

HARMONICS AND FOURIER ANALYSIS



HALF WAVE RECTIFIER SINE WAVE



FULL WAVE RECTIFIER SINE WAVE



SQUARE WAVE



PULSE WAVE



SQUARE WAVE WITH DISTORTION



EXPONENTIAL WAVE



SAWTOOTH WAVE



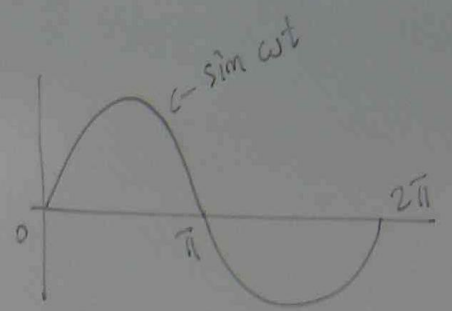
TRIANGULAR WAVE



DISTORTED SINE WAVE

FOURIER SERIES →

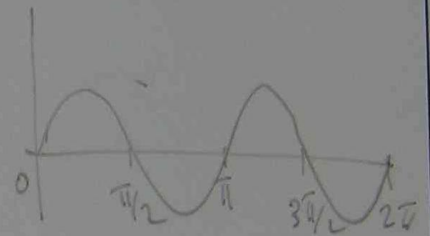
FUNDAMENTAL = BASE FREQUENCY ($\sin \omega t$) \rightarrow



2nd HARMONIC = 2x BASE FREQUENCY ($\sin 2\omega t$) \rightarrow

3rd HARMONIC = 3x BASE FREQUENCY ($\sin 3\omega t$) \rightarrow

n^{th} HARMONIC = $n \times$ BASE FREQUENCY ($\sin n\omega t$)



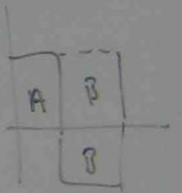
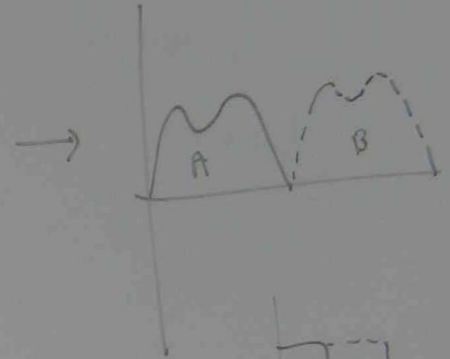
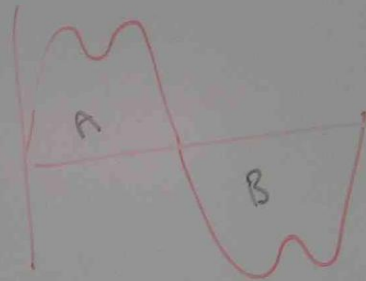
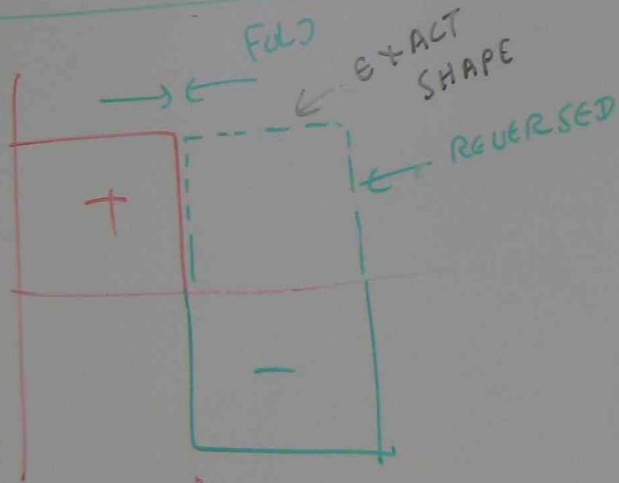
\rightarrow FUNDAMENTAL + HARMONIC = DISTORTED WAVE

