where E is the voltage applied to the galvanometer circuit. The term $G\frac{d\theta}{dt}$ represents a "back e.m.f." induced in the coil due to its motion through the magnet field, G being $NHlr$, N being the number of turns on the coil, l its active length, r its breadth and H the intensity of the permanent magnet field.

The equation of motion now becomes

$$a\frac{d^2\theta}{dt^2} + b'\frac{d\theta}{dt} + c\theta = \frac{G}{R}\left(E - G\frac{d\theta}{dt}\right)$$

or

$$a\frac{d^2\theta}{dt^2} + \left(\frac{G^2}{R} + b'\right)\frac{d\theta}{dt} + c\theta = G\frac{E}{R} = GI \quad . \quad . \quad . \quad . \qquad (199)$$

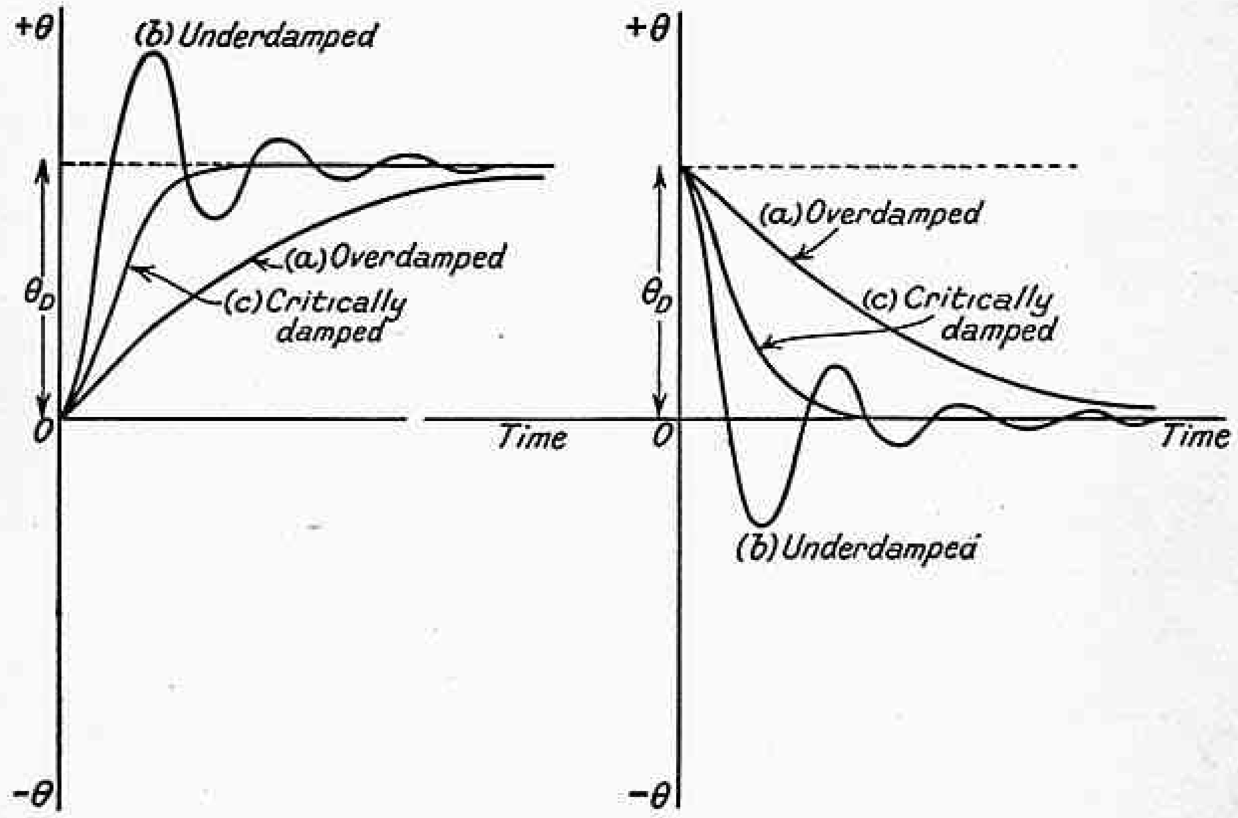

Fig. 189. Deflection-Time Curves for Different Degrees of Damping

where I is the final value of the galvanometer current when the coil has attained its steady deflection. The effective damping constant is now, therefore, $\left(\frac{G^2}{R} + b'\right)$, and this now replaces b in the original theory.

If b' is small compared with $\frac{G^2}{R}$ the galvanometer is critically damped when $\left(\frac{G^2}{R}\right)^2 = 4ac$, and since G, a, and c are constants for any particular instrument, critical damping may be obtained by variation of R.

It is important, in order to save both time and trouble in using such galvanometers, that the damping shall be properly adjusted, and also that the sensitivity of the galvanometer chosen for a particular measurement shall not greatly exceed that demanded by the work in hand.

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continuous variation of the capacitance is required, as is often the case in a.c. bridge work, a variable air capacitor of the parallel plate type is used. In some cases a combination of both of the above types may be most useful.

Continuously variable capacitors, having air as dielectric, consist of two sets of plates, usually semicircular—one set fixed and the other moving—arranged so that the moving plates can be rotated in the air gap between the fixed plates as shown in Figs. 147 and 148. The capacitance is varied by varying the area of the moving plates interleaving with the fixed plates. The plates, which are usually of aluminium or brass, are proportioned so that the capacitance varies in almost exact proportion to the angle turned through. The plates should be made fairly thick, so as to avoid bending, which would alter the calibration, and all corners should be carefully rounded. The bearings must be well fitted, so that the axial distance between the plates shall be definite and constant. The variable air capacitor shown in Fig. 148 is of the precision type for use as a laboratory standard or for a.c. bridge measurements. It has a slow-motion, worm-gear device permitting high accuracy in setting and reading. Its temperature coefficient is 30–40 parts in 10^6 per degree centigrade, the power factor being less than 0·0001 at 1,000 c/s, and residual self inductance between 0·04 and 0·07 μH, for all settings.

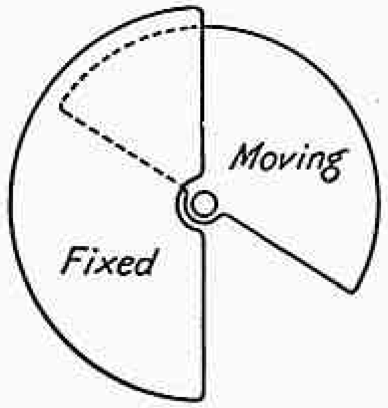


FIG. 147

Square Law Capacitors. Duddell (Ref. (26)) constructed a variable capacitor with the plates shaped so as to give a square law of capacitance variation. Such capacitors are of use in wavemeters for radio work where the wavelength is approximately proportional to the square root of the capacitance of the variable capacitor.

The plates were shaped as shown in Fig. 149. R is the inner radius of the fixed plates; r is a radius of the moving plates. The law of the curve bounding the moving plates is

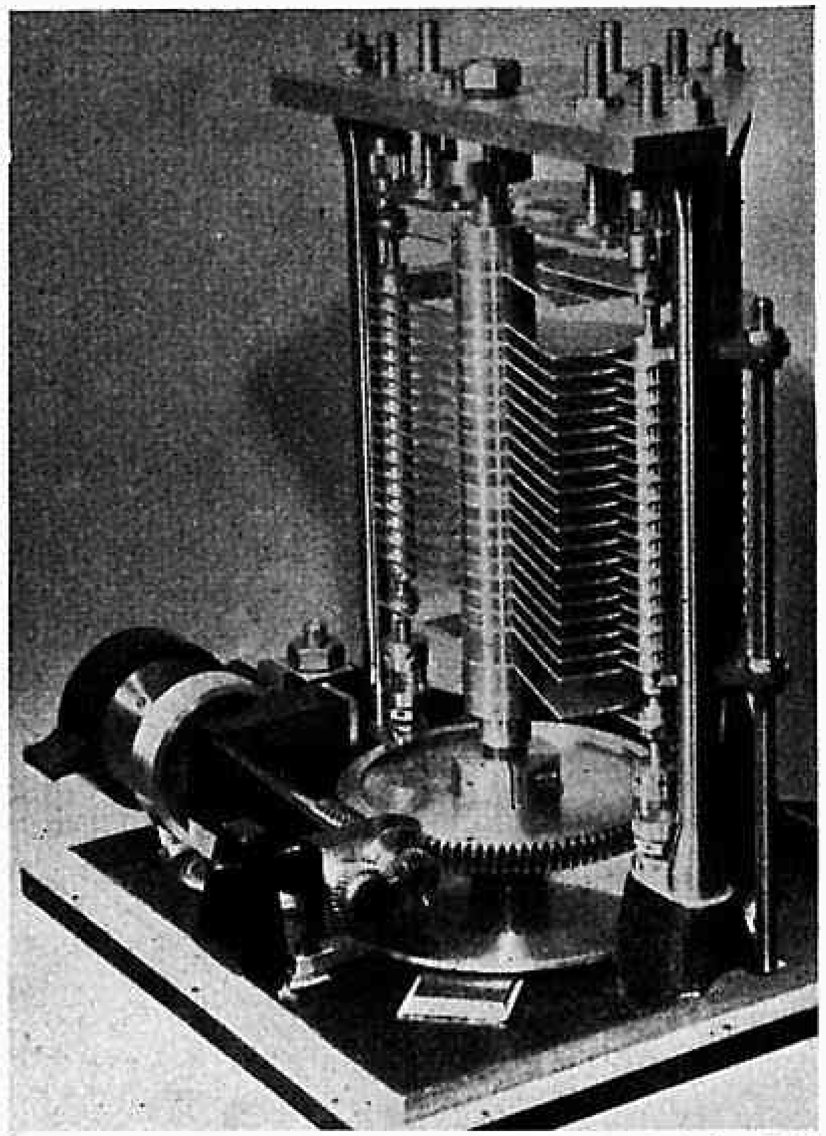
$$r^2 = 4K\theta + R^2 \quad . \quad . \quad . \quad . \quad (173)$$

K being a constant such that the interleaving area of the plates —shown shaded in the figure—is equal to $K\theta^2$. θ is the angle turned through by the moving plates from their zero position. Then, since the shaded area is proportional to θ^2 it follows that the capacitance also is very nearly proportional to θ^2. W. H. F. Griffiths* has investigated the laws of variable air capacitors with several different designs of plates. One of these, the Sullivan-Griffiths logarithmic variable air capacitor, covers a wide range of

* *Experimental Wireless and The Wireless Engineer*, Vol. III, No. 28, January, 1926, and Vol. III, No. 39, December, 1926. See also Refs. (44), (49).

capacitance, the logarithmic scale law giving the same accuracy of scale reading throughout the range for which the law holds. This is particularly important for very low values of capacitance.

The Sullivan precision variable air capacitor standard has fused silica insulation ensuring a high degree of permanence in the calibration. It has very low power losses, a scale accuracy of 1 part in 20,000 (or better) and a temperature coefficient less than 10 parts in 10^6 per degree centigrade.



(*Muirhead & Co., Ltd.*)

FIG. 148. VARIABLE AIR CAPACITOR

The decade air capacitor by the same maker consists of a number of variable capacitors. The sectors are introduced in such a manner that small angular movements, due to a possible uncertainty of "click" positioning, produce only very small edge capacitance changes on one sector only.

Self-contained Bridge Networks and Meters. It is of great convenience in a.c. bridge work to have some form of permanently connected bridge. Apart from the saving of time and labour in connecting up the bridge network, such a piece of apparatus, if properly designed, minimizes errors due to inductance and capacitance in the leads, and to leakage effects. A measurement can also be repeated if necessary, with the assurance that the distribution of the bridge will be the same as on that employed in the previous measurement.

A number of self-contained a.c. bridges for the measurement of inductance and capacitance have been developed by various manufacturing firms. An interesting example is the "Mufer" capacitance bridge made by the Baldwin Instrument Company. This employs the De Sauty Circuit (see page 221) the resistance arms

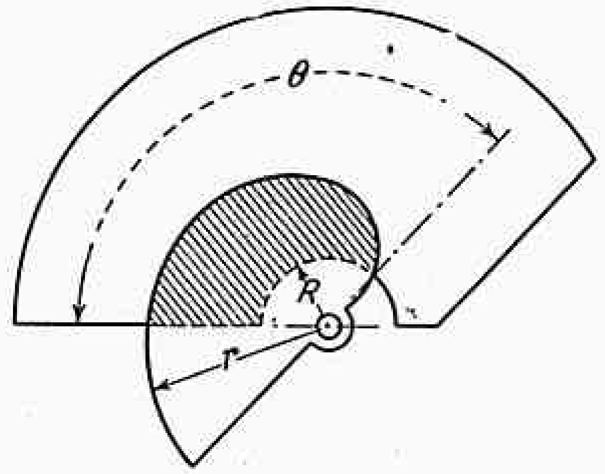


FIG. 149. DUDDELL SQUARE LAW CONDENSER

R_1 and R_2 being in the form of a potential divider the setting of which, for balance, indicates the value of the capacitance under test by means of an attached pointer and scale. Two standard capacitors are incorporated to give two scales (0·00005 to 0·016 microfarad and 0·015 to 4 microfarads) and the instrument also contains its own oscillator of the neon tube type, the only auxiliary apparatus required being a 120 volt dry battery and telephones. The makers claim an accuracy of within 2 per cent. While this may not be regarded as a precision instrument it has the advantages of portability, and great simplicity in use.

A Muirhead and Co. impedance bridge can be used for the measurement of resistance, inductance, capacitance, dissipation factor and Q-factor. It is completely self-contained, including a fixed frequency oscillator and an amplifier preceding the telephones used as detector; operation is entirely from the a.c. supply mains. Ranges covered are

Resistance, 0·001 ohm to 1 megohm (accuracy $\pm$ 1 per cent).

Inductance, 1 μH to 1,000 H (accuracy $\pm$ 1 per cent).

Capacitance, 1 $\mu\mu$F to 100 μF (accuracy $\pm$ 1 per cent).

Dissipation factor, 0 to 1·2 (accuracy $\pm$ 15 per cent or $\pm$ 0·005 whichever is the greater).

Q factor, 0 to 60 on two ranges (accuracy $\pm$ 15 per cent).

Messrs. Evershed and Vignoles make a capacitance meter based, essentially, on the well-known "Megger" Insulation Tester (see

p. 305). The capacitance meter, however, has an electro-magnet, instead of a permanent magnet, and an a.c. generator. The armature has a constant-speed centrifugal clutch and there are two independent windings displaced in phase relationship, the displacement being such that the current in the electro-magnet lags 90° behind the voltage across the measuring circuit. Thus, with capacitors present, the currents through the control and deflecting coils are 180° in phase ahead of the field and, by an appropriate arrangement of the connections, they act as if in phase. If the capacitor has imperfect insulation this gives rise to a current component in quadrature with the flux in the electro-magnet and hence this current develops no torque.

Another useful instrument is the Avo universal bridge (The Automatic Coil Winder and Electrical Equipment Co., Ltd.). This is a 50-cycle bridge, self contained, and having 20 calibrated ranges. It covers resistance measurements, in 8 ranges, from 0·5 Ω to 50 MΩ, inductance from 50 mH to 500 H in four ranges and capacitance from 5 $\mu\mu$F to 50 μF in 8 ranges. The accuracy at mid scale is of the general order of $\pm$ 2 per cent.

The Cintel wide-range capacitance bridge (Cinema-Television, Ltd.) is a self-contained equipment including its own oscillator (giving a fixed frequency of 1,592 c/s) and detector. It covers capacitances from 0·002 $\mu\mu$F to 100 μF in 18 steps and the accuracy is $\pm$ 1 per cent of full scale on all ranges. It can also be used for high resistance measurements up to 30,000 megohms. The balance indication for the bridge is a dual Electron Ray tube and a fine balance is obtainable by the slow-motion dials which have a 50 : 1 step-down ratio. A mutual and self inductance bridge by the same makers covers a range from 0·001 μH to 30 mH in 12 steps.

The Sullivan-Griffiths direct-reading inductance bridge has a range of 1 μH to 100 μH with an accuracy of 0·1 per cent. It has very small frequency and temperature errors and measures also capacitance from 0·0001 μF to 1 μF, resistance, and iron-cored or air-cored inductance without or with superposed d.c. up to 2 amperes flowing through the coil being measured. This bridge is based upon the use of a standard arm comprising an inductance standard of four decades tapped by rotary switches which also maintain the resistance of the whole standard arm constant for all values of inductance.

Another Sullivan inductance bridge has been specially designed for the measurement of iron-cored inductances from 10 mH to 1,000 mH without or with superposed d.c. up to 2 amperes. It may be used also for the measurement of air- or dust-cored coils. The network is that of the "Owen" bridge (see p. 224) but several novel features have been introduced by W. H. F. Griffiths to make the bridge entirely direct-reading for both inductance and resistance.

The Sullivan-Griffiths precision decade capacitance bridge has a wide range (0·1 $\mu\mu$F to 100 μF) and a direct-reading accuracy of

0·01 $\mu\mu$F to 0·01 per cent. The accuracy of direct-reading of power-factor measurement is 0·0001 and both this and the capacitance accuracy are maintained up to high frequencies.

Detectors. Electrodynamometer instruments have been used in a modified form as detectors in a.c. bridge measurements. Sumpner (Ref. (27)) introduced an electrodynamometer having an iron core giving very high sensitivity, and Weibel (Ref. (28)) describes several similar instruments designed for the same purpose.

The detectors in most common use for a.c. bridge measurements are, however, the telephone and the vibration galvanometer.

Telephones are widely used as detectors at frequencies of 500 cycles and over, up to 2,000 or 3,000 cycles, and are the most sensitive detectors available for such frequencies. The sensitivity of a telephone varies with the frequency of the supply, since the vibrating diaphragm which produces the sound has certain natural frequencies of vibration at which frequencies resonance is obtained, giving very high sensitivity. Wien (Ref. (29)), when investigating such resonance, found that for a Bell telephone resonance was obtained—with consequent highly increased sensitivity—at frequencies of 1,100, 2,800, and 6,500 cycles per second, and in the case of a Siemens telephone at frequencies of 720, 2,100, and 5,000 cycles.

The sensitivity of the observer's ear must also be taken into account when considering the sensitivity of the telephone as a detector. This varies with frequency. For most people a frequency of 800 cycles per second is a convenient one, since a note of this frequency is easily distinguished.

In selecting a telephone it is therefore best to choose one which has maximum sensitivity at the frequency at which it is to be used. The resistance of a telephone should match that of the bridge network. The range of resistances obtainable is roughly from 50 ohms to 7,000 or 8,000 ohms, a suitable telephone resistance for bridges of medium impedance being of the order of 200 ohms.

Transformers are sometimes used in conjunction with a low resistance telephone when the bridge network is of high impedance. The telephone is connected to the transformer secondary (low voltage side), the primary (high voltage side) being connected to the branch points of the network to which the detector is usually connected. In this way the voltage applied to the telephone is stepped down and the current stepped up.

Tuned Detectors. To improve the sensitivity of a detector it may be tuned so that resonance—and therefore maximum amplitude of vibration for a given current—is obtained. Such tuned detectors also have the advantage that the response to frequencies other than the fundamental frequency of the supply is very small. Errors due to harmonics in the supply waveform are thus minimized.

Campbell showed that an ordinary telephone can be tuned by means of a small screw pressing against the diaphragm at an eccentric point.

Amplifiers. Thermionic amplifiers are commonly used to increase the sensitivity in bridge measurements. Several forms are described by Hague (Ref. (1)).

An amplifier-detector made by Muirhead and Co., Ltd. and operated off 200–250 V a.c. mains is especially for use, in preference to an amplifier and telephones, where measurements are being made under noisy conditions and when the frequency is above or below the audible range. In the input circuit there are two balanced and screened transformers, with different input impedances, either of which may be selected by a low-capacitance key switch; this enables the detector to be used with most a.c. bridges. The linear amplifier, covering a wide range of frequencies, is followed by a bridge-connected metal rectifier and moving-coil milliameter indicating the rectified current. The amplifier itself is two-stage, resistance-capacitance coupled and uses R.F. pentode valves. Provision is made for backing-off the standing current in the anode circuit so that the milliammeter reads zero when no input is applied to the amplifier.

A power input of 4×10^{-14} watt causes a readable deflection of the meter corresponding to about 5 or 16 microvolts across the input according to which of the two transformers is used.

Vibration Galvanometers are the most widely-used tuned detectors. They are manufactured for various frequencies from 5 cycles per second up to 1,000 cycles, but are most commonly used below 200 cycles per second over which range they are considerably more sensitive than the telephone.

Vibration galvanometers are of two types—

(*a*) Moving-magnet. (*b*) Moving-coil.

The latter type is the more generally used, the moving-magnet type having the disadvantage of being seriously affected by magnetic fields of the resonant frequency, unless adequately screened. The moving-coil galvanometers are not appreciably affected by such fields.

Moving-magnet Type. The galvanometers of this type consist of a suspended system which carries one or more small, permanent magnets, and a light mirror about 2 or 3 mm diameter. The magnets are suspended between the poles of a magnet which is, in some forms, a permanent one, and in others is an electromagnet energized by coils carrying the current to be measured or detected. In the former the current is passed through coils whose magnetic field causes the suspended magnets to oscillate, the permanent magnet acting as the control. The control in other forms is supplied by torsion of the suspension. Air friction is the chief source of damping.

The moving system is tuned to the supply frequency either by altering the tension and length of the suspension or by varying the strength of the permanent magnet field, if such a magnet is included in the instrument.

A beam of light is thrown upon the mirror and, when current is passing through the instrument, the moving system oscillates, producing a band of light on the scale. In adjusting a bridge network to give zero deflection of the galvanometer, this band of light must, of course, be reduced until it again becomes a single spot, of the same diameter as when the supply is switched off. Some practice is necessary in observing when this condition has been attained. It is usually best to switch the galvanometer in and

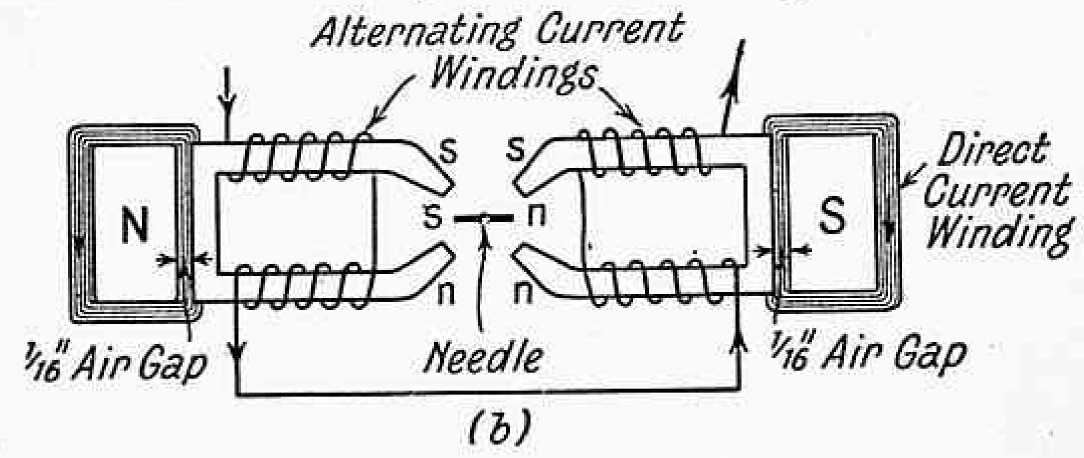


FIG. 150. CONSTRUCTION OF SCHERING AND SCHMIDT GALVANOMETER

out of circuit and to note if there is any observable difference in the size of the spot in the two cases.

Tuning. To tune the galvanometer, a small current of the supply frequency is passed through it, and the tuning adjustments (variation of the tension and length of suspension or otherwise) are continued until the reflected band of light reaches its maximum length.

It is often helpful, in tuning, to adopt some method such as the following: First vary the frequency of the supply until the instrument shows maximum deflection and note this frequency. Next adjust the galvanometer and again vary the supply frequency to give maximum deflection. Note this frequency and proceed thus by successive steps until maximum deflection is produced by a supply frequency equal to that at which the measurements are to be made.

When used in the bridge network, the galvanometer should be shunted by a variable resistance to protect it against excessive currents when the bridge is out of balance. The shunting can be removed in steps until balance is almost obtained, when the shunt may be entirely removed, so that maximum sensitivity is obtained.

Drysdale Galvanometer. The Drysdale instrument can be tuned to resonance over a range of frequency of from about 20 cycles per second to 200 cycles per second. The resonance curve is so steep that at 50 cycles per second a variation in frequency of 2 per cent up or down reduces the deflection for a given current by 80 per cent. The necessity of maintaining the supply frequency constant within very narrow limits, when such instruments are being used, is thus obvious.

In this instrument the coil carrying the current to be detected is situated behind the moving system. Control is by means of a permanent magnet. Tuning is effected by varying the magnetic field in the gap of this permanent magnet by means of a magnetic shunt, the position of which is altered by means of a screw.

The sensibility of this instrument at 50 cycles per second, when a 40 ohm coil is used, is 4 millimetres per microampere with the scale distant one metre from the instrument.

Advantages of the instrument are the ease with which it can be tuned, and the fact that coils of different impedances can be inserted, as required by the bridge network used.

Schering and Schmidt Galvanometer. In this instrument the suspension is a phosphor-bronze strip and carries a light piece of iron and a mirror. This moving system is enclosed in an ebonite tube which can be slipped in between the four poles of two U-shaped magnets as shown (Fig. 150). These magnets carry four magnetizing coils, connected in series, through which the alternating current is passed. The two U-shaped magnets themselves fit in between the two poles of another magnet excited by a winding which carries direct current. The resistance of this latter winding is about 20 ohms and it can be supplied from a 10 volt battery. It is for the purpose of polarizing the iron needle of the suspended system. Oscillation of the needle is produced by the distortion of the d.c. magnet field by the superposed alternating field.

The instrument is tuned by variation of the controlling magnetic field by adjustment of the current in the d.c. exciting winding. In vibration galvanometers generally, the smaller the damping, the sharper the resonance curve. If the supply frequency is not absolutely constant it may be convenient to make the tuning curve less sharp by increasing the damping. Provision for this is made, in this instrument, by supplying a small piece of copper, adjacent to the moving needle, the position of which can be adjusted by a screw in the suspension piece. Damping is effected due to eddy currents induced in the copper by the moving needle.

Various resistances of the coils carrying the alternating current can be used, a common value being 500 ohms. With a single moving system the frequency range of an instrument of this type is about 25 to 100 cycles per second. The sensitivity, as given by the makers, when a 500 ohm coil is used, varies from 90 mm per microampere at 25 cycles, to 25 mm per microampere at 70 cycles, the scale being distant 1 metre from the instrument.

The Schering instrument is largely used in capacitance bridges at high voltages, its advantages for such work being as follows—

1. High insulation between the alternating current system and the d.c. windings, owing to the fact that a $\frac{1}{16}$ in. air gap is left between the a.c. magnets and the control magnet.

2. It can be tuned from a distance by variation of the d.c. magnet exciting current.

3. The instrument has a very small self-capacitance.

Messrs. H. Tinsley and Co. make an instrument of this type.

Moving-coil Vibration Galvanometers. These galvanometers are of the d'Arsonval type, having a moving coil suspended between the poles of a strong, permanent magnet. The moving system is designed to have a very short, natural period of vibration; and the damping is very small, in order that the resonance curve shall be sharp—i.e. the deflection, for a given current passing through the instrument, is very much reduced by a small departure from the frequency to which the galvanometer is tuned. The alternating current to be detected is passed through the suspended coil, which consists of a few turns—or often of only a single loop—of wire. The moving system carries a small mirror, upon which a beam of light is cast. The system vibrates when an alternating current is passed through the coil, the reflected beam of light from the mirror thus throwing a band of light upon the scale.

