in the top right wind turbine power of electricity production, in the bottom left the rotor of a steam turbine for power production and a water pump is shown in the bottom right.

1.4 A Simple Turbine

There are many variants of turbine, here we describe the operation of a simple turbine so you get a feel for what is going on. An outline of a turbine is shown in Figure 1.2. From this view all we know about the device is that flow goes into it and as if by magic the shaft turns and produces a torque.

If we look at the device in an exploded view (Figure 1.3) we see that as well as a number of covers and bearings there is a row of aerodynamically shaped objects that don't move followed by a row of aerodynamically shaped objects that provide the torque to the shaft.

The objects are known various as blades, buckets, nozzles, aerofoils or airfoils. In this book we will generally refer to them as blades. The row of stationary blades is known as a stator and the row of rotating blades connected to the output shaft is known as the rotor.

The basic mechanism of operation is as follows (Figure 1.4):

1. the fluid flows directly into the device in an axial direction (in line with the machine)
2. the stator blades turn the flow so that it is lined up with the turbine blades
3. the turbine blades turn the flow back towards the axial direction and turn the output shaft.

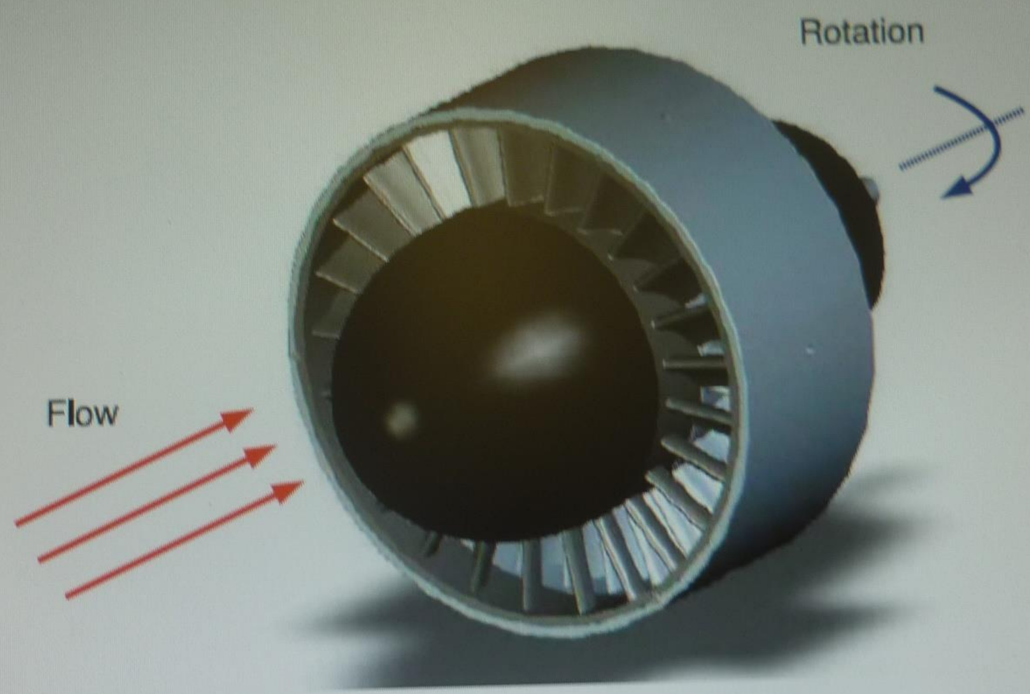

Figure 1.2: A Simple Turbine

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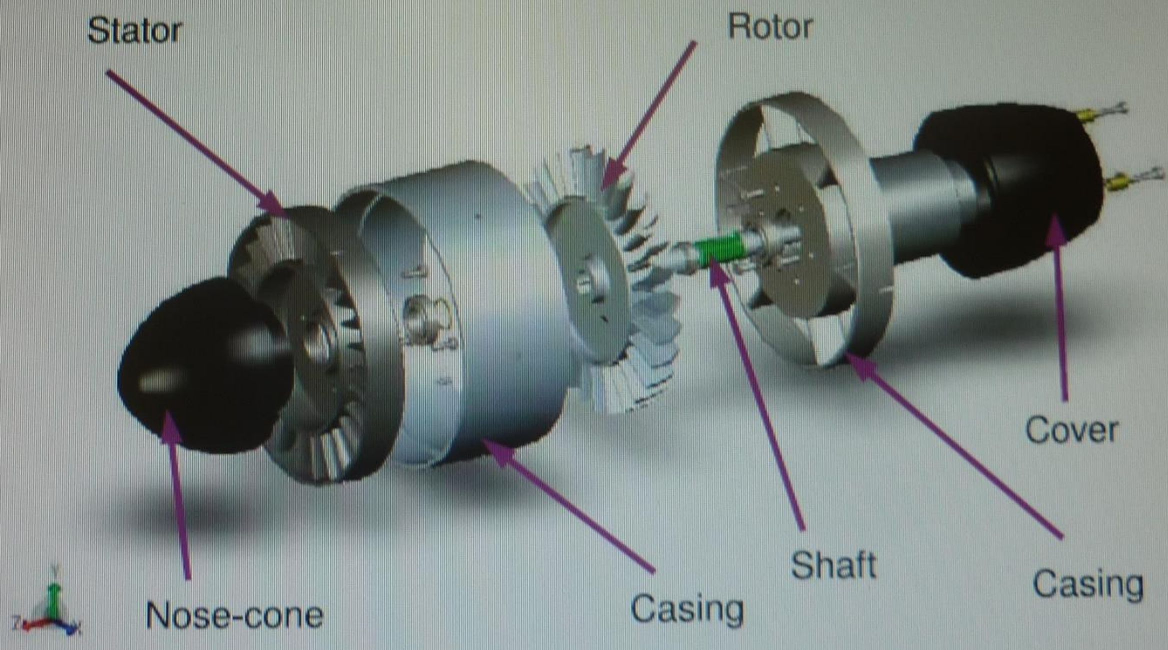


