5

CHAPTER

Interaction of Harmonics with Capacitors

Capacitors are extensively used in power systems for voltage control, power-factor correction, filtering, and reactive power compensation. With the proliferation of nonlinear loads and the propagation of harmonics, the possibility of parallel/series resonances between system and capacitors at harmonic frequencies has become a concern for many power system engineers.

Since the 1990s, there has been an increase of nonlinear loads, devices, and control equipment in electric power systems, including electronic loads fed by residential and commercial feeders, adjustable-speed drives and arc furnaces in industrial networks, as well as the newly developing distributed generation (DG) sources in transmission and distribution systems. This has led to a growing presence of harmonic disturbances and has deteriorated the quality of electric power. Moreover, some nonlinear loads and power electronic control equipment tend to operate at relatively low power factors, causing poor voltage regulation, increasing line losses, and forcing power plants to supply more apparent power. The conventional and practical procedure for overcoming these problems, as well as compensating reactive power, are to install (fixed and/or switching) shunt capacitor banks on either the customer or the utility side of a power system. Capacitor banks are also used in power systems as reactive power compensators and tuned filters. Recent applications are power system stabilizers (PSS), flexible AC transmission systems (FACTS), and custom power devices, as well as high-voltage DC (HVDC) systems.

Misapplications of power capacitors in today's modern and complicated industrial distribution systems could have negative impacts on both the customers (sensitive linear and nonlinear loads) and the utility (equipment), including these:

- amplification and propagation of harmonics resulting in equipment overheating, as well as failures of capacitor banks themselves,
- harmonic parallel resonances (between installed capacitors and system inductance) close to the frequencies of nearby harmonic sources,
- unbalanced system conditions which may cause maloperation of (ground) relays, and
- additional power quality problems such as capacitor in-rush currents and transient overvoltages due to capacitor-switching actions.

Capacitor or frequency scanning is usually the first step in harmonic analysis for studying the impact of capacitors on system response at fundamental and harmonic frequencies. Problems with harmonics often show up at capacitor banks first, resulting in fuse blowing and/or capacitor failure. The main reason is that capacitors form either series or parallel resonant circuits, which magnify and distort their currents and voltages. There are a number of solutions to capacitor related problems:

- altering the system frequency response by changing capacitor sizes and/or locations,
- altering source characteristics, and
- designing harmonic filters.

The last technique is probably the most effective one, because properly tuned filters can maintain the primary objective of capacitor application (e.g., power-factor correction, voltage control, and reactive power compensation) at the fundamental frequency, as well as low impedance paths at dominant harmonics.

This chapter starts with the main applications of capacitors in power systems and continues by introducing some power quality issues associated with capacitors. A section is provided for capacitor and frequency scanning techniques. Feasible operating regions for safe operation of capacitors in the presence of voltage and current harmonics are presented. The last section of this chapter discusses equivalent circuits of capacitors.

5.1 APPLICATION OF CAPACITORS TO POWER-FACTOR CORRECTION

The application of capacitor banks in transmission and distribution systems has long been accepted as a necessary step in the design of utility power systems. Design considerations often include traditional factors such as voltage and reactive power (VAr) control, power-factor correction, and released capacity. More recent applications concern passive and active filtering, as well as parallel and series (active and reactive) power compensation. Capacitors are also incorporated in many PSS, FACTS, and custom power devices, as well as HVDC systems.

An important application of capacitors in power systems is for power-factor correction. Poor power factor has many disadvantages:

- degraded efficiency of distribution power systems,
- decreased capacity of transmission, substation, and distribution systems,
- poor voltage regulation, and
- increased system losses.

Many utility companies reward customers who improve their power factor (PF) and penalize those who do not meet the prescribed PF requirements. There are a number of approaches for power factor improvement:

- Synchronous condensers (e.g., overexcited synchronous machines with leading power factor) are used in transmission systems to provide stepless (continuous) reactive power compensation for optimum transmission line performance under changing load conditions. This technique is mostly recommended when a large load (e.g., a drive) is added to the system;
- Shunt-connected capacitor banks are used for stepwise (discrete) full or partial compensation of system inductive power demands [1–2]; and
- Static VAr compensators and power converter-based reactive power compensation systems are employed for active power-factor correction (PFC). These configurations use power electronic switching devices to implement variable capacitive or inductive impedances for optimal power-factor correction. Many PFC configurations have been proposed [3–5] for residential and commercial loads, including passive, single-stage, and double-stage topologies for low (below 300 W), medium (300–600 W) and high (above 600 W) power ratings, respectively.

The capacitor-bank approach is most widely used owing to its low installation cost and large capacity [2]. Active power-factor circuits are relied on for relatively small power rating applications ...

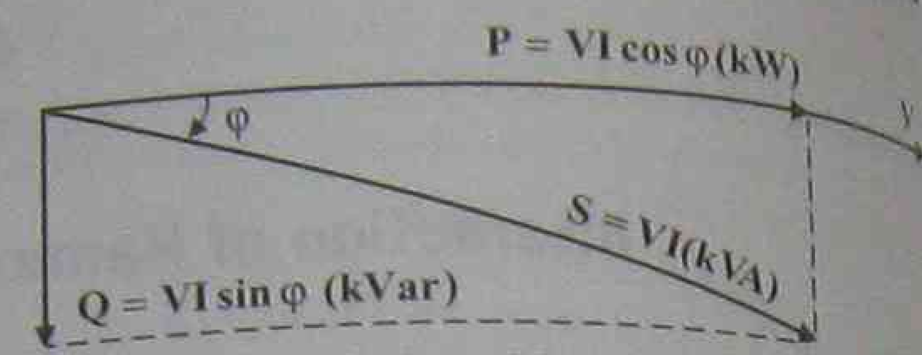

FIGURE 5.1 Definition of lagging displacement power factor (consumer notation).

5.1.1 Definition of Displacement Power Factor

Displacement power factor (DPF) is the ratio of the active or real fundamental power P (measured in W or kW) to the fundamental apparent power S (measured in VA or kVA). The reactive power Q (measured in VAr or kVAr) supplied to inductive devices is the vector difference between the real and apparent powers. The DPF is the cosine of the angle between these two quantities. It reflects how efficiently a facility uses electricity by comparing the amount of useful work that is extracted from the electrical apparent power supplied. Of course, there are many other measures of energy efficiency. The relationship between S, P, and Q is defined by the power triangle of Fig. 5.1.

$$S^2 = \sqrt{P^2 + Q^2}. \tag{5-1}$$

The displacement power factor, determined from $\varphi = \tan^{-1}(Q/P)$, measures the displacement angle between the fundamental components of the phase voltage and phase current:

$$\text{displacement power factor} = DPF = \cos\varphi = P/S. \tag{5-2}$$

The DPF varies between zero and one. A value of zero means that all power is supplied as reactive power and no useful work is accomplished. Unity DPF means that all of the power consumed by a facility goes to produce useful work, such as resistive heating and incandescent lighting.

Electric power systems usually experience lagging displacement power factors – based on the consumer notation of current – because a significant part consists of reactive devices that employ coils and requires reactive power to energize their magnetic circuits. Generally, motors that are operated at or near full nameplate ratings will have high DPFs (e.g., 0.90 to 0.95) whereas the same motor under no-load and/or light load conditions may exhibit a DPF in the range ... in large hydraulic

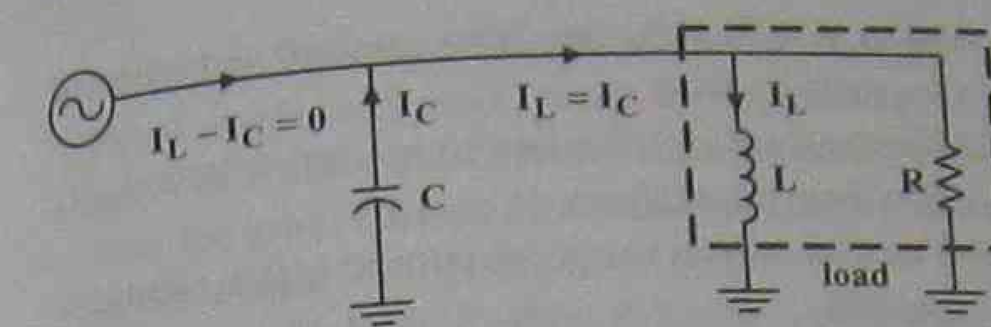

FIGURE 5.2 Current paths demonstrating the essential features of power-factor correction.

operate under lightly loaded conditions, contributing to an overall power factor of about 0.60.

The reactive power absorbed by electrical equipment such as transformers, electric motors, welding units, and static converters increases the currents of generators, transmission lines, utility transformers, switchgear, and cables. Electric utility companies must supply the entire apparent power demand. Because a customer only extracts useful work from the real power component, a high displacement power factor is important. Therefore, in most power systems lagging (underexcited) DPFs (larger than 0.8) are acceptable, and leading (overexcited) power factors should be avoided because they may cause resonance conditions within the power system.

Inductive loads with low DPFs require the generators and transmission or distribution systems to supply reactive current with the associated power losses and increased voltage drops. If a shunt capacitor bank is connected across the load, its (capacitive) reactive current component I_C can cancel the load (inductive) reactive current component I_L. If the bank is sufficiently large (e.g., $I_C = I_L$), it will supply all reactive power, and there will be no reactive current flow entering the power system as is indicated in Fig. 5.2.

The DPF can be measured either by a direct-reading *cosφ meter* indicating an instantaneous value or by a recording *VAr meter*, which allows recording of current, voltage, and power factor over time. Readings taken over an extended period of time (e.g., seconds, minutes) provide a useful means of estimating an average value of power factor for an installation.

5.1.2 Total Power Factor in the Presence of Harmonics

Equations 5-1 and 5-2 assume that system loads have linear voltage–current characteristics and harmonic distortions are not significant. Harmonic voltage and current distortions will change the expression for the total apparent power and the total power factor (TPF). Consider a voltage v(t) and a current i(t) expressed in terms of their rms harmonic components:

$$v(t) = V_0 + \sum_{h=1}^{\infty} \sqrt{2} V_h \sin(h\omega_0 t + \theta_h^v), \tag{5-3a}$$

$$i(t) = I_0 + \sum_{h=1}^{\infty} \sqrt{2} I_h \sin(h\omega_0 t + \theta_h^i). \tag{5-3b}$$

The active (real, average) and reactive power are given by

$$P_{total} = V_o I_o + \sum_{h=1}^{\infty} V_h I_h \cos(\theta_h^v - \theta_h^i), \tag{5-4a}$$

$$Q_{total} = \sum_{h=1}^{\infty} V_h I_h \sin(\theta_h^v - \theta_h^i), \tag{5-4b}$$

and the apparent (voltampere) power is

$$S_{total} = \sqrt{\left(\sum_{h=0}^{\infty} I_h^2\right)\left(\sum_{h=0}^{\infty} V_h^2\right)}. \tag{5-4c}$$

Therefore, for nonsinusoidal cases, Eq. 5-1 does not hold and must be replaced by

$$D = \sqrt{S_{total}^2 - P_{total}^2 - Q_{total}^2}, \tag{5-5}$$

where D is called the distortion power.

