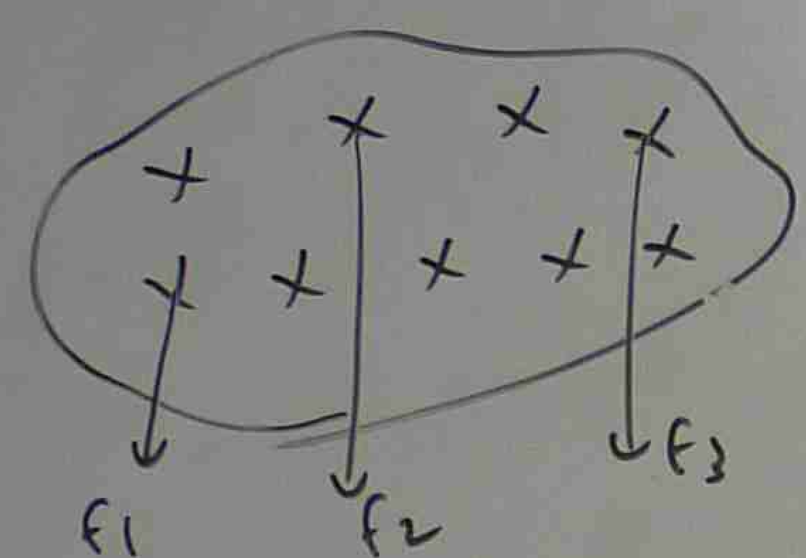
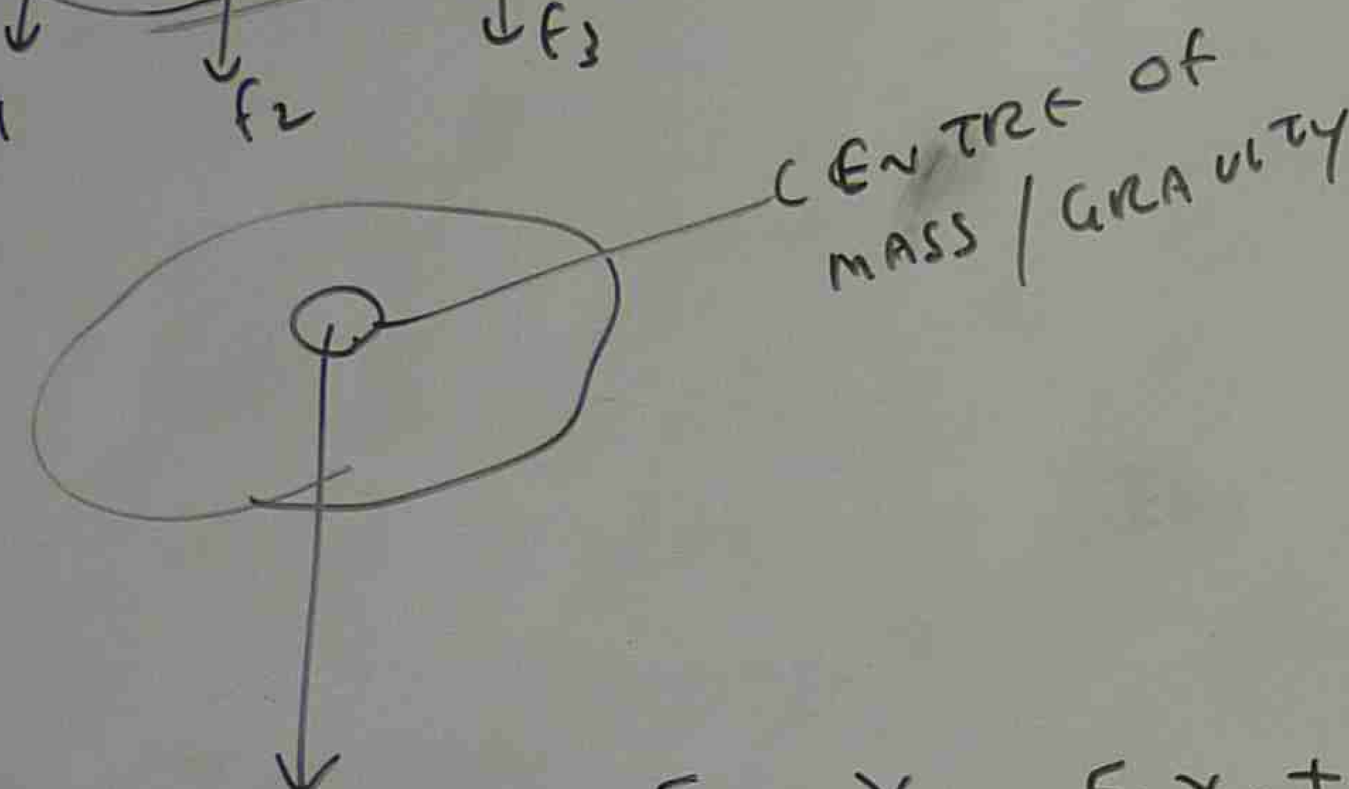
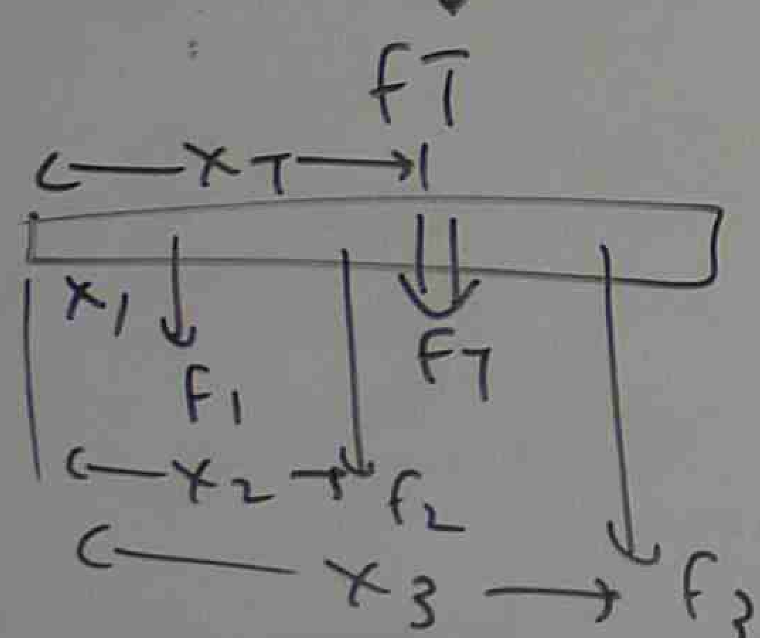
CENTRE OF MASS AND LINEAR MOMENTUM



