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Calculations

Piping Calculations Manual

E. SHASHI MENON

- Analyzing the capabilities of existing pipelines
- HP Calculation–Hydraulic horsepower vs. Brake horsepower
- Optimizing performance and estimating requirements for expanding throughput of pipelines

Piping Calculations Manual

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Dedicated to my mother

ABOUT THE AUTHOR

E. SHASHI MENON, P.E., has over 29 years' experience in the oil and gas industry, holding positions as design engineer, project engineer, engineering manager, and chief engineer for major oil and gas companies in the United States. He is the author of *Liquid Pipeline Hydraulics* and several technical papers. He has taught engineering and computer courses, and is also developer and co-author of over a dozen PC software programs for the oil and gas industry.

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Preface

This book covers piping calculations for liquids and gases in single phase steady state flow for various industrial applications. Pipe sizing and capacity calculations are covered mainly with additional analysis of strength requirement for pipes. In each case the basic theory necessary is presented first followed by several example problems fully worked out illustrating the concepts discussed in each chapter. Unlike a textbook or a handbook the focus is on solving actual practical problems that the engineer or technical professional may encounter in their daily work. The calculation manual approach has been found to be very successful and I want to thank Ken McCombs of McGraw-Hill for suggesting this format.

The book consists of ten chapters and three appendices. As far as possible calculations are illustrated using both US Customary System of units as well as the metric or SI units. Piping calculations involving water are covered in the first three chapters titled Water Systems Piping, Fire Protection Piping Systems and Wastewater and Stormwater Piping. Water Systems Piping address transportation of water in short and long distance pipelines. Pressure loss calculations, pumping horsepower required and pump analysis are discussed with numerous examples. The chapter on Fire Protection Piping Systems covers sprinkler system design for residential and commercial buildings. Wastewater Systems chapter addresses how wastewater and stormwater piping is designed. Open channel gravity flow in sewer lines are also discussed.

Chapter 4 introduces the basics of steam piping systems. Flow of saturated and superheated steam through pipes and nozzles are discussed and concepts explained using example problems.

Chapter 5 covers the flow of compressed air in piping systems including flow through nozzles and restrictions. Chapter 6 addresses transportation of oil and petroleum products through short and long distance pipelines. Various pressure drop equations used in the oil industry are reviewed using practical examples. Series and parallel piping configurations are analyzed along with pumping requirements and pump performance. Economic analysis is used to compare alternatives for expanding pipeline throughput.

Chapter 7 covers transportation of natural gas and other compressible fluids through pipeline. Calculations illustrate how gas piping are sized, pressures required and how compressor stations are located on long distance gas pipelines. Economic analysis of pipe loops versus compression for expanding throughput are discussed. Fuel Gas Distribution Piping System is covered in chapter 8. In this chapter low pressure gas piping are analyzed with examples involving Compressed Natural Gas (CNG) and Liquefied Petroleum Gas (LPG).

Chapter 9 covers Cryogenic and Refrigeration Systems Piping. Commonly used cryogenic fluids are reviewed and capacity and pipe sizing illustrated. Since two phase flow may occur in some cryogenic piping systems, the Lockhart and Martinelli correlation method is used in explaining flow of cryogenic fluids. A typical compression refrigeration cycle is explained and pipe sizing illustrated for the suction and discharge lines.

Finally, chapter 10 discusses transportation of slurry and sludge systems through pipelines. Both newtonian and nonnewtonian slurry systems are discussed along with different Bingham and pseudo-plastic slurries and their behavior in pipe flow. Homogenous and heterogeneous flow are covered in addition to pressure drop calculations in slurry pipelines.

I would like to thank Ken McCombs of McGraw-Hill for suggesting the subject matter and format for the book and working with me on finalizing the contents. He was also aggressive in followthrough to get the manuscript completed within the agreed time period. I enjoyed working with him and hope to work on another project with McGraw-Hill in the near future. Lucy Mullins did most of the copyediting. She was very meticulous and thorough in her work and I learned a lot from her about editing technical books. Ben Kolstad, Editorial Services Manager of International Typesetting and Composition (ITC), coordinated the work wonderfully. Neha Rathor and her team at ITC did the typesetting. I found ITC's work to be very prompt, professional, and of high quality.

Needless to say, I received a lot of help during the preparation of the manuscript. In particular I want to thank my wife Pramila for the many hours she spent on the computer typing the manuscript and meticulously proof reading to create the final work product. My father-in-law, A. Mukundan, a retired engineer and consultant, also provided

valuable guidance and help in proofing the manuscript. Finally, I would like to dedicate this book to my mother, who passed away recently, but she definitely was aware of my upcoming book and provided her usual encouragement throughout my effort.

E. Shashi Menon

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Chapter

1

Water Systems Piping

Introduction

Water systems piping consists of pipes, valves, fittings, pumps, and associated appurtenances that make up water transportation systems. These systems may be used to transport fresh water or nonpotable water at room temperatures or at elevated temperatures. In this chapter we will discuss the physical properties of water and how pressure drop due to friction is calculated using the various formulas. In addition, total pressure required and an estimate of the power required to transport water in pipelines will be covered. Some cost comparisons for economic transportation of various pipeline systems will also be discussed.

1.1 Properties of Water

1.1.1 Mass and weight

Mass is defined as the quantity of matter. It is measured in slugs (slug) in U.S. Customary System (USCS) units and kilograms (kg) in Système International (SI) units. A given mass of water will occupy a certain volume at a particular temperature and pressure. For example, a mass of water may be contained in a volume of 500 cubic feet (ft^3) at a temperature of 60°F and a pressure of 14.7 pounds per square inch (lb/in^2 or psi). Water, like most liquids, is considered incompressible. Therefore, pressure and temperature have a negligible effect on its volume. However, if the properties of water are known at standard conditions such as 60°F and 14.7 psi pressure, these properties will be slightly different at other temperatures and pressures. By the principle of conservation of mass, the mass of a given quantity of water will remain the same at all temperatures and pressures.

Weight is defined as the gravitational force exerted on a given mass at a particular location. Hence the weight varies slightly with the geographic location. By Newton's second law the weight is simply the product of the mass and the acceleration due to gravity at that location. Thus

$$W = mg \tag{1.1}$$

where W = weight, lb
m = mass, slug
g = acceleration due to gravity, ft/s^2

In USCS units g is approximately 32.2 ft/s^2, and in SI units it is 9.81 m/s^2. In SI units, weight is measured in newtons (N) and mass is measured in kilograms. Sometimes mass is referred to as pound-mass (lbm) and force as pound-force (lbf) in USCS units. Numerically we say that 1 lbm has a weight of 1 lbf.

1.1.2 Density and specific weight

Density is defined as mass per unit volume. It is expressed as slug/ft^3 in USCS units. Thus, if 100 ft^3 of water has a mass of 200 slug, the density is 200/100 or 2 slug/ft^3. In SI units, density is expressed in kg/m^3. Therefore water is said to have an approximate density of 1000 kg/m^{3}at room temperature.

Specific weight, also referred to as weight density, is defined as the weight per unit volume. By the relationship between weight and mass discussed earlier, we can state that the specific weight is as follows:

$$\gamma = \rho g \tag{1.2}$$

where γ = specific weight, lb/ft^3
ρ = density, slug/ft^3
g = acceleration due to gravity

The volume of water is usually measured in gallons (gal) or cubic ft (ft^3) in USCS units. In SI units, cubic meters (m^3) and liters (L) are used. Correspondingly, the flow rate in water pipelines is measured in gallons per minute (gal/min), million gallons per day (Mgal/day), and cubic feet per second (ft^3/s) in USCS units. In SI units, flow rate is measured in cubic meters per hour (m^3/h) or liters per second (L/s). One ft^3 equals 7.48 gal. One m^{3}equals 1000 L, and 1 gal equals 3.785 L. A table of conversion factors for various units is provided in App. A.

Example 1.1 Water at 60°F fills a tank of volume 1000 ft^3 at atmospheric pressure. If the weight of water in the tank is 31.2 tons, calculate its density and specific weight.

Solution

$$\text{Specific weight} = \frac{\text{weight}}{\text{volume}} = \frac{31.2 \times 2000}{1000} = 62.40\ \text{lb/ft}^3$$

From Eq. (1.2) the density is

$$\text{Density} = \frac{\text{specific weight}}{g} = \frac{62.4}{32.2} = 1.9379\ \text{slug/ft}^3$$

Example 1.2 A tank has a volume of 5 m^3 and contains water at 20°C. Assuming a density of 990 kg/m^3, calculate the weight of the water in the tank. What is the specific weight in N/m^3 using a value of 9.81 m/s^2 for gravitational acceleration?

Solution

$$\text{Mass of water} = \text{volume} \times \text{density} = 5 \times 990 = 4950\ \text{kg}$$

$$\text{Weight of water} = \text{mass} \times g = 4950 \times 9.81 = 48{,}559.5\ \text{N} = 48.56\ \text{kN}$$

$$\text{Specific weight} = \frac{\text{weight}}{\text{volume}} = \frac{48.56}{5} = 9.712\ \text{N/m}^3$$

1.1.3 Specific gravity

Specific gravity is a measure of how heavy a liquid is compared to water. It is a ratio of the density of a liquid to the density of water at the same temperature. Since we are dealing with water only in this chapter, the specific gravity of water by definition is always equal to 1.00.

1.1.4 Viscosity

Viscosity is a measure of a liquid's resistance to flow. Each layer of water flowing through a pipe exerts a certain amount of frictional resistance to the adjacent layer. This is illustrated in the shear stress versus velocity gradient curve shown in Fig. 1.1*a*. Newton proposed an equation that relates the frictional shear stress between adjacent layers of flowing liquid with the velocity variation across a section of the pipe as shown in the following:

$$\text{Shear stress} = \mu \times \text{velocity gradient}$$

or

$$\tau = \mu \frac{dv}{dy} \tag{1.3}$$

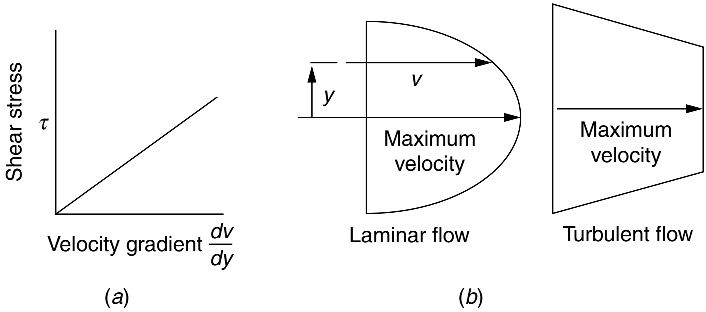

Figure 1.1 Shear stress versus velocity gradient curve.

where τ = shear stress

μ = absolute viscosity, $(\mathrm{lb \cdot s})/\mathrm{ft}^2$ or $\mathrm{slug}/(\mathrm{ft \cdot s})$

$\dfrac{dv}{dy}$ = velocity gradient

The proportionality constant μ in Eq. (1.3) is referred to as the *absolute viscosity* or *dynamic viscosity*. In SI units, μ is expressed in poise or centipoise (cP).

The viscosity of water, like that of most liquids, decreases with an increase in temperature, and vice versa. Under room temperature conditions water has an absolute viscosity of 1 cP.

Kinematic viscosity is defined as the absolute viscosity divided by the density. Thus

$$\nu = \frac{\mu}{\rho} \tag{1.4}$$

where ν = kinematic viscosity, ft^2/s

μ = absolute viscosity, $(\mathrm{lb \cdot s})/\mathrm{ft}^2$ or $\mathrm{slug}/(\mathrm{ft \cdot s})$

ρ = density, $\mathrm{slug}/\mathrm{ft}^3$

In SI units, kinematic viscosity is expressed as stokes or centistokes (cSt). Under room temperature conditions water has a kinematic viscosity of 1.0 cSt. Properties of water are listed in Table 1.1.

Example 1.3 Water has a dynamic viscosity of 1 cP at 20°C. Calculate the kinematic viscosity in SI units.

Solution

$$\text{Kinematic viscosity} = \frac{\text{absolute viscosity } \mu}{\text{density } \rho}$$

$$= \frac{1.0 \times 10^{-2} \times 0.1\ (\mathrm{N \cdot s})/\mathrm{m}^2}{1.0 \times 1000\ \mathrm{kg/m^3}} = 10^{-6}\ \mathrm{m^2/s}$$

since 1.0 N = 1.0 $(\mathrm{kg \cdot m})/\mathrm{s}^2$.

TABLE 1.1 Properties of Water at Atmospheric Pressure

Temperature °F	Density slug/ft^3	Specific weight lb/ft^3	Dynamic viscosity (lb · s)/ft^2	Vapor pressure psia
		USCS units		
32	1.94	62.4	3.75×10^{-5}	0.08
40	1.94	62.4	3.24×10^{-5}	0.12
50	1.94	62.4	2.74×10^{-5}	0.17
60	1.94	62.4	2.36×10^{-5}	0.26
70	1.94	62.3	2.04×10^{-5}	0.36
80	1.93	62.2	1.80×10^{-5}	0.51
90	1.93	62.1	1.59×10^{-5}	0.70
100	1.93	62.0	1.42×10^{-5}	0.96
Temperature °C	**Density kg/m^3**	**Specific weight kN/m^3**	**Dynamic viscosity (N · s)/m^2**	**Vapor pressure kPa**
		SI units		
0	1000	9.81	1.75×10^{-3}	0.611
10	1000	9.81	1.30×10^{-3}	1.230
20	998	9.79	1.02×10^{-3}	2.340
30	996	9.77	8.00×10^{-4}	4.240
40	992	9.73	6.51×10^{-4}	7.380
50	988	9.69	5.41×10^{-4}	12.300
60	984	9.65	4.60×10^{-4}	19.900
70	978	9.59	4.02×10^{-4}	31.200
80	971	9.53	3.50×10^{-4}	47.400
90	965	9.47	3.11×10^{-4}	70.100
100	958	9.40	2.82×10^{-4}	101.300

1.2 Pressure

Pressure is defined as the force per unit area. The pressure at a location in a body of water is by Pascal's law constant in all directions. In USCS units pressure is measured in lb/in^2 (psi), and in SI units it is expressed as N/m^2 or pascals (Pa). Other units for pressure include lb/ft^2, kilopascals (kPa), megapascals (MPa), kg/cm^2, and bar. Conversion factors are listed in App. A.

Therefore, at a depth of 100 ft below the free surface of a water tank the intensity of pressure, or simply the pressure, is the force per unit area. Mathematically, the column of water of height 100 ft exerts a force equal to the weight of the water column over an area of 1 in^2. We can calculate the pressures as follows:

$$\text{Pressure} = \frac{\text{weight of 100-ft column of area } 1.0\ \text{in}^2}{1.0\ \text{in}^2}$$

$$= \frac{100 \times (1/144) \times 62.4}{1.0}$$

In this equation, we have assumed the specific weight of water to be 62.4 lb/ft^3. Therefore, simplifying the equation, we obtain

$$\text{Pressure at a depth of 100 ft} = 43.33 \text{ lb/in}^2 \text{ (psi)}$$

A general equation for the pressure in a liquid at a depth h is

$$P = \gamma h \tag{1.5}$$

where P = pressure, psi
γ = specific weight of liquid
h = liquid depth

Variable γ may also be replaced with ρg where ρ is the density and g is gravitational acceleration.

Generally, pressure in a body of water or a water pipeline is referred to in psi above that of the atmospheric pressure. This is also known as the *gauge pressure* as measured by a pressure gauge. The *absolute pressure* is the sum of the gauge pressure and the atmospheric pressure at the specified location. Mathematically,

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}} \tag{1.6}$$

To distinguish between the two pressures, psig is used for gauge pressure and psia is used for the absolute pressure. In most calculations involving water pipelines the gauge pressure is used. Unless otherwise specified, psi means the gauge pressure.

Liquid pressure may also be referred to as *head pressure*, in which case it is expressed in feet of liquid head (or meters in SI units). Therefore, a pressure of 1000 psi in a liquid such as water is said to be equivalent to a pressure head of

$$h = \frac{1000 \times 144}{62.4} = 2308 \text{ ft}$$

In a more general form, the pressure P in psi and liquid head h in feet for a specific gravity of Sg are related by

$$P = \frac{h \times \text{Sg}}{2.31} \tag{1.7}$$

where P = pressure, psi
h = liquid head, ft
Sg = specific gravity of water

In SI units, pressure P in kilopascals and head h in meters are related by the following equation:

$$P = \frac{h \times \mathrm{Sg}}{0.102} \tag{1.8}$$

Example 1.4 Calculate the pressure in psi at a water depth of 100 ft assuming the specific weight of water is 62.4 lb/ft^3. What is the equivalent pressure in kilopascals? If the atmospheric pressure is 14.7 psi, calculate the absolute pressure at that location.

Solution Using Eq. (1.5), we calculate the pressure:

$$P = \gamma h = 62.4 \text{ lb/ft}^3 \times 100 \text{ ft} = 6240 \text{ lb/ft}^2$$

$$= \frac{6240}{144} \text{ lb/in}^2 = 43.33 \text{ psig}$$

$$\text{Absolute pressure} = 43.33 + 14.7 = 58.03 \text{ psia}$$

In SI units we can calculate the pressures as follows:

$$\text{Pressure} = 62.4 \times \frac{1}{2.2025}(3.281)^3 \text{ kg/m}^3 \times \left(\frac{100}{3.281}\text{m}\right)(9.81 \text{ m/s}^2)$$

$$= 2.992 \times 10^5 (\text{ kg} \cdot \text{m})/(\text{s}^2 \cdot \text{m}^2)$$

$$= 2.992 \times 10^5 \text{ N/m}^2 = 299.2 \text{ kPa}$$

Alternatively,

$$\text{Pressure in kPa} = \frac{\text{pressure in psi}}{0.145}$$

$$= \frac{43.33}{0.145} = 298.83 \text{ kPa}$$

The 0.1 percent discrepancy between the values is due to conversion factor round-off.

1.3 Velocity

The velocity of flow in a water pipeline depends on the pipe size and flow rate. If the flow rate is uniform throughout the pipeline (steady flow), the velocity at every cross section along the pipe will be a constant value. However, there is a variation in velocity along the pipe cross section. The velocity at the pipe wall will be zero, increasing to a maximum at the centerline of the pipe. This is illustrated in Fig. 1.1*b*.

We can define a bulk velocity or an average velocity of flow as follows:

$$\text{Velocity} = \frac{\text{flow rate}}{\text{area of flow}}$$

Considering a circular pipe with an inside diameter D and a flow rate of Q, we can calculate the average velocity as

$$V = \frac{Q}{\pi D^2/4} \tag{1.9}$$

Employing consistent units of flow rate Q in ft^3/s and pipe diameter in inches, the velocity in ft/s is as follows:

$$V = \frac{144Q}{\pi D^2/4}$$

or

$$V = 183.3461\frac{Q}{D^2} \tag{1.10}$$

where V = velocity, ft/s
Q = flow rate, ft^3/s
D = inside diameter, in

Additional formulas for velocity in different units are as follows:

$$V = 0.4085\frac{Q}{D^2} \tag{1.11}$$

where V = velocity, ft/s
Q = flow rate, gal/min
D = inside diameter, in

In SI units, the velocity equation is as follows:

$$V = 353.6777\frac{Q}{D^2} \tag{1.12}$$

where V = velocity, m/s
Q = flow rate, m^3/h
D = inside diameter, mm

Example 1.5 Water flows through an NPS 16 pipeline (0.250-in wall thickness) at the rate of 3000 gal/min. Calculate the average velocity for steady flow. (*Note*: The designation NPS 16 means nominal pipe size of 16 in.)

Solution From Eq. (1.11), the average flow velocity is

$$V = 0.4085\frac{3000}{15.5^2} = 5.10 \text{ ft/s}$$

Example 1.6 Water flows through a DN 200 pipeline (10-mm wall thickness) at the rate of 75 L/s. Calculate the average velocity for steady flow.

Solution The designation DN 200 means metric pipe size of 200-mm outside diameter. It corresponds to NPS 8 in USCS units. From Eq. (1.12) the average flow velocity is

$$V = 353.6777\left(\frac{75 \times 60 \times 60 \times 10^{-3}}{180^2}\right) = 2.95 \text{ m/s}$$

The variation of flow velocity in a pipe depends on the type of flow. In laminar flow, the velocity variation is parabolic. As the flow rate becomes turbulent the velocity profile approximates a trapezoidal shape. Both types of flow are depicted in Fig. 1.1*b*. Laminar and turbulent flows are discussed in Sec. 1.5 after we introduce the concept of the Reynolds number.

1.4 Reynolds Number

The Reynolds number is a dimensionless parameter of flow. It depends on the pipe size, flow rate, liquid viscosity, and density. It is calculated from the following equation:

$$R = \frac{VD\rho}{\mu} \tag{1.13}$$

or

$$R = \frac{VD}{\nu} \tag{1.14}$$

where R = Reynolds number, dimensionless
V = average flow velocity, ft/s
D = inside diameter of pipe, ft
ρ = mass density of liquid, slug/ft^3
μ = dynamic viscosity, slug/(ft · s)
ν = kinematic viscosity, ft^2/s

Since R must be dimensionless, a consistent set of units must be used for all items in Eq. (1.13) to ensure that all units cancel out and R has no dimensions.

Other variations of the Reynolds number for different units are as follows:

$$R = 3162.5\frac{Q}{D\nu} \tag{1.15}$$

where R = Reynolds number, dimensionless
Q = flow rate, gal/min
D = inside diameter of pipe, in
ν = kinematic viscosity, cSt

In SI units, the Reynolds number is expressed as follows:

$$R = 353{,}678 \frac{Q}{\nu D} \tag{1.16}$$

where R = Reynolds number, dimensionless
Q = flow rate, m^3/h
D = inside diameter of pipe, mm
ν = kinematic viscosity, cSt

Example 1.7 Water flows through a 20-in pipeline (0.375-in wall thickness) at 6000 gal/min. Calculate the average velocity and Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution Using Eq. (1.11), the average velocity is calculated as follows:

$$V = 0.4085 \frac{6000}{19.25^2} = 6.61 \text{ ft/s}$$

From Eq. (1.15), the Reynolds number is

$$R = 3162.5 \frac{6000}{19.25 \times 1.0} = 985{,}714$$

Example 1.8 Water flows through a 400-mm pipeline (10-mm wall thickness) at 640 m^3/h. Calculate the average velocity and Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution From Eq. (1.12) the average velocity is

$$V = 353.6777 \frac{640}{380^2} = 1.57 \text{ m/s}$$

From Eq. (1.16) the Reynolds number is

$$R = 353{,}678 \frac{640}{380 \times 1.0} = 595{,}668$$

1.5 Types of Flow

Flow through pipe can be classified as laminar flow, turbulent flow, or critical flow depending on the Reynolds number of flow. If the flow is such that the Reynolds number is less than 2000 to 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is said to be *turbulent*. *Critical flow* occurs when the Reynolds number is in the range of 2100 to 4000. Laminar flow is characterized by smooth flow in which no eddies or turbulence are visible. The flow is said to occur in laminations. If dye was injected into a transparent pipeline, laminar flow would be manifested in the form of smooth streamlines of dye. Turbulent flow occurs at higher velocities and is accompanied by eddies and other disturbances in the liquid. Mathematically, if R represents the Reynolds number of flow, the flow types are defined as follows:

Laminar flow:	$R \leq 2100$
Critical flow:	$2100 < R \leq 4000$
Turbulent flow:	$R > 4000$

In the critical flow regime, where the Reynolds number is between 2100 and 4000, the flow is undefined as far as pressure drop calculations are concerned.

1.6 Pressure Drop Due to Friction

As water flows through a pipe there is friction between the adjacent layers of water and between the water molecules and the pipe wall. This friction causes energy to be lost, being converted from pressure energy and kinetic energy to heat. The pressure continuously decreases as water flows down the pipe from the upstream end to the downstream end. The amount of pressure loss due to friction, also known as head loss due to friction, depends on the flow rate, properties of water (specific gravity and viscosity), pipe diameter, pipe length, and internal roughness of the pipe. Before we discuss the frictional pressure loss in a pipeline we must introduce Bernoulli's equation, which is a form of the energy equation for liquid flow in a pipeline.

1.6.1 Bernoulli's equation

Bernoulli's equation is another way of stating the principle of conservation of energy applied to liquid flow through a pipeline. At each point along the pipeline the total energy of the liquid is computed by taking into consideration the liquid energy due to pressure, velocity, and elevation combined with any energy input, energy output, and energy losses. The total energy of the liquid contained in the pipeline at any point is a constant. This is also known as the principle of conservation of energy.

Consider a liquid flow through a pipeline from point A to point B as shown in Fig. 1.2. The elevation of point A is Z_A and the elevation at B is Z_B above some common datum, such as mean sea level. The pressure at point A is P_A and that at B is P_B. It is assumed that the pipe diameter at A and B are different, and hence the flow velocity at A and B will be represented by V_A and V_B, respectively. A particle of the liquid of

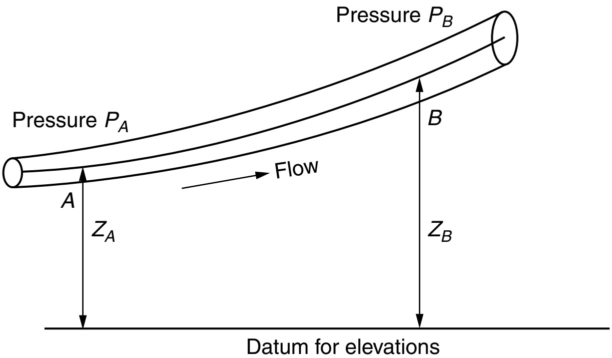

Figure 1.2 Total energy of water in pipe flow.

unit weight at point A in the pipeline possesses a total energy E which consists of three components:

$$\text{Potential energy} = Z_A$$

$$\text{Pressure energy} = \frac{P_A}{\gamma}$$

$$\text{Kinetic energy} = \left(\frac{V_A}{2g}\right)^2$$

where γ is the specific weight of liquid.

Therefore the total energy E is

$$E = Z_A + \frac{P_A}{\gamma} + \frac{V_A{}^2}{2g} \tag{1.17}$$

Since each term in Eq. (1.17) has dimensions of length, we refer to the total energy at point A as H_A in feet of liquid head. Therefore, rewriting the total energy in feet of liquid head at point A, we obtain

$$H_A = Z_A + \frac{P_A}{\gamma} + \frac{V_A{}^2}{2g} \tag{1.18}$$

Similarly, the same unit weight of liquid at point B has a total energy per unit weight equal to H_B given by

$$H_B = Z_B + \frac{P_B}{\gamma} + \frac{V_B{}^2}{2g} \tag{1.19}$$

By the principle of conservation of energy

$$H_A = H_B \tag{1.20}$$

Therefore,

$$Z_A + \frac{P_A}{\gamma} + \frac{V_A{}^2}{2g} = Z_B + \frac{P_B}{\gamma} + \frac{V_B{}^2}{2g} \tag{1.21}$$

In Eq. (1.21), referred to as Bernoulli's equation, we have not considered any energy added to the liquid, energy taken out of the liquid, or energy losses due to friction. Therefore, modifying Eq. (1.21) to take into account the addition of energy (such as from a pump at A) and accounting for frictional head losses h_f, we get the more common form of Bernoulli's equation as follows:

$$Z_A + \frac{P_A}{\gamma} + \frac{V_A{}^2}{2g} + H_p = Z_B + \frac{P_B}{\gamma} + \frac{V_B{}^2}{2g} + h_f \tag{1.22}$$

where H_P is the equivalent head added to the liquid by the pump at A and h_f represents the total frictional head losses between points A and B.

We will next discuss how the head loss due to friction h_f in Bernoulli's equation is calculated for various conditions of water flow in pipelines. We begin with the classical pressure drop equation known as the Darcy-Weisbach equation, or simply the Darcy equation.

1.6.2 Darcy equation

The Darcy equation, also called Darcy-Weisbach equation, is one of the oldest formulas used in classical fluid mechanics. It can be used to calculate the pressure drop in pipes transporting any type of fluid, such as a liquid or gas.

As water flows through a pipe from point A to point B the pressure decreases due to friction between the water and the pipe wall. The Darcy equation may be used to calculate the pressure drop in water pipes as follows:

$$h = f\frac{L}{D}\frac{V^2}{2g} \tag{1.23}$$

where h = frictional pressure loss, ft of head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = inside pipe diameter, ft
V = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

In USCS units, $g = 32.2$ ft/s^2, and in SI units, $g = 9.81$ m/s^2.

Note that the Darcy equation gives the frictional pressure loss in feet of head of water. It can be converted to pressure loss in psi using Eq. (1.7). The term $V^2/2g$ in the Darcy equation is called the velocity head, and it represents the kinetic energy of the water. The term *velocity head* will be used in subsequent sections of this chapter when discussing frictional head loss through pipe fittings and valves.

Another form of the Darcy equation with frictional pressure drop expressed in psi/mi and using a flow rate instead of velocity is as follows:

$$P_m = 71.16 \frac{fQ^2}{D^5} \tag{1.24}$$

where P_m = frictional pressure loss, psi/mi
f = Darcy friction factor, dimensionless
Q = flow rate, gal/min
D = pipe inside diameter, in

In SI units, the Darcy equation may be written as

$$h = 50.94 \frac{fLV^2}{D} \tag{1.25}$$

where h = frictional pressure loss, meters of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, m
D = pipe inside diameter, mm
V = average flow velocity, m/s

Another version of the Darcy equation in SI units is as follows:

$$P_{\text{km}} = (6.2475 \times 10^{10}) \frac{fQ^2}{D^5} \tag{1.26}$$

where P_{km} = pressure drop due to friction, kPa/km
Q = liquid flow rate, m^3/h
f = Darcy friction factor, dimensionless
D = pipe inside diameter, mm

In order to calculate the friction loss in a water pipeline using the Darcy equation, we must know the friction factor f. The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eq. (1.23). This friction factor is a nondimensional number between 0.0 and 0.1 (usually around 0.02 for turbulent flow) that depends on the internal roughness of the pipe, the pipe diameter, and the Reynolds number, and therefore the type of flow (laminar or turbulent).

For laminar flow, the friction factor f depends only on the Reynolds number and is calculated as follows:

$$f = \frac{64}{R} \tag{1.27}$$

where f is the friction factor for laminar flow and R is the Reynolds number for laminar flow ($R < 2100$) (dimensionless).

Therefore, if the Reynolds number for a particular flow is 1200, the friction factor for this laminar flow is $64/1200 = 0.0533$. If this pipeline has a 400-mm inside diameter and water flows through it at 500 m^3/h, the pressure loss per kilometer would be, from Eq. (1.26),

$$P_{\text{km}} = 6.2475 \times 10^{10} \times 0.0533 \times \frac{(500)^2}{(400)^5} = 81.3 \text{ kPa/km}$$

If the flow is turbulent ($R > 4000$), calculation of the friction factor is not as straightforward as that for laminar flow. We will discuss this next.

1.6.3 Colebrook-White equation

In turbulent flow the calculation of friction factor f is more complex. The friction factor depends on the pipe inside diameter, the pipe roughness, and the Reynolds number. Based on work by Moody, Colebrook-White, and others, the following empirical equation, known as the Colebrook-White equation, has been proposed for calculating the friction factor in turbulent flow:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{R\sqrt{f}}\right) \tag{1.28}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
R = Reynolds number, dimensionless

The absolute pipe roughness depends on the internal condition of the pipe. Generally a value of 0.002 in or 0.05 mm is used in most calculations, unless better data are available. Table 1.2 lists the pipe roughness for various types of pipe. The ratio e/D is known as the relative pipe roughness and is dimensionless since both pipe absolute roughness e and pipe inside diameter D are expressed in the same units (inches in USCS units and millimeters in SI units). Therefore, Eq. (1.28) remains the same for SI units, except that, as stated, the absolute pipe roughness e and the pipe diameter D are both expressed in millimeters. All other terms in the equation are dimensionless.

TABLE 1.2 Pipe Internal Roughness

Pipe material	Roughness	
	in	mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

It can be seen from Eq. (1.28) that the calculation of the friction factor f is not straightforward since it appears on both sides of the equation. Successive iteration or a trial-and-error approach is used to solve for the friction factor.

1.6.4 Moody diagram

The Moody diagram is a graphical plot of the friction factor f for all flow regimes (laminar, critical, and turbulent) against the Reynolds number at various values of the relative roughness of pipe. The graphical method of determining the friction factor for turbulent flow using the Moody diagram (see Fig. 1.3) is discussed next.

For a given Reynolds number on the horizontal axis, a vertical line is drawn up to the curve representing the relative roughness e/D. The friction factor is then read by going horizontally to the vertical axis on the left. It can be seen from the Moody diagram that the turbulent region is further divided into two regions: the "transition zone" and the "complete turbulence in rough pipes" zone. The lower boundary is designated as "smooth pipes," and the transition zone extends up to the dashed line. Beyond the dashed line is the complete turbulence in rough pipes zone. In this zone the friction factor depends very little on the Reynolds number and more on the relative roughness. This is evident from the Colebrook-White equation, where at large Reynolds numbers, the second term within the parentheses approaches zero. The friction factor thus depends only on the first term, which is proportional to the relative roughness e/D. In contrast, in the transition zone both R and e/D influence the value of friction factor f.

Example 1.9 Water flows through a 16-in pipeline (0.375-in wall thickness) at 3000 gal/min. Assuming a pipe roughness of 0.002 in, calculate the friction factor and head loss due to friction in 1000 ft of pipe length.

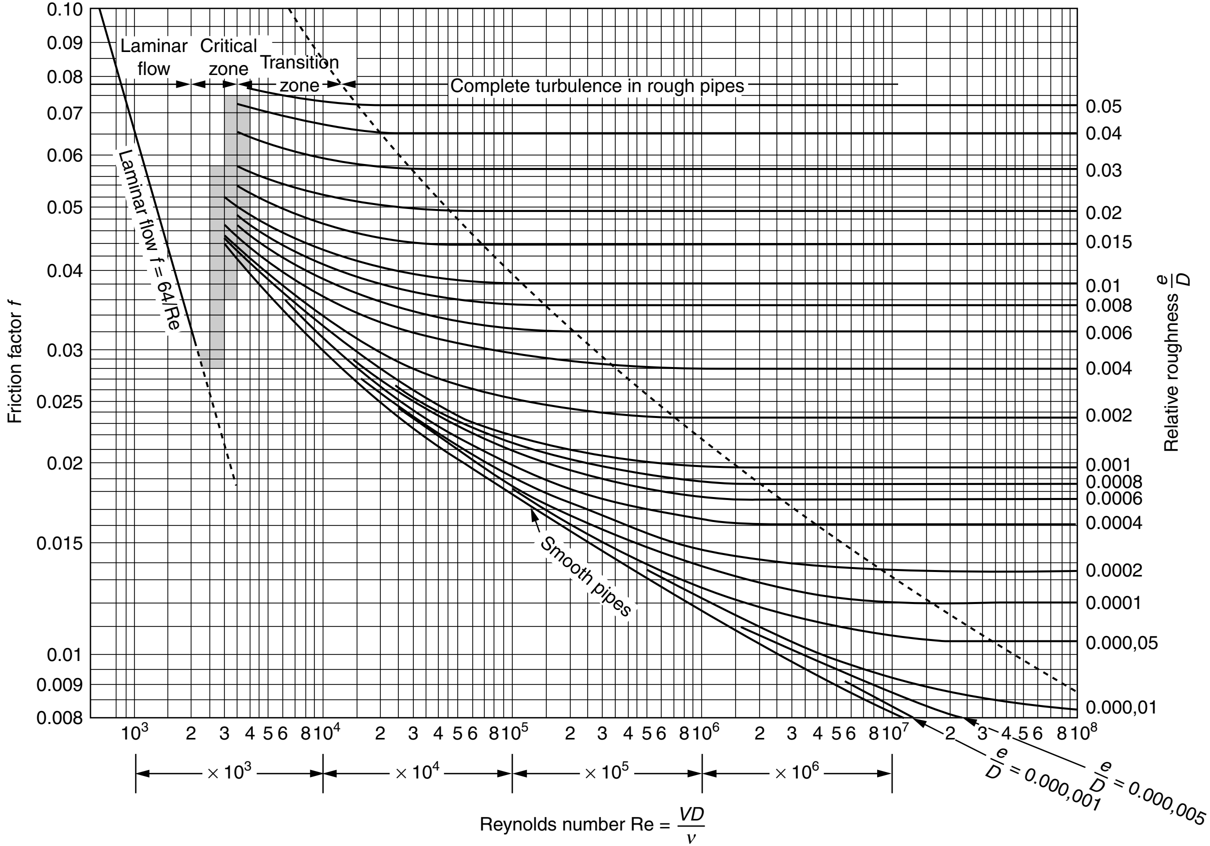

Figure 1.3 Moody diagram.

Solution Using Eq. (1.11) we calculate the average flow velocity:

$$V = 0.4085\frac{3000}{(15.25)^2} = 5.27 \text{ ft/s}$$

Using Eq. (1.15) we calculate the Reynolds number as follows:

$$R = 3162.5\frac{3000}{15.25 \times 1.0} = 622{,}131$$

Thus the flow is turbulent, and we can use the Colebrook-White equation (1.28) to calculate the friction factor.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{f}}\right)$$

This equation must be solved for f by trial and error. First assume that $f = 0.02$. Substituting in the preceding equation, we get a better approximation for f as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{0.02}}\right) \quad \text{or} \quad f = 0.0142$$

Recalculating using this value

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{(622{,}131\sqrt{0.0142}}\right) \quad \text{or} \quad f = 0.0145$$

and finally

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{0.0145}}\right) \quad \text{or} \quad f = 0.0144$$

Thus the friction factor is 0.0144. (We could also have used the Moody diagram to find the friction factor graphically, for Reynolds number $R = 622{,}131$ and $e/D = 0.002/15.25 = 0.0001$. From the graph, we get $f = 0.0145$, which is close enough.)

The head loss due to friction can now be calculated using the Darcy equation (1.23).

$$h = 0.0144\frac{1000 \times 12}{15.25}\frac{5.27^2}{64.4} = 4.89 \text{ ft of head of water}$$

Converting to psi using Eq. (1.7), we get

$$\text{Pressure drop due to friction} = \frac{4.89 \times 1.0}{2.31} = 2.12 \text{ psi}$$

Example 1.10 A concrete pipe (2-m inside diameter) is used to transport water from a pumping facility to a storage tank 5 km away. Neglecting any difference in elevations, calculate the friction factor and pressure loss in kPa/km due to friction at a flow rate of 34,000 m^3/h. Assume a pipe roughness of 0.05 mm. If a delivery pressure of 4 kPa must be maintained at the delivery point and the storage tank is at an elevation of 200 m above that of the pumping facility, calculate the pressure required at the pumping facility at the given flow rate, using the Moody diagram.

Solution The average flow velocity is calculated using Eq. (1.12).

$$V = 353.6777 \frac{34{,}000}{(2000)^2} = 3.01 \text{ m/s}$$

Next using Eq. (1.16), we get the Reynolds number as follows:

$$R = 353{,}678 \frac{34{,}000}{1.0 \times 2000} = 6{,}012{,}526$$

Therefore, the flow is turbulent. We can use the Colebrook-White equation or the Moody diagram to determine the friction factor. The relative roughness is

$$\frac{e}{D} = \frac{0.05}{2000} = 0.00003$$

Using the obtained values for relative roughness and the Reynolds number, from the Moody diagram we get friction factor $f = 0.01$.

The pressure drop due to friction can now be calculated using the Darcy equation (1.23) for the entire 5-km length of pipe as

$$h = 0.01 \frac{5000}{2.0} \frac{3.01^2}{2 \times 9.81} = 11.54 \text{ m of head of water}$$

Using Eq. (1.8) we calculate the pressure drop in kilopascals as

$$\text{Total pressure drop in 5 km} = \frac{11.54 \times 1.0}{0.102} = 113.14 \text{ kPa}$$

Therefore,

$$\text{Pressure drop in kPa/km} = \frac{113.14}{5} = 22.63 \text{ kPa/km}$$

The pressure required at the pumping facility is calculated by adding the following three items:

1. Pressure drop due to friction for 5-km length.
2. The static elevation difference between the pumping facility and storage tank.
3. The delivery pressure required at the storage tank.

We can also state the calculation mathematically.

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \tag{1.29}$$

where P_t = total pressure required at pump
P_f = frictional pressure head
P_{elev} = pressure head due to elevation difference
P_{del} = delivery pressure at storage tank

All pressures must be in the same units: either meters of head or kilopascals.

$$P_t = 113.14 \text{ kPa} + 200 \text{ m} + 4 \text{ kPa}$$

Changing all units to kilopascals we get

$$P_t = 113.14 + \frac{200 \times 1.0}{0.102} + 4 = 2077.92 \text{ kPa}$$

Therefore, the pressure required at the pumping facility is 2078 kPa.

1.6.5 Hazen-Williams equation

A more popular approach to the calculation of head loss in water piping systems is the use of the Hazen-Williams equation. In this method a coefficient C known as the Hazen-Williams C factor is used to account for the internal pipe roughness or efficiency. Unlike the Moody diagram or the Colebrook-White equation, the Hazen-Williams equation does not require use of the Reynolds number or viscosity of water to calculate the head loss due to friction.

The Hazen-Williams equation for head loss is expressed as follows:

$$h = \frac{4.73\, L(Q/C)^{1.852}}{D^{4.87}} \tag{1.30}$$

where h = frictional head loss, ft
L = length of pipe, ft
D = inside diameter of pipe, ft
Q = flow rate, ft^3/s
C = Hazen-Williams C factor or roughness coefficient, dimensionless

Commonly used values of the Hazen-Williams C factor for various applications are listed in Table 1.3.

TABLE 1.3 Hazen-Williams *C* Factor

Pipe material	C factor
Smooth pipes (all metals)	130–140
Cast iron (old)	100
Iron (worn/pitted)	60–80
Polyvinyl chloride (PVC)	150
Brick	100
Smooth wood	120
Smooth masonry	120
Vitrified clay	110

On examining the Hazen-Williams equation, we see that the head loss due to friction is calculated in feet of head, similar to the Darcy equation. The value of h can be converted to psi using the head-to-psi conversion [Eq. (1.7)]. Although the Hazen-Williams equation appears to be simpler to use than the Colebrook-White and Darcy equations to calculate the pressure drop, the unknown term C can cause uncertainties in the pressure drop calculation.

Usually, the C factor, or Hazen-Williams roughness coefficient, is based on experience with the water pipeline system, such as the pipe material or internal condition of the pipeline system. When designing a new pipeline, proper judgment must be exercised in choosing a C factor since considerable variation in pressure drop can occur by selecting a particular value of C compared to another. Because of the inverse proportionality effect of C on the head loss h, using $C = 140$ instead of $C = 100$ will result in a $[1 - (\frac{100}{140})^{1.852}]$ or 46 percent less pressure drop. Therefore, it is important that the C value be chosen judiciously.

Other forms of the Hazen-Williams equation using different units are discussed next. In the following formulas the presented equations calculate the flow rate from a given head loss, or vice versa.

In USCS units, the following forms of the Hazen-Williams equation are used.

$$Q = (6.755 \times 10^{-3})CD^{2.63}h^{0.54} \tag{1.31}$$

$$h = 10{,}460 \left(\frac{Q}{C}\right)^{1.852} \frac{1}{D^{4.87}} \tag{1.32}$$

$$P_m = 23{,}909 \left(\frac{Q}{C}\right)^{1.852} \frac{1}{D^{4.87}} \tag{1.33}$$

where Q = flow rate, gal/min
h = friction loss, ft of water per 1000 ft of pipe
P_m = friction loss, psi per mile of pipe
D = inside diameter of pipe, in
C = Hazen-Williams C factor, dimensionless (see Table 1.3)

In SI units, the Hazen-Williams equation is expressed as follows:

$$Q = (9.0379 \times 10^{-8})CD^{2.63} \left(\frac{P_{\mathrm{km}}}{\mathrm{Sg}}\right)^{0.54} \tag{1.34}$$

$$P_{\mathrm{km}} = 1.1101 \times 10^{13} \left(\frac{Q}{C}\right)^{1.852} \frac{\mathrm{Sg}}{D^{4.87}} \tag{1.35}$$

where Q = flow rate, m^3/h
D = pipe inside diameter, mm
P_{km} = frictional pressure drop, kPa/km
Sg = liquid specific gravity (water = 1.00)
C = Hazen-Williams C factor, dimensionless (see Table 1.3)

1.6.6 Manning equation

The Manning equation was originally developed for use in open-channel flow of water. It is also sometimes used in pipe flow. The Manning equation uses the Manning index n, or roughness coefficient, which like the Hazen-Williams C factor depends on the type and internal condition of the pipe. The values used for the Manning index for common pipe materials are listed in Table 1.4.

The following is a form of the Manning equation for pressure drop due to friction in water piping systems:

$$Q = \frac{1.486}{n} A R^{2/3} \left(\frac{h}{L}\right)^{1/2} \tag{1.36}$$

where Q = flow rate, ft^3/s
A = cross-sectional area of pipe, ft^2
R = hydraulic radius = $D/4$ for circular pipes flowing full
n = Manning index, or roughness coefficient, dimensionless
D = inside diameter of pipe, ft
h = friction loss, ft of water
L = pipe length, ft

TABLE 1.4 Manning Index

Pipe material	Resistance factor
PVC	0.009
Very smooth	0.010
Cement-lined ductile iron	0.012
New cast iron, welded steel	0.014
Old cast iron, brick	0.020
Badly corroded cast iron	0.035
Wood, concrete	0.016
Clay, new riveted steel	0.017
Canals cut through rock	0.040
Earth canals average condition	0.023
Rivers in good conditions	0.030

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In SI units, the Manning equation is expressed as follows:

$$Q = \frac{1}{n} A R^{2/3} \left(\frac{h}{L}\right)^{1/2} \tag{1.37}$$

where Q = flow rate, m^3/s
A = cross-sectional area of pipe, m^2
R = hydraulic radius = $D/4$ for circular pipes flowing full
n = Manning index, or roughness coefficient, dimensionless
D = inside diameter of pipe, m
h = friction loss, ft of water
L = pipe length, m

Example 1.11 Water flows through a 16-in pipeline (0.375-in wall thickness) at 3000 gal/min. Using the Hazen-Williams equation with a C factor of 120, calculate the pressure loss due to friction in 1000 ft of pipe length.

Solution First we calculate the flow rate using Eq. (1.31):

$$Q = 6.755 \times 10^{-3} \times 120 \times (15.25)^{2.63} h^{0.54}$$

where h is in feet of head per 1000 ft of pipe.

Rearranging the preceding equation, using $Q = 3000$ and solving for h, we get

$$h^{0.54} = \frac{3000}{6.755 \times 10^{-3} \times 120 \times (15.25)^{2.63}}$$

Therefore,

$$h = 7.0 \text{ ft per 1000 ft of pipe}$$

$$\text{Pressure drop} = \frac{7.0 \times 1.0}{2.31} = 3.03 \text{ psi}$$

Compare this with the same problem described in Example 1.9. Using the Colebrook-White and Darcy equations we calculated the pressure drop to be 4.89 ft per 1000 ft of pipe. Therefore, we can conclude that the C value used in the Hazen-Williams equation in this example is too low and hence gives us a comparatively higher pressure drop. Therefore, we will recalculate the pressure drop using a C factor = 140 instead.

$$h^{0.54} = \frac{3000}{6.755 \times 10^{-3} \times 140 \times (15.25)^{2.63}}$$

Therefore,

$$h = 5.26 \text{ ft per 1000 ft of pipe}$$

$$\text{Pressure drop} = \frac{5.26 \times 1.0}{2.31} = 2.28 \text{ psi}$$

It can be seen that we are closer now to the results using the Colebrook-White and Darcy equations. The result is still 7.6 percent higher than that obtained using the Colebrook-White and Darcy equations. The conclusion is that the

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C factor in the preceding Hazen-Williams calculation should probably be slightly higher than 140. In fact, using a C factor of 146 will get the result closer to the 4.89 ft per 1000 ft we got using the Colebrook-White equation.

Example 1.12 A concrete pipe with a 2-m inside diameter is used to transport water from a pumping facility to a storage tank 5 km away. Neglecting differences in elevation, calculate the pressure loss in kPa/km due to friction at a flow rate of 34,000 m^3/h. Use the Hazen-Williams equation with a C factor of 140. If a delivery pressure of 400 kPa must be maintained at the delivery point and the storage tank is at an elevation of 200 m above that of the pumping facility, calculate the pressure required at the pumping facility at the given flow rate.

Solution The flow rate Q in m^3/h is calculated using the Hazen-Williams equation (1.35) as follows:

$$\begin{aligned} P_{\text{km}} &= (1.1101 \times 10^{13}) \left(\frac{34{,}000}{140}\right)^{1.852} \times \frac{1}{(2000)^{4.87}} \\ &= 24.38 \text{ kPa/km} \end{aligned}$$

The pressure required at the pumping facility is calculated by adding the pressure drop due to friction to the delivery pressure required and the static elevation head between the pumping facility and storage tank using Eq. (1.29).

$$\begin{aligned} P_t &= P_f + P_{\text{elev}} + P_{\text{del}} \\ &= (24.38 \times 5) \text{ kPa} + 200 \text{ m} + 400 \text{ kPa} \end{aligned}$$

Changing all units to kPa we get

$$P_t = 121.9 + \frac{200 \times 1.0}{0.102} + 400 = 2482.68 \text{ kPa}$$

Thus the pressure required at the pumping facility is 2483 kPa.

1.7 Minor Losses

So far, we have calculated the pressure drop per unit length in straight pipe. We also calculated the total pressure drop considering several miles of pipe from a pump station to a storage tank. Minor losses in a water pipeline are classified as those pressure drops that are associated with piping components such as valves and fittings. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. A pipe nozzle exiting from a storage tank will have entrance and exit losses. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe.

Generally, minor losses are included in calculations by using the equivalent length of the valve or fitting or using a resistance factor or

TABLE 1.5 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

K factor multiplied by the velocity head $V^2/2g$. The term minor losses can be applied only where the pipeline lengths and hence the friction losses are relatively large compared to the pressure drops in the fittings and valves. In a situation such as plant piping and tank farm piping the pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses is really a misnomer. In any case, the pressure losses through valves, fittings, etc., can be accounted for approximately using the equivalent length or K times the velocity head method. It must be noted that this way of calculating the minor losses is valid only in turbulent flow. No data are available for laminar flow.

1.7.1 Valves and fittings

Table 1.5 shows the equivalent lengths of commonly used valves and fittings in a typical water pipeline. It can be seen from this table that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore, a 20-in-diameter gate valve may be replaced with a $20 \times 8 = 160$-in-long piece of pipe that will match the frictional pressure drop through the valve.

Example 1.13 A piping system is 2000 ft of NPS 20 pipe that has two 20-in gate valves, three 20-in ball valves, one swing check valve, and four

90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe and valves and fittings.

Solution Using Table 1.5, we can convert all valves and fittings in terms of 20-in pipe as follows:

$$\begin{aligned}
\text{Two 20-in gate valves} &= 2 \times 20 \times 8 = 320 \text{ in of 20-in pipe} \\
\text{Three 20-in ball valves} &= 3 \times 20 \times 3 = 180 \text{ in of 20-in pipe} \\
\text{One 20-in swing check valve} &= 1 \times 20 \times 50 = 1000 \text{ in of 20-in pipe} \\
\text{Four } 90^\circ \text{ elbows} &= 4 \times 20 \times 30 = 2400 \text{ in of 20-in pipe} \\
\text{Total for all valves and fittings} &= 4220 \text{ in of 20-in pipe} \\
&= 351.67 \text{ ft of 20-in pipe}
\end{aligned}$$

Adding the 2000 ft of straight pipe, the total equivalent length of straight pipe and all fittings is

$$L_e = 2000 + 351.67 = 2351.67 \text{ ft}$$

The pressure drop due to friction in the preceding piping system can now be calculated based on 2351.67 ft of pipe. It can be seen in this example that the valves and fittings represent roughly 15 percent of the total pipeline length. In plant piping this percentage may be higher than that in a long-distance water pipeline. Hence, the reason for the term *minor losses*.

Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation (1.23), the pressure drop in a straight length of pipe is given by

$$h = f\frac{L}{D}\frac{V^2}{2g} \tag{1.38}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K (also known as the resistance coefficient) and Eq. (1.38) then becomes

$$h = K\frac{V^2}{2g} \tag{1.39}$$

In Eq. (1.39), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $V^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(V^2/2g)$, where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 1.6. It can be seen that the K factor depends on the

TABLE 1.6 Friction Loss in Valves—Resistance Coefficient *K*

Description	L/D	Nominal pipe size, in											
		$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72

nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

From Table 1.6, it can be seen that a 6-in gate valve has a K factor of 0.12, while a 20-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be $0.12\,(V^2/2g)$ and that in the 20-in valve is $0.10\,(V^2/2g)$. The velocities in both cases will be different due to the difference in diameters.

If the flow rate was 1000 gal/min, the velocity in the 6-in valve will be approximately

$$V_6 = 0.4085\frac{1000}{6.125^2} = 10.89 \text{ ft/s}$$

Similarly, at 1000 gal/min, the velocity in the 20-in valve will be approximately

$$V_6 = 0.4085\frac{1000}{19.5^2} = 1.07 \text{ ft/s}$$

Therefore,

$$\text{Head loss in 6-in gate valve} = \frac{0.12\,(10.89)^2}{64.4} = 0.22 \text{ ft}$$

and

$$\text{Head loss in 20-in gate valve} = \frac{0.10\,(1.07)^2}{64.4} = 0.002 \text{ ft}$$

These head losses appear small since we have used a relatively low flow rate in the 20-in valve. In reality the flow rate in the 20-in valve may be as high as 6000 gal/min and the corresponding head loss will be 0.072 ft.

1.7.2 Pipe enlargement and reduction

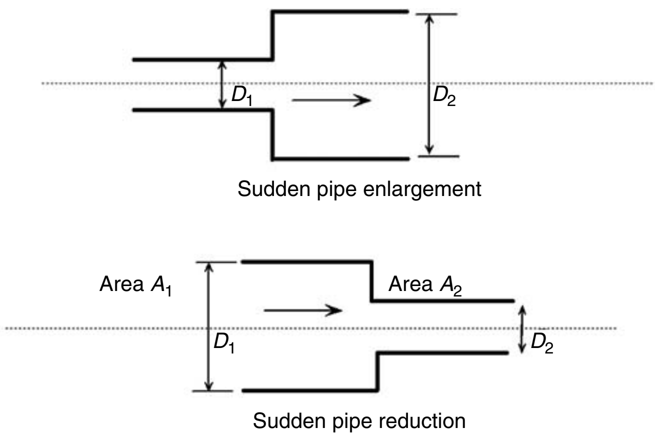
Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(v_1 - v_2)^2}{2g} \tag{1.40}$$

where v_1 and v_2 are the velocities of the liquid in the two pipe sizes D_1 and D_2 respectively. Writing Eq. (1.40) in terms of pipe cross-sectional areas A_1 and A_2,

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \left(\frac{{v_1}^2}{2g}\right) \tag{1.41}$$

for sudden enlargement. This is illustrated in Fig. 1.4.



A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 1.4 Sudden pipe enlargement and reduction.

For sudden contraction or reduction in pipe size as shown in Fig. 1.4, the head loss is calculated from

$$h_f = \left(\frac{1}{C_c} - 1\right)\frac{{v_2}^2}{2g} \tag{1.42}$$

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 1.4.

Gradual enlargement and reduction of pipe size, as shown in Fig. 1.5, cause less head loss than sudden enlargement and sudden reduction. For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c(v_1 - v_2)^2}{2g} \tag{1.43}$$

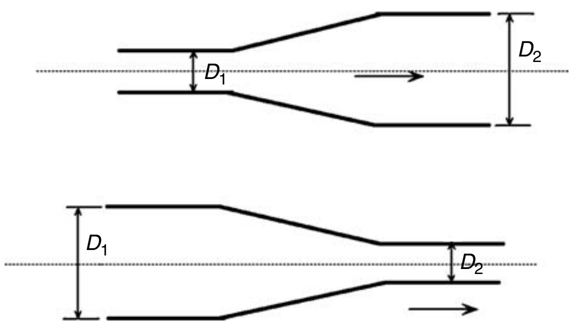


Figure 1.5 Gradual pipe enlargement and reduction.

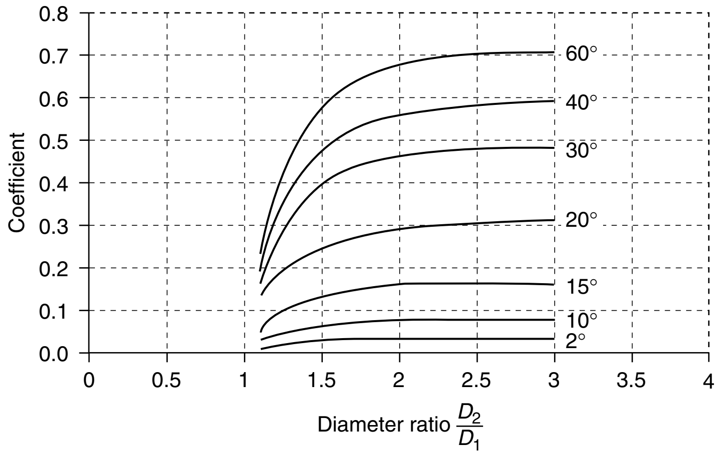


Figure 1.6 Gradual pipe expansion head loss coefficient.

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is shown in Fig. 1.6.

1.7.3 Pipe entrance and exit losses

The K factors for computing the head loss associated with pipe entrance and exit are as follows:

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

1.8 Complex Piping Systems

So far we have discussed straight length of pipe with valves and fittings. Complex piping systems include pipes of different diameters in series and parallel configuration.

1.8.1 Series piping

Series piping in its simplest form consists of two or more different pipe sizes connected end to end as illustrated in Fig. 1.7. Pressure drop calculations in series piping may be handled in one of two ways. The first approach would be to calculate the pressure drop in each pipe size and add them together to obtain the total pressure drop. Another approach is to consider one of the pipe diameters as the base size and convert other pipe sizes into equivalent lengths of the base pipe size. The resultant equivalent lengths are added together to form one long piece

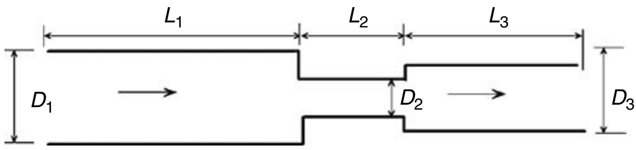


Figure 1.7 Series piping.

of pipe of constant diameter equal to the base diameter selected. The pressure drop can now be calculated for this single-diameter pipeline. Of course, all valves and fittings will also be converted to their respective equivalent pipe lengths using the L/D ratios from Table 1.5.

Consider three sections of pipe joined together in series. Using subscripts 1, 2, and 3 and denoting the pipe length as L, inside diameter as D, flow rate as Q, and velocity as V, we can calculate the equivalent length of each pipe section in terms of a base diameter. This base diameter will be selected as the diameter of the first pipe section D_1. Since equivalent length is based on the same pressure drop in the equivalent pipe as the original pipe diameter, we will calculate the equivalent length of section 2 by finding that length of diameter D_1 that will match the pressure drop in a length L_2 of pipe diameter D_2. Using the Darcy equation and converting velocities in terms of flow rate from Eq. (1.11), we can write

$$\text{Head loss} = \frac{f(L/D)(0.4085Q/D_2)^2}{2g} \tag{1.44}$$

For simplicity, assuming the same friction factor,

$$\frac{L_e}{D_1{}^5} = \frac{L_2}{D_2{}^5} \tag{1.45}$$

Therefore, the equivalent length of section 2 based on diameter D_1 is

$$L_e = L_2\left(\frac{D_1}{D_2}\right)^5 \tag{1.46}$$

Similarly, the equivalent length of section 3 based on diameter D_1 is

$$L_e = L_3\left(\frac{D_1}{D_3}\right)^5 \tag{1.47}$$

The total equivalent length of all three pipe sections based on diameter D_1 is therefore

$$L_t = L_1 + L_2\left(\frac{D_1}{D_2}\right)^5 + L_3\left(\frac{D_1}{D_3}\right)^5 \tag{1.48}$$

The total pressure drop in the three sections of pipe can now be calculated based on a single pipe of diameter D_1 and length L_t.

Example 1.14 Three pipes with 14-, 16-, and 18-in diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

14-in pipeline, 0.250-in wall thickness, 2000 ft long

16-in pipeline, 0.375-in wall thickness, 3000 ft long

18-in pipeline, 0.375-in wall thickness, 5000 ft long

One 16×14 in reducer

One 18×16 in reducer

Two 14-in 90° elbows

Four 16-in 90° elbows

Six 18-in 90° elbows

One 14-in gate valve

One 16-in ball valve

One 18-in gate valve

(a) Use the Hazen-Williams equation with a C factor of 140 to calculate the total pressure drop in the series water piping system at a flow rate of 3500 gal/min. Flow starts in the 14-in piping and ends in the 18-in piping.
(b) If the flow rate is increased to 6000 gal/min, estimate the new total pressure drop in the piping system, keeping everything else the same.

Solution

(a) Since we are going to use the Hazen-Williams equation, the pipes in series analysis will be based on the pressure loss being inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe, per Eq. (1.30).

We will first calculate the total equivalent lengths of all 14-in pipe, fittings, and valves in terms of the 14-in-diameter pipe.

$$\text{Straight pipe: 14 in., 2000 ft} = \text{2000 ft of 14-in pipe}$$

$$\text{Two 14-in 90° elbows} = \frac{2 \times 30 \times 14}{12} = \text{70 ft of 14-in pipe}$$

$$\text{One 14-in gate valve} = \frac{1 \times 8 \times 14}{12} = \text{9.33 ft of 14-in pipe}$$

Therefore, the total equivalent length of 14-in pipe, fittings, and valves = 2079.33 ft of 14-in pipe.

Similarly we get the total equivalent length of 16-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 16-in, 3000 ft} = \text{3000 ft of 16-in pipe}$$

$$\text{Four 16-in 90° elbows} = \frac{4 \times 30 \times 16}{12} = \text{160 ft of 16-in pipe}$$

$$\text{One 16-in ball valve} = \frac{1 \times 3 \times 16}{12} = \text{4 ft of 16-in pipe}$$

Therefore, the total equivalent length of 16-in pipe, fittings, and valve = 3164 ft of 16-in pipe.

Finally, we calculate the total equivalent length of 18-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 18-in, 5000 ft} = 5000 \text{ ft of 18-in pipe}$$

$$\text{Six 18-in } 90^\circ \text{ elbows} = \frac{6 \times 30 \times 18}{12} = 270 \text{ ft of 18-in pipe}$$

$$\text{One 18-in gate valve} = \frac{1 \times 8 \times 18}{12} = 12 \text{ ft of 18-in pipe}$$

Therefore, the total equivalent length of 18-in pipe, fittings, and valve = 5282 ft of 18-in pipe.

Next we convert all the preceding pipe lengths to the equivalent 14-in pipe based on the fact that the pressure loss is inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe.

$$2079.33 \text{ ft of 14-in pipe} = 2079.33 \text{ ft of 14-in pipe}$$

$$3164 \text{ ft of 16-in pipe} = 3164 \times \left(\frac{13.5}{15.25}\right)^{4.87} = 1748 \text{ ft of 14-in pipe}$$

$$5282 \text{ ft of 18-in pipe} = 5282 \times \left(\frac{13.5}{17.25}\right)^{4.87} = 1601 \text{ ft of 14-in pipe}$$

Therefore adding all the preceding lengths we get

$$\text{Total equivalent length in terms of 14-in pipe} = 5429 \text{ ft of 14-in pipe}$$

We still have to account for the 16 × 14 in and 18 × 16 in reducers. The reducers can be considered as sudden enlargements for the approximate calculation of the head loss, using the K factor and velocity head method. For sudden enlargements, the resistance coefficient K is found from

$$K = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \tag{1.49}$$

where d_1 is the smaller diameter and d_2 is the larger diameter.

For the 16 × 14 in reducer,

$$K = \left[1 - \left(\frac{13.5}{15.25}\right)^2\right]^2 = 0.0468$$

and for the 18 × 16 in reducer,

$$K = \left[1 - \left(\frac{15.25}{17.25}\right)^2\right]^2 = 0.0477$$

The head loss through the reducers will then be calculated based on $K(V^2/2g)$.

Flow velocities in the three different pipe sizes at 3500 gal/min will be calculated using Eq. (1.11):

$$\text{Velocity in 14-in pipe: } V_{14} = \frac{0.4085 \times 3500}{(13.5)^2} = 7.85 \text{ ft/s}$$

$$\text{Velocity in 16-in pipe: } V_{16} = \frac{0.4085 \times 3500}{(15.25)^2} = 6.15 \text{ ft/s}$$

$$\text{Velocity in 18-in pipe: } V_{18} = \frac{0.4085 \times 3500}{(17.25)^2} = 4.81 \text{ ft/s}$$

The head loss through the 16 × 14 in reducer is

$$h_1 = 0.0468 \frac{7.85^2}{64.4} = 0.0448 \text{ ft}$$

and the head loss through the 18 × 16 in reducer is

$$h_1 = 0.0477 \frac{6.15^2}{64.4} = 0.028 \text{ ft}$$

These head losses are insignificant and hence can be neglected in comparison with the head loss in straight length of pipe. Therefore, the total head loss in the entire piping system will be based on a total equivalent length of 5429 ft of 14-in pipe.

Using the Hazen-Williams equation (1.32) the pressure drop at 3500 gal/min is

$$h = 10{,}460 \left(\frac{3500}{140}\right)^{1.852} \frac{1.0}{(13.5)^{4.87}} = 12.70 \text{ ft per 1000 ft of pipe}$$

Therefore, for the 5429 ft of equivalent 14-in pipe, the total pressure drop is

$$h = \frac{12.7 \times 5429}{1000} = 68.95 \text{ ft} = \frac{68.95}{2.31} = 29.85 \text{ psi}$$

(b) When the flow rate is increased to 6000 gal/min, we can use proportions to estimate the new total pressure drop in the piping as follows:

$$h = \left(\frac{6000}{3500}\right)^{1.852} \times 12.7 = 34.46 \text{ ft per 1000 ft of pipe}$$

Therefore, the total pressure drop in 5429 ft of 14-in. pipe is

$$h = 34.46 \times \frac{5429}{1000} = 187.09 \text{ ft} = \frac{187.09}{2.31} = 81.0 \text{ psi}$$

Example 1.15 Two pipes with 400- and 600-mm diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

400-mm pipeline, 6-mm wall thickness, 600 m long

600-mm pipeline, 10-mm wall thickness, 1500 m long

One 600 × 400 mm reducer

Two 400-mm 90° elbows

Four 600-mm 90° elbows

One 400-mm gate valve

One 600-mm gate valve

Use the Hazen-Williams equation with a C factor of 120 to calculate the total pressure drop in the series water piping system at a flow rate of 250 L/s. What will the pressure drop be if the flow rate were increased to 350 L/s?

Solution The total equivalent length on 400-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 600 \text{ m}$$

$$\text{Two } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 400}{1000} = 24 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 400}{1000} = 3.2 \text{ m}$$

Thus,

$$\text{Total equivalent length on 400-mm-diameter pipe} = 627.2 \text{ m}$$

The total equivalent length on 600-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 1500 \text{ m}$$

$$\text{Four } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 600}{1000} = 72 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 600}{1000} = 4.8 \text{ m}$$

Thus,

$$\text{Total equivalent length on 600-mm-diameter pipe} = 1576.8 \text{ m}$$

Reducers will be neglected since they have insignificant head loss. Convert all pipe to 400-mm equivalent diameter.

$$\begin{aligned} 1576.8 \text{ m of 600-mm pipe} &= 1576.8 \left(\frac{388}{580}\right)^{4.87} \\ &= 222.6 \text{ m of 400-mm pipe} \end{aligned}$$

$$\text{Total equivalent length on 400-mm-diameter pipe} = 627.2 + 222.6 = 849.8 \text{ m}$$

$$Q = 250 \times 10^{-3} \times 3600 = 900 \text{ m}^3\text{/h}$$

The pressure drop from Eq. (1.35) is

$$\begin{aligned} P_m &= 1.1101 \times 10^{13} \left(\frac{900}{120}\right)^{1.852} \frac{1}{(388)^{4.87}} \\ &= 114.38 \text{ kPa/km} \end{aligned}$$

$$\text{Total pressure drop} = \frac{114.38 \times 849.8}{1000} = 97.2 \text{ kPa}$$

When the flow rate is increased to 350 L/s, we can calculate the pressure drop using proportions as follows:

$$\text{Revised head loss at 350 L/s} = \left(\frac{350}{250}\right)^{1.852} \times 114.38 = 213.3 \text{ kPa/km}$$

Therefore,

$$\text{Total pressure drop} = 213.3 \times 0.8498 = 181.3 \text{ kPa}$$

1.8.2 Parallel piping

Water pipes in parallel are set up such that the multiple pipes are connected so that water flow splits into the multiple pipes at the beginning and the separate flow streams subsequently rejoin downstream into another single pipe as depicted in Fig. 1.8.

Figure 1.8 shows a parallel piping system in the horizontal plane with no change in pipe elevations. Water flows through a single pipe *AB*, and at the junction *B* the flow splits into two pipe branches *BCE* and *BDE*. At the downstream end at junction *E*, the flows rejoin to the initial flow rate and subsequently flow through the single pipe *EF*.

To calculate the flow rates and pressure drop due to friction in the parallel piping system, shown in Fig. 1.8, two main principles of parallel piping must be followed. These are flow conservation at any junction point and common pressure drop across each parallel branch pipe.

Based on flow conservation, at each junction point of the pipeline, the incoming flow must exactly equal the total outflow. Therefore, at junction *B*, the flow *Q* entering the junction must exactly equal the sum of the flow rates in branches *BCE* and *BDE*.

Thus,

$$Q = Q_{BCE} + Q_{BDE} \tag{1.50}$$

where Q_{BCE} = flow through branch *BCE*
Q_{BDE} = flow through branch *BDE*
Q = incoming flow at junction *B*

The other requirement in parallel pipes concerns the pressure drop in each branch piping. Based on this the pressure drop due to friction

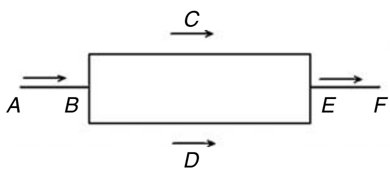


Figure 1.8 Parallel piping.

in branch *BCE* must exactly equal that in branch *BDE*. This is because both branches have a common starting point (B) and a common ending point (E). Since the pressure at each of these two points is a unique value, we can conclude that the pressure drop in branch pipe *BCE* and that in branch pipe *BDE* are both equal to $P_B - P_E$, where P_B and P_E represent the pressure at the junction points B and E, respectively.

Another approach to calculating the pressure drop in parallel piping is the use of an equivalent diameter for the parallel pipes. For example in Fig. 1.8, if pipe *AB* has a diameter of 14 in and branches *BCE* and *BDE* have diameters of 10 and 12 in, respectively, we can find some equivalent diameter pipe of the same length as one of the branches that will have the same pressure drop between points B and C as the two branches. An approximate equivalent diameter can be calculated using the Darcy equation.

The pressure loss in branch *BCE* (10-in diameter) can be calculated as

$$h_1 = \frac{f(L_1/D_1){V_1}^2}{2g} \tag{1.51}$$

where the subscript 1 is used for branch *BCE* and subscript 2 for branch *BDE*.

Similarly, for branch *BDE*

$$h_2 = \frac{f(L_2/D_2){V_2}^2}{2g} \tag{1.52}$$

For simplicity we have assumed the same friction factors for both branches. Since h_1 and h_2 are equal for parallel pipes, and representing the velocities V_1 and V_2 in terms of the respective flow rates Q_1 and Q_2, using Eq. (1.23) we have the following equations:

$$\frac{f(L_1/D_1){V_1}^2}{2g} = \frac{f(L_2/D_2){V_2}^2}{2g} \tag{1.53}$$

$$V_1 = 0.4085\frac{Q_1}{{D_1}^2} \tag{1.54}$$

$$V_2 = 0.4085\frac{Q_2}{{D_2}^2} \tag{1.55}$$

In these equations we are assuming flow rates in gal/min and diameters in inches.

Simplifying Eqs. (1.53) to (1.55), we get

$$\frac{L_1}{D_1}\left(\frac{Q_1}{{D_1}^2}\right)^2 = \frac{L_2}{D_2}\left(\frac{Q_2}{{D_2}^2}\right)^2$$

or

$$\frac{Q_1}{Q_2} = \left(\frac{L_2}{L_1}\right)^{0.5} \left(\frac{D_1}{D_2}\right)^{2.5} \tag{1.56}$$

Also by conservation of flow

$$Q_1 + Q_2 = Q \tag{1.57}$$

Using Eqs. (1.56) and (1.57), we can calculate the flow through each branch in terms of the inlet flow Q. The equivalent pipe will be designated as D_e in diameter and L_e in length. Since the equivalent pipe will have the same pressure drop as each of the two branches, we can write

$$\frac{L_e}{D_e}\left(\frac{Q_e}{D_e{}^2}\right)^2 = \frac{L_1}{D_1}\left(\frac{Q_1}{D_1{}^2}\right)^2 \tag{1.58}$$

where Q_e is the same as the inlet flow Q since both branches have been replaced with a single pipe. In Eq. (1.58), there are two unknowns L_e and D_e. Another equation is needed to solve for both variables. For simplicity, we can set L_e to be equal to one of the lengths L_1 or L_2. With this assumption, we can solve for the equivalent diameter D_e as follows:

$$D_e = D_1\left(\frac{Q}{Q_1}\right)^{0.4} \tag{1.59}$$

Example 1.16 A 10-in water pipeline consists of a 2000-ft section of NPS 12 pipe (0.250-in wall thickness) starting at point A and terminating at point B. At point B, two pieces of pipe (4000 ft long each and NPS 10 pipe with 0.250-in wall thickness) are connected in parallel and rejoin at a point D. From D, 3000 ft of NPS 14 pipe (0.250-in wall thickness) extends to point E. Using the equivalent diameter method calculate the pressures and flow rate throughout the system when transporting water at 2500 gal/min. Compare the results by calculating the pressures and flow rates in each branch. Use the Colebrook-White equation for the friction factor.

Solution Since the pipe loops between B and D are each NPS 10 and 4000 ft long, the flow will be equally split between the two branches. Each branch pipe will carry 1250 gal/min.

The equivalent diameter for section BD is found from Eq. (1.59):

$$D_e = D_1\left(\frac{Q}{Q_1}\right)^{0.4} = 10.25 \times (2)^{0.4} = 13.525 \text{ in}$$

Therefore we can replace the two 4000-ft NPS 10 pipes between B and D with a single pipe that is 4000 ft long and has a 13.525-in inside diameter.

The Reynolds number for this pipe at 2500 gal/min is found from Eq. (1.15):

$$R = \frac{3162.5 \times 2500}{13.525 \times 1.0} = 584{,}566$$

Considering that the pipe roughness is 0.002 in for all pipes:

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.525} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section BD is [using Eq. (1.24)]

$$P_m = 71.16 \frac{fQ^2}{D^5}$$

$$= 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(13.525)^5} = 14.45 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } BD = \frac{14.45 \times 4000}{5280} = 10.95 \text{ psi}$$

For section AB we have,

$$R = \frac{3162.5 \times 2500}{12.25 \times 1.0} = 645{,}408$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{12.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section AB is [using Eq. (1.24)]

$$P_m = 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(12.25)^5} = 22.66 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } AB = \frac{22.66 \times 2000}{5280} = 8.58 \text{ psi}$$

Finally, for section DE we have,

$$R = \frac{3162.5 \times 2500}{13.5 \times 1.0} = 585{,}648$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.5} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section DE is

$$P_m = 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(13.5)^5} = 14.58 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } DE = \frac{14.58 \times 3000}{5280} = 8.28 \text{ psi}$$

Finally,

$$\text{Total pressure drop in entire piping system} = 8.58 + 10.95 + 8.28$$
$$= 27.81 \text{ psi}$$

Next for comparison we will analyze the branch pressure drops considering each branch separately flowing at 1250 gal/min.

$$R = \frac{3162.5 \times 1250}{10.25 \times 1.0} = 385{,}671$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{10.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0158$. The pressure drop in section BD is [using Eq. (1.24)]

$$P_m = 71.16 \frac{0.0158 \times (1250)^2 \times 1}{(10.25)^5} = 15.53 \text{ psi/mi}$$

This compares with the pressure drop of 14.45 psi/mi we calculated using an equivalent diameter of 13.525. It can be seen that the difference between the two pressure drops is approximately 7.5 percent.

Example 1.17 A waterline 5000 m long is composed of three sections A, B, and C. Section A has a 200-m inside diameter and is 1500 m long. Section C has a 400-mm inside diameter and is 2000 m long. The middle section B consists of two parallel pipes each 3000 m long. One of the parallel pipes has a 150-mm inside diameter and the other has a 200-mm inside diameter. Assume no elevation change throughout. Calculate the pressures and flow rates in this piping system at a flow rate of 500 m^3/h, using the Hazen-Williams formula with a C factor of 1.20.

Solution We will replace the two 3000-m pipe branches in section B with a single equivalent diameter pipe to be determined. Since the pressure drop according to the Hazen-Williams equation is inversely proportional to the 4.87 power of the pipe diameter, we calculate the equivalent diameter for section B as follows:

$$\frac{Q_e{}^{1.852}}{D_e{}^{4.87}} = \frac{Q_1{}^{1.852}}{D_1{}^{4.87}} = \frac{Q_2{}^{1.852}}{D_2{}^{4.87}}$$

Therefore,

$$\frac{D_e}{D_1} = \left(\frac{Q_e}{Q_1}\right)^{0.3803}$$

Also $Q_e = Q_1 + Q_2$ and

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^{2.63} = \left(\frac{150}{200}\right)^{2.63} = 0.4693$$

Solving for Q_1 and Q_2, with $Q_e = 500$, we get

$$Q_1 = 159.7\text{m}^3/\text{hr} \qquad \text{and} \qquad Q_2 = 340.3\text{m}^3/\text{h}$$

Therefore, the equivalent diameter is

$$D_e = D_1\left(\frac{Q_e}{Q_1}\right)^{0.3803} = 150 \times \left(\frac{500}{159.7}\right)^{0.3803} = 231.52 \text{ mm}$$

The pressure drop in section A, using Hazen-Williams equation (1.35), is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(200)^{4.87}} = 970.95 \text{ kPa/km}$$

$$\Delta P_a = 970.95 \times 1.5 = 1456.43 \text{ kPa}$$

The pressure drop in section B, using Hazen-Williams equation, is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(231.52)^{4.87}} = 476.07 \text{ kPa/km}$$

$$\Delta P_b = 476.07 \times 3.0 = 1428.2 \text{ kPa}$$

The pressure drop in section C, using Hazen-Williams equation, is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(400)^{4.87}} = 33.20 \text{ kPa/km}$$

$$\Delta P_c = 33.2 \times 2.0 = 66.41 \text{ kPa}$$

Therefore,

$$\begin{aligned}\text{Total pressure drop of sections A, B, and C} &= 1456.43 + 1428.20 + 66.41 \\ &= 2951.04 \text{ kPa}\end{aligned}$$

1.9 Total Pressure Required

So far we have examined the frictional pressure drop in water systems piping consisting of pipe, fittings, valves, etc. We also calculated the total pressure required to pump water through a pipeline up to a delivery station at an elevated point. The total pressure required at the beginning of a pipeline, for a specified flow rate, consists of three distinct components:

1. Frictional pressure drop
2. Elevation head
3. Delivery pressure

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (1.29)}$$

The first item is simply the total frictional head loss in all straight pipe, fittings, valves, etc. The second item accounts for the pipeline elevation difference between the origin of the pipeline and the delivery terminus. If the origin of the pipeline is at a lower elevation than that of the pipeline terminus or delivery point, a certain amount of positive pressure is required to compensate for the elevation difference. On the other hand, if the delivery point were at a lower elevation than the beginning of the pipeline, gravity will assist the flow and the pressure required at the beginning of the pipeline will be reduced by this elevation difference. The third component, delivery pressure at the terminus, simply ensures that a certain minimum pressure is maintained at the delivery point, such as a storage tank.

For example, if a water pipeline requires 800 psi to take care of frictional losses and the minimum delivery pressure required is 25 psi, the total pressure required at the beginning of the pipeline is calculated as follows. If there were no elevation difference between the beginning of the pipeline and the delivery point, the elevation head (component 2) is zero. Therefore, the total pressure P_t required is

$$P_t = 800 + 0 + 25 = 825 \text{ psi}$$

Next consider elevation changes. If the elevation at the beginning is 100 ft and the elevation at the delivery point is 500 ft, then

$$P_t = 800 + \frac{(500 - 100) \times 1.0}{2.31} + 25 = 998.16 \text{ psi}$$

The middle term in this equation represents the static elevation head difference converted to psi. Finally, if the elevation at the beginning is 500 ft and the elevation at the delivery point is 100 ft, then

$$P_t = 800 + \frac{(100 - 500) \times 1.0}{2.31} + 25 = 651.84 \text{ psi}$$

It can be seen from the preceding that the 400-ft advantage in elevation in the final case reduces the total pressure required by approximately 173 psi compared to the situation where there was no elevation difference between the beginning of the pipeline and delivery point.

1.9.1 Effect of elevation

The preceding discussion illustrated a water pipeline that had a flat elevation profile compared to an uphill pipeline and a downhill pipeline. There are situations, where the ground elevation may have drastic peaks and valleys, that require careful consideration of the pipeline topography. In some instances, the total pressure required to transport

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a given volume of water through a long pipeline may depend more on the ground elevation profile than the actual frictional pressure drop. In the preceding we calculated the total pressure required for a flat pipeline as 825 psi and an uphill pipeline to be 998 psi. In the uphill case the static elevation difference contributed to 17 percent of the total pressure required. Thus the frictional component was much higher than the elevation component. We will examine a case where the elevation differences in a long pipeline dictate the total pressure required more than the frictional head loss.

Example 1.18 A 20-in (0.375-in wall thickness) water pipeline 500 mi long has a ground elevation profile as shown in Fig. 1.9. The elevation at Corona is 600 ft and at Red Mesa is 2350 ft. Calculate the total pressure required at the Corona pump station to transport 11.5 Mgal/day of water to Red Mesa storage tanks, assuming a minimum delivery pressure of 50 psi at Red Mesa. Use the Hazen-Williams equation with a C factor of 140. If the pipeline operating pressure cannot exceed 1400 psi, how many pumping stations, besides Corona, will be required to transport the given flow rate?

Solution The flow rate Q in gal/min is

$$Q = \frac{11.5 \times 10^6}{24 \times 60} = 7986.11 \text{ gal/min}$$

If P_m is the head loss in psi/mi of pipe, using the Hazen-Williams equation (1.33),

$$P_m = 23{,}909 \left(\frac{7986.11}{140}\right)^{1.852} \frac{1}{19.25^{4.87}} = 23.76 \text{ psi/mi}$$

Therefore,

$$\text{Frictional pressure drop} = 23.76 \text{ psi/mi}$$

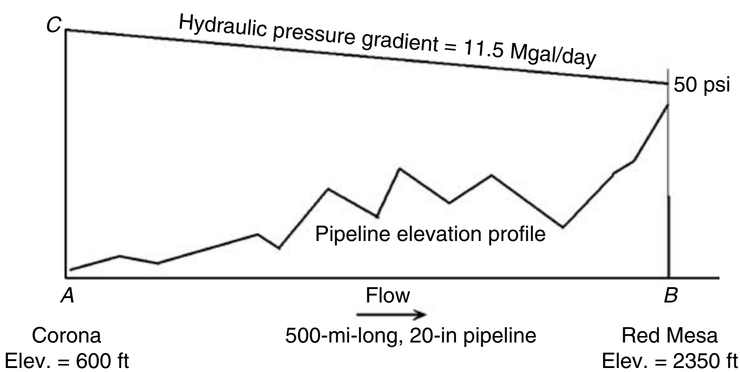

Figure 1.9 Corona to Red Mesa pipeline.

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The total pressure required at Corona is calculated by adding the pressure drop due to friction to the delivery pressure required at Red Mesa and the static elevation head between Corona and Red Mesa.

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (1.29)}$$

$$= (23.76 \times 500) + \frac{2350 - 600}{2.31} + 50$$

$$= 11{,}880 + 757.58 + 50 = 12{,}688 \text{ psi} \qquad \text{rounded off to the nearest psi}$$

Since a total pressure of 12,688 psi at Corona far exceeds the maximum operating pressure of 1400 psi, it is clear that we need additional intermediate booster pump stations besides Corona. The approximate number of pump stations required without exceeding the pipeline pressure of 1400 psi is

$$\text{Number of pump stations} = \frac{12{,}688}{1400} = 9.06 \text{ or } 10 \text{ pump stations}$$

With 10 pump stations the average pressure per pump station will be

$$\text{Average pump station pressure} = \frac{12{,}688}{10} = 1269 \text{ psi}$$

1.9.2 Tight line operation

When there are drastic elevation differences in a long pipeline, sometimes the last section of the pipeline toward the delivery terminus may operate in an open-channel flow. This means that the pipeline section will not be full of water and there will be a vapor space above the water. Such situations are acceptable in water pipelines compared to high vapor pressure liquids such as liquefied petroleum gas (LPG). To prevent such open-channel flow or slack line conditions, we pack the line by providing adequate back pressure at the delivery terminus as illustrated in Fig. 1.10.

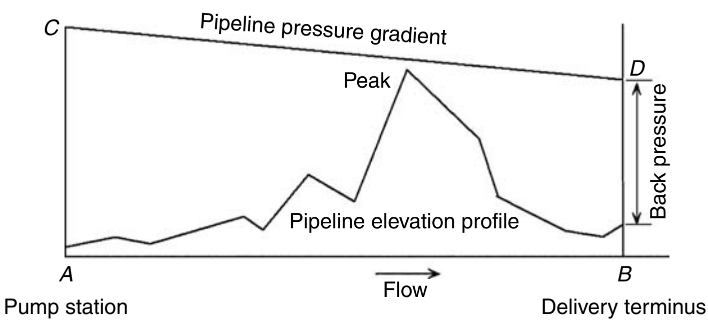

Figure 1.10 Tight line operation.

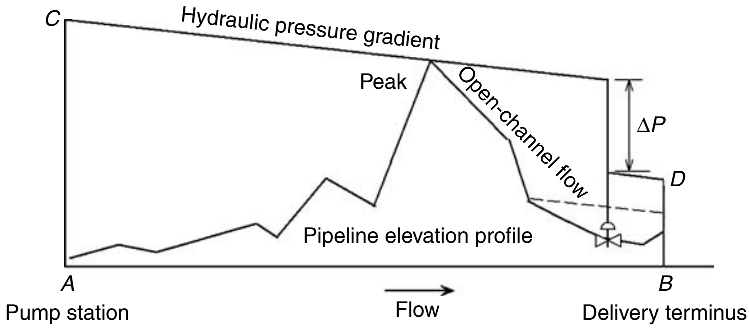

Figure 1.11 Slack line flow.

1.9.3 Slack line flow

Slack line or open-channel flow occurs in the last segment of a long-distance water pipeline where a large elevation difference exists between the delivery terminus and intermediate point in the pipeline as indicated in Fig. 1.11.

If the pipeline were packed to avoid slack line flow, the hydraulic gradient is as shown by the solid line in Fig. 1.11. However, the piping system at the delivery terminal may not be able to handle the higher pressure due to line pack. Therefore, we may have to reduce the pressure at some point within the delivery terminal using a pressure control valve. This is illustrated in Fig. 1.11.

1.10 Hydraulic Gradient

The graphical representation of the pressures along the pipeline, as shown in Fig. 1.12, is called the hydraulic pressure gradient. Since elevation is measured in feet, the pipeline pressures are converted to feet of head and plotted against the distance along the pipeline superimposed

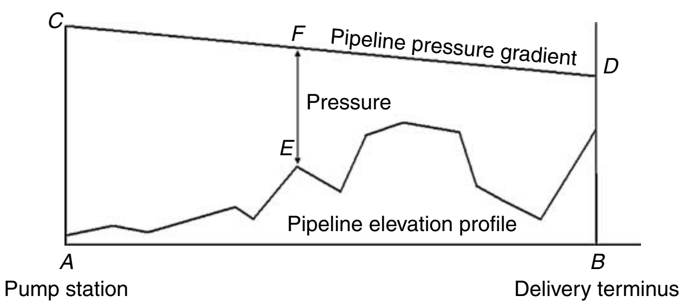

Figure 1.12 Hydraulic pressure gradient.

on the elevation profile. If we assume a beginning elevation of 100 ft, a delivery terminus elevation of 500 ft, a total pressure of 1000 psi required at the beginning, and a delivery pressure of 25 psi at the terminus, we can plot the hydraulic pressure gradient graphically by the following method.

At the beginning of the pipeline the point C representing the total pressure will be plotted at a height of

$$100 \text{ ft} + (1000 \times 2.31) = 2410 \text{ ft}$$

Similarly, at the delivery terminus the point D representing the total head at delivery will be plotted at a height of

$$500 + (25 \times 2.31) = 558 \text{ ft} \qquad \text{rounded off to the nearest foot}$$

The line connecting the points C and D represents the variation of the total head in the pipeline and is termed the *hydraulic gradient*. At any intermediate point such as E along the pipeline the pipeline pressure will be the difference between the total head represented by point F on the hydraulic gradient and the actual elevation of the pipeline at E.

If the total head at F is 1850 ft and the pipeline elevation at E is 250 ft, the actual pipeline pressure at E is

$$(1850 - 250)\text{ft} = \frac{1600}{2.31} = 693 \text{ psi}$$

It can be seen that the hydraulic gradient clears all peaks along the pipeline. If the elevation at E were 2000 ft, we would have a negative pressure in the pipeline at E equivalent to

$$(1850 - 2000)\text{ft} = -150 \text{ ft} = -\frac{150}{2.31} = -65 \text{ psi}$$

Since a negative pressure is not acceptable, the total pressure at the beginning of the pipeline will have to be higher by the preceding amount.

$$\text{Revised total head at } A = 2410 + 150 = 2560 \text{ ft}$$

This will result in zero gauge pressure in the pipeline at peak E. The actual pressure in the pipeline will therefore be equal to the atmospheric pressure at that location. Since we would like to always maintain some positive pressure above the atmospheric pressure, in this case the total head at A must be slightly higher than 2560 ft. Assuming a 10-psi positive pressure is desired at the highest peak such as E (2000-ft elevation), the revised total pressure at A would be

$$\text{Total pressure at } A = 1000 + 65 + 10 = 1075 \text{ psi}$$

Therefore,

$$\text{Total head at } C = 100 + (1075 \times 2.31) = 2483 \text{ ft}$$

This will ensure a positive pressure of 10 psi at the peak E.

1.11 Gravity Flow

Gravity flow in a water pipeline occurs when water flows from a source at point A at a higher elevation than the delivery point B, without any pumping pressure at A and purely under gravity. This is illustrated in Fig. 1.13.

The volume flow rate under gravity flow for the reservoir pipe system shown in Fig. 1.13 can be calculated as follows. If the head loss in the pipeline is h ft/ft of pipe length, the total head loss in length L is $(h \times L)$. Since the available driving force is the difference in tank levels at A and B, we can write

$$H_1 - (h \times L) = H_2 \tag{1.60}$$

Therefore,

$$hL = H_1 - H_2 \tag{1.61}$$

and

$$h = \frac{H_1 - H_2}{L} \tag{1.62}$$

where h = head loss in pipe, ft/ft
L = length of pipe
H_1 = head in tank A
H_2 = head in tank B

In the preceding analysis, we have neglected the entrance and exit losses at A and B. Using the Hazen-Williams equation we can then calculate flow rate based on a C value.

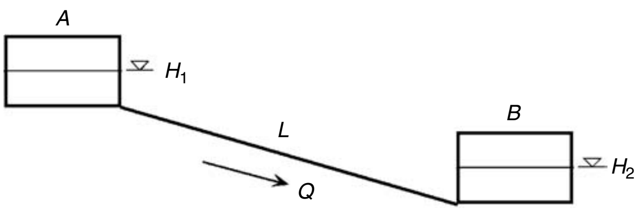

Figure 1.13 Gravity flow from reservoir.

Example 1.19 The gravity feed system shown in Fig. 1.13 consists of a 16-inch (0.250-in wall thickness) 3000-ft-long pipeline, with a tank elevation at $A = 500$ ft and elevation at $B = 150$ ft. Calculate the flow rate through this gravity flow system. Use a Hazen-Williams C factor of 130.

Solution

$$h = \frac{500 - 150}{3000} = 0.1167 \text{ ft/ft}$$

Substituting in Hazen-Williams equation (1.32), we get

$$0.1167 \times 1000 = 10{,}460 \times \left(\frac{Q}{130}\right)^{1.852} \left(\frac{1}{15.5}\right)^{4.87}$$

Solving for flow rate Q,

$$Q = 15{,}484 \text{ gal/min}$$

Compare the results using the Colebrook-White equation assuming $e = 0.002$.

$$\frac{e}{D} = \frac{0.002}{15.5} = 0.0001$$

We will assume a friction factor $f = 0.02$ initially. Head loss due to friction per Eq. (1.24) is

$$P_m = 71.16 \times \frac{0.02(Q^2)}{(15.5)^5} \text{ psi/mi}$$

or

$$\begin{aligned} P_m &= 1.5908 \times 10^{-6} Q^2 \quad \text{psi/mi} \\ &= \left(1.5908 \times 10^{-6}\, \frac{2.31}{5280}\right) Q^2 \quad \text{ft/ft} \\ &= (6.9596 \times 10^{-10}) Q^2 \quad \text{ft/ft} \\ 0.1167 &= (6.9596 \times 10^{-10}) Q^2 \end{aligned}$$

Solving for flow rate Q, we get

$$Q = 12{,}949 \text{ gal/min}$$

Solving for the Reynolds number, we get

$$\text{Re} = 3162.5 \times \frac{12{,}949}{15.5} \times 1 = 2{,}642{,}053$$

From the Moody diagram, $f = 0.0128$. Now we recalculate P_m,

$$\begin{aligned} P_m &= 71.16 \times 0.0128 \times \frac{Q^2}{(15.5)^5} \text{ psi/mi} \\ &= 4.4541 \times 10^{-10} Q^2 \quad \text{ft/ft} \end{aligned}$$

Solving for Q again,

$$Q = 16{,}186 \text{ gal/min}$$

By successive iteration we arrive at the final flow rate of 16,379 gal/min using the Colebrook-White equation. Comparing this with 15,484 gal/min obtained using the Hazen-Williams equation, we see that the flow rate is underestimated probably because the assumed Hazen-Williams C factor ($C = 130$) was too low.

Example 1.20 The two-reservoir system described in Fig. 1.13 is modified to include a second source of water from a tank located at C between the two tanks located at A and B and away from the pipeline AB. The tank at C is at an elevation of 300 ft and connects to the piping from A to B via a new 16-inch, 1000-ft-long pipe CD. The common junction D is located along the pipe AB at a distance of 1500 ft from the tank at B. Determine the flow rates Q_1 from A to D, Q_2 from C to D, and Q_3 from D to B. Use the Hazen-Williams equation with $C = 130$.

Solution At the common junction D we can apply the conservation of flow principle as follows:

$$Q_1 + Q_2 = Q_3$$

Also since D is a common junction, the head H_D at point D is common to the three legs AD, CD, and DB. Designating the head loss due to friction in the respective pipe segments AD, CD, and DB as h_{fAD}, h_{fCD}, and h_{fDB}, we can write the following pressure balance equations for the three pipe legs.

$$H_D = H_A - h_{fAD}$$

$$H_D = H_C - h_{fCD}$$

$$H_D = H_B + h_{fDB}$$

Since the pipe sizes are all 16 in and the C factor is 130, using the Hazen-Williams equation (1.32) we can write

$$h_{fAD} = 10{,}460 \times \frac{L_{AD}}{1000}\left(\frac{Q_1}{130}\right)^{1.852}\left(\frac{1}{15.5}\right)^{4.87} = KL_{AD} \times Q_1{}^{1.852}$$

where K is a constant for all pipes and is equal to

$$K = 10{,}460 \times \frac{1}{1000}\left(\frac{1}{130}\right)^{1.852}\left(\frac{1}{15.5}\right)^{4.87} = 2.0305 \times 10^{-9}$$

and

$$L_{AD} = \text{length of pipe from } A \text{ to } D = 1500 \text{ ft}$$

Similarly, we can write

$$h_{fCD} = KL_{CD} \times Q_2{}^{1.852}$$

and for leg DB

$$h_{fDB} = KL_{DB} \times Q_3^{1.852}$$

Substituting the values in the preceding H_D equations, we get

$$H_D = 500 - K \times 1500 \times Q_1^{1.852}$$

$$H_D = 300 - K \times 1000 \times Q_2^{1.852}$$

$$H_D = 150 + K \times 1000 \times Q_3^{1.852}$$

Simplifying these equations by eliminating H_D, we get the following two equations:

$$1.5Q_1^{1.852} - Q_2^{1.852} = \frac{0.2}{K} \qquad (A)$$

$$1.5Q_1^{1.852} + Q_3^{1.852} = \frac{0.35}{K} \qquad (B)$$

Also

$$Q_1 + Q_2 = Q_3 \qquad (C)$$

Solving for the three flow rates we get,

$$Q_1 = 16{,}677 \qquad Q_2 = 1000 \qquad \text{and} \qquad Q_3 = 17{,}677$$

1.12 Pumping Horsepower

In the previous sections we calculated the total pressure required at the beginning of the pipeline to transport a given volume of water over a certain distance. We will now calculate the pumping horsepower (HP) required to accomplish this.

Consider Example 1.18 in which we calculated the total pressure required to pump 11.5 Mgal/day of water from Corona to Red Mesa through a 500-mi-long, 20-in pipeline. We calculated the total pressure required to be 12,688 psi. Since the maximum allowable working pressure in the pipeline was limited to 1400 psi, we concluded that nine additional pump stations besides Corona were required. With a total of 10 pump stations, each pump station would be discharging at a pressure of approximately 1269 psi.

At the Corona pump station, water would enter the pump at some minimum pressure, say 50 psi and the pumps would boost the pressure to the required discharge pressure of 1269 psi. Effectively, the pumps would add the energy equivalent of 1269 – 50, or 1219 psi at a flow rate of 11.5 Mgal/day (7986.11 gal/min). The water horsepower (WHP) required is calculated as

$$\text{WHP} = \frac{(1219 \times 2.31) \times 7986.11 \times 1.0}{3960} = 5679 \text{ HP}$$

The general equation used to calculate WHP, also known as hydraulic horsepower (HHP), is as follows:

$$\text{WHP} = \frac{\text{ft of head} \times (\text{gal/min}) \times \text{specific gravity}}{3960} \tag{1.63}$$

Assuming a pump efficiency of 80 percent, the pump brake horsepower (BHP) required is

$$\text{BHP} = \frac{5679}{0.8} = 7099 \text{ HP}$$

The general equation for calculating the BHP of a pump is

$$\text{BHP} = \frac{\text{ft of head} \times (\text{gal/min}) \times (\text{specific gravity})}{3960 \times \text{effy}} \tag{1.64}$$

where effy is the pump efficiency expressed as a decimal value.

If the pump is driven by an electric motor with a motor efficiency of 95 percent, the drive motor HP required will be

$$\text{Motor HP} = \frac{7099}{0.95} = 7473 \text{ HP}$$

The nearest standard size motor of 8000 HP would be adequate for this application. Of course this assumes that the entire pumping requirement at the Corona pump station is handled by a single pump-motor unit. In reality, to provide for operational flexibility and maintenance two or more pumps will be configured in series or parallel configurations to provide the necessary pressure at the specified flow rate. Let us assume that two pumps are configured in parallel to provide the necessary head pressure of 1219 psi (2816 ft) at the Corona pump station. Each pump will be designed for one-half the total flow rate (7986.11) or 3993 gal/min and a head pressure of 2816 ft. If the pumps selected had an efficiency of 80 percent, we can calculate the BHP required for each pump as follows:

$$\begin{aligned}\text{BHP} &= \frac{2816 \times 3993 \times 1.0}{3960 \times 0.80} \qquad \text{from Eq. (1.64)}\\ &= 3550 \text{ HP}\end{aligned}$$

Alternatively, if the pumps were configured in series instead of parallel, each pump will be designed for the full flow rate of 7986.11 gal/min but at half the total pressure required, or 1408 ft. The BHP required per pump will still be the same as determined by the preceding equation. Pumps are discussed in more detail in Sec. 1.13.

1.13 Pumps

Pumps are installed on water pipelines to provide the necessary pressure at the beginning of the pipeline to compensate for pipe friction and any elevation head and provide the necessary delivery pressure at the pipeline terminus. Pumps used on water pipelines are either positive displacement (PD) type or centrifugal pumps.

PD pumps generally have higher efficiency, higher maintenance cost, and a fixed volume flow rate at any pressure within allowable limits. Centrifugal pumps on the other hand are more flexible in terms of flow rates but have lower efficiency and lower operating and maintenance cost. The majority of liquid pipelines today are driven by centrifugal pumps.

Since pumps are designed to produce pressure at a given flow rate, an important characteristic of a pump is its performance curve. The performance curve is a graphic representation of how the pressure generated by a pump varies with its flow rate. Other parameters, such as efficiency and horsepower, are also considered as part of a pump performance curve.

1.13.1 Positive displacement pumps

Positive displacement (PD) pumps include piston pumps, gear pumps, and screw pumps. These are used generally in applications where a constant volume of liquid must be pumped against a fixed or variable pressure.

PD pumps can effectively generate any amount of pressure at the fixed flow rate, which depends on its geometry, as long as equipment pressure limits are not exceeded. Since a PD pump can generate any pressure required, we must ensure that proper pressure control devices are installed to prevent rupture of the piping on the discharge side of the PD pump. As indicated earlier, PD pumps have less flexibility with flow rates and higher maintenance cost. Because of these reasons, PD pumps are not popular in long-distance and distribution water pipelines. Centrifugal pumps are preferred due to their flexibility and low operating cost.

1.13.2 Centrifugal pumps

Centrifugal pumps consist of one or more rotating impellers contained in a casing. The centrifugal force of rotation generates the pressure in the liquid as it goes from the suction side to the discharge side of the pump. Centrifugal pumps have a wide range of operating flow rates with fairly good efficiency. The operating and maintenance cost of a centrifugal pump is lower than that of a PD pump. The performance

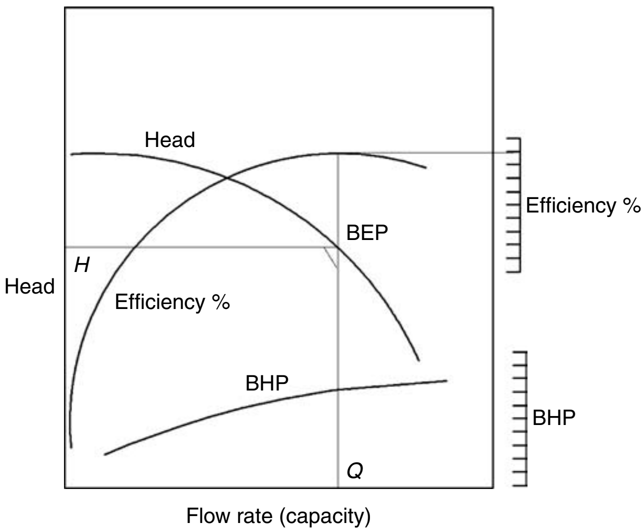

Figure 1.14 Performance curve for centrifugal pump.

curves of a centrifugal pump consist of head versus capacity, efficiency versus capacity, and BHP versus capacity. The term *capacity* is used synonymously with flow rate in connection with centrifugal pumps. Also the term *head* is used in preference to pressure when dealing with centrifugal pumps. Figure 1.14 shows a typical performance curve for a centrifugal pump.

Generally, the head-capacity curve of a centrifugal pump is a drooping curve. The highest head is generated at zero flow rate (shutoff head) and the head decreases with an increase in the flow rate as shown in Fig. 1.14. The efficiency increases with flow rate up to the best efficiency point (BEP) after which the efficiency drops off. The BHP calculated using Eq. (1.64) also generally increases with flow rate but may taper off or start decreasing at some point depending on the head-capacity curve.

The head generated by a centrifugal pump depends on the diameter of the pump impeller and the speed at which the impeller runs. The affinity laws of centrifugal pumps may be used to determine pump performance at different impeller diameters and pump speeds. These laws can be mathematically stated as follows:
For impeller diameter change:

Flow rate: $$\frac{Q_1}{Q_2} = \frac{D_1}{D_2} \tag{1.65}$$

Head: $$\frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \tag{1.66}$$

BHP: $$\frac{\text{BHP}_1}{\text{BHP}_2} = \left(\frac{D_1}{D_2}\right)^3 \tag{1.67}$$

For impeller speed change:

Flow rates: $$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \tag{1.68}$$

Heads: $$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{1.69}$$

BHP: $$\frac{\text{BHP}_1}{\text{BHP}_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{1.70}$$

where subscript 1 refers to initial conditions and subscript 2 to final conditions. It must be noted that the affinity laws for impeller diameter change are accurate only for small changes in diameter. However, the affinity laws for impeller speed change are accurate for a wide range of impeller speeds.

Using the affinity laws if the performance of a centrifugal pump is known at a particular diameter, the corresponding performance at a slightly smaller diameter or slightly larger diameter can be calculated very easily. Similarly, if the pump performance for a 10-in impeller at 3500 revolutions per minute (r/min) impeller speed is known, we can easily calculate the performance of the same pump at 4000 r/min.

Example 1.21 The performance of a centrifugal pump with a 10-in impeller is as shown in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	2355	0
1600	2340	57.5
2400	2280	72.0
3200	2115	79.0
3800	1920	80.0
4000	1845	79.8
4800	1545	76.0

(a) Determine the revised pump performance with a reduced impeller size of 9 in.

(b) If the given performance is based on an impeller speed of 3560 r/min, calculate the revised performance at an impeller speed of 3000 r/min.

Solution

(a) The ratio of impeller diameters is $\frac{9}{10} = 0.9$. Therefore, the Q values will be multiplied by 0.9 and the H values will be multiplied by $0.9 \times 0.9 = 0.81$.

Revised performance data are given in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	1907	0
1440	1895	57.5
2160	1847	72.0
2880	1713	79.0
3420	1555	80.0
3600	1495	79.8
4320	1252	76.0

(b) When speed is changed from 3560 to 3000 r/min, the speed ratio $=$ $3000/3560 = 0.8427$. Therefore, Q values will be multiplied by 0.8427 and H values will be multiplied by $(0.8427)^2 = 0.7101$. Therefore, the revised pump performance is as shown in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	1672	0
1348	1662	57.5
2022	1619	72.0
2697	1502	79.0
3202	1363	80.0
3371	1310	79.8
4045	1097	76.0

Example 1.22 For the same pump performance described in Example 1.21, calculate the impeller trim necessary to produce a head of 2000 ft at a flow rate of 3200 gal/min. If this pump had a variable-speed drive and the given performance was based on an impeller speed of 3560 r/min, what speed would be required to achieve the same design point of 2000 ft of head at a flow rate of 3200 gal/min?

Solution Using the affinity laws, the diameter required to produce 2000 ft of head at 3200 gal/min is as follows:

$$\left(\frac{D}{10}\right)^2 = \frac{2000}{2115}$$

$$D = 10 \times 0.9724 = 9.72 \text{ in}$$

The speed ratio can be calculated from

$$\left(\frac{N}{3560}\right)^2 = \frac{2000}{2115}$$

Solving for speed,

$$N = 3560 \times 0.9724 = 3462 \text{ r/min}$$

Strictly speaking, this approach is only approximate since the affinity laws have to be applied along iso-efficiency curves. We must create the new H-Q curves at the reduced impeller diameter (or speed) to ensure that at 3200 gal/min the head generated is 2000 ft. If not, adjustment must be made to the impeller diameter (or speed). This is left as an exercise for the reader.

Net positive suction head. An important parameter related to the operation of centrifugal pumps is the concept of net positive suction head (NPSH). This represents the absolute minimum pressure at the suction of the pump impeller at the specified flow rate to prevent pump cavitation. If the pressure falls below this value, the pump impeller may be damaged and render the pump useless.

The calculation of NPSH available for a particular pump and piping configuration requires knowledge of the pipe size on the suction side of the pump, the elevation of the water source, and the elevation of the pump impeller along with the atmospheric pressure and vapor pressure of water at the pumping temperature. The pump vendor may specify that a particular model of pump requires a certain amount of NPSH (known as NPSH required or NPSH_R) at a particular flow rate. Based on the actual piping configuration, elevations, etc., the calculated NPSH (known as NPSH available or NPSH_A) must exceed the required NPSH at the specified flow rate. Therefore,

$$\text{NPSH}_A > \text{NPSH}_R$$

If the NPSH_R is 25 ft at a 2000 gal/min pump flow rate, then NPSH_A must be 35 ft or more, giving a 10-ft cushion. Also, typically, as the flow rate increases, NPSH_R increases fairly rapidly as can be seen from the typical centrifugal pump curve in Fig. 1.14. Therefore, it is important that the engineer perform calculations at the expected range of flow rates to ensure that the NPSH available is always more than the required NPSH, per the vendor's pump performance data. As indicated earlier, insufficient NPSH available tends to cavitate or starve the pump and eventually causes damage to the pump impeller. The damaged impeller will not be able to provide the necessary head pressure as indicated on the pump performance curve. NPSH calculation will be illustrated using an example next.

Figure 1.15 shows a centrifugal pump installation where water is pumped out of a storage tank that is located at a certain elevation above that of the centerline of the pump. The piping from the storage tank to the pump suction consists of straight pipe, valves, and fittings. The NPSH available is calculated as follows:

$$\text{NPSH} = (P_a - P_v)\frac{2.31}{\text{Sg}} + H + E_1 - E_2 - h_f \tag{1.71}$$

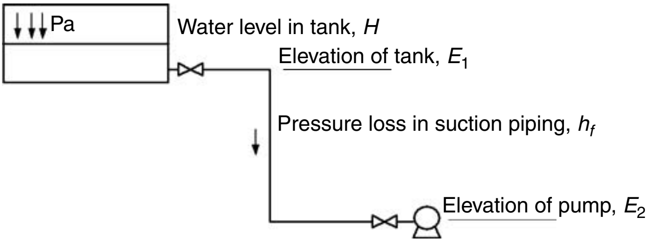

Figure 1.15 NPSH calculations.

where P_a = atmospheric pressure, psi
P_v = liquid vapor pressure at flowing temperature, psia
Sg = liquid specific gravity
H = liquid head in tank, ft
E_1 = elevation of tank bottom, ft
E_2 = elevation of pump suction, ft
h_f = friction loss in suction piping from tank to pump suction, ft

All terms in Eq. (1.71) are known except the head loss h_f. This item must be calculated considering the flow rate, pipe size, and liquid properties. We will use the Hazen-Williams equation with $C = 120$ for calculating the head loss in the suction piping. We get

$$P_m = 23{,}909\left(\frac{3000}{120}\right)^{1.852}\frac{1}{13.5^{4.87}} = 29.03 \text{ psi/mi}$$

The pressure loss in the piping from the tank to the pump $= \frac{29.03\times 500}{5280} =$ 2.75 psi. Substituting the given values in Eq. (1.71) assuming the vapor pressure of water is 0.5 psia at the pumping temperature,

$$\text{NPSH} = (14.7 - 0.5) \times 2.31 + 10 + 102 - 95 - 2.75 = 47.05 \text{ ft}$$

The required NPSH for the pump must be less than this value. If the flow rate increases to 5000 gal/min and the liquid level in turn drops to 1 ft, the revised NPSH available is calculated as follows.

With the flow rate increasing from 3200 to 5000 gal/min, the pressure loss due to friction P_m is approximately,

$$P_m = \left(\frac{5000}{3200}\right)^{1.852} \times 29.03 = 66.34 \text{ psi/mi}$$

$$\text{Head loss in 500 ft of pipe} = 66.34 \times \frac{500}{5280} = 6.3 \text{ psi}$$

Therefore,

$$\text{NPSH} = (14.7 - 0.5) \times 2.31 + 1 + 102 - 95 - 6.3 = 34.5 \text{ ft}$$

It can be seen that the NPSH available dropped off considerably with the reduction in liquid level in the tank and the increased friction loss in the suction piping at the higher flow rate.

The required NPSH for the pump (based on vendor data) must be lower than the preceding available NPSH calculations. If the pump data shows 38 ft NPSH required at 5000 gal/min, the preceding calculation indicates that the pump will cavitate since NPSH available is only 34.5 ft.

Specific speed. An important parameter related to centrifugal pumps is the specific speed. The specific speed of a centrifugal pump is defined as the speed at which a geometrically similar pump must be run such that it will produce a head of 1 ft at a flow rate of 1 gal/min. Mathematically, the specific speed is defined as follows

$$N_S = \frac{NQ^{1/2}}{H^{3/4}} \tag{1.72}$$

where N_S = specific speed
N = impeller speed, r/min
Q = flow rate, gal/min
H = head, ft

It must be noted that in Eq. (1.72) for specific speed, the capacity Q and head H must be measured at the best efficiency point (BEP) for the maximum impeller diameter of the pump. For a multistage pump the value of the head H must be calculated per stage. It can be seen from Eq. (1.72) that low specific speed is attributed to high head pumps and high specific speed for pumps with low head.

Similar to the specific speed another term known as *suction specific speed* is also applied to centrifugal pumps. It is defined as follows:

$$N_{SS} = \frac{NQ^{1/2}}{(\text{NPSH}_R)^{3/4}} \tag{1.73}$$

where N_{SS} = suction specific speed
N = impeller speed, r/min
Q = flow rate, gal/min
NPSH_R = NPSH required at the BEP

With single or double suction pumps the full capacity Q is used in Eq. (1.73) for specific speed. For double suction pumps one-half the value of Q is used in calculating the suction specific speed.

Example 1.23 Calculate the specific speed of a four-stage double suction centrifugal pump with a 12-in-diameter impeller that runs at 3500 r/min and generates a head of 2300 ft at a flow rate of 3500 gal/min at the BEP. Calculate the suction specific speed of this pump, if the NPSH required is 23 ft.

Solution From Eq. (1.72), the specific speed is

$$N_S = \frac{NQ^{1/2}}{H^{3/4}}$$

$$= \frac{3500(3500)^{1/2}}{(2300/4)^{3/4}} = 1763$$

The suction specific speed is calculated using Eq. (1.73):

$$N_{SS} = \frac{NQ^{1/2}}{\mathrm{NPSH}_R{}^{3/4}}$$

$$= \frac{3500(3500/2)^{1/2}}{(23)^{3/4}} = 13{,}941$$

1.13.3 Pumps in series and parallel

In the discussions so far we considered the performance of a single centrifugal pump. Sometimes, because of head limitations of a single pump or flow rate limits, we may have to use two or more pumps together at a pump station to provide the necessary head and flow rate. When more than one pump is used, they may be operated in series or parallel configurations. Series pumps are so arranged that each pump delivers the same volume of water, but the total pressure generated by the combination is the sum of the individual pump heads. Parallel pumps are configured such that the total flow delivered is the sum of the flow rates through all pumps, while each pump delivers a common head pressure. For higher pressures, pumps are operated in series, and when larger flow is required they are operated in parallel.

In Example 1.18 we found that the Corona pump station required pumps that would provide a pressure of 1219 psi at a flow rate of 7986.11 gal/min. Therefore we are looking for a pump or a combination of pumps at Corona that would provide the following:

$$\text{Flow rate} = 7986.11 \text{ gal/min} \qquad \text{and} \qquad \text{Head} = 1219 \times 2.31 = 2816 \text{ ft}$$

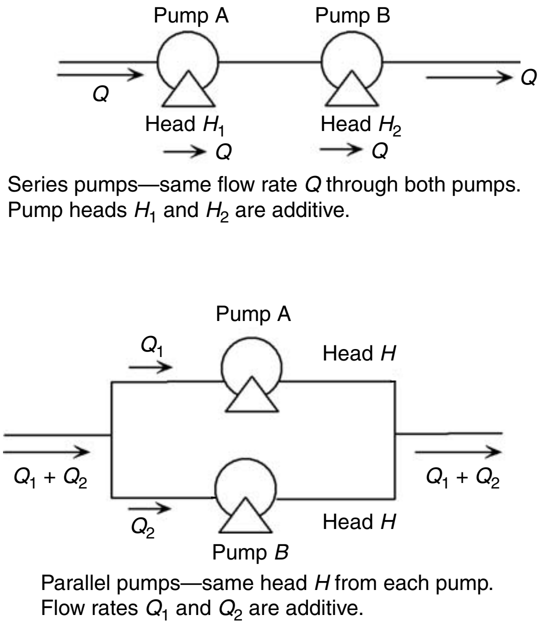

Figure 1.16 Pumps in series and parallel.

From a pump manufacturer's catalog, we can select a single pump that can match this performance. We could also select two smaller pumps that can generate 2816 ft of head at 3993 gal/min. We would operate these two pumps in parallel to achieve the desired flow rate and pressure. Alternatively, if we chose two other pumps that would each provide 1408 ft of head at the full flow rate of 7986.11 gal/min, we would operate these pumps in series. Example of pumps in series and parallel are shown in Fig. 1.16.

In some instances, pumps must be configured in parallel, while other situations might require pumps be operated in series. An example of where parallel pumps are needed would be in pipelines that have a large elevation difference between pump stations. In such cases, if one pump unit fails, the other pump will still be able to handle the head at a reduced flow rate. If the pumps were in series, the failure of one pump would cause the entire pump station to be shut down, since the single pump will not be able to generate enough head on its own to overcome the static elevation head between the pump stations. Figure 1.17 shows how the performance of a single pump compares with two identical pumps in series and parallel configurations.

Example 1.24 Two pumps with the head-capacity characteristics defined as follows are operated in series.

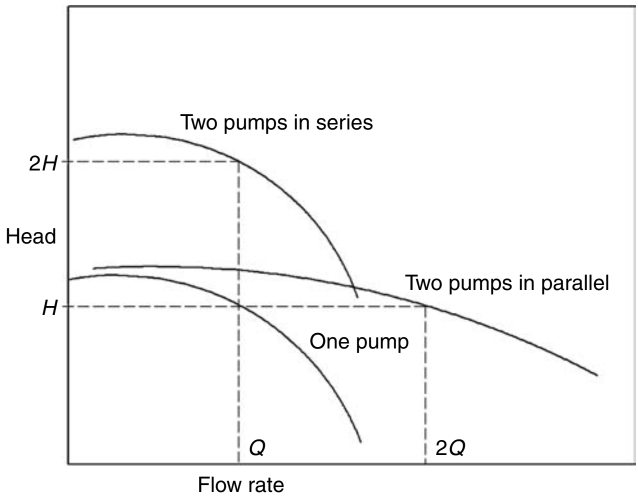

Figure 1.17 Pump performance—series and parallel.

Pump A:

Q, gal/min	0	600	1400	2200	3200
H, ft	2400	2350	2100	1720	1200

Pump B:

Q, gal/min	0	600	1400	2200	3200
H, ft	800	780	700	520	410

(a) Calculate the combined performance of the two operated in series.

(b) When operated in series, what impeller trims must be made to either pump, to meet the requirement of 2080 ft of head at 2200 gal/min?

(c) Can these pumps be operated in parallel configuration?

Solution

(a) Pumps in series cause the heads to be additive at the same flow rate. Therefore, at each flow rate, we add the corresponding heads to create the new H-Q curve for the combined pumps in series.

The combined performance of pump A and pump B in series is as follows:

Q, gal/min	0	600	1400	2200	3200
H, ft	3200	3130	2800	2240	1610

(b) Reviewing the combined pump curve, we see that the head generated at 2200 gal/min is 2240 ft. Since our requirement is 2080 ft of head at 2200 gal/min, clearly we must trim one of the pump impellers. We will leave the smaller pump B alone and trim the impeller of the larger pump A to achieve the total head of 2080 ft.

$$\text{Pump A head trim required} = 2240 - 2080 = 160 \text{ ft}$$

At the desired flow rate of 2200 gal/min, pump A produces 1720 ft. We must reduce this head by 160 ft, by trimming the impeller, or the head must become $1720 - 160 = 1560$ ft. Using the affinity laws, the pump trim required is

$$\left(\frac{1560}{1720}\right)^{1/2} = 0.9524 \quad \text{or} \quad 95.24 \text{ percent trim}$$

It must be noted that this calculation is only approximate. We must create the new pump performance curve at 95.24 percent trim and verify that the trimmed pump will generate the desired head of 1560 ft at a flow rate of 2200 gal/min. This is left as an exercise for the reader.

(c) For parallel pumps, since flow is split between the pumps at the common head, the individual pump curves should each have approximately the same head at each flow rate, for satisfactory operation. Reviewing the individual curves for pumps A and B, we see that the pumps are mismatched. Therefore, these pumps are not suitable for parallel operation, since they do not have a common head range.

Example 1.25 Two identical pumps with the head-capacity characteristic defined as follows are operated in parallel. Calculate the resultant pump performance.

Q, gal/min	0	600	1400	2200	3200
H, ft	2400	2350	2100	1720	1200

Solution Since the pumps operated in parallel will have common heads at the combined flow rates, we can generate the combined pump curve by adding the flow rates corresponding to each head value. The resulting combined performance curve is as follows:

Q, gal/min	0	1200	2800	4400	6400
H, ft	2400	2350	2100	1720	1200

1.13.4 System head curve

A *system head curve*, or a system head characteristic curve, for a pipeline is a graphic representation of how the pressure needed to pump water through the pipeline varies with the flow rate. If the pressures required at 1000, 2000, up to 10,000 gal/min are plotted on the vertical axis, with

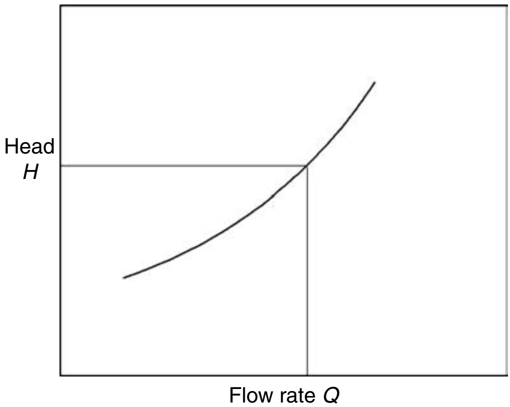

Figure 1.18 System head curve.

the flow rates on the horizontal axis, we get the system head curve as shown in Fig. 1.18.

It can be seen that the system curve is not linear. This is because the pressure drop due to friction varies approximately as the square of the flow rate, and hence the additional pressure required when the flow is increased 2000 to 3000 gal/min is more than that required when the flow rate increases from 1000 to 2000 gal/min.

Consider a pipeline used to transport water from point A to point B. The pipe inside diameter is D and the length is L. By knowing the elevation along the pipeline we can calculate the total pressure required at any flow rate using the techniques discussed earlier. At each flow rate we would calculate the pressure drop due to friction and multiply by the pipe length to get the total pressure drop. Next we will add the equivalent of the static head difference between A and B converted to psi. Finally, the delivery pressure required at B would be added to come up with the total pressure required similar to Eq. (1.29). The process would be repeated for multiple flow rates so that a system head curve can be constructed as shown in Fig. 1.18. If we plotted the feet of head instead of pressure on the vertical axis, we could use the system curve in conjunction with the pump curve for the pump at A. By plotting both the pump H-Q curve and the system head curve on the same graph, we can determine the point of operation for this pipeline with the specified pump curve. This is shown in Fig. 1.19.

When there is no elevation difference between points A and B, the system head curve will start at the point where the flow rate and head are both zero. If the elevation difference were 100 ft, B being higher than A, the system head curve will start at $H = 100$ ft and flow $Q = 0$.

This means at zero flow rate the pressure required is not zero. This simply means that even at zero flow rate, a minimum pressure must be present at A to overcome the static elevation difference between A and B.

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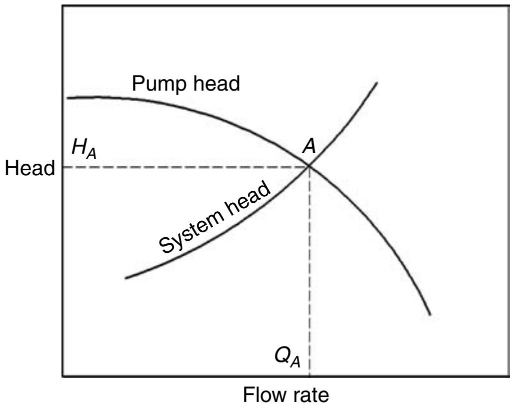

Figure 1.19 Pump head curve and system head curve.

1.13.5 Pump curve versus system head curve

The system head curve for a pipeline is a graphic representation of the head required to pump water through the pipeline at various flow rates and is an increasing curve, indicating that more pressure is required for a higher flow rate. On the other hand, the pump performance (head versus capacity) curve shows the head the pump generates at various flow rates, generally a drooping curve. When the required head per the system head curve equals the available pump head, we have a match of the required head versus the available head. This point of intersection of the system head curve and the pump head curve is the operating point for this particular pump and pipeline system. This is illustrated in Fig. 1.19.

It is possible that in some cases there may not be a point of intersection between a system head curve and a pump curve. This may be because the pump is too small and therefore the system head curve starts off at a point above the shutoff head of the curve and it diverges from the pump curve. Such a situation is shown in Fig. 1.20. It can be seen from this figure that even though there is no operating point between the system head curve and the single pump curve, by adding a second pump in series, we are able to get a satisfactory operating point on the system head curve.

When we use multiple pumps in series or parallel, a combined pump curve is generated and superimposed on the system head curve to get the operating point. Figure 1.21 shows how for a given pipeline system head curve, the operating point changes when we switch from a series pump configuration to a parallel pump configuration.

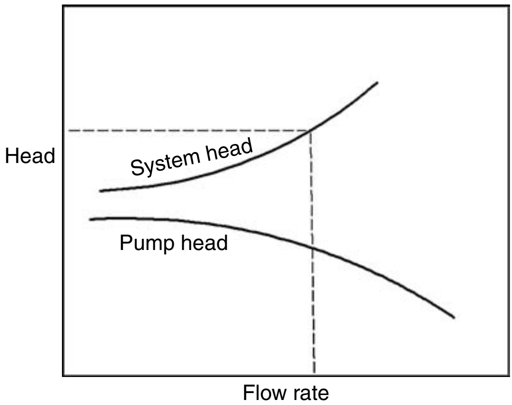

Figure 1.20 Diverging pump head curve and system head curve.

In Fig. 1.21, the pipeline system head curve is plotted along with the pump curves. Also shown are the combined pump curves for both series and parallel operation of two identical pumps. It can be seen that A represents the operating point with one pump, C the operating point for two pumps in series, and finally B the operating point with the two pumps in parallel. Corresponding to these points, the pipeline (and pump) flow rates are Q_A, Q_C, and Q_B, respectively.

The relative magnitudes of these flow rates would depend upon the nature of the system head curve. A steep system head curve will produce a higher flow rate with pumps in series, whereas a flat system head curve will produce a higher flow rate with parallel pumps.

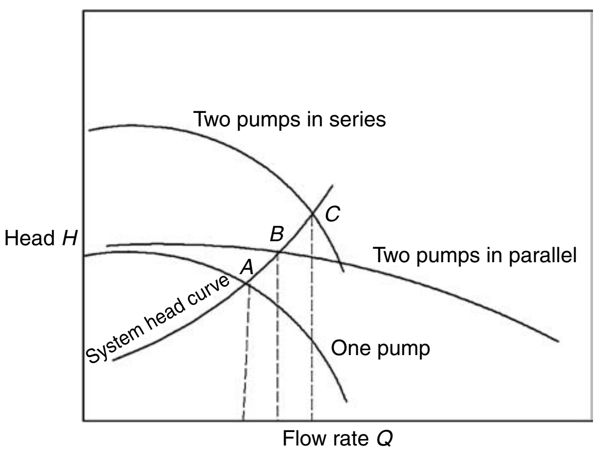

Figure 1.21 Multiple pumps with system head curve.

1.14 Flow Injections and Deliveries

So far we have discussed water pipelines with flow entering the pipeline at the beginning and exiting at the end of the pipeline. There was no flow injection or flow delivery along the pipeline between the entrance and exit. In many instances a certain volume of water would be pumped out of a storage tank and on its way to the destination several intermediate deliveries may be made at various points as shown in Fig. 1.22.

In Fig. 1.22 we see a pipeline that carries 10,000 gal/min from point A and at two intermediate points C and D delivers 2000 and 5000 gal/min, respectively, ultimately carrying the remainder of 3000 gal/min to the termination point B. Such a water pipeline would be typical of a small distribution system that serves three communities along the path of the pipeline. The hydraulic analysis of such a pipeline must take into account the different flow rates and hence the pressure drops in each segment. The pressure drop calculation for the section of pipe between A and C will be based on a flow rate of 10,000 gal/min. The pressure drop in the last section between D and B would be based on 3000 gal/min. The pressure drop in the intermediate pipe segment CD will be based on 8000 gal/min. The total pressure required for pumping at A will be the sum of the pressure drops in the three segments AC, CD, and DB along with adjustment for any elevation differences plus the delivery pressure required at B. For example, if the pressure drops in the three segments are 500, 300, and 150 psi, respectively, and the delivery pressure required at B is 50 psi and the pipeline is on a flat terrain, the total pressure required at A will be

$$500 + 300 + 150 + 50 = 1000 \text{ psi}$$

In comparison if there were no intermediate deliveries at C and D, the entire flow rate of 10,000 gal/min would be delivered at B necessitating a much higher pressure at A than the 1000 psi calculated.

Similar to intermediate deliveries previously discussed, water may be injected into the pipeline at some locations in between, causing additional volumes to be transported through the pipeline to the terminus B. These injection volumes may be from other storage facilities or

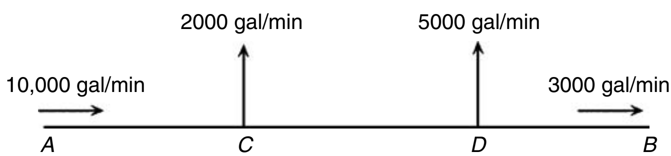

Figure 1.22 Water pipeline with multiple deliveries.

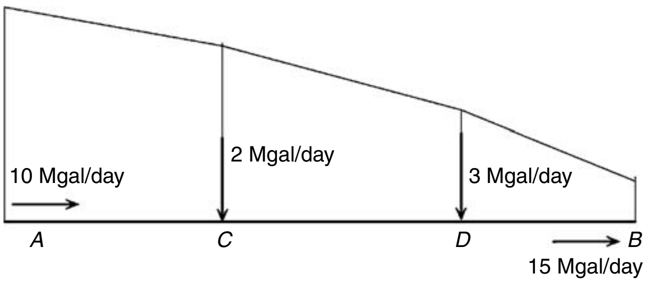

Figure 1.23 Hydraulic gradient with injections and deliveries.

water wells. The impact of the injections and deliveries on the hydraulic pressure gradient is illustrated in Fig. 1.23.

Because of the varying flow rates in the three pipe sections, the slope of the hydraulic gradient, which represents the pressure loss per mile, will be different for each section. Hence the hydraulic gradient appears as a series of broken lines. If the flow through the entire pipeline were a constant value as in previous examples, the hydraulic gradient will be one continuous line with a constant slope equal to the head loss per mile. We will illustrate injection and delivery in a water pipeline system using an example.

Example 1.26 An NPS 30 water pipeline (0.5-in wall thickness) 106 mi long from A to B is used to transport 10,000 gal/min with intermediate deliveries at C and D of 2000 and 3000 gal/min, respectively, as shown in Fig. 1.24. At E, 4000 gal of water is injected into the pipeline so that a total of 9000 gal/min is delivered to the terminus at B at 50 psi. Calculate the total pressure and pumping HP required at A based on 80 percent pump efficiency. Use the Hazen-Williams equation with $C = 120$. The elevations of points A through E are as follows:

$$A = 100 \text{ ft} \quad B = 340 \text{ ft} \quad C = 180 \text{ ft} \quad D = 150 \text{ ft} \quad \text{and} \quad E = 280 \text{ ft}$$

Solution Section AC has a flow rate of 10,000 gal/min and is 23 mi long. Using the Hazen-Williams equation (1.33), we calculate the pressure drop in

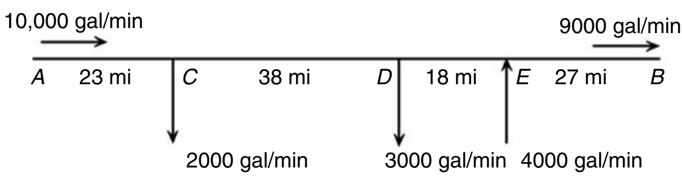

Figure 1.24 Example of water pipeline with injections and deliveries.

this section of pipe to be

$$P_m = 23{,}909\left(\frac{10{,}000}{120}\right)^{1.852}\left(\frac{1}{29.0}\right)^{4.87}$$
$$= 6.5169 \text{ psi/mi}$$

$$\text{Total pressure drop in } AC = 6.52 \times 23 = 149.96 \text{ psi}$$

$$\text{Elevation head for } AC = \frac{180 - 100}{2.31} = 34.63 \text{ psi}$$

Section CD has a flow rate of 8000 gal/min and is 38 mi long. Therefore, the pressure drop is

$$P_m = \left(\frac{8000}{10{,}000}\right)^{1.852} \times 6.5169 = 4.3108 \text{ psi/mi}$$

$$\text{Total pressure drop in } CD = 4.3108 \times 38 = 163.81 \text{ psi}$$

$$\text{Elevation head for } CD = \frac{150 - 180}{2.31} = -12.99 \text{ psi}$$

Section DE flows 5000 gal/min and is 18 mi long. We calculate the pressure drop in this section of pipe to be

$$P_m = \left(\frac{5000}{10{,}000}\right)^{1.852} \times 6.5169 \qquad \text{using proportions}$$
$$= 1.8052 \text{ psi/mi}$$

$$\text{Total pressure drop in } DE = 1.8052 \times 18 = 32.49 \text{ psi}$$

$$\text{Elevation head for } DE = \frac{280 - 150}{2.31} = 56.28 \text{ psi}$$

Section EB flows 9000 gal/min and is 27 mi long. We calculate the pressure drop in this section of pipe to be

$$P_m = \left(\frac{9000}{10{,}000}\right)^{1.852} \times 6.5169 = 5.3616 \text{ psi/mi}$$

$$\Delta P_{EB} = 5.3616 \times 27 = 144.76 \text{ psi}$$

$$\text{Elevation head for } EB = \frac{340 - 280}{2.31} = 25.97 \text{ psi}$$

Adding all the pressure drops and adjusting for elevation difference we get the total pressure required at A including the delivery pressure of 50 psi at B as follows:

$$P_A = (149.96 + 34.63) + (163.81 - 12.99) + (32.49 + 56.28)$$
$$+ (144.76 + 25.97) + 50$$

Therefore, $P_A = 644.91$ psi.

Approximately 645 psi is therefore required at the beginning of pipeline A to pump the given volumes through the pipeline system. The pump HP required at A is calculated next. Assuming a pump suction pressure of 50 psi

$$\text{Pump head} = (645 - 50) \times 2.31 = 1375 \text{ ft}$$

Therefore, the BHP required using Eq. (1.64) is

$$\text{BHP} = 1375 \times 10{,}000 \times \frac{1}{3960 \times 0.8} = 4341$$

Therefore, a 5000-HP motor-driven pump will be required at A.

1.15 Valves and Fittings

Water pipelines include several appurtenances as part of the pipeline system. Valves, fittings, and other devices are used in a pipeline system to accomplish certain features of pipeline operations. Valves may be used to communicate between the pipeline and storage facilities as well as between pumping equipment and storage tanks. There are many different types of valves, each performing a specific function. Gate valves and ball valves are used in the main pipeline as well as within pump stations and tank farms. Pressure relief valves are used to protect piping systems and facilities from overpressure due to upsets in operational conditions. Pressure regulators and control valves are used to reduce pressures in certain sections of piping systems as well as when delivering water to third-party pipelines which may be designed for lower operating pressures. Check valves are found in pump stations and tank farms to prevent backflow as well as separating the suction piping from the discharge side of a pump installation. On long-distance pipelines with multiple pump stations, the pigging process necessitates a complex series of piping and valves to ensure that the pig passes through the pump station piping without getting stuck.

All valves and fittings such as elbows and tees contribute to the frictional pressure loss in a pipeline system. Earlier we referred to some of these head losses as minor losses. As described earlier, each valve and fitting is converted to an equivalent length of straight pipe for the purpose of calculating the head loss in the pipeline system.

A control valve functions as a pressure reducing device and is designed to maintain a specified pressure at the downstream side as shown in Fig. 1.25.

If P_1 is the upstream pressure and P_2 is the downstream pressure, the control valve is designed to handle a given flow rate Q at these pressures. A coefficient of discharge C_v is typical of the control valve design and is related to the pressures and flow rates by the following equation:

$$Q = C_v A (P_1 - P_2)^{1/2} \tag{1.74}$$

where A is a constant.

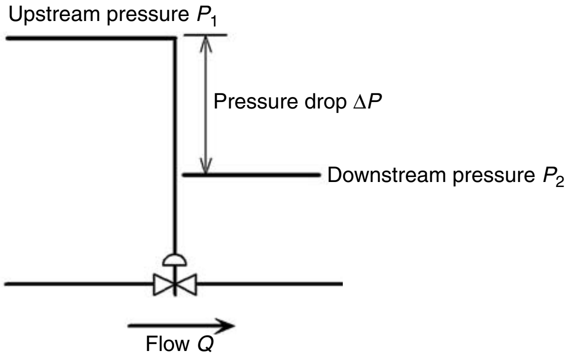

Figure 1.25 Control valve.

Generally, the control valve is selected for a specific application based on P_1, P_2, and Q. For example, a particular situation may require 800 psi upstream pressure, 400 psi downstream pressure, and a flow rate of 3000 gal/min. Based on these numbers, we may calculate a $C_v = 550$. We would then select the correct size of a particular vendor's control valve that can provide this C_v value at a specified flow rate and pressures. For example, a 10-in valve from vendor A may have a C_v of 400, while a 12-in valve may have a $C_v = 600$. Therefore, in this case we would choose a 12-in valve to satisfy our requirement of $C_v = 550$.

1.16 Pipe Stress Analysis

In this section we will discuss how a pipe size is selected based on the internal pressure necessary to transport water through the pipeline. If 1000 psi pressure is required at the beginning of a pipeline to transport a given volume of water a certain distance, we must ensure that the pipe has adequate wall thickness to withstand this pressure. In addition to being able to withstand the internal pressure, the pipeline also must be designed not to collapse under external loads such as soil loading and vehicles in case of a buried pipeline.

Since pipe may be constructed of different materials such as reinforced concrete, steel, wrought iron, plastic, or fiberglass, the necessary wall thickness will vary with the strength of the pipe material. The majority of pipelines are constructed of some form of material conforming to the American National Standards Institute (ANSI), American Society for Testing and Materials (ASTM), American Petroleum Institute (API), American Water Works Association (AWWA), Plastic Pipe Institute (PPI), or Federal Specification.

Barlow's equation is used to calculate the amount of internal pressure that a pipe can withstand, based on the pipe diameter, wall thickness, and the yield strength of the pipe material. Once we calculate this allowable internal operating pressure of the pipeline, we can then determine a hydrostatic test pressure, to ensure safe operation. The hydrostatic test pressure is generally 125 percent of the safe working pressure. The pipeline will be pressurized to this hydrostatic test pressure and the pressure held for a specified period of time to ensure no leaks and no pipe rupture. Generally, aboveground pipelines are hydrotested to 4 h minimum and underground pipelines for 8 h. Various local, city, state, and federal government codes may dictate more rigorous requirements for hydrotesting water pipelines.

Barlow's equation. Consider a circular pipe of outside diameter D and wall thickness T. Depending on the D/T ratio, the pipe may be classified as thin walled or thick walled. Most water pipelines constructed of steel are thin-walled pipes. If the pipe is constructed of some material (with a yield strength S psi) an internal pressure of P psi will generate stresses in the pipe material. At any point within the pipe material two stresses are present. The hoop stress S_h acts along the circumferential direction at a pipe cross section. The longitudinal or axial stress S_a acts along the length or axis of the pipe and therefore normal to the pipe cross section. It can be proved that the hoop stress S_h is twice the axial stress S_a. Therefore, the hoop stress becomes the controlling stress that determines the pipe wall thickness required. As the internal pressure P is increased, both S_h and S_a increase, but S_h will reach the yield stress of the material first. Therefore, the wall thickness necessary to withstand the internal pressure P will be governed by the hoop stress S_h generated in the pipe of diameter D and yield strength S.

Barlow's equation is as follows

$$S_h = \frac{PD}{2T} \tag{1.75}$$

The corresponding formula for the axial (or longitudinal) stress S_a is

$$S_a = \frac{PD}{4T} \tag{1.76}$$

Equation (1.75) for hoop stress is modified slightly by applying a design factor to limit the stress and a seam joint factor to account for the method of manufacture of pipe. The modified equation for calculating the internal design pressure in a pipe in U.S. Customary units is as follows:

$$P = \frac{2TSEF}{D} \tag{1.77}$$

where P = internal pipe design pressure, psi
D = pipe outside diameter, in
T = nominal pipe wall thickness, in
S = specified minimum yield strength (SMYS) of pipe material, psig
E = seam joint factor, 1.0 for seamless and submerged arc welded (SAW) pipes (see Table 1.7)
F = design factor, usually 0.72 for water and petroleum pipelines

The design factor is sometimes reduced from the 0.72 value in the case of offshore platform piping or when certain city regulations require buried pipelines to be operated at a lower pressure. Equation (1.77) for calculating the internal design pressure is found in the Code of Federal Regulations, Title 49, Part 195, published by the U.S. Department of Transportation (DOT). You will also find reference to this equation in ASME standard B31.4 for design and transportation of liquid pipelines.

TABLE 1.7 Pipe Design Joint Factors

Pipe specification	Pipe category	Joint factor E
ASTM A53	Seamless	1.00
	Electric resistance welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
ASTM A106	Seamless	1.00
ASTM A134	Electric fusion arc welded	0.80
ASTM A135	Electric Resistance Welded	1.00
ASTM A139	Electric fusion welded	0.80
ASTM A211	Spiral welded pipe	0.80
ASTM A333	Seamless	1.00
ASTM A333	Welded	1.00
ASTM A381	Double submerged arc welded	1.00
ASTM A671	Electric fusion welded	1.00
ASTM A672	Electric fusion welded	1.00
ASTM A691	Electric fusion welded	1.00
API 5L	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
API 5LX	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
API 5LS	Electric resistance welded	1.00
	Submerged arc welded	1.00

In SI units, the internal design pressure equation is the same as shown in Eq. (1.77), except the pipe diameter and wall thickness are in millimeters and the SMYS of pipe material and the internal design pressures are both expressed in kilopascals.

For a particular application the minimum wall thickness required for a water pipeline can be calculated using Eq. (1.77). However, this wall thickness may have to be increased to account for corrosion effects, if any, and for preventing pipe collapse under external loading conditions. For example, if corrosive water is being transported through a pipeline and it is estimated that the annual corrosion allowance of 0.01 in must be added, for a pipeline life of 20 years we must add $0.01 \times 20 = 0.20$ in to the minimum calculated wall thickness based on internal pressure. If such a pipeline were to be designed to handle 1000 psi internal pressure and the pipeline is constructed of NPS 16, SAW steel pipe with 52,000 psi SMYS, then based on Eq. (1.77) the minimum wall thickness for 1000 psi internal pressure is

$$T = 1000 \times \frac{16}{2 \times 52{,}000 \times 1.0 \times 0.72} = 0.2137 \text{ in}$$

Adding $0.01 \times 20 = 0.2$ in for corrosion allowance for 20-year life, the revised wall thickness is

$$T = 0.2137 + 0.20 = 0.4137 \text{ in}$$

Therefore, we would use the nearest standard wall thickness of 0.500 in.

Example 1.27 What is the internal design pressure for an NPS 20 water pipeline (0.375-in wall thickness) if it is constructed of SAW steel with a yield strength of 42,000 psi? Assume a design factor of 0.66. What would be the required hydrotest pressure range for this pipe?

Solution Using Eq. (1.77),

$$P = 2 \times 0.375 \times 42{,}000 \times 1.0 \times \frac{0.66}{20} = 1039.5$$

$$\text{Hydrotest pressure} = 1.25 \times 1039.5 = 1299.38 \text{ psi}$$

The internal pressure that will cause the hoop stress to reach the yield stress of 42,000 psi will correspond to $1039.5/0.66 = 1575$ psi. Therefore, the hydrotest pressure range is 1300 to 1575 psi.

1.17 Pipeline Economics

In pipeline economics we are concerned with the objective of determining the optimum pipe size and material to be used for transporting a given volume of water from a source to a destination. The criterion would be to minimize the capital investment as well as annual operating and maintenance cost. In addition to selecting the pipe itself to handle the flow rate we must also evaluate the optimum size of pumping equipment required. By installing a smaller-diameter pipe we may reduce the pipe material cost and installation cost. However, the smaller pipe size would result in a larger pressure drop due to friction and hence higher horsepower, which would require larger more costly pumping equipment. On the other hand, selecting a larger pipe size would increase the capital cost of the pipeline itself but would reduce the capital cost of pumping equipment. Larger pumps and motors will also result in increased annual operating and maintenance cost. Therefore, we need to determine the optimum pipe size and pumping power required based on some approach that will minimize both capital investment as well as annual operating costs. The least present value approach, which considers the total capital investment, the annual operating costs over the life of the pipeline, time value of money, borrowing cost, and income tax rate, seems to be an appropriate method in this regard.

In determining the optimum pipe size for a given pipeline project, we would compare three or four different pipe diameters based on the capital cost of pipeline and pump stations, annual operating costs (pump station costs, electricity costs, demand charges, etc.), and so forth. Taking into consideration the project life, depreciation of capital assets, and tax rate, along with the interest rate on borrowed money, we would be able to annualize all costs. If the annualized cost is plotted against the different pipe diameters, we will get a set of curves as shown in Fig. 1.26. The pipe diameter that results in the least annual cost would be considered the optimum size for this pipeline.

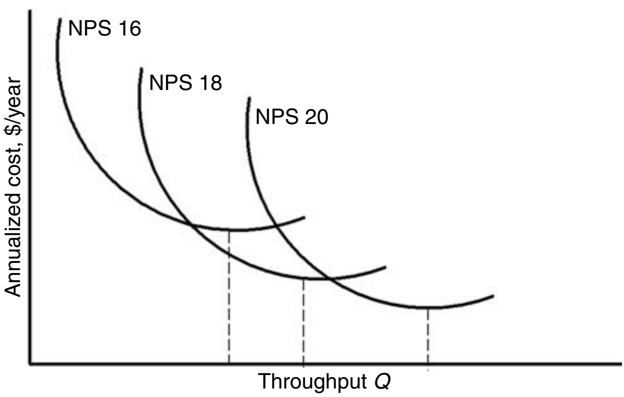

Figure 1.26 Pipeline costs versus pipe diameter.

Example 1.28 A 25-mi-long water pipeline is used to transport 15 Mgal/day of water from a pumping station at Parker to a storage tank at Danby. Determine the optimum pipe size for this application based on the minimum initial cost. Consider three different pipe sizes: NPS 20, NPS 24, and NPS 30. Use the Hazen-Williams equation with $C = 120$ for all pipes. Assume the pipeline is on fairly flat terrain. Use 85 percent pump efficiency. Use \$700 per ton for pipe material cost and \$1500 per HP for pump station installation cost. Labor costs for installing the three pipe sizes are \$100, \$120, and \$130 per ft, respectively. The pipeline will be designed for an operating pressure of 1400 psi. Assume the following wall thickness for the pipes:

NPS 20 pipe: 0.312 in

NPS 24 pipe: 0.375 in

NPS 30 pipe: 0.500 in

Solution First we determine the flow in gal/min:

$$15 \text{ Mgal/day} = \frac{15 \times 10^6}{(24 \times 60)} = 10,416.7 \text{ gal/min}$$

For the NPS 20 pipe we will first calculate the pressure and pumping HP required. The pressure drop per mile from the Hazen-Williams equation (1.33) is

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{19.376^{4.87}}$$

$$= 50.09 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 50.09 = 1252.25 \text{ psi}$$

Assuming a 50-psi delivery pressure at Danby and a 50-psi pump suction pressure, we obtain

$$\text{Pump head required at Parker} = 1252.25 \times 2.31 = 2893 \text{ ft}$$

$$\text{Pump flow rate} = 10{,}416.7 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = 2893 \times 10{,}416.7 \times \frac{1}{3960 \times 0.85}$$

$$= 8953 \text{ HP}$$

Therefore, a 9000-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 20 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.312\,(20 - 0.312) = 65.60 \text{ lb/ft}$$

$$\text{Total pipe tonnage for 25 mi} = 25 \times 65.6 \times \frac{5280}{2000} = 4330 \text{ tons}$$

Increasing this by 5 percent for contingency and considering \$700 per ton material cost, we get

$$\text{Total pipe material cost} = 700 \times 4330 \times 1.05 = \$3.18 \text{ million}$$

$$\text{Labor cost for installing NPS 20 pipeline} = 100 \times 25 \times 5280 = \$13.2 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 9000 = \$13.5 \text{ million}$$

Therefore, the total capital cost of NPS 20 pipeline = \$3.18 + \$13.2 + \$13.5 = \$29.88 million.

Next we calculate the pressure and HP required for the NPS 24 pipeline. The pressure drop per mile from the Hazen-Williams equation is

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{23.25^{4.87}}$$

$$= 20.62 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 20.62 = 515.5 \text{ psi}$$

Assuming a 50-psi delivery pressure at Danby and a 50-psi pump suction pressure, we obtain

$$\text{Pump head required at Parker} = 515.5 \times 2.31 = 1191 \text{ ft}$$

$$\text{Pump flow rate} = 10{,}416.7 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = 1191 \times 10{,}416.7 \times \frac{1}{3960 \times 0.85}$$

$$= 3686 \text{ HP}$$

Therefore a 4000-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 24 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.375\,(24 - 0.375) = 94.62 \text{ lb/ft}$$

$$\text{Total pipe tonnage for 25 mi} = 25 \times 94.62 \times \frac{5280}{2000} = 6245 \text{ tons}$$

Increasing this by 5 percent for contingency and considering \$700 per ton material cost, we obtain

$$\text{Total pipe material cost} = 700 \times 6245 \times 1.05 = \$4.59 \text{ million}$$

$$\text{Labor cost for installing NPS 24 pipeline} = 120 \times 25 \times 5280 = \$15.84 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 4000 = \$6.0 \text{ million}$$

Therefore, the total capital cost of NPS 24 pipeline = \$4.59 + \$15.84 + \$6.0 = \$26.43 million.

Next we calculate the pressure and HP required for the NPS 30 pipeline. The pressure drop per mile from the Hazen-Williams equation is

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{29.0^{4.87}}$$
$$= 7.03 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 7.03 = 175.75 \text{ psi}$$

Assuming a 50-psi delivery pressure at Danby and a 50-psi pump suction pressure, we obtain

$$\text{Pump head required at Parker} = 175.75 \times 2.31 = 406 \text{ ft}$$

$$\text{Pump flow rate} = 10{,}416.7 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = 406 \times 10{,}416.7 \times \frac{1}{3960 \times 0.85} = 1257 \text{ HP}$$

Therefore a 1500-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 30 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.500\,(30 - 0.500) = 157.53 \text{ lb/ft}$$

$$\text{Total pipe tonnage for 25 mi} = 25 \times 157.53 \times \frac{5280}{2000} = 10{,}397 \text{ tons}$$

Increasing this by 5 percent for contingency and considering \$700 per ton material cost, we obtain

$$\text{Total pipe material cost} = 700 \times 10{,}397 \times 1.05 = \$7.64 \text{ million}$$

$$\text{Labor cost for installing NPS 30 pipeline} = 130 \times 25 \times 5280 = \$17.16 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 1500 = \$2.25 \text{ million}$$

Therefore, the total capital cost of NPS 30 pipeline = \$7.64 + \$17.16 + \$2.25 = \$27.05 million.

In summary, the total capital cost of the NPS 20, NPS 24, and NPS 30 pipelines are

$$\text{NPS 20 capital cost} = \$29.88 \text{ million}$$

$$\text{NPS 24 capital cost} = \$26.43 \text{ million}$$

$$\text{NPS 30 capital cost} = \$27.05 \text{ million}$$

Based on initial cost alone, it appears that NPS 24 is the preferred pipe size.

Example 1.29 A 70-mi-long water pipeline is constructed of 30-in (0.375-in wall thickness) pipe for transporting 15 Mgal/day from Hampton pump station to a delivery tank at Derry. The delivery pressure required at Derry is 20 psi. The elevation at Hampton is 150 ft and at Derry it is 250 ft. Calculate the pumping horsepower required at 85 percent pump efficiency.

This pipeline system needs to be expanded to handle increased capacity from 15 Mgal/day to 25 Mgal/day. The maximum pipeline pressure is 800 psi. One option would be to install a parallel 30-in-diameter pipeline (0.375 wall thickness) and provide upgraded pumps at Hampton. Another option would require expanding the capacity of the existing pipeline by installing an intermediate booster pump station. Determine the more economical alternative for the expansion. Use the Hazen-Williams equation for pressure drop with $C = 120$.

Solution At 15 Mgal/day flow rate,

$$Q = \frac{15 \times 10^6}{24 \times 60} = 10,416.7 \text{ gal/min}$$

Using the Hazen-Williams equation,

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{29.25^{4.87}} = 6.74 \text{ psi/mi}$$

The total pressure required at Hampton is

$$P_t = P_f + P_{\text{elev}} + P_{\text{def}} \qquad \text{from Eq. (1.29)}$$

$$= (6.74 \times 70) + \frac{250 - 150}{2.31} + 20 = 535.1 \text{ psi}$$

Therefore the Hampton pump head required is $(535.1 - 50) \times 2.31 = 1121$ ft, assuming a 50-psi suction pressure at Hampton.

The pump HP required at Hampton [using Eq. (1.64)] is

$$\text{HP} = 1121 \times 10{,}416.7 \frac{1}{3960 \times 0.85} = 3470 \text{ HP}, \quad \text{say 4000 HP installed}$$

For expansion to 25 Mgal/day, the pressure drop will be calculated using proportions:

$$25 \text{ Mgal/day} = \frac{25 \times 10^6}{24 \times 60} = 17{,}361.11 \text{ gal/min}$$

$$P_m = 6.74 \times \left(\frac{25}{15}\right)^{1.852} = 17.36 \text{ psi/mi}$$

The total pressure required is

$$P_t = (17.36 \times 70) + \frac{250 - 150}{2.31} + 20 = 1279 \text{ psi}$$

Since the maximum pipeline pressure is 800 psi, the number of pump stations required

$$= 1279/800 = 1.6, \quad \text{or 2 pump stations}$$

With two pump stations, the discharge pressure at each pump station $= 1279/2 = 640$ psi. Therefore, the pump head required at each pump station $= (640 - 50) \times 2.31 = 1363$ ft, assuming a 50-psi suction pressure at each pump station.

The pump HP required [using Eq. (1.64)] is

$$\begin{aligned} \text{HP} &= 1363 \times 17{,}361.11 \frac{1}{3960 \times 0.85} \\ &= 7030 \text{ HP, say 8000 HP installed} \end{aligned}$$

$$\text{Increase in HP for expansion} = 2 \times 8000 - 4000 = 12{,}000 \text{ HP}$$

$$\begin{aligned} &\text{Incremental pump station} \\ &\text{cost based on \$1500 per HP} = 1500 \times 12{,}000 = \$18 \text{ million} \end{aligned}$$

This cost will be compared to looping a section of the pipeline with a 30-in pipe. If a certain length of the 70-mi pipeline is looped with 30-in pipe, we could reduce the total pressure required for the expansion from 1279 psi to the maximum pipeline pressure of 800 psi. The equivalent diameter of two 30-in pipes is

$$D_e = 29.25 \left(\frac{2}{1}\right)^{0.3803} = 38.07 \text{ in}$$

The pressure drop in the 30-in pipe at 25 Mgal/day was calculated earlier as 17.36 psi/mi. Hence,

$$P_m \text{ for the 38.07-in pipe} = 17.36 \times (29.25/38.07)^{4.87} = 4.81 \text{ psi/mi}$$

If we loop x miles of pipe, we will have x miles of pipe at $P_m = 4.81$ psi/mi and $(70 - x)$ mi of pipe at 17.36 psi/mi. Therefore, since the total pressure cannot exceed 800 psi, we can write

$$4.81x + 17.36(70 - x) + 43.3 + 20 \leq 800$$

Solving for x we get,

$$x \geq 38.13$$

Therefore we must loop about 39 mi of pipe to be within the 800-psi pressure limit.

If we loop loop 39 mi of pipe, the pressure required at the 25 Mgal/day flow rate is

$$(39 \times 4.81) + (31 \times 17.36) + 43.3 + 20 = 789.1 \text{ psi}$$

The cost of this pipe loop will be calculated based on a pipe material cost of \$700 per ton and an installation cost of \$120 per ft.

$$\text{Pipe weight per foot} = 10.68 \times 0.375 \times (30 - 0.375)$$

$$= 118.65 \text{ lb/ft}$$

$$\text{Material cost of 39 mi of 30-in loop} = \$700 \times 118.65 \times 5280 \times 39$$

$$= \$17.1 \text{ million}$$

$$\text{Pipe labor cost for installing 39 mi of 30-in loop} = \$120 \times 5280 \times 39 = \$24.7 \text{ million}$$

$$\text{Total cost of pipe loop} = \$17.1 + \$24.7 = \$41.8 \text{ million}$$

compared to

$$\text{Incremental pump station cost based on adding a booster pump station} = \$18 \text{ million}$$

Therefore, based on the minimum initial cost alone, looping is not the economical option.

In conclusion, at the expanded flow rate of 25 Mgal/day, it is more cost effective to add HP at Hampton and build the second pump station to limit pipe pressure to 800 psi.

Chapter 2

Fire Protection Piping Systems

Introduction

Fire protection piping is used to transport fire extinguishing substances such as water from the supply point to locations where it is used to fight fire and to provide fire protection. Generally, water is used as the fire extinguishing substance. In addition to water, other substances used for fire protection are foam, carbon dioxide, dry chemical, and other inert gases. Piping hydraulics in a fire protection system that transports water are handled similar to that in ordinary water pipelines, although the pressures encountered with fire protection water piping systems are lower.

2.1 Fire Protection Codes and Standards

In the United States most fire protection piping are governed by the National Fire Protection Association (NFPA) and insurance companies. The NFPA publishes almost 300 codes, standards, and recommended practices that are applicable for design and construction of fire protection systems. The standards are regularly revised and issued on a yearly basis. These codes include guidelines, mandatory requirements, and recommended practices for design, construction, and installation. Local, state, and city regulations may require additional stringent requirements for the design and operation of fire protection piping systems. A list of NFPA standards used for the protection of residential and commercial buildings is given in Table 2.1. In addition the following publications must be consulted for design and construction of fire protection systems.

TABLE 2.1 National Fire Protection Association (NFPA) Standards

Title	Description
NFPA 13	Standard for the Installation of Sprinkler Systems
NFPA 13D	Standard on the Installation of Sprinkler Systems in One and Two Family Dwellings and Manufactured Homes
NFPA 13R	Standard on the Installation of Sprinkler Systems in Residential Occupancies up to and Including Four Stories in Height
NFPA 14	Standard for the Installation of Standpipe and Hose Systems
NFPA 15	Standard for Water Spray Fixed Systems for Fire Protection
NFPA 20	Standard for the Installation of Centrifugal Fire Pumps
NFPA 22	Standard for the Installation of Water Tanks for Private Fire Protection
NFPA 24	Standard for Private Service Mains and Their Appurtenances
NFPA 61A	Standard for the Prevention of Fire and Dust Explosion in Facilities Manufacturing and Handling Starch
NFPA 61B	Standard for the Prevention of Fires and Explosions in Grain Elevators and Facilities Handling Bulk Raw Agriculture Commodities
NFPA 61C	Standard for the Prevention of Fire and Dust Explosions in Feed Mills
NFPA 61D	Standard for the Prevention of Fire and Dust Explosions in the Milling of Agricultural Commodities for Human Consumption
NFPA 68	Guide for Venting of Deflagrations
NFPA 69	Standard on Explosions Prevention Systems
NFPA 70	National Electrical Code
NFPA 72	National Fire Alarm Code
NFPA 77	Recommended Practice on Static Electricity
NFPA 170	Standard on Fire Safety Symbols
NFPA 214	Standard on Water Cooling Towers
NFPA 231	Standard on General Storage
NFPA 231C	Standard on Rack Storage of Materials
NFPA 231D	Standard for Storage of Rubber Tires
NFPA 231F	Standard for Storage of Rolled Paper
NFPA 321	Standard on Basic Classification of Flammable and Combustible Liquids
NFPA 325M	Fire Hazard Properties of Flammable Liquids, Gases and Volatile Solids
NFPA 495	Explosive Materials Code
NFPA 750	Standard for the Installation of Water Mist Fire Protection Systems

1. NFPA Handbook of Fire Protection
2. Factory Mutual Handbook of Industrial Loss Prevention
3. NFPA Standards: National Fire Codes. This is in 10 volumes covering
   a. Flammable liquids
   b. Gases
   c. Combustible solids, dust, and explosives
   d. Building, construction, and facilities
   e. Electrical
   f. Sprinklers
   g. Fire pumps
   h. Water tanks
   i. Alarms
   j. Special extinguisher system

2.2 Types of Fire Protection Piping

Fire protection piping may be classified as underground or aboveground. The underground piping system generally feeds the aboveground piping system. The underground piping system consists of water pipes from the city water supply to a hydrant and piping system connected to a storage tank that may be pressurized by compressed air. An aboveground piping system includes piping from a gravity tank that provides water by gravity flow. Sprinkler systems are also classified as aboveground piping systems.

2.2.1 Belowground piping

Underground or belowground piping systems are designed according to NFPA 24, Standard for Private Service Mains and Their Appurtenances. The following methods are used to supply water to a fire protection system:

1. City water supply
2. Gravity tank
3. Pressurized tank
4. Fire protection water pump

Generally, underground piping that brings in water from one of these sources will be installed and tested before being connected to an aboveground piping system that would serve a sprinkler system for a residential or commercial building.

The design and construction of underground fire protection piping must be checked to ensure the following criteria are met:

1. Depth of cover. The vertical distance from the top of the pipe to the ground surface must be sufficient to prevent freezing of the pipe. This minimum depth varies geographically. The designer must consult publications such as NFPA 24, which shows a chart indicating the recommended depth of cover in various parts of the United States. This publication shows contour lines that indicate the recommended depth of cover such as 2.5 to 3.0 ft in California and 6.5 to 8.0 ft in parts of Minnesota.
2. Conflict with other utility piping. Underground fire protection piping must be installed at locations where there will be no interference with existing utility pipelines such as gas lines or oil lines. Certain minimum clearances must exist between pipelines.
3. Avoiding physical damage to piping. To prevent damage from settling of buildings, underground piping must be routed away from building

slabs, footings, etc. Underground piping that is located under roads and railroads needs additional depth of cover and must be installed in casing or sleeve pipes for extra protection.

Underground piping materials used for fire water systems include ductile iron, class 50 and class 52 PVC piping, class 150 plastic pipe, cement-lined piping, and cast iron piping. The pipe fittings used include mechanical joint, push-on joint, and PVC plastic fittings.

Thrust blocks and piping restraint are required when installing elbows and bends, tees, etc., to counteract forces due to changes in the direction of flow through underground pipelines. As an example, a 12-inch (in) pipe elbow requires 18 square feet (ft^2) of bearing area for the thrust block. NFPA 24 lists the bearing area for concrete blocks for different pipe sizes and bend configurations. The size of the block depends on the nature of the soil, such as whether it is clay, sand, or gravel. The bearing area is proportionately increased depending upon the softness of the soil.

2.2.2 Aboveground piping

An aboveground fire protection piping system consists of all piping related to fire protection that is not buried. Piping from a city water system, private mains, and fire water pumps, that goes along the sides of a building or into a building and is connected to an automated sprinkler system is classified as aboveground piping.

NFPA 13, Standard for the Installation of Sprinkler Systems, is used for the design and construction of automatic sprinkler systems. Such sprinkler systems are installed in residential and commercial buildings. There are two types of sprinkler systems in use today, wet pipe systems and dry pipe systems. In wet pipe systems the heat responsive elements in the sprinklers activate the flow of water. When activated, the water in the pipe is immediately discharged through the sprinklers. Dry pipe systems are installed in areas where the temperature is low and water in the pipe could freeze. Therefore, the pipes in this system are pressurized with air, and when the sprinkler activates, water is discharged with a certain amount of delay since the pressurized air must escape first before the water can be discharged through the sprinkler heads.

The NFPA 13 standard limits the time delay to 60 seconds (s). This means that from the point of sprinkler actuation, water must reach the farthest sprinkler within 60 s. Because of the delay factor in dry pipe systems, the number of sprinklers required for a dry pipe system will be more than that for a wet pipe system with the same area to be protected.

Steel piping and copper tubing used in a sprinkler piping system are based on American Society for Testing and Materials (ASTM) and

American National Standards Institute (ANSI) specifications. Most sprinkler systems are designed for 175 pounds per square inch gauge (psig) maximum pressure consisting of schedule 5, schedule 10, and schedule 40 pipe. If pressures above 175 psi are required, schedule 80 pipe is used. Fittings used along with piping are cast iron and malleable iron. Cast iron fittings are brittle and hence are prone to cracks if accidentally hit, whereas malleable iron fittings can withstand considerable impact loading.

