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6
CHAPTER

Lifetime Reduction of Transformers and Induction Machines

The total installed power capacity within the Eastern, Western, and Texan power pools of the United States is 900 GW with about 70 GW of spinning reserve. Approximately 60% of the 900 GW is consumed by induction motors and 100% passes through transformers. Similar percentages exist in most countries around the world. For this reason transformers and induction motors are important components of the electric power system.

The lifetime of any device is limited by the aging of the insulation material due to temperature: the higher the activation energy of any material, the faster the aging proceeds. Iron and copper/aluminum have low activation energies and for this reason their aging is negligible. Insulation material – either of the organic or inorganic type – is most susceptible to aging caused by temperature. If a device is properly designed then the rated temperature results in the rated lifetime. Temperature rises above rated temperature result in a decrease of lifetime below its rated value. There are a few mechanisms by which the rated lifetime can be reduced:

1. Temperature rises above the rated temperature can come about due to overload and voltage or current harmonics.
2. Lifetime can be also decreased by intermittent operation. It is well known that generators of pumped storage plants must be rewound every 15 years as compared to 40 years for generators which operate at a constant temperature.
3. Vibration within a machine due to load variations (e.g., piston compressor) can destroy the mechanical properties of conductor insulation.
4. Faults – such as short-circuits – can impact the lifetime due to excessive mechanical forces acting on the winding and their insulation.

In this chapter we are concerned with aging due to elevated temperatures caused by harmonics. The presence of current and voltage harmonics in today's power systems causes additional losses in electromagnetic components and appliances, creating substantial elevated temperature rises and decreasing lifetime of machines and transformers. Therefore, estimating additional losses, temperature rises, and aging of power system components and loads has become an important issue for utilities and end users alike.

Three different phases are involved in the estimation (or determination) of the lifetime of magnetic devices:

1. modeling and computation of the additional losses due to voltage or current harmonics,
2. determination of the ensuing temperature rises, and
3. estimation of the percentage decrease of lifetime as compared with rated lifetime [1].

The literature is rich in documents and papers that report the effect of poor power quality on losses and temperature rises of power system components and loads; however, a matter that still remains to be examined in more detail is the issue of device aging under nonsinusoidal operating conditions. Earlier papers were mostly concerned about magnetic device losses under sinusoidal operating conditions [2–13]. However, more recent research has expanded the scope to magnetic device derating under nonsinusoidal operation [14–17]. Only a few papers have addressed the subjects of device aging and economics issues caused by poor power quality [18–23].

In the first sections of this chapter the decrease of lifetime of power system components and loads such as universal motors, single- and three-phase transformers, and induction motors are estimated based on their terminal voltage harmonics. After a brief review of temperature relations, the concept of weighted harmonic factors is introduced. This is a quantity that relates the device terminal voltage harmonic amplitudes to its temperature rise. Additional temperature rises and losses due to harmonics are discussed, and thereafter the weighted harmonic factors are employed to determine the decrease of lifetime of electrical appliances. Toward the end of this chapter the time-varying nature of harmonics is addressed with respect to their measurement, sum-

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Problem 6.10: Calculation of the Temperature Rise of a Drip-Proof 5 hp Induction Motor

a) Based on Application Example 6.11 calculate the temperature rise ΔT of a drip-proof induction motor provided the following data are given: Input power $P_{in} = 4355$ W, output power $P_{out} = 3730$ W, line frequency $f = 60$ Hz, thickness of stator slot insulation $\beta_i = 6 \cdot 10^{-4}$ m, thermal conductivity of stator slot insulation $\lambda_i = 0.148$ W/m°C, total area of the stator slots $S_i = 0.0701$ m^2, thermal conductivity of the stator laminations (iron core) $\lambda_c = 47.24$ W/m°C, radial height of stator back iron $h_{y1} = 0.0399$ m, cross-sectional area of the stator core at the middle of the stator back iron (stator yoke) $S_{y1} = 0.0325$ m^2, length of the small air gap between the stator core and the frame of the motor $\beta_\delta = 5 \cdot 10^{-6}$ m, thermal conductivity of the small air gap between stator core and frame $\lambda_\delta = 0.236$ W/m°C, outside surface area of the stator frame $S_\delta = 0.0403$ m^2, surface thermal coefficient for dissipation of heat to stationary air $\alpha_o = 18.70$ W/m^{2o}C, surface thermal coefficient for dissipation of heat to moving air at $v = 4.6$ m/s $\alpha_v = 58.81$ W/m^{2o}C, surface area in contact with stationary air $S_o = 0.181$ m^2, and surface area in contact with moving air $S_v = 0.696$ m^2. Table P6.10 details the percentage values of the various losses of the 5 hp induction motor. Note that the iron-core losses of the rotor are very small due to the low frequency in the rotor member (slip frequency).

b) Determine the rated temperature of the motor T_{rat} provided the ambient temperature is $T_{amb} = 40$°C.

TABLE P6.10 Percentage Losses of a 5 hp Three-Phase Induction Motor

P_{sut}	P_{y1}	P_{t1}	P_{sp1}	P_{AG}	P_{sp2}	P_{mech}
30%	8%	10%	2%	42%	2%	6%

Problem 6.11: Solution of the Partial Differential Equation Governing Heat Conduction

The solution of the heat equation in one spatial dimension (x)

$$\frac{\partial T(x,t)}{\partial t} = c^2 \frac{\partial^2 T(x,t)}{\partial x^2} \quad \text{(P6.11-1)}$$

is based on the separation of variables as was proposed by Fourier, where T is the temperature and x is the spatial coordinate as illustrated, for example, in Fig. P6.11, and t is the time. The boundary conditions are

$$T(0, t) = 0 \text{ and } T(\ell, t) = 0 \quad \text{(P6.11-2)}$$

for all times t, and the initial condition at $t = 0$ is

$$T(x, 0) = f(x), \quad \text{(P6.11-3)}$$

where $f(x)$ is a given function of x.

The temperature $T(x, t)$, is assumed separable in x and t.

$$T(x, t) = X(x) \cdot T(t) \quad \text{(P6.11-4)}$$

so that

$$X'' \cdot T = \frac{1}{c^2} X \cdot T', \quad \text{(P6.11-5)}$$

or after dividing by $X(x)T(t)$

$$\frac{X''}{X} = \frac{1}{c^2}\frac{T'}{T} = k, \quad \text{(P6.11-6)}$$

where k is the separation constant. We can expect k to be negative as can be seen from the time equation:

$$T' = c^2 kT \quad \text{(P6.11-7)}$$

or after integration

$$T(t) = T_0 e^{c^2 kT}. \quad \text{(P6.11-8)}$$

Thus $T(t)$ will increase in time if k is positive and decrease in time if k is negative. The increase of T in time is not possible without any additional heating mechanism and therefore we assume that k is negative. To force this we set $k = -p^2$. The spatial differential equation is now

$$X'' + p^2 X = 0, \quad \text{(P6.11-9)}$$

which is similar to the harmonic motion equation with the trigonometric solution

$$X(x) = A \cos px + B \sin px. \quad \text{(P6.11-10)}$$

Now, applying the boundary condition, we find that for

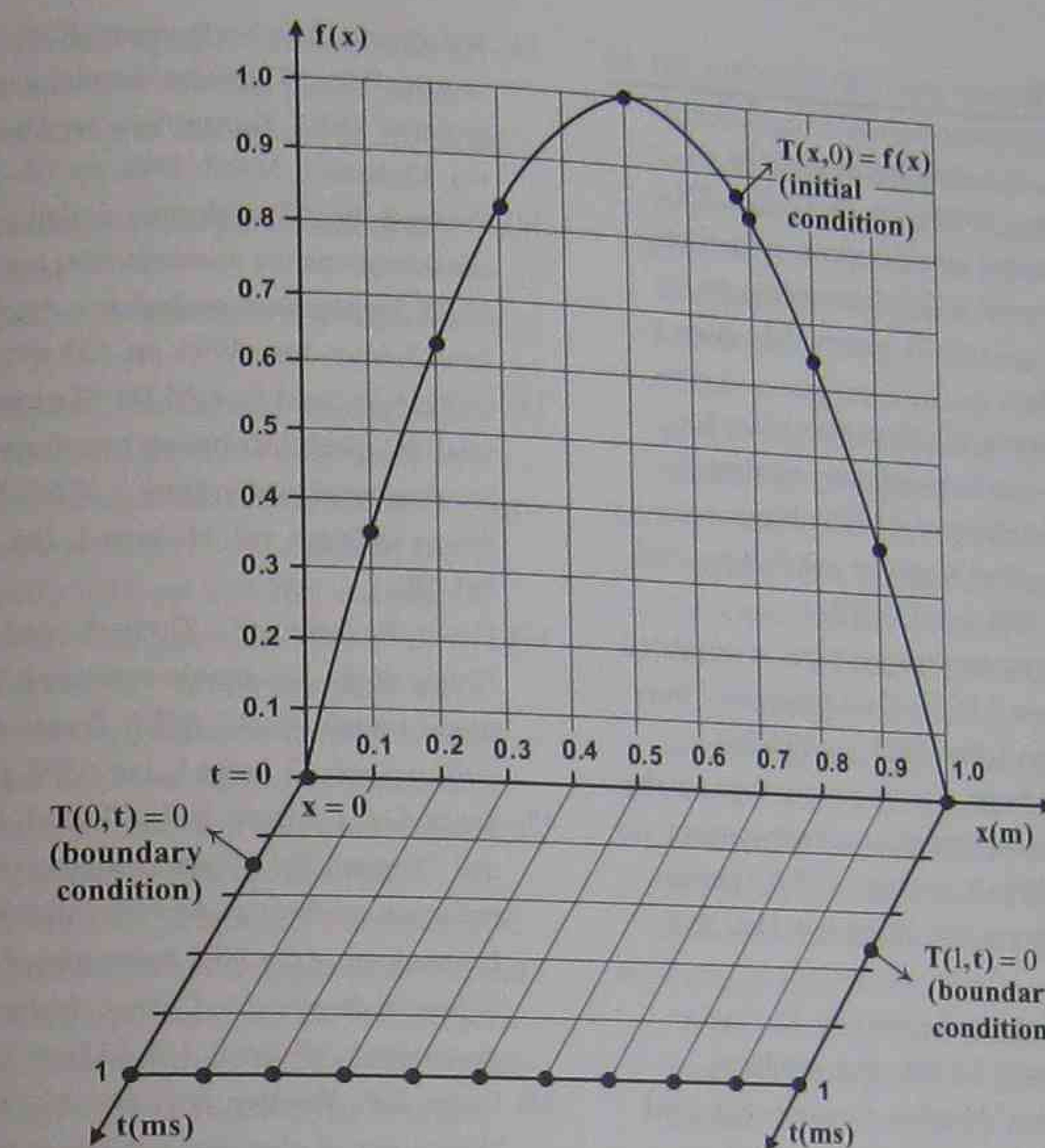


FIGURE P6.11 Heat conduction in a homogeneous one-dimensional body of length $\ell = 1$ m.

$$X(0) = 0, \quad X(\ell) = 0. \quad \text{(P6.11-11)}$$

$A = 0$ and $\sin(p\ell) = 0$, resulting in $p\ell = n\pi$. We take the most general solution by adding together all possible solutions, satisfying the boundary conditions, to obtain

$$T(x,t) = \sum_{n=0}^{\infty} B_n \sin\frac{n\pi}{\ell} e^{\frac{n^2\pi^2c^2}{\ell^2}t}. \quad \text{(P6.11-12)}$$

The final step is to apply the initial conditions

$$T(x,0) = \sum_{n=0}^{\infty} B_n \sin\frac{n\pi}{\ell} = f(x). \quad \text{(P6.11-13)}$$

Finding the Fourier coefficient of the above expression leads to

$$B_n = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\frac{n\pi}{\ell}x\,dx \quad \text{(P6.11-14)}$$

for all positive integers n.

Note that because every term in the solution for $T(x, t)$ has a negative exponential in it, the temperature must decrease in time, and the final solution will tend to $T = 0$.

a) For $c^2 = 1$, $\ell = 1$, that is, $\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2}$, the boundary conditions $T(0, t) = T(1, t) = 0$, and the initial condition $T(x, 0) = f(x) = -4x^2 + 4x$ find $T(x, t)$ for $0 < x < 1$. The initial conditions and boundary conditions are shown in Fig. P6.11 as a function of the spatial coordinate x and the time t. Calculate and plot $T(x, t)$ as a function of x for $t = 1$ ms; thereby it will be sufficient to take into account three terms of the Fourier series.

b) For $c^2 = 1$, $\ell = 1$, that is, $\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2}$, the boundary conditions $T(0, t) = T(1, t) = 0$, and the initial conditions $T(x, 0) = f(x) = x$ (for $0 < x < 1/2$) and $T(x, 0) = f(x) = 1 - x$ (for $1/2 < x < 1$) find $T(x, t)$. Calculate and plot $T(x, t)$ as a function of x for $t = 1$ ms; thereby it will be sufficient to take into account three terms of the Fourier series.

mation, and propagation. The probabilistic evaluation of the economical damage – that is, cost – due to harmonic losses in industrial energy systems is explored next, and the increase of temperature in a device as a function of time as well as intermittent operation of devices concludes this chapter.

6.1 RATIONALE FOR RELYING ON THE WORST-CASE CONDITIONS

Field measurements show that the harmonic voltages and currents of a distribution feeder are time-varying. This is due to

1. the changes in the loads, and
2. the varying system's configuration.

This means that steady-state harmonic spectra do not exist. However, loads such as variable-speed drives for air-conditioning, rolling mills, and arc furnaces operate during their daily operating cycle for at least a few hours at rated operation. It is for this operating condition that system components (e.g., transformers, capacitors) and loads must be designed. Most electromagnetic devices such as transformers and rotating machines have thermal time constants of 1 to 3 hours – depending on their sizes and cooling mechanisms. When these components continually operate at rated power, they are exposed to steady-state current and voltage spectra resulting at an ambient temperature of $T_{amb} = 40°C$ in the maximum temperature rise $T_{risemax}$, and the resulting temperature $T_{max} = T_{amb} + T_{risemax}$ must be less or equal to the rated temperature T_{rat}. The worst case is therefore when the power system components operate at rated load with their associated current or voltage spectra where individual harmonic amplitudes are maximum. It is not sufficient to limit the total harmonic distortion of either voltage (THD_v) or current (THD_i) because these parameters can be the same for different spectra – resulting in different additional temperature rises, insulation stresses, and mechanical vibrations. In Sections 6.2 to 6.10 worst-case conditions will be assumed.

6.2 ELEVATED TEMPERATURE RISE DUE TO VOLTAGE HARMONICS

Most electric appliances use electric motors and/or transformers. In all these cases the power system's sinusoidal voltage causes ohmic and iron-core losses resulting in a temperature rise, which approaches the rated temperature rise ($T_{riserat}$) at continuous operation:

$$T_{rat} = T_{amb} + T_{riserat}, \tag{6-1}$$

where $T_{amb} = 40°C$ is the ambient temperature. The lifetime of a magnetic device is greatly dependent on this temperature rise and the lifetime will be reduced if this rated temperature rise is exceeded over any length of time [1].

The presence of voltage harmonics in the power system's voltage causes harmonic currents in induction motors and transformers of electrical appliances, resulting in an elevated temperature rise ($T_{riserat} + \Delta T_h$) such that the device temperature is

$$T = T_{amb} + T_{riserat} + \Delta T_h, \tag{6-2}$$

where ΔT_h is the additional temperature rise due to voltage (integer, sub-, inter-) harmonics. Through a series of studies this additional temperature rise of various electric machines and transformers was calculated and measured as a function of the harmonic amplitude V_h and the phase shifts of the voltage harmonics φ_h with respect to the fundamental [18, 19].

The insulating material of electrical apparatus, as used in electrical appliances, is of either organic or inorganic origin. The deterioration of the insulation caused by the elevated temperature rise is manifested by a reduction of the mechanical strength and/or a change of the dielectric behavior of the insulation material. This thermal degradation is best represented by the reaction rate equation of Arrhenius. Plots which will be drawn based on this equation are called Arrhenius plots [24, 25]. The slopes of these plots are proportional to the activation energy E of the insulation material under consideration. Knowing the rated lifetime of insulation materials at rated temperature ($T_{rat} = T_{amb} + T_{riserat}$), and the elevated temperature rise (ΔT_h) due to given amplitudes V_h and phase shifts φ_h of voltage harmonics, one will be able to estimate from the Arrhenius plot the reduction of the lifetime of an electrical appliance due to ΔT_h.

Definition: The activation energy E is the energy transmitted in the form of heat to the chemical reaction of decomposition.

6.3 WEIGHTED-HARMONIC FACTORS

The harmonic voltage content (order h, amplitude V_h, and phase shift φ_h) of power system voltages varies with the type and size of harmonic generators and loads as well as with the topology of the system and it will hardly be the same for any two networks. Since voltage harmonics result in additional losses and temperature rises (ΔT_h) in electrical appliances, it would be desirable to derive one single criterion which limits – for a maximum allowable additional temperature rise – the individual amplitudes and phase shifts of the occurring harmonic voltages including their relative weight with respect to each other in contributing to the elevated temperature rise ΔT_h.

Also, this criterion should be simple enough so that the additional losses, temperature rises ΔT_h, and the reduction of lifetime of electrical appliances can easily be predicted as a function of the harmonic content of their terminal voltages. As most electrical appliances use transformers and motors as input devices, in this section weighted harmonic factors will be derived for single-phase and three-phase transformers and induction machines.

The ambient temperature is at the most $T_{amb} = 40°C$. The rated temperature rise depends on the class of insulation material used. $T_{riserat} = 80°C$ is a commonly permitted value so that

$$T_{rat} = T_{amb} + T_{riserat} = 40°C + 80°C = 120°C. \tag{6-3}$$

In transformers, iron-core sheets and copper (aluminum) conductors are insulated by paper, plastic material, or varnish.

The Swedish scientist Svante Arrhenius originated the so-called Arrhenius rule: a differential equation describing the speed of degradation or deterioration of any (organic, inorganic) material. This deterioration is not oxidation due to elevated temperatures.

The idea of a weighted harmonic factor for different magnetic devices [20] is based on the fact that the additional temperature rise ΔT_h is different for transformers, universal machines, and induction machines, although the harmonic content (V_h, φ_h) of the terminal voltages is the same.

6.3.1 Weighted-Harmonic Factor for Single-Phase Transformers

Figure 6.1 illustrates the equivalent circuit of a single-phase transformer for the fundamental ($h = 1$) and harmonic voltages of hth order.

The ohmic losses of the hth harmonic expressed in terms of the fundamental ohmic losses are

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} = \left(\frac{V_{ph}}{V_{p1}}\right)^2 \left(\frac{R_{ph} + R'_{sh}}{R_{p1} + R'_{s1}}\right)(\text{RATIO}), \tag{6-4}$$

where

$$\text{RATIO} = \frac{(R_{p1} + R'_{s1} + R'_{load,1})^2 + \omega^2 (L_{p1} + L'_{s1} + L'_{load,1})^2}{(R_{ph} + R'_{sh} + R'_{load,h})^2 + (h\omega)^2 (L_{ph} + L'_{sh} + L'_{load,h})^2}.$$

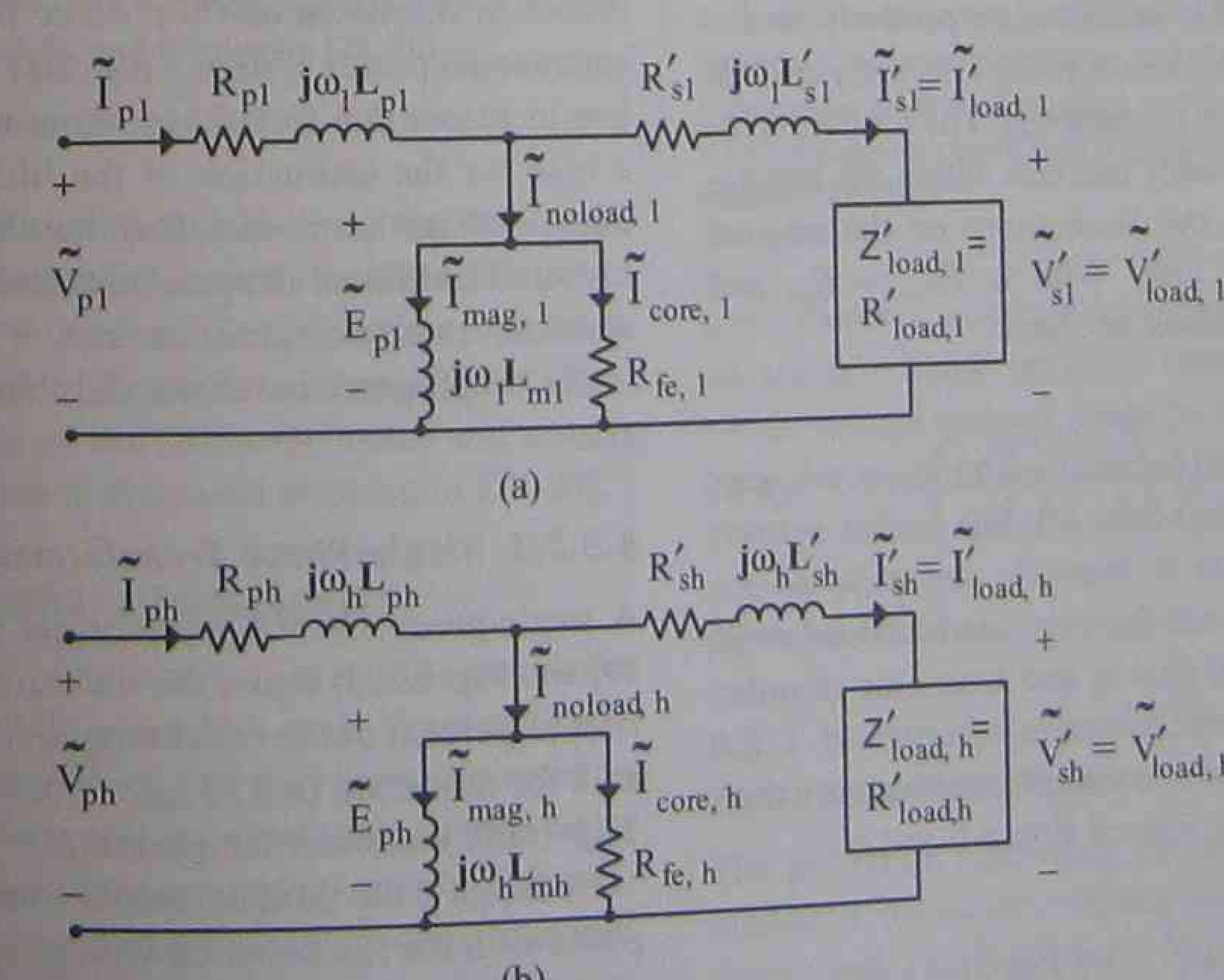


FIGURE 6.1 Fundamental (a) and harmonic (b) equivalent circuits of single-phase transformers.

