$$P_A = P_B$$

Pressure at point A is given by

$$P_A = h\rho \boldsymbol{g}$$

Therefore, the atmospheric pressure $P_{atm} = P_B = h\rho \boldsymbol{g}$ (2.13)

Since ρg is a constant, height of the mercury column could be used as a measure of the pressure. This is how mercury millimetre (Hg mm) became a unit of pressure. In honour of Torricelli, 1/760 of the standard atmospheric pressure is called a Torr.

mercury column in a barometer. Density of mercury is 13590kgm^{-3}.

2.5.2 U-tube manometer

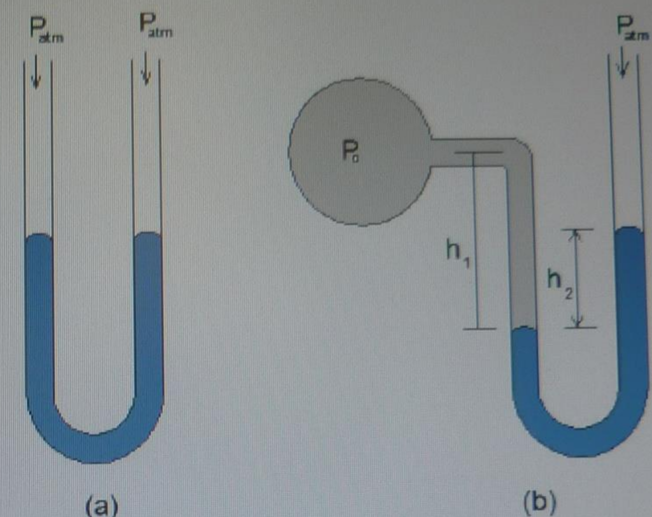

Figure 2.7 U-Tube manometer

Barometer has limited applications. It is widely used to measure atmospheric pressure. However, the principle could be used to measure pressure relative to the atmospheric pressure or pressure difference between two points. For this purpose, a U-tube partially filled with mercury is used. When the both ends of the U-tube are open to the atmosphere, the mercury column balances giving the same height in both arms of the U tube (see Figure 2.7(a)). When one end is open to the atmosphere and the other end to a vessel with a pressure different from the atmosphere, the mercury column moves to a new equilibrium position giving a height difference in the U-tube as shown in Figure 2.7 (b).

Suppose the fluid in the bulb has a density ρ_1 and the density of mercury to be ρ_M. Furthermore, assume the pressure inside the bulb to be P_0. Once the mercury column attains equilibrium, a simple force balance at a point just inside the static mercury meniscus will give

$$P_0 + h_1\rho_1 g = P_{atm} + h_2\rho_M g$$

Rearranging terms gives

$$P_0 - P_{atm} = h_2\rho_M g - h_1\rho_1 g \tag{2.14}$$

Manometers could be used to measure the pressure difference between two points. Consider an arrangement as shown in Figure 2.8. A U-tube partially filled with a heavier liquid, mercury in most cases, connected to a pipe across a restriction in the pipe. Density of the fluid in the pipe is ρ_L and the density of the heavy liquid is ρ_M. Pressure at two tapings to which the manometer arms are connected are P_1 and P_2 ($P_1 > P_2$).

2.7 Buoyancy

In a fluid, the pressure increases linearly with the depth. As a result, any submerged body feels an upward force due to the difference of pressure acting on it. To illustrate this point consider a cylindrical object immersed in a liquid as shown in Figure 2.12.

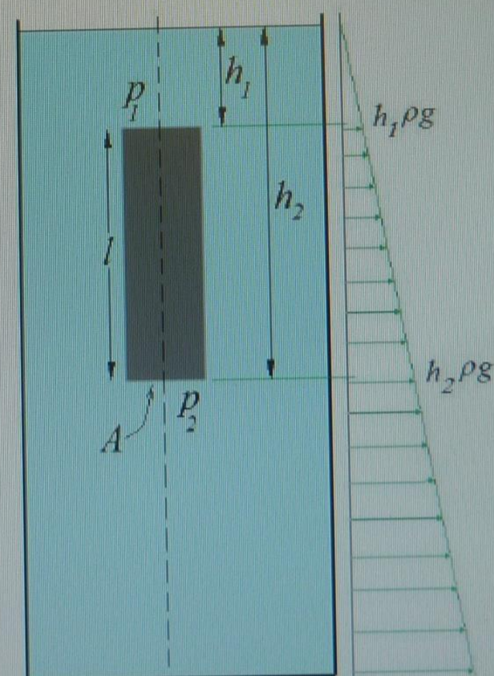

Figure 2.12.

