The Kaplan turbine is used for low head applications, the operation is very similar to that of a Francis turbine but the rotor design is different. The blades do not extend to the radial direction, so the flow is turned before it enters the rotor blade row. This can be seen in Figure 4.13. The Kaplan turbine impeller is a lot like that of a propeller. The bulb turbine is very similar to an axial flow turbine, except that the medium is water rather than a gas. Again both turbine types are shown in Figure 4.10.

4.6 Common Design Choices

One common design choice is to have *constant axial velocity*: $V_{1x} = V_{2x} = V_{3x}$

Hence the blade height for turbine must increase as density falls, as seen earlier. There is no fundamental physical law that produces this constraint it is a design *choice*, although it is often used.

For multi-stage machines, successive stages are often designed to have identical flow angles and velocities at corresponding positions in each stage:

$$V_1 = V_3 \implies \alpha_1 = \alpha_3 \text{ and } V_{1\theta} = V_{3\theta}$$

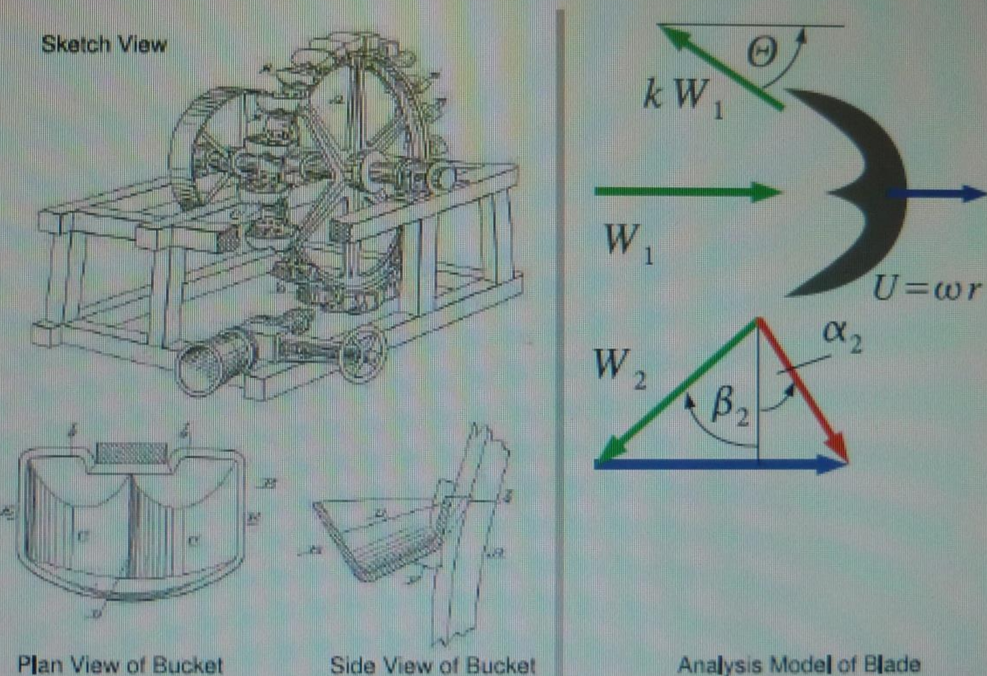

Figure 4.11: Pelton's Patent Application and Analysis Model

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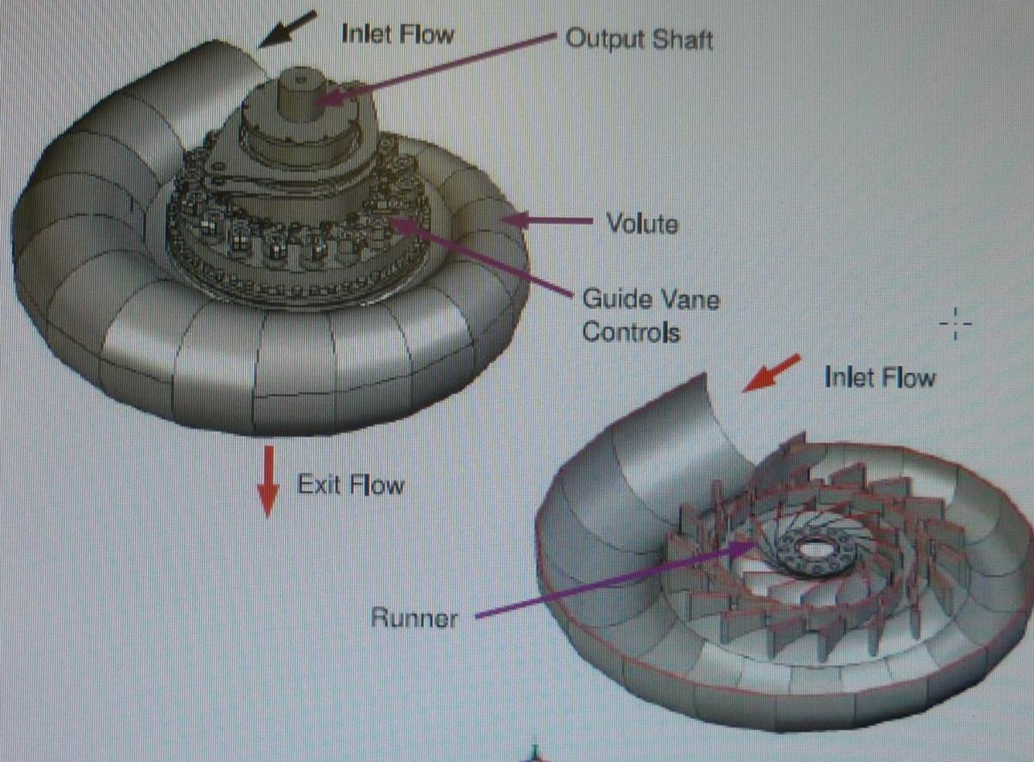

Figure 4.12: Three Dimensional Views of a Francis Turbine

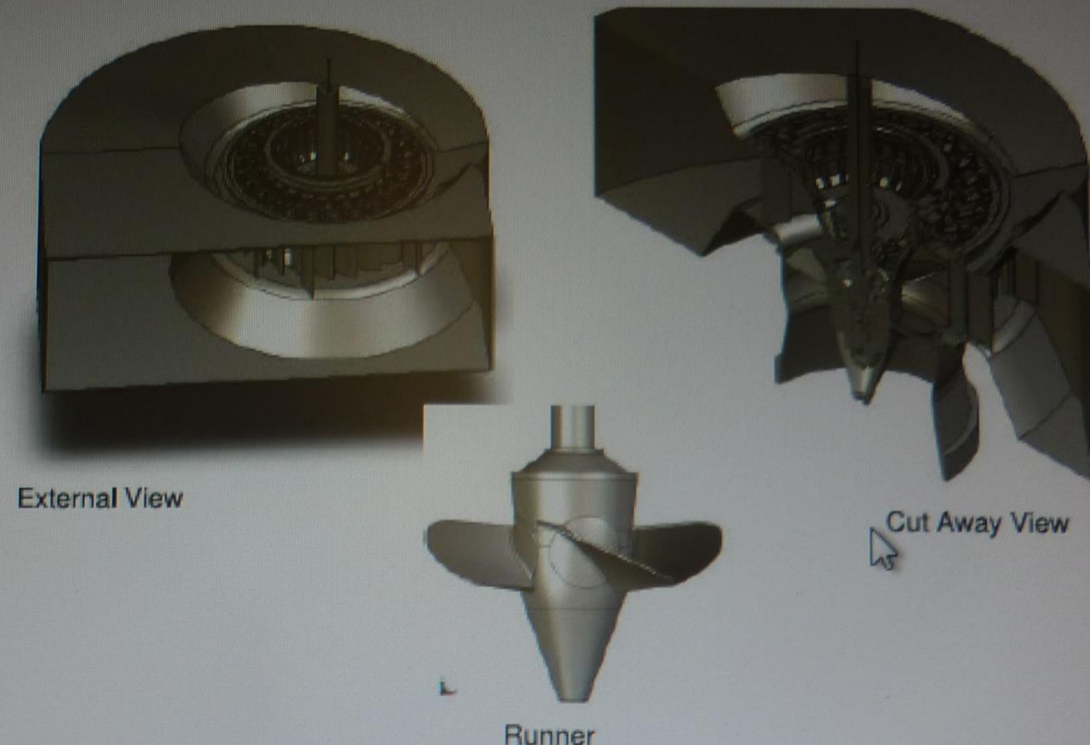

Figure 4.13: Three Dimensional Views of a Kaplan Turbine

This is know as a *Repeating Stage* where the velocity at inlet is equal to the velocity at exit.

