources and pollution 150
Glossary 150
Energy as a resource 150
Economically important natural energy resources 151
Pollution 151
The greenhouse effect 153
Test yourself 153
154

Technology 157
Glossary 157
Communications technology and electromagnetic radiation 157
Technology in the nuclear industry 158
Biotechnology 159
New materials (made resources) 160
Test yourself 161
End-chapter test 163
Summary 165

Chapter 6
Investigations and Problem Solving 166

Planning investigations 166
Glossary 166
Observations and the scientific method 166
Identifying data sources 166
Planning first-hand investigations and fair testing 167
Test yourself 169

Performing first-hand investigations 172
Designing an experimental procedure 172
Choosing equipment for experiments 173
Working individually or in teams 173
Assembling the apparatus and making observations safely 174
Test yourself 174

Gathering, processing, presenting and evaluating information 176
Gathering first-hand information 176
Gathering information from secondary sources 177
Processing information 179
Presenting information 180
Critical thinking strategies 182
Test yourself 182
End-chapter test 184
Summary 187

Chapter 7
Achieving High Levels in the School Certificate Test 188

Levels of achievement 188
Instruction words in the School Certificate Test 188
Test yourself 192

School Certificate Trial Revision Test 193

Answers
Chapter 1 204
Chapter 2 212
Chapter 3 216
Chapter 4 221
Chapter 5 225
Chapter 6 230
Chapter 7 233

School Certificate Trial Revision Test 235

Index 239

Periodic Table of the Elements . . . . Inside back cover

Using this book

What is in this book?

This study guide covers the complete Stage 5 NSW School Certificate Course in Science. It is an effective revision and study program for exams and class tests in Years 9 and 10, and will ensure that you are thoroughly prepared for the School Certificate Science examination.

- All the core knowledge content is covered in this book.
- All the required problem-solving skills are practised in this book.
- All aspects of the scientific method are treated in this study guide.
- Within each chapter there are many sets of revision questions for you to try.
- At the end of the book is a complete sample trial School Certificate examination paper.
- All revision questions and the trial SC exam have answers supplied.

The book is divided into sections based on the key areas of study in Science:

- Chapter 1—Physics
- Chapter 2—Chemistry
- Chapter 3—Biology
- Chapter 4—Earth and Space
- Chapter 5—Ecosystems, resources and technology
- Chapter 6—Investigations and problem solving

Preparing for class tests and examinations

Each topic in your school programs for Years 9 and 10 will have **integrated content** from more than one of these different key areas of study. It is unlikely that you will have studied the content in this order. To prepare for your school exams you will need to examine the **Contents** section at the front of the book and then identify the relevant sections of this book before you begin to revise and practise exam-style questions.

Preparing for the School Certificate examination

To prepare for the School Certificate examination you will need to review the content of the whole book. This process will need to start near the end of Term 3 of Year 10.

- You need to allow **6 weeks** for this thorough revision.
- Revise **one chapter each week** and attempt all questions. Use the supplied answers to identify the areas in which you need further work.
- At the end of the six weeks, attempt the trial School Certificate exam paper. Use the suggested answers to assess your mastery of the content and skills.

Answering multiple-choice questions

- Make sure you thoroughly read the stem of the question.
- Look at any diagrams, flow charts or tables and interpret them thoroughly.
- Choose the letter of the best response and not just a correct independent statement. If you do not know the answer, make the most logical choice you can.

Answering free-response questions

- Highlight the verbs and key words in the question. Ensure that you respond accordingly.

- Do not waste time by restating the question. Keep your answers concise.
- Make sure that any diagrams are fully labelled and drawn with pencil and ruler.
- Make sure that line graphs occupy more than 80% of the available grid space. All graphs have a caption and the axes should have linear scales and be appropriately labelled with titles and units.
- Make sure that any experimental methods are written as a series of numbered sentences in the present or past tense.
- Repeating the experiment at least five times or more can improve reliability of experimental results. Different groups can do these repetitions.
- For questions involving the scientific method, ensure that you state the dependent and independent variables, the variables you controlled (kept the same) and the experiment that you used as a control.
- When drawing a table of data, ensure that the table is fully bounded by lines to create columns and rows. The column headings and units should occupy the first row.

Chapter 1

Energy, Force and Motion

The wave model

Waves moving on the surface of the ocean or a lake are interesting to observe. Surfers often use these waves for recreation or sport. There are also other types of waves. The concept of waves and wave motion is examined in this section.

Glossary

Longitudinal wave—a compression wave in which the particles of the medium oscillate to-and-fro along the axis of energy transfer

Mechanical wave—a wave form in which the particles of the medium oscillate in order to transmit the energy

Medium—the material space in which a wave travels

Sonar—acronym for sound navigation and ranging; a technique in which reflected sound waves are used to measure distance

Spectrum—a range of frequencies or wavelengths

Vibration—an oscillation or shaking motion (a to-and-fro movement)

Mechanical waves

Waves can be classified into two categories:

- **mechanical waves**—waves requiring a medium in which to move
- **electromagnetic waves**—waves not requiring a medium in which to move.

In this section we examine examples of mechanical waves.

Ocean waves

Ocean waves are mechanical waves, as the water surface is the medium that transmits the energy.

ocean waves carry the energy

Ocean waves are formed by winds transferring kinetic energy to the surface of the water. This energy makes the water molecules **vibrate**. Some of this kinetic energy of vibration is transmitted to neighbouring water molecules, which also begin to move up and down.

- A **wave crest** begins to form where the particles move up relative to the normal undisturbed surface.
- When the particles move down, a **wave trough** is formed.

The crests and troughs move across the water surface because some of the energy of the vibrating water molecules is transferred away from the source of the original vibration.

We see the waves **travelling** across the water surface. They **carry energy** with them due to their motion.

Motion of particles in water waves

The waves on the water surface **carry energy but not matter away from the source.**

- The water particles move to-and-fro but **do not progress.**

If you watch a floating object in the ocean, it moves **up and down** as the wave passes by. The wave lifts it up and then it falls down as the wave passes. You can observe this using a cork floating in a bowl of water.

At the beach where **breaking waves** occur, the motion of the particles is different. Due to collisions of the vibrating water particles with the sand, the behaviour of the wave is altered and so there is some forward motion of the water particles. Surfboard riders use this phenomenon to ride breaking waves into the shore.

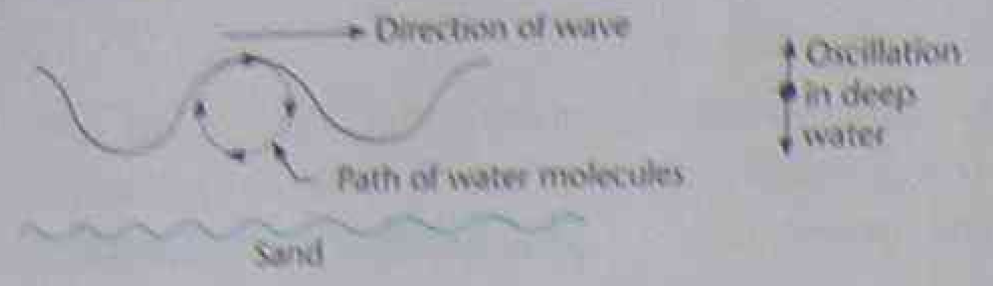


Figure 1.1 Motion of particles in a water wave near the shore

Sound waves

Sound is also an example of mechanical wave motion. Sound energy can travel through solids, liquids and gases.

- Sound waves in air are caused by **transfer of energy** from a vibrating source to the air molecules. For example, when a tuning fork is struck, the prongs vibrate and strike air molecules around them. The air particles also oscillate to-and-fro and energy radiates out from the source as a sound wave.
- Sound waves are different to water waves in that the particles of the air vibrate to-and-fro **in the direction of energy transfer** rather than at right angles to the energy transfer, as in water waves.
- As a result the air particles alternately bunch up to form **compressions** and then spread out to form **rarefactions**.

The sound wave thus consists of regions of compression and rarefaction. When the sound wave reaches our eardrum it causes the drum to vibrate and energy is then transferred to the inner ear and finally the brain.

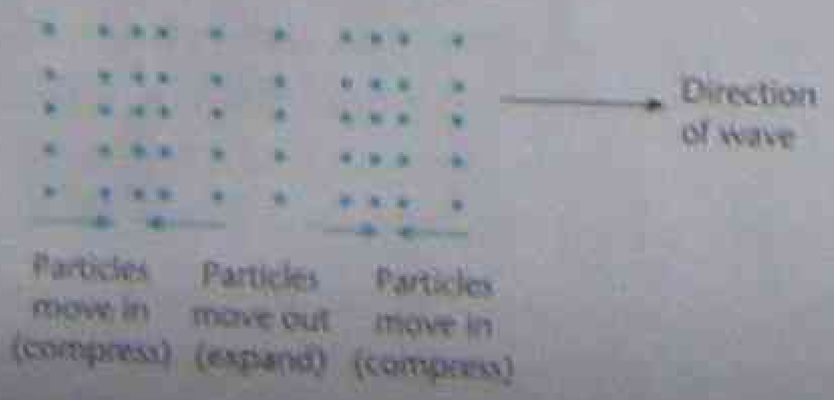


Figure 1.2 Motion of particles in a sound wave

Sound waves can be **reflected** from various surfaces. This property is used in a number of ways:

- **echo location**—animals such as dolphins and bats produce ultrasonic waves (~200 000 Hz) for navigation
- **sonar**—boats use ultrasonic waves to test water depth and detect schools of fish
- **medicine**—imaging of the fetus during pregnancy
- **seismic surveys**—using sound waves in Earth to determine its internal structure and to locate structures such as oil deposits.

Waves in strings and springs

Strings such as those in guitars and pianos can be made to vibrate by plucking or hitting them. As they vibrate, waves are formed in the string. Since the particles of the string vibrate at right angles to the string, the waves in the string disturb the air molecules around them, leading to the production of sound waves.

Mechanical waves can also be created in springs. If the spring is stretched and then one end is vibrated along the axis of the spring, then a series of compressions and rarefactions are observed to move down the spring from the source of the vibration. Vibrations can also be made at right angles to the spring axis, leading to travelling waves similar to water waves.

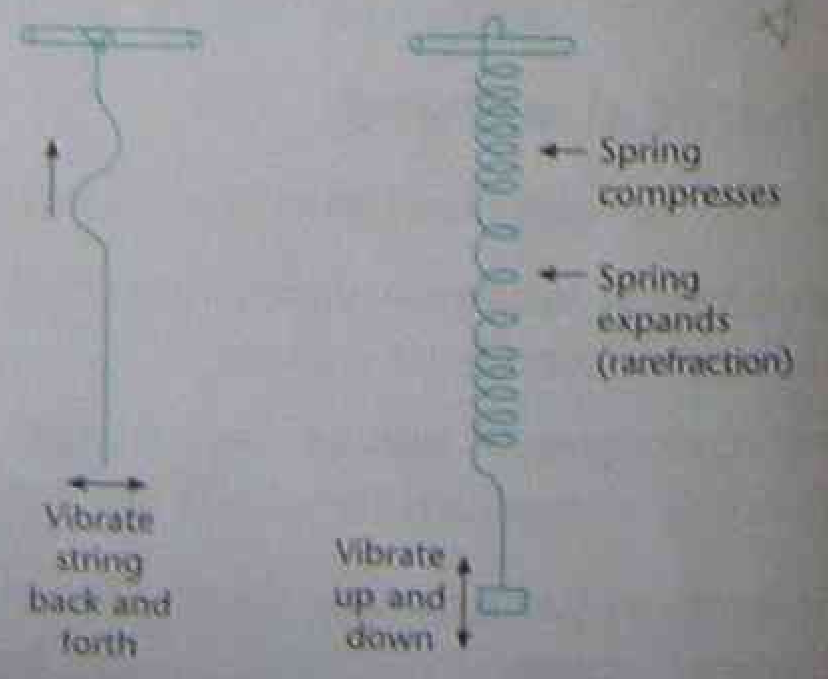


Figure 1.3 Waves in strings and springs

Transverse waves

Waves can be classified into two types:

- **transverse waves**—particles of the medium (or fields in electromagnetic waves) vibrate at right angles to the direction of wave motion
- **compression waves**—particles of the medium vibrate in the same direction as the wave motion. (These waves are also called longitudinal waves.)

A good example of a mechanical transverse wave is the wave in a vibrating string. Figure 1.4 shows how the particles in the string move as the wave moves along the string.

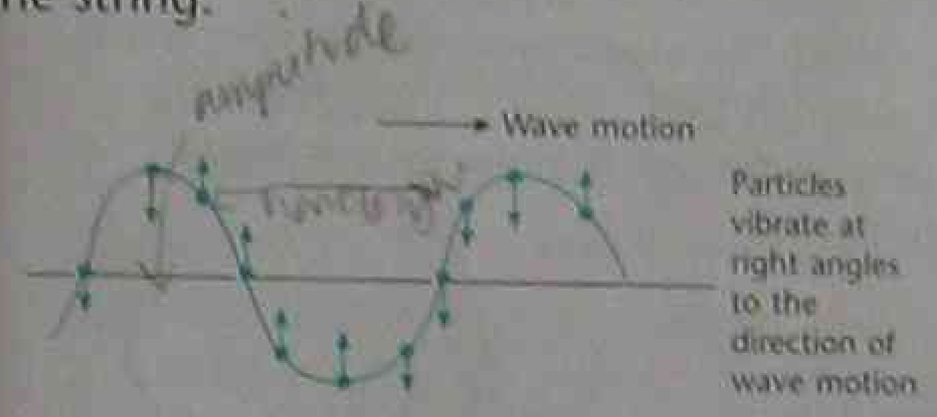


Figure 1.4 Motion of particles in a transverse wave

Some examples of transverse waves include:

- **water waves**
- **visible light waves**
- **radio waves**
- **vibrating strings**
- **surface earthquake waves.**

Mechanical transverse waves can only travel:

- **through solids**
- **over the surface of solids or liquids.**

Transverse waves can also be described in terms of a number of features:

- **wavelength** (λ)—the distance between two crests or two troughs (unit = metre)
- **amplitude** (A)—the height of a crest (or depth of a trough) measured from the midway point (central axis) (unit = metre)
- **frequency** (f)—the number of waves passing a fixed point in 1 second (unit = hertz, Hz)
- **period** (T)—the time for one complete wave to pass a fixed point (unit = second, s)
- **velocity (speed)** (v)—the distance moved by a crest (or trough) in 1 second (unit = metres per second, m/s).

Additional content—Mathematical extension:

The period (T) of a wave is the reciprocal of the frequency (f):

$$T = 1/f$$

The velocity of a wave is related to its wavelength and frequency via the **wave equation**.

Wave equation:

$$\textbf{velocity} = \textbf{frequency} \times \textbf{wavelength}$$

$$v = f\lambda$$

Figure 1.5 shows the features of a typical transverse wave.

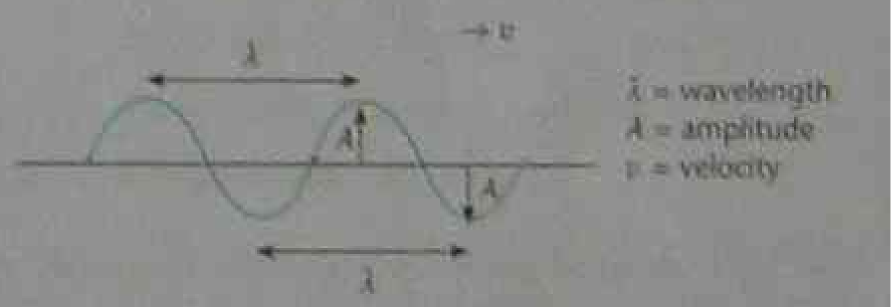


Figure 1.5 Features of a transverse wave

Compression waves

These waves are also called **longitudinal waves** as the vibration occurs along the axis of propagation of the energy.

Compression waves can travel through **all forms of matter.**

Examples of compression waves include:

- **sound waves**
- **longitudinal spring oscillations**
- **some earthquake waves (inside Earth).**

The features of a compression wave are the same as those of a transverse wave. Figure 1.6 illustrates the features of a compression wave.

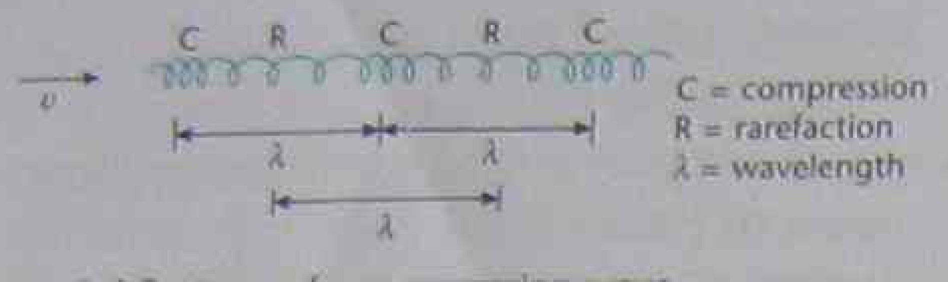


Figure 1.6 Features of a compression wave

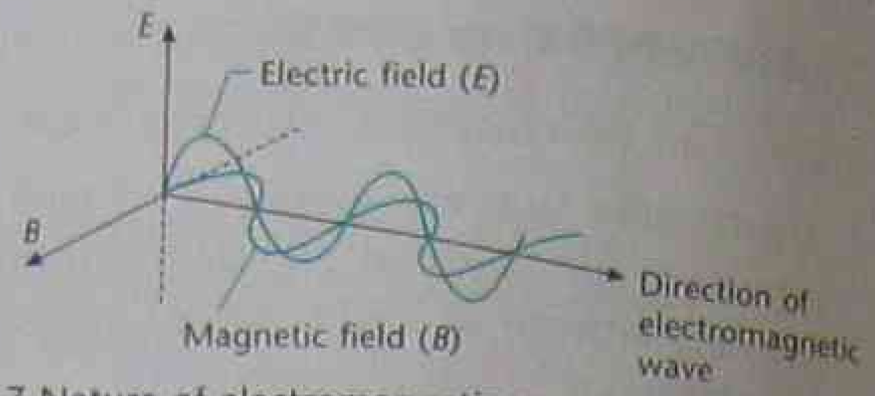


Figure 1.7 Nature of electromagnetic waves

Electromagnetic waves

Unlike a mechanical wave in which the particles in a medium are disturbed, electromagnetic rays **do not require a medium** for their propagation.

- Electromagnetic waves involve the propagation of **oscillating electric and magnetic fields**.
- Electromagnetic waves are **transverse** waves (Figure 1.7).
- Electromagnetic waves move at their highest velocity ($v = 3 \times 10^8$ m/s, the speed of light) in a vacuum. They travel slightly more slowly through matter (eg. air, glass).

Electromagnetic spectrum

Electromagnetic waves vary in their frequencies and wavelength even though their velocities are the same in a vaccum. The collection of different frequency waves is called the **electromagnetic spectrum**. Figure 1.8 and Table 1.1 illustrate the components of the electromagnetic spectrum.

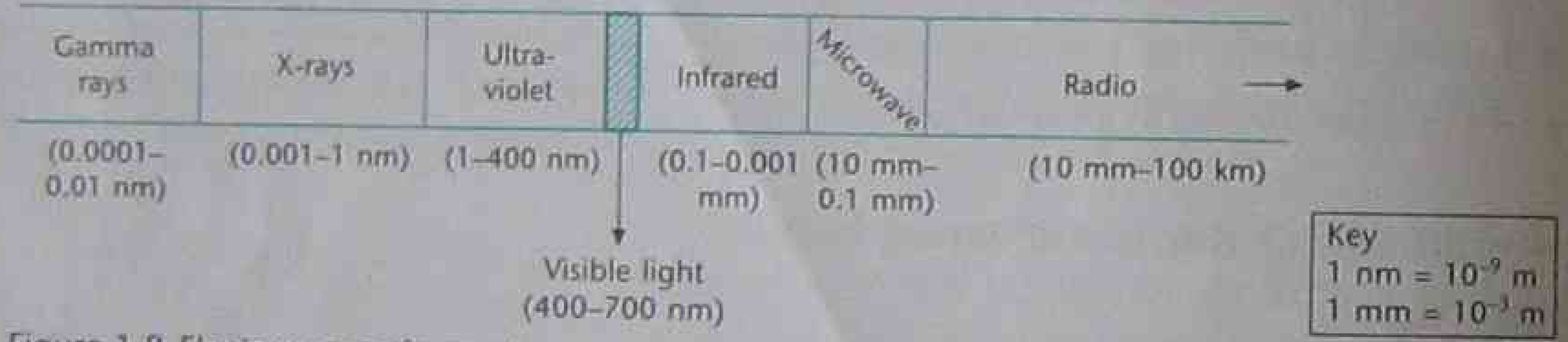


Figure 1.8 Electromagnetic spectrum

Table 1.1 Features of the electromagnetic spectrum

Band name	Wavelength band (approximate)	Sources of waves	Uses of waves
Radio/TV	100 km–10 mm	radio/TV transmitters	radio/TV communication, radio astronomy
Microwave	10 mm–0.1 mm	radar transmitters microwave ovens	satellite communication, cooking food
Infrared	0.1 mm–0.001 mm	electric radiators	heating rooms, medical heat treatments, night vision systems
Visible light	400 nm–700 nm	stars, electric lamps	human vision, photosynthesis, photography astronomy
Ultraviolet	400 nm–1 nm	UV lamps, stars	UV astronomy, sterilisation
X-rays	1 nm–0.001 nm	X-ray tubes, black holes	medical radiography (diagnosis and treatment), flaws in structural materials, X-ray astronomy
Gamma rays	0.01 nm–0.0001 nm	radioactive minerals	sterilisation, killing cancer cells

(1 nm = 1 nanometre = 1×10^{-9} m)

Test yourself (answers on pages 204–5)

Part A. Knowledge (answers on page 204)

1 Which of the following waves is **not** an example of a mechanical wave?
  [✓ a] Seismic wave
  b Gamma wave
  c Ultrasonic wave emitted by a dolphin
  d Vibrating violin string (1 mark)

2 Apollo astronauts on the surface of the Moon in the early 1970s communicated to each other using which type of waves?
  a Infrared rays and microwaves
  b Sound waves and ultraviolet rays
  [✓ c] Visible light rays and radio waves
  d High frequency sound waves and radio waves (1 mark)

3 Which statement is true of waves in deep ocean water?
  [✓ a] The water particles in surface water waves do not progress but simply oscillate up-and-down.
  b Water waves are examples of longitudinal waves.
  c Water particles oscillate along the axis of propagation of the wave.
  d Water waves have wavelengths similar in magnitude to infrared rays. (1 mark)

4 Water waves near a beach have a frequency of 0.4 Hz. Which statement is true about these waves?
  [✓ a] In one second the wave will travel a distance of 40 cm.
  b The distance between adjacent crests is 0.4 m.
  c These waves have similar frequencies to microwaves.
  d In ten seconds, four waves will pass a fixed point. (1 mark)

5 Select the statement that is true of infrared waves.
  a Infrared rays have much higher frequencies than radio waves.
  b Infrared rays are used in radiography for treating cancer.
  c Visible light rays travel at a much greater velocity than infrared rays in space.
  d Green plants utilise infrared rays for photosynthesis. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each)
  a A _____ wave is one in which the particles vibrate at right angles to the direction of wave propagation.
  b The height of a wave above the equilibrium level is called the _____.
  c Gamma waves have the highest frequency in the _____ spectrum.
  d Mechanical waves require a _____ in which to travel.
  e The region of a sound wave in which the particles of a medium are spread apart is called a _____.

7 Use the code letters to match the terms or phrases in each column. (1 mark each)

Column 1	Column 2
a trough	f distance between two troughs
b wavelength	g electromagnetic wave
c sound wave	h oscillation
d X-ray	i bottom of a wave
e vibration	j mechanical wave

8 The velocity of a sound wave is much greater in a solid such as a rock than it

is in air. Use particle theory to explain this observation. (2 marks)

9 Explain how two students could use a slinky spring to illustrate both transverse and longitudinal wave motion. (2 marks)

10 Describe the importance of electromagnetic waves in astronomy. (3 marks)

Part A. Skills (answers on pages 204–5)

1 Use the information in the scale diagram (Figure 1.9) to determine the wavelength of the transverse wave in metres. (2 marks)

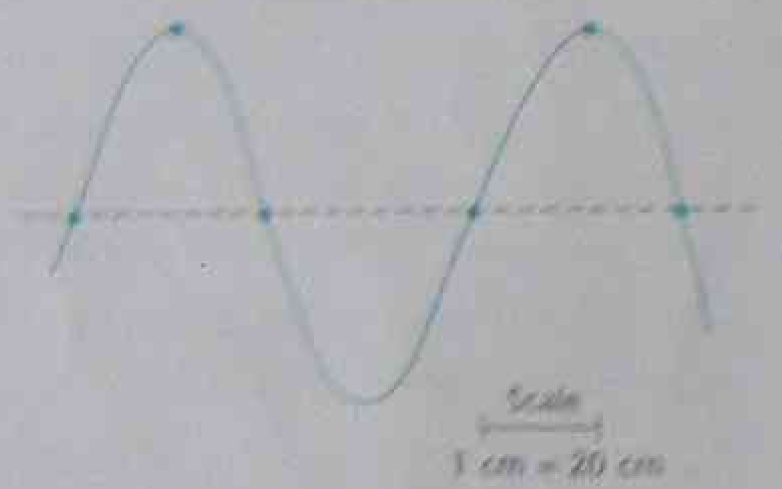


Figure 1.9 Scale diagram of transverse wave

2 The velocity of a sound wave in air was measured by experiment to be 320 m/s. The scale diagram (Figure 1.10) shows the positions of the air particles at a particular instant.

Figure 1.10 Scale diagram of a sound wave

- a Use the diagram to determine the wavelength of the sound wave in metres. (2 marks)
- b Calculate the frequency of this wave using the wave equation. (2 marks)

3 The following diagram (Figure 1.11) shows the screen of a cathode ray oscilloscope displaying two wave forms

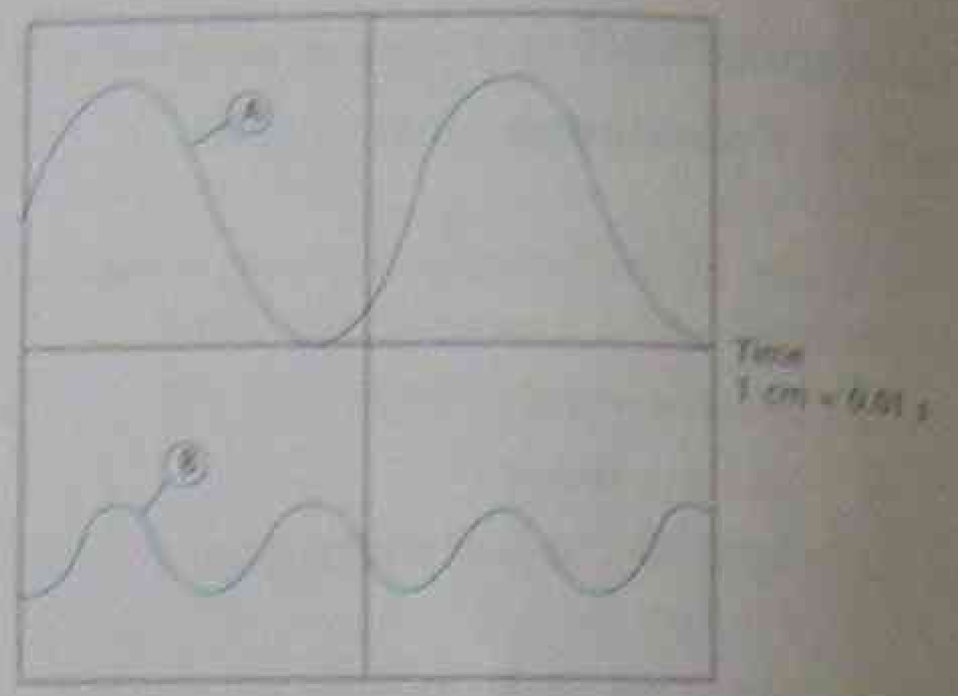


Figure 1.11 Waves A and B on a cathode ray oscilloscope

(A and B) produced by two different oscillators.

- a Which wave has a higher frequency? (1 mark)
- b Which wave has the higher amplitude? (1 mark)
- c Use the scale on the time axis (horizontal axis) to determine the period (*T*) and frequency (*f*) of each wave. (2 marks)

4 Two radio stations (A and B) transmit programs on 702 kHz and 954 kHz respectively in the AM band.

- a Which station transmits waves of greater wavelength? (1 mark)
- b A third station (C) transmits at 104.9 MHz in the FM band. How do the waves from this station compare with those transmitted from station A in terms of (i) wavelength? (ii) velocity? (2 marks)

5 The speed of sound waves is greater in solids than in liquids. The speed of sound is greater in liquids than in gases.

The following table (Table 1.2) provides data about a sound wave propagating from different frequency sound sources through three different materials (A, B and C). One material is a solid, one a liquid and one a gas. Which material is which? (2 marks)

Table 1.2 Propagation of sound in different states of matter

Material	Frequency (Hz)	Wavelength (m)
A	540	2.5
B	90	3.5
C	950	5.0

6 a The speed of sound was measured experimentally by measuring the time delay between producing the sound and detecting the sound echo produced by a wall 100 metres away from the source. The time delay was measured and found to be 0.59 seconds.

Use this information to determine the speed of sound in the air using the formula:

speed = distance/time (2 marks)

b The experiment was repeated on a very cold day and the time delay was found to be 0.64 seconds. What conclusion can be drawn about the speed of sound with increasing temperature? (2 marks)

7 The colours of the rainbow (red, orange, yellow, green, blue, indigo, violet) represent the spectrum of visible light. Table 1.3 lists typical wavelengths of three of these coloured waves.

Table 1.3 Visible spectrum

Colour	Typical wavelength (nm)
Red	700
Green	500
Violet	400

- a Which wave has the lowest frequency? (1 mark)
- b Predict the wavelength of a typical blue wave. (1 mark)
- c The velocity of these waves in a vacuum is 3×10^8 m/s. Calculate the frequency of the green wave. (2 marks)

8 The hearing and sound production ranges were measured for several cats and dogs. The results are in Table 1.4.

Table 1.4 Hearing range and sound production range for cats and dogs

Animal	Frequency range of hearing (Hz)	Frequency range of sound production (Hz)
Cat	60–65 000	700–1500
Dog	15–50 000	450–1000

- a Present this information in appropriate graphical formats for hearing and sound production. (3 marks)
- b Which of these animals can
  - i hear sounds of the higher pitch? (1 mark)
  - ii produce sounds of the higher pitch? (1 mark)
- c A bat emits a sound of frequency 90 000 Hz. Can a cat or dog hear such a sound? (1 mark)

Newton's laws of motion

The English scientist Isaac Newton (1642–1727) developed the laws of motion as well as theories of gravitation and light. He ranks as one of the world's greatest scientists. His laws of motion explain the effects of forces acting on bodies at rest or in motion.

Glossary

Acceleration—a type of motion in which the speed continues to increase (unit = m/s^2)

Deceleration—a type of motion in which the speed continues to decrease (unit = m/s^2)

Force—a push, pull or twist that changes the motion or shape of an object on which it acts (unit = newton, N)

Friction—a force that opposes motion when surfaces move over each other

Mass—the amount of matter in a body (unit = kg, g)

Velocity—a measure of speed in a fixed direction (unit = m/s)

Weight—a force acting on a body due to gravity (unit = newton, N)

Force, mass and acceleration

A force is a push or a pull that acts on a body.

The unit of force is the **newton (N)**. It is named in honour of Isaac Newton. Forces can be classified into two categories:

- **contact forces**—these are forces in which there is a direct contact between the force and the body (eg. the tensional force in a rope as it pulls an object along the ground; the frictional forces preventing an object sliding freely over a surface).
- **field forces**—these are non-contact forces in which a body experiences a force due to its presence in a field (such as a magnetic field, electric field or gravitational field). An iron nail is attracted to a bar magnet owing to the existence of the magnetic field around the magnet.

Balanced and unbalanced forces

In a tug-o-war the centre of the rope does not move if the pulling forces of each team are exactly equal (ie. balanced). This principle is true of all balanced forces. A computer sitting at rest on your desk is acted upon by a set of balanced forces. The force of gravity is pulling the computer downward. This is balanced by the table pushing equally upward on the base of the computer.

Balanced forces do not always imply that the object is stationary. Any object that is travelling at **constant velocity** is also acted upon by a set of balanced forces. What are these balanced forces?

- The force of gravity pulling the car down is balanced by the road pushing up on the car's tyres.
- The force of the engine driving the car forward is balanced by the sum total of all the frictional forces acting backward on the car.

However, for a car to **reach** a constant speed, it must experience an unbalanced force which accelerates it up to the desired speed. Once the desired speed is reached, the accelerator of the car is backed off a little so that the car can cruise under the action of balanced forces.

These observations about motion can be expressed by Newton's first law:

Newton's First Law of Motion: ***A body remains at rest or at constant velocity unless acted upon by an unbalanced force.***

Net (unbalanced) forces

A net force is the result of unbalanced forces. In order to make a car move from rest, a sufficient force must be applied to the road in order to overcome frictional forces that hinder movement. Let us examine a simple example.

Example

If a smooth rectangular block of metal (mass = 100 kg) is placed on a smooth, low friction surface (eg. ice) and pulled along by a rope with a constant net force (force = 200 N) the block gains speed as long as the net force acts on it. In this example the block experiences an acceleration of 2 m/s^2.

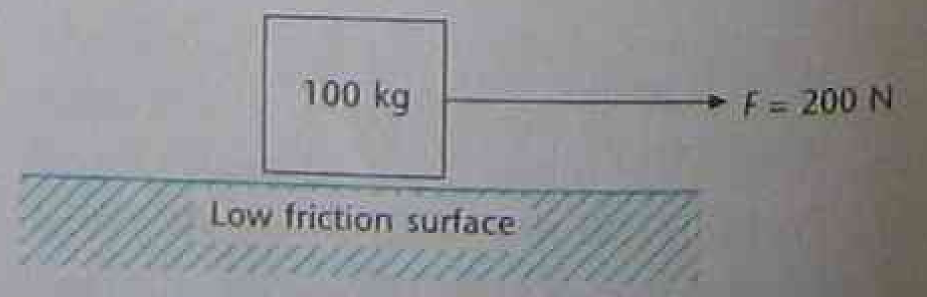


Figure 1.12 Net force acting on a metal block on a low friction surface

- As long as the net force is constant, the block will accelerate uniformly (ie. at 2 m/s^2).
- If the size of the pulling force is reduced (eg. reduced to 100 N) then the acceleration is also reduced proportionally (ie. reduced to 1 m/s^2).
- If the block of metal is replaced by a much heavier block (eg. 200 kg) and the experiment repeated with the same net force (ie. 200 N), the heavier block accelerates less than the lighter block. This lower acceleration is found to be 1 m/s^2.

The observations described above can be summarised using Newton's second law:

Newton's Second Law of Motion: ***The acceleration of a body depends directly on the size of the unbalanced force and inversely on the mass of the body*** (ie. the bigger the force the greater is the acceleration, the bigger the mass the smaller is the acceleration).

Additional content—Mathematical extension:

Another way of describing Newton's second law is in algebraic terms:

$$F = ma$$

force = mass x acceleration

Example:

Q If a net force of 1000 N acts on a 200 kg body, what is its acceleration?

A $F = 1000$ N

$m = 200$ kg

$F = ma$

$1000 = 200a$

$a = 5$ m/s^2

Action and reaction forces

Newton discovered that forces always occur **in pairs**. They are called action–reaction pairs. This idea is expressed in Newton's third law:

Newton's Third Law of Motion: ***To every action there is an equal and opposite reaction.***

Example 1. Ball fired from a cannon

A ball can be projected from a cannon by igniting gunpowder to produce an explosive force. This explosive force is the action force which acts on the ball. At the same time the ball exerts an equal and opposite reaction force on the cannon. Because a net force acts on the ball, it accelerates forward out of the cannon. Because of the reaction force acting backward on the cannon, the cannon recoils.

Example 2. Sprinting from rest in a 100 m race

The muscles in the runner's legs and feet exert an action force (backward) on the track. At the same time the track exerts an equal and opposite reaction force on the runner. Because a net force acts on the runner, he will accelerate out of the blocks and along the track. At each contact with the ground the same thing happens. Note that the action force applied to the track is actually applied to the whole Earth, which is so massive that its acceleration is essentially unobservable.

Distance, speed and time

The **average speed** of a moving body can be calculated by measuring the total distance travelled and the time taken to travel that distance.

Average speed = distance moved/time taken

Example

Q Calculate the average speed for a 150 km journey which takes 2 h 45 min.

2.75

A Distance = 150 km

Time = 2.75 h

Average speed = 150/2.75 = 54.6 km/h

54.5

The **speedometer** in a car does not measure the average speed. It measures the

instantaneous speed. This is the speed of the car at a particular moment of time. The instantaneous speed may be higher or lower than the average speed for a whole trip. When slowing down near traffic lights a car's instantaneous speed will drop below its average speed for the journey. Along an open section of road the car can travel above the average speed according to the speed limits of the road.

Velocity

it is useful to distinguish between the terms **speed** and **velocity**. Velocity is a term used by physicists to measure the change in motion of an object along a straight line in a particular **direction**. The speed of a body is independent of direction.

Example

Consider a straight road running east–west. A car travels along the road at a constant speed of 40 km/h. When it is travelling east, its velocity is said to be 40 km/h east. When it is travelling at the same speed west, its velocity is said to be 40 km/h west.

Acceleration

Whenever the velocity of an object changes by changing its speed and/or changing its direction, it is said to **accelerate**. The extent of this acceleration depends on the mass of the object and the size of the net force as described by Newton's second law.

The acceleration of an object often changes throughout its motion and so it is often useful to calculate the **average acceleration**.

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

Example

Q A car starts from rest and reaches a velocity of 15 m/s in 5 seconds. Calculate its average acceleration.

A Initial velocity = 0 m/s (ie. at rest).

Final velocity = 15 m/s.

Change in velocity

= final velocity – initial velocity

= 15 – 0 = 15 m/s.

Time taken = 5 s.

Average acceleration = 15/5 = 3 m/s^2.

Deceleration

If an object such as a car loses speed we say it is **decelerating**. This can happen when a motorist puts his foot on the brake.

Example

Q A car is travelling at 18 m/s, the brakes are applied and it comes to a stop in 4 seconds. Calculate its average deceleration.

A Initial velocity = 18 m/s.

Final velocity = 0 m/s (ie. at rest).

Change in velocity = final velocity – initial velocity = 0 – 18 = –18 m/s.

Time taken = 4 s.

Average acceleration = –18/4 = – 4.5 m/s^2.

The negative sign indicates that the acceleration is negative. That is, the car is decelerating with an average deceleration of 4.5 m/s^2.

Uniform acceleration

An object will undergo **uniform acceleration** if a constant net force acts on it. The size of this uniform acceleration can be calculated using Newton's second law or from a knowledge of the initial and final velocities and the time it takes to change the velocity.

Example

Q A 10 kg mass, initially moving along a flat surface at 10 m/s is uniformly accelerated for 3 seconds until its velocity is 16 m/s. Calculate its uniform acceleration.

A Initial velocity = 10 m/s.

Final velocity = 16 m/s.

Change in velocity = final velocity – initial velocity = 16 – 10 = 6 m/s.

Time taken = 3 s.

Uniform acceleration = 6/3 = 2 m/s^2.

Accelerating by changing *direction*

Acceleration can also be due to changes in direction rather than changes in speed. A car that is travelling at a constant velocity of 10 m/s north turns a corner and continues to travel at a constant velocity of 10 m/s towards the west. At all times its speed has stayed constant at 10 m/s but due to its change in direction the car must have experienced a net force. Consequently it must have accelerated to turn the corner. This example illustrates Newton's first and second laws. The first law tells us that the car will travel in a straight line at constant velocity unless acted upon by a force. As the car does not continue in a straight line, a net force has operated on the car. The second law tells us that net forces cause an object to accelerate (in this case to change direction).

Analysing motion

Analyse each of the following motion examples.

Example 1. Motion of a car along a straight road

A 1000 m straight road runs east–west. The most westerly point is labelled A and the most easterly point is labelled B. A car starts from rest at the mid-point (M) of the road AB and initially travels east. (Figure 1.13).

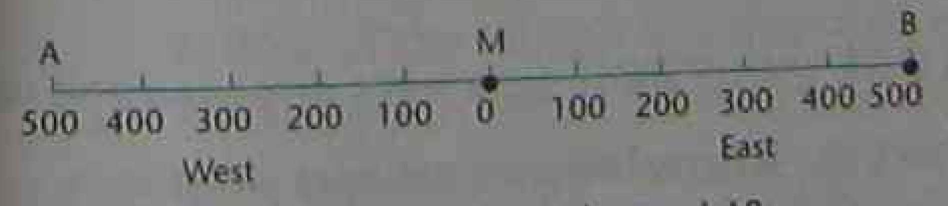


Figure 1.13 Motion of a car along the road AB

Table 1.5 shows the position of the car measured from the starting line (M) over a period of time.

Table 1.5 Data for the motion of a car along the road AB

Time (s)	Position of car relative to M (metres)
0	at M
10	100 east
20	200 east
30	300 east
40	500 east
50	500 east
60	300 east
70	100 east
80	100 west
90	200 west
100	250 west
110	250 west

Q Use the data in Table 1.5 to

a Plot the data points and join the points to form a line graph of position versus time.

b Discuss the motion of the car in relation to the shape of the graph.

A Answer

a Figure 1.14 shows the line graph of these data.

b Between 0 and 30 seconds the car is travelling east at a constant speed as

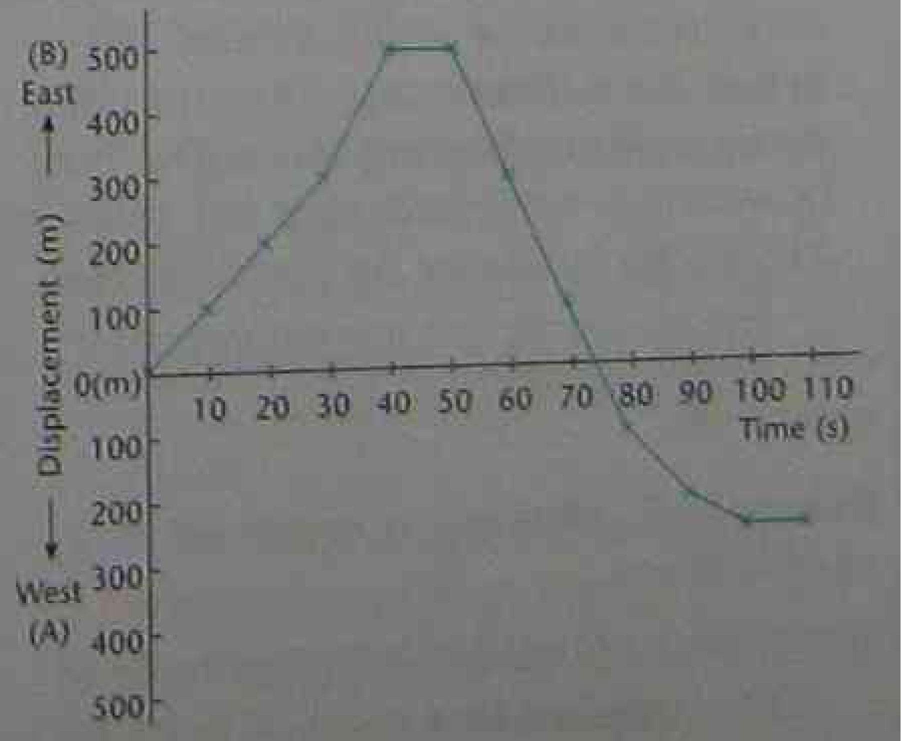


Figure 1.14 Line graph of data

it covers equal distances in equal time intervals (shown by the straight line in the graph).

After 30 seconds the car speeds up and travels at a higher speed in an easterly direction than before. By 40 seconds it has reached the end of the road at B (after 500 m). It remains stationary at B for 10 seconds.

Between 50 and 80 seconds after the start, the car travels west at a constant speed. It passes through M on the journey west.

Between 80 and 100 seconds the car gradually slows down until at 100 seconds it reaches a point 250 m west of M. It remains stationary at this point for 10 seconds.

Example 2. Emergency stop of a bus

Q Consider a bus containing seated and standing passengers that is forced to make a sudden emergency stop as a dog runs across the road. Explain what happens to the standing passengers in terms of Newton's first law.

A The standing passengers fall forward as the bus comes to a sudden stop. This is explained by Newton's first law which states that a body will continue in uniform motion unless acted upon by a net force. In this case the passengers are not attached to the bus and so continue on with the same speed as they did before the sudden stop. They do not experience the braking force. The bus, however, is subjected to a net force and its velocity suddenly decreases to zero.

Additional content—Mathematical extension:

Example 3. Moving a crate on a rough floor

Q Figure 1.15 shows an action force of 800 N applied to a crate of mass 80 kg which is resting on a rough floor. As the crate moves, the floor exerts a constant frictional force of 160 N on the crate. Calculate the acceleration of the crate.

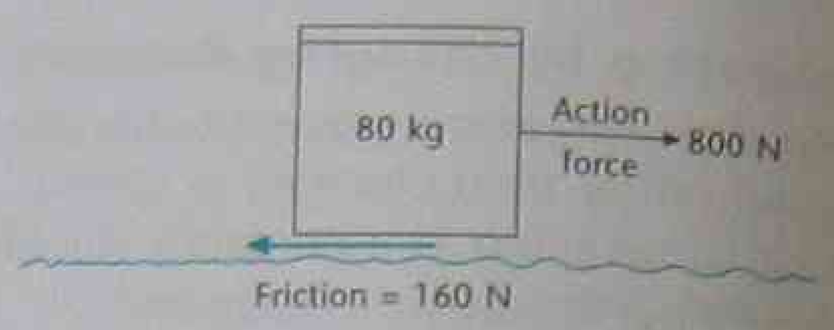


Figure 1.15 Moving a crate on a rough floor

A Net force = action force – frictional force
= 800 – 160 = 640 N.

Using Newton's second law:

$$F = ma$$

$$640 = 80a$$

$$a = 8 \text{ m/s}^2$$

Example 4. Skating on ice

Q A skater is standing still on an ice rink. She is holding a 10 kg bag. She throws the bag out and away in front of her. Use Newton's laws to explain what happens to the skater.

A The skater exerts an action force on the bag in order to throw it away from her. According to Newton's third law the bag will exert an equal and opposite reaction force on the skater. Therefore the skater will move backward as the bag moves forward. This effect is noticeable as the skater is standing on a very low friction surface. A similar observation would be made if an astronaut free-floating in space threw an object away from him. He would be propelled in the opposite direction.

Test yourself (answers on pages 205–6)

Part A. Knowledge (answers on pages 205–6)

1 A bicycle and its rider are moving along a smooth, straight road at a constant speed of 8 m/s. Which statement is true about the bicycle?

- a The motion of the bicycle illustrates Newton's first law.
- b The bicycle is subject to an unbalanced force.
- c The force of friction on the road is greater than the force being applied to the wheels.
- d The force of gravity acting on the bicycle is balanced by the force applied to the pedals. (1 mark)

2 A man applies a 50 N force to the side of a heavy crate which rests on the road. The crate does not move, even though the force is acting. Select the correct statement about the crate.

- a There is a net force acting on the crate.
- b The crate pushes back on the man with a force of 50 N.
- c The force of gravity acting on the crate is 50 N.
- d The frictional force between the floor and crate is much less than 50 N. (1 mark)

3 A heavy rock (weight = 800 N) is placed on a table and the table top sags under the weight of the rock. Eventually the table top stops sagging. Which statement is true?

- a The weight of the rock is greater than the force applied by the table on the rock.
- b There is an unbalanced force on the table top even when the sagging stops.
- c Once the sagging stops, the force acting on the table is less than 800 N.
- d After sagging stops, the force of the table top on the rock equals the force of the rock on the table top. (1 mark)

4 Figure 1.16 shows the position of a car on a road, travelling east, at intervals of 5 seconds.

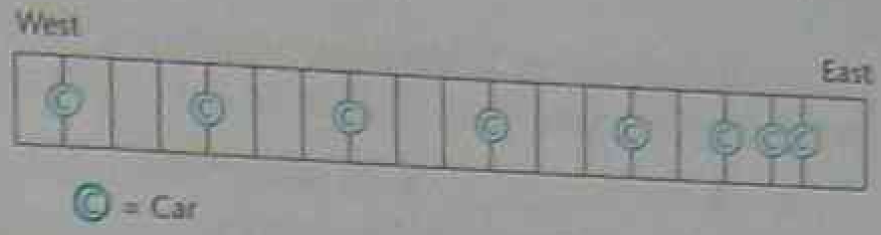


Figure 1.16 Position of a car at regular time intervals

Select the correct statement about the record of motion.

- a The car accelerated during the period of observation.
- b The car initially travelled at constant speed and then decelerated.
- c The car was moving at constant speed for the first 15 seconds and then accelerated uniformly for 10 seconds before stopping.
- d The car slowed down and then moved with constant speed in an easterly direction. (1 mark)

5 Which of the following objects undergoes uniform acceleration?

- a A parachutist gliding down to the ground
- b A skydiver
- c A steel ball dropped from 3 m above the Moon's surface
- d A car speeding away from a traffic light (1 mark)

6 Complete the following restricted response questions using the appropriate word.

- a A person could not walk along the ground without the assistance of _____ forces.
- b The force acting on an iron bar in a magnetic field is an example of a _____ force rather than a contact force.
- c The average speed of a moving vehicle is equal to the _____ travelled divided by the time taken.
- d The statement 'For every action there is an equal and opposite

reaction' is known as Newton's ______ law.

e A car is ______ if its speed changes from 20 m/s to 15 m/s in 3 seconds. (5 marks)

7 Use the code letters to match the terms or phrases in each column. (5 marks)

Column 1	Column 2
a acceleration	f field force
b gravity	g balanced forces
c newton	h reaction force
d stationary object	i change in velocity
e cannon recoil	j unit of force

8 Describe the experimental method and the analysis steps used by a student to measure the motion of a golf ball rolling from rest down a sloping wooden ramp. (4 marks)

9 a A tennis player volleys a ball back across the net for a winning shot. State the consequences of the force acting on the tennis racquet. (1 mark)

b A 10 year old boy sits at one end of a see-saw and a heavier 15 year old boy sits about half-way on the other side so that the see-saw is horizontal and does not move. What can be said about the rotational forces now acting on the see-saw? (1 mark)

c The space shuttle deploys a small parachute behind it on landing in Florida. Explain the purpose of this parachute. (1 mark)

d A skydiver in free fall will eventually reach a terminal velocity. This means that he no longer speeds up but falls with constant speed. Explain how this is possible. (1 mark)

10 a Two 60 kg skaters are seated on smooth sleds on the ice rink facing each other. They throw a heavy medicine ball back and forth to each other. Explain the motion of both skaters during this ball-throwing activity. (2 marks)

b A 50 kg skater and a 100 kg skater face each other on an ice rink such that the palms of their hands are touching. They push each other apart suddenly. Explain the motion of both skaters after the sudden push. (2 marks)

Part B. Skills (answers on page 206)

1 A car travels 200 km between two towns A and B in 240 minutes. Calculate the average speed for the journey. (2 marks)

a 0.83 km/h

b 1.2 km/h

c 20.0 km/h

d 50.0 km/h

$A = \frac{d}{t}$

$a = \frac{200}{240}$

$a = 0.83$

2 Use the tabulated data about 4 cars to determine which car has the greatest average acceleration. (2 marks)

Answer	Car	Initial velocity (m/s)	Final velocity (m/s)	Time taken (s)
a	W	30	40	2.1
b	X	50	80	8.0
c	Y	0	20	2.5
d	Z	20	55	3.8

3 The following data were collected for a car travelling along a straight road.

Time elapsed (s)	Velocity (m/s)
0	0
1	3
2	6
3	9
4	12

What can be concluded from the data? (1 mark)

a The car is moving at constant velocity.

b The car is travelling with uniform acceleration.

c There is no net force acting on the car.

d The car will reach a speed of 16 m/s after 5 seconds if it continues to move forward at the same rate.

4 A student conducts an experiment to measure the motion of two unknown masses (M and N) along a smooth table. A constant force of 1 N was applied by means of a string connected to a falling weight as shown in Figure 1.17.

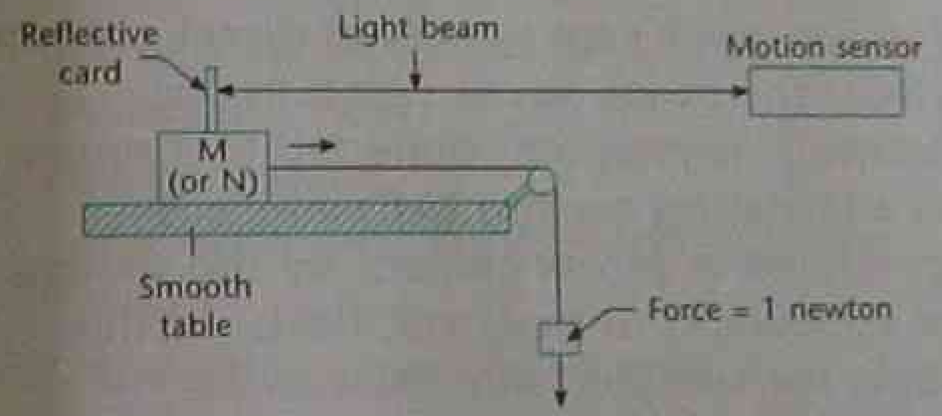


Figure 1.17 Motion experiment

A motion sensor recorded the position (measured from the starting line) of each mass at regular time intervals as they slid along the table. The results displayed by the data logger are tabulated.

Time elapsed (s)	0	0.4	0.8	1.2	1.6	2.0
Position of M (cm)	0	8	32	72	128	200
Position of N (cm)	0	4	16	36	64	100

a Describe the motion of each mass along the table. (2 marks)

b Explain which body has the greater mass. (2 marks)

c Calculate the average speed of M between 0.8 s and 1.6 s. Give the answer in cm/s. (2 marks)

5 The table shows the speeds of 5 objects (K, L, M, N and P) over a 5 second time interval.

Time (s)	Object's speed (m/s)				
	K	L	M	N	P
0	6	30	0	3	2
1	6	25	2	3	3
2	6	20	4	3	5
3	7	15	4	3	9
4	8	10	4	3	17
5	9	5	4	3	33

a Which object has no net force acting on it throughout the five seconds? (1 mark)

b Which object is decelerating? (1 mark)

c Which object is initially moving with constant speed and then slowly accelerates? (1 mark)

d Which object slowly accelerates to its desired speed and then maintains that speed for the rest of the time? (1 mark)

e Calculate the average acceleration of object M in the first 2 seconds of its motion. (2 marks)

6 The following performance figures were obtained for two cars (X and Y) based on a standing start.

Speed attained (km/h)	Time to reach specified speed (s)	
	Car X	Car Y
50	3.1	3.6
60	4.5	4.9
80	6.4	7.0
100	9.2	11.1

a Why is attainment of specified speed measured from a standing start? (1 mark) b Which vehicle has the higher performance? Explain. (2 marks)

7 Figure 1.18 gives information on the stopping distance on dry bitumen roads for a typical driver with a reaction time of about 1 second.

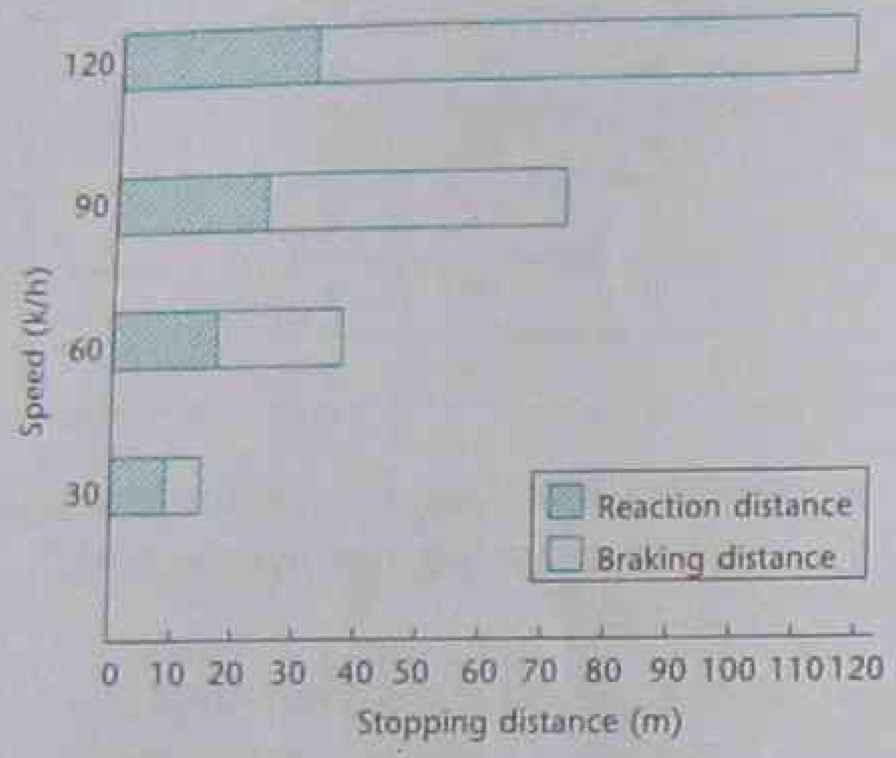


Figure 1.18 Stopping distance on dry bitumen road

The stopping distance is defined as:

Stopping distance = reaction distance + braking distance

The 'reaction distance' is the distance the car moves during the driver's reaction time. The reaction time varies between drivers but it is usually between 0.75 and 1 second.

a Explain what is meant by the 'braking distance'. (1 mark)

b How does the reaction distance change as the speed of the car increases? (1 mark)

c How does the braking distance change as the speed of the car increases? (1 mark)

d Calculate the reaction distance at 120 km/h (= 33.3 m/s) for a driver with a reaction time of 0.75 s. (2 marks)

e Predict how the reaction distance and braking distance will change at 60 km/h if the road is wet. (1 mark)

Electrical energy

Electrical energy is an important form of energy in modern society. It is used to power machines, including the majority of large and small household appliances. Electrical energy is transported very rapidly to these appliances via conducting wires in electrical circuits. It is important to remember that the individual free **electrons** in the wire move relatively slowly (a few millimetres per second) but the **electrical energy** is transmitted very rapidly along the conductor. In this way the energy reaches an appliance almost instantaneously when you turn the switch.

There are a number of sources of electrical energy. A **battery** and a **DC transformer** ('**power pack**') are sources of **direct current** (**DC**). The mains power points in our homes, however, are connected to a source of **alternating current (AC)** which is generated in power stations by the motion of conductors in magnetic fields. In this topic we consider only DC circuits.

Glossary

Battery—a portable electrical source consisting of a number of electrical cells in series

Current—the flow of electric charge (measured in amps, A)

Electric cell—a device that produces electrical energy by chemical reactions

Resistance—property of a conductor that restricts current flow (measured in ohms, Ω)

Resistor—a device that exhibits resistance to the flow of current

Voltage—the electrical pressure that causes currents to flow (measured in volts, V)

Electrical circuits

An electrical circuit is a complete conducting pathway for the electric current to flow from one terminal of an **electric cell** or **battery** (energy source) back to the other terminal. A battery consists of a number of simple electric cells joined in series with one another. The electrical energy is generated inside the electric cell by chemical reactions in each half of the cell. As the electric current flows around the circuit, its energy is transformed into heat and sometimes light. If a motor is connected into the circuit, the electrical energy is partly converted into mechanical energy (a form of kinetic energy).

A simple circuit

A simple electrical circuit consists of:

- **a DC energy source** (eg. battery that stores separate electric charges and gives energy to the charges as they leave it; it also sets up an electric field in the connecting wires)
- **circuit controls**
  —**switches** used to open and close the circuit
  —**branching connecting wires** that allow the current to divide into different parts of the circuit
- **connecting wires**—conductors (such as insulated copper wires) to transfer the current between each components of the circuit
- **resistor(s)**—materials that resist current flow (eg. filaments in light bulbs)
- **measuring devices**
  —**ammeters** (meters which measure the current flow in the circuit)
  —**voltmeters** (meters which measure the voltage or potential difference across resistors or other components).

In a household circuit there will also be **fuses** (or **circuit breakers**) included in the circuit. This is a safety feature in household circuits to prevent overheating and fires from currents that are too high.

Figure 1.19 shows a simple electrical circuit diagram.

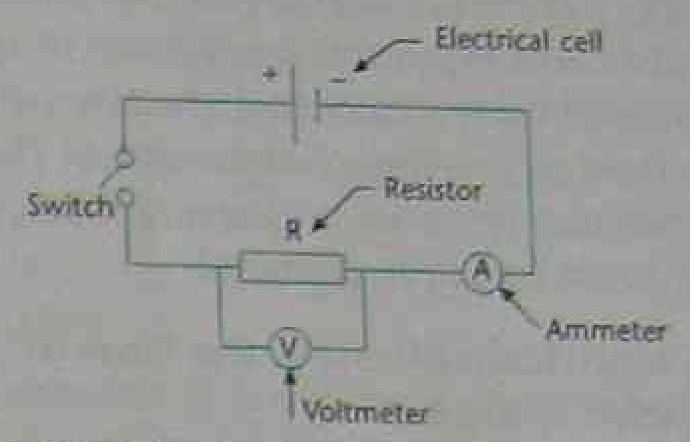


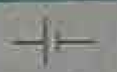
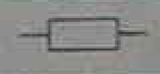
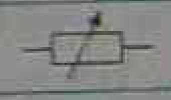
Figure 1.19 Simple electric circuit

The following terminology is used for electrical circuits.

- **Open circuit**—the switch is OFF and no current flows.
- **Closed circuit**—the switch is ON and current flows.

Electrical circuits use circuit symbols which are a type of shorthand. Table 1.6 shows examples of the symbols used in electrical circuit diagrams.

Table 1.6 Electrical circuit symbols

Component	Symbol	Component	Symbol
Conducting wire	——	Electric cell	
Battery		Resistor	
Lamp		Switch	
Ammeter		Voltmeter	
Fuse		Variable resistor	

Current, voltage and resistance

Current

Before electrons were discovered at the end of the nineteenth century, physicists believed that electrical currents consisted of moving positive charges. Eventually it was discovered that electrical currents were due to the motion of (negatively charged) electrons. Although electrons flow out of the negative terminal of an electric cell and through the circuit to the positive terminal, physicists and electricians continue to use the original convention that electric current is the flow of positive charges out of the cell's positive terminal and around the circuit to its negative terminal.

- **An electrical current is the flow of positive charge.**

As energised electrical charges flow out of the positive terminal of the battery and into one end of the circuit, energy-depleted charges move back into the battery via the negative terminal.

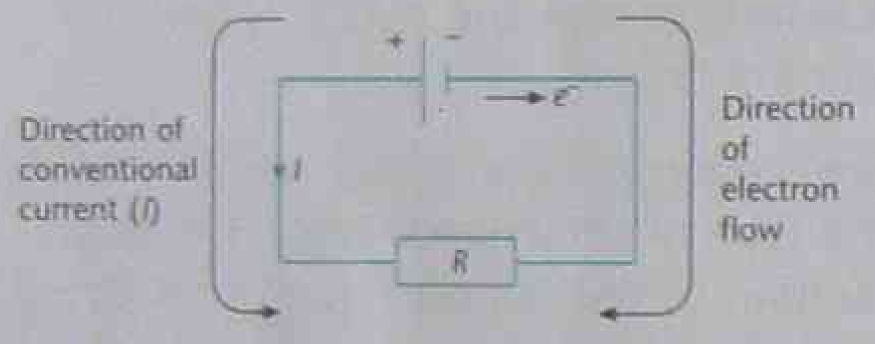


Figure 1.20 Conventional and electron currents

It is often useful to visualise electrical currents in terms of an analogy.

Observation: Current flow is almost instantaneous once the switch is turned on.

Analogy: A long line of marching students is ordered to begin marching along a circular road, starting at the long bridge which links the two ends of the road. They all start together and observers at each side of the bridge and at other points along the road observe immediate movement, even though individual students will take a long time to move around the whole road system and pass a fixed point.

- **Electrical currents are measured using ammeters that are placed in the circuit.**

These ammeters effectively count the number of electric charges passing any given point in one second. The ammeters themselves have low internal resistances so they do not disrupt the current flow.

- **Electrical current is measured in units called 'amperes' or 'amps' (unit symbol = A).**

This unit of current is named in honour of the French physicist André Ampère.

Voltage

Electrical charges flow around a circuit in response to an electrical force. The force is caused by the existence of an electric field that is established in the conducting wires when they are connected to the battery terminals. We can think of the charges emerging from the positive terminal of the battery as having a high potential energy. Those charges returning to the negative terminal have zero potential energy. In some parts of the circuit some of this energy is transformed into other forms and the potential energy decreases. This is particularly true across a resistor such as the filament in a light bulb. Potential differences therefore exist across various parts of the circuit. These **potential differences** are also called **voltage drops**.

- **Voltage is a measure of the potential energy differences between any two points in a circuit.**

Some people like to think of the voltage of a battery as the electrical pressure it puts on the charges that flow around the circuit.

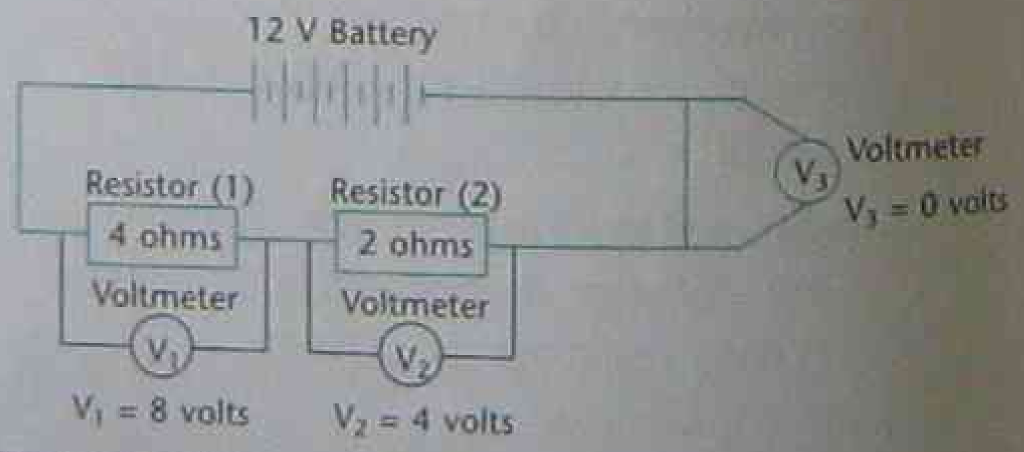


Figure 1.21 Voltage drop around a simple circuit

It is often useful to develop an understanding of the term 'voltage' in terms of an analogy.

Observation: High-voltage batteries make model electric motors spin faster than do batteries with low voltages.

Analogy: Water flows rapidly out of a tap if the water pressure behind the tap is high. If the water is stored in a tank and the tap is opened, the pressure of water flow decreases as the level of water drops in the tank. In this analogy the height of the water column behind the tap creates a pressure (due to the water's gravitational potential energy) which drives the water out when the tap is open. Similarly the high-voltage battery gives the charges a high electric potential energy. When the switch is closed, the charges begin to flow as a response to the electrical pressure.

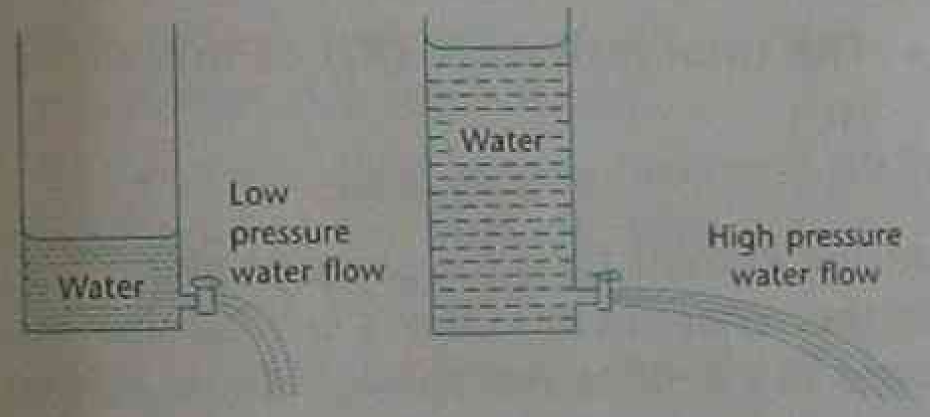


Figure 1.22 Tank of water analogy for voltage

- **Voltmeters are used to measure the voltage drop across any two points in a circuit.**

Because voltmeters have high internal resistance to current flow, they are placed in parallel to the circuit and so they only sample part of the current flow at the points being measured.

- **Voltage is measured in units called 'volts'. (Unit symbol = V)**

The unit of voltage is named in honour of the Italian physicist Alessandro Volta.

Resistance

Not all materials are excellent electrical conductors like silver and copper. Some materials reduce the flow of electric currents when they are present in a circuit. Nichrome alloy wires, for example, reduce the flow of a current, compared with the flow in similar copper wires. Such materials are said to offer **resistance** to current flow. Materials that completely block current flow, such as plastics and glass, are called **insulators**.

- **Resistance is a measure of the electrical conductivity of the conductor.**

A wire that is a good conductor has a low resistance. A wire that is a poor conductor has a higher resistance. The resistance of any wire also depends upon its length and diameter as well as its temperature.

- **Devices that are manufactured to provide resistance to current flow are called resistors.**

The tungsten filament in a light bulb is a resistor. As current flows though it, considerable energy is transformed into heat energy and light energy. In this case the resistor is designed to emit light. In other appliances, resistors are used to limit current levels in certain parts of a circuit.

- **Resistance is measured in units called 'ohms'. (Unit symbol = Ω)**

The unit of electrical resistance is named in honour of the German physicist Georg Ohm. Ohm also showed that the voltage, current and resistance are related mathematically. He showed that the resistance (*R*) of a conductor was equal to the voltage drop (*V*) across the conductor divided by the current (*I*) flowing through it.

Additional content—Mathematical extension:

Mathematically, **Ohm's law**, as it is known, is expressed as:

$$R = V/I$$

Example

Q A piece of Nichrome wire is included in a simple electrical circuit. The ammeter registers a current of 2.5 A flowing through the wire and a voltmeter registers a voltage drop of 7.5 V across the wire. Calculate the resistance of the Nichrome wire.

A $I = 2.5$ A

$V = 7.5$ V

$R = V/I = 7.5/2.5 = 3.0\ \Omega$

It is often useful to develop an understanding of the term 'resistance' in terms of an analogy.

Observation: Narrow-diameter wires have greater electrical resistance than wide-diameter wires.

Analogy: Narrow water pipes slow down the flow of water. In a garden hose, the nozzle can be narrowed to restrict the water flow. Water is transported from reservoirs to cities in huge pipes with large diameters to reduce the resistance to flow.

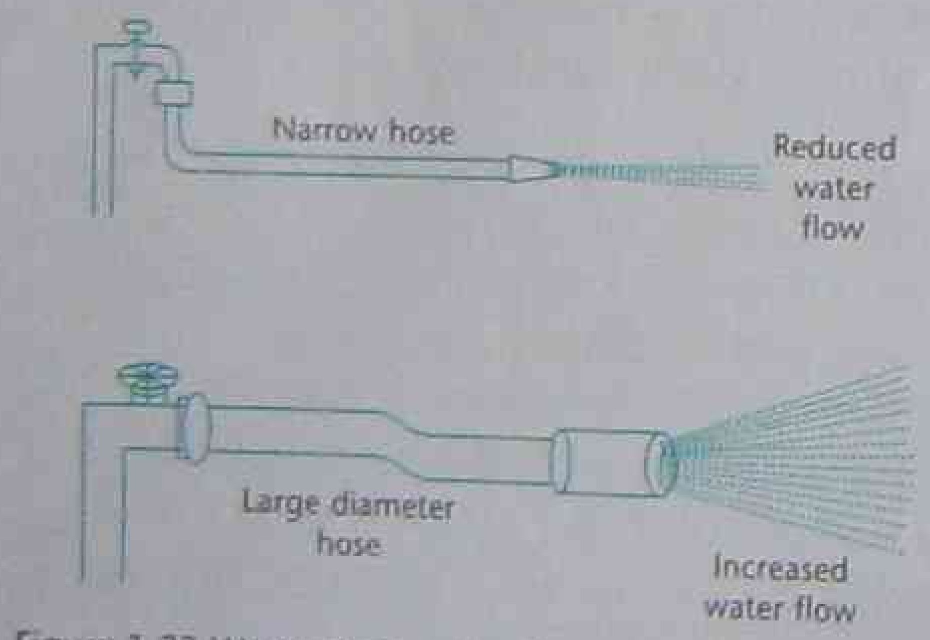


Figure 1.23 Water pipes and resistance analogy

Series and parallel circuits

Resistors can be connected in various arrangements in electrical circuits. They can be connected **in series** or **parallel** or in combinations of the two.

Series circuits

In a series circuit all the resistors are placed one after the other so that the electrical current passes in turn through each resistor as it flows around the circuit.

- **At all points in the series circuit the current is the same.**
- **The voltage drop across the two battery terminals is equal to the sum of the voltage drops across each resistor.**
- **The greater the number of resistors in series the greater is the total resistance.**

Additional content—Mathematical extension:

Figure 1.24 shows a typical series circuit for two resistors R_1 and R_2.

- **The total resistance (R_T) of this circuit is:**

$$R_T = R_1 + R_2$$

As the current (I) is the same at all points in the series circuit, Ohm's law can be used to calculate the voltage drops across each resistor.

Resistor 1: $V_1 = I{\cdot}R_1$

Resistor 2: $V_2 = I{\cdot}R_2$

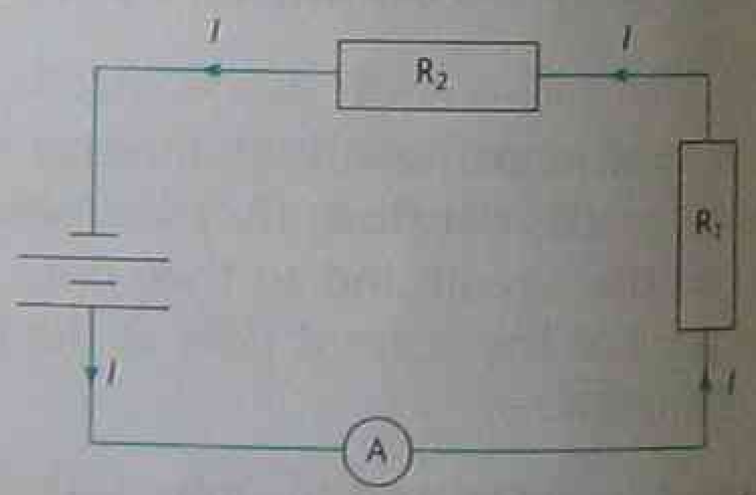


Figure 1.24 Two resistors in series

$I = \frac{V}{R}$

Example

$I = \frac{12}{20}$

$I = 0{\cdot}6$ A

Q An 8 Ω and a 12 Ω resistor are placed in series with a 12 V battery. Calculate:

a the current through each resistor

b the voltage drop across each resistor.

$V = I \times R$

$V = 0{\cdot}6 \times 8 = 4{\cdot}8$ $V = 0{\cdot}6 \times 12$

A a Total resistance $= R_T = R_1 + R_2$

$= 8 + 12 = 20\ \Omega$

Total current $= I_T = V/R_T$

$= 12/20 = 0.6$ A

The current flowing through each resistor is 0.6 A.

b Resistor 1: $V_1 = I{\cdot}R_1 = (0.6)(8)$

$= 4.8$ V

Resistor 2: $V_2 = I{\cdot}R_2 = (0.6)(12)$

$= 7.2$ V

(*Note:* $V_T = V_1 + V_2 = 4.8 + 7.2$

$= 12.0$ V)

Parallel circuits

In simple parallel circuits the resistors are arranged so that:

- **the battery supplies current to each resistor at the same time;**
- **the voltage drop across each resistor is the same as the voltage drop across the battery terminals;**

  Thus for two resistors (R_1 and R_2) in parallel:

  $$V_T = V_1 = V_2.$$

- **the total current which divides into each parallel resistor is equal to the sum of the currents in each resistor.**

  For two resistors (R_1 and R_2) in parallel:

  $$I_T = I_1 + I_2.$$

Two lamps in parallel are very bright (as bright as a single lamp), but two lamps in series are dimmer. They are, however, as bright as each other. Figure 1.25 shows a simple parallel circuit containing two resistors.

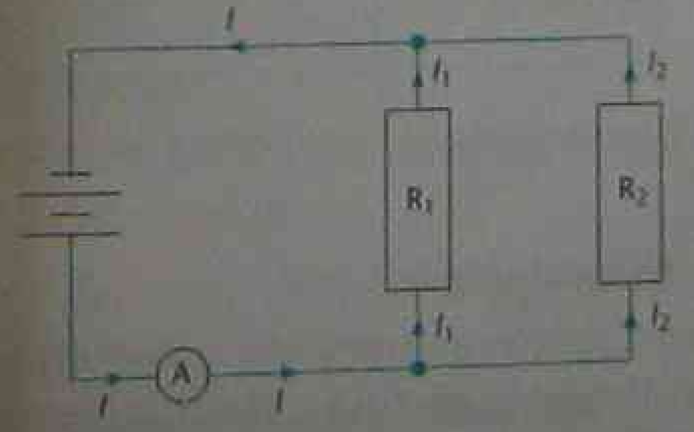


Figure 1.25 Simple parallel circuit with two resistors

Additional content—Mathematical extension:

Example

Q A 3 Ω and a 4 Ω resistor are placed in parallel with a 12 V battery. Calculate:

a the voltage drop across each resistor 12

b the current flowing through each resistor $I = \frac{V}{R} = \frac{12}{3} = 4A$ $\frac{12}{4} = 3A$

c the total current in the circuit 7A

d the total resistance of the circuit. $\frac{12\ V}{I = 7}$ 1·71

A a The voltage drop across each resistor is the same as the voltage drop across the battery.

Thus: $V_T = V_1 = V_2 = 12$ V

b Resistor 1: $I_1 = V_1/R_1 = 12/3 = 4$ A

Resistor 2: $I_2 = V_2/R_2 = 12/4 = 3$ A

c The total current is the sum of the currents in each branch.

$I_T = I_1 + I_2 = 4 + 3 = 7$ A

d The total resistance is the total voltage divided by the total current:

$R_T = V_T/I_T = 12/7 = 1.71\ \Omega$

This last calculation shows that the total resistance in a parallel circuit is less than the resistance of each individual resistor.

Modelling the resistance of series and parallel circuits

From the information and calculations above we can see that the total resistance of a circuit increases when the resistors are placed in series, but decreases when the resistors are placed in parallel. We can use a simple model or analogy to explain why this is so.

a. Modelling a series circuit

Figure 1.26 shows a crowd of people waiting to go into a football match at a poorly designed stadium. Initially, queuing for a ticket hinders the flow of people. Having purchased a ticket they are made to pass through one turnstile before reaching a holding area. The turnstile slows down the flow of people. Once in the holding area the fans have to move through a narrow tunnel under the stands to get to a point where they can go to their seats. The narrow tunnel also reduces the flow of people. Thus the total resistance to people flow is the sum of all the individual points of restricted flow.

$$R(\text{total}) = R(\text{ticket queue}) + R(\text{turnstile}) + R(\text{tunnel})$$

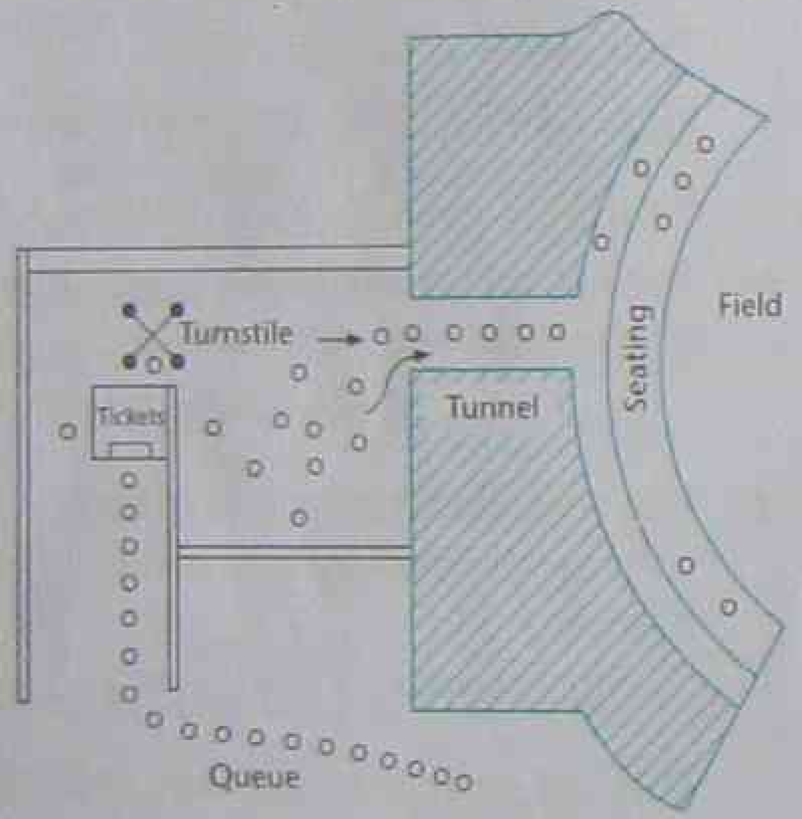


Figure 1.26 Series resistance: Modelling resistance to crowd flow at a football match

b. Modelling a parallel circuit

Figure 1.27 shows a remodelling of the football stadium access to speed up crowd movements. In the new design there are more ticket offices, to reduce waiting time for a ticket. The ticket offices are in parallel. There are also multiple parallel turnstiles so that the fans can move into the holding area faster. There are more parallel access tunnels that are wider than the old narrow tunnel to allow more people to reach their seats faster. The old narrow tunnel is still there and carries some people but the flow through it is not as great as in the wider tunnels. The wider tunnels therefore have lower resistance. The net result of these changes is a larger flow of people to their seats due to decreased resistance.

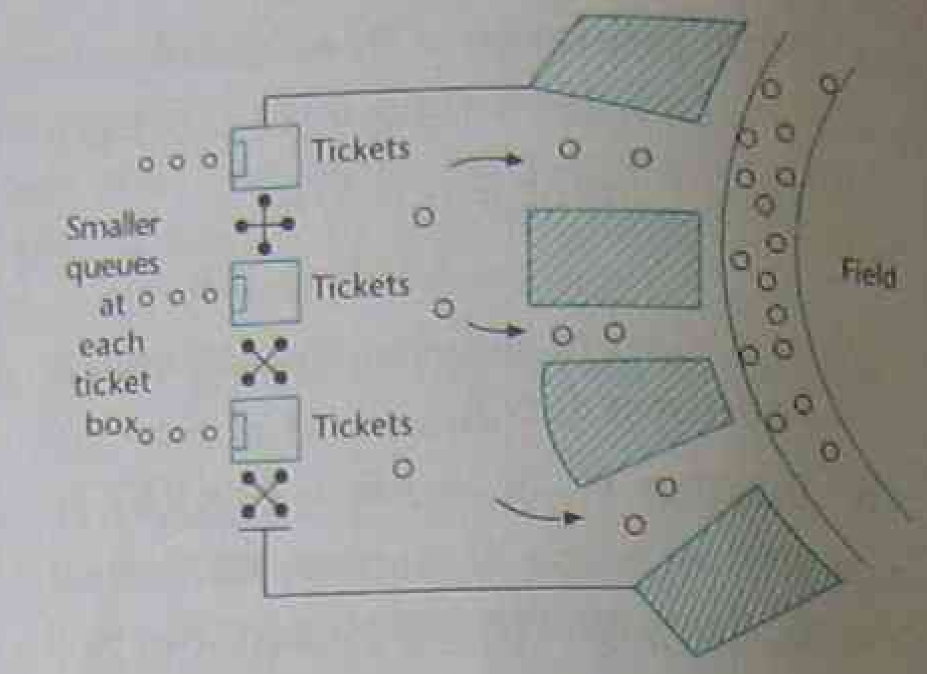


Figure 1.27 Parallel resistance: Modelling resistance to crowd flow in a redesigned stadium

Circuits in our homes

a. Lighting and power circuits

Parallel circuits are used for lighting and power circuits in our homes. This ensures that when the filament in one light breaks, the other lights in the house do not go out (as would happen in the case of a series circuit). In a parallel circuit individual switches control each light or power point. In a series circuit only one switch would be required to turn all lights on or off.

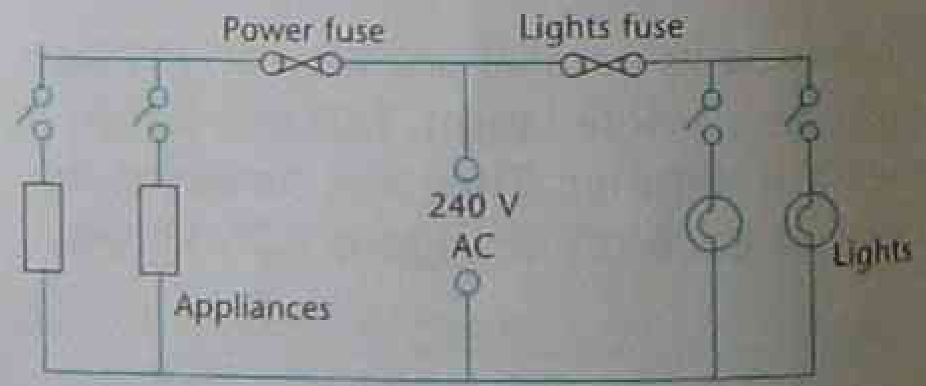


Figure 1.28 Parallel lights and switches in a home circuit

b. Christmas lights

Lights for Christmas trees are often arranged in series. If one bulb fuses, then it has to be replaced to allow them all to work.

c. Radiators and electric blankets

Radiators and electric blankets use resistors to generate heat. These appliances often use combinations of series and parallel circuits for the resistors to allow variable heat settings (high, medium or low). For a high heat setting the current flow must be high, and thus the resistance must be low. Low resistance is achieved by turning switches so that the resistors are in parallel. For a low heat setting the current flow must be low and thus the resistance must be high. High resistance is achieved by turning switches so that the resistors are in series.

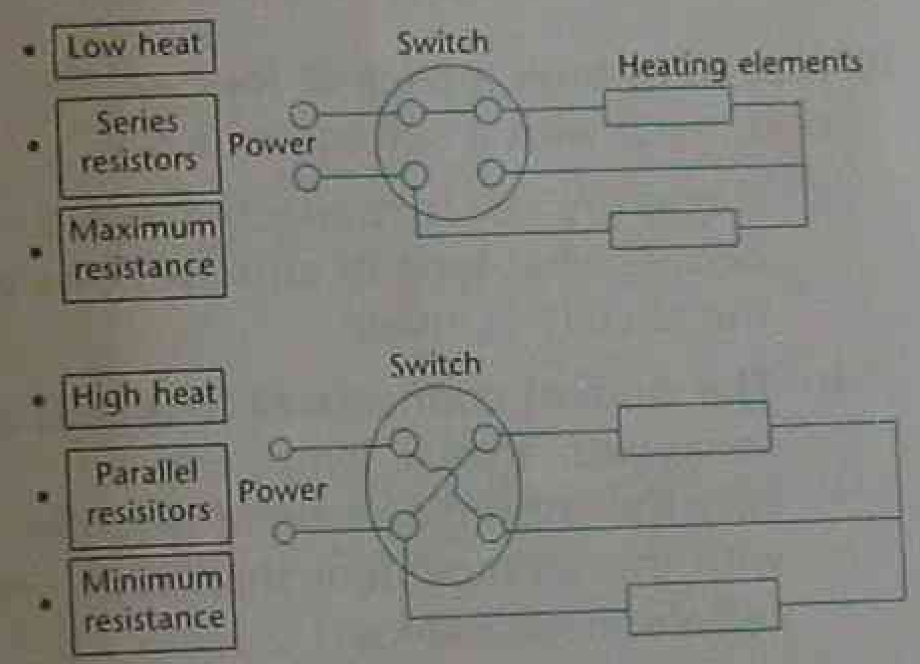


Figure 1.29 Circuits in an electric blanket

Test yourself (answers on pages 206–7)

Part A. Knowledge (answers on pages 206–7)

1 Which of the following materials would make the best connecting wires in an electric circuit? (1 mark)

- a plastic
- b glass fibres ✓
- c iron
- [✓ d] copper

2 Select the correct statements about an electric circuit.

- a Currents flow through the circuit when it is open. ✓
- [✓ b] Current flows around the connecting wires of a circuit from the positive terminal of the battery towards the negative terminal.
- c The filament in an electric light bulb is chosen so that little electrical energy is wasted as light and heat.
- d When two equal resistors are arranged in parallel, the total resistance is higher than when they are arranged in series. (1 mark)

3 A suitable device to measure the current flow in a circuit is:

- [✓ a] an ammeter
- b a voltmeter ✓
- c an ohmmeter
- d a battery. (1 mark)

4 Five resistors are arranged in parallel with a 12 V battery. Three resistors each have a resistance of 1 Ω and the other two each have a resistance of 2 Ω. Select the correct statement about this circuit.

- a The voltage drop across the two-ohm resistors is greater than that across the one-ohm resistors.
- b The current flow is the same through all resistors. ✓
- [✓ c] Each resistor has the same voltage drop across it.
- d The current is the same in all parts of the circuit. (1 mark)

5 An electric blanket contains two resistors. One resistor has a higher resistance than the other. A variable selector switch allows different combinations of resistors to be combined in a circuit with the power supply. What is needed to obtain the greatest heat from the blanket?

- a Arrange the two resistors in parallel.
- [✓ b] Arrange the two resistors in series.
- c Use only the resistor with the greater resistance.

d Use only the resistor with the lower resistance. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each)

a An electric current is actually due to the flow of electrons.

b A voltmeter is used to measure the drop in electric potential across a resistor.

c Ammeters are placed in series in a circuit to measure the current flowing in the circuit.

d Two resistors in series have a higher resistance than each individual resistor.

e The unit of electrical current is the ampere.

7 Use the code letters to match the terms or phrases in each column. (1 mark each)

Column 1	Column 2
a series circuit	f same drop in voltage across each resistor
b conductors	g resistance
c conventional current	h metals
d Georg Ohm	i same current through each resistor
e parallel circuit	j flow of positive charge

8 Figure 1.30 shows a water circuit containing a water pump, filter, water flow meter, pressure gauge and a tap. Draw the equivalent electric circuit using the same symbols for the devic as in the water flow circuit. (2 mark

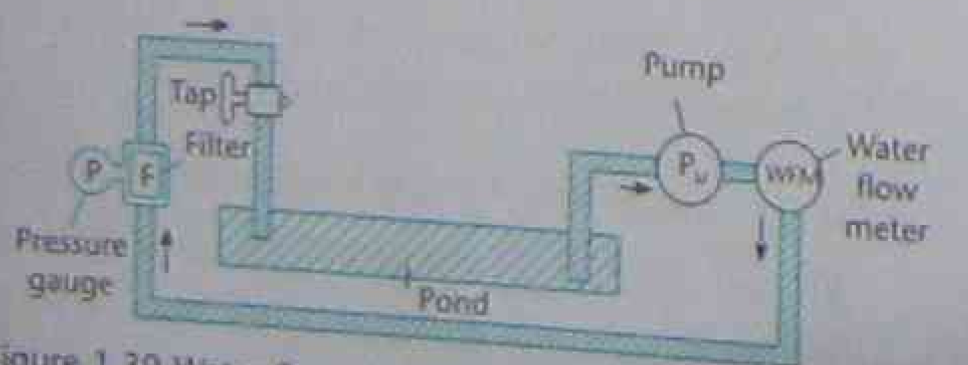
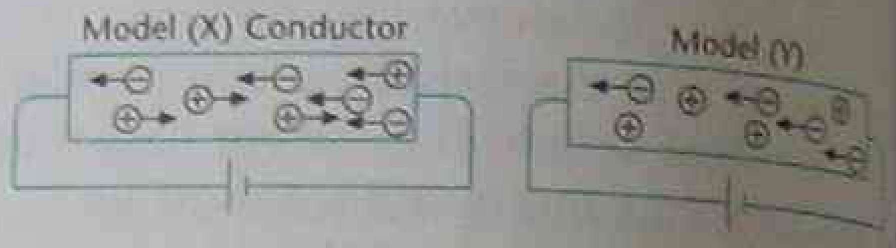


Figure 1.30 Water flow circuit

9 Figure 1.31 shows some different models for charge flow in a metallic conductor. Explain which diagram correctly shows the flow of charge in the conductor. (2 marks)



⊖ = Electron

⊕ = Positive metal ions

Figure 1.31 Models for charge flow in a conductor

10 A student buys a pack of four AA dry cells, each with a voltage of 1.5 V.

a If one dry cell is connected into a circuit, what type of current flows i the circuit? (1 mark)

b The student connects all four cells i series with a small lamp. Explain why the lamp glows more brightly with this arrangement than with one cell alone. (2 marks)

Part B. Skills (answers on page 207)

Note—Some additional content (mathematical extensions) is examined as indicated.

1 A student is asked to set up a circuit in which three identical lamps are arranged so that two lamps have the same brightness but the third lamp is the brightest. The equipment available is three identical lamps; 12 V battery; switch; connecting leads.

Draw a circuit diagram for the circuit constructed by the student. (2 marks)

2 A student is supplied with a lamp, 12 V battery, connecting leads and two switches.

a Design a circuit so that the lamp wil light when either switch is closed (on) but does not light when both switches are open (off). (2 marks)

b Name one practical application of this type of circuit. (1 mark)

3 **Mathematical extension:** A simple series circuit was set up as shown in Figure 1.32. A 6 V battery is in series with an ammeter and two resistors (AB and CD). Two voltmeters are connected as shown. The voltage drop across AB was 2 V. The ammeter reads 1.0 A.

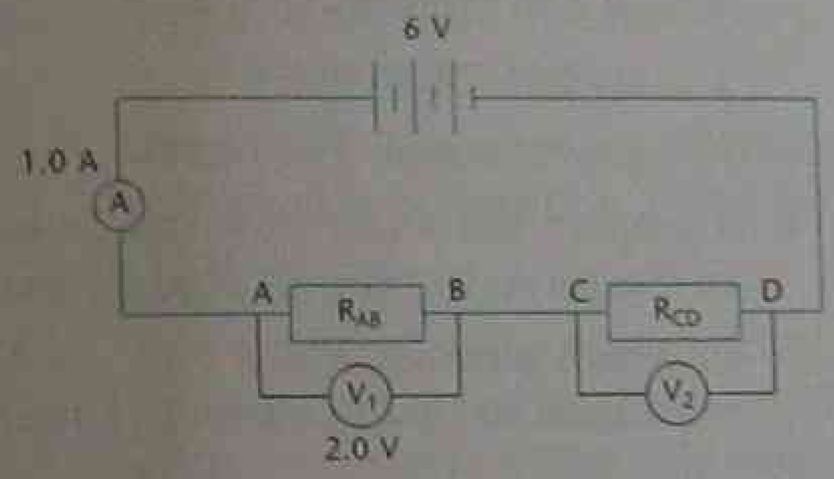


Figure 1.32 Series circuit

a What will the voltmeter across CD read? (1 mark)

b What is the resistance of AB? (1 mark)

c What is the resistance of CD? (1 mark)

d What is the total resistance of the circuit? (1 mark)

4 A 24 V battery is connected in series with three known and one unknown resistor (R), a switch and an ammeter. The three known resistors have the following resistances: $R_1 = 3\ \Omega$; $R_2 = 4\ \Omega$; $R_3 = 1\ \Omega$.

a Draw a circuit diagram. (2 marks)

b **Mathematical extension:** If the ammeter reads 1.5 A, calculate the value of resistance R. (2 marks)

c A voltmeter is placed in turn across each resistor. In which case will the lowest voltage drop reading be recorded? (1 mark)

5 A 3 Ω resistor and a 4 Ω resistor are arranged in parallel with a 12 V battery and three ammeters as shown in Figure 1.33. Three voltmeters are also connected: one across each resistor and across the battery terminals.

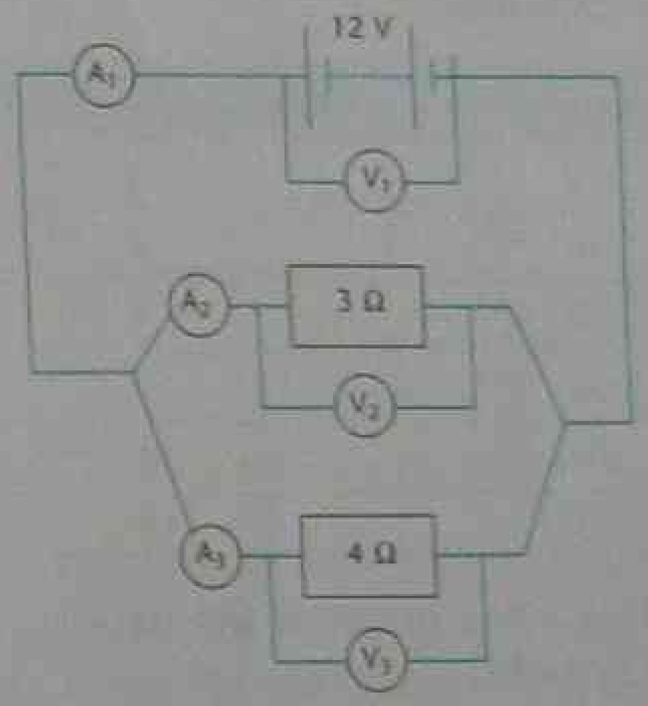


Figure 1.33 Parallel circuit

a **Mathematical extension:** Which ammeter will have the (i) highest, (ii) lowest, reading? Explain. (2 marks)

b Compare the readings on the voltmeters. (2 marks)

c **Mathematical extension:** A switch is inserted in the circuit to turn off the branch to the 3 Ω resistor. How will this affect the readings on each ammeter? (1 mark)

6 Figure 1.34 shows a 100 cm resistance wire AB connected in a series circuit with a 20 V DC transformer and an ammeter. Two wires allow a voltmeter to be

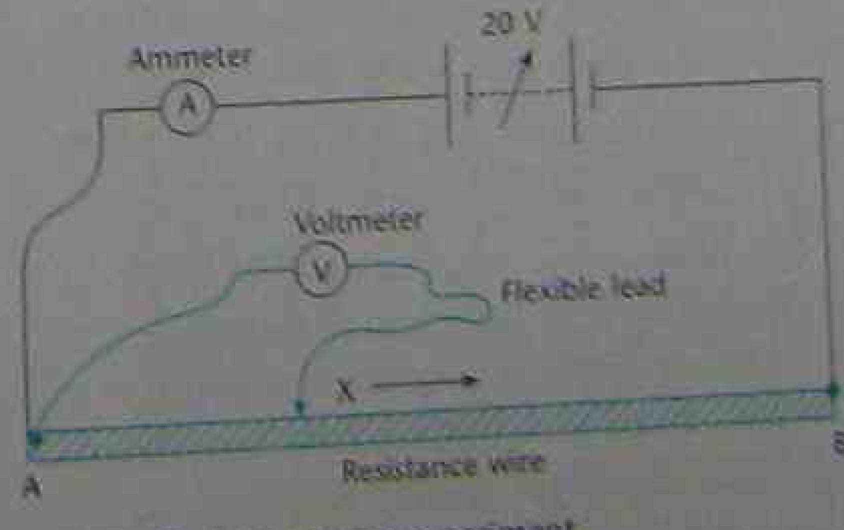


Figure 1.34 Resistance wire experiment

connected at any two points along the resistance wire AB. One end is permanently attached to A and the other slid along to a point X. Readings of the voltmeter are made at each position of X.

The following table shows the results of the experiment.

Distance AX (cm)	Voltmeter reading (V)
0	0
10	2
30	6
50	10
70	14
90	18

a Plot a line graph of the data. (3 marks)

b Use the graph to predict the voltage drop across AB. (1 mark)

c **Mathematical extension:** If the ammeter reads 4 A, calculate the resistance of the wire AB. (2 marks)

Mid-chapter test (answers on pages 208–9)

1 Figure 1.35 shows a visual display of a radio wave on a cathode ray oscilloscope. Draw the wave pattern of a radio wave with double the frequency and half the amplitude. (2 marks)

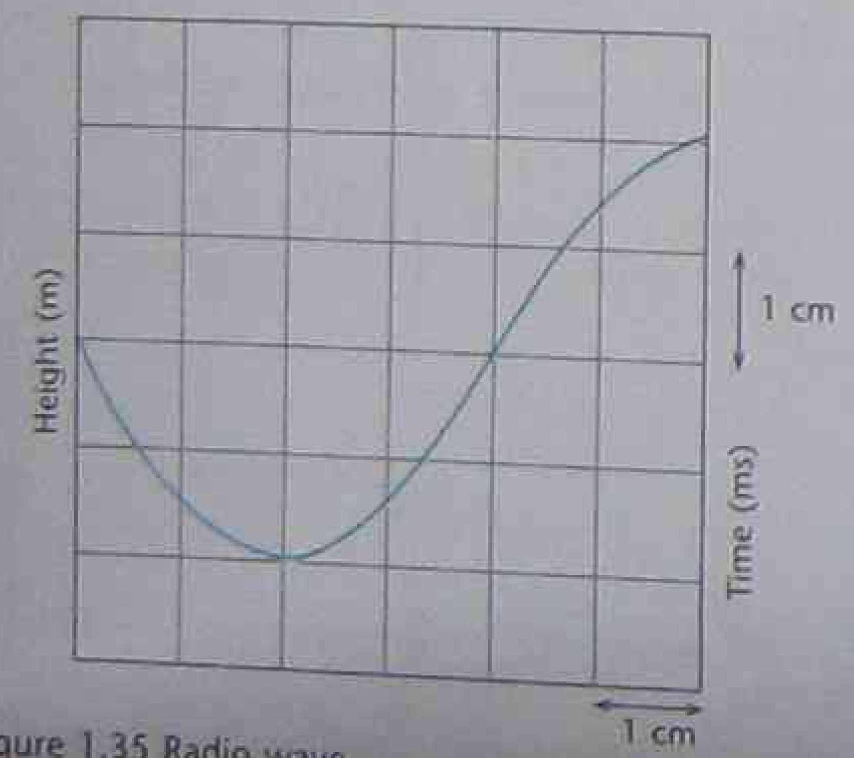


Figure 1.35 Radio wave

2 Water flows around a series of pipes that form a complete circuit. The circuit also includes a pump, tap and water-flow meters in each section of pipe. There are three types of pipes with different internal diameters (1 cm, 2 cm, 3 cm) which are arranged in series in the circuit. This water-flow circuit is compared to the operation of an electric circuit involving a three-speed electric fan.

a Which component of the water-flow circuit corresponds to the:

i switch? (1 mark)

ii power supply? (1 mark)

b Compare the water flow rate in each of the three sections of pipe. (1 mark)

c In which pipe section is the resistance to flow greatest? (1 mark)

3 Sonar sensors use sound waves to detect the depth of water under a boat. Sound waves are emitted from the base of the boat and return to a detector after reflection off the seabed.

The boat was anchored at sea and used sonar to measure the depth of water. The time for the return of the echo was 0.15 s. The speed of sound in seawater during the experiment was 1450 m/s.

a Calculate the depth of water under the boat. (1 mark)

b In the same depth of fresh water the echo returned after 0.16 s. Calculate the speed of sound in fresh water. (2 marks)

4 The following table shows the results of an experiment in which a student measured the current and voltage in various simple circuits involving different known resistors.

a Complete the following sentences by stating a conclusion that is consistent with the tabulated data: (2 marks)

(*R*) Resistance (ohms)	(*V*) Voltage (volts)	(*I*) Current (amps)
4	2	0.5
4	4	1
2	6	3
4	8	2
4	12	3

i If the resistance is constant, a higher voltage _____.

ii For a constant current, a lower resistance requires_____.

b Use the data to state an algebraic relationship between *R*, *V* and *I*. (1 mark)

5 a Two bodies (M and N) are acted upon by equal forces (F). Complete each statement so that it is correct. (2 marks)

i If the mass of M is greater than the mass of N, then the acceleration of M is_____.

ii If the mass of M is equal to the mass of N, then the acceleration of M is_____.

b Body X is twice as massive as body Y. Body X is acted upon by a force which is a quarter as strong as the force acting on Y. Compare the accelerations of X and Y. (2 marks)

6 The following table shows the speed of a manual racing car over 60 seconds as it starts from rest. It reaches its top speed at 60 seconds and stays at that speed thereafter.

a Plot the data as a line graph. (3 marks)

b During what period is the greatest force acting on the car? (1 mark)

c The car has reached top gear after 25 seconds. Compare the acceleration of the car in top gear to its acceleration in lower gears. (2 marks)

Time (s)	Speed (km/h)
0	0
5	45
10	90
15	100
20	110
25	120
40	130
50	138
60	145
70	145
80	145

d Once the car has reached its top speed and maintains that speed, what is the net force acting on the car? (1 mark)

7 Figure 1.36 shows a circuit containing two light globes (Y and Z), a battery, ammeter and two switches (A and B).

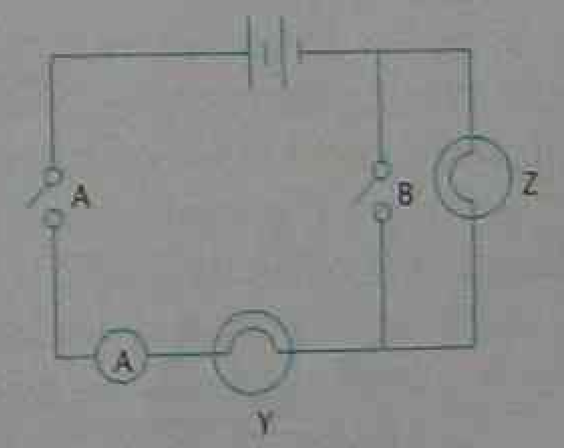


Figure 1.36 Simple electric circuit

a If both switches are closed, which globes will light up? Why? (2 marks)

b If only switch A is closed, which globes (if any) will light up? (1 mark)

c Switch B is moved so that it is in series with Y and Z. Switch B is closed and switch A is open. Will any of the globes light up? Explain. (2 marks)

8 The regions of the electromagnetic spectrum are (in alphabetical order):

gamma rays, infrared rays, microwaves, radio waves, ultraviolet rays, visible rays, X-rays.

a Which wave category has the longest wavelength? (1 mark)

b Which wave category has the greatest frequency? (1 mark)

c Which wave groups are commonly used for astronomical observations? (1 mark)

d True or false: Electromagnetic rays are examples of longitudinal waves. (1 mark)

e True or false: Electromagnetic waves do not require a medium in which to travel. (1 mark)

9 Figure 1.37 shows two resistors (40 Ω and 80 Ω) in parallel with a lamp (10 Ω) and a constant DC power supply. A selector switch is able to allow the current to flow through either resistor via the connecting points X and Y in the circuit.

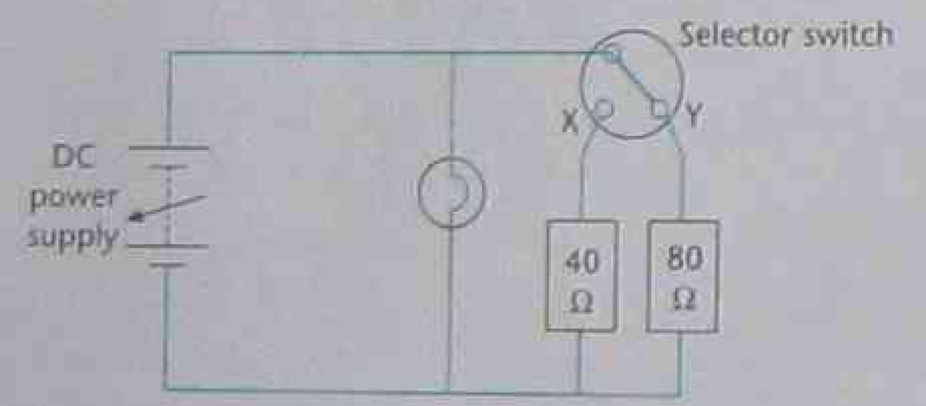


Figure 1.37 Circuit with selector switch

Explain how the brightness of the lamp will change as the selector switch is moved from Y to X. (2 marks)

10 a A hang-glider experiences a downward force of 500 N and an upthrust force of 1200 N. Describe the direction of the motion of the hang-glider. (1 mark)

b A toy cart moving along a horizontal bench is subjected simultaneously to the following horizontal forces:

- A string provides a pulling force of 10 N to the left.
- A frictional force of 3 N between the bench and wheels operates to the right.
- An air resistance of 2 N operates to the right.

Calculate the net force acting on the cart and describe its subsequent motion. (2 marks)

c Under what conditions will a parachutist fall with a constant terminal velocity? (1 mark)

Light energy

Light is the general name given to all forms of **electromagnetic waves**. (See page 4.)

Our **eyes** are able to detect only the **visible band** which has wavelengths in the approximate range 400 nm (violet end) to 700 nm (red end). Photographic film or light-meters are also able to detect light.

Light is emitted from a variety of **sources** such as the Sun, incandescent and fluorescent light bulbs, burning materials and lasers.

Light waves travel at **very high speeds** in **straight lines**. In a vacuum, light travels at a speed of 300 000 km/s. In other materials such as water it is slowed down.

Glossary

Converge—to come together

Diverge—to spread apart

Opaque—describes a material that does not allow light to pass through it

Refraction—the bending of light rays when they pass into different media at an angle

Transmission—passage of light rays into a medium and out the other side

Translucent—describes a material that partially transmits and partially scatters light rays

Transparent—describes a material that allows light to pass through it without scattering

Absorption and reflection

In this section we examine some of the common properties of light rays. When light rays strike an object they may:

- pass straight through the material with some small loss of energy. **Transparent** materials such as window glass or the clear glass in spectacles are like this. The thicker the glass the more light energy is absorbed;
- be **partially transmitted** through the material and **partially reflected or scattered** at the surfaces. **Translucent** materials such as frosted glass in bathroom windows are like this;
- be almost completely **absorbed** so that no light emerges on the other side. **Opaque** objects such as brick walls and wood absorb most of the light rays that fall on them. Some of the light that is not absorbed is **reflected** off the surface of opaque objects. This allows us to see the object. Shiny or lustrous objects such as polished metals or mirrors reflect more light than rough surfaces. **Scattering** of reflected light from an opaque object indicates that the surface is irregular or rough.

Reflection of light

A certain amount of light is always reflected from a surface that separates two different media. For example, sunlight reflects off the surface of window glass or off a smooth water surface. If it were not for this reflected light we could not see most objects. Only luminous objects (eg. the Sun and a candle) emit their own light, which allows us to see them. The Moon, however, does not emit its own light. We see it due to the reflection of the Sun's rays into our eyes.

- Smooth surfaces reflect light rays in one particular direction. This is called **regular reflection.**

If light from a torch is shone at an angle of 30° onto the surface of a mirror, the reflected light is brightest when viewed at 30° to the mirror's surface. (See Figure 1.39.)

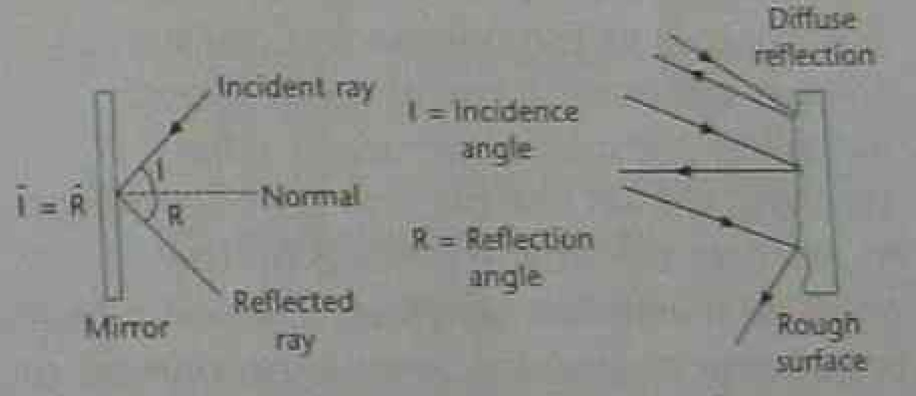


Figure 1.39 Reflection of light at an acute angle to a mirror and a rough surface

Rough surfaces cause the reflected light to be scattered in many directions. Light from a torch which reflects from a rough surface is bright when viewed from many directions. The surface of a white sheet of paper looks uniformly bright when illuminated by overhead lights, due to the scattering of light on reflection.

- When parallel rays of light reflect off a rough surface the scattering of the rays is called **diffuse reflection**.

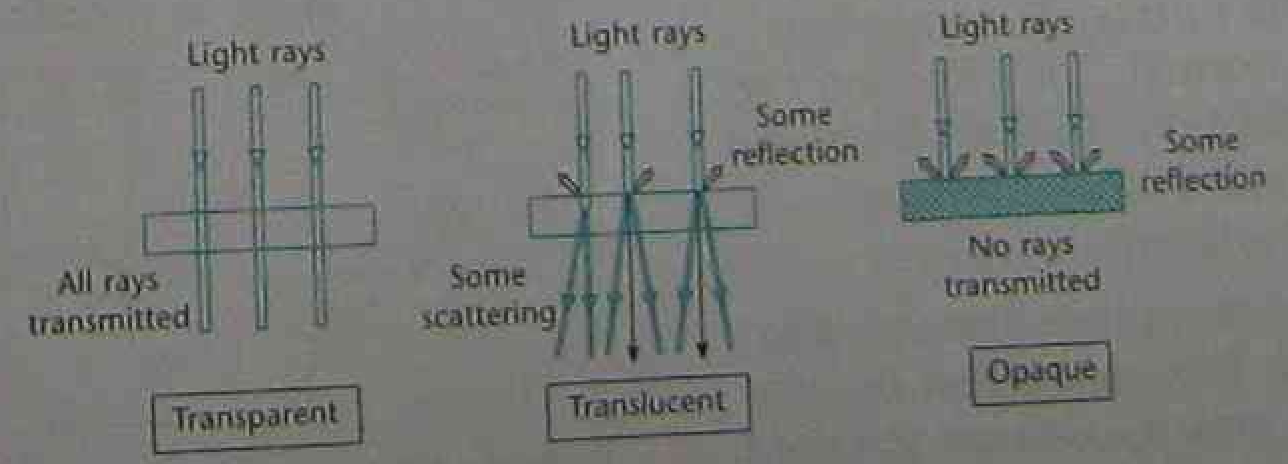


Figure 1.38 Properties of light rays

Curved surfaces reflect light rays in a different way to flat (or plane) surfaces. Figure 1.40 shows examples of plane, concave and convex reflecting surfaces. Parallel light rays are made to **converge** when they reflect off concave mirrors, whereas with convex mirrors the rays **diverge**.

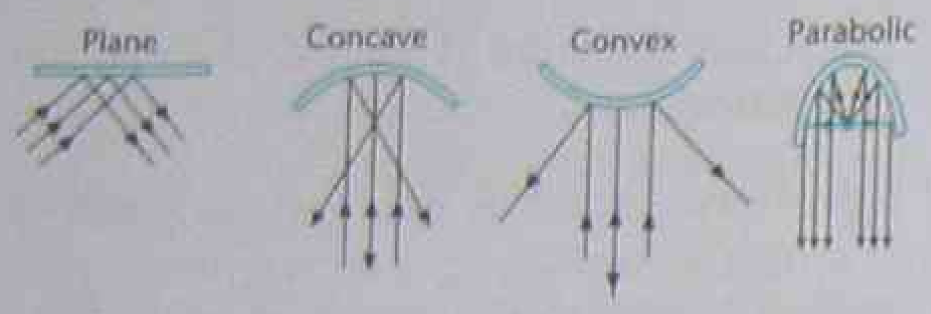


Figure 1.40 Reflection from plane, concave and convex surfaces

Plane mirrors are used in our homes in the bathroom and the bedroom. Concave mirrors are used in many applications, including telescopic mirrors to reflect starlight, dental mirrors and shaving mirrors. In all cases the image formed by the concave mirror is magnified. Convex mirrors are commonly used as rear-vision mirrors on the passenger side of a car. The images are reduced in size and the objects appear closer than they really are but such mirrors give a wide field of view. Parabolic shaped mirrors are useful in car headlights as they cause light rays to reflect off them to form parallel beams. When light rays reflect off a surface, they obey the **law of reflection**. This law states:

The angle of incidence is equal to the angle of reflection.

Figure 1.41 shows that the angle of incidence and the angle of reflection are both measured from an imaginary line at right angles to the surface. This imaginary line is called the **normal**. The incoming ray is called the incident ray. This law is obeyed when light rays reflect off any surface, including rough surfaces.

The image formed in a plane mirror is also **laterally inverted**. This means that the left side of the object now appears on the right

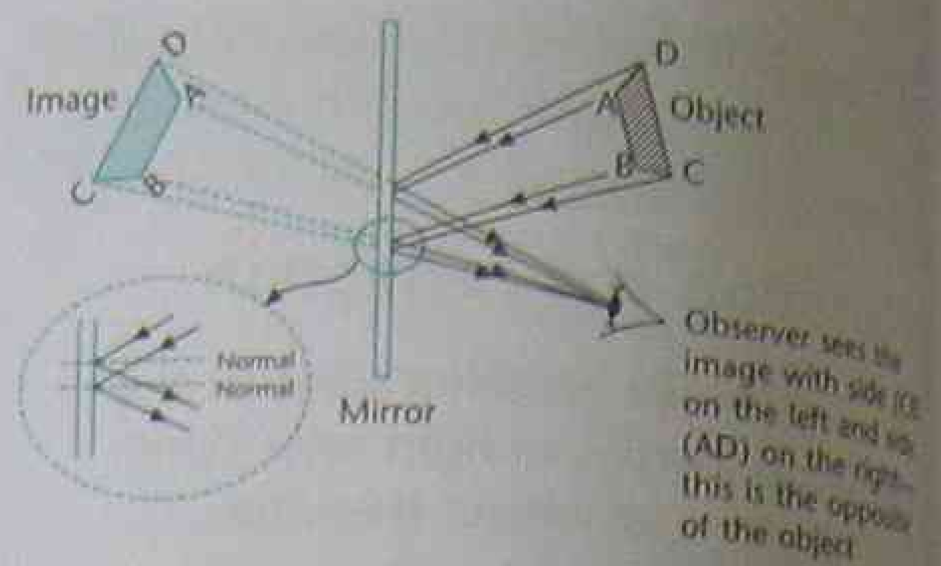


Figure 1.41 Law of reflection and lateral inversion

of the image. Because of this, some emergency vehicles such as ambulances and police cars have words written backward on the front of the vehicle so that a motorist in front can read it correctly when looking in the rear vision mirror.

Refraction

Light rays only travel at 300 000 km/s in a vacuum. In gases the speed is slightly reduced, but in liquids and transparent solids the speed of the light is considerably reduced. For example, in glass the speed drops to around 200 000 km/s.

If a ray of light passes from air into a block of glass it will slow down while in the glass. If the ray is incident at right angles to the surface, it will continue on into the glass and emerge into the air on the other side. The ray does not deviate from its original direction. If the incident ray strikes the glass surface at some other angle, the slowing of the ray while in the glass leads to a deviation or **bending** of the ray. This bending of the ray of light is called **refraction**. Figure 1.42 shows the path of two rays through a glass block.

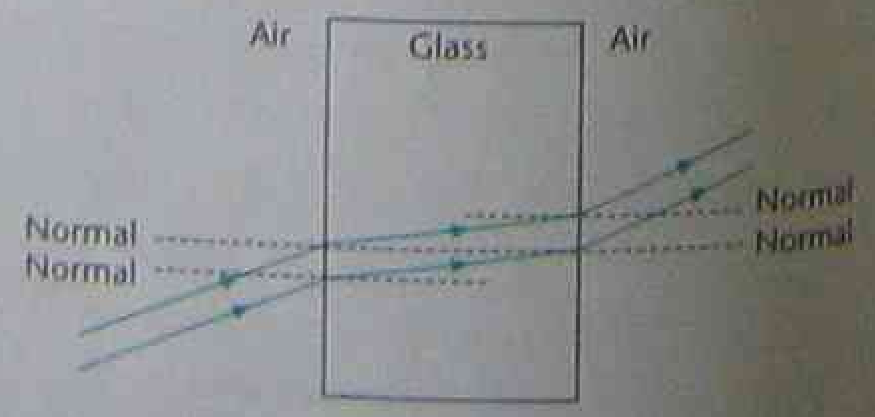


Figure 1.42 Refraction of light through a glass block

Generally when light rays are incident at an acute angle to a surface they:

- **bend towards the normal** when they pass from a less dense medium into a more dense medium (eg. from air to water, from water to glass);
- **bend away from the normal** when they pass from a more dense medium into a less dense medium (eg. from water to air, from glass to water).

The above generalisations help us to explain some common observations.

Example 1. A pool of water appears shallower than it really is

Figure 1.43 shows that the apparent depth of a pool is less than its real depth due to the bending of light rays away from the normal as they emerge from the water into the air.

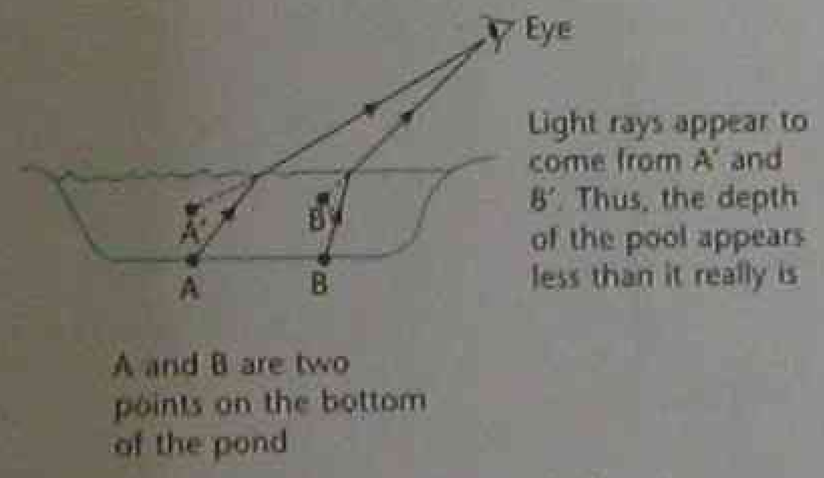


Figure 1.43 Apparent depth of a pool of water

Example 2. The apparent altitude of the Sun

Figure 1.44 shows the bending of rays of sunlight while they pass through different layers of atmosphere. Near the ground the layers of atmosphere. Near the ground the density of the atmosphere increases and the refraction increases. The Sun appears to be higher in the sky than it really is.

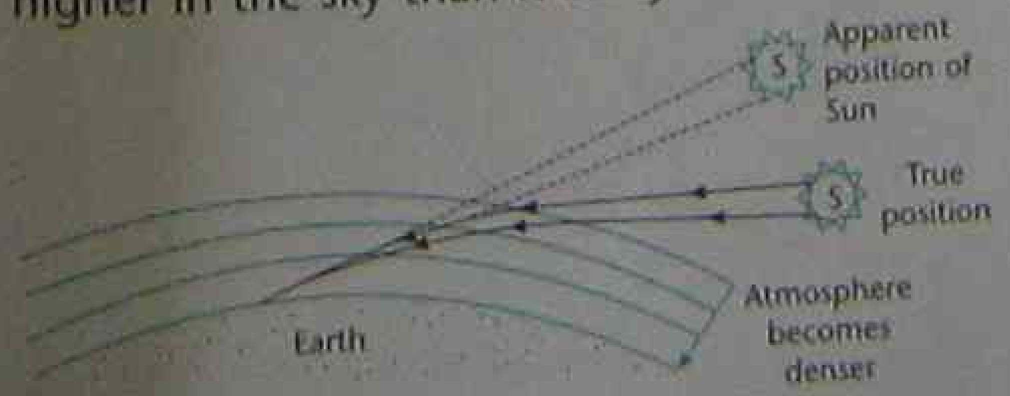


Figure 1.44 Apparent altitude of the Sun

Test yourself (answers on pages 209–10)

Part A. Knowledge (answers on page 209)

1 When light passes through a window pane it is:
  a transmitted completely without loss of energy.
  b scattered and the glass becomes translucent.
  c slowed down as glass is more dense than air.
  [✓ d] totally absorbed. (1 mark)

2 A piece of shiny black cardboard is placed under a fluorescent light. Select the best answer.
  [✓ a] The light is partly absorbed and partly reflected.
  b The light is completely absorbed as the cardboard is opaque.
  c The light is absorbed and refracted.
  d The light is scattered after reflection and little light is absorbed. (1 mark)

3 Select the correct statement about light.
  a The planets of the solar system are all luminous objects.
  b The Sun and the Moon emit light.
  c The Sun emits light and the Moon reflects and scatters light.
  [✓ d] The atmosphere of Earth absorbs most of the visible light that passes through it. (1 mark)

4 Refraction is the process in which light rays:
  a are scattered.
  b are reflected and absorbed.
  c converge after reflection from a concave mirror.
  [✓ d] are bent as they travel at an angle through media of different density. (1 mark)

5 A correct practical application of the reflection of light from a convex mirror is:

[✓ a] the passenger-side mounted rear-vision mirror.

b the mirror in the Hubble space telescope used to capture starlight.

c a shaving mirror.

d a car headlight reflector. (1 mark)

6 Complete the following restricted-response questions using an appropriate word. (1 mark each)

a When a light ray is reflected off the surface of water, the angle of incidence equals the angle of reflection.

b Light rays do not pass through gold and consequently gold is ~~transparent~~ opaque

c A rock pool is actually deeper than it looks due to the refraction of light.

d Translucent glass causes light rays to scatter when they pass through it.

e The image formed in a plane mirror is laterally inverted.

7 Use the code letters to match the terms or phrases in each column. (1 mark each)

Column 1	Column 2
a converge	f concave mirror
b smooth surface	g speed of light in a vacuum
c 300 000 km/s	h burning candle
d luminous	i line at right angles to surface
e normal	j regular reflection

8 Draw a ray diagram to show why opaque objects cast shadows on a screen. (2 marks)

9 Explain why algae and other photosynthetic plants do not grow at depths below about 300 metres in the ocean. (2 marks)

10 Mirrors are made from pieces of plane glass by silvering the back of the glass. The mirrors have various uses, including as a periscope. Periscopes are used in submarines to observe surface boats while the submarine is still submerged.

a Draw a diagram to show how two mirrors and various pieces of tubing or cardboard can be used to make a periscope. (2 marks)

b Explain why the periscope will not work well if the glass in the mirrors is too thick. (1 mark)

Part B. Skills (answers on pages 209–10)

1 Figure 1.45 shows two mirrors (A and B) at right angles to each other. A ray of light strikes mirror A. Complete the diagram to show the subsequent path of the light ray. (2 marks)

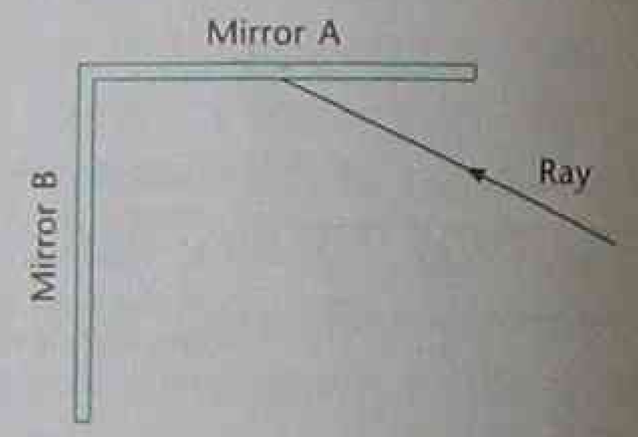


Figure 1.45 Two mirrors

2 Figure 1.46 shows a ray of light in air that is about to enter a semi-circular block of plastic. The angle between the ray and the plastic block is 30°.

a State the angle of incidence. (1 mark)

b Copy and complete the diagram to show the subsequent path of the ray into and out of the plastic block. (1 mark)

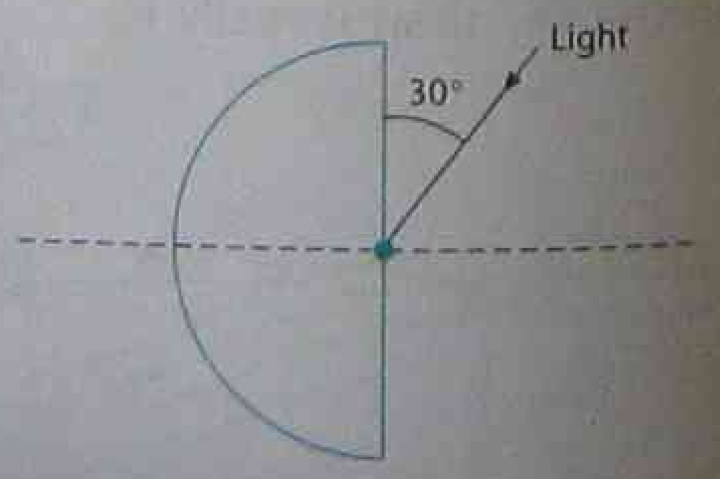


Figure 1.46 Semi-circular plastic block and light ray

c On your diagram, label the (i) angle of incidence, (ii) angle of refraction, (iii) refracted ray. (3 marks)

d When a student conducts this experiment she observes that the refracted ray has a lower brightness than the incident ray. Account for this observation. (2 marks)

3 Diamond has a much greater ability to refract light than glass. Figure 1.47 shows a slab of glass sandwiched between two diamond slabs. A ray of light in air is incident on one diamond surface as shown. Copy and complete the diagram to predict the subsequent path of the light ray. (2 marks)

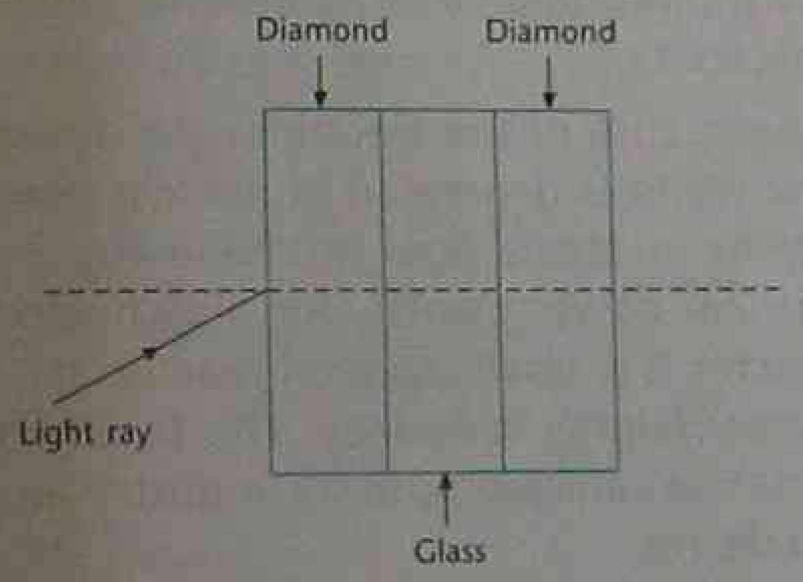


Figure 1.47 Glass and diamond slabs

4 Diverging light beams from a small lamp are incident on the surfaces of a convex and concave mirror as shown in Figure 1.48. Copy and complete the diagrams to show the subsequent paths of these beams. (2 marks)

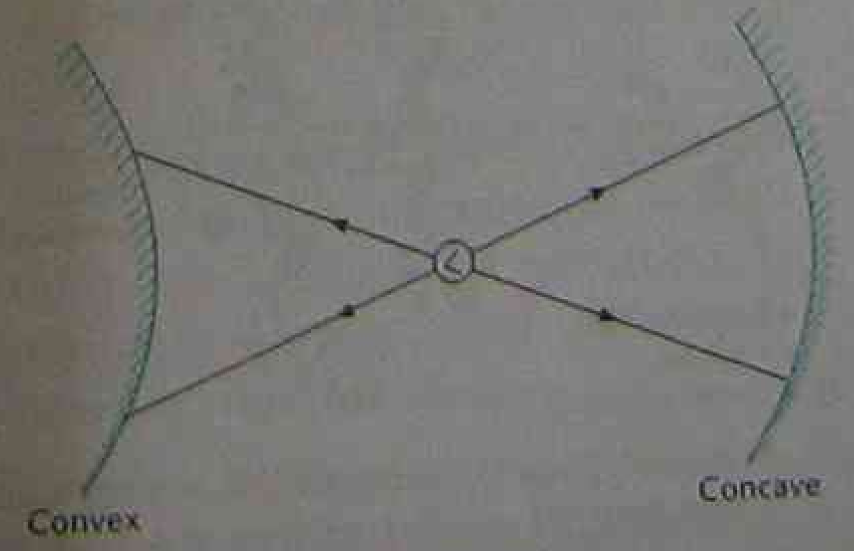


Figure 1.48 Convex and concave mirrors

5 A narrow beam of white light is incident at 45° on one side of an equilateral triangular glass prism. A student observes that as the beam passes through the prism and emerges from the other side, the light is no longer white. Instead a narrow, diverging rainbow of seven colours is observed. He observes that the red coloured beam is less refracted than the violet coloured beam. The remaining colours of the rainbow are refracted to intermediate extents. The rainbow forms an image on a white screen beyond the prism.

a State one deduction that can be made from this experiment. (1 mark)

b Draw a labelled diagram to illustrate this experimental observation. Show only the red and violet beams emerging. (2 marks)

6 A student conducted an experiment in which the angle of refraction of various light rays was measured as rays of variable angles of incidence passed from air into a rectangular tank of water. The results of the experiment are tabulated below.

Incident angle (°)	Refracted angle (°)
10	7.5
20	15
30	22
35	25.5
40	29
45	32

a Draw a labelled diagram of the experiment in which the incident angle was 30°. (2 marks)

b Plot the data as a line graph. (3 marks)

c What conclusions can be made from this experiment? (1 mark)

Nuclear energy

Nuclear energy is the energy that **binds nuclear particles** such as protons and neutrons into the nucleus. This energy is very large and it can be released for the use of humans. Since the 1940s this energy has been used for peaceful purposes such as generation of electricity in nuclear power stations and also in the treatment and diagnosis of disease. It has also been used in war (eg. atomic bombs dropped on Japan in 1945) and in nuclear missile testing programs.

Glossary

Alpha particle—radiation consisting of a positive particle composed of two protons and two neutrons

Beta particle—radiation consisting of a fast electron

Gamma rays—high energy electromagnetic rays

Fission—splitting of large nuclei into smaller nuclei by neutron bombardment with the release of energy

Fusion—joining of the nuclei of lightweight elements to form a heavier element with the release of energy

Radioactivity—emission of rays and/or particles due to the decay of an unstable nucleus

Energy release from the nucleus

Nuclear power

The nucleus of an atom is very small compared to the total volume of the atom. Most of the mass of the atom, however, is concentrated in this tiny space. The protons and neutrons are the particles that comprise the nucleus.

- Protons have a positive charge; neutrons have no charge.
- The protons and neutrons are bound strongly together by nuclear forces.
- In certain heavy atoms (such as uranium 235) some of this nuclear energy can be released by causing the nucleus to split into fragments. This process is called **nuclear fission**.
- Nuclear fission can be achieved by firing **neutrons** at the nucleus of a U-235 atom.
- As the nucleus splits into fragments, more neutrons are released that then cause other nuclei to split. This leads to a chain **reaction** and an explosion unless the process is controlled.
- In a nuclear power station the nuclear fission process is **controlled** so that no explosion can occur. The energy released by the fission process is used to generate electricity.
- About 20% of the electricity produced in the world is generated in nuclear power plants. Australia does not have any nuclear power plants. Our only nuclear reactor is a small research reactor at Lucas Heights in Sydney. This facility also provides radioisotopes for industry and medicine.
- One kilogram of uranium can release as much energy on fission as that released by the combustion of 2300 tonnes of coal.

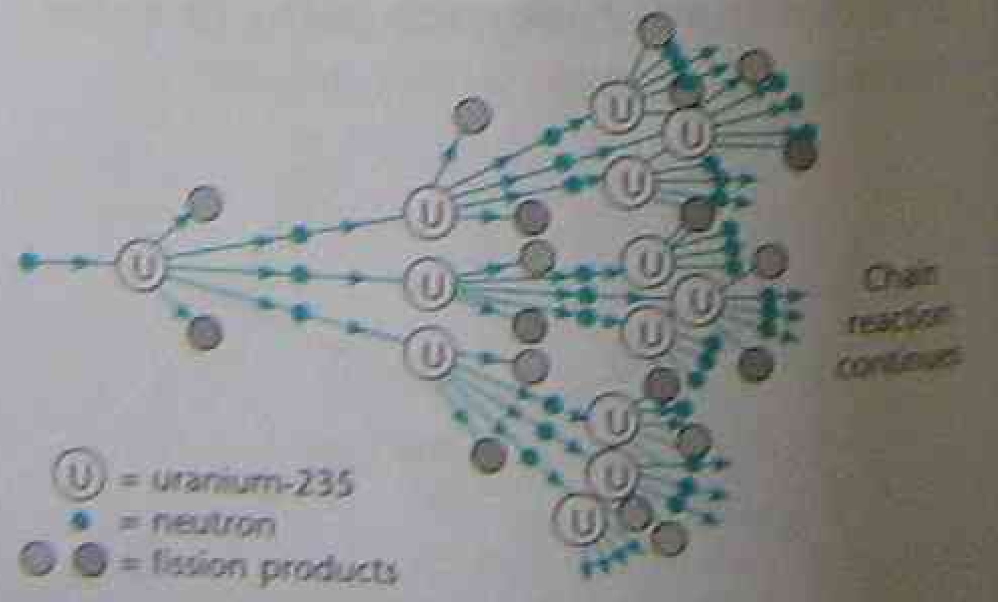


Figure 1.49 Nuclear fission of uranium-235 and the chain reaction

Energy from nuclear fusion

Energy is also released from the Sun and other stars by nuclear processes called **nuclear fusion**.

- Nuclear fusion involves the joining (fusing) together of the nuclei of lightweight atoms such as hydrogen.
- In the **Sun** and other stars the hydrogen nuclei fuse to form helium with the release of vast amounts of energy.
- On Earth, **hydrogen bombs** have been developed using the principle of hydrogen fusion. Such bombs release more energy than uranium fission bombs.
- Scientists have not yet succeeded in developing controlled nuclear fusion. The huge pressure and temperatures required to initiate and sustain controlled fusion have still not been achieved, despite many years of research.

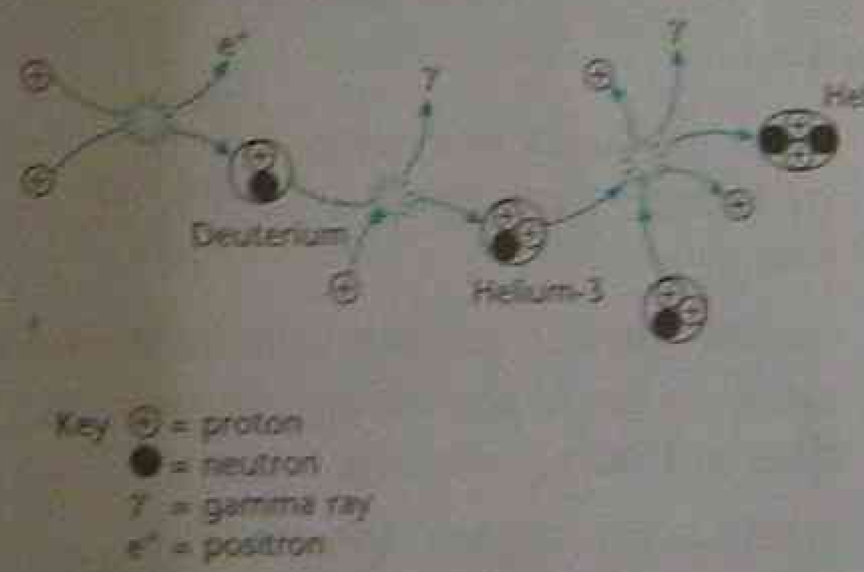


Figure 1.50 Nuclear fusion of hydrogen

Unstable nuclei

Many heavy atoms such as uranium have **unstable nuclei**. The nuclear forces are not strong enough to prevent the escape of nuclear particles or the release of radiant energy.

