instrument had 144 turns on each of its fixed coils and 288 turns on each of its moving coils, the total resistance being 17·5 ohms at 20° C.

Campbell and Butterworth-Tinsley Mutual Inductometers. Both of these instruments are, essentially, variable mutual inductances, such instruments having the advantage that their inductance can be reduced to zero or given negative values, while a variable self-inductance can only be reduced to some minimum value depending

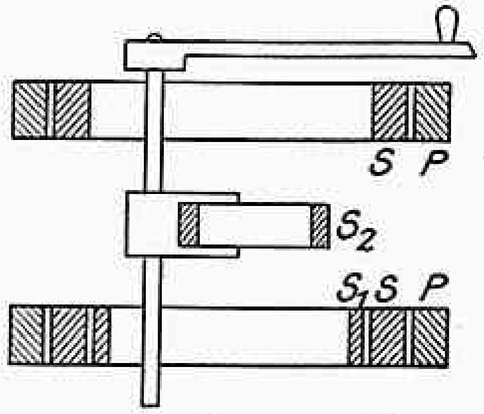

FIG. 146. CONSTRUCTION OF CAMPBELL MUTUAL INDUCTOMETER MANUFACTURED BY THE CAMBRIDGE INSTRUMENT CO.

upon the self-inductance of the coils and by the mutual-inductance between them.

The Campbell instrument, devised by A. Campbell (Ref. (10)) and manufactured by the Cambridge Instrument Co., has an arrangement of coils as shown in Fig. 146. *PP* are two equal coaxial fixed coils forming the primary winding. These are connected in series. The two coils *SS*, connected in series, form together one of the fixed secondary windings. S_1 is another secondary coil, also fixed, while S_2 is a movable secondary coil. The three secondary windings are connected in series. A link is provided for the purpose of reversing the connections to the moving coil S_2. Coils S and S_1 are each divided into ten sections of equal mutual inductance with the primary, and connections are taken from these sections to two dial switches. The mutual inductance of each of the sections of S_1 is, in one form of the instrument, 100 microhenries, and of coil S 1,000 microhenries, giving a total for the two coils of 11,000 microhenries. Fine adjustment is obtained by rotation of the secondary moving coil S_2, which is mounted midway between, and parallel to, the fixed coils as shown. An index mark on the handle arm serves the purpose of a pointer, readings being observed on a scale fixed under this arm on the lid of the instrument.

The coils are wound with stranded wire, and marble is used in the best instruments for the coil bobbins, and as the framework to which the coils are attached. These instruments have the advantage of high accuracy and simplicity, but possess considerable capacitance, which introduces errors at the higher frequencies.

Butterworth's mutual inductometer (Ref. (7)), manufactured by Messrs. H. Tinsley & Co., is designed so as to eliminate the defect of the Campbell and similar instruments, due to inter-capacitance between the windings. The makers claim that with this type of instrument a correction of only 0·07 per cent is necessary in the case of an instrument calibrated at 50 cycles and used at 1,000 cycles.

This instrument has a fixed primary coil and two sets of three secondary coils, also fixed. There is also a moving secondary coil for fine adjustment. Each set of fixed secondary coils consists of three coils having mutual inductances with the primary in the ratio of 6 : 3 : 1. Connections are made from each set to a commutator which is manipulated as a dial switch. The two dials are marked 1 to 10, the various inductances being obtained by connecting various combinations of the three coils, through the commutator, in series. In some cases one of the coils is reversed to give the required inductance. For example, 8 is obtained by the commutator connecting the 6 and 3 coils in series so that their magnetic effects are cumulative, and the 1 coil is reversed, thus giving the value $8 = 6 + 3 - 1$.

In a common form of the instrument one dial gives 10 millihenries in steps of 1, while the other dial has a total to 1 millihenry in steps of 0·1, the moving coil giving from $-0{\cdot}01$ to $+0{\cdot}11$ millihenry. The readings in the latter case are observed on a scale placed under the handle arm, as in the Campbell instrument. In this case the total range of the instrument is 11·11 millihenries.

In the Sullivan-Griffiths variable standard of self or mutual inductance the formers of both rotor and stator are constructed to have temperature compensation and so to give a high degree of stability to the windings. There are two ranges of self inductance and one range of mutual inductance, all three direct-reading in inductance, from a single calibration in terms of self inductance. Accuracies as high as 0·02 per cent are possible up to frequencies of a few kc/s but the instrument may be used at frequencies up to 50 kc/s by applying corrections reaching a maximum of 0·3 per cent at this frequency.

The Sullivan-Griffiths decade standards have a number of temperature-compensated standards, each having ten tappings taken to a rotary switch and thus providing a decade of inductance. The coils are all arranged geometrically at mutually zero magnetic coupling. In the standard having a maximum inductance of 1 henry, the finest subdivision on a sixth (continuously variable) dial is 0·05 μH. The decades are adjusted to be direct-reading to an accuracy of 0·03 per cent throughout their entire range and are provided with a calibration of 0·01 per cent.

Variable Capacitors. Variable capacitors may take the form of a subdivided fixed capacitor, various fractions of which can be obtained by movement of either a dial switch or by plugs. If

This may also be written

$$L = \frac{S^2}{10^9 l} \text{ (approx.)}$$

where S = the total length of wire on the coil = $2\pi Nr$.

These formulae must be regarded as approximate only, since the expression of the magnetic intensity is only true for an infinitely long solenoid. In practice, the *whole* of the flux produced does not link with *all* the turns, and this reduces the inductance of the solenoid. Nagaoka (Ref. (7)) has given the values of a factor by which the above expression may be multiplied in order to take into account the dispersion of the lines of force. This factor varies according to the ratio of length to diameter of the coil.

Equation (128) may be written

$$L = \frac{\pi^2 N^2 d^2}{10^9 l}$$

d being the diameter of the coil. Introducing Nagaoka's factor K, we have

$$L = \frac{\pi^2 N^2 d^2}{10^9 l} . K \quad . \quad . \quad . \quad . \quad . \quad . \quad (129)$$

which is considerably more exact than the previous equation (128).

If the wire on the solenoid is closely wound, so that adjacent turns are touching, this expression gives results which are sufficiently accurate for most purposes. A correction is necessary if the turns are widely spaced. Fig. 101 gives the values of the factor K for different ratios of length to diameter of coil. The curve refers to a single layer coil or to a coil whose depth of winding is small compared with its diameter.

Coursey (Ref. (8)) has given values of a second factor K_1, for use when the depth of winding on a coil is appreciable. This factor varies with the ratio $\frac{\text{depth of winding}}{\text{mean diameter of coil}}$ and also with the ratio $\frac{\text{length of coil}}{\text{depth of winding}}$. The inductance formula, when K_1 is used, becomes

$$L = (K - K_1) \frac{\pi^2 . N^2 d^2}{10^9 . l} \text{ henries} \quad . \quad . \quad . \quad (130)$$

A full table of Nagaoka's factors is given by Nottage (Ref. 5), where other tables for the calculation of the inductance of special coils are also given. For Coursey's curves and further tables for such calculations, see Refs. (5), (6), (8), (9), (10).

Fig. 102 gives values of Coursey's factor $(K - K_1)$ for various ratios of depth of winding to mean diameter of coil, and of length of coil to depth of winding.

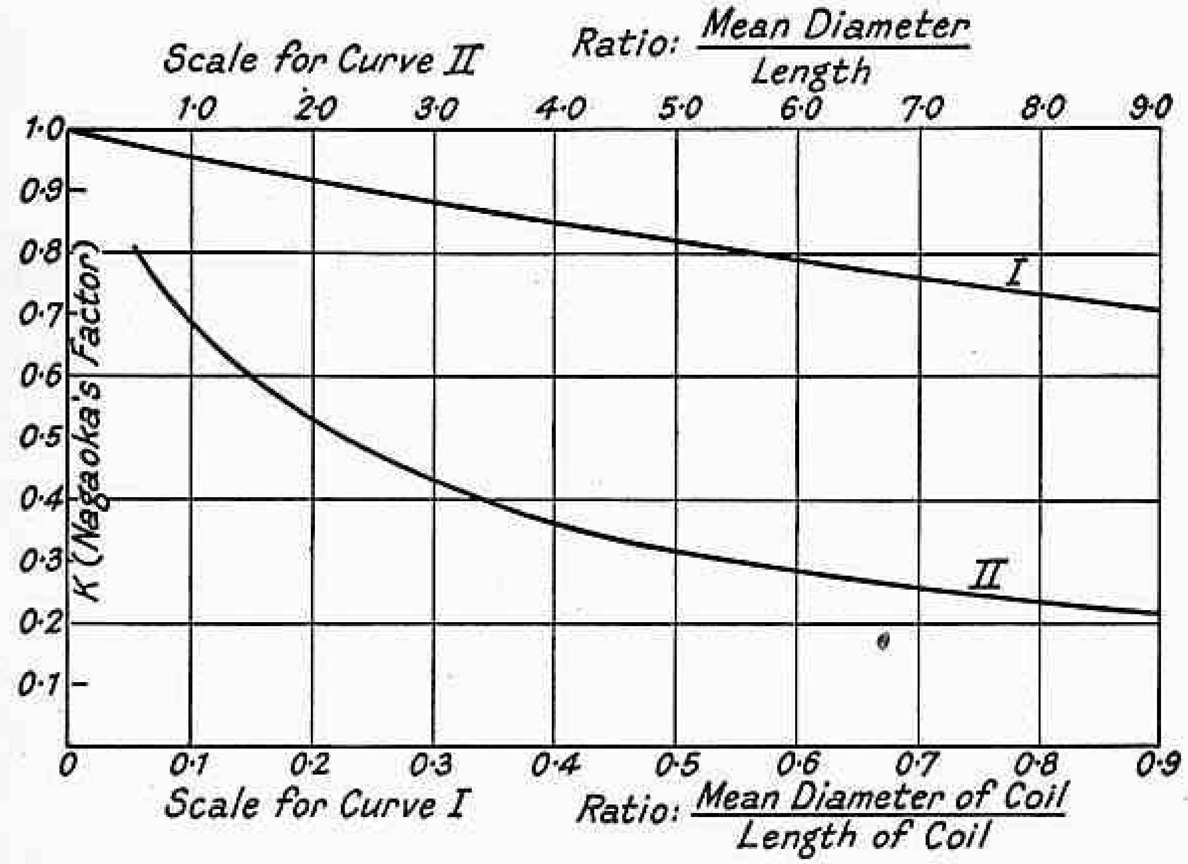

Fig. 101. Curves of Nagaoka Factors

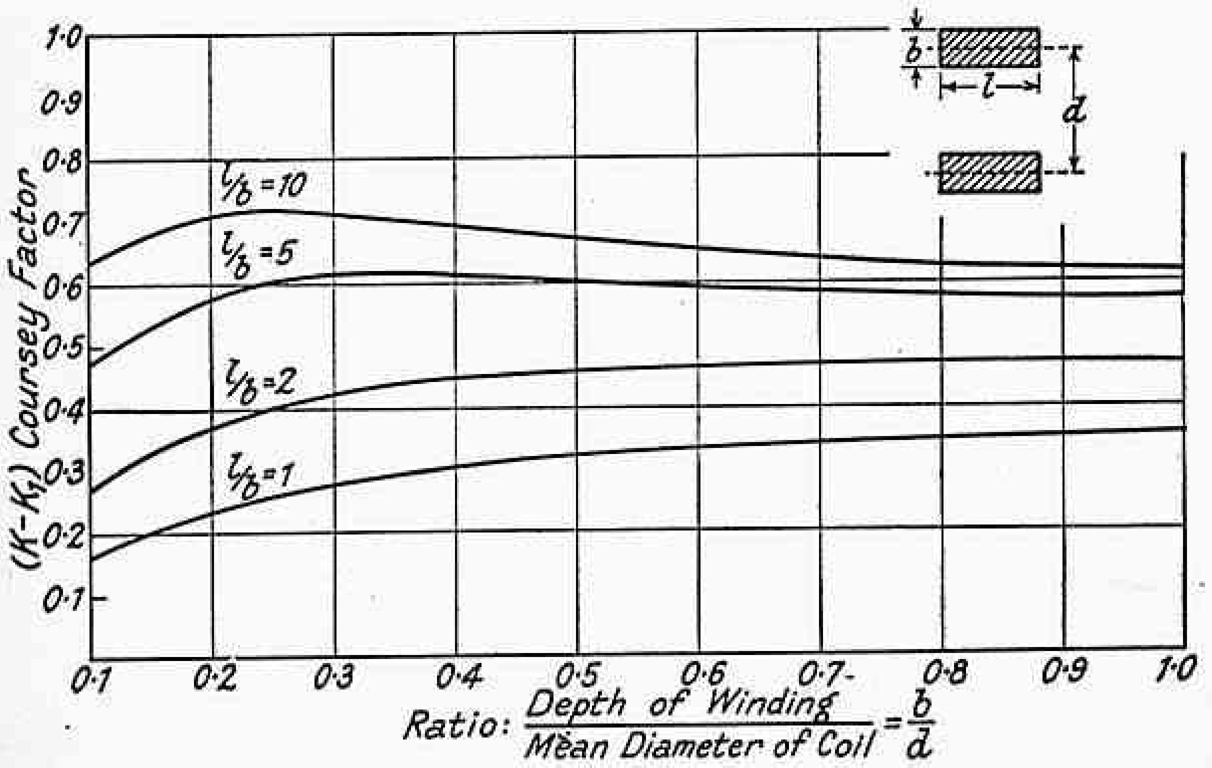

Fig. 102. Curves of Coursey Factors

Equations (129) and (130) are especially suited to long, circular coils whose depth of winding is small compared with their mean diameter. The assumption is made, in these formulae, that the distribution of the current over the cross-section is uniform.

Based on formulae derived by Rayleigh and Niven, Lyle, and Spielrein, Grover (Ref. 9) gives the formula

$$L = \frac{N^2dP}{2 \times 10^9} \text{ henries} \quad . \quad . \quad . \quad . \quad (131)$$

d being the mean diameter of the coil. P is a factor depending upon the ratios of the various dimensions of the coil and the values of P for different coil dimension ratios are given by Grover (*loc. cit.*).

This formula is more suited to the calculation of the inductances of short circular coils of rectangular cross-section whose depth of winding is comparatively large compared with their mean diameter, although it can be used also for the calculation of inductance in the same cases as Equation (130) with very little error.

Example. Calculate the inductance of a circular coil, of 500 turns, having a rectangular cross-section of winding. Given—

Axial length of coil = 10 cm.
Mean diameter of coil = 5 cm.
Depth of winding of coil = 1 cm.

(i) Using the Coursey curve (Fig. 102) in conjunction with equation (130),

Ratio $\frac{l}{b} = 10$. Ratio $\frac{b}{d} = \frac{1}{5} = 0{\cdot}2$.

From the curve $(K - K_1) = 0{\cdot}701$

$$L = \frac{0{\cdot}701\pi^2 \times 500^2 \times 5^2}{10^9 \times 10}$$

$$= \frac{4{,}325}{10^6} \text{ henries or 4,325 microhenries.}$$

(ii) Using Equation (131),

$$\frac{b}{l} = 0{\cdot}1 \quad \frac{b}{d} = 0{\cdot}2$$

From Grover's table, the value of P corresponding to these ratios is 6·92. Hence

$$L = \frac{500^2 \times 5 \times 6{\cdot}92}{2 \times 10^9}$$

$$= \frac{4{,}325}{10^6} \text{ henries, or 4,325 microhenries, as before}$$

Correction for Thickness of Insulation. As mentioned above, the formulae given take no account of the insulation between turns on the coil. For accurate calculations a correction for this insulation must be applied, although it is usually quite small.

This correction is made by subtracting the quantity $\frac{6{\cdot}283}{10^9}\, dN(A+B)$ henries

(Ref. (5)) from the calculated inductance, where d and N are as above, and A and B are constants depending upon the relative thickness of insulation and number of turns on the coil respectively. Values of these constants are given by Nottage (*lot. cit.*).

(7) **Self-inductance of Flat Coils.** By "flat" coils are meant those whose axial length is small compared with their mean diameter and depth of winding.

Spielrein gave the formula for such flat or "disc" coils of circular form as

$$L = \frac{N^2dQ}{2 \times 10^9} \text{ henries} \quad . \quad . \quad . \quad . \quad . \quad (132)$$

where N = No. of turns on the coil
d = mean diameter of the coil
and Q is a factor which can be calculated from the expression

$$Q = \frac{\left(1 + \frac{b}{d}\right)^3}{4\left(\frac{b}{d}\right)^2}\left[\begin{array}{l}6{\cdot}96957 - \beta^3\ 30{\cdot}3008 \log_{10}\frac{1}{\beta} + 9{\cdot}08008 \\ + 1{\cdot}48044\beta^5 + 0{\cdot}33045\beta^7 + 0{\cdot}12494\beta^9 + \ldots\end{array}\right]$$

where b = depth of winding

$$\beta = \text{the ratio } \frac{\text{inner radius of coil}}{\text{outer radius of coil}}$$

A table of values of the factor Q are given by Grover (Ref. (9)) for different values of $\frac{b}{d}$. If the axial length of the coil is appreciable Equation (131) (previous paragraph) applies.

To Correct for Insulation Thickness. In the case of a flat spiral wound with metal strip or ribbon of rectangular cross-section, the quantity

$$\frac{12{\cdot}57}{10^9} Nr(A_1 + B_1) \text{ henries (Ref. Grover, } \textit{loc. cit.}\text{)}$$

is added to the calculated inductance.

N = No. of turns on coil
r = mean radius of coil in centimetres

$$A_1 = \log_e \frac{\nu + 1}{\nu + \tau}$$

$$B_1 = -2\left[\frac{N-1}{N}\delta_{12} + \frac{N-2}{N}\delta_{13} + \frac{N-3}{N}\delta_{14} + \ldots \frac{1}{N}\delta_{1n}\right]$$

$\nu = \frac{w}{D}$ where w = axial length of strip
D = distance between adjacent turns

$\tau = \frac{t}{D}$ where t = thickness of strip

The factors δ_{12}, δ_{13}, etc., are given in tabular form by Grover for different values of τ and ν.

8. **Self-inductance in Other Cases.** (*a*) *Coils Wound on Polygonal Formers.* Grover (Ref. (10)), in a Bureau of Standards paper on this subject, gives a method of calculating the inductance of coils of this general form by obtaining, in each case, the "equivalent radius" of the coil, and then treating it as a circular coil having this radius. The formulae for calculation of the equivalent radii are somewhat complex, and reference should be made to the original paper for information on the subject.

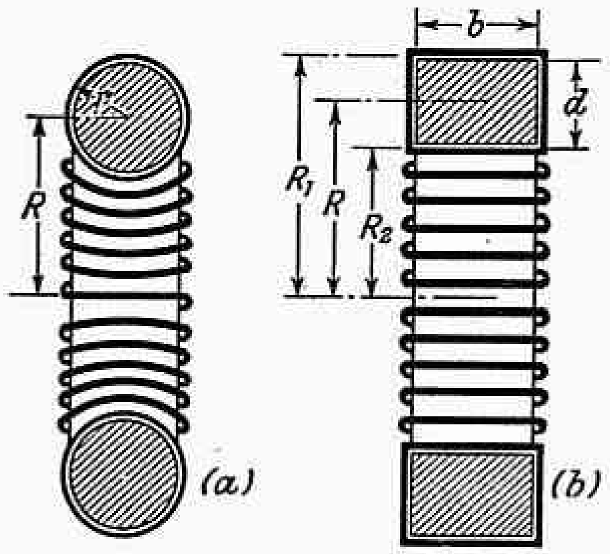

FIG. 103. SELF-INDUCTANCE OF TOROIDAL COILS

(*b*) *Toroidal Coils.* These are coils whose axis and cross-section are either both circular or the former circular and the latter rectangular.

(i) Axis circular, cross-section circular (torus) (Fig. 103 (*a*)).

Russell (*Alternating Currents*, Vol. I, p. 50) shows that the flux inside such a coil, of N turns, when a current of I amp. flows in it, is

$$\phi = \frac{4\pi}{10} NI(R - \sqrt{R^2 - r^2})$$

where R = mean radius of axis of coil in centimetres
r = radius of the cross-section of the coil in centimetres.

Thus, the inductance is given by

$$L = \frac{4\pi}{10^9} N^2(R - \sqrt{R^2 - r^2}) \text{ henries} \quad . \quad . \quad (133)$$

(ii) Axis circular, cross-section rectangular (Fig. 103 (*b*)).

Again, from Russell's expression for the flux within such a coil, we have

$$\phi = \frac{2NbI}{10} \log_e \frac{R + \frac{d}{2}}{R - \frac{d}{2}}$$

where b = the breadth of the coil
d = radial depth of the coil, both in centimetres
R = mean radius of axis of coil

Thus,
$$L = \frac{2bN^2}{10^9} \log_e \frac{R + \frac{d}{2}}{R - \frac{d}{2}}$$

or
$$L = \frac{2bN^2}{10^9} \log_e \frac{R_1}{R_2} \text{ henries} \quad . \quad . \quad . \quad (134)$$

where R_1 and R_2 are the outer and inner radii of the ring respectively.

Design of Inductance Coils for Maximum Time-constant. The ratio $\frac{\text{inductance}}{\text{resistance}}$ for any coil is spoken of as its "Time-constant." It is usually desirable, in designing inductance coils, to make this ratio as great as possible. This means that the dimensions must be such that the greatest possible inductance is obtained with a given length of wire. Since the resistance of coils increases considerably at high frequencies, compared with the continuous current, or low-frequency, resistance, it is difficult to give rules for the most economical design of coil to suit different frequencies when these are high.

Referring to low-frequency conditions, the maximum inductance for a given length of wire, using a coil of rectangular cross-section, is obtained when the cross-section is square, i.e. when the axial length of the coil is equal to the depth of winding on the coil.

Maxwell showed, also, that with a square section coil the inductance is maximum when the mean diameter of the coil is made 3·7 times the axial length of the coil, but, as already pointed out on page 92, later work by Shawcross and Wells (Ref. (26)) has shown that 2·95 (or more conveniently 3) is a better value than 3·7 for the ratio of mean diameter to axial length.

The maximum inductance for a given length of wire, if the coil is not limited as to shape, is obtained by making the coil of circular cross-section, with a ratio of $\frac{\text{radius of circular axis of coil}}{\text{radius of cross-section of the coil}}$ = 2·575. (Ref. (25)).

Experimental work on the most efficient shape of coil has been carried out also by Brooks and Turner (Ref. (11)) and a valuable paper by H. B. Brooks on the design of inductance coils is mentioned in Ref. (27)).

Iron-cored Inductances. The formulae for inductance so far considered have all been for coils with air, or non-magnetic, cores. The inductance of iron-cored coils cannot be calculated easily with great accuracy owing to the fact that the permeability of the iron core is not constant, but varies with the magnetizing force producing the flux. If an expression is to be given for the permeability under these conditions it must be some mean value of the different permeabilities occurring at different times throughout the current cycle. The question is further complicated by the fact that the value of the magnetizing force is not the same for all parts of the iron core, even for a given value of current in the coil. Thus the effective value to be assigned to the permeability of the core of such a coil is largely a matter for experimental determination under a given set of conditions.

The foregoing remarks apply especially to coils with open iron cores—say in the form of a straight bar. If the core is nearly closed, having a comparatively narrow air gap, the calculation of inductance can be carried out approximately as follows—

Let S_i = reluctance of iron path
S_a = reluctance of air gap
N = No. of turns on the coil
I = r.m.s. value of the current in the coil
ϕ = r.m.s. value of the flux produced

Then,
$$\phi = \frac{\frac{4\pi}{10} . NI}{S_i + S_a}$$

Now, obviously the value of the flux per ampere, i.e. $\frac{\phi}{I}$, would be constant if $S_i + S_a$ were constant. But, although the reluctance of the air gap S_a is constant whatever the value of the magnetizing force, the iron path reluctance S_i varies with varying current as pointed out above. If, however, the reluctance S_a is made large compared with S_i, the variation in the latter is negligible, since S_i may then be entirely neglected with very little error.

$$L = \frac{\phi N}{I \times 10^8}$$
$$= \frac{4\pi N^2}{10^9 S_a} = \frac{4\pi N^2 A}{10^9 l} \text{ benries} \qquad (135)$$
$$\text{since } S_a = \frac{l}{A} \times \frac{1}{\mu} \text{ and } \mu = 1 \text{ for air}$$

Under these circumstances, the iron core provides a low reluctance path for the flux, thus increasing the latter for a given magnetizing force, and hence increasing the inductance of the coil.

