Figure 6.1: *The ladder of Example 6.4 resting against a wall.*

Let θ be the angle the ladder makes with the horizontal. From the dimensions given, $\cos\theta = 2.5/6.5 = 5/13$, and θ is an angle in a $5, 12, 13$ Pythagorean triangle with $\sin\theta = 12/13$, $\tan\theta = 12/5$. The simplest moment balance comes from taking moments about the base of the ladder, which will eliminate both N and F_f from the resulting equation. Assuming the centre of gravity of the ladder to be at its mid-point, and balancing the moment about B of the ladder's weight with the moment of the reaction at the wall, R

$$200 \times \frac{1}{2} \times 6.5\cos\theta = R \times 6.5\sin\theta\ ,$$

giving $R = 200 \times 1/2 \times 5/12 = 41\frac{2}{3}$ newtons. And, resolving vertically and horizontally,

$$N = 200 \text{ newtons}\ ,\ F_f = R = 41\tfrac{2}{3} \text{ newtons}\ .$$

Since $\mu \geq F_f/N$, the limiting value for μ is $41\frac{2}{3} \div 200 = 5/24$, about 0.21.

Example 6.5

If, in *Example 6.4*, the actual value of μ is 0.35, how far up the ladder could Mr B climb before it starts to slip, if his weight is 800 N?

This looks a more complicated problem than *Example 6.4*, but the same equations apply. We need only to add in the extra contribution of Mr B. If he climbs a distance x up the ladder, the moments equation becomes (see *Figure 6.2*)

$$0.5 \times 200 \times 6.5\cos\theta + 800 \times x\cos\theta = R \times 6.5\sin\theta\ .$$

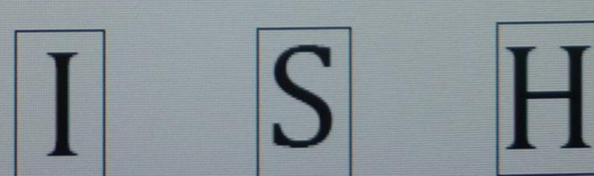
7 Centres of gravity

7.1 Using symmetry

In the examples of *Chapter 6*, we have assumed that the weights of objects like a straw bale or ladder act at centres of gravity which we take to be at their geometric centres. For rods and rectangles, cubes and circles, and many other shapes, the centre of gravity is obvious from the symmetry. Where they apply, arguments from symmetry are very pleasing and economical.

Example 7.1

Mr B, constructing a shop sign, wishes to determine the centres of gravity of large letters F, I, S, H, cut out from rectangular sheets of uniform density. He concludes that, because of symmetry, the centres of gravity for the letters I, S, H, must lie at the centre of their respective rectangles.

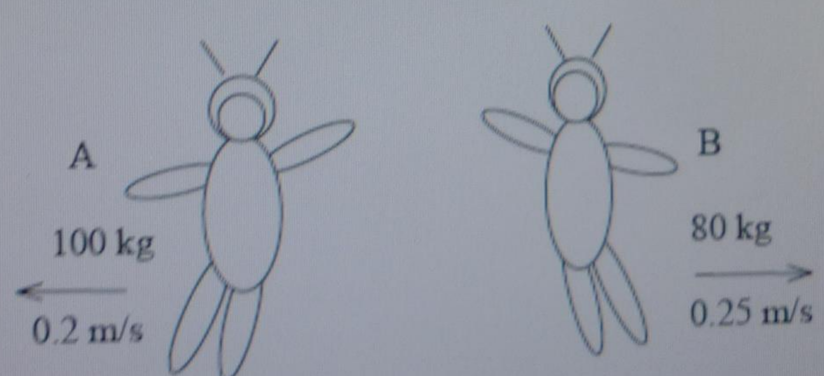


massive the object, and the greater its speed, the larger is its momentum. For a mass m, travelling with velocity v, the momentum is given by the simple formula mv. Since mass is measured in kg, and velocity in m/s, the unit of momentum is $\mathrm{kg\,m\,s^{-1}}$. Alternatively – we shall see why in *Section 8.4* – the equivalent form N s, standing for newton-seconds, is used.

8.2 Conservation of momentum

The formula, momentum equals mv, does more than just introduce another a piece of terminology. Momentum, so defined, obeys a far-reaching law, the law of conservation of momentum.

To illustrate, consider two astronauts, floating in space, facing each other, and both having zero velocity. A has mass 100 kg and B has mass 80 kg. A now pushes B, exerting a force of 20 newtons for a period of 1 second. According to Newton's second law, B now accelerates away from A, the acceleration being $a = F/m = 20/80 = 0.25\,\mathrm{m/s^2}$. After 1 second, B has velocity $0.25 \times 1 = 0.25\,\mathrm{m/s}$ and his momentum is $80 \times 0.25 = 20\,\mathrm{N\,s}$.