2.2.3 Hydrants and sprinklers

Hydrants are installed near buildings to provide the jet stream of fire protection water to fight fires in the buildings. The designs of hydrants are generally dictated by NFPA, American Water Works Association (AWWA), and other fire-testing laboratories. Generally hydrants are spaced 200 to 250 ft apart. In certain cases in hazardous locations this spacing may be reduced to 100 to 150 ft.

Sprinkler systems are installed inside buildings to provide firefighting water to protect the contents of the building from fire. Standards must be followed in the installation of the sprinkler system.

In this section we will discuss the configuration and design of automatic sprinkler systems. There are three main configurations used for sprinkler systems: tree system, grid system, and loop system. These are shown in Figs 2.1 through 2.3.

A tree system consists of a central pipe called the *crossmain*, which that is the main feed line that supplies water to the individual branch lines containing the sprinklers in a tree fashion as shown in Fig. 2.1. The crossmain is positioned so that it is located at the same distance from the ends of the branch lines. Tree systems may be center fed or end fed as shown in Fig. 2.1.

In the grid system the branch lines connect to a crossmain at each end in the form of a grid as shown in Fig. 2.2. A grid system is used only with wet pipe systems since the air cannot be pushed out quickly through the grid system with a dry pipe system.

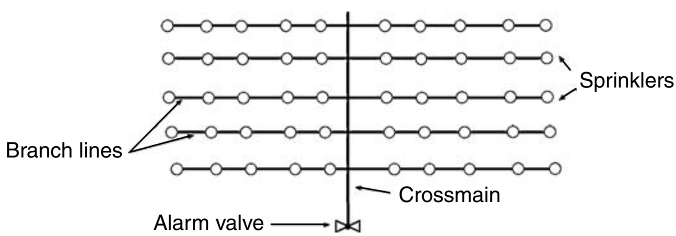

Figure 2.1 Tree sprinkler system.

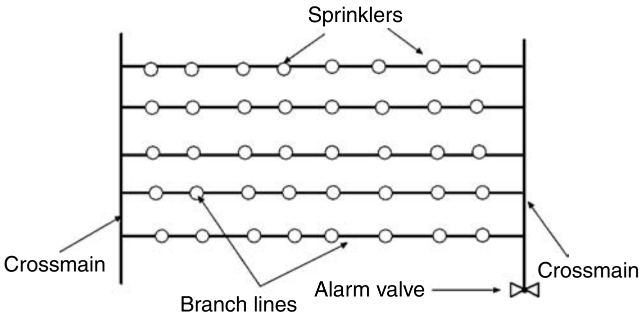

Figure 2.2 Grid sprinkler system.

A loop system may be a dry pipe or a wet pipe system. It is so configured that the crossmains are connected at two or more locations forming a loop. Compared to the tree system, the sprinklers are provided water from more than one location.

Occupancy and hazard class. In order to determine the spacing of the sprinklers we must first determine the hazard class of the occupancy. Occupancy depends on the expected level of severity of fire in a particular situation. It depends on the nature of the building use and its contents. The fire load density depends on the type of substances contained

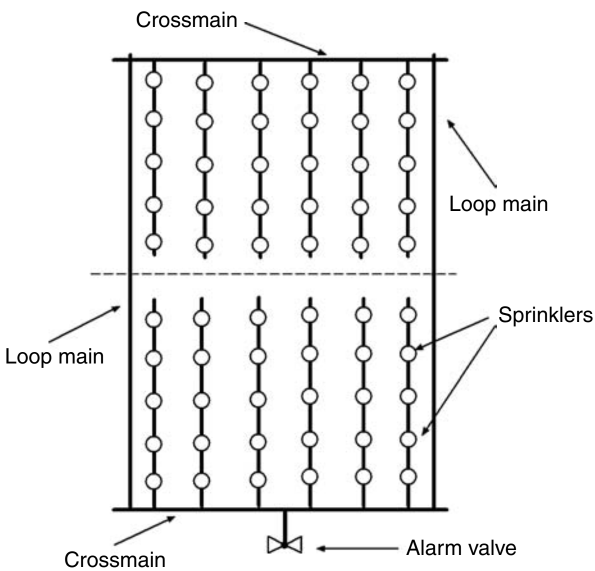

Figure 2.3 Loop sprinkler system.

within the building, how combustible these items are, and how they are arranged within the building. Occupancy is classified as follows:

1. Low
2. Moderate
3. Moderate to high
4. Very high

Low occupancy is considered a light hazard. It includes churches, clubs, educational institutions, hospitals, prisons, libraries, museums, nursing homes, offices, residences, restaurant seating areas, and theaters.

Moderate occupancy is referred to as ordinary hazard—Group I. It includes parking garages, car dealers, bakeries, dairies, laundries, and restaurant service areas.

Moderate to high occupancy is considered ordinary hazard—Group II. It includes cereal mills, chemical plants, confectionaries, distilleries, and machine shops.

Very high occupancy is referred to as extra hazard. It includes areas with flammable liquids, flammable metals, printing ink, solvent cleaning, varnish, and paint.

Once we determine the occupancy and the hazard classification, we must calculate the area protected by each sprinkler. NFPA 13 imposes a limitation of 52,000 ft^2 of area for the light hazard and ordinary hazard group of occupancy. For extra hazard occupancy the limitation is 40,000 ft^2. The sprinkler spacing is calculated from the following formula:

$$A = D_s \times D_b \tag{2.1}$$

where A = sprinkler coverage area, ft^2
D_s = distance from sprinkler to sprinkler on branch line, ft
D_b = branch line spacing, ft

NFPA 13 also limits the sprinkler coverage area according to the following:

1. Light hazard—200 to 225 ft^2
2. Light hazard—Buildings of combustible construction—168 ft^2
3. Ordinary hazard—130 ft^2
4. Extra hazard—100 ft^2

In addition NFPA 13 also limits the maximum distance between sprinklers (D_s) to 15 ft for light or ordinary hazard and 12 ft for extra hazard. Similar limits are also imposed on the spacing between branch lines (D_b).

Next we determine the number of branch lines required by dividing the width of the bay by the maximum branch line spacing (D_b). Therefore, the formula for the number of branch lines is

$$\text{Number of branch lines} = \frac{\text{bay width}}{\text{branch line spacing}} = \frac{W}{D_b} \tag{2.2}$$

where W is the bay width and D_b is the branch line spacing, both in feet. The calculated value is rounded up to the next whole number.

Once we determine the number of branch lines, we can calculate the actual spacing between the branch lines in the bay as follows:

$$D_b = \frac{\text{bay width}}{\text{number of branch lines}} \tag{2.3}$$

After determining the number of branch lines and their spacing, we calculate the spacing required between sprinklers on each branch line. This is calculated considering the NFPA limitation for the square footage coverage per sprinkler and the maximum allowable distance between sprinklers.

Example 2.1 For an ordinary hazard system, sprinklers have to be installed in a bay width of 32 ft. Determine the number of branch lines and the spacing between the branch lines.

Solution Since NFPA 13 limits the branch line spacing to 15 ft,

$$\text{Number of branch lines required} = \frac{32}{15} = 2.1$$

or three branch lines, rounding up to the next higher number. Therefore,

$$\text{Actual spacing between branch lines} = \frac{32}{3} = 10.67\,\text{ft}$$

Example 2.2 Determine the sprinkler spacing for Example 2.1 considering the 130-ft^2 coverage limitation per sprinkler for an ordinary hazard system.

Solution Since NFPA 13 allows 15-ft sprinkler spacing, from Eq. (2.1),

$$\text{Sprinkler spacing} = \frac{130}{10.67} = 12.18$$

This is less than the 15 ft allowed; therefore, 12.18-ft spacing is adequate.

Next we can determine the number of sprinklers on each branch line by considering the length of the area covered by the sprinkler and the sprinkler spacing calculated earlier. The number of sprinklers on the branch line is

$$N_s = \frac{\text{length of bay}}{D_s} \tag{2.4}$$

where N_s is the number of sprinklers and D_s is the distance in feet between sprinklers on the branch line.

From the preceding, we would round up to the next higher whole number to determine the number of sprinklers required on each branch line. For example, if the area to be protected had a bay length of 275 ft and a bay width of 32 ft, the number of sprinklers required for 12-ft spacing will be $275/12 = 22.91$, or 23 sprinklers. Once we determine the number of sprinklers, the actual distance between sprinklers can be recalculated by dividing the bay length by the number of sprinklers. In the current example the actual distance between the sprinklers will be $275/23 = 11.95$ ft.

After calculating the number of branch lines, branch spacing, number of sprinklers, and the sprinkler spacing, we can calculate and verify the sprinkler coverage area. In Example 2.2, the sprinkler coverage area is

$$\begin{aligned} A &= D_s \times D_b \\ &= 11.95 \times 10.67 = 127.51\,\text{ft}^2 \end{aligned} \tag{2.5}$$

where all symbols are as defined earlier.

2.3 Design of Piping System

In this section we will discuss the properties of water and its advantages and how the pressure required and the flow rates are calculated for a fire protection water piping system.

Water is the most common fluid used in fire protection because of its easy availability (compared to other fire suppression products) and its properties that help in extinguishing fire. Water is available in most instances because all commercial and residential buildings require a water supply and hence connections are already available from which the needed quantity can be taken for fire protection purposes. The properties of water include the following:

Freezing point:	32°F (0°C)
Boiling point:	212°F (100°C)
Density :	62.4 lb/ft^3 (1000 kg/m^3)

Absorbs heat from fire at a rate of 9330 Btu/lb.

2.3.1 Pressure

Pressure, also called the intensity of pressure, within a body of water is defined as the force per unit area. It is measured in psi in U.S. Customary System (USCS) units and kilopascals (kPa) in SI units.

Consider a storage tank 30 ft high containing water up to a level of 20 ft. If the tank has a rectangular cross section of 30 by 40 ft, the total weight of the water in the tank is

$$\text{Weight} = 30 \times 40 \times 20 \times 62.4 = 1{,}497{,}600\,\text{lb}$$

Since this weight acts on the tank bottom area of 30 × 40 ft, we can state that the intensity of pressure on the tank bottom is

$$P = \frac{1{,}497{,}600}{30 \times 40} = 1248\,\text{lb/ft}^2 = \frac{1248}{144} = 8.67\,\text{lb/in}^2\,(\text{psi})$$

This pressure of 8.67 psi acts on every square inch of the tank bottom. However, within the body of the water, say halfway into the tank (10 ft), the pressure will be less. In fact we can calculate the pressure within the water at a depth of 10 ft by considering the weight of half the quantity of water we calculated earlier. This means the pressure within the water at the halfway point is

$$P = \frac{1{,}497{,}600/2}{30 \times 40} = 624\,\text{lb/ft}^2 = 4.33\text{ psi}$$

The preceding demonstrates that the pressure within a liquid is proportional to the height of the column of liquid above it. In fact the pressure at a depth h below the free surface of water is calculated as

$$P = \frac{h \times 62.4}{144} = 0.433 \times h \quad \text{psi} \tag{2.6}$$

where P is the pressure (psi) and h is the depth of water (ft).

Equation (2.6) is an important relationship for calculating the pressure in psi from a water column height h ft. The water column height h that equates to the pressure P according to Eq. (2.6) is referred to as the *head of water*. The term *pressure head* is also used sometimes. To summarize, a head of 10 ft of water is equivalent to a pressure of 4.33 psi as calculated using Eq. (2.6).

Equation (2.6) is valid only for water. For other liquids such as gasoline or diesel, the pressure must be multiplied by the specific gravity of the liquid.

$$P = 0.433 \times h \times \text{Sg} \tag{2.7}$$

where P = pressure, psi
h = head, ft
Sg = specific gravity of liquid (for water, Sg = 1.00)

In SI units the pressure versus head equation becomes

$$P = h\frac{\text{Sg}}{0.102} \tag{2.8}$$

where P = pressure, kPa
h = head, m
Sg = specific gravity of liquid (for water, Sg = 1.00)

Generally, pressure in a body of water or a water pipeline is referred to in psi above that of the atmospheric pressure. This is also known as the *gauge pressure* as measured by a pressure gauge. The *absolute pressure* is the sum of the gauge pressure and the atmospheric pressure at the specified location. Mathematically,

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}} \tag{2.9}$$

To distinguish between the two pressures, psig is used for gauge pressure and psia is used for the absolute pressure. In most calculations involving fire protection water pipelines the gauge pressure is used. Unless otherwise specified, psi means the gauge pressure.

Water pressure may also be referred to as *head pressure*, in which case it is expressed in feet (or meters in SI units) of head of water. Therefore, a pressure of 100 psi in a liquid such as water is said to be equivalent to a pressure head of

$$h = \frac{100}{0.4333} = 231\,\text{ft}$$

Example 2.3 Calculate the pressure in psi at a water depth of 100 ft assuming the specific weight of water is 62.4 lb/ft^3. What is the equivalent pressure in kilopascals? If the atmospheric pressure is 14.7 psi, calculate the absolute pressure at that location.

Solution Using Eq. (2.6), we calculate the pressure:

$$P = 0.433 \times 100 \times 1.0 = 43.33\,\text{psig}$$

$$\text{Absolute pressure} = 43.33 + 14.7 = 58.03\,\text{psia}$$

In SI units we can calculate the pressure as follows:

$$\text{Pressure} = \frac{(100/3.281) \times 1.0}{0.102} = 298.8\,\text{kPa}$$

Alternatively,

$$\text{Pressure in kPa} = \frac{\text{pressure in psi}}{0.145}$$
$$= \frac{43.33}{0.145} = 298.83\,\text{kPa}$$

Example 2.4 A new sprinkler system is being installed for a 120-ft-high building. A 4-in sprinkler riser pipe is used to feed the top floor of the building. Assuming no pump pressure, calculate the pressure at the bottom of the riser.

Solution Pressure at the bottom of the 120-ft riser pipe is, per Eq. (2.6),

$$P = 120 \times 0.433 = 51.96\,\text{psi}$$

Example 2.5 A fire pump used in conjunction with a fire protection system has a pressure rating of 150 ft. Calculate the pressure developed by the pump.

Solution

$$\text{Pressure developed by pump} = 150\,\text{ft}$$
$$= 150 \times 0.433 = 64.95\,\text{psi}$$

2.3.2 Velocity

As water flows through fire protection piping at a constant flow rate, the velocity of flow can be calculated by the following equation:

$$\text{Flow rate} = \text{area} \times \text{velocity}$$

Therefore,

$$Q = A \times V \tag{2.10}$$

where Q = flow rate
A = pipe cross-sectional area
V = velocity of flow

Since flow rate is generally expressed in gal/min and pipe diameter is in inches, to obtain the velocity in ft/s we must use correct conversion factors.

$$A = 0.7854\left(\frac{D}{12}\right)^2$$

where A is the pipe cross-sectional area (ft^2) and D is the pipe inside diameter (in).

Therefore, the velocity is

$$V = \frac{Q}{A} = \frac{Q}{60 \times 7.48 \times 0.7854 \times (D/12)^2}$$

Simplifying,

$$V = 0.4085 \times \frac{Q}{D^2} \tag{2.11}$$

where V = velocity of flow, ft/s
Q = flow rate, gal/min
D = pipe inside diameter, in

In SI units, the velocity equation is as follows:

$$V = 353.6777 \frac{Q}{D^2} \tag{2.12}$$

where V = velocity, m/s
Q = flow rate, m^3/h
D = inside diameter, mm

Example 2.6 Water flows through an 8-in inside diameter fire protection water piping system at the rate of 1000 gal/min. Calculate the velocity of flow.

Solution From Eq. (2.11), the average flow velocity is

$$V = 0.4085 \frac{1000}{8^2} = 6.38 \text{ ft/s}$$

Therefore, velocity is 6.38 ft/s

The velocity of flow through a pipe depends upon the flow rate and the inside diameter of the pipe as shown by Eq. (2.11). On examining this equation we see that the velocity decreases as the pipe diameter increases, and vice versa. If at some point in the piping system the pipe diameter changes, the velocity will change in accordance with Eq. (2.11). We can calculate the velocity of flow through different sections of pipe with different diameters using the continuity equation. The continuity equation simply states that under steady flow the quantity of water Q passing through every cross section of pipe is the same. Using Eq. (2.10) we can write the following:

$$Q = A_1 V_1 = A_2 V_2 \tag{2.13}$$

where Q = flow rate
A_1, A_2 = pipe cross-sectional area at points 1 and 2, respectively, along pipeline
V_1, V_2 = velocities at points 1 and 2, respectively

Therefore, if we know the flow rate Q and the diameter of the pipe at points 1 and 2, we can calculate the velocity at points 1 and 2.

Example 2.7 Water flows through a fire protection water piping system at the rate of 450 gal/min. The diameter of the pipe starts at NPS 8, 0.250-in wall thickness and reduces to NPS 4, schedule 40, at a section 200 ft downstream. Calculate the velocity of water in both pipe sizes.

Solution

$$\text{Inside diameter for NPS 8, 0.250-in wall thickness} = 8.625 - (2 \times 0.250)$$
$$= 8.125\text{ in}$$
$$\text{Inside diameter for NPS 4, schedule 40} = 4.026\text{in}$$

$$\text{Velocity of water in NPS 8 pipe} = \frac{0.4085 \times 450}{8.125^2}$$
$$= 2.78\text{ ft/s}$$
$$\text{Velocity of water in NPS 4 pipe} = \frac{0.4085 \times 450}{4.026^2}$$
$$= 11.34\text{ ft/s}$$

2.4 Pressure Drop Due to Friction

As water flows through fire protection water piping there is a certain amount of friction between the water and the pipe wall. This causes the pressure to decrease in the direction of flow. If P_1 represents the pressure in the piping at some point A, and P_2 represents the pressure at some downstream point B, due to friction P_2 is less than P_1. The difference between P_1 and P_2 is the pressure drop due to friction, also known as head loss. The greater the distance between A and B, the greater will be the pressure drop $P_1 - P_2$.

If the pipe is horizontal with no elevation difference between points A and B, the pressure drop $P_1 - P_2$ will depend only on the following:

1. Flow rate
2. Pipe inside diameter
3. Internal condition of pipe, such as rough or smooth

If there is an elevation difference between points A and B, we must add a fourth item to the list:

4. Elevation difference between A and B

Most piping designs are such that the friction loss in the piping is minimized so as to provide the maximum flow rate with existing equipment and pipe size. Before we discuss the various formulas to calculate the pressure drop in fire protection water piping systems, we must introduce some general concepts of pipe flow, including the Reynolds number of flow.

2.4.1 Reynolds number

The Reynolds number is a dimensionless parameter of flow. It depends on the pipe size, flow rate, liquid viscosity (for water, viscosity = 1.0 cSt), and density. It is calculated from the following equation:

$$R = \frac{VD\rho}{\mu} \tag{2.14}$$

or

$$R = \frac{VD}{\nu} \tag{2.15}$$

where R = Reynolds number, dimensionless
V = average flow velocity, ft/s
D = pipe inside diameter, ft
ρ = mass density of liquid, slug/ft^3
μ = dynamic viscosity, slug/(ft · s)
ν = kinematic viscosity, ft^2/s

Since R must be dimensionless, a consistent set of units must be used for all items in Eq. (2.14) to ensure that all units cancel out and R has no dimensions. A more convenient version of the Reynolds number using USCS units in fire protection piping is as follows:

$$R = 3162.5\frac{Q}{D\nu} \tag{2.16}$$

where R = Reynolds number, dimensionless
Q = flow rate, gal/min
D = pipe inside diameter, in
ν = kinematic viscosity, cSt (for water, $\nu = 1.0$)

In SI units, the Reynolds number is expressed as follows

$$R = 353{,}678 \frac{Q}{\nu D} \tag{2.17}$$

where R = Reynolds number, dimensionless
Q = flow rate, m^3/h
D = pipe inside diameter, mm
ν = kinematic viscosity, cSt (for water, $\nu = 1.0$)

2.4.2 Types of flow

Flow through a pipe can be classified as laminar flow, turbulent flow, or critical flow depending on the Reynolds number. If the flow is such that the Reynolds number is less than 2000 to 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is said to be *turbulent*. *Critical flow* occurs when the Reynolds number is in the range of 2100 to 4000. Laminar flow is characterized by smooth flow in which there are no eddies or turbulence. The flow is said to occur in laminations. If dye was injected into a transparent pipe, laminar flow would be manifested in the form of smooth streamlines of dye. Turbulent flow occurs at higher velocities and is accompanied by eddies and other disturbances in the water. Mathematically, if R represents the Reynolds number of flow, the flow types are defined as follows:

Laminar flow: $R \leq 2100$

Critical flow: $2100 < R \leq 4000$

Turbulent flow: $R > 4000$

In the critical flow regime, where the Reynolds number is between 2100 and 4000, the flow is undefined as far as pressure drop calculations are concerned.

Example 2.8 Fire water flows through an NPS 8 pipeline, schedule 30 at 500 gal/min. Calculate the average velocity and the Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution Using Eq. (2.11), the average velocity is calculated as follows:

$$V = 0.4085 \frac{500}{8.071^2} = 3.14 \text{ ft/s}$$

From Eq. (2.16), the Reynolds number is

$$R = 3162.5 \frac{500}{8.071 \times 1.0} = 195,917$$

Since $R > 4000$, the flow is turbulent.

Example 2.9 Water flows through a DN 200 (6-mm wall thickness) pipe at 150 m^3/h. Calculate the average velocity and Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution From Eq. (2.12) the average velocity is

$$V = 353.6777 \frac{150}{188^2} = 1.50\,\text{m/s}$$

From Eq. (2.17) the Reynolds number is

$$R = 353{,}678 \frac{150}{188 \times 1.0} = 282{,}190$$

Since $R > 4000$, the flow is turbulent.

2.4.3 Darcy-Weisbach equation

Several formulas have been put forth to calculate the pressure drop in fire protection water piping. Among them, the Darcy-Weisbach and Hazen-Williams equations are the most popular.

We will first introduce the Darcy-Weisbach equation, also known simply as the Darcy equation, for calculating the friction loss in fire protection piping. The following form of the Darcy equation is the simplest used by engineers for a long time. In this version the head loss in feet (as opposed to pressure drop in psi) is given in terms of the pipe diameter, pipe length, and flow velocity.

$$h = f \frac{L}{D} \frac{V^2}{2g} \tag{2.18}$$

where h = frictional head loss, ft
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = pipe inside diameter, ft
V = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

In USCS units, $g = 32.2$ ft/s^2, and in SI units, $g = 9.81$ m/s^2. The friction factor f is a dimensionless value that depends upon the internal roughness of the pipe and the Reynolds number.

Note that the Darcy equation (2.18) gives the frictional pressure loss in feet of head of water. It can be converted to pressure loss in psi using Eq. (2.6). The term $V^2/2g$ in the Darcy equation is called the velocity head, and it represents the kinetic energy of the water. The term *velocity head* will be used in subsequent sections of this chapter when discussing frictional head loss through pipe fittings and valves.

Another form of the Darcy equation with frictional pressure drop expressed in psi/ft and using a flow rate instead of velocity is as follows:

$$P_f = 0.0135 \frac{f Q^2}{D^5} \tag{2.19}$$

where P_f = frictional pressure loss, psi/ft
f = Darcy friction factor, dimensionless
Q = flow rate, gal/min
D = pipe inside diameter, in

In SI units, the Darcy equation may be written as

$$h = 50.94 \frac{f L V^2}{D} \tag{2.20}$$

where h = frictional head loss, m
f = Darcy friction factor, dimensionless
L = pipe length, m
D = pipe inside diameter, mm
V = average flow velocity, m/s

Another version of the Darcy equation in SI units is as follows:

$$P_m = (6.2475 \times 10^7) \frac{f Q^2}{D^5} \tag{2.21}$$

where P_m = pressure drop due to friction, kPa/m
Q = flow rate, m^3/h
f = Darcy friction factor, dimensionless
D = pipe inside diameter, mm

In order to calculate the friction loss in a fire protection water pipeline using the Darcy equation, we must know the friction factor f. The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eq. (2.18). This friction factor is a dimensionless number between 0.0 and 0.1 (usually around 0.02 for turbulent flow) that depends on the internal roughness of the pipe, pipe diameter, and the Reynolds number and therefore the type of flow (laminar or turbulent).

For laminar flow, the friction factor f depends only on the Reynolds number and is calculated as follows:

$$f = \frac{64}{R} \tag{2.22}$$

where f is the friction factor for laminar flow and R is the Reynolds number for laminar flow ($R < 2100$) (dimensionless).

Therefore, if the Reynolds number for a particular flow is 1200, the flow is laminar and the friction factor according to Eq. (2.22) is

$$f = \frac{64}{1200} = 0.0533$$

If this pipeline has a 200-mm inside diameter and water flows through it at 100 m^3/h, the pressure loss per meter would be, from Eq. (2.21),

$$P_m = 6.2475 \times 10^7 \times 0.0533 \frac{100^2}{200^5} = 0.1041\,\text{kPa/m}$$

If the flow is turbulent ($R > 4000$), calculation of the friction factor is not as straightforward as that for laminar flow. We will discuss this next.

In turbulent flow the calculation of friction factor f is more complex. The friction factor depends on the pipe inside diameter, pipe roughness, and the Reynolds number. Based on work by Moody, Colebrook-White, and others, the following empirical equation, known as the Colebrook-White equation, has been proposed for calculating the friction factor in turbulent flow:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{R\sqrt{f}}\right) \tag{2.23}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
R = Reynolds number, dimensionless

The absolute pipe roughness e depends on the internal condition of the pipe. Generally a value of 0.002 in or 0.05 mm is used in most calculations, unless better data are available. Table 2.2 lists the pipe roughness for various types of pipe. The ratio e/D is known as the relative pipe roughness and is dimensionless since both pipe absolute

TABLE 2.2 Pipe Internal Roughness

Pipe material	Roughness	
	in	mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

roughness e and pipe inside diameter D are expressed in the same units (inches in USCS units and millimeters in SI units). Therefore, Eq. (2.23) remains the same for SI units, except that, as stated, the absolute pipe roughness e and the pipe diameter D are both expressed in mm. All other terms in the equation are dimensionless.

It can be seen from Eq. (2.23) that the calculation of the friction factor f is not straightforward since it appears on both sides of the equation. Successive iteration or a trial-and-error approach is used to solve for the friction factor.

Suppose $R = 300{,}000$ and $e/D = 0.002/8 = 0.0003$. To solve for the friction factor f from Eq. (2.23), we first assume a value of f and substitute that value on the right-hand side of the equation. This will give us a new value of f. Using the new value of f on the right-hand side of the equation again, we recalculate f. This process is continued until successive values of f are within a small tolerance, such as 0.001.

Continuing with the example, try $f = 0.02$ initially. Therefore,

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.0003}{3.7} + \frac{2.51}{300{,}000\sqrt{0.02}}\right)$$

Solving, $f = 0.0168$.

Using this value again in the preceding equation, we get the next approximation to f as

$$f = 0.017$$

And repeating the process, we finally get $f = 0.017$.

2.4.4 Moody diagram

The Moody diagram is a graphical plot of the friction factor f for all flow regimes (laminar, critical, and turbulent) against the Reynolds number at various values of the relative roughness of pipe. The graphical method of determining the friction factor for turbulent flow using the Moody diagram (see Fig. 2.4) is discussed next.

For a given Reynolds number on the horizontal axis, a vertical line is drawn up to the curve representing the relative roughness e/D. The friction factor is then read by going horizontally to the vertical axis on the left. It can be seen from the Moody diagram that the turbulent region is further divided into two regions: the "transition zone" and the "complete turbulence in rough pipes" zone. The lower boundary is designated as "smooth pipes," and the transition zone extends up to the dashed line. Beyond the dashed line is the complete turbulence in rough pipes zone. In this zone, the friction factor depends very little on the Reynolds number and more on the relative roughness. This is evident from the Colebrook-White equation, where at large Reynolds numbers,

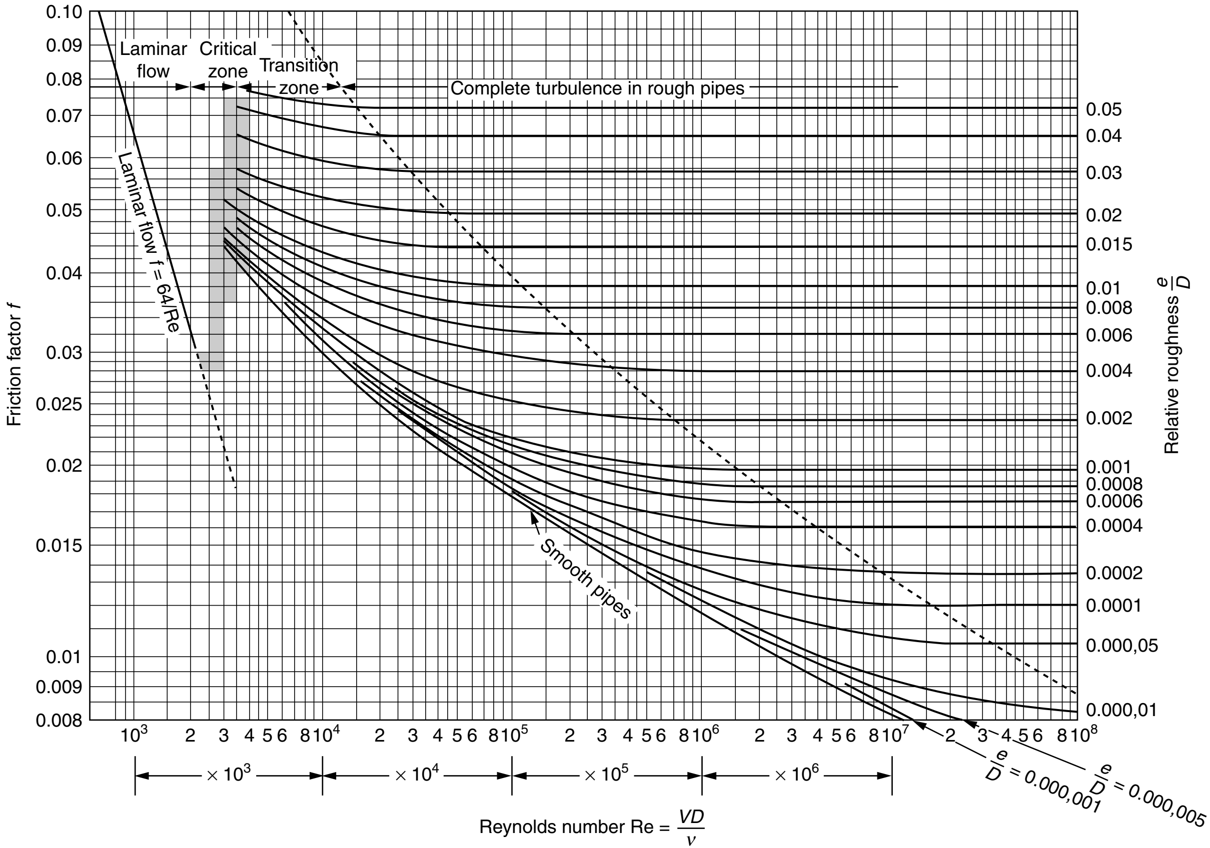

Figure 2.4 Moody diagram.

the second term within the parentheses approaches zero. The friction factor thus depends only on the first term, which is proportional to the relative roughness e/D. In contrast, in the transition zone both R and e/D influence the value of friction factor f.

Example 2.10 Water flows through an NPS 6 schedule 40 pipeline at 500 gal/min. Assuming a pipe roughness of 0.002 in, calculate the friction factor and head loss due to friction in 100 ft of pipe length, using the Colebrook-White equation.

Solution NPS 6, schedule 40 pipe has an inside diameter of 6.065 in. Using Eq. (2.11), we calculate the velocity as

$$V = 0.4085 \frac{500}{6.065^2} = 5.55\,\text{ft/s}$$

Using Eq. (2.16) we calculate the Reynolds number as follows:

$$R = 3162.5 \frac{500}{6.065 \times 1.0} = 260{,}717$$

Thus the flow is turbulent and we can use the Colebrook-White equation (2.23), to calculate the friction factor.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 6.065} + \frac{2.51}{260{,}717\sqrt{f}}\right)$$

Solving for f by trial and error, we get $f = 0.0152$. Thus the friction factor is 0.0152.

The head loss due to friction can now be calculated using the Darcy equation (2.18):

$$h = 0.0152 \frac{100 \times 12}{6.065} \frac{(5.55)^2}{64.4} = 1.44\,\text{ft of head of water}$$

Converting to psi, using Eq. (2.6), we get

$$\text{Pressure drop due to friction} = 1.44 \times 0.433 = 0.624\,\text{psi}$$

Example 2.11 A steel pipe DN 250 (8-mm wall thickness) is used to transport water from a fire pump to a fire protection water distribution piping system. Calculate the friction factor and pressure loss in kPa/m due to friction at a flow rate of 250 m^3/h. Assume a pipe roughness of 0.05 mm. Use the Moody diagram to calculate the pressure drop and determine the pumping pressure required if the pipe length is 198 m. If the delivery point is located at a height of 50 m, calculate the pump pressure.

Solution The DN 250 (8-mm wall thickness) pipe has an inside diameter,

$$D = 250 - 2 \times 8 = 234\,\text{mm}$$

The average flow velocity is calculated using Eq. (2.12):

$$V = 353.6777 \frac{250}{234^2} = 1.61\,\text{m/s}$$

Next using Eq. (2.17), we get the Reynolds number as follows:

$$R = 353{,}678 \frac{250}{1.0 \times 234} = 377{,}860$$

Therefore, the flow is turbulent. We can use the Colebrook-White equation or the Moody diagram to determine the friction factor.

$$\text{Relative roughness } \frac{e}{D} = \frac{0.05}{234} = 0.0002$$

Using the preceding values for relative roughness and the Reynolds number, from the Moody diagram we get friction factor $f = 0.0162$.

The pressure drop due to friction can now be calculated using the Darcy equation (2.18) for the entire 198-m length of pipe as

$$h = 0.0162 \frac{198}{0.234} \frac{1.61^2}{2 \times 9.81} = 1.81\,\text{m of head of water}$$

Using Eq. (2.8) we calculate the pressure drop in kPa as follows:

$$\text{Total pressure drop in 198 m} = 1.81 \frac{1.0}{0.102} = 17.75\,\text{kPa}$$

Therefore,

$$\text{Pressure drop in kPa/m} = \frac{17.75}{198} = 0.0897\,\text{kPa/m}$$

If the delivery point is at a height of 50 m,

$$\text{Pump pressure required} = 50 + 1.81 = 51.81\,\text{m}$$

or

$$\frac{51.81}{0.102} = 508\,\text{kPa}$$

2.4.5 Hazen-Williams equation

For water pipelines, generally the Hazen-Williams equation is found to give fairly accurate results compared to field data. Therefore, this method is used in fire protection piping as well. However, as will be seen shortly there are uncertainties associated with the C factor used in the Hazen-Williams formula and there is a tendency to fall back on classical equations such as the Darcy formula discussed earlier, especially for high-pressure and high-flow piping system.

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TABLE 2.3 Hazen-Williams *C* Factor

Pipe material	*C* factor
Smooth pipes (all metals)	130–140
Cast iron (old)	100
Cast iron (unlined new)	120
Iron (worn/pitted)	60–80
Polyvinyl chloride (PVC)	150
Brick	100
Smooth wood	120
Smooth masonry	120
Vitrified clay	110
Plastic	150

The Hazen-Williams equation for calculating the pressure drop due to friction for a given pipe diameter and flow rate is as follows

$$\Delta P = 4.524\left(\frac{Q}{C}\right)^{1.85}\frac{1}{D^{4.87}} \tag{2.24}$$

where ΔP = pressure loss due to friction, psi per ft of pipe length
Q = flow rate, gal/min
D = pipe inside diameter, in
C = Hazen-Williams roughness coefficient factor, dimensionless

Equation (2.24) has been specially modified for water (specific gravity = 1.00). The Hazen-Williams C factor depends on the type of pipe material and the internal condition of the pipe. Table 2.3 gives a list of C values used in practice.

In general an average value of $C = 100$ is used for most applications. A low value such as $C = 75$ may be used for pipe that is 10 to 15 years old. Steel pipe used in sprinkler systems is designed for $C = 100$, if the pipe size is 2 in or smaller or $C = 120$ for larger pipe.

In SI units the Hazen-Williams equation is as follows:

$$\Delta P = 1.1101 \times 10^{10}\left(\frac{Q}{C}\right)^{1.85}\frac{1}{D^{4.87}} \tag{2.25}$$

where ΔP = frictional pressure drop, kPa/m
Q = flow rate, m^3/h
D = pipe inside diameter, mm
C = Hazen-Williams C factor, dimensionless (see Table 2.3)

Example 2.12 A 4-in pipe is used to transport 300 gal/min of water in a fire protection piping system. Using a C value of 100 in the Hazen-Williams equation, calculate the friction loss in 650 ft of pipe.

Solution Assuming the given pipe size to be the inside diameter and using the Hazen-Williams equation, the pressure drop is

$$\Delta P = 4.524\left(\frac{300}{100}\right)^{1.85}\frac{1}{4^{4.87}} = 0.0404\,\text{psi/ft}$$

Total pressure drop for 650 ft of pipe $= 650 \times 0.0404 = 26.25\,\text{psi}$

2.4.6 Friction loss tables

Using the Hazen-Williams equation, friction loss tables have been constructed that provide the pressure drop in various pipe sizes and flow rates considering different C factors. Table 2.4 shows a typical friction loss table in abbreviated form. For a complete list of friction loss tables the reader is advised to refer to a handbook such as *Fire Protection Systems* by Robert M. Gagnon, Delmar Publishers, 1997. We will illustrate the use of the friction loss table to calculate the pressure drop in a fire protection piping system.

Consider, for example, a 4-in schedule 40 steel pipe (4.026-in inside diameter) with a water flow of 200 gal/min. The pressure drop with a C factor of 100 is found to be 0.0185 psi/ft from the friction loss table. Therefore, if the piping is 500 ft long, the total pressure drop due to friction will be $500 \times 0.0185 = 9.25$ psi.

We will now verify the preceding using the Hazen-Williams equation (2.24) as follows:

$$\Delta P = 4.524\left(\frac{200}{100}\right)^{1.85}\frac{1}{4.026^{4.87}} = 0.0185\,\text{psi/ft}$$

which is exactly what we found using the friction loss table. These friction loss tables are quite handy when we need to quickly check the pressure drop in various size piping used in fire protection systems.

2.4.7 Losses in valves and fittings

So far, we have calculated the pressure drop per unit length in straight pipe. Minor losses in a fire protection pipeline are classified as those pressure drops that are associated with piping components such as valves and fittings. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. A pipe nozzle exiting from a storage tank will have entrance and exit losses. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe.

Generally, minor losses are included in calculations by using the equivalent length of the valve or fitting (found from a table such as

TABLE 2.4 Friction Loss Table

Schedule 40 Steel Pipe																					Schedule 30 Steel Pipe		
1-in (ID = 1.049 in)			1.5-in (ID = 1.61 in)			2-in (ID = 2.067 in)			2.5-in (ID = 2.469 in)			3-in (ID = 3.068 in)			4-in (ID = 4.026 in)			6-in (ID = 6.065 in)			8-in (ID = 8.071 in)		
Q, gal/min	ΔP, psi/ft	V, ft/s	Q, gal/min	ΔP, psi/ft	V, ft/s	Q, gal/min	ΔP, psi/ft	V, ft/s	Q, gal/min	ΔP, psi/ft	V, ft/s	Q, gal/min	ΔP, psi/ft	V, ft/s	Q, gal/min	ΔP, psi/ft	V, ft/s	Q, gal/min	ΔP, psi/ft	V, ft/s	Q, gal/min	ΔP, psi/ft	V, ft/s
7	0.0261	2.6	15	0.0133	2.4	30	0.0142	2.9	40	0.0102	2.7	50	0.0053	2.2	100	0.0051	2.5	400	0.009	4.4	500	0.0034	3.1
10	0.0506	3.7	20	0.0226	3.2	40	0.0242	3.8	55	0.0183	3.7	70	0.0099	3.0	140	0.0095	3.5	500	0.0137	5.6	700	0.0063	4.4
15	0.1071	5.6	25	0.0342	3.9	50	0.0365	4.8	70	0.0286	4.7	90	0.0158	3.9	180	0.0152	4.5	600	0.0192	6.7	900	0.0101	5.6
20	0.1823	7.4	30	0.0479	4.7	60	0.0512	5.7	85	0.0410	5.7	110	0.0229	4.8	220	0.022	5.5	700	0.0255	7.8	1300	0.0199	8.2
25	0.2755	9.3	35	0.0637	5.5	70	0.0681	6.7	100	0.0554	6.7	130	0.0313	5.6	250	0.0279	6.3	800	0.0326	8.9	1700	0.0327	10.7
30	0.3860	11.1	40	0.0816	6.3	80	0.0871	7.7	115	0.0718	7.7	150	0.0407	6.5	280	0.0344	7.1	900	0.0406	10.0	1930	0.0414	12.1
35	0.5134	13.0	45	0.1015	7.1	90	0.1083	8.6	130	0.0900	8.7	170	0.0513	7.4	310	0.0415	7.8	1000	0.0493	11.1	2130	0.0496	13.4
40	0.6573	14.9	50	0.1233	7.9	100	0.1317	9.6	145	0.1102	9.7	190	0.0631	8.3	340	0.0493	8.6	1100	0.0588	12.2	2530	0.0683	15.9
45	0.8173	16.7	55	0.1471	8.7	110	0.1570	10.5	160	0.1322	10.7	210	0.0759	9.1	370	0.0576	9.3	1200	0.0691	13.3	2730	0.0786	17.1
50	0.9932	18.6	60	0.1728	9.5	120	0.1845	11.5	175	0.1560	11.7	233	0.0920	10.1	400	0.0666	10.1	1300	0.0801	14.4	2930	0.0896	18.4
55	1.1848	20.4	65	0.2003	10.2	130	0.2139	12.4	190	0.1817	12.7	253	0.1071	11.0	430	0.0761	10.8	1400	0.0919	15.6	3330	0.1135	20.9
60	1.3917	22.3	70	0.2298	11.0	140	0.2453	13.4	205	0.2091	13.7	273	0.1233	11.9	460	0.0862	11.6	1500	0.1044	16.7	3530	0.1264	22.1
65	1.6138	24.1	75	0.2611	11.8	150	0.2787	14.3	220	0.2383	14.8	293	0.1406	12.7	490	0.0969	12.4	1600	0.1176	17.8	3730	0.1400	23.4
70	1.8509	26.0	80	0.2942	12.6	160	0.3141	15.3	235	0.2692	15.8	313	0.1588	13.6	520	0.1081	13.1	1700	0.1315	18.9	4130	0.1690	25.9
75	2.1029	27.9	85	0.3291	13.4	170	0.3514	16.3	250	0.3018	16.8	333	0.1781	14.5	550	0.1200	13.9	1800	0.1462	20.0	4530	0.2005	28.4
80	2.3696	29.7	90	0.3658	14.2	180	0.3906	17.2	265	0.3362	17.8	353	0.1984	15.3	580	0.1324	14.6	1950	0.1696	21.7	4730	0.2172	29.7
85	2.6509	31.6	95	0.4043	15.0	190	0.4316	18.2	280	0.3722	18.8	373	0.2197	16.2	610	0.1453	15.4	2100	0.1945	23.3	4930	0.2345	30.9
89	2.8862	33.1	100	0.4445	15.8	200	0.4746	19.1	295	0.4100	19.8	393	0.2420	17.1	640	0.1588	16.1	2220	0.2155	24.7	5100	0.2497	32.7
			106	0.4951	16.7	210	0.5194	20.1	310	0.4493	20.8	416	0.2688	18.1	670	0.1728	16.9	2370	0.2432	26.3			
			121	0.6325	19.1	220	0.5661	21.0	325	0.4904	21.8	446	0.3058	19.4	700	0.1874	17.7	2460	0.2606	27.3			
			126	0.6817	19.9	230	0.6146	22.0	340	0.5331	22.8	476	0.3449	20.7	730	0.2026	18.4	2520	0.2725	28.0			
			131	0.7326	20.7	240	0.665	23.0	355	0.5774	23.8	506	0.3862	22.0	760	0.2182	19.2	2640	0.297	29.3			
			136	0.7851	21.4	250	0.7172	23.9	370	0.6234	24.8	536	0.4296	23.3	790	0.2344	19.9	2700	0.3096	30.0			
			146	0.8953	23.0	260	0.7711	24.9	390	0.6871	26.1	566	0.4752	24.6	830	0.2569	20.9	2880	0.3488	32.0			
			151	0.9528	23.8	270	0.8269	25.8	410	0.7537	27.5	596	0.5228	25.9	875	0.2832	22.1						
			161	1.0728	25.4	280	0.8844	26.8	430	0.8231	28.8	626	0.5726	27.2	920	0.3107	23.2						
			171	1.1993	27.0	305	1.0361	29.2	450	0.8954	30.2	656	0.6243	28.5	1010	0.3693	25.5						
			201	1.6174	31.7	335	1.2324	32.0	477	0.9973	32.0	686	0.6782	29.8	1055	0.4003	26.6						
												716	0.7341	31.1	1100	0.4325	27.7						
												736	0.7725	32.0	1160	0.4771	29.3						
															1205	0.5120	30.4						
															1250	0.5479	31.5						
															1310	0.5975	33.0						

NOTE: Based on $C = 100$.

TABLE 2.5 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	50
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

Table 2.5) or using a resistance factor K multiplied by the velocity head $V^2/2g$. The term minor losses can be applied only where the pipeline lengths and hence the friction losses are relatively large compared to the pressure drops in the fittings and valves. In fire protection piping, depending upon the pipe length, pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses is really a misnomer. In any case, the pressure losses through valves, fittings, etc., can be accounted for approximately using the equivalent length or K times the velocity head method. A table listing the equivalent lengths of valves and fittings along with the K factors is shown in Table 2.6.

As an example, if the total length of straight pipe were 250 ft and all valves, fittings, etc., amounted to an equivalent length of 40 ft, we would calculate the total pressure loss in this piping system as follows, considering a total equivalent length of 290 ft of pipe:

$$\text{Total friction loss in pipe and fittings} \\ = 290 \times \text{pressure drop per ft of pipe}$$

Table 2.5 shows the equivalent length of commonly used valves and fittings in fire protection water pipelines. It can be seen from this table that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore a 6-in-diameter gate valve may be replaced with $6 \times 8 = 48$-in-long

TABLE 2.6 Friction Loss in Valves—Resistance Coefficient *K*

		Nominal pipe size, in											
Description	L/D	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72

piece of pipe that will match the frictional pressure drop through the valve.

Example 2.13 A fire protection piping system is 500 ft of NPS 8 pipe, schedule 30 that has two 8-in gate valves and four NPS 8, 90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe and valves and fittings. What is the pressure drop due to friction at 900 gal/min? Use the Hazen-Williams equation with $C = 120$.

Solution Using Table 2.5, we can convert all valves and fittings in terms of 8-in pipe as follows:

$$\text{Two NPS 8 gate valves} = 2 \times 8 \times 8 = 132 \text{ in of NPS 8 pipe}$$

$$\text{Four NPS 8 } 90^\circ \text{ elbows} = 4 \times 8 \times 30 = 960 \text{ in of NPS 8 pipe}$$

$$\text{Total for all valves and fittings} = 132 + 960 = 1092 \text{ in} = 91 \text{ ft of NPS 8 pipe}$$

Adding the 500 ft of straight pipe,

$$\begin{aligned}&\text{Total equivalent length of straight pipe and all fittings}\\ &\quad = 500 + 91 = 591 \text{ ft of NPS 8 pipe}\end{aligned}$$

The pressure drop due to friction in the preceding piping system can now be calculated based on 591 ft of pipe. Using Hazen-Williams equation (2.24), we get

$$\Delta P = 4.524 \left(\frac{900}{120}\right)^{1.85} \frac{1}{8.071^{4.87}} = 0.0072 \text{ psi/ft}$$

where NPS 8, schedule 30 pipe is taken to have an 8.07-in inside diameter.

$$\text{Total pressure drop} = 591 \times 0.0072 = 4.26 \text{ psi}$$

Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation (2.18), the pressure drop in a straight length of pipe is given by

$$h = f \frac{L}{D} \frac{V^2}{2g}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K (also known as the resistance coefficient) and the preceding equation then becomes

$$h = K \frac{V^2}{2g} \tag{2.26}$$

where $K = f(L/D)$.

In Eq. (2.26), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $V^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(V^2/2g)$ where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 2.6. It can be seen that the K factor depends on the nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

From Table 2.6 it can be seen that a 6-in gate valve has a K factor value of 0.12, while a 10-in gate valve has a K factor of 0.11. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be $0.12(V^2/2g)$ and that in the 10-in valve is $0.11(V^2/2g)$. The velocities in both cases will be different due to the difference in diameters.

If the flow rate was 1000 gal/min, the velocity in the 6-in valve will be approximately

$$V_6 = 0.4085 \frac{1000}{6.125^2} = 10.89 \text{ ft/s}$$

Similarly, at 1000 gal/min, the velocity in the 10-in valve will be approximately

$$V_6 = 0.4085 \frac{1000}{10.25^2} = 3.89 \text{ ft/s}$$

Therefore,

$$\text{Head loss in 6-in gate valve} = \frac{0.12(10.89)^2}{64.4} = 0.22 \text{ ft}$$

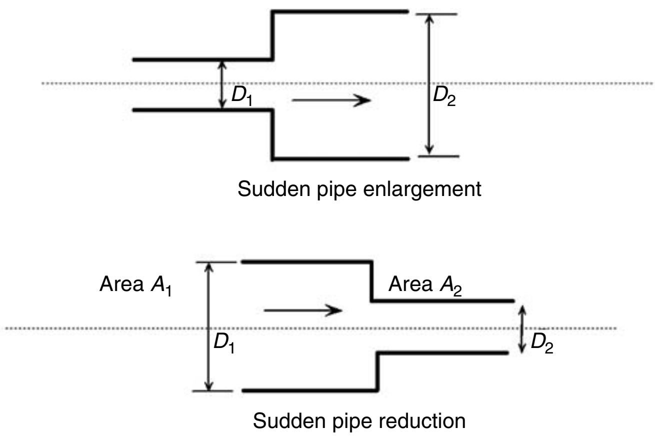
$$\text{head loss in 10-in gate valve} = \frac{0.11(3.89)^2}{64.4} = 0.026 \text{ ft}$$

It can be seen that the head loss in the 10-in valve is only about one-tenth of that in the 6-in valve. Both head losses are still very small compared to the head loss in straight 6-in pipe, about 0.05 psi/ft. One hundred feet of 6-in pipe will have a pressure drop of 5 psi compared to the very small losses in the 6-in and 10-in valves.

Pipe enlargement and reduction. Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(v_1 - v_2)^2}{2g} \tag{2.27}$$

where v_1 and v_2 are the velocities of the liquid in the two pipe sizes D_1 and D_2, respectively. Writing Eq. (2.27) in terms of pipe cross-sectional



A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 2.5 Sudden pipe enlargement and pipe reduction.

areas A_1 and A_2 (as illustrated in Fig. 2.5), we obtain

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{{v_1}^2}{2g} \tag{2.28}$$

for sudden enlargement.

For sudden contraction or reduction in pipe size as shown in Fig. 2.5, the head loss is calculated from

$$h_f = \left(\frac{1}{C_c} - 1\right) \frac{{v_2}^2}{2g} \tag{2.29}$$

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 2.5.

Gradual enlargement and reduction of pipe size, as shown in Fig. 2.6, cause less head loss than sudden enlargement and sudden reduction. For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c(v_1 - v_2)^2}{2g} \tag{2.30}$$

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is shown in Fig. 2.7.

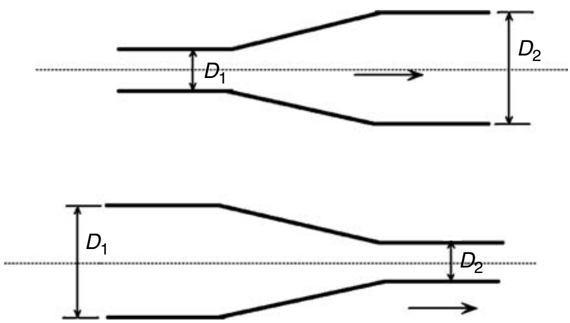


Figure 2.6 Gradual pipe enlargement and pipe reduction.

Pipe entrance and exit losses. The K factors for computing the head loss associated with pipe entrance and exit are as follows:

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

2.4.8 Complex piping systems

So far we have discussed straight length of pipe with valves and fittings. Complex piping systems include pipes of different diameters in series and parallel configuration. Fire protection piping is designed as a looped system or grid system. A loop system provides water supply from more than one location to any point. Sprinkler systems piping has simple

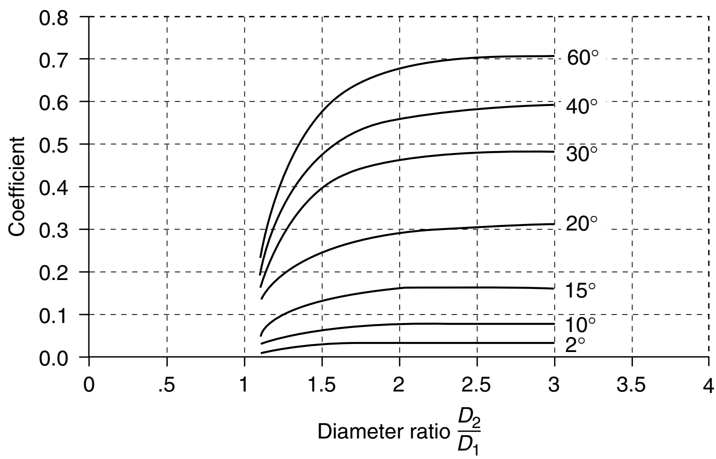


Figure 2.7 Gradual pipe expansion head loss coefficient.

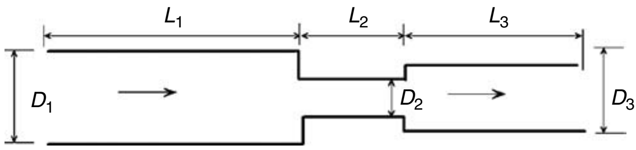


Figure 2.8 Series piping.

loops or complex loops depending on the piping arrangement. We will discuss both series and parallel piping next.

Series piping. Series piping in its simplest form consists of two or more different pipe sizes connected end to end as illustrated in Fig. 2.8. Pressure drop calculations in series piping may be handled in one of two ways. The first approach would be to calculate the pressure drop in each pipe size and add them together to obtain the total pressure drop. Another approach is to consider one of the pipe diameters as the base size and convert other pipe sizes into equivalent lengths of the base pipe size. The resultant equivalent lengths are added together to form one long piece of pipe of constant diameter equal to the base diameter selected. The pressure drop can now be calculated for this single-diameter pipeline. Of course, all valves and fittings will also be converted to their respective equivalent pipe lengths using the L/D ratios from Table 2.5.

Consider three sections of pipe joined together in series. Using subscripts 1, 2, and 3 and denoting the pipe length as L, inside diameter as D, and flow rate as Q, we can calculate the equivalent length of each pipe section in terms of a base diameter. This base diameter will be selected as the diameter of the first pipe section D_1. Since equivalent length is based on the same pressure drop in the equivalent pipe as the original pipe diameter, we will calculate the equivalent length of section 2 by finding that length of diameter D_1 that will match the pressure drop in a length L_2 of pipe diameter D_2. Using the Hazen-Williams equation (2.24) we can write the total pressure drop for a pipe with flow Q, diameter D, and length L as

$$\Delta P = 4.524 \left(\frac{Q}{C}\right)^{1.85} \frac{1}{D^{4.87}} L$$

For simplicity, assuming the same C factor for all pipes, since Q and C are the same for all series pipes,

$$\frac{L_e}{{D_1}^{4.87}} = \frac{L_2}{{D_2}^{4.87}} \tag{2.31}$$

Therefore, the equivalent length of section 2 based on diameter D_1 is

$$L_e = L_2 \left(\frac{D_1}{D_2} \right)^{4.87} \tag{2.32}$$

Similarly, the equivalent length of section 3 based on diameter D_1 is

$$L_e = L_3 \left(\frac{D_1}{D_3} \right)^{4.87} \tag{2.33}$$

The total equivalent length of all three pipe sections based on diameter D_1 is therefore

$$L_t = L_1 + L_2 \left(\frac{D_1}{D_2} \right)^{4.87} + L_3 \left(\frac{D_1}{D_3} \right)^{4.87} \tag{2.34}$$

The total pressure drop in the three sections of pipe can now be calculated based on a single pipe of diameter D_1 and length L_t.

Example 2.14 Three pipes of NPS 4, NPS 6, and NPS 8 (all standard wall thickness) are connected in series with pipe reducers, fittings, and valves as follows:

NPS 4 pipe, 0.237-in wall thickness, 200 ft long

Two 4-in 90° elbows and one 4-in gate valve

NPS 6 pipe, 0.280-in wall thickness, 300 ft long

Four 6-in 90° elbows and one 6-in gate valve

NPS 8 pipe, 0.277-in wall thickness, 500 ft long

Two 8-in 90° elbows and one 8-in gate valve

(a) Use Hazen-Williams equation with a C factor of 120 to calculate the total pressure drop in the series water piping system at a flow rate of 500 gal/min. Flow starts in the 4-in piping and ends in the 8-in piping.

(b) If the flow rate is increased to 600 gal/min, estimate the new total pressure drop in the piping system, keeping everything else the same.

Solution

(a) Since we are going to use the Hazen-Williams equation, the pipes in series analysis will be based on the pressure loss being inversely proportional to $D^{4.87}$ where D is the inside diameter of pipe, per Eq. (2.24).

We will first calculate the total equivalent lengths of all NPS 4 pipe, fittings, and valves in terms of the NPS 4 pipe. Using the equivalent length of values and fittings (Table 2.5),

$$\text{Straight pipe: NPS 4, 200 ft} = 200 \text{ ft of NPS 4 pipe}$$

$$\text{Two 4-in } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 4}{12} = 20 \text{ ft of NPS 4 pipe}$$

$$\text{One 4-in gate valve} = \frac{1 \times 8 \times 4}{12} = 2.67 \text{ ft of NPS 4 pipe}$$

Therefore, the total equivalent length of NPS 4 pipe, fittings, and valve = 222.67 ft of NPS 4 pipe.

Similarly we get the total equivalent length of NPS 6 pipe, fittings, and valve as follows:

$$\text{Straight pipe: NPS 6, 300 ft} = 300 \text{ ft of NPS 6 pipe}$$

$$\text{Four 6-in } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 6}{12} = 60 \text{ ft of NPS 6 pipe}$$

$$\text{One 6-in gate valve} = \frac{1 \times 8 \times 6}{12} = 4 \text{ ft of NPS 6 pipe}$$

Therefore, the total equivalent length of NPS 6 pipe, fittings, and valve = 364 ft of NPS 6 pipe.

Finally, we get the total equivalent length of NPS 8 pipe, fittings, and valve as follows:

$$\text{Straight pipe: NPS 8, 500 ft} = 500 \text{ ft of NPS 8 pipe}$$

$$\text{Two 8-in } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 8}{12} = 40 \text{ ft of NPS 8 pipe}$$

$$\text{One 8-in gate valve} = \frac{1 \times 8 \times 8}{12} = 5.33 \text{ ft of NPS 8 pipe}$$

Therefore, the total equivalent length of NPS 8 pipe, fittings, and valve = 545.33 ft of NPS 8 pipe.

Next we convert all the preceding pipe lengths to the equivalent NPS 4 pipe based on the fact that the pressure loss is inversely proportional to $D^{4.87}$ where D is the inside diameter of pipe, and all series pipes have the same flow rate.

$$222.67 \text{ ft of NPS 4 pipe} = 222.67 \text{ ft of NPS 4 pipe}$$

$$364.00 \text{ ft of NPS 6 pipe} = 364\left(\frac{4.026}{6.065}\right)^{4.87} = 49.48 \text{ ft of NPS 4 pipe}$$

$$545.33 \text{ ft of NPS 8 pipe} = 545.33\left(\frac{4.026}{8.071}\right)^{4.87} = 18.44 \text{ ft of NPS 4 pipe}$$

Therefore adding all the preceding lengths we get:

$$\text{Total equivalent length in terms of NPS 4 pipe} = 290.59 \text{ ft of NPS 4 pipe}$$

The head losses in the reducers are insignificant and hence can be neglected in comparison with the head loss in straight length of pipe. Therefore the total head loss in the entire piping system will be based on a total equivalent length 290.59 ft of NPS 4 pipe.

Using the Hazen-Williams equation (2.24) the pressure drop at 500 gal/min is

$$\Delta P = 4.524\left(\frac{500}{120}\right)^{1.85}\frac{1}{4.026^{4.87}} = 0.0718$$

Therefore for the 290.59 ft of equivalent NPS 4-in pipe,

$$\text{Total pressure drop} = 290.59 \times 0.0718 = 20.88 \text{ psi}$$

(b) When the flow rate is increased to 600 gal/min, we can use proportions to estimate the new total pressure drop in the piping as follows:

$$\Delta P = \left(\frac{600}{500}\right)^{1.85} \times 20.88 = 29.26 \text{ psi}$$

Example 2.15 DN 200 pipe and a DN 300 pipe are connected in series as follows:

DN 200 pipe, 6-mm wall thickness, 60 m long

DN 300 pipe, 8-mm wall thickness, 50 m long

Use the Hazen-Williams equation with a C factor of 100 to calculate the total pressure drop in the series fire protection water piping system at a flow rate of 30 L/s. What will the pressure drop be if the flow rate were increased to 45 L/s?

Solution The total equivalent length will be based on DN 200 pipe:

$$60 \text{ m of straight pipe} = 60 \text{ m of DN 200 pipe}$$

The total equivalent length of DN 300 pipe in terms of DN 200 pipe is

$$50 \text{ m of straight pipe} = 50 \times \left(\frac{188}{284}\right)^{4.87} = 6.71 \text{ m}$$

$$\text{Total equivalent length of both pipes} = 60 + 6.71 = 66.71 \text{ m}$$

$$Q = 30 \times 10^{-3} \times 3600 = 108 \text{ m}^3/\text{h}$$

The pressure drop from the Hazen-Williams equation (2.25) is

$$\Delta P = 1.1101 \times 10^{10}\left(\frac{108}{100}\right)^{1.85}\frac{1}{188^{4.87}} = 0.1077 \text{ kPa/m}$$

Total pressure drop in 66.71-m length of pipe $= 66.71 \times 0.1077 = 7.18$ kPa

When the flow rate is increased to 45 L/s, we can calculate the pressure drop using proportions as follows:

$$\text{Revised head loss at 45 L/s} = \left(\frac{45}{30}\right)^{1.85} \times 0.1077 = 0.228 \text{ kPa/m}$$

Therefore,

$$\text{Total pressure drop in 66.71-m length of pipe} = 66.71 \times 0.288 = 15.21 \text{ kPa}$$

Parallel piping. Fire protection water pipes in parallel are so configured that multiple pipes are connected so that water flow splits into the multiple pipes at the beginning and the separate flow streams subsequently rejoin downstream into another single pipe as depicted in Fig. 2.9. This is also called a looped piping system.

Figure 2.9 shows a parallel piping system in the horizontal plane with no change in pipe elevations. Water flows through a single pipe *AB*, and at the junction *B* the flow splits into two pipe branches *BCE* and *BDE*. At the downstream end at junction *E*, the flows rejoin to the initial flow rate and subsequently flow through the single pipe *EF*.

To calculate the flow rates and pressure drop due to friction in the parallel piping system, shown in Fig. 2.9, two main principles of parallel piping must be followed. These are flow conservation at any junction point and common pressure drop across each parallel branch pipe.

Based on flow conservation, at each junction point of the pipeline, the incoming flow must exactly equal the total outflow. Therefore, at junction *B*, the flow *Q* entering the junction must exactly equal the sum of the flow rates in branches *BCE* and *BDE*.

Thus,

$$Q = Q_{BCE} + Q_{BDE} \tag{2.35}$$

where Q_{BCE} = flow through branch *BCE*
Q_{BDE} = flow through branch *BDE*
Q = incoming flow at junction *B*

The other requirement in parallel pipes concerns the pressure drop in each branch piping. Based on this the pressure drop due to friction in branch *BCE* must exactly equal that in branch *BDE*. This is because

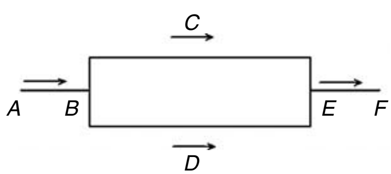


Figure 2.9 Parallel piping.

both branches have a common starting point (B) and a common ending point (E). Since the pressure at each of these two points is a unique value, we can conclude that the pressure drop in branch pipe BCE and that in branch pipe BDE are both equal to $P_B - P_E$, where P_B and P_E represent the pressure at the junction points B and E, respectively.

The pressure drop in branch BCE is calculated using the Hazen-Williams equation as

$$\Delta P_1 = 4.524\left(\frac{Q_1}{C}\right)^{1.85}\frac{1}{D_1{}^{4.87}}L_1 \tag{2.36}$$

where ΔP_1 = pressure loss due to friction in branch BCE
Q_1 = flow rate in branch BCE
D_1 = pipe inside diameter of branch BCE
L_1 = pipe length of branch BCE

Similarly the pressure drop in branch BDE is calculated using the Hazen-Williams equation as

$$\Delta P_2 = 4.524\left(\frac{Q_2}{C}\right)^{1.85}\frac{1}{D_2{}^{4.87}}L_2 \tag{2.37}$$

where ΔP_2 = pressure loss due to friction in branch BDE
Q_2 = flow rate in branch BDE
D_2 = pipe inside diameter of branch BDE
L_2 = pipe length of branch BDE

We have assumed a common C factor for the pressure drop calculations for both branches BCE and BDE.

Simplifying, since the two pressure drops just determined have to be equal for a looped system, we get

$$\Delta P_1 = \Delta P_2$$

Therefore,

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^{2.63}\left(\frac{L_2}{L_1}\right)^{0.54} \tag{2.38}$$

Also the total flow rate Q_t is the sum of the two flow rates Q_1 and Q_2. Therefore,

$$Q_1 + Q_2 = Q_t \tag{2.39}$$

Solving for Q_1 and Q_2 in terms of Q_t, we get

$$Q_1 = \frac{Q_t}{1 + (L_1/L_2)^{0.54}} \tag{2.40}$$

and

$$Q_2 = Q_t \left[\frac{(L_1/L_2)^{0.54}}{1 + (L_1/L_2)^{0.54}} \right] \tag{2.41}$$

We have thus calculated the flow split between the two branches *BCE* and *BDE*. The pressure drop ΔP_1 or ΔP_2 can be calculated using Eq. (2.36) or Eq. (2.37).

Another approach to calculating the pressure drop in parallel piping is the use of an equivalent diameter for the parallel pipes. For example in Fig. 2.9, if pipe *AB* were NPS 8 pipe and branches *BCE* and *BDE* were NPS 4 and NPS 6, respectively, we can find some equivalent diameter pipe of the same length as one of the branches that will have the same pressure drop between points *B* and *C* as the two branches. An approximate equivalent diameter can be calculated using the Hazen-Williams equation as follows.

The pressure drop in branch *BCE* is calculated using the Hazen-Williams equation as

$$\Delta P_1 = 4.524 \left(\frac{Q_1}{C} \right)^{1.85} \frac{1}{{D_1}^{4.87}} L_1 \tag{2.42}$$

Similarly the pressure drop in branch *BDE* is calculated using the Hazen-Williams equation as

$$\Delta P_2 = 4.524 \left(\frac{Q_2}{C} \right)^{1.85} \frac{1}{{D_2}^{4.87}} L_2 \tag{2.43}$$

where the subscript 1 is used for branch *BCE* and subscript 2 for branch *BDE*. For simplicity we have assumed the same *C* factors for both branches.

Similarly, the equivalent diameter pipe D_e with length L_e that will replace both branches *BCE* and *BDE* will have a pressure drop equal to

$$\Delta P_e = 4.524 \left(\frac{Q_e}{C} \right)^{1.85} \frac{1}{{D_e}^{4.87}} L_e \tag{2.44}$$

where Q_e is really the same as $Q_1 + Q_2$ or the total flow Q_t, and L_e may be chosen as equal to the length of one of the branches. Therefore, replacing L_e with L_1 and setting ΔP_1 equal to ΔP_e, the common pressure drop between B and E is

$$\frac{Q_1}{Q_t} = \left(\frac{D_1}{D_e}\right)^{2.63} \left(\frac{L_e}{L_1}\right)^{0.54} \tag{2.45}$$

Similarly,

$$\frac{Q_2}{Q_t} = \left(\frac{D_2}{D_e}\right)^{2.63} \left(\frac{L_e}{L_2}\right)^{0.54} \tag{2.46}$$

Combining Eqs. (2.45) and (2.46) with the equation for conservation of flow, $Q_1 + Q_2 = Q_t$, we get

$$Q_t \left(\frac{D_1}{D_e}\right)^{2.63} \left(\frac{L_e}{L_1}\right)^{0.54} + Q_t \left(\frac{D_2}{D_e}\right)^{2.63} \left(\frac{L_e}{L_2}\right)^{0.54} = Q_t \tag{2.47}$$

Simplifying by eliminating Q_t and setting $L_e = L_1$, we get for the equivalent diameter

$$D_e = \left[{D_1}^{2.63} + {D_2}^{2.63} \left(\frac{L_1}{L_2}\right)^{0.54} \right]^{1/2.63} \tag{2.48}$$

This is the equivalent diameter of a pipe of length L_1 that will completely replace both pipe loops for the same head loss.

As an example, if $D_1 = D_2 = 6$ and $L_1 = L_2 = 200$, the equivalent diameter of two 6-in loops, from Eq. (2.48), is

$$D_e = (2 \times 6^{2.63})^{0.38} = 7.8 \text{ in}$$

Thus two 6-in pipe loops, 200 ft long, can be replaced with one 200-ft long pipe that has an equivalent (inside) diameter of 7.8 in.

Example 2.16 A fire protection water pipeline consists of a 200-ft section of NPS 10 (0.250-in wall thickness) pipe starting at point A and terminating at point B. At point B, two pieces of pipe (each 400 ft long and NPS 6 pipe with 0.250-in wall thickness) are connected in parallel and rejoin at a point C. From point C 150 ft of NPS 10 pipe (0.250-in wall thickness) extends to point D. Using the equivalent diameter method calculate the pressures and flow rate throughout the system when transporting fire protection water at 5000 gal/min. Compare the results by calculating the pressures and flow rates in each branch. Use the Hazen-Williams equation with $C = 120$.

Solution Since the pipe loops between B and C are each NPS 10 and 400 ft long, the flow will be equally split between the two branches. Each branch pipe will carry 2500 gal/min.

The equivalent diameter for section *BC* is found from Eq. (2.48):

$$D_e = \left[10.25^{2.63} + 10.25^{2.63}\left(\frac{400}{400}\right)^{0.54}\right]^{1/2.63} = 13.34 \text{ in}$$

Therefore we can replace the two 400-ft NPS 10 pipes between *B* and *C* with a single pipe that is 400 ft long and has a 13.34-in inside diameter.

The pressure drop in section *BC*, using Hazen-Williams equation (2.24), is

$$\Delta P_e = 4.524\left(\frac{5000}{120}\right)^{1.85}\frac{1}{13.34^{4.87}} \times 400 = 5.95 \text{ psi}$$

Therefore, the total pressure drop in section *BC* is 5.95 psi.

For section *AB* we have,

$$D = 10.25 \text{ in} \qquad Q = 5000$$

The pressure drop in section *AB*, using Hazen-Williams equation, is

$$\Delta P = 4.524\left(\frac{5000}{120}\right)^{1.85}\frac{1}{10.25^{4.87}} \times 200 = 10.73 \text{ psi}$$

Therefore, the total pressure drop in section *AB* is 10.73 psi.

Finally, for section *CD*, the pressure drop, using the Hazen-Williams equation, is

$$\Delta P = 4.524\left(\frac{5000}{120}\right)^{1.85}\frac{1}{10.25^{4.87}} \times 150 = 8.05 \text{ psi}$$

Therefore, the total pressure drop in section *CD* is 8.05 psi.

Therefore,

$$\begin{aligned}\text{Total pressure drop in entire piping system} &= 5.95 + 10.73 + 8.05 \\ &= 24.73 \text{ psi}\end{aligned}$$

Next for comparison we will analyze the branch pressure drops assuming each branch separately carries 2500 gal/min.

$$\Delta P = 4.524\left(\frac{2500}{120}\right)^{1.85}\frac{1}{10.25^{4.87}} \times 400 = 5.96 \text{ psi}$$

This compares with the pressure drop of 5.95 psi/mi we calculated using an equivalent diameter of 13.34. It can be seen that both results are essentially the same.

2.5 Pipe Materials

Generally, fire protection piping systems are constructed of cast iron or steel. To prevent corrosion of underground steel piping due to soil, buried pipes are externally coated and wrapped. The maximum working pressure allowed in piping is determined by the pressure class or rating of the pipe. Class 150 pipe is suitable for pressures not exceeding 150 psi. Similarly class 200 pipe is for pressures not exceeding 200 psi. Cast iron and fittings used in fire protection systems use ANSI, AWWA, and federal specifications. To prevent internal corrosion when using corrosive water, cast iron pipes may be internally lined. Asbestos-cement (AC) pipe used in water pipelines is manufactured per AWWA standard and is constructed of asbestos fiber and portland cement. AC pipes are found to be more corrosion resistant than cast iron pipe.

Steel pipe used for fire protection water piping is manufactured to conform to ANSI and ASTM standards. Schedule standard weight pipe is used for pressures below 300 psi. Higher pressures require schedule 80 pipe.

2.6 Pumps

Pumps used in fire protection water piping are generally centrifugal pumps. Motors may be 1750 or 3600 r/min. Standard fire pumps range in capacity from 500 to 2500 gal/min. If suction lifts of more than 15 ft are required, a submerged multistage turbine-type centrifugal pump is used.

A typical fire pump installation showing water supply, pump bypass, and connection piping to a sprinkler system is shown in Fig. 2.10. NFPA 20, Standard for the Installation of Centrifugal Fire Pumps, must be consulted for application of a particular fire pump in fire protection service.

Referring to Fig. 2.10, the fire protection water pump receives water from the city water supply. A test header is installed on the discharge side of the pump. This is used to test the fire pump and verify that the pump can generate the specified pressure at the required flow rate.

Also on the discharge of the pump a relief valve is installed to prevent overpressure of the piping connected to the sprinklers. A bypass piping is also installed to route the city water directly to the sprinkler piping

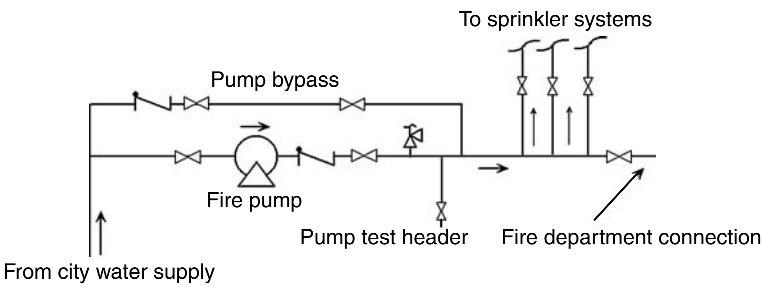


Figure 2.10 Typical fire protection water pump installation.

system, bypassing the fire pump, in the event the fire pump is shut down for maintenance.

2.6.1 Centrifugal pumps

Centrifugal pumps consist of one or more rotating impellers contained in a casing. The centrifugal force of rotation generates the pressure in the water as it goes from the suction side to the discharge side of the pump. Centrifugal pumps have a wide range of operating flow rates with fairly good efficiency. The performance curves of a centrifugal pump consist of head versus capacity, efficiency versus capacity, and brake horsepower (BHP) versus capacity. The term capacity is used synonymously with flow rate in connection with centrifugal pumps. Also the term head is used in preference to pressure when dealing with centrifugal pumps. Figure 2.11 shows a typical performance curve for a centrifugal pump.

Generally, the head-capacity curve of a centrifugal pump is a drooping curve. The highest head is generated at a zero flow rate (shutoff head), and the head decreases with an increase in flow rate as shown in Fig. 2.11. The efficiency increases with flow rate up to the best efficiency point (BEP) after which the efficiency drops off. The BHP also generally increases with flow rate but may taper off or start decreasing at some point depending on the head-capacity curve.

The head generated by a centrifugal pump depends on the diameter of the pump impeller and the speed at which the impeller runs. A larger

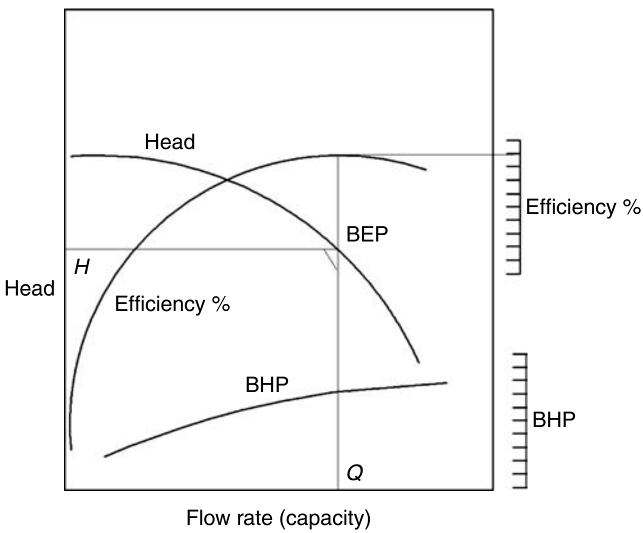


Figure 2.11 Performance curve for centrifugal pump.

impeller may be installed to increase the pump pressure, or a smaller impeller may be used where less pressure is needed.

2.6.2 Net positive suction head

An important parameter related to the operation of centrifugal pumps is the concept of net positive suction head (NPSH). This represents the absolute minimum pressure at the suction of the pump impeller at the specified flow rate to prevent pump cavitation. If the pressure falls below this value, the pump impeller may be damaged and render the pump useless.

The calculation of NPSH available for a particular pump and piping configuration requires knowledge of the pipe size on the suction side of the pump, the elevation of the water source, and the elevation of the pump impeller along with the atmospheric pressure and vapor pressure of water at the pumping temperature. The pump vendor may specify that a particular model of pump requires a certain amount of NPSH (known as NPSH required or NPSH_R) at a particular flow rate. Based on the actual piping configuration, elevations, etc., the calculated NPSH (known as NPSH available or NPSH_A) must exceed the required NPSH at the specified flow rate. Therefore,

$$\text{NPSH}_A > \text{NPSH}_R$$

If the NPSH_R is 25 ft at a 2000 gal/min pump flow rate, then NPSH_A must be 35 ft or more, giving a 10-ft cushion. Also, typically, as the flow rate increases, NPSH_R increases fairly rapidly as can be seen from the typical centrifugal pump curve in Fig. 2.11. Therefore, it is important that the engineer perform calculations at the expected range of flow rates to ensure that the NPSH available is always more than the required NPSH, per the vendor's pump performance data. As indicated earlier, insufficient NPSH available tends to cavitate or starve the pump and eventually causes damage to the pump impeller. The damaged impeller will not be able to provide the necessary head pressure as indicated on the pump performance curve.

2.6.3 System head curve

A *system head curve*, or a system head characteristic curve, for a fire water pipeline is a graphic representation of how the pressure needed to pump water through the pipeline varies with the flow rate. If the pressures required at 200, 400, up to 1000 gal/min are plotted on the vertical axis, with the flow rates on the horizontal axis, we get the system head curve as shown in Fig. 2.12.

It can be seen that the system curve is not linear. This is because the pressure drop due to friction varies approximately as the square of

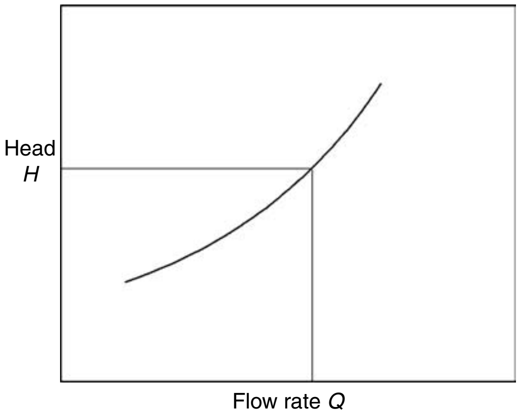


Figure 2.12 System head curve.

the flow rate (actually $Q^{1.85}$ according to the Hazen-Williams equation), and hence the additional pressure required when the flow is increased from 400 to 500 gal/min is more than that required when the flow rate increases from 200 to 300 gal/min.

Consider a fire protection water pipeline used to transport water from point A to point B. The pipe inside diameter is D and the length is L. By knowing the elevation along the pipeline we can calculate the total pressure required at any flow rate using the techniques discussed earlier. At each flow rate we would calculate the pressure drop due to friction using the Hazen-Williams equation and multiply by the pipe length to get the total pressure drop. Next we will add the equivalent of the static head difference between A and B converted to psi. Finally, the delivery pressure required at B would be added to come up with the total pressure required. The process would be repeated for multiple flow rates so that a system head curve can be constructed as shown in Fig. 2.12. If we plotted the feet of head instead of pressure on the vertical axis, we could use the system curve in conjunction with the pump curve for the pump at A. By plotting both the pump H-Q curve and the system head curve on the same graph, we can determine the point of operation for this pipeline with the specified pump curve. This is shown in Fig. 2.13.

When there is no elevation difference between points A and B, the system head curve will start at the point where the flow rate and head are both zero. If the elevation difference were 100 ft, B being higher than A, the system head curve will start at $H = 100$ ft and flow $Q = 0$. This simply means that even at zero flow rate, a minimum pressure must be present at A to overcome the static elevation difference between A and B.

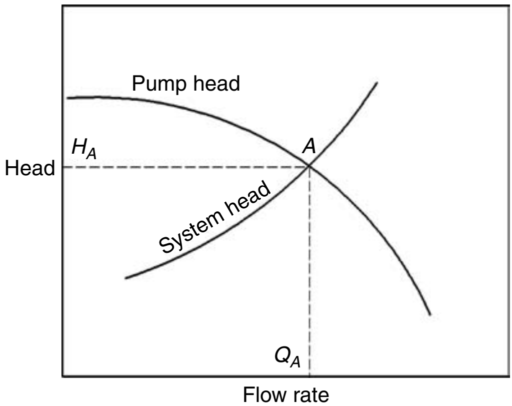


Figure 2.13 Pump head curve and system head curve.

2.6.4 Pump curve versus system head curve

The system head curve for a pipeline is a graphic representation of the head required to pump water through the pipeline at various flow rates and is an increasing curve, indicating that more pressure is required for a higher flow rate. On the other hand, the pump performance (head versus capacity) curve shows the head the pump generates at various flow rates, generally a drooping curve. When the required head per the system head curve equals the available pump head, we have a match of the required head versus the available head. This point of intersection of the system head curve and the pump head curve is the operating point for this particular pump and pipeline system. This is illustrated in Fig. 2.13.

2.7 Sprinkler System Design

The flow through a sprinkler head depends on its orifice design and pressure available. The flow rate Q and the pressure P are related by the equation

$$Q = K\sqrt{P} \tag{2.49}$$

where K is a coefficient called the K factor. It varies from 5.3 to 5.8 for half-inch sprinklers. The NFPA requires that the minimum pressure at any sprinkler shall be 7.0 psi. The minimum flow at the most demanding sprinkler may be specified as 20 gal/min. For this flow rate the pressure required at the sprinkler is calculated by Eq. (2.49):

$$20 = 5.6\sqrt{P}$$

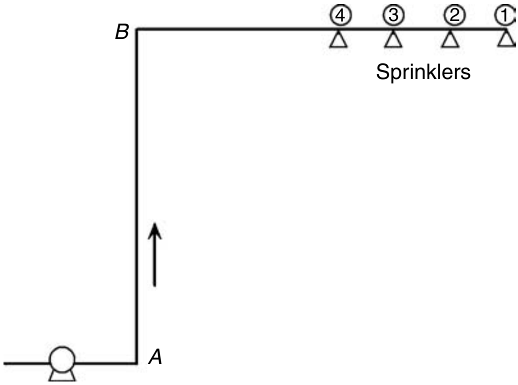


Figure 2.14 Sprinkler system.

using $K = 5.6$. Solving for pressure we get

$$P = 12.76 \text{ psi}$$

This is more than the NFPA 13 requirement of 7 psi.

Consider the sprinkler system shown in Fig. 2.14. If the remotest sprinkler (sprinkler 1) is to operate at 20 gal/min, then it will have a pressure of 12.76 psi, as calculated in the preceding. The next sprinkler closest to B (sprinkler 2) will have a pressure P_2 such that

$$P_2 = P_1 + \text{head loss between sprinklers 1 and 2} \tag{2.50}$$

The head loss between sprinklers 1 and 2 can be calculated since we know the flow in the pipe segment from sprinkler 2 to sprinkler 1 is equal to the discharge volume of sprinkler 1. Therefore, from Eq. (2.50) we can calculate the pressure at sprinkler 2. Then we can continue this process until we get to the sprinkler closest to B.

The pressure at the top of the riser at B can then be calculated. Next from the length of the riser pipe AB we can calculate the pressure drop in it, and considering the elevation difference between A and B we can calculate the pressure at the pump at A as follows:

$$\text{Pump pressure} = (H_B - H_A) \times 0.433 + \text{pressure at } B \tag{2.51}$$

Example 2.17 A sprinkler system for a small warehouse has three branch pipes with four sprinkler heads, each spaced 12 ft apart as shown in Fig. 2.15. The branch lines are spaced 15 ft apart and connect to a riser pipe 20 ft high from the fire pump. The riser pipe AB is 2-in schedule 40 pipe. The branch lines are 1-in schedule 40 pipe except for the section from the top of the riser to the first sprinkler on each branch line, which is 1.5-in schedule 40 pipe. The most remote sprinkler requires 20 gal/min. All sprinklers have a 0.5-in

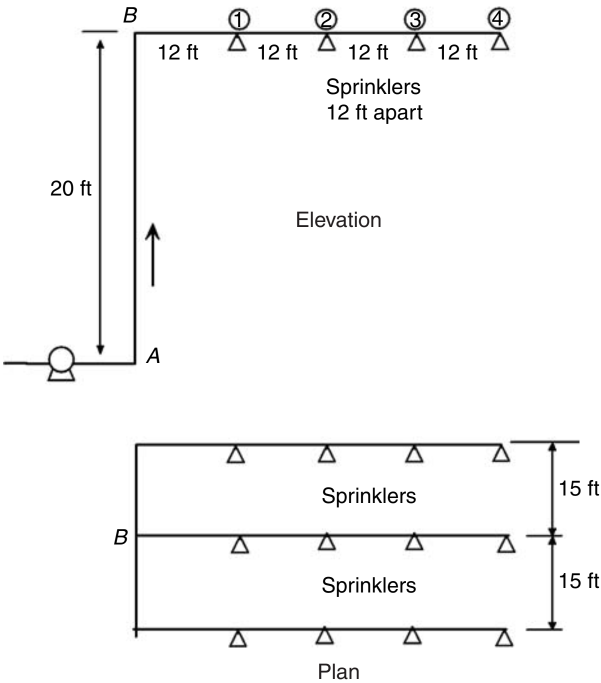


Figure 2.15 Sprinkler system—example problem.

orifice with $K = 5.6$. Use a Hazen-Williams C factor $= 100$. Calculate the flow through each branch line and the total pump flow rate and pressure required.

Solution There are three branch pipes, each with four sprinklers spaced 12 ft apart. Point B represents the top of the riser pipe, and the pipe diameters between sprinklers 1–2, 2–3, and 3–4 are 1-in schedule 40.

Using Eq. (2.49), the pressure at sprinkler 4 is

$$20 = 5.6(P_4)^{1/2}$$

$$P_4 = 12.76 \text{ psi}$$

The pressure at sprinkler 3 is found by adding the pressure drop in pipe section 3–4 to P_4. Using the friction loss table (Table 2.4) at a flow rate of 20 gal/min for 1-in schedule 40 pipe, the pressure drop in the 12-ft-long section of pipe is

$$P_3 = 0.1823 \times 12 + 12.76 = 14.95 \text{ psi}$$

The flow rate through sprinkler 3, using Eq. (2.49), is

$$Q_3 = 5.6\,(14.95)^{1/2} = 21.65 \text{ gal/min}$$

The pressure at sprinkler 2 is found by adding the pressure drop in pipe section 2–3 to P_3. Using Table 2.4 at a flow rate of 41.65 gal/min for 1-in schedule 40 pipe, the pressure drop in the 12-ft-long section of pipe is

$$P_2 = 0.7194 \times 12 + 14.95 = 23.58 \text{ psi}$$

The flow rate through sprinkler 2 is

$$Q_2 = 5.6\,(23.58)^{1/2} = 27.19 \text{ gal/min}$$

The pressure at sprinkler 1 is found by adding the pressure drop in pipe section 1–2 to P_2. Using Table 2.4 at a flow rate of 68.84 gal/min for 1-in schedule 40 pipe, the pressure drop in the 12-ft-long section of pipe is

$$P_1 = 1.802 \times 12 + 23.58 = 45.20 \text{ psi}$$

The flow rate through sprinkler 1 is

$$Q_1 = 5.6\,(45.20)^{1/2} = 37.65 \text{ gal/min}$$

The total flow from the top of the riser to branch line 1 is $37.65 + 68.84 = 106.5$ gal/min. This flow rate is through a 1.5-in schedule 40 pipe. Using Table 2.4 at a flow rate of 106.5 gal/min for 1.5-in schedule 40 pipe, 12 ft long,

$$\text{Pressure at top of riser (point } B) = 45.2 + 12 \times 0.5 = 51.2 \text{ psi}$$

This is the pressure at the common junction of the three branch lines.

$$\text{Total flow in riser pipe } AB = 3 \times 106.5 = 319.5 \text{ gal/min}$$

Considering 2-in schedule 40 riser pipe at this flow rate, head loss = 1.165 psi/ft.

$$\text{Total pressure drop in riser pipe} = 1.165 \times 20 = 23.3 \text{ psi}$$

Therefore,

$$\begin{aligned}\text{Total pressure required at pump} &= 23.3 + 20 \times 0.433 + 51.2 \\ &= 83.16 \text{ psi}\end{aligned}$$

For simplicity in this example we have used 1-in pipe between sprinklers 1 and 4 on each branch line. In reality the pipe size from sprinkler 1 to sprinkler 4 will reduce to compensate for the reduction in flow in each segment.

Chapter 3

Wastewater and Stormwater Piping

Introduction

Wastewater piping systems carry residential, commercial, and industrial wastes and waste products, using water as the transport medium, to sewage plants for subsequent treatment and disposal. Stormwater piping systems, on the other hand, carry stormwater and rainwater captured in basins and ponds to discharge points. These are also known as storm sewer systems. In some installations a single piping system is used to convey both wastewater and stormwater to treatment and disposal areas.

In this chapter, we will discuss the various wastewater and stormwater piping designs, show how to calculate flow rates and pipe sizes, and review pumping systems. Before we discuss sewer piping design and stormwater piping systems, we will briefly cover the basics of water pipelines, how pressure drop due to friction is calculated, and how series and parallel piping systems are analyzed for pressure drops and flow rates.

3.1 Properties of Wastewater and Stormwater

Pure water is an incompressible fluid with a specific gravity of 1.00 and a viscosity of 1.00 centipoise (cP) at normal temperature and pressure. Groundwater or stormwater, however, may consist of dissolved minerals, gases, and other impurities. Wastewater also contains minerals and gases in addition to dissolved solids. Commercial and industrial wastewater may contain more solids and therefore may have drastically

TABLE 3.1 Properties of Water at Atmospheric Pressure

Temperature °F	Density slug/ft^3	Specific weight lb/ft^3	Dynamic viscosity (lb · s)/ft^3	Vapor pressure psia
		USCS units		
32	1.94	62.4	3.75×10^{-5}	0.08
40	1.94	62.4	3.24×10^{-5}	0.12
50	1.94	62.4	2.74×10^{-5}	0.17
60	1.94	62.4	2.36×10^{-5}	0.26
70	1.94	62.3	2.04×10^{-5}	0.36
80	1.93	62.2	1.80×10^{-5}	0.51
90	1.93	62.1	1.59×10^{-5}	0.70
100	1.93	62.0	1.42×10^{-5}	0.96
Temperature °C	Density kg/m^3	Specific weight kN/m^3	Dynamic viscosity (N · s)/m^2	Vapor pressure kPa
		SI units		
0	1000	9.81	1.75×10^{-3}	0.611
10	1000	9.81	1.30×10^{-3}	1.230
20	998	9.79	1.02×10^{-3}	2.340
30	996	9.77	8.00×10^{-4}	4.240
40	992	9.73	6.51×10^{-4}	7.380
50	988	9.69	5.41×10^{-4}	12.300
60	984	9.65	4.60×10^{-4}	19.900
70	978	9.59	4.02×10^{-4}	31.200
80	971	9.53	3.50×10^{-4}	47.400
90	965	9.47	3.11×10^{-4}	70.100
100	958	9.40	2.82×10^{-4}	101.300

different physical properties such as specific gravity and viscosity. Because these differences can affect the hydraulic properties, laboratory testing may be needed to ascertain the gravity and viscosity of industrial wastewater. See Table 3.1 for typical properties of water at various temperatures.

3.1.1 Mass and weight

Mass is defined as the quantity of matter. It is measured in slugs (slug) in U.S. Customary System (USCS) units and kilograms (kg) in Système International (SI) units. A given mass of water will occupy a certain volume at a particular temperature and pressure. For example, a certain mass of water may be contained in a volume of 500 cubic feet (ft^3) at a temperature of 60°F and a pressure of 14.7 pounds per square inch (lb/in^2 or psi). Water, like most liquids, is considered incompressible. Therefore, pressure and temperature have a negligible effect on its volume. However, if the properties of water are known at standard conditions such as 60°F and 14.7 psi pressure, these properties will be slightly different at other temperatures and pressures. By the principle of conservation of mass, the mass of a given quantity of water will remain the same at all temperatures and pressures.

Weight is defined as the gravitational force exerted on a given mass at a particular location. Hence the weight varies slightly with the geographic location. By Newton's second law the weight is simply the product of the mass and the acceleration due to gravity at that location. Thus

$$W = mg \tag{3.1}$$

where W = weight, lb
m = mass, slug
g = acceleration due to gravity, ft/s^2

In USCS units g is approximately 32.2 ft/s^2, and in SI units it is 9.81 m/s^2. In SI units weight is measured in newtons (N) and mass is measured in kilograms. Sometimes mass is referred to as pound-mass (lbm) and force as pound-force (lbf) in USCS units. Numerically we say that 1 lbm has a weight of 1 lbf.

3.1.2 Density and specific weight

Density is defined as mass per unit volume. It is expressed as slug/ft^3 in USCS units. Thus, if 100 ft^3 of water has a mass of 200 slug, the density is 200/100 or 2 slug/ft^3. In SI units, density is expressed in kg/m^3. Therefore water is said to have an approximate density of 1000 kg/m^3 at room temperature.

Specific weight, also referred to as weight density, is defined as the weight per unit volume. By the relationship between weight and mass discussed earlier, we can state that the specific weight is related to density as follows:

$$\gamma = \rho g \tag{3.2}$$

where γ = specific weight, lb/ft^3
ρ = density, slug/ft^3
g = acceleration due to gravity, ft/s^2

3.1.3 Volume

The volume of water is usually measured in gallons (gal) or cubic ft (ft^3) in USCS units. In SI units, cubic meters (m^3) and liters (L) are used. Flow rate, also called discharge, is the rate at which volume is conveyed through a pipeline. The flow rate in water pipelines is measured in gallons per minute (gal/min), million gallons per day (Mgal/day), and cubic feet per second (ft^3/s) in USCS units. In SI units, flow rate is measured in cubic meters per hour (m^3/h) or liters per second (L/s). One ft^3 equals 7.4805 gal. One m^3equals 1000 L, and one U.S. gallon equals 3.785 L. A table of conversion factors for various units is provided in App. A.

Example 3.1 Water at 60°F fills a tank of volume 1000 ft^3 at atmospheric pressure. If the weight of water in the tank is 31.2 tons, calculate its density and specific weight.

Solution

$$\text{Specific weight} = \frac{\text{weight}}{\text{volume}} = \frac{31.2 \times 2000}{1000} = 62.40 \text{ lb/ft}^3$$

From Eq. (3.2) the density is

$$\text{Density} = \frac{\text{specific weight}}{g} = \frac{62.4}{32.2} = 1.9379 \text{ slug/ft}^3$$

Example 3.2 A tank has a volume of 5 m^3 and contains water at 20°C. Assuming a density of 990 kg/m^3, calculate the weight of the water in the tank. What is the specific weight in N/m^3 using a value of 9.81 m/s^2 for gravitational acceleration?

Solution

$$\text{Mass of water} = \text{volume} \times \text{density} = 5 \times 990 = 4950 \text{ kg}$$

$$\text{Weight of water} = \text{mass} \times g = 4950 \times 9.81 = 48{,}559.5 \text{ N} = 48.56 \text{ kN}$$

$$\text{Specific weight} = \frac{\text{weight}}{\text{volume}} = \frac{48.56}{5} = 9.712 \text{ N/m}^3$$

3.1.4 Specific gravity

Specific gravity is a measure of how heavy a liquid is compared to water. It is a ratio of the density of a liquid to the density of water at the same temperature. Since we are dealing with water only in this chapter, the specific gravity of pure water by definition is always equal to 1.00. However, wastewater contains dissolved solids and therefore the specific gravity of wastewater may be sometimes in the range of 1.00 to 1.20 or more depending on the solids content.

3.1.5 Viscosity

Viscosity is a measure of a liquid's resistance to flow. Each layer of water flowing through a pipe exerts a certain amount of frictional resistance to

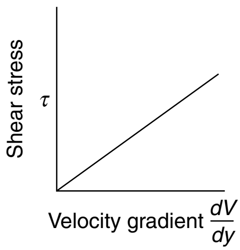


Figure 3.1 Shear stress versus velocity gradient curve.

the adjacent layer. This is illustrated in the shear stress versus velocity gradient curve shown in Fig. 3.1. Newton proposed an equation that relates the frictional shear stress between adjacent layers of flowing liquid with the velocity variation across a section of the pipe as shown in the following:

$$\text{Shear stress} = \mu \times \text{velocity gradient}$$

or

$$\tau = \mu \frac{dV}{dy} \tag{3.3}$$

where τ = shear stress
μ = absolute viscosity, (lb · s)/ft^2 or slug/(ft · s)
$\frac{dV}{dy}$ = velocity gradient

The proportionality constant μ in Eq. (3.3) is referred to as the *absolute viscosity* or *dynamic viscosity*. In SI units, μ is expressed in poise or centipoise (cP).

The viscosity of water, like that of most liquids, decreases with an increase in temperature, and vice versa. Under room temperature conditions water has an absolute viscosity of 1.00 cP.

Kinematic viscosity is defined as the absolute viscosity divided by the density. Thus

$$\nu = \frac{\mu}{\rho} \tag{3.4}$$

where ν = kinematic viscosity, ft^2/s
μ = absolute viscosity, slug/(ft · s)
ρ = density, slug/ft^3

In SI units, kinematic viscosity is expressed as stokes (St) or centistokes (cSt). Under room temperature conditions water has a kinematic viscosity of 1.00 cSt. Some useful conversions for viscosity in SI units are as follows:

$$1 \text{ poise} = 1 \text{ (dyne}\cdot\text{s)/cm}^2 = 1 \text{ g/(cm}\cdot\text{s)} = 10^{-1} \text{ (N}\cdot\text{s)/m}^2$$

$$1 \text{ centipoise} = 10^{-2} \text{ poise} = 10^{-3} \text{ (N}\cdot\text{s)/m}^2$$

Example 3.3 Water has a dynamic viscosity of 1.00 cP at 20°C and a density of 1000 kg/m^3. Calculate the kinematic viscosity in SI units.

Solution

$$\text{Kinematic viscosity} = \frac{\text{absolute viscosity } \mu}{\text{density } \rho} = \frac{1.0 \times 10^{-3} \text{ (N}\cdot\text{s)/m}^2}{1.0 \times 1000 \text{ kg/m}^3}$$

$$= 10^{-6} \text{ m}^2\text{/s}$$

since $1.0 \text{ N} = 1.0 \text{ (kg}\cdot\text{m)/s}^2$.

3.2 Pressure

Pressure is defined as the force per unit area. The pressure at a location in a body of water is by Pascal's law constant in all directions. In USCS units pressure is measured in lb/in^2 (psi), and in SI units it is expressed as N/m^2 or pascals (Pa). Other units for pressure include lb/ft^2, kPa, mega pascals (MPa), kg/cm^2, and bar. Conversion factors are listed in App. A.

At a depth of 100 ft below the free surface of a water tank (of height 150 ft) the intensity of pressure, or simply the pressure, is the force per unit area. Mathematically, the column of water of height 100 ft exerts a force equal to the weight of the water column over an area of 1 in^2. We can calculate the pressure as follows:

$$\text{Pressure} = \frac{\text{weight of 100-ft column of area 1.0 in}^2}{1.0 \text{ in}^2}$$

$$= \frac{100 \times (1/144) \times 62.4}{1.0}$$

In this equation, we have assumed the specific weight of water to be 62.4 lb/ft^3. Therefore, simplifying the equation, we obtain

$$\text{Pressure at a depth of 100 ft} = 43.33 \text{ psi}$$

Therefore, at a depth of 1 ft, the pressure will be 0.433 psi.

A general equation for the pressure in a liquid at a depth h is as follows:

$$P = \gamma h \tag{3.5}$$

where P = pressure, psi
γ = specific weight of liquid
h = liquid depth

Variable γ may also be replaced with ρg where ρ is the density and g is gravitational acceleration.

Generally, pressure in a body of water or a water pipeline is referred to in psi above that of the atmospheric pressure. This is also known as the *gauge pressure* as measured by a pressure gauge. The *absolute pressure* P_{abs} is the sum of the gauge pressure P_{gauge} and the atmospheric pressure P_{atm} at the specified location. Mathematically,

$$P_{abs} = P_{gauge} + P_{atm} \tag{3.6}$$

To distinguish between the two pressures, psig is used for gauge pressure and psia is used for the absolute pressure. In most calculations involving water pipelines the gauge pressure is used. Unless otherwise specified, psi means the gauge pressure.

Liquid pressure may also be referred to as *head pressure*, in which case it is expressed in feet of liquid head (or meters in SI units). Therefore, a pressure of 1000 psi in a liquid such as water is said to be equivalent to a pressure head of

$$h = \frac{1000 \times 144}{62.4} = 2308 \text{ ft}$$

In a more general form, the pressure P in psi and liquid head h in feet for a specific gravity of Sg are related by

$$P = \frac{h \times \text{Sg}}{2.31} \tag{3.7}$$

where P = pressure, psi
h = liquid head, ft
Sg = specific gravity of water

In SI units, pressure P in kilopascals and head h in meters are related by the following equation:

$$P = \frac{h \times \text{Sg}}{0.102} \tag{3.8}$$

Example 3.4 Calculate the pressure in psi at a water depth of 100 ft assuming the specific weight of water is 62.4 lb/ft^3. What is the equivalent pressure in kilopascals? If the atmospheric pressure is 14.7 psi, calculate the absolute pressure at that location.

Solution Using Eq. (3.5), we calculate the pressure:

$$P = \gamma h = 62.4 \text{ lb/ft}^3 \times 100 \text{ ft} = 6240 \text{ lb/ft}^2$$

$$= \frac{6240}{144} \text{ lb/in}^2 = 43.33 \text{ psig}$$

$$\text{Absolute pressure} = 43.33 + 14.7 = 58.03 \text{ psia}$$

In SI units we can calculate the pressures as follows:

$$\text{Pressure} = 62.4 \frac{1}{2.2025}(3.281)^3 \text{ kg/m}^3 \times \left(\frac{100}{3.281}\text{m}\right)(9.81 \text{ m/s}^2)$$

$$= 2.992 \times 10^5 \text{ (kg}\cdot\text{m)/(s}^2\cdot\text{m}^2)$$

$$= 2.992 \times 10^5 \text{ N/m}^2 = 299.2 \text{ kPa}$$

Alternatively,

$$\text{Pressure in kPa} = \frac{\text{Pressure in psi}}{0.145}$$

$$= \frac{43.33}{0.145} = 298.83 \text{ kPa}$$

The 0.1 percent discrepancy between the values is due to conversion factor round-off.

3.3 Velocity

The velocity of flow in a water pipeline depends on the pipe size and flow rate. If the flow rate is uniform throughout the pipeline (steady flow), the velocity at every cross section along the pipe will be a constant value. However, there is a variation in velocity along the pipe cross section. The velocity at the pipe wall will be zero, increasing to a maximum at the centerline of the pipe. This is illustrated in Fig. 3.2.

We can define a bulk velocity or an average velocity of flow as follows:

$$\text{Velocity} = \frac{\text{flow rate}}{\text{area of flow}}$$

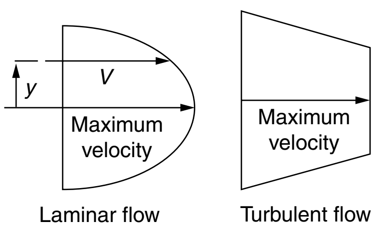


Figure 3.2 Velocity variation in pipe flow.

Considering a circular pipe with an inside diameter D and a flow rate of Q, we can calculate the average velocity as

$$V = \frac{Q}{\pi D^2/4} \tag{3.9}$$

Employing consistent units of flow rate Q in ft^3/s and pipe diameter in inches, the velocity in ft/s is as follows:

$$V = \frac{144Q}{\pi D^2/4}$$

or

$$V = 183.3461 \frac{Q}{D^2} \tag{3.10}$$

where V = velocity, ft/s
Q = flow rate, ft^3/s
D = inside diameter, in

Additional formulas for velocity in different units are as follows:

$$V = 0.4085 \frac{Q}{D^2} \tag{3.11}$$

where V = velocity, ft/s
Q = flow rate, gal/min
D = inside diameter, in

In SI units, the velocity equation is as follows:

$$V = 353.6777 \frac{Q}{D^2} \tag{3.12}$$

where V = velocity, m/s
Q = flow rate, m^3/h
D = inside diameter, mm

Example 3.5 Water flows through an NPS 16 (0.250-in wall thickness) pipeline at the rate of 3000 gal/min. Calculate the average velocity for steady flow. (*Note*: The designation NPS 16 means nominal pipe size of 16 in.)

Solution From Eq. (3.11), the average flow velocity is

$$V = 0.4085 \frac{3000}{15.5^2} = 5.10 \text{ ft/s}$$

Example 3.6 Water flows through a DN 200 (10-mm wall thickness) pipeline at the rate of 75 L/s. Calculate the average velocity for steady flow.

Solution The designation DN 200 means metric pipe size of 200-mm outside diameter. It corresponds to NPS 8 in USCS units. From Eq. (3.12) the average flow velocity is

$$V = 353.6777 \frac{75 \times 60 \times 60 \times 10^{-3}}{180^2} = 2.95 \text{ m/s}$$

The variation of flow velocity in a pipe depends on the type of flow. In laminar flow, the velocity variation is parabolic. As the flow rate becomes turbulent the velocity profile approximates a trapezoidal shape as depicted in Fig. 3.2. Laminar and turbulent flows are discussed in Sec. 3.5 after we introduce the concept of the Reynolds number.

3.4 Reynolds Number

The Reynolds number is a dimensionless parameter of flow. It depends on the pipe size, flow rate, liquid viscosity, and density. It is calculated from the following equation:

$$\text{Re} = \frac{VD\rho}{\mu} \tag{3.13}$$

or

$$\text{Re} = \frac{VD}{\nu} \tag{3.14}$$

where Re = Reynolds number, dimensionless
V = average flow velocity, ft/s
D = inside diameter of pipe, ft
ρ = mass density of liquid, slug/ft^3
μ = dynamic viscosity, slug/(ft · s)
ν = kinematic viscosity, ft^2/s

Since R must be dimensionless, a consistent set of units must be used for all items in Eq. (3.13) to ensure that all units cancel out and R has no dimensions.

Other variations of the Reynolds number for different units are as follows:

$$\text{Re} = 3162.5 \frac{Q}{D\nu} \tag{3.15}$$

where Re = Reynolds number, dimensionless
Q = flow rate, gal/min
D = inside diameter of pipe, in
ν = kinematic viscosity, cSt

In SI units, the Reynolds number is expressed as follows:

$$\mathrm{Re} = 353{,}678 \frac{Q}{\nu D} \tag{3.16}$$

where Re = Reynolds number, dimensionless
Q = flow rate, m^3/h
D = inside diameter of pipe, mm
ν = kinematic viscosity, cSt

Example 3.7 Water flows through a 20-in (0.375-in wall thickness) pipeline at 6000 gal/min. Calculate the average velocity and the Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution Using Eq. (3.11), the average velocity is calculated as follows:

$$V = 0.4085 \frac{6000}{19.25^2} = 6.61 \text{ ft/s}$$

From Eq. (3.15), the Reynolds number is

$$\mathrm{Re} = 3162.5 \frac{6000}{19.25 \times 1.0} = 985{,}714$$

Example 3.8 Water flows through a 400-mm pipeline (10-mm wall thickness) at 640 m^3/h. Calculate the average velocity and the Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution From Eq. (3.12) the average velocity is

$$V = 353.6777 \frac{640}{380^2} = 1.57 \text{ m/s}$$

From Eq. (3.16) the Reynolds number is

$$\mathrm{Re} = 353{,}678 \frac{640}{380 \times 1.0} = 595{,}668$$

3.5 Types of Flow

Flow through pipe can be classified as laminar flow, turbulent flow, or critical flow depending on the Reynolds number of flow. If the flow is such that the Reynolds number is less than 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is said to be *turbulent*. *Critical flow* occurs when the Reynolds number is in the range of 2100 to 4000. Laminar flow, also called viscous flow, is characterized by smooth flow in which no eddies or turbulence are visible. The flow is said to occur in laminations. If dye was injected into a transparent pipeline, laminar flow would be manifested in the form of smooth streamlines of dye. Turbulent flow occurs at higher velocities and is accompanied by eddies and other disturbances in the liquid.

Mathematically, if R represents the Reynolds number of flow, the flow types are defined as follows:

Laminar flow: $\text{Re} \leq 2100$

Critical flow: $2100 < \text{Re} \leq 4000$

Turbulent flow: $\text{Re} > 4000$

In the critical flow regime, where the Reynolds number is between 2100 and 4000, the flow is undefined as far as pressure drop calculations are concerned.

3.6 Pressure Drop Due to Friction

As water flows through a pipe there is friction between the adjacent layers of water and between the water molecules and the pipe wall. This friction causes energy to be lost, being converted from pressure energy and kinetic energy to heat. The pressure continuously decreases as water flows through the pipe from the upstream end to the downstream end. The amount of pressure loss due to friction, also known as head loss due to friction, depends on the flow rate, properties of water (specific gravity and viscosity), pipe diameter, pipe length, and internal roughness of the pipe. We will discuss several commonly used equations for calculating the head loss due to friction.

3.6.1 Manning equation

The Manning equation was originally developed for use in open-channel flow of water. It is also sometimes used in pipe flow. The Manning equation uses the Manning index, or roughness coefficient, n, which depends on the type and internal condition of the pipe. The values used for the Manning index for common pipe materials are listed in Table 3.2.

TABLE 3.2 Manning Index

Pipe material	Resistance factor
PVC	0.009
Very smooth cement	0.010
Cement-lined ductile iron	0.012
New cast iron, welded steel	0.014
Old cast iron, brick	0.020
Badly corroded cast iron	0.035
Wood, concrete	0.016
Clay, new riveted steel	0.017
Canals cut through rock	0.040
Earth canals average condition	0.023
Rivers in good conditions	0.030

The following is a form of the Manning equation for frictional pressure drop in water piping systems:

$$Q = \frac{1.486}{n} AR^{2/3} \left(\frac{h}{L}\right)^{1/2} \tag{3.17}$$

where Q = flow rate, ft^3/s
A = cross-sectional area of pipe, ft^2
R = hydraulic radius = $D/4$ for circular pipes flowing full
n = Manning roughness coefficient, dimensionless
D = inside diameter of pipe, ft
h = friction loss, ft of water
L = pipe length, ft

In SI units, the Manning equation is expressed as follows:

$$Q = \frac{1}{n} AR^{2/3} \left(\frac{h}{L}\right)^{1/2} \tag{3.18}$$

where Q = flow rate, m^3/s
A = cross-sectional area of pipe, m^2
R = hydraulic radius = $D/4$ for circular pipes flowing full
n = Manning roughness coefficient, dimensionless
D = inside diameter of pipe, m
h = friction loss, m of water
L = pipe length, m

The Manning equation will be discussed in more detail in sewer piping design in Sec. 3.9.

3.6.2 Darcy equation

The Darcy equation, also called the Darcy-Weisbach equation, is one of the oldest formulas used in classical fluid mechanics. It can be used to calculate the pressure drop in pipes transporting any type of fluid, such as a liquid or gas.

As water flows through a pipe from point A to point B the pressure decreases due to friction between the water and the pipe wall. The Darcy equation may be used to calculate the pressure drop in water pipes as follows:

$$h = f \frac{L}{D} \frac{V^2}{2g} \tag{3.19}$$

where h = frictional pressure loss, ft of head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = pipe inside diameter, ft
V = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

In USCS units, $g = 32.2$ ft/s^2 and in SI units, $g = 9.81$ m/s^2.

Note that the Darcy equation gives the frictional pressure loss in feet of head of water. It can be converted to pressure loss in psi using Eq. (3.7). The term $V^2/2g$ in the Darcy equation is called the velocity head, and it represents the kinetic energy of the water. The term *velocity head* will be used in subsequent sections of this chapter when discussing frictional head loss through pipe fittings and valves.

Another form of the Darcy equation with frictional pressure drop expressed in psi/mi and using a flow rate instead of velocity is as follows:

$$P_m = 71.16 \frac{fQ^2}{D^5} \tag{3.20}$$

where P_m = frictional pressure loss, psi/mi
f = Darcy friction factor, dimensionless
Q = flow rate, gal/min
D = pipe inside diameter, in

In SI units, the Darcy equation may be written as

$$h = 50.94 \frac{fLV^2}{D} \tag{3.21}$$

where h = frictional pressure loss, m of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, m
D = pipe inside diameter, mm
V = average flow velocity, m/s

Another version of the Darcy equation in SI units is as follows:

$$P_{km} = (6.2475 \times 10^{10}) \frac{fQ^2}{D^5} \tag{3.22}$$

where P_{km} = pressure drop due to friction, kPa/km
Q = liquid flow rate, m^3/h
f = Darcy friction factor, dimensionless
D = pipe inside diameter, mm

In order to calculate the friction loss in a water pipeline using the Darcy equation, we must know the friction factor f. The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eqs. (3.19) through (3.22). This friction factor is a nondimensional number between 0.0 and 0.1 (usually around 0.02 for turbulent flow) that depends on the internal roughness of the pipe, pipe diameter, and the Reynolds number, and therefore the type of flow (laminar or turbulent).

For laminar flow, the friction factor f depends only on the Reynolds number and is calculated from the following equation:

$$f = \frac{64}{\text{Re}} \tag{3.23}$$

where f is the friction factor for laminar flow and Re is the Reynolds number for laminar flow ($R < 2100$) (dimensionless).

Therefore, if the Reynolds number for a particular flow is 1200, the friction factor for this laminar flow is $64/1200 = 0.0533$. If this pipeline has a 400-mm inside diameter and water flows through it at 500 m^3/h, the pressure loss per kilometer would be, from Eq. (3.22),

$$P_{\text{km}} = 6.2475 \times 10^{10} \times 0.0533 \times \frac{(500)^2}{(400)^5} = 81.3 \text{ kPa/km}$$

If the flow is turbulent ($\text{Re} > 4000$), calculation of the friction factor is not as straightforward as that for laminar flow. We will discuss this next.

3.6.3 Colebrook-White equation

In turbulent flow the calculation of friction factor f is more complex. The friction factor depends on the pipe inside diameter, the pipe roughness, and the Reynolds number. Based on work by Moody, Colebrook-White, and others, the following empirical equation, known as the Colebrook-White equation, or simply the Colebrook equation, has been proposed for calculating the friction factor in turbulent flow:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{(\text{Re}\sqrt{f})}\right) \tag{3.24}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
Re = Reynolds number, dimensionless

TABLE 3.3 Pipe Internal Roughness

Pipe material	Roughness in	Roughness mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

The absolute pipe roughness depends on the internal condition of the pipe. Generally a value of 0.002 in or 0.05 mm is used in most calculations, unless better data are available. Table 3.3 lists the pipe roughness for various types of pipe. The ratio e/D is known as the relative pipe roughness and is dimensionless since both pipe absolute roughness e and pipe inside diameter D are expressed in the same units (inches in USCS units and millimeters in SI units). Therefore, Eq. (3.24) remains the same for SI units, except that, as stated, the absolute pipe roughness e and the pipe diameter D are both expressed in millimeters. All other terms in the equation are dimensionless.

It can be seen from Eq. (3.24) that the calculation of the friction factor f is not straightforward since it appears on both sides of the equation. A solution for f by successive iteration or a trial-and-error approach is used to solve for the friction factor.

3.6.4 Moody diagram

The Moody diagram is a graphical plot of the friction factor f for all flow regimes (laminar, critical, and turbulent) against the Reynolds number at various values of the relative roughness of pipe. The friction factor for turbulent flow can be found using the Moody diagram (Fig. 3.3) after first calculating the Reynolds number and the relative roughness e/D. For example, using the Moody diagram, we see that at Reynolds number Re = 1,000,000 and a relative roughness $e/D = 0.0002$, the Darcy friction factor is $f = 0.0147$.

Example 3.9 Water flows through a 16-in (0.375-in wall thickness) pipeline at 3000 gal/min. Assuming a pipe roughness of 0.002 in, calculate the friction factor and head loss due to friction in 1000 ft of pipe length.

Solution Using Eq. (3.11) we calculate the average flow velocity:

$$V = 0.4085 \times \frac{3000}{(15.25)^2} = 5.27 \text{ ft/s}$$

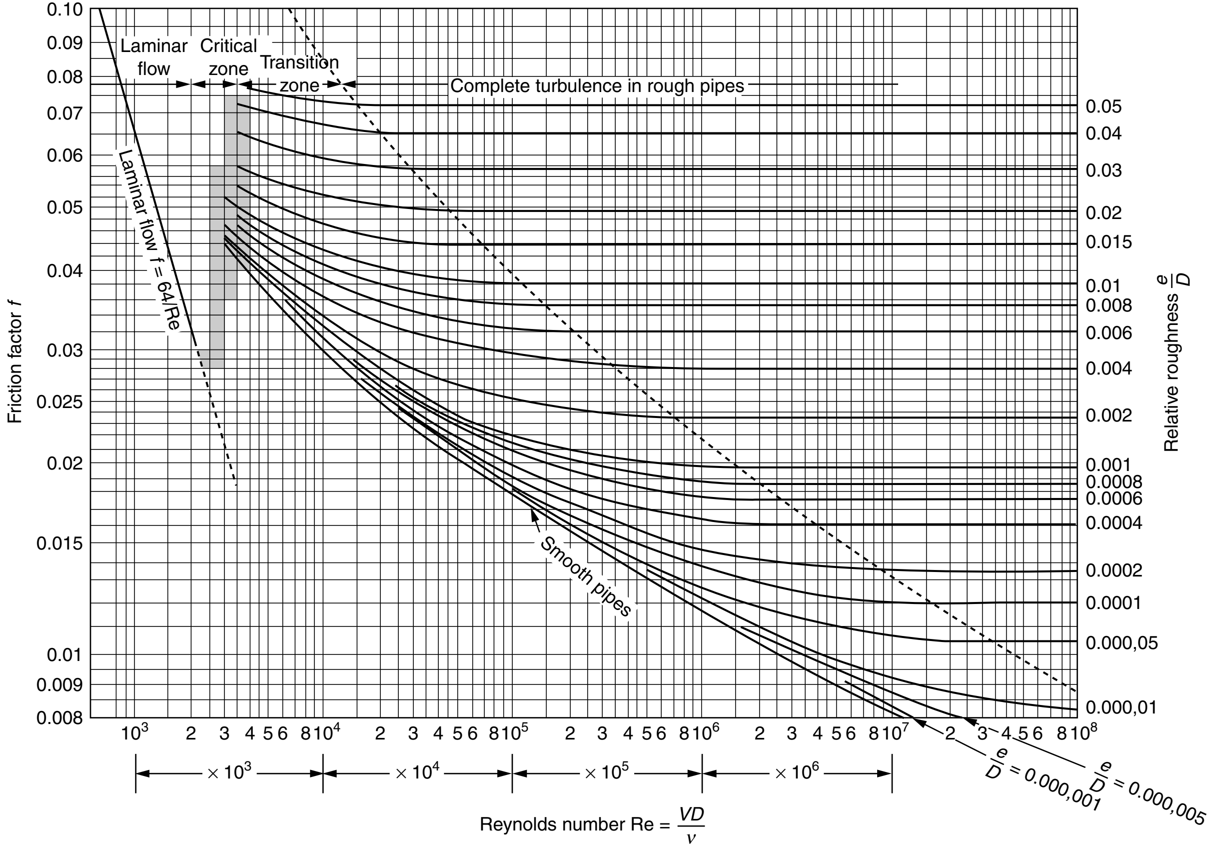


Figure 3.3 Moody diagram.

Using Eq. (3.15) we calculate the Reynolds number as follows:

$$\mathrm{Re} = 3162.5 \frac{3000}{15.25 \times 1.0} = 622{,}131$$

Thus the flow is turbulent, and we can use the Colebrook-White equation to calculate the friction factor.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{f}}\right)$$

This equation must be solved for f by trial and error. First assume that $f = 0.02$. Substituting in the preceding equation, we get a better approximation for f as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{0.02}}\right) = 0.0142$$

Recalculating using this value

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{0.0142}}\right) = 0.0145$$

and finally

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{0.0145}}\right) = 0.0144$$

Thus the friction factor is 0.0144. (We could also have used the Moody diagram to find the friction factor graphically, for Reynolds number $R = 622{,}131$ and $e/D = 0.002/15.25 = 0.0001$. From the graph, we get $f = 0.0145$, which is close enough.)

The head loss due to friction can now be calculated using the Darcy equation (3.18).

$$h = 0.0144 \frac{1000 \times 12}{15.25} \frac{5.27^2}{64.4} = 4.89 \text{ ft of head of water}$$

Converting to psi using Eq. (3.7), we get

$$\text{Pressure drop due to friction} = \frac{4.89 \times 1.0}{2.31} = 2.12 \text{ psi}$$

Example 3.10 A concrete (2-m inside diameter) pipe is used to transport water from a pumping facility to a storage tank 5 km away. Neglecting any difference in elevations, calculate the friction factor and pressure loss in kPa/km due to friction at a flow rate of 34,000 m^3/h. Assume a pipe roughness of 0.05 mm. If a delivery pressure of 4 kPa must be maintained at the delivery point and the storage tank is at an elevation of 200 m above that of the pumping facility, calculate the pressure required at the pumping facility at the given flow rate, using the Moody diagram.

Solution The average flow velocity is calculated using Eq. (3.12).

$$V = 353.6777 \frac{34{,}000}{(2000)^2} = 3.01 \text{ m/s}$$

Next using Eq. (3.16), we get the Reynolds number as follows:

$$\mathrm{Re} = 353{,}678 \times \frac{34{,}000}{1.0 \times 2000} = 6{,}012{,}526$$

Therefore, the flow is turbulent. We can use the Colebrook-White equation or the Moody diagram to determine the friction factor. The relative roughness is

$$\frac{e}{D} = \frac{0.05}{2000} = 0.00003$$

Using the obtained values for relative roughness and the Reynolds number, from the Moody diagram we get friction factor $f = 0.01$.

The pressure drop due to friction can now be calculated using the Darcy equation (3.19) for the entire 5-km length of pipe as

$$h = 0.01 \frac{5000}{2.0} \frac{3.01^2}{2 \times 9.81} = 11.54 \text{ m of head of water}$$

Using Eq. (3.8), we calculate the pressure drop in kPa as

$$\text{Total pressure drop in 5 km} = \frac{11.54 \times 1.0}{0.102} = 113.14 \text{ kPa}$$

Therefore,

$$\text{Pressure drop in kPa/km} = \frac{113.14}{5} = 22.63 \text{ kPa/km}$$

The pressure required at the pumping facility is calculated by adding the following three items:

1. Pressure drop due to friction for 5-km length.
2. The static elevation difference between the pumping facility and storage tank.
3. The delivery pressure required at the storage tank.

We can state the calculation mathematically,

$$P_t = P_f + P_{\mathrm{elev}} + P_{\mathrm{del}} \tag{3.25}$$

where P_t = total pressure required at pump
P_f = frictional pressure head
P_{elev} = pressure head due to elevation difference
P_{del} = delivery pressure at storage tank

All pressures must be in the same units: either meters of head or kilopascals.

$$P_t = 113.14 \text{ kPa} + 200 \text{ m} + 4 \text{ kPa}$$

Changing all units to kilopascals we get

$$P_t = 113.14 + \frac{200 \times 1.0}{0.102} + 4 = 2077.92 \text{ kPa}$$

Therefore, the pressure required at the pumping facility is 2078 kPa.

3.6.5 Hazen-Williams equation

A more popular approach to the calculation of head loss in water piping systems is the use of the Hazen-Williams equation. In this method a coefficient C known as the Hazen-Williams C factor is used to account for the internal pipe roughness or efficiency. Unlike the Moody diagram or the Colebrook-White equation, the Hazen-Williams equation does not require use of the Reynolds number or viscosity of water to calculate the head loss due to friction.

The Hazen-Williams equation for head loss is expressed as follows:

$$h = \frac{4.73\, L(Q/C)^{1.852}}{D^{4.87}} \tag{3.26}$$

where h = frictional head loss, ft
L = length of pipe, ft
D = inside diameter of pipe, ft
Q = flow rate, ft^3/s
C = Hazen-Williams roughness coefficient, dimensionless

Commonly used values of the Hazen-Williams C factor for various applications are listed in Table 3.4.

On examining the Hazen-Williams equation, we see that the head loss due to friction is calculated in feet of head, similar to the Darcy equation. The value of h can be converted to psi using the head-to-psi conversion equation (3.7). Although the Hazen-Williams equation appears to be simpler than using the Colebrook-White and Darcy equations to calculate the pressure drop, the unknown term C can cause uncertainties in the pressure drop calculation.

Usually, the C factor, or Hazen-Williams roughness coefficient, is based on experience with the water pipeline system, such as the pipe material or internal condition of the pipeline system. When designing a new pipeline, proper judgment must be exercised in choosing a C factor since considerable variation in pressure drop can occur by selecting a particular value of C compared to another.

TABLE 3.4 Hazen-Williams *C* Factor

Pipe material	C factor
Smooth pipes (all metals)	130–140
Cast iron (old)	100
Iron (worn/pitted)	60–80
Polyvinyl chloride (PVC)	150
Brick	100
Smooth wood	120
Smooth masonry	120
Vitrified clay	110

Other forms of the Hazen-Williams equation are shown next. In the following, the presented equations calculate the flow rate from a given head loss, or vice versa.

In USCS units, the following forms of the Hazen-Williams equation are used.

$$Q = 6.755 \times 10^{-3} C D^{2.63} h^{0.54} \tag{3.27}$$

$$h = 10{,}460 \left(\frac{Q}{C}\right)^{1.852} \frac{1}{D^{4.87}} \tag{3.28}$$

$$P_m = 23{,}909 \left(\frac{Q}{C}\right)^{1.852} \frac{1}{D^{4.87}} \tag{3.29}$$

where Q = flow rate, gal/min
h = friction loss, ft of water per 1000 ft of pipe
P_m = friction loss, psi per mile of pipe
D = inside diameter of pipe, in
C = Hazen-Williams factor, dimensionless.

In SI Units, the Hazen-Williams equation is expressed as follows:

$$Q = 9.0379 \times 10^{-8} C D^{2.63} \left(\frac{P_{\mathrm{km}}}{\mathrm{Sg}}\right)^{0.54} \tag{3.30}$$

$$P_{\mathrm{km}} = \left(1.1101 \times 10^{13}\right) \left(\frac{Q}{C}\right)^{1.852} \frac{\mathrm{Sg}}{D^{4.87}} \tag{3.31}$$

where Q = flow rate, m^3/h
D = pipe inside diameter, mm
P_{km} = frictional pressure drop, kPa/km
Sg = liquid specific gravity (water = 1.00)
C = Hazen-Williams factor, dimensionless

Example 3.11 Water flows through a 16-in (0.375-in wall thickness) pipeline at 3000 gal/min. Using the Hazen-Williams equation with a C factor of 120, calculate the pressure loss due to friction in 1000 ft of pipe length.

Solution First we calculate the flow rate using Eq. (3.27):

$$Q = 6.755 \times 10^{-3} \times 120 \times (15.25)^{2.63} h^{0.54}$$

where h is in feet of head per 1000 ft of pipe.

Rearranging the preceding equation, using $Q = 3000$ and solving for h, we get

$$h^{0.54} = \frac{3000}{6.755 \times 10^{-3} \times 120 \times (15.25)^{2.63}}$$

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Therefore,

$$h = 7.0 \text{ ft per 1000 ft of pipe}$$

$$\text{Pressure drop} = \frac{7.0 \times 1.0}{2.31} = 3.03 \text{ psi}$$

Compare this with the same problem described in Example 3.9. Using the Colebrook-White and Darcy equations we calculated the pressure drop to be 4.89 ft per 1000 ft of pipe. Therefore, we can conclude that the C value used in the Hazen-Williams equation in this example is too low and hence gives us a comparatively higher pressure drop. If we recalculate, using a C factor of 146 will get 5.26 ft per 1000 ft of pipe, which is closer to the 4.89 ft per 1000 ft we got using the Colebrook-White equation.

Example 3.12 A concrete pipe with a 2-m inside diameter is used to transport water from a pumping facility to a storage tank 5 km away. Neglecting differences in elevation, calculate the pressure loss in kPa/km due to friction at a flow rate of 34,000 m^3/h. Use the Hazen-Williams equation with a C factor of 140. If a delivery pressure of 400 kPa must be maintained at the delivery point and the storage tank is at an elevation of 200 m above that of the pumping facility, calculate the pressure required at the pumping facility at the given flow rate.

Solution The flow rate Q in m^3/h is calculated using the Hazen-Williams equation (3.31) as follows:

$$P_{\text{km}} = (1.1101 \times 10^{13}) \left(\frac{34{,}000}{140}\right)^{1.852} \times \frac{1}{(2000)^{4.87}}$$

$$= 24.38 \text{ kPa/km}$$

The pressure required at the pumping facility is calculated by adding the pressure drop due to friction to the delivery pressure required and the static elevation head between the pumping facility and storage tank using Eq. (3.25).

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}}$$

$$= (24.38 \times 5) \text{ kPa} + 200 \text{ m} + 400 \text{ kPa}$$

Changing all units to kPa we get

$$P_t = 121.9 + \frac{200 \times 1.0}{0.102} + 400 = 2482.68 \text{ kPa}$$

Thus the pressure required at the pumping facility is 2483 kPa.

3.7 Minor Losses

So far, we have calculated the pressure drop per unit length in straight pipe. We also calculated the total pressure drop considering several miles of pipe from a pump station to a storage tank. Minor losses in a water pipeline are classified as those pressure drops that are associated with piping components such as valves and fittings. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. A pipe nozzle exiting from a storage tank will have entrance and exit losses. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe.

Generally, minor losses are included in calculations by using the equivalent length of the valve or fitting or using a resistance factor or K factor multiplied by the velocity head $V^2/2g$. The term minor losses can be applied only where the pipeline lengths and hence the friction losses are relatively large compared to the pressure drops in the fittings and valves. In a situation such as plant piping and tank farm piping the pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses is really a misnomer. In any case, the pressure losses through valves, fittings, etc., can be accounted for approximately using the equivalent length or K times the velocity head method. It must be noted that this way of calculating the minor losses is valid only in turbulent flow. No data are available for laminar flow.

3.7.1 Valves and fittings

Table 3.5 shows the equivalent length of commonly used valves and fittings in a typical water pipeline. It can be seen from this table that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore, a 20-in-diameter gate valve may be replaced with a $20 \times 8 = 160$-in-long piece of pipe that will match the frictional pressure drop through the valve.

Example 3.13 A piping system is 2000 ft of NPS 20 pipe that has two 20-in gate valves, three 20-in ball valves, one swing check valve, and four 90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe and valves and fittings.

Solution Using Table 3.5, we can convert all valves and fittings in terms of 20-in pipe as follows:

Two 20-in gate valves	$= 2 \times 20 \times 8$	$= 320$ in of 20-in pipe
Three 20-in ball valves	$= 3 \times 20 \times 3$	$= 180$ in of 20-in pipe
One 20-in swing check valve	$= 1 \times 20 \times 50$	$= 1000$ in of 20-in pipe
Four 90° elbows	$= 4 \times 20 \times 30$	$= 2400$ in of 20-in pipe
Total for all valves and fittings	$= 4220$ in of 20-in pipe	$= 351.67$ ft of 20-in pipe

Adding the 2000 ft of straight pipe, the total equivalent length of straight pipe and all fittings is

$$L_e = 2000 + 351.67 = 2351.67 \text{ ft}$$

TABLE 3.5 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

The pressure drop due to friction in the preceding piping system can now be calculated based on 2351.67 ft of pipe. It can be seen in this example that the valves and fittings represent roughly 15 percent of the total pipeline length. In plant piping this percentage may be higher than that in a long-distance water pipeline. Hence, the reason for the term *minor losses*.

Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation, the pressure drop in a straight length of pipe is given by

$$h = f \frac{L}{D} \frac{V^2}{2g} \tag{3.32}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K (also known as the resistance coefficient) and Eq. (3.32) then becomes

$$h = K \frac{V^2}{2g} \tag{3.33}$$

In Eq. (3.33), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $V^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(V^2/2g)$, where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 3.6. It can be seen that the K factor depends on the nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

From Table 3.6, it can be seen that a 6-in gate valve has a K factor of 0.12, while a 20-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be 0.12 $(V^2/2g)$ and that in the 20-in valve is 0.10 $(V^2/2g)$. The velocities in both cases will be different due to the difference in diameters.

If the flow rate was 1000 gal/min, the velocity in the 6-in valve will be approximately

$$V_6 = 0.4085 \frac{1000}{6.125^2} = 10.89 \text{ ft/s}$$

Similarly, at 1000 gal/min, the velocity in the 20-in valve will be approximately

$$V_6 = 0.4085 \frac{1000}{19.5^2} = 1.07 \text{ ft/s}$$

Therefore,

$$\text{Head loss in 6-in gate valve} = \frac{0.12(10.89)^2}{64.4} = 0.22 \text{ ft}$$

$$\text{Head loss in 20-in gate valve} = \frac{0.10(1.07)^2}{64.4} = 0.002 \text{ ft}$$

These head losses appear small since we have used a relatively low flow rate in the 20-in valve. In reality the flow rate in the 20-in valve may be as high as 6000 gal/min and the corresponding head loss will be 0.072 ft.

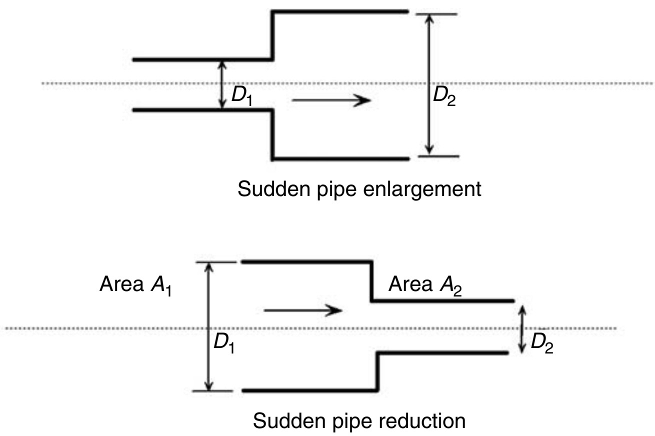
3.7.2 Pipe enlargement and reduction

Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(V_1 - V_2)^2}{2g} \tag{3.34}$$

TABLE 3.6 Friction Loss in Valves—Resistance Coefficient *K*

		Nominal pipe size, in											
Description	L/D	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72



A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 3.4 Sudden pipe enlargement and pipe reduction.

where V_1 and V_2 are the velocities of the liquid in the two pipe sizes D_1 and D_2, respectively. Writing Eq. (3.34) in terms of pipe cross-sectional areas A_1 and A_2,

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2g} \tag{3.35}$$

for sudden enlargement. This is illustrated in Fig. 3.4.

For sudden contraction or reduction in pipe size as shown in Fig. 3.4, the head loss is calculated from

$$h_f = \left(\frac{1}{C_c} - 1\right) \frac{V_2^2}{2g} \tag{3.36}$$

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 3.4.

Gradual enlargement and reduction of pipe size, as shown in Fig. 3.5, cause less head loss than sudden enlargement and sudden reduction. For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c(V_1 - V_2)^2}{2g} \tag{3.37}$$

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is shown in Fig. 3.6.

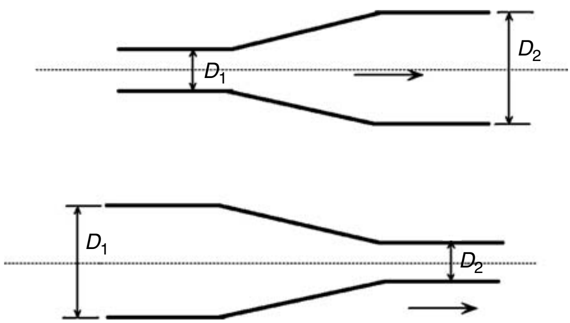


Figure 3.5 Gradual pipe enlargement and pipe reduction.

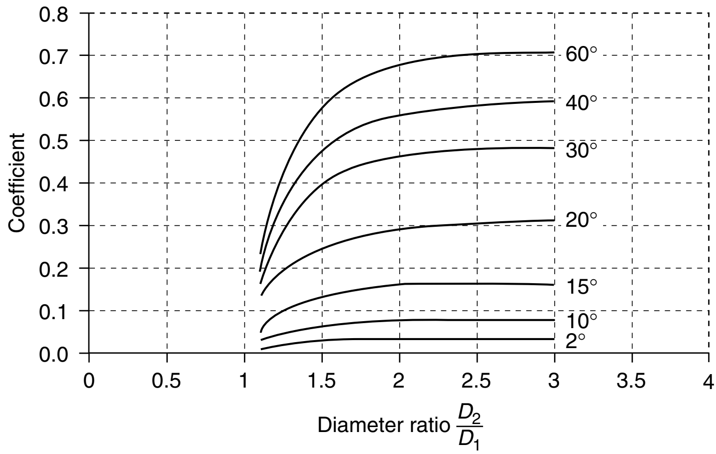


Figure 3.6 Gradual pipe expansion head loss coefficient.

3.7.3 Pipe entrance and exit losses

The K factors for computing the head loss associated with pipe entrance and exit are as follows

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

3.8 Sewer Piping Systems

So far we have discussed wastewater pipelines considering pressurized flow. Water is conveyed from point A to point B starting with a pressure higher than atmospheric. Because of frictional loss in pipe, the water pressure decreases until it reaches the destination at some minimum pressure sufficient to enter a storage tank. Gravity pipelines and open-channel flow pipelines are nonpressurized lines. The head loss at a certain flow rate occurs due to the elevation change between the upstream and downstream ends of the pipeline. Sewer piping systems are generally nonpressurized gravity flow pipelines. They may run partially full, as in open-channel flow, or sometimes they run full flow.

Sanitary sewer systems are composed of piping that is used to transport wastewater consisting of residential, commercial, and industrial waste. Some amount of stormwater, surface water, and groundwater may also be present in sanitary sewer systems.

Storm sewer systems are composed of those piping systems that carry only stormwater, surface water, and other waters that are drained into the storm sewer system. They do not carry residential, commercial, or industrial wastes.

A combined sewer system consists of a combination of a sanitary sewer system and a storm sewer system. Thus a combined sewer system carries both stormwater as well as wastewater.

Infiltration is defined as water that enters a sanitary sewer system from the ground through pipes, pipe joints, manholes, etc. *Inflow* is water that enters a sanitary sewer system from roof leaders, cellars, or other drains. Additionally, this will include water discharged from cooling systems, manhole covers, catch basins, storm sewers, and surface runoff. *Exfiltration* occurs when the wastewater from the sewer system flows out through pipe joints, cracks, etc., into the surrounding soil.

3.9 Sanitary Sewer System Design

In designing a sanitary sewer system we must first correctly estimate the quantity of wastewater that will be flowing through the system. The water consumed by residential and industrial facilities does not all end up in the sewer system. Part of the water consumed is lost into the ground when used for landscaping, car washing, etc. The average per capita water consumption in residential units ranges between 40 and 120 gal/day. Table 3.7 lists typical wastewater flow rates from residential sources.

Commercial and industrial sewage flow rates depend upon the type of activity and industry. Table 3.8 shows average commercial wastewater flows.

Several local, state, and federal regulations exist for designing sanitary sewer systems. The American Society of Civil Engineers' (ASCE) *Manual of Engineering Practice*, #37, Design and Construction of Sanitary and Storm Sewers, must be consulted when designing sanitary sewer systems.

Sewer systems are generally designed as gravity flow systems with a free water surface. This means that the sewer pipe may run full or partially full so that there is an air space above the water level.

TABLE 3.7 Typical Wastewater Flow Rates from Residential Units

Source	Unit	Flow rate: Range, gal/day	Flow rate: Typical gal/day
Apartment			
High-rise	Person	35–75	50
Low-rise	Person	50–80	65
Hotel	Guest	30–55	45
Individual residence			
Typical home	Person	45–90	70
Better home	Person	60–100	80
Luxury home	Person	75–150	95
Older home	Person	30–60	45
Summer cottage	Person	25–50	40
Motel			
With kitchen	Unit	90–180	100
Without kitchen	Unit	75–150	95
Trailer park	Person	30–50	40

This is known as open-channel flow. The advantage of open-channel flow includes ventilation of the sewer and maintenance of good velocities at low flow rates for cleaning the sewers. Pumps are also used to provide the lift necessary from deep sewer locations to force the sewage to a higher elevation from which point gravity flow can continue. When a sanitary sewer system is flowing full, minimum velocities range from 2 to 2.5 ft/s (0.6 to 0.75 m/s). Storm sewers generally have a minimum velocity range of 3 to 3.5 ft/s (1.0 to 1.2 m/s). The minimum velocity is required to prevent deposition of solids on the pipe wall. The velocity of flow ensures the solids will remain in suspension and move with the water. There is also a maximum velocity that must not be exceeded to prevent erosion of the sewer pipe. The maximum velocity is in the range of 9 to 10 ft/s (3 to 3.5 m/s) for both sanitary sewers and storm sewers.

Since sewer flow is open-channel flow, we can use the Manning equation for calculating the flows and pressure loss in sewer piping. The term

TABLE 3.8 Average Commercial Wastewater Flow

Type of establishment	Average flow, gal/day per capita
Stores, offices, and small businesses	12–25
Hotels	50–150
Motels	50–125
Drive-in theaters (3 persons per car)	8–10
Schools, no showers, 8 h	8–35
Schools with showers, 8 h	17–25
Tourists and trailer camps	80–120
Recreational and summer camps	20–25

slope is used to describe the hydraulic energy gradient in the sewer piping. The slope is a dimensionless parameter that can be referred to as ft/ft, m/m, or as a percentage. For example, the slope may be referred to as 0.003 ft/ft or 0.3 percent. It is also equal to the geometrical slope or gradient of the sewer pipe.

The Manning equation uses the Manning index n, or roughness coefficient, which depends on the type and internal condition of the pipe. The value of n ranges from 0.01 for smooth surfaces to 0.10 for rough surfaces. For sewer design, generally the Manning roughness coefficient of 0.013 is used. For older sewer pipes, a value of 0.015 may be used.

The general form of the Manning equation for open-channel flow is as follows:

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \tag{3.38}$$

where V = average velocity of flow, ft/s
n = roughness coefficient, dimensionless
R = hydraulic radius = (wetted cross-sectional area / wetted perimeter), ft [for a circular pipe flowing full, $R = (\pi D^2/4)/(\pi D) = D/4$]
S = slope of hydraulic energy gradient, ft/ft

In SI units, the Manning equation is

$$V = \frac{1}{n} R^{2/3} S^{1/2} \tag{3.39}$$

where V = average velocity of flow, m/s
n = roughness coefficient, dimensionless
R = hydraulic radius = (wetted cross-sectional area/ wetted perimeter), m [for a circular pipe flowing full, $R = (\pi D^2/4)/(\pi D) = D/4$]
S = slope of hydraulic energy gradient, m/m

Since, in general, we are dealing with sewer flow rates in ft^3/s and not velocities, Eqs. (3.38) and (3.39) are converted to the equivalent in flow rates for circular pipe as follows:

$$Q = \frac{0.463}{n} D^{8/3} S^{1/2} \tag{3.40}$$

where Q = flow rate, ft^3/s
n = roughness coefficient, dimensionless
D = inside diameter of pipe, ft
S = slope of hydraulic energy gradient, ft/ft

In SI units, the Manning equation is expressed as follows:

$$Q = \frac{0.312}{n} D^{8/3} S^{1/2} \tag{3.41}$$

where Q = flow rate, m^3/s
n = roughness coefficient, dimensionless
D = inside diameter of pipe, m
S = slope of hydraulic energy gradient, m/m

Another form of the Manning equation for calculating the slope S for full flow of circular pipes is as follows:

$$S = \frac{0.466}{D^{16/3}} n^2 Q^2 \tag{3.42}$$

and in SI units as follows

$$S = \frac{10.27}{D^{16/3}} n^2 Q^2 \tag{3.43}$$

All symbols are as defined previously.

It can be seen from the Manning equation that the slope of the sewer S, which represents the energy grade line, is directly proportional to the flow velocity or flow rate. Thus for a given pipe, flowing full, as the flow rate increases, the slope increases. In other words, as the physical slope of the sewer pipe is increased from, say, 1 in 500 to 1 in 200, the flow velocity and hence the flow rate increases. When the pipe is not flowing full, the hydraulic radius R has to be calculated based on the actual wetted area and the wetted perimeter. Figure 3.7 shows a partially full sewer pipe.

It can be seen from Fig. 3.7 that there is a relationship between the water depth d, the pipe diameter D, and the included angle θ, as

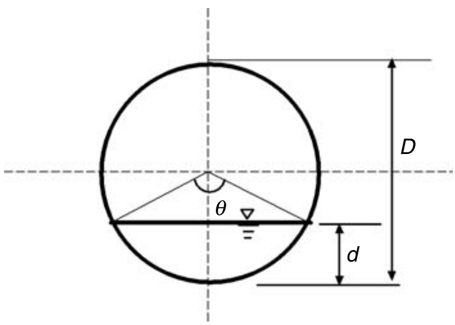


Figure 3.7 Partially full sewer pipe.

follows:

$$\cos\left(\frac{\theta}{2}\right) = \frac{D/2 - d}{D/2} = 1 - \frac{2d}{D} \tag{3.44}$$

The wetted area A is calculated from

$$A = \frac{\pi D^2}{4}\left(\frac{\theta}{360}\right) - \frac{1}{2}\left(\frac{D^2}{4}\right)\sin\theta = \frac{\pi D^2}{4}\left(\frac{\theta}{360} - \frac{\sin\theta}{2\pi}\right) \tag{3.45}$$

and the wetted perimeter P is

$$P = \frac{\theta}{360}\pi D \tag{3.46}$$

Finally the hydraulic radius R for the partially full sewer flow is calculated from

$$R = \frac{A}{P} = \frac{D}{4}\left[1 - \frac{180}{\pi}\left(\frac{\sin\theta}{\theta}\right)\right] \tag{3.47}$$

Table 3.9 shows the values of the wetted area ratio, wetted perimeter ratio, and the hydraulic radius ratio for circular pipes at various flow depths to pipe diameter ratio d/D, calculated using Eqs. (3.44) through (3.47). These ratios relate to the corresponding values for full pipe flow as illustrated in the following sample calculation.

It can be seen from Table 3.9 that at a water depth of 70 percent ($d/D = 0.70$), the hydraulic radius is 1.185 times that at full flow.

$$\text{hydraulic radius at 70\% depth} = 1.185 \times \frac{D}{4} = 0.2963D$$

TABLE 3.9 Hydraulic Radius for Partially Full Circular Pipes

d/D	Angle θ	Wetted area ratio	Wetted perimeter ratio	Hydraulic radius ratio
0.1	73.7398	0.0520	0.2048	0.2539
0.2	106.2602	0.1423	0.2952	0.4822
0.3	132.8437	0.2523	0.3690	0.6837
0.4	156.9261	0.3735	0.4359	0.8569
0.5*	180.0001	0.5000	0.5000	1.0000
0.6	203.0740	0.6265	0.5641	1.1106
0.7	227.1564	0.7477	0.6310	1.1850
0.8	253.7399	0.8577	0.7048	1.2168
0.9	286.2603	0.9480	0.7952	1.1922
1.0	360.0001	1.0000	1.0000	1.0000

*At $d/D = 0.5$, wetted area $= 0.5 \times 0.7854 \times D \times D$; wetted perimeter $= 0.5 \times 3.14159 \times D$; hydraulic radius $= 0.25 \times D = 1.00 \times D/4$.

Similarly, at 70 percent depth, from Table 3.9 the corresponding wetted area of flow is calculated as follows:

$$\text{Wetted area at 70\% depth} = 0.7477 \times (0.7854D^2) = 0.5872D^2$$

In a particular sewer piping with a given slope S, when flowing full, the Manning equation can be used to calculate the velocity of flow V and the flow rate Q. When the same sewer pipe (with the same slope S) is flowing partly full, the hydraulic radius is less and hence the flow rate Q_p is less. The partly full flow results in a velocity of flow of V_p. However, under both conditions, we must ensure the velocity is sufficient for the sewer to be self-cleansing. Thus the slope of the sewer must be checked for both conditions to ensure that this cleansing velocity requirement is met. The self-cleansing velocity is 2 ft/s to 2.5 ft/s (0.6 m/s to 0.75 m/s).

When pipes are flowing full, we can calculate the slope for a given flow rate very easily using the Manning equations previously discussed. Many times sewer pipes do not run full. The ratio of the depth of flow d to the pipe inside diameter D is an important parameter that relates to various other dimensionless parameters in partly full sewer pipes. Figure 3.8 shows the variation of d/D with other critical parameters such as velocity ratio and flow ratio.

Upon examining Fig. 3.8 it can be seen that, when the sewer depth is 50 percent or $d/D = 0.5$, the ratio of the partially full flow rate to the full pipe flow rate (Q/Q_f) is approximately 0.40. This is true, if the Manning roughness coefficient n is considered to be variable with depth. On the other hand, if n is assumed constant, the ratio Q/Q_f becomes 0.50.

Consider now the sewer to be 70 percent full, or $d/D = 0.7$. From Fig. 3.8 we find that the flow rate ratios are

$$\frac{Q}{Q_f} = \begin{cases} 0.70 & \text{approximately for variable } n \\ 0.85 & \text{approximately for constant } n \end{cases}$$

Figure 3.8 is very useful for calculations of partially full sewer pipes. We will illustrate this with several examples.

Example 3.14 A sewer pipe system is constructed of NPS 12 (0.3125-in wall thickness) pipe. Assuming the pipe is flowing full at 700 gal/min, calculate the slope of the energy gradient using the Manning equation with $n = 0.013$.

(a) If this pipe were flowing half full, what is the discharge rate and velocity?

(b) If the slope is changed to 0.005, what is the effect?

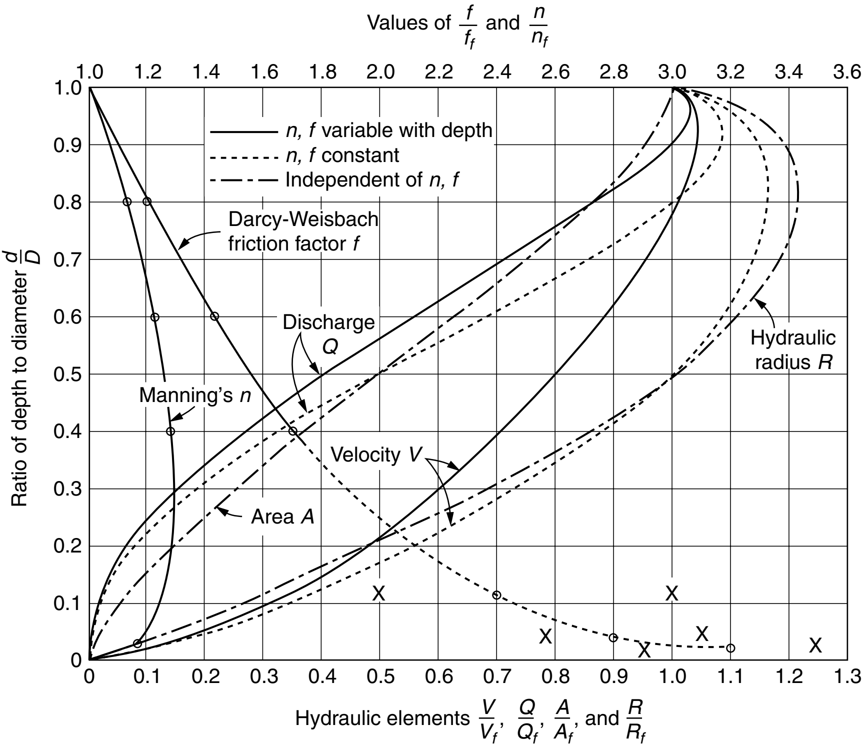


Figure 3.8 Hydraulic ratios of circular sewer pipes. (Courtesy: McGraw-Hill, *Water and Wastewater Calculations Manual*, Shun Dar Lin, 2001. Reproduced by permission.)

Solution

(a) We first calculate the pipe inside diameter:

$$D = 12.75 - 2 \times 0.3125 = 12.125 \text{ in} = \frac{12.125}{12} = 1.0104 \text{ ft}$$

$$\text{Discharge rate } Q = \frac{700 \times 1}{7.4805 \times 60} = 1.5596 \text{ ft}^3\text{/s}$$

Using the Manning equation (3.40), we get

$$1.5596 = \frac{0.463}{0.013} \times (1.0104)^{8/3} \times S^{1/2}$$

Solving for S, we get

$$S = \left[\frac{1.5596 \times 0.013}{0.463 \times (1.0104)^{8/3}}\right]^2 = 0.0018 \text{ ft/ft}$$

Therefore, the slope of the energy gradient is 0.0018 ft/ft or 0.18 percent.

The average velocity is

$$V = \frac{1.5596}{0.7854(1.0104)^2} = 1.95 \text{ ft/s}$$

If the pipe were flowing half full, then $d/D = 0.50$. From Fig. 3.8 we get

$$\frac{Q}{Q_f} = 0.4 \text{ to } 0.5$$

depending on whether n is constant or variable with depth.

Assuming a constant n value, from Fig. 3.8, we get

$$\frac{Q}{Q_f} = 0.4$$

and the velocity ratio is

$$\frac{V}{V_f} = 0.8$$

Therefore, when the pipe is flowing half full, the discharge is

$$Q = 0.4 \times 700 = 280 \text{ gal/min}$$

and the average velocity is

$$V = 0.8 \times 1.95 = 1.56 \text{ ft/s}$$

Since this velocity is less than 2 ft/s, self-cleansing will not occur. Either the flow rate or slope should be increased to ensure a velocity of at least 2 ft/s for self-cleansing.

(b) If the slope is changed to 0.005 for the half-full condition, we first calculate the full flow value of discharge at the higher slope. Since discharge is proportional to the square root of the slope, from the Manning equation, the new discharge is proportional to the square root of the slope. The new full flow discharge at a slope of 0.005 is

$$Q_f = \left(\frac{0.005}{0.0018}\right)^{1/2} \times 700 = 1166.7 \text{ gal/min} = 2.60 \text{ ft}^3\text{/s}$$

For the half-full condition, we have $d/D = 0.5$. From Fig. 3.8, we get

$$\frac{Q}{Q_f} = 0.4$$

and the velocity ratio is

$$\frac{V}{V_f} = 0.8$$

Then the full flow velocity is

$$V_f = \frac{2.60}{0.7854 \times (1.0104)^2} = 3.24 \text{ ft/s}$$

Therefore, the discharge for the half-full condition is

$$Q = 0.4 \times 2.60 = 1.04 \text{ ft}^3/\text{s} = 467 \text{ gal/min}$$

and the velocity is

$$V = 0.8 \times 3.24 = 2.59 \text{ ft/s}$$

Thus, at the higher slope of 0.005, at half depth, the velocity is high enough for self-cleansing.

Example 3.15 A sewer pipe with a 750-mm outside diameter (20-mm wall thickness) is flowing full at 2000 m^3/h. Assume $n = 0.013$.

(a) What is the slope of the energy gradient?

(b) Calculate the depth of flow and flow velocity when discharging at 1200 m^3/h.

(c) Calculate the flow velocities in both cases.

Solution

(a) The diameter is

$$D = 750 - 40 = 710 \text{ mm}$$

The discharge rate is

$$Q = 2000 \text{ m}^3/\text{h} = \frac{2000}{3600} \text{ m}^3/\text{s}$$

Using the full pipe flow version of the Manning equation (3.41),

$$\frac{2000}{3600} = \frac{0.312}{0.013} \times (0.71)^{8/3} \times S^{1/2}$$

Solving for the slope S we get

$$S = 0.0033 \text{ m/m}$$

Therefore, the slope of the energy gradient is 0.0033 m/m or 0.33 percent.

(b) When discharge drops to 1200 m^3/h, the ratio

$$\frac{Q}{Q_f} = \frac{1200}{2000} = 0.6$$

Assuming n is a constant, from Fig. 3.8, we get $d/D = 0.55$. Hence,

$$\text{Depth of flow} = 0.55 \times 710 = 391 \text{ mm}$$

If n is considered variable with depth, we get $d/D = 0.63$, or

$$\text{Depth of flow} = 0.63 \times 710 = 447 \text{ mm}$$

(c) Flow velocities are calculated for cases (a) and (b) as follows. For case (a), full flow in the pipe at 2000 m^3/h, the velocity is

$$V = \frac{2000/3600}{0.7854 \times (0.71)^2} = 1.40 \text{ m/s}$$

In case (b), discharging at 1200 m^3/h, we calculated a depth ratio of $d/D = 0.55$ when n is a constant, and from Table 3.9 the wetted area ratio is 0.5633 by interpolation. Therefore the velocity in this partly full sewer pipe is

$$V = \frac{1200/3600}{0.5633 \times 0.7854 \times (0.71)^2} = 1.49 \text{ m/s}$$

And, in case (b) where n is variable and $d/D = 0.63$, from Table 3.9 the wetted area ratio is 0.663 by interpolation. Then

$$V = \frac{1200/3600}{0.663 \times 0.7854 \times (0.71)^2} = 1.27 \text{ m/s}$$

Therefore, in summary,

(a) The slope of the energy gradient is 0.0033 m/m or 0.33 percent.

(b) The depth of flow when discharging at 1200 m^3/h is 391 mm if the roughness coefficient n is constant or 447 mm if the roughness coefficient n is variable with depth.

(c) For case (a) the flow velocity at 2000 m^3/h discharge is 1.4 m/s. For case (b) the flow velocities are 1.49 and 1.27 m/s at 1200 m^3/h discharge, respectively for depths of 391 and 447 mm.

Example 3.16 A 24-in-diameter concrete pipe is used as a sewer and has a slope of 2 in 1000. The depth of the liquid in the pipe is 11 in. What are the discharge rate and the average velocity using the Manning equation? Use $n = 0.013$. Will this system produce a sufficient flow velocity for self-cleansing?

Solution The pipe diameter is

$$D = 24 \text{ in}$$

The slope is

$$S = \frac{2}{1000} = 0.002 \text{ ft/ft}$$

The depth of liquid to the pipe diameter ratio is

$$\frac{d}{D} = \frac{11}{24} = 0.4583$$

For this ratio from Table 3.9 we get by interpolation a hydraulic radius of

$$R = 0.9403 \times \frac{24}{4} = 5.642 \text{ in}$$

and a wetted area of flow,

$$A = 0.4472 \times \left(0.7854 \times \frac{24^2}{144}\right) = 1.405 \text{ ft}^2$$

Using the Manning equation (3.38), we get the average velocity of flow as

$$V = \frac{1.486}{0.013} \times \left(\frac{5.642}{12}\right)^{2/3} (0.002)^{1/2} = 3.09 \text{ ft/s}$$

The discharge rate Q is given by

$$Q = \text{average velocity} \times \text{area of flow} = 3.09 \times 1.405 = 4.34 \text{ ft}^3\text{/s}$$

Since the velocity of 3.09 ft/s is greater than the minimum 2 ft/s required for self-cleansing, we can state that this flow will cause self-cleansing of the sewer pipe.

3.10 Self-Cleansing Velocity

Since sanitary sewers contain suspended solids that may deposit on the pipe wall, some minimum velocity is desirable to keep the solid particles suspended and in motion. This velocity that is necessary to prevent deposition of solids is known as the self-cleansing velocity. For a pipe flowing full, the ASCE formula for the self-cleansing velocity is as follows:

$$V = \frac{1.486R^{1/6}}{n}[B(\text{Sg} - 1)D_p]^{1/2} \tag{3.48}$$

or

$$V = \left[\frac{8B}{f}g(\text{Sg} - 1)D_p\right]^{1/2} \tag{3.49}$$

where V = average flow velocity, ft/s
R = hydraulic radius, ft
n = roughness coefficient, dimensionless
B = dimensionless constant (0.04 to 0.8)
Sg = specific gravity of particle
D_p = diameter of particle, ft
f = friction factor, dimensionless
g = acceleration due to gravity, ft/s^2

In SI units

$$V = \frac{R^{1/6}}{n}[B(\mathrm{Sg} - 1)D_p]^{1/2} \tag{3.50}$$

or

$$V = \left[\frac{8B}{f}g(\mathrm{Sg} - 1)D_p\right]^{1/2} \tag{3.51}$$

where V = average flow velocity, m/s
R = hydraulic radius, m
n = roughness coefficient, dimensionless
B = dimensionless constant (0.04 to 0.8)
Sg = specific gravity of particle
D_p = diameter of particle, m
f = friction factor, dimensionless
g = acceleration due to gravity, m/s^2

Figure 3.9 shows a graph that can be used for determining the self-cleansing velocity of partly full sewer pipes. Reviewing this figure, it can be seen that if the d/D ratio is 0.5 corresponding to the flow ratio of 0.4, the slope ratio for self-cleansing is

$$\frac{S}{S_f} = 1.8 \text{ approximately}$$

and the velocity ratio is

$$\frac{V}{V_f} = 0.8 \text{ approximately}$$

Therefore, if the full flow velocity is 3 ft/s and the slope is 0.0003, the corresponding velocity and slope for an equal self-cleansing property at 50 percent depth are

$$V = 0.8 \times 3 = 2.4 \text{ ft/s}$$

and

$$S = 1.8 \times 0.0003 = 0.0005$$

Example 3.17 A concrete sewer pipe is laid on a slope of 1 in 350 and carries a flow rate of 0.25 m^3/s. The flow is 70 percent full. What minimum diameter is required? Calculate the flow velocity and compare it with the minimum velocity required for self-cleansing. Use $n = 0.013$.

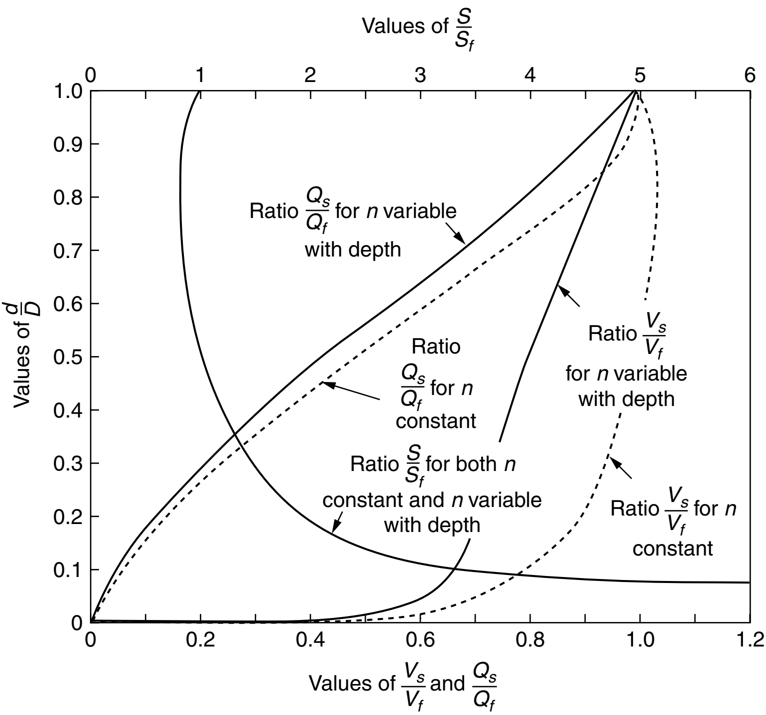


Figure 3.9 Self-cleansing velocity of partly full sewers. (Courtesy: McGraw-Hill, *Water and Wastewater Calculations Manual*, Shun Dar Lin, 2001. Reproduced by permission.)

