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CHAPTER IX

MAGNETIC MEASUREMENTS

MAGNETIC tests can be divided into two general classes: direct current tests and alternating current tests. Although they may be subdivided to a considerable extent, these are the two most distinctly defined classes of tests. The methods of testing magnetic specimens will therefore be dealt with under these two general headings.

Direct Current Tests. Such tests are most generally made upon solid (as distinct from laminated) materials, the alternating current test methods being used chiefly for laminated materials.

The two most important quantities to be measured in these tests are the flux density in a specimen and the magnetizing force producing this flux density.

Magnetometer Methods. These are fundamentally the simplest of all methods of magnetic testing, and were largely used in the early work on magnetism. Magnetometers are used for the measurement of magnetic field. Often the horizontal component of such a field is measured by them. They may be applied, also, to the measurement of flux density in bar specimens of magnetic material, their advantage for this purpose being that they measure the actual (or static) value of flux density in the specimen as distinct from methods such as those using a ballistic galvanometer, which measure a *change* in flux density. The intensity of magnetization J of a specimen is measured by the magnetometer, and the corresponding flux density B is obtained from the formula

$$B = 4\pi J + H$$

where H is the strength of the magnetic field producing this intensity of magnetization. For full details of such applications the reader should refer to the works given in Refs. (2) and (3).

Magnetometers consist essentially of a suspended magnetic needle or system of needles, the suspension itself having good torsional elastic qualities and exerting, usually, only a small torsional control upon the needle.

Referring to Fig. 208, let ns represent a magnetic needle, suspended at O, and of length l cm. Let n and s be its poles, of strength m units, and suppose that a horizontal control field of strength H exists in the direction XX'. If another horizontal field, of strength F, having a direction at right angles to XX', is made to act upon the needle, a deflection θ is produced. The couples acting upon the needle are $Hml \sin\theta$ clockwise, and $Fml \cos\theta$ anti-clockwise. When the needle is at rest in the deflected position,

$$Hml \sin\theta = Fml \cos\theta$$

from which the strength of the field F is given by

$$F = H \tan\theta$$

If H is the horizontal component of the earth's magnetic field—as it often is—its value can be easily found, from physical tables, for any point on the earth's surface.

If the deflecting field is due to a bar magnet in the neighbourhood of the magnetometer needle, the pole strength of this magnet can be obtained as follows—

Theory. Referring to Fig. 209, let ns be the magnetometer needle

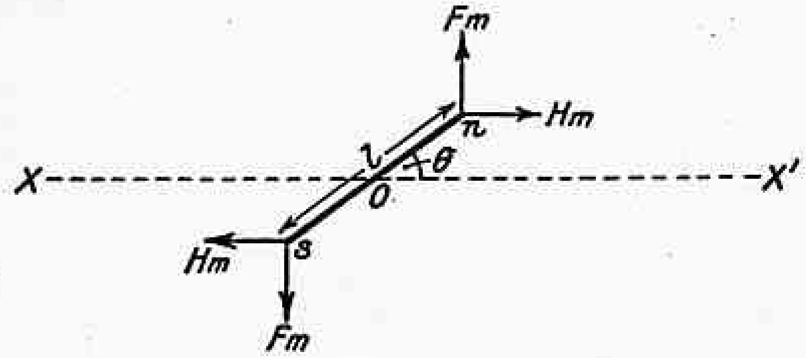


FIG. 208

of length l and pole strength m. Let NS be the bar magnet under test, of length L (between its poles) and pole strength m'. Suppose that both ns and NS are in the horizontal plane and are placed as shown. Then, the force upon pole n of the needle, due to pole N of the bar magnet, when resolved into the direction XX' is $\frac{mm'}{l_1^2}\cos\phi_1$ where l_1 and ϕ_1 are as shown. Pole N will exert the same force upon pole S of the needle, but in the opposite direction to the force on pole n. The pole S will exert forces of $\frac{mm'}{l_2^2}\cos\phi_2$ in direction XX' upon the two poles of the needle in the opposite directions to the forces due to N. Thus, the deflecting moment upon the needle due to the two poles of the bar magnet will be

$$mm'l\left[\frac{\cos\phi_1}{l_1^2} - \frac{\cos\phi_2}{l_2^2}\right]$$

Now, $\cos\phi_1 = \dfrac{r - \frac{L}{2}}{l_1}$ and $\cos\phi_2 = \dfrac{r + \frac{L}{2}}{l_2}$. The deflecting moment is, therefore

$$mm'l\left[\frac{r - \frac{L}{2}}{l_1^3} - \frac{r + \frac{L}{2}}{l_2^3}\right]$$

If the needle is situated in a horizontal control field H in a direction perpendicular to XX', and if the deflection of the needle is θ, then we have

$$Hml \sin\theta = mm'l\left[\frac{r-\frac{L}{2}}{l_1^3}-\frac{r+\frac{L}{2}}{l_2^3}\right]\cos\theta$$

Hence,
$$m' = \frac{H\tan\theta}{\left[\frac{r-\frac{L}{2}}{l_1^3}-\frac{r+\frac{L}{2}}{l_2^3}\right]}$$

from which the ferric induction in the magnet or sample may be determined (see Chapter I).

FIG. 209. MAGNETOMETER MEASUREMENT

It is assumed that the length l of the needle is small compared with the distance r.

Instead of using the horizontal component of the earth's field as the controlling field, a stronger control field may be used, this being obtained by using a permanent magnet, or otherwise. The strength of such a field may be measured by comparing it with that of the earth by the *oscillation method*, which forms a simple means of measuring field strength. The magnetic needle is allowed to oscillate freely whilst situated in the field whose strength is to be measured, and the time of one complete oscillation is measured. Let this time be T_1 sec. The needle is then placed in a known magnetic field such as that of the earth, and the time of free oscillation again measured. Let this time be T_2.

Then, from the expression for the time of one complete free oscillation, namely

$$T = 2\pi\sqrt{\frac{I}{MF}}$$

(where I is the moment of inertia of the needle about the axis of oscillation, M is the magnetic moment of the needle, and F the strength of the magnetic field in which it oscillates), we have

$$T_1 = 2\pi\sqrt{\frac{I}{MF_1}}$$

and
$$T_2 = 2\pi\sqrt{\frac{I}{MF_2}} \quad . \quad . \quad . \quad . \quad . \quad (202)$$

F_1 and F_2 being the strengths of the two fields, the latter being known.

Hence
$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{I}{MF_1}}}{2\pi\sqrt{\frac{I}{MF_2}}} = \sqrt{\frac{F_2}{F_1}}$$

or
$$F_1 = F_2 \cdot \frac{T_2^2}{T_1^2} \quad . \quad . \quad . \quad . \quad . \quad (203)$$

The magnetometer method has the advantage that it is an absolute method, but the disadvantage that it is susceptible to the influence of external magnetic fields, and also that it requires the samples under test to be in the form either of long, thin rods, or in the form of ellipsoids, owing to the demagnetizing effect of the ends of bar-shaped samples.*

During the second World War several accurate and sensitive magnetometers were developed in Great Britain. These took advantage of the high permeability of alloys, such as Mumetal (see p. 689) to concentrate the magnetic field. Differential arrangements could be made so that a very small movement of a coil removed it from the field to be measured. Another development in magnetometer design is based on the fact that, Mumetal becoming saturated at a relatively low flux density, it is possible to obtain high sensitivity by using this alloy in the region of saturation; a small change in the magnetic field causes a large change in permeability.

A mumetal-wire magnetometer is described by S. Whitehead (Ref. (49)).

The Ballistic Galvanometer. Before proceeding with the description of methods of testing bar- and ring-shaped samples of magnetic material, the ballistic galvanometer and fluxmeter—instruments which are largely used in such tests—will be described.

The ballistic galvanometer is used to measure a *quantity of electricity* passed through it. This quantity, in magnetic measurements, is the result of an e.m.f. instantaneously induced in a "search coil" connected to the galvanometer terminals, when the magnetic flux interlinking with the search coil is changed. Such a galvanometer is usually of the D'Arsonval type, since this type is least affected by external magnetic fields. It does not show a steady deflection when in use, owing to the transitory nature of the current passing through, but gives a "throw" which is proportional to the quantity of electricity instantaneously passed through it. This quantity—and hence the change in the flux producing it—is determined from the calibration of the galvanometer, as will be described later. The

* Several forms of magnetometer are described by D. W. Dye in the *Dictionary of Applied Physics*, p. 455, amongst which is F. E. Smith's magnetometer for the measurement of the intensity of the earth's magnetic field.

proportionality of the throw only holds if the discharge of the electricity through the galvanometer has been completed before any appreciable deflection of the moving system has taken place. For this reason the moving system of such a galvanometer must have a large moment of inertia—often obtained by the addition of weights to the moving system—compared with the restoring moment due to the suspension. This means that the galvanometer has a long period of vibration—usually from 10 to 15 seconds in practice. The damping of the galvanometer should also be small in order that the first deflection (or throw) shall be great.

For convenience in working, a galvanometer which is almost dead-beat is best, but the damping must be electromagnetic, so that it may be determined from the constants of the instrument. Appreciable air damping should not be present, as this is indeterminate. A key by which the galvanometer may be short-circuited saves time in bringing the moving system to rest.

Other important points in the construction of such galvanometers are that the moving coil should be free from magnetic material, and also that the suspension strip should be carefully chosen and mounted to avoid "set." The terminals, coil, and connections within the instrument, should be of copper, throughout, in order to avoid thermo-electric effects at the junctions. In the best instruments the suspension is non-conducting, the current being led into the coil by delicate spirals of very thin copper strip.

Theory. As already stated above, the quantity of electricity must be discharged through the galvanometer in a very short time, during which, the moment of inertia of the moving system being large, the movement from the zero position is negligibly small. The passage of the electricity through the instrument gives to the moving system energy which is dissipated gradually thereafter in friction and electromagnetic damping.

During the actual motion the deflecting torque is thus zero, and the equation of motion is

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = 0$$

a being the moment of inertia of the moving system, b the damping constant, c the control constant, θ the deflection in radians, and t the time in seconds.

As shown on page 261 the solution of this equation is

$$\theta = A\varepsilon^{m_1t} + B\varepsilon^{m_2t}$$

where A and B are constants and

$$m_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \quad m_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The damping, and therefore b, is small so that both m_1 and m_2 are imaginary. Under these conditions, as on page 262 the solution may be written

$$\theta = \varepsilon^{-\frac{bt}{2a}} F \,.\, \sin\left(\frac{\sqrt{4ac - b^2}}{2a} \,.\, t + \alpha\right)$$

F being a constant which may be evaluated from a knowledge of the initial conditions of the motion. Since b is small a justifiable simplification is

$$\theta = \varepsilon^{-\frac{bt}{2a}} \,.\, F \,.\, \sin\left(\sqrt{\frac{c}{a}}\, t + \alpha\right)$$

Initial Conditions. When $t = 0$ the deflection $\theta = 0$.

Again, if i is the current in amperes at any instant during the discharge of electricity through the instrument, the torque may be represented by Gi and hence

$$Gi = a\frac{d^2\theta}{dt^2}$$

from which $\int_0^\tau Gi\; dt = \int_0^\tau a\,\frac{d^2\theta}{dt^2}\,.\,dt$ where $\tau =$ total time of the discharge. Since $\int_0^\tau idt =$ the quantity of electricity discharged $= Q$ coulombs we may write

$$G\int_0^\tau idt = GQ = a\,\frac{d\theta}{dt}$$

$\frac{d\theta}{dt}$ is the velocity of the moving system at the end of time τ, i.e. at the beginning of the first deflection, since τ is very small.

We may thus write (as a close approximation) when $t = 0$, $\frac{d\theta}{dt} = \frac{G}{a}\,.\,Q.$

Now, differentiating the above expression for θ we have

$$\frac{d\theta}{dt} = -\frac{b}{2a}\,.\,\varepsilon^{-\frac{b}{2a}t}\, F \sin\left(\sqrt{\frac{c}{a}}\, t + \alpha\right)$$

$$+ \varepsilon^{-\frac{b}{2a}t}\,.\, F\sqrt{\frac{c}{a}} \cos\left(\sqrt{\frac{c}{a}}\, t + \alpha\right)$$

and, when $t = 0$

$$\frac{d\theta}{dt} = -\frac{b}{2a}\,.\,\varepsilon^{\circ}\,.\,F \sin\alpha + \varepsilon^{\circ}\,.\,F\sqrt{\frac{c}{a}} \cos\alpha$$

Again, since $\theta = 0$ when $t = 0$

$$0 = \varepsilon^{\circ} . F \sin (0 + \alpha) \text{ or, } \alpha = 0$$

$$\therefore \quad \frac{d\theta}{dt} = F\sqrt{\frac{c}{a}} = \frac{G}{a} . Q$$

$$\text{or,} \quad F = \frac{G}{a} . \sqrt{\frac{a}{c}} . Q$$

Substituting in the expression for θ we have

$$\theta = \frac{G}{a} . Q . \varepsilon^{-\frac{b}{2a}t} \sqrt{\frac{a}{c}} \sin\sqrt{\frac{c}{a}} . t$$

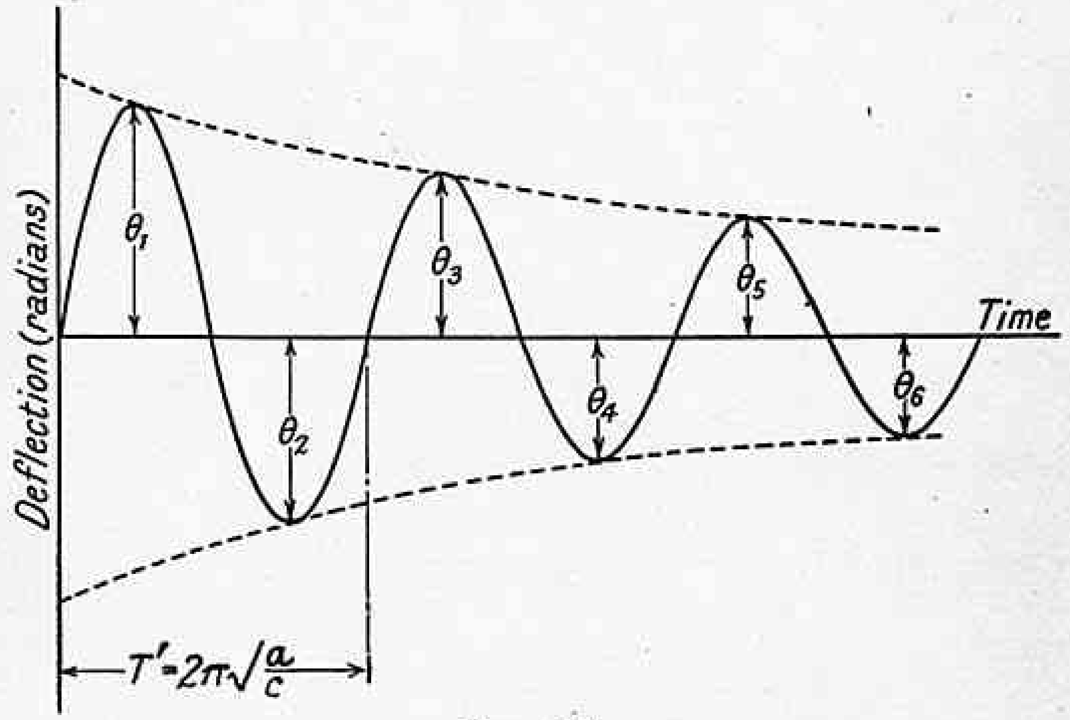


FIG. 210

The deflection at any time is thus proportional to Q and the motion is oscillatory, the frequency of the oscillation being

$$\frac{\omega}{2\pi} = \frac{\sqrt{\frac{c}{a}}}{2\pi}$$

The periodic time of the motion is thus

$$T' = \frac{1}{f} = 2\pi\sqrt{\frac{a}{c}}$$

The graph of deflection against time is shown in Fig. 210, the successive diminishing maxima corresponding to times

$$\frac{T'}{4}, \frac{3T'}{4}, \frac{5T'}{4}, \text{ etc.}$$

Substituting the values of time in the expression for θ gives

$$\theta_1 = \frac{G}{a} . Q . \varepsilon^{-\frac{\pi}{4}\frac{b}{\sqrt{ac}}} \sqrt{\frac{a}{c}}$$

$$\theta_2 = \frac{G}{a} . Q . \varepsilon^{-\frac{3\pi}{4}\frac{b}{\sqrt{ac}}} \sqrt{\frac{a}{c}}$$

$$. \quad . = . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$\theta_n = \frac{G}{a} . Q . \varepsilon^{-\frac{\pi}{4}(2n-1)\frac{b}{\sqrt{ac}}} \sqrt{\frac{a}{c}}$$

Without damping the amplitudes would all have been

$$\theta' = \frac{G}{a} . Q . \sqrt{\frac{a}{c}}$$

$$\text{Now} \quad \theta' = \frac{\theta_1}{\varepsilon^{-\frac{\pi}{4}\frac{b}{\sqrt{ac}}}} = \theta_1 \varepsilon^{\frac{\pi}{4}\frac{b}{\sqrt{ac}}}$$

$$\text{and} \quad \frac{\theta_1}{\theta_2} = \frac{\varepsilon^{-\frac{\pi}{4}\frac{b}{\sqrt{ac}}}}{\varepsilon^{-\frac{3\pi}{4}\frac{b}{\sqrt{ac}}}} = \varepsilon^{\frac{\pi}{2} \cdot \frac{b}{\sqrt{ac}}}$$

$$\text{Hence} \quad \sqrt{\frac{\theta_1}{\theta_2}} = \varepsilon^{\frac{\pi}{4} \cdot \frac{b}{\sqrt{ac}}} \text{ and } \theta' = \theta_1 \sqrt{\frac{\theta_1}{\theta_2}}$$

Logarithmic Decrement. The "logarithmic decrement" λ is a constant of the galvanometer which is proportional to the damping and depends upon the resistance of the galvanometer circuit.

From above we have

$$\frac{\theta_1}{\theta_2} = \varepsilon^{\frac{\pi}{2} \cdot \frac{b}{\sqrt{ac}}}$$

$$\text{so that} \quad \log_\varepsilon \frac{\theta_1}{\theta_2} = \frac{\pi}{2}\frac{b}{\sqrt{ac}} = \lambda$$

$$\text{Then} \quad \theta' = \theta_1 \varepsilon^{\frac{\pi}{4}\frac{b}{\sqrt{ac}}} = \theta_1 \varepsilon^{\frac{\lambda}{2}}$$

$$= \theta_1\left(1 + \frac{\lambda}{2}\right) \text{ approx.*}$$

Obviously, from the equations for θ_1, θ_2, etc.

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \quad . \; . \; . \; . = \frac{\theta_{n-1}}{\theta_n} = \varepsilon^{\frac{\pi}{2}\frac{b}{\sqrt{ac}}} = \varepsilon\lambda$$

$$\therefore \quad \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \quad . \; . \; . \; . \times \frac{\theta_{n-1}}{\theta n} = (\varepsilon^{\lambda})^{n-1}$$

* This follows since

$$\varepsilon^{\frac{\lambda}{2}} = 1 + \frac{\frac{\lambda}{2}}{\lfloor 1} + \frac{\left(\frac{\lambda}{2}\right)^2}{\lfloor 2} + \frac{\left(\frac{\lambda}{2}\right)^3}{\lfloor 3} + . .$$

$$\text{or} \quad \frac{\theta_1}{\theta_n} = \varepsilon^{\lambda(n-1)}$$

$$\therefore \log_\varepsilon \left(\frac{\theta_1}{\theta_n}\right) = \lambda(n-1)$$

$$\text{or} \quad \lambda = \frac{1}{n-1} \log_\varepsilon \frac{\theta_1}{\theta_n}$$

From the preceding equation for the ideal undamped deflection θ', namely, $\theta' = \frac{G}{a} \cdot Q\sqrt{\frac{a}{c}}$

we have $$Q = \frac{\sqrt{ac}}{G} \cdot \theta' \text{ coulombs}$$

showing that the quantity of electricity to be measured is directly proportional to the undamped deflection θ'.