These galvanometers are tuned by adjusting the length and tension of the suspended system.

*Duddell Moving-coil Vibration Galvanometer.** In this instrument the moving coil consists of a single loop of fine bronze or platinum-silver wire, this wire passing over a small pulley at the top and being pulled tight by a spring attached to the pulley (Fig. 151 (*a*)). The tension of this spring can be adjusted for tuning purposes by turning a milled head to which it is attached. The loop of wire is stretched over two ivory bridge pieces, the distance apart of these being adjustable in tuning the instrument. Variation of this distance apart obviously varies the length of the loop which is free to vibrate, and thus varies the natural period of the galvanometer. The galvanometer is roughly tuned by adjustment of the bridge pieces, fine adjustment of the tuning being obtained by varying the tension on the loop.

When a current passes through the loop a couple, tending to turn the loop about its vertical axis, is produced. When the current reverses this couple also reverses, thus causing oscillation of the loop when alternating current is passed through it.

This galvanometer can be used for frequencies between 100 and 1,800 cycles per second, the current sensitivity being about 50 mm per microampere, with a scale distance of 1 metre. The effective resistance is about 250 ohms. The sensitivity, if the loop is not too short, is almost inversely proportional to the frequency. In common with moving-coil vibration galvanometers generally, the instrument is not greatly affected by external magnetic fields. It has the disadvantage that the tuning can only be carried out by actually

* See Ref. (25).

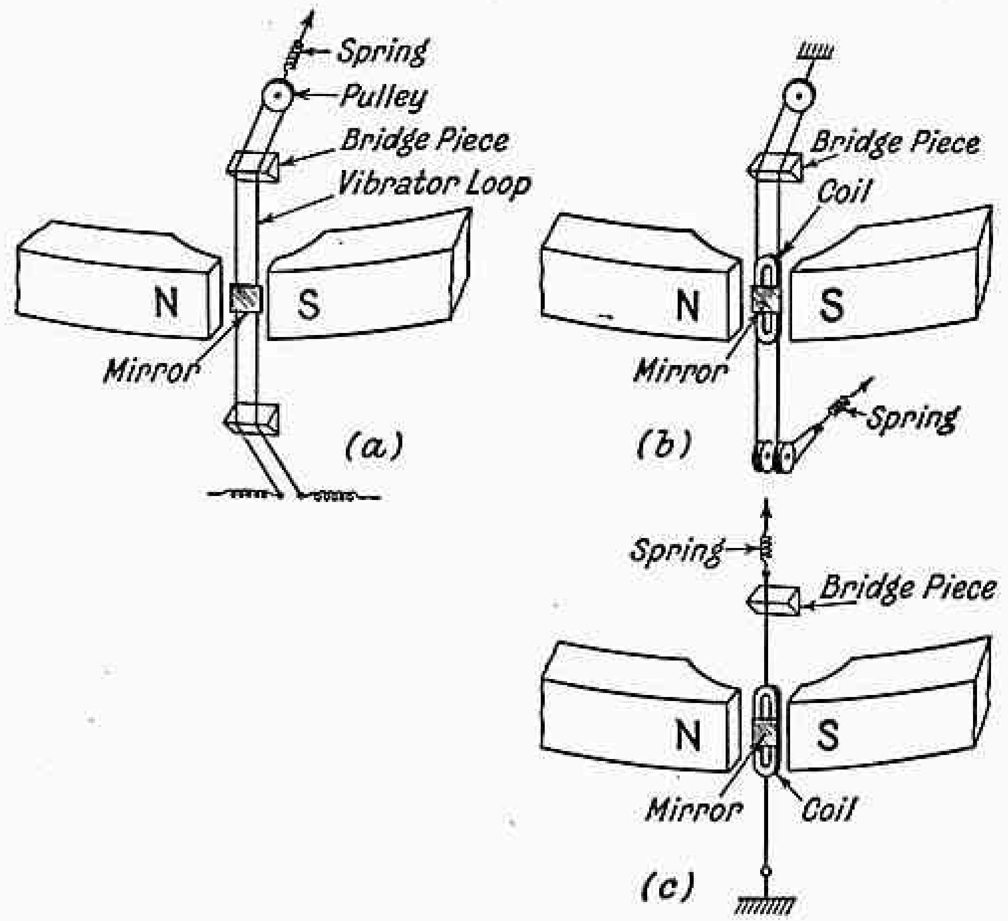
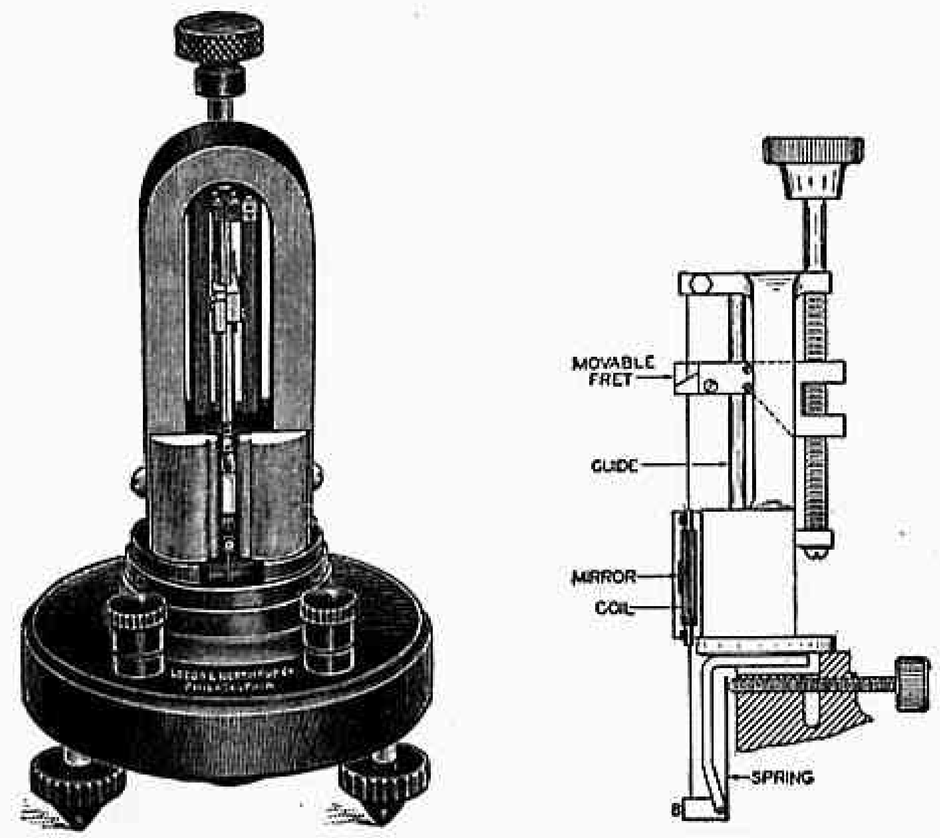


FIG. 151. CONSTRUCTION OF MOVING-COIL VIBRATION GALVANOMETERS



(*Leeds & Northrup Co.*)

FIG. 152. LEEDS AND NORTHRUP MOVING-COIL VIBRATION GALVANOMETER

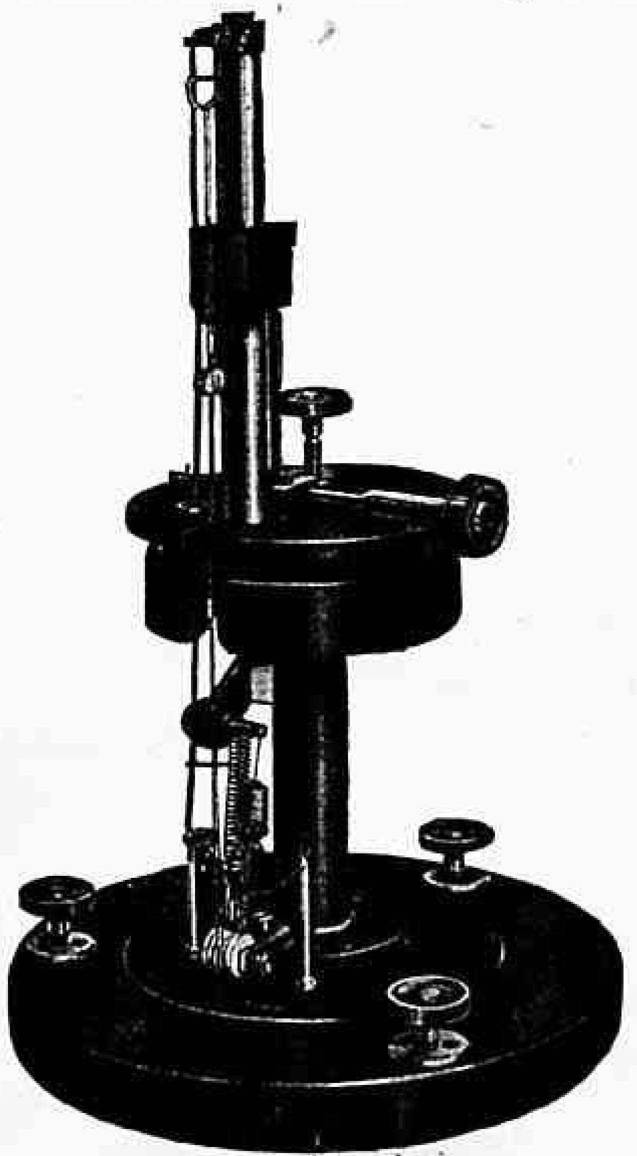
handling the instrument, and is, therefore, not very convenient for use in high-voltage work.

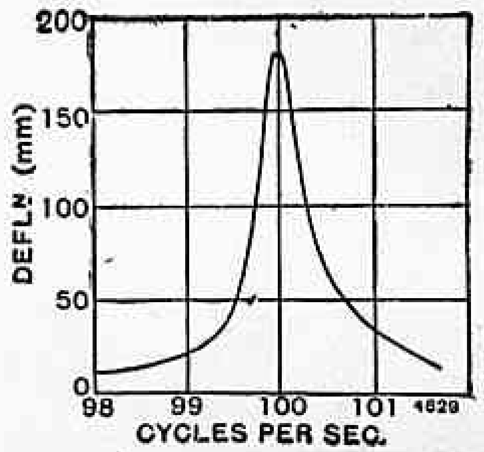
Other Moving-coil Vibration Galvanometers. A. Campbell (Refs. (15), (31), (32), (33)) has developed other moving-coil vibration galvanometers. Fig. 151 (*b*) shows the construction of his long-range instrument, and Fig. 151 (*c*) his short-range pattern. The former instrument has a bifilar suspension carrying a very light coil and a small mirror. The length of the suspension is varied, for tuning purposes, by the movement of a bridge above the coil, and the tension by means of a spiral spring at the bottom of the suspension. The range of frequency covered by such an instrument is from 50 to 1,000 cycles per second, and the sensitivity at 50 cycles is of the order of 60 mm per microampere at 1 metre scale distance, with an effective resistance of about 500 ohms, this sensitivity falling off at the higher frequencies to less than 1 mm per microampere.

The short-range instrument has a single strip suspension. Tuning is carried out in a similar way to that of the long-range instrument. The frequency range is from 10 cycles to 400 cycles per second, and has very high sensitivity at the lower frequencies (of the order of 400 mm per microampere at a scale distance of 1 metre when the frequency is 10 cycles per second).

Fig. 153 shows the construction and also a resonance curve of a Campbell moving-coil vibration galvanometer, manufactured by the Cambridge Instrument Co.

Fig. 152 gives details of the construction of a vibration galvanometer of a similar type, manufactured by the Leeds & Northrup Co. The screw adjustments for variation of the length of and tension on the suspension are clearly shown. The frequency range of this particular instrument is from 50 to 80 cycles per second, the sensitivity being stated by the makers as 40 mm per microampere at a scale distance of 1 metre and a frequency of 60 cycles per second, the resistance being 700 ohms.





(*Cambridge Instrument Co., Ltd.*)

FIG. 153. CAMPBELL MOVING-COIL VIBRATION GALVANOMETER AND RESONANCE CURVE

To avoid the falling off in sensitivity with increase of frequency, several suspensions are usually provided for use with the same instrument at different frequencies.

Vibration galvanometers, generally, are susceptible to mechanical vibrations whose frequency is of the same order as that to which they are tuned. For this reason it is often necessary to provide some form of support which acts as a protection from such vibrations. One method of support which has been found to be satisfactory is to stand the instrument on rubber feet which rest on a heavy block of slate suspended from the ceiling by springs. Underneath the slate may be fitted damping vanes dipping into an oil dash-pot.

Theory of the Vibration Galvanometer with One Degree of Freedom. All the vibration galvanometers described above have only one degree of freedom—i.e. their suspended system only rotates about the axis of suspension. The theory of galvanometers with one degree of freedom was first given by Wenner (Ref. 30), and the following is based upon his work on the subject.

Considering, first of all, the constants of the galvanometer considered—called by Wenner the "intrinsic constants"—we have

(*a*) The "*displacement constant.*" If the suspended coil is of length l cm (measured along the axis of the suspension), has a width r cm, and has N turns, then the couple displacing the coil, when it carries i e.m. units of current, and is situated in a magnetic field of strength H, is $NHilr \cos \theta$ dyne-cm (see Fig. 154), where θ is the angle (in radians) between the plane of the coil and the direction of the magnetic field. If θ is small, $\cos \theta \doteqdot 1$, and the deflecting couple is $NHilr$ dyne-cm. Assuming the coil to be rectangular, lr is the area of its plane. Let $lr = A$, then the expression for the couple may be written $NHAi = Gi$. The constant G is called the "displacement constant" of the galvanometer, and is equal to NHA.

(*b*) The "*constant of inertia.*" Of the three couples retarding the motion, one is dependent upon the moment of inertia of the suspended system and upon the angular acceleration of this system. This couple may be written $a\dfrac{d^2\theta}{dt^2}$ where a is the "constant of inertia" or moment of inertia of the system.

(*c*) The "*damping constant.*" Another couple, retarding motion, is that due to the damping effect of air friction and elastic hysteresis in the suspension. This is usually assumed proportional to the angular velocity, and may be written

$$b\,\frac{d\theta}{dt}$$

where b is the "damping constant."

(*d*) The "*control or restoration constant.*" The couple due to the elasticity of the suspension is proportional to the displacement, and may be written $c\theta$, where c is the "*restoring constant.*"

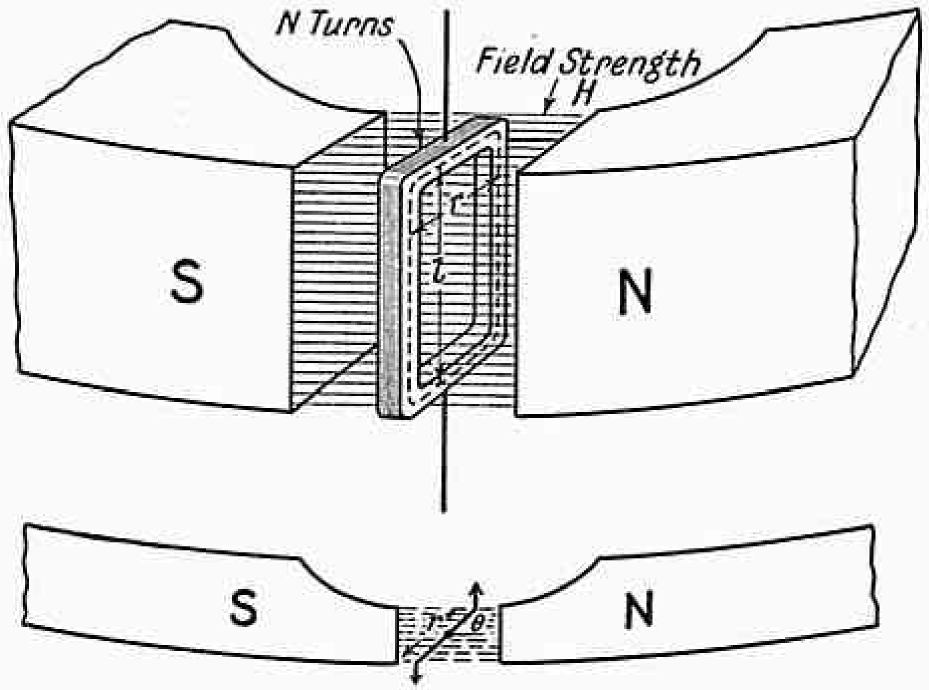


FIG. 154

We have, therefore, as the equation of motion of the system,

$$a\,\frac{d^2\theta}{dt^2} + b\,\frac{d\theta}{dt} + c\theta = Gi \quad . \quad . \quad . \quad . \quad (174)$$

Now, if the current i is alternating, and is given by the expression

$$i = I_{max} \cos \omega t$$

we have $$a\,\frac{d^2\theta}{dt^2} + b\,\frac{d\theta}{dt} + c\theta = GI_{max} \cos \omega t \quad . \quad . \quad (175)$$

The solution of this differential equation will be in two parts. The expression for θ will be the sum of a Particular Integral and the Complementary Function. The complementary function—obtained by solving the equation

$$a\,\frac{d^2\theta}{dt^2} + b\,\frac{d\theta}{dt} + c\theta = 0$$

—will, in this case, give the expression for the angle θ when the current in the coil is zero—i.e. it will represent the natural free

vibration of the coil. As will be shown later, this expression contains a factor of the form $\varepsilon^{-\alpha t}$ so that it represents a vibration of the coil which rapidly dies away when the current is switched on. This is, therefore, the transient part of the solution of the equation of motion.

The particular integral will give an expression for θ, which represents the steady vibration of the coil after the current has been switched on for some appreciable time. Proceeding, then, to obtain the complementary function, we have

$$a\,\frac{d^2\theta}{dt^2} + b\,\frac{d\theta}{dt} + c\theta = 0$$

The "auxiliary equation" is $am^2 + bm + c = 0$ and the roots of this equation are

$$m_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $$m_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Thus $$\theta = A\varepsilon^{m_1 t} + B\varepsilon^{m_2 t} \quad . \quad . \quad . \quad . \quad (176)$$

where A and B are constants to be determined from the initial conditions.

In vibration galvanometers the damping is small, and b^2 is less than $4ac$. Thus m_1 and m_2 are imaginary, and may be written

$$m_1 = -k_1 + jk_2$$
$$m_2 = -k_1 - jk_2$$

where $j = \sqrt{-1}$, $k_1 = \dfrac{b}{2a}$, and $k_2 = \dfrac{\sqrt{4ac - b^2}}{2a}$

$$\therefore\ \theta = A\varepsilon^{(-k_1 + jk_2)t} + B\varepsilon^{(-k_1 - jk_2)t}$$
$$= \varepsilon^{-k_1 t}[A\varepsilon^{jk_2 t} + B\varepsilon^{-jk_2 t}] \quad . \quad . \quad . \quad . \quad (177)$$

Since, from trigonometry,

$$\varepsilon^{jpx} = \cos px + j \sin px$$

and $$\varepsilon^{-jpx} = \cos px - j \sin px$$

we have

$$\theta = \varepsilon^{-k_1 t}[A(\cos k_2 t + j \sin k_2 t) + B(\cos k_2 t - j \sin k_2 t)]$$
$$= \varepsilon^{-k_1 t}[(A + B) \cos k_2 t + j(A - B) \sin k_2 t]$$
$$= \varepsilon^{-k_1 t}[P \cos k_2 t + Q \sin k_2 t]$$

where $P = A + B$ and $Q = j(A - B)$

i.e. $\theta = \varepsilon^{-k_1 t}[F \sin (k_2 t + \alpha)]$

where $F = \sqrt{P^2 + Q^2}$ and $\alpha = \tan^{-1} \dfrac{P}{Q}$

Thus $\theta = \varepsilon^{\frac{b}{2a}t}\left[F \sin\left(\frac{\sqrt{4ac-b^2}}{2a}t + \alpha\right)\right]$ . . . . (178)

this being the transient portion of the solution which rapidly decreases in value as t is increased. The constants F and α must be determined from the initial conditions—i.e. they depend upon the position of the coil at the instant corresponding to zero time.

In this expression $\frac{\sqrt{4ac-b^2}}{2a}$ is the angular velocity ω, and is equal to $2\pi \times$ the frequency of the vibratory motion.

Thus $$f = \frac{\sqrt{4ac-b^2}}{4\pi a}$$

where f is the "natural frequency" of the vibrating system.

If the damping is negligibly small, $b = 0$, and the "*undamped natural frequency*" is given by

$$f = \frac{\sqrt{4ac}}{4\pi a} = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$$

and obviously depends upon the moment of inertia of the system and upon the controlling forces.

The instrument is critically damped—i.e. it will not vibrate freely—when $f = 0$. This condition is fulfilled when $4ac = b^2$, or when the damping constant $b = 2\sqrt{ac}$.

Proceeding to find the particular integral, we have

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = GI_{max}\cos\omega t$$

or $$\frac{d^2\theta}{dt^2} + \frac{b}{a}\frac{d\theta}{dt} + \frac{c}{a}\theta = \frac{GI_{max}}{a}\cos\omega t$$

Employing the operator D, we have

$$(D^2 + hD + g)\theta = \frac{GI_{max}}{a}\cos\omega t$$

where $h = \frac{b}{a}$ and $g = \frac{c}{a}$

$$\therefore \theta = \frac{\frac{GI_{max}}{a}\cos\omega t}{(D^2 + hD + g)}$$

Multiplying numerator and denominator by $(D^2 - hD + g)$ gives

$$\theta = \frac{\frac{GI_{max}}{a}(D^2 - hD + g)\cos\omega t}{(D^2+g)^2 - h^2D^2}$$

$$= \frac{\frac{GI_{max}}{a}}{(D^2+g)^2 - h^2D^2}[-\omega^2\cos\omega t + h\omega\sin\omega t + g\cos\omega t]$$

$$= \frac{GI_{max}}{a}(g-\omega^2)\frac{1}{(D^2+g)^2-h^2D^2}\cos\omega t + \frac{GI_{max}}{a}h\omega\frac{1}{(D^2+g)^2-h^2D^2}\sin\omega t$$

$$= \frac{\frac{GI_{max}}{a}(g-\omega^2)\cos\omega t}{(g-\omega^2)^2 + h^2\omega^2} + \frac{\frac{GI_{max}}{a}h\omega\sin\omega t}{(g-\omega^2)^2+h^2\omega^2}$$

Substituting for h and g, we have

$$\theta = \frac{\frac{GI_{max}}{a}\left[\left(\frac{c}{a}-\omega^2\right)\cos\omega t + \frac{b}{a}\omega\sin\omega t\right]}{\left(\frac{c}{a}-\omega^2\right)^2 + \frac{b^2}{a^2}\omega^2}$$

$$= \frac{GI_{max}[(c-\omega^2 a)\cos\omega t + b\omega\sin\omega t]}{(c-a\omega^2)^2 + b^2\omega^2}$$

$$= \frac{GI_{max}\sqrt{(c-a\omega^2)^2+b^2\omega^2}\left[\frac{c-\omega^2 a}{\sqrt{(c-a\omega^2)^2+b^2\omega^2}}\cos\omega t + \frac{b\omega}{\sqrt{(c-a\omega^2)^2+b^2\omega^2}}\sin\omega t\right]}{(c-a\omega^2)^2+b^2\omega^2}$$

or $\theta = \frac{GI_{max}}{\sqrt{(c-a\omega^2)^2+b^2\omega^2}}[\cos(\omega t - \beta)]$ . . . . (179)

where $\beta = \tan^{-1}\frac{b\omega}{c-a\omega^2}$

This obviously represents a steady vibratory motion of amplitude $\frac{GI_{max}}{\sqrt{(c-a\omega^2)^2+b^2\omega^2}}$ and of frequency $\frac{\omega}{2\pi}$.

The complete solution for θ, being the sum of this expression and the expression derived previously as the complementary function, is thus

$$\theta = \varepsilon^{-\frac{b}{2a}\cdot t}\left[F\sin\left(\frac{\sqrt{4ac-b^2}}{2a}t + \alpha\right)\right] + \frac{GI_{max}}{\sqrt{(c-a\omega^2)^2+b^2\omega^2}}\cos(\omega t - \beta)] \quad . \quad (180)$$

Since the first expression is a transient which usually affects only the first few vibrations after switching on, we may neglect it and take as the law of the displacement simply

$$\theta = \frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}} \cos(\omega t - \beta) \qquad . \quad . \quad . \quad (181)$$

Example. Fig. 155 shows how the transient term, at the beginning of the vibration period, produces an unsteady state which gradually disappears, giving, finally, a steady vibration.

The curves shown are based on data given by Campbell* for a vibration galvanometer of the moving-coil type developed by him.

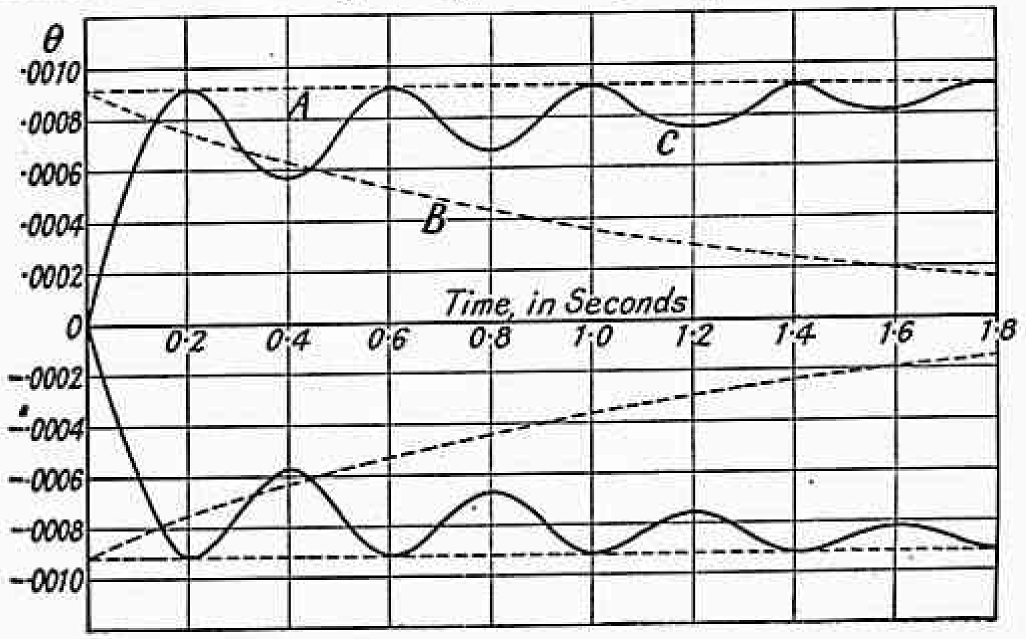


FIG. 155

The data given are as follows—

Number of turns	40
Mean area of turn	0·07 sq. cm.
Strength of magnetic field	2,700 lines per sq. cm.
Effective resistance	1,540 ohms
Resonance frequency	100 cycles per sec.
Inertia constant a	26×10^{-6}
Damping constant b	49×10^{-6}
Restoring constant c	10·4
G	8,600

Thus, in the expression for the deflection θ as derived above,

$$\frac{b}{2a} = 0{\cdot}942$$

$$\frac{\sqrt{4ac - b^2}}{2a} = 632$$

(This expression is equal to $\sqrt{\frac{c}{a}}$ to a very close approximation, and should therefore equal 628 when the instrument is tuned (see page 265); slight errors in the values of the constants probably being responsible for the discrepancy.)

$$\beta = 0^\circ\ 11'$$
$$\omega = 628\ (= 2\pi \times 100)$$

* *Dictionary of Applied Physics*, Vol. II, p. 974.

Assuming that I_{max} has the value 10 microamps, the expression

$$\frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$$

for the amplitude of the steady deflection has the value 0·000917.