Example. A coil of 500 turns is wound on a cylindrical former 10 cm. long and 1·5 cm. radius. This former is placed on a rectangular iron core of effective cross-section 2 sq. cm., and whose length of magnetic path is 30 cm. The core contains an air-gap 0·5 cm. long. A current of 0·1 amp. r.m.s. flows through the coil. Given that the mean permeability of the iron of the core under these conditions = 1,000, calculate the inductance of the coil.

Reluctance of air gap
$$S_a = \frac{0{\cdot}5}{2} = 0{\cdot}25$$

Reluctance of iron path
$$S_i = \frac{30}{2} \times \frac{1}{1{,}000} = 0{\cdot}015$$

$$\text{Flux} = \phi = \frac{\frac{4\pi}{10} \times 500 \times 0{\cdot}1}{0{\cdot}25 + 0{\cdot}015} = 226 \text{ lines}$$

$$\text{Inductance} = \frac{\phi \times N}{I \times 10^8} = \frac{226 \times 500}{0{\cdot}1 \times 10^8} = 0{\cdot}0113 \text{ henry}$$

Obviously, if the reluctance of the iron path had been entirely neglected, the calculated inductance would have been some 6 per cent larger than the above value. Thus, uncertainty as to the correct value of the permeability of the iron under working conditions causes a negligible error if the air gap is made comparatively large.

To illustrate the effect of the iron core in increasing the inductance, we will calculate the inductance of the same coil with an air core.

From the approximate equation (128) this inductance is

$$\frac{4\pi^2 \times 500^2 \times 1{\cdot}5^2}{10^9 \times 10} = 0{\cdot}0022 \text{ henry (approx.)}$$

Skin Effect. It was pointed out earlier in the chapter that there is internal flux inside a straight cylindrical conductor which is carrying current. Considering the conductor to be made up of an infinite number of small filaments, parallel to its axis, each carrying a small fraction of the total current, I amp., of the conductor, and assuming the current density to be uniform over the conductor cross-section (an assumption which is really justified only with unidirectional or low-frequency current), we have for the flux density at a radius r within the conductor

$$B_r = \frac{2I_r}{10r} \text{ where } I_r = \frac{r^2}{R^2} . I$$

R being the radius of the conductor itself.

$$\therefore B_r = \frac{2r^2 I}{10rR^2} = \frac{2rI}{10R^2}$$

Thus, $B_r \propto r$. In Fig. 104 B_r is shown plotted against radius r. The total flux surrounding the filaments of the conductor (including the flux external to the entire conductor), when plotted against radius r, gives the dotted curve of Fig. 104. From this curve it can be seen that the flux surrounding the filaments near the centre of the conductor is greater than that surrounding the filaments near its surface. Thus the centre filaments have greater inductance than the surface filaments.

If P is the resistance of one filament and L its inductance, then its impedance is $\sqrt{P^2 + \omega^2 L^2}$ where $\omega = 2\pi \times$ frequency. At low frequencies the term $\omega^2 L^2$ is small compared with P, so that if a voltage V is applied to the two ends of the conductor the current carried by any one filament is $\frac{V}{\sqrt{P^2 + \omega^2 L^2}} = \frac{V}{P}$ (very nearly), and the current density within the conductor is uniform over its cross-section. At high frequencies P^2 is small compared with $\omega^2 L^2$ and the current carried by a filament $= \frac{V}{\omega L}$ (very nearly). Under these conditions, then, the difference in inductance between central and surface filaments becomes very important. The central filaments carry only a very small current, due to their greater inductance, and the current in the conductor is almost entirely carried by the surface filaments, i.e. by the outer "skin" of the conductor. Hence the name "*skin effect*" given to this phenomenon.

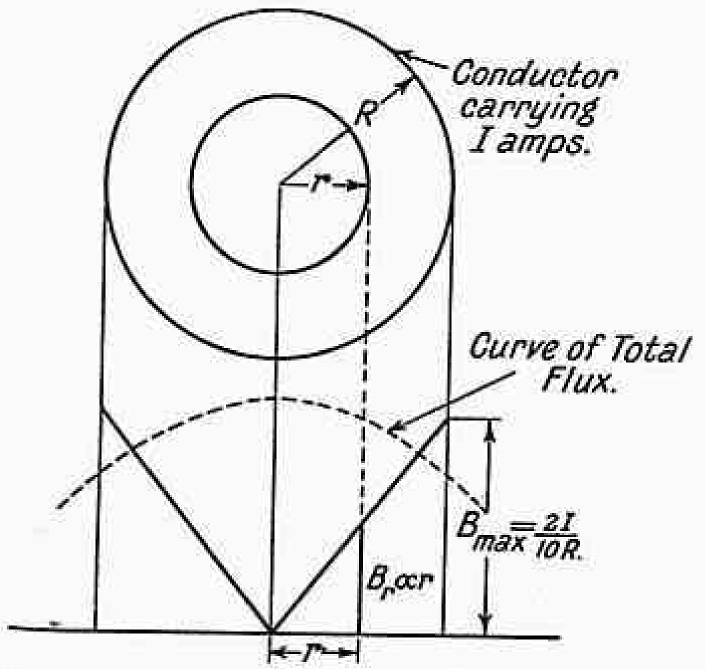

FIG. 104. DISTRIBUTION OF INTERNAL FLUX IN A CYLINDRICAL CONDUCTOR

The effective cross-section of the conductor at high frequencies is therefore only the area of an outer skin, and the resistance of the conductor is increased accordingly. Thus the "*high-frequency-resistance*" of a conductor is higher than its d.c. or low-frequency resistance, the difference depending upon the cross-section of the conductor, the frequency, and upon the permeability and resistivity of the material of the conductor. Since the material used for such conductors is usually non-magnetic, the permeability is almost always unity.

The high-frequency resistance of a conductor is given by

$$R_f = R\left[1 + \frac{1}{12}A^2 - \frac{1}{180}A^4 + \ldots\right] \quad . \quad . \quad . \quad (136)$$

where R is the steady-current resistance and

$$A = \frac{2\pi f l \mu}{10^9 R} = \frac{2\pi f l \mu}{10^9 \times \frac{l \times \rho}{\pi r^2}} = \frac{2\pi^2 r^2 f \mu}{10^9 \cdot \rho}$$

where f = frequency

l = length of conductor in centimetres

r = radius of conductor in centimetres

ρ = resistivity of conductor material in ohms per cm. cube

μ = permeability of the material of the conductor.

The inductance of the entire conductor is slightly reduced by the skin effect, since there is less internal flux.

The high frequency inductance is given by

$$L = \frac{l}{10^9}\left[M + \frac{1}{2} - \frac{1}{48}A^2 + \frac{13}{8640}A^4 - \ldots\right] \text{ henries} \quad . \quad (137)$$

M is a constant which depends upon the position of the return conductor of the circuit. The above two equations are due to Maxwell.

Reduction of Skin Effect. From Equation (136) it is obvious that the smaller the term A is made, the less the increase of resistance of the conductor with increasing frequency.

A can be kept small by making the radius r of the conductor as small as is consistent with current-carrying requirements and by using non-magnetic material (so that $\mu = 1$). If high-resistance material can be used, so that the resistivity ρ is large, this again will reduce A.

Other means which are adopted to reduce the effect are the employment of tubular conductors, or conductors consisting of two parallel discs with a number of parallel high-resistance rods, set at equal distances apart round their circumferences, joining them together, the whole forming a cage or barrel-shaped arrangement. In these cases the internal flux of the conductor is small and the conductors may be thought of as consisting merely of "skins" with hollow interiors.

Stranded conductors are used; these consist of a large number of fine strands, insulated from one another, and woven so that each strand lies as much at the centre of the conductor, and as much at the surface, as every other strand. In such conductors all the strands have the same surrounding flux and therefore have equal inductances.*

Skin Effect in Coils. Consider a cylindrical coil. The flux within such a coil is, of course, axial, and is distributed over the cross-section right up to the outer surface of the winding. Thus, in addition to the internal flux distribution previously considered, as

* For a number of curves relating to the high frequency resistance of straight conductors, the reader should consult Morecroft's *Principles of Radio Communication*, Chap. II.

affecting the inductance of the imaginary component filaments of the conductors, we have now a greater inductance of the radially outermost filaments of the coil, as compared with the filaments on the inner surface of the winding. This is due to the fact that they enclose the whole of the coil-flux, while the latter only enclose the flux within the winding (i.e. the flux in the core of the coil). A variation of the inductance between the various filaments is also caused by the proximity of other conductors.

Morecroft (Ref. (11)) has carried out a full investigation of the effect in various types of coils, and this work should be consulted for further information on the subject. The effect is usually negligibly small in coils when used at low frequencies for alternating-current measurement purposes.

In addition to the "skin effect" upon the actual self inductance of a coil, there is the effect of this current re-distribution upon the temperature coefficient of inductance. The extent of the re-distribution within a conductor (assuming unity permeability) depends upon its resistivity as well as upon the frequency. Therefore, as the resistivity changes with temperature, the inductance of a coil wound with that conductor varies with temperature to an extent which depends upon frequency. W. H. F. Griffiths* shows how to determine the more or less narrow band of frequencies in which this augmentation of temperature coefficient becomes appreciable.

Skin Effect in Iron Plates. In iron plates which are carrying alternating magnetic flux, the skin effect is of a different nature from that considered above. It is, in this case, the flux which is forced outwards so as to be carried almost entirely by the outer "skin" of the plate instead of being distributed uniformly over the cross-section.

The effect is due to the demagnetizing effect of "eddy currents" induced in the iron plates by the alternating flux. The plate itself acts as the short-circuited secondary winding of a transformer, and "eddy currents" flow in paths lying in a plane perpendicular to the axis of the plate, as shown in Fig. 105 (*a*), the currents being induced by the alternating flux.

The effect will be referred to later in Chapter XIV on "Eddy Currents."

Obviously the magnitude of the effect depends upon the thickness of the plate and upon the frequency, and it is for this reason that, in order to obtain uniform flux distribution—and hence economical utilization—of the iron cores of alternating current apparatus, it is necessary to limit the thickness of the laminations to be used, according to the supply frequency.

Fig. 105 (*b*) illustrates the diminution of flux density at the centre of an iron plate due to this demagnetizing effect. The figure refers to a fairly thick plate when used with a comparatively high frequency. In practice, with plates of the normal thickness (about

* *The Wireless Engineer*, Vol. XIX, No. 221, pp. 56–63.

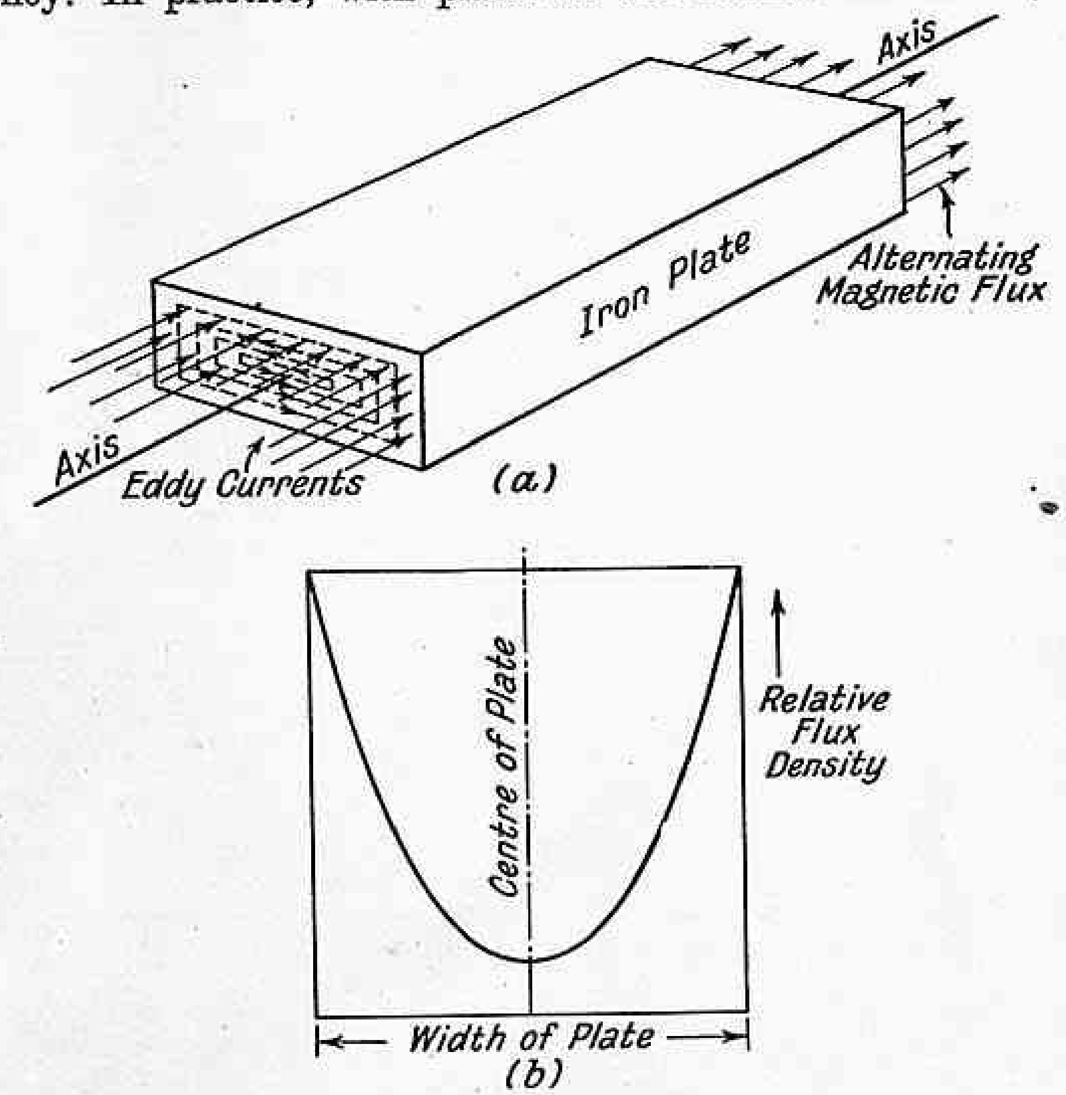

Fig. 105. Flux Distribution in Iron Plates

0·014 in.) and commercial frequencies, the variation in flux density over the cross-section is very much less than that shown.

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CHAPTER VI

MEASUREMENT OF INDUCTANCE AND CAPACITANCE

Self-inductance. Several approximate methods of measuring self-inductance are worthy of mention before the more precise methods —most of which are alternating current bridge methods—are described.

AMMETER AND VOLTMETER METHOD. Inductances of about 50 to 500 millihenries can be measured by this method. It is suitable for iron-cored coils, since the full normal current to be carried by the coil can be passed through it during the measurement.

A suitable current, of normal frequency, is passed through the coil, and this is measured by an a.c. ammeter while the voltage drop across the coil is measured by a high resistance voltmeter. The d.c. resistance R of the coil—which will be the same as the a.c. resistance, to a close approximation, if the frequency f is low—must also be measured. Then the inductance L of the coil is given by

$$L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} \text{ henries}$$

where $Z = \dfrac{\text{voltmeter reading}}{\text{ammeter reading}} = $ the impedance of the coil

Application of the A.C. Potentiometer to the Method. An improvement upon this simple method is the introduction of an alternating current potentiometer (see Chapter VIII) for the more precise measurement of the current and voltage drop.

A non-inductive resistance is then connected in series with the coil under test and the voltage drop across this, as well as that across the coil, is measured. The phase of the voltage drop across the coil, as well as its magnitude, is measured.

Let $R =$ the value of the non-inductive resistance

$\theta =$ the phase angle between the current and the voltage drop across the coil

$V' =$ the voltage drop across the coil

$I =$ the current (of frequency f)

Then $V' = I\sqrt{r^2 + (2\pi f L)^2}$

where r and L are the resistance and inductance of the coil under test.

Also $$I = \frac{V}{R}$$

where V = the voltage drop across the non-inductive resistance.

$$V' = \frac{V}{R}\sqrt{r^2 + (2\pi f L)^2}$$

or,
$$\sqrt{r^2 + (2\pi f L)^2} = \frac{V'R}{V}$$

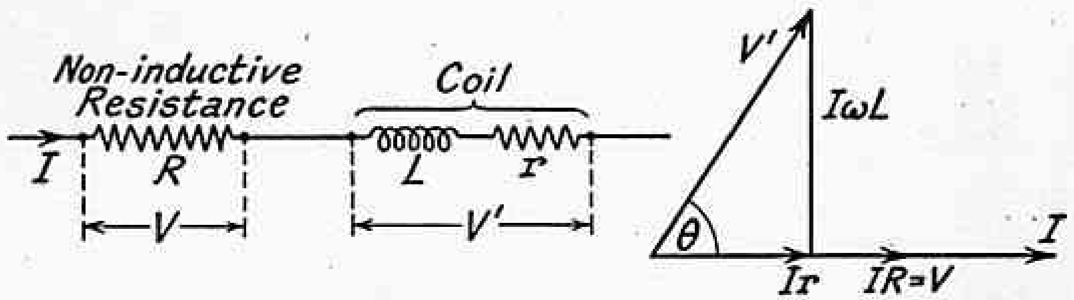


FIG. 106. VECTOR DIAGRAM FOR AMMETER AND VOLTMETER METHOD

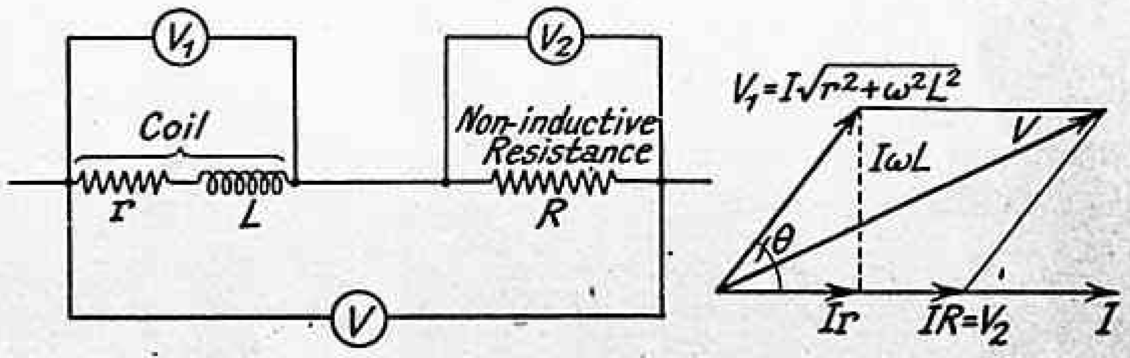


FIG. 107. VECTOR DIAGRAM FOR THREE-VOLTMETER METHOD

From the vector diagram of Fig. 106,

$$I \times 2\pi f L = V' \sin\theta = I\sqrt{r^2 + (2\pi f L)^2}\sin\theta$$

$$\therefore L = \frac{V'R}{V \times 2\pi f}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad (138)$$

Similarly, $\dfrac{Ir}{V'} = \cos\theta$

from which $$r = \frac{V'R}{V}\cos\theta \quad . \quad . \quad . \quad . \quad . \quad (139)$$

No measurement of the d.c. resistance is necessary, but the frequency and phase angle θ must be observed.

THREE-VOLTMETER METHOD. The connections of this method are shown in Fig. 107. A suitable current is passed through the coil, in series with a non-inductive resistance R, and the voltage drops across both parts of the circuit and across the whole circuit are measured as shown.

From the vector diagram in the figure

$$V^2 = V_1^2 + V_2^2 + 2V_1V_2\cos\theta$$

or $$\cos\theta = \frac{V^2 - V_1^2 - V_2^2}{2V_1V_2}$$

But $$\cos\theta = \frac{r}{\sqrt{r^2 + (2\pi f L)^2}}$$

where r and L are the resistance and inductance of the coil.

Thus $$\sqrt{r^2 + (2\pi f L)^2} = \frac{2rV_1V_2}{V^2 - V_1^2 - V_2^2}$$

or $$L = \frac{1}{2\pi f}\sqrt{\frac{4r^2V_1^2V_2^2}{(V^2 - V_1^2 - V_2^2)^2} - r^2} \quad . \quad . \quad (140)$$

The resistance r is measured on direct current.

THREE-AMMETER METHOD. The diagram of connections and vector diagram for this method are as shown in Fig. 108. In this case the non-inductive resistance R, together with an ammeter, is connected in parallel with the coil whose inductance is to be measured.

The theory of the method is exactly similar to that of the three-voltmeter method, but with currents I_1, I_2, and I replacing V_1, V_2, and V.

Thus $$L = \frac{1}{2\pi f}\sqrt{\frac{4r^2I_1^2I_2^2}{(I^2 - I_1^2 - I_2^2)^2} - r^2} \quad . \quad . \quad . \quad (141)$$

ALTERNATING CURRENT BRIDGE METHODS. The best, and most usual methods for the precise measurement of self- and mutual-inductance and capacity are those employing a bridge network with an alternating current supply. The supply may be of commercial frequency—when a vibration galvanometer is used as the detector—or it may be of higher frequency (say 500 to 2,000 ~ per second), when telephones or thermionic detectors (see Ref. (1)) are employed.

These networks are all, in general, modifications of the original Wheatstone bridge network and their operation is also similar.

In the Wheatstone bridge method of measuring resistance with direct current, the bridge is balanced (i.e. zero galvanometer deflection is obtained) when the voltage drops across the two arms connecting one of the supply terminals to the two ends of the galvanometer branch of the network are equal in magnitude. With a.c. bridge networks these volt drops must also be alike in *phase* as well as in magnitude, and for this reason the introduction of inductances or capacitances in other arms of the network is necessary when (say) an inductance to be measured is connected in one of the arms.

The bridge network to be chosen for the measurement of a given self-inductance depends upon the magnitude of that inductance and upon its "time-constant"—i.e. the ratio of inductance to resistance. In the following the magnitudes of the inductances to the measurement of which the various methods are best suited will be stated.

Maxwell's Method. In this method the unknown inductance is

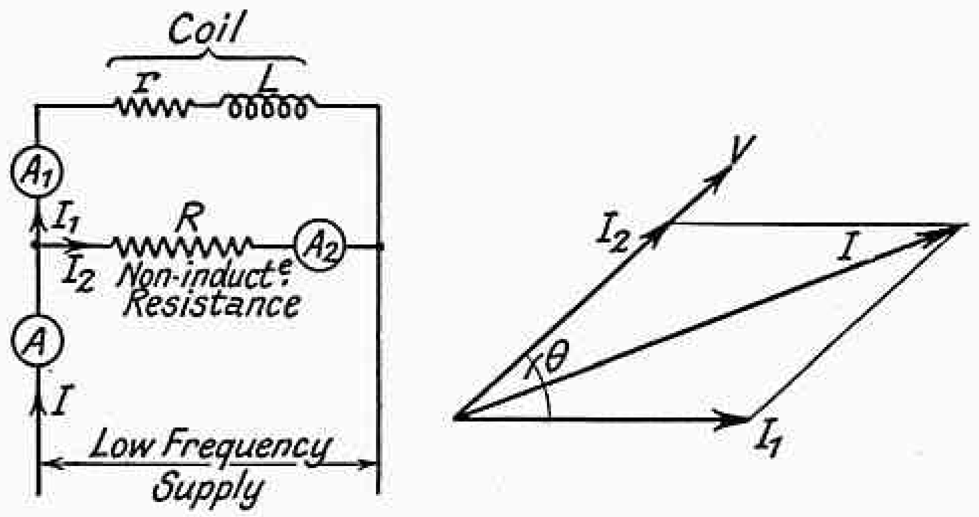


FIG. 108. THREE-AMMETER METHOD

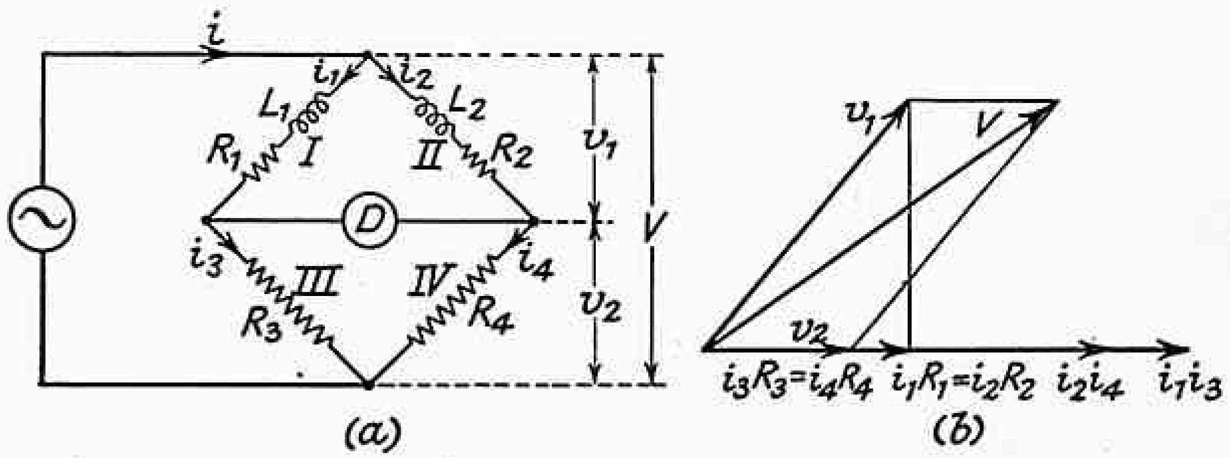


FIG. 109. MAXWELL'S METHOD FOR THE MEASUREMENT OF SELF-INDUCTANCE

compared with a known self-inductance. The connections for a.c. working, together with the vector diagram, are given in Fig. 109.