Since the periodic time $T' = 2\pi\sqrt{\frac{a}{c}}$ we have, by substitution,

$$Q = \frac{\sqrt{ac}}{G} \cdot \theta' = \frac{c}{G}\sqrt{\frac{a}{c}}\,\theta' = \frac{c}{G} \cdot \frac{T'}{2\pi} \cdot \theta'$$

To eliminate the quantities c and G, suppose that a direct current of I_g amperes passed through the galvanometer produces a steady deflection θ, then

$$G \cdot I_g = c \cdot \theta$$

$$\text{or} \quad \frac{c}{G} = \frac{I_g}{\theta}$$

Hence, finally

$$Q = \frac{T'}{2\pi} \cdot \frac{I_g}{\theta} \cdot \theta'$$

or $$Q = \frac{T'}{2\pi} \cdot \frac{I_g}{\theta}\left(1 + \frac{\lambda}{2}\right)\theta_1 \quad . \quad . \quad . \quad (204)$$

This equation may be written shortly as

$$Q = K \cdot \theta_1$$

where $$K = \frac{T'}{2\pi} \cdot \frac{I_g}{\theta}\left(1 + \frac{\lambda}{2}\right)$$

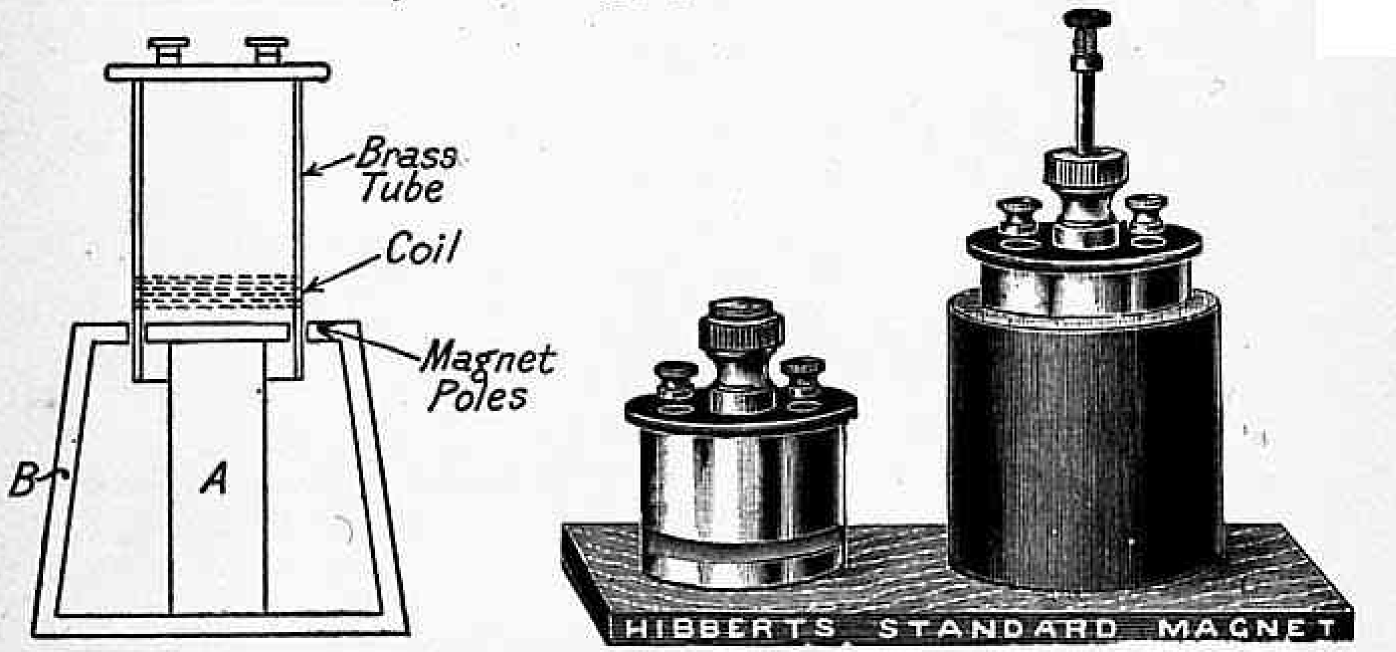
and must be found by calibration.

Since the value of K depends upon the damping and shunting of the galvanometer it is essential that the resistance of the galvanometer circuit during calibration shall be the same as when the galvanometer is being used for testing purposes.

Calibration of the Galvanometer. This may be carried out in a number of ways. Some of the methods used are—

(*a*) BY MEANS OF THE HIBBERT MAGNETIC STANDARD. The principle of this standard is illustrated diagrammatically in Fig. 211 (*a*), and its construction in

Fig. 211 (*b*). It is manufactured by W. G. Pye & Co. The circular bar magnet A and iron yoke B have a narrow annular air gap between them (about 2 mm. width) as shown. Down through this air gap a brass tube, carrying a single layer coil (of about 1 cm. axial length) in a shallow channel, can slide freely over a support—attached to A—which acts as a guide. The brass tube is released from a fixed position by a trigger and falls under gravity, thus ensuring that the coil always cuts through the magnetic field in the air gap at the same rate. The induced e.m.f. per turn on the coil is therefore constant. By the use of this standard the number of "line-turns"—i.e. the product of turns on the coil and lines of force through which these turns cut—which produce an observed throw on the ballistic galvanometer, can be determined in terms of the magnetic flux in the air gap of the standard and the number of turns on the coil. By means of tappings on the coil, the number of turns can



(*a*) (*b*) (*W. G. Pye & Co., Ltd.*)

FIG. 211. HIBBERT MAGNETIC STANDARD

be altered to give different numbers of line-turns. The number of turns obtainable is usually from 3 to 100, and the flux in the gap of the order of 20,000 lines, giving a maximum of 2,000,000 line-turns.

Such standards are reliable, and are easy to use, but have the disadvantage that only fixed numbers of line-turns—in multiples of (say) 20,000—can be obtained.

(*b*) BY MEANS OF A CAPACITOR. A capacitor which has been charged to a known voltage, by means of a standard cell, is discharged through the galvanometer. The quantity of electricity discharged can be calculated from the known voltage, and the capacitance of the capacitor. This method is not in general use owing to the difficulty of determining exactly the capacitance of the capacitor under all conditions, and also because of the fact that the damping of the galvanometer during calibration is different from that during testing.

(*c*) BY MEANS OF A STANDARD SOLENOID. This method is most commonly used for calibration purposes. A standard solenoid consists of a long coil of wire wound on a cylinder of insulating and non-magnetic material. There may be one or more layers of wire, but the design is such that the axial length of the solenoid is large compared with its mean diameter. Usually, the axial length is at least 1 metre, while the mean diameter is of the order of 10 cm. The winding must be uniform and the number of turns per centimetre axial length should be such that a strength of field H of 100 or more is obtained at the centre of the coil when carrying its maximum allowable current. If the axial length is great compared with the mean diameter, the field strength in the neighbourhood of the centre of the core of the solenoid is uniform and is given by

$$H = \frac{4\pi}{10}\frac{NI}{L}$$

where N = No. of turns on the solenoid,

I = the current in amperes flowing in the winding,

L = the axial length in centimetres.

If this condition as regards axial length and mean diameter is not fulfilled, the field strength at the centre is given by

$$H = k \cdot \frac{4\pi}{10}\frac{NI}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

where

$$k = \frac{L}{2d}\log_e\left[\frac{\left(r+\frac{d}{2}\right)+\sqrt{\left(r+\frac{d}{2}\right)^2+\frac{L^2}{4}}}{\left(r-\frac{d}{2}\right)+\sqrt{\left(r-\frac{d}{2}\right)^2+\frac{L^2}{4}}}\right]$$

where d = the radial depth of the winding on the solenoid in centimetres,

r = mean radius of the solenoid in centimetres.

If the radial depth, d, is small then, as seen in Chapter I, the field strength at the centre of the solenoid is given by

$$H = \frac{4\pi}{10}\frac{NI}{L}\cos\theta$$

where θ is the angle subtended at the centre of the coil by the mean radius r at one end of the coil, i.e. $\theta = \tan^{-1}\dfrac{r}{\frac{L}{2}}$

Thus,

$$H = \frac{4\pi}{10}\frac{NI}{L}\frac{\frac{L}{2}}{\sqrt{r^2+\left(\frac{L}{2}\right)^2}}$$

or

$$H = \frac{2\pi}{10}\frac{NI}{\sqrt{r^2+\left(\frac{L}{2}\right)^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (206)$$

At the centre of the solenoid is wound a small secondary coil, usually of several hundred turns of thin wire. The axial length of the secondary coil should be small, and it may either be wound over the solenoid, or placed within, and coaxial with it. In either case its dimensions must be accurately known.

This secondary coil is connected to the ballistic galvanometer, and a measured current is passed through the solenoid from a battery, through a reversing switch (Fig. 212). An e.m.f., producing a throw of the galvanometer, is induced in the secondary coil when the solenoid current is reversed. The number of line-turns producing this throw is obtained from the known value of H at the centre of the solenoid, and from the number of turns and dimensions of the secondary coil. For example, suppose that H at the centre of the solenoid is 60 when a certain current is flowing in it. Suppose, also, that the secondary coil has 400 turns, and that its mean area is 15 sq. cm.

Then, Flux threading through the secondary coil $= 60 \times 15 = 900$ lines

No. of line-turns $= 900 \times 400 = 360{,}000$

Change in the number of line-turns during reversal of the solenoid current $= 720{,}000$

Other methods of calibration of ballistic galvanometers, using a Duddell inductor or standard cell and a known current, are given by T. F. Wall (Ref. (2)) and by D. W. Dye (Ref. (3)).

Use of the Ballistic Galvanometer for the Measurement of Magnetic Flux. Referring to Fig. 213, in order to measure

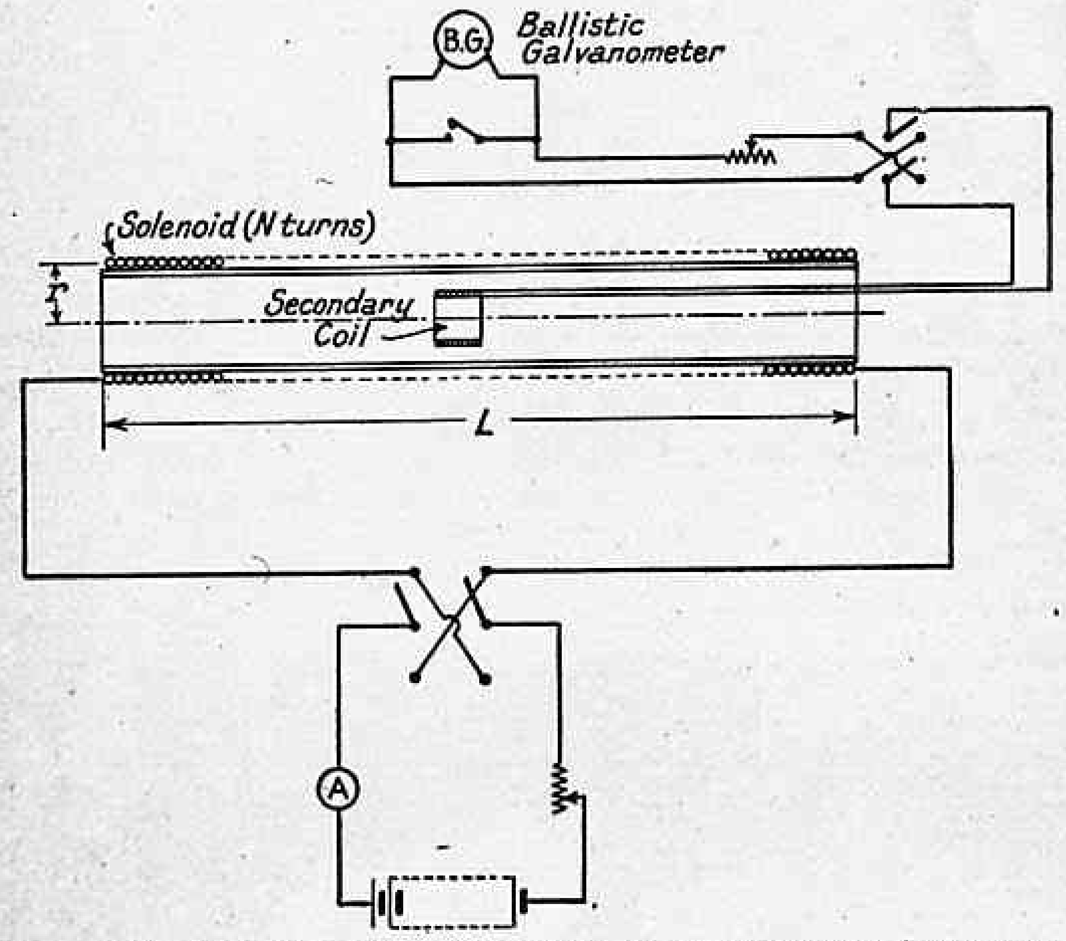


FIG. 212. BALLISTIC GALVANOMETER CALIBRATION BY STANDARD SOLENOID

the flux in the ring specimen of magnetic material corresponding to a given current I in the magnetizing winding which is uniformly wound on the specimen, a search coil of a convenient number of turns is wound on the specimen and connected, through a resistance and calibrating coil, to a ballistic galvanometer BG as shown. The magnetizing current I is reversed, and the galvanometer throw θ observed, the change in the flux produced by the current reversal being given by $kK\theta'$ where θ' is the undamped deflection (i.e. $\theta_1\left(1+\frac{\lambda}{2}\right)$) and K is the ballistic galvanometer constant, obtained by the use of the calibrating coil, which forms the secondary coil of a standard solenoid as previously described. k is a constant depending upon the resistance of the galvanometer circuit and number of search coil turns.

Theory.

Let N = No. of turns on search coil.

,, ϕ = flux embraced by the search coil.

,, R = resistance of the ballistic galvanometer circuit.

,, t = the time (in seconds) taken to reverse the magnetizing current (and hence the flux ϕ).

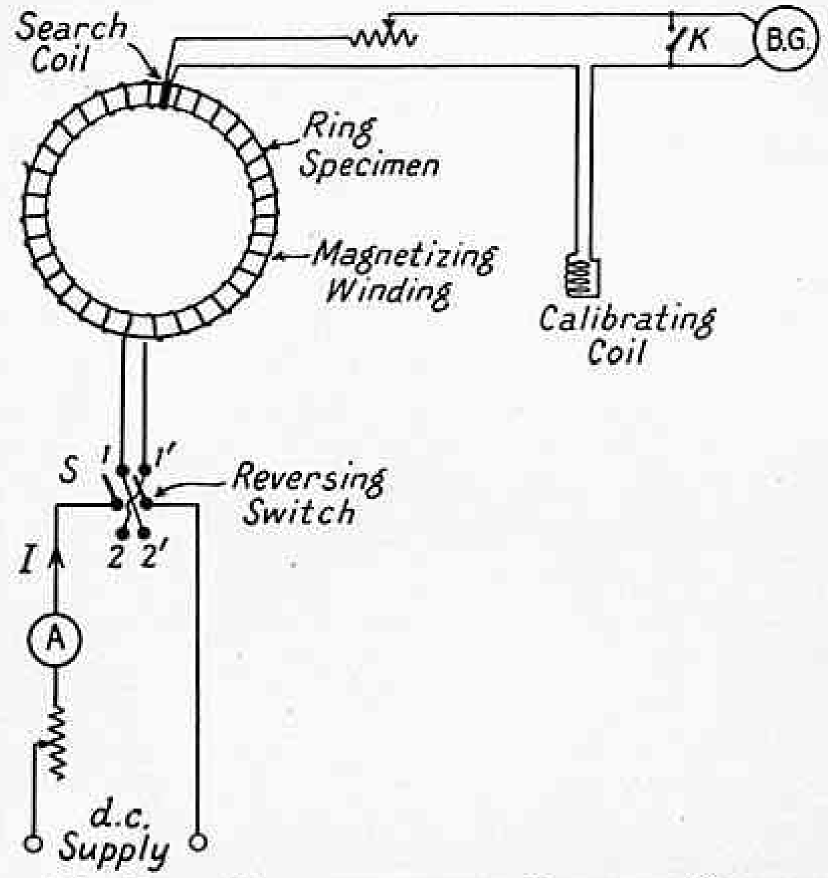


FIG. 213. FLUX MEASUREMENT BY BALLISTIC GALVANOMETER

Then, e.m.f. induced in search coil upon reversal of the flux

$$= N \frac{d\phi}{dt} \times 10^{-8} \text{ volts}$$

Average e.m.f. induced $= N \times \dfrac{2\phi}{t} \times 10^{-8}$ volts

Average current in the ballistic galvanometer circuit

$$= N \times \frac{2\phi}{tR} \times 10^{-8} \text{ amp}$$

Quantity of electricity discharged through the galvanometer during t sec.

$$= N \times \frac{2\phi}{tR} \times t \times 10^{-8} \text{ coulombs}$$

$$= 2\frac{N\phi}{R} \times 10^{-8}$$

But this equals $K\theta'$.

Hence, the flux in the specimen is given by

$$\phi = \frac{RK\theta' \times 10^8}{2N} \quad . \quad . \quad . \quad (207)$$

so that

$$k = \frac{R \times 10^8}{2N}$$

The Grassot Fluxmeter. This instrument is really a special type of ballistic galvanometer in which the controlling torque is very small and the electro-magnetic damping is heavy. The construction is illustrated by Fig. 214.

A coil of small cross-section is suspended from a spring support by means of a single thread of silk, and hangs with its parallel sides in the narrow air gaps of a permanent-magnet system, as shown. Current is led into the coil by spirals of very thin, annealed silver strips. By this construction the controlling torque is reduced to a minimum. The instrument is usually fitted with a pointer (attached to the moving system) and a scale, although it may also be used as a reflecting instrument. The scale is graduated in terms of line-turns.

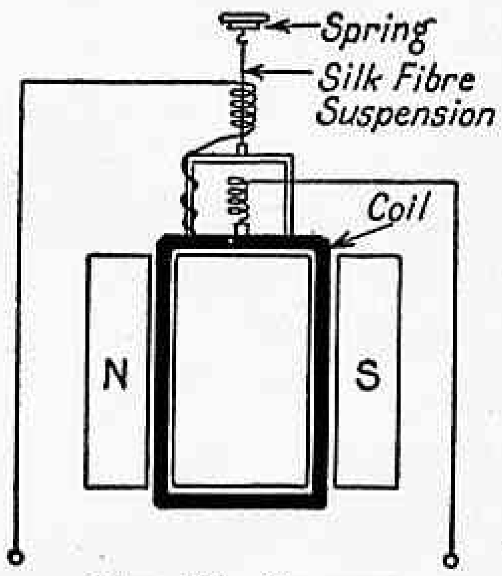


FIG. 214. GRASSOT FLUXMETER

The instrument is very portable and, although not so sensitive as a ballistic galvanometer, it has the great advantage that the length of time taken for the change in the flux producing the deflection need not be small. The deflection obtained, for a given change of flux interlinking with the search coil connected to the instrument, will, in a good instrument, be the same whether the time taken for the change be a fraction of a second or as much as one or two minutes.

If no controlling torque were present the instrument would remain in its deflected position indefinitely. Actually the pointer returns very slowly to zero, but readings may be taken by observing the difference in deflection at the beginning and end of the change in flux to be measured without waiting for the pointer to return to zero, the scale being uniform. The resistance of the search coil circuit connected to the fluxmeter should be fairly small, although its actual value is not important, a variation in this resistance of several ohms usually having a negligible effect upon the deflection. The inductance of the search coil circuit is also unimportant, and may be quite large, with negligible effect upon the deflection.

Theory of the Fluxmeter. Assume that the controlling torque is negligibly small, and also that air damping and friction are negligible.

Let N. = No. of turns on the search coil which is connected to the fluxmeter terminals.

r_s and l_s = the resistance and inductance of the search coil circuit.

R and L = the resistance and inductance of the fluxmeter.

e_s = the e.m.f. induced in the search coil at any instant.

e_f = the e.m.f. being induced at any instant in the fluxmeter coil due to its movement in the permanent-magnet field.

i = the current in the circuit at any instant.

Then, $$e_s = N \cdot \frac{d\phi}{dt}$$

where $\frac{d\phi}{dt}$ is the rate of change of flux linking with the search coil.

Also, $$e_f = K\frac{d\theta}{dt}$$

where K is a constant depending upon the dimensions of the fluxmeter coil, its number of turns, and upon the strength of the permanent magnet field; $\frac{d\theta}{dt}$ is the angular velocity of the fluxmeter coil.

The equation connecting the electrical quantities is, therefore,

$$e_s = e_f + (L + l_s)\frac{di}{dt} + (r_s + R)i \quad . \quad . \quad . \quad . \quad (208)$$

The term $(r_s + R)i$ may be neglected if r_s is small, since i is also very small.

Hence, $$N\frac{d\phi}{dt} = K.\frac{d\theta}{dt} + (L + l_s)\frac{di}{dt} \quad . \quad . \quad . \quad . \quad (209)$$

Integrating with respect to t we have

$$\int_o^T N\frac{d\phi}{dt}\,.\,dt = \int_o^T K\frac{d\theta}{dt}\,.\,dt + \int_o^T (L + l_s)\frac{di}{dt}\,.\,dt$$

T being the total time taken for the change in the flux.

Thus $$\int_{\phi_1}^{\phi_2} Nd\phi = \int_{\theta_1}^{\theta_2} Kd\theta + \int_{i_1}^{i_2} (L + l_s)di$$

ϕ_2 and ϕ_1 are the interlinking fluxes, θ_2 and θ_1 the deflections, and i_2 and i_1 the currents in the search-coil circuit, at the end and beginning of the change in the flux. Since i_2 and i_1 are both zero the value of $\int_{i_1}^{i_2}(L + l_s)di$ is also zero (which means that the inductance does not affect the deflection).