The value of $\frac{b}{2a}$ being somewhat small, the transient term persists for an appreciable time, taking 2 sec. to fall to 0·153 of its initial amplitude. For this reason it is impossible to show, in the figure, the full curves for the two terms in the expression for the deflection θ. Lines passing through successive maximum points of these curves are shown instead. The dotted lines A are the lines passing through the maximum points of the steady deflection curve given by $\theta_1 = 0{\cdot}000917 \cos(628t - 0^\circ\ 11')$, while the dotted curves B pass through the maximum points on the curve $\theta_2 = \varepsilon^{-0{\cdot}942t}[F \sin(628t + \alpha)]$. The full line curves C pass through the maximum points of the total deflection curve (given by the summation of curves θ_1 and θ_2).

The effect of the transient term is shown by the "beat" effect which gradually dies away as the transient terms disappear. In the figure these beats have been drawn approximately owing to the difficulty of showing the full curves with a scale which is, necessarily, very cramped.

Tuning. In tuning the galvanometer, the object is to make the amplitude $\frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$ as great as possible for a given current I_{max}, which means that $\frac{G}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$ must be made as great as possible. This expression can be increased by increasing the numerator G and by reducing the denominator $\sqrt{(c - a\omega^2)^2 + b^2\omega^2}$.

Since $G = NHA$, it may be made large by using a coil of large area A and with a large number of turns N. G is obviously increased also by increasing the strength H of the magnetic field in which the coil lies. This latter method is the more important, since increasing the area and number of turns on the coil will increase its moment of inertia so that a will be increased and, thereby, the denominator may be increased.

Considering the denominator: of the three constants a, b, and c contained in it, c is the only one which can usually be varied. The constant c is the control constant and is varied by adjusting the length and tension of the suspension of the moving system, or by variation of the polarizing field of the galvanometer, in the case of moving-magnet instruments.

If the supply frequency is fixed—as it usually is in bridge measurements—the tuning process consists of varying c until $c - a\omega^2$ is zero, thus making the denominator of the amplitude expression a minimum. Since $\omega = 2\pi \times$ the supply frequency, we have the condition $c - a(2\pi f)^2 = 0$ (f = supply frequency), which must be satisfied in tuning the instrument.

Thus, c must equal $a(2\pi f)^2$ or $f = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$

It should be noted that this expression for the supply frequency f, in terms of c and a, is the same as the expression for the frequency of the undamped vibration of the galvanometer (see page 262). This means that resonance occurs when the supply frequency is equal to the undamped natural frequency of the galvanometer. The amplitude under resonance conditions is obviously $\frac{GI_{max}}{b\omega}$.

Consider the case of the vibration galvanometer whose constants have already been given.

The constants are—

$$a = 26 \times 10^{-6}$$
$$b = 49 \times 10^{-6}$$
$$c = 10{\cdot}4$$

and the resonance frequency is given as 100 cycles per second. More exactly the frequency for resonance is $\frac{1}{2\pi}\sqrt{\frac{10{\cdot}4 \times 10^6}{26}}$ or 100·7 cycles per second. The deflection at resonance is

$$\frac{GI_{max}}{b\omega} = \frac{GI_{max}}{\frac{49}{10^6} \times 2\pi \times 100{\cdot}7} = GI_{max} \times 32{\cdot}27$$

The deflection at other frequencies is calculated from the expression $\frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$ since $(c - a\omega^2)$ is zero only at the resonance frequency. A table showing the deflection for a range of frequency from 98·5 to 102 cycles per second is given below, and these values are plotted in Fig. 156. The sharpness of the resonance curve of a vibration galvanometer is well illustrated in the curve obtained.

TABLE IX

Frequency	ω	Deflection
98·5	618·8	$GI_{max} \times$ 2·27
99	622	„ × 2·93
99·5	625·1	„ × 4·21
100	628·4	„ × 7·63
100·5	631·4	„ × 22·3
100·7	632·6	„ × 32·3
101	634·6	„ × 13
101·5	637·7	„ × 5·78
102	640·9	„ × 3·57

To compare the response of the galvanometer to harmonics in the supply waveform consider a third harmonic when the supply frequency is that to which the galvanometer is tuned—namely, 100·7 cycles per second. The frequency of the third harmonic is 302·1 cycles per second. At this frequency the value of the expression $\sqrt{(c - a\omega^2)^2 + b^2\omega^2}$ is 83·32, so that, even if the amplitude of the third harmonic were equal to that of the fundamental, the amplitude of the deflection would be only $\frac{GI_{max}}{83{\cdot}32} = 0{\cdot}012GI_{max}$.

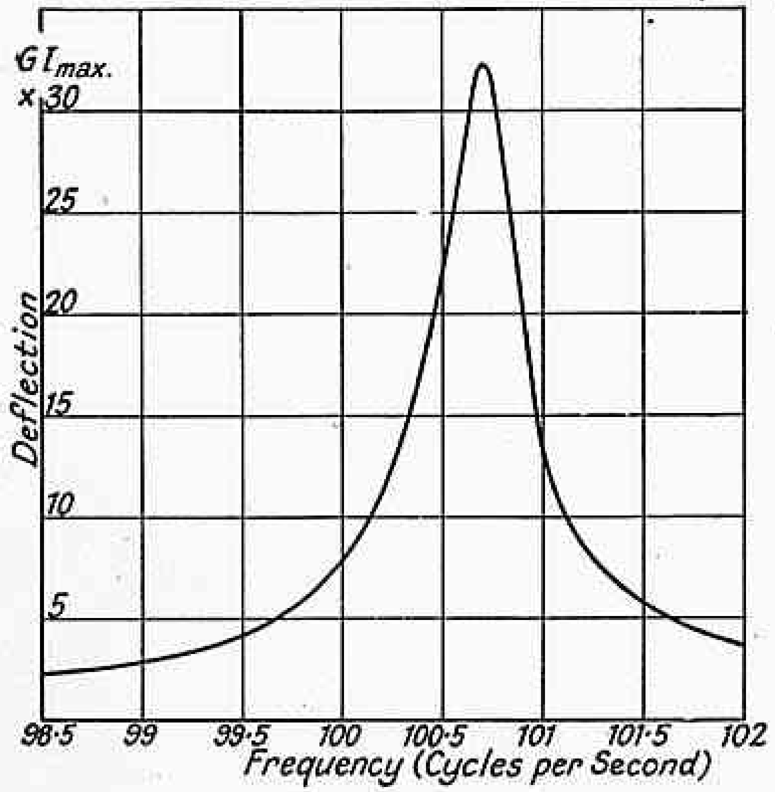


Fig. 156. Resonance Curve of Vibration Galvanometer

Thus the sensitivity to the fundamental compared with the sensitivity to the third harmonic is $\frac{32{\cdot}3}{0{\cdot}012} = 2{,}690$, showing that an entirely negligible error is introduced by the fact that the supply waveform contains harmonics.

The above theory assumes a current, $i = I_{max} \cos \omega t$, flowing through the galvanometer, and therefore refers to the "*current sensitivity*" of such instruments.

The "*voltage sensitivity*" may be determined by considering the case of a given voltage applied to the instrument terminals. In this consideration the voltage *induced* in the coil owing to the fact that, while vibrating, it is cutting through the magnetic field of strength H must be taken into account. The reader is referred to Hague's *Alternating Current Bridge Methods*, 4th Edition, p. 277, or to the *Dictionary of Applied Physics*, Vol. II, p. 971, for the full theory in this case.

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CHAPTER VII

MEASUREMENT OF RESISTANCE

FROM the point of view of measurement, resistances can be classified generally as follows—

(a) LOW RESISTANCES. All resistances of the order of 1 ohm and under may be classified thus. In practice such resistances may be met with in the armatures and series windings of large machines, in ammeter shunts, cable lengths, contacts, etc.

(b) MEDIUM RESISTANCES. This class includes resistances from about 1 ohm upwards to about 100,000 ohms. In practice the majority of the pieces of electrical apparatus used have resistances which lie between these limits.

(c) HIGH RESISTANCES. Resistances of 100,000 ohms and upwards must be so classified.

A classification such as the above is not rigid, but forms a guide as to the method of measurement to be adopted in any particular case.

Measurement of Low Resistance. Methods of measurement which are suitable for medium resistances are in most cases unsuitable for low resistance measurements, chiefly because contact resistances cause serious errors. It is clear that contact resistances of the order (say) of 0·001 ohm—negligible though they may be when a resistance of 100 or more ohms is to be measured—are of great importance when the resistance to be measured is of the order of 0·01 ohm.

Again, it is usually essential, with low resistances, that the two points between which the resistance is to be measured shall be very definitely defined. Thus the methods which are specially adapted to low resistance measurement employ *potential connections* —i.e. connecting leads which form no part of the circuit whose resistance is to be measured, but which connect two points, in this circuit, to the measuring circuit. These two points are spoken of as the *potential terminals*, and serve to fix, definitely, the length of the circuit under test. In the methods used for the precise measurement of low resistance, the "unknown" resistance is compared with a low-resistance standard of the same order as the unknown, and with which it is connected in series. Both resistances are fitted with four terminals—two "current terminals," to be connected to the supply circuit, and two "potential terminals" to be connected to the measuring circuit. This arrangement is shown in Fig. 157.

AMMETER AND VOLTMETER METHOD. This method, which is the simplest of all, is in very common use for the measurement of low resistances when an accuracy of the order of 1 per cent is sufficient.

It must be realized, however, that it is, essentially, a comparatively rough method, the accuracy being limited by those of the ammeter and voltmeter used, even if corrections are made for the "shunting" effect of the voltmeter. In Fig. 158, R is the resistance to be measured and V is a high-resistance voltmeter of resistance R_v. A current from a steady direct-current supply is passed through R in series with a suitable ammeter.

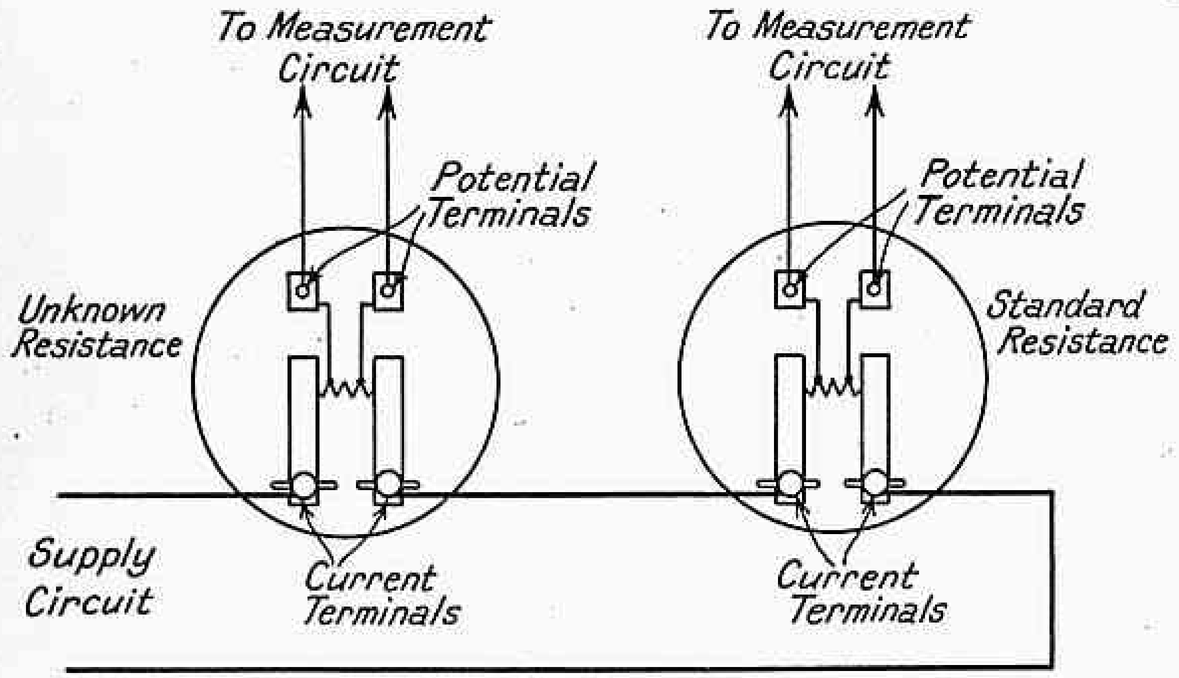


FIG. 157. MEASUREMENT OF LOW RESISTANCE

Then, assuming the current through the unknown resistance to be the same as that measured by the ammeter A, the former is given by

$$R = \frac{\text{voltmeter reading}}{\text{ammeter reading}}$$

If the voltmeter resistance is not very large compared with the resistance to be measured, the voltmeter current will be an appreciable fraction of the current I, measured by the ammeter, and a serious error may be introduced on this account.

Example. A resistance whose actual value is 1 ohm, is to be measured by the ammeter and voltmeter method. The carrying capacity of the resistance is 100 milliamperes, which is the current used in making the measurement. The voltmeter used has a resistance of 5 ohms, and reads up to 100 millivolts. What is the measured value of the 1 ohm resistance?

Let resistance of 1 ohm resistance and voltmeter in parallel $= r$.

Then
$$\frac{1}{r} = \frac{1}{1} + \frac{1}{5} = 1{\cdot}2$$
$$r = \frac{1}{1{\cdot}2} = 0{\cdot}833 \text{ ohm}$$

Volt drop across the resistance to be measured

$$= 0{\cdot}833 \times 0{\cdot}1$$
$$= 0{\cdot}0833 \text{ volt}$$
$$= \text{voltmeter reading}$$

Ammeter reading = 0·1 amp.

$$\text{Thus the measured value of the resistance} = \frac{0{\cdot}0833}{0{\cdot}1}$$
$$= 0{\cdot}833 \text{ ohm}$$

This means that, even if the ammeter and voltmeter give readings which are exactly correct, an error of 0·166 ohm, or 16·6 per cent,

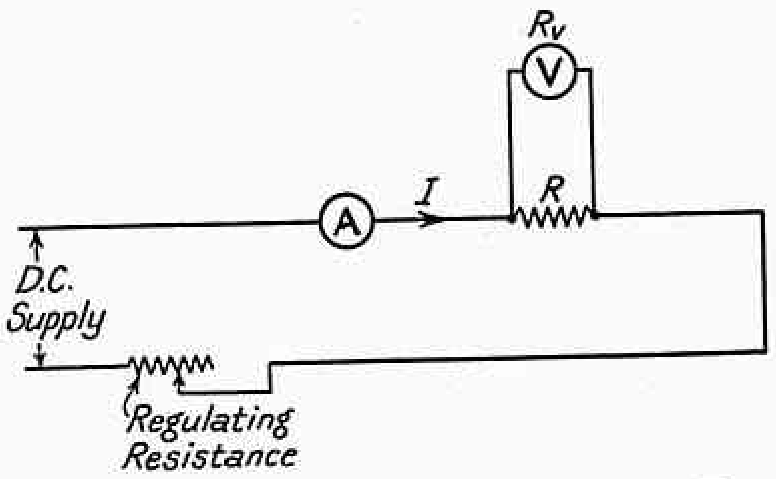


FIG. 158. AMMETER AND VOLTMETER METHOD OF RESISTANCE MEASUREMENT

is introduced by the fact that the voltmeter takes an appreciable fraction of the total current.

If, on the other hand, the current-carrying capacity of the 1 ohm resistance had been such that a much greater current could have been passed through it, so that a voltmeter of resistance (say) 500 ohms, reading up to 10 volts, could have been used, then the error introduced would have been only 0·2 per cent, as can be seen from a similar calculation to the above.

Correction for Shunting Effect of the Voltmeter. In general terms, if the actual value of the unknown resistance is R, and its measured value R_m, the voltmeter resistance being R_v, and the ammeter current I, we have—

$$\text{Resistance of voltmeter and } R \text{ in parallel} = \frac{RR_v}{R + R_v}$$

$$\text{Voltage drop across } R = \frac{RR_v\,I}{R + R_v}$$
$$= \text{voltmeter reading}$$

(assuming the voltmeter to read correctly).

Thus, upon the assumption that the ammeter reading also is exactly correct,

$$R_m = \frac{RR_v\,I}{(R + R_v)I} = \frac{RR_v}{R + R_v}$$

or

$$R = \frac{R_m R_v}{R_v - R_m} \quad . \quad . \quad . \quad . \quad . \quad (182)$$

This method is useful in practice in the measurement of such resistances as those of armatures, and of joints and contacts when the current-carrying capacity is fairly great and when the results are only required to within the limits of accuracy of the ammeter and voltmeter used.

POTENTIOMETER METHOD. In the potentiometer method of measuring a low resistance the unknown resistance is compared with a standard resistance of the same order of magnitude.

These standard low resistances are of the type described in Chapter II. The following table gives the resistances and current-carrying capacities of some of a range of standards as manufactured by Messrs. H. Tinsley and Co.

TABLE X

Resistance (Ohms)	Current-carrying capacity (Amp.)
10	1
5	1·4
2	2·2
1	3
0·5	4·5
0·1	22
0·01	150
0·001	700
0·0005	1000
0·0001	2250

The standards are adjusted to within 0·03 per cent of their nominal resistance (0·05 per cent for the last three).

The complete range of these resistance standards is from 10,000 Ω down to 0·0001 Ω. They are suitable for use at frequencies up to 1,000 c/s and their time constants for frequencies up to this value are 1×10^{-6} down to the 0·01 standard. For this and lower resistances the time constant is 3×10^{-6}. The ratings in watts are 10 W over the range 10,000 Ω to 0·2 Ω; 50 W between 0·2 Ω and 0·02 Ω; 200 W between 0·02 Ω and 0·005 Ω and 500 W between 0·001 Ω and 0·0001 Ω. Below 0·02 Ω water cooling, with a motor-driven stirrer, is used.

The unknown resistance and a standard of the same order of resistance are connected in series as in Fig. 157. A steady current is passed through them from a battery of the heavy-current type. The magnitude of this current should be chosen so that a voltage drop of the order of 1 volt is obtained, if possible, across each of the resistances.

The voltage drops across both the unknown resistance and the standard are then measured on the potentiometer (see Chapter VIII), several measurements being made, alternately, and with as small a time interval as possible between the measurements. The mean values of these are taken as the correct voltage drops across the two resistances. By carrying out the measurements in this way the error due to possible variation of the supply current is minimized.

The potential leads to the potentiometer carry no current when the potentiometer is balanced, and thus the current through the two resistances is the same. Then

$$\frac{\text{Resistance of the unknown}}{\text{Resistance of the standard}} = \frac{\text{Voltage drop across the unknown}}{\text{Voltage drop across the standard}}$$

from which the resistance of the unknown is obtained in terms of that of the standard resistance.

Precautions. When used for precise work resistance standards should be frequently checked against National Physical Laboratory standards, and the most recent calibration of the resistor should be used in calculating the resistance of the unknown. If the standard resistance is subject to appreciable variation with time, it may be necessary to estimate its probable variation from the time of the last calibration, on the assumption that its rate of variation is uniform, and the same as that between the dates of the two preceding calibrations. A standard resistor in which such variation with time is large is, of course, useless for precise work. The temperature of the two resistors, during the test, should be measured and the resistance of the standard, at the measured temperature, should be obtained from its resistance-temperature curve. The measured resistance of the unknown is that at the measured temperature, and this should be stated, as its temperature coefficient may be large, and hence its resistance may be appreciably different at other temperatures.

In precise work also, a second measurement should be made with the supply reversed (care must be taken to reverse the potential leads to the potentiometer at the same time), and thermo-electric effects should be taken into account as described in the next chapter.

When the necessary precautions are taken, and a good potentiometer and sensitive galvanometer are used, the accuracy obtainable by this method may be within a few parts in 100,000, or within 1 part in 10,000.

KELVIN DOUBLE BRIDGE. This method is one of the best available for the precise measurement of low resistances. It is a development of the Wheatstone Bridge by which the errors due to contact and leads resistances are eliminated. The connections of the bridge are shown in Fig. 159.

In the figure, X is the low resistance to be measured, and S is a standard resistance of the same order of magnitude. These are

connected in series with a low-resistance link r, connecting their adjacent current terminals. A current is passed through them from a battery supply. A regulating resistance and ammeter are connected in the circuit for convenience. Q, M, q, and m are four known, non-inductive resistances, one pair of which (M and m, or Q and q) are variable. These are connected to form two sets of ratio arms as shown, a sensitive galvanometer G connecting the dividing points of QM and qm. The ratio $\frac{Q}{M}$ is kept the same as $\frac{q}{m}$, these ratios being varied until zero deflection of the galvanometer

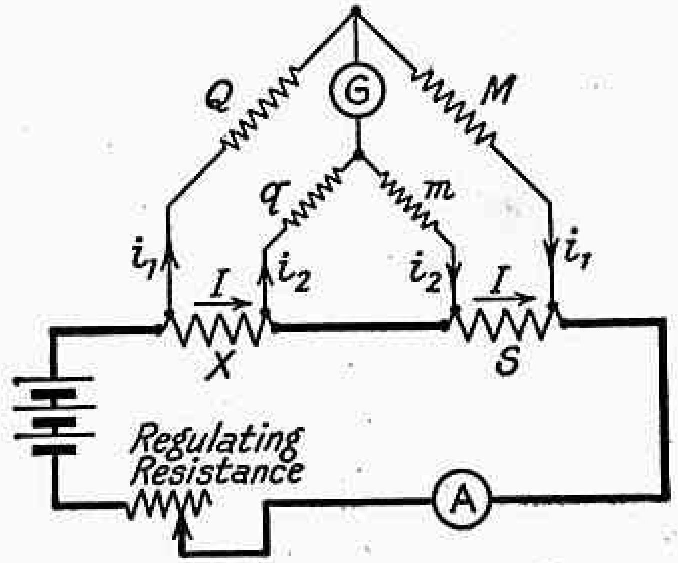


FIG. 159. KELVIN DOUBLE BRIDGE METHOD OF MEASURING LOW RESISTANCE

is obtained. Then $\frac{X}{S} = \frac{Q}{M} = \frac{q}{m}$, from which X is obtained in terms of S, Q, and M.

Theory. At balance of the bridge (i.e. zero galvanometer deflection) the current in arm Q = current in arm M. Let this current be i_1. Also, current in arm q = current in arm m. Let this current be i_2. Therefore, the current in X = current in S. Let this current be I.

Again, voltage drop across Q = voltage drop across X + voltage drop across q

i.e. $$i_1Q = IX + i_2q$$

In the same way, $$i_1M = IS + i_2m$$

Now, since q and m are in parallel with the resistance r, the current I in X divides so that $\frac{r}{r+q+m}I$ passes through q and m, i.e.

$$i_2 = \frac{r}{r+q+m}\cdot I$$

Substituting this value of i_2 in the above equations, we have

$$i_1Q = IX + \frac{rq}{r+q+m}\cdot I$$

and $$i_1M = IS + \frac{rm}{r+q+m}\cdot I$$

By division
$$\frac{Q}{M} = \frac{X + \dfrac{rq}{r+q+m}}{S + \dfrac{rm}{r+q+m}}$$

from which
$$MX = QS + \frac{Qmr}{r+q+m} - \frac{Mqr}{r+q+m}$$

$$X = \frac{QS}{M} + \frac{r}{r+q+m}\left(\frac{Qm}{M} - q\right)$$

or
$$X = \frac{QS}{M} + \frac{mr}{r+q+m}\left(\frac{Q}{M} - \frac{q}{m}\right) \quad . \quad . \quad (183)$$

The term $\dfrac{mr}{r+q+m}\left(\dfrac{Q}{M} - \dfrac{q}{m}\right)$ can be made very small by making the resistance of the link r very small, and also by making the ratio $\dfrac{Q}{M}$ as nearly as possible equal to $\dfrac{q}{m}$.

If this term is made negligibly small—which is not difficult to accomplish in practice—the expression for X becomes simply

$$X = \frac{Q}{M} . S$$

which gives the resistance of the unknown in terms of the resistance of the standard.

In order to take into account thermo-electric e.m.f.s (see next chapter), a measurement should also be made with the direction of the current reversed and the mean of the two readings should be taken as the correct value of X.

In a Kelvin double bridge manufactured by Messrs. H. Tinsley & Co. the range of resistance covered is 0·1 microhm to 1 ohm. Under specified conditions of test the accuracy is stated as

From 1,000 microhms to 1 ohm . . . 0·05 per cent.

From 100 microhms to 1,000 microhms . . . 0·2 per cent to 0·05 per cent.

From 10 microhms to 100 microhms . . . 0·5 per cent to 0·2 per cent, limited by thermo e.m.f.'s.

There are four internal resistance standards of 1 Ω, 0·1 Ω, 0·01 Ω and 0·001 Ω respectively.

H. W. Sullivan, Ltd. make a precision Kelvin and Wheatstone bridge covering a range of 1 microhm to 1 megohm.

Operation of Kelvin Double Bridge in Practice. The method of operation of the Kelvin double bridge in practice is often somewhat different from that described above, especially when precise measurements of low resistance are to be made.

Instead of varying the ratio arms, keeping the ratio $\dfrac{Q}{M}$ equal to $\dfrac{q}{m}$, to obtain balance of the bridge, the resistances Q, M, q, and m

are often made up of resistance coils whose resistances are fixed and are accurately known, together with their temperature coefficients. The ratios $\dfrac{Q}{M}$ and $\dfrac{q}{m}$ thus remain fixed during the test, and are made equal to one another, and roughly equal to the ratio $\dfrac{X}{S}$ assuming this to be known, approximately. If not known, the ratio can easily be determined approximately by measuring X, first of all, using a less accurate method such as the ammeter and voltmeter method, or, better, by the potentiometer method.

Adjustment of the bridge to obtain balance is then carried out by shunting either the unknown resistance or the standard by a variable resistance, such as a resistance box. Then, assuming balance to be obtained by shunting the unknown by a resistance x, let the resistance of X and x in parallel, at balance, be X', then

$$X' = \frac{Q}{M} . S$$

and also
$$\frac{1}{X'} = \frac{1}{X} + \frac{1}{x}$$

from which the value of X can be obtained. As before, measurements are made also with the supply current reversed, and the average value of the two results taken as the final value. The sensitivity of the bridge can be determined by noting the smallest variation of the shunting resistance x, which produces an observable deflection of the galvanometer. The difference in X' for such a variation of x can then easily be calculated, thus giving the sensitivity of the bridge. This method of obtaining fine adjustment, by shunting a low resistance by a resistance box of much greater resistance, will be found a very useful one in electrical measurements generally. It has the advantage, also, that the resistances of the coils in the resistance box need not be known to within any high degree of accuracy, since slight errors in their values introduce negligible errors in the resistance of the combination.

The ratios $\dfrac{Q}{M}$ and $\dfrac{q}{m}$ may be made exactly equal by adjustment of the resistances of the leads to be used in connecting up the bridge, since these leads resistances are obviously included in the arms as well as the resistances of the coils themselves. By suitably proportioning the resistances of these copper leads the ratios $\dfrac{Q}{M}$ and $\dfrac{q}{m}$ can also be made independent of temperature—if this is necessary—to a very close approximation. The resistances of the leads need only be known with an accuracy of (say) 1 per cent, since they are usually small compared with those of the coils which they connect.

Example.

Resistance of link r = 0·0001 ohm

Resistance of coil in arm Q = $10{\cdot}0027_0$ at 20° C.
(Temperature coefficient 0·00003)

Resistance of coil in arm M = $20{\cdot}0142_5$ at 20° C.
(Temperature coefficient 0·00002)

Resistance of coil in arm q = $10{\cdot}0027_1$ at 20° C.
(Temperature coefficient 0·00003)

Resistance of coil in arm m = $20{\cdot}0067_0$ at 20° C.
(Temperature coefficient 0·000025)

Resistances of copper leads—

$$\left.\begin{array}{lll} \text{In arm } Q = 0{\cdot}0146 & \text{at } 20^\circ \text{ C.} \\ \text{,, ,, } M = 0{\cdot}0660 & \text{,, ,,} \\ \text{,, ,, } q = 0{\cdot}0061_5 & \text{,, ,,} \\ \text{,, ,, } m = 0{\cdot}0122 & \text{,, ,,} \end{array}\right\} \text{Temperature coefficient } 0{\cdot}0043$$

Resistance of standard = $0{\cdot}0100120_2$ at 20° C.