According to Eq. 5-5, the total power factor is lower than the DPF (Eq. 5-2) in the presence of harmonic distortion of nonlinear devices such as solid-state or switched power supplies, variable-speed AC drives, and DC drives. Harmonic distortion essentially converts a portion of the useful energy into high-frequency energy that is no longer useful to most devices and is ultimately lost as heat. In this way, harmonic distortion further reduces the power factor. In the presence of harmonics, the total power factor is defined as

$$\text{total power factor} = TPF = \cos\theta = \frac{P_{total}}{S_{total}}, \tag{5-6}$$

where P_{total} and S_{total} are defined in Eq. 5-4. Since capacitors only provide reactive power at the fundamental frequency, they cannot correct the power factor in the presence of harmonics. In fact, improper capacitor sizing and placements can decrease the power factor by generating harmonic resonances, which increase the harmonic content of system voltage and current. $DPF = \cos\varphi$ (with $\varphi = \tan^{-1}(Q/P)$) is always greater than $TPF = \cos\theta$ (with $\theta = \cos^{-1}(P_{total}/S_{total})$) when harmonics are present. Displacement power factor is still very important to most industrial customers because utility billing with respect to power factor is almost universally based on the displacement power factor.

5.1.2.1 Application Example 5.1: Computation of Displacement Power Factor (DPF) and Total Power Factor (TPF)

Strong power systems have very small system impedances (e.g., $R_{syst} = 0.001\ \Omega$, and $X_{syst} = 0.005\ \Omega$) whereas weak power systems have fairly large system impedances (e.g., $R_{syst} = 0.1\ \Omega$, and $X_{syst} = 0.5\ \Omega$). Power quality problems are mostly associated with weak systems. Unfortunately, distributed generation (DG) inherently leads to weak systems because the source impedances of small generators are large, that is, they cannot supply a large current (ideally infinitely large) during transient operation. This application example studies the power-factor correction for a relatively weak power system where the difference between the displacement power factor and the total power factor is large due to the relatively large amplitudes of voltage and current harmonics.

a) Perform a PSpice analysis for the circuit of Fig. E5.1.1, where a three-phase thyristor rectifier – fed by a Y/Y transformer with turns ratio ($N_p/N_s = 1$) – via a filter (e.g., capacitor C_f) serves a load (R_{load}). You may assume $R_{syst} = 0.05\ \Omega$, $X_{syst} = 0.1\ \Omega$, $f = 60$ Hz, $v_{AN}(t) = \sqrt{2}\cdot 346\,\text{V}\cos\omega t$, $v_{BN}(t) = \sqrt{2}\cdot 346\,\text{V}\cos(\omega t - 120°)$, and $v_{CN}(t) = \sqrt{2}\cdot 346\,\text{V}\cos(\omega t - 240°)$. Thyristors T_1 to T_6 can be modeled by MOSFETs in series with diodes, $C_f = 500\ \mu\text{F}$, $R_{load} = 10\ \Omega$, and a firing angle of $\alpha = 0°$. Plot one period of voltage and current after steady state has been reached as requested in parts b and c.

b) Plot and subject the line-to-neutral voltage $v_{an}(t)$ to a Fourier analysis.

c) Plot and subject the phase current $i_a(t)$ to a Fourier analysis.

d) Repeat parts a to c for $\alpha = 10°, 20°, 30°, 40°, 50°, 60°, 70°$, and $80°$.

e) Calculate the displacement power factor (DPF) and the total power factor (TPF) based on the phase shifts of the fundamental and harmonics between $v_{an}(t)$ and $i_a(t)$ for all firing angles and plot the DPF and TPF as a function of α.

f) Determine the capacitance (per phase) C_{bank} of the power-factor correction capacitor bank such that the displacement power factor as seen by the power system is for $\alpha = 60°$ about equal to $DPF = 0.90$ lagging.

g) Explain why the total power factor cannot be significantly increased by the capacitor bank. Would the replacement of the capacitor bank by a three-phase tuned (R_{fbank}, L_{fbank}, and C_{fbank}) filter be a solution to this problem?

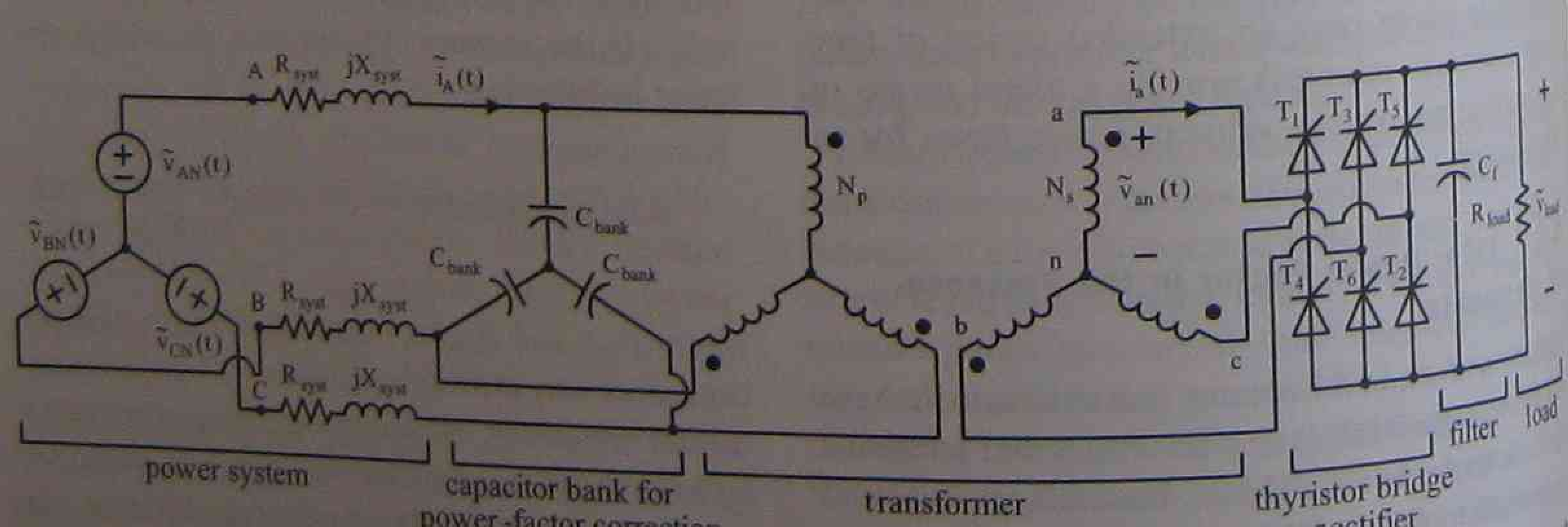

FIGURE E5.1.1 Connection of a wye/wye three-phase transformer with a thyristor rectifier, filter, load, and a bank of power-factor correction capacitors.

5.1.3 Benefits of Power-Factor Correction

Improving a facility's power factor (either DPF or TPF) not only reduces utility power factor surcharges, but it can also provide lower power consumption and offer other advantages such as reduced demand charges, increased load-carrying capabilities in existing lines and circuits, improved voltage profiles, and reduced power system losses. The overall results are lower costs to consumers and the utility alike, as summarized below:

- ***Increased Load-Carrying Capabilities in Existing Circuits.*** Installing a capacitor bank at the end of a feeder (near inductive loads) improves the power factor and reduces the current carried by the feeder. This may allow the circuit to carry additional loads and save costs for upgrading the network when extra capacity is required. In addition, the lower current flow reduces resistive losses in the circuit.
- ***Improved Voltage Profile.*** A lower power factor requires a higher current flow for a given load. As the line current increases, the voltage drop in the conductor increases, which may result in a lower voltage at the load. With an improved power factor, the voltage drop in the conductor is reduced.
- ***Reduced Power-System Losses.*** In industrial distribution systems, active losses vary from 2.5 to 7.5% of the total load measured in kWh, depending on hours of full-load and no-load plant operation, wire size, and length of the feeders. Although the financial return from conductor loss reduction alone is seldom sufficient to justify the installation of capacitors, it is an attractive additional benefit, especially in older plants with long feeders. Conductor losses are proportional to the current squared, whereas the current is reduced in direct proportion to the power-factor improvement; therefore, losses are inversely proportional to the square of the power factor.
- ***Release of Power-System Capacity.*** When capacitor banks are in operation, they furnish magnetizing current for electromagnetic devices (e.g., motors, transformers), thus reducing the current demand from the power plant. Less current means less apparent power, or less loading of transformers and feeders. This means capacitors can be used to reduce system overloading and permit additional load to be added to existing facilities. Release of system capacity by power-factor improvement and especially with capacitors has become an important practice for distribution engineers.
- ***Reducing Electricity Bills.*** To encourage more efficient use of electricity and to allow the utility to recoup the higher costs of providing service to customers with low power factor demanding a high amount of current, many utilities require that the consumers improve the power factors of their installations. This may be in the form of power-factor penalty (e.g., adjusted demand charges or an overall adjustment to the bill) or a rate structure (e.g., the demand charges or power rates are based on current or apparent power). Power-factor penalties are usually imposed on larger commercial and industrial customers when their power factors fall below a certain value (e.g., 0.90 or 0.95). Customers with current or apparent power-based demand charges in effect pay more whenever their power factor is less than about unity. More and more utilities are introducing power-factor penalties (or apparent power-based demand charges) into their rate structures, in part to comply with provisions of the Clean Air Act and deregulation, and to some extent in response to growing competition in a newly deregulated power market. Without these surcharges there would be no motivation for consumers to install power-factor correction facilities. Against the financial advantages of reduced billing, the consumer must balance the cost of purchasing, installing, and maintaining the power-factor-improvement capacitors. The overall result of enforcing incentives is lower costs to consumers and the utility companies. A high power factor avoids penalties imposed by utilities and makes better use of the available capacity of a power system.

Therefore, power-factor correction capacitors are usually installed in power systems to improve the efficiency at which electrical energy is delivered and to avoid penalties imposed by utilities, making better use of the available capacity of a power system.

5.2 APPLICATION OF CAPACITORS TO REACTIVE POWER COMPENSATION

Reactive power compensation plays an important role in the planning and improvement of power systems. Its aim is predominantly to provide an appropriate placement of the compensation devices, which ensures a satisfactory voltage profile and a reduction in power and energy losses within the system. It also maximizes the real power transmission capability of transmission lines, while minimizing the cost of compensation.

The increase of real power transmission in a particular system is restricted by a certain critical voltage level. This critical voltage depends on the reactive power support available in the system to meet the additional load at a given operating condition. Use of series and shunt compensation is one of the corrective measures suggested by various researchers. This is performed by implementing power capacitor banks or FACTS devices. Shunt-connected capacitor banks and shunt FACTS devices are implemented to produce an acceptable voltage profile, minimize the loss of the investment, and enhance the power transmission capability. Series-connected capacitors and series FACTS devices are incorporated in transmission lines to reduce losses and improve transient behavior as well as the stability of power systems.

