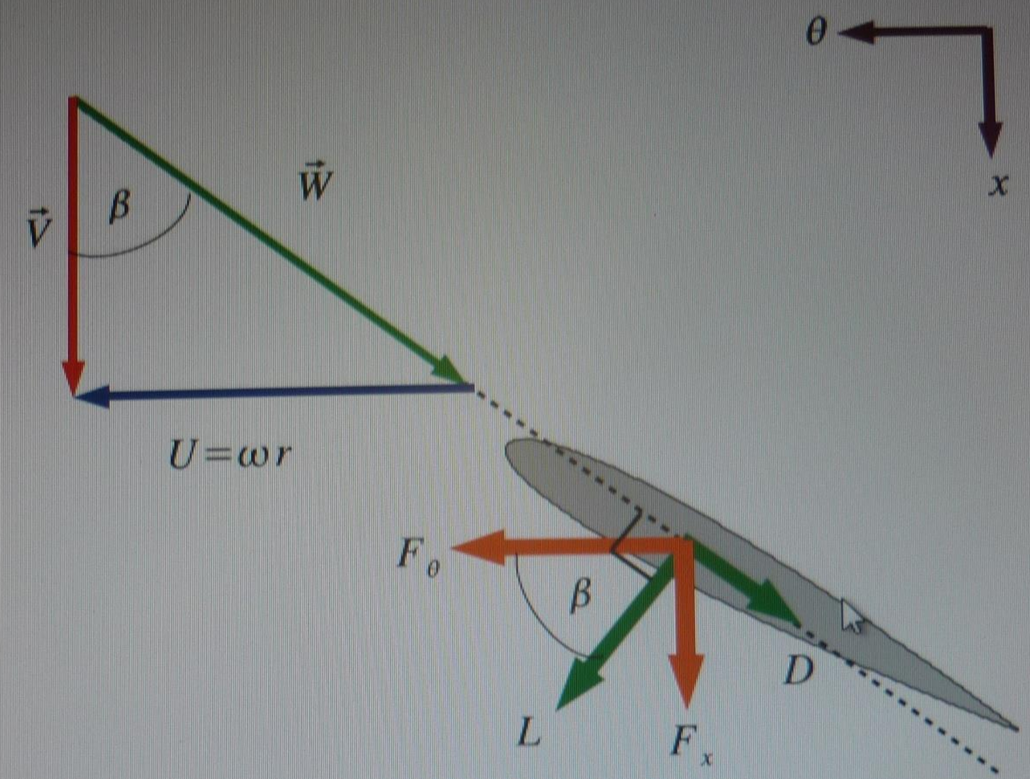

Figure 3.3: Forces on a Wind Turbine Blade

important for determining the loads on the wind turbine tower and wind turbine bearings.

All of this is shown in Figure 3.3 which shows the lift and drag forces on the blade. Note that these forces are perpendicular and parallel to the incoming flow $\vec{W}$ and not the axial chord line of the aerofoil. Some trigonometry shows the relationship between lift and drag and tangential and axial force to be:

$$F_x = L\sin\beta + D\cos\beta \tag{3.2}$$

If we consider F_θ and F_x to be forces per unit span. We can obtain a torque per unit span from the force times radius.

$$T = F_\theta \times r$$

The power per unit span is given by the torque multiplied by the rotational speed, this can then be integrated from the hub to the tip to give the total power on the blade.

$$P = \int_{r_h}^{r_t} F_\theta r \omega dr$$

and drag forces is required. This largely comes from test data obtained in wind tunnels, in the form a lift and drag plot against incidence. To use this a short digression into aerofoil performance is required.

3.1 Aerofoil Operation and Testing

In wind tunnel testing an aerofoil is placed in wind tunnel and the incidence of the aerofoil is changed, usually by rotating the aerofoil. A lift force perpendicular to the incoming flow and a drag force parallel to the incoming flow are measured. Figure 3.4 shows such an aerofoil at two incidences one of which is zero or aligned with the incoming flow. Note how the lift and drag remain in the same direction with changing incidence and that the aerofoil chord c also does not change with incidence. Since it is the flow relative to the aerofoil that produces the lift the incoming velocity is the relative velocity $\vec{W}$ and not the absolute velocity $\vec{V}$. The incidence i is defined as the angle made between the axial chord of the aerofoil and the incoming flow.

The lift and drag are usually expressed in turns of a non-dimensional coefficient so that they can be scaled for size, fluid density and incoming fluid velocity. These are given by:

$$C_L = \frac{L}{\frac{1}{2}\rho W^2 c} \tag{3.3}$$

where L is the lift force per unit length of aerofoil. The drag coefficient is given in a very similar form:

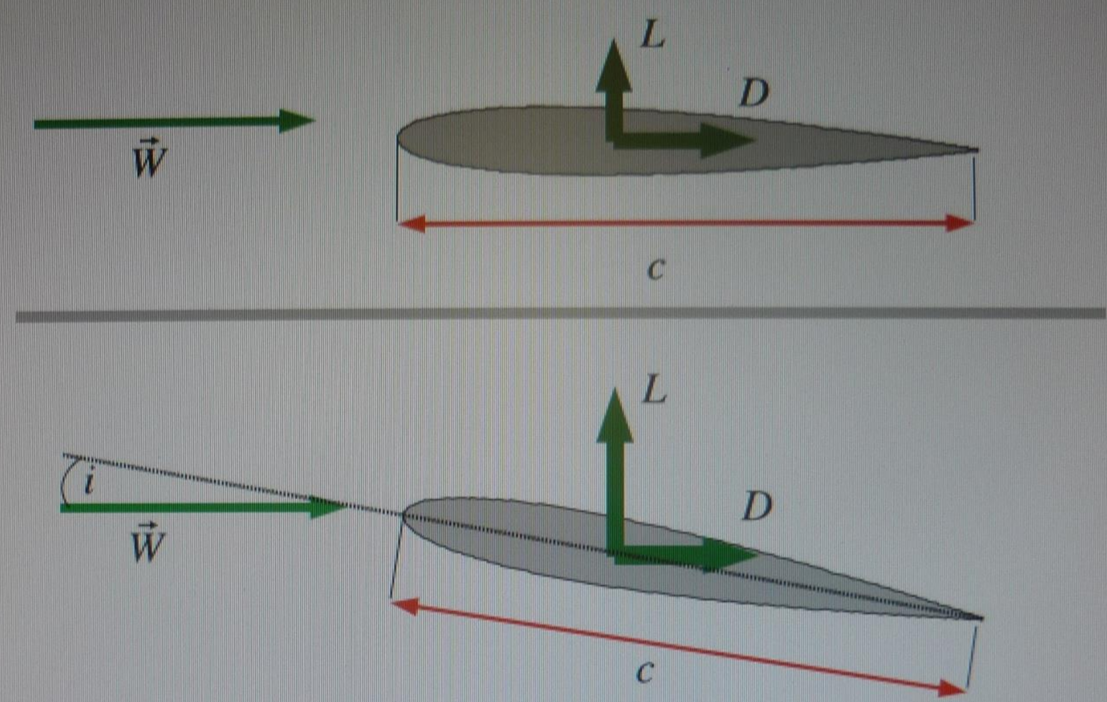

Figure 3.4: Aerofoil at Two Incidences

where D is the drag force per unit length of the aerofoil. For wind turbine analysis and design values of C_L and C_D as a function of incidence i are required, these are found in reference books such as Abbott and von Doenhoff (1959) A simplified example of aerofoil data is shown in Figure 3.5 which is for a NACA 0012 aerofoil. NACA was the predecessor of NASA and the four digit designation allows the aerofoil geometry to be determined. On-line coordinate generators are available.[1]

substantial reduction in lift along with a substantial increase in drag.

A common design choice is to place the design point at around 80% of the maximum lift to allow for some variation in incidence with stalling the aerofoil.

Example A wind turbine is designed to work at a condition with a wind speed of 10 m/s and an air density of 1.22 kg/m^3. The turbine has blades with a NACA 0012 profile and is rotating at one

[1] http://www.ppart.de/aerodynamics/profiles/NACA4.html

revolution per second. The blade chord length is 0.5 m. Taking the design point at 85% maximum lift condition and ignoring the drag on the aerofoil estimate the power output per unit blade span at a radius of 6 m for each blade.

Solution The maximum lift sh

radius of 6 m for each blade.

Solution The maximum lift shown in Figure 3.5 is around 1.3 and 85% of this value is around 1.1. Recall that:

$$L = C_L \frac{1}{2} \rho W^2 c$$

So we need to find W the magnitude of the relative velocity. To do this we consult a velocity triangle such as the one in Figure 3.2. From this triangle we can see that:

$$W = \sqrt{U^2 + V^2}$$

$$U = \omega r = 1 \times 2\pi \times 6 = 37.7\, m/s$$

V is the wind speed at 10 m/s so:

$$W = \sqrt{37.7^2 + 10^2} = 39\, m/s$$

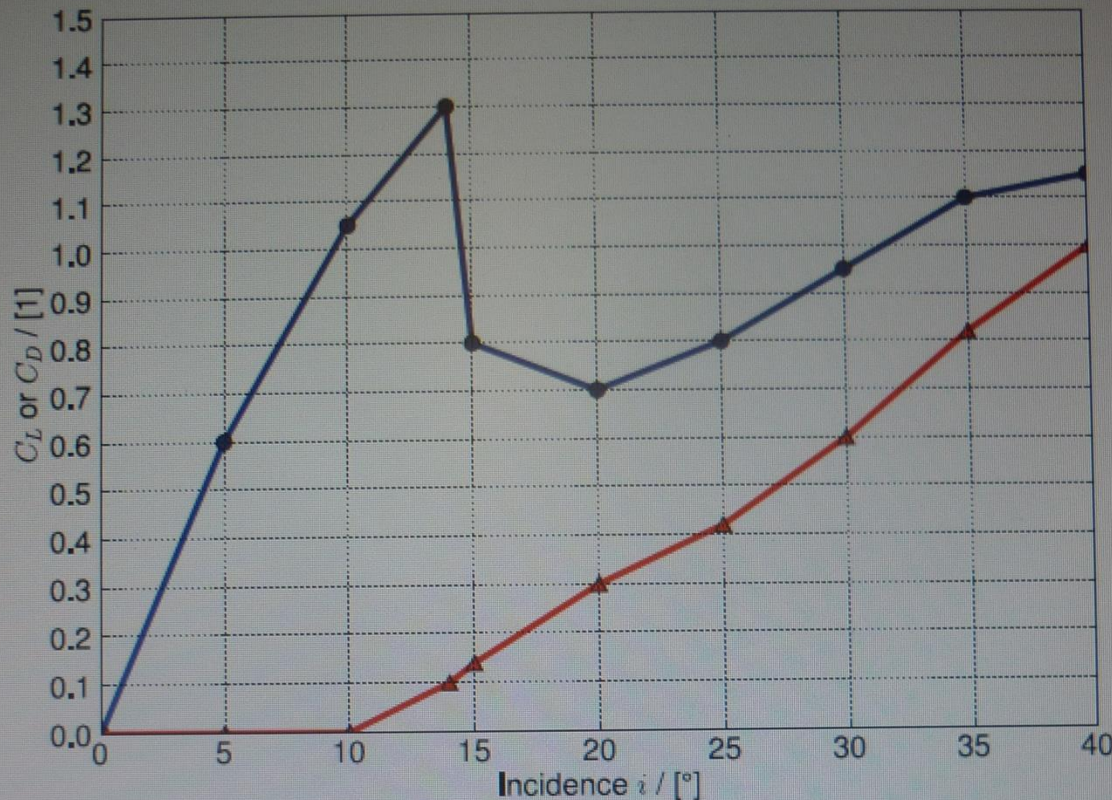

Figure 3.5: C_L and C_D for a NACA 0012 Aerofoil

The relative flow angle β may also be calculated:

$$\beta = \tan^{-1}\left(\frac{-U}{V}\right) = \tan^{-1}\left(\frac{-37.7}{10}\right) = -75.1^\circ$$