GENERALIZED MATHEMATICAL EQUATION FOR FUNDAMENTAL & HARMONIC WAVES

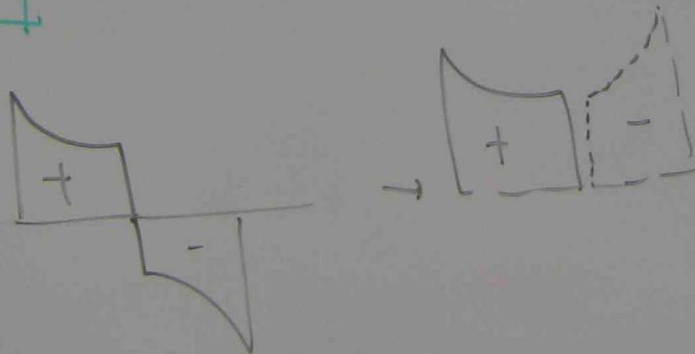
$$f(t) = \frac{1}{2}a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \dots + b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \dots$$

FOURIER SERIES SIMPLIFICATION USING WAVE FORM SYMMETRY

CONDITION (1) HALF WAVE SYMMETRY



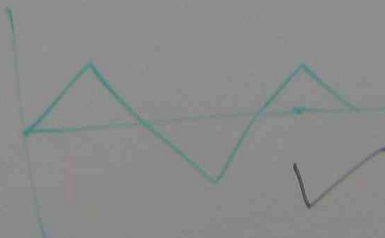
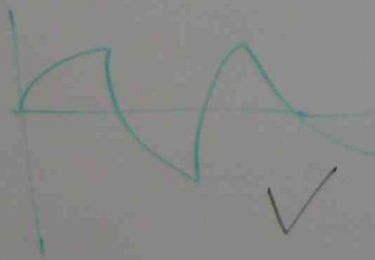
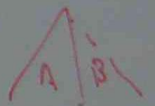
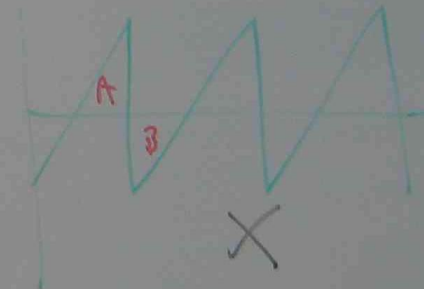
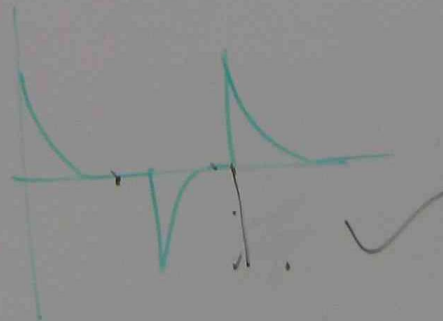
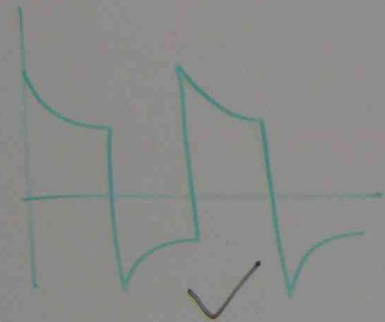
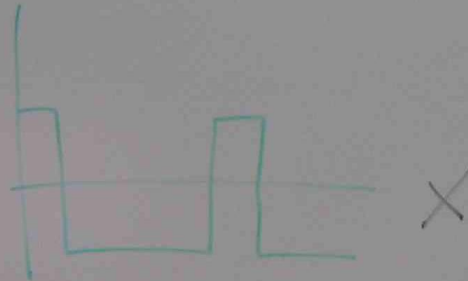
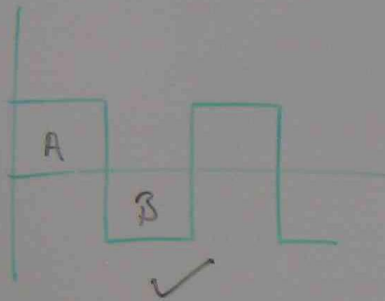
NOT HALF WAVE
SYMMETRY