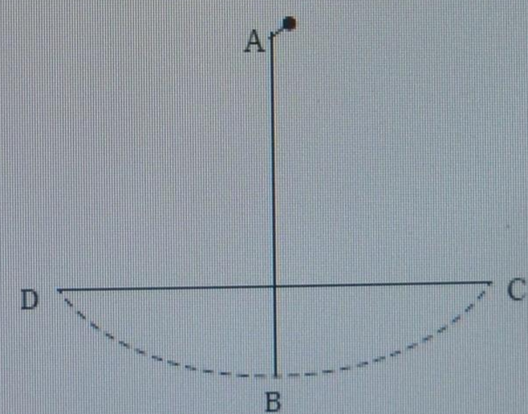


himself subject to a reaction force

9 Energy

9.1 Potential energy and kinetic energy

Imagine, said Galileo, that this page represents a vertical wall, with a nail driven into it at A, and from the nail let there be suspended a lead bullet by means of a fine vertical thread AB, say four to six feet long. On the wall let us draw a horizontal line CD at right angles to the vertical thread AB, which hangs about two finger breadths in front of the wall.



Then, if the thread is drawn aside till the bullet is in position C, and then released, it will descend along the arc CB, passing B, and rise again until it almost reaches the horizontal CD, any slight shortfall being caused by the resistance of the air or of the fixing of the

Suppose that the resistance force is constant and equal to R newtons. The work done by R against the dart is $R \times 0.01$ J which must be equal to its loss of kinetic energy, which is $\frac{1}{2} \times 0.025 \times 102$ J. Hence $R = 125$ newtons.

9.4 Power

Power is the rate of doing work. The power output P of an engine which delivers W joules of work in a time t is

$$P = \frac{W}{t} \,. \tag{9.3}$$

There is also an alternative formula which may be found by remembering that the work done W is equal to $F \times d$. Then

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{F \times d}{t} \\ &= F \times \frac{d}{t} \\ &= F \times v \,, \end{aligned} \tag{9.4}$$

since speed v equals distance divided by time.

Power is measured in units of watts. The units of power and energy, the watt and the

10 Circular motion

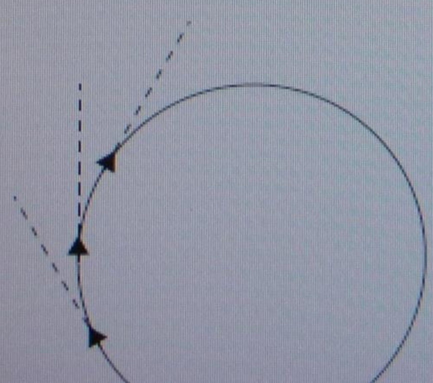
10.1 Centripetal acceleration

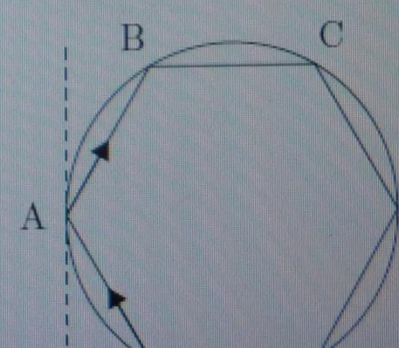
Consider a particle moving in a circle at constant speed. It is not in equilibrium, for this implies a state of rest, or motion in a straight line. But, if the speed is constant, in what sense is there an acceleration?

To resolve the paradox, we have to remember that acceleration is the rate of change of velocity and that velocity is a vector quantity, possessing both magnitude and direction. And though the speed remains constant, the direction is continuously changing, turning inwards away from the straight line direction along the tangent.

Because there is a change in direction towards the centre of the circle, there is what is called a centripetal acceleration, or literally a centre-seeking acceleration.

To calculate the acceleration, Newton visualised the particle moving at speed v on a many-sided path ABC... within a circular wall, radius r, undergoing a series of glancing impacts.