L_1 = unknown self-inductance of resistor R_1
L_2 = known self-inductance of resistor R_2
R_3 and R_4 = non-inductive resistors
D = detector

The resistances R_1, R_2, etc., include, of course, the resistances of the leads and contact resistances in the various arms. It is most convenient to use for the known inductance L_2, a variable self-inductance of constant resistance, its inductance being of the same order as that of L_1.

The bridge is balanced by varying L_2 and one of the resistors R_3 or R_4. Alternatively, R_3 and R_4 can be kept constant, and the resistance of one of the other two arms can be varied by connecting in the arm an additional resistor.

Theory. At balance the voltage drop v_1 across branch I = voltage drop across branch II, and the current i_1 in branch I = current i_3 in branch III. Similarly, volt drop v_2 across branch III = volt drop across branch IV, and $i_2 = i_4$, the volt drops being equal both in magnitude and phase.

Then, using the symbolic notation,

$$\frac{(R_1 + j\omega L_1)i_1}{R_3 i_3} = \frac{(R_2 + j\omega L_2)i_2}{R_4 i_4}$$

or

$$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3$$

Equating real and imaginary quantities, we have

$$R_1R_4 = R_2R_3$$

or

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

and also,

$$\frac{L_1}{L_2} = \frac{R_3}{R_4}$$

Thus

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{L_2} \quad . \quad . \quad . \quad . \quad . \quad (142)$$

The inductances L_1 and L_2 should be placed at a distance from one another and the leads used in the arms should be carefully twisted together to avoid loops. It should be remembered in this connection that a loop having an enclosed area of 1 sq. ft. has an inductance of roughly 1 microhenry.

The vector diagram of Fig. 109 (*b*) is for balance conditions, and shows i_1 and i_3 in phase with i_2 and i_4. This is obviously brought about by adjusting the impedances of the various branches so that these currents lag by the same phase angle behind the applied voltage V.

This method is very suitable for the measurement of inductances of medium magnitudes and can be arranged to give results of considerable precision.

Anderson Bridge. This method requires a standard capacitor in terms of which the self-inductance is expressed. It is actually a modification of Maxwell's method of comparing an inductance with a capacitance. The method is applicable to the precise measurement of inductances over a wide range of values, and is one of the commonest and best bridge methods.

Fig. 110 gives the diagram of connections and the vector diagram for balanced conditions.

L = self inductance to be measured
C = standard capacitor
R_1 = resistance of arm 1 (including the resistance of the self inductance)
r, R_2, R_3, R_4 = known non-inductive resistances

In the original method a battery and key were used instead of an alternating current supply. R_2, R_3, and R_4 were adjusted to give a balance for steady currents, with the battery key closed. The resistance r was then adjusted (without altering the original resistance settings) to give a balance when the battery key was opened or closed, the two balances being quite independent of one another.

When used with alternating currents, it is still convenient to obtain a preliminary balance for steady currents, using an ordinary galvanometer as detector, the alternating-current balance being then

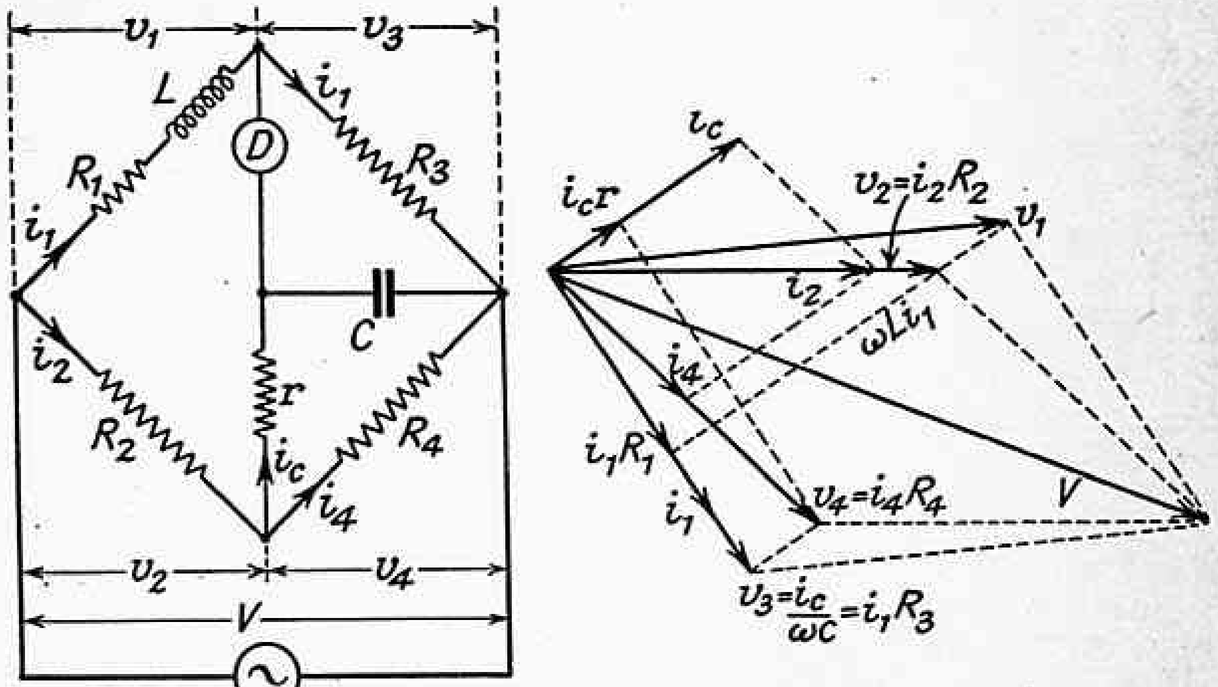


FIG. 110. ANDERSON BRIDGE FOR THE MEASUREMENT OF SELF-INDUCTANCE

obtained by varying r. Either telephones or a vibration galvanometer—according to the supply frequency—must be used for the detector when alternating currents are used.

When the bridge is finally balanced the self-inductance is given by

$$L = \frac{C \cdot R_3}{R_4}[r(R_2 + R_4) + R_2R_4] \quad . \quad . \quad . \quad (143)$$

Theory. Assume the capacitor C to be loss-free and the resistances completely non-inductive.

Referring to the simplified network diagram of Fig. 111, where the branch impedances are represented by Z_1, Z_2, etc., and the mesh currents by A, V, X, and $X + Y$, so that the detector current is Y, we have the mesh equations

Mesh I.

$$Z_1(X + Y) + Z_5(Y + X - X) + Z_6(X + Y - V) + Z_2(X + Y - A) = 0$$

or
$$X(Z_1 + Z_2 + Z_6) + Y(Z_1 + Z_2 + Z_5 + Z_6) - VZ_6 - AZ_2 = 0$$

Mesh II.

$$Z_3X + Z_7(X - V) + Z_5(X - X - Y) = 0$$

or
$$X(Z_3 + Z_7) - Z_5Y - Z_7V = 0$$

Mesh III.

$$Z_7(V - X) + Z_4(V - A) + Z_6(V - X - Y) = 0$$
$$-X(Z_6 + Z_7) - YZ_6 + V(Z_4 + Z_6 + Z_7) - AZ_4 = 0$$

Solving algebraically for Y and equating Y to zero (which is the condition at balance), we have

$$0 = Z_2 + \frac{Z_6Z_4}{(Z_4 + Z_6 + Z_7) - Z_7\dfrac{(Z_6 + Z_7)}{Z_3 + Z_7}}$$
$$- \frac{Z_4(Z_1 + Z_2 + Z_6)Z_7}{(Z_7 + Z_3)\left[(Z_4 + Z_6 + Z_7) - \dfrac{Z_7(Z_6 + Z_7)}{Z_3 + Z_7}\right]}$$

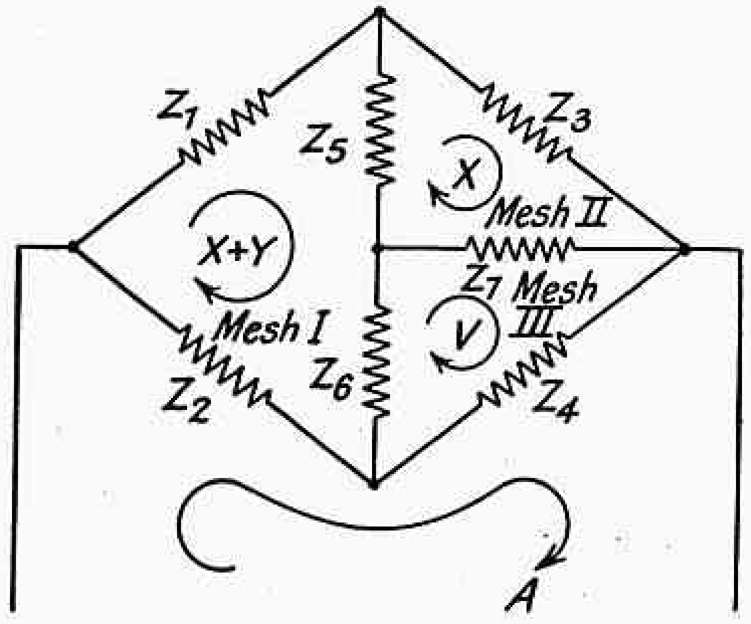


FIG. 111. SIMPLIFIED ANDERSON BRIDGE NETWORK

From which,

$$0 = Z_2Z_3Z_4 + Z_2Z_3Z_6 + Z_2Z_3Z_7 + Z_3Z_4Z_6 - Z_1Z_4Z_7$$

Expressing the impedances symbolically, we have

$$[Z_1] = R_1 + j\omega L \qquad [Z_4] = R_4$$
$$[Z_2] = R_2 \qquad [Z_6] = r$$
$$[Z_3] = R_3 \qquad [Z_7] = \frac{-j}{\omega C}$$

Substituting in the impedance equation gives,

$$0 = R_2R_3R_4 + R_2R_3r - R_2R_3\frac{j}{\omega C} + R_3R_4r - (R_1 + j\omega L)R_4\left(\frac{-j}{\omega C}\right)$$

Equating real and imaginary quantities,

$$0 = R_2R_3R_4 + R_2R_3r + R_3R_4r - R_4\frac{\omega L}{\omega C}$$

or
$$L = \frac{CR_3}{R_4}(R_2R_4 + R_2r + R_4r)$$

and $$\frac{jR_2R_3}{\omega C} = \frac{jR_1R_4}{\omega C}$$

or $$R_2R_3 = R_1R_4$$

which is the balance condition for steady currents.

Thus, the effective resistance of the self-inductance under test is

$$R_1 = \frac{R_2R_3}{R_4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

If the capacitor is not perfect, but has dielectric loss, the self-inductance value given by the above expression is unaltered, but the value of R_1 is affected.

If the self-inductance of the leads to the coil under test is appreciable, this may be measured by short-circuiting the coil and obtaining a second balance. The actual inductance of the coil may then be obtained by subtraction.

Theory at Balance. The above theory may be simplified by considering the case under balance conditions only.

Referring to Fig. 110 and taking $i_4 = i_2 - i_c$ (in symbolic notation), we have the equations

$$i_1(R_1 + j\omega L) = i_2R_2 + i_cr$$
$$i_1R_3 = i_c/j\omega C$$
$$i_cr + i_c/j\omega C = (i_2 - i_c)R_4$$

Eliminating the currents i_1, i_2, and i_c gives

$$R_1 + j\omega L - j \,.\, \omega C \,.\, \frac{R_2R_3r}{R_4} - \frac{R_2R_3}{R_4} - j\omega C \,.\, R_2R_3 - j\omega C \,.\, rR_3 = 0$$

The balance conditions, as obtained above, are then determined by equating real and imaginary terms.

The method can also be used to measure the capacitance of the capacitor C if a calibrated self-inductance is available.

Butterworth's Method. This method is especially suitable for the measurement of small inductances (e.g. a few microhenries).

The diagram of connections and the vector diagram are given in Fig. 112.

L = the self-inductance (of resistance R_1) to be measured

R_2 = a slide wire

C = fixed standard capacitor

R_3, R_4, r = non-inductive resistances.

The resistance balance, or the balance for steady current, can be obtained independently of the inductance balance by adjusting the resistances R_3 and R_4, while the inductance balance is obtained by adjustment of r and the slide wire setting.

At balance

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

and $$L = \frac{(R_2 - r_2)}{R_4}[(R_3 + R_4)r + R_3(R_4 + r_2)]C \quad . \quad . \quad (146)$$

(Ref. (6)). These conditions may be obtained from the mesh equations by a similar method to that used when considering the Anderson bridge.

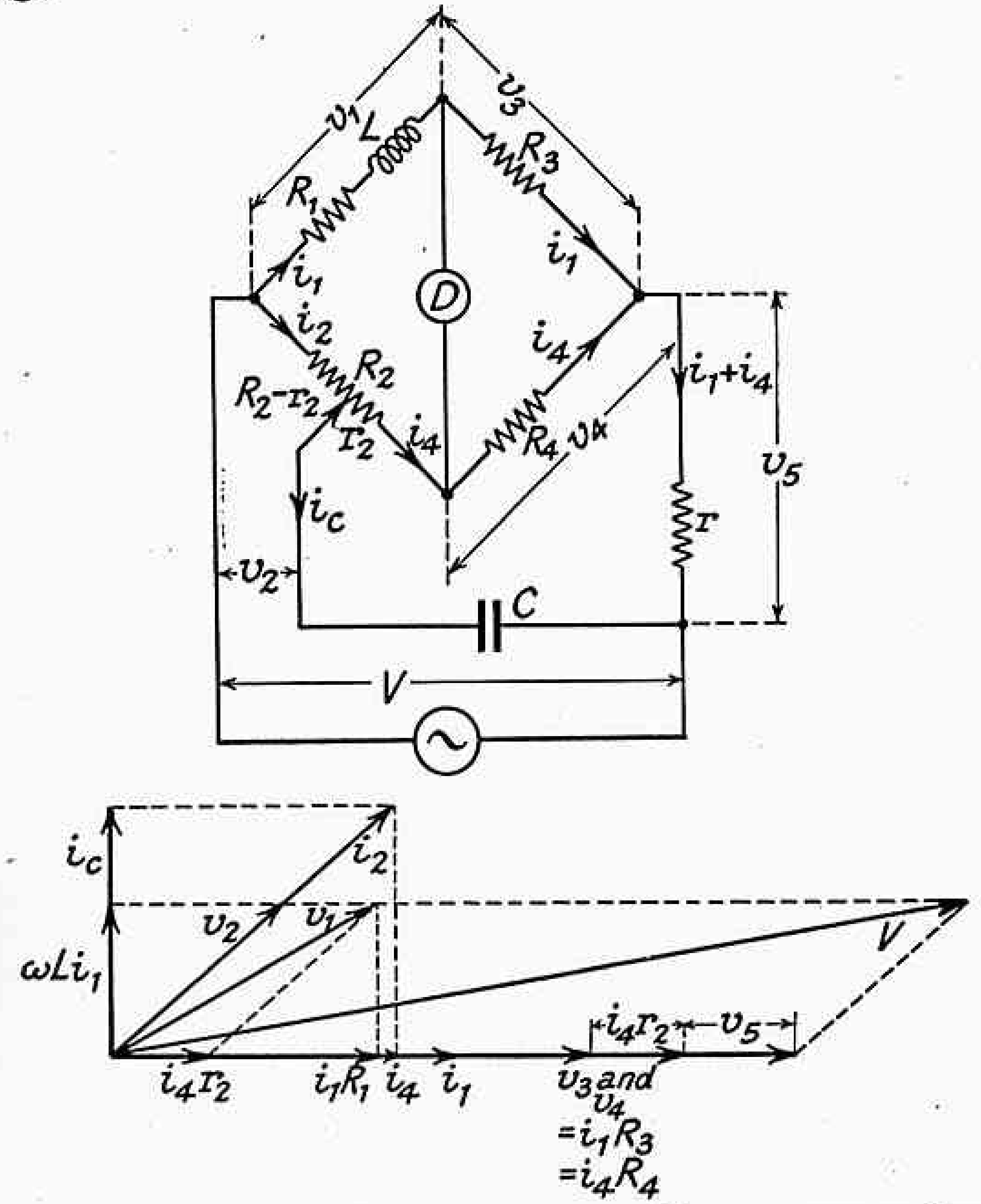


FIG. 112. BUTTERWORTH'S METHOD FOR THE MEASUREMENT OF SMALL SELF-INDUCTANCES

To obtain maximum sensitivity, Butterworth has shown that the following relationships should be fulfilled—

$$R_4 = \sqrt{S\,.\,D}, \qquad R_3 = \sqrt{R_1D\left(\frac{R_1 + S}{R_1 + D}\right)}$$

$$R_2 = \sqrt{R_1S\left(\frac{R_1 + D}{R_1 + S}\right)}$$

where S is the resistance of the alternator branch (including r) and D is the resistance of the detector branch. The resistance r, also, should be small.

Hay's Bridge. This method of measurement is particularly suited to the measurement of large inductances having a comparatively low resistance (i.e. having a large time-constant). The diagram of connections and the vector diagram are given in Fig. 113.

L is the inductance to be measured. R_1 is its resistance, C is a variable standard capacitor and R_2 R_3 and R_4 non-inductive resistances. Balance may be obtained by variation of C, R_4, and R_2.

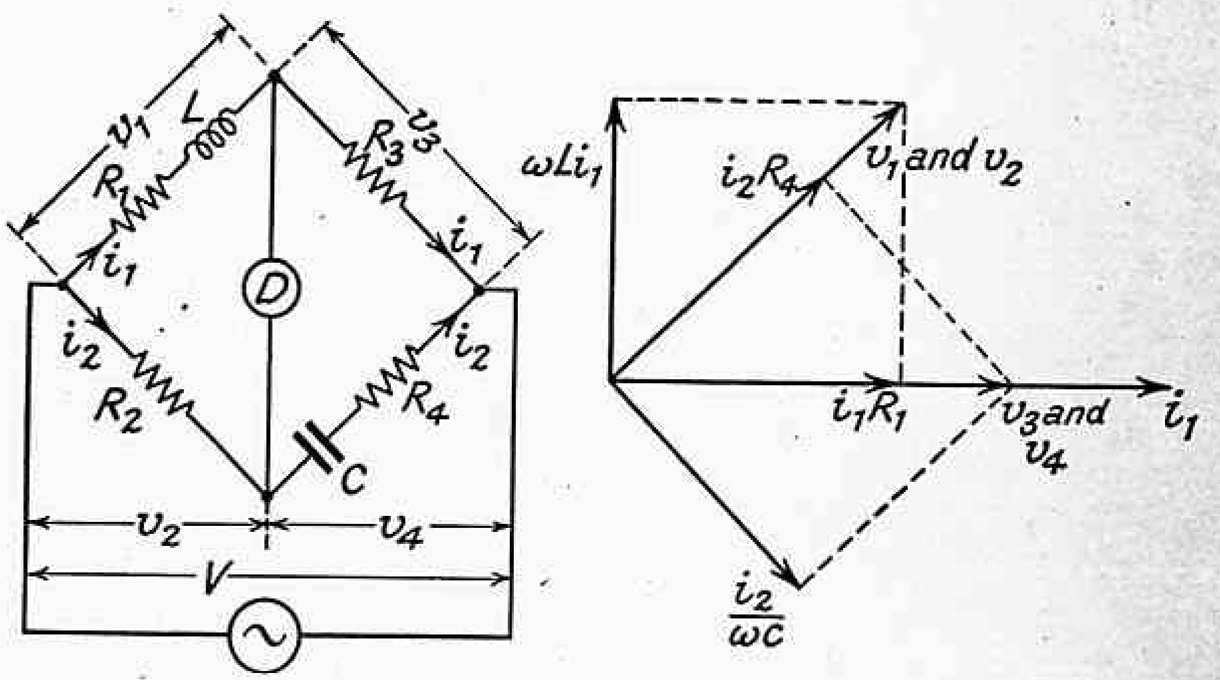


FIG. 113. HAY'S BRIDGE FOR THE MEASUREMENT OF LARGE SELF-INDUCTANCES

At balance, volt drop across arm I = volt drop across arm II, and volt drop across arm III = volt drop across arm IV.

Thus $$(R_1 + j\omega L)i_1 = R_2 i_2$$

$$R_3 i_1 = \left(R_4 - \frac{j}{\omega C}\right)i_2$$

From which $$L = \frac{R_2 R_3 C}{1 + \omega^2 R_4^2 C^2} \quad . \quad . \quad . \quad (147)$$

and the effective resistance R_1 of the coil is

$$R_1 = \frac{R_2 R_3 R_4 \omega^2 C^2}{1 + \omega^2 R_4^2 C^2} \quad . \quad . \quad . \quad (148)$$

Since the expressions for L and R_1 involve ω ($= 2\pi f$), the frequency must be accurately measured.

Measurement with Superposed D.C. and A.C. It has been shown by Landon and by Hartshorn (see Ref. 1, Fourth Edition, p. 391) that Hay's bridge may be used for the measurement of self-inductance in the case of iron-cored coils, in which both direct and alternating currents are flowing. The arrangement of the bridge for such a measurement is as shown in Fig. 114.

The direct current, which may be adjusted to the required value by r_1, passes through R_1, L, and R_3 only, the capacitors preventing its passage through the other branches. The magnitude of the alternating current passing through the coil L (under test) is regulated by C_2 and r_2. This current is obtained from the reading, when the bridge is balanced, of an electrostatic voltmeter connected

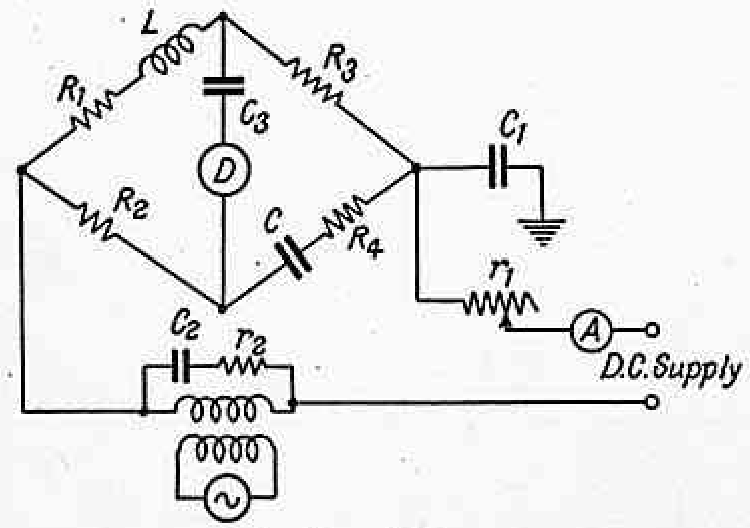


FIG. 114

across R_2. This reading gives the potential difference across R_1L. The voltmeter is removed before final balance of the bridge is made. To obtain the requisite sensitivity, the detector D consists of a vibration galvanometer supplied through a step-up transformer, the capacitor C_3 being adjusted to resonate this transformer.