SELF ASSIGNMENT (1)

INDICATE WHETHER THE FOLLOWING WAVES ARE HALF WAVE

SYMMETRIC



→ 3 PM - WEENEE 60324

→ 5 PM WEENEE WEENEE

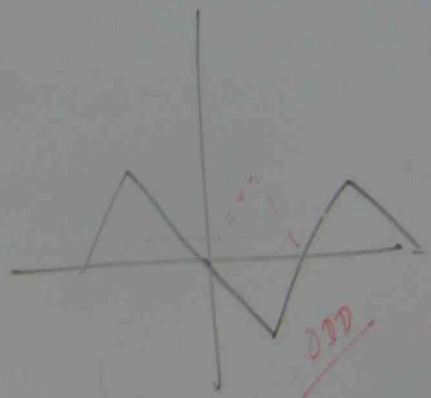
5:30-5:30 WEENEE WEENEE

EVEN FUNCTION

$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \dots$$

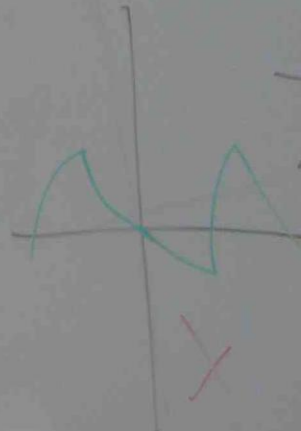
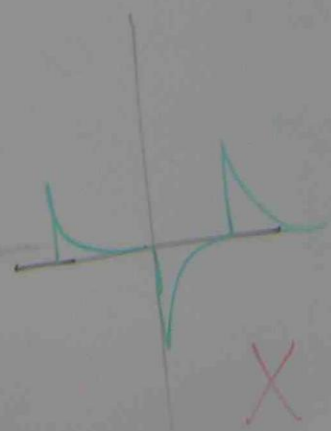
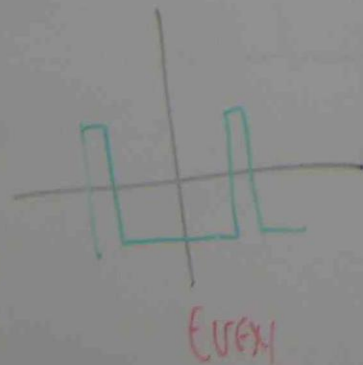
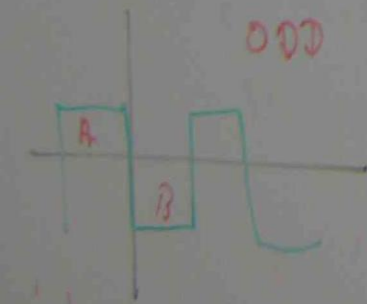
ODD FUNCTION

$$f(t) = b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \dots$$



EXERCISE (2)