Solution

$$\text{Slope} = \frac{1}{350} = 0.0029 \text{ m/m}$$

$$\text{Flow rate } Q = 0.25 \text{ m}^3/\text{s}$$

Since the depth is 70 percent, $d/D = 0.7$. From Table 3.9, the hydraulic radius ratio is

$$R = 1.185 \times \frac{D}{4} = 0.2963D$$

where D is the pipe inside diameter.

The area of flow from Table 3.9 is

$$A = 0.7477 \times \frac{\pi}{4} \times D^2 = 0.5872D^2$$

Using the Manning equation, we get

$$Q = 0.5872D^2 \times \frac{1}{n} \times (0.2963D)^{2/3}(0.0029)^{1/2}$$

or

$$0.25 = 1.0811 \times D^{8/3}$$

Solving for diameter,

$$D = \left(\frac{0.25}{1.0811}\right)^{3/8} = 0.5775 \text{ m}$$

Using 600-mm diameter pipe, we calculate the velocity of flow as

$$V = \frac{0.25}{0.5872D^2} = 1.277 \text{ m/s}$$

This is greater than the 0.6 to 0.75 m/s needed for self-cleansing.

Example 3.18 The sewer pipeline shown in Fig. 3.10 consists of four main pipes: *AB*, *BC*, *CD*, and *DE*. Lateral pipes *FB*, *GC*, and *HD* bring the wastewater in from three sources *F*, *G*, and *H*. The slopes of the pipes are as follows:

Pipe	Slope
AB	0.003
BC	0.002
CD	0.002
FB	0.003
GC	0.003
HD	0.002

Assume $n = 0.013$ and there is full flow in pipes *AB*, *FB*, *GC*, and *HD*.

(a) Calculate the pipe size required for section *BC*.

(b) Calculate the flow rates and sewage depth in section *CD*.

(c) Determine the slope required for full flow in section *DE*.

Solution

(a) Using the Manning equation we will calculate the flow through each pipe *AB*, *FB*, *BC*, *GC*, and *CD*.

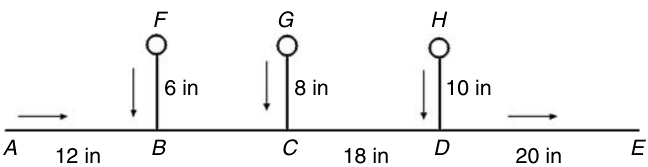


Figure 3.10 Sewer pipeline with branches.

Pipe *AB*

$$\text{Area } A = 0.7854 \times \left(\frac{12}{12}\right)^2 = 0.7854 \text{ ft}^2$$

$$\text{Discharge } Q_{AB} = 0.7854 \times \frac{1.486}{0.013} \times (0.25)^{2/3} \times (0.003)^{1/2} = 1.9513 \text{ ft}^3\text{/s}$$

Pipe *FB*

$$\text{Area } A = 0.7854 \times \left(\frac{6}{12}\right)^2 = 0.1964 \text{ ft}^2$$

$$\text{Hydraulic radius } R = \frac{D}{4} = \frac{6}{4 \times 12} = 0.125 \text{ ft}$$

$$\text{Discharge } Q_{FB} = 0.1964 \times \frac{1.486}{0.013} \times (0.125)^{2/3} \times (0.003)^{1/2}$$

$$= 0.3074 \text{ ft}^3\text{/s}$$

Pipe *BC*

$$\text{Flow } Q_{BC} = Q_{AB} + Q_{FB} = 1.95 + 0.3074 = 2.26 \text{ ft}^3\text{/s}$$

Assuming full flow in pipe *BC*, we can calculate the diameter using the Manning formula as

$$2.26 = 0.7854 D^2 \times \frac{1.486}{0.013} \left(\frac{D}{4}\right)^{2/3} (0.002)^{1/2}$$

Solving for diameter D we get

$$D^{8/3} = 1.4185 \qquad \text{or} \qquad D = 1.14 \text{ ft}(13.68 \text{ in})$$

Therefore, use NPS 16 diameter pipe.

Pipe *GC*

$$\text{Area } A = 0.7854 \times \left(\frac{8}{12}\right)^2 = 0.3491 \text{ ft}^2$$

$$\text{Hydraulic radius } R = \frac{D}{4} = \frac{8}{4 \times 12} = 0.1667 \text{ ft}$$

$$\text{Discharge } Q_{GC} = 0.3491 \times \frac{1.486}{0.013} \times (0.1667)^{2/3} \times (0.003)^{1/2}$$

$$= 0.662 \text{ ft}^3\text{/s}$$

Pipe *CD*

$$\text{Flow } Q_{CD} = Q_{GC} + Q_{BC} = 0.662 + 2.26 = 2.922 \text{ ft}^3/\text{s}$$

$$\text{Area } A = 0.7854 \times \left(\frac{18}{12}\right)^2 = 1.7672 \text{ ft}^2$$

$$\text{Hydraulic radius } R = \frac{18}{4 \times 12} = 0.375 \text{ ft}$$

For full flow in pipe *CD*, using the Manning formula, we get

$$Q_f = 1.7672 \times \frac{1.486}{0.013} \left(0.375\right)^{2/3} \left(0.002\right)^{1/2} = 4.684 \text{ ft}^3/\text{s}$$

Therefore, the flow ratio is

$$\frac{Q}{Q_f} = \frac{2.922}{4.684} = 0.6238$$

From Fig. 3.8 for this flow ratio, we get the depth ratio $d/D = 0.64$.

$$\text{Sewage depth in } CD = 0.64 \times 18 = 11.52 \text{ in}$$

Pipe *HD*

$$\text{Area } A = 0.7854 \times \left(\frac{10}{12}\right)^2 = 0.5454 \text{ ft}^2$$

$$\text{Hydraulic radius } R = \frac{D}{4} = \frac{10}{48} = 0.2083 \text{ ft}$$

$$\text{Discharge } Q_{HD} = 0.5454 \times \frac{1.486}{0.013} \times (0.2083)^{2/3} \times (0.002)^{1/2}$$

$$= 0.98 \text{ ft}^3/\text{s}$$

Pipe *DE*

$$\text{Flow } Q_{DE} = Q_{HD} + Q_{CD} = 0.98 + 2.922 = 3.90 \text{ ft}^3/\text{s}$$

$$\text{Area } A = 0.7854 \times \left(\frac{20}{12}\right)^2 = 2.1817 \text{ ft}^2$$

$$\text{Hydraulic radius } R = \frac{18}{4 \times 12} = 0.375 \text{ ft}$$

The requirement for pipe *DE* is to maintain full flow. The slope required for this is calculated from the Manning equation as follows:

$$3.9 = 2.1817 \times \frac{1.486}{0.013} \times (0.375)^{2/3} \times (S)^{1/2}$$

Solving for slope *S*, we get

$$S = 0.0009$$

Therefore, in summary,

(a) The pipe size required for section *BC* is 13.68-in inside diameter. Use NPS 16.

(b) The flow rate in section *CD* is 2.92 ft^3/s, and the sewage depth in *CD* is 11.52 in (64 percent).

(c) The slope required for full flow in section *DE* is 0.0009 ft/ft or 0.09 percent.

3.11 Storm Sewer Design

Stormwater piping design is similar to sanitary sewer design as far as determining the slope required for a given discharge volume using the Manning equation. However, the determination of the design flow to be used is different. Stormwater runoff and surface water resulting from precipitation, such as from rainfall or snow, are collected and transported through storm drains and storm sewer systems.

3.11.1 Time of concentration

An important parameter related to storm sewer design is the *time of concentration*. This is defined as the time taken for rainwater to flow from the most remote area of a drainage site to the storm drain inlet. The time taken from the storm drain inlet to the storm sewer through a branch sewer is added to the time taken for the rainwater to flow from the remote area to the inlet to obtain the total time of concentration. If t_i represents the inlet time from the remote location and t_s is the time of flow through the branch sewer, the total time is

$$t = t_i + t_s \tag{3.52}$$

The inlet time t_i, also known as the time of overland flow, depends upon the distance of the remote location of the storm drain inlet, the slope of the land, and the rainfall intensity in inches per hour. In addition, a coefficient, which depends upon the surface condition, such as whether it is a paved or nonpaved area, is used to account for the type of drainage land. The inlet time is calculated from the following equation:

$$t_i = C\left(\frac{L}{S}i^2\right)^{1/3} \tag{3.53}$$

where t_i = inlet time, min
C = coefficient, ranges from 0.5 to 2.5
L = distance of flow from remote point to sewer inlet, ft
i = rainfall intensity, in/h
S = land slope, ft/ft

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TABLE 3.10 Runoff Coefficient

Surface type	Flat slope (<2%)	Rolling slope (2%–10%)	Hilly slope (>10%)
Pavements, roofs	0.90	0.90	0.90
City business surface	0.80	0.85	0.85
Dense residential areas	0.60	0.65	0.70
Suburban residential areas	0.45	0.50	0.55
Unpaved areas	0.60	0.65	0.70
Grassed areas	0.25	0.30	0.30
Cultivated land, clay	0.50	0.55	0.60
Cultivated land, loam	0.50	0.55	0.60
Cultivated land, sand	0.25	0.30	0.35
Meadows and pasture lands	0.25	0.30	0.35
Forest and wooded areas	0.10	0.15	0.20

The coefficient C is equal to 0.5 for paved areas, 1.0 for bare earth, and 2.5 for turf.

3.11.2 Runoff rate

The rate of runoff of stormwater designated as Q ft^3/s is related to the drainage area A and the intensity of rainfall i as follows:

$$Q = CiA \tag{3.54}$$

where Q = stormwater runoff rate, ft^3/s
C = runoff coefficient, dimensionless
i = average rainfall intensity, in/h
A = drainage area, acres

In SI units, Eq. (3.54) is

$$Q = 10\,CiA \tag{3.55}$$

where Q = stormwater runoff rate, m^3/h
C = runoff coefficient, dimensionless
i = average rainfall intensity, mm/h
A = drainage area, hectares

The coefficient of runoff C for various surfaces is given in Table 3.10. It ranges from 0.1 for forest and wooded areas to 0.9 for pavements and roofs.

Example 3.19 Calculate the maximum stormwater runoff rate for a rolling suburban residential area of 1200 acres if the rainfall intensity duration is 5 in/h for a 20-min duration storm of 25 years.

Solution From Table 3.10 we determine the runoff coefficient as

$$C = 0.50$$

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The runoff rate Q is calculated using Eq. (3.54) as

$$Q = 0.5 \times 5 \times 1200 = 3000 \text{ ft}^3\text{/s}$$

The maximum runoff rate is 3000 ft^3/s.

Example 3.20 Consider a drainage system with two pipe sections AB and BC terminating at C, the inlet point to a storm sewer pipe. Section AB is a 1200-ft-long piece of 12-in pipe with a slope of 0.002 ft/ft. Section BC is a 1000-ft-long, 20-in-diameter pipe with a slope of 0.003 ft/ft. The roughness coefficient may be assumed to be 0.013. Assuming the pipes are running full, calculate the velocity in each pipe and the time of concentration. The flow from the most remote location in the drainage area can be considered to take 10 min to reach the entry to the sewer pipe at A.

Solution For pipe AB, the velocity of full flow in the pipe is calculated by using Eq. (3.38) as follows:

$$V = \frac{1.486}{0.013} \times \left(\frac{D}{4}\right)^{2/3} (S)^{1/2}$$

or

$$V_{AB} = \frac{1.486}{0.013} \times \left(\frac{12}{12 \times 4}\right)^{2/3} (0.002)^{1/2} = 2.03 \text{ ft/s}$$

Similarly, the average flow velocity in section BC is

$$V_{BC} = \frac{1.486}{0.013} \times \left(\frac{20}{12 \times 4}\right)^{2/3} (0.003)^{1/2} = 3.49 \text{ ft/s}$$

The time of flow for pipe section AB is

$$t_{AB} = \frac{\text{distance}}{\text{velocity}} = \frac{1200}{2.03 \times 60} = 9.85 \text{ min}$$

And the time of flow for section BC is

$$t_{BC} = \frac{1000}{3.49 \times 60} = 4.78 \text{ min}$$

Therefore, the time of concentration for the runoff to flow from the most remote area to point C is

$$10 + 9.85 + 4.78 = 24.63 \text{ min}$$

3.12 Complex Piping Systems

In this section we continue with some additional piping configurations that are mostly used in pressurized flow of wastewater pipelines. Some of this discussion will also apply to pressurized sewer systems that have multiple-size pipes connected together. Complex piping systems include pipes of different diameters in series and parallel configuration.

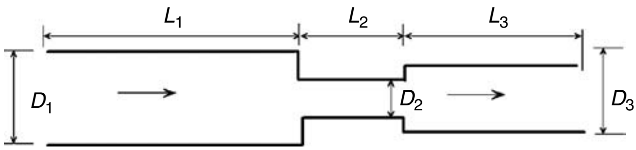

Figure 3.11 Series piping.

3.12.1 Series piping

Series piping in its simplest form consists of two or more different pipe sizes connected end to end as illustrated in Fig. 3.11. Pressure drop calculations in series piping may be handled in one of two ways. The first approach would be to calculate the pressure drop in each pipe size and add them together to obtain the total pressure drop. Another approach is to consider one of the pipe diameters as the base size and convert other pipe sizes into equivalent lengths of the base pipe size. The resultant equivalent lengths are added together to form one long piece of pipe of constant diameter equal to the base diameter selected. The pressure drop can now be calculated for this single-diameter pipeline. Of course, all valves and fittings will also be converted to their respective equivalent pipe lengths using the L/D ratios from Table 3.5.

Consider three sections of pipe joined together in series. Using subscripts 1, 2, and 3 and denoting the pipe length as L, inside diameter as D, flow rate as Q, and velocity as V, we can calculate the equivalent length of each pipe section in terms of a base diameter. This base diameter will be selected as the diameter of the first pipe section D_1. Since equivalent length is based on the same pressure drop in the equivalent pipe as the original pipe diameter, we will calculate the equivalent length of section 2 by finding that length of diameter D_1 that will match the pressure drop in a length L_2 of pipe diameter D_2. Using the Darcy equation and converting velocities in terms of flow rate from Eq. (3.11), we can write

$$\text{Head loss} = \frac{f(L/D)(0.4085Q/D_2)^2}{2g} \tag{3.56}$$

For simplicity, assuming the same friction factor,

$$\frac{L_e}{D_1^5} = \frac{L_2}{D_2^5} \tag{3.57}$$

Therefore, the equivalent length of section 2 based on diameter D_1 is

$$L_e = L_2\left(\frac{D_1}{D_2}\right)^5 \tag{3.58}$$

Similarly, the equivalent length of section 3 based on diameter D_1 is

$$L_e = L_3\left(\frac{D_1}{D_3}\right)^5 \tag{3.59}$$

The total equivalent length of all three pipe sections based on diameter D_1 is therefore

$$L_t = L_1 + L_2\left(\frac{D_1}{D_2}\right)^5 + L_3\left(\frac{D_1}{D_3}\right)^5 \tag{3.60}$$

The total pressure drop in the three sections of pipe can now be calculated based on a single pipe of diameter D_1 and length L_t.

Example 3.21 Three pipes with 14-, 16-, and 18-in diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

14-in pipeline, 0.250-in wall thickness, 2000 ft long

16-in pipeline, 0.375-in wall thickness, 3000 ft long

18-in pipeline, 0.375-in wall thickness, 5000 ft long

One 16 × 14 in reducer

One 18 × 16 in reducer

Two 14-in 90° elbows

Four 16-in 90° elbows

Six 18-in 90° elbows

One 14-in gate valve

One 16-in ball valve

One 18-in gate valve

(a) Use the Hazen-Williams equation with a C factor of 140 to calculate the total pressure drop in the series water piping system at a flow rate of 3500 gal/min. Flow starts in the 14-in piping and ends in the 18-in piping.

(b) If the flow rate is increased to 6000 gal/min, estimate the new total pressure drop in the piping system, keeping everything else the same.

Solution

(a) Since we are going to use the Hazen-Williams equation (3.26), the pipes in series analysis will be based on the pressure loss being inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe, per Eq. (3.26).

We will first calculate the total equivalent lengths of all 14-in pipe, fittings, and valves in terms of the 14-in-diameter pipe. Equivalent lengths are from Table 3.5.

$$\text{Straight pipe: 14 in, 2000 ft} = 2000 \text{ ft of 14-in pipe}$$

$$\text{Two 14-in } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 14}{12} = 70 \text{ ft of 14-in pipe}$$

$$\text{One 14-in gate valve} = \frac{1 \times 8 \times 14}{12} = 9.33 \text{ ft of 14-in pipe}$$

Therefore, the total equivalent length of 14-in pipe, fittings, and valves = 2079.33 ft of 14-in pipe.

Similarly we get the total equivalent length of 16-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 16-in, 3000 ft} = 3000 \text{ ft of 16-in pipe}$$

$$\text{Four 16-in } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 16}{12} = 160 \text{ ft of 16-in pipe}$$

$$\text{One 16-in ball valve} = \frac{1 \times 3 \times 16}{12} = 4 \text{ ft of 16-in pipe}$$

Therefore, the total equivalent length of 16-in pipe, fittings, and valve = 3164 ft of 16-in pipe.

Finally, we calculate the total equivalent length of 18-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 18-in, 5000 ft} = 5000 \text{ ft of 18-in pipe}$$

$$\text{Six 18-in } 90^\circ \text{ elbows} = \frac{6 \times 30 \times 18}{12} = 270 \text{ ft of 18-in pipe}$$

$$\text{One 18-in gate valve} = \frac{1 \times 8 \times 18}{12} = 12 \text{ ft of 18-in pipe}$$

Therefore, the total equivalent length of 18-in pipe, fittings, and valve = 5282 ft of 18-in pipe.

Next we convert all the preceding pipe lengths to the equivalent 14-in pipe based on the fact that the pressure loss is inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe.

$$2079.33 \text{ ft of 14-in pipe} = 2079.33 \text{ ft of 14-in pipe}$$

$$3164 \text{ ft of 16-in pipe} = 3164 \times \left(\frac{13.5}{15.25}\right)^{4.87} = 1748 \text{ ft of 14-in pipe}$$

$$5282 \text{ ft of 18-in pipe} = 5282 \times \left(\frac{13.5}{17.25}\right)^{4.87} = 1601 \text{ ft of 14-in pipe}$$

Therefore adding all the preceding lengths we get

$$\text{Total equivalent length in terms of 14-in pipe} = 5429 \text{ ft of 14-in pipe}$$

We still have to account for the 16 × 14 in and 18 × 16 in reducers. The reducers can be considered as sudden enlargements for the approximate calculation of the head loss, using the K factor and velocity head method. For sudden enlargements, the resistance coefficient K is found from

$$K = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \tag{3.61}$$

where d_1 is the smaller diameter and d_2 is the larger diameter.

For the 16 × 14 in reducer,

$$K = \left[1 - \left(\frac{13.5}{15.25}\right)^2\right]^2 = 0.0468$$

and for the 18 × 16 in reducer,

$$K = \left[1 - \left(\frac{15.25}{17.25}\right)^2\right]^2 = 0.0477$$

The head loss through the reducers will then be calculated based on $K(V^2/2g)$.

Flow velocities in the three different pipe sizes at 3500 gal/min will be calculated using Eq. (3.11):

$$\text{Velocity in 14-in pipe: } V_{14} = \frac{0.4085 \times 3500}{(13.5)^2} = 7.85 \text{ ft/s}$$

$$\text{Velocity in 16-in pipe: } V_{16} = \frac{0.4085 \times 3500}{(15.25)^2} = 6.15 \text{ ft/s}$$

$$\text{Velocity in 18-in pipe: } V_{18} = \frac{0.4085 \times 3500}{(17.25)^2} = 4.81 \text{ ft/s}$$

The head loss through the 16 × 14 in reducer is

$$h_1 = 0.0468 \frac{7.85^2}{64.4} = 0.0448 \text{ ft}$$

and the head loss through the 18 × 16 in reducer is

$$h_1 = 0.0477 \frac{6.15^2}{64.4} = 0.028 \text{ ft}$$

These head losses are insignificant and hence can be neglected in comparison with the head loss in straight length of pipe. Therefore, the total head loss in the entire piping system will be based on a total equivalent length of 5429 ft of 14-in pipe.

Using the Hazen-Williams equation the pressure drop at 3500 gal/min is

$$h = 10{,}460 \left(\frac{3500}{140}\right)^{1.852} \times \frac{1.0}{(13.5)^{4.87}} = 12.70 \text{ ft per 1000 ft of pipe}$$

Therefore, for the 5429 ft of equivalent 14-in pipe, the total pressure drop is

$$h = \frac{12.7 \times 5429}{1000} = 68.95 \text{ ft} = \frac{68.95}{2.31} = 29.85 \text{ psi}$$

(b) When the flow rate is increased to 6000 gal/min, we can use proportions to estimate the new total pressure drop in the piping as follows:

$$h = \left(\frac{6000}{3500}\right)^{1.852} \times 12.7 = 34.46 \text{ ft per 1000 ft of pipe}$$

Therefore, the total pressure drop in 5429 ft of 14-in pipe is

$$h = 34.46 \times \frac{5429}{1000} = 187.09 \text{ ft} = \frac{187.09}{2.31} = 81.0 \text{ psi}$$

Example 3.22 Two pipes with 400- and 600-mm diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

400-mm pipeline, 6-mm wall thickness, 600 m long

600-mm pipeline, 10-mm wall thickness, 1500 m long

One 600 × 400 mm reducer

Two 400-mm 90° elbows

Four 600-mm 90° elbows

One 400-mm gate valve

One 600-mm gate valve

Use the Hazen-Williams equation with a *C* factor of 120 to calculate the total pressure drop in the series water piping system at a flow rate of 250 L/s. What will the pressure drop be if the flow rate were increased to 350 L/s?

Solution The total equivalent length on 400-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 600 \text{ m}$$

$$\text{Two } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 400}{1000} = 24 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 400}{1000} = 3.2 \text{ m}$$

Thus,

$$\text{Total equivalent length on 400-mm-diameter pipe} = 627.2 \text{ m}$$

The total equivalent length on 600-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 1500 \text{ m}$$

$$\text{Four } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 600}{1000} = 72 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 600}{1000} = 4.8 \text{ m}$$

Thus,

$$\text{Total equivalent length on 600-mm-diameter pipe} = 1576.8 \text{ m}$$

Reducers will be neglected since they have insignificant head loss. Convert all pipe to 400-mm equivalent diameter.

$$\begin{aligned} 1576.8 \text{ m of 600-mm pipe} &= 1576.8 \left(\frac{388}{580}\right)^{4.87} \\ &= 222.6 \text{ m of 400-mm pipe} \end{aligned}$$

Total equivalent length on 400-mm-diameter pipe = 627.2 + 222.6 = 849.8 m

$$Q = 250 \times 10^{-3} \times 3600 = 900\ \text{m}^3/\text{h}$$

The pressure drop from Eq. (3.31) is

$$P_m = 1.1101 \times 10^{13} \left(\frac{900}{120}\right)^{1.852} \frac{1}{(388)^{4.87}}$$

$$= 114.38\ \text{kPa/km}$$

$$\text{Total pressure drop} = \frac{114.38 \times 849.8}{1000} = 97.2\ \text{kPa}$$

When the flow rate is increased to 350 L/s, we can calculate the pressure drop using proportions as follows:

$$\text{Revised head loss at 350 L/s} = \left(\frac{350}{250}\right)^{1.852} \times 114.38 = 213.3\ \text{kPa/km}$$

Therefore,

$$\text{Total pressure drop} = 213.3 \times 0.8498 = 181.3\ \text{kPa}$$

3.12.2 Parallel piping

Water pipes in parallel are set up such that the multiple pipes are connected so that water flow splits into the multiple pipes at the beginning and the separate flow streams subsequently rejoin downstream into another single pipe as depicted in Fig. 3.12.

Figure 3.12 shows a parallel piping system in the horizontal plane with no change in pipe elevations. Water flows through a single pipe AB, and at the junction B the flow splits into two pipe branches BCE and BDE. At the downstream end at junction E, the flows rejoin to the initial flow rate and subsequently flow through the single pipe EF.

To calculate the flow rates and pressure drop due to friction in the parallel piping system, shown in Fig. 3.12, two main principles of parallel piping must be followed. These are flow conservation at any junction point and common pressure drop across each parallel branch pipe.

Based on flow conservation, at each junction point of the pipeline, the incoming flow must exactly equal the total outflow. Therefore, at junction B, the flow Q entering the junction must exactly equal the sum of the flow rates in branches BCE and BDE.

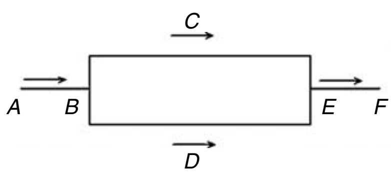

Figure 3.12 Parallel piping.

Thus,

$$Q = Q_{BCE} + Q_{BDE} \tag{3.62}$$

where Q_{BCE} = flow through branch BCE
Q_{BDE} = flow through branch BDE
Q = the incoming flow at junction B

The other requirement in parallel pipes concerns the pressure drop in each branch piping. Based on this the pressure drop due to friction in branch BCE must exactly equal that in branch BDE. This is because both branches have a common starting point (B) and a common ending point (E). Since the pressure at each of these two points is a unique value, we can conclude that the pressure drop in branch pipe BCE and that in branch pipe BDE are both equal to $P_B - P_E$, where P_B and P_E represent the pressure at the junction points B and E, respectively.

Another approach to calculating the pressure drop in parallel piping is the use of an equivalent diameter for the parallel pipes. For example in Fig. 3.12, if pipe AB has a diameter of 14 in and branches BCE and BDE have diameters of 10 and 12 in, respectively, we can find some equivalent diameter pipe of the same length as one of the branches that will have the same pressure drop between points B and C as the two branches. An approximate equivalent diameter can be calculated using the Darcy equation.

The pressure loss in branch BCE (10-in diameter) can be calculated as

$$h_1 = \frac{f(L_1/D_1)V_1^2}{2g} \tag{3.63}$$

where the subscript 1 is used for branch BCE and subscript 2 for branch BDE.

Similarly, for branch BDE

$$h_2 = \frac{f(L_2/D_2)V_2^2}{2g} \tag{3.64}$$

For simplicity we have assumed the same friction factors for both branches. Since h_1 and h_2 are equal for parallel pipes, and representing the velocities V_1 and V_2 in terms of the respective flow rates Q_1 and Q_2, using Eq. (3.11) we have the following equations:

$$\frac{f(L_1/D_1)V_1^2}{2g} = \frac{f(L_2/D_2)V_2^2}{2g} \tag{3.65}$$

$$V_1 = 0.4085\frac{Q_1}{D_1^2} \tag{3.66}$$

$$V_2 = 0.4085\frac{Q_2}{D_2^2} \tag{3.67}$$

In these equations we are assuming flow rates in gal/min and diameters in inches.

Simplifying Eqs. (3.65) to (3.67), we get

$$\frac{L_1}{D_1}\left(\frac{Q_1}{D_1^2}\right)^2 = \frac{L_2}{D_2}\left(\frac{Q_2}{D_2^2}\right)^2$$

or

$$\frac{Q_1}{Q_2} = \left(\frac{L_2}{L_1}\right)^{0.5}\left(\frac{D_1}{D_2}\right)^{2.5} \tag{3.68}$$

Also by conservation of flow

$$Q_1 + Q_2 = Q \tag{3.69}$$

Using Eqs. (3.68) and (3.69), we can calculate the flow through each branch in terms of the inlet flow Q. The equivalent pipe will be designated as D_e in diameter and L_e in length. Since the equivalent pipe will have the same pressure drop as each of the two branches, we can write

$$\frac{L_e}{D_e}\left(\frac{Q_e}{D_e^2}\right)^2 = \frac{L_1}{D_1}\left(\frac{Q_1}{D_1^2}\right)^2 \tag{3.70}$$

where Q_e is the same as the inlet flow Q since both branches have been replaced with a single pipe. In Eq. 3.70 there are two unknowns L_e and D_e. Another equation is needed to solve for both variables. For simplicity, we can set L_e to be equal to one of the lengths L_1 or L_2. With this assumption, we can solve for the equivalent diameter D_e as follows.

$$D_e = D_1\left(\frac{Q}{Q_1}\right)^{0.4} \tag{3.71}$$

Example 3.23 A 10-in water pipeline consists of a 2000-ft section of NPS 12 pipe (0.250-in wall thickness) starting at point A and terminating at point B. At point B, two pieces of pipe (4000 ft long each and NPS 10 pipe with 0.250-in wall thickness) are connected in parallel and rejoin at a point D. From D, 3000 ft of NPS 14 pipe (0.250-in wall thickness) extends to point E. Using the equivalent diameter method calculate the pressures and flow rate throughout the system when transporting water at 2500 gal/min. Compare the results by calculating the pressures and flow rates in each branch. Use the Colebrook-White equation for the friction factor.

Solution Since the pipe loops between B and D are each NPS 10 and 4000 ft long, the flow will be equally split between the two branches. Each branch pipe will carry 1250 gal/min.

The equivalent diameter for section BD is found from Eq. (3.71):

$$D_e = D_1 \left(\frac{Q}{Q_1} \right)^{0.4} = 10.25 \times (2)^{0.4} = 13.525 \text{ in}$$

Therefore we can replace the two 4000-ft NPS 10 pipes between B and D with a single pipe that is 4000 ft long and has a 13.525-in inside diameter.

The Reynolds number for this pipe at 2500 gal/min is found from Eq. (3.15):

$$\text{Re} = \frac{3162.5 \times 2500}{13.525 \times 1.0} = 584{,}566$$

Considering that the pipe roughness is 0.002 in for all pipes:

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.525} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section BD is [using Eq. (3.20)]

$$P_m = 71.16 \frac{fQ^2}{D^5}$$

$$= 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(13.525)^5} = 14.45 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } BD = \frac{14.45 \times 4000}{5280} = 10.95 \text{ psi}$$

For section AB we have,

$$\text{Re} = \frac{3162.5 \times 2500}{12.25 \times 1.0} = 645{,}408$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{12.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section AB is

$$P_m = 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(12.25)^5} = 22.66 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } AB = \frac{22.66 \times 2000}{5280} = 8.58 \text{ psi}$$

Finally, for section DE we have,

$$\text{Re} = \frac{3162.5 \times 2500}{13.5 \times 1.0} = 585{,}648$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.5} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section DE is

$$P_m = 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(13.5)^5} = 14.58 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } DE = \frac{14.58 \times 3000}{5280} = 8.28 \text{ psi}$$

Finally,

$$\begin{aligned}\text{Total pressure drop in entire piping system} &= 8.58 + 10.95 + 8.28 \\ &= 27.81 \text{ psi}\end{aligned}$$

Next for comparison we will analyze the branch pressure drops considering each branch separately flowing at 1250 gal/min.

$$\text{Re} = \frac{3162.5 \times 1250}{10.25 \times 1.0} = 385{,}671$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{10.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0158$. The pressure drop in section BD is

$$P_m = 71.16 \frac{0.0158 \times (1250)^2 \times 1}{(10.25)^5} = 15.53 \text{ psi/mi}$$

This compares with the pressure drop of 14.45 psi/mi, we calculated using an equivalent diameter of 13.525. It can be seen that the difference between the two pressure drops is approximately 7.5 percent.

Example 3.24 A waterline 5000 m long is composed of three sections A, B, and C. Section A has a 200-mm inside diameter and is 1500 m long. Section C has a 400-mm inside diameter and is 2000 m long. The middle section B consists of two parallel pipes each 3000 m long. One of the parallel pipes has a 150-mm inside diameter and the other has a 200-mm inside diameter. Assume no elevation change throughout. Calculate the pressures and flow rates in this piping system at a flow rate of 500 m^3/h, using the Hazen-Williams formula with a C factor of 1.20.

Solution We will replace the two 3000-m pipe branches in section B with a single equivalent diameter pipe to be determined. Since the pressure drop according to the Hazen-Williams equation is inversely proportional to the 4.87 power of the pipe diameter, we calculate the equivalent diameter for section B as follows:

$$\frac{Q_e^{1.852}}{D_e^{4.87}} = \frac{Q_1^{1.852}}{D_1^{4.87}} = \frac{Q_2^{1.852}}{D_2^{4.87}}$$

Therefore,

$$\frac{D_e}{D_1} = \left(\frac{Q_e}{Q_1}\right)^{0.3803}$$

Also $Q_e = Q_1 + Q_2$ and

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^{2.63} = \left(\frac{150}{200}\right)^{2.63} = 0.4693$$

Solving for Q_1 and Q_2, with $Q_e = 500$, we get

$$Q_1 = 159.7\ \text{m}^3/\text{h} \qquad \text{and} \qquad Q_2 = 340.3\ \text{m}^3/\text{h}$$

Therefore, the equivalent diameter is

$$D_e = D_1\left(\frac{Q_e}{Q_1}\right)^{0.3803} = 150 \times \left(\frac{500}{159.7}\right)^{0.3803} = 231.52\ \text{mm}$$

The pressure drop in section A, using the Hazen-Williams equation, is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(200)^{4.87}} = 970.95\ \text{kPa/km}$$

$$\Delta P_a = 970.95 \times 1.5 = 1456.43\ \text{kPa}$$

The pressure drop in section B, using the Hazen-Williams equation, is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(231.52)^{4.87}} = 476.07\ \text{kPa/km}$$

$$\Delta P_b = 476.07 \times 3.0 = 1428.2\ \text{kPa}$$

The pressure drop in section C, using the Hazen-Williams equation, is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(400)^{4.87}} = 33.20\ \text{kPa/km}$$

$$\Delta P_c = 33.2 \times 2.0 = 66.41\ \text{kPa}$$

Therefore,

$$\begin{aligned}\text{Total pressure drop of sections A, B, and C} &= 1456.43 + 1428.20 + 66.41\\ &= 2951.04\ \text{kPa}\end{aligned}$$

3.13 Total Pressure Required

So far we have examined the frictional pressure drop in water systems piping consisting of pipe, fittings, valves, etc. We also calculated the total pressure required to pump water through a pipeline up to a delivery station at an elevated point. The total pressure required at the beginning of a pipeline, for a specified flow rate, consists of three distinct components:

1. Frictional pressure drop
2. Elevation head
3. Delivery pressure

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (3.25)}$$

The first item is simply the total frictional head loss in all straight pipe, fittings, valves, etc. The second item accounts for the pipeline elevation difference between the origin of the pipeline and the delivery terminus. If the origin of the pipeline is at a lower elevation than that of the pipeline terminus or delivery point, a certain amount of positive pressure is required to compensate for the elevation difference. On the other hand if the delivery point were at a lower elevation than the beginning of the pipeline, gravity will assist the flow and the pressure required at the beginning of the pipeline will be reduced by this elevation difference. The third component, delivery pressure at the terminus, simply ensures that a certain minimum pressure is maintained at the delivery point, such as a storage tank.

For example, if a water pipeline requires 800 psi to take care of frictional losses and the minimum delivery pressure required is 25 psi, the total pressure required at the beginning of the pipeline is calculated as follows. If there were no elevation difference between the beginning of the pipeline and the delivery point, the elevation head (component 2) is zero. Therefore, the total pressure P_t required is

$$P_t = 800 + 0 + 25 = 825 \text{ psi}$$

Next consider elevation changes. If the elevation at the beginning is 100 ft and the elevation at the delivery point is 500 ft, then

$$P_t = 800 + \frac{(500 - 100) \times 1.0}{2.31} + 25 = 998.16 \text{ psi}$$

The middle term in this equation represents the static elevation head difference converted to psi. Finally, if the elevation at the beginning is 500 ft and the elevation at the delivery point is 100 ft, then

$$P_t = 800 + \frac{(100 - 500) \times 1.0}{2.31} + 25 = 651.84 \text{ psi}$$

It can be seen from the preceding that the 400-ft advantage in elevation in the final case reduces the total pressure required by approximately 173 psi compared to the situation where there as no elevation difference between the beginning of the pipeline and delivery point.

3.13.1 Effect of elevation

The preceding discussion illustrated a water pipeline that had a flat elevation profile compared to an uphill pipeline and a downhill pipeline. There are situations, where the ground elevation may have drastic peaks and valleys that require careful consideration of the pipeline topography. In some instances, the total pressure required to transport a given volume of water through a long pipeline may depend more on the ground elevation profile than the actual frictional pressure drop. In the preceding we calculated the total pressure required for a flat pipeline as 825 psi and an uphill pipeline to be 998 psi. In the uphill case the static elevation difference contributed to 17 percent of the total pressure required. Thus the frictional component was much higher than the elevation component. In some cases where the elevation differences in a long pipeline may dictate the total pressure required more than the frictional head loss.

Example 3.25 A 20-in (0.375-in wall thickness) water pipeline 500 mi long, has a ground elevation profile as shown in Fig. 3.13. The elevation at Corona is 600 ft and at Red Mesa is 2350 ft. Calculate the total pressure required at the Corona pump station to transport 11.5 Mgal/day of water to Red Mesa storage tanks, assuming a minimum delivery pressure of 50 psi at Red Mesa. Use the Hazen-Williams equation with a C factor of 140. If the pipeline operating pressure cannot exceed 1400 psi, how many

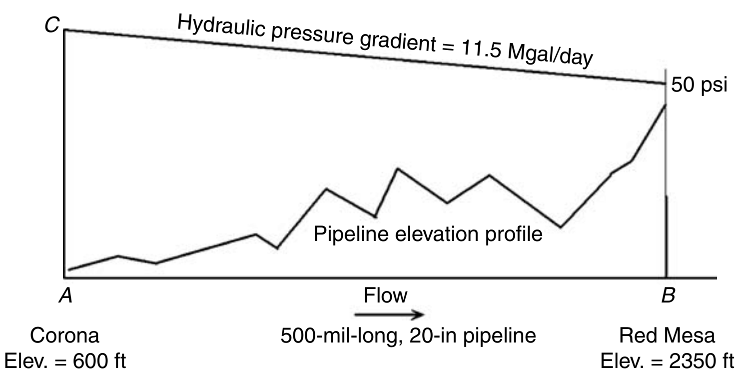

Figure 3.13 Corona to Red Mesa pipeline.

pumping stations, besides Corona, will be required to transport the given flow rate?

Solution The flow rate Q in gal/min is

$$Q = \frac{11.5 \times 10^6}{24 \times 60} = 7986.11 \text{ gal/min}$$

If P_m is the head loss in psi/mi of pipe, using the Hazen Williams equation,

$$P_m = 23{,}909 \left(\frac{7986.11}{140}\right)^{1.852} \frac{1}{19.25^{4.87}} = 23.76 \text{ psi/mi}$$

Therefore,

$$\text{Frictional pressure drop} = 23.76 \text{ psi/mi}$$

The total pressure required at Corona is calculated by adding the pressure drop due to friction to the delivery pressure required at Red Mesa and the static elevation head between Corona and Red Mesa.

$$\begin{aligned} P_t &= P_f + P_{\text{elev}} + P_{\text{del}} \\ &= (23.76 \times 500) + \frac{2350 - 600}{2.31} + 50 \\ &= 11{,}880 + 757.58 + 50 = 12{,}688 \text{ psi} \qquad \text{rounded off to the nearest psi} \end{aligned}$$

Since a total pressure of 12,688 psi at Corona far exceeds the maximum operating pressure of 1400 psi, it is clear that we need additional intermediate booster pump stations besides Corona. The approximate number of pump stations required without exceeding the pipeline pressure of 1400 psi is

$$\text{Number of pump stations} = \frac{12{,}688}{1400} = 9.06 \text{ or } 10 \text{ pump stations}$$

With 10 pump stations the average pressure per pump station will be

$$\text{Average pump station pressure} = \frac{12{,}688}{10} = 1269 \text{ psi}$$

3.13.2 Tight line operation

When there are drastic elevation differences in a long pipeline, sometimes the last section of the pipeline toward the delivery terminus may operate in an open-channel flow. This means that the pipeline section will not be full of water and there will be a vapor space above the water. Such situations are acceptable in water pipelines but not in pipelines transporting high vapor pressure liquids such as liquefied petroleum gas (LPG). To prevent such open-channel flow or slack line conditions,

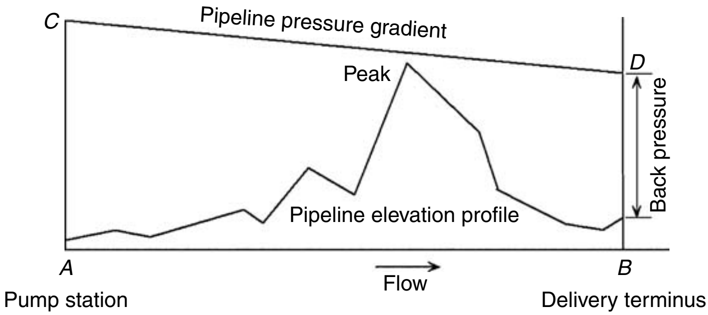

Figure 3.14 Tight line operation.

we pack the line by providing adequate back pressure at the delivery terminus as illustrated in Fig. 3.14.

3.13.3 Slack line flow

Slack line or open-channel flow occurs in the last segment of a long-distance water pipeline where a large elevation difference exists between the delivery terminus and intermediate point in the pipeline as indicated in Fig. 3.15.

If the pipeline were packed to avoid slack line flow, the hydraulic gradient is as shown by the solid line in Fig. 3.15. However, the piping system at the delivery terminal may not be able to handle the higher pressure due to line pack. Therefore, we may have to reduce the pressure at some point within the delivery terminal using a pressure control valve. This is illustrated in Fig. 3.15.

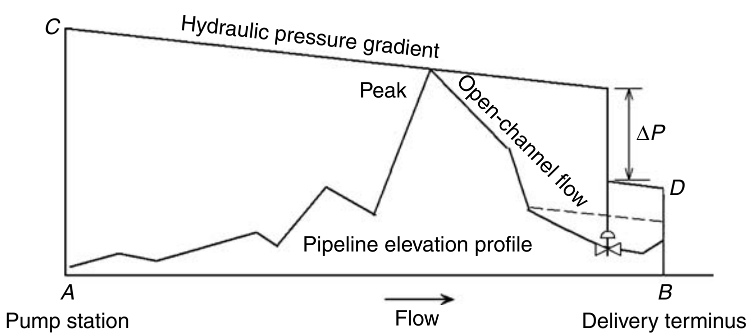

Figure 3.15 Slack line flow.

3.14 Hydraulic Gradient

The graphical representation of the pressures along the pipeline, as shown in Fig. 3.16, is called the hydraulic pressure gradient. Since elevation is measured in feet, the pipeline pressures are converted to feet of head and plotted against the distance along the pipeline superimposed on the elevation profile. If we assume a beginning elevation of 100 ft, a delivery terminus elevation of 500 ft, a total pressure of 1000 psi required at the beginning, and a delivery pressure of 25 at the terminus, we can plot the hydraulic pressure gradient graphically by the following method.

At the beginning of the pipeline the point C representing the total pressure will be plotted at a height of

$$100 \text{ ft} + (1000 \times 2.31) = 2410 \text{ ft}$$

Similarly, at the delivery terminus the point D representing the total head at delivery will be plotted at a height of

$$500 + (25 \times 2.31) = 558 \text{ ft} \qquad \text{rounded off to the nearest foot}$$

The line connecting the points C and D represents the variation of the total head in the pipeline and is termed the *hydraulic gradient*. At any intermediate point such as E along the pipeline the pipeline pressure will be the difference between the total head represented by point F on the hydraulic gradient and the actual elevation of the pipeline at E.

If the total head at F is 1850 ft and the pipeline elevation at E is 250 ft, the actual pipeline pressure at E is

$$(1850 - 250) \text{ ft} = \frac{1600}{2.31} = 693 \text{ psi}$$

It can be seen that the hydraulic gradient clears all peaks along the pipeline. If the elevation at E were 2000 ft, we would have a negative

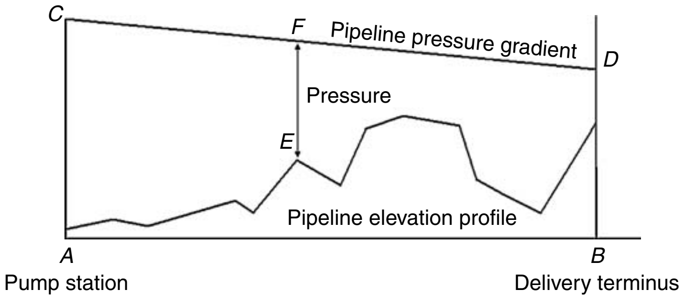

Figure 3.16 Hydraulic pressure gradient.

pressure in the pipeline at E equivalent to

$$(1850 - 2000)\text{ ft} = -150\text{ ft} = -\frac{150}{2.31} = -65\text{ psi}$$

Since a negative pressure is not acceptable, the total pressure at the beginning of the pipeline will have to be higher by the preceding amount.

$$\text{Revised total head at } A = 2410 + 150 = 2560\text{ ft}$$

This will result in zero gauge pressure in the pipeline at peak E. The actual pressure in the pipeline will therefore be equal to the atmospheric pressure at that location. Since we would like to always maintain some positive pressure above the atmospheric pressure, in this case the total head at A must be slightly higher than 2560 ft. Assuming a 10-psi positive pressure is desired at the highest peak such as E (2000-ft elevation), the revised total pressure at A would be

$$\text{Total pressure at } A = 1000 + 65 + 10 = 1075\text{ psi}$$

Therefore,

$$\text{Total head at } C = 100 + (1075 \times 2.31) = 2483\text{ ft}$$

This will ensure a positive pressure of 10 psi at the peak E.

3.15 Gravity Flow

Gravity flow in a water pipeline occurs when water flows from a source at point A at a higher elevation than the delivery point B, without any pumping pressure at A and purely under gravity. This is illustrated in Fig. 3.17.

The volume flow rate under gravity flow for the reservoir pipe system shown in Fig. 3.17 can be calculated as follows. If the head loss in the pipeline is h ft/ft of pipe length, the total head loss in length L is $(h \times L)$. Since the available driving force is the difference in tank levels at A and B, we can write

$$H_1 - (h \times L) = H_2 \tag{3.72}$$

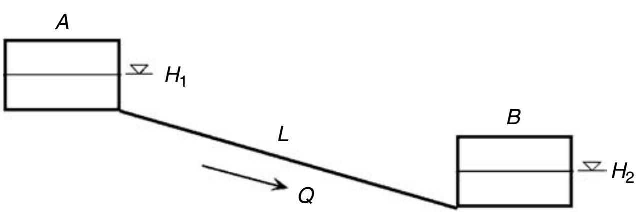

Figure 3.17 Gravity flow from reservoir.

Therefore,

$$h_L = H_1 - H_2 \tag{3.73}$$

and

$$h = \frac{H_1 - H_2}{L} \tag{3.74}$$

where h = head loss in pipe, ft/ft
L = length of pipe
H_1 = head loss in pipe A
H_2 = head loss in pipe B

In the preceding analysis, we have neglected the entrance and exit losses at A and B. Using the Hazen-Williams equation we can then calculate flow rate based on a C value.

Example 3.26 The gravity feed system shown in Fig. 3.17 consists of a 16-in (0.250-in wall thickness) 3000-ft-long pipeline, with a tank elevation at A = 500 ft and elevation at B = 150 ft. Calculate the flow rate through this gravity flow system. Use a Hazen-Williams C factor of 130.

Solution

$$h = \frac{500 - 150}{3000} = 0.1167 \text{ ft/ft}$$

Substituting in the Hazen-Williams equation, we get

$$0.1167 \times 1000 = 10{,}460 \times \left(\frac{Q}{130}\right)^{1.852} \left(\frac{1}{15.5}\right)^{4.87}$$

Solving for flow rate Q,

$$Q = 15{,}484 \text{ gal/min}$$

Compare the results using the Colebrook-White equation assuming $e = 0.002$.

$$\frac{e}{D} = \frac{0.002}{15.5} = 0.0001$$

We will assume a friction factor $f = 0.02$ initially. Head loss due to friction per Eq. (3.20) is

$$P_m = 71.16 \times \frac{0.02(Q^2)}{(15.5)^5} \text{ psi/mi}$$

or

$$P_m = 1.5908 \times 10^{-6} Q^2 \quad \text{psi/mi}$$

$$= \left(1.5908 \times 10^{-6} \frac{2.31}{5280}\right) Q^2 \quad \text{ft/ft}$$

$$= (6.9596 \times 10^{-10}) Q^2 \quad \text{ft/ft}$$

$$0.1167 = (6.9596 \times 10^{-10}) Q^2$$

Solving for flow rate Q, we get

$$Q = 12{,}949 \text{ gal/min}$$

Solving for the Reynolds number, we get

$$\text{Re} = 3162.5 \times \frac{12{,}949}{15.5} \times 1 = 2{,}642{,}053$$

From the Moody diagram, $f = 0.0128$. Now we recalculate P_m,

$$P_m = 71.16 \times 0.0128 \times \frac{Q^2}{(15.5)^5} \text{ psi/mi}$$

$$= 4.4541 \times 10^{-10} Q^2 \quad \text{ft/ft}$$

Solving for Q again,

$$Q = 16{,}186 \text{ gal/min}$$

By successive iteration we arrive at the final flow rate of 16,379 gal/min using the Colebrook-White equation. Comparing this with 15,484 gal/min obtained using the Hazen-Williams equation, we see that the flow rate is underestimated probably because the assumed Hazen-Williams C factor ($C = 130$) was too low.

3.16 Pumping Horsepower

In the previous sections we calculated the total pressure required at the beginning of the pipeline to transport a given volume of water over a certain distance. We will now calculate the pumping horsepower (HP) required to accomplish this.

Consider Example 3.25 in which we calculated the total pressure required to pump 11.5 Mgal/day of water from Corona to Red Mesa through a 500-mi-long, 20-in pipeline. We calculated the total pressure required to be 12,688 psi. Since the maximum allowable working pressure in the pipeline was limited to 1400 psi, we concluded that nine additional pump stations besides Corona were required. With a total of 10 pump stations, each pump station would be discharging at a pressure of approximately 1269 psi.

At the Corona pump station, water would enter the pump at some minimum pressure, say 50 psi and the pumps would boost the pressure to the required discharge pressure of 1269 psi. Effectively, the pumps would add the energy equivalent of 1269 − 50, or 1219 psi at a flow rate of 11.5 Mgal/day (7986.11 gal/min). The water horsepower (WHP) required is calculated as

$$\text{WHP} = \frac{(1219 \times 2.31) \times 7986.11 \times 1.0}{3960} = 5679 \text{ HP}$$

The general equation used to calculate WHP, also known as hydraulic horsepower (HHP), is as follows:

$$\text{WHP} = \frac{\text{ft of head} \times (\text{gal/min}) \times \text{specific gravity}}{3960} \qquad (3.75)$$

Assuming a pump efficiency of 80 percent, the pump brake horsepower (BHP) required is

$$\text{BHP} = \frac{5679}{0.8} = 7099 \text{ HP}$$

The general equation for calculating the BHP of a pump is

$$\text{BHP} = \frac{\text{ft of head} \times (\text{gal/min}) \times \text{specific gravity}}{3960 \times \text{effy}} \qquad (3.76)$$

where effy is the pump efficiency expressed as a decimal value.

If the pump is driven by an electric motor with a motor efficiency of 95 percent, the drive motor HP required will be

$$\text{Motor HP} = \frac{7099}{0.95} = 7473 \text{ HP}$$

The nearest standard size motor of 8000 HP would be adequate for this application. Of course this assumes that the entire pumping requirement at the Corona pump station is handled by a single pump-motor unit. In reality, to provide for operational flexibility and maintenance two or more pumps will be configured in series or parallel configurations to provide the necessary pressure at the specified flow rate. Let us assume that two pumps are configured in parallel to provide the necessary head pressure of 1219 psi (2816 ft) at the Corona pump station. Each pump will be designed for one-half the total flow rate (7986.11) or 3993 gal/min and a head pressure of 2816 ft. If the pumps selected had an efficiency of 80 percent, we can calculate the BHP required for each pump as follows:

$$\text{BHP} = \frac{2816 \times 3993 \times 1.0}{3960 \times 0.80} \qquad \text{from Eq. (3.76)}$$

$$= 3550 \text{ HP}$$

Alternatively, if the pumps were configured in series instead of parallel, each pump will be designed for the full flow rate of 7986.11 gal/min but at half the total pressure required, or 1408 ft. The BHP required per pump will still be the same as determined by the preceding equation.

3.17 Pumps

Pumps are installed on water pipelines to provide the necessary pressure at the beginning of the pipeline to compensate for pipe friction and any elevation head and provide the necessary delivery pressure at the pipeline terminus. Pumps used on water pipelines are either positive displacement (PD) type or centrifugal pumps.

PD pumps generally have higher efficiency, higher maintenance cost, and a fixed volume flow rate at any pressure within allowable limits. Centrifugal pumps on the other hand are more flexible in terms of flow rates but have lower efficiency and lower operating and maintenance cost. The majority of liquid pipelines today are driven by centrifugal pumps.

Since pumps are designed to produce pressure at a given flow rate, an important characteristic of a pump is its performance curve. The performance curve is a graphic representation of how the pressure generated by a pump varies with its flow rate. Other parameters, such as efficiency and horsepower, are also considered as part of a pump performance curve.

3.17.1 Positive displacement pumps

Positive displacement (PD) pumps include piston pumps, gear pumps, and screw pumps. These are used generally in applications where a constant volume of liquid must be pumped against a fixed or variable pressure.

PD pumps can effectively generate any amount of pressure at the fixed flow rate, which depends on its geometry, as long as equipment pressure limits are not exceeded. Since a PD pump can generate any pressure required, we must ensure that proper pressure control devices are installed to prevent rupture of the piping on the discharge side of the PD pump. As indicated earlier, PD pumps have less flexibility with flow rates and higher maintenance cost. Because of these reasons, PD pumps are not popular in long-distance and distribution water pipelines. Centrifugal pumps are preferred due to their flexibility and low operating cost.

3.17.2 Centrifugal pumps

Centrifugal pumps consist of one or more rotating impellers contained in a casing. The centrifugal force of rotation generates the pressure in

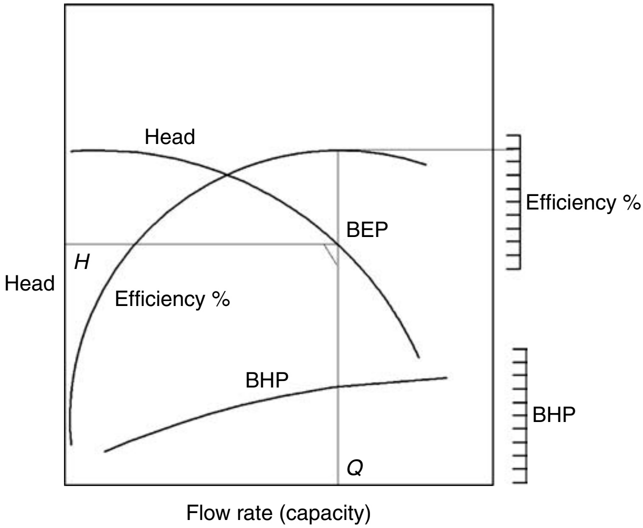

Figure 3.18 Performance curve for centrifugal pump.

the liquid as it goes from the suction side to the discharge side of the pump. Centrifugal pumps have a wide range of operating flow rates with fairly good efficiency. The operating and maintenance cost of a centrifugal pump is lower than that of a PD pump. The performance curves of a centrifugal pump consist of head versus capacity, efficiency versus capacity, and BHP versus capacity. The term *capacity* is used synonymously with flow rate in connection with centrifugal pumps. Also the term *head* is used in preference to pressure when dealing with centrifugal pumps. Figure 3.18 shows a typical performance curve for a centrifugal pump.

Generally, the head-capacity curve of a centrifugal pump is a drooping curve. The highest head is generated at zero flow rate (shutoff head) and the head decreases with an increase in the flow rate as shown in Fig. 3.18. The efficiency increases with flow rate up to the best efficiency point (BEP) after which the efficiency drops off. The BHP calculated using Eq. (3.76) also generally increases with flow rate but may taper off or start decreasing at some point depending on the head-capacity curve.

For further discussion on centrifugal pump performance, including operating in series and parallel configurations and system head analysis, refer to Chap. 1.

3.18 Pipe Materials

Pipes used for wastewater and stormwater may be constructed of different materials depending upon whether pressure flow or gravity flow is involved. Sewer pipes may be constructed of rigid pipe or flexible pipe. Types of rigid pipe include vitrified clay, asbestos-cement, concrete, and cast iron. Types of flexible sewer pipes include corrugated aluminum, steel, ductile iron, and thermoset plastic.

For gravity flow sewer pipes, diameters range from 4 to 42 in and lengths are of 10 to 14 ft. Vitrified clay pipe is manufactured to ASTM Standard C700. Diameter sizes range from 4 to 36 in. Joint types and materials are in accordance with ASTM C425, and construction and testing is done per ASTM C12, C828, and C1091. Vitrified clay pipes are used in corrosive environments.

Concrete pipe is defined by specifications given in ASTM C14. Construction and testing are in accordance with ASTM C924 and C969, respectively. The burial depth is limited to 10 to 25 ft.

Reinforced concrete pipe is specified in accordance with ASTM C76 and C361. Diameter sizes range from 12 to 120 in. Construction and testing standards are in accordance with ASTM C924 and C969, respectively. These pipes can be used for gravity sewers and pressure sewers. Burial depth is limited to 35 ft.

Ductile iron pipe is generally manufactured according to AWWA C151/ANSI A21.51 standards. Diameter sizes range from 4 to 36 in. The burial depth is limited to 32 ft. Ductile iron pipes are not used for gravity sewers.

Types of plastic pipe used in sewer systems include polyvinyl chloride (PVC), acrylonitrile-butadiene-styrene (ABS), and polyethylene (PE). These have good corrosion resistance and low-friction characteristics in addition to being lightweight. Plastic pipe diameter sizes range from 4 to 15 in.

3.19 Loads on Sewer Pipe

Sewer pipes must be able to withstand the vertical load arising from the soil above them and any vehicle loads that are superimposed on top of the soil loads. As the burial depth increases, the effect of the superimposed load decreases. Table 3.11 shows the percentage of vehicle loading

TABLE 3.11 Vehicle Loading on Buried Pipe

	Trench width at top of pipe, ft						
Depth of backfill over top of pipe, ft	1	2	3	4	5	6	7
1	17.0	26.0	28.6	29.7	29.9	30.2	30.3
2	8.3	14.2	18.3	20.7	21.8	22.7	23.0
3	4.3	8.3	11.3	13.5	14.8	15.8	16.7
4	2.5	5.2	7.2	9.0	10.3	11.5	12.3
5	1.7	3.3	5.0	6.3	7.3	8.3	9.0
6	1.0	2.3	3.7	4.7	5.5	6.2	7.0

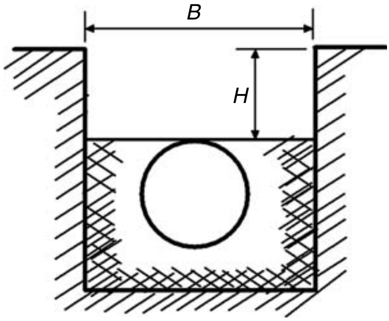

Figure 3.19 Buried pipe and trench dimensions.

transmitted to a buried pipe. It can be seen from the table that as the trench width increases, the load transmitted to the pipe increases. On the other hand as the depth of backfill over the pipe increases, the load on the pipe decreases.

Figure 3.19 shows a buried pipe in a trench. The width of the trench is B, and the depth of the trench is H. The load transmitted to the pipe from the backfill depends upon the weight of the surrounding soil, the width of the trench, and a dimensionless coefficient C. The following formula, referred to as Marston's formula, may be used for calculating the vertical soil load on a rigid pipe that is buried in the ground.

$$W = CwB^2 \tag{3.77}$$

where W = vertical load on pipe due to soil, per unit length, lb/ft
C = dimensionless coefficient
w = weight of backfill material on top of pipe, lb/ft^3
B = width of trench above pipe, ft

Table 3.12 lists the density of common backfill materials. The coefficient C depends upon the backfill material and the ratio H/B, where H is the height of the backfill material above the pipe. Table 3.13 gives the buried loading coefficient C for various backfill materials and trench dimensions.

TABLE 3.12 Density of Common Backfill Materials

Materials	Density, lb/ft^3
Dry sand	100
Ordinary (damp) sand	115
Wet sand	120
Damp clay	120
Saturated clay	130
Saturated topsoil	115
Sand and damp topsoil	100

TABLE 3.13 Buried Loading Coefficient

Ratio of depth to trench width	Coefficient C: Sand and damp topsoil	Saturated topsoil	Damp clay	Saturated clay
0.5	0.46	0.46	0.47	0.47
1.0	0.85	0.86	0.88	0.9
1.5	1.18	1.21	1.24	1.28
2.0	1.46	1.5	1.56	1.62
2.5	1.70	1.76	1.84	1.92
3.0	1.90	1.98	2.08	2.2
3.5	2.08	2.17	2.3	2.44
4.0	2.22	2.33	2.49	2.66
4.5	2.34	2.47	2.65	2.87
5.0	2.45	2.59	2.8	3.03
5.5	2.54	2.69	2.93	3.19
6.0	2.61	2.78	3.04	3.33
6.5	2.68	2.86	3.14	3.46
7.0	2.73	2.93	3.22	3.57
7.5	2.78	2.98	3.3	3.67
8.0	2.81	3.03	3.37	3.76
8.5	2.85	3.07	3.42	3.85
9.0	2.88	3.11	3.48	3.92
9.5	2.90	3.14	3.52	3.98
10.0	2.92	3.17	3.56	4.04
11.0	2.95	3.21	3.63	4.14
12.0	2.97	3.24	3.68	4.22
13.0	2.99	3.27	3.72	4.29
14.0	3.00	3.28	3.75	4.34
15.0	3.01	3.3	3.77	4.38
Very great	3.03	3.33	3.85	4.55

Example 3.27 A 24-in-diameter sewer pipe is installed in a trench of width 48 in. The top of the pipe is 6 ft below the ground surface. The topsoil is damp clay. What is the vertical loading due to the backfill material on the sewer pipe per linear foot?

Solution To determine the coefficient C in Marston's equation we need the ratio of trench height to trench width,

$$\frac{H}{B} = \frac{6 \times 12}{48} = 1.5$$

From Table 3.13 we get $C = 1.24$ for $H/B = 1.5$ and for damp clay.

From Table 3.12 the density of damp clay is

$$w = 120 \text{ lb/ft}^3$$

Therefore, using Marston's equation (3.77), we get the vertical loading on the pipe per linear foot as

$$W = 1.24 \times 120 \times 4^2 = 2381 \text{ lb/ft}$$

The load on the buried pipe due to the backfill material is 2381 lb/ft.

Chapter

4

Steam Systems Piping

Introduction

Steam systems piping is used in many industrial applications for creating the pressure and energy required to drive machines and other equipment and to convey the condensed steam back to the start of the process. Steam is used in heating and for converting the energy in water to beneficial use in industries. Steam is generally transported through piping systems and distributed to various locations with minimal noise and in the absence of air. Any air present in a steam piping system must be rapidly removed or the system will become inefficient.

4.1 Codes and Standards

The following American Society of Mechanical Engineers (ASME) codes and standards are used in the design and construction of steam piping systems.

1. ASME Boiler and Pressure Vessel Code—Section 3
2. ASME Code for Pressure Piping—B31.1
3. ASME Code for Pressure Piping—B31.3
4. ASME B36.10 M
5. ASME B36.19 M
6. ASME B16.9

Other codes include special regulations and standards imposed by individual state, city, and local agencies having jurisdiction over the installation and operation of steam piping.

4.2 Types of Steam Systems Piping

There are several types of steam systems piping in use today. They may be categorized as steam distribution systems, underground steam piping, fossil-fueled power plants, and nuclear fuel power plants.

The steam distribution systems consist of trunk line distribution systems and main and feeder distribution network systems. Underground piping consists of piping used in the district heating industry where steam piping is used to carry process steam. In fossil-fueled power plants superheated steam is supplied to turbines and for auxiliary services. In nuclear power plants steam is supplied from the boiler to the power plant for various services within the power plant.

4.3 Properties of Steam

Steam is produced by the evaporation of water. Water consists of hydrogen and oxygen and has the chemical formula H_2O. Considering the atomic weight of the two elements, the composition of water is two parts by weight of H_2 and eight parts by weight of O_2. In the solid form H_2O is called ice, and in the liquid form it is known as water. When water boils at 212°F (100°C) under normal atmospheric conditions, it is converted into vapor (or gaseous) form and is generally referred to as steam. The heat required to form steam from a unit weight of water is known as the *latent heat of vaporization*, and it will vary with the pressure. At an atmospheric pressure of 14.7 pounds per square inch absolute (psia), the latent heat of vaporization of dry steam is equal to 970 British thermal units per pound (Btu/lb).

When a quantity of water is heated to the point where vaporization occurs and a quantity of liquid and vapor are in equilibrium at the same temperature and pressure, we say that there is saturated vapor in equilibrium with saturated liquid. The particular temperature and pressure at which this occurs are called the saturation temperature and saturation pressure, respectively. As heat is applied and more liquid vaporizes to form steam, a point would be reached when the liquid will be uniformly dispersed within the steam. This mixture of vapor and liquid is referred to as wet saturated steam. The quality of steam, also known as the *dryness fraction*, S_x is defined as the ratio of the mass of saturated vapor (dry steam) to the mass of the total mixture of water and vapor (wet steam).

$$S_x = \frac{M_{\mathrm{sv}}}{M_t} \tag{4.1}$$

where S_x = steam quality
M_{sv} = mass of saturated vapor
M_t = total mass of liquid and vapor

Thus wet steam with a quality, or dryness fraction, of 0.9 has 10 percent moisture present. As more heat is applied to the wet steam, all liquid will be converted to vapor, and dry saturated steam is the final product. Under normal atmospheric conditions at 14.7 psia this happens at 212°F. At this point, the steam quality is 100 percent saturated and is also referred to as dry saturated steam. Further heating of the steam beyond the saturation point at constant pressure will result in an increase in temperature beyond 212°F, and then the steam becomes superheated.

As an example, if steam is heated to 320°F, it is said to be superheated steam at 14.7 psi and 320°F. The difference between the temperature of the superheated steam (320°F) and the boiling point (212°F) is referred to as 108°F of superheat. Superheated steam at any pressure is defined as steam that is heated to a higher temperature than the corresponding saturation temperature at that pressure. Therefore, at 14.7 psia, any steam that is at a temperature above 212°F is called superheated steam.

The boiling temperature of water occurs at 212°F when the pressure is 14.7 psia. As the pressure increases, the saturation temperature changes. As pressure increases, less heat is necessary to change the phase from liquid to vapor. Ultimately, at some pressure, known as the *critical pressure*, the least amount of heat is necessary to change the phase from liquid to vapor. The critical pressure of steam is approximately 3206 psia, and the corresponding critical temperature is 705.4°F.

4.3.1 Enthalpy

The amount of heat H at constant pressure needed to convert a unit mass of water at its freezing point into wet steam is the sum of the enthalpy of water and the fraction of the latent heat. Thus the enthalpy, or heat content, of wet steam is given by the following equation:

$$H_{\text{ws}} = H_w + xL \tag{4.2}$$

where H_{ws} = enthalpy of wet steam
H_w = enthalpy of water
x = dryness fraction or quality of steam, a number less than 1.0
L = latent heat of vaporization

For dry steam, $x = 1$ and

$$H_{\text{ds}} = H_w + L \tag{4.3}$$

where H_{ds} is the enthalpy of dry steam. Enthalpy or heat content is measured in Btu/lb in U.S. Customary System (USCS) units and kilojoules per kilogram (kJ/kg) in Systeme International (SI) units.

4.3.2 Specific heat

The *specific heat* of a substance is defined as the heat required per unit weight of the substance to increase its temperature by one degree. Solids, liquids, and gases have defined specific heats. The specific heats of gases change with temperature and pressure. Wet steam is considered to be partly liquid and partly gas. Hence, since wet steam contains water, it cannot be considered to have a specific heat. This is because, upon heating wet steam, the water evaporates and the steam quality approaches 1.0. Thus wet steam, unlike a pure gas, cannot have a C_p (specific heat at constant pressure) or C_v (specific heat at constant volume) property, since these values would continuously change as the steam quality changes.

Similarly, wet steam also cannot have a constant value of the specific heat ratio $\gamma = C_p/C_v$ or a gas constant R. When wet steam expands adiabatically, we can assume that it follows some type of polytropic expansion law $PV^n =$ constant as long as the range of pressure is fairly small. An average value of the polytrophic exponent n can be calculated from measured values of pressure and temperature. In most calculations, an average value of n equal to 1.13 can be used with a fair degree of accuracy. However, if the pressure drop is large, this value of n will not be correct.

Dry saturated steam and superheated steam do have defined specific heat values and specific heat ratios. Generally, the specific heat ratio $\gamma = 1.135$ for saturated steam and $\gamma = 1.3$ for superheated steam, are used in calculations.

4.3.3 Pressure

The pressure measured by a pressure gauge on a steam piping system is called the gauge pressure (lb/in^2 gauge or psig.) The absolute pressure (lb/in^2 absolute or psia) must be calculated by adding the atmospheric pressure at the location of the system to the gauge pressure. Therefore,

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}} \tag{4.4}$$

where P_{abs} = absolute pressure, psia
P_{gauge} = gauge pressure, psig
P_{atm} = atmospheric pressure, psia

As an example, if the steam pressure is 150 psig and the atmospheric pressure is 14.7 psia, the absolute pressure is $150 + 14.7 = 164.7$ psia.

In SI units, steam pressure may be measured in kilopascals (kPa) or bar. The atmospheric pressure may be 101 kPa or 1 bar. If the steam piping is at a pressure of 1000 kPa gauge, the absolute pressure of steam will be $1000 + 101 = 1101$ kPa absolute. Sometimes in SI units, megapascal (MPa) and pascal (Pa) are also used for pressure where 1 kPa = 0.145 psi. Conversion factors from various USCS units to SI units are given in App. A.

4.3.4 Steam tables

Many thermodynamic properties of steam, such as specific volume, enthalpy, and entropy at various saturation temperatures are listed in steam tables, such as the abbreviated version shown in Table 4.1. All pressures in the steam tables are listed in absolute pressures.

Steam tables are for dry steam only. When calculating properties for wet steam, we must consider the steam quality similar to the calculation of the enthalpy of wet steam discussed in Eq. (4.1).

Example 4.1 Calculate the enthalpy of 1 lb of steam at 60 psia and with 0.9 steam quality. How much heat would be required to raise 5 lb of this steam from water at 50°F?

Solution From Table 4.1, at 60 psia, the enthalpy of water is

$$H_w = 262.09 \text{ Btu/lb}$$

$$\text{Latent heat of vaporization } L = 915.5 \text{ Btu/lb}$$

Therefore, from Eq. (4.2), the enthalpy of wet steam is

$$\begin{aligned} H_s &= 262.09 + 0.9 \times 915.5 \\ &= 1086.04 \text{ Btu/lb} \end{aligned}$$

$$\text{Enthalpy of water at } 50^\circ\text{F} = 50 - 32 = 18 \text{ Btu/lb}$$

Therefore, the heat required to raise 5 lb of wet steam from water at 50°F is

$$H = 5 \times (1086.04 - 18) = 5340.2 \text{ Btu}$$

4.3.5 Superheated steam

The enthalpy of superheated steam can be calculated by considering it as a perfect gas. Since superheating is done at constant pressure, we can use the specific heat C_p for calculating enthalpy. The C_p for superheated steam varies from 0.48 to 3.5 and depends on the pressure and temperature. Steam tables also can be used to determine the enthalpy of superheated steam. If T_1 is the saturated temperature of steam at pressure P_1, and T_s is the temperature of the superheated steam, the heat absorbed per pound of steam during superheating is

$$\Delta H = C_p(T_s - T_1) \tag{4.5}$$

TABLE 4.1 Properties of Dry Steam

(a) Saturated Steam at Various Saturation Temperatures

Temperature, °F	Pressure, psia	Specific volume, ft^3/lb			Enthalpy, Btu/lb			Entropy, Btu/(lb · F)		
		Sat. liquid	Evaporation	Sat. vapor	Sat. liquid	Evaporation	Sat. vapor	Sat. liquid	Evaporation	Sat. vapor
32	0.08854	0.01602	3306	3306	0	1075.8	1075.8	0	2.1877	2.1877
35	0.09995	0.01602	2947	2947	3.02	1074.1	1077.1	0.0061	2.1709	2.177
40	0.1217	0.01602	2444	2444	8.05	1071.3	1079.3	0.0162	2.1435	2.1597
45	0.14752	0.01602	2036.4	2036.4	13.06	1068.4	1081.5	0.0262	2.1167	2.1429
50	0.17811	0.01603	1703.2	1703.2	18.07	1065.6	1083.7	0.0361	2.0903	2.1264
60	0.2563	0.01604	1206.6	1206.7	28.06	1059.9	1088	0.0555	2.0393	2.0948
70	0.3631	0.0606	867.8	867.9	38.04	1054.3	1092.3	0.0745	1.9902	2.0647
80	0.5069	0.01608	633.1	633.1	48.02	1048.6	1096.6	0.0932	1.9428	2.036
90	0.6982	0.0161	468	468.0	57.99	1042.9	1100.9	0.1115	1.8972	2.0087
100	0.9492	0.01613	350.3	350.4	67.97	1037.2	1105.2	0.1295	1.8531	1.9826
110	1.2748	0.01617	265.3	265.4	77.94	1031.6	1109.5	0.1471	1.8106	1.9577
120	1.6924	0.0162	203.25	203.27	87.92	1025.8	1113.7	0.1645	1.7694	1.9339
130	2.2225	0.01625	157.32	157.34	97.9	1020	1117.9	0.1816	1.7296	1.9112
150	3.718	0.01634	97.06	97.07	117.89	1008.2	1126.1	0.2149	1.6537	1.8685
160	4.741	0.01639	77.27	77.29	127.89	1002.3	1130.2	0.2311	1.6174	1.8485
170	5.992	0.01645	62.04	62.06	137.9	996.3	1134.2	0.2472	1.5822	1.8293
180	7.51	0.01651	50.21	50.23	147.92	990.2	1138.1	0.263	1.548	1.8109
190	9.339	0.01657	40.94	40.96	157.95	984.1	1142	0.2785	1.5147	1.7932
200	11.526	0.01663	33.62	33.64	167.99	977.9	1145.9	0.2938	1.4824	1.7762
210	14.123	0.0167	27.8	27.82	178.05	971.6	1149.7	0.309	1.4508	1.7598
212	14.696	0.01672	26.78	26.8	180.7	970.3	1150.4	0.312	1.4446	1.7566
220	17.186	0.01677	23.13	23.15	188.13	965.2	1153.4	0.3239	1.4201	1.744
230	20.78	0.01684	19.365	19.382	198.23	958.8	1137	0.3387	1.3901	1.7288
240	24.969	0.01692	16.306	16.323	208.34	952.2	1160.5	0.3531	1.3609	1.714
250	29.825	0.017	13.804	13.821	216.48	945.5	1164	0.3675	1.3223	1.6998
260	35.429	0.01709	11.746	11.763	228.64	938.7	1167.3	0.3817	1.3043	1.686
270	41.858	0.01717	10.044	10.061	238.84	931.8	1170.6	0.3958	1.2769	1.6727
280	49.203	0.01726	8.628	8.645	249.06	924.7	1173.8	0.4096	1.2501	1.6597

290	57.556	0.01735	7.444	7.461	259.31	917.5	1176.8	0.4234	1.2338	1.6472
300	67.013	0.01745	6.449	6.466	269.59	910.1	1179.7	0.4369	1.198	1.635
310	77.68	0.01755	5.609	5.626	279.92	902.6	1182.5	0.4504	1.1727	1.6231
320	89.66	0.01765	4.896	4.914	290.28	894.9	1185.2	0.4637	1.1478	1.6115
330	103.06	0.01776	4.289	4.307	300.68	887	1187.7	0.4769	1.1233	1.6002
340	118.01	0.01787	3.77	3.788	311.13	879	1190.1	0.49	1.0992	1.5891
350	134.63	0.01799	3.324	3.342	321.63	870.7	1192.3	0.5029	1.0754	1.5783
360	153.04	0.01811	2.939	2.957	332.18	862.2	1194.4	0.5158	1.0519	1.5677
370	173.37	0.01823	2.606	2.625	342.79	853.5	1196.3	0.5286	1.0287	1.5573
380	195.77	0.01836	2.317	2.335	353.45	844.6	1198.1	0.5413	1.0059	1.5471
390	220.37	0.0185	2.0651	2.0836	364.17	835.4	1199.6	0.5539	0.9832	1.5371
400	247.31	0.01864	1.8447	1.8633	374.97	826.0	1201	0.5664	0.9608	1.5272
410	276.75	0.01878	1.6512	1.6700	385.83	816.3	1202.1	0.5788	0.9386	1.5174
420	308.83	0.01894	1.4811	1.500	396.77	806.3	1203.1	0.5912	0.9166	1.5078
430	343.72	0.01910	1.3308	1.3499	407.79	796.0	1203.8	0.6035	0.8947	1.4982
440	381.59	0.01926	1.1979	1.2171	408.9	785.4	1204.3	0.6158	0.873	1.4887
450	422.6	0.0194	1.0799	1.0993	430.1	774.5	1204.6	0.628	0.8513	1.4793
460	466.9	0.0196	0.9748	0.9944	441.4	763.2	1204.6	0.6402	0.8298	1.4700
470	514.7	0.0198	0.8811	0.9009	452.8	751.5	1204.3	0.6523	0.8083	1.4606
480	566.1	0.0200	0.7972	0.8172	464.4	739.4	1203.7	0.6645	0.7868	1.4513
490	621.4	0.0202	0.7221	0.7423	476	726.8	1202.8	0.6766	0.7653	1.4419
500	680.8	0.0204	0.6545	0.6749	487.8	713.9	1201.7	0.6887	0.7438	1.4325
520	812.4	0.0209	0.5385	0.5594	511.9	686.4	1198.2	0.713	0.7006	1.4136
540	962.5	0.0215	0.4434	0.4649	536.6	656.6	1193.2	0.7374	0.6568	1.3942
560	1133.1	0.0221	0.3647	0.3868	562.2	624.2	1186.4	0.7621	0.6121	1.3742
580	1325.8	0.0228	0.2989	0.3217	588.9	588.4	1177.3	0.7872	0.5659	1.3532
600	1542.9	0.0236	0.2432	0.2668	617	548.5	1165.5	0.8131	0.5176	1.3307
620	1786.6	0.0247	0.1955	0.2201	646.7	503.6	1150.3	0.8398	0.4664	1.3062
640	2059.7	0.0260	0.1538	0.1798	678.6	452	1130.5	0.8679	0.411	1.2789
660	2365.4	0.0278	0.1165	0.1442	714.2	390.2	1104.4	0.8987	0.3485	1.2472
680	2708.1	0.0305	0.081	0.1115	757.3	309.9	1067.2	0.9351	0.2719	1.2071
700	3093.7	0.0369	0.0392	0.0761	823.3	172.1	995.4	0.9905	0.1484	1.1389
705.4	3206.2	0.0503	0	0.0503	902.7	0	902.7	1.0580	0	1.0580

TABLE 4.1 Properties of Dry Steam (*Continued*)

(b) Saturated Steam at Various Saturation Pressures

Pressure, psia	Temperature, °F	Specific volume, ft^3/lb		Enthalpy, Btu/lb			Entropy, Btu/(lb · °F)			Internal Energy, Btu/lb	
		Sat. liquid	Sat. vapor	Sat. liquid	Evaporation	Sat. vapor	Sat. liquid	Evaporation	Sat. vapor	Sat. liquid	Sat. vapor
0.491	79.03	0.01608	652.300	47.05	1049.20	1096.3	0.0914	1.9473	2.0387	47.05	1037.0
0.736	91.72	0.01611	444.900	59.71	1042.00	1101.7	0.1147	1.8894	2.0041	59.71	1041.1
0.982	101.14	0.01614	339.200	69.10	1036.60	1105.7	0.1316	1.8481	1.9797	69.10	1044.0
1.227	108.71	0.01616	274.900	76.65	1032.30	1108.9	0.1449	1.816	1.9609	76.65	1046.4
1.473	115.06	0.01618	231.600	82.99	1028.60	1111.6	0.156	1.7896	1.9456	82.99	1048.5
1.964	125.43	0.01622	176.700	93.34	1022.70	1116.0	0.1738	1.7476	1.9214	93.33	1051.8
2.455	133.76	0.01626	143.250	101.66	1017.70	1119.4	0.1879	1.715	1.9028	101.65	1054.3
5	162.24	0.01640	73.520	130.13	1001.00	1131.1	0.2347	1.6094	1.8441	130.12	1063.1
10	193.21	0.01659	38.420	161.17	982.10	1143.3	0.2835	1.5041	1.7876	161.14	1072.2
14.696	212	0.01672	26.800	180.07	970.30	1150.4	0.312	1.4446	1.7566	180.02	1077.5
15	213.03	0.01672	26.290	181.11	969.70	1150.8	0.3135	1.4415	1.7549	181.06	1077.8
16	216.32	0.01674	24.750	184.42	967.60	1152.0	0.3184	1.4313	1.7497	184.37	1078.7
18	222.41	0.01679	22.170	190.56	963.60	1154.2	0.3275	1.4128	1.7403	190.50	1080.4
20	227.96	0.01683	20.089	196.16	960.10	1156.3	0.3356	1.3962	1.7319	196.10	1081.9
25	240.07	0.01692	16.303	208.42	952.10	1160.6	0.3533	1.3606	1.7139	208.34	1085.1
30	250.33	0.01701	13.746	218.82	945.30	1164.1	0.368	1.3313	1.6993	218.73	1087.8
35	259.28	0.01708	11.898	227.91	939.20	1167.1	0.3807	1.3063	1.6870	227.80	1090.1
40	267.25	0.01715	10.498	236.03	933.70	1169.7	0.3919	1.2844	1.6763	235.90	1092.0
45	274.44	0.01721	9.401	243.36	928.60	1172.0	0.4019	1.265	1.6669	243.22	1093.7
50	281.01	0.01727	8.515	250.09	924.00	1174.1	0.411	1.2474	1.6585	294.93	1095.3
55	287.07	0.01732	7.787	256.30	919.60	1175.9	0.4193	1.2316	1.6509	256.12	1095.7
60	292.71	0.01738	7.175	262.09	915.50	1177.6	0.417	1.2168	1.6438	261.90	1097.9
65	297.97	0.01743	6.655	267.50	911.60	1179.1	0.4342	1.2032	1.6374	267.29	1099.1
70	302.92	0.01748	6.206	272.61	907.90	1180.6	0.4409	1.1906	1.6315	272.38	1100.2
75	307.6	0.01753	5.816	277.43	904.50	1181.9	0.4472	1.1787	1.6259	277.19	1101.2
80	312.03	0.01757	5.472	282.02	901.10	1183.1	0.4531	1.1676	1.6207	281.76	1102.1
85	316.25	0.01761	5.168	286.39	897.80	1184.2	0.4587	1.1571	1.6158	286.11	1102.9
90	320.27	0.01766	4.896	290.56	894.70	1185.3	0.4641	1.1471	1.6112	290.27	1103.7
100	327.81	0.01774	4.432	298.40	888.80	1187.2	0.474	1.1286	1.6026	298.08	1105.2

110	334.77	0.01782	4.049	305.66	883.20	1188.9	0.4832	1.1117	1.5948	305.30	1106.5
120	341.25	0.01789	3.728	312.44	877.90	1190.4	0.4916	1.0962	1.5878	312.05	1107.6
130	347.32	0.01796	3.455	318.81	872.90	1191.7	0.4995	1.0817	1.5812	318.38	1108.6
140	353.02	0.01802	3.220	324.82	868.20	1193.0	0.5069	1.0682	1.5751	324.35	1109.6
150	358.42	0.01809	3.015	330.51	863.60	1194.1	0.5138	1.0556	1.5694	330.01	1110.5
160	363.53	0.01815	2.834	335.95	859.20	1195.1	0.5204	1.0436	1.564	335.39	1111.2
170	368.41	0.01822	2.675	341.09	854.90	1196.0	0.5266	1.0324	1.559	340.52	1111.9
180	373.06	0.01827	2.532	346.03	850.80	1196.9	0.5325	1.0217	1.5542	345.42	1112.5
190	377.51	0.01833	2.404	350.79	846.80	1197.6	0.5381	1.0116	1.5497	350.15	1113.1
200	381.79	0.01839	2.288	355.36	843.00	1198.4	0.5435	1.0018	1.5453	354.68	1113.7
250	400.95	0.01865	1.844	376.00	825.10	1201.1	0.5675	0.9588	1.5263	375.14	1115.8
300	417.33	0.0189	1.543	393.84	809.00	1202.8	0.5879	0.9225	1.5104	392.79	1117.1
350	431.72	0.01913	1.326	409.69	794.20	1203.9	0.6056	0.891	1.4966	408.45	1118.0
400	444.59	0.0193	1.161	424.00	780.50	1204.5	0.6214	0.863	1.4844	422.60	1118.5
450	456.28	0.0195	1.032	437.20	767.40	1204.6	0.6356	0.8378	1.4734	435.50	1118.7
500	467.01	0.0197	0.928	449.40	755.00	1204.4	0.6487	0.8147	1.4634	447.60	1118.6
550	476.94	0.0199	0.842	460.80	743.10	1203.9	0.6608	0.7934	1.4542	458.80	1118.2
600	486.21	0.0201	0.770	471.60	731.60	1203.2	0.672	0.7734	1.4454	469.40	1117.7
650	494.9	0.0201	0.708	481.80	720.50	1202.3	0.6826	0.7548	1.4374	479.40	1117.1
700	503.1	0.0203	0.655	491.50	709.70	1201.2	0.6925	0.7371	1.4296	488.80	1116.3
750	510.86	0.0205	0.609	500.80	699.20	1200.0	0.7019	0.7204	1.4223	598.00	1115.4
800	518.23	0.0207	0.569	509.70	688.90	1198.6	0.7108	0.7045	1.4153	506.60	1114.4
850	525.26	0.0209	0.533	518.30	678.80	1197.1	0.7194	0.6891	1.4085	515.00	1113.3
900	531.98	0.0212	0.501	526.60	668.80	1195.4	0.7275	0.6744	1.402	523.10	1112.1
950	538.43	0.0214	0.472	534.60	659.10	1193.7	0.7355	0.6602	1.3957	530.90	1110.8
1000	544.61	0.0216	0.444	542.40	649.40	1191.8	0.743	0.6467	1.3897	538.40	1109.4
1100	556.31	0.022	0.400	557.40	630.40	1187.8	0.7575	0.6205	1.378	552.50	1106.4
1200	567.22	0.0223	0.362	571.70	611.70	1183.4	0.7711	0.5956	1.3667	566.70	1103.0
1300	577.46	0.0227	0.329	585.40	593.20	1178.6	0.784	0.5719	1.3559	580.00	1099.4
1400	587.1	0.0231	0.301	598.70	574.70	1173.4	0.7963	0.5491	1.3454	592.70	1095.4
1500	596.23	0.0235	0.277	611.60	556.30	1167.9	0.8082	0.5269	1.3351	605.10	1091.2
2000	635.82	0.0257	0.188	671.70	463.40	1135.1	0.8619	0.423	1.2849	662.20	1065.6
2500	668.13	0.0287	0.131	730.60	360.50	1091.1	0.9126	0.3197	1.2322	717.30	1030.6
3000	695.36	0.0346	0.086	802.50	217.80	1020.3	0.9731	0.1885	1.1615	783.40	972.7
3206.2	705.4	0.0503	0.050	902.70	0.00	902.7	1.0580	0	1.058	872.90	872.9

where ΔH = heat necessary to superheat steam from T_1 to T_s
C_p = specific heat of superheated steam
T_s = temperature of superheated steam
T_1 = saturated temperature of steam

The total enthalpy of superheated steam can now be calculated by adding the enthalpy of water, the latent heat of vaporization of steam, and the heat of superheating as follows:

$$H_s = H_w + L + C_p(T_s - T_1) \tag{4.6}$$

where H_s = enthalpy of superheated steam
H_w = enthalpy of water
L = latent heat of vaporization
C_p = specific heat of superheated steam
T_s = temperature of superheated steam
T_1 = saturated temperature of steam

Of course, to calculate the enthalpy of superheated steam we must know C_p. Using the steam tables avoids having to know the specific heat. We can in fact calculate the specific heat C_p by using the enthalpy from the steam tables in conjunction with Eq. (4.6).

Since superheated steam behaves fairly close to a perfect gas, we can say that adiabatic expansion of superheated steam follows the equation:

$$PV^{\gamma} = \text{constant} \tag{4.7}$$

where P = pressure
V = volume of steam
γ = ratio of specific heats for superheated steam

Variable V may be replaced by the specific volume. Since γ is 1.3 for superheated steam, the adiabatic expansion of superheated steam can be expressed by

$$PV^{1.3} = \text{constant} \tag{4.8}$$

Example 4.2 Calculate the amount of heat required to superheat 5 lb of dry saturated steam at a pressure of 160 psia to a temperature of 500°F. What is the specific heat of this steam?

Solution From Tables 4.1 and 4.2,

Enthalpy of superheated steam at 160 psia and 500°F = 1273.1 Btu/lb

Enthalpy of saturated steam at 160 psia or 500°F = 1195.1 Btu/lb

Saturation temperature = 363.53°F

The amount of heat required to superheat 5 lb of dry saturated steam is then

$$H = 5 \times (1273.1 - 1195.1) = 390 \text{ Btu}$$

The specific heat of steam can be found from the heat balance equation (4.5) as follows:

$$5 \times C_p(500 - 363.53) = 390$$

$$C_p = \frac{390}{5 \times 136.47} = 0.5716$$

Therefore, the specific heat of the superheated steam is 0.5716 Btu/(lb · °F).

When the steam properties are plotted such that entropy is on the horizontal axis and enthalpy is on the vertical axis at various temperatures and pressures, we get the Mollier diagram. This diagram is useful in calculations involving steam flow processes. A typical Mollier diagram is shown in Fig. 4.1. An abbreviated steam table of saturated and superheated steam is shown in Table 4.2*a* and *b*.

4.3.6 Volume

The volume of a unit weight of dry steam depends on the pressure and is determined experimentally. The steam tables include the specific volume (ft^3/lb) of dry steam at various saturation pressures and saturation temperatures. The density of dry steam is the reciprocal of the specific volume and is given by

$$\text{Density} = \frac{1}{v_s} \quad \text{lb/ft}^3 \tag{4.9}$$

where v_s is the specific volume (ft^3/lb).

Consider wet steam of quality x. One pound of this steam will contain x lb of dry steam and $(1 - x)$ lb of water. Since the volume of the wet steam is the sum of the volume of dry steam and that of the water, we can write

$$V_{\text{ws}} = V_{\text{ds}} + V_w \tag{4.10}$$

or

$$V_{\text{ws}} = xv_s + (1 - x)v_w \tag{4.11}$$

where V_{ws} = volume of 1 lb of wet steam
x = quality of steam, a number less than 1.0
v_s = specific volume of dry steam, ft^3/lb
v_w = specific volume of water, ft^3/lb
V_{ds} = volume of dry steam

Since the specific volume of water v_w is very small in comparison with the specific volume of steam v_s at low pressure, we can neglect the term

TABLE 4.2a Properties of Superheated Steam

Pressure, psia	Temperature, °F		200	300	400	500	600
1	101.74	v	392.6	452.3	512.0	571.6	631.2
		h	1150.4	1195.8	1241.7	1288.3	1335.7
		s	2.0512	2.1153	2.1720	2.2233	2.2702
5	162.24	v	78.16	90.25	102.3	114.22	126.16
		h	1148.8	1195.0	1241.0	1288.0	1335.4
		s	1.8718	1.9370	1.9942	2.0456	2.0927
10	193.21	v	38.85	45.00	51.04	57.05	63.03
		h	1146.6	1193.9	1240.6	1287.5	1335.1
		s	1.7927	1.8595	1.9172	1.9689	2.0160
14.696	212	v		30.53	34.68	38.78	42.86
		h		1192.8	1239.9	1287.1	1334.8
		s		1.8160	1.8743	1.9261	1.9734
20	227.96	v		22.36	25.43	28.46	31.47
		h		1191.6	1239.2	1286.6	1334.4
		s		1.7808	1.8396	1.8918	1.9392
40	267.25	v		11.04	12.628	14.168	15.688
		h		1186.8	1236.5	1284.8	1333.1
		s		1.6994	1.7608	1.8140	1.8619
60	292.71	v		7.2590	8.357	9.403	10.427
		h		1181.6	1233.6	1283.0	1331.8
		s		1.6492	1.7135	1.7678	1.8162
80	312.03	v			6.22	7.020	7.797
		h			1230.7	1281.1	1337.5
		s			1.6791	1.7346	1.7836
100	327.81	v			4.937	5.589	6.218
		h			1227.6	1279.1	1329.1
		s			1.6518	1.7085	1.7581
120	341.25	v			4.081	4.636	5.165
		h			1224.4	1277.2	1327.7
		s			1.6287	1.6869	1.7370
140	353.02	v			3.468	3.954	4.413
		h			1221.1	1275.2	1326.4
		s			1.6087	1.6683	1.7190
160	363.53	v			3.008	3.443	3.849
		h			1217.6	1273.1	1325
		s			1.5908	1.6519	1.7033
180	373.06	v			2.649	3.044	3.411
		h			1214.0	1271.0	1323.5
		s			1.5745	1.6373	1.6894
200	381.79	v			2.361	2.726	3.06
		h			1210.3	1268.9	1322.1
		s			1.5594	1.6240	1.6767
220	389.86	v			2.125	2.465	2.772
		h			1206.5	1266.7	1320.7
		s			1.5453	1.6117	1.6652
240	397.37	v			1.9276	2.247	2.533
		h			1202.5	1264.5	1319.2
		s			1.5319	1.6003	1.6546
260	404.42	v				2.063	2.33
		h				1262.3	1317.7
		s				1.5897	1.6447
280	411.05	v				1.9047	2.156
		h				1260.0	1316.2
		s				1.5796	1.6354
300	417.33	v				1.7675	2.005
		h				1257.6	1314.7
		s				1.5701	1.6268
350	431.72	v				1.4923	1.7036
		h				1251.5	1310.9
		s				1.5481	1.607
400	444.59	v				1.2881	1.477
		h				1245.1	1306.9
		s				1.5281	1.5894

Temperature, °F							
700	800	900	1000	1100	1200	1400	1600
690.8	750.4	809.9	869.5	869.5	988.7	1107.8	1227.0
1383.8	1432.8	1482.7	1533.5	1585.2	1637.7	1745.7	1857.5
2.2137	2.3542	2.3923	2.4283	2.4625	2.4192	2.5566	2.6137
138.10	150.03	161.95	173.87	185.79	197.71	221.60	245.40
1383.6	1432.7	1482.6	1533.4	1585.1	1637.7	1745.7	1857.4
2.1361	2.1767	2.2148	2.2509	2.2851	2.3178	2.3792	2.4363
69.01	74.98	80.95	86.92	92.88	98.84	110.77	122.69
1383.4	1432.5	1482.4	1533.2	1585.0	1637.6	1745.6	1857.3
2.0596	2.1003	2.1383	2.1744	2.2086	2.2413	2.3028	2.3598
46.94	51.00	55.07	59.13	63.19	67.25	75.37	83.48
1383.2	1432.3	1482.3	1533.1	1584.8	1637.5	1745.5	1857.3
2.0170	2.0576	2.0958	2.1319	2.1662	2.1989	2.2603	2.3174
34.47	37.46	40.45	43.44	46.42	49.41	55.37	61.34
1382.9	1432.1	1482.1	1533.0	1584.7	1637.4	1745.4	1857.2
1.9829	2.0	2.0618	2.0978	2.1321	2.1648	2.2263	2.2834
17.198	18.702	20.200	21.700	23.200	24.690	27.680	30.660
1381.9	1431.3	148.4	1532.4	1584.3	1637.0	1745.1	1857.0
1.9058	1.9467	1.9850	2.0212	2.0555	2.0883	2.1498	2.2069
11.441	12.449	13.452	14.454	15.453	16.451	18.446	20.440
1380.9	1430.5	1480.8	1531.9	1583.8	1636.6	1744.8	1856.7
1.8605	1.9015	1.9400	1.9762	2.0106	2.0434	2.1049	2.1621
8.562	9.322	10.077	10.830	11.582	12.332	13.830	15.523
1379.9	1429.7	1487.1	1531.3	1583.4	1636.2	1744.5	1856.5
1.8281	1.8694	1.9079	1.9442	1.9787	2.0115	2.0731	2.1303
6.835	7.446	8.052	8.656	9.259	9.860	11.060	12.258
1378.9	1428.9	1479.5	1530.8	1582.9	1635.7	1744.2	1856.2
1.8029	1.8443	1.8829	1.9193	1.9538	1.9867	2.0484	2.1056
5.683	6.195	6.207	7.207	7.710	8.212	9.214	10.213
1377.8	1428.1	1378.8	1530.2	1582.4	1635.3	1743.9	1856.0
1.8722	1.8237	1.8625	1.8990	1.9335	1.9664	2.0281	2.0854
4.861	5.301	5.738	6.172	6.604	7.035	7.895	8.752
1376.8	1427.3	1478.2	1529.7	1581.9	1634.9	1743.5	1855.7
1.7645	1.8063	1.8451	1.8817	1.9163	1.9493	2.0110	2.0683
4.244	4.631	5.015	5.396	5.775	6.152	6.906	7.656
1375.7	1426.4	1477.5	1529.1	1581.4	1634.5	1743.2	1855.5
1.7491	1.7911	1.8301	1.8667	1.9014	1.9344	1.9962	2.0535
4.764	4.11	4.452	4.792	5.129	5.466	6.136	6.804
1374.7	1425.6	1476.8	1528.6	1581.0	1634.1	1742.9	1855.2
1.7355	1.7776	1.8167	1.8534	1.8882	1.9212	1.9831	2.0404
3.38	3.693	4.002	4.309	4.613	4.917	5.521	6.123
1373.6	1424.8	1476.2	1528	1580.5	1633.7	1742.6	1855
1.7232	1.7655	1.8048	1.8415	1.8763	1.9094	1.9713	2.0287
3.066	3.352	3.634	3.913	4.191	4.467	5.017	5.565
1372.6	1424	1475.5	1527.5	1580	1633.3	1742.3	1854.7
1.712	1.7545	1.7939	1.8308	1.8656	1.8987	1.9607	2.0181
2.804	3.068	3.327	3.584	3.839	4.093	4.597	5.1
1371.5	1423.2	1474.8	1526.9	1579.6	1632.9	1742	1854.5
1.7017	1.7444	1.7839	1.8209	1.8558	1.8889	1.951	2.0084
2.582	2.827	3.067	3.305	3.541	3.776	4.242	4.707
1370.4	1422.4	1474.2	1526.3	1579.1	1632.5	1741.7	1854.2
1.6922	1.7352	1.7748	1.8118	1.8467	1.8799	1.942	1.9995
2.392	2.621	2.845	3.066	3.286	3.504	3.938	4.37
1369.4	1421.5	1473.5	1525.8	1578.6	1632.1	1741.4	1854.0
1.6834	1.7265	1.7662	1.8033	1.8383	1.8716	1.9337	1.9912
2.227	2.442	2.652	2.895	3.065	3.269	3.674	4.078
1368.3	1420.6	1472.8	1525.2	1578.1	1631.7	1741	1853.7
1.6751	1.7184	1.7582	1.7954	1.8305	1.8638	1.926	1.9835
1.898	2.084	2.266	2.445	2.622	2.798	3.147	3.493
1365.5	1418.5	1471.1	1523.8	1577	1630.7	1740.3	1853.1
1.6563	1.7002	1.7403	1.7777	1.813	1.8463	1.9086	1.9663
1.6508	1.8161	1.9767	2.134	2.29	2.445	2.751	3.055
1362.7	1416.4	1469.4	1522.4	1575.8	1629.6	1739.5	1852.5
1.6398	1.6842	1.7247	1.7623	1.7977	1.8311	1.8936	1.9513

TABLE 4.2b Properties of Superheated Steam

Pressure, psia	Temperature, °F		500	550	600	620	640	660
450	456.28	v	1.1231	1.2155	1.3005	1.3332	1.3652	1.3967
		h	1238.4	1272.0	1302.8	1314.6	1326.2	1337.5
		s	1.5095	1.5437	1.5735	1.5845	1.5951	1.6054
500	467.01	v	0.9927	1.0800	1.1591	1.1893	1.2188	1.2478
		h	1231.3	1266.80	1298.6	1310.7	1322.6	1334.2
		s	1.4919	1.5280	1.5588	1.5701	1.5810	1.5915
550	476.94	v	0.8852	0.9686	1.0431	1.0714	1.0989	1.1259
		h	1223.7	1261.20	294.3	1306.8	1318.9	1330.8
		s	1.4751	1.5131	1.5451	1.5568	1.5680	1.5787
600	486.21	v	0.7947	0.8753	0.9463	0.9729	0.9988	1.0241
		h	1215.7	1255.50	1289.90	1302.7	1315.2	1327.4
		s	1.4586	1.4990	1.5323	1.5443	1.5558	1.5667
700	503.1	v		0.0277	0.7934	0.8177	0.8411	0.8639
		h		1243.2	1280.6	1294.3	1307.5	1320.3
		s		1.4722	1.5084	1.5212	1.5333	1.5449
800	518.23	v		0.6154	0.6779	0.7006	0.7223	0.7433
		h		1229.80	1270.70	1285.4	1299.4	1312.9
		s		1.4467	1.4863	1.5000	1.5129	1.5250
900	531.98	v		0.5364	0.5873	0.6089	0.6294	0.6491
		h		1215.00	1260.10	1279.9	1290.9	1305.1
		s		1.4216	1.4653	1.4800	1.4938	1.5066
1000	544.61	v		0.4533	0.5140	0.535	0.555	0.5733
		h		1198.30	1248.80	1265.9	1297.0	1297.0
		s		1.3961	1.4450	1.4610	1.4757	1.4893
1100	556.31	v			0.4532	0.4738	0.4929	0.510
		h			1236.70	1255.3	1272.4	1288.5
		s			1.4251	1.4425	1.4583	1.4728
1200	567.22	v			0.4056	0.4222	0.4410	0.4586
		h			1223.5	1243.9	1262.4	1279.6
		s			1.4052	1.4243	1.4413	1.4568
1400	587.1	v			0.3174	0.339	0.358	0.3753
		h			1193.0	1218.4	1240.4	1260.3
		s			1.3639	1.3877	1.4079	1.4258
1600	604.9	v				0.2733	0.2936	0.3112
		h				1187.8	1215.2	1238.7
		s				1.3489	1.3741	1.3952
1800	621.03	v					0.2407	0.2597
		h					1185.1	1214.0
		s					1.3377	1.3638
2000	635.82	v					0.1936	0.2161
		h					1145.6	1184.9
		s					1.2945	1.3300
2500	668.13	v						
		h						
		s						
3000	695.36	v						
		h						
		s						
3206.2	705.4	v						
		h						
		s						
3500		v						
		h						
		s						
4000		v						
		h						
		s						
4500		v						
		h						
		s						
5000		v						
		h						
		s						
5500		v						
		h						
		s						

NOTE: v = specific volume; h = enthalpy; s = entropy.

Temperature, °F							
680	700	800	900	1000	1200	1400	1600
1.4278	1.4584	1.6074	1.7516	1.8928	2.1700	2.4430	2.7140
1348.8	1359.9	1414.3	1467.7	1521.0	1628.6	1738.7	1851.9
1.6153	1.6250	1.6699	1.7108	1.7486	1.8177	1.8803	1.9381
1.2763	1.3044	1.4405	1.5715	1.6996	1.9504	2.1970	2.4420
1345.7	1357.0	1412.1	1566.0	1519.6	1627.6	1737.9	1851.3
1.6016	1.6115	1.6571	1.6982	1.7363	1.8056	1.8683	1.9262
1.1523	1.7830	1.3038	1.4241	1.5414	1.7706	1.9957	2.2190
1342.5	1354.0	1409.9	1464.3	1318.2	1626.6	1737.1	1850.6
1.5890	1.5991	1.6452	1.6868	1.7250	1.7946	1.8675	1.9155
1.0489	1.0732	1.1899	1.3013	1.6208	1.6208	1.8279	2.0330
1339.3	1351.1	1407.7	1462.5	1516.7	1625.5	1736.3	1850.0
1.5773	1.5875	1.6343	1.6762	1.7147	1.7846	1.8476	1.9056
0.8860	0.9077	1.0108	1.1082	1.2024	1.3853	1.5641	1.7405
1332.8	1345.0	1403.2	1459.0	1315.9	1623.5	1734.8	1848.8
1.5559	1.5665	1.6147	1.6573	1.6963	1.7666	1.8299	1.8881
0.7635	0.7833	0.8763	0.9633	1.0470	1.2088	1.3662	1.5214
1325.9	1338.6	1398.6	1455.4	1511.4	1621.4	1733.2	1847.5
1.5366	1.5476	1.5972	1.6407	1.6801	1.7510	1.8146	1.8729
0.6680	0.6863	0.7716	0.8506	0.9262	1.0714	1.2124	1.3509
1318.8	1332.1	1393.9	1451.8	1508.1	1619.3	1731.6	1846.3
1.5187	1.5303	1.5814	1.6257	1.6656	1.7371	1.8009	1.8595
0.5912	0.6084	0.6878	0.7604	0.8294	0.9615	1.0893	1.2146
1311.4	1325.3	1389.2	1448.2	1505.1	1617.3	1730.0	1845.0
1.5021	1.5141	1.5670	1.6121	1.6525	1.7245	1.7885	1.8474
0.528	0.5445	0.619	0.687	0.7503	0.8716	0.9885	1.1031
1303.7	1318.3	1384.3	1444.5	1502.2	1615.2	1728.4	1843.8
1.4862	1.4989	1.5535	1.5995	1.6405	1.7130	1.7775	1.8363
0.4752	0.4909	0.5617	0.6250	0.6843	0.7967	0.9046	1.0101
1295.7	1311.0	1379.3	1440.7	1499.3	1613.1	1726.9	1842.5
1.4710	1.4843	1.5409	1.5879	1.6293	1.7025	1.7672	1.8263
0.3912	0.4062	0.4714	0.5281	0.5805	0.6789	0.7727	0.8640
1278.5	1295.5	1369.1	1433.1	1495.2	1608.9	1723.7	1840.0
1.4419	1.4597	1.5177	1.5666	1.6093	1.6836	1.7489	1.8083
0.3271	0.2417	0.4034	0.4553	0.5027	0.5904	0.6738	0.7545
1259.6	1278.7	1358.4	1425.3	1487.0	1604.6	1720.5	1837.5
1.4137	1.4303	1.4964	1.5476	1.5914	1.6669	1.7328	1.7926
0.276	0.2907	0.3502	0.3986	0.4421	0.5218	0.5968	0.6693
1238.5	1260.3	1347.2	1417.4	1480.8	1600.4	1717.3	1835.0
1.3855	1.4044	1.4765	1.5301	1.5712	1.652	1.7185	1.7786
0.2337	0.2489	0.3074	0.3532	0.3935	0.4668	0.5352	0.6011
1214.8	1240	1335.5	1409.2	1474.5	1596.1	1714.1	1832.5
1.3564	1.3783	1.4576	1.5139	1.5603	1.6384	1.7055	1.7660
0.1484	0.1686	0.2294	0.271	0.3061	0.3678	0.4244	0.4784
1132.3	1176.6	1303.6	1387.8	1458.4	1585.3	1706.1	1826.2
1.2687	1.3073	1.4127	1.4772	1.5273	1.6088	1.6775	1.7389
	0.0984	0.176	0.2159	0.2476	0.3018	0.3505	0.3966
	1060.7	1267.2	1365	1441.8	1574.3	1698	1819.9
	1.1966	1.369	1.4439	1.4984	1.5837	1.654	1.7163
		0.1583	0.1981	0.2288	0.2806	0.3267	0.3703
		1250.5	1355.2	1434.7	1569.8	1694.6	1817.2
		1.3508	1.4309	1.4874	1.5742	1.6452	1.708
	0.0306	0.1364	0.1762	0.2058	0.2546	0.2977	0.3381
	780.5	1224.9	1340.7	1424.5	1563.3	1689.8	1813.6
	0.9515	1.3241	1.4127	1.4723	1.5615	1.6336	1.6968
	0.0287	0.1052	0.1462	0.1743	0.2192	0.2581	0.2943
	763.8	1174.8	1314.4	1406.8	1552.1	1681.7	1807.2
	0.9347	1.2757	1.3827	1.4482	1.5417	1.6154	1.6795
	0.0276	0.0798	0.1226	0.1500	0.1917	0.2273	0.2602
	753.5	1115.9	1286.5	1388.4	1540.8	1673.5	1800.9
	0.9235	1.2204	1.3529	1.4253	1.5235	1.5990	1.6640
	0.0268	0.0593	0.1036	0.1303	0.1696	0.2027	0.2329
	746.4	1047.1	1256.5	1369.5	1529.5	1665.3	1794.5
	0.9152	1.1622	1.3231	1.4034	1.5066	1.5839	1.6499
	0.0262	0.0463	0.0880	0.1143	0.1516	0.1825	0.2106
	741.3	985.0	1224.1	1349.3	1518.2	1657.0	1788.1
	0.909	1.1093	1.293	1.3821	1.4908	1.5699	1.6369

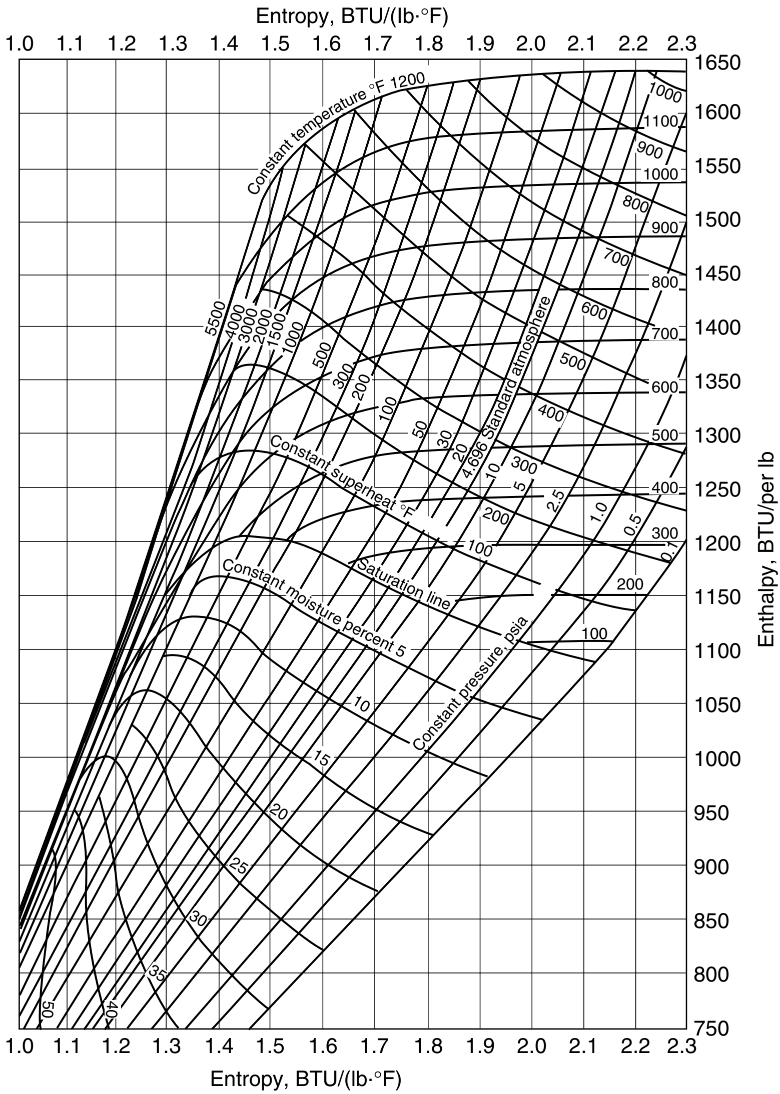

Figure 4.1 Mollier diagram.

containing v_w in Eq. (4.11), and therefore we can state that

$$V_{\mathrm{ws}} = xv_s \tag{4.12}$$

Thus, the density of wet steam is the reciprocal of the specific volume given in Eq. (4.12).

$$\text{Density of wet steam} = \frac{1}{xv_s} \tag{4.13}$$

where x is the quality of steam (a number less than 1.0) and v_s is the specific volume of dry steam (ft^3/lb).

It must be remembered that we neglected the second term in Eq. (4.11) at low pressures. For high-pressure steam or for low values of steam quality, we must include both terms in Eq. (4.11).

Example 4.3 Calculate the weight of 4 ft^3 of wet steam (quality = 0.7) at a pressure of 100 psia. Also calculate the enthalpy of 1 ft^3 of this steam.

Solution From Eq. (4.13),

$$\text{Density of wet steam} = \frac{1}{xv_s}$$

From (Table 4.1) at 100 psia the specific volume of dry saturated steam is 4.432 ft^3/lb. Therefore the density of wet steam of dryness fraction 0.7 is

$$\text{Density} = \frac{1}{0.7 \times 4.432} = 0.3223 \text{ lb/ft}^3$$

$$\text{Weight of 4 ft}^3 \text{ of wet steam} = 4 \times 0.3223 = 1.2893 \text{ lb}$$

The enthalpy of this wet steam is calculated using Eq. (4.2) as follows. From Table 4.1, at 100 psia,

$$\text{Enthalpy of water} = 298.4 \text{ Btu/lb} \qquad \text{and} \qquad L = 888.8 \text{ Btu/lb}$$

Therefore,

$$\text{Enthalpy of wet steam} = 298.4 + 0.7 \times 888.8 = 920.56 \text{ Btu/lb}$$

$$\text{Enthalpy per ft}^3 = 920.56 \times 0.3223 = 296.70 \text{ Btu/ft}^3$$

The volume of superheated steam may be calculated by two different methods. The first method is approximate and based on the assumption that steam behaves as a perfect gas during superheating. This is found to be accurate at low pressures and higher superheat temperatures. For high pressures and lower superheat temperatures the calculated volume will be inaccurate.

For ideal gases at low pressures we can apply the ideal gas equation as well as Boyle's law and Charles's law. Superheated steam behaves close to ideal gases at low pressures. The ideal gas law states that the pressure, volume, and temperature of a given quantity of gas are related by the ideal gas equation as follows:

$$PV = nRT \tag{4.14}$$

where P = absolute pressure, psia
V = gas volume, ft^3
n = number of lb moles of gas in a given mass
R = universal gas constant
T = absolute temperature of gas, °R (°F + 460)

In USCS units, R has a value of 10.732 (psia · ft^3)/(lb · mol · °R). Using Eq. (4.14) we can restate the ideal gas equation as follows:

$$PV = \frac{mRT}{M} \tag{4.15}$$

where m represents the mass and M is the molecular weight of gas. The ideal gas equation is only valid at pressures near atmospheric pressure. At high pressures it must be modified to include the effect of compressibility.

Two other equations used with gases are called Boyle's law and Charles's law. Boyle's law states that the pressure of a given quantity of gas varies inversely as its volume provided the temperature is kept constant. Mathematically, Boyle's law is expressed as

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

or

$$P_1V_1 = P_2V_2 \tag{4.16}$$

where P_1 and V_1 are the initial pressure and volume, respectively, at condition 1 and P_2 and V_2 refer to condition 2. In other words, PV = constant.

Charles's law relates to volume-temperature and pressure-temperature variations for a given mass of gas. Thus keeping the pressure constant, the volume of gas will vary directly with the absolute temperature. Similarly, keeping the volume constant, the absolute pressure will vary directly with the absolute temperatures. These are represented mathematically as follows:

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{for constant pressure} \tag{4.17}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \text{for constant volume} \tag{4.18}$$

Note that in the preceding discussion, the temperature is always expressed in absolute scale. In USCS units, the absolute temperature is stated as °R equal to (°F + 460). In SI units the absolute temperature is expressed in kelvin equal to (°C + 273).

Pressures used in Eq. (4.18) must also be in absolute units, such as psi absolute or kPa absolute.

If we know the pressure at which steam is superheated (P), the specific volume of dry steam at this pressure (v_s), the saturation temperature of steam at this pressure (T_1), and the final temperature of the superheated steam, (T_{sup}), then we can calculate the specific volume of the superheated steam (v_{sup}). Applying the ideal gas law, which becomes Charles's law since pressure is constant, we get

$$\frac{Pv_{sup}}{T_1} = \frac{Pv_s}{T_{sup}} \tag{4.19}$$

or

$$v_{sup} = \frac{v_s T_s}{T_1} \tag{4.20}$$

where v_{sup} = specific volume of superheated steam, ft³/lb
v_s = specific volume of dry saturated steam, ft³/lb
T_{sup} = final temperature of superheated steam, °R
T_1 = saturation temperature of steam, °R

Equation (4.20) gives an approximate value of the specific volume of superheated steam at a particular temperature and pressure.

A more accurate method is to use the following equation, known as Callendar's equation.

$$v_{sup} - 0.016 = \frac{0.1101JT}{P} - 1.192\left(\frac{273.1}{T}\right)^{10/3} \tag{4.21}$$

where v_{sup} = specific volume of superheated steam, ft³/lb
T = absolute temperature of steam, K
P = pressure of steam, psia
J = mechanical equivalent of heat, 1400 ft · lb per centigrade heat unit

Another equation for calculating the specific volume of superheated steam is as follows:

$$v_{sup} = \frac{1.253(H_s - 835)}{P} \tag{4.22}$$

where H_s is the enthalpy of superheated steam and P is the pressure of superheated steam (psia).

Example 4.4 Calculate the approximate volume of 4 lb of superheated steam at a pressure of 300 psia and a temperature of 500°F.

Solution From Table 4.2a, at 300 psia, the saturation temperature is

$$T_1 = 417.33°\text{F} + 460 = 877.33°\text{R}$$

Therefore the steam is superheated at a temperature of

$$T_s = 500 + 460 = 960°\text{R}$$

The specific volume of dry saturated steam from Table 4.1 is

$$v_s = 1.5433 \text{ ft}^3/\text{lb}$$

The specific volume of superheated steam, per Eq. (4.20), is

$$v_{\text{sup}} = \frac{1.5433 \times 960}{877.33} = 1.6887 \text{ ft}^3/\text{lb}$$

Example 4.5 Calculate the specific volume of superheated steam at a pressure of 120 psia and a temperature of 600°F using both the approximate method and the more exact method.

Solution Using the approximate method, at 120 psia the saturation temperature is

$$T_1 = 341.25°\text{F} = 341.25 + 460 = 801.25°\text{R}$$

Also from Table 4.1 the specific volume of dry steam is 3.728 ft^3/lb. Therefore using Eq. (4.20), the specific volume of superheated steam at 600°F is

$$v_{\text{sup}} = \frac{3.728}{801.25} \times (600 + 460) = 4.9319 \text{ ft}^3/\text{lb}$$

Using the more exact method, from Eq. (4.21), we calculate the specific volume of superheated steam at 600°F as follows:

$$600°\text{F} = \frac{600 - 32}{9} \times 5 = 315.56°\text{C} = 315.56 + 273 = 588.56 \text{ K}$$

The specific volume of superheated steam at 600°F is

$$v_{\text{sup}} - 0.0016 = \frac{0.1101 \times 1400 \times 588.56}{120 \times 144} - 1.192\left(\frac{273.1}{588.56}\right)^{10/3}$$

$$= 5.1594 \text{ ft}^3/\text{lb}$$

It can be seen that the difference between the valves obtained, respectively, by approximate method and the exact method is 4.4 percent.

4.3.7 Viscosity

Viscosity is defined as resistance to flow. It is found that as temperature increases, the viscosity of steam also increases. A similar behavior is exhibited with an increase in pressure. This is similar to most gases. Table 4.3 shows the variation of viscosity of steam with temperature and pressure. At 500 psia the viscosity of saturated steam is 1.9×10^{-5} lb/(ft · s) and 2.08×10^{-5} lb/(ft · s) at a temperature of 600°F. Viscosity is measured in lb/(ft · s) or Poise.

TABLE 4.3 Viscosity Variation with Temperature and Pressure

Pressure, psia	Saturated vapor (lb · s)/ft^2	Temperature°F					
		200	400	600	800	1000	1200
0		2.59	3.49	4.35	5.19	5.99	6.76
500	5.90			6.47	7.30	8.10	8.87
1000	8.17			8.41	9.24	10.00	10.80
1500	10.20			10.20	11.00	11.80	12.60
2000	11.90				12.60	13.40	14.20
2500	13.50				14.00	14.80	15.60
3000	14.80				15.20	16.00	16.80
3500					16.30	17.10	17.90

NOTE: Table values multiplied by 10^{-7} equal viscosity of steam in (lb · s)/ft^2.

4.4 Pipe Materials

Piping material used in steam piping generally conforms to national codes and standards published by the American National Standards Institute (ANSI) and the ASME. Other codes such as European (DIN), Japanese (JIS), British, and Canadian standards as applicable may be consulted for overseas projects. ASTM and ASME material specifications conforming to ASME Boiler and Pressure Vessel Codes are also consulted for steam piping.

Steel pipe used for steam piping may be welded or seamless pipe. ASTM A53 grades A and B and A106 grade B are used in many installations. The allowable design pressures must be adjusted downward for increased operating temperatures. For high-temperature operations, chrome-moly alloy steel is used.

Pipes are joined by means of welding or by screwed and flange fittings. Nowadays welding has mostly replaced all screwed joints. Flange connections are still necessary, and many installations have flanged valves in steam piping. For pressures not exceeding 250 psi and temperatures below 450°F, malleable, cast iron, or bronze fittings may be used. Cast or forged carbon steel fittings are used for higher temperatures and pressures. Welded fittings such as elbows, tees, and flanges must conform to ANSI B16.9 standards and ASTM A216, A234, or A105.

4.5 Velocity of Steam Flow in Pipes

The velocity of steam flowing through a pipe depends on the mass flow rate, pipe inside diameter, pressure, and steam properties.

$$\text{Mass flow rate} = \text{density} \times \text{pipe area} \times \text{velocity} \tag{4.23}$$

$$\text{Velocity} = \frac{\text{mass flow}}{\text{area} \times \text{density}} \tag{4.24}$$

Instead of density, we can use the reciprocal of the specific volume in Eqs. (4.23) and (4.24). For example, consider a 6-in pipe flowing 10,000 lb/h of dry saturated steam at 100 psia. At this pressure the specific volume of steam from Table 4.1 is 4.432 ft^3/lb. The cross-sectional area of 6-in schedule 40 pipe is

$$A = 0.7854 \left(\frac{6.065}{12}\right)^2 = 0.2006 \text{ ft}^2$$

Therefore, the velocity of steam, using Eq. (4.24), is

$$\text{Velocity} = \frac{10{,}000}{0.2006 \times 1/4.432} = 220{,}937 \text{ ft/h} = 3682 \text{ ft/min}$$

A higher steam velocity means a higher friction drop and increased noise and erosion of the pipe wall.

Table 4.4 lists some reasonable design velocities of steam flowing through pipes. The velocities are based on reasonable pressure drops that do not cause too much erosion in pipes. The velocity should be kept lower with wet steam than dry steam, since the former will tend to cause more erosion.

TABLE 4.4 Steam Velocities in Pipes

USCS units

Fluid	Pressure, psig	Use	Approximate velocity ft/min	Approximate velocity ft/s
Water	25–40	City water	120–300	2–5
Water	50–150	General service	300–600	5–10
Water	150+	Boiler feed	600–1,200	10–20
Saturated steam	0–15	Heating	4,000–6,000	67–100
Saturated steam	50+	Miscellaneous	6,000–10,000	100–167
Superheated steam	200	Large turbine and boiler leads	10,000–20,000	167–334

SI units

Fluid	Pressure, kPa gauge	Use	Approximate velocity m/min	Approximate velocity m/s
Water	172–276	City water	36–91	0.61–1.52
Water	345–1034	General service	91–183	1.52–3.05
Water	1034+	Boiler feed	183–366	3.05–6.10
Saturated steam	1–103	Heating	1,220–1,830	20.4–30.5
Saturated steam	345+	Miscellaneous	1,830–3,050	30.5–50.9
Superheated steam	1380+	Large turbine and boiler leads	3,050–4,570	50.9–76.2

The maximum velocity of steam in a pipe is equal to the speed of sound in the fluid. It is calculated as follows:

$$U_s = \sqrt{\gamma g R T} \tag{4.25}$$

where U_s = sonic velocity
γ = specific heat ratio of steam
g = acceleration due to gravity, 32.2 ft/s^2
R = gas constant
T = absolute temperature, °R

Example 4.6 What is the maximum (sonic velocity) of dry steam flowing through a 6-in schedule 40 pipe at 300°F? What is the corresponding velocity for superheated steam at 400°F and pressure at 100 psia?

Solution At 300°F saturation temperature, dry steam has a pressure of 67.013 psia and a specific volume of 6.449 ft^3/lb. The sonic velocity, using Eq. (4.25), is

$$U_s = \sqrt{\gamma g R T}$$

This equation can be rewritten using the ideal gas law as

$$U_s = \sqrt{\gamma g P v}$$

where P is the pressure in lb/ft^2 and v is the specific volume.
Using a specific heat ratio of dry steam $\gamma = 1.135$,

$$U_s = \sqrt{1.135 \times 32.2 \times 67.013 \times 144 \times 6.449} = 1508 \text{ ft/s}$$

Therefore, the sonic velocity of dry saturated steam = 1508 ft/s.
For superheated steam, we get the specific volume from Table 4.2*a* at 400°F and 100 psia pressure as

$$v_{\text{sup}} = 4.937 \text{ ft}^3\text{/lb}$$

Therefore, using a specific heat ratio of $\gamma = 1.3$, the sonic velocity of superheated steam is

$$U_s = \sqrt{1.3 \times 32.2 \times 100 \times 144 \times 4.937} = 1725 \text{ ft/s}$$

Thus, the sonic velocity of superheated steam is 1725 ft/s.

Example 4.7 A steam piping application requires 6000 lb of steam per hour at 100 psig. The velocity is limited to 4500 ft/min. What pipe size must be used?

Solution From Table 4.1, the specific volume of dry saturated steam at 100 psig is 4.049. From Eq. (4.24), the velocity is

$$\text{Velocity} = \frac{\text{mass}}{\text{density} \times \text{area}}$$

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Therefore,

$$4500 = \frac{6000/60}{(1/4.049) \times 0.7854 \times D^2}$$

Solving for the required diameter D,

$$D = 0.339 \text{ ft} = 4.06 \text{ in}$$

4.6 Pressure Drop

As steam flows through a pipe, energy is lost due to friction between the steam molecules and the pipe wall. This is evident in the form of a pressure gradient along the pipe. Before we introduce the various equations to calculate the amount of pressure drop due to friction, we will discuss an important parameter related to the flow of steam in a pipeline, called the Reynolds number.

The Reynolds number of flow is a dimensionless parameter that depends on the flow rate, pipe diameter, and steam properties such as density and viscosity. The Reynolds number is used to characterize flow type such as laminar flow and turbulent flow.

The Reynolds number is calculated as follows:

$$\mathrm{Re} = \frac{vD\rho}{\mu} \tag{4.26}$$

where Re = Reynolds number of flow, dimensionless
v = velocity of flow, ft/s
D = pipe inside diameter, ft
ρ = steam density, slug/ft^3
μ = steam viscosity, lb/(ft · s)

In steam flow, the following equation for the Reynolds number is more appropriate.

$$\mathrm{Re} = \frac{6.31W}{\mu D} \tag{4.27}$$

where W = steam flow rate, lb/h
D = pipe inside diameter, in
μ = steam viscosity, cP

In SI units the Reynolds number is given by

$$\mathrm{Re} = \frac{353.404W}{\mu D} \tag{4.28}$$

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where W = steam flow rate, kg/h
D = pipe inside diameter, mm
μ = steam viscosity, cP

Laminar flow is defined as flow that causes the Reynolds number to be below a threshold value such as 2000 to 2100. *Turbulent flow* is defined as flow that causes the Reynolds number to be greater than 4000. The range of Reynolds numbers between 2000 and 4000 characterizes an unstable flow regime known as *critical flow*.

Example 4.8 Steam at 500 psig and 800°F flows through a 6-in schedule 40 pipe at 20,000 lb/h. Calculate the Reynolds number.

Solution We need the viscosity of steam at 500 psig pressure and 800°F. From Table 4.3, we get

$$\text{Viscosity of steam} = 0.026 \text{ cP} \quad \text{approximately}$$

The inside diameter of 6-in schedule 40 pipe is

$$D = 6.625 - 2 \times 0.280 = 6.065 \text{ in}$$

Using Eq. (4.27), we get

$$\text{Re} = \frac{6.31 \times 20{,}000}{0.026 \times 6.065} = 800{,}304$$

The Reynolds number is 800,304. Since this is greater than 4000, the flow is turbulent.

4.6.1 Darcy equation for pressure drop

The Darcy equation, also called the Darcy-Weisbach equation, is one of the oldest formulas used in classical fluid mechanics. It can be used to calculate the pressure drop in pipes transporting any type of fluid, such as a liquid or gas.

As steam flows through a pipe from point A to point B, the pressure decreases due to friction between the steam and the pipe wall. The Darcy equation may be used to calculate the pressure drop in steam pipes as follows:

$$h = f \frac{L}{D} \frac{U^2}{2g} \tag{4.29}$$

where h = frictional pressure loss, in ft of head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = inside pipe diameter, ft
U = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

In USCS units, $g = 32.2$ ft/s^2, and in SI units, $g = 9.81$ m/s^2.

Note that the Darcy equation gives the frictional pressure loss in feet of head of flowing fluid. It can be converted to pressure loss in psi by multiplying by the density and a suitable conversion factor. The term $(U^2/2g)$ in the Darcy equation is called the velocity head, and it represents the kinetic energy of steam. The term *velocity head* will be used in subsequent sections of this chapter when discussing frictional head loss through pipe fittings and valves.

Another more convenient form of the Darcy equation with frictional pressure drop expressed in psi and using mass flow rate in lb/h of steam is as follows:

$$\Delta P = (3.3557 \times 10^{-6}) \frac{fLvW^2}{D^5} \tag{4.30}$$

where ΔP = frictional pressure loss, psi
f = Darcy friction factor, dimensionless
L = pipe length, ft
v = specific volume of steam, ft^3/lb
W = steam flow rate, lb/h
D = pipe inside diameter, ft

In SI units, the Darcy equation for steam flow may be written as

$$\Delta P = 62{,}511 \frac{fLvW^2}{D^5} \tag{4.31}$$

where ΔP = frictional pressure loss, kPa
f = Darcy friction factor, dimensionless
L = pipe length, m
v = specific volume of steam, m^3/kg
W = steam flow rate, kg/h
D = pipe inside diameter, mm

In order to calculate the friction loss in a steam pipeline using the Darcy equation, we must know the friction factor f. The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eq. (4.30). This friction factor is a dimensionless number between 0.0

and 0.1 (usually around 0.02 for turbulent flow) that depends on the internal roughness of the pipe, pipe diameter, and the Reynolds number, and therefore the type of flow (laminar or turbulent).

The friction factor may be calculated using the method described next or found from the Moody diagram shown in Fig. 4.2. The Moody diagram is a graphical plot of the friction factor f for all flow regimes (laminar, critical, and turbulent) against the Reynolds number at various values of the relative roughness of pipe. The internal roughness of the pipe is represented by e and is listed for various pipes in Table 4.5. The ratio of the pipe roughness to the inside diameter of the pipe (e/D) is a dimensionless term called the *relative roughness*. The graphical method of determining the friction factor for turbulent flow using the Moody diagram is discussed next.

For a given Reynolds number on the horizontal axis, a vertical line is drawn up to the curve representing the relative roughness e/D. The friction factor is then read by going horizontally to the vertical axis on the left. It can be seen from the Moody diagram that the turbulent region is further divided into two regions: the "transition zone" and the "complete turbulence in rough pipes" zone. The lower boundary is designated as "smooth pipes," and the transition zone extends up to the dashed line. Beyond the dashed line is the complete turbulence in rough pipes zone. In this zone the friction factor depends very little on the Reynolds number and more on the relative roughness. The Moody diagram method of finding the friction factor is easier than the calculation method using the Colebrook-White equation discussed next.

For laminar flow, the friction factor f depends only on the Reynolds number and is calculated from the following equation:

$$f = \frac{64}{\mathrm{Re}} \tag{4.32}$$

where f is the friction factor for laminar flow and Re is the Reynolds number for laminar flow ($R < 2100$) (dimensionless). Therefore, if the Reynolds number for a particular flow is 1200, the friction factor for this laminar flow is $64/1200 = 0.0533$.

4.6.2 Colebrook-White equation

If the flow is turbulent ($\mathrm{Re} > 4000$), calculation of the friction factor is not as straightforward as that for laminar flow. For turbulent flow, we can calculate the friction factor f, using the Colebrook-White equation as follows. The friction factor f is given for turbulent flow (for $\mathrm{Re} > 4000$) as:

$$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{e}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{4.33}$$

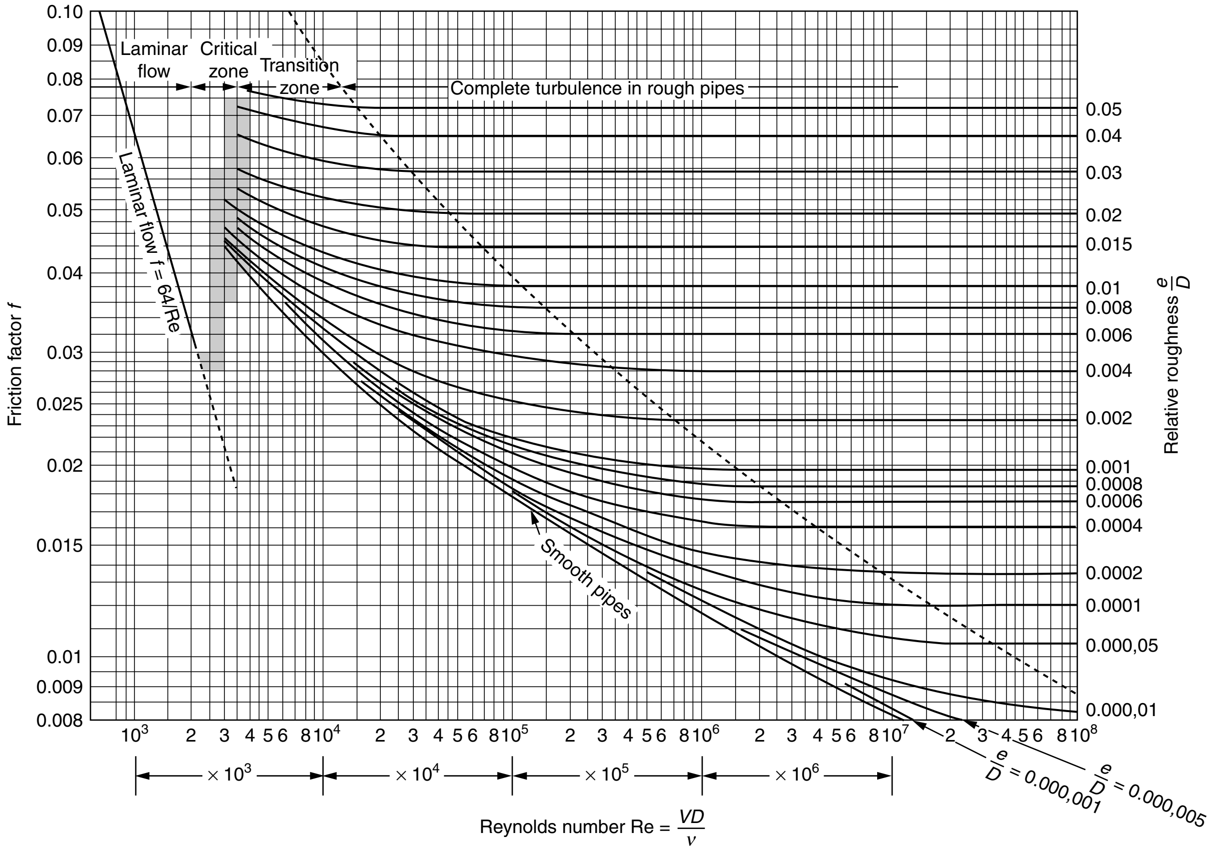


Figure 4.2 Moody diagram.

TABLE 4.5 Pipe Internal Roughness

Pipe material	Roughness in	Roughness mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

where f = Darcy friction factor
D = pipe inside diameter, in
e = absolute pipe roughness, in
Re = Reynolds number of flow, dimensionless

In SI units the friction factor equation is the same as Eq. (4.33), but with pipe diameter and absolute roughness of pipe both expressed in millimeters. The friction factor and Reynolds number are dimensionless and hence will remain the same.

It can be seen from Eq. (4.33) that the solution of friction factor f is not straightforward. This equation is implicit and therefore has to be solved by successive iteration. Once the friction factor is known, the pressure drop due to friction can be calculated using the Darcy equation (4.30).

Other formulas that have found popularity among engineers for friction loss calculations in steam pipes will be discussed next.

4.6.3 Unwin formula

The Unwin formula has been successfully used in steam piping calculations for many years. It is quite satisfactory for most purposes. However, at high flow rates, the pressure drops predicted by the Unwin formula are found to be higher than actual values. The Unwin formula in USCS units is as follows:

$$\Delta P = (3.625 \times 10^{-8}) v L W^2 \frac{1 + 3.6/D}{D^5} \tag{4.34}$$

where ΔP = pressure drop, psi
W = steam flow rate, lb/h
L = pipe length, ft
D = pipe inside diameter, in
v = specific volume, ft^3/lb

In SI units, the Unwin formula is as follows:

$$\Delta P = 675.2723 v L W^2 \frac{1 + 91.44/D}{D^5} \tag{4.35}$$

where ΔP = pressure drop, kPa
W = steam flow rate, kg/h
L = pipe length, m
D = pipe inside diameter, mm
v = specific volume, m^3/kg

4.6.4 Babcock formula

Another empirical equation for steam flow is the Babcock formula. It can also be used to calculate the pressure drop in steam piping. A version of the Babcock formula is as follows:

$$\Delta P = 0.47 \frac{D + 3.6}{D^6} w^2 L v \tag{4.36}$$

where ΔP = pressure drop, psig
D = pipe inside diameter, in
w = mass flow rate, lb/s
L = pipe length, ft
v = specific volume, ft^3/lb

Note that in Eq. (4.36), the steam flow rate is in lb/s, not in lb/h as in other equations discussed earlier.

In SI units the Babcock formula is

$$\Delta P = (8.755 \times 10^9) \frac{D + 3.6}{D^6} w^2 L v \tag{4.37}$$

where ΔP = pressure drop, kPa
D = pipe inside diameter, mm
w = mass flow rate, kg/s
L = pipe length, m
v = specific volume, m^3/kg

Several other pressure drop equations are used in steam piping calculations, including the Spitzglass and Fritzche formulas. Generally, because of their ease of use, charts are used to determine the pressure drop in steam piping. Thus a nomogram is available based on the Fritzche formula, and a chart using the Spitzglass formula is used for saturated steam calculations. Refer to *Standard Handbook of Engineering Calculations* by Tyler Hicks, McGraw-Hill, New York, 1995, for charts based on the Fritzche and Spitzglass formulas.

4.6.5 Fritzche's equation

This is another empirical equation for calculating pressure drop in steam piping. As indicated earlier, charts have been constructed based on this equation for quickly calculating the pressure drop. Fritzche's equation is as follows:

$$\Delta P = (2.1082 \times 10^{-7}) v L W^{1.85} \frac{1}{D^{4.97}} \tag{4.38}$$

where ΔP = frictional pressure loss, psi
v = specific volume of steam, ft^3/lb
L = pipe length, ft
W = steam flow rate, lb/h
D = pipe inside diameter, ft

In SI units Fritzche's equation is as follows:

$$\Delta P = 3165.38 v L W^{1.85} \frac{1}{D^{4.97}} \tag{4.39}$$

where ΔP = frictional pressure loss, kPa
v = specific volume of steam, m^3/kg
L = pipe length, m
W = steam flow rate, kg/h
D = pipe inside diameter, mm

Another equation that takes into account the compressibility of the steam, by using an expansion factor Y, is the modified Darcy formula applicable to steam and other compressible fluids. This equation is expressed as follows:

$$W = 1891\, Y d^2 \frac{\Delta P}{K v} \tag{4.40}$$

$$K = f \frac{L}{D} \tag{4.41}$$

where W = mass flow rate, lb/h
Y = expansion factor for pipe
D = pipe inside diameter, in
ΔP = pressure drop, psig
K = resistance coefficient
L = pipe length, ft
f = Darcy friction factor
v = specific volume of steam at inlet pressure, ft^3/lb

TABLE 4.6 Sonic Velocity Factors for $\gamma = 1.3$

K	$\Delta P/P_1$	Y
1.2	0.525	0.612
1.5	0.550	0.631
2.0	0.593	0.635
3.0	0.642	0.658
4.0	0.678	0.670
6.0	0.722	0.685
8.0	0.750	0.698
10.0	0.773	0.705
15.0	0.807	0.718
20.0	0.831	0.718
40.0	0.877	0.718
100.0	0.920	0.718

Using the equivalent length of valves and fittings, discussed in Sec. 4.9, the K values, of pipe, valves, and fittings may be calculated from Eq. (4.41) and added up to get the total value to be used in Eq. (4.40). The expansion factor Y must be found graphically or using a table. It depends on the specific heat ratio γ and the K value calculated for all pipe, valves, and fittings. Tables 4.6 and 4.7 list values to be used when sonic velocity occurs in pipes.

Example 4.9 Calculate the pressure drop in a 200-ft-long NPS 8 (0.250-in wall thickness) steam pipe flowing saturated steam at 50,000 lb/h. The initial pressure is 150 psia.

Solution From Table 4.1, at 150 psia, saturated steam has a specific volume of 3.015 ft^3/lb. The inside diameter of the pipe is

$$D = 8.625 - 2 \times 0.250 = 8.125 \text{ in}$$

TABLE 4.7 Sonic Velocity Factors for $\gamma = 1.4$

K	$\Delta P/P_1$	Y
1.2	0.552	0.588
1.5	0.576	0.606
2.0	0.612	0.622
3.0	0.662	0.639
4.0	0.697	0.649
6.0	0.737	0.671
8.0	0.762	0.685
10.0	0.784	0.695
15.0	0.818	0.702
20.0	0.839	0.710
40.0	0.883	0.710
100.0	0.926	0.710

Using the Unwin formula (4.34), we get the pressure drop as

$$\Delta P = 3.625 \times 10^{-8} \times 3.015 \times 200 \times (50{,}000)^2 \frac{(1 + 3.6/8.125)}{(8.125)^5} = 2.23 \text{ psi}$$

Therefore, the pressure drop is 2.23 psi.

Example 4.10 Steam flows through a 150-m-long DN 200 (6-mm wall thickness) pipe. If the steam velocity is limited to 40 m/s, what is the maximum flow rate permissible at an inlet pressure of 1000 kPa gauge? Calculate the pressure drop at this flow rate using the Unwin formula.

Solution At 1000 kPa, the specific volume of steam is found from Table 4.1 as follows:

$$1000 \text{ kPa gauge pressure} = 145 + 14.7 = 159.7 \text{ psia}$$

$$\text{Specific volume} = 2.839 \text{ ft}^3\text{/lb}$$

Therefore, the density is

$$\rho = \frac{1}{2.839} \times \frac{35.3147}{2.205} = 5.6413 \text{ kg/m}^3$$

The steam velocity is given by Eq. (4.24) as follows:

$$\text{Velocity} = \frac{\text{mass flow}}{\text{area} \times \text{density}}$$

The DN 200 (6-mm wall thickness) pipe has an inside diameter of

$$D = 200 - 2 \times 6 = 188 \text{ mm}$$

Limiting the velocity to 500 m/s, we get the mass flow rate as

$$W = 40 \times 0.7854 \times \left(\frac{188}{1000}\right)^2 \times 5.6413 = 6.26 \text{ kg/s} = 22{,}536 \text{ kg/h}$$

Next, using the Unwin formula, we get

$$\Delta P = 675.2723 \times \frac{1}{5.6413} \times 150 \times (22{,}536)^2 \frac{1 + 91.44/188}{(188)^5} = 57.71 \text{ kPa}$$

Therefore, the pressure drop is 57.71 kPa.

Example 4.11 A 50 ft-long, 2-in schedule 40 steam header pipe is flowing saturated steam at 200 psia. The piping includes two standard 90° elbows and a fully open globe valve. The exit pressure is atmospheric. Calculate the steam flow rate in lb/h using the Darcy equation.

Solution At 200 psia, from Table 4.1, we get

$$\text{Specific volume } v_s = 2.288 \text{ ft}^3\text{/lb}$$

We will use the K factor to account for the resistance in fittings, valves, and straight pipe. K is calculated from Eq. (4.41) for each component, such as

TABLE 4.8 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

pipe fittings, and added up to obtain the combined K factor. We will assume a friction factor of 0.02 since we do not know the Reynolds number as the flow rate is unknown.

For pipe,

$$K = 0.02 \times 50 \times \frac{12}{2.067} = 5.806$$

From a table of equivalent lengths of valves and fittings, Table 4.8, we get for two 90° elbows,

$$K = 2 \times 30 \times 0.02 = 1.2$$

and for one globe valve,

$$K = 340 \times 0.02 = 6.8$$

Adding one entrance loss of $K = 0.5$ and one exit loss of $K = 1.0$, we get

$$\text{Total } K \text{ for all components} = 5.806 + 1.2 + 6.8 + 1.5 = 15.31$$

$$\text{Pressure drop} = 200 - 14.7 = 185.3 \text{ psi}$$

$$\frac{\Delta P}{P_1} = \frac{185.3}{200} = 0.9265$$

For this pressure ratio, $\gamma = 1.3$, and $K = 15.31$, we get the maximum value of $\Delta P/P_1 = 0.81$ from Table 4.7. Since the actual pressure ratio is 0.9265, the sonic velocity exists at the pipe outlet. Therefore,

$$\Delta P = 0.81 \times 200 = 162 \text{ psi}$$

We also can obtain the expansion factor $Y = 0.718$ from Table 4.7. The steam flow rate can now be calculated from Eq. (4.40) as follows:

$$W = 1891 \times 0.718 \times (2.067)^2 \times \left(\frac{162}{15.31 \times 2.288}\right)^{0.5} = 12{,}475 \text{ lb/h}$$

4.7 Nozzles and Orifices

As steam flows through restrictions in a pipe, such as nozzles and orifices, the pressure drops and the velocity of flow increases. The required cross-sectional area of the nozzle will be based upon the properties of the steam, temperature, pressure, and mass flow rate. It has been found that for steam flow in nozzles, to handle a specific flow rate the shape of the nozzle must converge to a smaller diameter (known as a *throat*) and then increase in size. This is known as a convergent-divergent nozzle. If the divergent portion of the nozzle did not exist and the pressure P_2 at the discharge of the nozzle is decreased, keeping the inlet pressure P_1 fixed, the quantity of steam flowing through the nozzle will increase up to a point where P_2 reaches a critical pressure. A further decrease in P_2 will not increase the mass flow rate. The ratio of the critical pressure P_c to the inlet pressure P_1 is found to be a constant value that depends upon the specific heat ratio of steam. This ratio, known as the critical pressure ratio, is as follows:

$$\frac{P_c}{P_1} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)} \tag{4.42}$$

For saturated steam, $\gamma = 1.135$ and the critical pressure ratio becomes

$$\frac{P_c}{P_1} = 0.575 \tag{4.43}$$

For superheated steam, $\gamma = 1.3$ and the critical pressure ratio is

$$\frac{P_c}{P_1} = 0.545 \tag{4.44}$$

where P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
P_c = critical pressure, psia

Consider an orifice of area A_2 installed in a pipe of cross-sectional area A_1. If the upstream pressure is P_1 and the pressure at the orifice is P_2, then the mass flow rate is given by the following equation:

$$M = \frac{A_2}{\sqrt{1 - (P_2/P_1)^{2/\gamma}(A_2/A_1)^2}} \sqrt{\frac{2g\gamma}{\gamma - 1} P_1 \rho_1 \left[\left(\frac{P_2}{P_1}\right)^{2/\gamma} - \left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]} \tag{4.45}$$

where M = mass flow rate, lb/s
A_1 = upstream pipe cross-sectional area, ft^2
A_2 = nozzle throat area, ft^2
γ = ratio of specific heats of steam (usually 1.3), dimensionless
g = acceleration due to gravity, ft/s^2
ρ_1 = density of steam at upstream location, lb/ft^3
P_1 = upstream pressure, lb/ft^{2}absolute
P_2 = downstream pressure, lb/ft^{2}absolute

As steam flow approaches a smaller-diameter nozzle (see Fig. 4.3), the velocity increases and may equal the sonic velocity. At sonic velocity the Mach number (steam speed/sound speed) is 1.0. When this happens, the ratio of the pressure in nozzle P_2 to the upstream pressure P_1 is defined as the *critical pressure ratio*. This ratio is a function of the specific heat ratio γ of steam.

If the steam flow through the nozzle has not reached sonic velocity, the flow is termed *subsonic*. In this case the pressure ratio P_2/P_1 will be a larger number than the critical pressure ratio calculated from Eq. (4.42).

If the pressure drop ($P_1 - P_2$) increases such that the critical pressure ratio is reached, the flow through the nozzle will be sonic. The flow rate equation then becomes, after setting P_2/P_1 equal to the critical pressure

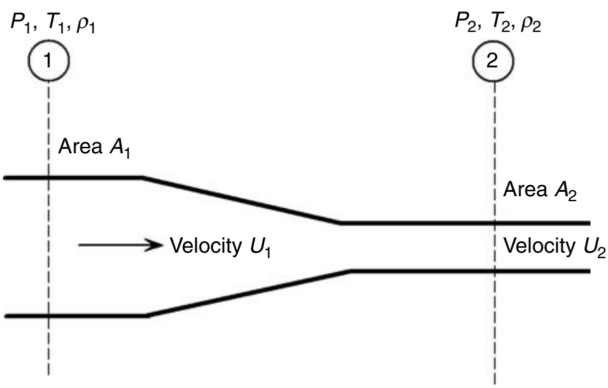


Figure 4.3 Steam flow through a restriction.

ratio from Eq. (4.42),

$$M = \frac{A_2 P_1}{\sqrt{T_1}} \sqrt{\frac{g\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} \tag{4.46}$$

A further increase in pressure drop causes the flow through the nozzle to remain sonic, and the pressure at the exit of the nozzle will increase. Even though the pressure drop has increased, there will be no change in mass flow rate. This is known as *choked flow*.

For pressure drops less than the critical ratio, the flow rate through a nozzle can also be calculated from

$$Q_1 = 31.5 C {D_2}^2 Y \left(\frac{\Delta P}{\rho_1}\right)^{0.5} \tag{4.47}$$

where Q_1 = upstream flow, ft^3/min
C = coefficient of discharge for nozzle, 0.94–0.96
D_1 = diameter of the upstream end of pipe
D_2 = diameter of throat
Y = expansion factor, depends on ratio of pressure P_2/P_1, ratio of diameters D_2/D_1, and specific heat ratio

Some values of Y are listed in Table 4.9. Equation (4.47) can also be used for orifices, but the coefficient of discharge C will range from 0.5 to 0.6.

TABLE 4.9 Expansion Factors for Nozzles

		Ratio of diameters, D_2/D_1				
Ratio of pressure P_2/P_1	k	0.30	0.40	0.50	0.60	0.70
0.95	1.40	0.973	0.972	0.971	0.968	0.962
	1.30	0.970	0.970	0.968	0.965	0.959
	1.20	0.968	0.967	0.966	0.963	0.956
0.90	1.40	0.944	0.943	0.941	0.935	0.925
	1.30	0.940	0.939	0.936	0.931	0.918
	1.20	0.935	0.933	0.931	0.925	0.912
0.85	1.40	0.915	0.914	0.910	0.902	0.887
	1.30	0.910	0.907	0.904	0.896	0.880
	1.20	0.902	0.900	0.896	0.887	0.870
0.80	1.40	0.886	0.884	0.880	0.868	0.850
	1.30	0.876	0.873	0.869	0.857	0.839
	1.20	0.866	0.864	0.859	0.848	0.829
0.75	1.40	0.856	0.853	0.846	0.836	0.814
	1.30	0.844	0.841	0.836	0.823	0.802
	1.20	0.820	0.818	0.812	0.798	0.776
0.70	1.40	0.824	0.820	0.815	0.800	0.778
	1.30	0.812	0.808	0.802	0.788	0.763
	1.20	0.794	0.791	0.784	0.770	0.745

For saturated steam when the back pressure past the orifice falls below the critical pressure, the flow rate depends upon the inlet pressure P_1 and the orifice area A_2. The mass flow rate w of saturated steam can then be calculated approximately using one of the following equations:

Napier's equation: $$w = \frac{P_1 \times A_2}{70} \tag{4.48}$$

Grashof equation: $$w = 0.0165 A_2 \times {P_1}^{0.97} \tag{4.49}$$

Rateau's equation: $$w = \frac{A_2 P_1 [16.367 - 0.96 \ \log_{10}(P_1)]}{1000} \tag{4.50}$$

where w = mass flow rate, lb/s
A_2 = orifice throat area, in^2
P_1 = upstream pressure, psia

The Grashof and Rateau's equations can be applied to well-rounded convergent orifices with a discharge coefficient of 1.0. For saturated steam calculation, for flow through convergent-divergent nozzles, the Grashof or Rateau equations may be used. When the back pressure P_2 is greater than the critical flow pressure P_c, the mass flow rate can be calculated from the general flow formula. Using steam tables or the Mollier chart we can determine the enthalpies H_1 and H_2 after the isentropic expansion. The velocity at throat U_2 is calculated from

$$U_2 = 223.7(H_1 - H_2)^{1/2} \tag{4.51}$$

where H_1 = enthalpy of steam at upstream location, Btu/lb
H_2 = enthalpy of steam at throat of nozzle, Btu/lb
U_2 = velocity of steam at throat of nozzle, ft/s

The mass flow rate is calculated from

$$W = \frac{A_2 U_2}{v_2} \tag{4.52}$$

where W = mass flow rate, lb/s
A_2 = throat area, ft^2
U_2 = velocity of steam at throat of nozzle, ft/s
v_2 = specific volume of steam at throat of nozzle, ft^3/lb

Example 4.12 Superheated steam at 400°F flows through a convergent-divergent nozzle that decreases in size from 2 in to 1 in at the throat.

(a) What is the mass flow rate of steam if the ratio of specific heat $\gamma = 1.3$, the pressure upstream is 160 psia, and the pressure at the throat is 120 psia?

(b) What is the maximum steam flow rate possible through this nozzle for critical pressure at the throat?

Solution

(a) At 160 psia, from Table 4.1 we get the following. The specific volume of superheated steam is

$$v_s = 3.008 \text{ ft}^3/\text{lb}$$

The cross-sectional area of the upstream section of pipe is

$$A_1 = 0.7854 \times \left(\frac{2}{12}\right)^2 = 0.0218 \text{ ft}^2$$

and the cross-sectional area at the nozzle throat is

$$A_2 = 0.7854 \times \left(\frac{1}{12}\right)^2 = 0.00545 \text{ ft}^2$$

Therefore, the ratio of the areas is

$$\frac{A_2}{A_1} = \frac{0.00545}{0.0218} = 0.25$$

The ratio of throat pressure to upstream pressure is

$$\frac{P_2}{P_1} = \frac{120}{160} = 0.75$$

For superheated steam, from Eq. (4.42), the critical pressure ratio is

$$\frac{P_c}{P_1} = 0.545$$

Therefore, the critical pressure ratio has not been reached.

Next we will calculate the various ratios needed in Eq. (4.45) for calculating the mass flow rate through the nozzle:

$$\frac{\gamma}{\gamma - 1} = \frac{1.3}{0.3} = 4.33$$

$$\frac{2}{\gamma} = \frac{2}{1.3} = 1.5385$$

$$\frac{\gamma + 1}{\gamma} = \frac{2.3}{1.3} = 1.7692$$

The mass flow rate can now be calculated from Eq. (4.45) as follows:

$$M = \frac{0.00545}{\sqrt{1 - (0.75)^{1.5385}(0.25)^2}} \times \sqrt{\frac{2 \times 32.2 \times 4.33}{3.008} \times 160 \times 144[(0.75)^{1.5385} - (0.75)^{1.7692}]}$$

$$= 1.6853 \text{ lb/s} = 1.6853 \times 3600 = 6067 \text{ lb/h}$$

Therefore, the mass flow rate of steam is 6067 lb/h.

(b) When the pressure at the throat reaches the critical value $P_c/P_1 = 0.545$, then using the ideal gas equation,

$$RT_1 = \frac{P}{\rho_1}$$

Therefore,

$$RT_1 = 160 \times 144 \times 3.008 = 69,304.32$$

The mass flow rate can be calculated by substituting values into Eq. (4.46):

$$M = 0.00545 \times 160 \times 144 \sqrt{\frac{32.2 \times 1.3}{69,304.32} \left(\frac{2}{2.3}\right)^{7.6667}}$$
$$= 1.806 \text{ lb/s} = 6502 \text{ lb/h}$$

Therefore, the maximum flow rate of steam at the critical pressure condition at the throat is 6502 lb/h.

Example 4.13 A saturated steam piping (200-mm inside diameter) operates at an inlet pressure of 1400 kPa absolute.

(a) What is the maximum flow rate if the velocity of the steam is limited to 1200 m/min?

(b) Calculate the pressure loss in a 200-m length of pipe. Use the Darcy equation with a friction factor of 0.02.

(c) What is the sonic velocity limit in this pipe?

Solution

(a) Converting kilopascals to psia,

$$1400 \text{ kPa} = 1400 \times 0.145 = 203 \text{ psia}$$

From Table 4.1, dry saturated steam has a specific volume of

$$v_s = 2.28 \text{ ft}^3\text{/lb} = 0.1424 \text{ m}^3\text{/kg}$$

The mass flow rate is

$$W = \frac{\text{area} \times \text{velocity}}{\text{specific volume}} = \frac{0.7854 \times (0.200)^2 \times 1200}{0.1424} = 264.7 \text{ kg/min}$$
$$= 15{,}844 \text{ kg/h}$$

Therefore, to limit the velocity to 1200 m/min, the steam flow rate must not exceed 15,844 kg/h.

(b) Using the Darcy equation (4.31), the pressure loss is

$$\Delta P = 62{,}511 \times 0.02 \times 200 \times 0.1424 \times \frac{(15{,}844)^2}{(200)^5} = 27.93 \text{ kPa}$$

Therefore, the pressure drop is 27.93 kPa.

(c) The sonic velocity limit is given by

$$U_s = \sqrt{\gamma g P v_s} = \sqrt{1.135 \times 9.81 \times 1400 \times 0.1424}$$
$$= 47.11 \text{ m/s} = 2827 \text{ m/min}$$

where we have used $g = 1.135$ for saturated steam. Therefore the sonic velocity limit is 2827 m/min.

Example 4.14 Steam at a flow rate of 20,000 lb/h is expanded in a convergent-divergent nozzle from an initial pressure of 300 psia at 700°F to a final pressure of 100 psia. Assuming the nozzle efficiency is 92 percent, calculate the areas of the exit and the throat. What inlet area would be required if the velocity of approach cannot exceed 90 ft/s?

Solution From the Mollier diagram (Fig. 4.1), at 300 psia and 700°F, the specific volume, enthalpy, and entropy are as follows:

$$v_1 = 2227 \text{ ft}^3/\text{lb}$$
$$h_1 = 1368.3 \text{ Btu/lb}$$
$$s_1 = 1.6751 \text{ Btu/lb R}$$

Drawing a vertical line for the isentropic process to 100 psia, the enthalpy for the Mollier diagram is $h_2 = 1250$ Btu/lb. The velocity at the outlet is

$$U_2 = \sqrt{90^2 + 2 \times 32.2 \times 778 \times 0.92(1368.3 - 1250)} = 2337 \text{ ft/s}$$

The actual enthalpy at the nozzle exit is calculated using the nozzle efficiency of 92 percent as

$$h_2 = 1368.3 - 0.92(1368.3 - 1250) = 1259.5 \text{ Btu/lb}$$

From Table 4.1 at 100 psia and above enthalpy h_2, the specific volume of steam, by interpolation, is

$$v_2 = 5.34 \text{ ft}^3/\text{lb}$$

The nozzle exit area is then, using the mass flow equation,

$$A_2 = \frac{20{,}000 \times 5.34}{2337 \times 3600} = 0.0127 \text{ ft}^2$$

To determine the throat area, assuming the critical pressure ratio is reached for superheated steam,

$$P_c = 0.55 \times 300 = 165 \text{ psia}$$

From the Mollier diagram, expansion to this pressure results in an enthalpy of

$$h_c = 1290 \text{ Btu/lb}$$

Applying the same nozzle efficiency of 92 percent, enthalpy at the throat is

$$h_t = 1368.3 - 0.92(1368.3 - 1290) = 1296.3 \text{ Btu/lb}$$

From Table 4.1, the specific volume for 165 psia and the obtained enthalpy h_t is, by interpolation,

$$v_t = 3.523 \text{ ft}^3/\text{lb}$$

The velocity at the throat is

$$V_t = 223.7\sqrt{1368.3 - 1296.3} = 1898.2 \text{ ft/s}$$

The area of the throat is

$$A_t = \frac{20{,}000}{3600} \times \frac{3.523}{1898.2} = 0.0103 \text{ ft}^2 = 1.48 \text{ in}^2$$

The inlet area required is

$$A_1 = \frac{20{,}000}{3600} \times \frac{2.227}{90} = 0.1375 \text{ ft}^2 = 19.8 \text{ in}^2$$

Example 4.15 Determine the pipe size required for 22,000 kg/h of saturated steam flowing at an inlet pressure of 1100 kPa absolute, if the pressure drop is limited to 20 percent in a 200-m length of pipe.

Solution From Table 4.1 at 1100 kPa $= 1100 \times 0.145 = 159.5$ psia, the specific volume of dry saturated steam is

$$v_s = 2.834 \text{ ft}^3/\text{lb} = 0.177 \text{ m}^3/\text{kg}$$

Using Unwin's equation 4.35, and letting pressure drop be $0.2 \times 1100 = 220$ kPa,

$$\Delta P = 220 = 675.2723 \times 0.177 \times 200 \times (22{,}000)^2 \frac{(1 + 91.44)/D}{D^5}$$

This equation will be solved for diameter d by successive iteration. First we will neglect the term $91.44/d$ and calculate a first approximation for the diameter as

$$D = 18.56 \text{ mm}$$

Substituting this value of d in the neglected term and recalculating the diameter we get

$$D = 26.49 \text{ mm}$$

Repeating the process a few more times we get a final value of diameter as

$$D = 25.21 \text{ mm}$$

Therefore, the pipe size required is 25.21-mm inside diameter.

4.8 Pipe Wall Thickness

The pipe wall thickness required to withstand the maximum operating pressure in a steel pipe is calculated using the ASME B31.1 Code for Pressure Piping as follows:

$$t = \frac{DP}{2(S + YP)} + C \tag{4.53}$$

where t = pipe wall thickness, in
D = pipe outside diameter, in
P = internal pressure, psig
S = allowable stress in pipe material, psig
Y = temperature coefficient
C = end condition factor, in

Values of S, Y, and C are taken from the ASME Code for Pressure Piping, Boiler and Pressure Vessel Code, and Code for Pressure Piping. For example, for a seamless Ferritic steel pipe with a 55,000-psi minimum tensile strength, the allowable pipe stress at 850°F is 13,150 psi.

Example 4.16 Calculate the pipe wall thickness required in an 8-in steel pipe used for steam at 900°F and 800 psig pressure. Assume allowable stress = 12,500 psi, $Y = 0.4$, and $C = 0.065$ in.

Solution Using Eq. (4.53), the pipe wall thickness required is

$$t = 8.625 \times \frac{800}{2(12{,}500 + 0.4 \times 800)} + 0.065 = 0.3341 \text{ in}$$

Allowing 12.5 percent for manufacturing tolerance, the wall thickness required = 0.3341 × 1.125 = 0.3759 in.

Example 4.17 Calculate the pressure loss in 500 ft of 4-in schedule 40 steel pipe used for conveying 300°F superheated steam at 10,000 lb/h and 60 psia pressure.

Solution The specific volume at 60 psia and 300°F from Table 4.1 is

$$v_s = 7.259 \text{ ft}^3/\text{lb}$$

From Unwin's formula

$$\Delta P = 3.625 \times 10^{-8} \times 7.259 \times 500 \times (10{,}000)^2 \frac{1 + 3.6/4.026}{(4.026)^5} = 23.56 \text{ psi}$$

Therefore, the pressure loss in 500 ft of pipe is 23.56 psi.

4.9 Determining Pipe Size

To calculate the size of pipe required to transport a given quantity of steam through a piping system we must take into account the initial pressure of the steam at the source and the total pressure drop allowable through the piping system. The velocity of steam affects noise and therefore is also an important consideration. Tables are available to use as a guide for pressure drops in steam piping such as shown in Table 4.10. As an example, if the initial steam pressure is 100 psig, the pressure drop recommended per 100 ft of pipe is 2 to 5 psi, and a total pressure drop in the steam supply piping should range between 15 and 25 psi. Charts are available from various HVAC sources that may be used for sizing steam piping and calculating pressure drops and velocities at different steam flow rates.