C_1 is a large capacitor through which one corner of the bridge is earthed. Its use avoids direct earthing of one side of the d.c. supply. The bridge is balanced in the usual way after the direct and alternating currents have been adjusted to the requisite values.

Heaviside-Campbell Bridge. This method employs a standard variable mutual-inductance, and can be used for the measurement of self-inductance over a very wide range. It is one of the best methods for general laboratory use. Fig. 115 shows the diagram of connections of Heaviside's bridge.

The primary of the mutual inductometer is in the supply circuit, and the secondary of self-inductance L_2 and resistance R_2 form arm II of the bridge. The inductance to be measured, of self-inductance L_1 and resistance R_1, is placed in arm I of the bridge. R_3 and R_4 are non-inductive resistances.

Balance may be obtained by varying the mutual-inductance and resistances, R_3 and R_4. At balance,

$$i_2(R_2 + j\omega L_2) + j\omega Mi = (R_1 + j\omega L_1)i_1$$

and $$i_2R_4 = i_1R_3$$

Since $$i = i_1 + i_2$$

$$i_2 [R_2 + j\omega(L_2 + M)] = i_1 [R_1 + j\omega(L_1 - M)]$$

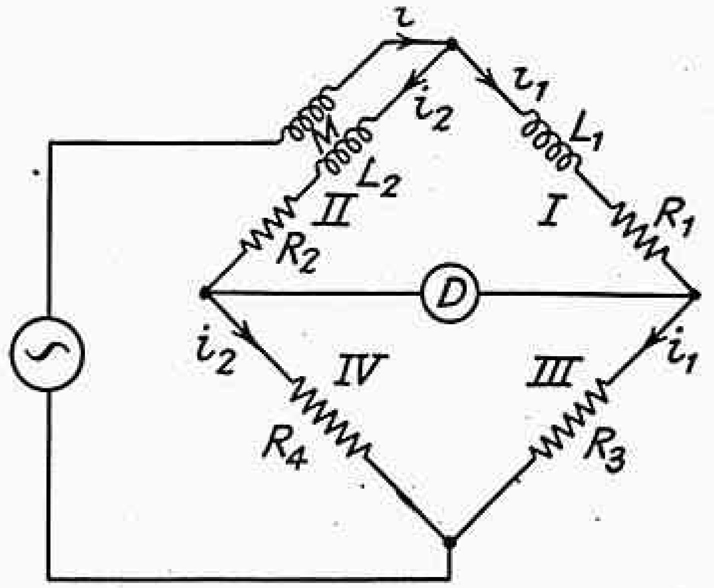


FIG. 115. HEAVISIDE BRIDGE

Thus $$\frac{R_2 + j\omega(L_2 + M)}{R_4} = \frac{R_1 + j\omega(L_1 - M)}{R_3}$$

or $$R_3 [R_2 + j\omega(L_2 + M)] = R_4 [R_1 + j\omega(L_1 - M)]$$

Equating real and imaginary quantities

$$R_2R_3 = R_1R_4 \quad . \quad . \quad . \quad . \quad . \quad (149)$$

and $$R_3(L_2 + M) = R_4(L_1 - M). \quad . \quad . \quad . \quad (150)$$

If the resistances R_3 and R_4 are equal,

$$L_2 + M = L_1 - M$$

or $$L_1 - L_2 = 2M$$

In *Campbell's Modification* of the bridge (Refs. (10) and (11)), the resistances R_3 and R_4 are made equal. A "balancing coil" of self-inductance equal to the self-inductance L_2 of the mutual-inductance secondary coil and of slightly greater resistance than the latter is introduced in arm I, in series with the inductance to be measured. A non-inductive resistance box and a "constant-inductance rheostat" are also introduced in arm II. These additions are shown in Fig. 116.

Balance is now obtained, by variation of the mutual inductometer and the variable resistance r, with the coil L_1, R_1, whose inductance and resistance are to be measured, in circuit. Suppose the readings of the mutual-inductance and resistance r are M_1 and r_1. The coil L_1R_1 is now removed, or short-circuited across its terminals, and balance is again obtained, giving, say, readings M_2 and r_2.

Then $$L_1 = 2(M_1 - M_2)$$

and $$R_1 = r_1 - r_2$$

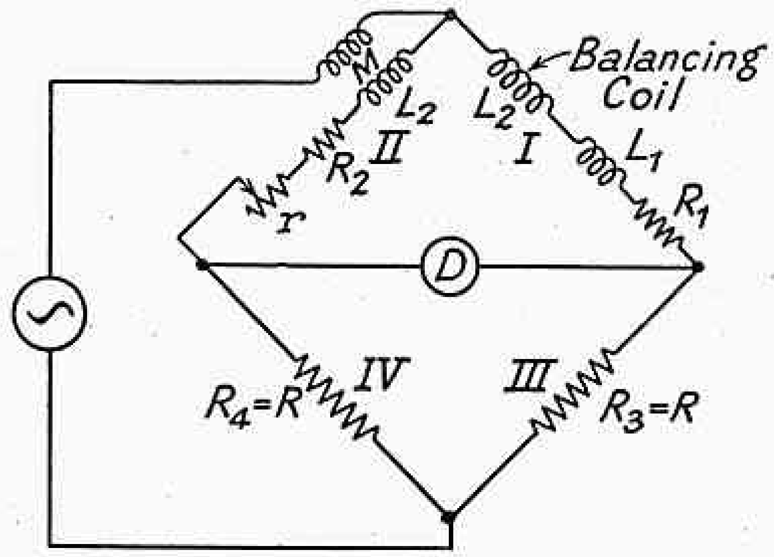


FIG. 116. CAMPBELL'S MODIFICATION OF THE HEAVISIDE BRIDGE

By this method of operation the self-inductance and resistance of the leads is eliminated and the inductance and resistance of the coil are obtained directly.

The use of a balancing coil in the above arrangement reduces the sensitivity of the bridge. Fig. 117 shows a better arrangement, which improves the sensitivity and eliminates the balancing coil. For this arrangement the secondary fixed coil of the inductometer must be made up of two equal coils LL, the primary coil reacting with both of them as shown. L_1 is the coil whose self-inductance is to be measured. The resistances R_3 and R_4 are equal ($R_3 = R_4 = R$). When so arranged the bridge is known as the *Heaviside-Campbell Equal Ratio Bridge*.

At balance—obtained by varying the constant-inductance rheostat r, and the mutual-inductance $M_1 + M_2$—we have the relationships

$$R_1 = R_2$$

and $$L_1 = 2(M_1 + M_2)$$

where R_1 and R_2 are the total resistances of arms I and II and $M_1 + M_2$ is the reading of the inductometer. With equal ratio arms R_3 and R_4 it is obvious that the magnitude of the self-inductance L_1 which can be measured is limited to twice the inductometer range.

If L_1 is greater than this value, unequal ratio arms are used with

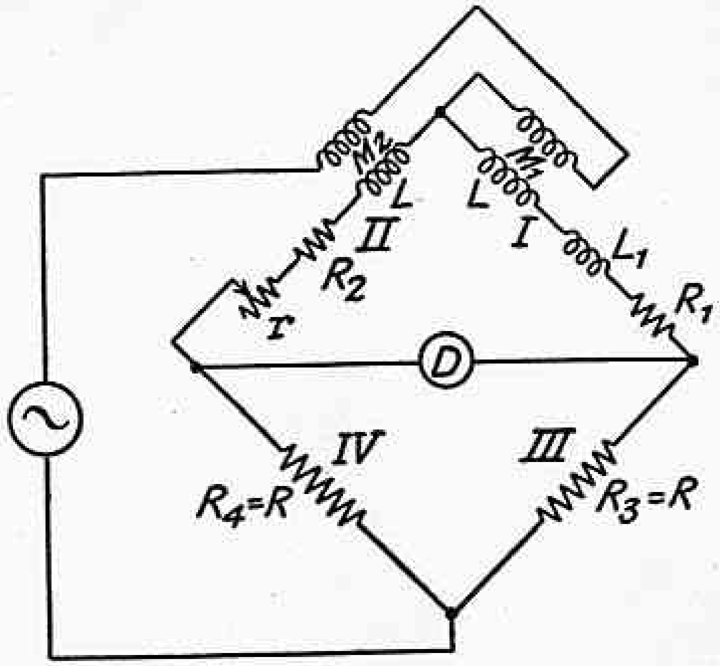


FIG. 117. HEAVISIDE-CAMPBELL EQUAL RATIO BRIDGE

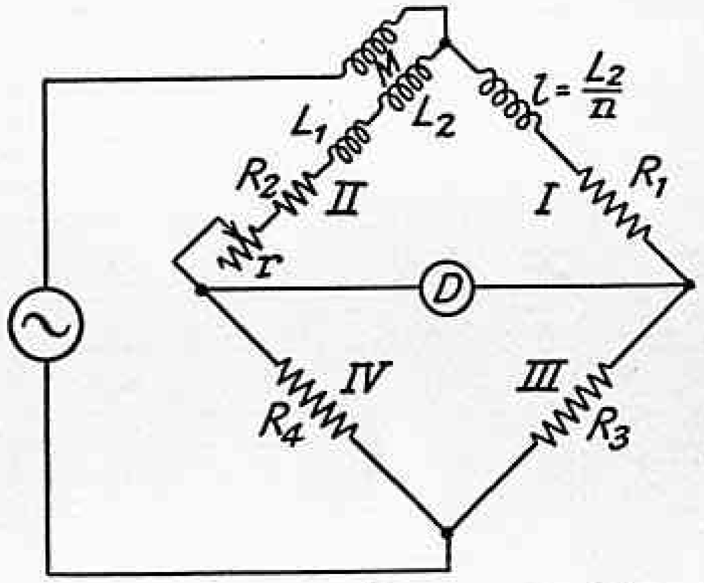


FIG. 118. HEAVISIDE-CAMPBELL BRIDGE WITH BALANCING COIL

a balancing coil l, the connections then being as shown (Fig. 118). Let the ratio

$$\frac{R_4}{R_3} = n$$

Then, if the inductance of the balancing coil is made equal to $\frac{L_2}{n}$

(where L_2 = inductance of inductometer secondary coil) the balance conditions are

$$\frac{R_2}{R_1} = n$$

and $$L_1 = (n + 1)M$$

When used as described above, the bridge can be used for the measurement of inductances varying from very low values to medium values. D. W. Dye (Ref. (9)) has modified the arrangement in order to make it suitable also for the measurement of large

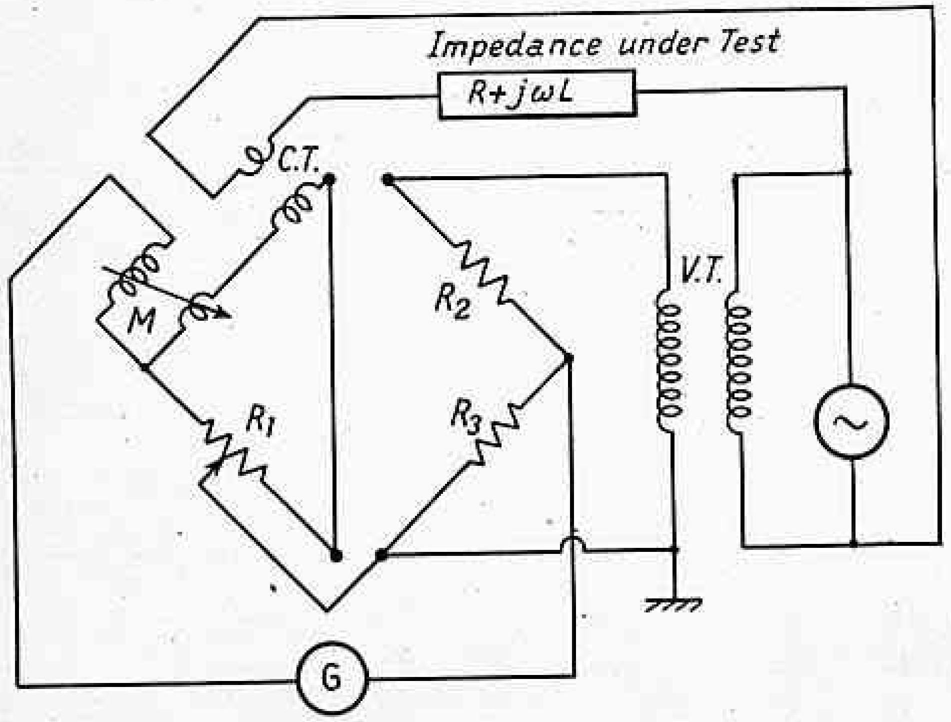


FIG. 119. KURIYAMA METHOD OF IMPEDANCE MEASUREMENT

inductances, and, when so modified, the method is a very good one for this purpose.

Kuriyama Method. B. Hague (Ref. (1), 4th Edition, p. 433) has described an interesting method, due to M. Kuriyama, of measuring the value of an impedance which is carrying a large current at high voltages. Instrument transformers are used to isolate the bridge from the high-voltage circuit. The connections of the method are given in Fig. 119 in which *C.T.* and *V.T.* are the current and voltage transformers whose ratios and phase angles (see Chapter XIX) are K_c and β (for *C.T.*) and K_v and γ (for *V.T.*).

If the impedance of the primary of the current transformer is negligible compared with that under test, it can be shown that

$$R \doteqdot \frac{K_v}{K_c} \cdot \frac{R_2 + R_3}{R_3} [R_1 + \omega M(\beta - \gamma)]$$

$$\doteqdot \frac{K_v}{K_c} \cdot \frac{R_2 + R_3}{R_3} \cdot R_1$$

and also $$L \doteqdot -\frac{K_v}{K_c}\cdot\frac{R_2+R_3}{R_3}\left[M - \frac{R_1}{\omega}(\beta-\gamma)\right]$$
$$\doteqdot -\frac{K_v}{K_c}\cdot\frac{R_2+R_3}{R_3}.M$$

Measurement of Mutual Inductance. The simplest method of measuring mutual inductance consists of passing an alternating current—measured by an ammeter—through the primary of the mutual inductance and observing the voltage induced in the secondary by means of an electrostatic voltmeter. It is important that

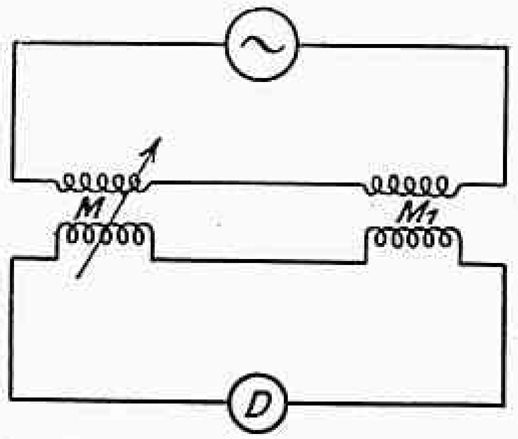


FIG. 120. FELICI'S METHOD OF MEASURING MUTUAL INDUCTANCE

the current shall have a purely sinusoidal wave-form, since harmonics may introduce serious errors.

If the current in the primary is given by

$$i = I_{max}\sin\omega t$$

then the induced voltage in the secondary will be

$$e = M\frac{di}{dt} = MI_{max}\,\omega\cos\omega t$$

or, taking r.m.s. values of current and voltage,

$$E = \omega MI$$

from which $$M = \frac{E}{\omega I}$$

Since $\omega = 2\pi \times$ frequency, the frequency of the supply should be accurately measured.

Felici's Method. If a variable standard mutual inductometer is available this method is an improvement upon the above. It can, however, only be used for the measurement of mutual inductance within the range of the standard inductometer. The connections are shown in Fig. 120, where M_1 is the mutual inductance to be measured and M is a variable standard mutual inductance.

An alternating current is passed through the two primary coils in series, and the secondary coils are connected in opposition. The standard inductance M is varied until the vibration galvanometer D shows no deflection. Then, the reading of the standard gives the mutual inductance of M_1.

Some difficulty may be encountered—especially at high frequencies—in obtaining zero deflection of the detector D. The self-capacity of the mutual inductances, and eddy currents induced in circuits, or metal parts, in the vicinity, may render it impossible to obtain exact balance, only a minimum deflection being obtainable.

Campbell (Ref. (3)) has discussed these effects very fully and shows how they may be taken into account.

MEASUREMENT OF A MUTUAL-INDUCTANCE AS A SELF-INDUCTANCE. It was pointed out in Chapter V that if two coils, of self-inductances

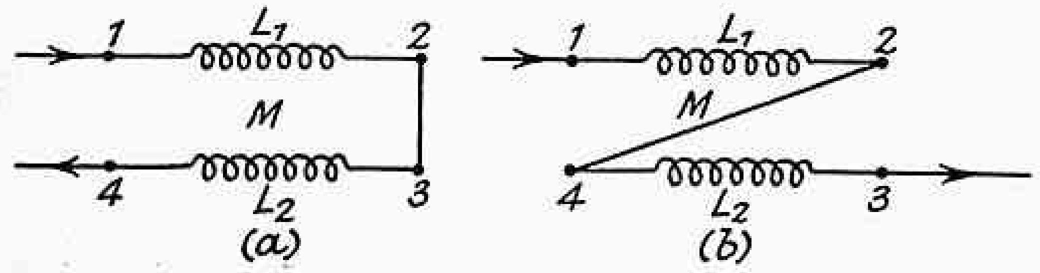


FIG. 121. MUTUAL INDUCTANCE CONNECTED AS A SELF-INDUCTANCE

L_1 and L_2, are connected in series, and if the mutual-inductance between them is M, then the self-inductance of the arrangement is given by $L = L_1 + L_2 \pm 2M$, the alternative sign depending upon the connections and on the relative positions of the two coils.

A simple method of obtaining the mutual-inductance M is to measure—by one of the methods already described—the self-inductance of the combination, first with the two coils connected in series, as in Fig. 121 (*a*), and then when connected as shown in Fig. 121 (*b*). In the first case $L = L_1 + L_2 + 2M$, and in the second $L' = L_1 + L_2 - 2M$, where L and L' are the measured self-inductances. By subtraction

$$L - L' = 4M \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

or $$M = \frac{L - L'}{4}$$

MEASUREMENT BY BALLISTIC GALVANOMETER. The secondary winding of the mutual-inductance is connected to a ballistic galvanometer, and a current of I amp. is passed through the primary winding. Upon reversal of the current I an average e.m.f. of $\frac{2MI}{t}$ volts is induced in the secondary winding, where M is the mutual-inductance in henries and t is the time in seconds taken for the reversal of the current I. The average current in the galvanometer circuit will be

$\frac{2MI}{tR}$ amp., where R is the total resistance of the ballistic galvanometer circuit. Thus the quantity of electricity passed through the ballistic galvanometer during the reversal is $\frac{2MI}{R}$ coulombs. Now, the equation giving the deflection of a ballistic galvanometer when a quantity of electricity Q passes through it is

$$Q = \frac{T}{\pi} \cdot K\left(1 + \frac{\lambda}{2}\right) \sin\frac{\theta}{2} \text{ (see Chap. IX)}$$

where T = time in seconds of one complete vibration of the galvanometer moving system

K = the galvanometer constant

θ = the "throw" of the galvanometer

λ = the logarithmic decrement of the galvanometer vibration

Hence, the mutual-inductance M is given by

$$M = \frac{RT}{2\pi I} \cdot K\left(1 + \frac{\lambda}{2}\right) \sin\frac{\theta}{2} \quad . \quad . \quad . \quad . \quad (152)$$

Maxwell's Method. The connections for the comparison of two

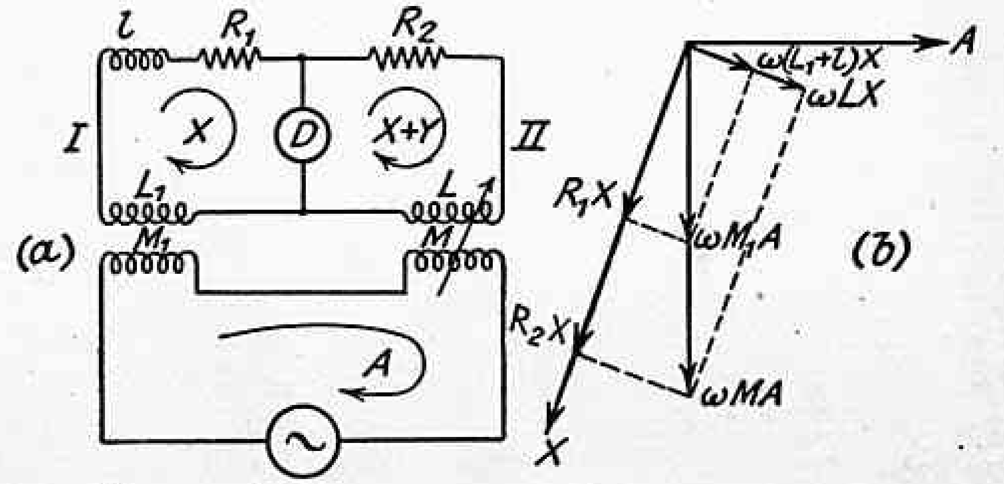


FIG. 122. MAXWELL'S METHOD FOR THE COMPARISON OF TWO MUTUAL-INDUCTANCES

unequal mutual-inductances are shown in Fig. 122 (*a*). M_1 is the mutual-inductance to be compared with the standard variable inductometer M. L_1 and L are the self-inductances of their secondary windings. R_1 and R_2 are the total resistances of the two branches, and l is a variable self-inductance inserted in either branch I or branch II to obtain exact balance. The final balance is obtained by successive adjustments of l, R_1, and R_2.

Theory. Using the mesh currents as in Fig. 122 (*a*), the mesh equations are—

Mesh I.

$$R_1X + j\omega(L_1 + l)X - DY + j\omega M_1A = 0$$

or

$$X[R_1 + j\omega(L_1 + l)] - DY + j\omega M_1A = 0$$

Mesh II.

$$(R_2 + j\omega L)(X + Y) + DY + j\omega MA = 0$$

or

$$(R_2 + j\omega L)X + (R_2 + D + j\omega L)Y + j\omega MA = 0$$

where D = impedance of the detector circuit.

Since the detector current Y is zero at balance, we have

$$X[R_1 + j\omega(L_1 + l)] + j\omega M_1A = 0 \quad . \quad . \quad . \quad \text{(i)}$$

$$X(R_2 + j\omega L) + j\omega MA = 0 \quad . \quad . \quad . \quad \text{(ii)}$$

Substituting in (i) for A from (ii),

$$X[R_1 + j\omega(L_1 + l)] - M_1\frac{(R_2 + j\omega L)X}{M} = 0$$

or

$$R_1 + j\omega(L_1 + l) = \frac{M_1}{M}(R_2 + j\omega L)$$

Equating real and imaginary quantities, we have

$$R_1 = \frac{M_1}{M}R_2 \quad \text{or} \quad \frac{R_1}{R_2} = \frac{M_1}{M}$$

and

$$L_1 + l = \frac{M_1}{M}L \quad \text{or} \quad \frac{L_1 + l}{L} = \frac{M_1}{M}$$

Hence

$$\frac{M_1}{M} = \frac{R_1}{R_2} = \frac{L_1 + l}{L} \quad . \quad . \quad . \quad . \quad (153)$$

Fig. 122 (*b*) gives the vector diagram for the network under balance conditions. Then, since $Y = 0$, the current in both of the branches I and II will be X.

Campbell's Method of Comparing Two Unequal Mutual-inductances. Fig. 123 gives the connections of the network for this method. M is the unknown mutual-inductance whose primary winding has self-inductance L. M_1 is a standard mutual inductometer with self-inductance L_1 in its primary winding. Suppose that M is greater than the maximum value of M_1, then a variable self-inductance, in series with the primary of the unknown, is inserted to make $L + l$ greater than L_1. R_1, R_2, R_3, and R_4 are the resistances of the four arms.

The switches S_1 and S_2 are first thrown on to contacts *aa*, so as to exclude from the detector circuit the secondaries of the mutual inductances. The bridge is then balanced by varying the resistances and the self-inductance l. Then, as shown in connection with Maxwell's method for the comparison of self-inductances

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{L + l} \quad . \quad . \quad . \quad . \quad (154)$$

The secondaries of the mutual-inductances are then connected in series with the detector, and in opposition to one another. This is done by throwing over switches S_1 and S_2 on to contacts *bb*. Balance is then again obtained by adjusting the variable standard M_1, the four branches I, II, III, and IV being left unaltered.