In most residential power transformers the influence of the skin effect can be neglected. However, the skin effect of an ohmic load cannot be ignored, because $R'_{load,h}$, $R'_{load,l} >> R_{ph}$, R_{p1}, R'_{sh}, R'_{s1}. Therefore, one obtains

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} = \left(\frac{V_{ph}}{V_{p1}}\right)^2 \frac{(R'_{load,1})^2 + \omega^2 (L_{p1} + L'_{s1} + L'_{load,1})^2}{(R'_{load,h})^2 + (h\omega)^2 (L_{ph} + L'_{sh} + L'_{load,h})^2}. \quad (6\text{-}5)$$

Due to $L_{ph} \approx L_{p1}$, $L'_{sh} \approx L'_{s1}$, $L'_{load,h} \approx L'_{load,1}$, and the fact that the load resistance $R'_{load,h}$ increases less than linearly with frequency [18], the following simplified expression is considered:

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} = \frac{1}{h^n}\left(\frac{V_{ph}}{V_{p1}}\right)^2, \quad (6\text{-}6)$$

where $0 < n < 2$ depends on the resistance and inductance values of the equivalent circuit of a load.

The iron-core losses of a single-phase transformer due to a voltage harmonic of order h are expressed in terms of the corresponding losses caused by the fundamental as

$$\frac{W_{iron,h}}{W_{iron,1}} = \left(\frac{B_{max,h}}{B_{max,1}}\right)^2 \frac{\left\{\sigma\left(\frac{fh}{100}\right)^2 k_m^2 + \varepsilon\left(\frac{fh}{100}\right)\right\}}{\left\{\sigma\left(\frac{f}{100}\right)^2 + \varepsilon\left(\frac{f}{100}\right)\right\}}, \quad (6\text{-}7)$$

where $B_{max,1}$ and $B_{max,h}$ define fundamental and harmonic maximum flux densities, respectively, and σ and ε are [26] coefficients related to eddy-current and hysteresis losses, respectively. The term k_m takes the reaction of the eddy currents within the laminations and between the laminations on the original field into account [18]. With $h\ B_{max,h} \propto E_{ph}$ and $B_{max,1} \propto E_{p1}$, the relation becomes

$$\frac{W_{iron,h}}{W_{iron,1}} = \left(\frac{E_{ph}}{hE_{p1}}\right)^2 \left(\frac{R_{fe,1}}{R_{fe,h}}\right), \quad (6\text{-}8)$$

where $R_{fe,1}$ and $R_{fe,h}$ are the iron-core loss resistances for the fundamental ($h = 1$) and harmonic of order h, respectively.

The total losses for a harmonic of order h are then determined from the sum of Eqs. 6-6 and 6-8:

$$\frac{W_{total,h}}{W_{total,1}} = \frac{1}{h^n}\left(\frac{V_{ph}}{V_{p1}}\right)^2 + \frac{1}{h^2}\left(\frac{E_{ph}}{E_{p1}}\right)^2 \left(\frac{R_{fe,1}}{R_{fe,h}}\right). \quad (6\text{-}9)$$

According to [27], the core-loss resistance ratio is $R_{fe,h}/R_{fe,1} = h^{0.6}$ and because of the leakage reactance being proportional to h, one obtains the inequality $(E_{ph}/E_{p1}) < (V_{ph}/V_{p1})$ or

$$\left(\frac{E_{ph}}{E_{p1}}\right)^2 = \left(\frac{V_{ph}}{V_{p1}}\right)^m, \quad (6\text{-}10)$$

where the exponent m is less than 2. Therefore, for single-phase transformers Eq. 6-9 becomes

$$\frac{W_{total,h}}{W_{total,1}} \approx \frac{1}{h^n}\left(\frac{V_{ph}}{V_{p1}}\right)^2 + \frac{1}{h^{2.6}}\left(\frac{V_{ph}}{V_{p1}}\right)^m. \quad (6\text{-}11)$$

Depending on the relative sizes of the ohmic and iron-core losses at full load, Eq. 6-11 can be rewritten for all occurring harmonics and is called the weighted harmonic factor [20]:

$$\frac{W_{total,h\Sigma}}{W_{total,1}} = K_1 \sum_{h=2}^{\infty} \frac{1}{(h)^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l, \quad (6\text{-}12)$$

where $W_{total,h\Sigma}$ and $W_{total,1}$ are the total harmonic losses and the total fundamental losses, respectively. Also $0 \le k \le 2.0$ and $0 \le l \le 2.0$. Average values for k and l are obtained from measurements as discussed in a later section.

6.3.2 Measured Temperature Increases of Transformers

Measured temperature rises in transformers and induction machines as they occur in a residential/commercial power system – together with calculated loss increases due to voltage harmonics – represent a base for the estimation of the lifetime reduction due to voltage harmonics. It is thereby assumed that for small additional temperature rises – as compared with the rated temperature rise – the additional losses are proportional to the additional temperature rise.

6.3.2.1 *Single-Phase Transformers*

A single-phase 150 VA transformer was tested [18, 19] and Fig. 6.2a,b shows the measured temperature rises in percent of the rated temperature rises at full load for moderate (e.g., $T_{amb} = 23°C$) and high (e.g., $T_{amb} = 40°C$) ambient temperatures.

Note that if the third harmonic voltage $v_3(t)$ superposed with the fundamental voltage $v_1(t)$ produces a peak-to-peak voltage $\{v_1(t) + v_3(t)\}$ that is maximum, then the peak-to-peak flux density in the core

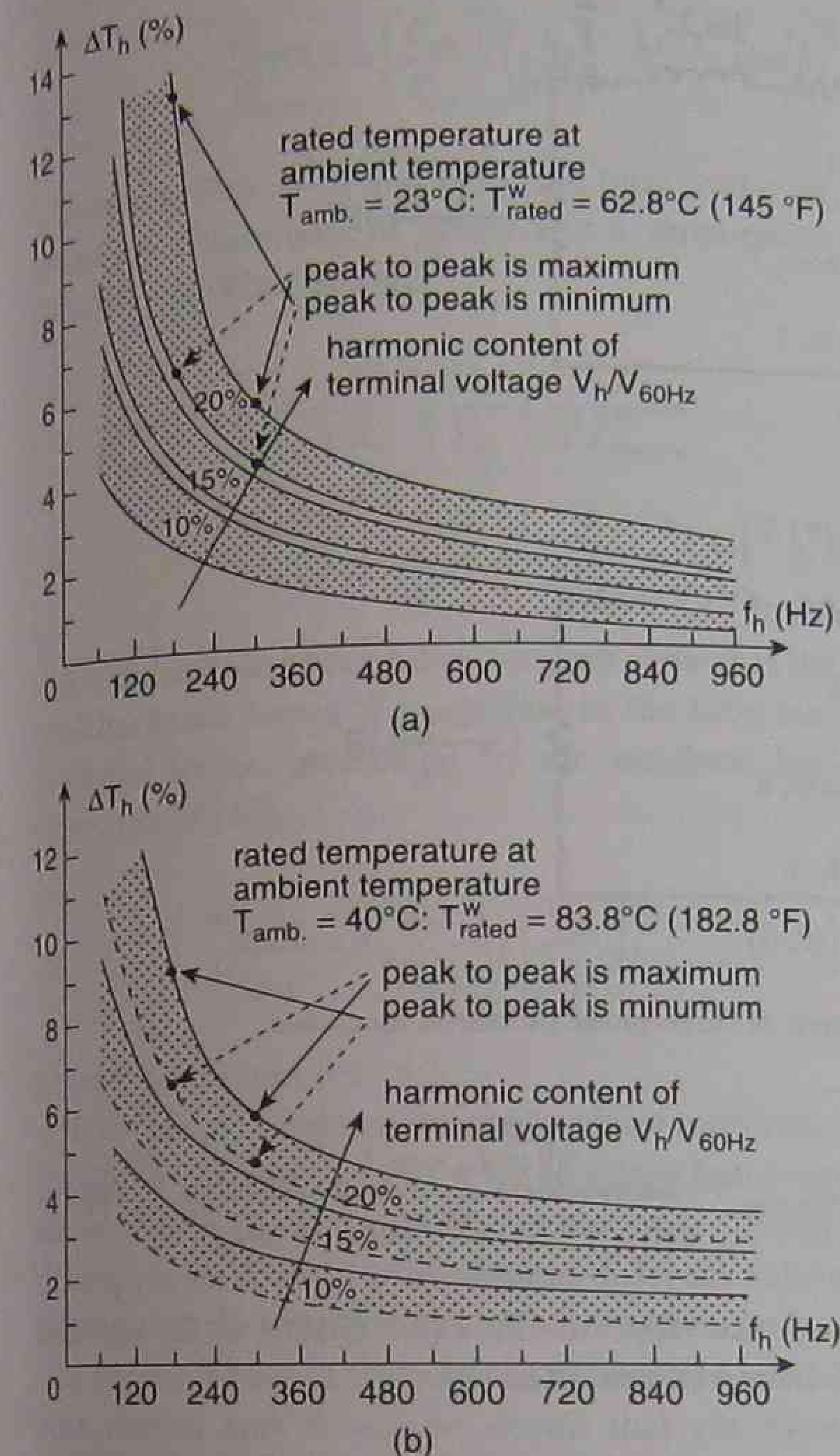


FIGURE 6.2 Measured additional temperature rise of the 150 VA single-phase transformer winding in % of the rated temperature rise at full load at an ambient temperature of (a) 23°C and (b) 40°C as a function of the harmonic voltage amplitude, phase shift, and frequency [29, 30].

$\{B_1(t) + B_3(t)\}$ is minimum. That is, the total harmonic losses are smaller than those when the peak-to-peak voltage is minimum, resulting in a maximum peak-to-peak value of the flux density. This "alternating" behavior of the harmonic losses and associated temperatures is discussed in detail in [27, 28].

6.3.2.2 *Three-Phase Transformers*

Three 150 VA single-phase transformers were assembled to Δ/Y-grounded and Y-grounded/Y-grounded configurations and they were subjected to the same harmonic voltage conditions as the single-phase transformer of the preceding section. The temperatures of the transformer windings were measured at the same location as for the single-phase transformer. The tests were limited to balanced operation. However, no phase-lock loop was available and no stable relation between fundamental and harmonic voltage systems could be maintained. Therefore, the temperature data of Fig. 6.3a,b represent average values and the alternating behavior could not be observed, although it also exists in three-phase transformers.

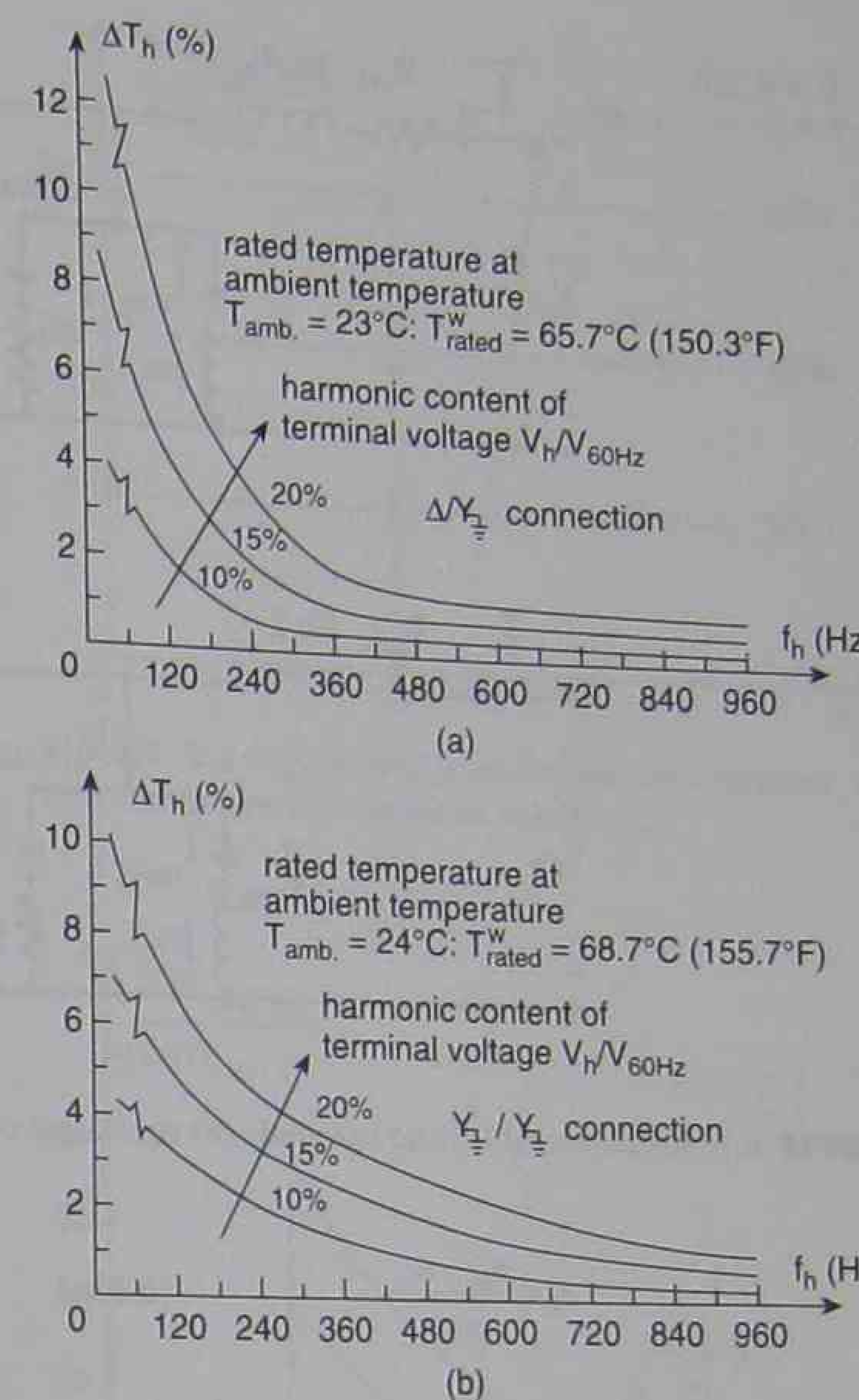


FIGURE 6.3 (a) Measured additional temperature rise of the transformer winding in % of the rated temperature rise at full load at an ambient temperature of 23°C as a function of the harmonic voltage amplitude and frequency for either forward- or backward-rotating harmonic voltage systems, if the transformer bank is in Δ/Y-grounded connection [29, 30]. (b) Measured additional temperature rise of the transformer winding in % of the rated temperature rise at full load at an ambient temperature of 24°C as a function of the harmonic voltage amplitude and frequency for either forward- or backward-rotating harmonic voltage systems, if the transformer bank is in Y-grounded/Y-grounded connection [29, 30].

6.3.3 Weighted-Harmonic Factor for Three-Phase Induction Machines

The per-phase equivalent circuits of a three-phase induction motor are shown for the fundamental ($h = 1$) and a (integer, sub-, inter-) harmonic of hth order in Fig. 6.4. In Fig. 6.4 the fundamental slip is

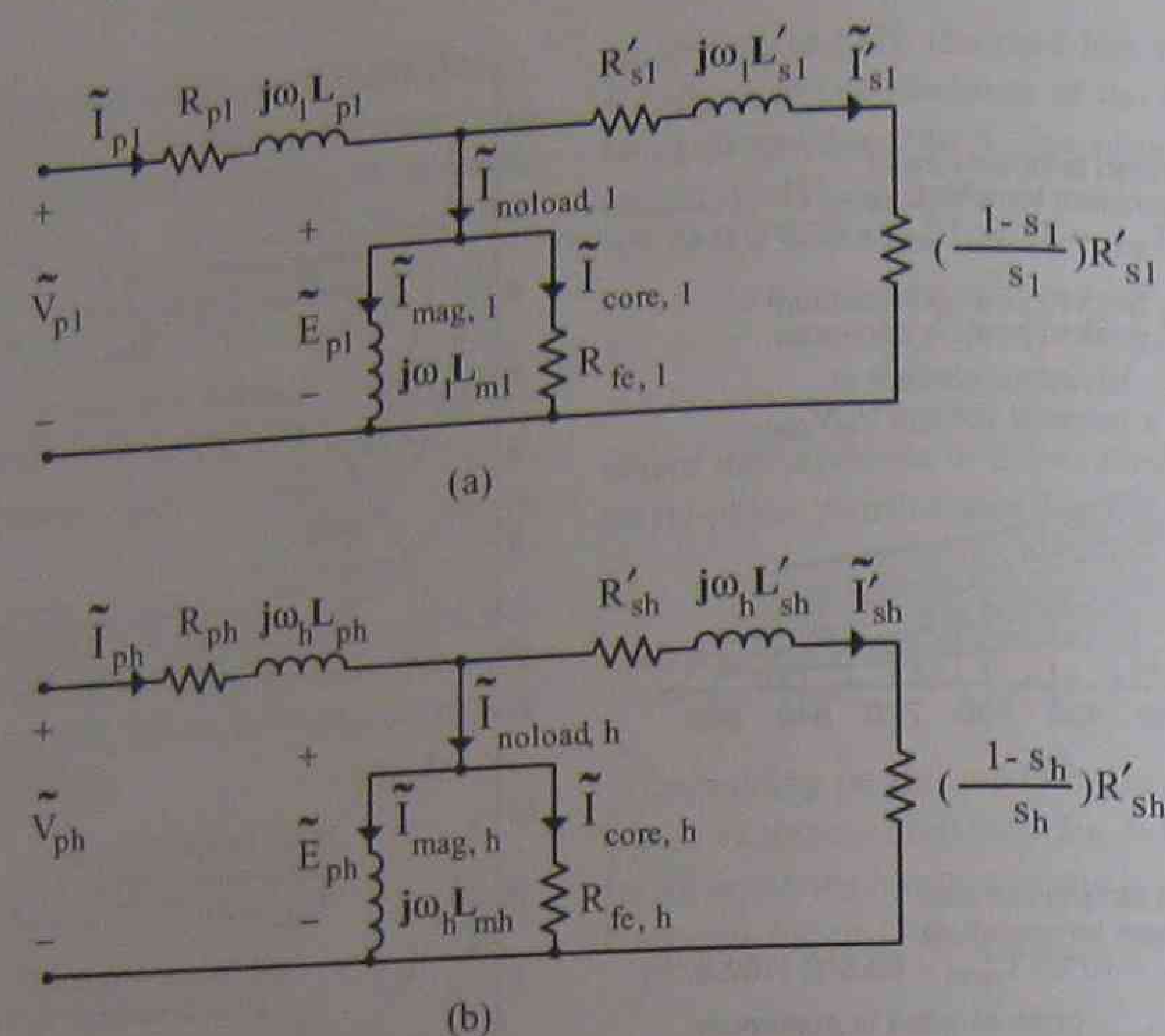


FIGURE 6.4 Fundamental (a) and harmonic (b) equivalent circuits of three-phase induction machines.

$$s_1 = \frac{n_{s1} - n_m}{n_{s1}}, \qquad (6\text{-}13a)$$

and the harmonic slip is

$$s_h = \frac{n_{sh} - n_m}{n_{sh}}. \qquad (6\text{-}13b)$$

The ohmic losses caused by either the fundamental or a harmonic of order h are proportional to the square of the respective currents. Therefore, one can write for the ohmic losses caused by voltage harmonics:

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} = \left(\frac{R_{ph} + R'_{sh}}{R_{p1} + R'_{s1}}\right)\left(\frac{V_{ph}}{V_{p1}}\right)^2 \left(\frac{Z_{p1}}{Z_{ph}}\right)^2, \qquad (6\text{-}14)$$

where

$$Z_{p1} = \sqrt{(R_{p1} + R'_{s1}/s_1)^2 + (\omega_1 L_p + \omega_1 L'_s)^2}$$

and

$$Z_{ph} = \sqrt{(R_{ph} + R'_{sh}/s_1)^2 + (h\omega_1 L_p + h\omega_1 L'_s)^2}.$$

For all time harmonics ($h = 2, 3, 4, \ldots$) an induction machine operating at rated speed can be considered being at standstill because $s_1 << s_h$. With R_{ph}, $R'_{sh}/s_h << h\omega_1 L_p$, $h\omega_1 L'_s$, the per-unit starting impedance

$$Z_{start\,pu} = \frac{\sqrt{(\omega_1 L_p + \omega_1 L'_s)^2}}{\sqrt{(R_{p1} + R'_{s1}/s_1)^2 + (\omega_1 L_p + \omega_1 L'_s)^2}}, \qquad (6\text{-}15)$$

and the slip $s_h = \{h \mp (1 - s_1)\}/h \approx (h \mp 1)/h$, Eq. 6-14 can be rewritten as

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} \approx \left(\frac{R_{ph} + R'_{sh}}{R_{p1} + R'_{s1}}\right)\left(\frac{V_{ph}}{V_{p1}}\right)^2 \frac{1}{h^2 Z^2_{start\,pu}}. \qquad (6\text{-}16)$$

The additional iron-core losses due to a voltage harmonic of order h referred to the iron-core losses at the fundamental frequency are (see Fig. 6.4)

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} \approx \left(\frac{V_{ph}}{V_{p1}}\right)^2 \left(\frac{Z_{fe,1}}{Z_{fe,h}}\right)^2 \frac{R_{fe,h}}{R_{fe,1}} = \left(\frac{E_{ph}}{E_{p1}}\right)^2 \frac{R_{fe,1}}{R_{fe,h}}, \qquad (6\text{-}17)$$

where

$$Z_{fe,1} = \frac{(j\omega_1 L_m + R_{fe,1})(R_{p1} + j\omega_1 L_p)}{j\omega_1 L_m} + R_{fe,1},$$

$$Z_{fe,h} = \frac{(jh\omega_1 L_m + R_{fe,h})(R_{ph} + jh\omega_1 L_p)}{jh\omega_1 L_m} + R_{fe,h}.$$

From [31] follows $\frac{R_{fe,h}}{R_{fe,1}} \approx h^{0.6}$, and therefore the iron-core losses are

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} \approx \left(\frac{E_{ph}}{E_{p1}}\right)^2 \left(\frac{1}{h^{0.6}}\right). \qquad (6\text{-}18)$$

Summing Eqs. 6-16 and 6-18, the total iron-core losses for a harmonic of order h of a three-phase induction machine become

$$\frac{W_{ohmic,h}}{W_{ohmic,1}} = \left(\frac{R_{ph} + R'_{sh}}{R_{p1} + R'_{s1}}\right)\left(\frac{V_{ph}}{V_{p1}}\right)^2 \frac{1}{h^2 Z^2_{start\,pu}} + \left(\frac{E_{ph}}{E_{p1}}\right)^2 \left(\frac{1}{h^{0.6}}\right), \qquad (6\text{-}19)$$

or for all occurring harmonics, one can show that the total harmonic losses $W_{totalh\Sigma}$ relate to the total fundamental losses, as defined by the weighted harmonic factor [32]:

$$\frac{W_{totalh\Sigma}}{W_{total1}} = K_2 \sum_{h=2}^{\infty} \frac{1}{(h)^k} \left(\frac{V_{ph}}{V_{p1}}\right)^l, \qquad (6\text{-}20)$$

where $0 \le k \le 2$ and $0 \le l \le 2$.