Resistance of "unknown" = 0·005 (about)

To balance the bridge the standard resistance is shunted by a resistance of 18·1 ohms, measurement being made at 20° C.

First, the values of the two ratios $\frac{Q}{M}$ and $\frac{q}{m}$ are calculated for a temperature of 20° C.

Then,
$$\frac{Q}{M} = \frac{10{\cdot}0027_0 + 0{\cdot}0146}{20{\cdot}0142_0 + 0{\cdot}0660} = 0{\cdot}49886_5$$
$$\frac{q}{m} = \frac{10{\cdot}0027_1 + 0{\cdot}0061_5}{20{\cdot}0067_0 + 0{\cdot}0122} = 0{\cdot}499970$$

To make these ratios more nearly equal the coil in arm q is shunted by some resistance y, the value of which is found as follows.

Let q' be the shunted value of this coil, then

$$\frac{q' + 0{\cdot}0061_5}{20{\cdot}0189_0} = 0{\cdot}49886_5$$
$$q' = 9{\cdot}9805_8$$

Now,
$$\frac{1}{q'} = \frac{1}{y} + \frac{1}{10{\cdot}0027_1}$$

from which y = 3,532 ohms

Since the two ratios have been thus made exactly equal at 20° C., if the measurement is made at 20° C. the value of the unknown resistance is given simply by

$$X = \frac{Q}{M} \cdot S'$$

where S' is the shunted value of the standard. Since the shunt for balance is 18·1 ohms,

$$\frac{1}{S'} = \frac{1}{0{\cdot}0100120_2} + \frac{1}{18{\cdot}1}$$

from which
$$S' = 0{\cdot}010006_5$$
$$\therefore X = 0{\cdot}010006_5 \times 0{\cdot}49886_5$$
$$= 0{\cdot}0049919_0 \text{ at } 20^\circ \text{ C.}$$

Consider, now, the effect upon the ratios $\frac{Q}{M}$ and $\frac{q}{m}$ of a rise in temperature of 5° C.

Then
$$\frac{Q}{M} = \frac{10{\cdot}0027_0[1 + 0{\cdot}00003 \times 5] + 0{\cdot}0146[1 + 0{\cdot}0043 \times 5]}{20{\cdot}0142_5[1 + 0{\cdot}00002 \times 5] + 0{\cdot}0660[1 + 0{\cdot}0043 \times 5]}$$
$$= 0{\cdot}49886_3$$

and
$$\frac{q}{m} = \frac{9{\cdot}9805_8[1 + 0{\cdot}00003 \times 5] + 0{\cdot}0061_5[1 + 0{\cdot}0043 \times 5]}{20{\cdot}0067_0[1 + 0{\cdot}00002_5 \times 5] + 0{\cdot}0122[1 + 0{\cdot}0043 \times 5]}$$
$$= 0{\cdot}49887_8$$

Then
$$\frac{Q}{M} - \frac{q}{m} = -\,0{\cdot}000010$$

and the correction term $\frac{mr}{r + q + m}\left(\frac{Q}{M} - \frac{q}{m}\right)$ is equal to

$$-\frac{0{\cdot}0001 \times 20{\cdot}0067_0}{10{\cdot}0027_1 + 20{\cdot}0067_0 + 0{\cdot}0001} \times 0{\cdot}00001$$

which is roughly $\frac{2}{3} \times 0{\cdot}000000001$ and is therefore entirely negligible.

Sensitivity. Suppose that in the above measurement the smallest change in the value of the shunt across the standard which can be detected is 0·5 ohm. Then, giving this shunt the value 18·6 instead of 18·1 we have

$$\frac{1}{S'} = \frac{1}{0{\cdot}0100120_2} + \frac{1}{18{\cdot}6}$$

from which $S' = 0{\cdot}010006_6$ instead of $0{\cdot}010006_5$, which means a change in S' of 1 part in 100,000, and therefore a change in the value of X of 1 part in 100,000. Thus the sensitivity of the bridge under the above conditions is 1 part in 100,000.

In general, if x is the value of the shunt across S, we have

$$\frac{1}{S'} = \frac{1}{S} + \frac{1}{x}$$

or
$$S' = \frac{Sx}{S + x}$$

Differentiating with respect to x,

$$\frac{dS'}{dx} = S\left[\frac{(S + x) - x}{(S + x)^2}\right] = \frac{S^2}{(S + x)^2} \doteqdot \frac{\Delta S'}{\Delta x}$$

where $\Delta S'$ is the change in S' for a given change Δx in x.

Thus,
$$\Delta S' = \frac{S^2}{(S + x)^2} \cdot \Delta x$$

Smith (Ref. 4) has shown that, if X is changed to $X + \delta X$, the galvanometer current is given by

$$\frac{I \,.\, \delta X}{G + \dfrac{qm}{q + m} + \left[\dfrac{(X + Q)(S + M)}{X + Q + S + M}\right]}\left[\frac{S + M}{X + Q + S + M}\right]$$

where G is the resistance of the galvanometer. Thus, if Y is the sensitivity of the galvanometer used, in millimetres per micro-ampere, the deflection for a change of δX in the value of X is given in millimetres by

$$D = \frac{YI\delta X \,.\, 10^6 (S + M)}{\left[G + \frac{qm}{q+m} + \frac{(X+Q)(S+M)}{X+Q+S+M}\right](X+Q+S+M)} \qquad (184)$$

The best value for the galvanometer resistance is

$$\frac{qm}{q+m} + \frac{(X+Q)(S+M)}{X+Q+S+M}$$

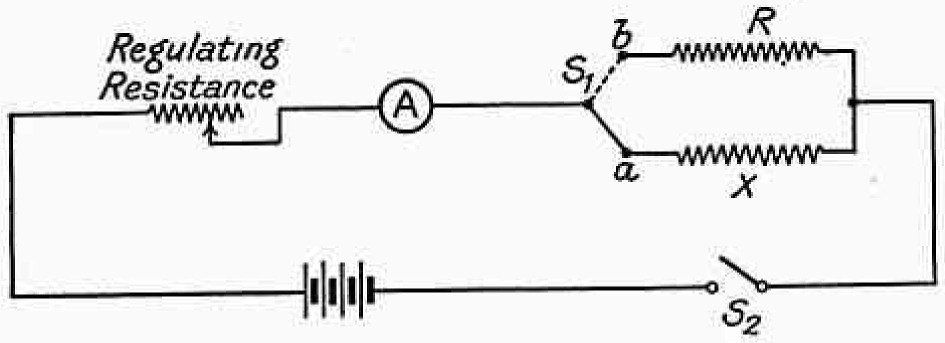


Fig. 160. Measurement of Resistance by Substitution

Measurement of Medium Resistance. The methods used for such measurements are—

(a) Ammeter and voltmeter method.
(b) Substitution method.
(c) Differential galvanometer method.
(d) Wheatstone bridge.

(a) This method has been considered in the section on low resistance measurements earlier in the chapter.

(b) Substitution Method. The diagram of connections for this method is given in Fig. 160. X is the resistance to be measured, while R is a variable known resistance. A battery of ample capacity is used for the supply, since it is important in this method that the supply voltage shall be constant. A is an ammeter of suitable range, or a galvanometer with a shunt which can be varied as required.

With switch S_2 closed, and with switch S_1 on stud a, the deflection of the ammeter or galvanometer is observed. S_1 is then thrown on to stud b and the variable resistance is adjusted until the same deflection is obtained on the indicating instrument. Then, the value of R which produces the same deflection gives the resistance of the unknown directly.

The resistances of R and X should be large compared with that of the rest of the circuit. The method is chiefly used—somewhat

modified—in the measurement of high resistance. The accuracy of the measurement obviously depends upon the constancy of the supply voltage, of the resistance of the circuit excluding X and R, and upon the sensitivity of the indicating instrument, as well as upon the accuracy with which the resistance R is known.

(c) Differential Galvanometer Method. A differential galvanometer has two similar, but separate, windings, insulated from one another. The windings are of equal resistance and are wound with twinned wire so that they may be as nearly coincident as possible and thus produce almost exactly equal and coincident magnetic fields when equal voltages are applied to their terminals. If equal and *opposite* voltages were applied to the two windings the resultant

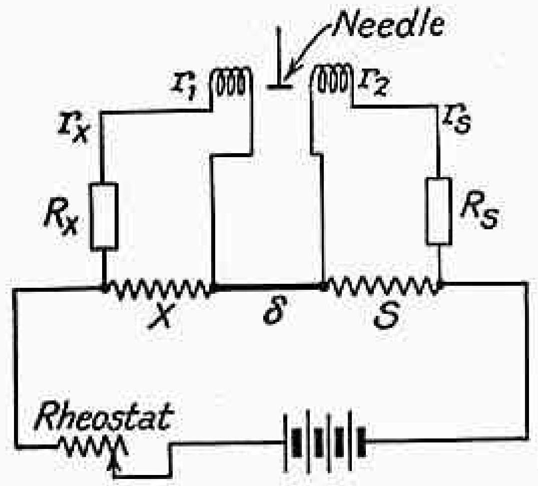


Fig. 161. Measurement of Resistance by Differential Galvanometer

magnetic field would be very nearly zero. This resultant field is made zero by the use of a small external coil connected in series with the coil having the weaker magnetic field and placed so that its magnetic effects add to that of this coil. This external coil is adjustable so that zero magnetic field may be obtained. A small magnetic needle, to which a pointer is attached, is pivoted in the field produced by the windings.

The circuit connections when the instrument is to be used for the measurement of resistance are shown in Fig. 161. X is the unknown resistance, while S is a standard resistor with which it is to be compared. R_s and R_x are resistance boxes connected in circuit as shown, their positions in the circuit being such that the two galvanometer coils, of resistances r_1 and r_2, are very nearly at the same potential, to avoid leakage effects between them. r_s and r_x represent the resistances of the leads and δ is the resistance of the connecting link between X and S, and should be very low.

The galvanometer is first adjusted to zero by connecting its two coils in series, so that their magnetic effects are in opposition, and moving the compensating coil until zero deflection is obtained. Then, with connections as in Fig. 161, R_x is given some suitable value and the resistance R_s adjusted until zero deflection of the galvanometer is obtained, when the resistance X is given by

$$X = \left(\frac{R_x + r_x + r_1}{R_s + r_s + r_2}\right) . S \qquad . \quad . \quad . \quad (185)$$

Theory. When zero deflection of the galvanometer is obtained, the currents in the two galvanometer coils are equal. Let i be this current, and let I be the current passing through X and S.

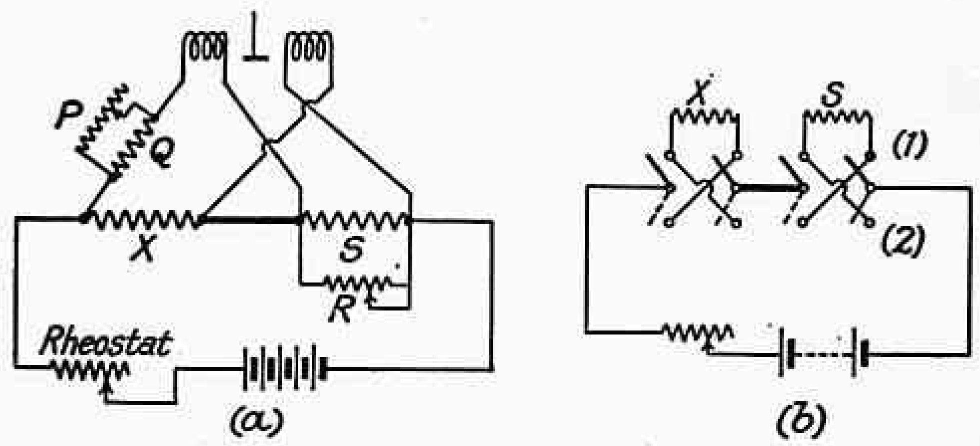


FIG. 162. KOHLRAUSCH METHOD OF USING THE DIFFERENTIAL GALVANOMETER

Then
$$i = \frac{IX}{R_x + r_x + r_1}$$
$$= \frac{IS}{R_s + r_s + r_2}$$

From which
$$\frac{X}{R_x + r_x + r_1} = \frac{S}{R_s + r_s + r_2}$$
$$X = \left(\frac{R_x + r_x + r_1}{R_s + r_s + r_2}\right) . S$$

It follows from this expression for X that the resistances of the connecting leads and of the two galvanometer coils must be known, or must be measured separately, although if R_x and R_s are fairly large the error introduced by neglecting the resistances of the leads will probably be small. The method has been displaced, for resistance measurements, by the Wheatstone bridge but the differential galvanometer is used, in conjunction with a portable bridge, for the location of faults in telephone cables.

Kohlrausch Method. This method, using a differential galvanometer, is more suited to precision measurements of resistance than the above. It eliminates errors arising from variation of the resistances of the galvanometer circuits due to contact resistances, etc.

The connections are shown in Fig. 162 (*a*), X being the unknown resistance and S a standard resistor approximately equal in resistance to X. This standard—assuming it to be larger than X—is shunted by a variable resistance R. P and Q are resistances in parallel in one of the galvanometer circuits, as shown, P being variable. Arrangements are made for reversing the battery connections to X and S by means of a specially designed switch or commutator. This has mercury contacts and is of heavy section to give low contact resistance. This arrangement is shown in Fig. 162 (*b*) and is used in conjunction with the circuit of Fig. 162 (*a*).

The galvanometer is first adjusted to give zero deflection. The resistances P and R are adjusted so that zero deflection is obtained with the switch in *both* of the positions (1) and (2).

Then $X = S'$ where S' is the shunted value of S and is given by $\dfrac{RS}{R + S}$. If X is shunted instead of S, then, at balance, $S =$ the shunted value of X.

If X and S are nearly equal the resistance R will be large compared with S and X, and its value need not be known with any high degree of precision.

The same relationship between X and S exists if, instead of zero deflection being obtained in both positions of the switch, the same deflection is obtained in both positions, the direction of the deflection being the same as well as its magnitude. This condition is obtained if the galvanometer is not adjusted exactly to zero initially, so that such exact adjustment is not necessary in this method.

(*d*) WHEATSTONE BRIDGE. This is the best and commonest method of measuring medium resistances. The general arrangement is shown in Fig. 163. P and Q are two known fixed resistances, S being a known variable resistance and R the unknown resistance. G is a sensitive D'Arsonval galvanometer shunted by a variable resistance N to avoid excessive deflection of the galvanometer when the bridge is out of balance. This shunt is increased as the bridge approaches balance, so that the shunting is zero—giving full sensitivity of the galvanometer—when balance is almost obtained. B is a battery of two or three cells and M is a reversing switch so that the battery connections to the bridge may be reversed and two separate measurements of the unknown resistance made in order to eliminate thermo-electric errors. K_B and K_G are keys fitted with insulating press-buttons, so that the hand does not come in contact with metal parts of the circuit, thus introducing thermo-electric e.m.f.s. The battery key, K_B, should be closed first, followed by the closing of K_G after a short interval. This avoids a sudden (possibly excessive) galvanometer deflection, due to self-induced e.m.f.s when the unknown resistance R has appreciable self-inductance.

At balance—obtained by adjustment of S—the same current i_1 flows in both of the arms P and Q, since the galvanometer takes no current, and the same current i_2 flows also in arms R and S.

Also, voltage drop across arm P = voltage drop across arm Q
and voltage drop across arm R = voltage drop across arm S

Thus, $$i_1P = i_2R$$
$$i_1Q = i_2S$$

By division $$\frac{P}{Q} = \frac{R}{S}$$

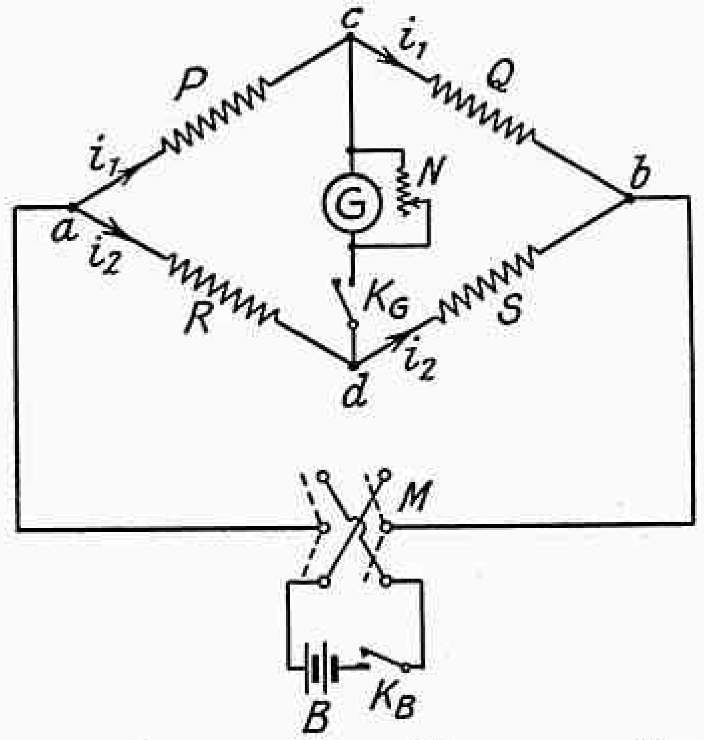


Fig. 163. Connections of Wheatstone Bridge

or $$R = \frac{P}{Q} . S \quad . \quad . \quad . \quad . \quad (186)$$

from which R is found in terms of P, Q, and S.

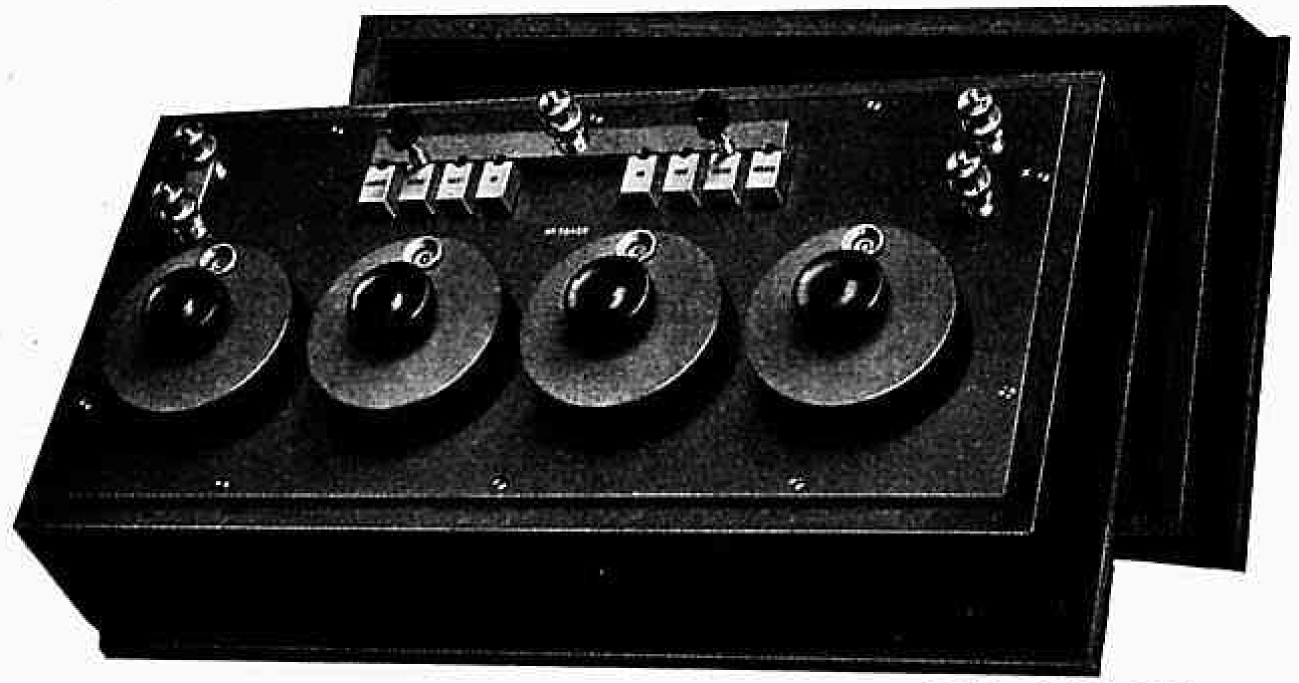
The arms P and Q are the "ratio arms" of the bridge and the ratio $\frac{P}{Q}$ may be varied as required to increase the range of the bridge.

In the elementary forms of the Wheatstone bridge the arms P and Q are replaced by a slide-wire of uniform cross-section whose resistance per unit length is constant. A scale is fitted under this slide-wire and a sliding contact—corresponding to the point c of Fig. 163—connects one terminal of the galvanometer to the wire.

S is a resistance of the same order of magnitude as the unknown. The sliding contact c is moved until zero deflection of the galvanometer is obtained. Then, since the slide-wire is of uniform cross-section, the ratio $\frac{P}{Q}$ is given by $\frac{\text{length of slide-wire } ac}{\text{length of slide-wire } cb}$, these lengths being obtained from the scale.

As before, $$R = \frac{P}{Q} . S$$

A precision form of Wheatstone bridge as manufactured by Messrs. Muirhead & Co. is shown in Fig. 164. Such forms of bridge contain either four or five pairs of ratio coils—tens, hundreds, thousands, and ten-thousands, in the bridge containing four pairs—and either four or five decades of resistance coils which constitute the variable arm S. These may be either of the plug pattern or sliding contact pattern. The internal connections of a Tinsley bridge are given, together with the external connections when the instrument is used in conjunction with a slide wire in Fig. 165.



(Muirhead & Co. Ltd.)

Fig. 164. Wheatstone Bridge, Sliding Contact Pattern

Operation of the Bridge. The method of operation can best be illustrated by an example.

Suppose that the actual value of the resistance to be measured is 57·63 ohms and that a four-dial Wheatstone bridge—the dials containing units, tens, hundreds, and thousands—is to be used in conjunction with a galvanometer of ample sensitivity.

The bridge is connected up according to the arrangement shown in Fig. 163, care being taken to ensure that all the connections are firmly made and that all the plugs in the bridge blocks are firmly pressed home so that contact resistances may be small and definite. The galvanometer is at first heavily shunted and the ratio arms are made equal (each 10 ohms, say). The battery supply switch is closed. Assuming the magnitude of the resistance to be measured to be entirely unknown, first set the variable resistance arm, S, to some small value—say 1 ohm. Depress the key K_B and then lightly press the galvanometer key K_G (immediately raising it again if the galvanometer deflection is excessive). Note the direction of the galvanometer deflection—right or left. Next set the arm S to some high resistance—say 10,000 ohms—and again note the direction of the deflection obtained. If this direction is opposite to the previous one, the unknown resistance has some value between 1 and 10,000 ohms, being nearest in value to the setting of S which gives the

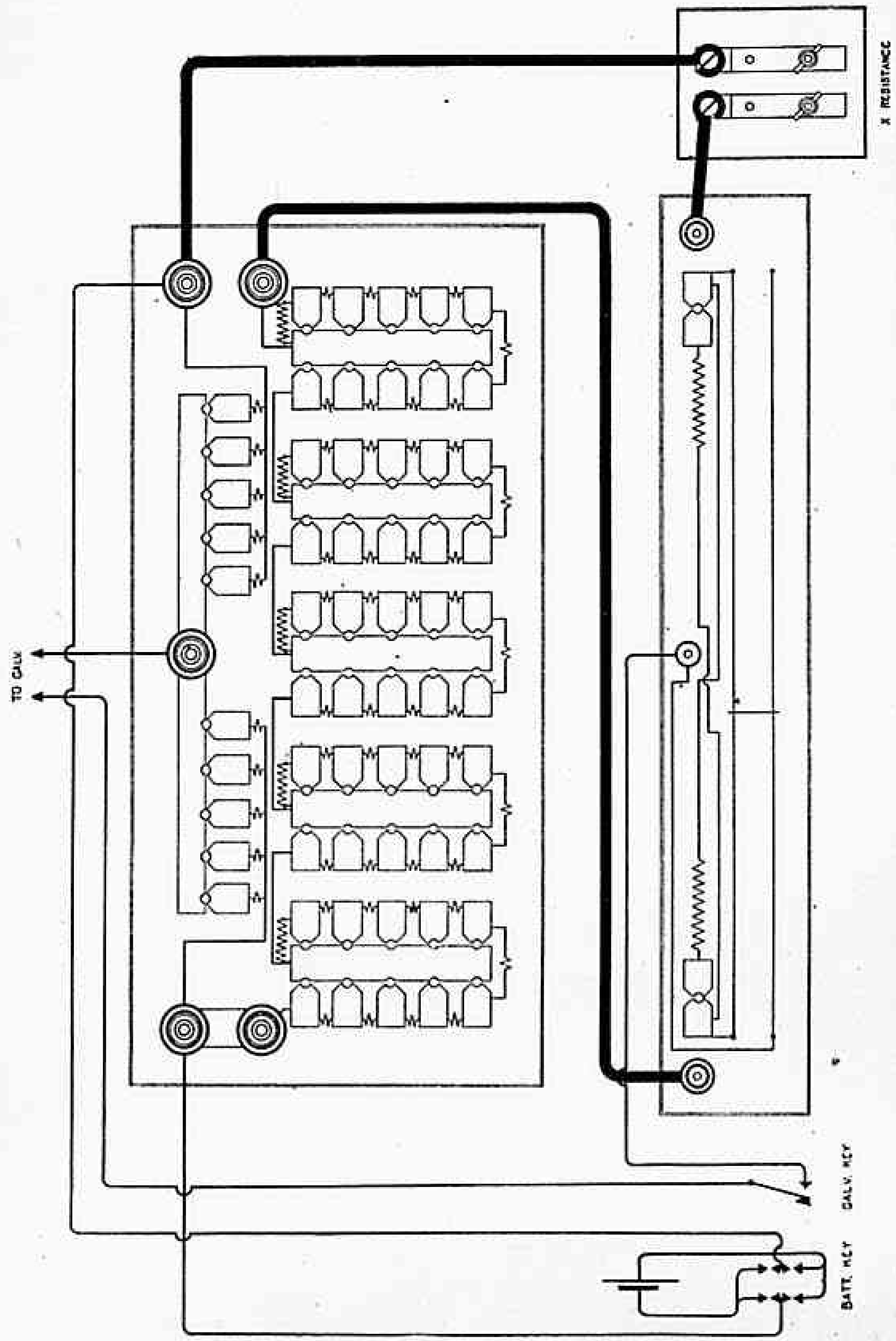


FIG. 165. CONNECTIONS FOR SETTING UP 5-DIAL BRIDGE AND SLIDE WIRE
(*H. Tinsley & Co., Ltd.*)

smaller deflection. Adjust the resistance S until approximate balance is obtained, and then remove the shunt from the galvanometer, in steps, adjusting S as required, until balance is obtained with the full galvanometer sensitivity. In the case under consideration ($R = 57{\cdot}63$), it will be found that the galvanometer deflects to the left (say) with S set at 57 and to the right with S set at 58, meaning that R lies between the two. This is the best that can be done with equal ratio arms.

To obtain greater accuracy, make Q 100 ohms, keeping P 10 ohms, at the same time altering S to 570. Then adjust S until approximate balance is obtained again. It will now be found that the balance point is between $S = 576$ and 577. Q should then be made 1,000 ohms (keeping P 10 ohms) and the process repeated. Final balance will be obtained when $S = 5{,}763$ ohms. Then

$$R = \frac{P}{Q} \, . \, S = \frac{10}{1000} \, . \, 5763 = 57{\cdot}63$$

The battery connections are then reversed by the switch M, and the measurement repeated. If it is found that an alteration of 1 ohm in S disturbs the balance, then the sensitivity is at least 1 part in 5,763. As a check on the balance it should always be ascertained that a slightly smaller value of S causes a galvanometer deflection to the left (say) and that a slight increase in S above the balance point causes a galvanometer deflection in the opposite direction.

