Worked Example 2.5
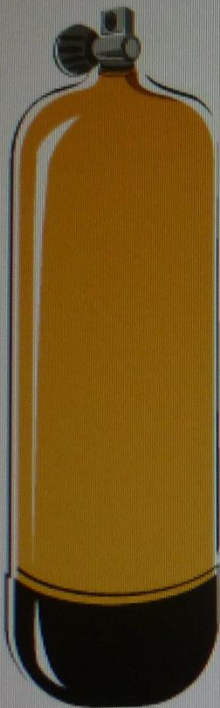
An unkown gas has a mass of 1.5 kg contained in a bottle of volume 1.17 m³ while at a temperature of 300 K, and a pressure of 200 kPa. Determine the ideal gas constant and deduce the gas?

Solution:

Assuming perfect gas behaviour:

$PV = mRT$

$200 \times 10^3 \times 1.17 = 1.5 \times R \times 300$

$\therefore R = 520 \text{ J/kgK}$

but $R = \dfrac{Ro}{M}$

hence $M = Ro/R = 8314.3/520 = 15.99$

The nearest gas with

The small difference may be attributed to measurements errors.

Worked Example 2.6

A 6 m^3 tank contains helium at 400K is evacuated form atmospheric pressure of 100kPa to a final pressure of 2.5kPa.

Determine

   a)   the mass of helium remaining in the tank;

   b)   the mass of helium pumped out;

   c)   if the temperature of the remaining helium falls to 10°C, what is the pressure in kPa?

a) $P_2V_2 = m_2 RT_2$

$$R = \frac{Ro}{M} = \frac{8314.3}{4.003} = 2077 \; J/kgK$$

The mass remaining is:

$$m_2 = \frac{2500 \times 6}{2077 \times 400} = 0.018 \; kg$$

b) initial mass is:

$$m_1 = \frac{100000 \times 6}{2077 \times 400} = 0.722 \; kg$$

∴ mass pumped out $= m_1 - m_2 = 0.704 \; kg$

since m3 = m2, then

c) $P_3 = \dfrac{m_3 .R.T_3}{V_3} = \dfrac{0.018 \times 2077 \times 283}{6} = 1763 \; Pa$

c) $P_3 = \dfrac{m_3.R.T_3}{V_3} = \dfrac{0.018 \times 2077 \times 283}{6} = 1763\ Pa$

Worked Example 2.7

A motorist equips his automobile tyres with a relief-type valve so that the pressure inside the tyre will never exceed 220 kPa (gauge). He starts the trip with a pressure of 200 kPa (gauge) and a temperature of 23°C in the tyres. During the long drive the temperature of the air in the tyres reaches 83°C. Each tyre contains 0.11 kg of air. Determine:

    a) the mass of air escaping each tyre,
    b) the pressure of the air inside the tyre when the temperature returns to 23°C.

a) $P_1V_1 = m_1RT_1$

$$m_1 = \frac{P_1V_1}{RT_1} = \frac{200 \times 10^3 \times V_1}{287 \times (273 + 23)} = 0.11\,kg$$