- Atoms are **radioactive** if their nuclei spontaneously emit particles or high energy electromagnetic radiation. This spontaneous emission of radiation is known as **nuclear decay**.
- There are three common types of **radiation** emitted from radioactive atoms.
  1. **Alpha particles**—these **positive** particles consist of 2 protons and 2 neutrons (ie. a helium nucleus).
  2. **Beta particles**—these **negative** particles are fast moving electrons that have formed by the breakdown (decay) of a neutron into a proton.
  3. **Gamma rays**—these are high energy electromagnetic rays released from a nucleus as it sheds excess energy.
- The different types of radiation have different abilities to **penetrate** matter. This is important because exposure to excessive radiation can cause disease and death in living things. The radiation can cause disruption to DNA. This disruption often leads to cancer.
  1. **Alpha** particles can travel through only several centimetres of air. They are absorbed by paper and thin metal foils.
  2. **Beta** particles are more penetrating and can travel up to several metres in air before losing their energy. Aluminium sheeting (~3 mm) will absorb beta particles.
  3. **Gamma** radiation is highly penetrating. It can travel thousands of metres through air and requires thick concrete (~1 m) or lead shields

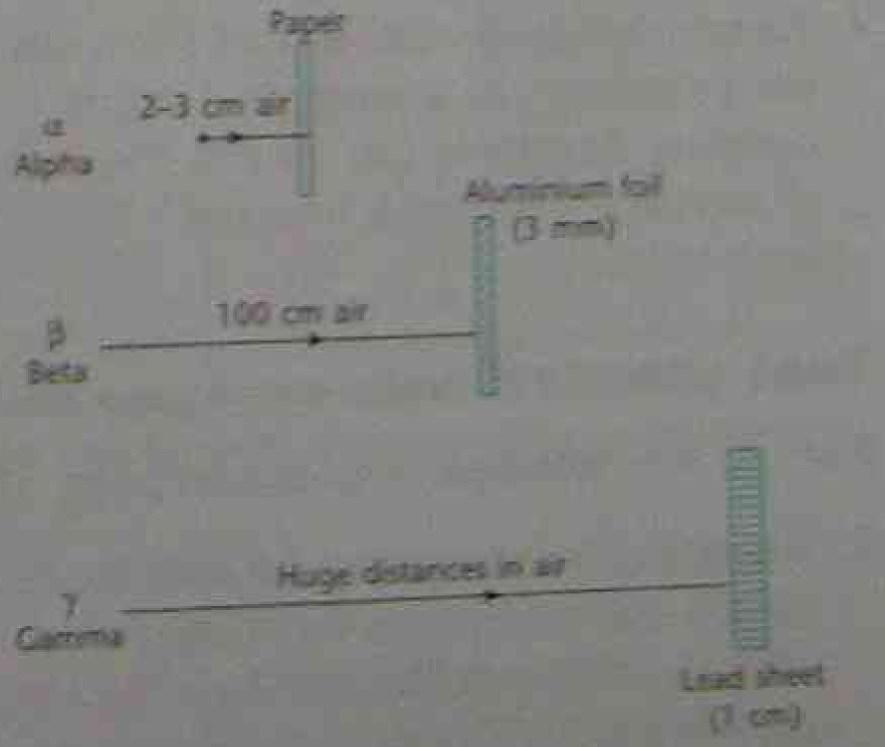


Figure 1.51 Types of radiation and their penetrating ability

(~1 cm) to contain it. It is highly dangerous to living cells.

Social and environmental issues

There are social and environmental advantages and disadvantages of using uranium as a fuel instead of chemical fuels such as coal and oil. Both types of fuel are non-renewable.

Advantages

- Large amounts of energy can be generated from small amounts of nuclear fuel. This makes nuclear fuel useful in powering nuclear submarines and some ships. It is also useful in countries that do not have coal and oil reserves.
- Unlike coal and oil combustion, few toxic gases and no greenhouse gases are released into the atmosphere by nuclear power stations.

Disadvantages

- Nuclear power plants produce highly radioactive waste products that must be stored safely in secure storage sites for thousands of years. The waste must not contaminate the biosphere.
- Nuclear power plants present a radiation problem when they reach the end of their useful lives. Methods need to be devised for safe storage of dismantled plants.
- Transportation of nuclear fuel from one site or country to another is a major concern. Accidents can lead to the escape of radioactive material into the environment.

Test yourself (answers on page 211)

Part A. Knowledge (answers on page 211)

1 Uranium-235 atoms can be split to obtain nuclear energy by firing a sub-atomic particle at the nucleus. This particle is:

a an electron.

b a proton.

[✓ c] a neutron.

d an alpha particle. (1 mark)

2 Neutrons are present in some subatom particles. Which particle contains one o more neutrons?

a An electron

b Beta particle

[✓ c] Alpha particle

d Gamma ray (1 mark)

3 Nuclear energy is released in the Sun and other stars by the process of:

[✓ a] nuclear fusion.

b nuclear fission.

c radioactive decay.

d chain reaction. (1 mark)

4 An advantage of nuclear energy over energy derived by the burning of fossil fuels is:

~~a~~ large amounts of energy can be generated from small amounts of fuel.

b nuclear power stations can be locate safely in large population centres as they are non-polluting.

c waste products are non-toxic.

[✓ d] nuclear fuel is renewable. (1 mark)

5 Select the correct statement about nuclear energy:

a Nuclear energy production is the main method of generating electricit in Australia.

b Nuclear energy is released when uranium atoms fuse together.

c Protons are used to split the nucleus of the uranium atom to release nuclear energy.

[✓ d] Energy released by nuclear fission of uranium can be used to generate electricity. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark for each part)

a Neutrons and _____ are the particles that comprise the nucleus of an atom.

b Protons and neutrons are bound together by _____ nuclear forces.

c The process in which hydrogen nuclei form helium nuclei under high pressures and temperatures is called nuclear _____.

d In a nuclear power station, nuclear fission is _____.

e Fusion bombs release more energy than _____ bombs.

7 Use the code letters to match the terms or phrases in each column. (1 mark each)

Column 1	Column 2
a controlled fission	f electron
b beta particle	g plutonium
c hydrogen fusion	h nuclear reactor
d radioactive element	i proton
e positive charge	j the Sun's source of energy

8 Explain why scientists have not yet developed nuclear fusion as a sustained and controlled source of energy. (2 marks)

9 NASA has proposed the use of nuclear powered spacecraft. State one disadvantage of the use of nuclear energy to power space vehicles. (1 mark)

Part B. Skills (answers on page 211)

1 Figure 1.52 shows three types of radiation (X, Y, Z) passing through electric fields in a vacuum. Use the information in the diagram to identify which radiation is: (2 marks)

a alpha particles

b beta particles

c gamma rays.

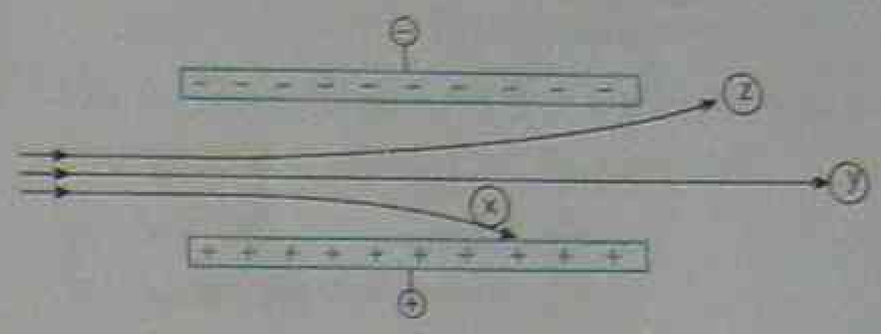


Figure 1.52 X, Y and Z and their motion in electric fields

2 Figure 1.53 shows a bar graph of the percentage of electricity derived from nuclear energy in six countries.

a In how many of the six countries is nuclear energy the major source of electrical power? (1 mark)

b What percentage of the Czech Republic's electrical energy is not derived from nuclear power plants? (1 mark)

c Convert the graphical data to tabular format. (2 marks)

d Why is Australia not shown on this graph? (1 mark)

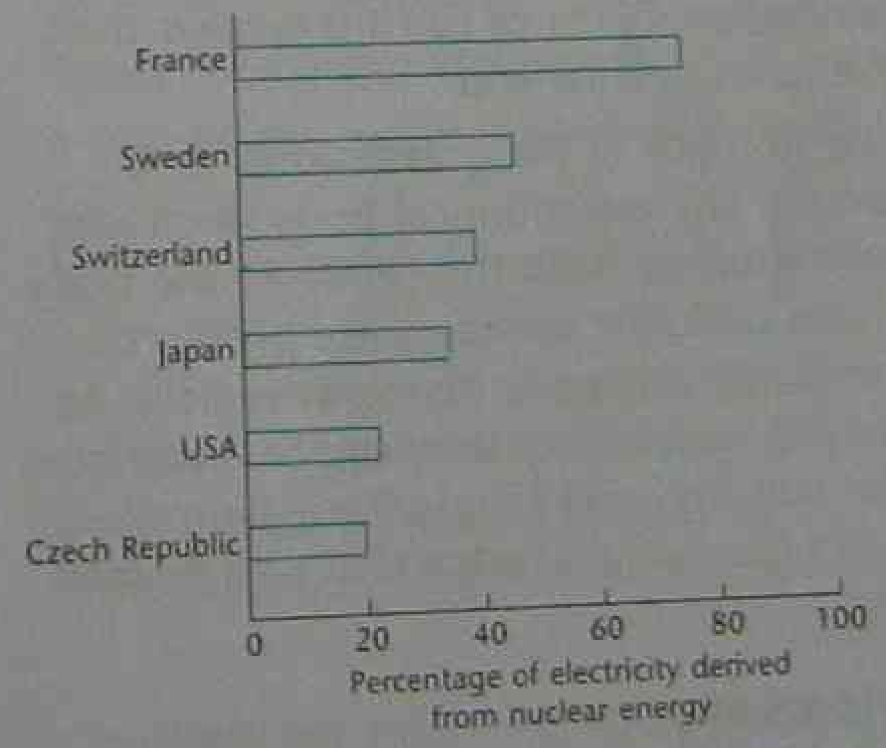


Figure 1.53 Bar graph of countries using nuclear energy

3 Hydrogen-3 is a radioactive isotope of hydrogen. Figure 1.54 shows the structure of the nucleus of ^3H and its decay products. Use the information in the diagram to deduce the origin of the

beta particle that is emitted during the decay. (2 marks)

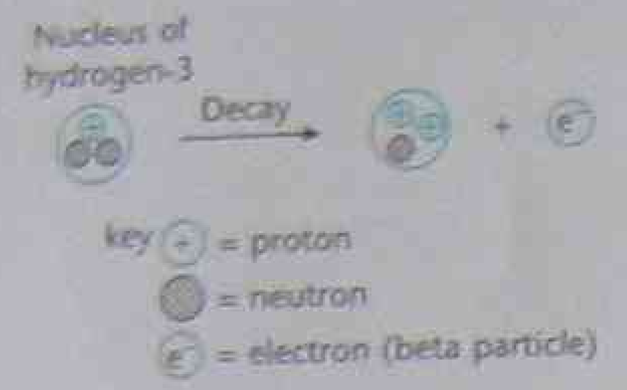


Figure 1.54 Subatomic diagram of beta decay of ^3H

Gravitational force

Isaac Newton developed a theory to explain the gravitational attraction between various objects of different mass. He was able to extend his gravitational theory to explain the motion of the planets in our solar system. The force of gravity is an example of a **field force**. A gravity field exists around all bodies. The more massive the body the stronger is the gravity field around it. The Sun is a very massive body and its gravitational field is very strong. Astronomical objects such as comets or planets become trapped in the Sun's gravitational field and orbit it. Earth is much less massive than the Sun. Therefore, the gravitational field of Earth is weaker than the Sun's gravity field.

The strength of the gravitational field around any astronomical body decreases with distance from the centre of the body. Given sufficient speed, space probes can eventually escape Earth's gravity field. As they move out into deep space away from the Sun they will eventually escape the Sun's gravitational attraction well beyond the orbit of Pluto.

Glossary

Gravitational acceleration—the acceleration experienced by a mass in a gravitational field

Mass—the amount of matter in a body

Weight—the force of gravity acting on a mass

Mass and weight

The **mass** of a body is the **amount of matter** present in the body. Mass is measured in units such as kilograms, grams or tonnes. The mass of an unknown body can be measured using a **beam balance**. By comparing it to bodies of known mass, the mass of the unknown is determined.

The **weight** of a body is different to its mass. Weight is a **force** experienced by a body when placed in a gravitational field. Weight is measured in units such as newtons (N). The weight of a body on Earth's surface is measured using spring balances called **newton meters**.

The mass of a body is always constant but its weight varies according to the strength of the gravity field. Thus a 10 kg body weighs more on Earth than on Mars as the gravity on Mars is much less than on Earth.

Additional content—Mathematical extension:

The weight (W) of a body of mass (m) in a gravitational field (g) is determined by the equation:

$$W = mg$$

Example

Q Calculate the weight of a 50 kg mass on the Moon's surface where the gravitational acceleration is 1.6 m/s^2.

A $m = 50$ kg

$g = 1.6$ m/s^2

$W = mg = (50)(1.6) = 80$ N.

Gravitational acceleration

The strength of the gravity field of a planet can be measured in terms of the **gravitational acceleration (g)**. The larger the planet or astronomical object the greater is the gravitational acceleration at its surface.

Table 1.7 gives some values of g at the surface of various planets. The value of g at the surface depends on the mass of the planet as well as its radius (ie. distance from the centre of mass).

Table 1.7 Gravitational acceleration at the surface of various planets

Planet	Mass (relative to Earth = 1)	Radius (relative to Earth = 1)	Gravitational acceleration (g) (m/s^2)
Earth	1	1	9.8
Mars	0.11	0.53	3.8
Jupiter	318	11	25.8
Saturn	95	9	11.5

Gravity and altitude

The strength of gravity decreases with altitude above the surface of a planet. In the case of Earth the surface gravity is 9.8 m/s^2. At 500 km from the surface the gravitational acceleration is 8.4 m/s^2. Table 1.8 shows the decrease in gravity with altitude above Earth's surface. At an altitude of 380 000 km, which corresponds to the orbit of the Moon, the gravitational acceleration due to Earth has dropped to 0.003 m/s^2. Although this is small it is sufficient to trap the Moon in Earth's gravitational field.

Table 1.8 Gravitational acceleration and altitude above Earth's surface

Altitude (km)	Gravitational acceleration (g) (m/s^2)
0	9.8
100	9.5
500	8.4
1000	7.3

Test yourself (answers on pages 211–12)

Part A. Knowledge (answers on page 211)

1 A 5 kg mass will have the greatest weight at the:
  - a the top of Mount Everest.
  - b the surface of the Moon.
  - [✓ c] the bottom of a deep gold mine in South Africa.
  - d the cruising altitude of commercial jets (~10 km). (1 mark)

2 On which planet will a 2 kg mass have the greatest weight at its surface?
  - [✓ a] Jupiter
  - b Saturn
  - c Earth
  - d Pluto (1 mark)

3 The mass of a body is a measure of:
  - a its volume.
  - [✓ b] the quantity of matter it contains.
  - c its density.
  - d the gravitational force that it experiences in the Sun's gravity field. (1 mark)

4 The weight of a 100 kg mass will be greatest on:
  - a the Moon's surface.
  - b the surface of Mars.
  - c the surface of an asteroid orbiting the Sun beyond Mars.
  - [✓ d] Earth's surface. (1 mark)

5 Select the correct statement:
  - a A newton meter is used to measure mass.
  - b A beam balance is used to measure weight.
  - c A 1 gram sample of gold is heavier than a 1 gram sample of feathers.
  - [✓ d] The shuttle while in Earth orbit experiences a gravitational force. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each)
  - a The force of gravity acting on a body is called its _____.
  - b The gravitational acceleration of a planet _____ with increasing altitude above its surface.

c The gravitational field around Neptune is _____ than the gravitational field around Jupiter.

d On Earth's surface a _____ kilogram mass has a weight of 98 newtons.

e Halley's comet is trapped in the Sun's _____ field.

7 Use the code letters to match the terms or phrases in each column. (1 mark each)

Column 1	Column 2
a mass	f newton meter
b gravity	g newtons
c Moon's gravity	h field force
d weight	i tonnes
e spring balance	j 1.6 m/s^2

8 A spring balance is taken to the Moon to measure the weight of rock samples. The rock samples are returned to Earth and reweighed. The data from the Moon do not agree with the Earth data. Explain. (2 marks)

9 An astronaut on a mission to Mars uses a beam balance to measure the mass of some rock samples on the surface of the planet. The samples are returned to Earth and the same beam balance is used to determine their mass. The results from both planets are identical. Explain. (2 marks)

Part B. Skills (answers on pages 211–12)

1 Figure 1.55 shows a graph of the weights of bodies of varying mass as measured by an astronaut on the surface of a planet which has the same radius as Earth.

a What is the weight of a 5 kg mass on the surface of this planet? (1 mark)

b What is the mass of a body that weighs 90 N on the surface of this planet? (1 mark)

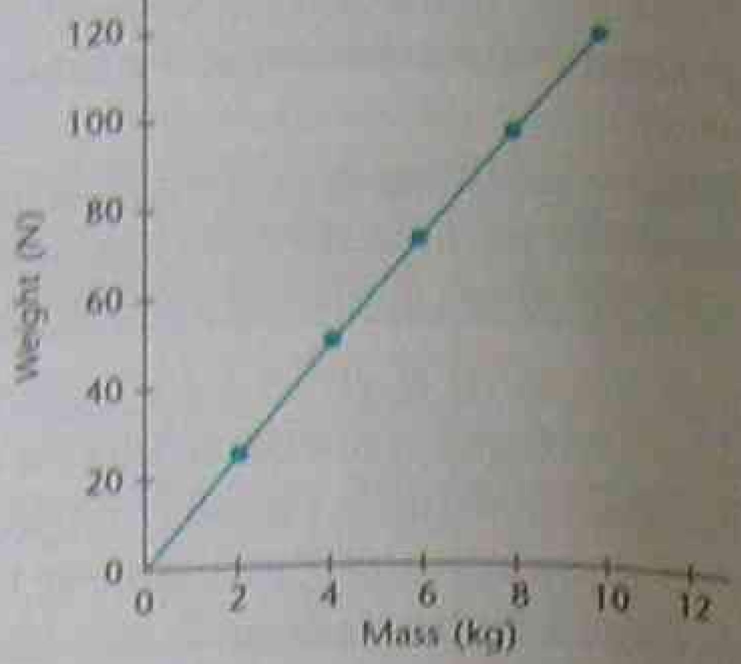


Figure 1.55 Graph of weight versus mass

c The slope of the line is equal to the gravitational acceleration on the surface of the planet. Calculate the gravitational acceleration. (2 marks)

d Is this planet more or less massive than Earth? (1 mark)

2 a A body weighs 0.6 N on the surface of Pluto. Calculate the mass of the body if the gravitational acceleration is 0.3 m/s^2. (1 mark)

b Calculate the weight of the body on the surface of Earth. (1 mark)

3 A 100 g steel ball is dropped in separate experiments from a height of 20 m onto the surface of Earth, Mars and the Moon. The time to reach the surface was measured and tabulated. The planets and the Moon are listed by code letters (X, Y, Z) in random order.

Planet or Moon	Time to reach the surface (s)
X	3.24
Y	5.00
Z	2.02

a Explain how the time to fall is related to the gravitational attraction of each astronomical body. (1 mark)

b Match the code letters to each planet or Moon. Explain your answer. (2 marks)

End-chapter test (answers on page 212)

1 Figure 1.56 shows two plane mirrors that are parallel, with their reflecting surfaces facing each other.

A ray of white light is incident on one mirror as shown.

a Copy and complete the diagram to show the subsequent path of the reflected beam. (2 marks)

b Predict and explain how the brightness of the reflected beam will change after each reflection. (2 marks)

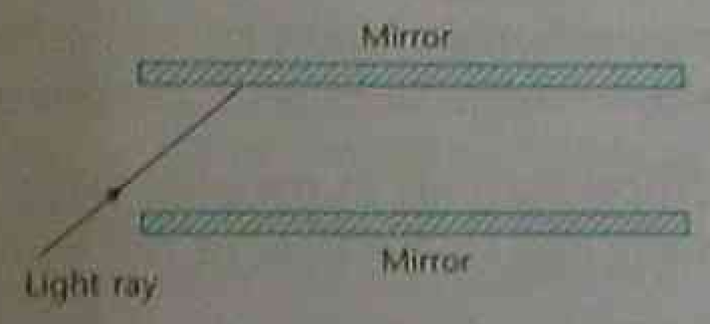


Figure 1.56 Reflection from two parallel mirrors

2 Three parallel beams of light travel upward at an angle through water until they reach the surface of the water. They are incident on the water–air surface at 30°. Carefully draw a diagram to show the subsequent path of the rays as they leave the water and enter the air. (2 marks)

3 Figure 1.57 shows two light beams from a fluorescent lamp incident on a convex lens. The rays pass through the lens and then reflect off the convex mirror. Copy path of one ray is partially shown. Copy and complete the diagram to show the path of both light rays. (2 marks)

4 The fission of uranium-235 releases 19 million megajoules (MJ) of energy for each 235 g of uranium-235 split.

a What subatomic particle is fired at the uranium-235 nucleus to cause it to fission? (1 mark)

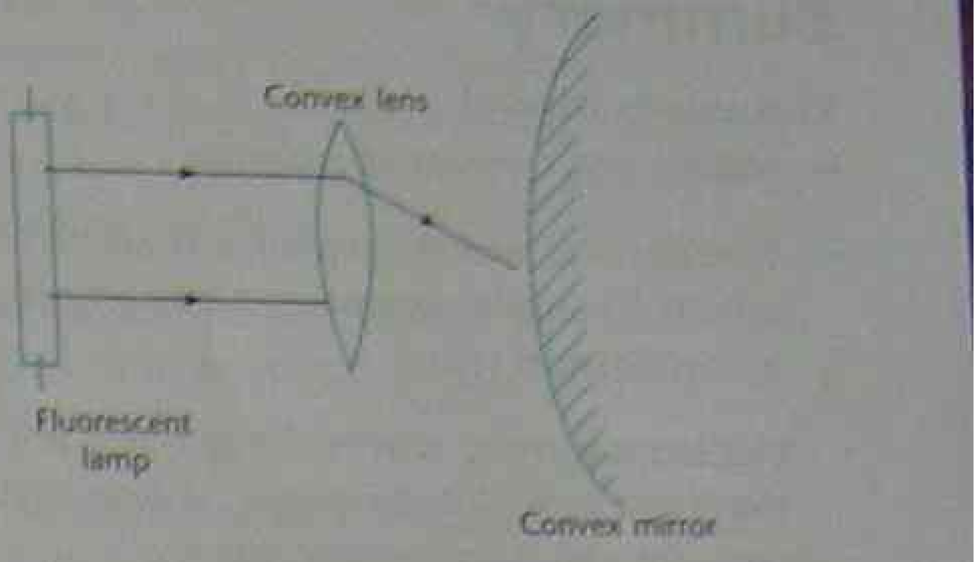


Figure 1.57 Convex mirror and convex lens

b Krypton-91 and barium-142 are products of the fission process. Why can't these products be released into the environment? (1 mark)

c Calculate the energy released in the fission of 1 kg of uranium-235. (2 marks)

d Why do some countries use a combination of nuclear power stations and coal-fired power stations to generate energy? (1 mark)

5 The weight of various bodies was measured at a very high fixed altitude above Earth's surface. The data are tabulated below.

Body	Mass (kg)	Weight (N)
A	5	47.5
B	8	76
C	7	66.5
D	4	37.4
E	12	114

a How do these data show that the measurements were made at a high altitude? (1 mark)

b One weight measurement could not have been made at the same altitude as the rest? Which is the odd measurement? (2 marks)

Summary

The wave model

- Waves are carriers of energy.
- Mechanical waves require a medium in which to move. Sound is an example of a mechanical wave.
- Electromagnetic waves do not require a medium in which to move. Visible light rays are electromagnetic waves.
- Waves can be classified as transverse waves or compression waves.
- In transverse waves the particles of the medium (or fields in an electromagnetic wave) vibrate at right angles to the wave direction.
- In compression waves the particles of the medium vibrate in the same direction as the wave motion.
- Waves are characterised by their wavelength, frequency and speed (velocity).
- The electromagnetic spectrum is a collection of electromagnetic waves that have different frequencies but which all travel at the speed of light.
- X-rays and gamma rays are high energy electromagnetic waves with high frequencies. Radio waves are low energy waves with low frequencies.

Newton's laws of motion

- Forces can be classified as contact forces or field forces.
- Force is measured in newtons (N).
- Unbalanced forces cause objects to accelerate. The greater the force (F) acting on a mass (m) the greater the acceleration (a) (Newton's second law). This is expressed by the equation: $F = ma$.
- For a fixed force, the larger the mass the lower is the acceleration.
- Balanced forces cause objects to rema at rest or in a state of linear uniform motion (Newton's first law).
- For every action there is an equal and opposite reaction. The action force and the reaction force operate on different bodies (Newton's third law).
- The average speed of a body is the tota distance travelled divided by the total time taken.
- The average acceleration of a body is th change in speed divided by the time taken.
- Objects accelerate if they change direction from straight-line motion.

Electrical energy

- A conventional current (I) is the flow of positive charge through a conductor.
- Electrical currents in metallic wires are actually due to the flow of electrons.
- Ammeters measure the size (in amps) of an electric current.
- Voltage (V) is a measure of the difference in potential energy between points in an electric circuit.
- Voltmeters measure the size (in volts) of the potential difference in a circuit.
- Resistance is a measure of the electrical conductivity of a conductor. Resistors with high resistance impede the flow of electric charge. Electrical energy can be transformed into heat in a resistor.
- Resistance (R) is measured in units calle ohms.
- Resistance can be calculated using the equation: $V = IR$.
- Resistors can be arranged in series in an electrical circuit. The total resistance of the circuit increases when the resistors are placed in series. The total current decreases.
- Resistors can be arranged in parallel in an electrical circuit. The total resistance of the circuit decreases when resistors are arranged in parallel. The total current increases.
- Lights and power points are arranged in parallel in our homes. If one component fuses then the other components will still operate.

Light energy

- Light is an electromagnetic wave. Light travels at 300 000 km/second in a vacuum.
- Light can be absorbed or reflected by matter.
- Objects that do not allow light to pass through them are opaque. If light passes through an object but is partially scattered, then the object is translucent. Transparent objects allow light to pass through them without scattering.
- Smooth surfaces reflect light in a particular direction.
- When light rays reflect off a surface, the angle of incidence is equal to the angle of reflection.
- Light waves slow down when they pass from a vacuum into matter. The denser the material, the more the light is slowed.
- Refraction is the bending of light rays caused by their slowing down in different media.
- When light rays pass at an angle from a less dense medium into a more dense medium they refract towards the normal.

Nuclear energy

- Nuclear energy binds protons and neutrons together in the nucleus of an atom.
- Nuclear energy can be released from heavy atoms such as uranium by nuclear fission. Nuclear fission occurs when the nucleus of a heavy atom is split by a fast moving neutron.
- A nuclear power station uses controlled fission to produce energy to generate electricity.
- Nuclear energy is released in nuclear fusion reactions in which nuclei of light elements fuse together. The Sun generates its energy by the nuclear fusion of hydrogen isotopes.
- Scientists have not been able to control nuclear fusion on Earth.
- Unstable nuclei are radioactive. They spontaneously decay and emit particles and rays.
- There are three common forms of radiation emitted during radioactive decay. Alpha particles are positive particles (helium nuclei) that have a low ability to penetrate matter. Beta particles are negative particles (electrons) that have a higher penetrating power than alphas. Gamma rays are high energy electromagnetic rays that have the highest penetrative ability.
- There are advantages and disadvantages in society's use of nuclear energy.

Gravitational force

- Gravitational fields exist around all bodies. The more massive the body the stronger is its gravitational field.
- Mass (m) is the amount of matter in a body. It is measured in kilograms.
- Weight (W) is the gravity force acting on a body. It is measured in newtons.
- Weight and mass are related by the equation: $W = mg$, where g = the gravitational acceleration.

Chapter 2

Atoms, Elements and Compounds

Atomic theory

The modern atomic theory was developed from John Dalton's simple atomic model of 1802. Dalton believed (like some philosophers in ancient Greece) that all matter was composed of small, indivisible particles called atoms. Dalton proposed that each element was composed of unique atoms with different atomic weights. It wasn't until the end of the nineteenth century and early twentieth century that the sub-components of the atom were identified.

Glossary

Atom—the smallest unit of an element; composed of protons, neutrons and electrons

Atomic number (Z)—the number of protons in the nucleus of an atom

Electron—a negatively charged sub-atomic particle located outside and moving around the nucleus

Electron configuration—the arrangement of electrons in their shells

Isotopes—atoms with the same atomic number but different mass numbers

Mass number (A)—the number of protons plus neutrons in the nucleus of an atom

Nucleus—the central positive core of an atom

Proton—a positively charged subatomic particle located in the nucleus

Neutron—a neutral subatomic particle found in the nucleus

Shells—energy levels (or orbits) around the nucleus occupied by electrons

Structure of the atom

Atoms are not solid bodies as believed by John Dalton. They have an internal structure.

Atoms are composed of a small, central positive core called the **nucleus** surrounded by diffuse outer shells of negative charge.

Most of the mass of the atom is concentrated in the nucleus. The nucleus is composed of varying numbers of **positive protons** and **neutral neutrons**. Protons and neutrons have very similar masses.

Outside the nucleus is a region where the **negatively charged electrons** are located. Electrons are very light compared with protons and neutrons. The mass of an electron is about 1840 times smaller than that of a proton.

There are various models that have been developed to describe the arrangement of electrons around the nucleus. One of these models is discussed on page 46.

Table 2.1 Subatomic components

Subatomic particle	Relative mass	Charge	Location in the atom
Proton (p)	1.008	+1	nucleus
Neutron (n)	1.009	0	nucleus
Electron (e)	0.000 55	–1	outside the nucleus

In all neutral atoms the total positive charge of the protons in the nucleus is equal to the total negative charge of the electrons.

Example

In a sodium atom, there are 11 protons and 12 neutrons in the nucleus. Surrounding the nucleus are 11 electrons. Thus:

Total nuclear charge = +11

Total electron charge = –11

Total charge on a neutral atom
= (+11) + (–11) = 0.

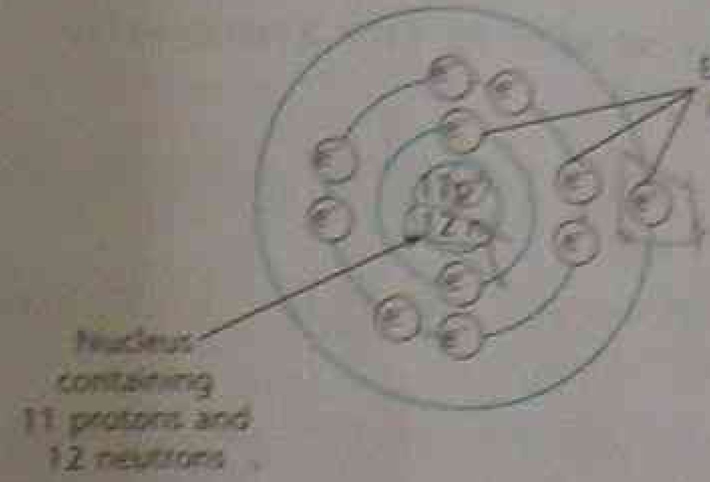


Figure 2.1 Sub-structure of a sodium atom

Elements

Atomic number

Each element has a unique proton number. This is called the **atomic number.**

- **The atomic number (Z)** is the number of protons in the nucleus of an atom
- For all neutral atoms the number of protons equals the number of electrons.

Table 2.2 Elements and atomic number

Element	Atomic number Z	Symbol
Hydrogen	1	H
Helium	2	He
Lithium	3	Li
Beryllium	4	Be
Boron	5	B
Carbon	6	C
Nitrogen	7	N

Mass number

Elements have variable numbers of neutrons. For light elements the number of neutrons is approximately equal to the number of protons. However, for heavy elements the number of neutrons is much greater than the number of protons.

Each element is also characterised by the number of protons plus neutrons. This is called the **mass number**.

- **The mass number (A)** is the number of protons + neutrons in the nucleus.

  The number of neutrons in the nucleus of an atom is therefore the difference between the mass number and the atomic number.
- **The neutron number (N) = A – Z.**

Each element can be represented symbolically using the atomic number and the mass number. For an element E, the symbol is written as:

$$^{A}_{Z}E$$

Example

The element sodium has an atomic number of 11 and a mass number of 23. This information is symbolically represented as:

$$^{23}_{11}Na$$

Thus, sodium atoms contain 23 – 11 = 12 neutrons. There are also 11 protons and 11 electrons.

Isotopes

Elements can exist in various **isotopic** forms. They have the same atomic number but different mass numbers. This is caused by the presence of a variable number of neutrons in the nucleus.

- **Isotopes of an element have the same number of protons but different numbers of neutrons.**

Example

Oxygen-16: $^{16}_{8}O$ consists of 8 protons, 8 neutrons and 8 electrons.

Oxygen-17: $^{17}_{8}O$ consists of 8 protons, 9 neutrons and 8 electons.

- Some isotopic forms of an element are radioactive. These are radioisotopes.

Example

Carbon-12 is a stable and non-radioactive isotope of carbon. Carbon-14 is a radioisotope.

$^{12}_{6}C = 6p + 6n + 6e$

$^{14}_{6}C = 6p + 8n + 6e$

Worked example

Q Complete the following table.

Element	Symbol	Z	A	N	Proton number	Electron number
Nitrogen		7	14			
	Be		9	5		
	B			5		5

A The number of protons = number of electrons = Z (for neutral atoms).

Element	Symbol	Z	A	N	Proton number	Electron number
Nitrogen	N	7	14	7	7	7
Beryllium	Be	4	9	5	4	4
Boron	B	5	10	5	5	5

Atomic weight

The weights of individual atoms of an element are very small. Consequently chemists refer to the **relative atomic weight** of an element. This weight is compared to a standard which is the carbon-12 isotope. By setting its atomic weight as 12 units (exactly), the relative atomic weights of all other elements can be compared. Because elements consist of mixtures of isotopes, the relative atomic weight (which is an average) is not a whole number.

Examples of relative atomic weight

Sulfur: 32.06

Chlorine: 35.45 35.5

Calcium: 40.08

Modelling subatomic structure

In the first half of the twentieth century various models of the atom were developed to explain the observed properties of elements.

One of the early models was developed by Niels Bohr in 1913. This is known as the **shell model** of the atom.

The Bohr model of the atom—the shell model

The main features of the Bohr atomic model are:

- Electrons occupy energy levels (or shells) around the nucleus.
- Electron shells are labelled with the letters K, L, M, N, with the K shell being closest to the nucleus.
- The maximum number of electrons that can occupy a shell increases with distance from the nucleus. The **maximum** number of electrons in each shell is:

  K shell—2 electrons

  L shell—8 electrons

  M shell—18 electrons

  N shell—32 electrons.
- The outer shell that contains electrons is called the **valence shell** and it can have a maximum of 8 electrons. Electrons in the valence shell are called **valence electrons**.
- The arrangement of electrons in shells around the nucleus is called the **electron configuration** of the atom.
- Eight electrons (an **octet**) in a valence shell provides additional stability.

Examples

1. **Hydrogen (Z = 1)**. This is the simplest element with only 1 electron in the K shell. This electron is the valence electron.
2. **Helium (Z = 2)**. This element has 2 electrons, both of which occupy the K shell. The shell is now full and helium is a very stable, unreactive element.
3. **Lithium (Z = 3)**. There are 2 electrons in the K shell and 1 electron in the L (valence) shell. The electron configuration is thus K2, L1, or more simply 2,1.
4. **Sodium (Z = 11)**. The 11 electrons are distributed with 2 electrons in the K shell, 8 electrons in the L shell and 1 electron in the M shell. The M shell in this example is the valence shell. The electron configuration of sodium is therefore K2, L8, M1, or 2, 8, 1. Like lithium, sodium has 1 electron in the valence shell.

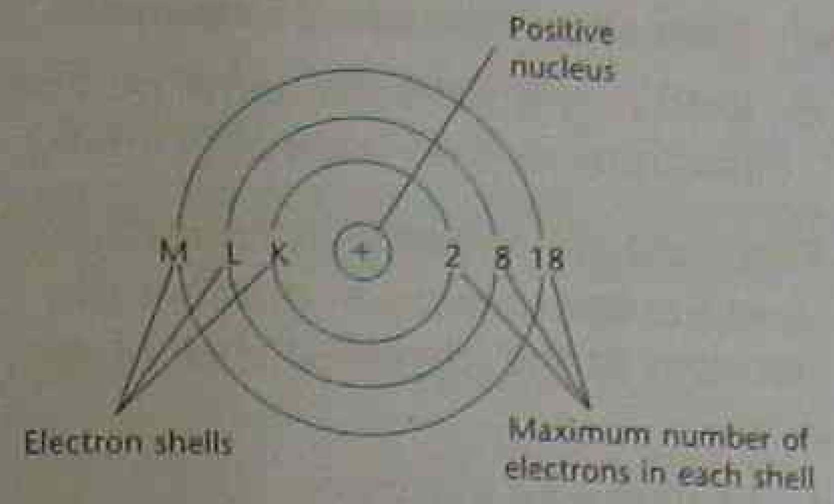


Figure 2.2 Bohr shell model of the atom

Electron configurations of elements beyond Z = 18

The need to keep the number of valence electrons at 8 or less leads to variations in the rules of filling shells with electrons.

Argon (Z = 18) has the electron configuration 2, 8, 8. The 8 electrons in the valence shell are the maximum number a valence shell can have. But the M shell can accommodate up to 18 electrons as long as it is *not* the valence shell. Thus, elements with atomic numbers between 19 and 30 have their valence electrons in the N shell and complete the filling of the M shell.

Examples

1. Potassium (Z = 19): 2,8,8,1
2. Calcium (Z = 20): 2, 8, 8, 2
3. Scandium (Z = 21): 2, 8, 9, 2
4. Titanium (Z = 22): 2, 8, 10, 2
5. Zinc (Z = 30): 2, 8, 18, 2

The Bohr model can be used to explain the physical and chemical properties of an element as well as the compounds formed when elements react together. This model is also consistent with the structure of the Periodic Table. (See page 50.)

Here are some general statements that will help you focus on the relationship between electrons, electron shells and the Periodic Table.

- **Metals are good electrical conductors.** Metals are elements that generally have 1, 2 or 3 electrons in their valence shell. These electrons are not bound to any one atom but are mobile in the metallic crystal. They are responsible for the conductivity of metals when a voltage is applied across the metal. (*Note*: Some metals have four valence electrons, eg. tin and lead.) Most elements in the Periodic Table are metals.
- **Non-metals are poor electrical conductors**. Non-metals are elements that generally have 5, 6, 7 or 8 electrons in their valence shell. These electrons are tightly bound to each atom and are not able to carry charges through the crystal when a voltage is applied across it. (*Note:* Carbon has 4 valence electrons and in the form of graphite it is a conductive non-metal due to the presence of some mobile electrons). There are very few non-metals in the Periodic Table.
- **Elements with eight valence electrons are highly unreactive**. The noble gases such as neon and argon have 8 electrons in their valence shell. They do not react with other elements, because an octet of electrons in the valence shell provides stability for the atom. (*Note:* Helium with a filled K shell is also very stable.)

- **Most metals with one valence electron are highly reactive.** Elements such as sodium and potassium have one valence electron. These elements are found on the far left column (group) of the Periodic Table. These are highly reactive metals that combine readily with non-metals such as oxygen and chlorine. During the reaction the valence electron is transferred into the valence shell of the non-metal. In this process the metal achieves a stable octet of electrons in its newly exposed outer shell.
- **Most non-metals with seven valence electrons are very reactive.** Elements such as fluorine and chlorine have seven valence electrons. They are highly reactive non-metals that combine readily with many metals. Electrons are transferred to the non-metals to complete an octet in their valence shell. This process stabilises the non-metal. Non-metals occupy the upper right-hand zone of the Periodic Table.

Test yourself (answers on pages 212–13)

Part A. Knowledge (answers on page 212)

1 Select the correct statement about the sub-structure of the atom.

- a Electrons are negative particles of similar mass to a neutron.
- [✓ b] Neutrons and protons are located inside the nucleus of an atom.
- c The number of neutrons equals the number of protons in all atoms.
- d Protons and neutrons are nuclear particles with identical masses. (1 mark)

2 An isotope of potassium has the symbol K-40. Select the correct statement about the atom of this isotope.

- [✓ a] Potassium-40 atoms contain 40 protons.
- b Each atom contains 19 protons, 19 electrons and 40 neutrons.
- Each atom contains 21 neutrons.
- d K-40 atoms have the same number of neutrons as K-39 atoms. (1 mark)

3 Select the statement that correctly identifies the maximum number of electrons in the identified shell of an atom.

- a M shell—32 electrons
- [✓ b] L shell—8 electrons
- c N shell—2 electrons
- d K shell—8 electrons (1 mark)

4 The mass of a proton is:

- a about the same as the mass of a neutron.
- [✓ b] similar to the mass of an electron.
- c much smaller than the mass of an electron.
- d positive, whereas the mass of an electron is negative. (1 mark)

5 An element with the electronic configuration 2, 8, 5:

- a is a metal.
- [✓ b] has an atomic number of 15.
- c has a mass number of 15.
- d is very unreactive. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each)

- a The outer electron shell of an atom is stable if it contains 8 electrons.
- b The mobile ______ in a metal are responsible for its high electrical conductivity.
- c Sodium is highly reactive because it has ______ valence electron.
- d All elements apart from hydrogen have ______ electrons in their K shell.
- e Carbon-14 is a ______ isotope of carbon-12.

7 Use the code letters to match the terms or phrases in each column. (1 mark each) ⑤

Column 1	Column 2
a K shell	f good electrical conductors
b metals	g valence shell octet
c argon	h seven valence electrons
d chlorine	i two electrons
e positive	j proton

8 Explain why an atom is electrically neutral. (1 mark)

9 a Explain the difference between the terms 'mass number' and 'atomic weight'. (1 mark)

b Explain why the atomic weights of elements are not whole numbers. (1 mark)

Part B. Skills (answers on page 213)

1 Copy and complete the following table. (5 marks)

Element	Symbol	Z	A	N	Proton number	Electron number
Oxygen	O	8	16	8	8	8
Carbon	C	6	12	6	6	6
Fluorine	F	9	19	10	9	9

2 Write the electron configurations for the following elements: (3 marks)

- a nitrogen (Z = 7)
- b magnesium (Z = 12)
- c argon (Z = 18).

3 For each of the following elements, use the information provided to determine the number of protons and number of neutrons: (2 marks)

- a Isotope = X-31; electron configuration = 2, 8, 5
- b Isotope = Y-60; electron configuration = 2, 8, 15, 2

4 Use the following symbols to determine the number of protons, neutrons and electrons in each element: (3 marks)

- a $^{207}_{82}Pb$
- b $^{222}_{86}Rn$
- c $^{99}_{43}Tc$.

5 Figure 2.3 shows a sample of atoms of the element boron (B). Boron consists of a mixture of isotopes of mass 10 units and mass 11 units.

Figure 2.3 Ten atoms of natural boron

- a Which isotope is in the greater abundance? (1 mark)
- b If the atomic weight of B-10 is 10 units and the atomic weight of B-11 is 11 units, would the average atomic weight of the natural mixture of boron isotopes be closest to (choose one): (1 mark)

a 10.0? b 10.5?
c 10.8? d 11.0?

Elements

Elements are the basic building blocks of matter. There are about 90 natural elements and about another 28 synthetic elements that have been produced by nuclear processes. Elements can be classified into families on the basis of their atomic sub-structure and their properties.

Glossary

Group—a column of the Periodic Table containing a family of related elements

Molecules—aggregates of two or more atoms joined by bonds

Period—a row of elements of the Periodic Table

Periodic Table—the arrangement of the elements according to increasing atomic number

Atoms and molecules

An atom is the smallest unit of an element.

Some elements consist of **single atoms** in the gaseous state at room temperature. These include the noble gases such as helium, neon and argon.

Metals can be considered to be single atoms arranged in a crystalline lattice.

Most non-metallic elements exist as **molecules** rather than single atoms.

Molecules are aggregates of atoms that are strongly joined together by chemical bonds. They are much more stable in the molecular form than as single atoms. Some elements that consist of molecules are shown in Table 2.3. Many of these non-metals form **diatomic** molecules. Sulfur forms an octa-atomic molecule whereas the white form of phosphorus is a tetra-atomic molecule (P_4). Oxygen is normally present as a diatomic molecule (O_2) but there is a toxic triatomic form called ozone (O_3).

Table 2.3 Molecular elements

Element	Molecular structure
Oxygen	O_2
Nitrogen	N_2
Hydrogen	H_2
Chlorine	Cl_2
Sulfur	S_8

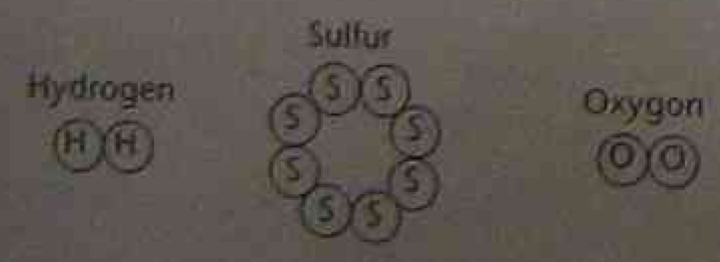


Figure 2.4 Examples of molecules

Periodic Table

The **Periodic Table** is a classification of elements. A full copy of the Periodic Table is reproduced on the **inside back cover of this study guide**.

- The elements are arranged in the Periodic Table according to their increasing atomic number (Z).

The elements are arranged in rows and columns.

- The rows are called **periods**. There are seven periods (see Figure 2.5). Not all periods have the same number of elements. The first period contains only 2 elements (hydrogen and helium) whereas the fourth period contains eighteen elements. The last two periods are very long because of the presence of two special series of elements (the lanthanides (Z = 57–71) and actinides (Z = 89–103) which are extracted to the bottom of the table to reduce its width.
- The columns are called **groups**. Each group represents a family of related elements. The group numbers are shown in Figure 2.5. The element hydrogen is sometimes placed with Group I, but in some tables it is not allocated to any group.

Metals, non-metals and semi-metals

The map of the Periodic Table also shows that the elements can be classified into three groups:

- **Metals**—the majority of elements are metals. Metals are usually shiny solids (except mercury) that are good conductors of electricity and heat. They are malleable and ductile.
- **Non-metals**—only 19 elements are classified as non-metals. Some are gases such as oxygen and argon. Bromine is a fuming red-brown liquid and some are soft, brittle solids such as sulfur and phosphorus. They are non-conductors of electricity and heat.
- **Semi-metals**—six elements are classified as semi-metals. Their properties are intermediate between metals and non-metals. Silicon and germanium are examples of semi-metals.

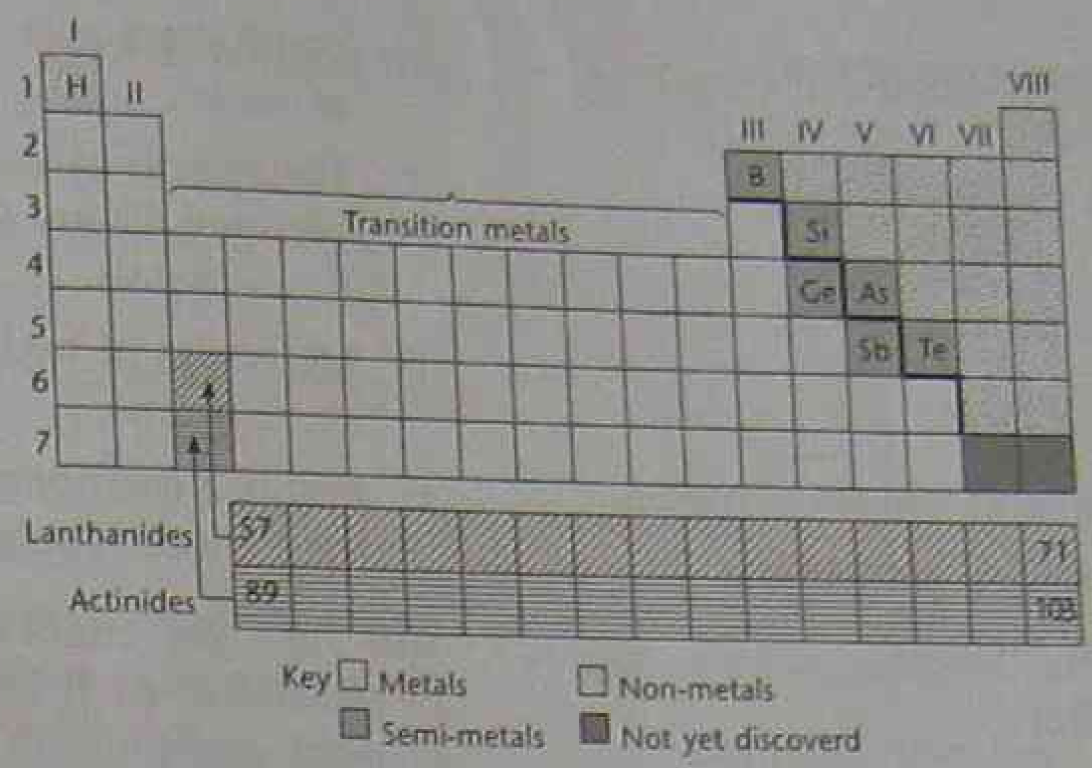


Figure 2.5 Map of the Periodic Table

Families of elements

Each vertical group of elements represents a family. The properties of elements in a family are similar. They show a gradation in properties down the group.

- **Group I—the alkali metal family**. These are soft, highly reactive metals that do not exist as free elements in nature. They react readily with water, oxygen and other non-metals, forming compounds. All Group I metals have one valence shell electron.
- **Group IV—the carbon family**. The elements of Group IV include carbon (a non-metal), silicon and germanium (semi-metals) as well as tin and lead (soft metals). All have four electrons in their valence shell.
- **Group VII—the halogen family**. These elements are reactive non-metals. Fluorine and chlorine are coloured gases, bromine is a red-brown fuming liquid and iodine is a purple-black solid. All these elements have seven electrons in their valence shell. They are non-conductors of electricity and heat.
- **Group VIII—the noble gases**. These elements are inert or very unreactive gases. Apart from helium they have eight electrons in their valence shell. They are non-conductors of heat and electricity.

Other important zones of the Periodic Table

- **The transition metals**. Between Group II and Group III is a central zone called the transition metals. Many of the important metals used in society (eg. iron, zinc, chromium, gold) are found in this zone. Each vertical group in the transition metal zone is a family of metals with the same number of electrons in their outer valence shell. Elements 104–112 are synthetic, unstable radioactive transition metals.
- **The lanthanide series (Z = 57–71)**. This is a special sub-group of the transition metals shown near the bottom of the Periodic Table. All metals in this series have similar chemical properties.
- **The actinide series (Z = 89–103)**. This is a group of natural and synthetic radioactive metals.

Test yourself (answers on page 213)

Part A. Knowledge (answers on page 213)

1 Which symbol shows the correct number

of atoms in the molecule of the nominated element?

[✓ a] Chlorine (Cl_2)
b Oxygen (O_4)
c Sulfur (S_3)
d Helium (He_3) (1 mark)

2 Select the set of elements that are all metals.
a Calcium, aluminium, germanium
[✓ b] Potassium, zinc, mercury
[✓ c] Gold, silver, sulfur
d Hydrogen, phosphorus, bromine (1 mark)

3 The element that belongs to Period 4 and Group VI of the Periodic Table is:
a Hf
[✓ b] Se
c Te
d Po (1 mark)

4 Select the set of elements that contains a metal and a semi-metal.
[✓ a] Mn, As
b K, U
c B, Xe
d He, Si (1 mark)

5 Select the set of elements that contains a non-metal and a member of the lanthanide series of elements.
a Kr, Np
b P, No
[✓ c] Br, Gd
d W, Sm (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each)
a The first element of Group VII in the Periodic Table is [illegible]
b The majority of elements in the Periodic Table are good electrical conductors and are classified as metals.
c The region of the Periodic Table between Group II and III is called the ______ metals.
d Molecules of hydrogen consist of ______ atoms.
e Iron is a member of Period ______ of the Periodic Table.

7 Use the code letters to match the terms or phrases in each column. (1 mark each)

Column 1	Column 2
a Period 1	f sodium
b Group I	g helium
c noble gas	h Be
d beryllium	i radioactive elements
e actinides	j two elements

8 a How many valence electrons do the elements of Group II each possess? (1 mark)
b Name the heaviest element in Group II. (1 mark)

9 a Define the term 'period' in relation to the Periodic Table. (1 mark)
b How many periods are shown in the Periodic Table? (1 mark)
c How many elements occupy Period 2? (1 mark)

Part B. Skills (answers on page 213)

1 a Copy and complete the following table using the Periodic Table of the elements. (3 marks)

Element	Atomic number Z	Symbol
helium		He
	8	
magnesium		
	20	
		Si
		Sn
	54	

b Identify the families of related elements in the completed table. (2 marks)

2 a Use the Periodic Table to identify the atomic weights of (i) neon, (ii) calcium, to the closest whole numbers. (2 marks)
b How many atoms of neon are required to balance the weight of 100 atoms of calcium? (1 mark)

3 Based on their position in the Periodic Table, classify the following elements as metals, non-metals or semi-metals. (5 marks)
a Rhodium
b Radon
c Rubidium
d Arsenic
e Tantalum

4 Use the Periodic Table to identify the following unknown elements. (2 marks)
a Element A: six valence electrons; member of Period 4
b Element Z: member of Period 3; yellow-green gas; reactive

5 Three **consecutive** elements of Period 3 (X, Y and Z) have the following properties.

solid solid gas

Element	X	Y	Z
Melting point (°C)	44	113	-101
Boiling point (°C)	280	445	-34
Colour	white	yellow	yellow-green

a Determine the physical state of each element at 25°C. (3 marks)
b By examining the pattern of solids, liquids and gases in the Periodic Table, identify X, Y and Z. (3 marks)

6 Figure 2.6 shows a blank diagram of the Periodic Table.

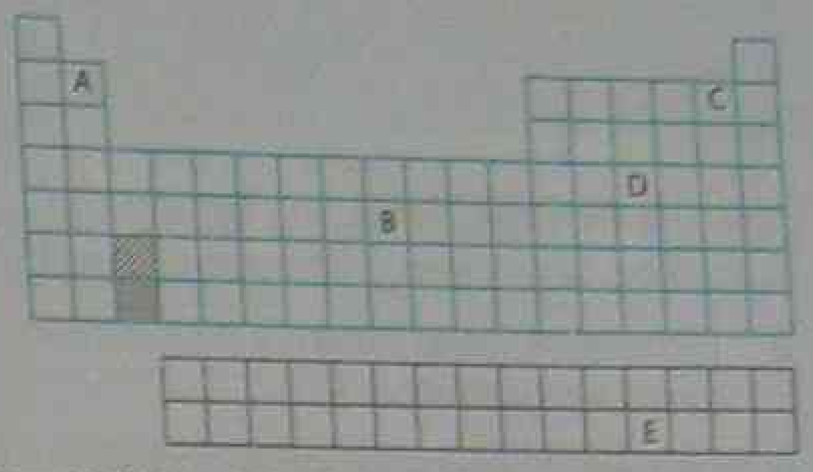


Figure 2.6 Map of the Periodic Table

Elements have been identified with code letters (A, B, C, D and E).
a Identify the period to which element (A) belongs. (1 mark) 2
b Classify element (B) as a metal, semi-metal or non-metal. (1 mark) metal
c Classify element (C) as a solid, liquid or gas at room temperature. (1 mark) gas
d Name another element that is a member of the same family as element (D) and which has a greater atomic weight than (D). (1 mark)
e Is element (E) natural or synthetic? (1 mark)

Mid-chapter test (answers on pages 213–14)

1 An isotope of element X has a mass number of 239.
The electron configuration of the element is 2, 8, 18, 32, 22, 9, 2.
a How many protons does this isotope contain? (1 mark)
b Use the Periodic Table to name this element. (1 mark)
c Is this element stable or radioactive? (1 mark)
d Calculate the number of neutrons in the nucleus of this isotope. (1 mark)

2 Use the Periodic Table to classify the following elements as metals, non-metals or semi-metals. (2 marks)

a Sb
b Cr
c Cs
d I

3 Use the Periodic Table to identify the following unknown elements: (2 marks)
  a Element X: four valence electrons; member of Period 5
  b Element Y: member of Period 2; colourless, unreactive gas, major component of air

4 Write the electron configuration of each of the following elements: (2 marks)
  a Fluorine
  b Magnesium

5 Phosphorus exists in a white form. The molecules of this white form are tetra-atomic.
  a Write the formula of a molecule of white phosphorus. (1 mark)
  b If the atomic weight of phosphorus is 31, calculate the weight of a phosphorus molecule. (1 mark)

6 Identify these elements from their descriptions. (4 marks)
  a A yellow-green gas that is used in solution to disinfect swimming pools
  b A colourless gas which is abundant in the atmosphere; it rekindles a glowing splint of wood
  c A colourless, very unreactive gas that is the third most abundant element in the atmosphere
  d A halogen that exists as a fuming red-brown liquid at room temperature and pressure

7 a Use the Periodic Table to determine the atomic weights of each of the listed elements to their closest whole numbers: (2 marks)
  i oxygen ii copper
  iii bromine iv molybdenum
  b How many times heavier than an oxygen atom is a: (2 marks)
    i copper atom?
    ii bromine atom?
    iii molybdenum atom?
    iv diatomic bromine molecule?

8 An isotope of element (E) has the following symbol: ${}^{40}_{18}E$.
  a Use the Periodic Table to identify E. (1 mark)
  b Calculate the number of neutrons in the nucleus of E. (1 mark)
  c Write the electron configuration for E. (1 mark)
  d How many electrons does E possess in its valence shell? (1 mark)
  e Discuss the reactivity of E. (1 mark)

Compounds and reactions

There are many millions of chemical compounds that can be formed when elements react. Some are natural and others are manufactured. Compounds can be classified according to the types of particles they contain.

Glossary

Anion—a negative ion

Cation—a positive ion

Combustion—the reaction of a fuel and oxidiser (oxygen) to release energy

Compound—a pure substance composed of two or more elements that are chemically combined

Corrosion—the degradation or wearing away of a metal on exposure to environmental agents such as air and water

Covalent bond—a chemical bond in which electron pairs are shared

Covalent compound—a compound in which the atoms are joined by covalent bonds

Decomposition—the breakdown of a substance into simpler substances

Effervescence—gas bubbles in a liquid

Indicators—dye molecules that change colour in the presence of an acid or base

Ion—a charged atom

Ionic bond—the electrostatic attraction between oppositely charged ions

Ionic compound—a compound composed of positive ions and negative ions

Neutralisation—the destruction of the properties of acids or bases when they react together

Precipitation—the formation of an insoluble solid on mixing solutions of ionic compounds

Valency—the combining power of an element in a compound

Compounds

Compounds are formed when the atoms of two or more elements combine chemically together.

- Compounds are *not* mixtures of elements.
- Compounds have their own unique properties that are different from the elements of which they are composed.

Example

Mercury oxide is a red powder that forms when silvery mercury is gently heated in the presence of oxygen gas. This reaction can be represented by the following word equation:

mercury + oxygen → mercury oxide

The colour difference is not the only indication that something new has formed. The physical properties of the red compound are quite different to the original elements, as shown in Table 2.4.

Table 2.4 Properties of mercury, oxygen and mercury oxide

Substance	Colour	State at 25°C	Melting point (°C)	Density (g/cm^3)
Mercury	silvery	liquid	−39	13.5
Oxygen	colourless	gas	−219	0.0013
Mercury oxide	red	solid	decomposes at 500°C	11.1

At higher temperatures (>500°C) the red mercury oxide can be decomposed back into its elements.

- Compounds are formed by the rearrangement of atoms of different elements.

Figure 2.7 shows some simple ball-and-stick models of the rearrangement of atoms of elements to form compounds. The balls represent the atoms of different elements. The sticks represent the chemical bonds (or attractive forces) that hold the particles together.

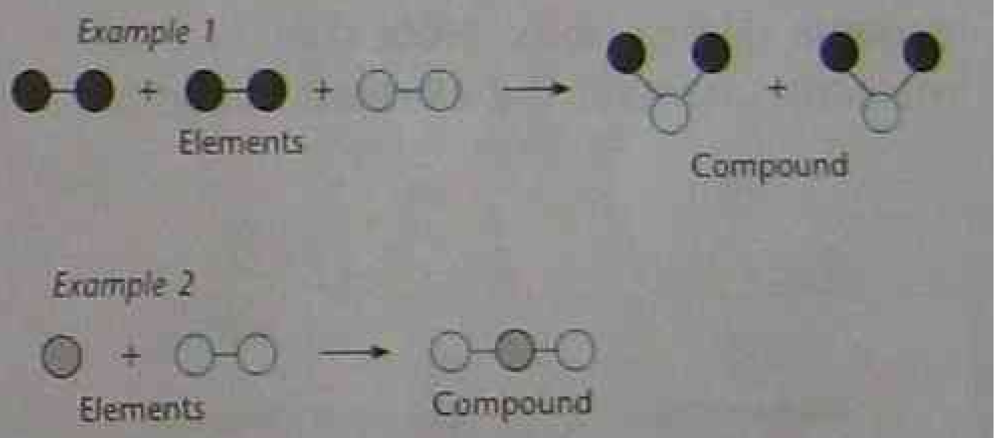


Figure 2.7 Ball-and-stick models of atom rearrangement to form compounds

- Compounds are composed of fixed numbers of atoms of each component element.

Examples

In the following examples the numbers of atoms of each type of element are shown as subscripts.

1. **Water.** Water is a compound composed of two atoms of hydrogen (H) and one atom of oxygen (O). Its chemical formula is H_2O (or HOH).
2. **Methane.** Methane is a compound composed of one atom of carbon (C) and

four atoms of hydrogen (H). Its chemical formula is CH_4.

3. **Carbon dioxide**. Carbon dioxide is a compound composed of one atom of carbon (C) and two atoms of oxygen (O). Its chemical formula is CO_2.

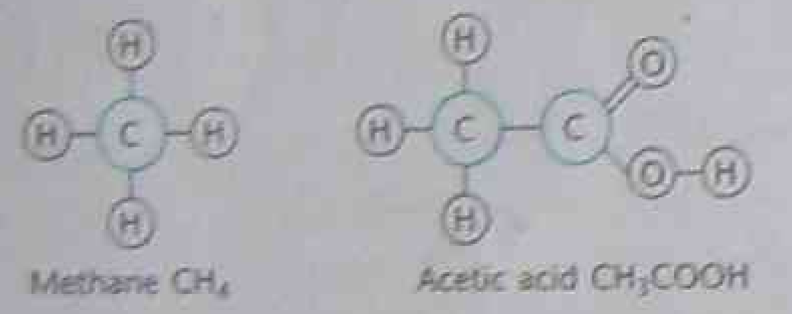
Classification of compounds

Compounds can be classified according to common chemical characteristics such as the types of particles they contain, as well as the type of bonds linking the component atoms. Some compounds belong to more than one classification scheme. Three examples of classification schemes are below.

a. Organic and inorganic compounds

This classification scheme is a simple way of distinguishing compounds that are formed by living organisms and those that are found in the rest of the environment.

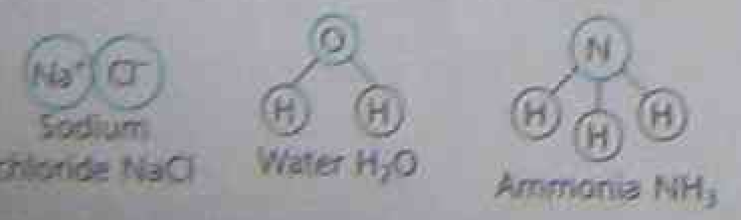
Figure 2.8 shows some models of typical **organic compounds**. Note that all these molecules contain the element **carbon**.


Methane CH_4

Acetic acid CH_3COOH

Figure 2.8 Models of simple organic compounds

Figure 2.9 shows some models of typical **inorganic compounds**. Note that some are composed of molecules and some are composed of charged atoms (called **ions**). The concept of an ion will be examined later.


Sodium chloride NaCl

Water H_2O

Ammonia NH_3

Figure 2.9 Models of simple inorganic compounds

b. Ionic compounds

Many inorganic compounds are composed of charged atoms.

- Charged atoms are called **ions**.
- Positive ions are called **cations**. They are formed when an atom of a **metal** loses one or more valence electrons. (They are attracted to the negative electrode or cathode.)
- Negative ions are called **anions**. They are formed when an atom of a **non-metal** gains one or more electrons into its valence shell. (They are attracted to the positive electrode or anode.)

The following table lists some of the common cations and anions and their Periodic Table group.

Table 2.5 Some common cations and anions in the Periodic Table

Group I	Group II	Group III	Group V	Group VI	Grou VI
sodium ion Na^+	magnesium ion Mg^{2+}	aluminium ion Al^{3+}	nitride ion N^{3-}	oxide ion O^{2-}	fluo io F^-
potassium ion K^+	calcium ion Ca^{2+}		phosphide ion P^{3-}	sulfide ion S^{2-}	chlo io Cl^-

- Table 2.5 shows that the metal ions have positive charges equal to their group number in the Periodic Table. This is also the number of electrons in their valence shell.

  The table also shows that non-metal ions have negative charges equal to their group number minus eight (eg. Group charge = 6 − 8 = −2).

  The table also shows that the name of the anion ends with the suffix 'ide'.
- Ionic compounds are composed of cations and anions. The attraction between these oppositely charged ions is called an **ionic bond**.

  Generally: Metal + Non-metal → Ionic compound

Example

A sodium atom (Na) readily forms a sodium ion (Na^+) by the loss of the single electron in its valence shell. This can be represented by a simple equation:

$$Na \rightarrow Na^+ + e^-$$

A chlorine atom (Cl) readily forms a chloride ion (Cl^-) by gaining one electron to make a stable octet in its valence shell. This can be represented by a simple equation:

$$Cl + e^- \rightarrow Cl^-$$

The sodium ion and chloride ion attract one another and form an ionic compound called sodium chloride (Na^+Cl^- or simply NaCl).

Figure 2.10 shows a model of this process.

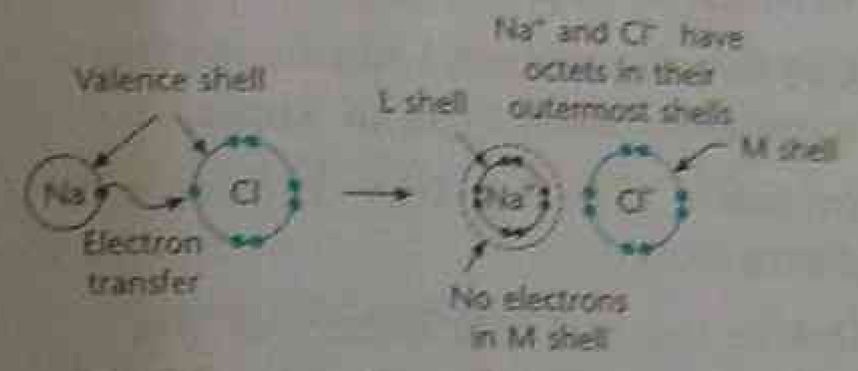


Figure 2.10 Formation of an ionic compound (NaCl)

Table 2.6 lists some common ionic compounds.

Table 2.6 Ionic compounds

Ionic compound	Chemical formula	Cation present	Anion present
Zinc oxide	ZnO	Zn^{2+}	O^{2-}
Magnesium chloride	$MgCl_2$	Mg^{2+}	Cl^-
Copper sulfide	CuS	Cu^{2+}	S^{2-}
Potassium bromide	KBr	K^+	Br^-
Calcium iodide	CaI_2	Ca^{2+}	I^-
Aluminium oxide	Al_2O_3	Al^{3+}	O^{2-}

c. Covalent compounds

When non-metals react with other non-metals to form a compound, there is no gain or loss of electrons. Instead electron pairs are shared between atoms.

- This sharing of electron pairs is called a **covalent bond**.
- The atoms in a covalent compound are linked by covalent bonds.

Generally: Non-metal + Non-metal → Covalent compound

Example

Hydrogen atoms have one valence electron and chlorine atoms have seven valence electrons. When hydrogen atoms bond with chlorine atoms they share an electron pair to form the covalent bond.

Figure 2.11 shows a model of the formation of the covalent bond in hydrogen chloride (HCl).

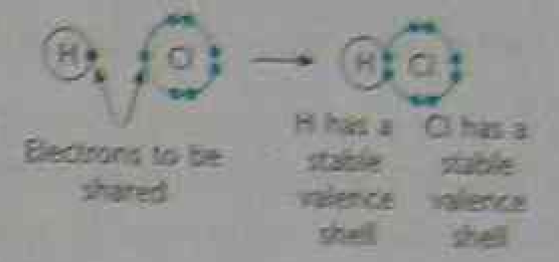


Figure 2.11 Covalent bonding in hydrogen chloride

Table 2.7 lists some common covalent compounds.

Table 2.7 Covalent compounds

Covalent compound	Chemical formula
Water	H_2O
Ammonia	NH_3
Ethylene	C_2H_4
Nitrogen dioxide	NO_2
Carbon monoxide	CO

Names and formulae of compounds

The names and formulae of compounds can be determined by following some simple rules.

a. Ionic compounds

We examine only the rules for simple ionic compounds composed of one metal and one non-metal.

Naming rules

- Name the metal first.
- Name the non-metal second.
- Delete the last few letters of the non-metal's name and substitute '-ide'.

Examples

1. metal = zinc
   non-metal = sulfur
   ionic compound = zinc sulfide.
2. metal = calcium
   non-metal = chlorine
   ionic compound = calcium chloride.

Valency rules for formulae

The chemical formula of an ionic compound can be determined using **valency rules**.

- Each metal and non-metal is assigned a valency.
- The valency is the *combining power* of the element.
- The valency of a metal is equal to the charge on its cation.
- The valency of a non-metal is equal to the charge on its anion.
- In an ionic compound the total positive charge of the cations must equal the total negative charge of the anions.

Examples

1. **Q** Determine the chemical formula of zinc sulfide.

   A Zinc sulfide: valency of zinc = +2 (Zn^{2+})
   valency of sulfur = –2 (S^{2-})

   The total positive charge equals the total negative charge.

   Thus there is one zinc ion and one sulfide ion in the compound.

   Thus, the formula of zinc sulfide = ZnS.

2. **Q** Determine the chemical formula of calcium chloride

   A Calcium chloride:

   valency of calcium = +2 (Ca^{2+})

   valency of chlorine = –1 (Cl^-)

   The positive charge is greater than the negative charge.

   Thus two chloride ions are needed to balance the charges.

   Thus there is one calcium ion and two chloride ions in the compound.

   Thus, the formula of calcium chloride = $CaCl_2$.

b. Covalent compounds

We examine only some simple examples of covalent compounds containing only two different elements.

Naming rules

- **Name the non-metal with the lower Periodic Group number first.**
- **Name the non-metal with the higher Periodic Group number second.**
- **Use Greek prefixes to indicate the number of each type of atom**

  (mono = 1; di = 2; tri = 3; tetra = 4; penta = 5).
- **Delete the last few letters of the second non-metal's name and substitute '-ide'.**

Examples

1. CO = carbon *mon*oxide
2. NO_2 = nitrogen *di*oxide
3. PCl_3 = phosphorus *tri*chloride
4. CF_4 = carbon *tetra*fluoride
5. N_2O_4 = *di*nitrogen *tetr*oxide

Valency rules for formulae

The valency rules are modified as there are no charged atoms in covalent compounds.

- Each non-metal atom is assigned a valency.
- For a simple covalent compound with two elements, the total valencies of each element must be equal.

Table 2.8 lists some of the common valencies of non-metals for Groups IV to VII. Note that some non-metals have more than one common valency.

- The normal valency of hydrogen is 1 and that of oxygen is 2.

Table 2.8 Common valencies of non-metal atoms in covalent compounds

Group IV	Group V	Group VI	Group VII
carbon 4	nitrogen 2, 3, 4, 5	oxygen 2	fluorine 1
silicon 4	phosphorus 3, 5	sulfur 2, 4	chlorine 1

Examples

1. **Q** Determine the chemical formula of silicon dioxide.

   A silicon dioxide: valency of Si = 4
   valency of O = 2

   To make the valencies of each element equal we need two atoms of oxygen (2 × 2 = 4). Thus the chemical formula of silicon dioxide = SiO_2.

2. **Q** Determine the chemical formula of dinitrogen trioxide.

   A dinitrogen trioxide: valency of N = 3
   valency of O = 2

   To make the valencies equal we need two atoms of N and three atoms of O (2 × 3 = 3 × 2).

   Thus, the chemical formula of dinitrogen trioxide = N_2O_3.

Reactions, observations and equations

Making accurate observations is an important skill when performing chemical reactions. Some events during a reaction cannot be detected using our senses, while others are easy to observe.

Common observations

- **A substance dissolves.**
- **Effervescence occurs (ie. gas bubbles form).**
- **The reaction mixture changes colour.**
- **The reaction mixture gets hot or cold.**
- **An insoluble substance (or precipitate) forms when solutions are mixed.**
- **Flames are produced or an explosion occurs.**

Observations should be consistent with the chemical equation for the reaction.

Chemical equations

A chemical equation summarises the events of a chemical reaction.

- The reacting chemicals (reactants) are written first.
- The new substances that form (products) are written last.
- An arrow separates the reactants and products. The arrow stands for the phrase 'reacts to form'.
- The equation shows that atoms are conserved in the reaction (ie. the number of atoms of reactants equals the number of atoms of products).
- The equation is consistent with the **law of matter conservation**.

Chemical equations can be written as a **word equation** or a **symbolic equation**.

Let us examine some simple cases.

Example 1. Burning magnesium in air

Observation: Silvery magnesium is heated in a Bunsen burner flame. The magnesium burns with a dazzling bright white light and a crumbly white powder is formed.

Explanation: The hot magnesium atoms react with oxygen molecules in the air to form a white ionic compound called magnesium oxide.

Word equation

magnesium + oxygen → magnesium oxide

Symbolic equation

Step 1. Write the correct symbols and formula for each substance.
magnesium = Mg
oxygen = O_2 (Remember that oxygen gas is a diatomic molecule.)

magnesium oxide = MgO
(Remember that the valency of Mg = +2 and of oxygen = –2.)

Step 2. Replace the words in the word equation by the chemical formula.

$$Mg + O_2 \rightarrow MgO$$

Step 3. Check that the atoms of each element balance on each side of the equation. (Atoms cannot disappear! All they do is get rearranged.)

Reactants: 1 magnesium atom

2 oxygen atoms (There are two atoms in the one molecule.)

Products: 1 magnesium atom (ion)

1 oxygen atom (oxide ion)

Conclusion: The magnesium atoms balance but the oxygen atoms do not.

Step 4. Place integers in front of the formula to achieve an atom balance.

In this case place a 2 in front of Mg and a 2 in front of MgO.

$$2Mg + O_2 \rightarrow 2MgO$$

Step 5. Re-check that the atoms now balance.

There are two magnesium atoms on each side and two oxygen atoms on each side.

Figure 2.12 shows a particle model of the reaction equation for magnesium plus oxygen.

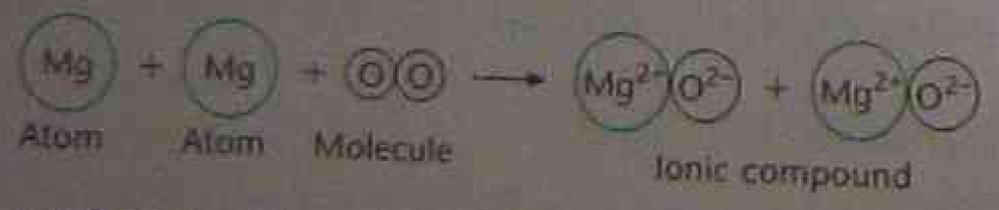


Figure 2.12 Particle model of magnesium + oxygen reaction

Example 2. Reaction of sodium and chlorine

Observation: Silvery sodium is heated until it forms a pool of molten sodium. The hot sodium is placed in a gas jar of yellow-green chlorine gas. A white smoke of very fine crystals is seen to form.

Explanation: The hot atoms of sodium rea with chlorine molecules to form white crystals of the ionic compound called sodium chloride.

Word equation

sodium + chlorine → sodium chloride

Symbolic equation

Step 1. Write the correct symbols and formulae for each substance.

sodium = Na

chlorine = Cl_2 (Remember that chlorine gas is a diatomic molecule.)

sodium chloride = NaCl. (Remember that the valency of Na = +1 and of chlorine = –1.)

Step 2. Replace the words in the word equation by the chemical formula.

$$Na + Cl_2 \rightarrow NaCl$$

Step 3. Check that the atoms of each element balance on each side of the equation. (Atoms cannot disappear! All they do is get rearranged.)

Reactants: 1 sodium atom

2 chlorine atoms (There are 2 atoms in the one molecule.)

Products: 1 sodium atom (ion)

1 chlorine atom (oxide ion)

Conclusion: The sodium atoms balance but the chlorine atoms do not.

Step 4. Place integers in front of the formula to achieve an atom balance.

In this case place a 2 in front of Na and a 2 in front of NaCl.

$$2Na + Cl_2 \rightarrow 2NaCl$$

Step 5. Re-check that the atoms now balance.

There are 2 sodium atoms on each side and 2 chlorine atoms on each side.

Combustion reactions

In a combustion reaction a **fuel** reacts with an **oxidiser** (eg. oxygen in the air) to produce **energy**, usually in the form of heat.

Fuel + Oxidiser → Products + Energy

Examples of combustion reactions are:

1. Burning a wax candle
2. Burning natural gas in a Bunsen burner
3. Combustion of petrol in a car's engine
4. A bushfire (wood cellulose and other fuels burn in oxygen)
5. Explosion of hydrogen in air
6. Burning sulfur in air

Each of these combustion reactions does not happen spontaneously when the reactants are combined. An **ignition source** is needed. We often use a spark or burning match to ignite other fuels. Once the combustion starts, the heat energy produced is sufficient to keep the reaction going. If the oxygen concentration is low, less heat is produced in the combustion and products such as black soot (carbon) or poisonous carbon monoxide are formed.

Worked example. Burning natural gas in a Bunsen burner

In this example we assume that natural gas is methane (CH_4).

Observations: With the valve (or hole) in the burner open, the gas burns with a blue flame. Heat and light energy are produced.

Explanation: The hot molecules of methane and oxygen combine to form carbon dioxide molecules and water molecules.

Word equation

methane + oxygen → carbon dioxide + water

Balanced symbolic equation

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

The balanced symbolic equation shows that one molecule of methane reacts with two molecules of oxygen to form one molecule of carbon dioxide and two molecules of water. A scientist can prove that carbon dioxide and water are formed by collecting the colourless gases coming off the flame and cooling them. The water vapour condenses to liquid water and the carbon dioxide can be confirmed because it turns limewater milky white.

Figure 2.13 shows a particle diagram of the combustion reaction of methane.

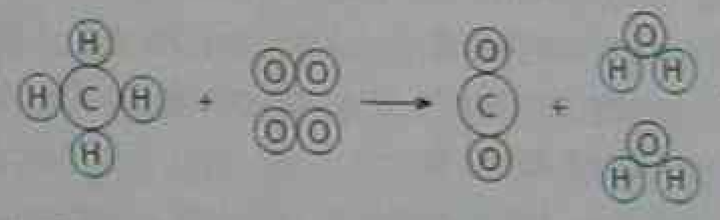


Figure 2.13 Particle diagram of methane combustion

Investigating combustion

Investigate the combustion of a wax candle in air and in a beaker containing oxygen (prepared by the catalytic decomposition of hydrogen peroxide with manganese dioxide). This experiment should show that the rate of combustion of the candle increases, as the candle burns faster when more oxygen is present.

Corrosion reactions

We are all familiar with the **rusting** of iron and steel objects. Steel bridges have to be regularly maintained to prevent them rusting. Rusted steel is weak and the steel structure will collapse if the rust penetrates deeply into the structure. In the home, wet steel wool left on the sink will quickly turn brown because it rusts. Rusting is an example of the process of corrosion.

- Corrosion is the process in which a metal **degrades** or wears away on exposure to the environment.

Iron corrodes because it reacts with oxygen and water in the environment to form a surface layer of a crumbly, brown iron compound that is called rust.

iron + oxygen + water → rust

Iron will not corrode if either oxygen or water is absent. Therefore, to protect iron from corrosion its surface must be coated with a material that prevents oxygen and/or water making contact. Paints and thin layers of metal (eg. zinc, tin) are commonly used to protect iron from corrosion. Coating of iron with zinc is called **galvanising.**

Aluminium is a common structural metal that has significant corrosion resistance. This is due to the natural layer of aluminium oxide that forms on the surface of aluminium when it is exposed to air. The oxide layer protects the aluminium below from further attack.

Investigating corrosion

The conditions under which corrosion occurs can be investigated in a series of controlled experiments. Equal quantities of fresh samples of steel wool can be placed in test tubes under different conditions. These conditions could include:

- dry air (use drying crystals in the stoppered test tube)
- moist air (place a drop of water in the stoppered tube)
- air and water present (cover half the steel wool with water)
- water only (use boiled water and immerse the steel wool; add a layer of oil to prevent air re-entering the water)
- a control tube with steel wool in an open tube.

These experiments should confirm that most rusting occurs in the presence of air and moisture.

Precipitation reactions

- A **precipitate** is an insoluble solid that forms when solutions of soluble ionic compounds are mixed.
- The precipitate is an ionic compound that has a low solubility in the solvent.

A knowledge of the solubilities of ionic compounds in water allows chemists to predict the solutions that will form precipitates on mixing. Many sulfide compounds are insoluble in water. There are many ores in nature that contain insoluble sulfide minerals.

Worked example. The reaction of copper chloride solution with sodium sulfide solution

Observation: Copper chloride solution ($CuCl_2$) is clear green. Sodium sulfide (Na_2S) solution is colourless. When the solutions are mixed, a black solid forms and falls as a sediment to the bottom of the container.

Explanation: The copper chloride solution contains copper ions (Cu^{2+}) and chloride ions (Cl^-). The sodium sulfide solution contains sodium ions (Na^+) and sulfide ions (S^{2-}). On mixing, the copper ions and the sulfide ions attract each other so strongly that they form an insoluble, black ionic solid called copper sulfide. The remaining ions do not precipitate because sodium chloride is very soluble in water.

Word equation

copper chloride + sodium sulfide → copper sulfide + sodium chloride

Balanced symbolic equation

$$CuCl_2 + Na_2S \rightarrow CuS_{(s)} + 2NaCl$$

Note that the subscript (*s*) is used to denote the solid precipitate that forms.

Figure 2.14 shows a particle diagram of the precipitation reaction.

Acids on metals and carbonates

There are a number of common laboratory acids that you should be familiar with. Their names and chemical formulae are listed in Table 2.9. They are normally used as dilute solutions for safety reasons.

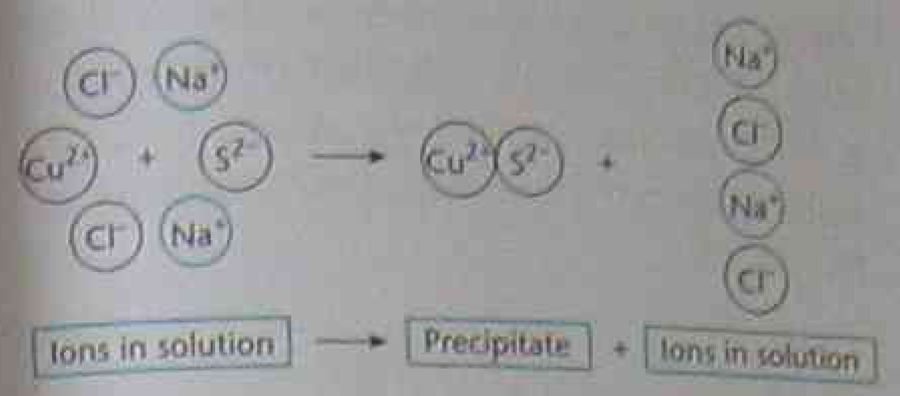


Figure 2.14 Particle diagram—precipitation

Table 2.9 Common strong laboratory acids

Acid	Formula	Ions present
Hydrochloric acid	HCl	H^+, Cl^-
Sulfuric acid	H_2SO_4	H^+, SO_4^{2-}
Nitric acid	HNO_3	H^+, NO_3^-

- These solutions are acidic due to the presence of **hydrogen ions (H^+)**.
- Sulfuric acid contains the sulfate anion (SO_4^{2-}). This is a polyatomic ion with a 2^- charge.
- Nitric acid contains the nitrate anion (NO_3^-). This is a polyatomic ion with a 1^- charge.
- There are many other acids in nature. Vinegar contains **acetic acid.** Citrus fruits contain **citric acid.** Milk contains **lactic acid.** All these acids are much weaker than the laboratory acids.

Acids reacting with metals

The reactions of dilute acids with a variety of different metals can be investigated in the school laboratory. Small, fresh samples of metals are placed in 2 mL samples of dilute hydrochloric or sulfuric acid. The rate of reaction depends on the:

- concentration of the acid—if the acid is too dilute the reaction is very slow;
- type of metal—some metals (eg. magnesium) react faster than other metals (eg. tin);
- surface area of the metal—powdered metals react faster than large lumps;
- temperature—warm acid solutions attack metals faster than cold solutions.

For acids such as dilute **hydrochloric acid** and dilute **sulfuric acid** the general equation for their reaction with reactive metals is:

Acid + Metal → Ionic compound (salt) + Hydrogen gas

- The ionic compound formed is usually called a **salt.** The word 'salt' refers to any ionic compound produced by the reaction of an acid (and not just cooking salt or table salt).
- Nitric acid behaves differently to the other acids. Its reactions do not produce hydrogen gas. Nitrogen monoxide or nitrogen dioxide is produced instead.
- The hydrogen gas can be collected and identified using the 'pop' test. (A small sample of hydrogen in the presence of air and a spark/flame will explode.)

Investigating metal reactivity with acids

The reactivity of small samples of different metals with dilute sulfuric acid can be easily investigated in the school laboratory. Tubes containing about 2 mL of the acid are used. Clean samples of each metal (eg. magnesium, zinc, iron, tin, lead, copper) can be tested. The more reactive the metal, the more rapidly hydrogen gas is evolved.

These tests should show that the order of **decreasing reactivity** of the metals in the acid is:

Mg > Zn > Fe > Sn > Pb > Cu

Lead and copper are fairly unreactive metals and little or no reaction should be observed.

Worked example

Q When zinc reacts with dilute sulfuric acid, hydrogen gas is released and the zinc dissolves to form a solution of zinc sulfate. Write a word equation and a balanced symbolic equation for this reaction.

A Refer to the general equation to complete the word equation.

Word equation

zinc + sulfuric acid → zinc sulfate + hydrogen gas

Balanced symbolic equation

Step 1. Write the correct formula for each reactant and product. (Note that the correct formula of zinc sulfate is $ZnSO_4$ as the zinc ion has a 2^+ charge and the sulfate ion has a 2^- charge.)

Step 2. Check whether the equation is balanced; if not, insert integers in front of the formulae to balance the equation.

$$Zn + H_2SO_4 \rightarrow ZnSO_4 + H_2$$

Acids reacting with carbonates

Carbonates are ionic compounds containing the carbonate anion (CO_3^{2-}).

Examples

- Zinc carbonate $ZnCO_3$
- Sodium carbonate Na_2CO_3
- Magnesium carbonate $MgCO_3$
- Calcium carbonate $CaCO_3$

Acids readily attack carbonates and dissolve them. Carbon dioxide, water and a 'salt' are formed.

Acid + Carbonate → Salt + Water + Carbon dioxide

The reaction of an acid and a carbonate is a special example of a **neutralisation** reaction.

Worked example

Q When sulfuric acid is added to a sample of green copper carbonate, the solid effervesces and dissolves in the acid to form a clear, blue solution of copper sulfate. Write a word equation and a balanced symbolic equation for this reaction.

A

Word equation

copper carbonate + sulfuric acid → copper sulfate + water + carbon dioxide

Balanced symbolic equation

Using the valency rules, the formulae for the two copper compounds can be determined.

Copper carbonate: $CuCO_3$ (as copper ions have a 2^+ charge and carbonate ions have a 2^- charge)

Copper sulfate: $CuSO_4$ (as copper ions have a 2^+ charge and sulfate ions have a 2^- charge)

Now write the equation and check that the atoms balance.

$$CuCO_3 + H_2SO_4 \rightarrow CuSO_4 + H_2O + CO_2$$

Neutralisation reactions

Neutralisation reactions involve acids and bases.

- **Acids** are substances that release **hydrogen ions** when they dissolve in water.
- **Bases** are substances that neutralise acids. Some common bases are compounds that contain oxide (O^{2-}), hydroxide (OH^-) or carbonate (CO_3^{2-}) ions.
- Acids also neutralise bases.
- When a base neutralises an acid, a solution of a salt is formed.
- The salt can be recovered from the solution by evaporating the water.
- Neutralisation reactions can be used to relieve indigestion. Weak bases such as sodium hydrogen carbonate or magnesium hydroxide in antacid tablets neutralise excess stomach acid.

Table 2.10 lists some common bases and their chemical formulae.

Table 2.10 Some common bases

Base	Formula
Sodium hydroxide	$NaOH$
Ammonium hydroxide	NH_4OH
Calcium oxide	CaO
Sodium carbonate	Na_2CO_3
Calcium carbonate	$CaCO_3$

The general reaction for neutralisation reactions involving **oxides and hydroxide** is:

Acid + Base → Salt + Water

Worked example

Q A student added hydrochloric acid to solid calcium oxide in a beaker. The calcium oxide dissolved to form a solution of calcium chloride. Write a word equation and a balanced symbolic equation.

A

Word equation

calcium oxide + hydrochloric acid → calcium chloride + water

Balanced symbolic equation

Calcium ions have a charge of 2^+.

Oxide ions have a charge of 2^-.

Chloride ions have a charge of 1^-.

Thus the formula of calcium oxide = CaO.

Calcium chloride = $CaCl_2$

$$CaO + 2HCl \rightarrow CaCl_2 + H_2O$$

Note: To balance this equation the number 2 has been inserted in front of the HCl.

Figure 2.15 shows a particle diagram of the neutralisation reaction.

The role of indicators

- **Indicators** are substances that **change colour** in the presence of an acid or base.

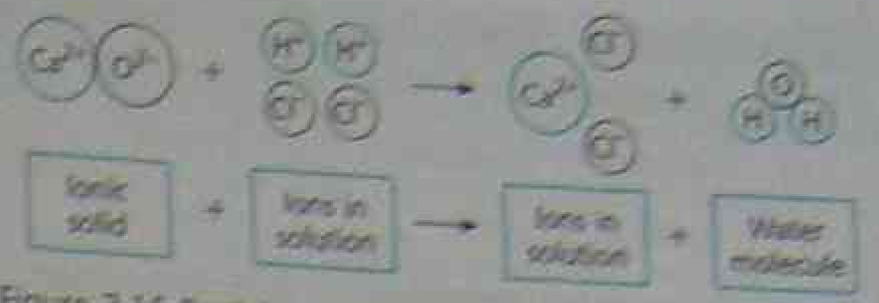


Figure 2.15 Particle model of the neutralisation reaction

- Indicators can be easily prepared from plant material such as **flower petals**. The petals can be soaked in water to dissolve the dye. The dye extract changes colour if acids or bases are added. These types of indicators are not stable for long-term use.
- Indicators can be used to determine whether a solution is **acidic, neutral** or **basic**. This is useful in the laboratory as well as domestically. Indicators are used to detect whether the water in a swimming pool is too acidic or basic.
- Indicators can be used in **neutralisation reactions** to determine the point where **exact neutralisation** occurs. This is important, especially when the reacting solutions are colourless.

Table 2.11 shows examples of some common indicators used in the school laboratory.

Table 2.11 Common indicators

Indicator	Colour in acidic solution	Colour in water (neutral)	Colour in basic solution
Litmus	red	purple	blue
Methyl orange	red	orange	yellow
Bromothymol blue	yellow	green	blue
Phenolphthalein	colourless	colourless	crimson

Worked example

Q Use the results of the following indicator experiments to classify the following solutions as acidic, basic or neutral.

Solution	Methyl orange	Phenolphthalein	Bromothymol blue
X	orange	colourless	green
Y	yellow	pink	blue
Z	orange-red	colourless	yellow-green

A Match the colours to the table of indicator colours.

- X is a neutral solution.
- Y is a slightly basic solution (the phenolphthalein is pink rather than crimson).
- Z is a slightly acidic solution (the bromothymol blue is yellow-green rather than yellow; the methyl orange is orange-red rather than red).

Universal indicator and pH

The acidity or basicity of a solution can be conveniently represented by the **pH scale**.

- pH is a measure of the acidity or basicity of a solution.
- **Neutral** solutions have a pH of 7.
- **Acidic** solutions have a pH **less than 7**. The lower the pH the more acidic is the solution.
- Strongly acidic solutions have pH around 0 to 2.
- **Basic** solutions have a pH **greater than 7**. The higher the pH the more basic is the solution.
- Strongly basic solutions have pH around 12–14.

The pH of a solution can also be correlated with the colour of indicators.

Universal indicator is a useful mixed indicator that shows a large range of colours over the pH scale.

Table 2.12 shows the correlation of the colour of Universal indicator and the pH scale.

Table 2.12 pH and the colour of Universal indicator*

pH range	0–4	4–5	5–6	7–8	8–9	9–10	10–14
Universal indicator colour	red	orange	yellow	green	blue-green	blue-violet	violet

*Note: The exact colours and ranges may vary in some different brands of Universal indicator. Always use the supplied colour card.

Worked example

Q A student dissolved some baking soda (sodium hydrogen carbonate) in water and tested the solution with Universal indicator and phenolphthalein. The Universal indicator turned green-blue and the phenolphthalein was very faintly pink.

a Determine the pH of the baking soda solution.

b State whether the solution is acidic, basic or neutral.

c Predict the colour that bromothymol blue would turn in this solution.

A a the pH is between 8 and 9. This is consistent with the Universal indicator colour chart.

b The solution is slightly basic. This is consistent with the phenolphthalein being faintly pink.

c The bromothymol blue should also be green-blue.

Table 2.13 shows some common acidic and basic substances. Universal indicator or other indicators can be used to show that these substances are acidic or basic.

Table 2.13 Common household acidic and basic substances (in decreasing order of strength)

Common acidic substances	Common basic substances
Car battery acid	Caustic soda (drain cleaner)
Vinegar (acetic acid)	Ammonia solution (window cleaner)
Lemon juice	Laundry detergents
Soft drinks	Baking soda

Decomposition reactions

Decomposition reactions involve the **breakdown** of a compound into its elements or into simpler compounds.

Decomposition can be achieved in a number of ways. These include:

- **thermal decomposition**

  Heating the substance with a Bunsen flame is a simple method used in a lab. Not all substances will decompose under these conditions.

- **electrical decomposition**

  Electrolysis is the process of decomposing a substance in solution using an electrical current. Metallic electrodes connected to a DC voltage source is a common method used. Water can be decomposed into hydrogen gas and oxygen gas by this method.

  water → hydrogen gas + oxygen gas

- **photochemical decomposition**

  Some substances are sensitive to decomposition by visible or ultraviolet light. For example, silver bromide in photographic film is decomposed into silver and bromine by light.

  silver bromide → silver + bromine

Thermal decomposition examples

In the school laboratory the decomposition of some ionic carbonates by heat can be tested. Blue-green copper carbonate can be heated in a test tube. The solid turns black, and colourless carbon dioxide is evolved. The black solid is copper oxide.

Word equation

copper carbonate → copper oxide + carbon dioxide

Balanced symbolic equation

$$CuCO_3 \rightarrow CuO + CO_2$$

Sodium carbonate and potassium carbonate are thermally stable. Carbonates of unreactive metals decompose to produce the metal. Silver carbonate decomposes to form silver metal, oxygen and carbon dioxide.

Test yourself (answers on pages 214–15)

Part A. Knowledge (answers on page 214)

1 Select the response that correctly names a compound and its chemical formula.

a Sodium chloride, $NaCl_2$

~~b~~ Calcium chloride, $CaCl_2$

[✓ c] Nitrogen trioxide, N_2O_3

d Sulfuric acid, HNO_3 (1 mark)

2 Consider the following word equation:

sulfur + oxygen → sulfur dioxide.

This reaction could be classified as a:

a precipitation reaction.

b neutralisation reaction.

c decomposition reaction.

[✓ d] combustion reaction. (1 mark)

3 Consider the following word equation:

magnesium oxide + sulfuric acid → magnesium sulfate + water

This reaction is classified as a:

[✓ a] neutralisation reaction.

~~b~~ precipitation reaction.

c decomposition reaction.

d combustion reaction. (1 mark)

NaCO3 HNO3

4 Sodium carbonate and nitric acid are allowed to react. A gas is evolved. The name of the gas evolved is:

a water.

[✓ b] carbon dioxide.

c nitrogen dioxide.

d hydrogen. (1 mark)

5 Figure 2.16 shows a particle model of a chemical reaction.

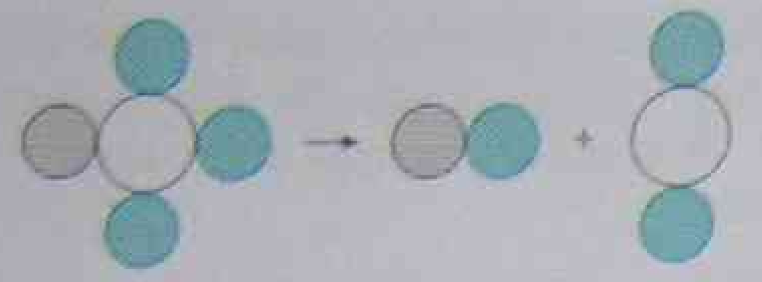


Figure 2.16 Particle model of a chemical reaction

The type of chemical reaction shown in this particle model is:

[✓ a] a decomposition reaction.

b an acid on a metal.

c an acid on a carbonate.

d a precipitation reaction. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each)

a Calcium bromide is an example of an _____ compound.

b The atoms in ammonia molecules are joined by _____ bonds.

c The correct name for N_2O_5 is dinitrogen _____ .

d An _____ is observed when magnesium is added to sulfuric acid.

e When magnesium burns in oxygen the product is called magnesium _____ .

7 Use the code letters to match the terms or phrases in each column. (1 mark each)

Column 1	Column 2
a covalent bond	f explosive combustion
b hydrogen burns in air	g shared electron pair
c rusting	h reactive metal
d magnesium	i strong base
e sodium hydroxide	j corrosion

8 State three ways in which the speed of the reaction between zinc and sulfuric acid solution can be increased. (3 marks)

9 Design a simple experiment to show that copper is a less reactive metal zinc. (3 marks)

10 Consider the following symbolic equation:

$K_2CO_3 + 2HCl \rightarrow 2KCl + H_2O + CO_2$

Convert this symbolic equation into word equation. (2 marks)

Part B. Skills (answers on pages 214-1

1 Figure 2.17 shows an atomic model a number of compounds. Use the ke provided to:

a name the compounds; (3 marks)

b write their chemical formulae; (3 marks)

c classify the compounds as ionic compounds or covalent compound (3 marks)

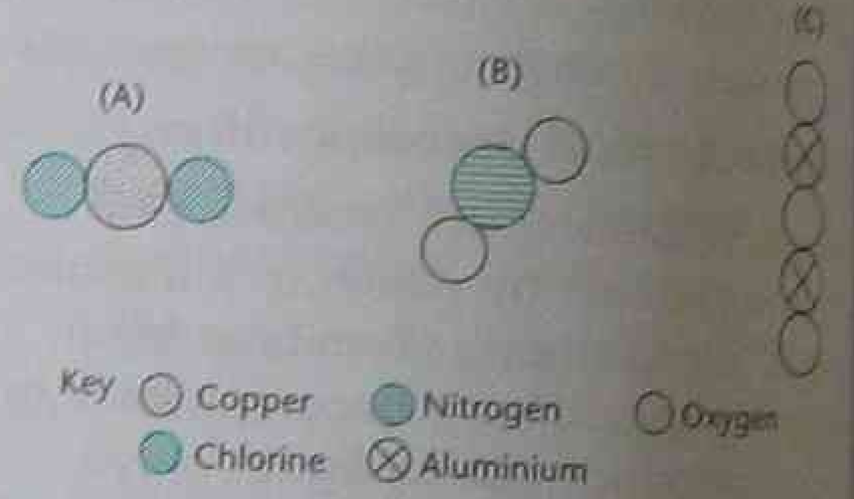


Figure 2.17 Atomic models of different compounds

2 The list below shows some common observations made during chemical reactions.

a A substance dissolves.

b Effervescence occurs.

c The reaction mixture changes colour.

d The reaction mixture gets hot or cold.

e A precipitate forms when solutions are mixed.

f Flames are produced or an explosio occurs.

Consider the reactions that occur when the following substances are combined. From the list above choose the observations that would be made in each case. Use the code letters.

a A piece of magnesium ribbon is placed in a beaker of dilute sulfuric acid. (1 mark)

b Ethanol is burnt using a spirit burner. (1 mark)

c Copper carbonate is heated strongly over a Bunsen burner flame. (1 mark)

3 The following information was collected concerning compounds of silver and sodium.

Silver chloride is an insoluble ionic compound whereas silver nitrate is soluble. Both sodium chloride and sodium nitrate are soluble in water.

All substances in the solid state are white.

All their solutions are colourless.

A solution of sodium chloride and a solution of silver nitrate are mixed together in a beaker.

a Describe what a student would observe on mixing these solutions. (1 mark)

b Classify the type of reaction occurring. (1 mark)

c Write a word equation for the reaction. (2 marks)

d Convert the word equation into a balanced symbolic equation. (2 marks)

4 Figure 2.18 shows the results of an experiment in which different iron nails are subjected to a range of environmental conditions.

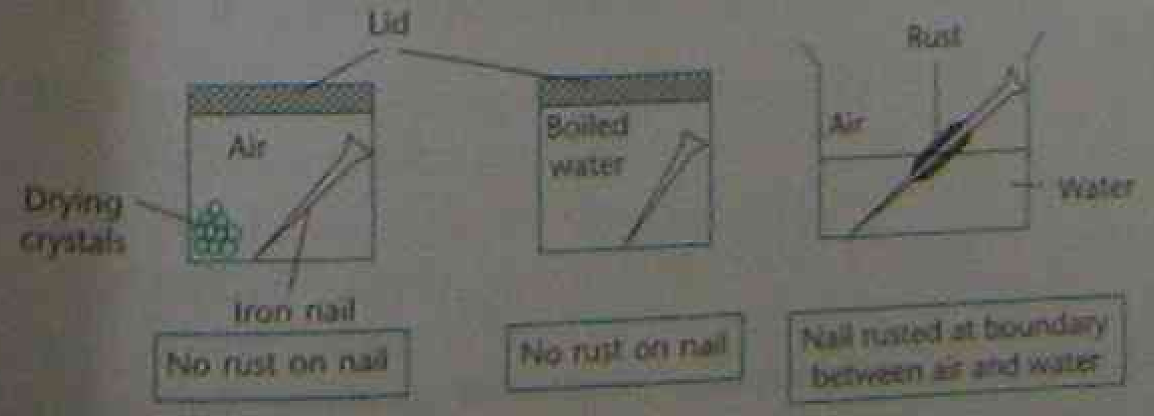


Figure 2.18 Results of the iron nail experiments

a What type of chemical change is the student investigating? (1 mark)

b What conclusions can be drawn from these experiments? (2 marks)

5 Two test tubes containing dilute hydrochloric acid and drops of Universal indicator were set up in a test-tube rack. Each tube also contained a thermometer. Two white powdered solids (X) and (Y) were slowly added separately to each tube until no further changes were observed. The results of the experiment are summarised below.

Powder	Observations
X	The white solid dissolves and the solution turns from red to green and then blue-violet. The temperature of the mixture increases.
Y	The white solid dissolves and the solution turns from red to green and then green-blue. Effervescence occurs. The temperature of the mixture increases.

a Classify the type of reaction occurring when X and Y combine with the hydrochloric acid. (1 mark)

b What is the purpose of the Universal indicator? (1 mark)

c Why does the temperature rise in each case? (1 mark)

d One substance is magnesium oxide and one is magnesium carbonate. Use the results to determine which substance is magnesium carbonate. Justify your answer. (2 marks)

e What can one conclude about the pH of the final mixture in each tube? (2 marks)

6 Figure 2.19 shows a ball-and-stick model of a chemical reaction involving molecules.

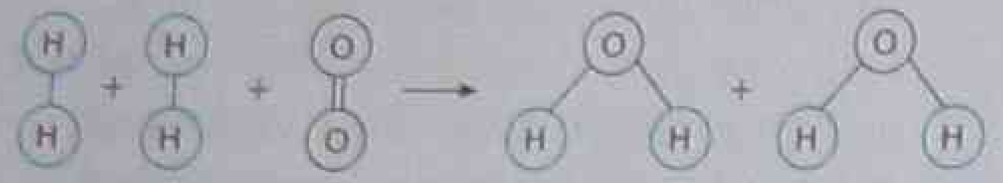


Figure 2.19 Ball-and-stick model of a chemical reaction

a What major scientific idea is illustrated by this equation? (1 mark)

b Classify the types of molecules according to the number of atoms per molecule. (2 marks)

c What type of reaction does this model represent? (1 mark)

7 Read the following description of substance Q.

Q is a colourless liquid.
Q reacts with calcium metal with the release of a colourless gas that explodes in the presence of a flame.
Q reacts with iron carbonate to produce a yellow solution and a colourless gas. The colourless gas turns limewater white.
Q conducts an electrical current.

a Suggest two chemical substances that could produce the reactions described. (2 marks)

b Write the chemical formulae for the substance named in (a). (2 marks)

c Name the gas that explodes in the presence of a flame. (1 mark)

d Name the gas that turned the limewater white. (1 mark)

8 When nitric acid is added to sodium sulfide (Na_2S) a foul smelling gas called hydrogen sulfide is released. The sodium sulfide dissolves to form a colourless solution of sodium nitrate.

The experiment was conducted with two equal mass samples (X and Y) of sodium sulfide. The volume and temperature of the acid were the same in each experiment. The volume of gas released was measured each minute and recorded, and the data were plotted as a line graph. Figure 2.20 shows the results of these experiments.

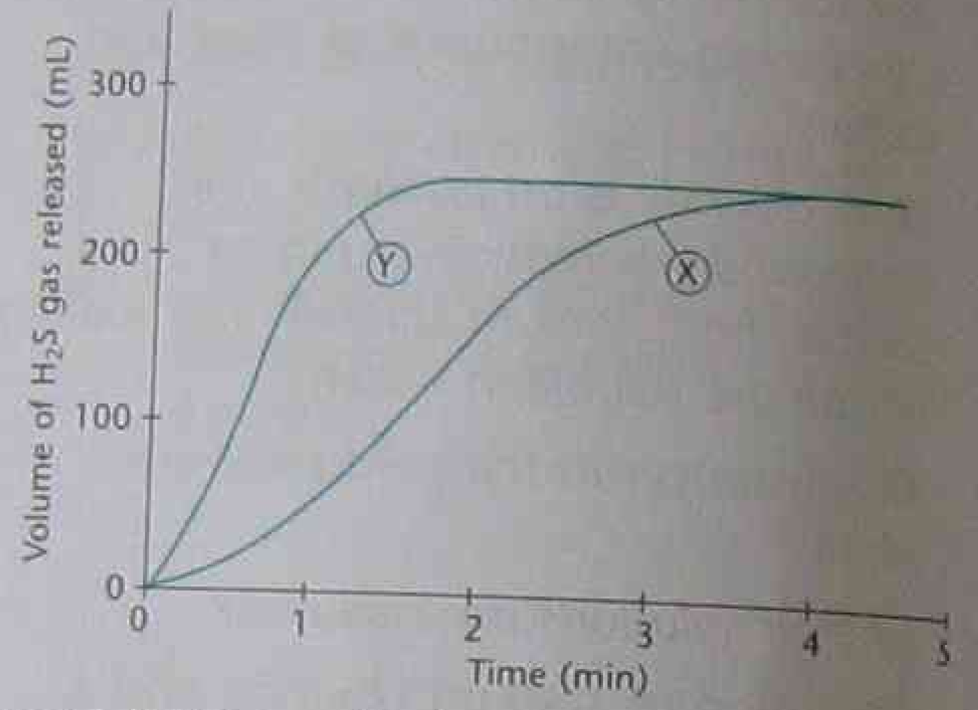


Figure 2.20 Volume of hydrogen sulfide gas released as a function of time

a Which reaction has the greater initial rate? (1 mark)

b Explain why the rates of the reaction were different in each container. (1 mark)

c Write a word equation for the reaction. (2 marks)

d Write a balanced symbolic equation for the reaction. (2 marks)

e Figure 2.21 shows four possible structures for the foul smelling gas released in the reaction. Which model is consistent with the experimental information and the symbolic equation?

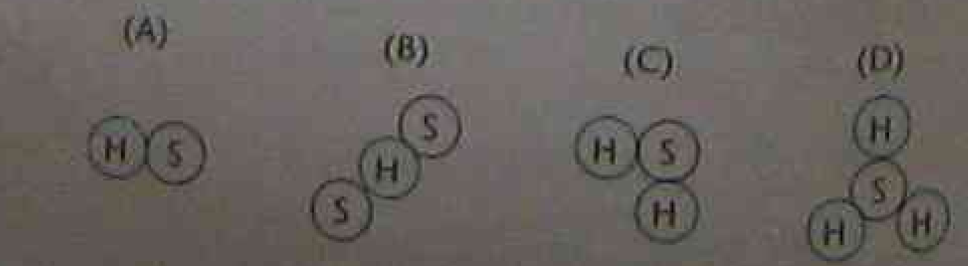


Figure 2.21 Possible models of the foul smelling gas

9 The following table lists some solubility rules for ionic compounds.

Solubility rules for some ionic salts

Type of salt	General solubility rule	Exceptions to rule
Group I salts	soluble	–
Nitrate salts	soluble	–
Chloride salts	soluble	silver chloride (AgCl) lead chloride ($PbCl_2$)
Sulfate salts	soluble	barium sulfate ($BaSO_4$) lead sulfate ($PbSO_4$) calcium sulfate ($CaSO_4$)
Carbonate salts	insoluble	Group I carbonates ammonium carbonate ($(NH_4)_2CO_3$)

Use this table to answer the following questions.

a Determine whether the following compounds are soluble or insoluble in water. (2 marks)

i calcium chloride

ii iron carbonate

iii barium nitrate

iv sodium sulfate

b Solutions of the following ionic compounds are mixed together. Predict whether a precipitate will form. (3 marks)

i silver nitrate and potassium chloride

ii ammonium sulfate and barium chloride

iii sodium nitrate and copper sulfate

10 Figure 2.22 shows an experiment in which a student passes an electric current (via platinum electrodes) into water containing a small amount of sodium sulfate. The sodium sulfate is added to make the water conductive

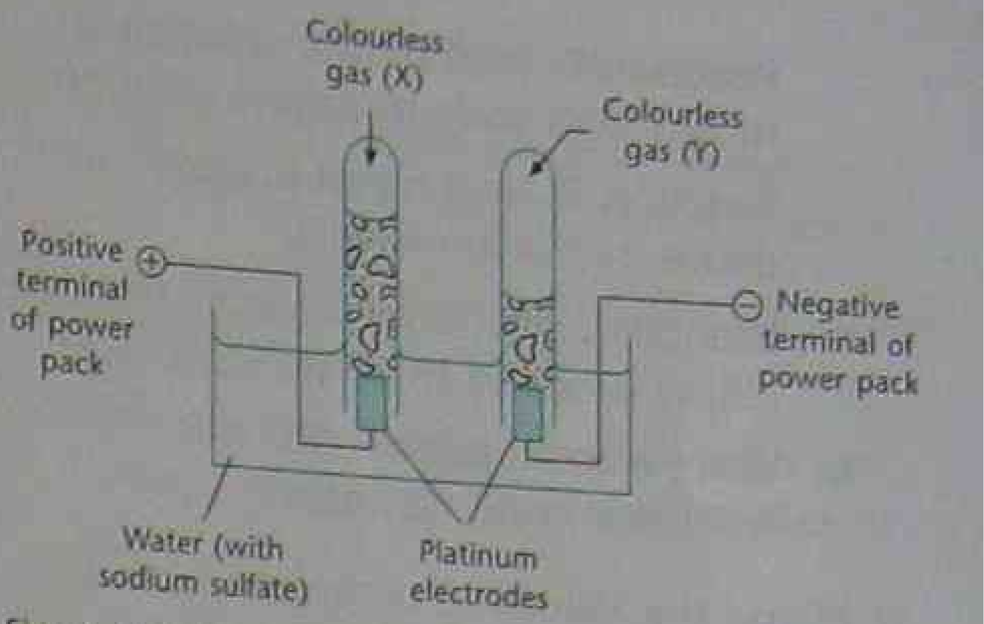


Figure 2.22 Passing an electric current into water

but it does not alter the reaction. (Sulfuric acid is also suitable for this purpose.) Gases collect in inverted test tubes (initially filled with water).

a Compare the volume of the gas released from the negative electrode (Y) to the volume of gas from the positive electrode (X). (1 mark)

b The gas (Y) from the negative electrode is tested and found to explode in the presence of air and a flame. What is this gas? (1 mark) hydroge

c The gas (X) from the positive electrode is tested and found to re-light a glowing wooden splint. What gas is this? (1 mark) oxygen

d Explain, with the aid of a word equation, the type of reaction that is occurring. (2 marks) ^ decomposition electrolysis

e Write a balanced symbolic equation for this reaction. (2 marks) H_2O + 2H

f How do the gas volume data support the balanced equation? (1 mark)

End-chapter test (answers on pages 215–16)

1 a For which of the following reactions does:

i effervescence occur? (1 mark)

ii a precipitate form? (1 mark)

If no reaction occurs, write 'no reaction'.

Reaction 1: lead nitrate solution is mixed with sodium sulfate solution.

Reaction 2: lead metal is added to dilute hydrochloric acid.

Reaction 3: calcium carbonate (marble chips) is added to dilute nitric acid.

b Write word equations for the reactions in (a). (2 marks)

2 Name the following compounds and classify them as ionic or covalent: (4 marks)

a CaI_2

b SCl_2

c HI

d HgO

3 Classify the following compounds as organic or inorganic: (4 marks)

a NaCl

b C_4H_{10}

c CH_3COOH

d Fe_2O_3

4 Write the symbols for the ions in the following ionic compounds and name each compound (refer to the Periodic Table): (3 marks)

a $BaCl_2$

b $Ga(NO_3)_3$

c RbF

5 Determine the valency of the indicated elements in each of the following covalent compounds: (3 marks)

a X in X_2O_3

b Y in YCl_5

c Z in H_2Z

6 Yellow sulfur burns in a gas jar of pure oxygen with a pale mauve flame. A colourless gas is formed. The gas dissolves in water and the solution formed turns blue litmus red.

a What are the indicators of a chemical change in the combustion process? (2 marks)

b Name the gas formed in the combustion reaction. (1 mark)

c Write a word equation for the combustion reaction. (2 marks)

d Write a balanced symbolic equation for the combustion reaction. (2 marks)

e What can one conclude about the acid–base properties of the colourless gas? (1 mark)

7 Ethane (C_2H_6) burns in oxygen to form carbon dioxide and water.

a How many atoms are present in one molecule of ethane? (1 mark)

b Write a word equation for the combustion reaction. (2 marks)

c Write a balanced symbolic equation for the combustion reaction. (2 marks)

d How could a student prove that carbon dioxide is released in the combustion process? (2 marks)

e Why is an ignition source not required after the combustion reaction has begun? (1 mark)

8 Name the salts produced in each of the following reactions with acids. (3 marks)

a Zinc dissolves in sulfuric acid.

b Calcium oxide dissolves in nitric acid.

c Sodium carbonate dissolves in hydrochloric acid.

9 A natural indicator was made by extracting the dye from flower petals into water. The pink extract was tested in a variety of solutions. The results of the experiment were:

Acetic acid: turns from pink to deep red

Sodium hydroxide solution: turns from pink to deep green

The extract was then used to test each of the following household substances. Predict the colour change that may occur. (2 marks)

a Ammonia window cleaner

b Lemonade

10 Magnesium carbonate is converted into magnesium oxide when the white solid is heated over a Bunsen flame.

a What type of chemical reaction is described above? (1 mark)

b What other product will form in this reaction? (1 mark)

c Write a word equation and a balanced symbolic equation for this reaction. (2 marks)

Summary

Atomic theory

- Atoms are composed of a central positive nucleus surrounded by shells of negative electrons.
- The nucleus contains positive protons and neutral neutrons.
- The atomic number (Z) of an element is the number of protons in the nucleus. electrons
- The mass number (A) of an element is the total number of protons plus neutrons in the nucleus.
- Isotopic forms of an element differ from one another by the number of neutrons.
- Electrons can be gained or lost from the outer electron shell (valence shell).
- The arrangement of electrons in the electron shells is called the electron configuration.

Elements

- Some elements consist of single atom molecules (monatomic) while others consist of multi-atomic molecules.
- The Periodic Table classifies elements according to increasing atomic number.
- The rows of the Periodic Table are called periods.
- The columns of the Periodic Table are called groups and represent families of elements.
- The majority of elements are metals. They are located to the left and centre of the Periodic Table.
- The 19 non-metals are located at the right side of the Periodic Table.
- A zone of six semi-metals separates the metals and non-metals.

Compounds

- Compounds are formed when fixed numbers of atoms of different elements join together chemically.
- Compounds can be classified in various ways. Ionic compounds and covalent compounds are just two ways of classifying compounds.
- Ionic compounds are composed of oppositely charged ions.
- Covalent compounds are composed of atoms linked by covalent bonds.
- Compounds are named according to fixed nomenclature rules.
- Valency rules can be used to determine the formula of a compound.

Chemical reactions

- Chemical reactions involve the formation of new substances.
- Chemical equations use symbolism to describe the conversion of reactants into products.
- Matter is conserved in all chemical reactions.

- Combustion reactions involve the combination of a fuel and an oxidiser with the release of energy.
- Combustion reactions require an ignition source.
- Corrosion reactions involve the degradation of a metal in contact with environmental agents such as oxygen and water. Rusting is an example of corrosion.
- Precipitation reactions involve the formation of an insoluble solid when solutions of ionic salts are mixed.
- Acids attack reactive metals, with the release of hydrogen gas. The metal will completely dissolve if the acid is in excess.
- Acids attack carbonate compounds with the production of carbon dioxide. This is an example of a neutralisation reaction.
- Acids and bases neutralise one another to forms salts and water.
- Indicators are dye molecules that change colour in the presence of acids or bases.
- The pH scale can be used to describe the acidity or basicity of solutions.
- Decomposition reactions involve the breakdown of a compound into elements and/or simpler compounds.
- Heat can be used to decompose compounds.

Chapter 3

Structure and Function of Living Things

Cell theory

Cells were first observed by Robert Hooke (1665) using a simple microscope. The development of better microscopes finally led to the development of the cell theory.

In the period 1838 to 1854 the major principles of the cell theory were proposed.

Cell theory

- **All living things are made up of one or more cells.**
- **Cells are the smallest units of living things.**
- **All cells come from pre-existing cells.**

Glossary

Chromosomes—structures in the nucleus that carry the genetic code (genes)

Differentiation—changes in a basic cell structure that produce specialised cells

Meiosis—special cell division that occurs to produce sex cells that have half the normal chromosome number

Mitosis—normal cell division in which a cell divides to produce two exact (but smaller) copies

Multicellular—an organism composed of many cells

Organ—a structure of the body composed of different tissue types; carries out specific functions for the organism

Tissue—cellular structures in which all the cells have similar shapes, sizes and functions

Systems serve the needs of cells

Multicellular organisms such as humans are composed of many different types of cells. Cells have a large variety of sizes and shapes. This is due to the **specialisation** of cells in a multicellular organism.

- These different cells are organised into structures called **tissues**. Cells that form a tissue have the same function and similar shapes and sizes.

  Examples: nerve tissue; smooth muscle tissue; plant photosynthetic tissue
- Different types of tissues are then organised into larger structures called **organs**. Each organ has its own specific functions.

  Examples: heart; stomach; lungs
- Organs are grouped into **body systems** or **organ systems** within the body of a multicellular organism.
- Body systems **serve the needs of the cells** that make up a multicellular organism.

Body systems

The following table provides information that summarises the major roles of some of the body systems. Some organs belong to more than one system. The diaphragm muscle, for example, is part of the muscular system as well as the respiratory system. The liver is part of the digestive as well as the excretory system. Other systems are examined on pages 94–104. (Humans).

Table 3.1 Roles of some of the major body systems

Body system	Major organs	Roles
Skeletal system	bones	supporting the body (legs; backbone) protecting organs (skull; rib-cage)
Muscular system	muscles	move the whole body (leg muscles) assist circulation (heart muscle) assist breathing (diaphragm muscle)
Digestive system	mouth	chews and breaks down food; mixes with saliva
	oesophagus	tube carrying food to stomach from mouth
	stomach	churns food with acid
	small intestine	completes digestion and nutrients are absorbed into the bloodstream
	liver	makes bile (for fat digestion) and stores glycogen
	pancreas	produces digestive enzymes; regulates sugar balance
	large intestine	removes water from waste
Circulatory system	heart	pumps blood (containing oxygen and nutrients) around the body
	blood vessels	transfer blood to organs, tissue and cells
Respiratory system	lungs	take in oxygen and remove carbon dioxide waste (gas exchange)
	diaphragm	muscle that contracts/ relaxes to change volume of chest cavity
Excretory system	kidneys	filter the blood and remove liquid waste
	bladder	stores urine
	liver	detoxifies blood and removes nitrogenous waste
	lungs	remove carbon dioxide

The table shows that our bodies contain a complex arrangement of tissues and organs. Their role is to ensure that **each cell**:

- receives the necessary **nutrients** (oxygen, food, minerals, vitamins, water) to live and carry out their functions; and
- has its **cellular products** (including energy) and **wastes** transported away to the appropriate cell or tissue for use or elimination.

Cell division

In multicellular organisms, cells divide to produce new cells. Cell division is necessary for:

- **growth** of the organism
- **repair** of tissues and organs
- **reproduction** by the formation of sex cells.

Figure 3.1 shows a simple model of normal cell division. In normal cell division the **genetic information** that is coded in the **chromosomes** inside the **nucleus** must be copied before the cell divides. In this way each new cell (daughter cell) has an **exact copy** of the genetic information.

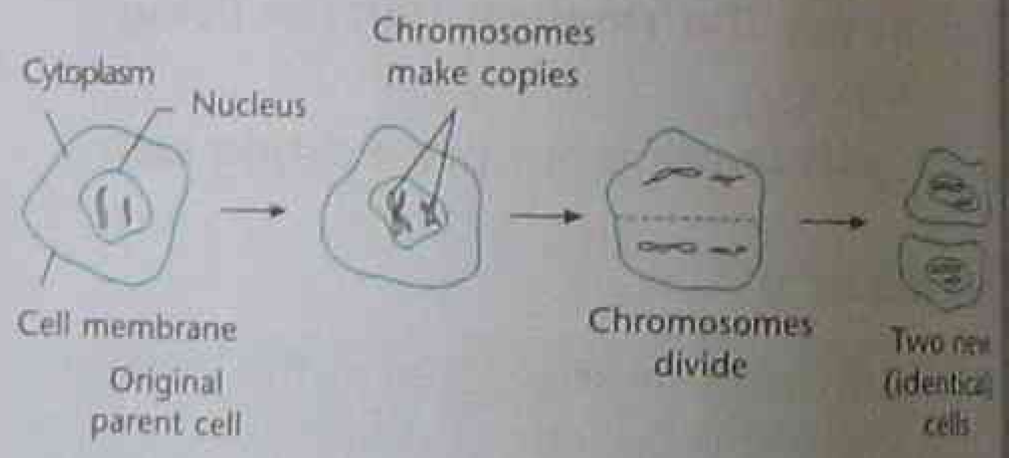


Figure 3.1 Simplified model of normal cell division

Growth

Living things begin their life as one cell. In multicellular organisms this first cell (the fertilised egg in sexually reproducing organisms) divides many times to produce all the cells that make up the adult organism. As an embryo grows, cells become specialised for different functions. Thus some cells will become muscle cells while others will become nerve cells.

An organism grows because:

- each new cell produced by the division of an old cell **grows to its maximum size**;
- cells continue to divide to **make sufficient cells** for the adult organism.

Repair

Cell division decreases as an organism ages. However, it does not stop. Throughout the life of a living organism, cells become damaged or die. These cells need to be **repaired** and in some cases **replaced** by new cells. Our skin cells eventually die and are shed. They are replaced by newly dividing cells formed in a layer below the surface. New red blood cells are constantly being formed by cell division in the bone marrow. In adult human males, sperm cells are constantly being produced by cell division in the testes.

Reproduction

In simple unicellular organisms, reproduction is achieved by simple cell division. This method of reproduction is called **asexual reproduction**. This type of cell division (called **mitosis**) is the same as the method used to produce new body cells in multicellular organisms. All cells produced by mitosis are genetically identical to the original cell from which they were formed. They have the same number and types of chromosomes present as were present in the original cell. Figure 3.2 shows some examples of asexual reproduction.

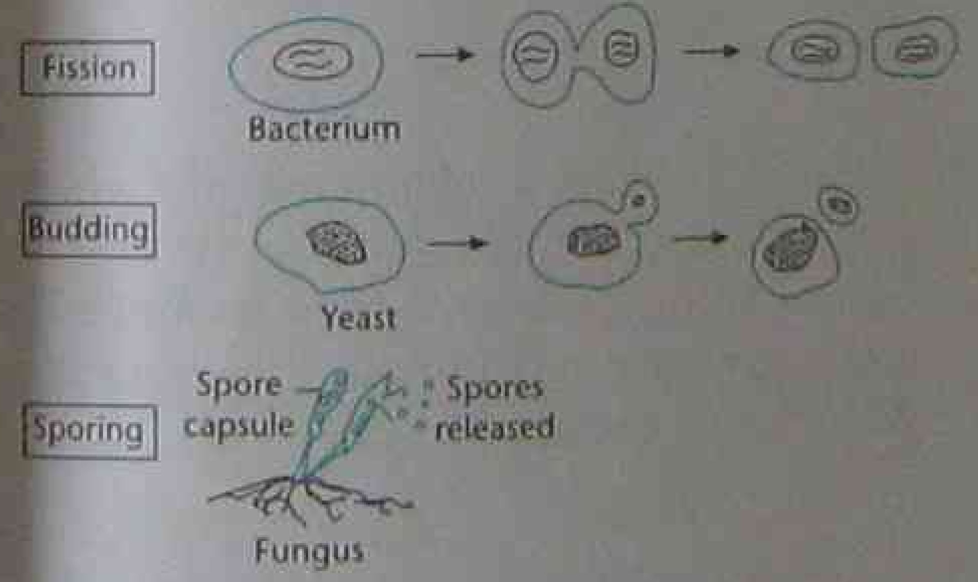


Figure 3.2 Types of asexual reproduction

In sexually reproducing organisms, specialised **sex cells (gametes—eggs and sperm)** are produced by a special type of cell division called **meiosis**.

Meiosis occurs in the testes or ovaries.

In meiosis there are two stages of cell division. At the end of these two stages, four new sex cells with **half the normal chromosomal number** are formed. Details of this complex process are beyond the scope of this course.

Test yourself (answers on page 216)

Part A. Knowledge (answers on page 216)

1 An important concept concerning the cell theory is that:
- a cells are composed of tissues.
- [✓ b] all living things are made up of one or more cells.
- c atoms are defined as the smallest units of living things and cells are the largest units.
- ~~d~~ cells are created at the time of fertilisation. (1 mark)

2 Chromosomes are:
- [✓ a] located in the nucleus of a cell.
- ~~b~~ formed only during meiosis.
- c present only in multicellular organisms.
- d involved only in the repair of cells. (1 mark)

3 A major role of the circulatory system of animals is to:
- a provide support for the body.
- b assist in breathing.
- [✓ c] absorb nutrients into the bloodstream.
- ~~d~~ ensure oxygen reaches all cells of the body. (1 mark)

4 A major role of the excretory system of a human is to:
- a produce enzymes to regulate the balance of water in the body.
- [✓ b] remove waste from cells and tissues.
- c remove carbon dioxide only.
- d make bile for fat metabolism. (1 mark)

5 An organism grows in size because:

- a the original embryonic cells grow increasingly larger and larger with age.
- b the body systems grow in size.
- [✓ c] cells divide to produce more cells.
- d the body is supplied with oxygen to promote growth. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

- a Cell division ______ as an organism grows older.
- b In a human male, sperm cells are produced by ______ in the testes.
- c Sex cells have ______ the normal chromosomal number.
- d Cells that undergo meiosis produce new cells with ______ the normal chromosome number.
- e ______ are cellular structures in which all the cells have similar shapes, sizes and functions.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a differentiation	f nutrient
b gamete	g egg
c asexual reproduction	h stomach
d vitamins	i specialisation
e digestive system	j mitosis

The body systems serve the needs of cells. State one role of each of the following systems that serves the need of a muscle cell in the leg. (3 marks)

- a Circulatory system
- b Excretory system
- c Digestive system

9 In multicellular organisms, cells divide produce new cells. State three reasons why cell division is necessary. (3 marks)

Part B. Skills (answers on page 216)

1 Figure 3.3 shows the jumbled stages of normal cell division. Use the code letters to place the stages in their correct sequence. (2 marks)

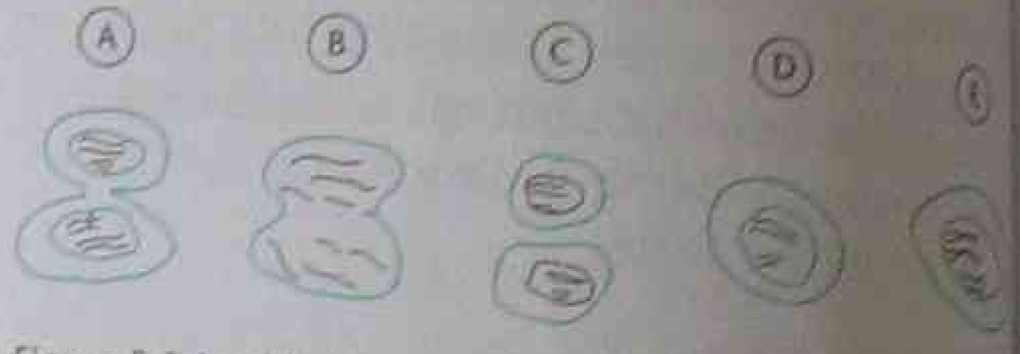


Figure 3.3 Jumbled stages of normal cell division

2 In the root tips of plants there are actively dividing cells. Following division, the new daughter cells grow until they are mature. Figure 3.4 shows a jumbled sequence of cells from a bean's root tip. Place these cells in their correct sequence along the root, starting at the tip. (2 marks)

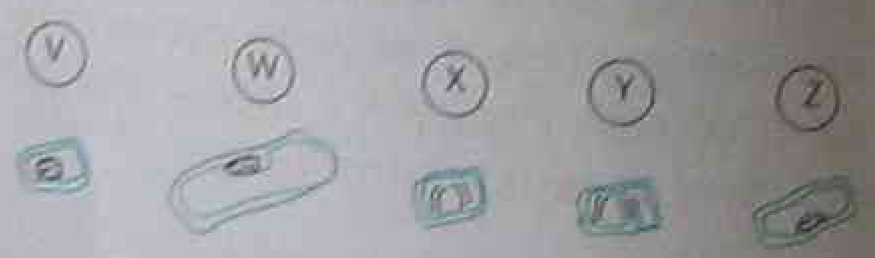


Figure 3.4 Jumbled cells in root tip

3 Figure 3.5 shows a millimetre grid when viewed using a microscope under low power. Some cells were then examined under low power. Estimate the average diameter of these cells. (2 marks)

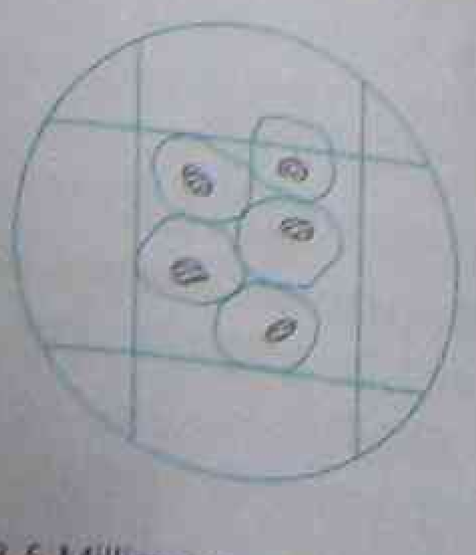


Figure 3.5 Millimetre grid and cells under low power

4 The following table compares the relative life span of various types of cells, compared with skin cells.

Cell	Relative life span
Skin	1
Red blood	6
Bone	43

- a Why do you think that bone cells have a much longer life span than skin cells? (1 mark)
- b If the average life span of skin cells is 4 weeks, calculate the life span of red blood cells and bone cells. (1 mark)

5 Figure 3.6 shows the cells and chromosomes of two organisms after several cell divisions.

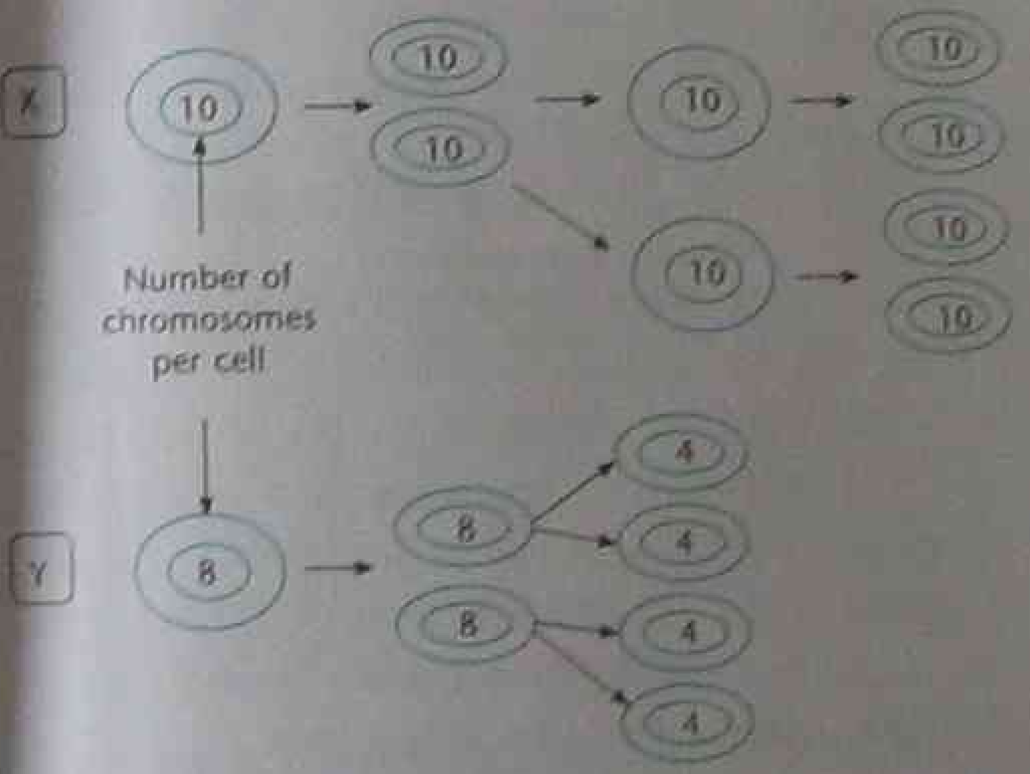


Figure 3.6 Cells and chromosomal number after several cell divisions

- a How has the chromosomal number changed after each cell division in each organism? (1 mark)
- b Which diagram shows cell division leading to gamete formation (ie. meiosis)? (1 mark)

6 The vascular system in plants contains two important types of tissue. Xylem cells in plants form tissues that serve the needs of the plant by transporting water and dissolved minerals from the root to all parts of the plant body. Phloem cells in plants form tissues that serve the needs of the plant by transporting food nutrients manufactured in the leaves to all living parts of the plant.

- a Identify the code letter in Figure 3.7 that refers to:
  - i xylem tissue
  - ii phloem tissue. (1 mark)

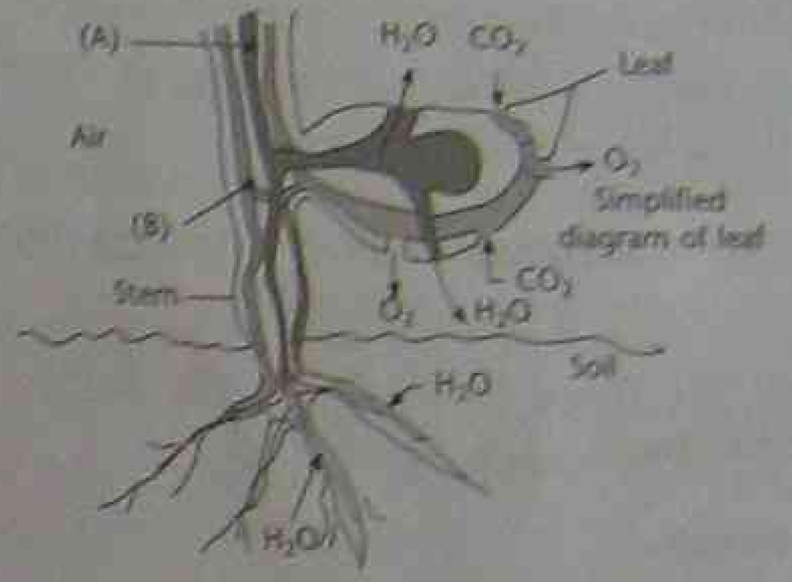


Figure 3.7 Coded diagram of phloem and xylem tissue in plants

- b Name an organ system in humans that could also be considered to contain vascular tissue. (1 mark)
- c The flow of water through xylem tissue and out of the leaf pores into the air was monitored in a mangrove plant. Between 12 am and 1 pm, 3 mg of water was lost for each square centimetre of leaf. Calculate the water lost from 100 leaves in 1 hour if the average area of each leaf is 60 cm^2. (2 marks)

The Watson-Crick model of DNA

In 1953 two scientists (James Watson and Francis Crick) in England used X-ray data obtained by Rosalind Franklin to discover the structure of the molecule that encodes our genetic information. This is the DNA molecule. They won a Nobel Prize for their discovery.

Glossary

DNA—deoxyribose nucleic acid

Gene—segment of a DNA chain that codes for an inheritable characteristic

Mutation—a change in the code sequence of a DNA molecule

Nucleotides—the basic components that make up a DNA molecule

Replication—The process by which the DNA molecule makes a copy of itself

Triplet code—a sequence of three nitrogen bases

DNA—structure and replication

DNA is an acronym for **deoxyribose nucleic acid**. The shape of the DNA molecule is a **double helix**. (See Figure 3.8.)

Structure

The two strands of the double helix resemble a spiral staircase. The separate strands are composed of molecules called **nucleotides**.

Nucleotides are composed of three parts:

- **a phosphate group**
- **a deoxyribose sugar molecule**
- **a nitrogen base.**

The 'spiral steps' are composed of the **nitrogen bases**. The sides of the 'spiral staircase' are composed of the **phosphate groups** and **deoxyribose sugar** molecules.

There are four types of nitrogen bases. These are:

- **cytosine (C)**
- **guanine (G)**
- **thymine (T)**
- **adenine (A).**

The 'spiral steps' are constructed by two nitrogen bases linking together.

- Cytosine (C) always bonds with guanine (G).
- Thymine (T) always bonds with adenine (A).

A simplified model of a short section of the DNA structure is shown in Figure 3.8.