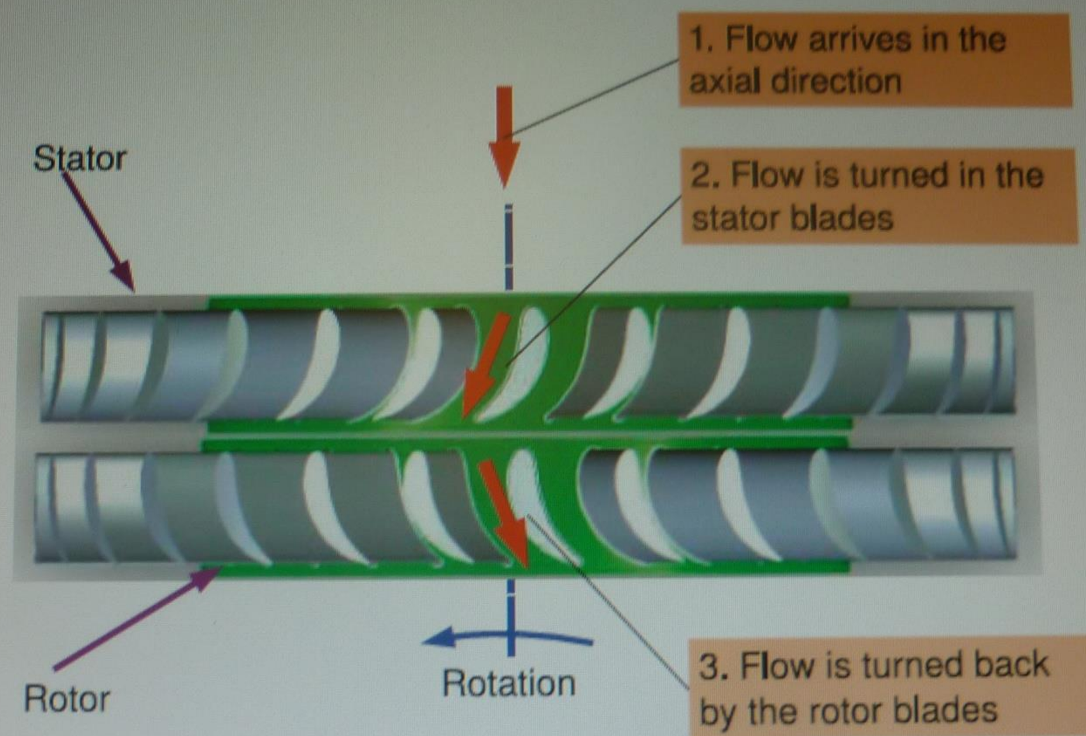


Figure 1.4: Simple Turbine Operation

Figure 1.4: Simple Turbine Operation

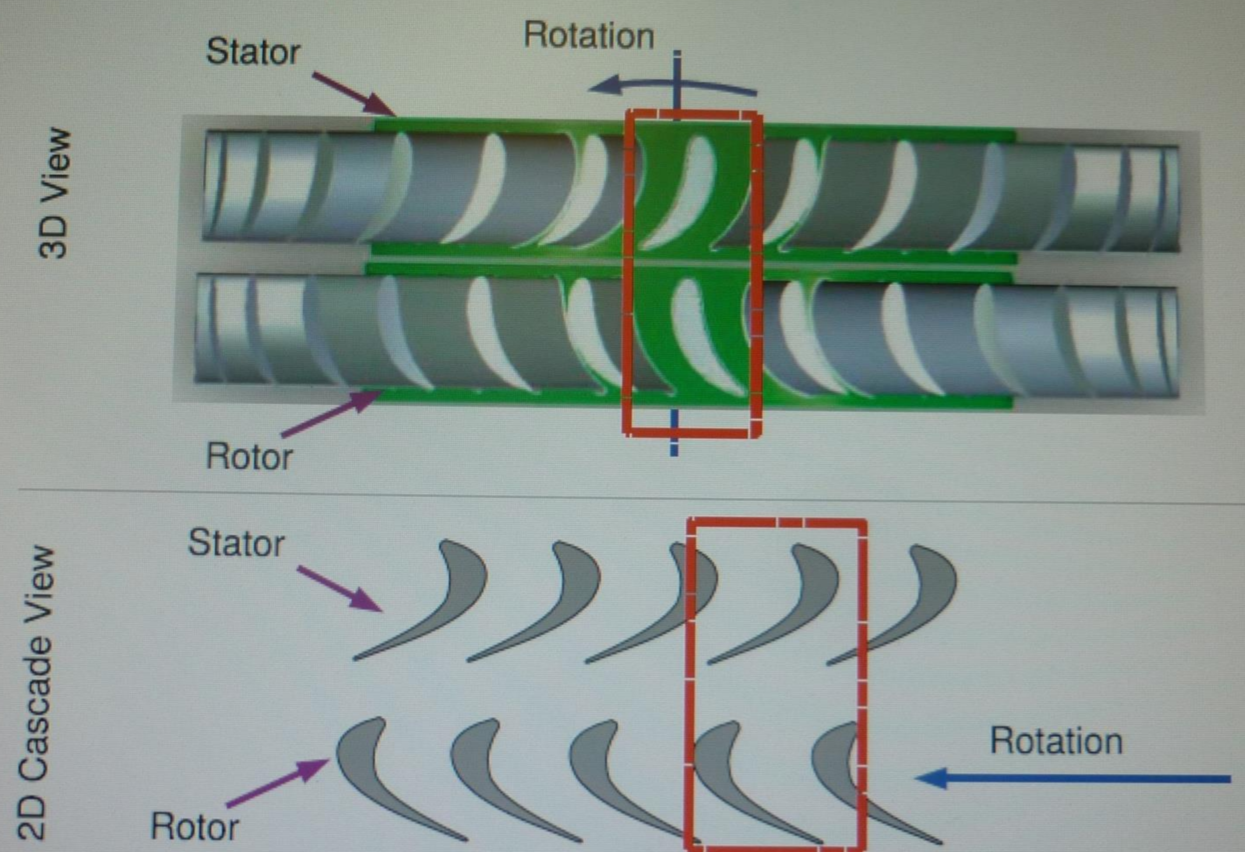


Figure 1.5: Cascade View

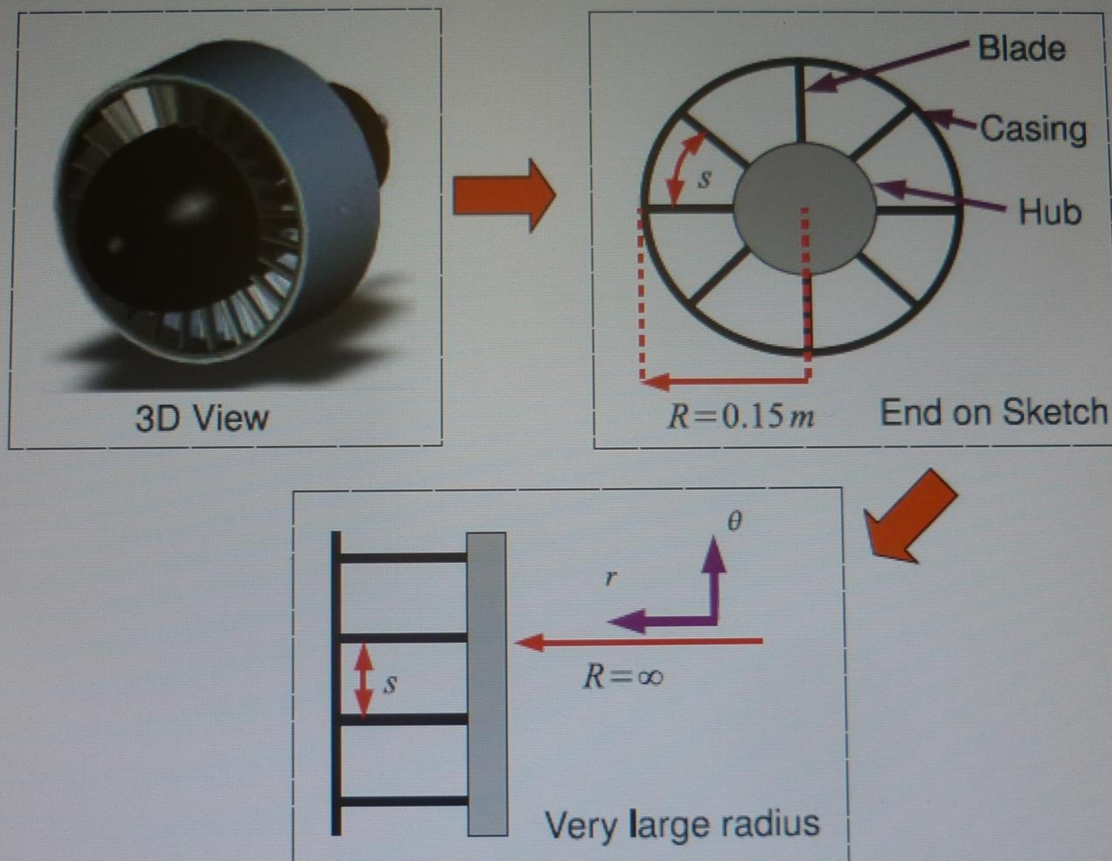


Figure 1.6: The Cascade View as a Large Radius Machine

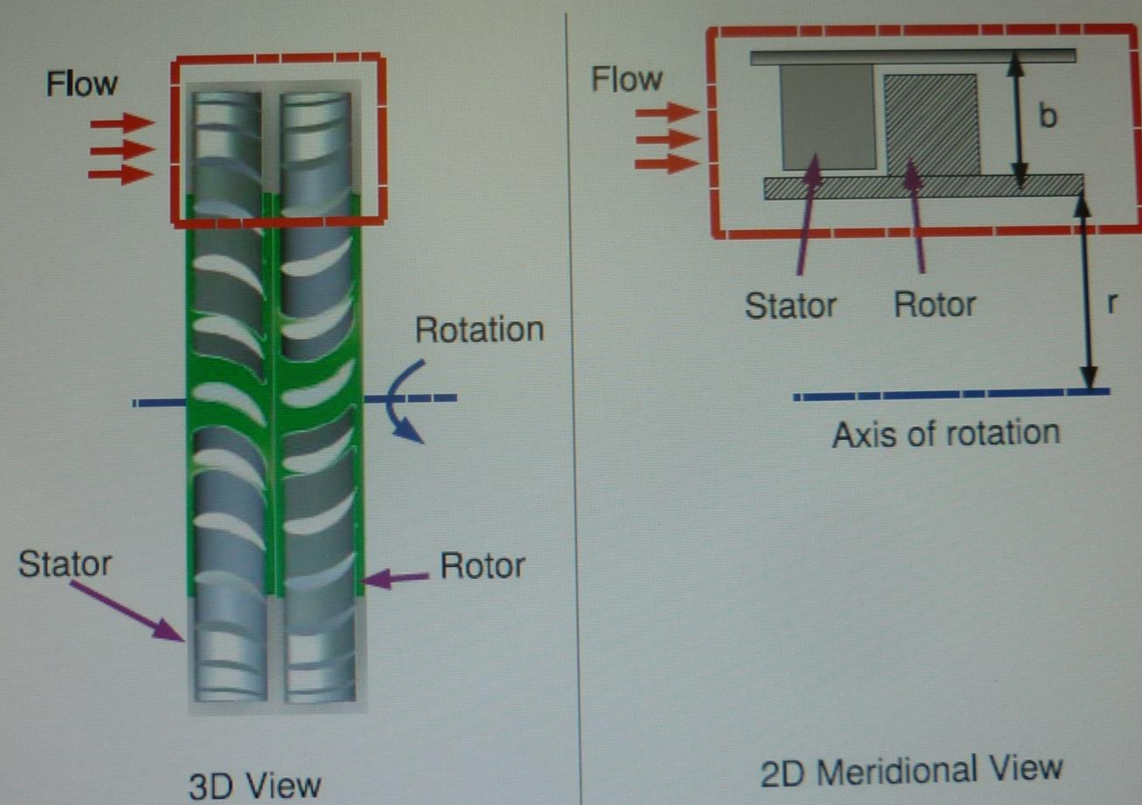


Figure 1.7: Meridional View

1.6 The Meridional View

The meridional view is much more straightforward than the cascade view and is illustrated in Figure 1.7. On the left of Figure 1.7 is the familiar 3D view of our simple turbine. For the meridional view instead of looking at the tip of the blade this time we take a side on view of the whole turbine and look at a cross section of the machine at the hub and tip radius. This is highlighted by a red box. On the right of Figure 1.7 is the actual meridional view which shows the stator followed by the rotor in cross section. The actual machine radius r is usually very large compared to the blade height b and so the axis of rotation is not always shown in the meridional view.