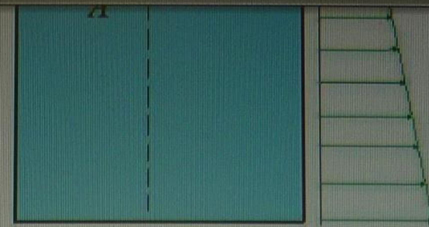

Figure 2.12.

The cylinder experiences a range of pressure values on its sides. The upper surface experiences a pressure p_1 and the lower surface experiences a pressure p_2. The cylindrical surface experiences an equal pressure at any cross horizontal cross section. Therefore, there is no resultant force acting on the cylindrical surface. However, the pressure exerted by the fluid is different at the top and the bottom surfaces. Upward force due to the hydrostatic pressure is given by

$$F_B = p_2 A - p_1 A$$

Using the fact that $\quad p_1 = h_1 \rho \boldsymbol{g}$ and $p_2 = h_2 \rho \boldsymbol{g}$

$$F_B = A(h_2 - h_1)\rho \boldsymbol{g}$$

Therefore,

$$F_B = Al\rho \boldsymbol{g} = V\rho \boldsymbol{g} \tag{2.17}$$

This equation does not depend on the geometric shape of the object immersed in the fluid but only the volume. *Therefore, the upward force (buoyancy) acting on a body immersed in a fluid is equal to the weight of an equivalent volume of the fluid.* This law is first discovered by the Greek philosopher, who anecdotally ran nakedly through the city when he discovered it. Archimedes

4 Basics of Fluid Flow

Introduction

In the previous lecture on fluid statics we have discussed the fluid at rest. In this section we discuss about fluid flow and some concepts associated with flowing fluids. Given that the gasses are incompressible, the concepts discussed here apply to both liquids and gasses.

Characteristics of flowing fluid have generated lot of interest over time. There are evidences that some of the ancient civilizations have running water and drainage systems in place. Much later, Romans built aqueducts to supply water to cities from where the sources were found. These magnificent engineering achievements were made simply making use of the observation that water runs down an elevation spontaneously. The explanation to why it happens came much later when the knowledge about fluid flow gathered over time. For instance, Leonardo da Vinci has drawn sketches of turbulent flows well before a proper description came into existence (see figure 4.1). Structured presentation of the knowledge gathered over last two millennia on the behaviour of fluid flow is called fluid dynamics.

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Figure 4.1 Turbulence

Kinematics explains the flow disregarding the forces that causes the flow and dynamics address the flow together with the forces that makes fluid to flow.

4.1 Velocity field

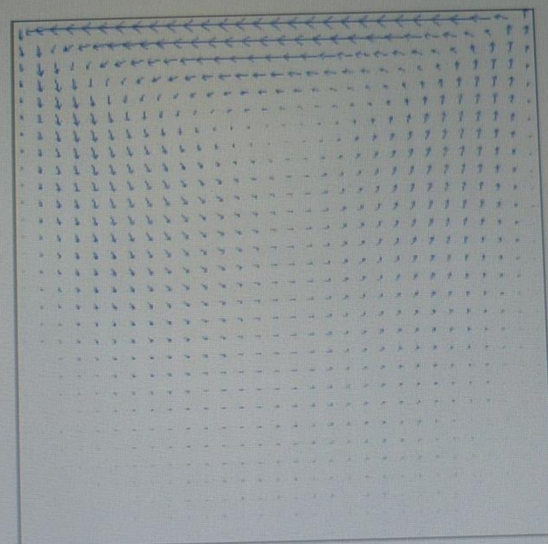

Figure 4.2. Velocity vectors

As we discussed earlier, a fluid is a continuous medium. Consider a fluid flowing in a pipe. If you trace the path a small fluid element would take during a period of time you can calculate the average velocity of that particle. If the period of observation is very small, then it gives the instantaneous velocity of the fluid element. Velocity is a vector quantity that has a magnitude and a direction. It could be represented by an arrow pointing towards the direction of the action and a length that corresponds to the magnitude.