Theory. Taking mesh currents X, $X + Y$, and A, as in the figure, we have

Mesh I.

$$(R_1 + j\omega L_1)(X - A) + [R_2 + j\omega(l + L)]X - DY + j\omega MX - j\omega M_1(X - A) = 0$$

or $(R_1 + j\omega L_1 - j\omega M_1)(X - A) + [R_2 + j\omega(l + L) + j\omega M]X - DY = 0$

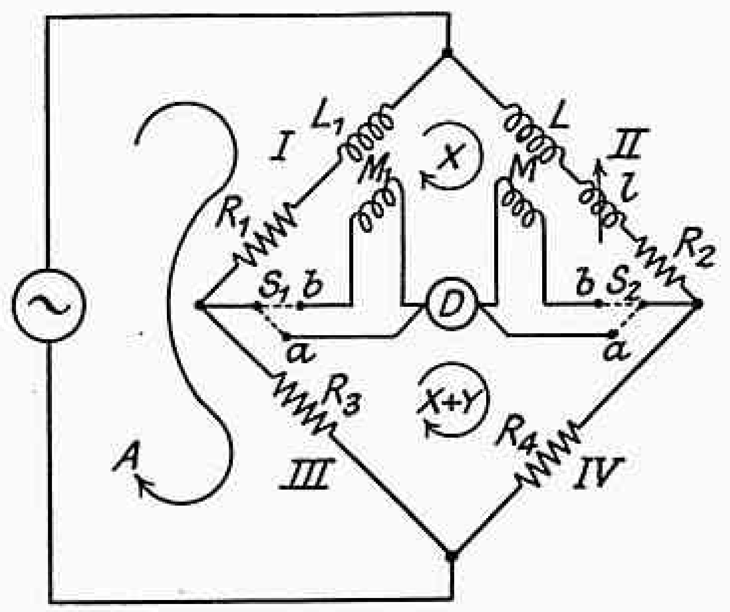


FIG. 123. CAMPBELL'S METHOD FOR THE COMPARISON OF TWO MUTUAL-INDUCTANCES

Mesh II.

$$R_4(X + Y) + R_3(X + Y - A) + YD - j\omega MX + j\omega M_1(X - A) = 0$$

or $(R_4 - j\omega M)X + (R_4 + R_3 + D)Y + (R_3 + j\omega M_1)(X - A) = 0$

where D = total impedance of the detector circuit.

When $Y = 0$,

$$(R_1 + j\omega L_1 - j\omega M_1)(X - A) = -X[R_2 + j\omega(l + L) + j\omega M]$$

and $(R_3 + j\omega M_1)(X - A) = -X(R_4 - j\omega M)$

By division

$$\frac{R_1 + j\omega L_1 - j\omega M_1}{R_3 + j\omega M_1} = \frac{R_2 + j\omega(l + L) + j\omega M}{R_4 - j\omega M}$$

Hence

$$R_1R_4 + j\omega L_1R_4 - j\omega M_1R_4 - j\omega MR_1 + \omega^2L_1M$$
$$= R_2R_3 + j\omega R_3(l + L) + j\omega MR_3 + j\omega M_1R_2 - \omega^2M_1(l + L)$$

Equating real and imaginary quantities

$$R_1R_4 + \omega^2L_1M = R_2R_3 - \omega^2M_1(l + L) \quad . \quad . \quad \text{(i)}$$
$$L_1R_4 - M_1R_4 - MR_1 = R_3(l + L) + MR_3 + M_1R_2 \quad . \quad \text{(ii)}$$

Using the conditions for the preliminary balance, viz. $\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{l + L}$ we have, from Equation (i),

$$L_1M = -M_1(l + L) \text{ or } -\frac{M}{M_1} = \frac{l + L}{L_1} = \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

and from Equation (ii),

$$-M_1(R_4 + R_2) = M(R_1 + R_3) \text{ or } -\frac{M}{M_1} = \frac{R_2 + R_4}{R_1 + R_3}$$

Hence, finally, the balance conditions are

$$-\frac{M}{M_1} = \frac{l + L}{L_1} = \frac{R_2 + R_4}{R_1 + R_3} = \frac{R_2}{R_1} = \frac{R_4}{R_3} \quad . \quad . \quad (155)$$

Heydweiller's Modification of Carey Foster's Method. The connections of the method are as in Fig. 124 (*a*). M is the mutual-inductance to be measured, having a self-inductance L in its secondary

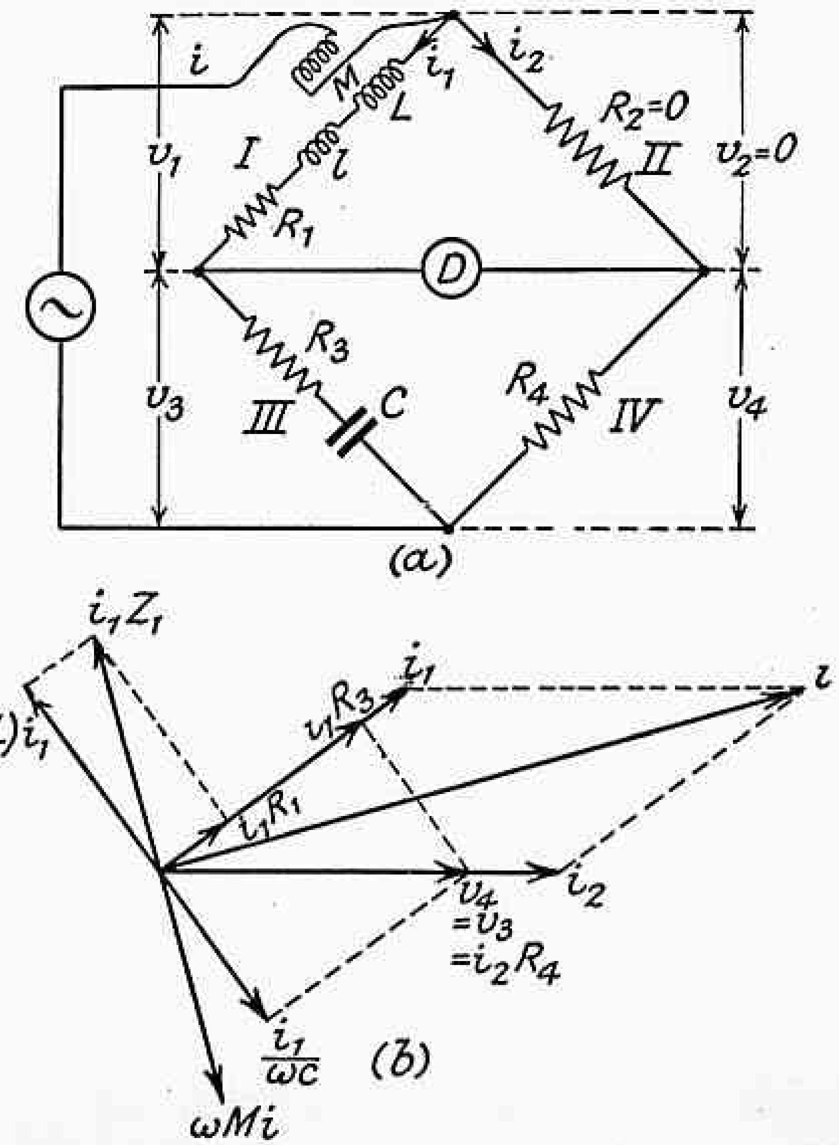


FIG. 124. HEYDWEILLER'S MODIFICATION OF CAREY FOSTER'S METHOD

winding; l is an additional self-inductance which may be necessary to obtain balance of the bridge. C is a standard capacitor. R_3 and R_4 are non-inductive resistances and R_1 the total resistance of arm I. The resistance R_2 is made zero in Heydweiller's modification of the original Carey Foster bridge.

When R_2 is zero the resistance R_4 is obviously connected directly across the supply (neglecting the primary of M). For this reason

R_4 is often a non-inductive, oil-cooled standard resistance. Balance is obtained by varying R_3, R_4, and C. The primary of the mutual-inductance must be connected so that the voltage induced by it in arm I neutralizes the volt drop due to the current i_1 in this branch, since, when $R_2 = 0$, the volt drop in branch I must be zero for balance of the bridge to be obtained.

Theory. At balance

$$i_1[R_1 + j\omega(l + L)] - j\omega Mi = i_2R_2$$

or, since $i = i_1 + i_2$

$$i_1[R_1 + j\omega(l + L) - j\omega M] = i_2[R_2 + j\omega M]$$

Also,

$$i_1\left[R_3 - \frac{j}{\omega C}\right] = i_2R_4$$

Therefore,

$$\frac{R_1 + j\omega(l + L) - j\omega M}{R_3 - \frac{j}{\omega C}} = \frac{R_2 + j\omega M}{R_4}$$

or,

$$R_1R_4 = R_2R_3 + \frac{M}{C}$$

$$\omega(l + L)R_4 - \omega MR_4 = \omega MR_3 - \frac{R_2}{\omega C}$$

Thus,

$$M = C(R_1R_4 - R_2R_3)$$

and

$$(l + L) = M\left(1 + \frac{R_3}{R_4}\right) - \frac{R_2}{\omega^2 CR_4}$$

When $R_2 = 0$, the expression for $(l + L)$ is made independent of frequency, since the second term is then zero and

$$(l + L) = M\left(1 + \frac{R_3}{R_4}\right) \quad . \quad . \quad . \quad . \quad (156)$$

Fig. 124 (*b*) gives the vector diagram for balance conditions, when $R_2 = 0$. The vector i_1z_1 representing the volt drop in the impedance (z_1) of branch I, is counterbalanced by the vector ωMi, representing the induced voltage in the secondary of the mutual-inductance, so that v_1 is zero.

Measurement of Capacitance. Although the commonest, and usually the best, methods of measuring capacitance are the alternating-current bridge methods, the apparatus required for such methods may not always be available. Under such circumstances one of the following methods might be used.

AMMETER AND VOLTMETER METHOD. If an alternating voltage of pure sine wave-form is applied to a capacitor of capacitance C farads, a current of ωCV amp. will flow, where V is the r.m.s. value of the applied voltage and $\omega = 2\pi \times$ frequency. If the current is measured by a low-reading ammeter and the voltage across the capacitor by an electrostatic voltmeter, the capacitance can be determined in terms of the readings of these instruments and of the frequency.

Instead of measuring the current by an ammeter, a non-inductive

resistance of known value may be connected in series with the capacitor, and the volt drop across this resistance measured by the voltmeter. The current is then given by the voltmeter reading divided by the series resistance.

If the voltage wave-form contains harmonics of appreciable magnitude a correction may be made for this by multiplying the measured value of the capacitance, obtained as above, by the factor

$$\sqrt{\frac{V_1^2 + V_3^2 + V_5^2 + \ldots}{V_1^2 + 9V_3^2 + 25V_5^2 + \ldots}}$$

where V_1, V_3, V_5, etc., are the values of the various components of

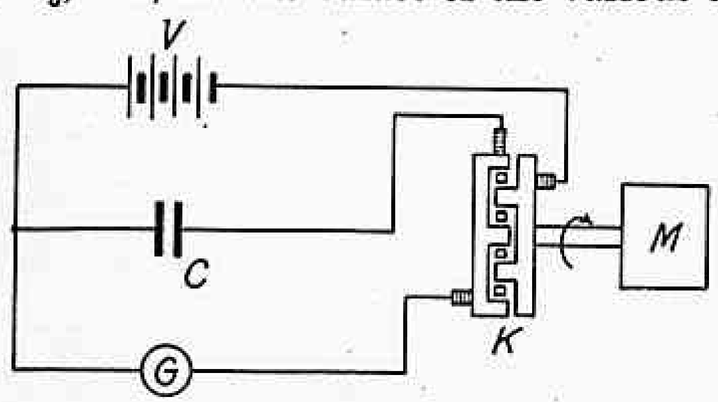


FIG. 125. FLEMING AND CLINTON'S COMMUTATOR METHOD FOR CAPACITANCE MEASUREMENTS

the voltage wave-form. It is important in measuring capacitance to bear in mind the fact that, since the capacitance reactance is $\frac{1}{2\pi fC}$, the reactance to the harmonics is less than the reactance to the fundamental of the voltage wave, and thus the current wave is not of the same shape as the voltage wave, the harmonics being accentuated (see Chapter XV).

FLEMING AND CLINTON'S COMMUTATOR METHOD. The connections for this method—which is a method using direct current—are shown in Fig. 125. C is the capacitor whose capacitance is to be measured, G is a moving-coil galvanometer whose natural period of vibration is large compared with the time of charge and discharge of the capacitor.

Commutator Construction. The latter operation is performed by the commutator K, the connections to which are as shown. This commutator consists of three metal barrels, insulated from one another, and mounted on one shaft as shown. The two outer barrels, which are connected, through brushes pressing on them, to one terminal of the battery and one terminal of the galvanometer respectively, each have the same even number of lugs on their peripheries. The inner barrel has teeth projecting radially and fitting in between these lugs. The commutator is driven at a constant speed by a small motor direct-coupled to it, a counter being geared to the shaft for the purpose of speed measurement. A third brush, connected to one terminal of the capacitor, presses on the rim of the commutator as shown. As the commutator rotates this brush makes contact, first with the barrel connected to the battery—which charges the capacitor to the voltage V of the battery—and then with the barrel connected to one terminal of the galvanometer—which discharges the capacitor. The third inner barrel is provided to ensure smooth running of this brush.

A commutator of this type, driven by a phonic motor to ensure a very steady speed, is manufactured by Messrs. Muirhead & Co.

Since the time of charge and discharge of the capacitor is small compared with the period of the galvanometer, the latter is continuously deflected.

Let this deflection correspond to a current of I amp. in the galvanometer. Then, if Q is the charge (in coulombs) given to the capacitor at each charge, and N is the number of charges per second, the quantity of electricity discharged through the galvanometer is NQ coulombs per second.

Thus the current $\quad I = NQ$

But $\quad Q = CV$ where V is the battery voltage.

Therefore $\quad I = NCV$

or
$$C = \frac{I}{NV} \text{ farads} \qquad (157)$$

Leakage in the capacitor may be detected by connecting the galvanometer in series with the battery to measure the charging current, a short-circuiting wire replacing the galvanometer in the discharge circuit. The capacitance of the capacitor, determined from $\frac{I'}{NV}$, where I' is the charging current, should be the same as the previously determined value if leakage is negligible.

MAXWELL'S COMMUTATOR BRIDGE METHOD has already been described in Chapter II, page 67.

BALLISTIC GALVANOMETER METHOD. In this method the capacitor is charged to a known voltage V by means of a battery, and then discharged through a ballistic galvanometer, the connections being the same as those of Fig. 125, except that a key replaces the commutator. The quantity of electricity (in coulombs) discharged by the capacitor is then given by

$$Q = \frac{T}{\pi} . K \left(1 + \frac{\lambda}{2}\right) \sin \frac{\theta}{2}$$

Then
$$C = \frac{Q}{V}$$

This method can also be used for the comparison of an unknown capacitance with a standard by comparing the quantities of electricity discharged through the Ballistic galvanometer when charged to the same voltage in each case.

With direct-current methods of measurement, the time of charge, and of discharge, is important in the case of absorptive capacitors, since the measured value of the capacitance will depend to some extent upon these times (see Chapter IV).

A.C. BRIDGE METHODS. *De Sauty Method.* This method is the simplest way of comparing two capacitances. When used on a.c. the connections are as in Fig. 126.

C_1 = capacitor whose capacitance is to be measured

C_2 = a standard capacitor

R_1 and R_2 = non-inductive resistances

Balance is obtained by varying either R_1 or R_2.

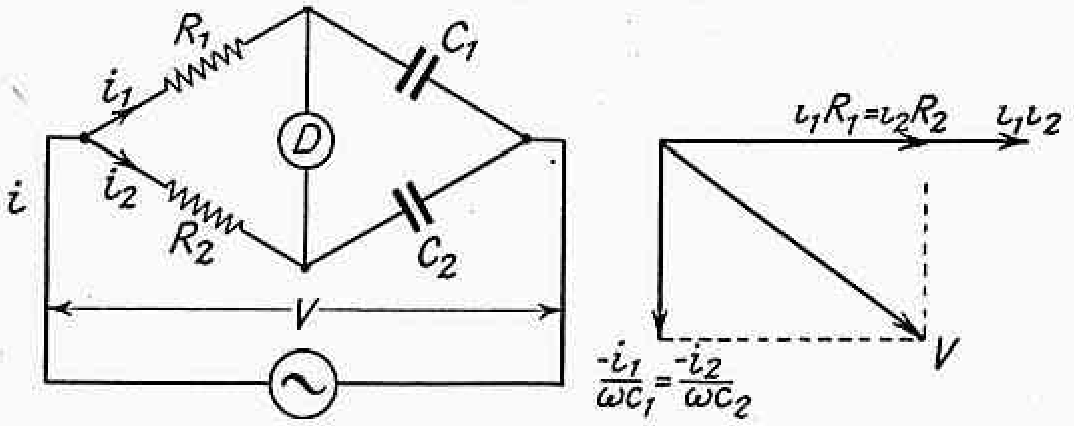


FIG. 126. DE SAUTY BRIDGE

At balance
$$i_1R_1 = i_2R_2$$

and
$$-\frac{j}{\omega C_1} i_1 = -\frac{j}{\omega C_2} i_2$$

Thus
$$\frac{R_1}{R_2} = \frac{C_2}{C_1}$$

or
$$C_1 = C_2 \frac{R_2}{R_1} \qquad (158)$$

For maximum sensitivity, C_2 should be equal to C_1. The advantage of the simplicity of this method is largely nullified by the fact that it is impossible to obtain a perfect balance if the capacitors are not both free from dielectric loss. Only in the case of air capacitors can a perfect balance be obtained.

If two imperfect capacitors are to be compared, the bridge is modified by connecting resistances in series with them, as in Fig. 127 (*a*). R_3 and R_4 are the series resistances, while r_1 and r_2 are small resistances representing the loss components of the capacitors. Balance is obtained by variation of the resistances R_1, R_2, R_3, R_4. At balance

$$i_1R_1 = i_2R_2$$

$$i_1\left[R_3 + r_1 - \frac{j}{\omega C_1}\right] = i_2\left[R_4 + r_2 - \frac{j}{\omega C_2}\right]$$

from which it follows that

$$\frac{R_1}{R_2} = \frac{R_3 + r_1}{R_4 + r_2} = \frac{C_2}{C_1}$$

The vector diagram of Fig. 127 (*b*) shows the relative positions of the vector quantities under balance conditions. The angles δ_1 and δ_2 are the phase angles of capacitors C_1 and C_2 respectively. Obviously

$$\tan \delta_1 = \frac{r_1}{\dfrac{1}{\omega C_1}} = r_1 \omega C_1$$

and $$\tan \delta_2 = r_2 \omega C_2$$

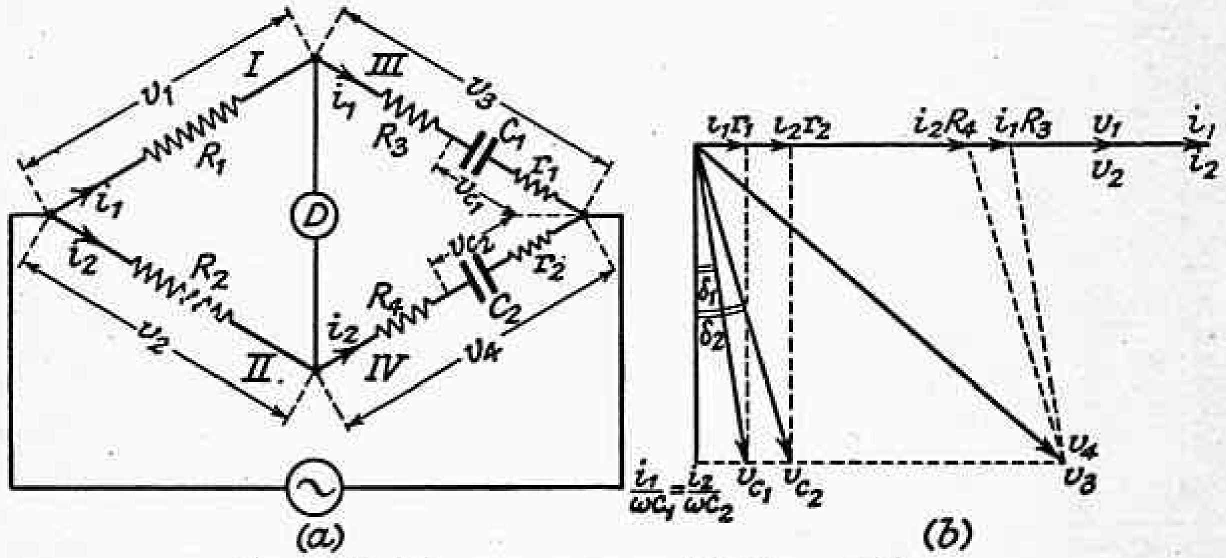


FIG. 127. MODIFICATION OF DE SAUTY BRIDGE

From the condition $\dfrac{C_2}{C_1} = \dfrac{R_3 + r_1}{R_4 + r_2}$

we have $$C_2 r_2 - C_1 r_1 = C_1 R_3 - C_2 R_4$$

or $$\omega C_2 r_2 - \omega C_1 r_1 = \omega(C_1 R_3 - C_2 R_4)$$

i.e. $$\tan \delta_2 - \tan \delta_1 = \omega(C_1 R_3 - C_2 R_4)$$

Since $$\frac{C_2}{C_1} = \frac{R_1}{R_2}$$

$$\tan \delta_2 - \tan \delta_1 = \omega C_1 \left(R_3 - \frac{R_1 R_4}{R_2} \right) \quad . \quad . \quad . \quad (159)$$

from which expression the phase angle of one capacitor can be found in terms of the phase angle of the other.

This method is due to Grover (Ref. (16)).

In the vector diagram the angles δ_2 and δ_1 are exaggerated for convenience in drawing. These angles are usually small, and thus it is usually a sufficiently good approximation to write

$$\tan (\delta_2 - \delta_1) = \omega C_1 \left(R_3 - \frac{R_1 R_4}{R_2} \right) \quad . \quad . \quad . \quad (160)$$

Grover's Series Inductance Method. This method is somewhat similar to the above, inductances being used instead of series resistances. It is a useful method for the determination of the capacitance and power factor of a small capacitor by comparison with a standard capacitor. The connections and vector diagram are given in Fig. 128.

L_1 and L_2 are variable standard inductances. R_1 and R_2 are the resistances of the arms in which these inductances are situated. Non-inductive resistances may be connected in series with L_1 and

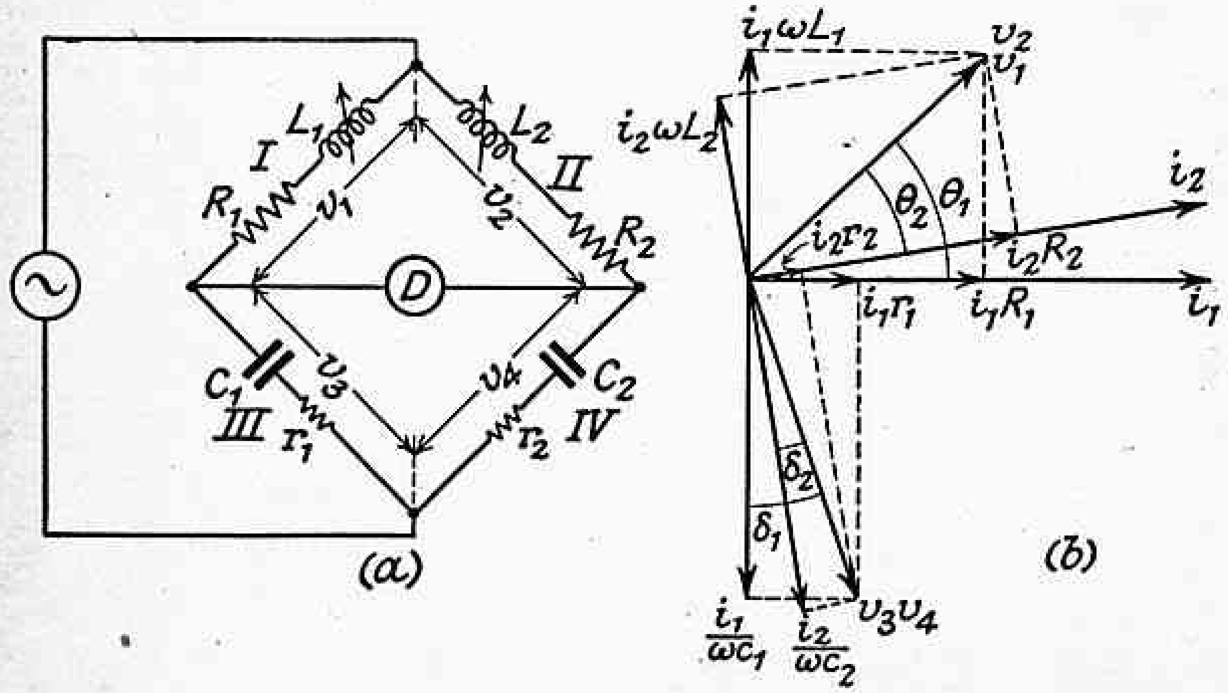


FIG. 128. GROVER'S SERIES INDUCTANCE METHOD

L_2, in which case R_1 and R_2 are the resistances of the arms, including the resistances of L_1 and L_2. C_1 is the unknown capacitance. C_2 is the standard capacitor, while r_1 and r_2 are resistances representing the loss components of these capacitors.