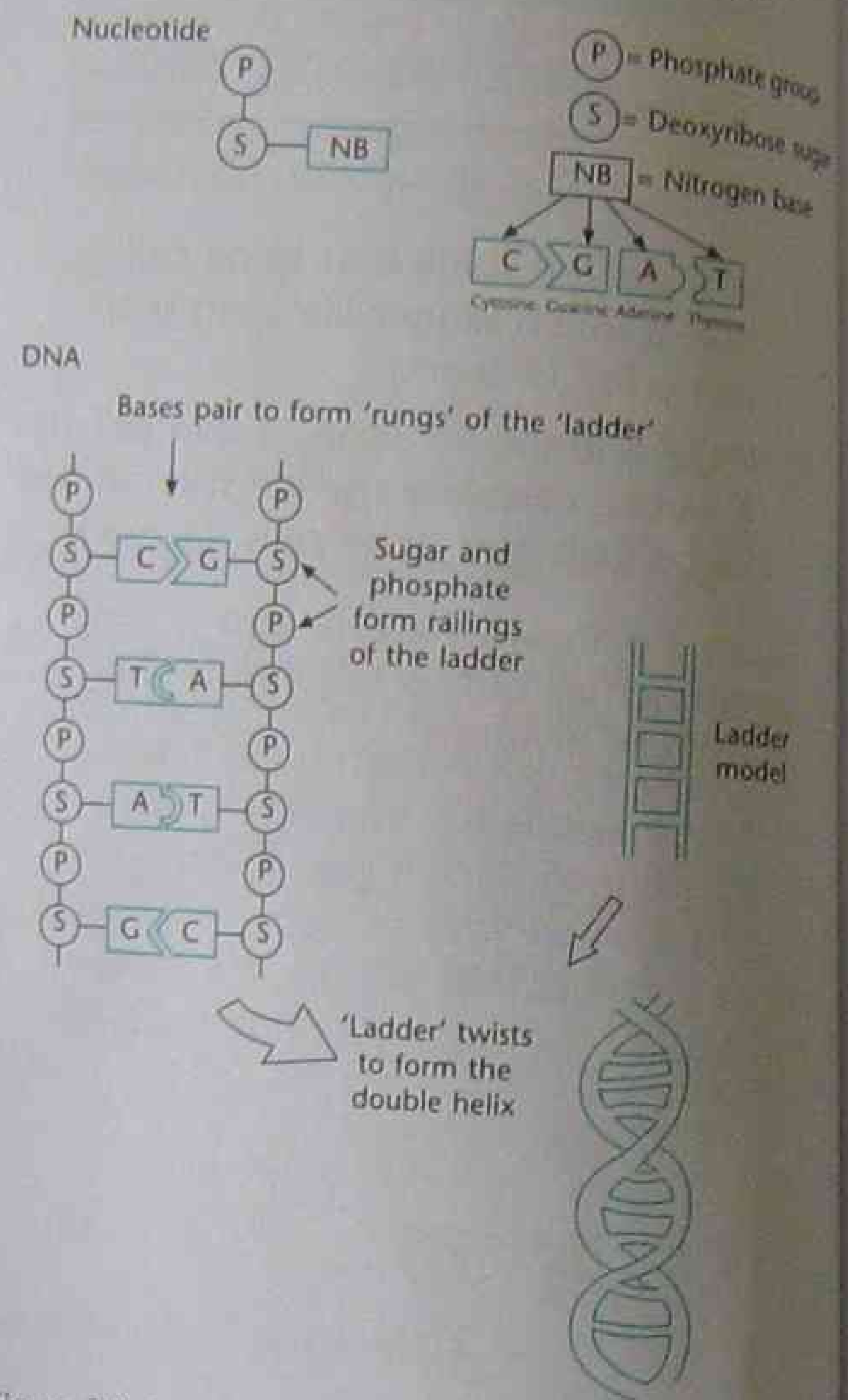


Figure 3.8 Structure of nucleotides and DNA

Replication

Prior to normal cell division, each DNA molecule must make a copy of itself. In this way each new cell will have an exact copy of each DNA molecule. This process is called **replication**.

The steps in replication are:

1. The nitrogen bases that form the 'spiral steps' start to split apart at one end of the DNA ladder.
2. New nucleotides are transported into place and linked together to produce a complementary strand according to the rule for base pairing (ie. C with G and T with A).
3. Two new double helices form when this process is completed.

Figure 3.9 shows a simplified model of the first stage of this replication process.

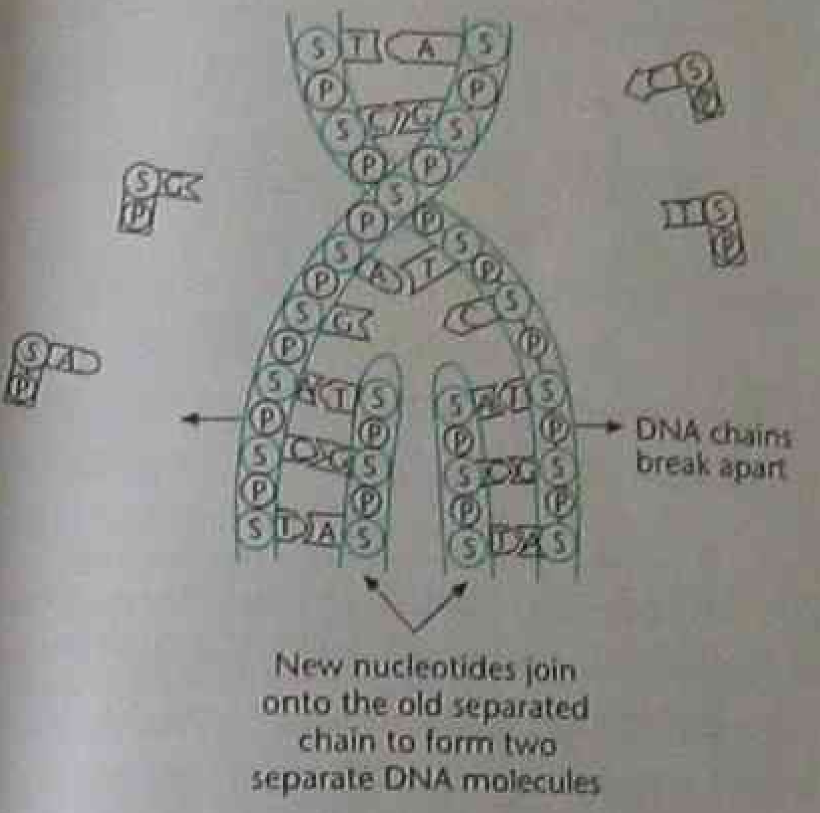


Figure 3.9 DNA replication

DNA mutations

The process of DNA replication may go wrong. This results in a change in the sequence of nitrogen bases in the DNA molecule. The sequence of the nitrogen bases is a **code**. The cell uses this code to carry out its normal functions. This change in the code is called a **DNA mutation.**

There are a number of ways in which DNA mutations can occur. Two of these ways are:

- **Deletion**—If a nitrogen base is omitted during replication, the nitrogen base sequence changes.

  Original sequence = CCT **GTC** GGA GCA ACA . . .

  C omitted in **GTC**

  New sequence = CCT GTG AGC AAC A . . .
- **Substitution**—Sometimes a nitrogen base is not copied properly and a different base is substituted.

  Original sequence = CAA CTA **CCC** ATA AAA . . .

  G is substituted for the first C in **CCC**.

  New sequence = CAA CTA GCC ATA AAA . . .

When the DNA code is changed in these ways, the information transmitted to the cell is also altered.

Chromosomes and genes

The DNA molecule is one of a number of molecules located in the chromosomes of the nucleus. The sequence of nitrogen bases in the DNA is a code that the cell uses to construct **protein** molecules required for life functions. Protein molecules are very large molecules (polymers) that are constructed from smaller molecules called **amino acids.** The amino acids link together to form the long chains of the protein.

- **Sets of three nitrogen bases form a triplet code.**
- **Each amino acid has a number of alternative triplet codes** (eg. CCC and CCT both code for the amino acid called **glycine**)

Example

The sequence of triplet codes

CCC TTT AAA GAG

leads to the following sequence of linked amino acids in a protein chain:—glycine—lysine—phenylalanine—leucine—

Genes

Genes are codes that contain information about **inheritance.**

- Genes are **segments** of the DNA molecule containing many triplet codes.
- Each gene is responsible for the production of a certain protein.
- Gene segments of the DNA strands are separated from one another by other non-coding segments.

- Genes do not function at all times. They can be turned 'on' or 'off'.
- Each gene may have two alternative forms. These alternative forms are called **alleles**. These alleles can be represented by code letters.

Example

Y = gene for **yellow**-coated pea seeds

y = gene for **green**-coated pea seeds

B = gene for **black** coats in guinea pigs

b = gene for **white** coats in guinea pigs

- Some alleles are **dominant**. They are shown by a capital letter (eg. Y and B). Only one copy of a dominant gene is needed for an organism to show this characteristic or trait.
- Some alleles are **recessive**. They are shown by lower-case letters (eg. y or b). Two copies of a recessive gene are needed for an organism to show this trait.

Features of an organism

The features or characteristics of an organism are determined by the:

- **genes** they inherit;
- interaction between the **environment** and the genes.

Inherited genes

As a result of sexual reproduction, each child inherits 23 chromosomes from their father and 23 chromosomes from their mother. These 23 pairs of chromosomes contain many pairs of genes for each characteristic. The features of the child will depend on the types of alleles inherited.

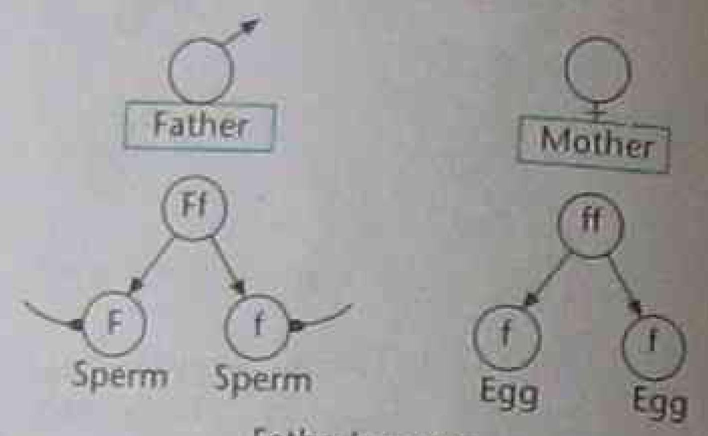
Example

A free ear lobe (F) is dominant to an attached ear lobe (f).

The genes are inherited in pairs—one from each parent.

The following table shows the features of three different children from different families.

Child	Gene from Parent 1	Gene from Parent 2	Gene pairs inherited by child (genotype)	Observed features (phenoty
1	F	F	FF	free ear lo
2	F	f	Ff	free ear lob
3	f	f	ff	attached ear lobe



		Father's genes	
		F	f
Mother's genes	f	Ff	ff
	f	Ff	ff

Ff = Free ear lobe
ff = Attached ear lobe
50% chance of each child having a free ear lobe

Figure 3.10 Diagram of inheritance of gene pairs

Environmental effects

The inheritance of particular gene combinations can lead to features that are more suitable in certain environments. The following environmental factors influence the characteristics of a population.

- Availability of food and water—poor diets and nutritional disease will affect the survival of weaker members of a population.
- Infectious disease—individuals with high natural immunity have a greater chance of survival when infectious diseases appear.
- Latitude and sunlight—individuals with darker skins can survive in equatorial latitudes where there is strong solar radiation.

Example 1. Light brown and dark coloured moths

Light brown moths have a mottled body that helps to camouflage them in their natural woodland environment. Dark coloured moths have a gene mutation that leads to dark coloured bodies. They are easily seen by birds in the woodlands and are therefore more likely to be eaten. Consequently, a woodland population of moths usually has more mottled light brown moths than dark moths. The features of the population of moths have been affected by the environment.

Example 2. Salt-tolerant plants

Some plant species are able to colonise coastal dunes because they have inherited genes to deal with high salinity environments. Seeds from non-salt-tolerant plants will quickly die after germination in such environments.

In some parts of Australia increases in soil salinity (due to poor farming practices) have led to a change in the characteristics of native vegetation. The proportion of salt-tolerant plants increases at the expense of non-salt-tolerant plants.

Test yourself (answers on pages 216–17)

Part A. Knowledge (answers on page 216)

1 The structure of DNA could be described as:

- a a large polymer molecule.
- [✓ b] a double helix.
- c a large protein molecule.
- d two of the above. (1 mark)

2 DNA stands for:

- a diribose nuclear acid.
- [✓ b] deoxyribose nucleic acid.
- c deoxyribose nuclear acid.
- d deoxyribulose nitrogenous acid. (1 mark)

3 In the DNA molecule the nitrogen bases:

- [✓ a] form the spiral backbone.
- b alternate with phosphate groups to form the 'railings' of the spiral.
- c form the connecting links that hold the two chains together.
- d bond so that adenine always links with cytosine. (1 mark)

4 Genes are:

- a the triplet code of nitrogen bases.
- [✓ b] segments of the DNA molecule that code for a particular protein.
- c found attached to one of each pair of chromosomes.
- d composed of five consecutive triplet codes. (1 mark)

5 The features of an organism:

- a are determined only by the genes they inherit.
- b are determined only by the environment in which they live.
- c that are acquired during their life can readily be inherited by their children.
- [✓ d] are determined both by its inherited genes and its interaction with the environment. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

- a Nucleotides are composed of ~~small~~ thee parts.
- b In DNA, guanine always links with Cytoine.
- c Protein molecules are constructed code using the genetic information located on the DNA molecules.
- d The alternative form of a gene is called an allele.
- e A dominant gene is represented by a capitl letter.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a DNA segment	f thymine
b CAT	g gene
c adenine	h change in genetic code
d mutation	i triplet code
e replication	j exact copy

8 Name the two scientists who discovered the structure of DNA in 1953. (1 mark)

9 The sequence of nitrogen bases along one strand of a DNA molecule is:

CCTGTGAGCAAC

Write the sequence of bases on the complementary DNA strand. (2 marks)

10 Name two ways in which the DNA code can be mutated. (2 marks)

Part B. Skills (answers on page 217)

1 The triplet codes in the DNA molecule are each associated with a particular amino acid.

a Use the table below to determine the amino acid sequence in a protein chain made by decoding the following set of triplet codes: (2 marks)

Triplet code set: TGA GAG CCC CTA AAA ACA

Triplet code	Amino acid	Triplet code	Amino acid
AAA	phenylalanine	GCT	arginine
AAT	leucine	TAA	isoleucine
AGA	serine	TGA	threonine
ACA	cysteine	CAA	valine
ACC	tryptophan	CGT	alanine
GAG	leucine	CTA	aspartic acid
GGA	proline	CTC	glutamic acid
TA	histidine	CCC	glycine

b A mutation in the DNA code occurs so that the previous sequence now becomes:

TGA GAG CCC CAA AAA ACA

i What type of mutation has occurred? (1 mark)

ii What change in amino acids wi occur in the protein chain? (1 mark)

2 A black-coated male guinea pig has two dominant genes (BB) inherited from its parents. This guinea pig mates with a white-coated female (bb).

a If all the sperm produced by the mal contains B genes, what gene do all the eggs from the female contain? (1 mark)

b All the baby pigs in the litter contain the same gene pairs for coat colour. What is this gene pair? (1 mark)

3 Figure 3.11 shows two cell nuclei, each with three pairs of chromosomes. Other chromosomes are not shown. Some genes are shown on each chromosome.

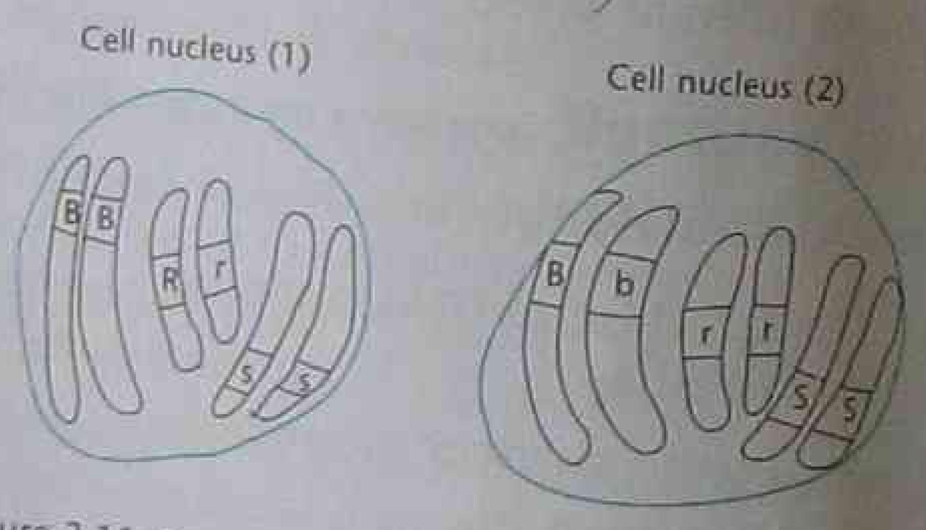


Figure 3.11 Two cell nuclei with three chromosomes

a Cell nucleus (1) has the following gene combination sequence (or genotype): BBRrss Write the gene combination sequence for cell nucleus (2). (1 mark)

b Use the following information about each gene to determine the appearance (phenotype) of people containing each type of cell. (2 marks)

B = brown eyes, b = blue eyes

R = round face, r = long face

S = straight hai

c What will be the appearance (phenotype) of a person with the following gene combination in their cells: bbrrss? (1 mark)

4 Chromosomes occur in pairs. They are called homologous pairs. In humans there are 23 pairs. Figure 3.12 shows a pair of homologous chromosomes. Bands along each chromosome represent genes.

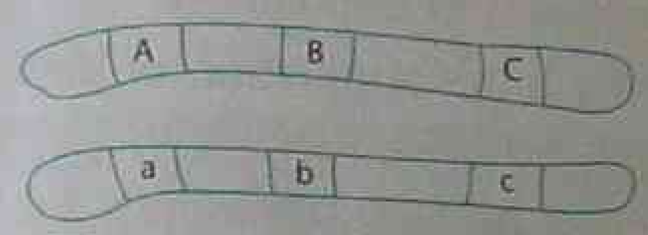


Figure 3.12 A homologous pair of chromosomes

a Gene 'b' is found on one of the chromosomes.

i What allele is found on the other chromosome? (1 mark)

ii If gene 'B' represents brown eyes and 'b' represents blue eyes, what coloured eyes will this person have? (1 mark)

b What generalisation can be made about the location of the gene pairs on the chromosome? (1 mark)

Mid-chapter test (answers on page 217)

1 a Name the scientist who first observed that living things were composed of cells. (1 mark)

b What instrument did this scientist use to make this discovery? (1 mark)

c Complete this sentence: (1 mark)

All living things are made up of ____.

2 Match the code letters of the body systems in column 1 to the code letters of their role (column 2) in maintaining a

Column 1	Column 2
a skeletal system	f breaks down food into nutrients required for body cells
b muscular system	g transfers blood around the body to deliver nutrients and remove wastes
c digestive system	h protects and supports organs
d circulatory system	i collects and removes waste products from cells
e excretory system	j allows movement of body parts

3 Complete these sentences concerning the role of body systems by inserting the missing word(s). (3 marks)

a Body systems are organised so that each ____ of the body receives the required ____ that allow it to live and carry out its functions.

b Metabolism in cells leads to the production of ____ that must be removed from the cells and the body.

4 Cell division is an important process in all living things. Match the code letters in column (1) to the example provided in column (2). (3 marks)

Purposes of cell division

Column 1	Column 2
a growth	d a cut on the skin gradually improves and eventually disappears; new red blood cells are generated in the bone marrow
b repair	e can occur by mitosis or meiosis
c reproduction	f cells multiply in numbers and grow to their maximum size

5 Figure 3.13 shows the components of a section of one strand of a DNA molecule. Z is a component known as a phosphate group.

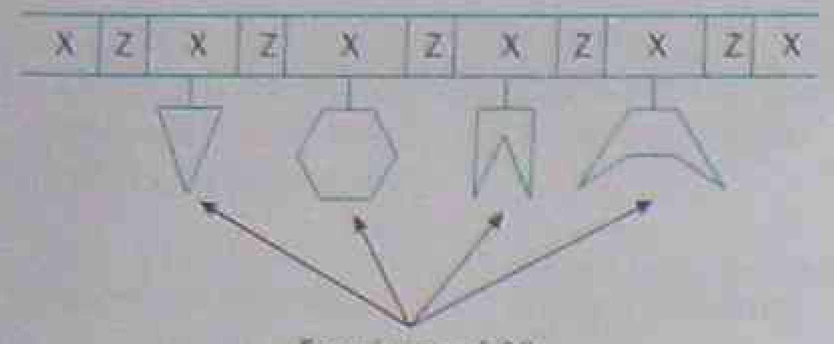


Figure 3.13 Section of a DNA molecule with coded sections X, Y and Z

a Identify the parts labelled by the code letters (X and Y). (2 marks)

b What general name is given to the structure comprising X, Y and Z? (3 marks)

c Why is DNA described as a polymer? (1 mark)

6 The following sequence of nitrogen bases was identified on one strand of the DNA extracted from a wolf:

ACGTCGTGCCACTGACCTA

a How many nitrogen bases are required to code for an amino acid? (1 mark)

b Write the base sequence of the complementary strand of this section of the wolf's DNA. (2 marks)

c The domestic dog is related to the wolf. The following sequence shows the same region of the dog's DNA.

ACGTCGTGCCACTGACTA

The dog shows a common type of mutation in the base sequence. Name this type of DNA mutation. (1 mark)

7 In garden peas the gene for yellow-coated seeds (Y) is dominant to the gene for green-coated seeds.

a What letter symbol is used for the gene for green-coated seeds? (1 mark)

b Write the genotype of a plant that produces seeds with green coats. (1 mark)

c Seeds from a green-coated pea plan are grown in a variety of environments and soil types. Will these different environments lead to the production of plants with yellow-coated seeds? Discuss. (2 marks)

8 In a natural woodland there exists a population of mottled, light brown moths and dark moths. The mottled, light brown moths are 90% of the total population. Moths are part of the diet of the native bird population.

a Why is the population of dark moths so low in this woodland? (1 mark)

b A coal-burning power plant is established near the woodland. Over many years the black soot from the power plant is blown by the wind into the woodland and the tree trunks gradually become quite black. How will this change in the environment affect the moth population? (2 marks)

9 A segment of a protein chain has the following sequence of amino acids:

–threonine—alanine—arginine—glutamic acid—histidine—

Use the table below to write the base sequence of the DNA molecule that coded to this section of the protein chain. (2 marks)

Triplet code	Amino acid
GCT	arginine
TAA	isoleucine
TGA	threonine
CAA	valine
CGT	alanine
CTA	aspartic acid
GTA	histidine
CTC	glutamic acid
CCC	glycine

The theory of evolution and natural selection

Charles Darwin (1809–1882) was the scientist who explained how living things had evolved (changed) over the millions of years of Earth's history. He developed a theory known as the theory of natural selection that provided an explanation for the evolution of species.

Glossary

Comparative anatomy—the science of comparing similar (homologous) structures in the bodies of animals

Embryology—the study of embryos

Evolution—the genetic change in organisms that leads to the production of new species

Extinction—the permanent disappearance of a species

Gene pool—the collection of all genes found in a population of organisms

Geographic isolation—habitats can be isolated from one another by geographical features such as oceans, rivers, cliffs, deserts, etc.

Half-life—the time for the radioactivity of a radioisotope to halve

Natural selection—the process in which species naturally reproduce and pass on to their offspring characteristics that make them more suited to their environment; this process is affected by the environment

Radiometric dating—a process of measuring the age of a rock, mineral or fossil by measuring the activity of various radioisotopes in the sample

Species—a group of organisms that can naturally breed to produce fertile offspring

Variation—differences in characteristics in a population

Evidence for evolution

There is considerable evidence that present-day organisms have developed from different organisms in the distant past. Let us examine some of this evidence.

1. Earth is extremely old

The primitive Earth formed about 4600 million years ago as matter condensed from the spinning disk of a newly formed planetary nebula. The hot Earth then cooled and the landmasses, oceans and atmosphere formed.

Using the technique of **radiometric dating**, scientists have determined the absolute age of various rocks and minerals. This technique is based on measuring the relative quantities of radioactive elements and their decay products. With this information and a knowledge of the **half-lives** of radioisotopes such as uranium-238, rubidium-87 and potassium-40, scientists can measure the age of the rock sample.

Example

Zircons. These minerals contain potassium-40 and its decay product (argon-40). The half-life of potassium-40 is 1300 million years. This means that it takes 1300 million years for half the original potassium-40 present in the zircon to decay into argon-40. The oldest zircons (dated by radiometric analysis) were formed 4200 million years ago.

Radiometric measurements of Earth rocks and Moon rocks (from the Apollo missions) give ages between 3300 and 4000 million years.

- **The great age of Earth has allowed sufficient time for the processes of evolutionary change to occur.**

2. Sedimentary strata and the law of superposition

Comparisons of rock sequences and sedimentary strata around the world led to the idea that the surface rock layers were younger than the deeper layers. William

Smith (1760–1839) expressed this observation in the **Law of Superposition:**

- **In a sequence of sedimentary strata, the layers are increasingly older with increasing depth from the surface.**

Radiometric measurements have since confirmed this law.

3. The fossil record over geological time

Fossils are the remains or record of ancient life. (See page 140, Natural Events.) The geologist Adam Sedgwick (1785–1873) was one of the first scientists to establish the great age of Cambrian fossils in Wales and Scotland. Sedgwick's work led to the idea that many fossils represented species that had become extinct in an ancient period of Earth's history.

As more fossils were collected and analysed from various strata, it soon became apparent that the fossils in deeper sedimentary strata were less complex in body structure than fossils in higher strata. Radiometric measurements showed that these lower layers and fossils were much older than fossils in strata closer to the surface. Thus the fossils of the most primitive fish are found in much deeper (and older) layers than the earliest mammal fossils. The oldest fossil layers contain impressions or traces of simple single-celled organisms. Stromatolite fossils (a type of cyanobacteria) found in the Pilbara region in Western Australia have been dated at 3500 million years old.

- **The fossil evidence therefore supports the view that the earliest life forms were very simple and that they changed into more complex organisms over geological time.**
- **Fossils reveal that the appearance of new life forms did not occur at an even rate over geological time.**
- **The fossil record has shown that more than one new species sometimes developed from a pre-existing specie**
- **The fossil record is incomplete since not all organisms become fossilised after death.**

4. Evidence from horse fossils

The evolution of the modern horse (*Equus*) has been firmly established from fossils dating back to 60 million years ago. The ancient ancestor of the horse (*Eohippus*) ha four toes (compared with one toe in *Equus*) and was as small as a dog. The sequence o fossil forms thus shows that the modern horse developed from a different organism in the distant past.

- **Fossil evidence describing the development of the horse supports evolution because it shows that organisms change over a long time.**

5. Evidence from comparative anatomy

Georges Cuvier (1769–1832) compared the anatomy of fossils and related living organisms. He showed that these fossils were different from those of living species.

Since then, comparative anatomy has shown that the bones at the ends of the forelimbs of many different vertebrates (including humans) are based on a common pattern or structure. Figure 3.14 compares the forelimbs of several different species.

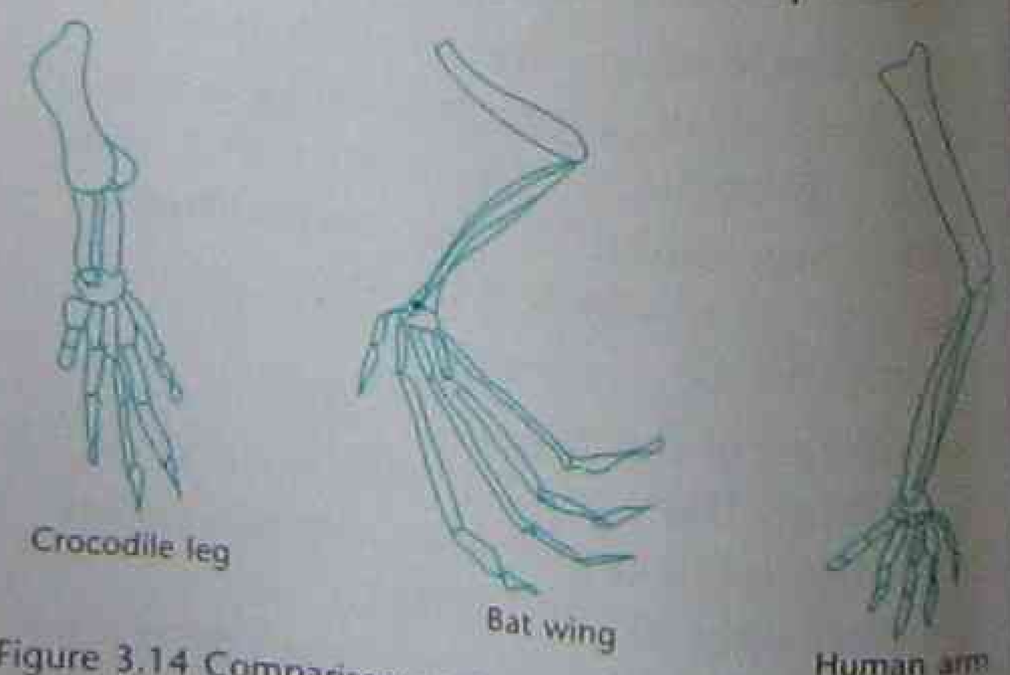


Figure 3.14 Comparison of the forelimbs of vertebrates

- **The comparative anatomy data are used as evidence of evolution from a common ancestor in the distant past.**

6. Evidence from comparative embryology

Vertebrate embryos in their early developmental stages from the fertilised egg show great similarity in structure. This suggests that the genes controlling this *early development* have been inherited from a common ancestor in the distant past. All the young vertebrate embryos show gill slits, even though fish are the only vertebrates to use gills in adult life.

- **The comparative embryology data are used as evidence of evolution because they show that ancient characteristics have been passed on as organisms evolved.**

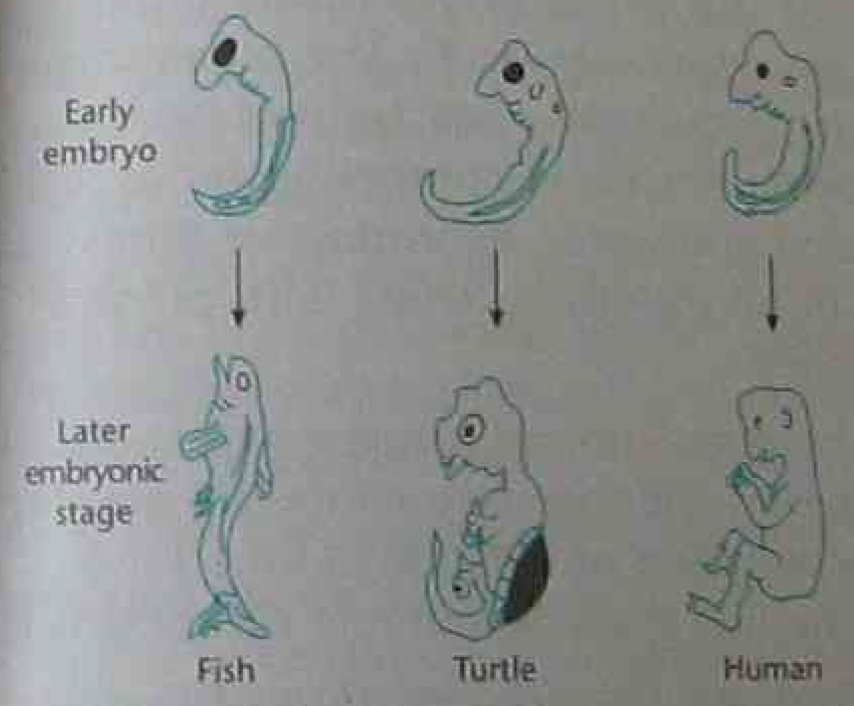


Figure 3.15 Comparison of early vertebrate embryos

7. Evidence from geographic distribution of living things

The theory of plate tectonics explains that the continents of Earth are in constant motion. (See page 130.) In the distant past the continents were joined and animals and plants were not geographically isolated as they are today. They were able to disperse across vast areas of land. As the giant continent (Pangaea) split up (about 225 million years ago) and the smaller landmasses moved to different latitudes, animals and plants became **geographically isolated**. Under these new environmental conditions, the animals and plants evolved to produce new species which were quite different to the common ancestral species, since the environments were so different.

The east coast of South America and the west coast of Africa were formerly joined in a large continental mass called Gondwanaland. Australia, India and Antarctica were also part of this landmass. It began to break apart 100 million years ago.

- **Comparisons of various fern and reptile fossils from along the margins of these separated continents show that many modern species shared a common ancestor.**

8. Catastrophes and extinction events

The fossil record shows many examples of sudden mass extinctions. These mass extinctions (which usually accounted for at least 75% of species alive at the time) seem to be related to worldwide catastrophes. The extinction of the dinosaurs at the end of the Mesozoic Era (65 million years ago) is a well-known example of a catastrophic event. Over 99.9% of all species that have existed on Earth are now extinct. Scientists have collected evidence of catastrophic events that may be related to extreme climate change, sea level changes and collisions of comets/meteorites with Earth.

- **The fossil record shows that following each mass extinction many new species appear.**

9. Evidence from biochemistry and DNA analysis

Chimpanzees and humans share identical amino acid sequences in several enzymes (proteins) found in their respiratory systems. This shows that the DNA base sequence code for these proteins is the same in both species.

Overall, humans and chimpanzees have 98.4% of DNA in common. This and other DNA evidence suggests that humans and chimpanzees shared a **common ancestor** about 4–5 million years ago. This common [extinct] ancestor was quite different from both modern humans and chimpanzees.

Biologists have established evolutionary trees of organisms based on DNA sequences. These evolutionary sequences show the importance of DNA mutations in the process of evolutionary change.

- **The great similarities of some enzymes and DNA of chimpanzees and humans supports evolution, as they point to a recent common ancestor that gave rise to humans and chimpanzees.**

Natural selection

In the 1830s Charles Darwin went on a 5 year voyage around the world on a naval ship (*HMS Beagle*). He visited South America, the Galapagos Islands, Australia and South Africa. During the voyage he collected specimens of animals and plants and studied the geology of the places he visited. He made numerous observations of animal anatomy and behaviour and he also collected many fossils. He became convinced that organisms had gradually changed over the long period of Earth's history. Over the next twenty years he used these studies and other collected information to develop a theory to explain the process of evolutionary change. This theory is called the **theory of natural selection.** Independently, Alfred Wallace developed a similar theory.

The theory of natural selection

The main points of Darwin's and Wallace's theories are:

1. There is a **natural variation in characteristics** within the population of any species. For example, humans have different hair colour, eye colour, height and many other characteristics. In kookaburras there is a natural variation in beak length, flight muscle strength and claw length.
2. In nature, organisms **struggle to survive.** A herbivore such as an antelope must eat sufficient grass each day to remain healthy. It must be fast enough to escape from hunting lions. Slow runners may be more readily captured. In times of drought the weaker individuals may die from starvation. This struggle for survival also keeps the population numbers in check. A rapid population rise leads to less grass for each individual. The rise in population also leads to more food for carnivores and they then begin to bring the population of antelopes under control. Disease also keeps a population in check.
3. Organisms with **favourable characteristics** in a given environment will **survive to reproduce**. An organism that fails to reproduce is said to be 'reproductively unfit'. The organisms with the favourable characteristics have a better chance in competing for available food and water. Reproductive fitness does not necessarily correlate with physical fitness.
4. The population of future generations of a species will therefore contain a **greater proportion** of individuals who have inherited these favourable characteristics.
5. Gradually the **preservation of favourable characteristics** leads to a change in the characteristics of the natural population. As long as the environment does not change, the species becomes better adapted to its environment. The environment has effectively selected certain characteristics for survival. This is also called *survival of the fittest.*

Worked example

Q Darwin studied the various finch populations on the different, distant islands of the Galapagos off the coast of South America. The finches were similar in many ways to the mainland finches but on each island there were different finch species. The different finch species varied in their beak shape, body size, habitat, food requirements and reproductive behaviour. How did Darwin account for these differences?

A Darwin explained the differences as follows:

- He concluded that the different species of finch on each island were the result of **different environmental selecting agents** in operation.
- He believed that the ancestors of these finches came from the mainland.
- Within this original population there was a natural variation in characteristics.
- As they migrated to different islands the ancestral finches found quite different environments. Some islands were drier, with tough, drought-resistant plants. Some had different types of shrubs and trees with different fruits and seeds. Each island had different insect populations or sources of food.
- These different environments selected different finch characteristics for survival. The fittest individuals on each island survived and reproduced.
- Over time the population of finches changed on each island. With limited chances of contact due to **geographical isolation**, the finches on each island eventually became different species with different body structures and breeding behaviour.

Modern views of evolution

Scientific theories change as new evidence becomes available. This has happened to Darwin's and Wallace's theories of natural selection. Since Darwin's time, considerable genetic evidence has been collected, which helps to explain how **gene mutations** can lead to evolutionary change.

Gene mutations that occur to the DNA in sex cells can be passed on to the offspring. Some of these gene mutations result in favourable characteristics, while others are unfavourable. These changes lead to further variation within a species. In some environments these new variations are beneficial and are selected for survival. Over time the population changes so the favourable genes are more common in the **gene pool**.

Test yourself (answers on pages 217–18)

Part A. Knowledge (answers on pages 217–18)

1 Earth is extremely old. The current estimate of the age of Earth is:
   a 65 million years.
   b 3500 million years.
   [✓ c] 4600 million years.
   d 12 billion years. (1 mark)

2 The most ancient fish fossils are:
   [✓ a] found in much older sedimentary strata than the earliest reptile fossils.
   b significantly older than the oldest stromatolite fossils.
   c located in sediments dated to 50 million years ago.
   d found in younger sedimentary strata than dinosaur fossils. (1 mark)

3 Comparative anatomy data have been used as evidence for evolution. Anatomical analysis shows:
   a that plants and animals have ancestors with similar body structures.
   [✓ b] that the forelimbs of different vertebrates are based on a common ancestral structure.
   c that the embryos of fish are similar to those of all other marine organisms.
   d that whales, seals and squid all share a recent common ancestor. (1 mark)

4 A major point in Darwin's theory of natural selection is:

a all members in a natural population of a species have identical genetic characteristics.

[✓ b] only the physically fit organisms survive to reproduce.

c living things with favourable characteristics in a given environment survive long enough to reproduce.

d disease is the major factor in an organism's struggle for existence. (1 mark)

5 Darwin studied the various species of finches on the Galapagos Islands in the Pacific Ocean. He concluded from this study that:

a the finches on each island had interbred with finches from other islands for millions of years.

b the ancestors of the finches from different islands were different species.

[✓ c] different environmental selecting agents on each island had led to the finches evolving into separate species.

d geographic isolation had led to new species only when the physically fit finches could fly between different islands. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

a The extinction of the dinosaurs was an ______ event that occurred at the end of the Mesozoic Era (65 million years ago).

b Evolution occurs when members of a species become ______ isolated.

c Fossils reveal that evolution of new species did not occur at an ______ rate over geological time.

d All young vertebrate embryos have gill slits, which suggests that these genes have been inherited from a ______ ancestor.

e Analysis of nitrogen base sequences in ______ show that chimpanzees and humans share a common ancestor.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a Charles Darwin	f age of strata increases with depth
b favourable characteristics	g incomplete
c environment	h Galapagos islands
d fossil record	i selecting agent
e law of superposition	j reproductive fitness

8 Bacteria reproduce very rapidly. Explain why there is a greater chance of rapid evolutionary change in such organisms than in vertebrates such as humans. (2 marks)

9 Darwin's theory of natural selection has been modified since his death. Explain why this modification has happened. (2 marks)

10 Explain the meaning of the phrase: 'survival of the fittest'. (2 marks)

Part B. Skills (answers on page 218)

1 Biogeography is the study of the distribution of organisms. Figure 3.16 shows the locations of continental masses that formed Gondwanaland at two different times in Earth's history. Use this information to explain why:

a fossil marsupials have been found in Australia and South America but none have been found in Africa; (1 mark)

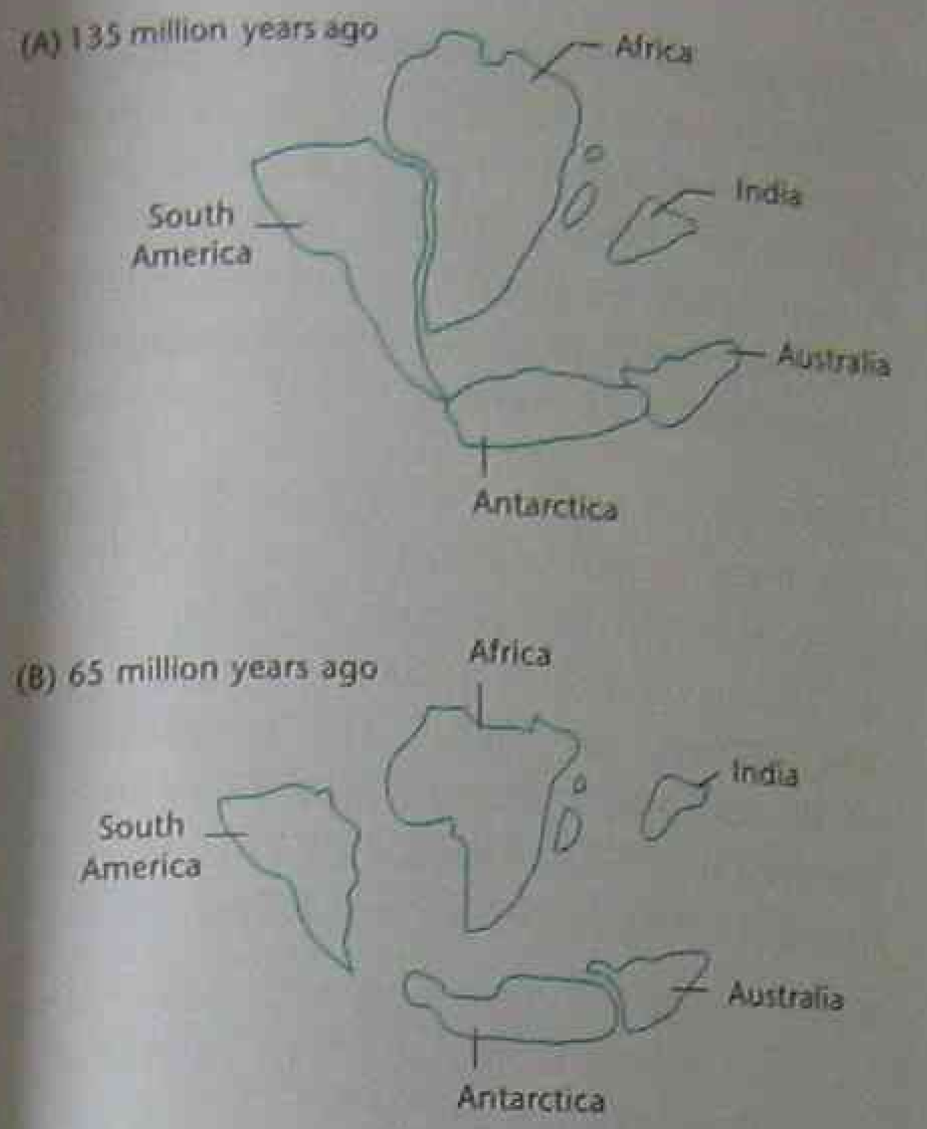


Figure 3.16 Gondwanaland breakup at (A) 135 million years ago (B) 65 million years ago

b it is likely that fossil marsupials will eventually be found in Antarctica; (1 mark)

c most living marsupials are found in Australia, with only two species found in South America. (1 mark)

2 Domesticated cats have many different breeds, whereas there are few natural breeds of wild cats. Use Darwin's theory to explain this observation. (2 marks)

3 Figure 3.17 shows a proposed evolutionary tree for five living species of frogs (A, B, C, D and E). Use this diagram to explain how modern DNA analysis could be used to support the hypothesis that only C, D and E have the extinct species X as their common ancestor. (2 marks)

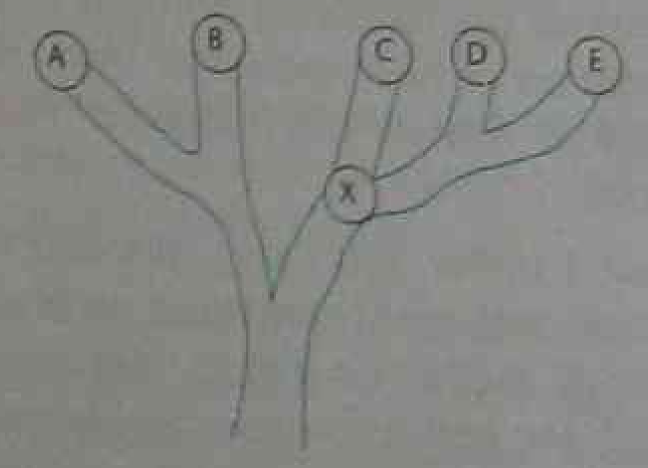


Figure 3.17 Evolutionary tree of 5 frog species and a common ancestor (X)

4 Figure 3.18 shows the evolutionary tree for the land plants. Use this diagram to answer the following questions.

a How long ago did ferns, tree ferns and horsetails share a common ancestor? (1 mark)

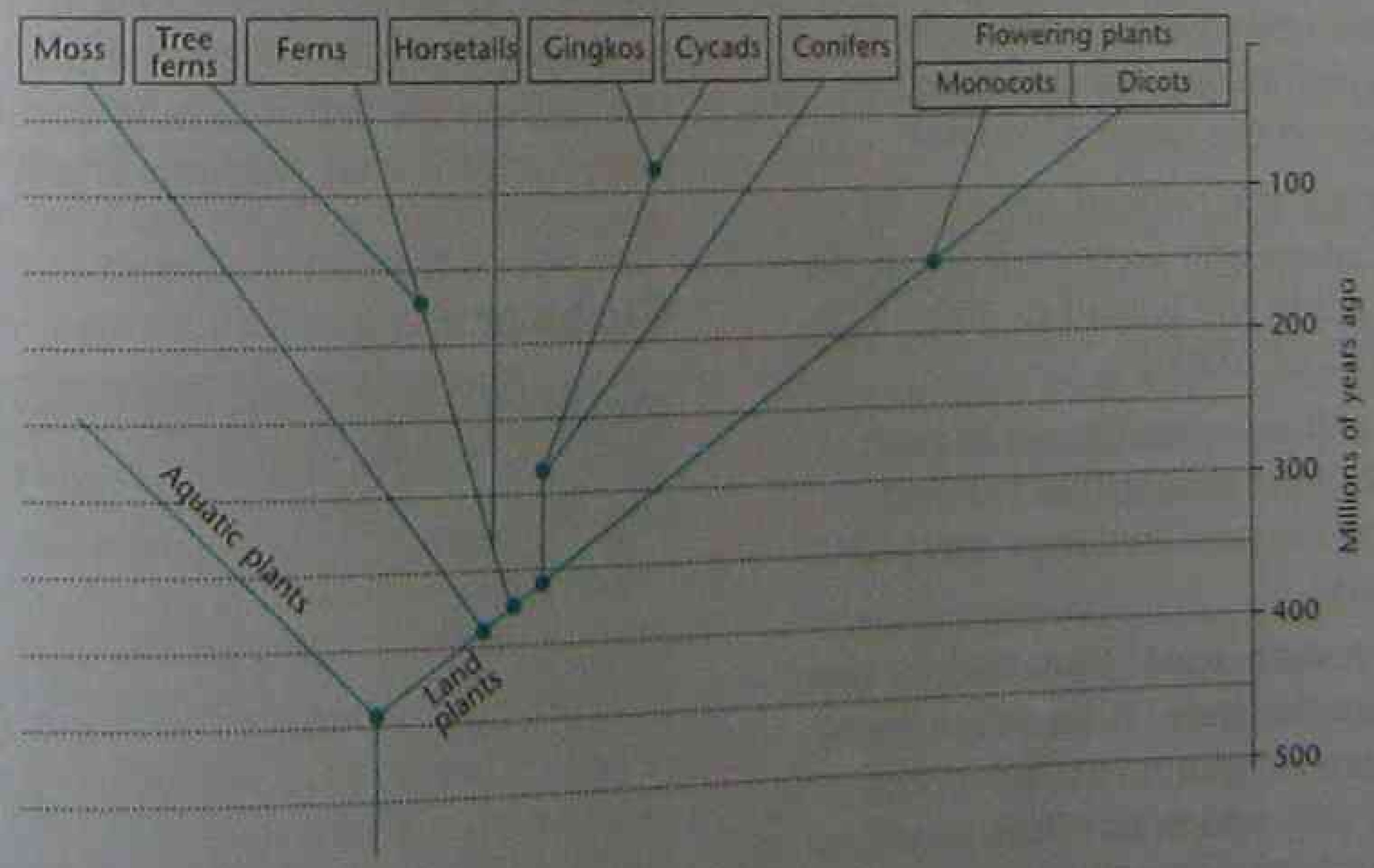


Figure 3.18 Evolutionary tree for land plants

b How long ago did ferns and conifers (pines) share a common ancestor? (1 mark)

c Which modern group of land plants was the first to evolve? (1 mark)

d How long ago did all land plants share a common ancestor with aquatic plants? (1 mark)

e Which major plant group was the last to evolve? (1 mark)

5 Consider the following account of the formation of new species of frogs in Australia. Answer the questions that follow.

About 1 million years ago, glacial periods created cool, moist conditions across the whole of southern Australia. Wet areas linking the eastern and western coastlines of southern Australia allowed a continuous population of frogs to interbreed. When the glacial period ended, the climate became much drier and this led to the formation of the Nullarbor Plain in southern central Australia. This desert zone prevented frogs in the south-east from interbreeding with the south-western frogs. Today the two populations show distinct differences in their appearance and in their reproductive behaviour.

a Explain how these environmental changes can lead to evolutionary change in the frogs. (2 marks)

b Compare the original gene pools of the eastern and western frogs at the time when the Nullarbor Plain formed. (2 marks)

c How will gene mutations in both populations ensure that the two groups become two new species? (1 mark)

d The western zone is much drier than the eastern zone. How will these different environments select the favourable characteristics in each population? (2 marks)

6 Species are often divided into sub-species called 'races'. We are familiar with the different racial characteristics in humans. These different races can interbreed. Suggest a mechanism for the development of races in a human population. (3 marks)

7 *Escherischia coli* is a common bacterium. If a natural population is cultured in a laboratory in the presence of an antibiotic, we find that most but not all the bacteria are killed. The survivors are collected and cultured again in the presence of the antibiotic. This time most survive and few are killed.

a What conclusion can one draw about the natural population of bacteria? (2 marks)

b Explain why the antibiotic is a selecting agent. (2 marks)

Humans

Humans or *Homo sapiens* are the only living members of the genus *Homo*. Together with chimpanzees and apes, they are classified as primates within the larger vertebral class called mammals. All placental mammals feed their young with milk produced from mammary glands. During pregnancy the foetus is attached to the wall of the uterus by a special tissue called the placenta. In this section we will examine some important body systems that humans share with other mammals.

Glossary

Antibodies—proteins produced by the immune system that immobilise and destroy pathogens

Antigens—substances or microbes that stimulate the body's immune response

Cerebellum—part of the brain (at the back of the head) controlling involuntary movements and balance

Cerebrum—part of the brain that controls higher order thinking, the emotions and voluntary movements

CNS—central nervous system, comprising the brain and spinal cord

Connector neurones—neurones in the spinal cord that transfer information between sensory and motor neurones

Endocrine system—a control and coordination system that produces chemical messengers called hormones in ductless glands

Lymphocytes—specialised white blood cells involved in the immune system; some produce antibodies and others produce memory cells

Motor neurones—carry nervous impulses to muscles or glands

Pathogen—disease-causing microorganism

Phagocytes—a type of white blood cells that engulf and destroy microbes

Placenta—special tissue linking a developing foetus to the uterine wall of its mother

Sensory neurones—carry electrical impulses from sense organs towards the CNS

Coordination systems in humans

The human body is a complex system of tissues, organs and organ systems. These organ systems collectively function to satisfy the needs of all cells. Such a complex system requires coordination, otherwise the human body cannot function efficiently. **Communication and control systems** are needed to ensure that all body systems are coordinated. The two control and coordination systems that we examine are:

- the **nervous system**—the brain is connected to the rest of the body by nerve fibres that transmit and receive information as electrical signals;
- the **endocrine system**—various glands produce chemical messenger molecules called hormones that control various bodily processes.

The nervous system

The nervous system consists of a network of nerve tissues which transmit electrical information from one site to another. Nervous tissues are composed of nerve cells called **neurones**. The nervous system is composed of two parts. These are:

- **the central nervous system (CNS)** which consists of the brain and spinal nerve cord;
- **the peripheral nervous system** which consists of nerves that connect the CNS to the rest of the body.

a. Structure and types of neurones

Each neurone consists of a **cell body** with radiating fibres called **dendrites**. In many types of neurones there is one longer and thicker fibre called the **axon**. The axon is covered in a fatty insulating layer called the myelin sheath. It prevents nervous impulses from crossing over to neighbouring neurones.

- Axons conduct electrical impulses *away* from the cell body.
- Dendrites conduct electrical impulses received from another neurone *towards* the cell body.

Neurones can be classified into three main types.

- **Sensory neurones**—these neurones carry electrical impulses towards the central nervous system. Our sense organs (eg. the tongue, eyes, ears, skin) have sensory receptors that contain many sensory neurones. They transmit information into the spinal cord.
- **Connector neurones**—these neurones are part of the CNS and are located in the spinal cord. They receive information from the sensory neurones. Some of this information is relayed up the spinal cord to the brain. Other information may be

relayed immediately to muscles (or glands) along motor neurones.

- **Motor neurones**—these neurones carry electrical impulses away from the CNS towards muscles or glands, which then respond. Muscles contract and glands release their hormones.

Figure 3.19 shows the typical structure of the three common types of neurones.

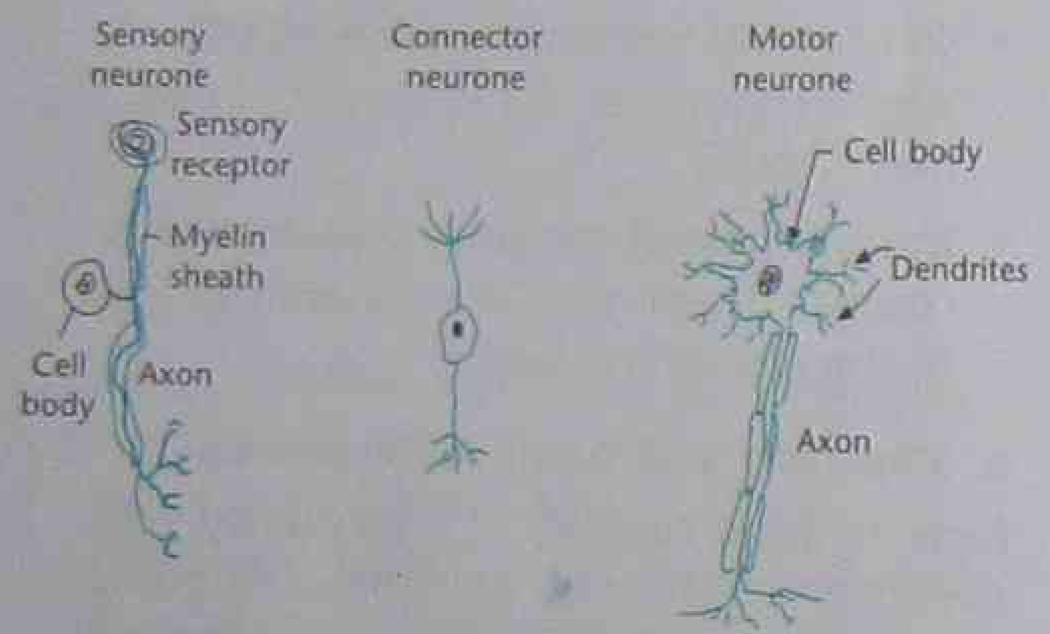


Figure 3.19 Three common types of neurones

b. Structure of the central nervous system

The central nervous system consists of the brain and spinal nerve cord. They are protected by layers of fluid, membranes and bone tissue.

- The brain is protected by the skull.
- The spinal nerve cord is protected by the vertebral bones—nerves from the body enter the spinal cord through small gaps between the vertebrae.

The brain is a control centre. Information received is processed and messages are sent by the spinal cord to various muscles and glands. Figure 3.20 shows some of the important parts of the brain.

Table 3.2 summarises the functions of the parts of the brain.

c. The peripheral nervous system

Information is relayed from the CNS to the body via the peripheral nervous system. Part of this system involves **voluntary** movements and part involves **involuntary** movements.

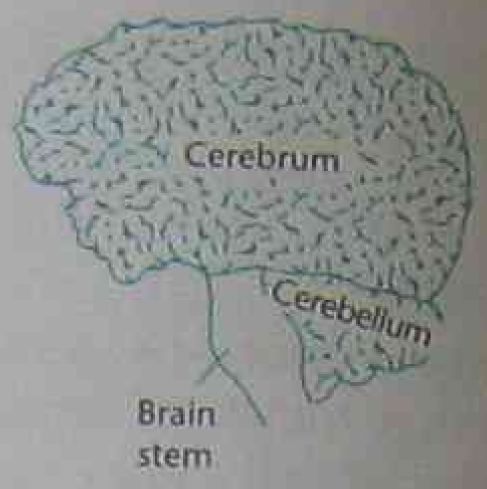


Figure 3.20 Parts of the brain

Table 3.2 Functions of the parts of the brain

Part	Function
Cerebrum	• composed of two halves • controls voluntary movements • controls memory, intelligence, behaviour, emotions, speech, vision, smell, touch, hearing
Cerebellum	• controls muscles involved in involuntary movements such as balance and fine motor control
Brain stem	• connects the brain to the spinal cord; information sorting centre • control centre for breathing, heart rate and swallowing • the hypothalamus (at the top of the brain stem) controls thirst and temperature

- The 43 pairs of **voluntary nerves** from the brain and spine connect to the muscles and sense organs in the head and body. Voluntary movements of the arms, leg and head are controlled by these nerves.
- The system of **involuntary nerves** regulates many functions including:
  - — heart muscle control
  - — iris muscle control
  - — bladder and bowel muscle control (can be controlled with training)
  - — responses to danger.
- The body has various automatic (involuntary) movements called **reflex arcs**. If a sensory neurone is stimulated, messages are sent via the connector neurones in the spinal cord direct to the motor neurone and an **effector** muscle or gland. This leads to a rapid response that is important in many situations.

Example

If you tread on a sharp thorn with bare feet your body immediately responds by raising your foot away from the danger. This response is not controlled by the brain, although the brain does register the event and the pain associated with it.

Figure 3.21 shows an example of the movement of nerve impulses in a reflex arc.

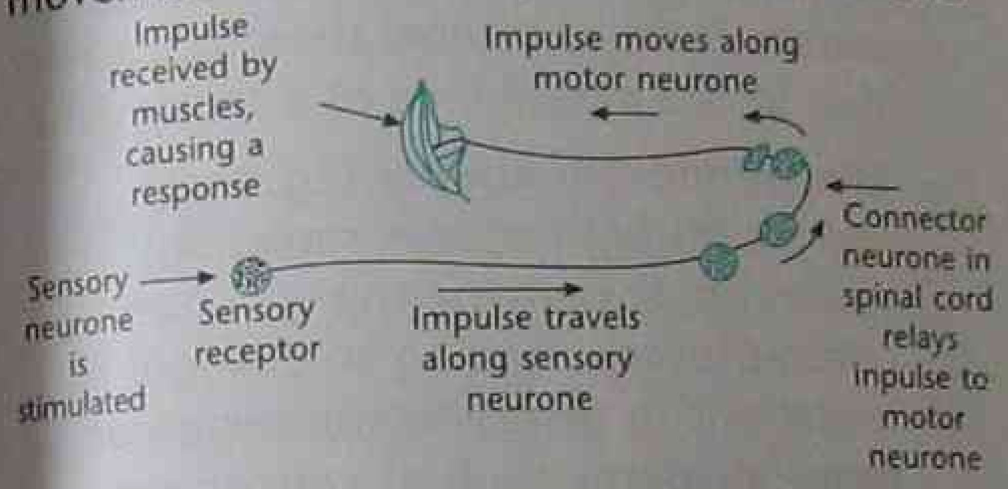


Figure 3.21 Nerve impulse movement in a reflex arc

d. The sense organs and receptors

Our body has sense organs that respond to different types of stimuli. These sense organs are listed in Table 3.3.

Table 3.3 Sense organs and their location

Sense organ	Location of sensory receptors
Taste	tongue surface
Smell	top of nasal cavity
Vision	retina in the eyes
Hearing	ear drum and auditory nerve
Touch/pressure	skin
Pain	skin; throughout body
Temperature	skin; throughout body

The endocrine system

The human body has a second system that is involved in control and coordination. This is the **endocrine** system. It consists of various **glands** that release **hormones** (special chemical messengers) directly into the bloodstream and bodily fluids (vascular system). The vascular system carries these hormones to various organs or cells around the body which are then stimulated to respond. The endocrine system is important in controlling growth, metabolism and reproduction. Figure 3.22 shows the location of some important endocrine glands. The **pituitary gland** at the base of the brain is an important gland in that it controls and stimulates many other glands. The pituitary gland releases many hormones, including those that regulate skin pigmentation, re-absorption of water in the kidneys and excretion of milk in nursing mothers.

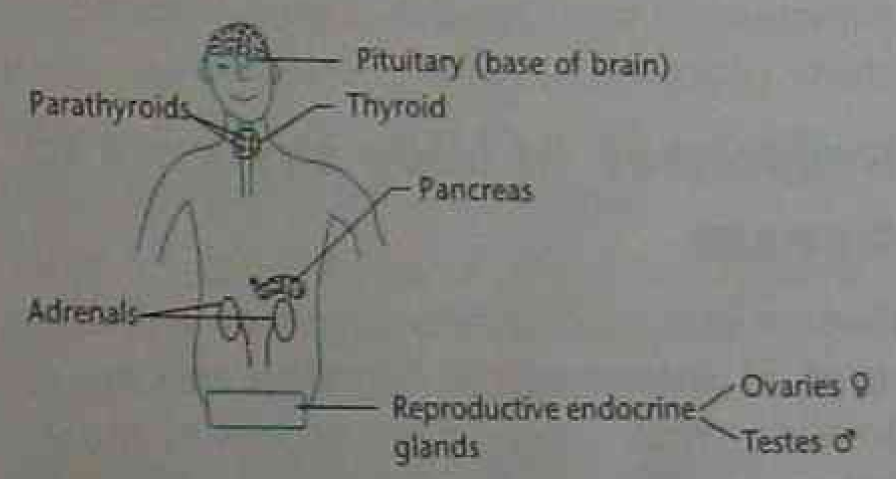


Figure 3.22 Locations of endocrine glands

Table 3.4 lists some important endocrine glands and some of the functions controlled or affected by the hormones they produce.

Table 3.4 Some important functions of the endocrine glands

Endocrine gland	Hormone	Function of hormone
Pituitary	growth hormone	stimulates growth and DNA synthesis
Pancreas	insulin	stimulates glucose uptake in all cells
	glucagon	stimulates the liver to break down glycogen into glucose
Thyroid	thyroxine	stimulates metabolism and heart rate
Adrenal	adrenalin	stimulates heart rate and blood pressure
Parathyroid	parathyroid hormone	stimulates calcium ion release in bones

Example. Control of glucose levels in the blood

The pancreas produces two important hormones called **insulin** and **glucagon**. Insulin reduces glucose levels in the blood by stimulating body cells to take up glucose. Glucagon acts in the reverse way. When blood glucose levels are too low, glucagon stimulates the liver to break down its stored glycogen into glucose, which is then released into the blood and body fluids. Together these two hormones keep glucose levels regulated. The disease called **diabetes** is caused by a failure of cells in the pancreas to produce the correct levels of these glucose-regulating hormones.

Responses of body systems to disease

Disease in a multicellular organism is the result of **abnormal cell function** and the breakdown of one or more organs or body systems.

Diseases of the human body can be classified as **infectious** or **non-infectious**.

- **Infectious diseases** are caused by **pathogens**. Pathogens are disease-producing microorganisms (microbes) such as bacteria, viruses, protozoa and some fungi. These pathogens enter the body in a variety of ways and their activities disrupt its proper functioning. Infectious diseases (eg. measles, HIV and diphtheria) can be passed from one person to another.
- **Non-infectious diseases** have a variety of causes.
  - **Genetic diseases** may have several causes. During the life of an organism the process of cell reproduction and cell differentiation may go wrong. This may occur because the **genetic code** on the chromosomes is **not copied correctly** prior to cell division. The new cells therefore receive faulty copies of the code. Sometimes **chromosomes do not divide** correctly and some cells inherit additional copies while others receive less than the normal number. The normal number of chromosomes in human cells is 46. Down syndrome is caused by cells receiving an extra copy of chromosome-21 attached to the end of chromosome-15. Thus Down syndrome sufferers have 47 chromosomes. Another genetic disease is haemophilia. It is a blood disease passed from one generation to the next due to the inheritance of a faulty gene.
  - **Environmental agents** (eg. toxic chemicals, high-energy radiation, viruses) may damage the chromosomes, leading to changes or mutations in genes. **Some organ or tissue diseases** such as **cancer** are caused by mutations of the genetic code by agents in the environment such as toxic chemicals, atomic radiation or viruses.
  - **Physiological diseases** such as giantism are also non-infectious. They are caused by organs failing to work properly.

Non-infectious diseases cannot be passed on to another person.

Infectious diseases

Figure 3.23 shows some examples of microbes. Not all microbes cause disease. Some have a beneficial role in the body (eg. in the production of vitamins) and in the environment (eg. as decomposers).

Bacteria belong to the **Kingdom Monera**. Their cells are very simple and their single coiled strand of DNA is not contained within a nuclear membrane like the cells of multicellular organisms. Bacteria may have many shapes, such as spheres, rods and spirals, as well as chains of spheres and rods. Pathogenic bacteria can damage the cells of the host by producing poisons (toxins). Bacteria multiply very rapidly at body temperature. Every 20–30 minutes, each bacterium cell divides by mitosis and produces two new bacteria. Over the course of a day the number of bacteria becomes quite large and their toxins start to disrupt the body.

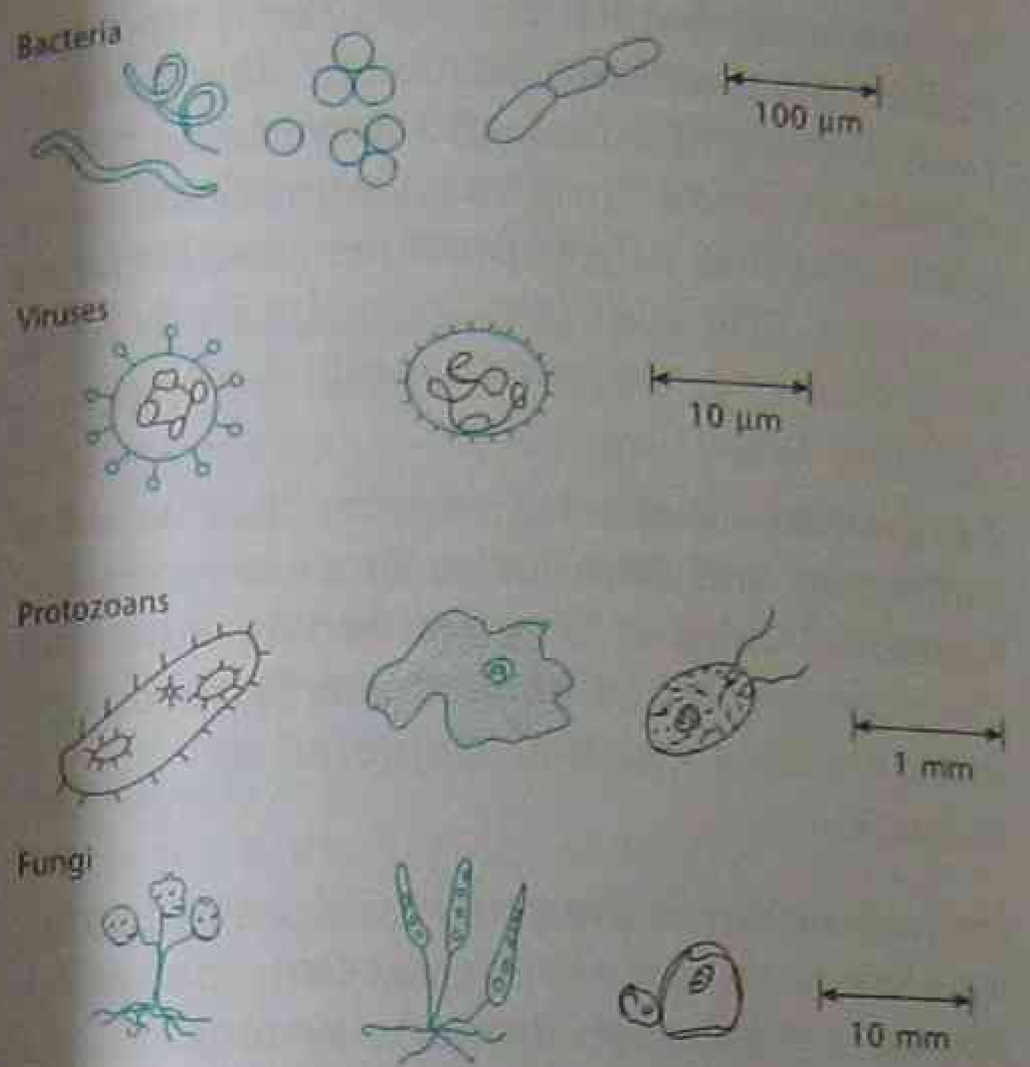


Figure 3.23 Examples of microbes

Viruses are not classified into the five major kingdoms of living things as they are not cellular life forms. They consist of particles that are usually made up of bundles of genetic material inside a protein coat. They invade a host and use the nucleus of the host to complete their reproduction. Eventually the host cell dies and splits open to release thousands of new viral particles, which then infect other cells.

Protozoans belong to the **Kingdom Protista.** They are single-celled microbes with their DNA inside a nucleus. They vary widely in shape, and mainly live in water or in moist soil. They can reproduce asexually or sexually. Not all protozoans are pathogenic.

Fungi are mostly multicellular organisms. The **Fungal Kingdom** is distinct from plants, as fungi do not produce their own food by photosynthesis. The parasitic fungi reproduce sexually or asexually. They are abundant in soil. The fungi that cause disease in humans tend to attack the skin, hair, nails and moist membranes.

Table 3.5 classifies some infectious diseases according to the type of microbe that causes the disease.

Resisting the attack of microbes

The human body resists attack by pathogenic microbes in a variety of ways. The body provides barriers to microbe entry. If microbes enter, then white blood cells of the immune system attack the invaders. Time is very important. The immune system must defeat the pathogens before they overwhelm the body. Disease and even

Table 3.5 Common infectious diseases

Disease	Pathogen	Transmission	Symptoms of the disease
Salmonella food poisoning	bacterium	eating contaminated food	diarrhoea; inflammation of the intestine; nausea
Tuberculosis	bacterium	inhaling infected droplets in the air; drinking contaminated milk	lesions form in lungs producing fever and coughing
Malaria	protozoan	protozoan enters human during a mosquito bite	severe fevers; shivering; sweating; nausea; reoccurs periodically
Mumps	virus	contact; inhalation of infected droplets	swollen salivary glands in cheek/neck; fever
Measles	virus	contact with infected people; inhalation of infected droplets	red spots (rash) on face and body; fever
Tinea	fungus	contact of the foot with infected areas (eg. communal bathing)	itching and blistering of skin between toes; cracking of skin

death may result if the body's defences cannot overcome the invasion.

1. Barriers to microbe entry

- **Acidic environments**. Acids are secreted onto the skin and acids are produced in the stomach and kill many of the microbes or slow down their reproduction. Urine is also acidic and this prevents microbes entering the body via the urethra. The highly acidic gastric juices in the stomach kill many microbes.
- **Mucus linings**. Our airways are lined with a sticky mucus that traps microbes that may be inhaled.
- **Hairs and cilia**. The respiratory tract is lined with cilia (microscopic hairs) that trap microbes.

2. Phagocytes

Phagocytes are **white blood cells** that attack foreign substances, including microbes. The phagocytes engulf and destroy the microbes. **Pus** is the remains of dead phagocytes and microbes.

A fever is often associated with the action of white blood cells. The increased temperature of the body helps to destroy some microbes.

3. Immunity

The body's third line of defence involves specialised white blood cells called **lymphocytes**. These are produced by the immune system as a response to infection. This process is called **naturally induced active immunity**.

Lymphocytes (which are made in lymph nodes that drain into veins) circulate in the body fluids and attack and immobilise the pathogen. Some lymphocytes produce antibodies and others produce special 'memory' cells that remain in the body for a long time and can react rapidly if re-infection occurs.

Antibodies are special proteins that bind to pathogens and immobilise or destroy them. Figure 3.24 shows how antibodies work. They are specific to an **antigen** (a specific foreign substance) that triggered its formation.

Immunisation is the process of introducing into the body a serum or **vaccine** that will stimulate the body's immune system to produce specific antibodies. Vaccination is an example of **artificially induced active immunity**.

Vaccines contain either:

- killed microbes
- harmless strains of living pathogens
- modified toxins that stimulate the immune system without destroying it.

Babies are born with antibodies transferred to them from the mother's own immune system. More are received while breast-feeding. This is an example of **naturally induced passive immunity.**

Antibodies for specific diseases can also be cultured in other animals (eg. horses) and then a vaccine is developed containing these specific antibodies. This is normally done for travellers who may come into contact with specific diseases such as cholera and yellow fever. This type of vaccination is an example of **artificially induced passive immunity.**

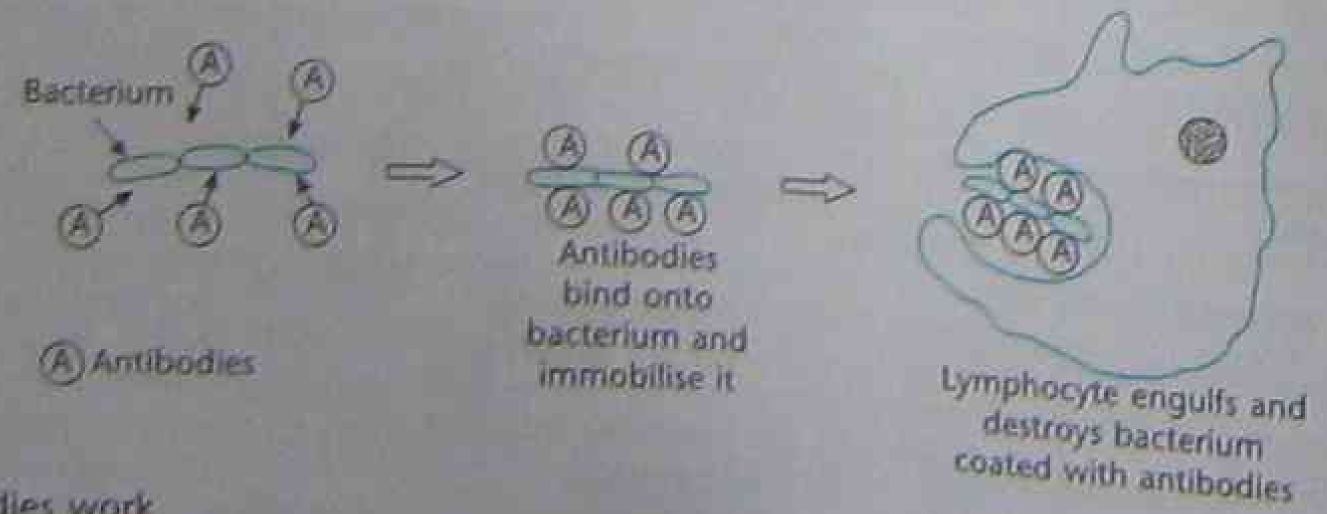


Figure 3.24 How antibodies work

Mass immunisation is an important program for various diseases such as whooping cough, diphtheria, polio, tetanus, measles and mumps. It ensures eradication of dangerous pathogens from the general population by ensuring that those that are more susceptible are protected.

Antibiotics

When the body cannot resist the attack of pathogenic bacteria, treatment with antibiotics may lead to a cure. Antibiotics are useful in many bacterial infections but cannot be used to cure viral infections. **Penicillin** was one of the earliest antibiotics that was developed in the late 1940s. It was extracted from a green mould growing on rockmelons. Since then many antibiotics have been produced from other moulds and bacteria, as well as being chemically synthesised, and millions of lives are saved annually.

Antibiotics work in a variety of ways, including:

- damaging the microbe's cell membrane
- interfering with the metabolism of the microbe
- preventing DNA replication
- preventing a cell wall forming around new microbes.

Unfortunately, bacteria can become resistant to antibiotics. This resistance is an example of the evolutionary principles previously discussed (see page 87). Other anti-microbial drugs continue to be developed to cure fungal and protozoan diseases.

Antibiotics should have certain properties. They should:

- kill (or inhibit) the pathogen
- not harm the person receiving the antibiotic
- cause minimal side-effects
- not disturb the balance of useful microbes in the body.

Reducing exposure to pathogens

Apart from the natural ability of the body to resist infection, humans need to ensure that their exposure to pathogens is reduced. Here are some examples.

- **Food**. Microbes such as bacteria and fungi can spoil food and make it unfit to eat. Food can be prevented from spoiling by:
  - —**freezing**, to slow down microbial growth
  - —**heating or cooking**, to kill any microbes present
  - —**drying**, so that there is insufficient water to allow microbes to grow
  - —**pickling** (in acidic juices such as vinegar) to kill microbes
  - —**salting**, to dry out microbes and prevent their growth.

  Food containers can also be **sterilised** by heating to high temperatures prior to placing food in the container.
- **Antiseptics**. Microbes often survive for long periods of time on surfaces in our homes. Antiseptics are solutions of various chemicals that are used to kill microbes on these surfaces. On the skin, milder antiseptics can also be used. Alcohol, for example, is a mild antiseptic that can be used on cuts to the skin.

Human reproduction

Human reproduction is an example of **sexual reproduction**, as humans produce specialised sex cells that combine to form the first cell of a new individual.

Gametes

Here are some important facts about sex cells.

- Sex cells are called **gametes**.

- Male gametes are called **spermatozoa** (or sperm).
- Female gametes are called **ova** (or eggs). (*Note*: Ovum is the singular form of ova.)
- Each sperm or egg contains **half the normal chromosomal number** found in all other body cells. In humans this half-number (or haploid number) is 23. Normal body cells contain 46 chromosomes (ie. 23 pairs, or diploid number).
- A cell division process called **meiosis** produces sperm and eggs.
- When a sperm cell **fertilises** an egg cell the new cell that is formed (called the **zygote**) has 23 pairs of chromosomes.
- Two of the 46 chromosomes are the **sex chromosomes**. In males these two chromosomes are labelled X and Y. In females the two sex chromosomes are both X chromosomes.
- All eggs produced by a female contain one X chromosome.
- Half the sperm produced by a male contain an X chromosome and half contain a Y chromosome.

Figure 3.25 shows the process of gamete formation and fertilisation to produce a zygote.

Male reproductive system

Figure 3.26 shows the major structures of the human male's reproductive system. The important functions of the major structures are listed in Table 3.6.

Female reproductive system

Figure 3.27 shows the major structures of the human female's reproductive system. Table 3.7 lists the functions of the important structures in the female reproductive system.

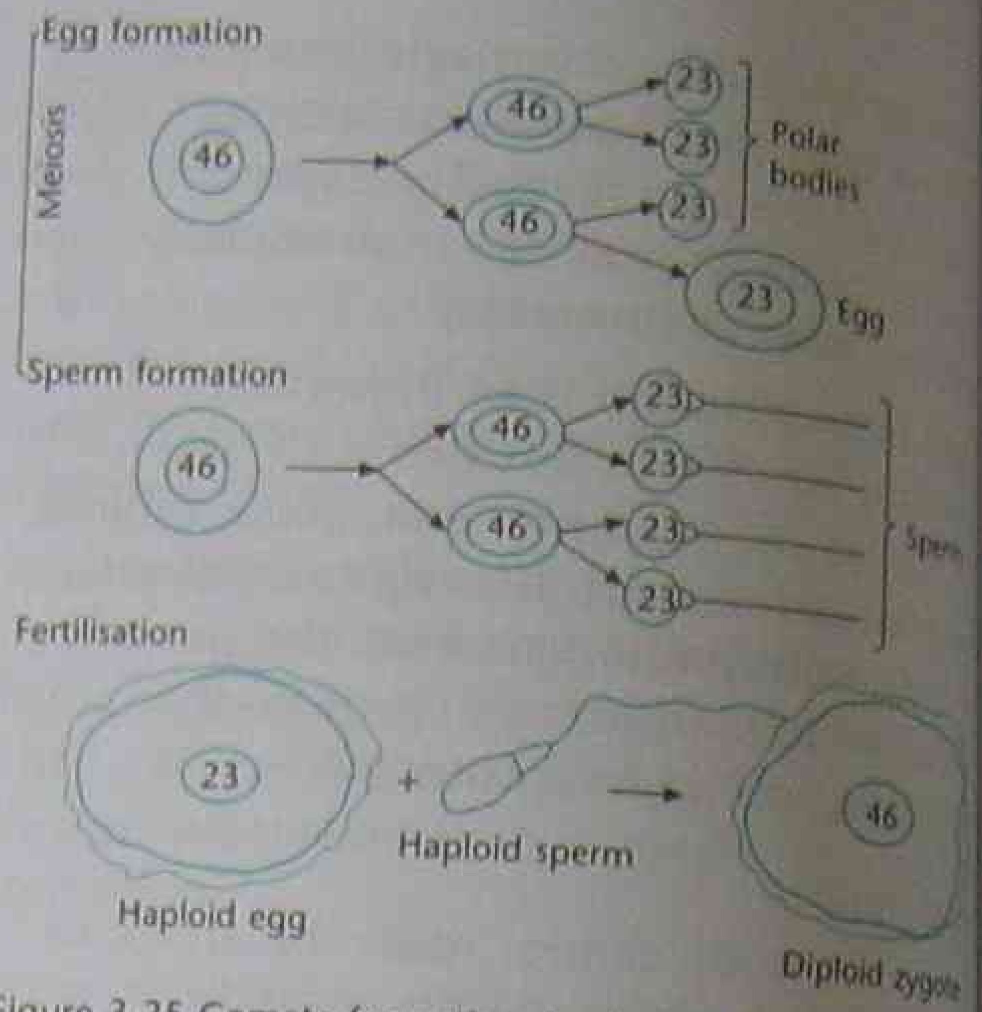


Figure 3.25 Gamete formation (meiosis) and fertilisation

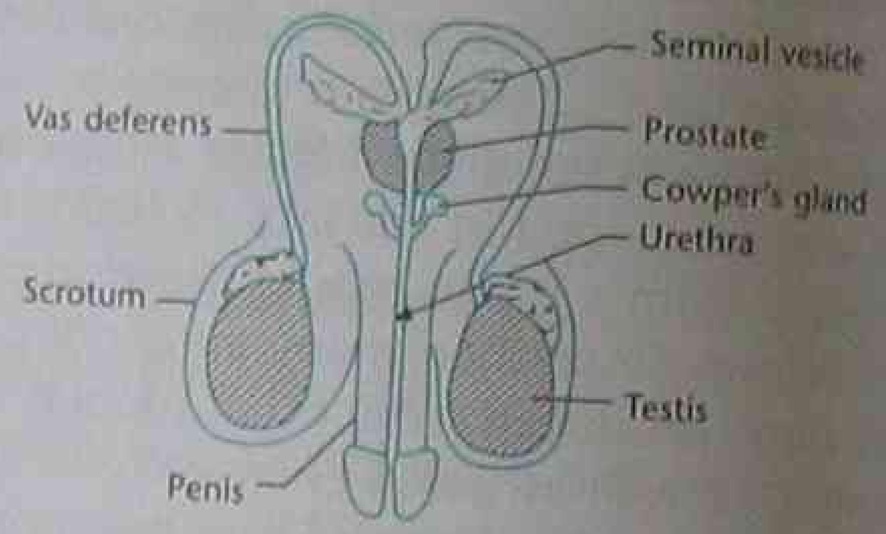


Figure 3.26 Male reproductive system

Table 3.6 Male reproductive system

Structures	Function
Scrotum	external sac holding the testes; helps to regulate the temperature of the testes
Testes	site of sperm production; glands that produce male hormones such as testosterone
Vas deferens (sperm duct)	tube that carries sperm away from the testes
Glands: prostate; seminal vesicles; Cowper's gland	produce protective and nutritive fluids for the sperm; the combination of sperm and these fluids is called **semen**
Urethra	tube that carries semen during intercourse; tube that discharges urine from the bladder
Penis	organ used to deposit semen in the vagina of the female during intercourse

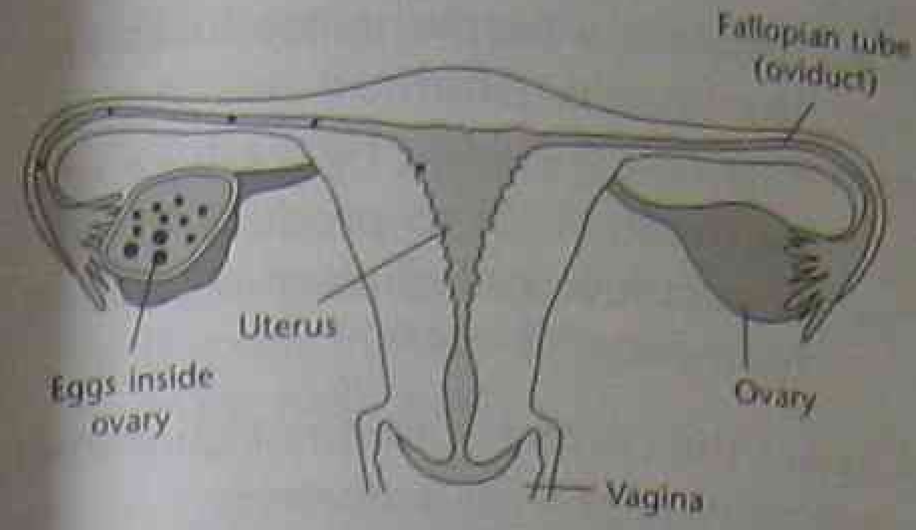


Figure 3.27 Female reproductive system

Table 3.7 Female reproductive system

Structures	Function
Ovaries	site of egg production; glands which produce female hormones such as oestrogen and progesterone
Fallopian tubes (oviducts)	tubes through which an egg moves following ovulation; site of egg fertilisation if sperm are present in the reproductive tract
Uterus (womb)	muscular organ in which a fertilised egg will implant and grow to produce a baby
Vagina	canal in which semen is deposited during intercourse; canal through which a baby is born

Puberty

Puberty is the time in the life of a child when it starts to become sexually mature. It is the beginning of adolescence. Hormones are released by glands such as the pituitary gland as well as the ovaries (female) and testes (male). These hormones stimulate the development of the sexual organs, as well as producing secondary sexual characteristics. Table 3.8 summarises some of the important changes at this time. The onset and length of puberty vary considerably from one person to another, and also between racial groups.

Fertilisation and pregnancy

The major events in the fertilisation of an egg and the development of a baby are summarised as follows:

Table 3.8 Characteristic changes at puberty

Males	Females
Puberty begins at 12–14 years of age	Puberty begins at 10–14 years of age
Growth of testes and penis	Growth of breasts, ovaries, uterus, vagina and hips
Growth of pubic hair and body hair; growth of facial hair	Growth of pubic hair and body hair
Production of sperm and semen	Production of eggs and menstrual periods begin
Thickening of vocal cords and deepening of the voice	Less thickening of vocal cords than males
Increased muscle development	Less muscle development than males; redistribution of fatty tissue to hips and thighs

- **Ovulation** is the process of egg release from an ovary. This occurs about every 28 days.
- The egg is transported through a Fallopian tube towards the uterus. If the egg is not fertilised by a sperm, it will pass out of the body in the **menstrual blood flow** produced by the breakdown of the thickened lining of the uterus. This blood flow lasts about 4 to 5 days. The uterine lining will re-grow in the next cycle.
- If the egg is **fertilised** by a sperm in the Fallopian tube it will begin to divide into a ball of cells. This ball of cells takes 8 days to travel into the uterus where it becomes **implanted** in the uterine wall. The pregnancy or **gestation period** has begun.
- The developing **embryo** is connected to its mother by a special tissue called the **placenta**. The **umbilical cord** is an extension of the placenta. Nutrients and oxygen pass across the placenta to the developing embryo. Wastes pass the other way and are eliminated by the mother. The blood of the baby and of the mother never mix.

- From 8 weeks until its birth the developing baby (or **foetus**) grows and matures.
- When the baby is mature it is expelled from the uterus by strong muscular contractions. It passes through the vagina (birth canal) and is born. This normally occurs 270 days after fertilisation.

Test yourself (answers on pages 218–19)

Part A. Knowledge (answers on pages 218–19)

1 Sensory neurones:

a transfer information inside the spinal cord.

b are stimulated by hormones and transmit information through the bloodstream.

[✓ c] carry electrical messages towards the central nervous system from sense organs.

d carry electrical impulses away from the central nervous system towards the peripheral nervous system. (1 mark)

2 An important function of the cerebellum is:

a to control muscles involved in involuntary movements and fine motor contol.

[✓ b] the control of memory and intelligence.

c control of behaviour.

d the control of breathing, temperature and thirst. (1 mark)

3 Select the statement that is true about the endocrine system.

a The endocrine system is involved only in reproduction and the control of sugar metabolism.

b The pituitary gland controls all other glands by sending electrical stimuli via the central nervous system.

[✓ c] Insulin is a hormone produced by the thyroid gland.

d Diabetes is caused by a failure of pancreatic cells to produce the correct levels of glucose-regulating hormones. (1 mark)

4 Select the correct statement concerning a common infectious disease.

a Tuberculosis is a bacterial disease that causes the formation of lesions in the lungs.

b Malaria and mumps are both infectious diseases caused by protozoans.

c Measles is a viral disease that responds readily to antibiotic treatment.

d Tinea is a fungal disease characterised by a red rash that covers most of the body. (1 mark)

5 Ovulation is the process in which:

a an egg is fertilised in the Fallopian tubes.

b a fertilised egg implants in the wall of the uterus.

[✓ c] an egg is released from the ovary.

d menstrual blood flow occurs because of the breakdown of the uterine lining. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

a The vas deferens is the tube that carries _____ away from the testes.

b Antibiotics can destroy a bacterium by damaging its cell _____ .

c Vaccination is an example of _____ induced active immunity.

d Nutritional diseases such as anorexia are examples of _____ diseases.

e In a reflex arc the electrical message travels from the sensory neurone to the connector neurones and finally the _____ neurone.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a central nervous system	f heart rate and blood pressure
b cerebrum	g white blood cells
c adrenal glands	h memory
d pathogens	i disease-causing microbes
e phagocytes	j brain and spinal cord

8 The body has various barriers to the entry of pathogenic microbes. Name three examples of these barriers. (3 marks)

9 State two functions of human testes. (2 marks)

10 Compare the number of chromosomes in the following human cells: (5 marks)

a white blood cell; a sperm cell; an egg cell; a zygote; a brain cell

Part B. Skills (answers on page 219)

1 Figure 3.28 shows two diagrams of an eye (X and Y) under different lighting conditions.

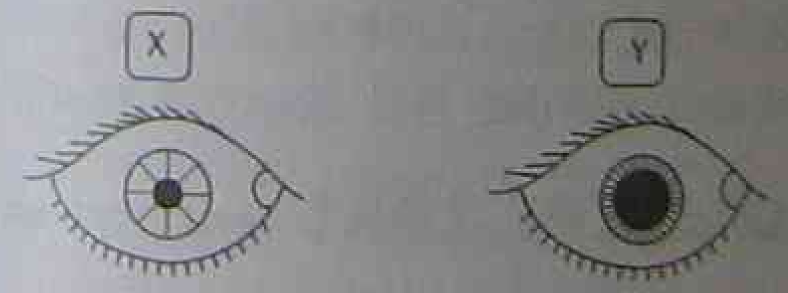


Figure 3.28 Diagrams of an eye under different lighting conditions

a Which eye corresponds to conditions of bright sunlight? (1 mark) X

b Is this change in the pupil of the eye an example of a voluntary or involuntary movement? (1 mark)

c Explain in terms of nerves the sequence of events that leads to a change in pupil size on walking from a darkened room into bright sunlight. (2 marks)

2 A person is involved in a motor vehicle accident that leads to a severing of his spinal cord in the middle of his back. A doctor conducts a reflex test on his knee.

a Will the knee produce a reflex arc when tested? (1 mark)

b Will the patient feel the pressure on his knee? (1 mark)

c Why can't the patient walk following this accident? (1 mark)

3 Match the stimulus in column 1 to the response in column 2. (5 marks)

Stimulus	Response
a smell or sight of food	f sneezing
b a sudden explosion	g production of saliva
c peppering food	h jerking of the body
d touching a hot surface	i shivering
e sudden drop in temperature to zero degrees	j hand pulls away suddenly

4 Figure 3.29 shows stages in the

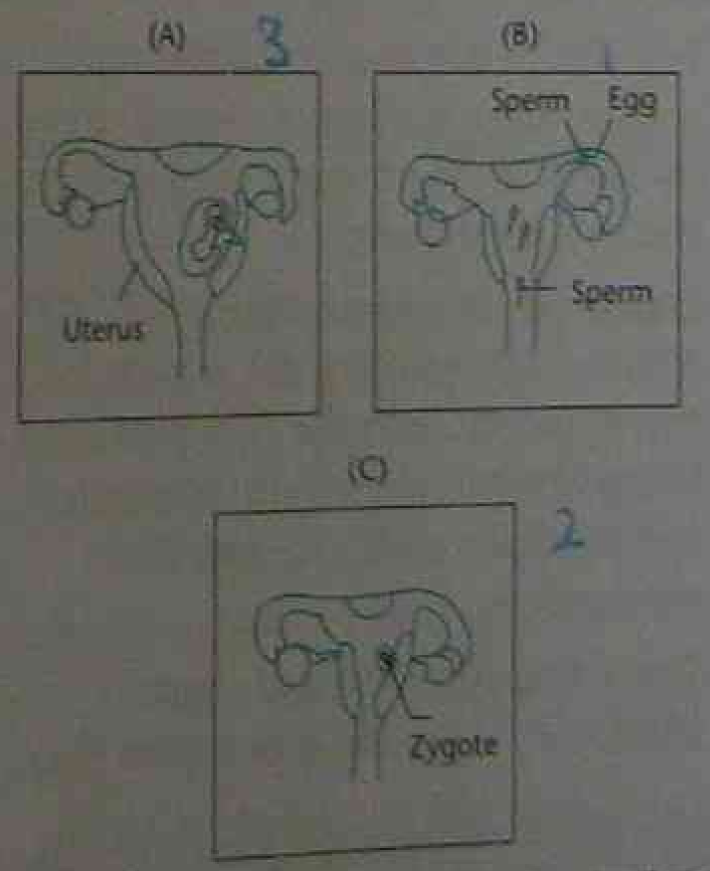


Figure 3.29 Jumbled stages of fertilisation, implantation and growth

fertilisation of an egg, implantation and growth of a human embryo. The stages are jumbled. Use the code letters to list the diagrams in their correct sequence. (3 marks)

5 Hormones are produced by various glands. Various feedback controls ensure that their production is regulated. Figure 3.30 shows the effects of two of these hormones on the control of the menstrual cycle. The organs and glands involved are the uterus, ovary and pituitary gland.

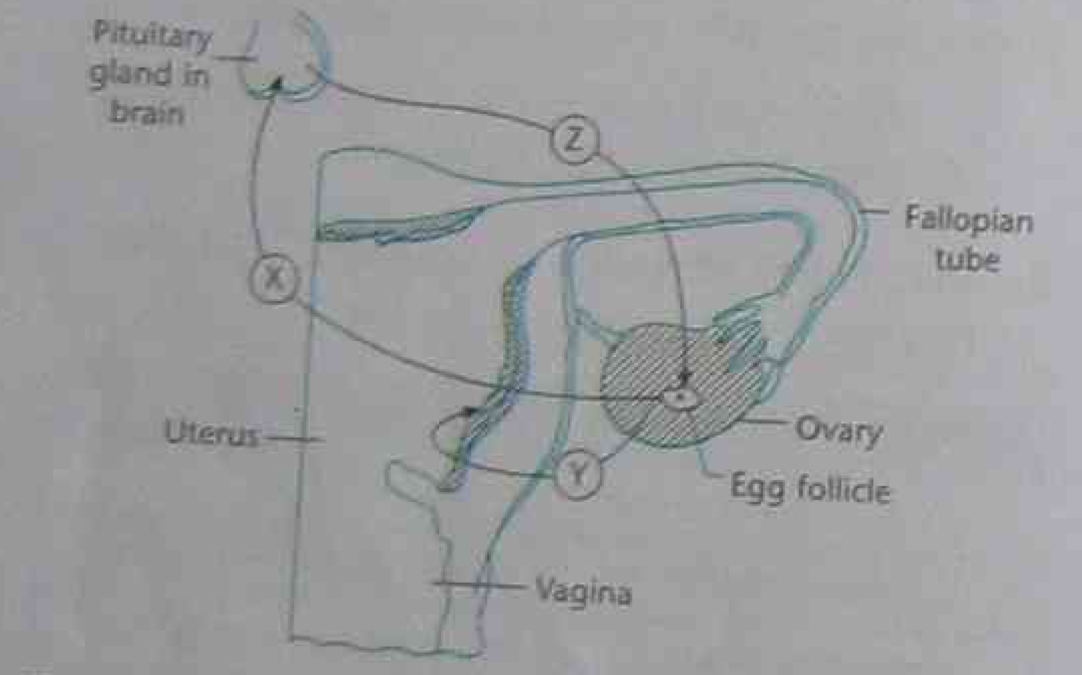


Figure 3.30 Code-labelled diagram of feedback controls in the menstrual cycle

Match the code labels in the diagram to one of the statements below. (3 marks)

Statements

a Follicles in the ovary containing unripe eggs are stimulated to grow by the FSH hormone released from the pituitary gland.

b Oestrogen is released into the bloodstream during the growth of the egg follicle in the ovary. The rising level of oestrogen stimulates the pituitary gland to stop FSH production.

c The rising level of oestrogen stimulates the uterus lining to grow.

6 Viruses are a non-cellular life form. They use the host's DNA for their reproduction. Figure 3.31 shows the jumbled steps in the reproductive cycle of a typical virus. List the jumbled diagrams in their correct sequence. (2 marks)

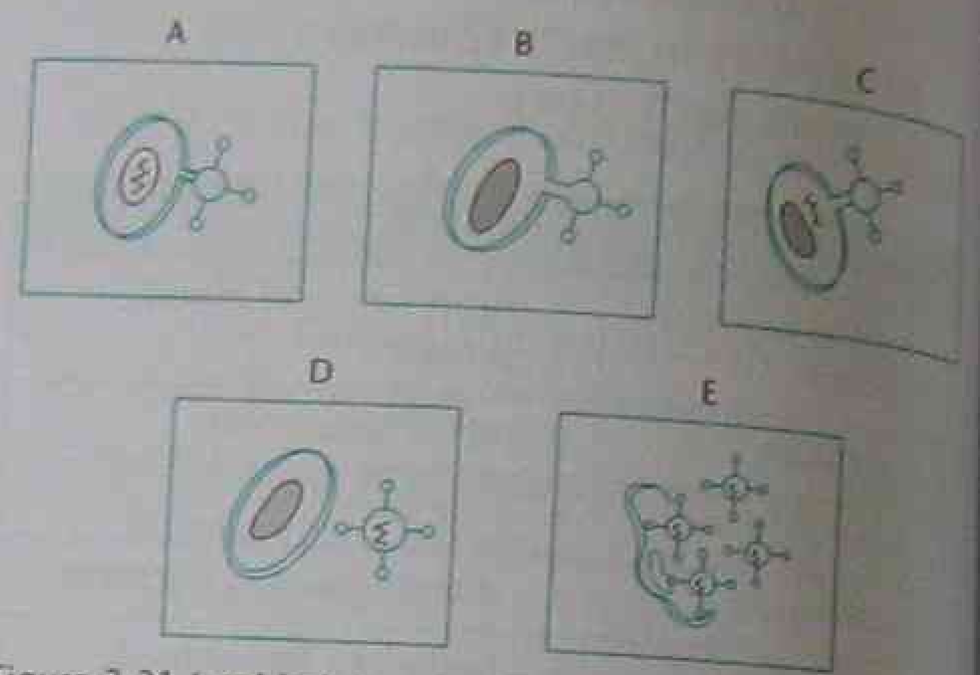
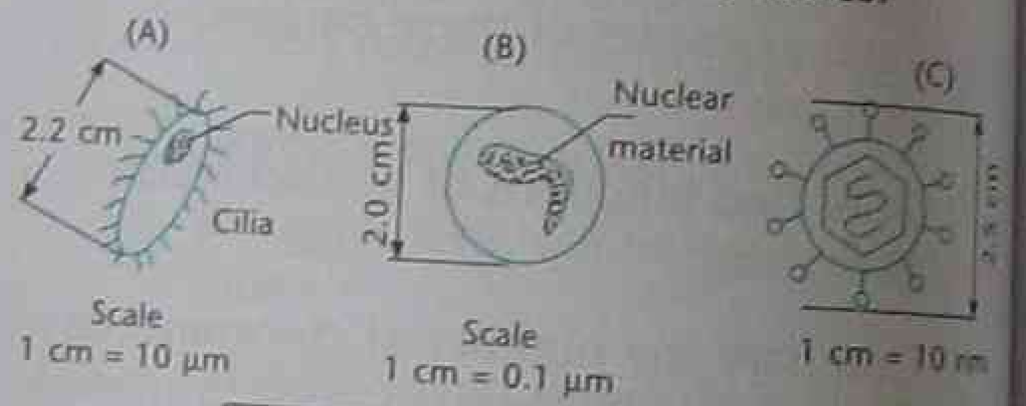


Figure 3.31 Jumbled steps in virus reproduction

7 Figure 3.32 shows diagrams of different microbes drawn to different scales.



1 nm = 1 nanometre = 1×10^{-9} m
1 µm = 1 micrometre = 1×10^{-6} m

Figure 3.32 Microbes drawn to different scales

a Use the scales to determine the maximum length of each microbe. (3 marks)

b Arrange the microbes in increasing order of size. (1 mark)

c Which microbe is a virus? (1 mark)

End-chapter test (answers on pages 219–21)

1 Use the following sub-headings to describe some evidence for evolution: (6 marks)

a Comparative embryology

b Comparative DNA analysis and biochemistry

c Geographic distribution and isolation

2 Explain the major points of Darwin's theory of natural selection. (4 marks)

3 The following comparative data were collected for changes in sea level over the last 300 million years. The scale is arbitrary from 0% (lowest sea level) to 100% (highest sea level).

Time (million years ago)	Relative sea level (%)
0	25
40	63
50	50
90	100
130	50
180	42
200	25
210	42
250	17
300	95

a Plot these data as a line graph. Join the points with straight lines. (3 marks)

b Three major extinction events have occurred in the last 300 million years. Over 75% of marine species became extinct at each event. These three events occurred at:

65 million years ago; 205 million years ago; 251 million years ago.

Use your graph to explain the relationship between these events and the relative sea level. (2 marks)

c Suggest a reason why sea level changes can lead to evolutionary changes. (2 marks)

4 Figure 3.33 is a diagram of neurones involved in a reflex arc. Code letters have been used to represent various structures.

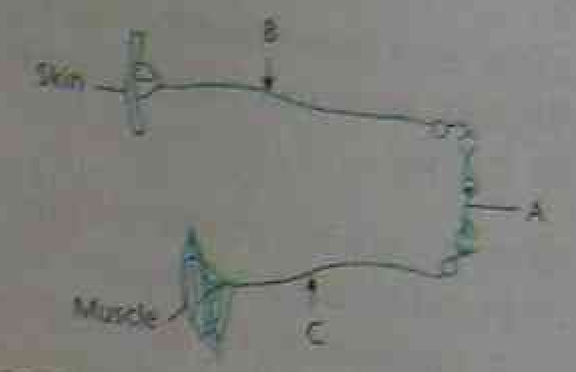


Figure 3.33 Code-labelled diagram of structures in a reflex arc

a Identify the structures labelled by the code letters. (3 marks)

b Use the code letters to state the sequence of neurones involved in the reflex arc. (1 mark)

c Why are reflex arcs important? (2 marks)

5 Figure 3.34 is a cross-section of the brain showing the location of various control centres.

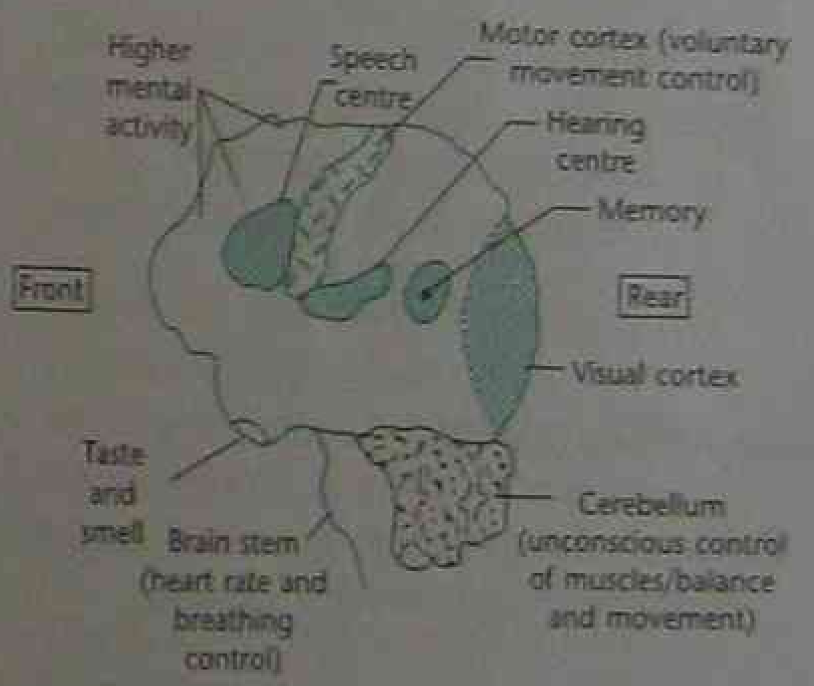


Figure 3.34 Brain cross-section showing various control centres

a If a student received a severe blow to the back of the head during a rock-climbing fall, what control centre in the cerebrum could be affected? (1 mark)

b An elderly person suddenly lost her sense of taste. Suggest a possible reason for this event. (1 mark)

c A boy fell down the steps and hit his forehead. Over the next few years he had trouble concentrating in class and remembering ideas. Explain why this has occurred. (2 marks)

6 a Various regions of the tongue respond to different tastes. Use the map diagram in Figure 3.35 to state what region of the tongue will be stimulated when a student eats: (3 marks)

i lemons

ii strong coffee

iii meringues

b How could a student establish the location of the bitter receptors of the tongue? (2 marks)

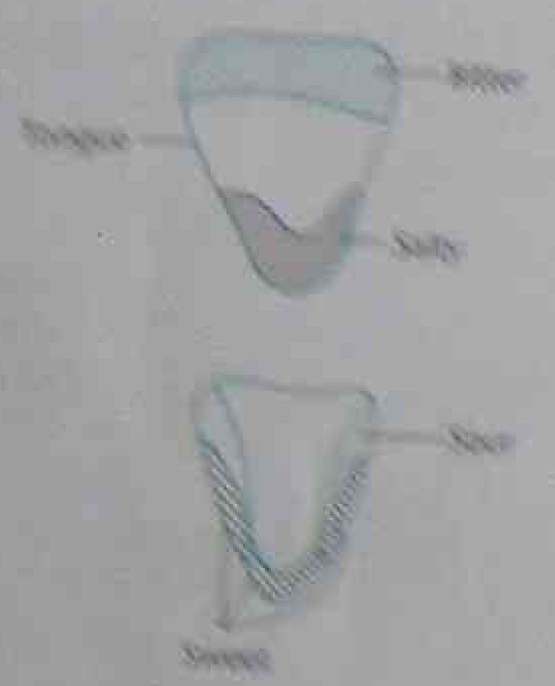


Figure 3.35 Map zones of the tongue

7 Match the glands in column 1 to their function in column 2. (5 marks)

Gland	Function
a pituitary	i produces adrenalin to make the heart beat faster and convert glycogen to glucose
b thyroid	ii produces testosterone which affects male secondary sexual characteristics
c pancreas	iii master gland that releases hormones to control/stimulate other glands
d adrenal	iv produces insulin which controls glucose metabolism
e testes	v produce thyroxin which controls the metabolic rate

8 a Is heart disease an infectious or non-infectious disease? (1 mark)

b The following table shows the relationship between smoking and heart disease. One value (X) is missing.

Cigarettes smoked (number per day)	Deaths from heart disease (number per 1000 people)
0	15
1–10	60
10–20	100
20–40	X
>40	225

i Describe the trend in the data. (1 mark)

ii Draw a column graph of the data. (3 marks)

iii Draw a trend line and determine a value for X. (2 marks)

9 During pregnancy the human embryo grows in length and mass. The following table shows some typical data on the approximate length and mass of an embryo over the 270 days of pregnancy.

Time (days)	Length (cm)	Mass (g)
30	1.3	0.02
60	2.5	1
90	9	15
120	18	100
150	25	Y
180	32	650
210	X	1000
240	43	1750
270	46	2500

a Plot line graphs of the data and use these graphs to estimate the missing values X and Y. (4 marks)

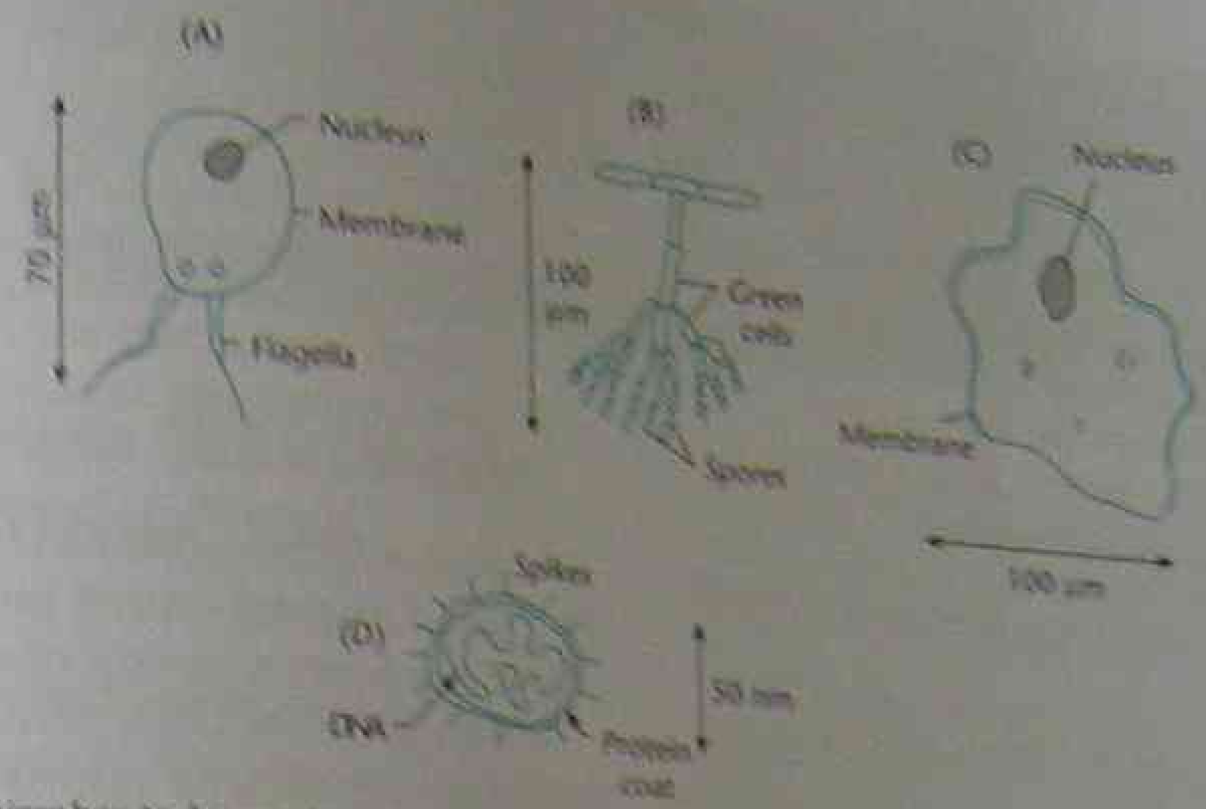


Figure 3.36 Selected microbes to be used to construct a key

b At what time will the foetus have a:

i length of 20 cm? (1 mark)

ii mass of 1.5 kg? (1 mark)

c Name the organ in which the foetus grows for 270 days. (1 mark)

10 Figure 3.36 shows various microbes identified by code letters. Use the visible features of these microbes to develop a written dichotomous key to classify and identify these selected microbes by their code letters. (4 marks)

Summary

Cells and body systems

- The cell theory states that all living things are composed of cells.
- All cells are derived from pre-existing cells.
- Multicellular organisms have cells that are organised into tissues and organs.
- Body systems serve the needs of cells (eg. by providing nutrients and removing wastes).
- Cell division is necessary for growth, repair and reproduction.
- Abnormal cell function leads to disease.

The Watson–Crick model of DNA

- DNA is a polymeric, double-helix molecule that contains the genetic code.
- During replications the DNA makes an exact copy of itself.
- DNA mutations can be caused by deletions and substitutions of nitrogen bases. There are advantages and disadvantages in DNA mutations.
- Genes are located on chromosomes. Genes are codes that contain information about inheritance. This information is transferred during reproduction.
- The features of an organism are determined by the genes it inherits and the interaction of the environment with these genes.

Evolution and natural selection

- The evidence for evolutionary change comes from the following studies:
  - — The age of Earth and radiometric studies of fossils
  - — Investigations of sedimentary strata, fossils and the law of superposition
  - — Comparative anatomy
  - — Comparative embryology

  - Geographic distribution and isolation of living things
  - Catastrophes and extinction events
  - Biochemistry and DNA analysis
- Darwin and Wallace developed the theory of natural selection to explain a mechanism for evolution.
- The major points of the theory of natural selection are:
  - There is a natural variation in a population.
  - There is a struggle to survive.
  - Organisms with favourable characteristics survive long enough to reproduce.
  - Subsequent generations contain a greater proportion of favourable characteristics.
- Gene mutations can lead to evolutionary change.

Humans

- The nervous system and endocrine system control and coordinate the functioning of the body.
- The nervous system consists of the central nervous system (brain and spinal cord) and the peripheral nervous system (nerves connecting the central nervous system to the rest of the body).
- The neurones of the nervous system consist of sensory, connector and motor neurones. Electrical impulses are connected along these neurones from a sense organ to a receptor such as a muscle or gland.
- The brain is divided into the cerebrum, cerebellum and brain stem. Each region has specific functions.
- The endocrine system uses chemical messengers called hormones (produced by ductless glands) to stimulate receptors around the body.
- The pituitary gland in the brain is a master gland.
- Diseases of the human body can be classified as infectious or non-infectious.
- Infectious diseases are caused by pathogens such as some types of bacteria, viruses, fungi and protozoa.
- Non-infectious diseases (eg. scurvy; cancer; heart disease) cannot be caught by others. They have a variety of causes including mutations of DNA, environmental effects and unbalanced nutrition.
- The body has various lines of defence against pathogenic microbes. These include:
  - barriers to microbe entry (eg. acidic environments; mucus; cilia)
  - phagocytes
  - immunity.
- Immunisation is the process of stimulating the body's immune system to produce specific antibodies.
- Sexual reproduction in humans involves the production and fertilisation of gametes.
- Gametes (eggs and sperm) have half the chromosomal number of normal body cells.
- Sperm are produced in the testes. As they travel through the male reproductive tract they are mixed with nutritive fluids from various glands to form semen. Semen is ejaculated into the female reproductive tract during sexual intercourse.
- An egg is produced once a month from an ovary. If sperm are present in a Fallopian tube, the egg may be fertilised. A zygote is formed.
- The zygote divides and implants in the uterine wall where its cells continue to divide and differentiate to form various tissues and organs.

Chapter 4
Earth and Space

The Big Bang theory and components of the universe

The early astronomers developed models to explain the motions of the stars and planets in the sky. Over time these models became refined until Newton developed a gravitational model that explained the motion of the heavenly bodies. It wasn't until the twentieth century, however, that answers to the problem of the origin of the elements of matter and the universe itself were developed. One theory, known as the Big Bang theory, eventually emerged as the most likely explanation of the origins of the universe. Like all theories, the Big Bang theory continues to be challenged as new evidence emerges from astronomical and theoretical studies.

Glossary

Astronomical unit—the distance between Earth and the Sun (1 AU)

Big Bang theory—a theory of the origin of the universe; it states that the universe came into existence about 13 billion years ago due to the creation of space-time and the partial conversion of energy into matter

Cosmic background radiation—the electromagnetic radiation in space that remains following the Big Bang; this radiation cooled as the universe expanded and is now at an average temperature of 3 degrees above absolute zero (−270°C)

Galaxy—vast collections of stars that are held together by gravitational forces

Light year—the distance light travels in one Earth year

Nuclear fusion—the process in which nuclei of light elements join together, with the release of large amounts of energy

Red shift—the shift in frequencies of visible spectral lines towards the red end (low frequency) of the electromagnetic spectrum; used as evidence of an expanding universe

Origin of the universe

The following information summarises the important ideas about the origin of the universe.

The equivalence of mass and energy

In 1915 Albert Einstein developed the Special Theory of Relativity. One important aspect of this theory was the famous equation:

$$E = mc^2$$

(E = energy; m = mass; c = velocity of light = 300 000 km/s)

This equation summarises the following major idea:

- **Mass and energy are equivalent.**
- Mass can be converted to energy.
- Energy can be converted to mass (matter).

The energy released by the Sun is an example of this process. Some of the matter of the Sun is being converted continually to energy in a process called **nuclear fusion**.

An expanding universe

In the first 30 years of the twentieth century various scientists (including Einstein, Lemaître and Friedmann) developed theories that suggested that the universe was expanding. No experimental evidence had been collected at the time to show this expansion. The concept of an expanding universe implied that the universe must have been originally much smaller than its current size. From this idea and Einstein's theory concerning the equivalence of mass and energy, the Big Bang theory developed.

Before we examine the Big Bang theory, we look at some of the evidence for an expanding universe.

- **Red shift of stars and galaxies.** Astronomers analysed the light from distant stars and galaxies with a spectroscope. When they looked at various elements in these sources, they found the characteristic frequencies of key lines was shifted towards the red end of the visible spectrum. This observation was made by **Edwin Hubble** in the late 1920s. This observation indicated that these stars and galaxies were moving away from us (and from each other). They are moving away because **space itself is expanding**.
- **Cosmic background radiation.** In 1965 two astronomers (Penzis and Wilson) detected uniform microwave radiation emanating from intergalactic space. This radiation was equivalent to a background temperature of –270°C (3 kelvin or 3 degrees above absolute zero). In 1989 the Cosmic Background Explorer satellite (COBE) studied this background radiation and found small amounts of matter irregularly scattered in the intergalactic spaces. These observations were consistent with the expansion and cooling of space following a very hot 'explosion' billions of years ago.

The steady state theory

Before the development of the Big Bang theory, another theory called the steady state theory (Gold, Hoyle, Bondi, 1948) proposed that the universe always existed and that it will forever continue to look the same as it did in the past. The universe expands because new matter and stars continue to form from a reservoir of energy. There is therefore a balance between expansion and star/galaxy formation. Opponents of the steady state theory have made the following criticisms of the theory:

- The discovery of the variable distribution of galactic radio sources and very distant and very bright quasars implies that the early universe looked different to the current universe. This is inconsistent with the steady state theory.
- The discovery of cosmic background radiation cannot be explained by the steady state theory.

However, the Hubble Space telescope (since 1996) has taken photographs of the most distant regions of space, showing mature galaxies similar to our local ones. This is consistent with the steady state theory.

The Big Bang theory

The Big Bang theory (Gamov, 1948) proposes that about **12–13 billion years ago** space and time came into existence in an 'explosion'. In terms of Einstein's equation, the energy that came into existence was partly converted to matter. Space became filled with hot matter that inflated and expanded rapidly and cooled as it expanded. The sequence of events that followed the 'explosion' can be summarised as follows (see Figure 4.1 on page 113):

- The early universe (<1 second old) was filled with radiation and subatomic particles but no atoms. It was very hot (100 billion degrees). It began to cool.
- One second after the Big Bang the primitive universe had a temperature of 10 billion degrees.
- By 3 minutes, the universe had cooled to 1 billion degrees and atomic nuclei formed from protons and neutrons.
- Over the next 300 000–700 000 years the temperature of the expanding universe dropped to 3000 K and atoms formed as nuclei and electrons combined to form hydrogen and helium. Light was able to escape from the hot matter.
- Gravitational forces led to the formation of stars, galaxies and planets. This process continues today. The universe continues to expand and cool. Today (~10^{10} years after the Big Bang) the intergalactic temperature is only 3 K.

The radiation from the cooling of the primitive expanding universe following the Big Bang still exists and has been detected by the COBE satellite.

Calculations of the relative proportions of hydrogen and helium in the universe based on the Big Bang theory have been confirmed by astronomical measurements. This is further evidence for the Big Bang theory.

The future of the universe

Various theories have been proposed concerning the fate of the universe.

- The **open universe theory**. This suggests that the universe will continue to expand and cool forever. (There is evidence that the distant regions of the universe are expanding at an ever-increasing rate.) Ultimately the stars will redden and die (~100 trillion years) as they exhaust their nuclear fuel and the universe will become very dark and very cold (>10^{100} years).
- **The closed (pulsating) universe theory.** The universe will expand for a time but will eventually stop expanding and contract as **gravity** draws matter back together. This scenario ends in a big 'crunch' followed by a new Big Bang. This process repeats itself forever.

Electromagnetic spectrum and astronomy

Electromagnetic waves and the electromagnetic spectrum were discussed on page 4 in Chapter 1. **Re-read that section now.**

Astronomers use various bands of the electromagnetic spectrum to investigate the universe. Table 4.1 shows some of the applications.

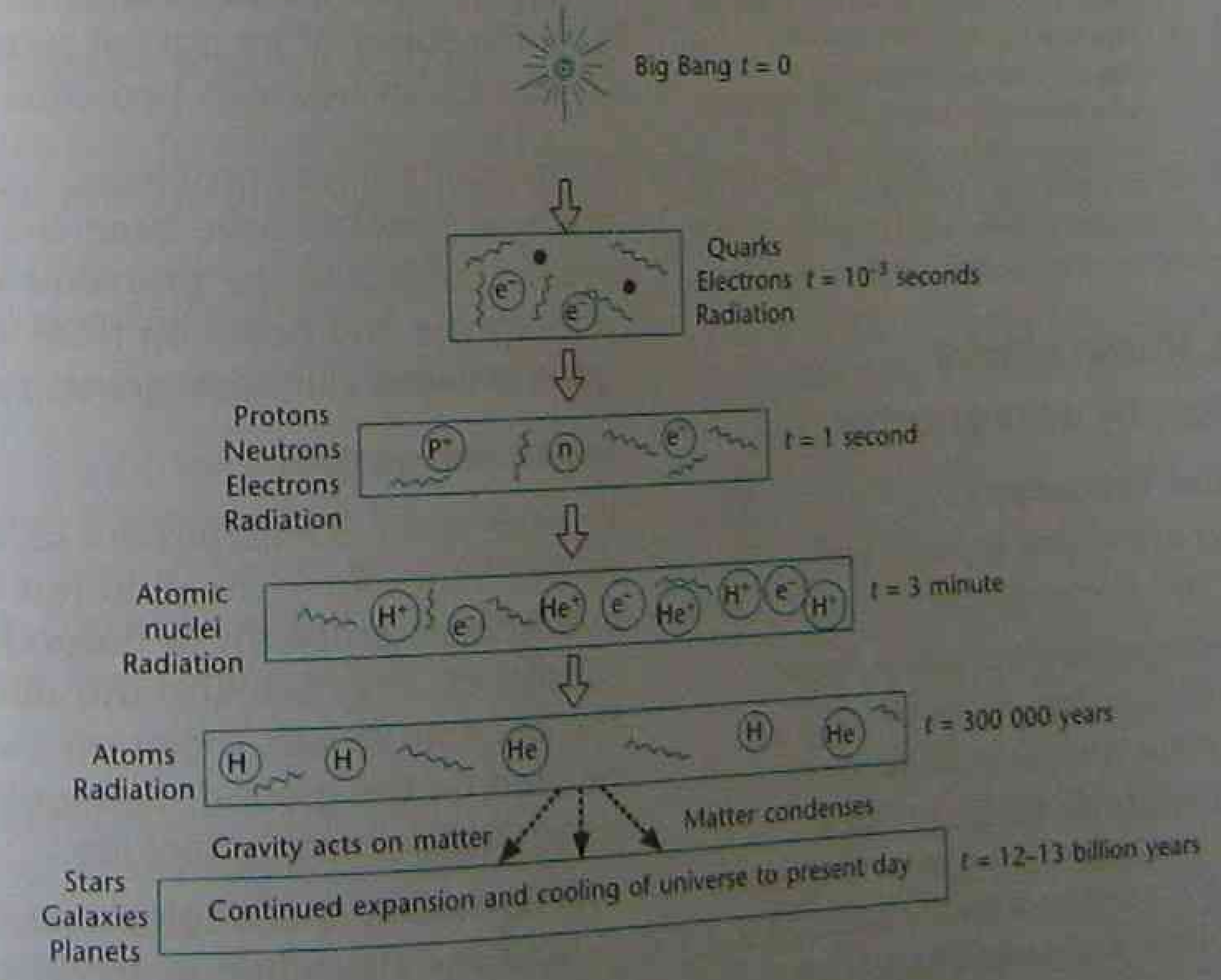


Figure 4.1 Events of the Big Bang

Table 4.1 Uses of the electromagnetic spectrum in astronomy

Electromagnetic band	Uses in astronomy
Radio	Radio telescopes are large dishes (eg. Parkes in NSW) that collect radio waves from space. The weak radio signals are amplified and analysed. Radio astronomy is used to: • observe objects that emit mainly radio waves rather than visible light (eg. dark nebulae; quasars; pulsars) • observe clouds of hydrogen in deep space
Infrared	Infrared telescopes and their spectrometers are used to: • detect objects that are too cool to emit visible light • measure the temperature of the atmosphere of solar system planets • determine the temperature of the background radiation in deep space
Visible	Ground-based optical telescopes and spectrometers as well as space telescopes such as the Hubble are used to: • observe and measure various optical sources such as planets, comets, stars and galaxies • measure the red shift of space objects such as galaxies to determine the extent of the expansion of the universe • measure the colour and temperature of stars

Problems in obtaining information in astronomy

Ground-based telescopes

Ground-based astronomy is faced with many difficulties. These include:

- **Earth's atmosphere.** The atmosphere absorbs various components of the electromagnetic spectrum to different extents. Infrared, UV and X-rays are significantly absorbed by the atmosphere. Visible light is scattered and refracted by the atmosphere and clear images are hard to obtain. Locating telescopes on high mountains and using modern adaptive optics improves the quality of the signals detected. The Keck telescopes (in a dormant volcano in Hawaii) have the largest computer-controlled mirrors in the world. At this site the air is very still. The Keck telescopes can see fainter sources than the Hubble space telescope.
- **Light pollution.** Cities emit so much visible light at night that telescopes have to be built (where possible) in sparsely populated areas where there is little visible light pollution.
- **Radio wave pollution.** Mobile phones, microwave sources and pay-TV transmissions make it more difficult for radio astronomers to detect weak radio signals from space.
- **Solar storms.** Solar flares release bursts of electromagnetic radiation that interfere with other electromagnetic sources from space.
- **Optical systems in telescopes.** Lenses and mirrors in telescopes produce some degree of distortion of images. Telescopes are limited by their resolution. Resolution is the ability of an optical system to distinguish between two close objects.

To overcome these problems, space satellites and space probes have been launched. They are not subject to the problems of Earth's atmosphere and pollution from various ground-based electromagnetic sources.

Space telescopes

Because space telescopes are located above Earth's atmosphere, they do not suffer the atmospheric problems of ground-based telescopes. They can also use other bands of the electromagnetic spectrum that cannot be used on the ground. Some of these new generation telescopes include:

- **The Hubble space telescope.** This telescope (launched in 1990) can detect fine detail of visible and ultraviolet sources in nearby stars and distant galaxies.
- **The Chandra X-ray observatory.** This facility was placed in orbit in 1999 and it can examine X-ray sources, such as black holes, in deep space.

Major features of the universe

From our perspective on Earth the observable universe has a radius of about 13 billion light years.

Distances in space

The normal units of distance measurement on Earth are replaced by larger units when describing distances in space outside our solar system.

The **astronomical unit** (AU) is defined as the distance between the Sun and Earth.

- **One astronomical unit**

  = 150 million kilometres

This unit is useful for measuring distances in the solar system.

Example: Sun–Mars distance = 1.5 AU
Sun–Jupiter distance = 5.2 AU
Sun–Neptune distance = 30 AU

The **light year** is a common unit used for measurements outside the solar system.

- One **light year** is the distance travelled by light in one Earth year.
- One light year = 9 461 000 000 000 km.

Our Sun is 8.5 light minutes from Earth. The Moon is about 1 light second from Earth. Stars that are part of the Southern Cross and pointers, together with their distances from our solar system, are shown in Figure 4.2.

The solar system

Earth is one of nine planets that orbit the Sun in elliptical orbits. The Sun is a yellow-white star. The planetary orbits are elliptical, although most are almost circular, except for Pluto's orbit which is much more elliptical and inclined in a different plane. Other bodies such as asteroids and comets also orbit the Sun. Comets have highly elliptical orbits.

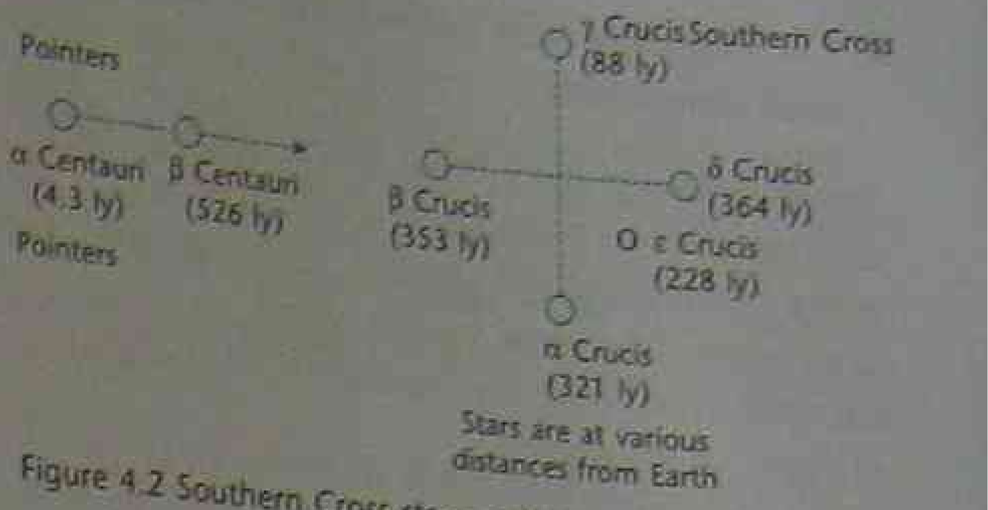


Figure 4.2 Southern Cross stars and the pointers

Figure 4.3 shows the location of these major features of the solar system.

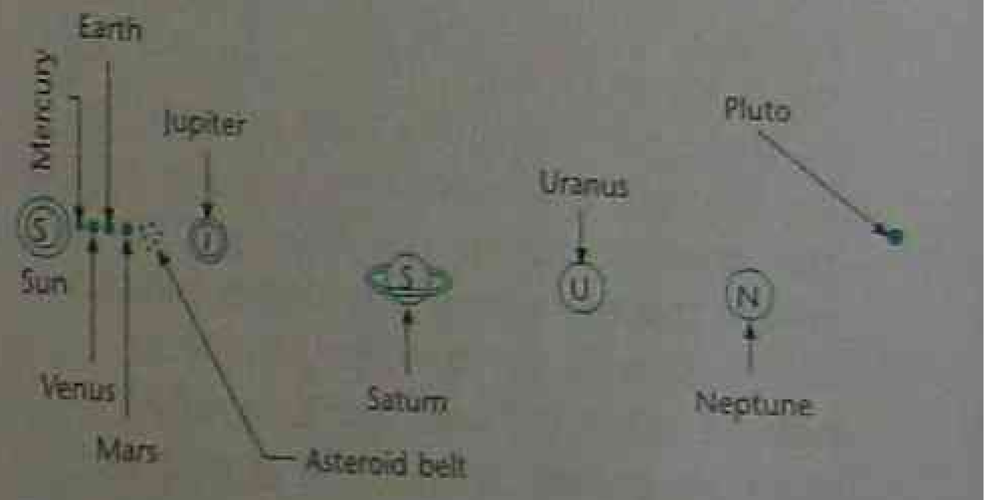


Figure 4.3 Structure of the solar system

Galaxies

The solar system is a small part of the **Milky Way** galaxy. The Milky Way is a **spiral galaxy** with a diameter of 100 000 light years. The solar system is located on one of its spiral arms as shown in Figure 4.4.

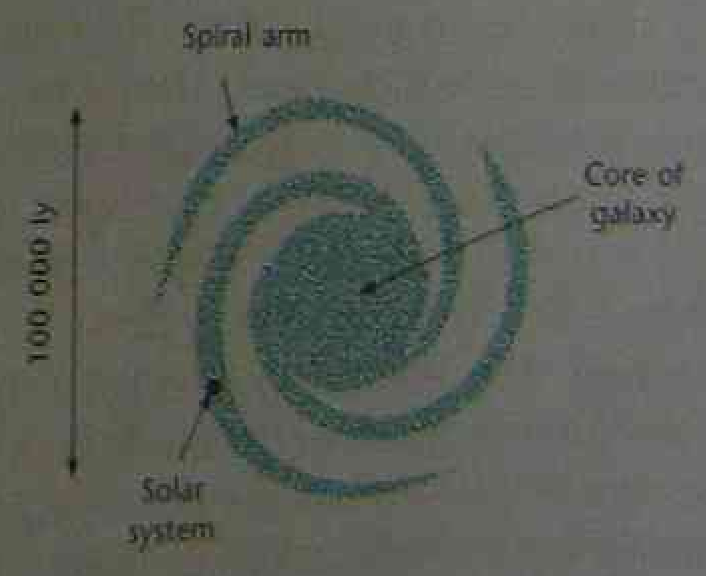


Figure 4.4 The Milky Way galaxy

- **Galaxies** are vast collections of stars, gas and stellar dust.

Some galaxies may have black holes at their centres. Some galaxies are elliptical and some have irregular shapes. Galaxies are organised into local clusters and superclusters that are hundreds of millions of light years across.

Deep space objects

Beyond the galaxy, superclusters are deep space objects. Some common objects found in deep space are:

- **Nebulae**. These are clouds of gas and stellar dust. Some glow brightly and others are dark.
- **Novae and supernovae**. Novae are formed by explosions that shear off the outer layers of stars. This causes the star to shine more brightly than normal. In supernovae the explosions tear the whole star apart and there is a short period where the exploding star shines billions of times more brightly. Eventually a **nebula** is left behind with a rapidly spinning **pulsar** or **neutron star** at its centre.
- **Pulsars** are very small (~20 km) and very dense objects that are about as heavy as the Sun. They emit pulses of radio waves as well as short bursts of X-rays and gamma rays along the direction of their magnetic axes. We detect the pulsar if its pulses are directed towards us.
- **Quasars and black holes.** Quasars are at vast distances in deep space. They are the centres of violent galaxies. They emit huge amounts of energy in jet streams at right angles to a spinning hydrogen gas disc. It is these bright jet streams of radio waves that we detect if they point in our direction. The spiralling gas is subjected to the intense gravity of a **black hole** at the centre. Black holes are extremely massive objects equivalent to billions of suns. Black holes are so massive that nothing, even light, can escape. This is why they look black.

Dark matter

Not all the matter in the universe emits light. It is called **dark matter.** Much of this dark matter is believed to be composed of cool non-radiating matter and subatomic particles such as neutrinos that are emitted in vast quantities by stars. Recent experiments have shown that each neutrino has a very small mass.

Relating features of the universe to the Big Bang theory

Following the Big Bang, the matter that formed spread out as space expanded. This matter was not uniformly spread but irregularly scattered. By 1 billion years after the Big Bang, **gravity** began to pull matter together to form various astronomical structures.

Gaseous clouds of hydrogen and dust collapsed under gravity to form **galaxies**. Inside these galaxies, **stars** began to form and illuminate space with their light. The Sun began to form from a planetary nebula about 5 billion years ago. The nuclear fusion reactions in its core were initiated when the gravitational heating reached 10 million degrees. Circling this young star were protoplanets which formed from the spinning disk of condensed matter. These protoplanets became the **planets** of our solar system about 4.6 billion years ago.

Star formation continues today in distant parts of the universe.

The life of stars

Stars have varying sizes. Some are smaller than the Sun and some are very much larger. They have different life cycles.

a. Stars like our sun

Star birth

The following account refers to stars that are similar in size to the Sun. It takes about 40 million years for a star like the Sun to form.

Vast clouds of hydrogen gas and interstellar dust are the birthplace of stars. This matter gravitationally condenses to form a dull red protostar. As it continues to condense, the material of the protostar heats up. At this stage the remaining matter of the star is still spread over a considerable amount of space. When the centre becomes hot enough due to gravitational heating, nuclear fusion reactions begin and helium is produced. This process generates considerable heat and yellow light. Some of the gaseous matter is ejected to form a rotating disc and over several million years it may lead to the formation of planets around the new star. The new star will shine for about 10 billion years.

Mature and ageing stars

The newly formed star continues to produce energy by the nuclear fusion of hydrogen into helium. The heavier helium sinks into the core of the star. This process generates heat and eventually causes the outer hydrogen shell to begin to fuse. This process is accompanied by a swelling of the star and an increase in its brightness.

The star continues to expand and its surface cools. It will form a **red giant star** which is about 100 times bigger than the original yellow star. At this stage helium fusion begins with the release of more energy. New elements such as carbon, nitrogen and oxygen start to form in the star's core. Eventually the star runs out of nuclear fuel and the core begins to shrink. Material is ejected to form a bright **planetary nebula** that drifts away. The remaining core finally turns into a small (yet heavy) **white dwarf** star. These white dwarfs are very hot. Over the next few billion years it will cool and eventually form a black crystalline object called a **black dwarf**.

The Sun is about half-way through its life cycle.

b. Larger stars

Stars that are five to ten times heavier than the Sun have a different evolutionary path. Their large mass creates rapid nuclear fusion and very high temperatures. These stars glow **blue-white** or **blue**. They have much shorter lifetimes than stars that are similar to the Sun.

These stars swell to form **red supergiants**, and in their cores heavier elements such as magnesium, sulfur and iron form. As nuclear fuel runs out, the star starts to collapse and the heating that is produced causes the outer layers of the star to explode, producing an incredibly bright **supernova**. The remaining core collapses to form a **neutron star** or **pulsar**.

Some stars are 30 to 50 times heavier than the Sun. They swell to form very large red supergiants. Following the explosion and supernova formation the core contracts to form a **black hole**.

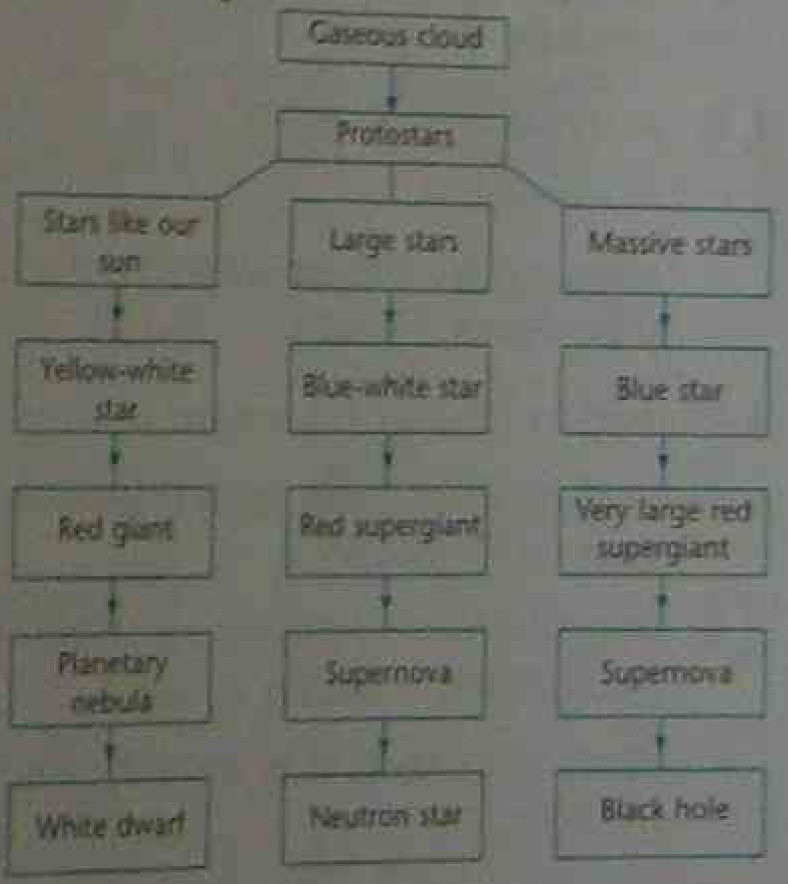


Figure 4.5 Star evolution

c. Very small stars

Stars that are less than half the size of the Sun are called **red dwarfs**. They have very long lives, and never evolve into red giants. Many of these common stars are almost as old as the universe. They fuse hydrogen into helium, but their small size prevents any further nuclear events. After they have exhausted their hydrogen, they cool and darken to form black dwarfs. Their lifetimes are estimated to be 100 billion years. **Proxima Centauri** (the closest star to the solar system) is a very small star with a mass about 15% of the Sun's mass.