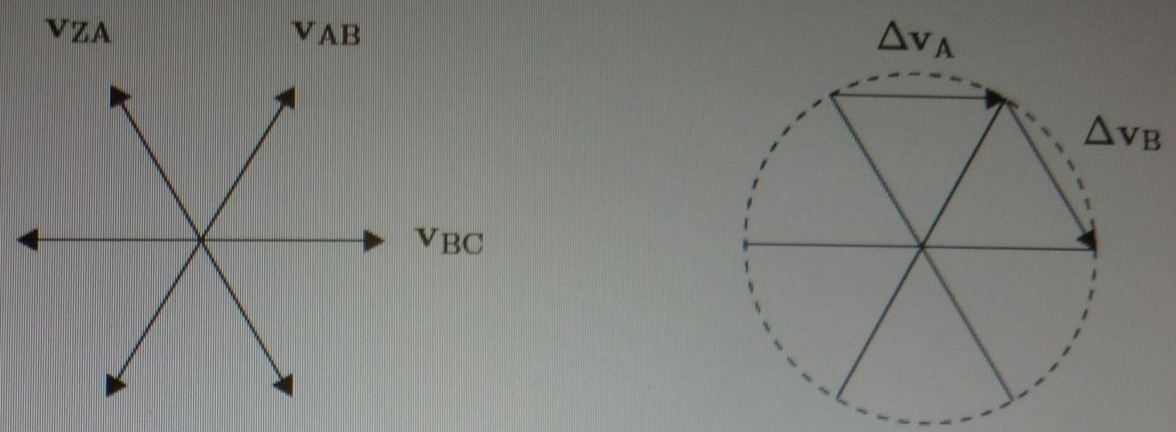


Figure 10.2: *The diagram on the left shows the velocity vectors anchored at a common origin, while that on the right shows the changes in velocity at* A, B, ...

travelling at speed v, is $2\pi r/v$, and the time taken to traverse each side is $\Delta t = 2\pi r/Nv$. Likewise the changes Δv at successive vertices A, B, . . . are approximated by arcs of the circumference of a circle, radius v, in the velocity diagram, and these are each of magnitude $2\pi v/N$. The magnitude of the acceleration, therefore, is

$$\begin{aligned} a &= \frac{\Delta v}{\Delta t} \\ &= \frac{2\pi v/N}{2\pi r/Nv} \\ &= \frac{v^2}{r}\,, \end{aligned} \tag{10.1}$$

and its direction is always towards the centre of the circle.

10.3 Motion in a vertical circle

Similar principles apply to the analysis of motion in a vertical circle. The main difference is that because there will be differences in gravitational potential energy at different positions on the circle, there will be differences in kinetic energy and the speed will therefore vary.

Example 10.4

A car drives over a hump-backed bridge. The profile of the humped surface is taken as circular with a radius of curvature of 40 metres. At what speed can the car be driven over the bridge without losing contact with the road?

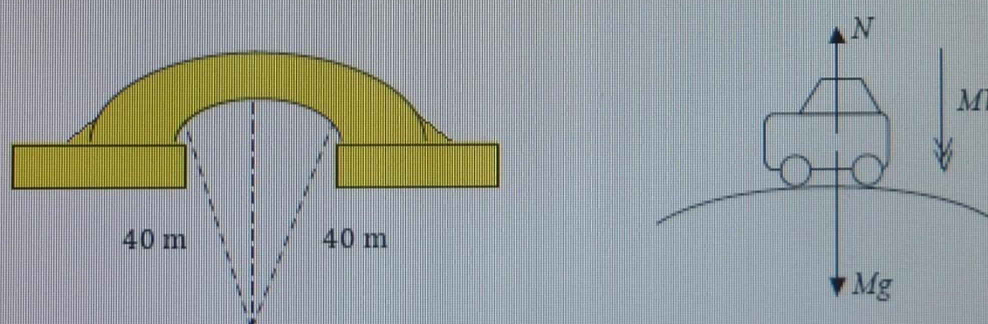


Figure 10.3: *A bridge with radius of curvature of* 40 m *(left) and the resultant forces and accelerations on the car (right).*

Suppose the car has mass M and is travelling at speed V. On a level road, the normal reaction from the road surface on to the car is equal to its weight Mg, but on top of the bridge it takes a lower value N. The difference $Mg-N$ constitutes a net downward force which ensures the car follows the profile of the road, accelerating downwards towards the centre of curvature. So

$$Mg - N = \frac{MV^2}{40} .$$

In the limiting case, when the car is on the point of becoming airborne, N reduces to zero, since clearly N must be zero once it is no longer in contact with the road. The critical value of V therefore satisfies the equation

$$Mg = \frac{MV^2}{40} ,$$

so that

$$\begin{aligned} V &= \sqrt{40g} \\ &= 19.8\,\mathrm{m/s} . \end{aligned}$$

In fact, the car would be difficult to handle even at lower speeds. The reduced normal reaction would lead to a lower limiting frictional force and – as in *Example 10.1* – a lesser capability of negotiating any bend.