Note that the relations for single-phase transformers (Eq. 6-12) and that for a three-phase induction motor (Eq. 6-20) are identical in structure, though the exponents k and l may assume different values for single-phase transformers and three-phase induction motors. These exponents will be identified in the next section and it will be shown that the same structure of the weighted harmonic factor will also be valid for three-phase transformers and single-phase induction machines as well as universal machines.

Depending on the values of k and l one obtains different loss or temperature dependencies as a function of the frequency. Unfortunately, these dependencies can vary within wide ranges from constant to hyperbolic functions. This is so because the iron-core losses depend on the electric steel (e.g., conductivity, lamination thickness, permeability) used and the winding configurations in primary or stator and secondary or rotor. These can exhibit different conductivities (e.g., Cu, Al) or skin and proximity effects. In order to study these dependencies various k and l combinations will be assumed, as is depicted in Figs. 6.5 to 6.7:

- $k = 0$ and $l = 1$ results in a linear frequency-independent characteristic (Fig. 6.5).
- $k = 1$ and $l = 1$ results in a inverse frequency-dependent characteristic (Fig. 6.6).
- $k = 2$ and $l = 2$ results in a quadratic inverse frequency-dependent characteristic (Fig. 6.7).

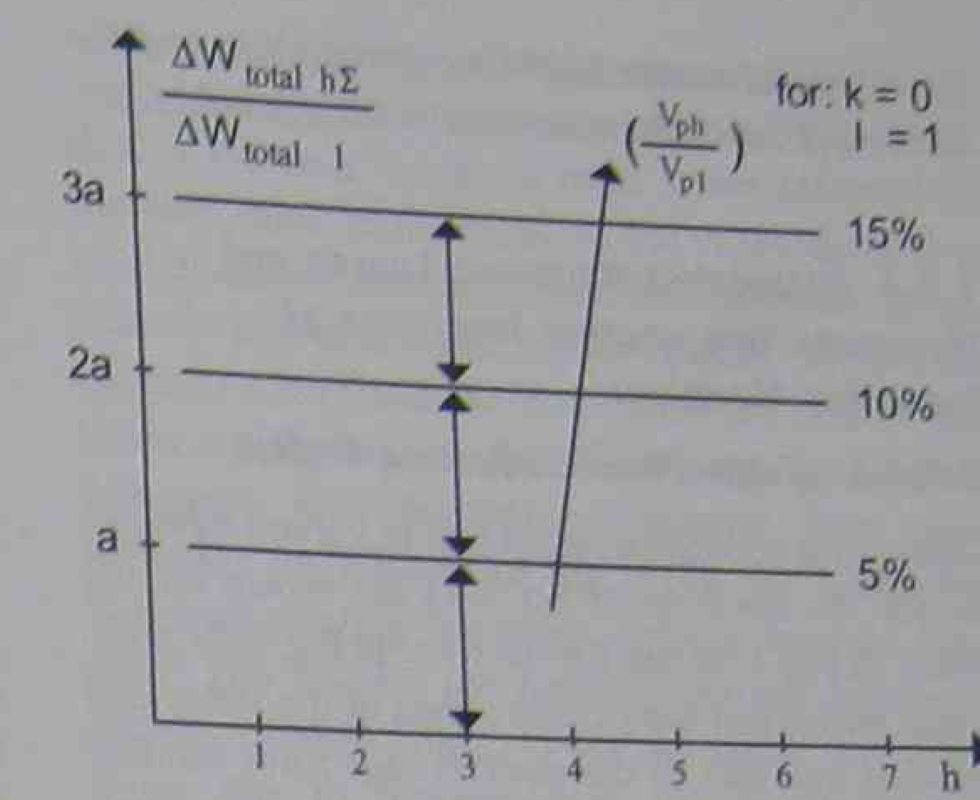


FIGURE 6.5 Linear frequency-independent increase of harmonic losses with harmonic voltage V_{ph}.

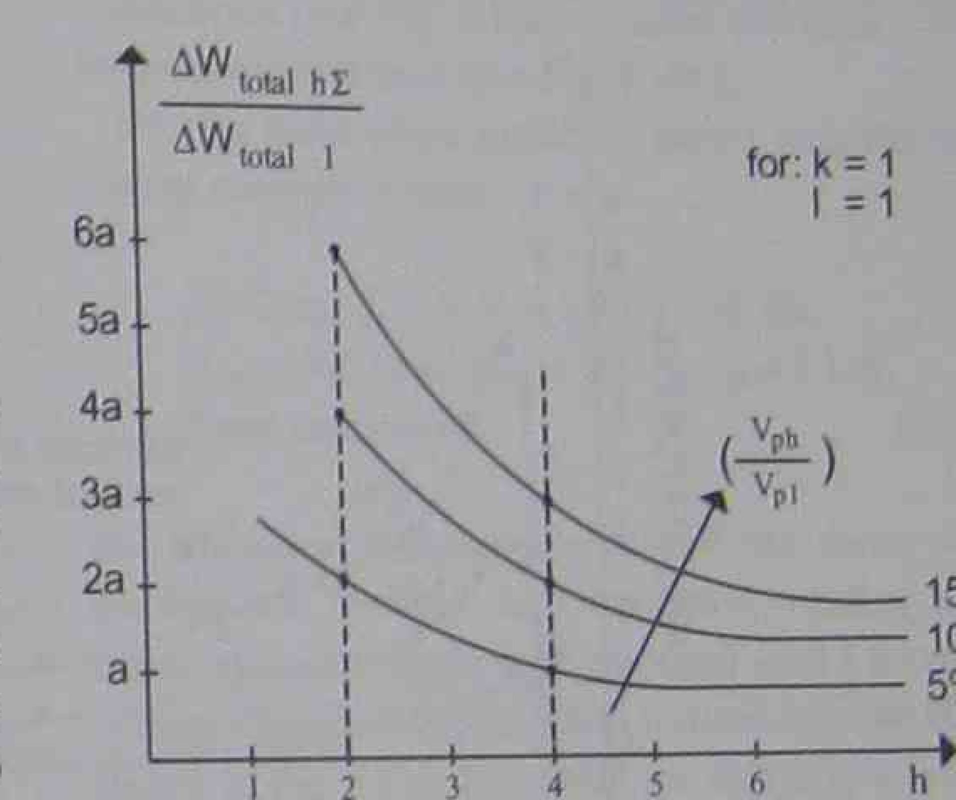


FIGURE 6.6 Inverse frequency-dependent increase of harmonic losses with harmonic voltage V_{ph}.

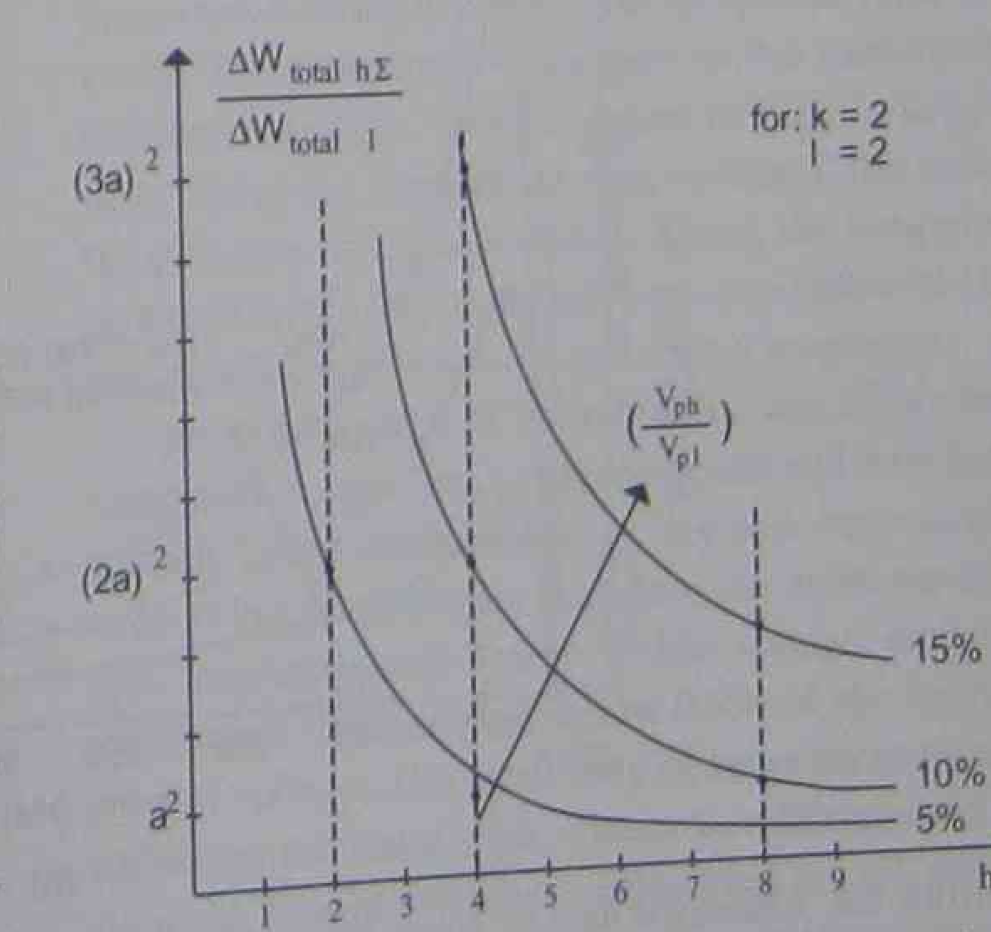


FIGURE 6.7 Quadratic inverse frequency-dependent increase of harmonic losses with harmonic voltage V_{ph}.

In real transformers and induction machines combinations of these characteristics will be possible.

6.3.4 Calculated Harmonic Losses and Measured Temperature Increases of Induction Machines

6.3.4.1 Single-Phase Induction Motors

Calculated additional losses due to voltage harmonics for the machine of Eq. 6-21 are shown for the stator in Fig. 6.8a and for the rotor in Fig. 6.8b. Measured additional temperature rises of the stator end winding and that of the squirrel-cage rotor winding at full load as a function of the harmonic voltage amplitude, phase shift, and frequency for the same single-phase induction motor are depicted in Fig. 6.9a and 6.9b, respectively. Note that the maximum additional temperature rises are obtained when the peak-to-peak value of the terminal voltage is a maximum as a function of the superposed harmonic voltage with the fundamental voltage.

$$P = 2\text{ hp},\ V_r = 115/208\text{ V},\ I_r = 24/12\text{ A},\ f = 60\text{ Hz},\ \eta = 0.73,\ n_{rat} = 1725\text{ rpm}. \qquad (6\text{-}21)$$

Figure 6.8a,b shows that the additional harmonic losses due to subharmonic voltages and interharmonics below the fundamental become very large even at low percentages and, therefore, the subharmonic voltages must be limited to below 0.5% of the fundamental. Note that at the slip frequency which corresponds to a slip of $s = 0$ the total stator losses reach a minimum (see Fig. 6.8a) because the total rotor losses are about zero (see Fig. 6.8b).

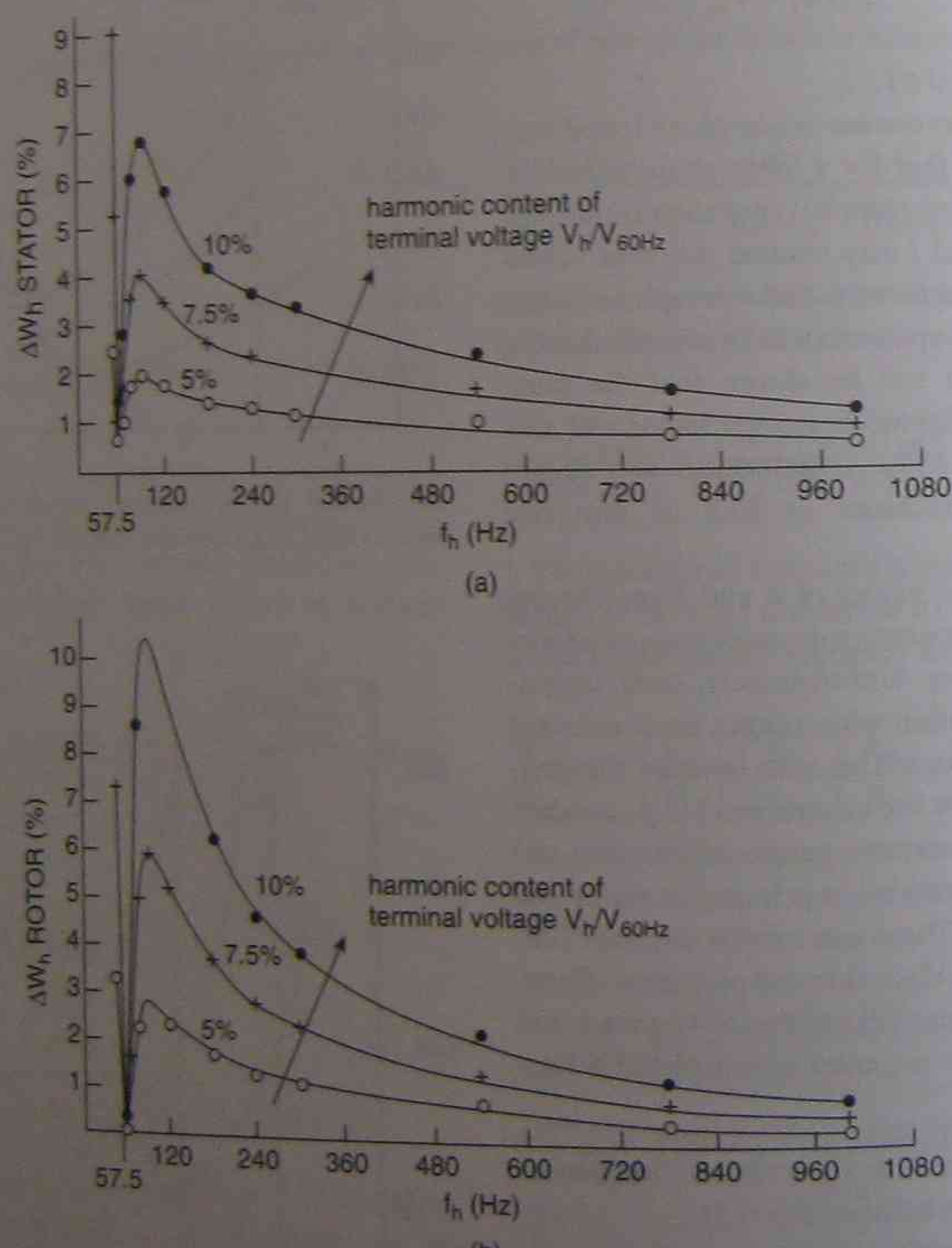


FIGURE 6.8 Calculated total harmonic losses of (a) the stator referred to the rated losses of the stator and (b) the rotor referred to the rated losses of the rotor as a function of the harmonic frequency h for the single-phase machine of Eq. 6-21 [33].

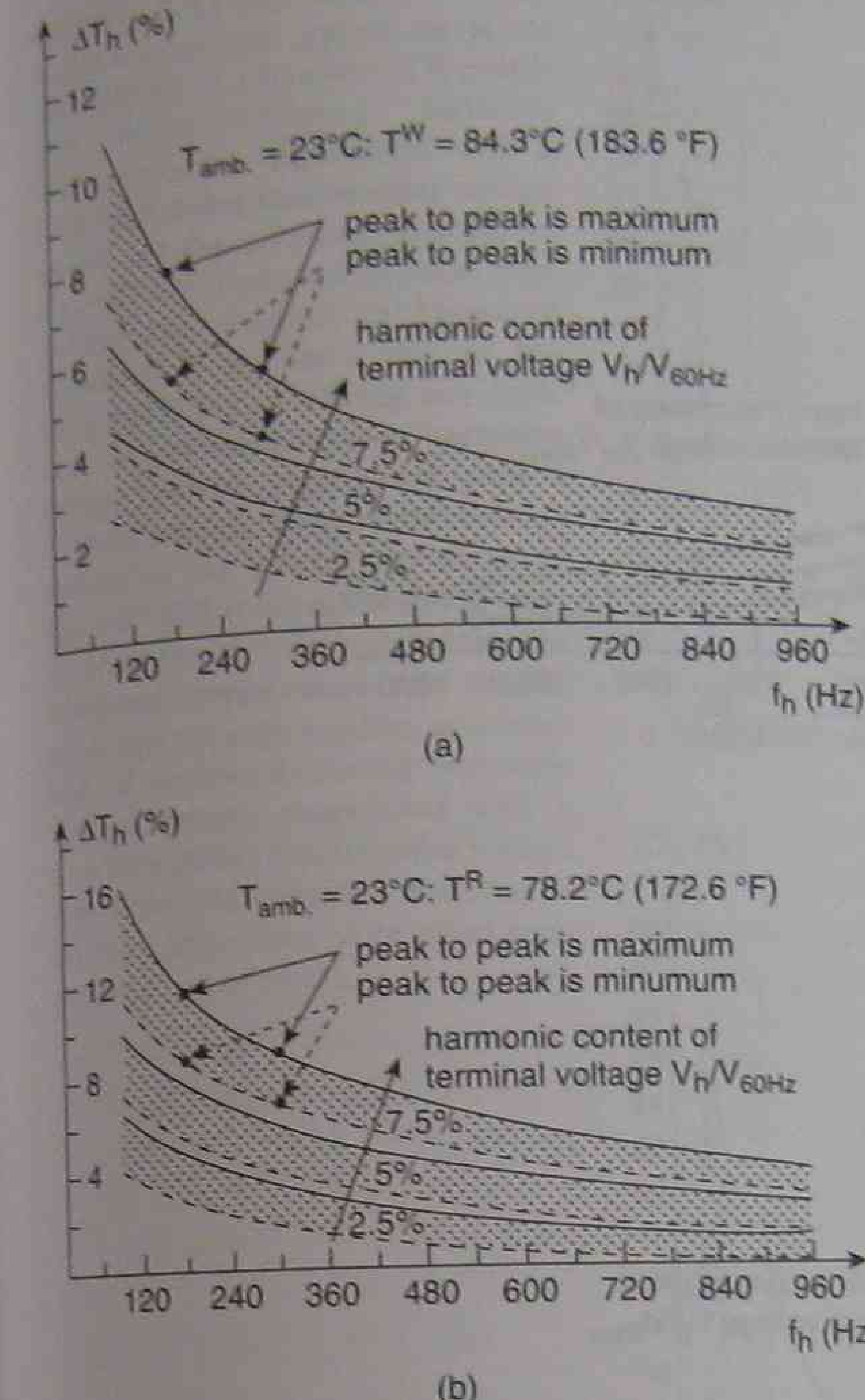


FIGURE 6.9 (a) Measured additional temperature rise of the stator-end winding at full load as a function of the harmonic voltage amplitude, phase shift, and frequency for the single-phase machine of Eq. 6-21 [33] (referred to rated temperature rise of stator). (b) Measured additional temperature rise of the squirrel-cage rotor winding at full load as a function of the harmonic voltage amplitude, phase shift, and frequency for the single-phase machine of Eq. 6-21 [33] (referred to rated temperature rise of rotor).

6.3.4.2 Three-Phase Induction Motors

Figure 6.4a,b illustrates the equivalent circuit of three-phase induction machines for the fundamental and time harmonics of order h (integer, sub-, and interharmonics). The total harmonic losses were calculated [18, 19] for a three-phase induction machine with the following nameplate data and equivalent circuit parameters:

$$P = 800\text{ W},\ p = 4\text{ poles},\ n_{rat} = 1738\text{ rpm},\ f = 60\text{ Hz},\ V_{L\text{-}L} = 220/380\text{ V},\ \Delta/Y\text{ connected},\ I_{ph} = 2.35\text{ A},\ R_{s1} = 7.0\ \Omega,\ X_{s1} = 8.0\ \Omega,\ X_{m1} = 110.0\ \Omega,\ R'_{r1} = 4.65\ \Omega,\ X'_{r1} = 7.3\ \Omega. \qquad (6\text{-}22)$$

Figure 6.10a shows the total harmonic stator losses referred to the total rated stator losses and Fig. 6.10b depicts the total harmonic rotor losses referred to the total rated rotor losses. In Fig. 6.10a there is a local maximum around 360 Hz; this maximum stems from the iron-core losses and its location depends on the lamination thickness of the iron-core sheets. The losses rapidly increase for subharmonics and interharmonics with decreasing frequency. The total harmonic rotor losses of Fig. 6.10b are larger for backward-rotating harmonic voltage systems (full lines) and smaller for forward-rotating harmonic systems (dashed lines). As for the stator, the rotor losses due to subharmonics and interharmonics below 60 Hz increase greatly with decreasing frequency. Note that at the slip frequency which corresponds to a slip of $s = 0$ the total stator losses reach a minimum (see Fig. 6.10a) because the total rotor losses are about zero (see Fig. 6.10b).

For the three-phase induction motor with the following nameplate data:

$$P = 2\text{ hp},\ q_1 = 3;\ N_o = \text{LPF},\ f = 60\text{ Hz},\ n_{rat} = 1725\text{ rpm},\ V_{L\text{-}L} = 200\text{ A},\ I_{ph} = 7.1\text{ A},\ \text{time: continuous}. \qquad (6\text{-}23)$$

the additional temperature rise of the stator end winding for forward- and backward-rotating harmonic voltage systems superimposed with a forward voltage system of fundamental frequency of 60 Hz is shown in Fig. 6.11a. Note that for subharmonic and interharmonics below 60 Hz the stator temperature increases rapidly. The corresponding temperature rises of the squirrel-cage winding of the rotor are depicted in Fig. 6.11b and 6.11c for forward- and backward-rotating harmonic voltage systems, respectively. The temperature rise due to the backward-rotating harmonic voltage system is slightly larger than that of the forward-rotating system, if the rotor temperature rise is considered. Again, the temperature rises due to sub- and interharmonics below 60 Hz are rapidly increasing with decreasing frequency.

Figures 6.10 and 6.11 illustrate that the additional temperature rises due to subharmonic and interharmonics below 60 Hz voltages become very large, even at low percentages; therefore, the subharmonic and interharmonics below 60 Hz voltage components must be limited to below 0.5% of the fundamental voltage. The sensitivity of large three-phase induction machines with respect to additional temperature rises has also been calculated in [34], and it is recommended to limit the subharmonic voltages to 0.1%.