Hence, $$N(\phi_2 - \phi_1) = K(\theta_2 - \theta_1)$$

or, if ϕ is the change in the flux and θ the change in the fluxmeter deflection

$$\phi = \frac{K}{N}\,.\,\theta \quad . \quad . \quad . \quad . \quad . \quad (210)$$

If the fluxmeter permanent-magnet field is uniform for all positions of the moving coil, K is constant, and the change in the flux is directly proportional to the change in the deflection.

Theory when Shunted. If a very large flux is to be measured (e.g. that in one of the poles of a large machine), the number of line-turns may be too

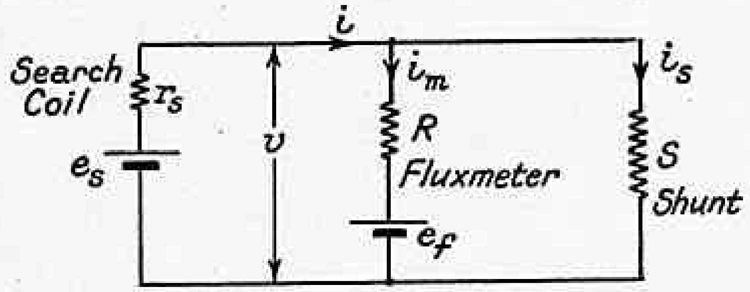


FIG. 215. FLUXMETER CIRCUIT WHEN SHUNTED

great to be measured on the fluxmeter, even though a search coil of only one turn is employed. The range of the fluxmeter can be extended for such measurements by the use of shunts.

Fig. 215 gives a diagram of the circuit when a shunt is used, the induced e.m.f's being shown as batteries, and the fluxmeter being represented by the resistance R. Let the resistance of the shunt be S, i being the current in the search coil, i_m that in the fluxmeter, and i_s that in the shunt.

Then, if v is the p.d., at any instant, across all three branches we have

$$v = i_m R + e_f \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$
$$v = i_s S = (i - i_m)S \quad . \quad . \quad . \quad . \quad . \quad (ii)$$
$$v = e_s - ir_s \quad . \quad . \quad . \quad . \quad . \quad . \quad (iii)$$
$$i = i_m + i_s \quad . \quad . \quad . \quad . \quad . \quad . \quad (iv)$$

Thus, $$e_s - ir_s = e_f + i_m R$$

or
$$\begin{aligned} e_s - e_f &= i_m R + ir_s \\ &= i_m R + (i_m + i_s)\,r_s \\ &= i_m(R + r_s) + r_s\frac{(i_m R + e_f)}{S} \end{aligned}$$

(from (i) and (ii)).

$$\therefore e_s - e_f - \frac{r_s}{S}\,.\,e_f = i_m(R + r_s) + \frac{r_s R}{S}\,.\,i_m$$
$$= 0$$

since the currents and resistances are small.

$$\therefore e_s = e_f\frac{(S + r_s)}{S} \quad . \quad . \quad . \quad . \quad (211)$$

From the previous theory $e_s = N\frac{d\phi}{dt}$ and $e_f = K\frac{d\theta}{dt}$

Thus, $$N\frac{d\phi}{dt} = K\frac{d\theta}{dt}\frac{(S + r_s)}{S}$$

Integrating with respect to t, we have

$$\int_0^T N\frac{d\phi}{dt}\,.\,dt = \int_0^T K\frac{(S+r_s)}{S}\frac{d\theta}{dt}\,.\,dt$$

or

$$N\phi = K\frac{(S+r_s)}{S}\,.\,\theta$$

where θ is the change in deflection caused by a change of ϕ in the flux interlinking the search coil. Hence,

$$\phi = \frac{K}{N}\frac{(S+r_s)}{S}\,\theta \quad . \quad . \quad . \quad . \quad (212$$

Unshunted, the expression is

$$\phi = \frac{K}{N}\,.\,\theta$$

Thus, the deflection for a given change in the flux, when shunted, is to the deflection for the same change when unshunted, as $\frac{S}{S+r_s}$.

It should be noted that it is the *resistance*, r_s, *of the search coil* which is important when shunting is used, and not the resistance of the fluxmeter itself.

MEASUREMENT OF LEAKAGE FACTOR BY MEANS OF THE FLUXMETER. In dynamo-electric machinery the magnetic flux per pole which crosses the air gap—the "useful flux"—is less than the flux in the body of the pole. This is due to the fact that some lines of force—referred to as "leakage flux"—pass from the pole to the adjacent poles without crossing the air gap to the armature. The flux at the root of the pole is called the "total" flux, and the ratio $\frac{\text{Total flux}}{\text{Useful flux}}$ is the "leakage factor" of the pole.

This factor can be measured by means of a fluxmeter, a ballistic galvanometer being unsuitable on account of the high inductance of the field winding, which results in a slow rate of increase of the flux when the voltage is switched on to the field winding.

The total flux may be measured by winding two search coils on the yoke of the machine—in the case of a direct current machine with a stationary field—one on either side of the pole (see Fig. 216). As the yoke carries half of the total flux, these search coils must be connected in series so that the fluxmeter measures the flux embraced by both of them. The flux so measured will be the total flux. Another search coil, placed on the (stationary) armature in such a position that it embraces the useful flux from the pole, is then connected to the fluxmeter and the useful flux measured, the leakage factor being obtained from the two measurements.

It will usually be found that search coils of one turn only will be most suitable, in which case the fluxmeter reading gives the flux directly. In the case of a large machine it may be necessary to use shunts across the search coils, as described previously.

The Chattock Magnetic Potentiometer. Before proceeding with the methods of testing magnetic specimens, this device for the measurement of the magnetic potential between any two points in a magnetic field will be described. The device consists of a uniform helix of thin wire, wound on a thin strip, or rod, of some flexible insulating and non-magnetic material. This can be used, in conjunction with a ballistic galvanometer, to measure magnetic potential differences.

Let the cross-sectional area—assumed uniform—of the strip upon which the helix is wound be A sq. cm., and the number of turns per

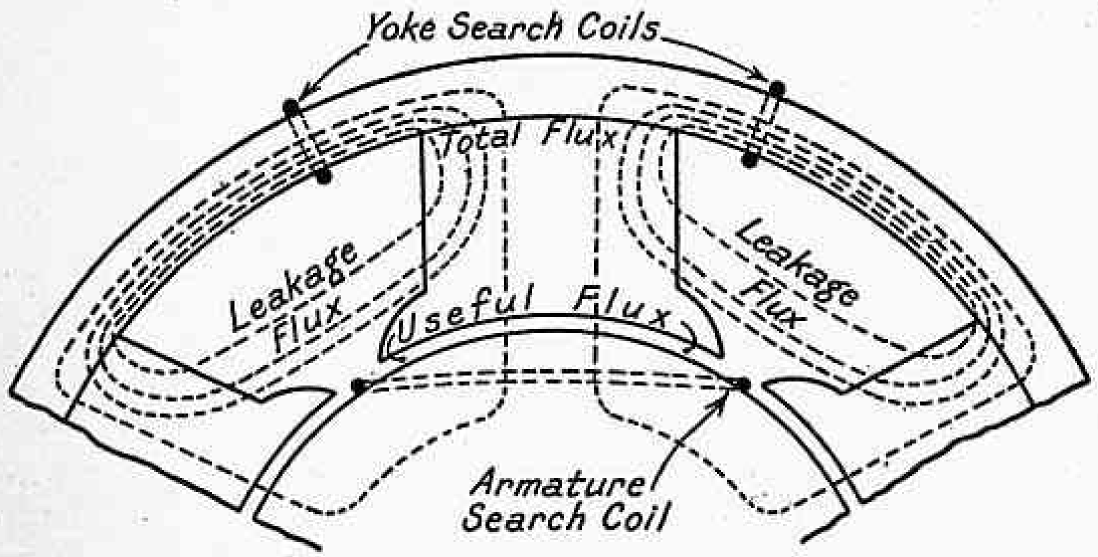


FIG. 216. MEASUREMENT OF LEAKAGE FACTOR

centimetre length be n. Suppose that, when the helix is situated in a magnetic field, p_{av} is the average magnetic potential difference between the two ends. Then $p_{av} = \int H dl$, where H is the field strength, at any point within the helix, in the direction of the element of length of path dl, the integral being taken between a certain "average" point on one end of the strip and a corresponding point on the other end.

The quantity

$$Ap_{av} = \int p\,.\,dA$$
$$= \int H\,.\,dV$$

where dV is an element of volume within the helix.

Now, if the strip upon which the helix is wound is of non-magnetic material, the permeability is unity and the flux crossing any given cross-section of the helix is given by $\int H\,.\,dA$ where H is the field strength at the point at which the cross-section is taken. Since the flux embraced at any cross-section of the helix is not constant, the e.m.f. induced in the helix when the strength of the magnetic field in which it is situated is altered must be expressed by

$$e = \frac{d}{dt}\int\int HdA \,.\, ndl$$

the integral term giving the effective "flux-turns" of the helix.

Thus, $$e = \frac{d}{dt}\, n \int HdV$$

or $$e = \frac{d}{dt}\,(n \,.\, A \,.\, p_{av}) = nA\,\frac{dp_{av}}{dt} \quad . \quad . \quad . \quad (213)$$

i.e., the induced e.m.f. is proportional to the rate of change of magnetic potential between the two ends of the helix. From the theory of the use of the ballistic galvanometer, given on page 356, it can be seen that the galvanometer deflection, when the magnetic potential is changed, will be proportional to this change, i.e. if Δp be the change in potential

$$\Delta p = K\theta'$$

where θ' is the corrected throw of the galvanometer and K is the galvanometer constant.

The change in potential Δp may be produced by a change in the magnetizing current producing the magnetic field in which the helix is situated, or it may be produced by the rapid movement of one end of the helix from one point in the field to another, the other end being kept in the same position.

Applications of this device are the measurement of the magnetic potential drop across a given part of a magnetic circuit, such as a joint, and the measurement of magnetic leakage. Measurements may be made upon alternating magnetic fields by using a vibration galvanometer instead of a ballistic galvanometer.

From the theory of the potentiometer it can be seen that the results are the same whether the strip upon which the helix is wound is straight or otherwise. This is a great advantage, as the use of a strip of flexible material is very convenient in investigations of this nature.

Other Methods of Measuring Magnetic Field Intensity. Several other methods of exploring magnetic fields, although not in very general use, deserve mention.

(*a*) Standard Search Coil. A search coil, consisting of a single layer of (say) 50 to 100 turns of fine silk-covered wire, wound upon a short cylinder of non-magnetic material—usually marble—may be used to investigate the variation in strength, from point to point, of a magnetic field in air. The marble cylinder must be carefully turned so that its cross-sectional area may be uniform throughout its axial length (about 2 or 3 cm.).

The product of cross-sectional area and number of turns for the search coil may be determined to within a few parts in 10,000 by the use of a standard solenoid.

(*b*) Bismuth Spiral. This method depends upon the fact that the resistance of a bismuth wire is increased when it is placed in a strong magnetic field. A curve, showing the order of this increase, is given in Fig. 217, the curve relating to a temperature of 20° C. A flat spiral of pure bismuth wire—about 1 mm. diameter—is used for the exploration of magnetic fields. The resistance of the spiral, when situated at a point in the magnetic field, is measured, and this

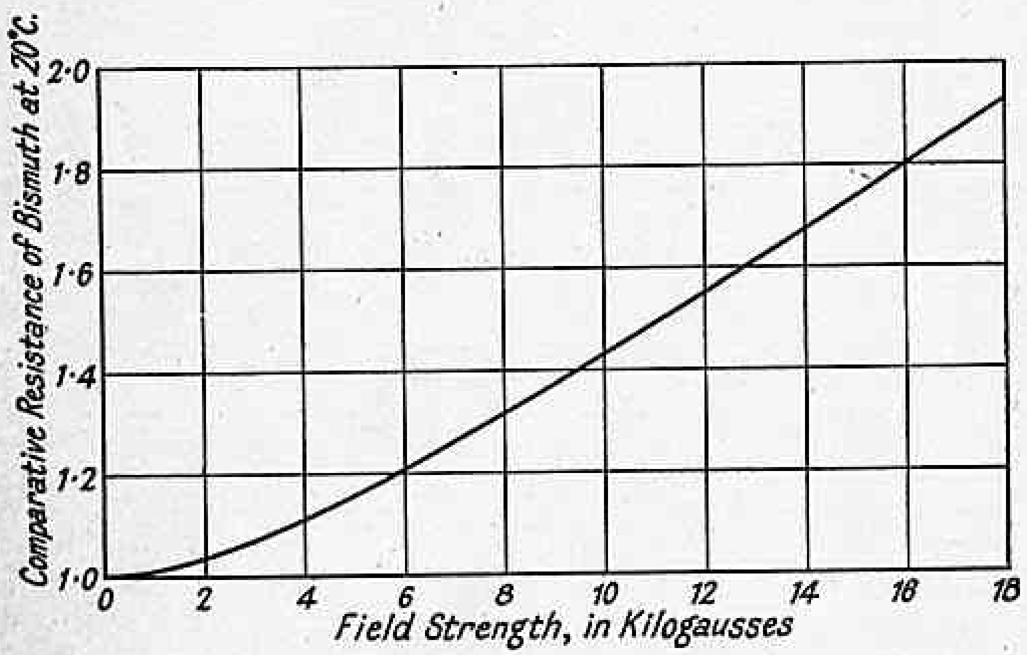


Fig. 217. Resistance–Field-strength Curve for Bismuth

resistance is compared with the resistance when the spiral is removed from the field, the temperature being the same in both cases. The field-strength is obtained from a curve such as that of Fig. 217, when the ratio $\dfrac{\text{Resistance of spiral whilst in the magnetic field}}{\text{Resistance of spiral when removed from the field}}$ has been determined.

The disadvantages of this method are that it is rather insensitive —the change in resistance per kilogauss change in field strength being comparatively small—and also that this change in resistance depends very largely upon the temperature.

Its advantages are that it is very simple to use, the resistance being conveniently measured by the Wheatstone bridge method, and that, since the spiral covers a small area—of the order of 1 sq. cm.—the exploration of the magnetic field can be carried out in greater detail than is possible with most other methods.

(*c*) Magnetic Balance and Deflecting-coil Methods. Such methods depend upon the fact that a force is exerted upon a current-carrying conductor when it is placed in a magnetic field, the force depending upon the magnitude of the current in the conductor and upon the strength of the magnetic field.

In the magnetic balance the force upon the conductor is balanced by weights on a pivoted beam, to one end of which the conductor is attached, the weight required for balance being used to determine the strength of the magnetic field in which the current-carrying conductor is situated.

In deflecting-coil methods a narrow coil, some 2 or 3 mm. wide and 1 or 2 cm. long, is used. This coil is suspended by a fine strip suspension from a torsion head, a mirror being attached to the suspension for use in conjunction with a lamp and scale to indicate deflection. A current is passed through the coil, which is placed in the magnetic field to be measured with its plane parallel to the field. If this current is known, the field strength corresponding to a certain deflection may be determined by calibration, using a magnet of known gap-flux density.

The reader is referred to the works given in Refs. (1), (3), and (7) for fuller information regarding such methods.

The Testing of Ring Specimens. Ring specimens are used, in preference to rods or strips, when accurate measurements of permeability, and hysteresis loss, up to a maximum value of field strength (H) of 200 or 250, are required. The use of such specimens has the disadvantage that they are more difficult to prepare than bar specimens, and also are more difficult to wind with the magnetizing winding which, when bar specimens are used, may be a permanently wound solenoid inside which the bar is slipped. The more reliable results obtainable with ring specimens on account of their freedom from self-demagnetizing effects may, in some cases, justify their use.

Such specimens are cut from a representative piece of the iron, the rings being machined so that their dimensions may be accurately determined. The radial thickness of such rings should be fairly small compared with their mean diameter.* If this condition is not fulfilled, most of the flux in the iron passes through the portion of the ring nearest to the inner circumference, thus causing a distribution of flux density across the cross-section of the ring which is far from uniform. The mean value of the flux density (as given by $\dfrac{\text{Total flux}}{\text{Cross-section}}$ under these conditions) will not correspond to the mean value of H for the cross-section of the ring, and the B-H curve obtained will be erroneous. For accurate results the ratio of outside diameter of the ring to the radial thickness should be at least 15.

If sheet material is to be tested, the ring specimen should be built up of ring punchings taken, if possible, from a number of different sheets. The punchings should be built up with the direction of rolling used in the manufacture of the sheet distributed radially to obtain a uniform distribution of reluctance round the ring. The

* This question is fully considered in a paper by E. Hughes (Ref. (8)).

permeability in a direction perpendicular to that of rolling is only some 75 per cent of the permeability, at the same flux density, in a direction parallel to the direction of rolling. During the shearing of the rings from the sheet, the material near the sheared edges (and for some distance inwards from the edge) is strained, the effect being to reduce its permeability. Unless the rings are to be annealed, after punching, to remove these strains, their radial width should be fairly large—say 2 or 3 cm.—in order that the strained portion shall not form an appreciable percentage of the whole cross-section.

Determination of the Magnetization, or B-H, Curve. (*a*) Method of Reversals. For this test a ballistic galvanometer is used as previously described. Before winding, the dimensions of the ring must be determined. When sheet material is being tested it may be necessary to determine the effective cross-section from the weight of the ring, taking the specific gravity as 7·8 (for soft sheet iron). This is necessary for accurate measurements on account of the air spaces between individual punchings, which cause measurements of thickness to be erroneous.

A layer of thin tape is then wound on the ring, and a search coil consisting of a few turns of thin wire, insulated by paraffined silk, is wound over the tape. The number of search-coil turns depends upon the sensitivity of the ballistic galvanometer. This number of turns must, of course, be noted. The search coil is protected by another layer of tape, over which the magnetizing winding is uniformly wound.

The connections for the test are shown in Fig. 213. Before commencing the test, it is essential that any residual magnetism which may be present in the specimen shall be removed by demagnetization. The short-circuiting key K of the galvanometer is left closed, and the current in the magnetizing winding is given such a value that the magnetizing force H acting upon the specimen is greater than the maximum value to be used in the test. This current is then very gradually reduced—the reversing switch S being continually thrown backwards and forwards meanwhile—in order to pass the iron specimen through as many cycles of magnetization as possible during the process. The minimum value of the current finally reached should give a magnetizing force in the specimen which is well below the smallest test value.

After demagnetization, the test is started by setting the magnetizing current at the lowest test value (such that $H = 1$ (say)). The galvanometer key K being closed, the iron specimen is then brought into a "reproducible cyclic magnetic state" by throwing the reversing switch S backwards and forwards some twenty or more times. The key K is next opened, and the flux in the specimen, corresponding to this value of H, is measured on the ballistic galvanometer as previously described. The change in flux, measured by the galvanometer, when the reversing switch S is quickly reversed, will be twice the flux in the specimen corresponding to the value of H which has been applied.

This value of H is given by $\frac{4\pi}{10} \cdot \frac{NI_1}{l}$ where N = number of turns on the magnetizing winding, I_1 = the magnetizing current, l = the length (in centimetres) of the mean circumference of the ring specimen.

The flux density B_1 corresponding to this value of H is obtained by dividing the value of the flux in the specimen (as measured by the ballistic galvanometer) by the cross-sectional area of the specimen.

In order to check whether the demagnetization of the specimens has been complete and also to determine whether a reproducible cyclic state has been attained, a second measurement of flux density—for the same value of H—may be made after subjecting the specimen to a further number of reversals of magnetization. The second value of B should agree with the first. As further checks upon the measurements, a reversing switch in the ballistic galvanometer circuit (not shown in Fig. 213) may be used as follows: Measurement of flux density for a given value of H may be made, with this reversing switch in one position, first by throwing the reversing switch S over from terminals 11′ to 22′ and the test repeated by throwing over from 22′ to 11′, having carried out a number of reversals of S in between the two measurements. This procedure may be repeated with the ballistic galvanometer reversing switch in its other position, four measurements of B being thus obtained. These should be very nearly equal to one another, the mean giving the value of the flux density.

The whole of this procedure is repeated for various increased values of H up to the maximum testing point, the 20 or more reversals of the magnetizing current at each value of H, before the measurement is made, being important. It should be noted also that if the resistance of the ballistic galvanometer circuit is changed during the test, the logarithmic decrement λ must be determined for each resistance value, in order that the observed galvanometer throws may be properly corrected, the deflection used in determining the flux being given by

$$\theta' = \theta_1\left(1 + \frac{\lambda}{2}\right)$$

where θ_1 is the observed throw.

The B-H curve may be plotted from the measured values of B corresponding to the various values of H.