Simultaneous compensation of losses and reactive powers is performed by series and shunt-connected FACTS devices to improve the static and dynamic performances of power systems.

Shunt capacitors applied at the receiving end of a power system supplying a load of lagging power factor have several benefits, which are the reason for their extensive applications:

- reduce the lagging component of circuit current,
- increase the voltage level at the load,
- improve voltage regulation if the capacitor units are properly switched,
- reduce I^2R real power loss (measured in W) in the system due to the reduction of current,
- reduce I^2X reactive power loss (measured in VAr) in the system due to the reduction of current,
- increase power factor of the source generators,
- decrease reactive power loading on source generators and circuits to relieve an overloaded condition or release capacity for additional load growth,
- by reducing reactive power load on the source generators, additional real power may be placed on the generators if turbine capacity is available, and
- to reduce demand apparent power where power is purchased. Correction to 1 pu power factor may be economical in some cases.

5.3 APPLICATION OF CAPACITORS TO HARMONIC FILTERING

Capacitors are widely used in passive and active harmonic filters. In addition, many static and converter-based power system components such as static VAr compensators, FACTS, and power quality and custom devices use capacitors as storage and compensation components.

Filters are the most frequently used devices for harmonic compensation. Passive and active filters are the main types of filters. Passive filters are composed of passive elements such as resistors (R), inductors (L), and capacitors (C). Passive harmonic filters are the most popular and effective mitigation method for harmonic problems. The passive filter is generally designed to provide a low-impedance path to prevent the harmonic current to enter the power system. They are usually tuned to a specific harmonic such as the 5th, 7th, 11th, etc. In addition, they provide displacement power-factor correction.

Filters are generally divided into passive, active, and hybrid filters. The hybrid structure uses a combination of active filters with passive filters. The passive part is used to reduce the overall filter rating and improve its performance. Classifications of passive and active filters are provided in Chapter 9.

5.3.1 Application Example 5.2: Design of a Tuned Harmonic Filter

A harmonic filter (consisting of a capacitor in series with a tuning reactor) is to be designed in parallel to a power-factor correction (PFC) capacitor to improve the power quality (as recommended by IEEE-519 [8]) and to improve the displacement power factor from 0.42 lag to 0.97 lag (consumer notation). Data obtained from a harmonic analyzer at 30% loading are 6.35 kV/phase, 618 kW/phase, DPF = 0.42 lag, $THD_i = 30.47\%$, and fifth harmonic (250 Hz) current $I_5 = 39.25$ A.

5.4 POWER QUALITY PROBLEMS ASSOCIATED WITH CAPACITORS

There are resonance effects and harmonic generation associated with capacitor installation and switching. Series and parallel resonance excitations lead to increase of voltages and currents, effects which can result in unacceptable stresses with respect to equipment installation or thermal degradation. Using joint capacitor banks for power-factor correction and reactive power compensation is a very well-established approach. However, there are power quality problems associated with using capacitors for such a purpose, especially in the presence of harmonics.

5.4.1 Transients Associated with Capacitor Switching

Transients associated with capacitor switching are generally not a problem for utility equipment. However, they could cause a number of problems for the customers, including

- voltage increases (overvoltages) due to capacitor switching,
- transients can be magnified in a customer facility (e.g., if the customer has low-voltage power-factor correction capacitors) resulting in equipment damage or failure (due to excessive overvoltages),
- nuisance tripping or shutdown (due to DC bus overvoltage) of adjustable-speed drives or other process equipment, even if the customer circuit does not employ any capacitors,
- computer network problems, and
- telephone and communication interference.

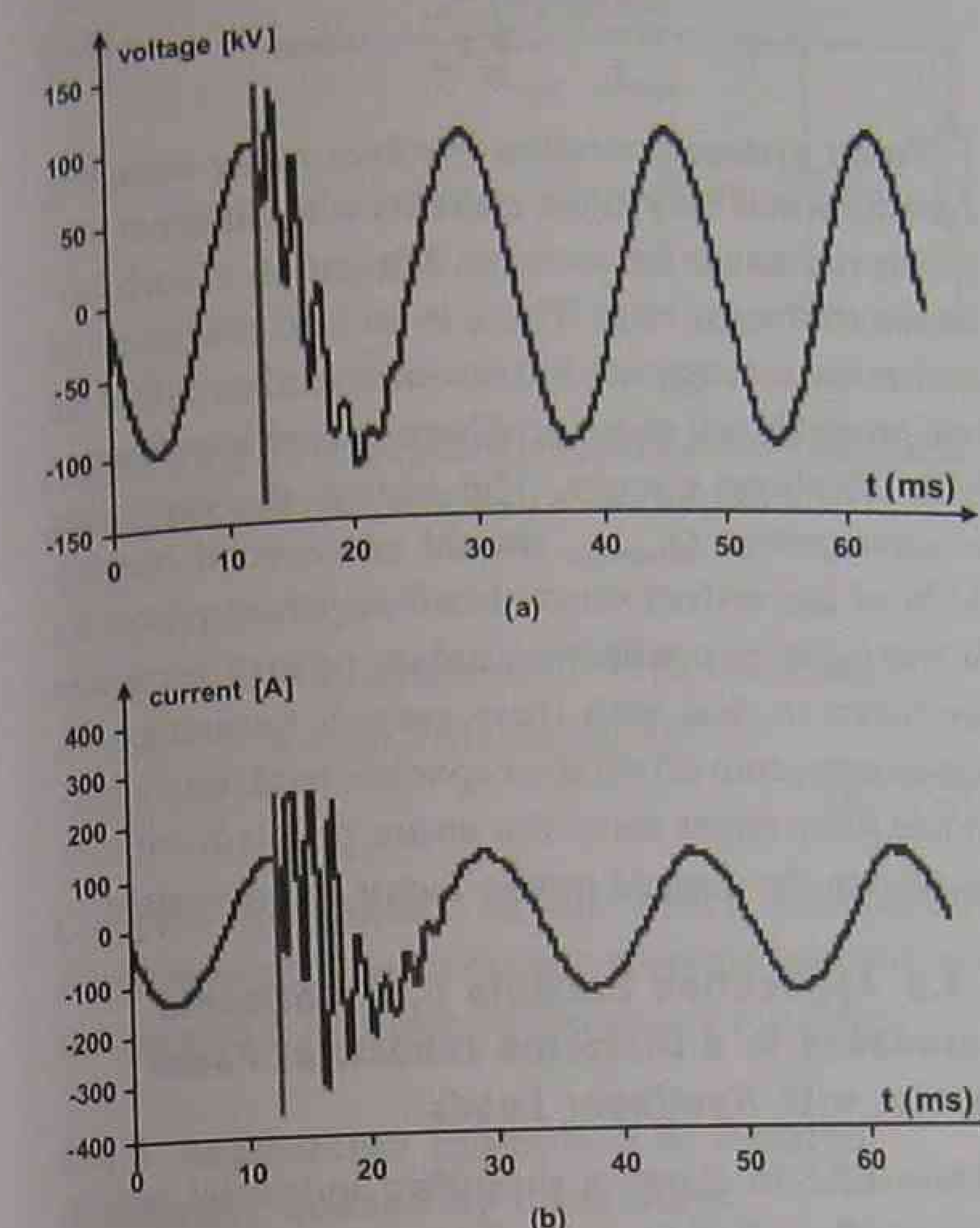

FIGURE 5.3 Typical distribution capacitor closing transient voltage (a) and current (b) waveforms [6].

Because capacitor voltage cannot change instantaneously, energizing a capacitor bank results in an immediate drop in system voltage toward zero, followed by a fast voltage recovery (overshoot), and finally an oscillating transient voltage superimposed with the fundamental waveform (Fig. 5.3). The peak-voltage magnitude (up to 2 pu with transient frequencies of 300–1000 Hz) depends on the instantaneous system voltage at the moment of capacitor connection.

Voltage increase occurs when the transient oscillation – caused by the energization of a capacitor bank – excites a series resonance formed by the leakage inductances of a low-voltage system. The result is an overvoltage at the low-voltage bus. The worst voltage transients occur when the following conditions are met:

- The size of the switched capacitor bank is significantly larger (>10) than the low-voltage power-factor correction bank;
- The source frequency is close to the series-resonant frequency formed by the step-down transformer and the power-factor correction-capacitor bank; and
- There is relatively small damping provided by the low-voltage load (e.g., for typical industrial plant configuration or primarily motor load with high efficiency).

Solutions to the voltage increase usually involve:

- Detune the circuit by changing capacitor bank sizes;
- Switch large banks in more than one section of the system;
- Use an overvoltage control method. For example, use switching devices that do not prestrike or restrike, employ synchronous switching of capacitors, and install high-energy metal-oxide surge arrestors to limit overvoltage and protect critical equipment;
- Apply surge arresters at locations where overvoltages occur;
- Convert low-voltage power-factor correction banks into harmonic filters (e.g., detune the circuit);
- Use properly designed and tuned filters instead of capacitors;
- Install isolation or dedicated transformers or series reactors for areas of sensitive loads; and
- Rely on transient-voltage-surge suppressors.

5.4.2 Harmonic Resonances

Improper placement and sizing of capacitors could cause parallel and/or series resonances and tune the system to a frequency that is excited by a harmonic source [7]. In industrial power systems, capacitor banks are normally specified for PFC, filtering, or reactive-power compensation without regard to resonances and other harmonic concerns. High overvoltages could result if the system is tuned to one harmonic only that is being supplied by a nonlinear load or saturated electromagnetic device such as a transformer (e.g., second, third, fourth, and fifth harmonics result from transformer inrush currents). Moreover, the capacitive reactance is inversely proportional to frequency; therefore, harmonic currents may overload capacitors beyond their limits. Thus, capacitor banks themselves may be affected by resonance, and may fail prematurely. This may even lead to plant or feeder shutdowns.

Resonance is a condition where the capacitive reactance of a system offsets its inductive reactance, leaving the small resistive elements in the network as the only means of limiting resonant currents. Depending on how the reactive elements are arranged throughout the system, the resonance can be of a series or a parallel type. The frequency at which this offsetting effect takes place is called the resonant frequency of the system.

Parallel Resonance. In a parallel-resonant circuit the inductive and the capacitive reactance impedance components are in parallel to a source of harmonic current and the resistive components of the impedances are small compared to the reactive components. The presence of a capacitor (e.g., for PFC) and harmonics may create such conditions and subject the system to failure. From the perspective of harmonic sources, shunt capacitor banks appear to be in parallel with the system short-circuit reactance. The resonant frequency of this parallel combination is

$$f_r = \frac{1}{2\pi\cdot\sqrt{LC}} = f_1\cdot\sqrt{\frac{1000\cdot S_{sc}}{Q_{cap}}}, \tag{5-7}$$

where f_r and f_1 are the resonant and the fundamental frequencies, respectively. S_{sc} and Q_{cap} are the system short-circuit apparent power – measured in MVA – at the bus and the reactive power rating – measured in kVAr – of the capacitor, respectively. Installation of capacitors in power systems modifies the resonance frequency. If this frequency happens to coincide with one generated by the harmonic source, then excessive voltages and currents will appear, causing damage to capacitors and other electrical equipment.