1.7 Assumptions used in the book

It is easy to see how the real turbomachinery flow field is three dimensional and unsteady now that

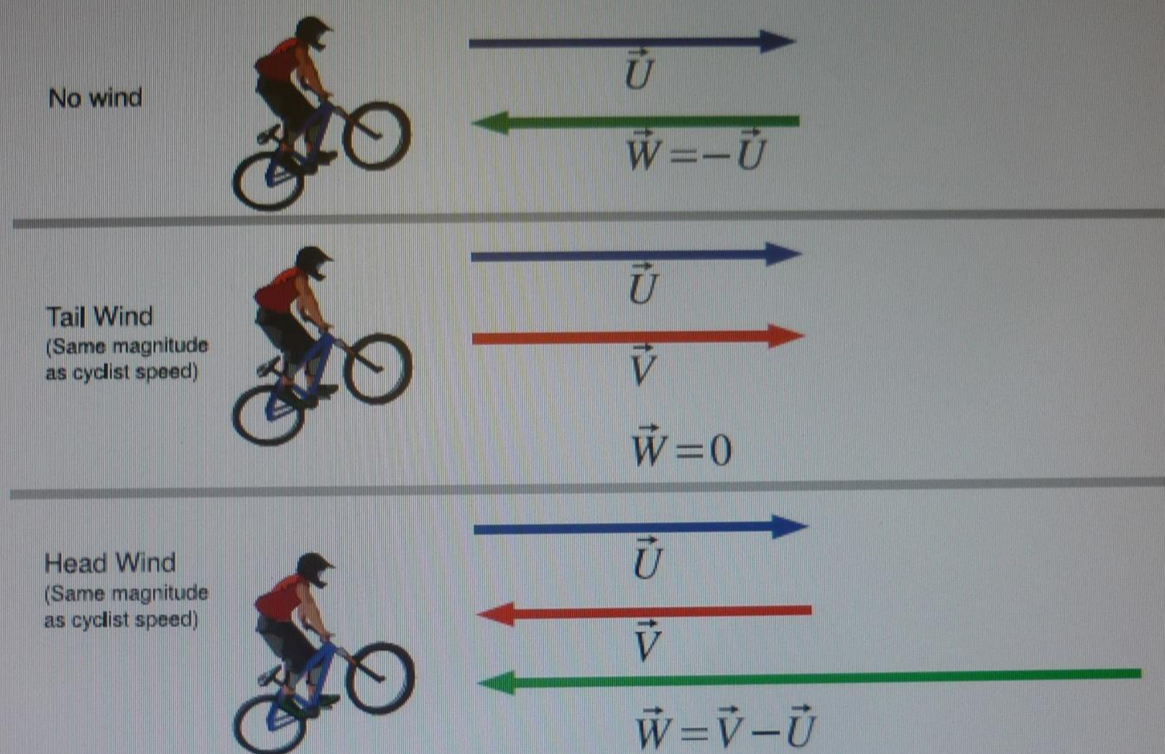


Figure 2.1: Relative and Absolute Velocities for a Cyclist

2.1 1D Motion

2.1 1D Motion

Consider the everyday activity of riding a bicycle with three cases one where there is no wind, the second with a tail wind and a third with a head wind. This is shown in Figure 2.1. The velocity of the bicycle we shall label $\vec{U}$ and call it the "frame velocity", the velocity of the wind we label $\vec{V}$ and call this the "absolute velocity". Clearly the absolute velocity $\vec{V}$ is the velocity that will be experienced by an observer watching the cyclist. The wind velocity experienced by the cyclist is called the "relative velocity" and given the symbol $\vec{W}$.

The first case shown at the top of Figure 2.1 shows the simplest case, if there is no wind the observer watching the cyclist will experience no wind and the cyclist will experience a relative velocity that is equal and opposite to that of the speed at which he or she is cycling. So the relative velocity $\vec{W} = -\vec{U}$.

The second case concerns a tail wind that is roughly equal in magnitude to the speed of the bicycle $\vec{U}$. This is shown in the middle of Figure 2.1. In this case a stationary observer would experience the wind velocity but since the cyclist is moving at the same speed as the air the relative velocity $\vec{W}$ will be around zero and the cyclist will experience no wind.

The third case concerns a head wind that is again roughly equal to the velocity $\vec{U}$ of the bicycle in magnitude but not in direction. This is shown at the bottom of Figure 2.1. A stationary observer would experience the same wind velocity as in the second case but in a different direction. The cyclist however has a very different experience. The relative velocity is made up of their own speed $-\vec{U}$ (that of the first case) added to that of the oncoming wind $\vec{V}$. By inspection we can see that $\vec{W} = \vec{V} - \vec{U}$. Since $\vec{V}$ is negative the cyclist now has to work much harder to maintain the same forward speed.

This suggests a generalisation of the relationship between relative and absolute velocity:

$$\vec{V} = \vec{U} + \vec{W} \tag{2.1}$$

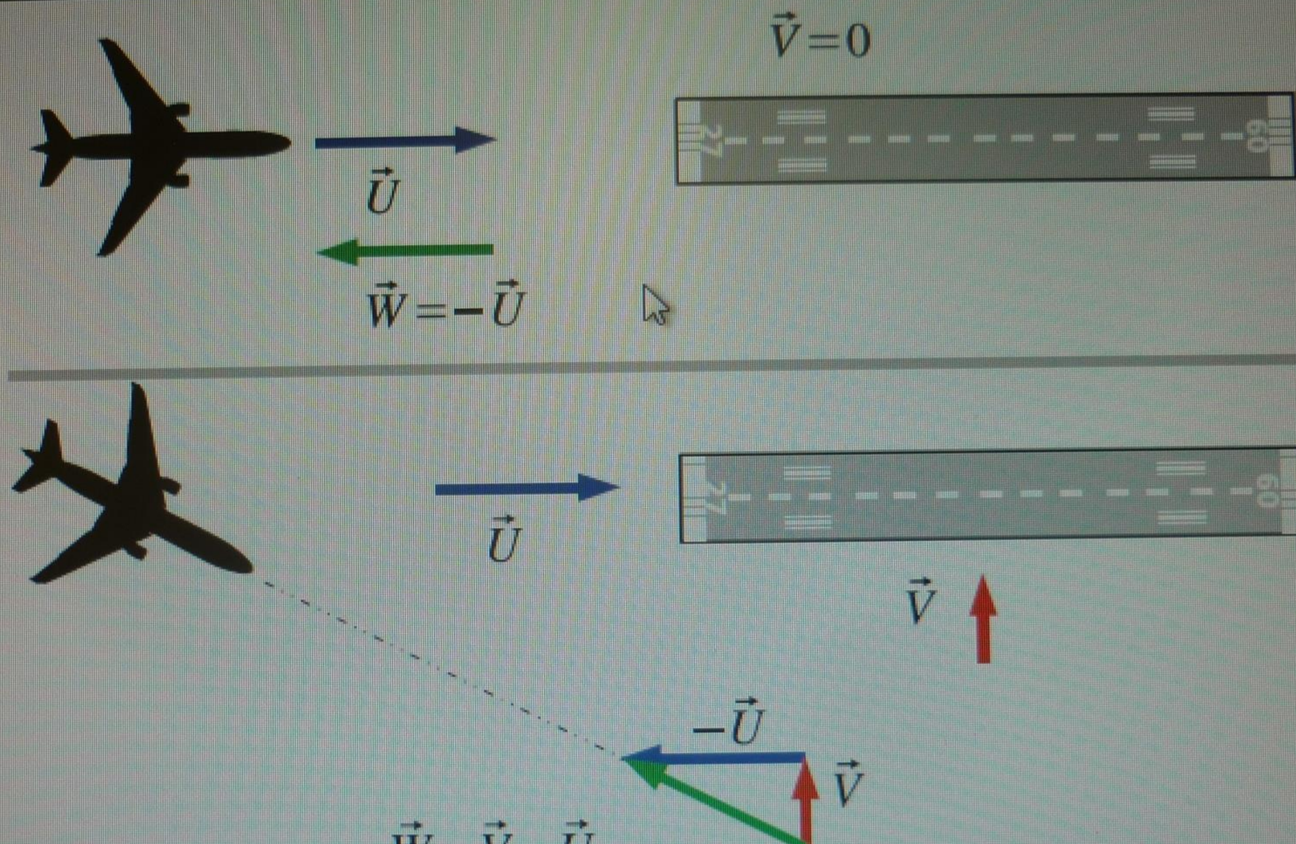


Figure 2.2: Velocity Triangles for an Aircraft Landing

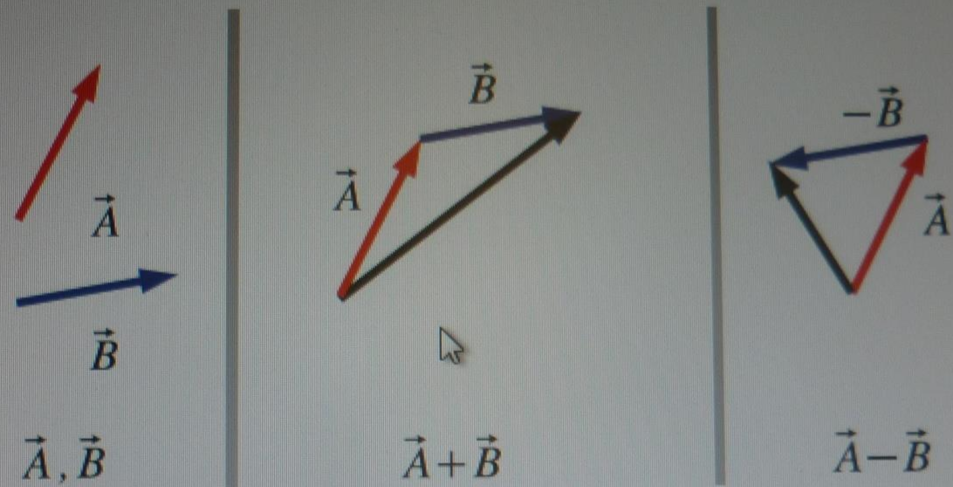


Figure 2.3: Graphical Addition and Subtraction of Vectors

- To subtract two vectors $\vec{A}-\vec{B}$ graphically: reverse the direction of $\vec{B}$ then proceed with addition of vectors as before.

same result if we drew the triangle with $-\vec{U}$ then $\vec{V}$.

All this may seem obvious but it is vitally important before we move onto turbomachinery that you are confident in how to draw a 2D vector and how to add and subtract vectors graphically.

2.3 Velocity Triangles in Turbomachinery

In this book we consider a Cartesian coordinate system consisting of an axial x, radial r and tangential θ set of coordinates. The velocity of the frame of motion is denoted by $\vec{U}$, velocities in the frame of motion are denoted with $\vec{W}$ and absolute velocities are denoted with $\vec{V}$. Consider a turbine consisting of a stator and a rotor, the cascade and meridional views are shown in Figure 2.4 along with the coordinate system.

There are three points that are of interest to us entry to the stator, the gap between the stator and the rotor and exit from the rotor, these are labelled 1,2 and 3 respectively in Figure 2.4. The