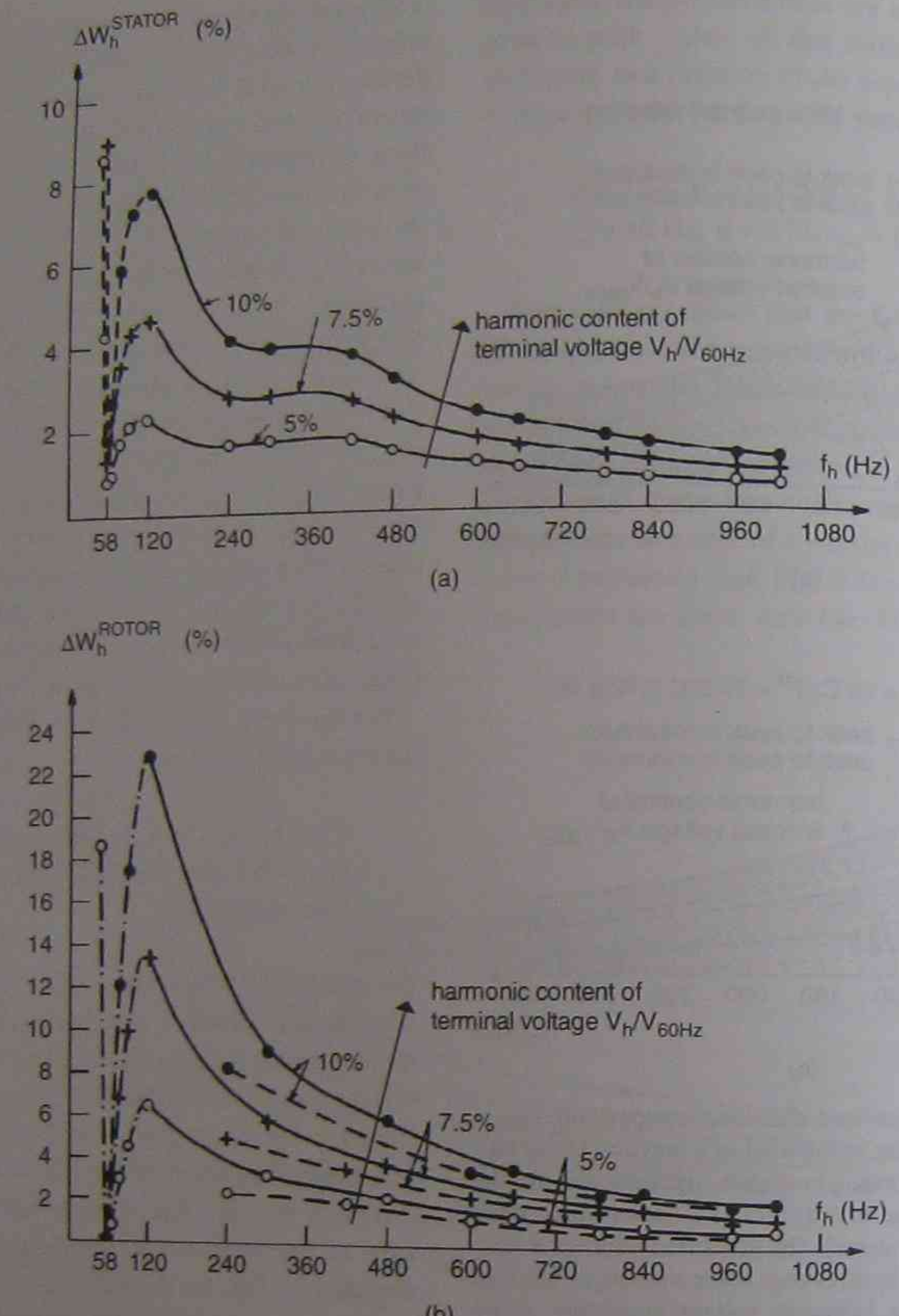


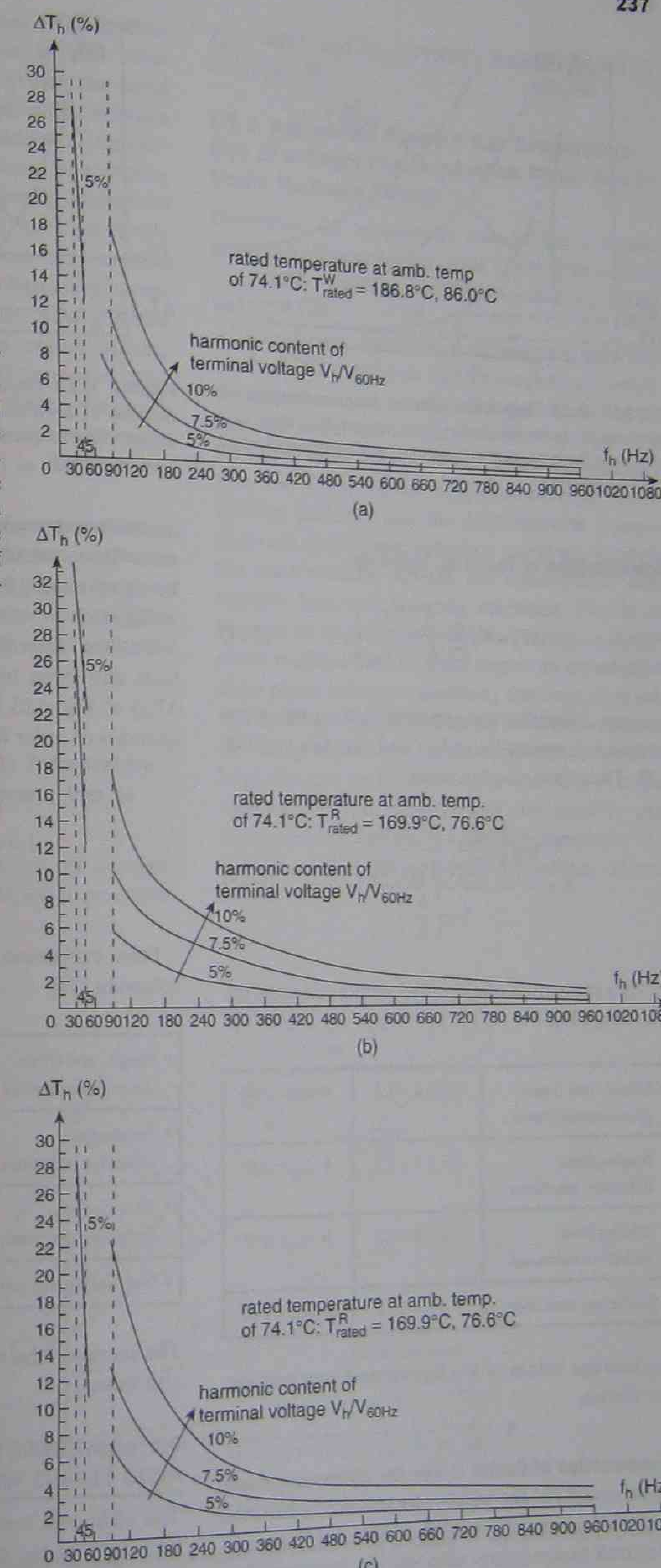
FIGURE 6.10 (a) Calculated total harmonic stator losses referred to the total rated stator losses for the machine of Eq. 6-22 [30]. (b) Calculated total harmonic rotor losses referred to the total rated rotor losses for the machine of Eq. 6-22 [30].

6.4 EXPONENTS OF WEIGHTED-HARMONIC FACTORS

In references [18, 19], the functional dependencies of the additional losses and the additional temperature rises due to harmonics of the terminal voltage are calculated for linear circuits and measured for actual machines, respectively. Thereby it is assumed that the additional temperature rise is proportional to the additional losses. Inspecting these loss and temperature functions, one notes that they can be approximated by hyperbolas for $h \geq 1.0$, as shown in Fig. 6.12.

For $h < 1$ corresponding to the slip frequency where the stator field is in synchronism with the mechanical rotor no voltage $\tilde{E}$ will be induced in the rotor and the rotor currents are zero; that is, the rotor loss is zero and the total motor loss is at a minimum, as is indicated in Fig. 6.12. At very low inter- or subharmonic frequencies, say, 3 Hz, the magnetizing reactance is very small and the 3 Hz current becomes very large, resulting in large total losses and harmonic torques as discussed in Chapter 3.

FIGURE 6.11 (a) Measured additional temperature rise of the stator end winding as a function of forward- and backward-rotating harmonic voltage systems superposed with a forward-rotating fundamental voltage system for the induction machine of Eq. 6-23 [30] (referred to rated temperature rise of stator). (b) Measured additional temperature rise of the rotor squirrel-cage winding as a function of forward-rotating harmonic voltage systems superposed with a forward-rotating fundamental voltage system for the induction machine of Eq. 6-23 [30] (referred to rated temperature rise of rotor). (c) Measured additional temperature rise of the rotor squirrel-cage winding as a function of backward-rotating harmonic voltage systems superposed with a forward-rotating fundamental voltage system for the induction machine of Eq. 6-23 [30] (referred to rated temperature rise of rotor).



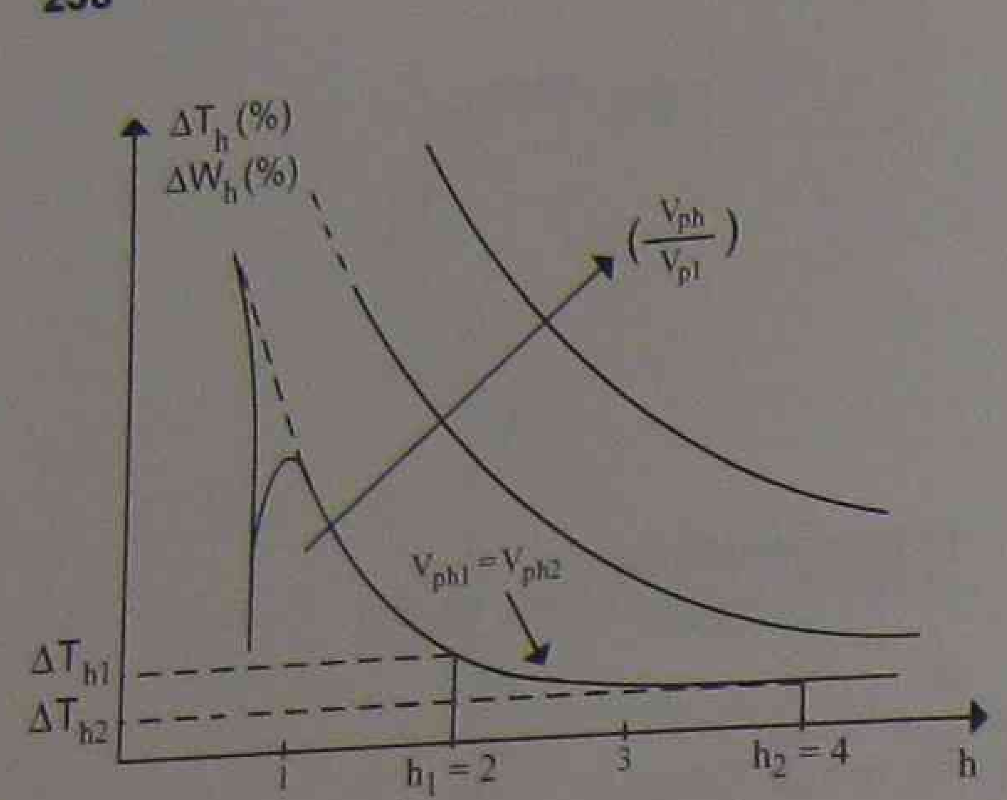


FIGURE 6.12 Hyperbolic inverse frequency-dependent increase of harmonic losses with harmonic voltage V_{ph}, except at the frequency corresponding to mechanical speed (slip frequency).

Determination of Factor k. Knowing

$$\Delta T_h = K_1 \sum_{h=2}^{\infty} \frac{1}{(h)^k} \left(\frac{V_{ph}}{V_{p1}} \right)^l, \tag{6-24}$$

one can determine the exponent k from two given (measured) points (h_1, ΔT_{h1}) and (h_2, ΔT_{h2}) of Fig. 6.12. Therefore, one can write

$$k = \frac{\log\left\{ \frac{\Delta T_{h2}}{\Delta T_{h1}} \left(\frac{(V_{ph1}/V_{p1})^l}{(V_{ph2}/V_{p1})^l} \right) \right\}}{\log\left(\frac{h_1}{h_2} \right)}. \tag{6-25}$$

From evaluation of measured points one finds the following [33]:

• Single- and three-phase transformers	$0.6 \le k \le 1.2$	▶ $k_{avg} = 0.90$
• Single-phase induction machines	$0.5 \le k \le 1.2$	▶ $k_{avg} = 0.85$
• Three-phase induction machines	$0.7 \le k \le 1.2$	▶ $k_{avg} = 0.95$
• Universal machines	$0.8 \le k \le 1.2$	▶ $k_{avg} = 1.00$

The average values of $k = k_{avg}$ for each category are also shown.

Determination of Factor *l*. For the determination of the exponent l of the weighted-harmonic factors, the functional dependencies of the additional losses and measured temperature rises on the harmonic fre-

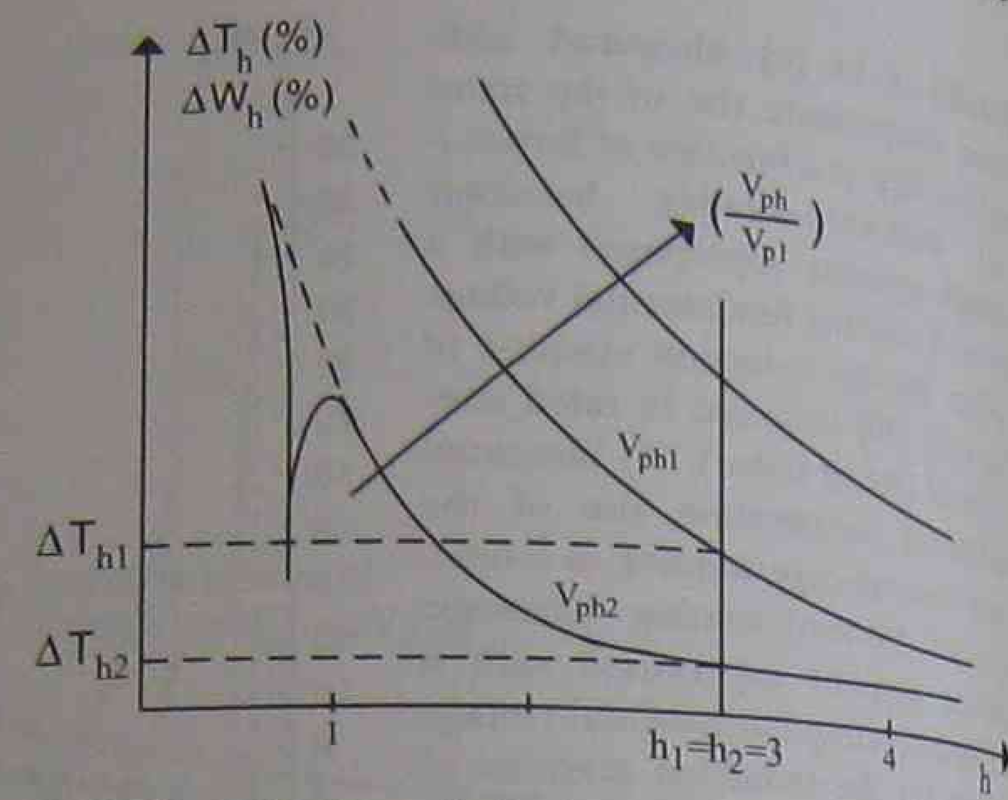


FIGURE 6.13 Hyperbolic inverse frequency-dependent increase of harmonic losses with harmonic voltage V_{ph}, except at the frequency corresponding to mechanical speed (slip frequency).

quencies and amplitude of references [18, 19] can be used. These calculated and measured functions can be approximated by a family of hyperbolas, as shown in Fig. 6.13.

One can determine the values of the exponent l from any given two points (V_{ph1}, ΔT_{h1}) and (V_{ph2}, ΔT_{h2}) of Fig. 6.13 for any given two harmonic frequencies of order h_1 and h_2:

$$l = \frac{\log\left\{ \frac{\Delta T_{h2}}{\Delta T_{h1}} \left(\frac{h_1}{h_2} \right)^k \right\}}{\log\left(\frac{V_{ph1}}{V_{ph2}} \right)}. \tag{6-26}$$

From evaluation of measured points one finds the following [33]:

• Single- and three-phase transformers	$1.50 \le l \le 2.0$	▶ $l_{avg} = 1.75$
• Single-phase induction machines	$1.0 \le l \le 1.80$	▶ $l_{avg} = 1.40$
• Three-phase induction machines	$1.2 \le l \le 2.0$	▶ $l_{avg} = 1.60$
• Universal machines	$1.5 \le l \le 2.5$	▶ $l_{avg} = 2.00$

The average values $l = l_{avg}$ for each category are also shown.

6.5 ADDITIONAL LOSSES OR TEMPERATURE RISES VERSUS WEIGHTED-HARMONIC FACTORS

The additional losses and temperature rises due to harmonics of the terminal voltage of transformers, induction machines, and universal machines are cal-

culated and measured in references [18, 19]. Provided these additional losses due to such time harmonics are small as compared with the rated losses ($W_{totalh\Sigma} \ll W_{total1}$), a proportionality between the additional losses and the additional temperature rises can be assumed because the cooling conditions are not significantly altered. Calculations and measurements show that the previously mentioned electromagnetic devices are sensitive to voltage harmonics in the frequency range $0 \le f_h \le 1500$ Hz. Harmonics of low order generate the largest additional loss. The correspondence of a given value of the weighted-harmonic factor (to an additional loss and temperature rise) leads to the additional temperature rise (or loss) versus the harmonic-factor function [20] as shown in Fig. 6.14.

With assumed values for the average exponents k_{avg} and l_{avg} as they apply to transformers, induction and universal machines, one can compute for given percentages of the harmonic voltages the weighted-harmonic factor and associate with it the additional losses or temperature rises.

6.5.1 Application Example 6.1: Temperature Rise of a Single-Phase Transformer Due to Single Harmonic Voltage

Determine the temperature rise ΔT_h of a single-phase transformer provided $V_3 = 0.10$ pu $= 10\%$, $T_{amb} = 40°C$, and $T_{rated} = 100°C$. Assume $k_{avg} = 0.90$ and $l_{avg} = 1.75$.

6.5.2 Application Example 6.2: Temperature Rise of a Single-Phase Induction Motor Due to Single Harmonic Voltage

Determine the temperature rise ΔT_h of a single-phase induction motor provided $V_3 = 0.10$ pu $= 10\%$, $T_{amb} = 40°C$, and $T_{rated} = 100°C$. Assume $k_{avg} = 0.85$ and $l_{avg} = 1.40$.

Transformers, induction machines, and universal machines have different loss or temperature sensitivities with respect to voltage harmonics and, therefore, the additional temperature rises are different for all five types of devices. The additional temperature rise (loss) versus weighted-harmonic factor function indicates that the most sensitive components are single-phase induction machines, whereas the least sensitive devices are transformers with resistive load and universal machines. This is so because for any harmonic terminal voltage, a single-phase machine (and to some degree an unbalanced three-phase induction machine) develops forward- and backward-rotating harmonic fields and responds like being under short-circuit conditions due to the large slip s_h of the harmonic field with respect to the rotating rotor. In transformers the resistive and inductive loads can never have zero impedance due to the nature of the frequency dependency of such loads.

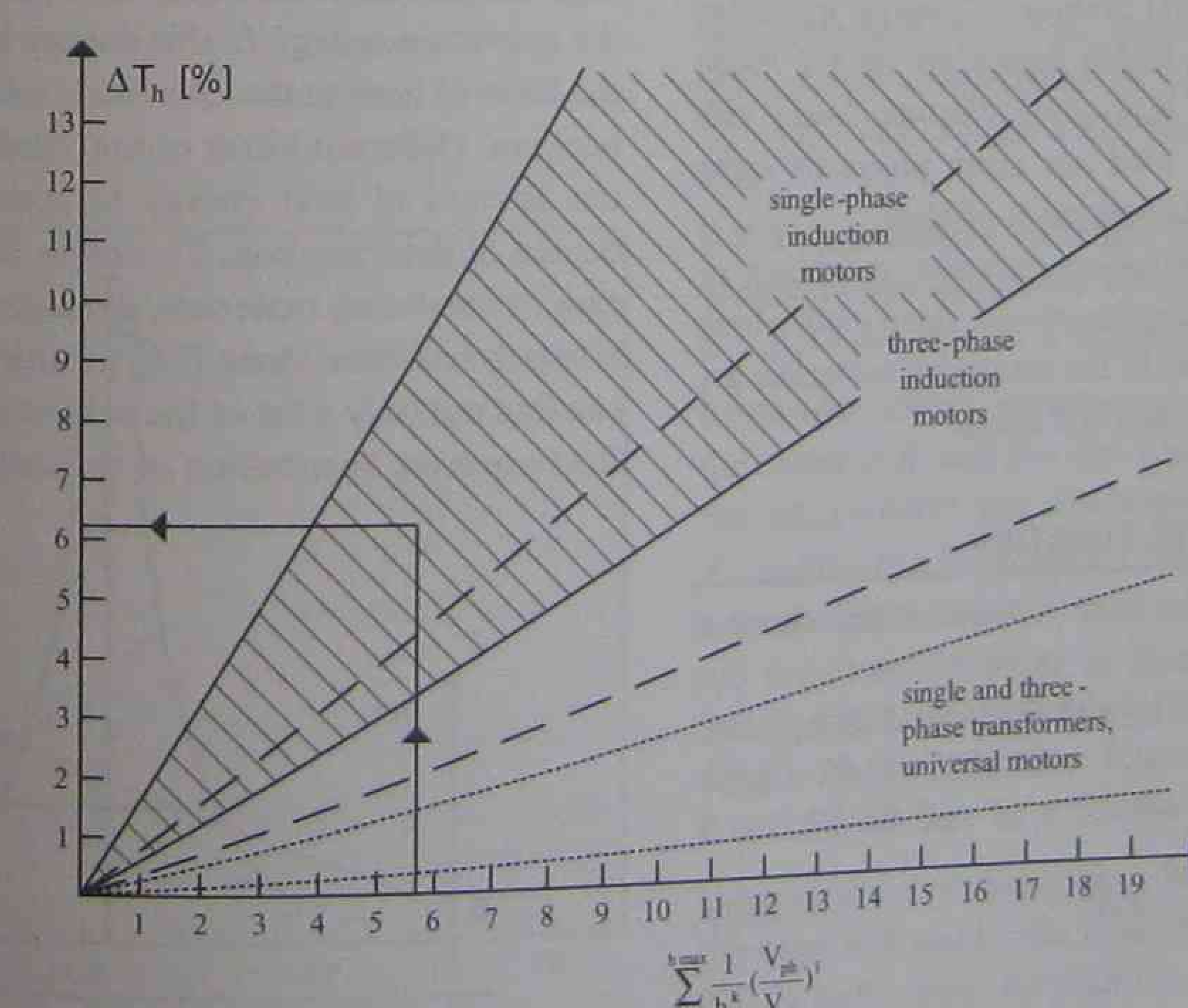


FIGURE 6.14 Additional temperature rise (or loss) versus weighted-harmonic factor function for universal motors, single- and three-phase transformers, and induction motors [33].

6.6 ARRHENIUS PLOTS

A series of studies have investigated the behavior of various electric machines and transformers as they occur in a residential distribution system and are exposed to harmonics of the terminal voltage [18, 19]. The influence of such harmonics expresses itself, among others (e.g., mechanical vibration), in an elevated (additional) temperature rise of the machine windings and iron cores. The question arises how the lifetime of the machines will be affected by such additional temperature rises.