Figure 3.5: C_L and C_D for a NACA 0012 Aerofoil

The relative flow angle β may also be calculated:

$$\beta = \tan^{-1}\left(\frac{-U}{V}\right) = \tan^{-1}\left(\frac{-37.7}{10}\right) = -75.1^{\circ}$$

Note that the angle is negative as the sign convention in this book is that angles are positive in the direction of rotation. With the relative velocity calculated the lift force can now be estimated:

$$L = C_L \frac{1}{2}\rho W^2 c = 1.1 \times \frac{1}{2} \times 1.22 \times 39^2 \times 0.5 = 510\ N/m$$

The tangential force per unit span can be calculated:

$$F_\theta = L\cos\beta = 510 \times \cos 75.1^{\circ} = 131\ N/m$$

and finally the power output per unit span can be calculated

$$P = F_\theta \times \omega r = 131 \times 37.7 = 4939\ W/m$$

3.2 Wind Turbine Design

In wind turbine design the turbine designer has three primary variables that he or she can change:

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3.2 Wind Turbine Design

In wind turbine design the turbine designer has three primary variables that he or she can change:

1. The type of aerofoil to be used. In this book the NACA 0012 aerofoil is used for simplicity but there are actually a wind variety of aerofoils available many designed specifically for wind turbine use.

2. The chord length c along the span of the blade.

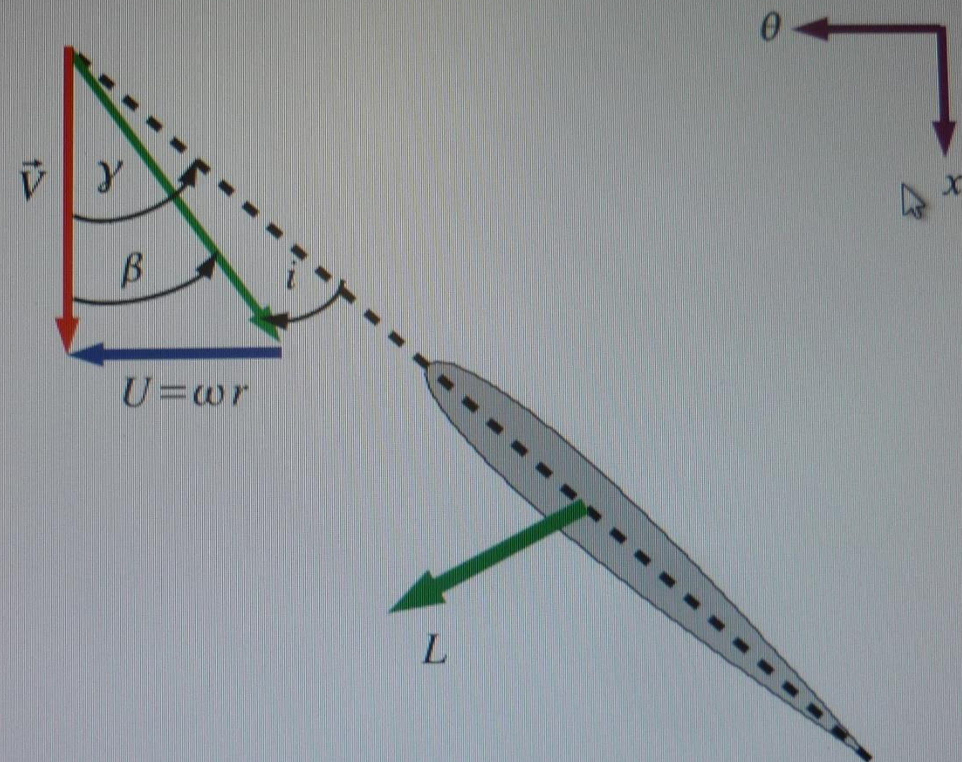

Figure 3.6: Relationship between γ and i

3. The angle that the blade is set to given the symbol γ, this is positive in the direction of rotation

Again using the velocity triangle to get the sign convention correct $\gamma = -50.1°$

3.3 Turbine Power Control

Too high a power output from wind turbines is actually very undesirable. Too high a power output can over-stress the blades causing structural failure and the generator in the turbine has only a finite capacity to absorb power. Two methods are commonly used to control the power output.

1. Stall Control. The blade is designed mechanically for maximum lift. At higher wind speeds, the blades will stall, causing a reduction in lift and hence tangential force. The drag will be increased significantly, which contributes to the axial force, so there is a need to ensure the structure can endure the increased axial loading. The drawback of this technique is that predicting the onset of stall and the flow around a stalled aerofoil is very difficult.

2. Pitch Control. The blade is provided with an actuated mechanism to vary the blade angle at different wind conditions. When the wind speed is too high, the blade angle is altered to reduce incidence. This is shown in Figure 3.7 in Case A on the left the wind turbine blade angle γ is set so that the incidence i is large. In Case B on the right the blade angle has been set so that the incidence i has been reduced. The inset in the bottom left of Figure 3.7 shows the two blade shapes superimposed to highlight the differences between them. In low wind conditions pitching can be used to increase the incidence and hence reduce the power output. The drawback here is that a complex actuation mechanism has to be provided.

clearer as the different machine types encountered in practise are described.