If, with equal ratio arms at the beginning of the measurement, the unknown resistance does not lie within 1 and 10,000, the ratio arms must be adjusted until an approximate balance is obtained between these limits of S—e.g. if R is greater than 10,000, arm P must be made greater than Q, and if smaller than 1 ohm, Q must be made greater than P.

An accuracy of a few parts in 10,000 is usually obtainable with a good bridge of the type described above.

Best Galvanometer Resistance. The current through the galvanometer for a given change δR in the unknown resistance is given by—

$$i_g = \frac{i_2 \, . \, \delta R}{G + \left[\dfrac{(R+P)(Q+S)}{R+P+Q+S}\right]} \, \frac{Q+S}{R+P+Q+S}$$

where G is the galvanometer resistance.

Let $i_2 \, \delta R \, \dfrac{Q+S}{R+P+Q+S} = K$, and $\dfrac{(R+P)(Q+S)}{R+P+Q+S} = A$

Then $$i_g = \frac{K}{G+A}$$

For given dimensions of the galvanometer coil

Number of turns on coil, N, $\propto \sqrt{G}$

$\therefore$ Deflecting torque $\propto N i_g \propto i_g \sqrt{G}$

Hence, deflection for a given change δR, and current i_2 is

$$\theta \propto i_g \sqrt{G} \propto \frac{K\sqrt{G}}{G+A}$$

Differentiating we have

$$\frac{d\theta}{dG} \propto \frac{A-G}{2\sqrt{G}(G+A)^2} \text{ which is zero when } G = A$$

Thus, maximum deflection for a given change in the resistance R, and a given current i_2, is obtained when the galvanometer resistance is given by

$$G = \frac{(R + P)(Q + S)}{R + P + Q + S}$$

Obviously, the sensitivity may be increased, also, by increasing the current i_2. The galvanometer may be of the D'Arsonval type and should be as nearly critically damped as possible.

Precision Modifications of the Wheatstone Bridge. There are three principal types of such bridges—

(*a*) The slide-wire type.
(*b*) The shunt type.
(*c*) The Reichsanstalt type.

Space is available here for the description of only one of these types. Full descriptions of the other types may be found in the *Dictionary of Applied Physics*, Vol. II, p. 714.

Carey Foster Slide-wire Bridge. The connections of this bridge are shown in Fig. 166A, a slide-wire of length L being included between R and S as shown. This bridge is specially suited to the comparison of two nearly equal resistances.

Resistances P and Q are first adjusted so that the ratio $\frac{P}{Q}$ is approximately equal to the ratio $\frac{R}{S}$. Exact balance is obtained by adjustment of the sliding contact on the slide-wire. Let l_1 be the distance of the sliding contact from the left-hand end of the slide-wire. The resistances R and S are then interchanged and balance again obtained. Let the distance now be l_2.

Then, for the first balance, $$\frac{P}{Q} = \frac{R + l_1 r}{S + (L - l_1)r}$$

where r is the resistance per unit length of the slide-wire.

For the second balance,

$$\frac{P}{Q} = \frac{S + l_2 r}{R + (L - l_2)r}$$

Now, $$\frac{P}{Q} + 1 = \frac{R + l_1 r + S + (L - l_1)r}{S + (L - l_1)r} = \frac{R + S + Lr}{S + (L - l_1)r}$$

also $$\frac{P}{Q} + 1 = \frac{S + l_2 r + R + (L - l_2)r}{R + (L - l_2)r} = \frac{S + R + Lr}{R + (L - l_2)r}$$

Hence $$S + (L - l_1)r = R + (L - l_2)r$$

or $$S - R = (l_1 - l_2)r \quad . \quad . \quad . \quad . \quad (187)$$

Thus the difference between S and R is obtained from the resistance per unit length of the slide-wire together with the difference $(l_1 - l_2)$ between the two slide-wire lengths at balance.

The slide-wire is calibrated—i.e. r is obtained—by shunting either S or R by a known resistance and again determining the difference in length $(l_1' - l_2')$.

Suppose that S is known and that S' is its resistance when shunted by a known resistance, then

$$S - R = (l_1 - l_2)r$$

and $$S' - R = (l_1' - l_2')r$$

$$\frac{S - R}{l_1 - l_2} = \frac{S' - R}{l_1' - l_2'}$$

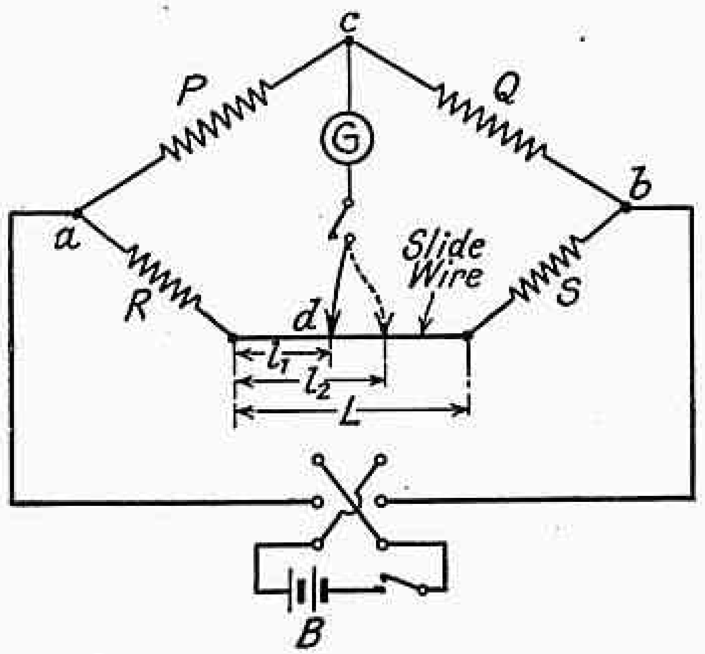


FIG. 166A. CAREY FOSTER SLIDE-WIRE BRIDGE

from which $$R = \frac{S(l_1' - l_2') - S'(l_1 - l_2)}{(l_1' - l_2' - l_1 + l_2)} \quad . \quad . \quad . \quad (188)$$

From this expression it can be seen that this method gives a direct comparison between S and R in terms of lengths only, the resistances of P and Q, contact resistances, and the resistances of connecting leads being eliminated.

As it is important that the two resistances R and S shall not be handled or disturbed during the measurement, a special switch is used to effect the interchanging of these two resistances during the test.

The Sullivan Wheatstone Bridge, which has an accuracy of 0·01 per cent, is designed (on a patented principle due to W. H. F. Griffiths) to measure with equal accuracy in either the new *Absolute* units or the older *International* units.

The principle is as follows—

Let P and Q be ratio arms each of four values, say, 10, 100, 1,000, 10,000 ohms, and let S be the standard (or rheostat) arms of four or five decades and X be the unknown resistance to be measured.

Q ratios are all adjusted to be correct to their nominal values in Absolute ohms.

P ratios are all adjusted to be $2{\cdot}5 \times 10^{-4}$ higher than nominal

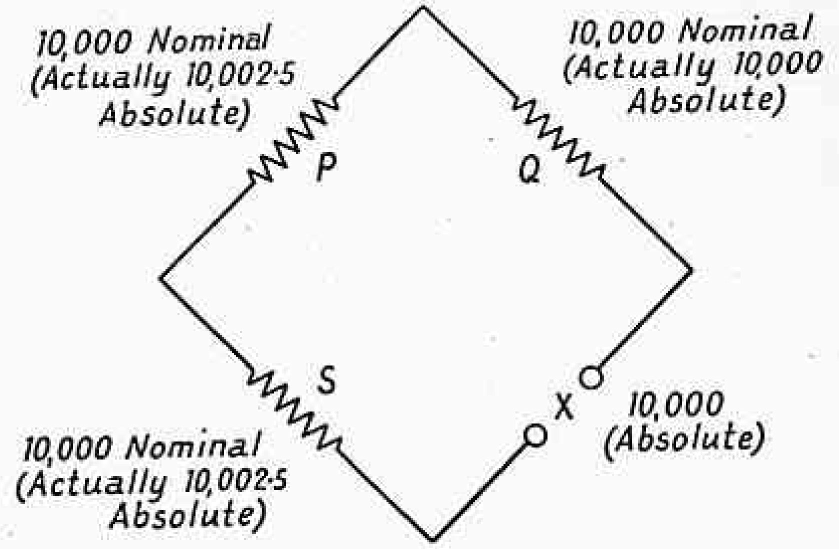


FIG. 166B

Absolute ohms, i.e. midway between Absolute and International ohms.

S is adjusted so that all its nominal values are midway between Absolute and International ohms, e.g. a nominal 1 ohm is actually

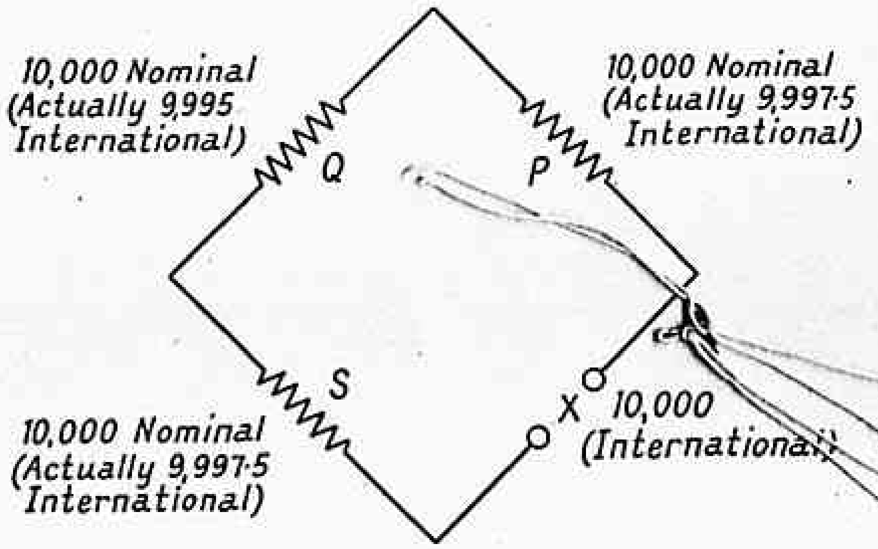


FIG. 166C

1·00025 ohm Absolute and is, therefore, 0·99975 ohm International.

When measuring in Absolute units, arms P and S are arranged to be adjacent as shown in Fig. 166B. Then, as an example, for the measurement of an X of 10,000 ohms (Absolute) in Absolute units with even ratios, we have the resistance values in the other three arms as shown.

It should be noted that the nominal values of all three "known" arms, P, Q and S, will be read as 10,000 notwithstanding their

actual values (which are given in brackets) so that X will be measured simply as 10,000 ohms (Absolute). The balance equation in Absolute units is actually—

$$X = \frac{QS}{P} = \frac{10{,}000 \times 10{,}002{\cdot}5}{10{,}002{\cdot}5} = 10{,}000$$

When using the same bridge for measuring in International units the ratio arms P and Q are transposed so that Q and S are now adjacent as shown in Fig. 166C. This transposition is effected automatically upon connecting the resistance to be measured to the pair of "X" terminals appropriate to the system of units required. Then, for the measurement of 10,000 ohms (International) in International units we have the resistance values shown.

The same three settings of the known arms P, Q and S are used and their nominal values are all read as 10,000 ohms as before, so that X will be read as 10,000 ohms (International), although the balance equation, in International ohms, is in fact—

$$X = \frac{PS}{Q} = \frac{9997{\cdot}5 \times 9997{\cdot}5}{9995}$$

The value of X from this expression is 10,000 to an accuracy of 1 part in 10^7.

Applications of Resistance Measurements by Wheatstone Bridge. Several quite different quantities can be measured through the change which they bring about in the resistance of some form of element adapted to the particular measurement concerned and used in conjunction with a Wheatstone bridge to indicate this change.

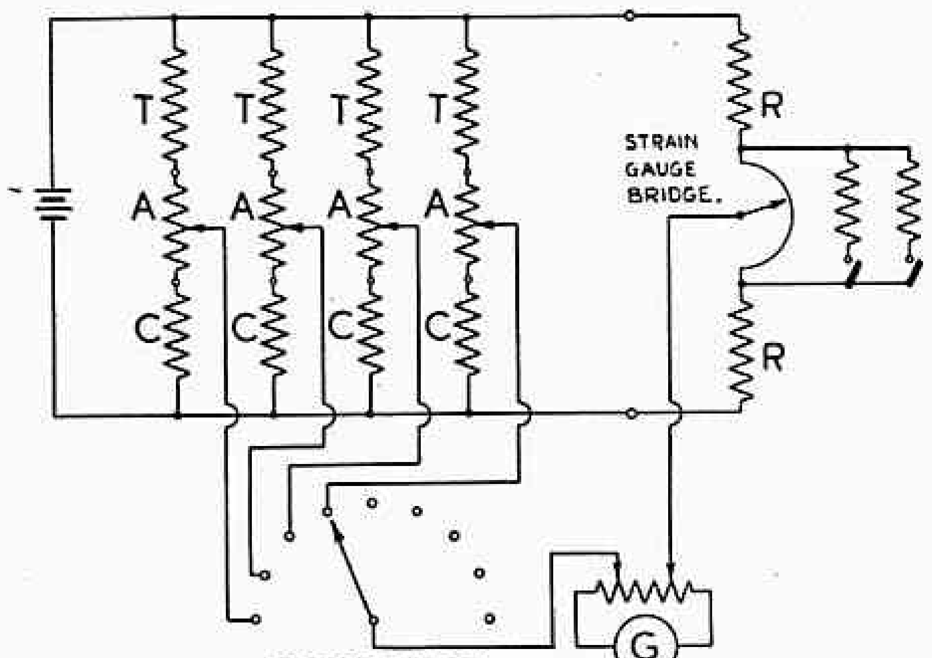
Examples in other chapters are the bismuth spiral (see p. 371) the resistance of which increases when it is placed in a magnetic field, the resistance thermometer (see p. 502) and the Ionic wind voltmeter (see p. 457).

Some important applications of resistance measurements which do not fall conveniently into the groundwork of other chapters are described below.

Strain Gauge Measurements. For the measurement of stresses and strains in structures and machines the technique of using electrical resistance strain gauges has been built up, particularly during World War II, in the aircraft industry. Thin copper-nickel alloy or nickel-chrome wires are used, these having a linear relationship—over the limited range required—between strain and electrical resistance. Nickel-chrome has the higher resistance but is less commonly used because it has some strain hysteresis and also a high temperature coefficient.

These gauges enable accurate measurements of strain to be made in any surface to which they can be cemented so that no slip occurs. They consist of grids of fine wires cemented to a thin paper membrane which is cemented, with nitro-cellulose cement, to the surface under test. The gauge is usually covered with a layer of felt to protect it and to reduce the effect of draughts. Particularly to eliminate electrolytic effects in the gauge, and also to avoid moisture absorption by the matrix, it is coated—when completely dry—with a protective wax such as Di Jell 171.

Care is necessary in preparing the surface to receive the strain gauge and in fixing the gauge itself and it is important to follow the manufacturer's instructions in this.



(*H. Tinsley & Co., Ltd.*)

FIG. 167. STRAIN GAUGE BRIDGE NETWORK

The gauge current may be as high as 25 mA for short periods but 10 mA is recommended for longer tests.

The ratio $\dfrac{\text{Percentage change of resistance}}{\text{Percentage strain}}$ is called the "gauge factor." The resistance change is greater than may be accounted for by the change in the wire dimensions and this factor is commonly about 2·2.

Sample gauges are made up and gauges are rejected if test results show a variation of more than $\pm$ 1 per cent from the mean.

In the measurements of strain, temperature effects are compensated for by using a dummy gauge which is connected in the appropriate arm of a Wheatstone bridge network (see Fig. 167). The dummy must be fixed to a piece of the same material as that under test but which is not, of course, under stress. The two gauges must be under the same conditions of ambient temperature.

The arrangement in Fig. 167, which is for static measurements, has a number of gauges T, each with its compensating gauge C. Zero balance is obtained by adjustment of the apex resistor A and calibration is by shunting one of the bridge arms R by a high resistance.

Measurement may be either by direct deflection, using a calibrated galvanometer, to measure the out-of-balance current, or by a null method through variation of the slide-wire setting. The selector switch shown must be free from thermo-electric effects. The slide wire is necessitated by the sensitivity required in such measurements, the change in gauge resistance being very small.

In some measurements all four arms of the bridge can be gauges mounted, for example, on the same specimen in tension and compression. Then, the zero adjusting and calibrating resistors can be located at a distance from the bridge.

For dynamic measurements, which are usually deflectional and use an oscilloscope or oscillograph, the bridge may be supplied at a carrier frequency from an ordinary a.c. amplifier having a flat response over the frequency range to be covered. Fig. 168 shows a bridge arrangement for carrier frequency operation. A capacitance balance is necessary in addition to a resistance balance and calibration and zero adjustment are effected by shunting the arms.

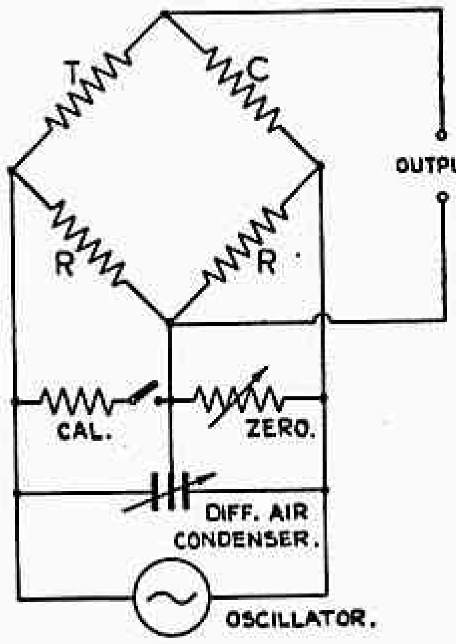


FIG. 168

Elliott Brothers (London), Ltd. have introduced a "ten-channel strain gauge equipment" to display, on one cathode-ray tube, the strain at ten different points in a test piece.

Fig. 169 shows the schematic layout and also the form in which the strain amplitudes and waveforms are displayed. Elliott publication No. G.121A gives a full description of the equipment.

The above is but a brief outline of the strain gauge technique. References (18), (19) and (20) deal with the subject comprehensively.

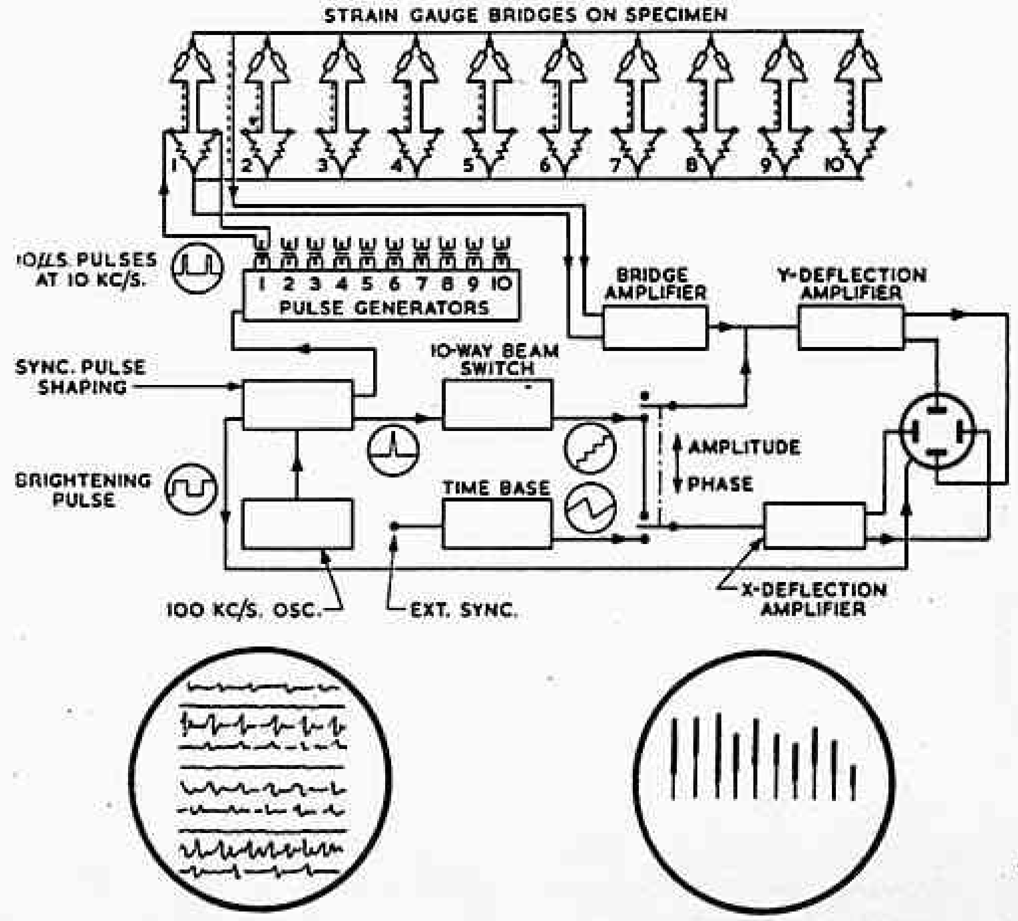
Electrical Weighing. H. I. Andrews (Ref. (21)) has described the application of the strain gauge principle to the measurement of weight or force in the testing of locomotives or trains under working conditions. For this purpose the apparatus must have reliability, rigidity, compactness and simplicity and must give reasonable accuracy during accelerations of several g with temperature changes up to 30° F.

In its development the aims were to obtain an increased output voltage and to enable robust measuring instruments to be used. The solution adopted was to employ, as the active element, a coil of enamel-insulated constantan wire wound directly on a round bar of nickel-chrome-molybdenum steel, or of mild steel or of duralumin. The coil is included in one arm of a Wheatstone bridge network and, as an example, with an applied voltage of 230 V and a coil on a mild steel bar stressed to 14 tons/in.[2] an output voltage of 0·034 V could be obtained. Constantan wire gives a good strain sensitivity, maintains its calibration indefinitely and has a low temperature coefficient.

When the active coil, of resistance R, forms one arm of a bridge of which the other three arms are each of resistance R also, the output voltage is given by

$$v = \frac{V}{4} \cdot \frac{P}{A} \cdot K \cdot \frac{m}{E}$$

where V is the voltage applied to the bridge, P is the applied load, A is the cross sectional area of the bar, m and E are Poisson's ratio



(*Elliott Bros. (London) Ltd.*)

FIG. 169. TEN-CHANNEL STRAIN GAUGE EQUIPMENT.

and Young's modulus, respectively, for the material of the bar and K is the sensitivity ratio which is the ratio between the proportional change of resistance of the wire and the circumferential strain in the bar.

The Measurement of Pressure. Another application of similar principles is to the measurement of pressure. Thus, for example, N. Gross and P. H. R. Lane (Ref. (22)) have described an accurate wire-resistance method of measuring pulsating pressures up to 6,000 lb./in.[2] As the pressure-sensitive elements, to replace the normal Bourdon tube types of gauge which are not sufficiently accurate for rapidly fluctuating pressures, cylindrical steel tubes with a wire winding are used. Two coils of 48 s.w.g. manganin wire

are wound side-by-side on the tube. These are dried and sealed against moisture. The coils are connected in two diagonally opposite arms of a Wheatstone bridge network of which the detector is a mirror galvanometer having a sensitivity of 100 mm/μA at 1 metre, and a periodic time of 0·1 sec. The bridge is supplied from a battery.

The proportionate change $\frac{\Delta R}{R}$ in the resistance of the pressure cell coils is related to the pressure by the expression

$$\frac{\Delta R}{R} = K(D - t)^2(1 - \frac{1}{2}m) \cdot \frac{p}{2EDt}$$

where K is the constant relating electrical strain to mechanical strain, m and E are Poisson's ratio and Young's modulus for the material of the pressure tube, p is the pressure and D and t are, respectively, the diameter of the middle layer and the thickness of the coils.

For recording the pressure pulsations the light reflected from the galvanometer mirror is allowed to fall on to 35-mm recording paper in an oscilloscope camera, the lens of the camera being replaced by a metal disc with a narrow slit parallel to the direction of movement of the light.

The accuracy is stated as within 0·5 per cent of the full scale deflection.

Hot-wire Anemometer. In this instrument air velocity is measured by its cooling effect upon a fine wire which is heated by the passage of a small current through it. L. F. G. Simmons and A. Bailey (Ref. (26)) described on early anemometer of this type developed at the National Physical Laboratory, and Simmons and J. A. Beavan (Ref. (27)) gave an account of its use for the measurement and recording of gusts. For this purpose advantage was taken of the quick response of the hot wire to the cooling effect. Two elements, each included in its own Wheatstone bridge network, were used to record both the velocity and direction of gusts the detectors in the networks being oscillograph elements.

King's formula (Ref. (28)) for the cooling of fine wires was

$$H = (a + b\sqrt{V})(T - T_a)$$

where H is the rate of heat loss per unit length of a circular wire which is maintained at a temperature T in a steady wind of velocity V, and T_a is the air temperature.

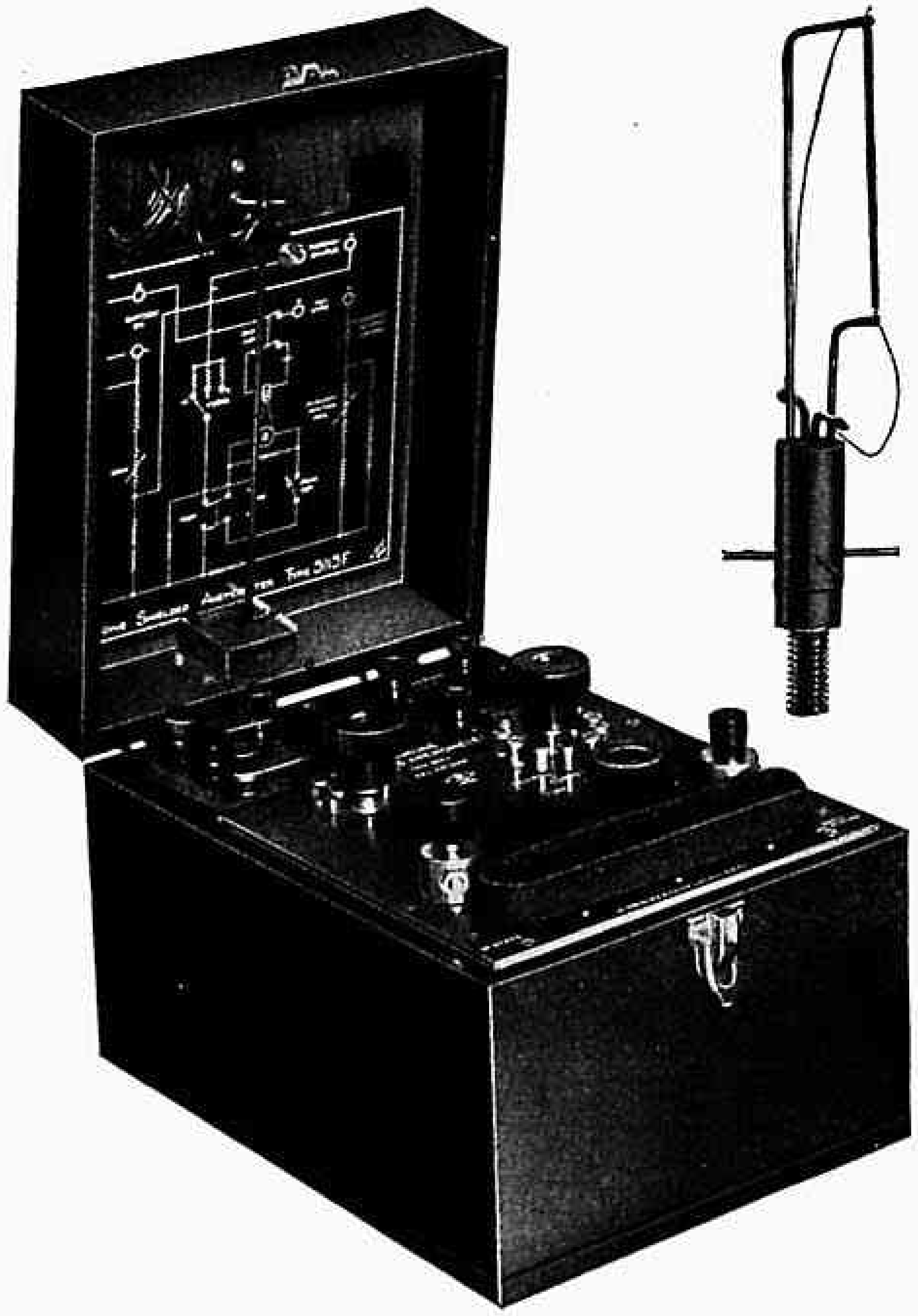
When current i flows in the wire the formula can be written

$$0{\cdot}24\, i^2r = (a + b\sqrt{V})(T - T_a)$$

The left-hand side represents the heat loss in calories per second; r is the resistance per unit length of wire: a and b do not remain constant over an appreciable range of temperature and each is expressable in the form $p + qT$ (Ref. (27)).