$$V_1 = \frac{0.11 \times 287 \times 296}{200 \times 10^3} = 0.04672\,m^3$$

$V_1 = V_2 = $ constant

$P_2V_2 = m_2\,RT_2$

$220 \times 10^3 \times 0.04672 = m_2 \times 287 \times (273 + 83)$

$\therefore m_2 = 0.1006$ kg

$\therefore dm = m_1 - m_2 = 0.11 - 0.1006 = 0.0094$ kg

b) $V_3 = V_2 = V_1$ and $m_3 = m_2$

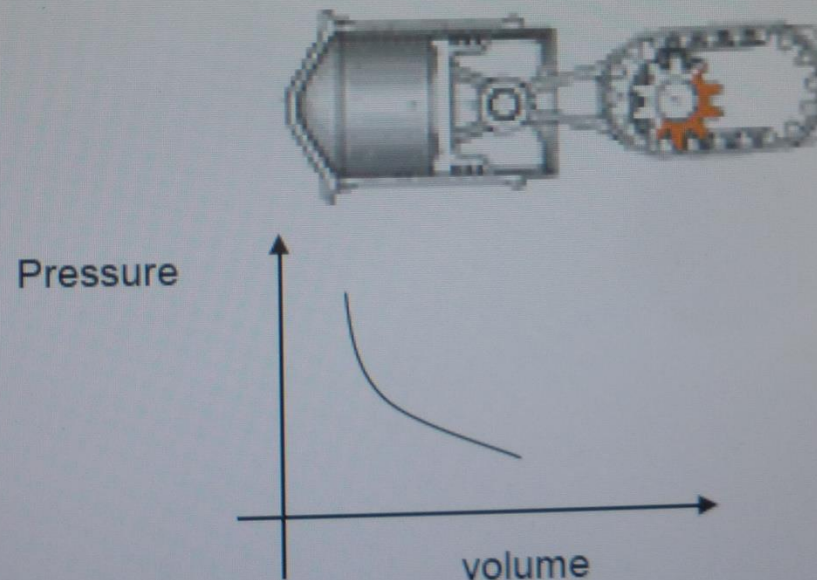$$P_3 = \frac{m.R.T_3}{V_4} = \frac{0.1006 \times 287 \times 296}{0.04672} = 183\,kPa$$

Worked Example 2.9

Self-ignition would occur in an engine using certain brand of petrol if the temperature due to compression reaches 350 °C; when the inlet condition is 1 bar, 27°C.
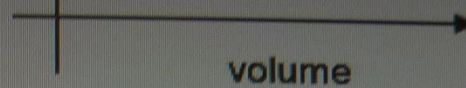
Calculate the highest compression ratio possible in order to avoif self-ignition, if the compression is according to

a) adiabatic, with index of 1.4; and
b) polytropic, with index of 1.3

Solution:

volume

The compression ratio is calculated as follows:

When n = 1.4, the volume ratio is :

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{1/(n-1)} = \left(\frac{349.9 + 273}{27 + 273}\right)^{1/0.4} = 6.2$$

and when n = 1.3, the volume ratio is :

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{1/(n-1)} = \left(\frac{349.9 + 273}{27 + 273}\right)^{1/0.3} = 11.4$$

$$= (1270 + 273)\left(\frac{1}{8}\right)$$

$$= 698K$$

Worked Example 2.11

Determine using Steam Tables, the volume occupied by 2 kg of steam at 500 kPa, under the following conditions and specify the state of steam.

a) pure liquid state
b) when it is in a pure vapour state
c) 20% moisture content
d) 20% dry.

p	t_s	v_f	v_g	h_f	h_g	s_f	s_g
(kPa)	(°C)	(m³/kg)	(m³/kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
500	151.86	0.00109	0.3749	640.23	2,749	1.8607	6.8213

Solution:

From Steam Table:- $v_f = 0.00109$ m³/kg and $v_g = 0.3749$ m³/kg

a) when pure liquid

$v = 0.00109$ m³/kg

∴ $V = 2 \times 0.00109 = 0.00218$ m³

b) when it is saturated vapour

$v = v_g = 0.3749$ m³/kg

∴ $V = 2 \times 0.3749 = 0.7498$ m³

c) The steam is obviously in its wet phase. X=0.8

∴ $v = (1-x)\, v_f + x v_g = 0.2 \times 0.00109 + 0.8 \times 0.3749 = 0.2785$ m³/kg

∴ $V = 2 \times 0.2785 = 0.557$ m³

d) The steam is obviously in its wet phase. X=0.2

∴ $v = (1-x)\, v_f + x v_g = 0.8 \times 0.00109 + 0.2 \times 0.3749 = 0.0758$ m³/kg

∴ $V = 2 \times 0.0758 = 0.152$ m³

Table 3.1 Example of Thermodynamic fixed temperatures

3.2 First Law of Thermodynamics

The first law of thermodynamics is the application of the conservation of energy principle.

3.2.1 First Law of Thermodynamics Applied to closed Systems

consider a closed system where there is no flow into or out of the system, and the fluid mass remains constant. For such system, the first law statement is known as the Non-Flow Energy Equation, or NFEE abbreviated, it can be summerised as follows:

$$\Delta U = Q - W$$

(25)

The first law makes use of the key concepts of internal energy (ΔU), heat (Q), and system work (W).