$$f_T = f_1 + f_2 + f_3$$





$$F_T \times X_T = F_1 x_1 + F_2 x_2 + F_3 x_3$$

$$X_T = \frac{F_1 x_1 + F_2 x_2 + F_3 x_3}{F_T}$$

$$F_T = F_1 + F_2 + F_3$$

$$X_T = \frac{F_1 x_1 + F_2 x_2 + F_3 x_3}{F_1 + F_2 + F_3}$$

F = force

$F = m\,g$ $\quad$ m = MASS

g = GRAVITY

$$X_T = \frac{m_1 g x_1 + m_2 g x_2 + m_3 g x_3}{m_1 g + m_2 g + m_3 g}$$

$$X_T = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

CENTRE OF MASS

$$X_T = \frac{\sum F x}{\sum F} \quad (\text{OR}) \quad \frac{\sum m x}{\sum m}$$

PD THREE PARTICLES OF MASSES $m_1 = 1.2\,kg$, $m_2 = 2.5\,kg$ AND $m_3 = 3.4\,kg$ FORM AN EQUILATERAL TRIANGLE OF EDGE LENGTH $a = 140\,cm$. WHERE IS THE CENTRE OF MASS OF THIS SYSTEM?

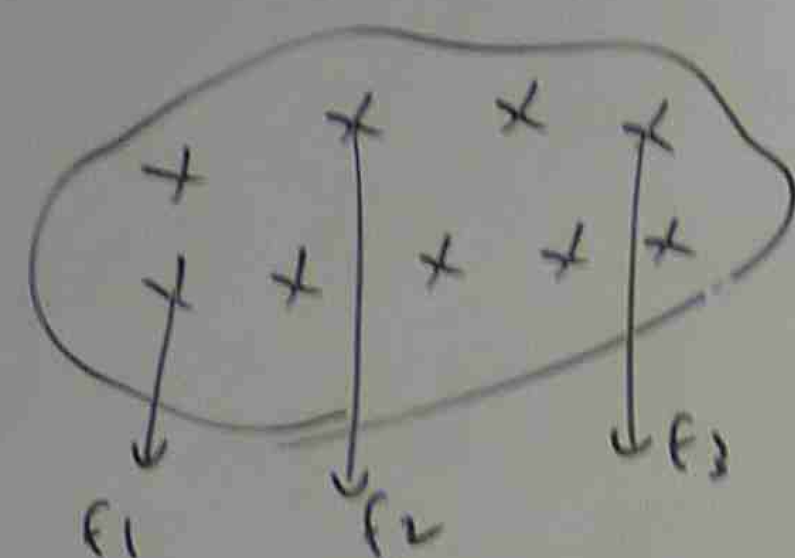
1.2

(0,

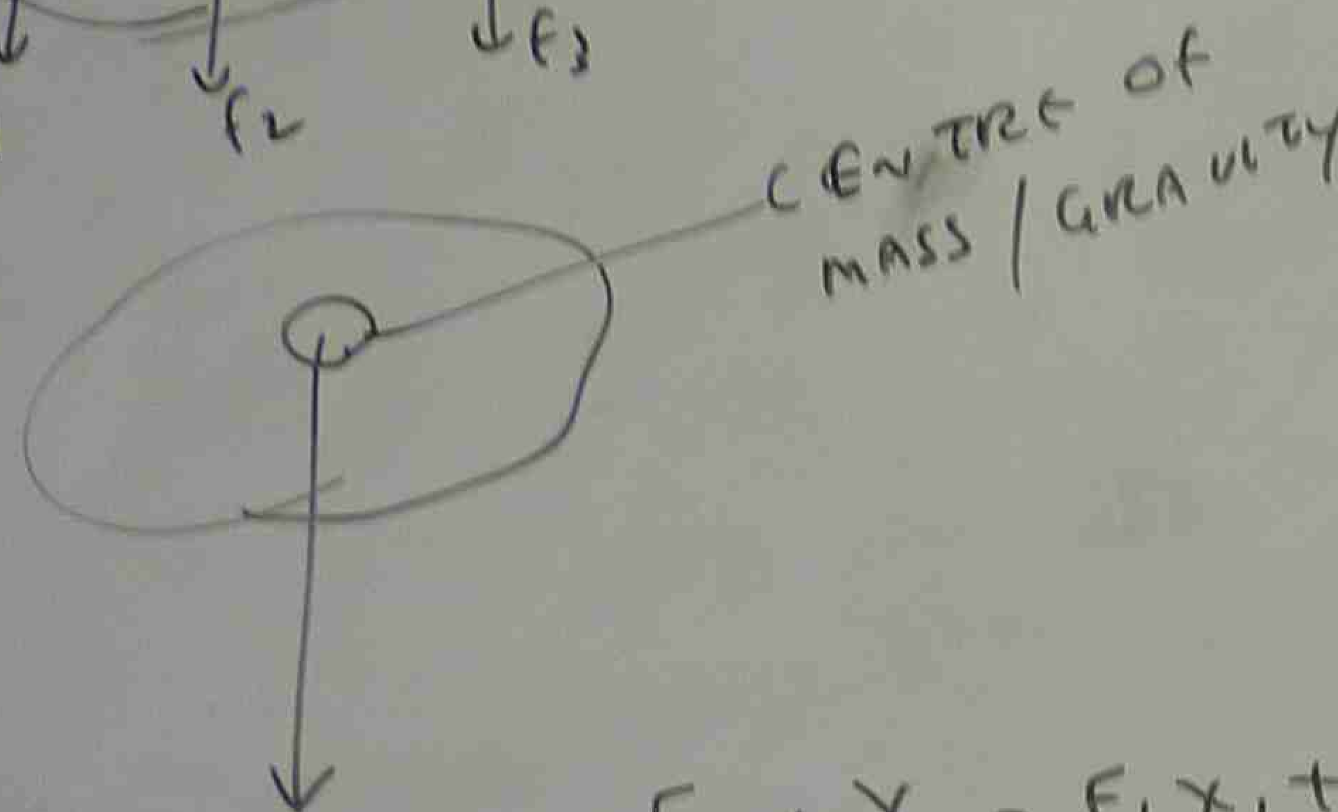
X_T

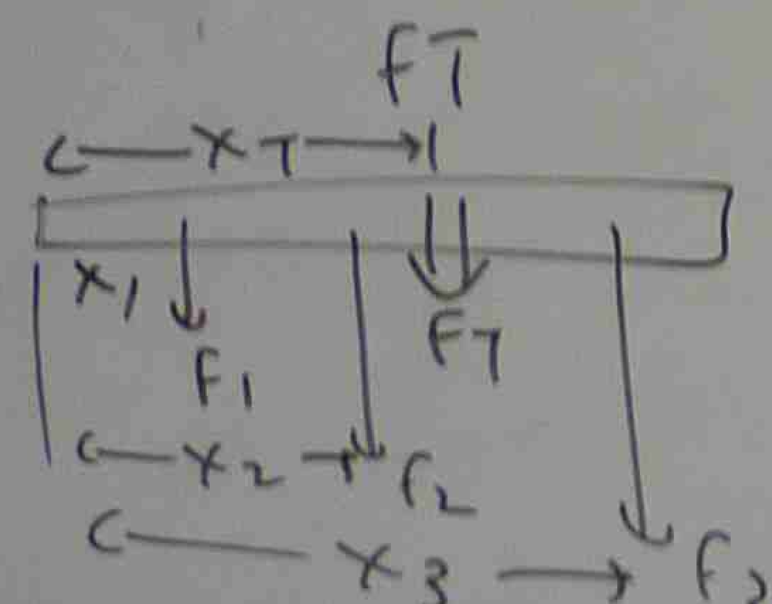
Y_T

CENTRE of MASS AND LINEAR MOMENTUM



$$f_T = f_1 + f_2 + f_3$$