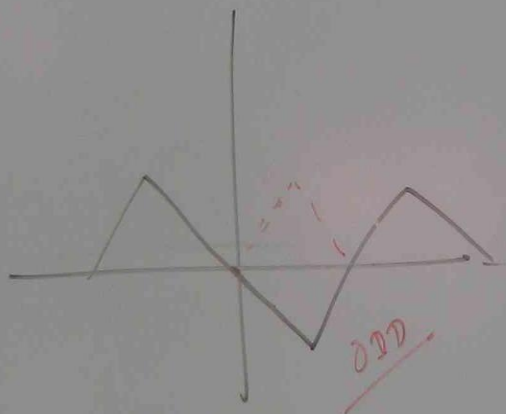
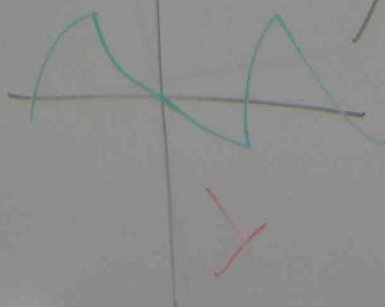
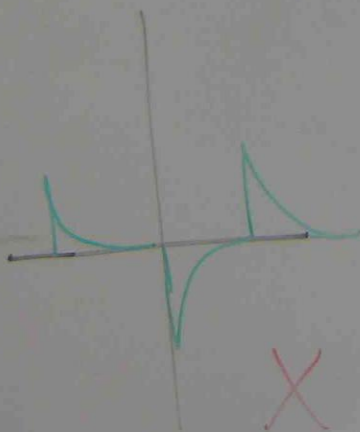
(1) INDICATE WHETHER THE FOLLOWING WAVES ARE ODD, EVEN (OR) NEITHER ODD NOR EVEN



$$a_2 \cos 2\omega t + a_3 \cos 3\omega t - \dots$$

$$b_1 \sin \omega t + b_3 \sin 3\omega t + \dots$$

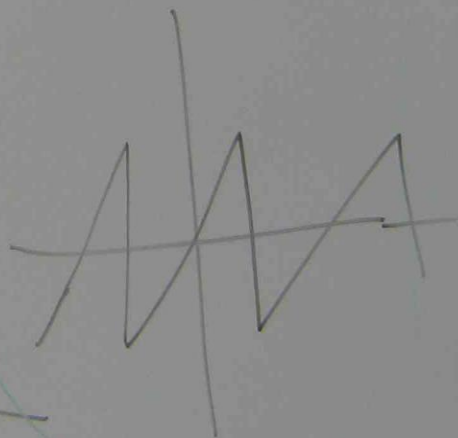
WAVES ARE ODD, EVEN (OR)



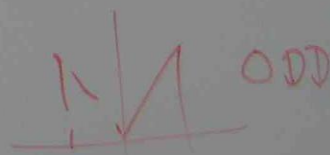
(2) INDICATE WHETHER THE
ARE EVEN (OR) ODD

odd (a) $f(t) = \frac{2}{\pi} \sin \omega t$

\times (b) $f(t) = 70 \sin \omega t + 20 \sin 3\omega t + 10 \sin 5\omega t$



(c) $f(t) = 3$



(2) INDICATE WHETHER THE FOLLOWING SERIES
ARE EVEN (OR) ODD

→ 3 PM - UENEE
6032 H

ODD (a) $f(t) = \frac{2}{\pi} \sin \omega t + \frac{3}{2\pi} \sin 3\omega t + \dots$