Often we design for *axial leaving velocity* from a turbomachine. Since for a given V_x the exit velocity V_3 is given by $V_3 = \sqrt{V_x^2 + V_{\theta 3}^2}$ the minimum exit kinetic energy at exit occurs when $V_{\theta 3} = 0$ or the flow is entirely in the axial direction. Clearly this condition occurs at $\alpha_3 = 0$ so an axial leaving condition specifies the shape of the exit velocity triangle for the stator or rotor row.

These three commonly used design conditions are design choices, although there are many successful turbomachines in use that do use these design choices there are some that do not.

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4.7 The Turbomachine and System

The turbomachine always operates as part of a system. Pumps are required to deliver fluid at a higher pressure for water delivery for example, steam turbines receive steam from a boiler and deliver it to a condenser or some industrial process. The torque from a turbine is either used to generate electricity or provide mechanical power for gearboxes. The mechanical work for a compressor or a fan comes from either electrical power (or in the case of a gas turbine engine is taken directly from the turbine). So both the fluid movement and the work changes from the machine fit into a system.

The apparent exception to this rule are devices such as wind turbines which although they supply electricity to some sort of load have varying and unpredictable inlet conditions. Even here a great deal of work is done to ensure that turbines are sited appropriately so that they extract the maximum amount of energy at a given location and have a minimal visual impact.

The inputs and outputs from the turbomachine are not arbitrary and are usually fixed by external parameters:

- For steam turbines the inlet conditions are fixed by the boiler conditions which are dependant on the type of fuel used in the cycle and the overall thermodynamic cycle that is chosen for the site. In a steam cycle the exit conditions are fixed by the temperature at which the condenser cooling fluid can operate at.

- For aircraft engines the inlet conditions are determined by the operating envelope of the aeroplane and the exit conditions largely determined by the amount of thrust required from the engine.
- For pumps the exit conditions are set by the flow rate required and pressure loss introduced by the piping system to which the pump is connected to. In the case of moving the fluid from one location to another the pump also has to supply a pressure equivalent to the change in height between the two locations as well as any losses in the system.
- For wind turbines the inlet conditions are set by the location of the wind turbine and the variability of the weather. Of all turbomachines wind turbines have the most variable inlet conditions and ensuring that the design performs over a wind range of inlet conditions is key to producing a successful turbine.
- For hydro-electric power systems the available pressure or head is determined by the height difference between the reservoir and the river into which the fluid is discharged. (See Figure 4.9) The flow-rate is determined by the rainfall for the region in which the hydro system is located and the capacity of the reservoir. A common strategy is to use the reservoir as an energy storage system and run the turbines when electrical demand is high, turn them off during periods of low demand and allow the reservoir level to rise.

Chapter 6

Efficiency and Reaction

6.1 Efficiency

There are over one hundred different ways of defining the efficiency of a turbomachine, though only one of them will be discussed in this book. Consider a turbine on a h-s diagram, Figure 6.1.

Three points are shown on this diagram 1 the entry condition to the machine, 3 the real exit condition and 3s the isentropic (or ideal) exit condition. (We use 3 rather than 2 as we will consider the stator/rotor interface as 2 later) Since the real exit condition will contain irreversibility's due to fluid friction, heat transfer over a finite temperature difference and so on the entropy in the real case increases meaning the line 1 to 3 is a diagonal. The ideal process is a straight line on the chart as the

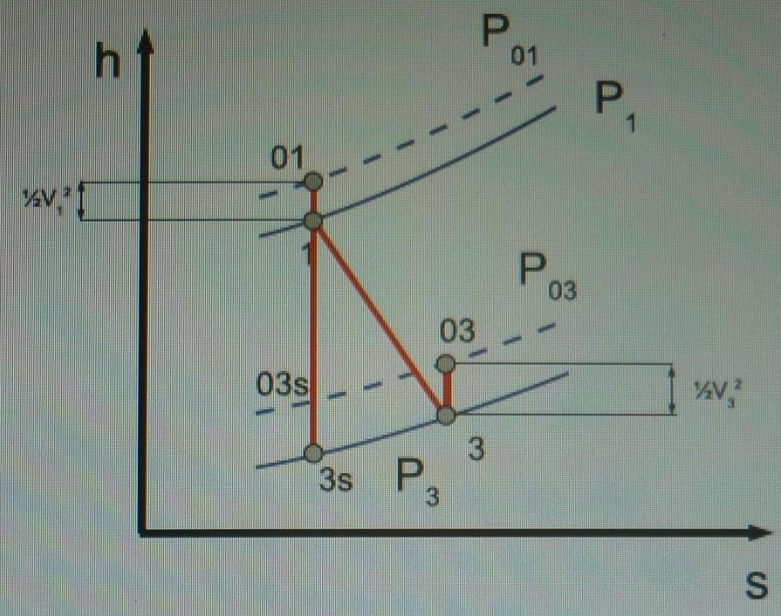

Figure 6.1: Enthalpy-Entropy Diagram for a Turbine

change in entropy is zero for an reversible process.