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$$X_T = \frac{m_1 g x_1 + m_2 g x_2 + m_3 g x_3}{m_1 g + m_2 g + m_3 g}$$

$$X_T = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

CENTRE of MASS

$$X_T = \frac{\Sigma F x}{\Sigma F} \quad (\text{or}) \quad \frac{\Sigma m x}{\Sigma m}$$

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r momentum

$+ f_3$

$f_2 x_2 + f_3 x_3$

$\frac{f_2 x_2 + f_3 x_3}{\ldots T}$

$+ f_3$

$\frac{\ldots_2 + f_3 x_3}{\ldots + f_3}$

Ty

$$X_T = \frac{m_1 g x_1 + m_2 g x_2 + m_3 g x_3}{m_1 g + m_2 g + m_3 g}$$

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CENTRE OF MASS

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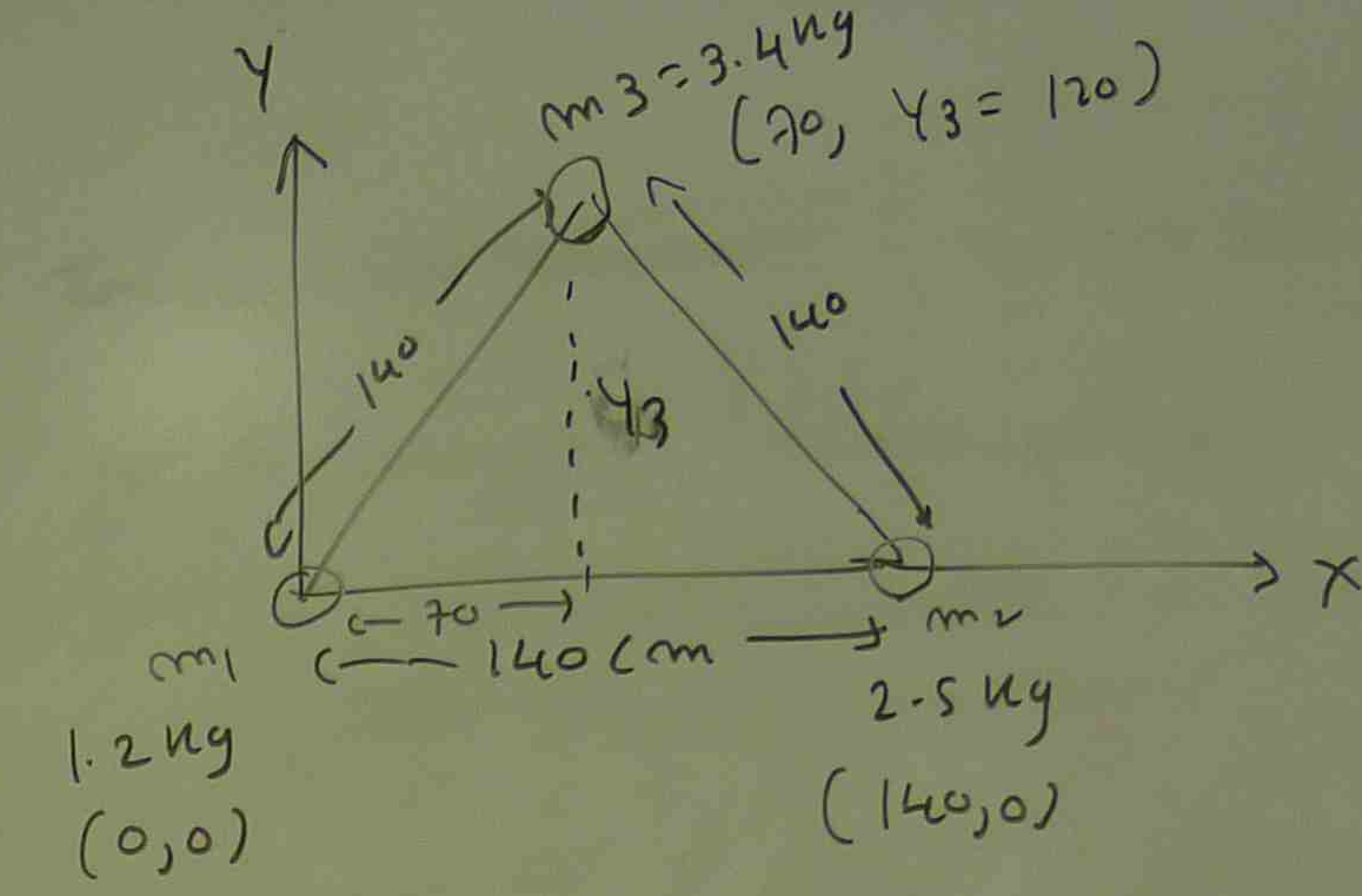
pb/ THREE PARTICLES OF MASSES $m_1 = 1.2$ kg, $m_2 = 2.5$ kg AND $m_3 = 3.4$ kg FORM AN EQUILATERAL TRIANGLE OF EDGE LENGTH $a = 140$ cm. WHERE IS THE CENTRE OF MASS OF THIS SYSTEM?