Thermal Aging. The insulating material of an electric apparatus as used in electrical appliances is of organic or inorganic origin. Due to the heating of these materials, caused by the loss within the machine, a deterioration of the insulating materials will occur. This deterioration is manifested either by

- lowering of the mechanical strength, and/or
- changing of the dielectric behavior of the insulating material.

It may be mentioned that not only the heat itself, but also small motions due to expansion and contraction of the wire and iron laminations, are causing deteriorations by mechanical friction. In addition, time harmonics induce small core vibrations that may aggravate the mechanical stresses. The mechanical failure of the insulation is a result of the decrease of the tensile strength or flexibility of the insulation material. The thermal lifetime of electric machines and transformers is highly dependent on the mode of utilization; there is no doubt that machines with frequently variable load are more prone to aging from a mechanical failure point of view.

All further investigations consider machines or transformers operating at constant rated load, where the chemical changes of the insulating materials are responsible only for thermal aging.

6.7 REACTION RATE EQUATION

For many years it has been recognized that thermal degradation of organic or inorganic materials can be best represented by the reaction rate equation [24, 25]

$$\frac{dR}{dt} = Ae^{(-E/KT)}, \qquad (6\text{-}27)$$

In this equation, dR/dt is the reduction in property R with respect to time, A is a constant of integration, K is the gas constant or, depending on the units, the Boltzmann constant, T is the absolute temperature in kelvins, and E is the activation energy of the aging reaction (large E leads to fast aging, small E leads to slow aging).

Equation 6-27 expresses the rule of chemical reactions which was derived by Svante Arrhenius in 1880. The form of the original Arrhenius formula can be obtained from the differential equation by integration as follows:

$$dR = Ae^{-(E/KT)}dt,$$

$$\int dR = A\int e^{-(E/KT)}dt,$$

$$R = Ae^{-(E/KT)}dt,$$

$$\frac{R}{t} = Ae^{-(E/KT)},$$

$$\ln(R/t) = \ln(Ae^{-(E/KT)}),$$

$$\ln R - \ln t = \ln A - E/KT,$$

or

$$\ln t = \left(\frac{E}{K}\right)\frac{1}{T} + B. \qquad (6\text{-}28)$$

The plot ln t versus $1/T$ is a straight line with the slope E/K, if for the lifetime t a logarithmic scale is employed, as shown in Fig. 6.15.

Equation 6-28 expresses that the logarithm of the degradation (decreased life) time t is proportional to the activation energy E: this energy is transmitted in the form of heat to the chemical reaction or decomposition. Different kinds of materials need a different amount of heat energy to arrive at the same degree of deterioration. To obtain these data in the case of insulating materials, extensive experimental research has been done [24]. In this reference, one can find not only a list of the activation energies, but the frequency distribution of the activation energies of various materials. Properties monitored for these materials include flexural strength, impact strength, and dielectric strength. In Fig. 4 of [24], one notes the number of materials for each increment of 0.1 eV activation energy (see Fig. 6.16). The peak of this distribution curve occurs at about $E = 1.15$ eV.

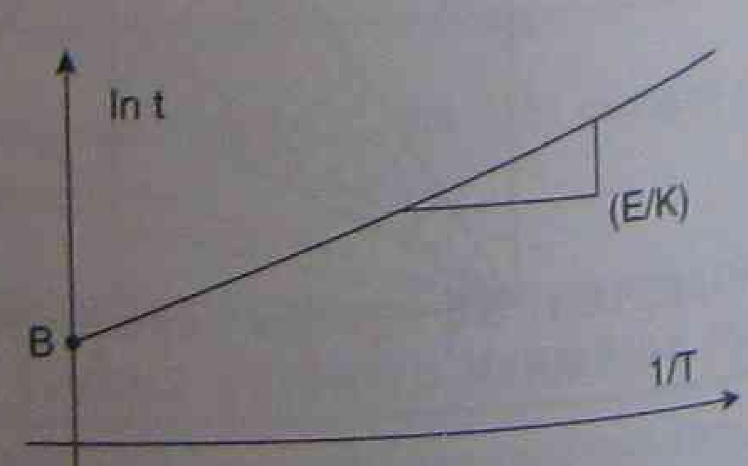


FIGURE 6.15 Lifetime t versus the inverse of the absolute temperature T.

The practical use of the Arrhenius plot and the consequences of the data of Fig. 4 of [24] will be discussed in the next section.

6.8 DECREASE OF LIFETIME DUE TO AN ADDITIONAL TEMPERATURE RISE

The slope of the Arrhenius plot based on

$$\ln t = \left(\frac{E}{K}\right)\frac{1}{T} + B \qquad (6\text{-}29)$$

is the most important quantity (proportional to the activation energy E) for our investigation; this is the only quantity that determines the aging of the insulation material. Therefore, we have to use the slope of the Arrhenius plot as a measure of aging.

If one knows from experiments two (different) points of the plot belonging to temperatures T_1 and T_2 and lifetimes t_1 and t_2, respectively, that is, (T_1, t_1) and (T_2, t_2), then one can obtain for a given insulation material

$$\ln t_1 - \ln t_2 = \left(\frac{E}{K}\right)\left(\frac{1}{T_1} - \frac{1}{T_2}\right). \qquad (6\text{-}30)$$

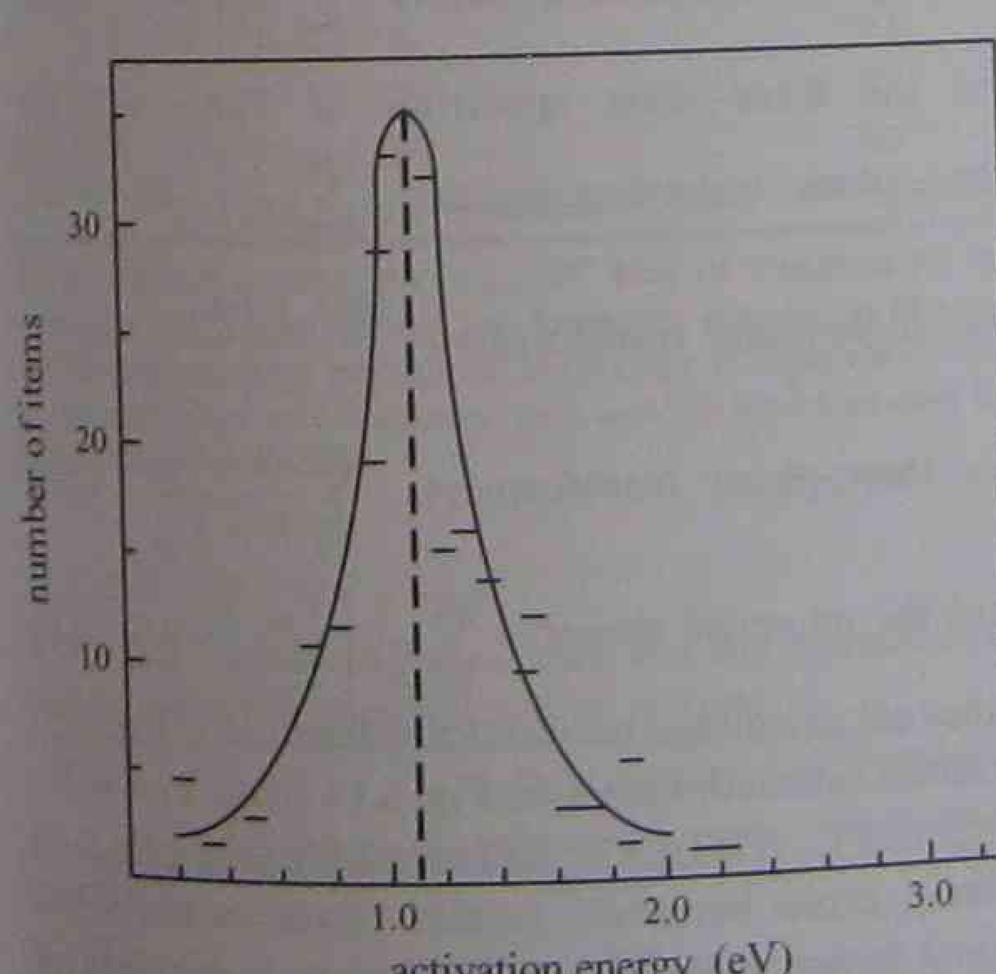


FIGURE 6.16 Frequency distribution of activation energies of various organic and inorganic materials [24].

It must be noted that if the activation energy E is measured in electron volts one has to use for K the Boltzmann constant: $K = 1.38 \cdot 10^{-23}$ J/Kelvin; in case the activation energy is given in kilocalories per mole the constant K will be the gas constant $K = R = 19.84 \cdot 10^{-4}$ kcal/mol. Note that 1 eV $= 1.602 \cdot 10^{-19}$ J or 1 $J = 0.624 \cdot 10^{19}$ eV.

Suppose one knows the rated lifetime t_2 of an apparatus and its rated (constant) temperature T_2 at which it is operating. The question arises to what extent the lifetime decreases provided the elevated temperature becomes $T_1 = T_2 + \Delta T$ where $\Delta T = \Delta T_h$. To answer this question one has to substitute in Eq. 6-30 T_1 by $T_2 + \Delta T$:

$$\ln t_1 - \ln t_2 = \left(\frac{E}{K}\right)\left(\frac{1}{T_2 + \Delta T} - \frac{1}{T_2}\right). \qquad (6\text{-}31)$$

After some manipulations, the new decreased lifetime is

$$t_1 = t_2 e^{-\left(\frac{E}{K}\right)\frac{\Delta T}{T_2(T_2+\Delta T)}}, \qquad (6\text{-}32)$$

where t_2 is the rated lifetime, T_2 is the rated temperature in kelvins, and ΔT is the (additional) temperature rise in degrees Celsius. Several examples will illustrate the use of the above relation as applied to the calculation of the decrease of the lifetime due to the additional temperature rise caused by the additional harmonic losses. In all following examples it is assumed that the rated lifetime of the apparatus is $t_2 = 40$ years, and the steady-state rated temperature of the hottest spot is $T_2 = 100°C \equiv 273 + 100 = 373$ °Kelvin.

6.8.1 Application Example 6.3: Aging of a Single-Phase Induction Motor with E = 0.74 eV Due to a Single Harmonic Voltage

Determine for an activation energy of $E = 0.74$ eV the slope E/K and for the additional temperature rise $\Delta T_{h=3} = 6.6°C$ (see Application Example 6.2 with $T_{amb} = 40°$, $T_2 = T_{rat} = 100°C$) the reduced lifetime of a single-phase induction motor.

6.8.2 Application Example 6.4: Aging of a Single-Phase Induction Motor with E = 0.51 eV Due to a Single Harmonic Voltage

Determine for an activation energy of $E = 0.51$ eV the slope E/K and for the additional temperature rise $\Delta T_{h=3} = 6.6°C$ (see Application Example 6.2 with $T_{amb} = 40°$, $T_2 = T_{rat} = 100°C$) the reduced lifetime of a single-phase induction motor.

6.9 REDUCTION OF LIFETIME OF COMPONENTS WITH ACTIVATION ENERGY E = 1.1 EV DUE TO HARMONICS OF THE TERMINAL VOLTAGE WITHIN RESIDENTIAL OR COMMERCIAL UTILITY SYSTEMS

Figure 6.14 shows the additional temperature rises (or losses) in percent of the rated temperature rises (or losses) as a function of the weighted-harmonic factor. These functions are obtained from calculations and measurements [18, 19]. Note that the ambient temperature has been eliminated. For the evaluation of the reduction of the lifetime one must base all calculations on an activation energy of about $E = 1.1$ eV because of Fig. 6.16, which confirms that the majority of (insulation) materials have such a distinct activation energy.

According to Fig. 6.14, a weighted harmonic-voltage factor of $\sum_{h=2}^{h\max} \frac{1}{h^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l = 5.8$ corresponds to additional temperature rises (referred to the rated temperature rises) on the average of 6.2% for single-phase induction machines, 3.2% for three-phase induction machines, and 0.85% for transformers and universal machines. These percentage values confirm that single-phase machines are very sensitive to harmonic voltages due to their forward- and backward-rotating fields and their short-circuited rotor, three-phase induction machines are sensitive to voltage harmonics due to their short-circuited rotor, transformers are not very sensitive to voltage harmonics because of their resistive load, and universal machines are not sensitive as well because the commutator transforms a voltage source to a current source where the current magnitudes are limited. A weighted-harmonic voltage factor of 5.8 results for the rated temperature $T_2 = 85°C$ at $T_{amb60Hz} \approx T_{ambh} = 23°C$ in temperature increases of 3.84°C for single-phase induction machines, 1.98°C for three-phase induction machines, and 0.53°C for transformers and universal machines. With these additional temperature increases one obtains at an activation energy of $E = 1.1$ eV – with $E/K = 12{,}769$ Kelvin – the decreased lifetime (using Eq. 6-32) of 31.5% for single-phase induction machines, 18% for three-phase induction machines, and 5% for transformers and universal machines.

Conclusion. It is believed that the weighted-harmonic voltage factor for single-phase and three-phase induction motors in the neighborhood of 5.8 represents a compromise which on the one hand promotes the installation of solid-state circuits by choosing generous permissible harmonic voltage levels, and on the other hand avoids severe detrimental reactions on the majority of the residential or commercial power system components including loads. Harmonic spectra of the residential or commercial power system voltage that satisfy for single-phase and three-phase induction motors

$$\sum_{h=2}^{h\max} \frac{1}{h^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l \approx 5.8. \qquad (6\text{-}33)$$

result in acceptable temperature rises as far as induction motors, transformers, and universal machines are concerned. Thus it is recommended not to rigidly fix the harmonic voltages but rather the additional temperature rises they generate.

6.10 POSSIBLE LIMITS FOR HARMONIC VOLTAGES

In order to illustrate the use of Eqs. 6-33 and 6-32, the harmonic voltage spectra for a single-phase and for a three-phase feeder (see Table 6.1) are proposed [33].

With average values of $k_{avg} = 0.85$ and $l_{avg} = 1.4$ for single-phase induction motors, $k_{avg} = 0.95$ and $l_{avg} = 1.6$ for three-phase induction motors, $k_{avg} = 0.90$ and $l_{avg} = 1.75$ for single-phase and three-phase transformers, and $k_{avg} = 1.0$ and $l_{avg} = 2.0$ for universal machines, one obtains for the single-phase spectrum of Table 6.1 the weighted harmonic-voltage factor for single-phase induction motors $\sum_{h=2}^{h\max} \frac{1}{h^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l \approx 5.7$, for the three-phase spectrum of Table 6.1 for three-phase induction motors $\sum_{h=2}^{h\max} \frac{1}{h^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l \approx 5.7$, for single-phase transformers $\sum_{h=2}^{h\max} \frac{1}{h^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l = 7.37$, for three-phase transformers $\sum_{h=2}^{h\max} \frac{1}{h^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l = 7.4$, and for universal motors $\sum_{h=2}^{h\max} \frac{1}{h^k}\left(\frac{V_{ph}}{V_{p1}}\right)^l \approx 8.5$. Any other set of voltage harmonics is feasible if Eq. 6-33 is about satisfied. Based on Fig. 6.14 (with $T_2 = 85°C$, $T_{amb} = 23°C$, $E = 1.1$ eV, and rated lifetime of $t_2 = 40$ years), those harmonic factors result in the additional temperature rises and lifetime reductions of Table 6.2.

6.10.1 Application Example 6.5: Estimation of Lifetime Reduction for Given Single-Phase and Three-Phase Voltage Spectra with High Harmonic Penetration with Activation Energy E = 1.1 eV

Estimate the lifetime reductions of induction machines, transformers, and universal machines for the single- and three-phase voltage spectra of Table E6.5.1 and their associated lifetime reduction for an activation energy of $E = 1.1$ eV. The ambient temperature is $T_{amb} = 23°C$, the rated temperature is $T_2 = 85°C$, and the rated lifetime of $t_2 = 40$ years can be assumed.

TABLE 6.1 Possible Voltage Spectra of Single-Phase and Three-Phase Feeders

h	$\left(\frac{V_h}{V_{60Hz}}\right)_{1\Phi}$ (%)	$\left(\frac{V_h}{V_{60Hz}}\right)_{3\Phi}$ (%)
1	100	100
2	0.5	0.5
3	4.0	2.0[a]
4	0.3	0.5
5	3.0	5.0
6	0.2	0.2
7	2.0	3.5
8	0.2	0.2
9	1.0	0.3
10	0.1	0.1
11	1.5	1.5
12	0.1	0.1
13	1.5	1.0
14	0.1	0.05
15	0.5	0.1
16	0.05	0.05
17	1.0	0.5
18	0.05	0.01
19	1.0	0.5
All higher harmonics < 0.5%		

[a]Under certain conditions (e.g., DC bias of transformers as discussed in Chapter 2, and the harmonic generation of synchronous generators as outlined in Chapter 4) triplen harmonics are not of the zero-sequence type and can therefore exist in a three-phase system.

6.10.2 Application Example 6.6: Estimation of Lifetime Reduction for Given Single-Phase and Three-Phase Voltage Spectra with Moderate Harmonic Penetration with Activation Energy E = 1.1 eV

Estimate the lifetime reductions for induction machines, transformers, and universal machines for the single- and three-phase voltage spectra of Table E6.6.1 and their associated lifetime reduction for an activation energy of $E = 1.1$ eV. The ambient temperature is $T_{amb} = 23°C$, the rated temperature is $T_2 = 85°C$, and the rated lifetime of $t_2 = 40$ years can be assumed.

TABLE E6.5.1 Possible Voltage Spectra with High-Harmonic Penetration

h	$\left(\frac{V_h}{V_{60Hz}}\right)_{1\Phi}$ (%)	$\left(\frac{V_h}{V_{60Hz}}\right)_{3\Phi}$ (%)
1	100	100
2	2.5	0.5
3	5.71	1.0
4	1.6	0.5
5	1.25	7.0
6	0.88	0.2
7	1.25	5.0
8	0.62	0.2
9	0.96	0.3
10	0.66	0.1
11	0.30	2.5
12	0.18	0.1
13	0.57	2.0
14	0.10	0.05
15	0.10	0.1
16	0.13	0.05
17	0.23	1.5
18	0.22	0.01
19	1.03	1.0
All higher harmonics < 0.2%		

TABLE 6.2 Additional Temperature Rise and Associated Lifetime Reduction of Induction Motors, Transformers, and Universal Motors Due to the Harmonic Spectra of Table 6.1

	Single-phase induction motors	Three-phase induction motors	Single-phase transformers	Three-phase transformers	Universal motors
ΔT_h (%)	6.2	3.2	1.2	1.2	1.3
ΔT_h (°C)	3.84	1.98	0.74	0.74	0.81
Lifetime reduction (%)	31.5	17.8	7.1	7.1	7.7

TABLE E6.6.1 Possible Voltage Spectra with Moderate-Harmonic Penetration

h	$\left(\frac{V_h}{V_{60Hz}}\right)_{1\Phi}$ (%)	$\left(\frac{V_h}{V_{60Hz}}\right)_{3\Phi}$ (%)
1	100	100
2	0.5	0.5
3	3.0	0.5
4	0.3	0.5
5	2.0	3.0
6	0.2	0.2
7	1.0	2.5
8	0.2	0.2
9	0.75	0.3
10	0.1	0.1
11	1.0	1.0
12	0.1	0.1
13	0.9	0.85
14	0.1	0.05
15	0.3	0.1
16	0.05	0.05
17	0.5	0.3
18	0.05	0.01
19	0.4	0.2
All higher harmonics < 0.2%		

6.11 PROBABILISTIC AND TIME-VARYING NATURE OF HARMONICS

As mentioned in Section 6.1, the harmonic current or voltage spectra within a distribution feeder are continually changing as a function of changes in the load and the system's condition. The rationale given for using the worst-case conditions, where loads are operating for at least a few hours at rated operation generating the "rated current/voltage spectra," is the basis for the IEEE [35] and IEC [36] harmonic guidelines and standards, respectively. Nevertheless it will be worthwhile to study the time-varying nature [37, 38] of harmonic spectra. In these publications continually changing current (THD_i) and voltage (THD_v) total harmonic distortions are plotted as a function of time. They indeed change widely from 2.5% to about 9% for THD_i, and from 1% to about 5% for the THD_v for one site, and for another site the changes are similar in magnitude. As mentioned in prior sections, the THDs are neither a good measure for temperature rises nor for vibrations; instead harmonic components should be used in any further studies. The measured THD data are characterized by statistical measures (e.g., minimum, maximum, average or mean values, standard deviation, probability distribution), histograms or probability density functions (pdf), probability distribution functions ($PX(x)$), statistical description of sub-time intervals, and combined deterministic or statistical description [41–45]. These publications address the harmonic summation and propagation by explaining the sum of random harmonic phasors via the representation of harmonic phasors, marginal pdf of x–y components, pdf of sum of projections of independent phasors, and pdf of magnitude of sum of random phasors. These papers apply the methods described to a 14-bus transmission system and provide general guidelines with respect to their applicability. The probabilistic evaluation of the economic cost due to harmonic losses in industrial energy systems is presented in the next section.

6.12 THE COST OF HARMONICS

The cost of harmonics can originate either in

- the complicated solid-state components necessary to maintain the current or voltage harmonics at a low level, for example, switched-mode power supplies operating at unity-power factor [39], or in
- the use of simple peak-rectifiers resulting in high harmonic amplitudes [40].

Both costs are not negligibly small and must be considered in the future. The first approach appears to be favored by the IEC [36] for low-, medium-, and high-voltage power systems, whereas the latter appears to be favored by IEEE [35] for single-phase systems only.

In references [41–45] the cost of the latter approach is discussed by defining the economic damage (cost) due to harmonic losses $D = D_w + D_a$, where D_w is the operating cost and D_a is the aging cost. This economic damage is a probabilistic quantity because the harmonic voltage or current spectra change in a random manner. The expected value of these costs is defined as $E(D) = E(D_w) + E(D_a)$, where $E(D_w)$ is the present-worth expected value of the operating costs due to harmonic losses and $E(D_a)$ is the present-worth expected value of the aging costs due to harmonic losses. Numerical results for a 20 kV, 150 MVA industrial feeder illustrate that the economic damage for induction motors is significant and within a lifetime of 35 years amounts to about the purchase cost ($7,625) of an induction motor.

6.13 TEMPERATURE AS A FUNCTION OF TIME

Rotating machines, transformers, and inductors must be designed so that their rated temperatures will not be exceeded. For rotating machines the maximum torques are not a design criterion as long as the machines continue to operate. The maximum permissible temperature (hot spot) must not be exceeded, otherwise the lifetime will be reduced. Note that any machine works most efficiently and cost-effectively if the permissible (maximum) rated temperature will be reached but not exceeded at any time. At steady state machines must be able to operate at rated torque. For a short time machines can be operated above rated torque. This leads to the concept of intermittent operation discussed in a later section.

A motor can be replaced by a radiating sphere as illustrated in Fig. 6.17. During a small time increment dt one can assume that the temperature of the sphere increases by the incremental temperature $d\theta$. For this reason the change in stored heat (increase) dQ_C during the time dt is

$$dQ_C = C \cdot d\theta, \tag{6-34}$$

where θ is the temperature rise and C is the heat-absorption capacity. Note that Eq. 6-34 is independent of time.