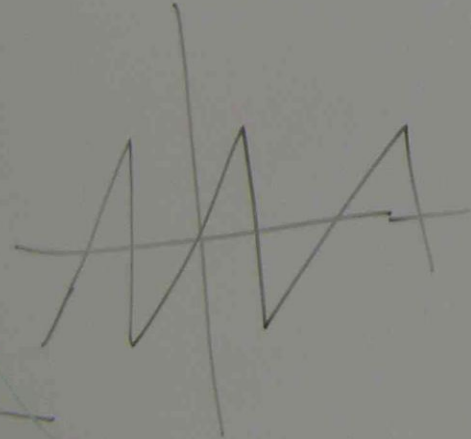
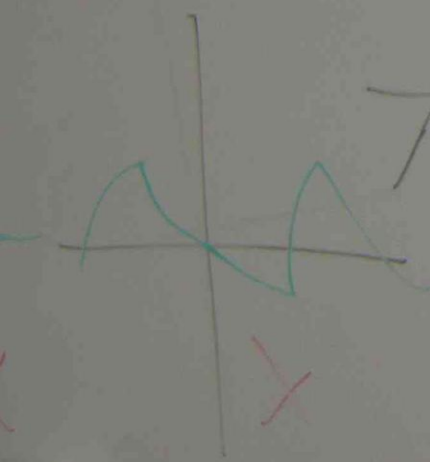
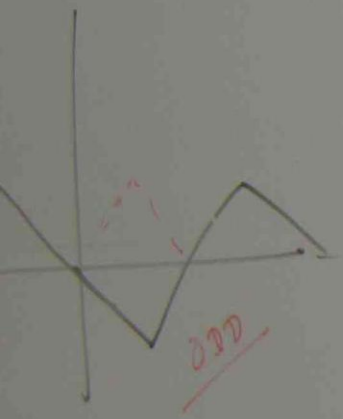
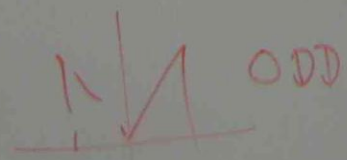
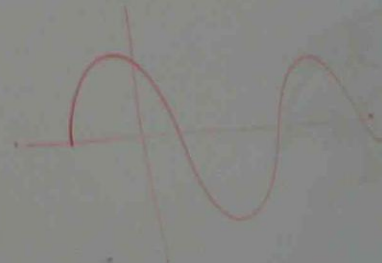
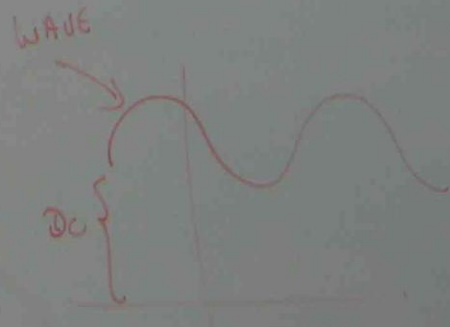
→ 5 PM UENEE
UENEE H

✗ (b) $f(t) = 70 \sin \omega t + 20 \sin 2\omega t$
 $+ 10 \sin 3\omega t + 40 \cos \omega t + 10 \cos 2\omega t$

5:30 - 5:45:30 UENEE H
UENEE

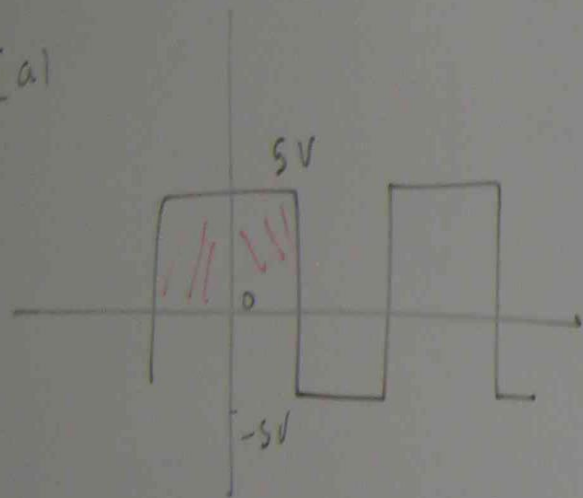
(c) $f(t) = 35 - \frac{10}{\pi} \sin \omega t - \frac{10}{2\pi} \sin 2\omega t$ ODD

↑
DC COMPONENT

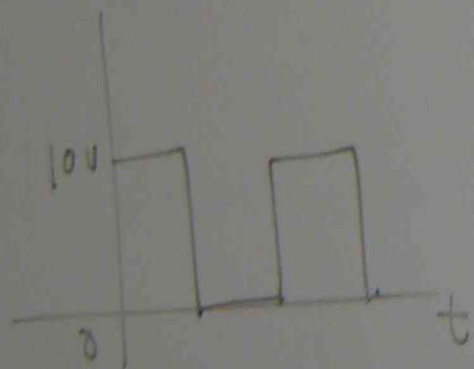


QD - CONSTRUCT BASIC FOURIER SERIES FOR THE FOLLOWING
WAVE SHAPES.