Balance is obtained by variation of the inductances L_1 and L_2, and of the series resistances in these arms if necessary.

At balance $(R_1 + j\omega L_1) i_1 = (R_2 + j\omega L_2) i_2$

and $$\left(r_1 - \frac{j}{\omega C_1} \right) i_1 = \left(r_2 - \frac{j}{\omega C_2} \right) i_2$$

From which we have

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} + \frac{\omega^2 C_1}{R_1} (L_1 r_2 - L_2 r_1) \quad . \quad . \quad (161)$$

In most cases, when L_1 and L_2 are not large, the second term may be neglected, whence

$$\frac{C_1}{C_2} \doteqdot \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad . \quad (162)$$

The phase angles δ_1 and δ_2 of the two capacitors may be obtained from the expressions

$$\tan \delta_1 = r_1 \omega C_1$$

and

$$\tan \delta_2 = r_2 \omega C_2$$

Substituting the relationship

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}$$

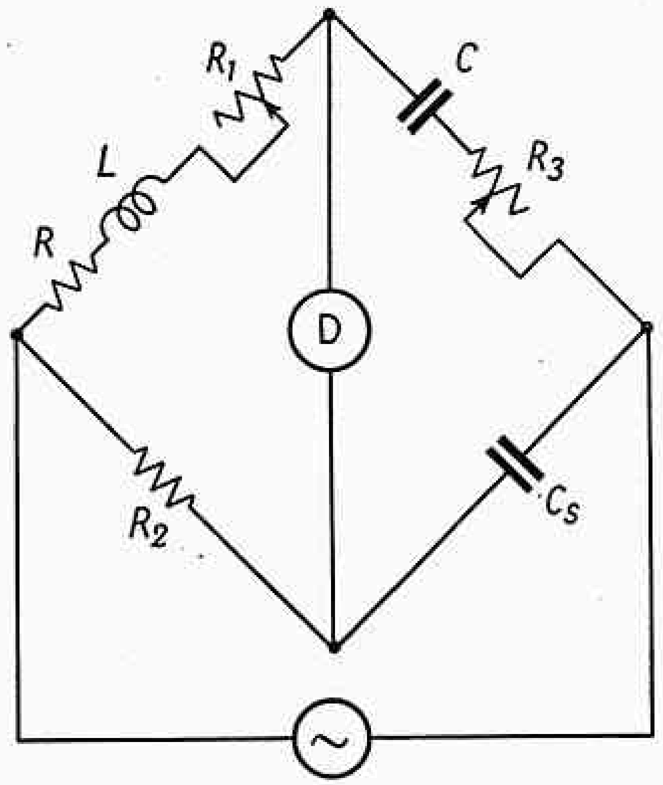


FIG. 129. OWEN'S BRIDGE

in the first of the balance conditions, we have

$$r_2 - \frac{R_2}{R_1} r_1 = \frac{L_2}{R_1 C_1} - \frac{L_1}{C_2 R_1}$$

or

$$r_2 - \frac{C_1}{C_2} r_1 = \frac{L_2}{R_1 C_1} - \frac{L_1}{C_2 R_1}$$

whence

$$\omega C_2 r_2 - \omega C_1 r_1 = \omega \left(\frac{L_2}{R_2} - \frac{L_1}{R_1} \right)$$

or

$$\tan \delta_1 - \tan \delta_2 = \omega \left(\frac{L_1}{R_1} - \frac{L_2}{R_2} \right)$$

$$= \tan \theta_1 - \tan \theta_2 \quad . \quad . \quad . \quad (163)$$

where θ_1 and θ_2 are the phase angles of the inductance arms I and II.

Owen's Bridge. This bridge, the connections for which are given in Fig. 129 is, in fact, a modification of Grover's method just described and provides a method of measuring self-inductance in terms of a standard capacitance.

The inductance under test, $R + j\omega L$, is connected in series with a variable non-inductive resistor R_1. The standard capacitance is C_s. At balance

$$\frac{R + R_1 + j\omega L}{R_2} = \frac{R_3 - \dfrac{j}{\omega C}}{-\dfrac{j}{\omega C_s}}$$

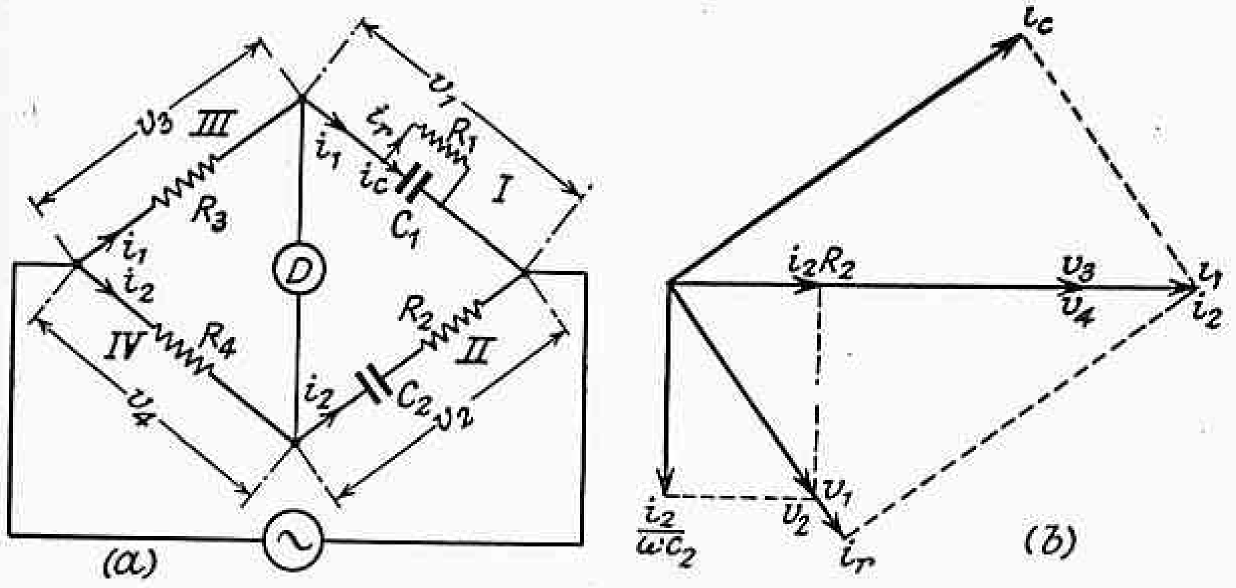


FIG. 130. WIEN BRIDGE

from which

$$R = R_2 \frac{C_s}{C} - R_1$$

and

$$L = R_2 R_3 C_s$$

This bridge, which can cover a wide range of inductance measurements with limited apparatus, is convenient to use since the balance, which is independent on frequency and wave-form, is obtained by successive adjustment of the two resistors R_1 and R_3.

Wien's Method. This method is a convenient one when an imperfect capacitor is shunted by a resistance as is the case in cable testing. In Fig. 130 (*a*), in which the connections of the bridge are shown, C_1 is the equivalent shunt capacitance of the capacitor, and R_1 the shunt resistance. The capacitor C_2 is a standard air capacitor and R_2, R_3, and R_4 are non-inductive resistances. If the unknown capacitor is not already shunted by a resistance, the resistance R_1 is placed in parallel with it. Balance is obtained by variation of the resistances R_2, R_3, and R_4.

At balance

$$i_1 \left(\frac{R_1}{1 + j\omega C_1 R_1} \right) = i_2 \left(R_2 - \frac{j}{\omega C_2} \right)$$

and

$$i_1 R_3 = i_2 R_4$$

Hence, it follows that

$$C_1 = \frac{\dfrac{R_4}{R_3} \times C_2}{1 + \omega^2 R_2^2 C_2^2} \quad . \quad . \quad . \quad (164)$$

and

$$R_1 = \frac{R_3(1 + \omega^2 R_2^2 C_2^2)}{\omega^2 R_2 R_4 C_2^2} \quad . \quad . \quad . \quad (165)$$

The vector diagram for balance conditions is shown in Fig. 130 (*b*).

OTHER BRIDGE METHODS OF MEASURING CAPACITANCE. The *Schering Bridge* method of measuring the capacitance and power factor of capacitors has already been described in Chapter IV.

Some of the methods already described earlier in this chapter for the measurement of self- or mutual-inductance in terms of capacitance form convenient methods of measuring the capacitance of a capacitor in terms of self- or mutual-inductance, if suitable inductance standards are available. Obviously, such methods may be used either way about without modification, the theory of the method remaining the same. Two such methods are *Anderson's Bridge* and the *Carey-Foster Bridge*.*

BRIDGE METHODS FOR SPECIAL PURPOSES. *Measurement of the Self-inductance of Alternating-current Resistance Standards.* It is often important that the self-inductance of heavy-current, low-resistance standards for use in alternating-current measurements should be known. The inductance of such standards is usually very small, and its measurement necessitates special methods. Wattmeter methods, using a reflecting-type wattmeter, have been devised but do not compare favourably with alternating current bridge methods, and the latter are therefore in more general use.

Campbell's Method. The connections of this method (Ref. (13)) are given in Fig. 131. The standard resistor R_s has four terminals, c_1c_2 being its "current terminals," and p_1p_2 its "potential terminals"—hence the name "four-terminal resistors" given to such standards. The self-inductance of R_s is l. M_1, M_2, and M are three variable mutual-inductances, the former two not necessarily being of known values, and the latter being a low-reading standard inductometer. R and L are the resistance and inductance of the circuit containing the secondary winding of M_1 and the primary of M_2. D is the detector. Balance is obtained by variation of the three mutual inductances as required. These should be well spaced in order to avoid mutual inductance effects between them.

Theory. At balance the sum of the voltages in the detector circuit is zero.

Thus,
$$(R_s + j\omega l)i + j\omega M i + j\omega M_2 i_1 = 0$$
or
$$i[R_s + j\omega(l + M)] + j\omega M_2 i_1 = 0$$

* Other methods of measuring both inductance and capacitance are given in Hague's *Alternating Current Bridge Methods* and in the *Dictionary of Applied Physics*, Vol. II, to which works the reader is referred for further information on the subject.

Also, in the circuit containing L and R,

$$j\omega M_1 i + i_1(R + j\omega L) = 0$$

Hence
$$\frac{R_s + j\omega(l + M)}{j\omega M_1} = \frac{j\omega M_2}{R + j\omega L}$$

$$RR_s - \omega^2 L(l + M) + j\omega L R_s + j\omega R(l + M) = -\omega^2 M_1 M_2$$

Equating real and imaginary quantities, we have

$$RR_s - \omega^2 L(l + M) = -\omega^2 M_1 M_2$$

$$RR_s = \omega^2[L(l + M) - M_1 M_2]$$

and
$$R(l + M) + LR_s = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (166)$$

Thus the self-inductance l can be found if L, R_s, R, and M are known, the values of M_1 and M_2 being unnecessary.

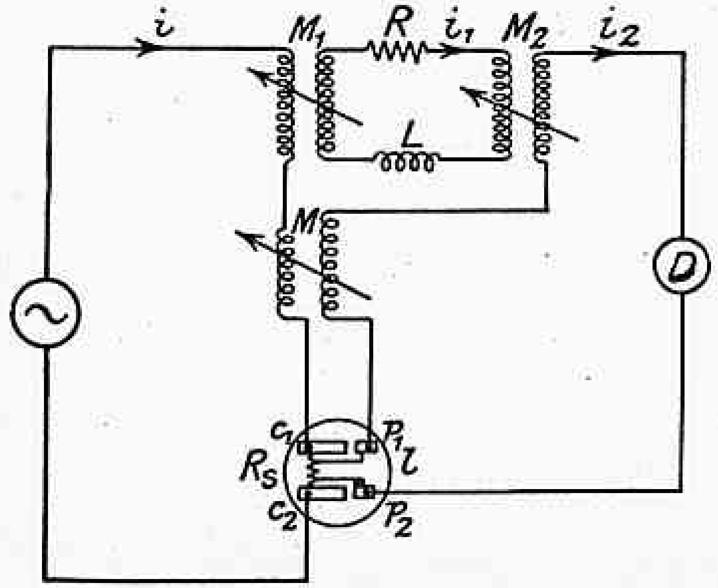


FIG. 131. CAMPBELL'S METHOD

If M_1 and M are reversed the expression for l becomes

$$\frac{M - l}{R_s} = \frac{L}{R}$$

or
$$l = M - L\frac{R_s}{R}$$

It is necessary for balance that M should be greater than l and $M_1 M_2 > \dfrac{L^2 R_s}{R}$

In the same paper Campbell describes the application of this method to the measurement of the capacitance and power factor of capacitors. For this purpose the connections of the network are the same, except that the capacitor replaces the standard resistance R_s. Balance is obtained in the same way, and it can be shown by the method used above that the balance conditions are

$$\frac{1}{C\omega^2} = M + \frac{rL}{R} \quad . \quad . \quad . \quad . \quad . \quad (167)$$

and
$$Rr = \left[M_1M_2 - \frac{L^2r}{R}\right]\omega^2 \quad . \quad . \quad . \quad . \quad (168)$$

where C is the capacitance and r the series resistance representing the loss component, of the capacitor under test. The power factor $r\omega C$ can be found from the two equations,

$$r = \frac{RM_1M_2\omega^2}{R^2 + L^2\omega^2} \quad . \quad . \quad . \quad . \quad (169)$$

and
$$\frac{1}{C\omega^2} = M + \frac{LM_1M_2\omega^2}{R^2+L^2\omega^2} \quad . \quad . \quad . \quad . \quad (170)$$

which follow from the balance conditions. If the time constant $\frac{L}{R}$ is small

$$C = \frac{1}{\omega^2 M} \quad . \quad . \quad . \quad . \quad . \quad . \quad (171)$$

to a very close approximation.

Hartshorn's Method for the Measurement of the Self-inductance of Low-resistance Standards. This method, described by L. Hartshorn (Ref. (18)), is essentially a modification of the Kelvin double bridge (see Chapter VII), and compares the phase angles of two low-resistance standards. To obtain the requisite sensitivity the supply is of telephonic frequency, and telephones are used as the detector. This is justifiable, since the frequency has little effect upon the inductance of such standards. The connections are given in Fig. 132.

S and X are the two low-resistance standards whose phase angles are to be compared, that of S being known and that of X unknown. Let their resistances be R_s and R_x and their self-inductances L_s and L_x. P, Q, p, and q are shielded non-reactive resistors, and r_1 and r_2 are low resistances—most suitably slide-wires—for fine adjustment of the resistances of the arms. C_1 and C_2 are variable air capacitors, shunting Q and q. The leads from the potential terminals of S and X are run close together to avoid inductive loops. R and R' are variable resistances with their connecting point earthed as shown. If the ratio $\frac{R}{R'}$ is made equal to $\frac{S}{X}\left(=\frac{P}{Q}=\frac{p}{q}\right)$ the potential of the detector D is that of earth, although it is not actually earthed. By this means earth-capacitance effects between the telephone detector D and the operator's head are eliminated. This point will be discussed further in connection with the Wagner earth device (see page 231).

Operation. In operation the following procedure is adopted—

1. Adjust r_1 and C_1, with connections as shown, until balance of the bridge—i.e. silence in the telephones—is obtained.
2. Remove the link M, connecting S and X, and obtain balance again by adjusting r_2 and C_2.

3. Replace the link and obtain balance again by adjusting r_1 and C_1.
4. Repeat the above procedure until balance is obtained with the link either in or out. Let the reading of C_1 for this condition be C_1'.
5. Remove the link, transfer the supply to the points T_1T_2, and adjust C_1 until balance is again obtained. Let the setting of C_1 now be C_1''.

Then the expression for the time constant of X is

$$\frac{L_x}{R_x} = \frac{L_s}{R_s} - \frac{1}{2}\,Q(C' + C'') - \frac{L_P}{P} + \frac{L_Q}{Q} \quad . \quad . \quad (172)$$

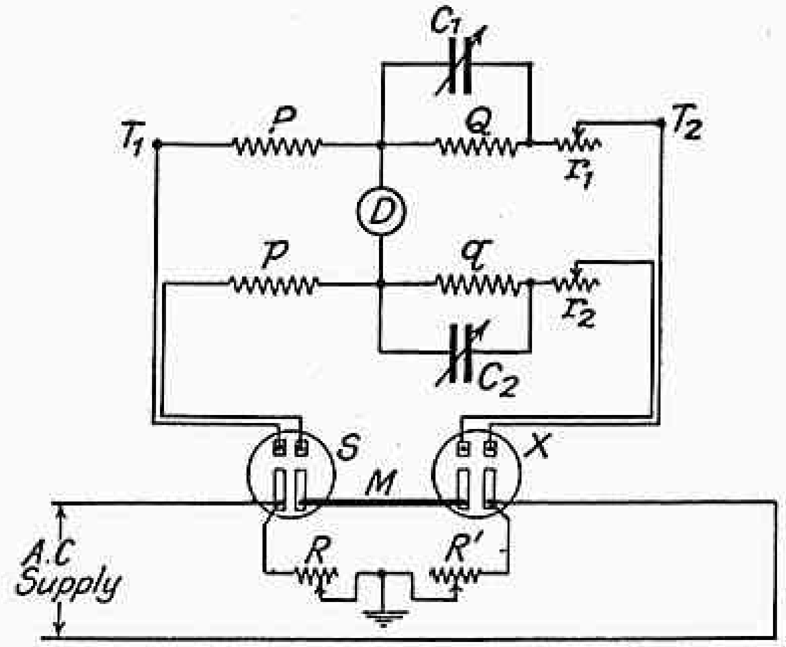


FIG. 132. HARTSHORN'S METHOD FOR THE MEASUREMENT OF SELF-INDUCTANCE OF LOW-RESISTANCE STANDARDS

where L_P and L_Q are the self-inductances, and P and Q the resistances of P and Q.

The original paper should be consulted for a fuller consideration of the method.

Double-ratio Bridges with Inductively-coupled Ratio Arms. The Wagner earth device (see p. 231), which is used to bring the terminals of the detector branch of a bridge to earth potential, has the disadvantage that its branches must be adjusted with every change of earth capacitance, i.e. with every bridge measurement. The two balances—that of the bridge network itself and that of the Wagner earth—are mutually dependent so that a series of alternate balances must be made to arrive at a final simultaneous balance.

As an alternative method of reducing earth capacitance effects A. D. Blumlein (Brit. Pat. No. 323037) proposed replacement of the resistive arms of the bridge network by a pair of tightly-coupled inductors. By this means it is possible to bring the three terminals of these two arms to almost the same potential so that, by connecting the centre terminal to earth, earth capacitance effects at all four corners of the bridge can be eliminated.

A development of this proposal is the use of double-ratio bridges with two sets of inductively-coupled ratio arms. A full description of the construction and application of such bridges has been given by H. A. M. Clark and P. B. Vanderlyn (Ref. (42)).

An interesting application of the double-ratio bridge is in a height indicator for aircraft. W. L. Watton and M. E. Pemberton (Ref. (43)) describe a direct-capacitance altimeter which uses the change in capacitance, with height, between the aircraft and the earth as a measure of low altitudes (up to about 200 feet). The sensitivity of the bridge for this purpose can be made sufficient to detect capacitance changes of 1 $\mu\mu\mu$F.

Sources of Error in Bridge Measurements, and Precautions. Although it is best, in considering the sources of error in a.c. bridge measurements, to treat each particular method separately, the space available here does not permit such consideration. The possible sources of error are considered in general below. For more detailed treatment the reader is referred to Dr. B. Hague's excellent book on *Alternating Current Bridge Methods*, to which work the author has referred for some of the information contained in this chapter.

Stray Field Effects. Errors may be caused by the fact that the various arms of the bridge network may be—unintentionally—either magnetically or electrostatically coupled, due to the "stray" magnetic or electrostatic fields existing round apparatus included in the network. When such effects are present the simple theory of the network—considering each arm as being entirely separate from the other arms except where intentionally coupled together—is no longer quite true. Under these conditions the detector may indicate balance—or zero deflection—when balance conditions have not really been obtained.

In networks containing two or more self- or mutual-inductances there may be mutual-inductance between two of them in different bridge arms. Usually, stray magnetic fields will be more important than electrostatic stray fields when inductances and resistances only are present. If the bridge contains capacitors the opposite is the case, errors then being caused by inter-capacitance between the various arms. Loops formed by the leads connecting a piece of apparatus to the bridge may also introduce errors owing to their inductance. In inductance measurements the leads should be twisted together to avoid such loops, while in capacitance measurements the leads should be separated from one another to avoid capacitance between them. As already pointed out, it is possible in some cases to eliminate the effects of the leads by making two measurements on the bridge—one with the apparatus under test in circuit, and one with the piece of apparatus short circuited—or by substituting a variable standard for the unknown and adjusting it to give balance with the same bridge settings as when the unknown was in circuit.

To avoid errors due to magnetic coupling between arms the inductance coils used should be wound astatically—i.e. having no appreciable stray magnetic field—or magnetic screening may be adopted. For such screening a thin sheet of high permeability material is placed so as to prevent the stray magnetic field from reaching the apparatus in the other arms. The inductance coils should, also, be arranged at some distance from one another and from the bridge. Mutual inductance between two pieces of apparatus can often be detected by altering their relative positions and observing if the bridge settings for balance are altered thereby.

In some cases errors are caused by direct induction effects between the supply to the bridge, and the detector circuit, which may cause the detector to indicate the passage of a current through it when the bridge is, in reality, balanced. Such effects may be eliminated by placing the supply alternator at some distance from the bridge and by supplying the bridge through an "inter-bridge transformer." The latter is a transformer having windings which are very well insulated from one another and which have an earthed metallic screen between them. Such transformers should have a closed magnetic circuit to avoid magnetic leakage, the core often consisting of sheet steel ring punchings. Usually there are several windings on the transformer to afford a choice in the working voltage.

When telephones are being used, direct induction between apparatus in the bridge arms and the telephone circuit may produce sound in the telephones when the bridge is balanced. The presence of such direct induction may be detected by moving the head, when the bridge is almost balanced, so as to alter the plane of the telephones relative to the bridge. If no difference in sound is detected when the telephones are thus moved about it may be assumed that such induction effects are negligible. To eliminate such effects, when present, the telephones must be disconnected from the bridge and moved so that silence is obtained when the bridge is supplied with power from the alternator. Adjustment of the bridge to obtain balance must be carried out with the telephones in this position.

Errors due to electrostatic coupling between the various arms of the bridge and to earth capacitances of the various pieces of apparatus may be guarded against by electrostatic screening. The use of such screens renders these capacitance effects definite in magnitude and independent of the distribution of the apparatus forming the bridge network. By this means the effect of such inter- and earth-capacitances upon the accuracy of the bridge may be made very small.

A similar purpose is served by the various earthing devices of which that due to Wagner (Ref. (21)) is commonly used.

Wagner Earthing Device. This device is used in conjunction with most of the bridge networks previously described, and is usually very necessary if accurate results are to be obtained. Fig. 133 shows the connections of the device for use in conjunction with the general form of bridge network. Z_1, Z_2, Z_3, and Z_4 are the impedances of the bridge arms. X and Y are the two variable impedances of the Wagner earth branch, the centre point of which is earthed as shown. These impedances may consist of variable resistances and self-inductances similar to those used in the arms of the bridge proper, but not necessarily of known value. D is the telephone detector.