Due to thermal radiation a part of the stored heat will be emitted to the surrounding environment. The change of the stored heat (reduction) dQ_A is time dependent:

$$dQ_A = A \cdot \theta \cdot dt, \tag{6-35}$$

where A is the heat-radiation capacity.

The sum of Eqs. 6-34 and 6-35 yields the ordinary differential equation

$$dQ = dQ_C + dQ_A = C \cdot d\theta + A \cdot \theta \cdot dt = P_{loss} \cdot dt, \tag{6-36}$$

where $P_{loss} \cdot dt$ is the (loss) energy absorbed by the machine during the incremental time dt. Dividing Eq. 6-36 by dt and A, one obtains the first-order ordinary differential equation

$$\frac{P_{loss}}{A} = \frac{C}{A} \cdot \frac{d\theta}{dt} + \theta = \tau_\theta \cdot \frac{d\theta}{dt} + \theta, \tag{6-37}$$

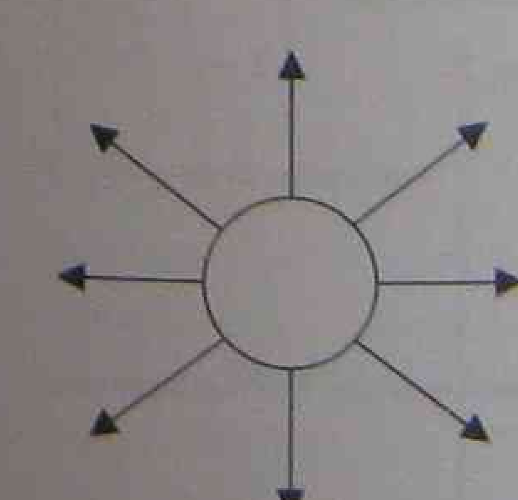


FIGURE 6.17 Rotating machine represented by radiating sphere.

where $\tau_\theta = (C/A)$ is the thermal time constant. The solution of this first-order differential equation is

$$\theta = \frac{P_{loss}}{A} + k \cdot e^{(-t/\tau_\theta)}. \tag{6-38}$$

For $t = 0$ and $\theta = \theta_o$ (initial temperature rise) and for $t = \infty$ with $\theta = \theta_f = P_{loss}/A$ (final temperature rise) one obtains for the temperature as a function of time

$$\theta = \theta_f - (\theta_f - \theta_o) \cdot e^{(-t/\tau_\theta)}. \tag{6-39}$$

Figure 6.18 illustrates the solution of Eq. 6-39 including its time constant. The temperature transient during a load cycle is schematically depicted in Fig. 6.19 with $\theta_f = P_{loss}/A$.

6.13.1 Application Example 6.7: Temperature Increase of Rotating Machine with a Step Load

An enclosed fan-cooled induction motor has a thermal time constant of $\tau_\theta = 3\ h$ and a steady-state rated temperature of $\theta_f = 120°C$ at an ambient temperature of $\theta_{amb} = 40°C$. The motor has at time $t = 0_-$ a temperature of $\theta = \theta_{amb}$, and the motor is fully loaded at $t = 0_+$. Calculate the time $t_{95\%}$ when the fully loaded motor reaches 95% of its final temperature $\theta_f = 120°C$.

6.14 VARIOUS OPERATING MODES OF ROTATING MACHINES

Depending on their applications, rotating machines can be subjected to various operating modes such as steady-state, short-term, steady-state with short-term, intermittent, and steady-state with intermittent operating modes.

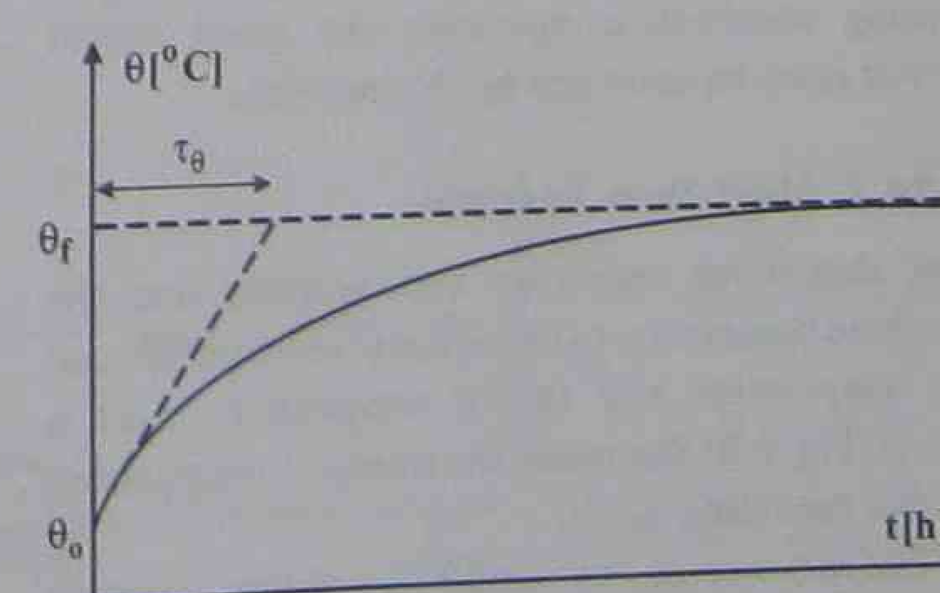


FIGURE 6.18 Solution of Eq. 6-39 including its time constant.

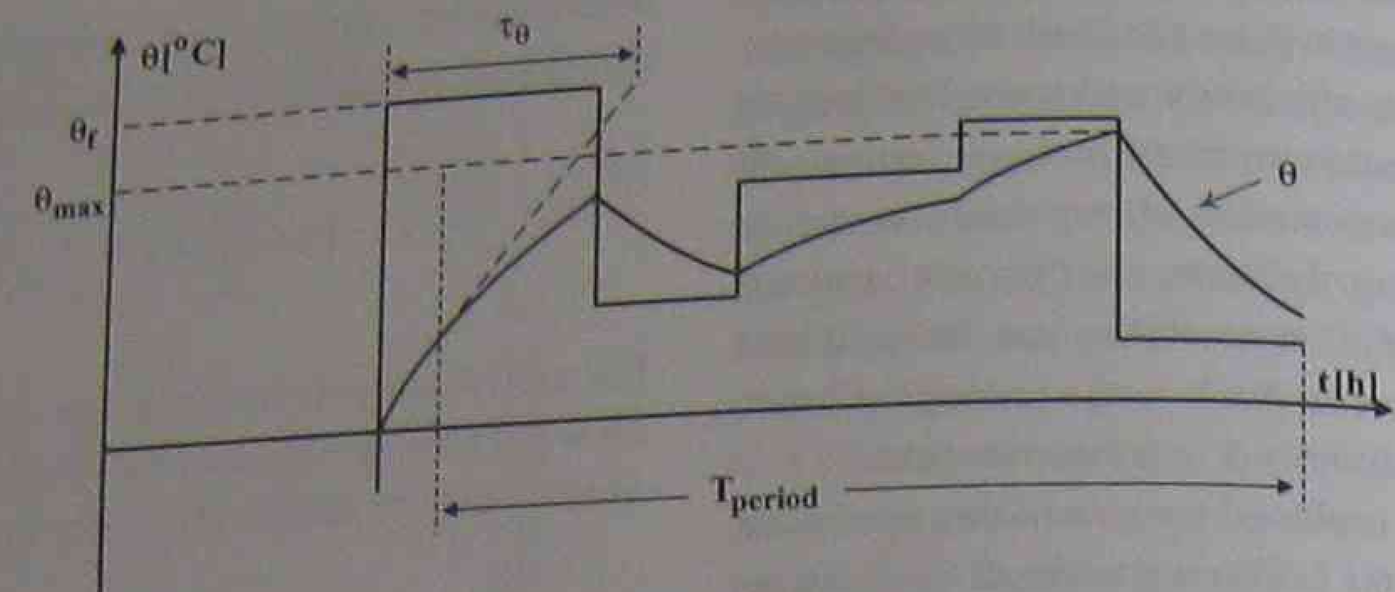


FIGURE 6.19 Temperature transient during a load cycle.

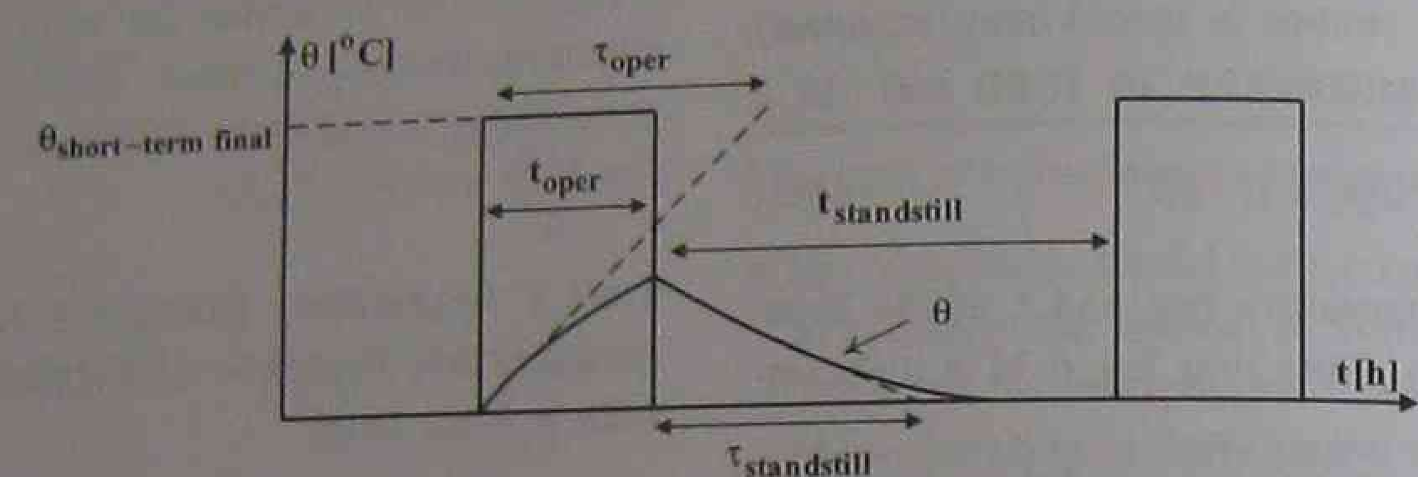


FIGURE 6.20 Transient temperature for short-term operation.

6.14.1 Steady-State Operation

Steady-state temperature is reached when the operating time of the machine t_{oper} is large as compared with the time constant τ_θ of the machine. That is,

$$t_{oper} \geq (3\text{–}4) \cdot \tau_\theta. \tag{6-40}$$

According to experience the thermal time constant for openly ventilated machines is

$$\tau_\theta \approx 1\ \text{h} \tag{6-41}$$

and for enclosed but ventilated machines

$$\tau_\theta \approx (3\text{–}4)\ \text{h}. \tag{6-42}$$

During steady-state operation the rated output power must be delivered by the machine.

6.14.2 Short-Term Operation

For short-term operation one assumes that the machine cools down to the ambient temperature and its temperature rise (initial temperature rise) is $\theta_o = 0$. Fig. 6.20 illustrates the transient temperature of this operating mode.

The absence of any ventilation during standstill (time $t_{standstill}$) requires that the time constant during standstill be longer than that during operation (time t_{oper}), that is, $\tau_{standstill} > \tau_{oper}$. For short-term operation the times t_{oper} and $t_{standstill}$ must relate to the time constants as follows:

$$t_{oper} < (3\text{–}4)\ \tau_{oper}, \tag{6-43}$$

$$t_{standstill} < (3\text{–}4)\ \tau_{standstill}. \tag{6-44}$$

The transient temperature is for short-term operation (see Fig. 6.20):

$$\theta_{short\text{-}term} = \theta_{rated} = \theta_{short\text{-}term\_final}\left(1 - e^{\left(-\frac{t_{oper}}{\tau_{oper}}\right)}\right). \tag{6-45}$$

The final short-term temperature is obtained from Eq. 6-45:

$$\theta_{short\text{-}term\_final} = \frac{\theta_{rated}}{\left(1 - e^{\left(-\frac{t_{oper}}{\tau_{oper}}\right)}\right)}. \tag{6-46}$$

with

$$\theta_{short\text{-}term\_final} \propto P_{loss\_short\text{-}term} \tag{6-47}$$

and

$$\theta_{rated} \propto P_{loss\_rated}. \tag{6-48}$$

It follows for the losses during short-term operation:

$$P_{loss\_short\text{-}term} = \frac{P_{loss\_rated}}{\left(1 - e^{\left(-\frac{t_{oper}}{\tau_{oper}}\right)}\right)}. \tag{6-49}$$

For machines whose losses consist of iron-core and copper losses (e.g., induction machines) the total losses at rated operation are

$$P_{loss\_rated} = p \cdot P_{rated} + q \cdot P_{rated}. \tag{6-50}$$

The first term of Eq. 6-50 pertains to the iron-core and the second term to the copper losses. The losses during short-term operation are

$$P_{loss\_short\text{-}term} = p \cdot P_{rated} + q \cdot P_{rated}\left(\frac{P_{loss\_short\text{-}term}}{P_{rated}}\right)^2. \tag{6-51}$$

Introducing Eqs. 6-50 and 6-51 into Eq. 6-49 yields the ratio

$$\frac{P_{loss\_short\text{-}term}}{P_{rated}} = \sqrt{\left(1 + \frac{p}{q}\right)\frac{1}{1 - e^{\left(-\frac{t_{oper}}{\tau_{oper}}\right)}} - \frac{p}{q}}, \tag{6-52}$$

where the ratio p/q is available from the manufacturer of the machine.

6.14.3 Steady State with Short-Term Operation

In this case the short-term load is superposed with the steady-state load (see Fig. 6.21). The operating time t_{oper} and the pause time t_{pause} relate to the time constant as follows:

$$t_{oper} < (3\text{–}4) \cdot \tau_\theta, \tag{6-53}$$

$$t_{pause} > (3\text{–}4) \cdot \tau_\theta, \tag{6-54}$$

and the ratio between the required power during steady-state with short-term load to the rated power is

$$\frac{P_{steady\text{-}state+short\text{-}time}}{P_{rated}} = \sqrt{\frac{1}{\left(1 - e^{\left(-\frac{t_{oper}}{\tau_\theta}\right)}\right)}}. \tag{6-55}$$

In Eq. 6-55 the iron-core losses are neglected ($p = 0$).

6.14.4 Intermittent Operation

The mode of intermittent operation occurs most frequently. One can differentiate between two cases:

- irregular load steps (see Fig. 6.22), and
- regular load steps.

The case with irregular load steps is discussed in Application Example 6.8.

The case with regular load steps can be approximated as follows based on Fig. 6.23:

$$\frac{P_{intermittent}}{P_{rated}} = \sqrt{\left(1 + \frac{p}{q}\right)\left(1 + \frac{t_{pause}}{t_{oper}} \cdot \frac{\tau_\theta}{\tau_{standstill}}\right) - \frac{p}{q}}. \tag{6-56}$$

6.14.5 Steady State with Intermittent Operation

Steady state with superimposed periodic intermittent operation (see Fig. 6.24) does not occur frequently. The ratio between the power required for this case and the rated power is

$$\frac{P_{steady\text{-}state+periodic\ intermittent}}{P_{rated}} = \sqrt{\left(1 + \frac{t_{pause}}{t_{oper}}\right)} \tag{6-57}$$

for $t_{oper} < (3\text{–}4)\tau\theta$ and $t_{pause} < (3\text{–}4)\tau\theta$. In summary one can state that for the motor sizing the maximum temperature (and not the torque) is important.

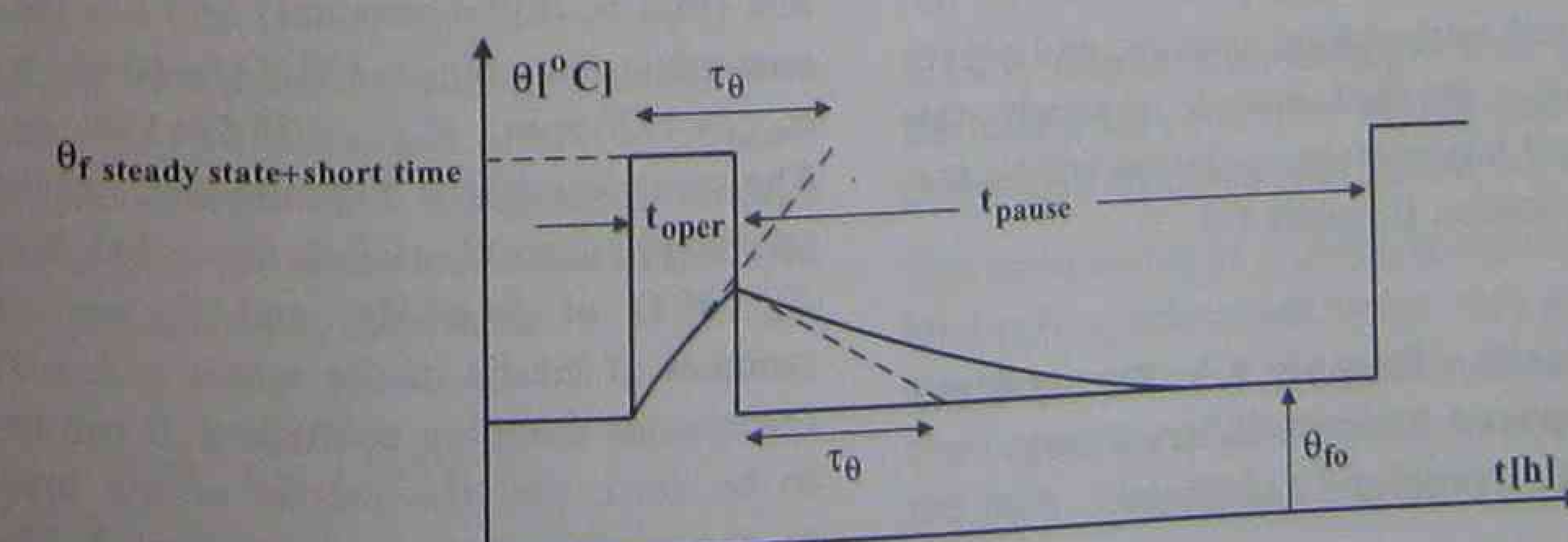


FIGURE 6.21 Transient temperature for steady-state load superposed with short-term load.

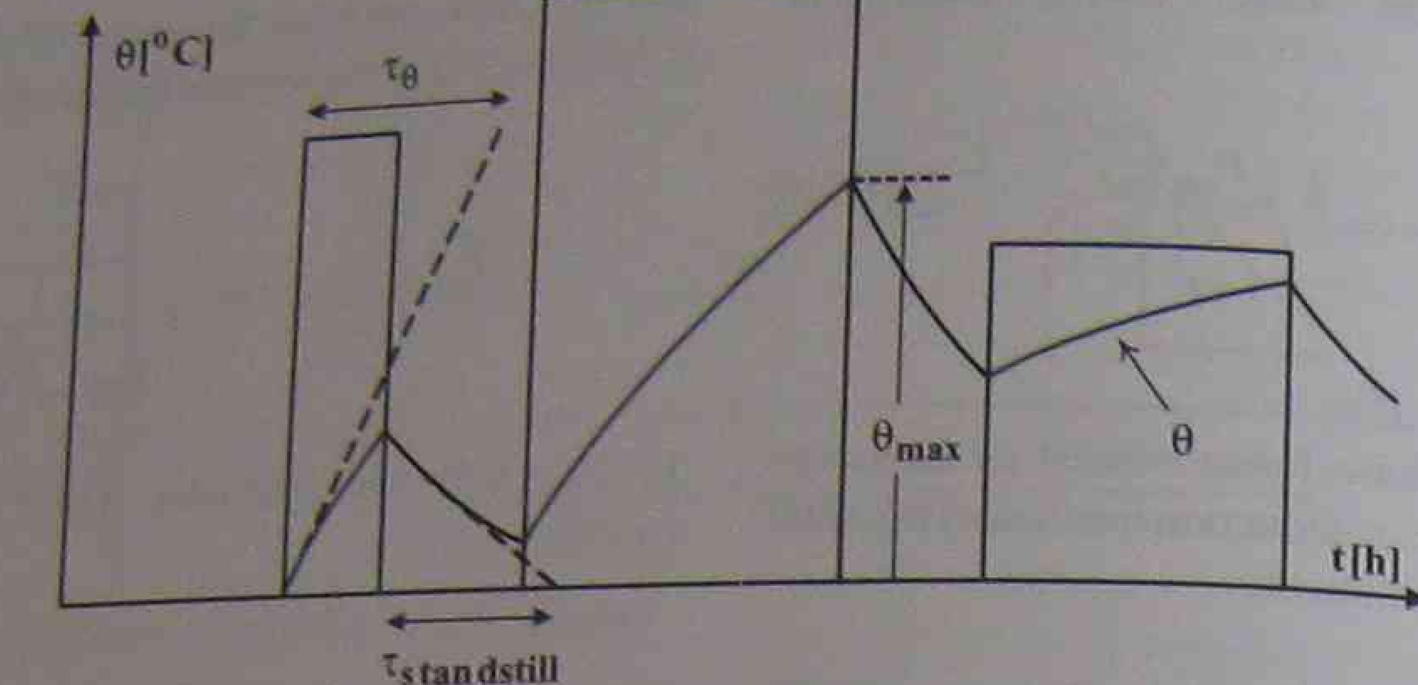


FIGURE 6.22 Intermittent operation with irregular load steps.