3.2.2 Internal Energy

Internal energy is defined as the energy associated with the random, disordered motion of molecules. It is separated in scale from the macroscopic ordered energy associated with moving objects; it refers to the invisible microscopic energy on the atomic and molecular scale. For example, a room temperature glass of water sitting on a table has no apparent energy, either potential or kinetic . But on the microscopic scale it is a seething mass of high speed molecules traveling at hundreds of meters per second. If the water were tossed across the room, this microscopic energy would not necessarily be changed when we superimpose an ordered large scale motion on the water as a whole.

$$\Delta U = m.Cv.\Delta T$$

(26)

Where Cv is the specific heat capacity of the fluid, and ΔT is the temperature difference during the process

3.2.3 Specific Heat

Heat may be defined as energy in transit from a high temperature object to a lower temperature object. An object does not possess "heat"; the appropriate term for the microscopic energy in an object is internal energy. The internal energy may be increased by transferring energy to the object from a higher temperature (hotter) object - this is properly called heating.

In order to heat or cool a given quantity of a gas in a given time, the following equation is used:

Quantity of Heat (Q)= mass(m) x specific heat capacity(C) x temperature difference

Since this heat exchange may take place

Either at constant pressure: $\qquad Q = \dot{m}\, Cp\, (T_2 - T_1)$ (27)

Or at constant volume: $\qquad Q = \dot{m}\, Cv\, (T_2 - T_1)$ (28)

Where:

Cp $\qquad$ specific heat at constant pressure (kJ/kg K), see Table 3.2

Cv $\qquad$ specific heat at constant volume (kJ/kg K) , see Table 3.2

Note that for a perfect gas $\qquad Cp = Cv + R \quad$ and $\quad n = Cp / Cv$ (29)

| Cv | specific heat at constant volume (kJ/kg K) , see Table 3.2 |

Note that for a perfect gas $\quad Cp = Cv + R \quad$ and $\quad n = Cp / Cv$ $\hfill$ (29)

Specific Heat at Constant Volume Cv

Consider a closed system of unit mass, the first law of thermodynamics applied to this system is:-

$$q - w = du$$

If the volume is constant, then $w = 0$, $\quad$ it follows that $\quad q = du$

But $\qquad\qquad q = Cv\, dT$

Hence $\qquad\quad du = C_v dT$

Or $\qquad\qquad Cv = du/dT$

This is known as Joule's Law of internal energy which states that "the internal energy of a perfect gas depends upon its temperature only".

Specific Heat at Constant Pressure Cp

Consider a constant pressure non-flow process, the first law:-

$$q - w = du$$

For a constant pressure process

$$W = p(v_2 - v_1) \qquad = (p_2 \, v_2 - p_1 \, v_1)$$

hence $q - (p_2 \, v_2 - p_1 \, v_1) = u_2 - u_1$

or $q = (u_2 + p_2 \, v_2) - (u_1 + p_1 \, v_1) \qquad = h_2 - h_1$

but $q = Cp \, dT$

hence $h_2 - h_1 = Cp \, (T_2 - T_1)$

or $Cp = dh/dT$

Where Q_s heat supplied

Q_r heat rejected

W_c work of compression

W_e work of expansion

3.2.6 First Law of Thermodynamics Applied to Open Systems

The first law of thermodynamics is based on the conversation of energy within a system. Open systems are associated with those, which have a steady flow, thus the first law applied to such systems is known as the Steady Flow Energy Equation (SFEE):-

$$Q - W = (\Delta H + \Delta KE + \Delta PE)$$

$$= m \left[(h_2 - h_1) + \frac{1}{2} \left(V_2^2 - V_1^2 \right) + g \left(Z_2 - Z_1 \right) \right]$$

(39)

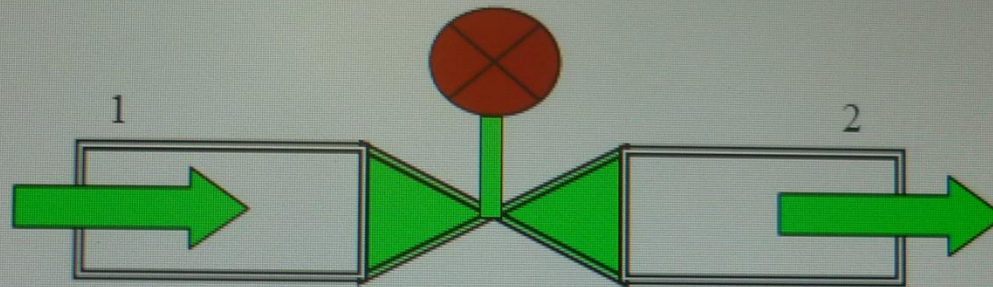| Variable	Symbol	Units
Heat transfer	Q	W
Work transfer	W	W
Mass flow rate	m	kg/s
Specific enthalpy	h	J/kg
Velocity	V	m/s
Gravitational acceleration	g	9.81 m/s²
Elevation above datum	z	m

hence for a boiler or a condensor, the amount of energy supplied or exctracted from the fluid "Heat " is equal to the enthalpy change for the fluid between inlet and outlet.