$$Y_3 = \sqrt{140^2 - 70^2} = 120$$

$$X_T = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{1.2 \times 0 + 2.5 \times 140 + 3.4 \times 70}{1.2 + 2.5 + 3.4}$$

$$= 83 \text{ cm}$$

$$Y_T = \frac{m_1 Y_1 + m_2 Y_2 + m_3 Y_3}{m_1 + m_2 + m_3}$$

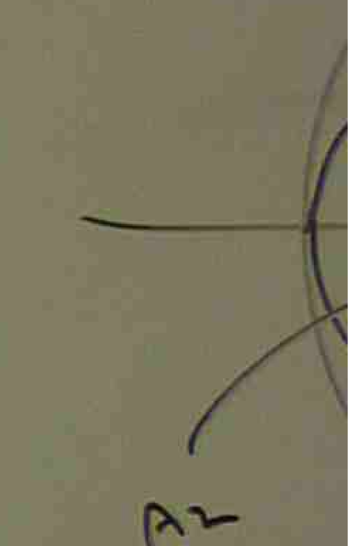
$$= \frac{1.2 \times 0 + 2.5 \times 0 + 3.4 \times 120}{1.2 + 2.5 + 3.4}$$

$$= 58 \text{ cm}$$

pb/ THE F…

A DIS…

Co-OR…



$\bar{X}_T =$

$m_3 = 3.4\,kg$
$(70,\ y_3 = 120)$

140°

m_2
2.5 kg
(140, 0)

120

$m_2 x_2 + m_3 x_3$
$m_2 + m_3$

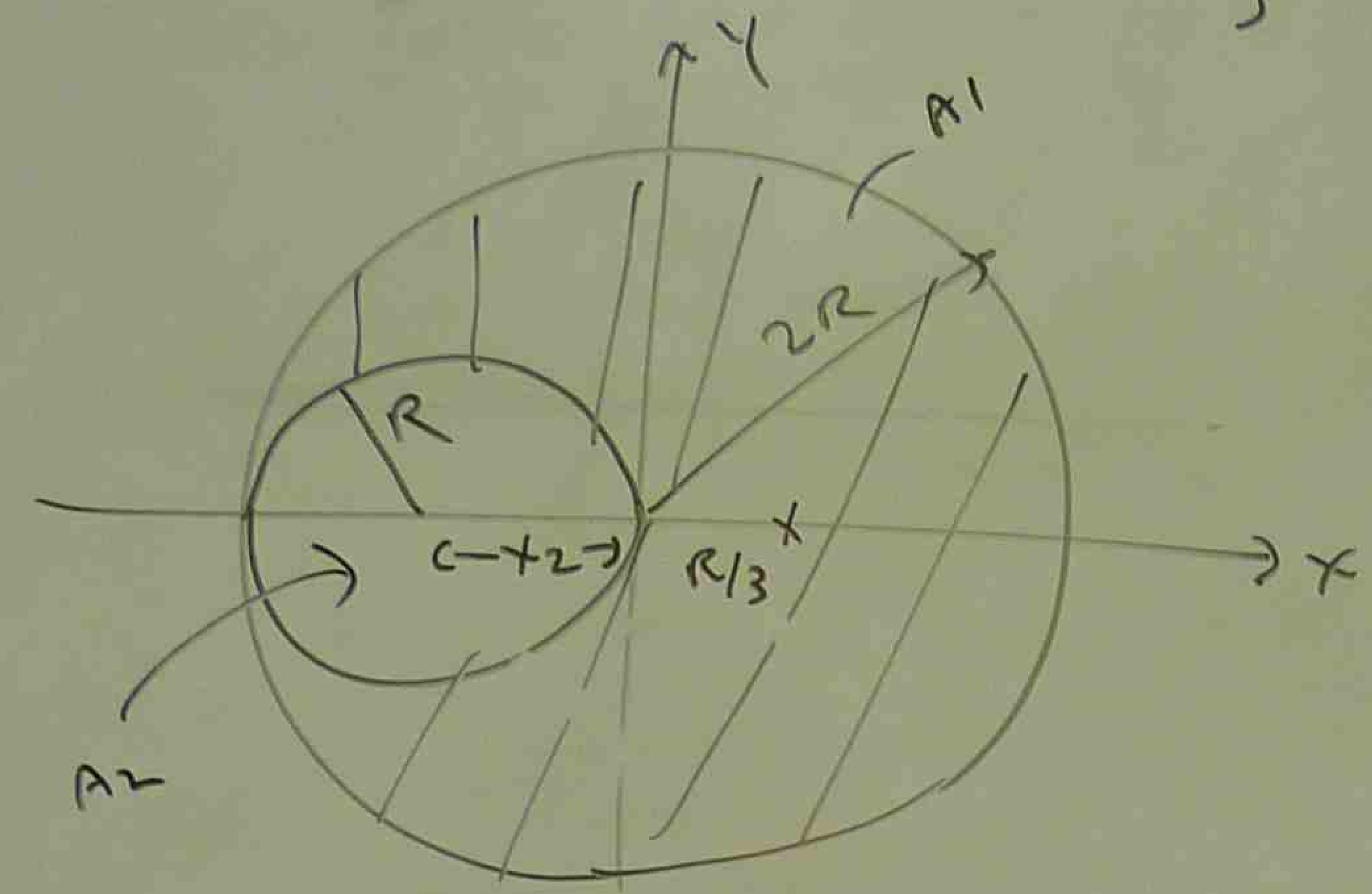
$2.5 \times 140 + 3.4 \times 70$
$2.5 + 3.4$

$m_2 y_2 + m_3 y_3$
$+ m_2 + m_3$

$+ 2.5 \times 0 + 3.4 \times 120$
$+ 2.5 + 3.4$

pb

THE FIGURE SHOWS A UNIFORM METAL PLATE "ρ" OF RADIUS (2R) FROM WHICH A DISK OF RADIUS R HAS BEEN STAMPED OUT. USING THE X-Y COORDINATE SYSTEM SHOWN, LOCATE THE CENTRE OF MASS OF THE PLATE.