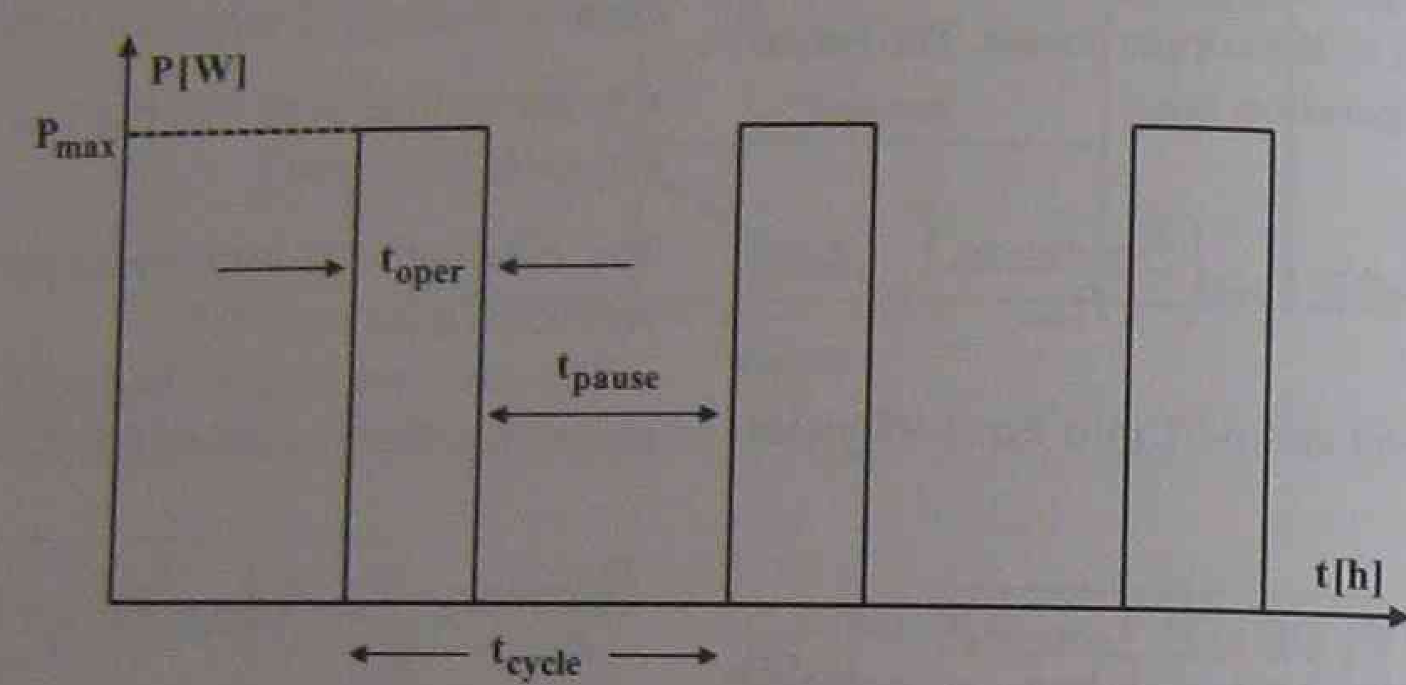


FIGURE 6.23 Intermittent operation with regular load steps.

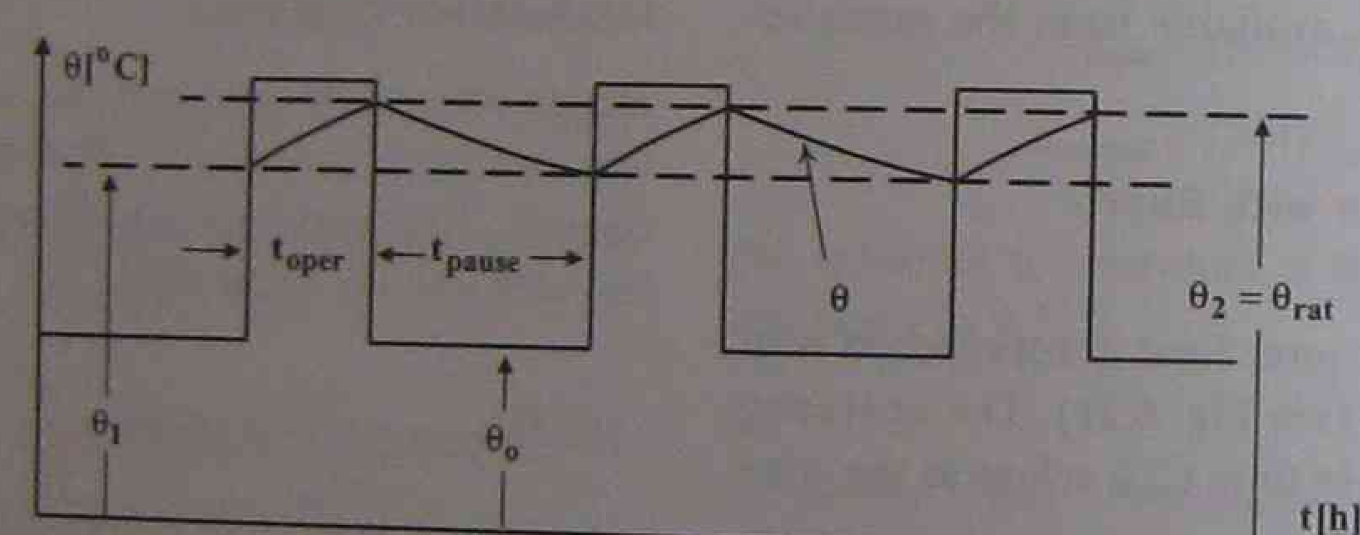


FIGURE 6.24 Steady state with superimposed periodic intermittent operation with regular load steps.

Intermittent operation occurs in drives such as used in hybrid cars, wind-power plants, air-conditioning and refrigeration systems, and others. For this reason a detailed analysis of steady state with superposed intermittent operation will be presented in Application Example 6.8.

6.14.6 Application Example 6.8: Steady State with Superimposed Periodic Intermittent Operation with Irregular Load Steps

A three-phase, squirrel-cage induction motor is fed by a voltage-source inverter. The induction motor is operated at variable frequency and constant (rated) flux (that is, $|\tilde{E}|/f = \text{constant}$) and has the following nameplate data: $V_{L-L} = 460$ V, $f = 60$ Hz, $p = 4$ poles, $n_{m\_rat} = 1720$ rpm, $P_{out\_rat} = 29.594$ kW or 39.67 hp. The stator winding is Y-connected, and the parameters per phase are $R_s = 0.5\ \Omega$, $R'_r = 0.2\ \Omega$, $X_s = X'_r = 1\ \Omega$, $X_m = 30\ \Omega$ at $f = 60$ Hz, and $R_{fe} \rightarrow \infty$. The axial moment of inertia of the motor is $J_m = 0.234$ kgm², the viscous damping coefficient B can be assumed to be zero, and the inertia of the load referred to the motor shaft is $J_{load} = 5$ kgm². The steady-state/intermittent load cycle is shown in Fig. E6.8.1.

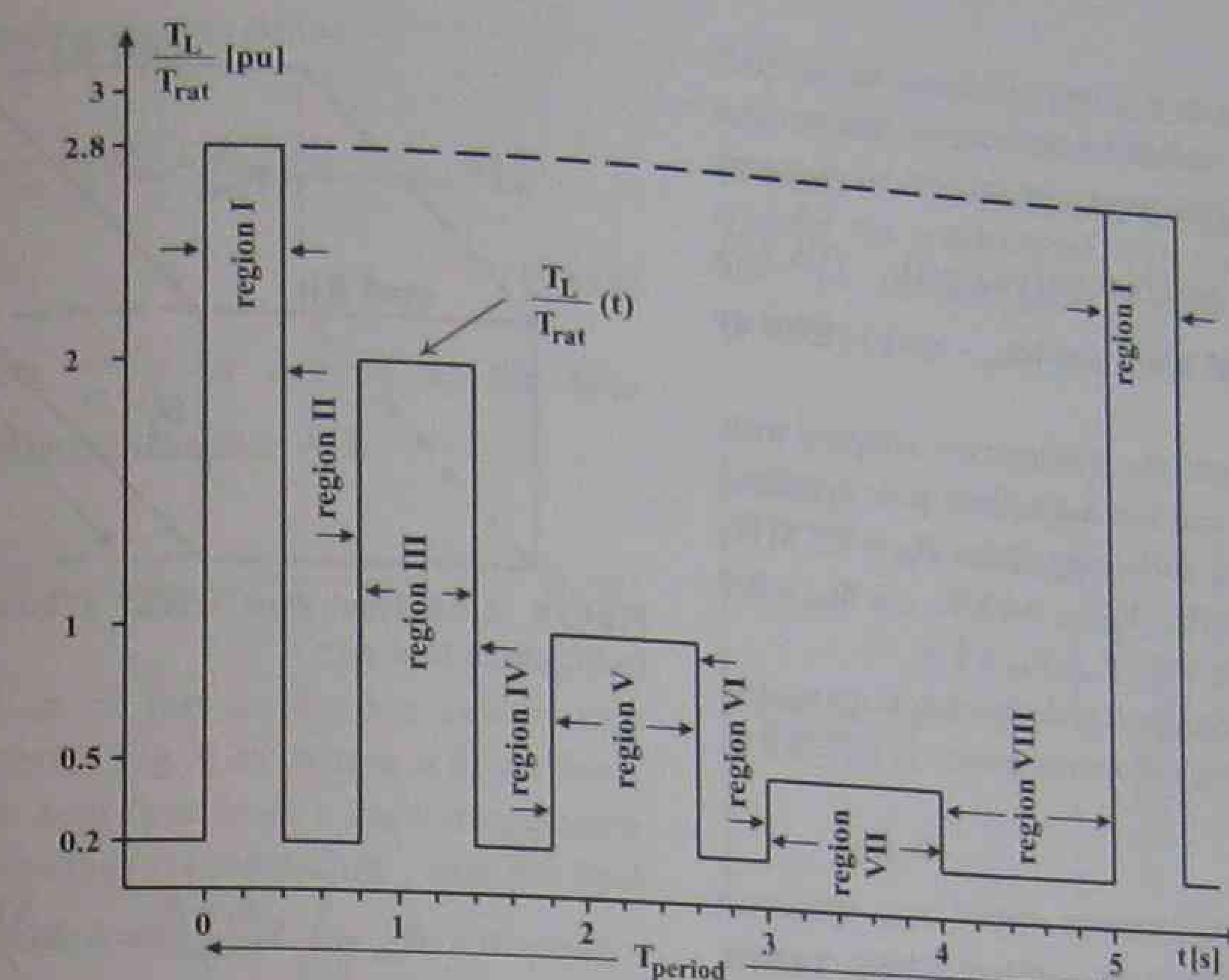


FIGURE E6.8.1 Steady-state/intermittent load cycle.

The induction motor is operated at 60 Hz (natural torque-speed characteristic).

a) Determine the rated synchronous speed, rated synchronous angular velocity, rated angular velocity, rated slip, rated stator current $|\tilde{I}_s|$, rated rotor current $|\tilde{I}'_r|$, and rated induced voltage $|\tilde{E}|$.
b) Calculate the maximum torque T_{max} at s_m and the fictitious maximum torque T_{max_fict} at $s = 1$.
c) Derive from the equation of motion the slip $s(t)$ as a function of the initial slip $s(0)$, the load torque $T_L(t)$, the maximum fictitious torque T_{max_fict}, the time constant τ_m, and the time t.
d) Apply the solution for $s(t)$ to the different regions of Fig. E6.8.1 and plot $s(t)$ from $t = 0$ to the maximum time $t = 5$ s of the load-cycle time period of $T_{period} = 5$ s.
e) Calculate the torque $T(t)$ and plot it from $t = 0$ to $t = 5$ s.
f) Based on the slip function compute pointwise the stator current $|\tilde{I}_s(t)|$ and plot $|\tilde{I}_s(t)|$ from $t = 0$ to $t = 5$ s.
g) Plot $|\tilde{I}_s(t)|^2$ as a function of time from $t = 0$ to $t = 5$ s.
h) Determine the rms value of the motor (stator) current $i_s(t)$ for the entire load cycle as specified in Fig. E6.8.1; that is, determine

$$I_{s\_rms} = \sqrt{\frac{1}{T}\int_{t}^{t+T} |\tilde{I}_s(t)|^2\, dt}.$$

i) Is this induction motor over- or underdesigned? If it is underdesigned, what is the reduction of lifetime?

6.14.7 Reduction of Vibrations and Torque Pulsations in Electric Machines

Vibrations and torque pulsations generate electromagnetic forces that mainly act on the end turns of stator windings. As it is well known the turns residing in the stator slots are not exposed to high magnetic fields and for this reason the magnetic forces acting on the turns within the slots are relatively small. Besides increased temperatures, vibrations and torque pulsations are a major reason for the deterioration of the insulation materials. Vibrations and pulsating torques can have their origin in motor asymmetries (e.g., rotor eccentricities, interturn faults, cogging due to permanent magnets, and a poor selection of stator and rotor slotting [47]) and loads (e.g., piston compressors). For this reason it will be important to demonstrate how torque pulsations caused by loads can be reduced to an acceptable level. This will be the subject of Application Example 6.9.

6.14.8 Application Example 6.9: Reduction of Harmonic Torques of a Piston-Compressor Drive with Synchronous Motor as Prime Mover

The drive motor of a piston compressor is a nonsalient-pole synchronous motor with an amortisseur. To compensate for the piston stroke an eccentric shaft is employed. The load (L) torque at the eccentric shaft consists of an average torque and two harmonic torques:

$$T_L = T_{Lavg} + T_{L1} + T_{L4}, \quad \text{(E6.9-1)}$$

where

$$T_{Lavg} = 265.53 \text{ kNm}, \quad \text{(E6.9-2)}$$

$$T_{L1} = 31.04 \text{ kNm} \sin(\omega_{ms}t - \pi), \quad \text{(E6.9-3)}$$

$$T_{L4} = 50.65 \text{ kNm} \sin(4\omega_{ms} - \pi/6). \quad \text{(E6.9-4)}$$

The drive motor and the flywheel are coupled with the eccentric shaft, and the nonsalient-pole synchronous motor has the following data: $P_{rat} = 7.5$ MW, $p = 28$ poles, $f = 60$ Hz, $V_{L\text{-}Lrat} = 6$ kV, $\cos \Phi_{rat} = 0.9$ overexcited, $\eta_{rat} = 96.9\%$, $T_{max}/T_{rat} = 2.3$.

The slip of the nonsalient pole synchronous motor when operating as an induction motor is $s_{rat} = 6.8\%$ at T_{rat}.

a) Calculate the synchronous speed n_{ms}, synchronous angular velocity ω_{ms}, rated stator current $\tilde{I}_{a\_rat}$, rated torque T_{rat}, the torque angle δ, stator current $\tilde{I}_a$ at T_{Lavg} and the synchronous reactance X_S (in ohms and in per unit).
b) Neglecting damping provided by induction-motor action, derive the nonlinear differential equation for the motor torque, and by linearizing this equation find an expression for the eigen angular frequency ω_{eigen} of the undamped motor.
c) Repeat part b, but do not neglect damping provided by induction motor action.
d) Solve the linearized differential equations of part b (without damping) and part c (with damping) and determine – by introducing phasors – the axial moment of inertia J required so that the first harmonic motor torque will be reduced to ±4% of the average torque (Eq. E6.9-2).
e) Compare the eigen frequency f_{eigen} with the pulsation frequencies occurring at the eccentric shaft.
f) State the total motor torque $T(t)$ as a function of time (neglecting damping) for one revolution of the eccentric shaft and compare it with the load torque $T_L(t)$. Why are they different?

6.14.9 Calculation of Steady-State Temperature Rise ΔT of Electric Apparatus Based on Thermal Networks

The losses of an electric apparatus (e.g., rotating machine, transformer, and inductor) heat up various parts of the apparatus, and the losses are dissipated via heat conduction, convection, and radiation.

Heat Flow Related to Conduction. Assume that the losses at end #2 with the temperature θ_2 of a homogeneous body (see Fig. 6.25) are

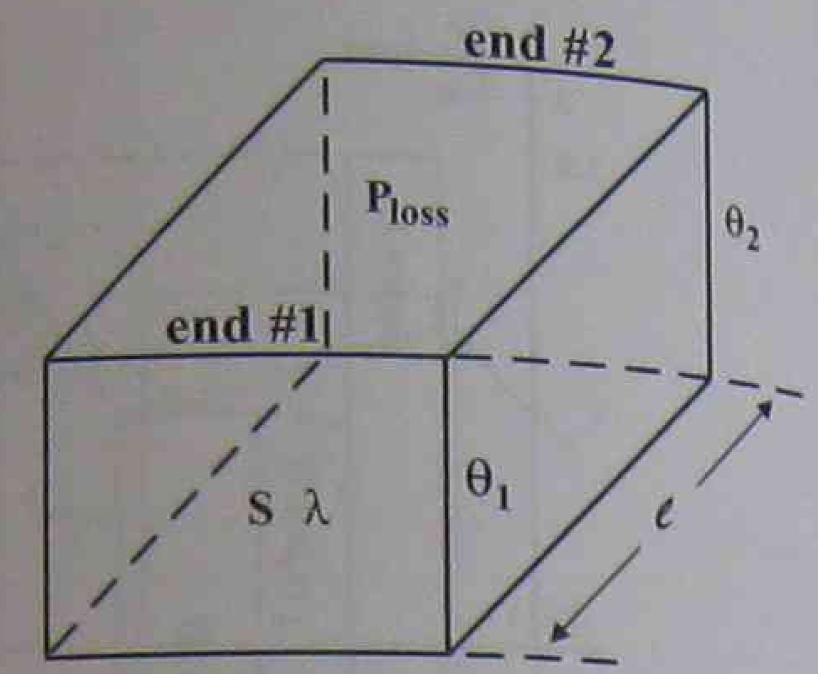


FIGURE 6.25 Heat flow within a homogeneous body (principle of heat pipe).

$$P_{loss} = \lambda \cdot S \frac{\theta_2 - \theta_1}{\ell} = \lambda \cdot S \frac{\Delta T}{\ell}, \quad \text{(6-58)}$$

where λ is the thermal conductivity of a homogeneous body measured in $\left[\frac{W}{m°C}\right]$, S is the surface of the homogeneous body perpendicular to the heat flow measured in m^2, and ℓ is the length of the homogeneous body along the direction of the heat flow measured in m.

The temperature at end #1 θ_1 is given by Eq. 6-58:

$$\Delta T = (\theta_2 - \theta_1) = \frac{\ell}{\lambda \cdot S} P_{loss} = R_{conduct} \cdot P_{loss}, \quad \text{(6-59)}$$

where the term $\left[\frac{\ell}{\lambda \cdot S}\right]$ is defined as the heat conduction resistance

$$R_{conduct} = \frac{\ell}{\lambda \cdot S} \quad \text{(6-60)}$$

Heat Flow Related to Radiation and Convection. The heat flow due to radiation and convection from the heat source to the cooling medium (e.g., ambient air) is proportional to the surface A of the body (e.g., winding) on which the heat source resides and its temperature difference $\Delta T = (\theta_2 - \theta_1)$ with respect to the cooling medium:

$$P_{loss} = \alpha \cdot A(\theta_2 - \theta_1), \quad \text{(6-61a)}$$

where α is the thermal conductivity measured in $\left[\frac{W}{m^2 °C}\right]$ between the heat source and the cooling medium.

Correspondingly, one can define

$$\Delta T = (\theta_2 - \theta_1) = \frac{1}{\alpha \cdot A} P_{loss} = R_{radiation/convection} \cdot P_{loss} \quad \text{(6-61b)}$$

where the term $\frac{1}{\alpha \cdot A}$ is defined as the heat radiation/convection resistance

$$R_{radiation/convection} = \frac{1}{\alpha \cdot A}. \quad \text{(6-62)}$$

The application of the conduction relationship (Eq. 6-59) generates Fig. 6.26, where a linear function leads to the heat flow from a high-temperature body #2 (e.g., winding) via insulation – with the heat conduction resistance $R_{conduct} = \frac{\ell}{\lambda \cdot S}$ – to a low-temperature body #1 (e.g., iron core).

The application of the radiation/convection relationship (Eq. 6-61) generates Fig. 6.27, where a step function leads to the heat flow from a high-temperature body #2 (e.g., frame) – with the heat radiation/convection resistance $R_{radiation/convection} = \frac{1}{\alpha \cdot A}$ – to a low-temperature body #1 (e.g., ambient air). An example for this case is the interface between the motor frame and the ambient air.

Figure 6.28 depicts the heat flow as it occurs from a winding via the insulation, the iron core, the small air gap between iron core and frame, and from the frame of an electrical apparatus to the ambient air serving as cooling medium. Note that the heat drop in the small air gap is a mixture between conduction and convection/radiation. The reason is that some of the iron core laminations have contact (heat transfer via conduction) with the frame and some have no contact (heat transfer via convection/radiation).

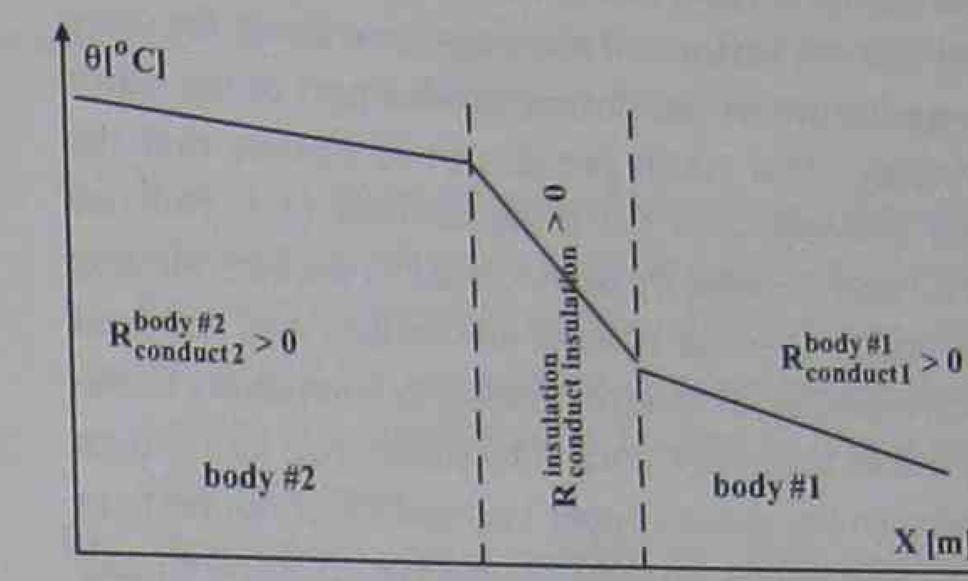


FIGURE 6.26 Linear heat flow through conduction from a high-temperature body #2 to a low-temperature body #1 with finite thermal resistances.

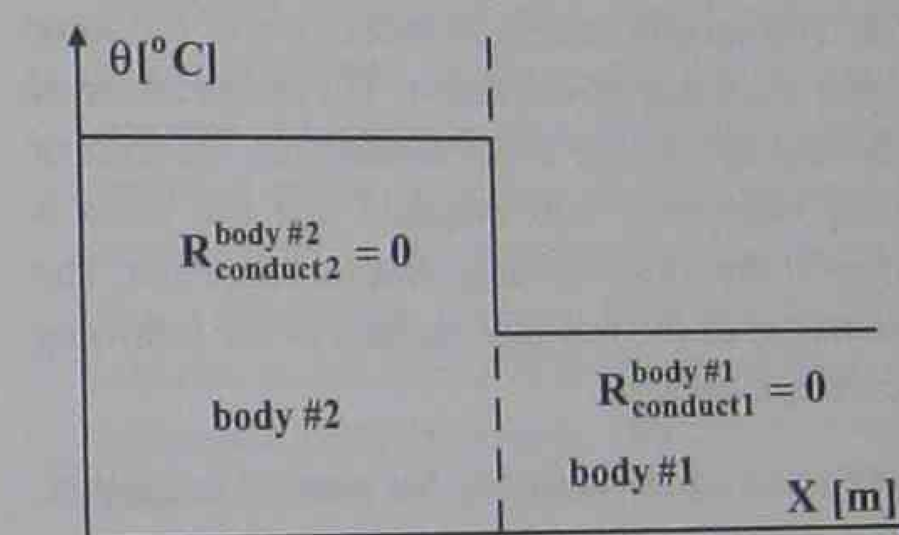


FIGURE 6.27 Step-function heat flow through convection/radiation from a high-temperature body to a low-temperature cooling medium.