(*b*) The "Step-by-step" Method is sometimes used. In this method there is no reversal of the magnetizing current, the procedure being as follows. The circuit shown in Fig. 213 is set up in the same way as for the test by the method of reversals, but the direct current supply to the magnetizing circuit is through a potential divider having a number of tappings, as shown in Fig. 218. The tappings are arranged so that the magnetizing force H may be increased, in a number of suitable steps, up to the desired maximum value. The specimen, after the application of the search coil and magnetizing winding, is first demagnetized. The tapping switch S_2 is then set

on tapping 1 and the switch S_1 closed, the galvanometer throw corresponding to this increase in the flux density in the specimen, from zero to some value B_1, being observed, and B_1 calculated as previously described. H_1, corresponding to this position of switch S_2, can be determined from the magnetizing current which then flows. The magnetizing force is then increased to H_2 by switching S_2 suddenly on to tapping 2, and the corresponding increase in flux density ΔB determined from the galvanometer throw observed. Then B_2—corresponding to H_2—is given by $B_1 + \Delta B$. This process is repeated for other values of H up to the maximum point, and the complete B-H curve is thus obtained without any reversal of the flux in the specimen.

Determination of the Hysteresis Loop. (*a*) Step-by-Step Method. The determination of the hysteresis loop by this method is carried out simply by continuing the procedure just described for the determination of the B-H curve. Having reached the point of maximum H—when S_2 (Fig. 218) is on tapping 10—the magnetizing current is next reduced, in steps, to zero by moving switch S_2 down through the tapping points, 9, 8, 7, etc. After the reduction of the magnetizing force to zero, negative values of H are obtained by reversing the reversing switch S (Fig. 213) and then moving the switch S_2 (Fig. 218), in steps as before.

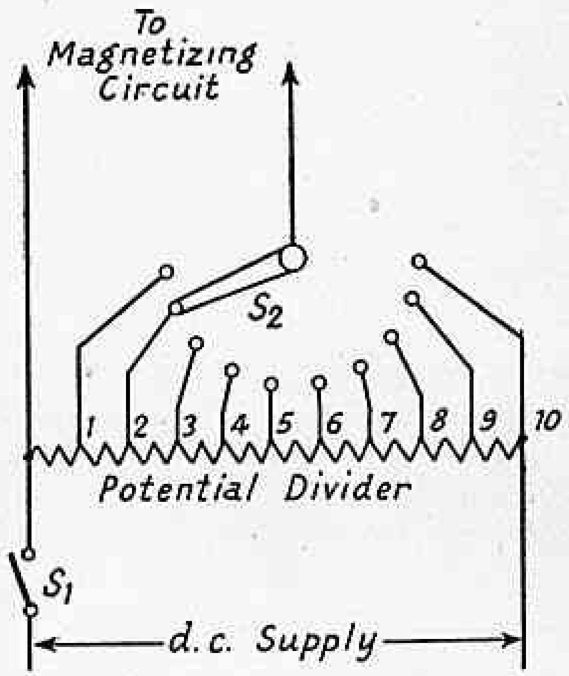


Fig. 218 Potential Divider for Step-by-step Method

(*b*) By the Method of Reversals. This test again is carried out by means of a number of steps, but the change in flux density measured at each step is the change from the maximum value $+ B_{max}$ down to some lower value, the iron specimen being passed through the remainder of the cycle of magnetization back to the flux density $+ B_{max}$ before commencing the next step in the test, thus preserving the cyclic state.

The connections for the test are shown in Fig. 219. R_1, R_2, and R_4 are resistances for the adjustment of the resistances of the magnetizing and ballistic galvanometer circuits. R_3 is a variable shunting resistance which is connected across the magnetizing winding by moving over the switch S_2, thus reducing the current in this winding from its maximum value down to any desired value—depending upon the value of R_3.

The procedure is as follows—

The value of H_{max} required to produce the value of B_{max} to be used during the test, is obtained from the previously determined B-H curve of the specimen. The resistances R_2 and R_4 are then adjusted so that the magnetizing current is such that this value of H is obtained when switch S_2 is in the "off" position. (H is, of course, given by $H = \frac{4\pi}{10} \cdot \frac{NI}{l}$, where I is the magnetizing current, N the number of turns on the magnetizing winding, and l the length of magnetic path, or mean circumference, of the specimen.) The resistance R_1 is adjusted so that a convenient deflection of the galvanometer is obtained when the maximum value of the magnetizing current is reversed. R_3 is adjusted to such a value that a

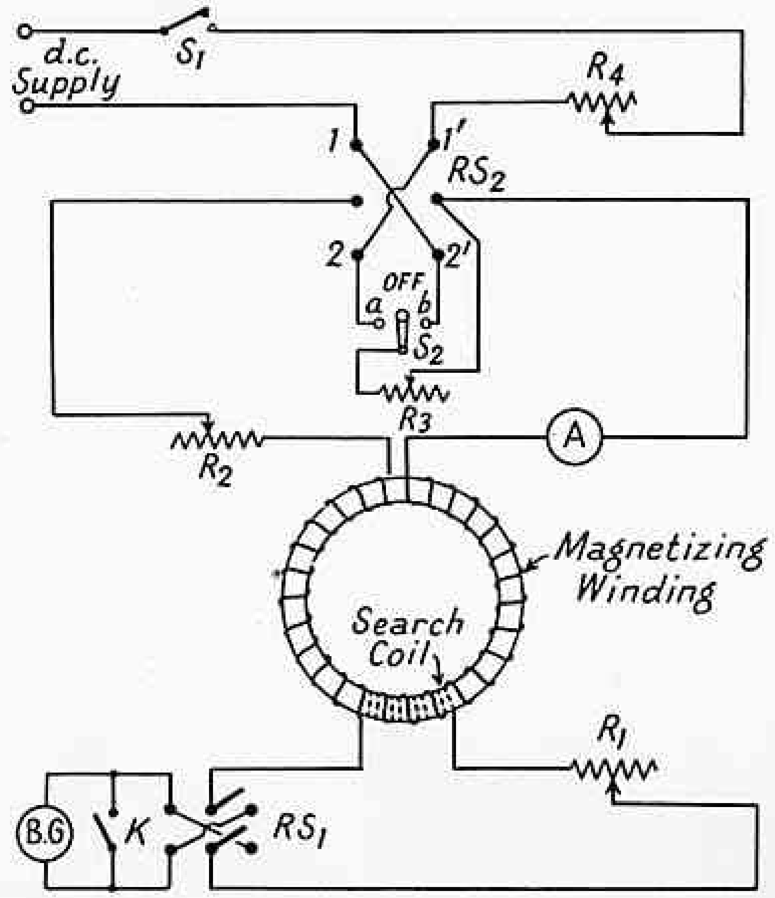


FIG. 219. CONNECTIONS FOR METHOD OF REVERSALS

suitable reduction of the current in the magnetizing winding is obtained when this resistance is switched in circuit.

Switch RS_2 is then placed on contacts 11′ and the short-circuiting key K opened. The magnetization of the specimen—since the maximum magnetizing current is now flowing—corresponds to point A on the loop (Fig. 220).

The next step is to throw switch S_2 quickly over from the "off" position to contact b, thus shunting the magnetizing winding by R_3 and reducing the magnetizing force to H_c (say). The corresponding reduction in flux density, $-\Delta B$, is obtained from the galvanometer throw, and hence the point C on the loop is obtained.

The key K is now closed, and switch RS_2 reversed on to contacts

22′. Switch S_2 is then opened and RS_2 moved back again to contacts 11′. This procedure passes the specimen through the cycle of magnetization and back to point A again, ready for the next step in the test. The section AD of the loop is obtained by continuation of this procedure.

To obtain the section DEF of the loop, with K closed and S_2 in the "off" position, place RS_2 on contacts 11′. Then place S_2 on contact a, open the key K, and rapidly reverse RS_2 on to contacts 22′, observing the corresponding galvanometer throw. From this throw the change in flux $\Delta B'$ (Fig. 220) may be obtained, since the switching

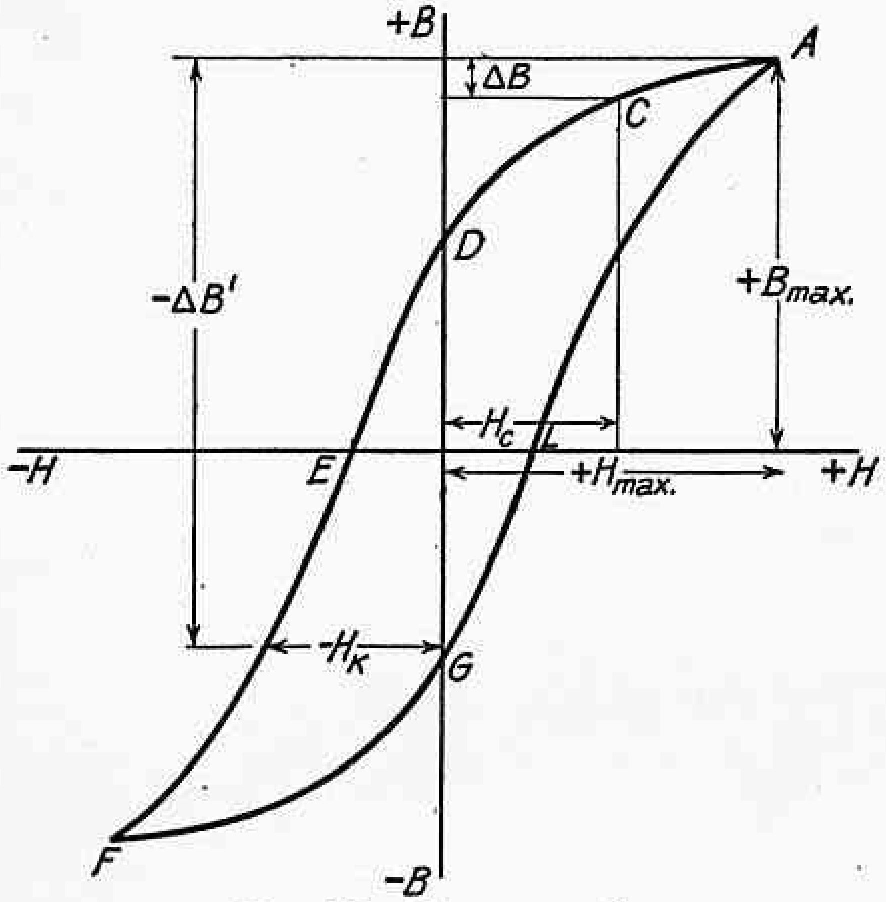


FIG. 220. HYSTERESIS LOOP

operations described cause H to be changed from $+H_{max}$ to $-H_K$ (say). To bring the magnetization of the specimen back to point A, close key K, open S_2 and reverse RS_2 on to contacts 11′.

By continuing this process, other points on the section DEF of the loop are obtained. The section $FGLA$ of the loop may be obtained by drawing in the reverse of $ADEF$, since the two halves are identical.

By measuring the area of the hysteresis loop so obtained—by means of a planimeter—and expressing this area in B-H units of area, the hysteresis loss for the material may be obtained, since

Hysteresis loss per cycle per cubic centimetre, in ergs

$$= \frac{\text{Area of loop in B-H units}}{4\pi}$$

The Testing of Specimens in the Form of Rods or Strips. It is obviously much easier to prepare a specimen in the form of a machined rod than to prepare a ring specimen as previously described. Such specimens suffer, however, from the disadvantage of "*self-demagnetization.*" When a rod is magnetized electromagnetically, poles are produced at its ends, and these poles produce, inside the rod, a magnetizing force from the north pole to the south which is in opposition to the applied magnetizing force, thus rendering the true value of H acting on the rod a somewhat uncertain quantity. For accurate results, therefore, if the methods of measurement

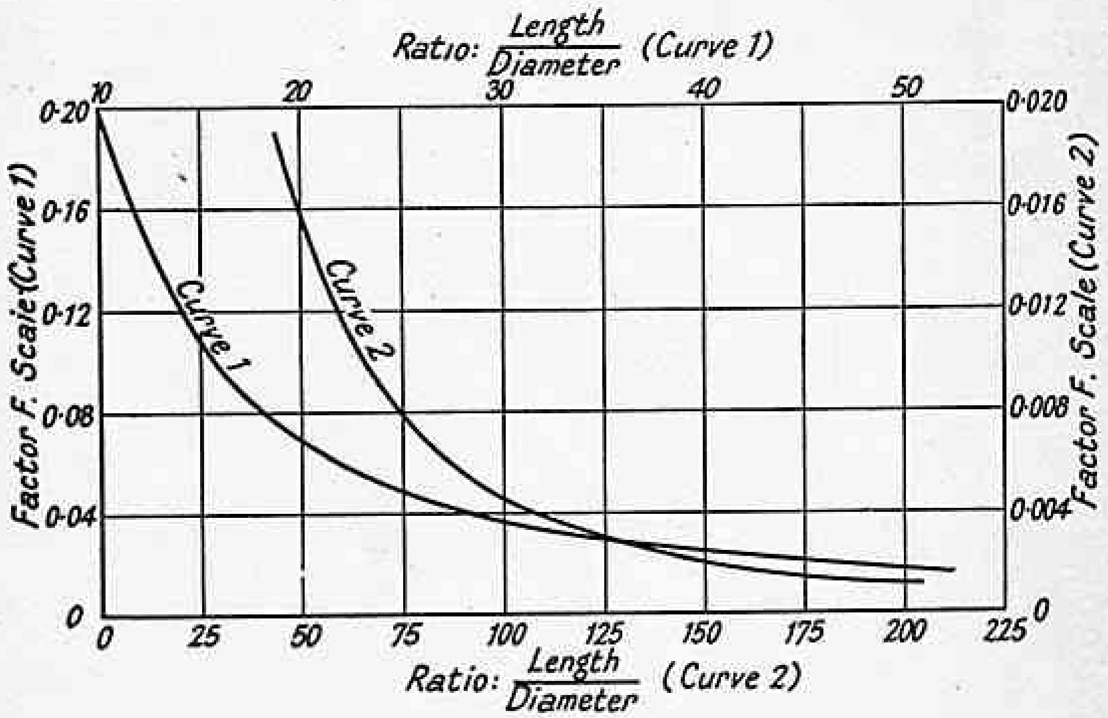


FIG. 221. CURVES OF DEMAGNETIZING FACTORS

using a ballistic galvanometer as described above, are used, this demagnetizing effect must be corrected for, or, since the effect is least when the ratio of diameter to length of the rod is small, the dimensions of the specimen should be chosen so that the effect is negligible.

The demagnetizing force due to this "end effect" is given by the expression

$$H_d = \frac{F}{4\pi} \cdot B_f \quad . \quad . \quad . \quad . \quad . \quad (214)$$

where B_f is the ferric induction, i.e. the flux density due to the magnetization of the iron itself, and F is a constant which depends upon the relative dimensions of the rod. The expression might also be written $H_d = F \cdot J$, where J is the intensity of magnetization. The value of F for various ratios of length to diameter for cylindrical rods may be determined from a curve such as that given in Fig. 221, which has been plotted from values given by Du Bois (Ref. (5)) and by Thompson and Moss (Ref. (9)).

For an ellipsoid or very long rod, the value of the coefficient F may be calculated from the expression

$$F = 4\pi\left[\frac{1}{2k}\log_e\frac{1+k}{1-k} - 1\right]\left(\frac{1}{k^2} - 1\right)$$

where $k = \sqrt{\left(1 - \frac{a^2}{b^2}\right)}$

a = the minor axis of the ellipsoid,
b = the major axis of the ellipsoid.

To obtain the true value of the magnetizing force H acting on a bar specimen H_d must be subtracted from the value of H calculated from the ampere-turns per centimetre length of the magnetizing

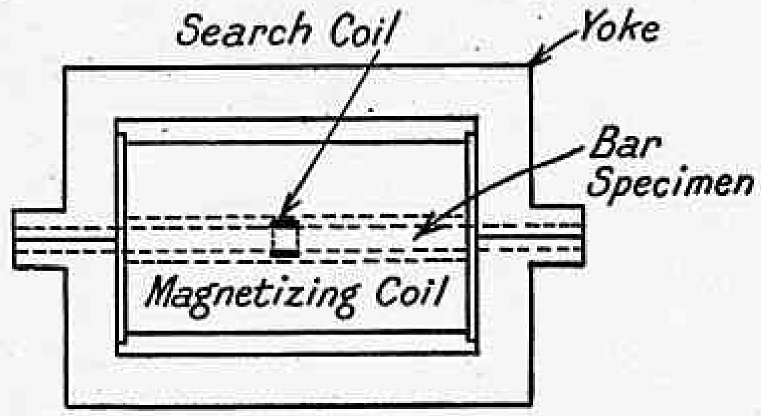


FIG. 222. BAR AND YOKE METHOD

winding. From the curves given it can be seen that the ratio $\frac{\text{Length}}{\text{Diameter}}$ of the specimen must be of the order of 25 or more for the effect to have a negligible influence upon the value of H.

On account of this demagnetizing effect the value of H is often measured by means of search coils wound on thin strips of glass and placed with the glass lying flat on the bar specimen (Refs. (2), (3)). The flux density in the air at the surface of the specimen (which is the same as H in the specimen) is measured by this means instead of relying upon calculated values and corrections.

Bar and Yoke Methods. Such methods are commonly used for the testing of bar specimens. They combine the advantages of both ring and bar specimens, the demagnetizing effect being largely eliminated by the use of heavy-section yokes, while the advantages of the bar specimen, as regards preparation and ease of application of the magnetizing force, are retained.

There are a number of such methods which are all essentially modifications of the original yoke method due to Hopkinson.

A search coil is wound upon the bar specimen at its centre, and the bar is then clamped between the two halves of a massive iron yoke, whose reluctance is small compared with that of the specimen, as shown in Fig. 222. The magnetizing winding is fixed inside the yoke, as shown, the specimen fitting inside it.

Let N = No. of turns on the magnetizing winding.
,, I = current in the magnetizing winding.
,, l = length of specimen between the two halves of the yoke.
,, A = cross-section of the specimen.
,, μ_s = permeability of the specimen when the magnetizing current is I.
,, S_y = the reluctance of the yoke.
,, S_g = the reluctance of the two joints between specimen and yoke.
,, ϕ = the total flux in the magnetic circuit.

Then
$$\phi = \frac{\frac{4\pi}{10} \cdot NI}{S_y + S_g + \frac{l}{A\mu_s}}$$

Now, if H is the actual magnetizing force acting on the specimen, and B is the flux density in it,

$$B = \mu_s \cdot H = \frac{\phi}{A}$$

$$\text{Hence, } H = \frac{\frac{4\pi}{10} \cdot NI}{\left(S_y + S_g + \frac{l}{A\mu_s}\right) A\mu_s} = \frac{\frac{4\pi}{10} \cdot NI}{S_y A\mu_s + S_g A\mu_s + l}$$

This may be written

$$H = \frac{\frac{4\pi}{10} \cdot NI}{(1 + m)l}, \text{ where } m = \frac{A\mu_s}{l}(S_y + S_g) \quad . \quad . \quad (215)$$

The quantity m is made small by carefully fitting the specimen into the yoke, and by making the yokes of very heavy section, thus reducing both S_g and S_y to small quantities. In preparing the specimen its dimensions must be very carefully adjusted so that it exactly fits the holes in the yoke to be used. The length of the specimen usually used is about 20 or 25 cm., and the diameter (if of rod form) about 1 cm.

If m is small,

$$H = \frac{4\pi}{10} \cdot \frac{NI}{l}(1 - m) \text{ approx.} \quad . \quad . \quad . \quad (216)$$

which means that the actual value of H in the specimen differs from the value calculated from the magnetizing ampere-turns and length of specimen by the amount

$$\frac{4\pi}{10} \cdot \frac{NI}{l} \cdot m$$

The flux density may be measured by ballistic galvanometer in the usual way.

Permeameters. Permeameters, of which there is a large number of different types, are essentially pieces of apparatus constructed for determination of the B-H curve of magnetic specimens, of bar form, by means of a test which is conveniently simple, and for the performance of which the time required is short. Only a small number of such permeameters can be described in the space available here. In the works given in Refs. (1), (3), (10), (11), (12), and (14), many other forms are described.

Ewing Double-bar Method. In this permeameter two exactly

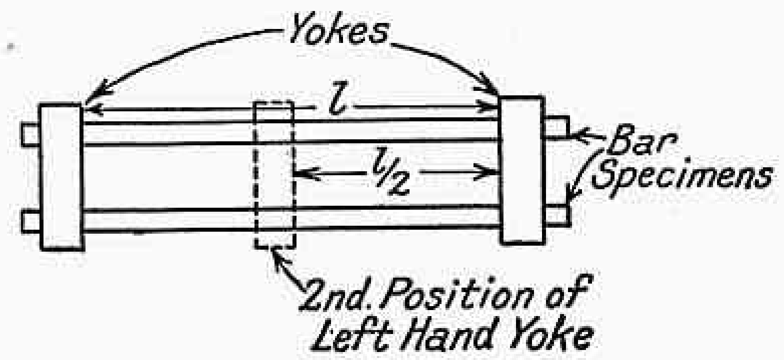


Fig. 223. Ewing Double-bar Method

similar bar specimens of the material under test are used, with two pairs of magnetizing coils, one pair of the latter being exactly half the length of the other pair. The number of turns, per centimetre axial length, is the same for both pairs of coil. Two yokes of annealed soft iron, with holes to receive the ends of the bar specimens—the fit being tight—are used. The arrangement of bars and yokes is shown in Fig. 223.