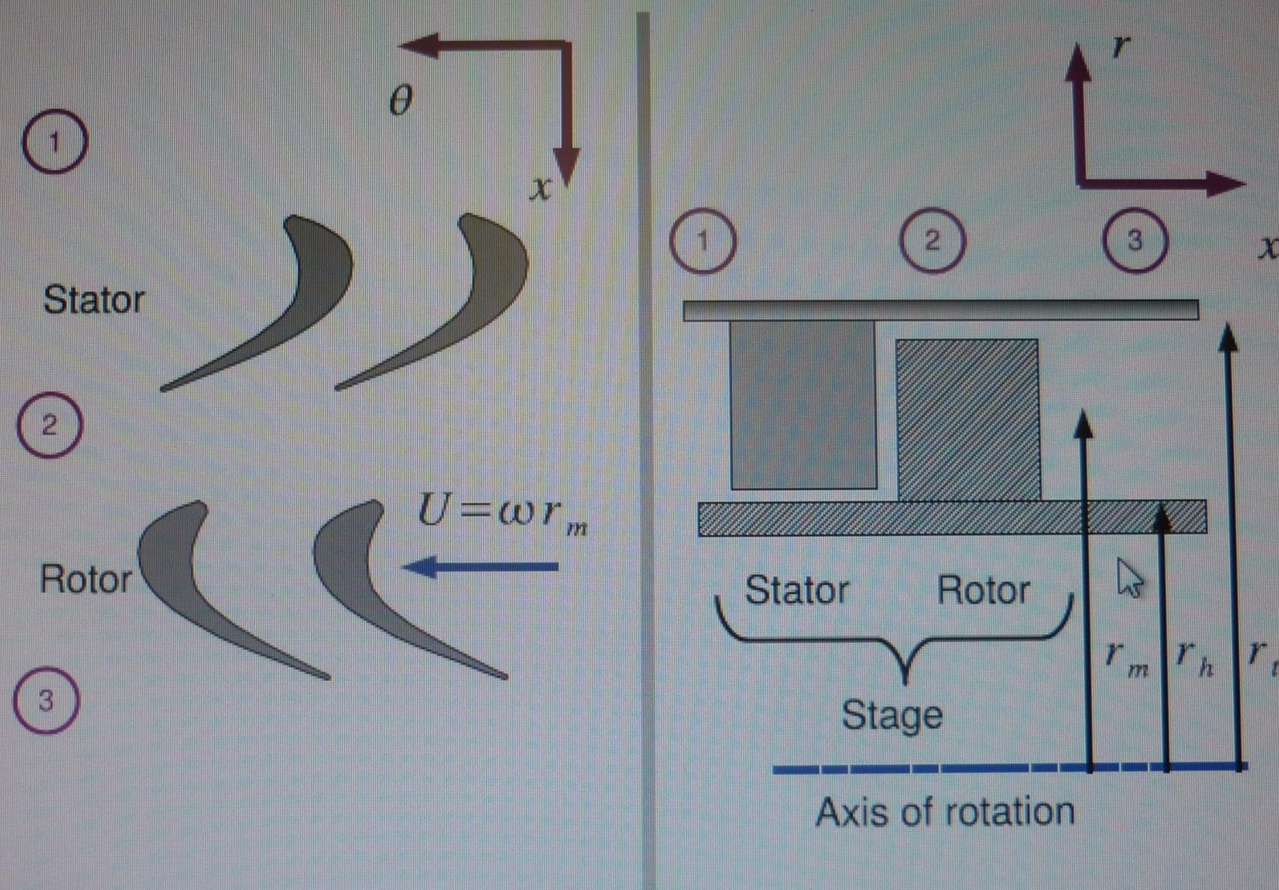


Figure 2.4: Cascade and Meridional Views of a Turbine Stage

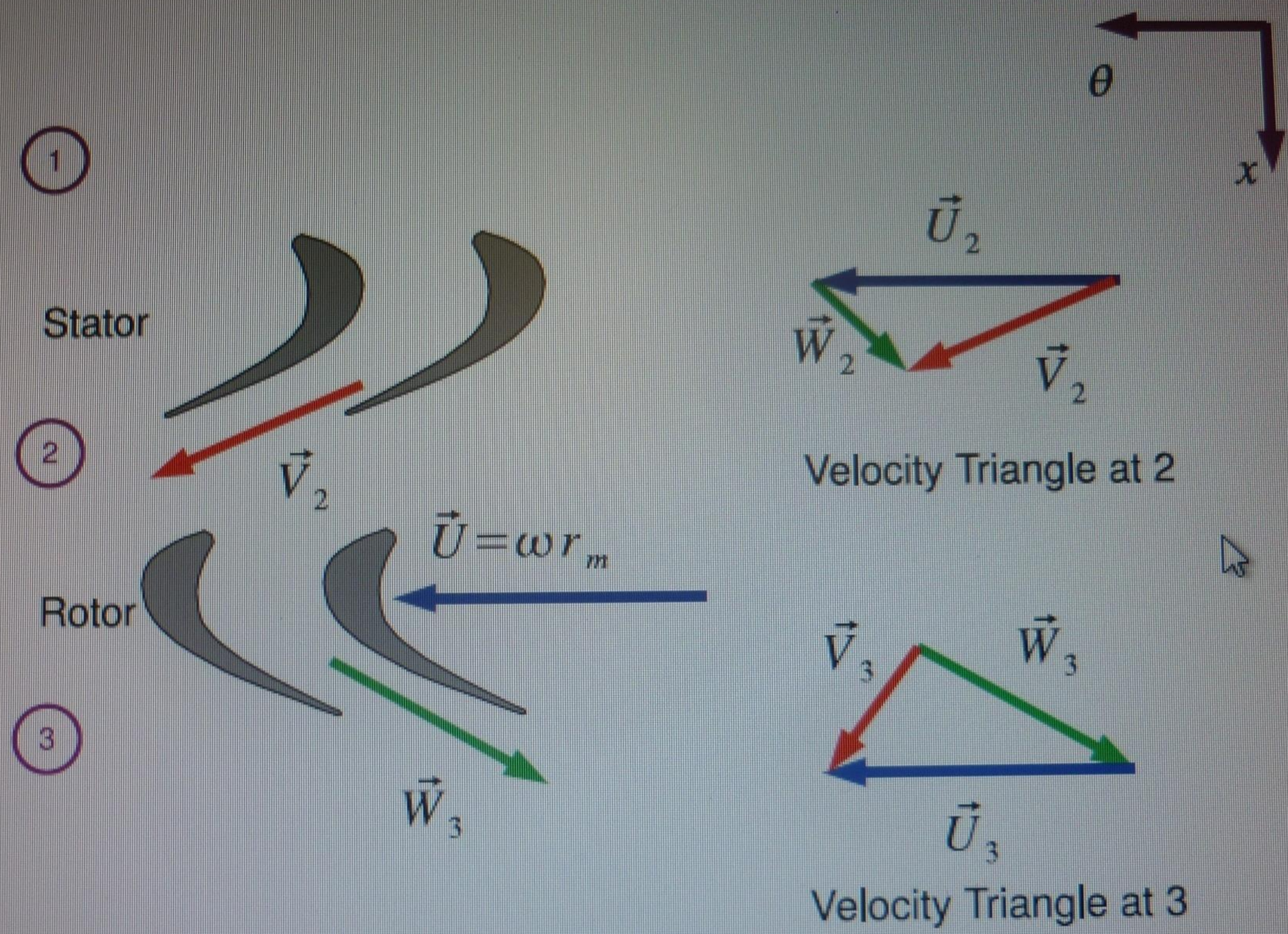


Figure 2.5: Velocity Triangles for a Turbine Stage

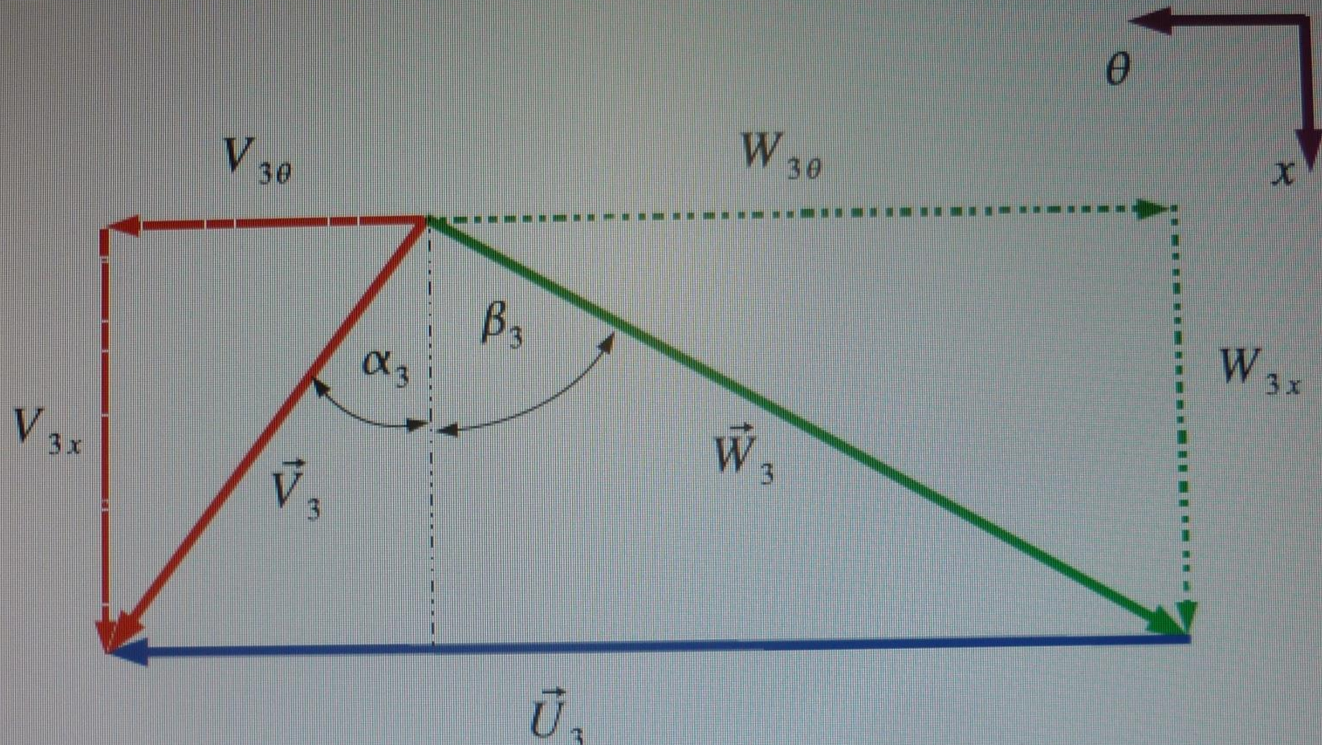


Figure 2.6: Velocity Triangles at Station 3 of a Turbine

Figure 2.6: Velocity Triangles at Station 3 of a Turbine

tangential plane and angles are measured from the radial direction. This will become clearer when radial and centrifugal machines are explained in Chapter 4.

The velocity triangle at station 3 in Figure 2.5 is shown in Figure 2.6 with the various components labelled, in order to indicate that we are dealing with station 3 a subscript 3 is added to all the symbols. The relative and absolute flow angles α and β are also shown.