In the previous sections, we introduced several formulas and tables to calculate the pressure drop in steam piping. Based on allowable steam velocities, the mass flow rate of steam is calculated. Next for the specified flow rate and allowable pressure drop a suitable pipe size is calculated using one of the Unwin, Darcy, or Fritzsche equations.

Example 4.18 A steam piping system transports 20,000 lb/h of dry saturated steam at 150 psia. If the velocity is limited to 3000 ft/min, what size pipe is required? Calculate the pressure loss due to friction in 500 ft of pipe using the Unwin and Darcy equations, and compare the answers obtained.

Solution At 150 psia, from Table 4.1, the specific volume of saturated steam is

$$v_s = 3.015 \text{ ft}^3/\text{lb}$$

TABLE 4.10 Pressure Drops in Steam Piping

Initial steam pressure, psig	Pressure drop per 100 ft	Total pressure drop in steam supply piping, psi
Subatmosphere	2–4 oz	1–2
0	0.5 oz	1
1	2 oz	1–4
2	2 oz	8
5	4 oz	1.5
10	8 oz	3
15	1 psi	4
30	2 psi	5–10
50	2–5 psi	10–15
100	2–5 psi	15–25
150	2–10 psi	25–30

The mass flow rate and velocity are related by Eq. (4.24). Therefore,

$$\frac{20{,}000}{60} = \text{Area} \times 3000 \times \frac{1}{3.015}$$

$$\text{Area required} = \frac{20,000 \times 3.015}{60 \times 3000} = 0.335 \text{ ft}^2$$

If the pipe inside diameter is D inches,

$$0.7854\left(\frac{D}{12}\right)^2 = 0.335$$

Solving for D, we get

$$D = 7.84 \text{ in}$$

The pressure loss due to friction per the Unwin formula is by Eq. (4.34).

$$\Delta P = 3.625 \times 10^{-8} \times 3.015 \times 500 \times (20{,}000)^2 \frac{(1 + 3.6/7.84)}{(7.84)^5} = 1.077 \text{ psi}$$

At the given conditions, the steam viscosity = 0.015 cP and the Reynolds number is

$$\text{Re} = \frac{6.31 \times 20{,}000}{0.015 \times 7.84} = 1.07 \times 10^6$$

From the Moody diagram $f = 0.0155$. Using the Darcy equation (4.30), we get

$$\Delta P = 3.3557 \times 10^{-6} \times 0.0155 \times 3.015 \times 500 \times \frac{20{,}000^2}{7.84^5} = 1.06 \text{ psi}$$

It can be seen from the calculations that the Unwin and Darcy eqations give close results.

4.10 Valves and Fittings

Valves of various types such as gate valves, globe valves, and check valves are used on steam piping systems to isolate piping and to provide connections to equipment. Gate valves are normally used in instances where the valve needs to be fully open or fully closed. For throttling purposes globe valves may be used. Check valves are used to prevent backflow such as on steam-feed lines. Control valves are used to provide pressure reduction to protect low-pressure equipment. Relief valves are installed to prevent overpressuring and rupture of piping and connected equipment. Safety and relief valves are designed in accordance with ASME codes.

Pressure loss through valves and fittings may be accounted for by using an equivalent length or resistance coefficient K. Table 4.8 lists the equivalent lengths and K factors for commonly used valves and fittings.

It can be seen from Table 4.8 that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore a 10-in-diameter gate valve may be replaced with a $10 \times 8 = 80$-in-long piece of pipe that will match the frictional pressure drop through the valve.

Example 4.19 A piping system is 2000 ft of NPS 20 pipe that has two 20-in gate valves, three 20-in ball valves, one swing check valve, and four 90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe, valves, and fittings.

Solution Using Table 4.8, we can convert all valves and fittings in terms of 20-in pipe as follows,

$$\begin{aligned}
\text{Two 20-in gate valves} &= 2 \times 20 \times 8 = 320 \text{ in of 20-in pipe} \\
\text{Three 20-in ball valves} &= 3 \times 20 \times 3 = 180 \text{ in of 20-in pipe} \\
\text{One 20-in swing check valve} &= 1 \times 20 \times 50 = 1000 \text{ in of 20-in pipe} \\
\text{Four } 90^\circ \text{ elbows} &= 4 \times 20 \times 30 = 2400 \text{ in of 20-in pipe} \\
\text{Total for all valves and fittings} &= 4220 \text{ in of 20 in-pipe} \\
&= 351.67 \text{ ft of 20-in pipe}
\end{aligned}$$

Adding the 2000 ft of straight pipe, the total equivalent length of straight pipe and all fittings is

$$L_e = 2000 + 351.67 = 2351.67 \text{ ft}$$

The pressure drop due to friction in this piping system can now be calculated based on 2351.67 ft of pipe. It can be seen in this example that the valves and fittings represent roughly 15 percent of the total pipeline length.

4.10.1 Minor losses

Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation the pressure drop in a straight length of pipe is given by

$$h = f \frac{L}{D} \frac{U^2}{2g} \tag{4.54}$$

The term f (L/D) may be substituted with a head loss coefficient K (also known as the resistance coefficient), and Eq. (4.54) then becomes

$$h = K\frac{U^2}{2g} \tag{4.55}$$

In Eq. (4.55), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $U^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(U^2/2g)$, where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 4.8. It can be seen that the K factor depends on the nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

From Table 4.8 it can be seen that a 6-in gate valve has a K factor of 0.12, while a 20-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be 0.12 $(U^2/2g)$ and that in the 20-in valve is 0.10 $(U^2/2g)$. The velocities in both cases will be different due to the difference in diameters.

4.10.2 Pipe enlargement and reduction

Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(U_1 - U_2)^2}{2g} \tag{4.56}$$

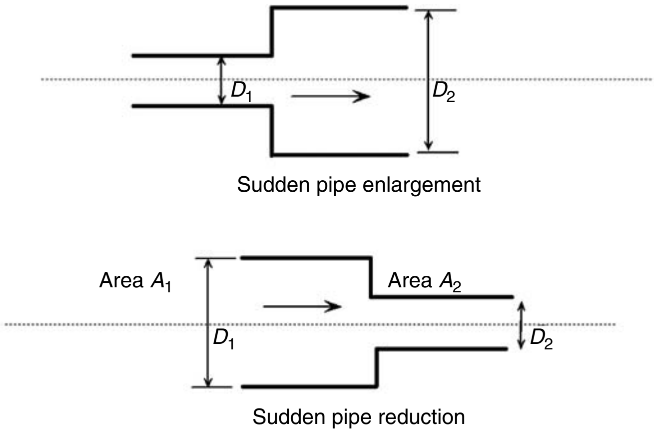
where U_1 and U_2 are the velocities of the liquid in the two pipe sizes D_1 and D_2, respectively. Writing Eq. (4.56) in terms of pipe cross-sectional areas A_1 and A_2,

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{{U_1}^2}{2g} \tag{4.57}$$

for sudden enlargement. This is illustrated in Fig. 4.4.

For sudden contraction or reduction in pipe size as shown in Fig. 4.4, the head loss is calculated from

$$h_f = \left(\frac{1}{C_c} - 1\right) \frac{{U_2}^2}{2g} \tag{4.58}$$



A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 4.4 Sudden pipe enlargement and pipe reduction.

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 4.4.

Gradual enlargement and reduction of pipe size, as shown in Fig. 4.5, cause less head loss than sudden enlargement and sudden reduction. For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c(U_1 - U_2)^2}{2g} \tag{4.59}$$

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is given in Fig. 4.6.

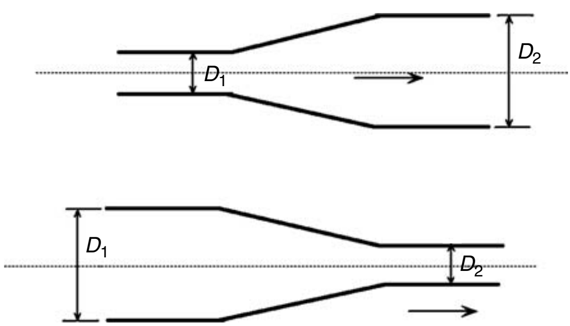


Figure 4.5 Gradual pipe enlargement and pipe reduction.

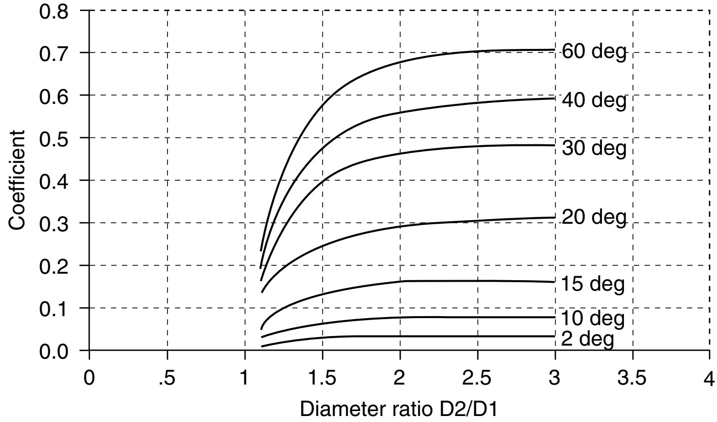


Figure 4.6 Gradual pipe expansion head loss coefficient.

4.10.3 Pipe entrance and exit losses

The K factors for computing the head loss associated with pipe entrance and exit are as follows:

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

Chapter

5

Compressed-Air Systems Piping

Introduction

Compressed air is clean and easily available. As an energy source it can be put to use in many different forms. However, the cost of producing compressed air must be compared against that of other forms of energy such as electricity. For several decades, despite the advent of new energy services, compressed air–driven equipment and tools have continued to be used in many industrial applications. In addition, the efficiency of these systems has increased in recent years.

Compressed air is used in food processing, material handling, and the operation of machines and tools. In plants that use compressed air the pressures range from 60 to 150 pounds per square inch (lb/in^2 or psi). Low-pressure compressed air, in the range of 10 to 25 psi, is used for the control of instruments. Low-pressure air is also used in heating, ventilating, and air-conditioning (HVAC) systems. Portable air compressors are used in construction, road building, painting, etc. The flow rates used in these applications range from 20 to 1500 cubic feet per minute (ft^3/min or CFM) with power ranging from 2 to 400 horsepower (HP).

5.1 Properties of Air

Air consists of approximately 78 percent nitrogen and 21 percent oxygen and small amounts of other gases such as argon, CO_2, and helium. Generally, for most calculations the composition of air is assumed to be 79 percent nitrogen and 21 percent oxygen on a volumetric basis. The corresponding values on a weight basis are 76.8 percent nitrogen and 23.2 percent oxygen. Air has a molecular weight of 28.97.

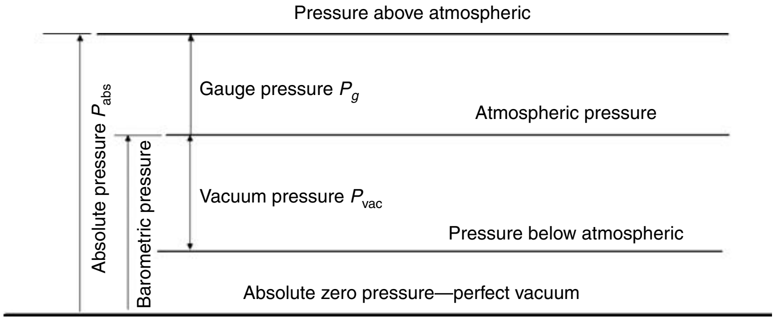


Figure 5.1 Absolute pressure and gauge pressure.

The gas constant R for air is 53.33 (ft · lb)/(lb · °R) [29.2 (N · m)/(N · K) in SI units].

In most instances, example problems are discussed in English units, also called U.S. Customary (USCS) units. However, Système International (SI) units are also illustrated in some examples.

The pressure of air in a vessel or pipe may be expressed as gauge pressure or absolute pressure. The gauge pressure, denoted by psig, is that which is measured by a pressure gauge or instrument that records the magnitude of pressure above the atmospheric pressure at a particular location. The absolute pressure, denoted by psia, includes the local atmospheric pressure. This is illustrated in Fig. 5.1. Mathematically, the gauge pressure and absolute pressure are related by the following equation:

$$\text{Absolute pressure} = \text{gauge pressure} + \text{atmospheric pressure}$$

All calculations involving air such as the perfect gas laws require knowledge of the local atmospheric pressure. The pressure drop due to friction, which represents the difference between the absolute pressure at two points along a compressed air pipeline, is expressed in psig. This is because the common pressure representing the atmospheric pressure cancels out when the downstream pressure is subtracted from the upstream pressure. Thus, if we denote the upstream pressure as P_1 in psia and the downstream pressure as P_2 in psia, the pressure loss is simply $P_1 - P_2$, measured in psig. Although sometimes pressure differences are indicated in absolute terms, gauge pressures are more appropriate.

In SI units, the pressures are measured in kilopascals (kPa) or megapascals (MPa), and we must clearly state whether the pressure is in absolute or gauge values. In USCS units, the psig and psia designations are self-explanatory. Other SI units for pressure are bar and kg/cm^2.

For many calculations air may be considered a perfect gas and, therefore, said to obey Boyle's law, Charles's law, and the ideal gas equation.

However, at high pressures the behavior of compressed air deviates from that of ideal gas, and hence compressibility effects must be considered.

Considering the perfect gas equation of state, we can calculate the density of air ρ at the standard conditions of 14.7 psia and 60°F as follows:

$$\frac{P}{\rho} = RT = \frac{1545}{\text{Mw}} T \tag{5.1}$$

where P = pressure, lb/ft^2
ρ = air density, lb/ft^3
R = gas constant for air
T = air temperature, °R (°F +460)
Mw = molecular weight of air, equal to 28.97
1545 = universal gas constant

In some books you will see the specific weight of air γ used instead of the density ρ. We will use the mass density ρ in this chapter. Care must be taken to use proper conversion factors to ensure that correct units are maintained.

Sometimes mass is expressed in slugs in USCS units. The unit of pound (lb) is reserved for force, including weight. Since it is more common to talk about mass flow rate (or weight flow rate) of air in lb/s or lb/min, we will use lb for mass throughout this chapter when using USCS units. In this regard, the mass flow and weight flow rates are interchangeable. Strictly speaking, mass is a scalar quantity while weight is a vector quantity, like force. Numerically 1 lb mass and 1 lb weight will be considered equal.

The mass flow rate of air in SI units may be expressed in kg/s, kg/min, kilonewtons/s (kN/s), or kN/min, even though the newton is actually defined as the force that is necessary to accelerate a mass of 1 kg at the rate of 1 m/s^2.

Standard conditions are an atmospheric pressure of 14.7 psia and a temperature of 60°F. Substituting these temperature and pressure values and the molecular weight of air into Eq. (5.1), we calculate the density of air at standard conditions (also referred to as base conditions) as

$$\rho = \frac{14.7 \times 144 \times 28.97}{1545 \times (460 + 60)} = 0.07633 \text{ lb/ft}^3$$

Thus, dry air has a density of 0.07633 lb/ft^3 at standard conditions (14.7 psia and 60°F). In SI units the base temperature and pressure used are 0°C and 760 mm pressure (1.033 kg/cm^2). Sometimes 15°C and 101 kPa are also used.

Even though temperatures are normally reported in °F or °C, calculations require that these temperatures be converted to absolute scale. In USCS units we use the absolute temperature scale of Rankine. In SI units the absolute temperature is denoted by the kelvin scale. The conversion from the ordinary temperatures of °F and °C to absolute scales are as follows:

$$°\mathrm{R} = °\mathrm{F} + 460$$

and

$$\mathrm{K} = °\mathrm{C} + 273$$

The temperature in kelvin is usually given without the degree symbol. Thus 60°F is 520°R and 20°C is 293 K.

The pressure of air may be expressed in psi in USCS units. To ensure proper units, the pressure in psi is multiplied by 144 to result in lb/ft^2 pressure as can be seen in the earlier calculation of the density of air using Eq. (5.1). In SI units, pressure may be expressed in kilopascals, megapascals, or bars.

The *critical temperature* is defined as the temperature above which, regardless of the pressure, a gas cannot be compressed into the liquid state. The *critical pressure* is defined as the least pressure at the critical temperature necessary to liquefy a gas. The critical temperature and critical pressure of air are −221°F and 546 psia, respectively. In comparison with a critical pressure and temperature, atmospheric air may be assumed to obey the perfect gas law fairly accurately.

The specific heat of air at constant pressure C_p is approximately 0.239 Btu/(lb · °R) at temperatures up to 400°R. The ratio of specific heat for air C_p/C_v is approximately 1.4. It is found that as temperature increases, C_p increases and the specific heat ratio denoted by k decreases. At 60°F, $C_p = 0.24$ and $k = 1.4$. Air tables (Tables 5.1 to 5.4) are used in calculations involving expansion and compression of air.

TABLE 5.1 Properties of Air for Temperatures in °F

Temperature, °F	Density, slug/ft^3	Specific weight, lb/ft^3	Kinematic viscosity, ft^2/s	Dynamic viscosity, (lb · s)/ft^2
0.0	0.00268	0.0862	12.6×10^{-5}	3.28×10^{-7}
20.0	0.00257	0.0827	13.6×10^{-5}	3.50×10^{-7}
40.0	0.00247	0.0794	14.6×10^{-5}	3.62×10^{-7}
60.0	0.00237	0.0764	15.8×10^{-5}	3.74×10^{-7}
68.0	0.00233	0.0752	16.0×10^{-5}	3.75×10^{-7}
80.0	0.00228	0.0736	16.9×10^{-5}	3.85×10^{-7}
100.0	0.00220	0.0709	18.0×10^{-5}	3.96×10^{-7}
120.0	0.00215	0.0684	18.9×10^{-5}	4.07×10^{-7}

TABLE 5.2 Properties of Air for Temperatures in °C

Temperature, °C	Density, kg/m^3	Specific weight, N/m^3	Kinematic viscosity, m^2/s	Dynamic viscosity, $N \cdot s/m^2$
0.0	1.29	12.7	13.3×10^{-6}	1.72×10^{-5}
10.0	1.25	12.2	14.2×10^{-6}	1.77×10^{-5}
20.0	1.20	11.8	15.1×10^{-6}	1.81×10^{-5}
30.0	1.16	11.4	16.0×10^{-6}	1.86×10^{-5}
40.0	1.13	11.0	16.9×10^{-6}	1.91×10^{-5}
50.0	1.09	10.7	17.9×10^{-6}	1.95×10^{-5}
60.0	1.06	10.4	18.9×10^{-6}	1.99×10^{-5}
70.0	1.03	10.1	19.9×10^{-6}	2.04×10^{-5}
80.0	1.00	9.8	20.9×10^{-6}	2.09×10^{-5}
90.0	0.972	9.53	21.9×10^{-6}	2.19×10^{-5}
100.0	0.946	9.28	23.0×10^{-6}	2.30×10^{-5}

TABLE 5.3 Correction Factor for Altitude

Altitude		
ft	m	Correction factor
0	0	1.00
1600	480	1.05
3300	990	1.11
5000	1500	1.17
6600	1980	1.24
8200	2460	1.31
9900	2970	1.39

TABLE 5.4 Correction Factor for Temperature

Temperature of intake		
°F	°C	Correction factor
−50	−46	0.773
−40	−40	0.792
−30	−34	0.811
−20	−28	0.830
−10	−23	0.849
0	−18	0.867
10	−9	0.886
20	−5	0.905
30	−1	0.925
40	4	0.943
50	10	0.962
60	18	0.981
70	22	1.000
80	27	1.019
90	32	1.038
100	38	1.057
110	43	1.076
120	49	1.095

The viscosity of air, like that of other gases, increases with a rise in temperature. At 40°F the viscosity of air is approximately 3.62×10^{-7} (lb · s)ft^2; at 240°F the viscosity increases to 4.68×10^{-7} (lb · s)/ft^2. The variation of viscosity of dry low-pressure air with temperature is listed in Tables 5.1 and 5.2. Although the viscosity variation is nonlinear for most calculations, we could use an average viscosity based on interpolation of values from the tables.

In many industrial processes we encounter air mixed with vapor. In the field of air-conditioning, air is mixed with water vapor. If we assume that each constituent obeys the perfect gas law, we can use Dalton's law of partial pressure in the calculations. Dalton's law of partial pressures states that in a mixture of gases, the pressure exerted by each gas is equal to the pressure that it would exert if it alone occupied the volume occupied by the gas mixture. Also the total pressure exerted by the mixture is equal to the sum of the pressures exerted by each component gas. The pressure exerted by each component is known as its partial pressure.

5.1.1 Relative humidity

Relative humidity is defined as the ratio of the actual vapor pressure to that of the saturated vapor at the current dry bulb temperature. The *dry bulb temperature* is the temperature of the atmospheric air measured by an ordinary thermometer. When the atmospheric air is cooled under constant total pressure, condensation of vapor occurs at a specific temperature. This temperature of condensation is called the *dew point*. It is the same as the saturation temperature or boiling point at the actual vapor pressure. When a thermometer bulb is covered with some absorbent material that is moistened with distilled water and exposed to atmospheric air, evaporation occurs from the moist cover that will cool the water and the bulb and the temperature will drop to the wet bulb temperature. Generally the *wet bulb temperature* is the temperature between the extremes of the dew point and the dry bulb temperature. The three temperatures, dew point, wet bulb temperature, and dry bulb temperature coincide when the air is saturated. Since atmospheric air is a mixture of air and water vapor, Dalton's law of partial pressures may be applied. The total atmospheric pressure P_t, also known as the *barometric pressure*, is composed of the vapor pressure of water and the air pressure as follows:

$$P_t = P_v + P_a \tag{5.2}$$

where P_t = total atmospheric pressure, psia
P_v = vapor pressure of water vapor, psia
P_a = air pressure, psia

Three vapor pressures correspond to the three temperatures previously discussed. At the dew point the vapor pressure P_v, called the actual vapor pressure, is used in calculations. At the dry bulb and wet bulb temperatures, the vapor pressures P_d and P_w, respectively, are used. Relative humidity is thus defined as

$$\mathrm{RH} = \frac{P_v}{P_d} \tag{5.3}$$

For all practical purposes the ratio of the vapor pressures may be replaced with the ratio of the vapor density:

$$\mathrm{RH} = \frac{\rho_v}{\rho_d} \tag{5.4}$$

5.1.2 Humidity ratio

The *humidity ratio*, also known as the specific humidity, is defined as the mass of water vapor per pound of air. Since the molecular weight of air is 28.97 and that of water is 18.0,

$$\text{Ratio of molecular weights} = \frac{28.97}{18.0} = 1.609$$

The humidity ratio can than be expressed using the relative humidity definition (5.3) as follows:

$$\text{Humidity ratio} = \frac{P_v}{1.609 P_a} \tag{5.5}$$

If the air density is represented by ρ_a and vapor density by ρ_v, then the density of the mixture is

$$\rho_m = \rho_a + \rho_v \tag{5.6}$$

5.2 Fans, Blowers, and Compressors

The pressure necessary to compress air and move it through pipes and equipment must be provided by some pressure-creating device such as a fan, blower, or compressor. The classification of these various devices is based on the pressure level that is produced. For small pressures, up to 2 psi, used in HVAC systems, fans are the most suitable. For pressures between 2 and 10 psi, blowers are used. For higher pressures, in hundreds or thousands of psi, compressors are used.

Several designs of fans, blowers, and compressors exist for specific applications. Propeller fans, duct fans, and centrifugal fans are used to circulate the air within a space or move air through ducts in HVAC systems. Fans are driven by electric motors and may deliver up to 50,000 CFM at low static pressure.

Centrifugal blowers are used for intermediate pressures and flow rates. These consist of a rotor with rotating blades that impart kinetic energy to the air and a mechanism that collects the air and discharges it through a duct system.

Centrifugal compressors are used for higher flow rates and pressures and may be driven by engines or turbines. The pressure is created by the conversion of kinetic energy due to centrifugal force. Larger pressures are created by employing multiple stages of compressor elements. Positive displacement compressors are also used to produce the necessary pressure in a compressed-air system. These include reciprocating and rotary compressors.

Example 5.1 The static pressure in a heating duct is measured as 4.5 inches of water (inH_2O). What is this pressure in psi?

Solution Using Eq. (5.11),

$$4.5 \text{ inH}_2\text{O} = \frac{4.5}{12} \times \frac{1}{2.31} = 0.162 \text{ psi}$$

5.3 Flow of Compressed Air

5.3.1 Free air, standard air, and actual air

Free air (also called standard air) represents the volume of air measured under standard conditions. As stated in Sec. 5.1 in USCS units, standard conditions are defined as a temperature of 60°F and an atmospheric pressure of 14.7 psia. In SI units 0°C and 101.3 kPa absolute pressure are used. The actual air volume, or flow rate, is defined as that volume at actual operating conditions of temperature and pressure. We can convert the volume of standard air, or free air, to that of actual air by using the perfect gas law equation.

$$\frac{PV}{T} = \text{constant} \tag{5.7}$$

Thus,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \tag{5.8}$$

where P_1, P_2 = pressure at initial and final conditions, respectively, psia

V_1, V_2 = volume at initial and final conditions, respectively, ft^3

T_1, T_2 = temperature at initial and final conditions, respectively, °R

Using subscript a for actual conditions and s for standard conditions,

$$\frac{P_a V_a}{T_a} = \frac{P_s V_s}{T_s} \tag{5.9}$$

Therefore,

$$V_a = V_s \frac{T_a}{T_s} \frac{P_s}{P_a} \tag{5.10}$$

Using the 60°F and 14.7 psia standard conditions, we get

$$V_a = V_s \frac{t_a + 460}{60 + 460} \frac{14.7}{P_a}$$

where t_a is the actual air temperature (°F) and P_a is the actual air pressure (psia). Remember that P_a is in absolute pressure and therefore includes the local atmospheric pressure.

When pressures are small, they are expressed in inches of water column (inH_2O). The head pressure due to a column of water can be converted to pressure in psi using the following equation:

$$\text{Pressure in psi} = \frac{\text{head of water in inches}}{2.31 \times 12} = 0.03608 \times h \tag{5.11}$$

where h represents the pressure in inches of water.

The factor 2.31 in Eq. (5.11) is simply the ratio 144/62.4 where the density of water is used as 62.4 lb/ft^3. For example a 2-in water column is equal to a pressure of

$$0.03608 \times 2 = 0.072 \text{ psi}$$

In many formulas in this chapter the pressure drop may be expressed in psi or sometimes in feet of head. Knowing the density of the flowing fluid and using Eq. (5.11) we can easily convert from feet of head to pressures in psi.

Example 5.2 A fan is rated at 5000 CFM at a static pressure of 0.75 inH_2O. Convert this in terms of SI units of flow rate (m^3/s) and pressure (Pa).

Solution

$$5000 \text{ CFM} = \frac{5000 \times (0.3048)^3}{60} = 2.36 \text{ m}^3/\text{s}$$

$$0.75 \text{ inH}_2\text{O} = \frac{0.75}{12} \times \frac{1}{2.31} = 0.02706 \text{ psi}$$

$$\frac{0.02706}{0.145} = 0.1866 \text{ kPa}$$

$$= 186.6 \text{ Pa}$$

Example 5.3 A compressor is used to pump dry air through a pipeline at 150 psig and a flow temperature of 75°F. The compressor is rated at 600 standard ft^3/min (SCFM). Calculate the airflow rate under actual conditions in actual ft^3/min (ACFM).

Solution Here we have 600 ft^3/min air at 14.7 psia and 60°F (standard conditions). We need to calculate the corresponding volume flow rate at the actual conditions of 150 psig and 75°F.

Using Eq. (5.10) and assuming the local atmospheric pressure is 14.7 psia, we get

$$V_a = 600 \times \frac{75 + 460}{60 + 460} \frac{14.7}{150 + 14.7} = 55.1 \text{ ft}^3/\text{min} \quad \text{or} \quad 55.1 \text{ ACFM}$$

It can be seen that the volume of air is drastically reduced at the higher pressure, even though the temperature is slightly higher than standard conditions.

Example 5.4 The flow rate of air at 21°C and a pressure of 700 kPa gauge is 100 m^3/h. What is the volume flow rate of free air at standard conditions (0°C and 101.3 kPa)? Assume the atmospheric pressure is 102 kPa.

Solution Substituting in Eq. (5.10), we get

$$100 = V_s \frac{21 + 273}{0 + 273} \frac{101.3}{700 + 102}$$

Solving for the standard volume flow rate

$$V_s = 100 \times \frac{273}{294} \frac{802}{101.3} = 735.16 \text{ m}^3/\text{h}$$

It must be noted that the standard pressure condition is 101.3 kPa, while the local atmospheric pressure is 102 kPa.

Airflow is expressed in terms of standard ft^3/min (SCFM) or standard ft^3/h, and in SI units as cubic meters per hour (m^3/h). This implies that the flow rate is measured at the standard conditions of 14.7 psia pressure and 60°F temperature. As seen from previous discussions, the flow rate at other temperatures and pressures will be different from that at standard conditions. If Q_1 represents the airflow at pressure P_1 and temperature T_1 corresponding to a standard volume of Q_s at standard pressure P_s and standard temperature T_s, using the perfect gas equation, we can state that

$$\frac{P_1 Q_1}{T_1} = \frac{P_s Q_s}{T_s} \tag{5.12}$$

This is similar to Eq. (5.9).

Sometimes we are interested in the mass flow rate of air. If the density of air is ρ, the mass flow rate can be calculated from

$$M = Q_s \times \rho_s \tag{5.13}$$

where M = mass flow rate, lb/h
Q_s = standard volume flow rate, ft^3/h
ρ_s = density of air, lb/ft^3

If the density of air is assumed to be 0.07633 lb/ft^3 at standard conditions, the mass flow rate corresponding to a volume flow rate of 1000 ft^3/min (SCFM) is

$$M = 1000 \times 0.07633 = 76.33 \text{ lb/min}$$

Since mass does not change with pressure or temperature, due to the law of conservation of mass, the mass flow rate defined in Eq. (5.13) can really be applied to any other pressure and temperature conditions. Therefore the mass flow rate at some condition represented by subscript 1 may be written as $M = Q_1 \times \rho_1$, where Q_1 and ρ_1 may correspond to the actual conditions of flow rate and density of air at some other temperature and pressure than that of the standard conditions.

Let's return to the recent example of air that flows at 1000 SCFM at 60°F and 14.7 psia, where the mass flow rate is 76.33 lb/min. When the actual condition of the air changes to 75°F and 100 psig pressure, the actual volume flow rate can be calculated from Eq. (5.10) as follows:

$$V_a = 1000 \times \frac{14.7}{114.7} \times \frac{75 + 460}{60 + 460} = 131.86 \text{ ft}^3\text{/min}$$

However, at these new conditions (75°F and 100 psig) the mass flow rate will still be the same: 76.33 lb/min. Because of the constancy of the mass flow rate we can state that

$$M = Q_s \times \rho_s = Q_1 \times \rho_1 = Q_2 \times \rho_2, \text{ etc.}$$

where the subscript s stands for standard conditions and subscripts 1 and 2 refer to two other conditions.

In flow through piping and nozzles, the preceding equation representing the conservation of mass flow rate will be used quite often.

Example 5.5 A compressor delivers 2900 CFM of free air. If the air flows through a pipe at an inlet pressure of 60 psig and a temperature of 90°F, what is the flow rate of air at actual conditions?

Solution Using Eq. (5.10),

$$V_a = V_s \frac{T_a}{T_s} \frac{P_s}{P_a} = 2900 \frac{90 + 460}{60 + 460} \frac{14.7}{60 + 14.7} = 603.6 \text{ CFM}$$

Example 5.6 Consider air at 70°F and 100 psig pressure to be an ideal gas. Calculate the specific weight of this air in lb/ft^3. The atmospheric pressure is 14.7 psia.

Solution Rearranging Eq. (5.1),

$$\frac{P}{\rho} = RT$$

we get

$$\rho = \frac{P}{RT} = \frac{(100 + 14.7) \times 144}{53.3 \times (460 + 70)} = 0.5847 \text{ lb/ft}^3$$

Example 5.7 Calculate the density of air in N/m^3, if the pressure is 700 kPa gauge and the temperature is 25°C. The atmospheric pressure is 101.3 kPa.

Solution Rearranging Eq. (5.1),

$$\frac{P}{\rho} = RT$$

we get

$$\rho = \frac{P}{RT} = \frac{(700 + 101.3) \times 10^3}{29.2 \times (273 + 25)} = 92.09 \text{ N/m}^3$$

5.3.2 Isothermal flow

Isothermal flow occurs at constant temperature. Thus the pressure, volume, and density of air change, but temperature remains the same. To maintain the constant temperature isothermal flow of air requires heat to be transferred out of the air. Compressed air flowing in long pipes can be analyzed considering isothermal flow. Under isothermal flow, the pressure, flow rate, and temperature of air flowing through a pipe are related by the following equation:

$$P_1^2 - P_2^2 = \frac{M^2 RT}{gA^2} \left(f \frac{L}{D} + 2 \log_e \frac{P_1}{P_2} \right) \tag{5.14}$$

where P_1 = upstream pressure at point 1, psia
P_2 = downstream pressure at point 2, psia
M = mass flow rate, lb/s
R = gas constant
T = absolute temperature of air, °R
g = acceleration due to gravity, ft/s^2

A = cross-sectional area of pipe, ft^2
f = friction factor, dimensionless
L = pipe length, ft
D = inside diameter of pipe, ft

Equation (5.14) can be used for small pressure drops and when elevation differences between points along the pipe are ignored. The friction factor f used in Eq. (5.14) will be discussed in detail in Sec. 5.4.

A consistent set of units must be used in Eq. (5.14). An example will illustrate the use of the isothermal flow equation.

Example 5.8 Air flows at 50 ft/s through a 2-in inside diameter pipe at 80°F at an initial pressure of 100 psig. If the pipe is horizontal and 1000 ft long, calculate the pressure drop considering isothermal flow. Use a friction factor $f = 0.02$.

Solution First calculate the density of air at 80°F. From Table 5.1

$$\text{Density at } 80^\circ\text{F} = 0.0736 \text{ lb/ft}^3$$

This density is at the standard condition of 14.7 psia. Using Eq. (5.1) we calculate the density at 100 psig as

$$\rho = \frac{100 + 14.7}{14.7} \times 0.0736 = 0.5743 \text{ lb/ft}^3$$

The cross-sectional area of the pipe is

$$A = 0.7854 \times \left(\frac{2}{12}\right)^2 = 0.0218 \text{ ft}^2$$

Next, the mass flow rate can be calculated from the density, velocity, and the pipe cross-sectional area using Eq. (5.13) as follows:

$$M = \rho A v = 0.5743 \times 0.0218 \times 50 = 0.6265 \text{ lb/s}$$

Using Eq. (5.14) we can write

$$[(100 + 14.7)^2 - P_2^{\,2}] \times (144)^2 = (0.6265)^2 \times 53.3 \times (80 + 460) \times \frac{(0.02 \times 1000 \times 12/2) + [2 \log_e(114.7/P_2)]}{32.2 \times 0.0218 \times 0.0218}$$

Simplifying we get

$$13{,}156.09 - P_2^{\,2} = 35.6016\left(120.0 + 2\log_e \frac{114.7}{P_2}\right)$$

We will first calculate P_2 by ignoring the second term containing P_2 on the right-hand side of the equation. This is acceptable since the term being ignored is a much smaller value compared to the first term 120.0 within the parentheses.

Therefore the first approximation to P_2 is calculated from

$$13{,}156.09 - P_2^{\,2} = 35.6016 \times 120$$

or

$$P_2 = 94.25 \text{ psia}$$

We can recalculate a better solution for P_2 by substituting the value just calculated in the preceding equation, this time including the $\log_e (114.7/P_2)$ term:

$$13{,}156.09 - P_2^{\,2} = 35.6016 \times \left(120 + 2\log_e \frac{114.7}{94.25}\right)$$

Solving for P_2 we get

$$P_2 = 94.18 \text{ psia}$$

which is quite close to our first approximation of $P_2 = 94.25$. Therefore

$$\text{Pressure drop} = P_1 - P_2 = 114.7 - 94.18 = 20.52 \text{ psig}$$

Example 5.9 Air flows through a 2000-ft-long NPS 6 pipeline at an initial pressure of 150 psig and a temperature of 80°F. If the flow is considered isothermal, calculate the pressure drop due to friction at a flow rate of 5000 SCFM. Assume smooth pipe.

Solution We start by calculating the Reynolds number (discussed in Sec. 5.4) from the flow rate. Assume a 6-inch inside diameter pipe.

$$\text{Area of cross section } A = 0.7854\left(\frac{6}{12}\right)^2 = 0.1964 \text{ ft}^2$$

$$\text{Velocity } v = \frac{\text{flow rate}}{\text{area}} = \frac{5000}{60 \times 0.1964} = 424.3 \text{ ft/s}$$

Next we need to find the density and viscosity of air at 80°F and 150 psig pressure. From Table 5.1, at 80°F

$$\text{Density } \rho = 0.0736 \text{ lb/ft}^3 \text{ at 14.7 psia}$$

and

$$\text{Viscosity } \mu = 3.85 \times 10^{-7} \text{ (lb} \cdot \text{s)/ft}^2$$

The density must be corrected for the higher pressure of 150 psig:

$$\rho = 0.0736 \times \frac{164.7}{14.7} = 0.8246 \text{ lb/ft}^3 \text{ at 150 psig}$$

The Reynolds number from Eq. (5.18) is

$$\text{Re} = \frac{424.3 \times 0.5 \times 0.8246}{32.2 \times 3.85 \times 10^{-7}} = 1.41 \times 10^7$$

From the Moody diagram (Fig. 5.2), for smooth pipe, the friction factor is

$$f = 0.0077$$

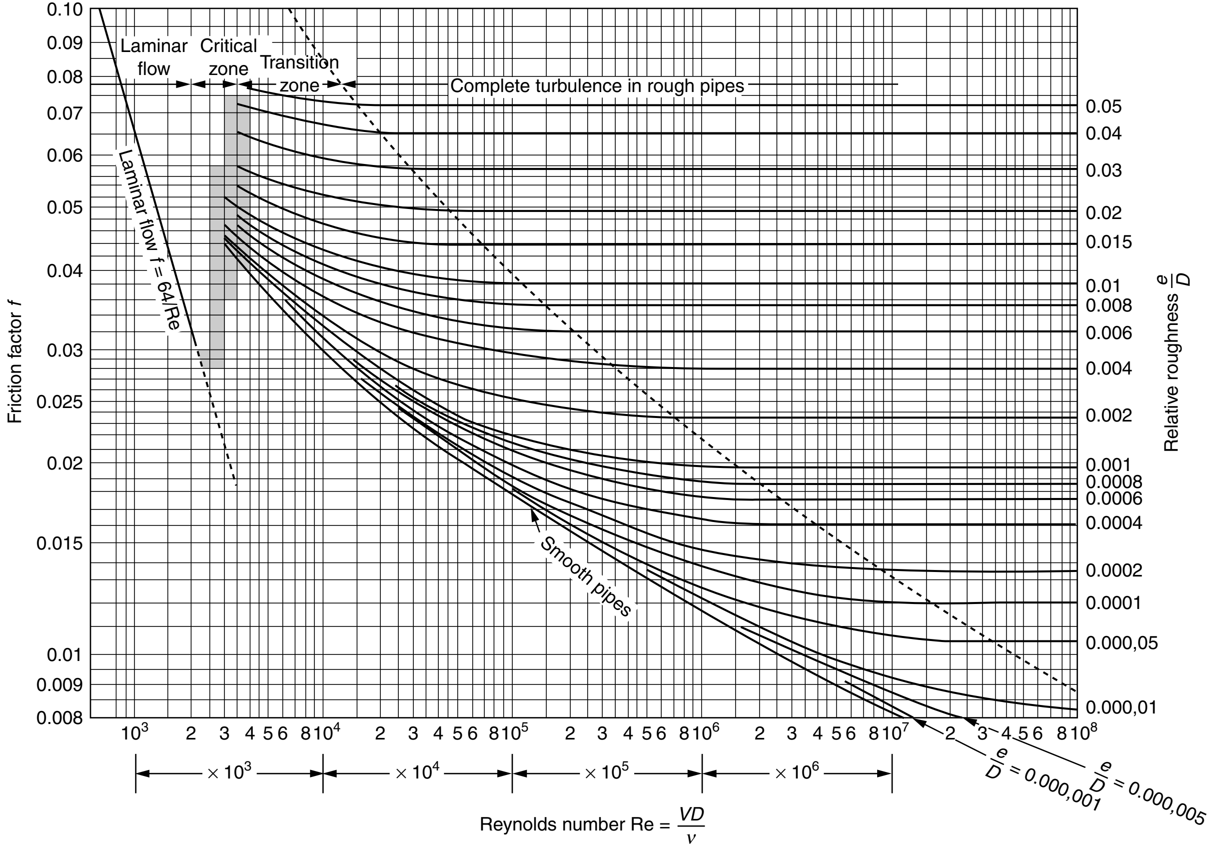


Figure 5.2 Moody diagram.

The mass flow rate will be calculated first from the given volume flow rate.

$$M = \text{volume rate} \times \text{density}$$

From Table 5.1 for density of air at 60°F (standard condition),

$$\text{Density} = 0.0764 \text{ lb/ft}^3$$

Therefore the mass flow rate is

$$M = 5000 \times 0.0764 = 382 \text{ lb/min} = 6.367 \text{ lb/s}$$

Using Eq. (5.14) for isothermal flow,

$$\left[(164.7)^2 - P_2^{\,2}\right] \times (144)^2 = \frac{(6.367)^2 \times 53.3 \times 540}{32.2 \times (0.1964)^2} \times \left(0.0077 \times \frac{2000}{0.5} + 2\log_e \frac{164.7}{P_2}\right)$$

This equation for P_2 must be solved by trial and error. Solving we get $P_2 =$ 160.4 psia. Thus

$$\text{Pressure drop due to friction} = (P_1 - P_2) = 164.7 - 160.4 = 4.3 \text{ psi}$$

Example 5.10 Air flows through a 500-m-long, 200-mm inside diameter pipeline at 20°C. The upstream and downstream pressures are 1035 and 900 kPa, respectively. Calculate the flow rate through the pipeline assuming isothermal conditions. Pressures are in absolute values, and the relative roughness of pipe is 0.0003.

Solution We will use the isothermal equation (5.14) for calculating the flow rate through the pipeline. The friction factor f depends on the Reynolds number which in turn depends on the flow rate which is unknown. Therefore, we will assume an initial value of the friction factor f and calculate the mass flow rate from Eq. (5.14). This mass flow rate will then be used to calculate the flow velocity and hence the corresponding Reynolds number. From this Reynolds number using the Moody diagram the friction factor will be found. The mass flow rate will be recalculated from the newly found friction factor. The process is continued until successive values of the mass flow rate are within 1 percent or less.

Assume $f = 0.01$ initially; from Eq. (5.14) we get,

$$(1035)^2 - (900)^2 = \frac{M^2 \times 29.3 \times 293}{9.81 \times (0.7854 \times 0.04)^2} \left(0.01 \times \frac{500}{0.2} + 2\log_e \frac{1035}{900}\right)$$

Solving for M, we get

$$M = 0.108 \text{ kN/s}$$

Next, calculate the density at 20°C from the perfect gas equation:

$$\rho = \frac{P}{RT} = \frac{1035}{29.3 \times 293} = 0.1206 \text{ kN/m}^3$$

The viscosity of air from Table 5.1 is

$$\mu = 1.81 \times 10^{-5}\ \mathrm{Pa \cdot s}$$

The flow velocity is calculated from the mass flow rate as follows:

$$M = \rho A v$$

Therefore,

$$0.108 = 0.1206 \times (0.7854 \times 0.04)v$$

Thus, velocity is

$$v = 28.505\ \mathrm{m/s}$$

The Reynolds number is calculated from Eq. (5.18) as

$$\begin{aligned}\mathrm{Re} &= \frac{0.1206}{9.81} \times 28.505 \times \frac{0.2}{1.81 \times 10^{-8}} \\ &= 3.87 \times 10^{6}\end{aligned}$$

For this Reynolds number, from the Moody diagram we get the friction factor for a relative roughness $(e/D) = 0.0003$ as

$$f = 0.0151$$

Using this value of f, we recalculate the mass flow rate as follows:

$$(1035)^2 - (900)^2 = \frac{M^2 \times 29.3 \times 293}{9.81 \times (0.7854 \times 0.04)^2}\left(0.0151 \times \frac{500}{0.2} + 2\log_e \frac{1035}{900}\right)$$

Solving for M, we get

$$M = 0.088\ \mathrm{kN/s}$$

The earlier value was $M = 0.108$ kN/s. This represents a 22 percent difference, and hence we must recalculate the friction factor and repeat the process for a better value of M.

Based on the recently calculated value of $M = 0.088$ we will recalculate the velocity and Reynolds number as follows. Using proportions, the new velocity is

$$v = \frac{0.088}{0.108} \times 28.505 = 23.226\ \mathrm{m/s}$$

The new Reynolds number is

$$\mathrm{Re} = \frac{23.226}{28.505} \times 3.87 \times 10^{6} = 3.15 \times 10^{6}$$

Next from the Moody diagram for the preceding Reynolds number we get a friction factor

$$f = 0.0152$$

Using this value of f in the isothermal flow equation we get a new value of mass flow rate as follows:

$$(1035)^2 - (900)^2 = \frac{M^2 \times 29.3 \times 293}{9.81 \times (0.7854 \times 0.04)^2}\left(0.0152 \times \frac{500}{0.2} + 2\log_e \frac{1035}{900}\right)$$

Solving for M, we get

$$M = 0.0877 \text{ kN/s}$$

The earlier value was $M = 0.088$ kN/s.

This represents a difference of 0.34 percent and hence we can stop iterating any further. The flow rate through the pipeline is 0.0877 kN/s.

Example 5.11 Air flows through a 1500-ft-long, NPS 10 (0.25-in wall thickness) pipeline, at a mass flow rate of 23 lb/s. What pressure is required at the upstream end to provide a delivery pressure of 80 psig? The airflow temperature is 80°F. Consider isothermal flow. Assume the friction factor is 0.02.

Solution The mass flow rate is $M = 23.0$ lb/s and the friction factor is $f = 0.02$. The cross-sectional area of pipe, with 10.75-in outside diameter and 0.25-in wall thickness, is

$$A = 0.7854\left(\frac{10.25}{12}\right)^2 = 0.573 \text{ ft}^2$$

From the isothermal flow equation (5.14), substituting the given values, we get

$$\left[P_1{}^2 - (94.7)^2\right] \times (144)^2 = \frac{23^2 \times 53.3 \times 540}{32.2 \times (0.573)^2}\left(0.02 \times \frac{1500 \times 12}{10.25} + 2\log_e \frac{P_1}{94.7}\right)$$

Assume $P_1 = 100$ psig initially and substitute this value on the right-hand side of the preceding equation to calculate the next approximation for P_1. Continue this process until successive values of P_1 are within 1 percent or less. Solving we get $P_1 = 106.93$ psia by successive iteration. Therefore the upstream pressure required is $106.93 - 14.7 = 92.23$ psig. The pressure loss in the 1500-ft-long pipe is $92.23 - 80 = 12.23$ psi.

Example 5.12 Consider isothermal flow of air in a 6-inch inside diameter pipe at 60°F. The upstream and downstream pressures for a 500-ft section of horizontal length of pipe are 80 and 60 psia, respectively. Calculate the mass flow rate of air assuming the pipe is smooth.

Solution From Eq. (5.14) for isothermal flow, we get

$$P_1^2 - P_2^2 = \frac{M^2 RT}{gA^2}\left(f\frac{L}{D} + 2\log_e \frac{P_1}{P_2}\right)$$

We must first calculate the Reynolds number Re and the friction factor f. Since Re depends on the flow rate (unknown), we will assume a value of f and calculate the flow rate from the preceding equation. We will then verify if the assumed f was correct. Some adjustment may be needed in the f value to get convergence.

Assume $f = 0.01$ in the preceding pressure drop equation. Substituting the given value, we get

$$(144)^2(80^2 - 60^2) = \frac{M^2 \times 53.3 \times 520}{32.2(0.7854 \times 0.5 \times 0.5)^2}\left(0.01\frac{500}{0.5} + 2\log_e\frac{80}{60}\right)$$

Solving for the mass flow rate,

$$M = 15.68 \text{ lb/s}$$

The gas density is

$$\rho = \frac{P}{RT} = \frac{80 \times 144}{53.3 \times 520} = 0.4156 \text{ lb/ft}^3$$

The mass flow rate is then calculated from Eq. (5.13),

$$\text{Mass flow} = \text{density} \times \text{volume flow rate} = \text{density} \times \text{area} \times \text{velocity}$$

Therefore,

$$M = \rho A v$$

Substituting the calculated values in Eq. (5.13), we get

$$15.68 = (0.4156)(0.7854 \times 0.5 \times 0.5)v$$

$$\text{Flow velocity } v = 192.15 \text{ ft/s}$$

The Reynolds number is then

$$\text{Re} = \frac{\rho d v}{\mu} = \frac{0.4156}{32.2}(0.5)\frac{192.15}{3.78 \times 10^{-7}}$$
$$= 3.28 \times 10^6$$

From the Moody diagram (Fig. 5.2), the Darcy friction factor $f = 0.0096$. We assumed $f = 0.01$ initially. This is quite close to the newly calculated value of f. If we use the value of $f = 0.0096$ and recalculate the mass flow rate, we get

$$M = 15.99 \text{ lb/s}$$

5.3.3 Adiabatic flow

Adiabatic flow of air occurs when there is no heat transfer between the flowing air and its surroundings. Adiabatic flow generally includes friction. When friction is neglected, the flow becomes isentropic.

5.3.4 Isentropic flow

When air flows through a conduit such that it is adiabatic and frictionless, the flow is termed *isentropic flow*. This type of flow also means that the entropy of the air is constant. If the flow occurs very quickly such that heat transfer does not occur and the friction is small, the flow may be considered isentropic. In reality, high-velocity flow occurring over short lengths of pipe with low friction and low heat transfer may be characterized as isentropic flow. The pressure drop that occurs in isentropic flow can be calculated from the following equation:

$$\frac{v_2^2 - v_1^2}{2g} = \frac{P_1}{\rho_1}\frac{k}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{(k-1)/k}\right] \tag{5.15}$$

or

$$\frac{v_2^2 - v_1^2}{2g} = \frac{P_2}{\rho_2}\frac{k}{k-1}\left[\left(\frac{P_1}{P_2}\right)^{(k-1)/k} - 1\right] \tag{5.16}$$

where v_1 = velocity at upstream location
v_2 = velocity at downstream location
P_1 = pressure at upstream location
P_2 = pressure at downstream location
k = specific heat ratio
g = acceleration of gravity
ρ_1 = density at upstream location
ρ_2 = density at downstream location

It can be seen from Eqs. (5.15) and (5.16) that the pressure drop $P_1 - P_2$ between the upstream and downstream locations in a pipe depends only on the pressures, velocities, and specific heat ratio of air. Unlike isothermal flow, discussed earlier, no frictional term exists in the isentropic flow equation. This is because, by definition, isentropic flow is considered to be a frictionless process.

Example 5.13 Isentropic flow of air occurs in a 6-inch inside diameter pipeline. If the upstream pressure and temperature are 50 psig and 70°F and the velocity of air at the upstream and downstream locations are 50 and 120 ft/s, respectively, calculate the pressure drop assuming $k = 1.4$.

Solution We will use Eq. (5.15) for isentropic flow of air. First let us calculate the ratio $k/(k-1)$ and its reciprocal.

$$\frac{k}{k-1} = \frac{1.4}{0.4} = 3.5$$

$$\frac{k-1}{k} = \frac{0.4}{1.4} = 0.2857$$

The term P_1/ρ_1, in Eq. (5.15) may be replaced with the term RT_1 using the perfect gas equation (5.1). Substituting the given values in Eq. (5.15) we get

$$\frac{(120)^2-(50)^2}{2\times 32.2}=53.3\times(70+460)\times 3.5\times\left[1-\left(\frac{P_2}{150+14.7}\right)^{0.2857}\right]$$

Simplifying and solving for P_2 we get

$$P_2 = 163.63 \text{ psia}$$

Therefore the pressure drop is

$$P_1 - P_2 = 164.7 - 163.63 = 1.07 \text{ psig}$$

5.4 Pressure Drop in Piping

The pressure drop due to friction for air flowing through pipes is generally calculated using one of the many formulas or empirical correlations. Charts have also been developed to approximately estimate the friction loss in compressed-air piping based on pipe size, pipe diameter, inlet pressure, flow temperature, and properties of air. These charts are shown in Tables 5.5 through 5.7. These tables list the friction loss in psi per 100 ft of pipe for 50 psi, 100 psi, and 125 psi, respectively. Table 5.8 lists typical pipe sizes for different flow rates.

Various formulas are also available to calculate the pressure drop, mass flow rate, and volume flow rate for specified pipe sizes. These will be discussed next.

5.4.1 Darcy equation

For both compressible fluids (such as air and other gases) and incompressible fluids (all liquids), the classical pressure drop formula, known as the Darcy-Weisbach equation or sometimes simply the Darcy equation, may be used. The Darcy equation is expressed as follows:

$$h_f = f\frac{L}{D}\frac{v^2}{2g} \tag{5.17}$$

where h_f = friction loss, ft of head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = pipe inside diameter, ft
v = flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

TABLE 5.5 Pressure Drop in psi/100 ft at a 50-psi Inlet Pressure

Flow rate, CFM (Standard conditions)	Pipe size (NPS)										
	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	4	5	6
1	0.006										
2	0.024	0.006									
3	0.055	0.012									
4	0.098	0.022	0.006								
5	0.153	0.034	0.009								
6	0.220	0.050	0.013								
8	0.391	0.088	0.023	0.006							
10	0.611	0.138	0.036	0.009							
15	1.374	0.310	0.082	0.020	0.009						
20	2.443	0.551	0.146	0.035	0.016						
25	3.617	0.861	0.227	0.055	0.024	0.007					
30	5.497	1.240	0.328	0.079	0.035	0.010					
35		1.688	0.446	0.108	0.047	0.013	0.005				
40		2.205	0.582	0.141	0.062	0.017	0.007				
45		2.791	0.737	0.178	0.078	0.021	0.009				
50		3.445	0.910	0.220	0.097	0.026	0.011				
60		4.961	1.310	0.317	0.140	0.038	0.016	0.005			
70			1.783	0.432	0.190	0.052	0.021	0.007			
80			2.329	0.564	0.248	0.068	0.028	0.009			
90			2.948	0.713	0.314	0.086	0.035	0.011			
100			3.639	0.881	0.388	0.106	0.044	0.014			
125			5.686	1.376	0.606	0.165	0.068	0.022			
150				1.982	0.872	0.238	0.098	0.031	0.007		
175				2.697	1.187	0.324	0.133	0.043	0.010		
200				3.523	1.550	0.423	0.174	0.056	0.013		
225				4.459	1.962	0.536	0.220	0.070	0.016		
250				5.505	2.423	0.662	0.272	0.087	0.020	0.006	
275					2.931	0.801	0.329	0.105	0.024	0.007	
300					3.489	0.953	0.392	0.125	0.029	0.009	
325					4.094	1.118	0.460	0.147	0.034	0.010	
350					4.748	1.297	0.533	0.17	0.039	0.012	
375					5.451	1.489	0.612	0.195	0.045	0.014	0.005
400					6.202	1.694	0.696	0.222	0.051	0.015	0.006
425						1.912	0.786	0.251	0.057	0.017	0.007
450						2.144	0.881	0.281	0.064	0.019	0.008
475						2.388	0.982	0.313	0.072	0.022	0.009
500						2.464	1.088	0.347	0.079	0.024	0.010
550						3.202	1.317	0.420	0.096	0.029	0.012
600						3.811	1.567	0.500	0.114	0.035	0.014
650						4.473	1.839	0.587	0.134	0.041	0.016

It must be noted that the Darcy equation (5.17) gives the head loss due to friction in terms of feet of head not psig. It needs to be converted to psig using the density of air at the flowing temperature.

The Darcy friction factor f in Eq. (5.17) must be calculated based on the dimensionless parameter known as Reynolds number of flow.

TABLE 5.6 Pressure Drop in psi/100 ft at a 100-psi Inlet Pressure

Flow rate, CFM (Standard conditions)	Pipe size (NPS)									
	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	4	5
1										
2	0.014									
3	0.031									
4	0.055	0.012								
5	0.086	0.019								
6	0.124	0.028								
8	0.220	0.050	0.013							
10	0.345	0.078	0.021							
15	0.775	0.175	0.046	0.011						
20	1.378	0.311	0.082	0.020						
25	2.153	0.486	0.128	0.031	0.014					
30	3.101	0.700	0.185	0.045	0.020					
35	4.220	0.952	0.251	0.061	0.027					
40	5.512	1.244	0.328	0.079	0.035					
45	6.976	1.574	0.416	0.101	0.044	0.012				
50	8.613	1.943	0.513	0.124	0.055	0.015				
60	12.402	2.799	0.739	0.179	0.079	0.021				
70		3.809	1.006	0.243	0.107	0.029	0.012			
80		4.975	1.314	0.318	0.14	0.038	0.016			
90		6.297	1.663	0.402	0.177	0.048	0.020			
100		7.774	2.053	0.497	0.219	0.060	0.025			
125		12.147	3.207	0.776	0.342	0.093	0.038	0.012		
150			4.619	1.118	0.492	0.134	0.055	0.018		
175			6.287	1.522	0.67	0.183	0.075	0.024		
200			8.211	1.987	0.875	0.239	0.098	0.031		
225			10.392	2.515	1.107	0.302	0.124	0.040		
250			12.830	3.105	1.367	0.373	0.153	0.049	0.011	
275				3.757	1.654	0.452	0.186	0.059	0.014	
300				4.471	1.968	0.537	0.221	0.071	0.016	
325				5.248	2.309	0.631	0.259	0.083	0.019	
350				6.086	2.678	0.731	0.301	0.096	0.022	
375				6.987	3.075	0.84	0.345	0.110	0.025	
400				7.949	3.498	0.955	0.393	0.125	0.029	
425				8.974	3.949	1.079	0.443	0.142	0.032	
450				10.061	4.428	1.209	0.497	0.159	0.036	0.011
475				11.210	4.933	1.347	0.554	0.177	0.040	0.012
500				12.421	5.466	1.493	0.614	0.196	0.045	0.014
550					6.614	1.806	0.743	0.237	0.054	0.016
600					7.871	2.150	0.884	0.282	0.064	0.020
650					9.238	2.523	1.037	0.331	0.076	0.023

The Reynolds number depends on the flow velocity, pipe size, and properties of air and is defined as

$$\mathrm{Re} = \frac{vD\rho}{\mu} \tag{5.18}$$

TABLE 5.7 Pressure Drop in psi/100 ft at a 125-psi Inlet Pressure

Flow rate, CFM (Standard conditions)	Pipe size (NPS)									
	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	4	5
3	0.025									
4	0.045									
5	0.071	0.016								
6	0.102	0.023								
8	0.181	0.041								
10	0.283	0.064	0.017							
15	0.636	0.144	0.038							
20	1.131	0.255	0.067	0.016						
25	1.768	0.399	0.105	0.025						
30	2.546	0.574	0.152	0.037	0.016					
35	3.465	0.782	0.206	0.050	0.022					
40	4.526	1.021	0.270	0.065	0.029					
45	5.728	1.292	0.341	0.083	0.036					
50	7.071	1.596	0.421	0.102	0.045					
60	10.183	2.298	0.607	0.147	0.065	0.018				
70	13.860	3.128	0.826	0.200	0.088	0.024				
80		4.085	1.079	0.261	0.115	0.031	0.013			
90		5.170	1.365	0.330	0.145	0.04	0.016			
100		6.383	1.685	0.408	0.180	0.049	0.020			
125		9.973	2.633	0.637	0.281	0.077	0.031			
150		14.361	3.792	0.918	0.404	0.110	0.045	0.014		
175			5.162	1.249	0.550	0.150	0.062	0.02		
200			6.742	1.632	0.718	0.196	0.081	0.026		
225			8.533	2.065	0.909	0.248	0.102	0.033		
250			10.534	2.550	1.122	0.306	0.126	0.040		
275			12.746	3.085	1.358	0.371	0.152	0.049		
300			15.169	3.671	1.616	0.441	0.181	0.058	0.013	
325				4.309	1.896	0.518	0.213	0.068	0.016	
350				4.997	2.199	0.601	0.247	0.079	0.018	
375				5.736	2.525	0.689	0.283	0.090	0.021	
400				6.527	2.872	0.784	0.323	0.103	0.024	
425				7.368	3.243	0.886	0.364	0.115	0.027	
450				8.260	3.635	0.993	0.408	0.130	0.030	
475				9.204	4.050	1.106	0.455	0.145	0.033	
500				10.198	4.488	1.226	0.504	0.161	0.037	
550				12.340	5.430	1.483	0.610	0.195	0.044	0.013
600				14.685	6.463	1.765	0.726	0.232	0.053	0.016
650					7.585	2.071	0.852	0.272	0.062	0.019

where Re = Reynolds number, dimensionless
v = average flow velocity, ft/s
D = inside diameter of pipe, ft
ρ = density of air
μ = dynamic viscosity of air

TABLE 5.8 Flow Rate versus Pipe Size

Flow rate		Pipe size	
ft^3/m	L/s	NPS	DN
50	24	$2\frac{1}{2}$	65
110	52	3	80
210	99	4	100
400	189	5	125
800	378	6	150

The units of ρ and μ in Eq. (5.18) must be chosen such that Re is dimensionless. Note that the diameter D in the Reynolds number equation (5.18) is in feet, whereas elsewhere in this chapter the pipe inside diameter, designated as d, is in inches.

Example 5.14 Air flows through an NPS 8 (0.250-in wall thickness) pipe at a flow rate of 6000 ft^3/min at 60°F and 14.7 psia. Calculate the Reynolds number of flow.

Solution The velocity of flow is first calculated.

$$\text{Velocity} = \frac{\text{flow rate } (\text{ft}^3/\text{min})}{\text{area } (\text{ft}^2)}$$

$$= \frac{6000}{0.7854\left(8.125/12\right)^2} = 16{,}664 \text{ ft/min} \quad \text{or} \quad 278 \text{ ft/s}$$

Where NPS 8 pipe has an outside diameter of 8.625 in and a wall thickness of 0.250 in, the inside diameter is 8.125 in. The density and viscosity of air from Table 5.1 are

$$\rho = 0.0764 \text{ lb/ft}^3$$

$$\mu = 3.74 \times 10^{-7} \text{ (lb} \cdot \text{s)/ft}^2$$

The Reynolds number of flow is

$$\text{Re} = \frac{278 \times \left(8.125/12\right) \times 0.0764}{3.74 \times 10^{-7} \times 32.2} = 1.2 \times 10^6$$

If the flow is such that the Reynolds number is less than 2000 to 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is said to be *turbulent*. *Critical* flow occurs when the Reynolds number is in the range of 2100 to 4000. Mathematically, the three regimes of flow are defined as

$$\begin{aligned} &\text{Laminar flow}: && \text{Re} \le 2100 \\ &\text{Critical flow}: && 2100 < \text{Re} \le 4000 \\ &\text{Turbulent flow}: && \text{Re} \ge 4000 \end{aligned}$$

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In the critical flow regime, where the Reynolds number is between 2100 and 4000 the flow is undefined as far as pressure drop calculations are concerned.

It has been found that in laminar flow the friction factor f depends only on the Reynolds number and is calculated from

$$f = \frac{64}{\text{Re}} \tag{5.19}$$

where f is the friction factor for laminar flow and Re is the Reynolds number for laminar flow (Re < 2100) (dimensionless).

For turbulent flow, the friction factor depends not only on the Reynolds number but also on the pipe inside diameter and the internal pipe roughness. It is either calculated using the Colebrook-White equation or read from the Moody diagram (Fig. 5.2). The Colebrook-White equation is as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7d} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \tag{5.20}$$

where f = Darcy friction factor, dimensionless
d = pipe inside diameter, in
e = absolute pipe roughness, in
Re = Reynolds number, dimensionless

The internal roughness of pipe e depends on the condition of the pipe. It ranges from 0.001 to 0.01. The term e/d is known as the relative roughness. Table 5.9 lists the internal pipe roughness values.

It can be seen from Eq. (5.20) that calculating the friction factor is not straightforward, since it appears on both sides of the equation. During the last 20 years many researchers have proposed explicit equations for the friction factor which are much easier to use than Eq. (5.20). Two such equations that are used to calculate the friction factor f include

TABLE 5.9 Pipe Internal Roughness

Pipe material	Roughness in	Roughness mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

the Churchill equation and the Swamee-Jain equation. These equations are explicit in friction factor calculations and therefore are easier to use than the Colebrook-White equation which requires solution of the friction factor by trial and error.

5.4.2 Churchill equation

This equation for the friction factor was proposed by Stuart Churchill and published in *Chemical Engineering* magazine in November 1977. This is an explicit equation for solving for the friction factor and is as follows:

$$f = \left[\left(\frac{8}{\mathrm{Re}}\right)^{12} + \frac{1}{(A+B)^{3/2}}\right]^{1/12} \tag{5.21}$$

where

$$A = \left[2.457 \log_e \frac{1}{(7/\mathrm{Re})^{0.9} + 0.27e/d}\right]^{16} \tag{5.22}$$

$$B = \left(\frac{37{,}530}{\mathrm{Re}}\right)^{16} \tag{5.23}$$

The Churchill equation for the friction factor yields comparable results with that obtained using the Colebrook-White equation.

5.4.3 Swamee-Jain equation

This is another explicit equation for calculating the friction factor. It was first presented by P. K. Swamee and A. K. Jain in 1976 in the *Journal of the Hydraulics Division of ASCE*. This equation is the easiest of all explicit equations for calculating the friction factor. The Swamee-Jain equation is expressed as

$$f = \frac{0.25}{\left[\log_{10}\left(e/3.7d + 5.74/\mathrm{Re}^{0.9}\right)\right]^2} \tag{5.24}$$

The friction factor obtained using the Churchill equation also correlates fairly well with that obtained from the Colebrook-White equation.

Since the Colebrook-White equation requires solution by trial and error, the Moody diagram (Fig. 5.2) is preferred by some, as the friction factor may be read off easily from the chart if the relative roughness e/d and the Reynolds number Re are known.

The Darcy equation (5.17) may be modified to calculate the pressure drop in psi as follows:

$$\Delta P = \frac{f \rho L Q^2}{82.76 d^5} \tag{5.25}$$

where ΔP = pressure drop, psi
f = Darcy friction factor, dimensionless
ρ = air density, lb/ft^3
L = pipe length, ft
Q = volume flow rate, ft^3/min (actual)
d = pipe inside diameter, in

The following equation can be used to calculate the flow rate for the given upstream and downstream pressures:

$$Q_s = 3.92 \frac{T_s}{P_s} \left[\frac{\left(P_1^2 - P_2^2\right) \times d^5}{fTL} \right]^{1/2} \tag{5.26}$$

where Q_s = volume flow rate at standard conditions, SCFM
T_s = temperature at standard conditions, °R
P_s = pressure at standard conditions, psia
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
d = pipe inside diameter, in
f = Darcy friction factor, dimensionless
T = temperature, °R
L = pipe length, ft

In terms of mass flow rate in lb/min, considering the standard conditions of 60°F and 14.7 psia, Eq. (5.26) becomes

$$M = 10.58 \left[\frac{\left(P_1^2 - P_2^2\right) \times d^5}{fTL} \right]^{1/2} \tag{5.27}$$

where M is the mass flow rate (lb/min). Other symbols are as defined earlier.

When pressures are low and slightly above atmospheric pressure, such as in ventilating systems, it is generally more convenient to express the pressure drop due to friction in inches of H_2O. Since 1 inch of water column equals $\frac{(1/12)62.4}{144} = 0.03613$ psi and considering pressures close to atmospheric pressure, the flow equation becomes

$$Q_s = \frac{T_s}{3.64} \left(\frac{hd^5}{fTL} \right)^{1/2} \tag{5.28}$$

where Q_s = volume flow rate at standard conditions, SCFM
T_s = temperature at standard conditions, °R
h = pressure drop, inH_2O column
d = pipe inside diameter, in
f = Darcy friction factor, dimensionless
T = temperature, °R
L = pipe length, ft

In ventilation work, standard conditions are 14.7 psia and 70°F. This results in the following equation for airflow:

$$Q = 145.6\left(\frac{hd^5}{fTL}\right)^{1/2} \tag{5.29}$$

where Q = volume flow rate, ft^3/min (actual)
h = pressure drop, inH_2O column
d = pipe inside diameter, in
f = Darcy friction factor, dimensionless
T = temperature, °R
L = pipe length, ft

Example 5.15 A pipe is to be designed to carry 150 CFM free air at 100 psig and 80°F. If the pressure loss must be limited to 5 psi per 100 ft of pipe, what is the minimum pipe diameter required?

Solution From Table 5.6 let us select 1-in pipe and from Table 5.1 at 80°F we get $\mu = 3.85 \times 10^{-7}$ (lb · s)/ft^2. Therefore, the density of air at 80°F and 100 psig is from the perfect gas equation (5.1):

$$\frac{P}{\rho} = RT$$

$$\rho = \frac{(100 + 14.7) \times 144}{53.3(80 + 460)} = 0.574 \text{ lb/ft}^3$$

The actual flow rate at 100 psig and 80°F is

$$Q_a = 150 \times \frac{14.7}{100 + 14.7}\,\frac{80 + 460}{60 + 460} = 19.96 \text{ ft}^3\text{/min}$$

Next, we calculate flow velocity (1-in pipe schedule 40 has an inside diameter of 1.049 in).

$$\text{Velocity} = \frac{\text{flow rate}}{\text{area}}$$

$$v = \frac{Q}{A} = \frac{19.96/60}{0.7854(1.049/12)^2} = 55.43 \text{ ft/s}$$

Therefore, the Reynolds number, using Eq. (5.18), is

$$\mathrm{Re} = \frac{vD\rho}{\mu} = \frac{55.43}{3.85 \times 10^{-7}} \times \frac{1.049}{12} \times \frac{0.574}{32.2} = 2.2435 \times 10^{5}$$

Using a pipe absolute roughness of $e = 0.0018$ in, the relative roughness is

$$\frac{e}{D} = \frac{0.0018}{1.049} = 0.00172$$

$$f = 0.0232$$

From the Darcy equation (5.17), the pressure drop in 100 ft of pipe is

$$h = f\frac{L}{D}\frac{v^2}{2g} = 0.0232\frac{100 \times 12}{1.049}\frac{55.43^2}{64.4} = 1266 \text{ ft}$$

The pressure drop in psi, using Eq. (5.11), is

$$\Delta P = 1266\frac{0.574}{144} = 5.05 \text{ psi}$$

This is close to the 5 psi per 100 ft limit.

Several other empirical formulas are used in the calculation of flow through ducts and pipes. Commonly used formulas include Harris, Fritzsche, Unwin, Spitzglass, and Weymouth. The Harris formula is similar to the Weymouth formula. In all these formulas, for a given pipe size and flow rate the pressure drop can be calculated directly without using charts or calculating a friction factor first. However, engineers today still use the well-known Darcy equation to calculate pressure drop in compressed-air piping in conjunction with the friction factor computed from the Colebrook-White equation or the Moody diagram.

5.4.4 Harris formula

The Harris formula for standard conditions is

$$\Delta P = \frac{LQ^2}{2390Pd^{5.31}} \tag{5.30}$$

where ΔP = pressure drop, psig
L = pipe length, ft
Q = volume flow rate at standard conditions, SCFM
P = average pressure, psia
d = pipe inside diameter, in

Also in terms of mass flow rate

$$\Delta P = \frac{LM^2}{13.95Pd^{5.31}} \tag{5.31}$$

where ΔP = pressure drop, psig
L = pipe length, ft
M = mass flow rate, lb/min
P = average pressure, psia
d = pipe inside diameter, in

In terms of flow rate Q and upstream and downstream pressures P_1 and P_2, the following formula is used.

$$Q = 34.5\left[\frac{(P_1^2 - P_2^2)d^{5.31}}{L}\right]^{1/2} \tag{5.32}$$

where Q = volume flow rate at standard conditions, SCFM
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
L = pipe length, ft
d = pipe inside diameter, in

5.4.5 Fritzsche formula

The Fritzsche formula uses the friction factor f calculated from the following equation:

$$f = 0.02993\left(\frac{T_s}{P_s Q_s}\right)^{1/7} \tag{5.33}$$

where f = friction factor
T_s = temperature at standard conditions, °R
P_s = pressure at standard conditions, psia
Q_s = volume flow rate at standard conditions, SCFM

The Fritzsche formula for pressure drop then becomes

$$\Delta P = \frac{(9.8265 \times 10^{-4})TL}{Pd^5}\left(\frac{P_s Q_s}{T_s}\right)^{1.857} \tag{5.34}$$

where ΔP = pressure drop, psi
L = pipe length, ft
d = pipe inside diameter, in
T = airflow temperature, °R
P = average air pressure, psia
Q_s = volume flow rate at standard conditions, SCFM
P_s = pressure at standard conditions, psia
T_s = temperature at standard conditions, °R

And in terms of flow rate and the upstream and downstream pressures, this becomes

$$Q_s = 29.167 \frac{T_s}{P_s} \left[\frac{(P_1^2 - P_2^2) d^5}{TL} \right]^{0.538} \tag{5.35}$$

where Q_s = volume flow rate at standard conditions, SCFM
P_s = pressure at standard conditions, psia
T_s = temperature at standard conditions, °R
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
L = pipe length, ft
d = pipe inside diameter, in
T = airflow temperature, °R

The preceding formulas can be used for the flow of air at standard conditions and any flowing temperatures. When standard conditions of 14.7 psia and 60°F are used along with a flowing temperature of 60°F, the preceding formulas can be simplified as follows:

$$\Delta P = \frac{L Q_s^{1.857}}{1480 P d^5} \tag{5.36}$$

where ΔP = pressure drop, psi
L = pipe length, ft
Q_s = volume flow rate at standard conditions, SCFM
d = pipe inside diameter, in
P = average air pressure, psia

$$Q_s = \frac{1}{35} \left[\frac{(P_1^2 - P_2^2) d^5}{L} \right]^{0.538} \tag{5.37}$$

where Q_s = volume flow rate at standard conditions, SCFM
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
L = pipe length, ft
d = pipe inside diameter, in

Where air pressures are low and close to the atmospheric pressure such as in ventilating work and in airflow through ducts, we can modify the Fritzsche formula to calculate the pressure drops in inH_2O. Since 1 in of water column is equal to 0.03613 psi, the pressure loss can be expressed as follows:

$$h = \frac{L{Q_s}^{1.857}}{785d^5} \tag{5.38}$$

where h is the pressure drop measured in inH_2O.

Another variation of Eq. (5.38) in terms of flow rate is

$$Q_s = \left(\frac{785hd^5}{L}\right)^{0.538} \tag{5.39}$$

5.4.6 Unwin formula

The Unwin formula is applicable for airflow in smooth pipes. This is based on tests conducted in Paris using compressed-air pipelines. In this formula the friction factor for airflow is represented by the following equation:

$$f = 0.0025\left(1 + \frac{3.6}{d}\right) \tag{5.40}$$

Using this friction factor under standard conditions we get the following equations for pressure drop, flow rate, and mass flow rate of air flowing through smooth pipes.

$$\Delta P = \frac{(1 + 3.6/d)L{Q_s}^2}{7400Pd^5} \tag{5.41}$$

$$Q_s = 86\sqrt{\frac{Pd^5/\Delta P}{(1 + 3.6/d)L}} \tag{5.42}$$

$$M = 6.56\sqrt{\frac{Pd^5/\Delta P}{(1 + 3.6/d)L}} \tag{5.43}$$

where ΔP = pressure drop, psi
L = pipe length, ft
Q_s = volume flow rate at standard conditions, SCFM
d = pipe inside diameter, in
P = average air pressure, psia
M = mass flow rate of air, lb/min

Example 5.16 Air flows in a 6-in inside diameter pipe at the rate of 3000 ft^3/min. If the upstream pressure is 100 psia, what is the downstream pressure and pressure drop for 1000 ft of pipe?

Solution From the Harris equation (5.30),

$$\Delta P = \frac{LQ^2}{2390Pd^{5.31}} = \frac{1000 \times 3000 \times 3000}{2390 \times 100 \times (6.0)^{5.31}} = 2.78 \text{ psi}$$

Using the Unwin formula (5.41), we get

$$\Delta P = \frac{1000 \times 3000 \times 3000(1 + 3.6/6.0)}{7400 \times 100(6.0)^5} = 2.5 \text{ psi}$$

5.4.7 Spitzglass formula

Spitzglass introduced this formula in 1912 based on tests conducted for the Peoples Gas Light and Coke Company of Chicago. This formula uses a friction factor as follows:

$$f = 0.0112\left(1 + \frac{3.6}{d} + 0.03d\right) \tag{5.44}$$

There are two versions of the pressure drop equation using the Spitzglass method. For low pressures up to 1 psig,

$$h = \frac{LQ_s^{\,2}}{1.26 \times 10^7 K^2} \tag{5.45}$$

$$Q_s = 3550K\sqrt{\frac{h}{L}} \tag{5.46}$$

$$K = \sqrt{\frac{d^5}{(1 + 3.6/d + 0.03d)}} \tag{5.47}$$

where h = frictional head loss, inH_2O
L = pipe length, ft
Q_s = volume flow rate at standard conditions, ft^3/h (SCFH)
K = A parameter that is a function of pipe diameter d
d = pipe inside diameter, in

For pressures greater than 1 psig,

$$\Delta P = \frac{LQ_s^{\,2}}{2.333 \times 10^7 PK^2} \tag{5.48}$$

$$Q_s = 4830K\sqrt{\frac{P\Delta P}{L}} \tag{5.49}$$

$$Q_s = 3415K\sqrt{\frac{\left(P_1^{\,2} - P_2^{\,2}\right)}{L}} \tag{5.50}$$

where P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
P = average pressure, psia

All other symbols are as defined earlier.

It has been found that the Spitzglass formula gives a lower value of flow rate for a given pressure drop and pipe size compared to the Weymouth formula (discussed next). Hence the Spitzglass formula is used in situations where a more conservative result is desired such as in pipes that are rough or rusty.

5.4.8 Weymouth formula

Thomas R. Weymouth presented this formula in 1912 for calculating gas flow through high-pressure pipelines. This formula is also used with the flow of compressed air. The Weymouth friction factor is as follows:

$$f = \frac{0.032}{d^{0.3333}} \tag{5.51}$$

The Weymouth formula for airflow at standard conditions is

$$\Delta P = \frac{(1.0457 \times 10^{-3})TL}{Pd^{5.3333}} \left(\frac{P_s Q_s}{T_s}\right)^2 \tag{5.52}$$

Also

$$Q_s = 21.8742\frac{T_s}{P_s}\sqrt{\frac{\left(P_1^{\,2} - P_2^{\,2}\right)d^{5.3333}}{TL}} \tag{5.53}$$

where all the symbols are as defined earlier.

Although many equations have been put forth for the flow of compressed air through pipes, such as those of Harris and Unwin, the classical method of calculating the pressure drop of a fluid using the Darcy equation (5.17) still finds popularity among engineers. Thus, knowing the pipe diameter, air properties, and flow rate the Reynolds number is calculated first. Next a friction factor is calculated from the Colebrook-White equation or read from the Moody diagram. Finally, using the Darcy equation the pressure drop due to friction is calculated. As mentioned before, for quick calculations of compressed-air systems the head loss may also be estimated from Tables 5.5 through 5.7.

Example 5.17 A pipeline 20,000 ft in length flows air at 4000 SCFM. The initial pressure is 150 psia, and the flow temperature is 60°F. If the pressure drop is limited to 50 psi, determine the approximate pipe diameter required. Compare solutions using the Harris, Fritzsche, and Weymouth formulas.

Solution

$$\text{Average pressure } P = \frac{150 + 100}{2} = 125 \text{ psia}$$

Harris formula: Using Eq. (5.30), we get

$$50 = \frac{20{,}000(4000)^2}{2390 \times 125 \times d^{5.31}}$$

Solving for diameter d, we get

$$d = 6.54 \text{ in}$$

Fritzsche formula: Using Eq. (5.34), we get

$$50 = \frac{9.8265 \times 10^{-4} \times (60 + 460) \times 20{,}000}{125 d^5} \left(\frac{14.7 \times 4000}{60 + 460} \right)^{1.857}$$

Solving for diameter d, we get

$$d = 6.39 \text{ in}$$

Weymouth formula: Using Eq. (5.52), we get

$$50 = \frac{1.0457 \times 10^{-3} \times 520 \times 20{,}000}{125 d^{5.333}} \left(\frac{14.7 \times 4000}{520} \right)^2$$

Solving for diameter d, we get

$$d = 6.53 \text{ in}$$

5.5 Minor Losses

Minor losses in a compressed-air piping system consist of those pressure drops that are caused by piping components such as fittings and valves. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe.

Generally, minor losses are included in calculations by using the concept of equivalent length of the device or using a K factor in conjunction with the velocity head $v^2/2g$. The term minor losses can be applied only when the pipeline lengths and hence the friction losses in the straight runs of pipe are relatively large compared to the friction loss in fittings and valves. In a situation such as plant piping the pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses may be incorrect. Regardless, pressure losses through valves and fittings can be approximated using the equivalent length or velocity head concept.

Table 5.10 gives the equivalent length of commonly used valves and fittings in a typical compressed-air piping system. For example, suppose we have a compressed-air piping system consisting of 500 ft of NPS 12 pipe with two 10-in gate valves and four standard 90° elbows of 12-in diameter.

TABLE 5.10 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

Using Table 5.10, we calculate the total equivalent length of pipe and fittings as follows:

500 ft of NPS 12 pipe		$= 500$ ft
Two 10-in gate valves	$= \dfrac{2 \times 8 \times 10}{12}$	$= 13.33$ ft
Four 12-in standard 90° elbows	$= \dfrac{4 \times 30 \times 12}{12}$	$= 120$ ft
Total equivalent length of pipe, valves, and fittings	$= 500 + 13.33 + 120$	$= 633.33$ ft

The pressure drop due to friction in the compressed-air piping system just described can now be calculated based on a total equivalent length of 633.33 ft of pipe. It can be seen in this example that the valves and fittings represent roughly 21 percent of the total pipe length. In plant piping this percentage may be higher than that in a long-distance pipeline. Hence, the reason for the term minor losses, when long lengths of piping are involved.

The K factor or head loss coefficient and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From Eq. (5.17) the pressure drop in a straight length of pipe is given by

$$h_f = f \frac{L}{D} \frac{v^2}{2g}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K. The preceding equation then becomes

$$h_f = K \left(\frac{v^2}{2g} \right) \tag{5.54}$$

where K = dimensionless head loss coefficient, also known as the K factor
v = flow velocity, ft/s
g = acceleration due to gravity

In this form, the head loss in a straight piece of pipe is represented as a multiple of the velocity head $v^2/2g$. It must be remembered that the factor K includes a friction factor and the L/D ratio of pipe. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(v^2/2g)$ where the coefficient K (also known as the resistance coefficient or head loss coefficient) is specific to the valve or fitting.

The K factor depends upon the specific design of the valve or fitting and must be obtained from the manufacturer of the valve or fitting.

However, for approximate calculations, charts are available for some of the more commonly used valves and fittings. Typical K factors for valves and fittings are listed in Table 5.11. It must be noted that the preceding analysis of representing the head loss through a valve or fitting using a K factor is applicable only for turbulent flows. No such data are available for laminar flow of compressed air.

From Table 5.11 it can be seen that a 6-in gate valve has a K factor of 0.12, while a 20-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be $0.12(v^2/2g)$, and that in the 20-in valve is $0.10(v^2/2g)$. The velocities in the two cases will be different due to the difference in diameters.

Suppose the compressed-air piping that consisted of the 6-in gate valve and the 20-in gate valve previously described had a volume flow rate of 2300 SCFM. The velocity of flow through the 6- and 20-inch valves will be calculated as follows:

$$\text{Flow velocity} = \frac{\text{flow rate (SCFM)}}{60 \times \text{pipe area (ft}^2\text{)}}$$

The velocity in the 6-in valve will be approximately

$$V_6 = \frac{2300}{0.7854 \times 0.5 \times 0.5 \times 60} = 195.23 \text{ ft/s}$$

Similarly, the velocity in the 20-in valve will be approximately

$$V_{20} = \frac{2300}{0.7854 \times 1.625 \times 1.625 \times 60} = 18.48 \text{ ft/s}$$

In the preceding, the 20-in valve is assumed to have an inside diameter of 19.5-in or 1.625 ft.

Therefore,

$$\text{Head loss in 6-in gate valve} = \frac{0.12(195.23)^2}{64.4} = 71.02 \text{ ft}$$

and

$$\text{Head loss in 20-in gate valve} = \frac{0.10(18.48)^2}{64.4} = 0.53 \text{ ft}$$

The head loss in the 20-in valve is insignificant compared to that in the 6-in valve, although the K value for the 20-in valve is 0.10 compared to 0.12 for the 6-in valve. The reason for the large difference in the head loss in the 20-in valve is because of the flow velocity. Care must be taken to use the right pipe size when computing the head loss based on Eq. (5.54).

TABLE 5.11 Friction Loss in Valves—Resistance Coefficient *K*

Description	L/D	Nominal pipe size, in											
		$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72

5.6 Flow of Air through Nozzles

In this section we will discuss the flow of compressed air through a nozzle making an assumption that the process follows a frictionless adiabatic flow. Such a process is termed *isentropic* where the entropy of the air remains the same throughout the process. In reality, there is always friction. However, for simplicity we will assume that the friction is negligible and therefore the process is isentropic. We will first consider an example of compressed air from a storage tank being released to the atmosphere through a pipe nozzle. Next we will analyze compressed air flowing through a pipeline with a restriction or reduced diameter at some point along the pipeline. We are interested in calculating the flow rate of air through a nozzle when a certain pressure difference exists between the upstream end of the system and the nozzle at the downstream end.

Consider a tank containing air at pressure P_1 and temperature T_1. A nozzle connected to this tank is opened in order to let the air flow out of the tank to the atmosphere as shown in Fig. 5.3. We will designate the pressure and temperature at the nozzle to be P_2 and T_2, respectively, as shown in the figure.

If we assume that the airflow through the nozzle is quite rapid, there is no time for any heat to be transferred between the air and the surroundings. Hence we can consider this process of airflow through the nozzle as an adiabatic process. The air in the tank is at rest (velocity $= 0$), and we are using the subscript 1 to represent the condition of the air in the tank and subscript 2 for the condition of the air in the nozzle.

Applying the adiabatic process equation P/ρ^k = constant between the air in the tank at point 1 and the air in the nozzle at point 2, we get

$$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^k \tag{5.55}$$

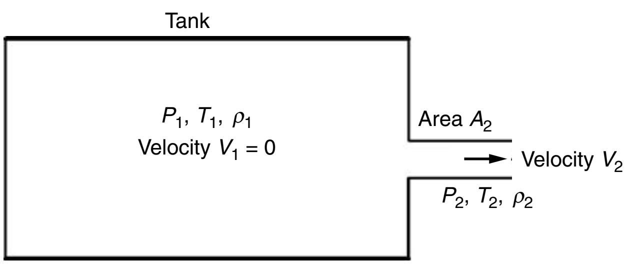

Figure 5.3 Discharge of air from tank through nozzle.

where P_1, ρ_1 = pressure and density, respectively, of air in tank
P_2, ρ_2 = pressure and density, respectively, of air at nozzle
k = ratio of specific heats of air (usually 1.4), dimensionless

The mass flow rate of air through the nozzle can be calculated if the flow velocity, the nozzle area, and the density of air at the nozzle are known:

$$M = \rho_2 v_2 A_2 \tag{5.56}$$

where M = mass flow rate of air, lb/s
ρ_2 = Density of air at nozzle, lb/ft^3
v_2 = flow velocity of air at nozzle, ft/s
A_2 = cross-sectional area at nozzle, ft^2

From thermodynamic analysis of the flow of air from the tank through the nozzle, it can be shown that the flow velocity of air in the nozzle is

$$v_2 = \sqrt{\frac{2gP_1}{\rho_1}\frac{k}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{(k-1)/k}\right]} \tag{5.57}$$

Thus, given the pressures P_1 and P_2 and the density of air in the tank, the velocity of flow of air at the nozzle can be calculated from Eq. (5.57).

Having calculated the velocity v_2 at the nozzle, the mass flow rate of air through the nozzle can be calculated using Eq. (5.56) and substituting the value of velocity v_2 as follows:

$$M = A_2\sqrt{\frac{2gk}{k-1}P_1\rho_1\left[\left(\frac{P_2}{P_1}\right)^{2/k} - \left(\frac{P_2}{P_1}\right)^{(k+1)/k}\right]} \tag{5.58}$$

By examining Eq. (5.57) for the velocity of flow through the nozzle we can conclude the following. As the pressure drop $P_1 - P_2$ between the tank and the nozzle increases, the pressure ratio P_2/P_1 decreases. Hence, the velocity in the nozzle increases until it reaches the sonic velocity. The *sonic velocity* is the velocity of sound in a fluid, in this case, air. When this happens, the air flows at a Mach number = 1.0. The *Mach number* is simply the ratio of the flow velocity to the velocity of sound. The pressure ratio P_2/P_1 when the velocity in the nozzle reaches the sonic velocity is termed the *critical pressure ratio*. This ratio is a function of the specific heat ratio k and is given by the following equation:

$$\text{Critical pressure ratio} = \frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{5.59}$$

From Eq. (5.58) after substituting the value of the critical ratio P_2/P_1 from Eq. (5.59), we can calculate the mass flow rate through the nozzle at the critical pressure ratio. This will represent the maximum possible flow through the nozzle. If the pressure drop $P_1 - P_2$ is increased further, by either increasing P_1 or reducing P_2, the velocity in the nozzle will remain sonic and no further increase in flow rate is possible. This is termed *choked flow*. The mass flow rate through the nozzle at the critical pressure ratio is calculated from the following equation, by substituting the critical pressure ratio P_2/P_1 in Eq. (5.58):

$$M = \frac{A_2 P_1}{\sqrt{T_1}} \sqrt{\frac{gk}{R}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} \tag{5.60}$$

where M = mass flow rate of air, lb/s
A_2 = cross-sectional area at nozzle, ft^2
P_1 = pressure in tank, psia
T_1 = absolute temperature of air in tank, °R
g = acceleration due to gravity
k = ratio of specific heats of air (usually 1.4), dimensionless
R = gas constant for air

In Eq. (5.60) we have introduced the temperature T_1 and gas constant R using the perfect gas equation (5.1). A similar analysis is presented next for compressed air flowing through a pipeline that has a restricted pipe size at a certain location in the pipeline.

5.6.1 Flow through a restriction

A convergent nozzle in a pipeline is a section of the pipe where the flow of air starts off initially in a larger-diameter section and is then made to flow through a smaller-diameter section. This is illustrated in Fig. 5.4.

Consider airflow through a pipe starting at a particular cross-sectional area A_1 at section 1 and becoming a smaller cross-sectional area A_2 at section 2 as shown in the figure. Let P_1, ρ_1, and T_1 represent the pressure, density, and temperature, respectively, at section 1 and the velocity of flow at section 1 be v_1. The corresponding values in section 2 of the pipe are denoted by P_2, ρ_2, T_2, and v_2. The mass flow rate for such a piping system can be calculated from the following equation:

$$M = \frac{A_2}{\sqrt{1-(P_2/P_1)^{2/k}(A_2/A_1)^2}} \sqrt{\frac{2gk}{k-1} P_1 \rho_1 \left[\left(\frac{P_2}{P_1}\right)^{2/k} - \left(\frac{P_2}{P_1}\right)^{(k+1)/k}\right]} \tag{5.61}$$

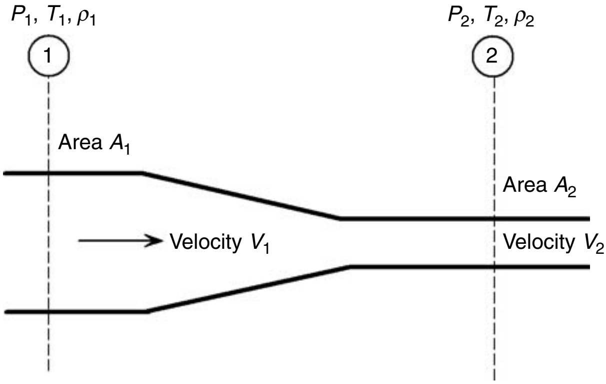

Figure 5.4 Airflow through a restriction.

where M = mass flow rate, lb/s
A_1 = upstream pipe cross-sectional area, ft^2
A_2 = nozzle throat area, ft^2
k = ratio of specific heats of air (usually 1.4), dimensionless
g = acceleration due to gravity, ft/s^2
ρ_1 = density of air at upstream location, lb/ft^3
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia

It may be seen from Eq. (5.61) that as A_1 increases such that the ratio A_2/A_1 is very small, it approximates the condition of a storage tank and nozzle described earlier. In this case Eq. (5.61) reduces to Eq. (5.58).

As airflow approaches the smaller-diameter nozzle (see Fig. 5.4), the velocity increases and may equal the sonic velocity. At sonic velocity the Mach number (air speed/sound speed) is 1.0. When this happens, the ratio of the pressure in nozzle P_2 to the upstream pressure P_1 is defined as the critical pressure ratio. This ratio is a function of the specific heat ratio k of air. This is similar to Eq. (5.59) for the discharge of air from a tank through a nozzle.

If the airflow through the nozzle has not reached sonic velocity, the flow is termed *subsonic*. In this case the pressure ratio P_2/P_1 will be a larger number than the critical pressure ratio calculated from Eq. (5.59).

If the pressure drop $P_1 - P_2$ increases such that the critical pressure ratio is reached, the flow through the nozzle will be sonic. The flow rate equation then becomes, after setting P_2/P_1 equal to the critical pressure ratio from Eq. (5.59),

$$M = \frac{A_2 P_1}{\sqrt{T_1}} \sqrt{\frac{gk}{R}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} \tag{5.62}$$

A further increase in pressure drop causes the flow through the nozzle to remain sonic and the pressure at the exit of the nozzle will increase. Even though the pressure drop has increased, there will be no change in the mass flow rate. This is known as choked flow, as discussed earlier under discharge of air from a tank through a nozzle.

Example 5.18 What is the critical pressure ratio for the flow of compressed air through a nozzle, assuming isentropic flow?

Solution When the airflow takes place under adiabatic conditions, with no heat transfer between the air and the surroundings and friction is neglected, it is said to be isentropic flow. The critical pressure ratio for air with the specific heat ratio $k = 1.4$ can be calculated from Eq. (5.59) as follows:

$$\text{Critical pressure ratio} = \frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$= \left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 0.5283$$

Thus the critical pressure ratio for compressed air flowing through a nozzle under isentropic conditions is 0.5283.

Example 5.19 Compressed air flows through a nozzle, and the upstream and downstream pressures were recorded as 2.75 and 1.75 MPa, respectively. Both pressures are in absolute values. Is the flow through the nozzle subsonic or sonic? What is the flow rate through the nozzle, if the nozzle size is 100 mm and the upstream pipe size is 200 mm? Assume the density of air is 0.065 kN/m^3 and the gas constant is 29.3.

Solution First we will calculate the critical pressure ratio:

$$\frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = \left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 0.5283$$

Next we will compare this with the ratio of given pressures.

$$\text{Pressure ratio} = \frac{1.75}{2.75} = 0.6364$$

Since the pressure ratio is higher than the critical pressure ratio, we conclude that the flow is subsonic.

We will use Eq. (5.61) to calculate the mass flow rate. The cross-sectional area of the nozzle is

$$A_2 = 0.7854 \times 0.1 \times 0.1 = 0.007854 \text{ m}^2$$

The cross-sectional area of the upstream end of the pipe is

$$A_1 = 0.7854 \times 0.2 \times 0.2 = 0.0314 \text{ m}^2$$

Therefore

$$\frac{A_2}{A_1} = \frac{0.007854}{0.0314} = 0.25$$

$$\frac{(k+1)}{k} = \frac{1.4+1}{1.4} = 1.7143$$

$$\frac{2}{k} = \frac{2}{1.4} = 1.4286$$

$$\frac{k}{k-1} = \frac{1.4}{0.4} = 3.5$$

Substituting the preceding ratios in Eq. (5.61), we get for mass flow rate,

$$M = \frac{0.007854}{\sqrt{1-(0.6364)^{1.4286}(0.25)^2}}$$

$$\sqrt{2 \times 9.81 \times 3.5 \times 2.75 \times 10^3 \times 0.065\,[(0.6364)^{1.4286} - (0.6364)^{1.7143}]}$$

$$= 0.223 \text{ kN/s}$$

Example 5.20 Consider air flowing through a 300-mm inside diameter pipe at 20°C, where the upstream pressure is 600 kPa and the downstream pressure 200 m away is 300 kPa. All pressures are in absolute value. Assume the pipe roughness to be 0.05 mm. Use a gas constant $R = 29.3$. Calculate the volume flow rate and mass flow rate.

Solution Assume a friction factor $f = 0.01$. Using the isothermal flow equation (5.14), we get

$$600^2 - 300^2 = \frac{M^2 \times 29.3(273+20)}{9.81(0.7854 \times 0.3 \times 0.3)^2}\left(0.01 \times \frac{200}{0.3} + 2\log_e \frac{600}{300}\right)$$

Solving for the mass flow rate:

$$M = 0.438 \text{ kN/s}$$

Using the perfect gas law from Eq. (5.1),

$$\text{Density } \rho = \frac{600}{29.3 \times 293} = 0.0699 \text{ kN/m}^3$$

From the mass flow rate equation (5.13),

$$\text{Velocity of flow } v = \frac{0.438}{(0.7854 \times 0.3 \times 0.3)(0.0699)} = 88.65 \text{ m/s}$$

Calculate the Reynolds number from Eq. (5.18):

$$\text{Re} = \frac{0.0699 \times 0.3 \times 88.65}{9.81 \times (1.81 \times 10^{-5} \times 10^{-3})} = 1.05 \times 10^7$$

where the viscosity of air $\mu = 1.81 \times 10^{-5}$ (N · s)/m^2 at 20°C from Table 5.2. The pipe relative roughness is

$$\frac{e}{d} = \frac{0.05}{300} = 1.667 \times 10^{-4}$$

Thus, from the Moody diagram at the calculated Reynolds number, the friction factor is found to be

$$f = 0.0134$$

Recalculating the flow rate M using this value of f we get

$$M = 0.387 \text{ kN/s}$$

Recalculating the velocity by proportions

$$V = \frac{0.387}{0.438} \times 88.65 = 78.33 \text{ m/s}$$

The revised Reynolds number then becomes by proportions

$$\text{Re} = 1.05 \times 10^7 \times \frac{78.33}{88.65} = 9.28 \times 10^6$$

Then from the Moody diagram at this Reynolds number, the friction factor is found to be

$$f = 0.01337$$

which is quite close to what we had before. Thus the calculations are complete, and the flow rate is

$$M = 0.387 \text{ kN/s}$$

The volume flow rate is equal to the mass flow rate divided by density:

$$\text{Volume rate } Q = \frac{0.387}{0.0699} = 5.536 \text{ m}^3\text{/s}$$

Example 5.21 Air flows at 50°F from a large storage tank through a convergent nozzle with an exit diameter of 1 in. The air discharges to the atmosphere (14.7 psia). The tank pressure is 400 psig. What is the airflow rate through the nozzle?

Solution The critical pressure ratio, from Eq. (5.59), is

$$\frac{P_2}{P_1} = \left(\frac{2}{1.4 + 1}\right)^{1.4/0.4} = 0.5283$$

$$\text{Actual pressure ratio} = \frac{14.7}{400 + 14.7} = 0.035$$

Since the actual pressure ratio is less than the critical value, the flow through the nozzle is sonic. The flow rate through the nozzle is found using Eq. (5.63).

First we calculate the nozzle area:

$$A_2 = 0.7854\left(\frac{1}{12}\right)^2 = 0.00545\ \text{ft}^2$$

Then,

$$M = \frac{0.00545 \times 414.7 \times 144}{\sqrt{460+50}}\sqrt{\frac{32.2 \times 1.4}{53.3}\left(\frac{2}{1.4+1}\right)^{2.4/0.4}} = 7.67\ \text{lb/s}$$

Note that to ensure a consistent set of units, the pressure (400 + 14.7) psia must be multiplied by 144 to convert to lb/ft^2.

Example 5.22 Air flows through a 4-in-diameter pipeline with a 2-in diameter restriction. The upstream pressure and temperature are 150 psig and 100°F, respectively. Calculate the flow rate of air if the pressure in the restriction is 75 psig. Assume an atmospheric pressure of 14.7 psia.

Solution To calculate the flow rate of air through a restriction using Eq. (5.61), we begin by solving the critical pressure ratio, cross-sectional areas and area ratio.

$$\frac{P_2}{P_1} = \frac{75+14.7}{150+14.7} = 0.5446$$

$$A_2 = 0.7854\left(\frac{2}{12}\right)^2 = 0.02182\ \text{ft}^2$$

$$A_1 = 0.7854\left(\frac{4}{12}\right)^2 = 0.08727\ \text{ft}^2$$

$$\frac{A_2}{A_1} = \frac{0.02182}{0.08727} = 0.25$$

Next, the density of air at the inlet is calculated using Eq. (5.1):

$$\rho_1 = \frac{P_1}{RT_1} = \frac{(150+14.7) \times 144}{53.3 \times (460+100)} = 0.7946\ \text{lb/ft}^3$$

Now the mass flow rate can be calculated easily by substituting in Eq. (5.61):

$$M = \frac{0.02182}{\sqrt{1-(0.5446)^{2/1.4}(0.25)^2}}$$
$$\sqrt{\frac{2 \times 32.2 \times 1.4}{0.4}(164.7 \times 0.7946 \times 144)\left[(0.5446)^{2/1.4} - (0.5446)^{2.4/1.4}\right]}$$

Solving we get $M = 11.79$ lb/s.

Chapter 6

Oil Systems Piping

Introduction

Oil systems piping includes those pipelines that transport oil and petroleum products from refineries and tank farms to storage facilities and end-user locations. We will discuss calculations that are required for sizing crude oil and petroleum products (diesel, gasoline, etc.) pipelines. Since oil is generally considered incompressible and therefore its volume does not change appreciably with pressure, its analysis is similar to that of other incompressible fluids such as water. We will begin our discussion with an exploration of the properties of crude oil and petroleum products and how they affect pipeline transportation. We will also cover pumping requirements such as the type of equipment and horsepower needed to transport these products from the various sources to their destinations. We will discuss short piping systems such as oil gathering lines as well as long-distance trunk lines. Throughout this chapter we will use the term *petroleum products* to refer to crude oil as well as refined petroleum products such as gasoline, kerosene, and diesel fuels.

6.1 Density, Specific Weight, and Specific Gravity

The *density* of a liquid is defined as its mass per unit volume. The *specific weight* is defined as weight per unit volume. Sometimes these two terms are used interchangeably. Density is expressed as slug/ft^3 and specific weight as lb/ft^3 in English, or U.S. Customary (USCS), units. For example, a typical crude oil may have a density of 1.65 slug/ft^3 and a specific weight of 53.0 lb/ft^3. In comparison water has a density of 1.94 slug/ft^3 and a specific weight of 62.4 lb/ft^3. Both the density and specific weight of petroleum products change with temperature. These two properties decrease as the temperature is increased, and vice versa.

The *volume* of a petroleum product is measured in gallons or barrels in USCS units and in cubic meters (m^3) or liters (L) in Système International (SI) units. One barrel of a petroleum product is equal to 42 U.S. gallons. Volume flow rates in oil pipelines are generally reported in gal/min, barrels per hour (bbl/h), or bbl/day in USCS units and in m^3/h or L/s in SI units. As indicated before, since liquids are incompressible, pressure has little effect on their volume or density.

Specific gravity is a measure of how heavy a liquid is compared to water at a particular temperature. Thus considering some standard temperature such as 60°F, if the density of petroleum product is 6 lb/gal and that of water is 8.33 lb/gal, we can say that the specific gravity Sg of the petroleum product is

$$\mathrm{Sg} = \frac{6}{8.33} = 0.72$$

Note that this comparison must use densities measured at the same temperature; otherwise it is meaningless. In USCS units, the standard temperature and pressure are taken as 60°F and 14.7 psi. In SI units the corresponding values are 15°C and 1 bar or 101 kPa. Typical specific gravities of common crude oils, diesel, gasoline, etc., are listed in Table 6.1.

In the petroleum industry a commonly used term is the *API gravity*, named after the American Petroleum Institute (API). The API gravity of a petroleum product is measured in the laboratory using the ASTM D1298 method. It is a measure of how heavy a liquid is compared to water and therefore has a correlation with specific gravity. However, the API scale of gravity is based on a temperature of 60°F and an API gravity of 10 for water. Liquids lighter than water have an API gravity greater than 10. Those liquids that are heavier than water will have

TABLE 6.1 Specific Gravities of Petroleum Products

Liquid	Specific Gravity at 60°F	API Gravity at 60°F
Propane	0.5118	N/A
Butane	0.5908	N/A
Gasoline	0.7272	63.0
Kerosene	0.7796	50.0
Diesel	0.8398	37.0
Light crude	0.8348	38.0
Heavy crude	0.8927	27.0
Very heavy crude	0.9218	22.0
Water	1.0000	10.0

N/A = not applicable.

an API gravity of less than 10. In comparison the specific gravity of a liquid lighter than water may be 0.85 compared to water with a specific gravity of 1.0. Similarly, brine, a heavier liquid, has a specific gravity of 1.26. It can thus be seen that the API gravity numbers increase as the product gets lighter than water whereas specific gravity numbers decrease. The API gravity is always measured at 60°F. It is incorrect to state that the API of a liquid is 37°API at 70°F. The phrase "37°API" automatically implies the temperature of measurement is 60°F.

The specific gravity of a liquid and its API gravity are related by the following two equations:

$$\mathrm{Sg} = \frac{141.5}{131.5 + \mathrm{API}} \tag{6.1}$$

$$\mathrm{API} = \frac{141.5}{\mathrm{Sg}} - 131.5 \tag{6.2}$$

Again, it must be remembered that in both Eqs. (6.1) and (6.2) the specific gravity Sg is the value at 60°F since by definition the API is always at 60°F. Thus, given the value of API gravity of a petroleum product we can easily calculate the corresponding specific gravity at 60°F using these equations.

Example 6.1

(a) A sample of crude oil when tested in a lab showed an API gravity of 35. What is the specific gravity of this crude oil?

(b) Calculate the API gravity of gasoline, if its specific gravity is 0.736 at 60°F.

Solution

(a) Using Eq. (6.1),

$$\mathrm{Sg} = \frac{141.5}{131.5 + 35} = 0.8498 \text{ at } 60^\circ\mathrm{F}$$

(b) Using Eq. (6.2),

$$\mathrm{API} = \frac{141.5}{0.736} - 131.5 = 60.76$$

It is understood that the above API value is at 60°F.

The specific gravity of a petroleum product decreases with an increase in temperature. Therefore, if the specific gravity of crude oil is 0.895 at 60°F, when the oil is heated to 100°F, the specific gravity will drop to some lower value, such as 0.825. The API gravity, on the other hand, still remains at the same value as before, since it is always referred to at 60°F.

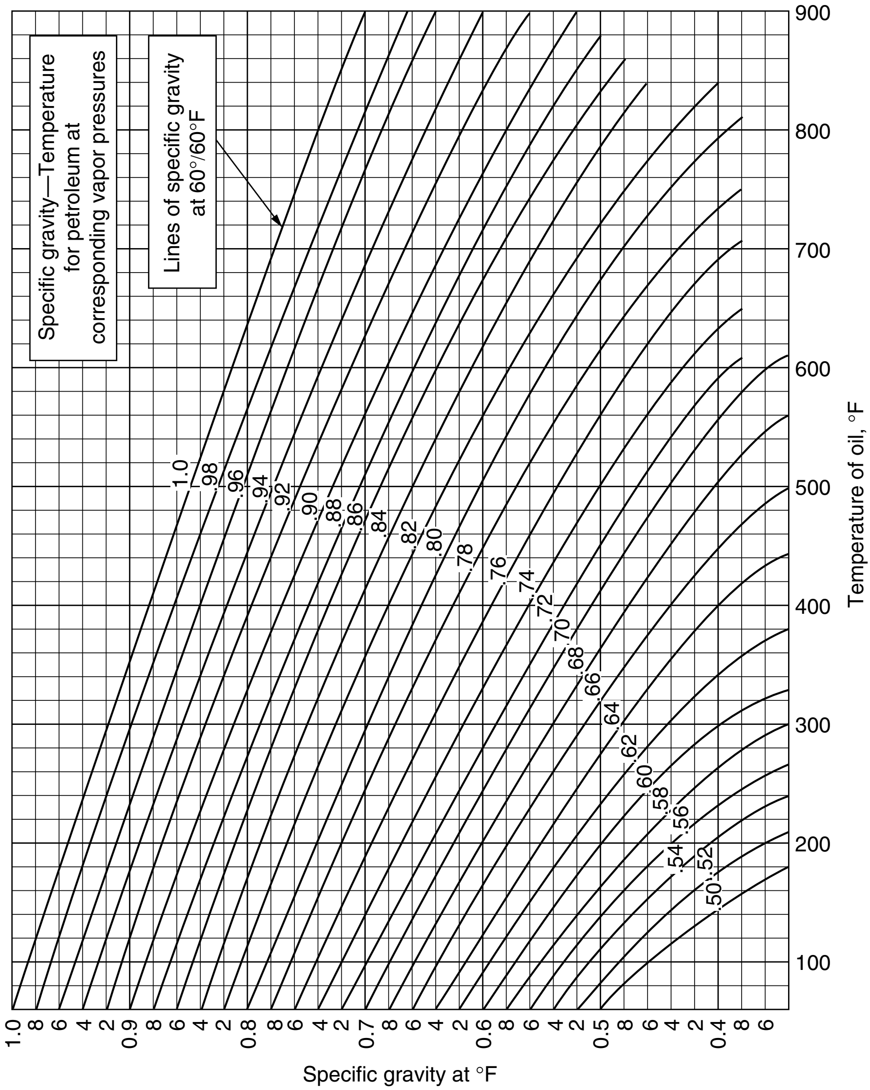

Figure 6.1 Variation of specific gravity with temperature for various petroleum liquids.

Let Sg_1 and Sg_2 represent the specific gravity at two different temperatures T_1 and T_2 . We find that an approximately linear relationship exists between specific gravity and temperature within the normal range of temperatures encountered in oil pipelines. Thus a probable relationship between the specific gravity and temperature may be expressed as

$$\mathrm{Sg}_1 - \mathrm{Sg}_2 = a(T_2 - T_1) + b \tag{6.3}$$

where a and b are constants.

It is more common to calculate the specific gravity of a petroleum product at any temperature from the specific gravity at the standard temperature of 60°F. We can then rewrite Eq. (6.3) in terms of the unknown value of specific gravity Sg_t at some given temperature T as follows:

$$Sg_t = Sg_{60} + a \times (60 - T) \tag{6.4}$$

The constant a in Eq. (6.4) depends on the particular liquid and represents the slope of the specific gravity versus temperature line for that product. Figure 6.1 shows the variation of specific gravity with temperature for various petroleum liquids.

Example 6.2 The specific gravity of kerosene at 60°F is 0.815. Calculate its specific gravity at 75°F, given that the constant a in Eq. (6.4) is 0.0001.

Solution Using Eq. (6.4) we calculate

$$Sg = 0.815 + 0.0001 \times (60 - 75) = 0.8135$$

Therefore, the specific gravity of kerosene at 75°F is 0.8135.

6.2 Specific Gravity of Blended Products

The specific gravity of a mixture of two or more petroleum products can be calculated fairly easily using the weighted-average method. Since weight is the product of volume and specific weight and the total weight of the mixture is equal to the sum of the component weights, we can write the following equation for the specific gravity of a blend of two or more products, assuming a homogenous mixture.

$$Sg_{\text{blend}} = \frac{(Sg_1 \times \text{pct}_1) + (Sg_2 \times \text{pct}_2) + \cdots}{100} \tag{6.5}$$

where Sg_1 and Sg_2 are the specific gravities, respectively, of the liquids with percentage volumes of pct_1 and pct_2 and Sg_{blend} is the specific gravity of the mixture.

Example 6.3 A mixture consists of 20 percent of light crude of 35 API gravity and 80 percent of heavy crude of 25 API gravity. Calculate the specific gravity and API gravity of the mixture.

Solution To use the specific gravity blending Eq. (6.5) we must convert API gravity to specific gravity,

$$\text{Specific gravity of light crude oil } Sg_1 = \frac{141.5}{131.5 + 35} = 0.8498$$

$$\text{Specific gravity of heavy crude oil } Sg_2 = \frac{141.5}{131.5 + 25} = 0.9042$$

Using Eq. (6.5), the specific gravity of the mixture is calculated as follows:

$$\mathrm{Sg}_{\mathrm{blend}} = \frac{(0.8498 \times 20) + (0.9042 \times 80)}{100} = 0.8933$$

The corresponding API gravity of the mixture, using Eq. (6.2), is

$$\mathrm{API}_{\mathrm{blend}} = \frac{141.5}{0.8933} - 131.5 = 26.9$$

6.3 Viscosity

Viscosity is a measure of a liquid's resistance to flow. Consider petroleum product flowing through a pipeline. Each layer of liquid flowing through the pipe exerts a certain amount of frictional resistance to the adjacent layer. This is illustrated in Fig. 6.2, where a velocity gradient is shown to exist across the pipe diameter.

According to Newton, the frictional shear stress between adjacent layers of the liquid is related to the flowing velocity across a section of the pipe as

$$\text{Shear stress} = \mu \times \text{velocity gradient}$$

or

$$\tau = \mu \frac{dv}{dy}$$

The *velocity gradient* is defined as the rate of change of liquid velocity along a pipe diameter. The proportionality constant μ in the preceding equation is referred to as the absolute, or dynamic viscosity. In SI units μ is expressed in poise [(dynes · s)/cm^2 or g/(cm · s)] or centipoise (cP). In USCS units absolute viscosity is expressed as (lb · s)/ft^2 or slug/(ft · s). However, centipoise is also used in calculations involving USCS units.

The viscosity of petroleum product, like the specific gravity, decreases with an increase in temperature, and vice versa. Typical viscosities of common petroleum products are listed in Table 6.2.

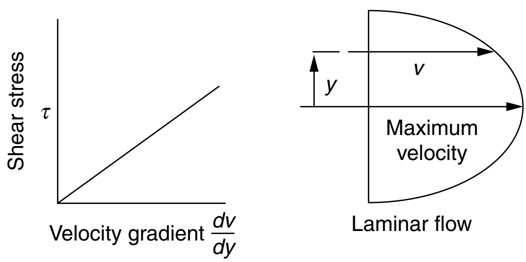

Figure 6.2 Viscosity and Newton's law.

TABLE 6.2 Viscosities of Petroleum Products

Product	Viscosity, cSt at 60°F
Regular gasoline	
Summer grade	0.70
Interseasonal grade	0.70
Winter grade	0.70
Premium gasoline	
Summer grade	0.70
Interseasonal grade	0.70
Winter grade	0.70
No. 1 fuel oil	2.57
No. 2 fuel oil	3.90
Kerosene	2.17
Jet fuel JP-4	1.40
Jet fuel JP-5	2.17

The absolute viscosity μ was defined earlier. Another term known as the *kinematic viscosity* of a liquid is defined as the absolute viscosity divided by the density. It is generally represented by the symbol ν. Therefore,

$$\text{Kinematic viscosity } \nu = \frac{\text{absolute viscosity } \mu}{\text{density } \rho}$$

In USCS units kinematic viscosity is measured in ft^2/s. In SI units, kinematic viscosity is expressed as m^2/s, stokes, or centistokes (cSt). However, centistoke units are also used in calculations involving USCS units. One stoke equals 1 cm^2/s. In SI units, absolute viscosity and kinematic viscosity are related simply by specific gravity as follows:

$$\text{Kinematic viscosity (cSt)} = \frac{\text{absolute viscosity (cP)}}{\text{specific gravity}}$$

In the petroleum industry kinematic viscosity is also expressed in terms of seconds Saybolt Universal (SSU) or seconds Saybolt Furol (SSF). These do not actually represent the physical concept of viscosity but rather a relative measure of how difficult or how easily the liquid flows. In fact both SSU and SSF represent the time taken for a fixed volume [usually 60 milliliters (mL)] of liquid to flow through a specified orifice as measured in a lab. Thus the viscosity of Alaskan North Slope (ANS) crude may be reported as 200 SSU at 60°F. This simply means that in a laboratory a 60-mL sample of ANS crude at 60°F took 200 seconds (s) to flow through a specified orifice. In comparison lighter crude may take only 80 seconds to flow through the same orifice at the same temperature. Therefore the lighter crude has a viscosity of 80 SSU.

The kinematic viscosity of a liquid may thus be expressed in cSt, SSU, or SSF. The equations to convert between these units are given here.

To convert viscosity from SSU to centistokes:

$$\text{Centistokes} = \begin{cases} 0.226 \times \text{SSU} - \dfrac{195}{\text{SSU}} & \text{for } 32 \leq \text{SSU} \leq 100 \quad (6.6) \\ 0.220 \times \text{SSU} - \dfrac{135}{\text{SSU}} & \text{for SSU} > 100 \quad (6.7) \end{cases}$$

To convert viscosity from SSF to centistokes:

$$\text{Centistokes} = \begin{cases} 2.24 \times \text{SSF} - \dfrac{184}{\text{SSF}} & \text{for } 25 \leq \text{SSF} \leq 40 \quad (6.8) \\ 2.16 \times \text{SSF} - \dfrac{60}{\text{SSF}} & \text{for SSF} > 40 \quad (6.9) \end{cases}$$

To convert viscosity from centistokes to SSU, we have to solve for SSU from Eqs. (6.6) or (6.7). It can be seen that this is not very straightforward. We have to solve a quadratic equation in the unknown quantity SSU, as follows:

$$0.226(\text{SSU})^2 - c(\text{SSU}) - 195 = 0 \qquad \text{for } 32 \leq \text{SSU} \leq 100 \qquad (6.10)$$

$$0.220(\text{SSU})^2 - c(\text{SSU}) - 135 = 0 \qquad \text{for SSU} > 100 \qquad (6.11)$$

In both Eqs. (6.10) and (6.11) the viscosity in centistokes is represented by the variable c.

For example, if the value of viscosity is 10 cSt and we want to convert it to SSU, we need to first guess the answer so we can choose which one of Eqs. (6.10) and (6.11) we should use. The SSU value is generally about 5 times the cSt value. So a viscosity of 10 cSt will be approximately 50 SSU. Therefore we must use Eq. (6.10) since that is for SSU values between 32 and 100. So the solution for the conversion of 10 cSt to SSU will be found from

$$0.226(\text{SSU})^2 - 10(\text{SSU}) - 195 = 0$$

An example will illustrate the method.

Example 6.4

(a) The kinematic viscosity of Alaskan North Slope (ANS) crude oil at 60°F is 200 SSU. Express this viscosity in cSt. The specific gravity of ANS at 60°F is 0.895.

(b) If a light crude oil has a kinematic viscosity of 5.9 cSt, what is this viscosity in SSU?

(c) A heavy fuel oil has a viscosity of 350 SSF. Convert this viscosity to kinematic viscosity in centistokes. If the specific gravity of the fuel oil is 0.95, what is the absolute viscosity in cP?

Solution

(a) From Eq. (6.7) we convert SSU to cSt,

$$\text{Centistokes} = 0.220 \times 200 - \frac{135}{200} = 43.33 \text{ cSt}$$

(b) First we guess the SSU as $5 \times \text{cSt} = 30$ SSU. Then using Eq. (6.6) we get

$$5.9 = 0.226(\text{SSU}) - \frac{195}{\text{SSU}}$$

Simplifying,

$$0.226(\text{SSU})^2 - 5.9(\text{SSU}) - 195 = 0$$

Solving the quadratic equation for SSU, we get

$$\text{SSU} = \frac{5.9 \pm \sqrt{(5.9)^2 + 4 \times 195 \times 0.226}}{2 \times 0.226} = \frac{5.9 \pm 14.53}{0.452}$$

or, taking the positive value of the solution,

$$\text{SSU} = 45.20$$

(c) Using Eq. (6.9) to convert SSF to centistokes,

$$\text{Centistokes} = 2.16(350) - \frac{60}{350} = 756 \text{ cSt}$$

The viscosity of a liquid decreases as the temperature increases, similar to the specific gravity. However, even in the normal range of temperature, unlike specific gravity, the viscosity variation with temperature is nonlinear. Several correlations have been proposed to calculate viscosity variation with temperature. The ASTM D341 method uses a log-log correlation that can be used to plot the viscosity versus temperature on a special graph paper. The temperatures and viscosities are plotted on a graph paper with logarithmic scales on each axis.

Sometimes, the viscosity ν in centistokes of a petroleum product and its absolute temperature T may be represented by the following equation:

$$\log_e \nu = A - B(T) \tag{6.12}$$

where A and B are constants that depend on the petroleum product and T is the absolute temperature in °R (°F + 460) or K (°C + 273).

Based on relationship (6.12), a graph of $\log_e \nu$ plotted against temperature T will be a straight line. The slope of the line will be represented by the constant B, and the intercept on the vertical axis would be the constant A. In fact, A would represent the log (viscosity) at the temperature $T = 0$.

If we are given two sets of viscosity values corresponding to two different temperatures, from lab data we could substitute those values in Eq. (6.12) and find the constants A and B for the particular petroleum product. Having calculated A and B, we will then be able to calculate the viscosity of the product at any other temperature using Eq. (6.12). We will explain this method using an example.

Example 6.5 A petroleum oil has the following viscosities at the two temperatures:

$$\text{Viscosity at } 60^\circ\text{F} = 43 \text{ cSt}$$

$$\text{Viscosity at } 100^\circ\text{F} = 10 \text{ cSt}$$

We are required to find the viscosity versus temperature correlation and calculate the viscosity of this oil at 80°F.

Solution Using Eq. (6.12), substituting the given pairs of temperature-viscosity data, we get two equations to solve for A and B as follows:

$$A - B(60 + 460) = \log_e 43$$

$$A - B(100 + 460) = \log_e 10$$

Solving these equations, we get the following values for the constants A and B:

$$A = 22.72 \qquad B = 0.0365$$

We can now calculate the viscosity of this liquid at any temperature from Eq. (6.12). To calculate the viscosity at 80°F, substitute the temperature in the equation as follows:

$$\log_e \nu = 22.72 - 0.0365(80 + 460)$$

Solving for viscosity, we get

$$\text{Viscosity at } 80^\circ\text{F} = 20.35 \text{ cSt}$$

In addition to the simple logarithmic relationship previously described for viscosity versus temperature, other empirical correlations have been put forth by several researchers. One of the more popular formulas is the ASTM method of calculating the viscosities of petroleum products. Using this approach, also known as the ASTM D341 method, a graph paper with logarithmic scales is used to plot the temperature versus viscosity of a liquid at two known temperatures. From two pairs of data plotted on the log-log paper, a straight line is drawn connecting them. The viscosity at any intermediate temperature can then be interpolated. Sometimes, viscosity may also be extrapolated from this chart, beyond

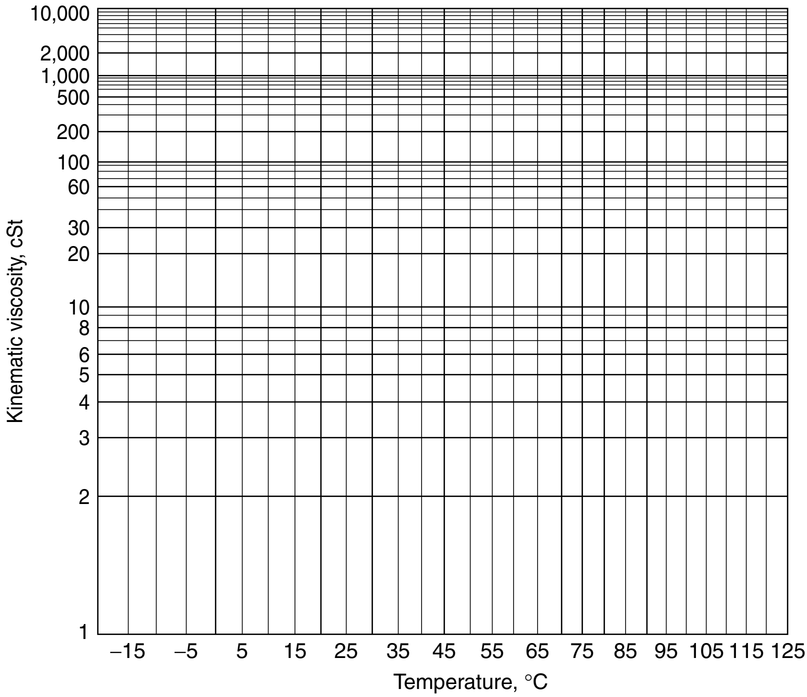

Figure 6.3 ASTM D341—Viscosity temperature chart.

the temperature range used. The ASTM viscosity versus temperature chart is shown in Fig. 6.3.

For viscosity variations with temperature, using the ASTM method, the following analytical method may be used. Here the relationship between viscosity and temperature is given by a log log equation as follows:

$$\log\log Z = A - B\ \log T \tag{6.13}$$

where log is the logarithm to base 10 and Z is a parameter that depends on the kinematic viscosity of the liquid ν in centistokes and T is the absolute temperature in °R or K. As before, the constants A and B depend on the specific petroleum product.

The parameter Z depends on the liquid viscosity as follows:

$$Z = \nu + 0.7 + C - D \tag{6.14}$$

where C and D are further parameters that depend on the viscosity as follows:

$$C = \exp(-1.14883 - 2.65868\nu) \tag{6.15}$$

$$D = \exp(-0.0038138 - 12.5645\nu) \tag{6.16}$$

where exp(x) represents the value of e^x where e is the base of natural logarithms and numerically $e = 2.71828$.

If we are given two sets of temperature-viscosity data, we can substitute those values in Eqs. (6.14) to (6.16) and calculate the pair of values for the parameters C, D, and Z. Next we can substitute the two sets of temperature and Z values in Eq. (6.13) to calculate the values of the constants A and B. Once we know A and B we can calculate the viscosity at any other temperature using Eq. (6.13). We will illustrate this method using an example.

Example 6.6 A certain petroleum product has temperature versus viscosity data obtained from a lab as follows:

Temperature, °F	60	180
Viscosity, cSt	750	25

(a) Determine the viscosity versus temperature relationship for this product based on the ASTM equations (6.14) to (6.16).

(b) Calculate the viscosity of this liquid at 110°F.

Solution

(a) First calculate the values of C, D, and Z at 60°F using Eqs. (6.14) through (6.16):

$$C_1 = \exp(-1.14883 - 2.65868 \times 750) = 0$$

$$D_1 = \exp(-0.0038138 - 12.5645 \times 750) = 0$$

$$Z_1 = 750 + 0.7 = 750.7$$

Next we repeat these calculations using the 180°F data. The values of C, D, and Z at 180°F are

$$C_2 = \exp(-1.14883 - 2.65868 \times 25) = 0$$

$$D_2 = \exp(-0.0038138 - 12.5645 \times 25) = 0$$

$$Z_2 = 25 + 0.7 = 25.7$$

Next, use the two sets of Z values at the two temperatures in Eq. (6.13) to produce two equations in A and B as follows:

$$\log\log 750.7 = A - B\ \log(60 + 460)$$

$$\log\log 25.7 = A - B\ \log(180 + 460)$$

Simplifying, these equations become,

$$0.4587 = A - 2.716B$$

and

$$0.1492 = A - 2.8062B$$

The values of A and B can now be found by solving the preceding two simultaneous equations, to yield

$$A = 9.78 \qquad B = 3.43$$

Therefore, the viscosity versus temperature relationship for this product is

$$\log\log Z = A - B \log T$$

where Z is a parameter that depends on viscosity in cSt, T is the absolute temperature in °F, and the logarithms are to base 10.

(b) At a temperature of 110°F using the equation generated in part (a), we get

$$\log\log Z = A - B \log(110 + 460)$$

Substituting the values of A and B, we have

$$\log\log Z = 9.78 - 3.43 \times 2.7559 = 0.3273$$

Solving for Z we get

$$Z = 133.26$$

The viscosity at 110°F is then found from Eq. (6.14) as

$$\text{Viscosity} = 133.26 - 0.7 = 132.56\,\text{cSt}$$

Example 6.7 A crude oil has a dynamic viscosity of 30 cP at 20°C. Calculate its kinematic viscosity in SI units. The density is 0.85 gram per cubic centimeter (g/cm^3).

Solution Since the density in g/cm^3 is numerically the same as specific gravity,

$$\begin{aligned}\text{Kinematic viscosity (cSt)} &= \frac{\text{absolute viscosity (cP)}}{\text{specific gravity}}\\ &= \frac{30.0}{0.85}\\ &= 35.29\ \text{cSt}\end{aligned}$$

Example 6.8 The viscosity of a typical crude oil was measured at two different temperatures as follows:

Temperature, °F	60	100
Viscosity, cSt	35	15

Using the ASTM method of correlation and the log log equations (6.14) to (6.16), calculate the viscosity of this oil at 75°F.

Solution First calculate the values of C, D, and Z at 60°F using Eqs. (6.14) through (6.16):

$$C_1 = \exp(-1.14883 - 2.65868 \times 35) = 0$$
$$D_1 = \exp(-0.0038138 - 12.5645 \times 35) = 0$$
$$Z_1 = 35 + 0.7 = 35.7$$

Next we repeat these calculations using the 100°F data. The values of C, D, and Z at 100°F are

$$C_2 = \exp(-1.14883 - 2.65868 \times 15) = 0$$
$$D_2 = \exp(-0.0038138 - 12.5645 \times 15) = 0$$
$$Z_2 = 15 + 0.7 = 15.7$$

Next, use the two sets of Z values at the two temperatures in Eq. (6.13) to produce two equations in A and B as follows:

$$\log\log 35.7 = A - B \log(60 + 460)$$
$$\log\log 15.7 = A - B \log(100 + 460)$$

Solving for A and B we get

$$A = 9.7561 \qquad \text{and} \qquad B = 3.5217$$

The viscosity of the oil at 75°F using Eq. (6.13) is

$$\log\log Z = 9.7561 - 3.5217 \times \log(75 + 460)$$

Solving for Z we get

$$Z = 25.406$$

Therefore the viscosity at 75°F using Eq. (6.14) is

$$\text{Viscosity} = Z - 0.7 = 24.71 \text{ cSt}$$

6.4 Viscosity of Blended Products

The viscosity of a mixture of two or more petroleum products can be calculated using one of two methods. Viscosity, unlike specific gravity, is a nonlinear property. Therefore we cannot use a weighted-average method to calculate the viscosity of a mixture of two or more liquids. For example, 20 percent of a liquid with 10 cSt viscosity when blended with 80 percent of a liquid of 20 cSt viscosity will not result in the following weight-averaged viscosity:

$$\text{Viscosity} = \frac{(10 \times 20) + (20 \times 80)}{100}$$
$$= 18 \text{ cSt}$$

This viscosity of mixture is incorrect. We will now show how to calculate the viscosity of the blend of two or more liquids using an empirical method. The viscosity of a mixture of petroleum products can be calculated using the following formula:

$$\sqrt{V_b} = \frac{Q_1 + Q_2 + \cdots}{(Q_1/\sqrt{V_1}) + (Q_2/\sqrt{V_2}) + \cdots} \tag{6.17}$$

where V_b = viscosity of blend, SSU
Q_1, Q_2, etc. = volumes of each liquid component
V_1, V_2, etc. = viscosity of each liquid component, SSU

Note that in Eq. (6.17) for calculating the viscosity of a mixture or a blend of multiple liquids, all viscosities must be in SSU. If the viscosities of the liquids are given in cSt, we must first convert the viscosities from cSt to SSU before using the equation to calculate the blended viscosity. Also the minimum viscosity that can be used is 32 SSU, equivalent to 1.0 cSt which happens to be the viscosity of water.

Another method for calculating the viscosity of a mixture of products is using the so-called *blending index*. It has been used in the petroleum pipeline industry for many years. Using this method involves calculating a parameter called the blending index for each liquid based on its viscosity. Next, from the component blending index, the blending index of the mixture is calculated using the weighted average of the composition of the mixture. Finally, the viscosity of the mixture is calculated from the blending index of the mixture. The calculation method is as follows:

$$H = 40.073 - 46.414 \log\log(\nu + A) \tag{6.18}$$

$$A = \begin{cases} 0.931(1.72)^{\nu} & \text{for } 0.2 < \nu < 1.5 \quad (6.19) \\ 0.6 & \text{for } \nu \geq 1.5 \quad (6.20) \end{cases}$$

$$H_m = \frac{H_1(\text{pct}_1) + H_2(\text{pct}_2) + \cdots}{100} \tag{6.21}$$

where H, H_1, H_2, etc. = blending index of the liquids
H_m = blending index of the mixture
A = constant in blending index equation
ν = viscosity, cSt
pct_1, pct_2, etc.... = percentage of liquids 1,2, etc., in the mixture
log = logarithm to base 10

Another method to calculate the blended viscosities of two or more petroleum products is the ASTM D341-77 method which employs a graphical approach. Two products at a time are considered and can be extended to more products, taking the blended properties of the first two products and combining with the third, etc. In this method, a special logarithmic graph paper with viscosity scales on the left and right sides of the paper and the percentage of the two products listed on the horizontal axis is used. This is shown in Fig. 6.4. This chart is also available in many handbooks such as the Hydraulic Institute's *Engineering Data Book*. Using this method requires that the viscosities of all products be in SSU and at the same temperature.

For more than two liquids, the blended viscosity of two product at a time is calculated and the process is then repeated for additional

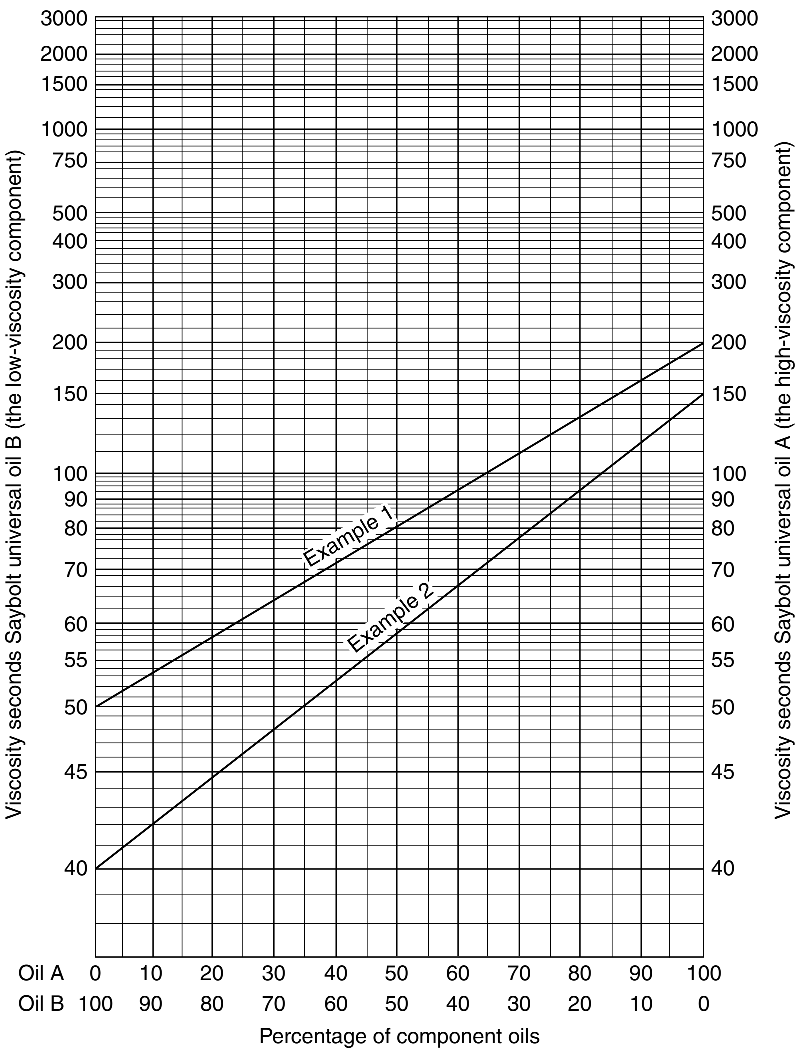

Figure 6.4 Viscosity blending chart.

products, combining the third product with the mixture of the first two products, and so on. Therefore if three products are to be blended in the ratios of 10, 30, and 60 percent, we would first calculate the viscosity of the blend of the first two liquids considering 10 parts of liquid A blended with 30 parts of liquid B. Therefore we would calculate the blend viscosity based on one-fourth of liquid A and three-fourths of liquid B. Next, we would calculate the blend of this mixture combined with liquid C in the proportions of 40 and 60 percent, respectively.

Example 6.9 Calculate the blended viscosity of a liquid consisting of a mixture of 15 percent of liquid A with 85 percent of liquid B. The liquids A and B have a viscosity of 12 and 23 cSt, respectively, at 60°F.

Solution For liquid A, the viscosity of 12 cSt is converted to SSU as follows. Since 12 cSt is estimated to be approximately 12 × 5 = 60 SSU, we use Eq. (6.6):

$$\text{Centistokes} = 0.226 \times \text{SSU} - \frac{195}{\text{SSU}} \qquad \text{for } 32 \leq \text{SSU} \leq 100$$

Substituting the 12 cSt in the preceding equation and rearranging, we get

$${\nu_A}^2 - \frac{12}{0.226}\nu_A - \frac{195}{0.226} = 0$$

Solving this quadratic equation;

$$\nu_A = 66.14 \text{ SSU}$$

Next the viscosity of liquid B (23 cSt) is converted to SSU using Eq. (6.7) as follows:

$${\nu_B}^2 - \frac{23}{0.22}\nu_B - \frac{135}{0.22} = 0$$

Solving we get

$$\nu_B = 110.12 \text{ SSU}$$

To calculate the blended viscosity we use Eq. (6.17):

$$\sqrt{\nu_{\text{blend}}} = \frac{15 + 85}{(15/\sqrt{66.14}) + (85/\sqrt{110.12})} = 10.06$$

Therefore the viscosity of the mixture is

$$\nu_{\text{blend}} = 101.12 \text{ SSU}$$

Converting this viscosity to cSt using Eq. (6.7),

$$\text{Centistokes} = 0.220 \times \text{SSU} - \frac{135}{\text{SSU}} \qquad \text{for SSU} > 100$$

$$= 0.22 \times 101.12 - \frac{135}{101.12} = 20.91$$

Thus the viscosity of the mixture is 20.91 cSt.

6.5 Bulk Modulus

The bulk modulus of a liquid indicates the compressibility of the liquid. Even though most petroleum liquids are incompressible for all practical purposes, this property becomes significant in some instances of liquid flow through pipelines. *Bulk modulus* is generally defined as the pressure required to produce a unit change in volume. If the volume is V and a pressure of ΔP causes a volume change of ΔV, the bulk modulus becomes

$$K = \frac{V \Delta P}{\Delta V} \tag{6.22}$$

where the ratio $\Delta V/V$ represents the change in volume divided by the original volume. In other words, it is the fractional change in volume generated by the pressure change ΔP. If the ratio $\Delta V/V$ becomes equal to 1.0, then numerically, the bulk modulus equals the value of ΔP from Eq. (6.22). For most petroleum products the bulk modulus K is in the range of 200,000 to 400,000 psi (29 to 58 GPa in SI units). There are two distinct values of bulk modulus defined in practice. The isothermal bulk modulus is measured at a constant temperature, while the adiabatic bulk modulus is based on adiabatic conditions (no heat transfer).

The bulk modulus is used in flow measurements of petroleum products and in line pack calculations of long-distance pipelines. The following equations are used to calculate the bulk modulus of a petroleum product, based on the API gravity, pressure, and temperature. Adiabatic bulk modulus K_a is calculated from

$$K_a = A + BP - C(T)^{1/2} - D(\mathrm{API}) - E(\mathrm{API})^2 + FT(\mathrm{API}) \tag{6.23}$$

where $A = 1.286 \times 10^6$
$B = 13.55$
$C = 4.122 \times 10^4$
$D = 4.53 \times 10^3$
$E = 10.59$
$F = 3.228$
$P =$ pressure, psig
$T =$ temperature, °R
API = API gravity of liquid

The isothermal bulk modulus K_i is calculated from

$$K_i = A + BP - C(T)^{1/2} + D(T)^{3/2} - E(\mathrm{API})^{3/2} \tag{6.24}$$

where $A = 2.619 \times 10^6$
$B = 9.203$
$C = 1.417 \times 10^5$

$$D = 73.05$$
$$E = 341.0$$
$$P = \text{pressure, psig}$$
$$T = \text{temperature, } ^\circ\text{R}$$
$$\text{API} = \text{API gravity of liquid}$$

Example 6.10 A typical crude oil has an API gravity of 35°. If the pressure is 1200 psig and the temperature of the crude is 75°F, calculate the bulk modulus.

Solution From Eq. (6.23), the adiabatic bulk modulus is

$$K_a = A + B(P) - C(T)^{1/2} - D(\text{API}) - E(\text{API})^2 + F(T)(\text{API})$$

Therefore,

$$K_a = 1.286 \times 10^6 + 13.55 \times 1200 - 4.122 \times 10^4 \times (75 + 460)^{1/2} - 4.53 \times 10^3 \times 35 - 10.59 \times (35)^2 + 3.228 \times (75 + 460)(35)$$

or

$$K_a = 237{,}760 \text{ psi}$$

From Eq. (6.24), the isothermal bulk modulus is

$$K_i = A + B(P) - C(T)^{1/2} + D(T)^{3/2} - E(\text{API})^{3/2}$$

Therefore,

$$K_i = 2.619 \times 10^6 + 9.203 \times (1200) - 1.417 \times 10^5 \times (75 + 460)^{1/2} + 73.05 \times (75 + 460)^{3/2} - 341.0 \times (35)^{3/2}$$

or

$$K_i = 186{,}868$$

In summary,

$$\text{Adiabatic bulk modulus} = 237{,}760 \text{ psi}$$
$$\text{Isothermal bulk modulus} = 186{,}868 \text{ psi}$$

6.6 Vapor Pressure

Vapor pressure is an important property of petroleum liquids when dealing with storage tanks and centrifugal pumps. Depending upon the location of petroleum product storage tanks, local air quality regulations require certain types of seals around floating roof tanks. These seal designs depend upon the vapor pressure of the liquid in the storage tank. Also, careful analysis of centrifugal pump suction piping used for higher vapor pressure liquids is required in order to prevent cavitation damage to pump impellers at low suction pressures.

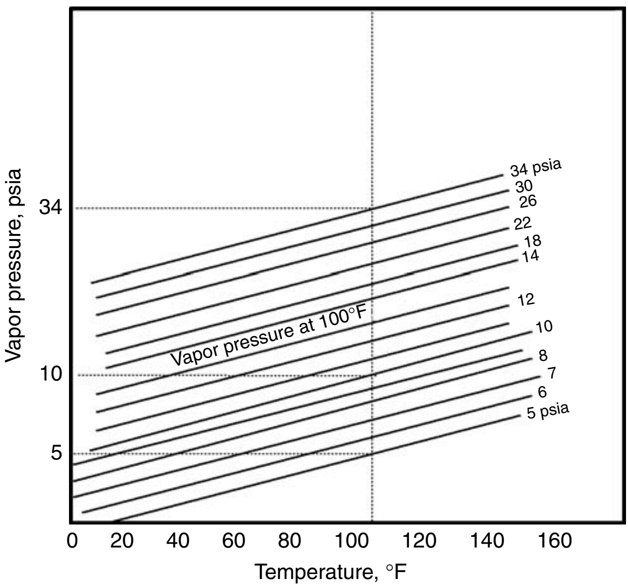

Figure 6.5 Vapor pressure chart for various petroleum products.

The *vapor pressure* may be defined as the pressure at a particular temperature when the liquid and its vapors are in equilibrium, under boiling conditions. When pumping petroleum products through a pipeline, the pressure at any point along the pipeline must be maintained above the vapor pressure of the liquid at the pumping temperature. This will ensure that the petroleum product will remain in the liquid phase throughout. Otherwise liquid may vaporize at some points and two-phase flow may occur that will cause damage to pumping equipment.

Vapor pressure is measured in the laboratory at a standard temperature of 100°F and is referred to as the Reid vapor pressure. ASTM specifications outline the laboratory method of determining this value. Once we know the Reid vapor pressure, we can calculate the vapor pressure at the operating temperature, such as 60°F or 70°F . Charts are available to determine the actual vapor pressure of a petroleum product at storage temperature from a given value of Reid vapor pressure. Figure 6.5 shows a sample vapor pressure chart for various petroleum products.

6.7 Pressure

Pressure within a body of fluid is defined as the force per unit area. In USCS units, pressure is measured in lb/in^2 (psi) and in SI units it is measured in N/m^2 or pascals (Pa). Other units for pressure include lb/ft^2, kPa, MPa, GPa, kg/cm^2, and bar.

The pressure at any point within a liquid is the same in all directions. The actual value of pressure at a point changes with the location of the point within the liquid. Consider a storage tank with the liquid surface exposed to the atmosphere. At all points along the surface of the liquid the pressure is equal to the atmospheric pressure (usually 14.7 psi at sea level or 1 bar in SI units). As we move vertically down through the liquid, the pressure at any point within the liquid is equal to the atmospheric pressure plus the intensity of pressure due to the depth below the free surface. This is defined as the *absolute pressure* since it includes the atmospheric pressure. If we neglect the atmospheric pressure, the pressure within the liquid is termed the *gauge pressure*. Since the atmospheric pressure is present everywhere, it is customary to ignore this and to refer to pressure in gauge pressure.

Returning to the example of the pressure within a storage tank, if the location is at a depth H below the free surface of the liquid, the pressure is equal to the column of liquid of height h acting over a unit cross-sectional area. If the specific weight of the liquid is γ lb/ft^3 and if we consider a cylindrical volume of cross-sectional area A ft^2 and height h ft the pressure at a depth of h is calculated as follows:

$$\text{Pressure } P = \frac{h \times A \times \gamma}{A} = \gamma H \quad \text{lb/ft}^2$$

Converting to the USCS unit of psi,

$$P = \frac{\gamma h}{144} \text{ psi}$$

This is the gauge pressure. The absolute pressure would be $(\gamma h/144) + P_{\text{atm}}$ where P_{atm} is the atmospheric pressure.

More generally we can state that the absolute pressure is

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}$$

The unit for absolute pressure is designated as psia, and the unit for gauge pressure is psig. Since the pressure for most petroleum product applications is measured by gauges, this unit is assumed. Unless otherwise specified, psi means gauge pressure.

Consider a numerical example based on the preceding. At a depth of 50 ft below the free surface of a petroleum (specific gravity = 0.85) storage tank the pressure in the liquid is calculated as follows:

$$\begin{aligned}\text{Pressure} &= \text{weight of 50-ft column of liquid acting on an area 1 in}^2\\ &= 50 \times \left(0.85 \times \frac{62.4}{144}\right) = 18.4 \text{ psig}\end{aligned}$$

we have assumed 62.4 lb/ft^3 as the specific weight of water.

Liquid pressure may also be expressed as head pressure, in which case it is expressed in feet of liquid head (or meters in SI units). Therefore, a pressure of 1000 psi in crude oil of specific gravity 0.895 is said to be equivalent to a pressure head of

$$h = \frac{1000 \times 144}{62.4 \times 0.895} = 2578.4 \text{ ft}$$

In a more general form, the pressure P in psi and liquid head h in feet for a specific gravity of Sg are related by

$$P = \frac{h \times \text{Sg}}{2.31} \tag{6.25}$$

In SI units, pressure P in kPa and head h in meters are related by the following equation:

$$P = \frac{h \times \text{Sg}}{0.102} \tag{6.26}$$

Example 6.11 Calculate the pressure in psi at a depth of 40 ft in a crude oil tank assuming 56.0 lb/ft^3 for the specific weight of crude oil. What is the equivalent pressure in kPa? If the atmospheric pressure is 14.7 psi, calculate the absolute pressure at that depth.

Solution Using Eq. (6.25),

$$\text{Pressure} = \frac{56.0/62.4 \times 40}{2.31} = 15.54 \text{ psig}$$

Thus,

$$\text{Pressure at depth 40 ft} = 15.54 \text{ psig}$$

$$\text{Absolute pressure} = 15.54 + 14.7 = 30.24 \text{ psia}$$

In SI units we can calculate the pressures as follows. Since 1 kPa = 0.145 psi,

$$\text{Pressure at depth 40 ft} = \frac{15.54 \text{ psig}}{0.145 \text{ psi/kPa}} = 107.2 \text{ Pa (gauge)}$$

6.8 Velocity

The speed at which a petroleum product flows through a pipeline, also referred to as velocity, is an important parameter in pipeline pressure drop calculations. The velocity of flow depends on the pipe diameter and flow rate. If the flow rate is constant throughout the pipeline (steady flow) and the pipe diameter is uniform, the velocity at every cross section along the pipe will be a constant value. However, there is a variation in velocity along the pipe cross section. The velocity at the pipe wall will be zero, increasing to a maximum at the centerline of the pipe. This is illustrated in Fig. 6.6.

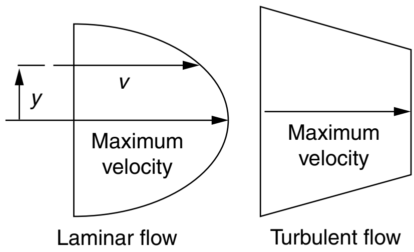

Figure 6.6 Velocity variation—laminar and turbulent.

We can define an average velocity of flow at any cross section of the pipe as follows:

$$\text{Velocity} = \frac{\text{flow rate}}{\text{area of flow}}$$

If the flow rate is in ft^3/s and the pipe cross-sectional area is in ft^2, the velocity from the preceding equation is in ft/s.

Consider liquid flowing through a circular pipe of internal diameter D at a flow rate of Q. Then the average flow velocity is

$$v = \frac{Q}{\pi D^2/4} \tag{6.27}$$

Employing commonly used units of flow rate Q in ft^3/s and pipe diameter in inches, the velocity in ft/s is as follows:

$$v = \frac{144Q}{\pi D^2/4}$$

Simplifying to

$$v = 183.3461\,\frac{Q}{D^2} \tag{6.28}$$

where the flow rate Q is in ft^3/s and the pipe inside diameter is in inches.

In petroleum transportation, flow rates are usually expressed in bbl/h, bbl/day, or gal/min. Therefore Eq. (6.28) for velocity can be modified in terms of more conventional pipeline units as follows. For flow rate in bbl/h:

$$v = 0.2859\,\frac{Q}{D^2} \tag{6.29}$$

where v = velocity, ft/s
Q = flow rate, bbl/h
D = pipe inside diameter, in

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For flow rate in bbl/day:

$$v = 0.0119 \frac{Q}{D^2} \tag{6.30}$$

where v = velocity, ft/s
Q = flow rate, bbl/day
D = pipe inside diameter, in

For flow rate in gal/min:

$$v = 0.4085 \frac{Q}{D^2} \tag{6.31}$$

where v = velocity, ft/s
Q = flow rate, gal/min
D = pipe inside diameter, in

In SI units, the velocity equation is as follows:

$$v = 353.6777 \frac{Q}{D^2} \tag{6.32}$$

where v = velocity, m/s
Q = flow rate, m^3/h
D = pipe inside diameter, mm

Example 6.12 Diesel flows through an NPS 16 (15.5-in inside diameter) pipeline at the rate of 4000 gal/min. Calculate the average velocity for steady-state flow. (*Note*: The designation NPS 16 means nominal pipe size of 16 in.)

Solution From Eq. (6.31) the average flow velocity is

$$v = 0.4085 \frac{4000}{15.5^2} = 6.80 \text{ ft/s}$$

Example 6.13 Gasoline flows through a DN 400 outside diameter (10-mm wall thickness) pipeline at 200 L/s. Calculate the average velocity for steady flow.

Solution The designation DN 400 in SI units corresponds to NPS 16 in USCS units. DN 400 means metric pipe size of 400-mm outside diameter. First convert the flow rate in L/s to m^3/h.

$$\text{Flow rate} = 200 \text{ L/s} = 200 \times 60 \times 60 \times 10^{-3} \text{ m}^3/\text{h} = 720 \text{ m}^3/\text{h}$$

From Eq. (6.32) the average flow velocity is

$$v = 353.6777 \frac{720}{380^2} = 1.764 \text{ m/s}$$

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The variation of flow velocity along the cross section of a pipe as depicted in Fig. 6.6 depends on the type of flow. In laminar flow, the velocity variation is parabolic. As the flow rate becomes turbulent, the velocity profile approximates a more trapezoidal shape as shown. Laminar and turbulent flows are discussed after we introduce the concept of the Reynolds number.

6.9 Reynolds Number

The Reynolds number of flow is a dimensionless parameter that depends on the pipe diameter liquid flow rate, liquid viscosity, and density. It is defined as follows:

$$R = \frac{vD\rho}{\mu} \tag{6.33}$$

or

$$R = \frac{vD}{\nu} \tag{6.34}$$

where R = Reynolds number, dimensionless
v = average flow velocity, ft/s
D = inside diameter of pipe, ft
ρ = mass density of liquid, slug/ft^3
μ = dynamic viscosity, slug/(ft · s)
ν = kinematic viscosity, ft^2/s

In terms of more commonly used units in the oil industry, we have the following versions of the Reynolds number equation:

$$R = 3162.5\,\frac{Q}{D\nu} \tag{6.35}$$

where R = Reynolds number, dimensionless
Q = flow rate, gal/min
D = inside diameter of pipe, in
ν = kinematic viscosity, cSt

In petroleum transportation units, the Reynolds number is calculated using the following equations:

$$R = 2213.76\,\frac{Q}{D\nu} \tag{6.36}$$

$$R = 92.24\,\frac{\text{BPD}}{D\nu} \tag{6.37}$$

where R = Reynolds number, dimensionless
Q = flow rate, bbl/h
BPD = flow rate, bbl/day
D = inside diameter of pipe, in
ν = kinematic viscosity, cSt

In SI units, the Reynolds number is expressed as follows

$$R = 353{,}678 \frac{Q}{\nu D} \tag{6.38}$$

where R = Reynolds number, dimensionless
Q = flow rate, m^3/h
D = inside diameter of pipe, mm
ν = kinematic viscosity, cSt

Example 6.14 A crude oil of specific gravity 0.85 and viscosity 10 cSt flows through an NPS 20 (0.375-in wall thickness) pipeline at 5000 gal/min. Calculate the average velocity and the Reynolds number of flow.

Solution The NPS 20 (0.375-in wall thickness) pipe has an inside diameter = $20.0 - 2 \times 0.375 = 19.25$ in. From Eq. (6.31) the average velocity is calculated first:

$$v = 0.4085 \frac{5000}{19.25^2} = 5.51 \text{ ft/s}$$

From Eq. (6.35) the Reynolds number is therefore

$$R = 3162.5 \frac{5000}{19.25 \times 10.0} = 82{,}143$$

Example 6.15 A petroleum product with a specific gravity of 0.815 and viscosity of 15 cSt flows through a DN 400 (10-mm wall thickness) pipeline at 800 m^3/h. Calculate the average flow velocity and the Reynolds number of flow.

Solution The DN 400 (10-mm wall thickness) pipe has an inside diameter = $400 - 2 \times 10 = 380$ mm. From Eq. (6.32) the average velocity is therefore

$$v = 353.6777 \frac{800}{380^2} = 1.96 \text{ m/s}$$

Next, from Eq. (6.38) the Reynolds number is

$$R = 353{,}678 \frac{800}{380 \times 15.0} = 49{,}639$$

6.10 Types of Flow

Flow through a pipeline is classified as laminar flow, turbulent flow, or critical flow depending on the magnitude of the Reynolds number of flow.

If the Reynolds number is less than 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is considered to be *turbulent*. *Critical flow* occurs when the Reynolds number is in the range of 2100 to 4000. Laminar flow is characterized by smooth flow in which no eddies or turbulence are visible. The flow is also said to occur in laminations. If dye was injected into a transparent pipeline, laminar flow would be manifested in the form of smooth streamlines of dye. Turbulent flow occurs at higher velocities and is accompanied by eddies and other disturbances in the liquid. More energy is lost in friction in the critical flow and turbulent flow regions as compared to the laminar flow region.

The three flow regimes characterized by the Reynolds number of flow are

$$\begin{aligned} &\text{Laminar flow}: && R \leq 2100 \\ &\text{Critical flow}: && 2100 < R \leq 4000 \\ &\text{Turbulent flow}: && R > 4000 \end{aligned}$$

In the critical flow regime, where the Reynolds number is between 2100 and 4000, the flow is undefined and unstable, as far as pressure drop calculations are concerned. In the absence of better data, it is customary to use the turbulent flow equation to calculate pressure drop in the critical flow regime as well.

6.11 Pressure Drop Due to Friction

As a liquid flows through a pipeline, energy is lost due to resistance between the flowing liquid layers as well as due to the friction between the liquid and the pipe wall. One of the objectives of pipeline calculation is to determine the amount of energy and hence the pressure lost due to friction as the liquid flows from the source to the destination. First we will introduce the equation for conservation of energy in liquid flow in a pipeline. After that we will cover the approach to calculating the frictional pressure drop or head loss calculations. We will begin by discussing Bernoulli's equation for the various forms of liquid energy in a flowing pipeline.

6.11.1 Bernoulli's equation

Bernoulli's equation is another way of stating the principle of conservation of energy applied to liquid flow through a pipeline. At each point along the pipeline the total energy of the liquid is computed by taking into consideration the liquid energy due to pressure, velocity, and elevation combined with any energy input, energy output, and energy losses. The total energy of the liquid contained in the pipeline at any

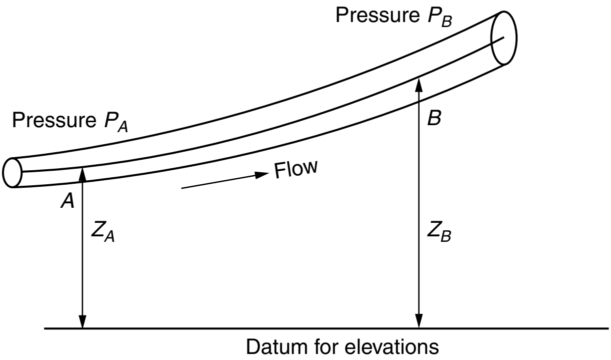

Figure 6.7 Total energy of liquid in pipe flow.

point is a constant. This is also known as the principle of conservation of energy.

Consider a liquid flow through a pipeline from point A to point B as shown in Fig. 6.7. The elevation of point A is Z_A and the elevation at B is Z_B above some common datum, such as mean sea level. The pressure at point A is P_A and that at B is P_B. It is assumed that the pipe diameter at A and B are different, and hence the flow velocity at A and B will be represented by V_A and V_B, respectively. A particle of the liquid of unit weight at point A in the pipeline possesses a total energy E which consists of three components:

$$\text{Potential energy} = Z_A$$

$$\text{Pressure energy} = \frac{P_A}{\gamma}$$

$$\text{Kinetic energy} = \frac{{v_A}^2}{2g}$$

where γ is the specific weight of liquid.

Therefore the total energy E is

$$E = Z_A + \frac{P_A}{\gamma} + \frac{{v_A}^2}{2g} \tag{6.39}$$

Since each term in Eq. (6.39) has dimensions of length, we refer to the total energy at point A as H_A in feet of liquid head. Therefore, rewriting the total energy in feet of liquid head at point A, we obtain

$$H_A = Z_A + \frac{P_A}{\gamma} + \frac{{v_A}^2}{2g} \tag{6.40}$$

Similarly, the same unit weight of liquid at point B has a total energy per unit weight equal to H_B given by

$$H_B = Z_B + \frac{P_B}{\gamma} + \frac{{v_B}^2}{2g} \tag{6.41}$$

By the principle of conservation of energy

$$H_A = H_B \tag{6.42}$$

Therefore,

$$Z_A + \frac{P_A}{\gamma} + \frac{{v_A}^2}{2g} = Z_B + \frac{P_B}{\gamma} + \frac{{v_B}^2}{2g} \tag{6.43}$$

In Eq. (6.43), referred to as Bernoulli's equation, we have not considered any energy added to the liquid, energy taken out of the liquid, or energy losses due to friction. Therefore, modifying Eq. (6.43) to take into account the addition of energy (such as from a pump at A) and accounting for frictional head losses h_f, we get the more common form of Bernoulli's equation as follows:

$$Z_A + \frac{P_A}{\gamma} + \frac{{v_A}^2}{2g} + H_p = Z_B + \frac{P_B}{\gamma} + \frac{{v_B}^2}{2g} + h_f \tag{6.44}$$

where H_P is the equivalent head added to the liquid by the pump at A and h_f represents the total frictional head losses between points A and B.

We will next discuss how the head loss due to friction h_f in Bernoulli's equation is calculated for various conditions of flow of petroleum products of water flow in pipelines. We begin with the classical pressure drop equation known as the Darcy-Weisbach equation, or simply the Darcy equation.

6.11.2 Darcy equation

As a petroleum product flows through a pipeline from point A to point B the pressure decreases due to frictional loss between the flowing liquid and the pipe. The extent of pressure loss due to friction, designated in feet of liquid, depends on various factors. These factors include the liquid flow rate, liquid specific gravity and viscosity, pipe inside diameter, pipe length, and internal condition of the pipe (rough, smooth, etc.). The Darcy equation may be used to calculate the pressure drop in a pipeline as follows:

$$h = f \frac{L}{D} \frac{v^2}{2g} \tag{6.45}$$

where h = frictional pressure loss, ft of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = inside pipe diameter, ft
v = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

Note that the Darcy equation gives the frictional pressure loss in feet of liquid head, which must be converted to pressure loss in psi using Eq. (6.25). The term $v^2/2g$ in the Darcy equation is called the velocity head, and it represents the kinetic energy of the liquid. The term *velocity head* will be used in subsequent sections of this chapter when discussing frictional head loss through pipe fittings and valves.

The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eq. (6.45). This friction factor is a nondimensional number between 0.0 and 0.1 that depends on the internal roughness of the pipe, the pipe diameter, and the Reynolds number of flow.

In laminar flow, the friction factor f depends only on the Reynolds number and is calculated from

$$f = \frac{64}{R} \tag{6.46}$$

where f is the friction factor for laminar flow and R is the Reynolds number for laminar flow ($R < 2100$) (dimensionless).

Therefore, if a particular flow has a Reynolds number of 1780 we can conclude that in this laminar flow condition the friction factor f to be used in the Darcy equation is

$$f = \frac{64}{1780} = 0.036$$

Some pipeline hydraulics texts may refer to another friction factor called the Fanning friction factor. This is numerically equal to one-fourth the Darcy friction factor. In this example the Fanning friction factor can be calculated as

$$\frac{0.036}{4} = 0.009$$

To avoid any confusion, throughout this chapter we will use only the Darcy friction factor as defined in Eq. (6.45).

In practical situations involving petroleum product pipelines it is inconvenient to use the Darcy equation in the form described in Eq. (6.45). We must convert the equation in terms of commonly used petroleum pipeline units. One form of the Darcy equation in pipeline units is as follows:

$$h = 0.1863 \, \frac{f L v^2}{D} \tag{6.47}$$

where h = frictional pressure loss, ft of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = pipe inside diameter, in
v = average flow velocity, ft/s

Another form of the Darcy equation with frictional pressure drop expressed in psi/mi and using a flow rate instead of velocity is as follows:

$$P_m = \text{const} \, \frac{f Q^2 \text{Sg}}{D^5} \tag{6.48}$$

where P_m = frictional pressure loss, psi/mi
f = Darcy friction factor, dimensionless
Q = flow rate, bbl/h
D = pipe inside diameter, in
Sg = liquid specific gravity
const = factor that depends on flow units

$$= \begin{cases} 34.87 & \text{for } Q \text{ in bbl/h} \\ 0.0605 & \text{for } Q \text{ in bbl/day} \\ 71.16 & \text{for } Q \text{ in gal/min} \end{cases}$$

In SI units, the Darcy equation may be written as

$$h = 50.94 \, \frac{f L v^2}{D} \tag{6.49}$$

where h = frictional pressure loss, m of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, m
D = pipe inside diameter, mm
v = average flow velocity, m/s

Another version of the Darcy equation in SI units is as follows:

$$P_{km} = (6.2475 \times 10^{10}) \left(\frac{f Q^2 \text{Sg}}{D^5} \right) \tag{6.50}$$

where P_{km} = pressure drop due to friction, kPa/km
Q = liquid flow rate, m^3/h
f = Darcy friction factor, dimensionless
Sg = liquid specific gravity
D = pipe inside diameter, mm

6.11.3 Colebrook-White equation

We have seen that in laminar flow the friction factor f is easily calculated from the Reynolds number as shown in Eq. (6.46). In turbulent flow, the calculation of friction factor f is more complex. It depends on the pipe inside diameter, the pipe roughness, and the Reynolds number. Based on work by Moody, Colebrook and White, and others, the following empirical equation, known as the Colebrook-White equation, has been proposed for calculating the friction factor in turbulent flow:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{R\sqrt{f}}\right) \tag{6.51}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
R = Reynolds number, dimensionless

The absolute pipe roughness, also known as internal pipe roughness, may range from 0.0 to 0.01 depending on the internal condition of the pipe. It is listed for common piping systems in Table 6.3. The ratio e/D is termed the relative roughness and is dimensionless. Equation (6.51) is also sometimes called simply the Colebrook equation.

In SI units, we can use the same form of the Colebrook equation. The absolute pipe roughness e and the pipe diameter D are both expressed in millimeters. All other terms in the equation are dimensionless.