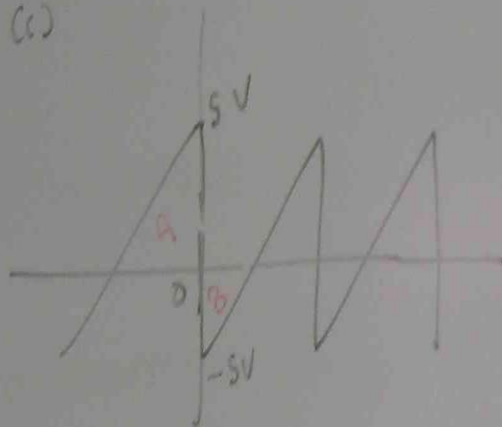
(a)



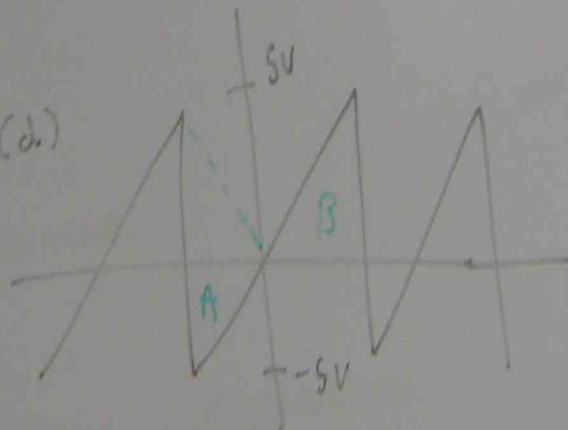
(b)



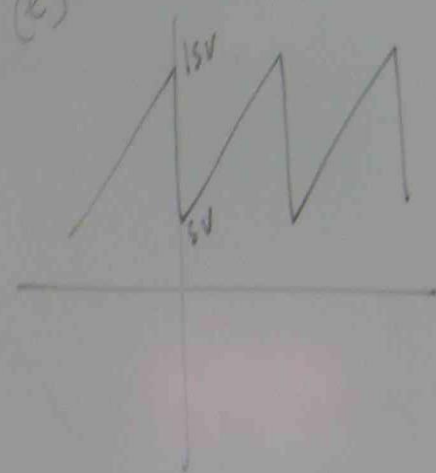
(c)



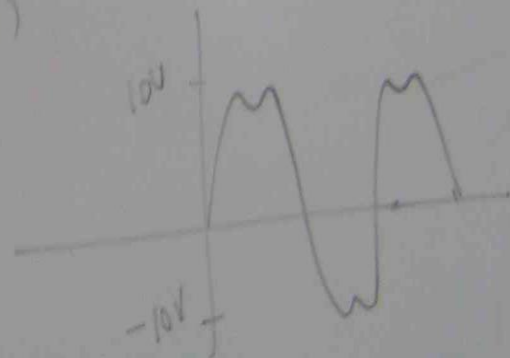
(d)



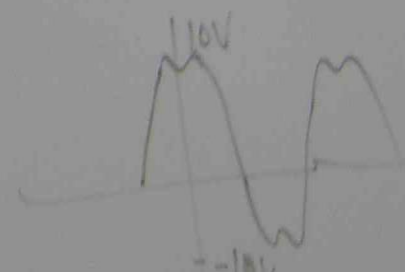
(e)



(f)



(g)



(a) (g) EVEN, NO DC

$$e(t) = \cancel{a_0} + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \dots$$

(b) NO EVEN, NO ODD, NO DC

$$e(t) = \cancel{a_0} + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \dots + b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \dots$$

(c) (d) (f) ODD, NO DC

$$e(t) = b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \dots$$

(e) NO EVEN, NO ODD WITH DC

$$e(t) = \underbrace{a_0}_{\substack{\uparrow \\ \text{DC}}} + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \dots + b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \dots$$