Series Resonance. In a series-resonance circuit the inductive impedance of the system and the capacitive reactance of a capacitor bank are in series to a source of harmonic current. Series resonance usually occurs when capacitors are located toward the end of a feeder branch. From the harmonic source perspective, the line impedance appears in series with the capacitor. At, or close to, the resonant frequency of this series combination, its impedance will be very low. If any harmonic source generates currents near this resonant frequency, they will flow through the low-impedance path, causing interference in communication circuits along the resonant path, as well as excessive voltage distortion at the capacitor.

Capacitor Bank Behaves as a Harmonic Source. There are many capacitor banks installed in industrial and overhead distribution systems. Each capacitor bank is a source of harmonic currents of order h, which is determined by the system short-circuit impedance (at the capacitor location) and the capacitor size. This order of harmonic current is given by

$$h = \sqrt{\frac{X_c}{X_{sc}}} \tag{5-8}$$

Power system operation involves many switching functions that may inject currents with different harmonic resonance frequencies. Mitigation of such harmonic sources is vital. There is no safe rule to avoid such resonant currents, but resonances above 1000 Hz will probably not cause problems except interference with telephone circuits. This means the capacitive reactive power $Q_{capacitor}$ should not exceed roughly 0.3% of the system short-circuit apparent power S_{sc} at the point of connection unless control measures are taken to deal with these current harmonics. In some cases, converting the capacitor bank into a harmonic filter might solve the entire problem without sacrificing the desired power-factor improvement.

5.4.3 Application Example 5.3: Harmonic Resonance in a Distorted Industrial Power System with Nonlinear Loads

Figure E5.3.1a shows a simplified industrial power system ($R_{sys} = 0.06\ \Omega$, $L_{sys} = 0.11$ mH) with a power factor correction capacitor ($C_{pf} = 1300\ \mu$F). Industrial loads may have nonlinear v–i characteristics that can be approximately modeled as (constant) decoupled harmonic current sources. The utility voltages of industrial distribution systems are often distorted due to the neighboring loads and can be approximately modeled as decoupled harmonic voltage sources (as shown in the equivalent circuit of Fig. E5.3.1b):

$$v_{sys}(t) = \sum_{h=1}^{\infty} v_{sys,h}(t) = \sum_{h=1}^{\infty} \sqrt{2}V_{sys,h}\sin(h\omega_1 t + \theta^v_{sys,h}), \tag{E5.3-1a}$$

$$i_{NL}(t) = \sum_{h=1}^{\infty} i_{NL,h}(t) = \sum_{h=1}^{\infty} \sqrt{2}I_{NL,h}\sin(h\omega_1 t + \theta^i_{NL,h}). \tag{E5.3-1b}$$

Fourier analyses of measured voltage and current waveforms of the utility system and the nonlinear loads can be used to estimate the values of $V_{sys,h}$, $\theta^v_{sys,h}$, $I_{NL,h}$, and $\theta^i_{NL,h}$. This is a practical approach because most industrial loads consist of a number of in-house nonlinear loads, and their harmonic models are not usually available.

For this example, assume

$$v_{sys}(t) = \sqrt{2}(110)\sin(\omega_1 t) + \sqrt{2}(5)\sin(5\omega_1 t) + \sqrt{2}(3)\sin(7\omega_1 t)[V], \tag{E5.3-2a}$$

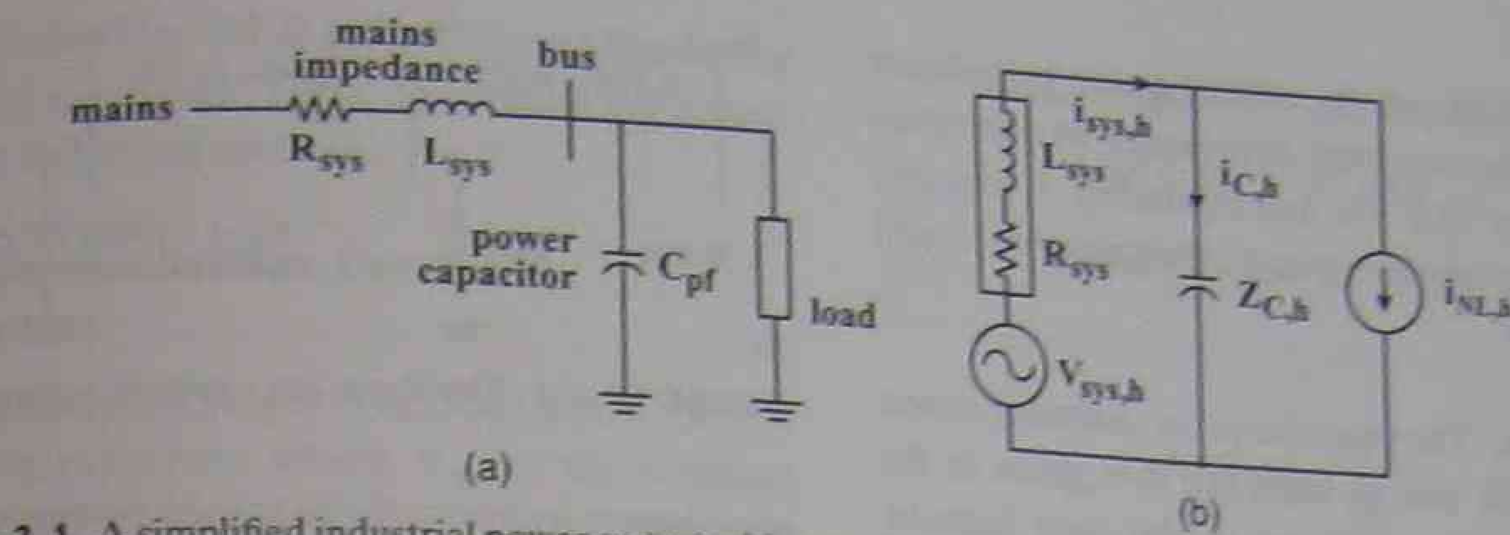

FIGURE E5.3.1 A simplified industrial power system with capacitor and nonlinear load: (a) configuration, (b) the harmonic equivalent circuit [2] with $Z_{sys,h} = R_{sys} + jhL_{sys}$.

$$i_{NL}(t) = \sqrt{2}(1.0)\sin(\omega_1 t) + \sqrt{2}(0.3)\sin(5\omega_1 t) + \sqrt{2}(0.2)\sin(7\omega_1 t)[A]. \tag{E5.3-2b}$$

Compute frequencies of the series and parallel resonances and the harmonic currents injected into the capacitor, and plot its frequency response.

5.4.4 Application Example 5.4: Parallel Resonance Caused by Capacitors

In the system of Fig. E5.4.1a, the source has the ratio $X/R = 10$. Assume $X/R = 4000$ (losses 0.25 W/kVAr) and $X/R = 5000$ (losses 0.2 W/kVAr) for low-voltage and medium-voltage high-efficiency capacitors, respectively. The harmonic current source is a six-pulse converter injecting harmonic currents of the order [8]

$$h = n(k \pm 1), \tag{E5.4-1}$$

where n is an integer (typically from 1 to 4) and k is the number of pulses (e.g., equal to 6 for a six-pulse converter). Find the resonance frequency of this circuit. Plot the frequency response and current amplification at bus 1.

5.4.5 Application Example 5.5: Series Resonance Caused by Capacitors

An example of a series resonance system is demonstrated in Fig. E5.5.1a. The equivalent circuit (neglecting resistances) is shown in Fig. E5.5.1b. Find the resonance frequency of this circuit. Plot the frequency response of bus 1 equivalent impedance, and the current amplification across the tuning reactor.

5.4.6 Application Example 5.6: Protecting Capacitors by Virtual Harmonic Resistors

Repeat Application Example 5.3 assuming a power converter is used to include a virtual harmonic resistor $R_{vir} = 0.5\ \Omega$ in series with the capacitor.

Suggested Solutions to Resonance Problems. As demonstrated in the above examples, harmonic currents and voltages that resonate with power system impedance are usually amplified and result in grave power quality problems such as destruction of capacitors, saturation of electromagnetic devices, and high losses and reduced lifetimes of appliances. It is difficult to come up with a single remedy for all resonance problems, as they highly depend on system configuration and load conditions. Some recommendations are

- Move resonance frequencies away from system harmonics;
- Perform system analysis and harmonic simulations before installing new capacitor banks. Optimal placement and sizing of capacitor banks in the presence of harmonic sources and nonlinear loads are highly recommended for all newly installed capacitor banks;
- Protect capacitors from harmonic destruction using damping circuits (e.g., passive or active resistors in series with the resonance circuit); and
- Use a power converter to include a virtual harmonic resistor in series with the power capacitor [2]. The active resistor only operates at harmonic frequencies to protect the capacitor. To increase system efficiency, the harmonic real power absorption of the virtual resistor can be stored on the DC capacitor of the converter, converted into fundamental real power, and regenerated back to the utility system.

5.5 FREQUENCY AND CAPACITANCE SCANNING

Resonances occur when the magnitude of the system impedance is extreme. Parallel resonance occurs when inductive and capacitive elements are in shunt and the impedance magnitude is a maximum. For a series-resonant condition, inductive and capacitive elements are in series and the impedance magnitude is a minimum. Therefore, the search for a resonant condition at a bus amounts to searching for extremes of the magnitude of its driving-point impedance $Z(\omega)$. There are two basic techniques to determine these extremes and the corresponding (resonant) frequencies for power systems: frequency scan and capacitive scan.

Frequency Scan. The procedures for analyzing power quality problems can be classified into those in the frequency domain and those in the time domain. Frequency-domain techniques are widely used for harmonic modeling and for formulations [9]. They are basically a reformulation of the fundamental load flow to include nonlinear loads and system response at harmonic frequencies.

Frequency scanning is the simplest and a commonly used approach for performing harmonic analysis, determining system resonance frequencies, and designing tuned filters. It is the computation (and plotting) of the driving-point impedance magnitude at a bus ($|Z(\omega)|$) for a range of frequencies and simply scanning the values for either a minimum or a maximum. One could also examine the phase of the driving-point impedance ($\angle Z(\omega)$) and search for its zero crossings (e.g., where the impedance is purely resistive). The frequency characteristic of impedance is called a frequency scan and the resonance frequencies are those that cause minimum and/or maximum values of the magnitude of impedance.

Capacitance Scan. In most power systems, the frequency is nominally fixed, but the capacitance (and/or the inductance) might change. Resonance conditions occur for values of C where the magnitude of impedance is a minimum or a maximum. Therefore, instead of sweeping the frequency, the impedance is considered as a function of C, and the capacitance is swept over a wide range of values to determine the resonance points. This type of analysis is called a capacitance scan.

It is important to note that frequency and capacitance scanning of the driving-point impedance at a bus may not reveal all resonance problems of the system. The plot of $Z(\omega)$ versus ω will only illustrate resonant conditions related to the bus under consideration. It will not detect resonances related to capacitor banks placed at locations relatively remote from the bus under study since it is not usually possible to check the driving-point impedances of all buses.