$A_1 = \pi\ (\text{RADIUS})^2$

$m_1 = A_1 \times \rho \leftarrow$ DENSITY

$= \pi\,(2R)^2 \times \rho = \pi \times 4R^2 \times \rho$

$X_1 = 0 \quad , Y_1 = 0$

$A_2 = \pi\ (\text{RADIUS})^2 = \pi\ (R)^2 = \pi R^2$

$m_2 = A_2 \times \rho = \pi R^2 \rho$

$X_2 = -R \ , \ Y_2 = 0$

$$\bar{X}_T = \frac{m_1 X_1 - m_2 X_2}{m_1 - m_2} = \frac{\pi 4R^2 \rho \times 0 - \pi R^2 \rho(-R)}{\pi \times 4R^2 \rho - \pi R^2 \rho} = \frac{+\pi R^3 \rho}{3\pi R^2 \rho} = \frac{R}{3}$$

$$\bar{Y}_T = \frac{m_1 Y_1 - m \ldots}{m_1 - m \ldots} = \frac{\pi \times 4R^2 \ldots}{\pi 4R^2 \ldots} = \frac{0}{3\pi R \ldots} = 0$$

CENTRE OF MASS $= R/3,\ \ldots$

pb THREE P... ARE INITIAL... EXPERIENCE... DUE TO BO... THREE PAR... THE DIRECT... AND THE M...

$F_2 = 12\,N$

WHAT IS THE ...

r^2 FROM WHICH

$x-y$

of THE PLATE

$R^2 \times \rho$

$$\bar{Y}_T = \frac{m_1 Y_1 - m_2 Y_2}{m_1 - m_2}$$

$$= \frac{\pi \times 4R^2 \rho \times 0 - \pi R^2 \rho \times 0}{\pi 4 R^2 \rho - \pi R^2 \rho}$$

$$= \frac{0}{3 \pi R^2 \rho}$$

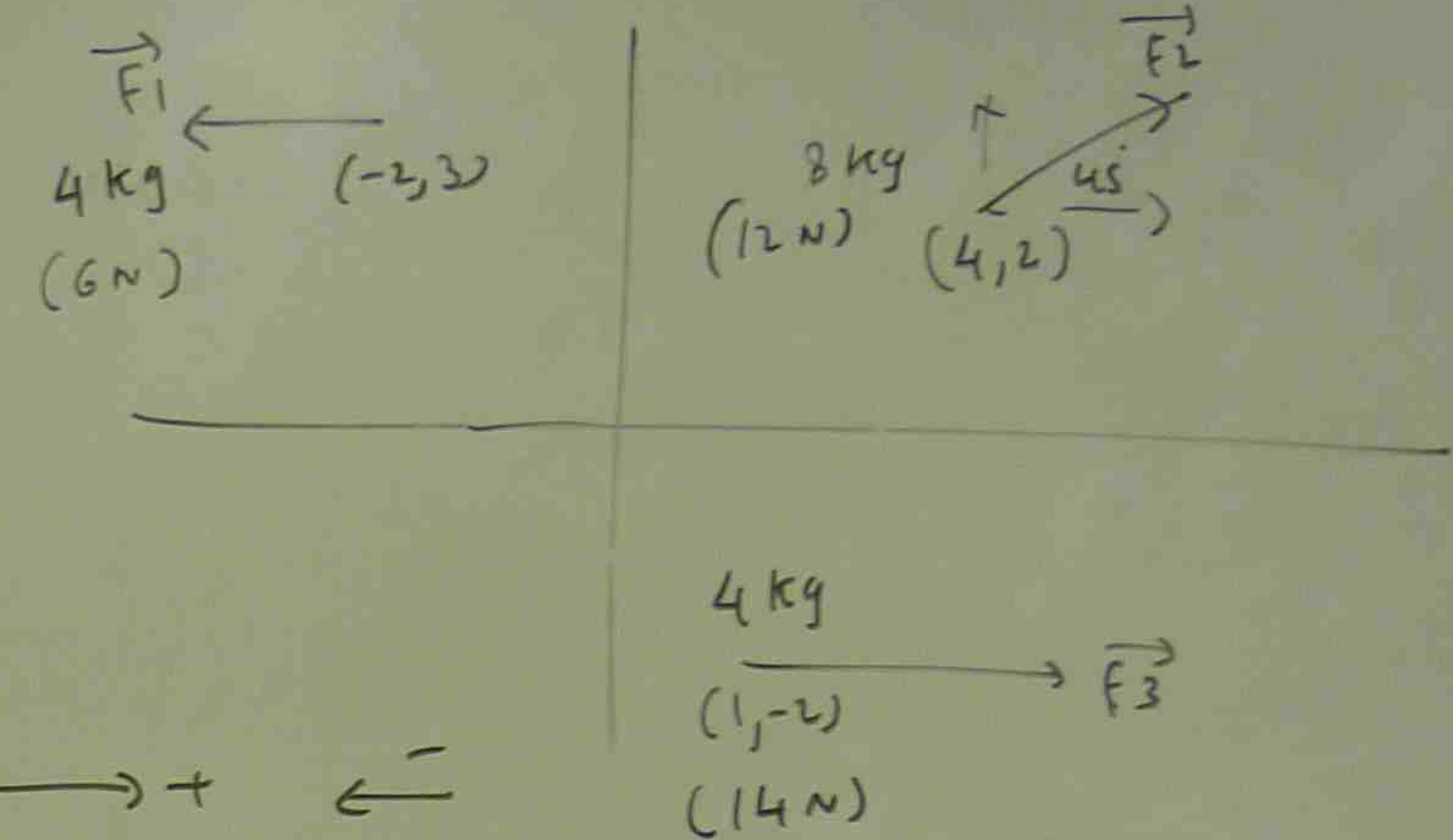
$$= 0$$

CENTRE OF MASS $= R/3, 0$

PB THREE PARTICLES IN FIGURE ARE INITIALLY AT REST. EACH EXPERIENCE THE EXTERNAL FORCE DUE TO BODIES OUTSIDE THE THREE PARTICLE SYSTEM.