MATHEMATICAL FUNCTION EXPRESSION OF EVEN AND ODD FUNCTION

EVEN

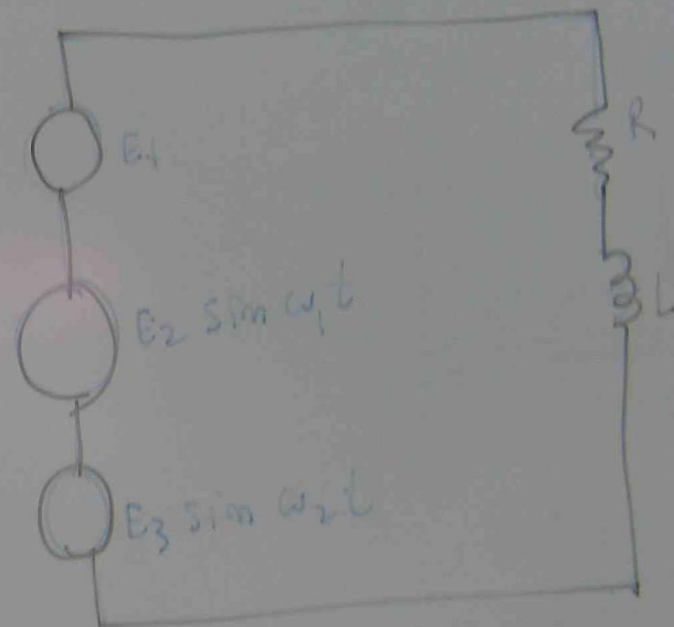
$$f(t) = f(-t)$$

ODD

$$f(t) = -f(-t)$$

↑
REVERSED

HARMONICS IN CIRCUIT ANALYSIS

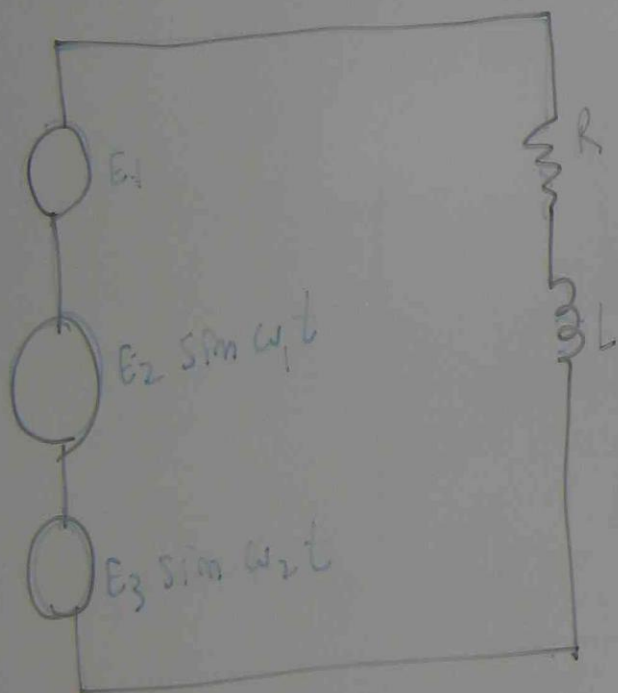


E_2 IS LEFT

$E_2 \sin \omega_1 t$

$$\sqrt{1.29^2 + 1.49^2 + 0.132^2}$$

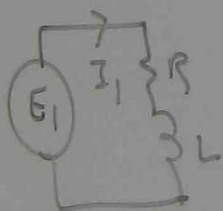
MONKS IN CIRCUIT ANALYSIS



$$E_{\text{rms TOTAL}} = \sqrt{E_1^2 + E_2^2 + E_3^2}$$

$$I_{\text{rms}} = \sqrt{I_1^2 + I_2^2 + I_3^2}$$

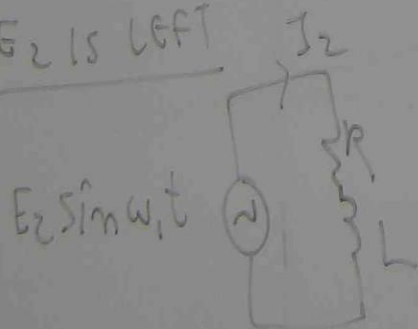
E1 IS LEFT



$$I_1 = \frac{E_1}{\sqrt{R^2 + \omega_1^2 L^2}} = \frac{E_1}{R}$$

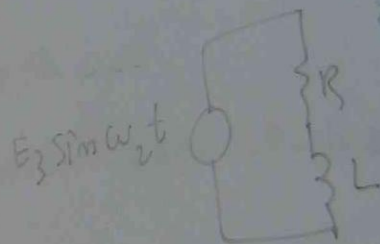
$\omega = 0$

E2 IS LEFT



$$I_2 = \frac{E_2}{\sqrt{R^2 + (\omega_1 L)^2}}$$

E3 IS LEFT



$$I_3 = \frac{E_3}{\sqrt{R^2 + (\omega_2 L)^2}}$$

pb

A SERIES R-L CIRCUIT HAS A RESISTANCE OF 20Ω AND AN INDUCTANCE OF 0.2 HENRY. THE APPLIED VOLTAGE IS