Test yourself (answers on pages 221–2)

Part A. Knowledge (answers on page 221)

1 The scientist(s) who proposed the connection between mass and energy was:

- a Newton.
- b Friedmann.
- [✓ c] Einstein.
- d Penzis and Wilson. (1 mark)

2 The observed red shift of stars and galaxies is evidence for:

- a the steady state theory of the universe.
- [✓ b] an expanding universe.
- c star evolution.
- d red giant stars about to become supernovae. (1 mark)

3 Infrared astronomy is used to:

- a observe deep space objects that emit radio waves.
- [✓ b] determine the temperature of the background radiation in space.
- c study the structure of distant galaxies.
- d observe hydrogen clouds in deep space. (1 mark)

4 Ground-based astronomy is faced with many difficulties. Select the statement that correctly identifies a problem and its cause.

- [✓ a] Earth's atmosphere strongly absorbs IR and UV light and consequently these emissions from space are difficult to study.
- b Cities emit too much light but astronomers must operate from cities where computing systems are available to analyse results.
- c Mobile phones emit so much infrared radiation that astronomers are experiencing interference with the infrared signals from space.
- d Clear images can only be obtained the telescopes are mounted on very high mountains where the air is very still for most of the year. (1 mark)

5 Select the statement that is true of the Milky Way.

- a The Milky Way is an elliptical galaxy with the Sun close to its centre.
- b The Milky Way is a supernova that exploded about 12 billion years ago.
- [✓ c] The Milky Way galaxy has a diameter of about 100 000 light years.
- d Pulsars and black holes are common in the Milky Way galaxy. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

- a Pulsars emit regular pulses of ____ waves.
- b The Sun was formed from vast clouds of ____ gas and interstellar dust.
- c When red supergiant stars use up their fuel, they collapse and explode, producing a bright ____.
- d A light year is the ____ light travels in one Earth year.
- e The open universe theory predicts that the universe will continue to ____ and cool forever.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a Big Bang theory	f X-ray sources
b Edwin Hubble	g ground-based astronomy
c light pollution	h red dwarf star
d Chandra observatory	i red shift of stars/galaxies
e Proxima Centauri	j expanding universe

8 Briefly outline how stars generate energy. (2 marks)

9 Briefly explain two pieces of evidence that support the view that the universe is expanding. (2 marks)

10 Outline the future stages in the evolution of a star such as the Sun. (3 marks)

Part B. Skills (answers on page 222)

1 a One astronomical unit is equal to 150 million kilometres. Calculate the distance of the following planets from the Sun in kilometres. (2 marks)

- i Venus ($d = 0.7$ AU)
- ii Saturn ($d = 9.5$ AU)

b The average distance of Pluto from the Sun is 5590 million kilometres. Calculate this distance in astronomical units. (2 marks)

2 One light year is equal to 9461 billion kilometres. Calculate the distance of the following stars from the Sun. (2 marks)

- a Wolf-359 ($d = 7.7$ ly (light years))
- b Beta Crucis ($d = 353$ ly)

3 The stars of the Southern Cross form a constellation. In the sky they appear to be close together. Their recently measured distances from the Sun are tabulated below.

Star	Distance (ly)
Alpha Crucis	321
Beta Crucis	353
Gamma Crucis	88
Delta Crucis	364
Epsilon Crucis	228

a Do these stars form a cluster in space? Explain. (2 marks)

b The stars are listed in order of decreasing brightness as seen from Earth.

- i Which star is the brightest as seen from Earth? (1 mark)
- ii Beta Crucis has a higher surface temperature than Alpha Crucis. Why does it appear less bright than Alpha Crucis? (1 mark)

4 Figure 4.6 is a scale diagram of the Milky Way galaxy shown in side view. The position of the Sun is also shown. Use the information to determine:

- a the distance of the Sun from the galactic core (1 mark)
- b the diameter of the Milky Way galaxy. (1 mark)

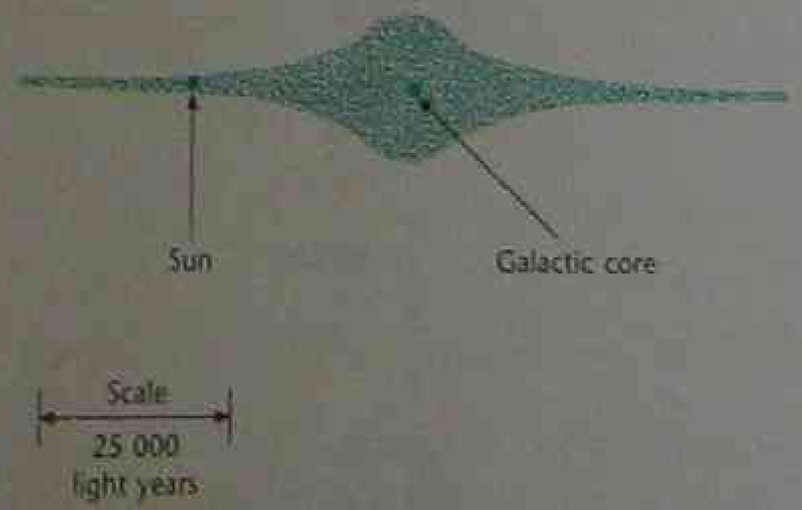


Figure 4.6 Scale diagram of the Milky Way

5 Figure 4.7 is a set of jumbled diagrams that show the evolution of a star such as the Sun. Use the code letters to place these diagrams in their correct sequence. (2 marks)

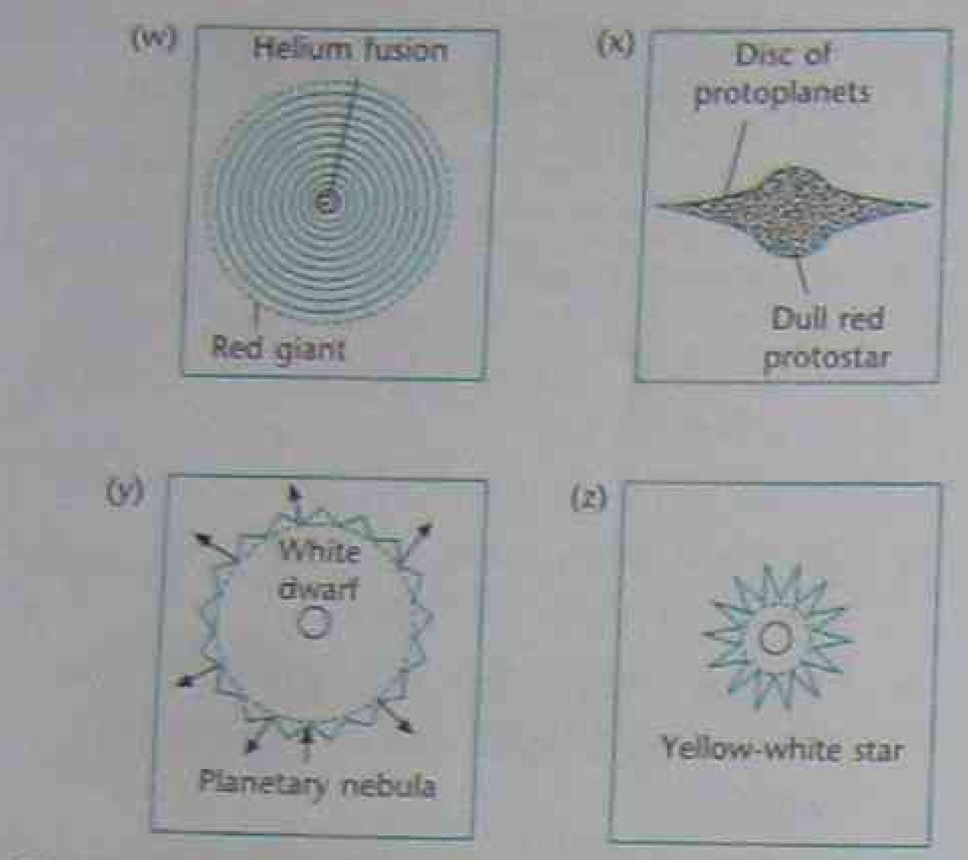


Figure 4.7 Jumbled diagrams of star evolution

6 Figure 4.8 shows simplified diagrams of the rotation of the Milky Way galaxy at various times. The position of the Sun is marked. Use this information to calculate the time for one complete revolution of the galaxy. (2 marks)

7 The surface temperature (T) of a star can be estimated from the wavelength (λ) using the following mathematical formula:

$$T.\lambda = 3\ 000\ 000$$

Temperature is measured in absolute

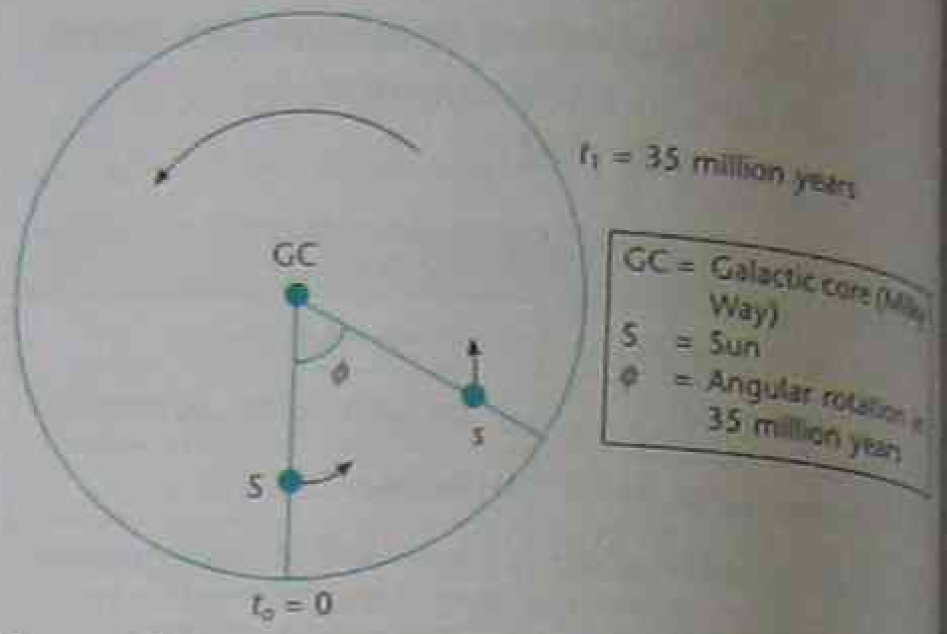


Figure 4.8 Time sequence for the rotation of the Milky Wa

units (kelvin, K) and wavelength is measured in nanometres (nm).

- a Light from the Sun has a wavelength of 510 nm. Calculate the surface temperature of the Sun in absolute units. (2 marks)
- b Given that 0°C = 273 K and 100°C = 373 K, determine the temperature of the surface of the Sun in degrees Celsius. (1 mark)
- c Betelgeuse is a red supergiant star. Its surface temperature is about 4300 K. Calculate the wavelength of the light emitted from its surface. (1 mark)

8 Figure 4.9 is a graph of the relative brightness of many stars compared with

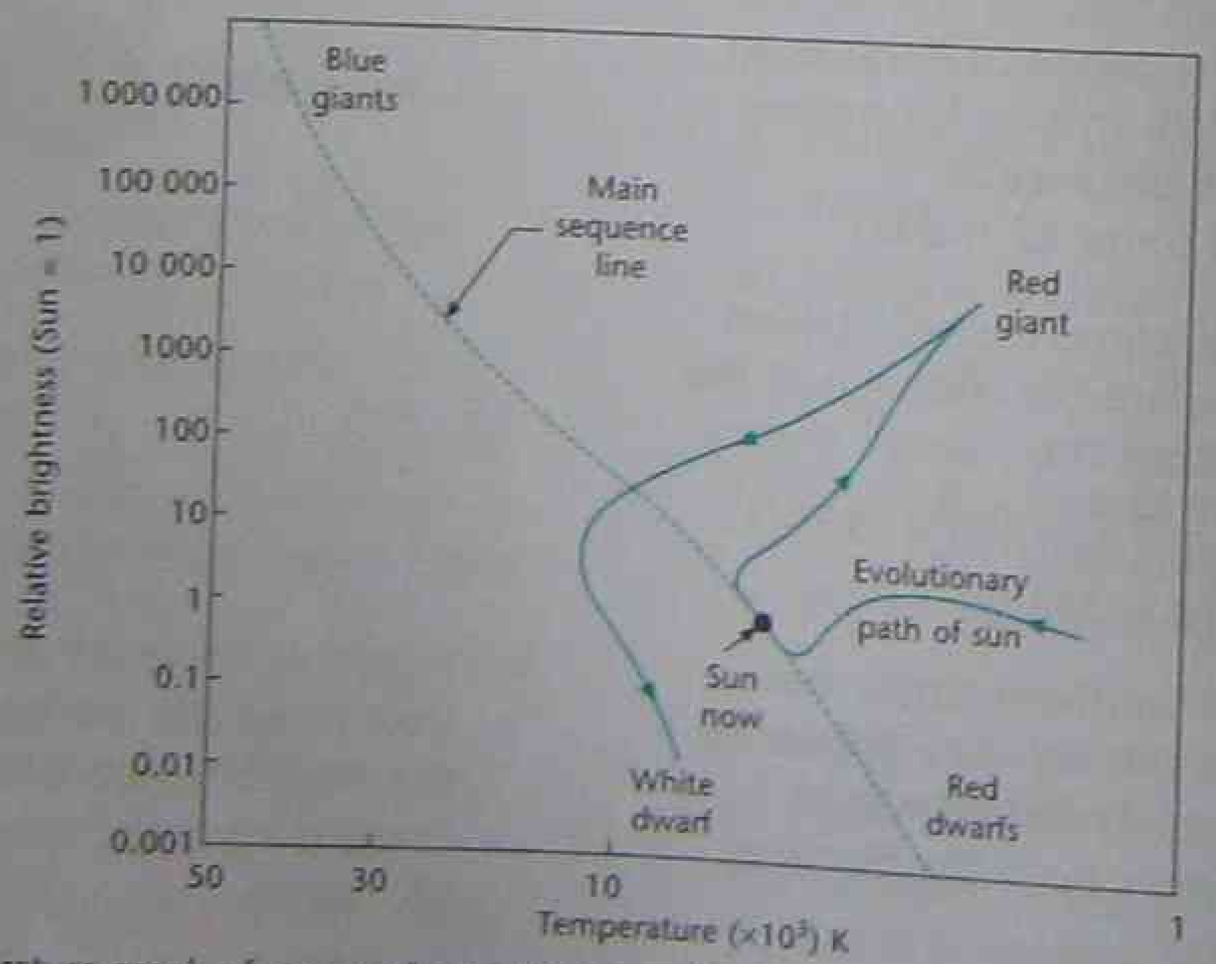


Figure 4.9 Brightness–temperature graph of stars and evolutionary path of the Sun

their surface temperatures. The dotted line is called the Main Sequence and links red dwarfs to blue giants. The graph also shows the evolutionary path of our Sun.

- a Are red dwarfs brighter or dimmer than the Sun? (1 mark)
- b Compare the brightness and surface temperature of a blue giant and the Sun. (2 marks)
- c The Sun will evolve into a red giant. Will the red giant be:
  - i brighter or dimmer than the present-day Sun? (1 mark)
  - ii hotter or colder on its surface than the present-day Sun? (1 mark)
- d Following the red giant stage, the Sun will evolve and cross the Main Sequence line. How will its brightness and surface temperature change? (2 marks)
- e What is the ultimate fate of the Sun? (1 mark)

Mid-chapter test (answers on pages 222–3)

1 Explain the evolution of very small stars such as Proxima Centauri that have a mass of 15% of the Sun. (2 marks)

2 Figure 4.10 is a graph of the penetration

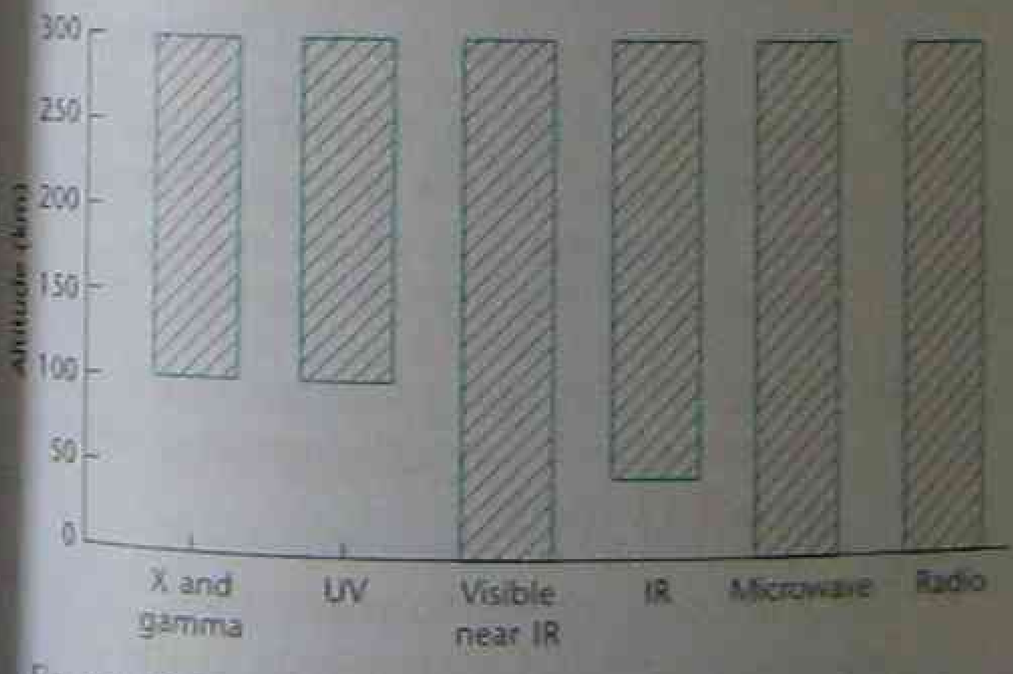


Figure 4.10 Penetration of electromagnetic waves through the atmosphere

of various electromagnetic waves through the atmosphere.

- a Which rays are quickly filtered out by the upper atmosphere? (1 mark)
- b Which rays are able to penetrate below 20 km altitude? (1 mark)
- c Use the graph to name the two common types of ground-based astronomy. (2 marks)

3 Figure 4.11A shows three lines in the visible spectrum produced by light emitted from a star that is rotating around another star. These three lines (X, Y and Z) correspond to different directions of motion of the star relative to an observer on Earth. Figure 4.11B shows the direction of motion of the star relative to Earth at different times in its cycle.

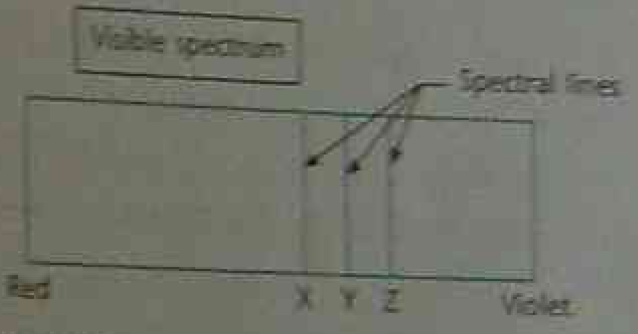


Figure 4.11A Spectrum with three lines

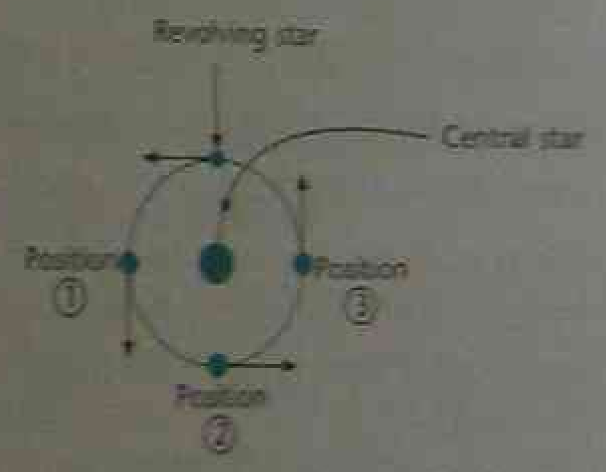


Figure 4.11B Motion of star relative to Earth

- a In which position in the star's orbit is the light from the star red-shifted? (1 mark)
- b Match the three spectral lines to the rotational direction of the star in Figure 4.11B. (3 marks)
- c Stars that are moving towards Earth emit light frequencies that are blue-shifted. In which position of the star's

orbit is the light from this star blue-shifted? (1 mark)

d If light from distant galaxies is red-shifted, what can one conclude about the motion of these galaxies relative to our own galaxy? (1 mark)

4 Figure 4.12 shows a model of the steps in the formation of one helium-4 nucleus from hydrogen (H-1) nuclei in a star such as the Sun.

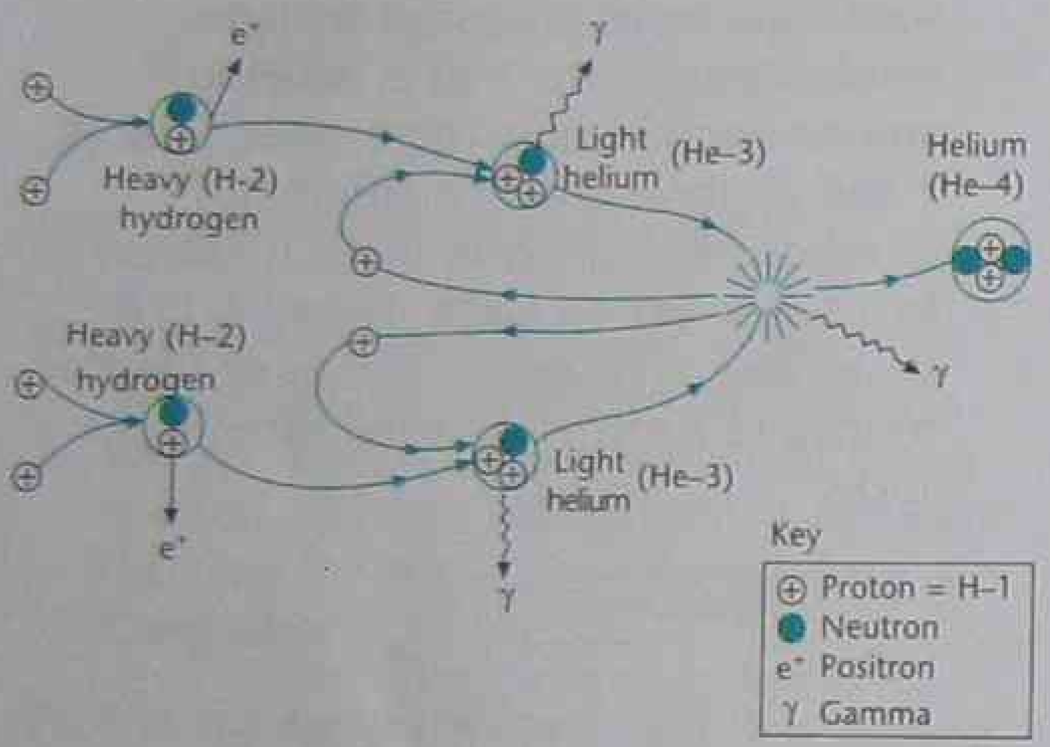


Figure 4.12 Steps of the fusion of hydrogen into helium in the Sun

a How many hydrogen nuclei fuse to form one nucleus of 'heavy hydrogen' in the first step? (1 mark)

b How is 'heavy hydrogen' different from normal hydrogen? (1 mark)

c What is the mass number (A) of the helium atom formed when hydrogen and heavy hydrogen fuse together? (1 mark)

d Write a word equation for the final step that produces helium-4. (2 marks)

e What type of highly penetrating radiation is released during these fusion reactions? (1 mark)

5 Figure 4.13 shows the evolutionary sequence of a star that has a mass ten times greater than the Sun. Use this diagram to explain how the brightness and surface temperature change as the star evolves. (2 marks)

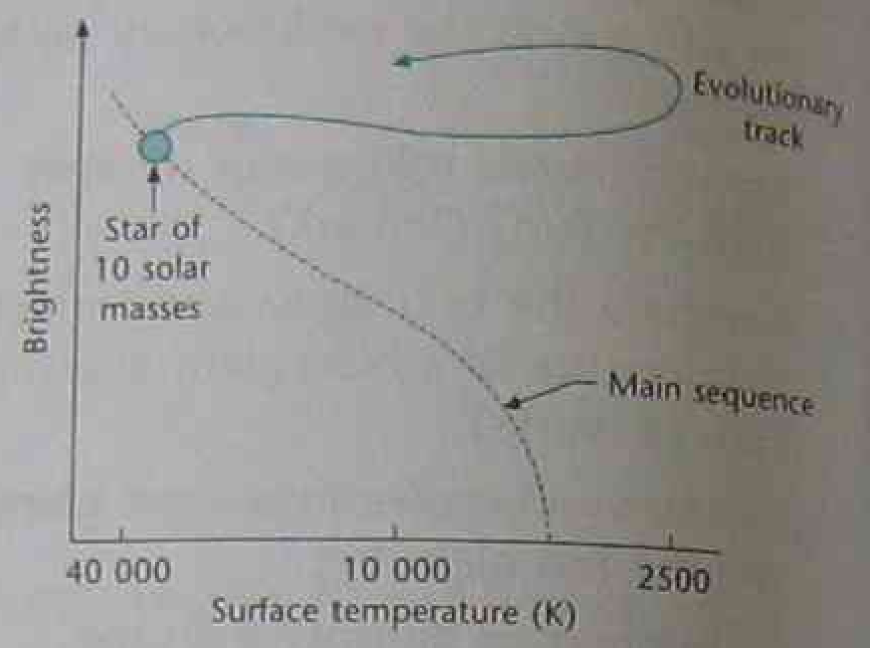


Figure 4.13 Evolutionary track of a large star

6 The planets vary in their orbital speed around the Sun. The following table shows this variation. One value (X) is missing.

Planet	Average distance from Sun (AU)	Average orbital speed (km/s)
Mercury	0.4	48
Venus	0.7	X
Earth	1.0	30
Mars	1.5	24
Jupiter	5.2	13

a Describe the trend in the data. (2 marks)

b Plot a line graph of the data and determine a value for X. (3 marks)

c Use your graph to determine the orbital speed of the asteroid Ceres that orbits the Sun at an average distance of 2.8 AU. (2 marks)

d Which of the two planets (Saturn and Neptune) will have the greater orbital speed? (1 mark)

7 Use the code letters to place the following jumbled statements in the correct sequence. (2 marks)

The Big Bang

a Gravitational forces cause matter to condense to form stars and galaxies.

b Some energy is converted to sub-atomic matter.

c The very hot primitive universe cools as it expands.

d Atomic nuclei start to form.

e The expansion and cooling of the universe continues to the present day.

f Space and time come into existence.

g Atoms of hydrogen and helium form as nuclei and electrons combine.

h Space inflates and expands.

8 True or false? (10 marks)

a Ground-based telescopes should be located in regions away from cities.

b Infrared astronomy can be used to measure the temperature of the atmospheres of the planets of the solar system.

c The open universe theory predicts that gravity will eventually draw matter back together.

d By three minutes after the Big Bang, atoms of hydrogen and helium had formed.

e Mass and energy are equivalent.

f The Sun generates energy by nuclear fusion.

g The Milky Way galaxy is a spiral galaxy that is about 10 light years in diameter.

h Pulsars are very dense objects formed at the end of the evolution of stars that have masses five to ten times the mass of the Sun.

i Black holes are holes in the fabric of space–time.

j Red dwarfs are very small stars that have very long lifetimes.

Natural events

Earth is a dynamic planet. Although the landscape may appear constant over a person's life, Earth is in a state of constant change. Many natural events shape the planet on which we live.

Glossary

Eons—the largest divisions of the geological time scale

Fossil—remains or impressions of past life forms

Igneous rocks—rocks that have formed from magma or lava

Lithosphere—the outer rigid layer of Earth that includes the crust and upper mantle

Plate tectonics—a study of the forces that cause the movement of the crustal plates

Pyroclastic—describes hot rock and ash fragments released in a volcanic explosion

Unconformity—a break in geological time between younger and older strata

Radiometric dating—determining the age of rocks or fossils using the known half-lives of radioisotopes

Seismology—the study of earthquakes

Strata—layers of rock (singular = stratum)

Geological history

The geological history of an area can often be determined by the sequence of rocks exposed in road cuttings, or through the analysis of cores obtained by drilling into the ground.

Sedimentary strata

Sedimentary rocks are formed from **sediments** that have been **compacted** and **cemented together**. This process takes millions of years. These sediments may have been deposited in a variety of environments, including oceans, lakes, rivers, dams, swamps and deserts. Table 4.2 lists some common sedimentary rocks, the sediments

Table 4.2 Sediments and sedimentary rocks

Sedimentary rock	Sediment	Environment of formation
Mudstone	clay/mud	sediments carried by quiet, slow-moving water and deposited in lakes, lagoons and on the ocean floor
Shale	silt/mud	sediments carried by quiet, slow-moving water and deposited in lakes, lagoons and on the ocean floor
Sandstone	sand	sediments carried by faster moving streams and rivers or ocean waves and deposited on the ocean floor
Conglomerate	gravel/pebbles	heavy sediments carried by fast-moving streams/rivers and deposited in these environments
Limestone	crushed shells/corals	marine invertebrates with shells, and corals, grow in shallow, warm seas; after death their exoskeletons are crushed and cemented together
Coal	plants	swamp plants die and are buried in mud on the floor of the swamps

that they are made from and the environments in which they form.

Sedimentary rocks are deposited in horizontal layers or **strata**. The more recent sediments are deposited on top of older sediments. When these sediments turn into sedimentary rocks they form a sequence in which **the oldest strata are on the bottom** and **the youngest are on the top**. This is known as the **law of superposition**. Figure 4.14 illustrates the law of superposition.

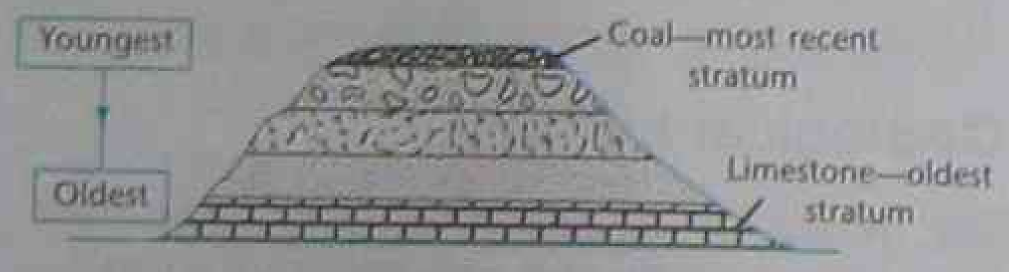


Figure 4.14 Law of superposition

The effect of igneous and metamorphic activity

Igneous rocks are formed from molten rock inside Earth (**magma**). Igneous rocks can be classified into groups based on their crystal size and rate of cooling. When the magma cools slowly deep inside Earth, the rock that forms has large crystals. Such rocks are called **plutonic igneous rocks**. Magma that flows out onto Earth's surface to form **lava** cools rapidly and forms rocks with very small crystals. Such rocks are called **volcanic igneous rocks**. Intermediate size crystals can form in rocks such as dolerite that solidify more rapidly than plutonic rocks.

Magma can intrude into existing strata to form various structures such as **plutons**, **dykes** and **sills**, as shown in Figure 4.15.

Table 4.3 Classification of some common igneous rocks

Type of igneous rock	Crystal size and rate of cooling	Examples
Plutonic	Large crystals due to very slow cooling of magma	granite (rich in light-coloured minerals such as feldspar and quartz) gabbro (rich in dark minerals such as hornblende and biotite mica)
Volcanic	Fine crystals (or none) due to rapid cooling of lava	rhyolite (rich in light-coloured minerals) basalt (rich in dark minerals) pumice (lava froth filled with gas cavities) volcanic glass (no crystals)

The presence of such **intrusions** can complicate the geological history of an area.

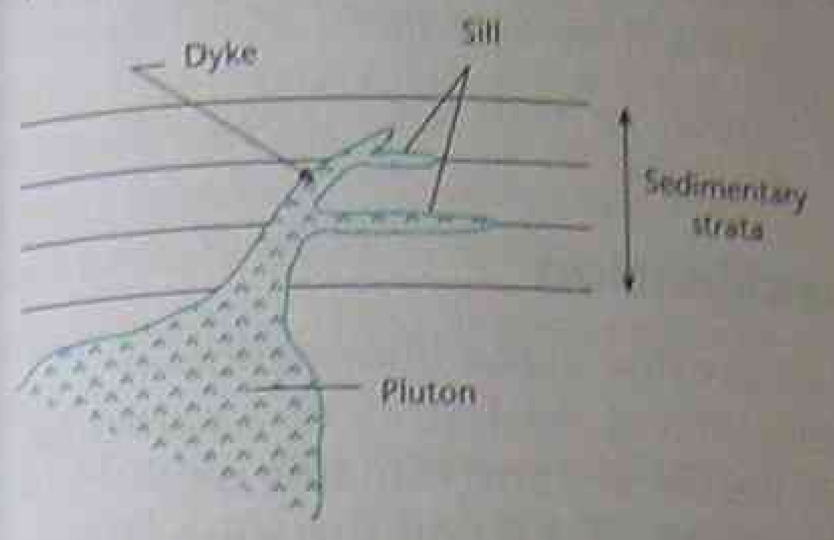


Figure 4.15 Intrusive structures

Changes in rock strata can also occur due to prolonged periods of heating and pressure caused by igneous activity or tectonic activity in the crust. This leads to the formation of **metamorphic rocks**. The presence of metamorphic rocks is therefore evidence of periods of tectonic or igneous activity.

Table 4.4 lists some of the common metamorphic rocks and the parent sedimentary rock from which they are formed. Hornfels is formed when shale is baked by contact with hot magma. Slate, schist and gneiss are formed by the heat and pressure associated with mountain building.

Table 4.4 Metamorphic rocks

Parent sedimentary rock	Metamorphic rock(s) formed
Quartz sandstone	Quartzite
Limestone	Marble
Shale	Hornfels, slate, schist, gneiss

The effect of folding and faulting

The geological history of an area may also be complicated by tectonic activity that leads to folding or faulting of strata. Figure 4.16 shows some of the structures that result from folding and faulting.

Interpreting geological histories

The following examples illustrate the steps involved in determining the geological history of an area. In each case the oldest rock needs to be established. This may be the lowest layer but earth movements may have caused some changes. Check that you agree with the sequence of steps in each history.

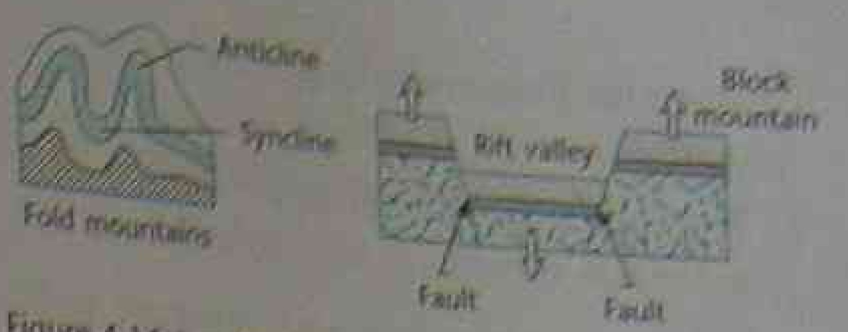


Figure 4.16 Landforms and structures formed from folding and faulting

Example 1

The following history refers to Figure 4.17. The stages in the diagram are listed from **oldest to most recent**. In the case of sedimentary strata the mode of sediment deposition and sedimentary rock formation is not discussed. The deposition events occur under water.

1. The limestone layer at the bottom of the section was deposited first.
2. Deposition of shale layer
3. Deposition of limestone layer
4. Deposition of sandstone layer
5. Faulting of the sedimentary strata occurs.
6. Intrusion of granite occurs and the heat from the cooling magma causes contact metamorphism of the sedimentary

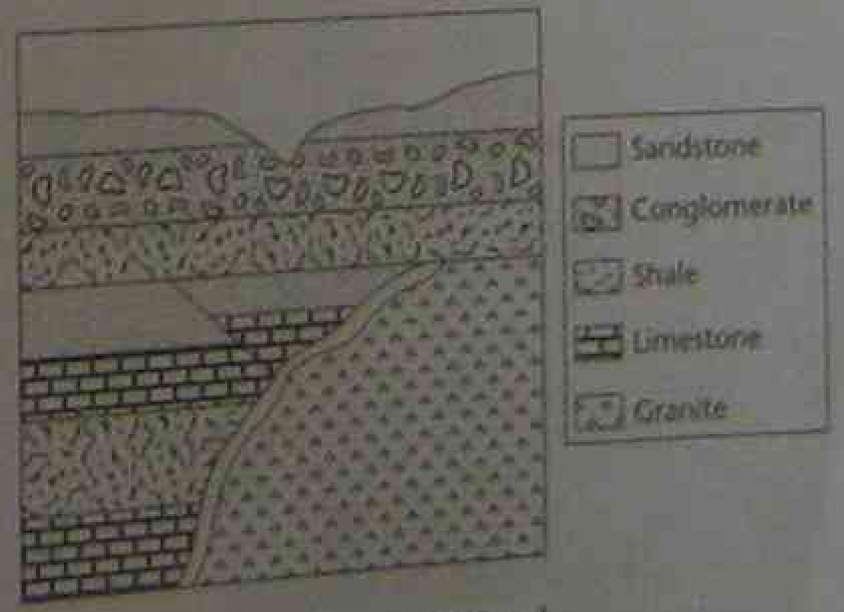


Figure 4.17 Geological cross-section 1

layers to form marble, hornfels and quartzite.

7. The region is uplifted and a period of extensive erosion occurs to produce a plain.
8. Water once again covers the area and shale is deposited.
9. Deposition of conglomerate
10. Deposition of sandstone
11. Erosion to produce the present-day landscape. Note the V-shaped valley cut by a river.

Note. The term **unconformity** is used to describe the break in time between younger rocks and older rocks. This has occurred in the example above as the shale deposited in stage 8 is very much younger than the sandstone layer below it.

Example 2

The following history refers to Figure 4.18.

1. Deposition of sandstone
2. Deposition of limestone
3. Deposition of conglomerate
4. Deposition of shale
5. Folding of strata to form an anticline and syncline
6. Uplift; extensive erosion to form a plain
7. Deposition of mudstone (note the unconformity that is now produced)
8. Deposition of sandstone
9. Deposition of coal
10. Intrusion of igneous rock (dolerite) to form a dyke and sill
11. Erosion to form the present-day landscape

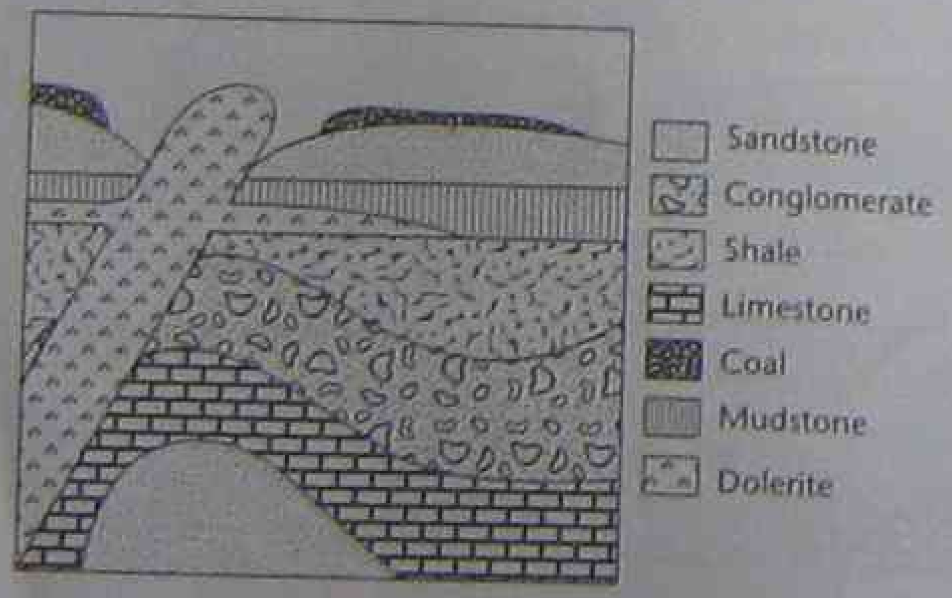


Figure 4.18 Geological cross-section 2

Fossilisation

Fossils provide useful information in reconstructing the history of a particular area. **Fossils are the remains or impressions of a living organism from Earth's past.**

Fossils will form only if the organism is preserved in some way before it decays. There are few fossils of the soft parts of organisms because they decay more readily than hard parts such as bones and shells. Organisms that live in water are more likely to be preserved than terrestrial organisms. Their remains sink to the floor of the body of water and quickly become covered with sediments. This helps to exclude oxygen that promotes decay.

The surrounding sediment gradually turns to sedimentary rock and this entombs the remains as a fossil. Sedimentary rocks are the sources of many fossils. Other types of rocks, such as igneous and metamorphic rocks, do not favour fossilisation as the fossils become destroyed quickly by the heat or pressure.

Examples of fossils

Examples of fossils include:

- **Whole organisms that remain almost unchanged**—Mammoths frozen in ice are examples of this recent group of fossils.
- **Unaltered hard parts**—In more recent fossils, the hard exoskeletons of insects are often preserved in amber, and mollusc shells are preserved in sedimentary rocks.
- **Altered hard parts**—In older fossils, bones and shells of some organisms become altered by minerals replacing existing minerals. Some bones become **opalised** in this process. Tree trunks may form fossils called **petrified wood,** due to minerals filling the intercellular spaces. Some fossils (eg. plant leaves) are turned to **black carbon,** leaving only an imprint. This is common in plant fossils and sedimentary rocks including coal.
- **Trace fossils**—These common fossils include:
  - (a) **Moulds and casts**—Sediments can pack hard around the remains of an organism to form a mould. A cast is formed if minerals fill and harden in the space left in the mould when the original remains dissolve away.
  - (b) **Footprints and imprints**—Footprints of ancient animals such as dinosaurs can be preserved if the mud on which they walked hardens before they are washed away. Soft-bodied invertebrates (eg. jellyfish) may also leave imprints in hardening sediments.

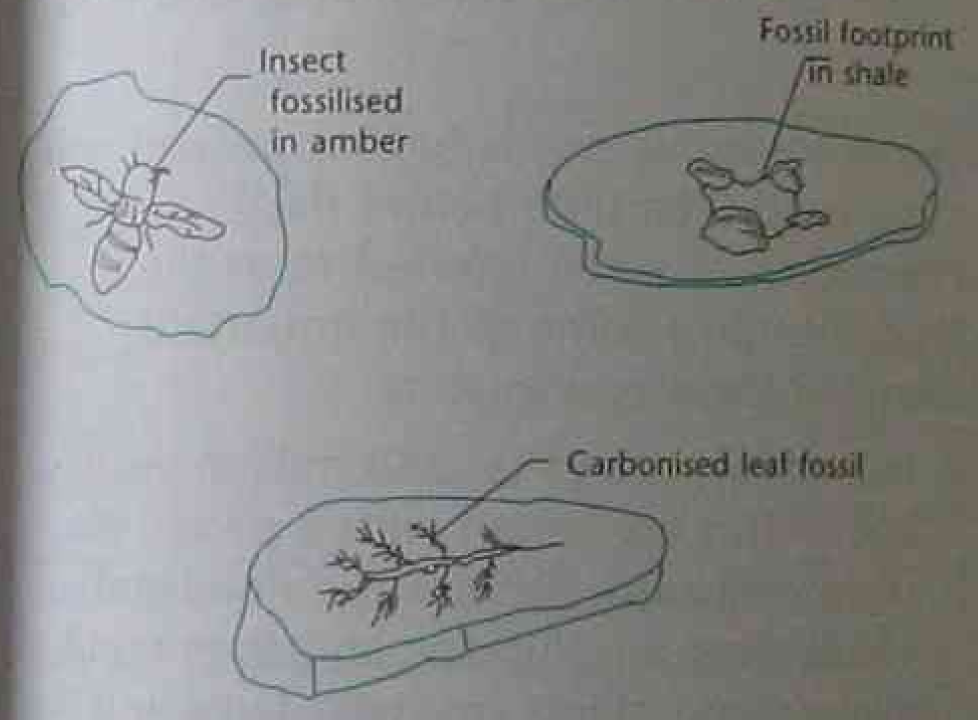


Figure 4.19 Examples of fossil formation

The fossil record and geological time

The presence of fossils in sedimentary rocks helps geologists to date the rock strata.

Relative dating

The law of superposition (see page 124) allows geologists to establish the relative age of strata. In undisturbed strata, the oldest layers are on the bottom. The presence of fossils in some of these layers therefore allows geologists to establish the relative ages of fossils.

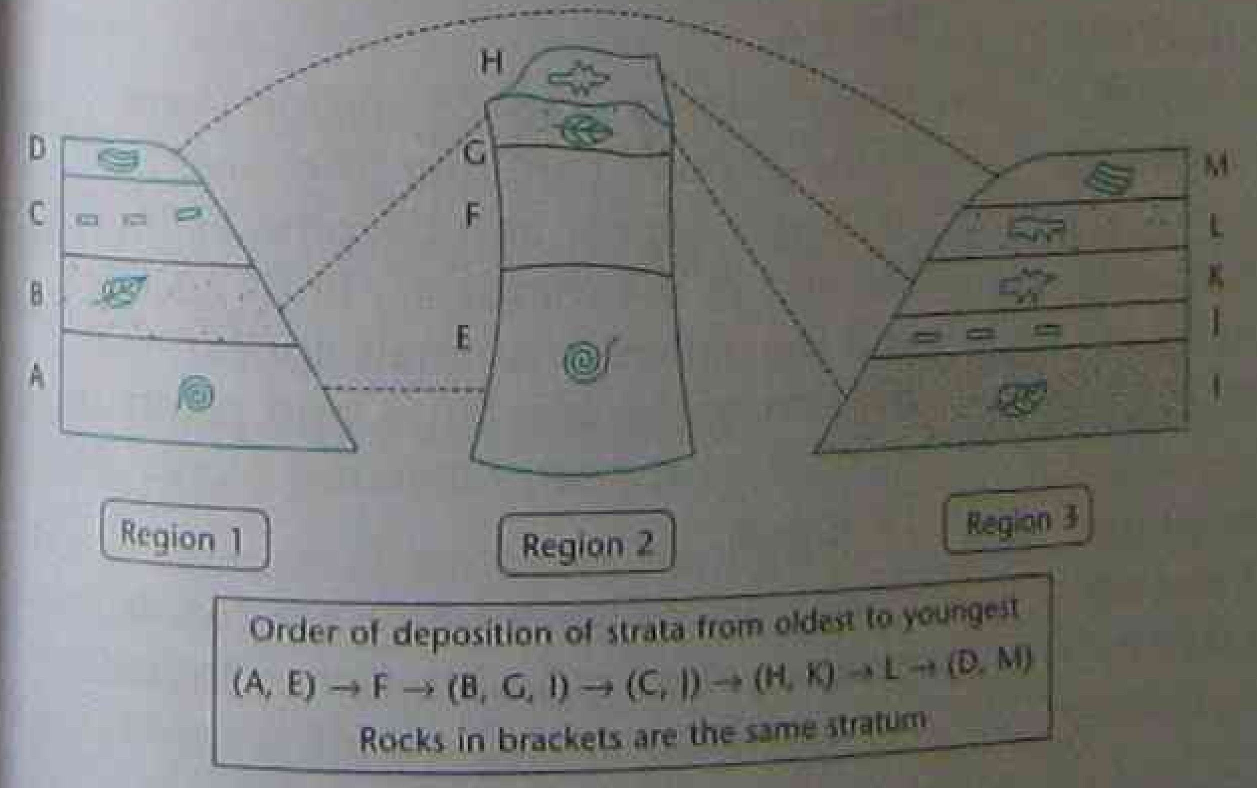
- **The simplest fossils are located in the deepest sedimentary layers.**

By comparing particular fossils in similar sedimentary strata around the world, geologists were able to construct a relative time scale for the evolution of living things.

Figure 4.20 shows an example of the use of fossil correlation to establish the order in which strata were deposited.

Absolute dating of rocks and fossils

The real (or absolute) age of rocks and fossils can be established by techniques such



Order of deposition of strata from oldest to youngest

(A, E) → F → (B, G, I) → (C, J) → (H, K) → L → (D, M)

Rocks in brackets are the same stratum

Figure 4.20 Example of fossil correlation to establish sequence of strata as **radiometric dating**. By measuring the amounts of certain radioactive elements and their decay products in a sample, a scientist can use the known half-life of the radioisotope to calculate the age of the sample.

Some useful radioisotopes are:

- **Carbon-14**—useful for dating carbon-containing fossils up to 60 000 years old. The age of the sedimentary layer is equal to the age of the fossil.
- **Potassium-40**—used to date the minerals in igneous rocks and feldspar minerals.
- **Rubidium-87**—used to date minerals in metamorphic rocks as well as micas and feldspars.

Some very old minerals, such as zircons, have been dated at 4.2 billion years old using K-40 and Rb-87 dating. Earth is believed to be 4.6 billion years old.

If a sedimentary layer is located between crystalline rocks (such as igneous and metamorphic rocks), the age of the sedimentary layer can be deduced by dating the surrounding crystalline rocks. The sedimentary layer cannot be directly dated because the process depends on radioactive minerals being trapped by crystals.

Geological time scale

The geological time scale divides the time between Earth's formation and the present into divisions and smaller sub-divisions according to major events and the types of life forms that appeared or became extinct in Earth's history. As scientists gain more knowledge about these events the placement of the divisions on the time scale is changed. The time scale shown in Figure 4.21 is an average of the current range of published data.

Note: You do not need to remember all the details shown in this time scale but you will be expected to process data based on it.

Some of the major features of the geological time scale include:

- The 4.6 billion years is divided into four **eons** of varying length.
- There is no fossil evidence of life forms in the **Hadean** eon (4.6–3.8 billion years ago).
- Simple life forms (bacteria) appeared in the **Archean** eon (3.8–2.5 billion years ago).
- The fossil record shows increasing complexity of life forms in the **Proterozoic** eon (2.5 billion—545 million years ago). Protozoans, aquatic plants and hard-shelled invertebrates (eg. corals) had appeared by the end of this eon.
- The current eon (**Phanerozoic**—545 million years ago to the present day) shows the continued evolution of animals, including vertebrates, and land plants.

The Phanerozoic eon is further divided into three **eras**. (The time before the Phanerozoic is often referred to as the **Precambrian**.) Some of the important events in these eras include:

- **Palaeozoic era** (545–248 million years ago)—This era began with a period of rapid evolution; fish are the dominant vertebrates; amphibians and then reptiles evolve; land plants (mosses, ferns and the earliest conifers) appear; largest mass extinction of marine invertebrates ends the era.
- **Mesozoic era** (248–65 million years ago)—Reptiles continue to evolve; dinosaurs appear and become extinct by the end of the Mesozoic; birds appear and the earliest mammals appear near the end of the era; large land plants such as conifers appear; flowering plants start to appear.
- **Cenozoic (or Cainozoic) era** (65–0 million years ago)—Modern mammals appear; flowering plants dominate the land; humans appear about 2 million years ago.

Eon	Era	Period	Fauna	Flora
Phanerozoic 545	0 Cenozoic (Cainozoic) 65	0 Time is divided into 7 epochs 65	• Humans appear 2 million years ago • Mammals increase in size • Mammals diversity • Birds diversify	• Modern flora • Forests develop • Grasslands • Flowering plants diversify
	Mesozoic 248	Cretaceous 146	• Dinosaurs flourish • Major extinctions at end of Cretaceous (including dinosaurs)	• First flowering plants (magnolias/palms)
		Jurassic 208	• Dinosaurs dominate • First birds	• Conifers and ferns dominate
		Triassic 248	• Many reptiles • First dinosaurs • First mammals	• Conifers • Cycads
	Palaeozoic 545	Permian 280	• Amphibians and reptiles are dominant	• Cone trees dominant
		Carboniferous 360	• Giant insects • Amphibians • Early reptiles	• Large tree ferns • Increasing cone plants
		Devonian 408	• Fish flourish • Insects	• First ferns • First seed-bearing plants
		Silurian 438	• Jawed fish • Freshwater fish	• First vascular plants
		Ordovician 500	• Primitive fish • Molluscs • Corals	• Red/green algae • First land plants (mosses)
		Cambrian 545	• Many marine invertebrates • Trilobites and arthropods	Algae No land plants
Proterozoic 2500	Precambrian	—	• Soft-bodied invertebrates (jellyfish, worms) • Algae in the oceans	
Archaean 3800		—	• Life forms appear in the fossil record • Simple cellular organisms including archaea, bacteria, cyanobacteria	
Hadean 4600		—	No life—Earth too hot	

(Dates in MyBP = million years before present day)

Figure 4.21 Geological time scale and major life forms

These eras are further subdivided into small sub-units called **periods**. The Cenozoic is now divided into **seven epochs** rather than two periods.

Appearance and extinction of life forms

The abundance of life forms has varied throughout geological time. This information can be displayed using a graph. Abundance is shown by the **thickness of the band** for each organism. The thicker the band the more abundant is the organism. When the thickness drops to zero the organism has become **extinct**.

Example

Trilobites are invertebrates (crawling and swimming arthropods) that thrived in shallow seas in the Palaeozoic era. They first appeared in the fossil record about 545 million years ago and became extinct about 245 million years ago. Their appearance in the fossil record is often used to signal the start of the Palaeozoic era. Their period of greatest abundance was in the first

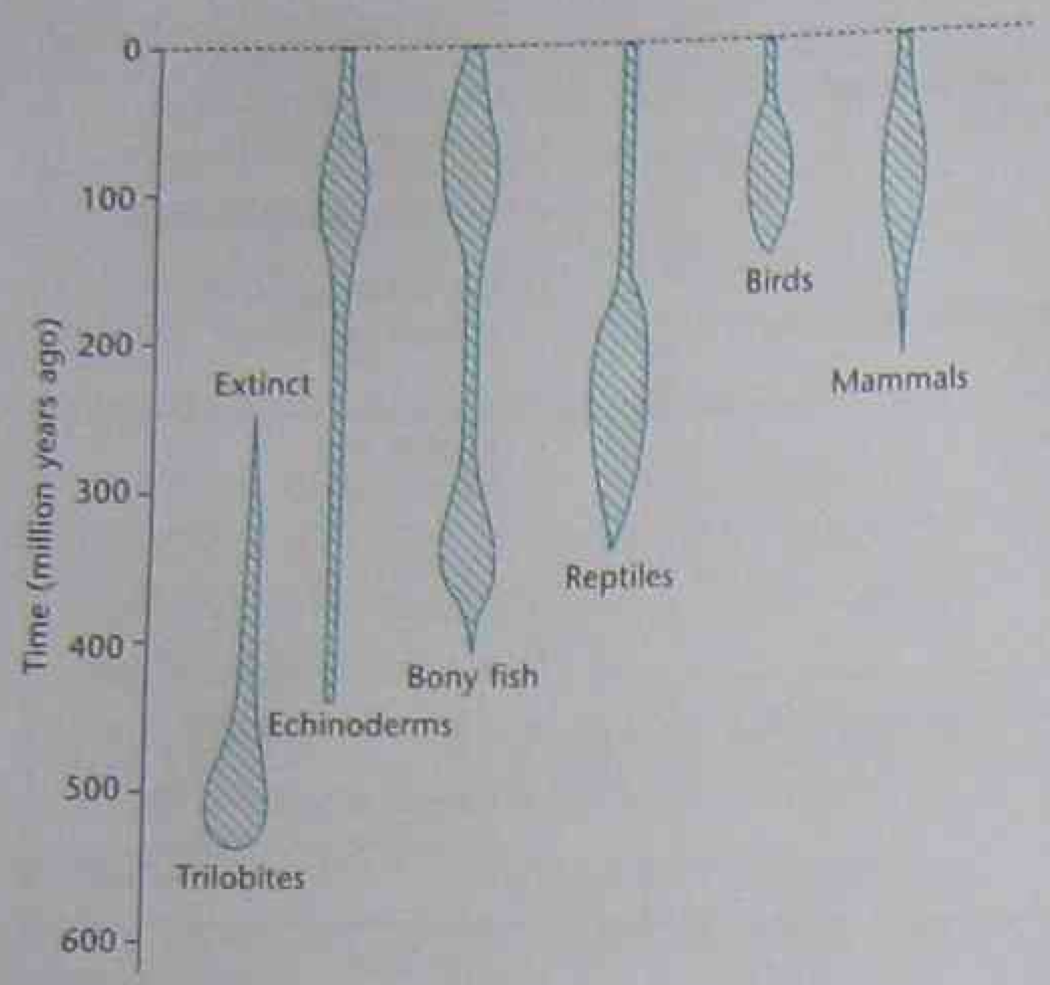


Figure 4.22 Abundance of selected invertebrates and vertebrates over geological time

40 million years of the Palaeozoic era. Their numbers then declined and they became extinct near the close of the Palaeozoic.

Plate tectonics and continental drift

The lithosphere is the outer rigid shell of Earth. It is between 50 and 100 km thick and includes the crust and the rigid upper mantle. The lithosphere is composed of about twelve rigid blocks or **plates**. Geologists suggest that these plates move in response to the combined effects of **convection currents** in the hot, partly molten region of the upper mantle (called the **asthenosphere)** that lies below the plates, and **gravitational forces** that help to pull heavy plate edges downward at subduction zones. **Plate tectonics** is the study of the forces leading to plate movement. Figure 4.23 shows these convection currents in the mantle below a mid-ocean ridge.

Plate interactions

The motion of plates can explain many natural events such as earthquakes and volcanoes. There are four ways in which the edges of plates can interact.

Collision zones

Continental plates can push into each other and this leads to the formation of **mountain ranges**. The Himalayas are formed as the Australian–Indian plate and the Eurasian plate collide. The mountains are rising at 8 cm per year as the land becomes folded upward due to the immense forces of the colliding plates.

Subduction zones

An ocean plate collides and slides underneath a continental plate at the edge of a continent. This is called a subduction zone. This movement is believed to be due to the combined action of convection currents and gravity pulling the plate edges downward into the mantle. Such a zone occurs at the deep ocean trench along the west coast of South America. **Mountain building, volcanic activity** and **earthquakes** can result.

Collision zones and subduction zones are examples of **convergent or destructive plate boundaries.** Oceanic crust is dragged down into the mantle at these sites.

Spreading zones

Spreading zones produce **mid-ocean ridges** and **rift valleys.** This is a site where two plates are moving apart. Such zones are also called **divergent plate boundaries.** Molten rock rises to the surface along these zones and new sea bed and chains of volcanoes form. The mid-Atlantic trench is an example of a spreading zone. Iceland is a volcanic island formed across this zone.

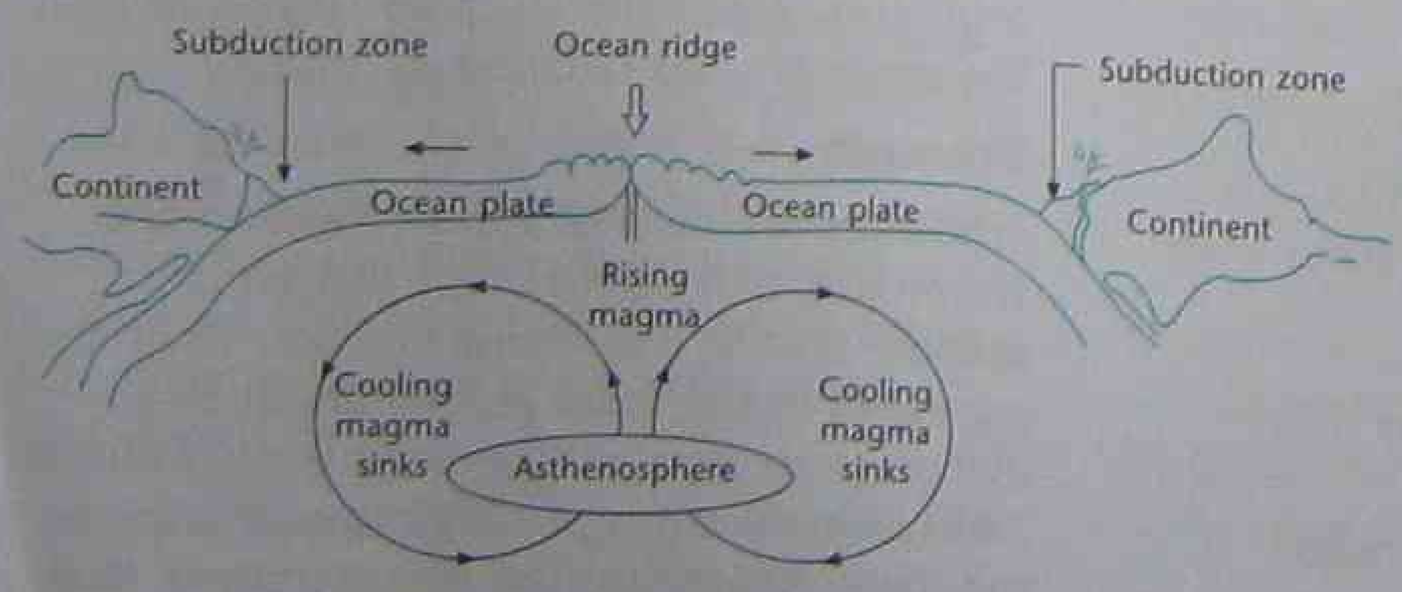
Transform fault zones

Plates can slide past each other along fault lines. The San Andreas fault in California is a transform fault line in which the south-moving Pacific plate slides past the north-moving North American plate. Many **earthquakes** have occurred along this fault line.

Figure 4.24 shows examples of these plate interactions.

Continental drift over geological time

Because of the motion of Earth's plates, the locations of the continents have changed over geological time. This is known as **continental drift.** Figure 4.25 shows the locations of the continents during two key times in the past. The giant continent Pangaea broke up into Laurasia in the north and Gondwanaland in the south. Australia was part of this great southern super-continent. Similar fossils collected at the edges of continents that were once joined



- Ocean plates dragged away from ocean ridge by convection currents in the asthenosphere
- At the subduction zone, gravity pulls the heavy plate edge downward

Figure 4.23 Convection currents below a mid-ocean ridge

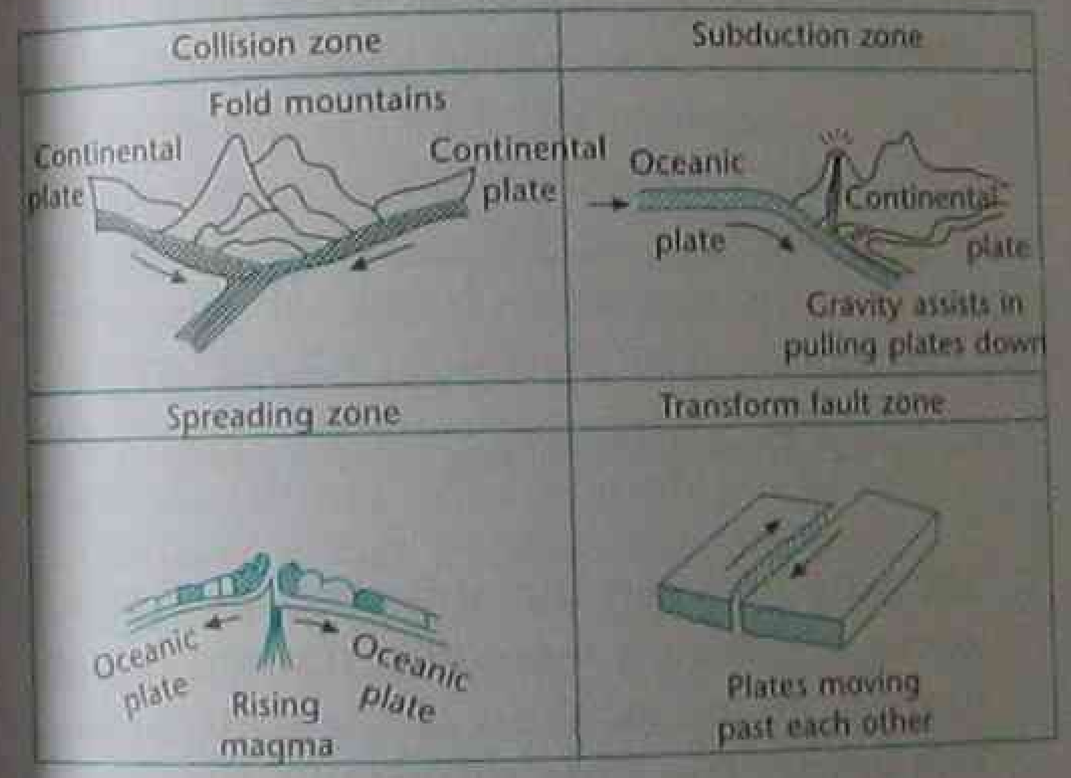


Figure 4.24 Plate interactions

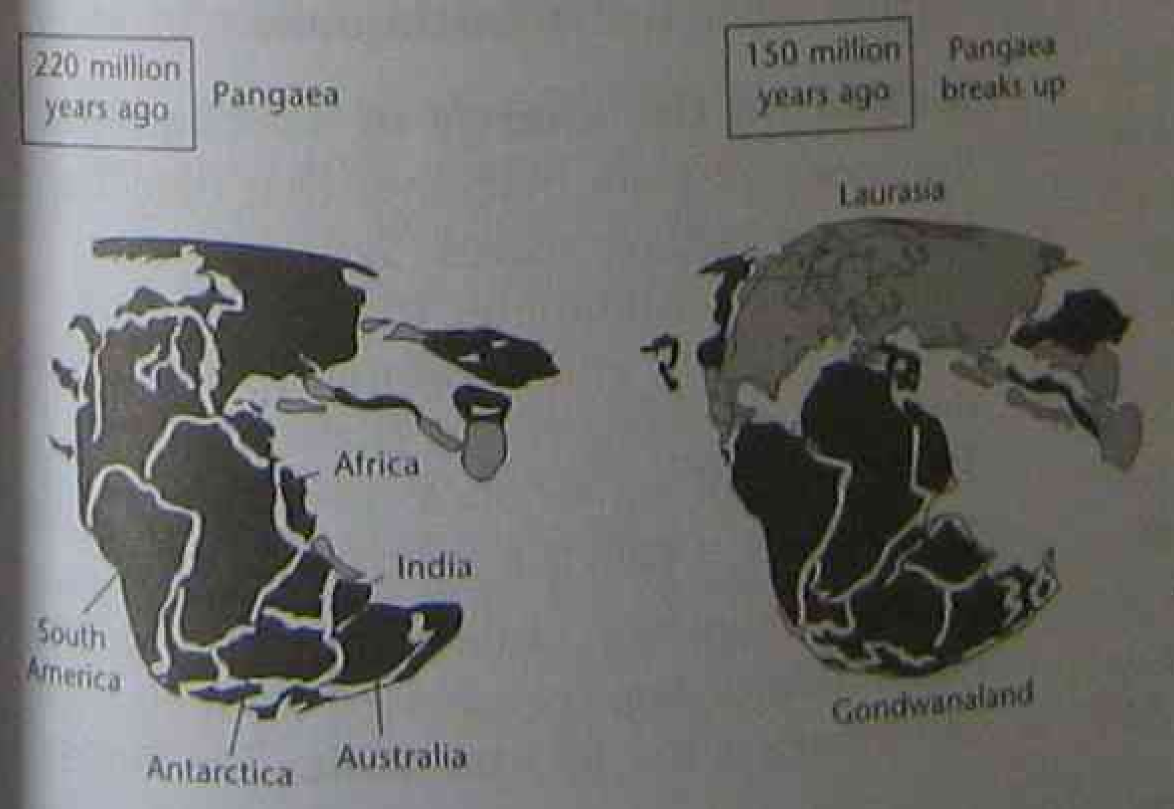


Figure 4.25 Location of the continents as Pangaea breaks up into Laurasia and Gondwanaland

provide further evidence for continental drift. The breaking apart and formation of new landmasses is related to the evolution of new species. When continents are separated, organisms are isolated and natural selection and gene mutation can lead to the emergence of new species.

Earthquakes and vulcanism

Seismology is the study of earthquakes.

Earthquakes occur at the four zones of interaction between tectonic plates.

- **Collision zone earthquakes**—Shallow earthquakes occur due to the intense compression of the colliding continental masses. Some deep-focus earthquakes are also produced in the Himalayas.
- **Subduction zone earthquakes**—These can occur at various depths and lead to various-strength earthquakes, mountain building and volcanic activity.
- **Spreading zone earthquakes**—The activity is low here and occurs at shallow depths as the lithosphere is very thin in these locations. Volcanic activity is found at these locations.
- **Transform fault zone earthquakes**—These occur at shallow depths but no volcanic activity occurs. Huge stresses build up which are released by sudden plate movements, leading to devastating earthquakes such as those in Turkey and California.

Earthquake waves

Earthquakes occur at various depths underground. The site of the earthquake is called the **focus**. The **epicentre** is a point on Earth's surface immediately above the focus.

Seismic (earthquake) waves can be classified into three groups.

- **Primary (P) waves**—These **push** waves are compression waves that can travel through solids, liquids or gases. These waves travel quite fast through Earth. As they pass through different materials the waves bend or refract.
- **Secondary (S) waves**—These transverse **shear** waves travel through the solid Earth but not through liquids or gases. Consequently they do not penetrate the outer molten core of Earth. Shear waves cause rock particles to oscillate at right angles to the direction of movement of the wave. These S waves travel more slowly and arrive at a given seismographic recording station later than P waves.
- **Land surface (L) waves**—These waves (which originate at the **epicentre** of the earthquake on Earth's surface) travel along the surface and cause the **greatest damage**. They are the slowest of all seismic waves.

Figure 4.26 shows **a seismogram**. This recording shows the three types of seismic waves.

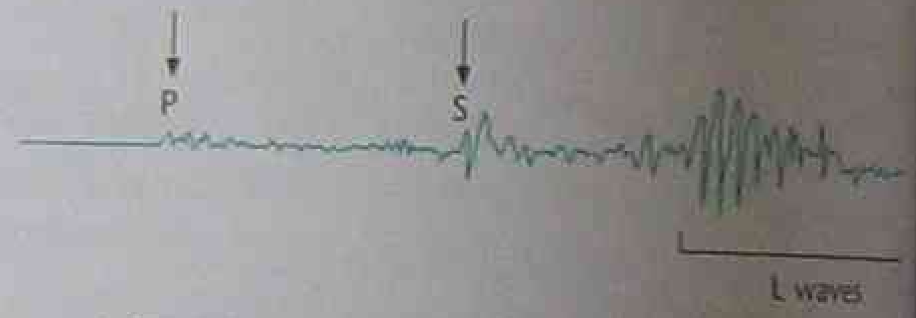


Figure 4.26 Seismogram showing types of waves

The time between the arrival of the P and S waves at different recording stations is used to measure the location of the earthquake. **The longer the time difference the further away is the earthquake.**

Measuring the energy of earthquakes

Seismographs are machines that detect and record earthquake waves. Seismographs consist of a seismometer that detects the ground motion, and an amplifier. The arrival time of the various waves is also recorded. The display of waves on a computer screen or on paper is called a seismogram.

The most common scale for measuring the magnitude or energy released from an earthquake is the **Richter scale**. The energy released is related to the amplitude of the seismic waves. The Richter scale is not a linear scale. A change of one unit on the scale represents about a **thirty-one** times change in energy released by an earthquake. Thus an earthquake of magnitude 7 releases about thirty-one times more energy than one of magnitude 6. The largest known earthquake had a magnitude of 8.9. The Richter scale does not measure damage because large earthquakes that happen in isolated areas cause little damage.

Earthquake	Richter magnitude
minor	0–3.9
light–moderate	4–5.9
strong	6–6.9
major	7–7.9
great	>8

Earthquakes are not common in Australia as the continent does not lie near the edge of a tectonic plate. The 1989 Newcastle earthquake had a magnitude of 5.5. Although it was classified as a moderate earthquake, it caused much damage and death because it happened in a populated area. Ocean trenches associated with subduction zones are common places where earthquakes occur. Earthquakes are much more common in New Zealand since these islands lie over the Pacific plate and the Indian-Australian plate subduction zone.

Types of volcanic activity

Tectonic activity at **spreading zones** and **subduction zones** is associated with volcanic activity. Volcanic activity also occurs away from the plate boundaries at **hot spots** or **volcanic plumes**. The Hawaiian Islands have formed from magma released from a hot spot that moved upward through cracks and faults to penetrate the crust. A hot spot under the Newcastle area is believed to have produced the stresses that triggered the 1989 earthquake.

Volcanic activity results when **magma**, **gases** and/or **ash** are released onto Earth's surface from chambers of magma deep underground. Once the magma is discharged onto the surface it is called **lava**.

Volcanoes vary in size, shape and composition.

Additional content

The following information is provided as *additional content* for students interested in volcanoes.

- **Lava shields**—These volcanoes are very wide and have very gentle slopes. They are formed by incandescent outpourings of very fluid (basaltic) lava from fissures. The Hawaiian Islands are examples of lava shields. Convex shaped **lava domes** may also occur within the shield.
- **Pyroclastic volcanoes**—These volcanoes explosively produce coarse and fine fragments of lava called scoria or tuff that build up around the vent, forming straight or gently concave slopes.
- **Strato-volcanoes**—These cones are built up by layers of lava flows and pyroclastic deposits. Mt Vesuvius and Mt Fuji are examples of this type. **Composite** volcanoes are like strato-volcanoes except that the lava and the pyroclastic deposits tend to be mixed up.

The most violent volcanic eruptions involve the explosive ejection of viscous lava. The Mt St Helens eruption of 1980 in western USA and the Mt Pinatubo eruption in 1991 in the Philippines are examples of this type. Ash and glowing volcanic gases were released with extreme explosive force from these strato-volcanoes. In the Mt St Helens eruption, a rising magma plug caused the production of superheated steam in overlying groundwater. The hydrothermal blast tore the side of the mountain away. Rising magma then suddenly degassed, producing an ejection of pyroclastic material. Solidifying magma eventually closed the vent.

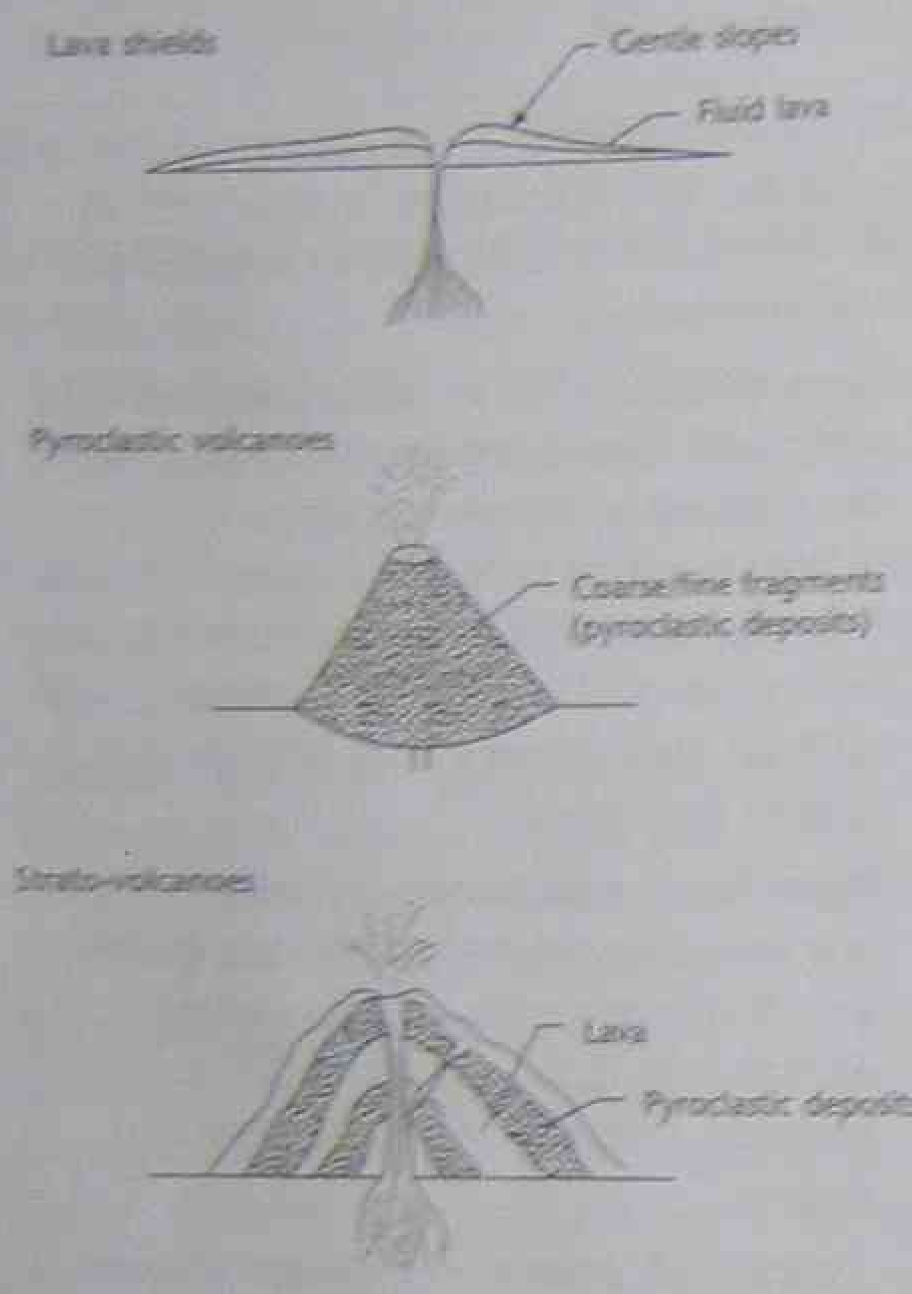


Figure 4.27 Types of volcanic structures

Impact of natural events on the spheres of Earth

Natural events have impacts on the environment in which we live. Our environment includes the atmosphere, hydrosphere, lithosphere and biosphere. Examples of the effects of natural events on these spheres are given below.

Atmosphere (the gaseous sphere around Earth)

The atmosphere is affected by many natural events including:

- release of **poisonous gases** (eg. sulfur dioxide) and **volcanic dust** from **volcanoes**. Dust clouds from volcanoes can block sunlight and cause atmosph cooling;
- release of **greenhouse gases** (eg. methane) from decaying vegetation;
- release of smoke and gases from **bushfires**. Extensive fires can reduce th air quality for humans;
- **lightning**, which generates nitrogen oxides that produce acidic rain;
- **dust storms** that pollute the air and make it difficult to breathe;
- **cyclones** (atmospheric storms) that damage the natural and built environment.

Hydrosphere (the sphere in which water is located around Earth)

The hydrosphere is affected by many natur events.

- Lava released from mid-ocean ridges and fissures increases the concentration of minerals in sea water and thermal lakes.
- Acid rain produced from volcanic eruptions can produce acidic lakes and streams.
- Erosion of the land during **floods** cause more minerals and sediments to enter rivers, lakes and seas.
- Earthquakes can produce **tsunamis**.

Lithosphere (the sphere in which rocky material is found around Earth)

The lithosphere is affected by many natura events.

- **Earthquakes** cause sudden earth movements leading to the formation of new landforms.
- **Vulcanism** produces new landforms.
- **Avalanches** lead to new landforms by depositing rocks and scraping out valleys.
- **Erosion of rocks by running water** releases valuable minerals to make the soil fertile.

- **Glaciers** erode rocks to produce new landforms.

Biosphere (the sphere in which life forms are found around Earth)

The biosphere is affected by many natural events.

- **Volcanoes** release poisonous gases and eject pyroclastic materials that kill living things.
- **Earthquakes** can kill living things that live in these zones.
- **Cyclones** can destroy the habitats of living things and cause the deaths of those that live there.
- **Bushfires** destroy living things, their habitats and food supply.

Test yourself (answers on pages 223–4)

Part A. Knowledge (answers on page 223)

1 A limestone layer was discovered during a geological expedition. The layer contained numerous coral fossils. The sediments that formed this layer were originally deposited in a:

a fast-flowing mountain stream.
b freshwater lagoon.
[✓ c] swamp.
~~d~~ warm, shallow sea. (1 mark)

2 A dinosaur footprint in an exposed shale layer is an example of what type of fossil?

~~a~~ Trace fossil
[✓ b] Petrified fossil
c Cast
d Carbonised impression (1 mark)

3 Select the correct response concerning the geological time scale.

[✓ a] The divisions of the scale are fixed and will never change.
b Fossils did not appear in t geological record until the beginning of the Phaneroz
~~c~~ Reptiles appeared in the fo record during the Palaeozoic era.
d The first mammals appear in the fossil record late in the Cenozoic era. (1 mark)

4 The volcanic activity and mountain building along the west coast of South America is an example of which type of plate tectonic activity?

~~a~~ A subduction zone
b A spreading zone
c A collision zone
[✓ d] A transform fault zone (1 mark)

5 An earthquake that registers 6.2 on the Richter scale would be described as:

a major.
b minor.
c light.
[✓ d] strong. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

a The law of superposition states that in a sequence of strata, the _____ layers of rock are on top.
b Granite has very large crystals as the magma from which it formed cooled _____ deep inside Earth.
c Tree trunks may form petrified fossils if the intercellular spaces become filled with _____.
d Carbon-14 is a useful _____ that can be used to date carbonaceous fossils up to 60 000 years old.
e Birds and mammals appeared in the fossil record near the end of the _____ era.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a asthenosphere	f transform fault line
b spreading zone	g convection currents
c San Andreas fault	h shear wave
d primary waves	i divergent plate boundary
e secondary waves	j push waves

8 Distinguish between the focus and epicentre of an earthquake. (2 marks)

9 Explain why lava shield volcanoes are very wide with gentle slopes, whereas strato-volcanoes are steep-sided cones. (2 marks)

10 Describe three natural events that lead to changes in the atmosphere of Earth. (3 marks)

Part B. Skills (answers on pages 223–4)

1 Figure 4.28 shows a section through sedimentary strata. Use the diagram to list the strata from youngest to oldest. (3 marks)

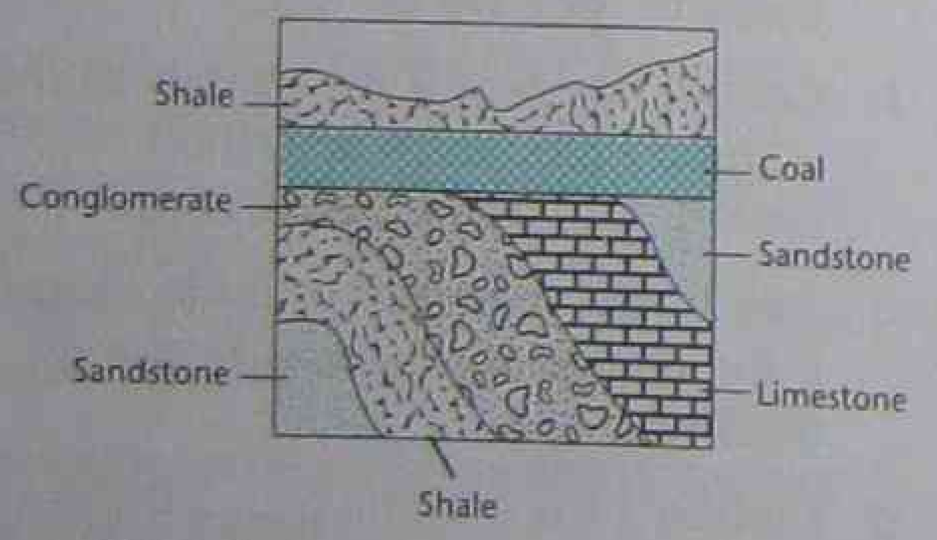


Figure 4.28 Sedimentary strata

2 Figure 4.29 shows a geological cross-section exposed by a road cutting. Write the geological history of this section. (4 marks)

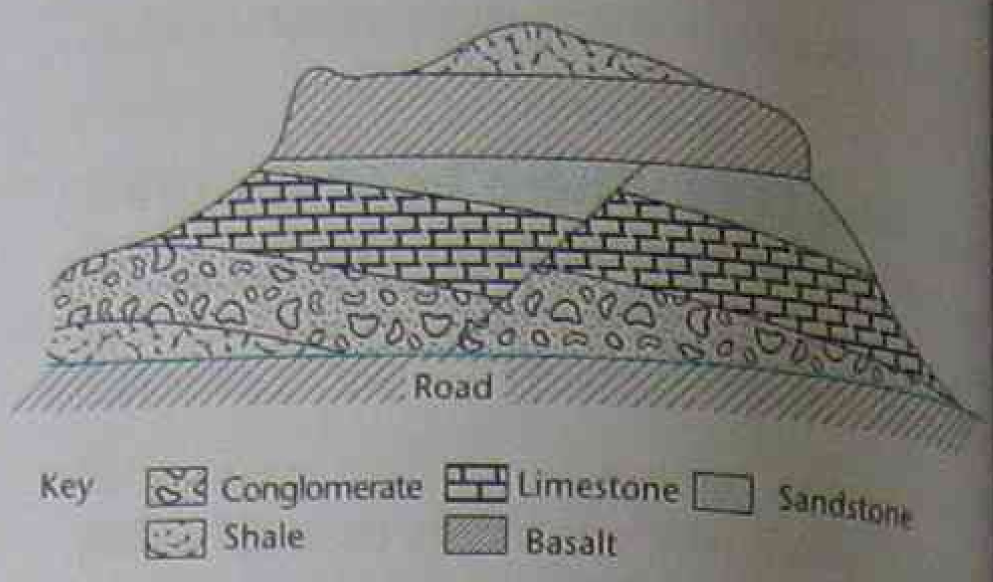


Figure 4.29 Road cutting

3 Figure 4.30 shows the jumbled steps in the formation of a fossil cast of a mollusc shell. Use the code letters to place the steps in their correct order. (2 marks)

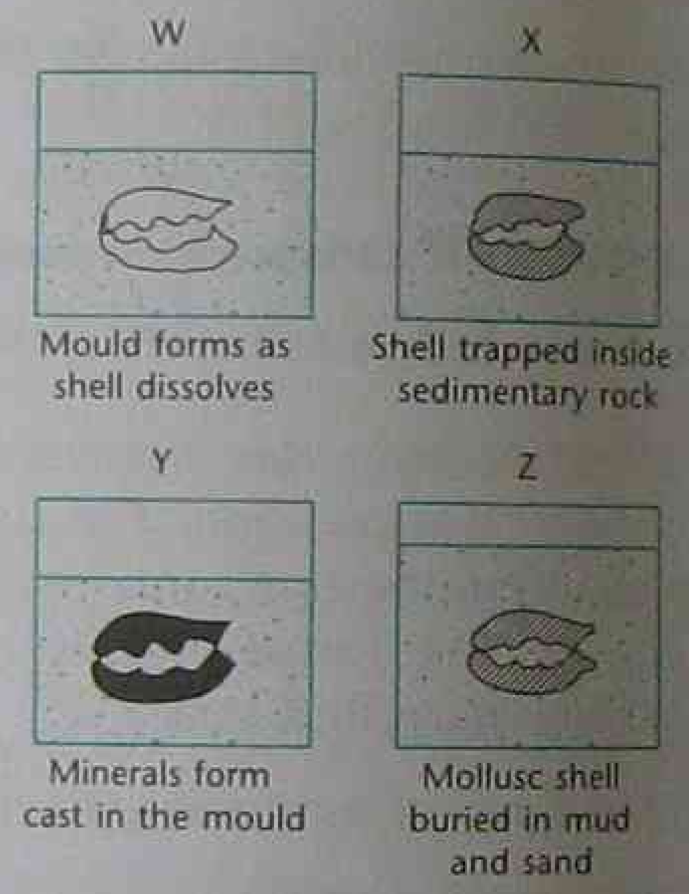


Figure 4.30 Jumbled steps in the formation of a fossil cast

4 Figure 4.31 shows various sedimentary strata containing fossils. Two lava flows are also shown. The age of these flows was determined radiometrically.

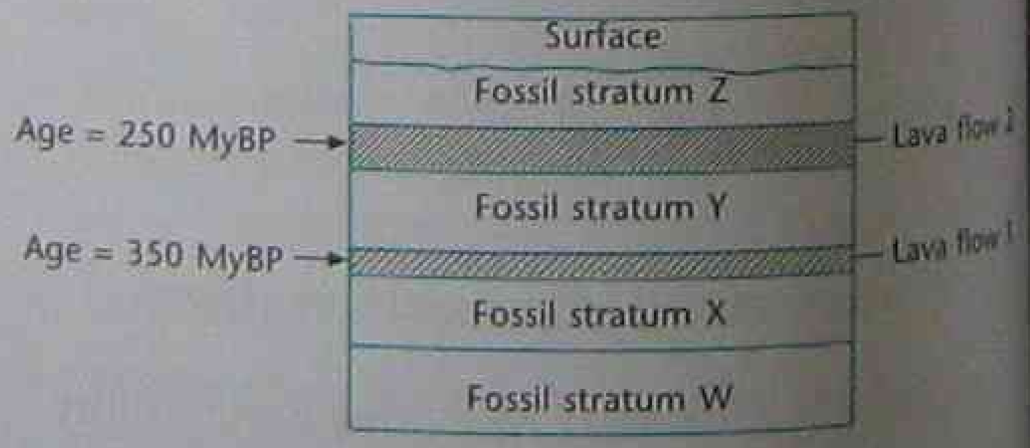


Figure 4.31 Fossilised strata and lava flows

a Name the geological era in which the sedimentary stratum (Y) between the lava flows formed. (1 mark)

b Which of the listed fossils would not be found in the sedimentary stratum (Y) between the lava flows formed? (1 mark)

List: coral; fish; algae; bird; trilobite; flowering plant

c Which fossil in the above list could be found in layer W but not in layer Z? (1 mark)

5 Figure 4.32 shows a drawing of a fossilised footprint. The real length of the footprint is 37 cm. What scale has been used in this drawing? (2 marks)

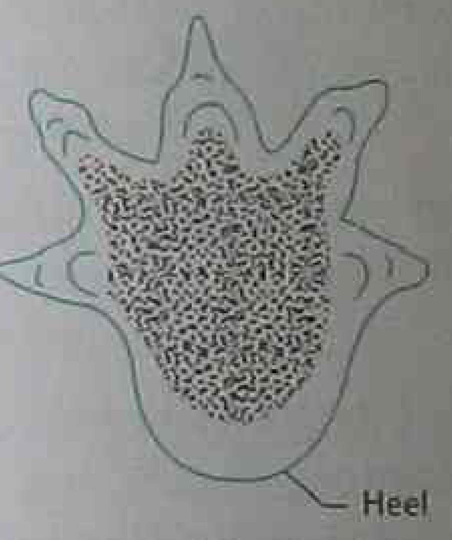


Figure 4.32 Scale diagram of fossilised footprint

6 Figure 4.33 shows plate boundaries between Australia and New Guinea, and Australia and Antarctica. The arrows show the direction of late movement.

Figure 4.33 Plate boundaries between Australia, New Guinea and Antarctica

a What type of plate interaction between the Indian–Australian and Antarctic plate is shown in this diagram? (1 mark)

b The Australian plate is moving north at an average speed of 6 cm per year. How far will it move in 1 million years? (2 marks)

c Explain why there are high mountain ranges in central New Guinea. (1 mark)

7 Use the theory of plate tectonics to account for the following observations.

a The rim of the Pacific Ocean is called the Ring of Fire, owing to the large number of earthquakes and volcanoes located there. (1 mark)

b Flightless birds such as the ostrich, emu and rhea are found in Africa, Australia and South America respectively. (1 mark)

c The climate of the Sydney region has changed from cold and icy to warm and temperate over the last 135 million years. (1 mark)

d Earthquakes and volcanic activity are common in the Mediterranean area and Middle East. (1 mark)

8 At the time of its death a portion of the leg bone of a bird produced 960 units of radiation per minute due to its carbon-14 content.

a Given that the half-life of carbon-14 is 5730 years, determine the radiation count of the leg bone after it has been fossilised for four half-lives (ie. 22 920 years later). (2 marks)

b Why is the carbon-14 method useful only for determining the age of very recent carbonaceous fossils? (2 marks)

9 Figure 4.34 shows the earthquake waves recorded at Sydney, Brisbane and Perth.

a Which city is
  i closest
  ii furthest, from the epicentre of the earthquake? (2 marks)

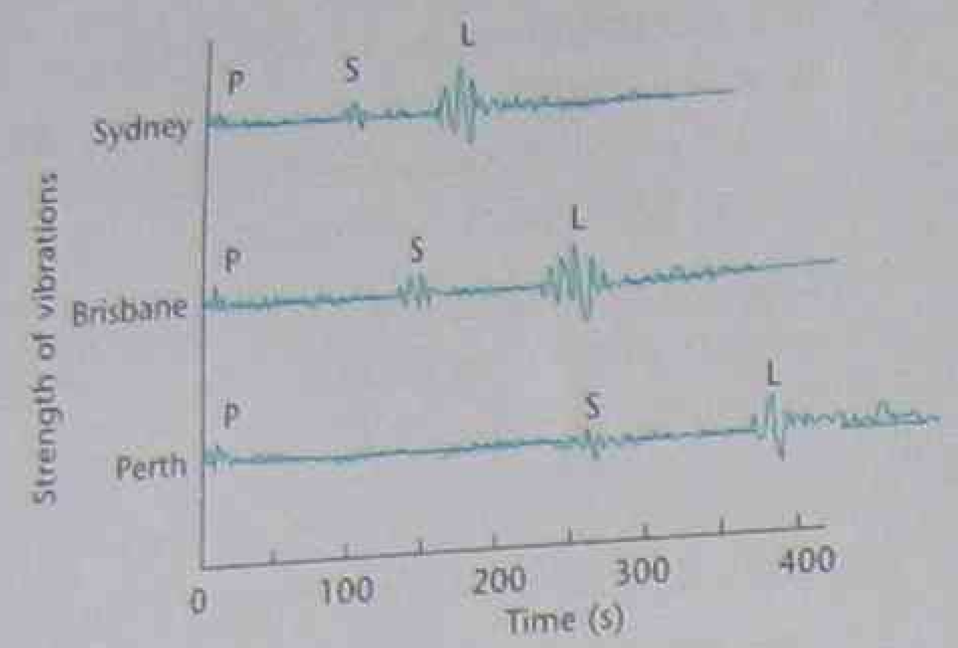


Figure 4.34 Earthquake waves recorded at Sydney, Brisbane and Perth

b The earthquake's epicentre was at one of the following locations. Which one? Justify your answer. (2 marks)

Possible locations: New Guinea (highlands); New Zealand (south island); Japan

End-chapter test (answers on pages 224–5)

1 Read the following information about the formation of the Australian continent from the break-up of Gondwanaland.

a Convert this information into a bar graph by plotting time along the horizontal axis. (3 marks)

(MYBP = million years before the present)

180 MYBP—Gondwanaland (Africa, South America, Australia, India and Antarctica) separates from Laurasia. This break-up is complete by 135 MYBP.

135 MYBP—Gondwanaland begins to break up with Antarctica–Australia–India moving away from the rest of Gondwanaland.

120 MYBP—India separates from Antarctica–Australia.

105 MYBP—South America and Africa separate.

80 MYBP—Australia moves apart from Antarctica.

b In which geological era did the events described above occur? (1 mark)

2 Figure 4.35 shows three geological cores from different areas (X, Y and Z). Some of the sedimentary strata contain fossils. Assuming that the layers with the same fossils are of the same age, determine which core has the most recent fossil. (2 marks)

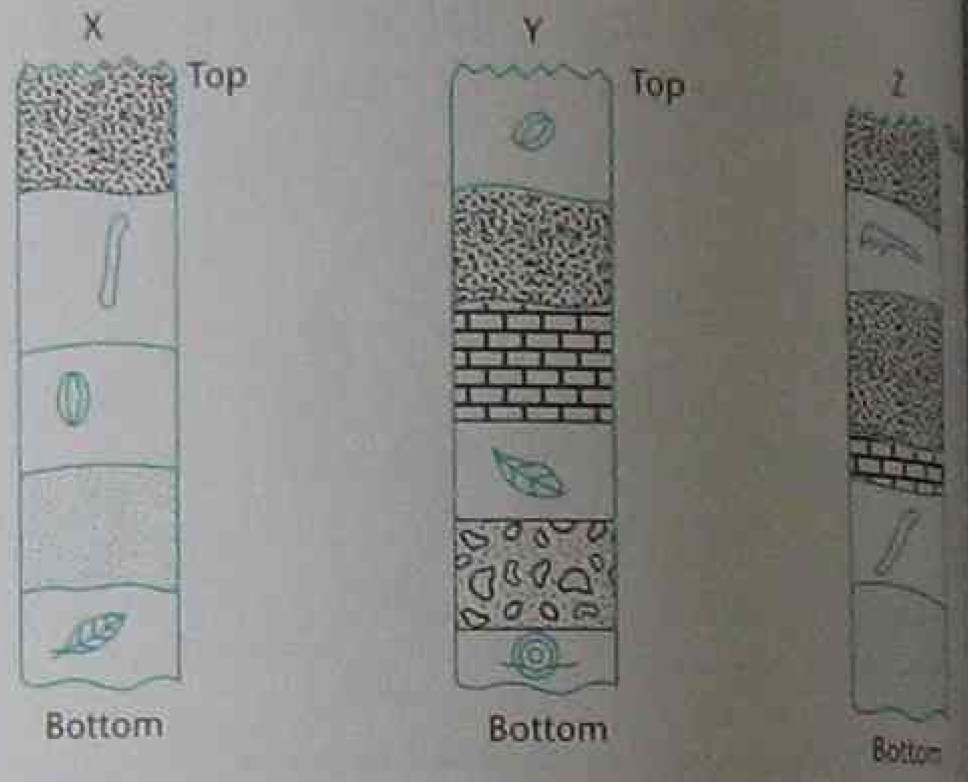


Figure 4.35 Core sections X, Y and Z

3 Figure 4.36 shows a geological cross-section involving dolerite intruding into various sedimentary strata.

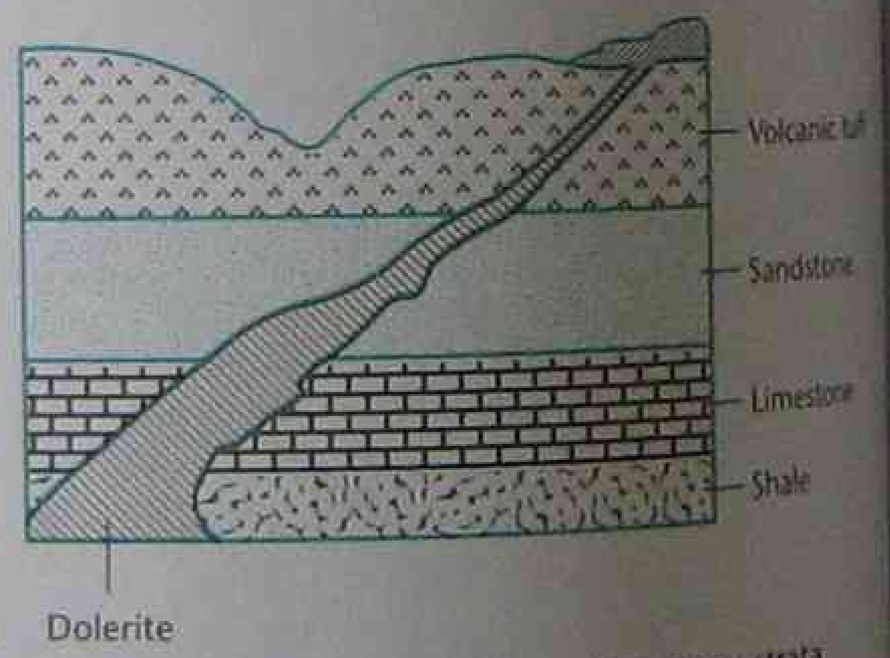


Figure 4.36 Dolerite intrusion in sedimentary strata

a Name the type of igneous structure shown. (1 mark)

b List the rocks in order from oldest to youngest. (2 marks)

c Describe the effect of the intruding magma on the surrounding sedimentary rocks. (1 mark)

d Compare the crystal size in the dolerite intrusion to the lava that flowed as an extrusion onto Earth's surface. (1 mark)

4 a Explain how a volcano is formed at a subduction zone. (2 marks)

b Name two locations in the Pacific rim where volcanic activity is due to plate movement at a subduction zone. (2 marks)

5 The potassium–argon method is commonly used to date volcanic rocks and minerals that are millions of years old. Zircons, for example, contain potassium-40.

The potassium-40 decays to argon-40. The half-life of potassium-40 is 1.3 billion years.

a A zircon was formed 3.9 billion years ago. Draw a graph of the amount of K-40 remaining as a function of time. Label the axes. (3 marks)

b Draw a graph of the amount of Ar-40 formed as a function of time. (2 marks)

c In which geological eon was the zircon formed? (1 mark)

d In which geological eon did the amount of K-40 remaining equal 20% of the original amount? (1 mark)

6 Figure 4.37 shows three seismograms (X, Y and Z) of an earthquake with an epicentre in New Zealand. The seismograms were recorded in Melbourne, Adelaide and Perth. Determine which seismogram was recorded in each city. (2 marks)

7 Describe, using examples, the impact of earthquake and volcanic activity on the:

a lithosphere (2 marks)

b hydrosphere (2 marks)

c atmosphere (2 marks)

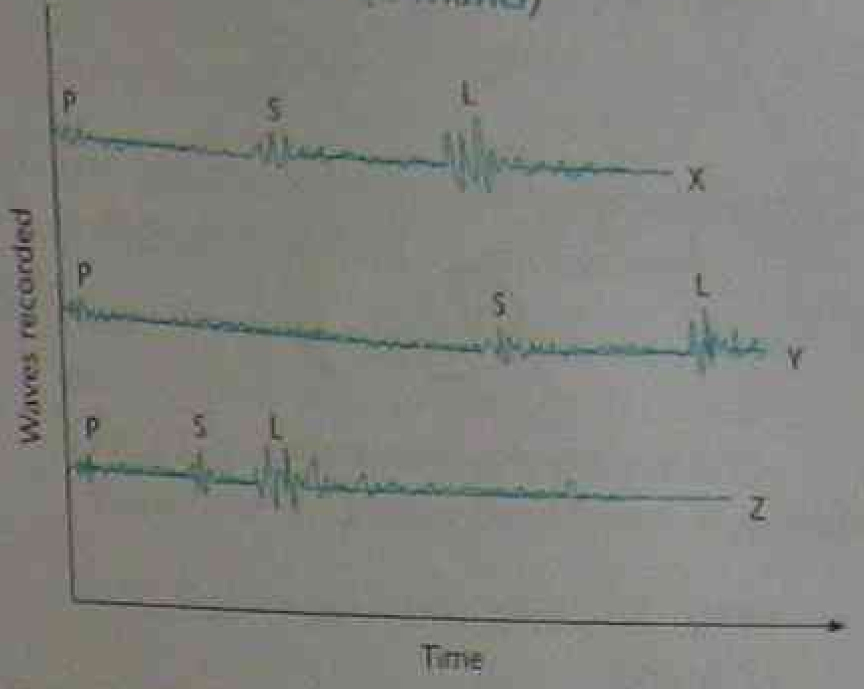


Figure 4.37 Seismograms X, Y and Z

8 Figure 4.38 shows geological activity at a zone between two oceanic plates.

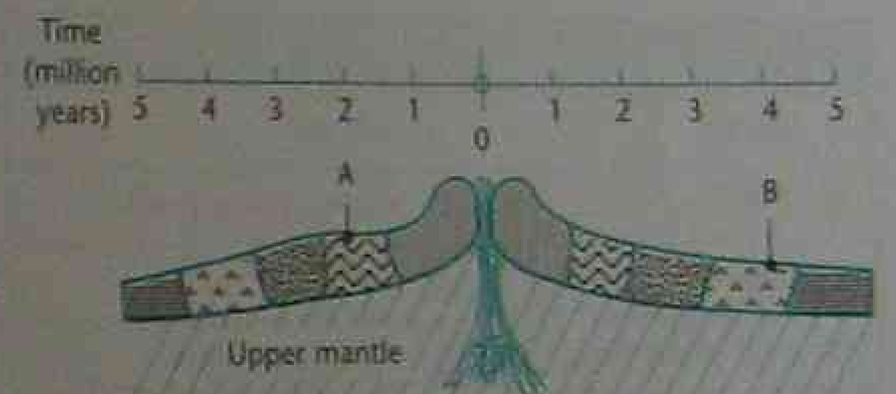


Figure 4.38 Geological activity at a plate boundary

a Name the type of plate tectonic activity shown. (1 mark)

b Use the scale to determine the age of the rocks at points A and B on the diagram. (2 marks)

c Explain why the rocks (A and B) have different ages. (1 mark)

d In which layer of Earth are the convection currents causing crustal movement found? (1 mark)

e If new crust is forming at these zones, why doesn't the crust of Earth expand? (1 mark)

9 Figure 4.39 shows a map of Gondwanaland before it split apart. Fossils of *Mesosaurus*, a freshwater reptile, were found at the locations marked on the map of Gondwanaland. Explain why the fossils are not found in

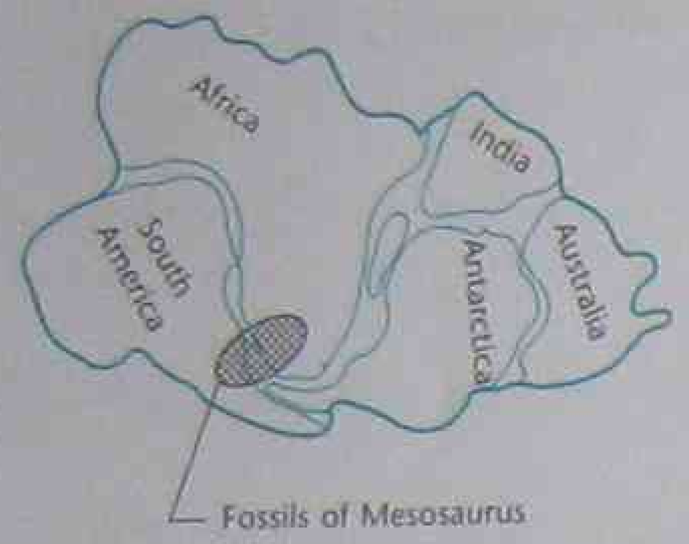


Figure 4.39 Map of Gondwanaland

India, Australia and Antarctica. (2 marks)

10 Match the fossils listed in column 1 to the past environments listed in column 2. (4 marks)

Column 1	Column 2
a coral	e swamp
b fish	f warm, shallow sea
c ferns	g rivers and seas
d coal plant fossils	h warm terrestrial

Summary

The Big Bang theory and components of the universe

- Astronomical measurements show that the universe is expanding.
- The Big Bang theory is the currently accepted theory for the origin of the universe. According to this theory the universe began in a giant explosion in which space and time came into existence about 12–13 billion years ago.
- Various bands of the electromagnetic spectrum can be used in astronomy to investigate the universe.
- Ground-based telescopes are affected by the atmosphere as well as radio and light pollution from cities.
- Distances in space are so large that they are measured in light years.
- Stars like the Sun form as hydrogen gas and interstellar dust condense and heat up until nuclear fusion begins.
- Stars that have a similar mass to the Sun will evolve into red giants before they eject material to form a planetary nebula. The remaining core is a white hot dwarf star that fades as it cools.
- Large stars evolve into red supergiants before they explode to form a supernova. A neutron star or black hole remains.

Natural events

- The law of superposition states that in a sequence of sedimentary strata the oldest layer is on the bottom and the youngest layer is on the top.
- Igneous activity and metamorphism can influence the geological history of an area.
- Fossils are the remains or impressions of living organisms from the past.
- Fossils include whole organisms that remain almost unchanged; unaltered hard parts; altered hard parts; trace fossils.
- In a sequence of sedimentary strata the simplest fossils are found in the deepest layers.
- Radioactive dating can be used to determine the age of rocks and fossils.
- The geological time scale divides the 4.6 billion years of Earth's history into large and small divisions called eons, eras and periods.
- The lithosphere is broken up into twelve plates that move due to convection currents in the asthenosphere.
- Plates interact in four main ways: collision zones; subduction zones; spreading zones; transform fault zones.
- Movement of tectonic plates has led to the breaking up and re-forming of Earth's continents.
- Earthquakes and volcanic activity are usually associated with interactions at plate boundaries.
- Earthquake wave analysis can be used to determine the location of earthquakes.
- The Richter scale is used to measure the energy released by an earthquake.
- Natural events such as cyclones, earthquakes and volcanic activity can affect the hydrosphere, atmosphere, lithosphere and biosphere.

Chapter 5

Ecosystems, Resources and Technology

Ecosystems

The places where living things such as plants and animals live are called **habitats.** In Australia there are many different habitats, including rivers, rainforests, muddy estuaries, paddocks and deserts. The sand, mud, rocks and water in these habitats are some of the non-living components of the habitat. The term **ecosystem** refers to both the living and non-living components of the environment.

Glossary

Competitors—living things that compete with others for available resources such as food and light

Consumers—living things that obtain their food by eating producers or other consumers

Decomposers—living things (such as bacteria, fungi and protista) that obtain their energy by decomposing the remains of dead organisms into basic nutrients

Ecosystem—community of living things within the environment that includes abiotic and biotic components

Habitats—places for living

Predators—living things that kill other living things for food

Producers—living things (such as green plants) that make their own food by photosynthesis

Scavengers—living things that feed on the dead remains of other organisms

Biotic and abiotic features of the local environment

The **abiotic** components of an ecosystem are the non-living components. These vary from one ecosystem to another and include:

- energy and light intensity
- water quality (eg. salinity, turbidity, pH, dissolved oxygen levels) and water temperature
- rainfall levels
- air quality, relative humidity and temperature
- wind speed and direction
- soil quality (eg. sandy, clayey), moisture levels, aeration and temperature.

The **biotic** components of an ecosystem involve the interactions of living things. These may include:

- competition for food and water
- competition for living space and shelter
- competition for mating
- predation, including relative abundance of predator and prey
- beneficial interactions (eg. different organisms living together for their mutual benefit) and parasitism (eg. one species living off the host without killing it).

The various ecosystems in Australia are distinguished by the variation in their abiotic and biotic components. Table 5.1 lists some common Australian ecosystems and some of their characteristics.

Table 5.1 Some Australian ecosystems

Ecosystem	Location	Characteristics
Desert	Central Australia and central Western Australia	Low and infrequent rainfall (<200 mm per year) High temperatures (35–50°C in summer) Hardy grasses and low vegetation; no trees
Scrubland	Southern South Australia and southern Western Australia	Low rainfall (200–400 mm per year) High temperatures (27–35°C in summer) Low shrubs and small trees (eg. Mallee eucalypts) and grasses
Woodland	Eastern and central NSW	Rainfall (500–700 mm per year) Warm to cool temperate (25–35°C in summer) Scattered trees, shrubs and grasses (eg. banksia, eucalypt, acacia)
Dry sclerophyll forest	Coastal eastern Australia	Rainfall (700–900 mm per year) Warm temperate (27–35°C in summer) Medium height eucalyptus forest with low shrubs and ferns
Wet sclerophyll forest	Coastal eastern Australia	Rainfall (>900 mm per year) Warm temperate (27–35°C in summer) Taller eucalyptus forest with low shrubs and ferns; restricted to gullies and sheltered areas

Measuring some abiotic features

Excursions to different ecosystems allow students to measure various abiotic features of the local environment.

a. Measuring relative humidity of the air

A wet and **dry thermometer** is used to measure relative humidity. The wet bulb thermometer contains a wet piece of fabric wrapped around the thermometer bulb. As the water evaporates it cools the bulb. The lower the humidity of the air the faster the water evaporates and the cooler the bulb. A table of relative humidity (Table 5.2) is then used to calculate the relative humidity.

Table 5.2 Relative humidity (%)

Dry bulb temperature (°C)	Wet bulb temperature (°C)						
	15	16	17	18	19	20	21
22	43	50	59	67	74	82	91
23	38	45	52	60	67	75	83
24	33	39	46	53	61	67	75
25	28	34	41	47	54	61	68
26	24	30	36	42	48	55	62
27	21	26	32	37	43	50	56
28	17	22	28	33	39	45	51

Example

Dry bulb temperature = 25°C

Wet bulb temperature = 20°C

Relative humidity (from table) = 61%

b. Measuring total dissolved solids in creek water

Rivers and creeks contain dissolved solids that are mainly ionic salts. In estuaries the total dissolved solid level is high owing to the mixing of fresh water with seawater. Conductivity meters can be used to measure the level of total dissolved solids since salts make the water more conductive. The meter must first be calibrated with solutions of known salt level.

Total dissolved solids (TDS) can also be measured by collecting samples of water and evaporating them to dryness. The weight of salts remaining is then determined and the TDS calculated.

Example

Five litres of creek water was filtered and then slowly evaporated. The partially evaporated sample was transferred to a smaller beaker and the evaporation continued until a dry residue was obtained.

The residue was weighed. The mass of the residue was found to be 0.485 g.

The TDS was calculated as follows:

TDS = mass of dissolved solid (in milligrams) per litre of water
= 485 mg/S L
= 97 mg/L

Unpolluted bushland streams typically have TDS values of less than 100 mg/L.

c. Measuring the acidity of water in a creek or river

Samples of water from streams and lakes can be collected in clean containers for later pH analysis using a pH meter in the laboratory. Alternatively a portable pH meter or a pH testing kit consisting of Universal indicator and an indicator–pH colour card can be used to test samples on site. The pH of water in bushland streams around Sydney is typically in the mildly acidic range (pH 6–7) owing to the presence of dissolved carbon dioxide, acidic products of plant decomposition and the absence of basic rocks such as limestone.

Cycles of materials in ecosystems

Feeding relationships

The living things in an ecosystem can be classified according to their feeding relationships.

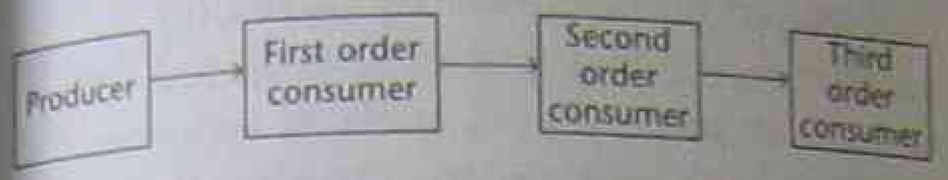
These include:

- **Producers:** Living things that can make their own food by **photosynthesis** are called producers. Land and water plants, algae and phytoplankton are examples of producers. Their bodies contain molecules such as chlorophyll that can absorb solar energy to power the production of energy-rich molecules such as glucose from raw materials.
- **Consumers:** Living things that obtain their food by consuming other living things are classified as consumers. There are various classes of consumers:
  a. **First order consumers**—These animals are often called **herbivores** as they eat only plant material. Cows, sheep, kangaroos and koalas are herbivores as they eat grass or leaves.
  b. **Higher (second and third) order consumers**—These animals eat other animals. They are often called **carnivores.** In a river estuary, for example, the seagrass is eaten by water snails (first order consumers). In turn the water snails are food for crabs (second order consumers). Crabs are eaten by birds such as egrets (third order consumers).

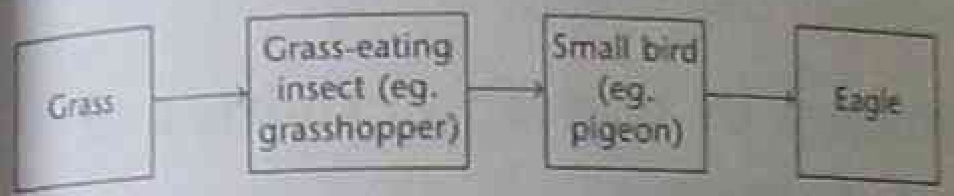
  Note: Some organisms eat plants as well as animal material. These are **omnivores.** Humans and pigs are omnivores.
- **Scavengers:** Many higher order consumers do not consume all the animal they kill. The remains may then be eaten by **scavengers.** Hyenas are common scavengers of animal remains left by African lions.
- **Decomposers:** The remains of organisms that are not eaten are recycled by the action of microbial decomposers such as some bacteria, moulds and protozoans. In this way valuable organic and inorganic nutrients are returned to the environment.

Food chains, food webs and food pyramids

Food chains and food webs represent ways in which energy can flow from one organism to another through an ecosystem. At each stage of the food chain, considerable energy (~80–90%) is converted to heat to keep the organism operating at the correct temperature, or lost to the environment. This occurs because some of the food energy is used for respiration and metabolism and some is lost by heat radiation to the environment. The following flow charts represent a general food chain and a common example of such a food chain in a grassy woodland. The arrow should be read as: 'is eaten by'. Thus the producer is eaten by the first order consumer.



Example. Food chain in a grassy woodland



Food chains are interconnected to form more complex food webs. This occurs because one living thing is often eaten by a number of different consumers. Tadpoles, for example, can be eaten by different species of fish. Lizards, birds and amphibians can eat insects. Figure 5.1 shows an example of a food web. Note that the decomposers are often omitted from such webs or chains.

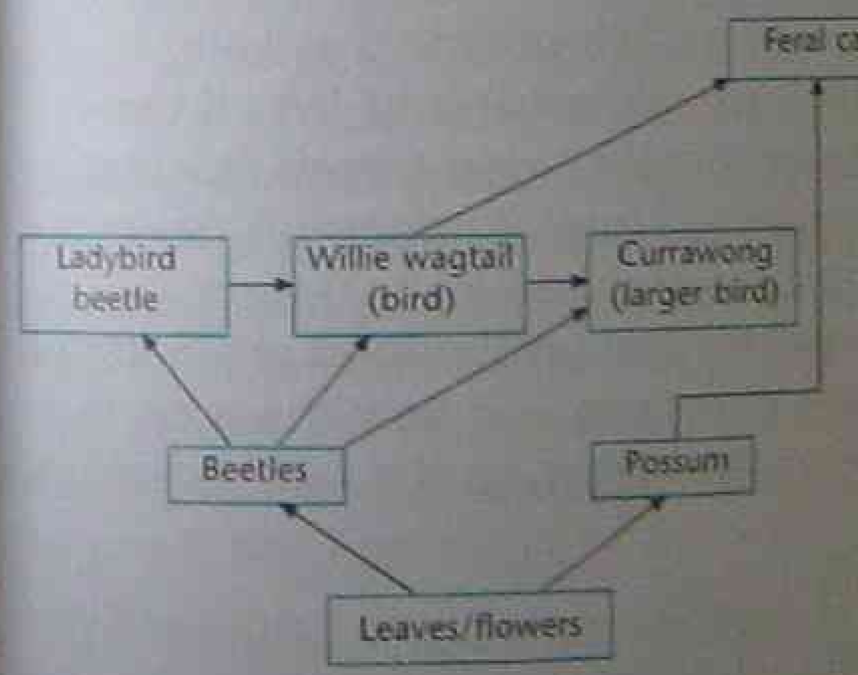


Figure 5.1 Food web

Changes to the population of organisms in a food web affect other parts of the web. If, for example, a disease causes the death of many first order consumers, the numbers of higher order consumers will drop because they no longer have a food source in sufficient numbers to sustain their population.

Because the energy that is passed on from a lower feeding level to a higher feeding level decreases at each feeding level in an ecosystem, the energy available to the top carnivores is relatively small. Thus the total population of organisms at each higher feeding level decreases. This is expressed in terms of a food (or energy) pyramid. The base of the pyramid is occupied by the producers because a large population mass is required to sustain all the animals higher on the pyramid.

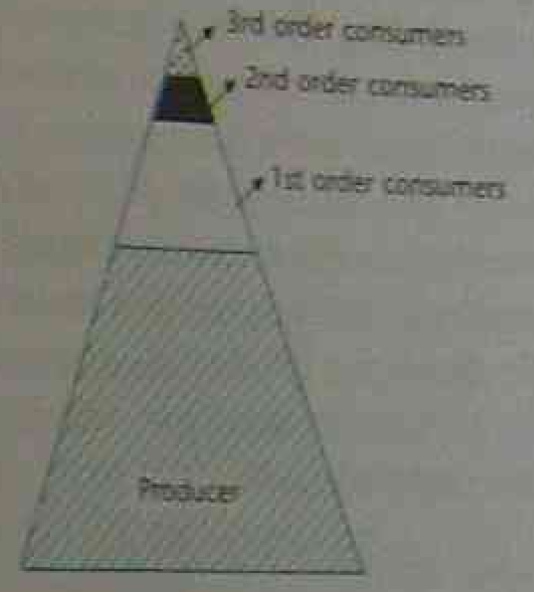


Figure 5.2 Food pyramid

Cycles

Apart from food and energy, other materials such as water, carbon, oxygen and nitrogen need to be transferred around an ecosystem.

a. Water cycle

Water is essential for life. Plants use water as an essential component of the photosynthetic process. Water also provides a medium for biochemical reactions in cells and a transport medium in various vascular systems.

Liquid water is present in the oceans, rivers and lakes. Water absorbs solar energy and evaporates into the atmosphere to form invisible water vapour. As the water vapour cools, clouds containing tiny water droplets form and eventually the water is returned to Earth as rain, snow or ice. Some water soaks into the ground and supports life in the soil; other water collects in oceans, rivers and lakes.

b. Carbon–oxygen cycle

Carbon and oxygen are important elements in ecosystems. The bodies of living things are composed of compounds of carbon, hydrogen, oxygen and nitrogen. Oxygen is used by animals and plants for cellular respiration. Plants use carbon dioxide and water, in the process of photosynthesis, to make energy-rich nutrients (eg. glucose). Oxygen is released into the atmosphere in this process. The carbon and oxygen atoms in the glucose are then converted to thousands of other chemical compounds in cellular reactions in plants and animals. On their death these carbon–oxygen compounds are recycled by decomposers back into the environment. Combustion of fossil fuels by humans also returns carbon (in the form of carbon dioxide and carbon monoxide) to the atmosphere. Figure 5.3 shows the carbon–oxygen cycle.

c. Nitrogen cycle

Nitrogen is also an important element in nature. Nitrogen is an essential element in proteins that form cell membranes. It is also a vital component of DNA in the nucleus of each cell. Nitrogen is present in the soil in various compounds including nitrate and ammonium salts. These are absorbed by plants through their roots. Nitrogen is incorporated into many nitrogen compounds in plants. Nitrogen compounds make their way through food chains via the various feeding relationships in ecosystems. After death, decomposers return nitrogen compounds to the soil. Nitrogen can be returned to the air by denitrifying bacteria in the soil. Nitrogen can also be converted to nitrogen compounds by nitrifying bacteria in the soil and in specialised root nodules in some plants such as legumes. Lightning can also cause nitrogen fixation in the atmosphere. The high energy of the lightning bolt leads to the formation of nitrogen compounds.

Impact of human activities in ecosystems

Humans often upset the balance of nature. As populations of humans grow, they require space for their houses and so natural ecosystems are disturbed or destroyed as new homes are built. Farms and mines also contribute to the destruction of the environment. Some examples of the impact of humans on ecosystems are:

- **Waste.** The wastes that humans generate can pollute the natural environment. Sewage outfalls pollute the water in which aquatic organisms live. If these wastes contain heavy metals (such as mercury) or toxic compounds (such as pesticides and herbicides), then organisms low on the food chain absorb these pollutants and pass them on to organisms higher up the food chain. In so doing their concentration becomes magnified. Many large fish species have been shown to contain toxic levels of heavy metals.

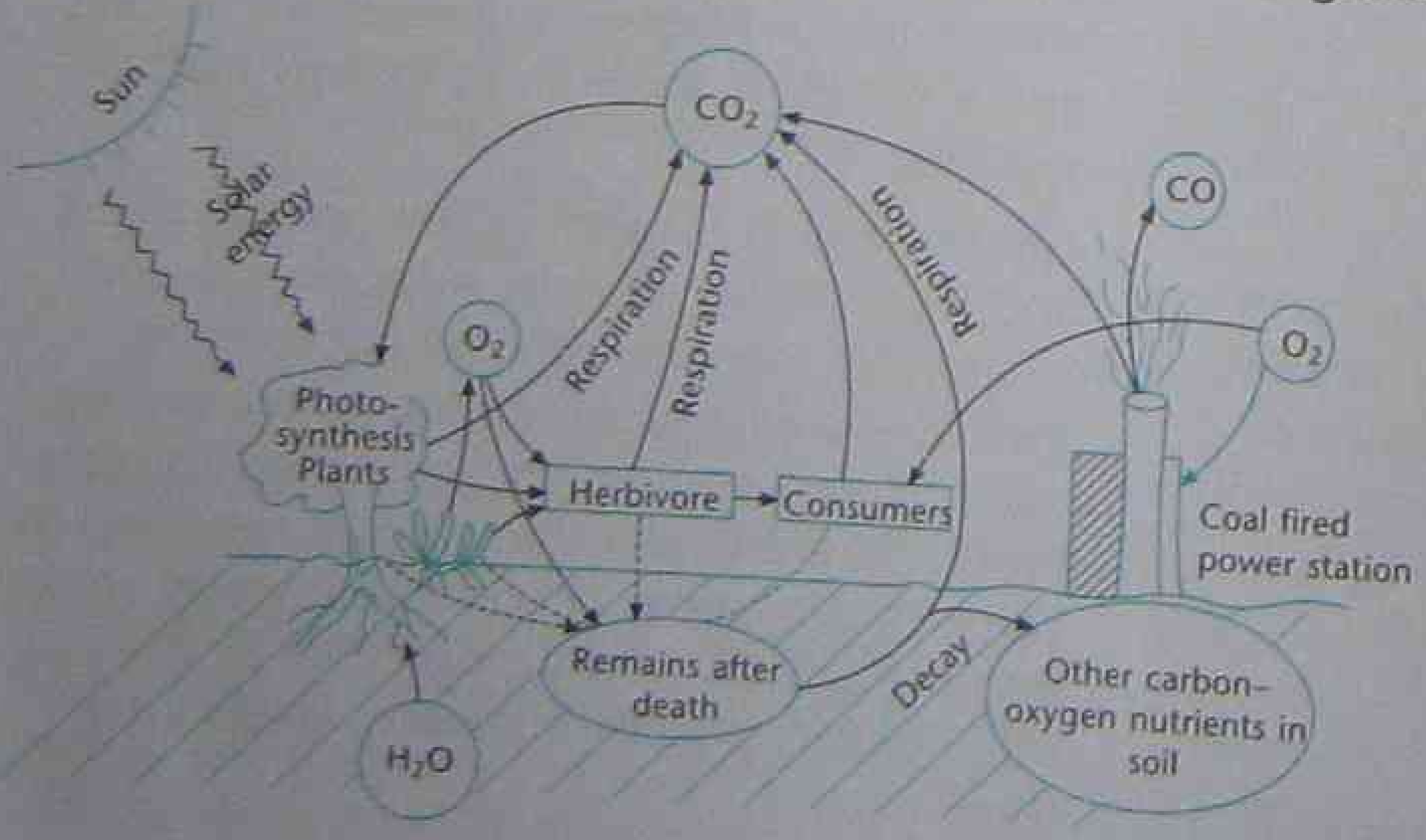


Figure 5.3 Carbon–oxygen cycle

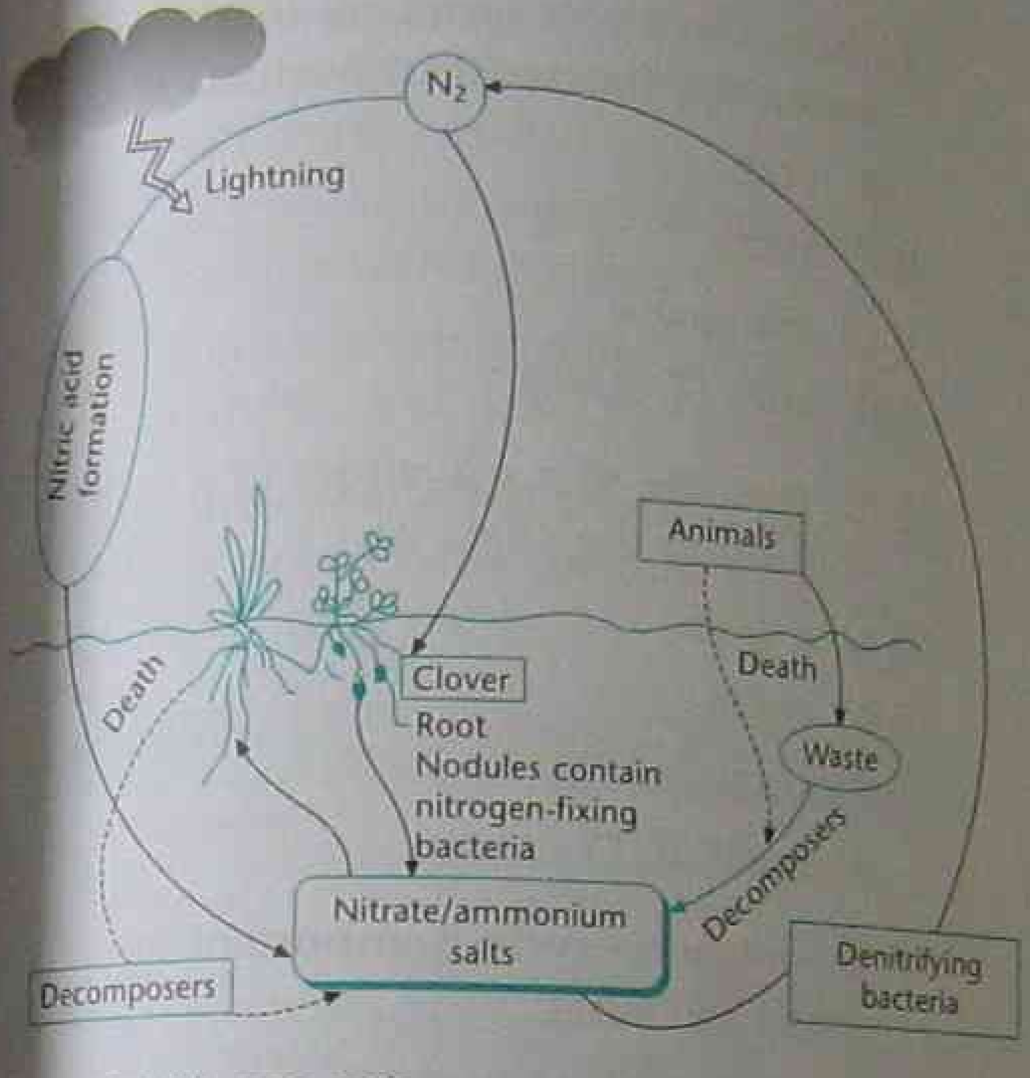


Figure 5.4 Nitrogen cycle

- **Acid rain.** Acidic gases (such as sulfur dioxide) from factories and metal smelters cause damage to ecosystems by the formation of acid rain. When acidic gases dissolve in raindrops, acid rain is formed. Acid rain can leach valuable minerals from soils and so plants will not grow. Acid rain damages tree leaves so that they cannot photosynthesise. Lakes and rivers can become so acidified that fish eggs will not hatch.
- **Algal blooms.** The use of fertilisers in farming can be detrimental to the surrounding environment. Rain can wash fertiliser into rivers and creeks. These nutrients cause algae to grow excessively, leading to an algal bloom. The algal blooms upset the normal balance in this ecosystem. When these algae die the decomposition of their bodies uses up the dissolved oxygen in the water and this leads to the death of other aquatic organisms.
- **Introduced animals and plants.** The introduction of non-native plants and animals can have a devastating impact on local ecosystems. The introduction of rabbits to Australia in the early 1800s led to widespread problems. Rabbits reproduced so quickly that their large numbers stripped the land bare of grass and so deprived sheep and native animals of food. The introduction of special viruses into wild rabbit populations has been the only way to control them.

  The introduction of the prickly pear cactus as a fodder plant also caused a similar problem. As the prickly pear had no natural predators it quickly proliferated and took over valuable land and made it unsuitable for farming. The plant was eventually controlled by the release of a moth that laid its eggs on the cactus. The caterpillars that hatched from the eggs ate its flesh and so controlled the numbers of cacti.
- **Loss of habitat.** The removal of trees by farmers has led to a loss of habitat for many communities. Some living things have become endangered or extinct because of these human activities.
- **Salination.** In many parts of rural Australia, mineral salts are rising to the surface as trees are removed by farmers. The salts rise with the rising water table. Salt kills plants and so renders the land useless for farming.
- **Farming and urban development.** Land has to be cleared so that farms and towns can exist. Many forests have been lost due to urbanisation.

Conservation of the environment

Conservation of the environment means the maintenance of our resources for future generations. Non-renewable resources, for example, need to be recycled wherever possible. We can conserve our environment in the following ways:

- **Maintenance of habitat.** It is important to preserve places for animals and plants to live. National parks are one way that this can be achieved. Without these wilderness habitats, organisms will not survive into the future.
- **Recycling.** Metals are non-renewable resources. They need to be recycled. This is also cheaper than mining new metals since less energy is involved in recycling than in mining and extraction. Recycling of plastics also reduces the need to process more oil.
- **Reducing consumption and waste.** Water is a scarce resource in Australia. It is important not to waste it. Governments and water authorities place restrictions on consumers to help reduce consumption and waste. Energy usage can also be reduced by using more energy-efficient machines.
- **Restoration.** Places can be returned to a state where plants and animals can live again. This is an expensive, long-term process. The Parramatta River in Sydney has been cleaned up and now fish are returning to live in the upper reaches of the river.
- **Biodegradable waste.** Scientists are developing biodegradable plastics that will decompose in garbage dumps. Existing plastics are non-biodegradable and contaminate the environment.

Test yourself (answers on pages 225–7)

Part A. Knowledge (answers on pages 225–6)

1 An example of an abiotic feature of the environment is:

a predation.
b competition for shelter.
[✓ c] wind speed.
d competition for food. (1 mark)

2 An example of a producer in a food web along the coast of Australia is a:

a crab.
b shark.
[✓ c] mangrove.
d cow. (1 mark)

3 An example of a scavenger in a rock pool along the coast is:

[✓ a] a crab.
b algae.
c coral.
d shellfish. (1 mark)

4 The nitrogen cycle is important in nature because:

a nitrogen is important in the processes of respiration and photosynthesis.
[✓ b] nitrogen atoms are required for the production of proteins.
c bacteria cannot survive in the soil without nitrogen molecules.
d nitrogen is a source of energy. (1 mark)

5 Recycling of resources such as metals is important because:

a metals are renewable resources.
b metals will contaminate the environment if they are not recycled.
[✓ c] metals are not biodegradable.
d less energy is used in recycling compared with mining and extraction. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

a Loss of hab itat can lead to the extinction of living things.
b In a food chain a first order consumer is eaten by a Second order Carnivore.
c The living thing at the ~~top~~ end of a food chain is known as the top carnivore.
d A ~~virus~~ parasite is an organism that lives off its host without killing it.
e Decomposers include protista, moulds, and ~~[illegible]~~. fungi bacteria

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a conservation	f shark
b herbivore	g resources
c carnivore	h infrequent rainfall
d photosynthesis	i giraffe
e desert	j producer

8 Account for the differences in vegetation in scrublands compared to wet sclerophyll forests. (2 marks)

9 Explain how the water temperature and pH in a small creek could be monitored over a 48 hour period. (3 marks)

10 Discuss three ways in which humans can conserve our environment. (3 marks)

Part B. Skills (answers on pages 226–7)

1 Use the data in Table 5.2 (page 143) to determine the relative humidity of the air on each of the following measurements: (2 marks)

a Dry bulb = 27°C; wet bulb = 19°C
b Dry bulb = 22°C; difference in temperature between wet and dry bulb = 2°

2 10 litres of water was collected from a river. The water was filtered and then slowly evaporated until a dry residue remained in the final vessel. The following data were collected:

Mass of empty vessel = 205.456 g
Mass of vessel + residue = 209.341 g

a Explain why the water was filtered. (1 mark)
b Calculate the TDS (total dissolved solids) for the river water. (2 marks)
c Suggest a reason why the TDS is relatively high. (1 mark)

3 Construct a food chain from each following set of living things. (3 marks)

Set A: leaf-eating insect; lizard; eucalyptus leaf; snake
Set B: shark; phytoplankton; small fish; zooplankton
Set C: grasshopper; hawk; goanna; grass

4 Construct a food web from the following information. (4 marks)

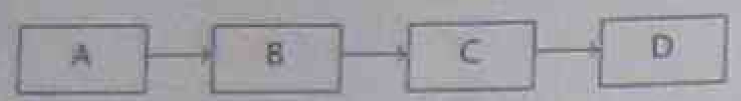
In a marine rock pool there are microscopic phytoplankton and macroscopic seaweeds that use solar energy to make food. Phytoplankton are a food source for microscopic zooplankton as well as sponges. Zooplankton are also a food for sponges. The seaweeds are eaten by a wide variety of herbivorous molluscs. These molluscs in turn are eaten by carnivorous molluscs, sea anemones and starfish.

5 Construct a food web from the following information: (2 marks)

A typical Australian paddock contains a wide variety of grasses and trees. The leaves of these plants are a food source for cattle, rabbits and grazing marsupials. The leaves are also eaten by various insects such as caterpillars and snails. The roots of these plants may also be eaten by rats and mice. The flowers of the grasses and trees produce nectar that is a food source for bees, moths and flies. The fruits and seeds that form from the flowers are a food source for ants, bats, birds and seed-eating insects. Insectivorous birds eat bees, insects. Insectivorous birds eat bees, moths, ants, flies, caterpillars and snails.

Eagles eat rabbits and insectivorous birds, as well as seed- and fruit-eating birds.

6 Consider the following food chain in a river estuary. Letter codes are used.



a Which letter(s) correspond to living things that could be classified as carnivores? (1 mark)

b Assume 200 MJ of energy is transferred to B from A. If 90% of the energy is lost as heat to the environment at each remaining stage, determine the energy received by D. (2 marks)

c A scientist investigates the levels of heavy metals in the tissues of A, B, C and D. The results are listed below:

Organism	A	B	C	D
Heavy metals (ppm)*	0.013	12	125	865

(*ppm = parts per million)

Explain the trend in the data. (2 marks)

7 The following observations were made by a biologist as she studied the populations of plants, water slugs and frogs in a swamp.

In the time of the study the population of:

- plants increased by 20%
- water slugs decreased by 50%
- frogs increased by 140%.

State whether each of the following statements is supported by the biologist's observations.

a A large number of tadpoles metamorphosed into frogs. (1 mark)

b Tadpoles consume large numbers of water slugs. (1 mark)

c The frogs consumed large numbers of plants. (1 mark)

d The decrease in water slugs allows the number of plants to increase. (1 mark)

8 The following table provides information on the level of dissolved oxygen at various depths in water in a lake.

Depth (m)	0	2	4	6	8	10	12
Dissolved oxygen (ppm)	10.0	9.8	9.4	5.2	2.2	1.5	12

A redfish requires 5.0 ppm of oxygen in water for it to survive for 1 day.

a Explain why redfish are never found in the deep water of the lake. (2 marks)

b Plot a line graph of these data and determine the dissolved oxygen level at 9 m. (3 marks)

Energy resources and pollution

Throughout human history various natural resources have been used to provide energy. The Sun is essential to life because it provides the energy to power photosynthesis, as well as the various material cycles. Over thousands of years, humans have burnt wood, coal, oil and gas to provide heat and energy for their personal needs and to power their machines. Humans are developing new technologies to harness solar energy, wind energy, geothermal energy and tidal energy.

Glossary

Acid rain—rain that has a pH less than normal rainwater (<pH 5.5); caused by the presence of acidic pollutants in the atmosphere

Fossil fuel—fuel derived from the fossilised remains of once-living things (eg. gas, petroleum, coal)

Incomplete combustion—combustion reactions in which there is insufficient oxygen present to completely burn the fuel into carbon dioxide. Less energy is also produced. Carbon (soot) and carbon monoxide often result

Renewable energy sources—energy sources that will not become depleted over time and which can be readily replaced

Energy as a resource

Energy can be stored in many ways. Stored energy is called potential energy. Stored energy is an important resource for humans.

The different forms of potential energy include:

- **Chemical energy**. Chemical substances such as **fuels** (petrol, diesel, gas, coal) and food (carbohydrates, fats, proteins) store energy in the chemical bonds that hold atoms together. This energy can be released by combustion reactions. Batteries also store chemical potential energy. Their energy can be released when the battery is connected into an electric circuit.
- **Nuclear energy.** The subatomic particles of the nucleus are held together by strong nuclear forces. This stored energy can be released by splitting heavy nuclei (eg. uranium fission) or by combining light nuclei (eg. hydrogen fusion). Nuclear power stations can fairly safely convert the nuclear energy in heavy nuclei into usable forms of energy.
- **Gravitational potential energy.** Massive bodies that are held above Earth's surface store gravitational potential energy. Thus a large reservoir of water above a hydroelectric power station stores gravitational potential energy.

Stored energy must be converted into other energy forms to allow the energy to be used by humans. Thus the chemical energy in fuels is often converted to heat energy by burning the fuels. The heat energy can be used to warm our homes, to cook food or to make electricity. The chemical reactions in a battery produce electrical energy which is used to power appliances. Nuclear energy is converted to heat in nuclear reactors. This heat energy is used to boil water and produce steam to drive turbines and produce electricity. In a hydroelectricity power station, the gravitational potential energy of the water is transformed into kinetic energy as the water falls down. This kinetic energy is used to drive turbines to produce electricity.