Later, Simmons described (Ref. (29)) a shielded form of hot-wire anemometer for low speeds, and an instrument of this type is now manufactured by H. Tinsley & Co.

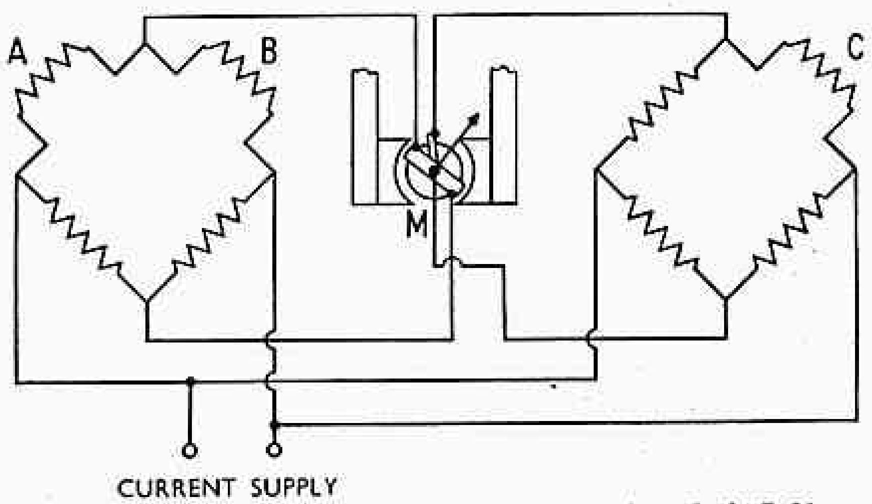
The hot-wire element, shown in Fig. 170, has a fine wire, about 3 cm long housed in one bore of a small twin-bore silica tube. The



(*H. Tinsley & Co., Ltd.*)
FIG. 170. SIMMONS HOT-WIRE ANEMOMETER

other bore contains a thermo-couple for the measurement of the hot-wire temperature. A standard heating current of 0·6 A is used and standardization is carried out by covering the element with a box and then adjusting the controlling resistors until the galvanometer used with the instrument gives a specified deflection. The standing e.m.f. of the thermo-couple is backed off by the zero setting control.

During the air-speed measurements the output from the thermocouple is read on the galvanometer scale. Calibration of the scale is necessary because the deflection/air-speed relationship is non-linear. The Tinsley instrument has 3 ranges, for 0 to 0·4, 0 to 2·0 and 0 to 5·0 feet per second. L. L. Fox, P. L. Palmer and D. Whittaker (Ref. (30)) describe a compensating circuit for this shielded hot-wire anemometer. This overcomes the difficulty that small changes in the wire-heating current cause large errors in air flow measurement. A small e.m.f. is injected from the heater-wire



(*Elliott Bros. (London), Ltd.*)
FIG. 171. ELECTRICAL HUMIDITY METER

circuit into the thermo-couple circuit so that the measured e.m.f. is largely independent of small current changes.

Electrical Humidity Meter. Yet another application of the Wheatstone bridge is the measurement of the humidity of the air by wet- and dry-bulb thermometers, the latter being of the electrical resistance type. Fig. 171 shows the circuit diagram of the Elliott Humidity meter. There are two, interconnected, Wheatstone bridges; one contains dry-bulb A and wet-bulb B thermometers and the other a second dry-bulb thermometer C. In the first, the galvanometer current is proportional to the temperature difference $T - T_1$ between the dry and wet bulbs while, in the second bridge, the galvanometer current is proportional to the dry-bulb temperature T. Instead of actual galvanometers two coils of a "cross-coil" indicator are used and the pointer reads the psychrometric difference $T - T_1$ corrected by the dry-bulb temperature T, i.e. the instrument reads the *relative humidity*.

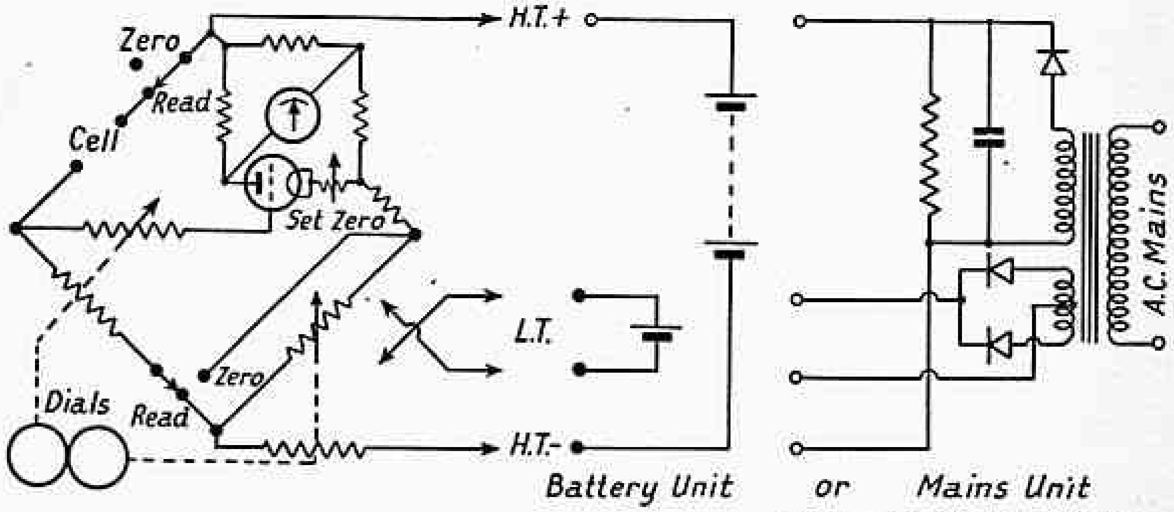
The three thermometers are mounted in a "humidity transmitter" through which the air under test is drawn, first over the two dry bulbs and then over the wet bulb.

The meter is independent of voltage variations in the supply since both bridges are supplied from the same source. A.C. mains can be used with a transformer and rectifier.

Moisture Meter. The principle of the Marconi moisture meter, designed for the measurement of the moisture content of grain and other hygroscopic materials, is shown in Fig. 172.

The circuit used (Brit. Patent No. 635, 674) is an extension of the Wheatstone bridge principle, there being a subsidiary bridge network in the arm opposite to that containing the sample under test. Out-of-balance voltage reduces or increases the resistance of the triode which, in turn, alters the effective resistance of the subsidiary bridge network. This self-balancing feature is included to render the reading of the meter independent of valve characteristics and supply voltage variations.

The sample is contained in a test cell in which a great mechanical



(*Marconi Instruments, Ltd.*)

FIG. 172. MARCONI MOISTURE METER

pressure is applied to the material to bring it to a uniform state and eliminate packing errors.

The range of moisture content covered corresponds to equilibrium with atmospheres having relative humidities from about 20 per cent to 90 per cent and the accuracy is $\pm$ 1 per cent m.c.

Marconi Instruments, Ltd. also make a moisture-in-timber meter though the construction and principle of operation are different from those of the meter just described.

Measurement of High Resistance. When the resistance to be measured is of the order of one or more megohms, the methods of measurement described in the foregoing pages are unsuitable. In such cases the resistance offered to the passage of current along the surface of the insulation is often comparable with the resistance itself, and special methods have to be adopted to take such "surface leakage" into account.

Amongst the high-resistance measurements which are required to be made in practice those of the insulation resistance of cables are very important. The absorption effects in dielectrics have already been mentioned in Chapter IV, and such effects are apt to destroy the value of insulation resistance measurements unless special precautions are taken.

A simple method of measuring insulation resistance is the direct deflection method. A very sensitive moving-coil galvanometer of high resistance (1,000 ohms or more) is connected in series with the resistance to be measured, and to a battery supply. The deflection of the galvanometer gives a measure of the insulation resistance. This method is, however, only sufficient to indicate whether the insulation is faulty or otherwise, and cannot be regarded as a precise method.

PRICE'S GUARD-WIRE METHOD. Fig. 173A gives the connections for a direct deflection method in which the errors due to surface

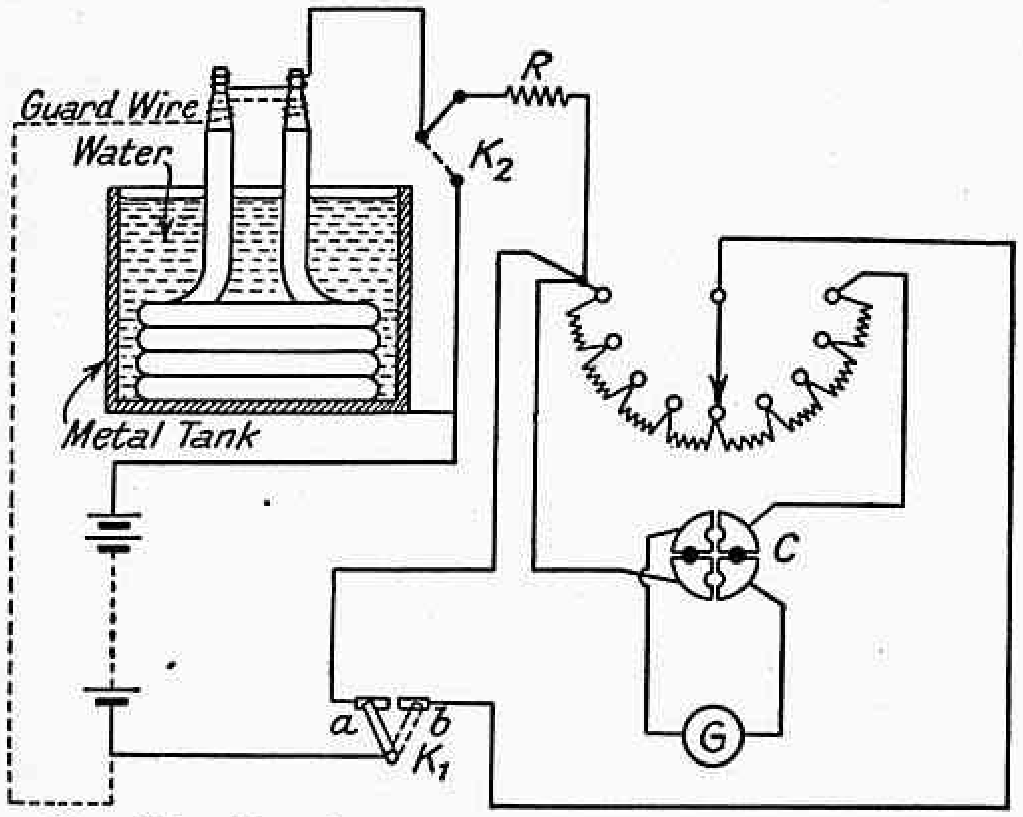


FIG. 173A. PRICE'S GUARD-WIRE METHOD OF MEASURING HIGH RESISTANCE

leakage are eliminated by the use of a guard wire. In the figure the resistance to be measured is the insulation resistance of a length of cable. The cable is immersed in water, which is contained in a metal tank, 24 hours being allowed to elapse—the temperature meanwhile being maintained constant—before the test is carried out. This enables the water to soak through any defects which may exist in the insulation, and also allows the insulation to attain the same temperature as the water. The ends of the cable are trimmed as shown, the outer protective covering being removed at these ends for a length of at least 12 in. A bare wire, twisted round the insulation near the end, is connected to the negative pole of the supply battery—the positive pole of which is connected to the metal tank—so that any current which leaks across the insulation surface is taken direct to the battery instead of passing through the galvanometer, and increasing its deflection. The galvanometer is shunted as shown, the shunt being of the Ayrton universal type.

The deflection of the galvanometer is observed, and its scale is afterwards calibrated by replacing the insulation resistance by a standard high resistance (usually 1 megohm), the galvanometer shunt being varied, as required, to give a deflection of the same order as before. The galvanometer, which is of the D'Arsonval type, should be very sensitive (at least 1,000 mm per microampere at a scale distance of 1 metre), should have high resistance, and, also, its deflection should be directly proportional to the current flowing through it. The resistance of the universal shunt across the galvanometer may be so chosen that the galvanometer is critically damped, thus saving time in making observations.

The battery should be of about 500 volts, and its e.m.f. should be constant. C is a four-part commutator for reversal of the galvanometer connections. R is a protective resistance of about 100,000 ohms in series with the galvanometer. K_1 is a key which is closed on contact "a" when the battery is first switched on. The galvanometer is thus protected from the sudden initial rush of current which charges the cable—the latter acting, of course, as a capacitor. Contact "b" is sufficiently close to "a" for the circuit to remain closed when the key is being moved over. K_2 is another key for the purpose of discharging the capacitance of the cable.

The galvanometer and its circuit, together with the keys, must be well insulated to prevent leakage currents.

Fig. 173B shows the connection diagram for insulation testing when a universal shunt on the Kelvin Varley slide principle (see p. 315) is used. The resistance of this (Sullivan) four-dial shunt is variable in steps of 1/10,000th part of the whole.

Loss of Charge Method. In this method the insulation resistance to be measured is connected in parallel with a capacitor and electrostatic voltmeter. The capacitor is charged, by means of a battery, to some suitable voltage, and is then allowed to discharge through the resistance, its terminal voltage being observed over a considerable period of time during discharge.

Then, assuming the capacitor to be perfect, if V is its terminal voltage at any time t, Q being the charge, in coulombs, still remaining in the capacitor, and C its capacitance, we have for the current i at time t,

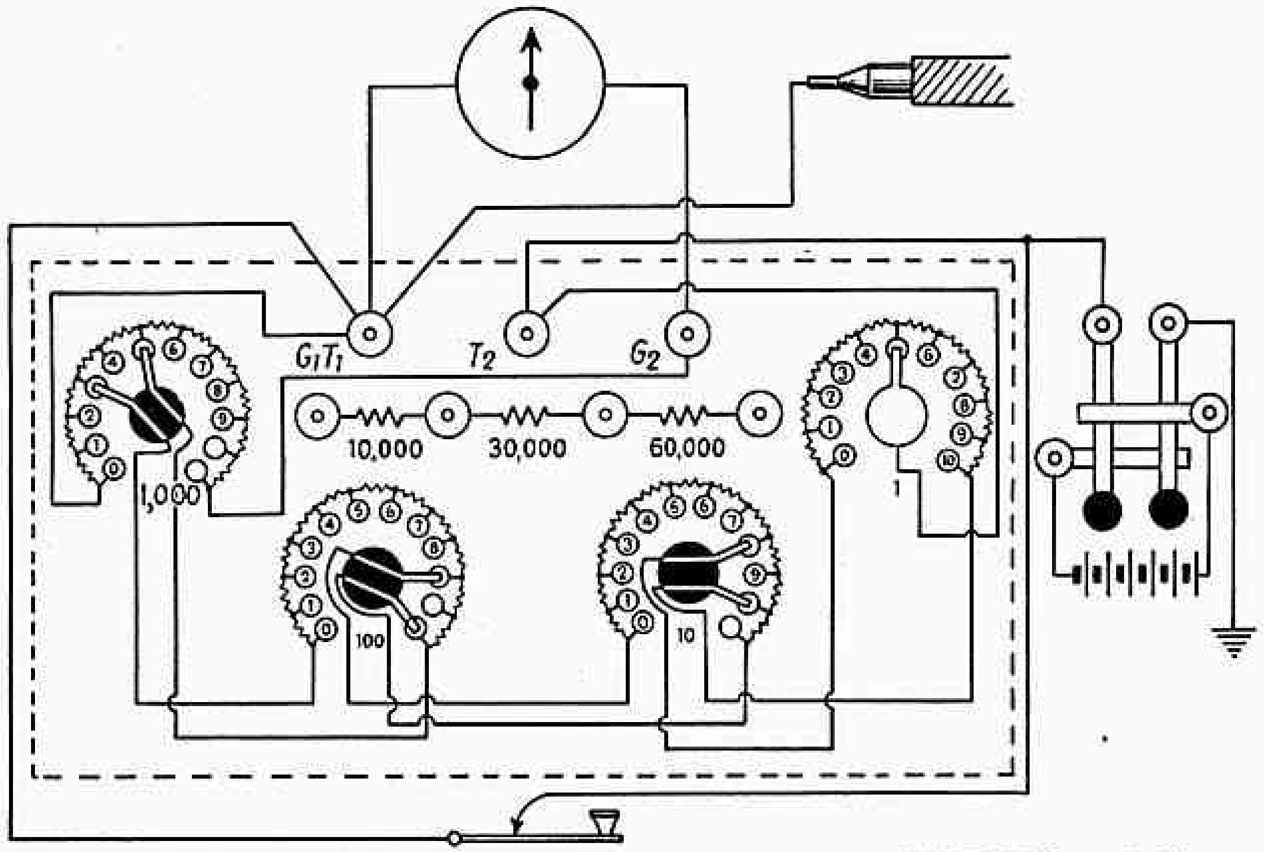
$$i = -\frac{dQ}{dt} = -C\frac{dV}{dt}$$

But $i = \frac{V}{R}$ where R is the resistance to be measured, and through which the capacitor is discharging.

$$\therefore \frac{V}{R} = -C\frac{dV}{dt}$$

or $$\frac{V}{R} + C\frac{dV}{dt} = 0$$

Solving this differential equation for V gives $V = E\varepsilon^{\frac{-t}{CR}}$ where E is the voltage when $t = 0$ (i.e. the voltage to which the capacitor was originally charged), and ε is the base of Napierian logarithms.



(H. W. Sullivan, Ltd.)

Fig. 173B. Four-dial Shunt for use in Insulation Testing

The box contains an independent and standard resistance of 100,000 ohms (between the inner line of terminals), which is used for taking "constants" in direct deflection tests, as also in the fall of potential method of localizing faults in a cable, where the potentials are measured by direct deflection, the galvanometer being earthed through this high resistance. In addition, the high resistance is specially subdivided into three sections, 10,000 ohms, 30,000 ohms, and 60,000 ohms, for conveniently observing earth current readings in Schaefer's test for locating breaks and faults in submarine cables.

Thus $$\log_\varepsilon V = \log_\varepsilon E - \frac{t}{CR}\log_\varepsilon \varepsilon = \log_\varepsilon E - \frac{t}{CR}$$

or $$R = \frac{t}{C\log_\varepsilon \frac{E}{V}} = \frac{0{\cdot}4343t}{C\log_{10}\frac{E}{V}} \quad . \quad . \quad . \quad (189)$$

R will be given in ohms if t is in seconds and C in farads.

Example. If $C = 0{\cdot}2$ microfarad, $E = 400$ volts, and the time taken for the capacitor terminal voltage to fall to 250 volts is $1\frac{1}{2}$ min.,

$$R = \frac{0{\cdot}4343 \times 90}{\frac{0{\cdot}2}{10^6} \times \log_{10}\frac{400}{250}} = 957{\cdot}6 \times 10^6 \text{ ohms or } 957{\cdot}6 \text{ megohms.}$$

If the resistance R is very large, the time for an appreciable fall in voltage is also large. The voltage-time curve will thus be very flat, and, unless great care is taken in measuring the voltages at the beginning and end of a given time t, a serious error may be made in the value of the ratio $\frac{E}{V}$, causing a corresponding error in the measured value of R. More accurate results may thus be obtained by measuring the *change* in the voltage $(E - V)$ directly. Calling this change v, the expression for R then becomes

$$R = \frac{0{\cdot}4343t}{C \log_{10} \frac{E}{E-v}}$$

Laws (Ref. (2)) gives details of a method of measuring this change in voltage v, using a ballistic galvanometer.

If the insulation resistance of a length of cable is being measured, the cable itself may be used as the capacitor, but its capacitance must be either known or must be determined.

Several serious difficulties are encountered when this method is used. Owing to absorption effects in the dielectric under test, the current actually flowing through the resistance is not simply dependent upon its resistance, since an absorption current also flows into the insulation. For this reason observations should be continued for a long period—several hours—if the results of such measurements are to be of value. Again, the insulation resistance of the voltmeter and of the capacitor must be exceedingly high in order that leakage effects shall be negligible.

Effect of the Time of Electrification. As absorption effects are present to a greater or less degree in all insulation resistance measurements, it is important that their influence upon the measured value of the resistance should be considered. In Fig. 81 a curve showing the variation of the absorption current in a capacitor with time was given. This current falls away fairly rapidly at first, the decrease thereafter being more gradual. Evershed (Ref. (5)) found, in the case of a length of rubber-covered cable, that the absorption current was some five or six times the true leakage current—dependent upon the resistance only—after a time of application of voltage of 1 min., and was about equal in value to the true leakage current after 7 min. After the voltage had been applied for 6 or 7 hours, the absorption current still formed some 5 to 10 per cent of the total current flowing through the insulation. From these results it is obvious that the insulation resistance as defined by

$$\frac{\text{Applied voltage}}{\text{Current flowing through the insulation}}$$

depends very much upon the time of application of the voltage. In commercial testing a time of application of 1 min. is normally specified, but the resistance value so obtained will be very much less than the true value. The magnitude of the insulation resistance itself will influence the effect which the absorption has upon the measured value. If this resistance is low, the absorption current may be negligible compared with the true leakage current, but the former will be of much greater importance in the case of very high resistance insulation.

Effect of Temperature upon Insulation Resistance. The resistance of insulating materials, generally, falls with increase of temperature, the change in resistance being, in some cases, very rapid. For this reason it is important in

such measurements that the result should be stated together with the temperature at which the test was carried out.

Koenigsberger and Reichenheim's formula (Ref. 6) for the variation of the resistance of hard insulating materials with temperature is as follows—

$$R_t = R_0 \varepsilon^{\frac{-kt}{(273+t)273}}$$

where R_t is the insulation resistance at temperature $t°$ C.

R_0 ,, ,, ,, ,, 0° C.

k is a constant for any given material.

ε is the base of Napierian logarithms.

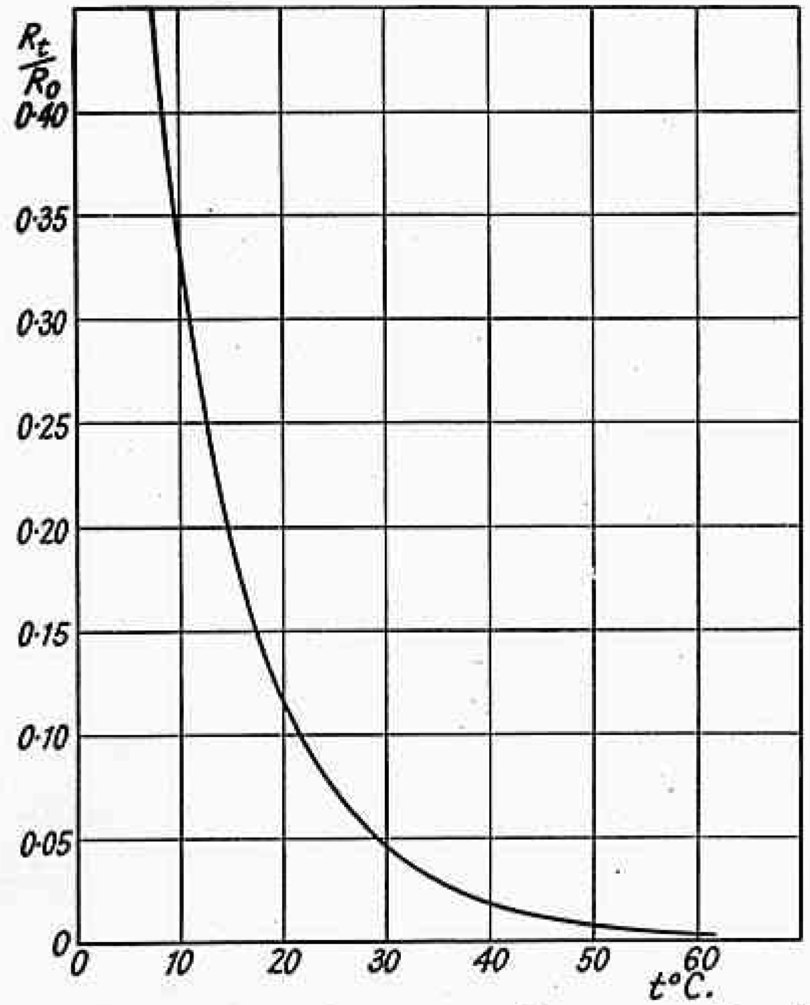


FIG. 174. RESISTANCE-TEMPERATURE CURVE FOR RED FIBRE

Fig. 174 shows the curve of R_t (expressed as a fraction of R_0) against temperature, for red fibre, for which k = 8,477. The very great reduction of resistance with increase of temperature can be observed. Curtis (Ref. (7)) gives a table of values of the resistivity at 30° C., and of the ratio $\frac{\text{resistivity at 20° C.}}{\text{resistivity at 30° C.}}$ for a number of hard insulating materials. The value of this ratio varies from 1 for India ruby mica to 16·0 for yellow beeswax.

Measurement of Resistance of Specimens of Insulating Materials. SURFACE AND VOLUME RESISTIVITY. In the above tests the resistance to be measured has been assumed to be that of the insulation of a length of cable. The direct deflection method is often applied to measurements of the resistance of insulating materials, small samples of the material, in sheet form, being used. In such cases it is necessary to distinguish between the "*surface resistivity*" and the "*volume resistivity*," or specific resistance of the material.

The "*surface resistivity*" is defined as the resistance between opposite edges of a unit square area of the surface of the material. This quantity depends upon the general condition of the surface and upon the humidity, and is not a constant.

The form of specimen used for such measurements is shown in Fig. 175. The sample rests in a pool of mercury to which the negative

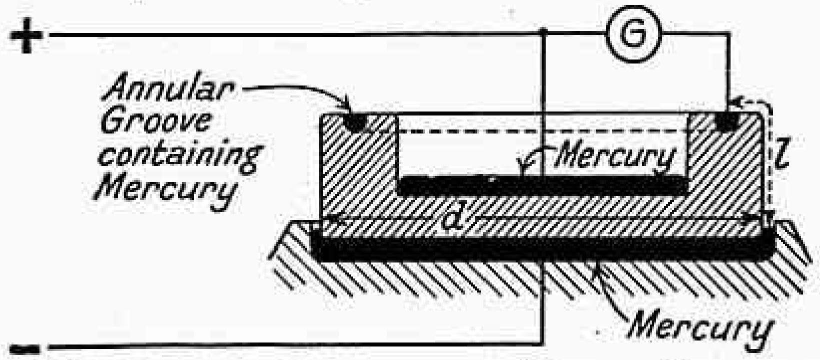


FIG. 175. FORM OF SPECIMEN FOR SURFACE RESISTIVITY MEASUREMENTS

pole of the battery is connected, an annular groove containing mercury forming the other electrode. There is a pool of mercury also inside the specimen, and a wire from this is taken to the positive side of the battery as shown, being connected so as to form a guard wire to prevent the leakage current which actually passes through the body of the specimen from passing through the galvanometer.