$$V(t) = 25 + 80 \sin \omega t + 20 \sin 3\omega t \quad \text{WHERE } \omega = 250 \text{ rad/sec.}$$

FIND

- THE INSTANTANEOUS CURRENT
- RMS VOLTAGE AND CURRENT
- AVERAGE POWER SUPPLIED TO CIRCUIT.

$$E_1 = 25V$$

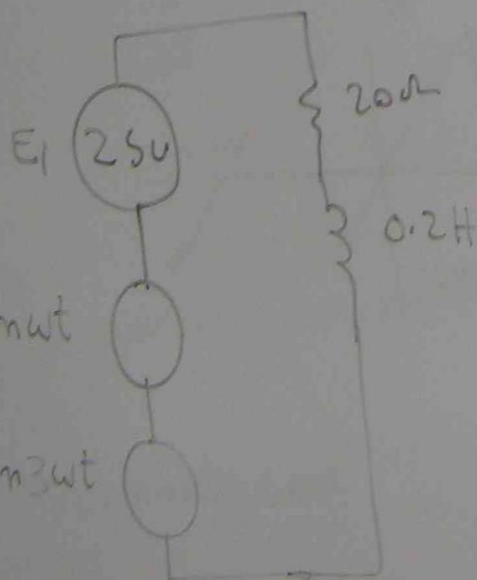
$$E_2 = 80V$$

$$E_3 = 20V$$

$$E_{rms} = \sqrt{25^2 + 80^2 + 20^2}$$

$$E_2 = 80 \sin \omega t$$

$$E_3 = 20 \sin 3\omega t$$



$$I_1 = \frac{E_1}{\sqrt{R^2 + \omega^2 L^2}} = \frac{25}{20} = 1.25 \text{ Amp.}$$

$$E_2 = 80 \sin \omega t \quad \omega = 250$$

$$I_2 = \frac{80}{\sqrt{20^2 + (250 \times 0.1)^2}}$$

$$= \frac{80}{\sqrt{20^2 + 50^2}} \angle \tan^{-1} \frac{50}{20} = \angle 68.2$$

$$= 1.49 \angle -68.2 \text{ amp.}$$

$$E_3 = 20 \sin 2\omega t$$

$$I_3 = \frac{20}{\sqrt{20^2 + (2 \times 250 \times 0.1)^2}}$$

$$= 0.132 \angle -82.4$$

$$I_{rms} = \sqrt{I_1^2 + I_2^2 + I_3^2}$$