TABLE 6.3 Pipe Internal Roughness

	Roughness	
Pipe material	in	mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

It can be seen from the Colebrook-White equation that the calculation of the friction factor f is not straightforward since it appears on both sides of the equation. This is known as an implicit equation in f, compared to an explicit equation. An explicit equation in f will have the unknown quantity f on one side of the equation. In the present case, a trial-and-error approach is used to solve for the friction factor. First an initial value for f is assumed (for example, $f = 0.01$) and substituted in the right-hand side of the Colebrook equation. This will result in a new calculated value of f, which is used as the next approximation and f recalculated based on this second approximation. The process is continued until successive values of f calculated by such iterations is within a small value such as 0.001. Usually three or four iterations will yield a satisfactory solution. There are other explicit equations for the friction factor proposed by many researchers, such as Churchill and Swamee-Jain that are easier to use than the Colebrook equation.

6.11.4 Moody diagram

A graphical method of determining the friction factor for turbulent flow is available using the Moody diagram shown in Fig. 6.8. First the Reynolds number is calculated based upon liquid properties, flow rate, and pipe diameter. This Reynolds number is used to locate the ordinate on the horizontal axis of the Moody diagram. A vertical line is drawn up to the curve representing the relative roughness e/D of the pipe. The friction factor is then read off of the vertical axis to the left. From the Moody diagram it is seen that the turbulent region is further divided into two regions: the "transition" zone and the "complete turbulence in rough pipes" zone. The lower boundary is designated as "smooth pipes." The transition zone extends up to the dashed line, beyond which is known as the zone of complete turbulence in rough pipes. In this zone, the friction factor depends very little on the Reynolds number and more on the relative roughness.

The *transmission factor* is a term that is used in conjunction with pressure drop and flow rate in pipelines. The transmission factor, a dimensionless number, is proportional to the flow rate, whereas the friction factor is inversely proportional to the flow rate. With a higher transmission factor, the flow rate is increased, whereas with a higher friction factor, flow rate decreases. The transmission factor F is inversely related to the Darcy friction factor f as follows:

$$F = \frac{2}{\sqrt{f}} \tag{6.52}$$

Examining the Moody diagram we see that the friction factor f ranges from 0.008 to 0.10. Therefore, from Eq. (6.52) we can conclude that

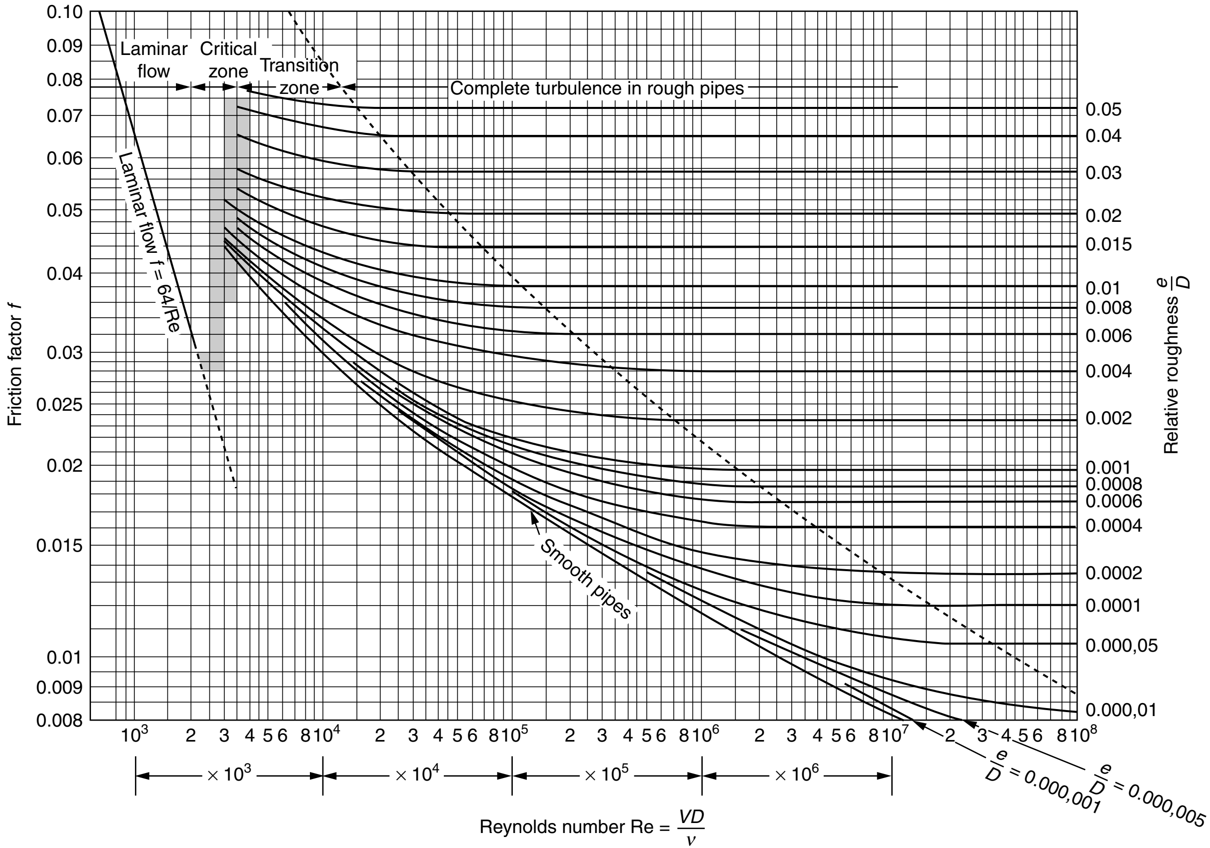

Figure 6.8 Moody diagram.

the transmission factor F will range between 6 and 22. Having introduced the transmission factor F we can now rewrite the Colebrook-White equation in terms of the transmission factor as

$$F = -4\log_{10}\left(\frac{e}{3.7D} + \frac{1.255F}{R}\right) \qquad \text{for turbulent flow } R > 4000 \quad (6.53)$$

As we did before with the friction factor f, the transmission factor F must also be calculated from Eq. (6.53) by successive iteration. We assume an initial value for F (for example, $F = 10.0$) and calculate a new value of F by substituting this initial value in the right-hand side of Eq. (6.53). This will result in a second approximation for F, which is then used to recalculate a better value of F. By successive iteration, a satisfactory value of F can be calculated.

The U.S. Bureau of Mines proposed a modified version of the Colebrook-White equation. This is expressed in terms of the transmission factor.

$$F = -4\log_{10}\left(\frac{e}{3.7D} + 1.4125\,\frac{F}{R}\right) \qquad \text{for turbulent flow } R > 4000 \quad (6.54)$$

By comparing the modified version in Eq. (6.54) with the original Colebrook-White equation (6.53), we see that the modified Colebrook-White equation uses the constant 1.4125 instead of 1.255. This modification causes a more conservative value of the transmission factor. In other words the modified Colebrook-White equation yields a higher pressure drop for the same flow rate compared to the original Colebrook-White equation.

Example 6.16 A petroleum oil with 0.85 specific gravity and 10 cSt viscosity flows through an NPS 16 (0.250-in wall thickness) pipeline at a flow rate of 4000 bbl/h. The absolute roughness of the pipe may be assumed to be 0.002 in. Calculate the Darcy friction factor and pressure loss due to friction in a mile of pipe length using the Colebrook-White equation. What is the transmission factor?

Solution The inside diameter of an NPS 16 (0.250-in wall thickness) pipe is

$$16.00 - 2 \times 0.250 = 15.50 \text{ in}$$

Next we will calculate the Reynolds number R to determine the flow regime (laminar or turbulent). The Reynolds number from Eq. (6.36) is

$$R = 2213.76\frac{4000}{15.5 \times 10.0} = 57{,}129$$

Since $R > 4000$, the flow is turbulent and we can use the Colebrook-White equation to calculate the friction factor. We can also use the Moody diagram to read the friction factor based on R and the pipe relative roughness e/D.

From the Colebrook-White equation (6.51), the friction factor f is

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.5} + \frac{2.51}{57{,}129\sqrt{f}}\right)$$

This equation must be solved for f by trial and error.

First assume that $f = 0.02$. Substituting in the preceding equation, we get a better approximation for f as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.5} + \frac{2.51}{57{,}129\sqrt{0.02}}\right) = 0.0209$$

Recalculating using this value

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.5} + \frac{2.51}{57{,}129\sqrt{0.0209}}\right) = 0.0208$$

And finally

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.5} + \frac{2.51}{57{,}129\sqrt{0.0208}}\right) = 0.0208$$

Thus $f = 0.0208$ is the solution. The transmission factor is

$$F = \frac{2}{\sqrt{f}} = 13.87$$

Next calculate the average flow velocity needed for the Darcy equation for head loss:

$$\text{Average flow velocity } V = 0.2859 \times \frac{4000}{(15.5)^2} = 4.76 \text{ ft/s} \qquad \text{from Eq. (6.29)}$$

The head loss due to friction can now be calculated using the Darcy equation (6.47), considering a mile of pipe:

$$h = 0.1863\left(0.0208 \times 5280 \times \frac{4.76^2}{15.5}\right)$$
$$= 29.908 \text{ ft of liquid head per mile of pipe}$$

Converting liquid head to pressure in psi using Eq. (6.25) we get

$$\text{Pressure drop } P_m = 29.908 \times \frac{0.85}{2.31} = 11.01 \text{ psi/mi}$$

We could have also calculated the pressure drop per mile directly in psi/mi using the version of the Darcy equation shown in Eq. (6.48).

$$P_m = 34.87 \times 0.0208 \times (4000)^2 \times \frac{0.85}{15.5^5}$$

Therefore,

$$P_m = 11.03 \text{ psi/mi}$$

The slight difference between the two values for P_m is due to rounding off in unit conversions. If we used the Moody diagram to find the friction factor, we would use the Reynolds number of 57,129 and the relative roughness $e/D = 0.002/15.5 = 0.000129$ and read the value of the friction factor $f = 0.021$ approximately. After that, the pressure drop calculation will still be the same as described previously.

Example 6.17 A DN 500 (10-mm wall thickness) steel pipe is used to transport gasoline from a refinery to a storage tank 15 km away. Neglecting any difference in elevations, calculate the friction factor and pressure loss due to friction (kPa/km) at a flow rate of 990 m^3/h. Assume an internal pipe roughness of 0.05 mm. A delivery pressure of 4 kPa must be maintained at the delivery point, and the storage tank is at an elevation of 200 m above that of the refinery. Calculate the pump pressure required at the refinery to transport the given volume of gasoline to the storage tank location. Assume the specific gravity of gasoline is 0.736 and the viscosity is 0.6 cSt.

Solution The DN 500 (10-mm wall thickness) pipe has an inside diameter of

$$D = 500 - 2 \times 10 = 480 \text{ mm}$$

First calculate the Reynolds number from Eq. (6.38):

$$R = \frac{353{,}678Q}{\nu D}$$

$$= \frac{353{,}678 \times 990}{0.6 \times 480} = 1{,}215{,}768$$

Therefore the flow is turbulent and we can use the Colebrook-White equation or the Moody diagram to determine the friction factor.

$$\text{Relative roughness } \frac{e}{D} = \frac{0.05}{480} = 0.0001$$

Using the preceding values for the relative roughness and Reynolds number, from the Moody diagram we get $f = 0.013$. The pressure drop due to friction can now be calculated using the Darcy equation (6.50):

$$P_{\text{km}} = (6.2475 \times 10^{10}) \left(0.013 \times 990^2 \times \frac{0.736}{480^5}\right)$$

$$= 22.99 \text{ kPa/km}$$

The pressure required at the pumping facility is calculated by adding the pressure drop due to friction to the delivery pressure required and the static elevation head between the pumping facility and storage tank. The static head difference is 200 m. This is converted to pressure in kPa, using Eq. (6.26),

$$\text{Pressure drop due to friction in 15 km of pipe} = 15 \times 22.99 = 344.85 \text{ kPa}$$

$$\text{Pressure due to elevation head} = 200 \times \frac{0.736}{0.102} = 1443.14 \text{ kPa}$$

$$\text{Minimum pressure required at delivery point} = 4 \text{ kPa}$$

Therefore adding all three numbers, the total pressure required at the refinery is

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}}$$

where P_t = total pressure required at pump
P_f = frictional pressure drop
P_{elev} = pressure head due to elevation difference
P_{del} = delivery pressure at storage tank

Therefore,

$$P_t = 344.85 + 1443.14 + 4.0 = 1792 \text{ kPa}$$

Thus the pump pressure required at the refinery is 1792 kPa.

6.11.5 Hazen-Williams equation

The Hazen-Williams equation has been used for the calculation of pressure drop in water pipelines and water distribution networks. This equation has also been successfully applied to the calculation of pressure drop in refined petroleum product pipelines, such as gasoline and diesel pipelines. Using the Hazen-Williams method a coefficient C, known as the Hazen-Williams C factor, is used to account for the internal pipe roughness or efficiency. Unlike the Moody diagram or Colebrook-White equation, the Hazen-Williams equation does not use the Reynolds number or viscosity of the liquid to calculate the pressure drop. The Hazen-Williams C factor is a number that is based on experience with a particular product and pipeline. For example, one product pipeline company may use $C = 125$ for diesel and $C = 150$ for gasoline. The higher the C factor, the higher will be the flow rate through the pipeline and the lower the pressure drop due to friction. It may be thought of as an opposite of the friction factor. The Hazen-Williams equation is not used for crude oil and heavier liquids. The Colebrook-White equation gives a better correlation with field data when applied to crude oil pipelines and heated oil pipelines.

The Hazen-Williams equation is generally expressed as follows

$$h = \frac{4.73\, L(Q/C)^{1.852}}{D^{4.87}} \tag{6.55}$$

where h = frictional head loss, ft of liquid head
L = length of pipe, ft
D = pipe inside diameter, ft
Q = flow rate, ft^3/s
C = Hazen-Williams C factor, dimensionless

TABLE 6.4 Hazen-Williams *C* Factor

Pipe material	*C* factor
Smooth pipes (all metals)	130–140
Cast iron (old)	100
Iron (worn/pitted)	60–80
Polyvinyl chloride (PVC)	150
Brick	100
Smooth wood	120
Smooth masonry	120
Vitrified clay	110

The values of the C factor for various applications are listed in Table 6.4. However, it must be noted that when applied to refined petroleum product pipelines these factors have to be adjusted based on experience, since these factors were originally intended for water pipelines.

On examining the Hazen-Williams equation, it can be seen that the head loss due to friction is calculated in feet of liquid head, similar to the Darcy equation. The value of the head loss h can be converted to psi using the head-to-psi conversion equation (6.25). Although using the Hazen-Williams equation appears to be simpler than using the Colebrook-White and Darcy equations to calculate the pressure drop, the unknown term C can cause uncertainties in the pressure drop calculation.

Usually, the C factor is determined based on experience with the particular liquid and the piping system. When designing a new petroleum product pipeline, using the Hazen-Williams equation, we must carefully select the C factor since considerable variation in pressure drop can occur by choosing a particular value of C compared to another. Because of the inverse proportionality effect of C on the head loss, using $C = 120$ instead of $C = 100$ will result in $[1 - (100/120)^{1.852}]$ or 29 percent less pressure drop. Therefore, it is important that the C value be chosen judiciously.

The Hazen-Williams equation (6.55) is not convenient to use when dealing with petroleum pipelines due to the units employed in the original form. Therefore, more acceptable forms of the Hazen-Williams equation have been used in practice. These modified versions of the equation use flow rates in gal/min, bbl/h, and bbl/day with pressure drops expressed in psi/mi and diameter in inches in USCS units. In the following formulas the presented Hazen-Williams equations have been rearranged to calculate the flow rate from a given pressure drop. The versions of the equations to calculate the pressure drop from a given flow rate are also shown.

A modified version of the Hazen-Williams equation in pipeline units is

$$Q = (6.755 \times 10^{-3})CD^{2.63}(h)^{0.54} \qquad (6.56)$$

where Q = flow rate, gal/min
h = friction loss, ft of liquid per 1000 ft of pipe
D = inside diameter of pipe, in
C = Hazen-Williams C factor, dimensionless

Other variants in petroleum pipeline units are as follows:

$$Q = (6.175 \times 10^{-3}) C D^{2.63} \left(\frac{P_m}{\mathrm{Sg}}\right)^{0.54} \tag{6.57}$$

$$P_m = 12{,}352 \left(\frac{Q}{C}\right)^{1.852} \frac{\mathrm{Sg}}{D^{4.87}} \tag{6.58}$$

and

$$P_f = 2339 \left(\frac{Q}{C}\right)^{1.852} \frac{\mathrm{Sg}}{D^{4.87}} \tag{6.59}$$

where Q = flow rate, bbl/h
D = pipe inside diameter, in
P_m = frictional pressure drop, psi/mi
P_f = frictional pressure drop, psi per 1000 ft of pipe length
Sg = liquid specific gravity
C = Hazen-Williams C factor, dimensionless

In SI units, the Hazen-Williams equation is expressed as follows:

$$Q = (9.0379 \times 10^{-8}) C D^{2.63} \left(\frac{P_{km}}{\mathrm{Sg}}\right)^{0.54} \tag{6.60}$$

and

$$P_{\mathrm{km}} = (1.1101 \times 10^{13}) \left(\frac{Q}{C}\right)^{1.852} \frac{Sg}{D^{4.87}} \tag{6.61}$$

where Q = flow rate, m^3/h
D = pipe inside diameter, mm
P_{km} = frictional pressure drop, kPa/km
Sg = liquid specific gravity (water = 1.00)
C = Hazen-Williams C factor, dimensionless

Example 6.18 Gasoline (specific gravity = 0.74 and viscosity = 0.7 cSt) flows through an NPS 16 (0.250-in wall thickness) pipeline at 4000 gal/min. Using the Hazen-Williams equation with a C factor of 150, calculate the pressure loss due to friction in a mile of pipe.

Solution The flow rate is

$$Q = 4000 \text{ gal/min} = \frac{4000 \times 60}{42} \text{ bbl/h} = 5714.29 \text{ bbl/h}$$

The NPS 16 (0.25-in wall thickness) pipeline has an inside diameter = $16 - 2 \times 0.25 = 15.5$ in

$$P_m = 12{,}352 \left(\frac{5714.29}{150}\right)^{1.852} \frac{0.74}{15.5^{4.87}} \quad \text{psi/mi} \qquad \text{from Eq. (6.58)}$$

Thus the pressure loss due to friction per mile of pipe is 12.35 psi/mi.

Example 6.19 A DN 400 (8-mm wall thickness) steel pipe is used to transport jet fuel (specific gravity = 0.82 and viscosity = 2.0 cSt) from a pumping facility to a storage tank 10 km away. Neglecting differences in elevations, calculate the pressure loss due to friction in bar/km at a flow rate of 700 m^3/h. Use the Hazen-Williams equation with a C factor of 130. If a delivery pressure of 3.5 bar must be maintained at the delivery point and the storage tank is at an elevation of 100 m above that of the pumping facility, calculate the pressure required at the pumping facility at the given flow rate.

Solution The inside diameter = $400 - 2 \times 8 = 384$ mm. Using the Hazen-Williams equation (6.61) we get

$$P_{\text{km}} = (1.1101 \times 10^{13}) \left(\frac{700}{130}\right)^{1.852} \times \frac{0.82}{(384)^{4.87}}$$

$$= 53.40 \text{ kPa/km}$$

$$\text{Pressure loss due to friction} = 53.4 \text{ kPa/km} = 0.534 \text{ bar/km}$$

$$\text{Total pressure drop in 10 km of pipe length} = 0.534 \times 10 = 5.34 \text{ bar}$$

The pressure required at the pumping facility is calculated by adding the pressure drop due to friction to the delivery pressure required and the static elevation head between the pumping facility and storage tank.

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \tag{6.62}$$

where P_t = total pressure required at pump
P_f = friction pressure
P_{elev} = pressure head due to elevation difference
P_{del} = delivery pressure at storage tank

$$P_t = 5.34 + \frac{100 \times 1.0/0.102}{100} + 3.5 = 18.64 \text{ bar}$$

Therefore the pressure required at the pumping facility is 18.64 bar, or 1864 kPa.

6.11.6 Miller equation

The Miller equation, or the Benjamin Miller formula, is used for calculating pressure drop in crude oil pipelines. Unlike the Colebrook-White equation this formula does not use the pipe roughness. It can be used to calculate the flow rate for a given pipe size and liquid properties, given the pressure drop due to friction. One form of the Miller equation is as follows:

$$Q = 4.06M\left(\frac{D^5 P_m}{\mathrm{Sg}}\right)^{0.5} \tag{6.63}$$

where the parameter M is defined as

$$M = \log_{10}\left(\frac{D^3 \mathrm{Sg} P_m}{\nu^2}\right) + 4.35 \tag{6.64}$$

and

where Q = flow rate, bbl/day
D = pipe inside diameter, in
P_m = pressure drop, psi/mi
Sg = liquid specific gravity
ν = liquid viscosity, cP

Rearranging the equation to solve for pressure drop, we get

$$P_m = \frac{0.0607(Q/M)^2 \mathrm{Sg}}{D^5} \tag{6.65}$$

where the symbols are as defined before.

In SI Units, the Miller equation has the following form:

$$Q = (3.996 \times 10^{-6})M\left(\frac{D^5 P_m}{\mathrm{Sg}}\right)^{0.5} \tag{6.66}$$

where the parameter M is calculated from

$$M = \log_{10}\left(\frac{D^3 \mathrm{Sg} P_m}{\nu^2}\right) - 0.4965 \tag{6.67}$$

and

where Q = flow rate, m^3/h
D = pipe internal diameter, mm
P_m = frictional pressure drop, kPa/km
Sg = liquid specific gravity
ν = liquid viscosity, cP

Reviewing the Miller equation, we see that to calculate the pressure drop P_m given a flow rate Q is not a straightforward process. The intermediate parameter M depends on the unknown pressure drop P_m. We have to solve the problem by successive iteration. We assume an initial value of the pressure drop P_m (say 5 psi/mi) and calculate a starting value for M. Using this value of M in Eq. (6.65), we calculate the second approximation for pressure drop P_m . Next using this newfound value of P_m we recalculate the new value of M and the process is continued until successive values of the pressure drop P_m are within some tolerance such as 0.001 psi/mi.

Example 6.20 An NPS 18 (0.375-in wall thickness) crude oil pipeline flows at the rate of 5000 bbl/h. Calculate the pressure drop per mile using the Miller equation. Assume the specific gravity of crude oil is 0.892 at 60°F and the viscosity is 20 cSt at 60°F. Compare the results using the Colebrook equation with a pipe roughness of 0.002.

Solution Since the Miller equation requires viscosity in centipoise, calculate that first:

$$\begin{aligned}\text{Liquid viscosity (cP)} &= \text{viscosity (cSt)} \times \text{specific gravity}\\ &= 20 \times 0.892 = 17.84 \text{ cP}\end{aligned}$$

The inside diameter of the pipe is

$$D = 18 - 2 \times 0.375 = 17.25 \text{ in}$$

Assume an initial value for the pressure drop of 10 psi/mi. Next calculate the parameter M from Eq. (6.64).

$$M = \log_{10}\left(\frac{17.25^3 \times 0.892 \times 10}{17.84^2}\right) + 4.35 = 6.5079$$

Substituting this value of M in Eq. (6.65) we calculate the pressure drop as

$$\begin{aligned}P_m &= 0.0607 \times \left(\frac{5000 \times 24}{6.5079}\right)^2 \times \frac{0.892}{17.25^5}\\ &= 12.05 \text{ psi/mi}\end{aligned}$$

Using this value of P_m a new value for M is calculated:

$$M = \log_{10}\left(\frac{17.25^3 \times 0.892 \times 12.05}{17.84^2}\right) + 4.35 = 6.5889$$

Recalculate the pressure drop with this value of M:

$$P_m = 0.0607 \times \left(\frac{5000 \times 24}{6.5889}\right)^2 \times \frac{0.892}{17.25^5} = 11.76 \text{ psi/mi}$$

Continuing the iterations a couple of times more, we get the final answer for $P_m = 11.79$. Thus the pressure drop per mile is 11.79 psi/mi.

Next, for comparison, we calculate the pressure drop using the Colebrook equation.

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{17.25} = 0.0001$$

Calculate the Reynolds number from Eq. (6.36):

$$R = 2213.76 \times \frac{5000}{17.25 \times 20} = 32{,}083$$

Using the Colebrook equation (6.51) we get the friction factor f as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.0001}{3.7} + \frac{2.51}{32{,}083\sqrt{f}}\right)$$

Solving for f by successive iteration, we get $f = 0.0234$. Using the Darcy equation (6.48) for pressure drop,

$$P_m = 34.87 \times \frac{0.0234 \times 5000^2 \times 0.892}{(17.25)^5}$$
$$= 11.91 \text{ psi/mi}$$

Therefore the pressure drop per mile using the Colebrook equation is 11.91 psi/mi. This compares with a pressure drop of 11.79 psi/mi using the Miller formula.

6.11.7 Shell-MIT equation

The Shell-MIT equation, also known as the MIT equation, was initially used by the Shell pipeline company for modeling the flow of high-viscosity heated crude oil pipelines. This equation for pressure drop uses a modified Reynolds number R_m, which is a multiple of the normal Reynolds number. From R_m a friction factor is calculated depending on whether the flow is laminar or turbulent. The calculation method is as follows. The Reynolds number of flow is first calculated from

$$R = \frac{92.24Q}{D\nu} \tag{6.68}$$

From the preceding, a modified Reynolds number is defined as

$$R_m = \frac{R}{7742} \tag{6.69}$$

where R = Reynolds number, dimensionless
R_m = modified Reynolds number, dimensionless
Q = flow rate, bbl/day
D = pipe inside diameter, in
ν = liquid kinematic viscosity, cSt

Next, a friction factor is calculated from one of the following equations:

$$f = \begin{cases} \dfrac{0.00207}{R_m} & \text{for laminar flow} \quad (6.70) \\ 0.0018 + 0.00662\left(\dfrac{1}{R_m}\right)^{0.355} & \text{for turbulent flow} \quad (6.71) \end{cases}$$

The laminar flow limit is the same as before: Reynolds number $R < 2100$ approximately.

The friction factor f in Eqs. (6.70) and (6.71) is not the Darcy friction factor we have used so far with the Colebrook equation. Therefore we cannot directly use it in the Darcy equation (6.45) to calculate the pressure drop.

The pressure drop due to friction with the Shell-MIT equation is then calculated as follows:

$$P_m = \frac{0.241(f\mathrm{Sg}Q^2)}{D^5} \tag{6.72}$$

where P_m = pressure drop due to friction, psi/mi
f = Shell-MIT equation friction factor, dimensionless
Sg = liquid specific gravity
Q = liquid flow rate, bbl/day
D = pipe inside diameter, in

With flow rate in bbl/h, the pressure drop due to friction is calculated using the following modified version of the Darcy equation:

$$P_m = \frac{138.82(f\mathrm{Sg}Q^2)}{D^5} \tag{6.73}$$

where P_m = pressure drop due to friction, psi/mi
f = Shell-MIT equation friction factor, dimensionless
Sg = liquid specific gravity
Q = liquid flow rate, bbl/h
D = pipe inside diameter, in

In SI units the MIT equation is expressed as follows:

$$P_m = (6.2191 \times 10^{10})\frac{f\mathrm{Sg}Q^2}{D^5} \tag{6.74}$$

where P_m = frictional pressure drop, kPa/km
f = Shell-MIT equation friction factor, dimensionless
Sg = liquid specific gravity
Q = liquid flow rate, m^3/h
D = pipe inside diameter, mm

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Example 6.21 A 400-mm outside diameter (8-mm wall thickness) crude oil pipeline is used for shipping a heavy crude oil between two storage terminals at a flow rate of 600 m^3/h at 80°C. Calculate, using the MIT equation, the frictional pressure drop assuming the crude oil has a specific gravity of 0.895 and a viscosity of 100 cSt at 80°C. Compare the result using the Moody diagram method.

Solution The inside diameter of pipe $D = 400 - 2 \times 8 = 384$ mm. From Eq. (6.38), the Reynolds number is first calculated:

$$R = \frac{353{,}678 \times 600}{100 \times 384} = 5526$$

Since $R > 2100$, the flow is in the turbulent zone. Calculate the Shell-MIT modified Reynolds number using Eq. (6.69).

$$R_m = \frac{5526}{7742} = 0.7138$$

Calculate the friction factor from Eq. (6.71).

$$\text{Friction factor} = 0.0018 + 0.00662\left(\frac{1}{0.7138}\right)^{0.355} = 0.0093$$

Finally, we calculate the pressure drop from Eq. (6.74) as follows:

$$\begin{aligned} P_m &= (6.2191 \times 10^{10})\frac{0.0093 \times 0.895 \times 600 \times 600}{(384)^5} \\ &= 22.23 \text{ kPa/km} \end{aligned}$$

6.11.8 Other pressure drop equations

Two other equations for friction factor calculations are the Churchill equation and the Swamee-Jain equation. These equations are explicit equations in friction factor calculation, unlike the Colebrook-White equation, which requires solution by trial and error.

Churchill equation. This equation for the friction factor was proposed by Stuart Churchill and published in *Chemical Engineering* magazine in November 1977. It is as follows:

$$f = \left[\left(\frac{8}{R}\right)^{12} + \frac{1}{(A+B)^{3/2}}\right]^{1/12} \tag{6.75}$$

where

$$A = 2.457 \log_e \left[\frac{1}{(7/R)^{0.9} + (0.27e/D)}\right]^{16} \tag{6.76}$$

$$B = \left(\frac{37{,}530}{R}\right)^{16} \tag{6.77}$$

The Churchill equation for the friction factor yields results that compare closely with that obtained using the Colebrook-White equation or the Moody diagram.

Swamee-Jain equation. This is another explicit equation for calculating the friction factor. It was first presented by P. K. Swamee and A. K. Jain in 1976 in *Journal of the Hydraulics Division of ASCE*. This equation is the easiest of all equations for calculating the friction factor. The Swamee-Jain equation is as follows:

$$f = \frac{0.25}{[\log_{10}(e/3.7D + 5.74/R^{0.9})]^2} \tag{6.78}$$

Similar to the Churchill equation friction factor, the Swamee-Jain equation correlates fairly well with the friction factor calculated using the Colebrook-White equation or the Moody diagram.

6.12 Minor Losses

So far, we have calculated the pressure drop per unit length in straight pipe. We also calculated the total pressure drop considering several miles of pipe from a pump station to a storage tank. Minor losses in a petroleum product pipeline are classified as those pressure drops that are associated with piping components such as valves and fittings. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. A pipe nozzle exiting from a storage tank will have entrance and exit losses. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe.

Generally, minor losses are included in calculations by using the equivalent length of the valve or fitting or using a resistance factor or K factor multiplied by the velocity head $v^2/2g$. The term minor losses can be applied only where the pipeline lengths and hence the friction losses are relatively large compared to the pressure drops in the fittings and valves. In a situation such as plant piping and tank farm piping the pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses is really a misnomer. Regardless, the pressure losses through valves, fittings, etc., can be accounted for approximately using the equivalent length or K times the velocity head method. It must be noted that this way of calculating the minor losses is valid only in turbulent flow. No data are available for laminar flow.

6.12.1 Valves and fittings

If Table 6.5 shows the equivalent lengths of commonly used valves and fittings in a petroleum pipeline system. It can be seen from this table

TABLE 6.5 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore, a 20-in-diameter gate valve may be replaced with a $20 \times 8 =$ 160-in-long piece of pipe that will match the frictional pressure drop through the valve.

Example 6.22 A piping system is 2000 ft of NPS 20 pipe that has two 20-in gate valves, three 20-in ball valves, one swing check valve, and four 90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe and valves and fittings.

Solution Using Table 6.5, we can convert all valves and fittings in terms of 20-in pipe as follows:

$$\begin{aligned}
\text{Two 20-in gate valves} &= 2 \times 20 \times 8 = 320 \text{ in of 20-in pipe} \\
\text{Three 20-in ball valves} &= 3 \times 20 \times 3 = 180 \text{ in of 20-in pipe} \\
\text{One 20-in swing check valve} &= 1 \times 20 \times 50 = 1000 \text{ in of 20-in pipe} \\
\text{Four } 90^\circ \text{ elbows} &= 4 \times 20 \times 30 = 2400 \text{ in of 20-in pipe} \\
\text{Total for all valves and fittings} &= 4220 \text{ in of 20-in pipe} \\
&= 351.67 \text{ ft of 20-in pipe}
\end{aligned}$$

Adding the 2000 ft of straight pipe, the total equivalent length of straight pipe and all fittings is

$$L_e = 2000 + 351.67 = 2351.67 \text{ ft}$$

The pressure drop due to friction in the preceding piping system can now be calculated based on 2351.67 ft of pipe. It can be seen in this example that the valves and fittings represent roughly 15 percent of the total pipeline length. In plant piping this percentage may be higher than that in a long-distance petroleum pipeline. Hence, the reason for the term *minor losses*.

Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation (6.45), the pressure drop in a straight length of pipe is given by

$$h = f\frac{L}{D}\frac{v^2}{2g}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K (also known as the resistance coefficient) and the preceding equation then becomes

$$h = K\frac{v^2}{2g} \tag{6.79}$$

In Eq. (6.79), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $v^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(v^2/2g)$, where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 6.6. It can be seen that the K factor depends on the nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

From Table 6.6, it can be seen that a 6-in gate valve has a K factor of 0.12, while a 20-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be $0.12\,(v^2/2g)$ and that in the 20-in valve is $0.10\,(v^2/2g)$. The velocities in both cases will be different due to the difference in diameters.

If the flow rate was 1000 gal/min, the velocity in the 6-in valve will be approximately

$$v_6 = 0.4085\,\frac{1000}{6.125^2} = 10.89 \text{ ft/s}$$

Similarly, at 1000 gal/min, the velocity in the 20-in valve will be approximately

$$v_6 = 0.4085\,\frac{1000}{19.5^2} = 1.07 \text{ ft/s}$$

TABLE 6.6 Friction Loss in Valves—Resistance Coefficient *K*

		Nominal pipe size, in											
Description	L/D	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72

Therefore,

$$\text{Head loss in 6-in gate valve} = \frac{0.12\,(10.89)^2}{64.4} = 0.22 \text{ ft}$$

and

$$\text{Head loss in 20-in gate valve} = \frac{0.10\,(1.07)^2}{64.4} = 0.002 \text{ ft}$$

These head losses appear small since we have used a relatively low flow rate in the 20-in valve. In reality the flow rate in the 20-in valve may be as high as 6000 gal/min and the corresponding head loss will be 0.072 ft.

6.12.2 Pipe enlargement and reduction

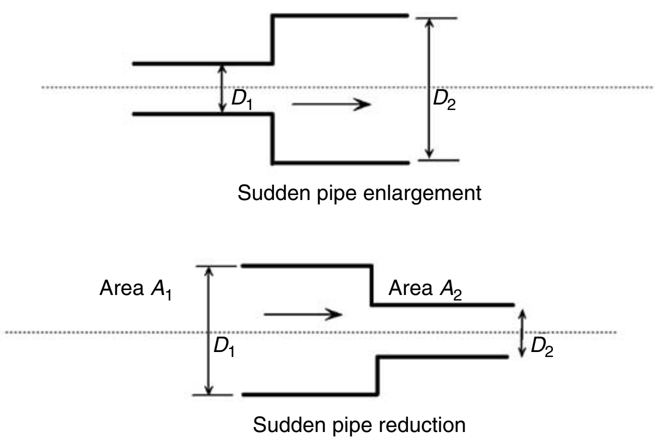
Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(v_1 - v_2)^2}{2g} \tag{6.80}$$

where v_1 and v_2 are the velocities of the liquid in the two pipe sizes D_1 and D_2, respectively. Writing Eq. (6.80) in terms of pipe cross-sectional areas A_1 and A_2,

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{{v_1}^2}{2g} \tag{6.81}$$

for sudden enlargement. This is illustrated in Fig. 6.9.



A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 6.9 Sudden pipe enlargement and pipe reduction.

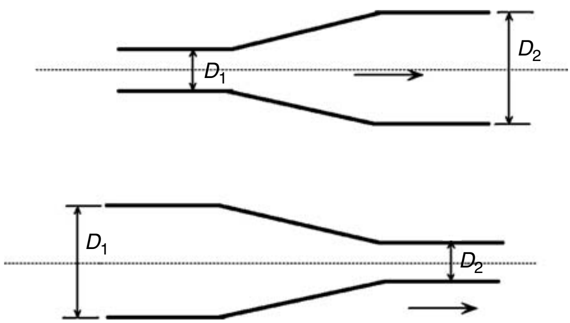


Figure 6.10 Gradual pipe enlargement and pipe reduction.

For sudden contraction or reduction in pipe size as shown in Fig. 6.9, the head loss is calculated from

$$h_f = \left(\frac{1}{C_c} - 1\right)\frac{{v_2}^2}{2g} \tag{6.82}$$

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 6.9.

Gradual enlargement and reduction of pipe size, as shown in Fig. 6.10, cause less head loss than sudden enlargement and sudden reduction. For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c(v_1 - v_2)^2}{2g} \tag{6.83}$$

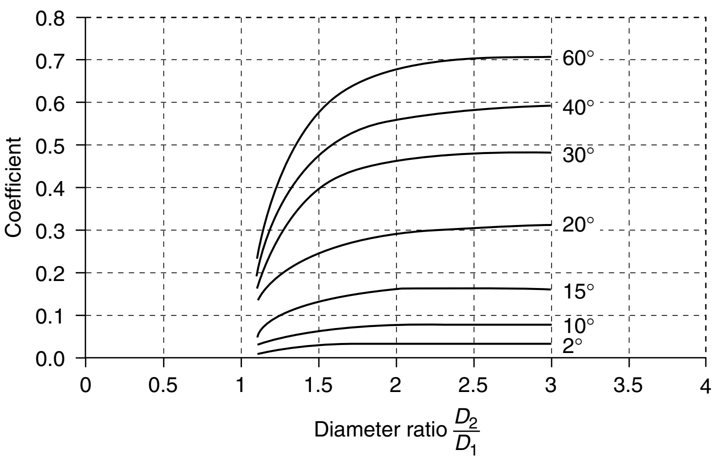


Figure 6.11 Gradual pipe expansion head loss coefficient.

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is shown in Fig. 6.11.

6.12.3 Pipe entrance and exit losses

The K factors for computing the head loss associated with pipe entrance and exit are as follows:

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

6.13 Complex Piping Systems

So far we have discussed straight length of pipe with valves and fittings. Complex piping systems include pipes of different diameters in series and parallel configuration.

6.13.1 Series piping

Series piping in its simplest form consists of two or more different pipe sizes connected end to end as illustrated in Fig. 6.12. Pressure drop calculations in series piping may be handled in one of two ways. The first approach would be to calculate the pressure drop in each pipe size and add them together to obtain the total pressure drop. Another approach is to consider one of the pipe diameters as the base size and convert other pipe sizes into equivalent lengths of the base pipe size. The resultant equivalent lengths are added together to form one long piece of pipe of constant diameter equal to the base diameter selected. The pressure drop can now be calculated for this single-diameter pipeline. Of course, all valves and fittings will also be converted to their respective equivalent pipe lengths using the L/D ratios from Table 6.6.

Consider three sections of pipe joined together in series. Using subscripts 1, 2, and 3 and denoting the pipe length as L, inside diameter as D, flow rate as Q, and velocity as V, we can calculate the equivalent length of each pipe section in terms of a base diameter. This base

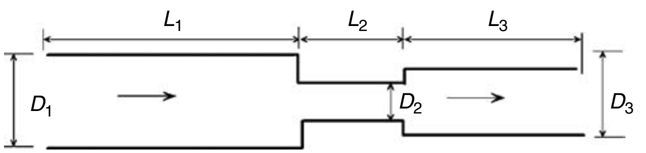


Figure 6.12 Series piping.

diameter will be selected as the diameter of the first pipe section D_1. Since equivalent length is based on the same pressure drop in the equivalent pipe as the original pipe diameter, we will calculate the equivalent length of section 2 by finding that length of diameter D_1 that will match the pressure drop in a length L_2 of pipe diameter D_2. Using the Darcy equation and converting velocities in terms of flow rate from Eq. (6.31), we can write

$$\text{Head loss} = \frac{f(L/D)(0.4085Q/D_2)^2}{2g}$$

For simplicity, assuming the same friction factor,

$$\frac{L_e}{{D_1}^5} = \frac{L_2}{{D_2}^5}$$

Therefore, the equivalent length of section 2 based on diameter D_1 is

$$L_e = L_2 \left(\frac{D_1}{D_2}\right)^5$$

Similarly, the equivalent length of section 3 based on diameter D_1 is

$$L_e = L_3 \left(\frac{D_1}{D_3}\right)^5$$

The total equivalent length of all three pipe sections based on diameter D_1 is therefore

$$L_t = L_1 + L_2 \left(\frac{D_1}{D_2}\right)^5 + L_3 \left(\frac{D_1}{D_3}\right)^5$$

The total pressure drop in the three sections of pipe can now be calculated based on a single pipe of diameter D_1 and length L_t.

Example 6.23 Three pipes with 14-, 16-, and 18-in diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

14-in pipeline, 0.250-in wall thickness, 2000 ft long

16-in pipeline, 0.375-in wall thickness, 3000 ft long

18-in pipeline, 0.375-in wall thickness, 5000 ft long

One 16 × 14 in reducer

One 18 × 16 in reducer

Two 14-in 90° elbows

Four 16-in 90° elbows

Six 18-in 90° elbows

One 14-in gate valve

One 16-in ball valve

One 18-in gate valve

(a) Use the Hazen-Williams equation with a C factor of 140 to calculate the total pressure drop in the series piping system at a flow rate of 3500 gal/min. The product transported is gasoline with a specific gravity of 0.74. Flow starts in the 14-in piping and ends in the 18-in piping.

(b) If the flow rate is increased to 6000 gal/min, estimate the new total pressure drop in the piping system, keeping everything else the same.

Solution

(a) Since we are going to use the Hazen-Williams equation, the pipes in series analysis will be based on the pressure loss being inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe, per Eq. (6.55).

We will first calculate the total equivalent lengths of all 14-in pipe, fittings, and valves in terms of the 14-in-diameter pipe.

$$\text{Straight pipe: 14 in, 2000 ft} = \text{2000 ft of 14-in pipe}$$

$$\text{Two 14-in } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 14}{12} = \text{70 ft of 14-in pipe}$$

$$\text{One 14-in gate valve} = \frac{1 \times 8 \times 14}{12} = \text{9.33 ft of 14-in pipe}$$

Therefore, the total equivalent length of 14-in pipe, fittings, and valves = 2079.33 ft of 14-in pipe.

Similarly we get the total equivalent length of 16-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 16-in, 3000 ft} = \text{3000 ft of 16-in pipe}$$

$$\text{Four 16-in } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 16}{12} = \text{160 ft of 16-in pipe}$$

$$\text{One 16-in ball valve} = \frac{1 \times 3 \times 16}{12} = \text{4 ft of 16-in pipe}$$

Therefore, the total equivalent length of 16-in pipe, fittings, and valve = 3164 ft of 16-in pipe.

Finally, we calculate the total equivalent length of 18-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 18-in, 5000 ft} = \text{5000 ft of 18-in pipe}$$

$$\text{Six 18-in } 90^\circ \text{ elbows} = \frac{6 \times 30 \times 18}{12} = \text{270 ft of 18-in pipe}$$

$$\text{One 18-in gate valve} = \frac{1 \times 8 \times 18}{12} = \text{12 ft of 18-in pipe}$$

Therefore, the total equivalent length of 18-in pipe, fittings, and valve = 5282 ft of 18-in pipe.

Next we convert all the preceding pipe lengths to the equivalent 14-in pipe based on the fact that the pressure loss is inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe.

$$2079.33 \text{ ft of 14-in pipe} = 2079.33 \text{ ft of 14-in pipe}$$

$$3164 \text{ ft of 16-in pipe} = 3164 \times \left(\frac{13.5}{15.25}\right)^{4.87} = 1748 \text{ ft of 14-in pipe}$$

$$5282 \text{ ft of 18-in pipe} = 5282 \times \left(\frac{13.5}{17.25}\right)^{4.87} = 1601 \text{ ft of 14-in pipe}$$

Therefore adding all the preceding lengths we get

$$\text{Total equivalent length in terms of 14-in pipe} = 5429 \text{ ft of 14-in pipe}$$

We still have to account for the 16 × 14 in and 18 × 16 in reducers. The reducers can be considered as sudden enlargements for the approximate calculation of the head loss, using the K factor and velocity head method. For sudden enlargements, the resistance coefficient K is found from

$$K = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2$$

where d_1 is the smaller diameter and d_2 is the larger diameter.

For the 16 × 14 in reducer,

$$K = \left[1 - \left(\frac{13.5}{15.25}\right)^2\right]^2 = 0.0468$$

and for the 18 × 16 in reducer,

$$K = \left[1 - \left(\frac{15.25}{17.25}\right)^2\right]^2 = 0.0477$$

The head loss through the reducers will then be calculated based on $K(V^2/2g)$.

Flow velocities in the three different pipe sizes at 3500 gal/min will be calculated using Eq. (6.31):

$$\text{Velocity in 14-in pipe: } V_{14} = \frac{0.4085 \times 3500}{(13.5)^2} = 7.85 \text{ ft/s}$$

$$\text{Velocity in 16-in pipe: } V_{16} = \frac{0.4085 \times 3500}{(15.25)^2} = 6.15 \text{ ft/s}$$

$$\text{Velocity in 18-in pipe: } V_{18} = \frac{0.4085 \times 3500}{(17.25)^2} = 4.81 \text{ ft/s}$$

The head loss through the 16 × 14 in reducer is

$$h_1 = 0.0468 \frac{7.85^2}{64.4} = 0.0448 \text{ ft}$$

and the head loss through the 18 × 16 in reducer is

$$h_1 = 0.0477 \frac{6.15^2}{64.4} = 0.028 \text{ ft}$$

These head losses are insignificant and hence can be neglected in comparison with the head loss in straight length of pipe. Therefore, the total head loss in the entire piping system will be based on a total equivalent length of 5429 ft of 14-in pipe.

Using the Hazen-Williams equation (6.59) the pressure drop at 3500 gal/min (equal to 3500/0.7 bbl/h) is

$$P_f = 2339 \left(\frac{5000}{140}\right)^{1.852} \frac{0.74}{(13.5)^{4.87}} = 4.07 \text{ psi per 1000 ft of pipe}$$

Therefore, for the 5429 ft of equivalent 14-in pipe, the total pressure drop is

$$P_f = 4.07 \frac{5429}{1000} = 22.1 \text{ psi}$$

(b) When the flow rate is increased to 6000 gal/min, we can use proportions to estimate the new total pressure drop in the piping as follows:

$$P_f = \left(\frac{6000}{3500}\right)^{1.852} \times 4.07 = 11.04 \text{ psi per 1000 ft of pipe}$$

Therefore, the total pressure drop in 5429 ft of 14-in. pipe is

$$P_f = 11.04 \times \frac{5429}{1000} = 59.94 \text{ psi}$$

Example 6.24 Two pipes with 400- and 600-mm diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

400-mm pipeline, 6-mm wall thickness, 600 m long

600-mm pipeline, 10-mm wall thickness, 1500 m long

One 600 × 400 mm reducer

Two 400-mm 90° elbows

Four 600-mm 90° elbows

One 400-mm gate valve

One 600-mm gate valve

Use the Hazen-Williams equation with a C factor of 120 to calculate the total pressure drop in the series oil piping system at a flow rate of 250 L/s. Liquid specific gravity is 0.82 and viscosity is 2.5 cSt.

Solution The total equivalent length on 400-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 600 \text{ m}$$

$$\text{Two } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 400}{1000} = 24 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 400}{1000} = 3.2 \text{ m}$$

Thus,

$$\text{Total equivalent length on 400-mm-diameter pipe} = 627.2 \text{ m}$$

The total equivalent length on 600-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 1500 \text{ m}$$

$$\text{Four } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 600}{1000} = 72 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 600}{1000} = 4.8 \text{ m}$$

Thus,

$$\text{Total equivalent length on 600-mm-diameter pipe} = 1576.8 \text{ m}$$

Reducers will be neglected since they have insignificant head loss. Convert all pipe to 400-mm equivalent diameter.

$$1576.8 \text{ m of 600-mm pipe} = 1576.8 \left(\frac{388}{580}\right)^{4.87}$$

$$= 222.6 \text{ m of 400-mm pipe}$$

$$\text{Total equivalent length on 400-mm-diameter pipe} = 627.2 + 222.6 = 849.8 \text{ m}$$

$$Q = 250 \times 10^{-3} \times 3600 = 900 \text{ m}^3\text{/h}$$

The pressure drop from Eq. (6.61) is

$$P_m = 1.1101 \times 10^{13} \left(\frac{900}{120}\right)^{1.852} \times \frac{0.82}{(388)^{4.87}}$$

$$= 93.79 \text{ kPa/km}$$

$$\text{Total pressure drop} = \frac{93.79 \times 849.8}{1000} = 79.7 \text{ kPa}$$

6.13.2 Parallel piping

Liquid pipelines in parallel configured such that the multiple pipes are connected so that the liquid flow splits into the multiple pipes at

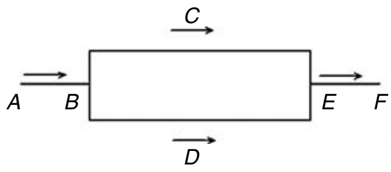


Figure 6.13 Parallel piping.

the beginning and the separate flow streams subsequently rejoin downstream into another single pipe as depicted in Fig. 6.13.

Figure 6.13 shows a parallel piping system in the horizontal plane with no change in pipe elevations. Liquid flows through a single pipe *AB*, and at the junction *B* the flow splits into two pipe branches *BCE* and *BDE*. At the downstream end at junction *E*, the flows rejoin to the initial flow rate and subsequently flow through the single pipe *EF*.

To calculate the flow rates and pressure drop due to friction in the parallel piping system, shown in Fig. 6.13, two main principles of parallel piping must be followed. These are flow conservation at any junction point and common pressure drop across each parallel branch pipe.

Based on flow conservation, at each junction point of the pipeline, the incoming flow must exactly equal the total outflow. Therefore, at junction *B*, the flow *Q* entering the junction must exactly equal the sum of the flow rates in branches *BCE* and *BDE*.

Thus,

$$Q = Q_{BCE} + Q_{BDE} \tag{6.84}$$

where Q_{BCE} = flow through branch *BCE*
Q_{BDE} = flow through branch *BDE*
Q = incoming flow at junction *B*

The other requirement in parallel pipes concerns the pressure drop in each branch piping. Based on this the pressure drop due to friction in branch *BCE* must exactly equal that in branch *BDE*. This is because both branches have a common starting point (*B*) and a common ending point (*E*). Since the pressure at each of these two points is a unique value, we can conclude that the pressure drop in branch pipe *BCE* and that in branch pipe *BDE* are both equal to $P_B - P_E$, where P_B and P_E represent the pressure at the junction points *B* and *E*, respectively.

Another approach to calculating the pressure drop in parallel piping is the use of an equivalent diameter for the parallel pipes. For example in Fig. 6.13, if pipe *AB* has a diameter of 14 in and branches *BCE* and *BDE* have diameters of 10 and 12 in, respectively, we can find some equivalent diameter pipe of the same length as one of the branches that will have the same pressure drop between points B and C as the two branches. An approximate equivalent diameter can be calculated using the Darcy equation.

The pressure loss in branch BCE (10-in diameter) can be calculated as

$$h_1 = \frac{f(L_1/D_1){v_1}^2}{2g} \tag{6.85}$$

where the subscript 1 is used for branch BCE and subscript 2 for branch BDE.

Similarly, for branch BDE

$$h_2 = \frac{f(L_2/D_2){v_2}^2}{2g} \tag{6.86}$$

For simplicity we have assumed the same friction factors for both branches. Since h_1 and h_2 are equal for parallel pipes, and representing the velocities v_1 and v_2 in terms of the respective flow rates Q_1 and Q_2, using Eq. (6.85) we have the following equations:

$$\frac{f(L_1/D_1){v_1}^2}{2g} = \frac{f(L_2/D_2){v_2}^2}{2g}$$

$$v_1 = 0.4085\,\frac{Q_1}{{D_1}^2}$$

$$v_2 = 0.4085\,\frac{Q_2}{{D_2}^2}$$

In these equations we are assuming flow rates in gal/min and diameters in inches.

Simplifying the equations, we get

$$\frac{L_1}{D_1}\left(\frac{Q_1}{{D_1}^2}\right)^2 = \frac{L_2}{D_2}\left(\frac{Q_2}{{D_2}^2}\right)^2$$

or

$$\frac{Q_1}{Q_2} = \left(\frac{L_2}{L_1}\right)^{0.5}\left(\frac{D_1}{D_2}\right)^{2.5} \tag{6.87}$$

Also by conservation of flow

$$Q_1 + Q_2 = Q \tag{6.88}$$

Using Eqs. (6.87) and (6.88), we can calculate the flow through each branch in terms of the inlet flow Q. The equivalent pipe will be designated as D_e in diameter and L_e in length. Since the equivalent pipe will have the same pressure drop as each of the two branches, we can write

$$\frac{L_e}{D_e}\left(\frac{Q_e}{D_e{}^2}\right)^2 = \frac{L_1}{D_1}\left(\frac{Q_1}{D_1{}^2}\right)^2 \tag{6.89}$$

where Q_e is the same as the inlet flow Q since both branches have been replaced with a single pipe. In Eq. (6.89), there are two unknowns L_e and D_e. Another equation is needed to solve for both variables. For simplicity, we can set L_e to be equal to one of the lengths L_1 or L_2. With this assumption, we can solve for the equivalent diameter D_e as follows.

$$D_e = D_1\left(\frac{Q}{Q_1}\right)^{0.4} \tag{6.90}$$

Example 6.25 A gasoline pipeline consists of a 2000-ft section of NPS 12 pipe (0.250-in wall thickness) starting at point A and terminating at point B. At point B, two pieces of pipe (4000 ft long each and NPS 10 pipe with 0.250-in wall thickness) are connected in parallel and rejoin at a point D. From D, 3000 ft of NPS 14 pipe (0.250-in wall thickness) extends to point E. Using the equivalent diameter method calculate the pressures and flow rate throughout the system when transporting gasoline (specific gravity = 0.74 and viscosity = 0.6 cSt) at 2500 gal/min. Compare the results by calculating the pressures and flow rates in each branch.

Solution Since the pipe loops between B and D are each NPS 10 and 4000 ft long, the flow will be equally split between the two branches. Each branch pipe will carry 1250 gal/min.

The equivalent diameter for section BD is found from Eq. (6.90):

$$D_e = D_1\left(\frac{Q}{Q_1}\right)^{0.4} = 10.25 \times (2)^{0.4} = 13.525 \text{ in}$$

Therefore we can replace the two 4000-ft NPS 10 pipes between B and D with a single pipe that is 4000 ft long and has a 13.525-in inside diameter.

The Reynolds number for this pipe at 2500 gal/min is found from Eq. (6.35):

$$R = \frac{3162.5 \times 2500}{13.525 \times 0.6} = 974{,}276$$

Considering that the pipe roughness is 0.002 in for all pipes:

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.525} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0138$. The pressure drop in section BD is [using Eq. (6.48)]

$$P_m = 71.16\,\frac{0.0138 \times (2500)^2 \times 0.74}{(13.525)^5} = 10.04 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } BD = \frac{10.04 \times 4000}{5280} = 7.61 \text{ psi}$$

For section AB we have,

$$R = \frac{3162.5 \times 2500}{12.25 \times 0.6} = 1{,}075{,}680$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{12.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0148$. The pressure drop in section AB is [using Eq. (6.48)]

$$P_m = 71.16 \frac{0.0148 \times (2500)^2 \times 0.74}{(12.25)^5} = 17.66 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } AB = \frac{17.66 \times 2000}{5280} = 6.69 \text{ psi}$$

Finally, for section DE we have,

$$R = \frac{3162.5 \times 2500}{13.5 \times 0.6} = 976{,}080$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.5} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0138$. The pressure drop in section DE is [using Eq. (6.48)]

$$P_m = 71.16 \frac{0.0138 \times (2500)^2 \times 0.74}{(13.5)^5} = 10.13 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } DE = \frac{10.13 \times 3000}{5280} = 5.76 \text{ psi}$$

Finally,

$$\begin{aligned}\text{Total pressure drop in entire piping system} &= 6.69 + 7.61 + 5.76 \\ &= 20.06 \text{ psi}\end{aligned}$$

Next for comparison we will analyze the branch pressure drops considering each branch separately flowing at 1250 gal/min.

$$R = \frac{3162.5 \times 1250}{10.25 \times 0.6} = 642{,}785$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{10.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.015$. The pressure drop in section BD is [using Eq. (6.48)]

$$P_m = 71.16 \frac{0.015 \times (1250)^2 \times 0.74}{(10.25)^5} = 10.65 \text{ psi/mi}$$

This compares with the pressure drop of 10.04 psi/mi we calculated using an equivalent diameter of 13.525. It can be seen that the difference between the two pressure drops is approximately 6 percent.

Example 6.26 A 5000-m-long crude oil pipeline is composed of three sections A, B, and C. Section A has a 200-m inside diameter and is 1500 m long. Section C has a 400-mm inside diameter and is 2000 m long. The middle section B consists of two parallel pipes each 1500 m long. One of the parallel pipes has a 150-mm inside diameter and the other has a 200-mm inside diameter. Calculate the pressures and flow rates in this piping system at a flow rate of 500 m^3/h. The specific gravity of the liquid is 0.87, the viscosity is 10 cSt, and the pipe roughness is 0.05 mm.

Solution For the center section B, the flow rates will be distributed between the two branches according to Eq. (6.87):

$$\frac{Q_1}{Q_2} = \left(\frac{L_2}{L_1}\right)^{0.5} \left(\frac{D_1}{D2}\right)^{2.5} = 1 \times \left(\frac{200}{150}\right)^{2.5}$$
$$= 2.053$$

Also

$$Q_1 + Q_2 = Q = 500$$

Solving for Q_1 and Q_2, we get

$$Q_1 = 336.23 \text{ m}^3\text{/h and } Q_2 = 163.77 \text{ m}^3\text{/h}$$

Therefore the flow rates in section B are 336.23 m^3/h through 200-mm-diameter pipe and 163.77 m^3/h through 150-mm-diameter pipe.

Section A consists of 200-mm-diameter pipe that flows at 500 m^3/h. The Reynolds number from Eq. (6.38) is

$$R = \frac{353{,}678 \times 500}{10 \times 200} = 88{,}420$$

Therefore flow is turbulent.

$$\text{Relative roughness} = \frac{e}{D} = \frac{0.05}{200} = 0.0003 \text{ in}$$

From the Moody diagram the friction factor $f = 0.0195$. The pressure drop from Eq. (6.50) is

$$P_m = 6.2475 \times \frac{10^{10} \times 0.0195 \times (500)^2 \times 0.87}{(200)^5} = 828.04 \text{ kPa/km}$$

Therefore the total pressure drop in section A is

$$\Delta P_a = 1.5 \times 828.04 = 1242 \text{ kPa}$$

Section B consists of 200-mm-diameter pipe that flows at 336.23 m^3/h (one branch). The Reynolds number from Eq. (6.38) is

$$R = \frac{353{,}678 \times 336.23}{10 \times 200} = 59{,}459$$

Therefore flow is turbulent.

$$\text{Relative roughness} = \frac{e}{D} = \frac{0.05}{200} = 0.0003 \text{ in}$$

From the Moody diagram the friction factor $f = 0.0205$. The pressure drop from Eq. (6.50) is

$$P_m = (6.2475 \times 10^{10}) \times \frac{0.0205 \times (336.23)^2 \times 0.87}{(200)^5} = 393.64 \text{ kPa/km}$$

Therefore the total pressure drop in section B is

$$\Delta P_b = 1.5 \times 393.64 = 590.46 \text{ kPa}$$

Finally section C consists of 400-mm-diameter pipe that flows at 500 m^3/h. The Reynolds number from Eq. (6.38) is

$$R = \frac{353{,}678 \times 500}{10 \times 400} = 44{,}210$$

Therefore flow is turbulent.

$$\text{Relative roughness} = \frac{e}{D} = \frac{0.05}{400} = 0.0001 \text{ in}$$

From the Moody diagram the friction factor $f = 0.022$. The pressure drop from Eq. (6.50) is

$$P_m = (6.2475 \times 10^{10}) \times \frac{0.022 \times (500)^2 \times 0.87}{(400)^5} = 29.19 \text{ kPa/km}$$

Therefore the total pressure drop in section C is

$$\Delta P_c = 2.0 \times 29.19 = 58.38 \text{ kPa}$$

Total pressure drop in entire pipeline system $= 1242 + 590.46 + 58.38 = 1891$ kPa.

6.14 Total Pressure Required

So far we have examined the frictional pressure drop in petroleum systems piping consisting of pipe, fittings, valves, etc. We also calculated the total pressure required to pump oil through a pipeline up to a delivery station at an elevated point. The total pressure required at the beginning of a pipeline, for a specified flow rate, consists of three distinct components:

1. Frictional pressure drop
2. Elevation head
3. Delivery pressure

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \tag{6.91}$$

The first item is simply the total frictional head loss in all straight pipe, fittings, valves, etc. The second item accounts for the pipeline elevation difference between the origin of the pipeline and the delivery terminus. If the origin of the pipeline is at a lower elevation than that of the pipeline terminus or delivery point, a certain amount of positive pressure is required to compensate for the elevation difference. On the other hand, if the delivery point were at a lower elevation than the beginning of the pipeline, gravity will assist the flow, and the pressure required at the beginning of the pipeline will be reduced by this elevation difference. The third component, delivery pressure at the terminus, simply ensures that a certain minimum pressure is maintained at the delivery point, such as a storage tank.

For example, if an oil pipeline requires 800 psi to compensate for frictional losses and the minimum delivery pressure required is 25 psi, the total pressure required at the beginning of the pipeline is calculated as follows. If there were no elevation difference between the beginning of the pipeline and the delivery point, the elevation head (component 2) is zero. Therefore, the total pressure P_t required is

$$P_t = 800 + 0 + 25 = 825 \text{ psi}$$

Next consider elevation changes. If the elevation at the beginning is 100 ft and the elevation at the delivery point is 600 ft, then

$$P_t = 800 + \frac{(600 - 100) \times 0.82}{2.31} + 25 = 1002.49 \text{ psi}$$

The middle term in this equation represents the static elevation head difference converted to psi. Finally, if the elevation at the beginning is 600 ft and the elevation at the delivery point is 100 ft, then

$$P_t = 800 + \frac{(100 - 600) \times 0.82}{2.31} + 25 = 647.51 \text{ psi}$$

It can be seen from the preceding that the 500-ft advantage in elevation in the final case reduces the total pressure required by approximately 178 psi compared to the situation where there was no elevation difference between the beginning of the pipeline and delivery point (825 psi versus 647.51 psi).

6.14.1 Effect of elevation

The preceding discussion illustrated a liquid pipeline that had a flat elevation profile compared to an uphill pipeline and a downhill pipeline. There are situations where the ground elevation may have drastic peaks and valleys that require careful consideration of the pipeline topography. In some instances, the total pressure required to transport a given volume of liquid through a long pipeline may depend more on the ground elevation profile than on the actual frictional pressure drop. In the preceding we calculated the total pressure required for a flat pipeline as 825 psi and an uphill pipeline to be 1002 psi. In the uphill case the static elevation difference contributed to 17 percent of the total pressure required. Thus the frictional component was much higher than the elevation component. We will examine a case where the elevation differences in a long pipeline dictate the total pressure required more than the frictional head loss.

Example 6.27 A 20-in, 500-mi-long (0.375-in wall thickness) oil pipeline has a ground elevation profile as shown in Fig. 6.14. The elevation at Corona is 600 ft and at Red Mesa is 2350 ft. Calculate the total pressure required at the Corona pump station to transport 200,000 bbl/day of oil (specific gravity = 0.895 and viscosity = 35 cSt) to the Red Mesa storage tanks, with a minimum delivery pressure of 50 psi at Red Mesa.

Use the Colebrook equation for calculation of the friction factor. If the pipeline operating pressure cannot exceed 1400 psi, how many pumping stations besides Corona will be required to transport the given flow rate? Use a pipe roughness of 0.002 in.

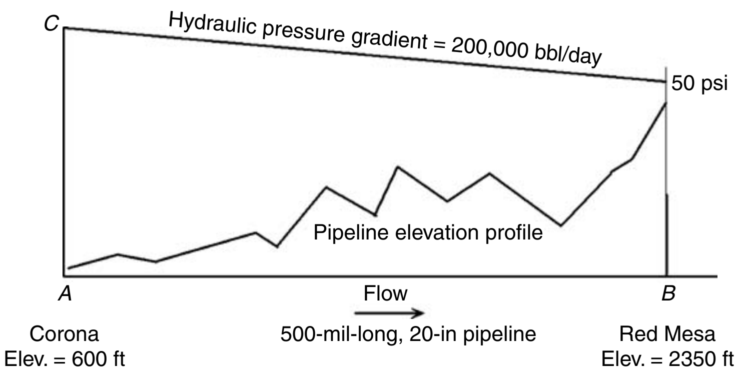


Figure 6.14 Corona to Red Mesa pipeline.

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Solution First, calculate the Reynolds number from Eq. (6.37):

$$R = \frac{92.24 \times 200{,}000}{19.25 \times 35} = 27{,}381$$

Therefore the flow is turbulent.

$$\text{Relative pipe roughness} = \frac{e}{D} = \frac{0.002}{19.25} = 0.0001$$

Next, calculate the friction factor f using the Colebrook equation (6.51).

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.0001}{3.7} + \frac{2.51}{27{,}381\sqrt{f}}\right)$$

Solving for f by trial and error, $f = 0.0199$. We can now find the pressure loss due to friction using Eq. (6.48) as follows:

$$P_m = 0.0605 \times \frac{0.0199 \times (200{,}000)^2 \times 0.895}{(19.25)^5}$$
$$= 16.31 \text{ psi/mi}$$

The total pressure required at Corona is calculated by adding the pressure drop due to friction to the delivery pressure required at Red Mesa and the static elevation head between Corona and Red Mesa.

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (6.91)}$$
$$= (16.31 \times 500) + (2350 - 600) \times \frac{0.895}{2.31} + 50$$
$$= 8155 + 678 + 50 = 8883 \text{ psi}$$

Since a total pressure of 8883 psi at Corona far exceeds the maximum operating pressure of 1400 psi, it is clear that we need additional intermediate booster pump stations besides Corona. The approximate number of pump stations required without exceeding the pipeline pressure of 1400 psi is

$$\text{Number of pump stations} = \frac{8883}{1400} = 6.35, \quad \text{or 7 pump stations}$$

Therefore, we will need six additional booster pump stations besides Corona. With seven pump stations the average pressure per pump station will be

$$\text{Average pump station discharge pressure} = \frac{8883}{7} = 1269 \text{ psi}$$

6.14.2 Tight line operation

When there are drastic elevation differences in a long pipeline, sometimes the last section of the pipeline toward the delivery terminus may operate in an open-channel flow. This means that the pipeline section will not be full of liquid and there will be a vapor space above the liquid.

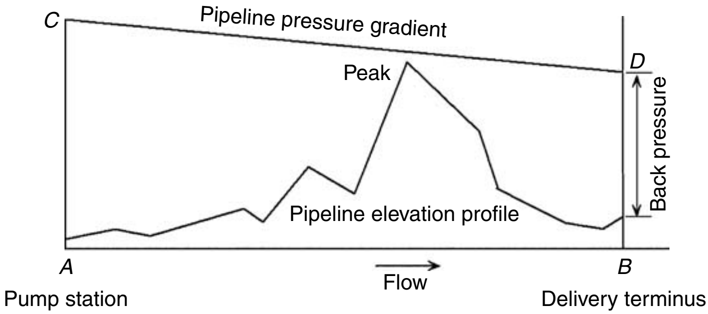

Figure 6.15 Tight line operation.

Such situations are acceptable in ordinary petroleum liquid (gasoline, diesel, and crude oil) pipelines compared to high vapor pressure liquids such as liquefied petroleum gas (LPG) and propane. To prevent such open-channel flow or slack line conditions, we must pack the line by providing adequate back pressure at the delivery terminus as illustrated in Fig. 6.15.

6.15 Hydraulic Gradient

The graphical representation of the pressures along the pipeline, as shown in Fig. 6.16, is the hydraulic gradient. Since elevation is measured in feet, the pipeline pressures are converted to feet of head of liquid and plotted against the distance along the pipeline superimposed on the elevation profile. If we assume a beginning elevation of 100 ft, a delivery terminus elevation of 500 ft, a total pressure of 1000 psi required at the beginning, and a delivery pressure of 25 psi at the terminus, we can plot the hydraulic pressure gradient graphically by the following method.

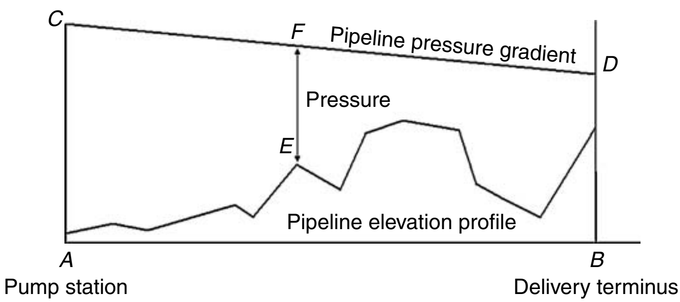

Figure 6.16 Hydraulic gradient.

At the beginning of the pipeline the point C representing the total pressure will be plotted at a height of

$$100\text{ ft} + \frac{1000 \times 2.31}{0.85} = 2818\text{ ft}$$

where the liquid specific gravity is 0.85. Similarly, at the delivery terminus the point D representing the total head at delivery will be plotted at a height of

$$500 + \frac{25 \times 2.31}{0.85} = 568\text{ ft}$$

The line connecting the points C and D represents the variation of the total head in the pipeline and is termed the *hydraulic gradient*. At any intermediate point such as E along the pipeline the pipeline pressure will be the difference between the total head represented by point F on the hydraulic gradient and the actual elevation of the pipeline at E.

If the total head at F is 1850 ft and the pipeline elevation at E is 250 ft, the actual pipeline pressure at E is

$$(1850 - 250)\text{ ft} = \frac{1600 \times 0.85}{2.31} = 589\text{ psi}$$

It can be seen that the hydraulic gradient clears all peaks along the pipeline. If the elevation at E were 2000 ft, we would have a negative pressure in the pipeline at E equivalent to

$$(1850 - 2000)\text{ ft} \quad \text{or} \quad -150\text{ ft} = \frac{-150 \times 0.85}{2.31} = -55\text{ psi}$$

Since a negative pressure is not acceptable, the total pressure at the beginning of the pipeline will have to be higher by 55 psi.

$$\text{Revised total head at } A = 2818 + 150 = 2968\text{ ft}$$

This will result in zero gauge pressure in the pipeline at peak E. The actual pressure in the pipeline will therefore be equal to the atmospheric pressure at that location. Since we would like to always maintain some positive pressure above the atmospheric pressure, in this case the total head at A will be slightly higher than 2968 ft. Assuming a 10-psi positive pressure is desired at the highest peak such as E (2000 ft elevation), the revised total pressure at A would be

$$\text{Total pressure at } A = 1000 + 55 + 10 = 1065\text{ psi}$$

Therefore,

$$\text{Total head at } C = 100 + \frac{1065 \times 2.31}{0.85} = 2994\text{ ft}$$

The difference between 2994 ft and 2968 ft is 26 ft, which is approximately 10 psi.

6.16 Pumping Horsepower

In the previous sections we calculated the total pressure required at the beginning of the pipeline to transport a given volume of liquid over a certain distance. We will now calculate the pumping horsepower (HP) required to accomplish this.

Consider Example 6.27 in which we calculated the total pressure required to pump 200,000 bbl/day of oil from Corona to Red Mesa through a 500-mi-long, 20-in pipeline. We calculated the total pressure required to be 8883 psi. Since the maximum allowable working pressure in the pipeline was limited to 1400 psi, we concluded that six additional pump stations besides Corona were required. With a total of seven pump stations, each pump station would be discharging at a pressure of approximately 1269 psi.

At the Corona pump station oil would enter the pump at some minimum pressure, say 50 psi, and the pumps would boost the pressure to the required discharge pressure of 1269 psi. Effectively, the pumps would add the energy equivalent of (1269 − 50) or 1219 psi at a flow rate of 200,000 bbl/day (5833.33 gal/min). The water horsepower (WHP) required is calculated as

$$\text{WHP} = \frac{(1219 \times 2.31/0.895) \times 5833.33 \times 0.895}{3960} = 4148 \text{ HP}$$

In general the WHP, also known as hydraulic horsepower (HHP), based on 100 percent pump efficiency, is calculated from the following equation:

$$\text{WHP} = \frac{\text{ft of head} \times \text{gal/min} \times \text{liquid specific gravity}}{3960}$$

Assuming a pump efficiency of 80 percent, the pump brake horsepower (BHP) required at Corona is

$$\text{BHP} = \frac{4148}{0.8} = 5185 \text{ HP}$$

The general formula for calculating the BHP of a pump is

$$\text{BHP} = \frac{\text{ft of head} \times \text{gal/min} \times \text{liquid specific gravity}}{3960 \times \text{effy}} \tag{6.92}$$

where effy is the pump efficiency expressed as a decimal value.

If the pump is driven by an electric motor with a motor efficiency of 95 percent, the drive motor HP required will be

$$\text{Motor HP} = \frac{5185}{0.95} = 5458 \text{ HP}$$

The nearest standard size motor of 6000 HP would be adequate for this application. Of course, this assumes that the entire pumping requirement at the Corona pump station is handled by a single pump-motor unit. In reality, to provide for operational flexibility and maintenance two or more pumps will be configured in series or parallel to provide the necessary pressure at the specified flow rate. Let us assume that two pumps are configured in parallel to provide the necessary head pressure of 1219 psi (3146 ft) at the Corona pump station. Each pump will be designed for one-half the total flow rate, or 2917 gal/min, and a pressure of 3146 ft. If the pumps selected had an efficiency of 80 percent, we could calculate the BHP required for each pump as follows:

$$\begin{aligned}\text{BHP} &= \frac{3146 \times 2917 \times 0.895}{3960 \times 0.80} \qquad \text{from Eq. (6.92)}\\ &= 2593 \text{ HP}\end{aligned}$$

Alternatively, if the pumps were configured in series instead of parallel, each pump would be designed for the full flow rate of 5833.33 gal/min but at half the total head required or 1573 ft. The BHP required per pump will still be the same as for the parallel configuration. Pumps are discussed in more detail in Sec. 6.17.

6.17 Pumps

Pumps are installed on petroleum products pipelines to provide the necessary pressure at the beginning of the pipeline to compensate for pipe friction and any elevation head and provide the necessary delivery pressure at the pipeline terminus. Pumps used on petroleum pipelines are either positive displacement (PD) type or centrifugal pumps.

PD pumps generally have higher efficiency, higher maintenance cost, and a fixed volume flow rate at any pressure within allowable limits. Centrifugal pumps on the other hand are more flexible in terms of flow rates but have lower efficiency and lower operating and maintenance cost. The majority of liquid pipelines today are driven by centrifugal pumps.

Since pumps are designed to produce pressure at a given flow rate, an important characteristic of a pump is its performance curve. The performance curve is a graphic representation of how the pressure generated by a pump varies with its flow rate. Other parameters, such as efficiency and horsepower, are also considered as part of a pump performance curve.

6.17.1 Positive displacement pumps

Positive displacement (PD) pumps include piston pumps, gear pumps, and screw pumps. These are used generally in applications where a constant volume of liquid must be pumped against a fixed or variable pressure.

PD pumps can effectively generate any amount of pressure at the fixed flow rate, which depends on its geometry, as long as equipment pressure limits are not exceeded. Since a PD pump can generate any pressure required, we must ensure that proper pressure control devices are installed to prevent rupture of the piping on the discharge side of the PD pump. As indicated earlier, PD pumps have less flexibility with flow rates and higher maintenance cost. Because of these reasons, PD pumps are not popular in long-distance and distribution liquid pipelines. Centrifugal pumps are preferred due to their flexibility and low operating cost.

6.17.2 Centrifugal pumps

Centrifugal pumps consist of one or more rotating impellers contained in a casing. The centrifugal force of rotation generates the pressure in the liquid as it goes from the suction side to the discharge side of the pump. Centrifugal pumps have a wide range of operating flow rates with fairly good efficiency. The operating and maintenance cost of a centrifugal pump is lower than that of a PD pump. The performance curves of a centrifugal pump consist of head versus capacity, efficiency versus capacity, and BHP versus capacity. The term *capacity* is used synonymously with flow rate in connection with centrifugal pumps. Also the term *head* is used in preference to pressure when dealing with centrifugal pumps. Figure 6.17 shows a typical performance curve for a centrifugal pump.

Generally, the head-capacity curve of a centrifugal pump is a drooping curve. The highest head is generated at zero flow rate (shutoff head) and the head decreases with an increase in the flow rate as shown in Fig. 6.17. The efficiency increases with flow rate up to the best efficiency point (BEP) after which the efficiency drops off. The BHP calculated using Eq. (6.92) also generally increases with flow rate but may taper off or start decreasing at some point depending on the head-capacity curve.

The head generated by a centrifugal pump depends on the diameter of the pump impeller and the speed at which the impeller runs. The affinity laws of centrifugal pumps may be used to determine pump performance at different impeller diameters and pump speeds. These laws can be mathematically stated as follows:

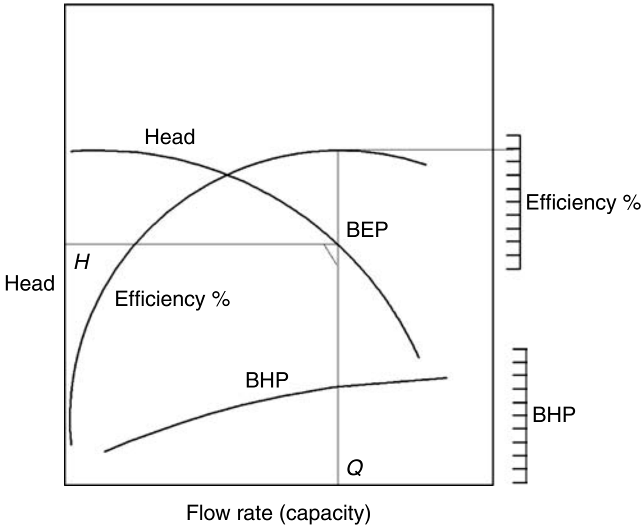

Figure 6.17 Performance curve for centrifugal pump.

For impeller diameter change:

$$\text{Flow rate:} \quad \frac{Q_1}{Q_2} = \frac{D_1}{D_2} \tag{6.93}$$

$$\text{Head:} \quad \frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \tag{6.94}$$

$$\text{BHP:} \quad \frac{\text{BHP}_1}{\text{BHP}_2} = \left(\frac{D_1}{D_2}\right)^3 \tag{6.95}$$

For impeller speed change:

$$\text{Flow rate:} \quad \frac{Q_1}{Q_2} = \frac{N_1}{N_2} \tag{6.96}$$

$$\text{Head:} \quad \frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{6.97}$$

$$\text{BHP:} \quad \frac{\text{BHP}_1}{\text{BHP}_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{6.98}$$

where subscript 1 refers to initial conditions and subscript 2 to final conditions. It must be noted that the affinity laws for impeller diameter change are accurate only for small changes in diameter. However, the affinity laws for impeller speed change are accurate for a wide range of impeller speeds.

Using the affinity laws, if the performance of a centrifugal pump is known at a particular diameter, the corresponding performance at a slightly smaller diameter or slightly larger diameter can be calculated very easily. Similarly, if the pump performance for a 10-in impeller at 3500 revolutions per minute (r/min) impeller speed is known, we can easily calculate the performance of the same pump at 4000 r/min.

Example 6.28 The performance of a centrifugal pump with a 10-in impeller is as shown in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	2355	0
1600	2340	57.5
2400	2280	72.0
3200	2115	79.0
3800	1920	80.0
4000	1845	79.8
4800	1545	76.0

(a) Determine the revised pump performance with a reduced impeller size of 9 in.

(b) If the given performance is based on an impeller speed of 3560 r/min, calculate the revised performance at an impeller speed of 3000 r/min.

Solution

(a) The ratio of impeller diameters is $\frac{9}{10} = 0.9$. Therefore, the Q values will be multiplied by 0.9 and the H values will be multiplied by $0.9 \times 0.9 = 0.81$. Revised performance data are given in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	1907	0
1440	1895	57.5
2160	1847	72.0
2880	1713	79.0
3420	1555	80.0
3600	1495	79.8
4320	1252	76.0

(b) When speed is changed from 3560 to 3300 r/min, the speed ratio $=$ $3000/3560 = 0.8427$. Therefore, Q values will be multiplied by 0.8427 and H values will be multiplied by $(0.8247)^2 = 0.7101$. Therefore, the revised pump performance is as shown in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	1672	0
1348	1662	57.5
2022	1619	72.0
2697	1502	79.0
3202	1363	80.0
3371	1310	79.8
4045	1097	76.0

Example 6.29 For the same pump performance described in Example 6.28, calculate the impeller trim necessary to produce a head of 2000 ft at a flow rate of 3200 gal/min. If this pump had a variable-speed drive and the given performance was based on an impeller speed of 3560 r/min, what speed would be required to achieve the same design point of 2000 ft of head at a flow rate of 3200 gal/min?

Solution Using the affinity laws, the diameter required to produce 2000 ft of head at 3200 gal/min is as follows:

$$\left(\frac{D}{10}\right)^2 = \frac{2000}{2115}$$

$$D = 10 \times 0.9724 = 9.72 \text{ in}$$

The speed ratio can be calculated from

$$\left(\frac{N}{3560}\right)^2 = \frac{2000}{2115}$$

Solving for speed,

$$N = 3560 \times 0.9724 = 3462 \text{ r/min}$$

Strictly speaking, this approach is only approximate since the affinity laws have to be applied along iso-efficiency curves. We must create the new H-Q curves at the reduced impeller diameter (or speed) to ensure that at 3200 gal/min the head generated is 2000 ft. If not, adjustment must be made to the impeller diameter (or speed). This is left as an exercise for the reader.

6.17.3 Net positive suction head

An important parameter related to the operation of centrifugal pumps is the net positive suction head (NPSH). This represents the absolute minimum pressure at the suction of the pump impeller at the specified flow rate to prevent pump cavitation. Below this value the pump impeller may be damaged and render the pump useless. The calculation of NPSH available for a particular pump and piping configuration requires knowledge of the pipe size on the suction side of the pump, the elevation of the liquid source and the pump impeller, along with the atmospheric pressure and vapor pressure of the liquid being pumped. This will be illustrated using an example.

Example 6.30 Figure 6.18 shows a centrifugal pump installation where liquid is pumped out of a storage tank which is located at an elevation of 25 ft above that of the centerline of the pump. The piping from the storage tank to the pump suction consists of straight pipe, valves, and fittings. Calculate the NPSH available at a flow rate of 3200 gal/min. The liquid being pumped has a specific gravity of 0.825 and a viscosity of 15 cSt. If flow rate increases to 5000 gal/min, what is the new NPSH available?

Solution The NPSH available is calculated as follows:

$$\text{NPSH} = (P_a - P_v)\frac{2.31}{\text{Sg}} + H + E_1 - E_2 - h_f \tag{6.99}$$

where P_a = atmospheric pressure, psi
P_v = liquid vapor pressure at flowing temperature, psia
Sg = liquid specific gravity
H = liquid head in tank, ft
E_1 = elevation of tank bottom, ft
E_2 = elevation of pump suction, ft
h_f = friction loss in suction piping from tank to pump suction, ft

All terms in Eq. (6.99) are known except the head loss h_f. This item must be calculated considering the flow rate, pipe size, and liquid properties. The Reynolds number at 3200 gal/min in the 16-in pipe, using Eq. (6.35), is

$$R = \frac{3162.5 \times 3200}{15.5 \times 15} = 43{,}527$$

The friction factor will be found from the Moody diagram. Assume the pipe absolute roughness is 0.002 in. Then

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{15.5} = 0.0001$$

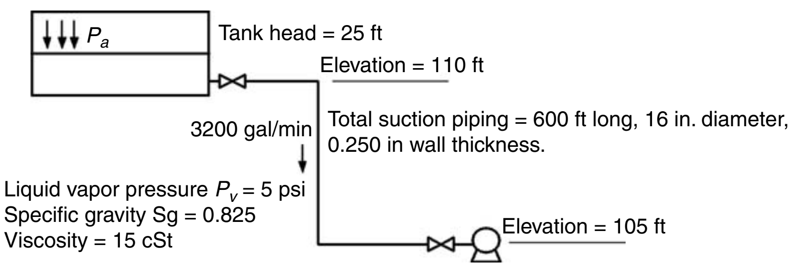

Figure 6.18 NPSH calculations.

From the Moody diagram $f = 0.0215$. The flow velocity from Eq. (6.31) is

$$v = \frac{0.4085 \times 3200}{(15.5)^2} = 5.44 \text{ ft/s}$$

The pressure loss in the suction piping from the tank to the pump will be calculated using the Darcy equation (6.47):

$$\begin{aligned} h_f &= \frac{0.1863\, fLv^2}{D} \\ &= \frac{0.1863 \times 0.0215 \times 600 \times (5.44)^2}{15.5} = 4.59 \text{ ft} \end{aligned}$$

Substituting these values in Eq. (6.99), we obtain

$$\begin{aligned} \text{NPSH} &= (14.73 - 5) \times \frac{2.31}{0.825} + 25 + 110 - 105 - 4.59 \\ &= 27.24 + 25 + 110 - 105 - 4.59 = 52.65 \end{aligned}$$

The required NPSH for the pump must be less than this value. If the flow rate increases to 5000 gal/min and the liquid level in turn drops to 1 ft, the revised NPSH available is calculated as follows. With flow rate increasing from 3200 to 5000 gal/min, the head loss due to friction h_f is approximately,

$$h_f = \left(\frac{5000}{3200}\right)^2 \times 4.59 = 11.2 \text{ ft}$$

Therefore,

$$\text{NPSH} = 27.24 + 1 + 110 - 105 - 11.2 = 22.04 \text{ ft}$$

It can be seen that the NPSH available dropped off dramatically with the reduction in liquid level in the tank and the increased friction loss in the suction piping at the higher flow rate.

The required NPSH for the pump (based on vendor data) must be lower than the available NPSH calculations just obtained. If the pump data show 30 ft NPSH is required at 5000 gal/min, the preceding calculation indicates that the pump will cavitate since the NPSH available is only 22.04 ft.

6.17.4 Specific speed

An important parameter related to centrifugal pumps is the specific speed. The *specific speed* of a centrifugal pump is defined as the speed at which a geometrically similar pump must be run such that it will produce a head of 1 ft at a flow rate of 1 gal/min. Mathematically, the specific speed is defined as follows:

$$N_S = \frac{NQ^{1/2}}{H^{3/4}} \tag{6.100}$$

where N_S = specific speed
N = impeller speed, r/min
Q = flow rate, gal/min
H = head, ft

It must be noted that in Eq. (6.100) for specific speed, the capacity Q and head H must be measured at the best efficiency point (BEP) for the maximum impeller diameter of the pump. For a multistage pump the value of the head H must be calculated per stage. It can be seen from Eq. (6.100) that low specific speed is attributed to high head pumps and high specific speed for pumps with low head.

Similar to the specific speed, another term known as *suction specific speed* is also applied to centrifugal pumps. It is defined as follows:

$$N_{SS} = \frac{NQ^{1/2}}{(\mathrm{NPSH}_R)^{3/4}} \tag{6.101}$$

where N_{SS} = suction specific speed
N = impeller speed, r/min
Q = flow rate, gal/min
NPSH_R = NPSH required at best efficiency point

With single or double suction pumps the full capacity Q is used in Eq. (6.101) for specific speed. For double suction pumps one-half the value of Q is used in calculating the suction specific speed.

Example 6.31 Calculate the specific speed of a four-stage double suction centrifugal pump with a 12-in-diameter impeller that runs at 3500 r/min and generates a head of 2300 ft at a flow rate of 3500 gal/min at the BEP. Calculate the suction specific speed of this pump, if the NPSH required is 23 ft.

Solution From Eq. (6.100), the specific speed is

$$N_S = \frac{NQ^{1/2}}{H^{3/4}}$$

$$= \frac{3500\,(3500)^{1/2}}{(2300/4)^{3/4}} = 1763$$

The suction specific speed is calculated using Eq. (6.101).

$$N_{SS} = \frac{NQ^{1/2}}{\mathrm{NPSH}_R{}^{3/4}}$$

$$= \frac{3500\,(3500/2)^{1/2}}{(23)^{3/4}} = 13{,}941$$

6.17.5 Effect of viscosity and gravity on pump performance

Generally pump vendors provide centrifugal pump performance based on water as the pumped liquid. Thus the head versus capacity, efficiency versus capacity, and BHP versus capacity curves for a typical centrifugal pump as shown in Fig. 6.17 is really the performance when pumping water. When pumping a petroleum product, the head generated at a particular flow will be slightly less than that with water. The degree of departure from the water curve depends on the viscosity of the petroleum product. For example, when pumping gasoline, jet fuel, or diesel, the head generated will practically be the same as that obtained with water, since these three liquids do not have appreciably high viscosity compared to water.

Generally, if the viscosity is greater than 10 cSt (50 SSU), the performance with the petroleum product will degrade compared to the water performance. Thus when pumping ANS crude with a viscosity of 200 SSU at 60°F, the head-capacity curve will be located below that for water as shown in Fig. 6.19. The Hydraulic Institute chart can be used to correct the water performance curve of a centrifugal pump when pumping high-viscosity liquid. It must be noted that with a high-viscosity

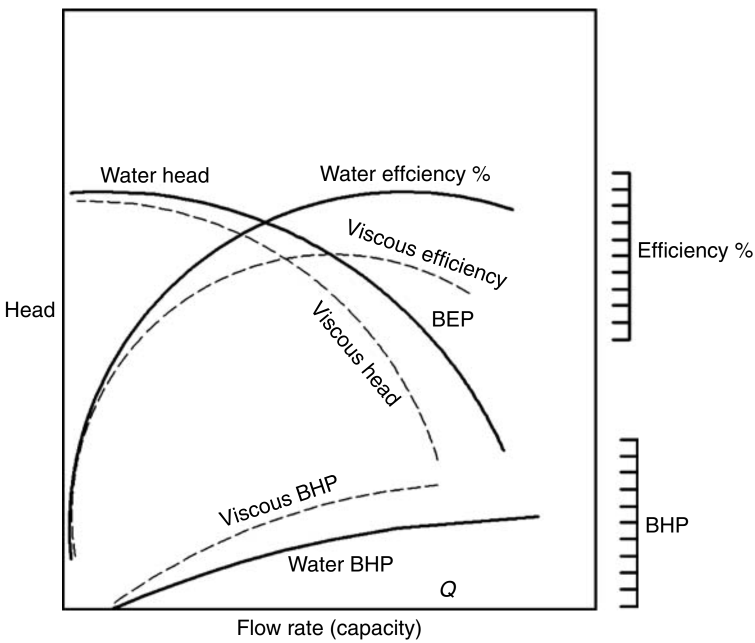

Figure 6.19 Head-capacity curves.

liquid, the pump efficiency degrades faster than the pump head. This can be seen in the comparative performance curve for water and high-viscosity liquid shown in Fig. 6.19.

Several software programs are available to calculate the performance of a centrifugal pump when pumping a high-viscosity liquid. These programs use the Hydraulic Institute chart method to correct the head, efficiency, and BHP from the water performance data. One such program is PUMPCALC published by SYSTEK Technologies, Inc. (*www.systek.us*). Appendix C includes a sample printout and graphic of a viscosity corrected pump performance curve using PUMPCALC.

Positive displacement pumps such as screw pumps and gear pumps tend to perform better with high-viscosity liquids. In fact the higher the viscosity of the pumped liquid, the less would be the slip in these types of pumps. For example, if a screw pump is rated at 5000 gal/min, the volume flow rate will be closer to this number with a 2000-SSU viscosity liquid compared to a 500-SSU viscosity liquid. In contrast centrifugal pump performance degrades from water to 500 SSU viscosity to the lowest performance with the 2000-SSU viscosity liquid.

The BHP required by the pump is a function of the liquid specific gravity, flow rate, head, and pump efficiency [from Eq. (6.92)]. We can therefore conclude that the BHP required increases with higher specific gravity liquids. Thus water (specific gravity = 1.0) may require a BHP of 1500 HP at a particular flow rate. The same pump pumping diesel (specific gravity = 0.85) at the same flow rate and head will require less BHP according to the pump curve. Actually, due to the higher viscosity of diesel (approximately 5.0 cSt compared to that of water at 1.0 cSt) the head required to pump the same volume of diesel will be higher than that of water. From this standpoint the BHP required with diesel will be higher than water. However, when reviewing the pump performance curve, the BHP required is directly proportional to the specific gravity and hence the BHP curve, for diesel will be below that of water. The BHP curve for gasoline will be lower than diesel since gasoline has a specific gravity of 0.74.

6.18 Valves and Fittings

Oil pipelines include several appurtenances as part of the pipeline system. Valves, fittings, and other devices are used in a pipeline system to accomplish certain features of pipeline operations. Valves may be used to communicate between the pipeline and storage facilities as well as between pumping equipment and storage tanks. There are many different types of valves, each performing a specific function. Gate valves and ball valves are used in the main pipeline as well as within pump stations and tank farms. Pressure relief valves are used to protect piping systems and facilities from overpressure due to upsets in operational conditions. Pressure regulators and control valves are used to reduce pressures in certain sections of piping systems as well as when delivering petroleum product to third-party pipelines that may be designed for lower operating pressures. Check valves are found in pump stations and tank farms to prevent backflow as well as separating the suction piping from the discharge side of a pump installation. On long-distance pipelines with multiple pump stations, the pigging process necessitates a complex series of piping and valves to ensure that the pig passes through the pump station piping without getting stuck.

All valves and fittings such as elbows and tees contribute to the frictional pressure loss in a pipeline system. Earlier we referred to some of these head losses as minor losses. As described earlier each valve and fitting is converted to an equivalent length of straight pipe for the purpose of calculating the head loss in the pipeline system.

A control valve functions as a pressure-reducing device and is designed to maintain a specified pressure at the downstream side as shown in Fig. 6.20. If P_1 is the upstream pressure and P_2 the downstream pressure, the control valve is designed to handle a given flow rate Q at these pressures. A coefficient of discharge C_v is typical of the control valve design and is related to the pressures and flow rates by the following equation:

$$Q = C_v A (P_1 - P_2)^{1/2} \tag{6.102}$$

where A is a constant.

Generally, the control valve is selected for a specific application based on P_1, P_2, and Q. For example, a particular situation may require 800 psi upstream pressure, 400 psi downstream pressure, and a flow rate of 3000 gal/min. Based on these numbers, we may calculate a $C_v = 550$. We would then select the correct size of a particular vendor's control valve that can provide this C_v value at a specified flow rate and pressures.

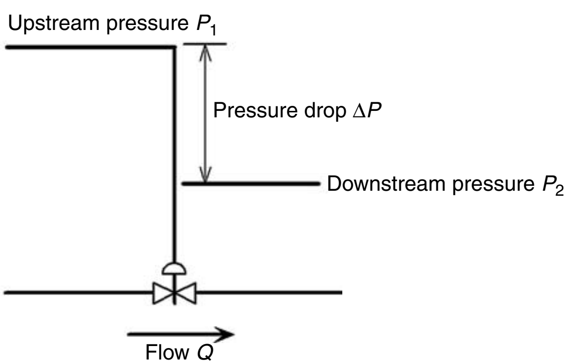

Figure 6.20 Control valve.

For example, a 10-in valve from vendor A may have a C_v of 400, while a 12-in valve may have a $C_v = 600$. Therefore, in this case we would choose a 12-in valve to satisfy our requirement of $C_v = 550$.

6.19 Pipe Stress Analysis

The pipe used to transport petroleum product must be strong enough to withstand the internal pressure necessary to move liquid at the desired flow rate. The wall thickness T necessary to safely withstand an internal pressure of P depends upon the pipe diameter D and yield strength of the pipe material and is generally calculated based upon Barlow's equation:

$$S_h = \frac{PD}{2T} \tag{6.103}$$

where S_h represents the hoop stress in the circumferential direction in the pipe material. Another stress, termed the axial stress or longitudinal stress, acts perpendicular to the cross section of the pipe. The axial stress is one-half the magnitude of the hoop stress. Hence the governing stress is the hoop stress from Eq. (6.103).

Applying a safety factor and including the yield strength of the pipe material, Barlow's equation is modified for use in petroleum piping calculation as follows:

$$P = \frac{2T \times S \times E \times F}{D} \tag{6.104}$$

where P = internal pipe design pressure, psig
D = pipe outside diameter, in
T = pipe wall thickness, in
S = specified minimum yield strength (SMYS) of pipe material, psig
E = seam joint factor = 1.0 for seamless and submerged arc welded (SAW) pipes (see Table 6.7 for other joint types)
F = design factor, usually 0.72 for liquid pipelines

The design factor is sometimes reduced from the 0.72 value in the case of offshore platform piping or when certain city regulations require buried pipelines to be operated at a lower pressure. Equation (6.104) for calculating the internal design pressure is found in the Code of Federal Regulations, Title 49, Part 195, published by the U.S. Department of Transportation (DOT). You will also find reference to this equation in ASME standard B31.4 for design and transportation of liquid pipelines.

In SI units, the internal design pressure equation is the same as shown in Eq. 6.104, except the pipe diameter and wall thickness are in

TABLE 6.7 Pipe Design Joint Factors

Pipe specification	Pipe category	Joint factor E
ASTM A53	Seamless	1.00
	Electric resistance welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
ASTM A106	Seamless	1.00
ASTM A134	Electric fusion arc welded	0.80
ASTM A135	Electric Resistance Welded	1.00
ASTM A139	Electric fusion welded	0.80
ASTM A211	Spiral welded pipe	0.80
ASTM A333	Seamless	1.00
ASTM A333	Welded	1.00
ASTM A381	Double submerged arc welded	1.00
ASTM A671	Electric fusion welded	1.00
ASTM A672	Electric fusion welded	1.00
ASTM A691	Electric fusion welded	1.00
API 5L	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
API 5LX	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
API 5LS	Electric resistance welded	1.00
	Submerged arc welded	1.00

millimeters. The SMYS of pipe material and the internal design pressures are both expressed in kilopascals.

Petroleum pipelines are constructed of steel pipe conforming to American Petroleum Institute (API) standards 5L and 5LX specifications. Some piping may also be constructed of steel pipe conforming to ASTM and ANSI standards. High-strength steel pipe may be designated as API 5LX-52, 5LX-60, or 5LX-80. The last two digits of the pipe specification denote the SMYS of the pipe material. Thus 5LX-52 pipe has a yield strength of 52,000 psi.

Example 6.32 Calculate the allowable internal design pressure for a 16-inch (0.250-in wall thickness) pipeline constructed of API 5LX-52 steel. What wall thickness will be required if an internal working pressure of 1400 psi is required?

Solution Using Eq. (6.104),

$$P = \frac{2 \times 0.250 \times 52{,}000 \times 0.72 \times 1.0}{16} = 1170 \text{ psi}$$

For an internal working pressure of 1400 psi, the wall thickness required is

$$1400 = \frac{2 \times T \times 52{,}000 \times 0.72 \times 1.0}{16}$$

Solving for T, we get

$$\text{Wall thickness } T = 0.299 \text{ in}$$

The nearest standard pipe wall thickness is 0.312 in.

6.20 Pipeline Economics

In pipeline economics we are interested in determining the most economical pipe size and material to be used for transporting a given volume of a petroleum product from a source to a destination. The criterion would be to minimize the capital investment as well as annual operating and maintenance cost. In addition to selecting the pipe itself to handle the flow rate we must also evaluate the optimum size of pumping equipment required. By installing a smaller-diameter pipe we may reduce the pipe material cost and installation cost. However, the smaller pipe size would result in a larger pressure drop due to friction and hence a higher horsepower, which would require larger, more costly pumping equipment. On the other hand, selecting a larger pipe size would increase the capital cost of the pipeline itself but would reduce the pump horsepower required and hence the capital cost of pumping equipment. Larger pumps and motors will also result in increased annual operating and maintenance cost. Therefore, we need to determine the optimum pipe size and pumping power required based on some approach that will minimize both capital investment as well as annual operating costs. The least present value approach, which considers the total capital investment, the annual operating costs over the life of the pipeline, time value of money, borrowing cost, and income tax rate, seems to be an appropriate method in this regard.

Example 6.33 A 25-mi-long crude oil pipeline is used to transport 200,000 bbl/day of light crude (specific gravity = 0.850 and viscosity = 15 cSt) from a pumping station at Parker to a storage tank at Danby. Determine the optimum pipe size for this application based on the least initial cost. Consider three different pipe sizes—NPS 16, NPS 20, and NPS 24. Use the Colebrook-White equation or the Moody diagram for friction factor calculations. Assume the pipeline is on fairly flat terrain. Use 85 percent pump efficiency, \$700 per ton for pipe material cost, and \$1500 per HP for pump station installation cost. The labor costs for installing the three pipe sizes are \$80, \$100, and \$110 per ft. The pipeline will be designed for an operating pressure of 1400 psi. The pipe absolute roughness $e = 0.002$ in.

Solution Based on a 1400-psi design pressure, the wall thickness of NPS 16 pipe will be calculated first. Assuming API 5LX-52 pipe, the wall thickness required for a 1400-psi operating pressure is calculated from Eq. (6.104):

$$T = \frac{1400 \times 16}{2 \times 52{,}000 \times 0.72} = 0.299 \text{ in}$$

The nearest standard size is 0.312 in. The Reynolds number is calculated from Eq. (6.37) as follows:

$$R = \frac{92.24 \times 200{,}000}{15.376 \times 15} = 79{,}986$$

Therefore, the flow is turbulent.

$$\frac{e}{D} = \frac{0.002}{15.376} = 0.0001$$

The friction factor f is found from the Moody diagram as

$$f = 0.0195$$

The pressure drop per mile per Eq. (6.48) is

$$P_m = 0.0605 \times \frac{0.0195 \times (200{,}000)^2 \times 0.85}{(15.376)^5} = 46.67 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 46.67 = 1167 \text{ psi}$$

Assuming a 50-psi delivery pressure and a 50-psi pump suction pressure,

$$\text{Pump head required at Parker} = \frac{1167 \times 2.31}{0.85} = 3172 \text{ ft}$$

$$\text{Pump flow rate} = \frac{200{,}000 \times 0.7}{24} = 5833.33 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = \frac{3172 \times 5833.33 \times 0.85}{3960 \times 0.85} = 4673 \text{ HP}$$

Therefore a 5000-HP pump unit will be required. Next we will calculate the total pipe required. The total tonnage of NPS 16 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.312(16 - 0.312) = 52.275$$

$$\text{Total pipe tonnage for 25 mi} = \frac{25 \times 52.275 \times 5280}{2000} = 3450 \text{ tons}$$

Increasing this by 5 percent for contingency and considering a material cost of \$700 per ton,

$$\text{Total pipe material cost} = 700 \times 3450 \times 1.05 = \$2.54 \text{ million}$$

$$\text{Labor cost for installing NPS 16 pipeline} = 80 \times 25 \times 5280 = \$10.56 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 5000 = \$7.5 \text{ million}$$

Therefore,

$$\text{Total capital cost of NPS 16 pipeline} = \$2.54 + \$10.56 + \$7.5 = \$20.6 \text{ million}$$

Next we calculate the pressure and HP required for the NPS 20 pipeline:

$$T = \frac{1400 \times 20}{2 \times 52{,}000 \times 0.72} = 0.374 \text{ in}$$

The nearest standard size is 0.375 in. The Reynolds number is calculated from Eq. (6.37) as follows:

$$R = \frac{92.24 \times 200{,}000}{19.25 \times 15} = 63{,}889$$

Therefore, the flow is turbulent.

$$\frac{e}{D} = \frac{0.002}{19.25} = 0.0001$$

The friction factor f is found from the Moody diagram as

$$f = 0.020$$

The pressure drop per mile per Eq. (6.48) is

$$P_m = 0.0605 \times \frac{0.020 \times (200{,}000)^2 \times 0.85}{(19.25)^5} = 15.56 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 15.56 = 389 \text{ psi}$$

Assuming a 50-psi delivery pressure and a 50-psi pump suction pressure,

$$\text{Pump head required at Parker} = \frac{389 \times 2.31}{0.85} = 1057 \text{ ft}$$

$$\text{Pump flow rate} = \frac{200{,}000 \times 0.7}{24} = 5833.33 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = \frac{1057 \times 5833.33 \times 0.85}{3960 \times 0.85} = 1557 \text{ HP}$$

Therefore a 1750-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 20 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.375\,(20 - 0.375) = 78.6$$

$$\text{Total pipe tonnage for 25 mi} = 25 \times 78.6 \times \frac{5280}{2000} = 5188 \text{ tons}$$

Increasing this by 5 percent for contingency and considering a material cost of $700 per ton,

$$\text{Total pipe material cost} = 700 \times 5188 \times 1.05 = \$3.81 \text{ million}$$

$$\text{Labor cost for installing NPS 20 pipeline} = 100 \times 25 \times 5280 = \$13.2 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 1750 = \$2.63 \text{ million}$$

Therefore,

$$\text{Total capital cost of NPS 20 pipeline} = \$3.81 + \$13.2 + \$2.63 = \$19.64 \text{ million}$$

Next we calculate the pressure and HP required for NPS 24 pipeline.

$$T = \frac{1400 \times 24}{2 \times 52{,}000 \times 0.72} = 0.449 \text{ in}$$

The nearest standard size is 0.500 in. The Reynolds number is calculated from Eq. (6.37) as follows:

$$R = \frac{92.24 \times 200{,}000}{23.0 \times 15} = 53{,}473$$

Therefore, the flow is turbulent.

$$\frac{e}{D} = \frac{0.002}{23.0} = 0.0001$$

The friction factor f is found from the Moody diagram as

$$f = 0.021$$

The pressure drop per mile per Eq. (6.48) is

$$P_m = 0.0605 \times \frac{0.021 \times (200{,}000)^2 \times 0.85}{(23.0)^5} = 6.71 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 6.71 = 167.8 \text{ psi}$$

Assuming a 50-psi delivery pressure and a 50-psi pump suction pressure,

$$\text{Pump head required at Parker} = \frac{167.8 \times 2.31}{0.85} = 456 \text{ ft}$$

$$\text{Pump flow rate} = \frac{200{,}000 \times 0.7}{24} = 5833.33 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = \frac{456 \times 5833.33 \times 0.85}{3960 \times 0.85} = 672 \text{ HP}$$

Therefore an 800-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 24 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.5\,(24 - 0.5) = 125.5$$

$$\text{Total pipe tonnage for 25 mi} = \frac{25 \times 125.5 \times 5280}{2000} = 8283 \text{ tons}$$

Increasing this by 5 percent for contingency and considering a material cost of \$700 per ton,

$$\text{Total pipe material cost} = 700 \times 8283 \times 1.05 = \$6.09 \text{ million}$$

$$\text{Labor cost for installing NPS 24 pipeline} = 110 \times 25 \times 5280 = \$14.52 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 800 = \$1.2 \text{ million}$$

Therefore,

$$\text{Total capital cost of NPS 24 pipeline} = \$6.09 + \$14.52 + \$1.2 = \$21.81 \text{ million}$$

In summary, capital costs of the NPS 16, NPS 20, and NPS 24 pipelines are

$$\text{NPS 16} = \$20.6 \text{ million}$$

$$\text{NPS 20} = \$19.64 \text{ million}$$

$$\text{NPS 24} = \$21.81 \text{ million}$$

Therefore, based on initial cost alone it appears that NPS 20 is the preferred pipe size.

Example 6.34 A 68-mi-long refined petroleum products pipeline is constructed of NPS 24 (0.375-in wall thickness) pipe and is used for transporting 10,000 bbl/h of diesel from Hampton pump station to a delivery tank at Derry. The delivery pressure required at Derry is 30 psi. The elevation at Hampton is 150 ft and at Derry it is 250 ft. Calculate the pumping horsepower required at 80 percent pump efficiency. This pipeline system needs to be expanded to handle increased capacity from 10,000 bbl/h to 20,000 bbl/h. One option would be to install a parallel NPS 24 (0.375-in wall thickness) pipeline and provide upgraded pumps at Hampton. Another option would require expanding the capacity of the existing pipeline by installing an intermediate booster pump station. Determine the more economical alternative for the expansion. Diesel has a specific gravity of 0.85 and a viscosity of 5.5 cSt.

Solution First calculate the Reynolds number from Eq. (6.36):

$$R = \frac{2213.76 \times 10{,}000}{23.25 \times 5.5} = 173{,}119$$

Assuming relative roughness $e/D = 0.0001$, from the Moody diagram we get the friction factor as

$$f = 0.017$$

Pressure drop is calculated using Eq. (6.48).

$$P_m = 34.87 \times \frac{0.017 \times (10{,}000)^2 \times 0.85}{(23.25)^5} = 7.42 \text{ psi/mi}$$

The total pressure required is the sum of friction head, elevation head, and delivery head using Eq. (6.91).

$$P_T = \frac{(68 \times 7.42) + (250 - 150) \times 0.85}{2.31} + 30 = 571.36 \text{ psi}$$

Assuming a 50-psi suction pressure, the pump head required at Hampton is

$$H = \frac{(571.36 - 50) \times 2.31}{0.85} = 1417 \text{ ft}$$

$$\text{Pump flow rate } Q = 10{,}000 \text{ bbl/h} = 7000 \text{ gal/min}$$

Therefore, the pump HP required using Eq. (6.92) is

$$\mathrm{BHP} = \frac{1417 \times 7000 \times 0.85}{3960 \times 0.8} = 2662$$

When the flow rate increases to 20,000 bbl/h from 10,000 bbl/h, the new Reynolds number is

$$R = 2 \times 173{,}119 = 346{,}238$$

Assuming relative roughness $e/D = 0.0001$, from the Moody diagram we get the friction factor as

$$f = 0.0154$$

The pressure drop is calculated using Eq. (6.48):

$$P_m = 34.87 \times \frac{0.0154 \times (20{,}000)^2 \times 0.85}{(23.25)^5} = 26.87 \text{ psi/mi}$$

The total pressure required at Hampton is

$$P_T = (68 \times 26.87) + \frac{(250 - 150) \times 0.85}{2.31} + 30 = 1894 \text{ psi}$$

Since this pressure is higher than a maximum allowable operating pressure (MAOP) of 1400 psi, we will need to install an intermediate booster pump station between Hampton and Derry.

Assuming the total HP required in this case is equally distributed between the two pump stations, we will calculate the pump HP required at each station as follows:

$$\text{Pump station discharge pressure} = \frac{1894 - 50}{2} = 922 \text{ psi}$$

$$\text{Pump head} = \frac{(922 - 50) \times 2.31}{0.85} = 2370 \text{ ft}$$

$$\text{Pump flow rate} = 20{,}000 \text{ bbl/h} = 14{,}000 \text{ gal/min}$$

Therefore, the pump HP required from Eq. (6.92) is

$$\mathrm{BHP} = \frac{2370 \times 14{,}000 \times 0.85}{3960 \times 0.8} = 8903$$

Thus each pump station requires a 9000-HP pump for a total of 18,000 HP.

If we achieve the increased throughput by installing an NPS 24 parallel pipe, the flow through each 24-in pipe will be 10,000 bbl/h, the same as before expansion. Therefore, comparison between the two options of installing a parallel pipe versus adding an intermediate booster pump station must be based on the cost comparison of 68 mi of additional NPS 24 pipe versus increased HP at Hampton and an additional 9000 HP at the new pump station.

Initially, at 10,000 bbl/h, Hampton required 2662, or approximately 3000, HP installed. In the second phase Hampton must be upgraded to 9000 HP and a new 9000-HP booster station must be installed.

$$\text{Incremental HP required for expansion} = 18{,}000 - 3000 = 15{,}000 \text{ HP}$$

$$\text{Capital cost of incremental HP at \$1500 per HP} = 1500 \times 15{,}000 = \$22.5 \text{ million}$$

Compared to installing the booster station, looping the existing NPS 24 line will be calculated on the basis of $700 per ton of pipe material and $100 per ft labor cost.

$$\text{Pipe weight per ft} = 10.68 \times 0.375 \times (24 - 0.375) = 94.62 \text{ lb/ft}$$

$$\text{Material cost for 68 mi of pipe} = \frac{700 \times 94.62 \times 5280 \times 68}{2000} = \$11.9 \text{ million}$$

$$\text{Labor cost for installing 68 mi of NPS 24 pipe} = 68 \times 5280 \times 100 = \$35.9 \text{ million}$$

$$\text{Total cost of NPS 24 pipe loop} = 11.9 + 35.9 = \$47.8 \text{ million}$$

Therefore, based on capital cost alone, it is more economical to install the booster pump station.

Chapter

7

Gas Systems Piping

Introduction

Gas systems piping consists of pipelines that are used to transport compressible fluids such as natural gas and other hydrocarbons. Examples include natural gas gathering systems, gas distribution, and transmission piping. The calculation methods discussed in this chapter are applicable to any compressible fluid including methane and ethane.

7.1 Gas Properties

7.1.1 Mass

Mass is defined as the quantity of matter. It is measured in slugs (slug) and pounds (lb) in U.S. Customary System (USCS) units and kilograms (kg) in Système International (SI) units. A given mass of gas will occupy a certain volume at a particular temperature and pressure. For example, a mass of gas may be contained in a volume of 500 cubic feet (ft^3) at a temperature of 60°F and a pressure of 100 pounds per square inch (lb/in^2 or psi). If the temperature is increased to 100°F, pressure remaining the same, the volume will change according to Charles's law. Similarly, if the volume remains the same, the pressure will increase with temperature. The mass always remains constant as long as gas is neither added nor subtracted from the system. This is referred to as *conservation of mass*.

7.1.2 Volume

Volume is defined as the space occupied by a given mass of gas at a specified temperature and pressure. Since gas expands to fill the container, it varies with pressure and temperature. Thus a large volume of a given mass of gas at low pressure and temperature can be compressed to a small volume at a higher pressure and temperature. Volume is measured in ft^3 in USCS units and cubic meters (m^3) in SI units.

7.1.3 Density

The density of gas is defined as mass per unit volume. Thus,

$$\rho = \frac{m}{V} \tag{7.1}$$

where ρ = density of gas
m = mass of gas
V = volume of gas

Density is expressed in slug/ft^{3}or lb/ft^3 in USCS units and kg/m^3 in SI units.

7.1.4 Specific gravity

The *specific gravity*, or simply the gravity, of gas is measured relative to the density of air at a particular temperature as follows:

$$\text{Gas gravity} = \frac{\text{density of gas}}{\text{density of air}}$$

Both densities are measured at the same temperature and pressure. For example, a sample of natural gas may be referred to as having a specific gravity of 0.65 (specific gravity of air = 1.00) at 60°F. This means that the gas is 65 percent as heavy as air.

The specific gravity of a gas can also be represented as a ratio of its molecular weight to that of air.

$$\text{Specific gravity} = \frac{M_g}{M_{\text{air}}}$$

or

$$G = \frac{M_g}{28.9625} \tag{7.2}$$

where G = specific gravity of gas
M_g = molecular weight of gas
M_{air} = molecular weight of air

In Eq. (7.2) we have used 28.9625 for the apparent molecular weight of air. Sometimes the molecular weight of air is rounded off to 29.0, and then the gas gravity becomes $M_g/29$. If the gas is composed of a mixture of several gases, the value of M_g in Eq. (7.2) is called the apparent molecular weight of the gas mixture.

Generally, a natural gas sample will consist of several components such as methane and ethane. The gravity of such a mixture can be calculated using the individual molecular weights of the component gases.

7.1.5 Viscosity

The *viscosity* of a fluid is defined as the resistance to flow. The viscosity of gases is very low compared to that of liquids. (For example, water has a viscosity of 0.01 poise compared to natural gas which has a viscosity of 0.00012 poise). However, the viscosity of a gas is an important property in the study of gas flow in pipe. The Reynolds number, explained in Sec. 7.2, is a dimensionless parameter that depends on the gas gravity and viscosity and is used to characterize flow through pipes. Two types of viscosities are used. Dynamic viscosity μ, also known as the absolute viscosity, is expressed in lb/(ft · s) in USCS units and poises (P) in SI units. The kinematic viscosity ν is calculated by dividing the dynamic viscosity by the density. Thus the relationship between the two viscosities is expressed as follows:

$$\text{Kinematic viscosity } \nu = \frac{\text{dynamic viscosity } \mu}{\text{density}} \tag{7.3}$$

Kinematic viscosity is measured in ft^2/s in USCS units and stokes (St) in SI units. Other units of viscosity include centipoises (cP) and centistokes (cSt). The viscosity of a pure gas such as air or methane depends only on its temperature and pressure. The viscosity of a gas mixture consisting of various gases such as C_1, C_2, etc., depends on the composition of the mixture, its temperature, and its pressure. If the viscosity of each component gas is known, we can calculate the viscosity of the gas mixture, knowing the mole percent of each component in the mixture, using the following formula:

$$\mu = \frac{\sum(\mu_i y_i \sqrt{M_i})}{\sum(y_i \sqrt{M_i})} \tag{7.4}$$

where y_i represents the mole fraction of each component gas with molecular weight M_i, and μ_i is the viscosity of the component. The viscosity of the mixture is μ_m. Viscosities of common gases at atmospheric conditions are shown in Fig. 7.1. Equation (7.4) is discussed in detail in Sec. 7.1.10.

Several correlations and charts for calculating the viscosity of a gas mixture are also available.

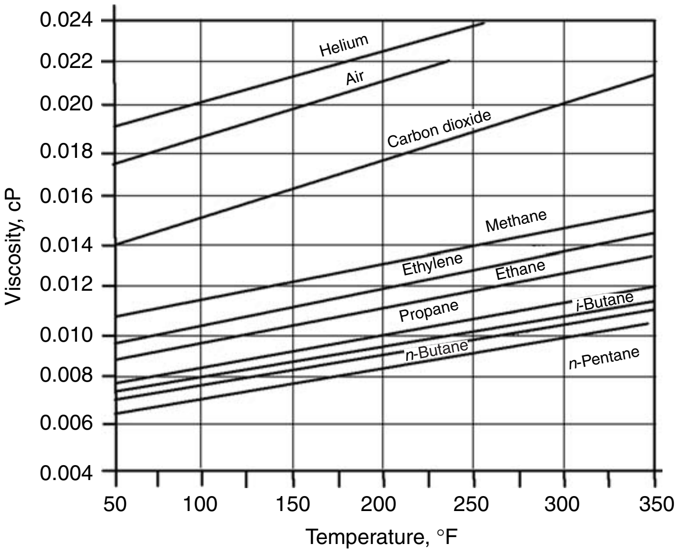

Figure 7.1 Viscosity of common gasses.

7.1.6 Ideal gases

An *ideal gas* is one in which the volume occupied by its molecules is negligible compared to that of the total gas. In addition there is no attraction or repulsion between the gas molecules and the container. The molecules of an ideal gas are considered to be perfectly elastic, and there is no loss in internal energy due to collision between the gas molecules. Ideal gases follow Boyle's law and Charles's law and can be represented by the ideal gas equation or the perfect gas equation. We will discuss the behavior of ideal gases first followed by that of real gases.

The molecular weight M of a gas represents the weight of one molecule of gas. The given mass m of gas will thus contain m/M number of moles. Therefore,

$$n = \frac{m}{M} \tag{7.5}$$

For example, the molecular weight of methane is 16.043 and that of nitrogen is 28.013. Then 100 lb of methane will contain approximately 6 moles of methane.

The ideal gas law states that the pressure, volume, and temperature of a given quantity of gas are related by the ideal gas equation as follows:

$$PV = nRT \tag{7.6}$$

where P = absolute pressure, psia
V = gas volume, ft^3
n = number of lb moles as defined in Eq. (7.5)
R = universal gas constant
T = absolute temperature of gas, °R (°F + 460)

In USCS units R has a value of 10.732 psia $ft^3/(lb \cdot mol \cdot {}^\circ R)$. Using Eq. (7.5) we can restate the ideal gas equation as follows:

$$PV = \frac{mRT}{M} \tag{7.7}$$

where m represents the mass and M is the molecular weight of gas. The ideal gas equation is only valid at pressures near atmospheric pressure. At high pressures it must be modified to include the effect of compressibility.

Two other equations used with gases are Boyle's law and Charles's law. Boyle's law states that the pressure of a given quantity of gas varies inversely as its volume provided the temperature is kept constant. Mathematically, Boyle's law is expressed as

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

or

$$P_1 V_1 = P_2 V_2 \tag{7.8}$$

where P_1 and V_1 are the initial pressure and volume, respectively, at condition 1 and P_2 and V_2 refer to condition 2. In other words, PV = constant.

Charles's law relates to volume-temperature and pressure-temperature variations for a given mass of gas. Thus keeping the pressure constant, the volume of gas will vary directly with the absolute temperature. Similarly, keeping the volume constant, the absolute pressure will vary directly with the absolute temperatures. These are represented mathematically as follows:

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \qquad \text{for constant pressure} \tag{7.9}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \qquad \text{for constant volume} \tag{7.10}$$

Note that in the preceding discussions, the gas temperature is always expressed in absolute scale. In USCS units, the absolute temperature is stated as °R, equal to °F + 460. In SI units the absolute temperature is expressed in kelvin (K), equal to °C + 273.

Pressures used in the preceding equations must also be in absolute units, such as psi absolute or kilopascals absolute. The absolute pressure is obtained by adding the atmospheric base pressure (usually 14.7 psia in USCS units or 101 kPa in SI units) to the gauge pressure.

$$\text{psia} = \text{psig} + \text{base pressure}$$

$$\text{kPa (abs)} = \text{kPa (gauge)} + \text{base pressure}$$

Example 7.1 A certain quantity of gas occupies a volume of 1500 ft^3 at 50 psig. If the temperature is kept constant and its pressure is increased to 100 psig, what is the final volume? Use 14.73 psi for the atmospheric pressure.

Solution Since the temperature is kept constant, Boyle's law can be applied. Using Eq. (7.8) the final volume is calculated as

$$V_2 = \frac{P_1 V_1}{P_2}$$

or

$$V_2 = \frac{(50 + 14.73) \times 1500}{100 + 14.73} = 846.29 \text{ ft}^3$$

Example 7.2 A certain quantity of gas occupies a volume of 1000 ft^3 at 50 psig and 60°F. If the volume is kept constant and its temperature is increased to 100°F, what is the final pressure? If the pressure is kept constant at 50 psig and the temperature is increased to 100°F, what is the final volume? Use 14.73 psi for the atmospheric pressure.

Solution Since the volume is kept constant in the first part of the problem, Charles's law per Eq. (7.10) can be applied as follows:

$$\frac{50 + 14.73}{P_2} = \frac{60 + 460}{100 + 460}$$

Solving for P_2, we get

$$P_2 = 69.71 \text{ psia} \quad \text{or} \quad 54.98 \text{ psig}$$

In the second part of the problem, the pressure is kept constant, and therefore Charles's law per Eq. (7.9) can be applied.

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\frac{1000}{V_2} = \frac{60 + 460}{100 + 460}$$

Solving for V_2, we get

$$V_2 = 1076.92 \text{ ft}^3$$

Example 7.3 An ideal gas is contained in a 200-ft^3 tank at a pressure of 60 psig and a temperature of 100°F.

(a) What is the volume of this quantity of gas at standard conditions of 14.73 psia and 60°F? Assume the atmospheric pressure is 14.6 psia.

(b) If the tank is cooled to 70°F, what would be the pressure in the tank?

Solution

(a) Using the ideal gas law, we can state

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

where $P_1 = 60 + 14.6 = 74.6$ psia
$V_1 = 200$ ft^3
$T_1 = 100 + 460 = 560$°R
$P_2 = 14.73$
$V_2 =$ unknown
$T_2 = 60 + 460 = 520$°R

Substituting the numerical values into the equation, we obtain

$$\frac{74.6 \times 200}{560} = \frac{14.73 \times V_2}{520}$$

$$V_2 = 940.55 \text{ ft}^3$$

(b) When the tank is cooled to 70°F, the final conditions are

$$T_2 = 70 + 460 = 530°\text{R}$$

$$V_2 = 200 \text{ ft}^3$$

$$P_2 = \text{unknown}$$

The initial conditions are

$$P_1 = 60 + 14.6 = 74.6 \text{ psia}$$

$$V_1 = 200 \text{ ft}^3$$

$$T_1 = 100 + 460 = 560°\text{R}$$

It can be seen that we are keeping the volume of the gas constant and simply reducing the temperature from 100°F to 70°F. Therefore, Charles's law applies in this case. Using Eq. (7.10),

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\frac{74.6}{P_2} = \frac{560}{530}$$

$$P_2 = \frac{74.6 \times 530}{560} = 70.60 \text{ psia}$$

So the final pressure will be

$$70.6 - 14.6 = 56.0 \text{ psig}$$

7.1.7 Real gases

The ideal gas equation is applicable only when the pressure of the gas is very low or near atmospheric pressure. When gas pressures and temperatures are higher, the ideal gas equation will not give accurate results. The calculation errors may be as high as 500 percent. An equation of state is generally used for calculating the properties of gases at higher temperatures and pressures.

Real gases behave according to a modified version of the ideal gas law [Eq. (7.6)]. The modifying factor is known as the compressibility factor Z. This is also called the gas deviation factor. Z is a dimensionless number less than 1.0 and varies with temperature, pressure, and physical properties of the gas.

The real gas equation can be written as follows:

$$PV = ZnRT \tag{7.11}$$

where P = absolute pressure, psia
V = gas volume, ft^3
Z = gas deviation factor or compressibility factor, dimensionless
T = absolute temperature of gas, °R
n = number of lb moles as defined in Eq. (7.5)
R = universal gas constant, 10.732 $(\text{psia} \cdot \text{ft}^3)/(\text{lb} \cdot \text{mol} \cdot {}^\circ\text{R})$

The calculation of the compressibility factor will be discussed in Sec. 7.19.

7.1.8 Natural gas mixtures

The *critical temperature* of a pure gas is the temperature above which it cannot be liquefied regardless of the pressure. The *critical pressure* of a pure substance is defined as the pressure above which liquid and gas cannot coexist, regardless of the temperature. With multicomponent mixtures these properties are referred to as the *pseudo critical temperature* and *pseudo critical pressure*. If the composition of the gas mixture is known, we can calculate the pseudo critical pressure and the pseudo critical temperature of the gas mixture using the critical pressure and temperature of the pure components.

The *reduced temperature* is simply the temperature of the gas divided by its critical temperature. Similarly, the *reduced pressure* is simply the pressure of the gas divided by its critical pressure, both temperature and pressure being in absolute units. Similar to the pseudo critical temperature and pressure, we can calculate the pseudo reduced temperature and the pseudo reduced pressure for a gas mixture.

Example 7.4 Calculate the pseudo critical temperature and the pseudo critical pressure of a natural gas mixture consisting of 85 percent methane, 10 percent ethane, and 5 percent propane. From Table 7.1 for properties of gases, we find that the components C_1, C_2, and C_3 have the following critical properties:

Component	Critical Temperature, °R	Critical Pressure, psia
C_1 (methane)	343	666
C_2 (ethane)	550	707
C_3 (propane)	666	617

Some numbers have been rounded off for simplicity.

Solution From the given mole fractions of components, we use Kay's rule to calculate the average pseudo critical temperature and pressure of gas.

$$T_{\mathrm{pc}} = \sum yT_c \tag{7.12}$$

$$P_{\mathrm{pc}} = \sum yP_c \tag{7.13}$$

where T_c and P_c are the critical temperature and pressure of the pure component (C_1, C_2, etc.) and y represents the mole fraction of the component. The calculated values T_{pc} and P_{pc} are the average pseudo critical temperature and pressure of the gas mixture.

Using the given mole fractions, the pseudo critical properties are

$$T_{\mathrm{pc}} = (0.85 \times 343) + (0.10 \times 550) + (0.05 \times 666) = 379.85^\circ\mathrm{R}$$

and

$$P_{\mathrm{pc}} = (0.85 \times 666) + (0.10 \times 707) + (0.05 \times 617) = 667.65 \text{ psia}$$

Example 7.5 The temperature of the gas in Example 7.4 is 80°F and the average pressure is 1000 psig. What are the pseudo reduced temperature and pressure? The base pressure is 14.7 psia.

Solution

$$\text{Pseudo reduced temperature } T_{\mathrm{pr}} = \frac{80 + 460}{379.85} = 1.4216$$

$$\text{Pseudo reduced pressure } P_{\mathrm{pr}} = \frac{1000 + 14.7}{667.65} = 1.5198$$

Pseudo critical properties from gravity. If the gas composition data are not available, we can calculate an approximate value of the pseudo critical temperature and pressure of the gas from the gas gravity as

TABLE 7.1 Properties of Gases

(a) Molecular Weight and Critical Constants

				Critical constants			
Compound	Formula	Molecular weight	Vapor pressure, psia at 100°F	Pressure, psia	Temp., °F	Volume, ft^3/lb	Compressibility factor, 14.696 psia, 60°F
Methane	CH_4	16.0430	(5000)	666.0	−116.66	0.0988	0.998
Ethane	C_2H_6	30.0700	(800)	707.0	90.07	0.0783	0.9919
Propane	C_3H_8	44.0970	188.65	617.0	205.93	0.0727	0.9825
Isobutane	C_4H_{10}	58.1230	72.581	527.9	274.4	0.0714	0.9711
n-butane	C_4H_{10}	58.1230	51.706	548.8	305.52	0.0703	0.9667
Iso-pentane	C_5H_{12}	72.1500	20.443	490.4	368.96	0.0684	
n-pentane	C_5H_{12}	72.1500	15.575	488.1	385.7	0.0695	
Neo-pentane	C_5H_{12}	72.1500	36.72	464.0	321.01	0.0673	0.9582
n-hexane	C_6H_{14}	86.1770	4.9596	436.9	453.8	0.0688	
2-methyl pentane	C_6H_{14}	86.1770	6.769	436.6	435.76	0.0682	
3-methyl pentane	C_6H_{14}	86.1770	6.103	452.5	448.2	0.0682	
Neo hexane	C_6H_{14}	86.1770	9.859	446.7	419.92	0.0667	
2,3-dimethylbutane	C_6H_{14}	86.1770	7.406	454.0	440.08	0.0665	
n-Heptane	C_7H_{16}	100.2040	1.621	396.8	512.8	0.0682	
2-Methylhexane	C_7H_{16}	100.2040	2.273	396.0	494.44	0.0673	
3-Methylhexane	C_7H_{16}	100.2040	2.13	407.6	503.62	0.0646	
3-Ethylpentane	C_7H_{16}	100.2040	2.012	419.2	513.16	0.0665	
2,2-Dimethylpentane	C_7H_{16}	100.2040	3.494	410.8	476.98	0.0665	
2,4-Dimethylpentane	C_7H_{16}	100.2040	3.294	397.4	475.72	0.0667	
3,3-Dimethylpentane	C_7H_{16}	100.2040	2.775	427.9	505.6	0.0662	
Triptane	C_7H_{16}	100.2040	3.376	427.9	496.24	0.0636	
n-octane	C_8H_{18}	114.2310	0.5371	360.7	564.15	0.0673	
Di isobutyl	C_8H_{18}	114.2310	1.1020	361.1	530.26	0.0676	
Iso-octane	C_8H_{18}	114.2310	1.7090	372.7	519.28	0.0657	
n-Nonane	C_9H_{20}	128.2580	0.17155	330.7	610.72	0.0693	
n-Decane	$C_{1}0H_{22}$	142.2850	0.06088	304.6	652.1	0.0702	
Cyclopentane	C_5H_{10}	70.1340	9.917	653.8	461.1	0.0594	
Methylcyclopentane	C_6H_{12}	84.1610	4.491	548.8	499.28	0.0607	
Cyclohexane	C_6H_{12}	84.1610	3.267	590.7	536.6	0.0586	
Methylcyclohexane	C_7H_{14}	98.1880	1.609	503.4	570.2	0.0600	

Ethylene	C_2H_4	28.0540	(1400)	731.0	48.54	0.0746	0.9936
Propylene	C_3H_6	42.0810	232.8	676.6	198.31	0.0717	0.9844
Butylene	C_4H_8	56.1080	62.55	586.4	296.18	0.0683	0.9699
Cis-2-butene	C_4H_8	56.1080	45.97	615.4	324.31	0.0667	0.9665
Trans-2-butene	C_4H_8	56.1080	49.88	574.9	311.8	0.0679	0.9667
Isobutene	C_4H_8	56.1080	64.95	580.2	292.49	0.0681	0.9700
1-Pentene	C_5H_{10}	70.1340	19.12	509.5	376.86	0.0674	0.9487
1,2-Butadene	C_4H_8	54.0920	36.53	(656)*	(354)	(0.070)	(0.969)
1,3-Butadene	C_4H_8	54.0920	59.46	620.3	306	0.0653	0.9723
Isoprene	C_5H_8	68.1190	16.68	(582)*	403	0.066	
Acetylene	C_2H_2	26.0380		890.4	95.29	0.0693	0.993
Benzene	C_6H_8	78.1140	3.225	710.4	552.15	0.0531	
Toluene	C_7H_8	92.1410	1.033	595.5	605.5	0.0549	
Ethyl-benzene	C_8H_{10}	106.1670	0.3716	523	651.22	0.0564	
o-Xylene	C_8H_{10}	106.1670	0.2643	541.6	674.85	0.0557	
m-Xylene	C_8H_{10}	106.1670	0.3265	512.9	650.95	0.0567	
p-Xylene	C_8H_{10}	106.1670	0.3424	509.2	649.47	0.0572	
Styrene	C_8H_8	104.1520	0.2582	587.8	(703)	0.0534	
Isopropylbenzene	C_9H_{12}	120.1940	(0.188)	465.4	676.2	0.0569	
Methyl alcohol	CH_4O	32.0420	4.631	1174	463.01	0.059	
Ethyl alcohol	C_2H_6O	46.0690	2.313	891.7	465.31	0.0581	
Carbon monoxide	CO	28.0100		506.8	−220.51	0.0527	0.9996
Carbon dioxide	CO_2	44.0100		1071	87.73	0.0342	0.9964
Hydrogen sulfide	H_2S	34.0820	394.59	1306	212.4	0.0461	0.9846
Sulfur dioxide	SO_2	64.0650	85.46	1143	315.7	0.0305	0.9802
Ammonia	NH_3	17.0305	211.9	1647	270.2	0.0681	0.9877
Air	$N_2 + O_2$	28.9625		546.9	−221.29	0.0517	0.9996
Hydrogen	H_2	2.0159		187.5	(−400.3)	0.5101	1.0006
Oxygen	O_2	31.9988		731.4	−181.4	0.0367	0.9992
Nitrogen	N_2	28.0134		493	−232.48	0.051	0.9997
Chlorine	Cl_2	70.9054	157.3	1157	290.69	0.028	(0.9875)
Water	H_2O	18.0153	0.95	3200.1	705.1	0.04975	
Helium	He	4.0026		32.99	−450.31	0.23	1.0006
Hydrogen chloride	HCl	36.4606	906.71	1205	124.75	0.0356	0.9923

*Values in parentheses are estimates.

TABLE 7.1 Properties of Gases (*Continued*)

(b) Density and Specific Heat

Compound	Density of Liquid, 14.696 psia, 60°F: Specific gravity 60°F/60°F	lb/gal*	gal/(lb · mol)	Ideal Gas, 14.696 psia, 60°F: Specific gravity (air = 1.00)	ft³/lb gas	ft³/gal liquid	Specific heat, Btu/lb · °F 14.696 psia, 60°F: Ideal gas	Liquid
Methane	(0.3)†	(2.5)	(6.4172)	0.5539	23.654	(59.135)	0.52676	
Ethane	(0.35542)	2.9632	10.148	1.0382	12.62	37.396	0.40789	0.97225
Propane	(0.50694)	4.2265	10.433	1.5226	8.6059	36.373	0.38847	0.61996
Isobutane	(0.56284)	4.6925	12.386	2.0068	6.5291	30.638	0.38669	0.57066
n-butane	0.58400	4.8689	11.938	2.0068	6.5291	31.790	0.39500	0.57272
Iso-pentane	0.62441	5.2058	13.86	2.4912	5.2596	27.38	0.38448	0.53331
n-pentane	0.63105	5.2612	13.714	2.4912	5.2596	27.672	0.38831	0.54363
Neo-pentane	0.59665	4.9744	14.504	2.4912	5.2596	26.163	0.39038	0.55021
n-hexane	0.66404	5.5362	15.566	2.9755	4.4035	24.379	0.38631	0.53327
2-methyl pentane	0.65788	5.4849	15.712	2.9755	4.4035	24.153	0.38526	0.52732
3-methyl pentane	0.66909	5.5783	15.449	2.9755	4.4035	24.564	0.37902	0.51876
Neo hexane	0.65408	5.4532	15.803	2.9755	4.4035	24.013	0.38231	0.51367
2,3-dimethylbutane	0.6663	5.5551	15.513	2.9755	4.4035	24.462	0.37762	0.51308
n-Heptane	0.68805	5.7364	17.468	3.4598	3.7872	21.725	0.38449	0.52802
2-Methylhexane	0.68316	5.6956	17.593	3.4598	3.7872	21.57	0.38170	0.52199
3-Methylhexane	0.69165	5.7664	17.377	3.4598	3.7872	21.838	0.37882	0.51019
3-Ethylpentane	0.70284	5.8597	17.101	3.4598	3.7872	22.192	0.38646	0.51410
2,2-Dimethylpentane	0.67842	5.6561	17.716	3.4598	3.7872	21.421	0.38651	0.51617
2,4-Dimethylpentane	0.67721	5.6460	17.748	3.4598	3.7872	21.382	0.39627	0.5244
3,3-Dimethylpentane	0.69690	5.8102	17.246	3.4598	3.7872	22.004	0.38306	0.50194
Triptane	0.69561	5.7994	17.278	3.4598	3.7872	21.963	0.37724	0.4992
n-octane	0.70678	5.8926	19.385	3.9441	3.322	19.575	0.38334	0.52406
Di Isobutyl	0.69804	5.8197	19.628	3.9441	3.322	19.333	0.37571	0.51130
Iso-octane	0.69629	5.8051	19.678	3.9441	3.322	19.285	0.38222	0.49006
n-Nonane	0.72193	6.0189	21.309	4.4284	2.9588	17.808	0.38248	0.52244
n-Decane	0.73417	6.1209	23.246	4.9127	2.6671	16.325	0.38181	0.52103
Cyclopentane	0.75077	6.2593	11.205	2.4215	5.411	33.869	0.27122	0.42182
methylcyclopentane	0.75467	6.2918	13.376	2.9059	4.509	28.37	0.30027	0.44126
Cyclohexane	0.78339	6.5313	12.886	2.9059	4.509	29.449	0.29012	0.43584
Methylcyclohexane	0.77395	6.4526	15.217	3.3902	3.8649	24.939	0.31902	0.44012

Ethylene				0.9686	13.527		0.35789	
Propylene	0.52098	4.3435	9.6883	1.4529	9.0179	39.169	0.35683	0.57201
Butylene	0.60035	5.0052	11.210	1.9373	6.7636	33.853	0.35535	0.52581
Cis-2-butene	0.62858	5.2406	10.706	1.9373	6.7636	35.445	0.33275	0.5298
Trans-2-butene	0.61116	5.0954	11.012	1.9373	6.7636	34.463	0.35574	0.54215
Isobutene	0.60153	5.0151	11.188	1.9373	6.7636	33.920	0.36636	0.54839
1-Pentene	0.64538	5.3807	13.034	2.4215	5.411	29.115	0.35944	0.51782
1,2-Butadene	0.65798	5.4857	9.8605	1.8677	7.0156	38.485	0.34347	0.54029
1,3-Butadene	0.62722	5.2293	10.344	1.8677	7.0156	36.687	0.34223	0.53447
Isoprene	0.68614	5.7205	11.908	2.3520	5.571	31.869	0.35072	0.51933
Acetylene				0.8990	14.574		0.39754	
Benzene	0.88458	7.3749	10.592	2.6971	4.8581	34.828	0.24295	0.40989
Toluene	0.87191	7.2693	12.675	3.1814	4.1184	29.938	0.26005	0.40095
Ethyl-benzene	0.87168	7.2674	14.609	3.6657	3.5744	25.976	0.27768	0.41139
o-Xylene	0.88467	7.3757	14.394	3.6657	3.5744	26.363	0.28964	0.4162
m-Xylene	0.86894	7.2445	14.655	3.6657	3.5744	25.894	0.27427	0.40545
p-Xylene	0.86570	7.2175	14.71	3.6657	3.5744	25.798	0.2747	0.40255
Styrene	0.91069	7.5926	13.718	3.5961	3.6435	27.664	0.26682	0.41261
Isopropylbenzene	0.86635	7.2229	16.641	4.1500	3.1573	22.805	0.30704	0.42053
Methyl alcohol	0.79620	6.6381	4.827	1.1063	11.843	78.618	0.32429	0.59192
Ethyl alcohol	0.79395	6.6193	6.9598	1.5906	8.2372	54.525	0.33074	0.56381
Carbon monoxide	0.78938	6.5812	4.2561	0.9671	13.548	89.163	0.24847	
Carbon dioxide	0.81801	6.8199	6.4532	1.5196	8.6229	58.807	0.19909	
Hydrogen sulfide	0.80143	6.6817	5.1008	1.1768	11.134	74.397	0.23838	0.50415
Sulfur dioxide	1.3974	11.650	5.4991	2.2120	5.9235	69.008	0.14802	0.32458
Ammonia	0.61831	5.1550	3.3037	0.5880	22.283	114.87	0.49678	1.12090
Air	0.87475	7.2930	3.9713	1.0000	13.103	95.557	0.2398	
Hydrogen	0.071069	0.59252	3.4022	0.06960	188.25	111.54	3.4066	
Oxygen	1.14210	9.5221	3.3605	1.1048	11.859	112.93	0.21897	
Nitrogen	0.80940	6.7481	4.1513	0.9672	13.546	91.413	0.24833	
Chlorine	1.4243	11.875	5.9710	2.4482	5.3519	63.554	0.11375	
Water	1.00000	8.3372	2.1608	0.62202	21.065	175.62	0.44469	0.99974
Helium	0.12510	1.0430	3.8376	0.1382	94.814	98.891	1.2404	
Hydrogen chloride	0.85128	7.0973	5.1372	1.2589	10.408	73.869	0.19086	

*Weight in vacuum.
†Values in parentheses are estimates.

follows:

$$T_{pc} = 170.491 + 307.344G \tag{7.14}$$

$$P_{pc} = 709.604 - 58.718G \tag{7.15}$$

where G = gas gravity (air = 1.00)
T_{pc} = pseudo critical temperature of gas
P_{pc} = pseudo critical pressure of gas

Example 7.6 Calculate the gas gravity of a natural gas mixture consisting of 85 percent methane, 10 percent ethane, and 5 percent propane. Using the gas gravity, calculate the pseudo critical temperature and pressure for this natural gas.

Solution Using Kay's rule for the molecular weight of a gas mixture and Eq. (7.2),

$$\text{Gas gravity } G = \frac{(0.85 \times 16.04) + (0.10 \times 30.07) + (0.05 \times 44.10)}{29.0}$$

$$= 0.6499$$

Using Eqs. (7.14) and (7.15), we get for the pseudo critical properties,

$$T_{pc} = 170.491 + 307.344 \times (0.6499) = 370.22^\circ\text{R}$$

$$P_{pc} = 709.604 - 58.718 \times (0.6499) = 671.44 \text{ psia}$$

Comparing these calculated values with the more accurate solution in Example 7.5, we see that the T_{pc} is off by 2.5 percent and P_{pc} is off by 0.6 percent. These discrepancies are acceptable for most engineering calculations dealing with natural gas pipeline transportation.

Adjustment for sour gas and nonhydrocarbon components. The Standing-Katz chart for compressibility factor calculation (discussed in Sec. 7.1.9) can be used only if there are small amounts of nonhydrocarbon components, up to 50 percent by volume. Adjustments must be made for sour gases containing carbon dioxide and hydrogen sulfide. The adjustments are made to the pseudo critical temperature and pressure as follows. First an adjustment factor ε is calculated based on the amounts of carbon dioxide and hydrogen sulfide present in the sour gas as follows:

$$\varepsilon = 120\,(A^{0.9} - A^{1.6}) + 15\,(B^{0.5} - B^{4.0}) \tag{7.16}$$

where A = sum of mole fractions of CO_2 and H_2S
B = mole fraction of H_2S
ε = adjustment factor, °R

We can then apply this adjustment to the pseudo critical temperature to get the adjusted pseudo critical temperature T'_{pc} as follows:

$$T'_{pc} = T_{pc} - \varepsilon \tag{7.17}$$

Similarly, the adjusted pseudo critical pressure P'_{pc} is

$$P'_{pc} = \frac{P_{pc} \times T'_{pc}}{T_{pc} + B(1 - B)\varepsilon} \tag{7.18}$$

7.1.9 Compressibility factor

The concept of the compressibility factor or gas deviation factor was briefly mentioned in Sec. 7.1.7. It is a measure of how close a real gas is to an ideal gas. The compressibility factor Z is a dimensionless number close to 1.00. It is independent of the quantity of gas. It depends on the gravity, temperature, and pressure of the gas. For example, a sample of natural gas may have a Z value of 0.8595 at 1000 psia and 70°F. Charts are available that show the variation of Z with temperature and pressure. A related term called the *supercompressibility factor* F_{pv} is defined as follows:

$$F_{pv} = \frac{1}{Z^{1/2}} \tag{7.19}$$

or

$$Z = \frac{1}{(F_{pv})^2} \tag{7.20}$$

Several methods are available to calculate the value of Z at a temperature T and pressure P. One approach requires knowledge of the critical temperature and pressure of the gas mixture. The reduced temperature and pressure are calculated from the critical temperatures and pressures as follows:

$$\text{Reduced temperature} = \frac{T}{T_c} \tag{7.21}$$

$$\text{Reduced pressure} = \frac{P}{P_c} \tag{7.22}$$

where temperatures and pressures are in absolute units. The value of the compressibility factor Z is calculated using one of the following methods:

1. Standing and Katz method
2. Hall-Yarborough method
3. Dranchuk, Purvis, and Robinson method
4. AGA method
5. CNGA method

Standing and Katz method. This method uses a chart based on binary mixtures and saturated hydrocarbon vapor data. This approach is reliable for sweet natural gas compositions. Corrections must be applied for hydrogen sulfide and carbon dioxide content of natural gas, using the adjustment factor ε discussed earlier. See Fig. 7.2 for the compressibility factor chart.

Hall-Yarborough method. This method was developed using the equation of state proposed by Starling and Carnahan and requires knowledge of the pseudo critical temperature and pseudo critical pressure of the gas. At a given temperature T and pressure P, we first calculate the pseudo reduced temperature and pseudo reduced pressure. Next, a parameter y, known as the reduced density, is calculated from the following equation:

$$-0.06125 P_{\mathrm{pr}} t e^{-1.2(1-t)^2} + \frac{y + y^2 + y^3 - y^4}{(1-y)^3} - Ay^2 + By^{(2.18+2.82t)} = 0 \tag{7.23}$$

where $A = 14.76t - 9.76t^2 + 4.58t^3$
$B = 90.7t - 242.2t^2 + 42.4t^3$
P_{pr} = pseudo reduced pressure
T_{pr} = pseudo reduced temperature
$t = 1/T_{\mathrm{pr}}$
y = reduced density, dimensionless

It can be seen that the calculation of y is not straightforward and requires a trial-and-error approach. Once y is calculated, the compressibility factor Z is found from the following equation:

$$Z = \frac{-0.06125 P_{\mathrm{pr}} t e^{-1.2(1-t)^2}}{y} \tag{7.24}$$

Dranchuk, Purvis, and Robinson method. In this method the Benedict-Webb-Rubin equation of state is used to correlate the Standing-Katz Z factor chart. Eight coefficients A_1, A_2, etc., are used in this equation as

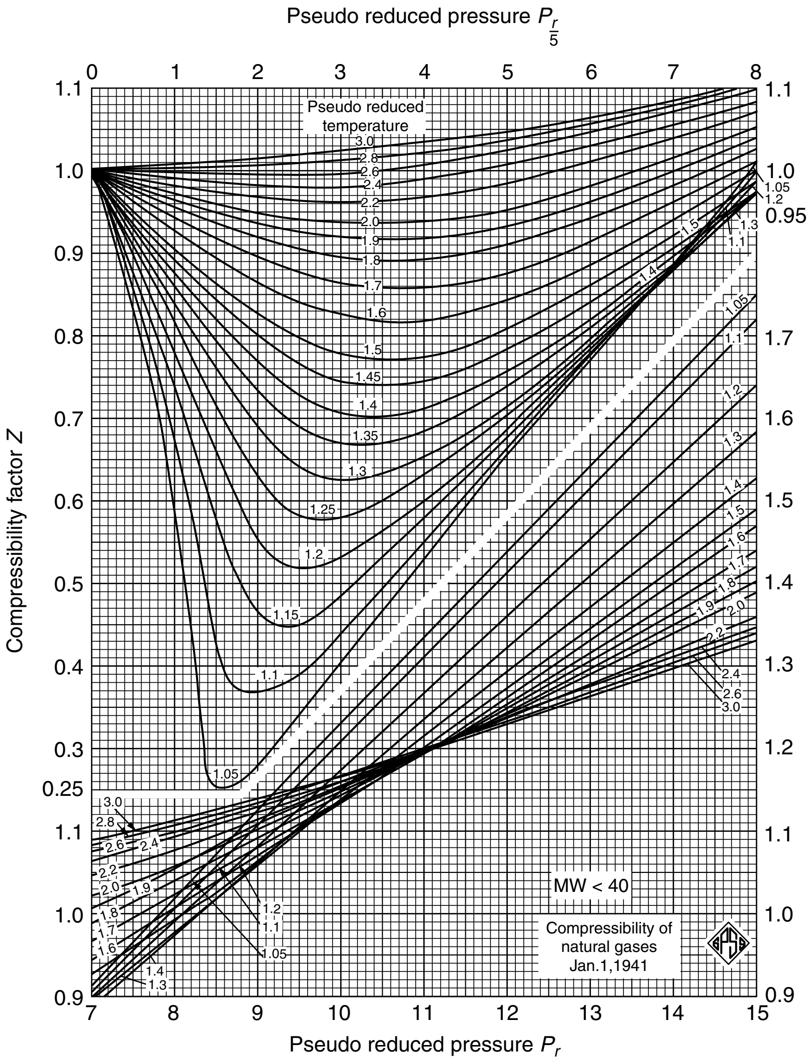

Figure 7.2 Compressibility factor chart. (From Gas Processors Assoc. Eng. Data Book, Vol II, reproduced with permission.)

shown:

$$Z = 1 + \left(A_1 + \frac{A_2}{T_{\text{pr}}} + \frac{A_3}{T_{\text{pr}}{}^3}\right)\rho_r + \left(A_4 + \frac{A_5}{T_{\text{pr}}}\right)\rho_r{}^2 + \frac{A_5 A_6 \rho_r{}^5}{T_{\text{pr}}} + \frac{A_7 \rho_r{}^3}{T_{\text{pr}}{}^3\left(1 + A_8\rho_r{}^2\right)\exp\left(-A_8\rho_r{}^2\right)} \tag{7.25}$$

where ρ_r and the constants A_1 through A_8 are given as follows:

$$\rho_r = \frac{0.27 P_{\mathrm{pr}}}{Z T_{\mathrm{pr}}} \tag{7.26}$$

where $A_1 = 0.31506237$ $\quad A_2 = -1.04670990$
$A_3 = -0.57832729$ $\quad A_4 = 0.53530771$
$A_5 = -0.61232032$ $\quad A_6 = -0.10488813$
$A_7 = 0.68157001$ $\quad A_8 = 0.68446549$

American Gas Association (AGA) method. The AGA method of calculating the compressibility factor Z involves a complicated mathematical approach using the gas properties. A computer program is necessary to calculate the Z factor. It may be stated as follows:

$$Z = \text{function (gas properties, pressures, temperature)} \tag{7.27}$$

The AGA method for calculating Z is outlined in AGA-IGT, Report No. 10. This correlation is valid for gas temperatures ranging from 30°F to 120°F and for gas pressures up to 1380 psig. The calculated values are fairly accurate and within 0.03 percent of the chart method in this range of temperatures and pressures. With higher temperatures and pressures, the difference between the AGA method and the chart method may be as high as 0.07 percent.

For details of other methods of compressibility calculations refer to the American Gas Association publication, Report No. 8, 2nd ed., November 1992.

California Natural Gas Association (CNGA) method. This is one of the easiest equations for calculating the compressibility factor from given gas gravity, temperature, and pressure values. Using this method the compressibility factor Z is calculated from the following formula:

$$Z = \frac{1}{1 + P_{\mathrm{avg}}\,(344{,}400)(10)^{1.785G} / T_f^{\,3.825}} \tag{7.28}$$

where P_{avg} = average gas pressure, psig
T_f = average gas temperature, °R
G = gas gravity (air = 1.00)

This formula is valid for the average gas pressure $P_{\mathrm{avg}} > 100$ psia. When $P_{\mathrm{avg}} \le 100$, we can assume that $Z = 1.00$.

In the case of a gas flowing through a pipeline, since the pressure varies along the pipeline, the compressibility factor Z must be calculated based on an average pressure at a particular location on the pipeline. If two locations have pressures of P_1 and P_2, we could use a simple average pressure of $(P_1 + P_2)/2$. However, a more accurate value of the average pressure is calculated using the following equation:

$$P_{\text{avg}} = \frac{2}{3}\left(P_1 + P_2 - \frac{P_1 \times P_2}{P_1 + P_2}\right) \tag{7.29}$$

Example 7.7 Using the Standing-Katz chart and the calculated values of T_{pc} and P_{pc}, calculate the compressibility factor for the gas in Example 7.6 at 80°F and 100 psig.

Solution From Example 7.6 we get

$$\text{Pseudo reduced temperature } T_{\text{pr}} = 1.4216°\text{R}$$

$$\text{Pseudo reduced pressure } P_{\text{pr}} = 1.5198 \text{ psia}$$

Using the Standing-Katz chart (Fig. 7.1), we read the value of Z as

$$Z = 0.83$$

Example 7.8 A natural gas sample has the following molecular composition:

Component	y
C_1	0.780
C_2	0.005
C_3	0.002
N_2	0.013
CO_2	0.016
H_2S	0.184

where y represents the mole fraction.

(a) Calculate the molecular weight of the gas, its gravity, and the pseudo critical temperature and pressure.

(b) Determine the compressibility factor of this gas at 100°F temperature and 1000 psia pressure.

Solution From the properties of hydrocarbon components (Table 7.1b), we create the following spreadsheet showing the molecular weight M, critical temperature T_c, and critical pressure P_c for each of the component gases, and calculate the molecular weight of the mixture and the pseudo critical temperature and pressure using Kay's rule [Eqs. (7.12) and (7.13)].

Component	y	M	yM	T_c	P_c	yT_c	yP_c
C_1	0.780	16.04	12.5112	343	666	267.54	519.48
C_2	0.005	30.07	0.1504	550	707	2.75	3.54
C_3	0.002	44.10	0.0882	666	617	1.33	1.23
N_2	0.013	28.01	0.3641	227	493	2.95	6.41
CO_2	0.016	44.01	0.7042	548	1071	8.77	17.14
H_2S	0.184	34.08	6.2707	672	1306	123.65	240.30
Total	1.000		20.0888			406.99	788.10

Therefore, the molecular weight of the natural gas sample is

$$\text{Mw} = \sum yM = 20.09$$

and the gas gravity is

$$G = \frac{\text{Mw}}{29.0} = \frac{20.09}{29.0} = 0.6928$$

Also from the preceding,

$$\text{Pseudo critical temperature} = \sum yT_c = 406.99°\text{R}$$

$$\text{Pseudo critical pressure} = \sum yP_c = 788.1 \text{ psia}$$

Since this is a sour gas that contains more than 5 percent nonhydrocarbons, we must adjust the pseudo critical temperature and pressure using Eq. (7.16). The temperature adjustment factor ε is calculated from Eq. (7.16) as follows:

$$A = 0.016 + 0.184 = 0.20 \qquad \text{and} \qquad B = 0.184$$

Therefore,

$$\varepsilon = 120[(0.2)^{0.9} - (0.2)^{1.6}] + 15[(0.184)^{0.5} - (0.184)^{4.0}] = 25.47°\text{R}$$

Therefore, the adjusted pseudo critical temperature and pressure are

$$T'_{\text{pc}} = 406.99 - 25.47 = 381.52°\text{R}$$

$$P'_{\text{pc}} = \frac{788.1 \times 381.52}{406.99 + 0.184 \times (1 - 0.184) \times 25.47} = 731.90 \text{ psia}$$

We can now calculate the compressibility factor Z at 100°F and 1000 psia pressure using the pseudo reduced temperature and pressure as follows:

$$\text{Pseudo reduced temperature} = \frac{100 + 460}{381.52} = 1.468$$

$$\text{Pseudo reduced pressure} = \frac{1000}{731.9} = 1.366$$

Then using these values and the Standing-Katz chart, we get

$$Z = 0.855$$

Example 7.9 The gas gravity of a sample of natural gas is 0.65. Calculate the compressibility factor of this gas at 1000 psig pressure and a temperature of 80°F using the CNGA method. Use a base temperature of 60°F.

Solution

$$\text{Gas temperature } T_f = 80 + 460 = 540°\text{R}$$

Using Eq. (7.28), with slight simplification, the Z factor is given by

$$\frac{1}{Z} = 1 + \frac{1000 \times 344{,}400 \times (10)^{1.785 \times 0.65}}{540^{3.825}} = 1.1762$$

Solving for Z, we get

$$Z = 0.8502$$

7.1.10 Heating value

The *heating value* of a gas represents the thermal energy available per unit volume of the gas. For natural gas, the heating value ranges from 900 to 1000 Btu/ft^3. Two heating values are used in practice: lower heating value (LHV) and higher heating value (HHV). The gross heating value of a gas mixture is calculated from the heating value of the component gases using the following equation:

$$H_m = \sum yH \tag{7.30}$$

where y represents the percentage of each component gas with the corresponding heating value H.

7.1.11 Calculating properties of gas mixtures

The specific gravity and viscosity of gas mixtures may be calculated from that of the component gases as follows. The specific gravity of a mixture of gases is calculated from the percentage composition of each component gas and its molecular weight. If the gas mixture consists of three components with molecular weights, M_1, M_2, M_3, and the respective percentages are pct_1, pct_2, pct_3, then the apparent molecular weight of the mixture is

$$M_m = \frac{\text{pct}_1 M_1 + \text{pct}_2 M_2 + \text{pct}_3 M_3}{100}$$

or

$$M_m = \frac{\sum yM}{100} \tag{7.31}$$

where y represents the percentage of each component gas with molecular weight M.

The specific gravity G_m of the gas mixture (relative to air $= 1.00$) is

$$G_m = \frac{M_m}{28.9625} \tag{7.32}$$

Example 7.10 A typical natural gas mixture consists of 85 percent methane, 10 percent ethane, and 5 percent butane. Assuming the molecular weights of the three component gases to be 16.043, 30.070, 44.097, respectively, calculate the specific gravity of this natural gas mixture. Use 28.9625 for the molecular weight of air.

Solution Applying the percentages to each component in the mixture we get the molecular weight of the mixture as

$$(0.85 \times 16.043) + (0.10 \times 30.070) + (0.05 \times 44.097) = 18.8484$$

$$\text{Specific gravity of gas} = \frac{\text{molecular weight of gas}}{\text{molecular weight of air}}$$

$$G = \frac{18.8484}{28.9625} = 0.6508$$

The viscosity of a mixture of gases at a specified pressure and temperature can be calculated if the viscosities of the component gases in the mixture are known. The following formula can be used to calculate the viscosity of a mixture of gases:

$$\mu = \frac{\sum(\mu_i y_i \sqrt{M_i})}{\sum(y_i \sqrt{M_i})} \tag{7.33}$$

Example 7.11 The viscosities of components C_1, C_2, C_3, and C_4 of a natural gas mixture and their percentages are as follows:

Component	y
C_1	0.8500
C_2	0.0900
C_3	0.0400
nC_4	0.200
Total	1.000

Determine the viscosity of the gas mixture.

Solution

Component	y	M	$M^{1/2}$	$yM^{1/2}$	μ	$\mu y M^{1/2}$
C_1	0.8500	16.04	4.00	3.4042	0.0130	0.0443
C_2	0.0900	30.07	5.48	0.4935	0.0112	0.0055
C_3	0.0400	44.10	6.64	0.2656	0.0098	0.0026
nC_4	0.0200	58.12	7.62	0.1525	0.0091	0.0014
Total	1.000			4.3159		0.0538

The viscosity of the gas mixture is calculated using Eq. (7.33) as follows:

$$\text{Viscosity of gas mixture} = \frac{0.0538}{4.3158} = 0.0125$$

7.2 Pressure Drop Due to Friction

As gas flows through a pipeline, energy is lost due to friction between the gas molecules and the pipe wall. This is evident in the form of a pressure gradient along the pipeline. Before we introduce the various equations to calculate the amount of pressure drop due to friction we will discuss a couple of important parameters related to the flow of gas in a pipeline. The first of these is the velocity of flow, and the other is the Reynolds number.

7.2.1 Velocity

As gas flows at a particular volume flow rate Q, through a pipeline of diameter D, the velocity of the gas can be calculated using the cross-sectional area of pipe as follows:

$$v = \frac{Q}{A} \tag{7.34}$$

Since the flow rate Q is a function of gas pressure and temperature, we must relate the velocity to volume flow at standard conditions. If the density of gas at flowing temperature is ρ and the density at standard conditions is ρ_b from the law of conservation of mass, the mass flow rate at standard conditions must equal the mass flow rate at flowing conditions. Therefore,

$$\rho_b Q_b = \rho Q \tag{7.35}$$

Using the real gas equation, Eq. (7.35) can be simplified as

$$\rho_b = \frac{P_b M}{Z_b R T_b} \tag{7.36}$$

$$\frac{\rho_b}{\rho} = \frac{P_b}{P} \frac{Z}{Z_b} \frac{T}{T_b} \tag{7.37}$$

$$Q = Q_b \frac{P_b}{P} \frac{T}{T_b} \frac{Z}{Z_b} = Q_b \frac{T}{P} \frac{P_b}{T_b} \frac{Z}{Z_b} \tag{7.38}$$

$$\begin{aligned} v &= \frac{4}{86{,}400\pi (D/12)^2} Q_b \frac{T}{P} \frac{P_b}{T_b} \frac{Z}{Z_b} \\ &= (2.653 \times 10^{-3}) \frac{Q_b}{D^2} \frac{T}{P} \frac{P_b}{T_b} \frac{Z}{Z_b} \end{aligned} \tag{7.39}$$

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where v = velocity of flowing gas, ft/s
D = pipe inside diameter, in
T = temperature of flowing gas, °R
P = pressure of gas, psia
Q_b = flow rate, million standard ft^3/day (MMSCFD)
P_b = base pressure, psia
T_b = base temperature, °R

Example 7.12 Calculate the gas velocity in a pipeline at 1000 psig pressure and 80°F temperature. The pipeline is NPS 16 (0.250-in wall thickness). Flow rate = 80 MMSCFD. Use $Z = 0.89$.

Solution

$$\text{Diameter } D = 16 - 0.5 = 15.5 \text{ in}$$

$$P = 1000 + 14.7 = 1014.7 \text{ psia}$$

$$T = 80 + 460 = 540°\text{R}$$

The gas velocity is calculated from Eq. (7.39) as

$$v = (2.653 \times 10^{-3}) \frac{80 \times 10^6}{(15.5)^2} \frac{540}{1014.7} \frac{14.7}{520} \frac{0.89}{1.0} = 11.83 \text{ ft/s}$$

7.2.2 Reynolds number

The Reynolds number of flow is a dimensionless parameter that depends on the flow rate, pipe diameter, and gas properties such as density and viscosity. The Reynolds number is used to characterize the flow type such as laminar flow and turbulent flow.

The Reynolds number is calculated as follows:

$$\text{Re} = \frac{vD\rho}{\mu} \tag{7.40}$$

where Re = Reynolds number of flow, dimensionless
v = velocity of flowing gas, ft/s
D = pipe inside diameter, ft
ρ = gas density, slug/ft^3
μ = gas viscosity, lb/(ft · s)

In gas flow, the following equation for the Reynolds number is more appropriate:

$$\text{Re} = 0.0004778 \frac{P_b}{T_b} \frac{GQ}{\mu D} \tag{7.41}$$

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where P_b = base pressure, psia
T_b = base temperature, °R
G = gas gravity (air = 1.0)
Q = gas flow rate, standard ft^3/day (SCFD)
D = pipe internal diameter, in
μ = Gas viscosity, lb/(ft · s)

In SI units the Reynolds number is given by

$$\text{Re} = 0.5134 \frac{P_b}{T_b} \frac{GQ}{\mu D} \tag{7.41a}$$

where P_b = base pressure, kPa
T_b = base temperature, K
G = gas gravity (air = 1.0)
Q = gas flow rate, m^3/day
D = pipe internal diameter, mm
μ = gas viscosity, P

Laminar flow is defined as flow that causes the Reynolds number to be below a threshold value such as 2000 to 2100. *Turbulent flow* is defined as flow that causes the Reynolds number to be greater than 4000. The range of Reynolds numbers between 2000 and 4000 characterizes an unstable flow regime known as *critical flow*.