If the switch S is on contact b, balance of the bridge may be obtained by adjustment of the impedances Z_1, Z_2, etc. At balance, the points a and b are at the same potential, and no current should flow in the detector branch. These two points are not, however, necessarily at earth potential, and it is found that a capacitance current flows from the detector branch to the observer's head through the telephones, thus rendering complete silence in the telephones unattainable, although a point of minimum sound can be obtained.

After adjusting the bridge to give minimum sound, the switch S is thrown on to contact c, so that the telephones are then connected between point a and earth. X and Y are next adjusted until silence is obtained. Under these conditions point a must be at the same potential as earth although it is not permanently earthed.

The telephones are next connected, by the switch, back on to ab, and Z_3 and Z_4 are adjusted to give minimum sound again. The process is continued until silence is obtained with the switch on either of the positions b and c, without further adjustment of the impedances being required when the switch is thrown over from one to the other. Then all three points a, b, and c must be at earth potential. Thus the telephones are at earth potential and the capacitance effect with the observer's head is eliminated. In the double-ratio a.c. bridges mentioned on p. 230 some of the disadvantages of the use of the Wagner earth arrangement are overcome.

Leakage Errors. If the insulation between the various pieces of apparatus forming a bridge network is not good, trouble may arise through leakage currents from one arm to another. This is especially true in the case of high impedance bridges. To avoid this the apparatus used may be mounted on insulating stands.

Eddy Current Errors. Standard resistances and inductances used in bridge networks should be so constructed as to avoid variation of their values due to eddy currents when the frequency is varied. The effective resistance and

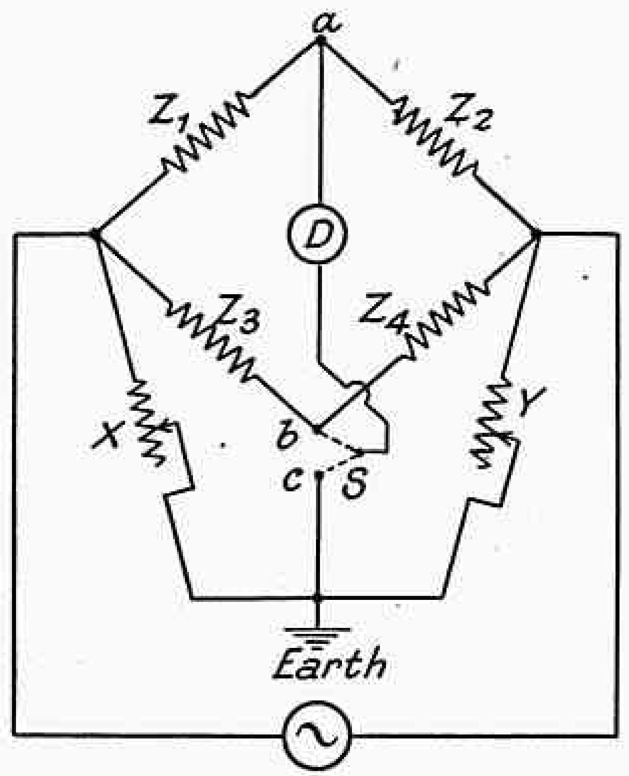


FIG. 133. CONNECTIONS OF WAGNER EARTH DEVICE

inductance of a piece of apparatus under test may vary with frequency due to this cause. Large masses of metal in the vicinity of the bridge should be avoided, as the flux produced by eddy currents induced in them may set up troublesome e.m.f.'s in parts of the network. In the case of mutual inductances eddy current effects, if present, cause the induced voltage to lag by some angle less than 90° behind the inducing current, in which case the simple theory of the network does not hold.

Residual Errors. In speaking of the resistors used in the various bridge networks the description "non-inductive" or "non-reactive" has been applied to them, indicating that their inductance and capacitance are both zero. Although resistances for such purposes are constructed so that these quantities are very small, it cannot always be assumed that they are zero. The term "residual" is used to indicate the small inherent inductance or capacitance of a resistance coil. In precise work it is sometimes necessary to take these residuals into account—for which purpose they must either be measured or calculated—in order that errors due to them shall be avoided. The self-capacitance of coils is usually only important when the coil has many turns and the supply frequency is high. The resistance and inductance of such coils are increased and reduced, respectively, due to this cause by amounts which are proportional to the square of the frequency (Refs. (19), (20)).

Frequency and Wave-form Errors. Some of the bridges previously described are independent of the frequency of the supply in the sense that the balance conditions do not involve the frequency. In such cases, therefore, the frequency of the supply is only important in its effect upon the effective resistance and inductance of the apparatus under test; and the fact that the supply wave-form contains harmonics is only important for the same reason.

In the case of networks in which the balance conditions do involve the frequency, the latter is important and must be carefully measured. The wave-form of the supply is also obviously of importance, since the bridge cannot be balanced both for the fundamental and the harmonics in the wave-form (if any) simultaneously. If telephones are employed in such bridges it will be found impossible to obtain complete silence, only a point of minimum sound being obtainable.

There are two means of circumventing this difficulty. The first is by using some form of "wave filter" such as those described by Campbell (Ref. (14)), and the second is by using a tuned detector, such as a vibration galvanometer, instead of telephones. Such detectors will not respond appreciably to frequencies other than that of the fundamental of the supply.

It is advantageous, if possible, to use in the bridge network such values of the impedances that the frequency term in the expression for the quantity to be measured is reduced to zero.

Apparatus Used in Conjunction with A.C. Bridge Networks. 1. SOURCES OF CURRENT. These may be conveniently divided into three classes: (*a*) microphone hummers, (*b*) alternators, (*c*) oscillators.

Oscillators are nowadays almost universally used but, for completeness, the other two methods will be described briefly.

(*a*) *Microphone Hummers* provide a means of obtaining a supply of constant frequency, and of reasonably pure sine wave form, by the use of comparatively simple apparatus. The power obtainable from such sources is, however, usually small. Fig. 134 illustrates the principle and connections of such a piece of apparatus due to Campbell.

A steel bar, 2·5 cm. diameter, and of length depending upon the desired frequency $\left(\text{given by length} \doteqdot \dfrac{1{,}075}{\sqrt{\text{frequency}}}\right)$ is supported at two nodal points by knife edges. This bar carries a microphone M at one end, connected to one terminal of a battery as shown. T is a transformer having three windings, an earthed screen being placed between the winding supplying the bridge and the other two windings. C_1 and C_2 are capacitors. An electromagnet, energized by two coils as shown, is placed under the centre of the bar. Coil A is for the purpose of polarizing this magnet and carries the microphone current, while coil B maintains the bar in vibration when once such vibration is started.

The frequency obtained by this means is very constant, and the wave-form is good, the disadvantage being the small amount of power available. H. W. Sullivan Ltd. make a reed hummer requiring an input of 100 mA at 6 to 8 volts.

Details of a number of other microphone hummers and of interrupters, etc., are given by Campbell (Ref. (3)) and by Hague (Ref. (1)).

(b) *Alternators* of the inductor type have the advantage of a comparatively large output, although the frequency obtainable is not so constant, nor, in general, is the wave-form so good as that obtained by other methods of supply. For measurements at commercial frequencies a motor-alternator set of the ordinary type may be used. For higher frequencies (of the order of 500 to 2,000 cycles per second) an alternator such as that due to Duddell (Ref. (24)) may be used. Different frequencies may be obtained by varying the speed

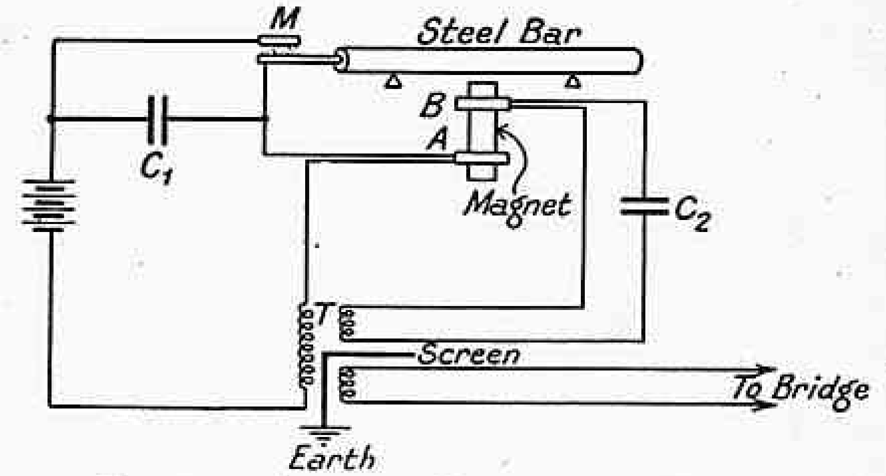


FIG. 134. CAMPBELL MICROPHONE HUMMER

of the driving motor, but constancy of frequency may require the use of an automatic speed regulator.

The most suitable type of alternator, giving a comparatively large output and good wave-form (slight 3rd harmonic), is the Duddell alternator referred to above. This machine has a rotating field system and a stator which is in the form of a smooth ring—unslotted —carrying a Gramme ring winding to avoid tooth ripples. The rotor is a steel disc of 20 cm. diameter, with 30 projecting pole pieces each carrying a magnetizing coil, these coils being held in position by wedges in the slots separating the poles, and being supplied with direct current from a battery. The alternator is driven by a motor and a link belt. Frequencies up to 2,000 cycles per second are obtainable by variation of the motor speed, the alternator speed for this frequency being 8,000 r.p.m. A motor running at 2,000 r.p.m. may be used, the required alternator speed being obtained by gearing by means of the pulleys upon which the link belt runs.

The speed of the motor may be controlled by connecting a low fixed resistance in the armature circuit, for starting purposes, and a diverter resistance in parallel with the armature itself for speed control purposes. This method, especially at the lower speeds, gives much greater stability of speed with variation of load than the more usual variable armature-series-resistance method.

As stability of speed (and therefore frequency) is of great importance, it is often a useful precaution to include in series with the bridge network, a variable resistance and a low reading ammeter, so that the load resistance may be

maintained constant by adjustment of this variable resistance whenever the impedance of the bridge network is altered. By this means the load on the driving motor is maintained constant and the speed remains steady.

The Duddell alternator as described above has an output varying from a few watts at 100 cycles to about half a kilowatt at 2,000 cycles. In a test upon a machine of this type, the following results were obtained—

Alternator field d.c. volts .	. 100
Alternator armature voltage	. from 3 volts at 100 cycles up to 55 volts at 1,000 cycles
Alternator output . .	. from 5 watts at 100 cycles to 400 watts at 1,200 cycles

Bridge Current. In connection with the supply of current to a bridge network it must be pointed out that the current-carrying capacity of the impedances forming the arms of the network should be carefully considered. It may happen that the total impedance of the two arms forming one "side" of the network (such as arms Z_1 and Z_2 in Fig. 133) is low compared with the impedances of the other two "sides." In such a case the low-impedance side may take an excessive current, with consequent damage to the apparatus in these arms. A rough rule in connection with current-carrying capacity in resistance boxes is to allow from $\frac{1}{2}$ to 1 watt per coil in circuit in the box. For example, in the case of a 10 ohm coil, allowing $\frac{1}{2}$ watt per coil, the current-carrying capacity is $\sqrt{\frac{0 \cdot 5}{10}} = 0 \cdot 22$ amp., since $I^2R = 0 \cdot 5$.

From this point of view it may be best to make up any required resistance by using as many coils as possible to obtain a greater radiation surface; thus 10 ohms may be made up of 5 ohms, 2 ohms, 2 ohms, and 1 ohm, instead of using one 10 ohm coil.

(c) *Oscillators*. The advantages of this method of supply are that the frequency is absolutely constant and determinable with great accuracy. The power available is sufficient for most bridge measurements, and the wave form is very close to a pure sine wave. For these reasons this method of supply has very largely displaced other methods.

Such oscillators depend upon the fact that a circuit containing inductance L and capacitance C has a natural frequency of oscillation given by $f = \frac{1}{2\pi\sqrt{LC}}$. If such a circuit has an e.m.f. induced in it and is then left to oscillate, the frequency of oscillation of the current in the circuit will be given by the above expression. The natural dying away of these oscillations, due to resistance in the circuit, is prevented by supplying energy from another circuit containing (say) a three electrode valve and hence oscillations of this frequency are continuously produced.

Fig. 135 gives the connections of a triode valve oscillator, as commonly used for purposes of bridge supply. B_1 is a battery of about 200 volts to obtain the required anode potential in the valve

V. B_2 is a 6 volt battery, with regulating resistance R, for the purpose of heating the filament of the valve. L_1 and L_2 are coils inductively coupled with coil L_3, the latter being part of a tuned circuit of which the other part is a variable capacitor C. The tuned circuit is connected in the valve anode circuit and coil L_1 in the grid circuit, as shown, while coil L_2 supplies the bridge network. The tuning is carried out by variation of the capacitance C by means of which any desired frequency within the range of the apparatus may be obtained.

In operation, when the circuit is first closed a current is produced in the anode circuit, and oscillations are set up in the tuned circuit.

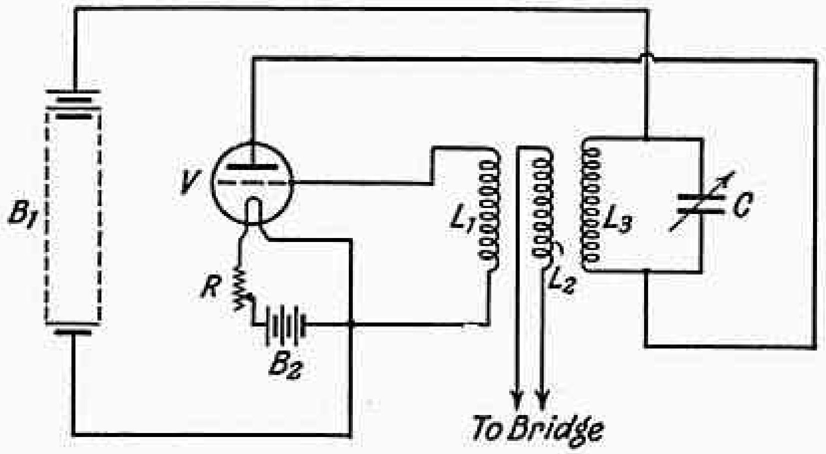


FIG. 135. CONNECTIONS OF TRIODE VALVE OSCILLATOR

Owing to the inductive coupling between coils L_3 and L_1 these oscillations cause variations of grid potential which produce currents in the anode circuit of the valve and so the oscillations are maintained, the energy required for the supply of the losses being obtained from the batteries.

Any frequency up to the extreme limit of audibility can be obtained by suitably choosing the values of the inductance L_3 and the capacitor C, e.g. if L_3 is $\frac{1}{36}$ henry and C is 1 microfarad,

$$f = \frac{1}{2\pi\sqrt{\frac{1}{36} \times \frac{1}{10^6}}} = \frac{3{,}000}{\pi}$$

$$= 955 \text{ cycles per second}$$

Fig. 136 gives the connections of a valve oscillator which has been developed by the Cambridge Instrument Company. In this generator the oscillation frequency is determined by the characteristics of a Wien bridge network. The output from a 2-stage amplifier, having negative feed-back, is connected back to the input circuit through the Wien bridge so that a suitable amount of positive feed-back is obtained at the characteristic frequency of the bridge.

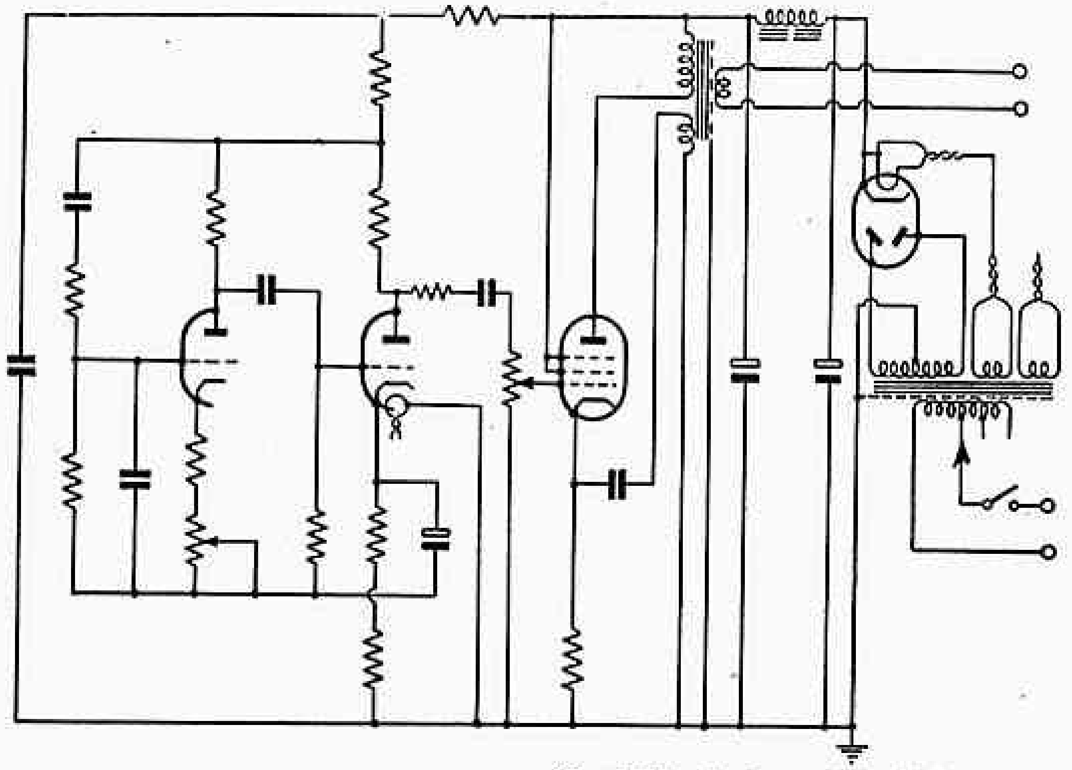
The amplifier output is also taken to the output valve via a suitable

network and variable attenuator. In order to maintain a good wave-form produced by the oscillator, considerable negative feed-back is also applied over the output stage.

The oscillator can be constructed for any frequency from a few cycles per second up to 100 kc/sec. or more. Outputs of up to 2 watts with practical sinusoidal wave-form can be obtained.

A multi-frequency bridge oscillator made by H. W. Sullivan, Ltd. gives a frequency range of 300 to 10,000 c/s or 1,000 to 30,000 c/s each to within $\pm$ 1 per cent.

An amplitude control valve ensures constant output and purity of wave-form, the output being up to 4 or 5 watts.



(Cambridge Instrument Co. Ltd.)

FIG. 136. MAINS FIXED FREQUENCY OSCILLATOR CIRCUIT DIAGRAM

An audio signal generator made by Advance Components, Ltd. covers the range 15 c/s to 50,000 c/s. The bridge-type resistance-capacitance oscillator used incorporates a stabilizing circuit and is followed by two stages of amplification with heavy negative feed-backs so that there is stability of frequency and negligible distortion. Either sine-wave or square-wave output can be used, the former being variable from 200 μV to 20 V and the latter from 400 μV to 40 V.

Some of the H. W. Sullivan valve oscillators covering frequency ranges of 50–170,000 c/s are precise enough to be used as frequency standards of 0·1 per cent accuracy. They have a frequency stability of a few parts in a million during a period of measurement. Although for ordinary bridge measurements of inductance and capacitance a fixed-frequency simple valve oscillator of, say, 1,000 c/s and output of about 1 watt is adequate, for more specialized bridge measurements, oscillators of the heterodyne or resistance-capacitance type, continuously variable from "supply" frequencies through the telephonic range and even including the "carrier" frequency range are preferable. These have output powers up to 5 watts. Although on occasions such high power is necessary for supply to a bridge, it is usually better practice, wherever possible, to limit the power supplied

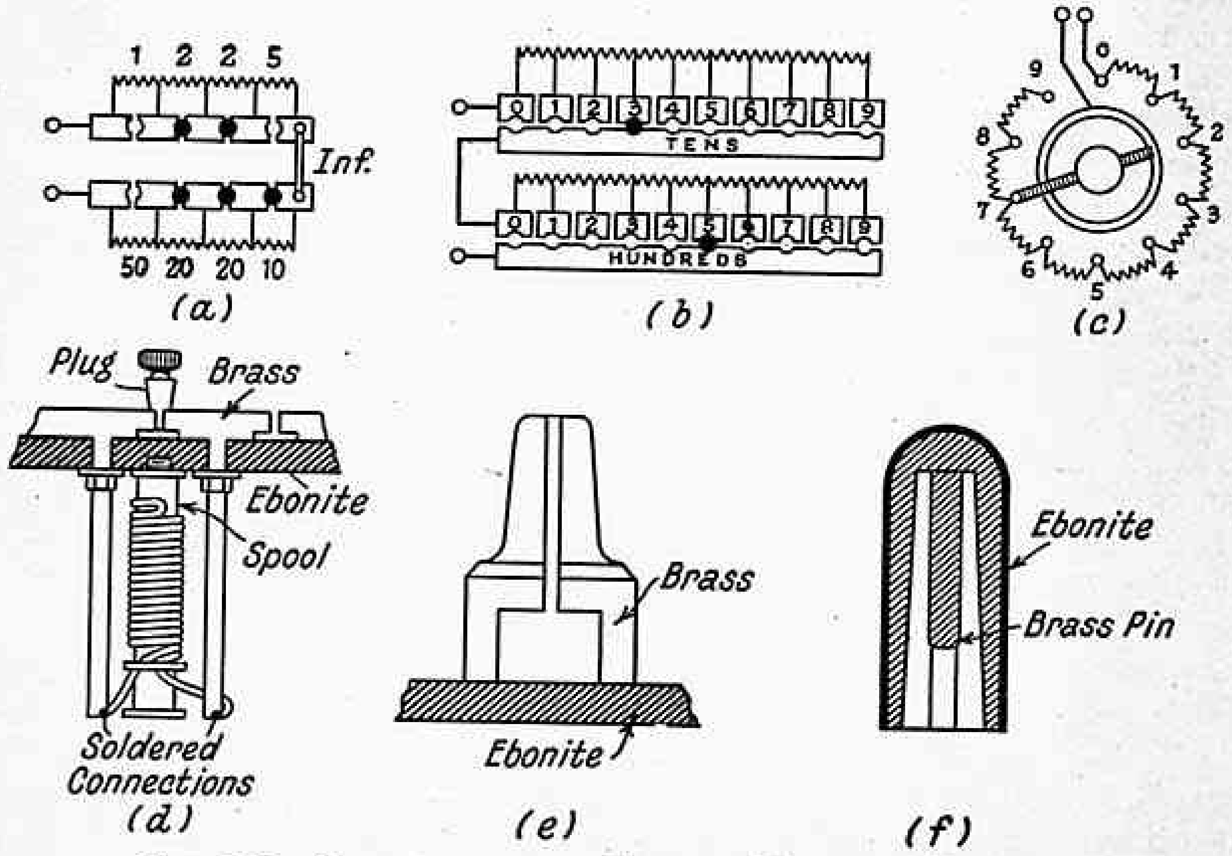


FIG. 137. CONSTRUCTION OF VARIOUS TYPES OF VARIABLE RESISTANCES

to a bridge and employ a valve detector-amplifier of an aperiodic type in which balance detection is sensed both aurally by telephones and visually by a pointer galvanometer having a logarithmic deflection (to avoid damage which may be caused by a greatly unbalanced bridge).

2. VARIABLE RESISTANCES, INDUCTANCES, AND CAPACITORS. Fixed standards of resistance, inductance, and capacitance have been described in Chapter II.

Variable Resistances. These are usually in the form of resistance boxes. Fig. 137 illustrates the various methods of arranging and mounting the coils in such boxes. Fig. 137 (*a*) shows the simplest arrangement, in which the removal of a plug puts a coil in circuit. The reading, as shown, is 56 ohms. An open-circuiting link or plug is usually provided. Fig. 137(*b*) shows the "straight-decade" arrangement. One plug only is necessary for each decade, the reading with

plugs as shown being 530 ohms. The dial arrangement, the value of the resistance in which is varied by the rotation of a laminated copper brush or arm, is shown in Fig. 137 (*c*). Fig. 137 (*d*) illustrates a common method of mounting the resistance coils, the coil being non-inductively wound on a brass spool. These brass tubes have usually a double layer of silk ribbon wound on them, and are coated with shellac varnish and baked to remove moisture. Manganin wire is used in all high-grade resistance coils, the coils being impregnated with shellac after winding, and being annealed by baking for 10 hours at 140° C. After annealing, the coils are usually boiled in paraffin wax to prevent the absorption of moisture. The ends of the coil are soldered to the terminal blocks with silver solder. Two brass rods, connected to the brass contacts on the top of the box, serve as terminals for the coil. The Leeds and Northrup Co. use an improved method, which consists of attaching the coil spool to the contact blocks and soldering one end of the coil to the spool, the other end being soldered to the spool of an adjacent coil.