From basic trigonometry the follow relationships apply for any station in a turbomachine.

$$V^2 = V_\theta^2 + V_x^2 \tag{2.3}$$

$$V_x = V \cos\alpha \tag{2.4}$$

$$V_\theta = V \sin\alpha \tag{2.5}$$

$$V_\theta = V_x \tan\alpha \tag{2.6}$$

$$W^2 = W_\theta^2 + W_x^2 \tag{2.7}$$

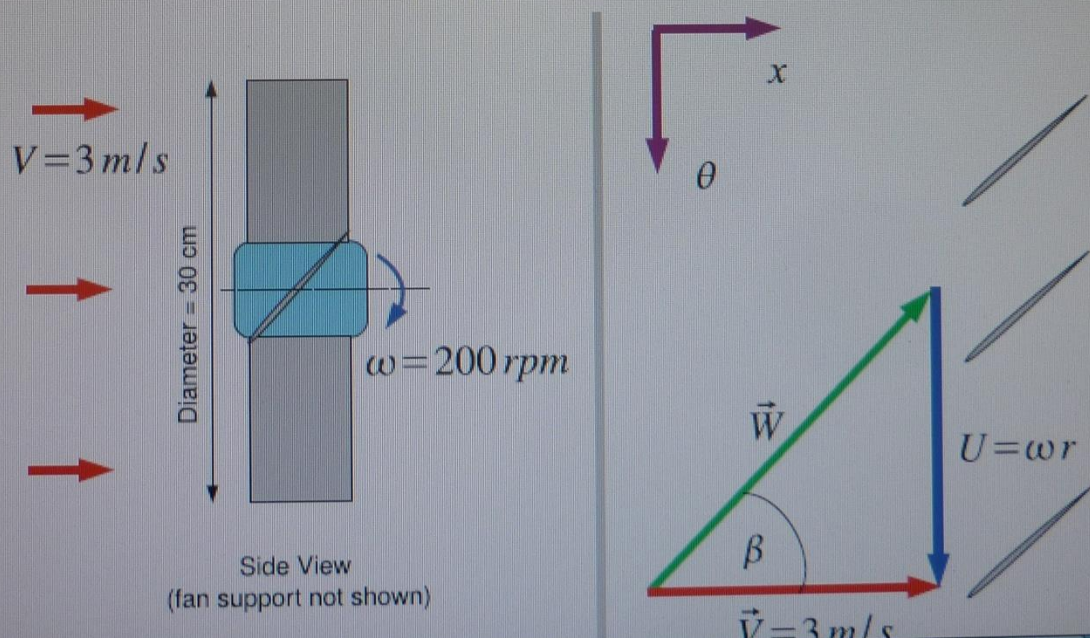
$$W_x = W \cos\beta \tag{2.8}$$

$$W_\theta = W \sin\beta \tag{2.9}$$

$$W_\theta = W_x \tan\beta \tag{2.10}$$

Aside from trigonometry we can also work out that $W_x = V_x$ for all turbomachinery. The reason

Example Consider an Office Desk Fan. It rotates at 200 rpm and has a diameter of 30 cm. Air enters the fan at 3 m/s, parallel to the axis of rotation. Calculate the relative velocity ($\vec{W}$) at the tip of the fan.