Example 7.13 Calculate the Reynolds number of flow for an NPS 16 (0.375-in wall thickness) gas pipeline at a flow rate of 150 MMSCFD. Flowing temperature = 80°F, gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), base pressure = 14.73 psia, and base temperature = 60°F.

Solution Using Eq. (7.41) the Reynolds number is

$$\begin{aligned}\text{Re} &= 0.0004778 \frac{P_b}{T_b} \frac{GQ}{\mu D} \\ &= 0.0004778 \frac{14.73}{460 + 80} \times \frac{0.6 \times 150 \times 10^6}{0.000008 \times 15.25} = 9{,}614{,}746\end{aligned}$$

Therefore, the flow is turbulent since Re > 4000.

7.2.3 Pressure drop equations

Pressure drop in a gas pipeline is calculated using one of several formulas, each of which will be discussed.

1. General flow equation
2. Colebrook-White equation

3. Modified Colebrook-White equation
4. AGA equation
5. Panhandle A equation
6. Panhandle B equation
7. Weymouth equation

The general flow equation, also referred to as the fundamental flow equation, relates flow rate, gas properties, pipe size, and flowing temperature to the upstream and downstream pressures in a pipeline segment. The internal roughness of the pipe is used to calculate a friction factor using the Colebrook-White, modified Colebrook-White, or AGA equation. The friction factor is then used in the general flow equation.

In a steady-state flow of a gas in a pipeline, pressure loss occurs due to friction between the pipe wall and the flowing gas. The general flow equation can be used to calculate the pressure drop due to friction between two points along the pipeline. Since gas properties change with pressure and temperature, the general flow equation must be applied for short segments of the pipeline at a time. The total pressure drop will be the same of the individual pressure drops.

General flow equation. The general flow equation for the steady-state isothermal flow in a gas pipeline is as follows:

$$Q = 38.77F\left(\frac{T_b}{P_b}\right)\left(\frac{P_1{}^2 - P_2{}^2}{GT_f LZ}\right)^{0.5} D^{2.5} \tag{7.42}$$

where Q = volume flow rate, SCFD
F = transmission factor, dimensionless
P_b = base pressure, psia
T_b = base temperature, °R
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
G = gas gravity (air = 1.00)
T_f = average gas flow temperature, °R
L = pipe segment length, mi
Z = gas compressibility factor, dimensionless
D = pipe inside diameter, in

The transmission factor F is related to the friction factor in an inverse way. It will be discussed in detail shortly.

Since the pressure at the inlet of the pipe segment is P_1 and that at the outlet is P_2, an average pressure must be used to calculate the gas compressibility factor Z at the average flowing temperature T_f. Instead of an arithmetic average $(P_1 + P_2)/2$, the following formula is used to calculate the average gas pressure in the pipe segment.

$$P_{\text{avg}} = \frac{2}{3}\left(P_1 + P_2 - \frac{P_1 P_2}{P_1 + P_2}\right) \tag{7.43}$$

It must be noted that Eq. (7.42) does not include any elevation effects. The effect of elevation difference between the upstream and downstream ends of the pipe segment is taken into account by modifying the pipe segment length L and the term $P_1{}^2 - P_2{}^2$ in Eq. (7.42). If the elevation of the upstream end is H_1 and at the downstream end is H_2, the length of the pipe segment L is replaced with an equivalent length L_e as follows:

$$L_e = \frac{L(e^s - 1)}{s} \tag{7.44}$$

where L_e = equivalent length of pipe, mi
L = length of pipe between upstream and downstream ends, mi
s = elevation correction factor, dimensionless

The parameter s depends on the elevation difference $H_2 - H_1$, and in USCS units is calculated as follows:

$$s = \frac{0.0375G(H_2 - H_1)}{T_f Z} \tag{7.45}$$

The calculation for L_e shown in Eq. (7.44) is correct only if we assume a single slope between point 1 (upstream) and point 2 (downstream). If instead a series of slopes are to be considered, we define a parameter j as follows:

$$j = \frac{e^s - 1}{s} \tag{7.46}$$

The term j must be calculated for each slope of each pipe segment of length L_1, L_2, etc., that make up the length L. The equivalent length then must be calculated as

$$L_e = j_1 L_1 + j_2 L_2 e^{s1} + j_3 L_3 e^{s2} + \cdots \tag{7.47}$$

where j_1, j_2, etc., are calculated for each rise or fall in the elevation for pipe segments between the upstream and downstream ends. The parameters s_1, s_2, etc., are calculated for each segment in accordance with Eq. (7.45).

Finally, the term $P_1{}^2 - P_2{}^2$ in Eq. (7.42) is modified to $P_1{}^2 - e^s P_2{}^2$ as follows:

$$Q = 38.77F\left(\frac{T_b}{P_b}\right)\left(\frac{P_1{}^2 - e^s P_2{}^2}{GT_f L_e Z}\right)^{0.5} D^{2.5} \tag{7.48}$$

The transmission factor F in this equation may also be replaced with the Darcy friction factor f defined by the equation

$$f = \frac{4}{F^2} \tag{7.49}$$

Some texts refer to a Fanning friction factor that is one-fourth the Darcy friction factor defined in Eq. (7.49). Throughout this chapter, we will only use the Darcy friction factor. The general flow equation (7.42) may be rewritten in terms of the Darcy friction factor f as follows:

$$Q = 77.54\frac{1}{\sqrt{f}}\frac{T_b}{P_b}\left(\frac{P_1{}^2 - P_2{}^2}{GT_f LZ}\right)^{0.5} D^{2.5} \tag{7.50}$$

With the correction for elevation, considering the pipeline subdivided into short segments, and by substituting 1 for upstream and substituting 2 for downstream, the general flow equation becomes

$$Q = 38.77F\frac{T_b}{P_b}\left(\frac{P_1^2 - e^s P_2^2}{GT_f L_e Z}\right)^{0.5} D^{2.5} \tag{7.51}$$

and

$$Q = 77.54\frac{1}{\sqrt{f}}\frac{T_b}{P_b}\left(\frac{P_1^2 - e^s P_2^2}{GT_f L_e Z}\right)^{0.5} D^{2.5} \tag{7.52}$$

where s and L_e are defined by Eqs. (7.44) and (7.45) as

$$s = \frac{0.0375G(H_2 - H_1)}{T_f Z} \tag{7.53}$$

$$L_e = \frac{L(e^s - 1)}{s} \tag{7.54}$$

In SI units, Eqs. (7.51) and (7.52) become

$$Q = (5.7473 \times 10^{-4})F\frac{T_b}{P_b}\left(\frac{P_1{}^2 - e^s P_2{}^2}{GT_f L_e Z}\right)^{0.5} D^{2.5} \tag{7.55}$$

and

$$Q = (11.4946 \times 10^{-4}) \frac{1}{\sqrt{f}} \frac{T_b}{P_b} \left(\frac{P_1{}^2 - e^s P_2{}^2}{G T_f L_e Z} \right)^{0.5} D^{2.5} \tag{7.56}$$

and the elevation adjustment term s is given by

$$s = \frac{0.0684 G (H_2 - H_1)}{T_f Z} \tag{7.57}$$

where Q = gas flow rate at standard conditions, m^3/day
T_b = base temperature, K (273 + °C)
P_b = base pressure, kPa
T_f = average gas flow temperature, K (273 + °C)
P_1 = upstream pressure, kPa
P_2 = downstream pressure, kPa
H_1 = upstream elevation, m
H_2 = downstream elevation, m
L_e = equivalent length of pipe, km
L = pipe length, km

Other terms are the same as those for USCS units.

Reynolds number and friction factor. The friction factor f, introduced earlier, depends on the type of flow (such as laminar or turbulent) and on the pipe diameter and internal roughness. For laminar flow, for Re $\leq$ 2000, the friction factor is calculated from

$$f = \frac{64}{\text{Re}} \tag{7.58}$$

Depending on the value of Re, flow is laminar or turbulent.

For laminar flow: Re $\leq$ 2000

For turbulent flow: Re $>$ 4000

The region for Re between these two values is termed the critical flow regime.

The turbulent flow region is further subdivided into three separate regions

1. Turbulent flow in smooth pipes
2. Turbulent flow in fully rough pipes
3. Transition flow between smooth pipes and rough pipes.

This is shown in the Moody diagram (Fig. 7.3).

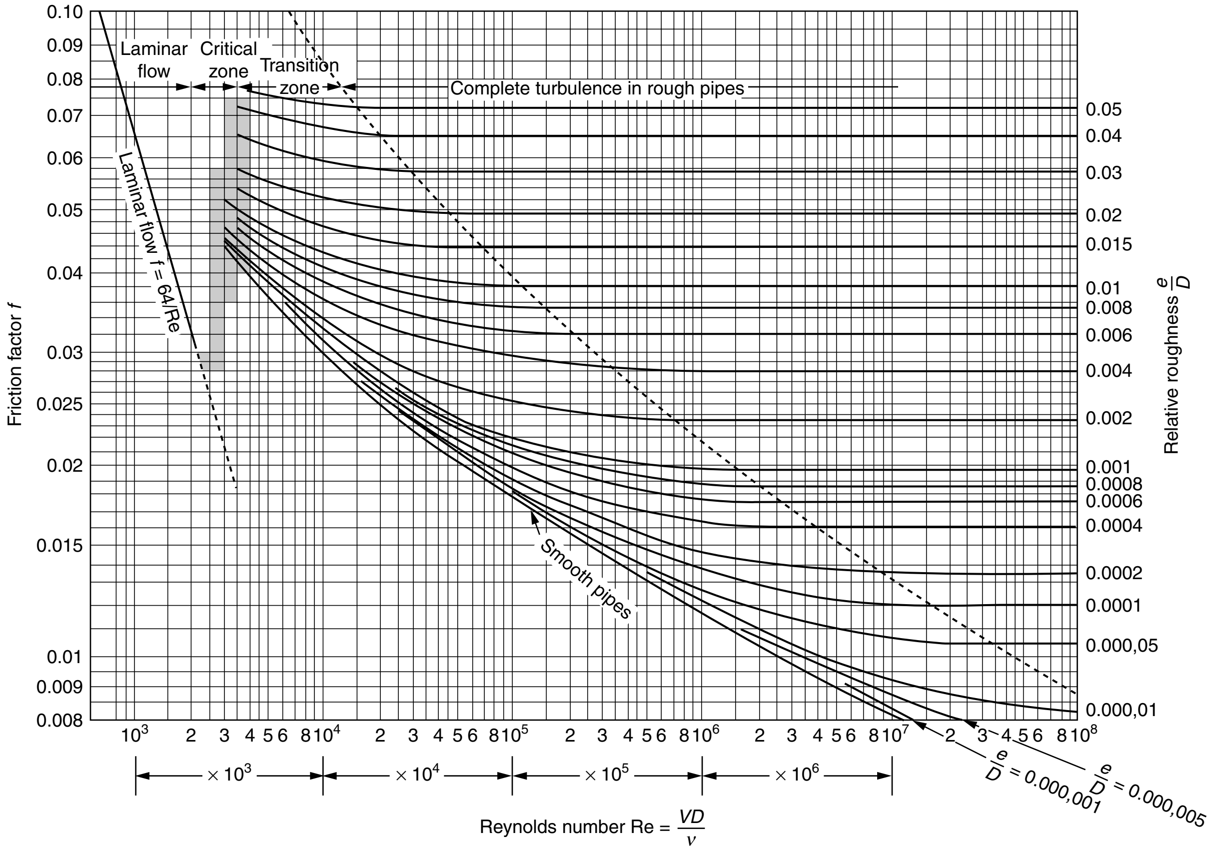

Figure 7.3 Moody diagram.

In the smooth pipe zone of turbulent flow, the pipe friction factor is not affected significantly by the pipe internal roughness. The friction factor f in this region depends only on the Reynolds number Re according to the following equation:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{7.59}$$

In the zone of turbulent flow of fully rough pipes the friction factor f depends less on the Reynolds number and more on the pipe roughness and diameter. It is calculated using the following equation:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D}\right) \tag{7.60}$$

where f = Darcy friction factor
D = pipe inside diameter, in
e = absolute pipe roughness, in

Table 7.2 lists typical pipe roughness values to be used.

In the transition zone between the smooth pipes zone and fully rough pipes zone, the friction factor is calculated using the Colebrook-White equation as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{7.61}$$

Again, see Table 7.2 for typical values of pipe roughness.

For laminar flow the friction factor f is calculated from Eq. (7.58). It can be seen from Eq. (7.58) that the friction factor for laminar flow depends only on the Reynolds number and is independent of pipe diameter or roughness. It must be noted that the Reynolds number does depend on the pipe diameter and gas properties.

TABLE 7.2 Pipe Internal Roughness

Pipe material	Roughness in	Roughness mm
Riveted steel	0.0354–0.354	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.0102	0.26
Galvanized iron	0.0059	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, Drawn tubing, Glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

The friction factor is calculated using either the Colebrook-White equation or the AGA equation (discussed next), and then is used in the general flow equation to calculate the pressure drop. The last three equations listed earlier, Panhandle A and B and Weymouth, do not use a friction factor or the general flow equation. Instead these three equations directly calculate the flow rate for a given pressure drop in a gas pipeline.

7.2.4 Transmission factor and friction factor

The transmission factor F is a measure of how much gas can be transported through the pipeline. Hence it has an inverse relationship to the friction factor f. As the friction factor increases, the transmission factor decreases and the flow rate reduces. Conversely, the higher the transmission factor, the lower the friction factor and hence the higher the flow rate.

The transmission factor F and the friction factor f are related by the following equations:

$$f = \frac{4}{F^2} \tag{7.62}$$

$$F = \frac{2}{\sqrt{f}} \tag{7.63}$$

The friction factor f is actually the Darcy friction factor discussed in classical books on fluid mechanics. A similar friction factor called the Fanning friction factor is also used in industry. The Darcy friction factor and the Fanning friction factor are related as follows:

$$\text{Darcy friction factor} = 4 \times \text{Fanning friction factor} \tag{7.64}$$

We will only use the Darcy friction factor in this book.

Colebrook-White equation. The Colebrook-White equation for obtaining the friction factor is applicable for a wide range of flow in gas pipelines. Friction factor f is given for turbulent flow as:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{7.65}$$

for Re > 4000.

where f = Darcy friction factor
D = pipe inside diameter, in
e = absolute pipe roughness, in
Re = Reynolds number of flow

In terms of the transmission factor F, discussed earlier, Eq. (7.65) may be written as

$$F = -4\log_{10}\left(\frac{e}{3.7D} + \frac{1.255F}{\text{Re}}\right) \tag{7.66}$$

for turbulent flow Re > 4000.

It can be seen from Eqs. (7.65) and (7.66) that the solutions of friction factor f and the transmission factor F are not straightforward. These equations are implicit equations and therefore have to be solved by successive iteration.

Example 7.14 Calculate the flow rate through a 20-mi-long NPS 20 (0.500-in wall thickness) pipeline using the general flow equation. Gas gravity = 0.6, flowing temperature = 80°F, inlet pressure = 1000 psig, outlet pressure = 800 psig, compressibility factor = 0.85, base temperature = 60°F, and base pressure = 14.7 psia. Assume the friction factor is 0.02.

Solution

$$\begin{aligned}
P_1 &= 1000 + 14.7 = 1014.7\,\text{psia} \\
P_2 &= 800 + 14.7 = 814.7\,\text{psia} \\
T_f &= 80 + 460 = 540°R \\
T_b &= 60 + 460 = 520°R \\
Z &= 0.85 \\
P_b &= 14.7\,\text{psia}
\end{aligned}$$

The transmission factor F is found from Eq. (7.49) as

$$F = \frac{2}{\sqrt{f}} = \frac{2}{\sqrt{0.02}} = 14.14$$

From the general flow equation (7.42), we calculate the flow rate as

$$\begin{aligned}
Q &= 38.77 \times 14.14\,\frac{520}{14.7}\left[\frac{(1014.7)^2 - (814.7)^2}{0.6 \times 540 \times 20 \times 0.85}\right]^{0.5} (19.0)^{2.5} \\
&= 248{,}706{,}761 \quad \text{SCFD} \\
&= 248.71 \quad \text{MMSCFD}
\end{aligned}$$

Example 7.15 Calculate the friction factor and transmission factor using the Colebrook-White equation for a 16-in (0.250-in wall thickness) gas pipeline at a flow rate of 100 MMSCFD. Flowing temperature = 80°F, gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), base pressure = 14.73 psia, and base temperature = 60°F. Assume a pipe internal roughness of 600 microinches (μin).

Solution Using Eq. (7.41) the Reynolds number is

$$\mathrm{Re} = 0.0004778\,\frac{P_b}{T_b}\,\frac{GQ}{\mu D}$$

$$= 0.0004778\,\frac{14.73}{460+80} \times \frac{0.6 \times 100 \times 10^6}{0.000008 \times 15.5} = 6{,}306{,}446$$

Since the flow is turbulent we use the Colebrook-White equation (7.61) to calculate the friction factor as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

$$= -2\log_{10}\left(\frac{0.0006}{3.7 \times 15.5} + \frac{2.51}{6{,}306{,}446\sqrt{f}}\right)$$

This equation must be solved by trial and error. Initially, assume $f = 0.02$ and calculate the next approximation as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.0006}{3.7 \times 15.5} + \frac{2.51}{6{,}306{,}446 \times (0.02)^{1/2}}\right) = 9.7538$$

or

$$f = 0.0105$$

Using this value of f, the next approximation is

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.0006}{3.7 \times 15.5} + \frac{2.51}{6{,}306{,}446 \times (0.0105)^{1/2}}\right)$$

$$f = 0.0107$$

After a few more trials we get

$$f = 0.0107$$

The transmission factor is calculated from Eq. (7.63) as follows:

$$F = \frac{2}{\sqrt{f}} = \frac{2}{(0.0107)^{1/2}} = 19.33$$

Modified Colebrook-White equation. In 1956, the U.S. Bureau of Mines published a report proposing a modified version of the Colebrook-White equation. The modified equation tends to produce a higher friction factor and hence a more conservative solution. It is represented by the following equation:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.825}{\mathrm{Re}\sqrt{f}}\right) \tag{7.67}$$

for turbulent flow $\mathrm{Re} > 4000$.

In terms of the transmission factor, Eq. (7.67) may be written as

$$F = -4\log_{10}\left(\frac{e}{3.7D} + \frac{1.4125F}{\mathrm{Re}}\right) \tag{7.68}$$

for turbulent flow Re > 4000.

Example 7.16 Calculate the friction factor and transmission factor using the modified Colebrook-White equation for a 16-in (0.250-in wall thickness) gas pipeline at a flow rate of 100 MMSCFD. Flowing temperature = 80°F, gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), base pressure = 14.73 psia, and base temperature = 60°F. Assume a pipe internal roughness of 600 μin.

Solution Using Eq. (7.41) the Reynolds number is

$$\mathrm{Re} = 0.0004778\,\frac{P_b}{T_b}\frac{GQ}{\mu D}$$

$$= 0.0004778\,\frac{14.73}{460+80} \times \frac{0.6 \times 100 \times 10^6}{0.000008 \times 15.5} = 6{,}306{,}446$$

Since the flow is turbulent, we use the modified Colebrook-White equation (7.67) to calculate the friction factor as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.825}{\mathrm{Re}\sqrt{f}}\right)$$

$$= -2\log_{10}\left(\frac{0.0006}{3.7 \times 15.5} + \frac{2.825}{6{,}306{,}446\sqrt{f}}\right)$$

As before, solving by trial and error for friction factor we get

$$f = 0.02$$

The transmission factor is then calculated from Eq. (7.63) as follows:

$$F = \frac{2}{\sqrt{f}} = \frac{2}{(0.02)^{1/2}} = 14.14$$

It can be seen from the preceding that the friction factor is higher than that calculated using the original Colebrook-White equation in Example 7.15.

AGA equation. The AGA NB-13 method is based on a report published under the sponsorship of the American Gas Association (AGA) in 1964 and 1965. Based on this report, the transmission factor F is calculated using two different equations. The first one is based on the rough pipe law, and the second one is based on the smooth pipe flow. The smaller of the two values of F is used in the general flow equation (7.42) to calculate the pressure drop. For fully turbulent flow:

$$F = 4\log_{10}\left(\frac{3.7D}{e}\right) \tag{7.69}$$

For partially turbulent flow:

$$F = 4D_f \log_{10}\left(\frac{\mathrm{Re}}{1.4125F_t}\right) \tag{7.70}$$

$$F_t = 4\log_{10}\left(\frac{\mathrm{Re}}{F_t}\right) - 0.6 \tag{7.71}$$

where F_t is the smooth pipe transmission factor and D_f is the pipe drag factor that depends on the bend index (BI) of the pipe.

The drag factor D_f is used to account for bends, fittings, etc., and ranges in value from 0.90 to 0.99. The bend index (BI) is the sum of all the angles of all bends in the pipe segment. The drag factor D_f can be estimated from Table 7.3.

Example 7.17 Calculate the transmission factor using the AGA method for a 20-in (0.50-in wall thickness) pipeline at a flow rate of 250 MMSCFD. Absolute pipe roughness = 0.0007 in, bend index = 60°, gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), base pressure = 14.73 psia, and base temperature = 60°F.

Solution From Eq. (7.41) the Reynolds number is calculated first.

$$\mathrm{Re} = 0.0004778\,(250 \times 10^6) \times 0.6 \times \frac{14.73}{19.0 \times 0.000008 \times 520} = 13{,}356{,}517$$

The fully turbulent transmission factor using Eq. (7.69) is

$$F = 4\log_{10}\left(\frac{3.7D}{e}\right) = 4\log_{10}\left(\frac{3.7 \times 19}{0.0007}\right) = 20.01$$

TABLE 7.3 Bend Index and Drag Factor

	Bend Index		
	Extremely low (5°–10°)	Average (60°–80°)	Extremely high (200°–300°)
Bare steel	0.975–0.973	0.960–0.956	0.930–0.900
Plastic lined	0.979–0.976	0.964–0.960	0.936–0.910
Pig burnished	0.982–0.980	0.968–0.965	0.944–0.920
Sand-blasted	0.985–0.983	0.976–0.970	0.951–0.930

NOTE: Values of the drag factor given are pipelines with 40-ft joints at 10-mi spacing of mainline block valves.

For the smooth pipe zone using Eq. (7.71),

$$F_t = 4\log_{10}\left(\frac{\text{Re}}{F_t}\right) - 0.6$$

Solving the preceding equation by trial and error we get $F_t = 22.49$.

For the partially turbulent flow zone using Eq. (7.70), the transmission factor is

$$F = 4D_f \log_{10}\left(\frac{\text{Re}}{1.4125F_t}\right) = 4 \times 0.96\log_{10}\left(\frac{13{,}356{,}517}{1.4125 \times 22.49}\right) = 21.6$$

We have used a drag factor of 0.96, taken from Table 7.2.

Therefore, using the smaller of the two values, the AGA transmission factor is 20.01.

Panhandle A equation. The Panhandle A equation for flow rate and pressure drop in a gas pipeline does not use pipe roughness or a friction factor. Instead an efficiency factor E is used as described.

$$Q = 435.87E\left(\frac{T_b}{P_b}\right)^{1.0788}\left(\frac{P_1{}^2 - P_2{}^2}{G^{0.8539}T_f LZ}\right)^{0.5394} D^{2.6182} \tag{7.72}$$

where Q = volume flow rate, SCFD
E = pipeline efficiency, a decimal value less than 1.0
P_b = base pressure, psia
T_b = base temperature, °R
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
G = gas gravity (air = 1.00)
T_f = average gas flow temperature, °R
L = pipe segment length, mi
Z = gas compressibility factor, dimensionless
D = pipe inside diameter, in

In SI Units, the Panhandle A equation is

$$Q = (4.5965 \times 10^{-3})E\left(\frac{T_b}{P_b}\right)^{1.0788}\left(\frac{P_1{}^2 - P_2{}^2}{G^{0.8539}T_f LZ}\right)^{0.5394} D^{2.6182} \tag{7.72a}$$

where Q = gas flow rate, standard condition m^3/day
E = pipeline efficiency, a decimal value less than 1.0
T_b = base temperature, K (273 + °C)
P_b = base pressure, kPa
T_f = average gas flow temperature, K (273 + °C)
P_1 = upstream pressure, kPa
P_2 = downstream pressure, kPa
L = pipe length, km

Example 7.18 Using the Panhandle A equation, calculate the pressure drop in a 10-mi segment of a 16-in (0.250-in wall thickness) gas pipeline at a flow rate of 100 MMSCFD. The inlet pressure at the beginning of the pipe segment is 1000 psia. Gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), flowing temperature of gas in pipeline = 80°F, base pressure = 14.73 psia, and base temperature = 60°F. Use the CNGA method for the compressibility factor Z and a pipeline efficiency of 0.95.

Solution The average pressure P_{avg} needs to be calculated before the compressibility factor can be determined. Since the inlet pressure $P_1 = 1000$ psia and the outlet pressure P_2 is unknown, we will have to assume a value of P_2 (such as 800 psia) and calculate P_{avg} and hence the value of Z. Once Z is known using the Panhandle A equation we can calculate the outlet pressure P_2. Using this value of P_2, a better approximation for Z is calculated. This process is repeated until successive values of P_2 are within allowable tolerance limits, such as 0.1 psia.

Assume P_2 = 800 psia. The average pressure from Eq. (7.43) is

$$P_{avg} = \frac{2}{3}\left(P_1 + P_2 - \frac{P_1 P_2}{P_1 + P_2}\right) = \frac{2}{3}\left(1000 + 800 - \frac{1000 \times 800}{1000 + 800}\right)$$

$$= 903.7\,\text{psia} = 888.97\,\text{psig}$$

Next we calculate the compressibility factor Z using the CNGA method. From Eq. (7.28)

$$Z = \frac{1}{1 + [888.97 \times 344{,}400(10)^{(1.785\times0.6)}/(540)^{3.825}]} = 0.8869$$

From Eq. (7.72) substituting the given values, we get

$$100 \times 10^6 = 435.87\,(0.95)\left(\frac{520}{14.73}\right)^{1.0788}$$

$$\times \left[\frac{P_1^{\,2} - P_2^{\,2}}{(0.6)^{0.8539} \times 540 \times 10 \times 0.8869}\right]^{0.5394} (15.5)^{2.6182}$$

$$P_1^{\,2} - P_2^{\,2} = 39{,}530$$

Solving for P_2 we get

$$P_2 = 980.04 \text{ psia}$$

Since this is different from the assumed value of $P_2 = 800$, we recalculate the average pressure and Z using $P_2 = 980.04$ psia. After a few iterations, we calculate the final outlet pressure to be

$$P_2 = 980.3 \text{ psia}$$

Therefore, the pressure drop in the 10-mi segment = $P_1 - P_2 = 1000 - 980.3 =$ 19.7 psi.

Panhandle B equation. Similar to the Panhandle A equation, the Panhandle B equation calculates the flow rate for a given pressure drop in a gas pipeline and does not use pipe roughness or a friction factor. Instead an efficiency factor E is used as described.

$$Q = 737E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{P_1{}^2 - P_2{}^2}{G^{0.961}T_f LZ}\right)^{0.51} D^{2.53} \tag{7.73}$$

All symbols are as defined before.

In SI units, the Panhandle B equation is

$$Q = (1.002 \times 10^{-2})E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{P_1{}^2 - P_2{}^2}{G^{0.961}T_f LZ}\right)^{0.51} D^{2.53} \tag{7.73a}$$

where Q = gas flow rate, standard condition m^3/day
E = pipeline efficiency, a decimal value less than 1.0
T_b = base temperature, K (273 + °C)
P_b = base pressure, kPa
T_f = average gas flow temperature, K (273 + °C)
P_1 = upstream pressure, kPa
P_2 = downstream pressure, kPa
L = pipe length, km

Example 7.19 Using the Panhandle B equation, calculate the pressure drop in a 10-mi segment of a 16-in (0.250-in wall thickness) gas pipeline at a flow rate of 100 MMSCFD. The inlet pressure at the beginning of the pipe segment is 1000 psia. Gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), flowing temperature of gas in pipeline = 80°F, base pressure = 14.73 psia, and base temperature = 60°F. Use the CNGA method for the compressibility factor Z and a pipeline efficiency of 0.95.

Solution The average pressure P_{avg} needs to be known before the compressibility factor can be calculated. Since the inlet pressure P_1 = 1000 psia and the outlet pressure P_2 is unknown, we will have to assume a value of P_2 (such as 800 psia) and calculate P_{avg} and hence the value of Z. Once Z is known using the Panhandle A equation, we can calculate the outlet pressure P_2. Using this value of P_2, a better approximation for Z is recalculated. This process is repeated until successive values of P_2 are within allowable tolerance limits, such as 0.1 psia. Assume P_2 = 800 psia.

The average pressure from Eq. (7.43) is

$$P_{\text{avg}} = \frac{2}{3}\left(P_1 + P_2 - \frac{P_1 P_2}{P_1 + P_2}\right) = \frac{2}{3}\left(1{,}000 + 800 - \frac{1000 \times 800}{1000 + 800}\right)$$

$$= 903.7\,\text{psia} = 888.97\,\text{psig}$$

Next we calculate the compressibility factor Z using the CNGA method. From Eq. (7.28),

$$Z = \frac{1}{1 + [888.97 \times 344{,}400(10)^{(1.785\times0.6)}]/(540)^{3.825}} = 0.8869$$

From Eq. (7.73) substituting given values, we get

$$Q = 737E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{P_1{}^2 - P_2{}^2}{G^{0.961}T_f LZ}\right)^{0.51} D^{2.53}$$

$$100 \times 10^6 = 737(0.95)\left(\frac{520}{14.73}\right)^{1.02}$$

$$\times \left[\frac{P_1{}^2 - P_2{}^2}{(0.6)^{0.961} \times 540 \times 10 \times 0.8869}\right]^{0.51} (15.5)^{2.53}$$

$$P_1{}^2 - P_2{}^2 = 35{,}000$$

Solving for P_2 we get

$$P_2 = 981 \text{ psia}$$

Since this is different from the assumed value of $P_2 = 800$, we recalculate the average pressure and Z using $P_2 = 981$ psia. After a few iterations we calculate the final outlet pressure to be

$$P_2 = 981.3 \text{ psia}$$

Therefore, the pressure drop in the 10-mi segment $= P_1 - P_2 = 1000 - 981.3 =$ 18.7 psi.

Example 7.20 For the gas pipeline system shown in Fig 7.4, calculate the pressure required at A if $P_c = 300$ psig. Use the Panhandle B equation with 90 percent pipeline efficiency. Gas gravity is 0.70 and viscosity is 8×10^{-6}

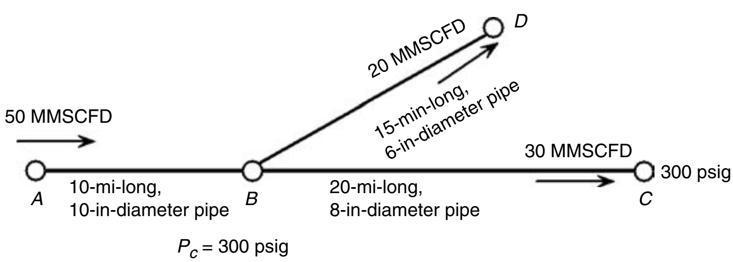

Figure 7.4 Gas pipeline with a branch.

lb/(ft · s). What is the pressure at D? Compressibility factor $Z = 0.85$ and $T_f = 60°F$.

Solution We need to first calculate the pressure at junction B. Consider the pipe section BC transporting 30 MMSCFD through NPS 8 (0.250-in wall thickness) pipe. The upstream pressure P_B is calculated from Panhandle B equation (7.73) as follows:

$$30 \times 10^6 = 737 \times 0.9 \times \left(\frac{520}{14.7}\right)^{1.02} \times \left[\frac{P_B{}^2 - 314.7^2}{(0.7)^{0.961} \times 520 \times 20 \times 0.85}\right]^{0.51} \times (8.125)^{2.53}$$

Therefore, the pressure at junction B is $P_B = 552.80$ psia.

Again, using the Panhandle B equation (7.73) for pipe section BD, we calculate the pressure at D as follows:

$$20 \times 10^6 = 737 \times 0.9 \times \left(\frac{520}{14.7}\right)^{1.02} \times \left[\frac{(552.8)^2 - P_D{}^2}{(0.7)^{0.961} \times 520 \times 15 \times 0.85}\right]^{0.51} \times (6.125)^{2.53}$$

Solving for P_D we get

$$P_D = 146.30\,\text{psia}$$

Finally we calculate the pressure required at A as follows:

$$20 \times 10^6 = 737 \times 0.9 \times \left(\frac{520}{14.7}\right)^{1.02} \times \left[\frac{P_A{}^2 - (552.8)^2}{(0.7)^{0.961} \times 520 \times 10 \times 0.85}\right]^{0.51} \times (10.25)^{2.53}$$

Solving for P_A we get

$$P_A = 628.01\,\text{psia}$$

Weymouth equation. This formula is generally used for short pipelines and gathering systems. Like the Panhandle equations, this equation also uses an efficiency factor.

$$Q = 433.5E\frac{T_b}{P_b}\left(\frac{P_1{}^2 - P_2{}^2}{GT_fLZ}\right)^{0.5} D^{2.667} \tag{7.74}$$

P_1 is the upstream pressure and P_2 is the downstream pressure, both in psia. All other symbols are as defined before.

In SI units, the Weymouth equation is

$$Q = (3.7435 \times 10^{-3})E\frac{T_b}{P_b}\left(\frac{P_1{}^2 - P_2{}^2}{GT_fLZ}\right)^{0.5} D^{2.667} \tag{7.74a}$$

All symbols are as defined before.

Example 7.21 Using the Weymouth equation, calculate the flow rate in a 5-mi-long, 12.75-in-diameter (0.250-in wall thickness) gas gathering pipeline system. The upstream pressure is 1000 psia and the delivery pressure is 800 psia at the downstream end. Gas gravity = 0.6 and viscosity = 0.000008 lb/(ft · s). Flowing temperature of gas in pipeline = 80°F, base pressure = 14.73 psia, and base temperature = 60°F. Assume the Z factor to be 0.92 and a pipeline efficiency of 0.90.

Solution Using Eq. (7.74), substituting given values, we get the flow rate as follows:

$$Q = 433.5(0.9)\frac{520}{14.73}\left(\frac{1000^2 - 800^2}{0.6 \times 540 \times 5 \times 0.92}\right)^{0.5}(12.25)^{2.667}$$

$$= 170.84 \quad \text{MMSCFD}$$

Example 7.22 A natural gas transmission pipeline is used to transport 36 million m^3/day of gas from a refinery to a compressor station site 150 km away. The pipeline terrain may be assumed to be essentially flat. Determine the pipe diameter required if the operating pressure is limited to 8000 kPa. The delivery pressure must be at least 5000 kPa. Consider a pipe roughness factor of 0.02 mm. The gas gravity is 0.64 and the flowing temperature is 20°C. Compare results using the Panhandle A, Panhandle B, and Weymouth equations. Base temperature = 15°C, base pressure 101 kPa, compressibility factor Z = 0.85, and pipeline efficiency = 0.95.

Solution

$$\frac{T_b}{P_b} = \frac{15 + 273}{101} = 2.8515$$

$$T_f LZ = (20 + 273) \times 150 \times 0.85 = 37{,}357.5$$

Panhandle A

Substituting in Eq. (7.72*a*), we get

$$36 \times 10^6 = (4.5965 \times 10^{-3}) \times 0.95 \times \left(\frac{288}{101}\right)^{1.0788}$$

$$\times \left[\frac{8000^2 - 5000^2}{(0.64)^{0.8539} \times 293 \times 150 \times 0.85}\right]^{0.5394} \times D^{2.6182}$$

Solving for D we get

$$D = 878.78\,\text{mm}$$

Panhandle B

Using Eq. (7.73*a*), we get

$$36 \times 10^6 = (1.002 \times 10^{-2}) \times 0.95 \times (2.8515)^{1.02} \times \left[\frac{8000^2 - 5000^2}{(0.64)^{0.961} \times 37{,}357.5}\right]^{0.51} \times D^{2.53}$$

Solving for D we get

$$D = 903.92\,\text{mm}$$

Weymouth

Using Eq. (7.74*a*), we get

$$36 \times 10^6 = (3.7435 \times 10^{-3}) \times 0.95 \times 2.8515 \times \left[\frac{8000^2 - 5000^2}{(0.64)^{0.961} \times 37{,}357.5}\right]^{0.5} \times D^{2.667}$$

Solving for D we get

$$D = 951.96\ \text{mm}$$

Thus, we see that the largest diameter is calculated using the Weymouth equation, and the smallest using the Panhandle A equation. Weymouth is therefore the most conservative equation.

7.3 Line Pack in Gas Pipeline

Consider a section of a gas pipeline between points A and B. The upstream end A is at a pressure of P_1 psia and that at the downstream end B is at P_2 psia. The length of the pipe segment is L miles. The gas temperature is T_f. The inside diameter of the pipe is D inches. The volume of gas in packed condition at an average pressure P_{avg} will be calculated as follows. The average pressure in the pipeline is calculated from the upstream and downstream pressures using Eq. (7.43):

$$P_{\text{avg}} = \frac{2}{3}\left(P_1 + P_2 - \frac{P_1 P_2}{P_1 + P_2}\right)$$

where P_{avg}, P_1, and P_2 are all in absolute pressures.

The physical volume V contained in L miles of circular pipe can be calculated as

$$V = \text{area} \times \text{length}$$

or

$$V = \text{const1}\frac{\pi}{4}D^2L \tag{7.75}$$

where V = volume, ft^3
const1 = conversion constant that depends on units used
D = pipe inside diameter, in
L = pipe length, mi

This is the volume of the packed gas at temperature T_f and pressure P_{avg}. Under standard conditions this gas will have a volume designated as V_b. Using the perfect gas law [Eq. (7.11)], modified by the compressibility factor, we can write the following equation:

$$\frac{P_b V_b}{Z_b T_b} = \frac{P_{\text{avg}} V}{Z_{\text{avg}} T_f} \tag{7.76}$$

where P_b = base pressure, 14.7 psia, in USCS units
T_b = base temperature, °R (60°F + 460), in USCS units
Z_b = gas compressibility factor at base conditions, dimensionless
Z_{avg} = gas compressibility factor at P_{avg} and T_f conditions, dimensionless

Other symbols are as defined earlier.

Rearranging and solving for V_b we get

$$V_b = V\frac{P_{\text{avg}}}{T_f}\frac{T_b}{P_b}\frac{Z_b}{Z_{\text{avg}}} \tag{7.77}$$

Substituting the value of the physical pipe volume V according to Eq. (7.75) we get the line pack volume in the pipeline in standard ft^3 as follows:

$$\text{Line pack} = V_b = \text{const1}\frac{\pi}{4}D^2L\frac{P_{\text{avg}}}{T_f}\frac{T_b}{P_b}\frac{Z_b}{Z_{\text{avg}}} \tag{7.78}$$

In this equation, the line pack V_b will be in standard ft^3 in USCS units and standard m^3 in SI units and all other symbols are as defined before. The term const1 depends on the units used and is defined as

$$\begin{aligned} \text{const1} &= 36.6667 \qquad \text{in USCS units} \\ &= 0.001 \qquad \text{in SI units} \end{aligned}$$

It must be noted that in the line pack equation (7.78), the compressibility factors Z_b and Z_{avg} must be computed at the standard conditions and the pipeline conditions (T_f and P_{avg}), respectively. We can use either the Standing-Katz chart or the CNGA method to calculate the Z factors.

Example 7.23 Calculate the line pack in a 5-mi section of NPS 16 (0.250-in wall thickness) pipe at an average pressure of 950 psig. The gas temperature is 80°F and gas gravity is 0.68. Use the CNGA method for calculation of the compressibility factor. Base temperature = 60°F and base pressure = 14.7 psia.

Solution The compressibility factor using the CNGA method is

$$Z = \frac{1}{1 + [950 \times 344{,}400 \times (10)^{(1.785 \times 0.68)}]/(460 + 80)^{3.825}} = 0.8408$$

$$\text{Line pack} = V_b = 36.667 \times 0.7854 \times (15.5)^2 \times 5 \, \frac{964.7}{540} \, \frac{520}{14.7} \, \frac{1}{0.8408}$$

$$= 2.60 \quad \text{MMSCF}$$

Example 7.24 A 10-mm-thick, DN 500 natural gas pipeline operates at a pressure of 7000 kPa (absolute). Estimate the line pack in 1 km length of this pipe at a flowing temperature of 20°C.

Base temperature = 15°C and base pressure = 101 kPa. Assume gas composition as follows, taken from Example 7.8:

Component	y
C_1	0.780
C_2	0.005
C_3	0.002
N_2	0.013
CO_2	0.016
H_2S	0.184

where y is the mole fraction.

Solution From Example 7.8, $Z = 0.855$

$$\text{Line pack} = V_b = (1 \times 10^{-3}) \times \frac{\pi}{4}(480)^2(1.0)\frac{7{,}000}{273 + 20} \, \frac{15 + 273}{101} \, \frac{1}{0.855}$$

$$= 14{,}418 \text{ standard m}^3$$

7.4 Pipes in Series

So far we have discussed pipelines that have the same pipe diameter throughout the length. Many gas pipelines are constructed with different pipe sizes and wall thicknesses to handle different volumes through the pipeline. An example would be the following. A certain volume, say

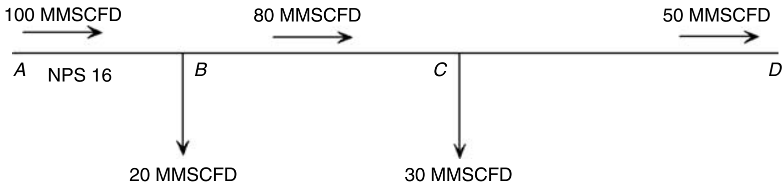

Figure 7.5 Series piping with multiple flow rates.

100 MMSCFD, enters a 16-in pipeline at *A*. Twenty miles downstream at *B* a portion of the inlet volume such as 20 MMSCFD may be delivered to a customer with the remaining 80 MMSCFD proceeding down the line. Then 30 MMSCFD would be delivered to a second customer at point *C*, and finally, the remaining 50 MMSCFD would be delivered to the final destination at the end of the pipeline at *D*. This is illustrated in Fig. 7.5.

Since section *AB* handles the largest volume (100 MMSCFD) and section *CD* handles the least volume (50 MMSCFD), it is clear that both *AB* and *CD* need not be the same pipe size. For reasons of economy it would be preferable to size section *CD* as a smaller-diameter pipe compared to *AB*. Suppose *AB* is NPS 16, section *BC* may be NPS 14, and section *CD* may be designed as NPS 12 pipeline. Here we have essentially pipes in series, *AB*, *BC*, and *CD* together comprising the entire pipeline *A* to *D*. By reducing the pipe size as the flow reduces we are saving on material and labor cost. It would be foolish to install the same NPS 16 pipe for *CD* when that section transports only one-half of the flow rate that section *AB* is required to handle.

A slightly different scenario would be if at point *E* between *C* and *D*, additional volumes of gas enters the pipeline, maybe from another pipeline. This is illustrated in Fig. 7.6 where both deliveries out of the pipeline and injection into the pipeline are shown.

It is clear that in this case section *ED* must be designed to handle the larger volume ($40 + 50 = 90$ MMSCFD) due to the 40-MMSCFD injection at *E*. In fact, we may have to size *ED* as an NPS 16 pipe. How do you decide on the required pipe size for such a pipeline? One way

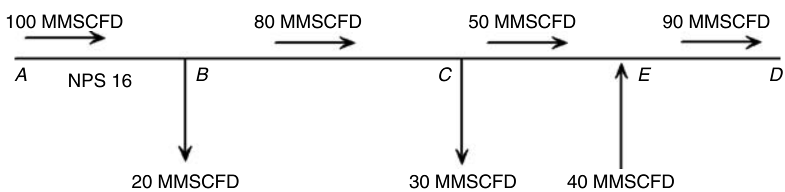

Figure 7.6 Series piping with injection and deliveries.

would be to allow approximately the same gas velocity and pressure drop in each segment of pipe. This would necessitate increasing the pipe diameter in proportion to the flow rate. Recalling that the flow rate is proportional to $D^{2.5}$ and the pressure drop is proportional to D^5 we can approximately estimate the different pipe diameters required to handle the different flow rates as follows:

$$\frac{Q_1{}^2 L_1}{D_1{}^5} = \frac{Q_2{}^2 L_2}{D_2{}^5} = \frac{Q_3{}^2 L_3}{D_3{}^5} = \cdots \tag{7.79}$$

where Q_1 = flow rate through section AB
Q_2 = flow rate through section BC
Q_3 = flow rate through section CE
L_1 = length of section AB
L_2 = length of section BC
L_3 = length of section CE
D_1 = pipe inside diameter of section AB
D_2 = pipe inside diameter of section BC
D_3 = pipe inside diameter of section CE

We pick a pipe size D_1 for the first section AB and calculate an estimate for the pipe size D_2 for section BC as follows using Eq. (7.79):

$$D_1 = D_2 \left(\frac{Q_1}{Q_2}\right)^{0.4} \left(\frac{L_1}{L_2}\right)^{0.2} \tag{7.80}$$

Similarly, we can determine the pipe diameters for relationships of the other sections CD, DE, etc. Consider now a simplified case of pipes in series as shown in Fig. 7.7. In this pipeline we have the same flow rate Q flowing through three sections AB, BC, and CD of pipes of different diameters and pipe lengths. We are interested in calculating the pressure drop through this pipeline using the easiest approach. One way to solve the problem would be to treat this series piping system as three different pipes and calculate the pressure drop through each pipe diameter separately and add the pressure drops together. Thus starting with an inlet pressure P_A at A, we would calculate the downstream

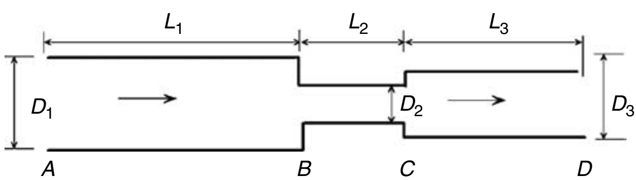

Figure 7.7 Series piping with uniform flow rates.

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pressure P_B at B by considering the flow rate Q through a pipe of diameter D_1 and length L_1. This would establish the pressure at B, which would form the starting point of calculations for section BC. Using P_B we would calculate the pressure P_C at C considering a flow rate of Q through a pipe diameter D_2 and length L_2. Finally, starting with PC we can calculate the pressure P_D at D considering a flow of Q through a pipe diameter D_3 and length L_3.

Another easier way to calculate the pressure drop in a series piping system is using the concept of equivalent pipe length. In this approach we assume the same flow rate Q through the same pipe diameter D_1 as the first section and calculate an equivalent length for each section in terms of D_1 such that the pressure drop in section BC of diameter D_2 and length L_2 will be the same as if BC were of diameter D_1 and length $L_{e_{BC}}$. The length $L_{e_{BC}}$ is called the equivalent length of BC in terms of the diameter D_1. Thus we can replace section BC with a piece of pipe with diameter D_1 and length $L_{e_{BC}}$ which will have the same pressure drop as the original section BC of diameter D_2 and length L_2. Similarly the section CD can be replaced with a piece of pipe with diameter D_1 and length $L_{e_{CD}}$ which will have the same pressure drop as the original section BC of diameter D_2 and length L_2. We can continue this process for each piece of pipe in series. Finally, we have a pipeline system of constant diameter D_1 having a length of $(L_1 + L_{e_{BC}} + L_{e_{CD}} + \cdots)$ that will have the same pressure drop characteristic of the multiple diameter pipes in series. This is illustrated in Fig. 7.8.

The equivalent length for each pipe section in terms of diameter D_1 is calculated using the following formula:

$$L_e = L_2 \left(\frac{D_1}{D_2}\right)^5 \tag{7.81}$$

An example will illustrate this approach.

Example 7.25 A series piping system consists of 20 mi of NPS 16 (0.250-in wall thickness) pipe connected to 20 mi of NPS 14 (0.250-in wall thickness) pipe and 20 mi of NPS 12 (0.250-in wall thickness) pipe. Using the equivalent length concept calculate the total pressure drop in this pipeline system for a gas flow rate of 80 MMSCFD. Inlet pressure = 1000 psia, gas gravity = 0.6,

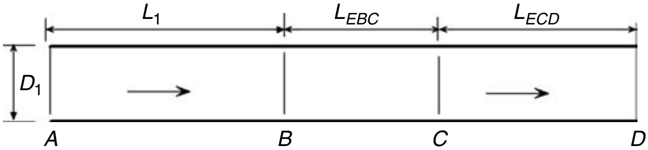

Figure 7.8 Equivalent length of series piping.

viscosity = 0.000008 lb/(ft · s), and flowing temperature = 60°F. Assume the compressibility factor = 0.95. Use the general flow equation with a Darcy friction factor = 0.02. Base temperature = 60°F and base pressure = 14.7 psia. Compare results calculating individual pressure drops in the three pipe sections.

Solution Using the base diameter D_1 as the diameter of the first section of NPS 16 pipe the equivalent length of the NPS 14 pipe section is from Eq. (7.81):

$$L_e = 20\left(\frac{15.5}{13.5}\right)^5 = 39.9\,\text{mi}$$

Similarly, the equivalent length of NPS 12 is

$$L_e = 20\left(\frac{15.5}{12.25}\right)^5 = 64.86\,\text{mi}$$

Therefore, the given series pipeline system can be replaced with a single NPS 16 pipe of length

$$20.00 + 39.90 + 64.86 = 124.76\,\text{mi}$$

Using the general flow equation (7.42), substituting given values we get

$$80 \times 10^6 = 77.54\frac{1}{\sqrt{0.02}}\frac{520}{14.7}\left(\frac{1000^2 - P_2{}^2}{0.6 \times 520 \times 124.76 \times 0.95}\right)^{0.5} 15.5^{2.5}$$

Transposing and solving for P_2, we get

$$P_2 = 544.79\,\text{psia}$$

7.5 Pipes in Parallel

Many times pipelines are installed in parallel. Such installations are necessary sometimes to reduce pressure drop in a bottleneck section due to pressure limitations or for expansion of an existing pipeline without adding expensive compression equipment. A typical parallel piping system is illustrated in Fig. 7.9.

Gas pipelines in parallel are configured such that the multiple pipes are connected together so that the gas flow splits into the multiple pipes at the beginning and the separate flow streams subsequently rejoin

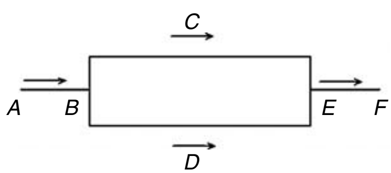

Figure 7.9 Parallel piping.

downstream into another single pipe as shown in Fig. 7.9. In this figure we assume that the parallel piping system is in the horizontal plane with no change in pipe elevations. Gas flows through a single pipe *AB*, and at the junction *B* the flow splits into two pipe branches *BCE* and *BDE*. At the downstream end at junction *E*, the flows rejoin to the initial flow rate and subsequently flow through the single pipe *EF*.

To calculate the flow rates and pressure drop due to friction in the parallel piping system, two main principles of parallel piping must be followed. These are flow conservation at any junction point and common pressure drop across each parallel branch pipe.

Based on flow conservation, at each junction point of the pipeline, the incoming flow must exactly equal the total outflow. Therefore, at junction *B*, the flow *Q* entering the junction must exactly equal the sum of the flow rates in branches *BCE* and *BDE*. Thus

$$Q = Q_{BCE} + Q_{BDE} \tag{7.82}$$

where Q_{BCE} = flow through branch *BCE*
Q_{BDE} = flow through branch *BDE*
Q = incoming flow at junction *B*

The other requirement in parallel pipes concerns the pressure drop in each branch piping. Based on this, the pressure drop due to friction in branch *BCE* must exactly equal that in branch *BDE*. This is because both branches have a common starting point (*B*) and a common ending point (*E*). Since the pressure at each of these two points is a unique value, we can conclude that the pressure drop in branch pipe *BCE* and that in branch pipe *BDE* are both equal to $P_B - P_E$ where P_B and P_E represent the pressure at the junction points *B* and *E*, respectively.

Another approach to calculating the pressure drop in parallel piping is the use of an equivalent diameter for the parallel pipes. For example in Fig. 7.9, if pipe *AB* were NPS 14 and branches *BCE* and *BDE* were NPS 10 and NPS 12, respectively, we can find some equivalent diameter pipe of the same length as one of the branches that will have the same pressure drop between points *B* and *C* as the two branches. An approximate equivalent diameter can be calculated using the general flow equation.

The pressure loss in branch *BCE* which is NPS 10 can be calculated as

$$P_B{}^2 - P_E{}^2 = \frac{K_1 L_1 Q_1{}^2}{D_1{}^5} \tag{7.83}$$

where the term *K* (resistance) depends on gas gravity, compressibility factor, flowing temperature, base temperature, base pressure, and friction factor. P_B and P_E are the pressures at the junctions *B* and *E*, respectively. The subscript 1 is used for branch BCE and subscript 2 for branch BDE.

Similarly, we have for branch BDE,

$$P_B{}^2 - P_E{}^2 = \frac{K_2 L_2 Q_2{}^2}{D_2{}^5} \tag{7.84}$$

Suppose we replace the two branches BCE and BDE with a single piece of pipe of diameter D_e and length L_e between B and E. For hydraulic equivalence, the pressure drop in the equivalent diameter pipe must equal the pressure drop in either branch BCE or BDE from Eq. (7.84). Therefore,

$$P_B{}^2 - P_E{}^2 = \frac{K_e L_e Q^2}{D_e{}^5} \tag{7.85}$$

where K_e represents the resistance coefficient for the equivalent diameter pipe of length L_e flowing the full volume $Q = Q_{BCE} + Q_{BDE}$. We can also choose $L_e = L_1$, and Eq. (7.85) then reduces to

$$P_B{}^2 - P_E{}^2 = \frac{K_e L_1 Q^2}{D_e{}^5} \tag{7.86}$$

From Eqs. (7.83) through (7.85), we have

$$\frac{K_1 L_1 Q_1{}^2}{D_1{}^5} = \frac{K_2 L_2 Q_2{}^2}{D_2{}^5} = \frac{K_e L_e Q^2}{D_e{}^5} \tag{7.87}$$

Also the flow conservation equation (7.82) can be written as

$$Q = Q_1 + Q_2 \tag{7.88}$$

We can solve Eqs. (7.87) and (7.88) for Q_1, Q_2, and D_e in terms of all other known quantities:

$$\left(\frac{Q_1}{Q_2}\right)^2 = \frac{K_2}{K_1}\left(\frac{D_1}{D_2}\right)^5 \frac{L_2}{L_1}$$

$$Q_1 = Q_2\sqrt{\frac{K_2}{K_1}\left(\frac{D_1}{D_2}\right)^5 \frac{L_2}{L_1}} \tag{7.89}$$

$$Q_1 = \text{const1}\,(Q_2) \tag{7.90}$$

where

$$\text{const1} = \sqrt{\frac{K_2}{K_1}\left(\frac{D_1}{D_2}\right)^5 \frac{L_2}{L_1}}$$

Substituting the value of Q_1 from Eq. (7.90) into Eq. (7.88) we get

$$Q_2 = \frac{Q}{1 + \text{const1}} \tag{7.91}$$

and

$$Q_1 = \frac{\text{const1}\,Q}{1 + \text{const1}} \tag{7.92}$$

Next from Eq. (7.86) we calculate D_e as follows:

$$\left(\frac{D_e}{D_1}\right)^5 = \frac{K_e}{K_1}\frac{L_e}{L_1}\left(\frac{Q}{Q_1}\right)^2 \tag{7.93}$$

Substituting the value of Q_1 in Eq. (7.93) using $L_e = L_1$, we get

$$D_e = D_1\left[\frac{K_e}{K_1}\left(\frac{1 + \text{const1}}{\text{const1}}\right)^2\right]^{1/5} \tag{7.94}$$

where

$$\text{const1} = \sqrt{\frac{K_2}{K_1}\left(\frac{D_1}{D_2}\right)^5 \frac{L_2}{L_1}} \tag{7.95}$$

and K_1, K_2, and K_e are parameters that depend on the gas gravity, compressibility factor, flowing temperature, base temperature, base pressure, and friction factor. We will illustrate this by means of an example.

Example 7.26 The parallel piping system shown in Fig. 7.9 is to be designed for a flow rate of 100 MMSCFD.

AB is 10 mi long and is NPS 16 (0.250-in wall thickness)

BCE is 20 mi long and is NPS 14 (0.250-in wall thickness)

BDE is 15 mi long and is NPS 12 (0.250-in wall thickness)

EF is 20 mi long and is NPS 16 (0.250-in wall thickness)

If the gas gravity is 0.6, calculate the outlet pressure and flow rate in the two parallel pipes. Other given values are inlet pressure at $A = 1000$ psia, flowing temperature $= 60°\text{F}$, base temperature $= 60°\text{F}$, base pressure $=$ 14.7 psia, compressibility factor $Z = 0.90$, and friction factor $f = 0.02$.

Solution Using Eq. (7.94) we calculate the equivalent diameter and the flow rates Q_1 and Q_2 in the branches:

$$\text{const1} = \sqrt{1\left(\frac{13.5}{12.25}\right)^5 \frac{15}{20}} = 1.1041 \qquad \text{from Eq. (7.95)}$$

Using Eq. (7.91),

$$Q_2 = \frac{100}{1 + 1.1041} = 47.53 \quad \text{MMSCFD}$$

$$Q_1 = 100 - 47.53 = 52.47 \quad \text{MMSCFD}$$

$$\text{Flow rate in NPS 14 branch} = 52.47 \quad \text{MMSCFD}$$

$$\text{Flow rate in NPS 12 branch} = 47.53 \quad \text{MMSCFD}$$

The equivalent diameter D_e is calculated from Eq. (7.94):

$$D_e = 13.5\left[1 \times \left(\frac{1 + 1.1041}{1.1041}\right)^2\right]^{1/5} = 17.47\,\text{in}$$

Therefore, D_e is NPS 18, 0.265-in wall thickness.

We now have the pipeline reduced to three pipes in series: 10 mi of NPS 16, 20 mi of NPS 18, and 20 mi of NPS 16. The middle section will be converted to an equivalent length of NPS 16 pipe using the theory of pipes in series. From Eq. (7.81), the equivalent length of midsection in terms of NPS 16 is

$$L_e = 20\left(\frac{15.5}{17.47}\right)^5 = 11.0\ \text{mi of NPS}\,16$$

Therefore, we now have a single NPS 16 pipe of equivalent length

$$10 + 11 + 20 = 41\,\text{mi}$$

Since the friction factor $f = 0.02$, we get a transmission factor

$$F = \frac{2}{\sqrt{0.02}} = 14.14$$

Using the general flow equation (7.42) we get

$$100 \times 10^6 = 38.77 \times 14.14\,\frac{520}{14.7}\left(\frac{1000^2 - P_2{}^2}{0.6 \times 520 \times 41 \times 0.9}\right)^{0.5} \times (15.5)^{2.5}$$

Solving for P_2 we get the outlet pressure at F as

$$P_2 = 811.06\,\text{psia}$$

The pressures at B and D may now be calculated considering sections AB and DF separately as follows. For AB, applying the general flow equation, we get

$$100 \times 10^6 = 38.77 \times 14.14 \frac{520}{14.7}\left(\frac{1000^2 - P_B{}^2}{0.6 \times 520 \times 10 \times 0.9}\right)^{0.5} \times (15.5)^{2.5}$$

Solving for P_B we get

$$P_B = 958.92\,\text{psia}$$

Similarly considering section EF, we get

$$100 \times 10^6 = 38.77 \times 14.14 \frac{520}{14.7}\left(\frac{P_E{}^2 - 811.06^2}{0.6 \times 520 \times 20 \times 0.9}\right)^{0.5} \times (15.5)^{2.5}$$

Solving for P_E we get

$$P_E = 904.86\,\text{psia}$$

Therefore the pressures and flow rates are

$$P_A = 1000\,\text{psia} \qquad Q = 100 \quad \text{MMSCFD}$$

$$P_B = 958.92\,\text{psia} \qquad Q_{BCE} = 52.47 \quad \text{MMSCFD}$$

$$P_E = 904.86\,\text{psia} \qquad Q_{BDE} = 47.53 \quad \text{MMSCFD}$$

$$P_F = 811.06\,\text{psia}$$

Example 7.27 A DN 500 (10-mm wall thickness) pipeline is 50 km long. Gas flows at 6.0 Mm^3/day at 20°C. If the inlet pressure is 8 MPa, what is the delivery pressure, using the Colebrook-White equation? Pipe roughness = 0.0152 mm. If the entire line is looped with a DN 400 (10-mm wall thickness) pipeline, estimate the delivery pressure at an increased flow of 10 Mm^3/day. Calculate the line pack volume in both cases. Gas gravity = 0.65, viscosity = 0.000119 P, compressibility factor $Z = 0.9$, base temperature = 15°C, and base pressure = 101 kPa.

Solution

$$D = 500 - 2 \times 10 = 480\,\text{mm}$$

$$Q = 6.0 \times 10^6\,\text{m}^3/\text{day}$$

$$T_f = 20 + 273 = 293\,\text{K}$$

$$P_1 = 8000\,\text{kPa}$$

The Reynolds number, using Eq. (7.41*b*) is

$$\text{Re} = 0.5134 \frac{101}{288}\frac{0.65 \times 6 \times 10^6}{0.000119 \times 480} = 12.293 \times 10^6$$

From the Colebrook-White equation (7.66),

$$F = -4\log_{10}\left(\frac{0.0152}{3.7 \times 480} + \frac{1.255F}{12.293 \times 10^6}\right)$$

Solving by successive iteration, we get

$$F = 19.71$$

Using the general flow equation,

$$6 \times 10^6 = (5.7473 \times 10^{-4}) \times 19.71 \frac{273 + 15}{101} \left(\frac{8000^2 - P_2{}^2}{0.65 \times 293 \times 50 \times 0.9} \right)^{0.5} \times (480)^{2.5}$$

Solving for P_2 we get

$$P_2 = 7316 \text{ kPa} = 7.32 \text{ MPa}$$

If the entire line is looped with a DN 400 pipeline, the equivalent diameter, according to Eq. (7.94), is

$$\text{const1} = \sqrt{1 \times \left(\frac{480}{380} \right)^5 (1)} = 1.7933$$

$$D_e = 480 \left[1 \times \left(\frac{1 + 1.7933}{1.7933} \right)^2 \right]^{1/5} = 573.09 \text{ mm}$$

Now we have a single 573.09-mm-diameter pipeline flowing at 10 Mm^3/day.
Next we determine the Reynolds number:

$$\text{Re} = \frac{0.5134 \times 10 \times 10^6 \times 0.65 \times 101}{573.09 \times 0.000119 \times (15 + 273)} = 17.16 \times 10^6$$

From the Colebrook-White equation (7.66),

$$F = -4 \log_{10} \left(\frac{0.0152}{3.7 \times 573.09} + \frac{1.255F}{17.16 \times 10^6} \right)$$

Solving, $F = 20.25$. Using the general flow equation (7.55),

$$10 \times 10^6 = 5.7473 \times 10^{-4} \times 20.25 \frac{273 + 15}{101} \left(\frac{8000^2 - P_2{}^2}{0.65 \times 293 \times 50 \times 0.9} \right)^{0.5} \times (573.09)^{2.5}$$

Solving for P_2, we get

$$P_2 = 7.24 \text{ MPa}$$

Example 7.28 A natural gas distribution system (NPS 16, 0.250-in wall thickness) is described in Fig. 7.10. The inlet flow rate is 75 MMSCFD. The plant at Davis must be supplied with 20 MMSCFD at a minimum pressure of 500 psig. Calculate the inlet pressure required at Harvard.

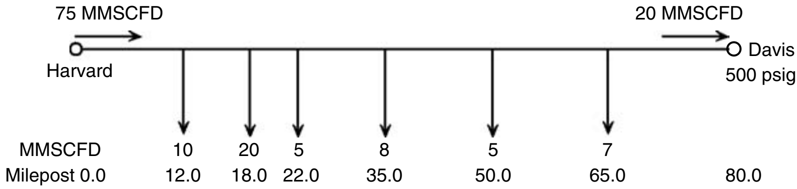

Figure 7.10 Harvard to Davis distribution pipeline.

Use the AGA equation. Assume compressibility factor = 0.95, gas gravity = 0.6, viscosity = 8×10^{-6} lb/(ft · s), flowing temperature = 70°F, pipe roughness = 700 μin, base temperature = 60°F, and base pressure = 14.7 psia.

Solution For each section of piping such as Harvard to A, AB, etc., we must calculate the pressure drop due to friction at the correct flow rates and then determine the total pressure drop for the entire pipeline.

Using the AGA turbulent equation (7.69), we get

$$\text{Transmission factor } F = 4\log_{10}\left(3.7 \times \frac{15.5}{0.0007}\right) = 19.65$$

Using the general flow equation, for the last milepost 65 to milepost 80, we get

$$20 \times 10^6 = 38.77 \times 19.65\,\frac{520}{14.7}\left[\frac{P_F{}^2 - 514.7^2}{0.6 \times 530 \times 15 \times 0.95}\right]^{0.5} \times (15.5)^{2.5}$$

Solving for pressure at F,

$$P_F = 517.40\,\text{psia}$$

Next we will use this pressure P_F to calculate the upstream pressure P_E from the 15-mi section of pipe EF flowing 27 MMSCFD.

$$27 \times 10^6 = 38.77 \times 19.65\,\frac{520}{14.7}\left(\frac{P_E{}^2 - 517.4^2}{4531.5}\right)^{0.5} \times (15.5)^{2.5}$$

Solving for pressure at E,

$$P_E = 522.29\,\text{psia}$$

Repeating the process we get the pressures at D, C, etc., as follows,

$$P_D = 529.08\,\text{psia}$$

$$P_C = 538.14\,\text{psia}$$

$$P_B = 541.64\,\text{psia}$$

$$P_A = 552.41\,\text{psia}$$

And at Harvard, $P_1 = 580.12$ psia.

$$\text{Inlet pressure required at Harvard} = 580.12\,\text{psia}$$

7.6 Looping Pipelines

From Sec. 7.5, it is clear that by installing a parallel pipeline on an existing pipeline the pressure drop can be reduced for a particular flow rate. Alternatively, if we keep the inlet and outlet pressures the same, we can realize a higher flow rate. The installation of parallel pipes in certain segments of a pipeline is also referred to as *looping*. Figure 7.11 shows a 50-mi-long NPS 20, (0.500-in wall thickness) pipeline transporting 200 MMSCFD.

At an inlet pressure of 1000 psig, the delivery pressure is 818 psig, using the AGA equation. If the flow rate is increased to 250 MMSCFD, the delivery pressure drops to 696 psig. If we need to keep the delivery pressure the same as before, we must either increase the inlet pressure from 1000 to 1089 psig or install a loop in the pipeline as shown by the dashed line in Fig. 7.10. If we are already at the maximum allowable operating pressure (MAOP) of the pipeline, we cannot increase the inlet pressure; therefore to keep the delivery pressure at 818 psig starting at an inlet pressure of a 1000 psig at 250 MMSCFD, we must install a loop of length x and diameter D. If we choose the loop to be the same diameter as the main pipe, NPS 20 (0.500-in wall thickness), we can calculate the looping length x by equating the pressure drop ($1000 - 818 = 182$ psig) in the unlooped pipe case to the looped pipe case. We will use the equivalent diameter concept to determine the miles of loop needed.

The equivalent diameter D_e from Eq. (7.94) is

$$D_e = D_1 \left[\frac{K_e}{K_1} \left(\frac{1 + \text{const1}}{\text{const1}} \right)^2 \right]^{1/5} \tag{7.96}$$

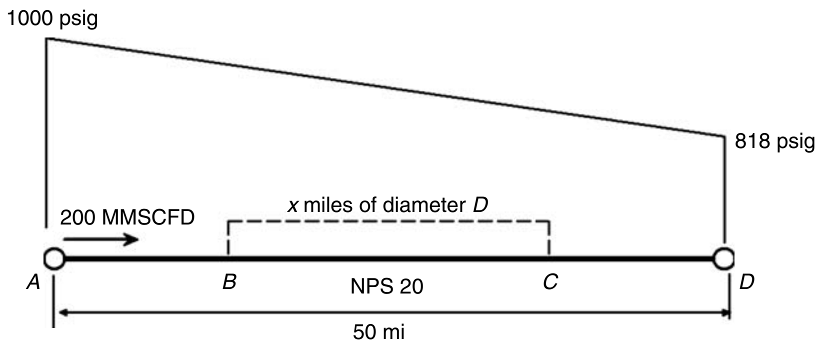

Figure 7.11 Looping a pipeline.

where

$$\text{const1} = \sqrt{\frac{K_2}{K_1}\left(\frac{D_1}{D_2}\right)^5 \frac{L_2}{L_1}} \tag{7.97}$$

Example 7.29 A DN 500 (10-mm wall thickness) pipeline transports 5 Mm³/day of natural gas (gravity = 0.60) from Tapas to Benito, a distance of 200 km. Average flowing temperature is 15°C, base temperature is 15°C, and base pressure is 101 kPa. Assume Z = 0.90. If inlet pressure is 9000 kPa, what is the delivery pressure at Benito? Use the Panhandle A equation with an efficiency of 0.9. If the first 100 km is looped with the same pipe size, what is the revised pressure at Benito?

Solution Using the Panhandle A equation (7.72*a*), we get

$$5 \times 10^6 = (4.5965 \times 10^{-3}) \times 0.9 \times \left(\frac{288}{101}\right)^{1.0788} \times \left[\frac{9000^2 - P_2{}^2}{(0.6)^{0.8539} \times 288 \times 200 \times 0.9}\right]^{0.5394} \times (480)^{2.6182}$$

Solving for the outlet pressure P_2 we get

$$P_2 = 7319\,\text{kPa}$$

If the first 100 km is looped, the equivalent diameter from Eq. (7.94) is

$$\text{const1} = \sqrt{(1)\left(\frac{1}{1}\right)^5 \frac{1}{1}} = 1$$

$$D_e = 480\left[1 \times \left(\frac{1+1}{1}\right)^2\right]^{1/5} = 1.3195 \times 480 = 633.36 \text{ mm}$$

Now we have 100 km of 633.36-mm inside diameter pipe in series with 100 km of DN 500 pipe. Reducing this to the same diameter (DN 500), we get the equivalent length as

$$L_e = 100 + 100\left(\frac{480}{633.36}\right)^5 = 125.0 \text{ km}$$

Therefore the system reduces to one 125-km-long section of DN 500 pipe. Applying the Panhandle A equation as before we get,

$$5 \times 10^6 = (4.5965 \times 10^{-3}) \times 0.9 \times \left(\frac{288}{101}\right)^{1.0788} \times \left[\frac{9000^2 - P_2{}^2}{(0.6)^{0.8539} \times 288 \times 125 \times 0.9}\right]^{0.5394} \times (480)^{2.6182}$$

Solving for P_2 we get

$$P_2 = 7991 \text{ kPa}$$

7.7 Gas Compressors

Compressors are required to provide the pressure in gas pipelines to transport a given volume of gas from source to destination. During the process of compressing the gas from inlet conditions to the necessary pressure at the discharge side, the temperature of the gas increases with pressure. Sometimes the discharge temperature may increase to levels beyond the maximum that the pipeline coating can withstand. Therefore, cooling of the compressed gas will be necessary to protect the pipeline coating. Cooling also has a beneficial effect on the gas transported, since cooler gas results in a lower pressure drop at a given flow rate. This in turn will reduce the compressor horsepower required.

Compressors are classified as positive displacement (PD) compressors or centrifugal compressors. PD compressors may be reciprocating or rotary compressors. Generally centrifugal compressors are more commonly used in natural gas transportation due to their flexibility and reduced operating costs. The drivers for the compressors may be internal combustion engines, electric motors, steam turbines, or gas turbines.

The work done to compress a given quantity of gas from a suction pressure P_1 to the discharge pressure P_2, based upon isothermal compression or adiabatic compression can be calculated as demonstrated in Sec. 7.7.1.

7.7.1 Isothermal compression

The work done in isothermal compression of 1 lb of natural gas is calculated using the following equation:

$$W_i = \frac{53.28}{G} T_1 \log_e \frac{P_2}{P_1} \tag{7.98}$$

where W_i = isothermal work done, (ft · lb)/lb of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, °R
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia
$\log_e$ = natural logarithm to base e($e = 2.718\ldots$)

The ratio P_2/P_1 is called the *compression ratio*.

In SI units the isothermal compression equation is as follows:

$$W_i = \frac{159.29}{G} T_1 \log_e \frac{P_2}{P_1} \tag{7.98a}$$

where W_i = isothermal work done, J/kg of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, K
P_1 = suction pressure of gas, kPa
P_2 = discharge pressure of gas, kPa
$\log_e$ = natural logarithm to base $e(e = 2.718\ldots)$

7.7.2 Adiabatic compression

In the adiabatic compression process the pressure and volume of gas follow the adiabatic equation $PV^\gamma =$ constant where γ is the ratio of the specific heats C_p and C_v, such that

$$\gamma = \frac{C_p}{C_v} \tag{7.99}$$

The work done in adiabatic compression of 1 lb of natural gas is given by the following equation:

$$W_a = \frac{53.28}{G} T_1 \frac{\gamma}{\gamma - 1} \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \tag{7.100}$$

where W_a = adiabatic work done, (ft · lb)/lb of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, °R
γ = ratio of specific heats of gas, dimensionless
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia

In SI units the adiabatic compression equation is as follows:

$$W_a = \frac{159.29}{G} T_1 \frac{\gamma}{\gamma - 1} \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \tag{7.100a}$$

where W_a = adiabatic work done, J/kg of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, K
γ = ratio of specific heats of gas, dimensionless
P_1 = suction pressure of gas, kPa
P_2 = discharge pressure of gas, kPa

Example 7.30 A compressor compresses natural gas ($G = 0.6$) from the suction temperature of 60°F and 800 to 1400 psia discharge. If isothermal compression is assumed, what is the work done by the compressor?

Solution Using Eq. (7.98) for isothermal compression, the work done is

$$W_i = \frac{53.28}{0.6}(520) \times \log_e \frac{1400}{800} = 25{,}841 \quad (\text{ft}\cdot\text{lb})/\text{lb}$$

Example 7.31 In Example 7.30, if the compression were adiabatic ($\gamma = 1.29$), calculate the work done per pound of gas.

Solution From Eq. (7.100) for adiabatic compression, the work done is

$$W_a = \frac{53.28}{0.6} \times 520 \times \frac{1.29}{1.29 - 1}\left[\left(\frac{1400}{800}\right)^{(1.29-1)/1.29} - 1\right] = 27{,}537 \quad (\text{ft}\cdot\text{lb})/\text{lb}$$

It can be seen by comparing results with those of Example 7.31 that the adiabatic compressor requires more work than an isothermal compressor.

7.7.3 Discharge temperature of compressed gas

When gas is compressed adiabatically according to the adiabatic process PV^γ = constant, the discharge temperature of the gas can be calculated as follows:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \tag{7.101}$$

where T_1 = suction temperature of gas, °R
T_2 = discharge temperature of gas, °R
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia
γ = ratio of specific heats of gas, dimensionless

Example 7.32 What is the final temperature of gas in Example 7.31 for adiabatic compression?

Solution We get the discharge temperature by using Eq. (7.101):

$$T_2 = 520 \times \left(\frac{1400}{800}\right)^{0.29/1.29} = 589.7^\circ\text{R} \quad \text{or} \quad 129.7^\circ\text{F}$$

7.7.4 Compressor horsepower

Compressor head measured in (ft · lb)/lb of gas is the energy added to the gas by the compressor. In SI units it is referred to in J/kg. The horsepower necessary for compression is calculated from

$$\text{HP} = \frac{\text{mass flow of gas} \times \text{head}}{\text{efficiency}}$$

It is common practice to refer to compression HP per MMSCFD of gas. Using the perfect gas equation modified by the compressibility factor [Eq. (7.11)], we can state that the compression HP is

$$\text{HP} = 0.0857 \frac{\gamma}{\gamma - 1} T_1 \frac{Z_1 + Z_2}{2} \frac{1}{\eta_a} \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \tag{7.102}$$

where HP = compression HP per MMSCFD
γ = ratio of specific heats of gas, dimensionless
T_1 = suction temperature of gas, °R
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia
Z_1 = compressibility of gas at suction conditions, dimensionless
Z_2 = compressibility of gas at discharge conditions, dimensionless
η_a = compressor adiabatic (isentropic) efficiency, decimal value

In SI units, the power equation is as follows:

$$\text{Power} = 4.0639 \frac{\gamma}{\gamma - 1} T_1 \frac{Z_1 + Z_2}{2} \frac{1}{\eta_a} \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \tag{7.102a}$$

where Power = compression power, kW per Mm^3/day
γ = ratio of specific heats of gas, dimensionless
T_1 = suction temperature of gas, K
P_1 = suction pressure of gas, kPa
P_2 = discharge pressure of gas, kPa
Z_1 = compressibility of gas at suction conditions, dimensionless
Z_2 = compressibility of gas at discharge conditions, dimensionless
η_a = compressor adiabatic (isentropic) efficiency, decimal value

The adiabatic efficiency η_a is usually between 0.75 and 0.85. We can incorporate a mechanical efficiency η_m of the driver unit to calculate the brake horsepower (BHP) of the driver as follows:

$$\mathrm{BHP} = \frac{\mathrm{HP}}{\eta_m} \tag{7.103}$$

The driver efficiency η_m may range from 0.95 to 0.98. The adiabatic efficiency η_a may be expressed in terms of the suction and discharge pressures and temperatures and the specific heat ratio g as follows:

$$\eta_a = \frac{T_1}{T_2 - T_1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \tag{7.104}$$

All symbols in Eq. (7.104) are as defined earlier.

It can be seen from the preceding that the efficiency term η_a modifies the discharge temperature T_2 given by Eq. (7.101).

Example 7.33 Calculate the compressor HP required in Example 7.32 if $Z_1 = 1.0$, $Z_2 = 0.85$, and $\eta_a = 0.8$. What is the BHP if the mechanical efficiency of the driver is 0.95?

Solution From Eq. (7.102), the HP required per MMSCFD is

$$\mathrm{HP} = 0.0857\frac{1.29}{0.29}(520)\frac{1+0.85}{2}\frac{1}{0.8}\left[\left(\frac{1400}{800}\right)^{0.29/1.29} - 1\right]$$

$$= 30.73 \text{ per MMSCFD}$$

Using Eq. (7.103)

$$\text{BHP required} = \frac{30.73}{0.95} = 32.35 \text{ HP per MMSCFD}$$

7.8 Pipe Stress Analysis

The pipe used to transport natural gas must be strong enough to withstand the internal pressure necessary to move the gas at the desired flow rate. The wall thickness T necessary to safely withstand an internal pressure of P depends upon the pipe diameter D and yield strength of the pipe material. It is generally calculated from Barlow's equation as

$$S_h = \frac{PD}{2T} \tag{7.105}$$

where S_h represents the hoop stress in the circumferential direction in the pipe material. Another stress, termed the axial stress, or longitudinal stress, acts perpendicular to the cross section of the pipe. The axial stress is one-half the magnitude of the hoop stress. Hence the governing stress is the hoop stress from Eq. (7.105).

Applying a safety factor and including the yield strength of the pipe material, Barlow's equation is modified for use in gas pipeline calculation as follows:

$$P = \frac{2t \times S \times E \times F \times T}{D} \tag{7.106}$$

where P = internal design pressure, psig
D = pipe outside diameter, in
t = pipe wall thickness, in
S = specified minimum yield strength (SMYS) of pipe material, psig
E = seam joint factor, 1.0 for seamless and submerged arc welded (SAW) pipes (see Table 7.4 for other joint types)
F = design factor, usually between 0.4 and 0.72 for natural gas pipelines
T = temperature derating factor, 1.00 for temperature below 250°F (121.1°C)

The design factor F in Eq. (7.106) depends upon the type of construction. There are four construction types: A, B, C, and D. Corresponding to these, the design factors are as follows:

$$F = \begin{cases} 0.72 & \text{for type A} \\ 0.60 & \text{for type B} \\ 0.50 & \text{for type C} \\ 0.40 & \text{for type D} \end{cases}$$

The construction type depends upon the population density and corresponds to class 1, 2, 3, and 4 as defined by DOT standards, Code of Federal Regulations, Title 49, Part 192.

TABLE 7.4 Pipe Design Joint Factors

Pipe specification	Pipe category	Joint factor E
ASTM A53	Seamless	1.00
	Electric resistance welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
ASTM A106	Seamless	1.00
ASTM A134	Electric fusion arc welded	0.80
ASTM A135	Electric resistance welded	1.00
ASTM A139	Electric fusion welded	0.80
ASTM A211	Spiral welded pipe	0.80
ASTM A333	Seamless	1.00
ASTM A333	Welded	1.00
ASTM A381	Double submerged arc welded	1.00
ASTM A671	Electric fusion welded	1.00
ASTM A672	Electric fusion welded	1.00
ASTM A691	Electric fusion welded	1.00
API 5L	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
API 5LX	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
API 5LS	Electric resistance welded	1.00
	Submerged arc welded	1.00

The temperature derating factor T depends upon the operating temperature of the pipeline. It is equal to 1.00 as long as the temperature does not exceed 250°F (121.1°C). When the operation temperature exceeds 250°F, the value of T is less than 1.00. ASME B31.8 Code for Pressure Piping lists the temperature derating factors. See Table 7.5.

Equation (7.106) for calculating the internal design pressure is found in the Code of Federal Regulations, Title 49, Part 192, published by the

TABLE 7.5 Temperature Derating Factors

Temperature		
°F	°C	Derating factor T
250 or less	121 or less	1.000
300	149	0.967
350	177	0.033
400	204	0.900
450	232	0.867

U.S. Department of Transportation (DOT). You will also find reference to this equation in ASME standard B31.8 for design and transportation of natural gas pipelines.

In SI units, the internal design pressure equation is the same as shown in Eq. (7.106), except the pipe diameter and wall thickness are in millimeters. The SMYS of pipe material and the internal design pressures are both expressed in kilopascals.

Natural gas pipelines are constructed of steel pipe conforming to American Petroleum Institute (API) standard 5L and 5LX specifications. Some piping may also be constructed of steel pipe conforming to ASTM and ANSI standards. High-strength steel pipe may be designated as API 5LX-52, 5LX-60, or 5LX-80. The last two digits of the pipe specification denote the SMYS of the pipe material. Thus 5LX-52 pipe has a yield strength of 52,000 psi. The pipe material is also referred to as the grade of pipe. Thus grade 52 means X-52 pipe. Refer to Table 7.6 for various commonly used grades of pipe.

A useful formula in calculating pipe costs is the one for determining the weight per foot of steel pipe. Pipe vendors use this handy formula for quickly calculating the tonnage of pipe needed for a particular application. In USCS units pipe weight is referred to as lb/ft and can be calculated from a given diameter and wall thickness as follows:

$$w = 10.68 \times t \times (D - t) \tag{7.107}$$

where D = pipe outside diameter, in
t = pipe wall thickness, in
w = pipe weight, lb/ft

The constant 10.68 includes the density of steel.

In SI units, the following equation can be used to calculate the pipe weight in kg/m:

$$w = 0.0246 \times t \times (D - t) \tag{7.107a}$$

TABLE 7.6 Grades of Pipes

Pipe sizes API 5LX grade	Specified minimum yield strength (SMYS), psig
X42	42,000
X46	46,000
X52	52,000
X56	56,000
X60	60,000
X65	65,000
X70	70,000
X80	80,000
X90	90,000

where D = pipe outside diameter, mm
t = pipe wall thickness, mm
w = pipe weight, kg/m

Example 7.34 Calculate the allowable internal design pressure for a 16-inch (0.250-in wall thickness) pipeline constructed of API 5LX-52 steel. What wall thickness will be required if an internal working pressure of 1400 psi is required? Use class 1 construction with design factor $F = 0.72$ and for operating temperatures below 200°F.

Solution Using Eq. (7.106),

$$P = \frac{2 \times 0.250 \times 52000 \times 0.72 \times 1.0 \times 1.0}{16} = 1170\,\text{psi}$$

For an internal working pressure of 1400 psi, the wall thickness required is

$$1400 = \frac{2 \times t \times 52{,}000 \times 0.72 \times 1.0}{16}$$

Solving for t, we get

$$\text{Wall thickness } t = 0.299\,\text{in}$$

The nearest standard pipe wall thickness is 0.312 in.

Example 7.35 A DN 1000 natural gas pipeline is 1000 km long and has an operating pressure of 9.7 MPa. Compare the cost of using X-70 or X-80 steel pipe for this application. The material cost of the two grades of steel are as follows:

Pipe grade	Material cost, $/ton
X-70	800
X-80	1000

Use a design factor of 0.72 and temperature deration factor of 1.00.

Solution We will first determine the wall thickness of pipe required to withstand the operating pressure of 9.7 MPa. Using Eq. (7.106), the pipe wall thickness required for X-70 pipe (70,000 psi = 482 MPa) is

$$t = \frac{9.7 \times 1000}{2 \times 482 \times 1.0 \times 0.72 \times 1.0} = 13.98\,\text{mm}, \quad \text{say } 14\,\text{mm}$$

Similarly, the pipe wall thickness required for X-80 pipe (80,000 psi = 552 MPa) is

$$t = \frac{9.7 \times 1000}{2 \times 552 \times 1.0 \times 0.72 \times 1.0} = 12.2\,\text{mm}, \quad \text{say } 13\,\text{mm}$$

Pipe weight in kg/m will be calculated using Eq. (7.107a). For X-70 pipe,

$$\text{Weight per meter} = 0.0246 \times 14 \times (1000 - 14) = 339.58\,\text{kg/m}$$

Therefore the total cost of a 1000-km pipeline at \$800 per ton of X-70 pipe is

$$\text{Total cost} = 800 \times 339.58 \times 1000 \times \frac{1000}{1000} = \$271.66\,\text{million}$$

Similarly, the pipe weight in kg/m for X-80 pipe is

$$\text{Weight per meter} = 0.0246 \times 13 \times (1000 - 13) = 315.64\,\text{kg/m}$$

Therefore, the total cost of a 1000-km pipeline at \$1000 per ton of X-80 pipe is

$$\text{Total cost} = 1000 \times 315.64 \times 1000 \times \frac{1000}{1000} = \$315.64\,\text{million}$$

Therefore the X-80 pipe will cost more than the X-70 pipe. The difference in cost is \$315.64 − \$271.66 = \$43.98 million.

7.9 Pipeline Economics

In pipeline economics we are interested in determining the most economical pipe size and material to be used for transporting a given volume of a gas from a source to a destination. The criterion would be to minimize the capital investment as well as annual operating and maintenance cost. In addition to selecting the pipe itself to handle the flow rate we must also evaluate the optimum size of compression equipment required. By installing a smaller-diameter pipe we may reduce the pipe material cost and installation cost. However, the smaller pipe size would result in larger pressure drop due to friction and hence higher horsepower, which would require larger more costly compression equipment. On the other hand selecting a larger pipe size would increase the capital cost of the pipeline itself but would reduce the compression horsepower required and hence the capital cost of compression equipment. Larger compressors and drivers will also result in increased annual operating and maintenance cost. Therefore, we need to determine the optimum pipe size and compression equipment required based on some approach that will minimize both capital investment as well as annual operating costs. The least present value approach, which considers the total capital cost and the annual operating costs over the life of the pipeline, time value of money, borrowing cost, and income tax rate, seems to be an appropriate method in this regard.

Example 7.36 A 250-mi-long is transmission pipeline is used to transport 200 MMSCFD of natural gas [specific gravity = 0.650, viscosity = 0.000008 lb/(ft · s)] from a gathering plant at Bloomfield to a compressor station at Topock. The flowing temperature is 60°F. Use $Z = 0.89$ and $\gamma = 1.29$.

Determine the optimum pipe size for this application based on the least initial cost. Consider three different pipe sizes: NPS 20, NPS 24, and NPS 30. Use the Colebrook-White equation or the Moody diagram for friction factor calculations. Assume the pipeline is on fairly flat terrain. Use 85 percent adiabatic efficiency and 95 percent mechanical efficiency for centrifugal compression at Bloomfield. Use \$700 per ton for pipe material cost and \$1500 per HP for compressor station installation cost. Labor costs for installing the three pipe sizes are \$100, \$120, and \$140 per ft. The pipeline will be designed for an operating pressure of 1400 psig. Pipe absolute roughness $e = 700\ \mu$in.

Solution Based on a 1400 psi design pressure, the wall thickness of NPS 20 pipe will be calculated first. Assuming API 5LX-52 pipe, the wall thickness required for a 1400-psi operating pressure is calculated from Eq. (7.106), assuming design factor $F = 0.72$.

$$\text{Wall thickness } t = \frac{1400 \times 20}{2 \times 52{,}000 \times 0.72} = 0.374\,\text{in}$$

The nearest standard size is 0.375 in. Therefore, the NPS 20 pipe will have an inside diameter of

$$D = 20 - 2 \times 0.375 = 19.25\,\text{in}$$

Next we calculate the Reynolds number using Eq. (7.41):

$$\text{Re} = 0.0004778\,\frac{14.7}{520} \times \frac{0.65 \times 200 \times 10^6}{0.000008 \times 19.25} = 1.1402 \times 10^7$$

Using the Colebrook equation (7.66), the transmission factor is

$$F = -4\log_{10}\left(\frac{0.0007}{3.7 \times 19.25} + \frac{1.255F}{1.1402 \times 10^7}\right)$$

Solving by iteration, $F = 19.68$.

The pressure drop, using the general flow equation (7.42), is

$$200 \times 10^6 = 38.77 \times 19.68\,\frac{520}{14.7}\left(\frac{1414.7^2 - P_{\text{del}}^{\,2}}{0.65 \times 520 \times 250 \times 0.89}\right)^{0.5} \times (19.25)^{2.5}$$

Solving for P_{del}, the delivery pressure at Topock,

$$P_{\text{del}} = 662.85\,\text{psia}$$

We will assume a compression ratio of 1.50. Therefore,

$$\frac{P_2}{P_1} = \frac{1414.7}{P_1} = 1.5$$

and

$$P_1 = 943.13\,\text{psia}$$

The NPS 20 pipeline will require one compressor station discharging at 1400 psig. The compressor HP required from Eq. (7.102) is

$$\text{BHP} = \frac{0.0857 \times 200}{0.95} \frac{1.29}{0.29} (520) \frac{1 + 0.89}{2} \frac{1}{0.85} \left[\left(\frac{1414.7}{943.13} \right)^{0.29/1.29} - 1 \right]$$

$$\text{BHP} = 4428, \quad \text{say 5000 HP installed.}$$

$$\text{Capital cost of compressor station} = \$1500 \times 5000 = \$7.5 \text{ million}$$

Next the pipe material cost can be determined using Eq. (7.107):

$$\$700 \times 10.68 \times 0.375(20 - 0.375) \times 5280 \times \frac{250}{2000} = \$35.62 \text{ million}$$

The labor cost for installing 250 mi of NPS 20 pipe is

$$\$100 \times 5280 \times 250 = \$132 \text{ million}$$

Therefore the total capital cost of the NPS 20 pipeline system is

$$\$7.5 + \$35.62 + \$132.0 = \$175.12 \text{ million}$$

Similarly, we will repeat calculations for the NPS 24 and NPS 30 systems. For the NPS 24 system:

$$\text{Wall thickness } t = \frac{1400 \times 24}{2 \times 52{,}000 \times 0.72} = 0.449 \text{ in}, \quad \text{say 0.500 in}$$

$$D = 24 - 2 \times 0.5 = 23.00 \text{ in}$$

$$R = 9.543 \times 10^6 \quad \text{and} \quad F = 19.86$$

The compressor HP = 5000 as before.

$$\text{Capital cost of compressor station} = \$1500 \times 5000 = \$7.5 \text{ million}$$

The pipe material cost is

$$\$700 \times 10.68 \times 0.500(24 - 0.500) \times 5280 \times \frac{250}{2000} = \$57.98 \text{ million}$$

The labor cost for installing 250 mi of NPS 24 pipe is

$$\$120 \times 5280 \times 250 = \$158.4 \text{ million}$$

Therefore the total capital cost of the NPS 24 pipeline system is

$$\$7.5 + \$57.98 + \$158.4 = \$223.88 \text{ million}$$

Finally, we repeat the calculations for the NPS 30 system.

$$\text{Wall thickness } t = \frac{1400 \times 30}{2 \times 52{,}000 \times 0.72} = 0.561 \text{ in}, \quad \text{say 0.600 in}$$

$$D = 30 - 2 \times 0.6 = 28.800 \text{ in}$$

$$R = 7.621 \times 10^6 \quad \text{and} \quad F = 20.02$$

The compressor HP = 5000 as before.

$$\text{Capital cost of compressor station} = \$1500 \times 5000 = \$7.5\,\text{million}$$

The pipe material cost is

$$\$700 \times 10.68 \times 0.600\,(30 - 0.600) \times 5280 \times \frac{250}{2000} = \$87.04\,\text{million}$$

The labor cost for installing 250 mi of NPS 30 pipe is

$$\$140 \times 5280 \times 250 = \$184.8\,\text{million}$$

Therefore the total capital cost of the NPS 30 pipeline system is

$$\$7.5 + \$87.04 + \$184.8 = \$279.34\,\text{million}$$

The summary of the total capital cost is

Pipe size	Total cost, $ million
NPS 20	175.12
NPS 24	223.88
NPS 30	279.34

From the preceding it appears that NPS 20 is the most economical of the three pipe sizes since it has the least initial cost.

Example 7.37 A natural gas transmission pipeline is being constructed to serve a central distribution system in San Jose. The pipeline is 500 km long and originates at a Santa Fe compressor station (elevation 1200 m). The pipeline MAOP is limited to 9.5 MPa (gauge). The delivery pressure required at San Jose is 4.5 MPa. San Jose is at an elevation of 2500 m. During the first phase of the project, 15 million m^3/day of natural gas (specific gravity = 0.60, viscosity = 0.000119 P) will be transported at a 95 percent availability factor. What is the most economical pipe size for this project? The pipe material cost is estimated at $800/ton, and the labor cost for pipe installation is $800 per mm diameter per km pipe length. The compressor station cost is $2500 per kilowatt installed. Consider three different pipe sizes, DN 800, DN 1000, and DN 1200, of API 5LX-65 grade. Use the Colebrook-White equation or the Moody diagram for friction factor calculations. Use 80 percent adiabatic efficiency and 98 percent mechanical efficiency for centrifugal compressors at Santa Fe. Pipe absolute roughness $e = 0.02$ mm, base temperature = 15°C, base pressure = 101 kPa, flowing temperature = 20°C, and compressibility factor = 0.9.

Solution Consider DN 800 pipe. The wall thickness required for 9.5 MPa pressure for X65 (65,000/ 145 = 448 MPa) pipe is

$$t = \frac{9.5 \times 800}{2 \times 448 \times 0.72} = 11.78\,\text{mm}, \quad \text{use 12 mm}$$

$$\text{Weight per meter of pipe} = 0.0246 \times 12 \times (800 - 12)$$
$$= 232.62\,\text{kg/m}$$
$$\text{Cost of pipe for 500 km at \$800/ton} = 800 \times 232.62 \times 500$$
$$= \$93.05\,\text{million}$$
$$\text{Installation cost} = \$800 \times 800 \times 500$$
$$= \$320\,\text{million}$$

Next we will calculate the pressure and HP required.

$$\text{Reynolds number Re} = 0.5134 \times \frac{101}{288} \times \frac{0.6 \times 15 \times 10^6}{0.000119 \times 776} = 1.755 \times 10^7$$

The Colebrook-White transmission factor is

$$F = -4 \log\left(\frac{0.02}{3.7 \times 776} + \frac{1.255F}{1.755 \times 10^7}\right)$$

Solving by iteration, $F = 20.3$. The elevation correction factor is

$$s = \frac{0.0684 \times 0.6 \times (2500 - 1200)}{293 \times 0.9} = 0.2023$$

The equivalent length is

$$L_e = \frac{500\,(e^{0.2023} - 1)}{0.2023} = 554.17\,\text{km}$$

Using the general flow equation, the pressure at Santa Fe is given by

$$15 \times 10^6 = (5.7473 \times 10^{-4})(20.3)\frac{288}{0.101}\left(\frac{P_1{}^2 - 1.2242 \times 4.601^2}{0.6 \times 293 \times 554.17 \times 0.9}\right)^{0.5} 776^{2.5}$$

Solving for P_1,

$$P_1 = 9.45 \quad \text{MPa (absolute)}$$

We will assume a compression ratio of 1.5. Therefore,

$$\text{Suction pressure at Santa Fe} = \frac{9.45}{1.5} = 6.3 \quad \text{MPa}$$

The power required at Santa Fe is

$$\text{Power} = 15 \times \frac{4.0639}{0.98} \times \frac{1.29}{0.29} \times 288 \times \frac{1 + 0.9}{2} \frac{1}{0.8}\left[(1.5)^{0.29/1.29} - 1\right] = 9031\,\text{kW}$$

Assume 10,000 kW installed.

$$\text{Cost of compression station} = \$2500 \times 10{,}000 = \$25\,\text{million}$$

Finally the total capital cost of DN 800 pipe is

$$\$93.05 + \$320 + \$25 = \$438.05\,\text{million}$$

Next consider DN 1000 pipe. The wall thickness required for 9.5 MPa pressure for X65 (65,000/145 = 448 MPa) pipe is

$$t = \frac{9.5 \times 1000}{2 \times 448 \times 0.72} = 14.73\,\text{mm}, \quad \text{use 15 mm}$$

$$\begin{aligned}\text{Weight per meter of pipe} &= 0.0246 \times 15 \times (1000 - 15)\\ &= 363.47\,\text{kg/m}\\ \text{Cost of pipe for 500 km at \$800/ton} &= 800 \times 363.47 \times 500\\ &= \$145.39\,\text{million}\\ \text{Installation cost} &= \$800 \times 1000 \times 500 = \$400\,\text{million}\end{aligned}$$

Next we will calculate the pressure and HP required.

$$\text{Reynolds number Re} = 0.5134 \times \frac{101}{288} \times \frac{0.6 \times 15 \times 10^6}{0.000119 \times 970} = 1.404 \times 10^7$$

The Colebrook-White transmission factor is

$$F = 20.52$$

Using the general flow equation, the pressure at Santa Fe is given by

$$15 \times 10^6 = (5.7473 \times 10^{-4})(20.52)\frac{288}{0.101}\left(\frac{P_1{}^2 - 1.2242 \times 4.601^2}{0.6 \times 293 \times 554.17 \times 0.9}\right)^{0.5} 970^{2.5}$$

Solving for P_1,

$$P_1 = 6.8 \quad \text{MPa (absolute)}$$

$$\text{Compression ratio} = \frac{6.8}{6.3} = 1.08$$

The power required at Santa Fe is

$$\begin{aligned}\text{Power} &= 15 \times \frac{4.0639}{0.98} \times \frac{1.29}{0.29} \times 288 \times \frac{1+0.9}{2}\frac{1}{0.8}\left[(1.08)^{0.29/1.29} - 1\right]\\ &= 1652\,\text{kW}\end{aligned}$$

Assume 2000 kW installed.

$$\text{Cost of compression station} = \$2500 \times 2000 = \$5\,\text{million}$$

Finally, the total capital cost of DN 1000 pipe is

$$\$145.39 + \$400 + \$5 = \$550.39\,\text{million}$$

Finally we consider DN 1200 pipe. The wall thickness required for 9.5 MPa pressure for X65 (65,000/145 = 448 MPa) pipe is

$$t = \frac{9.5 \times 1200}{2 \times 448 \times 0.72} = 17.67\,\text{mm}, \quad \text{use 18 mm}$$

$$\text{Weight per meter of pipe} = 0.0246 \times 18 \times (1200 - 18)$$
$$= 523.39\,\text{kg/m}$$

$$\text{Cost of pipe for 500 km at \$800/ton} = 800 \times 523.39 \times 500 = \$209.36\,\text{million}$$

$$\text{Installation cost} = \$800 \times 1200 \times 500 = \$480\,\text{million}$$

Next we will calculate the pressure and HP required.

$$\text{Reynolds number Re} = 0.5134 \times \left(\frac{101}{288}\right) \times \frac{0.6 \times 15 \times 10^6}{0.000119 \times 1164} = 1.17 \times 10^7$$

The Colebrook-White transmission factor is

$$F = 20.65$$

Using the general flow equation, the pressure at Santa Fe is given by

$$15 \times 10^6 = (5.7473 \times 10^{-4})(20.65)\frac{288}{0.101}\left(\frac{P_1{}^2 - 1.2242 \times 4.601^2}{0.6 \times 293 \times 554.17 \times 0.9}\right)^{0.5}$$
$$\times 1164^{2.5}$$

Solving for P_1,

$$P_1 = 5.83 \quad \text{MPa (absolute)}$$

Since the suction pressure is 6.3 MPa, no compression is needed. Finally the total capital cost of DN 1200 pipe is

$$\$209.36 + \$480 = \$689.36\,\text{million}$$

Since the total capital cost is least using the DN 800 pipe, this is the most economical pipe size.

Chapter 8

Fuel Gas Distribution Piping Systems

Introduction

Fuel gas distribution piping systems are used to supply fuel gas for heating and lighting purposes. The more commonly used fuel gases are natural gas (NG), liquefied petroleum gas (LPG), and propane. Other gases include acetylene and butane. In this chapter we will discuss the more commonly used fuel gas piping systems such as for NG and LPG. We will look at how a typical fuel gas distribution piping system is sized based on customer demand. These are low-pressure piping systems. For a detailed discussion of the transportation of NG and other compressible gases at high pressures, refer to Chap. 7.

8.1 Codes and Standards

Several design codes and standards regulate the design, manufacture, and installation of NG and LPG fuel gas systems. The more commonly used standards are as follows:

ASME Section VIII	American Society of Mechanical Engineers—Pressure Vessels Code
ANSI/NFPA 30	American National Standards Institute/National Fire Protection Association—Flammable and Combustible Liquids Code
ANSI Z223.1/NFPA 54	American National Standards Institute/National Fire Protection Association—National Fuel Gas Code

ANSI Z83.3	American National Standards Institute—The Standard for Gas Utilization Equipment in Large Boilers
ANSI/UL 144	Pressure Regulating Valves for LPG
NFPA 58	National Fire Protection Association—Standard for the Storage and Handling of LPG
SBCCI	International Fuel Gas Code

8.2 Types of Fuel Gas

Natural gas, LPG, and propane are commonly used fuel gases. LPG is a mixture of propane and butane. It is generally transported and stored in liquid form. Other gases may also be used as fuel, but cost and availability may dictate the use of a specific gas over another. Table 8.1 lists commonly available fuel gases and their properties such as heating value and density.

Since NG, LPG, and propane are the most common fuel gases, detailed properties of these fuels are listed in Table 8.2. LPG is the commercial term for a liquid under pressure that contains varying proportions of propane (C_3H_8) and butane (C_4H_{10}). It is generally transported and

TABLE 8.1 Physical and Combustion Properties of Fuel Gases

Gas name	Heating value Btu/ft^3 Gross	Heating value Btu/ft^3 Net	Heating value Btu/lb Gross	Heating value Btu/lb Net	Specific gravity	Density, lb/ft^3	Specific volume, ft^3/lb
Acetylene	1,498	1,447	21,569	21,837	0.91	0.070	14.4
Blast furnace gas	92	92	1,178	1,178	1.02	0.078	12.8
Butane	3,225	2,977	21,640	19,976	1.95	0.149	6.71
Butylene	3,077	2,876	20,780	19,420	1.94	0.148	6.74
Carbon monoxide	323	323	4,368	4,368	0.97	0.074	13.5
Carbureted gas	550	508	11,440	10,566	0.63	0.048	20.8
Coke oven gas	574	514	17,048	15,266	0.44	0.034	29.7
Sewage gas	690	621	11,316	10,184	0.80	0.062	16.3
Ethane	1,783	1,630	22,198	20,295	1.06	0.060	12.5
Hydrogen	325	275	61,084	51,628	0.07	0.0054	186.9
Methane	1,011	910	23,811	21,433	0.55	0.042	23.8
Natural gas, California, U.S.	1,073	971	20,065	18,158	0.70	0.054	18.4
Propane	2,572	2,365	21,500	19,770	1.52	0.116	8.61
Propylene	2,332	2,181	20,990	19,630	1.45	0.111	9.02
Water gas (bituminous)	261	239	4,881	4,469	0.71	0.054	18.7

TABLE 8.2 Properties of Natural Gas and Propane

	Propane	Natural gas
Formula	C_3H_8	CH_4
Molecular weight	44.097	16.402
Melting point, °F	−305.84	−3.54
Boiling point, °F	−44.0	−258.7
Specific gravity of gas (air = 1.00)	1.52	0.60
Specific gravity of liquid 60°F/60°F (water = 1.00)	0.588	0.30
Latent heat of vaporization at normal boiling point, Btu/lb	183.0	245.0
Vapor pressure, lb/in^2, gauge at 60°F (15.6°C)	92.0	
Liquid		
lb/gal at 60°F	4.24	2.51
gal/lb at 60°F	0.237	
Gas		
Btu/lb (gross)	21591	23,000
Btu/ft^3 at 60°F and 30 in mercury	2516	1050+/−
Btu/gal at 60°F	91,547	
ft^3 at 60°F, 30 in mercury/gal of liquid	36.39	59
ft^3 at 60°F, 30 in mercury/lb of liquid	8.58	23.6
Air in ft^3 required to burn 1 ft^3 of gas	23.87	9.53
Flame temperature, °F	3,595	3,416
Octane number (iso-octane = 100)	125	
Flammability limit in air		
Upper	9.5	15.0
Lower	2.87	5.0

stored as a liquid under pressure ranging from 200 to 300 pounds per square inch (lb/in^2 or psi). As a liquid it is approximately half as heavy as water. When the pressure is reduced, LPG vaporizes to form a gas with a specific gravity of approximately 1.52 (air = 1.00).

8.3 Gas Properties

Natural gas consists of hydrocarbon gases such as methane, ethane, etc. Generally a sample of NG will contain a majority (85 to 95 percent) of methane. The specific gravity of NG relative to air is approximately 0.6 indicating that NG is lighter than air and is about 60 percent as heavy as air. The physical properties of natural gas are listed in Table 8.2.

LPG on the other hand, which consists of a mixture of propane and butane, is treated as a liquid because it is normally stored under pressure in liquid form. Therefore, the specific gravity of LPG is compared to the density of water and is approximately 0.50. As LPG vaporizes, depending upon the composition, the vapor will be heavier than air since propane has a gravity of 1.52 (air = 1.00) and butane has a gravity of

1.95 (air = 1.00). LPG, therefore, settles on the ground as it vaporizes and may flow along the ground surfaces and potentially be ignited by a source considerably far from the leakage location. NG, on the other hand, being lighter than air rises above the ground and disperses into the surrounding air. When LPG is mixed with air in the right proportions, a flammable mixture is formed. At normal ambient temperature and pressure, between 2 and 10 percent of LPG vapor in air is the range for an explosive mixture. Beyond this range the mixture is too weak or too rich to cause flame propagation. At higher pressures the upper explosive limit increases. LPG vapor is also an anesthetic and will cause asphyxiation in large quantities by reducing the amount of available oxygen. Commercial LPG is generally odorized by the addition of ethyl mercaptan or dimethyl sulfide. This will enable small leaks to be detected fairly quickly due to the smell resulting from the odorant. As LPG leaks from storage tanks the resulting vaporization causes a cooling effect of the surroundings, and hence condensation and even freezing of water vapor will occur. This may be manifested in the form of ice in the vicinity of the leak. Because of the rapid vaporization of LPG and the resulting drop in temperature, LPG contact with human skin must be avoided as it will result in severe frost burn. Proper eye and hand protection must be worn when handling and being in the vicinity of LPG storage vessels and piping systems. The physical properties of LPG are listed in Table 8.2.

8.4 Fuel Gas System Pressures

Compared to trunk lines carrying natural gas, fuel gas distribution systems operate at low pressures. A typical pressure in a public utility main piping is in the range of 25 to 50 psig. The pressure downstream of the gas meter is as low as 4 to 7 in of water column (WC). This is equivalent to 0.14 to 0.25 psi. Because we are dealing with low pressures stated in inches of water column or inches of mercury, a convenient table such as Table 8.3 may be used to determine the pressure in psi from inches of water column.

The maximum allowable operating pressure of fuel gas piping inside a building is regulated by NFPA 54 or other more stringent local city codes or insurance carrier requirements. Generally, NG piping is limited to 5 psig. The local codes may allow higher pressures if the entire fuel gas piping system is of welded construction, piping is enclosed for protection, and the system is located in well-ventilated areas such that there will be no accumulation of fuel gas in the event of a leak. Higher pressures of up to 20 psig are allowed for LPG piping systems provided piping is run within industrial buildings constructed in accordance with NFPA 58.

TABLE 8.3 Pressures in Inches of Water Column, psi, and kPa at 60°F

Inches		Pressure	
Water	Mercury	psi	kPa
0.10	0.007	0.0036	0.02
0.20	0.015	0.0072	0.05
0.30	0.022	0.0108	0.07
0.40	0.029	0.0144	0.10
0.50	0.037	0.0180	0.12
0.60	0.044	0.0216	0.15
0.70	0.051	0.0253	0.17
0.80	0.059	0.0289	0.20
0.90	0.066	0.0325	0.22
1.00	0.074	0.0361	0.25
1.50	0.110	0.0541	0.37
2.00	0.147	0.0722	0.50
2.50	0.184	0.0902	0.62
3.00	0.221	0.1082	0.75
4.00	0.294	0.1443	1.00
5.00	0.368	0.1804	1.24
6.00	0.441	0.2165	1.49
7.00	0.515	0.2525	1.74
8.00	0.588	0.2886	1.99
9.00	0.662	0.3247	2.24
10.00	0.735	0.3608	2.49
12.00	0.882	0.4329	2.99
14.00	1.029	0.5051	3.48
16.00	1.176	0.5772	3.98
18.00	1.324	0.6494	4.48
20.00	1.471	0.7215	4.98
22.00	1.618	0.7937	5.47
24.00	1.765	0.8658	5.97
27.72	2.038	1.0000	6.90

8.5 Fuel Gas System Components

Filters in fuel gas systems are necessary to prevent dirt and other foreign matter from entering meters and pressure regulators and causing damage to these components. Depending upon the quality of fuel gas, such filters may be necessary. Gas meters are installed in fuel distribution systems to measure the quantity of fuel gas being supplied from the utility company's service mains to the residential or commercial consumer. A complete gas metering system will consist of a filter, a pressure regulator, and relief valves. Pressure regulators are installed to reduce the utility fuel gas pressure down to that required for a residential or commercial service. Direct-acting and pilot-operated pressure regulators are in common use. Sometimes a two-step regulation is used to cut the pressure from the comparatively high utility pressure (25 to 50 psig) to the lower pressure required to operate appliances, etc. A pressure relief valve is installed to protect the piping downstream of the meter and pressure regulator in the event of a malfunction of the pressure regulator.

8.6 Fuel Gas Pipe Sizing

As fuel gas flows through a pipeline, energy is lost due to friction between the gas molecules and the pipe wall. Therefore, there is a pressure gradient or pressure loss from the inlet of the pipe to the outlet. This frictional pressure drop depends on the flow rate, pipe inside diameter, and gas gravity. It has been found that for an efficient fuel gas distribution piping system, the pressure drop must be limited to about 10 percent of the inlet pressure. Therefore, if the pipe inlet pressure is 20 psig, the total pressure drop in the entire pipe length must be limited to 2 psig. The pipe size required for a particular flow rate and equivalent length (of all pipes, fittings, and valves) of pipe will be based on this pressure drop. Suppose the size selected is NPS 4 for a certain capacity and length of pipe. If the flow rate is increased, the pressure drop will increase. In order to keep the total pressure loss to within 10 percent of the inlet pressure, we may have to choose a larger pipe size. (*Note*: The designation NPS 4 means nominal pipe size of 4 in.)

Pipe sizing in fuel gas distribution systems is generally done using tables that list capacity in cubic feet per hour (ft^3/h) for different pipe sizes and lengths based upon the available fuel gas pressure. As indicated earlier, in determining the pipe diameter required for a particular flow rate, the pressure drop is limited to about 10 percent of the available pressure over the length of the piping. Table 8.4 shows the capacity of horizontal gas piping for different pipe diameters and lengths at an inlet pressure of 20 psi.

It can be seen from Table 8.4 that for 100 ft of NPS 2 pipe the capacity is 21,179 ft^3/h or 21.179 thousand ft^3/h (MCF/h). This particular pipe size at this capacity and inlet pressure will produce a pressure drop of 2 psig over the 100-ft length. The length to be used is the total equivalent length of pipe, and it includes the straight run of pipe, valves, and fittings. To determine the equivalent length of valves and fittings, we can use a table similar to Table 8.5. As an example, using Table 8.5 we can determine the total equivalent length of NPS 2 pipe consisting of 100 ft of straight pipe, four elbows, and one plug valve as follows:

$$\text{Straight pipe, NPS2} = 100 \text{ ft}$$

$$\text{Four NPS 2 elbows} = \frac{4 \times 30 \times 2}{12} = 20 \text{ ft}$$

$$\text{One NPS 2 plug valve} = \frac{1 \times 18 \times 2}{12} = 3 \text{ ft}$$

TABLE 8.4 Pipeline Capacities at 20 psig Inlet Pressure and 2 psig Pressure Drop

	Nominal pipe size (actual inside diameter), inches of schedule 40 Pipe											
Length, ft	0.5 (0.622)	0.75 (0.824)	1 (1.049)	1.25 (1.380)	1.5 (1.610)	2 (2.067)	2.5 (2.469)	3 (3.068)	3.5 (3.548)	4 (4.026)	5 (5.047)	6 (6.065)
10	2,723	5,765	10,975	22,804	34,398	66,973	107,577	191,989	282,890	396,270	724,020	1,181,799
20	1,926	4,076	7,760	16,125	24,323	47,357	76,068	135,757	200,034	280,205	511,959	838,658
25	1,722	3,646	6,941	14,422	21,755	42,358	68,037	121,424	178,915	250,623	457,910	747,435
30	1,572	3,328	6,336	13,166	19,860	38,667	62,109	110,845	163,327	228,787	418,013	682,312
35	1,456	3,082	5,866	12,189	18,386	35,799	57,502	102,622	151,211	211,815	387,005	631,698
40	1,362	2,883	5,487	11,402	17,199	33,487	53,788	95,994	141,445	198,135	362,010	590,900
45	1,284	2,718	5,174	10,750	16,215	31,572	50,712	95,504	133,356	186,804	341,306	557,105
50	1,218	2,578	4,908	10,198	15,383	29,951	48,110	85,860	126,512	177,217	323,791	528,517
60	1,112	2,354	4,480	9,310	14,043	27,342	43,918	78,379	115,489	161,777	295,580	482,467
70	1,029	2,179	4,148	8,619	13,001	25,314	40,660	72,565	106,922	149,776	273,654	446,678
80	963	2,038	3,880	8,062	12,161	23,679	38,034	67,878	100,017	140,103	255,980	417,829
90	908	1,922	3,658	7,601	11,466	22,324	35,859	63,996	94,297	132,090	241,340	393,933
100	861	1,823	3,471	7,211	10,878	21,179	34,019	60,712	89,458	125,312	228,955	373,718
125	770	1,631	3,104	6,450	9,729	18,943	30,427	54,303	80,013	112,082	204,784	334,263
150	703	1,489	2,834	5,888	8,881	17,292	27,776	49,571	73,042	102,317	186,941	305,139
200	609	1,289	2,454	5,099	7,692	14,976	24,055	42,930	63,256	88,609	161,896	264,258
300	497	1,053	2,004	4,163	6,280	12,228	19,641	35,052	51,648	72,349	132,187	215,766
400	431	912	1,735	3,606	5,439	10,589	17,009	30,356	44,729	62,656	114,478	186,859
500	385	815	1,552	3,225	4,865	9,471	15,214	27,151	40,007	56,041	102,392	167,132
1,000	272	577	1,097	2,280	3,440	6,697	10,758	19,199	28,289	39,627	72,402	118,180
1,500	222	471	896	1,862	2,809	5,468	8,784	15,676	23,098	32,355	59,116	96,493
2,000	193	408	776	1,612	2,432	4,736	7,607	13,576	20,003	28,021	51,196	83,566

NOTE: Natural gas flow rates in standard ft^3/h and specific gravity = 0.6.
SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

TABLE 8.5 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

Therefore,

$$\text{Total equivalent length} = 123 \text{ ft of NPS 2 pipe}$$

It must be noted that Table 8.4 lists the capacity of horizontal pipes carrying natural gas at an inlet pressure of 20 psig. Since some piping may be vertical, the pressure drop in the vertical pipes should also be accounted for. Generally when calculating the capacity of NG systems, the vertical runs of piping are ignored because NG is lighter than air and expands as it rises in a vertical section of pipe. This argument is applicable only to NG. On the other hand LPG, when vaporized, is a gas that is heavier than air (specific gravity = 1.52), and therefore vertical runs of pipe are included in the total equivalent length. When the initial pressure is 50 psig, with a 10 percent allowable pressure drop, a table such as Table 8.6 may be used to determine the capacity of a NG piping system.

For example from Table 8.6, NPS 2 pipe with a 100-ft equivalent length has a capacity of 45,494 ft^3/h. This is based on an initial gas pressure of 50 psig and a total pressure drop of 5 psig in the 100-ft length of NPS 2 pipe.

The table method of calculating the capacity of a pipe for fuel gas flow is only approximate. More accurate formulas are available to calculate

TABLE 8.6 Pipeline Capacities at 50 psig Inlet Pressure and 5 psig Pressure Drop

Length, ft	Nominal pipe size (actual inside diameter), inches of schedule 40 Pipe											
	0.5 (0.622)	0.75 (0.824)	1 (1.049)	1.25 (1.380)	1.5 (1.610)	2 (2.067)	2.5 (2.469)	3 (3.068)	3.5 (3.548)	4 (4.026)	5 (5.047)	6 (6.065)
10	5,850	12,384	23,575	48,984	73,889	143,864	231,083	412,407	607,670	851,220	1,555,251	2,538,598
20	4,137	8,757	16,670	34,637	52,248	101,727	163,400	291,616	429,688	601,903	1,099,729	1,795,060
50	2,616	5,538	10,543	21,906	33,044	64,338	103,343	184,434	241,758	380,677	695,529	1,135,295
100	1,850	3,916	7,456	15,490	23,336	45,494	73,075	130,415	192,162	269,179	491,814	802,775
200	1,308	2,769	5,271	10,953	16,522	32,169	51,672	92,217	135,879	190,339	347,765	567,648
300	1,068	2,261	4,304	8,943	13,490	26,266	42,190	75,295	110,945	155,411	283,949	463,482
400	925	1,958	3,727	7,745	11,683	22,747	36,537	65,207	96,081	134,590	245,907	401,388
500	827	1,751	3,334	6,927	10,450	20,345	32,680	58,323	85,938	120,381	219,946	359,012
1,000	585	1,238	2,357	4,898	7,389	14,386	23,108	41,241	60,767	85,122	155,525	253,860
1,500	478	1,011	1,925	4,000	6,033	11,746	18,868	33,673	49,616	69,502	126,986	207,276
2,000	414	876	1,667	3,464	5,225	10,173	16,340	29,162	42,969	60,190	109,973	179,506

NOTE: Natural gas flow rates in standard ft^3/h and specific gravity = 0.6.
SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

the pressure drop in a specific pipe size at a certain gas flow rate. These are called the Spitzglass and Weymouth formulas for pressure drop.

In what follows, psi and psig both refer to gauge pressures. Absolute pressures (inclusive of the base atmospheric pressure) is referred to as psia.

For low-pressure (less than or equal to 1 psi) calculations, the Spitzglass formula is used. This formula is expressed in U.S. Customary System (USCS) units as follows:

$$Q_s = 3550K\sqrt{\frac{h}{GL}} \tag{8.1}$$

and

$$K = \sqrt{\frac{d^5}{1 + 3.6/d + 0.03d}} \tag{8.2}$$

where Q_s = gas flow rate at standard conditions (60°F or 15.6°C), ft^3/h
K = parameter that is a function of pipe diameter, d
h = frictional head loss, in of WC
L = equivalent pipe length, ft
G = fuel gas specific gravity (air = 1.00), dimensionless
d = pipe inside diameter, in

In SI units the Spitzglass formula is expressed as follows, for pressures less than 6.9 kilopascals (kPa):

$$Q_s = 11.0128K\sqrt{\frac{h}{GL}} \tag{8.3}$$

and

$$K = (3.075 \times 10^{-4})\sqrt{\frac{d^5}{1 + 91.44/d + 0.001181d}} \tag{8.4}$$

where Q_s = gas flow rate at standard conditions (15.6°C), m^3/h
K = parameter that is a function of pipe diameter, d
h = frictional head loss, mm of WC
L = equivalent pipe length, m
G = fuel gas specific gravity (air = 1.00), dimensionless
d = pipe inside diameter, mm

The value of h in millimeters of water column in Eq. (8.3) may be converted to pressure in kilopascals as follows:

$$\text{Pressure in kPa} = \frac{h}{25.4} \times \frac{0.0361}{0.145}$$

or

$$\text{Pressure in kPa} = \frac{h}{102} \tag{8.4a}$$

where h is in millimeters of water column.

For pressures greater than 1.0 psig, the Weymouth equation is used. This equation in USCS units is expressed as follows:

$$Q_s = 3550K\sqrt{\frac{P_{\text{avg}}\ \Delta P}{GL}} \tag{8.5}$$

where P_{avg} is the average pressure (psig) and ΔP is the pressure drop (psig). All other symbols are as defined earlier and K is calculated using Eq. (8.2).

In SI units the Weymouth formula is expressed as follows, for inlet pressures greater than 6.9 kPa:

$$Q_s = 8.0471K\sqrt{\frac{P_{\text{avg}}\ \Delta P}{GL}} \tag{8.6}$$

where P_{avg} is the average pressure (kPa) and ΔP is the pressure drop (kPa). All other symbols are as defined earlier and K is calculated using Eq. (8.4).

Tables 8.7 and 8.8 show the capacities of different pipe sizes based on low pressures (1.0 psig) and higher pressures (2.0 to 10.0 psig), respectively. Equivalent tables in SI units with gas capacity in liters per second (L/s) and pressures in kilopascals are given in Tables 8.9 and 8.10, respectively. These tables are based on the Spitzglass and Weymouth equations.

Example 8.1 Calculate the fuel gas capacity of NPS 4 pipe with an inside diameter of 4.026 in and a total equivalent length of 150 ft. The inlet pressure is 1.0 psig. Consider a pressure drop of 0.6 in water column and assume the specific gravity of the gas is 0.6.

Solution Since this is low pressure, we will use the Spitzglass formula. First we will calculate the parameter K from Eq. (8.2).

$$K = \sqrt{\frac{4.026^5}{1 + (3.6/4.026) + (0.03 \times 4.026)}} = 22.91$$

and from Eq. (8.1), the capacity in ft^3/h is

$$Q_s = 3550 \times 22.91\sqrt{\frac{0.6}{0.6 \times 150}} = 6641\ \text{ft}^3/\text{h}$$

TABLE 8.7 Pipeline Capacities at Low Pressures (1.0 psig) and Pressure Drop of 0.5 in Water Column

Length, ft	Nominal pipe size (actual inside diameter), inches of schedule 40 Pipe											
	0.5 (0.622)	0.75 (0.824)	1 (1.049)	1.25 (1.380)	1.5 (1.610)	2 (2.067)	2.5 (2.469)	3 (3.068)	3.5 (3.548)	4 (4.026)	5 (5.047)	6 (6.065)
10	120	272	547	1,200	1,860	3,759	6,169	11,225	16,685	23,479	42,945	69,671
20	85	192	387	849	1,315	2,658	4,362	7,938	11,798	16,602	30,367	49,265
30	69	157	316	693	1,074	2,171	3,562	6,481	9,633	13,556	24,794	40,225
40	60	136	273	600	930	1,880	3,084	5,613	8,342	11,740	21,473	34,835
50	54	122	244	537	832	1,681	2,759	5,020	7,462	10,500	19,206	31,158
60	49	111	223	490	759	1,535	2,518	4,583	6,811	9,585	17,532	28,443
70	45	103	207	454	703	1,421	2,332	4,243	6,306	8,874	16,232	26,333
80	42	96	193	424	658	1,329	2,181	3,969	5,899	8,301	15,183	24,632
90	40	91	182	400	620	1,253	2,056	3,742	5,562	7,826	14,315	23,224
100	38	86	173	379	588	1,189	1,951	3,550	5,276	7,425	13,581	22,032
150	31	70	141	310	480	971	1,593	2,898	4,308	6,062	11,088	17,989
200	27	61	122	268	416	841	1,379	2,510	3,731	5,250	9,603	15,579
400	19	43	86	190	294	594	975	1,775	2,638	3,712	6,790	11,016
500	17	38	77	170	263	532	872	1,588	2,360	3,320	6,073	9,853
1,000	12	27	55	120	186	376	617	1,123	1,668	2,348	4,295	6,967
1,500	10	22	45	98	152	307	504	917	1,362	1,917	3,506	5,689
2000	8	19	39	85	132	266	436	794	1180	1660	3037	4926

NOTE: Flow rates in standard ft^3/h with gas specific gravity = 0.6.
SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

TABLE 8.8 Pipeline Capacities at Higher Pressures (2.0–10.0 psig)

Inlet pressure, psig	Pipe size, in	Pressure drop per 100 ft as percent of inlet pressure: 2%	6%	10%
2	1	340	590	760
5		590	1,030	1,320
10		930	1,610	2,070
2	$1\frac{1}{4}$	710	1,230	1,590
5		1,230	2,130	2,740
10		1,950	3,370	4,330
2	$1\frac{1}{2}$	1,080	1,870	2,410
5		1,860	3,220	4,140
10		2,940	5,080	6,530
2	2	2,100	3,640	4,700
5		3,630	6,270	8,070
10		5,740	9,890	12,720
2	$2\frac{1}{2}$	3,390	5,880	7,580
5		5,850	10,100	13,010
10		9,240	15,940	20,500
2	3	6,060	10,500	13,540
5		10,450	18,050	23,240
10		16,510	28,480	36,610
2	4	12,480	21,620	27,890
5		21,520	37,180	47,880
10		34,000	58,650	75,410
2	6	37,250	64,560	83,270
5		64,240	111,010	142,950
10		101,520	175,120	225,150

NOTE: Flow rates in standard ft^3/h of natural gas with specific gravity = 0.6.
SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

Example 8.2 Calculate the fuel gas capacity of DN 100 (6-mm wall thickness) pipe for a total equivalent length of 50 m. The inlet pressure is 6 kPa. Consider a pressure drop of 25 mm of water column and assume the specific gravity of the gas is 0.6.

Solution Since this is low pressure, we will use Spitzglass formula. First we will calculate the parameter K from Eq. (8.4).

Inside diameter of pipe = 100 − 2 × 6 = 88 mm

$$K = (3.075 \times 10^{-4})\sqrt{\frac{88^5}{1 + (91.44/88) + 0.001181 \times 88}}$$
$$= 15.26$$

and from Eq. (8.3), the capacity in m^3/h is

$$Q_s = 11.0128 \times 15.26\sqrt{\frac{25}{0.6 \times 50}} = 153.41 \text{ m}^3/\text{h}$$

TABLE 8.9 Pipeline Capacities at Low Pressures (up to 6.9 kPa) and Pressure Drop of 1.2 kPa

Length, m	Capacity in L/s for horizontal gas piping for DN sizes																
	6	10	15	20	25	32	40	50	65	80	90	100	125	150	200	250	300
3	0.20	0.49	0.96	2.15	4.38	9.60	15.00	30.0	49.0	90.0	133.0	188.0	344.0	557	1135	2022	3134
6	0.13	0.34	0.67	1.53	2.06	6.79	11.00	21.0	35.0	64.0	94.0	133.0	243.0	394	802	1430	2216
9	0.12	0.29	0.55	1.24	1.30	5.54	9.00	17.0	28.0	52.0	77.0	108.0	198.0	322	655	1168	1809
12	0.10	0.24	0.47	1.08	1.10	4.80	7.50	15.0	25.0	45.0	67.0	94.0	172.0	279	567	1011	1567
15	0.08	0.22	0.42	0.97	0.98	4.30	6.60	13.0	22.0	40.0	60.0	84.0	154.0	249	507	904	1401
18	0.08	0.22	0.39	0.87	0.92	3.92	6.00	12.0	20.0	37.0	54.0	77.0	140.0	228	463	826	1279
21	0.07	0.18	0.35	0.82	0.84	3.36	5.60	11.0	19.0	34.0	50.0	71.0	130.0	211	429	764	1184
24	0.07	0.17	0.34	0.76	0.78	3.39	5.20	11.0	17.0	32.0	47.0	66.0	121.0	197	401	715	1108
27	0.07	0.15	0.32	0.72	0.73	3.20	5.00	10.0	16.0	30.0	44.0	63.0	115.0	186	378	674	1045
30	0.07	0.15	0.30	0.69	0.69	3.03	4.70	10.0	15.0	28.0	42.0	59.0	109.0	176	359	640	991
45	0.05	0.13	0.25	0.55	0.58	2.43	3.80	8.0	13.0	23.0	34.0	48.0	89.0	144	293	522	809
60	0.05	0.12	0.22	0.49	0.54	2.14	3.30	7.0	11.0	20.0	30.0	42.0	77.0	125	254	452	701
90	0.03	0.10	0.18	0.42	0.46	1.92	3.00	6.0	10.0	18.0	27.0	38.0	69.0	111	227	404	627
120	0.03	0.08	0.15	0.34	0.41	1.52	2.30	5.0	8.0	14.0	21.0	30.0	54.0	88	179	320	495
150	0.03	0.07	0.13	0.30	0.33	1.36	2.10	4.0	7.0	13.0	19.0	27.0	49.0	79	160	286	443
300	0.02	0.05	0.10	0.22	0.29	0.96	1.50	3.0	5.0	9.0	13.0	19.0	34.0	56	113	202	313
450	0.02	0.03	0.08	0.17	0.21	0.78	1.20	2.0	4.0	7.0	11.0	15.0	28.0	46	93	165	256
600	0.02	0.03	0.07	0.15	0.17	0.68	1.00	2.0	3.0	6.0	9.0	13.0	24.0	39	80	143	222

NOTE: Flow rates in L/s with gas specific gravity = 0.6.
SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

TABLE 8.10 Pipeline Capacities at Higher Pressures (13.8–69 kPa)

kPa	Pipe size, DN	Pressure drop kPa/m as percent of inlet pressure 2%	6%	10%
13.8	25	2.72	4.00	5.60
34.5	25	4.72	8.24	10.56
69.0	25	7.44	12.88	16.56
13.8	32	5.68	9.84	12.72
34.5	32	9.84	17.04	21.92
69.0	32	15.60	26.72	34.64
13.8	40	8.64	14.96	19.28
34.5	40	14.88	25.76	33.12
69.0	40	23.52	40.64	52.24
13.8	50	16.80	29.12	37.60
34.5	50	29.04	50.16	64.56
69.0	50	45.92	79.12	101.76
13.8	65	27.12	47.04	60.64
34.5	65	46.80	80.80	104.08
69.0	65	73.92	127.52	164.00
13.8	80	48.48	84.00	108.32
34.5	80	83.60	144.40	185.92
69.0	80	132.08	227.84	292.88
13.8	100	99.84	172.96	222.40
34.5	100	172.16	297.44	383.04
69.0	100	272.00	469.20	603.28
13.8	150	298.00	516.48	666.16
34.5	150	513.92	888.08	1143.60
69.0	150	812.16	1400.96	1801.20

NOTE: Flow rates in L/s with gas specific gravity = 0.6.
SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

Example 8.3 A fuel gas pipeline is 250 ft in equivalent length and is constructed of NPS 6 pipe, with an inside diameter of 6.065 in. For an inlet pressure of 10.0 psig, calculate the total pressure drop at a flow rate of 60,000 standard cubic feet per hour (SCFH). Specific gravity of gas is 0.6.

Solution Since this is not low pressure, we will use the Weymouth equation (8.5). First we will calculate the parameter K from Eq. (8.2).

$$K = \sqrt{\frac{6.065^5}{1 + (3.6/6.065) + (0.03 \times 6.065)}} = 67.99$$

The flow rate and pressure drop are related by Eq. (8.5).

$$60{,}000 = 3550 \times 67.99\sqrt{\frac{10\ \Delta P}{0.6 \times 250}}$$

Solving for ΔP, we get

$$\Delta P = 0.93 \text{ psig}$$

In the preceding we used the inlet pressure in place of the average pressure. The average pressure can now be calculated, since the pressure drop has been calculated:

$$\text{Average pressure} = \frac{10 + (10 - 0.93)}{2} = 9.54 \text{ psi}$$

We can recalculate the pressure drop using this average pressure. This process can be repeated until the successive values of ΔP are within 0.1 psi.

Example 8.4 A fuel gas pipeline is 70 m in equivalent length and is constructed of DN 150 (6-mm wall thickness) pipe. The inlet pressure is 50 kPa and the flow rate is 300 L/s. Calculate the pressure drop if the specific gravity of gas is 0.65.

Solution

$$\text{Pipe inside diameter} = 150 - 2 \times 6 = 138 \text{ mm}$$

Since the pressure is higher than 6.9 kPa, the Weymouth formula will be used. First we calculate the value of the parameter K using Eq. (8.4):

$$K = (3.075 \times 10^{-4})\sqrt{\frac{138^5}{1 + (91.44/138) + 0.001181 \times 138}} = 50.91$$

From Eq. (8.6), converting the flow rate from L/s to m^3/h;

$$\frac{300 \times 60 \times 60}{1000} = 8.0471 \times 50.91\sqrt{\frac{50\ \Delta P}{0.65 \times 70}}$$

Solving for ΔP, we get

$$\Delta P = 6.32 \text{ kPa}$$

It must be noted that in Eq. (8.6) we used 50 kPa for the average pressure since we did not know how much the pressure drop was going to be. We can calculate the average pressure based on the ΔP obtained and recalculate the corresponding ΔP from Eq. (8.6) as follows:

$$\text{Average pressure} = \frac{50 + (50 - 6.32)}{2} = 46.84$$

$$\frac{300 \times 60 \times 60}{1000} = 8.0471 \times 50.91\sqrt{\frac{46.84\ \Delta P}{0.65 \times 70}}$$

$$\Delta P = 6.75 \text{ kPa}$$

The process is repeated until successive values of ΔP are within 0.1 kPa. This is left as an exercise for the reader.

Example 8.5 A typical NG fuel gas distribution system for a building is illustrated schematically in Fig. 8.1. Three fuel consumption devices A, B,

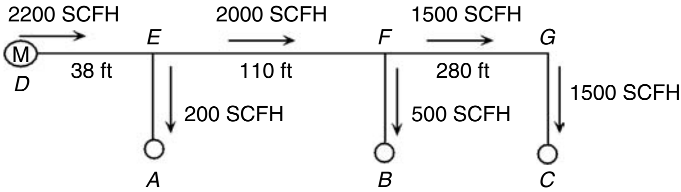

Figure 8.1 Sample fuel gas distribution system.

and C are shown requiring NG in the amounts of 200, 500, and 1500 ft^3/h, respectively. The equivalent lengths of piping are as shown in Fig. 8.1. Determine the pipe size required for each of the sections DE, EF, FG and the branch piping EA, FB, GC to handle the required fuel gas volumes. Assume the pressure available downstream of the utility meter at D is 6 in of water column.

Solution The total equivalent length will be calculated based on the length from the meter at D to the most remote point C. Accordingly,

$$\text{Total length} = 38 + 110 + 280 + 50 = 478 \text{ ft}$$

We will round this up to 500 ft equivalent length.

In order to size the various sections of the fuel gas distribution system shown in the figure, we will use Table 8.7 based on the inlet pressure of 1 psig and a pressure drop of 10 percent of inlet pressure.

$$\text{Total flow rate for all devices} = 200 + 500 + 1500 = 2200 \text{ ft}^3/\text{h}$$

From Table 8.7, for a length of 500 ft, we find that NPS 3.5 pipe has a capacity of 2360 ft^3/h. This flow rate is quite close to our requirement of 2200 ft^3/h that will flow through section DE. Therefore, section DE will require NPS 3.5 pipe.

Similarly, section EF has a flow of 2000 ft^3/h which also requires NPS 3.5 pipe. Section FG and GC both require a capacity of 1500 ft^3/h. From Table 8.7 this requires NPS 3 pipe which has a capacity of 1588 ft^3/h.

Next, we will select pipe sizes for branches EA and FB. Branch EA requires 200 ft^3/h, which according to Table 8.7 requires NPS 1.5 pipe (263 ft^3/h). Finally, branch FB carries 500 ft^3/h, which requires NPS 2 pipe that has a capacity of 532 ft^3/h according to Table 8.7.

It must be noted that the table method demonstrated here is fairly easy but only approximate. A more accurate approach would be to select a pipe size for the entire length from D to C and calculate the pressure drop using the Spitzglass formula. Section DE will have a flow rate of 2200 ft^3/h, EF will have a flow rate of 2000 ft^3/h, and sections FG and GC will each have a flow rate of 1500 ft^3/h. Similarly, branches EA and FB will be sized to handle flow rates of 200 and 500 ft^3/h, respectively.

8.7 Pipe Materials

Pipe materials used in NG piping systems include carbon steel, copper tubing, and high-density polyethylene (HDPE). Pipe materials are specified in NFPA 54 and other codes listed in Sec. 8.1. The working pressures of the fuel gas piping system must be lower than the pressure rating of the pipe, fittings, and valves used. Class 150 pipe and fittings are specified for carbon steel and are suitable for working pressures of up to 285 psig at 100°F. As the temperature of services increases, the allowable working pressure decreases. Underground fuel gas distribution piping is often constructed of plastic pipe (HDPE). These pipes are buried to a minimum depth of 3 ft. For safety reasons a corrosion-resistant tracer wire is buried with the plastic pipe so that the fuel gas line may be located using a metal detector. Warning signs must be installed indicating the existence of an underground natural gas pipeline. Steel pipes used for underground distribution piping systems generally conform to ASTM A106 or A53. Steel pipe and fittings are welded, and the pipe exterior is coated and wrapped to prevent pipe corrosion. Aboveground pipes are always constructed with carbon-steel material. Plastic piping is not allowed for aboveground installation. In order to isolate appliances from each other, valves are used. Small valves used in conjunction with domestic appliances are referred to as gas cocks. Check valves are used to prevent backflow of the fuel gas and are constructed of a cast iron body with stainless steel trim. Screwed fittings are used with NPS 3 and smaller valves. Larger size valves are constructed of flanged connections. Special valves are used in earthquake zones. These valves automatically shut down the fuel gas supply in the event that the horizontal or vertical displacements (due to earthquakes) exceed predetermined design values.

8.8 Pressure Testing

Fuel gas distribution piping must be pressure tested before being put into service. Compressed air is used for the test. After satisfactory pressure testing, all air in the piping must be purged by using an inert gas such as nitrogen, before filling the piping with natural gas. The test pressure is 150 percent of the highest pressure in the main fuel gas piping. The duration of the test depends upon the length and total volume of the pipe. For example, the test must be held for 6 h if the pipe length is 700 ft. The testing is reduced to 2 h for a pipe length of 200 ft of NPS 6 plastic pipe. For piping inside a building consisting of low-pressure gas (8 in of mercury or less), testing is done with air or fuel gas at a test pressure of 3 psi for a minimum period of 1 h. When the operating pressure is between 9 in of water column and 5 psig, the pressure test is conducted using air at 50 psig for a period of 4 h. When pressure is greater than 5 psig, the test is done using air at 100 psig for a minimum period of 4 h. No pressure drop is allowed for the duration of the test. Refer to design codes for details.

8.9 LPG Transportation

LPG is economically transported as a compressed fluid in the liquid phase. When used as a fuel, LPG is vaporized and distributed as a gas through the distribution piping system similar to the NG piping system discussed earlier. In this section we will first discuss LPG transportation (at high pressure) and pipe sizing. Next we will discuss storage of LPG and subsequent distribution as a fuel in vapor form.

Pressure within an LPG transportation piping system must be maintained at some minimum level to prevent vaporization during transport. The vapor pressure of the components propane and butane will determine this minimum pressure. In general most LPG transportation systems are maintained at a minimum of 200 to 250 psig (1.38 to 1.72 MPa) depending upon the ambient temperature and the percentage of propane in LPG.

Sometimes, we need to convert the pressure in psi to head of liquid in feet, and vice versa. If the specific weight of the liquid is γ lb/ft^3, a pressure of P in psig and the equivalent head of liquid H ft are related by the following equation

$$P = \frac{\gamma H}{144} \tag{8.7}$$

This is the gauge pressure. The absolute pressure would be $(\gamma H/144) + P_{\text{atm}}$ where P_{atm} is the atmospheric pressure.

More generally we can state that the absolute pressure is

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} \tag{8.8}$$

The unit of pressure designated as psia is for absolute pressure and that designated as psig is for gauge pressure. Unless otherwise specified, psi means gauge pressure or psig. The variable γ may also be replaced with ρg, where ρ is the density in slug/ft^3 and g is gravitational acceleration in ft/s^2.

In a more general form, the pressure P in psi and liquid head h in feet for a specific gravity of Sg are related by

$$P = \frac{h \times \text{Sg}}{2.31} \tag{8.9}$$

In SI units, pressure P in kilopascals and head h in meters are related by the following equation:

$$P = \frac{h \times \mathrm{Sg}}{0.102} \tag{8.10}$$

Example 8.6 Calculate the pressure in psi in an LPG piping system if the pressure in feet of head is 2500 ft and LPG specific gravity is 0.5. What is the equivalent pressure in kilopascals? If the atmospheric pressure is 14.7 psi, calculate the absolute pressure.

Solution Using Eq. (8.9),

$$\text{Pressure} = \frac{2500 \times 0.5}{2.31} = 541.13 \text{ psig}$$

Thus,

$$\text{Pressure at 2500 ft head} = 541.13 \text{ psig}$$

$$\text{Absolute pressure} = 541.13 + 14.7 = 555.83 \text{ psia}$$

In SI units we can calculate the pressures as follows. Since 1 kPa = 0.145 psi (see App. A for various conversion factors),

$$\text{Pressure at 2500 ft head} = \frac{541.13 \text{ psig}}{0.145 \text{ psi/kPa}} = 3732 \text{ kPa} \quad \text{or} \quad 3.73 \text{ MPa}$$

8.9.1 Velocity

The velocity at which LPG flows through a pipeline depends on the pipe diameter and flow rate. If the flow rate is constant (steady flow) and the pipe diameter is uniform, the velocity at every cross section along the pipe will be a constant value. However, there is a variation in velocity along the pipe cross section. The velocity at the pipe wall will be zero, increasing to a maximum at the centerline of the pipe.

We can define an average velocity of flow at any cross section of the pipe as follows:

$$\text{Velocity} = \frac{\text{flow rate}}{\text{area of flow}} \tag{8.11}$$

If the flow rate is in ft^3/s and the pipe cross-sectional area is in ft^2, the velocity from Eq. (8.11) is in ft/s.

Employing commonly used units of flow rate Q in gallons per minute (gal/min) and pipe diameter in inches, the velocity in ft/s is as follows:

$$V = 0.4085 \frac{Q}{D^2} \tag{8.12}$$

where V = velocity, ft/s
Q = flow rate, gal/min
D = pipe inside diameter, in

Sometimes, in the petroleum transportation industry, flow rates are expressed in barrels per hour (bbl/h) or bbl/day. Therefore, Eq. (8.12) for velocity can be modified as follows. For flow rate in bbl/h:

$$V = 0.2859 \frac{Q}{D^2} \tag{8.13}$$

where V = velocity, ft/s
Q = flow rate, bbl/h
D = pipe inside diameter, in

For the flow rate in bbl/day:

$$V = 0.0119 \frac{Q}{D^2} \tag{8.14}$$

where V = velocity, ft/s
Q = flow rate, bbl/day
D = pipe inside diameter, in

In SI units, the velocity equation is as follows:

$$V = 353.6777 \frac{Q}{D^2} \tag{8.15}$$

where V = velocity, m/s
Q = flow rate, m^3/h
D = internal diameter, mm

Example 8.7 LPG flows through an NPS 16 (15.5-in inside diameter) pipe at the rate of 4000 gal/min. Calculate the average velocity for steady-state flow.

Solution From Eq. (8.12) the average flow velocity is

$$V = 0.4085 \frac{4000}{15.5^2} = 6.80 \text{ ft/s}$$

Example 8.8 LPG flows through a DN 400 outside diameter (10-mm wall thickness) pipeline at 200 L/s. Calculate the average velocity for steady flow.

Solution The designation DN 400 in SI units corresponds to NPS 16 in USCS units. DN 400 means a metric pipe size of 400-mm outside diameter.

$$\text{Inside diameter of pipe} = 400 - 2 \times 10 = 380 \text{ mm}$$

First convert flow rate in L/s to m^3/h.

$$\text{Flow rate} = 200 \text{ L/s} = 200 \times 60 \times 60 \times 10^{-3} \text{ m}^3/\text{h} = 720 \text{ m}^3/\text{h}$$

From Eq. (8.15) the average flow velocity is

$$V = 353.6777 \frac{720}{380^2} = 1.764 \text{ m/s}$$

8.9.2 Reynolds number

The Reynolds number of flow is a dimensionless parameter that depends on the pipe diameter, liquid flow rate, liquid viscosity, and density. It is defined as follows:

$$R = \frac{VD\rho}{\mu} \tag{8.16}$$

or

$$R = \frac{VD}{\nu} \tag{8.17}$$

where R = Reynolds number, dimensionless
V = average flow velocity, ft/s
D = inside diameter of pipe, ft
ρ = mass density of liquid, slug/ft^3
μ = dynamic viscosity, slug/(ft · s)
ν = kinematic viscosity, ft^2/s

In terms of more commonly used units in the petroleum industry, we have the following version of the Reynolds number equation:

$$R = 3162.5 \frac{Q}{D\nu} \tag{8.18}$$

where R = Reynolds number, dimensionless
Q = flow rate, gal/min
D = inside diameter of pipe, in
ν = kinematic viscosity, centistokes (cSt)

When the flow rate is given in bbl/h or bbl/day, the following forms of the Reynolds number are used:

$$R = 2213.76 \frac{Q}{D\nu} \tag{8.19}$$

$$R = 92.24 \frac{\text{BPD}}{D\nu} \tag{8.20}$$

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where R = Reynolds number, dimensionless
Q = flow rate, bbl/h
BPD = flow rate, bbl/day
D = inside diameter of pipe, in
ν = kinematic viscosity, cSt

In SI units, the Reynolds number is expressed as follows:

$$R = 353{,}678 \, \frac{Q}{\nu D} \tag{8.21}$$

where R = Reynolds number, dimensionless
Q = flow rate, m^3/h
D = inside diameter of pipe, mm
ν = kinematic viscosity, cSt

Example 8.9 An LPG (specific gravity = 0.5 and viscosity = 0.15 cP) pipeline is composed of NPS 20 pipe with 0.375-in wall thickness. At a flow rate of 5000 gal/min, calculate the average velocity and the Reynolds number of flow.

Solution The NPS 20 (0.375-in wall thickness) pipe has an inside diameter = $20.0 - 2 \times 0.375 = 19.25$ in. From Eq. (8.12) the average velocity is calculated first:

$$V = 0.4085 \, \frac{5000}{19.25^2} = 5.51 \text{ ft/s}$$

$$\text{Kinematic viscosity of LPG} = \frac{0.15 \text{ cP}}{0.5} = 0.30 \text{ cSt}$$

From Eq. (8.18) the Reynolds number is therefore

$$R = 3162.5 \, \frac{5000}{19.25 \times 0.3} = 2{,}738{,}095$$

Example 8.10 LPG (specific gravity = 0.5 and viscosity = 0.3 cSt) flows through a DN 400 (10-mm wall thickness) pipeline at the rate of 800 m^3/h. Calculate the average flow velocity and the Reynolds number of flow.

Solution The DN 400 (10-mm wall thickness) pipe has an inside diameter = $400 - 2 \times 10 = 380$ mm. From Eq. (8.15) the average velocity is therefore

$$V = 353.6777 \, \frac{800}{380^2} = 1.96 \text{ m/s}$$

Next, from Eq. (8.21), the Reynolds number is

$$R = 353{,}678 \, \frac{800}{380 \times 0.3} = 2{,}481{,}951$$

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8.9.3 Types of flow

Flow through a pipeline is classified as laminar flow, turbulent flow, or critical flow depending on the magnitude of the Reynolds number of flow. If the Reynolds number is less than 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is considered to be *turbulent*. *Critical flow* occurs when the Reynolds number is in the range of 2100 to 4000. Laminar flow is characterized by smooth flow in which no eddies or turbulence exist. The flow is also said to occur in laminations. If dye was injected into a transparent pipeline, laminar flow would be manifested in the form of smooth streamlines of dye. Turbulent flow occurs at higher velocities and is accompanied by eddies and other disturbances in the liquid. More energy is lost in friction in the critical flow and turbulent flow regions as compared to the laminar flow region.

The three flow regimes characterized by the Reynolds number of flow are

$$\begin{aligned} &\text{Laminar flow:} && R \leq 2100 \\ &\text{Critical flow:} && 2100 < R \leq 4000 \\ &\text{Turbulent flow:} && R > 4000 \end{aligned}$$

In the critical flow regime, where the Reynolds number is between 2100 and 4000, the flow is undefined and unstable, as far as pressure drop calculations are concerned. In the absence of better data, it is customary to use the turbulent flow equation to calculate the pressure drop in the critical flow regime as well.

8.9.4 Pressure drop due to friction

As LPG flows through a pipeline, energy is lost due to resistance between the flowing liquid layers as well as due to the friction between the liquid and the pipe wall. One of the objectives of pipeline calculation is to determine the amount of energy and hence the pressure lost due to friction as the liquid flows from the source to the destination. The Darcy equation can be used to determine the head loss due to friction in LPG pipelines for a given flow rate, LPG properties, and pipe diameter.

8.9.5 Darcy equation

As LPG flows through a pipeline from point A to point B the pressure along the pipeline decreases due to frictional loss between the flowing liquid and the pipe. The extent of pressure loss due to friction, designated in feet of liquid head, depends on various factors. These factors include the liquid flow rate, liquid specific gravity and viscosity, pipe inside diameter, pipe length, and internal condition of the pipe (rough, smooth, etc.) The Darcy equation may be used to calculate the pressure drop in a pipeline as follows:

$$h = f\frac{L}{D}\frac{V^2}{2g} \tag{8.22}$$

where h = frictional pressure loss, ft of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = inside diameter of pipe, ft
V = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

The Darcy equation gives the frictional pressure loss in feet of liquid head, which must be converted to pressure loss in psi using Eq. (8.9). The term $V^2/2g$ in the Darcy equation is the velocity head, and it represents the kinetic energy of the liquid. The term velocity head will be used in subsequent sections of this chapter when analyzing frictional loss through pipe fittings and valves.

The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eq. (8.22). This friction factor is a nondimensional number between 0.0 and 0.1 that depends on the internal roughness of the pipe, the pipe diameter, and the Reynolds number of flow.

In laminar flow, the friction factor f depends only on the Reynolds number and is calculated from

$$f = \frac{64}{R} \tag{8.23}$$

where f is the friction factor for laminar flow and R is the Reynolds number for laminar flow ($R \leq 2100$) (dimensionless). Therefore, if a particular flow has a Reynolds number of 1780, we can conclude that in this laminar flow condition the friction factor f to be used in the Darcy equation is

$$f = \frac{64}{1780} = 0.036$$

Some pipeline hydraulics texts may refer to another friction factor called the Fanning friction factor. This is numerically equal to one-fourth the Darcy friction factor. In this example the Fanning friction factor can be calculated as

$$\frac{0.036}{4} = 0.009$$

To avoid any confusion, throughout this chapter we will use only the Darcy friction factor as defined in Eq. (8.22).

For LPG pipelines, it is inconvenient to use the Darcy equation in the form described in Eq. (8.22). We must convert the equation in terms of commonly used petroleum pipeline units. One form of the Darcy equation in pipeline units is as follows:

$$h = 0.1863 \frac{fLV^2}{D} \tag{8.24}$$

where h = frictional pressure loss, ft of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = pipe inside diameter, in
V = average flow velocity, ft/s

Another form of the Darcy equation with frictional pressure drop expressed in psi/mi and using flow rate instead of velocity is as follows:

$$P_m = \text{const} \frac{fQ^2\text{Sg}}{D^5} \tag{8.25}$$

where P_m = frictional pressure loss, psi/mi
f = Darcy friction factor, dimensionless
Q = flow rate
D = pipe inside diameter, in
Sg = liquid specific gravity
const = factor that depends on flow units

$$= \begin{cases} 34.87 & \text{for } Q \text{ in bbl/h} \\ 0.0605 & \text{for } Q \text{ in bbl/day} \\ 71.16 & \text{for } Q \text{ in gal/min} \end{cases}$$

In SI units, the Darcy equation may be written as

$$h = 50.94 \frac{fLV^2}{D} \tag{8.26}$$

where h = frictional pressure loss, m of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, m
D = pipe inside diameter, mm
V = average flow velocity, m/s

In terms of flow rate, the Darcy equation in SI units is as follows:

$$P_{\text{km}} = (6.2475 \times 10^{10}) fQ^2 \frac{\text{Sg}}{D^5} \tag{8.27}$$

where P_{km} = pressure drop due to friction, kPa/km
Q = liquid flow rate, m^3/h
f = Darcy friction factor, dimensionless
Sg = liquid specific gravity
D = pipe inside diameter, mm

8.9.6 Colebrook-White equation

We have seen that in laminar flow the friction factor f is easily calculated from the Reynolds number using Eq. (8.23). In turbulent flow, the calculation of friction factor f is more complex. It depends on the pipe inside diameter, the pipe roughness, and the Reynolds number. Based on work by Moody, Colebrook and White, and others, the following empirical equation, known as the Colebrook-White equation, is used for calculating the friction factor in turbulent flow:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{R\sqrt{f}}\right) \tag{8.28}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
R = Reynolds number, dimensionless

The absolute pipe roughness, or internal pipe roughness, may range from 0.0 to 0.01 depending on the internal condition of the pipe. It is listed for common piping systems in Table 8.11. The ratio e/D is termed the relative roughness and is dimensionless. Equation (8.28) is also sometimes called simply the Colebrook equation.

In SI units, we can use the same form of the Colebrook equation. The absolute pipe roughness e and the pipe diameter D are both expressed in millimeters. All other terms in the equation are dimensionless.

TABLE 8.11 Pipe Internal Roughness

Pipe material	Roughness	
	in	mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

It can be seen from the Colebrook-White equation that the calculation of the friction factor f is not straightforward since it appears on both sides of the equation. This is known as an implicit equation in f, compared to an explicit equation. An explicit equation in f will have the unknown quantity f on one side of the equation. In the present case, a trial-and-error approach is used to solve for the friction factor. First an initial value for f is assumed (for example, $f = 0.01$) and substituted in the right-hand side of the Colebrook equation. This will result in a new calculated value of f, which is used as the next approximation, and f is recalculated based on this second approximation. The process is continued until successive values of f calculated by such iterations are within a small value such as 0.001. Usually three or four iterations will yield a satisfactory solution.

8.9.7 Moody diagram

A graphical method of determining the friction factor for turbulent flow is available using the Moody diagram as shown in Fig. 8.2. First the Reynolds number is calculated based upon liquid properties, flow rate, and pipe diameter. This Reynolds number is used to locate the ordinate on the horizontal axis of the Moody diagram. A vertical line is drawn up to the curve representing the relative roughness e/D of the pipe. The friction factor is then read off of the vertical axis to the left. From the Moody diagram it is seen that the turbulent region is further divided into two regions: the "transition" zone and the "complete turbulence in rough pipes" zone. The lower boundary is designated as "smooth pipes." The transition zone extends up to the dashed line, beyond which is known as the zone of complete turbulence in rough pipes. In the zone of complete turbulence in rough pipes, the friction factor depends very little on the Reynolds number and more on the relative roughness.

Example 8.11 LPG (specific gravity = 0.5 and viscosity = 0.3 cSt) flows through an NPS 16 (0.250-in wall thickness) pipeline at a flow rate of 3000 gal/min. The absolute roughness of the pipe may be assumed to be 0.002 in. Calculate the Darcy friction factor and pressure loss due to friction in a mile of pipe length, using the Colebrook-White equation.

Solution The inside diameter of an NPS 16 (0.250-in wall thickness) pipe is

$$16.00 - 2 \times 0.250 = 15.50 \text{ in}$$

Next we will calculate the Reynolds number R to determine the flow regime (laminar or turbulent). The Reynolds number from Eq. (8.18) is

$$R = 3162.5 \times \frac{3000}{15.5 \times 0.3} = 2{,}040{,}323$$

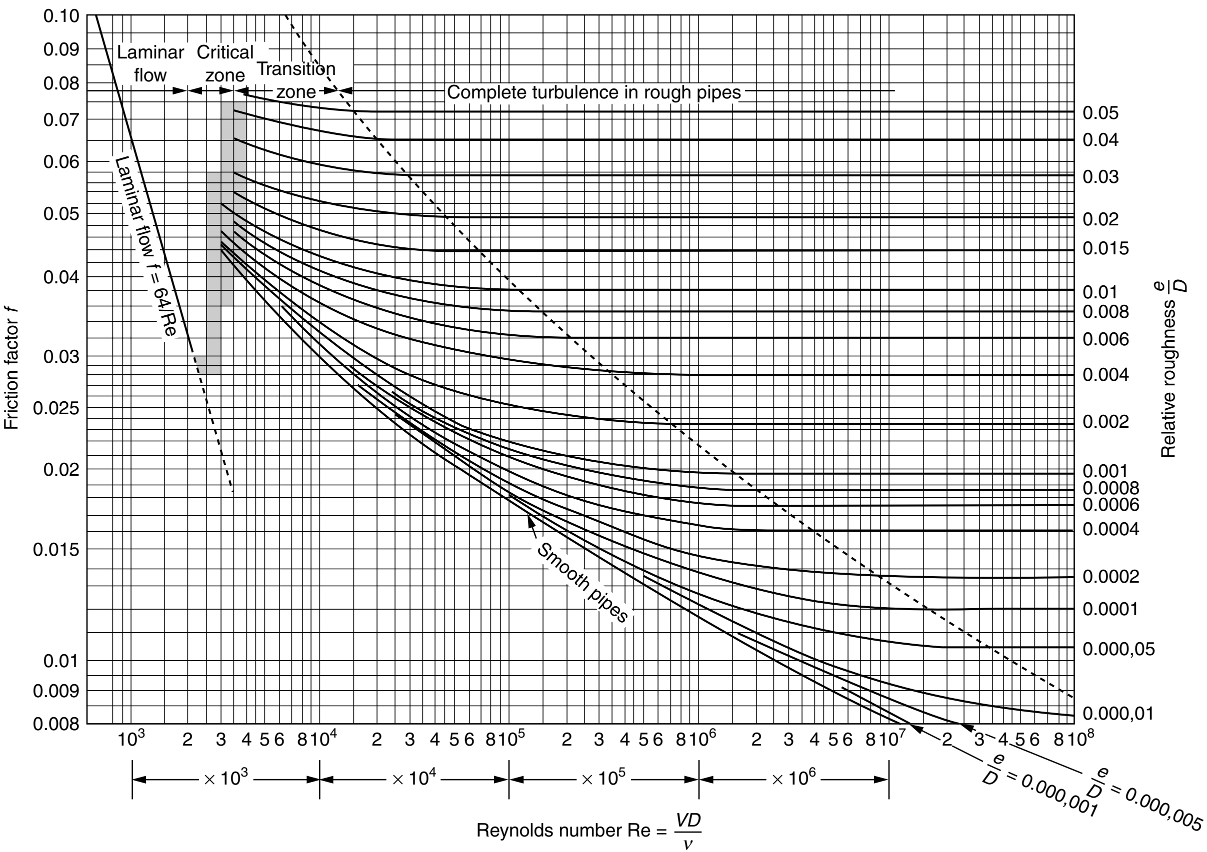


Figure 8.2 Moody diagram.

Since $R > 4000$, the flow is turbulent and we can use the Colebrook-White equation to calculate the friction factor. We can also use the Moody diagram to read the friction factor based on R and the pipe relative roughness e/D. Using Eq. (8.28), the friction factor is

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.5} + \frac{2.51}{2{,}040{,}323\sqrt{f}}\right)$$

Solving by trial and error, we get the Darcy friction factor

$$f = 0.0133$$

Next calculate the pressure drop due to friction using the Darcy equation (8.25) as follows:

$$P_m = \frac{71.16 \times 0.0133 \times (3000)^2 \times 0.5}{15.5^5}$$
$$= 4.76 \text{ psi/mi}$$

Therefore, pressure loss due to friction in a mile of pipe is 4.76 psi/mi.

Example 8.12 A DN 500 (10-mm wall thickness) steel pipe is used to transport LPG from a refinery to a storage tank 15 km away. Neglecting any difference in elevations, calculate the friction factor and pressure loss due to friction (kPa/km) at a flow rate of 990 m^3/h. Assume an internal pipe roughness of 0.05 mm. A delivery pressure of 1800 kPa must be maintained at the delivery point, and the storage tank is at an elevation of 200 m above that of the refinery. Calculate the pump pressure required at the refinery to transport the given volume of LPG to the storage tank location. Specific gravity of LPG = 0.5 and viscosity = 0.3 cSt.

Solution The pipe designated as DN 500 and 10-mm wall thickness has an inside diameter of

$$D = 500 - 2 \times 10 = 480 \text{ mm}$$

First calculate the Reynolds number from Eq. (8.15):

$$R = 353{,}678 \frac{Q}{\nu D}$$
$$= 353{,}678 \times \frac{990}{0.3 \times 480} = 2{,}431{,}536$$

Therefore, the flow is turbulent, and we can use the Colebrook-White equation or the Moody diagram to determine the friction factor.

$$\text{Relative roughness } \frac{e}{D} = \frac{0.05}{480} = 0.0001$$

Using the determined values for relative roughness and the Reynolds number, from the Moody diagram we get

$$f = 0.0128$$

The pressure drop due to friction can now be calculated using the Darcy equation (8.27):

$$P_{\text{km}} = (6.2475 \times 10^{10}) \frac{0.0128 \times 990^2 \times 0.5}{480^5}$$
$$= 15.38 \text{ kPa/km}$$

The pressure required at the pumping facility is calculated by adding the pressure drop due to friction, the delivery pressure required, and the static elevation head between the pumping facility and storage tank, all expressed in same unit of pressure.

Pressure drop due to friction in 15 km of pipe = $15 \times 15.38 = 230.7$ kPa

The static head difference is 200 m. This is converted to pressure in kilopascals. Using Eq. (8.10),

$$\text{Pressure due to elevation head} = \frac{200 \times 0.5}{0.102} = 980.39 \text{ kPa}$$

Minimum pressure required at delivery point = 1800 kPa

Therefore, adding all three numbers, the total pressure required at the refinery is

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}}$$

where P_t = total pressure required at refinery pump
P_f = frictional pressure drop
P_{elev} = pressure head due to elevation difference
P_{del} = delivery pressure at storage tank at destination

Therefore

$$P_t = 230.7 + 980.39 + 1800.0 = 3011.1 \text{ kPa}$$

Therefore, the pump pressure required at the refinery is 3011 kPa.

8.9.8 Minor losses

So far, we have calculated the pressure drop per unit length in straight pipe. We also calculated the total pressure drop considering several miles of pipe from a pump station to a storage tank. Minor losses in an LPG pipeline are classified as those pressure drops that are associated with piping components such as valves and fittings. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. A pipe nozzle exiting from a storage tank will have entrance and exit losses. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe. Generally, minor losses are included in calculations by using the equivalent length of the valve or fitting or using a resistance factor or K factor multiplied by the velocity head $V^2/2g$ discussed earlier. The term minor losses can be applied only where the pipeline lengths and hence the friction losses are relatively large compared to the pressure drops in the fittings and valves. In a situation such as plant piping and tank farm piping the pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses is really a misnomer. Regardless, the pressure losses through valves, fittings, etc., can be accounted for approximately using the equivalent length or K times the velocity head method.

8.9.9 Valves and fittings

Table 8.5 shows the equivalent length ratios of commonly used valves and fittings in a petroleum pipeline system. It can be seen from this table that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore, a 20-in-diameter gate valve may be replaced with a $20 \times 8 = 160$ in long piece of pipe that will match the frictional pressure drop through the valve.

Example 8.13 A piping system is 2000 ft of NPS 20 pipe that has two 20-in gate valves, three 20-in ball valves, one swing check valve, and four 90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe and valves and fittings.

Solution Using Table 8.5 for equivalent length ratios, we can convert all valves and fittings in terms of 20-in pipe as follows:

Two 20-in gate valves $= 2 \times 20 \times 8 = 320$ in of 20-in pipe
Three 20-in ball valves $= 3 \times 20 \times 3 = 180$ in of 20-in pipe
One 20-in swing check valve $= 1 \times 20 \times 50 = 1000$ in of 20-in pipe
Four 90° elbows $= 4 \times 20 \times 30 = 2400$ in of 20-in pipe
Total for all valves
and fittings $= 3900$ in of 20-in pipe $= 325$ ft of 20-in pipe

Adding the 2000 ft of straight pipe, the total equivalent length of straight pipe and all fittings $= 2000 + 325 = 2325$ ft.