Then, if R is the measured value of the resistance, the surface resistivity is given by $\frac{R \times \pi d}{l}$ where d is the diameter of the specimen and l is the length of the surface path from the annular groove to the outer mercury pool (see figure).

This arrangement and shape of specimen are recommended in the British Electrical and Allied Industries Research Association Report (Ref. (11)). A number of such reports, dealing with the measurement of volume and surface resistivity of insulating materials and the effect of temperature humidity, etc., upon these quantities, have been published (Refs. (8) to (13)). These reports give very full directions for the study of many different forms of insulating materials—vulcanized fibre, hard composite dielectrics, unvarnished textile fabrics, insulating oils, papers, etc., and also lay down the conditions under which the various tests should be made. Recommendations for testing procedures are also given in British Standard specifications dealing with various classes of insulating materials.

Portable Resistance Testing Sets. A portable, and reasonably accurate, form of testing set is often necessary in order that insulation tests may be made on cables and wiring systems after

installation. A number of such sets have been developed and are manufactured by various instrument makers. Most of these sets are modifications of the ohm-meter originally designed by Ayrton and Perry. The principle of this instrument is illustrated by Fig. 176.

Two coils, C and P, are fixed at right angles to one another, and so that their magnetic fields—when current flows through them—both exert a turning moment upon the pivoted magnetic needle M, to which a pointer is attached. SS are the supply terminals, the supply usually being obtained from a small generator, giving about 500 volts, which is turned by hand. P is the pressure coil and is

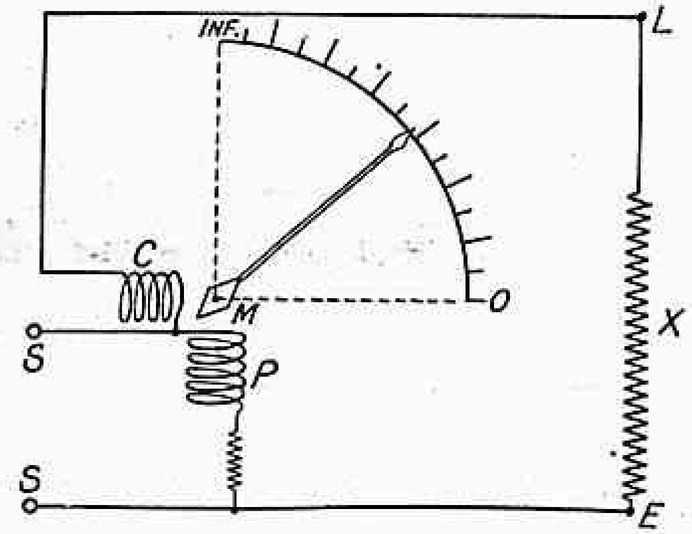


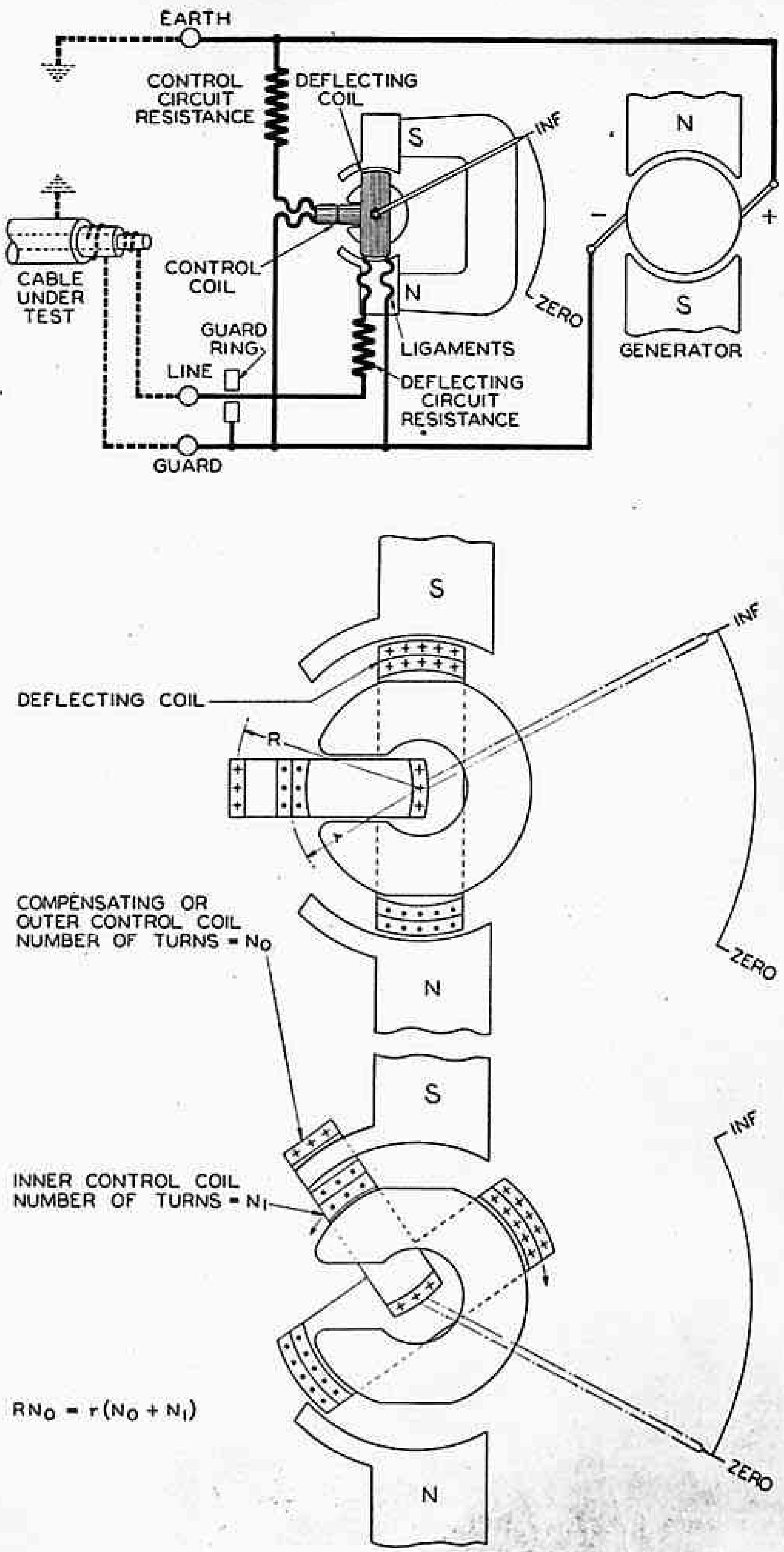
FIG. 176. CONNECTIONS OF SIMPLE AYRTON AND PERRY OHMMETER

connected, in series with a resistance, across the supply terminals. C is the current coil, connected in series with the resistance X, to be measured. This coil carries a current which is inversely proportional to the resistance X.

The current in coil P, which is directly proportional to the supply voltage, is fixed, and is independent of the resistance to be measured. The magnetic field of this coil tends to turn the needle in an anti-clockwise direction, while the field of coil C tends to cause clockwise rotation.

The balance position of the needle is such that these two turning moments are equal. If X is infinite, there is no current in C, and the needle sets along the axis of coil P, whereas if the resistance X is very low, the turning moment of C is far greater than that of P, and the needle sets along the axis of C. The scale is graduated in resistance values (usually megohms), the intermediate points between infinity and zero being obtained by calibration.

The commonest of the more modern testing sets is the "Megger" insulation tester, manufactured by Messrs. Evershed and Vignoles, the construction and connections of which are shown in Fig. 177. The moving system consists of two coils—the "control coil" and the "deflecting coil"—rigidly mounted at an angle to one another and connected, in parallel across a small generator, with polarities such



(*Evershed & Vignoles, Ltd.*)

FIG. 177. INTERNAL CONNECTIONS OF THE MEGGER

that the torques produced by them are in opposition. The coils move in the air gap of a permanent magnet. The control coil is in series with a fixed control circuit; the deflecting coil is connected in series with a fixed deflecting circuit resistance and the resistance under test. If this last is infinitely high no current flows in the deflecting coil and the control coil sets itself perpendicular to the magnetic axis, the pointer indicating "Infinity." A lower test resistance allows current to flow in the deflecting coil and turns the movement clockwise. The control torque produces a restoring torque which progressively increases with the angular deflection, and the equilibrium position of the movement is attained when the two opposing torques balance.

The control coil is actually in two parts, in series, the outer part being a compensating coil. The two parts are arranged with numbers of turns and radii of action such that, for external magnetic fields of uniform intensity, their torques cancel one another thus giving an astatic combination.

The instrument has a small permanent magnet d.c. generator (developing 100, 250, 500, 1,000 or 2,500 V in different instruments). This may be hand-driven, through gearing and a centrifugally controlled clutch which slips at a predetermined speed so that a steady voltage can be obtained, or it may be motor-driven.

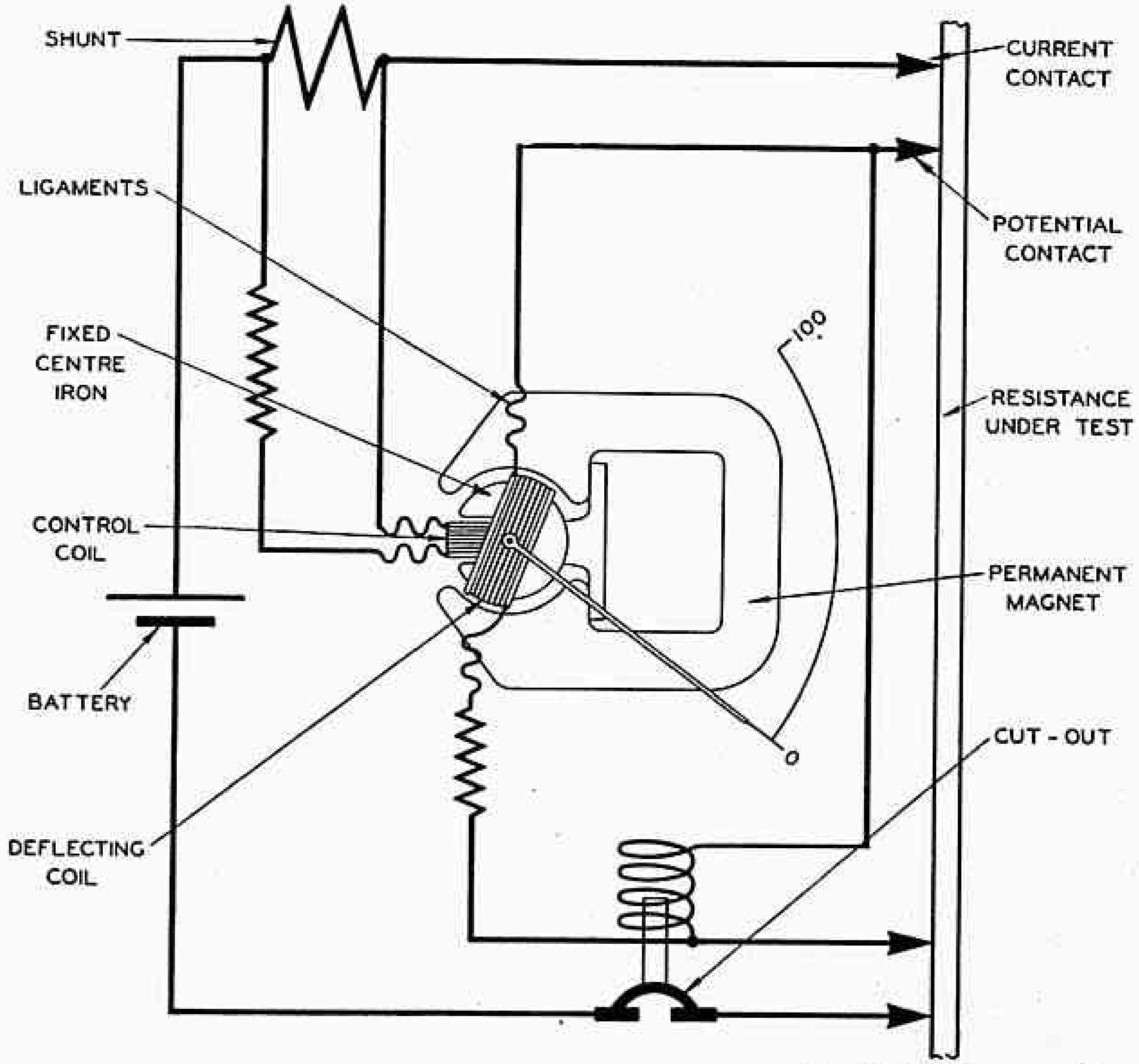
The "Bridge-Meg" and "Bridge-Megger" testing sets, also made by Evershed and Vignoles, Ltd., can be used for insulation resistance measurements, as a Wheatstone bridge for a wide range of resistance values or for fault localization by the Varley loop method (see p. 491). Variable ratio arms and a four-decade resistance are included for these uses.

Two other instruments made by the same manufacturers should be mentioned. The first is the "Ducter" ohmmeter, a low-resistance testing set for resistances from a few ohms down to one microhm, and the other is the "Megger Capacity Meter."

In the former, the connections for which are shown in Fig. 178, the control coil is connected across a shunt in the main circuit, so that the torque produced by it is proportional to the current in the resistance under test, while the deflecting coil is connected across this resistance and so carries a current proportional to the potential drop. The position taken up by the moving system is thus dependent on the ratio of potential drop and current, i.e. on the resistance being measured.

The capacity meter resembles the Megger Insulation Tester in general form but contains a hand-driven alternating current generator and an a.c. ratiometer movement calibrated in microfarads. It compares the capacitance under test with a standard capacitance and covers a range from 0 to 0·1 μF up to 0 to 10 μF.

Measurement of Insulation Resistance when the Power is On. It may be necessary, in some cases, to measure the insulation resistance to earth of a distribution system, while the power is on. Such a measurement may be made as follows. The voltage E between the two mains—positive and negative—is measured, together with the voltage V_1 from the positive main to earth, and the voltage V_2 from the negative main to earth. These measurements are made with a



(*Evershed & Vignoles, Ltd.*)
FIG. 178. EVERSHED'S "DUCTER" OHMMETER

high resistance voltmeter whose resistance R_v should be comparable with the insulation resistances to be measured.

Let R_1 = resistance between + ve main and earth

R_2 = " " − ve " "

Fig. 179 (*a*) shows, diagrammatically, the system when the voltmeter is connected between the positive main and earth, and Fig. 179 (*b*) the system with the voltmeter between the negative main and earth.

If I_1 is the current flowing from the positive main to the negative

main, through R_2 and R_1—the latter being in parallel with R_v—in the first case we have

$$V_1 = \frac{R_1 R_v}{R_1 + R_v} \cdot I_1$$

and $$E - V_1 = R_2 I_1$$

Thus, $$\frac{E - V_1}{V_1} = \frac{R_2}{\dfrac{R_1 R_v}{R_1 + R_v}} \text{ or } \frac{E}{V_1} = \frac{R_1 R_2 + R_v(R_1 + R_2)}{R_1 R_v}$$

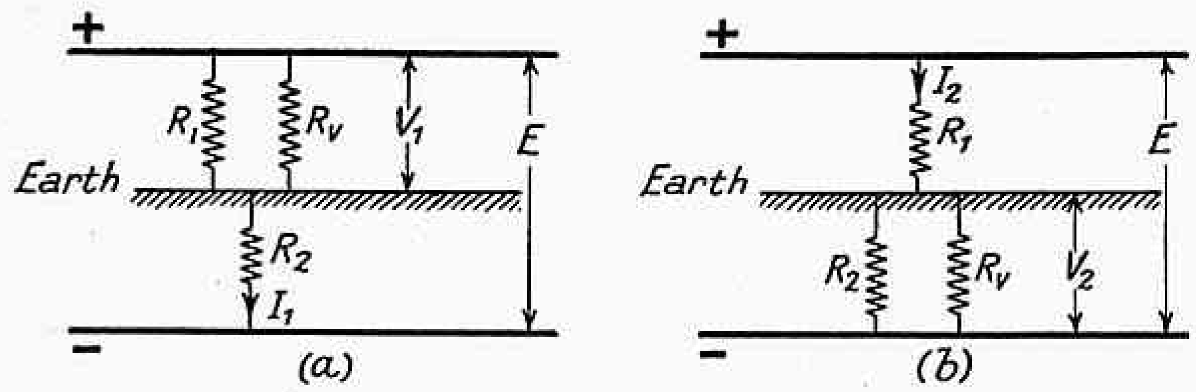


FIG. 179. MEASUREMENT OF INSULATION RESISTANCE WHEN THE POWER IS ON

By similar reasoning, in the second case we have

$$\frac{E}{V_2} = \frac{R_1 R_2 + R_v(R_1 + R_2)}{R_2 R_v}$$

Hence, $$\frac{\dfrac{E}{V_1}}{\dfrac{E}{V_2}} = \frac{\dfrac{R_1 R_2 + R_v(R_1 + R_2)}{R_1 R_v}}{\dfrac{R_1 R_2 + R_v(R_1 + R_2)}{R_2 R_v}} = \frac{R_2}{R_1} = \frac{V_2}{V_1}$$

Substituting $R_2 = R_1 \cdot \dfrac{V_2}{V_1}$ in the expression

$$\frac{E}{V_1} = \frac{R_1 R_2 + R_v(R_1 + R_2)}{R_1 R_v}$$

we have $$\frac{E}{V_1} = \frac{R_1 \dfrac{V_2}{V_1}(R_1 + R_v) + R_1 R_v}{R_1 R_v} = \left(\frac{R_1 + R_v}{R_v}\right)\frac{V_2}{V_1} + 1$$

from which $$R_1 = \left[\frac{E - (V_1 + V_2)}{V_2}\right] R_v \quad . \quad . \quad . \quad (190)$$

Similarly, $$R_2 = \left[\frac{E - (V_1 + V_2)}{V_1}\right] R_v \quad . \quad . \quad . \quad (191)$$

This method cannot be used if one of the mains is earthed, and is generally only applicable if the insulation resistances to be measured are not more than 1 or 2 megohms.

The Measurement of the Resistance of Earth Connections. The resistance between an earthing plate and the surrounding ground is often an important quantity in distribution systems. It is usually measured by a fall-of-potential method as illustrated by Fig. 180.

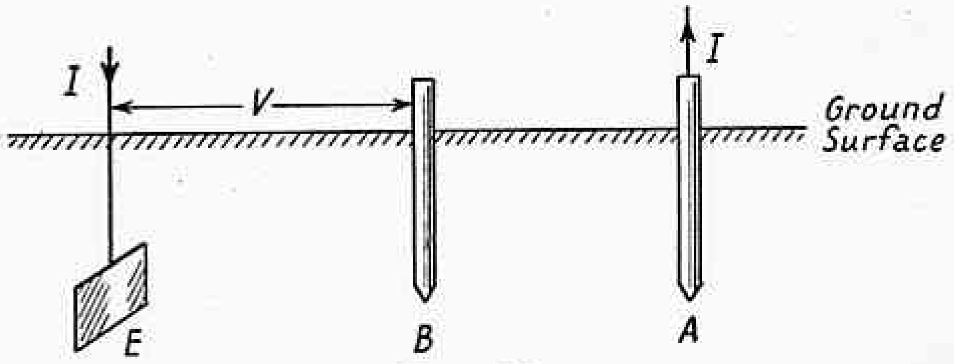


FIG. 180

A current is passed through the plate E to an auxiliary electrode A in the earth at a distance away from the plate. A second auxiliary electrode B is inserted between E and A and the potential difference V, between E and B is measured for a given current I so that the

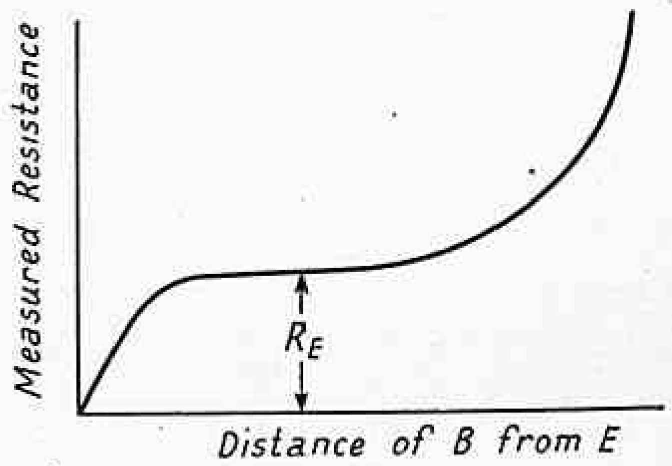


FIG. 181

resistance of the earth connection is $\frac{V}{I}$. The placing of the auxiliary electrodes is, however, important and G. F. Tagg (Ref. (31)) has called attention to the errors which can arise from incorrect placing of these electrodes, especially when the earth resistance is low. He gives a curve, as in Fig. 181, for the resistance measured when B is at various distances from E. The correct value for the resistance of the earth connection is that measured (R_E) when B is at such a distance that the resistance lies on the horizontal part of the curve. Tagg points out that, when the earthing resistance is low the spacing

between the earth plate and auxiliary electrodes may need to be hundreds of feet.

Following these principles, Messrs. Evershed and Vignoles, Ltd. make a "Megger Earth Tester" containing a direct-reading ohm-meter and a hand-driven generator.

Measurement of Resistance of Electrolytes. Owing to the fact that a polarization e.m.f. is produced whenever a current passes

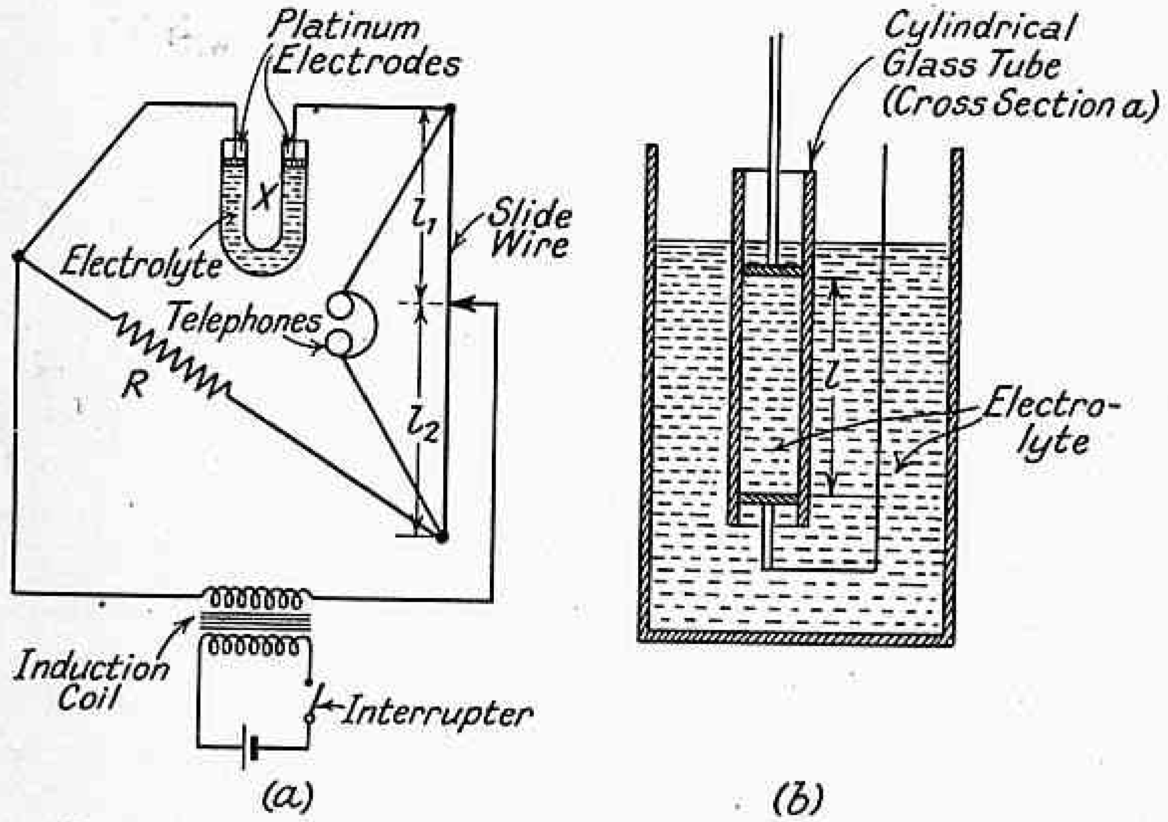


FIG. 182. MEASUREMENT OF THE RESISTANCE OF ELECTROLYTES

through an electrolyte, the usual methods of measuring resistance cannot be used to measure the resistance of electrolytes.

Kohlrausch devised a method of measuring their resistivity. The electrolyte is contained in a glass tube having two platinum electrodes dipping into it. This cell is connected in one arm of a Wheatstone bridge network as shown in Fig. 182 (*a*). The bridge is of the slide-wire form, and is supplied from an induction coil, a telephone being used as the detector. The slide-wire may be of special form consisting of a long manganin wire wound spirally in a groove cut in a marble cylinder, the cylinder being stationary and the contact —of hard steel mounted in a manganin rod to avoid thermo-electric e.m.f.'s—sliding round the cylinder. The spindle carrying the contact has a thread of the same pitch as that of the groove in which the slide-wire lies.

R is a known resistance of the same order as that of the electrolyte. Balance is obtained by adjusting the sliding contact until no sound can be detected in the telephone.

Then, if l_1 and l_2 are the two lengths into which the slide-wire is divided by the sliding contact, the electrolyte resistance X is given by

$$X = \frac{l_1}{l_2} R \qquad . \quad . \quad . \quad . \quad . \quad (192)$$

If the resistivity of the electrolyte is to be measured it is best to use a cylindrical glass tube, of uniform cross-section, supported vertically in a vessel containing the electrolyte, and with its upper end above the surface of the liquid (Fig. 182 (*b*)). The electrodes should be of platinum and should be circular, fitting tightly inside the cylinder. The lower electrode may be pierced to allow liquid to flow through it, and the glass tube may be graduated so that the length of the column of electrolyte between the two electrodes can be accurately determined. Then, if a is the cross-sectional area of the column of electrolyte and l is its length, the resistivity is given by $\frac{Xa}{l}$ where X is the measured resistance of the column.

The temperature should be carefully observed when making such measurements, and this temperature stated when the results are given.

Measurement of Water Purity. Closely connected with the measurement of the resistance of electrolytes is that of the purity of water in terms of its electrical conductivity.

The Evershed and Vignoles "Dionic" Water Purity Meters (or Electric Salinometers) are calibrated in conductivity units taking as unity water having a resistance, at 20° C., of 1 megohm between the opposite faces of a centimetre cube. Then, water having a resistance of $\frac{1}{2}$ megohm per cm.^{3} has a conductivity of 2 and so on. These conductivity measures can be interpreted in terms of water purity from known data concerning the influence of various concentrations of salt upon conductivity.