Barometer has limited applications. It is widely used to measure atmospheric pressure. However, the principle could be used to measure pressure relative to the atmospheric pressure or pressure difference between two points. For this purpose, a U-tube partially filled with mercury is used. When the both ends of the U-tube are open to the atmosphere, the mercury column balances giving the same height in both arms of the U tube (see Figure 2.7(a)). When one end is open to the atmosphere and the other end to a vessel with a pressure different from the atmosphere, the mercury column moves to a new equilibrium position giving a height difference in the U-tube as shown in Figure 2.7 (b).

Suppose the fluid in the bulb has a density ρ_1 and the density of mercury to be ρ_M. Furthermore, assume the pressure inside the bulb to be P_0. Once the mercury column attains equilibrium, a simple force balance at a point just inside the static mercury meniscus will give

$$P_0 + h_1\rho_1 g = P_{atm} + h_2\rho_M g$$

Rearranging terms gives

$$P_0 - P_{atm} = h_2\rho_M g - h_1\rho_1 g \tag{2.14}$$

Manometers could be used to measure the pressure difference between two points. Consider an arrangement as shown in Figure 2.8. A U-tube partially filled with a heavier liquid, mercury in most cases, connected to a pipe across a restriction in the pipe. Density of the fluid in the pipe is ρ_L and the density of the heavy liquid is ρ_M. Pressure at two tapings to which the manometer arms are connected are P_1 and P_2 ($P_1 > P_2$).

Chapter 3

Simple Analysis of Wind Turbines

This chapter provides an immediate application of the principles of relative motion by using the horizontal axis wind turbine as an example. Such a wind turbine is shown in Figure 3.1 as can be see from Figure 3.1 the blades are far apart so the influence between them is very small. In turbomachinery jargon the pitch s is very large. The interactions between the different blades can be ignored in a simple treatment. The wind turbine is one of the only examples in turbomachinery where each blade can be considered in isolation and this is one of the reasons that a simple analysis is easy.

Consider one of the turbine blades shown in Figure 3.1, the speed of each of the three blades will be the same and assuming that the wind does not vary over the area of the machine, an analysis need only be conducted on a single blade and multiplied as necessary. The interaction of the blades and the tower is ignored but since this occupies a small fraction of the $360°$ rotation of the turbine this

Real Wind Turbine

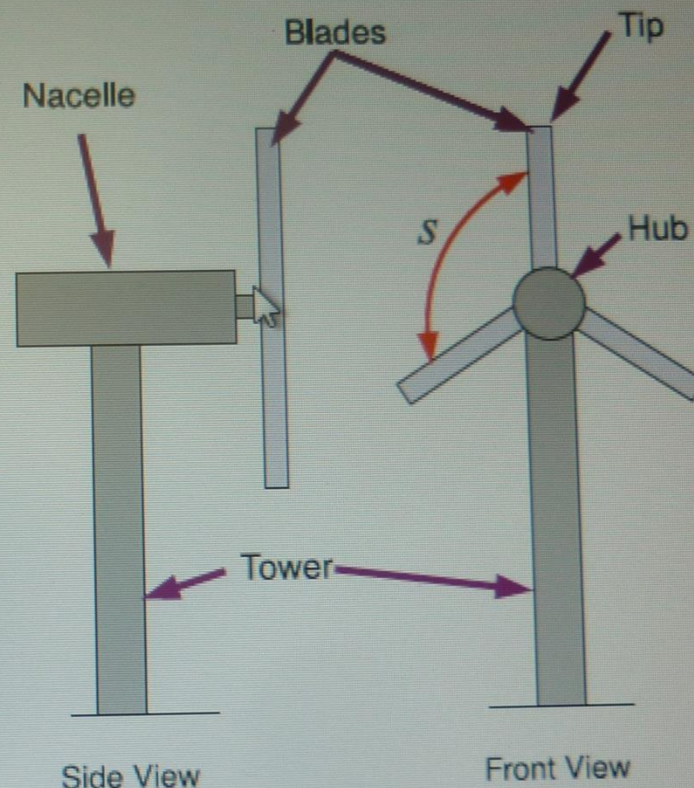


Figure 3.1: Wind Turbine Picture and Sketch

will not influence the analysis greatly.

Consider what happens if an observer was positioned on the turbine blade about half way along the span and a virtual cut made through the blade. If the observer looked towards the hub of the machine, they would see an aerofoil profile rotating around the hub. Since the radius of the machine is large the rotational motion of the turbine blade can be approximated by a linear motion - in much the same way as the earth is round but looks flat as the radius is very large. Such a view of a turbine blade is shown in Figure 3.2. The rotational motion of speed ω is translated to a linear motion of ωr the tangential velocity of the rotating blade. If the turbine is facing into the wind the incoming wind $\vec{V}$ will be perpendicular to the rotating blade.

The velocity triangle for the wind turbine blade can then be drawn according to the four step procedure and is shown on the left hand side of Figure 3.2. The flow that is known is the incoming wind velocity $\vec{V}$ which is in the absolute frame of reference. The blade speed, $\vec{U}$ is then drawn and the triangle is closed by the relative velocity $\vec{W}$. The triangle can then be checked by following the relative velocity vector and the blade speed to ensure that the same point is arrived at if $\vec{V}$ alone was followed.

The blade therefore experiences a velocity of magnitude W and angle β which will produce a force on the blade.

Recall that from basic mechanics that a force in two dimensions can be resolved into two perpendicular components of any orientation. Two particularly convenient directions are found to be perpendicular and parallel to the incoming flow. The force perpendicular to the incoming flow is known as the lift force L and the force parallel to the incoming flow is known as the drag force D. The principle reason that these are useful directions is that a large body of data on aerofoil performance is available in this form.