The pressure drop due to friction in the preceding piping system can now be calculated based on 2325 ft of pipe. It can be seen in this example the valves and fittings represent roughly 14 percent of the total pipeline length. In plant piping this percentage may be higher than that in a long-distance petroleum pipeline. Hence, the reason for the term minor losses.

Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation (8.22), the pressure drop in a straight length of pipe is given by

$$h = f\frac{L}{D}\frac{V^2}{2g}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K (also known as the resistance coefficient) and Eq. (8.28) then becomes

$$h = K\,\frac{V^2}{2g} \tag{8.29}$$

In Eq. (8.29), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $V^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(V^2/2g)$, where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 8.12. It can be seen that the K factor depends on the nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

From the K factor table it can be seen that a 6-in gate valve has a K factor value of 0.12, while a 20-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be 0.12 $(V^2/2g)$ and that in the 20-in valve is 0.10 $(V^2/2g)$. The velocities in both cases will be different due to the difference in diameters.

If the flow rate was 1000 gal/min, the velocity in the 6-in valve will be approximately

$$V_6 = 0.4085\,\frac{1000}{6.125^2} = 10.89 \text{ ft/s}$$

Similarly, at 1000 gal/min, the velocity in the 20-in valve will be approximately

$$V_6 = 0.4085\,\frac{1000}{19.5^2} = 1.07 \text{ ft/s}$$

TABLE 8.12 Friction Loss in Valves—Resistance Coefficient *K*

		Nominal pipe size, in											
Description	L/D	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72

Therefore

$$\text{Head loss in 6-in gate valve} = \frac{0.12\,(10.89)^2}{64.4} = 0.22\ \text{ft}$$

$$\text{Head loss in 20-in gate valve} = \frac{0.10\,(1.07)^2}{64.4} = 0.002\ \text{ft}$$

These head losses appear small since we have used a relatively low flow rate in the 20-in valve. In reality the flow rate in the 20-in valve may be as high as 6000 gal/min and the corresponding head loss will be 0.072 ft.

8.9.10 Pipe enlargement and reduction

Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(V_1 - V_2)^2}{2g} \tag{8.30}$$

where V_1 and V_2 are the velocities of the liquid in the two pipe sizes D_1 and D_2 and h_f is the head loss in feet of liquid. Writing the above in terms of pipe cross-sectional areas A_1 and A_2, we get for sudden enlargement:

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{{V_1}^2}{2g} \tag{8.31}$$

This is illustrated in Fig. 8.3.

For sudden contraction or reduction in pipe size as shown in Fig. 8.3 the head loss is calculated from

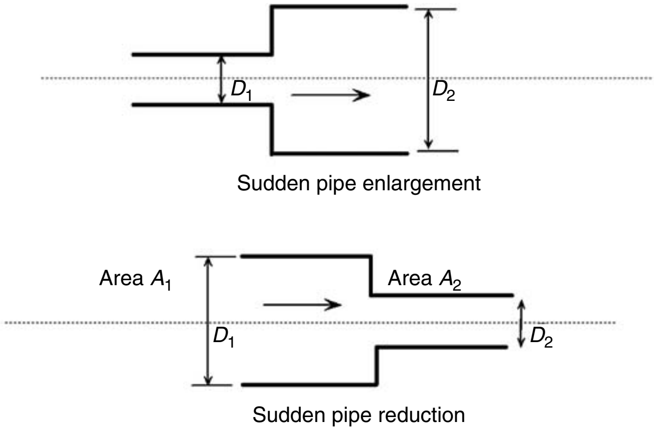
$$h_f = \left(\frac{1}{C_c} - 1\right) \frac{{V_2}^2}{2g} \tag{8.32}$$

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 8.3.

Gradual enlargement and reduction of pipe size, as shown in Fig. 8.4, cause less head loss than sudden enlargement and sudden reduction. For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c\,(V_1 - V_2)^2}{2g} \tag{8.33}$$

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is shown in Fig. 8.5.



A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 8.3 Sudden pipe enlargement and pipe reduction.

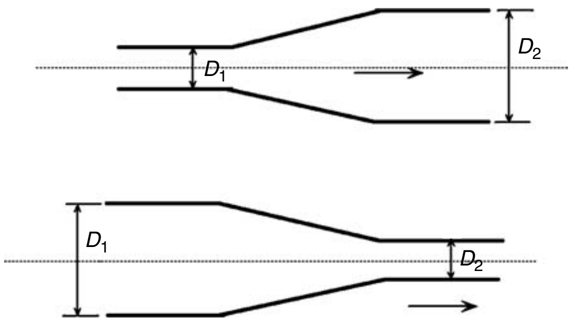


Figure 8.4 Gradual pipe enlargement and pipe reduction.

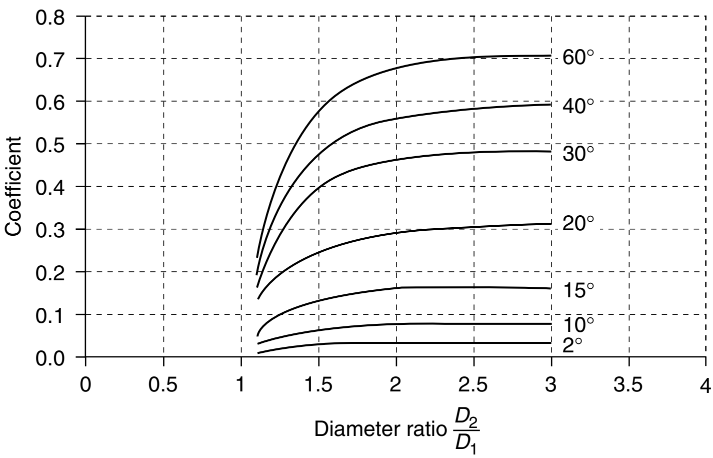


Figure 8.5 Gradual pipe expansion head loss coefficient.

8.9.11 Pipe entrance and exit losses

The K factors for computing the head loss associated with the pipe entrance and exit are as follows:

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

8.9.12 Total pressure required

So far we have examined the frictional pressure drop in an LPG pipeline consisting of pipe, valves, fittings, etc. We also calculated the total pressure required to pump LPG through a pipeline up to a delivery station at an elevated point. The total pressure required at the beginning of a pipeline, for a specified flow rate consists of three distinct components:

1. Frictional pressure drop
2. Elevation head
3. Delivery pressure

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \tag{8.34}$$

The first item is simply the total frictional head loss in all straight pipe, fittings, valves, etc. The second item accounts for the pipeline elevation difference between the origin of the pipeline and the delivery terminus. If the origin of the pipeline is at a lower elevation than that of the pipeline terminus or delivery point, a certain amount of positive pressure is required to compensate for the elevation difference. On the other hand if the delivery point were at a lower elevation than the beginning of the pipeline, gravity will assist the flow and the pressure required at the beginning of the pipeline will be reduced by this elevation difference. The third component, delivery pressure at the terminus, simply ensures that a certain minimum pressure is maintained at the delivery point, such as a storage tank. In addition due to the high vapor pressure of LPG compared to other petroleum liquids, we must also make sure that the pressure in the pipeline at any point does not drop below the vapor pressure of LPG. In a pipeline with drastic elevation changes at high points the pipeline pressure must be maintained above LPG vapor pressure. An example will be used to illustrate this.

Suppose an LPG pipeline requires 800 psi to compensate for frictional losses and the minimum delivery pressure required is 300 psi,

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the total pressure required at the beginning of the pipeline is calculated as follows. If there were no elevation difference between the beginning of the pipeline and the delivery point, the elevation head (component 2) is zero. Therefore, the total pressure P_t required is

$$P_t = 800 + 0 + 300 = 1100 \text{ psi}$$

Next consider elevation changes. If the elevation at the beginning is 100 ft, the elevation at the delivery point is 600 ft, and the specific gravity of LPG is 0.5,

$$P_t = 800 + \frac{(600 - 100) \times 0.5}{2.31} + 300 = 1208.23 \text{ psi}$$

The middle term in this equation represents the static elevation head difference converted to psi. Finally, if the elevation at the beginning is 600 ft and the elevation at the delivery point is 100 ft, then

$$P_t = 800 + \frac{(100 - 600) \times 0.5}{2.31} + 300 = 991.77 \text{ psi}$$

It can be seen from the preceding that the 500-ft advantage in elevation in the final case reduces the total pressure required by approximately 108.23 psi compared to the situation where there was no elevation difference between the beginning of the pipeline and delivery point (1100 psi versus 991.77 psi).

8.9.13 Effect of elevation

The preceding discussion illustrated an LPG pipeline that had a flat elevation profile compared to an uphill pipeline and a downhill pipeline. There are situations, where the ground elevation may have drastic peaks and valleys that require careful consideration of the pipeline topography. In some instances, the total pressure required to transport a given volume of liquid through a long pipeline may depend more on the ground elevation profile than on the actual frictional pressure drop. In the preceding we calculated the total pressure required for a flat pipeline as 1100 psi and that for an uphill pipeline to be 1208.23 psi. In the uphill case the static elevation difference contributed to 9 percent of the total pressure required. Thus the frictional component was much higher than the elevation component. We will examine a case where the elevation differences in a long pipeline dictate the total pressure required more than the frictional head loss.

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8.9.14 Pump stations required

In a long pipeline the pressure required at the beginning for pumping a certain volume may exceed the maximum allowable operating pressure (MAOP) of the pipeline. Therefore, the necessary pressure may have to be provided in stages at two or more pump stations. For example, consider a 500-mi pipeline pumping LPG from a refinery to a storage site. The pressure required at the delivery point is 300 psi and the MAOP of the pipeline is limited to 1400 psi. Suppose calculations show that taking into account friction losses and elevation difference and the minimum delivery pressure required, the pressure required at the beginning of the pipeline is 3600 psi at a certain flow rate. Since pipe pressure is limited to 1400 psi, we need to provide the required 3600 psi by installing two intermediate pump stations in addition to the pump station at the origin. The first pump station will operate at 1400 psi and by the time the LPG arrives at the second pump station its pressure would have dropped to the minimum required pressure of 300 psi (to prevent vaporization of LPG). At this second pump station the LPG pressure is boosted to 1400 psi which then drops to 300 psi at the third pump station. Finally, the LPG is boosted to 1400 psi at the third station for eventual delivery at the required pressure of 300 psi at the storage site.

In general the equation for calculating the approximate number of pump stations based upon total pressure required, MAOP, and minimum delivery pressure is as follows:

$$n = \frac{P_t - P_s}{\text{MAOP} - P_s} \tag{8.35}$$

where n = number of pump stations required
P_t = total pressure required calculated from Eq. (8.34), psi
P_s = minimum suction pressure required at each pump station, psi
MAOP = maximum allowable operating pressure of pipe, psi

The calculated value of n from Eq. (8.35) is rounded up to the nearest whole number. It must be noted that the preceding analysis assumes that the entire pipeline has the same MAOP and the same minimum suction pressure at all pump stations.

Using the example discussed earlier, we have

$$P_t = 3600 \text{ psi}$$

$$P_s = 300 \text{ psi}$$

$$\text{MAOP} = 1400 \text{ psi}$$

Therefore, the number of pump stations required from Eq. (8.35) is

$$n = \frac{3600 - 300}{1400 - 300} = 3$$

Thus, three pump stations are required.

If the total pressure required had been 3400 psi, everything else remaining the same, the number of pump stations required from Eq. (8.35) would be

$$n = \frac{3400 - 300}{1400 - 300} = 2.82 \quad \text{or} \quad 3 \text{ pump stations}$$

With three pump stations, the adjusted discharge pressure at each station becomes

$$\text{Discharge pressure} = \frac{3400 - 300}{3} + 300 = 1333.33 \text{ psi}$$

Example 8.14 A 20-in (0.375-in wall thickness) LPG pipeline 500 mi long has a ground elevation profile as shown in Fig. 8.6. The elevation at Corona is 600 ft and at Red Mesa is 2350 ft.

(a) Calculate the total pressure required at the Corona pump station to transport 200,000 bbl/day of LPG (specific gravity = 0.5 and viscosity = 0.3 cSt) to Red Mesa storage tanks, with a minimum delivery pressure of 300 psi at Red Mesa. Use the Colebrook equation for friction factor calculation.

(b) If the pipeline operating pressure cannot exceed 1400 psi, how many pumping stations, besides Corona will be required to transport the above flow rate? Use a pipe roughness of 0.002 in.

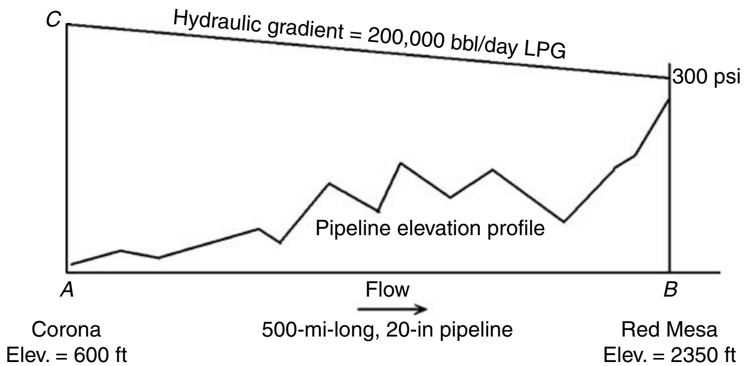

Figure 8.6 Corona to Red Mesa pipeline.

Solution

(a) First, calculate the Reynolds number from Eq. (8.20):

$$R = 92.24 \times \frac{200{,}000}{19.25 \times 0.3} = 3{,}194{,}459$$

Therefore the flow is turbulent.

$$\text{Relative pipe roughness} = \frac{e}{D} = \frac{0.002}{19.25} = 0.0001$$

Next, calculate the friction factor f using the Colebrook equation (8.28):

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.0001}{3.7} + \frac{2.51}{3{,}194{,}459\sqrt{f}}\right)$$

Solving for f by trial and error, $f = 0.0125$. We can now find the pressure loss due to friction using Eq. (8.25) as follows:

$$P_m = 0.0605 \times \frac{0.0125 \times (200{,}000)^2 \times 0.5}{(19.25)^5}$$
$$= 5.72 \text{ psi/mi}$$

The total pressure required at Corona is calculated by adding the pressure drop due to friction to the delivery pressure required at Red Mesa and the static elevation head between Corona and Red Mesa.

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (8.34)}$$
$$P_t = (5.72 \times 500) + \frac{(2350 - 600) \times 0.5}{2.31} + 300$$
$$= 2860 + 378.79 + 300 = 3539 \text{ psi}$$

Since a total pressure of 3539 psi at Corona far exceeds the maximum operating pressure of 1400 psi, it is clear that we need additional intermediate booster pump stations besides Corona.

(b) The approximate number of pump stations required without exceeding the pipeline pressure of 1400 psi according to Eq. (8.35) is

$$\text{Number of pump stations} = \frac{3539 - 300}{1400 - 300} = 2.95 \quad \text{or} \quad 3 \text{ pump stations}$$

Therefore, we will need two additional booster pump stations besides Corona. With three pump stations the average discharge pressure per pump station will be

$$\text{Average pump station discharge pressure} = \frac{3539 - 300}{3 + 300} = 1380 \text{ psi}$$

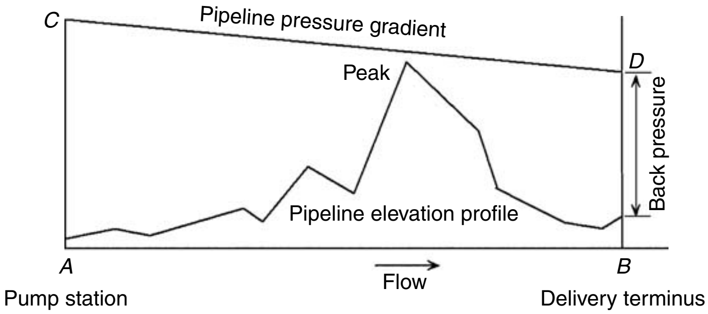

Figure 8.7 Tight line operation.

8.9.15 Tight line operation

When there are drastic elevation differences in a long pipeline, sometimes the last section of the pipeline toward the delivery terminus may operate in an open-channel flow. This means that the pipeline section will not be full of liquid and there will be a vapor space above the liquid. Such situations are acceptable in ordinary petroleum liquid (gasoline, diesel, and crude oils) pipelines compared to high vapor pressure liquids such as LPG. In LPG pipelines the pressure cannot be allowed to fall below the vapor pressure of LPG. Hence slack line conditions or open-channel flow conditions cannot be allowed. We must therefore pack the line by providing adequate back pressure at the delivery terminus as illustrated in Fig. 8.7.

8.9.16 Hydraulic gradient

The graphical representation of the pressures along the pipeline as shown in Fig. 8.8 is the hydraulic gradient. Since elevation is measured in feet, the pipeline pressures are converted to feet of head of LPG and plotted against the distance along the pipeline, superimposed on the

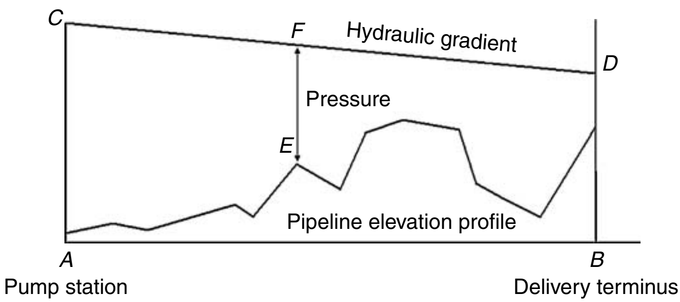

Figure 8.8 Hydraulic gradient.

elevation profile. If we assume a beginning elevation of 100 ft, a delivery terminus elevation of 500 ft, a total pressure of 1000 psi required at the beginning, and a delivery pressure of 250 psi at the terminus, we can plot the hydraulic pressure gradient graphically by the following method.

At the beginning of the pipeline the point C representing the total pressure will be plotted at a height of

$$100\ \text{ft} + \frac{1000 \times 2.31}{0.5} = 4720\ \text{ft}$$

where the liquid specific gravity = 0.5 has been assumed. Similarly, at the delivery terminus the point D representing the total head at delivery will be plotted at a height of

$$500 + \frac{250 \times 2.31}{0.5} = 1655\ \text{ft}$$

The line connecting points C and D represents the variation of the total head in the pipeline and is termed the hydraulic gradient. At any intermediate point such as E along the pipeline the pipeline pressure will be the difference between the total head represented by point F on the hydraulic gradient and the actual elevation of the pipeline at E.

If the total head at F is 2500 ft and the pipeline elevation at E is 250 ft, the actual pipeline pressure at E is

$$(2500 - 250)\ \text{ft} = \frac{2250 \times 0.5}{2.31} = 487\ \text{psi}$$

It can be seen that the hydraulic gradient clears all peaks along the pipeline. If the elevation at E were 3000 ft, we would have a negative pressure in the pipeline at E equivalent to

$$(2500 - 3000)\ \text{ft} \quad \text{or} \quad -500\ \text{ft} = \frac{-500 \times 0.5}{2.31} = -108\ \text{psi}$$

A negative pressure is not acceptable for LPG, and the minimum pressure anywhere in the pipeline must be higher than the vapor pressure of LPG. Otherwise vaporization of LPG will occur.

Therefore, the total pressure at the beginning of the pipeline will have to be higher by 108 psi, and the vapor pressure of LPG will have to be at the flowing temperature. If the latter is taken as 250 psig, the revised pressure at A becomes

$$\text{Revised pressure at } A = 1000 + 108 + 250 = 1358\ \text{psi}$$

Correspondingly,

$$\text{Revised total head at } A = \frac{1358 \times 2.31}{0.5} + 100 = 6374 \text{ ft}$$

and the revised total head at F becomes

$$2500 + \frac{(108 + 250) \times 2.31}{0.5} = 4154 \text{ ft}$$

Calculating the revised pressure at peak E, we get

$$\text{Pressure at peak } E = (4154 - 3000) \text{ ft} \quad \text{or} \quad 1154 \text{ ft} = \frac{1154 \times 0.5}{2.31}$$
$$= 250 \text{ psi}$$

which is the minimum pressure required for LPG, and therefore the pressures are fine.

8.9.17 Pumping horsepower

In the previous sections we calculated the total pressure required at the beginning of the pipeline to transport a given volume of LPG over a certain distance. We will now calculate the pumping horsepower (HP) required to accomplish this.

The water horsepower (WHP), also known as the hydraulic horsepower (HHP), based on 100 percent pump efficiency, is calculated from the following equation:

$$\text{WHP} = \frac{\text{ft of head} \times \text{gal/min} \times \text{liquid specific gravity}}{3960} \tag{8.36}$$

The brake horsepower (BHP) of a pump takes into account the pump efficiency and is calculated as follows:

$$\text{BHP} = \frac{\text{ft of head} \times \text{gal/min} \times \text{liquid specific gravity}}{3960 \times \text{effy}} \tag{8.36a}$$

where effy is the pump efficiency expressed as a decimal value.

In SI units, the pumping power is expressed in kW. If pressures are in kPa and the liquid flow rate is in m^3/h, the pumping power required is calculated from the following:

$$\text{Power in kW} = \frac{\text{pressure in kPa} \times \text{flow rate in m}^3\text{/h}}{3600}$$

Therefore, the power equation for pumping a liquid [Eq. (8.36a)] can be modified for SI units as follows:

$$\text{Power} = \frac{(P_d - P_s) \times Q}{3600 \times \text{effy}} \tag{8.36b}$$

where Power = pump power required, kW
P_d = pump discharge pressure, kPa
P_s = pump suction pressure, kPa
Q = liquid flow rate, m^3/h
effy = pump efficiency, decimal value

Consider Example 8.14 in which we calculated the total pressure required to pump 200,000 bbl/day of LPG from Corona to Red Mesa through a 500-mi-long, 20-in pipeline. We calculated the total pressure required to be 3539 psi. Since the maximum allowable working pressure in the pipeline was limited to 1400 psi, we concluded that two additional pump stations besides Corona were required. With a total of three pump stations, each pump station would be discharging at a pressure of approximately 1380 psi.

At the Corona pump station LPG would enter the pump at some minimum suction pressure, say 300 psi, and the pumps would boost the pressure to the required discharge pressure of 1380 psi. Effectively, the pumps would add the energy equivalent of (1380 − 300) or 1080 psi at a flow rate of 200,000 bbl/day (5,833.33 gal/min). The water horsepower (WHP) required is calculated as follows:

$$\text{WHP} = \left(1080 \times \frac{2.31}{0.5}\right) \times \frac{5833.33 \times 0.5}{3960} = 3675 \text{ HP}$$

Assuming a pump efficiency of 80 percent, the pump brake horsepower (BHP) required at Corona is

$$\text{BHP} = \frac{3675}{0.8} = 4594 \text{ HP}$$

If the pump is driven by an electric motor with a motor efficiency of 95 percent, the drive motor HP required will be

$$\text{Motor HP} = \frac{4594}{0.95} = 4836 \text{ HP}$$

The nearest standard size motor of 5000 HP would be adequate for this application. Of course, this assumes that the entire pumping requirement at the Corona pump station is handled by a single pump-motor unit. In reality, to provide for operational flexibility and maintenance two or more pumps will be configured in series or parallel to provide the necessary pressure at the specified flow rate. Let us assume that two pumps are configured in parallel to provide the necessary head pressure of 1080 psi (4990 ft of LPG) at the Corona pump station. Each pump will be designed for one-half the total flow rate, or 2917 gal/min, and a pressure of 4990 ft. If each pump selected had an efficiency of 80 percent, we can calculate the BHP required for each pump as follows:

$$\text{BHP} = \frac{4990 \times 2917 \times 0.5}{3960 \times 0.80} \qquad \text{from Eq. } (8.36a)$$
$$= 2298 \text{ HP}$$

Alternatively, if the pumps are configured in series instead of parallel, each pump will be designed for the full flow rate of 5833.33 gal/min but at half the total head required, or 2495 ft. The BHP required per pump will still be the same as for the parallel configuration. Pumps are discussed in more detail in Chap. 6.

8.10 LPG Storage

LPG is usually stored as a liquid in steel storage tanks. These tanks may be aboveground or belowground. Underground tanks have the advantage of a constant temperature of LPG in the tank and therefore minimal vaporization. Aboveground tanks are less expensive to install, but the LPG will be subject to temperature fluctuations and therefore different evaporation rates. These tanks are designed in accordance with the ASME Boiler and Pressure Vessel Code, Section VIII. The vapor pressure that is developed in the tanks depends upon the outside air temperature. For example, 100 percent propane at 50°F has a vapor pressure of approximately 80 psig. When the temperature increases to 80°F the vapor pressure becomes 150 psig. On the other hand 100 percent butane has a vapor pressure of 7 psig at 50°F and increases to 24 psig at 80°F.

Commercial LPG, being a mixture of propane and butane, will have vapor pressures between the values for propane and butane just given. LPG tank capacities range from 6000 to 30,000 gal, and the tanks weigh between 11,000 and 50,000 lb. Smaller standard size tanks are available in a capacity range from 120 to 1000 gal. LPG cylinders are manufactured in capacities from 1 to 420 lb. Underground tanks must be protected from potential traffic loads by installing them at a depth of at least 2 ft below the ground surface. If LPG tanks are located in remote areas and no traffic or potential for damage from construction equipment is anticipated, the tank burial depth can be reduced to as low as 6 in. Before filling a storage tank with LPG, the tank must be completely purged of any water and air, usually with an inert gas, such as nitrogen. The maximum allowable amount of air is limited to 6 percent.

Pressure regulators and pressure relief valves are installed on the LPG tanks to reduce pressure to that required in fuel distribution piping and to protect piping from excessive pressures.

8.11 LPG Tank and Pipe Sizing

The size of the LPG tank is determined by the demand (in ft^3/h) for the fuel. The vaporization rate of propane determines the amount of fuel available from a particular size tank at a certain ambient temperature. The tank must be large enough to provide the vaporization rate when the ambient temperature is minimum. The rate of vaporization can be calculated considering the wetted area of LPG in the tank. The following formula can be used to calculate the vaporization rate for an aboveground tank based on the ambient temperature and the temperature of LPG in the tank.

$$Q = U \times A \times \Delta T \tag{8.37}$$

where Q = heat transfer rate to vaporize a given quantity of LPG, Btu/h
U = overall heat transfer coefficient for the tank, Btu/(h · ft^2 · °F)
A = wetted surface area of the aboveground tank, ft^2
ΔT = temperature difference between ambient air and LPG temperature in tank

For a belowground tank, A may be taken as the entire surface area of the tank.

Generally, the difference between the coldest outside temperature and the warmest LPG temperature is used to calculate ΔT. Depending upon the relative humidity of the air, frost formation may occur on the outside of the tank. Frost must be avoided since it acts as an insulation and therefore inhibits the vaporization of the LPG. Table 8.13 shows the temperature difference to be used at different humidity levels.

For example, from the table when the relative humidity is 50 percent and the outside temperature is 40°F, ΔT equals 16.5. For aboveground tanks a value of $U = 2.0$ may be used. For underground tanks $U = 0.5$

TABLE 8.13 Temperature Difference and Relative Humidity

Air temperature		Relative humidity							
°C	°F	20	30	40	50	60	70	80	90
−34.4	−30.0					8.0	5.0	2.5	1.0
−28.9	−20.0		20.0	15.0	11.5	8.5	5.0	3.0	1.5
−23.3	−10.0	27.5	20.5	16.0	12.0	9.0	6.0	3.0	1.5
−17.8	0.0	29.0	21.5	16.5	12.5	9.0	6.0	4.0	2.0
−12.2	10.0	30.0	22.5	17.0	13.0	9.5	6.5	4.0	2.0
−6.7	20.0	31.5	24.0	18.0	14.0	10.0	7.0	4.0	2.0
−1.1	30.0	33.0	25.0	19.5	15.0	11.0	8.0	5.0	3.0
4.4	40.0	35.0	27.0	21.0	16.5	12.0	9.0	8.0	8.0

TABLE 8.14 Latent Heat of Vaporization of Propane

Ambient air temperature		Propane	
°C	°F	Btu/lb	Btu/gal
−40.0	−40.0	180.8	765
−34.4	−30.0	178.7	755
−28.9	−20.0	176.2	745
−23.3	−10.0	173.9	735
−17.8	0.0	171.5	725
−12.2	10.0	169.0	715
−6.7	20.0	166.3	704
−1.1	30.0	163.4	691
4.4	40.0	160.3	678
10.0	50.0	156.5	662
15.6	60.0	152.6	645

is used. After calculating the vaporization rate Q using Eq. (8.37), we can calculate the quantity of LPG vaporized in gal/h as follows:

$$V = \frac{Q}{L} \tag{8.38}$$

where V = volume of LPG vaporized, gal/h
Q = heat transfer rate to vaporize a given quantity of LPG, Btu/h
L = latent heat of vaporization of propane, Btu/gal

The latent heat of vaporization for propane is listed in Table 8.14 for various ambient temperatures.

Example 8.15 An aboveground LPG storage tank is installed at a location where the relative humidity is 70 percent and the lowest expected ambient temperature is 40°F. The continuous demand for LPG is at the rate of 150,000 Btu/h. Calculate the vaporization rate required in gal/h and the minimum surface area of tank required.

Solution Since the LPG requirement is 150,000 Btu/h, we will determine the flow rate out of the tank in gal/h as follows. From Table 8.2 the heat content of LPG is 91,547 Btu/gal. Then

$$\text{LPG vaporization rate} = \frac{150{,}000 \text{ Btu/h}}{91{,}547 \text{ Btu/gal}} = 1.6385 \text{ gal/h}$$

Also from Table 8.14, the latent heat of vaporization at 40°F is 160.3 Btu/lb or 678 Btu/gal. Therefore the heat transfer rate to vaporize 1.6385 gal/h of LPG at 40°F, using Eq. (8.38), is

$$1.6385 = \frac{Q}{678}$$

Solving for heat transfer Q,

$$Q = 1.6385 \times 678 = 1111 \text{ Btu/h}$$

From Table 8.14 at a relative humidity of 70 percent and an ambient temperature of 40°F, the temperature difference ΔT for heat transfer is 9°F. Therefore, the minimum tank area required to vaporize LPG at this rate, using Eq. (8.37), is

$$1111 = 0.2 \times A \times 9$$

Solving for A, we get

$$A = 617.22 \text{ ft}^2$$

This is the minimum wetted surface area of the aboveground tank required to vaporize LPG and provide the required demand of 150,000 Btu/h at an LPG flow rate of 1.6385 gal/h.

Finally, from the manufacturer's catalog we can select a tank that will provide the minimum wetted area previously calculated for the minimum level of LPG in the tank.

When LPG is supplied as a fuel gas through distribution piping, the pressures are limited to that allowed by the code for fuel gas distribution piping. It was mentioned earlier that an NG fuel distribution piping system is limited to 5 psig and a LPG piping system is limited to 20 psig. The 20-psig limitation for LPG fuel gas distribution piping is allowed only if the building containing the LPG distribution piping is constructed in compliance with NFPA 58 fuel gas code and the buildings are used exclusively for industrial applications or laboratories. In all other instances LPG distribution piping is limited to 5 psig as with NG fuel gas distribution piping.

For low-pressure LPG distribution piping we can use the same methods for determining the pipe size and capacity as with NG pipe sizing. Therefore, the Spitzglass equation (less than or equal to 1 psi) and the Weymouth equation (greater than 1.0 psi) can be used. The NG piping capacity tables (Tables 8.7 through 8.10) may also be used for an LPG distribution piping system provided adjustments are made to the capacities to account for the difference in specific gravities between LPG vapor and natural gas. The table values are based on NG with a specific gravity of 0.60 (air = 1.0), whereas LPG vapor has a specific gravity of approximately 1.52 (air = 1.0). Since the capacity is inversely proportional to the square root of the specific gravity from Eqs. (8.1) and (8.5), the multiplication factor for the capacity from Tables 8.7 through 8.10 is

$$\text{Multiplication factor} = \left(\frac{0.6}{1.52}\right)^{0.5} = 0.6283$$

Sometimes propane is mixed with air in varying proportions to use in place of NG. One such mixture has a specific gravity of 1.30 and a heating value of 1450 Btu/ft^3. In such a case the multiplication factor for capacity becomes

$$\text{Multiplication factor} = \left(\frac{0.6}{1.3}\right)^{0.5} = 0.6794$$

Example 8.16 Calculate the LPG capacity of fuel gas distribution piping consisting of NPS 4 pipe, with an inside diameter of 4.026 in and a total equivalent length of 150 ft. The inlet pressure is 1.0 psig. Consider a pressure drop of 0.6 in of water column and a specific gravity of gas = 1.52.

Solution Since this is low pressure, we will use the Spitzglass formula. First we will calculate the parameter K from Eq. (8.2):

$$K = \sqrt{\frac{4.026^5}{1 + (3.6/4.026) + (0.03 \times 4.026)}} = 22.91$$

and from Eq. (8.1), the capacity in ft^3/h is

$$Q_s = 3550 \times 22.91\sqrt{\frac{0.6}{1.52 \times 150}} = 4172 \text{ ft}^3/\text{h}$$

Thus the LPG capacity of the NPS 4 pipe is 4172 SCFH.

Example 8.17 Calculate the LPG capacity of a fuel gas distribution pipeline consisting of DN 100 (6-mm wall thickness) pipe with a total equivalent length of 50 m. The inlet pressure is 6 kPa. Consider a pressure drop of 0.5 kPa and a specific gravity of gas = 1.52.

Solution Since this is low pressure, we will use the Spitzglass formula. First we will calculate the parameter K from Eq. (8.4):

$$K = 3.075 \times 10^{-4}\sqrt{\frac{88^5}{1 + (91.44/88) + 0.001181 \times 88}}$$

$$= 15.26$$

$$\text{Pressure drop of 0.5 kPa} = 0.5 \times 0.145 = 0.0725 \text{ psi}$$

$$= 0.0725 \times 2.31 \times 12 = 2 \text{ in of water column}$$

$$= 2 \times 25.4 = 50.8 \text{ mm of water column}$$

and from Eq. (8.3), the capacity in m^3/h is

$$Q_s = 11.0128 \times 15.26\sqrt{\frac{50.8}{1.52 \times 50}} = 137.4 \text{ m}^3/\text{h}$$

Therefore, the LPG capacity of the DN 100 pipe is 137.4 m^3/h at standard conditions.

Example 8.18 An LPG fuel gas distribution pipeline is 210 ft of straight NPS 6 pipe with an inside diameter of 6.065 in and two NPS 6 elbows and two NPS 6 plug valves.

(a) Calculate the total equivalent length of all pipe valves and fittings.

(b) Consider an inlet pressure of 10.0 psig and calculate the total pressure drop at a flow rate of 50,000 SCFH. The specific gravity of the gas is 1.52.

Solution

(a) The total equivalent length will be calculated using Table 8.5 for valves and fittings:

$$\text{Two NPS 6 } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 6}{12} = 30 \text{ ft of NPS 6 pipe}$$

$$\text{Two NPS 6 plug valves} = \frac{2 \times 18 \times 6}{12} = 18 \text{ ft of NPS 6 pipe}$$

$$\text{Total for all valves and fittings} = 48 \text{ ft of NPS 6 pipe}$$

Adding the 210 ft of straight pipe, the total equivalent length of straight pipe and all fittings

$$L_e = 210 + 48 = 258 \text{ ft}$$

(b) Since this is not low pressure, we will use the Weymouth equation (8.5). First we will calculate the parameter K from Eq. (8.2):

$$K = \sqrt{\frac{6.065^5}{1 + (3.6/6.065) + (0.03 \times 6.065)}} = 67.99$$

The flow rate and pressure drop are related by Eq. (8.5):

$$50{,}000 = 3550 \times 67.99 \sqrt{\frac{10\,\Delta P}{1.52 \times 258}}$$

In the preceding we have used the inlet pressure as the average pressure since we need to calculate ΔP in order to determine the average pressure.

Solving for ΔP, we get

$$\Delta P = 1.68 \text{ psig}$$

With this pressure drop, the average pressure is

$$\frac{10 + 10 - 1.68}{2} = 9.16 \text{ psig}$$

Recalculating ΔP based on this average pressure, we get

$$50{,}000 = 3550 \times 67.99 \sqrt{\frac{8.995\,\Delta P}{1.52 \times 258}} = 1.87 \text{ psig}$$

The process is repeated until successive values of ΔP are within 0.1 psi. This is left as an exercise for the reader.

Example 8.19 An LPG fuel gas distribution pipeline is 50 m of straight DN 150 (6-mm wall thickness) pipe. The inlet pressure is 60 kPa and the flow rate is 180 L/s. The piping includes four DN 150 elbows and two DN 150 plug valves.

(a) Calculate the total equivalent length of all pipe valves and fittings.

(b) Calculate the pressure drop if the specific gravity of gas is 1.52.

(c) If the quantity of LPG required is increased to 250 L/s and the inlet pressure remains the same, what pipe size is required to limit the pressure drop to 10 percent of the inlet pressure in a total equivalent length of 110 m of piping?

Solution

(a) The total equivalent length will be calculated using Table 8.5 for valves and fittings:

$$\text{Four DN 150 } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 150}{1000} = 18.00 \text{ m of DN 150 pipe}$$

$$\text{Two DN 150 plug valves} = \frac{2 \times 18 \times 150}{1000} = 5.4 \text{ m of DN 150 pipe}$$

$$\text{Total for all valves and fittings} = 23.4 \text{ m of DN 150 pipe}$$

Adding the 50 m of straight pipe, the total equivalent length of straight pipe, valves, and fittings is

$$L_e = 50 + 23.4 = 73.4 \text{ m of DN 150 pipe}$$

(b) Since the pressure is higher than 6.9 kPa, the Weymouth formula will be used. First we calculate the value of the parameter K using Eq. (8.4):

$$K = (3.075 \times 10^{-4})\sqrt{\frac{138^5}{1 + (91.44/138) + 0.001181 \times 138}} = 50.91$$

We will assume a 10 percent pressure drop and calculate the average pipeline pressure as

$$\text{Average pressure} = \frac{60 + 54}{2} = 57 \text{ kPa}$$

From Eq. (8.6),

$$\frac{180 \times 60 \times 60}{1000} = 8.0471 \times 50.91\sqrt{\frac{57\Delta P}{1.52 \times 73.4}}$$

Solving for ΔP, we get

$$\Delta P = 4.90 \text{ kPa}$$

This is almost 9 percent of the inlet pressure we assumed at the start.

(c) When the flow rate is increased from 180 to 250 L/s, keeping the pressure loss at 10 percent of the inlet pressure and increasing the equivalent length from 73.4 to 110 m, we will have to select a larger pipe size. Since calculation of the diameter from the Weymouth equation is not straightforward, we will assume a pipe size and check for the pressure drop to be within 10 percent of the inlet pressure.

Initially, choose DN 200 pipe with 6-mm wall thickness.

$$\text{Pipe inside diameter } d = 200 - 12 = 188 \text{ mm}$$

Next we calculate the value of the parameter K using Eq. (8.4):

$$K = (3.075 \times 10^{-4})\sqrt{\frac{188^5}{1 + (91.44/188) + 0.001181 \times 188}} = 114.01$$

We will assume a 10 percent pressure drop and calculate the average pipeline pressure as

$$\text{Average pressure} = \frac{60 + 54}{2} = 57 \text{ kPa}$$

From Eq. (8.6),

$$\frac{250 \times 60 \times 60}{1000} = 8.0471 \times 114.01\sqrt{\frac{57\,\Delta P}{1.52 \times 110}}$$

Solving for ΔP, we get

$$\Delta P = 2.83 \text{ kPa}$$

This is almost 5 percent of the inlet pressure and therefore is acceptable. Hence, the pipe size required for the increased flow rate is DN 200.

Chapter

9

Cryogenic and Refrigeration Systems Piping

Introduction

Cryogenic piping systems are those installations where the operating temperature is below 20°F. This limit is established on the basis of the embrittlement point of most carbon-steel materials. Many industrial gases such as oxygen, nitrogen, and argon are stored and transported in cryogenic containers and piping systems, since this is more efficient compared to storage in gaseous form that requires high pressures and therefore stronger vessels and pipes, which increases costs. Although cryogenic vessels do not have to withstand higher pressures, the low temperatures cause embrittlement problems, resulting in larger expansion and contraction of piping systems. These storage containers and piping are subject to larger temperature differentials which cause structural problems. Nevertheless, cryogenic piping and storage are preferred for many industrial gases since they are more efficient and more economical in the long run.

Refrigeration piping systems are used with refrigeration equipment to produce temperatures lower than normal for industrial and residential use. A refrigerant fluid is used to create the low temperature by absorbing heat from the surroundings and in the process it evaporates. The evaporated vapor is compressed and condensed by using a compressor in the system. The condensed liquid is then reduced in pressure through an expansion valve after which it enters the evaporator to start the cycle over again. Many volatile substances such as ammonia are used as refrigerants to produce the lower temperatures required.

Several halogenated hydrocarbons are also used as refrigerants. Ethylene glycol, propylene glycol, and brine are also used to produce lower temperatures as secondary coolants. These fluids do not change from the liquid to the vapor phase, however, as do other common refrigerants.

9.1 Codes and Standards

Cryogenic piping systems are designed and constructed in accordance with the ASME B31.3 Process Piping Code. This code presents methods to size pipe considering stresses due to internal pressure, weight of pipe, weight of liquid, and thermal expansion and contraction of piping. Piping material used for cryogenic piping systems must conform to ASTM specifications which list material to be used based on operating temperature and pressure.

Refrigeration piping is designed to the American Standard Safety Code for Mechanical Refrigeration. This standard is sponsored by the American Society of Heating, Refrigerating, and Air-Conditioning Engineers (ASHRAE). Many state, city, and local codes also regulate refrigeration piping, but most of these adopt the ASHRAE standards. This code is also referred to as ANSI/ASHRAE 15. The American National Standard Code for Pressure Piping, ASME B31.5, is also used in structural design, construction, and testing of refrigeration piping.

9.2 Cryogenic Fluids and Refrigerants

Various cryogenic fluids such as helium and hydrogen are used in industrial processes. Table 9.1 lists the properties of some common cryogenic fluids.

Enthalpy and entropy versus pressure and temperature charts are also used in conjunction with cryogenic piping calculations. One of the properties used for cryogenic piping calculations is the density, which is also the reciprocal of the specific volume. As an example, for nitrogen at a temperature of 200 K and a pressure of 0.1 MPa the density is 1.75 kg/m^3. When a cryogenic liquid flows through a throttle valve, flashing may occur. This flashing produces vapors resulting in two-phase flow. Two-phase flow results in a larger pressure drop compared to that of single-phase flow. Larger pressure drops require a larger pipe size, and hence two-phase flow must be avoided. As far as possible, cryogenic piping systems must be maintained in single-phase flow.

Refrigeration systems use secondary coolants and refrigerants. Brine and glycol solutions such as ethylene glycol and propylene glycol are secondary coolants. Refrigerants include ammonia and halogenated hydrocarbons. Table 9.2 lists commonly used refrigerants in refrigeration systems.

TABLE 9.1 Properties of Common Cryogenic Fluids

	Helium	Hydrogen normal	Nitrogen	Carbon monoxide	Air	Argon	Oxygen	Methane	R-14	Carbon dioxide	Propane	Ammonia
Formula	He	n-H_2	N_2	CO	Mixture	Ar	O_2	CH_4	CF_4	CO_2	C_3H_8	NH_3
Molecular weight	4	2.02	28.01	28.01	28.96	39.95	32	16.04	88.01	44.01	44.1	17.03
Triple point												
Temperature, K		13.95	63.15	68.15		83.81	54.36	90.68	89.52	216.58	85.47	195.41
Pressure, kPa		7.2	12.5	15.4		69.1	0.15	11.7	0.11	518	3.00E-07	6.1
Heat of fusion, J/g		58.1	25.74	30.0		29.58	13.9	58.6	7.95	204.9	79.9	332
Normal boiling point												
Temperature, K	4.22	20.38	77.35	81.7	78.7/81.7	87.29	90.19	111.64	145.09	194.67	231.08	239.72
Density, kg/m^3												
Liquid	124.9	70.7	805.4	789	875.4	1394	1134	42.3	1633		581	682
Vapor	16.89	1.329	4.6	4.4	4.51	5.77	4.49	1818	7.74		2.42	0.89
Heat of vaporization, J/g	20.4	448	199.7	215.8	201.1	160.78	212.1	510	134.1	573	428	1371
Specific heat, J/(g · K)												
Liquid	4.52	9.75	2.042	2.15	1.966	1.07	1.737	3.43	0.91		2.246	4.43
Vapor	9.08	12.2	1.34	1.22	1.13	0.56	0.971	2.15	0.51		1.46	2.24
Viscosity g/(m · s)												
Liquid	0.0036	0.0133	0.17	0.17	0.18	0.27	0.189	0.12	0.32		0.199	0.262
Vapor	0.0012	0.0011	0.0052	0.0056	0.01	0.007	0.0074	0	0.01		0.0064	0.0081
Thermal conductivity, W/(m · K)												
Liquid	0.026	0.119	0.140	0.140	0.14	0.12	0.15	0.193	0.09		0.129	0.587
Vapor	0.009	0.017	0.0070	0.0069	0.01	0.0057	0.0076	0.01	0.01	0.01	0.114	0.0175
Critical point												
Temperature, K	5.19	33.25	126.2	132.85	132.5	150.65	154.58	190.55	227.6	304.12	369.8	405.5
Pressure, kPa	227.5	1297	3400	3494	3766	4898	5043	4599	3740	7374	4240	11353
Density, kg/m^3	69.64	31.0	313.1	303.9	316.5	535.7	436.2	162.7	629	467.8	220.5	235.2
Gas at 101.3 kPa, 294.6 K												
Density, kg/m^3	0.17	0.08	1.160	1.161	1.2	1.66	1.33	0.665	3.66	1.832	1.861	0.713
Specific heat, J/(g · K)	5.19	14.29	1.041	1.039	1.01	0.52	0.92	2.226	0.690	0.839	1.67	2.09
Specific heat ratio	1.67	1.407	1.401	1.402	1.4	1.67	1.4	1.31	1.16	1.316	1.14	1.32
Viscosity, g/(m · s)	0.02	0.0089	0.0174	0.0176	0.0183	0.02	0.0204	**0.01**	0.017	0.015	0.01	0.0101
Thermal conductivity, W/(m · K)	0.15	0.183	0.0254	0.0247	0.0261	0.02	0.0263	0.033	0.0155	0.159	0.017	0.023

TABLE 9.2 Commonly Used Refrigerants

ASHRAE refrigerant number	Chemical name	Chemical formulas	Molecular weight	Normal boiling point, °F at 14,696 psia	Critical temperature, °F	Critical pressure, psia	Freezing point, °F at 14,696 psia	Specific heat ratio $k = C_p/C_v$
11	Trichlorofluoromethane	CCl_3F	137.4	74.8	388.4	640	−168	1.13
114	Dichlorotetrafluoroethane	$CClF_2OClF_2$	170.0	38.4	294.3	474	−137	1.09
12	Dichlorodifluoromethane	CCl_2F_2	120.9	−21.6	233.6	597	−252	1.14
22	Chlorodifluoromethane	$CHClF_2$	86.5	−41.4	204.8	716	−256	1.18
600	*n*-Butane	C_4H_{10}	58.1	31.1	305.6	550.7	−217	1.09
290	Propane	C_3H_8	44.1	−43.7	206	616.3	−305	1.14
1270	Propylene	C_3H_6	42.1	−53.9	197.1	667.2	−301	1.15
170	Ethane	C_2H_6	30.1	−127.4	90.09	707.8	−297	1.19
1150	Ethylene	C_2H_4	28.1	−154.8	48.6	731.1	−272	1.24
50	Methane	CH_4	16.0	−258.7	−111.7	667.8	−296	1.305
717	Ammonia	NH_3	17.0	−28.0	270.4	1636.0	−108	1.29

9.3 Pressure Drop and Pipe Sizing

Pressure drop in cryogenic piping may be calculated based on single-phase (liquid or gas) or two-phase flow (liquid and gas) depending upon whether a single-phase or two-phase flow exists in the pipeline. Single-phase liquid calculations are similar to that of water and oil piping systems. Single-phase gas calculation systems follow the methods used with flow of compressed gases in pipes. We will first address pressure drop in cryogenic piping systems for the liquid phase followed by that for the gas phase and finally that for two-phase flow. For more details of single-phase liquid or gas flow, please refer to Chaps. 6 and 7.

9.3.1 Single-phase liquid flow

The density and viscosity of a liquid are important properties required to calculate the pressure drop in liquid flow through pipes. The *density* is the mass per unit volume of a liquid. For example, the density of water is 62.4 lb/ft^3 at 60°F. The density of liquid oxygen is 1134 kg/m^3 at 54 K.

Viscosity is a measure of a liquid's resistance to flow. Consider a liquid flowing through a circular pipe. Each layer of liquid flowing through the pipe exerts a certain amount of frictional resistance to the adjacent layer. This is illustrated in Fig. 9.1, where a velocity gradient is shown to exist across the pipe diameter.

According to Newton, the frictional shear stress between adjacent layers of the liquid is related to the flowing velocity across a section of the pipe as

$$\text{Shear stress} = \mu \times \text{ velocity gradient}$$

or

$$\tau = \mu \frac{dv}{dy} \tag{9.1}$$

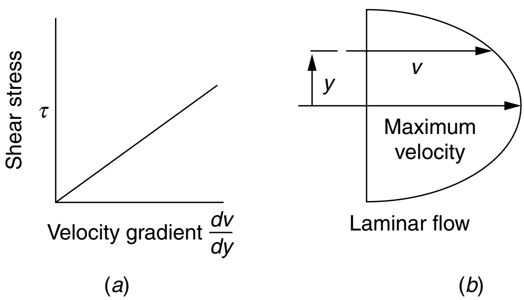

Figure 9.1 Viscosity and Newton's law.

The *velocity gradient* is defined as the rate of change of liquid velocity along the pipe diameter. The proportionality constant μ in Eq. (9.1) is referred to as the *absolute viscosity* or *dynamic viscosity*. In SI units μ is expressed in poise [(dyne · s)/cm^2 or g/(cm · s)] or centipoise (cP). In U.S. Customary System (USCS) units absolute viscosity is expressed as (lb · s)/ft^2 or slug/(ft · s).

For example, water has a viscosity of 1 cP at 60°F and liquid oxygen has a viscosity of 0.189 cP. Another term known as the *kinematic viscosity* of a liquid is defined as the absolute viscosity divided by the density. It is generally represented by the symbol ν. Therefore,

$$\text{Kinematic viscosity } \nu = \frac{\text{absolute viscosity } \mu}{\text{density } \rho} \tag{9.2}$$

In USCS units, kinematic viscosity is measured in ft^2/s. In SI units, kinematic viscosity is expressed as m^2/s, stokes (St), or centistokes (cSt). One stoke equals 1 cm^2/s.

We will next discuss some important parameters relating to liquid flow and how they affect the pressure loss due to friction. Velocity of liquid in a pipe, the dimensionless parameter known as the Reynolds number, and the various flow regimes will be covered first. Next we will introduce the Darcy equation and the Moody diagram for determining the friction factor. The analytical method of calculating the friction factor using the Colebrook-White equation will be discussed, and examples of pressure drop calculation and pipe sizing for single-phase liquid flow will be shown.

Velocity. The speed at which a liquid flows through a pipe, also referred to as *velocity*, is an important parameter in pressure drop calculations. The velocity of flow depends on the pipe diameter and flow rate. If the flow rate is constant through the pipeline (steady flow) and the pipe diameter is uniform, the velocity at every cross section along the pipe will be a constant value. However, there is a variation in velocity along the pipe cross section. The velocity at the pipe wall will be zero, increasing to a maximum at the centerline of the pipe. This is illustrated in Fig. 9.2.

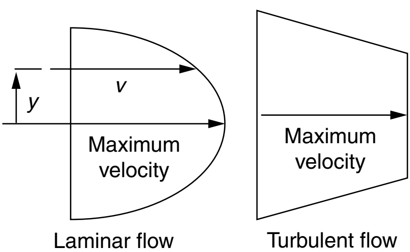

Figure 9.2 Velocity variation—laminar and turbulent flow.

We can define an average velocity of flow at any cross section of the pipe as follows:

$$\text{Average velocity} = \frac{\text{flow rate}}{\text{area of flow}}$$

If the flow rate is in ft^3/s and the pipe cross-sectional area is in ft^2, the velocity from the preceding equation is in ft/s.

Considering liquid flowing through a circular pipe of internal diameter D at a flow rate of Q, the average flow velocity is

$$v = \frac{Q}{\pi D^2/4} \tag{9.3}$$

where v = velocity, ft/s
Q = flow rate, ft^3/s
D = pipe inside diameter, ft

Employing commonly used units of flow rate Q in ft^3/s and pipe diameter in inches, the velocity in ft/s is as follows:

$$v = \frac{144Q}{\pi D^2/4}$$

simplifying to

$$v = 183.3461 \frac{Q}{D^2} \tag{9.4}$$

where the flow rate Q is in ft^3/s and the pipe inside diameter is in inches.

Equation (9.4) for velocity can be modified in terms of flow rate in gal/min as follows:

$$v = 0.4085 \frac{Q}{D^2} \tag{9.5}$$

where v = velocity, ft/s
Q = flow rate, gal/min
D = pipe inside diameter, in

In SI units, the velocity equation is as follows:

$$v = 353.6777 \frac{Q}{D^2} \tag{9.6}$$

where v = velocity, m/s
Q = flow rate, m^3/h
D = internal diameter, mm

Example 9.1 Liquid flows through an NPS 16 (15.5-in inside diameter) pipe at the rate of 4000 gal/min. Calculate the average velocity for steady-state flow. (*Note*: The designation NPS 16 means nominal pipe size of 16 in.)

Solution From Eq. (9.5) the average flow velocity is

$$v = 0.4085 \frac{4000}{15.5^2} = 6.80 \text{ ft/s}$$

Example 9.2 A liquid flows through a DN 400 outside diameter (10-mm wall thickness) pipeline at 200 L/s. Calculate the average velocity for steady flow.

Solution The designation DN 400 in SI units corresponds to NPS 16 in USCS units. DN 400 means a metric pipe size of 400-mm outside diameter. First convert the flow rate in L/s to m^3/h.

$$\text{Flow rate} = 200 \text{ L/s} = 200 \times 60 \times 60 \times 10^{-3} \text{ m}^3\text{/h} = 720 \text{ m}^3\text{/h}$$

From Eq. (9.6) the average flow velocity is

$$v = 353.6777 \frac{720}{380^2} = 1.764 \text{ m/s}$$

The variation of flow velocity along the cross section of a pipe as depicted in Fig. 9.2 depends on the type of flow. In laminar flow, the velocity variation is parabolic. As the flow rate becomes turbulent, the velocity profile approximates a more trapezoidal shape as shown. Laminar and turbulent flows are discussed after we introduce the concept of Reynolds number.

Reynolds number. The Reynolds number of flow is a dimensionless parameter that depends on the pipe diameter, liquid flow rate, liquid viscosity, and density. It is defined as follows:

$$\text{Re} = \frac{vD\rho}{\mu} \tag{9.7}$$

or

$$\text{Re} = \frac{vD}{\nu} \tag{9.8}$$

where Re = Reynolds number, dimensionless
v = average flow velocity, ft/s
D = inside diameter of pipe, ft
ρ = mass density of liquid, slug/ft^3
μ = dynamic viscosity, slug/(ft · s)
ν = kinematic viscosity, ft^2/s

In terms of more commonly used units, we have the following versions of the Reynolds number equation:

$$\mathrm{Re} = 3162.5\,\frac{Q}{D\nu} \tag{9.9}$$

where Re = Reynolds number, dimensionless
Q = flow rate, gal/min
D = inside diameter of pipe, in
ν = kinematic viscosity, centistokes (cSt)

In SI units, the Reynolds number is expressed as follows:

$$\mathrm{Re} = 353{,}678\,\frac{Q}{\nu D} \tag{9.10}$$

where Re = Reynolds number, dimensionless
Q = flow rate, m^3/h
D = inside diameter of pipe, mm
ν = kinematic viscosity, cSt

Example 9.3 A liquid having a density of 70 lb/ft^3 and a viscosity of 0.2 cP flows through an NPS 10 (0.250-in wall thickness) pipeline at 1000 gal/min. Calculate the average velocity and Reynolds number of flow.

Solution The NPS 10 (0.250-in wall thickness) pipeline has an inside diameter = 10.75 − 2 × 0.25 = 10.25 in. From Eq. (9.5) the average velocity is calculated first:

$$v = 0.4085\,\frac{1000}{10.25^2} = 3.89 \text{ ft/s}$$

$$\begin{aligned}\text{Liquid viscosity in cSt} &= \frac{\text{viscosity in cP}}{\text{density}} = \frac{0.2\times 6.7197\times 10^{-4}}{70}\\ &= 1.9199\times 10^{-6}\ \text{ft}^2/\text{s}\\ &= 1.9199\times 10^{-6}\times (0.3048)^2\ \text{m}^2/\text{s}\\ &= 1.7837\times 10^{-7}\ \text{m}^2/\text{s} = \frac{1.7837\times 10^{-7}}{10^{-6}}\ \text{cSt}\\ &= 0.1784\ \text{cSt}\end{aligned}$$

using conversion factors from App. A.

From Eq. (9.9) the Reynolds number is therefore

$$\mathrm{Re} = 3162.5\,\frac{1000}{(10.25\times 0.1784)} = 1.73\times 10^6$$

Example 9.4 A liquid having a density of 1120 kg/m^3 and a viscosity of 0.2 cSt flows through a DN 200 (6-mm wall thickness) pipeline at 200 m^3/h. Calculate the average flow velocity and the Reynolds number of flow.

Solution The DN 200 (6-mm wall thickness) pipe has an inside diameter = $200 - 2 \times 6 = 188$ mm. From Eq. (9.6) the average velocity is therefore

$$v = 353.6777 \frac{200}{188^2} = 2.00 \text{ m/s}$$

Next, from Eq. (9.10) the Reynolds number is

$$\text{Re} = 353{,}678 \frac{200}{188 \times 0.2} = 1.88 \times 10^6$$

Types of flow. Flow through a pipe is classified as laminar flow, turbulent flow, or critical flow depending on the magnitude of the Reynolds number of flow. If the Reynolds number is less than 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is considered to be *turbulent*. *Critical flow* occurs when the Reynolds number is in the range of 2100 to 4000. Laminar flow is characterized by smooth flow in which no eddies or turbulence is visible. The flow is also said to occur in laminations. If dye was injected into a transparent pipeline, laminar flow would be manifested in the form of smooth streamlines of dye. Turbulent flow occurs at higher velocities and is accompanied by eddies and other disturbances in the liquid. More energy is lost in friction in the critical flow and turbulent flow regions as compared to the laminar flow region.

The three flow regimes characterized by the Reynolds number of flow are

Laminar flow:	$\text{Re} \leq 2100$
Critical flow:	$2100 < \text{Re} \leq 4000$
Turbulent flow:	$\text{Re} > 4000$

In the critical flow regime, where the Reynolds number is between 2100 and 4000, the flow is undefined and unstable, as far as pressure drop calculations are concerned. In the absence of better data, it is customary to use the turbulent flow equation to calculate pressure drop in the critical flow regime as well.

Pressure drop due to friction. As a liquid flows through a pipe, energy is lost due to resistance between the flowing liquid layers as well as due to the friction between the liquid and the pipe wall. One of the objectives of pipeline calculation is to determine the amount of energy and hence the pressure lost due to friction as the liquid flows from the source to the destination. We will begin by discussing the Darcy equation for pressure drop calculations.

Darcy equation. As a liquid flows through a pipe from point A to point B the pressure along the pipe decreases due to frictional loss between the flowing liquid and the pipe. The extent of pressure loss due to friction depends on various factors such as the liquid flow rate, liquid density, liquid viscosity, pipe inside diameter, pipe length, and internal condition of the pipe (rough, smooth, etc.) The Darcy equation is used to calculate the pressure drop in a pipeline as follows:

$$h = f \frac{L}{D} \frac{v^2}{2g} \tag{9.11}$$

where h = frictional pressure loss, ft of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = inside diameter of pipe, ft
v = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

The Darcy equation gives the frictional pressure loss in feet of liquid head, which can be converted to pressure loss in psi using the following equation:

$$\Delta P = \frac{h \times \rho}{144} \tag{9.12}$$

where ΔP = pressure loss, psi
h = pressure loss, ft of liquid head
ρ = liquid density, lb/ft^3

In SI units Eq. (9.12) becomes

$$\Delta P = \frac{h \times \rho}{101.94} \tag{9.13}$$

where ΔP = pressure loss, kPa
h = pressure loss, m of liquid head
ρ = liquid density, kg/m^3

The term $v^2/2g$ in the Darcy equation is the velocity head, and it represents the kinetic energy of the liquid. The term *velocity head* will be used in subsequent sections of this chapter when analyzing frictional loss through pipe fittings and valves.

The following form of the Darcy equation is represented in terms of commonly used units.

$$h = 0.1863 \frac{fLv^2}{D} \tag{9.14}$$

where h = frictional pressure loss, ft of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = pipe inside diameter, in
v = average flow velocity, ft/s

Another form of the Darcy equation with frictional pressure drop expressed in psi/ft and using the flow rate instead of velocity is as follows:

$$P_f = (2.1635 \times 10^{-4}) \frac{fQ^2\rho}{D^5} \tag{9.15}$$

where P_f = frictional pressure loss, psi/ft
f = Darcy friction factor, dimensionless
Q = flow rate, gal/min
D = pipe inside diameter, in
ρ = liquid density, lb/ft^3

In SI units, the Darcy equation may be written as

$$h = 50.94 \frac{fLv^2}{D} \tag{9.16}$$

where h = frictional pressure loss, m of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, m
D = pipe inside diameter, mm
v = average flow velocity, m/s

Another version of the Darcy equation in SI units is as follows:

$$P_m = (6.2475 \times 10^4) \left(fQ^2 \frac{\rho}{D^5} \right) \tag{9.17}$$

where P_m = frictional pressure loss, kPa/m
Q = liquid flow rate, m^3/h
f = Darcy friction factor, dimensionless
ρ = liquid density, kg/m^3
D = pipe inside diameter, mm

The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eqs. (9.14) through (9.17). This friction factor is a nondimensional number between 0.0 and 0.1 that depends on the internal roughness of the pipe, the pipe diameter, and the Reynolds number of flow.

In laminar flow, the friction factor f depends only on the Reynolds number and is calculated from

$$f = \frac{64}{R} \tag{9.18}$$

where f is the friction factor for laminar flow and Re is the Reynolds number for laminar flow (Re $\leq$ 2100) (dimensionless). Therefore, if a particular flow has a Reynolds number of 1800, we can conclude that in this laminar flow condition the friction factor f to be used in the Darcy equation is

$$f = \frac{64}{1800} = 0.0356$$

Some pipeline hydraulics texts may refer to another friction factor called the Fanning friction factor. This is numerically equal to one-fourth the Darcy friction factor. In the preceding example the Fanning friction factor can be calculated as

$$\frac{0.0356}{4} = 0.0089$$

To avoid any confusion, throughout this chapter we will use only the Darcy friction factor as defined in Eq. (9.11).

Example 9.5 A cryogenic liquid with a density of 70 lb/ft^3 flows through an NPS 6 (0.250-in wall thickness) pipeline at a flow rate of 500 gal/min. Calculate the average flow velocity and pressure loss due to friction in 200 ft of pipe length, using the Darcy equation. Assume a friction factor $f = 0.02$.

Solution

$$\text{Pipe inside diameter} = 6.625 - 2 \times 0.250 = 6.125 \text{ in}$$

Using Eq. (9.5), the velocity is

$$v = \frac{0.4085 \times 500}{6.125^2} = 5.44 \text{ ft/s}$$

The pressure drop is calculated using Eq. (9.15) as follows:

$$P_f = (2.1635 \times 10^{-4})\frac{0.02 \times 500^2 \times 70}{6.125^5} = 0.0088 \text{ psi/ft}$$

Therefore, the total pressure drop in 200 ft of pipe is

$$\Delta P = 200 \times 0.0088 = 1.75 \text{ psi}$$

Colebrook-White equation. We have seen that in laminar flow (Re $\leq$ 2100) the friction factor f is easily calculated from the Reynolds number as shown in Eq. (9.18). In turbulent flow (Re $>$ 4000), the friction factor f depends on the pipe inside diameter, the pipe roughness, and the Reynolds number. The following empirical equation, known as the Colebrook-White equation (also simply called the Colebrook equation) is used to calculate the friction factor in turbulent flow.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{9.19}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
Re = Reynolds number, dimensionless

The absolute pipe roughness or internal pipe roughness may range from 0.0 to 0.01 depending on the internal condition of the pipe. It is listed for common piping systems in Table 9.3. The ratio e/D is termed the relative roughness and is dimensionless.

In SI units, we can use the same form of the Colebrook equation. The absolute pipe roughness e and the pipe diameter D are both expressed in millimeters. All other terms in the equation are dimensionless.

It can be seen from the Colebrook-White equation that the calculation of the friction factor f is not straightforward since it appears on both sides of the equation. This is known as an implicit equation in f, compared to an explicit equation. An explicit equation in f will have the unknown quantity f only on one side of the equation. In the present case, a trial-and-error approach is used to solve for the friction factor. First an initial value for f is assumed (for example, $f = 0.02$) and substituted in the right-hand side of the Colebrook equation. This will result in a new calculated value of f, which is used as the next approximation, and f will be recalculated based on this second approximation. The process is continued until successive values of f calculated by such iterations are within a small value such as 0.001. Usually three or four

TABLE 9.3 Pipe Internal Roughness

Pipe material	Roughness in	Roughness mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

iterations will yield a satisfactory solution. Example 9.6 illustrates the method.

Moody diagram. A graphical method of determining the friction factor for turbulent flow is available using the Moody diagram as shown in Fig. 9.3. This graph is based on the Colebrook equation and is much easier to use compared to calculating the value of the friction factor from the implicit equation (9.19).

First the Reynolds number is calculated from the liquid properties, flow rate, and pipe diameter. This Reynolds number is used to locate the ordinate on the horizontal axis of the Moody diagram. A vertical line is drawn up to the curve representing the relative roughness e/D of the pipe. The friction factor is then read off on the vertical axis to the left. From the Moody diagram it is seen that the turbulent region is further divided into two regions: the "transition" zone and the "complete turbulence in rough pipes" zone. The lower boundary is designated as "smooth pipes." The transition zone extends up to the dashed line, beyond which is known as the zone of complete turbulence in rough pipes. In the zone of complete turbulence in rough pipes, the friction factor depends very little on the Reynolds number and more on the relative roughness.

Example 9.6 A cryogenic liquid with a density of 70 lb/ft^3 and 0.2 cSt viscosity flows through an NPS 10 (0.250-in wall thickness) pipeline at a flow rate of 1500 gal/min. The absolute roughness of the pipe may be assumed to be 0.002 in. Calculate the Darcy friction factor and pressure loss due to friction in 500 ft of pipe length, using the Colebrook-White equation.

Solution The inside diameter of an NPS 10 (0.250-in wall thickness) pipe is

$$10.75 - 2 \times 0.250 = 10.25 \text{ in}$$

Next we will calculate the Reynolds number Re to determine the flow regime (laminar or turbulent). The Reynolds number from Eq. (9.9) is

$$\text{Re} = 3162.5 \frac{1500}{10.25 \times 0.2} = 2.31 \times 10^6$$

Since Re > 4000, the flow is turbulent and we can use the Colebrook-White equation to calculate the friction factor. We can also use the Moody diagram to read the friction factor based on Re and the pipe relative roughness e/D.

From the Colebrook-White equation (9.19), the friction factor f is calculated from

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{0.002}{3.7 \times 10.25} + \frac{2.51}{(2.31 \times 10^6)\sqrt{f}}\right]$$

This equation must be solved for f by trial and error.

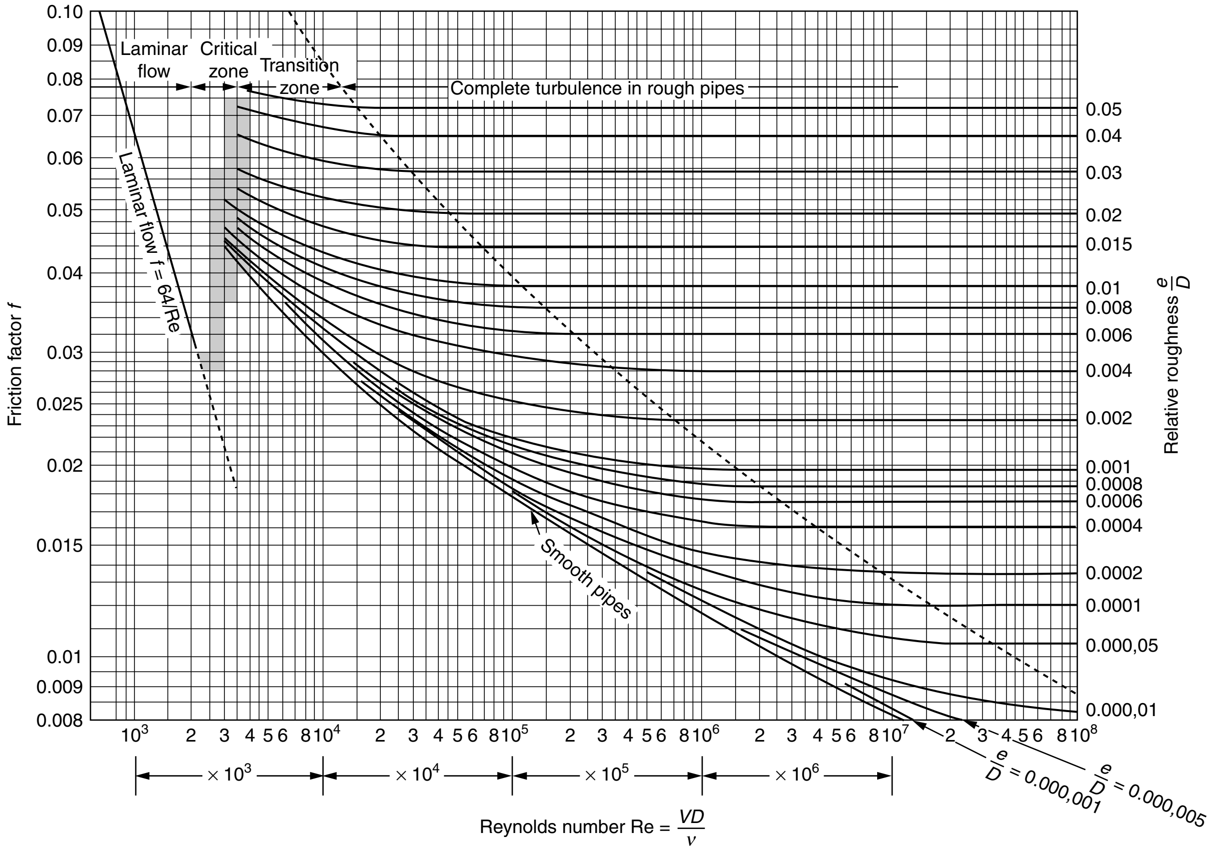

Figure 9.3 Moody diagram.

First assume that $f = 0.02$. Substituting in the preceding equation, we get a better approximation for f as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{0.002}{3.7 \times 10.25} + \frac{2.51}{(2.31 \times 10^6)\sqrt{0.02}}\right] = 0.0140$$

Recalculating using this value

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{0.002}{3.7 \times 10.25} + \frac{2.51}{(2.31 \times 10^6)\sqrt{0.0140}}\right] = 0.0141$$

And finally

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{0.002}{3.7 \times 10.25} + \frac{2.51}{(2.31 \times 10^6)\sqrt{0.0141}}\right] = 0.0141$$

Thus $f = 0.0141$ is the solution.

The pressure loss due to friction can now be calculated using the Darcy equation (9.15), considering a 500-ft length of pipe:

$$\begin{aligned}\Delta P &= (2.1635 \times 10^{-4})\frac{0.0141 \times 1500^2 \times 70}{10.25^5} \times 500 \\ &= 150.59 \text{ psi in 500 ft of pipe length}\end{aligned}$$

Example 9.7 A DN 300 (8-mm wall thickness) steel pipe is used to transport a cryogenic liquid from a plant to a storage facility 1500 m away. Calculate the friction factor and pressure loss due to friction (kPa/m) at a flow rate of 190 m^3/h. Assume an internal pipe roughness of 0.05 mm. A delivery pressure of 140 kPa must be maintained at the delivery point which is at an elevation of 200 m above that of the plant. Calculate the pump pressure required at the plant to transport the given volume of liquid to the storage facility. Density of liquid = 800 kg/m^3 and viscosity = 0.17 cSt.

Solution The pipe designated as DN 300 and 8-mm wall thickness has an inside diameter of

$$D = 300 - 2 \times 8 = 284 \text{ mm}$$

First calculate the Reynolds number from Eq. (9.10):

$$\text{Re} = 353{,}678\,\frac{190}{284 \times 0.17} = 1.39 \times 10^6$$

Therefore the flow is turbulent and we can use the Colebrook-White equation or the Moody diagram to determine the friction factor.

$$\text{Relative roughness } \frac{e}{D} = \frac{0.05}{284} = 0.0002$$

Using the determined values for relative roughness and the Reynolds number, from the Moody diagram we get $f = 0.0142$.

The pressure drop due to friction can now be calculated using the Darcy equation (9.17):

$$P_m = (6.2475 \times 10^4)\left(0.0142 \times 190^2 \frac{800}{284^5}\right) = 0.0139 \text{ kPa/m}$$

$$\text{Total pressure loss in 1500 m} = 0.0139 \times 1500 = 20.8 \text{ kPa}$$

The pressure required at the plant is calculated by adding the pressure drop due to friction to the delivery pressure required and the static elevation head between the plant and storage facility.

The static head difference is 200 m. This is converted to pressure in kilopascals, using Eq. (9.13):

$$\text{Pressure drop due to friction in 1500 m of pipe} = 20.8 \text{ kPa}$$

$$\text{Pressure due to elevation head} = \frac{200 \times 800}{101.94} = 1569.6 \text{ kPa}$$

$$\text{Minimum pressure required at delivery point} = 140 \text{ kPa}$$

Therefore adding all three numbers, the total pressure required at the plant is

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}}$$

where P_t = total pressure required at plant
P_f = frictional pressure drop
P_{elev} = pressure head due to elevation difference
P_{del} = delivery pressure at storage facility

Therefore,

$$P_t = 20.8 + 1569.6 + 140.0 = 1730.4 \text{ kPa}$$

Thus, the pump pressure required at the plant is 1730.4 kPa.

Minor losses. So far, we have calculated the pressure drop per unit length in straight pipes. We also calculated the total pressure drop considering several feet of pipe from a plant to a storage facility. Minor losses in a liquid pipeline are classified as those pressure drops that are associated with piping components such as valves and fittings. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. A pipe nozzle exiting from a storage tank will have entrance and exit losses. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe. Generally, minor losses are included in calculations by using the equivalent length of the valve or fitting or using a resistance factor K multiplied by the velocity head $v^2/2g$ discussed earlier. The term minor losses can be applied only where the pipeline lengths and the friction losses are relatively large compared to the pressure drops in the fittings and valves. In a situation such as plant piping and tank farm piping the pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses is really a misnomer. Regardless, the pressure losses through valves, fittings, etc., can be accounted for approximately using the equivalent length or K times the velocity head method.

Valves and fittings. Table 9.4 shows the equivalent lengths of commonly used valves and fittings in a liquid pipeline system. It can be seen from this table that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore a 14-in-diameter gate valve may be replaced with a $14 \times 8 = 112$ in long piece of pipe that will have the same frictional pressure drop as the valve.

Example 9.8 A piping system is 600 ft of NPS 14 pipe with two 14-in gate valves, three 14-in ball valves, and four 90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe, valves, and fittings.

TABLE 9.4 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

Solution Using Table 9.4, we can convert all valves and fittings in terms of 14-in pipe as follows,

$$\begin{aligned}
\text{Two 14-in gate valves} &= 2 \times 14 \times 8 = 224 \text{ in of 14-in pipe} \\
\text{Three 14-in ball valves} &= 3 \times 14 \times 3 = 126 \text{ in of 14-in pipe} \\
\text{Four } 90^{\circ}\text{elbows} &= 4 \times 14 \times 30 = 1680 \text{ in of 14-in pipe} \\
\text{Total for all valves and fittings} &= 2030 \text{ in of 14-in pipe} \\
&= 169.17 \text{ ft of 14-in pipe}
\end{aligned}$$

Adding the 600 ft of straight pipe, the total equivalent length of straight pipe and all fittings is

$$L_e = 600 + 169.17 = 769.17 \text{ ft}$$

The pressure drop due to friction in the preceding piping system can now be calculated based on 769.17 ft of NPS 14 pipe. It can be seen in this example that the valves and fittings represent roughly 22 percent of the total pipeline length.

Resistance coefficient. Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating the pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation (9.11), the pressure drop in a straight length of pipe is given by

$$h = f \frac{L}{D} \frac{v^2}{2g}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K (also known as the resistance coefficient) and the preceding equation then becomes

$$h = K \frac{v^2}{2g} \tag{9.20}$$

In Eq. (9.20), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $v^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(v^2/2g)$, where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 9.5. It can be seen that the K factor depends on the nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

TABLE 9.5 Friction Loss in Valves—Resistance Coefficient *K*

		Nominal pipe size, in											
Description	L/D	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72

From Table 9.5 it can be seen that a 6-in gate valve has a K value of 0.12, while a 14-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be $0.12\,(v^2/2g)$ and that in the 14-in valve is $0.10\,(v^2/2g)$. The velocity v in both cases will be different due to the difference in diameters.

If the flow rate was 1000 gal/min, the velocity in the 6-in valve will be approximately

$$v_6 = 0.4085\,\frac{1000}{6.125^2} = 10.89 \text{ ft/s}$$

Similarly, at 1000 gal/min, the velocity in the 14-in valve will be approximately

$$v_6 = 0.4085\,\frac{1000}{13.5^2} = 2.24 \text{ ft/s}$$

Therefore,

$$\text{Head loss in 6-in gate valve} = \frac{0.12\,(10.89)^2}{64.4} = 0.22 \text{ ft}$$

and

$$\text{Head loss in 14-in gate valve} = \frac{0.10\,(2.24)^2}{64.4} = 0.008 \text{ ft}$$

These head losses appear small since we have used a relatively low flow rate in the 14-in valve. In reality the flow rate in the 14-in valve may be as high as 3000 gal/min and the corresponding head loss will be 0.07 ft.

Pipe enlargement and reduction. Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(v_1 - v_2)^2}{2g} \tag{9.21}$$

where v_1 and v_2 are the velocities of the liquid in the two pipe sizes, D_1 and D_2, respectively. This is illustrated in Fig. 9.4. Writing Eq. (9.21) in terms of pipe cross-sectional areas A_1 and A_2 for sudden enlargement, we get

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 {v_1}^2 \sqrt{2g} \tag{9.22}$$

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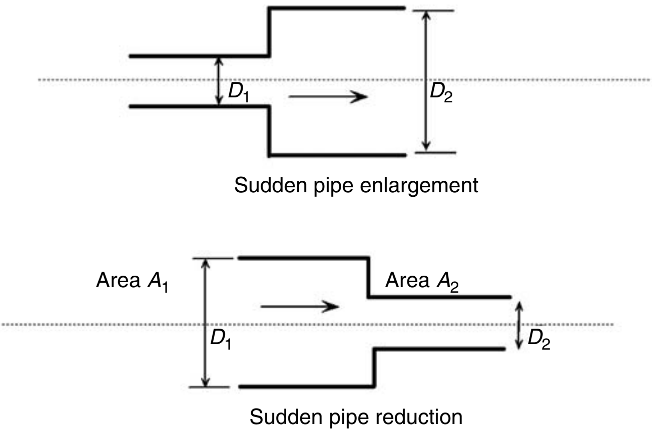

A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 9.4 Sudden pipe enlargement and pipe reduction.

For sudden contraction or reduction in pipe size as shown in Fig. 9.4, the head loss is calculated from

$$h_f = \left(\frac{1}{C_c} - 1\right)^2 \frac{v_2{}^2}{2g} \tag{9.23}$$

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 9.4.

Gradual enlargement and reduction of pipe size, as shown in Fig. 9.5, cause less head loss than sudden enlargement and sudden reduction.

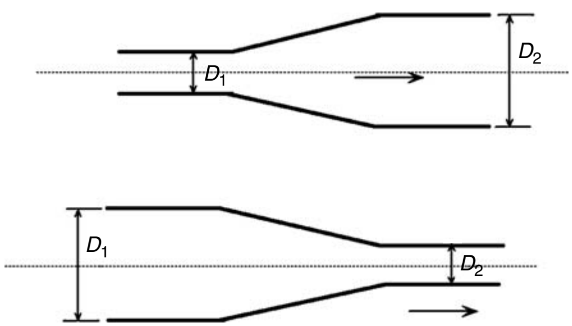

Figure 9.5 Gradual pipe enlargement and pipe reduction.

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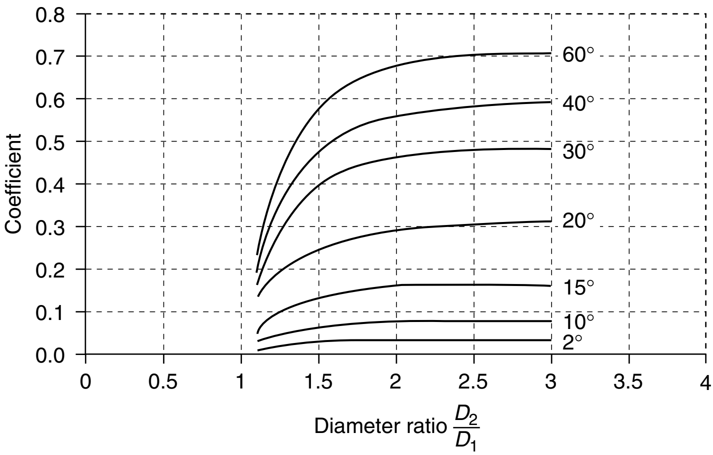

Figure 9.6 Gradual pipe expansion head loss coefficient.

For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c(v_1 - v_2)^2}{2g} \tag{9.24}$$

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is shown in Fig. 9.6.

Pipe entrance and exit losses. The K factors for computing the head loss associated with the pipe entrance and exit are as follows

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

Complex piping systems. So far we have discussed straight length of pipe with valves and fittings. Complex piping systems include pipes of different diameters in series and parallel configurations.

Series piping. Series piping in its simplest form consists of two or more different pipe sizes connected end to end as illustrated in Fig. 9.7. Pressure drop calculations in series piping may be handled in one of two ways. The first approach is to calculate the pressure drop in each pipe size and add them together to obtain the total pressure drop. Another method is to consider one of the pipe diameters as the base size and convert other pipe sizes into equivalent lengths of the base pipe size. The resultant equivalent lengths are added together to form one long piece

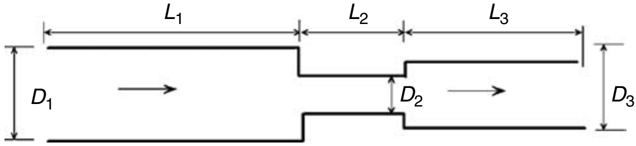

Figure 9.7 Series piping.

of pipe of constant diameter equal to the base diameter selected. The pressure drop can then be calculated for this single-diameter pipeline. Of course, all valves and fittings will also be converted to their respective equivalent pipe lengths using the L/D ratios from Table 9.4.

Consider three sections of pipes joined together in series. Using subscripts 1, 2, and 3 and denoting the pipe length as L, inside diameter as D, flow rate as Q, and velocity as v, we can calculate the equivalent length of each pipe section in terms of a base diameter. This base diameter will be selected as the diameter of the first pipe section D_1. Since equivalent length is based on the same pressure drop in the equivalent pipe as in the original pipe, the equivalent length of section 2 is calculated by finding that length of diameter D_1 that will match the pressure drop in a length L_2 of pipe diameter D_2. Using the Darcy equation, the equivalent lengths of the two pipes with diameter D_2 and D_3 are calculated in terms of the diameter D_1. For series pipes, the flow rate is the same through each pipe. The pressure drop is inversely proportional to the fifth power of the diameter, from the Darcy equation (9.15). Considering friction factors to be approximately the same for all pipes, we get the total equivalent length L_t of all three pipe sections based on diameter D_1 as follows:

$$L_t = L_1 + L_2\left(\frac{D_1}{D_2}\right)^5 + L_3\left(\frac{D_1}{D_3}\right)^5 \tag{9.25}$$

The total pressure drop in the three sections of pipe can now be calculated based on a single pipe of diameter D_1 and length L_t.

Example 9.9 Three pipes, 10-, 12-, and 8-in diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

10-in pipeline, 0.250-in wall thickness, 200 ft long

12-in pipeline, 0.250-in wall thickness, 300 ft long

8-in pipeline, 0.250-in wall thickness, 500 ft long

Two 10-in 90° elbows

Four 12-in 90° elbows

Six 8-in 90° elbows

One 10-in gate valve

One 12-in ball valve

One 8-in gate valve

(a) Use the Darcy equation to calculate the total pressure drop in the series piping system at a flow rate of 1200 gal/min. The liquid transported has a density of 50 lb/ft^3 and a viscosity of 0.20 cSt. Flow starts in the 10-in piping and ends in the 8-in piping. Assume a friction factor $f = 0.02$.
(b) If the flow rate is increased to 1500 gal/min, estimate the new total pressure drop in the piping system, keeping everything else the same.

Solution

(a) Since we are going to use the Darcy equation, the pipes in series analysis will be based on the pressure loss being inversely proportional to D^5 where D is the inside diameter of pipe, per Eq. (9.25).

We will first calculate the total equivalent lengths of all 10-in pipe, fittings, and valve in terms of the 10-in diameter pipe.

$$\text{Straight pipe: 10 in, 200 ft} = 200 \text{ ft of 10-in pipe}$$

$$\text{Two 10-in } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 10}{12} = 50 \text{ ft of 10-in pipe}$$

$$\text{One 10-in gate valve} = \frac{1 \times 8 \times 10}{12} = 6.67 \text{ ft of 10-in pipe}$$

Therefore, the total equivalent length of 10-in pipe, fittings, and valve = 256.67 ft of 10-in pipe.

Similarly we get the total equivalent length of 12-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 12-in, 300 ft} = 300 \text{ ft of 12-in pipe}$$

$$\text{Four 12-in } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 12}{12} = 120 \text{ ft of 12-in pipe}$$

$$\text{One 12-in ball valve} = \frac{1 \times 3 \times 12}{12} = 3 \text{ ft of 12-in pipe}$$

Therefore, the total equivalent length of 12-in pipe, fittings, and valve = 423 ft of 12-in pipe.

Finally, we calculate the total equivalent length of 8-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 8-in, 500 ft} = 500 \text{ ft of 8-in pipe}$$

$$\text{Six 8-in } 90^\circ \text{ elbows} = \frac{6 \times 30 \times 8}{12} = 120 \text{ ft of 8-in pipe}$$

$$\text{One 8-in gate valve} = \frac{1 \times 8 \times 8}{12} = 5.33 \text{ ft of 8-in pipe}$$

Therefore, the total equivalent length of 8-in pipe, fittings, and valve = 625.33 ft of 8-in pipe.

Next we convert all the preceding pipe lengths to the equivalent 10-in pipe based on the fact that the pressure loss is inversely proportional to D^5 where

D is the inside diameter of pipe, according to Eq. (9.25):

$$L_t = 256.67 + \left(\frac{10.25}{12.25}\right)^5 \times 423 + \left(\frac{10.25}{8.125}\right)^5 \times 625.33$$
$$= 2428.24 \text{ ft of 10-in pipe}$$

Total equivalent length in terms of 10-in pipe = 2428.24 ft of 10-in pipe

We still have to account for the 12 × 10 in and 12 × 8 in reducers. The reducers can be considered as sudden enlargements for approximate calculation of the head loss, using the K factor and velocity head method. For sudden enlargements, the resistance coefficient K is found from Eq. (9.22):

$$K = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2$$

where the area ratios have been replaced with the square of the ratio of the diameters and D_1 is the smaller diameter and D_2 is the larger diameter.

For the 12 × 10 in reducer,

$$K = \left[1 - \left(\frac{10.25}{12.25}\right)^2\right]^2 = 0.0899$$

And for the 12 × 8 in reducer,

$$K = \left[1 - \left(\frac{8.125}{12.25}\right)^2\right]^2 = 0.3137$$

The head loss through the reducers will then be calculated based on $K(v^2/2g)$.

Flow velocities in the three different pipe sizes at 1200 gal/min will be calculated using Eq. (9.5):

$$\text{Velocity in 10-in pipe:} \quad v_{10} = \frac{0.4085 \times 1200}{(10.25)^2} = 4.67 \text{ ft/s}$$

$$\text{Velocity in 12-in pipe:} \quad v_{12} = \frac{0.4085 \times 1200}{(12.25)^2} = 3.27 \text{ ft/s}$$

$$\text{Velocity in 8-in pipe:} \quad v_8 = \frac{0.4085 \times 1200}{(8.125)^2} = 7.43 \text{ ft/s}$$

The head loss through the 12 × 10 in reducer is

$$h_f = 0.0899 \times \frac{4.67^2}{64.4} = 0.0304 \text{ ft}$$

and the head loss through the 12 × 8 in reducer is

$$h_f = 0.3137 \times \frac{7.43^2}{64.4} = 0.2689 \text{ ft}$$

These head losses in the reducers are insignificant and hence can be neglected in comparison with the head loss in straight length of pipe. Therefore, the total head loss in the entire piping system will be based on a total equivalent length of 2428.24 ft of 10-in pipe.

Using the Darcy equation (9.15) the pressure drop at 1200 gal/min is

$$P_f = (2.1635 \times 10^{-4}) \frac{0.02 \times (1200)^2 \times 50}{(10.25)^5} = 0.0028 \text{ psi per ft of pipe}$$

Therefore,

$$\text{Total pressure drop in 2428.24 ft} = 0.0028 \times 2428.24 = 6.8 \text{ psi}$$

(b) When the flow rate is increased to 1500 gal/min, we can use proportions to estimate the new total pressure drop in the piping as follows:

$$P_f = \left(\frac{1500}{1200}\right)^2 \times 0.0028 = 0.0044 \text{ psi per ft of pipe}$$

Therefore,

$$\text{Total pressure drop in 2428.24 ft} = 0.0044 \times 2428.24 = 10.62 \text{ psi}$$

Parallel piping. Liquid pipelines in parallel are so configured that multiple pipes are connected so that the liquid flow splits into the multiple pipes at the beginning and the separate flow streams subsequently rejoin downstream into another single pipe as depicted in Fig. 9.8.

Figure 9.8 shows a parallel piping system in the horizontal plane with no change in pipe elevations. Liquid flows through a single pipe *AB*, and at the junction *B* the flow splits into two pipe branches *BCE* and *BDE*. At the downstream end at junction *E*, the flows rejoin to the initial flow rate and subsequently flow through the single pipe *EF*.

To calculate the flow rates and pressure drop due to friction in the parallel piping system, shown in Fig. 9.8, two main principles of parallel piping must be followed. These are flow conservation at any junction point and common pressure drop across each parallel branch pipe.

Based on flow conservation, at each junction point of the pipeline, the incoming flow must exactly equal the total outflow. Therefore, at junction *B*, the flow *Q* entering the junction must exactly equal the sum of the flow rates in branches *BCE* and *BDE*. Thus,

$$Q = Q_{BCE} + Q_{BDE} \tag{9.26}$$

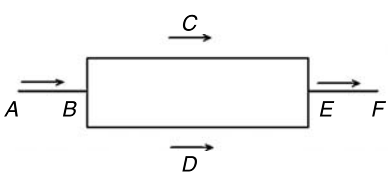

Figure 9.8 Parallel piping.

where Q_{BCE} = flow through branch BCE
Q_{BDE} = flow through branch BDE
Q = incoming flow at junction B

The other requirement in parallel pipes relates to the pressure drop in each branch piping. Accordingly, the pressure drop due to friction in branch BCE must exactly equal that in branch BDE. This is because both branches have a common starting point (B) and a common ending point (E). Since the pressure at each of these two points is a unique value, we can conclude that the pressure drop in branch pipe BCE and that in branch pipe BDE are both equal to $P_B - P_E$, where P_B and P_E represent the pressure at the junction points B and E, respectively.

Using these principles the flow rate in each branch and the common pressure drop for both branches can be calculated from a set of simultaneous equations in Q_{BCE} and Q_{BDE} by substituting the pipe diameters, lengths, etc., in the Darcy equation.

Another approach to calculating the pressure drop in parallel piping is the use of an equivalent diameter for the parallel pipes. For example in Fig. 9.8, if pipe AB has a diameter of 14 in and branches BCE and BDE have diameters of 10 and 12 in, respectively, we can find some equivalent diameter pipe of the same length as one of the branches that will have the same pressure drop between points B and C as the two branches. An approximate equivalent diameter can be calculated using the Darcy equation.

We can solve for the equivalent diameter D_e as follows:

$$D_e^{2.5} = D_1^{2.5} + D_2^{2.5}\left(\frac{L_1}{L_2}\right)^{0.5} \tag{9.27}$$

If both branches are of equal length, Eq. (9.27) reduces to

$$D_e^{2.5} = D_1^{2.5} + D_2^{2.5} \tag{9.27a}$$

Example 9.10 A cryogenic liquid pipeline consists of 800 ft of NPS 10 (0.250-in wall thickness) pipe starting at point A and terminating at point B. At point B, two pieces of pipe (each 400 ft long and NPS 8 pipe with 0.250-in wall thickness) are connected in parallel and rejoin at point D. From D, 500 ft of NPS 10 (0.250-in wall thickness) pipe extends to point E. Using the equivalent diameter method calculate the pressures and flow rate through the system when transporting a liquid (density = 55 lb/ft^3 and viscosity = 0.12 cSt) at 1200 gal/min. Use a pipe roughness of 0.002 in. Compare the results by calculating the pressures and flow rates in each branch.

Solution Since the pipe loops between B and D are each NPS 8 and 400 ft long, the flow will be equally split between the two branches. Each branch pipe will carry 600 gal/min. Each branch has an inside diameter $= 8.625 - 2 \times 0.25 = 8.125$ in.

The equivalent diameter for section BD is found from Eq. (9.27a):

$$D_e{}^{2.5} = 8.125^{2.5} + 8.125^{2.5} = 376.347$$

Therefore,

$$D_e = 10.72 \text{ in}$$

Thus, we can replace the two 400-ft NPS 8 pipes between B and D with a single 400-ft-long pipe with a 10.72-in inside diameter.

The Reynolds number for this pipe at 1200 gal/min is found from Eq. (9.9):

$$\text{Re} = \frac{3162.5 \times 1200}{10.72 \times 0.12} = 2.95 \times 10^6$$

Considering that the pipe roughness is 0.002 in for all pipes:

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{10.72} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0141$. The pressure drop in section BD, using Eq. (9.15), is

$$P_f = (2.1635 \times 10^{-4})\frac{0.0141 \times 1200^2 \times 55}{10.72^5} = 0.0017 \text{ psi/ft}$$

Therefore,

$$\text{Total pressure drop in } BD = 0.0017 \times 400 = 0.68 \text{ psi}$$

For section AB we have,

$$\text{Re} = \frac{3162.5 \times 1200}{10.25 \times 0.12} = 3.085 \times 10^6$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{10.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0140$. The pressure drop in section AB is

$$P_f = (2.1635 \times 10^{-4})\frac{0.0140 \times 1200^2 \times 55}{10.25^5} = 0.0021 \text{ psi/ft}$$

Therefore,

$$\text{Total pressure drop in } AB = 0.0021 \times 800 = 1.697 \text{ psi}$$

Finally, for section DE,

$$\text{Re} = \frac{3162.5 \times 1200}{10.25 \times 0.12} = 3.085 \times 10^6$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{10.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0140$. The pressure drop in section DE is

$$P_f = (2.1635 \times 10^{-4})\frac{0.0140 \times 1200^2 \times 55}{10.25^5} = 0.0021 \text{ psi/ft}$$

Therefore,

$$\text{Total pressure drop in } DE = 0.0021 \times 500 = 1.05 \text{ psi}$$

Finally,

$$\text{Total pressure drop in entire piping system} = 0.68 + 1.697 + 1.05$$
$$= 3.43 \text{ psi}$$

Next, for comparison we will analyze the branch pressure drops considering each 8-in branch separately flowing at 600 gal/min.

$$\text{Re} = \frac{3162.5 \times 600}{8.125 \times 0.12} = 1.946 \times 10^6$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{8.125} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0148$. The pressure drop in section BD is

$$P_f = (2.1635 \times 10^{-4})\frac{0.0148 \times 600^2 \times 55}{8.125^5} = 0.00179 \text{ psi/ft}$$

This compares with the pressure drop of 0.0017 psi/ft we calculated using an equivalent diameter of 10.72. It can be seen that the difference between the two pressure drops is approximately 5 percent.

Total pressure required. In the previous sections, we examined the frictional pressure drop in a liquid piping system consisting of pipe, valves, fittings, etc. The total pressure required at the beginning of a pipeline for a specified flow rate consists of three distinct components:

1. Frictional pressure drop, P_f
2. Elevation head P_{elev}
3. Delivery pressure P_{del}

This can be stated as follows

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \tag{9.28}$$

where P_t is the total pressure required at the beginning of the pipe.

The first item on the right-hand side of Eq. (9.28) is simply the total frictional head loss in all straight pipe, fittings, valves, etc. The last item is the delivery pressure required at the end of the pipeline to satisfy tank or some other back pressure requirement at the terminus.

The second item accounts for the pipeline elevation difference between the origin of the pipeline and the delivery terminus. If the origin of the pipeline is at a lower elevation than that of the pipeline terminus or delivery point, a certain amount of positive pressure is required to compensate for the elevation difference. On the other hand, if the delivery point were at a lower elevation than the beginning of the pipeline, gravity will assist the flow and the pressure required at the beginning of the pipeline will be reduced by this elevation difference. The third component, delivery pressure at the terminus, simply ensures that a certain minimum pressure is maintained at the delivery point, such as a storage tank.

For example, if a liquid pipeline requires 50 psi to compensate for frictional losses and the minimum delivery pressure required is 20 psi, the total pressure required at the beginning of the pipeline is calculated as follows. If there were no elevation difference between the beginning of the pipeline and the delivery point, the elevation head is zero. Therefore, the total pressure P_t required is

$$P_t = 50 + 0 + 20 = 70 \text{ psi}$$

Next consider elevation changes. If the elevation at the beginning is 100 ft, the elevation at the delivery point is 150 ft, and the density of liquid is 50 lb/ft^3, the total pressure required is

$$P_t = 50 + \frac{(150 - 100) \times 50}{144} + 20 = 87.36 \text{ psi}$$

The middle term represents the static elevation head difference converted to psi, using Eq. (9.12). Finally, if the elevation at the beginning is 100 ft and the elevation at the delivery point is 70 ft,

$$P_t = 50 + \frac{(70 - 100) \times 50}{144} + 20 = 59.58 \text{ psi}$$

It can be seen from the preceding that the 30-ft advantage in elevation in the final case reduces the total pressure required by approximately 10.42 psi compared to the situation where there was no elevation difference between the beginning of the pipeline and delivery point (70 versus 59.58 psi).

Pumping horsepower. In the section on total pressure required, we calculated the total pressure required at the beginning of the pipeline to transport a given volume of liquid over a certain distance. We will now calculate the pumping horsepower (HP) required to accomplish this.

The brake horsepower (BHP) required to pump a liquid can be calculated from the following equation:

$$\text{BHP} = \frac{(P_d - P_s) \times Q}{1714 \times \text{effy}} \tag{9.29}$$

where BHP = brake horsepower
P_d = pump discharge pressure, psi
P_s = pump suction pressure, psi
Q = liquid flow rate, gal/min
effy = pump efficiency, decimal value

We define the hydraulic horsepower (HHP) as the horsepower when the pump efficiency is taken as 100 percent.

Consider an example in which the total pressure required to pump liquid from a pump station to a storage tank is 129 psi. If the flow rate is 500 gal/min and the liquid density is 50 lb/ft^3, we can calculate the pumping horsepower at 75 percent pump efficiency as follows:

$$\text{BHP} = \frac{(129 - 25) \times 500}{1714 \times 0.75} = 40.45$$

(The hydraulic horsepower for this result is $40.45 \times 0.75 = 30.34$ HP.)

In the preceding calculation we assumed the suction pressure at the pump to be 25 psi. If the pump is driven by an electric motor with a motor efficiency of 95 percent, the drive motor HP required will be

$$\text{Motor HP} = \frac{40.45}{0.95} = 42.58 \text{ HP}$$

The nearest standard size motor of 50 HP would be adequate for this application.

In SI units, the pumping power is expressed in kilowatts. If pressures are in kilopascals and the liquid flow rate is in m^3/h, the pumping power required is calculated from the following:

$$\text{Power in kW} = \frac{\text{pressure (kPa)} \times \text{ flow rate (m}^3\text{/h)}}{3600}$$

Therefore, the power equation for pumping a liquid, Eq. (9.29) can be modified for SI units as follows:

$$\text{Power} = \frac{(P_d - P_s) \times Q}{(3600 \times \text{effy})} \tag{9.29a}$$

where Power = pump power required, kW
P_d = pump discharge pressure, kPa
P_s = pump suction pressure, kPa
Q = liquid flow rate, m^3/h
effy = pump efficiency, decimal value

9.3.2 Single-phase gas flow

Cryogenic fluids may be treated as pure gas if no liquid or two-phase condition exists. In such instances the flow of cryogenic fluid may be treated as that of any other compressible fluid such as air or gas. At low pressures the gas may be assumed to obey the ideal gas equation and Boyle's and Charles's laws. At higher pressures, calculations must account for compressibility effects. The fundamental gas flow equation may be used to calculate the friction loss using a friction factor based on the Moody diagram or the Colebrook-White equation.

We will briefly introduce the concepts for calculating pressure drop using the preceding methods for a cryogenic fluid in the gaseous state. For details of compressible fluid flow please refer to Chaps. 5 and 7.

Gas properties

Mass. *Mass* is defined as the quantity of matter. It is measured in slugs (slug) and pounds (lb) in USCS units and kilograms in SI units. A given mass of gas will occupy a certain volume at a particular temperature and pressure. For example, a mass of gas may be contained in a volume of 500 ft^3 at a temperature of 60°F and a pressure of 100 psi. If the temperature is increased to 100°F, pressure remaining the same, the volume will change according to Charles's law. Similarly, if the volume remains the same, the pressure will increase with temperature. The mass always remains constant as long as gas is neither added nor subtracted from the system. This is referred to as conservation of mass.

Volume. *Volume* is defined as the space occupied by a given mass of gas at a specified temperature and pressure. Since gas expands to fill the container, volume varies with pressure and temperature. Thus a large volume of a given mass of gas at low pressure and temperature can be compressed to a small volume at a higher pressure and temperature. Volume is measured in ft^3 in USCS units and m^3 in SI units.

Density. *Density* of gas is defined as mass per unit volume. Thus,

$$\text{Density } \rho = \frac{m}{V} \tag{9.30}$$

where ρ = density of gas
m = mass of gas
V = volume of gas

Density is expressed in slug/ft^{3}or lb/ft^3 in USCS units and kg/m^3 in SI units.

When a gas flows through a pipe under steady flow conditions, the mass flow rate at any cross section of the pipe is constant as long as no gas enters or leaves the pipe between the inlet and the outlet of the pipe. Mass is the product of volume and density. The volume flow rate may be expressed as the average velocity times the cross-sectional area of the pipe. Therefore,

$$\text{Volume flow rate} = \text{average velocity} \times \text{pipe cross-sectional area} \tag{9.31}$$

$$\text{Mass flow rate} = \text{volume flow rate} \times \text{density} \tag{9.32}$$

or

$$M = \rho A v \tag{9.33}$$

where M = mass flow rate, lb/s
ρ = density of gas, lb/ft^3
A = pipe cross-sectional area, ft^2
v = velocity of flow, ft/s

If the density is in slug/ft^3, the mass flow rate is in slug/s.

Specific gravity. The *specific gravity*, or simply the gravity, of gas is measured relative to the density of air at a particular temperature as follows:

$$\text{Gas gravity} = \frac{\text{density of gas}}{\text{density of air}}$$

Both densities are measured at the same temperature and pressure. For example, a gas may be referred to as having a specific gravity of 0.65 (air = 1.00) at 60°F. This means that the gas is 65 percent as heavy as air.

The specific gravity of a gas can also be represented as a ratio of its molecular weight to that of air.

$$\text{Specific gravity} = \frac{M_g}{M_{\text{air}}}$$

or

$$G = \frac{M_g}{28.9625} \tag{9.34}$$

where G = specific gravity of gas
M_g = molecular weight of gas
M_{air} = molecular weight of air

In Eq. (9.34) we have used 28.9625 for the apparent molecular weight of air. Sometimes the molecular weight of air is rounded off to 29.0 and therefore the gas gravity becomes $M_g/29.0$. Nitrogen has a molecular weight of 28.0134. Therefore, the gravity of N_2 is 28.0134/28.9625 = 0.9672 relative to air = 1.00.

Viscosity. The *viscosity* of a fluid is defined as its resistance to flow. For gases, the viscosity is very low compared to that of liquids. (For example, water has a viscosity of 1.0 cP compared to a nitrogen gas viscosity of 0.0174 cP). However, the viscosity of a gas is an important property in the study of gas flow in pipes. Two types of viscosities are used. Dynamic viscosity μ, also known as the absolute viscosity, is expressed in lb/(ft · s) in USCS units and poise (P) in SI units. The kinematic viscosity ν is calculated by dividing the dynamic viscosity by the density. Thus the relationship between the two viscosities is expressed as follows:

$$\text{Kinematic viscosity } \nu = \frac{\text{dynamic viscosity } \mu}{\text{density}}$$

Kinematic viscosity is measured in ft^2/s in USCS units and stokes (St) in SI units. Other units of viscosity include centipoise (cP) and centistokes (cSt). The viscosity of a pure gas such as air or methane depends only on its temperature and pressure. Viscosities of common gases at atmospheric conditions are shown in Fig. 9.9.

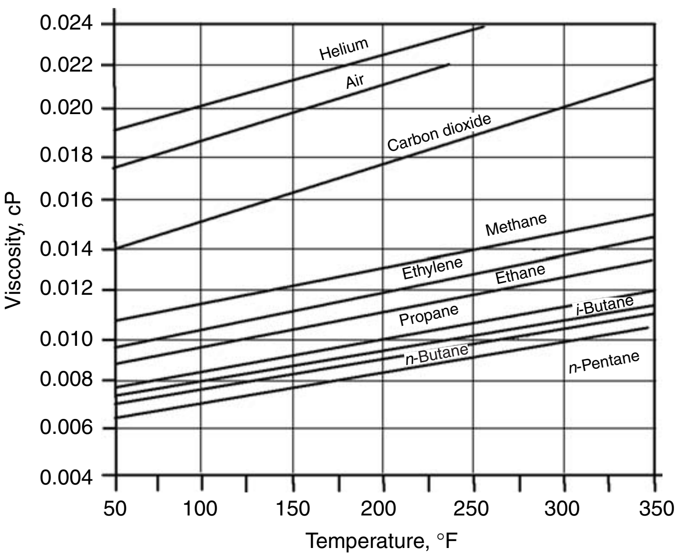

Figure 9.9 Viscosities of common gases.

Ideal gases. An ideal gas is one in which the volume occupied by its molecules is negligible compared to that of the total gas. In addition there is no attraction or repulsion between the gas molecules and the container. The molecules of an ideal gas are considered to be perfectly elastic, and there is no loss in internal energy due to collision between the gas molecules. Ideal gases follow Boyle's and Charles's laws and can be represented by the ideal gas equation or the perfect gas equation. We will discuss the behavior of ideal gases first followed by that of real gases.

The molecular weight M of a gas represents the weight of one molecule of gas. The given mass m of gas will thus contain m/M number of moles. Therefore,

$$n = \frac{m}{M} \tag{9.35}$$

For example, the molecular weight of methane is 16.043 and that of nitrogen N_2 is 28.0134. Then 100 lb of N_2 will contain approximately 4 moles.

The ideal gas law states that the pressure, volume, and temperature of a given quantity of gas are related by the ideal gas equation as follows:

$$PV = nRT \tag{9.36}$$

where P = absolute pressure, psia
V = gas volume, ft^3
n = number of lb moles as defined in Eq. (9.35)
R = universal gas constant
T = absolute temperature of gas, °R (°F + 460)

In USCS units R has a value of 10.732 psia ft^3/(lb · mol · °R)

Using Eq. (9.35) we can restate the ideal gas equation as follows:

$$PV = \frac{mRT}{M} \tag{9.37}$$

where m represents the mass and M is the molecular weight of gas. The ideal gas equation is only valid at pressures near atmospheric pressure. At high pressures it must be modified to include the effect of compressibility.

Two other equations used with gases are called Boyle's law and Charles's law. Boyle's law states that the pressure of a given quantity of gas varies inversely as its volume provided the temperature is kept constant. Mathematically, Boyle's law is expressed as

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

or

$$P_1V_1 = P_2V_2 \tag{9.38}$$

where P_1 and V_1 are the initial pressure and volume, respectively, at condition 1, and P_2 and V_2 refer to condition 2. In other words, $PV =$ constant.

Charles's law relates to volume-temperature and pressure-temperature variations for a given mass of gas. Thus keeping the pressure constant, the volume of gas will vary directly with the absolute temperature. Similarly, keeping the volume constant, the absolute pressure will vary directly with the absolute temperatures. These are represented mathematically as follows:

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \qquad \text{for constant pressure} \tag{9.39}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \qquad \text{for constant volume} \tag{9.40}$$

Note that in the preceding discussions, the gas temperature is always expressed in absolute scale. In USCS units, the absolute temperature is stated as degrees Rankine (°R) equal to °F + 460.

In SI units the absolute temperature is expressed in kelvin (K) equal to °C + 273.

Thus 60°F is 60 + 460 = 520°R and 20°C is 20 + 273 = 293 K.

Pressures used in Eqs. (9.38) and (9.40) must also be in absolute units, such as lb/in^{2}absolute or kilopascals absolute. The absolute pressure is obtained by adding the atmospheric base pressure (usually 14.7 psia in USCS units or 101 kPa in SI units) to the gauge pressure. Other units of pressure in SI units include megapascals (MPa) and bars. Refer to App. A for conversion factors between various units.

$$\text{psia} = \text{psig} + \text{base pressure}$$

$$\text{kPa (abs)} = \text{kPa (gauge)} + \text{base pressure}$$

Real gases. The ideal gas equation is applicable only when the pressure of the gas is very low or near atmospheric pressure. When gas pressures and temperatures are higher, the ideal gas equation will not give accurate results. The calculation errors may be as high as 500 percent. An equation of state is generally used for calculating the properties of gases at higher temperatures and pressures.

Real gases behave according to a modified version of the ideal gas law discussed earlier. The modifying factor is known as the compressibility factor Z. This is also called the gas deviation factor. Z is a dimensionless number less than 1.0 and varies with the temperature, pressure, and physical properties of the gas.

The real gas equation can be written as follows:

$$PV = ZnRT \tag{9.41}$$

where P = absolute pressure, psia
V = gas volume, ft^3
Z = gas deviation factor or compressibility factor, dimensionless
T = absolute temperature of gas, °R
n = number of lb moles as defined in Eq. (9.35)
R = universal gas constant, 10.732 psia ft^3/(lb · mol · °R)

Critical properties. The *critical temperature* of a pure gas is the temperature above which it cannot be liquefied regardless of the pressure. The *critical pressure* of a pure substance is defined as the pressure above which liquid and gas cannot coexist, regardless of the temperature. The *reduced temperature* is simply the temperature of the gas divided by its critical temperature. Similarly, the *reduced pressure* is simply the pressure of the gas divided by its critical pressure, both temperature and pressure being in absolute units. Table 9.1 lists critical properties of common gases used in cryogenic piping systems.

The reduced temperature is defined as the ratio of the temperature of the gas to its critical temperature. Similarly, the reduced pressure is defined as the pressure of the gas divided by its critical pressure. Both reduced temperature and reduced pressure are dimensionless terms. For example, nitrogen has a critical temperature of −232.48°F and a critical pressure of 492.8 psia. If the gas temperature and pressure are 100°F and 200 psia, respectively, the reduced temperature and pressure are calculated as follows:

$$T_r = \frac{100 + 460}{-232.48 + 460} = 2.46$$

$$P_r = \frac{200}{492.8} = 0.406$$

Compressibility factor. The compressibility factor, or gas deviation, factor is a measure of how close a real gas is to an ideal gas. The compressibility factor Z is a dimensionless number close to 1.00. It is independent of the quantity of gas. It depends on the gravity of gas, its temperature, and pressure. For example, a sample of natural gas may have a Z value of 0.8595 at 1000 psia and 70°F. Charts are available that show the variation of Z with temperature and pressure. At pressures close to atmospheric pressure the value of Z is almost 1.00.

Many researchers have correlated the compressibility factor Z against reduced temperatures and pressures. It has been found that gases such as carbon dioxide and nitrogen follow approximately the same variation in compressibility factors with respect to the reduced temperatures and pressures as shown in Fig. 9.10.

Several methods are available to calculate the value of Z at temperature T and pressure P. One approach requires knowledge of the critical temperature and critical pressure of the gas. The reduced temperature

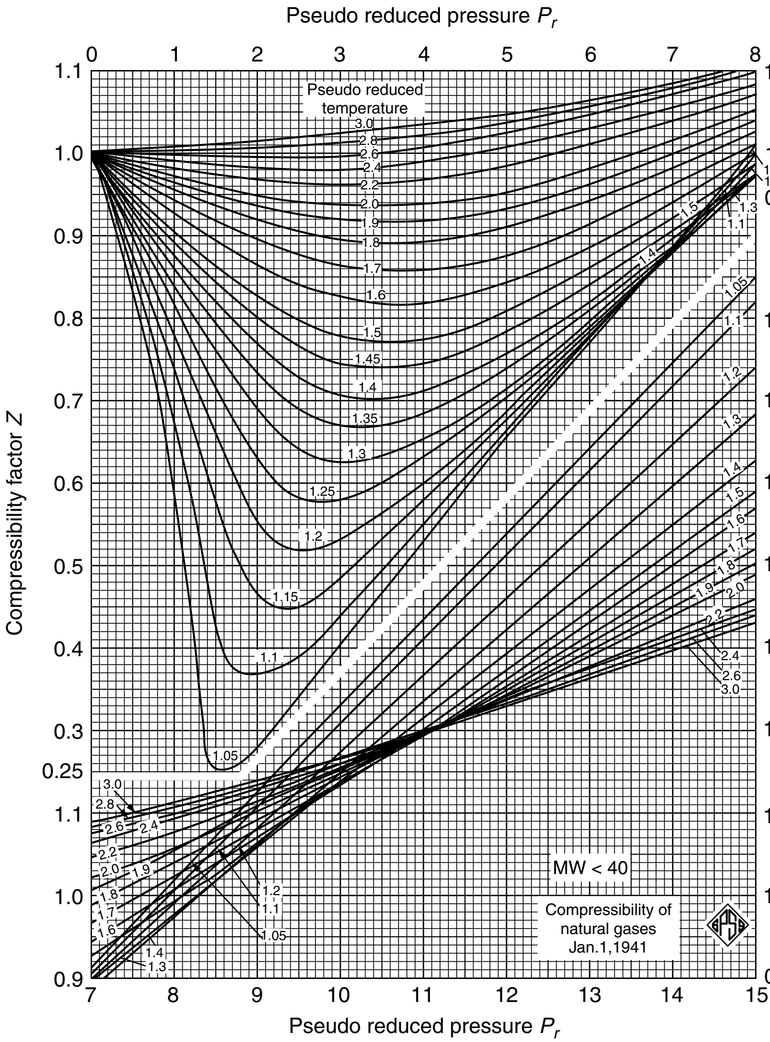

Figure 9.10 Compressibility factor. (Reproduced with permission from Gas Processors Association, *Engineering Data Book*, vol. II, Tulsa, Oklahoma, 1994.)

T_r and pressure P_r are then calculated from the critical temperature and pressure as follows:

$$T_r = \frac{T}{T_c} \tag{9.42}$$

$$P_r = \frac{P}{P_c} \tag{9.43}$$

Temperatures and pressures in these equations are in absolute units.

The value of the compressibility factor Z is then calculated from the reduced pressures P_r using Fig. 9.10. For example, if the reduced temperature and pressure of the gas calculated above is $T_r = 1.5$ and $P_r = 2.0$, from the chart we get $Z = 0.825$.

In the case of a gas flowing through a pipeline, since the pressure varies along the pipeline, the compressibility factor Z must be calculated based on an average pressure at a particular location on the pipeline. If two locations have pressures of P_1 and P_2, we could use a simple average pressure of $(P_1 + P_2)/2$. However, a more accurate value of the average pressure is calculated using the following equation:

$$P_{\mathrm{avg}} = \frac{2}{3}\left(P_1 + P_2 - \frac{P_1 \times P_2}{P_1 + P_2}\right) \tag{9.44}$$

Pressure drop due to friction. As gas flows through a pipeline, energy is lost due to friction between the gas molecules and the pipe wall. This is evident in the form of a pressure gradient along the pipeline. Before we introduce the various equations to calculate the amount of pressure drop due to friction, we will discuss a couple of important parameters related to the flow of gas in a pipeline. The first of these is the velocity of flow and the other is the Reynolds number.

Velocity. When gas flows at a particular volume flow rate Q through a pipeline of inside diameter D, the average velocity of the gas can be calculated knowing the cross sectional area of the pipe as follows:

$$v = \frac{Q}{A} \tag{9.45}$$

Since the flow rate Q is a function of the gas pressure and temperature, we must relate the velocity to the volume flow at standard conditions (such as 60°F and 14.7 psia). If the density of gas at the flowing temperature is ρ and the density at standard conditions is ρ_b from the law of conservation of mass, the mass flow rate at standard conditions must equal the mass flow rate at flowing conditions. Therefore,

$$\rho_b Q_b = \rho Q \tag{9.46}$$

Using the real gas equation, Eq. (9.46) can be simplified as

$$\rho_b = \frac{P_b M}{Z_b R T_b} \tag{9.47}$$

$$\frac{\rho_b}{\rho} = \frac{P_b}{P}\frac{Z}{Z_b}\frac{T}{T_b} \tag{9.48}$$

$$Q = Q_b \frac{P_b}{P}\frac{T}{T_b}\frac{Z}{Z_b} = Q_b \frac{T}{P}\frac{P_b}{T_b}\frac{Z}{Z_b} \tag{9.49}$$

$$v = \frac{4 \times 10^{-6}}{86{,}400\pi(D/12)^2} Q_b \frac{T}{P}\frac{P_b}{T_b}\frac{Z}{Z_b} = (2.653 \times 10^{-9})\frac{Q_b}{D^2}\frac{T}{P}\frac{P_b}{T_b}\frac{Z}{Z_b} \tag{9.50}$$

where v = velocity of flowing gas, ft/s
D = pipe inside diameter, in
T = temperature of flowing gas, °R
P = pressure of gas, psia
Q_b = flow rate at standard conditions, standard ft^3/day (SCFD)
P_b = base pressure, psia
T_b = base temperature, °R

Example 9.11 Calculate the gas velocity in a pipeline at 1000 psig pressure and 80°F temperature. The pipeline is NPS 16, 0.250-in wall thickness. The flow rate is 80 million SCFD (MMSCFD). Use $Z = 0.89$.

Solution

$$\text{Diameter } D = 16 - 0.5 = 15.5 \text{ in}$$

$$P = 1000 + 14.7 = 1014.7 \text{ psia}$$

$$T = 80 + 460 = 540°\text{R}$$

The gas velocity is calculated from Eq. (9.50) as

$$v = (2.653 \times 10^{-3})\frac{80 \times 10^6}{(15.5)^2}\frac{540}{1014.7}\frac{14.7}{520}\frac{0.89}{1.0} = 11.83 \text{ ft/s}$$

Reynolds number. The Reynolds number of flow was introduced earlier in Sec. 9.3.1. It is a dimensionless parameter that depends on the flow rate, pipe diameter, and gas properties such as density and viscosity. The Reynolds number is used to characterize flow type such as laminar flow and turbulent flow. It was defined in Eq. (9.7) as $\text{Re} = vD\rho/\mu$. The Reynolds number for gas flow is calculated from a modified version of this equation as follows:

$$\text{Re} = 0.0004778\,\frac{P_b}{T_b}\frac{GQ}{\mu D} \tag{9.51}$$

where P_b = base pressure, psia
T_b = base temperature, °R
G = gas gravity (air = 1.0)
Q = gas flow rate, SCFD
D = pipe internal diameter, in
μ = gas viscosity, lb/(ft · s)

In SI units the Reynolds number is given by

$$\mathrm{Re} = 0.5134 \frac{P_b}{T_b} \frac{GQ}{\mu D} \tag{9.52}$$

where P_b = base pressure, kPa
T_b = base temperature, K
G = gas gravity (air = 1.0)
Q = gas flow rate, m^3/day
D = pipe internal diameter, mm
μ = gas viscosity, P

Laminar flow is defined as flow that causes the Reynolds number to be below a threshold value such as 2000 to 2100. *Turbulent flow* is defined as a Reynolds number greater than 4000. The range of Reynolds numbers between 2000 and 4000 characterizes an unstable flow regime known as *critical* flow.

Example 9.12 Calculate the Reynolds number of flow for an NPS 16 (0.375-in wall thickness) gas pipeline at a flow rate of 150 MMSCFD. Flowing temperature = 80°F, gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), base pressure = 14.73 psia, and base temperature = 60°F.

Solution Using Eq. (9.51) the Reynolds number is

$$\mathrm{Re} = 0.0004778 \frac{P_b}{T_b} \frac{GQ}{\mu D}$$

$$= 0.0004778 \frac{14.73}{460 + 80} \times \frac{0.6 \times 150 \times 10^6}{0.000008 \times 15.25} = 9{,}614{,}746$$

Therefore, the flow is turbulent since Re > 4000.

Isothermal flow. Isothermal gas flow occurs at constant temperature. Therefore, the gas pressure, volume, and density change, but the gas temperature remains the same. To maintain the constant temperature, isothermal flow requires heat to be transferred out of the gas. Gas flowing in long pipes can be considered to be under isothermal flow. In such cases, the pressure, flow rate, and temperature of a gas flowing through a pipe are related by the following equation:

$$P_1^2 - P_2^2 = \frac{M^2 RT}{gA^2}\left(f\frac{L}{D} + 2\log_e \frac{P_1}{P_2}\right) \tag{9.53}$$

where P_1 = upstream pressure at point 1
P_2 = downstream pressure at point 2
M = mass flow rate of gas
R = gas constant
T = absolute temperature of gas
g = acceleration due to gravity
A = cross-sectional area of pipe
f = friction factor, dimensionless
L = pipe length
D = inside diameter of pipe

Equation (9.53) can be used for small pressure drops and when elevation differences between points along the pipe are ignored. The friction factor f used in Eq. (9.53) is a dimensionless number that depends upon the pipe diameter, the pipe roughness, and the Reynolds number of flow. Knowing the Reynolds number, the friction factor is found from the Moody diagram (Fig. 9.3).

A consistent set of units must be used in Eq. (9.53). An example will illustrate the use of the isothermal flow equation.

Example 9.13 Air flows at 50 ft/s through a 2-in inside diameter pipe at 80°F at an initial pressure of 100 psig. If the pipe is horizontal and 1000 ft long, calculate the pressure drop considering isothermal flow. Use a friction factor $f = 0.02$.

Solution First calculate the density of air at 80°F. From Chap. 5, Table 5.1,

$$\text{Density of air at } 80^\circ\text{F} = 0.0736 \text{ lb/ft}^3$$

This density is at the standard condition of 14.7 psia. Using Eq. (9.38) we calculate the density at 100 psig as

$$\rho = \frac{100 + 14.7}{14.7} \times 0.0736 = 0.5743 \text{ lb/ft}^3$$

The cross-sectional area of the pipe is

$$A = 0.7854 \times \left(\frac{2}{12}\right)^2 = 0.0218 \text{ ft}^2$$

Next, the mass flow rate can be calculated from the density, velocity, and the pipe cross-sectional area using Eq. (9.33) as follows:

$$M = \rho A v = 0.5743 \times 0.0218 \times 50 = 0.6265 \text{ lb/s}$$

Using Eq. (9.53) we can write

$$[(100 + 14.7)^2 - P_2{}^2] \times (144)^2 = (0.6265)^2 \times (53.3) \times (80 + 460) \times \frac{(0.02 \times 1000 \times 12/2) + 2\log_e(114.7/P_2)}{32.2 \times 0.0218 \times 0.0218}$$

Simplifying we get

$$13{,}156.09 - P_2{}^2 = 35.6016\left(120.0 + 2\log_e \frac{114.7}{P_2}\right)$$

We will first calculate P_2 by ignoring the second term containing P_2 on the right-hand side of the equation. This is acceptable since the term being ignored is a much smaller value compared to the first term of 120.0 within the parentheses.

Therefore the first approximation to P_2 is calculated from

$$13{,}156.09 - P_2{}^2 = 35.6016 \times 120$$

or

$$P_2 = 94.25 \text{ psia}$$

We can recalculate a better solution for P_2 by substituting the value just calculated in the preceding equation, this time including the $\log_e(114.7/P_2)$ term:

$$13{,}156.09 - P_2{}^2 = 35.6016 \times \left(120 + 2\log_e \frac{114.7}{94.25}\right)$$

Solving for P_2 we get

$$P_2 = 94.18 \text{ psia}$$

which is quite close to our first approximation of $P_2 = 94.25$. Therefore

$$\text{Pressure drop} = P_1 - P_2 = 114.7 - 94.18 = 20.52 \text{ psig}$$

Example 9.14 Air flows through a 2000-ft-long NPS 6 pipeline at an initial pressure of 150 psig and a temperature of 80°F. If the flow is considered isothermal, calculate the pressure drop due to friction at a flow rate of 5000 SCFM. Assume smooth pipe.

Solution We start by calculating the Reynolds number from the flow rate. Assuming a 6-in inside diameter pipe:

$$\text{Cross-sectional area } A = 0.7854\left(\frac{6}{12}\right)^2 = 0.1964 \text{ ft}^2$$

$$\text{Velocity } v = \frac{\text{flow rate}}{\text{area}} = \frac{5000}{60 \times 0.1964} = 424.3 \text{ ft/s}$$

Next we need to find the density and viscosity of air at 80°F and 150 psig pressure. From Chap. 5, Table 5.1, at 80°F, the density of air $\rho = 0.0736$ lb/ft^3 at 14.7 psia and the viscosity $\mu = 3.85 \times 10^{-7}$ lb/ft^2.

The density must be corrected for the higher pressure of 150 psig.

$$\rho = 0.0736 \times \frac{164.7}{14.7} = 0.8246 \text{ lb/ft}^3 \text{ at 150 psig}$$

The Reynolds number from Eq. (9.7) is

$$\text{Re} = \frac{424.3 \times 0.5 \times 0.8246}{32.2 \times 3.85 \times 10^{-7}} = 1.41 \times 10^7$$

From the Moody diagram, for smooth pipe, the friction factor is

$$f = 0.0077$$

The mass flow rate will be calculated first from the given volume flow rate and density:

$$M = \text{volume rate} \times \text{density}$$

From Chap. 5, Table 5.1, the density of air at 60°F (standard condition) is

$$\text{Density} = 0.0764 \text{ lb/ft}^3$$

Therefore, the mass flow rate is

$$M = 5000 \times 0.0764 = 382 \text{ lb/min} = 6.367 \text{ lb/s}$$

Using Eq. (9.53) for isothermal flow

$$\left[(164.7)^2 - {P_2}^2\right] \times (144)^2 = \frac{(6.367)^2 \times 53.3 \times 540}{32.2 \times (0.1964)^2} \times \left(0.0077 \times \frac{2000}{0.5} + 2\log_e \frac{164.7}{P_2}\right)$$

This equation for P_2 must be solved by trial and error, as in Example 9.14. Solving, we get $P_2 = 160.4$ psia. Thus

$$\text{Pressure drop due to friction} = P_1 - P_2 = 164.7 - 160.4 = 4.3 \text{ psi}$$

Example 9.15 Air flows through a 500-m-long, 200-mm inside diameter pipeline at 20°C. The upstream and downstream pressures are 1035 and 900 kPa, respectively. Calculate the flow rate through the pipeline assuming isothermal conditions. Pressures are in absolute values, and the relative roughness of pipe is 0.0003.

Solution We will use the isothermal equation (9.53) for calculating the flow rate through the pipeline. The friction factor f depends on the Reynolds number which in turn depends on the flow rate which is unknown. Therefore, we will assume an initial value for the friction factor f and calculate the mass flow rate from Eq. (9.53). This mass flow rate will then be used to calculate the flow velocity and hence the corresponding Reynolds number. From this Reynolds number using the Moody diagram the friction factor will be found. The mass flow rate will be recalculated from the newly found friction factor.

The process is continued until successive values of the mass flow rate are within 1 percent or less.

Assume $f = 0.01$ initially. From Eq. (9.53) we get

$$(1035)^2 - (900)^2 = \frac{M^2 \times 29.3 \times 293}{9.81 \times (0.7854 \times 0.04)^2} \left(0.01 \times \frac{500}{0.2} + 2 \log_e \frac{1035}{900}\right)$$

Solving for M, we get

$$M = 0.108 \text{ kN/s}$$

Next, calculate the density at 20°C from the perfect gas equation.

$$\rho = \frac{P}{RT} = \frac{1035}{29.3 \times 293} = 0.1206 \text{ kN/m}^3$$

The viscosity of air from Chap. 5, Table 5.1,

$$\mu = 1.81 \times 10^{-5} \text{ Pa} \cdot \text{s}$$

The flow velocity is calculated from the mass flow rate as follows:

$$M = \rho A v$$

Therefore

$$0.108 = 0.1206 \times (0.7854 \times 0.04)\, v$$

Thus, velocity is

$$v = 28.505 \text{ m/s}$$

The Reynolds number is calculated from Eq. (9.7) as

$$\text{Re} = \frac{0.1206}{9.81} \times 28.505 \times \frac{0.2}{1.81 \times 10^{-8}} = 3.87 \times 10^6$$

For this Reynolds number, from the Moody diagram we get the friction factor for a relative roughness $e/d = 0.0003$ as follows,

$$f = 0.0151$$

Using this value of f, we recalculate the mass flow rate as follows:

$$(1035)^2 - (900)^2 = \frac{M^2 \times 29.3 \times 293}{9.81 \times (0.7854 \times 0.04)^2} \left(0.0151 \times \frac{500}{0.2} + 2 \log_e \frac{1035}{900}\right)$$

Solving for M, we get

$$M = 0.088 \text{ kN/s}$$

The earlier value was $M = 0.108$ kN/s. This represents a 22 percent difference, and hence we must recalculate the friction factor and repeat the process for a better value of M. Based on the recently calculated value of $M = 0.088$ we will recalculate the velocity and the Reynolds number as follows. Using proportions, the new velocity is

$$v = \frac{0.088}{0.108} \times 28.505 = 23.226 \text{ m/s}$$

The new Reynolds number is

$$\text{Re} = \frac{23.226}{28.505} \times 3.87 \times 10^6 = 3.15 \times 10^6$$

Next from the Moody diagram for this Reynolds number we get a friction factor

$$f = 0.0152$$

Using this value of f in the isothermal flow equation, we get a new value of mass flow rate as follows:

$$(1035)^2 - (900)^2 = \frac{M^2 \times 29.3 \times 293}{9.81 \times (0.7854 \times 0.04)^2} \left(0.0152 \times \frac{500}{0.2} + 2 \log_e \frac{1035}{900}\right)$$

Solving for M, we get

$$M = 0.0877 \text{ kN/s}$$

The earlier value was $M = 0.088$ kN/s. This represents a difference of 0.34 percent, and hence we stop iterating any further. The flow rate through the pipeline is 0.0877 kN/s.

Example 9.16 Air flows through a 1500-ft-long NPS 10 (0.25-in wall thickness) pipeline at a mass flow rate of 23 lb/s. What pressure is required at the upstream end to provide a delivery pressure of 80 psig? The air flow temperature is 80°F. Consider isothermal flow. Assume the friction factor is 0.02.

Solution

$$\text{Mass flow rate } M = 23.0 \text{ lb/s} \qquad f = 0.02$$

The cross-sectional area of pipe, with a 10.75-in outside diameter and a 0.25-in wall thickness, is

$$A = 0.7854 \left(\frac{10.25}{12}\right)^2 = 0.573 \text{ ft}^2$$

From the isothermal flow Eq. (9.53) and substituting the given values, we get

$$[(P_1)^2 - (94.7)^2] \times (144)^2 = \frac{23^2 \times 53.3 \times 540}{32.2 \times (0.573)^2} \times \left(0.02 \times \frac{1500 \times 12}{10.25} + 2 \log_e \frac{P_1}{94.7}\right)$$

Assume $P_1 = 100$ psig initially and substitute this value on the right-hand side of the preceding equation to calculate the next approximation for P_1.

Continue this process until successive values of P_1 are within 1 percent or less. Solving we get $P_1 = 106.93$ psia by successive iteration.

Therefore, the upstream pressure required is $106.93 - 14.7 = 92.23$ psig. The pressure loss in the 1500-ft-long pipe is $92.23 - 80 = 12.23$ psi.

Example 9.17 Consider isothermal flow of air in a 6-in inside diameter pipe at 60°F. The upstream and downstream pressures for a 500-ft section of horizontal length of pipe are 80 and 60 psia, respectively. Calculate the mass flow rate of air assuming the pipe is smooth.

Solution From Eq. (9.53) for isothermal flow, we get

$$P_1^2 - P_2^2 = \frac{M^2RT}{gA^2}\left(f\frac{L}{D} + 2\log_e\frac{P_1}{P_2}\right)$$

We must first calculate the Reynolds number Re and the friction factor f. Since Re depends on the flow rate (unknown), we will assume a value of f and calculate the flow rate from the preceding equation. We will then verify if the assumed f was correct. Some adjustment may be needed in the f value to get convergence.

Assume $f = 0.01$ in the preceding pressure drop equation. Substituting the given value, we get

$$(144)^2(80^2 - 60^2) = \frac{M^2 \times 53.3 \times 520}{32.2\,(0.7854 \times 0.5 \times 0.5)^2}\left(0.01\,\frac{500}{0.5} + 2\log_e\frac{80}{60}\right)$$

Solving for the mass flow rate,

$$M = 15.68 \text{ lb/s}$$

The gas density ρ is

$$\rho = \frac{P}{RT} = \frac{80 \times 144}{53.3 \times 520} = 0.4156 \text{ lb/ft}^3$$

The mass flow rate is then calculated from Eq. (9.33).

$$\text{Mass flow} = \text{density} \times \text{volume flow rate} = \text{density} \times \text{area} \times \text{velocity}$$

Therefore,

$$M = \rho A v$$

Substituting the calculated values in Eq. (9.33) we get

$$15.68 = (0.4156)(0.7854 \times 0.5 \times 0.5)\,v$$

$$\text{Flow velocity } v = 192.15 \text{ ft/s}$$

The Reynolds number is then

$$\text{Re} = \frac{\rho d v}{\mu} = \frac{0.4156}{32.2}(0.5)\frac{192.15}{3.78 \times 10^{-7}} = 3.28 \times 10^6$$

From the Moody diagram (Fig. 9.3), the Darcy friction factor $f = 0.0096$. We assumed $f = 0.01$ initially. This is quite close to the newly calculated value of f. If we use the value of $f = 0.0096$ and recalculate the mass flow rate, we get $M = 15.99$ lb/s.

Adiabatic flow. *Adiabatic flow* of gas occurs when there is no heat transfer between the flowing gas and its surroundings. Adiabatic flow generally includes friction. When friction is neglected, the flow becomes isentropic.

Isentropic flow. When gas flows through a conduit such that it is adiabatic and frictionless, the flow is termed *isentropic flow*. This type of flow also means that the entropy of the gas is constant. If the flow occurs very quickly such that heat transfer does not occur and the friction is small, the flow may be considered isentropic. In reality, high-velocity flow occurring over short lengths of pipe with low friction and low heat transfer may be characterized as isentropic flow. The pressures, velocities, and gas density in isentropic flow are related by the following equation:

$$\frac{v_2{}^2 - v_1{}^2}{2g} = \frac{P_1}{\rho_1}\frac{k}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{(k-1)/k}\right] \tag{9.54}$$

or

$$\frac{v_2{}^2 - v_1{}^2}{2g} = \frac{P_2}{\rho_2}\frac{k}{k-1}\left[\left(\frac{P_1}{P_2}\right)^{(k-1)/k} - 1\right] \tag{9.55}$$

where v_1 = velocity at upstream location
v_2 = velocity at downstream location
P_1 = pressure at upstream location
P_2 = pressure at downstream location
k = specific heat ratio
g = acceleration of gravity
ρ_1 = density at upstream location
ρ_2 = density at downstream location

It can be seen from Eqs. (9.54) and (9.55) that the pressure drop $P_1 - P_2$ between the upstream and downstream locations in a pipe depends only on the pressures, velocities, and specific heat ratio of air. Unlike isothermal flow, discussed earlier, no frictional term exists in the isentropic flow equation. This is because, by definition, isentropic flow is considered to be a frictionless process.

Example 9.18 Isentropic flow of air occurs in a 6-in inside diameter pipeline. If the upstream pressure and temperature are 50 psig and 70°F, respectively, and the velocity of air at the upstream and downstream locations are 50 and 120 ft/s, respectively, calculate the pressure drop assuming $k = 1.4$.

Solution We will use Eq. (9.54) for isentropic flow of air. First let us calculate the ratio $k/(k-1)$ and its reciprocal.

$$\frac{k}{k-1} = \frac{1.4}{0.4} = 3.5$$

$$\frac{k-1}{k} = \frac{0.4}{1.4} = 0.2857$$

The term P_1/ρ_1 in Eq. (9.54) may be replaced with the term RT_1 using the perfect gas equation. Substituting the given values in Eq. (9.54), we get

$$\frac{(120)^2 - (50)^2}{2 \times 32.2} = 53.3 \times (70 + 460) \times 3.5 \times \left[1 - \left(\frac{P_2}{150 + 14.7}\right)^{0.2857}\right]$$

Simplifying and solving for P_2 we get

$$P_2 = 163.63 \text{ psia}$$

Therefore the pressure drop is

$$P_1 - P_2 = 164.7 - 163.63 = 1.07 \text{ psig}$$

Pressure drop calculations. In the previous sections we discussed flow and pressure drops considering ideal gas and low pressures. In reality at high pressures, the ideal gas equation is not correct. We must include the effect of the compressibility factor in the flow equation. This section will introduce pressure drop calculations in a flowing gas pipeline, using the general flow equation. This is also referred to as the fundamental flow equation. It relates the flow rate, gas properties, pipe size, and flowing temperature to the upstream and downstream pressures in a pipeline segment. The internal roughness of the pipe is used to calculate a friction factor using the Moody diagram or the Colebrook equation based on the Reynolds number.

General flow equation. The general flow equation for the steady-state isothermal flow in a gas pipeline is as follows;

$$Q = 77.54 \frac{1}{\sqrt{f}} \frac{T_b}{P_b} \left(\frac{P_1^2 - P_2^2}{GT_f LZ}\right)^{0.5} D^{2.5} \tag{9.56}$$

where Q = volume flow rate, SCFD
P_b = base pressure, psia
T_b = base temperature, °R
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia

f = Darcy friction factor, dimensionless
G = gas gravity (air = 1.00)
T_f = average gas flow temperature, °R
L = pipe segment length, mi
Z = gas compressibility factor, dimensionless
D = pipe inside diameter, in

Since the pressure at the inlet of the pipe segment is P_1 and that at the outlet is P_2, an average pressure must be used to calculate the gas compressibility factor Z at the average flowing temperature T_f. Instead of an arithmetic average $(P_1 + P_2)/2$ the following formula is used to calculate the average gas pressure in the pipe segment:

$$P_{\text{avg}} = \frac{2}{3}\left(P_1 + P_2 - \frac{P_1 P_2}{P_1 + P_2}\right) \tag{9.57}$$

It must be noted that Eq. (9.56) does not include any elevation effects. The effect of elevation difference between the upstream and downstream ends of the pipe segment is taken into account by modifying the pipe segment length L and the term $P_1{}^2 - P_2{}^2$ in Eq. (9.56). If the elevation of the upstream end is H_1 and at the downstream end is H_2, the length of the pipe segment L is replaced with an equivalent length L_e as follows:

$$L_e = \frac{L(e^s - 1)}{s} \tag{9.58}$$

where L_e = equivalent length of pipe, mi
L = length of pipe between upstream and downstream ends, mi
s = elevation correction factor, dimensionless

The parameter s depends on the elevation difference $H_2 - H_1$ and in USCS units is calculated as follows:

$$s = \frac{0.0375G(H_2 - H_1)}{T_f Z} \tag{9.59}$$

The calculation for L_e shown in Eq. (9.58) is correct only if we assume a single slope between point 1 (upstream) and point 2 (downstream). If instead a series of slopes are to be considered, we define a parameter j as follows:

$$j = \frac{e^s - 1}{s} \tag{9.60}$$

The term j must be calculated for each slope of each pipe segment of length L_1, L_2, etc., that make up the length L. The equivalent length then must be calculated as follows:

$$L_e = j_1 L_1 + j_2 L_2 e^{s1} + j_3 L_3 e^{s2} + \cdots \tag{9.61}$$

where j_1, j_2, etc., are calculated for each rise or fall in the elevation for pipe segments between the upstream and downstream ends. The parameters s_1, s_2, etc., are calculated for each segment in accordance with Eq. (9.59).

Finally, the term $P_1{}^2 - P_2{}^2$ in Eq. (9.56) is modified to $P_1{}^2 - e^s P_2{}^2$ as follows:

$$Q = 77.54 \frac{1}{\sqrt{f}} \frac{T_b}{P_b} \left(\frac{P_1{}^2 - e^s P_2{}^2}{G T_f L_e Z} \right)^{0.5} D^{2.5} \tag{9.62}$$

where s and L_e are defined by

$$s = \frac{0.0375\, G (H_2 - H_1)}{T_f Z} \tag{9.63}$$

$$L_e = \frac{L(e^s - 1)}{s} \tag{9.64}$$

In SI units, Eq. (9.62) becomes

$$Q = (11.4946 \times 10^{-4}) \frac{1}{\sqrt{f}} \frac{T_b}{P_b} \left(\frac{P_1{}^2 - e^s P_2{}^2}{G T_f L_e Z} \right)^{0.5} D^{2.5} \tag{9.65}$$

and the elevation adjustment term s is given by

$$s = \frac{0.0684 G (H_2 - H_1)}{T_f Z} \tag{9.66}$$

where Q = gas flow rate at standard conditions, m^3/day
T_b = base temperature, K (273 + °C)
P_b = base pressure, kPa
T_f = average gas flow temperature, K (273 + °C)
P_1 = upstream pressure, kPa
P_2 = downstream pressure, kPa
H_1 = upstream elevation, m
H_2 = downstream elevation, m
L_e = equivalent length of pipe, km
L = pipe length, km

Other terms are the same as those for USCS units.

Friction factor. The friction factor f introduced earlier depends on the type of flow (such as laminar or turbulent) and on the pipe diameter and internal roughness. For laminar flow (Re $\leq$ 2100) the friction factor is calculated from

$$f = \frac{64}{\mathrm{Re}} \tag{9.67}$$

Depending on the value of Re, flow is laminar or turbulent.

For laminar flow: $\mathrm{Re} \leq 2100$

For turbulent flow: $\mathrm{Re} > 4000$

The region for Re between the above two values is termed the critical flow regime.

The turbulent flow region is further subdivided into three separate regions:

1. Turbulent flow in smooth pipes
2. Turbulent flow in fully rough pipes
3. Transition flow between smooth pipes and rough pipes

This is shown in the Moody diagram (Fig. 9.3).

In the smooth pipe zone of turbulent flow, the pipe friction factor is not affected significantly by the pipe internal roughness. The friction factor f in this region depends only on the Reynolds number Re according to the following equation:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{9.68}$$

In the zone of turbulent flow of fully rough pipes the friction factor f depends less on the Reynolds number and more on the pipe roughness and diameter. It is calculated using the equation

$$\frac{1}{\sqrt{f}} = -2\log_{10}\frac{e}{3.7D} \tag{9.69}$$

where f = Darcy friction factor
D = pipe inside diameter, in
e = absolute pipe roughness, in

See Table 9.3 for typical values of pipe roughness.

In the transition zone between the smooth pipes zone and fully rough pipes zone, the friction factor is calculated using the Colebrook-White equation as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\frac{e}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}} \tag{9.70}$$

It can be seen from Eq. (9.70) that the solution of friction factor f is not straightforward. This equation is an implicit equation and therefore has to be solved by successive iteration.

It can be seen that the friction factor for laminar flow depends only on the Reynolds number and is independent of pipe roughness. It must be noted that the Reynolds number does depend on the pipe diameter and gas properties.

The friction factor is calculated using either the Colebrook-White equation (9.7) or is found from the Moody diagram. It is then used in the general flow equation (9.56) to calculate the pressure drop.

Example 9.19 Calculate the flow rate through a 20-mi-long NPS 20 (0.500-in wall thickness) pipeline using the general flow equation. Gas gravity = 0.6, flowing temperature = 80°F, inlet pressure = 1000 psig, outlet pressure = 800 psig, compressibility factor = 0.85, base temperature = 60°F, and base pressure = 14.7 psia. Assume the friction factor is 0.02.

Solution

$$\text{Pipe inside diameter } D = 20 - 2 \times 0.5 = 19.00 \text{ in}$$

$$P_1 = 1000 + 14.7 = 1014.7 \text{ psia}$$

$$P_2 = 800 + 14.7 = 814.7 \text{ psia}$$

$$T_f = 80 + 460 = 540°\text{R}$$

$$T_b = 60 + 460 = 520°\text{R}$$

$$Z = 0.85$$

$$P_b = 14.7 \text{ psia}$$

$$L = 20 \text{ mi}$$

From the general flow equation (9.56), we calculate the flow rate as

$$Q = 77.54 \times \left(\frac{1}{0.02}\right)^{0.5} \frac{520}{14.7} \left[\frac{(1014.7)^2 - (814.7)^2}{0.6 \times 540 \times 20 \times 0.85}\right]^{0.5} (19.0)^{2.5}$$

$$= 248{,}744{,}324 \quad \text{SCFD}$$

$$= 248.74 \quad \text{MMSCFD}$$

Example 9.20 Calculate the friction factor using the Colebrook-White equation for a 16-in (0.250-in wall thickness) gas pipeline at a flow rate of 100 MMSCFD. Flowing temperature = 80°F, gas gravity = 0.6, viscosity = 0.000008 lb/(ft · s), base pressure = 14.73 psia, and base temperature = 60°F. Assume a pipe internal roughness of 600 microinches (μin).

Solution Using Eq. (9.51), the Reynolds number is

$$\text{Re} = 0.0004778 \frac{P_b}{T_b} \frac{GQ}{\mu D}$$

$$= 0.0004778 \frac{14.73}{460 + 80} \times \frac{0.6 \times 100 \times 10^6}{0.000008 \times 15.5} = 6{,}306{,}446$$

Since the flow is turbulent, we use the Colebrook-White equation (9.70) to calculate the friction factor f as follows:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{e}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

$$= -2 \log \left(\frac{0.0006}{3.7 \times 15.5} + \frac{2.51}{6{,}306{,}446\sqrt{f}} \right)$$

This equation must be solved by trial and error. Initially, assume $f = 0.02$ and calculate the next approximation as follows:

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{0.0006}{3.7 \times 15.5} + \frac{2.51}{6{,}306{,}446 \times (0.02)^{1/2}} \right] = 9.7538$$

$$f = 0.0105$$

Using this value of f, the next approximation is

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{0.0006}{3.7 \times 15.5} + \frac{2.51}{6{,}306{,}446 \times (0.0105)^{1/2}} \right]$$

$$f = 0.0107$$

After a few more trials we get

$$\text{friction factor } f = 0.0107$$

Work done in compressing gas. The work done to compress a given quantity of gas from a suction pressure P_1 to the discharge pressure P_2, based upon isothermal compression or adiabatic compression, can be calculated as follows.

Isothermal compression. The work done in isothermal compression of 1 lb of gas is calculated using the following equation:

$$\text{Work done } W_i = \frac{53.28}{G} T_1 \log_e \frac{P_2}{P_1} \tag{9.71}$$

where W_i = isothermal work done, (ft · lb)/lb of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, °R
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia
$\log_e$ = natural logarithm to base e ($e = 2.71828$)

The ratio P_2/P_1 is called the compression ratio.

In SI units the isothermal compression equation (9.71) is as follows:

$$\text{Work done } W_i = \frac{159.29}{G} T_1 \log_e \frac{P_2}{P_1} \tag{9.72}$$

where W_i = isothermal work done, J/kg of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, K
P_1 = suction pressure of gas, kPa
P_2 = discharge pressure of gas, kPa
$\log_e$ = natural logarithm to base e($e = 2.71828$)

Adiabatic compression. In the adiabatic compression process the pressure and volume of gas follow the adiabatic equation PV^γ = constant where γ is the ratio of the specific heats C_p and C_v, such that

$$\gamma = \frac{C_p}{C_v} \tag{9.73}$$

The work done in adiabatic compression of 1 lb of gas is given by the following equation:

$$W_a = \frac{53.28}{G} T_1 \frac{\gamma}{\gamma - 1} \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \tag{9.74}$$

where W_a = adiabatic work done, (ft · lb)/lb of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, °R
γ = ratio of specific heats of gas, dimensionless
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia

In SI units the adiabatic compression equation is as follows:

$$W_a = \frac{159.29}{G} T_1 \frac{\gamma}{\gamma - 1} \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \tag{9.75}$$

where W_a = adiabatic work done, J/kg of gas
G = gas gravity, dimensionless
T_1 = suction temperature of gas, K
γ = ratio of specific heats of gas, dimensionless
P_1 = suction pressure of gas, kPa
P_2 = discharge pressure of gas, kPa

Example 9.21 A compressor compresses a gas ($G = 0.6$) from the suction temperature of 60°F and 800 to 1400 psia discharge. If isothermal compression is assumed, what is the work done by the compressor?

Solution Using Eq. (9.71) for isothermal compression, the work done is

$$W_i = \frac{53.28}{0.6}(520) \times \log_e \frac{1400}{800} = 25{,}841\,(\text{ft} \cdot \text{lb})/\text{lb}$$

Example 9.22 In Example 9.21, if the compression were adiabatic ($\gamma = 1.29$), calculate the work done per pound of gas.

Solution From Eq. (9.74) for adiabatic compression, the work done is

$$W_a = \frac{53.28}{0.6} \times 520 \times \frac{1.29}{1.29 - 1}\left[\left(\frac{1400}{800}\right)^{(1.29-1)/1.29} - 1\right] = 27{,}537\ (\text{ft} \cdot \text{lb})/\text{lb}$$

It can be seen by comparing results with those of Example 9.21 that the adiabatic compressor requires more work than an isothermal compressor.

Discharge temperature of compressed gas. When gas is compressed adiabatically according to the adiabatic process PV^γ = constant, the discharge temperature of the gas can be calculated as follows:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \tag{9.76}$$

where T_1 = suction temperature of gas, °R
T_2 = discharge temperature of gas, °R
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia
γ = ratio of specific heats of gas, dimensionless

Example 9.23 What is the final temperature of gas in Example 9.22 for adiabatic compression?

Solution We get the discharge temperature by using Eq. (9.76):

$$T_2 = 520 \times \left(\frac{1400}{800}\right)^{0.29/1.29} = 589.7°\text{R} \quad \text{or} \quad 129.7°\text{F}$$

Compressor horsepower. Compressor head measured in (ft · lb)/lb of gas is the energy added to the gas by the compressor. In SI units it is referred to in J/kg. The horsepower necessary for compression is calculated from

$$\text{HP} = \frac{\text{mass flow of gas} \times \text{head}}{\text{efficiency}}$$

It is common practice to refer to compression HP per MMSCFD of gas. Using the perfect gas equation modified by the compressibility factor [Eq. (9.41)], we can state that the compression HP is

$$\text{HP} = 0.0857 \frac{\gamma}{\gamma - 1} T_1 \frac{Z_1 + Z_2}{2} \frac{1}{\eta_a} \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \tag{9.77}$$

where HP = compression HP per MMSCFD
γ = ratio of specific heats of gas, dimensionless
T_1 = suction temperature of gas, °R
P_1 = suction pressure of gas, psia
P_2 = discharge pressure of gas, psia
Z_1 = compressibility of gas at suction conditions, dimensionless
Z_2 = compressibility of gas at discharge conditions, dimensionless
η_a = compressor adiabatic (isentropic) efficiency, decimal value

In SI units, the power equation is as follows:

$$\text{Power} = 4.0639 \frac{\gamma}{\gamma - 1} T_1 \frac{Z_1 + Z_2}{2} \frac{1}{\eta_a} \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \tag{9.78}$$

where HP = compression power, kW per Mm^3/day (million m^3/day)
γ = ratio of specific heats of gas, dimensionless
T_1 = suction temperature of gas, K
P_1 = suction pressure of gas, kPa
P_2 = discharge pressure of gas, kPa
Z_1 = compressibility of gas at suction conditions, dimensionless
Z_2 = compressibility of gas at discharge conditions, dimensionless
η_a = compressor adiabatic (isentropic) efficiency, decimal value

The adiabatic efficiency η_a is usually between 0.75 and 0.85. We can incorporate a mechanical efficiency η_m of the driver unit to calculate the brake horsepower (BHP) of the driver as follows:

$$\text{BHP} = \frac{\text{HP}}{\eta_m} \tag{9.79}$$

The driver efficiency η_m may range from 0.95 to 0.98. The adiabatic efficiency η_a may be expressed in terms of the suction and discharge pressures and temperatures and the specific heat ratio γ as follows:

$$\eta_a = \frac{T_1}{T_2 - T_1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \tag{9.80}$$

All symbols in Eq. (9.80) are as defined earlier.

It can be seen from the preceding that the efficiency term η_a modifies the discharge temperature T_2 given by Eq. (9.76).

Example 9.24 Calculate the compressor HP required in Example (9.23) if $Z_1 = 1.0$, $Z_2 = 0.85$, and $\eta_a = 0.8$. What is the BHP if the mechanical efficiency of the driver is 0.95?

Solution From Eq. (9.77), the HP required per MMSCFD is

$$\text{HP} = 0.0857\,\frac{1.29}{0.29}(520)\frac{1+0.85}{2}\,\frac{1}{0.8}\left[\left(\frac{1400}{800}\right)^{0.29/1.29} - 1\right]$$

$$= 30.73 \text{ per MMSCFD}$$

Using Eq. (9.79) for a mechanical efficiency of 0.95, we get

$$\text{BHP required} = \frac{30.73}{0.95} = 32.35 \text{ HP per MMSCFD}$$

9.3.3 Two-phase flow

One of the problems of cryogenic piping systems is heat leakage due to the absorption of heat. A portion of the cryogenic liquid may evaporate resulting in both liquid and vapor present in the piping system. Also, throttling of the cryogenic liquid through a valve can cause flashing or formation of vapor. In both cases two-phase flow would result. The calculation of pressure drop in two-phase flow is more complex than in single-phase liquid flow discussed in Sec. 9.3.2. It is found that the pressure drop in two-phase flow is larger than that of single-phase liquid flow. Larger pressure drop means larger pipe size and hence more cost. Because of these reasons, we must as far as possible maintain single-phase flow of cryogenic fluids. Subcooling of the liquid before a throttle valve can prevent flashing. Use of proper insulation around the cryogenic piping can minimize heat leaks into the system, thereby preventing vaporization of the liquid.

When two-phase flow is present, we must calculate the pressure drop due to friction using one of the many correlations and empirical formulas such as Lockhart-Martinelli or Dukler. These correlations are only approximate, and the calculated results may be off by 20 to 30 percent compared to actual pressure drops measured in the field. Therefore, to be conservative, pipe sizing for two-phase flow is based on increasing the calculated value of pressure drop by as much as 30 percent in some cases. In this section we will discuss the approach to calculating the pressure drop using the Lockhart-Martinelli method, as described in M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

The total pressure drop in two-phase flow can be considered to be the sum of three components.

1. Frictional
2. Gravitational
3. Accelerational

or mathematically,

$$\left(\frac{\Delta P}{\Delta z}\right)_T = \left(\frac{\Delta P}{\Delta z}\right)_F + \left(\frac{\Delta P}{\Delta z}\right)_G + \left(\frac{\Delta P}{\Delta z}\right)_A \tag{9.81}$$

where $\left(\frac{\Delta P}{\Delta z}\right)_T$ = total pressure drop per unit length in two-phase flow, psi/ft

$\left(\frac{\Delta P}{\Delta z}\right)_F$ = frictional pressure drop per unit length, psi/ft

$\left(\frac{\Delta P}{\Delta z}\right)_G$ = gravitational pressure drop per unit length, psi/ft

$\left(\frac{\Delta P}{\Delta z}\right)_A$ = accelerational pressure drop per unit length, psi/ft

The frictional component is calculated from the individual pressure drops considering each phase (liquid or gas) flowing separately and alone in the pipe.

The gravitational component depends on the elevation of the pipe with respect to the horizontal and is calculated taking into account the inclination of the pipe, the gas density, and the void fraction (explained later). Obviously, if the pipe is horizontal, the gravitational component is zero.

The acceleration component of the pressure drop in two-phase flow is generally negligible for cryogenic fluids. However, it can be calculated from a complex equation that depends on the void fraction, liquid and gas densities, and the flow rates.

The frictional component is calculated, as indicated earlier, by treating the liquid flow separately from the gas flow. We calculate the Reynolds number for the liquid and gas phase separately using the following equations:

$$\mathrm{Re}_L = \frac{M_L D}{A_L \mu_L} \quad \text{for liquid phase} \tag{9.82}$$

$$\mathrm{Re}_g = \frac{M_g D}{A_g \mu_g} \quad \text{for gas phase} \tag{9.83}$$

where subscripts L and g refer to the liquid and gas phases, respectively, and

where Re = Reynolds number, dimensionless
M = mass flow rate
D = pipe inside diameter
A = pipe cross-sectional area
μ = dynamic viscosity

Consistent units must be used for all of these terms so as to make the Reynolds number dimensionless. If the pipe diameter is in feet, pipe area is in ft^2, and mass flow rate is in lb/s, then the viscosity must be in lb/(ft · s).

Knowing the Reynolds number for each phase from Eqs. (9.82) and (9.83), we can calculate the parameters k, n, and m for each phase from the values shown in Table 9.6.

Laminar flow is said to occur for Reynolds numbers less than 1000 and turbulent flow for Reynolds numbers larger than 2000. This is slightly different from the Reynolds number boundaries for single-phase flow.

TABLE 9.6 Parameters for Two-Phase Flow Pressure Drop

Liquid	Vapor	R_L	R_g	k_L	k_g	n	m
Turbulent	Turbulent	>2000	>2000	0.046	0.046	0.2	0.2
Laminar	Turbulent	<1000	>2000	16.0	0.046	1.0	0.2
Turbulent	Laminar	>2000	<1000	0.046	16.0	0.2	1.0
Laminar	Laminar	<1000	<1000	16.0	16.0	1.0	1.0

SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

The individual pressure drop due to friction for each phase is then calculated using the following equations:

$$\left(\frac{\Delta P_f}{\Delta z}\right)_L = \frac{2k_L(\mathrm{Re}_L)^{-n}\rho_L}{D}\left(\frac{M_L}{A\rho_L}\right)^2 \tag{9.84}$$

$$\left(\frac{\Delta P_f}{\Delta z}\right)_g = \frac{2k_g(\mathrm{Re}_g)^{-n}\rho_g}{D}\left(\frac{M_g}{A\rho_g}\right)^2 \tag{9.85}$$

where subscripts L and g refer to the liquid and gas phases, respectively; $\Delta P_f/\Delta z$ is the frictional pressure drop per unit length of pipe; and ρ is the density of fluid. Other symbols are as defined earlier.

Once we calculate the frictional pressure drop for the liquid phase and gas phase separately, the Lockhart-Martinelli parameter X is found by the following equation:

$$X^2 = \frac{(\Delta P_f/\Delta z)_L}{(\Delta P_f/\Delta z)_g} \tag{9.86}$$

Using the Lockhart-Martinelli parameter X calculated from Eq. (9.86), we go to Fig. 9.11 to obtain the parameters ϕ_L, ϕ_g and the void fraction α.

The frictional pressure drop for two-phase flow is then calculated from one of the following equations:

$$\left(\frac{\Delta P}{\Delta z}\right)_F = {\phi_L}^2\left(\frac{\Delta P_f}{\Delta z}\right)_L \tag{9.87}$$

$$\left(\frac{\Delta P}{\Delta z}\right)_F = {\phi_g}^2\left(\frac{\Delta P_f}{\Delta z}\right)_g \tag{9.88}$$

As indicated earlier, the calculated frictional pressure drop using Eqs. (9.87) or (9.88) is within plus or minus 30 percent of actual field test results.

The gravitational component of the total pressure drop in two-phase flow, the second item on the right-hand side of Eq. (9.81), is calculated as follows:

$$\left(\frac{\Delta P}{\Delta z}\right)_G = g\,\sin\theta\,[\alpha\rho_g + (1-\alpha)\rho_L] \tag{9.89}$$

where $\left(\frac{\Delta P}{\Delta z}\right)_G$ = gravitational pressure drop per unit length, psi/ft

g = acceleration due to gravity, ft/s^2

α = void fraction, from the graph in Fig. 9.11

θ = angle of inclination of pipe to the horizontal, degrees

ρ = density of fluid

Subscripts L and g refer to the liquid and gas phases, respectively.

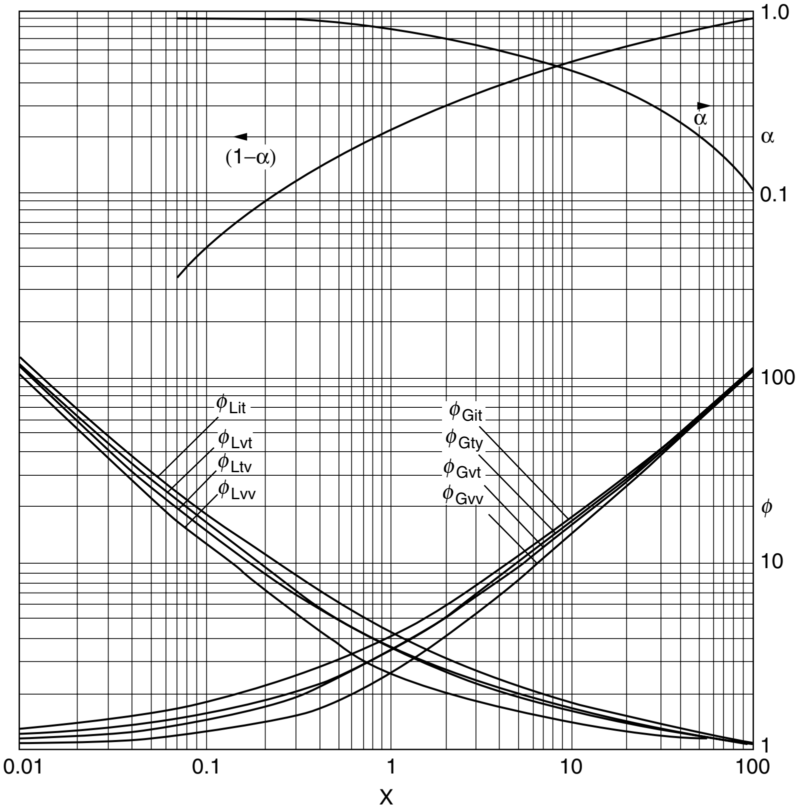

Figure 9.11 Lockhart-Martinelli correlation parameters.

The accelerational pressure drop component, the last term on the right-hand side of Eq. (9.81), can be calculated from

$$\left(\frac{\Delta P}{\Delta z}\right)_A = \frac{1}{A^2}\frac{d}{dz}\left(\frac{{M_g}^2}{\alpha\rho_g} + \frac{{M_L}^2}{(1-\alpha)\rho_L}\right) \tag{9.90}$$

The maximum mass flow rate of the two-phase flow mixture must be compared with the critical flow rate for choked flow at the downstream end of the pipe. Several equations are available for determining the choked or maximum flow rate. These require knowledge of the thermodynamic properties of the cryogenic fluid. One such equation is as follows:

$$M_c = \left[\frac{A^2}{(\partial v/\partial p)_S}\right]^{0.5} \tag{9.91}$$

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where M_c = critical mass flow rate of fluid, lb/s
v = specific volume of fluid mixture, ft^3/lb
A = pipe cross-sectional area, ft^2

The partial derivative in the denominator is calculated for isentropic conditions at the downstream end of the pipe from the thermodynamic properties.

If the actual flow rate is greater than the critical mass flow rate, the pipe size must be increased or the mass flow rate should be reduced and the pressure drop recalculated.

Example 9.25 A cryogenic piping system is composed of a 200-ft-long (horizontal) NPS 6 (0.250-in wall thickness) pipe. Calculate the pressure drop in two-phase flow, considering an inlet pressure of 300 psia, liquid viscosity of 0.2 cP, and vapor viscosity of 0.012 cP. The liquid flow rate is 12 ft^3/min, and the vapor flow rate is 200 ft^3/min. The liquid and vapor densities are 50 and 0.8 lb/ft^3, respectively, at the operating temperature.