c. Throttling valve

Consider the flow of fluid through a small valve as shown

if the SFEE is applied between sections 1 and 2 :

$$Q - W = m (\Delta h = \Delta ke + \Delta Pe)$$

$$Q = 0 \qquad \text{Assuming adiabatic}$$

3.4 Third Law

The entropy of a perfect crystal is zero when the temperature of a the crystal is equal to absolute zero (0 K).

- At 0 K, there is no thermal motion, and if the crystal is perfect, there will be no disorder

- Once the temperature begins to rise above 0, the particles begin to move and entropy gradually increases as the average kinetic energy of the particles increases.

- When temperature reaches the melting point of the substance (T_m), there is an abrupt increase in entropy as the substance changes from a solid to a more disordered liquid.

Again the entropy increases gradually as the motion of the particles increases until the temperature reaches the boiling point of the substance (T_b). At this point, there is another drastic increase in entropy as the substance changes from a confined liquid particles to radom motion gas particles.

Process	Constant Volume	Constant Pressure	Constant Temperature	Polytropic	Reversible Adiabatic or Isentropic
Law	$\dfrac{P}{T} = const$	$\dfrac{V}{T} = const$	$PV = const$	$PV^n = const$	$PV^\gamma = const$
P, V, T. Relation	$\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$	$\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$	$P_1V_1 = P_2 V_2$	$P_1V_1^n = P_2V_2^n$ $\dfrac{P_1}{P_2} = \left(\dfrac{V_2}{V_1}\right)^n$ $= \left(\dfrac{T_1}{T_2}\right)^{\frac{n}{n-1}}$	$P_1V_1^\gamma = P_2V_2^\gamma$ $\dfrac{P_1}{P_2} = \left[\dfrac{V_2}{V_1}\right]^\gamma$ $= \left[\dfrac{T_1}{T_2}\right]^{\frac{\gamma}{\gamma-1}}$
Change in Internal Energy ΔU	$mC_v(T_2 - T_1)$	$mC_v(T_2 - T_1)$	0	$mC_v(T_2 - T_1)$	$mC_v(T_2 - T_1)$
Work Transfer $W = \int pdv$	0	$P(V_2 - V_1)$ Or $mR(T_2 - T_1)$	$PV \ln\dfrac{V_2}{V_1}$ Or $mRT \ln\dfrac{V_2}{V_1}$	$\dfrac{P_1V_1 - P_2V_2}{(n-1)}$ $\dfrac{mR(T_1 - T_2)}{(n-1)}$	$\dfrac{P_1V_1 - P_2V_2}{(\gamma-1)}$ $\dfrac{mR(T_1 - T_2)}{(\gamma-1)}$ $mC_v(T_1 - T_2)$
Heat Transfer Q	$mC_v(T_1 - T_2)$	$mC_p(T_1 - T_2)$	$PV \ln\dfrac{V_2}{V_1}$ Or $mRT \ln\dfrac{V_2}{V_1}$	$W + (U_2 - U_1)$	0

Internal Energy ΔU	$mC_v(T_2-T_1)$	$mC_v(T_2-T_1)$	0	$mC_v(T_2-T_1)$	$mC_v(T_2-T_1)$
Work Transfer $W=\int pdv$	0	$P(V_2-V_1)$ Or $mR(T_2-T_1)$	$PV\,\ln\dfrac{V_2}{V_1}$ Or $mRT\,\ln\dfrac{V_2}{V_1}$	$\dfrac{P_1V_1-P_2V_2}{(n-1)}$ $\dfrac{mR(T_1-T_2)}{(n-1)}$	$\dfrac{P_1V_1-P_2V_2}{(\gamma-1)}$ $\dfrac{mR(T_1-T_2)}{(\gamma-1)}$ $mC_v(T_1-T_2)$
Heat Transfer Q	$mC_v(T_1-T_2)$	$mC_p(T_1-T_2)$	$PV\,\ln\dfrac{V_2}{V_1}$ Or $mRT\,\ln\dfrac{V_2}{V_1}$	$W+(U_2-U_1)$	0
Change in Entropy $\Delta S=S_2-S_1$	$mC_v\ln\dfrac{T_2}{T_1}$ or $mC_v\ln\dfrac{P_2}{P_1}$	$mC_p\ln\dfrac{T_2}{T_1}$ or $mC_p\ln\dfrac{V_2}{V_1}$	$mR\ln\dfrac{V_2}{V_1}$ or $mR\ln\dfrac{P_1}{P_2}$	$m\left[R\ln\dfrac{V_2}{V1}+C_v\ln\dfrac{T_2}{T_1}\right]$ $m\left[R\ln\dfrac{P_1}{P_2}+C_p\ln\dfrac{T_2}{T_1}\right]$ $m\left[C_p\ln\dfrac{V_2}{V_1}+C_v\ln\dfrac{P_2}{P_1}\right]$	0