Some forms of energy cannot be stored, but they are an important resource. Here are some examples:

- **Wind energy**—Moving air contains kinetic energy. It can be used to drive windmills and wind turbines.
- **Wave energy**—Moving water also contains kinetic energy. Wave energy is used in some locations to generate electrical energy using special turbines.
- **Solar energy**—The energy of the Sun's electromagnetic waves can be trapped by solar collectors and transformed into heat energy or electrical energy. Some of this energy can be temporarily stored in rechargeable batteries.
- **Geothermal energy**—The heat from hot rocks inside Earth can be used to boil water and generate electricity.

Economically important natural energy resources

Over 80% of our energy in western society is derived from the combustion of non-renewable resources such as coal, oil and gas. Coal, oil and gas are called **fossil fuels**. Table 5.3 shows the relative proportions of Australia's non-renewable fuel resources. Our known coal reserves are so vast that they are expected to last for 1000 years at the current rate of use. Uranium is exported to Europe and the USA for use as a fuel in nuclear reactors.

Table 5.3 Non-renewable fuel resources

Non-renewable fuel	%
Black coal	57
Brown coal	24
Uranium	16
Oil and natural gas	3

Although black coal is our most abundant non-renewable energy resource, it is oil and its refined products (eg. petrol, kerosene) that represent the greatest proportion of the total energy used in Australia. Table 5.4 shows the relative contributions of various energy resources used in Australia. These proportions have changed over the last century.

Table 5.4 Percentage of total energy sources used in Australia

Energy resource	% used
Oil & oil products	37
Black coal	29
Natural gas	15
Brown coal	12
Other*	7

* includes wood and waste fibre/hydroelectricity/solar

Coal

Coal is a fossil fuel that formed from the remains of swamp plants. In NSW the black coal was formed about 250 million years ago. The coal seams in Victoria are composed of brown coal (lignite) which is geologically much younger. There are four ranks of coal. Each rank represents a stage in the fossilisation process in which plant material is heated and pressurised. This process destroys the cells and chemical compounds. Volatile matter and moisture are gradually removed.

Plant remains → peat (55% C) → brown coal (67% C) → black coal (85% C) → anthracite (93% C)

Black coal is burnt in power stations in NSW to generate electricity. When black coal is burnt, most of the coal's stored energy is lost to the environment. Only about 35% of the original energy is converted to electrical energy. About 45% of the original energy is absorbed by the cooling water that prevents the turbines overheating. The remainder is lost as heat up the chimneys.

Oil and gas

The decay and fossilisation (over millions of years at high pressures and temperatures) of marine organisms such as algae and plankton lead to the formation of oil and natural gas. As the complex soup of oil and gas formed, it migrated upward from the sediment until it was trapped by impervious rock layers to form an oil and gas deposit. The natural gas is mainly composed of methane (CH_4). The oil is mined and converted to a wide range of products such as petrol and kerosene by fractional distillation.

Replacements for non-renewable fuels

The increasing world population and the high energy usage of westernised society has placed considerable demands on our non-renewable energy resources. Over the next century, humans will need to conserve fossil fuels. This can be achieved in a number of ways.

- Reduce wastage and develop new technologies to extract the maximum amount of energy from these fuels.
- Develop renewable energy technologies such as wind, wave, tidal and solar power.
- Production of renewable liquid fuels such as ethanol from plants. Countries such as Brazil have used ethanol as a major vehicle fuel for many decades.

Pollution

The combustion of fossil fuels leads to the production of pollutants.

Incomplete combustion

When hydrocarbon fuels are burnt in a restricted air (ie. oxygen) supply, a mixture of products including carbon monoxide, carbon dioxide, carbon (soot) and water are formed.

hydrocarbon + air → carbon monoxide + carbon dioxide + carbon + water

Carbon dioxide production will accelerate global warming (see next column). Carbon monoxide is a dangerous pollutant because it can lead to suffocation and death. Carbon monoxide is absorbed by red blood cells in preference to oxygen. Soot particles may be deposited in the environment, leading to the blackening of buildings and trees. Soot particles may also contain dangerous cancer-causing agents (carcinogens).

Release of acidic oxides

Fossil fuels often contain sulfur impurities. When these fuels are burnt, the sulfur is also burnt to form sulfur dioxide gas. In the atmosphere, the sulfur dioxide can also be oxidised to sulfur trioxide. These oxides of sulfur are acidic oxides. They combine with moisture in the air to form droplets of acid rain. (See page 147.)

sulfur + oxygen → sulfur dioxide (SO_2)

sulfur dioxide + oxygen → sulfur trioxide (SO_3)

sulfur dioxide + water → sulfurous acid (H_2SO_3)

sulfur trioxide + water → sulfuric acid (H_2SO_4)

Acidic oxides of nitrogen can also be formed in the engines of cars and trucks. These oxides are emitted in the exhaust and contaminate the environment. They also contribute to acid rain.

nitrogen + oxygen → nitrogen monoxide (NO)

nitrogen monoxide + oxygen → nitrogen dioxide (NO_2)

nitrogen dioxide + water → nitrous acid + nitric acid (HNO_2, HNO_3)

The greenhouse effect

Gases such as carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and water vapour (H_2O) are very important in our atmosphere. They are responsible for trapping solar radiation and warming our atmosphere. Without these gases Earth would be very cold, like the planet Mars.

Since the Industrial Revolution, the amount of carbon dioxide and methane has been increasing. Today there is 25% more carbon dioxide in the atmosphere than 200 years ago. In this same time the amount of methane in the atmosphere has doubled. Nitrous oxide has increased because of the large increase in the use of fertilisers and the burning of organic matter. This has led to a slight rise in global warming. This is

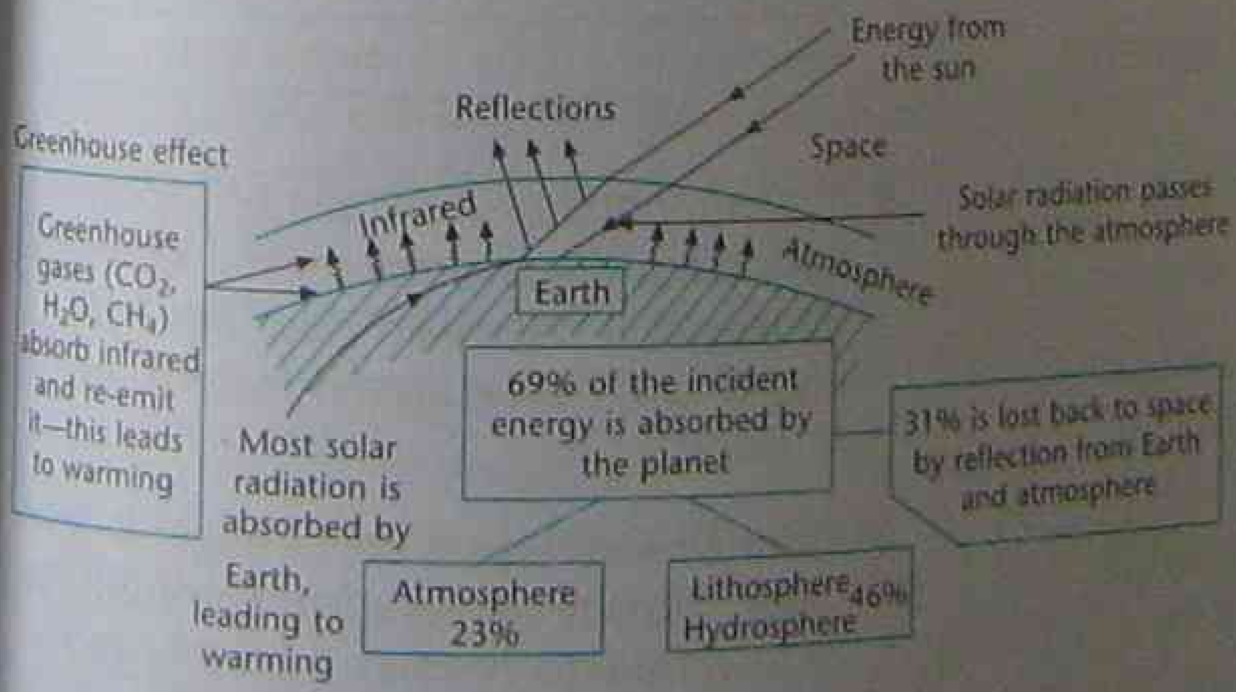


Figure 5.5 Greenhouse effect

called the **enhanced greenhouse effect** because the atmosphere is trapping heat just like the glass of a gardener's greenhouse traps heat to keep plants alive in cold climates. Even though both processes cause warming, the mechanisms of the two processes are quite different.

Test yourself (answers on pages 227–8)

Part A. Knowledge (answers on page 227)

1 Which of the following is a renewable fuel?

- ~~a~~ Wood
- b Coal
- c Oil
- [✓ d] Uranium (1 mark)

2 Batteries store:

- a electrical energy.
- b nuclear energy.
- [✓ c] chemical energy.
- d gravitational potential energy. (1 mark)

3 In a hydroelectric power station the sequence of energy transformations is:

- a heat energy → electrical energy.
- ~~b~~ gravitational potential energy → kinetic energy → electrical energy.
- [✓ c] gravitational potential energy → kinetic energy → heat energy → electrical energy.
- d solar energy → kinetic energy → electrical energy. (1 mark)

4 The fossil fuel that is the major source of energy for the production of electricity in Australia is:

- a oil.
- b gas.
- c uranium.
- [✓ d] coal. (1 mark)

5 The energy resource that has the greatest use in Australia is:

- [✓ a] brown coal.
- b natural gas.
- ~~c~~ electricity.
- ~~d~~ oil and oil products. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

- a The black coal in NSW was formed from _____ plants about 250 million years ago.
- b Brown coal is also called _____.
- c The rank of coal that has the highest percentage of _____ is called anthracite.
- d Carbon monoxide is a dangerous pollutant gas as it can cause _____ and death.
- e Sulfur dioxide is a gaseous pollutant that can lead to the formation of _____ rain.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a nitrogen dioxide	f geothermal
b global warming	g carbon dioxide
c carbon	h acidic oxide
d hot rocks	i soot
e waves	j kinetic energy

8 In some parts of Australia hot rocks have been discovered several kilometres underground. Explain how this heat energy can be extracted for our use. (2 marks)

9 Write a word equation to explain how sulfur trioxide can lead to the formation of acid rain. (2 marks)

10 Why has the amount of carbon dioxide in the atmosphere increased significantly in the last 200 years? (2 marks)

Part B. Skills (answers on pages 227–8)

1 The following table shows the percentage of total energy sources used in Australia.

Energy resource	% used
Oil and oil products	37
Black coal	29
Natural gas	15
Brown coal	12
Wood/plant fibre	5
Hydroelectricity/solar	2

- a What percentage of energy usage is derived from coal? (1 mark)
- b What percentage of energy is derived from renewable sources? (2 marks)
- c Construct a pie graph of these data. (3 marks)

2 The following data represent the energy consumption per person per day in three countries:

Country	Energy usage (MJ/person/day)
USA	970
Australia	500
India	35

- a Calculate the yearly energy usage of an Australian family of four people. (2 marks)
- b Explain the differences in energy usage of these three countries. (2 marks)

3 In a technological society such as the USA the energy usage per person per day can be broken down into contributions from various sectors. For example, agriculture and industry require energy to provide products and services to each person. The following table shows these contributing sectors.

Sector	Agriculture/ industry	Homes/ offices	Transport	Food consumption
Energy usage (MJ/ person/ day)	380	280	265	45

- a What percentage of energy usage is required for transportation? (1 mark)
- b Which of these sectors is strongly reliant on energy derived from the combustion of fuels derived from oil? (1 mark)
- c Plot these tabulated data as a bar graph. (3 marks)

4 Two black powders (A and B) were known to be samples of black coal and anthracite but the labels were missing. A scientist decided to determine the identity of the samples by burning them in air and using the heat released to heat samples of water. The initial and final temperatures of each sample were measured. The results are tabulated below.

	Sample A	Sample B
Mass of sample burnt (g)	1.0	1.0
Mass of water heated by burning sample (g)	500	500
Initial temperature of water (°C)	21.0	24.5
Final temperature of water (°C)	35.2	36.2

- a Which sample is black coal and which is anthracite? Justify your answer. (2 marks)
- b Suggest one way of improving the design of the experiment. (1 mark)

5 Figure 5.6 shows a model of the combustion of hexane vapour in oxygen.

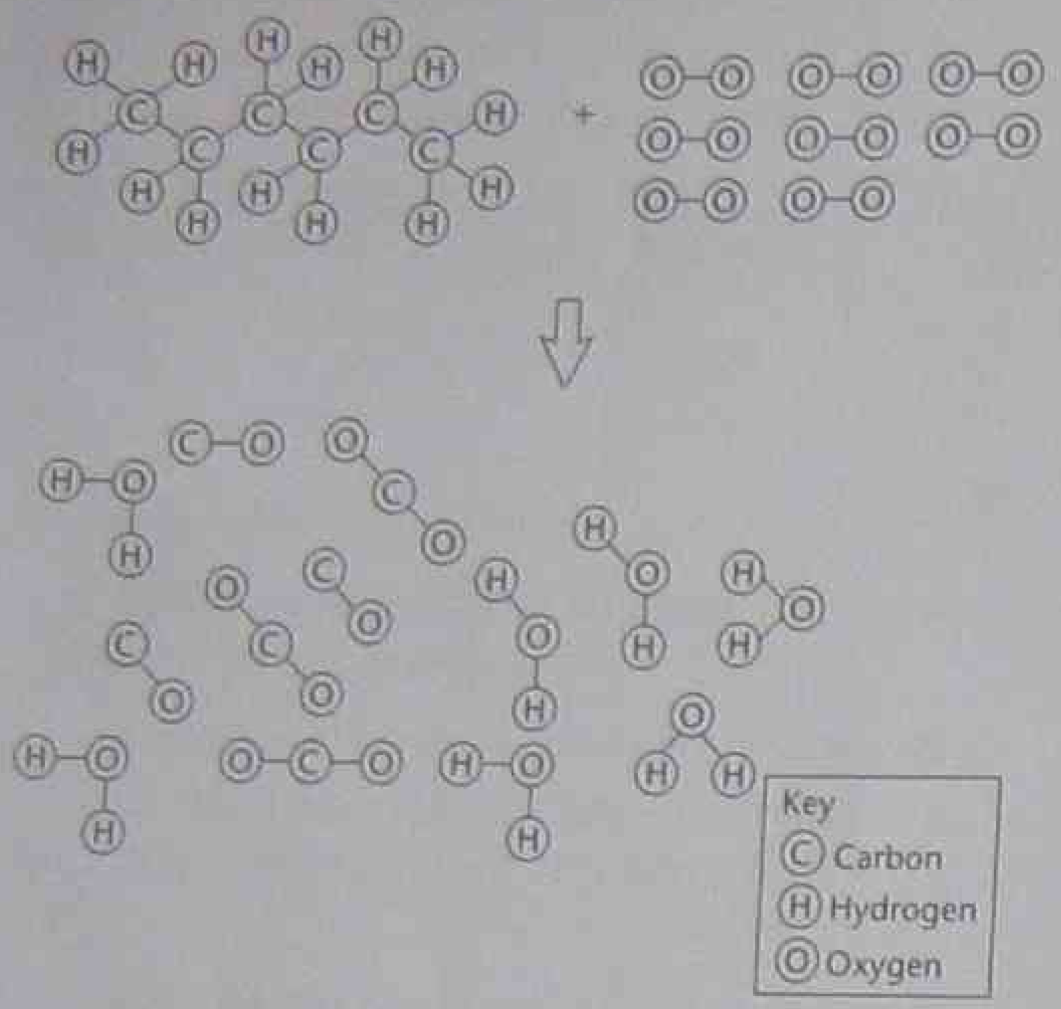


Figure 5.6 Model of hexane combustion in oxygen

a Use the model to determine whether the combustion is complete or incomplete. (1 mark)

b Write a word equation for the combustion. (2 marks)

c Write the chemical formula for hexane. (1 mark)

6 Figure 5.7 shows the concentration of carbon dioxide in the atmosphere over time.

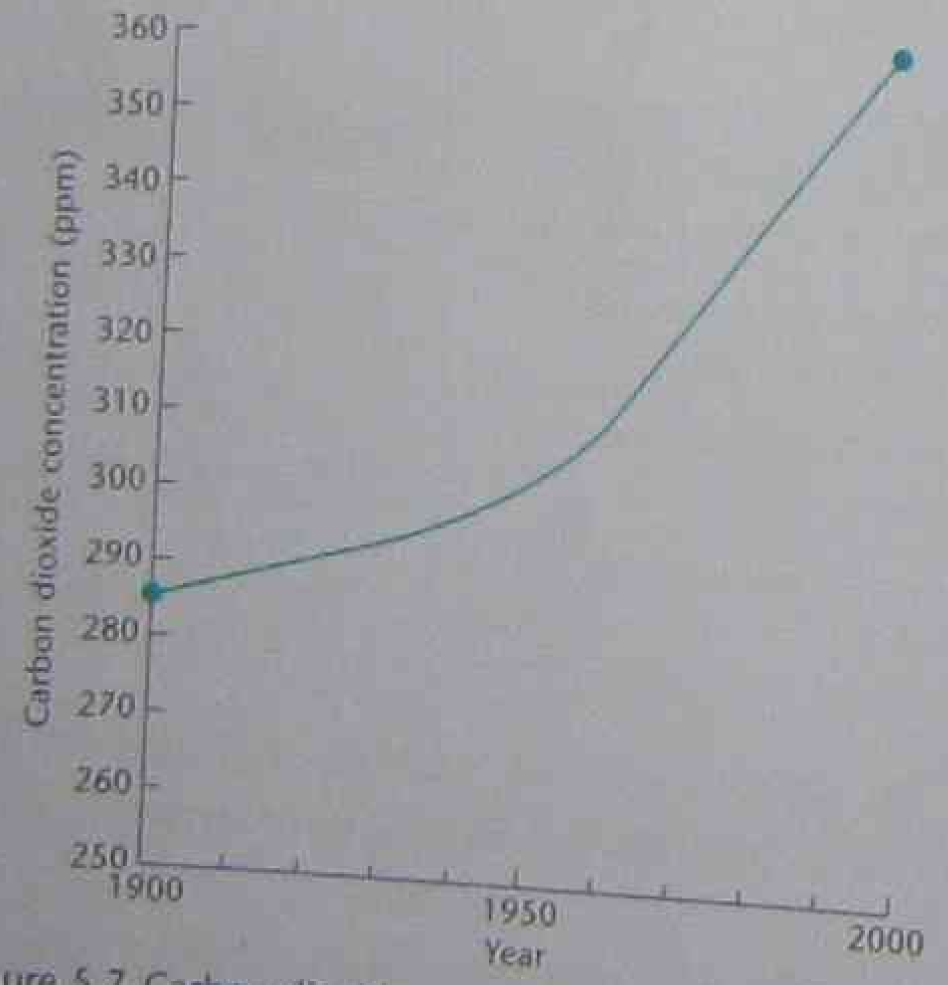


Figure 5.7 Carbon dioxide concentration in the atmosphere over time

a Describe the trend in this graph. (1 mark)

b What is the percentage increase in carbon dioxide in the atmosphere over the hundred years of the 20th century? (2 marks)

7 Transportation is responsible for significant carbon dioxide emissions in Australia. The following table shows the percentage contribution of various transport sectors to these carbon dioxide emissions.

Transportation sector	Percentage of CO_2 emissions
Aeroplanes	12
Rail transport	5
Cars	58
Buses and trucks	19
Shipping	6

a The data in this table have been tabulated poorly. Redraw the table so that the data are provided in descending order. (2 marks)

b Use your table to plot a divided bar graph of contributions of each sector to the carbon dioxide emissions. (3 marks)

8 A steam engine uses black coal as a fuel. A scientist measures the heat energy input per hour from the burning coal and the mechanical energy output per hour of the engine. The results (in megajoules, MJ) are:

- heat energy input = 15 MJ
- mechanical energy output = 3 MJ

a Why is the energy output less than the energy input? (1 mark)

b Calculate the percentage efficiency of the steam engine. (2 marks)

Technology

Australians live in a technological age in which new machines, devices and materials have been and continue to be created to make our lives easier and more enjoyable. Technology requires the use of large amounts of energy and raw materials. It can also lead to pollution unless properly regulated.

Glossary

Cyclotron—a device used to accelerate ions to high velocity; used in research and in the production of radioisotopes

Microwaves—part of the electromagnetic spectrum between short radio waves and infrared waves

Modulation—the process of altering a wave to carry information; an audio or video signal is combined with a carrier wave so that the wave form varies in its amplitude or frequency

Communications technology and electromagnetic radiation

On page 4, we learnt about electromagnetic waves and some of their uses. Re-read that page now before looking at the following examples.

Radio and TV waves

Radio waves are produced when electrons are made to oscillate in a wire. Radio stations have tall aerials from which these waves radiate. Radio waves travel at the speed of light. Prior to transmission these waves are modulated (altered) by combining a carrier wave (specific to each station) and an audio signal. They are generated in two bands—the AM band and the FM band. The lower the frequency the greater is the distance that the wave can be transmitted. Very low frequency waves can be bounced off the ionosphere and transmitted around Earth.

- **AM band (about 500–1600 kHz frequency).** The radio waves transmitted on the AM band are **amplitude modulated.** This means that their amplitudes vary with time. This variation in amplitude encodes information that is transmitted from the transmitter to the receiver. The information is demodulated at the receiver. The typical wavelength of these waves is 600–200 m. These wavelengths fall into the low to medium radio frequency (LF–MF) bands.
- **FM band (about 85–110 MHz frequency).** These waves are **frequency modulated.** The quality of transmission by FM is superior to AM. The typical wavelengths of these waves are 4 – 0.3 m. These wavelengths fall into the very high frequency (VHF) bands.

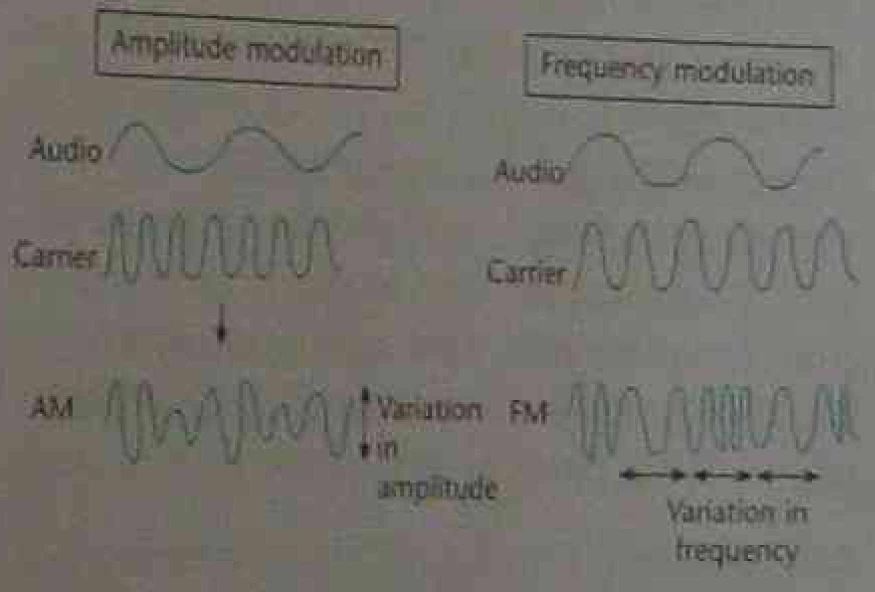


Figure 5.8 Modulation of waves

Television communication involves the transmission of both audio and video signals. The very high frequency (VHF: 300–30 MHz)) and ultrahigh frequency (UHF: 3000–300 MHz) bands are used in TV transmission. These high frequency waves suffer less interference than lower frequency waves but they are limited to shorter transmission distances (<100 km). Repeater stations are needed to relay the signal.

- The video transmission is an AM signal. It encodes colour and brightness information.
- The audio transmission is an FM signal.

- The TV set synchronises the video and sound signals.

Digital transmission

Modern communication (eg. telephone, computers, TV) involves conversion of the wave signal (known as an analogue signal) to digital format. The digital signal encodes information using the binary digits (bits) '1'(switch is on) and '0' (switch is off). Larger units called bytes are composed of a combination of 8 bits.

To convert an AM analogue signal to a digital signal the height of the wave is sampled and a '1' allocated to a non-zero amplitude and a '0' allocated to a zero amplitude. In FM conversion a '1' is allocated where the frequency changes and a '0' is allocated where the frequency does not change. A **modem** is a device that allows conversion between analogue and digital signals.

Digital information is stored in CD technologies. Digital information is encoded as a spiral track of 'pits' in the polycarbonate disc. The laser light reflects off the aluminium layer that coats the 'pits' or the 'land' disc surface.

Microwaves

Microwaves are electromagnetic waves with wavelengths in the range 10 mm – 0.1 mm. They are commonly used in radar, mobile phone and satellite communication. Microwaves travel in straight lines.

- **Satellite communication.** Microwaves can be relayed from repeater stations about 40 km apart or they can be transmitted to a **geostationary satellite** that then relays the signal (after amplification) back to a receiver on Earth. Thus a message can be sent from Sydney up to a satellite and then from the satellite down to a receiver ground station in Perth.
- **Radar** uses microwaves. Police use Doppler radar to determine the speed of vehicles. Radar towers at airports use pulses of microwaves to determine the distance of aeroplanes from the airport. The time between emission of the pulse and its return after reflection from the aeroplane is a measure of its distance.
- **Mobile phones** and mobile phone towers (base stations) transmit and receive signals in the 890–960 MHz range in Australia. This range of frequencies is classified as very short radio waves or long microwaves. A mobile phone transmits a signal to an antenna at the local base station, which then transfers the information to a computerised switching centre for relay to another cell network or a landline phone. A city is divided into cells so that switching can occur if the user moves from one location to another. There are 124 channels for transmitting and 124 channels for receiving. Each channel can handle 8 calls simultaneously. The signal from the mobile phone to the base station differs from the return signal by 45 MHz.

Technology in the nuclear industry

Nuclear energy and radioisotopes were discussed on pages 34–38. Re-read that section before you continue with the following additional examples.

Radioisotopes are radioactive forms of an element. Some radioactive isotopes exist naturally (eg. uranium-235, potassium-40), whereas others can be synthesised by nuclear reactions.

Radioisotopes can be produced by:

- bombarding non-radioactive elements with neutrons produced in a fission reactor;
- bombarding non-radioactive elements with ions that have been accelerated to high velocity in a cyclotron or linear accelerator.

The radioisotopes produced are used in many fields including medicine, industry, agriculture and scientific research.

Medicine

- **Diagnostic nuclear imaging:** In this technique **gamma**-emitting radioactive isotopes are attached to a pharmaceutical chemical and then given orally, or injected into, or inhaled by a patient. The radiopharmaceutical travels in the blood to a specific target organ where it concentrates. A gamma camera then scans the patient and obtains an image of the organ containing the radiopharmaceutical. An example of this technique is the drug dexetimide to which radioactive **iodine-123** (produced in a cyclotron) is attached. This radiopharmaceutical binds to certain parts of the brain so that epilepsy can be diagnosed and studied.
- **Therapeutic radioisotopes:** Cancerous growths can be attacked using radiotherapy techniques. **Iodine-131** (a reactor-produced radioisotope) can be used internally to detect and treat thyroid cancer. Iodine-131 emits beta particles that kill cancer cells. Radiotherapy can also be conducted externally. **Cobalt-60** produces gamma rays that can be used to kill cancerous tissue.

Industry

- **Gamma radiography:** Flaws in welded metallic joints (eg. in aircraft engines) can be detected by passing gamma rays from a radioisotope (eg. cobalt-60; iridium-192) through the material onto a photographic film.
- **Gauging:** Variations in the thickness of plastic film can be monitored by passing radiation emitted from a radioisotope (eg. beta rays from strontium-90) through the film as it passes at high speed through the detector.

Biotechnology

Advances in biology and medicine have come about by the development of new technologies. What is 'biotechnology'?

- Biotechnology – the use of biological discoveries to produce modified organisms and products as well as the development of industrial, agricultural or medical processes.

Some of these technologies are controversial. Here are some examples.

In-vitro fertilisation (IVF)

The IVF technique developed from infertility research in the 1960s. Researchers developed techniques to retrieve eggs and fertilise them under special conditions in the laboratory. The term 'in-vitro' literally means 'in glass'. Research continued to determine the best culture media and improve the egg maturation process. In Victoria a method was developed to produce multiple eggs by hormonally inducing the menstrual cycle. Fertilised eggs can then be implanted in the womb to allow the embryo to develop. Fertilised eggs and young embryos can be frozen and stored for later use.

Genetically modified (GM) foods

Biochemists have developed techniques to transfer genes from one organism to another. This is called gene splicing or genetic engineering.

Genetic engineering has been applied to agriculture to grow transgenic crops with new traits such as improved insect or virus resistance. There are now over 40 countries growing transgenic crops such as cotton, canola, corn, soybeans and sweet potatoes. There are both benefits and concerns associated with this technology. Here are some examples:

Benefits: increased crop yields; improved taste and quality of crops; better yields of meat in cattle; increased resistance to disease in livestock and crops

Concerns: transfer of antibiotic resistance to organisms in the environment; unknown effects on other organisms; development of new allergies in humans; objections by vegetarians to eating plant products with animal genes

Stem cell research

What are 'stem' cells?

- **Stem cells** are unspecialised cells that can be induced under certain experimental conditions to turn into specialised cells such as heart muscle cells or pancreatic cells.

There are two types of stem cells: embryonic stem cells and adult stem cells. Three- to five-day-old embryos contain a mass of stem cells that will eventually become specialised into the cells of each tissue and organ. Adult stem cells can be found in the brain, heart, bone marrow and nose.

In 1998, methods were discovered for isolating and growing embryonic stem cells from donated, unused embryos in IVF programs.

Research into the properties of embryonic stem cells has been influenced by ethical and religious concerns from various groups in society. Scientists want to continue researching both types of stem cells in the hope that they can develop ways of replacing damaged tissues and organs by the cloning of stem cells. It is hoped that diseases such as Parkinson's disease can be cured by replacing the damaged neurones in the brain. People with spinal cord damage may also be able to be cured. Stem cells could also be used to test new drugs.

New materials (made resources)

Chemists have developed a wide range of new materials over the last 100 years. Over the course of history, humans have used natural products such as wood and stone to construct homes and tools. The development of smelting techniques led to the production of metals and alloys. The discovery of oil deposits led to the petrochemical industry and the production of plastics. Plastics have replaced many wooden and metallic objects in homes and in industry. Here are some examples of new materials that are being produced and studied.

- **Semi-conducting organic molecules**—Chemists have developed rod-shaped organic molecules containing sulfur and fluorine atoms that behave like p-type and n-type silicon semi-conductors. These have the potential to be used in solid-state devices such as transistors. The use of semi-conducting plastics in electronic circuits on 'smart cards' is one proposed application.
- **New metal oxide composite polymers for dentistry**—A composite material made of silica and tantalum oxide nanoparticles in a liquid crystal matrix is being developed for use in restorative dentistry. These new materials combine the properties of ceramics and polymers and it is hoped that they will replace traditional restorative materials such as amalgams. Mechanical testing is currently being carried out.
- **Carbon nanotube fibres**—Carbon nanotubes are hollow cylindrical tubes made of carbon atoms. Their diameters are typically only a few nanometres. Chemists have developed techniques to construct long fibres of these nanotubes by binding them together into a cross-linked polymer matrix. These fibres are very much stronger than the existing carbon fibres used in tennis racquets and racing cars.
- **Super-elastic alloys**—A new type of alloy containing titanium, zirconium, vanadium, niobium, tantalum and oxygen has unusual elastic properties. They can be stretched up to 2.5 times their original length before springing back to their original size. The alloy expands very little on heating. They also have great tensile strength. They can be bent and straightened repeatedly without cracking. Their ability to resist denting and to be moulded without heating makes them very useful in the car industry.
- **Ceramic superconductors**—Some novel new metal oxide ceramics have superconducting properties at relatively high temperatures. They are oxides of mercury, thallium, barium, calcium and copper. The current record-holder has a critical temperature of −135°C. When cooled with liquid nitrogen it loses its electrical resistance and becomes superconducting below −135°C.

Superconductors have many uses including superconducting electromagnets used in high energy particle accelerators; superconducting wires in electrical generators; ultrahigh-performance filters for use in the electronics industry; magnetic resonance imaging (MRI) of the body; magnetically levitated trains.

Test yourself (answers on pages 228–9)

Part A. Knowledge (answers on pages 228–9)

1 Select the statement that is true of AM radio waves:
  - [✓ a] The AM band has frequencies in the range 80–100 MHz.
  - b Information is encoded by variations in frequency.
  - c AM radio waves have wavelengths of several hundred metres.
  - d The AM band is classified as very high frequency (VHF) radio waves. (1 mark)

2 Select the statement that is true about microwaves and communication:
  - a Microwaves can be relayed to geostationary satellites.
  - b Microwaves can travel thousands of kilometres around the surface of Earth.
  - c Microwaves travel at slower speeds than radiowaves.
  - [✓ d] Mobile phones use microwaves in the 200–250 MHz range. (1 mark)

3 The petrochemical industry is associated with the production of:
  - a computer chips.
  - b new dental materials.
  - [✓ c] plastics, fuels and solvents.
  - d nanofibres. (1 mark)

4 Select the statement that is true about the practical use of radioisotopes:
  - [✓ a] Gamma rays produced from cobalt-60 can be used to discover flaws or cracks in joints or fabricated alloys.
  - b Only radioisotopes with very short half-lives can be used in industrial gauging.
  - c Long-lived radioisotopes such as uranium-235 can be used to investigate diseases of the body.
  - d All therapeutic radioisotopes are produced in cyclotrons. (1 mark)

5 Select the statement that is true about biotechnology:
  - a Stem cell research involves the insertion of foreign genes into human chromosomes.
  - b GM foods are produced by irradiating food with gamma rays.
  - c In-vitro fertilisation refers to multiple births.
  - [✓ d] The genetic engineering of crops can produce varieties with increased resistance to disease. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

a Ceramic superconductors have been used in magnetic resonance _____ of the body.

b Biotechnology uses _____ discoveries to produce modified organisms and products.

c Imaging using radiation from _____ often avoids the need for invasive surgery.

d Superconducting magnets have been used in particle _____ .

e Pulses of microwaves are used by _____ towers at airports to determine the positions of aircraft.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part).

Column 1	Column 2
a Doppler radar	f GM foods
b digital signals	g analogue/digital converter
c frequency modulation	h VHF band
d modem	i bits
e genetic engineering	j vehicle speeds

8 Describe two examples of the importance of the development of made resources (new materials) in our society. (2 marks)

9 Distinguish between diagnostic and therapeutic radiotherapy. (2 marks)

Part B. Skills (answers on page 229)

1 The following table provides information about radiofrequency bands.

The wave equation can be used to calculate the frequency of a wave given its wavelength.

$v = 3 \times 10^8 = f \cdot \lambda$

Radiofrequency band	Frequency
Ultra high frequency (UHF)	3000 – 300 MHz
Very high frequency (VHF)	300 – 30 MHz
High frequency (HF)	30 – 3 MHz
Medium frequency (MF)	3 – 0.3 MHz
Low frequency (LF)	300 – 30 kHz

For each of the waves (X and Y) in Figure 5.9, determine their wavelength and their frequency band. (5 marks)

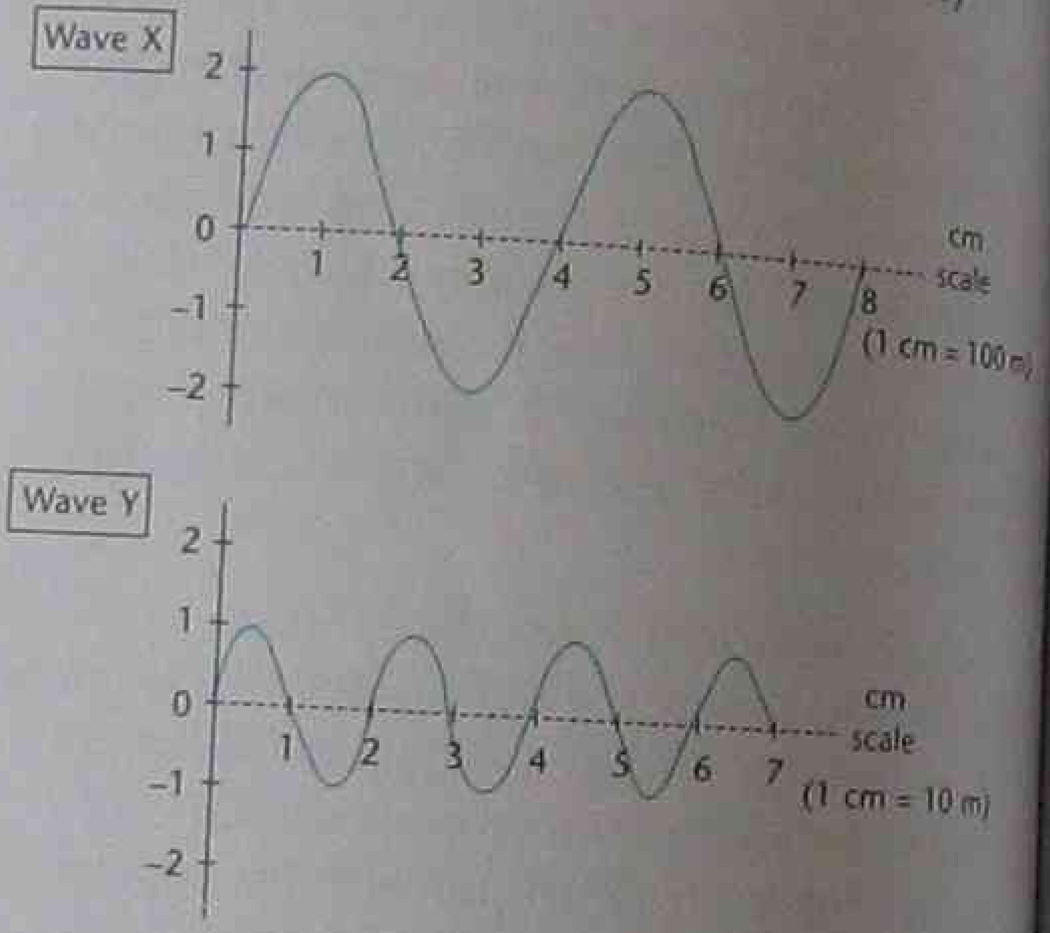


Figure 5.9 Radiowaves

2 Figure 5.10 shows two wave forms (X and Y) from two different regional radio stations.

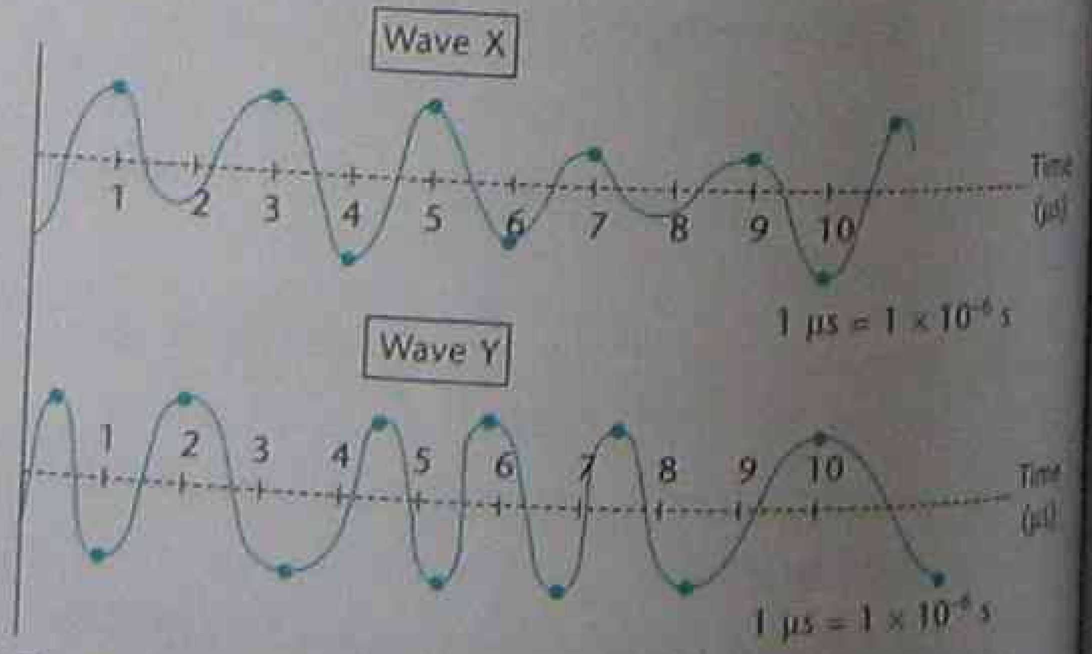


Figure 5.10 Waveforms (X and Y) from two different radio stations

a Which waveform corresponds to an AM station? (1 mark)

b What is the transmission frequency of the AM station, given the following equation that relates the period (*T*) of the wave and its frequency (*f*)? (2 marks)

$$f = \frac{1}{T}$$

3 Carbon-11 is a cyclotron-produced radioisotope that is used in a type of nuclear imaging known as positron emission tomography (PET). Emitted positrons collide with electrons, leading to the formation of gamma rays that are detected by a gamma camera.

Figure 5.11 shows a particle diagram of the production of carbon-11 from nitrogen-14. Use the diagram to identify:

a the particle that collides with the nitrogen atom (1 mark)

b the particle produced along with carbon-11. (1 mark)

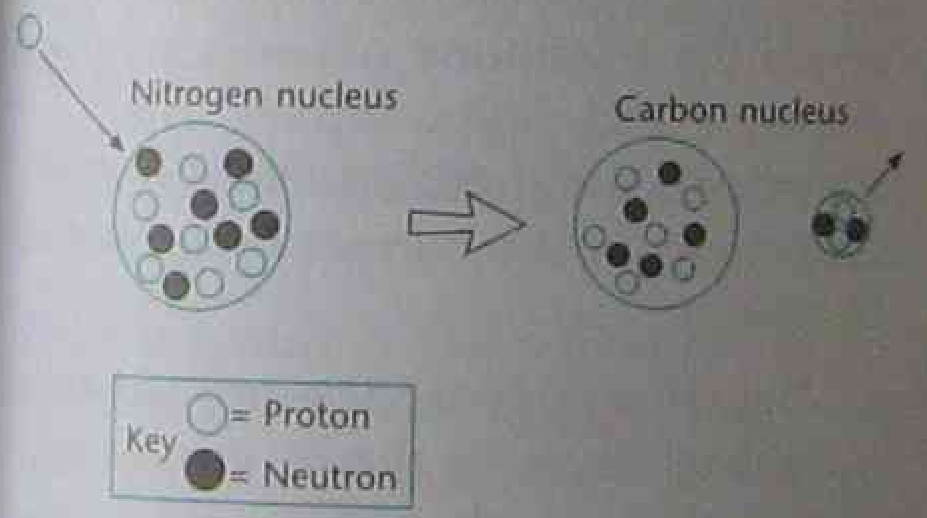


Figure 5.11 Particle diagram of the production of carbon-11

End-chapter test (answers on pages 229–30)

1 The table shows the world energy usage in the decade from 1981 to 1990.

Year	1981	1983	1985	1987	1990
Energy usage*	6.4	6.5	7.0	7.3	7.8

*($\times 10^9$ tonnes of oil equivalent per day)

a Plot a line graph of these data. (3 marks)

usage first exceed 7.5×10^9 tonnes of oil equivalent? (1 mark)

c Explain the meaning of the unit of energy used. (1 mark)

d Would it be valid to extrapolate this graph to determine the daily world energy usage this year? Explain. (1 mark)

2 Convert the following information into an energy pyramid. (3 marks)

A small family of humans is living on a small island which has such poor soil that they can only eat food obtained from the sea. They eat several varieties of fish that they catch in the lagoon. These fish feed on smaller fish. The small fish consume phytoplankton and algae.

3 Carbon monoxide is a toxic pollutant.

a State two common sources of this pollutant. (2 marks)

b Write the chemical formula of this compound. (1 mark)

c The table on page 164 compares the time of exposure to develop a range of symptoms of carbon monoxide poisoning. No symptoms are evident until they are classified as 'slight'. (2 marks)

i Will symptoms of carbon monoxide poisoning be evident after 30 minutes at the following carbon monoxide concentrations?

(A) 75 mg/m^3 (B) 150 mg/m^3

ii State one conclusion that can be made from these data.

d At carbon monoxide concentrations less than 15 mg/m^3, there are no symptoms. Despite this, health authorities recommend that the maximum exposure level over an 8 hour period be 11 mg/m^3. Explain why they have made this recommendation. (2 marks)

Carbon monoxide concentration (mg/cubic metre of air)	Symptoms	Exposure time to exhibit symptom (hours)
75	slight	0.3
	nausea/ headache	1.0
	severe	1.5
	death	2.6
150	slight	0.1
	nausea/ headache	0.4
	severe	0.9
	death	1.3

4 In large cities the roads often become highly congested during peak hours as people drive to work. The following information was collected to compare the energy needed per person to travel one kilometre by three different modes of transport.

Mode of transport	Energy required ($\times 10^6$) J
Car	1350
Bus	480
Train	390

What recommendation would you make to town planners in planning for future transport options in cities? (2 marks)

5 Hydrogen is often described as an energy resource for the future. When hydrogen burns in air it produces water and large amounts of energy.

a Explain why hydrogen can be considered an eco-friendly fuel. (1 mark)

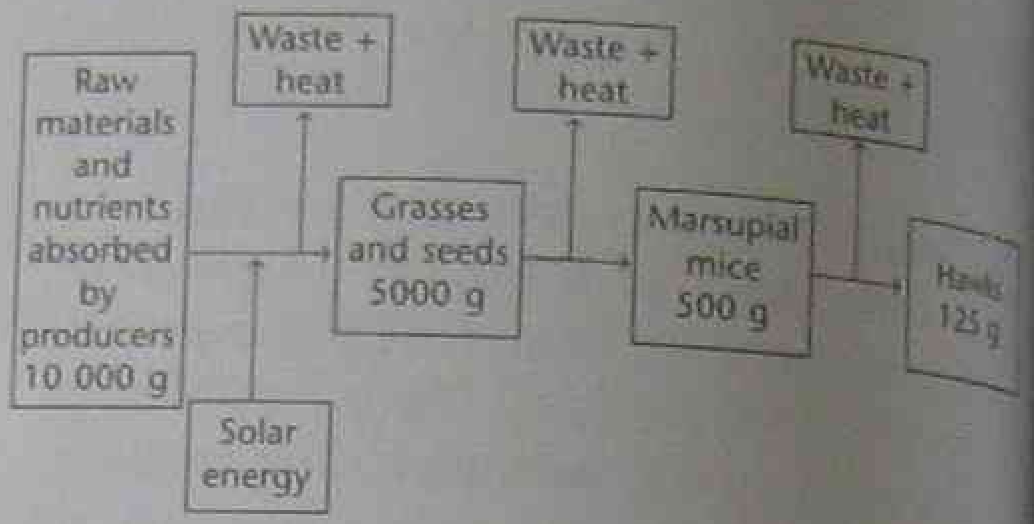
b Hydrogen can be made by electrolysing water. Oxygen is also produced in this process.

i Write a word equation for this reaction. (2 marks)

ii What use could be made of the oxygen produced? (1 mark)

iii Suggest an environmentally friendly source of electricity for the electrolysis. (1 mark)

6 Consider the following data for the food chain involving marsupial mice and hawks. The mass data are relative numbers.



a Name the first-order consumer in this chain. (1 mark)

b Explain why the mass of tissue decreases along the chain. (2 marks)

c At which stage does photosynthesis occur? (1 mark)

7 The following series of statements concern the transfer of information across the telephone network involving analogue and digital signals. The statements are jumbled. Place the statements in the correct order. (3 marks)

a Digital signals are transmitted along optical fibres to the receiver's telephone exchange.

b Electrical analogue signals are converted to sound waves in the speaker of the receiver's telephone.

c At the receiver's telephone exchange the digital signals are converted back to analogue signals.

d The microphone in the mouthpiece converts sound waves into analogue electrical signals.

e At the local exchange the analogue signals are converted to digital laser signals.

f Analogue signals travel to the receiver's telephone.

g Analogue signals are transmitted along copper wires to a local telephone exchange.

Summary

Ecosystems

- The local environment can be described in terms of abiotic and biotic features.
- The biotic components of an ecosystem involve interactions with living things.
- The abiotic components involve the non-living features of the environment.
- Living things in ecosystems can be classified according to their feeding relationships.
- Food chains and food webs represent ways in which energy flows from one organism to another.
- Materials such as water, carbon, oxygen and nitrogen move in cycles within an ecosystem.
- Human activities can upset the balance of nature.
- There are various ways to conserve our environment including recycling, maintenance of habitat and reducing waste.

Energy resources and pollution

- Energy can be stored in many ways including chemical energy, nuclear energy and gravitational potential energy.
- Coal, oil, natural gas and uranium are economically important energy resources.
- Developing energy resources include wind, wave, solar and geothermal energy.
- The combustion of fossil fuels leads to pollution of the environment.
- The enhanced greenhouse effect is caused by increases in gases such as carbon dioxide and methane in the atmosphere.

Technology

- Electromagnetic waves such as radio waves and microwaves can be used for communication.
- Information can be encoded in radio waves by amplitude or frequency modulation.
- Information can be transmitted as analogue waves or digital waves.
- The production and use of radioisotopes has led to new procedures to detect and cure diseases. In industry, radioisotopes can be used to monitor processes and to investigate flaws in materials.
- Biotechnology is the application of biological discoveries to industrial, agricultural and medical processes.
- Some biotechnologies are controversial.

Chapter 6

Investigations and Problem Solving

Planning investigations

The first step of a scientific investigation takes place when an observation is made. This might be an observation of an event such as a pine cone falling from a tree. It might be an observation of some feature of an animal such as the presence of webbing between the toes of a bird. These observations may then lead you to think about the cause or reason behind the observation.

Louis Pasteur wrote, 'In the field of observation, chance favours only the prepared mind'. This means that you will only discover something new if your mind is prepared to understand what you observe. This is done by reading and planning.

Glossary

Controlled variables—factors that are held constant during an investigation

Dependent variable—a factor that the investigator is observing or measuring in response to the independent variable

Generalisation—a statement that is true in the majority of cases

Hypothesis—a logical explanation of an observation or solution to an investigation that is made after all available evidence and information has been collected

Independent variable—a factor that the investigator selects to change or vary

Inference—a logical explanation of an individual observation

Observations—things that can be seen, heard, felt, smelt or measured using a device

Placebo—a substance that does not contain the active material to be tested

Observations and the scientific method

The scientific method is generally described as a series of steps in which an observation or problem is investigated. These steps are:

- making observations (using your senses or with the aid of instruments)
- making inferences
- reviewing the known facts using data sources
- making predictions
- planning, designing and experimenting to test predictions
- formulating an hypothesis based on an analysis of collected data
- testing and modifying the hypothesis by further investigations
- discussing collected results
- making conclusions
- planning for further investigations about the problem.

Identifying data sources

To prepare for an investigation of some problem or observation you have made you need to review what is currently known about this topic. This involves identifying:

- what information or data needs to be collected to help you understand the issues;
- the possible sources of this information or data – eg. library books, textbooks, dictionaries, encyclopedias, periodicals and multimedia resources including the internet;
- the **relevant** parts of the gathered information by doing some background reading, highlighting key ideas and summarising.

In this way you will develop an understanding of the problem to be investigated.

Planning first-hand investigations and fair testing

Some problems will be investigated by conducting research in the laboratory or in the field. Some problems will be investigated by second-hand data research.

Inferences

Following an observation, a scientist may make an **inference**. This is a logical explanation of the observation.

Predictions

The next step is to test the inference by making a **prediction** and performing an experiment to test the prediction. These experiments may support the inference or they may not. A new explanation may need to be sought.

Generalisations

As more supporting data are collected, the investigator may see a pattern emerging. At this point he may be able to make a **generalisation** about the topic under investigation.

Generalisations are statements that are true in the great majority of cases.

Example

Q Read each of the following statements and decide whether they are examples of observations, inferences, predictions or generalisations.

1. The crack in the glass window was caused by a sudden change in temperature.
2. Most compounds of copper are blue.
3. The flight feathers of the bird were a different colour to the tail feathers.
4. If a more concentrated acid is added to zinc, the zinc will dissolve faster.

A 1. **Inference**—the observed crack could have been caused by many things (eg. a stone, ball, etc.). A change in temperature is just one possibility.

2. **Generalisation**—examination of a wide variety of copper compounds shows that most of them are blue and so this is a useful generalisation.

3. **Observation**—the colour of the different feathers can be observed.

4. **Prediction**—the statement suggests that the student has already tried the dilute acid and is now predicting that a more concentrated solution will react even faster.

Hypotheses

The next step in the scientific method is to generate an hypothesis. What is an hypothesis? An explanation of the observation(s) or solution to a problem being investigated.

An hypothesis is *not* a wild guess.

Hypotheses are temporary statements that lead towards the development of scientific theories.

Hypotheses are only made after looking at *all* the available evidence and information that has been collected. This is why background reading of identified data sources is so important. The work of others may help you to produce a good hypothesis. Hypotheses are the result of a scientist making inferences, making predictions and developing generalisations.

The hypothesis must be able to:

- predict the results of future investigation about the problem;
- suggest ways in which it can be tested to determine whether it is true or not.

Hypotheses are not always correct. If the data collected during the investigation do not support the hypothesis, a new hypothesis must be developed.

Example

Q Read each of the following statements and decide whether each statement is an hypothesis. Justify your answer.

1. Snails prefer to live in moist, dark places rather than dry, lit places.
2. Neanderthals were exclusively cave dwellers but the ancestors of modern humans were not.
3. Jazz music is more pleasant to the ear than rock and roll.

A 1. This is an hypothesis. It is suggesting an answer to a problem and it can be experimentally tested. It predicts the outcome of the research.

2. This is NOT an hypothesis. It does suggest an answer to a problem. It cannot, however, be tested directly as there are no living Neanderthals or ancestors of modern humans. The finding of Neanderthal bones in caves does not provide sufficient evidence to prove the hypothesis.

3. This is NOT an hypothesis. It involves personal opinions and this will vary from one group to another.

Theories

A *scientific theory* is the result of very extensive research that supports an hypothesis, or a group of related hypotheses. Over a longer period of time the theory may become so universally accepted that it becomes known as a scientific **law**. Sometimes old theories are replaced by new theories. The **law of mass–energy conservation** is an example of an old theory (the law of energy conservation) being replaced by a newer law when Albert Einstein showed the equivalence of mass and energy. No experiments have ever been able to disprove the conservation of mass–energy theory and so it has been elevated to the status of a law of science. The **theory of evolution** may become a law in the future. This will depend on the results of ongoing research.

Fair testing and variables

Many factors can influence the results of an experiment. These factors are called **variables.** To investigate a problem experimentally all the variables must be controlled so that only the variable being investigated can affect the result. This is known as **fair testing.**

- **Experimental results are only reliable if the tests are fair.**

During an investigation the experimenter tests what happens when he alters one variable. The effect of a change in this factor (or **independent** variable) on another variable (the **dependent variable**) is investigated. All other variables are kept constant (ie. **controlled**).

- The **independent variable** is the one that is systematically allowed to change by the researcher. It is the 'change' that you allow to happen when conducting an experiment.
- The **dependent variable** is affected by the change in the independent variable. It is the 'what happens' part of the investigation.
- The **controlled variables** are the other factors that can alter the result if they are not held constant.

Example 1

Hypothesis: Dark-coloured objects heat up faster than light-coloured objects when exposed to sunlight.

Independent variable: Colour of the object

Dependent variable: Temperature of the object

Controlled (constant) variables: Size of objects; shape of objects; initial temperature of objects; material composition of objects; time of exposure to sunlight; position in the sunlight

Example 2

Hypothesis: The speed at which limestone dissolves in acid increases with temperature.

Independent variable: Temperature

Dependent variable: Rate of limestone dissolving in acid

Controlled (constant) variables: Size of limestone pieces; mass of limestone pieces; surface area of limestone pieces; concentration of the acid; volume of the acid; initial temperature of the limestone; type of acid used; container in which reaction occurs

Using a control

Measurements may only be reliable in some experiments if a **control** is used. This is particularly true in biological or biochemical systems where many factors can alter a result. A control is an experiment that is performed as a **comparison** with those in which the independent variable is allowed to change.

Example

A gardener wishes to test which fertiliser leads to the greatest plant growth in potted geraniums. Unless she uses a potted geranium that has no fertiliser applied, she has no means of comparison. This unfertilised potted plant is otherwise treated in exactly the same way (ie. all other variables are the same) as the plants being tested. The unfertilised plant is the control, while the other plants in the various fertilisers are tested.

Experiments with living things

In many experiments involving human subjects, scientists adopt special methods to ensure that **bias** does not influence the result. In addition, all living things of the same species have such variability that it is difficult to control the variables. Two common methods used to overcome these problems are the blind and double-blind experiments.

a. Blind experiments

When testing whether a particular medication works on humans it is important that the subjects do not know whether they are the **test** group or the **control** group. The control group receives a **placebo**. This is a tablet or capsule or other medication that looks the same but has no active or test ingredient. Only the test group receives the medication with the active agent being tested. This procedure overcomes problems in attitude and wishful thinking.

b. Double-blind experiments

This technique goes one step further than the blind experiment. In this case the possible bias of the scientist is removed because he/she does not know which medication samples contain the test ingredient. Only the person preparing the samples knows the code and that person does not communicate with the scientist.

Test yourself (answers on pages 230–1)

Part A. Knowledge (answers on page 230)

1 An inference is:

a a statement that is true in the majority of cases.

b a statement that predicts what will happen in the next set of experiments.

c a theory.

[✓ d] a logical explanation of an observation. (1 mark)

2 The independent variable in an investigation is:

[✓ a] the one that is allowed to change systematically.

b affected by the changes made during the experiment.

- c the control
- d controlled so that its value does not change. (1 mark)

3 In experiments involving the testing of new drugs on humans, a placebo is a:

- a tablet containing the new drug to be tested.
- b person who volunteers to take the drug.
- c blind person who cannot see what is being done.
- [✓ d] control that is used for comparison. (1 mark)

4 Which of the following statements is not a scientific hypothesis?

- [✓ a] Wine is a preferable drink to beer.
- b Cricketers wear light-coloured clothes rather than dark clothes when playing in the summer.
- c Sugar dissolves more readily when the water is warmer.
- d Roosters crow when they see the first light of day. (1 mark)

5 A prediction is a:

- a new observation.
- ~~b~~ statement that suggests the result of a future experiment.
- c theory that accounts for all the observations.
- [✓ d] logical explanation for an observation. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark for each part)

- a Further experiments may ______ an inference or they may not.
- b Generalisations are statements that are ______ in the majority of cases.
- c Hypotheses are ______ statements that lead eventually to the development of scientific theories.
- d The factors that influence the results of experiments are called ______.
- e The dependent variable is affected by a change in the ______ variable.

7 Use the code letters to match the terms or phrases in each column. (1 mark for each part)

Column 1	Column 2
a control	f predictive explanation
b human control group	g comparison
c evolution	h placebo
d hypothesis	i theory
e inference	j logical explanation

Part B. Skills (answers on pages 230–1)

1 Suggest a logical inference that could be made for each of the following observations. (5 marks)

- a Betty noticed that moss only grew in the garden on the southern side of her house.
- b Eight boys suddenly turned up at sick bay after lunch at school last Tuesday. They all had stomach cramps.
- c The label on the brown bottle of hydrogen peroxide said to store it in the refrigerator after opening.
- d Kerosene floats on water.
- e All six children of a family have pale skin and blue eyes.

2 Use the following data to predict the time (X) for the reaction between A and B at 50°C. (2 marks)

Temperature (°C)	Time (s)
10	400
20	200
30	100
40	50
50	X

3 The electrical conductivity of a variety of aqueous solutions was measured. The results are shown in the table.

Aqueous solution	Conductivity
Sodium chloride	high
Sucrose	nil
Potassium nitrate	high
Copper sulfate	high
Alcohol	nil
Nickel chloride	high
Glycerine	nil

- a Use the observations in the table to construct an hypothesis. (1 mark)
- b How would this hypothesis be tested? (1 mark)
- c What results would support the hypothesis? (1 mark)

4 Billy observed that magnesium metal dissolved in dilute hydrochloric acid with the release of a colourless gas. He noticed that on some days the reaction seemed faster than on other days. He decided to investigate the reaction between the acid and magnesium. He made the following hypothesis which he then tested:

Magnesium dissolves faster in dilute hydrochloric acid as the temperature of the acid increases.

- a Name the independent variable in Billy's experiment. (1 mark)
- b Name the dependent variable. (1 mark)
- c Name the variables that need to be kept constant. (2 marks)

5 A research scientist investigated whether a new medication could successfully stop people smoking. Initial studies suggested that the medication may be useful. Two groups, each of 50 smokers, were tested. One group received tablets containing the test substance. The others received a placebo. The total number of cigarettes they smoked each day was recorded for 10 days. The results are tabulated.

Total number of cigarettes smoked each day

Day	Group A	Group B
1	1020	1005
2	950	850
3	960	700
4	935	580
5	955	450
6	920	500
7	910	475
8	945	460
9	935	440
10	965	470

- a Which group is likely to be taking the placebo? (1 mark)
- b Does the test suggest that the drug is useful in helping people to quit smoking? (1 mark)
- c Why should such tests be conducted with large numbers of people? (1 mark)
- d Would this test be better conducted as a double-blind experiment? Explain. (2 marks)

6 The solubility of a large number of chloride salts in water was examined. The results are tabulated below.

Chloride	Solubility
Sodium chloride	soluble
Calcium chloride	soluble
Lead chloride	insoluble
Potassium chloride	soluble
Copper chloride	soluble
Nickel chloride	soluble
Cobalt chloride	soluble
Silver chloride	insoluble
Barium chloride	soluble
Zinc chloride	soluble

Write a generalisation about the solubility of chloride salts in water. (2 marks)

Performing first-hand investigations

It is important when undertaking a practical first-hand investigation that the procedure adopted is safe and that modifications are made as problems arise.

Designing an experimental procedure

It is important to develop a planned procedure before undertaking experimentation. The planned procedure will:

- identify the variables that need to be kept the same
- specify the independent variable
- specify the dependent variable
- ensure that the selected procedure is valid
- ensure that the measurements are valid and accurate
- ensure that the measurements are reliable.

Example

Hypothesis: Sugar dissolves more rapidly in water as the temperature increases.

Experimental design:

- Procedure:
  1. Weigh a 2.0 g sample of table sugar onto a glossy square of paper.
  2. Measure a 100 mL sample of tap water into a 250 mL beaker.
  3. Place the beaker in a water bath held at a fixed temperature.
  4. Set up a mechanical stirrer to allow the water sample to be stirred at a constant rate.
  5. Allow the water and stirrer to reach the desired temperature. Measure this temperature.
  6. Add the sugar (by tipping the weighed sample off the paper) to the stirred water at zero time.
  7. Record the time when all the sugar has dissolved.
  8. Repeat the procedure **at least five times** at the selected temperatures. Take an average of the readings.
  9. Repeat the procedure several times at selected higher temperatures.

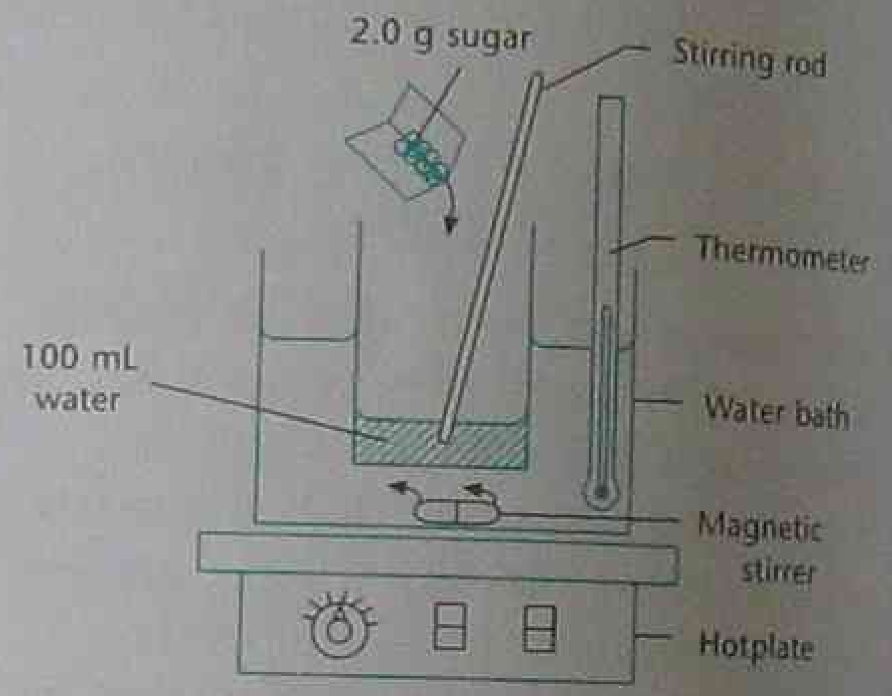


Figure 6.1 Experimental apparatus to determine the time for sugar to dissolve in water

- **Constant variables:** mass of sugar; grain size of sugar; volume of water; rate of stirring; size of vessel
- **Independent variable:** temperature of the water
- **Dependent variable:** time for the sugar to dissolve
- **Validity of procedure:** The procedure is valid as it measures the time it takes for the sugar to dissolve at a measured temperature. The use of a mechanical stirrer that rotates at a constant rate ensures that the mixing rate is kept constant. The use of a constant-temperature water bath ensures that the temperature is known for each run.
- **Measurements are valid and accurate:** The measurements are **valid** if they are 'trustworthy' and the method is one that is commonly used by expert scientists. The **accuracy** of the measurements depends on the equipment selected and how carefully the experiment is conducted. The accuracy in determining the mass of the crystals can be improved by using an electronic balance that weighs to 2 or 3 decimal places. The accuracy of measuring volume can be improved by measuring volumes from a pipette or burette rather than a measuring cylinder or beaker. The accuracy of the temperature measurement can be improved by using a temperature probe connected to a data logger.
- **Measurements are reliable:** The measurements are reliable if repeated experiments lead to consistent results. Sometimes in a large set of repeated measurements, an individual measurement may differ widely from the other measurements. You may be justified in ignoring this measurement before calculating an average. Measurements cannot be reliable if they are not performed with high precision and accuracy.
- Safety: Take great care in using the hotplate; do not touch it or the beaker of water as they are hot.

Choosing equipment for experiments

The equipment chosen for a practical investigation must be appropriate for the task and such as to ensure that the experiment can be done safely.

Examples

1. A thermometer whose scale is divided into fifths or tenths of a degree is more suitable to determine the variation in body temperature of a student over 2 hours than a thermometer marked in whole degrees.
2. A 100 mL measuring cylinder marked with millilitre scale divisions is more suitable to measure exactly 73 mL of water than a 100 mL beaker with 10 mL divisions.
3. The reaction of solid iron sulfide with dilute hydrochloric acid should be performed in a beaker in a fume cupboard, as the evolved rotten-egg gas is very poisonous. The student should also wear safety glasses to avoid splashing of acid into the eyes.
4. The increasing speed of a trolley moving down a ramp can more accurately be measured with a motion sensor connected to a data logger than with a ruler and stopwatch.

Working individually or in teams

Students need to be able to work individually on some occasions, as well as in teams on others. There are advantages in both ways of working. There are also responsibilities that must be considered.

Working individually requires students to:

- appreciate the importance of taking personal responsibility in planning and performing a task
- work to realistic timelines and goals
- persevere with an activity to achieve a reasonable endpoint
- accept personal responsibility for a safe working environment for themselves and others in the laboratory
- evaluate the effectiveness of their work in completing the task.

Working in teams requires students to:

- identify the different roles of each team member
- negotiate and allocate responsibility as a team member
- accept and perform the roles decided by the team

- collaborate with others in setting and working to agreed timelines and goals
- accept personal responsibility for maintaining a safe working environment for all team members.
- work cooperatively to monitor progress of the investigation
- evaluate the process used by the team and the effectiveness of the team in completing the task.

Assembling the apparatus and making observations safely

The selected equipment needs to be assembled to produce an apparatus that will perform the required task.

Assembling pieces of equipment and using the apparatus requires good manipulation skills and attention to safety.

Examples

1. When using a Bunsen burner to heat a beaker of water supported on a tripod and gauze, ensure that a blue flame is used (the inlet hole is open) and that the Bunsen burner is not placed under the gauze until all the apparatus has been assembled. After the heating is completed, the Bunsen burner should either be turned off at the tap or placed at the back of the bench with the hole closed to produce a yellow safety flame. Do not touch the hot equipment until it has cooled down.
2. When assembling a distillation apparatus, ensure that several retort stands and clamps are used to support the flask as well as the condenser. Ensure that water is flowing from the tap through the jacket of the condenser both before heating commences and after heating has finished.
3. When making observations using chemicals, ensure that safety glasses are worn and that no chemicals touch the skin. Chemicals should never be ingested. Smelling of some chemicals can also be dangerous.

Test yourself (answers on pages 231–2)

Part A. Knowledge (answers on page 231)

1 An important step in designing an experimental procedure as part of a fair test is to:

[✓ a] ensure that the selected procedure is valid.

b control only two variables at a time.

c control the dependent variable and measure the change in the independent variable.

d persevere with the experiment until an endpoint is attained. (1 mark)

2 To ensure that measurements are reliable in a first-hand investigation involving the measurement of air temperature at midday inside cars of different colours, the student should:

a use a new thermometer for each car.

[✓ b] repeat the experiment five or more times for each coloured car.

c conduct the experiment on one day only.

d ensure that cars of different sizes are compared. (1 mark)

3 Which piece of equipment would be appropriate for measuring 22 mL of water?

a A 100 mL beaker

b A 25 mL beaker

[✓ c] A 25 mL measuring cylinder

d A 1 litre measuring cylinder (1 mark)

4 Which of the following is *not* an advantage of working in teams during a first-hand investigation?

a Students learn to work cooperatively.

b Students learn to accept responsibility for their allocated task.

c Teams allow greater input in designing the experiment.

[✓ d] Time can be wasted unless all students work to realistic timelines. (1 mark)

5 A student group is asked to design an experiment to determine whether a candle or a methylated spirits burner produces more heat per second as it burns. The students plan the investigation and collect a set of equipment to conduct the experiment. Which set contains the necessary equipment to conduct the investigation?

[✓ a] candle, methylated spirits burner, tripod, gauze, beaker, thermometer, measuring cylinder, stopwatch, stirring rod, matches

b candle, methylated spirits burner, beam balance, measuring cylinder, thermometer, matches

c candle, methylated spirits burner, tripod, gauze, measuring cylinder, thermometer, matches, conical flask, ruler

d candle, methylated spirits burner, Bunsen burner, tripod, beaker, thermometer, measuring cylinder, stopwatch, stirring rod, matches (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

a When conducting a first-hand investigation, students must accept personal responsibility for a ______ working environment.

b The accuracy of a measurement depends on the ______ that is selected.

c The ______ variable is the one that the scientist allows to change.

d The set of variables that need to be kept the same are the ______ variables.

e At the end of an experiment the team should ______ the process and their effectiveness.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a valid	f repeatable
b manipulation skills	g sense organs
c safety issues	h trustworthy
d reliable	i personal responsibility
e observations	j construction of apparatus

Part B. Skills (answers on pages 231–2)

1 Write the procedure for an experiment that will test the hypothesis 'Limestone is more rapidly eroded by acidic solutions than marble'. (3 marks)

2 Consider the hypothesis 'Iron nails rust faster when exposed to water that contains high quantities of salt'. A student designs an experiment to test this hypothesis. He decides to prepare water solutions containing different concentrations of salt. He places each of these solutions in a test tube with a clean nail and observes the nails over many days.

a Name the independent variable in the student's experiment. (1 mark)

b Name the dependent variable in the student's experiment. (1 mark)

c Name the variables that should be held constant. (2 marks)

d What control will he use? (1 mark)

e How could the student improve the reliability of his experimental results? (1 mark)

3 Frederick read in his chemistry textbook that chlorine gas can be generated by adding concentrated hydrochloric acid to black manganese dioxide powder. If this experiment were performed in the laboratory, what safety precautions would Frederick need to employ? (2 marks)

4 Kristy is asked by her teacher to collect and assemble the necessary equipment for an experiment in which she will show that hot water cools faster when placed in a black can than in a shiny can. List the equipment that Kristy will select, and write a method for her experiment so that it is a fair test. (2 marks)

5 A group of four students is asked to work as a team to design and perform an experiment to test the hypothesis 'The temperature of garden soil decreases with depth'.

The team designs the experiment and then negotiates and accepts responsibility for various parts of the experiment.

a Suggest a suitable procedure that the team could adopt for their investigation. (3 marks)

b Suggest the responsibilities of the four team members. (2 marks)

Gathering, processing, presenting and evaluating information

Gathering and processing second-hand data relevant to the problem to be investigated normally precedes investigations involving practical experimentation. This helps to clarify the problem and to direct further first-hand investigations.

Gathering first-hand information

First-hand information is gathered in a variety of ways. The following points should be noted concerning the gathering of first-hand information:

- Making **accurate measurements** is important when conducting an investigation. This involves choosing the most accurate measuring devices available. It is also important to avoid **parallax error** when reading the scale on the instrument (ie. have your eye directly in line with the measurement).
- Always record the **metric unit** of the measurement. Thus, if you are measuring the mass of magnesium in an experiment, record the unit next to the value (eg. 4.30 g).
- Some instruments have more than one **scale**. For example, voltmeters often have a 0–2 V and a 0–20 V scale. Ensure that you read the correct scale.
- All measuring instruments are accurate only within certain limits. There is a limit to the number of decimal places to which you can read a scale. The **limit of reading** is normally half a scale division.
- Reliability can be improved by making **repeated measurements** over a number of trials (minimum of 5 trials).
- **New technologies** are gradually replacing older methods of collecting data. Sensitive probes connected to data loggers can be used to collect data accurately. The collected data can be downloaded to computer programs for analysis.
- Collected data should be displayed in tables, graphs, diagrams or photographic records.

Examples

Figure 6.2 shows how the size of different objects can be measured using a ruler, a vernier caliper and a micrometer screw gauge. Figure 6.3 shows the way to avoid parallax error. Figure 6.4 shows how to read different scales on the same instrument and how accurately the reading can be stated.

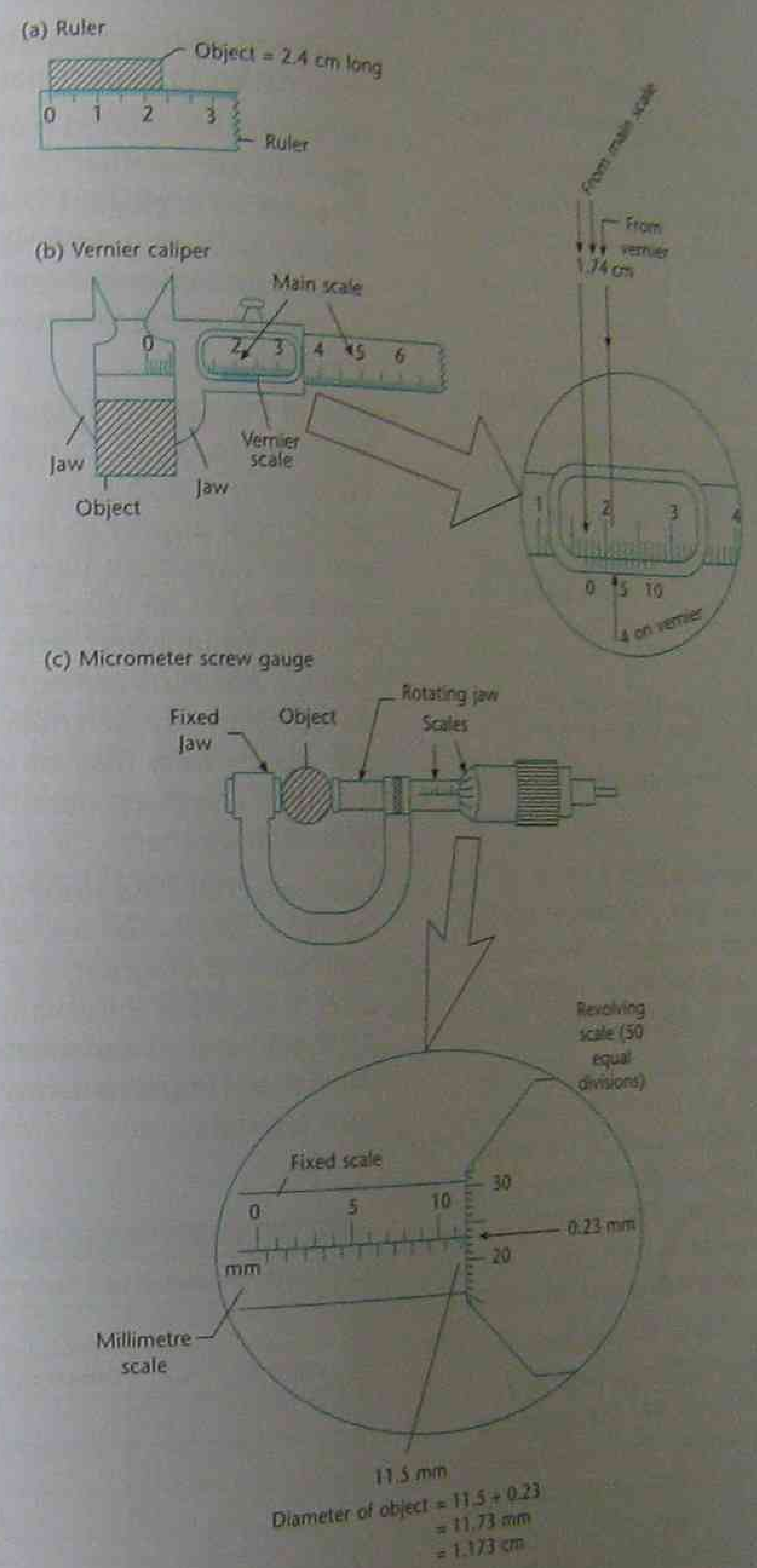


Figure 6.2 Measuring size

Gathering information from secondary sources

The following points should be noted when gathering information from secondary sources:

- Use a **wide range of resources** to gather secondary information. Do not just rely on the internet. Not all the information on the internet is accurate or correct. Much of it is not peer reviewed. CD ROMs, periodicals, library books and

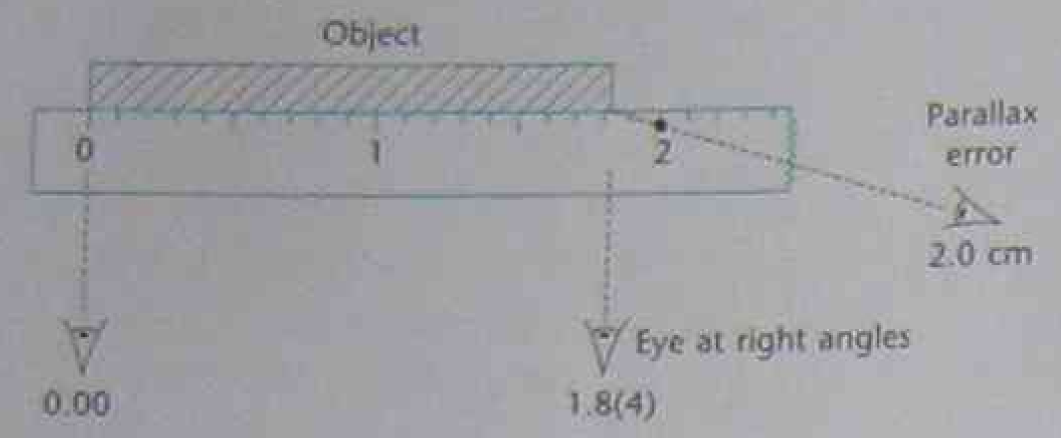
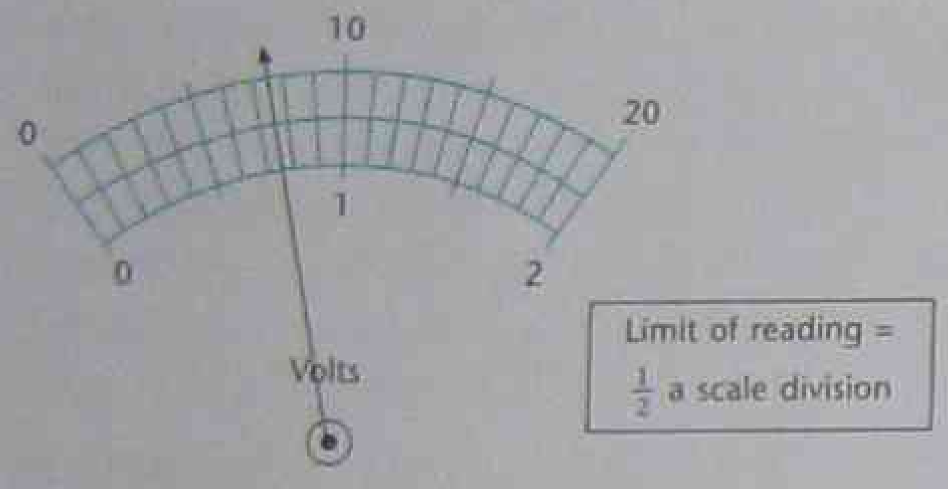


Figure 6.3 Avoiding parallax error



On 0–2 V scale: reading = 0.75 V (limit of reading = 0.05 V)
On 0–20 V scale: reading = 7.5 V (limit of reading = 0.5 V)

Figure 6.4 Instruments with several scales, and limits of reading

newspapers are also valuable sources of information. Ensure that you reference sources from which you have collected information in the appropriate manner (Author, Year, Title, Publisher, Place).

- When using library books or textbooks to locate information you should look for key words in the **index**. Look at the **table of contents** at the front of the book to locate relevant chapters. There may also be a **glossary** of terms to help you understand difficult technical words.
- Once you have gathered the second-hand data you will need to **collate** and **summarise** it. You must not plagiarise the work collected. You must read and make notes from this collected information into a scaffold/table which you have constructed earlier and then use these note summaries to construct your own text.
- Information from secondary sources may contain various types of **graphs** (eg. histograms, sector graphs, line graphs, divided bar graphs), **flow charts** and **diagrams**. You can use this type of information to extract data.
- As you gather second-hand data you must decide whether the information is **relevant** or **irrelevant**. For example, if you are gathering information on water movement in flowering plants, any data on non-flowering plants would be irrelevant.
- Not all the gathered data will be **reliable**. You need to check its reliability by comparing it with other sources. Science articles that appear in respected journals and that have been peer reviewed are more likely to be reliable than an article that is self-published. Articles written by creation scientists are not reliable as they do not follow the scientific method. Some companies may be unethical in that they only keep data that is favourable to them.
- Your collected data need to be **organised** for easy analysis. Tables, proformas, scaffolds and diagrams are commonly used to organise information. Computer databases and spreadsheets are becoming increasingly important ways of organising collected data.

	A	B	C
1	**Distance travelled by a toy truck versus time**		
2			
3	**Time (s)**	**Distance (cm)**	
4	0.0	0.0	
5	1.0	5.5	
6	2.0	12.0	
7	3.0	18.3	
8	4.0	25.0	
9	5.0	28.3	
10	6.0	30.4	
11	7.0	32.5	
12	8.0	33.7	

Figure 6.5 Organising information on a spreadsheet

Processing information

The following points should be noted when processing information that has been gathered from various first-hand and second-hand sources:

- Where applicable, use **mathematical procedures** to analyse the data.

Examples

1. If you have collected data on the velocity of a ball falling from rest as a function of time, then the formula: $v = u + at$, can be used to determine the acceleration of the ball.
2. If you have plotted v as a function of t, then your data should fall on a straight line that passes through the origin. The slope of this line will be equal to the acceleration a.
3. Data can also be determined from a graph by **interpolation** or **extrapolation**. When extrapolating a graph it is important to determine whether the extrapolation is valid.
4. Reliability is improved by making repeated measurements. The collected measurements should then be averaged.

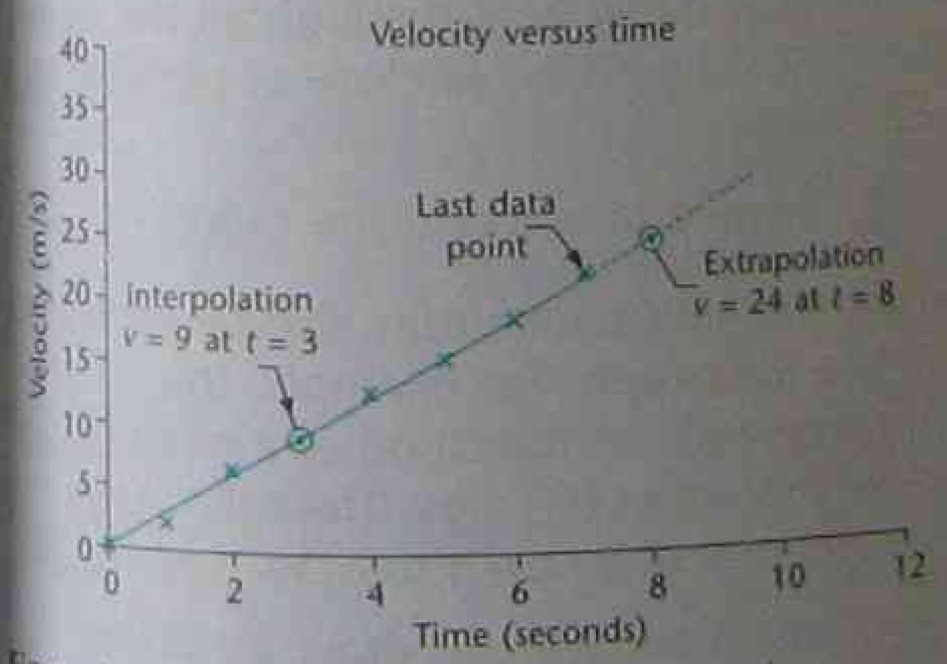


Figure 6.6 Obtaining data by interpolation and extrapolation of a graph

- When conducting a fair test, your data will either **support** or **discount** the **hypothesis**. If the hypothesis is supported, it does not mean that it is necessarily correct, as further experiments may be contradictory and not support it.
- As you collect data during an investigation, you may notice **trends** or **patterns** starting to emerge.

Example

If you are plotting a graph of the radii of atoms as a function of their atomic weight you will begin to notice that there is a repeating pattern where the radius drops from Group I to Group VIII across any one period.

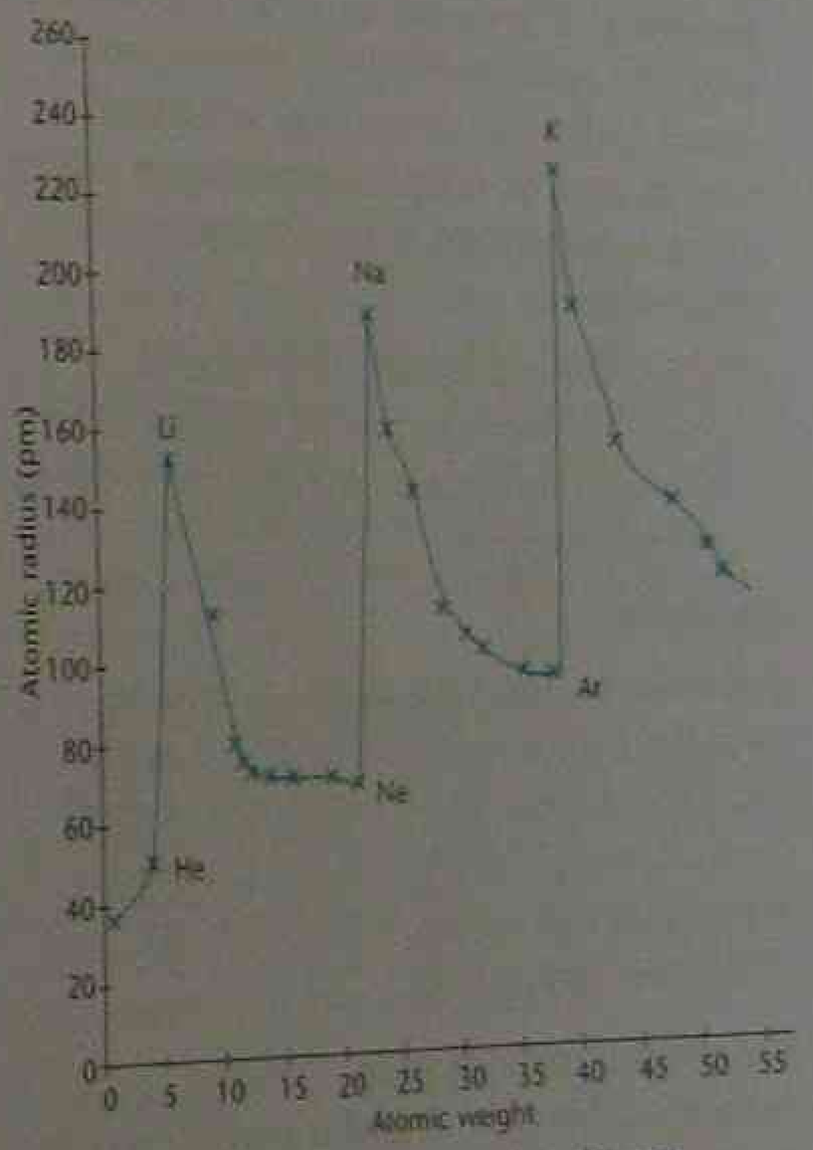


Figure 6.7 Patterns in data emerge when data are processed

- Once you have processed your results you will need to draw **valid conclusions**. The conclusions must relate directly to the experimental results. You cannot make a conclusion about information that you have not gathered or measured.
- As part of the scientific method, you will then provide plausible **explanations** of the phenomena being investigated. These possible explanations will be written in the **discussion section** of the report. There is also an opportunity at this point

to justify any **inferences** you have made about the collected data. In the discussion, you may make some **predictions** that will lead to further experimentation.

- As more experiments are completed, you may be able to make some **generalisations** about your area of investigation.

Presenting information

Collected and processed data must be presented to an audience in an appropriate way. The presentation of an experimental report is different to the way you would present an exposition. Here are some points to follow to present the processed information.

- Select the appropriate **text type** to present information. Note that the report on your investigation may be made up of several text types.

Examples

Procedure—This type of text is used to describe how an experiment that has not yet been done is to be performed. Present tense is used. The steps of the procedure are usually numbered or presented as sequential dot-points.

Procedural recount (experimental record)—This type of text is used to record the procedure of the experiment that has already been performed. It is also used for the results and conclusion. Past tense is used in a recount.

Report—This type of text is used to present information on a particular topic (eg. the different types of frogs in Australia) that has been investigated using first-hand and/or second-hand data. Factual and descriptive information is presented.

Discussion—This type of text identifies issues and presents points for and against. The report that you write for an investigation of a problem will have a discussion at the end. The discussion will often include **explanations** where causes and effects are discussed.

- Mathematical data can be presented graphically. A **line graph** is a very common method of plotting experimental data. The following rules should be followed:
  1. Use grid paper to draw your graph.
  2. Your graph should occupy at least 80% of the available grid space.
  3. The variable that you control (ie. the independent variable) is placed on the horizontal axis. Number the grid lines along this axis. Label this axis and its units.
  4. The variable being observed (ie. the dependent variable) is placed on the vertical axis. Number the grid lines along this axis. Label this axis and its units.
  5. Choose a suitable scale for each axis to ensure that the graph fills most of the grid space.
  6. Plot the data points as small crosses (x) using a pencil.
  7. Draw the 'line of best fit' through the data points. This line will not necessarily pass through all the points but should be as close as possible to them.
  8. Write a title above the graph.
- Mathematical data may also be presented as a **pie graph**. For example, the percentage composition of a mixture can be presented this way. The following rules should be used to draw a pie graph.
  1. Use a compass to draw a suitably sized circle that will fit on your page. A circle of radius 5 cm may be a good option.
  2. Convert the percentage data to degrees. Thus, if one component is 30% then 30/100 × 360 = 108°.

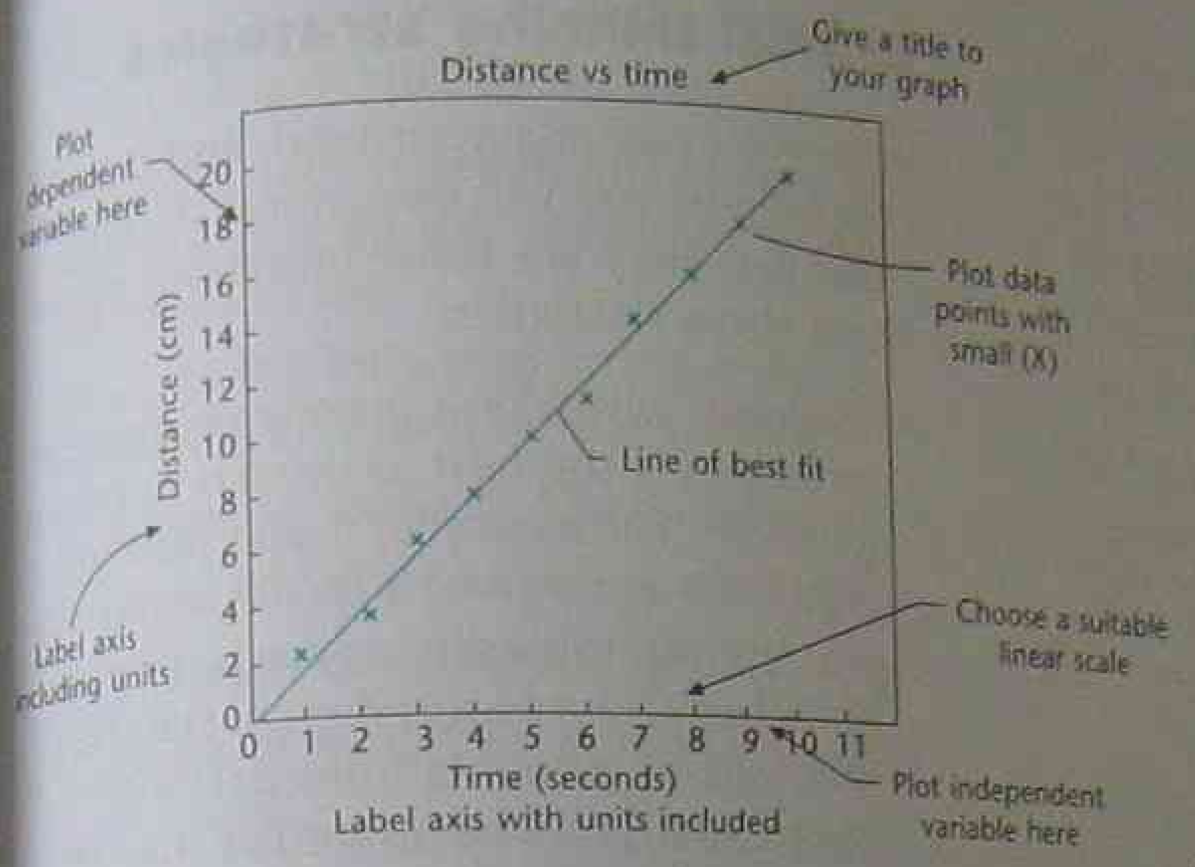


Figure 6.8 Drawing a line graph

  Repeat this with the remaining percentage data.
  3. Draw a radius on the circle using a ruler and pencil.
  4. Use your protractor to measure the first angle (eg. 108°) and draw a second radius to produce the first sector of the pie graph.
  5. Repeat this procedure to measure the remaining sectors.
  6. Label each sector and give the graph a title.

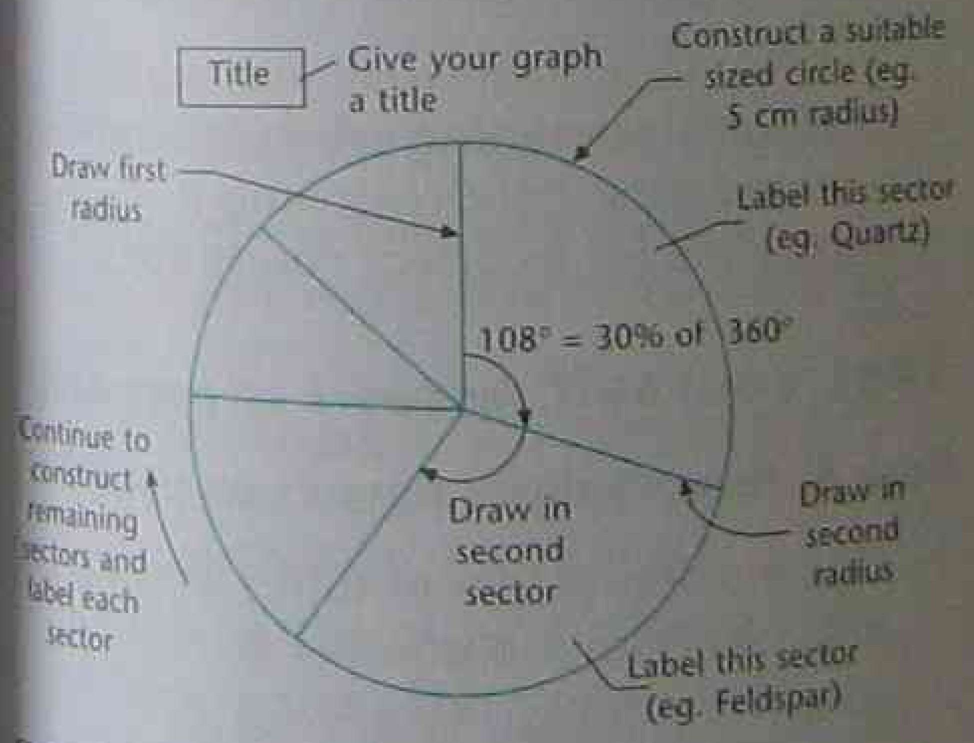


Figure 6.9 Drawing a pie graph

- Information can also be presented in a variety of ways including a written report, photos, sketches, videos, computer slide show, debates or orally.
- You must acknowledge all sources of information in your **bibliography.** Use an accepted method of citing references. This will include the author, name of the article/book, chapter or page reference, publisher and date of publication. Even for sources on the internet you should state as much of this information as you can.
- Use **standard symbolism** for mathematical and scientific information. Thus '*c*' is used in physics to stand for the 'velocity of light' and '*E*' stands for 'energy'. For very large or very small numbers use scientific notation.

Example

0.000 023 should be expressed as 2.3×10^{-5}

10 500 000 should be expressed as 1.05×10^{7}

- Diagrams and maps often have **different scales**. The scale should be stated on the map or diagram. The scale should be easy to use.
- Computer software can be used to generate graphs, tables, flow charts and diagrams. Graphing software often allows users to produce a variety of different graphs of the same data.

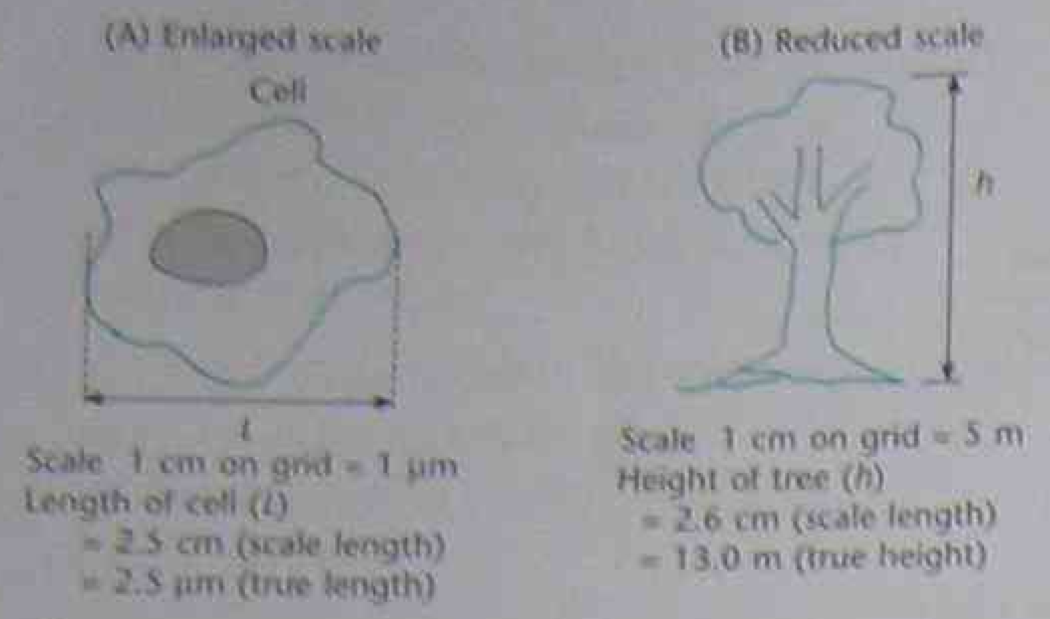


Figure 6.10 Using scales on maps and diagrams

Composition of an alloy

Metal	%
Pb	65
Sb	20
Sn	10
Bi	5

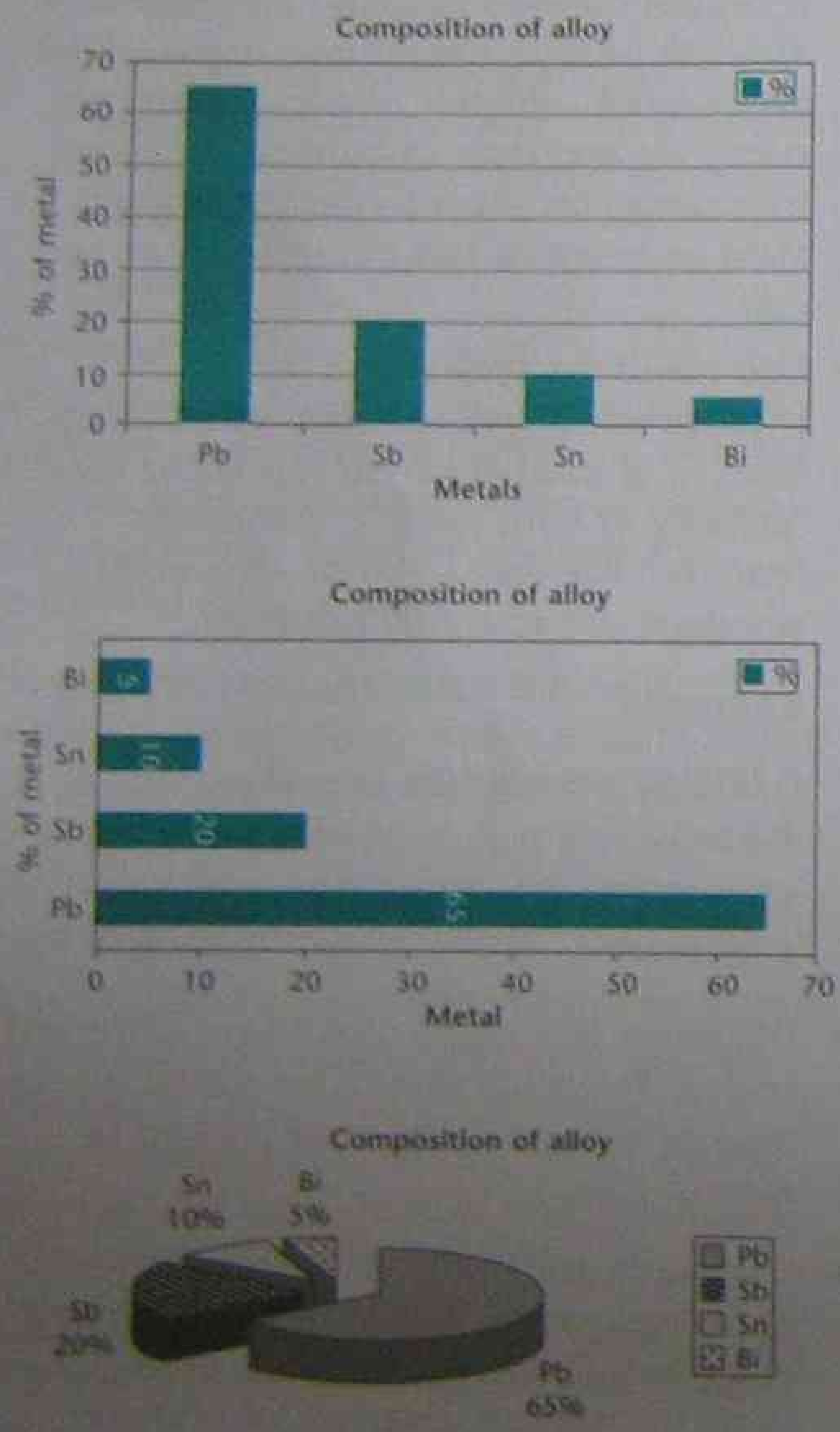


Figure 6.11 Computer graphing

Critical thinking strategies

When you evaluate your work and draw conclusions, you will use critical thinking strategies. Here are some important points about these strategies:

- **Evaluate** which of a number of possible strategies is the best approach to solve a problem. For example, if you plan to study the ecology of a local swamp, will you use photographic records, physical and chemical measurements, statistical methods or recounts from personal observations? Some strategies may be more useful than others.
- Be **flexible** and willing to change your view when confronted with contradictory evidence. Your hypothesis is not necessarily correct and you need to be prepared to change it if collected evidence discounts it.
- Think **creatively** when solving a problem.
- Distinguish between **facts** and **opinions**. Facts are pieces of information that can be tested to check that they are correct. Opinions are personal attitudes and feelings that individuals have about an issue. In science, opinions carry no validity.
- Use **logical reasoning** when interpreting collected data. Poor reasoning is responsible for incorrect conclusions.

Test yourself (answers on page 232)

Part A. Knowledge (answers on page 232)

1 One major problem of using information obtained from the internet is:

- a There is usually little relevant information on science topics.
- [✓ b] Not all the information is reliable as it is not always peer reviewed.
- c The author's name is not recorded on the articles.
- d The information in many articles is untrue and unreliable. (1 mark)

2 A student gathers second-hand data to write a report on the marsupials found outside Australia.

Which of the following article titles would not be useful for the student?

- [✓ a] Kangaroo fossils in Queensland
- b Types of South American mammals
- c Evolution of mammals in New Guinea
- d Fossils of mammals around the world (1 mark)

3 A suitable type of text to use when recording the experimental method used in a series of investigations would be:

- a narrative.
- b discussion.
- [✓ c] procedural recount.
- d exposition. (1 mark)

4 Which of the following is not an example of a critical thinking strategy?

- a Logical reasoning
- b Willingness to change one's point of view when new evidence is presented
- c Evaluating possible strategies
- [✓ d] Selecting experimental data that supports the hypothesis (1 mark)

5 Which of the following is **not** a satisfactory method of citing a science reference?

- a Waterhouse, G.R. (1846–48). *A Natural History of the Mammalia.* 2 vols. Bailliere, (SU) London.
- b Russell, T & Watt. D (1990) Evaporation and Condensation. *Science Processes and Concept Exploration*(SPACE) *Research Report*. University of Liverpool Press. Liverpool.
- [✓ c] Mason, Stephen. New York. F. *A History of the Sciences* New. Collier.
- d Fermi, Laura (1954) *Atoms in the Family*. University of Chicago Press. Chicago. (1 mark)

6 Complete the following restricted-response questions using the appropriate word. (1 mark each part)

- a The weight of the boy was measured and found to be 459 _____ .
- b Once the results of an investigation are processed, the scientist will need to draw _____ conclusions.
- c Tables and scaffolds are ways in which _____ can be organised.
- d Once information has been gathered from secondary sources it needs to be _____ and summarised.
- e The limit of reading of a measuring instrument is normally _____ a scale division.

7 Use the code letters to match the terms or phrases in each column. (1 mark each part)

Column 1	Column 2
a glossary	f factual information
b database	g testable information
c report text type	h organisation of data
d facts	i identification of issues
e discussion text type	j technical words

Part B. Skills (answers on page 232)

1 Figure 6.12 shows a number of scales. Read the value on each scale. (3 marks)

2 The following table shows the percentage of different components of a sandy loam soil. Use this information to construct a pie graph. (3 marks)

Soil type	% sand	% silt	% clay
Sandy loam	60	30	10

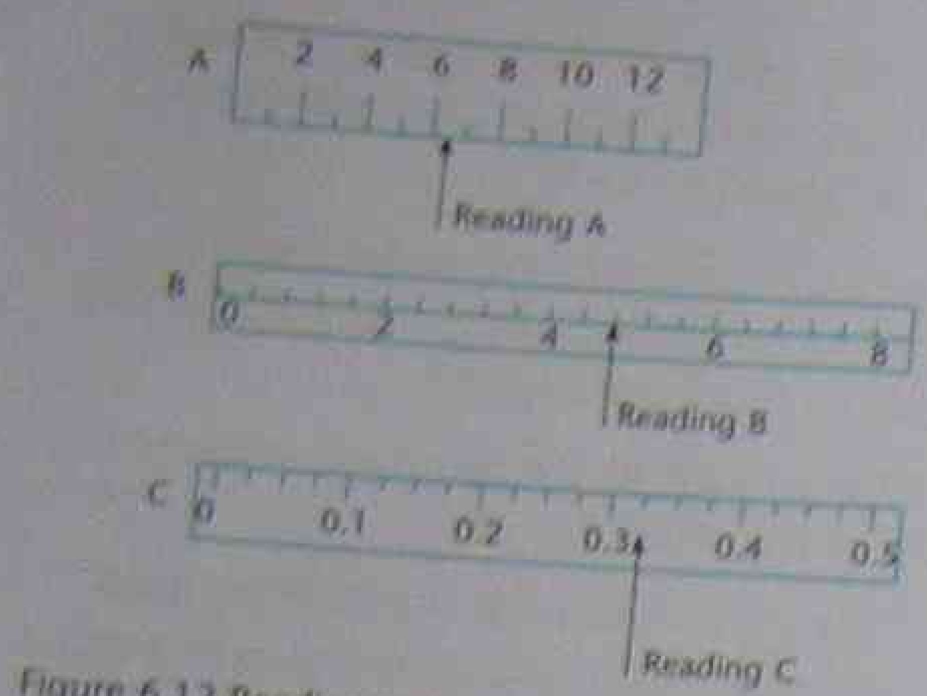


Figure 6.12 Reading scales

3 The following data were collected from an experiment involving light refraction in a plastic prism.

Angle of incidence (*i*) (degrees)	Angle of refraction (*r*) (degrees)
20	7
40	13
50	17
60	20
70	23

a Plot a line graph of the incidence angle (horizontal axis) versus the refracted angle. Draw the line of best fit. (3 marks)

b Use your graph to interpolate the refracted angle when the incidence angle is 30°. (1 mark)

c Extrapolate your graph to determine the incidence angle when the refracted angle is 26°. (1 mark)

4 Use the scale on the diagram to determine the true length of the insect. (2 marks)

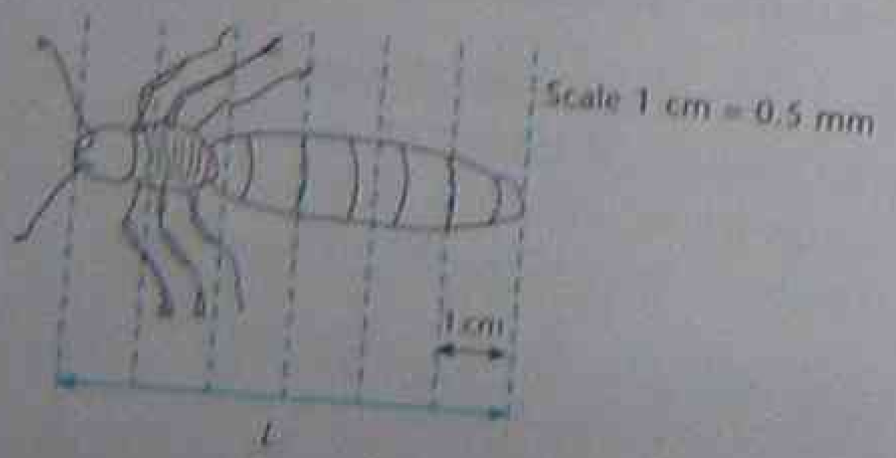


Figure 6.13 Insect drawn to an enlarged scale

5 Figure 6.14 shows the graph of a uniformly accelerated body.

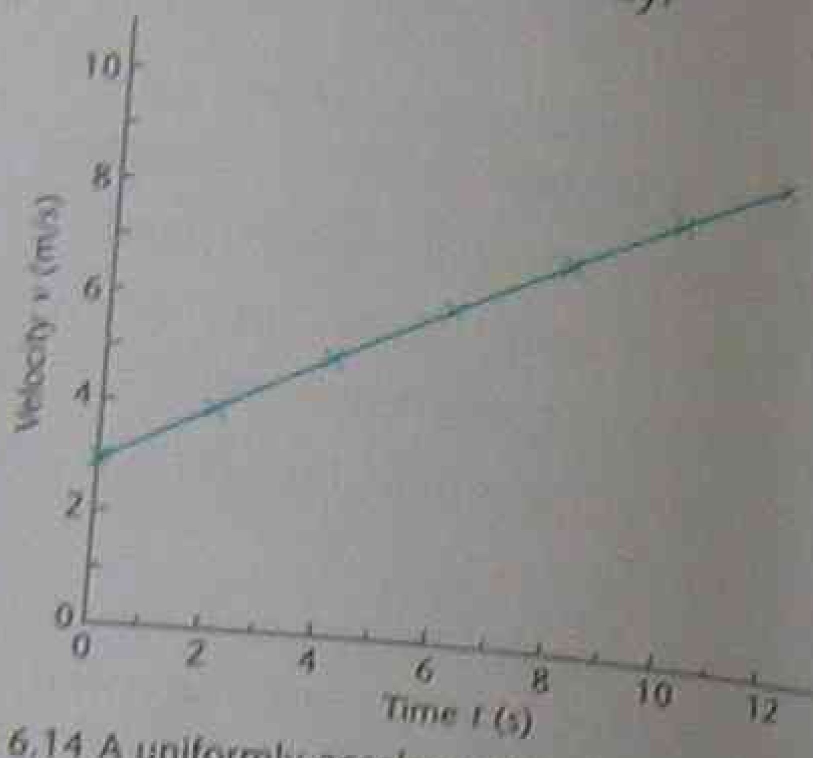


Figure 6.14 A uniformly accelerated body

a Show that these graphical data satisfy the equation: $v = u + at$,

where, u = initial velocity; v = final velocity; a = acceleration; t = time (2 marks)

b What is the initial velocity of the body? (1 mark)

c What is its acceleration? (1 mark)

End-chapter test (answers on pages 232–3)

1 A company that manufactures alternative medicines has made some claims about its latest product *Oxideze*. Which of the following claims are opinions that cannot be easily tested scientifically? (3 marks)

a *Oxideze* prevents the build-up of damaging free-radicals in the blood.

b *Oxideze* is made only from natural ingredients.

c *Oxideze* is the best alternative medical product on the market.

2 Bertram investigated the effect of different gases (X and Y) on the growth of cress seedlings. He placed 5 wheat seeds on damp cotton wool in each of two glass vessels as shown in Figure R1. He waited until they germinated before introducing the selected gas into containers through the tubes and taps. Plants 1 to 5 were exposed to gas X and plants 6 to 10 were exposed to gas Y.

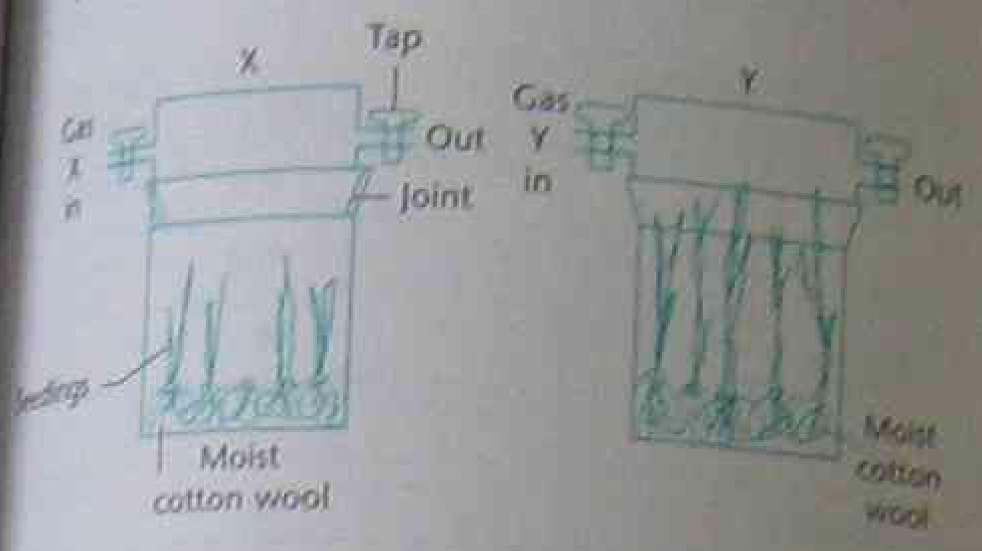


Figure R1 Bertram's experiment

After 6 days he measured the length of the shoot for each plant. His results are tabulated.

Plant number	Length of shoot (cm)	Plant number	Length of shoot (cm)
1	5.3	6	6.6
2	4.9	7	6.8
3	no growth	8	7.0
4	5.5	9	6.8
5	4.8	10	7.2

a Calculate the average length of the wheat shoots in each experimental condition. (5 marks)

b In order to conduct a fair experiment, Bertram tried to control his variables. Name the variables that needed to be controlled. (1 mark)

c What other experiment did Bertram need to do to determine whether these gases (X and Y) had an effect on the growth of the wheat seeds? (1 mark)

d Bertram concluded in his report that gas Y promoted the growth of the seedlings more than gas X. Is this a valid conclusion? Explain. (1 mark)

3 Figure R2 shows a graph of the results of an experiment in which Mary measured the pH of 10 mL of a dilute acid solution as 0.5 mL samples of a dilute base were added.

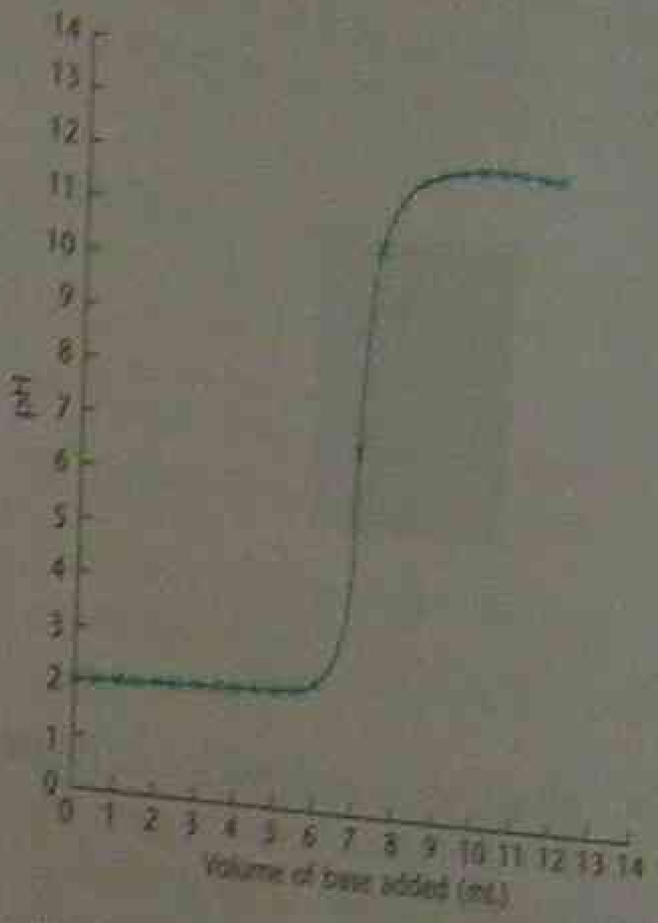


Figure R2 pH graph

a What volume of base solution did Mary add over the whole experiment? (1 mark)

b Describe how the pH of the mixture changed throughout the experiment. (1 mark)

c At what volume of added base did the solution become neutral. (1 mark)

d Mary did not record the pH of the mixture when 6.5 mL of base had been added. What would the pH of the mixture have been at this point? (1 mark)

e The base solution was placed in an accurate piece of glassware known as a burette. By opening a tap, small volumes of base can be added to the acid in the flask. Figure R3 shows the scale on the burette that Mary used to add the base to the acid. What reading is shown on the burette at this point. (1 mark)

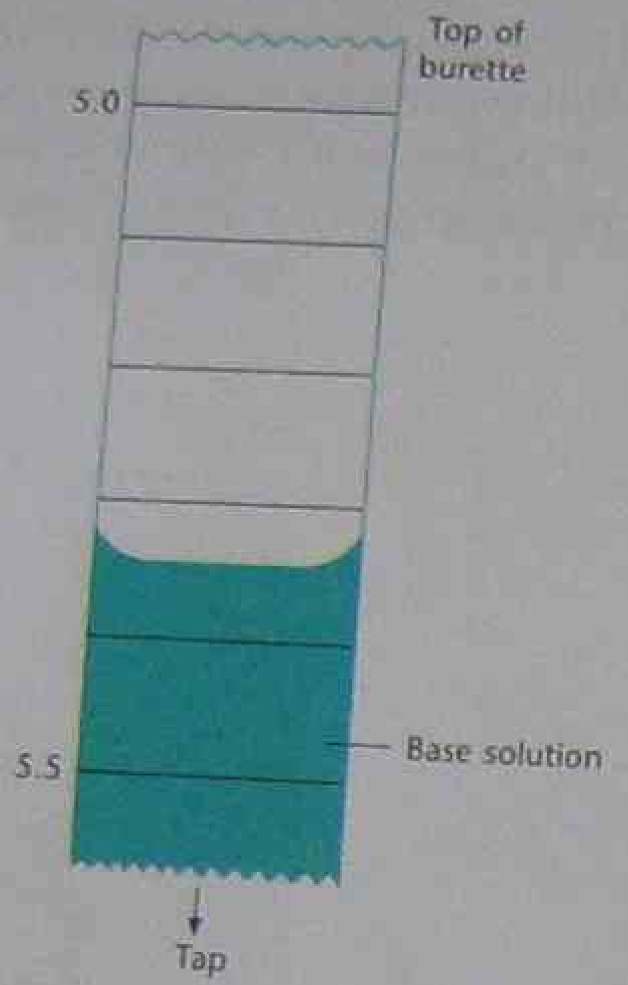


Figure R3 Close-up of a burette scale

4 A rock sample is analysed for its mineral content. The table below records the percentage by mass of each mineral.

Mineral	% (by mass)
Quartz	47
Feldspar	40
Ferromagnesian	
Mica	4

a Calculate the percentage of ferromagnesian minerals in the rock. (1 mark)

b Present this composition data in the form of a divided bar graph. (2 marks)

5 The following table provides information about the number of hours of daylight in different months of the year in Sydney.

a Plot these data as a line graph. (3 marks)

b Day length is an important factor in the flowering of plants. Plants can be classified into 'short-day plants' and 'long-day plants'.

Month	Hours of daylight/day
January	17
February	15
March	13
April	11.5
May	10
June	8.5
July	8.5
August	10
September	11.5
October	13
November	15
December	17

Short-day plants only flower if the plant is exposed to low sunlight levels in the month prior to flowering.

Long-day plants only flower if the plant is exposed to high sunlight levels in the month prior to flowering.

i Plant X is a short-day plant that requires exposure to less than 10 hours of sunlight in the month before flowering. In what season(s) is this plant most likely to flower. (1 mark)

ii Plant Y is a long-day plant that requires exposure to a minimum of 13 hours in the month prior to flowering. In what months of the year is this plant able to flower? (1 mark)

6 Solutions of cobalt chloride are pink. The higher the concentration of cobalt ions, the more pink is the solution. Clement made up four solutions of cobalt chloride by dissolving different masses of the cobalt chloride crystals in different volumes of water.

Use the information in the table to answer the following questions:

a Calculate the concentration of cobalt chloride in solution A in the units mg/L. (2 marks)

b Rank the solutions in decreasing order of pink colour. (1 mark)

Solution	Mass of solute (g)	Volume of water (mL)
A	3.6	150
B	4.4	200
C	2.8	120
D	3.3	130

Summary

- The scientific method involves: observations; inferences; predictions; experimentation; formulating hypotheses; testing and modifying hypotheses; making conclusions; generalising; and developing theories and laws.
- Fair testing involves the control of variables. The independent variable is allowed to change and the dependent variable is affected by this change. Other factors are the controlled or constant variables.
- Blind- and double-blind experiments are often used in testing medications on humans so as to remove bias and wishful thinking.
- Experiments should be carefully designed to ensure that they are valid and that they are safe.
- There are advantages and disadvantages in working individually or in teams.
- Data can be gathered by first-hand experimentation and/or by second-hand data analysis.
- Repeated measurements improve reliability.
- Data can be presented in a variety of ways, including graphs and tables.
- Critical thinking strategies should be used when conducting investigations.

Chapter 7

Achieving High Levels in the School Certificate Test

Levels of achievement

Your result in the School Certificate in Science is reported in terms of **levels of achievement**. The highest level of achievement is **Level 6**.

Students who achieve at Level 6 are able to:

- evaluate the impact of scientific research on science, society, technology and the environment
- explain scientific phenomena in terms of models, theories and laws
- explain the scientific methods used to validate various models, theories and laws
- explain interactions within and between structures and systems in the living and non-living world
- locate, gather and process data and draw valid conclusions
- communicate their scientific understanding in a variety of ways
- creatively solve scientific problems
- confidently work independently or in teams.

To achieve at Level 6, you must therefore have:

- deep knowledge of the Science content you have studied
- high skill levels in processing and analysing supplied data in a variety of formats including text, models, diagrams tables and graphs
- the ability to use higher order verbs in responding to open-ended questions.

The first two of the above criteria have been extensively developed throughout this study guide. The ability to respond appropriately to open-ended questions is covered below.

Instruction words in the School Certificate test

Some of the following **verbs** or **instruction words** are used each year in the School Certificate test. It is important that you understand what each instruction term requires you to do. The following table lists these verbs and gives examples of how they can be used. They are loosely arranged from **lower order** (requiring simple responses) to **higher order** (requiring more complex responses) words.

<table>
<tr><th>Verb</th><th>Meaning</th><th>Example</th></tr>
<tr><td>identify</td><td>recognise and name</td><td>Question: <i>Identify</i> the part of the reproductive system that transfers semen through the penis.
Answer: Urethra</td></tr>
<tr><td>recall</td><td>present remembered facts and ideas</td><td>Question: <i>Recall</i> the law of superposition.
Answer: In a series of sedimentary strata, the youngest layer is on top and the layers get older with increasing depth.</td></tr>
<tr><td>describe</td><td>provide characteristics and features</td><td>Question: <i>Describe</i> the features of a plutonic rock such as granite.
Answer: Plutonic rocks have quite large crystals of millimetre or centimetre dimensions. These crystals have grown together and are composed of some common minerals such as glassy quartz, white or pink feldspar and black mica.</td></tr>
<tr><td>outline</td><td>indicate the main features of; sketch in general terms only</td><td>Question: <i>Outline</i> the main steps in filtering a sample of sandy water.
Answer:
1. Set up the filter funnel, filter ring, stand and beaker ready for filtration.
2. Place the folded filter paper in the funnel.
3. Slowly pour the sand/water mixture into the filter paper.
4. Allow the water to collect in the beaker below.</td></tr>
<tr><td>compare</td><td>show how things are similar or different</td><td>Question: <i>Compare</i> two common theories concerning the fate of our universe.
Answer: The open universe theory predicts that the universe will continue to expand forever. This is supported by current measurements that show the outer reaches of the visible universe are expanding at increasingly greater rates. This theory differs from the closed (or pulsating) universe theory which proposes that the observed expansion will eventually stop as gravity pulls matter back inwards to an ultimate 'big crunch'.</td></tr>
<tr><td>classify</td><td>arrange or include in categories or classes</td><td>Question: <i>Classify</i> the following nine substances as solids, liquids or gases at room temperature and pressure: water, oxygen, mercury, granite, gold, carbon dioxide, zinc, lead, neon.
Answer:
<i>Solids</i>: granite, gold, zinc, lead
<i>Liquids</i>: water, mercury
<i>Gases</i>: oxygen, carbon dioxide, neon
(Note that this question involves <i>recall</i> as well as the ability to group things.)</td></tr>
<tr><td>organise</td><td>form different parts into a unified whole</td><td>Question: <i>Organise</i> the following data concerning the melting points of some common elements into a logical tabular format.
<i>Data</i>: Cs (669°C), Ba (725°C), Pb (327°C), Cd (321°C), Ra (700°C)
Answer: The data is arranged in a logical sequence according to increasing melting point. Note the column headings in the table.
<table>
<tr><th>Element symbol</th><th>Melting point (°C)</th></tr>
<tr><td>Cd</td><td>321</td></tr>
<tr><td>Pb</td><td>327</td></tr>
<tr><td>Cs</td><td>669</td></tr>
<tr><td>Ra</td><td>700</td></tr>
<tr><td>Ba</td><td>725</td></tr>
</table></td></tr>
<tr><td>deduce</td><td>draw conclusions</td><td>Question: A car is uniformly accelerating along a road. <i>Deduce</i> the speed (<i>x</i>) of the car after 3 seconds.
<table>
<tr><th>Time (s)</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td>Speed (m/s)</td><td>4</td><td>6</td><td><i>x</i></td><td>10</td><td>12</td></tr>
</table>
Answer: One can conclude from the data that the speed increases by 2 m/s each second. Thus the missing value <i>x</i> is 8 m/s.</td></tr>
</table>

Verb	Meaning	Example
account for	give reasons for	**Question**: *Account for* the use of fuses in electrical circuits. **Answer**: Fuses are used to protect the electrical components of the circuit from being damaged if there is a power surge or short circuit. A simple fuse consists of a thin wire that melts when the current is too high. The melted wire breaks and the power is cut off.
explain	relate cause and effect; provide why and/or how	**Question**: *Explain* why granulated sugar dissolves more slowly than caster sugar in hot coffee. **Answer**: Granulated sugar has a larger crystal size than finely powdered caster sugar. The caster sugar dissolves much faster because it has a much greater surface area for the water to collide with and dislodge sugar molecules from the crystal lattice.
calculate	mathematically determine a result from data	**Question**: *Calculate* the frequency of a wave with a wavelength of 2.5 m and a velocity of 300 m/s given the wave equation $\mathbf{v = f.\lambda}$ **Answer**: Insert the values into the equation and solve: $300 = f\ (2.5)$ $f = 120$ Hz (Note that the answer has a unit.)
predict	use available information to suggest what might happen	**Question**: A farmer purchases a wooded section of land in southern NSW. He removes all the trees to create a pasture to establish a dairy farm. *Predict* the consequences of the farmer's removal of all the trees on his property. **Answer:** Trees are important to stabilise land against erosion. Erosion and gully formation are likely to follow. Trees also keep the water table low and prevent salts rising to the surface. Their removal will likely lead to salination of the land, making it ultimately unsuitable for farming.
discuss	identify issues and provide arguments for and against	**Question**: *Discuss* ways in which we can conserve our environment. **Answer:** Our environment can be conserved in a number of ways. The major issues involve habitat destruction, the finite nature of our resources and dealing with waste. 1. *Maintenance of habitat for wild animals and plants*: If parks are not set aside then many wild animals and plants will not survive into the future. The establishment of parks, however, prevents the growth of agriculture and may harm jobs. 2. *Recycling of materials such as glass, metals and paper*: This will ensure that the environment is not spoiled by larger dump sites and it also minimises the number of new mines that need to be established. Fewer mines, however, will mean fewer jobs in that industry. 3. *Biodegradable landfills*: Wastes should ideally only be placed in landfills if they are biodegradable. This is not currently being achieved, although scientists are developing biodegradable plastics.
assess	make a judgment of value, result, size or quality	**Question**: *Assess* the value of using models in chemistry. **Answer:** There are various types of models used in chemistry. One of the most common types is a structural model of atoms and molecules. These models are useful as they help to visualise the arrangement of subatomic particles in the atom or the ways in which atoms combine in three dimensions to form a molecule. Molecular models are also helpful in visualising how particles interact in a chemical reaction. The way that ions or atoms are arranged in a crystal lattice is best shown by making a 3-D model based on X-ray analysis. These models have their limitations, however, as they are constructed from wood, plastic and metal and consequently cannot represent the true nature of atoms, particularly the low density electron cloud. The simple ball and stick models of compounds do not show how electrons interact between atoms to produce bonds. Space-filling models are better indicators of the shape of the molecule but do not show the bond angles easily. Some models in chemistry are mathematical. The kinetic theory of matter contains a mathematical description of the way matter behaves. Mathematical models and theories help us to calculate chemical properties or behaviour. The experimental measurements are then compared with theoretical models to see if they are consistent. These models are limited in that they often only approximate the true behaviour of matter. Without models, the observations made by chemists in their experiments would be difficult to interpret. Computer modelling and visualisations have improved the ways chemists can interpret and understand chemical processes.
justify	support an argument or conclusion	**Question**: Carbon is classified as a non-metal. *Justify* this classification. **Answer**: Carbon has many properties typical of all non-metals, although in some forms it has properties that are contradictory. The graphite form of carbon is dull and black. This is typical of non-metals as it is shiny and 'metallic' in appearance. Graphite is brittle and shatters when hammered and therefore cannot be beaten into thin sheets as metals can. This is also true of the diamond form of carbon, although it is very much harder than graphite. Diamond does not conduct electricity, which is typical of non-metals. Contrary to this, the graphite form is a good electrical conductor. When all carbon's properties are considered, the weight of evidence suggests it is best to classify it as a non-metal.
evaluate	make a judgment based on criteria; determine the value of	**Question**: *Evaluate* a proposal that electricity in NSW should be generated using solar energy rather than the chemical energy in coal. **Answer**: Coal is currently the major energy source for electricity production in NSW. This is because coal is an abundant resource in NSW and the coal seams are near the surface and easy to mine, thus reducing costs. The infrastructure has been set up to produce electricity in coal-fired power stations for over a century. Coal stations do, however, produce significant pollution and greenhouse gases. To replace coal with solar energy is currently impractical. The technology to produce highly efficient solar panels is still at the research level. This technology is currently very expensive (compared with coal technology) and to set up sufficient panels for all the cities in NSW is not yet possible. Storage of excess electricity produced by solar panels is also a problem. Solar electricity, however, is environmentally friendly because no poisonous emissions are produced. In this way it is superior to coal. Solar research should be strongly funded so that it can gradually replace our current methods.
analyse	identify components and the relation between them; draw out and relate implications	**Question**: *Analyse* the tabulated data concerning the shoot and root growth of pine seedlings and draw appropriate conclusions.

Night temperature (°C)	Day temperature (°C)	Shoot growth (cm)	Root growth (cm)
15	21	3	37
21	28	16	13

Verb	Meaning	Example
		Answer: High root growth and little shoot growth occur when the night temperature and day temperatures are low. When the night and day temperatures are high, the shoot growth is high but the root growth is not as great. This data suggests that in the winter (low night and day temperatures) the pine seedlings will use their energy to grow roots but not shoots. In the summer, however, when there is abundant sunshine as the days are longer, the plants will grow shoots and leaves to maximise photosynthesis and food production.

Test yourself (answers on pages 233–5)

For each of the following open-ended questions, construct a suitable response. Take particular note of the verb used in each case.

1 Assess some of the scientific ideas that different cultures have contributed to science throughout history. (5 marks)

2 Evaluate the potential impact of some issues raised in the mass media that require some scientific understanding. (5 marks)

3 Justify research which involves sending unmanned missions to the planets of our solar system. (5 marks)

School Certificate Trial Revision Test

Answers on pages 235–8

General instructions

- Reading time: 10 minutes
- Working time: 1.5 hours
- This test has two sections
- Attempt all questions.
- Draw diagrams using pencil.
- Calculators may be used.

Section 1. Multiple choice (50 marks)

- Answer all questions.
- Select the letter of the best response.

Section 2. Questions 51–66 (50 marks)

- Answer all questions.
- This section has FOUR PARTS
  - Part A. Questions 51–60 (10 marks)
  - Part B. Questions 61–62 (14 marks)
  - Part C. Questions 63–64 (12 marks)
  - Part D. Questions 65–66 (14 marks)

Section 1. Multiple choice

1 What is the reading on the thermometer scale shown in Figure SC1?

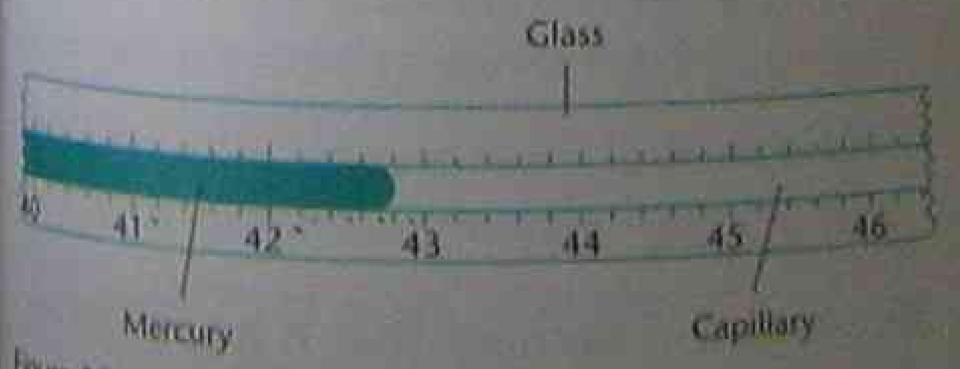


Figure SC1 Thermometer scale

[✓ a] 42.8°C ✓

b 42.4°C

c 40.8°C

d 43.2°C

2 Determine the true length (L) of the cell shown in Figure SC2.

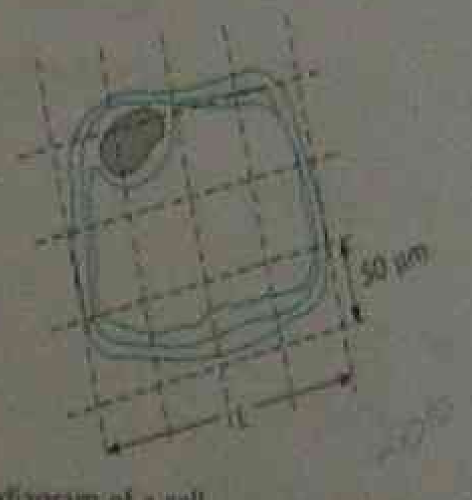


Figure SC2 Scale diagram of a cell

a 4 centimetres

b 100 micrometres ✓

c 20 micrometres

[✓ d] 200 micrometres

3 Use the divided bar graph in Figure SC3 to determine the percentage of zinc in the alloy.

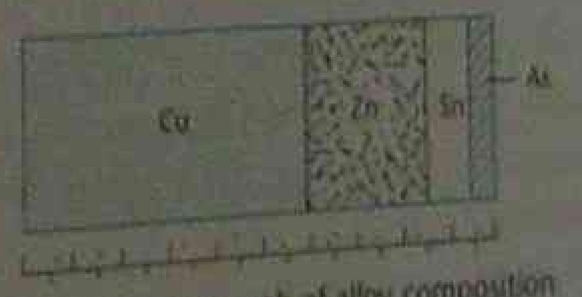


Figure SC3 Divided bar graph of alloy composition

a 5 %

b 10% ✓

[✓ c] 25%

d 60%

4 Figure SC4 shows a line graph of velocity of a vehicle starting from rest, versus time. Use the graph to determine the velocity of the vehicle 5 seconds after it started its journey.

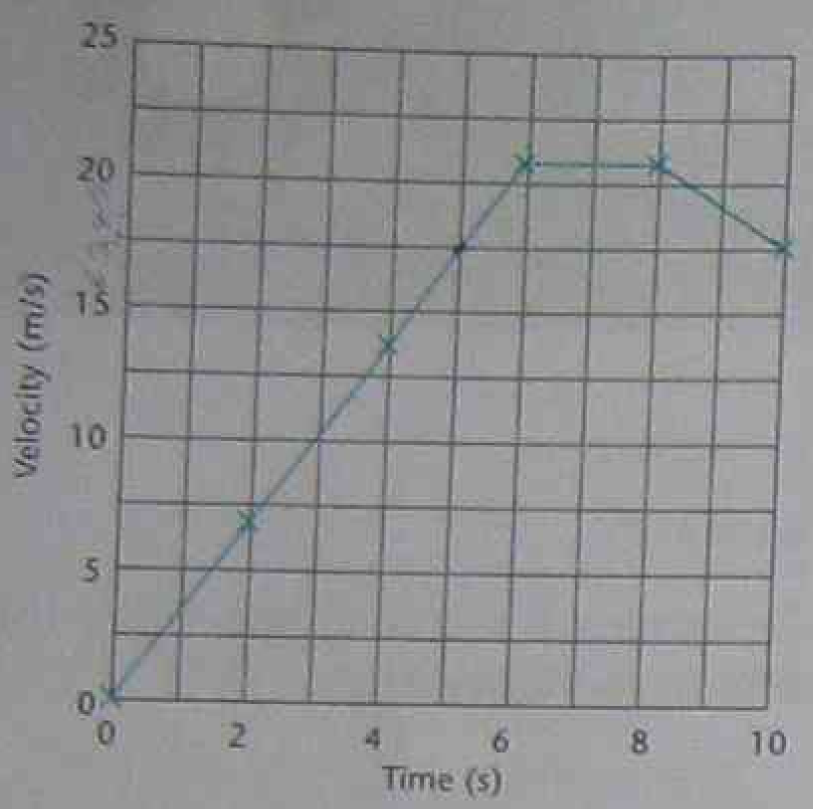


Figure SC4 Line graph of a vehicle's velocity versus time

- a 12.5 m/s
- [✓ b] 17 m/s ✗
- c 17.5 m/s
- d 20.0 m/s

5 A suitable unit for measuring the distance to a star in a galaxy outside the Milky Way would be:

- a Astronomical units (AU)
- b Kilometres (km)
- [✓ c] Light years (ly) ✓
- d Years (y)

6 A falling body accelerates uniformly in the absence of air friction. The following table shows the velocity of a body that is dropped from rest in a vacuum. Determine the value of the velocity after 4 seconds.