One of the reasons there are hundreds of definitions of efficiency is that there are various ways of accounting for the kinetic of the flow at inlet and outlet. Recall that for each location in a fluid machine there is a static condition and a corresponding stagnation condition. The stagnation condition is the condition that occurs when the fluid is brought reversibly to rest so the entropy is the same between the static and stagnation conditions. For the turbine stage on the h-s diagram (Figure 6.1) at each condition 1,3 and 3s there is a corresponding stagnation condition 01,03 and 03s which are also plotted.

Chapter 7

Dimensionless Parameters for Turbomachinery

Dimensionless coefficients are a very useful technique to bring out trends between groups of variables in fluid mechanics. Further details can be found in basic fluid mechanics texts Massey (1989) but essentially the technique works by constructing physically plausible relationships between variables. Consider the relationship between static pressure, dynamic head and stagnation pressure for example:

$$p_o = p + \frac{\rho V^2}{2}$$

For this relationship to be physically possible ρV^2 must have the same units as pressure, in simple terms one cannot count two oranges and expect to end up with an apple!

Pressure can be explained as a force per unit area, force can be expressed as a mass times an acceleration so the units of pressure will be given by:

$$\frac{mass \times acceleration}{area} = \frac{mass \times \frac{length}{time^2}}{length^2} = \frac{mass}{time^2 \times length}$$

The units of ρV^2 can similarly be expressed:

$$density \times velocity = \frac{mass}{length^3} \frac{length^2}{time^2} = \frac{mass}{time^2 \times length}$$

Therefore the equation is said to be dimensionally correct. Using this principle one can construct a number of dimensionless parameters or dimensionless coefficients. For example reaction is a dimensionless coefficient as the units on the top of the expression cancel the units on the bottom of the expression.

Dimensionless parameters provide a number of advantages:

1. With an appropriate choice of dimensionless parameters the performance of machines can be characterised using only a few key variables.
2. Given data on one size of machine performance can be predicted at different sizes.
3. Given data on one set of operating conditions behaviour at different operating conditions can be predicted
4. Enable designers to pick a particular machine shape of maximum efficiency.

Which dimensionless parameters are important depends very much on the application area several

Case	Flow Coefficient	Stage Loading Coefficient
Aircraft Engine Compressor	0.4 to 0.70	0.35 to 0.50
Aircraft Engine HP Turbine	0.5 to 0.65	1.0 to 2.0
Aircraft Engine LP Turbine	0.9 to 1.0	1.0 to 2.0