For example, at 20° C

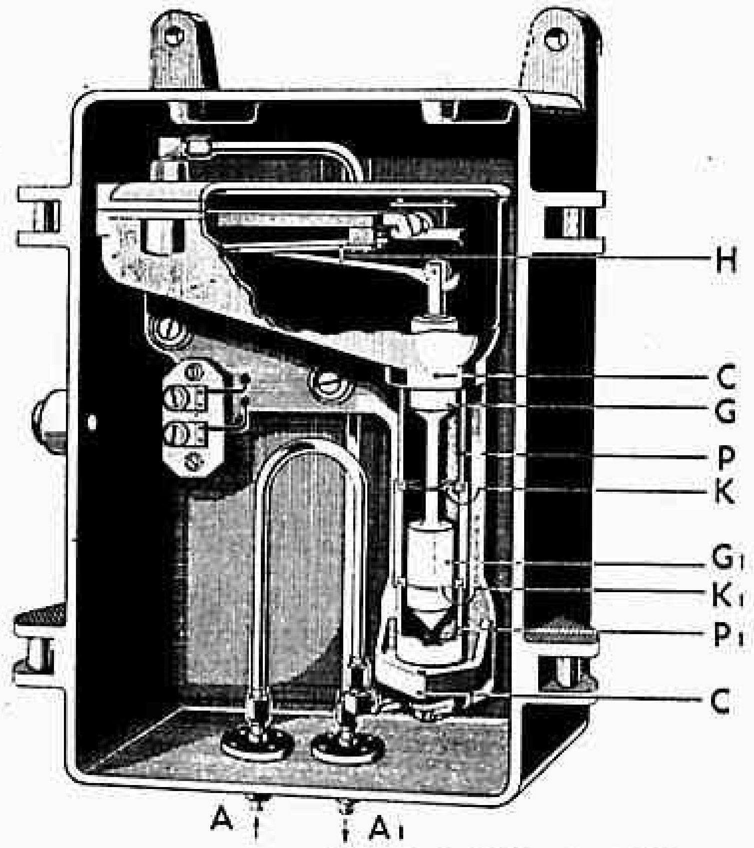
0·823 grain of common salt per gallon gives conductivity	23 units
5 grains per gallon give conductivity	139 units
7 grains per gallon give conductivity	192 units

The apparatus consists of a water conductivity tube (see Fig. 183) which is used in conjunction with an indicating (or recording) ohmmeter having a moving system of two rigidly fixed and magnetically opposing coils moving in a permanent magnet field as in the "Megger Insulation Tester" (see p. 305). The apparatus is, however, operated from a d.c. supply.

Allowance is made in the calibration for the back e.m.f. and there is an automatic device to compensate for changes in temperature

over the range 60° to 160° F. This device consists of plungers, G and G_1, which are lowered into the water tubes, P and P_1, by the bimetal levers H. This varies the effective cross sections of the water columns under test.

These water columns are contained in two insulating tubes, P and P_1, which are separated by a third insulating tube containing two platinum electrodes K and K_1. The testing current flows upwards through P and downwards through P_1 and the columns of water are of accurately known cross section. Electrically the



(*Evershed & Vignoles, Ltd.*)

FIG. 183. "DIONIC" WATER PURITY METER

columns are in parallel because the electrodes K, K_1 are at the same positive potential, the negative electrode being the gunmetal case C, C which carries the insulating tubes. (This case is of course insulated from the case of the instrument itself and from the water supply.) The water flows through the tube continuously, entering at A and discharging at A_1.

Apparatus Used in Resistance Measurements. Resistance standards and resistance boxes have already been discussed and their construction described. Several other pieces of apparatus, used in connection with resistance measurements, merit description.

AYRTON UNIVERSAL SHUNT. This is an important accessory in galvanometer work. Fig. 184 (*a*) shows the connections of such a shunt in diagrammatic form.

The galvanometer G, of resistance R_G, is connected across the outer terminals ac of the shunt, whose total resistance is S; b is a moving contact and x the resistance between points ab and depends upon the position of b.

Let I be the current flowing in the main circuit (i.e. into the parallel combination of G and S).

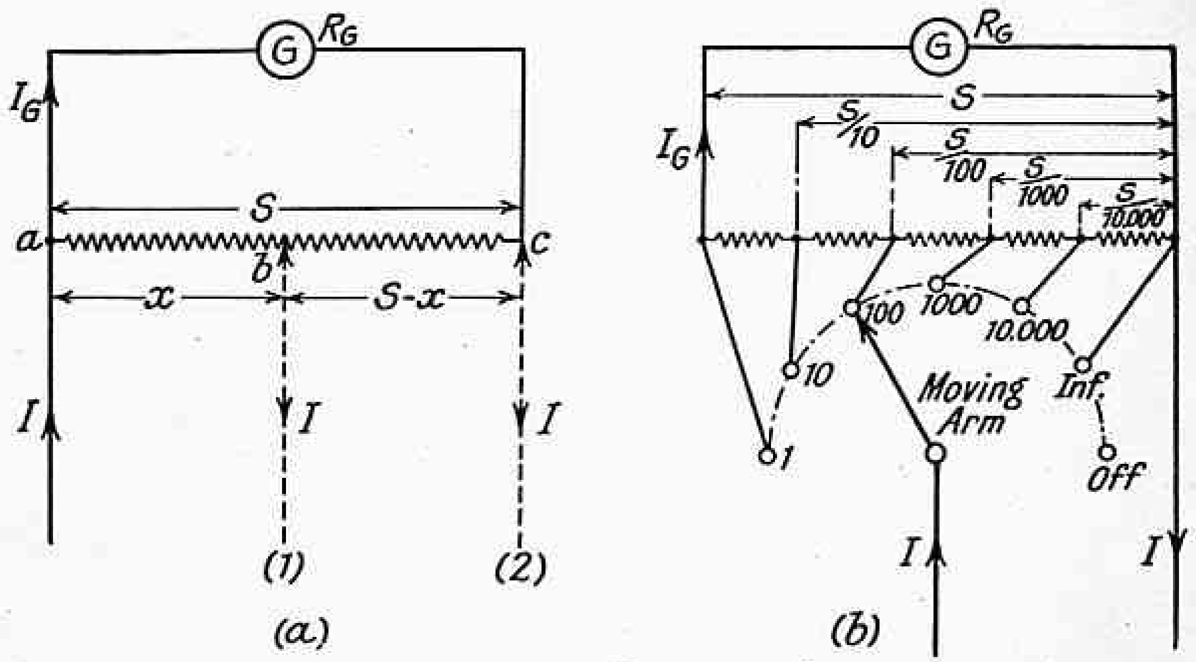


FIG. 184. AYRTON UNIVERSAL SHUNT

Then, with b in position (1) the galvanometer current

$$I_G = \frac{x}{x + S - x + R_G} \cdot I = \frac{x}{S + R_G} \cdot I$$

Again, with b in position (2)—when $x = S$—the galvanometer current is

$$I_G' = \frac{S}{S + R_G} \cdot I$$

Thus, in moving b from position (2) to position (1), the galvanometer current is reduced in the ratio $\frac{x}{S}$, this ratio being, therefore, independent of the resistance of the galvanometer.

The "multiplying power" of the shunt—i.e. the ratio

$$\frac{I}{\text{Galvanometer current}}$$

—for any given position, is $\dfrac{S + R_G}{x}$

$\frac{I_g}{I_g'} = \frac{x}{S}$

By arranging the contact b to be moved in steps (by means of a moving arm and studs connected to tapping points on the shunt S), the galvanometer current for any position of b may be made a definite fraction of the current which flows through the galvanometer when b is on the point c. Thus, if b is in such a position that $x = \frac{1}{10} \cdot S$, the galvanometer current is $\frac{1}{10}$th of the current flowing when $x = S$.

These shunts are often made up so that the ratios of $\frac{x}{S}$ obtainable are $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc. (see Fig. 184 (b)).

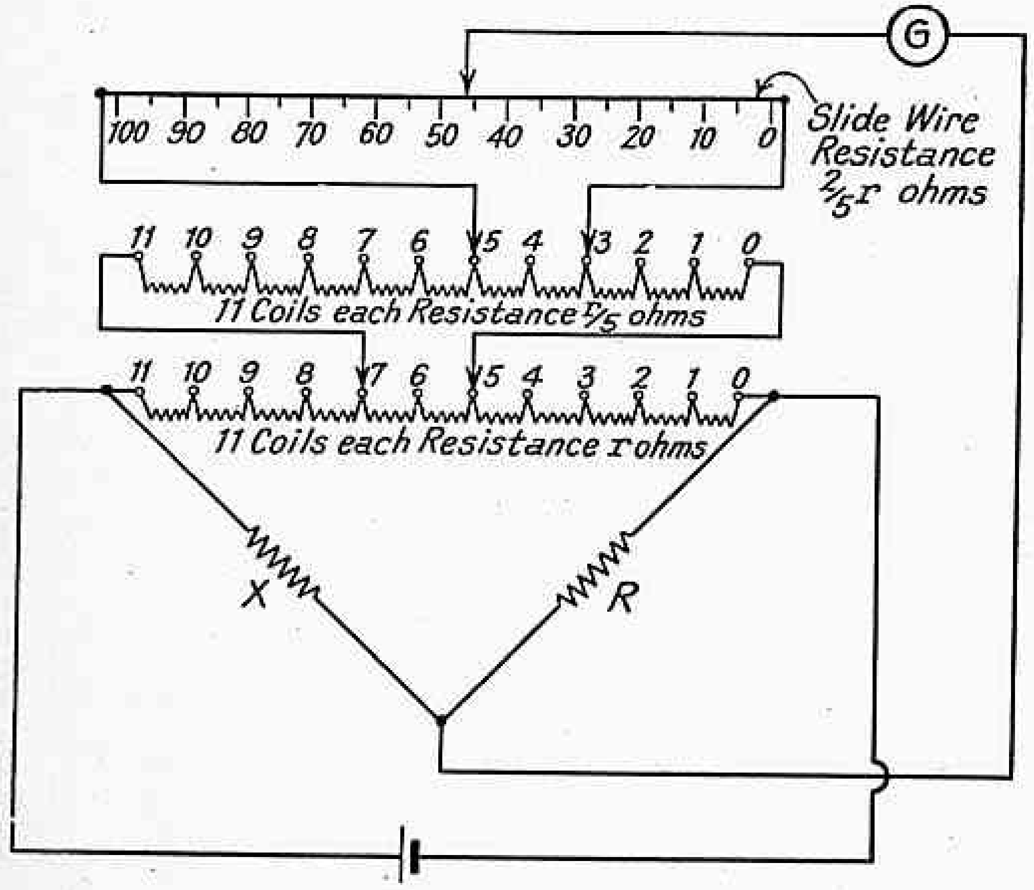
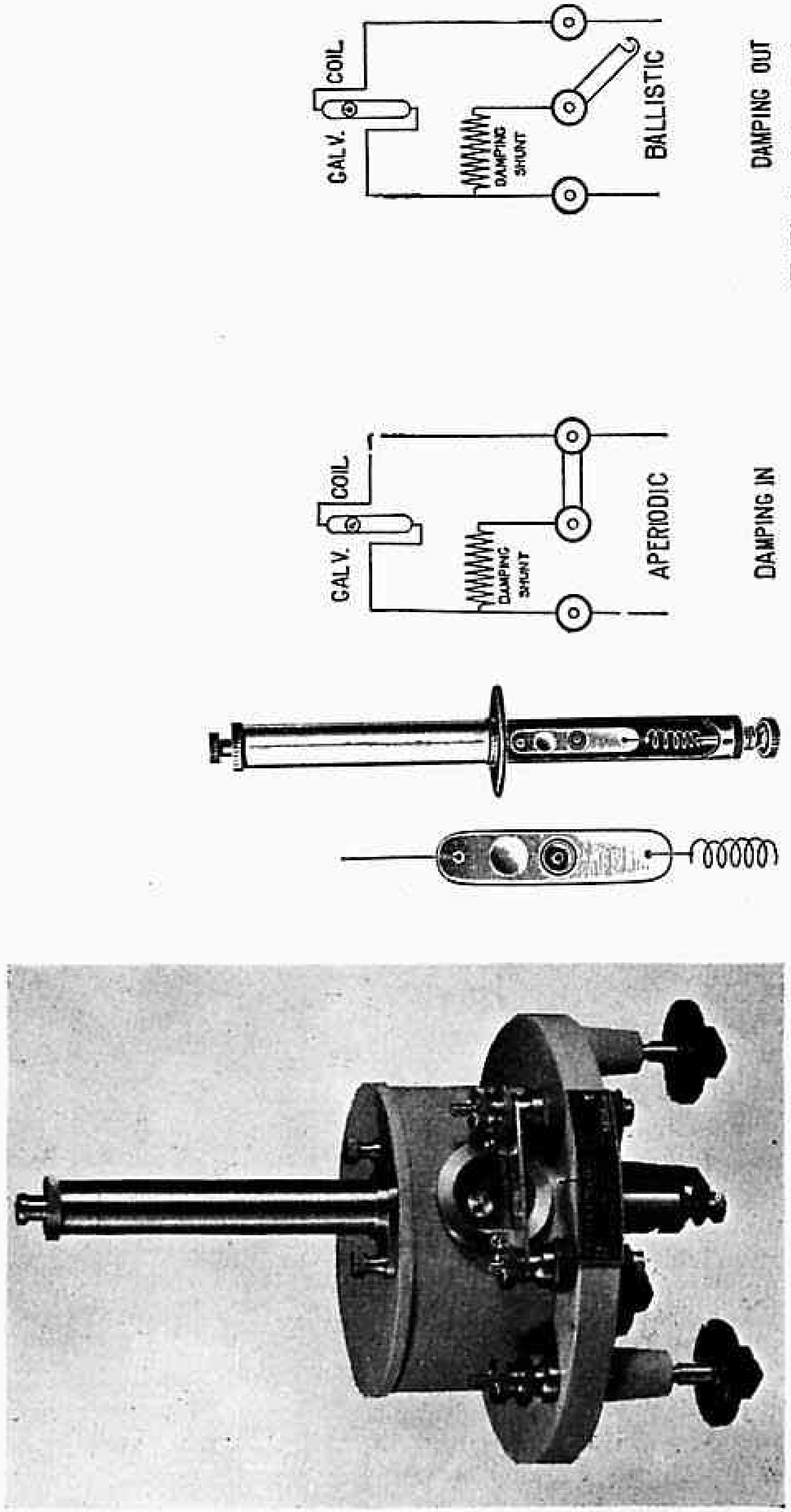


FIG. 185. KELVIN AND VARLEY SLIDE

It should be noted that the resistance across the galvanometer terminals is S, no matter what the position of the contact b. The resistance S of the shunt should be chosen about 10 times that of the galvanometer, so that the latter may not be over-damped, and that its sensibility may not be appreciably reduced by being thus shunted.

KELVIN AND VARLEY SLIDE. This device may be used to replace a simple slide-wire in a Wheatstone bridge network. The principle is used, also, in the construction of potentiometers and universal shunts. It consists of a slide-wire and a number of resistance coils connected as shown in Fig. 185, where the apparatus is inserted in a Wheatstone bridge network.

The lower row of coils consists of eleven coils, each of resistance



(H. Tinsley & Co., Ltd.)

FIG. 186. D'ARSONVAL GALVANOMETER

r ohms. Shunting two of the coils is another row of eleven coils, each of resistance $\frac{r}{5}$ ohms, the shunting connections being made through two sliding contacts which always move together, so that two coils in the bottom row are shunted by the row above, throughout. Again, two of the coils in this second row are shunted in the same way by a slide-wire whose resistance is equal to that of the two coils which it shunts, namely $\frac{2}{5}r$. Obviously, the number of rows of coils can be increased as far as is justifiable, taking into consideration contact resistance errors, etc.

Since in each case the two coils are shunted by a resistance equal to their own, the resistance of the combination is the same as the resistance of one coil, so that the total effective resistance of each of the rows of coils is in each case that of ten coils only.

The reading of the slide in the figure is 5,346. Thus $\frac{R}{X} = \frac{5{,}346}{4{,}654}$

THE D'ARSONVAL GALVANOMETER. This instrument, which is largely used in the various methods of measurement of resistance, and also for potentiometer work, consists essentially of a circular or rectangular coil of many turns of fine wire suspended between the poles of a permanent magnet. There is often a fixed cylindrical iron core inside the coil, the coil wires being situated in the two air gaps between this core and the permanent magnet. The length of the air gaps between the coil and pole faces, and between the coil and core, is usually about $\frac{1}{16}$ in., and the pole faces are shaped so as to give a radial field. The suspension is a single fine strip of phosphor-bronze, and serves as one lead to the coil, the other lead taking the form of a loosely coiled spiral of fine wire leading downwards from the bottom of the coil. The suspension carries a small mirror upon which a beam of light is cast through a glass window in the outer brass case surrounding the instrument. The beam of light is reflected on to a scale—usually at a distance of 1 metre from the mirror—upon which the deflection is measured. A torsion head is provided for adjustment of the coil position and zero setting.

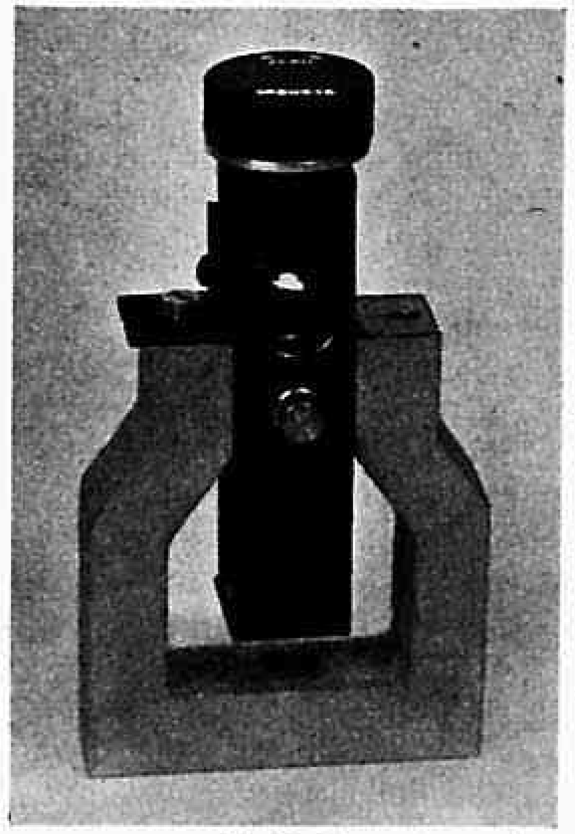
In order to save time in using the galvanometer, damping is provided by winding the coil on a light metal former. The damping is produced by the torque—opposing motion—due to the permanent magnet field in conjunction with currents which are induced in the metal former when it rotates in this magnetic field.

Damping may also be obtained by connecting a fairly low resistance across the galvanometer terminals. The damping being then dependent upon the magnitude of this resistance, it is possible, by suitably adjusting this resistance, to make the damping critical.

Fig. 186 shows a D'Arsonval galvanometer with a freely-suspended coil. The sensitivities and periodic times for three resistances are given below. This type of instrument is very suitable for ballistic measurements (see Chapter IX).

Resistance (Ohms)	Sensitivity at 1 metre		Period Time (sec.)
	mm/microamp	mm/microcoulomb	
500	250	310	5
1,000	1,100	860	8
4,000	3,500	2,000	11

Fig. 187 shows a portable galvanometer of high sensitivity (up to 1,500 mm/microamp at 1 metre) with a moving coil which is tautly

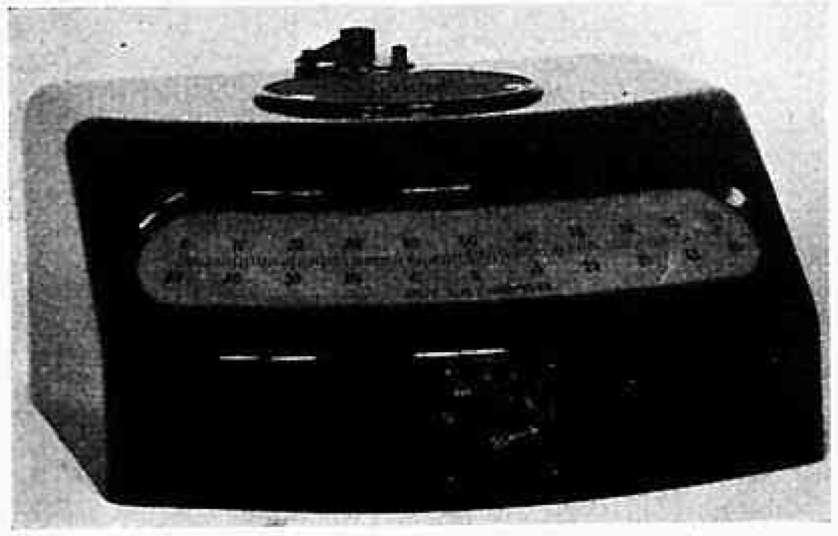


(*H. Tinsley & Co., Ltd.*)
FIG. 187. SUSPENSION TYPE GALVANOMETER

suspended at both ends by silver-gilt suspension strip. This form of suspension is independent of level.

The Cambridge Instrument Co. make a very robust form of galvanometer which requires no levelling or clamping. This is the Cambridge "Pot" Galvanometer. It is fitted with a pointer as well as a mirror for use with lamp and scale and gives a deflection per micro-ampere of 12 millimetres at a scale distance of 1 metre. Its resistance is 50 ohms and its period 1·3 seconds.

K. Copeland, A. C. Downing and A. V. Hill (Ref. (25)) describe a moving-coil galvanometer of extreme sensitivity. Used with photelectric amplification, the instrument is capable, in a few seconds, of reading to a few thousandths of a microvolt in a 50-Ω circuit. It is stable enough to allow 20-fold to 50-fold magnification under laboratory conditions and the magnified deflection is read on a microammeter. Two of these authors have also described (*Jour.*



(*Cambridge Instrument Co., Ltd.*)
FIG. 188. SPOT GALVANOMETER

Sci. Insts., Vol. 25, pp. 225 and 230) a rapid galvanometer which can record to within 0·02 μV in 0·01 sec.

H. W. Sullivan, Ltd. have introduced a very compact, sensitive, yet robustly suspended galvanometer fitted with shock-absorbing stops which permit overloads of up to 100 times full-scale current without risk of damage to the movement. Sensitivities up to 2,300 mm/μA can be obtained with a periodic time not greater than 2 seconds.

Theory. Let i be the current (assumed constant) flowing through the galvanometer coil. Then the equation of motion of the galvanometer is, from Equation (174) (Chapter VI),

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = Gi$$

where θ = the deflection in radians
t = time in seconds
a = moment of inertia of the moving system
b = the damping constant
c = the restoring constant
G = the displacement constant

(see theory of the vibration galvanometer, Chapter VI).

Referring to Equations (176) and (179) (*loc. cit.*), we have for the solution of the above equation

$$\theta = A\varepsilon^{m_1 t} + B\varepsilon^{m_2 t} + \frac{Gi}{c} \quad . \quad . \quad . \quad . \quad (193)$$

since the current is now constant and equal to i, ω in Equation (179) being zero. A and B are constants to be determined from the initial conditions.

Let θ_D be the final steady deflection of the galvanometer. Then $\theta_D = \frac{Gi}{c}$ the expression $A\varepsilon^{m_1t} + B\varepsilon^{m_1t}$ representing a motion which may, or may not, be oscillatory, according to the relative values of the constants a, b, and c.

To determine A and B, suppose that when t is zero, the deflection θ is zero and also $\frac{d\theta}{dt} = 0$ (i.e. the galvanometer moving system is stationary in its zero position).

Then, since when $t = 0$, $\theta = 0$,

$$0 = A + B + \frac{Gi}{c} = A + B + \theta_D$$

Also, since when $t = 0$, $\frac{d\theta}{dt} = 0$

$$0 = Am_1 + Bm_2$$

Hence, $$A = \frac{m_2\theta_D}{m_1 - m_2} \text{ and } B = \frac{-m_1\theta_D}{m_1 - m_2}$$

Hence, the equation for θ becomes

$$\theta = \theta_D - \theta_D\left[\frac{m_1}{m_1 - m_2}\varepsilon^{m_2t} - \frac{m_2}{m_1 - m_2}\varepsilon^{m_1t}\right] \quad . \quad . \quad . \quad (194)$$

Now, $m_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $m_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Thus, if $b^2 > 4ac$, both m_1 and m_2 are real and negative. The motion of the galvanometer is thus non-oscillatory, the deflection gradually rising from zero to its maximum value θ_D. The galvanometer under these conditions is said to be "*over-damped.*" If $b^2 < 4ac$, both m_1 and m_2 are imaginary. Then referring to Equation (180), the equation of motion is

$$\theta = \theta_D + \varepsilon^{-\frac{b}{2a}t} F \sin\left(\frac{\sqrt{4ac - b^2}}{2a}t + \alpha\right) \quad . \quad . \quad . \quad . \quad (195)$$

This equation represents an oscillatory motion, the oscillations dying away gradually as the time t is increased, giving finally a steady deflection θ_D.

If f is the frequency of these oscillations, then

$$2\pi f = \frac{\sqrt{4ac - b^2}}{2a}$$

or $$f = \frac{\sqrt{4ac - b^2}}{4\pi a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (196)$$

The galvanometer, under these conditions, is *under-damped*. If there is no damping $b = 0$ and $T = 2\pi\sqrt{\frac{a}{c}}$ (where T = the periodic time of the oscillations).

If $b^2 = 4ac$, then $m_1 = m_2 = -\frac{b}{2a}$

In this case of equal roots ($m_1 = m_2$), the general solution for θ takes the form

$$\theta = \theta_D + \varepsilon^{-\frac{b}{2a}t}[A + Bt] \quad . \quad . \quad . \quad . \quad (197)$$

To find A and B, let $\theta = 0$ and $\frac{d\theta}{dt} = 0$ when $t = 0$.

Then $$0 = \theta_D + A$$

or $$A = -\theta_D$$

and $$0 = B - \frac{b}{2a} \cdot A$$

or $$B = -\frac{b}{2a} \cdot \theta_D$$

Hence, $$\theta = \theta_D - \theta_D\,\varepsilon^{-\frac{bt}{2a}}\left[1 + \frac{b}{2a}t\right] \quad . \quad . \quad (198)$$

Under these conditions the motion of the galvanometer is just non-oscillatory and the damping is said to be "*critical.*"

The undamped natural frequency of the instrument is $f = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$ (see p. 262)

and, with critical damping, $b^2 = 4ac$ so that $f = \frac{1}{2\pi}\sqrt{\frac{b^2}{4a^2}}$ or $\frac{b}{2a} = 2\pi f$. The equation for the deflection becomes

$$\theta = \theta_D(1 - \varepsilon^{-2\pi ft} - 2\pi ft\varepsilon^{-2\pi ft})$$

Now, when $t = \frac{1}{f}$, the value of the factor in brackets is

$$1 - \varepsilon^{-2\pi} - 2\pi\varepsilon^{-2\pi}$$
$$= 1 - 0{\cdot}00185 - 0{\cdot}0116 = 0{\cdot}9865$$

Thus, in a time equal to the undamped periodic time, the instrument will, when critically damped, reach a deflection equal to 98·7 per cent of its final deflection when a current I is suddenly applied.

Fig. 189 shows the forms of the deflection-time curves in the three cases when the galvanometer is (*a*) over-damped, (*b*) under-damped, (*c*) critically damped. The curves on the left show the rise of the deflection, starting at $\theta = 0$ when $t = 0$, while those on the right show the dying away of the deflection, starting at $\theta = \theta_D$ when $t = 0$.

Influence of the Resistance of the Galvanometer Circuit upon the Damping. In the above, the damping constant b was assumed to be dependent merely upon air friction and elastic hysteresis in the suspension. If the coil is wound upon a metal former, an e.m.f. will be induced in this former when it moves through the magnetic field of the permanent magnet. A current will flow in a closed circuit in this former, and will produce damping even though the galvanometer circuit may be open. The induced e.m.f. is proportional to the angular velocity of the coil $\frac{d\theta}{dt}$ and this additional damping may be taken into account by making the original damping b, now b'.

If the galvanometer circuit is closed, R being the resistance of this circuit, the current flowing through the galvanometer—neglecting its inductance—will be given by

$$Ri = E - G \cdot \frac{d\theta}{dt}$$