Time (s)	Velocity (m/s)
0	0
1	9.8
2	19.6
3	29.4
4	
5	49.0
6	58.8

- a 34.3 m/s
- [✓ b] 39.2 m/s ✓
- c 44.1 m/s
- d 48.0 m/s

7 What is the correct symbol for the element iridium?

- a I
- b In
- [✓ c] Ir ✓
- d Id

8 The following information was collected about the chemical formulae for a number of compounds containing the element phosphorus:

- phosphorus trioxide: P_2O_3
- phosphine: PH_3
- potassium phosphide: K_3P

Which of the following formulae represents a likely compound formed between phosphorus and chlorine?

- a PCl
- [✓ b] PCl_2 ✗
- c PCl_3
- d PCl_4

9 Newton's second law of motion states that the acceleration *a* of a body of mass *m* is directly proportional to the net force *F* applied. Which mathematical relationship is consistent with this information?

- a $a = Fm$ ✓
- b $m = Fa$
- c $F = m/a$
- [✓ d] $F = ma$

10 Select the element that is a semi-metal.

- a calcium
- b fluorine
- [✓ c] silicon ✓
- d argon

11 Figure SC5 shows the structure of a magnesium atom.

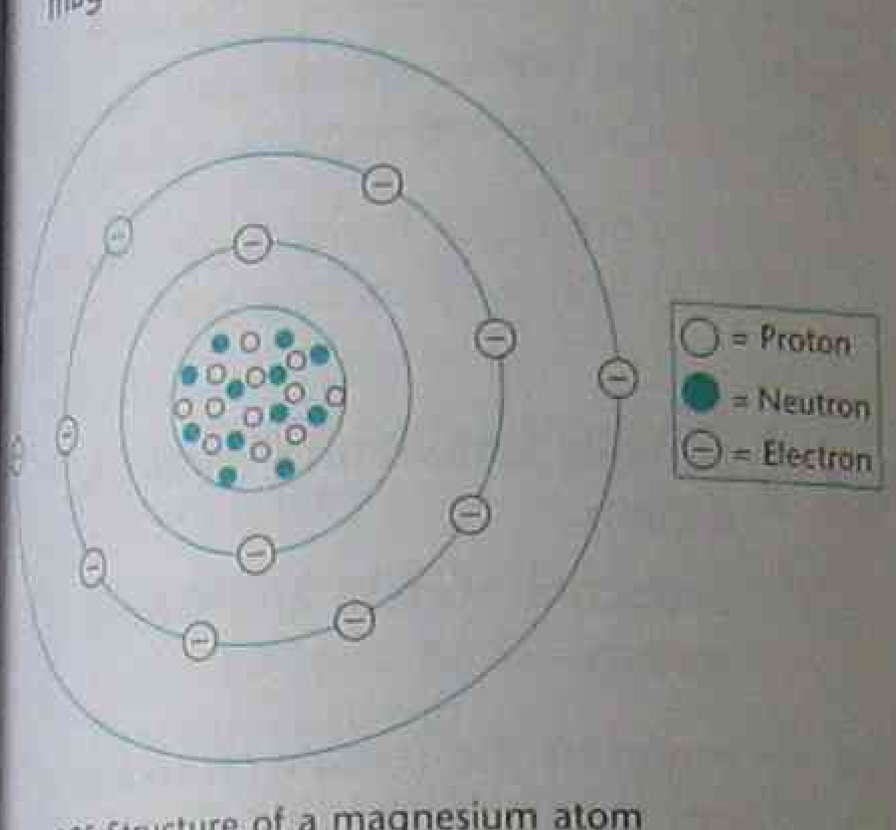


Figure SC5 Structure of a magnesium atom

Select the correct response about magnesium.

- [✓ a] Magnesium has two outer-shell electrons. ✓
- b The magnesium nucleus contains 24 protons.
- c There are more protons than neutrons in the nucleus.
- d The charge on the nucleus is –12.

12 A useful substance that is used to determine the pH of the water in a swimming pool is:

- a chlorine
- [✓ b] Universal indicator ✓
- c sodium chloride
- d iodine solution

13 Which of the answers in Figure SC6 shows a correctly wired electric circuit? D ✓

14 Classify the wave shown in Figure SC7 and determine its wavelength.

- a transverse wave; wavelength = 4.0 m
- b longitudinal wave; wavelength = 4.0 m
- [✓ c] transverse wave; wavelength = 400 mm ✓

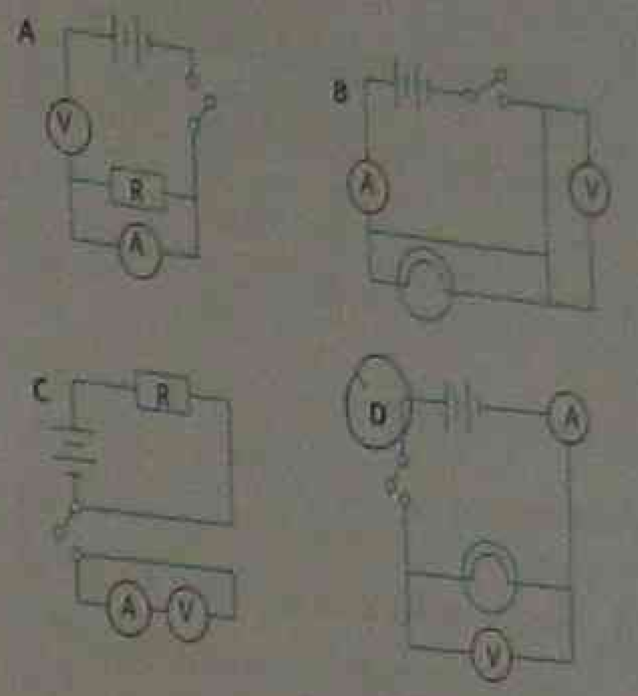
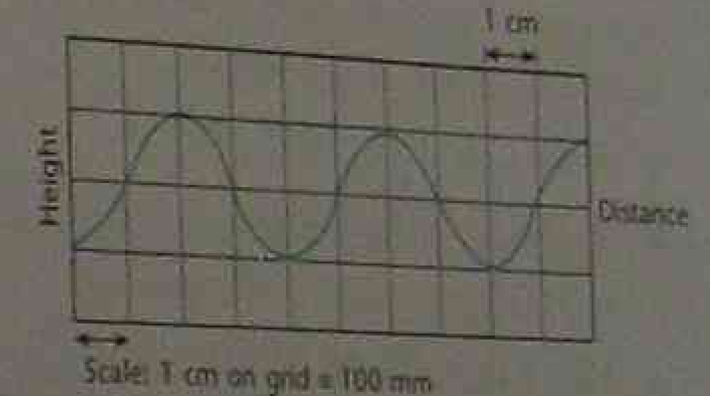


Figure SC6 Electric circuit



Scale: 1 cm on grid = 100 mm

Figure SC7 Wave

- d longitudinal wave; wavelength = 400 mm

15 Select the response that correctly identifies a component wave of the electromagnetic spectrum and its use:

Answer	Wave	Use
[✓ A]	sound	sonar
B	seismic	location of mineral deposits
C	gamma	sterilisation of instruments
D	infrared	detection of bone fractures

✗

a

16 Select the response in Figure SC8 that correctly identifies the path of a light ray as it travels through the glass. D ✗

17 Which of the following word equations describes a decomposition reaction?

- a methane + oxygen → carbon dioxide + water
- [✓ b] copper carbonate → copper oxide + carbon dioxide ✓
- c sodium chloride + silver nitrate → silver chloride + sodium nitrate

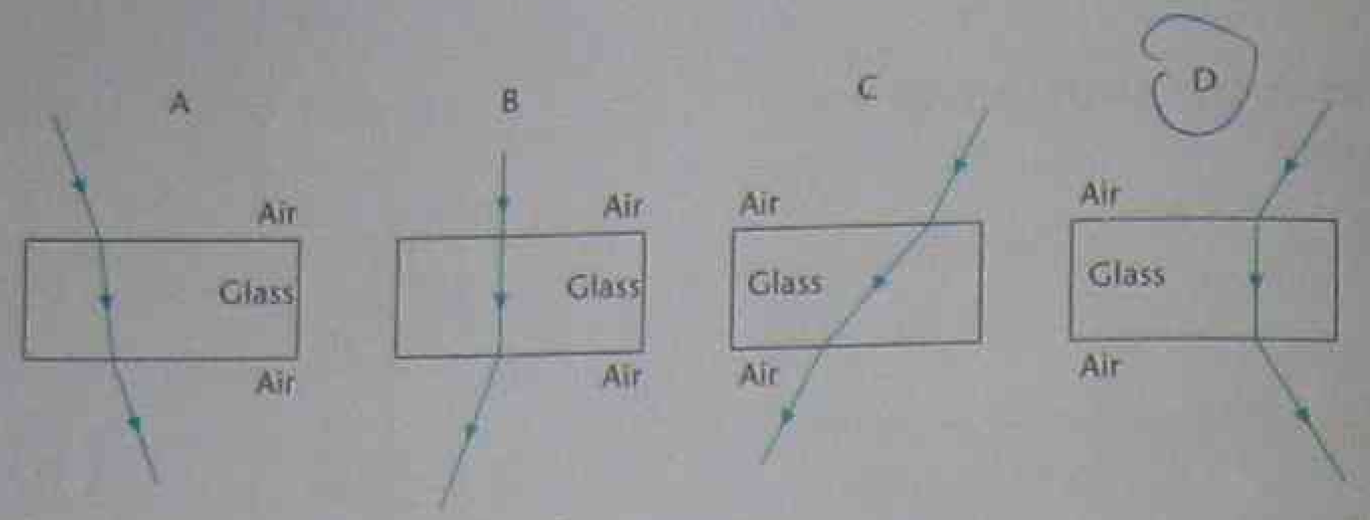


Figure SC8 Path of a light ray

d zinc + sulfuric acid → zinc sulfate + hydrogen

18 Select the set of compounds that represent common components of the exhaust from a motor vehicle.

a H_2O; CH_4; CO_2; NH_3

~~b~~ H_2O; CO; CO_2; C_8H_{18}

[✓ c] CO; CO_2; C; NO

d NO; CO; CuO; Pb

19 Which of the listed processes is not a role of cell division in multicellular organisms?

a growth ✓

b repair

c reproduction

[✓ d] respiration

20 Select the statement that is true of the DNA molecule.

[✓ a] DNA has a double helix structure.

b DNA is composed entirely of sequences of four different nitrogen bases. ✓

c The DNA backbone consists of phosphate groups alternating with nitrogen bases.

d DNA is a protein molecule.

21 Which of the following systems are components of the coordination systems of the body?

a digestive system and respiratory system

b endocrine system and circulatory system

[✓ c] nervous system and endocrine system ✓

d endocrine system and skeletal system

22 An organ of the female reproductive system that produces gametes is the:

a uterus

b Fallopian tube ✓

c vagina

[✓ d] ovary

23 Select the statement that is *not* used as evidence that present-day organisms have developed from different organisms in the distant past.

a Horse fossils show significant changes over the last 60 million years.

b The bones at the ends of the limbs of vertebrates are based on a common structure. ✓

c Following extreme climate changes, fossil evidence shows the appearance of many new species.

[✓ d] Breeding experiments with dogs show that new breeds can be developed.

24 A major problem faced by ground-based astronomers is: A

~~a~~ Our atmosphere absorbs many components of the electromagnetic spectrum. X

[✓ b] Radio waves cannot pass through the stratosphere.

c There is no part of Earth that does not suffer from visible light pollution.

d The atmosphere is too hot to obtain reliable images from space.

25 Identify the correct example of fossilisation and an example of this process.

a altered hard parts—mammoths frozen in ice ✓

[✓ b] trace fossils—imprints of jellyfish

c unaltered hard parts—opalised dinosaur skeleton

d moulds and casts—insect trapped in amber

26 Which diagram in Figure SC9 shows a subduction zone at plate boundaries? C

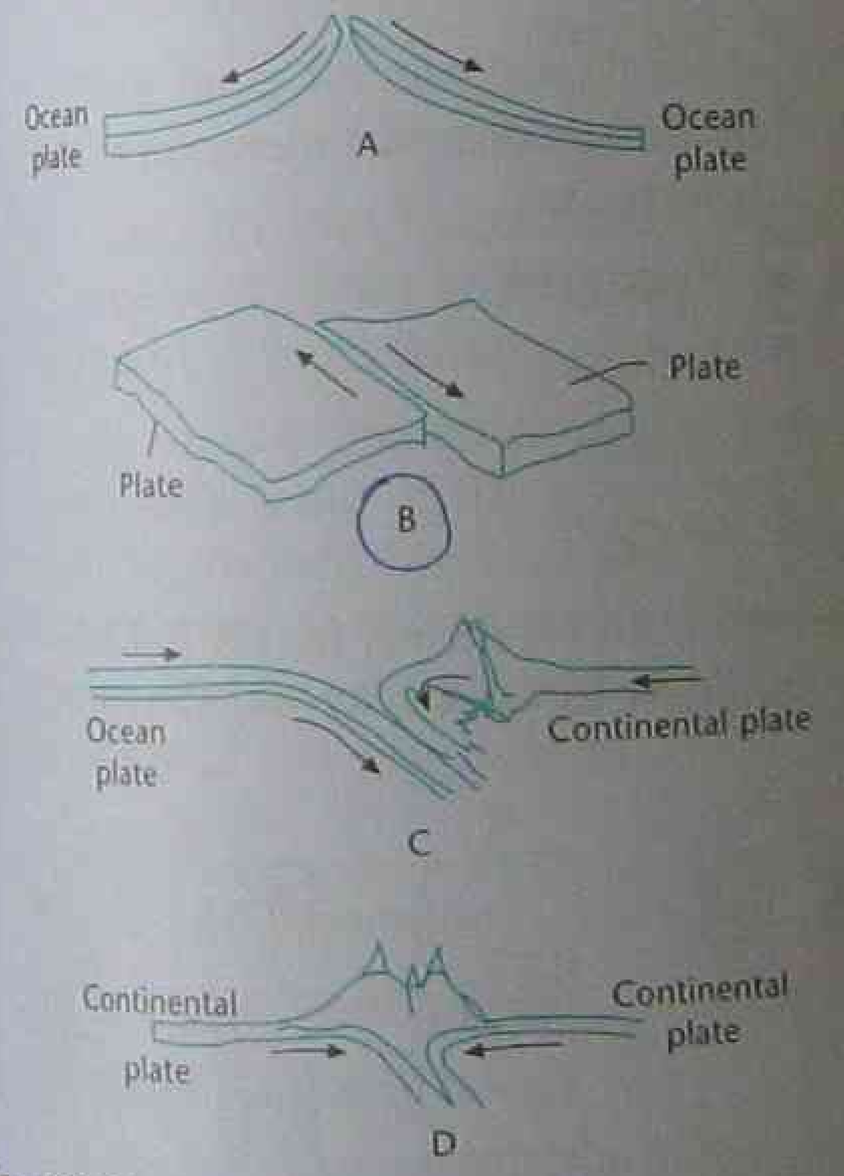


Figure SC9 Diagrams of plate boundaries

27 Which response contains both a biotic and abiotic feature of a rainforest ecosystem?

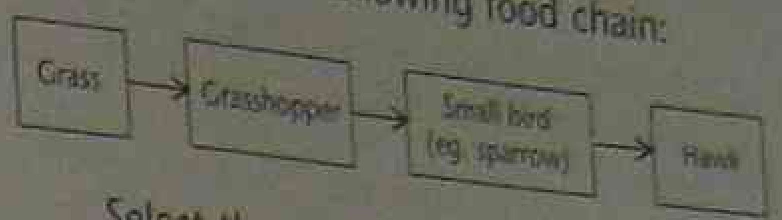
a rainfall; wind speed

b competition for mating; predation ✓

[✓ c] competition for food; light intensity

d light intensity; water quality

28 Consider the following food chain:



Select the correct response concerning this food chain.

[✓ a] The grasshopper is a first order consumer.

b The small bird is a producer.

c The food chain contains two herbivores. ✓

d The hawk is a fourth order consumer.

29 A common location for earthquakes and volcanoes is:

[✓ a] the Pacific rim ✓

b the south Pacific Ocean

c Antarctica

d Western Europe

30 The following information was gathered by a student:

- The Palaeozoic Era is older than the Mesozoic Era.
- Dinosaurs flourished in the Mesozoic Era.
- The Pre-Cambrian Era came before the Palaeozoic Era.
- Mammal fossils are found in Mesozoic rocks but not in Palaeozoic rocks.

Which of the following statements is true of the Palaeozoic Era?

a The Palaeozoic era is more recent than the Mesozoic era.

[✓ b] Fish and amphibians were the dominant vertebrate life forms during the Palaeozoic era. ✓

c Mammals first evolved during the Palaeozoic era.

d The Palaeozoic era dates back to the formation of a solid crust on Earth.

31 As the Sun ages, it will evolve into a giant star. What colour is that giant star?

a yellow
[✓ b white]
c blue
d red

32 Which of the following activities will increase global warming?

a Using nuclear energy to generate electricity
b Using battery-powered cars
c Release of chlorofluorocarbons into the atmosphere ✓
[✓ d Burning of coal and oil]

33 The blue colour of the sky is due to what process involving light rays from the Sun?

a reflection
[✓ b scattering]
c absorption ✓
d condensation

34 Name the common chemical substance represented by the following formula: H_2SO_4.

a hydrochloric acid ✓
b sulfur dioxide
c sulfur trioxide
[✓ d sulfuric acid]

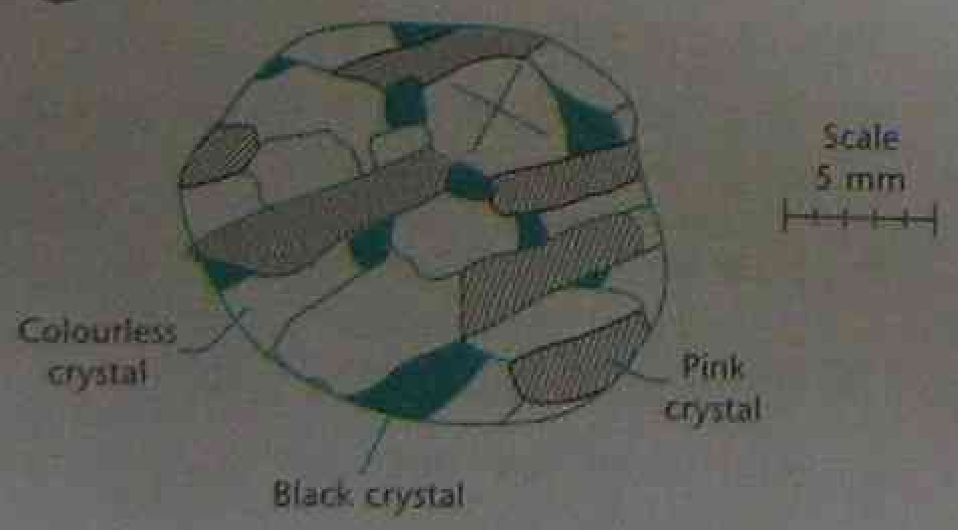


Figure SC10 Scale diagram of a section through a rock

The following two questions (35 & 36) refer to Figure SC10.

35 This rock consists entirely of inter-growing crystals. This rock could have been formed:

a by compaction and cementation of sediments
b from sediments collecting in a deep lake fed by mountain streams
[✓ c from magma cooling inside Earth] ✓
d gaseous lava cooling on Earth's surface

36 The true length of crystal (X) in the diagram is:

[✓ a 4 mm] ✓
b 2 cm
c 4 micrometres
d 0.2 cm

37 The most common element in the universe is:

[✓ a hydrogen] ✓
b helium
c carbon
d oxygen

38 Figure SC11 shows fossils in four different locations M, N, O and P.

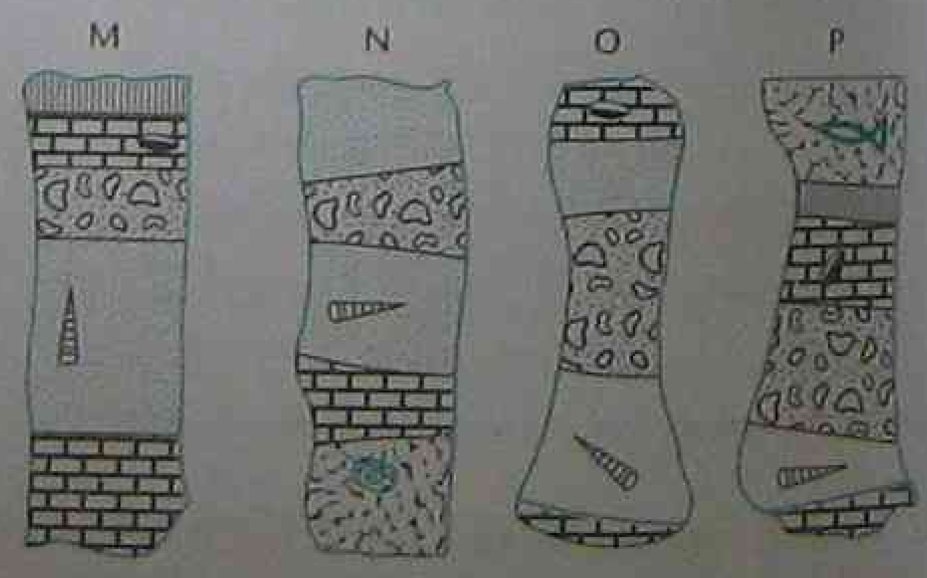


Figure SC11 Fossil layers

In which location is the oldest fossil located? ✓

a M
[✓ b N]
c O
d P

39 The Australian scientist Howard Florey won the Nobel Prize for isolating the antibiotic *penicillin* from a mould. This antibiotic has been useful because of its ability to treat diseases caused by:

a viruses
b fungi ✓
[✓ c bacteria]
d mosquitoes

40 Which of the following statements is *not* part of Charles Darwin's theory of natural selection?

a Organisms with favourable variations survive to reproduce.
b Within a species there is a natural variation.
c Over time, favourable characteristics are preserved in a population. ✓
[✓ d Characteristics acquired during an organism's life can be passed on to the next generation.]

41 A 5 kg mass is placed on a sheet of paper on a table. If the paper is very suddenly moved horizontally, the mass is left almost at the same position on the table. This observation is an example of the principle of:

a inertia
b mass conservation
c energy conservation
[✓ d action and reaction]

42 The fossil evidence for the dinosaur *Tyrannosaurus rex* has traditionally been interpreted in terms of it being a predator. A recent exhibit in the Natural History Museum in London puts forward an alternative theory that *T. rex* was a scavenger. Reasons in support of this theory include:

- the jaw shape and teeth are consistent with an animal that crunches bones rather than tearing flesh;
- because it was a large animal, it would be too slow moving to be an effective predator;
- its small front limbs were unsuitable for catching prey.

This change in the interpretation of the fossil evidence is an example of:

a poor quality scientific work by earlier scientists
b scientific theories being modified or rejected as a result of available evidence
c the establishment of a scientific law
[✓ d new observations producing new generalisations]

43 One of the worst mass extinctions on Earth occurred at the end of the Ordovician period 438 million years ago. One organism that was particularly affected was the marine trilobite. The extinction rate was higher for trilobites that lived in surface waters than for trilobites that were deep water dwellers. One recent theory put forward to explain these extinctions is that Earth at this time suffered periods of gamma-ray bursts emitted from the collapse of a nearby giant star and the formation of a black hole. According to the theory, the gamma rays converted some gases in the atmosphere into toxic nitrogen oxides that temporarily destroyed the protective ozone layer. For over a year, life on Earth would have been exposed to dangerous radiation that was normally blocked by the ozone layer. This dangerous radiation that is normally absorbed by the ozone layer is:

a infrared
b microwave

[✓ c] ultraviolet ✓
d X-rays

44 On 21 September 2003 the *Galileo* spacecraft, which was launched in 1989, was intentionally allowed to crash into the planet Jupiter. Gravitational forces and frictional melting tore the craft apart. During its operational life *Galileo* sent over 14 000 images of Jupiter and its moons to Earth.

These images were relayed back to Earth by: ✓

a compressional waves
b ultraviolet waves
c visible light waves
[✓ d] radio waves

45 The following information was gathered about tropical cyclones.

- The air pressure at the centre of a cyclone is quite low.
- Wind speed meters show increasing wind speeds with altitude around a tropical cyclone.
- Wind direction vanes show that air is moving outward at high altitude in a tropical cyclone.
- The air in tropical cyclones is very moist.

Select the statement that is true about tropical cyclones.

a Tropical cyclones form over dry land. x
[✓ b] In a tropical cyclone, air spirals inward at low levels and flows outward at high levels. ✓
c A tropical cyclone is also called a high pressure system. x
d Strong winds that blow outward are created at the base of a tropical cyclone.

46 A reflex arc is important because it:

a ensures that the brain has time to coordinate a response
b involves coordination between the endocrine and nervous systems
[✓ c] allows the body to respond quickly in time of danger ✓
d is a conscious and coordinated response to signals arriving from multiple sites

47 The element most chemically similar to sulfur is:

[✓ a] phosphorus
b chlorine
c carbon
d selenium ✓

48 Which of the following examples best illustrates Newton's third law of action and reaction?

[✓ a] Rockets move forward by expelling gases produced in the combustion chamber. ✓
b A ball falls down to Earth because of the pull of gravity.
c Heavy bodies tend to remain at rest.
d A tennis ball has greater acceleration when it is struck by a greater force.

49 Figure SC12 shows four electric circuits involving identical lamps. In which circuit does the lamp glow brightest?

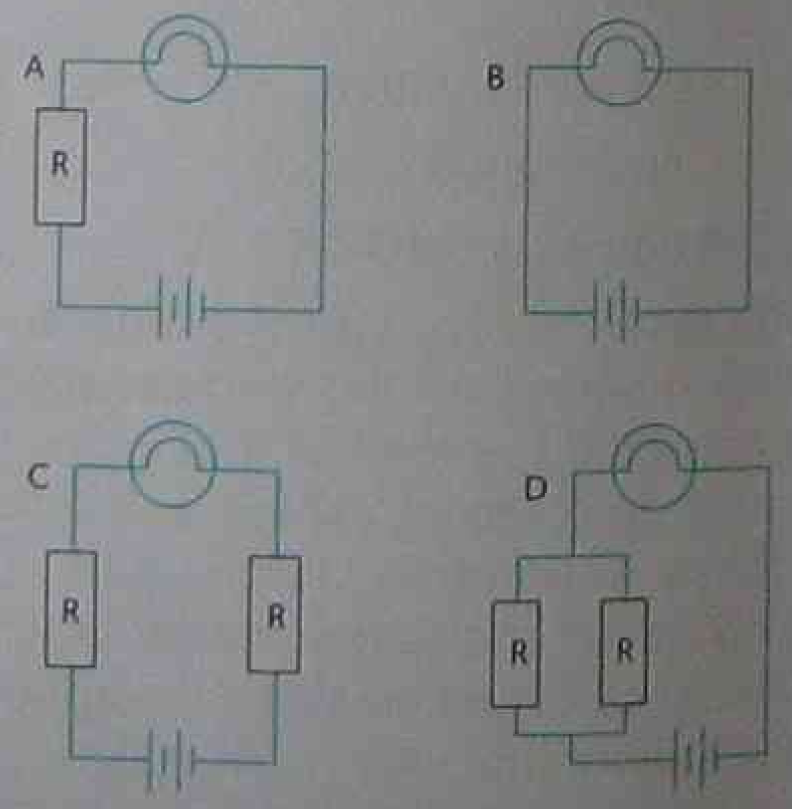


Figure SC12 Four circuits with identical lamps

a circuit A
[✓ b] circuit B ✓
c circuit C
d circuit D

50 Fossils are most unlikely to form in:

a sediment off the east coast of Australia
b the mud at the bottom of a swamp ✓
c the sand of a desert
[✓ d] hot lava pools

39/50

Section 2. Questions 51–66

Part A. Restricted response (10 marks)

Select the correct word to complete the statement.

51 Nuclear energy can be released from some heavy elements such as ______ (Z = 92) by fission reactions. (1 mark)

52 The instrument that measures the size of the electric current flowing in a circuit is an ammeter. (1 mark) ✓

53 When hydrochloric acid is added to a solid such as sodium carbonate, the gas that is released is called carbon dioxide. (1 mark) ✓

54 The rusting of iron is an example of the chemical process called corrosion. (1 mark) ✓

55 The non-living parts of an ecosystem are called abiotic components. (1 mark) ✓

56 Measles, tuberculosis, mumps and whooping cough are examples of infectious diseases. (1 mark) ✓

57 The movement of tectonic plates results from convection currents in the asthenosphere as well as gravitational forces. (1 mark) ✓

58 The excessive use of fossil fuels is believed to be a contributing factor to the enhanced greenhouse effect. (1 mark) ✓

59 Developments in the production of made materials', or new materials, has led to the manufacture of many new devices and products. (1 mark) x

60 Scientific theories can only be judged on the basis of scientific observations. (1 mark) x

7

Part B

61 Classify the following substances as acidic, basic or neutral. Present your answer in the form of a table.

Substances: salt; vinegar; ammonia cleanser; caustic soda; soda water; sugar (4 marks)

neutral acidic basic basic acidic neutral

62 a Figure SC13 shows a nuclear reaction involving uranium. Large amounts of energy are released in this reaction.

Figure SC13 Nuclear reaction

i Identify the particle (X) in the diagram. (1 mark)
ii Explain how the energy released from this reaction can be converted into electrical energy. (3 marks)

b Evaluate the environmental impact of using nuclear energy compared with other forms of energy generation used in our society. (6 marks)

Part C

63 When calcium salts, such as calcium chloride, are dissolved in water the water becomes 'hard'. Hard water is water that will not lather (form bubbles) when shaken vigorously with soap solution.

Propose an experimental investigation to test the following hypothesis:

The hardness of water increases as the concentration of calcium salts increases. (6 marks)

64 Figure SC14 shows a graph of the concentration of dissolved oxygen and carbon dioxide as a function of depth in the ocean. The concentrations are reported in the units 'parts per million' or 'ppm'.

a Identify the gas that has the higher concentration in surface waters. (1 mark)

b Describe the depth at which the oxygen concentration begins to rise again rather than fall with increasing depth. (1 mark)

c Explain how the graph supports the following statement:

'Biological respiration and decomposition of organic remains increases with depth'. (2 marks)

d Explain how the graph supports the following statement:

'Photosynthesis in aquatic plants and phytoplankton is confined to the upper 150 m of the ocean where light can penetrate.' (2 marks)

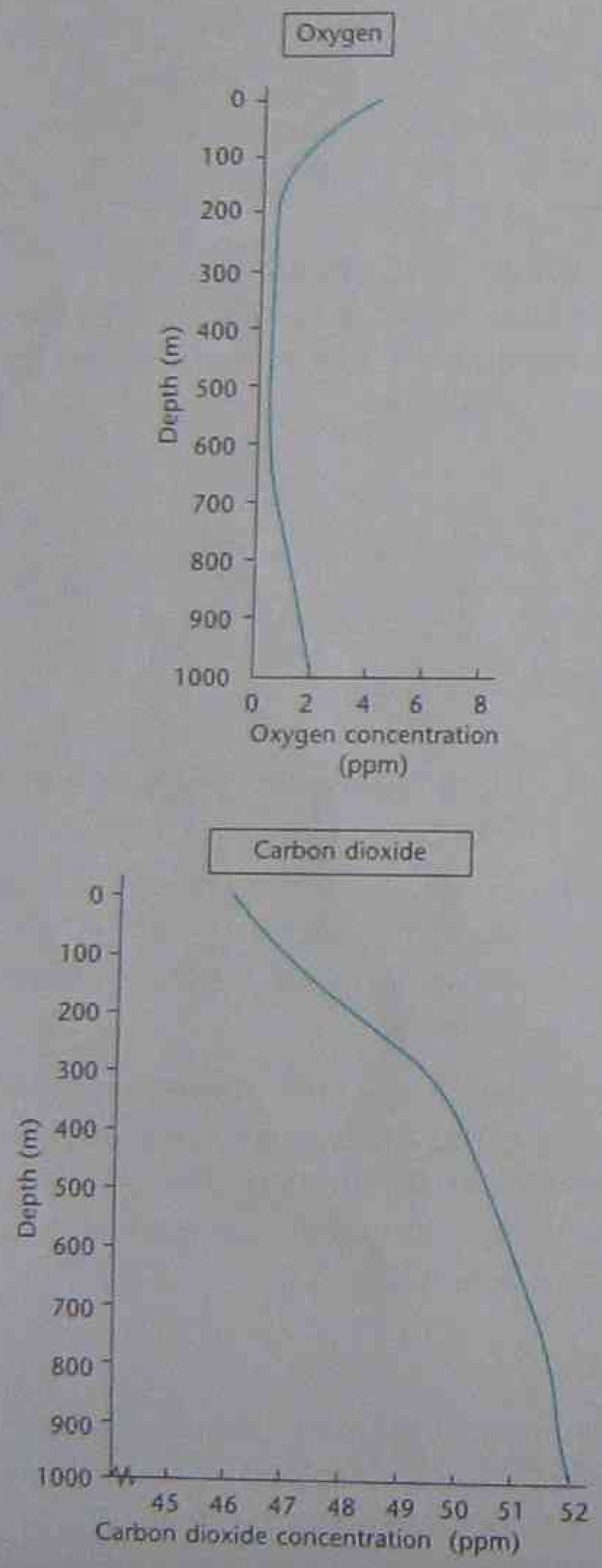


Figure SC14 Concentration of oxygen and carbon dioxide with ocean depth

Part D

65 Information about Earth's history can be obtained by investigating fossils and stratigraphic analysis.

a The following information was collected about certain types of fossils:

- Invertebrates appear in the fossil record before vertebrates.
- The earliest vertebrate forms lived in the sea.
- Amphibian fossils predate reptile fossils.
- Mammal fossils are the most recent.

Process and analyse this information and place the following organisms in the correct evolutionary sequence based on the appearance of their fossils in the fossil record.

Organisms: snake; frog; whale; dragonfly; shark (2 marks)

b Figure SC15 shows rock strata in a road cutting.

i Identify the oldest rock in the sequence. (1 mark)

ii The following geological history is jumbled. Re-organise the jumbled statements to produce the geological history of this area. Use the code numbers in your re-organisation. (3 marks)

Jumbled statements:

1. Folding to form an anticline
2. Deposition of shale
3. Erosion by a river to form existing landscape
4. Deposition of conglomerate
5. Deposition of sandstone
6. Deposition of limestone
7. Deposition of mudstone
8. Extrusion of lava to form a basalt lava plain. Erosion
9. Subsidence and deposition of sandstone
10. Uplift and erosion to form a plain

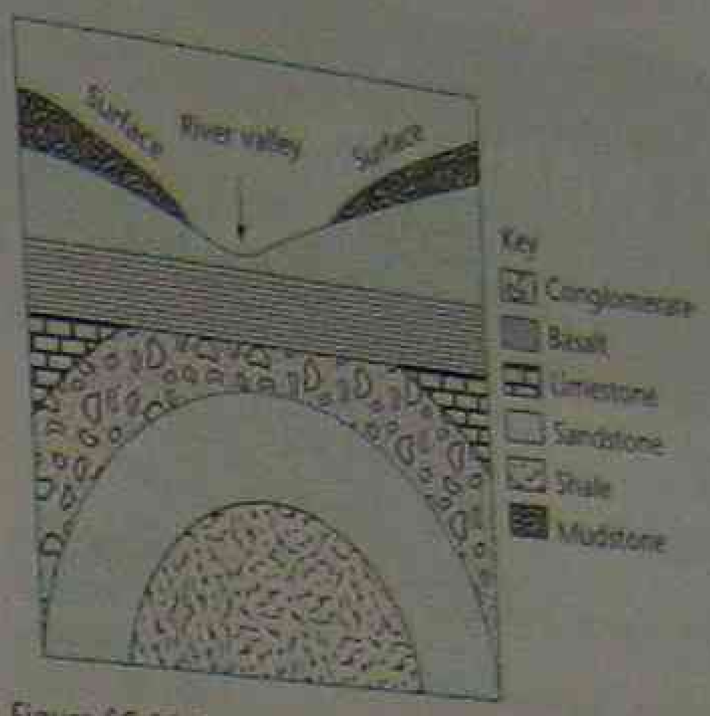


Figure SC 15 Geological section

66 The table shows the results obtained during an experiment in which the temperature of a test tube of melted wax was recorded each two minutes as the wax was allowed to cool and solidify.

Time (min)	0	2	4	6	8	10	12	14
Temperature (°C)	60	56	52	52	52	50	47	44

a Construct a line graph of the experimental data. (4 marks)

b Identify the dependent variable in this investigation. (1 mark)

c Suggest an explanation for the observation that the temperature remained constant between the 4th and 8th minutes. (1 mark)

d Predict and explain whether or not the results of the experiment would be different if the melted wax sample was allowed to cool in a 50 mL beaker rather than a test tube. (2 marks)

Answers

Chapter 1

The wave model pages 5–7

Part A. Knowledge pages 5–6

1 b Gamma waves are electromagnetic. ✓

2 c There is no air on the moon and so communications are by radio waves and visible light. ✓

3 a The water particles at the surface essentially oscillate up and down; water waves are not compression waves. ✓

4 d 0.4 Hz means 0.4 waves per second or 4 waves in 10 seconds ✓

5 a Infrared rays have shorter wavelengths and higher frequencies. ✓

6 a transverse ✓
  b amplitude ✓
  c electromagnetic ✓
  d medium ✓
  e rarefaction ✓ (5 marks)

7 a/i ✓ ; b/f ✓ ; c/j ✓ ; d/g ✓ ; e/h ✓ (5 marks)

8 In a rock the particles are much closer together than they are in air. As sound is a compression wave, it takes longer for air particles to move forward to collide and pass on their energy. ✓ In rocks the sound energy is rapidly transmitted as the particles move only an extremely small distance before colliding with the next particle. ✓ (2 marks)

9 The students stretch the spring along the floor and hold each end. One student moves one end of the spring sideways in a left-to-right motion. ✓ This generates a transverse wave down the spring. To generate a longitudinal wave, one student pushes and pulls the end of the spring along the axis of the spring. ✓ (2 marks)

10 Visible light is used by astronomers to detect bodies such as stars and planets. ✓ Radiowaves that are emitted from deep-space objects such as pulsars can be studied using radiotelescopes. ✓ Infrared sources in space can be observed using spectroscopes and telescopes fitted to orbiting spacecraft. ✓ (3 marks)

Part B. Skills pages 6–7

1 Wavelength = 3.6 cm × 20 = 72 cm ✓
= 0.72 m ✓
(2 marks)

2 a Wavelength = 4.6 × 10 = 46 cm ✓
= 0.46 m ✓
(2 marks)

  b f = velocity/wavelength= 320/0.46 ✓
= 696 Hz ✓
(2 marks)

3 a B ✓
  b A ✓
  c Wave A: T = 3.4 × 0.01 = 0.034 s; $f = 1/T = 1/0.034 = 29.41$ Hz ✓ Wave B: $T = 1.7 \times 0.01 = 0.017$ s; $f = 1/0.017 = 58.8$ Hz ✓ (4 marks)

4 a Station A (702 kHz)
  b i Much shorter wavelength than station A (and also B) ✓
    ii Same velocity (speed of light) for all electromagnetic waves. ✓ ✓ (3 marks)

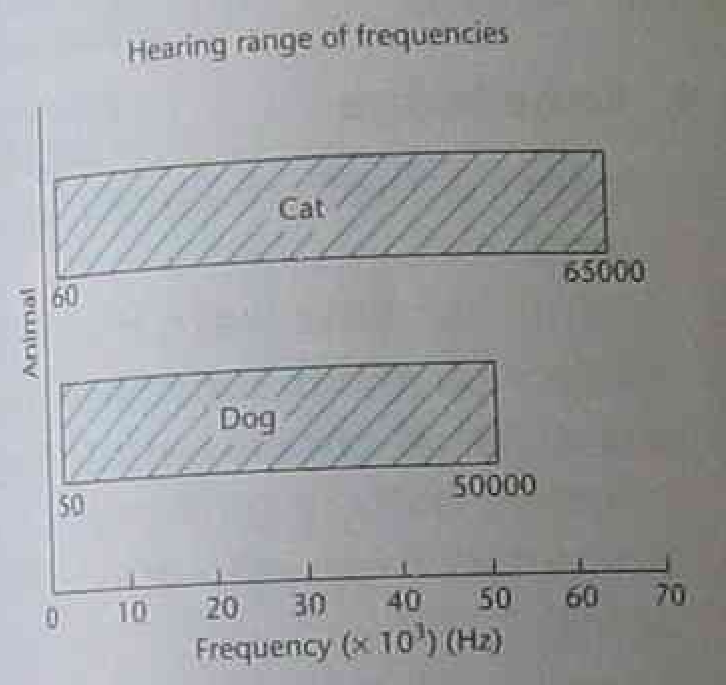
5 Calculate the velocity in each medium using the wave equation.
  a $v = (540)(2.5) = 1350$ m/s
  b $v = (90)(3.5) = 315$ m/s
  c $v = (950)(5.0) = 4750$ m/s
Thus, A = liquid; B = gas (slowest); C = solid (fastest) (2 marks)

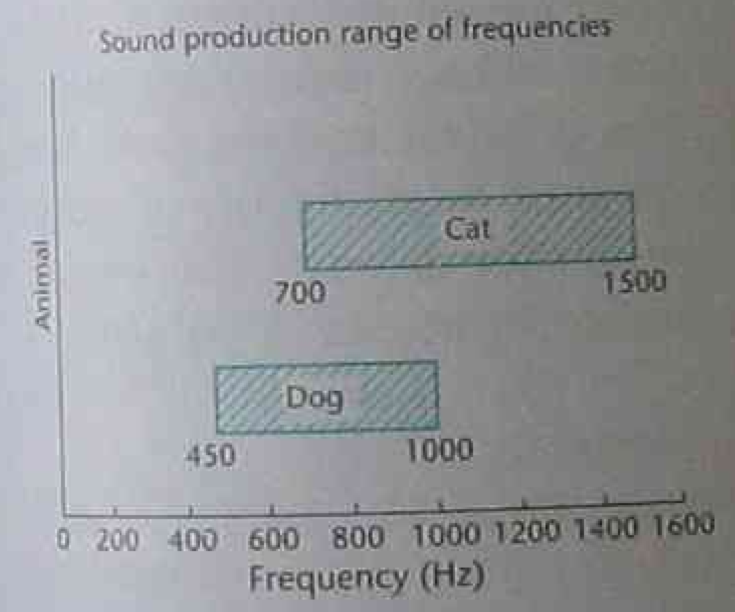
a Total distance travelled by sound wave = 2 × 100 = 200 m
speed = 200/0.59 = 339 m/s
(2 marks)

  b New speed = 200/0.64 = 313 m/s
The speed of sound decreases as the air temperature decreases. (2 marks)

a red ✓
  b 450 nm ✓
  c $v = f \cdot \lambda$; f = velocity/wavelength = $(3 \times 10^8)/(500 \times 10^{-9}) = 6 \times 10^{14}$Hz ✓
(2 marks)

a





  b i cats ✓
    ii cats ✓
  c No—this sound is above their hearing ranges. ✓ (2 marks)

Newton's laws of motion pages 12–16

Part A. Knowledge pages 12–14

1 a A body stays at constant speed unless acted upon by a net force. ✓

2 b Newton's third law (action equals reaction) ✓

3 d There is no further motion and so the forces are balanced. ✓

4 b The distances between the first five points are equal and so the car is moving with constant velocity to the east. After that, it slows down as the distances between each car position become smaller. ✓

5 c There is no air on the Moon (no air resistance)—the ball falls under gravity and so is uniformly accelerated. ✓

6 a frictional ✓ (5 marks)
  b field ✓
  c distance ✓
  d third ✓
  e decelerating ✓ (5 marks)

7 a/i ✓ ; b/f ✓ ; c/j ✓ ; d/g ✓ ; e/h ✓
(5 marks)

8 Place a metre rule along the ramp. Use a stopwatch to time how long it takes the golf ball to roll different distances (eg. 10 cm, 20 cm, 30 cm, etc.) from rest down the ramp. Repeat the experiment 5 to 10 times and tabulate the results. Calculate an average. Analyse the results by plotting a distance versus time graph. Calculate average speed between pairs of data points by dividing the distance travelled by the time taken. The average speed increases with time as the ball accelerates longer down the slope. ✓ (4 marks)

9 a The force of the ball on the strings of the racquet will stretch the strings. The extra tension in the stretched strings provides a reaction force that acts on the ball and it rebounds back across the net. ✓
  b The forces of rotation (torque) are balanced because the see-saw is not moving. ✓
  c To increase frictional drag to rapidly slow the spacecraft ✓
  d Frictional forces increase as the speed of fall increases. Eventually the frictional drag equals the action force down and the skydiver then

moves with constant speed, since the forces are balanced. ✓ (4 marks)

10 a As the first skater throws the ball, he moves backward (Newton's third law). The second skater catches the ball and is pushed backward, because there is little friction on the ice. They get further and further apart, as they throw the ball back and forth. ✓ ✓

b The lighter skater is accelerated more than the heavier skater as the same force acts on different masses. ✓ ✓ (4 marks)

Part B. Skills

pages 14–16

1 240 min = 4 hours ✓ ✓
Average speed = 200/4 = 50 km/h
Answer = d

2 Car W, $a = (40 - 30)/2.1 = 4.76\ \text{m/s}^2$;
car X, $a = (80 - 50)/8.0 = 3.75\ \text{m/s}^2$;
car Y, $a = (20 - 0)/2.5 = 8.0\ \text{m/s}^2$;
car Z, $a = (55 - 20)/3.8 = 9.21\ \text{m/s}^2$
Answer = d ✓ ✓ (2 marks)

3 b Speed increases regularly ✓ (1 mark)

4 a Mass M and Mass N both accelerate along the table; M has the greater acceleration. ✓ ✓ (2 marks)

b N has the greater mass since its acceleration is less than that of M. (2 marks)

c Average speed of M = (128 – 32)/(1.6 – 0.8) = 120 cm/s ✓ ✓ (2 marks)

5 a N, its speed does not change. ✓

b L, its speed is getting smaller. ✓

c K, constant for 25; accelerates for 35 ✓

d M, accelerates for 25; then constant speed ✓

e Average acceleration = (4 – 0)/2 = $2\ \text{m/s}^2$ ✓ ✓ (6 marks)

6 a There are a number of factors that will affect the initial acceleration result, including the mass of the car and the power of its engine. A standing start allows all these factors to be considered. ✓

b Car X has the higher performance because it takes less time to reach the specified speeds. ✓ ✓ (3 marks)

7 a The distance required to bring the car to a stop once the brakes are applied. ✓

b Reaction distance increases. ✓

c Braking distance increases. ✓

d $d = (33.3)(0.75) = 24.98 = 25$ m ✓ ✓

e The reaction distance will not change but the braking distance will increase because the road has less friction. ✓ (6 marks)

Electrical energy

pages 23–6

Part A. Knowledge

pages 23–4

1 d Copper is the best conductor of the four. ✓

2 b Electric currents leave the positive battery terminal and travel through the external circuit back to the negative battery terminal. ✓

3 a Ammeters measure current (in amps). ✓

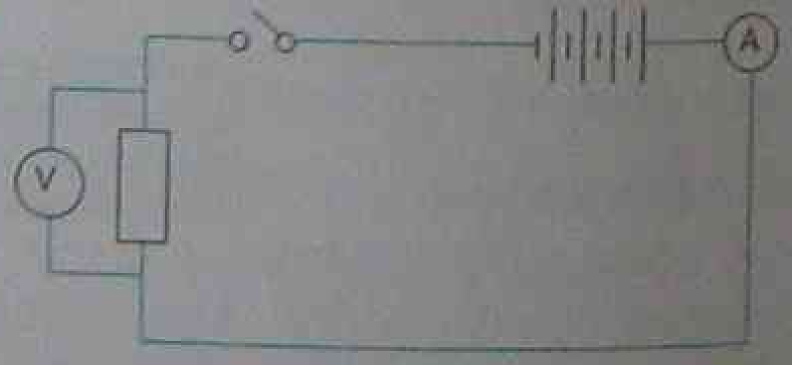
4 c They are all in parallel, so there is the same voltage drop across them all. ✓

5 a The greatest current (and therefore greatest heat) occurs when the resistance is least. This occurs when the resistors are in parallel. ✓ (5 marks)

6 a electrons ✓
b voltmeter ✓
c series ✓
d higher ✓
e ampere (amp) ✓ (5 marks)

7 a/i ✓ ; b/h ✓ ; c/j ✓ ; d/g ✓; e/f ✓ (5 marks)

8



✓
✓
(2 marks)

9 Model Y shows the movement of negative electrons only. The positive metal ions do not move, as shown in Model X. ✓ ✓ (2 marks)

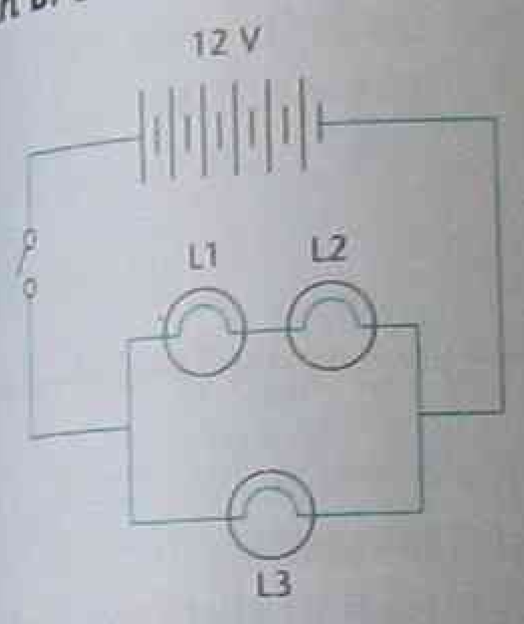
a direct current (DC) ✓

b The four batteries provide a total voltage of 6 V. The total current flow through the lamp will therefore be 4 times greater and thus the lamp will glow more brightly. ✓ ✓ (3 marks)

Part B. Skills

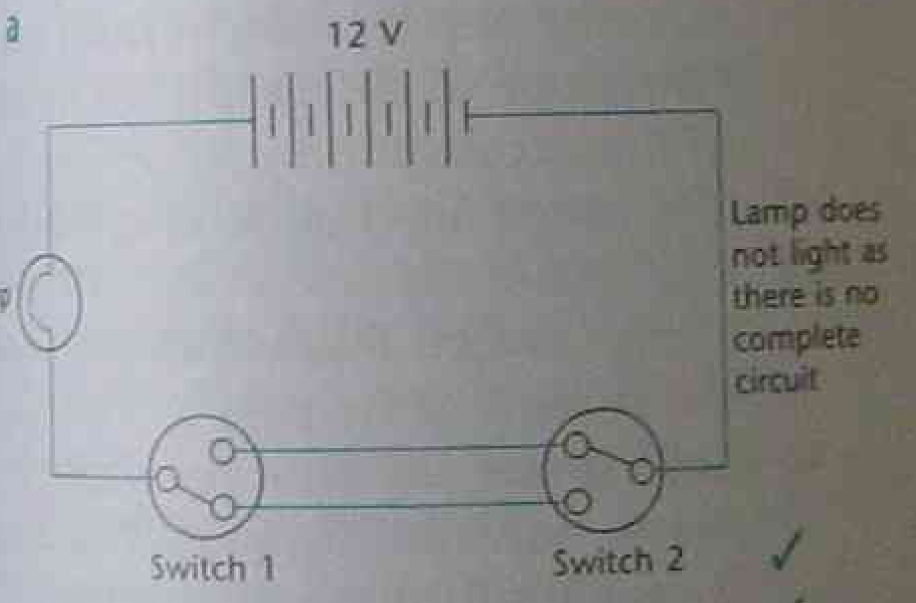
pages 24–6

1



(2 marks)

2 a



✓
✓

b These two-way switches are used in corridors or long rooms so that you can switch lights on/off as you enter/leave at either end. ✓ (3 marks)

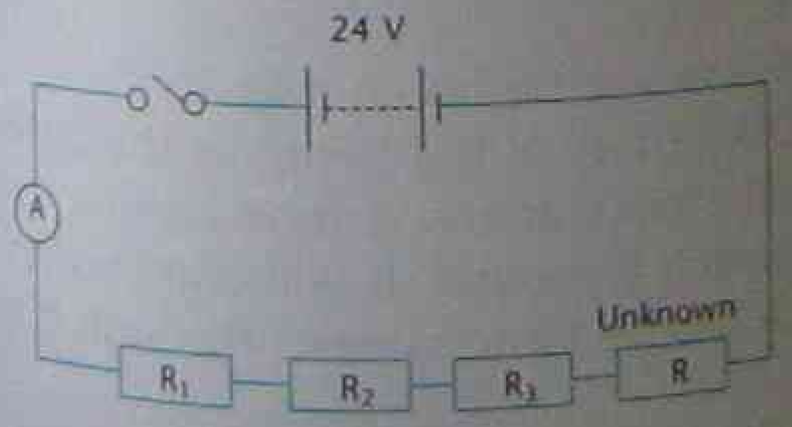
3 a 6.0 – 2.0 = 4.0 V ✓

b $R_{AB} = V_{AB}/I = 2.0/1.0 = 2.0$ ohms ✓

c $R_{CD} = V_{CD}/I = 4.0/1.0 = 4.0$ ohms ✓

d $R_T = R_{AB} + R_{CD} = 2.0 + 4.0 =$ 6.0 ohms ✓ (4 marks)

4 a



✓
✓

b Total resistance = 3 + 4 + 1 + R = 8 + R ohms
V = 24 V; I = 1.5 A
From Ohm's Law: V = IR
24 = 1.5(8 + R). Solve for R
R = 12/1.5 = 8 ohms ✓ ✓

c The lowest voltage drop will be across R_3 as it has the lowest resistance. (5 marks)

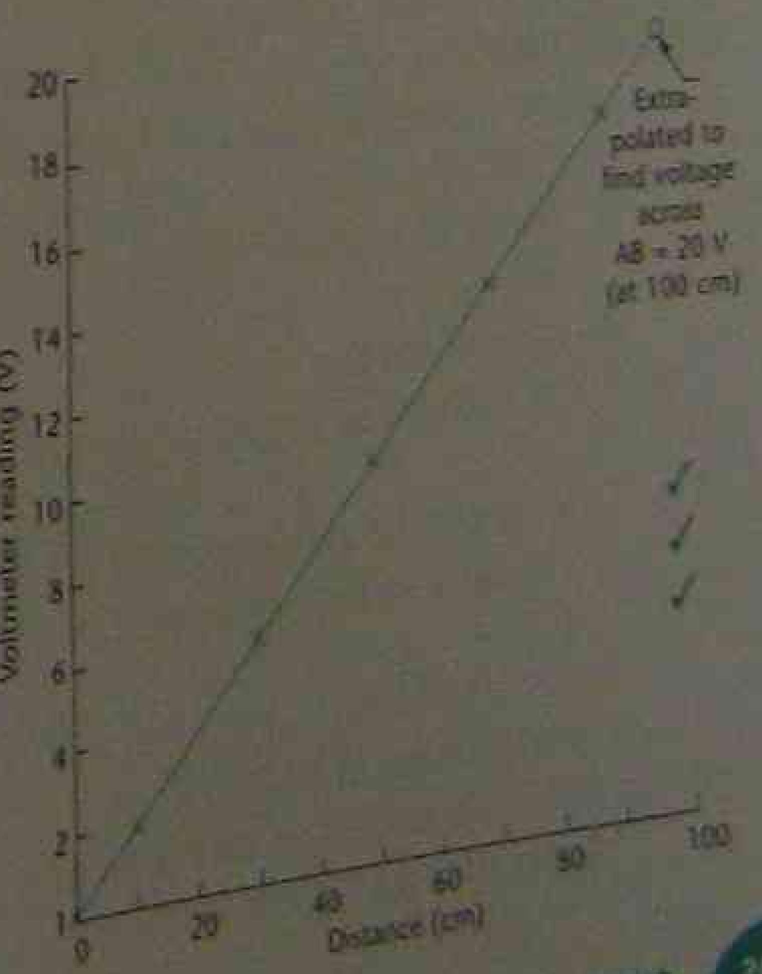
5 a i A_1 has the highest reading as the total circuit current flows through this ammeter.
$[V_1 = V_2 = V_3 = 12\text{V}; V_2 = I_2R_2; 12 = (I_2)(3); I_2 = 4 \text{ amps} = A_2$
$V_3 = (I_3)(R_3); 12 = I_3(4); I_3 = 3 \text{ amps} = A_3$
$I_T = 4 + 3 = 7 \text{ amps} = A_1]$

ii A_3 has the lowest reading as less current flows through the larger resistor.

b The readings are the same in all voltmeters as this is a parallel circuit. ✓ ✓

c The resistance increases and consequently the total current will decrease. A_1 and A_3 will now read the same value (3 A) and A_2 will read zero as no current flows through this arm when the new switch is off. $[R_t = 4 \text{ ohms}; I_t = 12/4 = 3 \text{ A}; A_1 = 3 \text{ A}; A_3 = 3 \text{ A}; A_2 = 0 \text{ A}]$ ✓ (5 marks)

6 a

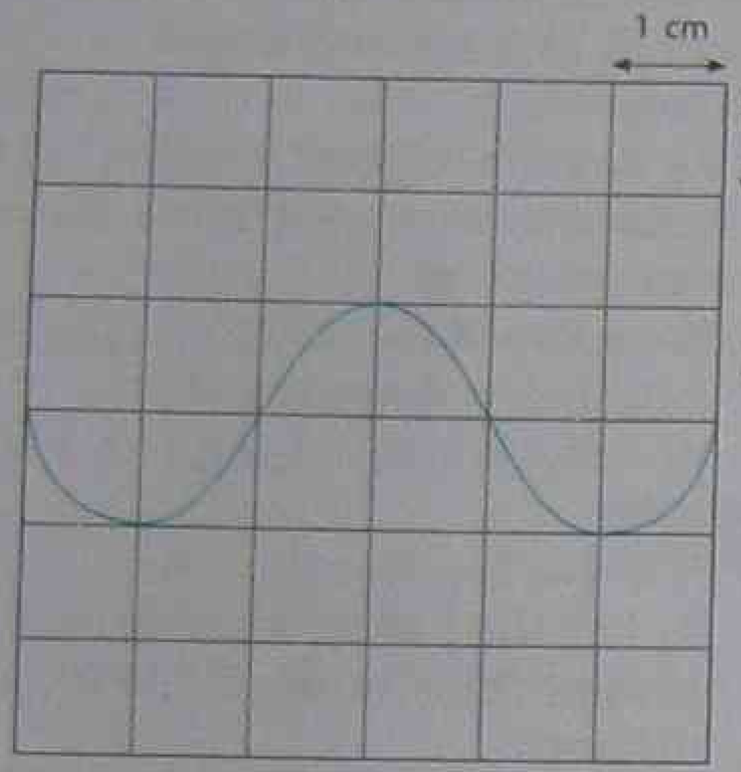


✓
✓
✓

b 20 V (see graph) ✓
c $R = V/I = 20/4 = 5\ \Omega$ ✓✓ (6 marks)

Mid-chapter test pages 26–8

1 (wavelength for double the frequency = 4 cm; amplitude = 1 cm) (2 marks)



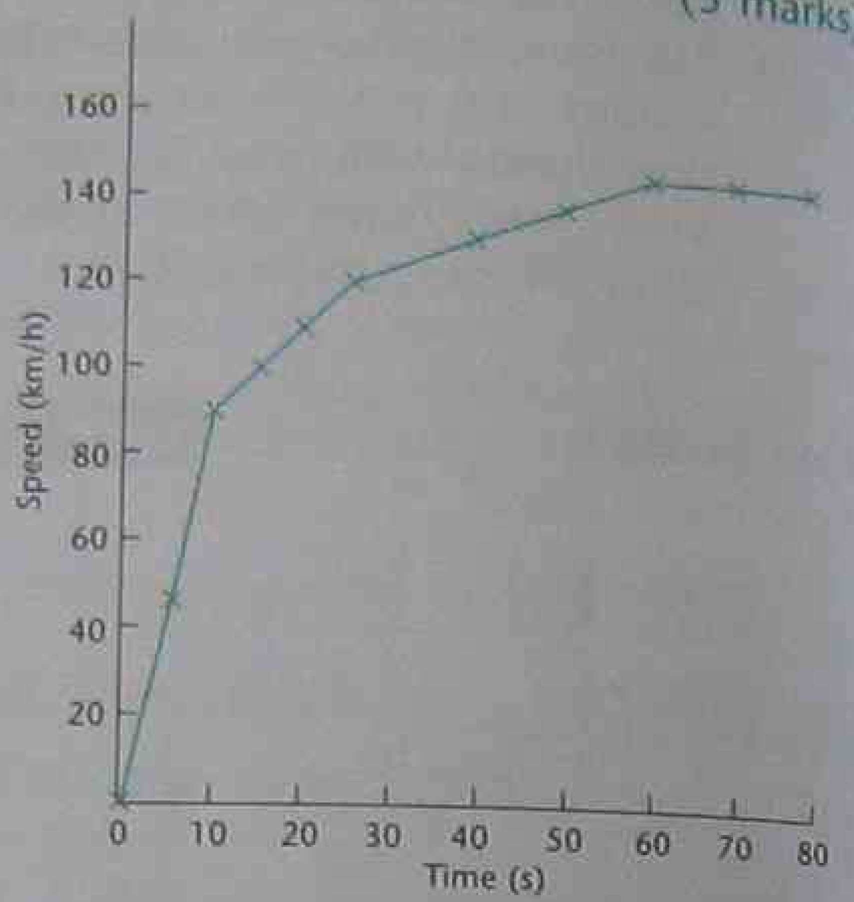
2 a i tap ✓
ii pump ✓
b The rate of water flow will decrease as the diameter of the pipe decreases. ✓
c 1 cm diameter pipe. Narrow pipes have greater resistance. ✓
(4 marks)

3 a Distance = speed × time
= 1450 × 0.15 = 217.5 m
depth of water = distance/2
= 108.75 m ✓
b speed = distance/time = 217.5/0.16
= 1359 m/s ✓✓ (3 marks)

4 a i produces a greater current flow. ✓
ii a lower voltage. ✓
b $R = V/I$ (check by substitution) ✓
(3 marks)

5 a i less than the acceleration of N ✓
ii the same as the acceleration of N ✓
b $X = 2m$; $Y = m$; $F_X = F$; $F_Y = 4F$ acceleration of $X = F/2m = 0.5F/m$ acceleration of $Y = 4F/m$
The acceleration of Y is 8 times greater than the acceleration of X. ✓✓
(4 marks)

6 a (3 marks)



b In the first 10 seconds ✓
c The acceleration in top gear is less than the acceleration in lower gears. (The slope of the graph is a measure of the acceleration.) ✓✓
d 0 N (since no change in speed) ✓
(7 marks)

7 a Y only; switch B when closed allows the current to by-pass Z. ✓✓
b Both lamps will light up. Current has to flow also through Z as there is no path through open switch B. ✓
c Neither lamp will light as the current from the battery cannot pass through the open switch A. ✓✓
(5 marks)

8 a radio waves ✓
b gamma rays ✓ (5 marks)
c visible, radio (infrared and X-rays with space telescopes) ✓
d False ✓
e True ✓ (2 marks)

9 There will be no change in the brightness of the lamp, as the potential difference across it stays constant as the switch is moved. It will continue to draw the same current through it even though the total current in the whole circuit drops as the switch moves from Y to X. ✓✓
(2 marks)

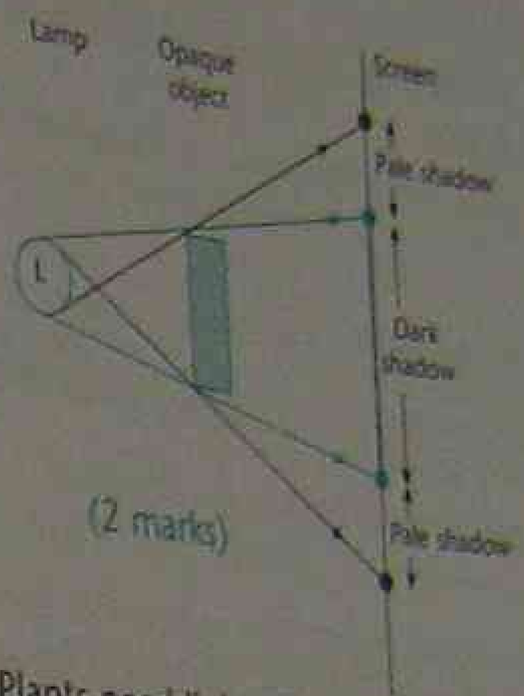
9 a Net force = 700 N up. The glider will move up. ✓
b Net force = 10 − 3 − 2 = 5 N in the direction of the pulling force. The cart accelerates in the direction of the net pulling force. ✓✓
c If the net force is zero he will move with constant (terminal) velocity. The upward drag forces must balance the weight force downward. ✓
(4 marks)

Light energy pages 31–3

Part A. Knowledge pages 31–2

1 c Light rays travel slower in matter than in a vacuum. ✓
2 a The cardboard is shiny and so reflects some light but as it is black, it absorbs most of the incident light. ✓
c The Sun is a star and so makes its own light; the Moon can only reflect or absorb light. ✓
4 d Light slows down in more dense media; if it enters the more dense medium at an angle, it will bend away from its former path. ✓
5 a Convex mirrors provide a good wide-angle view. ✓ (5 marks)
6 a incidence ✓
b opaque ✓
c refraction ✓
d scatter ✓
e laterally ✓ (5 marks)
7 a/f ✓ ; b/j ✓ ; c/g ✓ ; d/h ✓ ; e/i ✓
(5 marks)

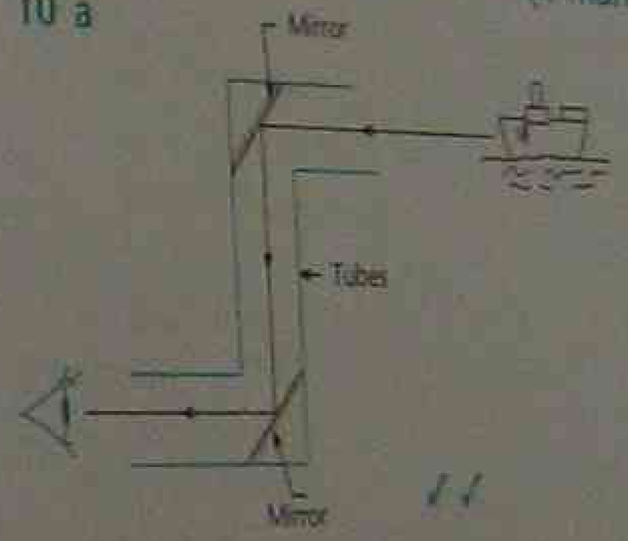
8



(2 marks)

9 Plants need light. The light is absorbed and refracted by water. Some light is reflected by the surface. Eventually the intensity drops to zero with increasing depth.
(4 marks)

10 a

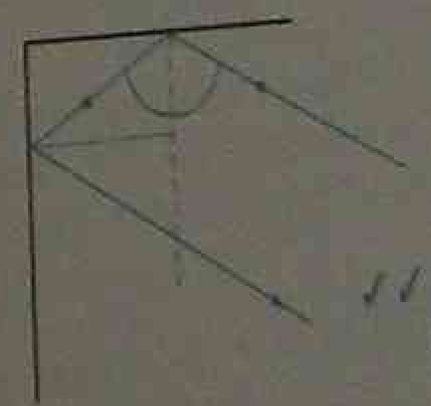


✓✓

b The thicker glass will refract and absorb some light and so the image will be blurred and fainter. ✓
(4 marks)

Part B. Skills pages 32–3

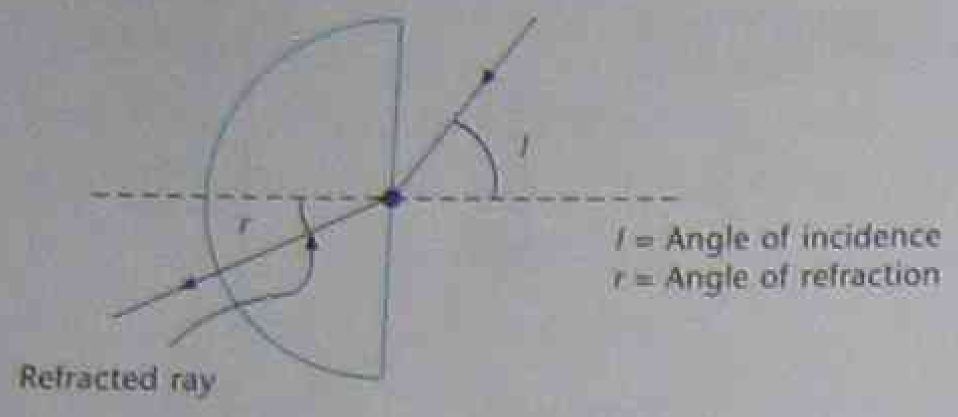
1



✓✓
(2 marks)

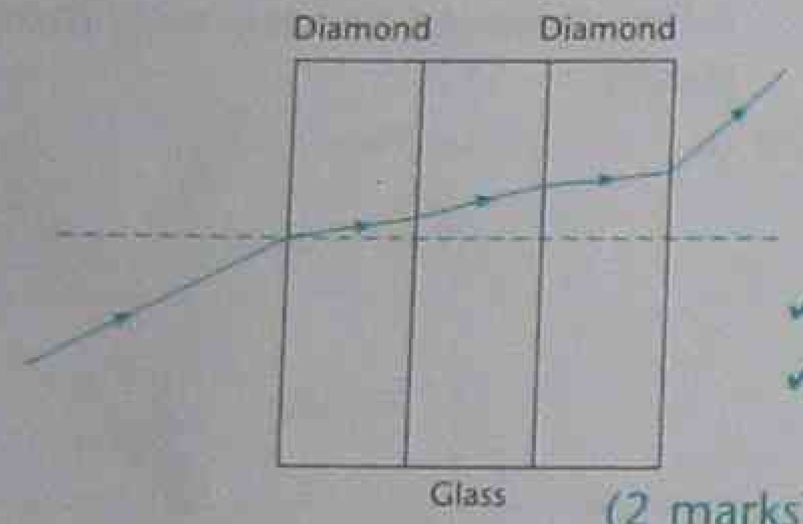
2 a 60° ✓

b & c ✓



d Some of the light energy has been absorbed by the plastic and some light may have been reflected at each surface. ✓ ✓ (7 marks)

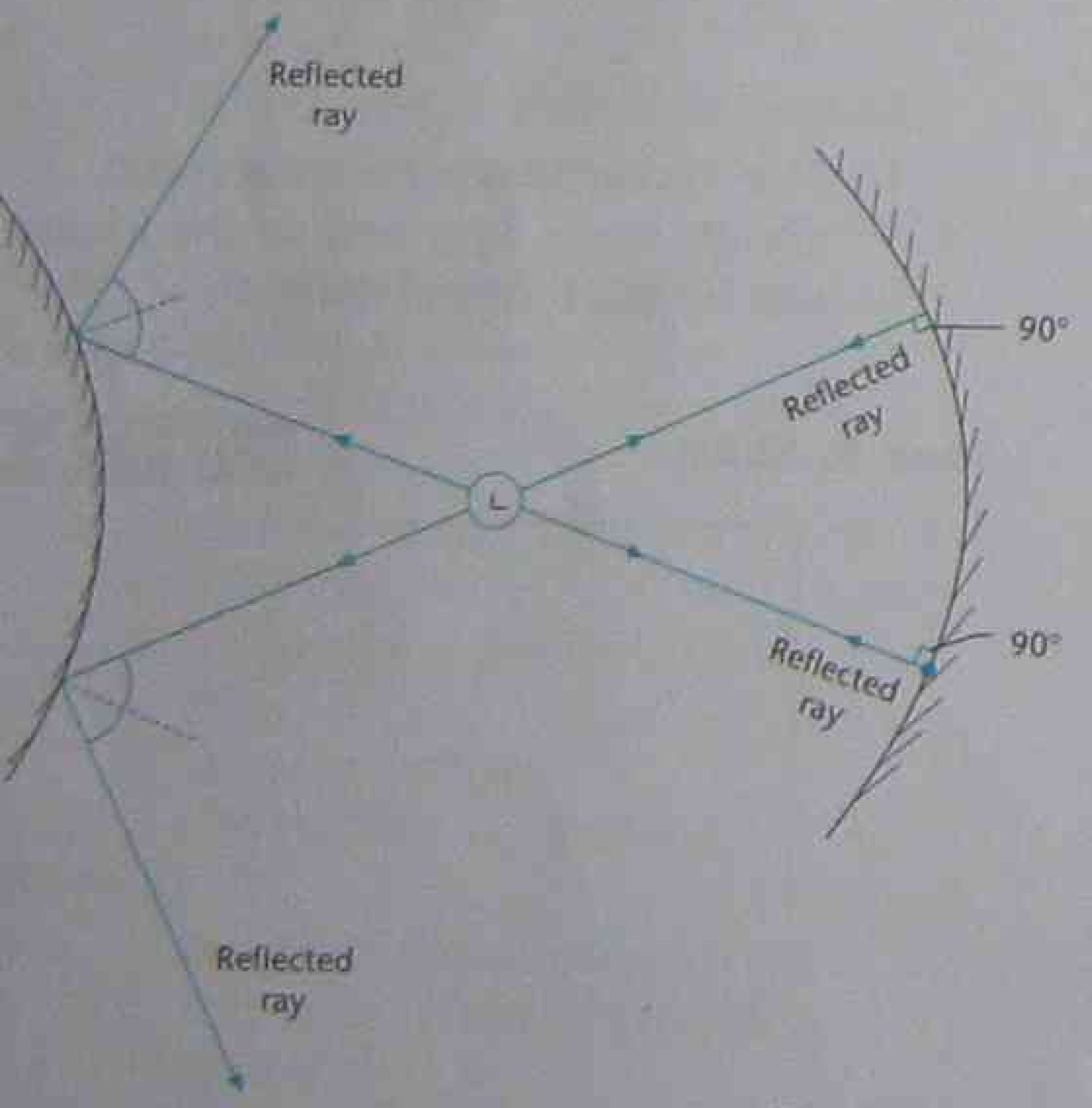
3



✓
✓

(2 marks)

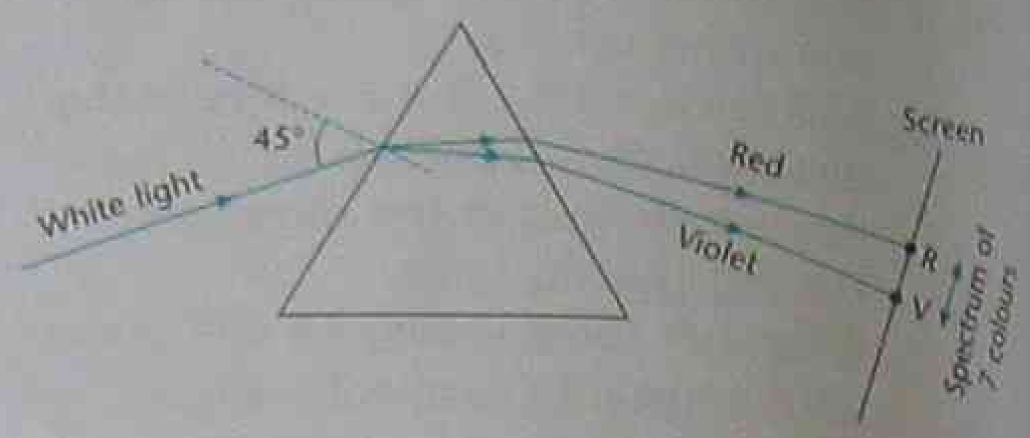
4



(2 marks)

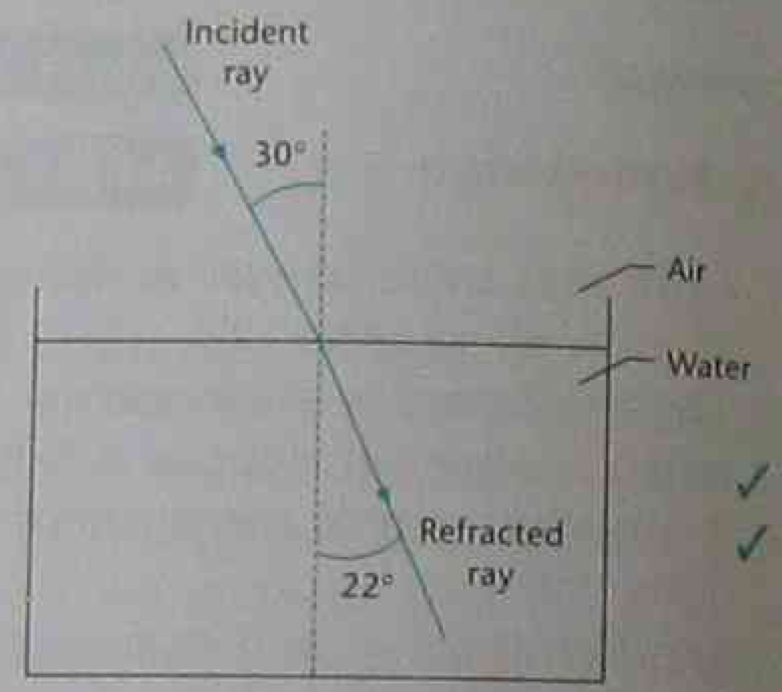
5 a White light is composed of various colours. ✓

b ✓ ✓



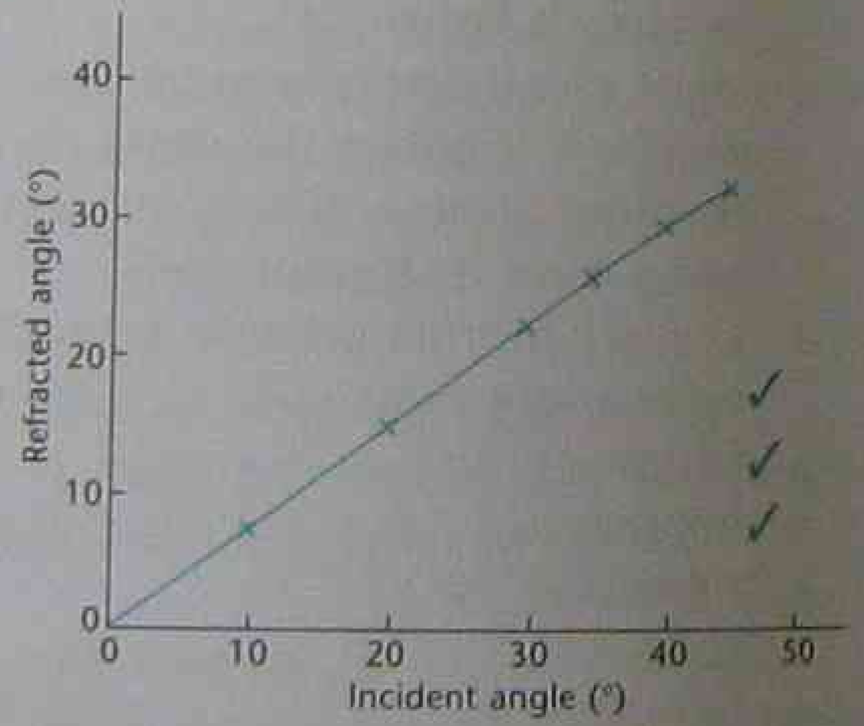
(3 marks)

6 a



✓
✓

b



✓
✓
✓

c The angle of refraction in water is less than the angle of incidence in air; light slows down as it enters water; between 10° and 35° the ratio of the angle of incidence to the angle of refraction is constant. ✓ (6 marks)

Nuclear energy

pages 36–8

Part A. Knowledge

pages 36–7

1 c Neutrons are neutral and are able to enter the nucleus to cause fission. ✓

2 c Alphas contain two protons and two neutrons. ✓

3 a Hydrogen atoms fuse together to form helium. ✓

4 a Small masses of uranium can produce vast amounts of energy by fission reactions. ✓

5 d The production of electricity is a common use of nuclear energy. ✓ (5 marks)

6 a protons ✓
- b strong ✓
- c fusion ✓
- d controlled ✓
- e fission ✓ (5 marks)

7 a/h ✓ ; b/f ✓ ; c/j ✓ ; d/g ✓ ; e/i ✓ (5 marks)

8 The high temperature and pressures to self-sustain a hydrogen plasma and nuclear fusion have not been achieved. (2 marks)

9 If there were an accident on launch, radioactivity would be spread over Earth. ✓ (1 marks)

Part B. Skills (pages 37–8)

1 a Z (protons attracted by negative) ✓
- b X (electrons attracted by positive) ✓
- c Y (e.m. waves not influenced) ✓ (3 marks)

2 a one (France) ✓

b 80% ✓

c

Country	% of electricity derived from nuclear energy
France	75
Sweden	47
Switzerland	40
Japan	35
USA	23
Czech Republic	20 ✓ ✓

d None of Australia's electricity is generated from nuclear power. ✓ (5 marks)

3 The beta particle is formed by the decay of a neutron into a proton. ✓✓ (2 marks)

Gravitational force

pages 39–41

Part A. Knowledge

pages 39–40

1 c The closer a body to the centre of Earth the greater is the pull of gravity. ✓

2 a Jupiter is the most massive planet with the largest gravitation. ✓

3 b Mass does not depend on gravity. ✓

4 d Earth is the largest of the four astronomical bodies. ✓

5 d The acceleration due to gravity is less at several hundred km altitude than on the surface, but it still exists and so a body weighs less at these altitudes. ✓ (5 marks)

6 a weight ✓
- b decreases ✓
- c less ✓
- d ten ✓
- e gravitational ✓ (5 marks)

7 a/i ✓; b/h ✓; c/j ✓; d/g ✓; e/f ✓ (5 marks)

8 The gravitational acceleration on the Moon is only one-sixth of that on Earth. The same sample will therefore have a different weight on the Moon compared with on Earth. ✓✓ (2 marks)

9 Beam balances are used to compare masses by balancing an unknown mass with a known mass on opposite sides of a pivot. Mass is independent of gravity. Thus the mass results will be identical in all locations. ✓✓ (2 marks)

Part B. Skills

pages 40–1

1 a 60 N ✓

b 7.5 kg (from graph) ✓

c $g = 120/10 = 12.0\ m/s^2$ ✓✓

d More massive as 'g' is greater than 'g' for Earth ✓ (5 marks)

2 a $m = W/g = 0.6/0.3 = 2.0$ kg ✓

b $W = m\,g = (2.0)(9.8) = 19.6$ N ✓ (2 marks)

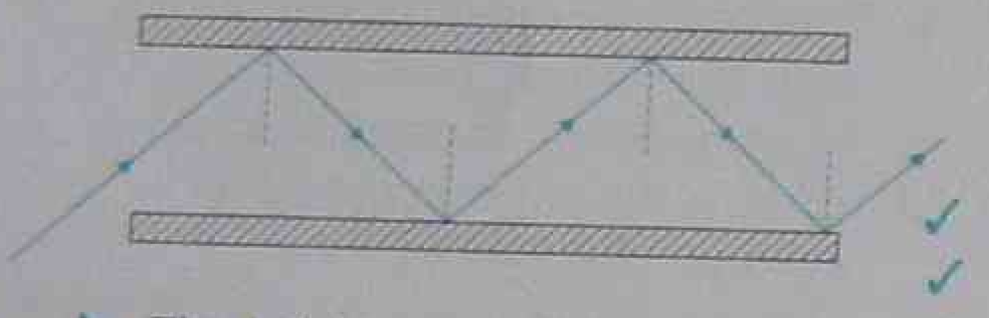
3 a Greater gravitational acceleration is due to the greater force acting on the falling body. A body that accelerates more will take a shorter time to cover the same distance. ✓

b Earth = Z; Mars = X ; Moon = Y; Earth has the greatest value of '*g*' of these three bodies and the Moon has the lowest value. ✓✓ (5 marks)

End-chapter test

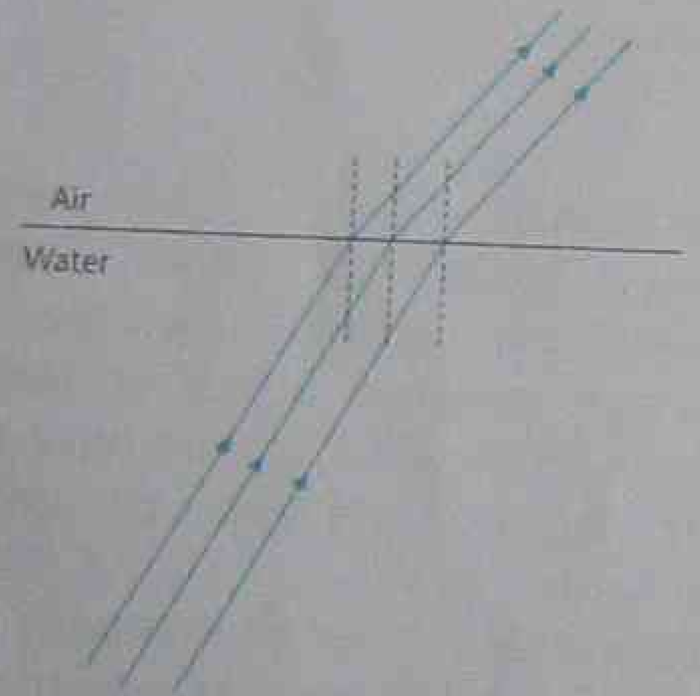
page 41

1 a



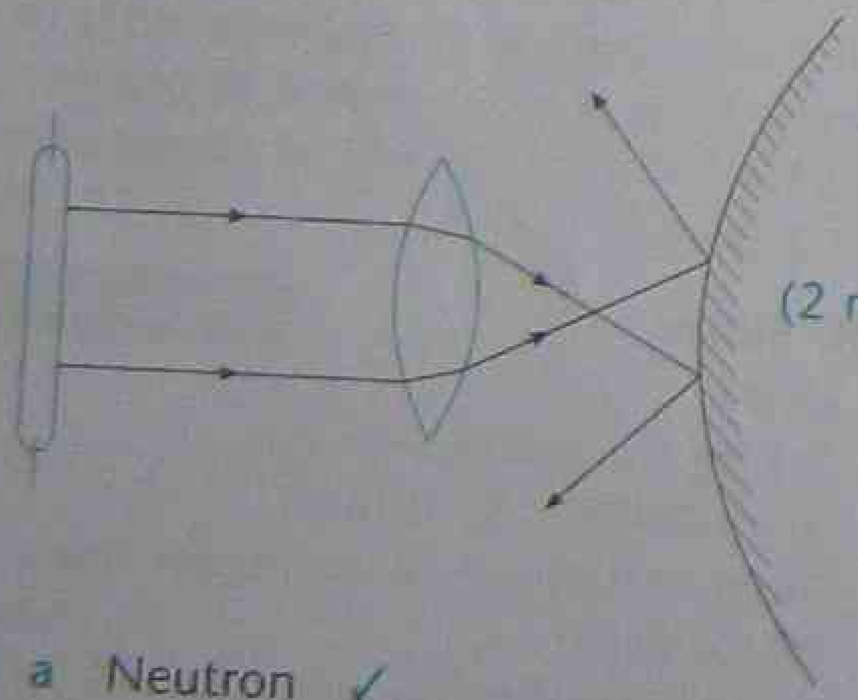
b The brightness will decrease on each reflection as some light is absorbed into the glass and some may be scattered by small imperfections. ✓✓ (4 marks)

2



(2 marks)

3



(2 marks)

4 a Neutron ✓

b They are both radioactive. ✓

c Energy released from 1 kg of uranium = 1/0.235 × 19 million megajoules = 81 million MJ ✓✓

d Various reasons including: insufficient uranium to use it alone for electricity generation; environmental and political issues have led to the phasing out of nuclear power in some countries. ✓ (5 marks)

5 a Divide weight by the mass for each body; in each case the value of '*g*' is less than 9.8; this shows that the readings were not made at ground level. ✓

b Values of *W*/*m* = A(9.5); B(9.5); C(9.5); D(9.35); E(9.5). Thus D is not measured at the same altitude. ✓✓ (3 marks)

Chapter 2

Atomic theory

pages 48–9

Part A. Knowledge

pages 48–9

1 b The nucleus contains both protons and neutrons; electrons exist outside the nucleus. ✓

2 c 40 – 19 = 21 neutrons. ✓

3 b The L shell (n = 2) can contain up to 8 electrons. ✓

4 a Protons and neutrons have very similar masses (neutrons are slightly heavier). ✓

5 b 2 + 8 + 5 = 15 electrons = 15 protons in a neutral atom; Z = 15 ✓ (2 marks)

6 a maximum (see Question 3) ✓

b electrons ✓

c one ✓

d two ✓

e radioactive ✓ (5 marks)

7 a/i ✓; b/f ✓; c/g ✓; d/h ✓; e/j ✓ (5 marks)

8 There is an equal number of positively charged protons and negatively charged electrons. ✓ (1 marks)

9 a The mass number refers to the number of protons and neutrons in the nucleus; the atomic weight is the mass of the whole atom measured in atomic mass units. ✓ (2 marks)

Part B. Skills

page 49

1

Element	Symbol	Z	A	N	Proton number	Electron number
Oxygen	O	8	16	8	8	8
Carbon	C	6	12	6	6	6
Fluorine	F	9	19	10	9	9

(5 marks)

2 a 2,5 ✓

b 2,8,2 ✓

c 2,8,8 ✓ (3 marks)

3 a protons = 15; neutrons = 16 ✓

b protons = 27; neutrons = 33 ✓ (2 marks)

4 a $p = 82$; $n = 125$; $e = 82$ ✓

b $p = 86$; $n = 136$; $e = 86$ ✓

c $p = 43$; $n = 56$; $e = 43$ ✓ (3 marks)

5 a Boron-11 ✓

b C—20% of the atoms have a mass of 10 and 80% have a mass of 11. The average of these numbers is equal to: 20/100 × 10 + 80/10 × 11 = 10.8 ✓ (2 marks)

Elements

pages 51–3

Part A. Knowledge

pages 51–2

1 a Chlorine is a diatomic gas. ✓

2 b Potassium is an active metal, zinc is less reactive and mercury is a low reactivity liquid metal. ✓

3 b Group VI is the oxygen group (O, S, Se, Te, Po). Oxygen is in Period 2, sulfur in Period 3, selenium in Period 4 ✓

4 a Manganese is a transition metal; arsenic is a semi-metal. ✓

5 c Bromine is a non-metal; gadolinium is a lanthanide metal. ✓ (2 marks)

6 a fluorine ✓

b metals ✓

c transition metals ✓

d two ✓ (5 marks)

e 4 ✓

7 a/j ✓ ; b/f ✓ ; c/g ✓ ; d/h ✓ ; e/i ✓ (5 marks)

8 a two ✓ (2 marks)

b radium ✓

9 a Periods are horizontal rows in the Periodic Table. ✓

b seven ✓

c eight ✓ (3 marks)

Part B. Skills

pages 52–3

1 a

Element	Atomic number Z	Symbol
Helium	2	He
Oxygen	8	O
Magnesium	12	Mg
Calcium	20	Ca
Silicon	14	Si
Tin	50	Sn
Xenon	54	Xe

(3 marks)

b Magnesium and calcium are both members of Group II; helium and xenon are both members of Group VIII; silicon and tin are both members of Group IV. ✓✓ (5 marks)

2 a i 20 ✓

ii 40 ✓

b 200 atoms of neon ✓ (3 marks)

3 a metal ✓

b non-metal ✓

c metal ✓

d semi-metal ✓

e metal ✓ (5 marks)

4 a selenium (Group VI, Period 4) ✓

b chlorine (argon is not a reactive gas) ✓ (2 marks)

5 a X = solid; Y = solid; Z = gas ✓✓✓

b X = phosphorus; Y = sulfur; Z = chlorine ✓✓✓ (5 marks)

6 a 2 ✓

b metal ✓

c gas ✓

d antimony or bismuth ✓

e synthetic ✓ (5 marks)

Mid-chapter test

pages 53–4

1 a 2 + 8 + 18 + 32 + 22 + 9 + 2 = 93 protons ✓

b neptunium ✓

c radioactive (all actinides are) ✓

d N = 239 – 93 = 146 ✓ (4 marks)

2 a semi-metal ✓✓
b metal ✓
c metal ✓
d non-metal ✓ (2 marks)
3 a X = tin ✓
b Y = nitrogen ✓ (2 marks)
4 a 2,7 ✓
b 2,8,2 ✓ (2 marks)
5 a P_4 ✓
b 4 × 31 = 124 atomic units ✓ (2 marks)
6 a chlorine ✓
b oxygen ✓
c argon ✓
d bromine ✓ (4 marks)
7 a i 16
ii 64
iii 80
iv 96 ✓✓
b i 4 times
ii 5 times
iii 6 times
iv 10 times ✓✓ (4 marks)
8 a argon ✓
b 40–18 = 22 ✓
c 2,8,8 ✓
d 8 ✓
e Very unreactive as it has a stable octet of electrons in the valence shell. ✓ (5 marks)

Compounds and reactions pages 67–71

Part A. Knowledge pages 67–8

1 b Calcium has a valence of +2 and chlorine has a valence of –1; thus the chemical formula of calcium chloride is $CaCl_2$. ✓
2 d Sulfur burns or combusts in oxygen to produce sulfur dioxide. ✓
3 a Magnesium oxide is a base that is neutralised by sulfuric acid. ✓
4 b Acids attack carbonates and release carbon dioxide gas. ✓
5 a A compound breaks down or decomposes into two new compounds. ✓ (5 marks)
6 a ionic
b covalent ✓
c pentoxide ✓
d effervescence ✓
e oxide ✓ (5 marks)
7 a/g ✓; b/f ✓; c/j ✓; d/h ✓; e/i ✓ (5 marks)
8 Increase the temperature; powder the zinc to increase its surface area; increase the concentration of the acid. (3 marks)
9 *Method*:

1. Place 5 mL of sulfuric acid in two separate test tubes. ✓
2. Add a piece of copper to one tube and a piece of zinc to the other tube. ✓
3. Note the rate at which bubbles come off the metal. ✓

Results:

Bubbles of gas will quickly form on the zinc, as the acid attacks and dissolves it. These bubbles will rise to the surface. No change will occur with the copper. Thus zinc is more active than copper. (3 marks)

10 Potassium carbonate + hydrochloric acid → potassium chloride + water + carbon dioxide (2 marks)

Part B. Skills pages 68–71

1 a A = copper chloride; B = nitrogen dioxide; C = aluminium oxide ✓✓✓
b $CuCl_2$; NO_2; Al_2O_3 ✓✓✓
c A = ionic; B = covalent; C = ionic ✓✓✓ (9 marks)
2 a A; B; D ✓
b D; F ✓
c C; D ✓ (3 marks)
3 a A white precipitate would be observed to form when the colourless solutions are mixed. ✓
b Precipitation ✓
c Sodium chloride + silver nitrate → silver chloride + sodium nitrate ✓✓
d $NaCl + AgNO_3 \rightarrow AgCl + NaNO_3$ ✓✓ (6 marks)
4 a Corrosion (rusting) or oxidation ✓
b Both air and water are required for a nail to rust. Air or water alone do not cause rusting. ✓✓ (3 marks)
5 a Neutralisation ✓
b To monitor the pH changes during the neutralisation ✓
c The reactions are exothermic since heat is released. ✓
d Y is magnesium carbonate because the neutralisation leads to the formation of a gas. Acids on carbonates release carbon dioxide gas. ✓✓
e In X the final pH is high (>10) whereas in Y the final pH is lower (between 8 and 9). ✓✓ (7 marks)
6 a Atoms and matter are conserved in a chemical change. ✓
b Reactants are all diatomic molecules; products are triatomic molecules. ✓✓
c Combustion reaction (H burns in O_2) ✓ (4 marks)
7 a Sulfuric acid and hydrochloric acid ✓✓
b H_2SO_4; HCl ✓✓
c Hydrogen ✓
d Carbon dioxide ✓
8 a Y (slope of graph is greater) ✓
b The acid was more concentrated in the reaction with Y. ✓
c Sodium sulfide + nitric acid → sodium nitrate + hydrogen sulfide ✓✓
d $Na_2S + 2HNO_3 \rightarrow 2NaNO_3 + H_2S$ ✓✓
e The molecule in C has two atoms of hydrogen and one of sulfur—that is consistent with the equation. ✓ (7 marks)
9 a i soluble
ii insoluble
iii soluble
iv soluble
b i yes—silver chloride
ii yes—barium sulfate
iii no (5 marks)
10 a The volume of gas Y is twice that of gas X. ✓
b hydrogen gas (pop test) ✓
c oxygen gas (glowing splint test) ✓
d decomposition ✓
water → hydrogen + oxygen ✓
e $2H_2O \rightarrow 2H_2 + O_2$ ✓
f The 2:1 ratio for the gas volume data is consistent with the 2:1 coefficients in the balanced equation. ✓ (8 marks)

End-chapter test pages 71–3

1 a i Reaction 3 ✓
ii Reaction 1. ✓
b Reaction 3: calcium carbonate + nitric acid → calcium nitrate + water + carbon dioxide. ✓
Reaction 1: Lead nitrate + sodium sulfate → lead sulfate + sodium nitrate ✓ (4 marks)
2 a calcium iodide (ionic) ✓
b sulfur dichloride (covalent) ✓
c hydrogen iodide (covalent) ✓
d mercury oxide (ionic) ✓ (4 marks)
3 a inorganic ✓
b organic ✓
c organic ✓
d inorganic ✓ (4 marks)
4 a Ba^{2+}, Cl^-; barium chloride ✓
b Ga^{3+}, NO_3^-; gallium nitrate ✓
c Rb^+, F^-; rubidium fluoride ✓ (3 marks)
5 a $X = 3^+$ ✓
b $Y = 5^+$ ✓
c $Z = 2^+$ ✓ (3 marks)
6 a Heat released; colour change ✓✓
b Sulfur dioxide ✓
c Sulfur + oxygen → sulfur dioxide ✓✓
d $S + O_2 \rightarrow SO_2$ ✓✓
e The gas is acidic, because the indicator turned red. ✓ (8 marks)
7 a 8 ✓
b Ethane + oxygen → carbon dioxide + water ✓✓
c $2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$ ✓✓
d Pass the gas into limewater which will turn white if it is carbon dioxide. ✓✓
e The heat released from the combustion keeps the combustion process going. ✓ (8 marks)
8 a zinc sulfate ✓
b calcium nitrate ✓
c sodium chloride ✓ (3 marks)
9 a pink to green (ammonia is a base) ✓
b pink to red (lemonade is acidic) ✓ (2 marks)

10 a decomposition ✓
b carbon dioxide ✓
c magnesium carbonate → magnesium oxide + carbon dioxide ✓✓ (4 marks)

Chapter 3

Cell theory

pages 77–9

Part A. Knowledge

pages 77–8

1 b The cell theory states that all life forms are composed of cells. ✓
2 a Chromosomes are small bodies containing DNA that are found in the nucleus. ✓
3 d The blood circulation ensures that oxygen for respiration reaches all body cells. ✓
4 b The excretory system removes waste (eg. urine). ✓
5 c For an organism to get larger it must make more cells; this is achieved by cell division and growth of the new cells. ✓ (5 marks)
6 a decreases meiosis ✓
b cell division/meiosis ✓
c half ✓
d half ✓
e tissues ✓ (5 marks)
7 a/i ✓; b/g ✓; c/j ✓; d/f ✓; e/h ✓ (5 marks)
8 a Carries oxygen to the muscle in the leg ✓
b Carbon dioxide waste from the muscle cell is excreted from the body via the lungs. ✓
c Nutrients such as glucose are produced by the breakdown of food in the digestive system. This glucose is required by muscle cells as a fuel. ✓ (3 marks)
9 Repair damaged cells; form new cells for growth; form sex cells for reproduction ✓✓✓ (3 marks)

Part B. Skills

pages 78–9

1 D, E, B, A, C ✓✓ (2 marks)
2 V, X, Y, Z, W ✓✓ (2 marks)
3 Typical average diameter = 0.4 mm ✓✓ (2 marks)
4 a Skin cells are constantly being damaged and need to be replaced; bone cells are needed for support and are less likely to be damaged. They have a longer life span as they can be repaired. ✓
b Red blood cells = 6 × 4 = 24 weeks; bone cells = 43 × 4 = 172 weeks. ✓ (2 marks)
5 a X – no change in chromosome number per cell in all divisions; Y – no change in the first division but a halving of chromosomes in the second stage ✓
b Y (chromosome numbers halved) ✓ (2 marks)
6 a A = xylem; B = phloem ✓
b Circulatory system (with blood vessels) ✓
c Water lost = 100 × 60 × 3 = 18 000 mg = 18 g ✓✓ (4 marks)

The Watson-Crick model of DNA

pages 83–5

Part A. Knowledge

pages 83–4

1 d DNA is a polymer that has a double helix shape. ✓
2 b D stands for deoxyribose (a sugar); NA stands for nucleic acid. ✓
3 c The two strands of the double helix are held together by forces between the nitrogen bases on each strand. ✓
4 b Genes are coding segments of the DNA. ✓
5 d Genes and their interaction with the environment decide the characteristics of an organism. ✓ (5 marks)
6 a three ✓
b cytosine ✓
c code ✓
d allele ✓
e capital ✓ (5 marks)
7 a/g ✓ ; b/i ✓ ; c/f ✓ ; d/h ✓ ; e/j ✓ (5 marks)
8 Watson and Crick ✓ (1 mark)
9 GGACACTCGTTG ✓✓ (2 marks)
10 Deletion and substitution of bases. ✓✓ (2 marks)

Part B. Skills

pages 84–5

1 a threonine-leucine-glycine-aspartic acid-phenylalanine-cysteine ✓
b i substitution
ii Aspartic acid will be replaced by valine. ✓ (2 marks)
2 a b genes ✓ (2 marks)
b Bb ✓
3 a BbrrSS ✓
b 1 = brown eyes; round face; peaked hairline
2 = brown eyes; long face; straight hairline ✓✓
c blue eyes; long face; peaked hairline ✓ (4 marks)
4 a i B ✓
ii brown eyes ✓
b Gene pairs are at the same location (ie. opposite) on each homologous chromosome. ✓ (3 marks)

Mid-chapter test

pages 85–6

1 a Robert Hooke ✓
b Microscope ✓
c cells ✓ (3 marks)
2 a/h ✓; b/j ✓; c/f ✓; d/g ✓; e/i ✓ (5 marks)
3 a cell; nutrients ✓
b wastes ✓ (3 marks)
4 a/f ✓; b/d ✓; c/e ✓ (3 marks)
5 a X = deoxyribose sugar; Y = nitrogen bases ✓✓
b nucleotide ✓✓✓
c It consists of many nucleotides (monomers) joined together to form very large molecules ✓ (6 marks)
6 a three bases ✓
b TGCAGCACGGTGACTGGAT ✓✓
c Insertion ✓ (4 marks)
7 a 'y' ✓
b yy ✓
c No, plants with yellow-coated seeds will be produced unless the gene for seed coat colour is mutated by agents in the environment. ✓✓ (4 marks)
8 a They are easily seen and are an easy meal for the birds. ✓
b The black moths will now not be easy to see when they are on the tree trunks. The mottled moths, however, will be much easier to see and the birds will eat them. The population will change over time to favour the dark moths. ✓✓ (4 marks)
9 TGACGTGCTCTCGTA ✓✓ (2 marks)

The theory of evolution and natural selection

pages 91–4

Part A. Knowledge

pages 91–2

1 c Earth is believed to be 4.6 billion years old (evidence from rock dating and fossil dating). ✓
2 a Fish evolved before reptiles. ✓
3 b The pentadactyl limb structure is common to vertebrate classes. ✓
4 c Surviving to reproduce and pass on favourable characteristics is the important concept in Darwin's theory ✓
5 c Each island had different selecting agents. ✓ (5 marks)
6 a extinction ✓
b geographically ✓
c even ✓
d common ✓
e DNA ✓ (5 marks)
7 a/h ✓; b/j ✓; c/i ✓; d/g ✓; e/f ✓ (5 marks)
8 There is a greater chance of DNA mutation as the DNA strands are copied more quickly and deletions or substitutions of bases may occur more often. New environments select out fitter bacteria to reproduce and so bacterial populations can evolve faster. (2 marks)
9 New information about DNA and how it can mutate has provided a mechanism for evolutionary change. (2 marks)
10 The environment selects certain characteristics for survival. Organisms of a species that have these characteristics

will survive to reproduce and pass their genes onto the next generation. (2 marks)

Part B. Skills

pages 92–4

1 a Marsupials evolved after Africa split away from the other continental masses of Gondwanaland, but before South America detached from the combined mass of Australia–Antarctica–South America. ✓

b Antarctica is the land bridge between Australia and South America. Thus fossil marsupials should eventually be found there. ✓

c The environment of South America (including the presence of predators) may have led to the extinction of most South American marsupials. Australia was isolated and so marsupials survived in this environment. ✓ (3 marks)

2 In nature, only a few breeds are reproductively fit to survive. Nature has eliminated the less fit. In domestic cats, many breeds survive as artificial selection is used to produce new types. Domesticated cats have few predators. ✓✓ (2 marks)

3 A and B (according to the proposed evolutionary tree) evolved from their common ancestor before X evolved. Their sequences of nitrogen bases should be different to the other three living frog species. Analysis of the DNA from the fossil remains of X (if DNA is available) will show more nitrogen base sequences in common with C, D and E. ✓✓ (3 marks)

4 a About 330 million years ago ✓

b About 375 million years ago ✓

c Mosses ✓

d About 445 million years ✓

e Flowering plants ✓ (5 marks)

5 a Geographic isolation due to the formation of the Nullarbor plain caused the two isolated groups to evolve separately. New species formed as a result. ✓✓

b The gene pool was the same, as the frogs could interbreed. ✓✓

c Mutations can occur in these separated populations. Favourable mutations in one group will not be transmitted to the other group and so different species will eventually arise. ✓

d Frogs that can survive the dry conditions will be fitter to reproduce and pass these favourable characteristics on to their offspring. Over time the western frog population will be more dry-weather tolerant than the eastern frogs. ✓✓ (7 marks)

6 Races develop when human populations are geographically isolated and mutations occur, and the environment acts as selecting agent. ✓✓✓ (3 marks)

7 a In the natural population there are some bacteria that have resistance to antibiotics but this ability does not give them any advantage. ✓✓

b In the presence of the antibiotic the resistant bacteria are selected and they survive to reproduce. Their offspring are now more numerous in subsequent generations. ✓✓ (4 marks)

Humans

pages 104–6

Part A. Knowledge

pages 104–5

1 c Sensory neurones join our sense organs to the connecting neurones in the CNS. ✓

2 a The cerebellum controls involuntary movements as well as fine motor responses. ✓

3 d Diabetes is caused by problems in regulating glucose metabolism. ✓

4 a Tuberculosis is a bacterial infection of the airways. ✓

5 c During ovulation an egg bursts out of a follicle in the ovary. ✓ (5 marks)

6 a sperm ✓

b membrane ✓

c artificially ✓

d nutritional ✓

e motor ✓ (5 marks)

7 a/j ✓ ; b/h ✓ ; c/f ✓ ; d/i ✓ ; e/g ✓ (5 marks)

8 Acidic environments (on skin/stomach); mucus linings (airways); hairs and cilia (respiratory tract) ✓✓✓ (3 marks)

9 Production of sperm; production of testosterone ✓✓ (2 marks)

10 White blood cell (46); sperm (23); egg (23); zygote (46); brain cell (46) (5 marks)

Part B. Skills

pages 105–6

1 a X with its small pupil ✓

b involuntary movement, a reflex ✓

c In the darkened room the pupil is wide open. On entering the bright sunlight the photoreceptors in the eye are stimulated and the message is conveyed along the sensory neurones (optical nerve), then the connector neurones (brainstem) and along motor neurones to the iris constrictor muscles that respond and cause the iris to enlarge and the pupil to reduce in size. ✓✓ (4 marks)

2 a Yes (spinal cord still connected) ✓

b No (brain disconnected from knee) ✓

c The severing of the spinal cord prevents electrical messages from the brain reaching the leg muscles. ✓ (3 marks)

3 a/g✓; b/h✓; c/f✓ ; d/j✓; e/i✓ (5 marks)

4 B, C, A ✓✓✓ (3 marks)

5 a = Z; b = X; c = Y ✓ (3 marks)

6 D, B, C, A , E ✓✓ (2 marks)

7 a A = 2.2 × 10 = 22 micrometres
B = 2 × 0.1 = 0.2 micrometres
C = 2.5 × 10 = 25 nanometres ✓✓✓

b C, B, A ✓

c C—The virus is the smallest of the organisms. ✓ (5 marks)

End-chapter test

pages 106–9

1 a Comparative embryology. Vertebrate embryos in their early developmental stages are remarkably similar. They all have gill slits. This suggests that all vertebrates shared a common ancestor. ✓✓

b Comparative DNA analysis and biochemistry. The amino acid sequences and base sequences in DNA are very similar in closely related species (ie. humans and chimpanzees). This suggests a common ancestor. ✓✓

c Geographic distribution and isolation. The continents were joined in the past and organisms dispersed across these zones. As the giant continent broke up, the organisms became isolated and selecting agents and mutations led to changes and new species. The fossils along the margins of continents are consistent with the idea of a common ancestor. ✓✓ (6 marks)

2 Theory of Natural Selection.

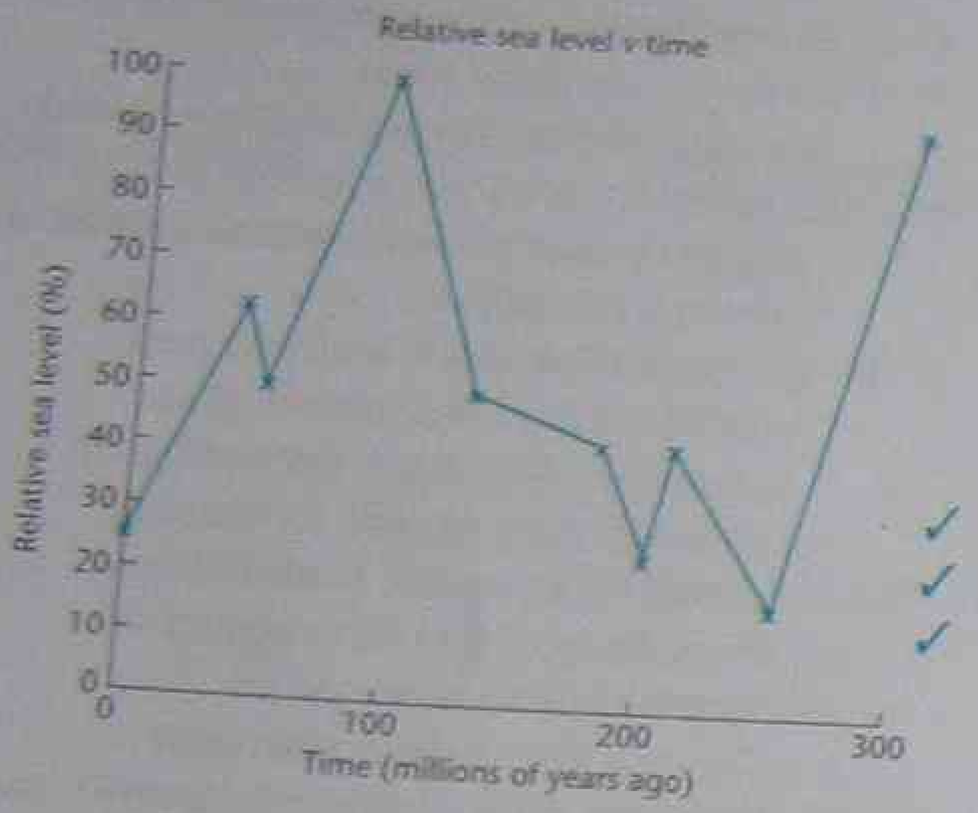
a There is a natural variation in characteristics within a population. ✓

b Organisms struggle to survive in nature and the strongest survive and the weakest are killed and eaten. Disease keeps populations in check. ✓

c Organisms that have favourable characteristics survive long enough to reproduce and pass their characteristics on to their offspring. These offspring will have a better chance of survival. ✓

d The future generations will have a greater proportion of organisms with these favourable characteristics. ✓ (4 marks)

3 a



b The extinction events occurred when the sea level was relatively low (particularly the event at 251 million years ago). The very low sea levels would have a major effect on marine species, especially those that lived in shallow areas. ✓✓

c Sea level changes can lead to geographic isolation and changes in the environment of habitats. Mutations occurring in communities that have been isolated lead to the formation of new species as interbreeding is prevented. Changes in the environment can act as selecting agents in the process of natural selection. ✓✓ (7 marks)

4 a A = connector neurone; B = sensory neurone; C = motor neurone ✓✓✓

b B, A, C ✓

c They allow rapid response without the need for processing the information in the brain; this helps to avoid damage to the body. ✓✓ (6 marks)

a Vision centre ✓

b Disease of this region of the brain (eg. a tumour) ✓

c He has damaged the forebrain which is a region controlling higher mental activities and thought. ✓✓ (4 marks)

6 a i sides where the sour receptors are located ✓

ii back of the tongue where the bitter receptors are located ✓

iii tip and front sides of the tongue where the sweet receptors are located ✓

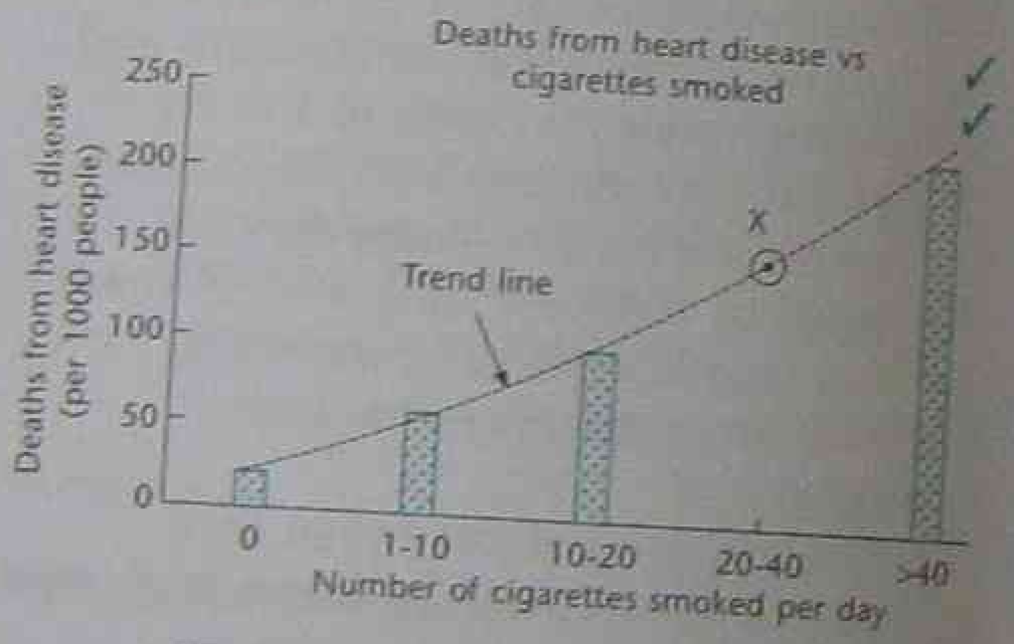
b Prepare a strong cup of coffee. Use a cotton bud to touch various regions of the tongue. The only regions that will respond to the bitter taste will be at the back of the tongue. ✓✓ (5 marks)

7 a/h ✓; b/j ✓; c/i ✓; d/f ✓; e/g ✓ (5 marks)

8 a Non-infectious disease ✓

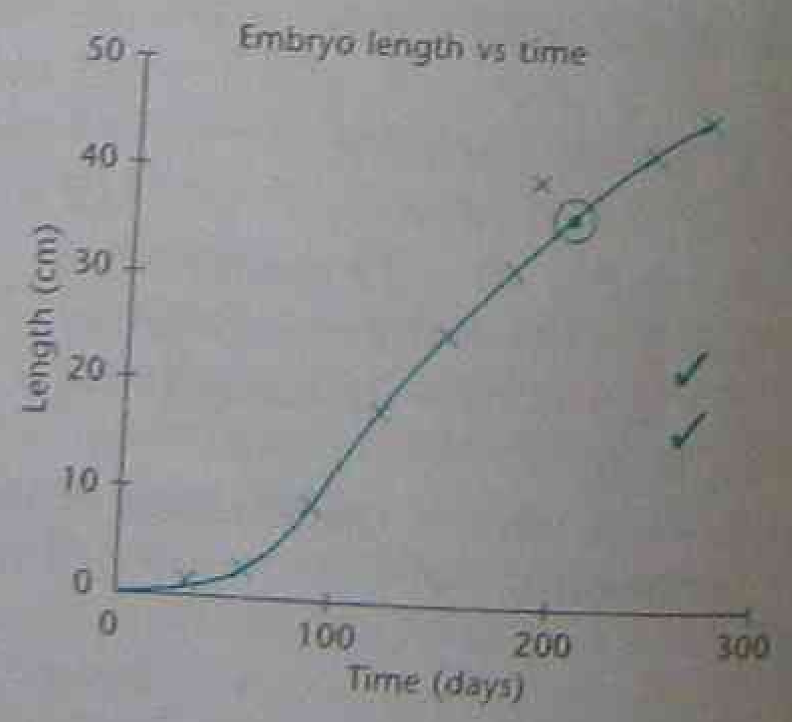
b i The greater the number of cigarettes smoked per day the greater is the number of deaths from heart disease. ✓

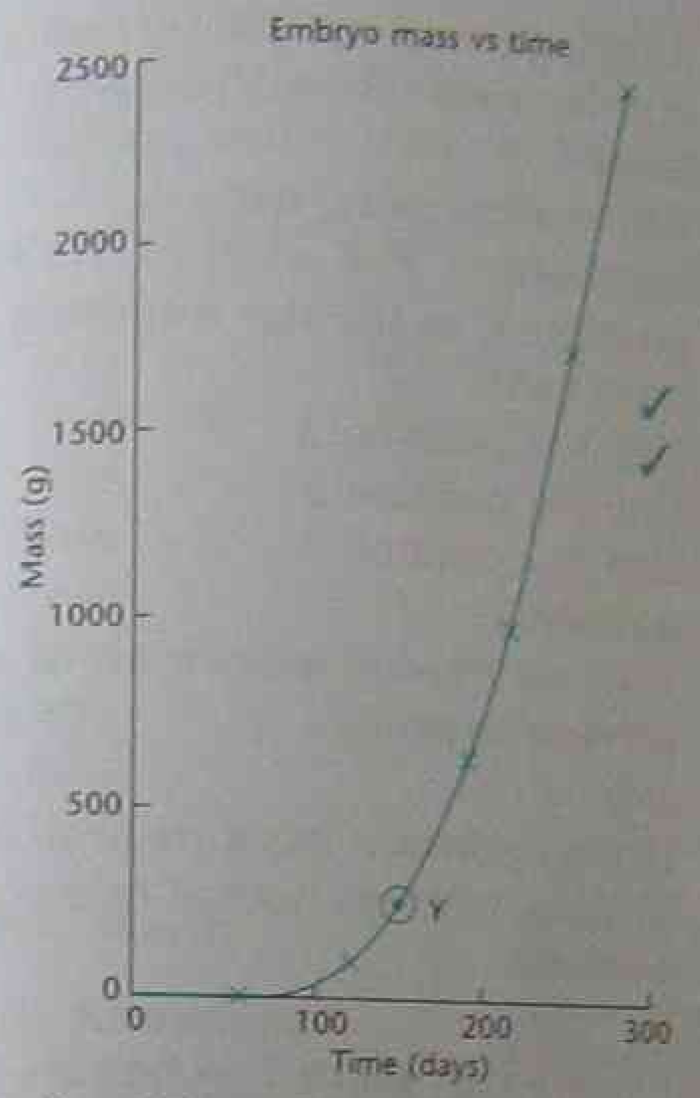
ii



iii X = approx. 150 ✓✓ (6 marks)

9 a





X = 37 cm; Y = 250 g

b i about 130 days (from graph); ✓

ii about 230 days (from graph) ✓

c Uterus (womb) ✓ (7 marks)

10 Many keys can be correct.
Example:

1a Microbe is less than 50 micrometres in size and its body is surrounded by a protein coat...D.
1b Microbe is larger than 50 micrometres in size ....go to 2.
2a Microbe is green and produces spores ...B.
2b Microbe is not green and does not produce spores ...go to 3.
3a Microbe has flagella ...A.
3b Microbe does not have flagella ...C.

(7 marks)

Chapter 4

The Big Bang theory and components of the universe pages 118–21

Part A. Knowledge pages 118–19

1 c Einstein developed the equation that links mass and energy ($E = mc^2$). ✓

2 b Light is red shifted if the source of light is moving away from the observer on Earth. ✓

3 b Infrared rays cause a rise in temperature when absorbed by matter. ✓

4 a Molecules in the atmosphere absorb UV and IR. Scientists must use telescopes outside the atmosphere to observe these types of rays from distant sources. ✓

5 c The Milky Way is a spiral galaxy with a diameter of about 100 000 ly. It is not a supernova. ✓ (5 marks)

6 a radio ✓

b hydrogen ✓

c supernova ✓

d distance ✓

e expand ✓ (5 marks)

7 a/j ✓; b/i ✓; c/g ✓; d/f ✓; e/h ✓ (5 marks)

8 Stars generate energy by nuclear fusion. Hydrogen nuclei fuse together to form helium with the release of considerable amounts of energy. In very large stars, helium can fuse to form heavier elements such as carbon and oxygen with further release of energy. ✓✓ (2 marks)

9 Red shift of stars: The spectrum of light from stars and galaxies shows red-shifting. This is interpreted as evidence for these sources moving away from us and from each other.