The final classification is that some turbomachines absorb power from the fluid (turbines) and some deliver energy to the fluid (compressors,fans or pumps).

4.1 Axial Flow Machines

A simple axial flow machine was used to introduce the idea of a turbomachine in Chapter 1 essentially an axial flow machine is one in which the fluid remains parallel to the axis of rotation as it passes through the machine. There are many examples of this type most aircraft engines use axial flow devices as do all large power station equipment and wind turbines are the most visually striking example.

The complication with axial flow machines compared to the simple one see in Figure 1.2 is that more than one stage may be present. This idea allows energy to be extracted in "small bites" from the machine allowing very large pressure ratios to be used in a single device - indeed it was this concept that is the key to the superiority of steam turbines compared to reciprocating engines. A single stage is sufficient when the fluid does not have a great deal of energy to be extracted for a turbine or when the requirement is for a modest pressure rise such as in most fans.

4.2 Radial and Centrifugal Flow Machines

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4.2 Radial and Centrifugal Flow Machines

To introduce the idea of a radial flow machine we will discuss a specific instance, that of a radial flow pump. A general arrangement drawing of the pump is shown in Figure 4.1, on the left the cross section of the pump is shown and on the right a plan view. Two components are shown in each view the outer casing also called the "volute" or "scroll casing" the aim of which is to ensure an even distribution of flow out of the machine, the area is therefore gradually increased up to the discharge pipe. The second component is the rotor also called the "impeller".

The operation of the machine is such that flow approaches the device through the inlet pipe in the axial direction, the fluid is then turned through 90° to the radial direction where it enters the impeller. The rotor then increases the angular momentum of the fluid and it exits in the radial direction into the volute. The pump shown in Figure 4.1 has no stator following the rotor, the fluid slows and experiences a rise in static pressure simply by the action of the increased cross sectional area of the