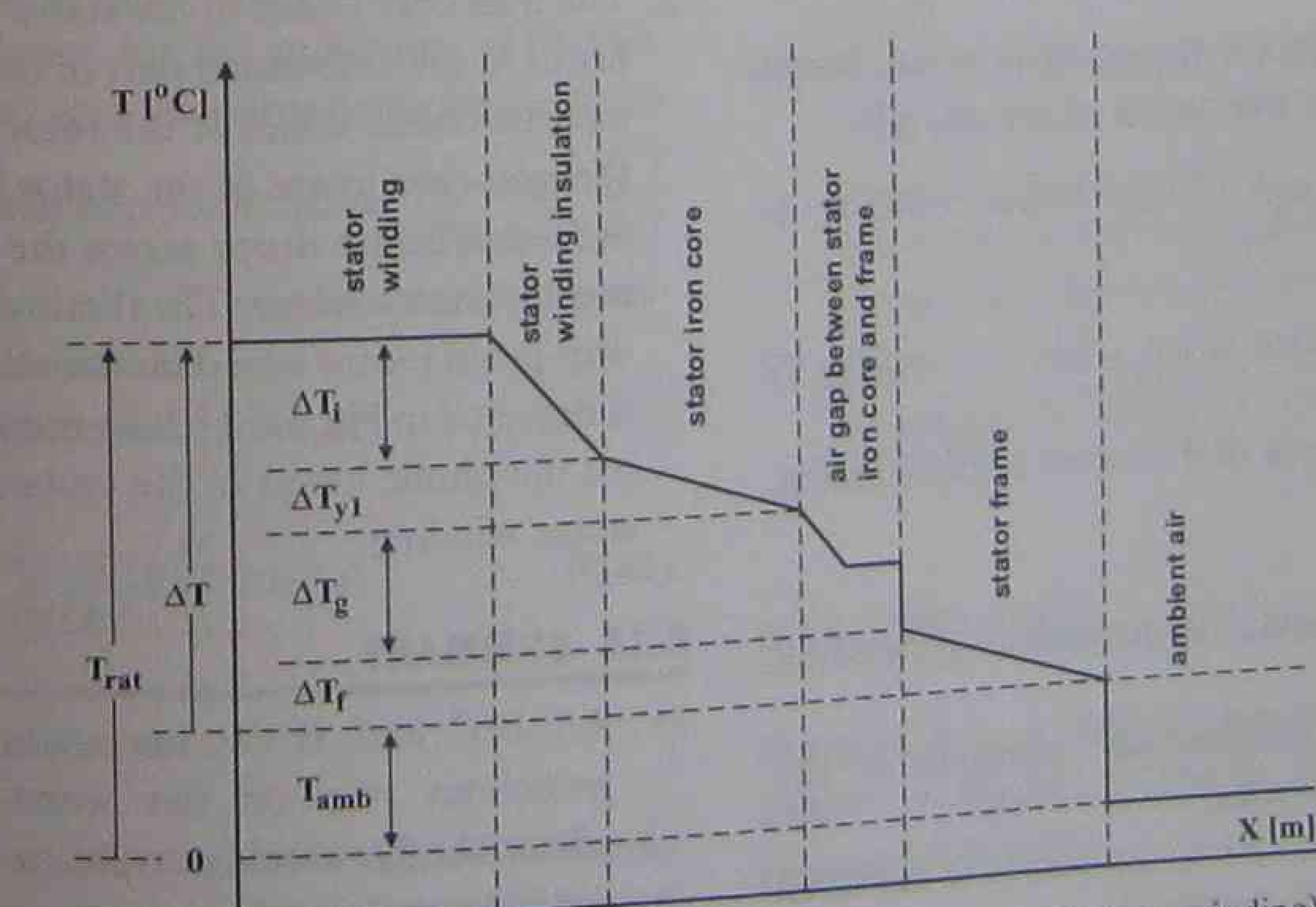


FIGURE 6.28 Heat flow within electrical apparatus from a high-temperature body (e.g., winding) to a low-temperature cooling medium (e.g., ambient air).

6.14.10 Application Example 6.10: Temperature-Rise Equations for a Totally Enclosed Fan-Cooled 100 hp Motor

For a totally enclosed fan-cooled motor [48] the surface of the external frame dissipates the heat. The ribs are located along the axis of the machine where the hot air is removed by a fan. Test results show [49, 50] that the stator end windings have about the same temperature as the slot-embedded part of the stator winding. This result permits us to assume that the end winding does not dissipate heat (e.g., radiates and removes heat by convection) to the surrounding air inside the end bells of the motor, and the heat developed by the stator winding is completely transferred to the stator core. The stator iron loss is then added to the stator copper loss and the resultant heat is transferred to the stator frame via the small air gap between core and frame from which it is removed by the fan through radiation and convection. A second assumption refers to the heat of the rotor winding. It is assumed that this heat is transferred to the stator. This means that the temperature of the rotor is higher than that of the stator. The temperature of the hottest spot in the stator determines the class of winding insulation. It is assumed that this point is residing in the stator winding inside a stator slot. The temperature of the hot spot consists of the following temperature drops:

- temperature drop across stator winding insulation, ΔT_i
- temperature drop across stator back iron (yoke), ΔT_{y1}
- temperature drop across the small air gap between the stator back iron and the frame of the motor, ΔT_g
- difference between the temperature of the frame and the ambient temperature of the air, ΔT_f.

The temperature rise of the stator winding is therefore

$$\Delta T = \Delta T_i + \Delta T_{y1} + \Delta T_g + \Delta T_f. \qquad \text{(E6.10-1)}$$

The thermal network of the motor is shown in Fig. E6.10.1, where

R_i is the thermal conductivity resistance of the stator slot insulation
R_{y1} is the thermal conductivity resistance of the stator back iron or yoke
R_g is the thermal conductivity resistance of the small air gap between the stator core and the frame of the motor

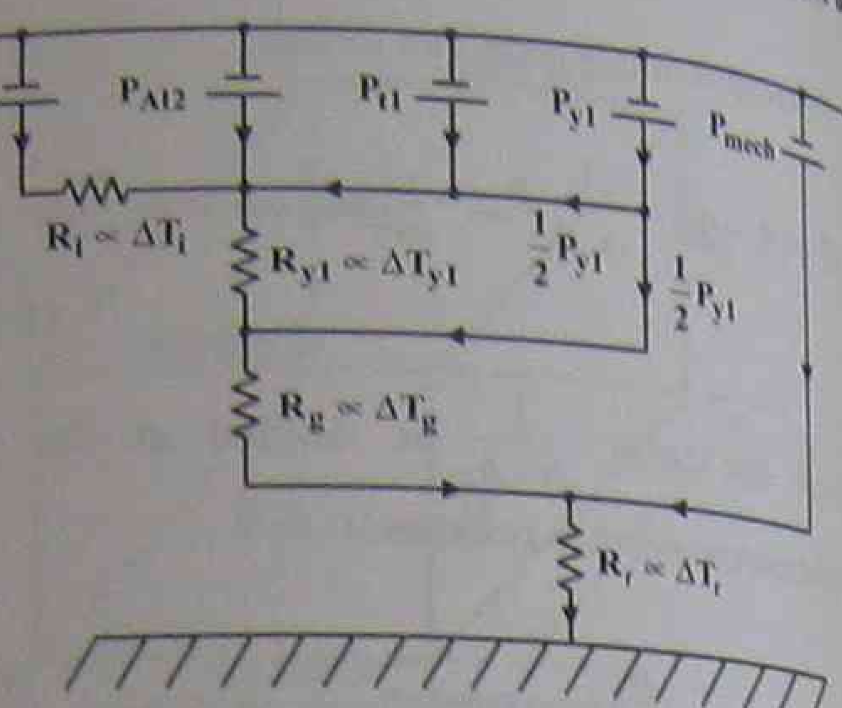


FIGURE E6.10.1 Thermal network of a totally enclosed fan-cooled induction motor.

R_f is the thermal radiation/convection resistance of the outside of the stator frame and the moving air
P_{cu1} is the stator ohmic (copper) loss
P_{Al2} is the rotor ohmic (aluminum) loss
P_{t1} is the stator teeth (iron-core) loss
P_{y1} is the stator yoke (iron-core) loss
P_{mech} is the friction (bearing, windage) loss

6.14.11 Application Example 6.11: Temperature-Rise Equations for a Drip-Proof 5 hp Motor

The temperature rise equation is written for a drip-proof motor [48]. This equation is based on the following assumptions:

1. The ohmic losses in the embedded part of the stator winding produce a temperature drop across the slot insulation.
2. The iron-core losses of the stator teeth, the ohmic losses in the embedded part of the stator winding, and the ohmic losses of the rotor bars and half of the iron-core losses of the stator back iron generate temperature drops across the stator back iron and the motor frame. The thermal network of this drip-proof motor based on the above assumptions is depicted in Fig. E6.11.1. In this figure P_{em} stands for the ohmic losses in the embedded part of the stator winding.

6.15 SUMMARY

1) All IEC and IEEE harmonic standards and guidelines rely on the worst-case condition where the harmonic current or voltage spectra are independent of time.
2) The weighted-harmonic voltage factor is based on the same assumption and estimates the additional temperature rise for rated operation of transformers, induction motors, and universal motors for the voltage spectrum with maximum individual harmonic voltage amplitudes.
3) The weighted-harmonic voltage factor

$$\sum_{h=2}^{\infty} \frac{1}{(h)^k}\left(\frac{V_h}{V_1}\right)^l \qquad \text{(6-63)}$$

described in this chapter is related to the square of the total harmonic distortion of the voltage

$$THD_v = \sqrt{\sum_{h=2}^{\infty}\left(\frac{V_h}{V_1}\right)^2} \qquad \text{(6-64)}$$

by introducing the weighting function $\left(\frac{1}{h}\right)^k$ and replacing the exponent 2 of the ratio by $\sum_{h=2}^{\infty} \frac{1}{(h)^k}\left(\frac{V_h}{V_1}\right)^2$ by the exponent l.
4) $\sum_{h=2}^{\infty} \frac{1}{(h)^k}\left(\frac{V_h}{V_1}\right)^l$ can be used as a measure for the losses and temperature rises of electromagnetic apparatus due to harmonic voltages.
5) Acceptable limits for the reduction of lifetime due to thermal aging requires limits for the value of the weighted-harmonic factor $\sum_{h=2}^{\infty} \frac{1}{(h)^k}\left(\frac{V_h}{V_1}\right)^l$ and indirectly limits the amplitude of the occurring voltage harmonics.
6) A weighted harmonic factor of

$$\sum_{h=2}^{\infty} \frac{1}{(h)^k}\left(\frac{V_h}{V_1}\right)^l = 5.8 \qquad \text{(6-65)}$$

is recommended.
7) It is evident that for a given weighted-harmonic voltage factor, single- and three-phase induction machines are more susceptible to aging than transformers and universal machines.
8) Strictly speaking, all current or voltage harmonic spectra are continually changing as a function of time due to changing system configuration and loads, and the assumption of time-independent spectra is a simplification leading to a first-cut approach only.
9) The cost of harmonics incurs by either purchasing more expensive solid-state equipment (e.g., PWM inverters, switched-mode power supplies), which generates a reduced amount of harmonics, or by using less-sophisticated but harmonic generating equipment (e.g., six-step inverters, diode rectifiers), which generate a greater amount of harmonics than the more expensive switched-mode equipment. In the first case the losses occur in the switched-mode equipment (e.g., efficiencies ranging from 80 to 98%), whereas in the second case of less-sophisticated solid-state equipment (e.g., efficiencies ranging from 90 to 98%) the harmonic losses incur in the loads (e.g., induction motors) and their reduction of lifetime must be accounted for due to temperature increase.
10) Design considerations for intermittent operation are presented so that no lifetime reduction occurs.
11) Most drives are subjected to pulsating loads that cause vibrations within the drive motor, in particular the motor windings. To limit vibrations to a certain value the required axial moment of inertia J of a flywheel is calculated.
12) The temperature rises for totally enclosed fan-cooled and for drip-proof induction motors are derived.

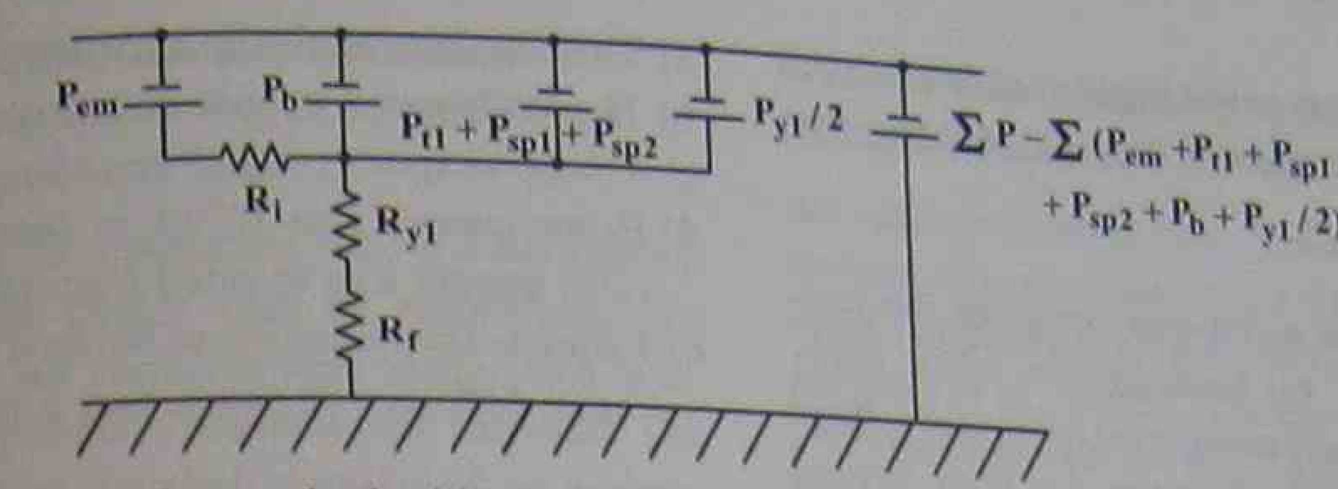


FIGURE E6.11.1 Thermal network of a drip-proof induction motor.

6.16 PROBLEMS

Problem 6.1: Aging of Transformers and Induction Machines Due to Single Harmonic Voltage

The terminal voltage of a single-phase transformer (rated resistive load) and a single-phase induction machine (operation at rated load) contains a third-harmonic voltage component of $V_{3}/V_{1} = 2.5\%$, 5%, and 7.5%.

a) Determine for the worst-case condition the temperature rises ΔT for both devices, which occur due to the previously mentioned harmonic voltage components. You may assume for both devices a rated temperature of $T_{rat} = T_2 = 125°C$ at an ambient temperature of $T_{amb} = 40°C$.
b) Calculate for the temperature rises ΔT of part a the decrease of lifetime for both devices under the given three (not simultaneously occurring) harmonic voltage components for an activation energy of $E = 1.15$ eV.

Problem 6.2: Operation of Induction Motor at Reduced Line-to-Line Voltage

A 100 hp, 480 V_{L-L}, 60 Hz, 6-pole three-phase induction motor runs at full load and rated voltage with a slip of 3%. Under conditions of stress on the power supply system, the line-to-line voltage drops to 420 V_{L-L}. If the load is of the constant torque type, then compute for the lower voltage

a) The slip (use small slip approximation).
b) The shaft speed in rpm.
c) The hp output.
d) The rotor copper loss ($I_2'^2 r_2'$) in terms of the rated rotor copper loss at rated voltage.
e) Estimate the reduction of lifetime due to reduced voltage operation provided the rated temperature is $T_{rat} = 125°C$, the ambient temperature is $T_{amb} = 40°C$, rated lifetime is $t_2 = 40$ years, and $E = 1.15$ eV.

Problem 6.3: Operation of Induction Motor at Increased Line-to-Line Voltage

Repeat Problem 6.2 for the condition when the line-to-line voltage of the power supply system increases to 520 V_{L-L}, and estimate the reduction of lifetime.

Problem 6.4: Reduced Voltage and Frequency Operation of Induction Machines

A 100 hp, 480 V_{L-L}, 60 Hz, 6-pole three-phase induction motor runs at full load and rated voltage and frequency with a slip of 3%. Under conditions of stress on the power supply system the terminal voltage and the frequency drops to 420 V_{L-L} and 54 Hz, respectively. If the load is of the constant torque type, then compute for the lower voltage and frequency:

a) The slip (use small slip approximation).
b) The shaft speed in rpm.
c) The hp output.
d) Rotor copper loss ($I_2'^2 r_2'$) in terms of the rated rotor copper loss at rated frequency and voltage.
e) Estimate the reduction of lifetime for $T_{rat} = 125°C$, $T_{amb} = 40°C$, $t_2 = 40$ years, and $E = 1.15$ eV.

Problem 6.5: Lifetime Reduction of Induction Motors and Transformers for Given Voltage Spectra

Estimate the lifetime reductions of induction machines, transformers, and universal machines for the single- and three-phase voltage spectra of Table P6.5, and their associated lifetime reductions for an activation energy of $E = 1.15$ eV. The ambient temperature is $T_{amb} = 40°C$, the rated temperature is $T_2 = 120°C$, and the rated lifetime of $t_2 = 40$ years can be assumed.

Problem 6.6: Calculate the Stator rms Current of an Induction Motor at Steady-State Operation with Superimposed Intermittent, Irregular Load Steps

Repeat the analysis of Application Example 6.8 provided the inverter-motor drive is operated at $f = 30$ Hz with constant flux, $(E/f) = \text{constant}$, control.

TABLE P6.5 Possible voltage spectra with high-harmonic penetration

h	$\left(\frac{V_h}{V_{60Hz}}\right)_{1\Phi}$ (%)	$\left(\frac{V_h}{V_{60Hz}}\right)_{3\Phi}$ (%)
1	100	100
2	0.5	0.5
3	7.0	0.5
4	1.6	0.5
5	5.0	7.5
6	0.88	0.2
7	3.0	5.0
8	0.62	0.2
9	2.0	0.3
10	0.66	0.1
11	1.0	4.0
12	0.18	0.1
13	0.8	3.0
14	0.10	0.05
15	0.10	0.1
16	0.13	0.05
17	0.23	1.5
18	0.22	0.01
19	1.03	1.0

All higher harmonics < 0.2%

Problem 6.7: Calculate the Reduction of Harmonic Synchronous Motor Torques for a Given Time-Dependent Load

The drive motor of a pulsating load is a nonsalient-pole synchronous motor with an amortisseur. To compensate for the changing load torque as a function of time a flywheel is coupled with the shaft. The load (L) torque at the shaft consists of an average torque and two harmonic torques:

$$T_L = T_{Lavg} + T_{L1} + T_{L3}, \quad \text{(P6.7-1)}$$

where

$$T_{Lavg} = 260 \text{ kNm}, \quad \text{(P6.7-2)}$$

$$T_{L1} = 30 \text{ kNm} \sin(\omega_m t - \pi), \quad \text{(P6.7-3)}$$

$$T_{L3} = 50 \text{ kNm} \sin(3\omega_m - \pi/6). \quad \text{(P6.7-4)}$$

The nonsalient-pole synchronous motor has the following data: $P_{rat} = 5$ MW, $p = 40$ poles, $f = 60$ Hz, $V_{L-Lrat} = 6$ kV, $\cos\Phi_{rat} = 0.8$ overexcited, $\eta_{rat} = 95\%$, $T_{max}/T_{rat} = 2.5$. The slip of the nonsalient pole synchronous motor when operating as an induction motor is $s_{rat} = 6\%$ at T_{rat}.

a) Calculate the synchronous speed n_{ms}, synchronous angular velocity ω_{ms}, rated stator current $\tilde{I}_{s,rat}$, rated torque T_{rat}, the torque angle δ, stator current $\tilde{I}_s$ at T_{Lavg} and the synchronous reactance X_s (in ohms and in per unit).
b) Neglecting damping provided by induction-motor action, derive the nonlinear differential equation for the motor torque, and by linearizing this equation find an expression for the eigen angular frequency ω_{eigen} of the undamped motor.
c) Repeat part b, but do not neglect damping provided by induction motor action.
d) Solve the linearized differential equations of part b (without damping) and part c (with damping) and determine – by introducing phasors – the axial moment of inertia J required so that the first harmonic order motor torque will be reduced to ±2% of the average torque (Eq. P6.7-2).
e) Compare the eigen frequency f_{eigen} with the pulsation frequencies occurring at the eccentric shaft.
f) State the total motor torque $T(t)$ as a function of time (neglecting damping) for one revolution of the eccentric shaft and compare it with the load torque $T_L(t)$. Why are they different?

Problem 6.8: Temperature Transient of an Induction Machine

An enclosed fan-cooled induction motor has a thermal time constant of $\tau_\theta = 4$ h and a steady-state rated temperature of $\theta_f = 140°C$ at an ambient temperature of $\theta_{amb} = 40°C$. The motor has at time $t = 0-$ a temperature of $\theta = \theta_{amb}$, and the motor is fully loaded at $t = 0+$. Calculate the time $t_{95\%}$ when the fully loaded motor reaches 95% of its final temperature $\theta_f = 140°C$.

Problem 6.9: Calculation of the Temperature Rise of a Totally Enclosed Fan-Cooled 100 hp Induction Motor

a) Based on Application Example 6.10 calculate the temperature rise ΔT of a totally enclosed fan-cooled induction motor provided the following data are given: Input power $P_{in} = 79{,}610$ W, output power $P_{out} = 74{,}600$ W, line frequency $f = 60$ Hz, thickness of stator slot insulation $\beta_i = 9 \cdot 10^{-4}$ m, thermal conductivity of stator slot insulation $\lambda_i = 0.109$ W/m°C, total area of the stator slots $S_i = 2.006$ m², thermal conductivity of the stator laminations (iron core) $\lambda_c = 47.24$ W/m°C, radial height of stator back iron $h_{s1} = 0.049$ m, cross-sectional area of the stator core at the middle of the stator back iron (stator yoke) $S_{s1} = 0.482$ m², length of the small air gap between the stator core and the frame of the motor $\beta_g = 3 \cdot 10^{-5}$ m, thermal conductivity of the small air gap between stator core and frame $\lambda_g = 0.028$ W/m°C, outside surface area of the stator frame $S_g = 0.563$ m², surface thermal coefficient for dissipation of heat to stationary air $\alpha_o = 14.30$ W/m²°C, surface thermal coefficient for dissipation of heat to moving air at $v = 16$ m/s $\alpha_v = 88.66$ W/m²°C, surface area in contact with stationary air $S_o = 0.841$ m², and surface area in contact with moving air $S_v = 3.36$ m². Table P6.9 details the percentage values of the various losses of the 100 hp induction motor. Note that the iron core losses of the rotor are very small due to the low frequency in the rotor member (slip frequency).
b) Determine the rated temperature of the motor T_{rat} provided the ambient temperature is $T_{amb} = 40°C$.

TABLE P6.9 Percentage Losses of a 100 hp Three-Phase Induction Motor

P_{cu1}	P_{y1}	P_{t1}	P_{fe1}	P_{Al2}	P_{fe2}	P_{mech}
30%	8%	10%	2%	45%	2%	3%