The object of this arrangement is the elimination of the reluctance of the yokes and air gaps—assumed to be the same, for a given flux density, in all positions of the yokes—by making two tests, one with a length of specimen l and the other with a length $\frac{l}{2}$. The difference in m.m.f. required to produce the same flux density in the two cases being that required for a length of l in each specimen. Thus,

Let n = No. of turns per centimetre length for both pairs of magnetizing coils.
,, I_1 = the current in the coils when the specimen length is l.
,, I_2 = the current in the coils for length $\frac{l}{2}$.
,, H_1 = the apparent magnetizing force for length l.
,, H_2 = ,, ,, ,, ,, $\frac{l}{2}$.

Let a = the m.m.f. required for the yokes and air gaps in each case.

„ B = the flux density in the specimen (the same in each case).

Then
$$H_1 = \frac{4\pi}{10} \cdot \frac{nI_1 l}{l} = \frac{4\pi}{10} \cdot nI_1$$

$$H_2 = \frac{4\pi \cdot nI_2 \frac{l}{2}}{10 \frac{l}{2}} = \frac{4\pi}{10} \cdot nI_2$$

Then, if H be the true magnetizing force in the iron for a flux density B,

$$Hl = \frac{4\pi}{10} \cdot nI_1 l - a = H_1 l - a$$

and
$$H\frac{l}{2} = \frac{4\pi}{10} \cdot nI_2 \cdot \frac{l}{2} - a = H_2 \frac{l}{2} - a$$

Hence,
$$a = l(H_2 - H_1) \qquad (217)$$

and
$$H = \frac{H_1 l - l(H_2 - H_1)}{l} = 2H_1 - H_2 \qquad (218)$$

The flux density corresponding to this actual value of H is measured by means of search coils and ballistic galvanometer in the ordinary way. The complete test is carried out by first obtaining and plotting a B-H curve for the specimens with a length of l—the apparent values of H being plotted. The specimens are then demagnetized, and a second B-H curve is obtained, and plotted, with a specimen length of $l/2$, the two curves being plotted on the same axes. The true B-H curve is obtained from these two, obtaining the true value of H corresponding to any value of B, from the expression above.

The disadvantages of the method are that the joint- and yoke-reluctances are not quite the same for two different positions of the yokes. The test requires, also, two carefully prepared specimens, and is somewhat lengthy in operation.

Ewing Permeability Bridge. In this apparatus a standard bar, whose B-H curve is known, is used, the value of H required to produce a certain flux density in the bar specimen under test being compared with that required by the standard bar for the same flux density. The two bars—each of which is placed inside a magnetizing coil—are joined together at the ends by yokes. The number of turns on the coil which magnetizes the standard bar is fixed, and the magnetizing force H which acts upon this bar, for a given current in this coil, is known. The same current flows through both coils, but the number of turns on the coil which magnetizes the test bar can be altered by dial switches, an arrangement for keeping

the circuit resistance constant being provided. The adjustment of the number of turns on this winding is continued until there is no difference of magnetic potential between the two yokes, when the flux density is the same in the two bars. The equality of potential of the two yokes is detected by means of a length of iron in the shape of an inverted U, having an air gap containing a pivoted magnetic needle at its centre. The two lower ends of this U are in contact with the two yokes, and if these are at different potentials flux will pass through the U piece and cause the needle to deflect from its

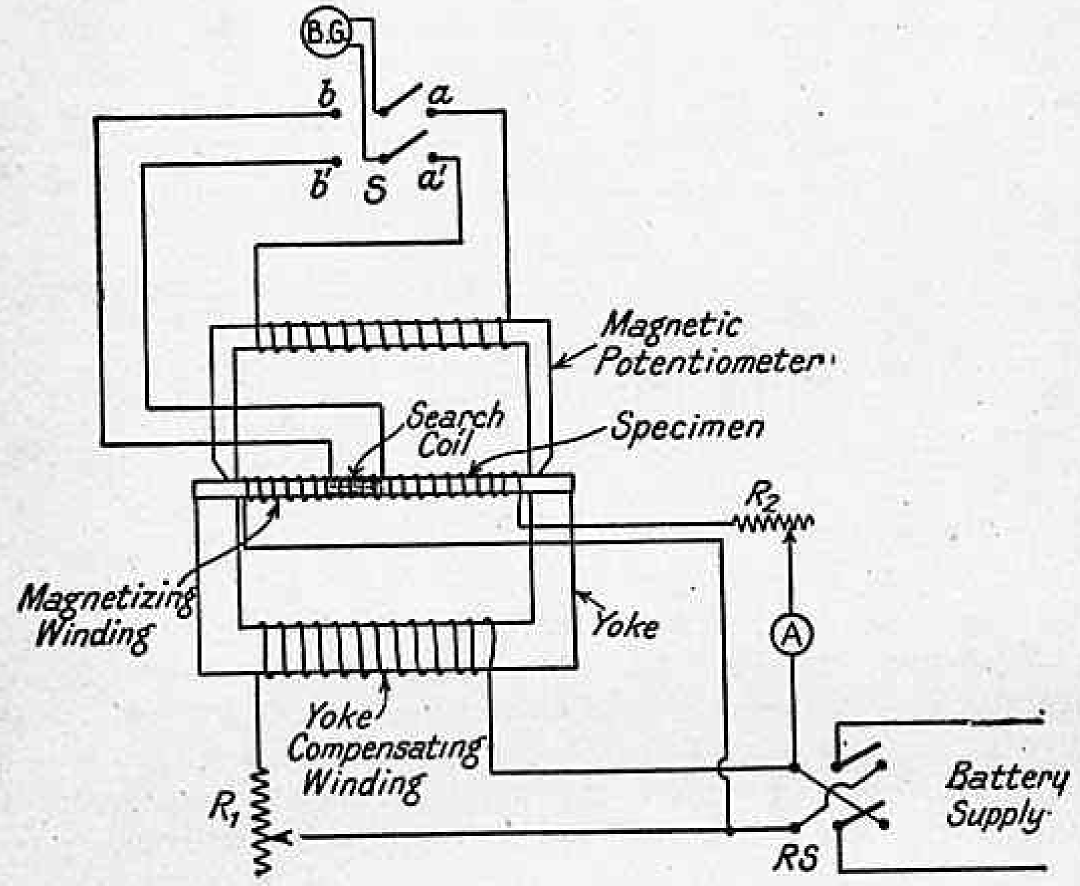


Fig. 224. Illiovici Permeameter

zero position. When the magnetizing turns on the test bar coil have been adjusted until this needle ceases to deflect, the flux density in the test bar is the same as that in the standard bar. The value of H for the test bar is obtained from the number of turns on its magnetizing winding compared with the number on the magnetizing winding of the standard bar, and the B-H curve of this bar is then constructed from that of the standard. This apparatus is not very widely used owing to its somewhat limited scope, and to inaccuracies which are chiefly due to variations of the reluctance of the joints in the magnetic circuit.

Illiovici Permeameter. The arrangement is shown in Fig. 224. There are two magnetizing windings connected in parallel, one uniformly distributed on the specimen, and the other—which acts as a compensating winding—on the yoke. Each has a variable resistance in series with it, the winding on the specimen having, also, an ammeter in series with it. The operation of the permeameter depends upon the fact that when the magnetomotive forces of the two magnetizing windings—on the yoke and specimen—have been adjusted until they are just sufficient to drive the existing flux through the reluctance of that part of the circuit upon which they are wound, there will be no magnetic potential difference across either of these parts. The magnetic potentiometer is used to indicate when this condition has been attained. Thus, when the magnetic potential drop across the specimen is zero, the m.m.f. in its magnetizing winding is just sufficient to drive the existing flux through its own reluctance. The true value of H in the specimen is then given by the m.m.f. then existing in this winding divided by the length of the specimen between the two arms of the yoke. The effect of yoke reluctance is thus eliminated. The flux density in the specimen is measured by a search coil and ballistic galvanometer in the usual way.

In carrying out a test the bar specimen is placed on the yoke with the search coil and magnetizing winding over it, the magnetic potentiometer being then clamped in position and the specimen demagnetized.

The current in the magnetizing winding of the test bar is then adjusted to give the value of H required, and the throw-over switch S is placed on contacts aa'. Resistance R_1 is then adjusted until no galvanometer throw is observed, when the reversing switch RS is reversed. The value of H in the specimen is then given by

$$H = \frac{4\pi}{10} \cdot \frac{NI}{l}$$

where N is the number of turns on the magnetizing coil on the specimen, l the length (in centimetres) of the specimen between the arms of the yoke, and I is the current indicated by the ammeter. The switch S is now thrown on to contacts bb' and the flux density in the specimen corresponding to this value of H is measured by observing the galvanometer throw when RS is reversed.

Tests up to $H = 400$ can be made with this apparatus, its principal advantages being its simplicity and its independence of the reluctance of the yoke.

Koepsel Permeameter. This piece of apparatus will be described because its principle is different from that of most other permeameters of the bar and yoke type. Fig. 225(a) shows the construction, from which it can be seen that the apparatus resembles a D'Arsonval instrument, the permanent magnet being replaced by the bar specimen and the heavy section yokes YY' in which the specimen is clamped. The moving coil C swings in a narrow air gap and is supplied with a known current from a battery, through a milliammeter. This coil carries a pointer moving over a scale,

and the deflection, for a given current, is obviously proportional to the flux density in the air gap, which, again, is proportional to the flux density in the specimen. The m.m.f. absorbed in the yokes is compensated for by the two coils AA'. M is the magnetizing coil surrounding the specimen, the proportions and number of turns on this coil being such that H in the specimen is 100 times the current in the coil (in amperes). The moving coil is so designed that the scale gives the flux density B in the specimen, directly, if the current in the moving coil in milliamperes is $\frac{50}{S}$, where S is the area of cross-section of the specimen in square centimetres. The compensating

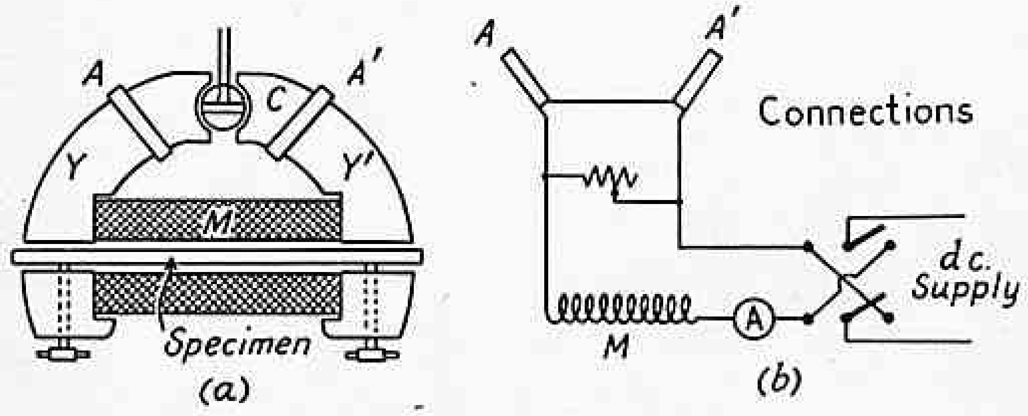


Fig. 225. Koepsel Permeameter

coils AA' are connected in series with coil M, and are shunted by a variable resistance. Before starting a test this resistance is adjusted until the moving coil shows no deflection with a heavy current in M, and with no specimen between the yokes.

A number of different bushings for use in the holes in the yokes allow different sizes of specimens to be used.

In carrying out a test the apparatus is set up so that the axis of the specimen is perpendicular to the magnetic meridian. The specimen is then clamped in position and demagnetized, the resistance shunting coils AA' having been previously adjusted for compensation. The current in M is then adjusted to give the required value of H, and the deflections with this current—both direct and reversed—are observed, the moving-coil current being adjusted in accordance with the cross-section of the specimen. The mean of the two deflections—which may be appreciably different—gives the flux density in the specimen.

The apparatus may be used for the determination of both B-H curves and hysteresis loops. The values of H for given values of flux density were found by C. W. Burrows, when investigating the characteristics of this type of permeameter, to be erroneous by an amount which depended upon the quality and size of the magnetic specimen. Hysteresis loops obtained by its use gave too low a value of residual flux density and too high a value of coercive force.

These errors may be corrected for by checks with standard bars whose magnetization curves are known.

TRACTION PERMEAMETERS. This type of permeameter makes use of the fact that the pull between two magnetized surfaces is given by

$$F = \frac{B^2A}{11{,}200{,}000} \text{ lb. wt.}$$

where B is the flux density in lines per square centimetre and A is the cross-section in square centimetres.

In the Thompson form of permeameter the pull required to separate a bar specimen from a yoke is measured by a spring balance

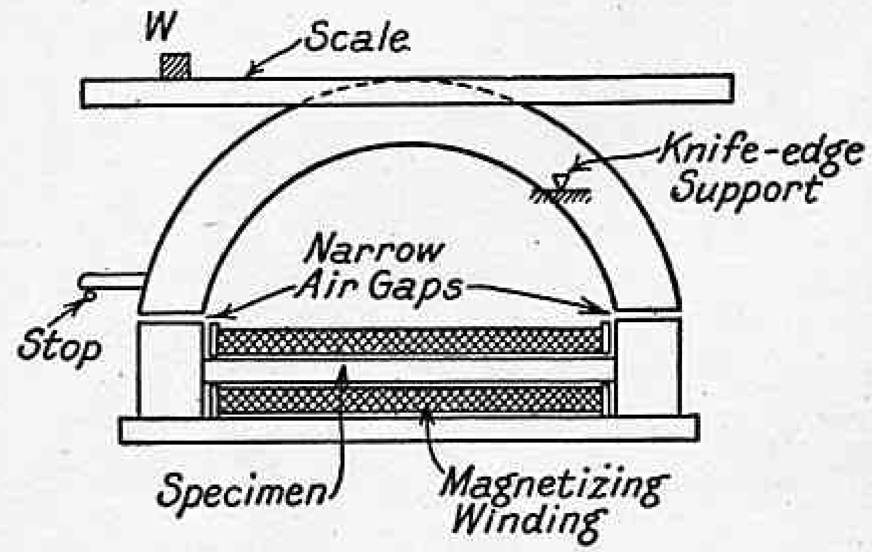


FIG. 226. DU BOIS MAGNETIC BALANCE

and the flux density obtained from the above expression. H is obtained from the constants of the magnetizing coil—which surrounds the bar—and from the current in this coil.

Another form of traction permeameter—the Du Bois magnetic balance—consists of a semicircular yoke which is divided into three parts by two air gaps near the lower ends (see Fig. 226). The surfaces at the air gaps are carefully faced to form plane surfaces. The bar specimen is placed inside a magnetizing coil and fits into the two lower fixed parts of the yoke. The weight W is slid along the scale carried by the pivoted upper part of the yoke, until the right-hand side of the yoke is pulled over by the pull across the air gap. The position of the weight, when this occurs, gives a measure of the flux density in the specimen. The apparent value of the magnetizing force acting on the specimen must be corrected by calibration of the apparatus, using a standard bar or otherwise. The correction required depends upon material which is being tested.

The inaccuracy of such permeameters causes them to be little used except for rough tests upon bar specimens.

BURROWS DOUBLE-BAR AND YOKE PERMEAMETER. This permeameter, which was first developed by Dr. C. W. Burrows (Ref. (12)), has been adopted as the standard apparatus for the testing of bar specimens in America, and is used by the Bureau of Standards. The effect of magnetic leakage at the joints between yoke and specimen are eliminated in this permeameter by the use of a number of compensating coils which apply compensating m.m.f's at different parts of the magnetic circuit, these m.m.f's being just sufficient to drive the flux through the reluctance of the part upon which the coils are placed.

Fig. 227 shows the arrangement of the magnetic circuit and coils. S_1 is the bar specimen to be tested, S_2 being a bar of similar dimensions to S_1. These bars are surrounded by magnetizing windings M_1 and M_2, which are uniformly wound along the lengths of the

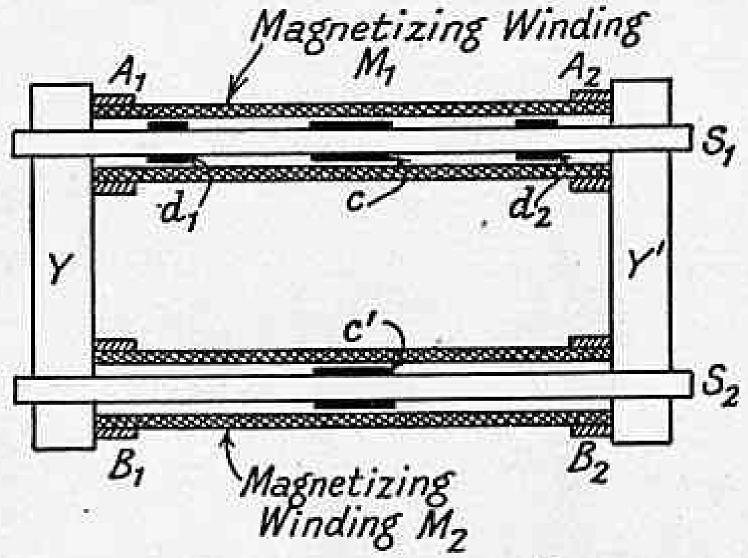


FIG. 227. MAGNETIC CIRCUIT OF BURROWS DOUBLE-BAR AND YOKE PERMEAMETER

bars. A_1, A_2, B_1, and B_2 are compensating coils for the elimination of leakage effects at the joints between the two bars and the massive yokes YY' into which the bars fit; c and c' are two exactly similar search coils wound at the centres of the two bars, while d_1 and d_2 are two similar search coils wound in the positions shown, on the test bar, and each having exactly half the number of turns of search coil c. Coils d_1 and d_2 are connected permanently in series, as are also the four coils A_1, A_2, B_1, B_2. The dimensions of the coils M_1, M_2 are such that H in the specimen is approximately 100 times the current in the windings, the maximum value of H for which the apparatus is used being from 300 to 400. The dimensions of the bar specimens are approximately 30 cm. long and 1 cm. in diameter.

The coils A_1, A_2, B_1, B_2, are supplied from a separate battery supply, coil M_1 from another separate supply, and M_2 from another. In carrying out the test it is necessary, first of all, to ensure that, for a given value of the current in M_1—i.e. of H in the test bar—the flux threading through all four search coils c, c', d_1, and d_2, is the same, the current in the compensating coils, and in M_2, being adjusted until this condition is fulfilled. If the flux threading coils

d_1 and d_2 is the same as that threading c and c', there can be no appreciable leakage of flux through the air in the neighbourhood of the joints. This means that the m.m.f. for the joints is being supplied by the compensating coils, and that the m.m.f. in coil M_1 is used up merely in driving the flux through the bar specimen S_1 inside it. Thus H in the specimen is given by $\frac{4\pi}{10} \cdot \frac{NI}{l}$ where

N = No. of turns on coil M_1
I = current (in amperes) in M_1
l = length of specimen in centimetres

It may, in some cases, be necessary to make corrections for the fact that the magnetizing solenoids are not infinitely long, but such corrections are usually negligible.

The procedure for obtaining equal flux threading all four search coils is as follows.

First, the specimen having been demagnetized, the current in the magnetizing coil M_1 is set at the required testing value. Search coils c and c' are then connected in series, but in opposition, to a ballistic galvanometer. The currents in coil M_1 and M_2 are then simultaneously reversed. A throw will be observed on the galvanometer. The current in M_2 is adjusted until no throw is obtained when the two currents are reversed. Since search coils c and c' have equal numbers of turns, this means that equal fluxes are now threading through them. Next, search coil c is connected in series with, but in opposition to, coils d_1 and d_2, and then to the ballistic galvanometer. The current in the compensating coils A_1, A_2, B_1, B_2, is then adjusted until no galvanometer throw is obtained upon simultaneous reversal of the currents in these coils and in M_1 and M_2. Then, since d_1 and d_2 together have the same number of turns as coil c, the flux threading all three coils is the same.

The flux density corresponding to the value of H in coil M_1 (obtained as described above) can be measured by connecting coil c alone to the ballistic galvanometer, and noting the throw when the currents in the two magnetizing coils and the compensating coils are simultaneously reversed.

Fig. 228 gives a diagram of connections showing how the switching may be arranged for convenience in carrying out the test as described above.*

FAHY SIMPLEX PERMEAMETER. This permeameter, which has been used by the Bureau of Standards, and is being used quite commonly in this country for the routine testing of bar specimens, has the advantage of simplicity both in construction and operation, while its accuracy is of the same order as that of the Burrows apparatus.