Table 7.1: Typical Values of Ψ and Φ

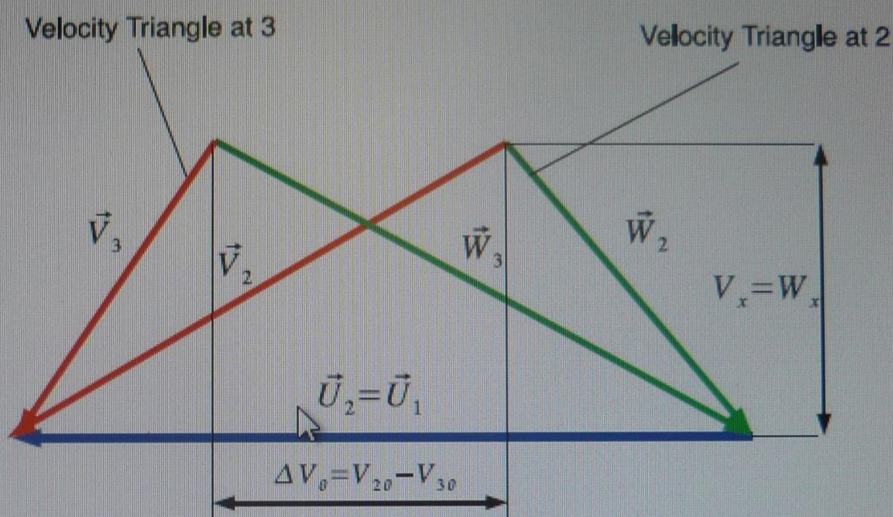

Figure 7.1: Velocity triangles for exit and inlet combined

In general the designer of turbomachinery blading does not have complete freedom to determine the dimensionless coefficients that the design will operate at. For example the mean blade speed is often fixed by mechanical considerations, or the rotation frequency of the machine is fixed by the frequency of the electrical supply and the number of poles in the electrical generator. The stage loading is often fixed by the number of stages and the required enthalpy drop in the machine.

and α_3 will reduce. There is a direct link between machine geometry and dimensionless coefficients.

Finally an important simplification with constant blade speed is that since $V_\theta = U_m + W_\theta$ then $\Delta V_\theta = \Delta W_\theta$

Example Given a turbine blade row with constant axial velocity of $150 m/s$ at $5000 rpm$ on a mean radius of $0.7\ m$ and an absolute flow angle at exit from the stator of $70°$. The turbine operates with axial leaving flow and is a repeating stage. Calculate the flow coefficient, stage loading coefficient and reaction.

Solution This problem is a straightforward application of dimensionless coefficients. First calculate the mean blade speed:

$$U_m = \omega r = 5000 \times \frac{2\pi}{60} \times 0.7 = 366.5\ m/s$$

$$\phi = \frac{V_x}{U_m} = \frac{150}{366.5} = 0.409$$

$$\psi = \frac{w}{U_m^2} = \frac{U_m(V_{3\theta} - V_{2\theta})}{U_m^2} = \frac{V_{3\theta} - V_{2\theta}}{U_m}$$

Now the turbine row is axial leaving so $V_{3\theta} = 0$ from the generic velocity triangle:

$$V_{2\theta} = V_x \tan\alpha_2 = 150 \tan 70 = 412.12\ m/s$$

so the stage loading coefficient is:

$$\psi = \frac{412.12}{366.5} = 1.124$$

Reaction is given by $R = \Delta h_{ROTOR}/\Delta h_{STAGE}$. The stage enthalpy drop is given by the work output as the turbine has repeating stages so $V_1 = V_3$.

$$\Delta h_{STAGE} = \Delta h_0 = w = U_m(\Delta V_\theta) = 366.5(412.12) = 151.04\,kJ/kg$$

From Figure 6.5 we see that the static enthalpy drop across the rotor is given by:

$$\Delta h_{ROTOR} = \frac{W_3^2 - W_2^2}{2} = \frac{W_{3\theta}^2 - W_{2\theta}^2}{2}$$

Since the turbine axial velocity is constant $W_{2x} = W_{3x}$ and so:

$$\Delta h_{ROTOR} = \frac{W_{3\theta}^2 - W_{2\theta}^2}{2}$$

However since we have constant blade speed $\Delta W_\theta = \Delta V_\theta$ so:

$$\Delta h_{ROTOR} = \frac{412.12^2 + 0^2}{2} = 84.92\,kJ/kg$$

Which means that the reaction is given by $R = 84.92/151.04 = 0.56$

7.2 Coefficients for Wind Turbines

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7.2 Coefficients for Wind Turbines

For wind turbines two coefficients are in widespread use, the first C_P or power coefficient is essentially an efficiency by a different name:

$$C_P = \frac{\text{Power from Turbine}}{\text{Power in Wind}} = \frac{P}{\frac{1}{2}\rho V^3 A} \tag{7.3}$$

where P is the power output, V is the velocity of the wind and A is the swept area of the machine. The second coefficient is the tip speed ratio, which is the ratio of the wind speed to the tangential velocity of the tip of the turbine:

$$\lambda = \frac{\omega r_{tip}}{V} \tag{7.4}$$

Performance of wind turbines is usually presented as Cp vs λ curve, this is often called a "power curve". A power curve for a Nordex N80 turbine [1] which produces 2.5MW at full power is shown in Figure 7.2. The power curve shows a characteristic of many turbomachinery devices, that there is distinct peak in efficiency with a lower device effectiveness away from the design point.

Example Given the power curve shown in Figure 7.2. Calculate the power output of the device at 10 rpm in a 10 m/s wind given that the turbine is 80m in diameter and the density of air can be taken as $1.15kg/m^3$.