THE DIRECTIONS ARE INDICATED. AND THE MAGNITUDES ARE $F_1 = 6N$ $F_2 = 12N$ AND $F_3 = 14N$.

WHAT IS THE ACCELERATION OF THE CENTRE OF MASS OF THE SYSTEM AND IN WHAT DIRECTION DOES IT MOVE?



$\longrightarrow +$ $\longleftarrow -$

$$F_x = \sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= (\overleftarrow{-6}) + (\overrightarrow{F_2 \cos 45}) + (\overrightarrow{F_3})$$

$$= -6 + 12 \cos 45 + 14$$

$$= -6 + 12 \times 0.707 + 14$$

$$= 16.48 \ N$$

$\uparrow +$ $\downarrow -$

$$F_y = \sum F_y$$

$$= F_{1y} + F_{2y} + F_{3y}$$

$$= 0 + F_2 \sin 45 + 0$$

$$= 12 \sin 45 = 8.48 N$$

$$F_T = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{16.48^2 + 8.48^2}$$

$$= 18.53 \ N$$

$$F_T = m_T \times a_T$$

$$18.53 = (m_1 + m_2 + m_3)\ a$$

$$18.53 = (4 + 8 + 4)\ a_T$$

$$18.53 = 16 \ a_T$$

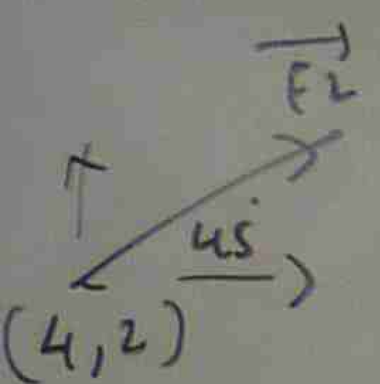
$$a_T = \frac{18.53}{16} = 1.1$$

$$X_T = \frac{F_{1x} x_1 + F_{2x} x}{F_{1x} + F_{2x} +}$$

$$= \frac{(-6) \times (-2) +}{-6 + 12}$$

$$= \frac{12 + 8.48 +}{-6 + 8.48 + 1}$$

IN WHAT DIRECTION DOES IT MOVE?



$+ f_{2x} + f_{3x}$

$\ldots os45) + (\vec{f_3})$

$+ 14$

$= 14$

$+ f_3 y$

$\ldots n45 \times 0$

$= 8.48\,N$

$$f_T = \sqrt{f_x^2 + f_y^2}$$
$$= \sqrt{16.48^2 + 8.48^2}$$
$$= 18.53\,N$$

$$f_T = m_T \times a_T$$
$$18.53 = (m_1 + m_2 + m_3)\, a_T$$
$$18.53 = (4 + 8 + 4)\, a_T$$
$$18.53 = 16\, a_T$$
$$a_T = \frac{18.53}{16} = 1.158\,m/s^2$$

$$x_T = \frac{f_{1x}x_1 + f_{2x}x_2 + f_{3x}x_3}{f_{1x} + f_{2x} + f_{3x}}$$
$$= \frac{(-6)\times(-2) + 12\cos45\times4 + 14(1)}{-6 + 12\cos45 + 14}$$
$$= \frac{12 + 8.48 + 14}{-6 + 8.48 + 14} = \frac{34.48}{16.48} = 2.09$$

$$y_T = \frac{f_{1y}y_1 + f_{2y}y_2 + f_{3y}y_3}{F_{1y} + F_{2y} + F_{3y}}$$
$$= \frac{0\times3 + 12\sin45\times2 + 0\times(-2)}{0 + 12\sin45 + 0}$$
$$= \frac{12\sin45\times2}{12\sin45}$$
$$= 2$$

DIRECTION $(x = 2.09,\ y = 2)$

$$\theta = \tan^{-1}\frac{y}{x}$$
$$= \tan^{-1}\frac{2}{2.09}$$
$$= \tan^{-1} 0.956$$
$$= 43.7^\circ$$

$$\frac{y_1 + \frac{f_2}{y} y_2 + \frac{f_3}{y} y_3}{f_1 + \frac{f_2}{y} + \frac{f_3}{y}}$$

$$= \frac{3 + 12 \sin 45 \times 2 + 0 \times (-2)}{0 + 12 \sin 45 + 0}$$

$$= \frac{12 \sin 45 \times 2}{12 \sin 45}$$

$$= 2$$

DIRECTION ($x = 2.09$

$y = 2$

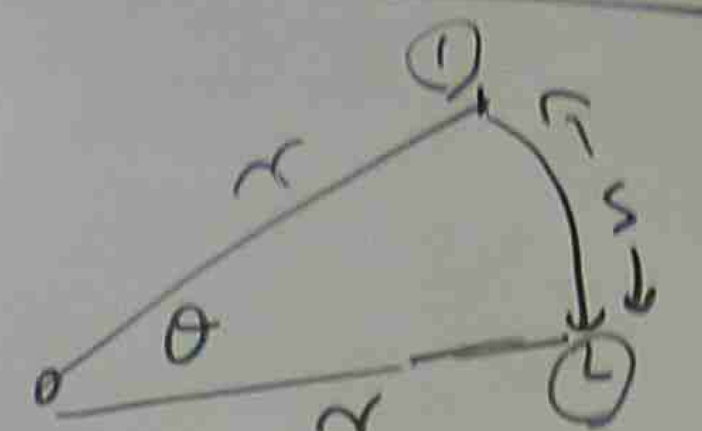
$$\theta = \tan^{-1} \frac{y}{x}$$

$$= \tan^{-1} \frac{2}{2.09}$$

$$= \tan^{-1} 0.456$$

$$= 43.7^{\circ}$$

ROTATION



r = RADIUS

S = ANGULAR PATH

1 REVOLUTION = 360° = 2π RADIAN

1 RADIAN = $\frac{1}{2\pi}$ = 0.159 REVOLUTION

ANGULAR VELOCITY = $\omega = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$

$\omega_2 = \omega_1 + \alpha \Delta t$

α = ANGULAR ACCELERATION (RAD s^{-2}

ω = ANGULAR VELOCITY RAD $\times s^{-1}$

$\alpha = \frac{\omega_2 - \omega_1}{\Delta t}$

ANGULAR DISPLACEMENT $S = \omega_1 t + \frac{1}{2} \alpha t^2$

Pb/ THE ANGULAR POSITION $\theta(t)$ OF A REFERENCE LINE ON THE DISK IS GIVEN BY

$$\theta = -1 - 0.6\,t + 0.250\,t^2$$

(a) GRAPH THE ANGULAR POSITION OF THE DISK VERSUS TIME (−3 TO 5.4 SEC)

(b) AT WHAT TIME DOES $\theta(t)$ REACH MINIMUM VALUE ?