An outline of its construction is given in Fig. 229. Two iron

* Fuller descriptions of the apparatus, testing methods, and the application of corrections, will be found in the works given in Refs. (1), (2), (3), (12).

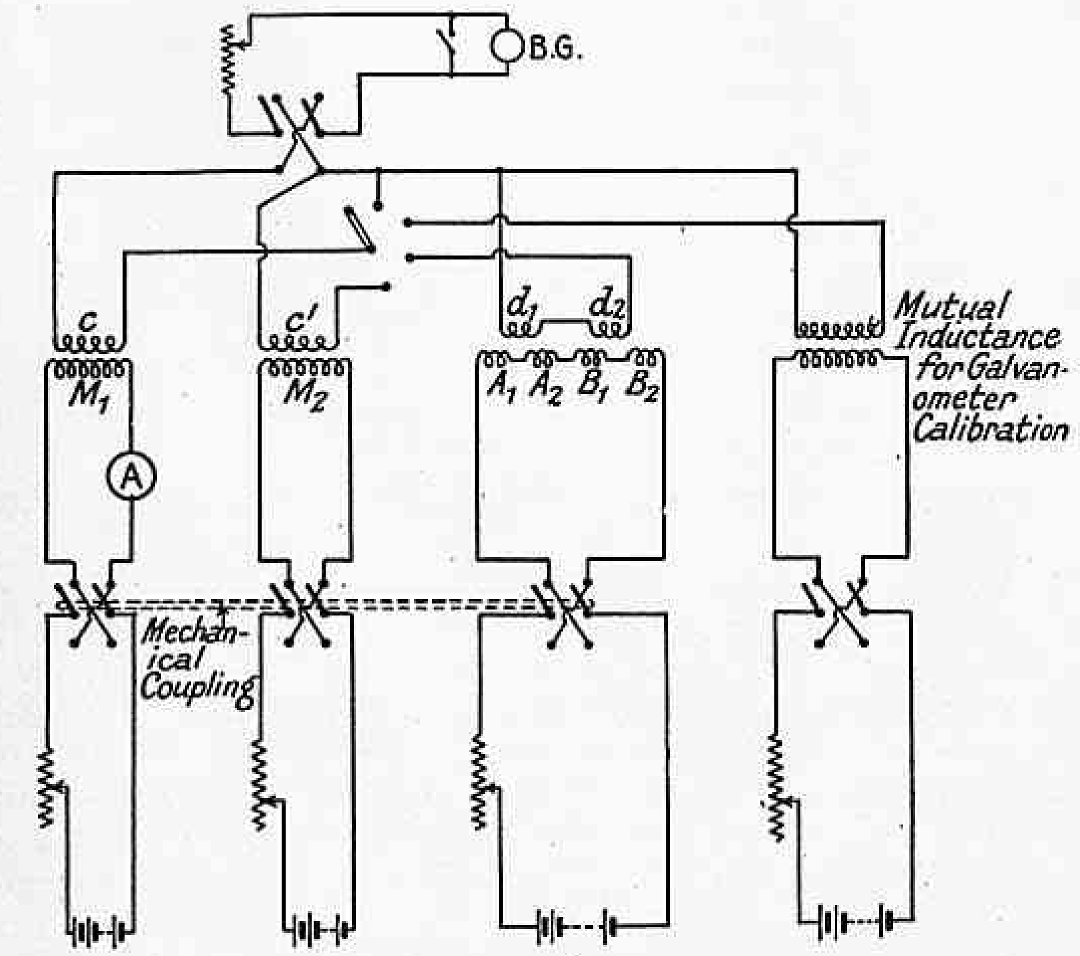


FIG. 228. ELECTRICAL CIRCUITS OF BURROWS DOUBLE-BAR AND YOKE PERMEAMETER

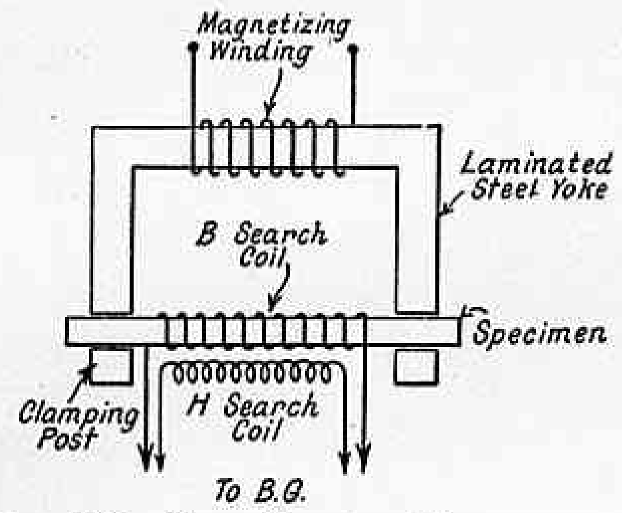


FIG. 229. FAHY SIMPLEX PERMEAMETER

clamping posts clamp the specimen (of cross-section $\frac{3}{4}$ in. by $1\frac{3}{4}$ in.) against a laminated steel yoke. The latter carries the magnetizing winding, and the specimen is surrounded by a search coil for the measurement of the flux density in it. The magnetizing force H acting on the specimen is measured, like the flux density, by a ballistic galvanometer, utilizing an air-cored search coil of several thousand turns located between the two clamping posts as shown. The values of H so measured are corrected by calibration of the H search coil, utilizing a specimen of known magnetic characteristics in place of the test specimen.

DRYSDALE PLUG PERMEAMETER. This permeameter, devised by

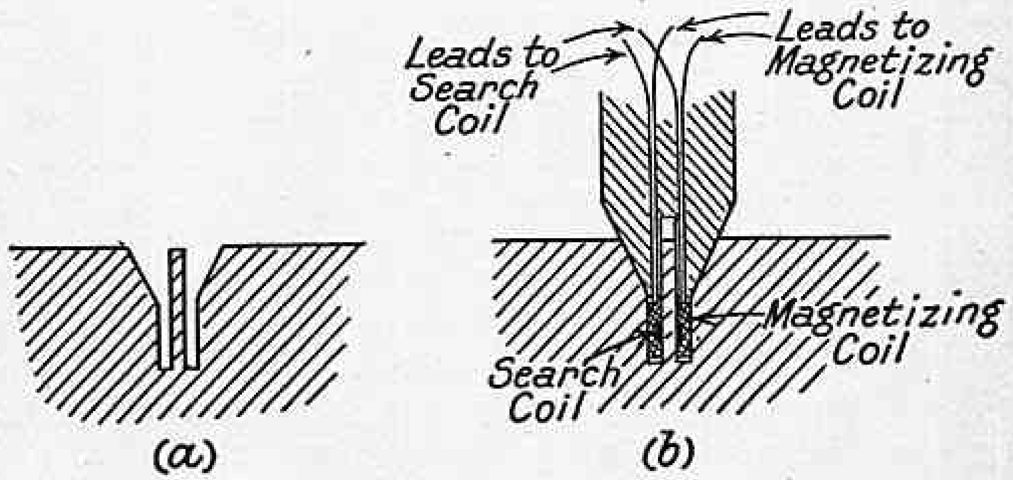


FIG. 230. DRYSDALE PLUG PERMEAMETER

Dr. C. V. Drysdale, is for the testing of a large mass of magnetic material. A special drill is used to cut a cylindrical hole in the mass of iron to be tested, the drill being so formed that it leaves a thin rod or pin of the metal (about 0·1 in. diameter) standing in the centre of the hole. A rose cutter at the upper end of the drill cuts a conical hole at the surface of the iron, so that after drilling, a section through the hole is as shown in Fig. 230 (*a*). A split conical plug, having a small magnetizing coil and search coil fixed to its lower end, is then pressed into the hole so that it grips the centre pin tightly and also makes good contact with the side of the conical hole in the iron mass (Fig. 230(*b*)). The whole then forms a bar and yoke permeameter on a small scale, the centre pin of material being the bar specimen and the mass of the material, together with the conical plug, forming the yoke. Two holes in the plug allow leads from the magnetizing coil and search coil to be brought out.

A measured current is passed through the magnetizing coil from a battery through a reversing switch, H being obtained from the characteristics of the magnetizing coil. The corresponding flux density is measured by the search coil and ballistic galvanometer, or fluxmeter, in the usual way.

Magnetic Testing with Intense Fields. The methods of testing so far described have been suitable for testing with values of the

magnetizing force H up to 400 or 500 in most cases. For tests at greater field strengths than these special methods have to be used.

Various "isthmus" methods, most of which are modifications of the original isthmus method due to Ewing, are used for this purpose, the title being derived from the fact that the specimen, in such methods, forms a narrow "isthmus" between the two specially-shaped poles of an electromagnet.

EWING'S ISTHMUS METHOD. Ewing carried out tests up to $H = 24{,}500$, and B about 45,000, by the use of the apparatus illustrated in Fig. 231.

The electromagnet carries magnetizing windings as shown. The

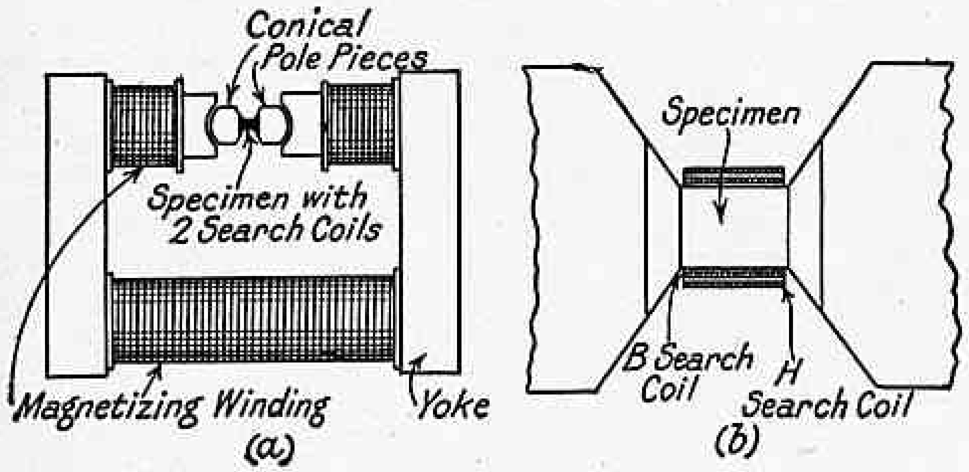


FIG. 231. EWING'S ISTHMUS METHOD

pole pieces are conical and have a cylindrical seating, so that the specimen may be rotated through 180° during the test. The specimen itself is turned down to a bobbin shape, having a cylindrical portion in the middle and conical ends which abut against faces on the pole pieces, the latter forming a continuation of the conical ends of the specimen. The whole of the flux in the pole pieces is thus forced to pass through the specimen of very much smaller section, thus giving a very high flux density in the specimen. The flux density obtainable depends upon the area of cross-section of the isthmus, or cylindrical part, of the specimen.

Two search coils, having equal known numbers of turns, and of known cross-sections, are fitted on the cylindrical portion of the specimen, as shown in Fig. 231. The inner coil—for the measurement of B—fits the specimen closely, while the outer coil—for the measurement of H—is separated from the inner coil by a small annular space.

In carrying out a test a magnetizing current is passed through the winding on the electromagnet, the inner search coil being connected to a ballistic galvanometer. The specimen—with its search coils—is then quickly rotated through 180°, this being equivalent, as regards interlinking flux, to a reversal of the magnetizing current. The galvanometer throw is observed and the flux density obtained therefrom. The outer search coil encloses some air flux as well as the flux in the specimen, and the difference between the flux which it encloses, and that enclosed by the inner search coil, is measured. This difference in flux, when divided by the difference in cross-section of the two search coils, gives the flux density in the air surrounding the specimen, and this air flux density is equal to the magnetizing force H in the specimen. The difference in the flux enclosed by the two search coils may be measured by connecting the two coils in series—opposing one another—and to the ballistic galvanometer, when the throw produced by a rotation of the specimen through 180° gives a measure of the flux between the coils. Otherwise, the total flux enclosed by the outer coil may be measured and the difference in enclosed flux obtained by subtraction. The inner search coil must of necessity enclose some air flux as well as the flux in the specimen. This air flux is taken into account, using the measured value of H as the air flux density. The small cross-section of the specimens used in such tests renders this correction necessary in most cases.

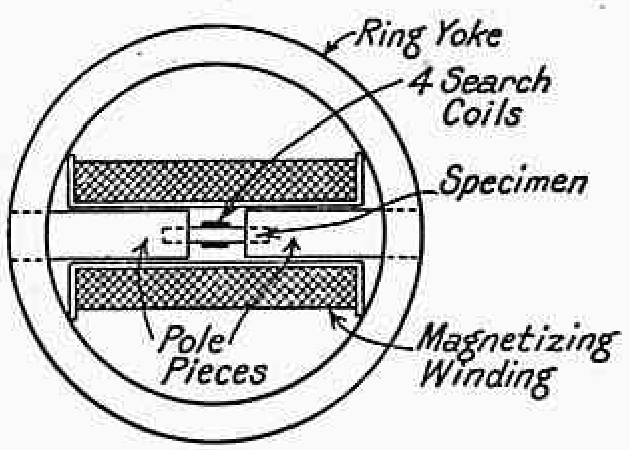


FIG. 232. GUMLICH'S METHOD

The slope of the conical pole and specimen end pieces for maximum concentration of flux is about 60° (between the cone side and axis), whilst the angle for the most uniform field within the specimen is about 39°. A compromise is usually made in practice.

GUMLICH'S METHOD. This method is the same in principle as the method described above, but employs an improved method of determining H in the specimen. The arrangement is shown in Fig. 232.

A laminated ring yoke is used with two diametrically opposite pole pieces PP'. These are 25 mm. in diameter, and have a central air gap of 12 mm. The pole pieces are bored centrally to take a specimen of diameter 6 mm. A magnetizing coil M surrounds the pole pieces and specimen, and upon the latter are wound four search coils in four layers. The radial spaces between the layers—and hence between the search coils—are known, and the coils have equal numbers of turns.

The flux density in the specimen is measured by connecting the inner coil—of known cross-section—to a ballistic galvanometer, and reversing the magnetizing current. Then, by connecting the search coils in pairs, opposing one another, to the ballistic galvanometer, and reversing the magnetizing current in each case, the air flux density in inter-coil spaces can be measured. These are plotted vertically on a graph, taking radial distances from the surface of the specimen as abscissae. By extrapolation the flux density at the

surface of the specimen—and hence H in the specimen—can be obtained. A correction can be made also, from this graph, for the air flux enclosed by the inner search coil.

This apparatus can be used up to $H = 6{,}000$, and can be adapted to tests upon sheet material by using pole pieces with rectangular holes. Other methods of carrying out tests at high inductions are described in the publications given in Refs. (1), (2), (3), (13).

The Testing of Feebly Magnetic Materials. For measurements upon material which is distinctly magnetic, but whose permeability is low (of the order of unity), the circuit shown in Fig. 233 may be

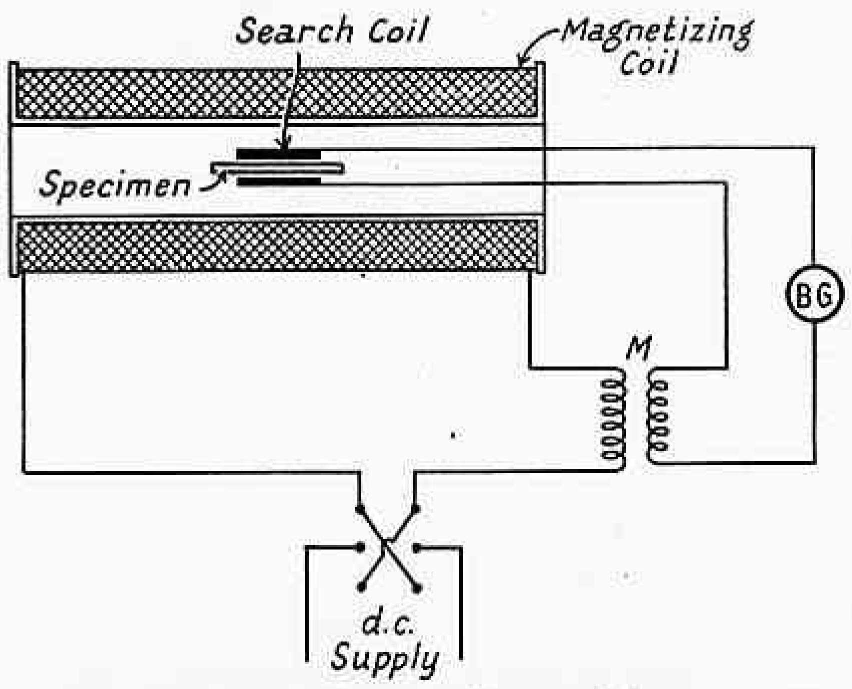


FIG. 233. APPARATUS FOR TESTING FEEBLY MAGNETIC MATERIALS

used. The magnetizing winding has a search coil at its centre, this search coil being fixed so that the specimen can be slipped inside it or withdrawn without disturbing the search coil itself. This search coil is connected, through a ballistic galvanometer, to the secondary of a mutual inductance, the primary of which is connected in the magnetizing circuit.

Since the material under test is only feebly magnetic, it is assumed that there is no demagnetization due to end effect. The procedure in testing is as follows.

Current is passed through the magnetizing coil to give the required field strength H, which is given simply by $H = \frac{4\pi}{10} \cdot \frac{NI}{l}$ (NI being the number of ampere-turns on the magnetizing coil and l its axial length), since the demagnetizing effect of the ends of the specimen is negligible. Before placing the specimen inside the search coil the mutual inductance is adjusted to give no galvanometer throw when the magnetizing current is reversed. The specimen is then inserted, and the galvanometer throw upon reversal of the current observed. The flux density obtained from this throw will be that due to the ferric induction in the specimen. Calling this flux density B_f, we have for the total flux density, when the field strength is H, $B = B_f + H$, and for the permeability $\mu = \frac{B_f + H}{H}$

The Curie balance, and various modifications of it (Refs. (3), (15)), may be used for the measurement of the susceptibility of materials such as ores and rock specimens. A simplified drawing of the arrangement is shown in Fig. 234.

The specimen is placed in a glass tube which is suspended from a

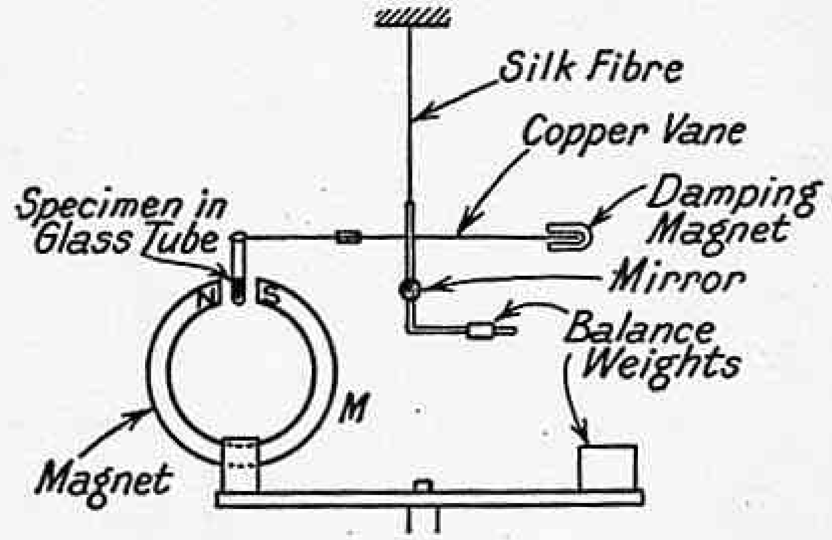


FIG. 234. THE CURIE BALANCE

light arm which also carries a balance weight and a copper vane, the latter moving in the gap of a permanent magnet for damping purposes. The moving system also carries a mirror and another balance weight, as shown. A permanent magnet of ring shape is mounted on an arm which may be rotated so that the air gap of the magnet may be moved towards or away from the tube containing the specimen.

In carrying out the test a measurement is first made upon the glass tube alone. The magnet M is rotated until maximum deflection of the moving system is obtained, this deflection θ' being observed. The specimen is then placed in the tube and the maximum deflection θ_1 of the moving system with different positions of M again observed. A third measurement is then made with the specimen replaced by some material whose susceptibility is known (distilled water, whose susceptibility is $0{\cdot}79 \times 10^{-6}$ is often used for this purpose).

Let the deflection now be θ_2. Then

$$\frac{\kappa_x}{\kappa_s} = \frac{m_s(\theta_1 - \theta')}{m_x(\theta_2 - \theta')} \quad . \quad . \quad . \quad . \quad (219)$$

where κ_x and κ_s are the susceptibilities, and m_x and m_s the masses, of the unknown and standard materials respectively.