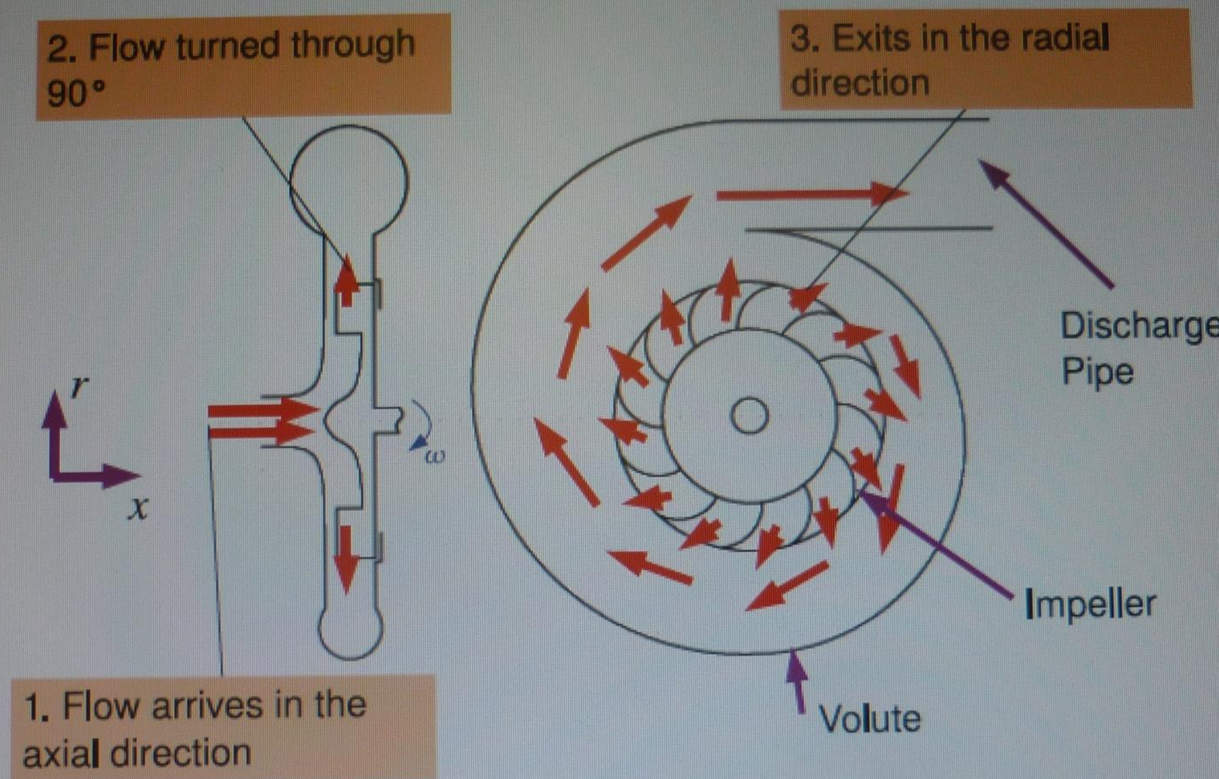


Figure 4.1: Radial Pump

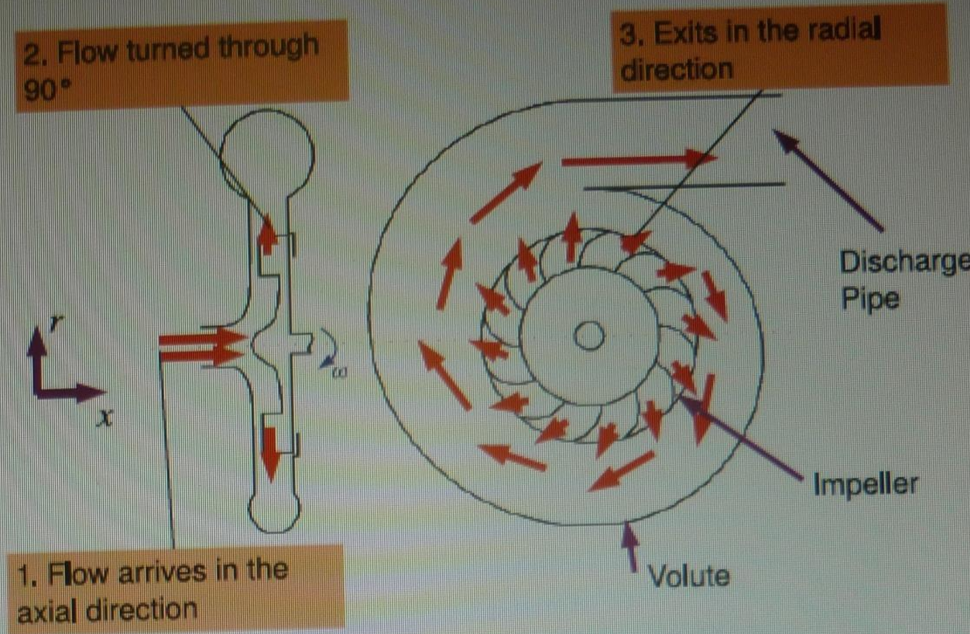


Figure 4.1: Radial Pump

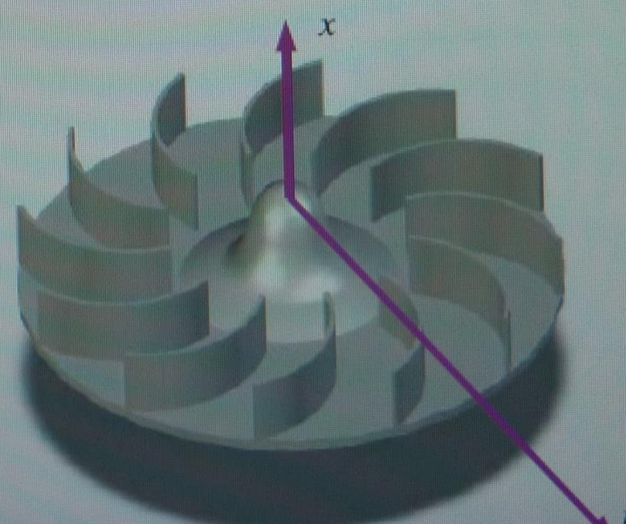


Figure 4.3: Centrifugal Impeller

widely used in small gas turbines such as the auxiliary power unit found on most aeroplanes and centrifugal designs and are used exclusively for turbochargers on internal combustion engines.

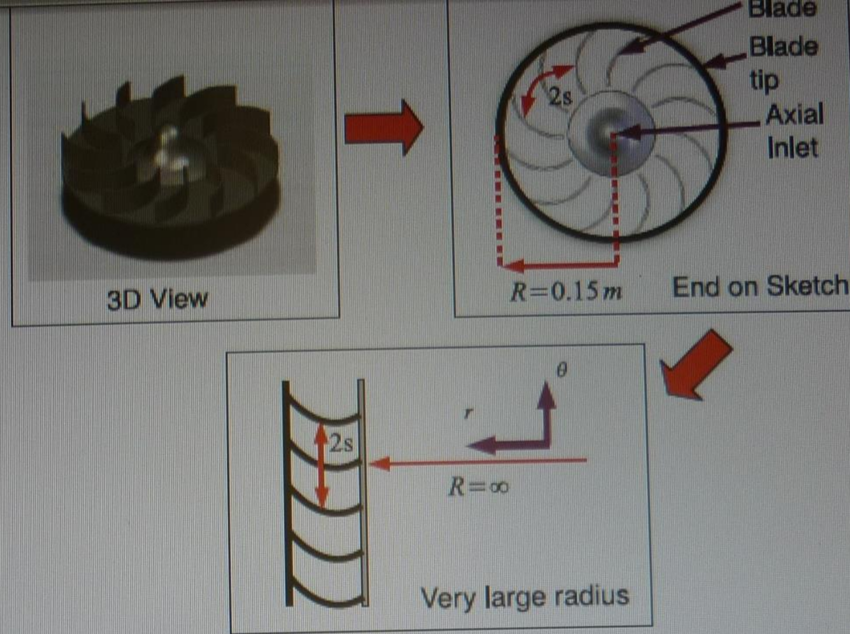
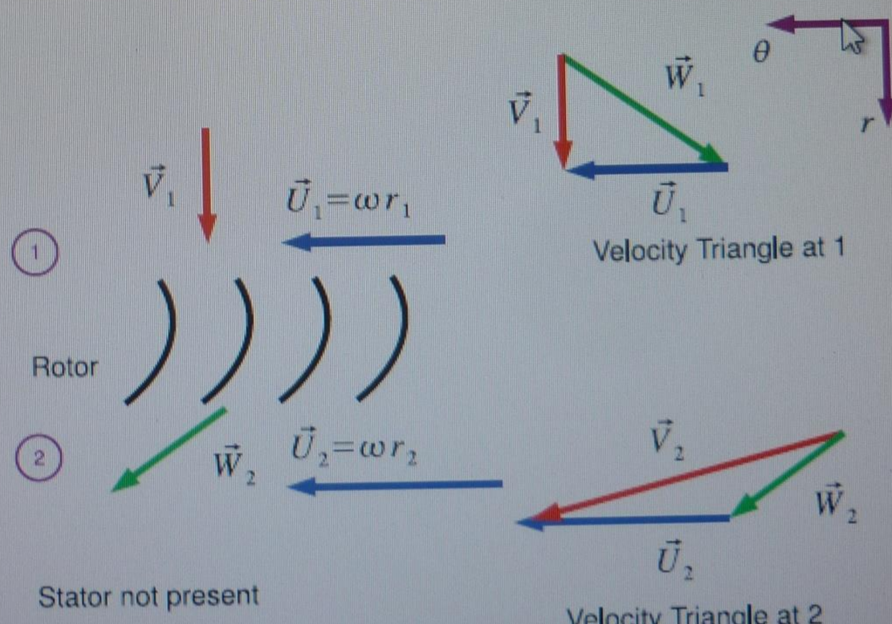