Solution This problem reduces to finding the tip speed ratio λ looking up the power coefficient on the graph and then calculating the power output. The tip speed ratio λ gives:

$$\lambda = \frac{\omega r_{tip}}{V} = \frac{\frac{10\times 2\pi}{60} \times \frac{80}{2}}{4} = 10.47$$

So look up the power coefficient that corresponds to 10.47 on Figure 7.2 which is approximately $Cp = 0.37$. The area of the turbine is given by $A = \pi r_{tip}^2 = \pi \times 40^2 = 5026\ m^2$, the assumption is that the area of the nacelle is negligible. Putting this all together:

$$Cp = \frac{P}{\frac{1}{2}\rho V^3 A} \implies P = Cp\frac{1}{2}\rho V^3 A = 0.37 \times \frac{1}{2} \times 1.15 \times 10^3 \times 5026 = 1.069\ MW$$

that the area of the nacelle is negligible. Putting this all together:

$$C_P = \frac{P}{\frac{1}{2}\rho V^3 A} \implies P = C_P \frac{1}{2}\rho V^3 A = 0.37 \times \frac{1}{2} \times 1.15 \times 10^3 \times 5026 = 1.069\, MW$$

7.3 Coefficients for Hydraulic Machines

Hydraulic machines are those turbomachines that operate with liquid (most often water) as the working fluid. These are enormously significant devices which produce up to 20% of the worlds electricity in large and small hydroelectric plants. Pumps are vital to the infrastructure of water and fuel supply.

Dimensional analysis of hydraulic machines yields a different set of coefficients, these are the *flow coefficient*:

$$\Pi_1 = \frac{Q}{ND^3} \tag{7.5}$$

where N is the number of revolutions per second made by the machine and D is the machine diameter. We give the coefficients the symbol Π. The next dimensionless parameter is the *head coefficient*:

$$\Pi_2 = \frac{gH}{N^2 D^2} \tag{7.6}$$

where H is the head produced or absorbed by the hydraulic machine and g is the acceleration due to gravity. Finally the power coefficient is given by:

[1] http://www.nordex-online.com/en/products-services/wind-turbines/n80-25-mw/

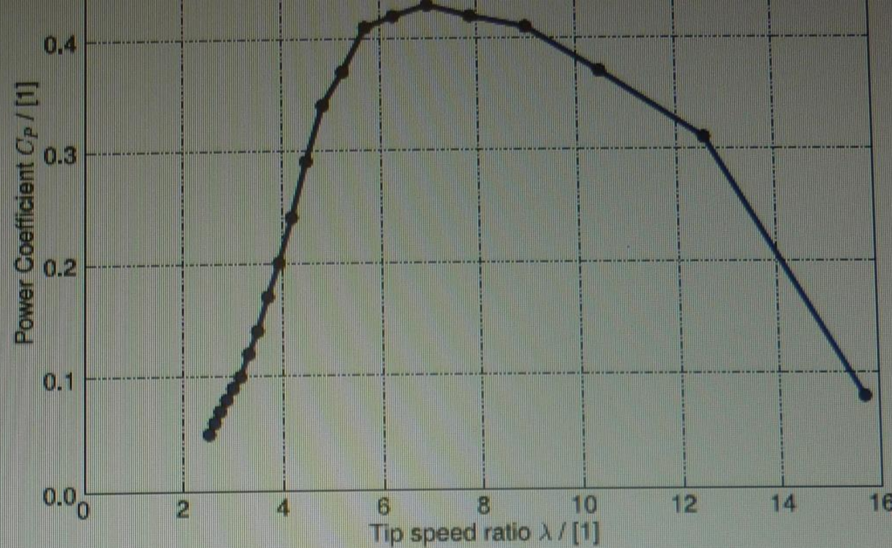
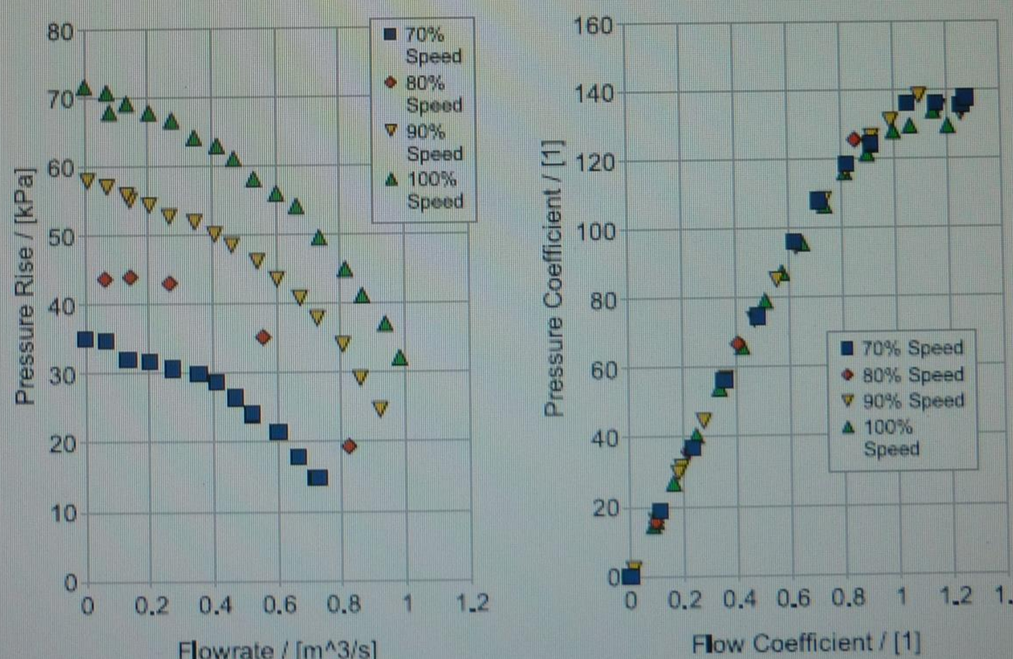