It is recommended to perform scanning and to check driving-point impedances of buses near harmonic sources (e.g., six- and twelve-pulse converters and adjustable drives) and to examine the transfer

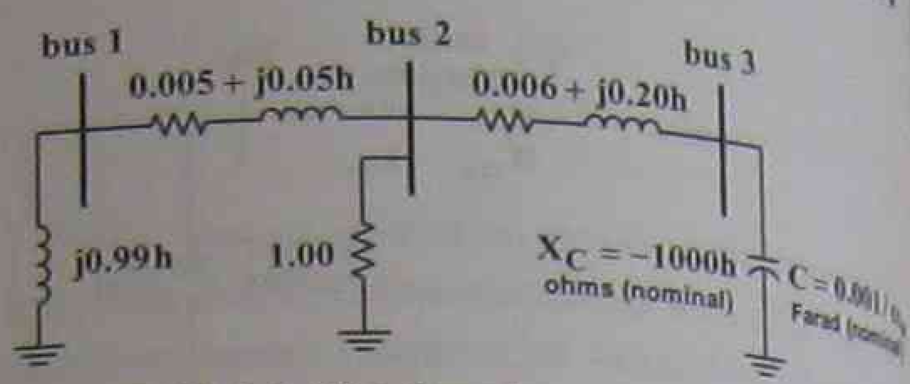

FIGURE E5.7.1 The three-bus example system [9].

impedances between the harmonic sources and a few nearby buses.

5.5.1 Application Example 5.7: Frequency and Capacitance Scanning [9]

Perform frequency and capacitance scans at bus 3 of the three-bus system shown in Fig. E5.7.1. The equivalent circuit of the external power system is represented as 0.99 h at bus 1 (h is the order of harmonics). The nominal design value of the shunt capacitor at bus 3 is $0.001/\omega_o$ farads and its reactance at harmonic h is $(-1000/h)$.

5.6 HARMONIC CONSTRAINTS FOR CAPACITORS

Nonlinear loads and power electronic-based switching equipment are being extensively used in modern power systems. These have nonlinear input characteristics and emit high harmonic currents, which may result in waveform distortion, resonance problems, and amplification of voltage and current harmonics of power capacitors. Therefore, the installation of any capacitor at the design and/or operating stages should also include harmonic and power quality analyses. Some techniques have been proposed for solving harmonic problems of capacitors. Most approaches disconnect capacitor banks when their current harmonics are high. Other techniques include harmonic filters, blocking devices, and active protection circuits. These approaches use the total harmonic distortion of capacitor voltage (THD_v) and current (THD_i) as a measure of distortion level and require harmonic voltage, current, and reactive power constraints for the safe operation of capacitor banks.

The reactive power of a capacitor is given by its capacitance C, the rms terminal voltage V_{rms}, and the line frequency f. This reactive power is valid for sinusoidal operating conditions with rated rms terminal voltage V_{rms_rat} and rated line frequency f_{rat}. If the terminal voltage and the line frequency deviate from their rated values, that is, if the capacitor contains voltage and current harmonics (V_h and I_h for $h = 2, 3, 4, 5, \ldots$), then appropriate constraints for voltage V_{rms}, current I_{rms}, and reactive power Q must be satisfied, as indicated by IEEE [10], IEC, and European [e.g., 11, 12] standards. These constraints are examined in the following sections.

5.6.1 Harmonic Voltage Constraint for Capacitors

Assume harmonic voltages (Eq. 5-3a) across the terminals of a capacitor, where V_{rms} is the terminal voltage. In the following relations h is the harmonic order of the voltage, current, and reactive power: $h = 3^*, 5, 7, 9^*, 11, 13\ 15^*, 17, \ldots$ We have

$$V_{rms}^2 = V_1^2 + \sum_{h=2}^{\infty} V_h^2, \tag{5-9a}$$

or

$$V_{rms} = \sqrt{V_1^2 + \sum_{h=2}^{\infty} V_h^2}. \tag{5-9b}$$

Normalized to V_{rms_rat},

$$\frac{V_{rms}}{V_{rms\_rat}} = \sqrt{(V_1^2 + \sum_{h=2}^{\infty} V_h^2)/V_{rms\_rat}^2} \leq 1.10 \text{ or } 1.15^{a}. \tag{5-9c}$$

5.6.2 Harmonic Current Constraint for Capacitors

Assume harmonic currents (Eq. 5-3b) through the terminals of a capacitor, where I_{rms} is the total current:

$$I_{rms}^2 = I_1^2 + I_2^2 + I_3^2 + \ldots \tag{5-10a}$$

For sinusoidal current components $I_1, I_2, I_3, \ldots$, one obtains with $\omega_1 = 2\pi f_1$

$$I_{rms}^2 = (\omega_1 C V_1)^2 + (2\omega_1 C V_2)^2 + (3\omega_1 C V_3)^2 + \ldots, \tag{5-10b}$$

or

$$I_{rms}^2 = \omega_1^2 C^2 (V_1^2 + 4V_2^2 + 9V_3^2 + \ldots),$$
$$V_1^2 = V_{rms}^2 - V_2^2 - V_3^2 - \ldots$$
$$I_{rms}^2 = \omega_1^2 C^2 (V_{rms}^2 + 3V_2^2 + 8V_3^2 + \ldots),$$

or

$$I_{rms}^2 = \omega_1^2 C^2 \left(V_{rms}^2 + \sum_{h=2}^{\infty} (h^2 - 1) V_h^2 \right), \tag{5-10c}$$

$$I_{rms}^2 = \omega_1^2 C^2 V_{rms}^2 \left(1 + \sum_{h=2}^{\infty} (h^2 - 1) \left(\frac{V_h}{V_{rms}} \right)^2 \right). \tag{5-10d}$$

With $I_{rms\_rat} = \omega_{rat} \cdot C \cdot V_{rms\_rat}$ one can write ($\omega_{rat} = 2\pi f_{rat}$)

$$\frac{I_{rms}}{I_{rms\_rat}} = \frac{f_1}{f_{rat}} \cdot \frac{V_{rms}}{V_{rms\_rat}} \sqrt{1 + \sum_{h=2}^{\infty} (h^2 - 1) \left(\frac{V_h}{V_{rms}} \right)^2} \leq 1.5 \text{ or } 1.8^{b}. \tag{5-10e}$$

5.6.3 Harmonic Reactive-Power Constraint for Capacitors

In the presence of both voltage and current harmonics, the reactive power will also contain harmonics (Eq. 5-4b):

$$\begin{aligned} Q &= Q_1 + Q_2 + Q_3 + \ldots \\ &= I_1 V_1 + I_2 V_2 + I_3 V_3 + \ldots \\ &= \omega_1 C V_1^2 + 2\omega_1 C V_2^2 + 3\omega_1 C V_3^2 + \ldots \\ &= \omega_1 C[(V_{rms}^2 - V_2^2 - V_3^2 - \ldots) + 2V_2^2 + 3V_3^2 + \ldots, \\ &= \omega_1 C[V_{rms}^2 + V_2^2 + 2V_3^2 + \ldots] \\ &= \omega_1 C[V^2 + \sum_{h=2}^{\infty} (h-1) V_h^2] \end{aligned} \tag{5-11a}$$

and with $Q_{rat} = \omega_{rat} C V_{rms\_rat}^2$ one obtains

$$\frac{Q}{Q_{rat}} = \frac{f_1}{f_{rat}} \cdot \frac{V_{rms}^2}{V_{rms\_rat}^2} \left[1 + \sum_{h=2}^{\infty} (h-1) \frac{V_h^2}{V_{rms}^2} \right] \leq 1.35 \text{ or } 1.45^{a}. \tag{5-11b}$$

*The triplen harmonics (which in most cases are of the zero-sequence type) are neglected for three-phase capacitor banks – these harmonics might not be zero but are usually very small. Note that not all triplen harmonics are necessarily of the zero-sequence type. All even harmonics are usually small in three- or single-phase power systems, and therefore they are neglected.

[a] The value of 1.15 for voltages and the value of 1.45 for reactive power are acceptable if the harmonic load is limited to 6 h during a 24 h period at a maximum ambient temperature of $T_{amb} = 35°C$ [13].

[b] The value 1.8 for harmonic currents is permissible if it is explicitly stated on the nameplate of the capacitor [13].

5.6.4 Permissible Operating Region for Capacitors in the Presence of Harmonics

The safe operating region for capacitors in a harmonic environment can be obtained by plotting of the corresponding constraints for voltage, current, and reactive power.

Safe Operating Region for the Capacitor Voltage

$$\frac{V_{rms}}{V_{rms\_rat}} = \frac{\sqrt{V_1^2+\sum_{h=2}^{\infty} V_h^2}}{V_{rms\_rat}} \le 1.10 \text{ or } 1.15^a, \qquad (5\text{-}12a)$$

or

$$\frac{V_1}{V_{rms\_rat}}\sqrt{1+\left(\frac{\sum_{h=2}^{\infty} V_h^2}{V_{rms\_rat}^2}\right)} = 1.10 \text{ or } 1.15^a \qquad (5\text{-}12b)$$

with $V_1 = V_{rms} = V_{rms\_rat}$ and relying on the lower limit in Eq. 5-12b (e.g., 1.10) one obtains

$$\sum_{h=2}^{\infty}\left(\frac{V_h^2}{V_{rms}^2}\right) = (1.1)^2 - 1 = 0.21, \qquad (5\text{-}12c)$$

or the voltage constraint becomes

$$\sum_{h=2}^{\infty}\frac{V_h}{V_{rms}} = 0.458, \qquad (5\text{-}12d)$$

which is independent of h.

Safe Operating Region for the Capacitor Current

$$\frac{I_{rms}}{I_{rms\_rat}} = \frac{f_1}{f_{rat}}\cdot\frac{V_{rms}}{V_{rms\_rat}}\sqrt{1+\sum_{h=2}^{\infty}(h^2-1)\left(\frac{V_h}{V_{rms}}\right)^2} = 1.5 \text{ or } 1.8^b, \qquad (5\text{-}13a)$$

for $f_1 = f_{rat}$ and $V_{rms} = V_{rms\_rat}$. Relying on the lower limit in Eq. 5-13a (e.g., 1.5) one obtains

$$(1.5)^2 - 1 = \sum_{h=2}^{\infty}(h^2-1)\left(\frac{V_h}{V_{rms}}\right)^2 \qquad (5\text{-}13b)$$

or

$$\sum_{h=2}^{\infty}\left(\frac{V_h}{V_{rms}}\right) = \sqrt{\frac{1.25}{h^2-1}}, \qquad (5\text{-}13c)$$

which is a hyperbola.