It should be noted that if θ' is in the opposite direction to θ_1 and θ_2 it must be treated as negative.

W. Sucksmith and R. R. Pearce (Ref. (43)) have described a method, using a balance, for the measurement of magnetic susceptibilities in a controlled atmosphere, or *in vacuo* at temperatures up to 1,500° C.

The Testing of Permanent Magnets. The details of the method of testing permanent magnets, when manufactured, depend upon the shape of the magnet, this varying greatly according to the purpose for which the magnet is to be used. Owing to the self-demagnetizing force, it is necessary, in testing such magnets, to measure H by means of search coils, of small cross-section, laid flat

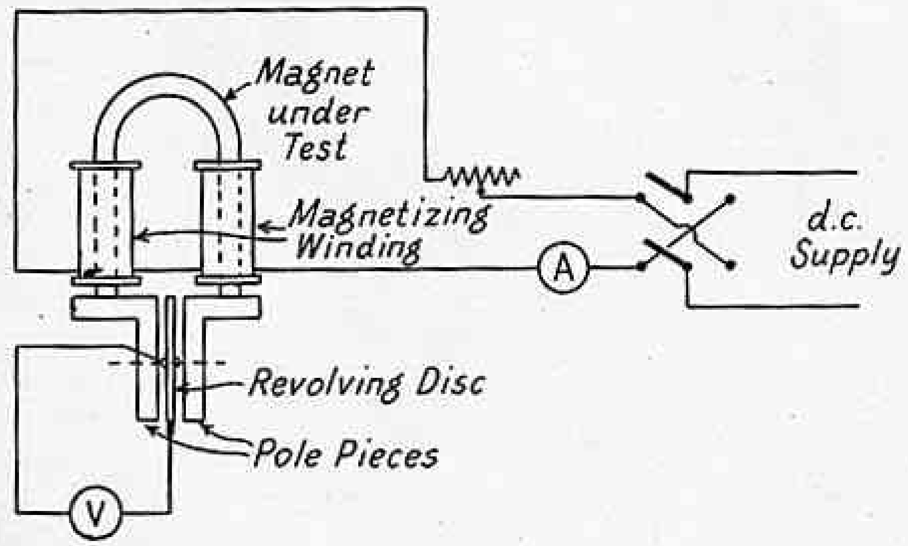


FIG. 235. BETTERIDGE APPARATUS

against the surface of the magnet as previously described. These search coils give the value of the air flux density at the surface of the magnet at different points, and thus measure H for these points. The flux density B is measured by search coils in the ordinary way.*

BETTERIDGE APPARATUS FOR MAGNET TESTING (Ref. (17)). The principle of this apparatus for the commercial testing of permanent magnets may be understood from Fig. 235. The magnet to be tested is placed with its straight ends inside two magnetizing coils and so that its ends press against two pole pieces, in the air gap between which a thin iron disc, plated with copper, is mounted. This disc is mounted in ball bearings and the clearances between it and the pole pieces are small. It is driven round at constant speed by a small motor, and has two small brushes making contact with its spindle and its rim. When a current flows in the magnetizing windings flux crosses the gap between the pole pieces, and the flux density in the gap will be proportional to that in the magnet. An e.m.f. will be induced in the revolving disc, this e.m.f. being proportional to the gap flux density and being measured by the millivoltmeter V.

* Details of tests upon different shapes of magnets, upon magnet steels, and of tests for the determination of temperature effects upon magnets, will be found in the works given in Refs. (3), (13), (14), and (16).

Thus, the voltmeter reading gives a measure of the flux density in the specimen, while H is obtained by the usual formula from the constants of the magnetizing coils and from the current in these coils.

Some leakage exists between the two arms of the magnet, but with proper design of the apparatus its effect upon the results is small.

British Standard Specification No. 406—1931 gives a complete specification for this apparatus, and also for another apparatus of the moving-coil pattern which uses a fluxmeter of special design.

Magnetic Testing with Alternating Current. When iron is subjected to an alternating magnetic field a loss of power occurs due to hysteresis effects in the iron. Power is absorbed, also, due to eddy currents, which are set up in the iron due to the fact that it is electrically conducting material, and that the flux threading through it is changing. Such eddy currents will be discussed more fully in Chapter XIV.

Although the hysteresis loss per cycle in iron may be determined from the hysteresis loop obtained in a d.c. test, this loss may be somewhat different under the actual alternating magnetization conditions with which it will be used in practice. Also, eddy current losses can only be measured by the use of alternating current. For these reasons, inspection tests upon sheet steel which is to be used in the manufacture of transformers and other a.c. apparatus are very commonly a.c. tests.

Separation of Iron Losses. It is often sufficient, in acceptance tests of sheet material, to measure the total loss in the steel at the standard frequency (50 cycles) and with a maximum flux density of about 10,000 lines per square centimetre. The separation of the losses into their two components—i.e. hysteresis loss and eddy current loss—involves a rather more lengthy test.

Hysteresis loss, as already seen, is given by the expression

$$W_h = k \, . \, f \, . \, B_{max}{}^{1 \cdot 6}$$

where W_h is the loss in watts per cubic centimetre of material, f being the number of cycles of magnetization per second (i.e. the frequency), B_{max} the maximum flux density, and k a constant for any given material. This law holds approximately for values of B_{max} between 1,000 and 12,000 lines per sq. cm.

Eddy current loss, provided the sheets are sufficiently thin for "skin effect" (see Chapter XIV) to be negligible, is given by the expression

$$W_e = k' K_f{}^2 f^2 t^2 B_{max}{}^2$$

where W_e is the loss in watts per cubic centimetre, f the frequency, t the thickness of the sheet, and B_{max} the maximum flux density. K_f is the "form factor" of the alternating wave of flux and depends upon the shape of this wave.

Thus, if the form factor remains constant throughout a test, and the maximum flux density B_{max} is kept constant, the total power loss being measured at different frequencies, then this total loss may be written

$$W_t = Mf + Nf^2$$

where $\quad M = kB_{max}{}^{1 \cdot 6}$

and $\quad N = k' K_f{}^2 \, . \, t^2 \, . \, B_{max}{}^2$

both M and N being constant for this test.

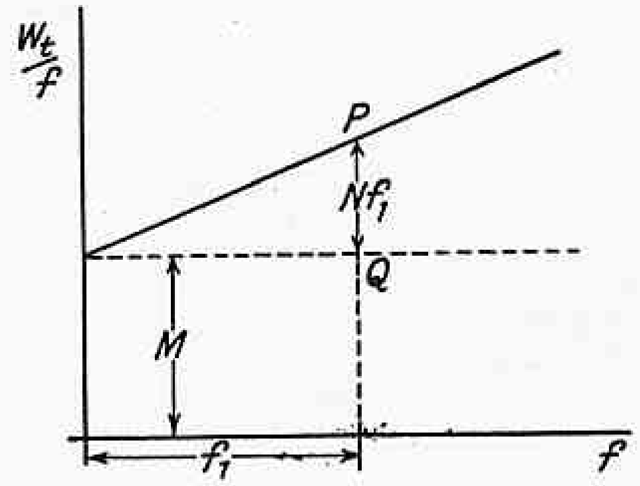


FIG. 236. SEPARATION OF IRON LOSSES

These constants may be determined—and the total loss thus split up into its two components, for any given frequency—by plotting $\dfrac{W_t}{f}$ against f as shown in Fig. 236.

Then, $\dfrac{W_t}{f} = M + Nf$, so that the intercept on the vertical axis gives M; and N can be obtained from the slope of the graph. M is the hysteresis loss per cycle. The eddy-current loss for any frequency f_1 is given by the intercept PQ, as shown in Fig. 236. In this way the two components of the total loss for any given form factor and maximum flux density, can be separated.

Again, if the frequency and B_{max} are kept constant and the form factor varied, the total loss being measured for various values of form factor, then we have

$$W_t = C + DK_f{}^2 \quad . \quad . \quad . \quad . \quad . \quad (220)$$

If W_t is now plotted vertically against values of $K_f{}^2$ horizontally the constants C and D may be obtained. The intercept upon the vertical axis (Fig. 237) gives C, and the slope of the line gives D.

Thompson and Walmsley (Ref. (20)) have described a method of measuring and of separating the iron losses of a transformer, using thermionic valve rectification to obtain the form factor of the voltage wave.

The effects of variation of both form factor of the applied voltage and of temperature are considered in this paper.

Methods of Measurement. WATTMETER METHOD. This is perhaps the commonest method of measuring the total loss in sheet steel with alternating current. The sheet material to be tested is arranged in the form of a "magnetic square," of which there are several forms, Epstein being the originator of the arrangement. In this square there are four bundles of strips of the sheet material. These are bound with tape to form four cores to fit inside four magnetizing coils, the individual strips being insulated from one another by thin tissue paper. The ends of the cores, projecting beyond the magnetizing windings, are, in the Epstein apparatus, interleaved and clamped at the corners.

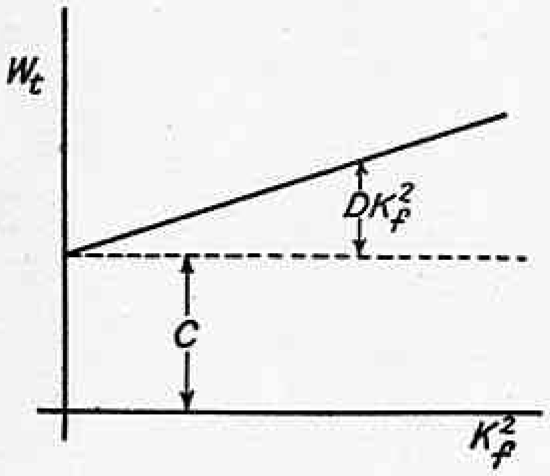


FIG. 237. SEPARATION OF IRON LOSSES

LLOYD-FISHER MAGNETIC SQUARE. Fig. 238 shows a magnetic square as used by Lloyd and Fisher and now in use at the National Physical Laboratory and Bureau of Standards. The strips of material—cut half in the direction of rolling of the sheet during manufacture and half perpendicular to this direction—are about 25 cm. long and 5 or 6 cm. wide. They are built up into four bundles and assembled to form a complete magnetic circuit with the aid of bent corner pieces and clamps, as shown in the figure. These corner pieces should be of the same material as the strips, or at least of material having similar magnetic properties. The overlap at the corners should be only a few millimetres and a correction may be applied, if necessary, to account for the fact that the cross-section of the magnetic circuit is doubled at the overlapping points.

A very small number of strips is shown in the figure for the sake of clearness. The bundles of strips are placed inside four similar magnetizing coils of heavy wire, connected in series to form the primary winding. Each of these coils has, underneath it, two single-layer coils of thin wire and having equal numbers of turns. These secondary coils are connected in series in groups of four —one on each core—to form two separate and similar secondary windings.

The primary winding is connected either directly, or through a transformer having a variable secondary, to an alternator having a waveform which is as nearly as possible sinusoidal. By this means regulation of the magnetizing current by means of resistance, with

consequent alteration of waveform, is avoided. If the total loss in the sample is to be measured, the connections are as shown in Fig. 239, the wattmeter pressure coil being supplied from one secondary winding and an electrostatic voltmeter from the other.

The iron specimen must be weighed and its cross-section determined before assembly. The magnetizing current is adjusted to give the value of B_{max} required, the frequency of the supply being previously adjusted to the correct value. The wattmeter and voltmeter readings are observed.

Theory of the Method. Then, the voltage induced in secondary

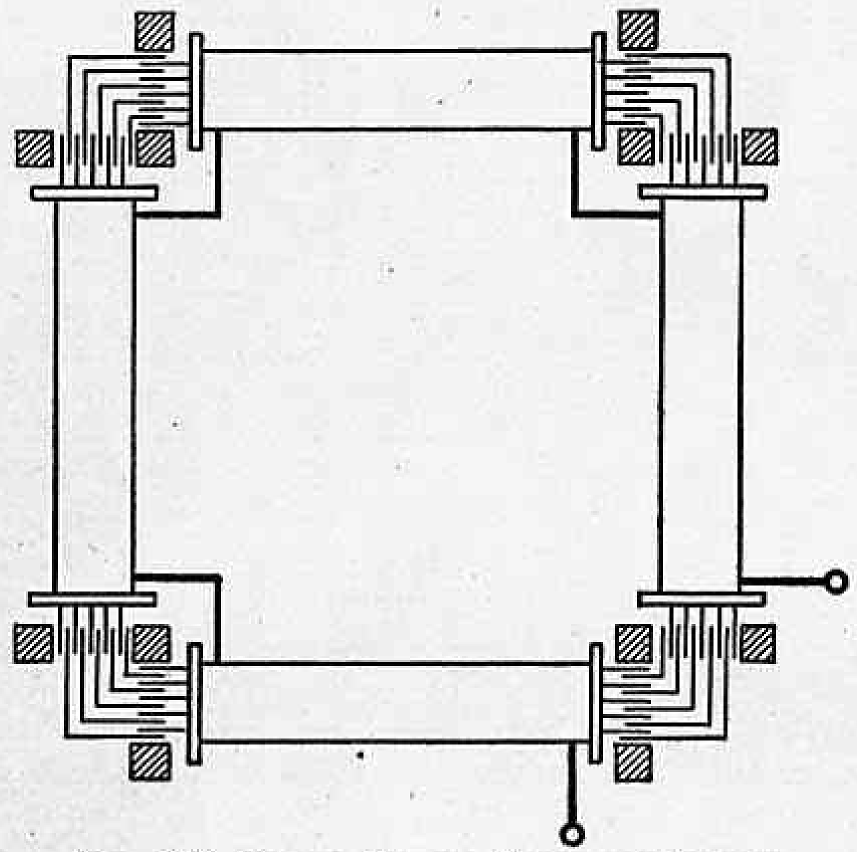


FIG. 238. LLOYD-FISHER MAGNETIC SQUARE

winding S_2, whose r.m.s. value is measured by the voltmeter, is given by the expression

$$E = \frac{4K_f \,.\, B_{max} A \,.\, f \,.\, N_2}{10^8} \text{ volts} \quad . \quad . \quad . \quad (221)$$

where K_f is the form factor, A the cross-section of the specimen, in which the maximum flux density is B_{max}, f the frequency, and N_2 the number of turns on secondary winding S_2.

Hence the value of B_{max} may be obtained,

$$B_{max} = \frac{E \times 10^8}{4\, K_f \,.\, A \,.\, f \,.\, N_2} \quad . \quad . \quad . \quad (222)$$

It may be necessary to correct this expression, especially at high values of B_{max}, for the fact that the coil S_2 encloses some air flux as well as the flux in the sample, since the cross-sectional area of the coil must be greater than that of the sample itself.

Let A_c = the cross-sectional area of the coil.
" A_s = the cross-sectional area of the sample.
" H_{max} = the maximum magnetizing force (equal to the flux density in the air space within the coil).
" B_{max} = the actual maximum flux density in the sample.

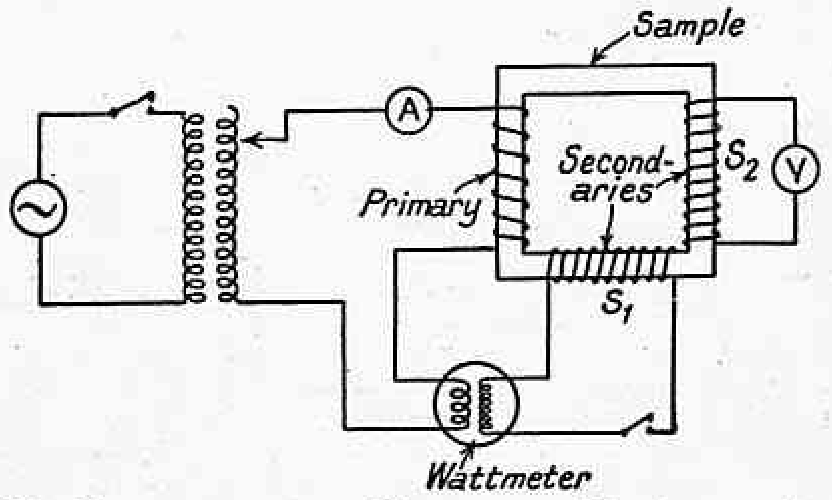


FIG. 239. CONNECTIONS FOR WATTMETER METHOD OF IRON-LOSS MEASUREMENT

Then the total flux within the coil is

$$B_{max}\,A_s + H_{max}\,(A_c - A_s) = B'_{max}\,A_s$$

where B'_{max} is the apparent maximum flux density in the sample. Thus,

$$E = 4K_f\,.\,fN_2\,[B_{max}\,A_s + H_{max}\,(A_c - A_s)] \times 10^{-8}$$
$$= 4K_f\,.\,f\,.\,N_2\,.\,A_s\,B'_{max} \times 10^{-8} \text{ volts}$$

where $$B'_{max} = B_{max} + H_{max}\left(\frac{A_c - A_s}{A_s}\right)$$

H_{max} may be determined from the permeability curve of the sample.

As regards the power loss in the iron—

Let W_t = total iron loss.
" W = wattmeter reading.
" V = voltage applied to wattmeter pressure coil.
" E = voltmeter reading
= voltage induced in coil S_1, since S_1 and S_2 have equal numbers of turns.
" r_p = resistance of wattmeter pressure coil.
" r_c = resistance of coil S_1.
" i_p = current in the pressure coil circuit.

Then $$E = i_p\,(r_p + r_c)$$
$$V = i_p r_p$$

Hence, power loss in the iron, together with the copper loss in the winding S_1 and in the wattmeter pressure coil

$$= W\,.\,\frac{E}{V} = W\,\frac{i_p\,(r_p + r_c)}{i_p r_p} = W\,\frac{(r_p + r_c)}{r_p}$$
$$= W\left(1 + \frac{r_c}{r_p}\right)$$

Again, the copper losses in r_p and r_c are

$$i_p^2\,(r_p + r_c) = \left(\frac{E}{r_p + r_c}\right)^2 (r_p + r_c) = \frac{E^2}{r_p + r_c}$$

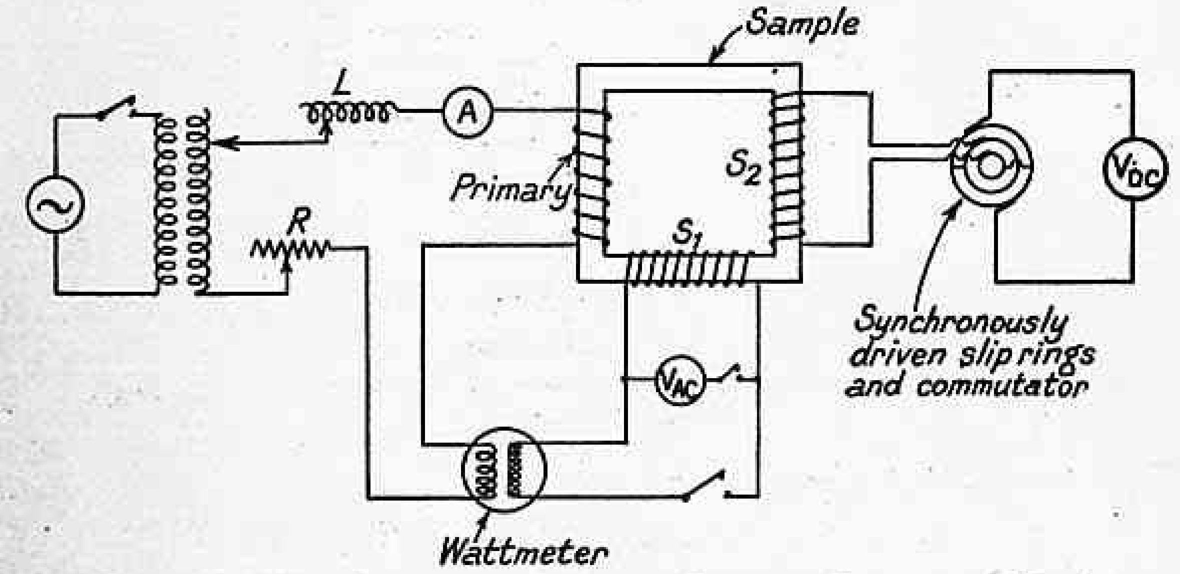


FIG. 240. MEASUREMENT OF POWER LOSS AT DIFFERENT FORM FACTORS

Therefore, $$W_t = W\left(1 + \frac{r_c}{r_p}\right) - \frac{E^2}{r_p + r_c} \quad . \quad . \quad . \quad (223)$$

As mentioned previously, the hysteresis and eddy current components of the loss can be graphically determined from the results of power measurements such as the above, at different frequencies.

Fig. 240 gives the connections for the measurement of power loss at different form factors, the form factor being obtained from measurements of r.m.s. and mean voltages induced in the two windings S_1 and S_2, which have equal numbers of turns. Variation of form factor is obtained by adjustment of R and L—variable resistance and inductance—and of the number of turns on the secondary of the supply transformer. The voltmeter V_{AC} measures the r.m.s. voltage, and voltmeter V_{DC}, in conjunction with a synchronously-driven two-part commutator and slip rings, measures the mean, or average, voltage.