Figure 7.2: C_P vs λ for 2.5MW Wind Turbine



Clearly efficiency is a non-dimensional parameter for hydraulic machines and again it varies between pumps and turbines as follows:

$$\eta_{pump} = \frac{\text{Ideal Work}}{\text{Actual Work}}, \eta_{turbine} = \frac{\text{Actual Work}}{\text{Ideal Work}}$$

7.3.1 Specific Speed for Turbines

Note that for a turbine:

$$\frac{\Pi_3}{\Pi_1 \Pi_2} = \frac{P}{\rho Q g H} = \eta$$

For a pump the reciprocal applies. So if for a model turbine we obtain test data and plot the dimensional parameters against each other the performance of larger machines of the same type can be predicted. For a particular machine the operating conditions are expressed by values of N, P and H. The rotational speed N is normally fixed by the frequency of electricity supply, P and H are set by the flow rate and the height drop of the physical location that the hydro-electric scheme is proposed for. It would therefore be useful to have a dimensional group which includes N, P and H but not D so that it would be independent of the machine size. This can be obtained by manipulating the dimensionless groups for a turbine to obtain a new dimensionless coefficient:

$$\Pi_4 = \frac{\Pi_3^{1/2}}{\Pi_2^{5/4}} = \frac{N P^{1/2}}{\rho^{1/2} (gH)^{5/4}}$$

Chapter 9

Hydraulic Turbines

Hydro-electric power accounts for up to 20% of the world's electrical generation. Hydraulic turbines come in a variety of shapes determined by the available head and a number of sizes determined by the flow rate through the device. This chapter introduces no new concepts but instead allows the analysis techniques learnt over the preceding chapters to be applied by means of analysis examples.

Three three basic hydraulic machines (Pelton, Francis and Kaplan) are discussed and with all hydraulic machines the simplification for analysis and operation is that the fluid density is constant but the complication is that cavitation may occur in the machine. Cavitation is effectively the boiling of the liquid as it is exposed to extremely low pressures in certain parts of the turbine.