It is of great importance in accurate work that the contact resistance in such resistance boxes shall be small and constant in value. The various manufacturing firms have developed different methods of attaining this, one such method being shown in Figs. 137(*e*) and 137(*f*). These show the special type of plug developed by Messrs. Gambrell Bros. This takes the form of a hollow brass cylinder with brass centre pin, the outer surface being coated with ebonite, milled so that a good grip on the plug may be obtained. This plug fits on to two contact blocks, of the shape shown on the left in the figure. They are slightly conical and have a centre hole to take the plug pin. When the plug is inserted the conical shape of the contact blocks serves to clamp it, and a very good contact is obtained. This type of plug has the advantage that it is independent of the other contacts on the lid of the box, which is not the case with the form of contact shown in Fig. 137 (*a*), where the insertion of a plug in a hole adjacent to a plug already inserted tends to tighten the original plug in its hole.

The sliding "Dual" contact shown in Fig. 138 is used by Messrs. H. Tinsley & Co. for dial pattern resistance boxes and bridges. The contact blocks are cut away as shown, a multiple-leaf brush being fitted to make contact on both the top and bottom surfaces of the slots cut in these blocks. This method gives a very good contact on all studs, and is capable of giving a degree of precision in measurement which would normally be expected only with a plug type of contact.

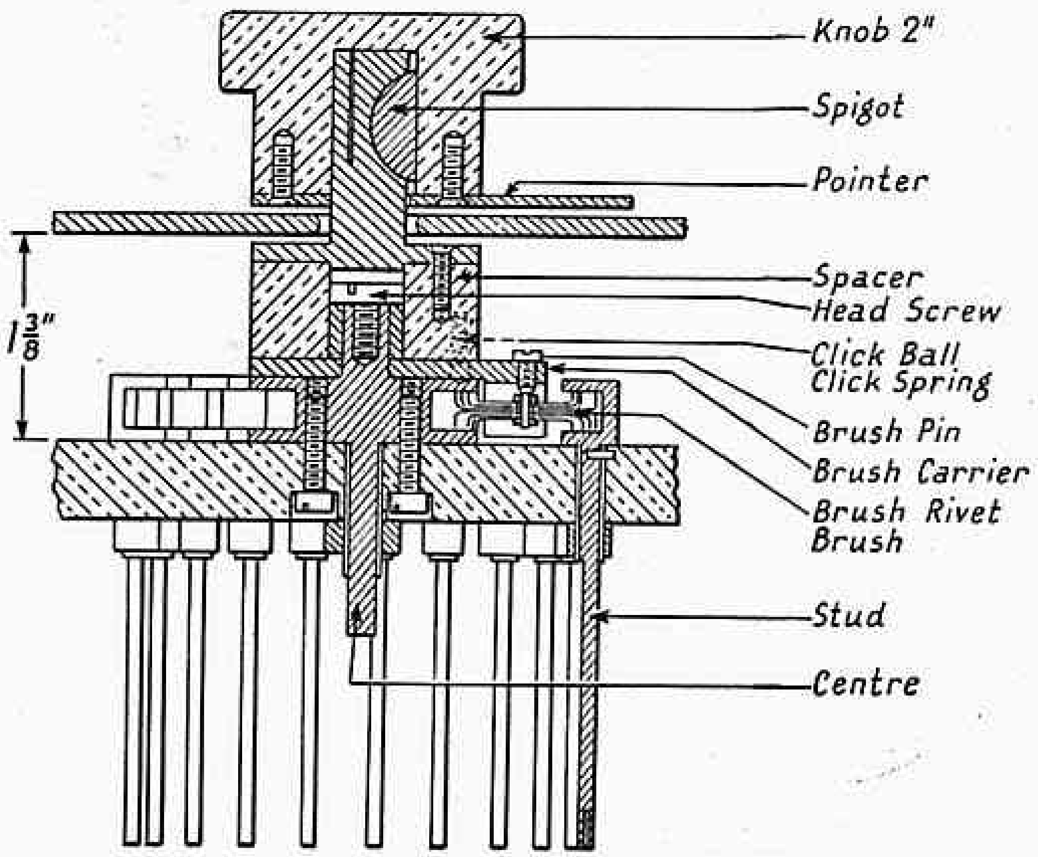
Fig. 139 shows the construction of a dial-pattern resistance box by the Cambridge Instrument Co. for use in a.c. Bridge measurements.

H. W. Sullivan a.c. Decade Resistance Standards have 0·01 per cent direct-reading accuracy and permanence of frequencies up to

50 kc/s. For bridge work, the lower decades are of constant inductance to 0·01 μH, and the time constants of the various decades are—

100 ohms	2.10^{-9}
1,000 ohms	2.10^{-9}
10,000 ohms	5.10^{-9}

A patented pre-set switching device due to W. H. F. Griffiths is employed to minimize the residual inductance and the high frequency



(*H. Tinsley & Co. Ltd.*)

FIG. 138

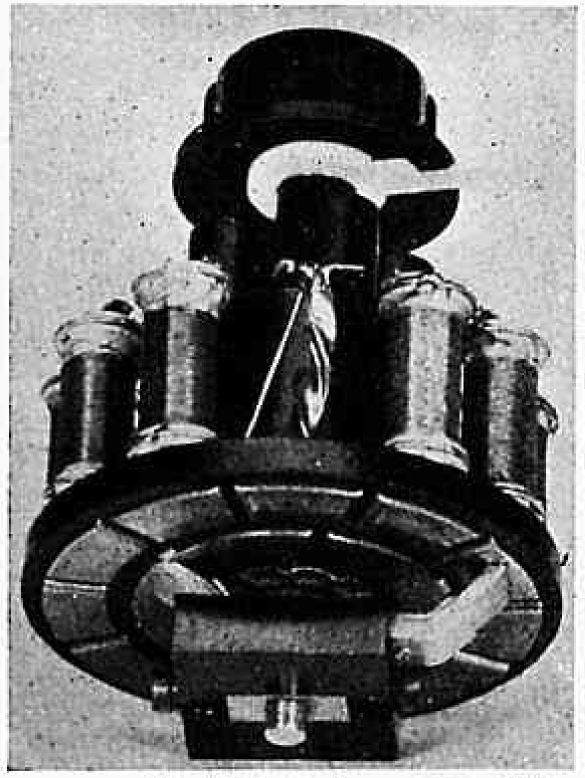
resistance error; the screen capacitance is compensated automatically and so the effective residual inductance is rendered sensibly independent of screen connection.

When plugs are used, as in the first two methods shown in Fig. 137, care should be taken to ensure that these are firmly pressed home before measurements are made, as appreciable contact resistance may result if any of the plugs are loosely inserted. Such plugs should be inserted and withdrawn with a rotational, or screwing, motion, and not simply pushed in, or pulled out, directly. The former method of insertion ensures good contact and prevents the plug from fastening tight in the contact hole, while, if the plugs are pushed directly in, considerable difficulty may be experienced in removing them and the contact blocks may thus become loosened.

Obviously the fewer the number of plugs used, the better, and for this reason the arrangement shown in Fig. 137 (*b*) is an improvement upon that of Fig. 137 (*a*).

Resistance boxes are made up in a large variety of ranges, varying from a total of 11 ohms variable in steps of 0·1 ohm up to a total of 1 megohm. The arrangement of the coil resistance values shown in Fig. 137 (*a*)—5, 2, 2, 1; 50, 20, 20, 10, etc.—is most usual.

The adjustment of the resistance values of the coils varies according to the type of box considered. For ordinary purposes, these values may be adjusted to 1 part in 1,000, while in the case of boxes for precision purposes, the adjustment is usually to 1 part in 10,000.



(*Cambridge Instrument Co. Ltd.*)

FIG. 139

It is essential that resistance coils for use in a.c. bridge measurements shall have very small residual inductance and capacitance. Special methods of winding are necessary to fulfil this condition. To obtain a very small inductance the coil is wound so that adjacent parts of it carry currents in opposite directions. In this way the magnetic field of the coil is kept very small. The self capacitance of the coil is kept small by subdividing it so that adjacent parts have a very small capacitance and also have only a small potential difference between them.

Since the self-capacitance of coils wound on bobbins is usually quite small, the reduction of their inductance is often the most important question. The winding of alternate turns of the coil in reverse directions is one method used for this purpose. This method is illustrated in Fig. 140 (*a*), the arrangement shown being due to Grover and Curtis (Ref. (17)). The wire is wound on a cylindrical former having an axial slit along the greater part of its length. The wire passes through this slit once in every turn, so as to give reversal of winding direction. The arrow heads show the directions of current in various parts of the winding. Obviously the magnetic effects of adjacent turns neutralize one another. This type of coil has a very small inductance, but is somewhat difficult to wind.

Fig. 140 (*b*) shows the Chaperon (Ref. (22)) method of winding. This winding is really an extension of the bifilar principle, the currents in adjacent wires neutralizing one another as regards resultant magnetic field, as shown. Both the inductance and capacitance of coils wound in this way are small.

Modifications of this method have been used in which the winding is divided into a number of sections connected in series.

Campbell Constant-inductance Rheostat. There are several forms of low-resistance rheostats designed to have a very low and calculable inductance. These are usually constructed on the bifilar principle, and are often of the slide-wire form. Fig. 141 (*a*) illustrates

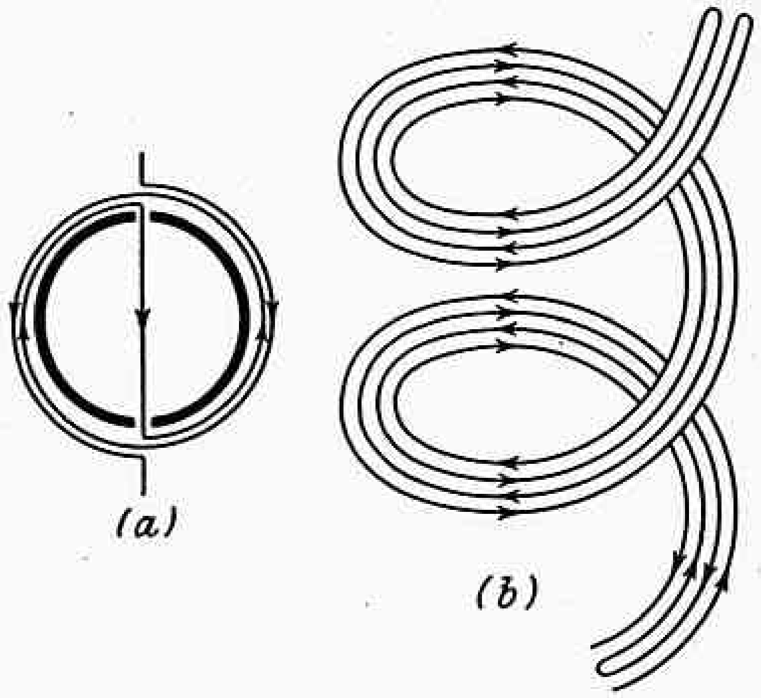


FIG. 140. NON-INDUCTIVE WINDINGS

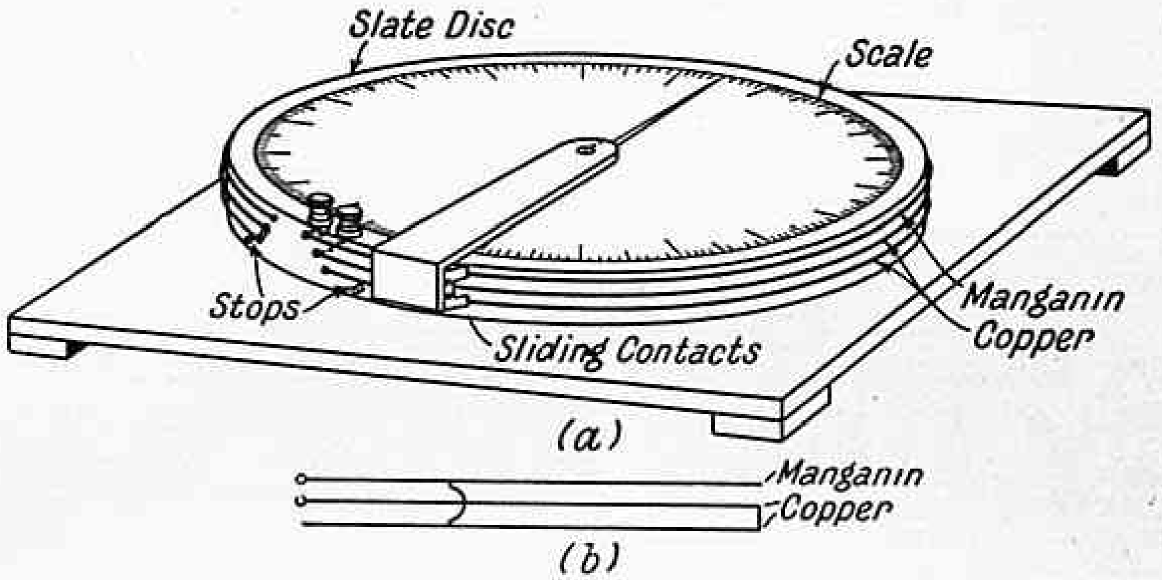


FIG. 141. CAMPBELL CONSTANT-INDUCTANCE RHEOSTAT

the construction of Campbell's constant-inductance rheostat. This is a very useful piece of apparatus for use in certain bridge measurements, since it enables a fine adjustment of the resistance of a bridge arm to be made without altering the inductance settings.

The resistance is increased by moving the sliding contact to the right (Fig. 141 *b*). Such a movement increases the length of manganin wire in circuit and reduces the length of copper wire in circuit. Whatever the position of the slider, the total length of wire in

circuit is always the same, and forms a bifilar loop, thus maintaining the inductance small and constant.

A precision slide wire of this type, made by Muirhead and Co., Ltd., may have any resistance from 0·5 to 10 Ω. One made by the Cambridge Instrument Co., Ltd. has a total resistance of 0·12 Ω with a residual inductance of 0·2 μH.

A precision decade non-reactive slide resistance designed on the Kelvin-Varley slide principle (see p. 315) is made by H. W. Sullivan, Ltd. It has manganin coils. One type, having a total resistance of 1,000 ohms, has each of its five dials brought out to separate terminals so that it can also be used as a slide resistance of 200, 40, 8 or 1·6 ohms.

Variable Inductances. Such pieces of apparatus should have as high an inductance as possible compared with their resistance, i.e. their time-constant should be great. Their inductance should be continuously variable and should cover as great a range as possible between maximum and minimum settings. In addition, it is highly desirable that the variation of inductance with position of the moving part should obey a straight line law, and also that the coils shall be astatically wound. The inductance for a given position should not, of course, vary with time, and variation of frequency should not cause appreciable variation of inductance.

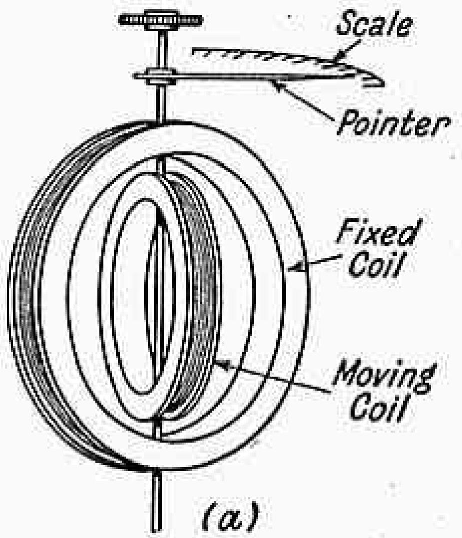


FIG. 142. AYRTON-PERRY INDUCTOMETER

Most variable-inductances are so constructed that they can be used as either self- or mutual-inductances. When used as self-inductances the fixed and moving coils are connected in series and the inductance is given by

$$L = L_1 + L_2 \pm 2M$$

where L_1 and L_2 are the self-inductances of the fixed and moving coils respectively, and M is the mutual-inductance (variable) between them.

In order to eliminate frequency errors the coils are usually wound with stranded wire and the use of metal parts in the construction is avoided as far as possible.

Ayrton-Perry Inductometer. Fig. 142 shows, diagrammatically, the construction of a simple form of variable inductance (self or mutual) due to Ayrton and Perry. The moving coil is mounted inside the fixed coil and is carried by a spindle which also carries a pointer and handle at the top as shown. Movements of the pointer indicate the variation in the angle between the planes of the coils, but the scale may be graduated to read the inductance directly.

When constructed for use in accurate measurements, the coils are wound on mahogany formers whose surfaces are spherical and great care is necessary in fixing the coils so as to ensure constancy of inductance with time.

This form of inductometer can be cheaply and easily constructed for use as a variable self-inductance in cases where the inductance must be variable but not necessarily known, e.g. for use in one arm of the Wagner earth device.

Its disadvantages are that the instrument produces an external magnetic field, which may be troublesome if the inductometer is

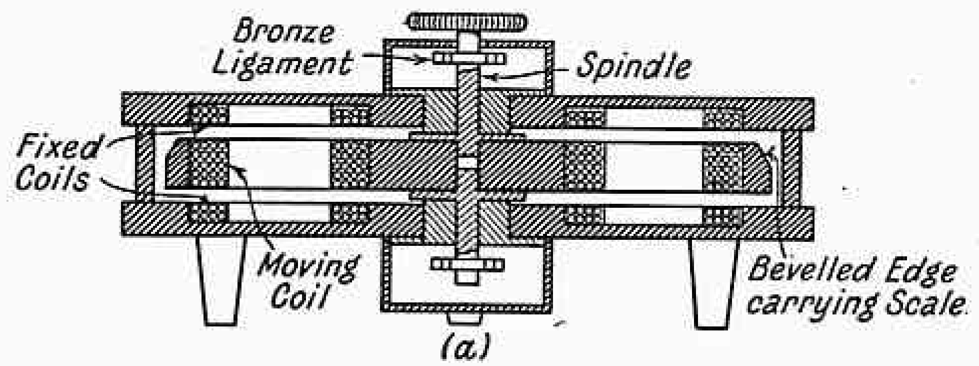


Fig. 143. Construction of Brooks and Weaver Inductometer

placed near to the bridge network, and that the scale is not linear.

Brooks and Weaver Inductometer. This form of inductometer (Ref. (23)) is one of the best forms for general purposes. The coils are wound and connected astatically, the time-constant is high, and the scale is uniform throughout the greater part of the range. It is also fairly easily constructed, and its calibration remains reasonably constant with time even when in continuous use. The current carrying capacity, also, is high for this type of apparatus.

The construction of the inductor is shown in Fig. 143. There are, in all, six link-shaped coils—four fixed and two moving. These are wound with stranded wire. The moving coils have twice as many turns as the fixed. These coils are embedded in ebonite or Bakelite discs which are about 15 in. diameter. Bakelite has the advantage that it has less tendency to warp than ebonite. The top and bottom discs, which are fixed, are about $\frac{1}{2}$ in. thick, and are separated by ebonite or Bakelite pillars. The centre disc is thicker—to carry the larger coils—and is of slightly smaller diameter. It has a bevelled edge, upon which a scale is marked out over 180° of its circumference, this scale being used in conjunction with an index mark on the lower fixed disc. Connections to the moving coils are made through copper or phosphor-bronze ligaments soldered to the two halves of the spindle.

The dimensions of the coils are specially chosen to give a uniform scale, and also to obtain as great an inductance as possible for a given length of wire. The relationships between the various dimensions are given in Fig. 144, in terms of the mean radius R of the

semicircular ends of the coils. The depth of the moving coils should be the same as their width of winding, i.e. $0{\cdot}78R$, and the depth of the fixed coils $0{\cdot}39R$.

A great advantage of this method of construction is that small

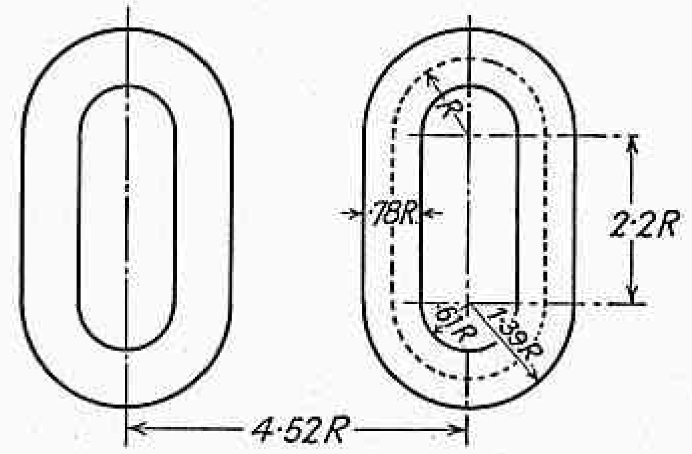


Fig. 144. Relative Dimensions of Brooks and Weaver Inductometer

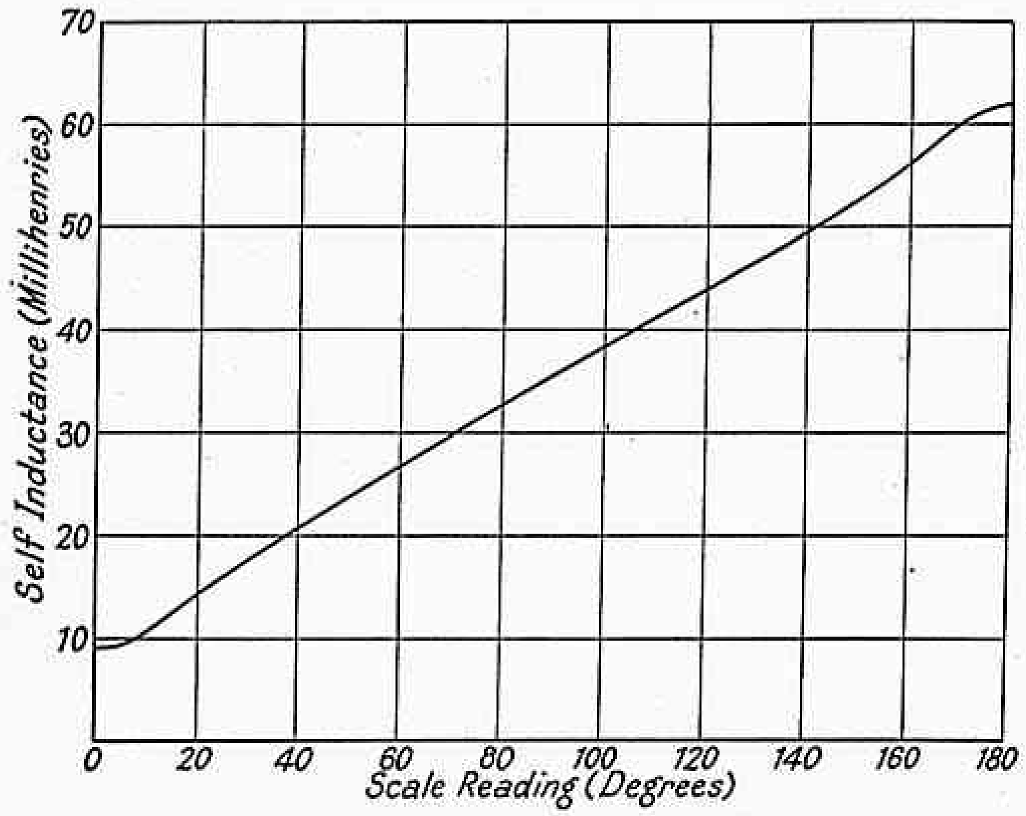


Fig. 145. Calibration Curve of Brooks and Weaver Inductometer

variations of the length of gap between (say) the moving disc and the upper fixed disc, due to warping of the former or to wear of the bearings, have no appreciable effect upon the inductance of the instrument, since movement away from the upper fixed disc means movement towards the lower one, thus maintaining the inductance the same within narrow limits.

Fig. 145 shows the calibration curve for an inductometer of this type when used as a self-inductance (all six coils in series). The