WHAT IS THE MINIMUM VALUE ?

(a) $\theta(-3) = -1 - 0.6(-3) + 0.25(-3)^2$

$(-2) = -1 - 0.6(-2) + 0.25(-2)^2$

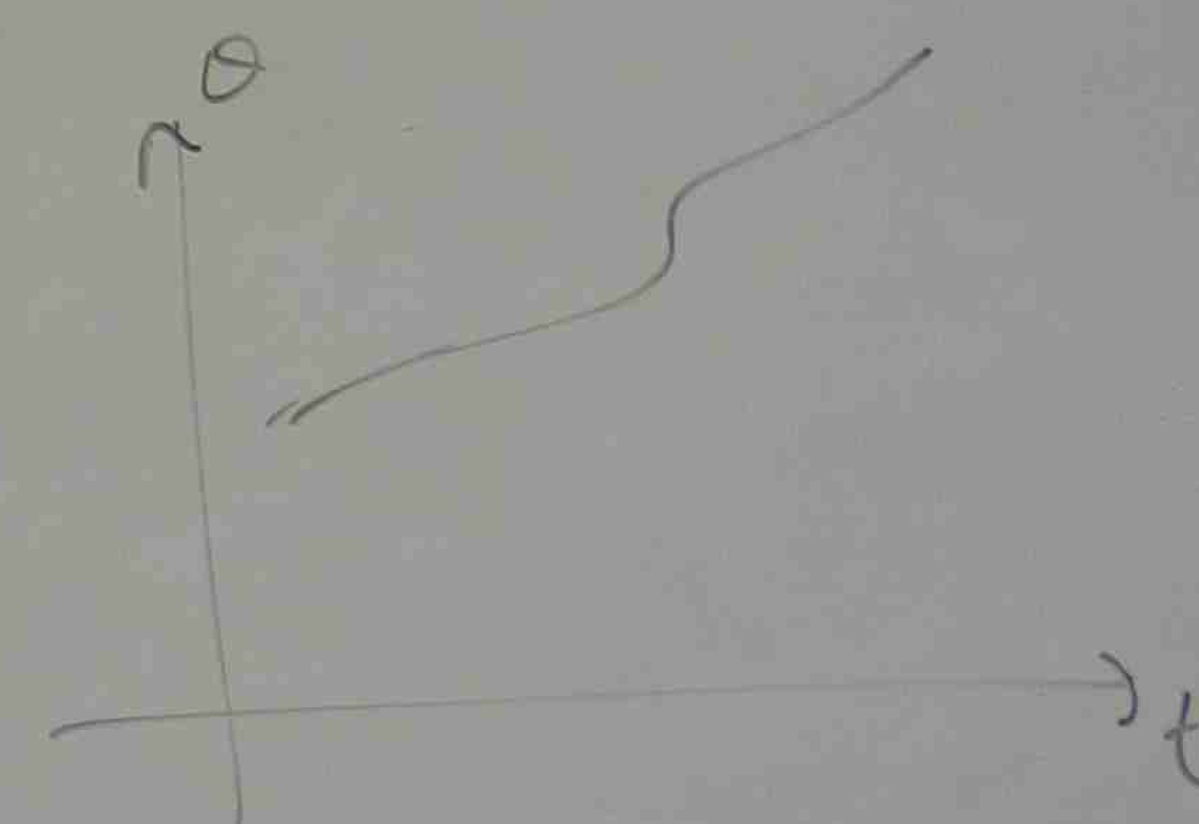
$(-1) = -1 - 0.6(-1) + 0.25(-1)^2$

$(0) = -1 - 0.6(0) + 0.25(0)^2$

|

|

(5.4)



(b) MINIMUM $\dfrac{d\theta}{dt} = 0 \rightarrow t = ?$

$$\theta = -1 - 0.6\,t + 0.25\,t^2$$

$$\frac{d\theta}{dt} = \frac{d}{dt}(-1) - \frac{d}{dt}(0.6t) + \frac{d}{dt}(0.25t^2)$$

$\dfrac{d\theta}{dt} =$

$\dfrac{d\theta}{dt} =$

$0 =$

$0.5t =$

t

$\theta = -1$

$= -$

$=$

180

REFERENCE LINE ON THE

of THE DISK VERSUS TIME (−3 TO 5.4 SEC)

REACH MINIMUM VALUE ?