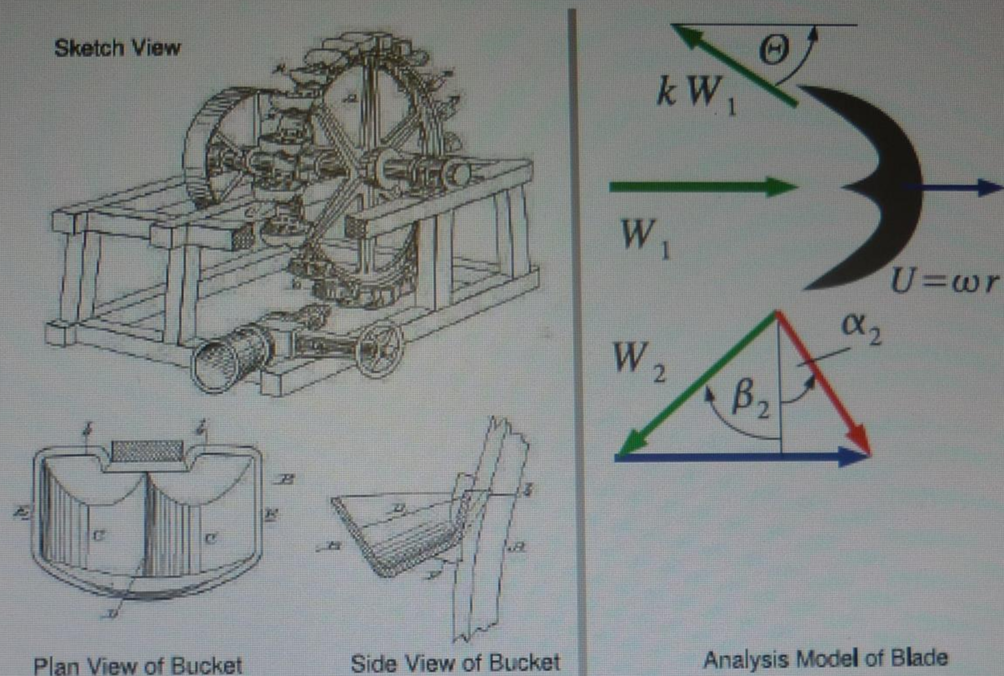


Figure 9.1: Pelton's Patent Application and Analysis Model

9.1 Pelton Wheel

This device invented around 1880 by Lester Pelton is used in high head locations with 300m to 4000m head, it has a peak efficiency of around 90%.

Figure 9.1 shows the Pelton wheel on the left and the sketch on the right shows a diagram of operation with velocity triangles.

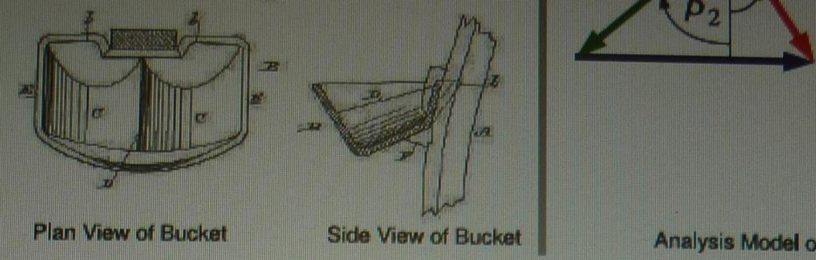


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Figure 9.1 shows the Pelton wheel on the left and the sketch on the right shows a diagram of operation with velocity triangles.

The analysis of this machine is straightforward and can be found in any basic fluid mechanics textbook (e.g. Massey (1989)) as a simple control volume can be drawn around the rotor blade and the force determined from a simple analysis. Alternatively the results can be obtained from the Euler equation as follows.

Recall that $P = \dot{m}\omega(V_{2\theta}r_2 - V_{1\theta}r_1)$ so the key parameters to work out are the inlet tangential velocity and the exit tangential velocity.

At inlet the situation is straightforward $V_{1\theta} = V_j$ the jet velocity, as the nozzle directs the flow only in the tangential direction. At exit the velocity triangle shown in Figure 4.11 must be used.

Note that : $W_1 = V_j - U$ so $W_2 = kW_1 = k(V_j - U)$ where k is an empirical coefficient for frictional losses in the bucket. From the velocity triangle:

9.2 Analysis Approach

The analysis of the Pelton wheel is extremely straightforward, other hydraulic machines required a more nuanced approach. This is a three step process:

1. Given the *flow rate* apply the *continuity* equation to get the radial or axial velocity.
2. The *geometry* (blade angles and radii) will yield absolute and relative velocities by means of velocity triangles.
3. *Power* is obtained from the Euler Equation.

Empirically obtained values of efficiency (or estimated values based on prior experience) link the power produced and the head drop across the machine. There are two methods of doing this depending of what information is available.

1. If the total head is given this yields the ideal power:

$$P_{ideal} = \rho \dot{Q} g H = \dot{m} g H$$

   Given the ideal power, we have the actual power (from the Euler equation above) this yields the efficiency.

2. If loss information is given, the actual head is obtained from:

$$H_{actual} = \frac{P_{actual}}{\rho \dot{Q} g} + H_{loss}$$