Safe Operating Region for the Capacitor Reactive Power

$$\frac{Q}{Q_{rat}} = \frac{f_1}{f_{rat}}\cdot\frac{V_{rms}^2}{V_{rms\_rat}^2}\left[1+\sum_{h=2}^{\infty}(h-1)\frac{V_h^2}{V_{rms}^2}\right] = 1.35 \text{ or } 1.45^c \qquad (5\text{-}14a)$$

for $f_1 = f_{rat}$ and $V_{rms} = V_{rms\_rat}$. Relying on the lower limit in Eq. 5-14a (e.g., 1.35) one obtains

$$\sum_{h=2}^{\infty}\left(\frac{V_h}{V_{rms}}\right) = \sqrt{\frac{0.35}{(h-1)}}, \qquad (5\text{-}14b)$$

which is also a hyperbola.

If the above listed limits are exceeded for a capacitor with the rated voltage V_{rms_rat1} then one has to select a capacitor with a higher rated voltage V_{rms_rat2} and a higher rated reactive power satisfying the relation

$$Q_{rat2} = Q_{rat1}\left(\frac{V_{rms\_rat2}}{V_{rms\_rat1}}\right)^2. \qquad (5\text{-}15)$$

Note all triplen and all even harmonics are small in three-phase power systems. However, triplen harmonics can be dominant in single-phase systems.

Feasible Operating Region for the Capacitor Reactive Power. Based on Eqs. 5-12d, 5-13c, and 5-14b, the safe operating region for capacitors in the presence of voltage and current harmonics is shown in Fig. 5.4.

5.6.5 Application Example 5.8: Harmonic Limits for Capacitors when Used in a Three-Phase System

The reactance of a capacitor decreases with frequency, and therefore the capacitor acts as a sink for

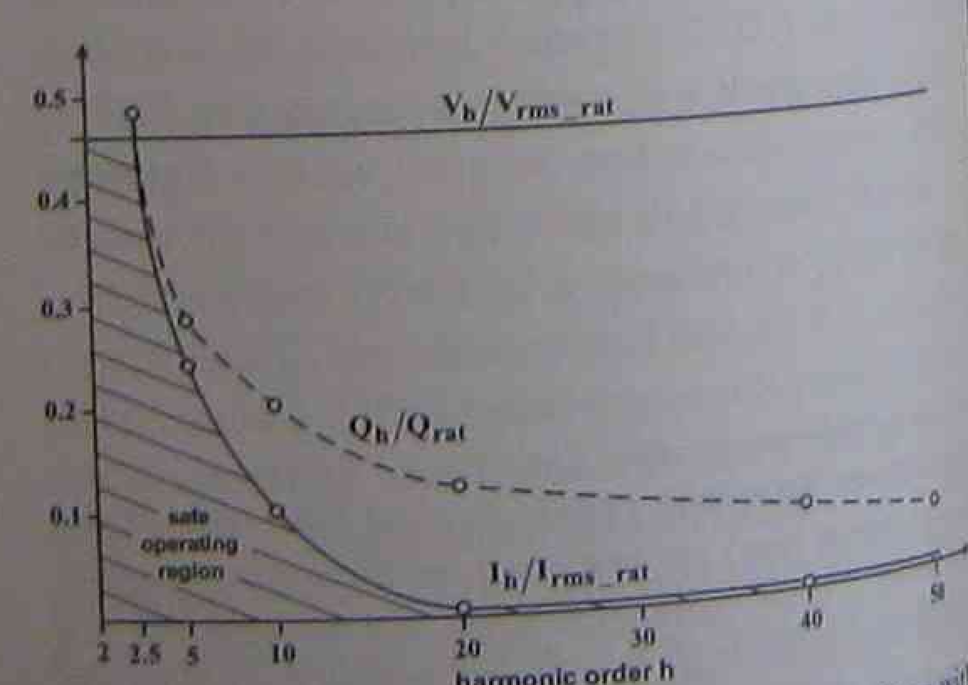

FIGURE 5.4 Feasible operating region of capacitors with voltage, current, and reactive power harmonics (only one harmonic is present at any time).

higher harmonic currents. The effect is to increase the heating and dielectric stresses. ANSI/IEEE [10], IEC, and European [e.g., 11–13] standards list limits for voltage, currents, and reactive power of capacitor banks. These limits can be used to determine the maximum allowable harmonic levels. The result of the increased heating and voltage stress brought about by harmonics is a shortened capacitor life due to premature aging.

The following constraints must be satisfied:

$$V_{rms}/V_{rated} = \sqrt{\left(V_1^2+\sum_{h=2}^{h\max} V_h^2\right)\Big/V_{rated}^2} \le 1.10, \qquad (E5.8\text{-}1)$$

$$I_{rms}/I_{rated} = (f_1/f_{rated})(V_{rms}/V_{rated}) = \sqrt{1+\sum_{h=2}^{h\max}(h^2-1)\left(\frac{V_h}{V_{rms}}\right)^2} \le 1.3, \qquad (E5.8\text{-}2)$$

$$Q/Q_{rated} = (f_1/f_{rated})(V_{rms}/V_{rated})^2 = 1+\sum_{h=2}^{h\max}(h-1)(V_h/V_{rms})^2 \le 1.35, \qquad (E5.8\text{-}3)$$

where V_{rms_rat} is the rated terminal voltage, V_{rms} is the applied effective (rms) terminal voltage, f_1 is the line frequency, and V_1 is the fundamental (rms) voltage.

Plot for $V_1 = V_{rms} = V_{rms\_rat}$ and $f_1 = f_{rat}$ the loci for V_h/V_{rms}, where $(V_{rms}/V_{rms\_rat}) = 1.10$, $(I_{rms}/I_{rms\_rat}) = 1.3$, and $(Q/Q_{rat}) = 1.35$, as a function of the harmonic order h for $5 \le h \le 49$ (only one harmonic is present at any time within a three-phase system).

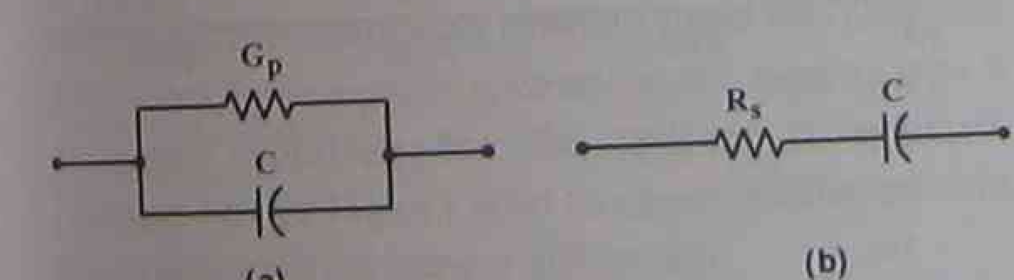

FIGURE 5.5 Equivalent circuits of a capacitor with losses; (a) parallel connection, (b) series connection.

5.7 EQUIVALENT CIRCUITS OF CAPACITORS

The impedance of a capacitor with conduction and dielectric losses can be represented by the two equivalent circuits of Fig. 5.5a (representing a parallel connection of capacitance C and conductance G_p) and Fig. 5.5b (representing a series connection of capacitance C and resistance R_s). The phasor diagrams for the admittance $G_p + j\omega C$ and the impedance $R_s - j/\omega C$ are depicted in Fig. 5.6a,b.

The loss factor $\tan\delta$ or the dissipation factor $DF = \tan\delta$ – which is frequency dependent – is defined for the parallel connection

$$\tan\delta = \frac{G_p}{\omega C}, \qquad (5\text{-}16)$$

and for the series connection

$$\tan\delta = R_s\omega C. \qquad (5\text{-}17)$$

Most AC capacitors have two types of losses:

- conduction loss caused by the flow of the actual charge through the dielectric, and
- dielectric loss due to the movement or rotation of the atoms or molecules of the dielectric in an alternating electric field.

The two equivalent circuits can be combined and expanded to the single equivalent circuit of Fig. 5.7, where $L_s = ESL$ = equivalent series inductance,

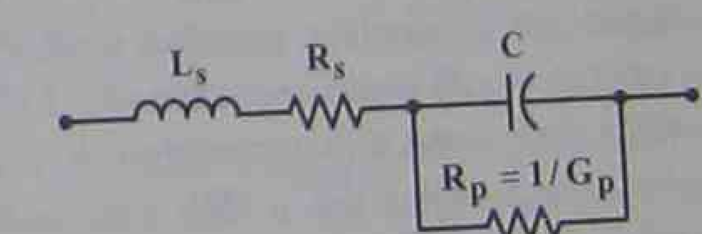

FIGURE 5.7 Detailed equivalent circuit for a capacitor.

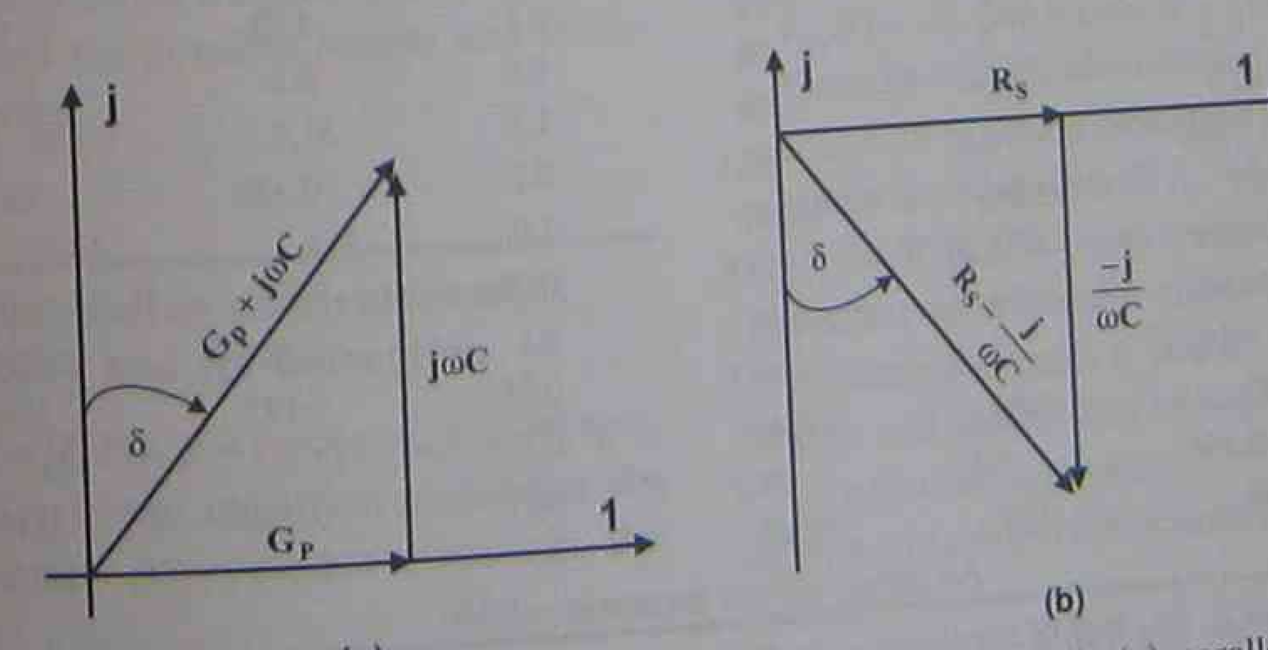

FIGURE 5.6 Phasor diagrams for admittance $G_p + j\omega C$ and impedance $R_s - (j/\omega C)$; (a) parallel connection, (b) series connection.