RICHTER APPARATUS. This apparatus, designed for the testing of complete sheets of steel instead of small strips cut from them, does not involve a different method of measurement but only a difference in the arrangement and assembly of the sample. The wattmeter method of measurement of losses is used. The sample consists of four sheets of steel having dimensions 100 cm. wide and 200 cm. long approximately. These sheets are placed inside a hollow wooden drum having a lattice arrangement inside to hold the sheets, and wooden clamps to clamp the ends of each sheet together. The drum is wound with about 100 turns of thick copper wire to carry a heavy magnetizing current.

The disadvantages of the apparatus are that corrections for the air space flux must be made for all values of B_{max}, the copper losses

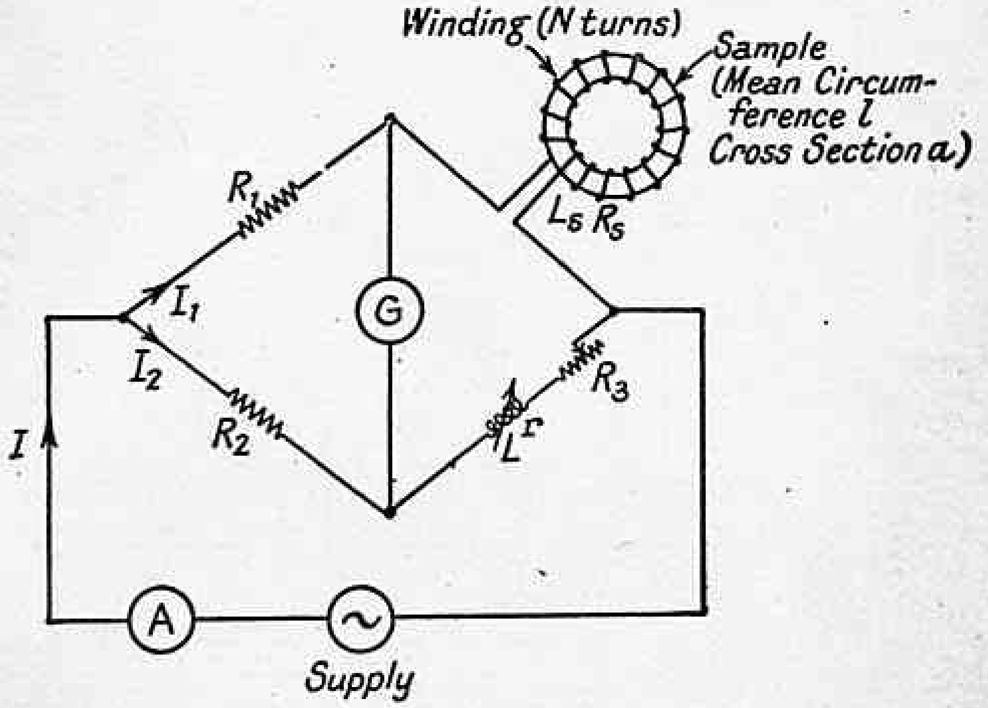


FIG. 241A. MAXWELL BRIDGE FOR IRON-LOSS MEASUREMENTS

are large, and the sheets are in a strained condition during the test, which means that the losses measured do not give the true losses under unstrained conditions.

A.C. BRIDGE METHODS. The a.c. bridge network can be adapted to the measurement of iron loss and effective permeability of magnetic samples. Such methods are very useful when the available samples are small, and when the test is to be carried out at commercial- or audio-frequencies, and with fairly low values of flux density. Fig. 241A shows the connections of the Maxwell bridge as applied to iron loss and permeability measurements.

R_1, R_2, and R_3 are resistances, the latter being variable. In series with R_3, in the same arm, is a variable inductance L and resistance r. It may be necessary to connect R_3 in series with the winding on the sample if the resistance of the latter is small. The specimen, in ring form, is wound with a winding whose inductance is L_s and effective resistance R_s, this effective resistance containing an iron-loss component. R_w is the actual resistance of the winding on the ring. G is

a vibration galvanometer or telephone, and A an ammeter. The supply is from an alternator having a pure sine waveform.

Balance of the bridge is obtained by adjustment of L and R_3.

Theory. At balance

$$I_1R_1 = I_2R_2$$

and $$I_1(R_s + j\omega L_s) = I_2[(r + R_3) + j\omega L]$$

using the symbolic notation. $\omega = 2\pi \times$ frequency.

Then $$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$I_1R_s = I_2(r + R_3).$$

and $$I_1L_s = I_2L$$

from which $$\frac{R_2}{R_1} = \frac{r + R_3}{R_s}$$

or $$R_s = (r + R_3)\frac{R_1}{R_2}$$

and $$L_s = L\frac{R_1}{R_2}$$

The iron loss in the sample is given by $R_sI_1^2 - R_wI_1^2$.

Now, the current $I = I_1 + I_2$ (I_1 and I_2 being in phase)

Thus $$I = I_1 + \frac{R_1}{R_2}I_1 = I_1\left(\frac{R_1 + R_2}{R_2}\right)$$

or $$I_1 = \frac{R_2}{R_1 + R_2} \cdot I$$

Hence, the iron loss W_t is given by

$$W_t = I^2\left(\frac{R_2}{R_1 + R_2}\right)^2(R_s - R_w) \qquad (224)$$

If N = No. of turns on the specimen,
l = length of mean circumference of specimen (in centimetres),
a = cross-section of specimen in square centimetres,
μ = effective permeability of specimen,

then, the inductance is

$$L_s = \frac{4\pi \,.\, NI_1}{10^9\dfrac{l}{a\mu}} \times \frac{N}{I_1} = \frac{4\pi N^2 a}{10^9 l}\,\mu \text{ henries}$$

from which μ may be calculated when L_s has been measured.

CAMPBELL BRIDGE METHOD. Fig. 241B shows the connections of a bridge method due to Campbell (Ref. (19)), this method being one of the best known for the purpose of iron-loss measurement by a bridge network.

The ring specimen carries two windings, a primary and a secondary, having N_1 and N_2 turns respectively, and having the same cross-sectional area. M is a variable mutual inductance connected as shown. R_1 and R_2 are variable resistances, and A is an ammeter.

G is a vibration galvanometer or telephone according to the supply frequency. The waveform of the supply voltage should be sinusoidal, and R_1 should be made sufficiently large to ensure that the current waveform is also sinusoidal. M and R_2 are adjusted until the galvanometer G shows no deflection or until the telephone gives minimum sound. Under balance conditions the vector diagram is as shown in the figure: ϕ is the flux in the sample and I the current in the primary winding, leading the flux by a small angle on account of iron loss; e_1, e_2, and e_m are the *induced* voltages in the primary winding, the secondary winding, and in the secondary of the mutual

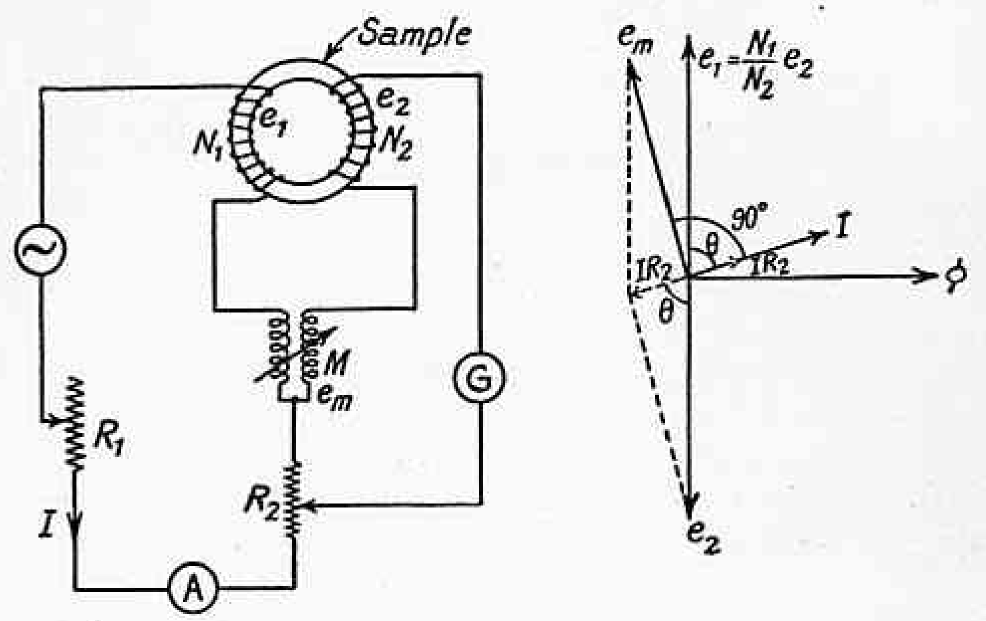


FIG. 241B. CAMPBELL BRIDGE METHOD

inductance M respectively. Balance is obtained when the vector sum of e_m and e_2 is equal, and opposite, to the vector IR_2 representing the voltage drop in resistance R_2.

From this vector diagram (since $e_m = \omega MI$ and $e_2 = \omega mI$, m being the mutual inductance between the primary and secondary windings on the specimen), we have,

$$e_m = e_2 \text{ (very nearly)}$$

or

$$m = M$$

and also, the iron loss

$$W_t = e_1 I \cos\theta = e_1 I \cdot \frac{IR_2}{e_2} = I^2 R_2 \frac{e_1}{e_2}$$

Now $e_1 = \dfrac{N_1}{N_2} e_2$, therefore the iron loss in the specimen is given by

$$W_t = \frac{N_1}{N_2} R_2 \cdot I^2 \qquad \ldots \qquad (225)$$

Let a be the cross-section of the specimen and l its mean circumference, a' being the cross-section of the windings on the specimen.

Then, flux per ampere flowing in the primary winding, linking the two windings

$$= \frac{4\pi N_1 I}{10\left(\dfrac{l}{a\mu + a' - a}\right) I}$$

or

$$\phi = \frac{4\pi N_1 (a\mu + a' - a)}{10l}$$

where μ is the permeability of the specimen. Hence, the mutual inductance

$$m = \frac{4\pi N_1 N_2 (a\mu + a' - a)}{10^9 l} = M \qquad \ldots \qquad (226)$$

If the current I is measured as an r.m.s. value, and is of sinusoidal waveform,

$$H_{max} \text{ in the specimen} = \frac{4\pi \cdot N_1 \cdot I_{max}}{10l} = \frac{4\pi N_1 I\sqrt{2}}{10l}$$

If, also, B_{max} is the maximum flux density in the specimen, the flux threading the secondary, per ampere in the primary, is

$$\frac{aB_{max} + (a' - a)H_{max}}{\sqrt{2}I}$$

or

$$m = \frac{N_2\,[aB_{max} + (a' - a)H_{max}]}{\sqrt{2}I \times 10^8} \qquad \ldots \qquad (227)$$

For the description of other bridge methods of iron-loss measurement, see Ref. (3).

Measurements on Iron Powders. T. H. Oddie (Ref. (44)) has described researches on the permeability and magnetic loss coefficients of carbonyl- and hydrogen-reduced iron powders, at frequencies from 200 c/s to 40 Mc/s. The method used, over the range 200 to 100,000 c/s, employs the resonance bridge circuit shown in Fig. 242. The powders under test were moulded in the form of (*a*) cylindrical cores with an axial hole or (*b*) toroidal cores of rectangular cross section.

The resonance bridge network is used in combination with a beat-frequency type oscillator-analyser. The apparent resistance and inductance of the coil containing the test core are obtained from the settings of the standard capacitor, of the variable resistances and the potential drop across R_3 for balance. These apparent values for the coil have to be corrected for losses in the mica standard capacitor, the self capacitance of the coil (found from the apparent increase of inductance with frequency) and the small inductance due to the air path in the magnetic circuit in parallel with the core.

The corrected value of inductance

$$L_m = 4{\cdot}61 \times 10^{-9} \,.\, \mu_m \,.\, N^2 \,.\, h \log_{10} \frac{d_1}{d_2}$$

where N = number of turns on the coil, d_1 and d_2 are external and internal diameters of the core, h is the length of the core.

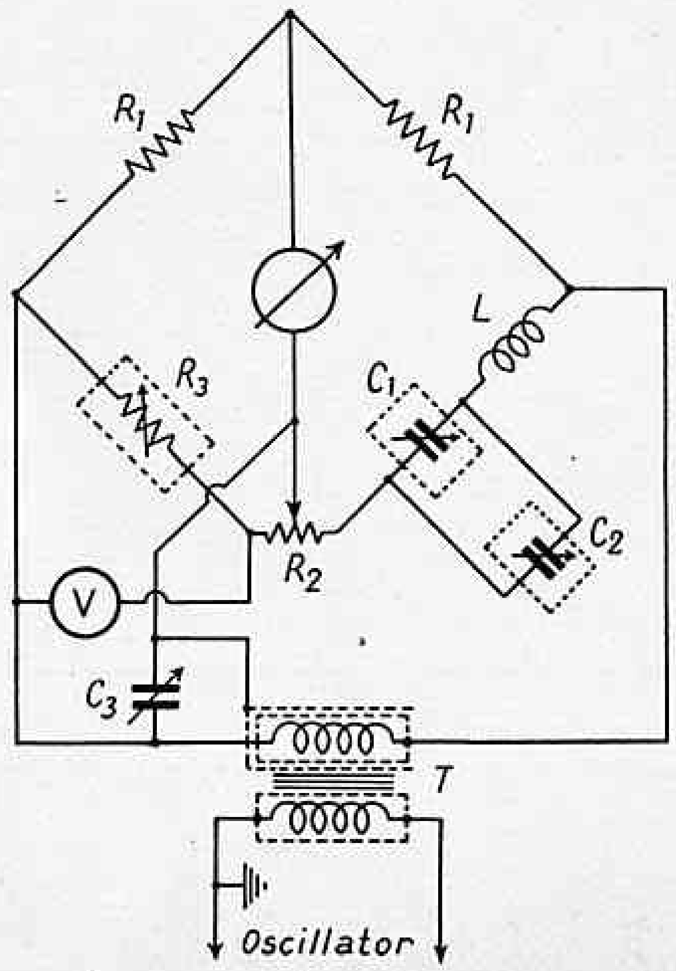


FIG. 242

C_1 = air capacitor, C_2 = mica capacitor, C_3 = trimmer capacitor, L = test coil, R_1 = ratio arms, R_2 = fine adjustment slide wire, R_3 = resistance box, T = input transformer, V = valve voltmeter.

The permeability of the core μ_m can be found from this expression. The maximum flux density

$$B_m = \frac{2{\cdot}83 \times 10^8 \,.\, I \,.\, L_m}{Nh(d_1 - d_2)}$$

where I is the r.m.s. current (in amperes) in the windings.

The magnetic losses in the core are found from analysis of the measured resistance R into component losses by methods given in Refs. (44), (45), and (46).

Measurement of Iron Loss by A.C. Potentiometer. The a.c. potentiometer, which has already been described, may be used for the measurement of the power loss in samples of iron at low flux

densities and forms a very satisfactory method. The connections are given in Fig. 243, in which the potentiometer is assumed to be of the Tinsley-Gall pattern. This potentiometer is very suitable for such measurements, since it measures the magnetizing- and iron-loss components of the exciting current separately.

The sample—of ring form—carries two windings. The primary winding has N_1 turns and the secondary N_2 turns, the supply to the former being through a regulating transformer, from an alternator

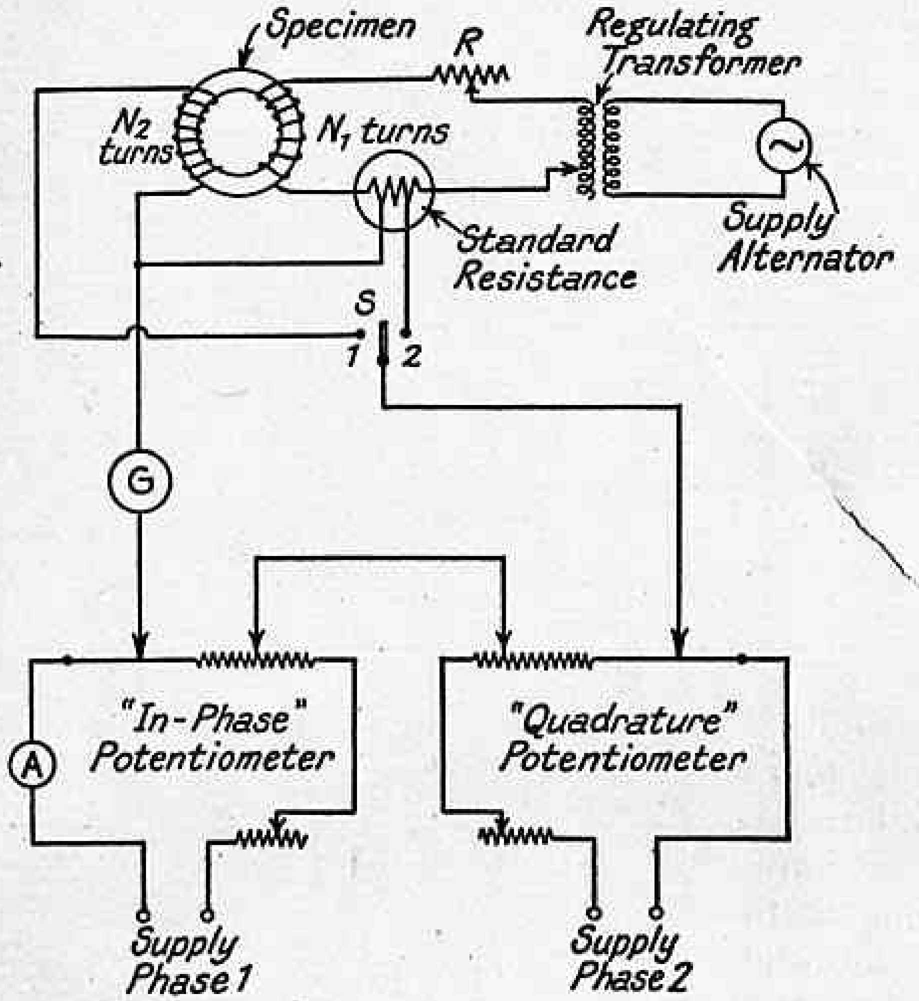


FIG. 243. IRON-LOSS MEASUREMENT BY A.C. POTENTIOMETER

which also supplies current to the two potentiometer slide-wire circuits. The alternator should have a sinusoidal voltage waveform in which case the current waveform will be approximately sinusoidal, if the flux density in the sample is low. A variable resistor R and a standard resistor are connected in series with the primary winding, the latter being for the purpose of measuring the current in the primary winding by measuring the magnitude and phase of the voltage drop across it. G is a vibration galvanometer.

The flux density B_{max} in the sample is obtained from the expression

$$B_{max} = \frac{E \times 10^8}{4K_f \,.\, A \,.\, f \,.\, N_2}$$

where E is the e.m.f. (r.m.s. value) induced in the secondary winding, A is the cross-section of the sample and f the supply frequency. If the form factor K_f is 1·11—i.e. if the supply voltage wave is sinusoidal—B_{max} is given by $\frac{E \times 10^8}{4{\cdot}44AfN_2}$.

The voltage E is measured by moving the switch S on to contact 1, setting the "quadrature" potentiometer at zero, and adjusting the "in-phase" potentiometer until the vibration galvanometer shows no deflection, it being assumed that the necessary standardizing adjustments of the potentiometer have first of all been made as described in Chapter VIII. The setting of the "in-phase" potentiometer for balance then gives the value of the voltage E directly.

The switch S is then thrown over to contact 2, and both the "in-phase" and "quadrature" potentiometers adjusted to give zero galvanometer deflection. The reading of the "in-phase" potentiometer then gives the value of I_eR_s, where I_e is the loss component of the current in the primary winding and R_s is the value of the standard resistance. The reading of the "quadrature" potentiometer gives the value I_MR_s, where I_M is the magnetizing component of the primary current.

Hysteresis Loss in Small Specimens. In development work on electrical sheet steels, it may be necessary to determine both the rotational and alternating hysteresis loss in the sheet when the available samples are quite small. Owing to the considerable variation in magnetic properties in different directions relative to the direction of rolling, it is also convenient to be able to carry out tests upon the same sample with different directions of magnetization. F. Brailsford (Refs. (39) and (40)) has described methods of measurement of such rotational and alternating hysteresis losses, utilizing for the purpose a modified torque magnetometer. The samples used are in the form of discs of $1\frac{1}{4}$ in. diameter, three such discs being used as one sample. These are mounted on a brass rod which is suspended, by a phosphor-bronze suspension, in the air gap of an electromagnet with which field strengths up to $H = 450$ can be obtained. Since the method of testing does not involve continuous rotation, eddy-current losses are absent.

The reader is referred to Brailsford's papers for a full description of the apparatus and for the theory of the method.

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