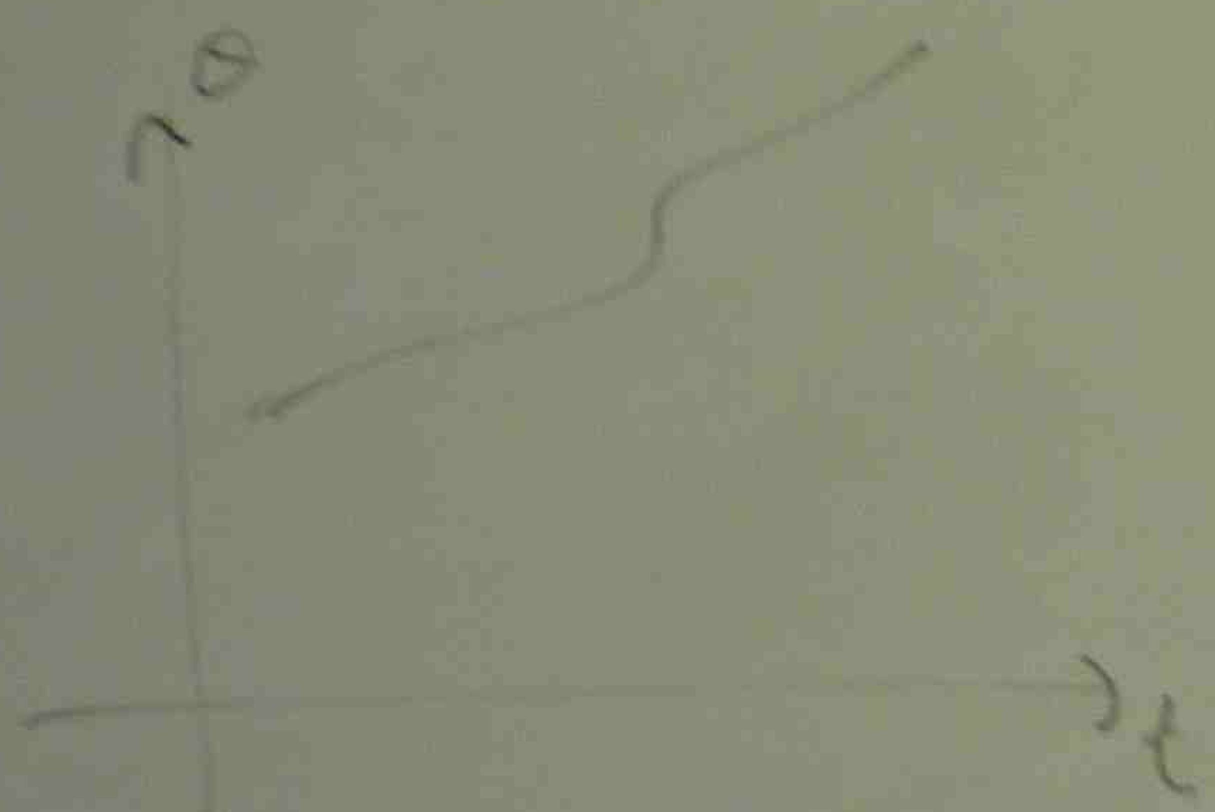
LUE ?

$0.25(-3)^2$

$0.25(-2)^2$

$.25(-1)^2$

$25(0)^2$



$\to t = ?$

$0.25t^2$

$\frac{d}{dt}(0.5t) + \frac{d}{dt}(0.25t^2)$

$\frac{d\theta}{dt} = 0 - 0.6 + 0.25 \times 2 t^{2-1}$

$\frac{d\theta}{dt} = -0.6 + 0.5t$

$0 = -0.6 + 0.5t$

$0.5t = 0.6$

$t = \frac{0.6}{0.5} = 1.2$ SEC

$\theta = -1 - 0.6t + 0.25t^2$

$= -1 - 0.6(1.2) + 0.25 \times (1.2)^2$

$= -1.36$ RAD

$180^\circ = \pi$ RADIAN

$\pi = 3.14$ RAD $\to 180^\circ$

$-1.36 \longrightarrow ? = \frac{180 \times (-1.36)}{3.14}$

$= -77.9^\circ$

P.4 A GRIN

$\alpha = 0.$

$\omega_0 = -4$

AT THE A

(a) AT

AT

(b)

(c) A

(a) $\theta_0 =$

10π

314

$0.175t^2$

Ax^2

$x =$

$2t^{2-1}$

^{2}

$(1 \cdot 2)^2$

180

$= \frac{180 \times (-1.36)}{3.14}$

$= -77.9$

pb A GRIND STONE ROTATES AT A CONSTANT ANGULAR ACCELERATION $\alpha = 0.35\ RAD/s^2$ AT TIME $t = 0$, IT HAS ANGULAR VELOCITY $\omega_0 = -4.6\ RAD/s$ AND A REFERENCE LINE ON IT IS HORIZONTAL AT THE ANGULAR POSITION $\theta_0 = 0$.

(a) AT WHAT TIME AFTER $t=0$, IS THE REFERENCE LINE AT ANGULAR POSITION $\theta = 5$ REV

(b) DESCRIBE THE ROTATION BETWEEN $t=0$ & $t=32$ SEC

(c) AT WHAT TIME t, DOES THE GRIND STONE MOMENTARILY STOP?

(a) $\theta_0 = 0$, $\theta = 5\ REV = 5 \times 2\pi\ RAD = 10\pi$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$10\pi = (-4.6)t + \frac{1}{2} \times 0.35 \times t^2$$

$$31.4 = -4.6t + 0.175t^2$$

$$0.175t^2 - 4.6t - 31.4 = 0$$

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{-(-4.6) \pm \sqrt{(-4.6)^2 - 4 \times 0.175 \times (-31.4)}}{2 \times 0.175}$$

$$= 32\ SEC$$

(b) $t=0 \rightarrow$

$\alpha =$

DECE

(c) $\omega_0 = -$

$\omega =$

$t = ?$

$t = \frac{\omega - \omega_0}{\alpha} =$

STONE ROTATES AT A CONSTANT ANGULAR ACCELERATION

RAD/s^2 AT TIME $t = 0$, IT HAS ANGULAR VELOCITY

S RAD/S AND A REFERENCE LINE ON IT IS HORIZONTAL

GULAR POSITION $\theta_0 = 0$.

HAT TIME AFTER $t = 0$, IS THE REFERENCE LINE

ANGULAR POSITION $\theta = 5$ REV

CRIBE THE ROTATION BETWEEN $t = 0$ & $t = 32$ SEC

WHAT TIME t, DOES THE GRIND STONE MOMENTARILY

OP?

$\theta = 5$ REV $= 5 \times 2\pi$ RAD $= 10\pi$

$\omega_0 t + \frac{1}{2} \alpha t^2$

$(-4.6)t + \frac{1}{2} \times 0.35 \times t^2$

$= -4.6t + 0.175t^2$

$4.6t - 31.4 = 0$

$Bx + c = 0$

$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$t = \frac{-(-4.6) \pm \sqrt{(-4.6)^2 - 4 \times 0.175 \times (-31.4)}}{2 \times 0.175}$

$= 32$ SEC

(b) $t = 0 \longrightarrow t = 32$

$\alpha = -4.6$ RAD/s^2

DECELERATION

(c) $\omega_0 = -4.6$

$\omega = 0$

$t = ?$

$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (-4.6)}{0.35}$

$= \frac{4.6}{0.35}$

$= 13$ SEC

OF MASS OF THE SYS

$\vec{F_1}$

4 kg (−3, 3)

(6 N)

$\longrightarrow +$ $\longleftarrow -$

$F_x = \sum F$

$= (-6$

$= -6$

$= -6$

$= 16$

$\uparrow + \downarrow -$

$F_y = \{$

=

=

=