$R_s = ESR$ = equivalent series resistance, and $R_p = EPR$ = equivalent parallel resistance. For most power system applications such a detailed equivalent circuit is not required. This circuit indicates that every capacitor has a self-resonant frequency, above which it becomes an inductor. R_s is readily measured by applying this frequency to a capacitor, measuring voltage and current, and calculating their ratio. The resistance R_p will always be much larger than the capacitive reactance at the resonant frequency and therefore this parallel resistance can be neglected at this resonant frequency.

The presence of voltage distortion increases the dielectric loss of capacitors; the total loss is for the frequency-dependent loss factor $\tan\delta_h = R_{sh}\omega_h C$ at h_{max} harmonic voltages:

$$P_{C\_total} = \sum_{h=1}^{h=h_{max}} C\cdot(\tan\delta_h)\omega_h V_h^2, \qquad (5\text{-}18)$$

where $\omega_h = 2\cdot\pi\cdot f_h$ and V_h is the rms voltage of the hth harmonic. For capacitors used for low-frequency applications the relation becomes

$$P_{C\_total} = \sum_{h=1}^{h=h_{max}} C^2\cdot R_{sh}\cdot\omega_h^2\cdot V_h^2. \qquad (5\text{-}19)$$

For 60 Hz applications inexpensive AC capacitors can be used, whereas for power electronics applications and filters low-loss (e.g., ceramic or polypropylene) capacitors must be relied on.

5.7.1 Application Example 5.9: Harmonic Losses of Capacitors

For a capacitor with $C = 100\,\mu F$, $V_{rat} = 460\,V$, $R_{s1} = 0.01\,\Omega$ (where R_{s1} is the series resistance of the capacitor at fundamental ($h = 1$) frequency of $f_{rat} = 60$ Hz), compute the total harmonic losses for the harmonic spectra of Table E5.9.1 (up to and including the 19th harmonic) for the following conditions:

a) R_{sh} is constant, that is, $R_{sh} = R_{s1} = 0.01\,\Omega$.
b) R_{sh} is proportional to frequency, that is, $R_{sh} = R_{s1}(f/f_{rat}) = 0.01\,h\,\Omega$.
c) R_{sh} is proportional to the square of frequency, $R_{sh} = R_{s1}(f/f_{rat})^2 = 0.01\,h^2\,\Omega$.

5.8 SUMMARY

Capacitors are important components within a power system: they are indispensable for voltage control, power-factor correction, and the design of filters.

TABLE E5.9.1 Possible Voltage Spectra and Associated Capacitor Losses

h	$\left(\frac{V_h}{V_{60Hz}}\right)_{1\Phi}$ (%)	P_h for part a (mW)	P_h for part b (mW)	P_h for part c (mW)	$\left(\frac{V_h}{V_{60Hz}}\right)_{3\Phi}$ (%)	P_h for part a (mW)	P_h for part b (mW)	P_h for part c (mW)
1	100	3005	3005	3005	100	3005	3005	3005
2	0.5	0.3	0.6	1.2	0.5	0.3	0.6	1.2
3	4.0	40	120	360	2.0[a]	10.8	32.4	97.2
4	0.3	0.4	1.6	6.4	0.5	1.2	4.8	19.2
5	3.0	67.7	339	1695	5.0	188	940	4700
6	0.2	0.43	2.6	15.6	0.2	0.43	2.6	15.6
7	2.0	58.9	412	2880	3.5	181	1267	8869
8	0.2	0.77	6.16	49.3	0.2	0.77	6.16	49.3
9	1.0	24.4	220	1980	0.3	2.19	19.71	177.4
10	0.1	0.3	3	30	0.1	0.3	3	30
11	1.5	81.9	900	9900	1.5	81.9	900	9900
12	0.1	0.433	5.2	62.4	0.1	0.433	5.2	62.4
13	1.5	114	1480	19240	1.0	114	1482	19270
14	0.1	0.59	8.26	115.6	0.05	0.15	2.58	28.81
15	0.5	16.9	254	3810	0.1	0.68	10.2	153.0
16	0.05	0.192	3.07	49.1	0.05	0.192	3.07	49.1
17	1.0	87	1480	25140	0.5	22	374	6358
18	0.05	0.244	4.4	79.2	0.01	0.0097	0.175	3.143
19	1.0	109	2070	39200	0.5	27.1	513	9750

All higher voltage harmonics < 0.5%

[a]Under certain conditions (e.g., DC bias of transformers as discussed in Chapter 2, and the harmonic generation of synchronous generators as outlined in Chapter 4) triplen harmonics are not of the zero-sequence type and can therefore exist in a three-phase system.

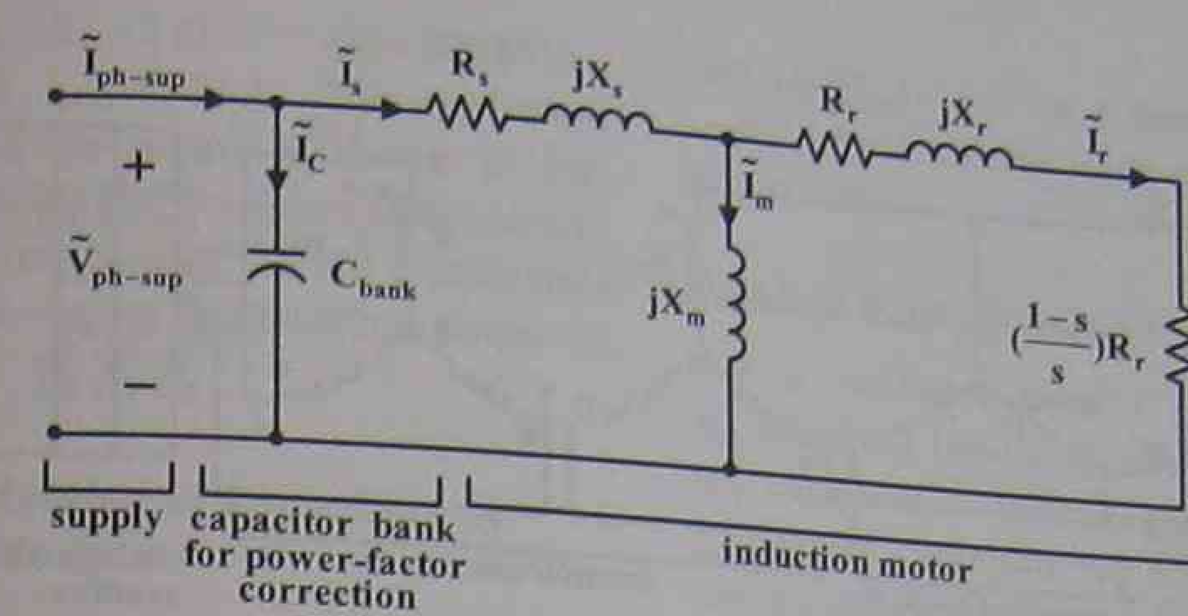

FIGURE P5.1 Per-phase equivalent circuit of induction motor with displacement power-factor correction capacitor bank.

Their deployment may cause problems associated with capacitor switching and series resonance. Capacitor failures are induced by too large voltage, current, and reactive power harmonics. In most cases triplen (multiples of 3) and even harmonics do not exist in a three-phase system. However, there are conditions where triplen harmonics are not of the zero-sequence type and they can occur within three-phase systems. Triplen harmonics are mostly dominant in single-phase systems, whereas even harmonics are mostly negligibly small within single- and three-phase systems. In this chapter the difference between displacement power factor (DPF) – pertaining to fundamental quantities only – and the concept of the total power factor (TPF) – pertaining to fundamental and harmonic quantities – is explained. For the special case when there are no harmonics present these two factors will be identical; otherwise, the TPF is always smaller than the DPF. The TPF concept is related to the distortion power D and the DPF and TPF must be in power systems of the lagging type if consumer notation for the current is assumed. Leading power factors tend to cause oscillations and instabilities within power systems.

The losses of capacitors can be characterized by the loss factor or dissipation factor (DF) $\tan\delta$, which is a function of the harmonic frequency. AC capacitor losses are computed for possible single- and three-phase voltage spectra.

5.9 PROBLEMS

Problem 5.1: Calculation of Displacement Power Factor (DPF) and Its Correction

A $V_{L\text{-}L} = 460$ V, $f = 60$ Hz, $p = 6$ pole, $n_m = 1180$ rpm, Y-connected squirrel-cage induction motor has the following parameters per phase referred to the stator:

$R_s = 0.19\,\Omega$, $X_s = 0.75\,\Omega$, $X_m = 20\,\Omega$, $R_r' = 0.070\,\Omega$, $X_r' = 0.380\,\Omega$, and $R_{fe} \to \infty$.

a) Calculate the full-load current $\tilde{I}_s$ and the displacement power factor $\cos\varphi$ of the motor (without capacitor bank).
b) In Fig. P5.1 the reactive current component of the motor is corrected by a capacitor bank C_{bank}. Find C_{bank} such that the displacement power factor as seen at the input of the capacitor is $\cos\varphi = 0.95$ lagging based on the consumer notation of current flow.

Problem 5.2: Calculation of the Total Power Factor (TPF) and the DPF Correction

The following problem studies the power-factor correction for a relatively weak power system where the difference between the displacement power factor (DPF) and the total power factor (TPF) is large due to the relatively large amplitudes of voltage and current harmonics.

a) Perform a PSpice analysis for the circuit of Fig. P5.2, where a three-phase diode rectifier – fed by a Y/Y transformer with turns ratio $N_p/N_s = 1$ – is combined with a self-commutated switch (e.g., MOSFET or IGBT) and a filter (e.g., capacitor C_f) serving a load (R_{load}). You may assume $R_{syst} = 0.1\,\Omega$, $X_{syst} = 0.5\,\Omega$, $f = 60$ Hz, $v_{AN}(t) = \sqrt{2}\cdot 346\,V\cos\omega t$, $v_{BN}(t) = \sqrt{2}\cdot 346\,V\cos(\omega t - 120°)$, $v_{CN}(t) = \sqrt{2}\cdot 346\,V\cos(\omega t - 240°)$, ideal diodes D_1 to D_6, $C_f = 500\,\mu F$, $R_{load} = 10\,\Omega$, and a switching frequency of the self-commutated switch of $f_{sw} = 600$ Hz at a duty cycle of $\delta = 50\%$. Plot one period of the voltage and current after steady state has been reached as requested in parts b and c.
b) Plot and subject the line-to-neutral voltage $v_{an}(t)$ to a Fourier analysis.
c) Plot and subject the phase current $i_a(t)$ to a Fourier analysis.
d) Repeat parts a to c for $\delta = 10\%, 20\%, 30\%, 40\%, 60\%, 70\%, 80\%,$ and 90%.