Figure 4.4: The Cascade View for a Radial Impeller



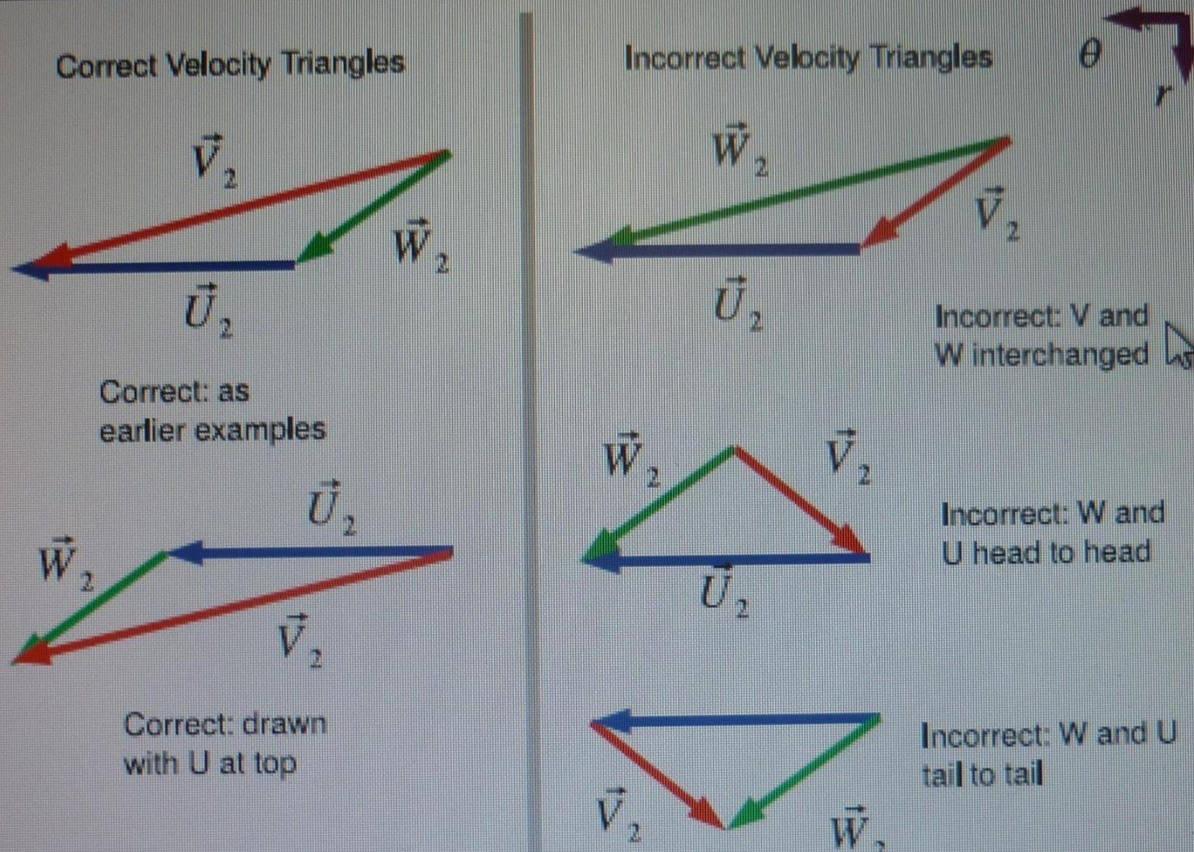


Figure 4.6: Common errors in Velocity Triangles

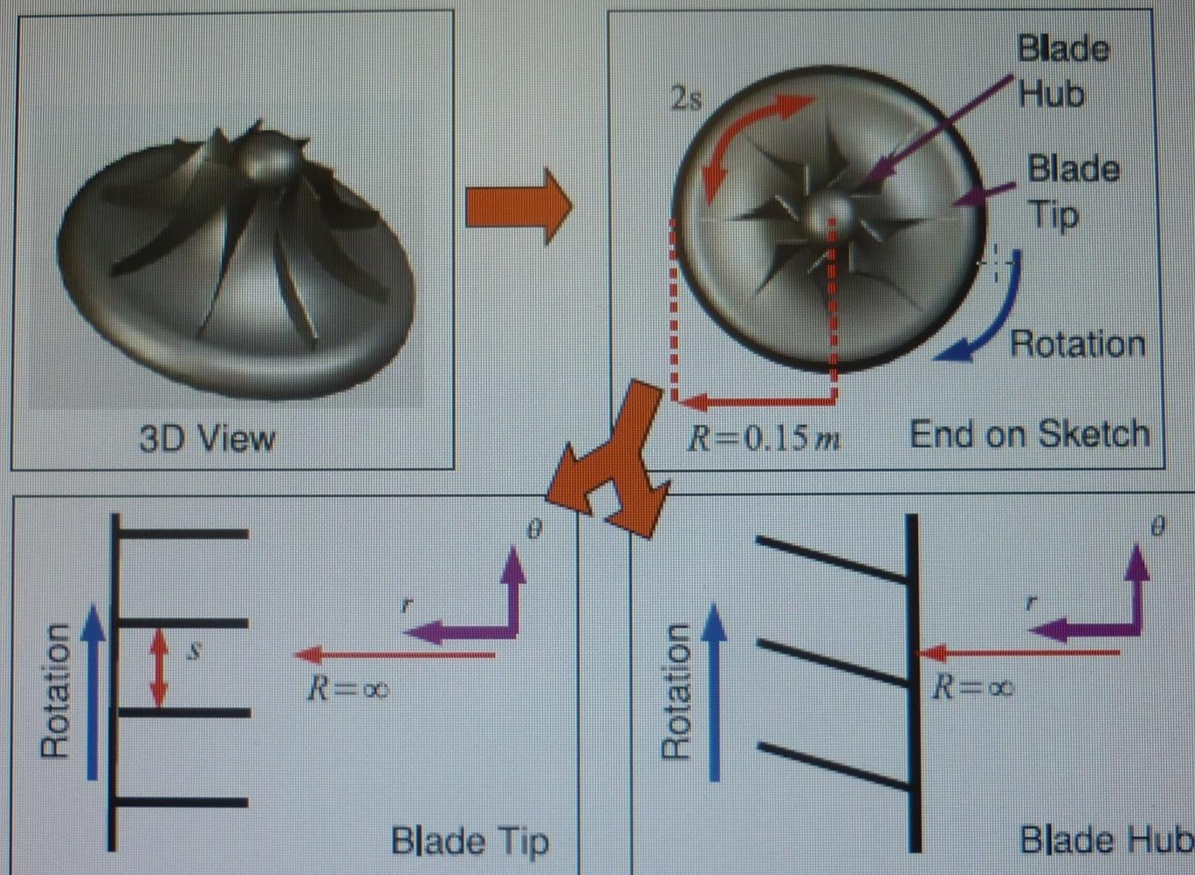


Figure 4.7: Constructing the Cascade View for a Centrifugal Impeller

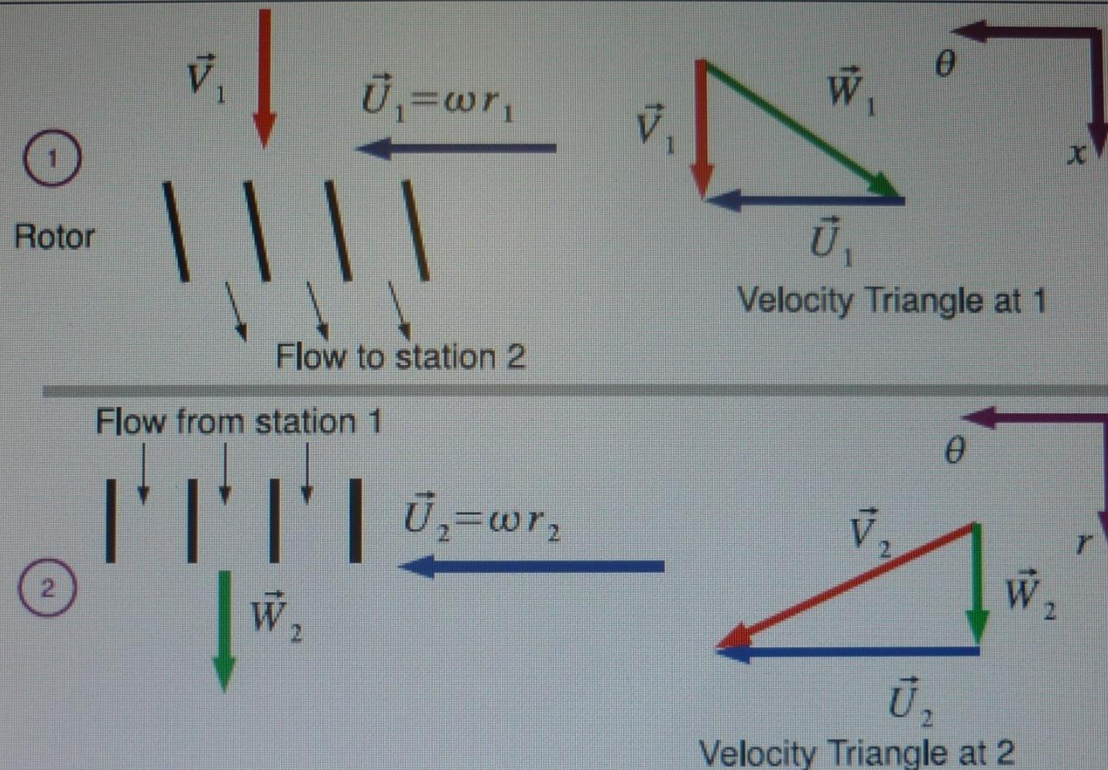


Figure 4.8: Velocity Triangles for a Centrifugal Impeller

Note that the analysis of radial flow machines is not restricted to pumps, the technique is equally applicable to hydraulic turbines, turbochargers and so on.

4.5 Hydraulic Turbines

Hydraulic turbines are devices for extracting energy from a reservoir of fluid usually obtained from rainfall above sea level. Water is collected in a reservoir at a height then is carried by a series of pipes to a turbine which discharges into a river (Figure 4.9). Hydraulic turbines are universally used to generate electricity and there are broadly three types of hydraulic machine in use: The Pelton wheel, Francis turbine and the Kaplan turbine, each of which were named after their inventors are are in common use around the world today.

Using the relationship $\Delta p = \Delta h \rho g$ it is possible to express any pressure in terms of height of a particular fluid. This is an approach commonly used in hydraulic turbines as pressures directly relate to the height of the reservoir or the height above river level that the turbine is situated at.