Cosmic background radiation: The microwave background radiation in space shows that space has cooled to 3 degrees above zero. This is interpreted as evidence of the cooling and expanding of an initially very hot universe after the big bang to its current cold state. ✓✓ (2 marks)

10 As more helium is formed by nuclear fusion, it sinks to the core of the sun and the outer hydrogen shell swells and increases in brightness. The star cools and becomes a red giant. Helium fusion begins to produce heavier elements. When the nuclear fuel runs out, the core shrinks and outer layers are ejected as a planetary nebula. A hot white core (white dwarf) remains. Over billions of years it cools to form a black dwarf. ✓✓✓ (3 marks)

Part B. Skills

pages 119–21

1 a i Sun–Venus = (0.7)(150) = 105 million kilometres ✓
ii Sun–Saturn = (9.5)(150) = 1425 million kilometres ✓
b Sun–Pluto distance = 5590/150 = 37.3 AU ✓✓ (4 marks)

2 a Distance = (7.7)(9461) = 72 849.7 billion kilometres ✓
b Distance = (353)(9461) = 3 339 733 billion kilometres ✓ (2 marks)

3 a No. Cluster refers to the organisation of local galaxies. These stars are all in one galaxy (the Milky Way). They are a constellation, because they appear to be together in the sky. ✓✓
b i Alpha Crucis ✓
ii Although Beta Crucis is brighter in absolute terms, it appears dimmer, as it is further from Earth. ✓ (4 marks)

4 a ~30 000 light years ✓
b ~100 000 light years ✓ (2 marks)

5 X, Z, W, Y ✓✓ (2 marks)

6 Angle = 60° (= one sixth of one complete revolution).
Time for one complete revolution = 6 × 35 = 210 million years. ✓✓ (2 marks)

7 a $T = 3\,000\,000/510 = 5882$ K ✓✓
b $T = 5882 - 273 = 5609$°C ✓
c Wavelength = $3\,000\,000/T$ = $3\,000\,000/4300 = 697.7$ nm = 698 nm (red end of visible spectrum). ✓ (5 marks)

8 a Dimmer ✓
b Greater brightness and greater surface temperature than the sun ✓✓
c i Brighter; ✓ ii colder ✓
d It will become cooler and dimmer. ✓✓
e It will form a less bright white dwarf. Then it will slowly cool to form a black dwarf (not shown on diagram). ✓ (5 marks)

Mid-chapter test

pages 121–3

Very small stars have very long lives. After using up all their hydrogen and converting it to helium, they cool and darken to form a black dwarf. ✓✓ (2 marks)

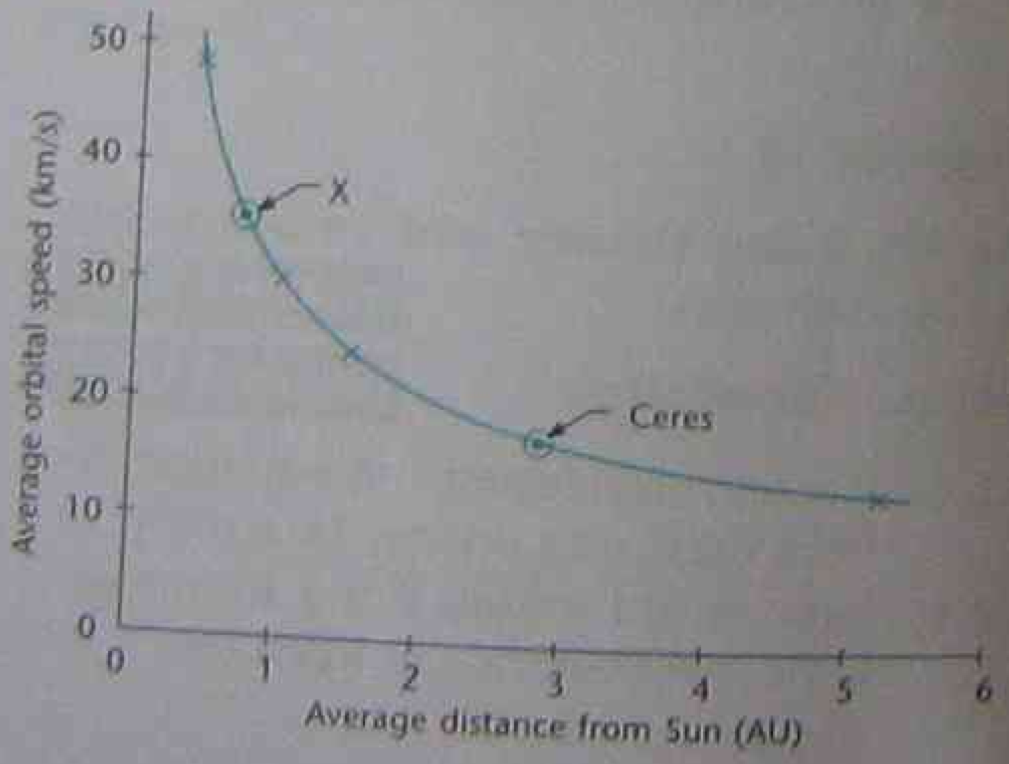
2 a X-rays, gamma rays and UV (some IR) ✓
b visible; near IR; microwaves; radio waves ✓
c visible astronomy; radio astronomy ✓✓ (2 marks)

3 a position 3, as the star is moving away from Earth ✓
b Line X = position 3 ✓
Line Y = position 2 ✓
Line Z = position 1 ✓
c position 1 ✓
d They are moving away from us as the universe expands. ✓ (6 marks)

4 a two ✓
b Heavy hydrogen has a neutron in its nucleus, whereas normal hydrogen does not. ✓
c A = 3 ✓
d helium-3 + helium-3 → helium-4 + hydrogen-1 + hydrogen-1 + γ-rays ✓✓
e gamma rays ✓ (6 marks)

5 Brightness: The brightness remains very high although there are small rises and decreases along the evolutionary path. The star is brightest at the super-red-giant stage.
Surface temperature: The surface temperature decreases as the blue giant evolves to form a red super giant. The temperature increases once more as the star moves back to the left on the evolutionary path. ✓✓ (2 marks)

6 a As the average distance from the Sun increases, the orbital speed decreases. ✓✓
b



X = 35 km/s) ✓✓✓
c 17.5 km/s (= 18 km/s) ✓✓
d Saturn, as it is closer to the Sun ✓ (8 marks)

7 F; B; H; C; D; G; A; E (2 marks)

8 a true ✓
b true ✓
c false ✓
d false ✓
e true ✓
f true ✓
g false ✓
h true ✓
i false ✓
j true ✓ (10 marks)

Natural events

pages 135–8

Part A. Knowledge

pages 135–6

1 d Corals grow in shallow, warm seawater. ✓
2 a A footprint is only a trace. ✓
3 c Mammals appeared in the Mesozoic era and reptiles first appeared in the Palaeozoic. ✓
4 a The Pacific ocean plate dives down beneath the South American plate along that coast line. ✓
5 d Richter values between 6 and 7 are indicative of strong earthquakes. ✓ (5 marks)

6 a youngest ✓
b slowly ✓
c minerals ✓
d radioisotope ✓
e Palaeozoic ✓ (5 marks)

7 a/g ✓; b/i ✓; c/f ✓; d/j ✓; e/h ✓ (5 marks)

8 The focus of an earthquake is the location in the lithosphere where the earthquake has actually occurred. The epicentre is the point on Earth's surface directly above the focus. (2 marks)

9 Lava shield volcanoes are made from very fluid lava that flows out a great distance from the vent before freezing. Thus their sides are gently sloping. Strato-volcanoes are built from more viscous lava and cinders. They pile up around the vent because the lava freezes quickly. A steep-sided volcano results. (2 marks)

10 Various possible answers.
Decaying vegetation releases greenhouse gases such as methane into the atmosphere; this increases the ability of the atmosphere to retain heat.
Lightning storms generate nitrogen oxides that will form acid rain. ✓
Cyclones are formed in the atmosphere and cause considerable damage.
Volcanoes release gases (including poisonous gases) and ash into the atmosphere. ✓
Bushfires release gases and smoke into the atmosphere. ✓ (3 marks)

Part B. Skills

pages 136–8

1 shale; coal; sandstone; limestone; conglomerate; shale; sandstone (3 marks)

2 1. Deposition of shale
2. Deposition of conglomerate
3. Deposition of limestone
4. Deposition of sandstone
5. Layers folded and faulted
6. Erosion to form a plain
7. Basaltic lava flows across a plain and solidifies to form basalt.
8. Deposition of shale
9. Erosion to current landscape (4 marks)

3 Z; X; W; Y (2 marks)

4 a Palaeozoic ✓
b bird; flowering plants – these did not appear until the Mesozoic era.
c trilobites—they became extinct in the late Palaeozoic. ✓ (3 marks)

5 Scale = 1 : 10 ✓✓ (2 marks)

6 a Spreading zone (divergent boundary) ✓
b 6 × 1 = 6 million centimetres = 60 km ✓✓
c There is a collision zone between the two plates. This leads to mountain building. ✓ (4 marks)

7 a The rim of the Pacific contains many plate boundaries (eg. subduction zones; transform fault zones). Earthquakes and volcanoes are produced due to these plate interactions. ✓

b These three continents were once joined as part of Gondwanaland. The ancestors of these birds were isolated when the continents split apart. They then evolved in isolation to their current forms. ✓

c Australia was once joined to Antarctica and the combined land mass occupied cold Antarctic latitudes. The continents then split apart and Australia moved north towards the equator. The climate became warmer and temperate. ✓

d The plate boundary between the African plate, Arabian plate and the Eurasian plate runs through this area. Consequently earthquakes and volcanic activity occur there. ✓

(2 marks)

8 a Count = 6.25/100 × 960 = 60 ✓✓

b The short half-life means that little radioactivity remains after 60 000 years. Thus, only recent fossils can be dated by this technique. ✓✓

(4 marks)

a i Sydney; ✓ ii Perth ✓

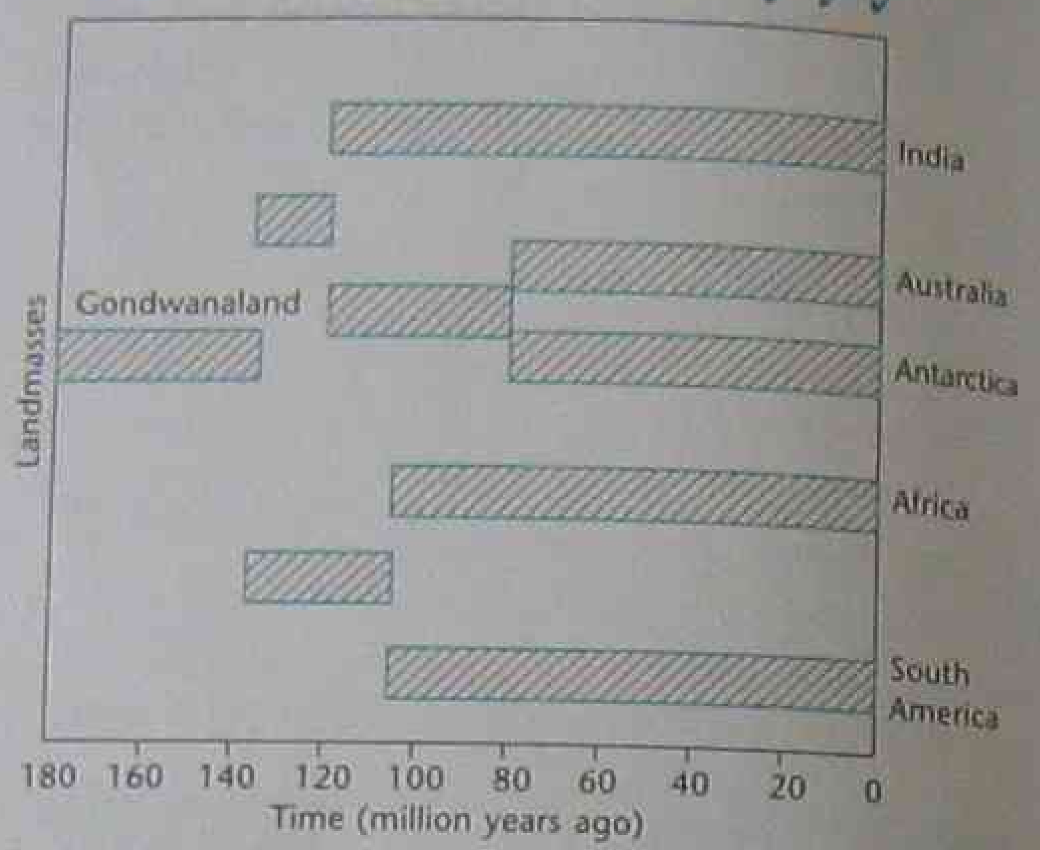
b New Zealand (south island). This location is closer to Sydney than Brisbane. Perth is much further away and the seismic waves take longer to reach the western side of Australia. If the earthquake had been in Japan, the P and S waves would have arrived at Sydney last. If the quake was in New Guinea then Brisbane would have recorded them first. ✓✓

(4 marks)

End-chapter test

pages 138–40

1 a ✓✓✓



b Mesozoic era ✓

2 Core Z has the most recent fossil. ✓✓

(2 marks)

3 a dyke ✓

b shale; limestone; sandstone; volcanic tuff; dolerite ✓✓

c Heat from the cooling magma will bake the surrounding rocks, leading to contact metamorphism. The sandstone will turn to quartzite, limestone to marble and shale to hornfels at the contact zone. ✓

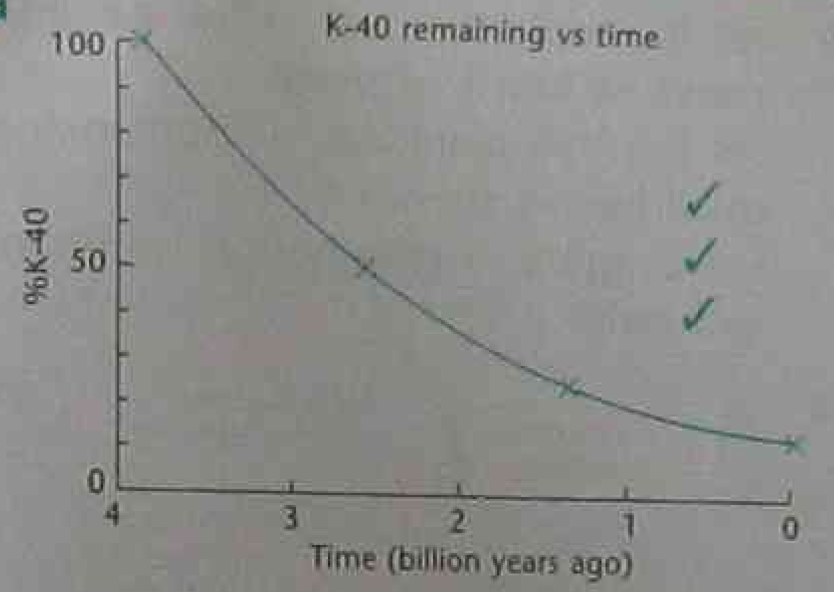
d The crystal size is much larger in the dyke than in the extrusion onto the surface. The slower the cooling of the magma the larger are the crystals that form. ✓ (5 marks)

4 a At a subduction zone an oceanic plate collides with a continental plate. Due to differing densities the oceanic plate moves down beneath the continental plate to form the subduction zone and ocean trench. Frictional heating occurs as the oceanic plate moves down towards the asthenosphere. Earthquakes result from this frictional contact. Magma is also formed from the heating and it moves upward under pressure through faults. The release of the hot melted rock produces vulcanism. ✓✓

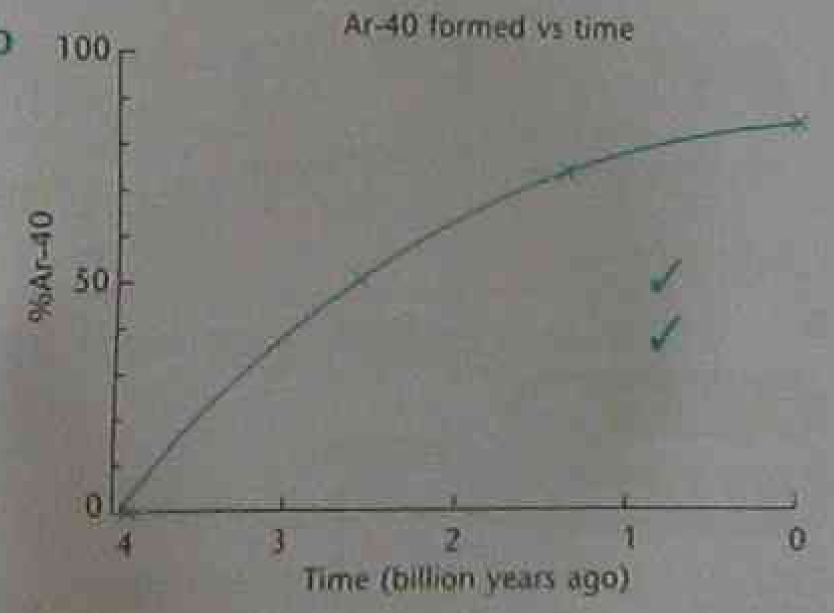
b West coast of South America where the Nazca plate collides with the South American plate; Japan where the Pacific and Philippine plates and the Eurasian plates collide. ✓✓

(4 marks)

5 a



b



c Hadean eon ✓

d Proterozoic eon ✓ (7 marks)

6 X = Adelaide
Y = Perth
Z = Melbourne ✓✓ (2 marks)

7 a Lithosphere: Sudden earth movements can lead to the formation of rift valleys and block mountains. Avalanches may occur in mountainous areas. New volcanic landforms such as cinder cones, lava shields and volcanoes may be formed. ✓✓

b Hydrosphere: Earthquakes can lead to tsunamis; lava emerging from ocean vents can lead to mineralisation of the ocean. Volcanic gases can produce acidic rain. ✓✓

c Atmosphere: Volcanic ash and gases are released into the atmosphere. The ash can block sunlight and cause atmospheric cooling. ✓✓

(6 marks)

8 a Spreading zone (divergent plate boundary) ✓

b A = 2 million years old; B = 4 million years old ✓✓

c New oceanic crust is forming at the mid-ocean ridge. The closer the rock to the mid-ocean ridge the younger is the rock. ✓

d Asthenosphere ✓

e The old oceanic crust is removed and recycled at the subduction zones. This prevents Earth's crust from expanding. ✓ (6 marks)

9 India, Australia and Antarctica had already split away from the rest of Gondwanaland before the reptile colonised these areas. This split occurred 135 million years ago. Africa and South America remained as one land mass until 105 million years ago. ✓✓ (2 marks)

10 a/f; b/g; c/h; d/e

Chapter 5

Ecosystems

pages 148–50

Part A. Knowledge

pages 148–9

1 c Wind speed does not depend on living things. ✓

2 c Mangroves are trees that live in saline water along our coast. ✓

3 a Crabs scavenge for the remains of animal tissue that is left over after a carnivore has eaten. ✓

4 b Nitrogen is an important element in the manufacture of amino acids that are the building blocks of proteins. ✓

5 d Considerable energy is used to mine and extract metals from minerals. It is much cheaper to recycle metals. ✓ (5 marks)

6 a habitat ✓
b carnivore ✓
c end ✓
d parasite ✓
e bacteria ✓ (5 marks)

7 a/g ✓; b/i ✓; c/f ✓; d/j ✓; e/h (5 marks)

8 In scrublands the rainfall is relatively low compared with wet sclerophyll forests. The scrubland can only support grasses, low shrubs and a few trees. The organic material in the soil is low. In a wet sclerophyll forest the higher rainfall allows larger trees to grow, together with ferns and small shrubs. Grasses tend to be absent as little light penetrates to the ground level. ✓✓ (5 marks)

9 A data logger with a pH probe and a temperature probe could be set up to monitor the pH and temperature each hour over a 48 hour period. The collected data could then be displayed and plotted on a computer application. (3 marks)

10 We can conserve our environment in many ways including:

- maintenance of habitat—establish national parks for endangered species; ✓
- recycling—ensure non-renewable resources are collected and recycled as this is a cheaper option than mining and extracting the material; ✓
- reduce consumption—many resources can be made to last longer if they are not wasted; the consumption of water can be reduced by installing water-saving devices. ✓ (3 marks)

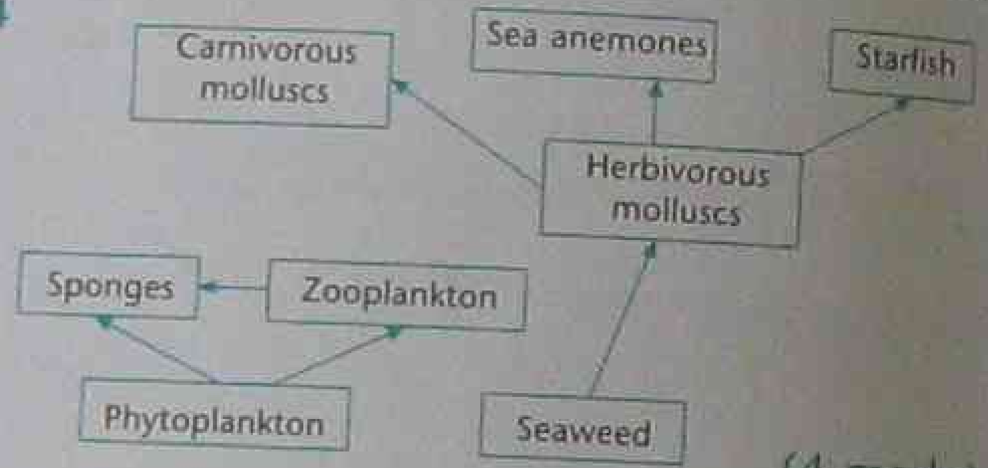
Part B. Skills pages 149–50

a 43% ✓
b 82% ✓ (2 marks)

2 a The water may contain some suspended matter that must be removed before analysis of the total dissolved solids. ✓
b Mass of residue = 209.341 − 205.456 = 3.885 g = 3885 mg. TDS = 3885/10 = 388.5 mg/L. ✓✓
c The relatively high (>100 mg/L) value could be due to the presence of salt if the water was collected close to an estuary or if there is runoff of fertiliser from nearby farms. ✓ (4 marks)

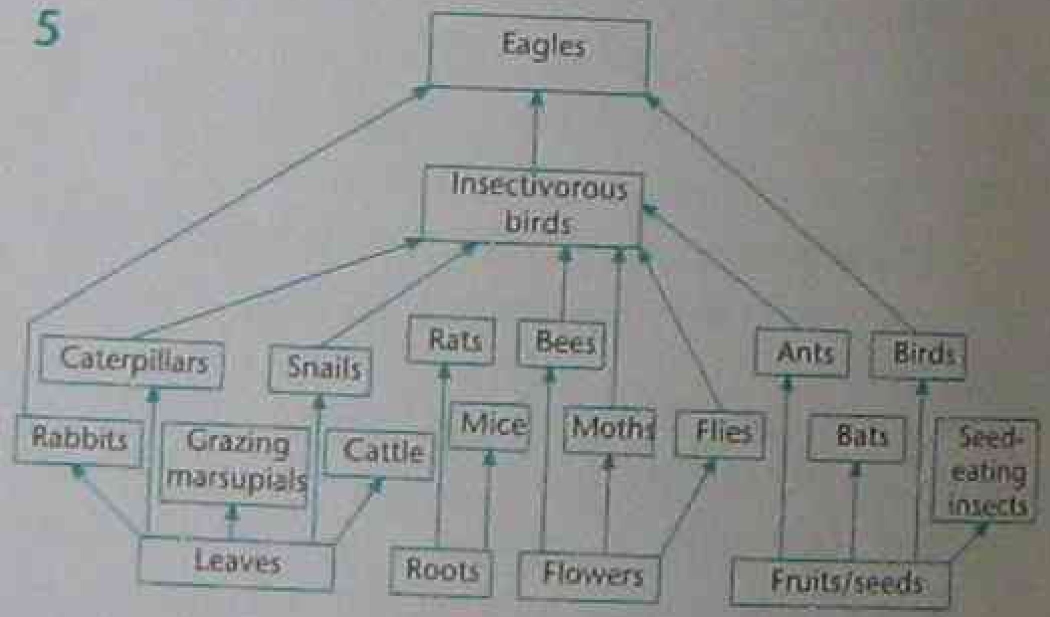
3 Set A: eucalyptus leaf → leaf-eating insect → lizard → snake
Set B: phytoplankton → zooplankton → small fish → shark
Set C: grass → grasshopper → goanna → hawk ✓✓✓ (3 marks)

4



(4 marks)

5

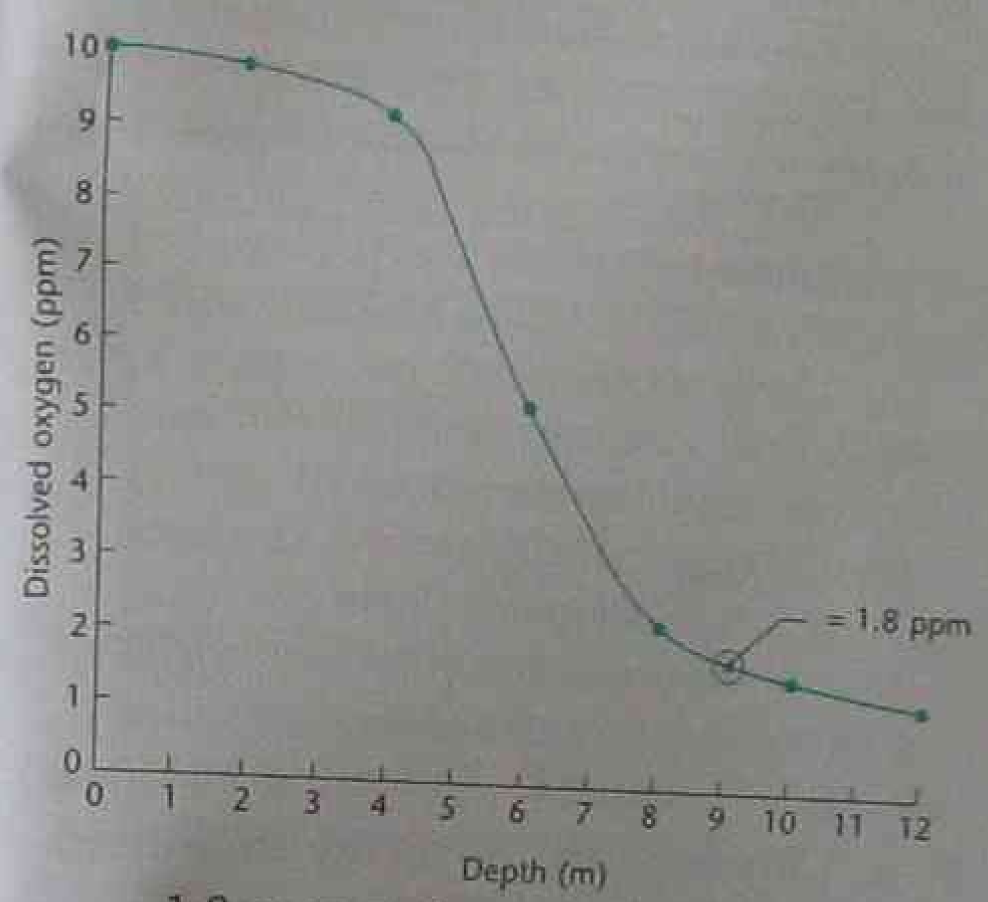


(2 marks)

6 a C and D ✓
b C receives only 10% of 200 MJ (ie. 20 MJ). D receives only 10% of 20 MJ = 2 MJ. ✓✓
c The concentration of heavy metals increases along the food chain as heavy metals are not eliminated and become concentrated in the tissues. This comes about because a large biomass of A is needed to support a small biomass of D. ✓✓ (5 marks)

7 a supported; ✓
b supported; ✓
c not supported; ✓
d supported ✓ (4 marks)

8 a The redfish required fairly oxygenated water. Below 6–7 m the levels of oxygen are too low for the fish. ✓✓
b



1.8 ppm at 9 m ✓✓✓ (3 marks)

Energy resources and pollution

pages 154–6

Part A. Knowledge pages 154–5

1 a Wood comes from trees that can be continually grown. ✓
2 c Chemical substances store energy in a battery. ✓
3 b Water in a high dam has gravitational PE and when it falls down, the energy is converted to kinetic energy and then finally to electrical energy in the turbines. ✓
4 d Australia relies on black and brown coal for most of its electricity production. ✓
5 d Petroleum is used to make petrol, kerosene and the products of the plastics industry. ✓ (5 marks)
6 a swamp ✓
b lignite ✓
c carbon ✓
d suffocation ✓
e acid ✓ (5 marks)

7 a/h ✓; b/g ✓; c/i ✓; d/f ✓; e/j ✓ (5 marks)

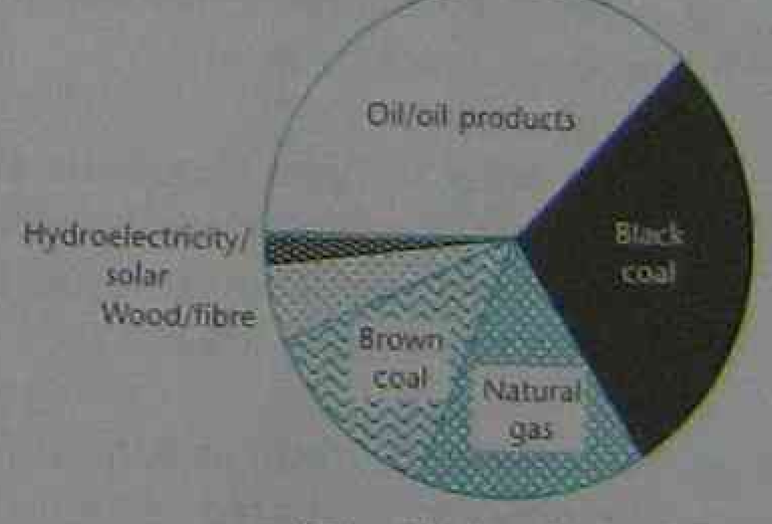
8 Bore a hole down to the hot rocks and pump cold water down to the rocks; the heat is transferred to the water and it is heated and boils. The water is then pumped out and passed through a heat exchanger, or the steam is used to drive turbines to produce electricity. ✓✓ (2 marks)

9 sulfur trioxide + water → sulfuric acid ✓✓ (2 marks)

10 The industrial revolution required fuels such as coal and oil to power machines. Cars require petrol. In all these cases the combustion of these fuels produces carbon dioxide. Large population increases have led to more fossil fuels being burnt to provide electricity. Consequently the levels of carbon dioxide have increased greatly. ✓✓ (2 marks)

Part B. Skills pages 155–6

1 a 29 + 12 = 41% ✓
b 5 + 2 = 7% ✓
c Pie graph



37% = Oil/oil products = 133°
29% = Black coal = 105°
15% = Natural gas = 54°
12% = Brown coal = 43°
5% = Wood/fibre = 18°
2% = Hydroelectnicity/solar = 7°

(3 marks)

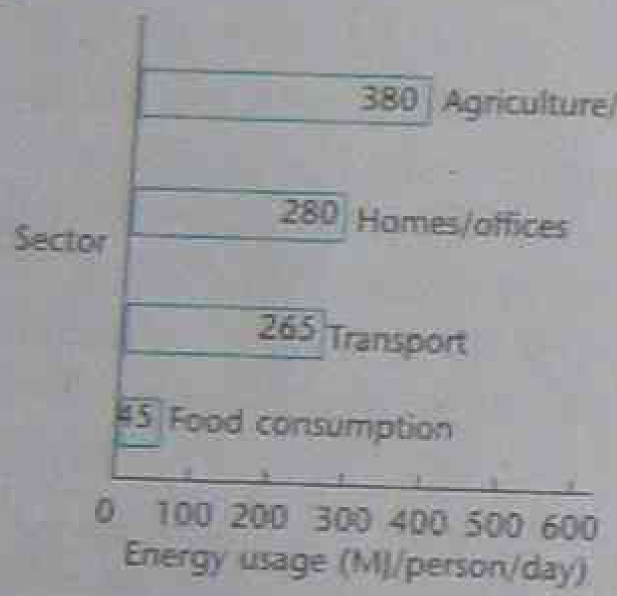
2 a Annual energy usage = 500 × 4 × 365 = 730 000 MJ ✓✓

b The USA has the greatest energy usage as it is the largest technological and industrialised society. People in the USA rely heavily on motor vehicles and electrical appliances. Australians are less dependent on energy and are less wasteful. In India many people live in rural and agricultural communities and do not drive cars. Their use of energy is much reduced. ✓✓ (4 marks)

3 a 265/970 × 100 = 27.3% ✓

b Agriculture/industry and transport use oil-based products such as petrol and diesel. (Homes and offices obtain electrical energy from coal combustion.) ✓

c



✓
✓
✓ (5 marks)

4 a Sample A is anthracite. The largest temperature rise occurred for sample A and as anthracite has a greater percentage of carbon, it will produce more heat on combustion. ✓✓

b Both water samples should be at the same initial temperature in each experiment; reduce heat losses to the environment using insulation. ✓ (3 marks)

a Incomplete (Carbon dioxide is not the only carbon-based combustion product. There is also monoxide.) ✓

b hexane + oxygen → carbon monoxide + carbon dioxide + water ✓✓

c C_6H_{14} ✓ (4 marks)

a The concentration of carbon dioxide has increased with time; the increase has accelerated in the second half of the 20th century. ✓

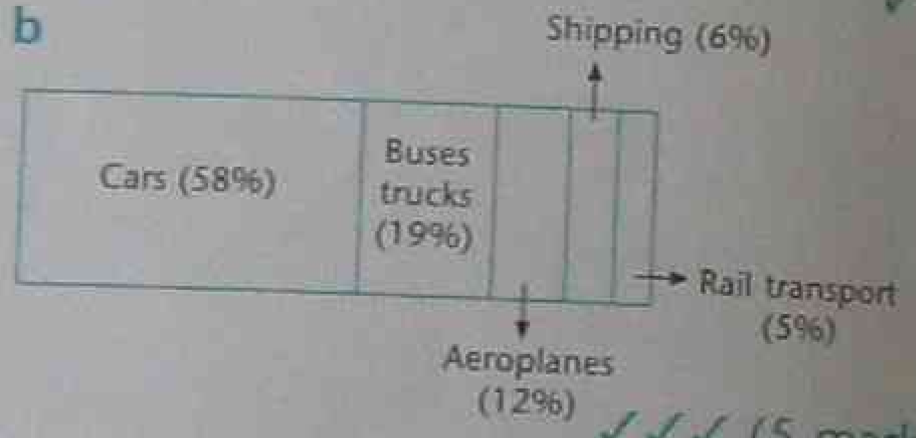
b rise in carbon dioxide = 360 – 285 = 75 ppm
% increase since 1900 = 75/285 × 100 = 26% ✓✓ (3 marks)

7 a

Transportation sector	Percentage of CO_2 emissions
Cars	58
Buses and trucks	19
Aeroplanes	12
Shipping	6
Rail transport	5

✓✓

b



✓✓✓ (5 marks)

8 a Large amounts of energy are wasted as heat that is lost to the environment. ✓

b % efficiency = 3/15 × 100 = 20% ✓

Technology

pages 161–3

Part A. Knowledge

pages 161–2

1 c The AM band (~1000 kHz radio waves) has wavelengths in the hundreds of metres. ✓

2 a Microwaves penetrate the atmosphere and can be transmitted to and from satellites. ✓

3 c The petrochemical industry processes petroleum to make fuels such as petrol, and uses gases such as ethylene to make plastics. ✓

4 a Gamma rays are highly penetrating. ✓

5 d GM foods are not produced by irradiation; the purpose of genetic engineering in crops is to produce resistance against disease and to improve yields. ✓ (5 marks)

6 a imaging ✓
b biological ✓
c radioisotopes ✓
d accelerators ✓
e radar ✓ (5 marks)

7 a/j ✓; b/i ✓; c/h ✓; d/g ✓; e/f ✓ (5 marks)

8 New material production allows the replacement of scarce or limited natural resources. It also allows new devices to be manufactured that could not have been made with existing materials. New materials such as heat-resistant alloys or ceramics have many applications including space travel. Polymers that are biodegradable help to reduce the impact of plastics on the environment. Superconducting ceramics have allowed energy to be transferred with reduced losses as well as producing very high powered electromagnets. ✓✓ (2 marks)

9 Diagnostic radiotherapy uses radioisotopes to determine whether a tissue or organ is diseased. Therapeutic radiotherapy is used to treat the diseased tissue by killing it with prescribed doses of more intense radiation. ✓✓ (2 marks)

Part B. Skills

pages 162–3

1 X: wavelength from graph = 400 m; f = $3 \times 10^8/400$ = 750 000 Hz = 0.75 MHz; this is the medium frequency band (MF).
Y: wavelength from graph = 20 m; f = $3 \times 10^8/20$ = 15 000 000 Hz = 15 MHz; this is in the high frequency band (HF). (5 marks)

2 a Wave X (amplitude is varying with time) ✓

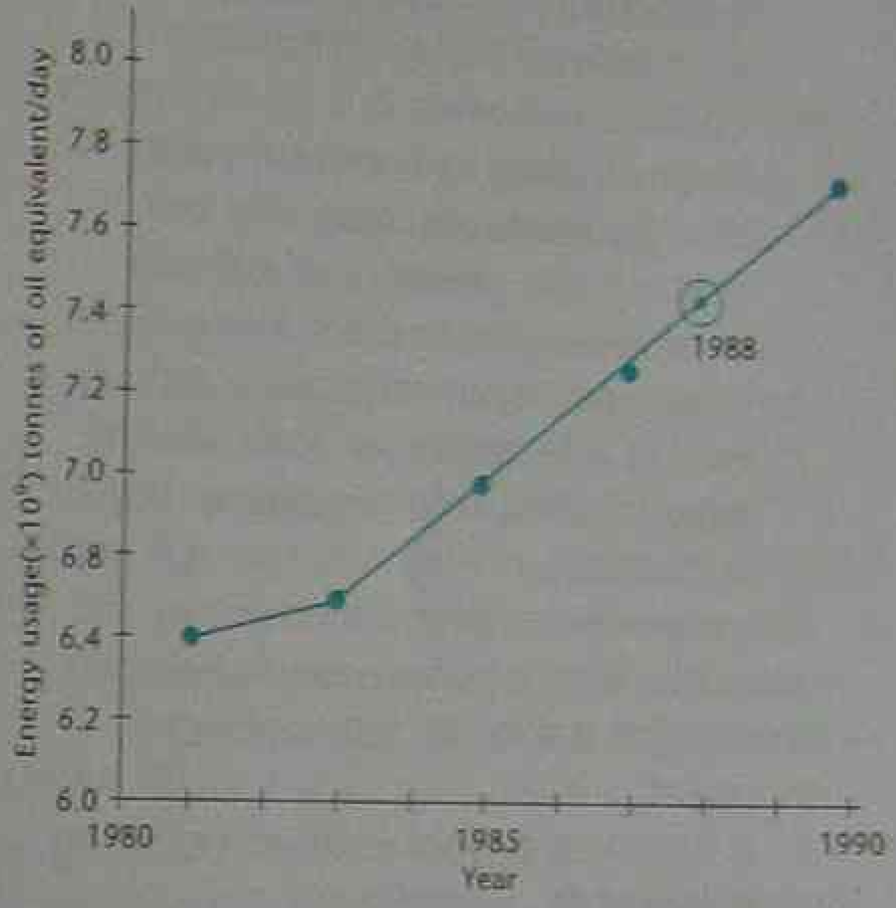
b From Graph X, the period (T) equals 2 microseconds.
$T = 2 \times 10^{-6}$s
$f = 1/T = 1/(2 \times 10^{-6)} = 500\,000$ Hz
= 500 kHz
Transmission frequency = 500 kHz (3 marks)
✓✓

3 a proton (hydrogen ion) ✓
b helium (alpha particle) ✓ (2 marks)

End-chapter test

pages 163–5

1 a ✓✓✓

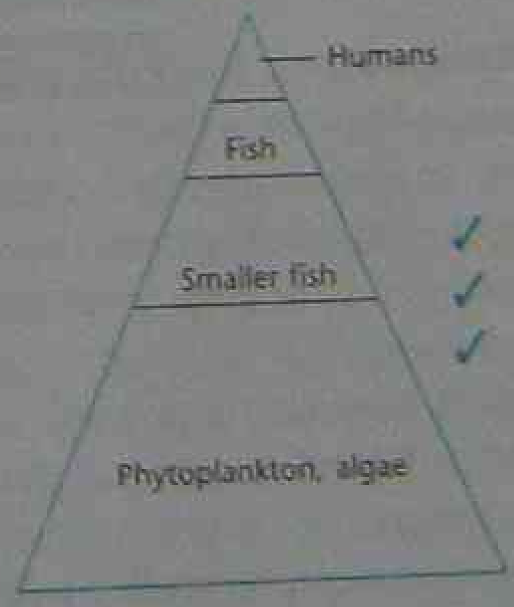


b By 1988 the daily energy usage had reached 7.5×10^9 tonnes of oil equivalent. Thus 1989 is the first year that the usage exceeded that value. ✓

c Not all energy is derived from oil but it can be expressed in terms of the amount of oil that would be equivalent to a certain mass of coal or gas. ✓

d No, it would not be valid as the usage rate may have changed considerably since the 1980s. ✓ (4 marks)

2 Pyramid



✓
✓
✓ (3 marks)

3 a Car exhaust; coal-fired power stations ✓✓

b CO ✓

c i (A) Yes—slight symptoms ✓
(B) Yes—nausea and headache ✓
ii Symptoms more severe faster in higher CO concentration.
d Carbon monoxide is a poison to the cardiovascular system and even though symptoms may not be evident, the presence of CO will reduce oxygen uptake. Also people who suffer from respiratory diseases will be affected more than healthy people. Long-term exposure may be cumulative. ✓✓ (7 marks)
4 Plan more extensive train and bus networks where people can leave their cars at the station to take public transport. ✓✓ (2 marks)
5 a It burns to produce a non-polluting material (ie. water). ✓
b i water → hydrogen + oxygen ✓✓
ii oxygen for hospitals, steelworks, research. ✓
iii solar cells ✓ (5 marks)
6 a Marsupial mouse ✓
b Mass is lost as waste; much food energy is lost as heat during respiration. ✓✓
c The first stage when solar energy and nutrients are combined in green plants. ✓ (4 marks)
7 d/g/e/a/c/f/b (3 marks)

Chapter 6

Planning investigations pages 169–72

Part A. Knowledge pages 169–70

d Inferences try to explain observations—they may not always be correct. ✓
a The experimenter allows the independent variable to change in an ordered way. ✓
d Placebos contain no active ingredients and so can be used as a comparison. ✓
a Wine and beer preferences depend on individual differences between people. ✓
b Predictions allow scientists to suggest what may happen in a future experiment. ✓ (5 marks)
6 a support ✓
b true ✓
c temporary ✓
d variables ✓
e independent ✓ (5 marks)
7 a/g✓; b/h✓; c/i✓; d/f✓; e/j✓ (5 marks)

Part B. Skills pages 170–2

1 a The southern side of the house is more moist and shady. ✓
b The boys all purchased and consumed contaminated food at the canteen during lunch. ✓
c Hydrogen peroxide breaks down as the temperature increases. ✓
d Kerosene is less dense than water. ✓
e The children's parents have pale skin and blue eyes. ✓ (2 marks)
2 25 seconds (time decreases by half for each ten degrees rise) ✓✓ (5 marks)
3 a Solutions of ionic salts conduct electric currents. ✓
b Obtain more samples of other ionic salts. Prepare aqueous solutions of these and determine their conductivity. Then check with solutions of non-ionic salts to see if there is a difference. ✓
c If all other ionic salt solutions also conduct electricity, the hypothesis would be supported. ✓ (3 marks)
4 a Temperature of the acid ✓
b The time for the magnesium to dissolve ✓
c Volume of acid; concentration of acid; rate of stirring; size of magnesium pieces; surface area of magnesium pieces; reaction vessel ✓✓ (4 marks)
5 a Group A—as there is little effect compared with group B ✓
b It suggests it is useful but not a cure, as thehsmoking does not drop to zero. ✓
c There is such variability between people that large numbers are required to make it a fair test. Some people might assume that they are taking the active ingredient and reduce their smoking even though they are taking the placebo. This is called the 'placebo effect'. ✓
d Yes. It is better that the research worker does not know which is the control group and which is the test group. ✓✓ (5 marks)
6 Most chloride salts are soluble in water. ✓✓

Performing first-hand investigations pages 174–6

Part A. Knowledge pages 174–5

1 a If the method is invalid, you will not be measuring what you think you are. ✓
2 b Repetition improves reliability. ✓
3 c Measuring cylinders have more accurate scales than beakers; choose a measuring cylinder with a size similar to the volume being measured. ✓
4 d Members of a team can waste time if they are not working cooperatively and are on task all the time. ✓
5 a Balances and rulers are not essential; a Bunsen is not needed; measuring cylinders are used to measure out the same volume of water into each beaker. ✓ (5 marks)
6 a safe ✓
b equipment ✓
c independent ✓
d controlled ✓
e evaluate ✓ (5 marks)
7 a/h✓; b/j✓; c/i✓; d/f✓; e/g ✓ (5 marks)

Part B. Skills pages 175–6

1 1. Obtain a piece of limestone and a piece of marble of the same dimensions (eg. cut them to the same size with a saw). Weigh each sample. ✓✓✓ (3 marks)
2. Place the limestone and marble in separate 100 mL beakers. (1 mark)
3. Measure 50 mL of dilute hydrochloric acid (using two measuring cylinders) and transfer the acid simultaneously into each beaker. This is time 'zero'. ✓✓
4. Allow the reaction to proceed for the same time in each beaker. ✓✓ (2 marks)
5. After a fixed time (say 10 min) tip off the acid carefully and wash each sample with water.
6. Dry the rock samples and reweigh.
7. Calculate the change in weight of each rock sample.

2 a The concentration of salt water
b The extent of rusting of the iron nails
c Volume of salty solutions used; temperature of solutions; volume of solutions; size of tubes; same position in the lab.
d Water containing no salt
e Repeat the whole experiment several times. This can be achieved by different groups conducting the experiment and pooling results.
3 Wear safety glasses; use a fume cupboard as chlorine is poisonous; use only small quantities to avoid large amounts of chlorine; have a portable shower ready in case acid is spilt on the skin.
4 **Equipment:** a black can and shiny can of the same size and shape; thermometers inserted through corks that fit in the mouth of each can; jug of hot water; large measuring cylinders; stopwatch.

Method

1. Measure equal amounts of hot water into each can using the jug and measuring cylinders.
2. Place the thermometer and cork in place in each can.
3. Place the cans near each other but not touching (but not in the sunlight) and record the initial temperature at time zero.
4. Record the temperature of each sample of water every minute for (say) 30 minutes.
5. Tabulate and graph the data.

5 a **Procedure**

1. Use a garden bed that has a large depth of good quality soil.
2. Push thin stakes (similar diameter to a thermometer) into the soil to various depths (eg. 1 cm; 5 cm; 10 cm; 15 cm; 20 cm, etc.).
3. Insert (carefully) stout alcohol thermometers (stirring type) and leave at each depth and monitor the temperature.
4. Repeat the procedure at other locations.
5. Tabulate all data.

b Team member responsibilities: student 1 drills holes in soil; students 2 and 3 measure the temperatures at various depths; student 4 is the recorder; team members can switch roles in repeat experiments.

Gathering, processing, presenting and evaluating information pages 182–4

Part A. Knowledge pages 182–3

1 b Anyone can publish material on the internet; it is not always checked by reliable people. ✓

2 a The information must come from outside Australia. ✓

3 c The procedural recount allows a student to use the past tense to describe how the experiment was performed. ✓

4 d Selecting only data that fit an hypothesis is poor science. ✓

5 c No date of publication; place (New York) is not at the end. ✓ (5 marks)

6 a newtons ✓
b valid ✓
c data or information ✓
d collated ✓
e half ✓ (5 marks)

7. A/J ✓; B/H ✓; C/F ✓; D/G ✓; E/I ✓ (5 marks)

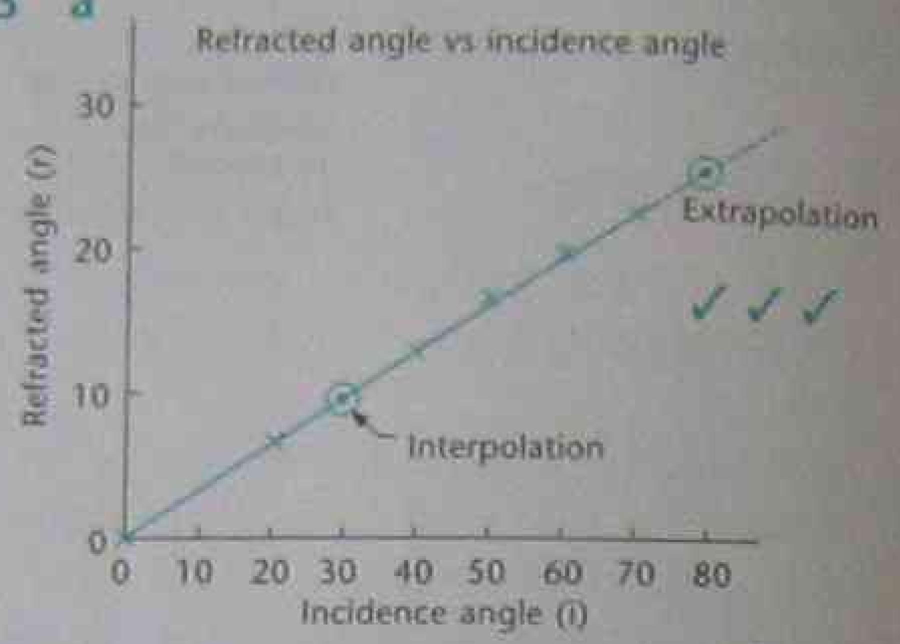
Part B. Skills pages 183–4

1 A = 6.5; B = 4.8; C = 0.325 ✓ ✓ ✓ (5 marks)

2

✓ ✓ ✓ (5 marks)

3 a



✓ ✓ ✓

b 10° ✓
c 79° (79–80) ✓ (5 marks)

4 $L = 3.5 \times 0.5 = 1.75$ mm ✓ (2 marks)

5 a This is a straight line graph $v = u + at$.
Substitute values from the graph to show that the equation is consistent.
eg. At $t = 4$, $v = 5$ and $u = 3$
$5 = 3 + a4$; solve for a.
$a = (5 - 3)/4 = 0.5$
eg. At $t = 8$, $v = 7$ and $u = 3$
$7 = 3 + a8$
$= (7 - 3)/8 = 0.5$ ✓ ✓

b $u = 3$ m/s from intercept on vertical axis ✓

c $a = 0.5$ m/s^2 as calculated in part (a) ✓ (4 marks)

End-chapter test pages 184–7

1 Claims a and b can be tested scientifically. The concentration of free radicals in the blood can be monitored in the presence of *Oxideze* and without it. The ingredients can be determined to be natural or synthetic. Claim c is not easy to test as it would be impossible to test all other alternative products on the market. (3 marks)

2 a Condition X: Average length = 5.1 cm
Condition Y: Average length = 6.9 cm ✓

b Same age of seedlings; same amount of water; same amount of exposure to light; same temperature; same concentration of each gas tested. ✓

c He needed to do a control experiment in which the seedlings were not exposed to either gas. ✓

d No. It could be due to natural variablity. He tested very few seedlings and he did not do a control. ✓ (5 marks)

3 a 12 mL ✓

b The pH was initially low and rose slowly as the base was added. At the point of neutralisation the pH changed rapidly upward. With excess base there was a rise in pH but there was a slower change in pH. ✓

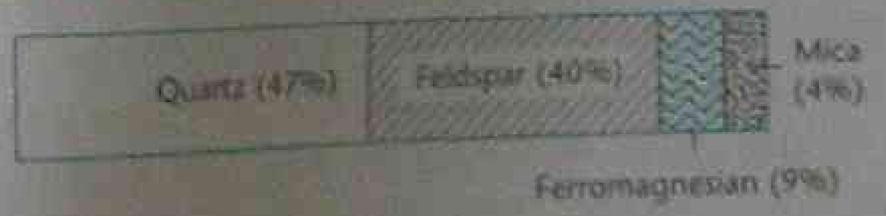
c 7 mL because the pH = 7 ✓

d 3.0 ✓

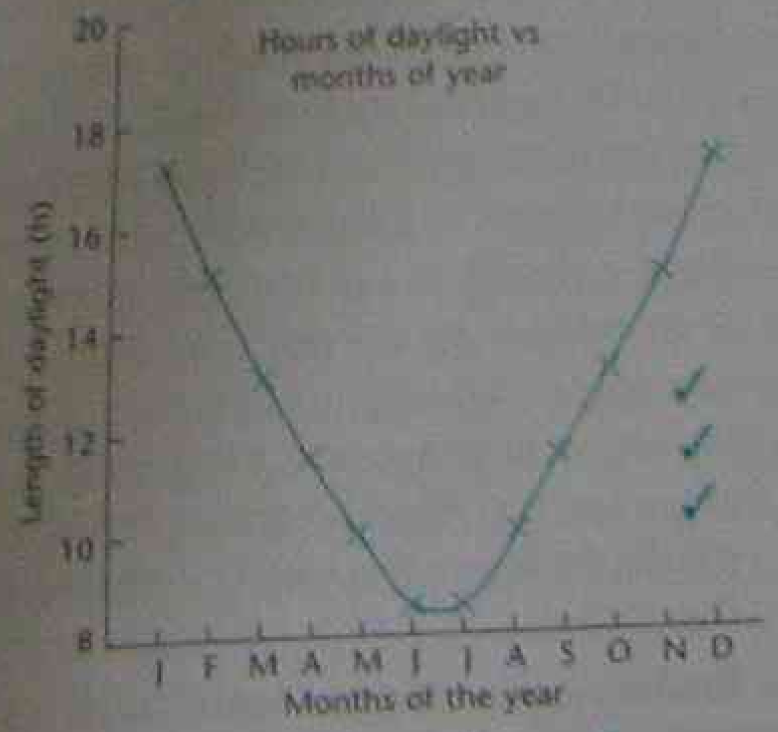
e 5.35 mL ✓

4 a 9% ✓

b ✓ ✓ (3 marks)



5 a



✓ ✓ ✓

b i Winter or early spring ✓

ii October, November, December, January, February, March ✓ (5 marks)

6 a concentration of A = 3.6 g/150 mL = 3600 mg/0.150 L = 24 000 mg/L = 24 g/L ✓ ✓

b D; A; C; B ✓ (3 marks)

Chapter 7

Sample answers page 192

1 Different cultures have made significant contributions to science throughout history. On all continents, the native populations discovered things about the world that improved the quality of their lives. Here are some examples.

African culture: Egyptians invented units of measurements and standards as well as geometry, trigonometry and algebra. The calendar of 365 days was based on early Egyptian astronomy. The Egyptians also developed sundials and wrote astronomical texts. The Moroccans experimented with light rays and developed the compass for navigation. They developed food preservation techniques and studied the properties of herbs.

Asian culture: The early Chinese developed decimal mathematics, matches and paper money. They also invented simple machines such as umbrellas and wheelbarrows. Smelting technologies allowed Chinese metallurgists to produce cast iron over 2500 years ago. The ancient Sumerians of Western Asia made extensive astronomical observations and plotted the movements of the planets across the zodiac. In addition, the Sumerians developed the way we tell time in terms of hours, minutes and seconds. Metallurgy was an important industry in western Asia. Bronze and other alloys of copper were developed in these areas. Iron and steel alloys were also developed across Asia.

European culture: Many early Greek scientists made important contributions to mathematics and astronomy. Eratosthenes, for example, wrote on astronomy and calculated the diameter of the Earth. Aristotle and Theophrastus developed the science of plant classification. Hippodamus developed a system of public water works to ensure better sanitation. Hero was the first hydraulic engineer and developed the siphon. Many scientific ideas that had their roots in the Middle East spread throughout Europe. This was particularly true of metallurgy, which became an important industry throughout Europe in the first millennium AD.

American culture: Native north Americans developed many botanical medications including those to treat heart problems. Central Americans developed vanilla as a flavouring agent in foods. They also had developed metallurgy and worked extensively with platinum using a new sintering process. Native Americans excelled in plant breeding and cultivated new breeds of plants.

Following the Dark Ages in Europe, a period known as the Renaissance led to greater freedom for people to pursue scientific discoveries. The astronomical discoveries of Copernicus, Galileo and Kepler put astronomy on a firm theoretical foundation. European chemists of the seventeenth and eighteenth centuries (eg. Lavoisier and Dalton) developed the concepts of elements and compounds by revitalising the atomic theory first proposed by the ancient Greeks. In most branches of science in the seventeenth and eighteenth centuries, it was the European scientists who dominated. This was largely due to the increasing wealth of European society which, in turn, allowed more complex experimentation to be undertaken. In poorer regions of the world there were insufficient funds to allow the luxury of scientific discovery. Gradually, over the next two centuries, the process of scientific discovery spread to North America, Australia and eastern Asia. In the modern world, science is a worldwide enterprise which is not dominated by any particular cultural group. It is universal, and ideas are freely exchanged through the internet. The major stumbling block to high-level research is funding, which comes from both private institutions and governments. (5 marks)

2 There are many scientific issues raised by the mass media that require people to have knowledge of science in order to fully understand the issues involved. Some of these issues are listed below.

Genetically modified (GM) food: To understand the potential impact of GM technology, we need to understand the processes involved and their likely consequences. GM food technology involves the insertion of foreign genes into an organism. These genes are derived from a completely different species. For example, a gene for resistance to cold can be extracted from an Antarctic fish and placed into a tomato to allow tomatoes to be grown in cold climates without damage. This technology assumes that the new hybrid tomato is safe to eat, which may or may not be true. Current testing shows it to be safe but critics are concerned that the presence of the foreign gene may produce changes that we currently do not understand. If these GM tomatoes are allowed to be cultivated near other crops, the gene may find its way (via pollen) to other plants. This issue is important in GM crops where herbicide-resistant genes have been inserted in crop plants such as canola to help prevent insect attack. The question focused on by the media is whether these plants are safe to eat and whether we should have a choice to eat non-GM crops. Without legislation to keep GM and non-GM products separate, consumers can never know what they are consuming. Balanced arguments should be presented by the mass media to keep the public fully informed of the issues.

Embryonic stem cells and cloning: Embryonic stem cells are undifferentiated cells that are present at an early developmental stage of an embryo. These cells go on to form the tissues and organs of our body. Current research suggests that one way of curing diseased organs would be to grow new replacement organs using embryonic stem cells. This would overcome the rejection of donor organs and also provide a greater supply of organs. For example, the technology may be able to restore movement to quadriplegics by repairing spinal cord damage. Critics of this technology usually base their objections on ethical, moral and religious grounds. In order to harvest embryonic stem cells, scientists need to destroy human embryos. Many of these embryos come from discarded IVF procedures or embryos that have not survived. This concept of growing and destroying embryos is repugnant to people who believe that life begins at conception rather than at birth. The mass media have presented both sides of this argument and politicians have been forced to debate the issue. Governments have responded in different ways. Forums such as letters pages in newspapers and internet chatrooms allow input on this issue from many citizens.

Citizens who are not scientifically literate cannot give proper consideration to the issues in these cases. Without scientific knowledge they can only argue from a position of ignorance. (5 marks)

3 The ability of nations to send unmanned space probes to the planets is largely dependent on their national wealth. Poorer nations cannot devote funds to such expensive research. The first missions were sent by the USA and the former USSR in an attempt to gain superiority in space. This was part of the Cold War and each nation was afraid that unmanned satellites could be used as military weapons. With the end of the Cold War, the USA has continued to send missions (such as the Voyager probes) to the outer planets. The European Space Agency (ESA) uses funds from member nations to continue space research, such as sending unmanned craft to Mars to investigate its geology and the possibility of life. The USA is also involved in Martian missions.

Many critics of these space programs say that such research is a waste of money. They say that this money should be directed to improving human health and lifestyle. Proponents of these programs argue that humans must continue to look outside our own domain to understand our place in the universe. Some of the fundamental questions of science can only be solved by leaving the Earth. Sending missions to other planets to investigate their geology and chemistry can improve our understanding of the evolution of the universe. This data can then be compared with theoretical models of planetary formation. As with the Apollo missions to the moon, there are spin-offs from such research. New materials (eg. fibre composites and alloys) that can be used in our everyday lives will be developed. New and more efficient means of communication which can then be used on Earth for various applications will be developed also. Apart from the practical benefits, there are intellectual developments that ensue. This research has an uplifting effect on the human spirit. It is the new frontier of exploration. (5 marks)

School Certificate Trial Revision Test

pages 193–203

Section 1. Multiple choice

(1 mark per correct answer)

1 a Each subdivision of the scale is 0.2.

2 d 4 × 50 = 200

3 c Each large scale division is 10% and each small division is 5%.

4 c Read the vertical axis when time = 5 s.

5 c AU is only used for bodies in our solar system, whereas ly (light years) is used for more distant objects.

6 **b** Each row increases by 9.8.

7 **c** Examine the Periodic table to find iridium (element 77).

8 **c** Phosphorus has a valency of 3 and chlorine has a valency of 1.

9 **d** $F = ma$ is the normal form of Newton's second law.

c Silicon is a semi-metal.

a Magnesium's electron configuration is 2, 8, 2 (Z = 12).

b Universal indicator is a common acid–base indicator.

d The ammeter is in series and the voltmeter is in parallel; the circuit is complete.

c The diagram shows a transverse wave; the wavelength is equal to 4 grid spaces (4 × 100 = 400 mm).

c Gamma rays have high energy that destroys microbes.

a Ray bends towards the normal as it enters the glass and bends away from the normal as it re-enters the air.

b One compound decomposes into two other compounds.

b Water, carbon monoxide, carbon dioxide and unburnt octane would be found in the exhaust—they are all gases.

d Respiration occurs in all cells to provide energy—it is not specific to dividing cells.

DNA consists of two strands wound around each other into a double helix.

Nerves and hormones are involved in body coordination.

Female gametes are eggs; they are produced in the ovaries.

Modern day experiments with dog breeding cannot prove what happened in nature in the past.

The atmosphere absorbs some types of radiation, including UV.

Jellyfish have no hard parts but their imprint in mud or sand can lead to trace fossil formation.

26 **c** The ocean plate dives under the continental plate at a subduction zone.

27 **c** Competition for food is biotic and light intensity is abiotic.

28 **a** First-order consumers are herbivores.

29 **a** The Pacific Rim has many subduction zones that generate earthquakes and volcanoes.

30 **b** Use the clues to determine that the Palaeozoic was dominated by fish and amphibians; reptiles evolved after amphibians.

31 **d** Yellow stars become very large and red as they evolve.

32 **d** Burning coal and oil produces carbon dioxide which is a greenhouse gas.

33 **b** Blue light scatters but red light is transmitted.

34 **d** The SO_4 radical is called sulfate.

35 **c** Rocks composed of crystals are usually igneous.

36 **a** The crystal's length is 80% of the 5 micrometres.

37 **a** Hydrogen is the simplest element and the first one formed after the Big Bang.

38 **b** Correlate similar fossils in different layers; layers containing the same indicator fossil are of the same age

39 **c** Antibiotics kill bacteria but not viruses and fungoid animals.

40 **d** D is the Lamarck theory that suggests that characteristics acquired in life can be passed on.

41 **a** Inertia is the tendency of a body to remain at rest or to continue with its constant motion.

42 **b** Theories survive as long as there are no observations that discount them.

43 **c** UV radiation is absorbed by the ozone layer.

44 **d** Radiowaves are the normal means of communication through space.

45 **b** The clues show that air spirals inward at low levels and outward at high levels.

46 **c** Reflex arcs by-pass the brain to provide a quick response.

47 **d** Sulfur and selenium are in the same group of the Periodic table.

48 **a** The force on the gases is equal to the force acting on the rocket.

49 **b** B has the lowest resistance and the highest current. Therefore the lamp glows the most.

50 **d** Fossils are destroyed by molten rock.

Section 2. Questions 51–66

Part A (1 mark per correct answer)

51 uranium
52 ammeter
53 carbonate
54 corrosion
55 abiotic
56 infectious
57 gravitational
58 fossil
59 resources
60 evidence

Part B (ticks indicate where marks are allocated)

61 1 mark for tabular format ✓
1 mark for headings of tables ✓
2 marks for all substances correctly classified ✓✓
(1 mark if one or two are wrong)

Acidic substances	Basic substances	Neutral substances
vinegar	ammonia cleanser	salt
soda water	caustic soda	sugar

62 a i neutron ✓
ii The energy released in the form of heat energy ✓ is used to boil water to produce steam. ✓ This steam is used to drive turbines that produce electricity. ✓

b **Environmental impact of nuclear energy**: Nuclear fuels produce energy and radioactive waste. ✓ This waste must be contained and stored for thousands of years so that it does not contaminate the environment. ✓ Many environmental groups are concerned that these storage facilities may ultimately fail or be subject to terrorism and that dangerous radioisotopes will contaminate the environment. ✓

Environmental impact of fossil fuels: Burning fossil fuels such as coal and oil to produce energy leads to the emission of carbon dioxide, carbon monoxide, soot and ash into the atmosphere. ✓ Carbon dioxide is a greenhouse gas and its rapid increase in atmospheric concentration over the last century has been linked to global warming. ✓ Carbon monoxide is a dangerous pollutant as it interferes with oxygen absorption in the blood. Ash and soot cause allergies and visual pollution. ✓

Part C

63 Method in numbered or bulleted steps ✓
Equipment and chemicals correctly identified ✓✓
Scientific method of fair testing used ✓✓
Controls mentioned ✓

1. Prepare 5 mL samples of pure water and **solutions of calcium chloride of increasing concentration** (eg. 0.5%; 1%; 2%; 4%) (independent variable). Place the samples in stoppered test tubes.
2. Prepare a standard soap solution by dissolving a known mass of soap flakes or liquid soap in a known volume of water.
3. Place 2 mL of soap solution in each test tube of calcium chloride solution and the pure water control.
4. Keep the tubes at the same constant temperature.

5. Stopper the tubes and shake each vigorously for the same time (say 1 minute).
6. Compare the **height of soap bubbles** (dependent variable) formed in each test tube.

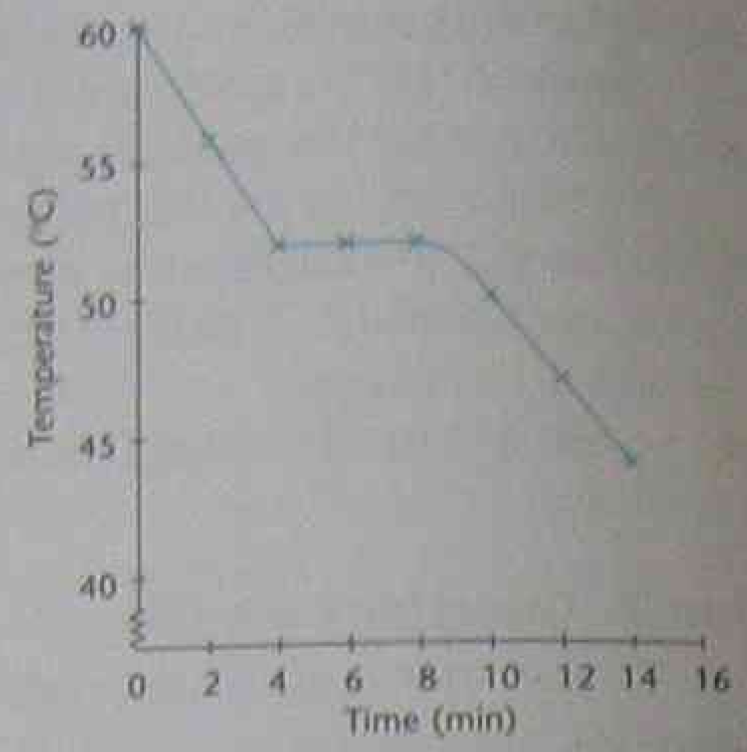
If the hypothesis is supported there will be less soap foam produced as the hardness (ie. calcium concentration) of the water increases.

4 a carbon dioxide ✓
  b after 600 m ✓
  c Respiration and decomposition produce carbon dioxide. ✓ The graph shows that carbon dioxide concentration increases with depth. ✓
  d The oxygen levels drop for the first 150 m indicating that the oxygen production continually decreases. ✓ After this the oxygen level does not change much. This is consistent with light penetration occurring only to a depth of 150 m. ✓

rt D

5 a dragonfly; shark; ✓ frog; snake; whale ✓
  b i shale ✓
    ii Order of events = 2, 5, 4, ✓ 6, 1, 10, ✓ 8, 9, 7, 3 ✓

66 a



1 mark for axes, titles and units correctly shown ✓
1 mark for correctly plotting points ✓
1 mark for drawing line of best fit ✓
1 mark for graph occupying at least 80% of available grid space provided. ✓

  b temperature of the wax ✓
  c The liquid wax was solidifying to form solid wax. The loss of heat to the surroundings was balanced by heat lost, as the solid formed kept the wax at a fairly constant temperature. ✓
  d Yes, the rate of cooling would be different and the shape of the graph would change. ✓ The temperature at which the liquid wax solidifies (52° C) should not change as it is a constant property of the material (ie. its freezing point). ✓

Index

A
abiotic components 142
absolute dating 127
AC (alternating current) 16
acceleration 7, 10, 38
accuracy of measurement 173
acetic acid 63
acid rain 134, 147–148
acids 62–66
  oxides 153
  water 144
actinides 51
action and reaction 9
adrenals 97
algal blooms 147
alkali metals 51
alleles 82
alloys, super-elastic 160
alpha particles 34–35
altitude and gravity 39, 113
AM (amplitude modulated) band 157
amino acids 81
ammeter 17–18
amplitude of vibration 3
anatomy, comparative 87–88
ancestor, common 89
anions 54, 56
Antarctica 89
antibiotics/antibodies 94, 100–101
anticlines 125
antigens 94, 100
antiseptics 101
Archaen eon 128
asexual reproduction 76–77
asthenosphere 130
astronomical unit 111, 115
atomic theory 44–46, 73
atoms 50, 113
atoms, elements and compounds 44–74
Australian ecosystems 143
avalanches 134
axon 95

B
bacteria 98–100
balancing equations 61
bases 64–66
  DNA 80
  see also acids
battery 16–18, 151
Beagle, HMS 90
beta particles 34–35
bias, statistical 169
big bang theory 111–112, 116, 140
biosphere 135
biotechnology 159
biotic component 142
black dwarf 117
black holes/quasars 116–117
blind experiments 169
body systems 75
Bohr, N. 46
bonds, atomic 54–55
brain stem 96
bromothymol blue 65
budding 77
Bunsen burner 61
bushfires 134–135

C
cancer 98
carbon 51, 56, 127
  isotopes 128
  nanotubes 160
carbon-oxygen cycle 146
carbonates 64
carnivores 144
casts and moulds 127
cations 54, 56
CBR (cosmic background radiation) 111–112
cell body 95–96
cell, electric 17
cell theory 75–76, 109
Cenozoic era 128
cerebellum/cerebrum 94, 96
chain reactions 34
Chandra X-ray observatory 115
charge, electrical 18
chemical energy 151
chemistry 44–74
chromosomes 75–77, 81, 102
cilia and hairs 100
circuits, electrical 16–17, 20–22
circulatory system 75
citric acid 63
closed (pulsating) universe theory 113
CNS (Central Nervous System) 94, 96
coal 152
COBE satellite 113
collision zones 132
combustion 54, 61, 151
common ancestor 89
communications technology 157
comparative anatomy and embryology 87–89
competitors, ecological 142
composite volcanoes 133, 135
compounds 44, 54–58, 73
compression waves 2–4
conductors 19, 47
connector neurones 95
conservation, environment 147
consumers 142, 144
contact forces 8
continental drift/plate tectonics 130–131
controlled variables 166
convection currents 130
convergent light rays 28–30
coordination 95
corrosion 54, 61–62
cosmic rays 111
covalent bonds 54–55
covalent compounds 57–58
Crick-Watson model of DNA 79, 109
critical thinking 182
current, convection 130
current, electricity 16–17
Cuvier, G. 87–88
cycles in nature 145–146
cyclones 134–135
cyclotron 157

D
Dalton, J. 44
dark matter 116
Darwin, C. 87, 90–91
dating 87, 123, 127–128
DC (direct current) 16–17
deceleration 7, 10
decomposers 142
decomposition reactions 55, 67
deletions in DNA 81
dendrites 95
deoxyribose 80
dependent variable 166
deserts 143
diagnostic nuclear imaging 159
differentiation 75
digestive system 75
digital transmission 157
disease 98
divergent light 28–30
DNA 79–80, 89–90, 98, 101
dominant alleles 82
double helix 80, 109

double-blind experiments 169
drift, continental 130–131
dust storms 134
dwarf stars 117
dykes 124

E
Earth 87
  and atmosphere 114
  and space 111–141
earthquakes 3, 131–135
echo location 2
ecosystems, resources and technology 142–165
effervescence 55
eggs/sperm 77, 102
Einstein, A. 111–112
electric cell 16
electric blankets/radiators 22
electric field 4
electrical energy 16, 42
electrolysis 67
electromagnetic spectrum 4, 113–114
electromagnetic waves 1, 4, 28, 157
electrons 16, 44, 113
  configuration 44, 46–47
  elements 44–45, 47, 49, 73
embryology 87, 89, 103
endocrine system 95, 97
energy
  batteries/chemical 151
  electrical 42
  equivalence 111
  force and motion 1–43
  geothermal/gravitational 151
  light 42
  nuclear 34, 43, 151
  renewable 151
  particles 1–2
energy shells/orbitals 44
environmental/social issues 36
Eohippus 88
eons 123, 128
epicentre of earthquake 132
epoch 129
equations 3, 59, 61, 64, 67
equivalence/energy 111
Equus 88
eras 128
erosion 134
error, parallax 176, 178
evolution 87, 91, 109
excretory system 75
expanding universe theory 111
experimental design 169, 172
extinction 89
extrapolation/interpolation 179

F
fair testing 167–168
farming 147
faulting/folding rocks 125, 131
feeding relationships 144
female reproductive system 102–103
ferns 89
fertilisation 102–103
field force 8, 38
film, photographic 28
finches, Galapagos 90–91
fission, reproductive 77
fission, nuclear 34
floods 134
FM (frequency modulated) 157
focus of earthquake 132
foetus 104
folding/faulting rocks 125, 131
food pyramids/webs 144–145
footprints and imprints 127
force 7–8
  field 38
  gravitational 38, 43, 113
formulae, valency rules 58
fossil fuels 150–152
fossilisation 88, 123, 126–127
frequency 3
friction 7
fuel, fossil 2, 150–151
fungi 99
fuse 17, 22
fusion, nuclear 34–35, 111

G
Galapagos finches 90–91
galaxies and stars 111–116
galvanising 62
gametes/sex cells 77, 101–102
gamma radiography 159
gamma rays 4, 34–35
gases 134
genetics 76, 79–82, 91, 98
geographic isolation 87, 89, 91
geological history 123, 128–129
geostationary satellite 158
geothermal energy 151
gestation 103
glaciers 135
glands 97
glucagon 97–98
glucose 98
GM foods 159–160
Gondwanaland 89, 131, 140
graphs 180–181
gravitation 38–39, 43, 112–113, 130, 151
greenhouse gases 134, 153–154
groups, elements 49–50, 56
growth 75–76

H
habitats 142, 148
Hadean eon 128
hairs and cilia 100
half-life 87
halogens 51
herbivores 144
Homo sapiens 94
hormones 97
Hubble Space Telescope 112, 114
humans 94, 110
hydrogen bomb 35
hydrogen ions 64
hydrosphere 134
hypotheses 166–169

I
igneous rocks 123–124, 128
immunity and immunisation 100–101
imprints and footprints 127
incomplete combustion 151
independent variable 166
indicators 55, 65
infectious disease 98
inference 166–167, 180
information, scientific 176, 180
inheritance 82
inorganic compounds 56
insulators 19
insulin 97–98
interpolation/extrapolation 179
introduced fauna and flora 147
intrusions 125
investigation and problem solving 166–187
involuntary nerves 96
ions 55–57, 64
IR (infrared) waves 4, 114
isotopes 44–45
IVF (in-vitro fertilisation) 159–160

K/L
Keck telescope 114
kingdoms 98–99
lactic acid 63
lamp, electrical 17
lanthanides 51
Laurasia 131

lava 124, 133–134
law of superposition 87–88, 124
light 3, 28–29, 43
  pollution 114
light year 111, 115
lighting and power circuits 22
lightning 134
lithosphere 123, 130, 134
litmus 65
longitudinal waves 3
loss of habitat 147
lymphocytes/phagocytes 95, 100

M
magma 124, 133
magnetic fields 4
male reproductive system 102
mass 7, 38, 43, 111
mass number 44–45
measurement, accuracy of 173
mechanical waves 1–2
medical use of waves 2
meiosis 102
menstruation 103
Mesozoic era 128
metals 50, 56
  reaction with acids 63
  conductors 47
metamorphic rocks 124–125, 128
meters, light 28
methyl orange 65
microbes 99
microwaves 157–158
mid-ocean ridges 131
Milky Way 115
mitosis 75–77
mobile phone 158
modem 157
modulation 157
molecules 49–50
Monera 98
moths 82
motion 11
motor neurones 96
moulds and casts 127
mountains 130–131
mucus 100
multicellular organisms 75
muscular system 75
mutations 80–81, 91
myelin sheath 96

N
naming chemicals 57
nanotubes, carbon 160
natural events 123, 140
natural selection 87, 90, 109
nebulae 116–117
nervous system 95
neurone 95–96
neutralisation 55, 64–66
neutron 34–35, 44, 113
neutron star/pulsar 116–117
newton (N) 8
Newton's laws of motion 7–16, 42
Newton, I. 38
nitrogen cycle 146
noble gases 51
non-metals 47, 50, 56
novae/supernovae 116
nucleotides 80
nuclear energy/power 34–36, 43, 151
nuclear fission/fusion 34–35, 111
nuclear technology 158–159
nucleus of atom 44
nucleus of cell 76
nutrients 75

O
ocean ridges 131
ocean waves 1
octet of electrons 46
Ohm's law 19
oil and gas 152
omnivores 144
opalescence 127
open universe theory 113
optical telescopes 114
organ systems 75, 97
organic compounds 56
ovaries 103
oxidisers 61
oxygen-carbon cycle 146

P
Palaeozoic era 128
pancreas 97
Pangaea 131
parallax error 176, 178
parallel circuits 19
parathyroid 97
particles, alpha/beta 34–35
pathogens 95, 98, 101
patterns/trends in data 179
penicillin 101
period, geological 129
period, wavelength 3
periodic table 49–50, 56
peripheral nervous system 95–96
petrified wood 127
pH 66
pH meter 144
phagocytes/lymphocytes 95, 100
Phanerozoic eon 128
phenolphthalein 65
phenotype 82
phone, mobile 158
photochemical decomposition 67
photographic film 28
photosynthesis 142, 144, 148
physics 1–43
physiological disease 98
pituitary gland 97
placebo 166
placenta 95
planet 116
plate tectonics 123, 130–131
plutons 124
pollution 153
positron 35
potassium-40 128
potential difference 18
potential energy 151
power circuits/lighting 22
power, nuclear 34
Precambrian era 128
precipitation reactions 55, 62
predators 142
predictions, scientific 167, 180
primary (P) waves 132
processing information 179
producers 142, 144
proteins 81
Protista 99
protons 35, 44, 113
protozoans 99
Proxima Centauri 118
puberty 103
pulsar/neutron star 116
pyroclastic rocks 123
pyroclastic volcanoes 133–134
pyramid, food 144–145

Q/R
quark 113
quasars and black holes 116–117
radar 158
radiators/electric blankets 22
radio waves 4, 114, 157
radioactivity 34
radioisotopes 158–159
radiometric dating 87, 123, 128
rain, acid 134, 147–148
ranges, mountain 130
rarefaction, wave theory 2–4
rays, gamma 4, 34–35
reaction/action 9

- reactions, chemical 54–55, 59, 73
  - combustion/corrosion 61–62
  - decomposition 55, 67
- reactivity 63
- receptors 97
- recessive alleles 82
- recycling 148
- red stars/supergiant 117
- red shift 111–112
- reflectance/translucence 29
- reflex arc 96
- refraction 28–30
- relative dating 127
- renewable energy 151
- replication of DNA 80
- reproduction 76–77, 101–103
- resistance/resistor 16–18
- respiratory system 75
- restoration of habitats 148
- Richter scale 132–133
- ridges, mid-ocean 131
- rift valley 131
- rocks 123–125, 128
- rubidium-87 128
- rusting 61

S
- salination 147
- salts 65
- satellite communication 158
- satellite, COBE 113
- scattered light 29
- scavengers 144
- scientific method 166
- sclerophyll forests 143
- scrublands 143
- secondary (S) waves 132
- secondary sources of information 177
- Sedgwick, Adam. 88
- sedimentary strata 87, 123, 126–128
- seismology 2, 123, 132
- selection, natural 87, 90, 109
- semi-conductors 160
- semi-metals 50–51
- sense organs 97
- sensory neurones 95
- series circuits 19
- sex cells/gametes 77, 101
- shells, energy 44
- shields 134
- sills 124
- skeletal system 75
- Smith, W. 87–88
- social/environmental issues 36
- solar energy 151
- solar system 114–115
- sonar 2
- sound waves 2–3
- space and Earth 111–141
- space telescopes 114
- species 87
- speed 3, 8, 10
- speedometer 9
- sperm/eggs 77, 102
- spores 77
- spreading zones 131
- springs/string vibrations 2
- stars and galaxies 111–117
- steady state theory 112
- stem cells 160
- sterilisation 101
- strata of rocks 123–124, 126
- strato-volcanoes 133
- stromatolites 88
- subatomic particles 44
- subduction zones 130, 132–133
- substitutions in DNA 81
- super-elastic alloys 160
- superconductors, ceramic 161
- supergiant stars 117
- supernovae 116–117
- superposition, law of 87–88, 124
- survival of the fittest 90
- switches, electric 17
- symbolic equations 59
- synclines 125
- systems, body 75
  - physiological 75, 95–97
  - reproductive 102–103
  - solar 115

T
- TDS (total dissolved solids) 143
- technology 157, 165
  - nuclear 158
- tectonics, plate 123, 130
- telescopes, optical 114
- testes 102
- theories, 168
  - astronomical 111–113, 116, 140
  - atomic 44, 73
  - cell 75
  - evolution/natural selection 87, 90
- therapeutic radioisotopes 159
- thermal decomposition 67
- thyroid 97
- tissues 75
- transition metals 51
- translucence 28–29
- transmission, digital 157
- transverse waves 3–4
- trends/patterns in data 179
- trilobites 129
- triplet code 80
- tsunamis 134
- tungsten filament 19
- TV waves 4, 157

U
- unconformity 123, 126
- units 176
  - astronomical 115
- universal indicator 66
- universe theories 111–113, 116, 123
- uranium 34
- urban development and farming 147
- UV (ultraviolet) waves 4

V
- valency 46–48, 55
- valleys, rift 131
- variables 166, 168
- variation 87
- velocity 3, 8, 10
- Vernier scale 177
- vertebrates 88–89
- vibrating strings and springs 2
- viruses 99
- visible light 28, 114
- volcanoes 124, 133–135
- voltage/voltmeter 16–19
- voluntary nerves 96
- vulcanism 3, 131–135

W
- Wallace, A. 90
- wastes 75, 146, 148
- water 3, 144–145
- Watson-Crick model of DNA 79, 109
- waves
  - energy 151
  - model 1, 42
  - (P) and (S) 132
- wavelength 1–4
- web, food 144–145
- weight 8, 38, 43
- white blood cells (phagocytes) 100
- white dwarf 117
- wind energy 151
- wires, electrical 17
- woodlands 143
- word equations 59, 61, 64, 67

X/Z
- X-rays 4, 114
- zones, geological 130–133
- zygote 102, 110

Excel

Science Study Guide

Years 9–10

The ***Excel*** *Science Study Guide* for Years 9–10 comprehensively covers all the topic areas in the Years 9–10 Science course in NSW. It also contains information relevant to students studying Science in other states.

The book contains:

- ✓ Coverage of five major topic areas in the NSW School Certificate test:
  - Energy, Force and Motion
  - Atoms, Elements and Compounds
  - Structure and Function of Living Things
  - Earth and Space
  - Ecosystems, Resources and Technology
- ✓ A chapter on Investigations and Problem Solving in Science to help with practical skills
- ✓ Revision questions and chapter tests to help you remember important information
- ✓ A glossary and summary in each section of the book
- ✓ Diagrams and illustrations to help your understanding
- ✓ A section to help you prepare for the School Certificate test
- ✓ A sample School Certificate test paper with answers
- ✓ Answers to all questions.

About the author

Will Marchment, BSc DipEd, has over 20 years' experience in teaching Science in NSW. He is also the author of *Excel Revise in a Month HSC Chemistry* (Core).

ISBN 1-74125-005-6

9 781741 250053

- Parents: visit our website for a complete list of books at **www.pascalpress.com.au**
- Our address is **Pascal Press, PO Box 250, Glebe NSW 2037, ph (02) 8585 4044**

Excel

PRELIMINARY

INFORMATION PROCESSES AND TECHNOLOGY

Your Step by Step Guide to Exam Success

MARY O'CONNOR-NICKEL

Complete Course

Excel

PRELIMINARY

INFORMATION PROCESSES AND TECHNOLOGY

MARY O'CONNOR-NICKEL
BA DipEd, MEd

PASCAL
PRESS

ISBN 1 74020 321 6

Pascal Press
PO Box 250
Glebe NSW 2037
(02) 8585 4044
www.pascalpress.com.au

Publisher: Vivienne Petris-Joannou
Editor: Ian Rohr
Reviewer/consultant: Joy Andrews
Cover by DiZign
Typeset by Typecellars Pty Ltd
Printed in Singapore by Green Giant Press

Students:
All care has been taken in the preparation of this study guide, but please check with your teacher or the Board of Studies about the exact requirements of the course you are studying as they can change from year to year.

Contents

INTRODUCTION v

Chapter 1 — INTRODUCTION TO INFORMATION SKILLS AND SYSTEMS 1
1.1 The Impact of Information Technology 2
1.2 Information Technology in Context 2
1.3 Computer Hardware 3
1.4 Computer Software 4
1.5 Participants 4
1.6 Information Processes 4
1.7 Data 5
1.8 Social and Ethical Issues 9
— Chapter Summary 11
— Your Checklist 11
— Review Questions 12
— Answers to Review Questions 14
— Glossary 18

Chapter 2 — TOOLS FOR INFORMATION PROCESSES 20
2.1 Collecting Data 21
2.2 Organising Data 25
2.3 Analysing Data 27
2.4 Storing and Retrieving Data 28
2.5 Processing Data 33
2.6 Transmitting and Receiving Data 37
2.7 Displaying Data 44
— Chapter Summary 47
— Your Checklist 49
— Review Questions 50
— Answers to Review Questions 52

Chapter 3 — PLANNING, DESIGNING AND IMPLEMENTATION 56
3.1 Understanding the Problem 58
3.2 Making Decisions About Solutions 63
3.3 Designing Solutions 64
3.4 Implementating Solutions 65
3.5 Testing, Evaluating and Maintaining Solutions 66
3.6 Social and Ethical Solutions 67
— Chapter Summary 69
— Your Checklist 70
— Review Questions 71
— Answers to Review Questions 73
— Glossary 76

Chapter 4 — PERSONAL AND GROUP SYSTEMS AND PROJECTS 79
4.1 Computer Technology Projects 80
4.1 Personal Information Systems 81
4.2 Group Information Systems 82
— Chapter Summary 84
— Your Checklist 84
— Review Questions 85
— Answers to Review Questions 87
— Glossary 91

Chapter 5 — WORD PROCESSING 92
5.1 Word Processing Concepts 95
5.2 Word Processing Skills 95
— Your Checklist 99

Chapter 6 — DATABASES 100
6.1 Database Skills 103
— Your Checklist 115

Chapter 7 — SPREADSHEETS 116
7.1 Spreadsheet Skills 118
7.2 Built-in Calculation Functions 119
— Your Checklist 121

Chapter 8 — GRAPHICS 122
8.1 Graphics Skills 125
— Your Checklist 128

Chapter 9 — DESK TOP PUBLISHING 129
9.1 Desk Top Publishing Skills 132
— Your Checklist 134

Chapter 10 — MULTIMEDIA 135
10.1 Multimedia Skills 138
— Your Checklist 143

Chapter 11 — THE INTERNET 144
11.1 Web Authoring Software 148
— Your Checklist 148

Chapter 12 — SAMPLE EXAMINATION PAPER 149
— Questions 151
— Answers 161

Introduction

The Preliminary IPT course is made up of two parts:

Part 1, *Information Processes and Technology*, includes the theory about information systems and computing technology, the manipulation of data and the tasks involved in the development of an information system.

Part 2 requires hands-on skill development in the use of a series of software applications. Students are required to undertake two projects, one concerning an information system for personal use and the other, an information system for group use. These projects are designed to provide a pivot for the development of technical, communication, management and problem solving skills.

Course Structure

The arrangement and relationship between components of the Preliminary course and the HSC course for Information Process and Technology Stage 6 are outlined below. The percentage values refer to suggested course time.

Preliminary Course

Introduction to Information Skills and Systems (20%)

- Information systems in context
- Information processes
- Digital representation of data
- Classification of information systems
- Social and ethical issues

Tools for Information Processes (40%)

- Collecting
- Organising
- Analysing
- Storing and retrieving
- Processing
- Transmitting and receiving
- Displaying

Planning, Design and Implementation (20%)

- Understanding the problem to be solved
- Making decisions
- Designing solutions
- Implementing
- Testing, evaluating and maintaining
- Social and ethical issues

Personal and Group Systems and Projects (20%)

- Personal information systems
- Group information systems

HSC Course

Project Work (20%)

- Understanding the problem
- Making decisions
- Designing solutions
- Project management
- Social and ethical design
- Implementing
- Testing, evaluating and maintaining

Information Systems and Databases (20%)

- Information systems
- Examples of database information systems
- Organisation methods
- Storage and retrieval
- Other information processes
- Issues related to information systems

Communication Systems (20%)

- Characteristics of communication systems
- Examples of communication systems
- Transmitting and receiving in communication systems
- Other information processes in communication systems
- Issues related to communication systems

Option Strands (40%)

Students will select TWO of the following options:
- Transaction processing systems
- Decision support systems
- Automated manufacturing systems
- Multimedia systems

1

Introduction to Information Skills and Systems

Outcomes

By studying this chapter and completing the exercises students should be able to:

- Describe the nature of information processes and information technology
- Classify the functions and operations of information processes and information technology
- Identify the information processes within a system
- Recognise the interdependence between each of the information processes
- Identify social and ethical issues
- Describe the historical developments of information systems and relate these to current and emerging technologies.

Source: Information Processes and Technology Stage 6 Syllabus © NSW Board of Studies, 1999.

KEY TERMS AND CONCEPTS

analysing
ASCII
binary number
collecting
data
data accuracy
data security
data validation
digital communication
displaying
digitisation
EBCDIC
ethical issues
environment
hardware
hexadecimal
information
information process
information system
input
integrated circuit
LAN
output
processing
software
storage
WAN

1.1 THE IMPACT OF INFORMATION TECHNOLOGY

Information technology has had an enormous impact on our lives in the way we write letters, do our banking, shop, design, manufacture and communicate with businesses and friends overseas.

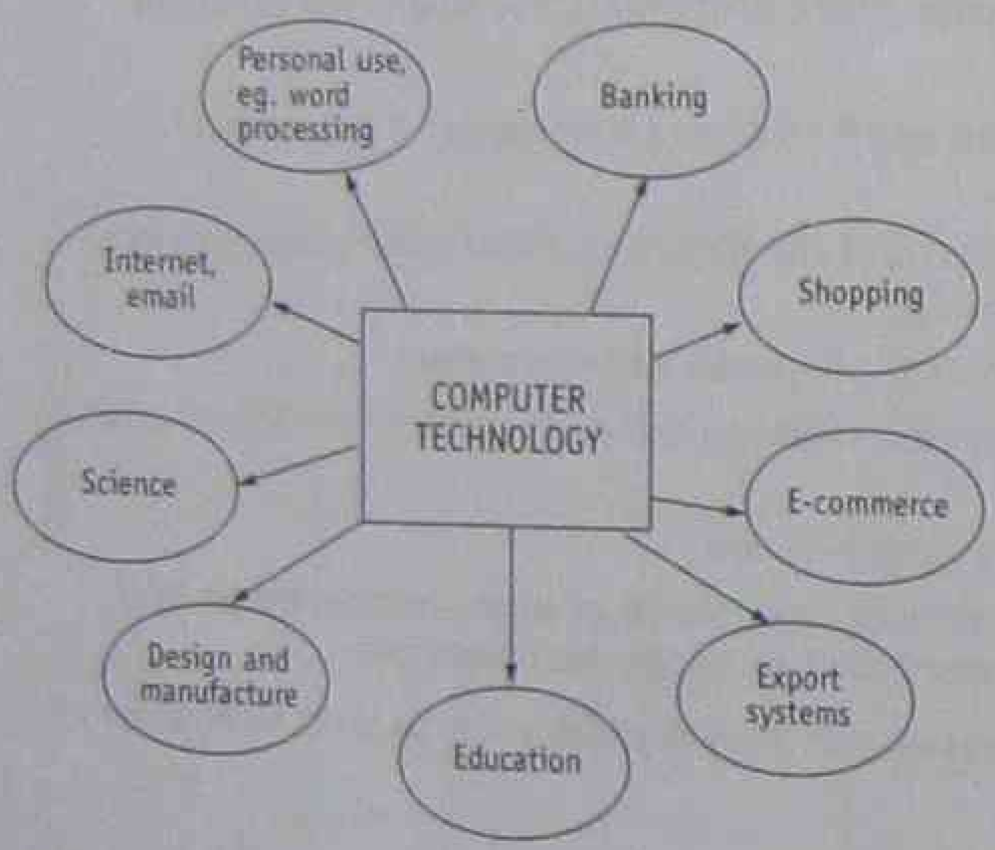


Figure 1.1 Impact of IT

1.2 INFORMATION TECHNOLOGY IN CONTEXT

Information technology involves the processing of information within a system and includes electronic hardware and software. The processes are carried out within a given environment and they need to have a specified purpose for being carried out.

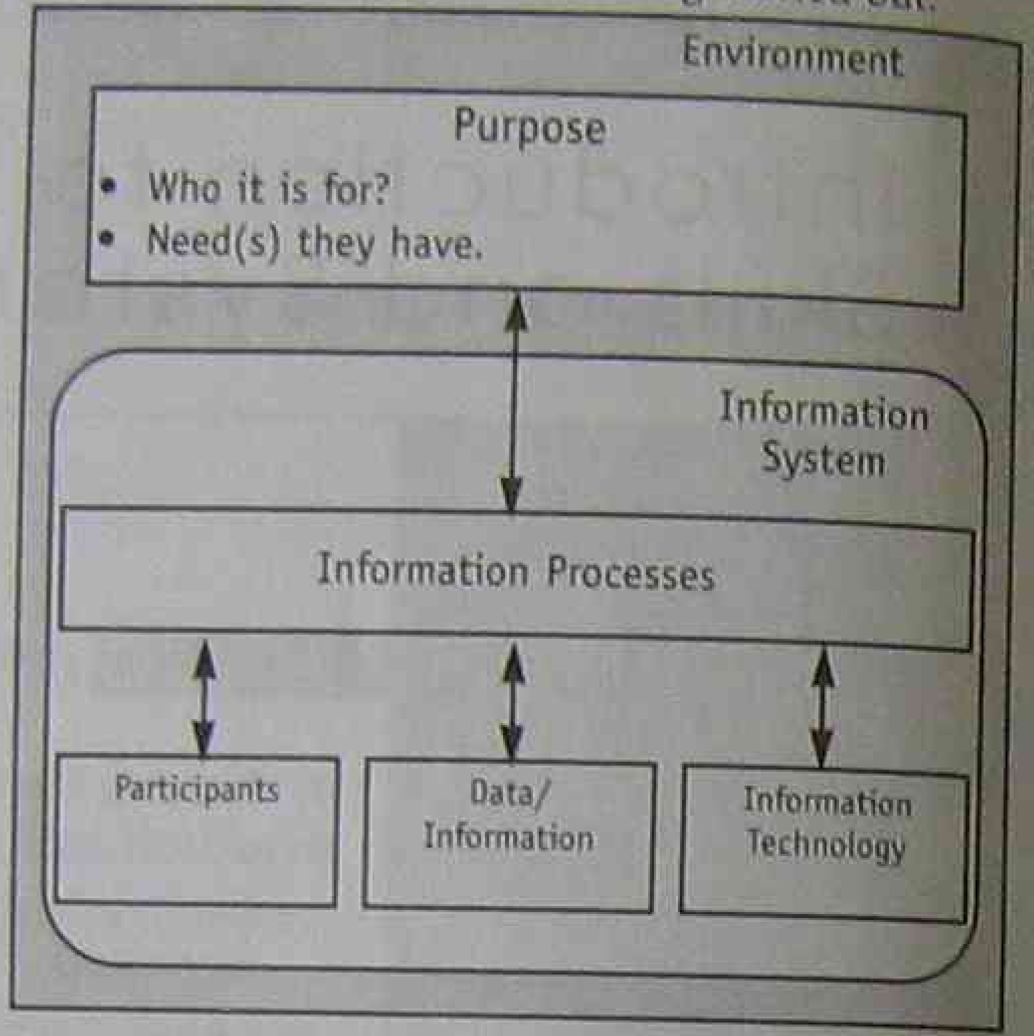


Figure 1.2 Main components of an information system

The above diagram represents the main components of an information system in context.

Within the lower box the information system affects and is affected by the information processes.

The information processes are impacted by and also impact on the participants, the input and output (data/information) as well as the hardware and software that makes up the information technology.

The information system is surrounded by a particular environment and its structure and function are directly related to the purpose of the system or the needs of the participants.

Environment:
Everything that influences and is influenced by the information system, eg. climate, electrical power, furniture, personnel, network cables and connected networks, people's lifestyles.

Purpose:
For what need and for whom is the information system designed? For example, banking system, personal computing (WP, DTP), design and manufacture, e-commerce, for individuals and/or organisations.

Information Systems:

- Users (individuals/organisations), input data, processing, output information.
- Can be computerised using hardware and software.
- Can be non-computerised: libraries of books, newspapers, radio.

Information Processes:

- Computerised, eg. WP on computer, accessing Internet.
- Non-computerised, eg. reading newspaper, listening to the news.

Participants:
People who carry out the information processes within the information system. Participants can also be called **direct users** (operators, technicians, programmers, managers, systems analysts).

Data/Information:
The raw material that information processes deal with is the data, ie. **input**.
After the data has been processed it becomes information, ie. **output**.

Information Technology:
The **hardware** and **software** used in information processes, eg. monitors, CPU, printers, applications, network cables, modems, hardware and software making up the Internet.

1.3 COMPUTER HARDWARE

Hardware comprises the physical components that make up a computer.

These include:

- The peripheral devices which can be input or output devices.
- Processing devices such as the CPU which contains the ALU, control unit and clock and temporary memory registers — processing turns data that is input into information that is output.
- Storage devices such as the floppy disk, hard disk, CD ROM disks, DVD disks, magnetic tape and ROM.

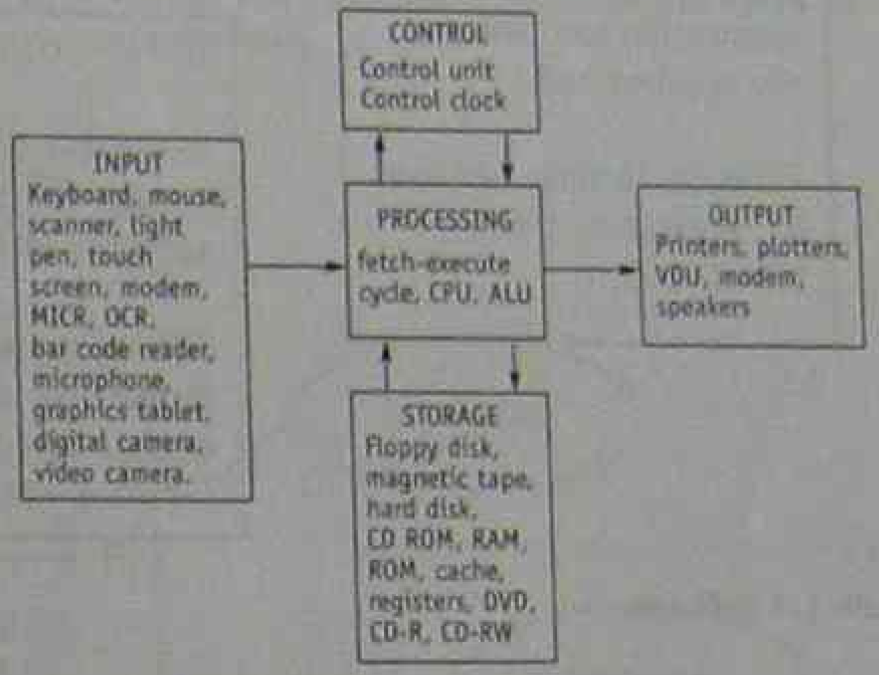


Figure 1.3 Computer hardware

Types of Computers

Computers range in size and power from the smallest personal (PC) or desktop computer to the largest and most powerful supercomputer, where the speed of the processing is measured in MIPS (millions of instructions per second). Examples are given in the table below.

Type of computer	Speed of processing in MIPS	Description of use	Examples of use	Size and portability
Supercomputer	> 200	Very fast calculations such as in simulation and modelling of complex phenomena.	Cray2 at NASA, USA. Janus built by Intel in USA.	Very large and not portable. Stored in a specially designed and air-conditioned room.
Mainframe	100–200	Central computer (server) for a large number of user terminals.	Universities, banks and large-sized businesses.	Large and not portable. Stored in a specially designed and air-conditioned toom.
Minicomputer	5–50	Central computer (server) for a large number of input/output user terminals.	Universities, banks and medium-sized businesses.	Can be portable with some difficulty. Are being replaced by groups of PCs.
Microcomputer or PC	0.1–16	Used by individuals as stand-alones or can be connected to a network.	Individuals and networks.	Portable, especially laptops.

1.4 COMPUTER SOFTWARE

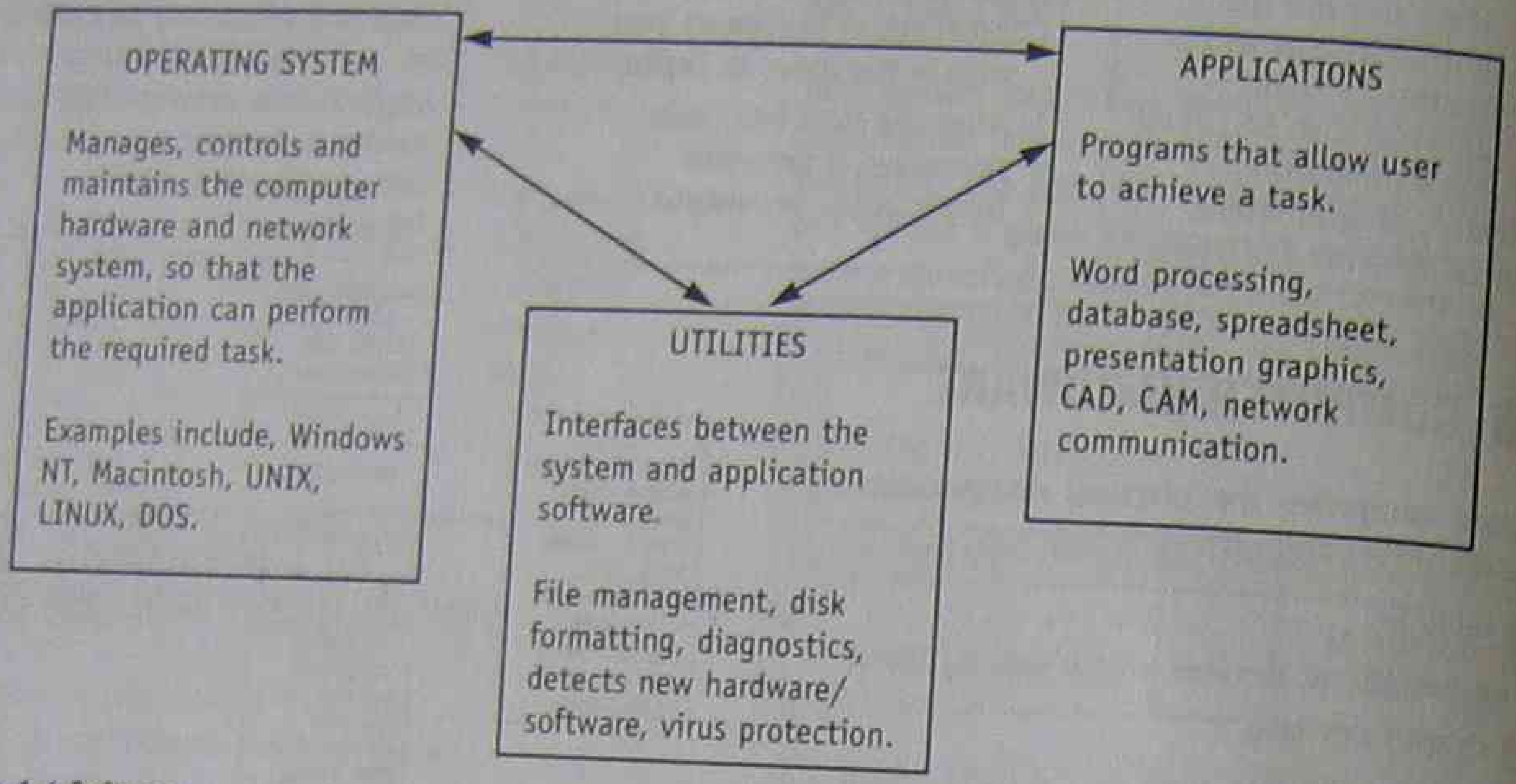


Figure 1.4 Software

1.5 PARTICIPANTS

Participants are the people who carry out the information processes within the information system.

The role of participants varies from computer operators through to systems administrators and engineers as show below.

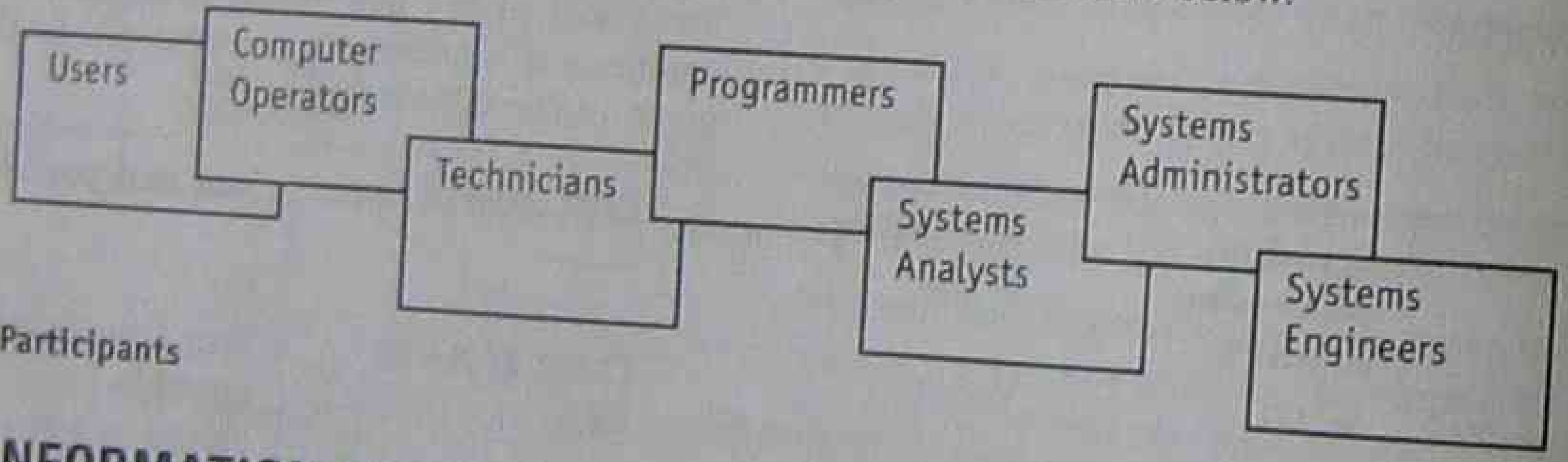


Figure 1.5 Participants

1.6 INFORMATION PROCESSES

Information technology is concerned with the manipulation of data. Data must be:

- Collected — by deciding what, where and how data will be gathered.
- Organised — by arranging, representing and formatting data so that it can be used by the information system.
- Analysed — interpreting and transforming data into a useful form so that it can be easily processed. Charts and graphs or models are often used to show how the data has been transformed.
- Stored and Retrieved — data is stored before, during and after processing in temporary memory and then in permanent memory so that it can be retrieved later.
- Processed — the manipulation of data into information. Processing occurs when the data is modified and updated, sorted or categorised. Processing is carried out within the CPU.
- Transmitted and Received — the transfer of information from one computer to another. Data transmission can be serial (along 1 wire in single file) or parallel (along 8 wires simultaneously). Two communicating computers need to agree on how the data will be transferred during the 'handshake' which is part of transmission protocol that specifies how the receiving computer will acknowledge the sending devices. A modem modulates or changes digital data understood by the computer into analog form traveling along phone lines and t demodulates it back into digital form for receiving computer to read. Modem is s for **mo**dulate-**dem**odulate.
- Displayed — the information is presen on the screen as a soft copy, printed a hard copy or heard through speakers synthesisers.

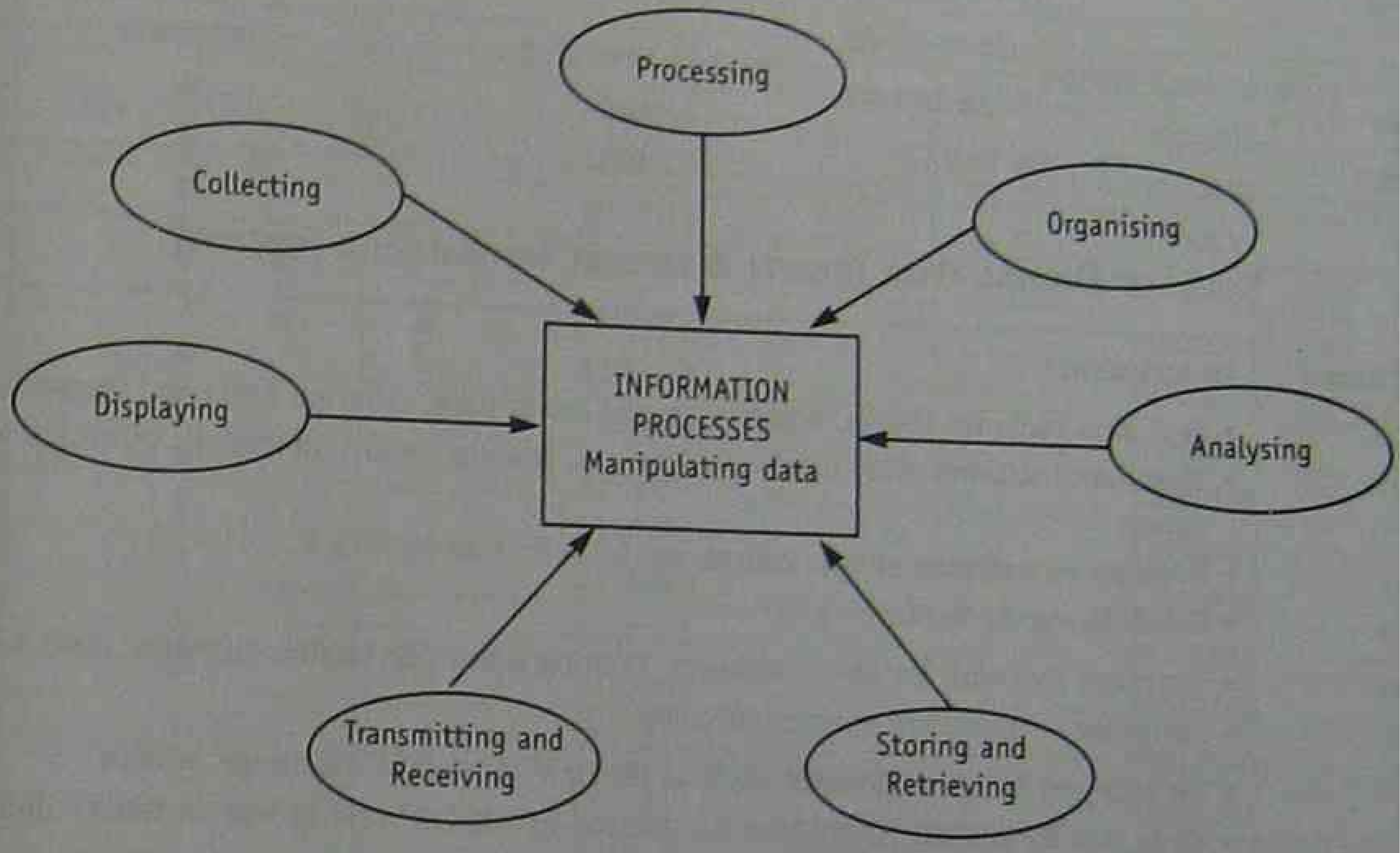


Figure 1.6 Information processes

1.7 DATA

The table below compares the storage of data before and after the introduction of computer tec including the advantages and disadvantages of digitisation.

Historical data storage	Digital data storage
Data stored in: • Filing systems • Catalogue systems (eg. library) • Journals and ledgers • Microfiche • Printed media, books.	Benefits are ease and efficiency of: • Editing • Storage • Presentation (graphs, DTP) • Search, sort and retrieval • Calculations • Transmission. Disadvantages include: • Cost of hardware and software • Compatibility with existing technology • Participants' training • Social and ethical issues of deskilling, co crime such as hacking, invasion of privac ease of breech of copyright.

Data Storage Comparative Sizes

Storage medium	Capacity
floppy disk	1.44 Mb to 2.8 Mb ... depends on density and sides used
hard disk	2 Gb to 80 Gb +
magnetic tape	QIC tapes 80 Mb to 5 Gb DAT cartridges 2 Gb to 24 Gb 8 mm cartridges 5 Gb to 50 Gb
CD ROM	650 Mb
DVD	5 Gb to 17 Gb
Zip disk	100 to 250 Mb
Jaz disk	up to 2 Gb

Types of Digital Data (input) & Digital Information (output)

Type of data	Description
Text	• Text data includes letters, upper case and lower case, different fonts and languages. • Numbers included with letters — integers, floating points (decimal), currency, dates, times. • Boolean — a choice of two values, eg. M or F; Y or N; 0 or 1. • Symbols, eg. $; %; &; ?; }.
Numbers	• Numbers not with letters — integers, floating point (decimals), currency, date, time. • Formulae — numbers used in calculations.
Images	• Bit mapped or vector images such as pictures, drawings, paintings, photos. • They can be stored, edited and transferred in similar ways to text as binary digits (bit mapped) or end point coordinates (vector).
Audio	• Audio data resulting in sound output can be stored in **wave format** (analog form) or **sampled format** (digitised).
Video	• Video data includes a combination of audio and image data. • It may also include text as subtitles.

Digital data is represented using 1's or 0's in the binary number system.

Computers can easily read 1's or 0's as an electrical impulse (1) or no electrical impulse (0).

- Each 0 or 1 is called a BIT (BInary digiT).
- A group of 8 bits is called a Byte.
- A group of 1024 (2^{10}) bytes is 1 Kilobyte (Kb).
- A group of 1024 kilobytes is 1 Megabyte (Mb).
- A group of 1024 megabytes is 1 Gigabyte (Gb).
- A group of 1024 gigabytes is 1 Terabyte (Tb).

ASCII and EBCDIC Code

American Standard Code for Information Interchange (ASCII) is the standard conversion code for all personal computers. Each letter, number or symbol, including white spaces, have a unique ASCII code. Using a code based on combinations of 0's and 1's reduces the number of digits needed to represent all required characters.

ASCII code usually has 7 bits, which allows for 128 distinct codes. Extended ASCII code has 8 bits which allows for 256 distinct codes.

Similarly, Extended Binary Coded Decimal Interchange Code is used on large IBM computers and was adapted by IBM from the early punch card system in 1960. EBCDIC code is composed of 8 bits.

A sample of characters with their ASCII and EBCDIC codes

Character	ASCII Code (7 bits)	EBCDIC Code (8 bits)
A	1000001	11000001
B	1000010	11000010
C	1000011	11000011
D	1000100	11000100
a	1100001	10000001
b	1100010	10000010
c	1100011	10000011
d	1100100	10000100
1	0110001	11110001
2	0110010	11110010
3	0110011	11110011
4	0110101	11110100
•	0101010	01011100
?	0111111	01101111
>	0111110	01101110
@	1000000	01111100
white space	0100000	01000000

Notice that in consecutive letters or numbers, eg. ABCD; abcd; or 1234, each ASCII or EBCDIC code increases by 1 bit. Thus you could be expected to work out what D is if you are told that A is 1000001.

You do not have to memorise all the codes.

The Representation of Data as Number Systems

The Decimal or Base 10 number system

Power of 10	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
Decimal value	10 000 000	1 000 000	100 000	10 000	1 000	100	10	1

The Binary or Base 2 number system

Power of 2	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal value	128	64	32	16	8	4	2	1

Binary and Decimal Conversions

CONVERTING AN 8 BIT BINARY NUMBER (01001110) INTO ITS DECIMAL EQUIVALENT

Conversion table

Power of 2	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal value	128	64	32	16	8	4	2	1
Binary number	0	1	0	0	1	1	1	0
Decimal value	0 × 128 = 0	1 × 64 = 64	0 × 32 = 0	1 × 16 = 0	1 × 8 = 8	1 × 4 = 4	1 × 2 = 2	0 × 1 = 0
Add these	0	64	0	0	8	4	2	0

The equivalent decimal value for the binary number 01001110_2 is 78_{10}.

CONVERTING A DECIMAL NUMBER (98_{10}) INTO ITS EQUIVALENT BINARY NUMBER

Method 1 using the conversion table

Power of 2	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal value	128	64	32	16	8	4	2	1
Binary number	0	1	1	0	0	0	1	0

Using the conversion table choose the largest power of 2 (decimal equivalent) that is less than or equal to the decimal number in question.

Ninety-eight is the decimal number in question and 64 is the largest number in the table that is less than or equal to 98.

Subtract it from the decimal number (ie. 98 - 64).

Choose the largest power of 2 that is less that the remainder of 98 - 64 (ie. 34).

Thirty-four is the decimal remainder in question and 32 is the largest number in the table that is less than or equal to 34.

Repeat this process until you reach a remainder of 0.

The largest decimal number less than 98 is 64 (2^6), thus there will be a 0 in the 2^7 column and a 1 in the 2^6 column.

98 - 64 = 34 (1 lot of 2^6)

34 - 32 = 2 (1 lot of 2^5)

2 - 2 = 0 (1 lot of 2^1) all other columns contain 0s.

(0 × 27) + (1 × 26) + (1 × 25) + (0 × 24) + (0 × 23) + (0 × 22) + (1 × 21) + (0 × 20) = 0 1 1 0 0 0 1 0

Method 2

Divide 98 by 2 and use the **remainder** as the binary number.

$2\overline{)\,98}$ 49 r0

$2\overline{)\,49}$ 24 r1

$2\overline{)\,24}$ 12 r0

$2\overline{)\,12}$ 6 r0

$2\overline{)\,6}$ 3 r0

$2\overline{)\,3}$ 1 r1

The last 1 becomes the first digit in the binary number followed by all the remainders above in order.

1 1 0 0 0 1 0* = 01100010 (as in the previous method).

* *To make the binary number up to 8 bits add 0 to the front.*

1.8 SOCIAL AND ETHICAL ISSUES

The following table includes examples of social and ethical issues showing the positive and negative effects of IT.

Social issue	Description	Example – Positive/Negative
Privacy	The right of individuals or organisations to have information protected.	**P** Police can trace known offenders quickly. **N** Government departments inadvertently sending private information to the wrong people due to computer operator error. The sale of databases of names and addresses for advertising and other unauthorised purposes.
Security	Measures taken to prevent inappropriate or unauthorised access to and/or manipulation of data.	**P** Computer systems should have security systems, eg. levels of access, passwords, PIN, biometric devices, firewalls, shredding of sensitive documents, screening of personnel, encryption, error detection. **N** Hackers may illegally access computer systems such as bank accounts, corporate databases, NASA and so on if security measures are not adequate.
Accuracy	The correctness of data entered. Accuracy checks include: • Data range checks • List checks • Type checks • Digit checks.	**P** Computer databases make it easy to cross-check information. **N** Inaccurate credit details can affect a person's loan application or credit rating.
Changing nature of work	Computer technology has changed the way people work.	**P** Telecommuting from home or in transit, architects doing more designing and less drawing, typists become more efficient and productive by using techniques such as cutting and pasting and mail merge. **N** Deskilling, eg. printers and draftsmen's skills can be replicated by an unskilled person with the necessary software.
Health and safety	Using computers creates some health issues such as RSI, CTS, eyestrain and so on.	**P** Computer users have instruction in ergonomic use of computers. **N** RSI (repetitive stain injury), CTS (carpel tunnel syndrome), tendonitis.
Copyright	The right of an author to have control over their intellectual property.	**P** Computer technology makes it easier to trace and prove breaches of copyright. **N** Downloading information from the Internet and using it as one's own without permission (plagiarism). Piracy of audio and video CD ROM material. Abuse of computer licenses by copying, unauthorised multiple use or on-selling software.

Social issue	Description	Example – Positive/Negative
Crime	Networked computers have made it very easy to commit crime including stealing information, accessing bank accounts inappropriately and destroying property without the perpetrator needing to be present on site.	**P** Computers make it easier to trace banking transactions. Computer security measures need to be regularly updated. **N** Computer databases make it relatively easy for a hacker to access, steal or destroy information, rather than physically looking through papers in filing cabinets.
Equity	The right of all persons to have equal access to the information made available by computer technology. This applies particularly to certain groups based on disability, gender, race, religion, socioeconomic status or age.	**P** People with disabilities (or mothers with young children) can now hold down a job and/or work from home, using equipment such as braille keywords. **N** People who cannot or do not use computer technology are disadvantaged in the workforce (knowledge). The cost of purchasing computer equipment may preclude some people from accessing the technology (economic).

CHAPTER SUMMARY

- Information systems and information technology had a large impact on the lives of all people during the second half of the twentieth century and continue to do so in this century.
- An information system includes all the hardware, software, personnel, data and processes that work together to meet the stated need of the user.
- An information system involves the collecting, organising, analysing, processing, storing and retrieving, displaying and transmitting and receiving of information.
- For information processes to occur, people (participants) interact with information technologies (hardware and software).
- The purpose of an information system is related to who the users are and what are their needs.
- An information system is both affected by and affects its environment.
- Data can be input into a computer system in the form of text, numbers, images or video.
- Computers read data as binary digits or base 2 numbers. Base 2 numbers can be converted into base 10 or decimal numbers.
- The way information systems affect the participants creates social and ethical issues that need to be considered before, during and after the development of an information system.

YOUR CHECKLIST

After studying this chapter you should have acquired the following skills:

	✓
Diagrammatically represent a given scenario that involves an information system.	☐
Explain how an information system impacts on its environment and how the environment in turn impacts on the information system.	☐
Describe the environment and purpose of an information system for a given context.	☐
Explain how a given need can be supported by an information system.	☐
Describe an information system in terms of its purpose.	☐
For a given scenario, identify the people who are: • in the environment • users of the information system • participants in the information system.	☐
Distinguish between and categorise the activities within an information system in terms of the seven information processes: • collecting • organising • analysing • storing and retrieving • processing • transmitting and receiving • displaying.	☐

Review Questions

Multiple Choice Questions

A Page 14

(1 mark each)

1. Information processing involves the:
   - (A) Processing of information into data within a system
   - (B) Processing of data into information within a system
   - (C) Input of information into a system
   - (D) Output of binary digits from a computer.
2. The environment of a system is defined as:
   - (A) The ergonomics of the system's hardware
   - (B) The software that influences the way data is processed
   - (C) All hardware and software that affect the system
   - (D) All the elements, not including the system itself, that have an effect on the system.
3. The main functions of hardware in a computer system are:
   - (A) Storage, manipulation, display, communication and output of data
   - (B) Input, processing, control, storage and output of data
   - (C) Collection, retrieval, manipulation, and display of data
   - (D) Input, processing, display and output of information.
4. Which of the following contains a list of computers in ascending order of processing speed and capacity:
   - (A) Minicomputer, PC, mainframe, supercomputer
   - (B) PC, mainframe, minicomputer, supercomputer
   - (C) Supercomputer, minicomputer, mainframe, PC
   - (D) PC, minicomputer, mainframe, supercomputer.
5. Another name for system software is:
   - (A) Machine software
   - (B) The operating system
   - (C) Application utility software
   - (D) Programs.
6. Which of the following is the best description of information system participants:
   - (A) A user who word processes three times a week
   - (B) All those who carry out the processing of information
   - (C) All users who can do programming
   - (D Technicians and systems analysts who are affected by data.
7. Which of the following is **not** an example of an information process:
   - (A) Selecting hardware to purchase
   - (B) Collecting data for a database
   - (C) Doing a calculation in a spreadsheet
   - (D) Saving data to a back-up tape.
8. How many bits of data are in one kilobyte?
   - (A) 8
   - (B) 8 × 1024
   - (C) 1024 or 1000
   - (D) 1024 × 1024.
9. Binary digits are examples of :
   - (A) Base 10 numbers
   - (B) Decimal numbers
   - (C) Octal numbers
   - (D) Base 2 numbers.
10. The equivalent decimal number for the binary number 01101100 is:
   - (A) 54
   - (B) 4
   - (C) 256
   - (D) 108.

Short Answer Questions

A Page 14

(25 marks)

1. Distinguish between data and information. (2 marks)
2. List and describe the components of the CPU. (4 marks)
3. Differentiate between volatile memory and non-volatile memory. (2 marks)
4. The PC or personal computer can process information at a rate between 1 and 16 MIPS. What does 'MIPS' stand for? (1 mark)
5. What is the difference between application software and system software? (2 marks)
6. What is the purpose of ASCII code? If the ASCII code for the capital letter 'D' is 1000100, how do you calculate the ASCII code for the capital letter B? (2 marks)
7. Name four methods for increasing data security in a computer system. (4 marks)
8. Discuss one positive and one negative example of changes in 'equity' with the introduction of information technology. (2 marks)
9. Name and describe four social and ethical issues that have become more prominent with the introduction of information technology. (4 marks)
10. What is the equivalent binary number for the decimal number 68? (2 marks)

Longer Response Questions

A Page 15

(40 marks)

1. Using a diagram, show the interaction between the following components in an information system: (7 marks)
   - Environment
   - Purpose
   - Information system
   - Information processes
   - Participants
   - Data/information
   - Information technology.

   Give an example of each component.
2. Explain, using a specific example, how an information system impacts on its environment. (3 marks)
3. Using a bank as an example of an information system, give examples of participants who are:
   - (i) In the environment
   - (ii) Users of the information
   - (iii) Participants in the information system. (3 marks)
4. Give an example of the activities involved in each of the following information processes with reference to a doctors' surgery (7 marks)
   - Collecting
   - Organising
   - Analysing
   - Storing and retrieving
   - Processing
   - Transmitting and receiving
   - Displaying.
5. Give an example of one non-computerised information system and one computerised information system. Explain how data is stored in each system. (4 marks)
6. Name and give examples of four different types of data that may be input into a computer system. (4 marks)
7. Name and describe four different input devices. (4 marks)
8. Complete the following table of number conversions.

Decimal	Binary
105_{10}	
	11110010_2
	001110101_2
134_{10}	

9. Explain the difference between a base 10 number and a base 2 number. (2 marks)
10. Describe one similarity and one difference between a WAN and a LAN. (2 marks)

Review Questions

Multiple Choice Questions

A Page 14

(1 mark each)

1. Information processing involves the:
   (A) Processing of information into data within a system
   (B) Processing of data into information within a system
   (C) Input of information into a system
   (D) Output of binary digits from a computer.
2. The environment of a system is defined as:
   (A) The ergonomics of the system's hardware
   (B) The software that influences the way data is processed
   (C) All hardware and software that affect the system
   (D) All the elements, not including the system itself, that have an effect on the system.
3. The main functions of hardware in a computer system are:
   (A) Storage, manipulation, display, communication and output of data
   (B) Input, processing, control, storage and output of data
   (C) Collection, retrieval, manipulation, and display of data
   (D) Input, processing, display and output of information.
4. Which of the following contains a list of computers in ascending order of processing speed and capacity:
   (A) Minicomputer, PC, mainframe, supercomputer
   (B) PC, mainframe, minicomputer, supercomputer
   (C) Supercomputer, minicomputer, mainframe, PC
   (D) PC, minicomputer, mainframe, supercomputer.
5. Another name for system software is:
   (A) Machine software
   (B) The operating system
   (C) Application utility software
   (D) Programs.
6. Which of the following is the best description of information system participants:
   (A) A user who word processes three times a week
   (B) All those who carry out the processing of information
   (C) All users who can do programming
   (D Technicians and systems analysts who are affected by data.
7. Which of the following is **not** an example of an information process:
   (A) Selecting hardware to purchase
   (B) Collecting data for a database
   (C) Doing a calculation in a spreadsheet
   (D) Saving data to a back-up tape.
8. How many bits of data are in one kilobyte?
   (A) 8
   (B) 8 × 1024
   (C) 1024 or 1000
   (D) 1024 × 1024.
9. Binary digits are examples of :
   (A) Base 10 numbers
   (B) Decimal numbers
   (C) Octal numbers
   (D) Base 2 numbers.
10. The equivalent decimal number for the binary number 01101100 is:
    (A) 54
    (B) 4
    (C) 256
    (D) 108.

Short Answer Questions

A Page 14

(25 marks)

1. Distinguish between data and information. (2 marks)
2. List and describe the components of the CPU. (4 marks)
3. Differentiate between volatile memory and non-volatile memory. (2 marks)
4. The PC or personal computer can process information at a rate between 1 and 16 MIPS. What does 'MIPS' stand for? (1 mark)
5. What is the difference between application software and system software? (2 marks)
6. What is the purpose of ASCII code? If the ASCII code for the capital letter 'D' is 1000100, how do you calculate the ASCII code for the capital letter B? (2 marks)
7. Name four methods for increasing data security in a computer system. (4 marks)
8. Discuss one positive and one negative example of changes in 'equity' with the introduction of information technology. (2 marks)
9. Name and describe four social and ethical issues that have become more prominent with the introduction of information technology. (4 marks)
10. What is the equivalent binary number for the decimal number 68? (2 marks)

Longer Response Questions

A Page 15

(40 marks)

1. Using a diagram, show the interaction between the following components in an information system: (7 marks)
   - Environment
   - Purpose
   - Information system
   - Information processes
   - Participants
   - Data/information
   - Information technology.

   Give an example of each component.
2. Explain, using a specific example, how an information system impacts on its environment. (3 marks)
3. Using a bank as an example of an information system, give examples of participants who are:
   (i) In the environment
   (ii) Users of the information
   (iii) Participants in the information system. (3 marks)
4. Give an example of the activities involved in each of the following information processes with reference to a doctors' surgery (7 marks)
   - Collecting
   - Organising
   - Analysing
   - Storing and retrieving
   - Processing
   - Transmitting and receiving
   - Displaying.
5. Give an example of one non-computerised information system and one computerised information system. Explain how data is stored in each system. (4 marks)
6. Name and give examples of four different types of data that may be input into a computer system. (4 marks)
7. Name and describe four different input devices. (4 marks)
8. Complete the following table of number conversions.

Decimal	Binary
105_{10}	
	11110010_2
	001110101_2
134_{10}	

9. Explain the difference between a base 10 number and a base 2 number. (2 marks)
10. Describe one similarity and one difference between a WAN and a LAN. (2 marks)

Answers to Review Questions

Multiple Choice

Page 12

1. (B) The act of processing in a computer system involves the changing of data that has been input into information that is either output or stored for later processing.
2. (D) The surroundings of a computer system that have an effect on its performance such as climate, other computers and networks, people, noise and so on are the 'environment' of the system. Ergonomics is the study of the way humans and computers interact; the software affects the ways data is processed, but is inside the computer not outside. The computer system hardware itself is not included in the environment. The computer system in turn has an effect on the environment.
3. (B) A computer system's hardware must have input and output which is processed, controlled and stored. The personnel and information are not hardware items and communication is what happens when data is exchanged.
4. (D) The PC, personal computer or microcomputer, is the smallest in the hierarchy of computer technology with a processing speed now around 1 GHz from 133 MHz a few years ago. The next most powerful computer is the minicomputer which can perform the processing for many workstations connected to a network in offices or businesses. The mainframe is larger and more powerful again and can be found doing the processing for very large numbers of workstations at places like universities and big businesses. The supercomputer is the fastest and most powerful and operates where high volume, very fast processing (including lots of calculations) is required, such as at NASA.
5. (B) There are three types of software. Firstly, the software that makes the computer operate is called the operating system or system software. Secondly, the software that contains the programs that 'run' on the computer is called application software. Finally, the software that carries out a variety of tasks by communicating between the operating system and the application software is the utility software.
6. (B) A, C and D include some participants of information technology but B includes all of them.
7. (A) Information processing can include one or more of the following manipulations of data: collecting; organising; analysing; storing and retrieving; processing; transmitting and receiving and displaying.
8. (B) There are 8 bits in each byte and 2^{10} (1024) bytes in each kilobyte.
9. (D) Base 10 numbers are called decimals and base 2 numbers are called binary digits or bits, for short. Base 2 numbers can be either 0 or 1.
10. (D) To convert a binary number into a decimal number, use the conversion table and add the decimal values for the 1's in the binary value position. In this case, $4 + 8 + 32 + 64 = 108_{10}$.

Short Answers

Page 13

1. Data is the raw material entered into a computer before it has been processed, while information is the data after it has been changed by the processing of the system.
2. ALU (Arithmetic Logic Unit), CU (control unit, including the system clock), registers (temporary storage for the CPU).
3. Volatile memory may be lost when the power is switched off and the data is not saved to secondary storage. RAM is volatile memory. Non-volatile memory is either stored in secondary memory or embedded on the motherboard as a ROM chip.
4. MIPS stands for millions of instructions per second and is a useful measure of the processing speed when comparing the categories of computers.
5. Application software are computer programs used for achieving specific goals, eg. word processing, spreadsheet design, database management, desk top publishing. System software manages and controls the hardware so that the application software can operate or perform its task.
6. ASCII code is the American Standard Code for Information Interchange and it provides an international computer standard for converting keyboard input of characters into machine code.

   If the letter D is 1000100, the letter B can be calculated by subtracting 2 (subtract 1 two times) from 1000100. The letter B will have the ASCII code of 1000010.

7. Data security may be increased within an information system by:
   - Safeguarding physical access to computer hardware by locking access doors or using access cards, biometric fingerprinting or iris detection
   - Using passwords for various levels of access
   - Using encryption for scrambling data
   - Creating firewalls within networked systems.
8. Changes in equity with IT.

Positive change	Negative change
• People with disabilities affecting their physical mobility can undertake employment from home.	• People who cannot afford computers may be unable to access as much information as those who own technology.

9. Social and ethical issues increasing with IT.

Crime	Bank accounts have been unlawfully accessed by hackers.
Privacy	Private information has been sent to incorrect individuals as a result of an incorrect mail merge by a government department.
Deskilling	Highly trained draftsmen have been replaced by a PC with CAD software.
Control	People who have access to IT knowledge can more easily exert power and control over those without it. For example, secretaries who know how to use computers while their bosses do not.

10. The equivalent binary number for the decimal number 68 can be calculated using either of the two methods:

Method 1

	Remainder
2) 68	
2) 34	0
2) 17	0
2) 8	1
2) 4	0
2) 2	0
1	0

1000100_2 or 01000100_2

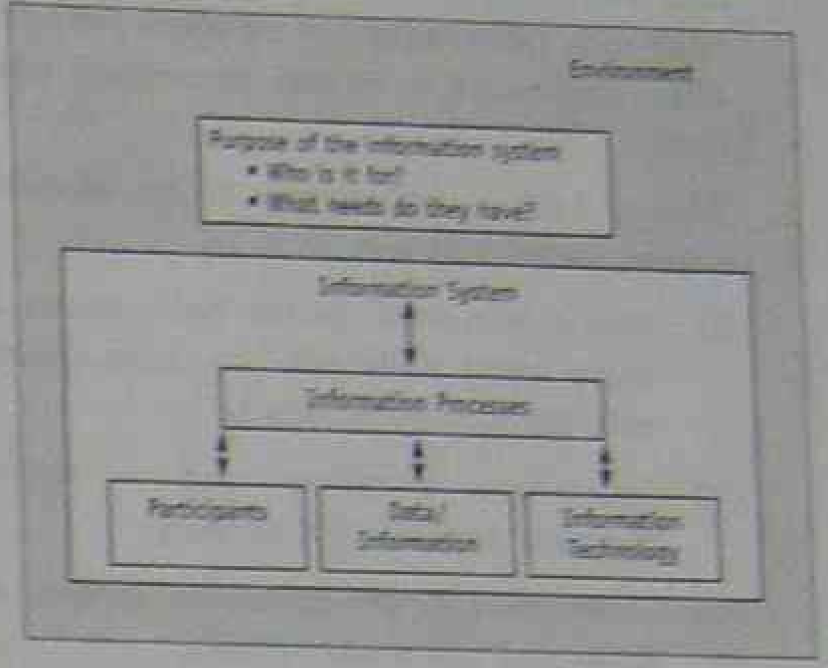
Method 2

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
0	1	0	0	0	1	0	0

Long Answers

Page 13

1.



Component	Example
Environment	Doctor's surgery.
Purpose	Collect and store patient information for use by doctors.
Information system	Computers, peripherals, personnel, patient data sheet, patient invoices, medical companies, hospitals and so on.
Information processes	Sorting and searching data in a database, collecting patient details, entering data into database and so on.
Participants	Doctors, administration staff, computer technicians.
Data/information	Patient information sheets, patient invoices.
Information technology	Computer hardware and medical software, patient database, printers.

2. An example of the impact of an information system on the environment is how the use of the Internet has changed the global environment of communication, eg. email, e-commerce, on-line shopping, telecommuting.

   Physically we have satellites and dedicated cabling lines and computer hardware whereas before there were only telephones and telephone lines. The people in the environment do things differently. People do not have to go to shopping centres; they order their groceries on the Internet and they are delivered. People do not post letters or make phone calls when they can combine the longevity of the letter with the immediacy of the phone call into the email. Commercial deals and transactions can be faster.

3. Using a bank as an example of an information system, participants are:
   (i) People who work in the bank, tellers and those using ATMs who are communicating with the bank electronically
   (ii) Technicians installing and maintaining computer hardware and software
   (iii) System analysts and software engineers who design and implement systems.

4. Examples of the activities involved in each of the following information processes with reference to a doctor's surgery:

Collecting	Patients filling in personal details inventory
Organising	Data entered into a database
Analysing	Patient information sorted into alphabetical order
Storing and retrieving	Database saved and then recalled on subsequent visits
Processing	Database searched for all patients using a particular drug
Transmitting and receiving	Patient information sent to hospital before admission
Displaying	Patient information displayed on the screen by the doctor and patient invoice printed

5. Examples of non-computerised information systems and computerised information systems are:

Information system	Example	Data storage
Non-computerised information system	Library book card catalogue	Filed in card catalogue drawers and kept in the library
Computerised infromation system	Database	Stored on magnetic disks on a computer system

6. Four different types of data that may be input into a computer system are:

Data type	Example
Text/numbers	Word processed letter
Image	Pictures from a scanner or digital camera or icons
Audio	Sounds in digitised music
Video	Pictures and sounds in a digitised movie

7. Examples and descriptions of input devices are:

Input devices	Description
Mouse/ joystick/ light pen	Movement sensor with clicking mechanism for selection
Scanners	Electronic capture of text or images using light
Digital cameras	Images captured as light enters the camera lens
Video cameras	Images and sound are captured through camera lens and microphone. Usually stored as MPEG.

8. Number system conversions table

Decimal	Binary
105_{10}	01101001_2
242_{10}	**11110010_2**
117_{10}	**001110101_2**
134_{10}	010000110_2

9. Base 10 numbers, also known as decimals, can contain the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. They are grouped into columns of powers of 10, namely 10^5 10^4 10^3 10^2 10^1 10^0 (100000; 10000; 1000; 100; 10; 1).

   As each column gains 10 digits they are bundled together and moved to the next column as a 1. For example, 10 units will be bundled to make 1 ten and the 1 is moved into the tens column, leaving a 0 in the units column.

   Base 2, numbers also known as binary digits, can contain the numbers 0, 1. They are grouped into columns of powers of 2, namely, 2^8, 2^7, 2^6, 2^5, 2^4, 2^3, 2^2, 2^1, 2^0 (256; 128; 64; 32; 16; 8; 4; 2; 1).

   As each column gains 2 digits they are bundled together and moved to the next column as a 1. For example, 2 units in column 2^0 will be bundled and the 1 (bundle) is moved into the 2^1 column leaving a 0 in the units (2^0) column.

10. One similarity and one difference between a WAN and a LAN is:

	LAN	WAN
Similarity	Both are networks (Local Area)	Both are networks (Wide Area)
Difference	Located in local geographical area, eg. single site such as school	Located in wide geographical area eg. sites across the country or the world such as on the Internet

Glossary

Key words and concepts: a working dictionary and glossary

ATM	An acronym for automatic teller machine which contains a computer processor that can electronically transfer data to and from banks and credit unions, so that money can be deposited or withdrawn at the ATM site (providing it is stocked up with money).
CAD	An acronym for computer aided design software which is used to make specialised drawings.
Control	The regulation of data and processes within the CPU by the Control Unit (CU). The CU directs and coordinates the movement of data within the computer, especially with regard to the fetch-execute cycle. The clock in the CU determines the processing speed (or the number of electrical impulses per second) of the computer, eg. 600 mHz means that the computer can process 600 million electrical impulses per second.
Data	The input to an information system such as: • Image • Audio • Video • Text • Numbers.
Digital communication	The transfer of data as 0's and 1's or as binary digits (bits) from one computer to another within a LAN (local area network) or within a WAN (wide area network).
EFTPOS	An acronym for electronic funds transfer at point of sale which is the process in which data can be electronically transferred to and from banks and credit unions so that purchases can be made at businesses such as shops. The purchaser's information, which is stored on a magnetic strip on a plastic card, and the product information, which is stored on a bar code, are read electronically into the computer system linking the shops and the banks.
Environment	Includes everything that is influenced by or influences the system, not including the system itself. For a computer system this includes the surrounding room, climate, furniture, noise and so on.
Facsimile	Also called fax, it allows images to be changed into digital form and sent electronically to another fax machine or computer.
Hardware	Includes any physical equipment that takes part in the input, processing, control, storage and output of data/information within a computer system. This includes the CPU, screen and all input, output and storage devices.
Information	The output from an information system after it has been transformed via the information processes. Output (information) from one information system can be input (data) for another information system.
Information processing	A set of information processes requiring participants, data/information and information technology.
Information system	(i) A set of information processes requiring participants, data/information and information technology to take place within a particular environment. The processes include collecting, organising, analysing, storing and retrieving, processing, transmitting and receiving, and displaying which function together to produce information. (ii) A group of components (including hardware, software, data, personnel).
Information technology	The hardware and software that are used in information processes within an information system.
Input and output devices	Input is the data that is entered by the user via one of the many input devices such as the keyboard, mouse, scanner, bar code reader, light pen, graphics tablet, MICR, OCR, joystick, touch screen, digital camera, video camera, microphone or modem (input and output). Output is the information that is sent to an output device such as a VDU (visual display unit), printer, plotter, speakers or modem (input and output).
Integrated circuits	Silicon chips containing electrical circuits that can process large amounts of digital data. The first IC was made in the 1960s and since then the capacity to store data has increased dramatically. ICs are found in many devices from the household dishwasher to the lift systems in multi-storey buildings, car engines and aircraft navigational instruments. The Y2K bug caused concern because it was thought that ICs could malfunction.
LAN	An acronym for local area network, which is a group of computers that are connected together by cables or microwaves, within a geographically restricted area. These computers usually share a fileserver for processing and storage space. The computer network within a school is an example of a LAN.
Participants	The people who carry out the information processes within the information system.
Processing	The changing of data into information when the CPU (Central Processing Unit) carries out some task on the incoming data such as sorting, doing arithmetic calculations, converting the key stroke into ASCII code for display on the screen or changing the text colour. The results of the process are sent to an output device and/or sent to a storage device.
Social implications	Issues that arise because computers/technology are used to complete tasks which were previously done without computers. These issues include security of data, privacy of information, copyright, changing nature of work, computer crime, equity, data accuracy and bias.
Software	Any instructions that are given to the computer as computer programs. These can be divided into: **System software** (such as Windows 2000, NT, Macintosh OS, DOS, Unix, Linux) which manages and controls the hardware and allows application software to operate. **Application software** (such as MS Office, Claris Works, Corel Draw, MYOB) which includes the 'programs' that allow the user to interact with the computer. **Utility software** which performs special tasks to help the operating system and the applications such as formatting a new disk, spell checking, diagnostic checks, detecting new devices and many other varied tasks.
Storage	The place where processed data or information and instructions are saved. It is also known as memory. ROM (read only memory, also called Firmware) holds data and instructions that are fixed at the time when the computer is produced. The operating system or system software is located on the silicon chip. **Primary storage** is contained within the computer and is directly accessible to the CPU. This includes the non-volatile ROM which is on the motherboard and the volatile RAM, which is used as temporary working memory. **Secondary storage** includes magnetic floppy disks, hard disks, tape, optical CD ROM. This memory is non-volatile. Temporary storage is memory that includes RAM (random access memory, also called disk cache), also SRAM, DRAM, cache (used to store frequently requested data or instructions) and CPU registers (storing instructions, data addresses, ALU calculation results). This memory is volatile.
System	A group of components or processes that function together to achieve a purpose, eg. the RTA's traffic light system (SCATS) has a purpose as well as many components and processes. A computer system is a collection of components and processes that make a computer or a computer network achieve a purpose.
WAN	An acronym for wide area network, which is a group of computers that are connected together by cables or microwaves, over a wide geographical area, eg. across a city, state or the world. The Internet is an example of a WAN.

2 Tools for Information Processes

Outcomes

By studying this chapter and completing the exercises students should be able to:

- Describe the hardware tools used in each of the seven information processes
- Identify and use the most appropriate input device to collect data
- Describe and use hardware devices for storage and retrieval
- Describe the concepts and tools used to transmit and receive data
- Describe and use a range of hardware to display data
- Outline the software tools used in each of the seven information processes
- Describe some non-computer tools used in each of the seven information processes
- Identify the social and ethical issues involved in the information processes.

Source: Information Processes and Technology Stage 6 Syllabus © NSW Board of Studies, 1999.

2.1 COLLECTING DATA

Collecting data includes all the activities involved in deciding what data to collect and the collection techniques used so that it can be entered into the computer or other information system.

Hardware	Software	Non-computer tools
Input devices for collection of data: • Pointing devices • Scanners • Digital cameras • Video cameras • Microphones • Keyboards • OCR devices. Emerging trends in hardware collection devices: • Data entry by voice • Portable scanning and electronic communication • Mobile phones connecting to computer networks • Digital cameras capturing images for direct input into computer hardware.	Operating systems: • MS-DOS • Win '95/2000 • Mac-OS • UNIX • Linux • GUI interface and WIMP environment • DOS interface.	Examples include: • Data counters, eg. traffic monitors • Literature • Surveys • Interviews • Manual observation/data forms.

Social Issues Related to Collecting Data

When data is collected and entered into an electronic database, it becomes very easy to access and reproduce. If, for some reason, this data is inaccurate or duplicated the problem is escalated further than if the data were not so easily accessible.

Following are some of the social issues that can arise with the introduction of computerisation for data collection.

Data bias occurs when the attitude of the person collecting the data affects what data is included in the sample. For example, collecting data from mainly white, Anglo-Saxon males about a general population issue would result in biased data. The data would not be truly representative and thus not really accurate.

Data integrity refers to the accuracy of the data. Data may be misspelt, entered incorrectly or be inaccurate for some other reason.

Copyright is the right of an author or the originator of an idea or other intellectual property to make decisions on what happens to their work. No one else is able to copy it without the permission of the author. Electronic communication makes breaching copyright much easier and probably more prevalent than before computerisation.

Ergonomics is the study of the way in which people interact with their work environment, in terms of both their efficiency and comfort. This includes furniture, lighting, temperature, noise level and work practices.

Appropriateness of data collection forms refers to the adequacy of the layout and content of data collection forms to ensure that correct and meaningful data is collected. For example, if the right questions are not included on the forms, the necessary data will not be collected.

Error detection of inaccurate data is often difficult as data collection becomes more computer driven. Computerisation has meant that people rely more on computers and less on their own judgement.

HOW HARDWARE DEVICES WORK

Collection (input) device	How it works
Mouse/trackball	• Is a pointing device that moves a cursor around on the screen and when clicked can choose an object on the screen. • On the bottom of the mouse (or top of a trackball) is a moveable ball and its movement is translated into digital signals using sensors touching the ball. • Connected to the computer via a cable connecting to a serial port. • There can be up to four buttons and a scrolling wheel located on top of the mouse. The first button is for selecting and dragging. The function of the others will vary depending on the software being used.
Keyboard	• The most common form of data input device. • Each key (when pressed) sends a specific message to the computer in digital form, eg. when the letter 's' is pressed the ASCII code for lower case s is sent to the computer and the letter 's' will be displayed on the screen unless it is a hot key for another function. (*This presumes that caps lock or shift keys are not pressed.*)
Scanner	• Uses light beams and measures the intensity of the reflected light to translate images of text, drawings, photos and other graphics into digital form. Scanners usually do three passes of light, building up the image from three different primary light colours. • The reflected light is converted into a bit mapped image which is saved in the computer's memory. • Scanners offer a variety of resolution, eg. 9600 dpi. The higher the resolution of the original scanned image, the higher the quality of the processed output, but more memory is needed as the resolution increases.
Light pen	• The light pen is a pointing device made up of a small stylus connected to the computer by a cable. • The tip of the stylus contains light sensors which detect the intensity of light emitted from the screen. • The user places the light pen at a particular place on the screen and as the pen is wiped across the screen, the computer is able to detect its position. • The effect is that the light pen selects an icon or hot spot on the screen or can draw directly via the screen.
Graphics tablet	• The graphics tablet or digitising tablet consists of a specialised electronic pad and a stylus. • The graphic tablet is connected by cable to the computer. • The fine grid of sensors on the electronic pad detect the position of the stylus and send this information in binary form to the computer. • Some are pressure sensitive allowing for thicker lines to be drawn when greater pressure is exerted on the stylus.
Touch screen	• The touching device, eg. finger, interrupts the matrix of infra-red sensors on the screen. • Useful for the selection of menu items and icons but less effective than graphics tablets for the precision input of drawings or handwriting.
Digital camera	• Uses a light sensitive processing chip to capture photographic images in digital form. • Contains a view finder and a lens to focus the image as well as a disk or memory card to store images. • After a picture is taken it is transferred directly to the computer using a fire wire and can be manipulated as a bit mapped image.
Video cameras	• Conventional video cameras capture analog images on video tape which are then transformed into digital images using digitising cards such as frame grabber cards or full motion video cards. • Digital video cameras capture images and store them directly in a compressed digital form such as MPEG on video tape. • Digital video software is used to edit the video.
Microphone	• Captures sound, including voices, in analog form. • The computer's sound card converts the analog sound into digital form. • The technique of changing voice sound into digital form is called voice recognition.
MIDI	• Musical instrument digital interface devices allow a musician to play an instrument where the notes are directly converted into digital form using the sound card, and then saved onto the computer.
OCR devices	• Optical character readers are specialised scanners that read pre-printed characters in a particular font. • The text is scanned and the computer matches it with a set of individual ASCII code characters.
MICR	• Magnetic ink character readers are scanners that read characters that have been printed using magnetic ink. • These magnetic characters are most commonly found at the bottom of cheques and show the account details.

SOFTWARE EXPLANATIONS (WIMP ENVIRONMENT)

Operating system software	• Also called system software, it is built into the motherboard when the computer is made. • Examples of operating systems are: ◆ MS DOS (Disk Operating System) uses a command line interface which was very popular before the GUI interface became common ◆ Apple Macintosh, eg. O/S 10, developed by Apple Corporation and uses the GUI interface ◆ Windows '95, '98, CE, NT and 2000 uses a GUI (graphical user interface) ◆ Novell Netware is a UNIX-like operating system especially designed for networking and is comparable to UNIX and Windows NT ◆ UNIX is one of the oldest operating systems used for multi-tasking by multiple users and has built-in networking capability ◆ LINUX is a more recently developed UNIX-like system that is freely available over the Internet. It is becoming very popular and is compatible with other operating systems such as Windows and Macintosh.

Application software	• Programs designed to perform specific tasks • Work in conjunction with an operating system • Can be bought as packaged (standard) or customised for a specific customer needs.
Examples of application software	• **Word processor:** ◆ Creates, edits and stores text-based documents ◆ Is used for letters, reports, articles, assignments and books. • **Database:** ◆ A collection of data organised into files, records, fields, and characters ◆ Data structure is described in a data dictionary ◆ Data can be sorted and searched using a database ◆ Databases can appear in table format as rows and columns or record format with the information for one record. • **Spreadsheets:** ◆ Composed of a grid of cells made up of intersecting rows and columns ◆ Contain values (number), labels (text) and formulas for doing calculations on the values ◆ What-if predictions or simulations can be done using spreadsheets ◆ Spreadsheet templates contain all labels, formulas and page layout. Values can be added each time the template is reused ◆ Charts and graphs can be produced using spreadsheet data. • **Desktop publishing:** ◆ Used to combine text and graphics to produce a document ◆ Data is organised and moved around the page using text frames and graphics frames for a variety of elements such as headings, columns of text and graphics. • **Multimedia:** ◆ Used for the presentation of information using text, graphics, animation, audio and video ◆ Graphics programs can be used to paint or draw ◆ Animation software is used for video and sound software is used for audio. • **Telecommunications:** ◆ Manage the transmission of data between computers ◆ Communications software supports on-line connections, eg. Internet electronic mail (email); facsimile; remote access connection; file transfer; automatic dial-up and answering.
Utility software	• Used to support, enhance or expand application and system software • Can perform a range of tasks such as: ◆ Formatting a disk ◆ File management, file conversion, back-up and data recovery ◆ Disk fragmentation ◆ Virus protection ◆ Data compression ◆ Memory management and so on.

2.2 ORGANISING DATA

Organising data includes all the activities that get data ready to be used by the information system. These include formatting, digitising, sampling and other methods that enable data to be analysed, processed, stored and retrieved, transmitted and displayed.

Hardware	Software	Non-computer tools
Data including text, numbers, images, sound and video is digitised as it is converted to ASCII code and executed within the CPU.	• Word processor, spreadsheet, database, multimedia • Paint and draw • Operating system for ASCII conversion.	Books, magazines, filing cabinet, manual library catalogues, other hardcopy structures.

Social Issues Related to Data Organisation

Data has traditionally been collected and entered directly into computers using input devices. However, data is increasingly being accessed by means of the **Internet** and **hyperlinks** from remote locations typically using the Internet. Software has been developed that allows for improved methods of **accessing different types of data** such as voice, fingerprint and even using the image of the eye's iris.

Electronic databases have made data much **easier to access**. However, if data that is held in those databases is inaccurate it can have serious implications. For example, if the police department holds inaccurate data on an individual it may cause them unnecessary conflict with the law.

Computer **display** is a further area where the effect of computerisation on the individual has been marked. Laptop computers are now widely used due to developments in LCD technology. Palm top computers and mobile phones are used to communicate with computer networks.

The population of today has become increasingly **dependent on electronic data**. If this data is not complete or stored appropriately there can be serious repercussions. One example of this is the occurrence of the **'Millennium Bug'** at the close of the twentieth century. Although none of the feared disasters came to pass, the fact that some microchips contained a date in two digits instead of four fuelled a concern that they would malfunction as the year changed from 99 (for 1999) to 00 (for 2000). Among the fears were that airplanes would crash, medical apparatus would fail, lifts would be stuck between floors and so on. The fact is the populace just did not know what was going to happen and spent many millions of dollars guarding against possible disasters.

DATA ORGANISING EXPLANATIONS

Organising issue	Explanation
Text	• Text is collected via input devices such as the keyboard or scanner with OCR (optical character reader) software and then converted into computer-readable ASCII code.
Images	• Images are usually saved as a series of pixels (short for picture elements) called **bit mapped** images. • The more pixels used to create an image the greater the resolution or clarity of the image on the screen. • The amount of memory required to save an image is determined by the number of pixels on the screen and the number of colours available to display each pixel. • The current image displayed on the screen is stored as a 'bit map' inside the frame buffer of the computer. • Images that are stored in bit mapped format are called **raster** graphics and can be created using paint programs. • Images can be stored using just the end coordinates and equations of lines forming the image. These are called **vector** graphics and are created using draw programs.

	• Bit mapped images take more memory to store in the frame buffer than vector images. However, as most personal computers have a raster screen, vector images must be **rasterised** before they are displayed on the screen, unless they are being used on specialist hardware that has a vector screen such as those used by draftsmen, CAD (computer aided design) users or other drawing specialists. • Bit mapped images are often compressed so that they take up less memory space. • **Lossy** compression involves the discarding of every third or fourth pixel or shades of colour that would not be missed. The image loses resolution but if the discarding is done well the loss will not be too noticeable. Lossy compression can shrink an image down to as little as 5% of its original size. For example, JPEG can compress images from 20:1. • **Lossless** compression uses mathematical techniques to replace repeated patterns of pixels with a 'coded' summary. During decompression these coded summaries are replaced with the original pixels and thus no change in the original resolution occurs, eg. GIF.
Audio	• Sound is digitised using **sampling**, which involves taking a series of sound wave samples at regular intervals. The number of samples selected is called the **sampling rate**. Each sample is assigned a number of bits and this is called the **sampling size**. • Better quality sound is achieved by increasing the sampling rate or sampling size.
Multimedia	• Multimedia is the presentation of information using at least three of text, hypertext, graphics, animation, audio or video. • These different formats can be combined to create a more interesting production that may be interactive. • Where there is more than one frame, as in animation, the navigation between each frame is shown in a storyboard
File converters	• File converters change a file from one format into another so that it can be used in another application. • They are often available within an application. For example, word processors allow the user to save a document as a HTML file so that it can be viewed on the Internet or a bit mapped graphics file can be saved in a compressed format such as a JPEG file.
HTML	• Hypertext markup language is the format used on the world wide web. • It consists of a set of special instructions called tags or markups that are used to specify formatting document structure and links to other documents.
Y2K bug	• The year 2000 bug (more commonly known as the Millennium Bug) was expected to create havoc in Windows-based computer systems and microchips around the world as the year date changed from 1999 to 2000. • The year 1999 had been stored in many computers as a two digit number '99' so that when the year 2000 arrived the two digit number would become '00' and unrecognisable or problematic to computer systems. • Macintosh systems did not have a problem with the Millennium Bug as their years have always been stored as four digit numbers.