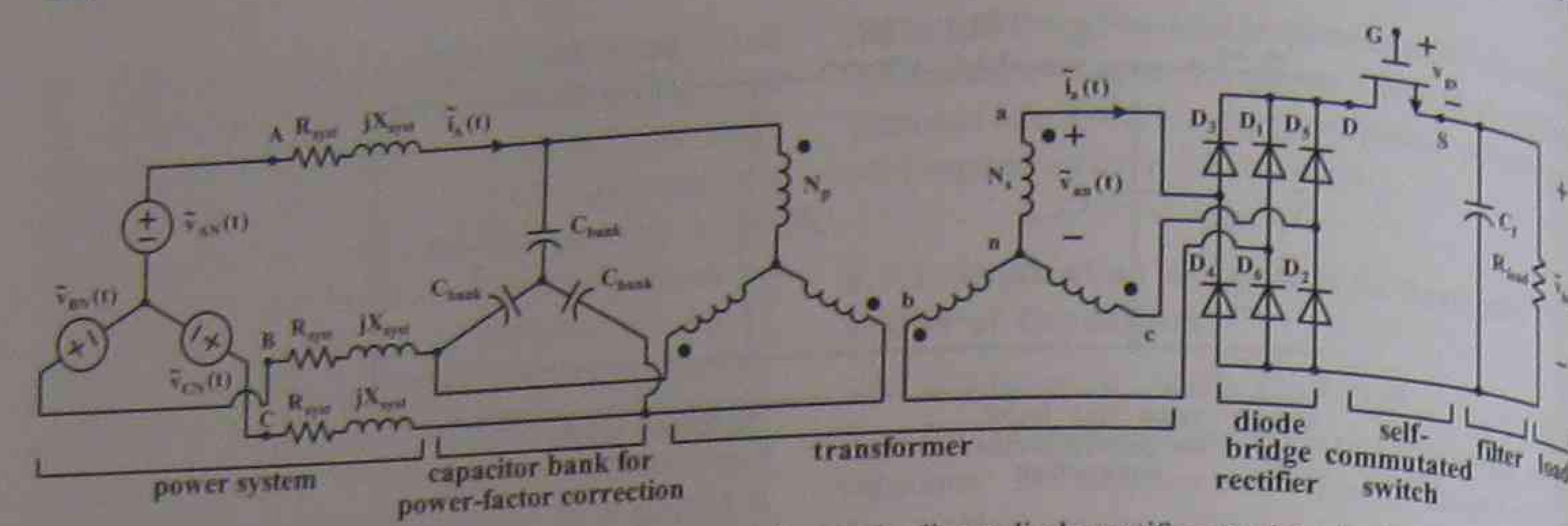

FIGURE P5.2 Connection of a wye/wye three-phase transformer feeding a diode rectifier combined with self-commutated switch, filter, load, and a bank of power-factor correction capacitors.

e) Calculate the displacement power factor (DPF) and the total power factor (TPF) based on the phase shifts of the fundamental and harmonics between $v_{an}(t)$ and $i_a(t)$ for all duty cycles and plot the DPF and TPF as a function of δ.

f) Determine the capacitance (per phase) C_{bank} of the power-factor correction capacitor bank such that the displacement power factor as seen by the power systems is for $\delta = 50\%$ about equal to DPF = 0.95 lagging.

Problem 5.3: Relation between Total and Displacement Power Factors

For the circuit of Fig. E5.1.1 (without taking into account power-factor correction capacitors), calculate with PSpice the Fourier components (magnitude and phase) of the phase voltage $v_{an}(t)$ and the phase current $i_a(t)$ up to the 10th harmonic. Compute and plot the ratio TPF/DPF as a function of the firing angle α.

Problem 5.4: Design of a Tuned Passive Filter

A harmonic filter (consisting of a capacitor and a tuning reactor) is to be designed in parallel to a power-factor correction (PFC) capacitor to improve the power quality (as recommended by IEEE-519 [8]) and to improve the displacement power factor *DPF* from 0.5 lag to 0.95 lag (consumer notation). Data obtained from a harmonic analyzer at 100% loading are 6.35 kV/phase, 600 kW/phase, $THD_i = 50\%$, and fifth harmonic (300 Hz) current $I_5 = 50$ A. It is desirable to reduce the THD_i to 10%.

Problem 5.5: Harmonic Resonance

Figure E5.3.1a shows a simplified industrial power system ($R_{sys} = 0.01\ \Omega$, $L_{sys} = 0.05$ mH) with a power-factor correction capacitor ($C_{pf} = 2000\ \mu$F). Industrial loads may have nonlinear v–i characteristics that can be approximately modeled as (constant) decoupled harmonic current sources. The utility voltages in industrial distribution systems are often distorted due to the neighboring loads and can be approximately modeled as decoupled harmonic voltage sources:

$$\begin{aligned} v_{sys}(t) &= \sqrt{2}\cdot 460\sin(\omega_1 t) + \sqrt{2}\cdot 25\sin(5\omega_1 t) \\ &\quad + \sqrt{2}\cdot 15\sin(7\omega_1 t) \\ i_{NL}(t) &= \sqrt{2}\cdot 10\sin(\omega_1 t) + \sqrt{2}\cdot 2\sin(5\omega_1 t) \\ &\quad + \sqrt{2}\cdot 1\sin(7\omega_1 t). \end{aligned} \qquad \text{(P5.5-1)}$$

Compute frequencies of the series and parallel resonances and the harmonic currents injected into the capacitor, and plot its frequency response.

Problem 5.6: Parallel Harmonic Resonance

In the system of Fig. E5.4.1a, the source has the ratio $X/R = 20$. Assume $X/R = 2000$ (losses 0.5 W/kVAr) and $X/R = 10{,}000$ (losses 0.1 W/kVAr) for low-voltage and medium-voltage high-efficiency capacitors, respectively. The harmonic current source is a six-pulse converter injecting harmonic currents of the order

$$h = n\,(k+1), \qquad \text{(P5.6-1)}$$

where n is an integer (typically from 1 to 4) and k is the number of pulses (e.g., equal to 6 for a six-pulse converter). Find the resonance frequency of this circuit. Plot the frequency response and current amplification at bus 1.

Problem 5.7: Series Harmonic Resonance

An example of a series resonance system is demonstrated in Fig. E5.5.1a, where $S = 200$ MVA, $X/R = 20$, $V_{bus\text{-}rms} = 7.2$ kV, $X_L = 1\ \Omega$, $Q_C = 900$ kVAr, $V_{Crms} = 7.2$ kV, and $I_{Lrms} = 100$ A. The equivalent circuit (neglecting resistances) is shown in Fig. E5.5.1b. Find the resonance frequency of this circuit. Plot the frequency response of the bus 1 equivalent impedance, and the current amplification across the tuning reactor.

Problem 5.8: Protection of Capacitors by Virtual Harmonic Resistors

Repeat Application Example 5.3 assuming a power converter is used to include a virtual harmonic resistor $R_{vir} = 1.0\ \Omega$ in series with the capacitor.

Problem 5.9: Harmonic Current, Voltage, and Reactive Power Limits for Capacitors When Used in a Single-Phase System

The reactance of a capacitor decreases with frequency and therefore the capacitor acts as a sink for higher harmonic currents. The effect is to increase the heating and dielectric stress. ANSI/IEEE [10], IEC, and European [e.g., 11, 12] standards provide limits for voltage, currents, and reactive power of capacitor banks. This can be used to determine the maximum allowable harmonic levels. The result of the increased heating and voltage stress brought about by harmonics is a shortened capacitor life due to premature aging.

According to the nameplate of the capacitors the following constraints must be satisfied:

$$V_{rms}/V_{rms\text{-}rat} = \sqrt{\left(V_1^2 + \sum_{h=2}^{h\max} V_h^2\right)\Big/ V_{rms\_rat}^2} \le 1.15, \qquad \text{(P5.9-1)}$$

$$I_{rms}/I_{rat} = (f_1/f_{rat})(V_{rms}/V_{rat})\sqrt{1 + \sum_{h=2}^{h\max}(h^2-1)\left(\frac{V_h}{V_{rms}}\right)^2} \le 1.3, \qquad \text{(P5.9-2)}$$

$$Q/Q_{rat} = (f_1/f_{rat})(V_{rms}/V_{rat})^2\left[1 + \sum_{h=2}^{h\max}(h-1)\left(\frac{V_h}{V_{rms}}\right)^2\right] \le 1.45, \qquad \text{(P5.9-3)}$$

where V_{rms_rat} is the rated terminal voltage, V_{rms} is the applied effective (rms) terminal voltage, f_1 is the line frequency, and V_1 is the fundamental (rms) voltage.

Plot for $V_1 \approx V_{rms} = V_{rms\_rat}$ and $f_1 = f_{rat}$ the loci for V_h/V_{rms}, where $(V_{rms}/V_{rms\_rat}) = 1.15$, $(I_{rms}/I_{rms\_rat}) = 1.3$, and $(Q/Q_{rat}) = 1.45$ as a function of the harmonic order h for $3 \le h \le 49$ (only one harmonic is present at any time).

Problem 5.10: Harmonic Losses of Capacitors

For a capacitor with $C = 100\ \mu$F, $V_{rat} = 1000$ V, $R_{s1} = 0.005\ \Omega$ (where R_{s1} is the series resistance of the capacitor at fundamental ($h = 1$) frequency of $f_{rat} = 60$ Hz), compute the total harmonic losses for the harmonic spectra of Table P5.10 (up to and including the 19th harmonic) for the following conditions:

a) R_{sh} is constant, that is, $R_{sh} = R_{s1} = 0.005\ \Omega$.

b) R_{sh} is proportional to frequency, that is,

$$R_{sh} = R_{s1}\left(\frac{f}{f_{rat}}\right) = 0.005\cdot h\,\Omega.$$

c) R_{sh} is proportional to the square of frequency,

$$R_{sh} = R_{s1}\left(\frac{f}{f_{rat}}\right)^2 = 0.005\cdot h^2\,\Omega.$$

d) Plot the calculated losses of parts a to c as a function of the harmonic order h.

TABLE P5.10 Possible Voltage Spectra with High Harmonic Penetration

h	$\left(\frac{V_h}{V_{60\text{Hz}}}\right)_{1\phi}$ (%)	$\left(\frac{V_h}{V_{60\text{Hz}}}\right)_{3\phi}$ (%)
1	100	100
2	2.5	0.5
3	5.71	1.0[a]
4	1.6	0.5
5	1.25	7.0
6	0.88	0.2
7	1.25	5.0
8	0.62	0.2
9	0.96	0.3
10	0.66	0.1
11	0.30	2.5
12	0.18	0.1
13	0.57	2.0
14	0.10	0.05
15	0.10	0.1
16	0.13	0.05
17	0.23	1.5
18	0.22	0.01
19	1.03	1.0
	All higher harmonics < 0.2%	

[a]Under certain conditions (e.g., DC bias of transformers as discussed in Chapter 2, and the harmonic generation of synchronous generators as outlined in Chapter 4) triplen harmonics are not of the zero-sequence type, and they can therefore exist in a three-phase system.