Hydraulic turbines are usually classified in terms of *total head*. For any position in a fluid system the total head is given by the following equation:

$$H = \frac{p}{\rho g} + \frac{V^2}{2g} + z \tag{4.1}$$

If we examine the total head at the dam of a reservoir (Station 0 in Figure 4.9) where the height datum is the river level at exit from the turbine. The surface area is very large so $V \approx 0$ and since the pressure is at atmospheric pressure $p = 0$. The total head is therefore given entirely by the height term z. At entry to the turbine however the height term $z \approx 0$ but the pressure and velocity terms will be much larger - essentially gravitational potential energy has been traded for pressure and velocity.

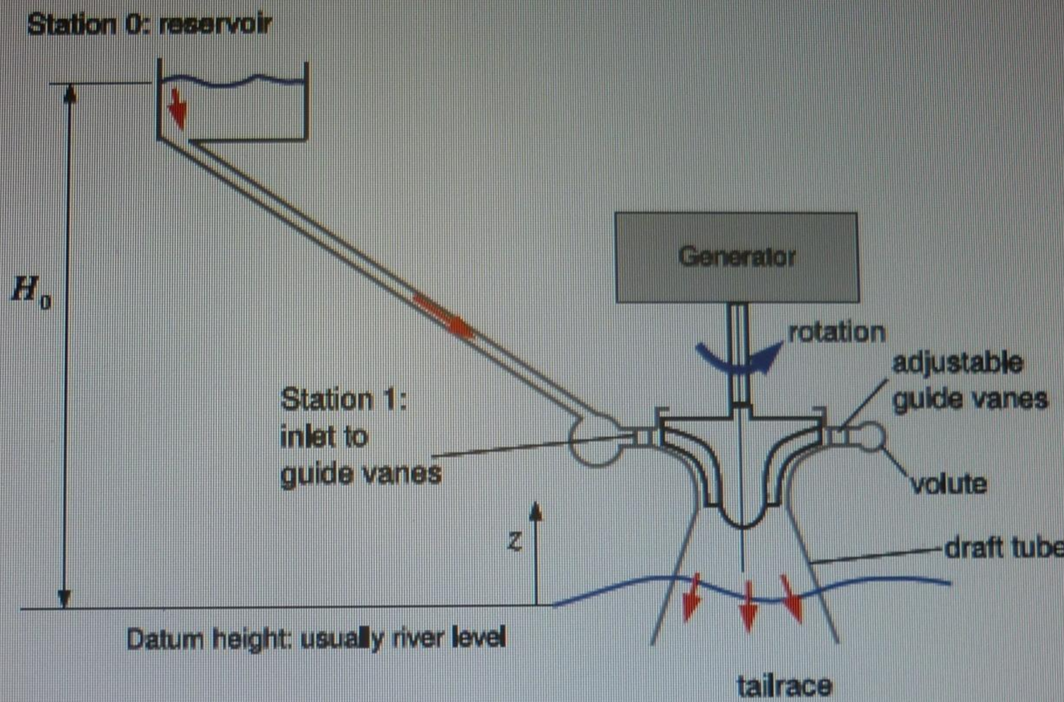


Figure 4.9: Schematic of Hydro-Electric Scheme

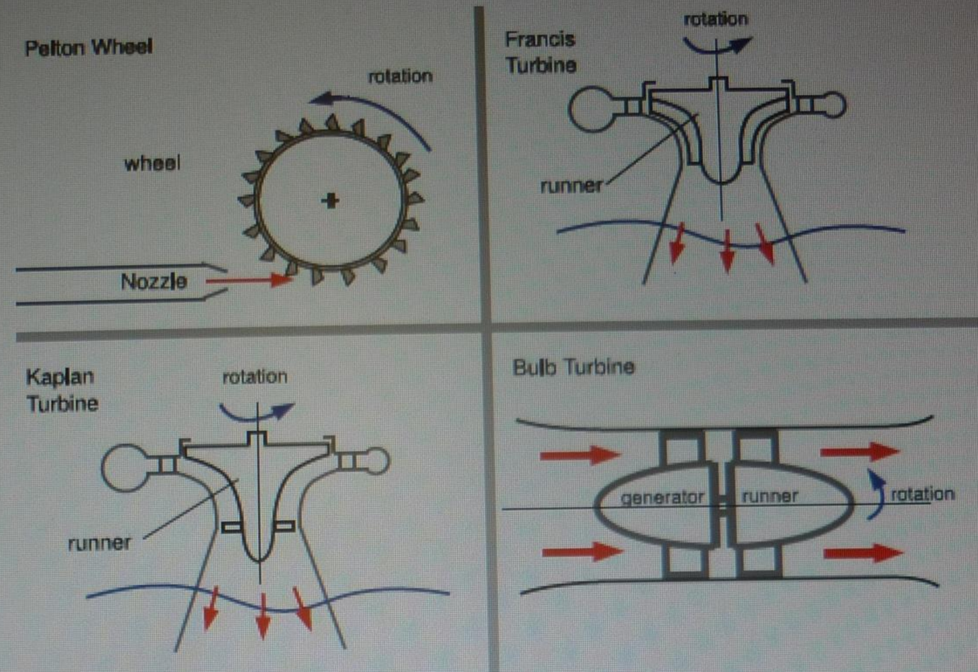


Figure 4.10: The Four Major Types of Hydraulic Turbine

Assuming that there are no losses the maximum velocity that would result from a nozzle placed next to the turbine open to the atmosphere can easily be found:

$$\frac{p_0}{\rho g} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

But $z_1 \approx 0$ and $p_0 = p_1 = 0$ and $v_0 \approx 0$ therefore:

$$z_0 = \frac{V_1^2}{2g} \implies V_1 = \sqrt{2gz_0}$$