2.3 ANALYSING DATA

Analysing data involves the interpretation of the way the data is input and preparing it for processing. For example, text data is different from graphical or audio data and must be processed differently.

DATA ANALYSING TOOLS/ISSUES

Hardware	Software (examples of analysis features)	Non-computer tools	Social and ethical issues
• Analysing data requires large primary and secondary storage. • High processing speed allows many fast calculations.	**Utility software:** • File comparisons. **Database:** • Searching/selecting • Sorting. **Spreadsheets:** • Modeling/simulation • What-if scenarios • Charts and graphs.	• Manually searching files • Non-computer models and simulations.	• Unauthorised analysis of data • Incorrect data analysis • Privacy threat through linking databases which make it easy to search vast stores of data quickly.

DATA ANALYSING EXPLANATIONS

Learning requirement	Examples
• Identify hardware requirements to carry out a particular type of analysis.	• Large amounts of data and complex calculations require large RAM and secondary storage and fast processing. • Scientific analysis requiring complex calculations, such as weather forecasting, may require a mainframe or supercomputer.
• Describe the best organisation of data for a particular type of analysis.	• **Searching** will require the use of a database application or at least a find or search command as in a word processor. • **Sorting** data alphabetically or numerically in ascending or descending order can be done using a database application as well as in the table of a word processor and within a spreadsheet. • The use of programs incorporating graphics and animations, that can be used for **modelling** and **simulation**, are ways to make predictions about decisions. Examples of these programs are flight simulators and computer games. Other simulations can be done using spreadsheet and financial/statistical software. • More sophisticated modeling and simulations can be done on mainframes and supercomputers. For example, stress testing on rocket components. • **What-if predictions** can be done on spreadsheets where values are changed within a template to reveal changing outcomes as in a loan repayment simulator. • Spreadsheet **charting** facilities can be used to provide a graphical representation of numerical data.

• Use software analysis features in a range of software applications to analyse image, audio, video, text and numeric data.	• Word processors can be used to analyse/process text and graphics. • Databases are use to store, search and sort data and create which manipulate the data in a specific manner. • Spreadsheets are used to store values, labels and formulae create charts and answer what-if predictions using values • Desk top publishers are used to store and manipulate text graphics to create a required page layout. • Multimedia software is used to create, store and manipulate graphics, animations and sound.
• Compare and contrast computer and non-computer tools for analysis on the basis of speed, volume of data that can be analysed and cost.	• Searching is done on a computer using query language in database or the 'find' facility of a word processor. • In a non-computerised system, searching can be done on files a filing cabinet or cards in a library catalogue. • The computerised search is usually faster and more accurate • File storage takes up less space in a computerised system consequently, depending on the cost of hardware and volume data, is less expensive to store. • Privacy can be more easily eroded using computer databases to their ease of access.
• Analyse data on individuals for the purpose it was collected.	• Incorrect analysis or the poor organisation of data can lead output information being inaccurate. • If data is analysed for purposes other than those for which it collected the resulting information may be inaccurate.

2.4 STORING AND RETRIEVING DATA

Storing of data involves placing it in secondary memory and is referred to as 'writing' data. Retrieving data occurs when it is 'read' from storage and moved from secondary storage into the working memory RAM.

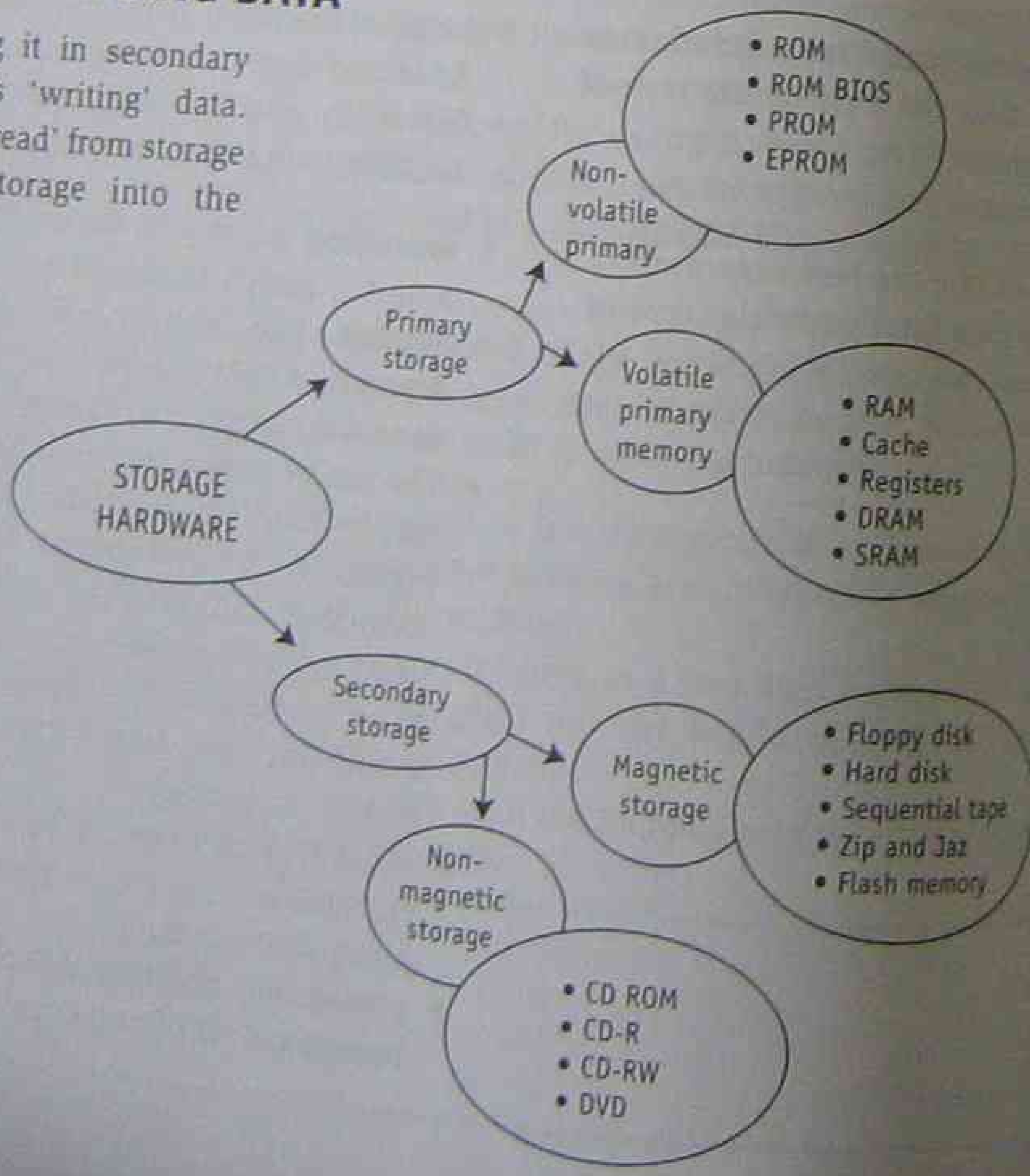


Fig 2.1 Storage devices

Storage/Retrieval Tools and Issues

Software	Non-computer tools	Social and ethical issues
Utility • Hardware interface • File management • Database management • File formats **Internet browser** • Machine independent browsers • Search engines	• Paper-based storage • Microfiche • Libraries	• Data security • Unauthorised access • Advances in storage and retrieval with data matching

Data Storage and Retrieval Explanations

Writing data	Is the process of saving data to secondary storage.
Reading data	Is the process of retrieving data from secondary storage.
Back-up copy	Data storage should always include a second copy, called a 'back-up copy', which is stored separately for security purposes.
Primary storage	Primary storage, often called 'memory', refers to either: • RAM (random access memory), the temporary, volatile or working memory. • ROM (read only memory), the non-volatile memory that is found on the motherboard and includes the operating system.
Secondary storage	Secondary storage is non-volatile or permanent memory. Information can be saved to secondary storage using peripheral devices. Examples of secondary storage are: • Floppy disk (magnetic) • Hard disk (magnetic) • Removable cartridges such as Zip disks, Jaz disks • Sequential tape (magnetic) • CD ROM (optical) • DVD (digital versatile disk).
Volatile data	Volatile data is temporary data and is lost if the computer is switched off before it is saved to secondary storage.
Non-volatile data	Non-volatile data is not lost if the computer is switched off. Secondary storage and ROM are non-volatile. RAM is volatile.
Magnetic disks	Magnetic disks are the most widely used form of data storage. They consist of a circular piece of metal covered with a plastic casing. The metal disk is coated with a thin layer of magnetic material and the disk is divided into a series of circular tracks and radial sectors. The data are stored in between the tracks and sectors. Floppy disks, hard disks and sequential storage tapes are magnetic.

Formatting magnetic disks	Formatting a magnetic disk means creating and organising the tracks and sectors of the magnetic coated disk so that data can be stored. This process removes any data on the disk, analyses the disk for faults and creates a directory called a FAT (file allocation table). The FAT stores information about the file size, time and date of last modification and the address (track and sector number) of the file on the disk. If a sector is faulty it is called a 'bad sector' and data will not be stored on that part of the disk.
RAM	Random Access Memory is data that can be accessed directly anywhere on the memory site (disk or chip) without having to traverse all preceding data.
Magnetic tape	Magnetic tape is covered with a layer of magnetic material but unlike magnetic disks access to data is sequential rather than direct.
Sequential access	Although it provides slower access, data is accessed in order of its storage. It is used in magnetic tape because it is a cheaper form of storage for data that is not accessed frequently or for data that usually needs to be accessed in order.
Disk controller	This is responsible for the transfer of data between the computer and a disk drive. Examples of disk controllers are: • EIDE (enhanced integrated drive electronics) which supports storage capacity up to 8 Gb with a transfer rate of 66 Mb per second. • Ultra DMA (direct memory access) with transfer rate of 66 Mb per second and improved error checking using CRC (cyclic redundancy checking). • SCSI (small computer system interface), usually on an expansion card with transfer rates up to 80 MB per second. Supports multiple disk drives.
ROM	Read Only Memory is also called firmware, which is programmed during the computer's manufacture and cannot be changed.
ROM BIOS	Read Only Memory for the Basic Input/Output System. These are the instructions located in ROM that tell the computer how to interact with the peripheral devices.
PROM	Programmable ROM. PROM is manufactured as blank memory and can be programmed once after manufacture.
EPROM	Erasable Programmable ROM can be erased and reprogrammed by the manufacturer. This allows the manufacturer to change the firmware to allow for upgrades, or new versions. EPROM is often used for computer games where changes of ROM are required.
Cache	A high speed memory located between the CPU and RAM that can be accessed quickly to be processed by the CPU. The most frequently used instructions are cached for easy and fast access, which allows the CPU to run faster because it does not have to take time to find instructions in the primary memory.
Registers	Temporary memory places within the CPU which are important in the fetch-execute machine cycle. Types of registers include: • **Address register** which holds the addresses of data and instructions in the primary memory. • **Accumulator** which holds the results of the last instruction executed. • **Storage** where the data to be executed is stored before execution. • **Instruction register** where the instruction is stored after it has been fetched from primary memory and before it is used in the ALU.
DRAM	Dynamic RAM is constantly refreshed 1000 times per second. DRAM chips are small and inexpensive.
SRAM	Static RAM is faster than DRAM as it is not constantly updated but is more expensive.
CD ROM	Compact Disk, Read Only Memory is a 12 cm wide plastic circle which can hold 650 Mb of data in a series of pits which reflect light back at varying frequencies. Once the data is stamped to the CD it is read only. Although the reading access is slower than for a hard drive, the CD provides useful and portable secondary storage. CD-R Recordable allows data to be written only once but read more than once. This is called a WORM environment, meaning 'write once read many'. A CD burner is used to write on a CD-R. CD-RW on which rewriteable data can be overwritten. The original data on the surface is erased when the disk is heated and cooled quickly.
DVD-ROM	Although it is the same size as a CD ROM, a digital versatile disk can store between 4 Gb and 17 Gb of memory. Unlike other forms of storage the structure of DVD data, video and audio have the same file structure called UDF (Universal Disc Format). This overcomes file incompatibilities in multimedia applications that slow down the retrieval of data on other forms of storage. DVDs can store full-length movies.
Flash memory	Is a non-volatile credit card size portable memory chip that retains its data when power is removed. It is used primarily in notebook computers and digital cameras.
Fileserver (networks)	A high speed computer in a LAN that stores the programs and files that are shared by the users who log in to the network.
File formats	The file format refers to the way data is stored. Each different type of data has its own method of storage. Some file formats are: • Data files such as: .DOC; .DAT; .DBF; .MDB; .XLS • ASCII files are text only files, eg. .TXT • Graphics files, eg. .BMP; .JPG; .GIF; .TIF; .EPS • Audio files, eg. .WAV; .MIDI • Video files, eg. .AVI; .MPG.
DBMS	Database Management System is the software that allows the user to enter, store and manipulate a database by controlling the structure of the data and the access to it. The value of a DBMS includes: • Reduced data redundancy • Improved data integrity • More program independence • Increased user productivity • Increased security.

Internet browsers	Software that allows users to access documents that are stored on other computers connected to the world wide web. Each web site has a unique address called a URL (uniform resource locator). A search engine is a database of web sites that can be searched by the web browser by using key words.
Passwords	Words that are used to control levels of access to computers or computer systems.
Microfiche	Transparent sheets of plastic that store about 200 sheets of miniaturised printed text. This is a non-computerised form of data storage, which is magnified using a microfiche reader in order to be read.
Frame buffer	The memory of the computer required to store the image of a single screen frame. The size of the frame buffer is greater when storing bit mapped or raster screens (as in paint programs) than for vector screens (as in draw programs).

Storing Graphical Data

Graphical data can be stored as a **bit mapped (raster image)** in the frame buffer or as a series of end points coordinated as a **vector image**.

If the image is stored as a bit mapped image it is possible to determine how much memory is required by the frame buffer by:

- First calculating the number of pixels being displayed on the screen.
- Multiplying that number by the number of bits required for each pixel, depending on the number of colours to be displayed.
- The number of colours to be displayed is equal to the index of the power of 2 that equates to the number of colours. For example:
  - For 2 colours ($2^1 = 2$), 1 bit for each pixel is required.
  - For 4 colours($2^2 = 4$), 2 bits for each pixel is required.
  - For 16 colours ($2^4 = 16$), 4 bits for each pixel is required.
  - For 32 colours ($2^5 = 32$), 5 bits for each pixel is required.

Note: Two colours is monochrome, that is, a screen of one colour with contrasting display such as black or white. Shades of black, such as grey, are counted as separate colours or tones.

- As the answer will be in 'bits' this must be converted to bytes and then kilobytes and megabytes:
  - Divide the number of **bits by 8** to convert it to bytes.
  - Divide the number of **bytes by 1024** to convert them to kilobytes.
  - Divide the number of **kilobytes by 1024** to convert them to megabytes.

Note: Rather than 1000 bytes in a kilobyte there are 2^{10}, which is 1024 bytes.

Example:

Calculate the size of the frame buffer (amount of memory) required to display an image on a screen with a resolution of 640 × 480 with 16 colours.

- For a resolution of 640 pixels across × 480 rows of pixels down, the total number of pixels on the screen is 640 × 480 = 307200.
- To display 16 colours ($16 = 2^4$) you need 4 bits for each pixel 307200 × 4 = 1229800 bits
- 1229800 bits/8 = 153825 bytes
- 153825 bytes/1024 = 150 kilobytes.

2.5 PROCESSING DATA

Processing manipulates data into information.

Types of processing include:

- Centralised
- Distributed
- Parallel.

ELEMENTS RELATED TO DATA PROCESSING

	Hardware	Software	Non-computer tools	Social and ethical issues
Processing	• CPU; ALU; CU; CU clock; registers. • Fetch-execute cycle. • Fast processing speed: ◦ Large RAM ◦ Large secondary memory for image, video and audio processing ◦ Affected by clock speed, bus capacity, word size, response time, CPU utilisation • Specialised hardware for centralised, distributed and parallel processing.	Utilities for text, numbers, images, video and audio data.	Documentation: • User manual • Data flow diagrams • System flow charts.	• Flexibility of use with distributed systems as more than one fileserver carries the processing responsibility within a network. • Security issues with centralised systems. • Bias in the way participants in the system process data. • Copyright/ownership of processed data.

Data is processed within the CPU (Central processing Unit) in the computer.

The completion of a single process occurs during the machine cycle or fetch-execute cycle.

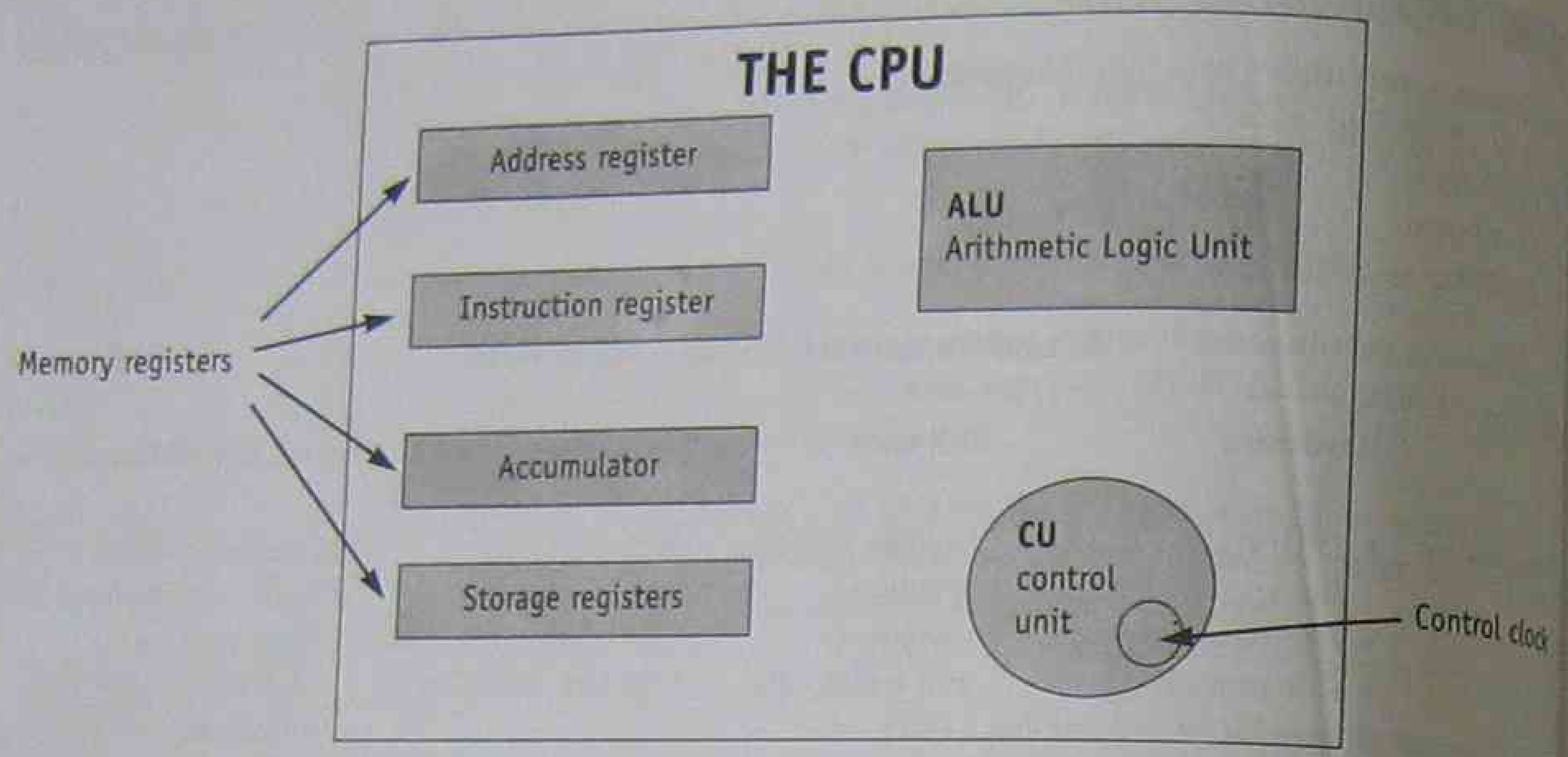


Figure 2.2 The components of the central processing unit

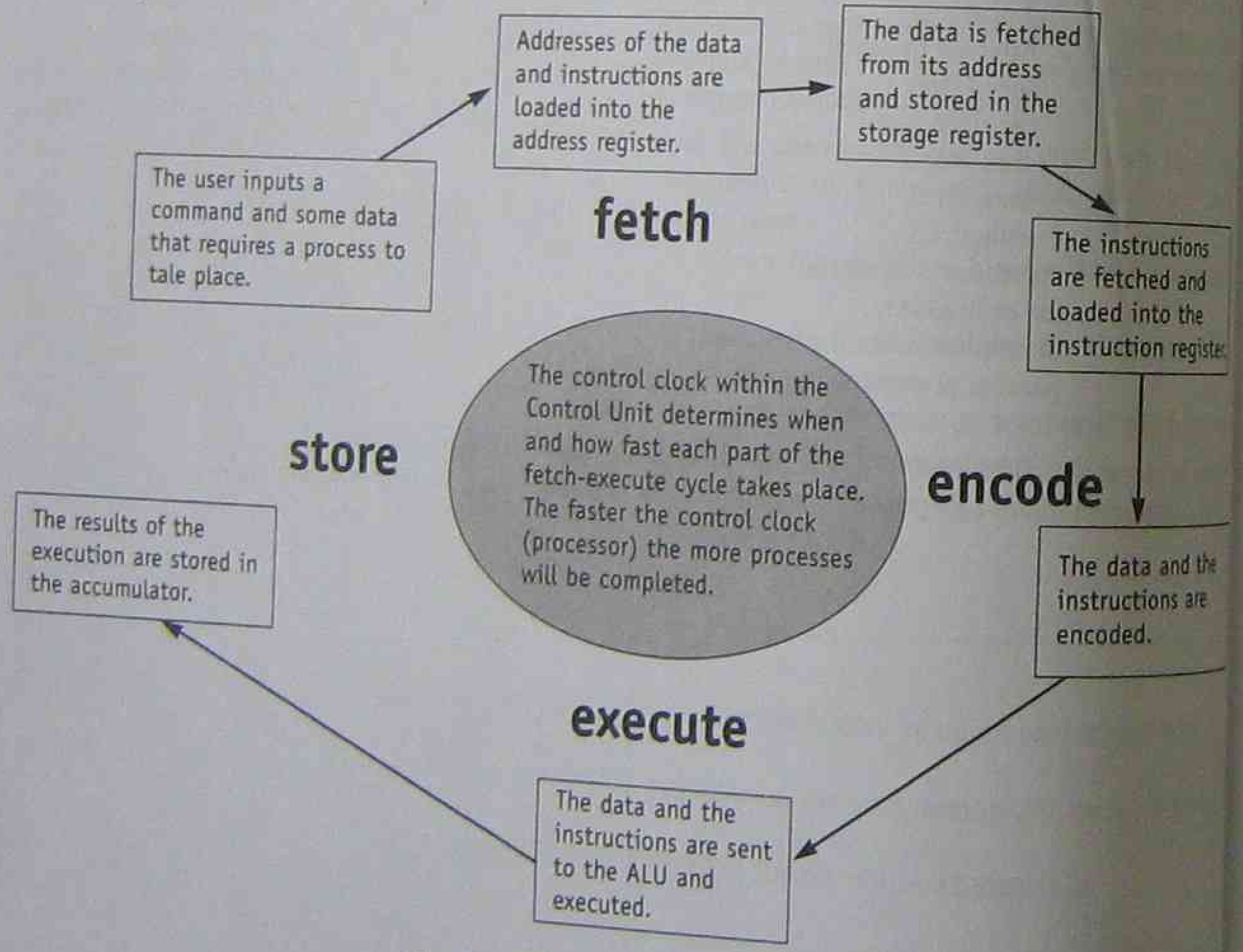


Figure 2.3 The fetch-execute cycle

Processing Data Definitions

CPU	The **Central Processing Unit** is where all the processing is carried out. The CPU contains the ALU, CU, control clock and memory registers. Each of these is involved in the machine cycle or fetch-execute cycle. The CPU is on the motherboard that is built into the computer when it is purchased.
ALU	The **Arithmetic Logic Unit** (ALU) is located in the CPU and is the site of all calculations and data processing. Calculations include the use of logical (AND, OR, NOT) and relational (=, <, >, < >) operators to manipulate the data. The result of the calculation is sent to the accumulator register. The address from where the data is fetched in primary storage is held in the address register and once it is fetched the data itself is stored in the storage register until it is sent to the ALU.
CU and control clock	The control unit directs the movement of data (before processing) and information (after processing) within the CPU. That is, back and forward to primary storage and back and forward to the registers and ALU.
Processing speed	The speed at which data can be processed within a computer. It is measured in Mega Hertz. The speed of a computer's control clock is a measure of its stated processing speed as advertised. For example, 800 MHz (800 million cycles per second).
Address register	A high speed temporary memory space within the CPU where the address of the data and instructions are stored, so that they can be fetched from primary memory during the machine cycle.
Accumulator register	A high speed temporary memory space within the CPU where the results of an execution are stored before being transferred to storage or used in another execution or process during the machine cycle.
Storage register	A high speed temporary memory space within the CPU where the data is stored, so that it can be executed using the instructions in the instruction register during the machine cycle.
Instruction register	A high speed temporary memory space within the CPU where the instructions are stored, so that they can be used to process the data that is stored in the storage register during the machine cycle.
Microprocessor	A single integrated circuit made up of a silicon chip that contains a CPU. The microprocessor and other components that make it work are mounted in the main board, called the motherboard.
Motherboard	The main board used to hold the microprocessor that is built into the computer when it is manufactured.
Logical operators	AND, OR or NOT are logical operators that are used in the mathematical calculations carried out in the ALU.
Relational operators	Mathematical symbols that show a relationship between two values and are used for mathematical calculations carried out in the ALU. These include: = ; < (smaller than); > (larger than); = < ; = > ; < > (not equal to).
Boolean operators	Same as logical operators AND, OR, NOT in which the outcome can have one of two values.

Word size	The number of bits that can travel along a bus pathway at the same time. The wider the bus width the more bits can travel together. The size of the bus is often called a bandwidth. The bandwidth is measured in bits. For example, Nintendo 64 means that it has a bandwidth or word size of 64 bits of data.
Response time	The amount of time it takes the computer to respond to a command given by the user.
Centralised processing	Processing that is performed by a central computer in a network so that the workstations connected to the network do little or none of their own processing.
Distributed processing	Splitting of the workload for processing tasks between several different workstations on a network. Each workstation completes a task that contributes to the whole network's processing.
Parallel processing	A type of multiprocessing where several processors within one fileserver work on the same task, sharing memory and other network resources, and data is sent to multiple processors along parallel paths. This is a fault tolerant system because if one CPU crashes the others will take over its workoad, eg. RAID technology.
Thrashing	Occurs when there is not enough RAM or virtual RAM (memory borrowed from the hard disk) so that the computer spends more time sending data back and forth to secondary memory than it does in processing.
Documentation	A written description of any details about an information system that helps the user to interact with the software. For example, user manual, on-line help, tutorials troubleshooting guide.
Data flow diagrams	A graphical presentation of the flow or movement of data through a system.
System flow charts	A graphical representation of the major inputs, outputs and processes within a system

2.6 TRANSMITTING AND RECEIVING DATA

Transmitting involves the sending of data/information within a computer on buses or between computers via links such as network cables and modems. Receiving involves the acceptance of data from other computers.

Data can be transferred in

- **Serial** form with 8 bits of data traveling in sequence on one wire
- **Parallel** form with 1 bit traveling on each of 8 wires
- **Simplex** form where data travels in one direction only, eg. TV transmission
- **Half Duplex** form where data can travel in both directions but only one way at a time, eg. two-way radio
- **Duplex** form where data can travel in both directions at all times, eg. telephone
- **Asynchronous** form where data includes a start bit before and stop bit after data transmission and no handshake is necessary
- **Synchronous** form where data has no start or stop bit because computers handshake before transmission.

	Hardware	Software	Non-computer tools	Social and ethical issues
Transmitting and receiving	• Terminal/CPU • Dumb terminal • Smart terminals • Modem (MOdulate/ DEModulate) • Facsimile (fax) • Scanner • Automatic answer • LAN (Local Area Network) • WAN (Wide Area Network) • PSTN (Public Switched Telephone Networks).	• Communication software • Protocol • Handshaking • Data compression • Electronic mail • Data encryption.	• 'Snail' mail • Telephone • Fax machines • Radio • Television.	• Data accuracy (integrity) • Data security • Netiquette • Authorship (acknowledgement of data source) • Privacy • Changing nature of work • E-commerce.

Examples of Social and Ethical Issues Related to Transmitting and Receiving

Data accuracy (integrity)

When data is entered into a database or other computer application incorrectly, or if incorrect data was collected originally, the data is said to lack integrity. In other words, it is not correct. This can lead to serious implications for a user applying for a credit rating if a database indicated they were a poor credit risk. This mistake can be passed onto several further databases and the user would have a difficult time proving themselves to be a good credit risk. In the movie *The Net*, Sandra Bullock's character has her social security and criminal records changed, causing serious difficulties in many areas of her life.

Data security

If data is not secured from unauthorised users it may be corrupted or changed. Again in *The Net*, criminals were able to bypass security measures by introducing a program that allowed them easy access into the computer system. As a result they were able to perform many criminal activities, thus causing havoc to many people.

Computer systems usually have several levels of security ranging from the restriction of physical access to computers themselves, to the requirement of passwords for accessing various parts of the system.

Netiquette

Netiquette refers to the unwritten expectation for acceptable and responsible behaviour when using the Internet, especially emails. Examples of poor netiquette include sending unsolicited junk mail (spamming); using capital letters throughout a document (shouting); using derogatory, offensive or obscene language (flaming); or breaching copyright by downloading and using someone else's intellectual property and calling it your own. Giving out your name or personal details in chat rooms is also considered poor netiquette.

Emoticons are special symbols that send an emotive message, for example, :-) is a happy face; :- (is a sad face; :-o is shock; ;-) is a wink. Other shortcuts: FYI is 'for your information' and IMHO is 'in my humble opinion'.

Before a person uses the Internet they should make themselves aware of the rules of netiquette.

Authorship

Authorship is another name for the intellectual property of an author or artist. Intellectual property is protected by the laws of copyright which give only the author/artist the right to copy or control their work. If anyone uses someone else's intellectual property they should at least acknowledge the source or better still gain permission from the author to use it.

Privacy

All individuals have the right to privacy. With the introduction of computers, and especially large databases, it is very easy for anyone with access to a computer to breach privacy. There have been numerous examples of this in the media such as government department letters being sent to the wrong people and unauthorised persons accessing police records that contain sensitive information and then misusing the data.

Changing nature of work

The introduction of computers has meant that many people have had to change the way they do their work. All commercial companies have had pressure placed on them to computerise or else become uncompetitive in the marketplace. Many skilled jobs such as plan drafting and graphic design can now be completed by relatively unskilled persons. Even movie making and cartoon drawing now rely heavily on the computer.

E-commerce

One of the biggest impacts of computers in the workplace is in the field of commerce. Large numbers of commercial transactions now occur over the Internet. Those companies that have not yet chosen to become computerised find that they are quickly becoming uncompetitive. Individuals can now do their shopping, book plane tickets, buy and sell shares, do their banking and many other everyday things using the Internet rather than exchanging cash in person. The downside of this ease of access is that commercial crime is made easier, necessitating the constant updating of security measures. There is also a very real concern that people just don't meet each other face to face resulting in isolation and dehumanising, again as in the case of Sandra Bullock's character in *The Net*.

Synchronous vs Asynchronous Serial Transmission Modes

Serial transmission can be synchronous or asynchronous

- Synchronous transmission occurs when two computers synchronise their time clocks and agree on transfer protocol before transmission so that all data is sent at once without the receiving computer having to acknowledge receipt before the next packet of data is sent.
- Synchronous transmission is faster than asynchronous and is used in large computer systems.
- Asynchronous transmission includes a start bit before the data packet followed by a stop bit after the data packet as well as error checking bits such as parity bits.
- Asynchronous transmission is commonly used in PC data transmissions.

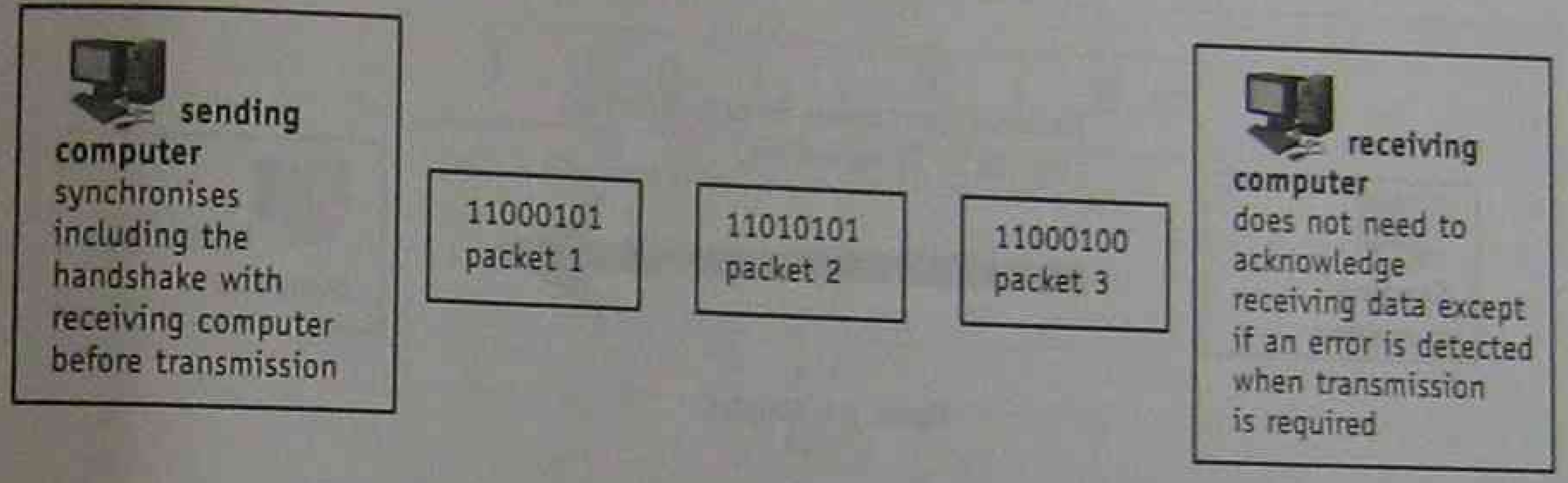


Figure 2.4 Synchronous

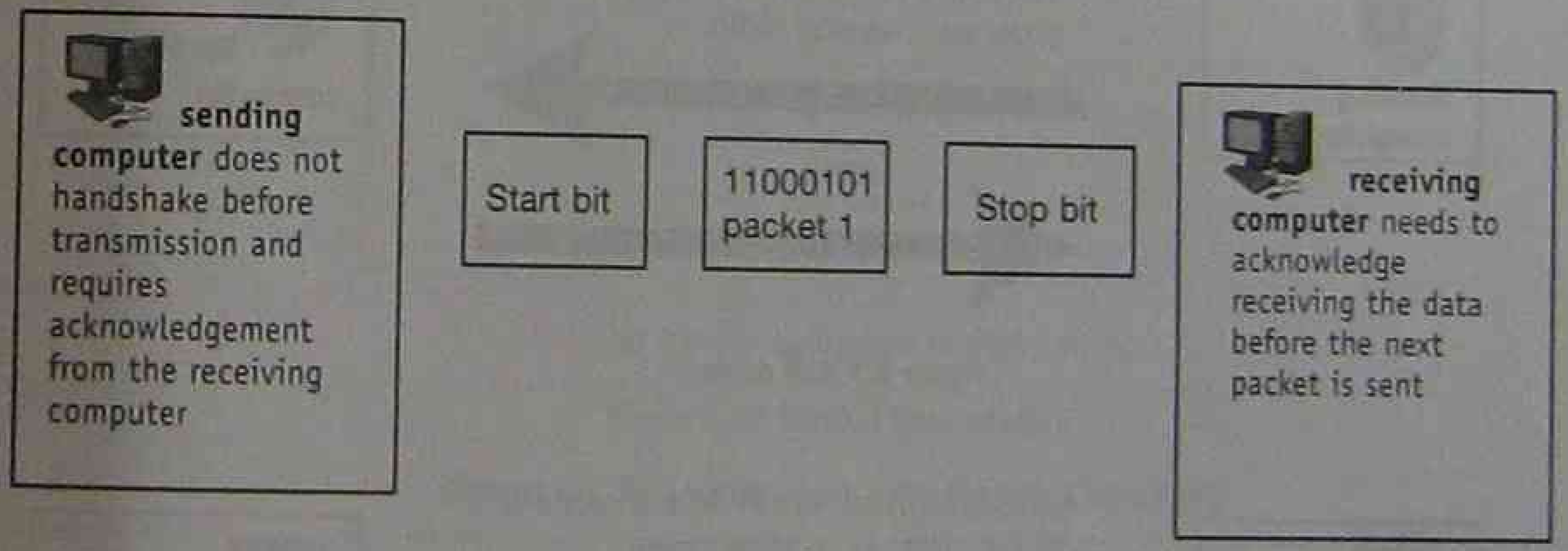
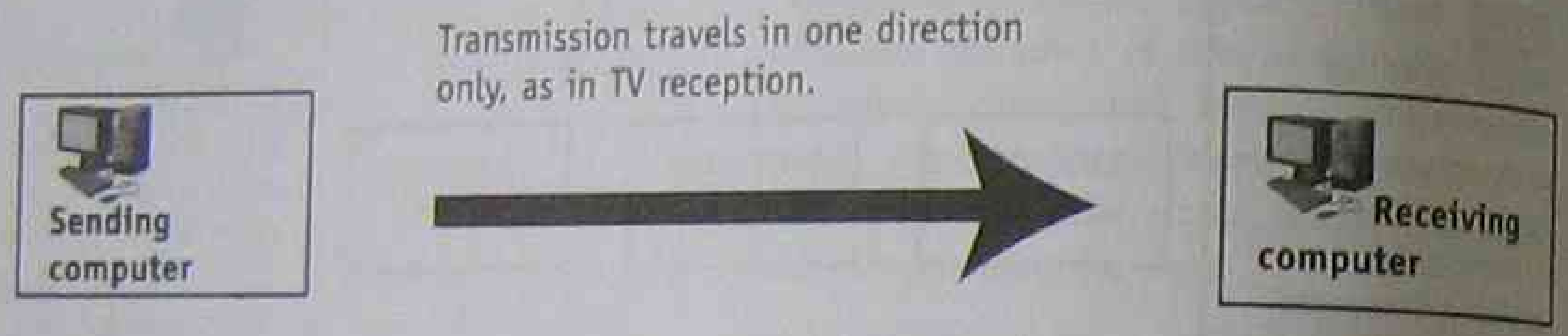


Figure 2.5 Asynchronous

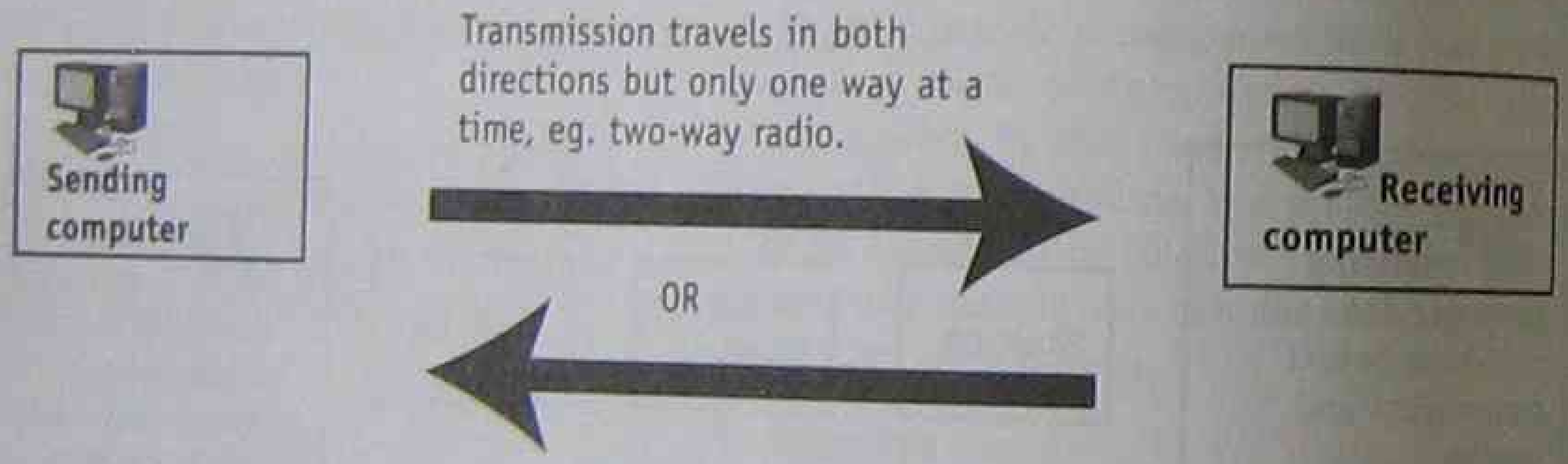
Simplex vs Half Duplex vs Full Duplex Serial Transmission Modes

	Serial transmission can be **simplex**, **half duplex** or **full duplex** depending on the direction of data flow during transmission: • Simplex transmission is where the data can flow one way from the sending device to the receiving device as in television transmission. • Half duplex transmission is where data can be transmitted in two directions but only in one direction at a time such as in two-way radio transmission. • Full duplex is where data can flow in both directions at the same time as in telephone transmission. In full duplex mode, data can be transmitted using two different audio frequencies. The sending frequency is called originate mode and the receiving frequency is called answer mode.



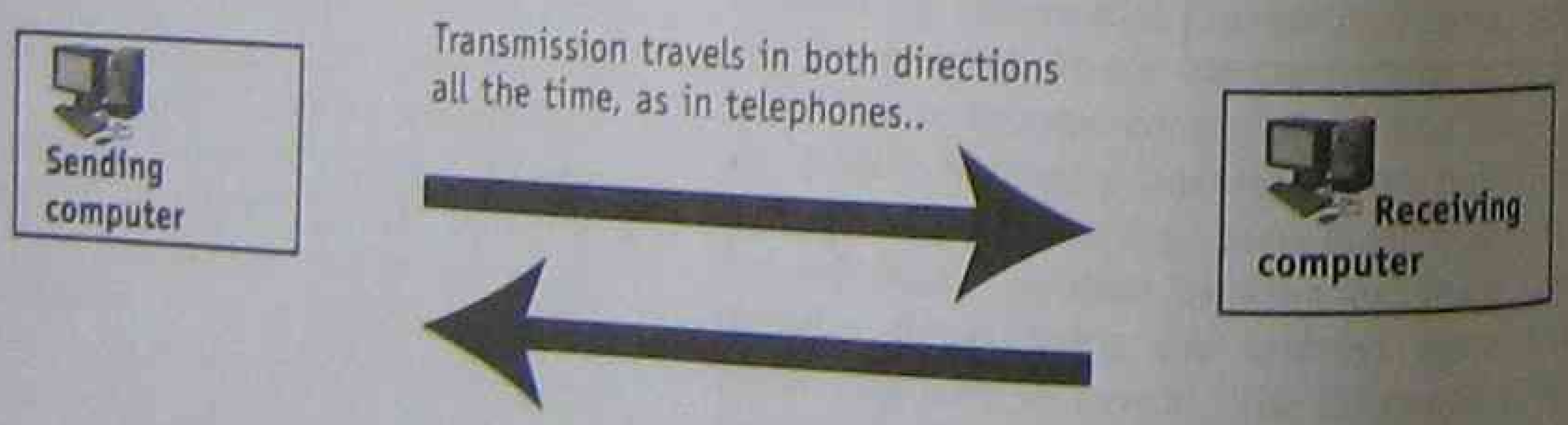
Transmission travels in one direction only, as in TV reception.

Figure 2.6 Simplex



Transmission travels in both directions but only one way at a time, eg. two-way radio.

Figure 2.7 Half duplex



Transmission travels in both directions all the time, as in telephones..

Figure 2.8 Full duplex

TRANSMITTING AND RECEIVING CONCEPTS

Types of digital transmission	• Digital data can be transmitted in **serial** form or **parallel** form. • Serial transmission involves the movement of 8 bits (1 byte) of data along a single bus, in sequence. • Parallel transmission occurs when each of the 8 bits travel along its own wire at the same time so 8 wires form a parallel pathway. • Parallel transmission is faster than serial but only occurs over short distances to avoid errors and the expense of lots of cabling. • Parallel transmission is used to connect peripheral devices, such as printers and disk drives, to the CPU, where larger volumes of data are transmitted. • Serial transmission can be used for peripheral devices such as the mouse but it is most commonly used for long distance transmissions via a modem.

1 0 0 1 1 1 1 0 ⇨

Bits move in single file along one wire.

Figure 2.9 Serial transmission

1 ⇨
0 ⇨
0 ⇨
1 ⇨
1 ⇨
1 ⇨
1 ⇨
0 ⇨

One bit travels along one of eight wires.

Figure 2.10 Parallel transmission

SPEED OF TRANSMISSION AND ERROR CHECKING

Error checking	• A **parity bit** is an extra bit added to the transmission. • Error checking parity can be **odd** or **even**. • If parity is to be set at **even parity** either a 0 or 1 parity bit is added to the transmission so that there are an even number of 1's, including the parity bit. • If parity is to be set at **odd parity** either a 0 or 1 parity bit is added to the transmission so that there are an odd number of 1's, including the parity bit. • Parity bit error detection is the least reliable because if there are an even number of errors the parity will still be correct. • **Checksum parity** is where extra data is added to the end of the data packet representing the total of all the 1's in the packet. The receiving computer also calculates the total number of 1's which should correspond to the sent checksum. • **CRC (cyclic redundancy check)** is where the checksum is divided by a prime number which is sent along with the remainder. CRC is the most reliable method of error checking and the most commonly used.

Speed of data transmission	• **Bits per second** is a measure of the number of bits that pass a particular point every second. Actual transmission speed is measured as bps. • **Baud rate** is the number of times per second the signal changes between 0 and 1. Modem transmission speed is stated as baud rate.
Communication within the computer	• A **bus** is the pathway for data to move along between peripheral devices and the CPU. • The **bus width** indicates the number of lanes or pathways available for bits of data to pass along the bus at once. For example, a 64 bit bus width allows 64 bits of data to move from the CPU to peripheral devices at the one time. • **Internal buses** are the pathways, inside the computer, between the motherboard (CPU) and the computer's memory. • **Expansion buses** are the pathways that lead to the external peripheral devices and are usually in the form of 'cards in slots' inside the computer connecting to the outside via ports. • Expansion buses can be ISA, EISA, VESA and PCI.
Communication outside the computer	• A **port** is an external socket connected to an expansion card inside the computer for the connection of peripheral devices. • **Parallel ports** allow for parallel transmission, usually to printers and disk drives. • **Serial ports** allow for serial transmission, usually to the mouse, keyboard, modem and plotter. Extra serial ports are called COM (communication) ports and most computers have two. Examples of serial ports are: ▪ RS-232 (recommended standard) ▪ DB-25 (25 pin connector) ▪ DB-9 (9 pin connector).
Modems	• A **modem** is a device that MOdulates/DEModulates data from digital to analog for transmission between computers over an analog telephone line. • **Internal modems** are placed into expansion cards fitting into expansion slots inside thecomputer. • **External modems** connect to a COM port and require a power supply. • The maximum speed at which a modem can transmit is its **baud rate**. A common modem baud rate is 56 K, meaning up to 56 000 changes of signal per second can be transmitted. • **Cable** modem allows data to travel from one computer to the next in digital form so that the modulation and demodulation of data between analog and digital is unnecessary. Transmission is faster and likely to have less errors. • Modems have a **facsimile** capability and are sometimes called fax modems, as they can send and receive transmissions from a fax machine. • Some fax machines have **OCR software** which enables incoming documents to be sent to word processing files for editing.

Networks

Types of networks	A network is a group of computers that are physically joined so that they can share data, programs and other network services such as printing and Internet access. Network topology refers to the design of the network and the relative positions of the workstations and fileserver. Some topologies are: • **Bus**, where computers called workstations are connected in a sequential line. Data is transferred between the fileserver and the workstations using CSMA/CD (carrier sense multiple access/collision detection). • **Star**, where computers are connected to a central fileserver in a star-shaped arrangement. CSMA/CD is used for star networks also. Many workstations can access the fileserver but sometimes data travelling on the same bus can collide. The collision is detected by the fileserver. • **Token ring**, where computers are in a ring structure, with or without a fileserver, and a token is passed around the ring.
Terminals on a network	Computer workstations are classified as dumb, smart or intelligent based on how much dependence they have on the fileserver: • Dumb terminals are workstations that do not contain a processor or storage and usually consist of a keyboard, mouse and monitor. • Smart workstations can do some low level processing such as text editing. • Intelligent terminals do all their own processing and storage. Most personal computers are classified as intelligent and are called workstations on a network.
Locations of networks	• **LANs (Local Area Networks)** operate on a small geographical site such as within one building or school. Terminals and fileservers are usually linked with cables or infra-red/radio waves. • **WANs (Wide Area Networks)** operate over a large geographical area such as across the country or the world. These networks are usually linked with telephone lines, leased lines, large coaxial cables or satellites and repeaters.

CSMA/CD transmission protocol

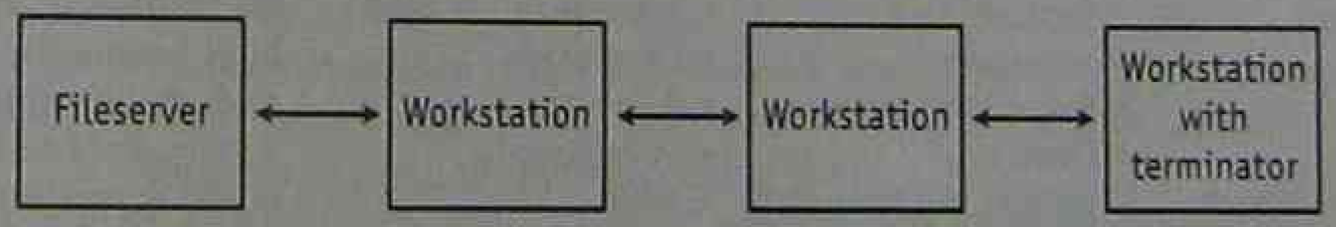


Figure 2.11 Bus network

CSMA/CD transmission protocol

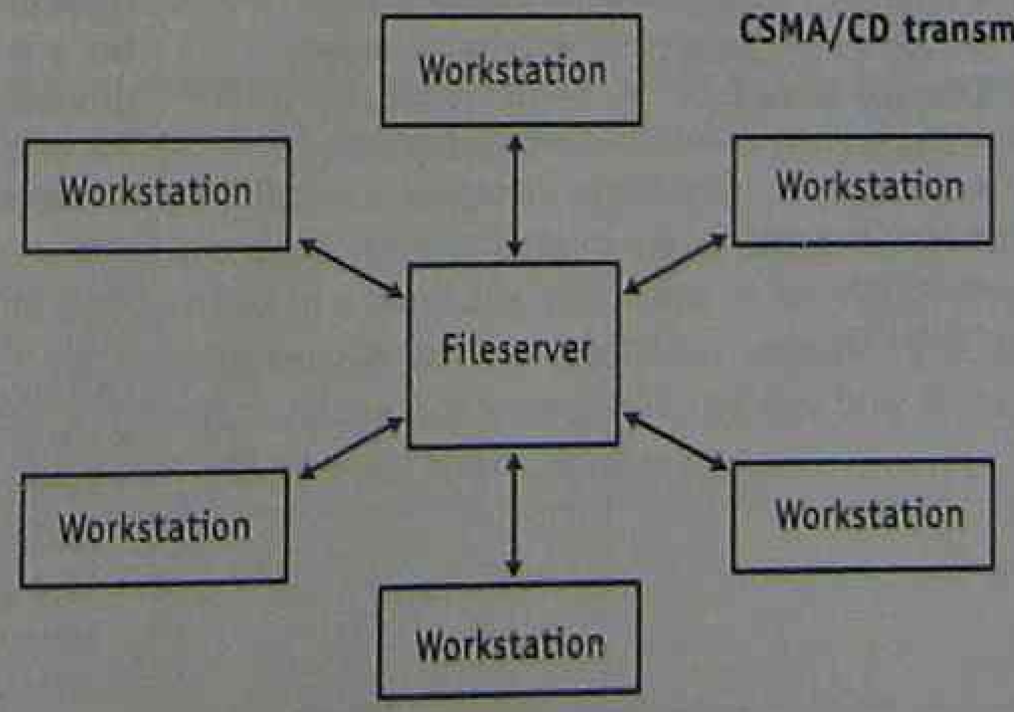


Figure 2.12 Star network

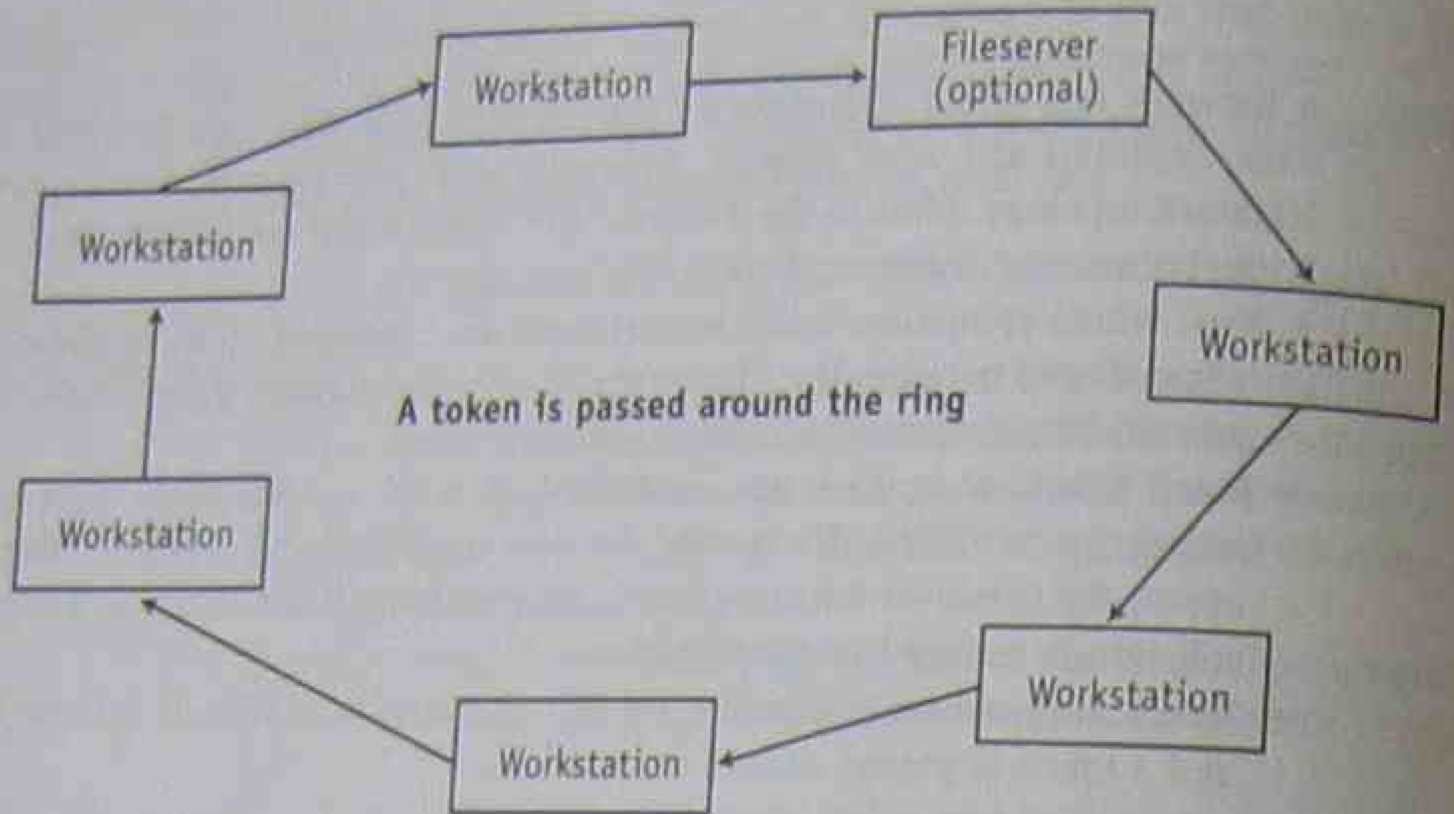


Figure 2.13 Token ring network

2.7 DISPLAYING DATA

Displaying involves a decision about what **FORM** the output information is to take and then **OUTPUTS** it a a hard copy, soft copy or sound.

Hardware	Software	Non-computer tools	Social and ethical issues
Screens For displaying text, numbers, images and video: • CRT • LCD. **Printers** For creating hard copies of text, numbers, images and video: • Dot matrix • Inkjet • Laser • Impact vs non-impact • Character vs bit mapped. **Plotters** For producing images and charts: • Pen plotters – flat bed or drum • Electrostatic plotters. **Speakers** Outputs sounds, music and voice synthesis or processed voice input.	All application software has its own built-in screen design for data display. **Specialised displays:** • Database reports • Word processed formats and templates • Excel templates • Desk top publishing formats • Presentation display formats • Graphics formats • Web page design. **Display issues:** • Formatting • Spacing • Mail merge • Tables • Charts.	**Moving graphics:** • Storyboards. **Text and graphics:** • Newspapers • Magazines • Books • Black/white boards • Overhead projectors • Billboards • T-shirts • Drawings • Paintings • Photographs. **Audio:** • Musical instruments • Radio • Television • Tape recorders • CD ROM and DVD • Video players.	**Communication skills:** Presenters need to be good communicators and information needs to be well organised and attractively displayed. **Current trends:** Screen size is larger and screens are more sophisticated, eg. flat and high resolution. **Visually and aurally impaired:** Alternatives to text, eg. using audio or voice recognition in web design as well as larger text for visually impaired. **Appropriateness of displays for a wide audience – offensive material:** Pornographic, racist or violent material on the web. Net nanny will restrict offensive sites from young children. • Chat lines can be used by criminals such as pedophiles. • Use great care if disclosing personal or banking details over the Internet.

How Displaying Hardware Works

Output device	How it works
CRT monitor	• The cathode ray tube screen is coated on the inside with a layer of **phosphor** which emits light when struck by electrons. • In a colour monitor, red, green and blue **filters** cover three separate **electron guns** which fire at the back of the screen. • The beams pass through a metal plate called a **shadow mask** which aligns them with the required spot on the screen. • As the phosphor is hit with an electron it glows and thus the image is created on the screen. • After a while the phosphor will stop glowing so it is **refreshed** by being hit again. • The process of **interlacing** refreshes every second line on the screen in turn. This is older technology designed to save on memory and speed up the refreshing rate. Most modern screens are **non-interlaced**. • The electron beams move in a zig zag pathway across the back of the screen completing a **raster scan** so that the screen will stay refreshed. • Each glowing spot represents a **pixel** or picture element. • The more pixels on a screen the better the screen **resolution**. • The space between each pixel is called the **dot pitch**. The smaller the dot pitch the greater the resolution.
LCD screens	• Liquid crystal display screens are flat and composed of a layer of **liquid crystalline** material lying between two **polarised** sheets. • Light is passed through the crystal and an electric current is produced at particular points. This causes the appropriate crystals to glow. • LCD screens are very small and light, produce no radiation, glare or heat and require less power, enabling them to run on batteries and be appropriate for use on laptops.
Laser printer	• The information being output from the computer is created as a laser beam and fired at a positively charged rotating drum. • The places where the laser strikes the drum become electrostatically charged and match the pattern for printing. • The drum is passed over a tray of black powder called toner and the toner is attracted to the charged areas on the drum thus creating the printed image. • The toner is fused to the paper using heat. • The toner is stored in a cartridge and needs to be replaced periodically. • Laser printers print a whole page at once and are usually of higher resolution (between 600 dpi and 1200 dpi) than other printers.
Inkjet printer	• Produce characters by spraying fine drops of ink onto paper. • The print head contains a nozzle with between 50 to several hundred holes that allow the ink to pass through in specified places. • Bubble jet printers form a bubble of ink and heat forces this onto the paper. • The ink is stored in a cartridge and needs to be replaced periodically. • Ink jet printers usually have a resolution between 600 dpi and 1000 dpi but unlike laser printouts they are not waterproof and can blur if the images are touched before the ink dries. They may 'bleed' or smudge.

Dot matrix printers	• Dot matrix printers are impact printers that make the image as a pin hits paper through a carbon ribbon. • Each character is formed using a series of dots. • They are reliable and cheap but noisy and not often used today except when the impact nature for making multiple carbon copies is required, for example, when printing dockets and receipts in conjunction with carbon paper, or where draft quality, large volume printing is required as in busy programming departments. • Dot matrix printers have low resolution between 60 and 180 dpi.
Pen plotter	• A pen plotter produces images using coloured pens which move across a flat bed (flat bed plotter) or a drum (drum plotter). • The pen moves in two dimensions across the static paper in flat bed plotters while the drum moves around the static pen in a drum plotter. • The movement of the pen is controlled by the plotter software. • Flat bed plotters are often used by draftsmen, architects or engineers to create computer aided drawings.

CHAPTER SUMMARY

Collecting data

Hardware for collecting data includes all devices that are used to input data into the computer. A variety of input devices are used to input different types of data such as text, (keyboard), graphics (scanner, digital camera), animation (digital video camera), and sound (microphone).

Non-computerised data may be collected as **qualitative** data or **quantitative** data. Qualitative data mechanisms may include interviews that canvas opinions or ideas. Quantitative data refers to numbers of items and may be collected using counting devices or surveys.

In conjunction with hardware devices, a variety of **software** is also required for data collection, including different types of operating systems. Specialised application software is also required to capture images, text, video and audio data. The Internet and its requisite software may also be thought of as specialised collection software.

Non-computer mechanisms that can be compared to computerised hardware and software include literature searches, surveys, interviews, televisions, radio and so on.

Some **ethical issues** are apparent when discussing the collection of data such as ensuring that the data is collected without **bias**; that it is accurate or has **integrity**; that **authorship** is respected and copyright is not breached; that individual's **privacy** is not invaded and that data is collected with regard to good **ergonomic** practices.

Organising data

Once data is input into a computer, using a variety of hardware devices, it is organised by the operating system software so that it can be further manipulated by the applications software. **Applications** may include paint and draw, multimedia, word processing, desk top publishing, spreadsheets and database software.

Non-computerised organising tools may include hard copies and filing systems.

The way that data is organised will determine its useability and accessibility. How the data needs to be used affects which application is used. For example, data is organised differently by spreadsheets, databases and desk top publishers. In a non-computerised system data which is not filed using logical methods may be difficult to find.

Analysing data

Hardware is involved in the analysis of data in processing (CPU) and storage (storage media). Data analysis is also carried out by application software such as databases (searching and sorting), spreadsheets ('what-if' predictions and charts) and modeling and simulation.

Non-computing tools for data analysis include filing systems and cabinets, colour coding systems and so on.

Social and ethical issues relating to analysis include making data easily available to unauthorised access, resulting in possible invasion of privacy, stealing or changing of data to make it inaccurate.

Storing and retrieving

The hardware involved in storage includes devices associated with temporary or volatile memory (RAM, CPU registers), non-volatile primary (ROM) and secondary storage devices (hard disk, floppy disk, magnetic tape, CD ROM, DVD). Primary and secondary memory is said to be non-volatile because it is not lost when the computer is accidentally turned off.

It is important to **back-up** data as well as saving it to secondary storage. Examples of **back-up devices** include Zip and Jaz disks; QIC (quarter inch cartridges); DAT (dual audio tape); 8 mm cartridges; CD ROM; CD-R (compact disk readable); CD-RW (compact disk readable and writable); DVD (digital video disk) and flash memory.

Memory or storage devices such as magnetic disks are **formatted** to create **sectors and tracks** onto which data is saved. File storage is done using **file management** software which is achieved more easily when files have **file extensions** indicating their **file format**. Specialist file management is done in databases using **DBMS**.

It is advisable that files are stored to make retrieval easier. However, files may then become easy to access making it possible for **privacy invasion** and breaches of security leading to **data stealing** or **corruption**.

Processing

Data is processed in the **CPU** of the computer, where it is executed in the **ALU** directed by the **CU** and stored temporarily in the **registers**. The processing speed of a computer is determined by the **control clock** within the CPU. The cycle in which one process is carried out is called the **machine cycle** or the **fetch-execute cycle**. The **word size** of a computer is the number of bits that can travel along a bus (pathway) at once. The larger the word size the faster the data processing. The processing speed of a computer affects the **response time**.

The equivalent non-computerised tools for processing data are described in a variety of documentation such as user manuals, data flow diagrams and system flow charts. Social issues related to processing include reliability of software and hardware and response time.

Transmitting and receiving (communication)

Data can be transferred along **serial** or **parallel** pathways. Data can include start and stop bits (**asynchronous** data) or involve the synchronisation of the sending and receiving computers (**synchronous** data). **Full duplex** data can travel along transmission pathways in two directions at the same time; **half duplex** data can travel along transmission pathways in two directions but not at the same time; **simplex** data can only travel along transmission pathways in one direction. The speed of data is measured by the **baud rate** and **bits per second**. Hardware devices are connected into serial or parallel **ports** to enable transmission. Internal or external **modems** modulate and demodulate data to and from (digital) computers and (analog) telephone lines. Communication between computers occurs when they are linked together to form **networks**. Networks can be LANs or WANs. A variety of communication software is used for accessing the Internet, data compression, email and data encryption. **Non-computerised** examples of communication tools include: 'snail' mail; telephones; fax machines; radio and TV.

Social and ethical issues related to data communication include data accuracy; security; netiquette; authorship; privacy; changing nature of work and e-commerce.

Displaying (output)

Hardware used for output includes **displaying** (VDU; monitor; LCD; pixel; resolution; refreshing; raster scan); **printing** (impact/non impact; dot matrix/inkjet/laser; plotters; flat bed; drum; electrostatic) and **sound** output (speakers; voice synthesis). It is useful to have software that provides for the required output such as reporting, formatting, mail merge, tables and charts.

Non-computer output tools include storyboards, magazines, newspapers, reports, videos, radios, billboards, T-shirts and so on.

Social and ethical output issues may include the communication skills of those presenting displays and can relate to past, present and emerging trends in displays. It is important to have appropriate displays for a wide range of audiences including standards for display for the visually impaired and displays suitable for young children.

YOUR CHECKLIST

After studying this chapter you should have acquired the following skills:

✓

- For a given scenario, identify alternative devices for data collection and choose the most appropriate one. ☐
- Describe the operation of a range of hardware collection devices. ☐
- Make predictions about new and emerging trends in data collection based on past practices. ☐
- Recognise personal bias and explain its impact on data collection. ☐
- Identify the privacy implications of particular situations and propose strategies to ensure they are respected. ☐
- Predict errors that might flow from data that is inaccurately collected. ☐
- Predict collection issues that might arise when data is subsequently analysed and processed. ☐
- Describe how different types of data are digitised by collection hardware. ☐
- Use data dictionaries to describe the organisation of data within a given system. ☐
- Identify hardware requirements to carry out particular types of analysis. ☐
- Describe the best organisation of data for a particular type of analysis. ☐
- Compare and contrast computer and non-computer tools for analysis on the basis of speed, cost and volume of data that can be analysed. ☐
- Describe the storage and retrieval processes in an information system. ☐
- Compare different file formats for storing the same data, explaining the features and benefits of each. ☐
- Estimate the storage capacity required to store a file. ☐
- Choose between centralised and distributed processing for a given scenario. ☐
- Identify and describe situations in which parallel processing would be an advantage over centralised or distributed processing. ☐
- Describe the operation of the central processing unit. ☐
- Diagrammatically represent data processing. ☐
- Compare and contrast the use of networked and distributed processing. ☐
- Distinguish between dumb and intelligent terminals. ☐
- Identify examples of human bias in data processing. ☐
- Compare the requirements of a LAN and a WAN. ☐
- Compare and contrast computer and non-computer based communication systems. ☐
- Predict and discuss possible future trends in communication and the impact they are likely to have on the transmitting and receiving of data/information. ☐
- Justify the choice of display of information for a given scenario. ☐
- Describe the operation of hardware display devices. ☐
- Compare and contrast computer and non-computer methods of displaying information. ☐
- Describe the concept of mail merge. ☐
- Describe the concept and use of a web page. ☐

Review Questions

Multiple Choice Questions

A Page 52

(1 mark each)

1. When a file is saved:
   (A) A copy of the file is sent to secondary storage
   (B A copy of the file is sent to RAM
   (C) The file is removed from RAM and sent to the CPU
   (D) A copy of the file is sent to a hard copy device.
2. Personal details of competition entrants are collected and stored in a database. This database is then sold to a third party and used for an advertising mail out. The social issue concerned is one of:
   (A) Equity
   (B) Privacy
   (C) Changing nature of work
   (D) Ergonomics.
3. Which of the following is **not** an input device?
   (A) Scanner
   (B) Magnetic ink reader
   (C) Touch screen monitor
   (D) Flat bed plotter.
4. The Nintendo 64® computer game is so called because it has a 64 bit word length. This means that:
   (A) 64 bytes of data can travel along a single bus
   (B) 64 bits of data can travel along a single bus
   (C) 64 words can fit in one memory register
   (D) The computer uses extended ASCII code.
5. The type of display that uses a bit map to store an image in memory is called:
   (A) Vector
   (B) LCD
   (C) Raster
   (D) Soft copy.
6. Data that is non-volatile:
   (A) Will be lost if the computer is accidentally switched off
   (B) Is likely to be found in secondary storage
   (C) Is part of the RAM
   (D) Is probably compressed in lossy form.
7. Microfiche is:
   (A) A non-computer equivalent of a spreadsheet
   (B) Used for searching a database
   (C) A non-computer way of storing large quantities of data
   (D) Part of the microprocessor in the ROM BIOS.
8. Simplex data transmission is where data travels:
   (A) In two directions but only in one direction at a time
   (B) In one direction only from transmitter to receiver
   (C) In two directions simultaneously if necessary
   (D) As a form of analog transmission.
9. Which of the following output devices uses light and toner as part of its function?
   (A) Inkjet printer
   (B) Laser printer
   (C) Laser plotter
   (D) LCD screen.
10. The simultaneous processing of data within multiple processing units is an example of:
   (A) Centralised processing
   (B) Synchronised processing
   (C) Parallel processing
   (D) Distributed processing.

Short Answer Questions

A Page 52

(24 marks)

1. What is one difference and one similarity between synchronous and asynchronous data transmission? (2 marks)
2. What size frame buffer would a computer need to store an image with 16 colours on a screen with a resolution of 640 × 480 (show **all** working). (4 marks)

   Hint 1. To display 4 colours ($4 = 2^2$) you need 2 bits of memory for each pixel and for 8 colours ($8 = 2^3$) you need 3 bits of memory for each pixel.

   Hint 2. A screen with a resolution of 30 pixels across and 60 rows down ($30 \times 60 = 1800$) has a total of 1800 pixels.
3. A busy office is acquiring some new computers. The needs of the office include:
   - High quality black and white printing
   - Storage of large amounts of graphical data
   - Efficient searching and sorting of large amounts of data
   - A portable memory chip for storing up to 250 Mb of data.

   Name four items that should be included in the hardware or software specifications. (2 marks)
4. Compare and contrast the organising and analysing of data. (2 marks)
5. Describe the function of the ALU and the control clock during the machine cycle. (2 marks)
6. Describe how the following types of data are digitised by the hardware that collects them:
   - Alphanumeric and numeric text
   - Still images
   - Moving images
   - Sound. (4 marks)
7. Differentiate between lossy and lossless data compression. (2 marks)
8. Explain how a data dictionary organises data. (2 marks)
9. Explain what is meant by netiquette and why it is necessary. (2 marks)
10. Describe the function of the registers in the fetch-execute cycle. (2 marks)

Longer Response Questions

A Page 55

(21 marks)

Consider the social issues for each of the following aspects of the information process. Give an example for each of the following:

1. Social and ethical issues in collecting (3 marks)
2. Social and ethical issues associated with organising (3 marks)
3. Social and ethical issues associated with analysis (3 marks)
4. Social and ethical issues associated with storing (3 marks)
5. Social and ethical issues associated with processing (3 marks)
6. Social and ethical issues associated with transmitting and receiving (3 marks)
7. Social and ethical issues associated with displays. (3 marks)

Answers to Review Questions

Multiple Choice

Page 50

1. (A) When a file is saved, a copy of it is sent to secondary storage.
RAM is temporary working memory which is lost when the computer is switched off.
A hard copy device is created by a printer or plotter that produces images on paper.
Data is sent to the CPU for processing.

2. (B) Personal details of competition entrants are collected and stored in a database. This database is then sold to a third party and used for an advertising mail-out. The social issue concerned is one of privacy because the personal details were collected from people and then used for other purposes without their knowledge.

3. (D) A flat bed plotter is **not** an input device.
A scanner, magnetic ink reader and touch screen monitor are all input devices.

4. (B) The Nintendo 64' computer game is so called because it has a 64 bit word length. This means that 64 bits of data can travel along a single bus.

5. (C) A raster display uses a bit map to store an image in memory.
A vector display is used for drawings and is stored as end point coordinates in the frame buffer.

6. (B) Data that is non-volatile is found in secondary storage.
Volatile memory (RAM) is lost if the computer is accidentally switched off before it is saved to secondary storage.
Compression has nothing to do with data being volatile.

7. (C) Microfiche is a non-computerised way of storing large quantities of data where transparent sheets of plastic store about 200 sheets of printed text.
These sheets must be magnified using a microfiche reader in order to read the data.

8. (B) Simplex data transmission is where data travels in one direction only from transmitter to receiver as in TV transmission and reception.
Half duplex data travels in two directions but only in one direction at a time.
Full duplex data travels in two directions simultaneously if necessary.

9. (B) The laser printer uses a laser beam to 'etch' an electrostatic image onto a drum. The black powdery toner is attracted to the electrostatically charged image on the drum and the image is then transferred to paper.

10. (C) The simultaneous processing of data within multiple processing units is an example of parallel processing.
Centralised processing is controlled by a central computer such as a fileserver within a LAN.
Distributed processing is done at workstations connected to shared resources such as in a token ring network with or without a fileserver.

Short Answers

Page 50

1. The similarity between asynchronous and synchronous transmission is that they are both forms of serial transmission.
The difference between them is that asynchronous transmission involves the use of a start and stop bit and data is retransmitted if an error occurs.
In synchronous transmission the sending and receiving computers synchronise their time clocks and exchange information about transmission protocols before transmission.

2. Calculating the frame buffer memory for the screen display is done in the following steps:
   (i) Calculate the bit plane for the number of colours to be stored.
   $16 = 2^4$ The power of two is 4 and this index is called the bit plane.
   The bit plane is the number of bits required to store each pixel to allow for 16 colours
   (ii) Calculate the number of pixels as determined by the resolution:
   A resolution of 640 × 480 means 480 rows of 640 pixels in each row = 307200 pixels.
   (iii) Calculate the total number of bits required:
   307200 × 4 = 1228800 bits
   (iv) Change the bits to bytes:
   1228800/8 = 153600 bytes
   (v) Change the bytes to kilobytes:
   153600/1024 = 150 kilobytes
   (vi) Change the kilobytes to megabytes (if necessary)
   150/1024 = 0.15 megabytes.

3. To satisfy the needs of the office personnel suggest:
   - A laser printer to produce fast high quality black and white printing
   - A large hard disk, eg. 20 Gb+ to store large amounts of graphical data (graphical data uses a lot of memory)
   - A database application will provide efficient searching and sorting of large amounts of data
   - Flash memory cards will provide a portable memory chip for storing up to 250 Mb of data.

4. One similarity is that both organising and analysing of data are done after the data has been collected and before the data is processed.
One difference is that organising is the activity that arranges, represents and formats collected data for later use. For example, choosing to represent letters of the alphabet as images rather than text will affect the software used to process the data. Organising includes the way in which collection devices create input. For example, a digital camera inputs different data than a keyboard.
Analysing data involves interpreting and understanding the meaning of the data in the context of the project. For example, displaying the data in different forms such as charts or data dictionaries helps give meaning to the purpose of the data. This is not necessarily the same as processing the data to provide the outcome. Although data does become information, it is only for the purposes of data interpretation.
Analysing requires a large amount of memory and fast processing whereas in organising, all the activity is done by the collecting device.

5. Arithmetic Logic Unit (ALU) is located in the CPU and is the site of all calculations and data processing. Calculations include the use of logical (AND, OR, NOT) and relational (=, +, −, *, <, >, <>) operators to manipulate the data.
The result of the calculation is sent to the accumulator register.
The address from where the data is fetched in primary storage is held in the address register and once it is fetched the data itself is stored in the storage register until it is sent to the ALU.
The control unit directs the movement of data (before processing) and information (after processing) within the CPU. That is, back and forward to primary storage, back and forward to the registers and ALU.
It contains the control clock which regulates the speed of all processing.

6.

Type of data	Process of digitising by collecting hardware
Alphanumeric and numeric text	**Keyboard** – using the ASCII (American Standard Code for Information Interchange) code the computer converts the keyboard input to ASCII (binary code) which is understood by the computer. **Scanner** – uses beams of light which are reflected off the items being scanned to translate images of text, drawings, photos, and other graphics into digital form. The reflected light is converted into a bit mapped image which is saved in the computer's memory. **OCR** (optical character readers) are specialised scanners that read pre-printed characters. The text is scanned and the computer matches it with a set of individual ASCII code characters. **MICR** (magnetic ink character readers) are specialised scanners that read characters that have been printed using magnetic ink in a particular font.
Still images	The **digital camera** uses a light sensitive processing chip to capture photographic images in digital form either on a memory chip or a floppy disk. The **graphics tablet** or digitising tablet consists of a specialised electronic pad and a stylus. The fine grid of sensors on the electronic pad detect the position of the stylus and send this information in binary form to the computer. Some graphics tablets are pressure sensitive allowing for thicker lines to be drawn with greater pressure on the stylus.

Moving images	**Non-digital video cameras** capture analog images on video tape which are then transformed into digital images using digitising cards such as frame grabber cards or full motion video cards. **Digital video cameras** capture samples of images approximately 25 times per second using a CCD (charge coupled device) which is a photosensitive silicon grid. The image is bit mapped onto the grid and usually compressed then stored. An example of compressed moving image storage is MPEG format (motion picture experts group).
Sound	**Microphones** capture sound, including voices, in analog form. The computer's sound card converts the analog sound into digital form. Microphones capture sound in WAVE format which involves sampling parts of the sound waves. The larger the sampling rate and/or the more samples the better the sound reproduction but also the more memory required. **MIDI** (musical instrument digital interface) devices allow a musician to play an instrument whose notes are directly converted into digital form using the sound card and then saved onto the computer.

7. **Lossy** compression involves the discarding of every third or fourth pixel or shades of colour that would be not be missed. The image loses resolution but if the discarding is done well, the loss will rarely be noticeable. However, the higher the compression the more noticeable the loss. Lossy compression can shrink an image down to as little as 5% of its original size. JPEG uses lossy compression.

   Lossless compression uses mathematical techniques to replace repeated patterns of pixels with a 'coded' summary. During decompression these coded summaries are replaced with the original pixels. Thus no change in the original resolution occurs. GIF uses lossless compression.

8. A data dictionary is a structured listing of all files and fields in a database. Its purpose is to describe the characteristics of all the data in the database. It looks like the list view of a database with 'field headings': field name; field width; data type; and a description or purpose of the data in each field. The data dictionary can be used to monitor the data being entered into a specific field making sure it conforms to the rules set out in the dictionary. For example, in the table below only numbers can be entered into the age and height fields. The data dictionary may also be used to protect the security of the database by indicating who has the right to gain access to it.

DATA DICTIONARY FOR A DATABASE WITH PERSONAL DETAILS

Field name	Data type	Field size	Description
Family name	Text	25	Person's family name
Given name	Text	20	Person's first name
Age	Numeric*	2	Person's age
Height	Numeric*	3	Person's height in cm

9. Netiquette is an undocumented way of behaving when communicating on the Internet. Netiquette is important so that the Internet does not become jammed with unsolicited files and so Internet users are not constantly being offended by rude and inappropriate language. Examples of poor netiquette include: sending unsolicited junk mail (spamming); using capital letters throughout a document (shouting); using derogatory, offensive or obscene language (flaming).

10. The registers provide enough temporary memory for one process. The address register holds the addresses of the data to be fetched; the instruction register holds the instruction to be executed; the storage register holds the data after it has been fetched and the accumulator holds the result of the execution.

Longer Answers

Page 51

1. **Social and ethical issues in collecting:**
   - Bias in the choice of what data to collect and where it is collected
   - Inaccuracy of the collected data as a result of inaccurate typing
   - Breach of copyright and lack of acknowledgment of source data when collecting (plagiarism)
   - The rights to privacy of individuals on whom data is collected was breached when private phone numbers were published
   - Ergonomic issues for participants entering large volumes of data into an information system when people sue their employer because they develop RSI.
2. **Social and ethical issues associated with organising:**
   - Current trends in organising data, such as:
     - The increase in hypermedia as a result of the world wide web
     - The ability of software to access different types of data
     - A greater variety of ways to organise resulting from advances in display technology.
   - The cost of poorly organised data, such as redundant data in a database used for mailouts
   - The appropriateness of a two digit date field at a time when storage and processing was more expensive, versus the current inappropriateness.
3. **Social and ethical issues associated with analysis:**
   - Unauthorised analysis of data
   - Erosion of privacy from linking databases for analysis.
4. **Social and ethical issues associated with storing:**
   - The security of stored data
   - Unauthorised retrieval of data
   - Advances in storage and retrieval technologies and new uses such as data matching.
5. **Social and ethical issues associated with processing:**
   - Types of computers on networks:
     - Flexibility from the distributed processing of personal computers on networks
     - Security from the centralised processing of network computers (terminals).
   - Ownership of processed data
   - Bias in the way participants in the system process data.
6. **Social and ethical issues associated with transmitting and receiving:**
   - Accuracy of data received from the Internet
   - Security of data being transferred
   - Netiquette
   - Acknowledgment of data source
   - Global network issues, time zones, date fields, exchange rates
   - Changing nature of work for participants, such as working from home and telecommuting
   - Current developments and future trends in digital communications, radio and television
   - The impact of the Internet on traditional business.
7. **Social and ethical issues associated with displays:**
   - Communication skills of those presenting displays
   - Past, present and emerging trends in displays
   - Appropriate displays for a wide range of audiences, including:
     - Standards for display for the visually impaired
     - Displays suitable for young children.

3 Planning, Design and Implementation

Outcomes

By studying this chapter and completing the exercises students should be able to:

- Describe the nature of information processes and information technology
- Classify the functions and operations of information processes and information technology
- Identify the information processes within information systems
- Recognise the interdependence between each of the information processes
- Identify social and ethical issues
- Describe the historical developments of information systems and relate these to current and emerging technologies
- Select and ethically use computer based and non-computer based resources and tools to process information
- Analyse and describe an identified need
- Generate ideas, consider alternatives and develop solutions for a defined need.

Source: Information Processes and Technology Stage 6 Syllabus © *NSW Board of Studies, 1999.*

KEY TERMS AND CONCEPTS

Understanding the problem

Problem	Existing system	System requirements	User needs	Analysing
Gantt chart	Data flow diagrams	System flow charts	Funding	Communication management plan (project plan)
Interview	Survey	Requirement report	Task scheduling	Journal and diary

Making decisions about solutions

Budget feasibility	Technical feasibility	Schedule feasibility	Operational feasibility	Requirements/feasibility/analysis report
Constraints	Analysis	Design tools	Organisational charts	Gantt chart
Data flow diagrams (DFD)	System flow chart	Design specification	New techniques	System development cycle

Designing solutions

Context diagrams	Diagrammatic view of solutions	Gantt chart	Data flow diagrams (DFD)	System flow charts
Technical specifications	Documentation	Test data (beta test)	Evaluation	Future maintenance
Top-down design	System design	Software package	Custom software	Design report
User interface	Design tools	Prototypes	Programmers	Participants concerns/needs

Implementing solutions

Implementation	Conversion	Parallel conversion	Direct conversion	Pilot conversion
Phased conversion	Training	Testing	Training specialists	Computer operators

Testing, evaluating and maintaining solutions

Testing the solution	Evaluating the solution	Maintaining the solution	Performance measures	Comparison with original stated requirements

Social and ethical issues related to solutions

Crime	Copyright	Equity	Power and control	OH&S (Occupational Health and Safety)
Ergonomics	Repetitive strain injury (RSI)	Carpel tunnel syndrome (CTS)	Tenosynovitis	Deskilling
Machine-centred systems	Human-centred systems	Changing nature of work	Changing relationships between participants	Safe working environment
Job routine	Multi-skilling	Telecommuting	Security	Privacy

The System Development Cycle/Planning Design and Implementation

The planning, design and implementation of a new information system usually involves collaboration between a systems analyst and the management and users of a company. Each of the following steps in this process involves a set of typical activities and the analyst works methodically through them. This process is known as the **system development cycle** and includes:

- Understanding the problem
- Making decisions about solutions
- Designing solutions
- Implementing solutions
- Testing, evaluating and maintaining solutions.

3.1 UNDERSTANDING THE PROBLEM

Understanding the Problem

Understanding the 'problem' is often easier to comprehend if it is rephrased 'understanding the requirements' for a new system/program/technological change of any kind. The 'problem' is that an individual or company is unable to achieve the outcomes they want to achieve with the present set of hardware, software, personnel, data and input and output mechanisms.

Some reasons for wanting to change an information (computer) system may include:

- Release of new software requiring updated hardware
- Release of new hardware techniques
- The need to remain competitive in a changing market environment
- New management
- Expansion of company.

In establishing the requirements of the new system the analyst or project leader will consider the following:

- Who are the participants? Who are the people involved with the information processing by interacting with the information system at any level, from data entry to system management?
- What data (input) and information (output) is involved in the system?
- What hardware and software is required in the system?
- What processes are completed on the data to transform it into information within the system?

Approaches to Identifying Problems with Existing Systems

Before a new solution is designed it is important that the existing system (if any) is thoroughly examined or analysed. The analyst analyses the existing system to find out **how it works**, **what it does** and **who uses it**. He or she will then be able to establish why the existing system does not suit the needs of the company at the present time. It may be that a quick fix to the existing system is possible or alternatively a whole new system may be required. The **preliminary investigation** considers the needs and concerns of all participants. Information about the existing system is collected from participants using surveys, questionnaires, observation and so on.

The table on the next page shows a variety of ways that information about the existing system can be collected and the advantages and disadvantages of each method.

Data collection technique	**Features**	**Advantages**	**Disadvantages**
Interview For example, an analyst interviews company management to find out the requirements of a new computer system.	• Specific questions about a person's feelings, opinions, ideas and knowledge. • Questions can be open ended allowing for a free response or closed for a specific type of response. • Usually conducted face-to-face, or on the telephone.	• Directed questions can vary with interviewee. • Allows for further questions following certain types of answers. • Data is immediate. • Body language or voice intonation data can be gained.	• Time consuming as is one-on-one. • Personality differences may interfere with perceived answers. • Training of interviewers needed. • Questions may vary. • Possible smaller sample size.
Questionnaires/surveys For example, users complete written questions about their computer use habits and opinions about the system they use.	• Specific questions about a person's feelings, opinions, ideas and knowledge. • Mostly closed questions with minimal free response questions, eg. T or F, multiple choice.	• Allows for large sample size. • Less expensive to deliver. • Standarised questions with set selection of answers for easy analysis. • Anonymity is more likely. • Data may or may not be immediate.	• Mailed responses have low response rates. • Wording of question set may be restrictive, resulting in non-comprehensive answers. • Time consuming to complete.
Observation For example, computer users are observed to determine their reactions to new software.	• Gathering data about people's behaviours, reactions, occurrences of specific activities.	• Data is immediate. • Body language or voice intonation data can be gained. • Provides a record of actual happenings.	• Time consuming. • Due to lack of structure, limited data is collected.

An important consideration for data collection is to ensure that it is **reliable** (accurate), free from **bias** (data inclusion, affected by the collector's prejudice) and **valid** (truly representative of the population being surveyed). Collected data is usually analysed for common trends and other pointers that provide the analyst with accurate insights into the true needs of the company in relation to a new information system.

Diagrammatically Representing the Existing System

An important part of evaluating the requirements of a company for a new system is the need to examine the existing system. This gives the analyst insights into what the present system can and cannot do and what needs to be changed in order for the desired outcomes to be achieved. The analyst uses a number of graphical tools to demonstrate how the existing system operates. These include the:

- Context diagram
- Data flow diagram (DFD)
- System flow chart.

The context diagram is a small 'version' of the data flow diagram showing a single process, with its input and output. As the name implies the diagram is designed to put each process into 'context' as it contributes to the whole system.

Three of the DFD symbols are used in a context diagram:

- The square (external entity) for input and output
- The circle for process
- The arrow for direction of data flow.

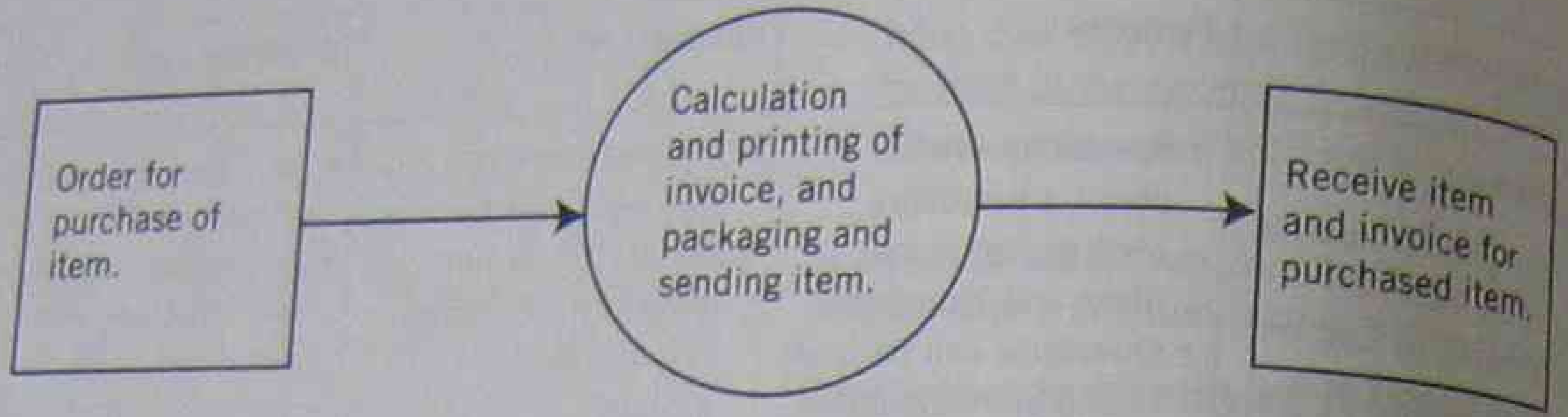


Figure 3.1 Context diagram

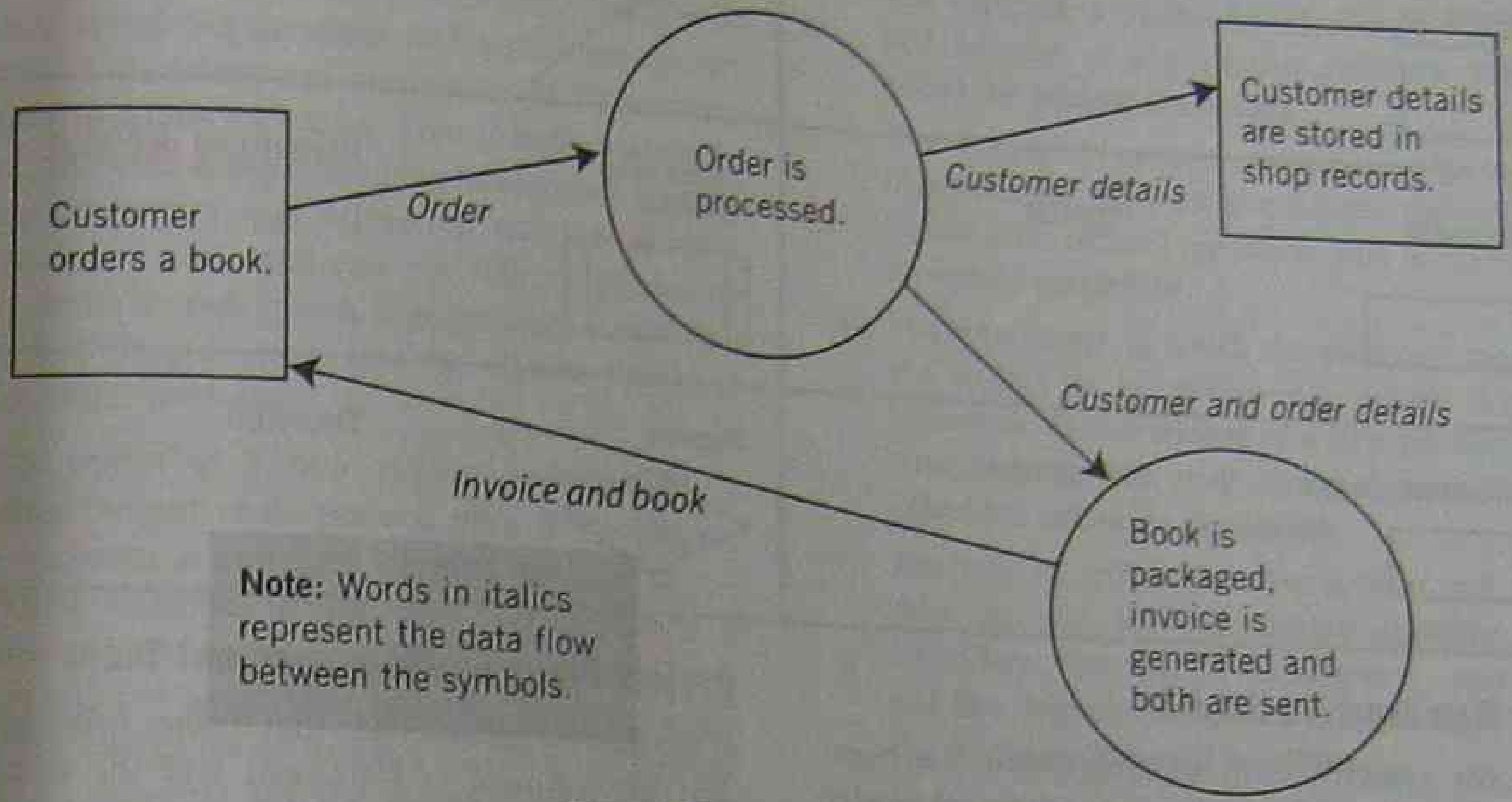
Figure 3.1 above illustrates a context diagram for purchasing a book, including the customer placing the order, the book shop printing the invoice and packaging the book and finally, the customer receiving the book and invoice.

The **data flow diagram** (see table below) is a diagrammatic method of representing a system by showing the logical flow of data through the system by including a series of processes, inputs and outputs as well as storage.

The symbols used in a data flow diagram are:

- A square (external entity) for input and output
- A circle for process
- An arrow for direction of data flow
- A rectangle opened at the right-hand side for data storage.

Symbol	**Function**
Square 	The square or external entity represents the origin or destination of data/information. This can be a person or organisation that sends data or the destination of processed information.
Circle 	A circle represents the processes or actions that change data into information. This can include searching a database, completing a calculation or printing out the results of a calculation.
Arrow 	The arrow shows the direction of data flow between processes, external entities and data storage.
Open ended rectangle 	An open rectangle shows where the data is stored. This could be a magnetic disk, storage tape, CD or filing cabinet.

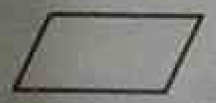


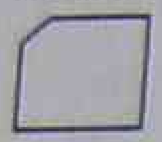
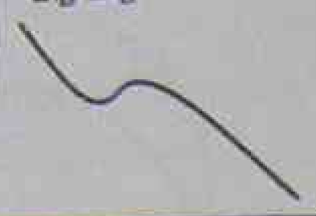
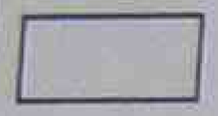
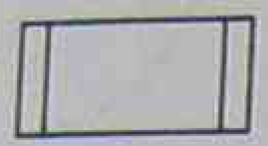
Note: Words in italics represent the data flow between the symbols.

Figure 3.2 A data flow diagram

The **system flow chart** is a graphical method that represents both the flow of data and the logic of the system, including the hardware and software and manual operations involved.

SYSTEM FLOW CHART SYMBOLS AND THEIR FUNCTIONS

Symbol	**Function**	**Symbol**	**Function**
Parralelogram 	Input/ouput	*Upside down trapezium* 	Manual operation
Rectangle with one curved side 	Paper document	*Whistle shape* 	Magnetic tape
Oval with triangular end 	On-line display	*Cylinder* 	Direct access
Rectangle with one sloping side 	On-line input	*Arrows with and without a head* 	Flowlines

Rectangle with one corner cut 	Punched card	*Zig Zag* 	Telecommunications link
Rectangle 	Process	*Double lined rectangle* 	Predefined decision
Rounded rectangle 	Terminator	*Diamond* 	Decision

User Requirement Report

Once the analysis of the existing system has been completed, the analyst writes a **requirement report** which summarises the needs of the company based on the findings from the surveys and so on. The requirement report typically includes:

- A statement of the user requirements of the new system
- An explanation of how the new system will help the organisation achieve stated aims and objectives
- A brief overview of the proposed new system in terms of input, output, processing hardware and software.

The requirement report is used to develop feasible solutions to the company's problem.

One of the most important parts of the requirement report is the **recommendation** by the analyst whether to leave the system as is, improve the system or develop a new system.

Project Plans and Management Tools

If the analyst recommends that further investigation and development is warranted and the company accepts the recommendation the analyst will develop a **project plan**.

The project plan is a method of organising 'who, what, when, where and how' a solution will be developed. Several standard tools are used to represent important information during this stage of the cycle:

- Gantt charts
- Journals and diaries
- Budget plan
- Communication management plan.

The **Gantt chart** is diagram showing the **time frame** for the **scheduling of tasks** during the systems development cycle (project). It depicts the list of tasks, proposed timing of tasks and the proposed sequence of tasks. The analyst or project leader can use a Gantt chart to schedule other tasks within the cycle.

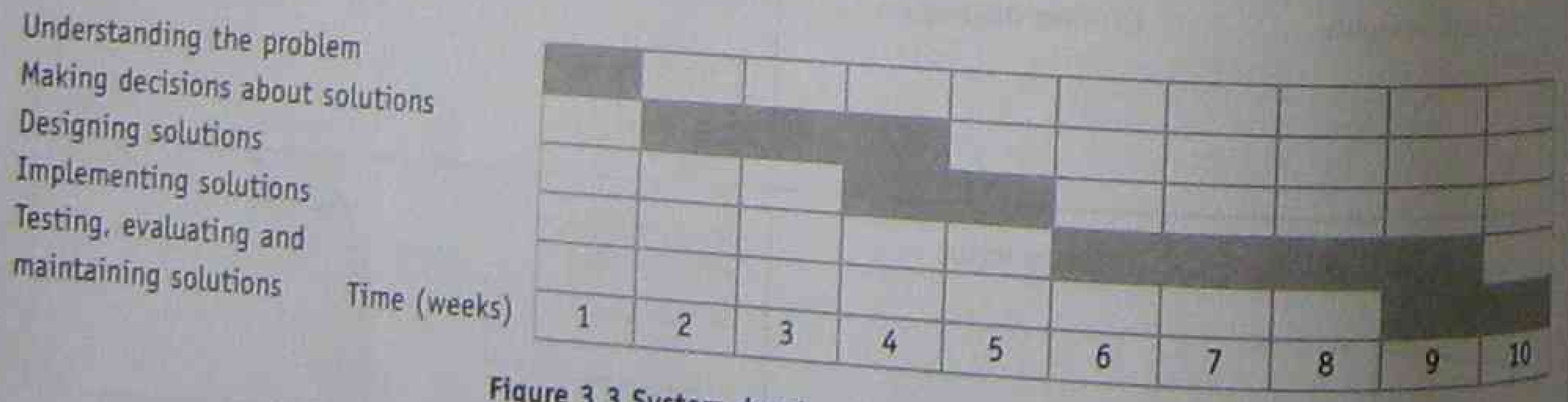


Figure 3.3 System development cycle task

Journal and diary entries are an essential part of the formal documentation of a project. These will include scheduled meetings and significant events as well as decisions and summaries of discussions leading to those decisions. This documentation is best completed using management software such as electronic diaries and calendars, word processors spreadsheets and databases for journal entries as they can be stored, edited, manipulated, sorted and searched. Progress can also be shown by using presentation software.

At the outset of a new system development, a **funding/budget management plan** is developed. This includes a cost and benefits analysis of the proposed changes in the new system. This is important because it needs to be determined if costs will or will not be offset by benefits. There are many different categories of benefits that may be derived such as speeding up the system and improving efficiency resulting from reduction of redundancy or errors in input. Probably the most desired benefit is improved cash flow from increased production, however, benefits can also include greater efficiency, ease of work load for staff and other social and environmental gains. The analyst selects software that will make it easy to give an overview of budget considerations. The costs of a system involve more than just dollars. They may include time, environment, social changes and so on.

Throughout the development of a new system it is important that communication between key personnel is maintained. This requires a formal plan of scheduled meetings and planned activities such as presentations and discussions at key points in the project. A **communication management plan** is established to ensure that adequate communication is maintained throughout the project.

3.2 MAKING DECISIONS ABOUT SOLUTIONS

Feasibility

A feasibility study involves evaluation of a variety of possible solutions based on their ability to meet the stated required needs of the company. There are four main constraints or important factors that influence the final selection of a solution. These are:

- The company's budget in relation to the costs and benefits of a particular solution. This is called the **budget feasibility**.
- The availability or even existence of particular hardware and software to complete the required information technology tasks. This is called the **technical feasibility**.
- The time frame in which the company requires the system to be completed, which may be influenced by the release of a new product by a competitor, or the new financial year. This is called the **schedule feasibility**.
- The way in which the new system will fit in with the way the organisation operates. This considers the staff, customers, company policies, mission statement and so on. This is called the **operational feasibility**.

Analysis

The project leader or analyst guides the company management through the selection process using the above 'constraints' as selection criteria. A solution is chosen and then further analysis is completed to examine how the solution will operate within the company. **Gantt charts** are used to show the proposed lists of tasks, time frames and sequences of activities within the proposed solution. **Organisational charts** can be used to show a top-down structure of the proposed system's personnel.

The analyst/project leader writes a report at the conclusion of the feasibility study detailing the characteristics of the favoured option with explanations as to the reasons it was chosen over other options including discussion of the advantages and disadvantages of rejected alternatives. The report usually contains the following:

- Project title page
- List of contents
- Problem definition/requirements definition
- Requirement report
- Summary of investigation, feasibilities/constraints
- Alternative solutions
- Recommendations (including design specifications for the next stage)
- Project plan (schedule for new system development)
- Appendix (supplementary materials, references, glossary of terms).

3.3 DESIGNING SOLUTIONS

Top-Down Design

Once the analysis of the existing system and the proposed new system is completed, the analyst or project leader designs the new system by identifying large modules of the problem to be solved and then breaking each of these down into smaller parts. This technique is called a **top-down approach**.

Hardware, Software, Input/Output, Processes, Personnel

The new solution will probably require new hardware or software and this needs to be specified at this stage. As the technical feasibility of this hardware and software was determined previously it will be incorporated into the design. The hardware obviously needs to be available as determined in the technical feasibility study and at this stage the actual **technical specifications** need to be documented. Once these specifications are in order, specific quotations can be obtained from various suppliers of hardware.

There are a few different aspects of the software that need to be decided or designed. Firstly, is it satisfactory to purchase software **'off the shelf'** using packaged software such as Microsoft Office Professional® or Claris Works® or will it be necessary to have **custom software** written by a programmer to meet the exact specifications? Secondly, the way in which the software will be used to produce the user interface needs to be considered. The **design of the interface** affects the user friendliness and the potential benefit to the users. Within the design stage not only do the **benefits to the users** need to be clarified and clearly documented but also consideration must be given to the 'human' needs, including the constraints due to organisational policy and goals.

The following screen design principles need to be considered in developing the user interface:

- Consistency in design so that the user becomes familiar and comfortable with the screen layout
- Use of appropriate messages for unambiguous and positive communication between the user and the computer both as instructions and error messages
- Legibility of all screen elements including suitable fonts and colour
- Justification, alignment and the use of borders where applicable
- Spacing and layout of all screen elements.

The screen design will also impact on the input and output of data and information.

The type and format of data that is to be input and the type and format of the information that will be output is confirmed. This will impact on the type of peripheral devices that need to be purchased. For example, if a receipt docket is required, a dot matrix receipt printer may need to be purchased if carbon copy receipt papers are used. If colour graphics will be scanned and high quality output is required, a high resolution colour scanner and printer will need to be purchased.

The type of processing that is to be done on the data will determine the software to be purchased. The skills, experience and adaptability of the personnel should be considered as part of the design process.

During the design stage a **prototype** of the new design is often developed. A prototype is a working model of the new system, and may be used for training, trialling to see if the new system will work and/or for demonstration purposes. As a working model which can be used for training purposes, it usually incorporates relevant snapshots of the hardware and software, screen design, input, output and processing.

Design Tools

The design of the hardware, software, input/output processes and personnel can be shown diagrammatically using the design tools that were used during the investigation stage. In the design stage, however, they refer to the new system rather than the existing system. These **design tools** include the following:

- Context diagram
- Data flow diagram
- System flow chart.

Documentation

It is most important for the analyst to document all design decisions in line with the previous reports written in the investigation and feasibility/analysis stages. During the design stage the new system should be tested thoroughly before implementation. **Test data** is carefully selected to demonstrate that the system operates smoothly as planned and/or to identify any potential problems. Before the system is completely released **beta tests** are done by selected participants. If errors are discovered in software, program patches (small 'fix up' programs) can be produced.

To ensure that the system is easy to maintain in the future, it is important that the analyst completes thorough documentation, including user documentation for on-line help with software as well as technical specifications for programmers and computer operators.

3.4 IMPLEMENTING SOLUTIONS

The implementation of the new system includes the installation of hardware and either packaged or custom programmed software, and the final 'in-situ' testing phase.

Methods of Implementation

The implementation or **conversion** from the old system to the new system can occur in one of four ways. It is important for the analyst to clarify the implications of each conversion for the ease of the user.

Conversion choices are:

- Parallel
- Direct
- Phased
- Pilot.

The following table shows a description of each conversion with their advantages and disadvantages.

Conversion method	Features	Advantages	Disadvantages
Parallel	The old system is left functioning while the new system is installed. Both systems operate simultaneously for a short time until the new system is confirmed to be working satisfactorily.	• Staff can learn the new system while still operating the old system. • Less likely to lose data. • Staff less likely to be stressed because they can refer to the old system while taking time to learn the new one. • Any problems with the new system can be solved before the old system is removed. • Staff can re-enter data if required.	• Expensive. • Time consuming. • Takes up double the space. • Likelihood of data redundancy or loss of data integrity. • Staff need to operate on two different systems. • Staff may be overworked and/or confused by a completely new system.
Direct	The old system is completely removed and the new system begins operation immediately.	• Least expensive unless a major error occurs. • Least time consuming. • Staff need to deal with one system only.	• Staff have little time to learn the new system. • More likely to lose data between systems as staff have limited time to re-enter data if required. • Staff more likely to be stressed. • Any problems with new system cannot be solved before the old system is removed.

Phased	The new system is installed and begins operation in one section of the company at a time until the entire system is installed.	• Staff can learn the new system while still operating the old system. • Staff less likely to be stressed. • Any problems with new system can be solved before the old system is removed. • Staff have time to re-enter data if required. • Staff have more time to plan and be trained.	• Staff may become confused working across two parts of the company. • Indecision may make staff uneasy.
Pilot	The new system is installed in one section of the company only and if it is suitable then the rest of the system will be installed.	• Decisions can be made to better meet staff needs over time. • Any problems with new system can be solved before the old system is removed.	• Indecision may make staff uneasy.

Training

When a new system is installed, especially if drastically different software is involved, it is important that management allow time and money to train staff in using the new system. Adequate and timely training ensures that staff efficiency will not be compromised and that they will not be placed under excessive and unnecessary stress. It is most efficient and least stressful for staff if some training is completed before the new system is installed, with on-going training where needed. An important part of training is establishing exactly what training is required and by whom.

Staff preparation/training for using a new system can include:

- **Hardware vendors** explaining the features and methods of use of their hardware, including on-going technical help desk support
- **Software vendors** (**programmers**) explaining how to use their software, especially its 'built-in' **tutorials**, on-line help and **user manuals**
- Most companies have on-line and/or **help desk** support for specified periods of time
- Software **training specialists** teach staff how to use specific software to meet the company goals as expressed in the first stage.

3.5 TESTING, EVALUATING AND MAINTAINING SOLUTIONS

Once the new system has been successfully installed, it is important that the analyst and the company management **evaluate** not only the **efficiency of the new system**, (in particular if it is in fact more efficient and performs better than the old system) but also if it **meets the needs** that were specified in the initial stages of the development.

The new system is tested using carefully selected test data (usually fictitious) and is often tested in a beta test form by small groups of users (usually with real data). Test data can be chosen to test all possible types of scenarios, called **scenario testing**, or large amounts of miscellaneous data can be tested to see how the system operates under **volume testing**.

At this stage of the development cycle consideration should be made of the effects that the introduction has had on the users, participants and all those in the environment who are affected by the introduction of the new system.

Maintaining the new system includes modifying problematic parts of the system, applying software patches, undertaking virus protection, installing new software where required and keeping the system running smoothly.

3.6 SOCIAL AND ETHICAL ISSUES

One of the most important and often least considered aspects of introducing a new information technology system is the social and ethical impacts on the user and on society.

As society becomes increasingly dependent on technology for its survival it is important to be aware of the impact of a change from a 'human-centred' work environment and society to a 'machine-centred' work environment and society.

Machine-centred vs Human-centred

A **machine-centred system** is designed to simplify what computers have to do, sometimes at the expense of participants. For example, new and complicated software may do fast reliable calculations but be user-unfriendly.

A **human-centred system** makes participant's work as efficient and satisfying as possible. For example, using software that is user-friendly even if it is not as fast and sophisticated as other software.

The table below outlines a comparison between machine-centred and human-centred systems in terms of specific social and ethical considerations or issues.

Environment	Advantages	Disadvantages
Machine-centred	• Efficient processing (**efficiency**) • Less data redundancy • Easy to search and sort data • Easy to access data • Easy to store and edit • Specialised programs, eg. graphics CAD, don't require expert skills • Speed of electronic communication • Persons with **disabilities** have access to information (**equity**) • Flexibility in work environment/ conditions, telecommuting (**changing nature of work**) • Increased access to information for all minority groups (**equity**) • E-commerce and shopping on-line (**competitive and efficient**).	• Easy to invade privacy (**privacy**) • Easy to collect, share and misuse unauthorised data (**ethics**) • Skilled persons can be replaced by unskilled persons (**deskilling**). (**changing nature of work**) • Access to unauthorised data, hackers, criminals (**crime**) • Persons with technology skills can control information (**power and control**) • Non-ergonomic environment can cause health problems (**OH&S issues**) • Ease of unauthorised access and copying data (**breach of copyright**) • Inequitable access to information for the economically disadvantaged or computer illiterate (**equity**) • E-commerce and shopping on-line (**disrupted social relationships**) • Work could become meaningless, repetitive and boring (changing nature of work, deskilling).
Human-centred	• Judgement on special cases or exceptions • Logical judgements reducing redundancy (same person with changed address in a database) • Human contact with clients (**social relationships**) • Considerations about ergonomics (**OH&S issues**) • Retraining and multi-skilling (**changing nature of work**) • Expanding/changing career prospects (**changing nature of work**) • Provision of meaningful/challenging work that matches skills.	• Less efficient processing because more importance placed on software being user-friendly • Less competitive and efficient without e-commerce and shopping on-line (**efficiency and competitive**).

Safety in the Workplace

Safety in the information technology environment has two important aspects:

- Ergonomic hardware and software, furniture and environment
- User work routine.

The following table describes the function of key aspects of an IT workplace and work routine.

Work environment aspect	Ergonomic features	Consequence of poor ergonomic features
Furniture		
Chair	• Adjustable height • Back support • Stable base, preferably with five star base and on wheels.	• Neck and back strain • Muscular aches • Reduced blood flow in legs if feet not touching the floor.
Desk	• Adequate width for computer, keyboard and wrists • Approximately 670 mm in height • Footrest if required.	• Neck and back strain • Muscular aches • Stress from inadequate room on desk.
Hardware		
Screen	• Tilted backwards about 15° • Adjustable angle, brightness and contrast • Anti-glare screen or filter • Positioned so that the eyes are level with the top of the screen.	• Neck strain • Eye strain • Fatigue.
Keyboard	• Positioned so that forearms are parallel to the floor • Slight angle to desk.	• Arm and wrist muscle strain.
Mouse	• Fits the hand • Easy to manipulate.	• Arm and wrist muscle strain.
Software	• User-friendly and ergonomically designed using design principles.	• Fatigue • Stress.
Environment	• Lighting sufficiently bright, uniform and non-glare • Climate temperature about 20° without excessive humidity • Noise should not exceed 55 decibels.	• Eye strain • General discomfort leading to inefficiency.
Work routine	• Varied to prevent RSI • Meaningful without undue pressure (eg. information overload).	• RSI • Tenosynovitis • CTS • Fatigue • Stress.

CHAPTER SUMMARY

There are five stages in the system development cycle and an acronym to help remember them is Ugly Dogs Dance In Tapshoes (UDDIT). The five stages are:

- Understanding the problem
- Making Decisions
- Designing solutions
- Implementing
- Testing and evaluating.

Understanding the problem is another way of expressing the requirements definition. The process of understanding the problem involves **data collection** — using surveys, interviews and observation; report writing; creating a project plan; and obtaining and using information management software and project management software. The Gantt chart is used at this stage to graphically represent the sequence and time frame of the project.

Making decisions involves conducting feasibility studies to examine a series of optional solutions. Each possible solution is considered using the four constraints: economic (budget), technical, scheduling and organisation. The solution that is recommended is examined in more detail using analysis tools such as data flow diagrams, context diagrams, system flow charts and organisational charts. A report is written outlining the findings of the analysis and generating a design specification.

The **designing solutions** stage is concerned with developing the design of the new system using the graphical tools of data flow diagrams, context diagrams, system flow charts and organisational charts. The design stage includes internal and external specifications for the new hardware and software and consideration is given to choices such as the use of **off-the-shelf** software packages versus **custom designed** software. It is important that adequate documentation is completed during the design stage for both users and technical personnel.

During the **implementation** stage the new system is installed at the company site. There are four ways in which a new system can be installed **direct**; **parallel**; **phased** and **pilot conversions**. Immediately after implementation the staff are trained to use the new system.

The new system is thoroughly **tested** to ensure that it is functioning correctly and **evaluated** to ensure that it meets the needs of the management as expressed in the problem definition stage. The system is **maintained** to keep it working correctly

A **social and ethical issue** involved in the planning, implementation and testing stage is, for example, ergonomical furniture. Ergonomic environmental factors such as adequate lighting and minimal noise are also necessary considerations. Management need to be aware of potential **health hazards** including **RSI** (**repetitive strain injury**); **CTS** (**carpal tunnel syndrome**) and **tenosynovitis** as well as **Australian standards AS3590.2** and **WorkSafe Australia**. The work routine is another important factor in maintaining good health for workers.

Adequate rest breaks as well as changes of routine minimise the risk of RSI and so on.

Management need also to consider other factors such as job design and information overload.

Social issues involved in the implementation stage of the development cycle include: skilling and deskilling, privacy, copyright, changing nature of work, crime, equity, occupational health and safety issues, social relationships, efficiency and competition and power and control.

YOUR CHECKLIST

After studying this chapter you should have acquired the following skills: ✓

- Select and apply surveys and interviews with the support of information technology, in order to understand the problem to be solved. ☐
- Diagrammatically represent existing systems using context diagrams, data flow diagrams and systems flow charts. ☐
- Define the requirements for a new systems in terms of: ☐
  - The needs of the users of the information systems
  - Who the participants are
  - The data/information to be used
  - Required information technology
  - Information processes.
- Produce a report stating the need for an information system and how that information system will meet the need. ☐
- Create Gantt charts to show the implementation time frame. ☐
- Investigate/research new information and technologies that could form part of the system. ☐
- Consider and justify the feasibility of a solution based on: ☐
  - Cost
  - Technical feasibility
  - Available time for implementation
  - Its alignment with the current system goals.
- Represent the new system diagrammatically. ☐
- Document the relationship between the new system, users of the information systems and their need(s). ☐
- Analyse and customise user interfaces and other tasks in application software that form part of the solution. ☐
- Compare and contrast conversion methods. ☐
- Justify the selected conversion method for a given situation. ☐
- Implement an appropriate information technology. ☐
- Demonstrate participant training in relation to the new system. ☐
- Identify the training needs of users of the information system. ☐
- Evaluate information processes in relation to adequate performance and design and implement modifications. ☐
- Compare the new system to the requirement report. ☐
- Design human-centred information systems. ☐
- Identify human-centred and machine-centred information systems and describe the impact each has on its participants. ☐
- Develop systems that pay as much attention to the needs of the participants as they do to information technology. ☐

Review Questions

Multiple Choice Questions

A Page 73

(1 mark each)

1. Which of the following contains **only** tasks carried out during the preliminary investigation stage of the systems development cycle?
   (A) Requirements definition, feasibility studies and project plan
   (B) Requirements definition, Gantt chart analysis and interviews
   (C) Surveys, problem definition and project plan
   (D) Design of context diagram, organisational report and selection of software management technique.
2. The purpose of the Gantt chart is to show:
   (A) Time frame, sequence and scheduling of tasks
   (B) Time frame of tasks and schedule feasibility
   (C) Task sequence and operational feasibility
   (D) Documentation for the management plan.
3. The main purpose of the analysis report is to:
   (A) Document the feasibility studies
   (B) Analyse potential new hardware and software
   (C) Clarify the needs of the existing system by examining the new system
   (D) Clarify the needs of the new system by examining the existing system.
4. Technical feasibility is a measure of the:
   (A) Availability of the required hardware and software
   (B) Time constraints
   (C) Cost of available hardware and software
   (D) Expertise of the technical support staff.
5. Which of the following contain **only** tools used during the design stage:
   (A) IPO chart, data flow flow chart and system diagram
   (B) Top-down chart, feasibility report and hardware specification report
   (C) Requirements report, organisational chart and data flow diagram
   (D) Context diagram, data flow diagram and systems flow chart.
6. Which of the following is **not** part of the implementation stage of system development?
   (A) Reviewing budget feasibility
   (B) Training staff
   (C) Choosing direct implementation
   (D) Installing packaged or custom software.
7. The main reason for testing a new information system is to:
   (A) Decide if it is sufficiently user-friendly
   (B) Ensure that it meets the specified requirements and review the effects on the analyst
   (C) Measure the impact on the environment
   (D) Ensure that it meets the specified requirements and review the effects on participants.
8. Which of the following types of system conversion is the least time consuming?
   (A) Direct
   (B) Parallel
   (C) Pilot
   (D) Phased.
9. Which of the following **best** describes machine-centred systems?
   (A) Allows for easy searching of data and may multi-skill users
   (B) Provides great efficiency and well-adjusted participants
   (C) Make participant's work as effective and satisfying as possible
   (D) Simplifies what computers do, sometimes at the expense of participants.
10. Which of the following contains **only** social and ethical issues related to information technology?
    (A) Changing nature of work, data accuracy, hardware
    (B) Copyright, privacy, deskilling
    (C) Software, power and control, equity
    (D) Distributed processing, telecommuting, RSI.

Short Answer Questions

A Page 73

(26 marks)

1. Why is it important that the analysts work through the five stages of the systems development cycle in the correct sequence? (2 marks)
2. Compare and contrast surveys and interviews for information collection. (2 marks)
3. Briefly outline the possible contents of the analysis report. (2 marks)
4. Describe each of the four feasibilities examined in the feasibility study and give an example of one constraint that would affect feasibility. (4 marks)

5. Use the four shapes of the data flow diagram to diagrammatically represent the borrowing of a library book. (4 marks)
6. Describe five principles of user-friendly screen interfaces. (4 marks)
7. A company needs to install a new computer system before the end of the financial year, which is a few months away. The staff have been with the company for many years and are feeling stressed about the prospect of having to manage a completely new set of hardware and software in a short period of time. Discuss the advantages and disadvantages of a direct conversion method of installing the new computer system in the company. (2 marks)
8. Describe the use of a prototype for training staff to use a new system. (2 marks)
9. Compare and contrast a machine-centred system and a human-centred system. (2 marks)
10. Explain five social and ethical issues arising from the introduction of information technology. (2 marks)

Answers to Review Questions

Multiple Choice

Page 71

1. (C) Surveys, problem definition and project plan contains **only** tasks carried out during the preliminary investigation stage of the systems development cycle.
2. (A) The purpose of the Gantt chart is to show time frame, sequence and scheduling of tasks.
3. (D) The main purpose of the analysis report is to clarify the needs of the new system by examining the existing system.
4. (A) Technical feasibility is a measure of the availability of the required hardware and software.
5. (D) Context diagram, data flow diagram and systems flow chart contain **only** tools used during the design stage.
6. (A) Reviewing budget feasibility is **not** part of the implementation stage of system development.
7. (D) The main reason for testing a new information system is to ensure that it meets the specified technical requirements and review the effects on participants.
8. (A) Direct system conversion is the least time consuming.
9. (D) The best description of machine-centred systems is to simplify what computers do, sometimes at the expense of participants.
10. (B) Copyright, privacy and deskilling are all social and ethical issues related to information technology.

Short Answers

Page 71

1. It is important that the analyst works through the five stages of the systems development cycle in the correct sequence because each stage has components which are necessary for the completion of the subsequent stage. For example, a feasibility study cannot be completed before the investigation of the problem is done. The design of a new system cannot be successfully completed until the existing system's deficiencies are understood.
2. Comparison and contrast of surveys and interviews as data collection techniques.

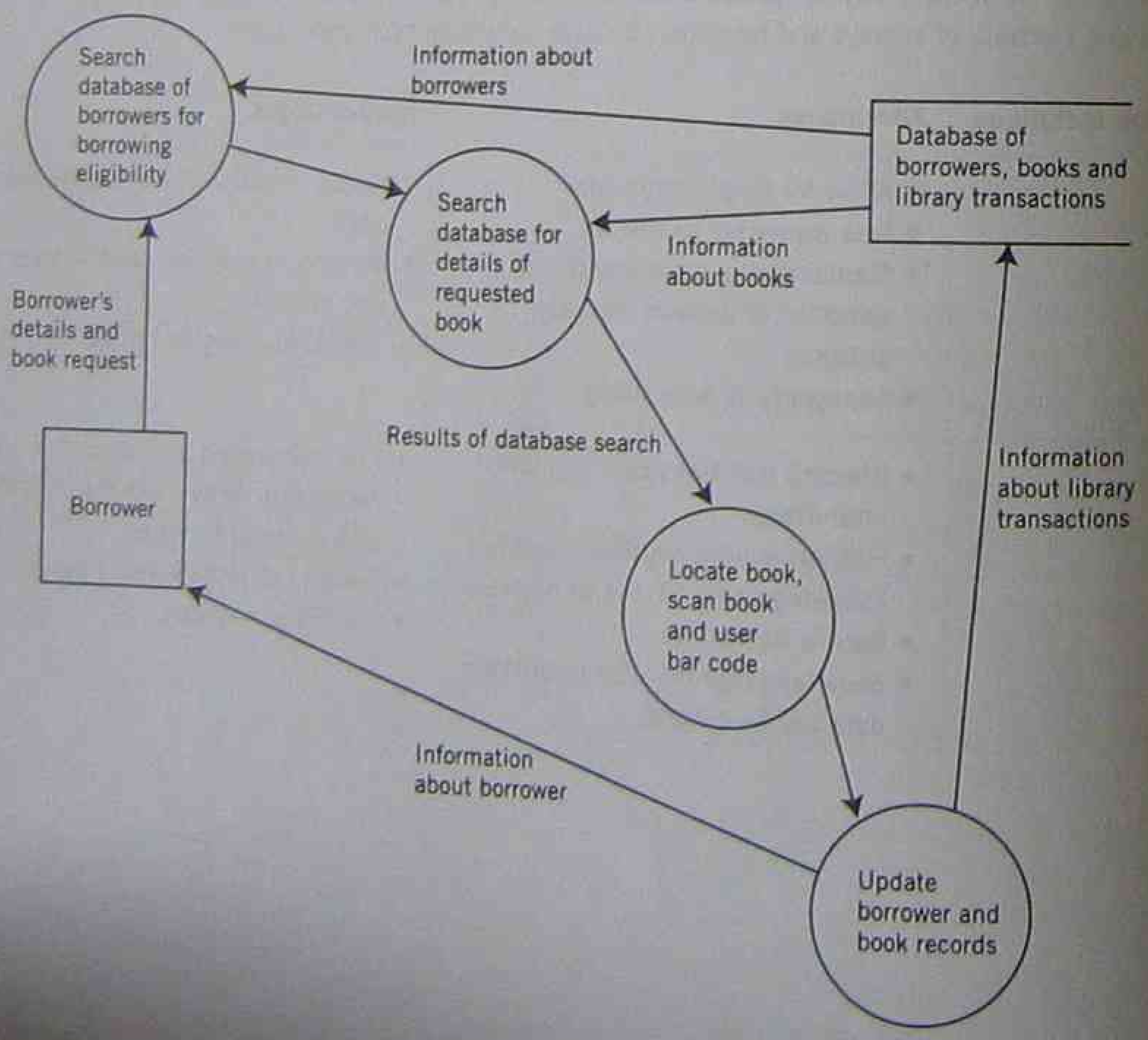
Data collection technique	Advantages	Disadvantages
Survey	• Allow for large sample size • Less expensive to deliver • Standarised questions with set selection of answers for easy analysis • Anonymity is more likely.	• Mailed responses have low response rates • Wording of questions and answer set may be restrictive • Time consuming to complete.
Interview	• Directed questions can vary with interviewee • Further probing questions possible following certain types of answers • Data is immediate • Body language or voice intonation data can be gained.	• Time consuming as one-on-one • Personality differences may interfere with perceived answers • Training of interviewers needed • Questions may vary.

3. The analysis report usually contains the following:
   - Project title page
   - List of contents
   - Problem definition/requirements definition
   - Requirement report
   - Summary of investigation, feasibilities/constraints
   - Alternative solutions
   - Recommendations (including design specifications for the next stage)
   - Project plan (schedule for new system development)
   - Appendix (supplementary materials, references, glossary of terms).

4. Examples of constraints that would effect each of the four feasibilities:

Feasibility	Example of a constraint
Budget	The company has a budget of $20K
Technical	Speech recognition software may not be efficient for multiple users
Schedule	The end of the financial year
Operational	Occupational health and safety (OH&S) is a high priority on the company's mission statement.

5. A data flow diagram diagrammatically representing the borrowing of a library book.



6. Principles for user-friendly screen interfaces include:
   - Consistency in design so that the user becomes familiar and comfortable with the screen layout
   - Use of appropriate messages for unambiguous and positive communication between the user and the computer
   - Legibility of all screen elements including suitable fonts and colour
   - Justification, alignment and the use of borders where applicable
   - Spacing and layout of all screen elements.

7. Advantages and disadvantages of the direct conversion method.

	Advantages	Disadvantages
Direct conversion	• Least expensive and time consuming. The direct conversion would suit the company if the conversion needed to be completed in a short time frame.	• Staff have little time to learn the new system. The staff may already be stressed and a direct conversion would increase their stress levels because they would need to learn new methods of computer use without time to become familiar with the new system.

8. A prototype can be used for training staff to use a new system. A prototype consists of a limited working model that can be used to test the system and train staff to use new hardware and software. Workstations, design tools and applications can be used to construct these working models.

9. A machine-centred system is designed to simplify what computers have to do, sometimes at the expense of participants. For example, new complicated software may do fast reliable calculations but be difficult for a user to follow.

   A human-centred system makes the participant's work as efficient and satisfying as possible. For example, using software that is user-friendly even if it is not as fast as other software.

Environment	Advantages	Disadvantages
Machine-centred	• Efficient processing (**efficiency**).	• Work could become meaningless, repetitive and boring.
Human-centred	• Considerations about ergonomics (**OH&S issues**).	• Less efficient processing.

10. Social and ethical issues arising from the introduction of information technology include:
    - The invasion of privacy is a social and ethical issue made 'worse' by the use of technology. Information about people, including large databases containing confidential details about addresses, phone numbers, marital status and so on, can now be easily accessed using computers. For example, government departments have very strict guidelines about access to confidential files. There have been cases where these guidelines have been breached and confidential documents have been sent to the wrong people.
    - Copyright is the right of an author to determine the fate of their intellectual property. With computer access it has become very simple to copy other people's work and thus breach copyright.
    - The nature of work has changed for some people where computers can output the same or better quality products. For example, printers and draftsmen can now be 'replaced' by an unskilled person with a word processor and a printer.
    - Computers have made it easier to commit white collar crime. Hackers have been known to access bank accounts and transfer millions of dollars into other accounts.
    - Equity refers to the equality of access to information. People who are either not computer literate or have no physical or economic access to computer technology are at a decided disadvantage to those who do have access.

Glossary

Key words and concepts: a working dictionary and glossary

Analysis or making decisions	The examination of the existing system to further understand the requirements for a new system.
Analysis report	Completed by the analyst after the analysis stage. It includes a full description of the feasibility studies, elaborating on the advantages and disadvantages of each proposed solution. One solution is recommended and a discussion is included outlining how that solution will benefit the company and meet the stated needs.
Australian Standard AS3590.2 and WorkSafe Australia	A checklist of standards for work practices relating to the use of technology in the workplace. They relate to standards in ergonomic practices.
Beta test	A testing procedure carried out by selected participants using real data after the initial alpha testing is completed using specially selected test data.
Computer operators	Participants who perform tasks on the computer system to help it run smoothly. Tasks include starting up, monitoring performance, running jobs and completing back-ups. An operations manual is an essential part of the system documentation and is used by operators.
Constraint	An aspect of feasibility studies where there is a restriction placed on the feasibility of a new system due to one or more budget, technical, organisational or scheduling reasons.
Context diagram	A smaller version of the data flow diagram that includes all symbols except the storage. It represents the new system as a single process with its inputs and outputs.
CTS	Carpal tunnel syndrome is pressure on the median nerve in the wrist and causes damage and pain to nerves and tendons in the hand.
Custom software	Software that is written by a programmer to meet specialised requirements.
Data bias	Data collected from a non-representative sample of the population.
Data collection	Data can be collected from users and participants using surveys, interviews or observation. Data is analysed and used to help define the problem.
Data flow diagram	Is also known as DFD, a graphical method of representing the movement of data within a system (either existing or proposed) showing inputs, outputs, processes and storage. The four symbols used in the DFD are: • External entity • Process • Flow direction arrow • Storage.
Data reliability	A check is done on the data to ensure it is accurate and not biased.
Data validity	A check is done to ensure that the data is within the range of expected data or of the correct data type.
Design report	A report completed by the analyst at the end of the design stage. This report includes documentation about the new system.
Designing solutions	Plan the hardware, software, input/output and process of the new system, consider the needs of the users. At least three design or case tools are used during the design stage.
Design tools	Diagrams are part of the documentation used to present information about the proposed new system. Design tools used during the design stage include: • The data flow diagram • Context diagram • The system flow chart.
Direct conversion	The old system is removed and the new system is installed in its place. The old and the new system never operate at the same time.
Documentation	A written description of the system design used to explain the development and how the system will work. Includes **user documentation** for individuals using the new system.
Economic (budget) feasibility	The likelihood that a solution will meet budget constraints (restrictions).
Ergonomics	The study of the interaction between participants and the technology of a system. Ergonomic furniture, work routine and appropriate software makes it easy, comfortable, healthy and safe for a participant to use a technology system.
Evaluating	Ensuring that the new system is meeting the requirements that were stated in the preliminary stage. Reviewing the effect on the users, participants and others in the environment.
Evaluation	Evaluating the system performance in terms of response time, processing time, storage capacity and the way in which the system meets the needs of the user and organisation.
Feasibility	The likelihood of a successful solution, determined by examining the costs and benefits in conjunction with the constraints of a proposal.
Feasibility study	The study of budget, technical, operational and scheduling constraints and costs and benefits to select the most appropriate solution for a new system.
Gantt chart	A diagram showing the sequence and scheduling of tasks during the systems development cycle.
Human-centered systems	Are concerned with making participant's work effective and satisfying (usually less efficient but user-friendly hardware and software).
Implementation	The delivery and installation of the new system hardware and software for the users to operate.
Implementation	Use one of the four conversion methods to install a new system and train the staff in using the hardware and software.
Machine-centered systems	Concerned with the efficiency and procedures of the computer, sometimes at the expense of the participants. Are efficient but non user-friendly hardware and software.
Maintaining	Making changes to the system when problems are identified.
Maintenance	Modifying the system where problems or inefficiencies have arisen. Maintenance can also include installing new hardware and software (e.g. downloading updated virus protection programs).
Making decisions	Making decisions on the new system by examining the requirements and the existing system, as well as undertaking feasibility studies. Design or case tools are used to diagrammatically represent findings. These include organisational charts, DFDs, context diagrams and system flow charts.
Managers	People responsible for the effective use of the computer system by overseeing the operators and data entry participants.
Organisational chart	Is a diagram showing the top down organisational structure of a system. It illustrates how control within the system passes from the top module down to the lower modules. Each module contains a single entry and exit point.
Organisational feasibility	The likelihood that a solution will meet organisational constraints (restrictions) determined by the organisational goals and the mission statement.
Parallel conversion	The old system remains in place for a short time while the new system is installed. For a short time both systems operate together.
Project plan	An outline of what, by whom, how and when tasks are to be completed. Management tools incorporated into the project plan include: • Gantt charts • Scheduling of tasks • Journals and diaries • Funding management plan • Communication management plan.
Phased conversion	To install the new system but in one section of the company at a time.
Pilot conversion	The new system is trialled in a small section of the company. If it is successful then the system is installed throughout the whole company.
Requirement report	A statement about the needs of a new system, including its aims and objectives and how it will help the organisation to meet its goals. The requirement report is based on data collected from participants.
RSI	Repetitive strain injury caused by prolonged use of a computer keyboard or other repetitive use of the muscles for a long period of time.

Schedule feasibility	The likelihood that a solution will meet time constraints (restrictions).
Software packages	Also called 'off the shelf' software because it is commercially available with standarised applications.
System conversion	The method in which the new system is installed. Can be one of four methods.
System flow chart	Is a graphical method of representing the flow of data as well as the hardware, software and processes within a system (either existing or proposed), showing inputs, outputs, processes, hardware, software and storage. There are fourteen symbols that can used in a system flow chart.
System development cycle	Five stages through which an analyst and company management work to provide a new system. These stages include: • Understanding the problem • Making decisions • Designing a solution • Implementing the solution • Testing, evaluating and maintaining the solution.
Technical feasibility	The likelihood that a solution will meet technical constraints (restrictions), such as availability of the proposed hardware and software.
Technical specifications	A description of the technical aspects of new hardware.
Technical support staff	People who assist participants to use the hardware, software (apart from applications) and peripherals This help is usually done over the phone and is called 'help desk' support.
Tenosynovitis	A narrowing and inflammation of the sheath around the tendon in the wrist. It is caused by repetitive use of the wrist past the point where the sheath can lubricate the tendon.
Test data	A set of fictitious input data designed to identify potential problems with the new system.
Testing	Ensuring that the new system's hardware and software are working correctly.
Training specialists	People who teach participants how to use/operate the applications within their system. These people are employed during the implementation stage of the system development cycle.
Understanding the problem	Defining the requirements for a new system, explaining why the existing system does not meet those requirements.
User-friendly	Usually refers to software but can include hardware and peripherals, being user-friendly means being easy for participants to use and learn.

4

Personal and Group Systems and Projects

Outcomes

By studying this chapter and completing the exercises students should be able to:

- Describe the nature of information processes and information technology
- Classify the functions and operations of information processes and information technology
- Identify the information processes within an information system
- Recognise the interdependence between each of the information processes
- Identify social and ethical issues within personal and group information systems
- Select and ethically use computer based and non-computer based resources and tools to process information
- Analyse and describe an identified need
- Generate ideas, consider alternatives and develop solutions for a defined need
- Recognise and apply management and communication techniques to project work
- Use technology to support group work.

Source: Information Processes and Technology Stage 6 Syllabus © *NSW Board of Studies, 1999.*

KEY TERMS AND CONCEPTS

Information system	Personal system	Group system	Project management
Analysis report	Context diagram	Data flow diagram	Systems flow chart
Documentation	Conversion methods	Training	Testing
Evaluation	Maintenance	Good design principles	Bibliography
File management	Report	Systems development cycle	Social and ethical issues
Gantt chart	Constraints	Feasibility study	Analysis
Problem statement	Data collection	Requirements report	Project plan
System design	Specifications	Information technology	Test data

4.1 COMPUTER TECHNOLOGY PROJECTS

An important part of the course is undertaking **two** projects preferably using **real** situations/companies. The two projects are:

- Personal information systems
- Group information systems.

Each project must contain the structural headings and theoretical knowledge from the system development cycle as described in Chapters 1, 2 and 3, as well as skills from each of the computer applications as described in Chapters 5 to 11:

Ch 5 – Word processing
Ch 6 – Databases
Ch 7 – Spreadsheets
Ch 8 – Graphics
Ch 9 – Desk top publishing
Ch 10 – Multimedia
Ch 11 – The Internet.

These projects may be undertaken at any time during the Preliminary courses.

Developing a solution

Use the following headings to carry out each of the stages in the systems development cycle:

- Understanding the problem
- Making decisions about solutions
- Designing solutions
- Implementing solutions
- Testing, evaluating and maintaining solutions.

Your own personal **project management** should be comparable to that of a real systems analyst. You must work through **all** the steps in each stage before moving to the next stage. You especially need to set yourself a Gantt chart time frame showing the sequence of tasks and time schedule.

As these projects will require the use of several different files and applications, it is important that you create a straight forward **file management** system and back-up your work at all times (preferably keeping a copy in two different places apart from on the computer).

Questionaires and surveys and all other **documentation** should be included as appendices as well as being referred to in your reports for each stage. You should demonstrate the appropriate use of all relevant design tools for each stage.

Writing a Report

The following is a suggested set of headings and subheadings:

Title page
Table of contents
Project plan
Understanding the problem (requirements definition)
  Data collection (surveys, interviews, observation)
  Report writing
  Project plan
  Information management software
  Project management software
  Gantt chart
Making decisions (feasibility study)
  Feasibility study:
    Economic (budget) constraints
    Technical constraints
    Schedule constraints
    Organisational constraints
  Analysis tools:
    Data flow diagrams;
    Context diagrams
    System flow charts
    Organisational charts
    Analysis report/Design specification
Designing solutions
  Design tools:
    Data flow diagrams;
    Context diagrams
    System flow charts
  System design
  Internal/external specifications
  Software packages vs custom designed software
Documentation
  User documentation
  Technical documentation (beta testing/test data)
Implementation
  Conversions:
    Direct
    Parallel
    Phased
    Pilot
  Training
Testing, evaluating and maintaining
  Testing and evaluation
    Original requirements comparison
    Technical operation (test data)
  Maintenance
Social and ethical issues
  Ergonomics:
    Health hazards (RSI, CTS, etc)
      Chair, backrest, footrest, desk
      Australian standards AS3590.2 and WorkSafe Australia
    Work routine:
      Adequate breaks/change of routine
      Job design/information overload
    Furniture:
      Environment:
      Light, noise, temperature
    Software user-friendliness (screen design principles)
  Skilling and deskilling
  Privacy
  Copyright
  Changing nature of work
  Crime
  Equity
  Social relationships
  Efficiency and competition
  Power and control
Appendix
Bibliography

4.2 PERSONAL INFORMATION SYSTEMS

Personal information systems are those where the participant is an **individual** and uses information technology to complete a project that does not directly involve another person.

The following table outlines some information system processes that may be carried out by individuals with varied needs.

Individual	Possible information technology processes
Scriptwriter	A word processor to produce an episode of a television program
Farmer	A spreadsheet to record income and expenditure figures and make what-if predictions from them
Doctor	A database of patient records
Teacher	Electronic mark book for storing and calculating marks, letters to parents, brochure about camps, presentations, Internet research, email
Student	A student project on the environment, report, graphs, statistics, presentation, Internet research
Retail business operator	Price lists, calculation of costs, invoices, advertising, parts inventory, e-commerce, email
Personnel manager	Wages calculations, personnel files, job descriptions, sales statistics (graphs), presentations, e-commerce, email
Basketball team manager	Team member's personal details, scheduled team games, information brochures, calculation of game points, email.