Solution

$$\text{Mass flow rate of liquid } M_L = \frac{12 \times 50}{60} = 10 \text{ lb/s}$$

$$\text{Mass flow rate of vapor } M_g = \frac{200 \times 0.8}{60} = 2.667 \text{ lb/s}$$

$$\text{Pipe inside diameter} = 6.625 - 2 \times 0.250 = 6.125 \text{ in}$$

$$\text{Pipe cross-sectional area } A = 0.7854 \left(\frac{6.125}{12}\right)^2 = 0.2046 \text{ ft}^2$$

$$\begin{aligned}\text{Liquid viscosity} &= 0.2 \text{ cP} = 0.2 \times 6.7197 \times 10^{-4} \\ &= 1.3439 \times 10^{-4} \text{ lb/(ft} \cdot \text{s)} \\ \text{Gas viscosity} &= 0.012 \text{ cP} = 0.012 \times 6.7197 \times 10^{-4}) \\ &= 8.064 \times 10^{-6} \text{ lb/(ft} \cdot \text{s)}\end{aligned}$$

The Reynolds number for liquid and gas are calculated next, using Eqs. (9.82) and (9.83):

$$\text{Re}_L = \frac{10 \times (6.125/12)}{0.2046 \times 1.3439 \times 10^{-4}} = 1.856 \times 10^5$$

$$\text{Re}_g = \frac{2.667 \times (6.125/12)}{0.2046 \times 8.064 \times 10^{-6}} = 8.25 \times 10^5$$

Since both Reynolds numbers are greater than 2000, the flow is turbulent in both phases. From Table 9.6 we get the following values:

$$k_L = k_g = 0.046 \qquad \text{and} \qquad n = m = 0.2$$

Next, the frictional pressure drop for each phase flowing alone in the pipe is calculated using Eqs. (9.84) and (9.85) as follows:

$$\left(\frac{\Delta P_f}{\Delta z}\right)_L = \frac{2 \times 0.046 \times (1.856 \times 10^5)^{-0.2} \times 50}{6.125/12}\left(\frac{10}{0.2046 \times 50}\right)^2 = 0.7610$$

$$= \frac{0.7610}{32.2 \times 144} = 0.0002 \text{ psi/ft}$$

$$\left(\frac{\Delta P_f}{\Delta z}\right)_g = \frac{2 \times 0.046 \times (8.25 \times 10^5)^{-0.2} \times 0.8}{6.125/12}\left(\frac{2.667}{0.2046 \times 0.8}\right)^2 = 2.5103$$

$$= \frac{2.5103}{32.2 \times 144} = 0.0005 \text{ psi/ft}$$

Next we calculate the Lockhart-Martinelli correlation parameter X using Eq. (9.86):

$$X = \left(\frac{0.7610}{2.5103}\right)^{0.5} = 0.5506$$

Using Fig. 9.11 we get

$$\phi_L = 6.0$$

Therefore, the frictional pressure drop for the two-phase flow from Eq. (9.87) is

$$\left(\frac{\Delta P}{\Delta z}\right)_F = 36 \times 0.002 = 0.0072 \text{ psi/ft}$$

The total pressure drop in 200 ft of pipe is

$$200 \times 0.0072 = 1.44 \text{ psi}$$

9.3.4 Refrigeration piping

In this section we will briefly review a typical refrigeration cycle and discuss the approach used in sizing the suction, discharge, and liquid line piping in a typical refrigeration system. The refrigeration capacity is generally measured in tons or British thermal units per hour (Btu/h). In SI units it is measured in kilowatts (kW). One ton of refrigeration is defined as the equivalent of 200 Btu/min or 12,000 Btu/h. This is also equal to 3.517 kW.

In a typical compression refrigeration cycle, shown in Fig. 9.12, the refrigerant absorbs heat from the area to be cooled and evaporates in the process. This is shown as 1–2 in the figure.

From 2–3 the suction line carries the refrigerant vapor into the compressor where it is compressed to point 4 at a higher temperature and pressure. The compressed gas then flows through the discharge piping

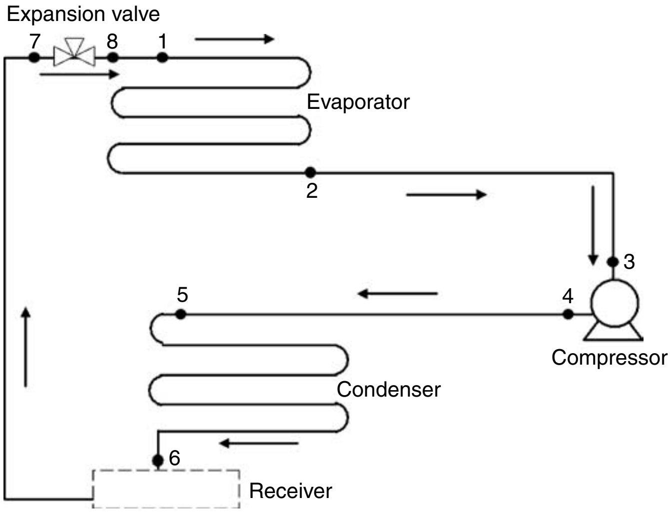

Figure 9.12 Typical compression refrigeration cycle.

from 4–5 and enters the condenser. In the condenser, the hot gas is condensed to liquid refrigerant at the condensing temperature. The condensed liquid then flows from receiver 6 to the expansion valve at 7, where the liquid expands to a lower pressure at 8 and then enters the evaporator at point 1 to start the refrigeration cycle all over. The difference in enthalpy of the refrigerant before the expansion valve and after the evaporator represents the amount of refrigeration per unit weight of refrigerant.

The suction line 2–3 is designed as a low-pressure, single-phase gas line. The discharge piping 4–5 is designed as a high-pressure, single-phase gas line. The liquid line 6–7 from the receiver to the inlet of the expansion valve is designed as a single-phase, high-pressure liquid line.

In order to determine the pipe sizes for the suction and discharge lines of a refrigeration piping system, tables are generally used to calculate the pressure drop based on the total equivalent length of pipe, valves, and fittings. Tables 9.7 and 9.8 show the pipeline capacity at specified temperatures for R-717 (ammonia) and R-22 refrigerants. As an example, using ammonia as the refrigerant, an NPS 4 suction line at a saturation temperature of $-60°F$ has a pressure drop of 0.046 psi per 100 ft and a temperature gradient of 0.025°F per 100 ft with a line capacity of 14.77 tons.

Generally, suction lines are sized such that the pressure drop equates to a temperature loss of 4°F in saturation temperature for most refrigerants except ammonia. For ammonia, the recommended temperature drop is 2°F in the suction line.

TABLE 9.7 Suction Piping Capacity for R-717

Steel line size		Saturated suction temperature, °F											
		−60		−40		−20		0		20		40	
NPS	SCH	$\Delta T =$ 0.25°F $\Delta P =$ 0.046	$\Delta T =$ 0.50°F $\Delta P =$ 0.092	$\Delta T =$ 0.25°F $\Delta P =$ 0.077	$\Delta T =$ 0.50°F $\Delta P =$ 0.155	$\Delta T =$ 0.25°F $\Delta P =$ 0.123	$\Delta T =$ 0.50°F $\Delta P =$ 0.245	$\Delta T =$ 0.25°F $\Delta P =$ 0.184	$\Delta T =$ 0.50°F $\Delta P =$ 0.368	$\Delta T =$ 0.25°F $\Delta P =$ 0.265	$\Delta T =$ 0.50°F $\Delta P =$ 0.530	$\Delta T =$ 0.25°F $\Delta P =$ 0.366	$\Delta T =$ 0.50°F $\Delta P =$ 0.582
$\frac{3}{8}$	80	0.03	0.05	0.06	0.09	0.11	0.16	0.18	0.26	0.28	0.40	0.41	0.53
$\frac{1}{2}$	80	0.06	0.10	0.12	0.18	0.22	0.32	0.36	0.52	0.55	0.80	0.82	1.05
$\frac{3}{4}$	80	0.15	0.22	0.28	0.42	0.50	0.73	0.82	1.18	1.26	1.83	1.87	2.38
1	80	0.30	0.45	0.57	0.84	0.99	1.44	1.62	2.34	2.5	3.60	3.68	4.69
$1\frac{1}{4}$	40	0.82	1.21	1.53	2.24	2.65	3.84	4.30	6.21	6.63	9.52	9.76	12.42
$1\frac{1}{2}$	40	1.25	1.83	2.32	3.38	4.00	5.80	6.49	9.34	9.98	14.34	14.68	18.64
2	40	2.43	3.57	4.54	6.59	7.79	11.26	12.57	18.12	19.35	27.74	28.45	36.08
$2\frac{1}{2}$	40	3.94	5.78	7.23	10.56	12.50	18.03	20.19	28.94	30.98	44.30	45.37	57.51
3	40	7.10	10.30	13.00	18.81	22.23	32.09	35.87	51.35	54.98	78.50	80.40	101.93
4	40	14.77	21.21	26.81	38.62	45.66	65.81	73.56	105.17	112.34	160.57	164.44	208.34
5	40	26.66	38.65	48.68	70.07	82.70	119.60	133.12	190.55	203.53	289.97	296.88	376.18
6	40	43.48	62.83	79.18	114.26	134.37	193.44	216.05	308.62	329.59	469.07	480.96	609.57
8	40	90.07	129.79	163.48	235.38	277.80	397.55	444.56	633.82	676.99	962.47	985.55	1250.34
10	40	164.26	236.39	297.51	427.71	504.98	721.08	806.47	1148.72	1226.96	1744.84	1786.55	2263.99
12	40	264.07	379.88	477.55	686.10	808.93	1157.59	1290.92	1839.28	1964.56	2790.37	2862.23	3613.23

TABLE 9.8 Suction Piping Capacity for R-22 (Single- or High-Stage Applications)

Line size type L		Saturated suction temperature, °F									
		−40		−20		0		20		40	
NPS	SCH	$\Delta T = 1.0°F$ $\Delta P = 0.393$	$\Delta T = 0.50°F$ $\Delta P = 0.197$	$\Delta T = 1.0°F$ $\Delta P = 0.577$	$\Delta T = 0.50°F$ $\Delta P = 0.289$	$\Delta T = 1.0°F$ $\Delta P = 0.813$	$\Delta T = 0.50°F$ $\Delta P = 0.406$	$\Delta T = 1.0°F$ $\Delta P = 1.104$	$\Delta T = 0.50°F$ $\Delta P = 0.552$	$\Delta T = 1.0°F$ $\Delta P = 1.455$	$\Delta T = 0.50°F$ $\Delta P = 0.727$
Copper (outside diameter)											
$\frac{1}{2}$		0.07	0.05	0.12	0.08	0.18	0.12	0.27	0.19	0.4	0.27
$\frac{5}{8}$		0.13	0.09	0.22	0.15	0.34	0.23	0.52	0.35	0.75	0.51
$\frac{3}{4}$		0.22	0.15	0.37	0.25	0.58	0.39	0.86	0.59	1.24	0.85
$\frac{7}{8}$		0.35	0.24	0.58	0.4	0.91	0.62	1.37	0.93	1.97	1.35
$1\frac{1}{8}$		0.72	0.49	1.19	0.81	1.86	1.27	2.77	1.9	3.99	2.74
$1\frac{3}{8}$		1.27	0.86	2.09	1.42	3.25	2.22	4.84	3.32	6.96	4.78
$1\frac{5}{8}$		2.02	1.38	3.31	2.26	5.16	3.53	7.67	5.26	11	7.57
$2\frac{1}{8}$		4.21	2.88	6.9	4.73	10.71	7.35	15.92	10.96	22.81	15.73
$2\frac{5}{8}$		7.48	5.13	12.23	8.39	18.97	13.04	28.19	19.40	40.38	27.84
$3\frac{1}{8}$		11.99	8.22	19.55	13.43	30.31	20.85	44.93	31	64.3	44.44
$3\frac{5}{8}$		17.89	12.26	29.13	20	45.09	31.03	66.81	40.11	95.68	66.09
$4\frac{1}{8}$		25.29	17.36	41.17	28.26	63.71	43.85	94.25	65.12	134.81	93.22
Steel											
$\frac{3}{8}$	80	0.06	0.04	0.1	0.07	0.15	0.1	0.21	0.15	0.3	0.21
$\frac{1}{2}$	80	0.12	0.08	0.19	0.13	0.29	0.2	0.42	0.3	0.6	0.42
$\frac{3}{4}$	80	0.27	0.18	0.43	0.3	0.65	0.46	0.95	0.67	1.35	0.95
1	80	0.52	0.36	0.84	0.59	1.28	0.89	1.87	1.31	2.64	1.86
$1\frac{1}{4}$	40	1.38	0.96	2.21	1.55	3.37	2.36	4.91	3.45	6.93	4.88
$1\frac{1}{2}$	40	2.08	1.45	3.32	2.33	5.05	3.55	7.38	5.19	10.42	7.33
2	40	4.03	2.81	6.41	4.51	9.74	6.85	14.22	10.01	20.07	14.14
$2\frac{1}{2}$	40	6.43	4.49	10.23	7.19	15.56	10.93	42.65	15.95	31.99	12.53
3	40	11.38	7.97	18.11	12.74	27.47	19.34	40.1	28.23	56.52	39.79
4	40	23.24	16.3	36.98	26.02	56.12	39.49	81.73	57.53	115.24	81.21
5	40	42.04	29.5	66.73	47.05	101.16	71.27	147.36	103.82	207.59	146.38
6	40	68.04	47.86	108.14	76.15	163.77	115.21	238.29	168.07	335.71	236.7
8	40	139.48	98.06	221.17	155.78	334.94	236.21	488.05	344.19	686.71	484.74
10	40	252.38	177.75	400.53	282.05	606.74	427.75	881.59	622.51	124.64	876.79
12	ID	403.63	284.69	639.74	451.09	969.02	683.22	1410.3	995.8	1987.29	1402.63

Discharge lines are sized such that the temperature loss is 2°F. Tables 9.9 and 9.10 list refrigeration capacities for different pipe sizes based on a 1°F temperature drop per 100 ft of pipe. Based on the actual total length of piping, which includes the equivalent length of all pipe, fittings, and valves, the table value for the temperature drop is adjusted in direct proportion to the pipe length. The adjustment for refrigeration capacity from the table values is performed in accordance with the following equation for temperature drop in steel pipe:

$$\Delta T = (\text{table } \Delta T) \times \frac{L_e}{100} \times \left(\frac{Q_A}{Q_T}\right)^{1.96} \tag{9.92}$$

where ΔT = actual temperature drop, °F
table ΔT = temperature drop from table, °F per 100 ft length of pipe
L_e = equivalent length of pipe including valves and fittings, ft
Q_A = actual refrigeration capacity, tons
Q_T = table refrigeration capacity, tons

For copper tubing, the index 1.96 in Eq. (9.92) is replaced with 1.85. For example, suppose the table shows a temperature drop of 2°F per 100 ft and a capacity of 125 tons for a particular pipe size. If the actual capacity is 100 tons and the pipe equivalent length is 250 ft, the actual temperature drop will be

$$\Delta T = 2.0 \times \frac{250}{100} \times \left(\frac{100}{125}\right)^{1.96} = 3.23^\circ\text{F}$$

Tables 9.10 and 9.11 also list the pressure drop per 100 ft of pipe and corresponding capacities for a particular pipe size. Similar to the adjustment for temperature drop previously discussed, the actual pressure drop for steel pipe can be calculated from the following:

$$\Delta P = (\text{table } \Delta P) \times \frac{L_e}{100} \times \left(\frac{Q_A}{Q_T}\right)^{1.96} \tag{9.93}$$

where ΔP is the actual pressure drop (psi) and table ΔP is the pressure drop from the table (psi per 100-ft length of pipe). Other symbols are as defined earlier.

In the preceding example if the corresponding pressure drop from the table is 3.5 psi, the adjusted value for equivalent length and capacity is

$$\Delta P = 3.5 \times \frac{250}{100} \times \left(\frac{100}{125}\right)^{1.96} = 5.65 \text{ psi}$$

TABLE 9.9 Suction, Discharge, and Liquid Line Capacity for R-717

Steel line size		Suction lines ($\Delta T = 1°F$ per 100 ft) at saturated suction temperature, °F					Discharge lines	Steel line size		Liquid lines	
		−40	−20	0	20	40	($\Delta T = 1.0°F$,			Velocity =	$\Delta T = 0.7°F$
NPS	SCH	$\Delta P = 0.31$	$\Delta P = 0.49$	$\Delta P = 0.73$	$\Delta P = 1.06$	$\Delta P = 1.46$	$\Delta P = 2.95$)	IPS	SCH	100 ft/min	$\Delta P = 2.0$ psi
$\frac{3}{8}$	80							$\frac{3}{8}$	80	8.6	12.1
$\frac{1}{2}$	80						3.1	$\frac{1}{2}$	80	14.2	24
$\frac{3}{4}$	80				2.6	3.8	7.1	$\frac{3}{4}$	80	26.3	54.2
1	80		2.1	3.4	5.2	7.6	13.9	1	80	43.8	106.4
$1\frac{1}{4}$	40	3.2	5.6	8.9	13.6	19.9	36.5	$1\frac{1}{4}$	80	78.1	228.6
$1\frac{1}{2}$	40	4.9	8.4	13.4	20.5	29.9	54.8	$1\frac{1}{2}$	80	107.5	349.2
2	40	9.5	16.2	26.0	39.6	57.8	105.7	2	40	204.2	811.4
$2\frac{1}{2}$	40	15.3	25.9	41.5	63.2	92.10	168.5	$2\frac{1}{2}$	40	291.1	1292.6
3	40	27.1	46.1	73.5	111.9	163.00	297.6	3	40	449.6	2287.8
4	40	55.7	94.2	150.1	228.7	333.0	606.2	4	40	774.7	4662.1
5	40	101.1	170.4	271.1	412.4	600.90	1095.2	5	40		
6	40	164.0	276.4	439.2	667.5	971.6	1771.2	6	40		
8	40	337.2	566.8	901.1	1366.6	1989.4	3623.0	8	40		
10	40	611.6	1027.2	1634.3	2474.5	3598.0		10	40		
12	40	981.6	1644.5	2612.4	3963.5	5764.6		12	ID		

TABLE 9.10 Suction, Discharge, and Liquid Line Capacity for R-22

Line size type L		Suction lines ($\Delta T = 2°F$ per 100 ft) at saturated suction temperature, °F (Corresponding, ΔP in psi per 100 ft)					Discharge lines ($\Delta T = 1.0°F$, $\Delta P = 3.05$ psi) at saturated suction temperature, °F		Liquid lines	
		−40 ($\Delta P = 0.79$)	−20 ($\Delta P = 1.15$)	0 ($\Delta P = 1.16$)	20 ($\Delta P = 2.22$)	40 ($\Delta P = 2.91$)	−40	40	Velocity = 100 fpm	$\Delta T = 0.1°F$ $\Delta P = 3.05$ psi
Copper (Outside diameter)										
$\frac{1}{2}$					0.4	0.6	0.75	0.85	2.3	3.6
$\frac{5}{8}$			0.32	0.51	0.76	1.1	1.4	1.6	3.7	6.7
$\frac{7}{8}$		0.52	0.86	1.3	2	2.9	3.7	4.2	7.8	18.20
$1\frac{1}{8}$		1.1	1.7	2.7	4	5.8	7.5	8.5	13.2	37
$1\frac{3}{8}$		1.9	3.1	4.7	7	10.1	13.1	14.8	20.2	64.7
$1\frac{5}{8}$		3	4.8	7.5	11.1	16	20.7	23.4	28.5	102.50
$2\frac{1}{8}$		6.2	10	15.6	23.1	33.1	42.8	48.5	49.6	213
$2\frac{5}{8}$		10.9	17.8	27.5	40.8	58.3	75.4	85.4	76.5	376.90
$3\frac{1}{8}$		17.5	28.4	44	65	92.9	120.2	136.2	109.2	601.50
$3\frac{5}{8}$		26	42.3	65.4	96.6	137.8	178.4	202.1	147.8	895.70
$4\frac{1}{8}$		36.8	59.6	92.2	136.3	194.3	251.1	284.4	192.1	1263.20
Steel										
$\frac{1}{2}$	40		0.38	0.58	0.85	1.2	1.5	1.7	3.8	5.7
$\frac{3}{4}$	40	0.5	0.8	1.2	1.8	2.5	3.3	3.7	6.9	12.8
1	40	0.95	1.5	2.3	3.4	4.8	6.1	6.9	7.5	25.2
$1\frac{1}{4}$	40	2	3.2	4.8	7	9.9	12.6	14.3	20.6	54.1
$1\frac{1}{2}$	40	3	4.7	7.2	10.5	14.8	19	21.5	28.3	82.6
2	40	5.7	9.1	13.9	20.2	28.5	36.6	41.4	53.8	192
$2\frac{1}{2}$	40	9.2	14.6	22.1	32.2	45.4	58.1	65.9	76.7	305.8
3	40	16.2	25.7	39	56.8	80.1	102.8	116.4	118.5	540.3
4	40	33.1	52.5	79.5	115.9	163.2	209.5	237.3	204.2	1101.2

For a particular refrigeration capacity we can determine the quantity of refrigerant required in lb/min from a table such as Table 9.11. For example, using R-717 (ammonia) with a 14°F evaporating temperature and a 104°F condensing temperature, the quantity of refrigerant required is 0.434 lb/min per ton of refrigeration. Therefore, for a 100-ton refrigeration capacity, the mass flow rate of R-717 is 43.4 lb/min. The volume flow rate of refrigerant vapor in the suction and discharge piping can be calculated from the mass flow rate using the specific volume of the vapor. The thermodynamic properties of refrigerants are available in charts, similar to Table 9.12 for R-717. From the table value of specific volume of the saturated vapor and the mass flow rate we can calculate the volume flow rate of the refrigerant in the suction line as follows:

$$\text{Volume flow rate} = \text{mass flow rate} \times \text{specific volume}$$

or

$$V_s = M \times v_s \tag{9.94}$$

where V_s = suction line volume flow rate, ft^3/min
M = refrigerant mass flow rate, lb/min
v_s = specific volume of refrigerant at evaporation temperature, ft^3/lb

The discharge volume flow rate in the discharge line can be calculated approximately from the following equation:

$$V_d = V_s \times \frac{P_1}{P_2} \times 1.2 \tag{9.95}$$

where V_d = discharge line volume flow rate, ft^3/min
V_s = suction line volume flow rate, ft^3/min
P_1 = compressor suction pressure, psia
P_2 = compressor discharge pressure, psia

Once the suction and discharge piping are selected and the pressure drops calculated, we can calculate the actual compressor suction and discharge pressures by adjusting the pressures P_1 and P_2 as follows:

$$\text{Compressor suction pressure} = P_1 - \Delta P_s \tag{9.96}$$

$$\text{Compressor discharge pressure} = P_2 + \Delta P_d \tag{9.97}$$

where ΔP_s and ΔP_d represent the suction and discharge piping losses, respectively.

TABLE 9.11 Refrigerant Flow Rates for Condensing Temperatures

		Refrigerant (chemical formula) [common name]								
		R717 (NH_3) [Ammonia]			R134a (CH_2FCF_3) [1,1,1,2-Tetrafluoroethane]			R22 ($CHCLF_2$) [Chlorodifluoromethane]		
		30°C (86°F)	35°C (95°F)	40°C (104°F)	30°C (86°F)	35°C (95°F)	40°C (104°F)	30°C (86°F)	35°C (95°F)	40°C (104°F)
Evaporating temp.		Flow rate in lb/min per ton								
0°C	(32°F)	0.410	0.420	0.429	2.89	3.02	3.17	2.70	2.81	2.92
−10°C	(14°F)	0.415	0.424	0.434	3.00	3.14	3.31	2.76	2.87	3.00
−20°C	(−4°F)	0.420	0.429	0.44	3.12	3.28	3.46	2.84	2.95	3.08
−30°C	(−22°F)	0.422	0.432	0.442	3.26	3.44	3.63	2.91	3.04	3.17
		Flow rate in CFM/ton (suction line)								
0°C	(32°F)	1.95	2.0	2.04	3.23	3.38	3.56	2.09	2.80	2.27
−10°C	(14°F)	2.85	2.9	2.99	4.87	5.11	5.38	2.97	3.09	3.22
−20°C	(−4°F)	4.31	4.4	4.51	7.52	7.91	8.37	4.32	4.50	4.69
−30°C	(−22°F)	6.68	6.8	7.00	12.04	12.69	13.43	6.50	6.77	7.08

TABLE 9.12 Thermodynamic Properties of Refrigerant R717 (Ammonia)

Temperature, °F	Pressure, psia	Specific volume, ft^3/lbm		Enthalpy, Btu/lbm			Entropy, Btu/(lbm · R)		
		Saturated liquid	Saturated vapor	Saturated liquid	Evap.	Saturated vapor	Saturated liquid	Evap.	Saturated vapor
−60	5.55	0.02278	44.73	−21.2	610.8	589.6	−0.0517	1.5286	1.4769
−50	7.67	0.02299	33.08	−10.6	604.3	593.7	−0.0256	1.4753	1.4497
−40	10.41	0.02322	24.86	0	597.6	597.6	0.0	1.4242	1.4242
−30	13.90	0.02345	18.97	10.7	590.7	601.4	0.0250	1.3751	1.4001
−20	18.30	0.02369	14.68	21.4	583.6	605.0	0.0497	1.3277	1.3774
−10	23.74	0.02393	11.50	32.1	576.4	608.5	0.0738	1.282	1.3558
0	30.42	0.02419	9.116	42.9	568.9	611.8	0.0975	1.2377	1.3352
5	34.27	0.02432	8.150	48.3	565.0	613.3	0.1092	1.2161	1.3253
10	38.51	0.02446	7.304	53.8	561.1	614.9	0.1208	1.1949	1.3157
20	48.21	0.02474	5.910	64.7	553.1	617.8	0.1437	1.1532	1.2969
30	59.74	0.02503	4.825	75.7	544.8	620.5	0.1663	1.1127	1.279
40	73.32	0.02533	3.971	86.8	536.2	623.0	0.1885	1.0733	1.2618
50	89.19	0.02564	3.294	97.9	527.3	625.2	0.2105	1.0348	1.2453
60	107.60	0.02597	2.751	109.2	518.1	627.3	0.2322	0.9972	1.2294
70	128.80	0.02632	2.312	120.5	508.6	629.1	0.2537	0.9603	1.2140
80	153.00	0.02668	1.955	132.0	498.7	630.7	0.2749	0.9242	1.1991
86	169.20	0.02691	1.772	138.9	492.6	631.5	0.2875	0.9029	1.1904
90	180.60	0.02707	1.661	143.5	488.5	632.0	0.2958	0.8888	1.1846
100	211.90	0.02747	1.419	155.2	477.8	633.0	0.3166	0.8539	1.1705
110	247.00	0.02790	1.217	167.0	466.7	633.7	0.3372	0.8194	1.1566
120	286.40	0.02836	1.047	179.0	455.0	634.0	0.3576	0.7851	1.1427

Example 9.26 Calculate the pipe sizes required for the suction line, discharge line, and liquid line for a refrigeration system using R-22 refrigerant. The refrigeration capacity is 120 tons. The evaporating temperature is 25°F, and the water cooled condensing temperature is 104°F. The suction piping consists of 65-ft-long pipe, two angle valves, one swing check valve, and two elbows. The discharge line consists of 100 ft of pipe with one globe valve, one check valve, and two elbows. The liquid line is 200 ft long and has two angled valves and two elbows.

Solution From Table 9.11 for R-22 refrigerant the flow rate is by interpolation 2.95 lb/(min · ton). Therefore, for a 120-ton system, the refrigerant flow rate is

$$w = 120 \times 2.95 = 354 \text{ lb/min}$$

At suction conditions, 25°F the specific volume of R-22 from Table 9.13 is

$$V_s = 0.8532 \text{ ft}^3\text{/lb}$$

Therefore, the suction volume flow rate is

$$V_s = 354 \text{ lb/min} \times 0.8532 \text{ ft}^3\text{/lb} = 302.03 \text{ ft}^3\text{/min}$$

The inlet and outlet pressures of the compressor are found by interpolation from Table 9.14:

$$P_1 = 63.66 \text{ psia at } 25°\text{F}$$

$$P_2 = 222.5 \text{ psia at } 104°\text{F}$$

The approximate discharge volume due to compression can be calculated as follows:

$$\text{Discharge volume} = 302.03 \times \frac{63.66}{222.5} \times 1.2 = 103.7 \text{ ft}^3\text{/min}$$

At a condensing temperature of 104°F, using Table 9.13 for R-22, the liquid density by interpolation is 70.44 lb/ft^3. Therefore,

$$\text{Liquid volume} = \frac{354 \text{ lb/min}}{70.44 \text{ lb/ft}^3} = 5.03 \text{ ft}^3\text{/min}$$

For the suction line we will assume NPS 4 steel pipe. First we will calculate the equivalent length of the suction line including pipe, valves, and fittings:

Pipe	65 ft
Two angle valves	94 ft
One swing check valve	40 ft
Two elbows	13.4 ft
Total	212.4 ft

TABLE 9.13 Thermodynamic Properties of Refrigerant R22

Temperature, °F	Pressure, psia	Specific volume, ft^3/lbm Saturated liquid	Specific volume, ft^3/lbm Saturated vapor	Enthalpy, Btu/lbm Saturated liquid	Enthalpy, Btu/lbm Evap.	Enthalpy, Btu/lbm Saturated vapor	Entropy, Btu/(lbm · °R) Saturated liquid	Entropy, Btu/(lbm · °R) Saturated vapor
−40	15.222	0.0114	3.2957	0.0	100.257	100.257	0	0.2389
−30	19.573	0.0115	2.6049	2.547	98.801	101.348	0.0060	0.2359
−20	24.845	0.0116	2.0826	5.131	97.285	102.415	0.0119	0.2332
−10	31.162	0.0118	1.6825	7.751	95.704	103.455	0.0178	0.2306
0	38.657	0.0119	1.3723	10.409	94.056	104.465	0.0236	0.2282
10	47.464	0.0121	1.1290	13.104	92.338	105.442	0.0293	0.2259
20	57.727	0.0123	0.9363	15.837	90.545	106.383	0.0350	0.2238
30	69.591	0.0124	0.7821	18.609	88.674	107.284	0.0407	0.2218
40	83.206	0.0126	0.6575	21.422	86.720	108.142	0.0463	0.2199
50	98.727	0.0128	0.5561	24.275	84.678	108.953	0.0519	0.2180
60	116.31	0.0130	0.4727	27.172	82.540	109.712	0.0575	0.2163
70	136.12	0.0133	0.4037	30.116	80.298	110.414	0.0630	0.2146
80	158.33	0.0135	0.3462	33.109	77.943	111.052	0.0685	0.2129
90	183.09	0.0138	0.2979	36.158	75.461	111.619	0.0739	0.2112
100	210.60	0.0140	0.2570	39.267	72.838	112.105	0.0794	0.2096
110	241.04	0.0144	0.2222	42.446	70.052	112.498	0.0849	0.2079
120	274.60	0.0147	0.1924	45.705	67.077	112.782	0.0904	0.2061
130	311.50	0.0151	0.1666	49.059	63.877	112.936	0.0960	0.2043
140	351.94	0.0155	0.1442	52.528	60.403	112.931	0.1016	0.2024
150	396.19	0.0160	0.1245	56.143	56.585	112.728	0.1074	0.2002
160	444.53	0.0166	0.1070	59.948	52.316	112.263	0.1133	0.1978
170	497.26	0.0174	0.0913	64.019	47.419	111.438	0.1196	0.1949

TABLE 9.14 Temperature Pressure Data for Refrigerants

	Pressure, psia						
Temp, °C	R717	R134a	R22	R507	R290	R744	Temp, °F
−90		0.222	0.696	1.049	0.933		−130
−80		0.535	1.501	2.197	1.887		−112
−70	1.5825	1.162	2.965	4.228	3.532		−94
−65	2.2608	1.738	4.2	5.704	4.856		−85
−60	3.1688	2.312	5.434	7.571	6.18		−76
−55	4.3636	3.294	7.394	9.899	8.2	80.47	−67
−50	5.9112	4.276	9.354	12.76	10.22	90.03	−58
−45	7.8870	5.852	12.02	16.24	13.16	120.8	−49
−40	10.376	7.429	15.26	20.42	16.1	145.8	−40
−35	13.473	9.832	19.16	25.41	19.9	174.5	−31
−30	17.281	12.26	23.76	31.27	24.32	207.1	−22
−25	21.915	15.45	29.23	38.16	29.5	244.1	−13
−20	27.498	19.25	35.58	46.11	35.45	285.7	−4
−15	34.163	23.77	42.99	55.30	42.29	332.3	5
−10	42.05	29.08	51.46	65.12	50.07	484.2	14
−5	51.311	35.3	61.21	77.75	58.9	441.8	23
0	62.102	42.45	72.24	91.23	68.79	505.5	32
5	74.591	50.72	84.77	106.4	79.94	575.7	41
10	88.95	60.12	98.8	123.4	92.32	653	50
15	105.36	70.84	115.9	142.5	106.1	737.7	59
20	124.01	82.9	132	163.5	121.3	803.8	68
25	145.09	96.52	151.6	186.8	138.1	933.2	77
30	168.80	111.7	172.9	212.5	156.5	1045.9	86
35	195.35	128.6	196.7	240.8	176.7		95
40	224.94	147.4	222.5	271.8	198.6		104
45	257.80	168.2	250.9	305.7	222.5		113
50	294.15	191.2	281.8	342.7	248.6		122
55	334.21	216.4	315.5	383.3	276.6		131
60	378.23	243.9	352	427.7	306.9		140
65	426.45	247.1	391.7	476.3	339.8		149
70	479.12	306.9	434.7		375.1		158
80	598.88	381.9	531.3		454.3		176
90	739.77	470.5	644.3		545.9		194
100	904.32	576.1					212
110	1095.5						230

From Table 9.10 by interpolation, the capacity for a suction line temperature drop of 2°F per 100 ft for NPS 4 steel pipe at 25°F is

$$115.9 + \frac{(163.2 - 115.9) \times 5}{20} = 127.73 \text{ tons}$$

We will now adjust the temperature drop from Table 9.10 for the equivalent length of 212.4 previously calculated and for 120-ton actual capacity.

$$\Delta T = 2 \times \frac{212.4}{100} \times \left(\frac{120}{127.73}\right)^{1.96} = 3.67°\text{F}$$

This temperature drop is less than the 4°F recommended. Hence this pipe size is fine.

Similarly, the pressure drop is interpolated from Table 9.10 at 25°F as follows:

$$2.22 + \frac{(2.91 - 2.22) \times 5}{20} = 2.4 \text{ psi}$$

Adjusting for the equivalent length and capacity we get

$$\Delta P = 2.4 \times \frac{212.4}{100} \times \left(\frac{120}{127.73}\right)^{1.96} = 4.51 \text{ psi}$$

Next, we will choose NPS 3 steel pipe for the discharge line. We will calculate the equivalent length of the discharge line including pipe, valves, and fittings:

Pipe	100 ft
One globe valve	84 ft
One check valve	84 ft
Two elbows	10 ft
Total	278 ft

From Table 9.10, the capacity of NPS 3 steel pipe at a temperature drop of 1°F per 100 ft and a condensing temperature of 105°F is 116.4 tons. Adjusting for the equivalent length and capacity we get

$$\Delta T = 1.0 \times \frac{278}{100} \times \left(\frac{120}{116.4}\right)^{1.96} = 2.95°\text{F}$$

This is larger than the 2°F recommended for discharge piping. We can reduce the equivalent length and thereby reduce the temperature difference using angle valves and a swing check valve as follows:

$$\text{Revised equivalent length} = 100 + 2 \times 35 + 30 + 10 = 210 \text{ ft}$$

Therefore, the revised temperature drop is

$$\Delta T = \frac{2.95 \times 210}{278} = 2.23°\text{F}$$

which is close enough to the 2°F recommended for discharge piping.

The pressure drop in the discharge line is calculated by using Table 9.10 and adjusting for the equivalent length and capacity as follows. From Table 9.10, the pressure drop is 3.05 psi per 100 ft at a capacity of 116.4 tons. Adjusting the pressure drop from Table 9.10 for the equivalent length of 278 previously calculated and for 120-ton actual capacity, we obtain

$$\Delta P = 3.05 \times \frac{210}{100} \times \left(\frac{120}{116.4}\right)^{1.96} = 6.8 \text{ psi}$$

Next, the condenser drain according to Table 9.10 at a velocity of 100 ft/min for NPS 3 steel pipe has a capacity of 118.5 tons. Since our capacity is 120 tons, using NPS 3 pipe would cause the velocity to increase to $100 \times 120/118.5 = 102$ ft/min, which is acceptable. Therefore, use NPS 3 steel pipe for the condenser drain.

Next we will size the liquid line from the receiver to the evaporator. Assume NPS 2 steel pipe and calculate the total equivalent length as follows:

Pipe	200 ft
Two angle valves	48 ft
Two elbows	6.6 ft
Total	254.6 ft

From Table 9.10 the liquid line had a 1°F temperature drop per 100 ft for NPS 2 pipe and a capacity of 192 tons. Adjusting for equivalent length and capacity, we get

$$\Delta T = 1.0 \times \frac{254.6}{100} \times \left(\frac{120}{192}\right)^{1.96} = 1.01^{\circ}\mathrm{F}$$

Also the pressure drop from Table 9.10 is 3.05 psi per 100 ft. Adjusting for equivalent length and capacity, we get

$$\Delta P = 3.05 \times \frac{254.6}{100} \times \left(\frac{120}{192}\right)^{1.96} = 3.1 \text{ psi}$$

The compressor suction and discharge pressures have to be adjusted for the pressure losses in the suction and discharge piping.

$$\text{Compressor suction pressure} = 63.66 - 4.51 = 59.15 \text{ psia}$$

$$\text{Compressor discharge pressure} = 222.5 + 6.8 = 229.3 \text{ psia}$$

$$\text{Compression ratio} = \frac{229.3}{59.15} = 3.88$$

$$\text{Flow rate at suction} = 302.03 \text{ ft}^3/\text{min}$$

Suction piping: Use NPS 4.

Discharge piping: Use NPS 3.

Condenser drain line: Use NPS 3.

Receiver to the evaporator: Use NPS 2.

9.4 Piping Materials

The ASME Code for Pressure Piping, Section B31.5, covers the requirement for design, construction, installation, and testing of refrigeration piping systems. Because of the nature of different refrigerants certain piping material cannot be used with some refrigerants. For example, carbon-steel material can be used with refrigerants such as R-22, 134A, 290, 400, 500, 717, and 744. Wrought iron may also be used with these refrigerants. Cast iron pipe is not allowed in any refrigerant system. Copper or brass piping may be used with all refrigerants except R-717 (ammonia). Table 9.15 lists the materials used and the levels of compatibility.

TABLE 9.15 Piping Materials and Refrigerant Compatibility

Material	Refrigerant number 22	134a	290	All 400s	All 500s	717	744
Carbon steel	S	S	S	S	S	S	S
Wrought iron	S	S	S	S	S	S	S
Cast iron pipe	NP	NP	NP	NP	NP	NP	NP
Copper or brass	S	S	S	S	S	NS	S
Aluminum	Q	Q	Q	Q	Q	Q	NS
Zinc	NS	NS	NS	NS	NS	NS	NS
Magnesium	NS	NS	NS	NS	NS	NS	NS
ASHRAE 15-94 group	A1	A1	A1	A1	A1	B2	A1

NOTES: NP—not permitted by ASME B31.5 Code; NS—not satisfactory; Q—qualified, moist refrigerant may corrode (consult supplier); S—satisfactory.
SOURCE: Reproduced from M. L. Nayyar, *Piping Handbook*, 7th ed., New York, McGraw-Hill, 2000.

TABLE 9.16 Allowable Working Pressures for Carbon-Steel Piping

Pipe size NPS	Pipe size DN	Schedule number	Allowable internal working pressure, psig	Allowable external working pressure, psig
$\frac{1}{8}$	3	40	1890	2070
		80	3510	2860
$\frac{1}{4}$	6	40	1490	2000
		80	2880	2700
$\frac{3}{8}$	10	40	1300	1660
		80	2500	2280
$\frac{1}{2}$	15	40	1126	1580
		80	2210	2140
$\frac{3}{4}$	20	40	994	1320
		80	1890	1800
1	25	40	866	1210
		80	1680	1670
$1\frac{1}{4}$	32	40	773	980
		80	1470	1410
$1\frac{1}{2}$	40	40	740	890
		80	1390	1270
2	50	40	670	750
$2\frac{1}{2}$	65	40	665	810
3	80	40	624	700
$3\frac{1}{2}$	90	40	600	640
4	100	40	580	580
6	150	40	534	450
8	200	40	515	390
10	250	40	496	340
12	300	40	436	260

TABLE 9.17 Allowable Working Pressures for Copper Tubing

Pipe size, in	Wall thickness, in	Rated internal working pressure, psig: 100°F (38°C) Annealed	100°F (38°C) Drawn	200°F (93°C) Annealed	200°F (93°C) Drawn	300°F (149°C) Annealed	300°F (149°C) Drawn	400°F (204°C) Annealed or Drawn
$\frac{1}{8}$	0.030	3130		3090		2620		1310
$\frac{3}{16}$	0.030	1990		1950		1650		820
$\frac{1}{4}$	0.030	1450		1420		1200		600
$\frac{15}{16}$	0.032	1230		1200		1020		510
$\frac{3}{8}$	0.030	900	1350	880	1300	740	1180	370
$\frac{3}{8}$	0.032	1010		990		840		420
$\frac{1}{2}$	0.032	740		730		610		300
$\frac{1}{2}$	0.035	800	1200	780	1150	660	1060	330
$\frac{5}{8}$	0.035	640		630		530		260
$\frac{5}{8}$	0.040	740	1110	720	1060	610	980	300
$\frac{3}{4}$	0.042	650	980	630	930	530	850	260
$\frac{7}{8}$	0.045	590	890	570	840	480	770	240
$1\frac{1}{8}$	0.050	510	770	490	720	420	670	210
$1\frac{3}{8}$	0.030	460	690	440	650	370	590	180
$1\frac{5}{8}$	0.030	430	650	410	600	350	560	170
$2\frac{1}{8}$	0.030	370	560	360	530	300	480	150
$2\frac{5}{8}$	0.030	350	530	340	500	280	450	140
$3\frac{1}{8}$	0.030	330	500	320	470	270	430	130
$3\frac{5}{8}$	0.030	320	480	300	440	260	420	130
$4\frac{1}{8}$	0.030	300	450	290	430	240	380	120

Steel pipe used must be schedule 40 or heavier. Butt-welded carbon-steel pipe is not used. However, electric resistance welded (ERW) pipe is allowed. Copper or brass tubing, type K, L, or ACR, can be used with any refrigerants.

Tables 9.16 and 9.17 show the allowable working pressures for carbon steel and copper tubing type ACR used in refrigerant piping systems. For more details on piping materials and allowable pressures refer to the ASHRAE and ASME standards discussed in Sec. 9.1.

Chapter

10

Slurry and Sludge Systems Piping

Introduction

A slurry consists of solid particles suspended in a liquid. A slurry pipeline is used to transport slurries from the source such as a coal mine to its destination such as a coal power plant. In this case the coal slurry will be a mixture of coal and water, which is a transportation medium used to propel the combined solid-liquid mass through the pipeline using centrifugal pumps to provide the required pressure. Slurry pipelines pose many challenges including daunting rheological issues, the availability of water as a medium of transport, and pumping equipment. Slurries may be newtonian or nonnewtonian in nature. When the particle concentration of solid within the liquid is less than 10 percent by volume, the slurry may be considered newtonian. When the slurry concentration is higher than 10 percent, it is generally regarded as nonnewtonian.

10.1 Physical Properties

Since the slurry consists of solid particles suspended in a liquid, the properties of a slurry mixture will depend upon those of the constituents. The density of slurry can be calculated from the following equation:

$$\rho_m = \frac{100}{(C_w/\rho_s) + [(100 - C_w)/\rho_L]} \tag{10.1}$$

where ρ_m = density of slurry mixture, lb/ft^3
C_w = solids concentration by weight, %

ρ_s = density of solid in mixture, lb/ft^3
ρ_L = density of liquid in mixture, lb/ft^3

The variable C_w represents the amount of solid in the mixture by weight. The term C_v is a corresponding value in terms of volume. Thus C_w may be 50 percent solids by weight, whereas C_v may be 15 percent solids by volume. The term *volume fraction* represented by the symbol Φ is equal to $C_v/100$. The term *volume ratio* represents the ratio of the volume of solid to the volume of liquid. Thus we get the following equations for the volume fraction and volume ratio:

$$\text{Volume fraction } \Phi = \frac{C_v}{100} \tag{10.2}$$

$$\text{Volume ratio} = \frac{\Phi}{1 - \Phi} \tag{10.3}$$

where C_v is the concentration of solids by volume (%) and Φ is the volume fraction.

The concentration of solids by volume C_v and the concentration of solids by weight C_w are related to the solid density and the mixture density by the following equation:

$$C_v = C_w \frac{\rho_m}{\rho_s} \tag{10.4}$$

where C_v = solid concentration by volume, %
C_w = solid concentration by weight, %
ρ_m = density of slurry mixture, lb/ft^3
ρ_s = density of solid, lb/ft^3

The viscosity of a dilute suspension consisting of solids in a liquid can be calculated approximately from the volume fraction Φ and the viscosity of the liquid using the following equation:

$$\mu_m = \mu_L(1 + 2.5\Phi) \tag{10.5}$$

where μ_m = viscosity of slurry mixture, centipoise (cP)
μ_L = viscosity of liquid, cP
Φ = volume fraction of slurry, dimensionless

The preceding calculation of the mixture viscosity applies only to laminar flow and to spherical particles. Also Eq. (10.5) does not apply for solid concentrations exceeding 1 percent by volume.

For higher-concentration suspensions the viscosity of the mixture can be calculated using a modified form of Eq. (10.5) attributed to D. G. Thomas.

$$\mu_m = \mu_L[1 + 2.5\Phi + 10.05\Phi^2 + 0.00273\ \exp(16.6\Phi)] \tag{10.6}$$

where μ_m = viscosity of slurry mixture, cP
μ_L = viscosity of liquid, cP
Φ = volume fraction of slurry, dimensionless

Example 10.1 If the volume concentration of a slurry mixture is 15 percent and the solid viscosity is 3 cP, calculate the mixture viscosity if the liquid is water and the flow is laminar.

Solution

$$C_v = 15\% \qquad \mu_s = 3.0$$

Since the solid concentration is 15 percent, we need to use Eq. (10.6) to calculate the mixture viscosity μ_m. First, calculate the volume fraction,

$$\Phi = \frac{C_v}{100} = \frac{15}{100} = 0.15$$

$$\mu_m = 3[1 + 2.5(0.15) + 10.05(0.15)^2 + 0.00273\exp(16.6 \times 0.15)]$$

Solving we get

$$\mu_m = 4.902\ \text{cP}$$

Example 10.2 A slurry mixture consisting of magnetite in water has a concentration of 65 percent solids by weight, and the specific gravity of the solids is 5.2. Calculate the specific gravity, volume fraction, and volume ratio of the slurry mixture.

Solution

$$C_w = 65\% \qquad \rho_s = 5.2$$

The specific gravity of the slurry mixture is calculated from Eq. (10.1) as follows:

$$\text{Sg}_m = \frac{100}{(65/5.2) + (35/1.0)} = 2.10$$

Therefore, the specific gravity of the slurry mixture is 2.10.

The concentration by volume C_v is given by Eq. (10.4).

$$C_v = 65\left(\frac{2.1}{5.2}\right) = 26.25\%$$

Next we calculate the volume fraction,

$$\Phi = \frac{26.25}{100} = 0.2625$$

and then we obtain

$$\text{Volume ratio} = \frac{0.2625}{1 - 0.2625} = 0.3559$$

Example 10.3 A slurry consists of raw salt in a brine solution. Experiments indicate that this slurry weighs 95 pounds per cubic foot (lb/ft^3). Calculate the concentration of solids by weight and by volume and the volume ratio. Use 130 lb/ft^3 for the density of salt and 80 lb/ft^3 for the density of brine.

Solution

$$\text{Slurry density } \rho_m = 95 \text{ lb/ft}^3$$
$$\text{Liquid density } \rho_L = 80 \text{ lb/ft}^3$$
$$\text{Solid density } \rho_s = 130 \text{ lb/ft}^3$$

From the slurry density Eq. (10.1) we get

$$95 = \frac{100}{(C_w/130) + [(100 - C_w)/80]}$$

Solving for the solids concentration by weight,

$$\frac{C_w}{130} + \frac{100}{80} - \frac{C_w}{80} = \frac{100}{95}$$

$$C_w\left(\frac{1}{130} - \frac{1}{80}\right) = 100\left(\frac{1}{95} - \frac{1}{80}\right)$$

$$C_w\,(0.000481) = 0.1974$$

$$C_w = 41.05\%$$

The concentration by volume C_v from Eq. (10.4) is

$$C_v = 41.05\,\frac{95}{130} = 29.998\%$$

Using C_v we calculate

$$\text{Volume fraction } \Phi = \frac{29.998}{100} = 0.30$$

$$\text{Volume ratio} = \frac{0.3}{1 - 0.3} = \frac{0.3}{0.7} = 0.4286$$

Example 10.4 Calculate the viscosity of a slurry mixture consisting of salt (50 percent by weight) in saturated brine assuming a newtonian fluid. The viscosity of brine is 2.0 cP, and the density of brine is 75 lb/ft^3 and that of salt is 130 lb/ft^3.

Solution We calculate the density of slurry first from Eq. (10.1)

$$\rho_m = \frac{100}{(50/130) + (50/75)} = 95.12 \text{ lb/ft}^3$$

The volume fraction Φ is calculated from C_v. Using Eq. 10.4,

$$C_v = 50\,\frac{95.12}{130} = 36.58\%$$

Therefore,

$$\text{Volume fraction } \Phi = \frac{36.58}{100} = 0.3658$$

The viscosity of the slurry mixture can then be calculated from Eq. (10.6) as follows:

$$\begin{aligned}\mu_m &= 2.0\,[1 + 2.5\,(0.3658) + 10.05\,(0.3658)^2 + 0.00273\,\exp\,(16.6 \times 0.3658)] \\ &= 13.55 \text{ cP}\end{aligned}$$

10.2 Newtonian and Nonnewtonian Fluids

Fluids may be characterized as newtonian or nonnewtonian. The distinction is based on how the fluid viscosity (resistance to flow) varies with the velocity gradient in a pipe. A newtonian fluid is one in which the shear stress between adjacent layers of the flowing fluid is proportional to the velocity gradient. The constant of proportionality is known as the absolute or dynamic viscosity μ of the fluid. This is illustrated in Fig. 10.1.

In Fig. 10.1 we have plotted the velocity gradient dV/dy along the x axis and shear stress τ along the y axis. The velocity gradient is defined as the rate of change of velocity V with respect to distance y measured along the pipe radius as shown in Fig. 10.1. The velocity at the pipe wall ($y = 0$) is zero. This linear relationship between shear stress and velocity gradient (sometimes also known as the shear rate)

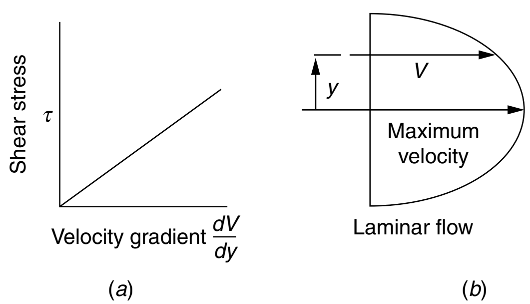

Figure 10.1 Newtonian flow.

is the classical Newton's equation for newtonian fluids as follows:

$$\tau = \mu \frac{dV}{dy} \tag{10.7}$$

where τ = shear stress at a vertical distance y from pipe wall
μ = viscosity of fluid
$\frac{dV}{dy}$ = velocity gradient, also known as shear rate

Since the viscosity μ of a newtonian fluid is constant at a particular temperature and pressure, the slope of the shear stress versus velocity gradient is constant. Also for newtonian fluids, at zero shear rate, the shear stress is zero and hence the straight line passes through the origin as indicated in Fig. 10.1. For nonnewtonian fluids this is not the case. Hence the shear stress versus velocity gradient will not be a straight line and will not pass through the origin for most nonnewtonian fluids.

Graphs showing the shear stress τ versus velocity gradient dV/dy are known as *rheograms*. The rheogram of a nonnewtonian fluid is not a straight line and may not start at $y = 0$, $dV/dy = 0$. It is generally curved and may have a positive shear stress at a zero velocity gradient. This is illustrated in Fig. 10.2, which shows rheograms of both newtonian and nonnewtonian fluids.

It can be seen from Fig. 10.2 that newtonian, pseudo-plastic, and dilatant fluids all have zero shear stress at a zero velocity gradient. Bingham plastic, yield pseudo-plastic, and yield dilatant fluids have positive shear stress at a zero velocity gradient. The pseudo-plastic and dilatant fluids have curved rheograms indicating that the dynamic viscosity is not a constant, unlike newtonian or Bingham plastic fluids.

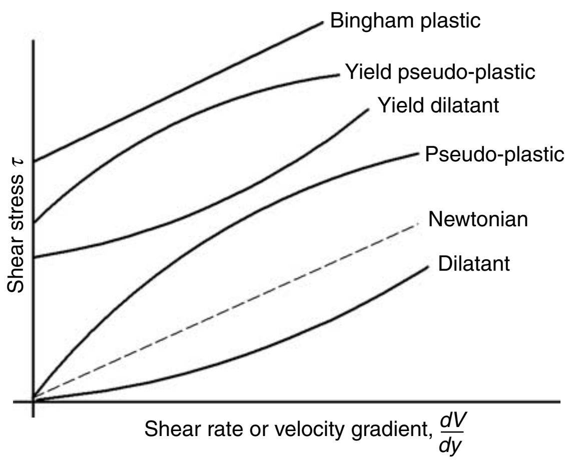

Figure 10.2 Newtonian and nonnewtonian fluids.

Since a minimum shear stress is required to cause flow in Bingham plastic, yield pseudo-plastic, and yield dilatant fluids, it is said that these fluids exhibit yield stress. As the shear stress is increased beyond the yield stress (τ_0) the Bingham fluid behaves like a newtonian fluid exhibiting constant slope or constant dynamic viscosity. Fluids that have curved rheograms are said to have an apparent viscosity μ equal to the slope of the rheograms at a particular velocity gradient. Thus, these fluids have an apparent viscosity which depends on the shear rate.

Petroleum liquids (such as oil, gasoline, and diesel) and water are considered newtonian liquids. Slurries are generally considered nonnewtonian; however, depending on the concentration, some slurries may behave as a newtonian fluid. Thus slurries at low concentration may be considered newtonian and become nonnewtonian as the slurry concentration increases.

10.2.1 Bingham plastic fluids

The shear stress τ for a Bingham plastic fluid can be represented in terms of the apparent viscosity and the velocity gradient by the following equation:

$$\tau = \tau_0 + \eta \frac{dV}{dy} \tag{10.8}$$

where τ = shear stress at distance y from pipe wall
τ_0 = yield stress
η = coefficient of rigidity
$\frac{dV}{dy}$ = velocity gradient

The term η is referred to as the coefficient of rigidity or plastic viscosity and has units of absolute viscosity μ. Examples of Bingham plastic fluids are clay suspended in water, sewage sludge, fly ash, paint, and coal slurry.

10.2.2 Pseudo-plastic fluids

For nonnewtonian fluids other than Bingham plastic fluids, the power law model is used to define the shear stress versus velocity gradient relationship. The power law equation is as follows:

$$\tau = K\left(\frac{dV}{dy}\right)^n \tag{10.9}$$

where τ = shear stress at distance y from pipe wall
K = power law coefficient or consistency index
n = power law exponent or flow behavior index
$\frac{dV}{dy}$ = velocity gradient

In Eq. (10.9) the term K is known as the power law coefficient or consistency index and the constant n is known as the power law exponent or flow behavior index. For pseudo-plastic fluids $n < 1$, and for dilatant fluids $n > 1$. Since the apparent viscosity μ is the slope of the shear stress τ versus velocity gradient plot, we can calculate the apparent viscosity of a nonnewtonian fluid that follows the power law from Eq. (10.9) as follows:

$$\mu = K\left(\frac{dV}{dy}\right)^{n-1} \qquad (10.10a)$$

where μ = absolute viscosity of liquid

K = power law coefficient

n = power law exponent

$\frac{dV}{dy}$ = velocity gradient

Pseudo-plastic fluids include water mixtures of limestone and hydrocarbon grease.

10.2.3 Yield pseudo-plastic fluids

Yield pseudo-plastic and yield dilatant fluids obey the power law and have a positive intercept on the τ axis, representing the yield stress τ_0. The following shear stress versus velocity gradient relationship exists for these fluids:

$$\tau = \tau_0 + K\left(\frac{dV}{dy}\right)^{n} \qquad (10.10b)$$

where τ = shear stress at distance y from pipe wall

τ_0 = yield stress

K = power law coefficient or consistency index

n = power law exponent or flow behavior index

$\frac{dV}{dy}$ = velocity gradient

It can be seen from Eq. (10.10b) that $n = 1$ and $K = \eta$ will give us the Bingham plastic fluid equation (10.8).

Most nonnewtonian fluids have apparent viscosity that does not change with time. However, the viscosity of thixotropic fluids decreases with time. In the petroleum industry the drilling fluid known as bentonitic clay is considered a thixotropic fluid. Also some crude oils may exhibit thixotropic behavior at low temperatures. Some nonnewtonian fluids show another form of thixotropic behavior, where the viscosity of fluid under shear increases with time. Such fluids with viscosity that increases with time are termed rheopectic fluids. These fluids are also said to be negative thixotropic fluids.

As discussed earlier, newtonian fluids at a particular pressure and temperature have a fixed viscosity that represents the proportionality constant in the shear stress versus velocity gradient relationship.

Example 10.5 A slurry mixture follows the power law model and has the following shear stress versus shear rate characteristics: 20 dynes per square centimeter (dyne/cm^2) at a shear rate of 10 and 39 dyne/cm^2 at a shear rate of 25. Calculate the index n and the coefficient K.

Solution Representing shear stress by τ and the velocity gradient (shear rate) by γ,

$$\tau_1 = 20 \text{ dyne/cm}^2 \qquad \gamma_1 = 10 \text{ s}^{-1}$$

$$\tau_2 = 39 \text{ dyne/cm}^2 \qquad \gamma_2 = 25 \text{ s}^{-1}$$

The power law model [Eq. (10.9)] is

$$\tau = K\left(\frac{dv}{dy}\right)^n$$

Substituting the two pairs of values for shear stresses and shear rates given, we get

$$20 = K(10)^n \tag{10.11a}$$

$$39 = K(25)^n \tag{10.11b}$$

Dividing one equation by the other, eliminating K we get

$$\left(\frac{25}{10}\right)^n = \frac{39}{20} = 1.95$$

Solving for n by taking the log on both sides,

$$n \log 2.5 = \log 1.95$$

$$n = \frac{\log 1.95}{\log 2.5} = \frac{0.29003}{0.39794} = 0.7288$$

The coefficient K can then be found from either Eq. (10.11a) or (10.11b):

$$K = \frac{20}{10^n} = \frac{20}{10^{0.7288}} = 3.7345$$

The power law equation is then

$$\tau = 3.7345\left(\frac{dv}{dy}\right)^{0.7288}$$

10.3 Flow of Newtonian Fluids

We will first discuss the flow of newtonian fluids and how pressure loss due to friction is calculated from the pipe size and fluid properties. For a newtonian fluid, under steady-state flow through a pipeline, the average velocity is given by

$$V = \frac{Q}{A} \tag{10.12}$$

where V = average velocity of flow
Q = flow rate, ft^3/s
A = area, ft^2

The dimensionless parameter called the Reynolds number of flow is defined as a function of flow velocity, pipe inside diameter, liquid density, and viscosity as follows:

$$\mathrm{Re} = \frac{VD\rho}{\mu} \tag{10.13}$$

where Re = Reynolds number of flow, dimensionless
V = velocity of flow
D = pipe inside diameter
ρ = density of liquid
μ = absolute viscosity of liquid

Consistent units are used to ensure that the resulting value of Re is dimensionless.

When the Reynolds number < 2100 approximately, the flow is termed *laminar*. When the Reynolds number > 4000, the flow is considered *turbulent*. If Re falls between these two numbers (2100 and 4000), the flow is termed *critical*. In laminar flow, the friction factor f (also known as the Darcy friction factor) depends only on the Reynolds number as follows:

$$f = \frac{64}{\mathrm{Re}} \tag{10.14}$$

where f is the Darcy friction factor and Re is the Reynolds number (dimensionless).

For turbulent flow, f depends not only on the Reynolds number but on the pipe diameter and internal pipe roughness as well. The friction factor for turbulent flow is generally calculated from the Colebrook-White formula as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{10.15}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
Re = Reynolds number, dimensionless

The Moody diagram, based on the Colebrook-White formula, shown in Fig. 10.3 can also be used to determine the friction factor for turbulent flow. In the critical flow regime ($2100 < \text{Re} < 4000$), the flow is unstable and hence there is no reliable method to calculate the friction factor. Sometimes, the turbulent flow friction factor is used in the critical flow regime also.

Once the friction factor is known, the pressure drop due to friction in a newtonian flow can be calculated using the Darcy equation, sometimes also called the Darcy-Weisbach equation, as follows:

$$h = \frac{fLV^2}{2gD} \tag{10.16}$$

where h = frictional pressure loss, ft of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, ft
V = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2
D = inside pipe diameter, in

It must be noted that the friction factor discussed earlier is more correctly called the Darcy friction factor. Some texts refer to another friction factor called the Fanning friction factor which is numerically equal to one-fourth the Darcy friction factor. Throughout this chapter we will use the Darcy friction factor except in a few instances where experimental data are plotted against the Fanning friction factor.

The Darcy equation is used to calculate the pressure loss due to friction in feet of liquid head. The corresponding pressure drop in U.S. Customary System (USCS) units of in pounds per square inch (lb/in^2 or psi), or in Système International (SI) units of kilopascals, may be calculated by factoring in the liquid specific gravity as follows:

$$\Delta P = \frac{h \times \text{Sg}}{2.31} \text{ psi} \tag{10.17}$$

where ΔP = pressure drop, psi
h = head loss due to friction, ft of liquid
Sg = liquid specific gravity

Similarly, in SI units,

$$\Delta P = 9.7955 \times h \times \text{Sg} \tag{10.18}$$

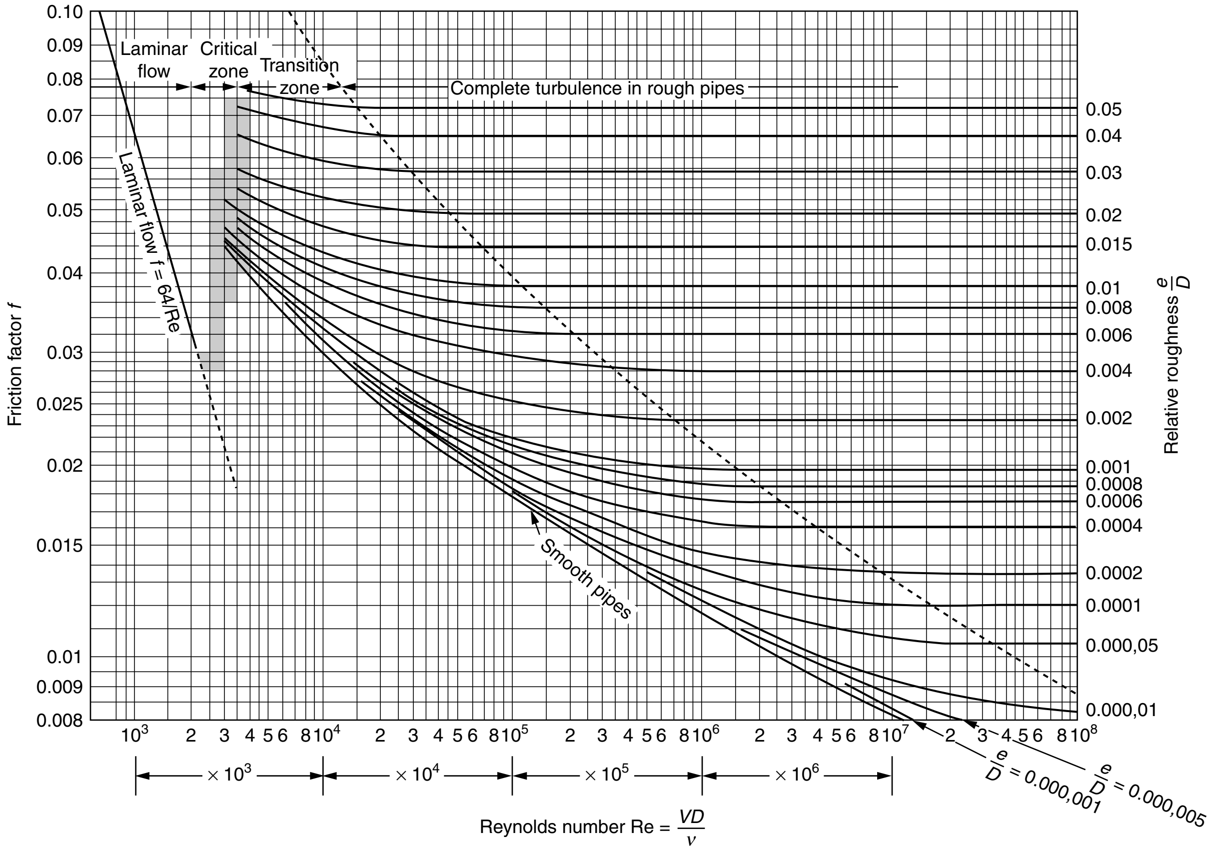

Figure 10.3 Moody diagram.

where ΔP = pressure drop, kPa
h = head loss due to friction, m
Sg = liquid specific gravity

10.4 Flow of Nonnewtonian Fluids

For nonnewtonian fluids, the discussion that follows will employ equations similar to that used for newtonian fluids. However, we will introduce several modified versions of the Reynolds number and friction factor to calculate the friction loss in laminar and turbulent flows of nonnewtonian fluids, such as slurries.

10.4.1 Laminar flow of nonnewtonian fluids

Since nonnewtonian fluids may be Bingham plastic, pseudo-plastic, or yield pseudo-plastic, we have to treat each type separately. As the velocity of flow of a fluid in a pipeline is increased, the pressure loss due to friction increases. The critical velocity at which the flow ceases to be laminar is termed the *transition velocity*. For newtonian fluids this velocity corresponds to that at which the Reynolds number equals approximately 2100.

Thus for newtonian fluids the transition velocity V_T can be calculated from

$$2100 = \frac{V_T D \rho}{\mu} \tag{10.19}$$

where V_T = laminar-turbulent transition velocity
D = pipe inside diameter
ρ = density
μ = viscosity

If a slurry mixture is approximately newtonian, we can calculate the transition velocity using Eq. (10.19) by substituting the slurry mixture density ρ and the slurry viscosity μ.

Bingham plastic fluids. The transition velocity for Bingham plastic fluids must be calculated using both the Reynolds number and another dimensionless parameter called the Hedstrom number. The Reynolds number is first calculated from

$$\text{Re} = \frac{VD\rho}{\eta} \tag{10.20}$$

where Re = Reynolds number, dimensionless
V = flow velocity, ft/s

D = pipe inside diameter
ρ = density of fluid
η = plastic viscosity or coefficient of rigidity as defined in Eq. (10.8).

Next, the dimensionless Hedstrom number He is defined as

$$\mathrm{He} = \frac{\rho D^2 \tau_0}{\eta^2} \tag{10.21}$$

where He = Hedstrom number, dimensionless
D = pipe inside diameter
ρ = density of fluid
τ_0 = yield stress
η = plastic viscosity of liquid

Consistent units are used to ensure that the resulting value of He is dimensionless.

Once He is calculated based upon pipe size and fluid properties using Eq. (10.21) the critical Reynolds number Re_c (corresponding to the laminar-turbulent transition velocity) is found using the graph shown in Fig. 10.4. Having found the critical Reynolds number Re_c we can then calculate the transition velocity V_T using Eq. (10.20).

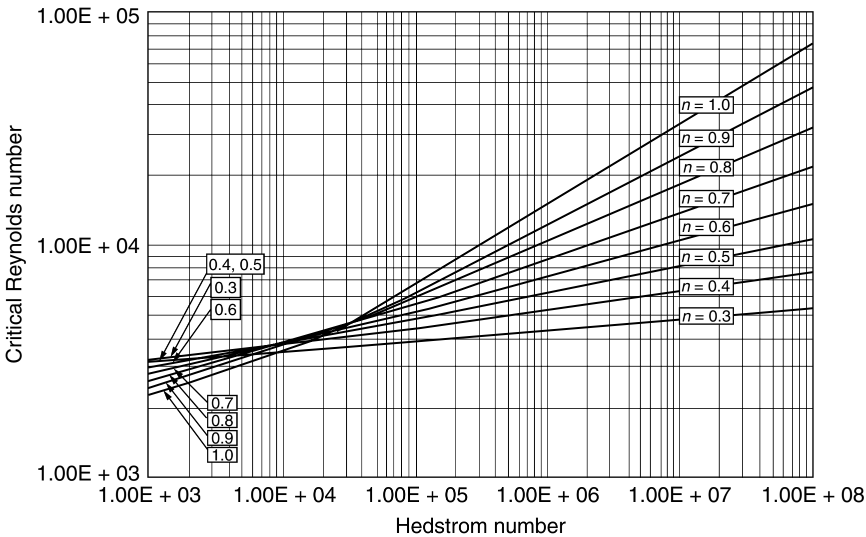

Figure 10.4 Critical Reynolds number versus Hedstrom number for Bingham plastic fluids.

Another approach to calculating the transition velocity V_T for Bingham plastics is using an effective viscosity calculated as follows:

$$\mu_e = \eta \left(1 + \frac{\tau_0 D}{6\eta V}\right) \tag{10.22}$$

where μ_e = effective viscosity, cP
η = coefficient of rigidity
τ_0 = yield stress
D = pipe inside diameter, in
V = flow velocity, ft/s

Neglecting the number 1 in comparison with the much larger second term in Eq. (10.22) and using V_T as the transition velocity, the effective viscosity becomes

$$\mu_e = \frac{\tau_0 D}{6V_T} \tag{10.23}$$

where μ_e = effective viscosity
τ_0 = yield stress
D = pipe inside diameter
V_T = transition velocity

We can now calculate the transition velocity V_T from the critical Reynolds number as follows:

$$V_T = \sqrt{\frac{\tau_0(\mathrm{Re}_c)}{6\rho}} \tag{10.24}$$

A variation of Eq. (10.24) for calculating transition velocity V_T is as follows:

$$V_T = 19\sqrt{\frac{\tau_0}{\rho}} \tag{10.25}$$

where V_T = transition velocity
τ_0 = yield stress
Re_c = critical Reynolds number at velocity V_T
ρ = fluid density

Equations (10.24) and (10.25) assume that the transition occurs at a Reynolds number of 2100. If we use a critical Reynolds number of 3000 instead, the constant 19 in Eq. (10.25) must be changed to 22.

Another equation proposed by R. Durand for calculating the transition velocity is

$$V_T = \frac{1000}{D\rho}\left[1 + \sqrt{\frac{1 + D^2\tau_0\rho}{3000\eta}}\right] \tag{10.26}$$

All symbols in Eq. (10.26) are as defined before.

Comparison of results shows that a better correlation with field data is found using the Hedstrom number approach than using Eq. (10.26). Nevertheless, Eqs. (10.24) and (10.25) do give an idea of the approximate value of transition velocity for Bingham plastic fluids. The friction factor in laminar flow of Bingham plastic fluids is calculated from the Reynolds number and Hedstrom number using the following equation.

$$\frac{f}{64} = \frac{1}{\mathrm{Re}} + \frac{\mathrm{He}}{6\mathrm{Re}^2} - \frac{64\mathrm{He}^4}{3f^3\mathrm{Re}^8} \tag{10.27}$$

where f is the Darcy friction factor. Since f appears on both sides of Eq. (10.27), it must be calculated by trial and error. A plot of the Fanning friction factor (equal to $f/4$) versus the Reynolds number for various values of the Hedstrom number is shown in Fig. 10.5.

It must be noted that the Fanning friction factor obtained from Fig. 10.5 is equal to one-fourth the Darcy friction factor we have used

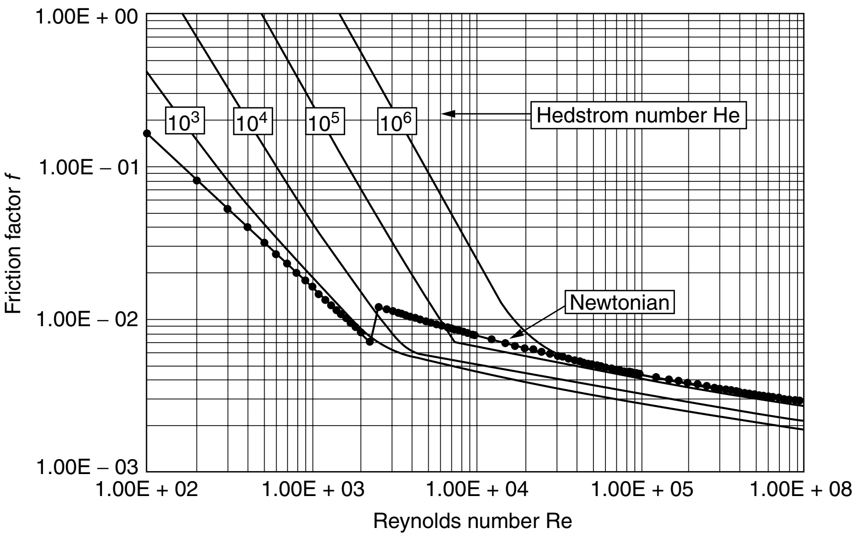

Figure 10.5 Fanning friction factor versus Reynolds number for Bingham plastic fluids.

so far. Once the friction factor is calculated either from Eq. (10.27) or from Fig. 10.5, we can calculate the head loss due to friction using the Darcy equation (10.16).

Pseudo-plastic fluids. Pseudo-plastic fluids that obey the power law model have the shear stress versus shear rate relationship described by Eq. (10.9). The Reynolds number for a nonnewtonian fluid that exhibits the power law behavior is calculated from the following equation:

$$\mathrm{Re} = \rho D^n \frac{V^{2-n}}{K} \tag{10.28}$$

where Re = Reynolds number of flow, dimensionless
ρ = density of fluid
D = pipe inside diameter
V = velocity of flow
K = power law coefficient or consistency index
n = power law exponent or flow behavior index

It can be seen that when $n = 1$ and $K = \mu$ for newtonian fluids, the Reynolds number equation (10.28) reduces to the familiar form $VD\rho/\mu$. However, for pseudo-plastic fluids, we define a modified Reynolds number Re_m as follows:

$$\mathrm{Re}_m = 8\left(\frac{n}{6n+2}\right)^n \rho D^n \left(\frac{V^{2-n}}{K}\right) \tag{10.29}$$

where Re_m = modified Reynolds number of flow, dimensionless
ρ = density of fluid
D = pipe inside diameter
V = velocity of flow
K = power law coefficient
n = power law exponent

Using the modified Reynolds number Re_m defined in Eq. (10.29), we get the Darcy friction factor f for laminar flow of a pseudo-plastic fluid that obeys the power law model as follows:

$$f = \frac{64}{\mathrm{Re}_m} \tag{10.30}$$

The transition from laminar flow to turbulent flow for pseudo-plastic fluids may be assumed to happen at $\mathrm{Re}_m = 2100$. Studies made by researchers N. W. Ryan and M. M. Johnson found that laminar to turbulent flow transition occurs when the critical Reynolds number becomes as defined in the following equation:

$$\mathrm{Re}_{mc} = 6464n \frac{(2+n)^{(2+n)/(1+n)}}{(1+3n)^2} \tag{10.31}$$

For newtonian fluids, $n = 1$ and by substitution in Eq. (10.31), the transition Reynolds number occurs at $\mathrm{Re}_{mc} = 2100$ as expected.

The variation of the critical Reynolds number versus the flow behavior index n is shown in Fig. 10.6. The transition velocity can now be calculated corresponding to the critical Reynolds number Re_{mc} using Eq. (10.29).

Having determined the critical Reynolds number, the friction factor f for the transition point can now be calculated from

$$f = \frac{(1+3n)^2}{101n}\left(\frac{1}{2+n}\right)^{(2+n)/(1+n)} \tag{10.32}$$

Note that the friction factor calculated in Eq. (10.32) is for the transition point only. At any other Reynolds number less than the critical Reynolds number Re_{mc} calculated from Eq. (10.31), f for laminar flow must be calculated using Eq. (10.30). The head loss due to friction in laminar flow of pseudo-plastic fluids can now be calculated using the Darcy equation (10.16).

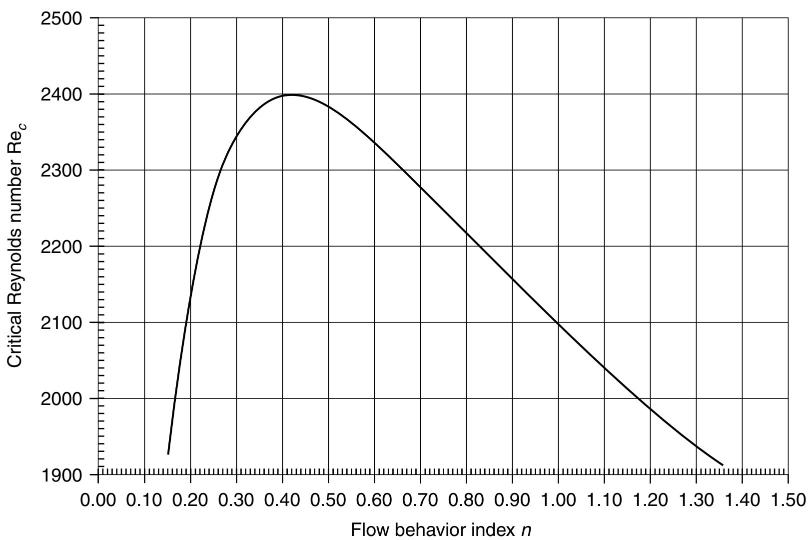

Figure 10.6 Critical Reynolds number versus flow behavior index for pseudo-plastic fluids.

Yield pseudo-plastic fluids. For yield pseudo-plastic fluids the calculation of the critical Reynolds number is more complex. Similar to Bingham plastic fluids we must first calculate the Hedstrom number from the following equation:

$$\mathrm{He} = \frac{\rho D^2 \tau_0}{K^2} \left(\frac{\tau_0}{K}\right)^{2/n-2} \tag{10.33}$$

where He = Hedstrom number of flow, dimensionless
D = pipe inside diameter
ρ = density of fluid
τ_0 = yield stress
K = power law coefficient
n = power law exponent

As before, consistent units are used to ensure that the resulting value of He is dimensionless.

Next we define a parameter x as follows:

$$x = \frac{\tau_0}{\tau_w} \tag{10.34}$$

where τ_0 is the yield stress and τ_w is the wall shear stress. This parameter x must be calculated from the Hedstrom number He as follows:

$$\mathrm{He} = \frac{3232}{n}(2+n)^{(2+n)/(1+n)} \left[\frac{x}{(1-x)^{1+n}}\right]^{(2-n)/n} \left(\frac{1}{1-x}\right)^{n} \tag{10.35}$$

The parameter Φ depends on x and is found from the following equation:

$$\Phi = \frac{\left[\dfrac{(1-x)^2}{1+3n} + \dfrac{2x\,(1-x)}{1+2n} + \dfrac{x^2}{1+n}\right]^{2-n}}{(1-x)^n} \tag{10.36}$$

After calculating Φ from Eq. (10.36) we calculate the critical Reynolds number for yield pseudo-plastic fluids as follows:

$$\mathrm{Re}_{mc} = 6464n \, \frac{(2+n)^{(2+n)/(1+n)}}{(1+3n)^n} \Phi \tag{10.37}$$

The variation of the critical Reynolds number Re_{mc} with the Hedstrom number for various values of n is shown in Fig. 10.7. The transition velocity can now be calculated corresponding to the critical Reynolds number Re_{mc} using Eq. (10.29).

Having determined the critical Reynolds number, the friction factor f for the transition point can now be calculated from Eq. (10.30) as

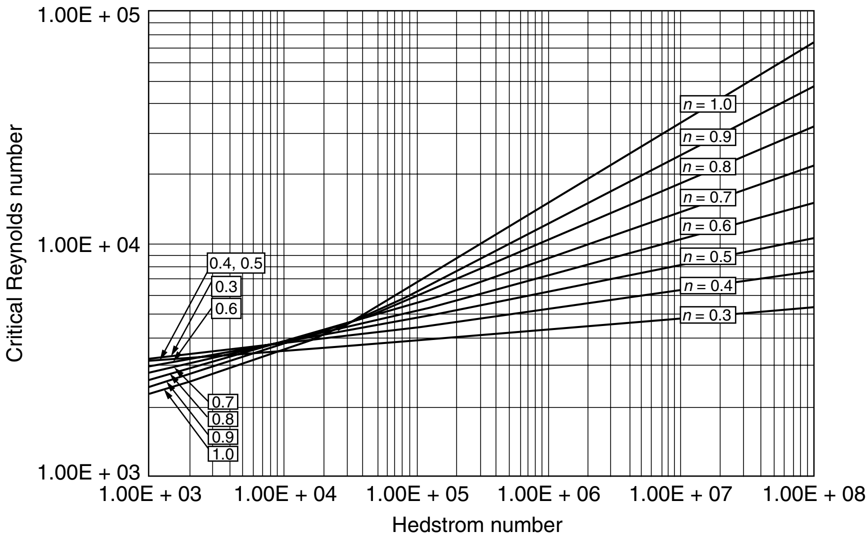

Figure 10.7 Critical Reynolds number versus Hedstrom number for yield pseudoplastic fluids.

follows:

$$f = \frac{64}{\mathrm{Re}_{mc}} \tag{10.38}$$

Note that the friction factor calculated in Eq. (10.38) is for the transition point only. At any other Reynolds number less than the critical Reynolds number Re_{mc} calculated from Eq. (10.37), f for laminar flow must be calculated using Eq. (10.30). The head loss due to friction in laminar flow of yield pseudo-plastic fluids can now be calculated using the Darcy equation (10.16).

A graphical method of determining the friction factor f in laminar flow of yield pseudo-plastic fluids is as follows. A parameter known as the flow function parameter Ψ is defined first. This parameter depends upon the Hedstrom number He and the power law exponent n. A graph showing the variation of Ψ with He and n is shown in Fig. 10.8. From the calculated value of He and exponent n we will determine the value of the flow function parameter Ψ from Fig. 10.8.

Next calculate the friction factor from

$$f = \frac{64}{\Psi \mathrm{Re}_m} \tag{10.39}$$

where Re_m is calculated from Eq. (10.29).

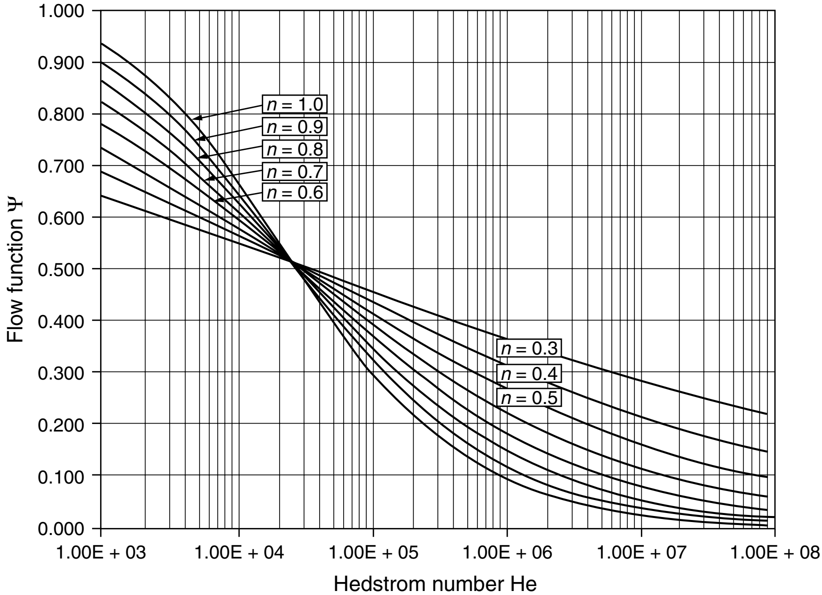

Figure 10.8 Flow function versus Hedstrom number for yield pseudo-plastic fluids.

Example 10.6 A slurry contains 60 percent solids by weight in water at 60°F and has a plastic viscosity of 7 cP and a yield stress of 40 dyne/cm^2. The specific gravity of solids is 3.0. Calculate the value of the Hedstrom number and the transition velocity for pipe inside diameters of 20 and 50 mm.

Solution

$$\tau_0 = 40 \text{ dyne/cm}^2 \qquad \eta = 7 \text{ cP} \qquad \rho_s = 3.0 \qquad D = 20 \text{ mm, } 50 \text{ mm}$$

First we calculate the slurry mixture density for Eq. (10.1):

$$\rho_m = \frac{100}{(60/3.0) + (40/1.0)} = 1.667$$

The Hedstrom number is calculated from Eq. (10.21). For 20-mm pipe,

$$\text{He} = \frac{\tau_0 \rho D^2}{\eta^2} = \frac{40 \times 1.667 \times (2.0)^2}{(0.07)^2} = 5.44 \times 10^4$$

and for 50-mm pipe,

$$\text{He} = \frac{\tau_0 \rho D^2}{\eta^2} = \frac{40 \times 1.667 \times (5.0)^2}{(0.07)^2} = 3.4 \times 10^5$$

The critical Reynolds number from Fig. 10.4 is

$$\text{Re}_c = \begin{cases} 6 \times 10^3 & \text{for He} = 5.44 \times 10^4 \\ 9.5 \times 10^3 & \text{for He} = 3.4 \times 10^5 \end{cases}$$

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The transition velocity can be calculated from Eq. (10.20).

$$\text{Re} = \frac{VD\rho}{\eta}$$

After transposing to solve for V_T, we get

$$V_T = \begin{cases} \dfrac{6 \times 10^3 \times 0.07}{2.0 \times 1.667} = 126.0 \text{ cm/s} & \text{for 20-mm pipe} \\[2ex] \dfrac{9.5 \times 10^3 \times 0.07}{5.0 \times 1.667} = 79.8 \text{ cm/s} & \text{for 50-mm pipe} \end{cases}$$

Example 10.7 A 52 percent by weight suspension of clay in water has a plastic viscosity of 8.5 cP and a yield stress of 0.9 lb/ft^2 (specific gravity of solid is 2.5). Assume yield pseudo-plastic fluid with power law coefficient $K = 1.6$ and exponent $n = 0.4$. Calculate the laminar-turbulent transition velocity for flow in NPS 12 pipe with 0.250-in wall thickness.

Solution Calculate the slurry density from Eq. (10.1).

$$\text{Sg}_m = \frac{100}{(52/2.5) + (48/1.0)} = 1.4535$$

Therefore, the density of the mixture is

$$\rho_m = 1.4535 \times 62.4 = 90.7 \text{ lb/ft}^3$$

$$K = 1.6 \qquad n = 0.4$$

The critical Reynolds number from Fig. 10.6 is

$$\text{Re}_c = 2400$$

The NPS 12 (0.250-in wall thickness) pipe has an inside diameter of

$$D = 12.75 - (2 \times 0.25) = 12.25$$

Therefore using Eq. (10.29) for the modified Reynolds number,

$$2400 = 8\left(\frac{n}{6n+2}\right)^n \frac{\rho D^n V^{2-n}}{K}$$

Solving for transition velocity V_T,

$$V_T{}^{1.6} = \frac{1.6 \times 2400}{8\left(\dfrac{0.4}{6 \times 0.4 + 2}\right)^{0.4} (90.7)\left(\dfrac{12.25}{12}\right)^{0.4}} = 13.6965$$

Therefore, solving for V_T, the transition velocity is

$$V_T = 5.13 \text{ ft/s}$$

Example 10.8 A slurry containing 75 percent solids by weight (solid specific gravity is 2.5) is transported through a 20-cm inside diameter pipe. If a thinning agent were used to reduce the yield stress from 100 to 50 dyne/cm^2 with the plastic viscosity remaining the same at 50 cP, what will be the impact on the transition velocity?

Solution

$$\rho_m = \frac{100}{(75/2.5) + (25/1.0)} = 1.818 \text{ g/cm}^3$$

$$\tau_0 = 100 \text{ dyne/cm}^2 \qquad D = 20 \text{ cm}$$

Next, the Hedstrom number is calculated as

$$\text{He} = \frac{\tau_0 \rho D^2}{\eta^2} = \frac{100 \times 1.818 \times (2.0)^2}{(0.50)^2} = 2.909 \times 10^5$$

From Fig. 10.4,

$$\text{Critical Reynolds number Re}_c = 10{,}000$$

The transition velocity is then calculated as follows:

$$10{,}000 = \frac{V_T D \rho}{\eta}$$

$$V_T = \frac{10{,}000 \times 0.5}{20 \times 1.818} = 137.5 \text{ cm/s} = 1.375 \text{ m/s}$$

Repeating calculations for $\tau_0 = 50$ dyne/cm^2:

$$\text{He} = 1.455 \times 10^5$$

$$\text{Re}_c = 7500$$

$$V_T = 1.03 \text{ m/s}$$

Therefore, the impact of a thinning agent is to reduce the transition velocity from 1.375 to 1.03 m/s.

10.4.2 Turbulent flow of nonnewtonian fluids

For turbulent flow in nonnewtonian fluids, similar to laminar flow, various correlations exist for calculating the friction factor from the Reynolds number and Hedstrom number depending on whether the fluid behaves as a Bingham plastic fluid or one that obeys the power law model.

Bingham plastic fluids. A correlation between the friction factor and the Reynolds number for Bingham plastic fluids was proposed by Hanks and Dadia. The results of their study are depicted in the graph shown in Fig. 10.9. It must be noted that this figure shows the variation of the

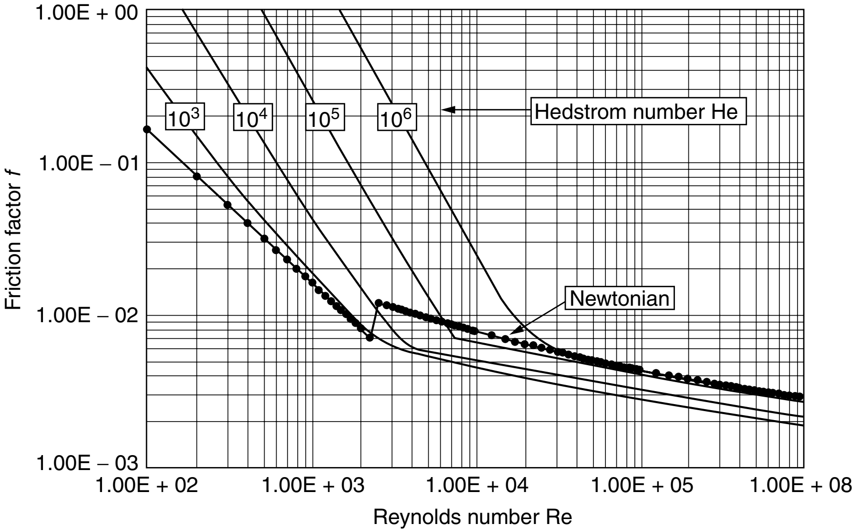

Figure 10.9 Fanning friction factor versus Reynolds number for Bingham plastic fluids.

Fanning friction factor with the Reynolds number for various values of the Hedstrom number. For example, at a Reynolds number of 10^5 and a Hedstrom number of 10^6 for a Bingham plastic fluid, the Fanning friction factor is approximately 0.0045. The Darcy friction factor is therefore 0.018. The head loss due to friction can be calculated as before using the Darcy equation (10.16).

Pseudo-plastic fluids and yield pseudo-plastic fluids. For pseudo-plastic fluids in turbulent flow the Dodge-Metzner equation can be used to calculate the Fanning friction factor as follows:

$$\frac{1}{\sqrt{f}} = \frac{4}{n^{0.75}} \log\left(\mathrm{Re}_m f^{1-n/2}\right) - \frac{0.4}{n^{1.2}} \qquad (10.40)$$

where f = Fanning friction factor, dimensionless
n = power law exponent
Re_m = modified Reynolds number as defined in Eq. (10.29)
log = logarithm to base 10

Equation (10.40) for f is used to plot the friction factor against the Reynolds number as shown in Fig. 10.10 for various values of the behavior index.

Another equation for the Fanning friction factor was proposed by Torrance for turbulent flow of pseudo-plastic fluids and yield

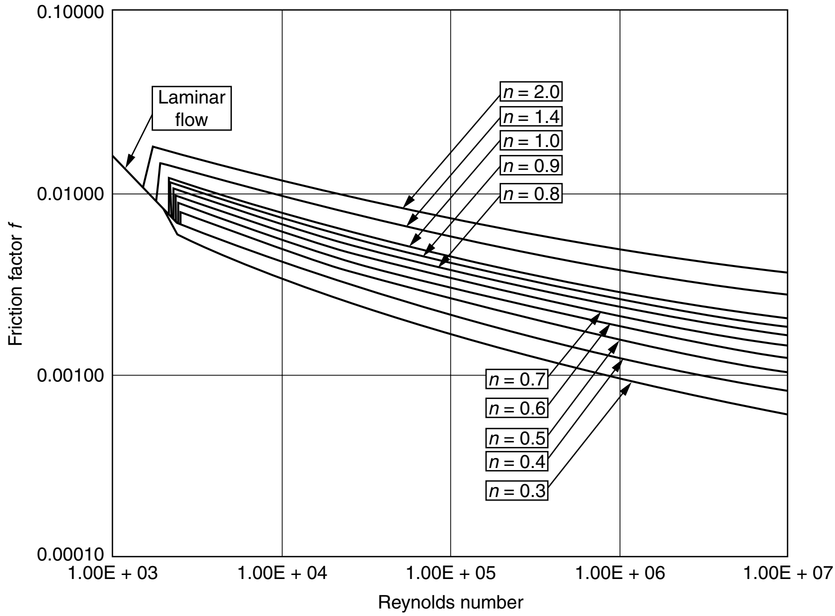

Figure 10.10 Fanning friction factor versus Reynolds number for pseudo-plastic fluids.

pseudo-plastic fluids. The following is applicable for smooth pipes only.

$$\frac{1}{\sqrt{f}} = \frac{2.69}{n} - 2.95 + \frac{4.53}{n}\log(1-x) - \frac{4.53}{n}\log\left(\mathrm{Re}_m\sqrt{f^{2-n}}\right) + \frac{0.68}{n} \tag{10.41}$$

where log represents the logarithm to base 10 and x is the parameter defined before ($x = \tau_0/\tau_w$).

The Reynolds number Re_m in Eq. (10.41) is calculated from

$$\mathrm{Re}_m = \rho D^n \frac{V^{2-n}}{8^{n-1}K} \tag{10.42}$$

Torrance also proposed an equation for rough pipes as follows

$$\frac{1}{\sqrt{f}} = 4.07 \log \frac{D}{2e} + 6.0 - \frac{2.65}{n} \tag{10.43}$$

where log = logarithm to base 10
f = Fanning friction factor for turbulent flow, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
n = power law exponent

This is applicable for rough pipes and may be used for the calculation of the friction factor in fully turbulent flow of Bingham, pseudo-plastic, and yield pseudo-plastic fluids.

Example 10.9 A 20-inch-diameter (0.25-in-wall thickness) coal slurry pipeline is 50 mi long and transports a 50/50 slurry mixture by weight of coal and water at the flow rate of 1800 tons/h. The density of slurry is 80 lb/ft^3. The yield stress is 25 dyne/cm^2, and the coefficient of rigidity is 30 cP. The pipeline availability is 95 percent per year.

(a) Calculate the average flow velocity.

(b) Determine the flow regime.

(c) Calculate the pressure drop per mile due to friction.

(d) What is the pumping HP required at an overall efficiency of 75 percent?

(e) Calculate the cost of transporting in \$/ton assuming an electrical cost of \$0.10 per kilowatt-hour (kWh).

Solution

(a)

$$\text{Average flow velocity} = \frac{\text{Mass flow rate}}{\rho A}$$

$$= \frac{1800 \times 2000}{(19.5/12)^2 \times 0.7854} \times \frac{1}{80 \times 3600} = 6.03 \text{ ft/s}$$

$$\text{Transitional velocity } V_T = 19\sqrt{\frac{25}{(80/62.4) \times 981}} = 2.68 \text{ ft/s}$$

Therefore the flow is turbulent.

$$\text{Reynolds number Re} = \frac{VD\rho}{\mu} = 6.03 \times \frac{19.5}{12} \times 80 \times \frac{100}{0.3 \times 6.7197} = 38{,}886$$

where the constant 6.7197×10^{-2} is used to convert a viscosity of 30 cP (0.3 P) to English units.

The Hedstrom number is

$$\text{He} = \frac{\rho D^2 \tau_0}{\mu^2} = \frac{80 \times (19.5/12)^2 \times 25}{(0.3 \times 6.7197 \times 10^{-2})^2} = 1.3 \times 10^7$$

From Fig. (10.9) we get the Fanning friction factor $= 6 \times 10^{-3}$.

$$\text{Darcy friction factor } f = 4 \times 6 \times 10^{-3} = 0.024$$

$$\text{Pressure drop due to friction} = \frac{fLV^2}{2gD} = \frac{0.024}{64.4}\left(\frac{5280}{19.5/12}\right)(6.03)^2$$

$$= 44.03 \text{ ft/mi}$$

$$= 44.03 \times \frac{80}{62.4} \times \frac{1}{2.31} = 24.44 \text{ psi/mi}$$

The total pressure drop in 50 mi of pipe is

$$\Delta P = 24.44 \times 50 = 1222 \text{ psi}$$

Neglecting elevation effects, the pumping HP required is

$$\text{HP} = \frac{1222 \times 144}{0.75} \frac{1800 \times 2000}{3600 \times 80} \frac{1}{550} = 5333$$

$$\text{Transport cost per ton} = \frac{5333 \times 0.746 \times 24 \times 365 \times 0.10}{0.95 \times 1800 \times 24 \times 365} = \$0.233/\text{ton}$$

Example 10.10 A nonnewtonian fluid that obeys the power law has $n = 0.8$ and $K = 0.00025\ (\text{lb} \cdot \text{s}^n)/\text{ft}^2$. The fluid specific gravity is 1.3. This fluid flows through a pipe with a 1.5-in inside diameter at 1 ft/s. Calculate the pressure drop due to friction at this velocity in a pipe length of 500 ft.

Solution This fluid will be considered as a pseudo-plastic slurry with $K =$ 0.00025 and $n = 0.8$.

$$\rho_m = 1.3 \qquad V = 1.0 \text{ ft/s} \qquad D = \frac{1.5}{12} = 0.125 \text{ ft}$$

The effective viscosity is calculated using Eq. (10.35):

$$\mu_e = K \left(\frac{8V}{D}\right)^{n-1} \left(\frac{4n}{3n+1}\right)^{n}$$

$$= 0.00025 \left(\frac{8 \times 1.0}{0.125}\right)^{-0.2} \left(\frac{4 \times 0.8}{3 \times 0.8 + 1}\right)^{0.8}$$

$$= 0.00025\ (0.4353)(0.9527) = 0.00010\ (\text{lb} \cdot \text{s})/\text{ft}^2$$

$$\text{Reynolds number Re} = \frac{VD\rho}{\mu} = \frac{1 \times 0.125 \times 1.3 \times 62.4}{32.2 \times 0.0001} = 3149$$

From Fig. 10.6, for $n = 0.8$, the critical Reynolds number is $\text{Re}_c = 2230$. Hence the flow is turbulent.

The Fanning friction factor is calculated from the Dodge-Metzner equation:

$$\text{Fanning friction factor } \frac{1}{\sqrt{f}} = \frac{4}{n^{0.75}} \log\left(\text{Re} f^{1-n/2}\right) - \frac{0.4}{n^{1.2}}$$

$$= 4.7287 \log\left(3149 f^{0.6}\right) - 0.5228$$

$$= 0.00946$$

$$\text{Darcy friction factor } f = 4 \times 0.00946 = 0.0378$$

$$\text{Friction head loss} = \frac{fLV^2}{2gD} = \frac{0.0378 \times 500 \times 1^2}{2 \times 32.2 \times 0.125} = 2.348 \text{ ft}$$

$$= \frac{2.348 \times 2.31}{1.3} = 4.17 \text{ psi}$$

Example 10.11 A coal slurry pipeline transports a 50/50 slurry mixture by weight of coal and water at the flow rate of 7 m^3/h. The density of slurry is 1300 kg/m^3. The yield stress is 25 dyne/cm^2, and the coefficient of rigidity is 30 cP. If the pipe inside diameter is 50 mm, determine

(a) Average flow velocity

(b) Flow regime

(c) Pressure drop due to friction for 100 m of pipe

Solution

(a)

$$\text{Average flow velocity} = \frac{Q}{A} = \frac{7}{\frac{\pi}{4}\left(\frac{50}{1000}\right)^2} = 3565.06 \text{ m/h}$$

$$= 0.9903 \text{ m/s} \quad (3.25 \text{ ft/s})$$

(b) The transitional velocity V_T according to Eq. (10.25) is

$$V_T = 19\sqrt{\frac{\tau_0}{\rho}} = 19\sqrt{\frac{25}{1.3 \times 981}}$$

Using proper conversions, 981 is the acceleration due to gravity in cm/s^2.

$$V_T = 2.66 \text{ ft/s}$$

Since the slurry velocity of 3.25 ft/s at the given flow rate exceeds the transitional velocity of 2.66 ft/s, the flow is in the turbulent region.

(c)

$$\text{Reynolds number Re} = \frac{VD\rho}{\mu} = \frac{99.03 \times 5 \times 1.3}{0.3} = 2145$$

$$\text{He} = \frac{\rho D^2 \tau_0}{\mu^2} = \frac{1.3 \times (5)^2 \times 25}{(0.3)^2} = 9028$$

From Fig. 10.4, the critical Reynolds number $\text{Re}_c = 4000$. Therefore, the Darcy friction factor is

$$f = 9.5 \times 10^{-3} \times 4 = 0.038$$

The pressure loss due to friction is therefore calculated from the Darcy equation as

$$\Delta P = \frac{fV^2L}{2gD} = \frac{0.038 \times (0.99)^2 \times 100}{2 \times (9.81) \times (50/1000)} = 3.7965 \text{ m per 100 m of pipe}$$

Example 10.12 A slurry consisting of a suspension of clay in water, 52 percent by weight, has a plastic viscosity of 8.5 cP and a yield stress of 0.9 lb/ft^2 (the specific gravity of the solid is 2.5). Assume yield pseudo-plastic fluid with a power law coefficient $K = 1.6$ and exponent $n = 0.4$. Pipe size is 12-in

NPS with 0.250-in wall thickness. Calculate the pressure drop due to friction using both the Bingham plastic and power law models. The slurry flow rate is 25,000 lb/min.

Solution The flow velocity at 25,000 lb/min is

$$V = \frac{\text{mass flow rate}}{\mathrm{A}\rho}$$

where A is the pipe cross-sectional area and ρ is the slurry density. Therefore

$$V = \frac{25{,}000/60}{0.7854 \times (12.25/12)^2 \times 90.7} = 5.61 \text{ ft/s}$$

Since this velocity is greater than the transition velocity calculated earlier (5.13 ft/s), the flow is turbulent. The Reynolds number is calculated from Eq. (10.29).

$$\begin{aligned} R_m &= 8\left(\frac{n}{6n+2}\right)^n \rho D^n \left(\frac{V^{2-n}}{K}\right) \\ &= 8\left(\frac{0.4}{6 \times 0.4 + 2}\right)^{0.4} \frac{90.7}{1.6}\left(\frac{12.25}{12}\right)^{0.4} (5.61)^{1.6} = 2769 \end{aligned}$$

Using the Dodge-Metzner equation (10.40), we get the Fanning friction factor as

$$\frac{1}{\sqrt{f}} = \frac{4}{(0.4)^{0.75}} \log\left(2769 f^{0.8}\right) - \frac{0.4}{(0.4)^{1.2}}$$

Solving by trial and error we get $f = 0.0066$. Therefore, the Darcy friction factor $f = 4 \times 0.0066 = 0.0264$. The pressure drop can be calculated from the Darcy equation (10.16) as follows:

$$\begin{aligned} h_f &= \frac{fLV^2}{2gD} = \frac{0.0264 \times 1000 \times (5.61)^2}{2 \times 32.2 \times (12.25/12)} \text{ ft per 1000 ft of pipe} \\ &= 12.64 \text{ ft per 1000 ft of pipe} \end{aligned}$$

The pressure drop in psi is

$$\Delta P = \frac{12.64}{2.31} \times \frac{90.7}{62.4} = 7.95 \text{ psi per 1000 ft of pipe}$$

The preceding analysis was based on the power law model for pseudo-plastic fluid.

With a Bingham plastic model we must calculate the Hedstrom number first:

$$\mathrm{He} = \frac{\tau_0 \rho D^2}{\eta^2} = \frac{0.9 \times 90.7 \times (12.25/12)^2}{32.2\,(0.085 \times 2.0886 \times 10^{-3})^2}$$

Therefore He $= 8.38 \times 10^7$. The critical Reynolds number Re_c is found from Fig. 10.4 as follows:

$$\mathrm{Re}_c = 32{,}000$$

Therefore the transition velocity is calculated as follows:

$$\frac{VD\rho}{\eta} = 32{,}000$$

$$\frac{V_T \times (12.25/12) \times (90.7/32.2)}{0.085 \times 2.0886 \times 10^{-3}} = 32{,}000$$

Therefore transition velocity $V_T = 1.98$ ft/s. Also using the simplified formula from Eq. (10.25), we get

$$V_T = 19\sqrt{\frac{\tau_0}{\rho}} = 19\sqrt{\frac{0.9}{90.7}} = 1.89 \text{ ft/s}$$

which is quite close to what we got using the Hedstrom number.

We will now calculate the pressure drop due to friction using the Bingham plastic model. Since the velocity is 5.61 ft/s, the flow is turbulent. For the Bingham plastic model, we calculate the friction factor from He and Re:

$$\text{Re} = \frac{VD\rho}{\eta} = 5.61 \times \frac{12.25}{12} \times \frac{90.7}{32.2} \times \frac{1}{0.085 \times 2.0886 \times 10^{-3}} = 65{,}759$$

The Hedstrom number He $= 8.38 \times 10^7$ from before. From Fig. 10.9,

$$\text{Darcy friction factor} = 0.005 \times 4 = 0.020$$

The pressure drop from the Darcy equation is

$$h_f = \frac{fLV^2}{2gD} = \frac{0.020 \times 1000 \times (5.61)^2}{2 \times 32.2 \times (12.25/12)}$$

$$= 9.58 \text{ ft per 1000 ft of pipe}$$

$$\text{Pressure drop } \Delta P = \frac{9.58}{2.31} \times \frac{90.7}{62.4} = 6.03 \text{ psi per 1000 ft of pipe}$$

Example 10.13 A slurry consists of limestone and water (solid specific gravity is 2.7) flowing in an 8-in inside diameter pipe. The solid concentration is 60 percent by weight. The plastic viscosity is 20 cP and the yield stress is 50 dyne/cm^2. The slurry flows at 5 ft/s. Calculate the pressure drop due to friction in psi per 1000 ft of pipe.

Solution Calculate the slurry density from Eq. (10.1):

$$\text{Sg}_m = \frac{100}{(60/2.7) + (40/1.0)} = 1.607$$

Therefore, the mixture density is

$$\rho_m = 1.607 \times 62.4 = 100.277 \text{ lb/ft}^3$$

Using the Bingham plastic model,

$$\tau_0 = 50 \text{ dyne/cm}^2 \qquad \eta = 20 \text{ cP} \qquad V = 5 \text{ ft/s}$$

We can calculate the Hedstrom number using Eq. (10.21):

$$\mathrm{He} = \frac{\tau_0 \rho D^2}{\eta^2} = \left(\frac{8}{12}\right)^2 \frac{50 \times 2.0886 \times 10^{-3}}{(0.2 \times 2.0886 \times 10^{-3})^2} \frac{100.277}{32.2} = 8.28 \times 10^5$$

Therefore, the critical Reynolds number from Fig. 10.7 is

$$\mathrm{Re}_c = 15{,}000$$

The transition velocity is then calculated as

$$15{,}000 = \frac{V_T D \rho}{\eta}$$

$$V_T = \frac{15{,}000 \times (0.2 \times 2.0886 \times 10^{-3})}{(8/12) \times (100.277/32.2)} = 3.02 \text{ ft/s}$$

Since the flow velocity is 5 ft/s, the flow is in the turbulent zone.

$$\mathrm{Re} = \frac{VD\rho}{\eta} = \frac{5}{3.02} \times 15{,}000 \text{ by proportions}$$

$$= 24{,}834$$

From Fig. 10.9, we get Fanning's friction factor

$$F = 0.006$$

The Darcy friction factor = 4 × 0.006 = 0.024. The head loss due to friction

$$h_f = \frac{fLV^2}{2gD} = \frac{0.024 \times 1000 \times (5)^2}{2 \times 32.2 \times (8/12)}$$

$$= 13.975 \text{ ft per 1000 ft of pipe.}$$

$$\text{Pressure drop } \Delta P = 13.975 \times \frac{1.607}{2.31} = 9.72 \text{ psi per 1000 ft of pipe}$$

10.5 Homogenous and Heterogeneous Flow

We now discuss the calculation methodology for homogenous and heterogeneous suspension in slurry pipelines.

10.5.1 Homogenous flow

Slurry flow consists of solid particles moved by means of a liquid transport medium. The solid particles are suspended in the liquid and depending upon the flow velocity and the particle size may be homogenously distributed or might settle to the lowest point due to gravity. When particle size is very small, it is said to cause Brownian motion which causes random motion of the particles. Since the particles are small and they have very small terminal velocity due to gravity, they remain suspended in the liquid transportation medium. In such cases, the slurry mixture is said to be homogenous. As indicated earlier depending upon the concentration of solid particles in the liquid, slurries may be treated as newtonian or nonnewtonian. When the concentration of the solids is less than 10 percent by volume, newtonian flow may be assumed. At higher concentrations, nonnewtonian equations apply. As the particle size increases, to prevent settlement, the flow velocity must be large enough to cause turbulence and hence keep the particles suspended. If particles settle, the liquid simply moves past the settled particles and hence slurry transportation will not occur.

In connection with slurry pipelines, the solid-liquid mixture is divided into the following categories of flow. Depending upon the particle size and the average velocity, pipe flows are categorized into homogenous (also called pseudo-homogenous), heterogeneous, moving-bed flow, and stationary-bed flow. These four flow zones are graphically illustrated in Fig. 10.11.

In connection with slurry flow, a term called *settling velocity* must be understood. The settling velocity is also known as the terminal velocity of a particle when it falls under gravity in the liquid which is at rest. The terminal velocity depends upon the particle size, the liquid density, and other factors and is calculated using Stokes's law as follows:

$$V_s = \frac{(\rho_s - \rho_m)gd^2}{18\mu} \tag{10.44}$$

where V_s = settling velocity or terminal velocity, ft/s
ρ_s = density of solids, lb/ft^3
ρ_m = density of liquid, lb/ft^3
d = particle diameter, ft

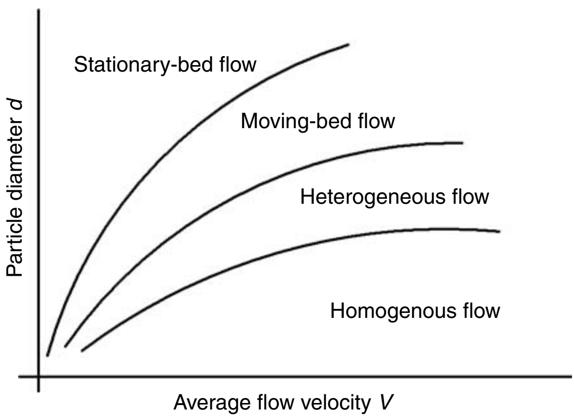

Figure 10.11 Flow regimes in slurry pipeline.

g = acceleration due to gravity
μ = viscosity of liquid, lb/(ft · s)

When the transporting medium is in motion and turbulence occurs, the vertical component of the liquid velocity due to turbulence must counteract and exceed the settling velocity V_s in order to keep the solid particles suspended in the liquid. The concentration of the particles across the pipe cross section will vary with higher concentration present in the lower half of the pipe compared to the upper half. To classify slurry flows the following equation has been proposed by slurry researchers such as E. J. Wasp and T. C. Aude:

$$\log \frac{C_T}{C_A} = -1.8 \frac{V_S}{k u_S} \tag{10.45}$$

where log = log to base 10
C_A = solids concentration by volume at pipe centerline
C_T = solids concentration by volume at top of pipe at distance 92 percent diameter from bottom
V_S = settling velocity
k = von Karman constant, 0.35 to 0.40
u_S = shear velocity at pipe wall given by

$$u_S = \sqrt{\frac{\tau_w}{\rho}} = V\sqrt{\frac{f}{8}} \tag{10.46}$$

where τ_w = shear stress at pipe wall
f = Darcy friction factor
V = flow velocity
ρ = average density

From Eq. (10.45) the ratio C_T/C_A, known as the concentration ratio, may be calculated if the flow velocity, friction factor, etc., are known. Once this ratio is known, the slurry flow can be categorized approximately based on the following ranges for the concentration ratio:

Homogenous flow: $\frac{C_T}{C_A} > 0.8$

Heterogeneous flow: $\frac{C_T}{C_A} < 0.1$

Intermediate flow: $0.1 < \frac{C_T}{C_A} < 0.8$

Sometimes the terms *settling* and *nonsettling* are used with slurries to distinguish between different slurry behavior. A settling slurry may behave homogenous at high-velocities and high-solid concentrations. Likewise at low velocities and low concentrations, it might behave heterogeneous. A homogenous slurry may be defined as one in which the concentration of solids remains fairly constant along the cross section of the pipe. This will happen when the inertia of the suspended particles is fairly negligible and hence they remain dispersed uniformly throughout the liquid. For solids with sizes less than 100 microns the slurry may be considered homogenous because inertial effects would be negligible. In such cases $C_T/C_A = 1$. As the particle size of the suspended solids increases beyond 100 microns the value of C_T/C_A decreases up to about a particle size of 600 microns. Even though at this particle size inertial effects are significant, they are still small in comparison with viscous and turbulent forces. As particle size increases beyond 600 microns, the inertial forces become more significant compared to viscous and turbulent forces. Figure 10.12 illustrates the variation of C_T/C_A with particle size.

It can be inferred from Fig. 10.12 that below 600 microns the slurry is fairly homogenous. However, the nature of the curve in Fig. 10.12 would change with the velocity of flow and the particle size, necessitating another break point for homogenous versus heterogeneous flow. A better approach would be to establish a C_T/C_A ratio that will dictate homogeneity. Strictly speaking there are three zones: homogenous, intermediate, and heterogeneous. At a low flow velocity, and hence under the laminar flow condition, a pressure drop is fairly flat up to some point where the velocity reaches a transitional value V_T known as the viscous transition velocity. As the velocity increases (and hence the flow rate) beyond V_T, the pressure drop increases at a faster rate in the turbulent zone.

Most slurry pipelines operate in the turbulent flow zone, and therefore the velocity will be beyond the transition velocity. However, there

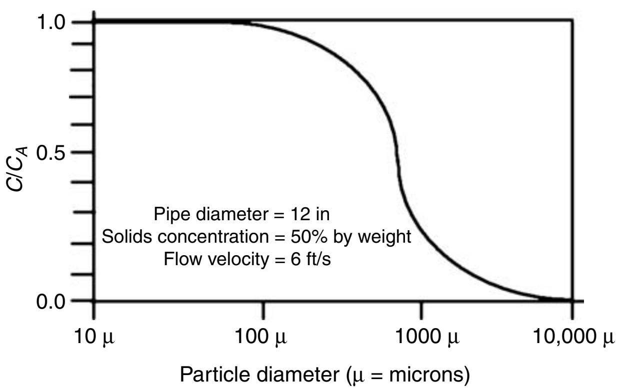

Figure 10.12 Particle size versus concentration ratio.

are exceptions such as a very fine limestone slurry that may at times be operated at velocities below V_T and hence in the laminar zone.

Example 10.14 A coal water slurry consisting of a 50 percent concentration by weight has a solid specific gravity of 1.3 and a slurry viscosity of 22 cP. This slurry flows through a 300-mm inside diameter pipeline at a velocity of 2 m/s. The shear stress at the wall is 10 N/m^2. Calculate the maximum particle size if the C_T/C_A ratio is 0.75.

Solution

$$C_w = 50\% \qquad \rho_s = 1300\ \text{kg/m}^3 \qquad \mu_m = 22\ \text{cP}$$

$$D = 300\text{mm} \qquad V = 2\ \text{m/s} \qquad \tau_w = 10\ \text{N/m}^2$$

$$\frac{C_T}{C_A} = 0.75$$

From Eq. (10.1)

$$\text{Specific gravity of mixture} = \frac{100}{(50/1.3) + [100 - 50/1.0]} = 1.13\ \text{lb/ft}^3$$

Therefore, the density of the slurry mixture is

$$\rho_m = 1.13 \times 1000 = 1130\ \text{kg/m}^3$$

The shear velocity at the wall is calculated from Eq. (10.46):

$$u_S = \sqrt{\frac{\tau_w}{\rho_m}} = \sqrt{\frac{10}{1130}} = 0.0941\ \text{m/s}$$

Next, using Eq. (10.45)

$$\log(0.75) = -1.8\,\frac{Vs}{0.35u_s}$$

The settling velocity is then

$$V_s = -0.35 \times 0.0941 \log 0.75 = 4.1149\ \text{m/s}$$

From the settling velocity we can calculate the maximum particle size using Stokes's law:

$$V_s = \frac{(\rho_s - \rho_m)gd^2}{18\mu}$$

Solving for d, we get

$$d = \sqrt{\frac{18 \times 22 \times 10^{-3} \times 4.1149}{9.81(1300 - 1130)}} = 0.03125\ \text{m}$$

$$= 3.125\ \text{cm}$$

The maximum particle size is 3.125 cm.

Example 10.15 A slurry consisting of iron sand in water in an 8-in inside diameter pipe has a velocity of 15 ft/s. The solids concentration is 50 percent by weight and the mixture viscosity is 1.5 cP. The particles have a diameter of 0.1 mm and a specific gravity of 4.8. The friction pressure drop is 4.5 psi per 1000 ft of pipe. What is the C_T/C_A ratio for this slurry? Use a von Karman constant of 0.38.

Solution

$$\text{Specific gravity of mixture} = \frac{100}{(50/4.8) + (50/1.0)} = 1.655$$

$$\text{Frictional head loss} = 4.5 \text{ psi per 1000 ft}$$

Using the Darcy equation,

$$h_f = \frac{fLV^2}{2gD}$$

for head loss, we get

$$\frac{4.5 \times 2.31}{1.655} = \frac{f}{64.4}\left(\frac{1000}{8/12}\right)15^2$$

Now solving for the Darcy friction factor f,

$$f = \frac{4.5 \times 2.31 \times 64.4 \times 8}{1.655 \times 1000 \times 225 \times 12} = 0.01198$$

From Eq. (10.46) we calculate the shear velocity:

$$u_s = V\sqrt{\frac{f}{8}} = 15\sqrt{\frac{0.01198}{8}} = 0.5806 \text{ ft/s}$$

Then using Eq. (10.44) we get the settling velocity:

$$V_s = \frac{g}{18\mu}(\rho_s - \rho_L)d^2 = \frac{32.2\,(4.8 - 1.0)\,62.4 \times \left(\dfrac{0.1}{25.4 \times 12}\right)^2}{18 \times 1.5 \times 6.7197 \times 10^{-4}} = 0.0453 \text{ ft/s}$$

From Eq. (10.45) we get C_T/C_A.

$$\log\frac{C_T}{C_A} = \frac{-1.8 \times 0.0453}{0.38 \times 0.5806} = -0.3696 = 0.427$$

10.5.2 Heterogeneous flow

Heterogeneous flow is characterized by a nonuniform solids concentration across the pipe cross section. Unlike a homogenous suspension, the volume concentration at the axis of the pipe will be different from that at the top of the pipe or the bottom of the pipe. The more heterogeneous the slurry is, the more the solids concentration tends to increase in the bottom half of the pipe compared to the top. As the flow velocity increases, the solids tend to move up and hence reduce the settling of the solids, which if not controlled, will eventually block the pipeline flow. Only horizontal or nearly horizontal slurry pipelines will be discussed. As the flow velocity decreases, the solid particles move downward and tend to settle at the bottom of the pipe.

An important parameter called *deposition velocity* is defined as the minimum velocity below which solid settlement takes place and hinders pipe flow. If the pipeline is operated for a long period of time at velocities below this critical deposition velocity, eventually the cross section of flow would be reduced and the pipeline will be blocked, and this is not desirable. Therefore, heterogeneous pipelines should be operated at speeds above the deposition velocity. The deposition velocity may also be referred to as the minimum velocity required to keep the solid in suspension. It has been found that the deposition velocity depends to some extent upon the Froude number of the slurry. An equation attributed to Durand for calculating deposition velocity V_L is as follows:

$$F_L = \frac{V_L}{\sqrt{2gD(S-1)}} \tag{10.47}$$

$$S = \frac{\rho_S}{\rho_L} \tag{10.48}$$

where F_L = Froude number
V_L = deposition velocity
g = acceleration due to gravity
D = pipe inside diameter
S = relative density of solid (density of solid/density of liquid medium)

The value of F_L depends upon the particle size d_s and the concentration of solids in slurry by volume. Figures 10.13 and 10.14 show curves for obtaining F_L from the particle size and slurry concentration.

Two sets of curves are shown for estimating F_L. The curve in Fig. 10.13 is for slurries in which the particle size is uniform, and the curve in Fig. 10.14 is for slurries containing nonuniform size particles. Since the deposition velocity is the minimum velocity at which a heterogeneous slurry mixture must be operated, we must ensure that in such a slurry pipeline the actual flowing velocity is at least 10 to 20 percent higher than the deposition velocity so that solid particles do not settle out.

Example 10.16 Calculate the deposition velocity of a heterogeneous slurry with a solid specific gravity of 3.0 in water, for a pipeline with an 8-in internal diameter. The particle size = 1 mm, and volume concentration = 15 percent.

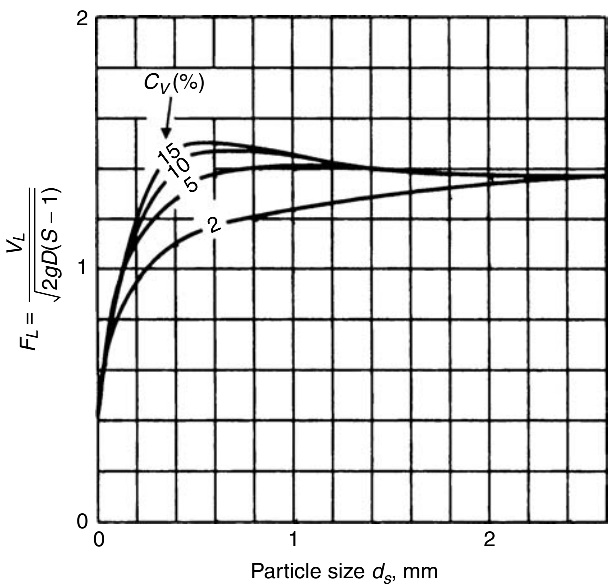

Figure 10.13 Froude number versus particle size for uniform particles.

Solution

$$\rho_s = 3.0 \qquad \rho_L = 1.0 \qquad D = 8.0 \text{ in}$$

From Fig. 10.11 for a uniform particle size of 1.0 mm and $C_v = 0.15$ we get the Froude number $F_L = 1.45$. Therefore, from Eq. (10.47)

$$F_L = \frac{V_L}{\sqrt{2gD(S-1)}} = 1.45$$

$$V_L = 1.45\sqrt{2 \times 32.2 \times \frac{8}{12}(3-1)} = 13.44 \text{ ft/s}$$

The deposition velocity is 13.44 ft/s.

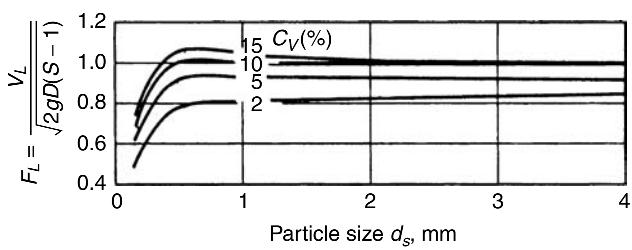

Figure 10.14 Froude number versus particle size for nonuniform particles.

If the solid particles were of nonuniform size, we use the curve in Fig. 10.13 and get $F_L = 1.05$. Then the deposition velocity becomes

$$V_L = 1.05\sqrt{2 \times 32.2 \times \frac{8}{12}(3 - 1)} = 9.73 \text{ ft/s}$$

10.6 Pressure Loss in Slurry Pipelines with Heterogeneous Flow

Many researchers have put forward correlations and equations to calculate the pressure loss due to friction in heterogeneous flows. The following equation proposed by Durand has found acceptance in analyzing heterogeneous slurries:

$$\Phi = \frac{i_m - i}{iC_v} = 67\left(\frac{\sqrt{gD(S-1)}}{V}\right)^3\left(\frac{V_s}{\sqrt{g(S-1)d_s}}\right)^{3/2} \tag{10.49}$$

where Φ = dimensionless parameter
i_m = pressure loss per unit length of pipe for slurry mixture
i = pressure loss per unit length of pipe for liquid medium at same velocity as slurry
C_v = volume concentration of solids, decimal value
g = acceleration due to gravity
D = pipe inside diameter, ft
S = relative density of solid (density of solid/density of liquid medium)
V = average flow velocity
V_s = settling velocity of solid in slurry, obtained by laboratory tests
d_s = particle size, ft

Once the numbers are substituted into the right-hand side of Eq. (10.49) we can calculate the value of the dimensionless parameter Φ. Since the pressure gradient i for the liquid is at the same velocity, the mixture can be easily calculated considering newtonian flow. We can then calculate the value of the pressure loss for slurry i_m as follows:

$$i_m = i(1 + \Phi C_v) \tag{10.50}$$

where Φ = dimensionless parameter
i_m = pressure loss per unit length of pipe for slurry mixture
i = pressure loss per unit length of pipe for liquid medium at the same velocity as slurry
C_v = Volume concentration of solids, decimal value

It must be noted that Durand's equation for pressure drop calculation of a heterogeneous slurry mixture is applicable only at velocities higher than the deposition velocity, which is the required speed for satisfactory operation of a heterogeneous flow anyway. From Durand's equation after some rearrangement we can represent the pressure loss i_m for the slurry mixture as a function of the velocity V. This function, when analyzed to produce the least pressure drop, results in an optimum velocity given by the following equation:

$$V_0 = 3.22\,[g(S-1)]^{1/4} C_v^{1/3} (DV_s)^{1/2} d_s^{-1/4} \tag{10.51}$$

where V_0 = optimum velocity of slurry
C_v = volume concentration of solids, decimal value
g = acceleration due to gravity
D = pipe inside diameter, ft
S = relative density of solid (density of solid/density of liquid medium)
V_s = settling velocity of solid in slurry, obtained by laboratory tests
d_s = particle size, ft

Since slurry flow may be classified as homogenous, intermediate, and heterogeneous, the pressure loss calculation based on these flow regimes will be different in each case. It is important to determine the flow regime first before attempting to calculate the pressure loss due to friction. It has been found that when the concentration ratio C_T/C_A is less than 0.1, the flow can be classified as heterogeneous, and when the ratio is above 0.8, it is classified as homogenous. Therefore in the intermediate flow regime we find that C_T/C_A lies between 0.1 and 0.8.

Several methods have been proposed to calculate pressure loss in intermediate flow regimes. One approach consists of dividing the slurry into several fractions and computing the head loss for a certain portion of the slurry based on a homogenous mixture and the remainder based on a heterogeneous flow. The sum of the two will be the actual pressure loss in the intermediate flow regime.

Another methodology of calculating pressure drop in slurries with intermediate flow was put forth by a Chinese researcher X. J. Fei and reported in *Pipeline Engineering* by Henry Liu (see References). This is expressed by the following equation:

$$i_{mh} = \frac{i_m}{\rho g} = \frac{\Delta P_m}{\rho g L} = \alpha \frac{f V^2 S_m}{2gD} + 11 \eta_s C_v (S - S_m) \frac{V_{sa}}{V} \tag{10.52}$$

where i_{mh} = pressure loss gradient, m/m or dimensionless
i_m = pressure loss per unit length of pipe for slurry mixture, Pa/m
ρ = density of water, kg/m^3
g = acceleration due to gravity, 9.81 m/s^2
ΔP_m = pressure drop for slurry mixture, Pa
L = pipe length, m
α = correction factor, dimensionless
f = Darcy friction factor, dimensionless
V = average flow velocity, m/s
S_m = relative density of slurry mixture compared to liquid, dimensionless
S = relative density of solids compared to liquid, dimensionless
D = pipe inside diameter, m
η_s = contact friction coefficient between solid particles and pipe, dimensionless
C_v = volume concentration of solids, decimal value, dimensionless
V_{sa} = weighted-average settling velocity of solids, m/s

The correction factor α in Eq. (10.52) is a function of μ_r, the relative viscosity of the slurry (slurry viscosity/liquid viscosity), and is calculated from the following equation:

$$\alpha = 1 - 0.4\,(\log \mu_r) + 0.2\,(\log \mu_r)^2 \tag{10.53}$$

Example 10.17 A sand slurry mixture in water is transported through 100 mi of fairly horizontal pipeline with an 8-in inside diameter at a velocity of 10 ft/s. The particle size is 2 mm, and the solid specific gravity is 2.5. The laboratory tests show the average settling velocity to be 0.1 ft/s. The von Karman constant is 0.38. Assume the viscosity of water = 1 cP and density = 62.4 lb/ft^3. Pipe roughness = 0.002 in. The slurry concentration is 50 percent by weight. Calculate the concentration ratio C_T/C_A and determine if the flow is heterogeneous. Calculate the deposition velocity and the pressure gradient for the slurry using Durand's equation.

Solution

$$\text{Density of mixture} = \rho_m = \frac{100 \times 62.4}{(50/2.5) + (50/1.0)} = 89.14 \text{ lb/ft}^3$$

The concentration of solids by volume is from Eq. (10.4):

$$C_v = C_w \frac{\rho_m}{\rho_s} = \frac{50 \times 89.14}{62.4 \times 2.5} = 28.57$$

$$F_L = 1.38 \qquad \text{from Fig. 10.13}$$

Using Eq. (10.47), we get

$$\text{Deposition velocity } V_L = 1.38\sqrt{2 \times 32.2 \times \frac{8}{12}(2.5 - 1)} = 8.02 \text{ ft/s}$$

Since the actual flow velocity is 10 ft/s, the flow is heterogeneous with fully suspended solids.

Next calculate the pressure loss using Durand's equation (10.49).

$$\frac{i_m - i}{iC_v} = 67\left(\frac{\sqrt{32.2 \times \frac{8}{12}(2.5-1)}}{10}\right)^3 \left(\frac{0.1}{\sqrt{32.2(2.5-1)\frac{2}{25.4 \times 12}}}\right)^{1.5}$$

$$= 67 \times 0.1827 \times 0.0749 = 0.9168$$

$$i_m = i(1 + 0.9168 \times 0.2857) = 1.262\, i$$

where i is the pressure loss per unit length for water only. Variable i is calculated as follows:

$$\text{Re} = \frac{VD\rho}{\mu} = 10 \times \frac{8}{12} \times \frac{62.4}{32.2} \times \frac{1}{0.01 \times 2.0886 \times 10^{-3}} = 618{,}560$$

The Darcy friction factor from the Moody diagram for Re = 618,560 and a relative roughness of 0.002/8.00 = 0.00025 is $f = 0.0175$. The pressure gradient for water is

$$i = \frac{fV^2}{2gD} = \frac{0.0175 \times (10)^2}{64.4 \times (8/12)} = 0.0408 \text{ ft/ft}$$

Therefore, $i_m = 1.262 \times 0.0408 = 0.0515$ ft/ft.

The friction factor for the slurry is calculated from the Darcy equation:

$$i_m = 0.0515 = \frac{fV^2}{2gD}$$

For slurry,

$$f = 1.262 \times 0.0175 = 0.0221$$

The shear velocity u_s, from Eq. (10.46), is given by

$$u_S = V\sqrt{\frac{f}{8}} = 10\sqrt{\frac{0.0221}{8}} = 0.5256 \text{ ft/s}$$

From Eq. (10.45) we get

$$\log\frac{C_T}{C_A} = \frac{-1.8 \times 0.1}{0.38 \times 0.5256} = -0.9012 = 0.1255$$

Since the ratio is between 0.10 and 0.80, the flow is in the intermediate zone.

Appendix A

Units and Conversions

Item	USCS units*	SI units†	USCS to SI conversion	SI to USCS conversion
Mass	slug (slug)	kilogram (kg)	1 lb = 0.45359 kg	1 kg = 0.0685 slug
	pound mass (lbm)		1 slug = 14.594 kg	1 kg = 2.205 lb
	1 U.S. ton = 2,000 lb	metric tonne (t) = 1,000 kg	1 U.S. ton = 0.9072 t	1 t = 1.1023 U.S. ton
	1 long ton = 2,240 lb		1 long ton = 1.016 t	1 t = 0.9842 long ton
Length	inch (in)	millimeter (mm)	1 in = 25.4 mm	1 mm = 0.0394 in
	1 foot (ft) = 12 in	1 meter (m) = 1,000 mm	1 ft = 0.3048 m	1 m = 3.2808 ft
	1 mile (mi) = 5,280 ft	1 kilometer (km) = 1,000 m	1 mi = 1.609 km	1 km = 0.6214 mi
Area	square foot (ft^2)	square meter (m^2)	1 ft^2 = 0.0929 m^2	1 m^2 = 10.764 ft^2
	1 acre = 43,560 ft^2	1 hectare = 10,000 m^2	1 acre = 0.4047 hectare	1 hectare = 2.4711 acre
Volume	cubic inch (in^3)	cubic millimeter (mm^3)	1 in^3 = 16387.0 mm^3	1 mm^3 = 6.1×10^{-5} in^3
	cubic foot (ft^3)	1 liter (L) = 1,000 cm^3 (cc)	1 ft^3 = 0.02832 m^3	1 m^3 = 35.3134 ft^3
	1 U.S. gallon (gal) = 231 in^3	1 cubic meter (m^3) = 1,000 L	1 gal = 3.785 L	1 L = 0.2642 gal
	1 barrel (bbl) = 42 gal		1 bbl = 158.97 L = 0.15897 m^3	1 m^3 = 6.2905 bbl
	1ft^3 = 7.4805 gal			
	1 bbl = 5.6146 ft^3			
Density	slug per cubic foot (slug/ft^3)	kilogram/cubic meter (kg/m^3)	1 slug/ft^3 = 515.38 kg/m^3	1 kg/m^3 = 0.0019 slug/ft^3
Specific weight	pound per cubic foot (lb/ft^3)	newton per cubic meter (N/m^3)	1 lb/ft^3 = 157.09 N/m^3	1 N/m^3 = 0.0064 lb/ft^3
Viscosity (Absolute or dynamic)	lb/(ft · s)	1 poise (P) = 0.1 Pa · s		1 cP = 6.7197×10^{-4} lb/(ft · s)
	(lb · s)/ft^2	1 centipoise (cP) = 0.01 P	1 (lb · s)/ft^2 = 47.88 (N · s)/m^2	1 (N · s)/m^2 = 0.0209 (lb · s)/ft^2
		1 poise = 1 (dyne · s)/cm^2	1 (lb · s)/ft^2 = 478.8 poise	1 poise = 0.00209 (lb · s)/ft^2
		1 poise = 0.1 (N · s)/m^2		
Viscosity (kinematic)	ft^2/s	m^2/s	1 ft^2/s = 0.092903 m^2/s	1 m^2/s = 10.7639 ft^2/s
	SSU†, SSF‡	stoke (S), centistoke (cSt)		1 cSt = 1.076×10^{-5} ft^2/s
Flow rate	cubic foot/second (ft^3/s)	liter/minute (L/min)	1 gal/min = 3.7854 L/min	1 L/min = 0.2642 gal/min
	gallon/minute (gal/min)	cubic meter/hour (m^3/h)	1 bbl/h = 0.159 m^3/h	1 m^3/h = 6.2905 bbl/h
	barrel/hour (bbl/h)			
	barrel/day (bbl/day)			

Force	pound (lb)	newton (N) = (kg · m)/s^2	1 lb = 4.4482 N	1 N = 0.2248 lb
Pressure	pound/square inch, lb/in^2(psi)	pascal (Pa) = N/m^2	1 psi = 6.895 kPa	1 kPa = 0.145 psi
	1 lb/ft^2 = 144 psi	1 kilopascal (kPa) = 1,000 Pa 1 megapascal (MPa) = 1,000 kPa		
		1 bar = 100 kPa	1 psi = 0.069 bar	1 bar = 14.5 psi
		kilogram/square centimeter (kg/cm^2)	1 psi = 0.0703 kg/cm^2	1 kg/cm^2 = 14.22 psi
Velocity	foot/second (ft/s) mile/hour (mi/h) = 1.4667 ft/s	meter/second (m/s)	1 ft/s = 0.3048 m/s	1 m/s = 3.281 ft/s
Work and energy	foot-pound (ft · lb) British thermal unit (Btu) 1 Btu = 778 ft · lb	joule (J) = N · m	1 Btu = 1055.0 J	1 kJ = 0.9478 Btu
Power	(ft · lb)/min Btu/hour (Btu/h)	joule/second (J/s) Watt (W) = J/s	1 Btu/h = 0.2931W	1 W = 3.4121 Btu/h
	Horsepower (HP) 1 HP = 33,000 (ft · lb)/min	1 kilowatt (kW) = 1,000 W	1 HP = 0.746 kW	1 kW = 1.3405 HP
Temperature	degree Fahrenheit (°F) 1 degree Rankine (°R) = °F + 460	degree Celsius (°C) Kelvin (K) = °C + 273	1°F = $\frac{9}{5}$°C + 32 1°R = 1.8 K	1°C = (°F − 32)/1.8 1 K = °R/1.8
Thermal conductivity	Btu/(h · ft · °F)	W/(m · °C)	1 Btu/(h · ft · °F) = 1.7307 W/(m · °C)	1 W/(m · °C) = 0.5778 Btu/(h · ft · °F)
Heat transfer coefficient	Btu/(h · ft^2 · °F)	W/(m^2 · °C)	1 Btu/(h · ft^2 · °F) = 5.6781 W/(m^2 · °C)	1 W/(m^2 · °C) =0.1761 Btu/(h · ft^2 · °F)
Specific heat	Btu/(lb · °F)	kJ/(kg · °C)	1 Btu/(lb · °F) = 4.1869 kJ/(kg · °C)	1 kJ/(kg · °C) = 0.2388 Btu/(lb · °F)

*USCS = U.S. Customary System.
†SI = Système International (modified metric).
‡Kinematic viscosity in SSU and SSF are converted to viscosity in cSt using Eqs. (6.6) through (6.9).

Appendix B

Pipe Properties (U.S. Customary System of Units)

Nominal pipe size (NPS)	Outside diameter, in	Schedule a	Schedule b	Schedule c	Wall thickness, in	Inside diameter, in	Inside area, in^2	Surface area, ft^2/ft	Volume, gal/ft	Pipe weight, lb/ft	Water weight, lb/ft
$\frac{1}{2}$	0.84			5S	0.065	0.710	0.3957	0.22	0.02	0.54	0.17
	0.84			10S	0.083	0.674	0.3566	0.22	0.02	0.67	0.15
	0.84	40	Std.	40S	0.109	0.622	0.3037	0.22	0.02	0.85	0.13
	0.84	80	XS	80S	0.147	0.546	0.2340	0.22	0.01	1.09	0.10
	0.84	160			0.187	0.466	0.1705	0.22	0.01	1.30	0.07
	0.84		XXS		0.294	0.252	0.0499	0.22	0.00	1.71	0.02
$\frac{3}{4}$	1.05			5S	0.065	0.920	0.6644	0.27	0.03	0.68	0.29
	1.05			10S	0.083	0.884	0.6134	0.27	0.03	0.86	0.27
	1.05	40	Std.	40S	0.113	0.824	0.5330	0.27	0.03	1.13	0.23
	1.05	80	XS	80S	0.154	0.742	0.4322	0.27	0.02	1.47	0.19
	1.05	160			0.218	0.614	0.2959	0.27	0.02	1.94	0.13
	1.05		XXS		0.308	0.434	0.1479	0.27	0.01	2.44	0.06
1	1.315			5S	0.065	1.185	1.1023	0.34	0.06	0.87	0.48
	1.315			10S	0.109	1.097	0.9447	0.34	0.05	1.40	0.41
	1.315	40	Std.	40S	0.330	0.655	0.3368	0.34	0.02	3.47	0.15
	1.315	80	XS	80S	0.179	0.957	0.7189	0.34	0.04	2.17	0.31
	1.315	160			0.250	0.815	0.5214	0.34	0.03	2.84	0.23
	1.315		XXS		0.358	0.599	0.2817	0.34	0.01	3.66	0.12
$1\frac{1}{2}$	1.900			5S	0.065	1.770	2.4593	0.50	0.13	1.27	1.07
	1.900			10S	0.109	1.682	2.2209	0.50	0.12	2.08	0.96
	1.900	40	Std.	40S	0.145	1.610	2.0348	0.50	0.11	2.72	0.88
	1.900	80	XS	80S	0.200	1.500	1.7663	0.50	0.09	3.63	0.77
	1.900	160			0.281	1.338	1.4053	0.50	0.07	4.86	0.61
	1.900		XXS		0.400	1.100	0.9499	0.50	0.05	6.41	0.41
2	2.375			5S	0.065	2.245	3.9564	0.62	0.21	1.60	1.71
	2.375			10S	0.109	2.157	3.6523	0.62	0.19	2.64	1.58
	2.375	40	Std.	40S	0.154	2.067	3.3539	0.62	0.17	3.65	1.45
	2.375	80	XS	80S	0.218	1.939	2.9514	0.62	0.15	5.02	1.28
	2.375	160			0.343	1.689	2.2394	0.62	0.12	7.44	0.97
	2.375		XXS		0.436	1.503	1.7733	0.62	0.09	9.03	0.77

$2\frac{1}{2}$	2.875			5S	0.083	2.709	5.7609	0.75	0.30	2.47	2.50
	2.875			10S	0.12	2.635	5.4504	0.75	0.28	3.53	2.36
	2.875	40	Std.		0.203	2.469	4.7853	0.75	0.25	5.79	2.07
	2.875	80	XS		0.276	2.323	4.2361	0.75	0.22	7.66	1.84
	2.875	160			0.375	2.125	3.5448	0.75	0.18	10.01	1.54
	2.875		XXS		0.552	1.771	2.4621	0.75	0.13	13.69	1.07
3	3.5			5S	0.083	3.334	8.7257	0.92	0.45	3.03	3.78
	3.5			10S	0.120	3.260	8.3427	0.92	0.43	4.33	3.62
	3.5	40	Std.	40S	0.216	3.068	7.3889	0.92	0.38	7.58	3.20
	3.5	80	XS	80S	0.300	2.900	6.6019	0.92	0.34	10.25	2.86
	3.5	160			0.437	2.626	5.4133	0.92	0.28	14.30	2.35
	3.5		XXS		0.600	2.300	4.1527	0.92	0.22	18.58	1.80
4	4.5			5S	0.083	4.334	14.7451	1.18	0.77	3.92	6.39
	4.5			10S	0.120	4.260	14.2459	1.18	0.74	5.61	6.17
	4.5	40	Std.	40S	0.237	4.026	12.7238	1.18	0.66	10.79	5.51
	4.5	80	XS	80S	0.337	3.826	11.4910	1.18	0.60	14.98	4.98
	4.5	120			0.437	3.626	10.3211	1.18	0.54	18.96	4.47
	4.5	160			0.531	3.438	9.2786	1.18	0.48	22.51	4.02
	4.5		XXS		0.674	3.152	7.7991	1.18	0.41	27.54	3.38
6	6.625			5S	0.109	6.407	32.2240	1.73	1.67	7.59	13.96
	6.625			10S	0.134	6.357	31.7230	1.73	1.65	9.29	13.75
	6.625	40	Std.	40S	0.280	6.065	28.8756	1.73	1.50	18.97	12.51
	6.625	80	XS	80S	0.432	5.761	26.0535	1.73	1.35	28.57	11.29
	6.625	120			0.562	5.501	23.7549	1.73	1.23	36.39	10.29
	6.625	160			0.718	5.189	21.1367	1.73	1.10	45.30	9.16
	6.625		XXS		0.864	4.897	18.8248	1.73	0.98	53.16	8.16
8	8.625			5S	0.109	8.407	55.4820	2.26	2.88	9.91	24.04
	8.625			10S	0.148	8.329	54.4572	2.26	2.83	13.40	23.60
	8.625	20			0.250	8.125	51.8223	2.26	2.69	22.36	22.46
	8.625	30			0.277	8.071	51.1357	2.26	2.66	24.70	22.16
	8.625	40	Std.	40S	0.322	7.981	50.0016	2.26	2.60	28.55	21.67
	8.625	60			0.406	7.813	47.9187	2.26	2.49	35.64	20.76
	8.625	80	XS	80S	0.500	7.625	45.6404	2.26	2.37	43.39	19.78

(continued)

(*Continued*)

Nominal pipe size (NPS)	Outside diameter, in	Schedule a	Schedule b	Schedule c	Wall thickness, in	Inside diameter, in	Inside area, in^2	Surface area, ft^2/ft	Volume, gal/ft	Pipe weight, lb/ft	Water weight, lb/ft
	8.625	100			0.593	7.439	43.4409	2.26	2.26	50.87	18.82
	8.625	120			0.718	7.189	40.5702	2.26	2.11	60.63	17.58
	8.625	140			0.812	7.001	38.4760	2.26	2.00	67.76	16.67
	8.625		XXS		0.875	6.875	37.1035	2.26	1.93	72.42	16.08
	8.625	160			0.906	6.813	36.4373	2.26	1.89	74.69	15.79
10	10.75			5S	0.134	10.482	86.2498	2.81	4.48	15.19	37.37
	10.75			10S	0.165	10.420	85.2325	2.81	4.43	18.65	36.93
	10.75	20			0.250	10.250	82.4741	2.81	4.28	28.04	35.74
	10.75				0.279	10.192	81.5433	2.81	4.24	31.20	35.34
	10.75	30			0.307	10.136	80.6497	2.81	4.19	34.24	34.95
	10.75	40	Std.	40S	0.365	10.020	78.8143	2.81	4.09	40.48	34.15
	10.75	60	XS	80S	0.500	9.750	74.6241	2.81	3.88	54.74	32.34
	10.75	80			0.593	9.564	71.8040	2.81	3.73	64.33	31.12
	10.75	100			0.718	9.314	68.0992	2.81	3.54	76.93	29.51
	10.75	120			0.843	9.064	64.4925	2.81	3.35	89.20	27.95
	10.75	140			1.000	8.750	60.1016	2.81	3.12	104.13	26.04
	10.75	160			1.125	8.500	56.7163	2.81	2.95	115.64	24.58
12	12.75			5S	0.156	12.438	121.4425	3.34	6.31	20.98	52.63
	12.75			10S	0.180	12.390	120.5070	3.34	6.26	24.16	52.22
	12.75	20			0.250	12.250	117.7991	3.34	6.12	33.38	51.05
	12.75	30			0.330	12.090	114.7420	3.34	5.96	43.77	49.72
	12.75		Std.	40S	0.375	12.000	113.0400	3.34	5.87	49.56	48.98
	12.75	40			0.406	11.938	111.8749	3.34	5.81	53.52	48.48
	12.75		XS	80S	0.500	11.750	108.3791	3.34	5.63	65.42	46.96
	12.75	60			0.562	11.626	106.1036	3.34	5.51	73.15	45.98
	12.75	80			0.687	11.376	101.5895	3.34	5.28	88.51	44.02
	12.75	100			0.843	11.064	96.0935	3.34	4.99	107.20	41.64
	12.75	120			1.000	10.750	90.7166	3.34	4.71	125.49	39.31
	12.75	140			1.125	10.500	86.5463	3.34	4.50	139.67	37.50
	12.75	160			1.312	10.126	80.4907	3.34	4.18	160.27	34.88

14	14.00			5S	0.156	13.688	147.0787	3.67	7.64	23.07	63.73
	14.00			10S	0.188	13.624	145.7065	3.67	7.57	27.73	63.14
	14.00	10			0.250	13.500	143.0663	3.67	7.43	36.71	62.00
	14.00	20			0.312	13.376	140.4501	3.67	7.30	45.61	60.86
	14.00	30	Std.		0.375	13.250	137.8166	3.67	7.16	54.57	59.72
	14.00	40			0.437	13.126	135.2491	3.67	7.03	63.30	58.61
	14.00		XS		0.500	13.000	132.6650	3.67	6.89	72.09	57.49
	14.00				0.562	12.876	130.1462	3.67	6.76	80.66	56.40
	14.00	60			0.593	12.814	128.8959	3.67	6.70	84.91	55.85
	14.00				0.625	12.750	127.6116	3.67	6.63	89.28	55.30
	14.00				0.687	12.626	125.1415	3.67	6.50	97.68	54.23
	14.00	80			0.750	12.500	122.6563	3.67	6.37	106.13	53.15
	14.00				0.875	12.250	117.7991	3.67	6.12	122.65	51.05
	14.00	100			0.937	12.126	115.4263	3.67	6.00	130.72	50.02
	14.00	120			1.093	11.814	109.5629	3.67	5.69	150.67	47.48
	14.00	140			1.250	11.500	103.8163	3.67	5.39	170.21	44.99
	14.00	160			1.406	11.188	98.2595	3.67	5.10	189.11	42.58
16	16.00			5S	0.165	15.670	192.7559	4.19	10.01	27.90	83.53
	16.00			10S	0.188	15.624	191.6259	4.19	9.95	31.75	83.04
	16.00	10			0.250	15.500	188.5963	4.19	9.80	42.05	81.73
	16.00	20			0.312	15.376	185.5908	4.19	9.64	52.27	80.42
	16.00	30	Std.		0.375	15.250	182.5616	4.19	9.48	62.58	79.11
	16.00				0.437	15.126	179.6048	4.19	9.33	72.64	77.83
	16.00	40	XS		0.500	15.000	176.6250	4.19	9.18	82.77	76.54
	16.00				0.562	14.876	173.7169	4.19	9.02	92.66	75.28
	16.00				0.625	14.750	170.7866	4.19	8.87	102.63	74.01
	16.00	60			0.656	14.688	169.3538	4.19	8.80	107.50	73.39
	16.00				0.687	14.626	167.9271	4.19	8.72	112.35	72.77
	16.00				0.750	14.500	165.0463	4.19	8.57	122.15	71.52
	16.00	80			0.843	14.314	160.8391	4.19	8.36	136.46	69.70
	16.00				0.875	14.250	159.4041	4.19	8.28	141.34	69.08
	16.00	100			1.031	13.938	152.5003	4.19	7.92	164.82	66.08

(continued)

(*Continued*)

Nominal pipe size (NPS)	Outside diameter, in	Schedule a	Schedule b	Schedule c	Wall thickness, in	Inside diameter, in	Inside area, in^2	Surface area, ft^2/ft	Volume, gal/ft	Pipe weight, lb/ft	Water weight, lb/ft
	16.00	120			1.218	13.564	144.4259	4.19	7.50	192.29	62.58
	16.00	140			1.437	13.126	135.2491	4.19	7.03	223.50	58.61
	16.00	160			1.593	12.814	128.8959	4.19	6.70	245.11	55.85
18	18.00			5S	0.165	17.670	245.0997	4.71	12.73	31.43	106.21
	18.00			10S	0.188	17.624	243.8252	4.71	12.67	35.76	105.66
	18.00	10			0.250	17.500	240.4063	4.71	12.49	47.39	104.18
	18.00	20			0.312	17.376	237.0114	4.71	12.31	58.94	102.70
	18.00		Std.		0.375	17.250	233.5866	4.71	12.13	70.59	101.22
	18.00	30			0.437	17.126	230.2404	4.71	11.96	81.97	99.77
	18.00		XS		0.500	17.000	226.8650	4.71	11.79	93.45	98.31
	18.00	40			0.562	16.876	223.5675	4.71	11.61	104.67	96.88
	18.00				0.625	16.750	220.2416	4.71	11.44	115.98	95.44
	18.00				0.687	16.626	216.9927	4.71	11.27	127.03	94.03
	18.00	60			0.750	16.500	213.7163	4.71	11.10	138.17	92.61
	18.00				0.875	16.250	207.2891	4.71	10.77	160.03	89.83
	18.00	80			0.937	16.126	204.1376	4.71	10.60	170.75	88.46
	18.00	100			1.156	15.688	193.1990	4.71	10.04	207.96	83.72
	18.00	120			1.375	15.250	182.5616	4.71	9.48	244.14	79.11
	18.00	140			1.562	14.876	173.7169	4.71	9.02	274.22	75.28
	18.00	160			1.781	14.438	163.6378	4.71	8.50	308.50	70.91
20	20.00			5S	0.188	19.624	302.3046	5.24	15.70	39.78	131.00
	20.00			10S	0.218	19.564	300.4588	5.24	15.61	46.06	130.20
	20.00	10			0.250	19.500	298.4963	5.24	15.51	52.73	129.35
	20.00				0.312	19.376	294.7121	5.24	15.31	65.60	127.71
	20.00	20	Std.		0.375	19.250	290.8916	5.24	15.11	78.60	126.05
	20.00				0.437	19.126	287.1560	5.24	14.92	91.30	124.43
	20.00	30	XS		0.500	19.000	283.3850	5.24	14.72	104.13	122.80
	20.00				0.562	18.876	279.6982	5.24	14.53	116.67	121.20
	20.00	40			0.593	18.814	277.8638	5.24	14.43	122.91	120.41
	20.00				0.625	18.750	275.9766	5.24	14.34	129.33	119.59

	20.00				0.687	18.626	272.3384	5.24	14.15	141.70	118.01
	20.00				0.750	18.500	268.6663	5.24	13.96	154.19	116.42
	20.00	60			0.812	18.376	265.0767	5.24	13.77	166.40	114.87
	20.00				0.875	18.250	261.4541	5.24	13.58	178.72	113.30
	20.00	80			1.031	17.938	252.5909	5.24	13.12	208.87	109.46
	20.00	100			1.281	17.438	238.7058	5.24	12.40	256.10	103.44
	20.00	120			1.500	17.000	226.8650	5.24	11.79	296.37	98.31
	20.00	140			1.750	16.500	213.7163	5.24	11.10	341.09	92.61
	20.00	160			1.968	16.064	202.5709	5.24	10.52	379.00	87.78
22	22.00			5S	0.188	21.624	367.0639	5.76	19.07	43.80	159.06
	22.00			10S	0.218	21.564	365.0298	5.76	18.96	50.71	158.18
	22.00	10			0.250	21.500	362.8663	5.76	18.85	58.07	157.24
	22.00	20	Std.		0.375	21.250	354.4766	5.76	18.41	86.61	153.61
	22.00	30	XS		0.500	21.000	346.1850	5.76	17.98	114.81	150.01
	22.00				0.625	20.750	337.9916	5.76	17.56	142.68	146.46
	22.00				0.750	20.500	329.8963	5.76	17.14	170.21	142.96
	22.00	60			0.875	20.250	321.8991	5.76	16.72	197.41	139.49
	22.00	80			1.125	19.750	306.1991	5.76	15.91	250.81	132.69
	22.00	100			1.375	19.250	290.8916	5.76	15.11	302.88	126.05
	22.00	120			1.625	18.750	275.9766	5.76	14.34	353.61	119.59
	22.00	140			1.875	18.250	261.4541	5.76	13.58	403.00	113.30
	22.00	160			2.125	17.750	247.3241	5.76	12.85	451.06	107.17
24	24.00			5S	0.188	23.624	438.1033	6.28	22.76	47.81	189.84
	24.00	10		10S	0.218	23.564	435.8807	6.28	22.64	55.37	188.88
	24.00				0.250	23.500	433.5163	6.28	22.52	63.41	187.86
	24.00	20			0.312	23.376	428.9533	6.28	22.28	78.93	185.88
	24.00		Std.		0.375	23.250	424.3416	6.28	22.04	94.62	183.88
	24.00				0.437	23.126	419.8273	6.28	21.81	109.97	181.93
	24.00	30	XS		0.500	23.000	415.2650	6.28	21.57	125.49	179.95
	24.00				0.562	22.876	410.7994	6.28	21.34	140.68	178.01
	24.00	40			0.593	22.814	408.5757	6.28	21.22	148.24	177.05
	24.00				0.625	22.750	406.2866	6.28	21.11	156.03	176.06

(continued)

(*Continued*)

Nominal pipe size (NPS)	Outside diameter, in	Schedule			Wall thickness, in	Inside diameter, in	Inside area, in^2	Surface area, ft^2/ft	Volume, gal/ft	Pipe weight, lb/ft	Water weight, lb/ft
		a	b	c							
	24.00	60			0.812	22.376	393.0380	6.28	20.42	201.09	170.32
	24.00	80			1.031	21.938	377.8015	6.28	19.63	252.91	163.71
	24.00	100			1.281	21.438	360.7765	6.28	18.74	310.82	156.34
	24.00	120			1.500	21.000	346.1850	6.28	17.98	360.45	150.01
	24.00	140			1.750	20.500	329.8963	6.28	17.14	415.85	142.96
	24.00	160			1.968	20.064	316.0128	6.28	16.42	463.07	136.94
26	26.00				0.250	25.500	510.4463	6.81	26.52	68.75	221.19
	26.00	10			0.312	25.376	505.4940	6.81	26.26	85.60	219.05
	26.00		Std.		0.375	25.250	500.4866	6.81	26.00	102.63	216.88
	26.00	20	XS		0.500	25.000	490.6250	6.81	25.49	136.17	212.60
	26.00				0.625	24.750	480.8616	6.81	24.98	169.38	208.37
	26.00				0.750	24.500	471.1963	6.81	24.48	202.25	204.19
	26.00				0.875	24.250	461.6291	6.81	23.98	234.79	200.04
	26.00				1.000	24.000	452.1600	6.81	23.49	267.00	195.94
	26.00				1.125	23.750	442.7891	6.81	23.00	298.87	191.88
28	28.00				0.250	27.500	593.6563	7.33	30.84	74.09	257.25
	28.00	10			0.312	27.376	588.3146	7.33	30.56	92.26	254.94
	28.00		Std.		0.375	27.250	582.9116	7.33	30.28	110.64	252.60
	28.00	20	XS		0.500	27.000	572.2650	7.33	29.73	146.85	247.98
	28.00	30			0.625	26.750	561.7166	7.33	29.18	182.73	243.41
	28.00				0.750	26.500	551.2663	7.33	28.64	218.27	238.88
	28.00				0.875	26.250	540.9141	7.33	28.10	253.48	234.40
	28.00				1.000	26.000	530.6600	7.33	27.57	288.36	229.95
	28.00				1.125	25.750	520.5041	7.33	27.04	322.90	225.55
30	30.00			5S	0.250	29.500	683.1463	7.85	35.49	79.43	296.03
	30.00	10		10S	0.312	29.376	677.4153	7.85	35.19	98.93	293.55
	30.00		Std.		0.375	29.250	671.6166	7.85	34.89	118.65	291.03
	30.00	20	XS		0.500	29.000	660.1850	7.85	34.30	157.53	286.08
	30.00	30			0.625	28.750	648.8516	7.85	33.71	196.08	281.17
	30.00	40			0.750	28.500	637.6163	7.85	33.12	234.29	276.30

	30.00			0.875	28.250	626.4791	7.85	32.54	272.17	271.47
	30.00			1.000	28.000	615.4400	7.85	31.97	309.72	266.69
	30.00			1.125	27.750	604.4991	7.85	31.40	346.93	261.95
32	32.00			0.250	31.500	778.9163	8.38	40.46	84.77	337.53
	32.00	10		0.312	31.376	772.7959	8.38	40.15	105.59	334.88
	32.00		Std.	0.375	31.250	766.6016	8.38	39.82	126.66	332.19
	32.00	20	XS	0.500	31.000	754.3850	8.38	39.19	168.21	326.90
	32.00	30		0.625	30.750	742.2666	8.38	38.56	209.43	321.65
	32.00	40		0.688	30.624	736.1961	8.38	38.24	230.08	319.02
	32.00			0.750	30.500	730.2463	8.38	37.93	250.31	316.44
	32.00			0.875	30.250	718.3241	8.38	37.32	290.86	311.27
	32.00			1.000	30.000	706.5000	8.38	36.70	331.08	306.15
	32.00			1.125	29.750	694.7741	8.38	36.09	370.96	301.07
34	34.00			0.250	33.500	880.9663	8.90	45.76	90.11	381.75
	34.00	10		0.312	33.376	874.4565	8.90	45.43	112.25	378.93
	34.00		Std.	0.375	33.250	867.8666	8.90	45.08	134.67	376.08
	34.00	20	XS	0.500	33.000	854.8650	8.90	44.41	178.89	370.44
	34.00	30		0.625	32.750	841.9616	8.90	43.74	222.78	364.85
	34.00	40		0.688	32.624	835.4954	8.90	43.40	244.77	362.05
	34.00			0.750	32.500	829.1563	8.90	43.07	266.33	359.30
	34.00			0.875	32.250	816.4491	8.90	42.41	309.55	353.79
	34.00			1.000	32.000	803.8400	8.90	41.76	352.44	348.33
	34.00			1.125	31.750	791.3291	8.90	41.11	394.99	342.91
36	36.00			0.250	35.500	989.2963	9.42	51.39	95.45	428.70
	36.00	10		0.312	35.376	982.3972	9.42	51.03	118.92	425.71
	36.00		Std.	0.375	35.250	975.4116	9.42	50.67	142.68	422.68
	36.00	20	XS	0.500	35.000	961.6250	9.42	49.95	189.57	416.70
	36.00	30		0.625	34.750	947.9366	9.42	49.24	236.13	410.77
	36.00	40		0.750	34.500	934.3463	9.42	48.54	282.35	404.88
	36.00			0.875	34.250	920.8541	9.42	47.84	328.24	399.04
	36.00			1.000	34.000	907.4600	9.42	47.14	373.80	393.23
	36.00			1.125	33.750	894.1641	9.42	46.45	419.02	387.47

(continued)

(*Continued*)

Nominal pipe size (NPS)	Outside diameter, in	Schedule a	Schedule b	Schedule c	Wall thickness, in	Inside diameter, in	Inside area, in^2	Surface area, ft^2/ft	Volume, gal/ft	Pipe weight, lb/ft	Water weight, lb/ft
42	42.00				0.250	41.500	1351.9663	11.00	70.23	111.47	585.85
	42.00		Std.		0.375	41.250	1335.7266	11.00	69.39	166.71	578.81
	42.00	20	XS		0.500	41.000	1319.5850	11.00	68.55	221.61	571.82
	42.00	30			0.625	40.750	1303.5416	11.00	67.72	276.18	564.87
	42.00	40			0.750	40.500	1287.5963	11.00	66.89	330.41	557.96
	42.00				1.000	40.000	1256.0000	11.00	65.25	437.88	544.27
	42.00				1.250	39.500	1224.7963	11.00	63.63	544.01	530.75
	42.00				1.500	39.000	1193.9850	11.00	62.03	648.81	517.39

Appendix C

Viscosity Corrected Pump Performance

The following is a report from PUMPCALC software comparing the water performance against the viscous performance of a centrifugal pump. Graphic plots of comparison are shown in Fig. C.1.

PUMPCALC—Centrifugal Pump Analysis Program (*www.systek.us*)

Water performance SpGrav: 1.00 Viscosity: 1.00 cSt				Viscous performance SpGrav: 0.985 Viscosity: 850.00 SSU			
Flow rate	Head	Efficiency	BHP	Flow rate	Head	Efficiency	BHP
456	113.8	72.98	17.96	433.6	109.35	48.64	24.25
608	107.65	80.21	20.61	578.14	101.63	53.46	27.34
760	99.28	82.01	23.23	722.67	92.4	54.66	30.39
912	84.6	78.96	24.67	867.2	76.45	52.63	31.33

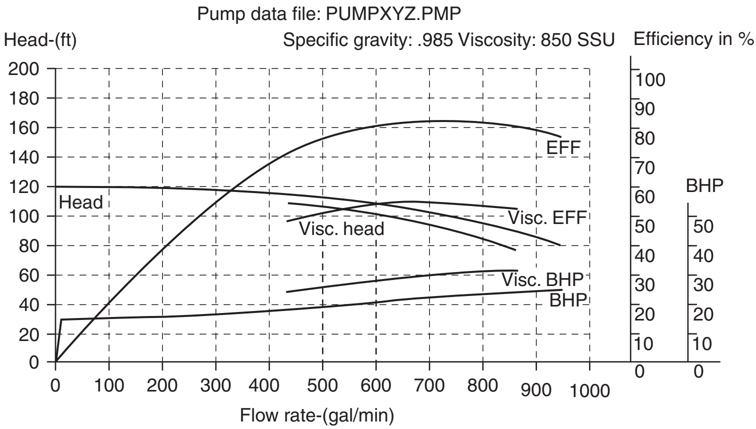

Figure C.1 Water and viscous pump performance.

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