Table 3.3: Perfect Gas Process-Relation

Worked Example 3.1

A closed rigid container has a volume of 1 m³ and holds air at 345 kPa and 20°C. Heat is added until the temperature is 327°C. Determine the change in Internal Energy:-

a) Using an average value of the specific heat.
b) Taking into account the variation of specific heat with temperature.

Solution:

a) $\Delta U = mC_v \Delta T$

$$Cv = \frac{764 + 718}{2} = 741 \, J/kgK$$

$$m = \frac{PV}{RT} = \frac{345 \times 10^3 \times 1}{287 \times 293} = 4.1026 \, kg$$

Therefore

$$\Delta U = 4.1026 \times 741 \, (327 - 20) = 932 \text{ kJ}$$

b) $\quad \Delta U = m.\int_{T1}^{T2} Cv.T$

$$\Delta u = \int_{T1}^{T2} CvT = Cv_2 T_2 - Cv_1 T_1$$

$$= 764 \times 600 - 718 \times 293$$

$$= 248{,}319 \text{ J/kg}$$

$$m = \frac{PV}{RT} = \frac{345 \times 10^3 \times 1}{287 \times 293} = 4.1026 kg$$

Therefore,

$$\Delta U = m \times \Delta u = 1018.7 \quad \text{kJ}$$

$$\text{Power} = \text{m x W} = (600/60)\text{x } 431 = 4.31 \text{ MW}$$

Worked Example 3.6

A boiler receives feed water at 40°C and delivers steam at 2 MPa and 500°C. If the furnace is oil fired, the calorific value of oil being 42000 kJ/kg and 4000 kg oil are burned while 45000 kg of steam are produced, determine :
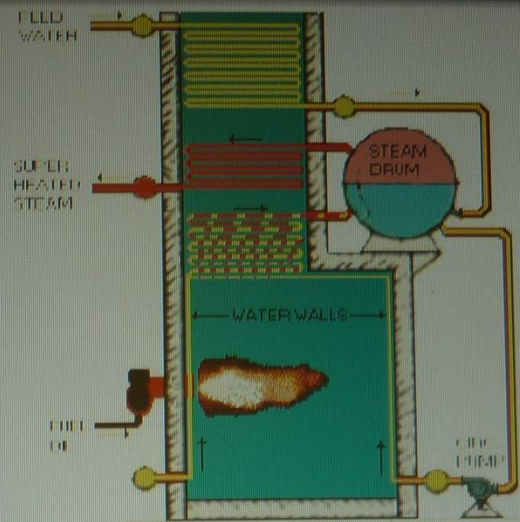
a) the heat supplied in the boiler.
b) the efficiency of the boiler.

Assume the values of enthalpies at the two state points as :

$h_1 = h_{f@40^\circ C} = 169.33 \text{ kJ/kg}$ 　　　　　　　　at 2 MPa , 500 C, $h_2 = 3467.6 \text{ kJ/kg}$

a) Constant pressure process.

$h_1 = h_{f@40°C} = 169.33$ kJ/kg $\qquad\qquad h_2 = 3467.6$ kJ/kg

SFEE ignoring W, Δke and ΔPe:

$Qs \qquad = m_s (h_2 - h_1) \qquad = 45000 (3467.6 - 169.33) \qquad = 1.484 \times 10^8$ kJ

b) The heat generated by burning oil in the furnace is

$= $ mass of oil burned x calorific value

$= 4000 \times 42000 = 1.68 \times 10^8$ kJ

$\therefore$ *Boiler efficiency* $= \dfrac{\text{Energy Output}}{\text{Energy Input}} = \dfrac{1.484 \times 10^8}{1.68 \times 10^8} = 88\%$