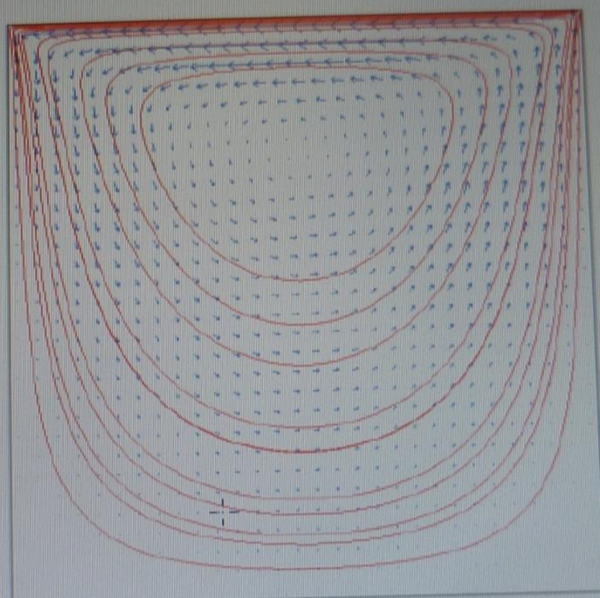

Figure 4.3 Streamlies with velocoity vectors

Fluid that flows consist of a large number of fluid elements that has various velocities. If all the vectors are mapped at a particular moment, it gives a snapshot of the velocity distribution within the flow domain. This velocity vector distribution is called the velocity field.

$$\frac{U_1}{U_2} = \frac{A_2}{A_1} \quad (4.2)$$

Example

A pipe A with ID 100mm is connected to a pipe B with ID 65 mm through a converging cone (see figure Q41). Kerosene (ρ=800kg/m³) is pumped through the pipe. If the average velocity in pipe A is 0.75 m/s, calculate

a) Velocity in pipe B
b) Mass flow rate in A
c) Mass flow rate in B

Answer

Concept tested: Continuity of mass flow.

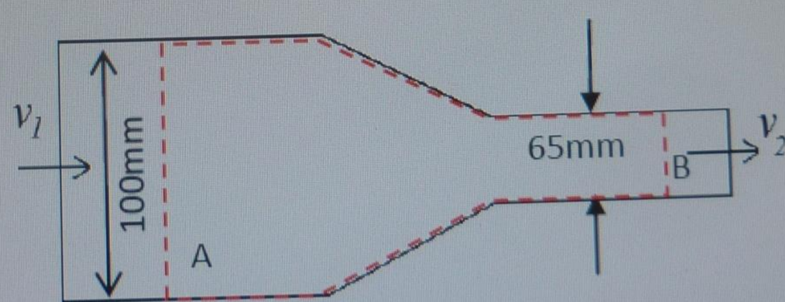

Figure Q4.1 control volume on a converging pipe section

Data given

V1 =0.75m/s

Density of kerosene $\rho = 800\ kg/m^3$

Diameter of pipe A D = 0.1 m

Diameter of pipe B d = 0.065 m

a) Velocity in pipe B

From continuity of mass flow suggest that in the absence of accumulation

Mass flow in A = mass flow in B (1)

Mass flow in A = density × volumetric flowrate

$$= \rho \times v_1 \times \frac{\pi D^2}{4} \quad (2)$$

Similarly mass flow in B

$$= \rho \times v_2 \times \frac{\pi d^2}{4} \qquad (3)$$

Substituting (2) and (3) in (1)

$$\rho \times v_1 \times \frac{\pi D^2}{4} = \rho \times v_2 \times \frac{\pi d^2}{4}$$

$$v_1 \times D^2 = v_2 \times d^2$$

$$\therefore v_2 = v_1 \times \frac{D^2}{d^2}$$

$$v_2 = 0.75 \times \left[\frac{0.1}{0.065}\right]^2 = 1.775\ m/s$$

b) Mass flow rate in A

From equation 3 the mass flowrate in A =

$$A = \rho \times v_1 \times \frac{\pi D^2}{4}$$

$$= 800 \times 0.75 \times \frac{\pi (0.1)^2}{4}$$

$$= 4.71 kg/s$$

c) Mass flowrate in B = mass flow in A

4.4 Types of flow

Observing the general characteristics of fluid flow, several flow types can be identified.

4.4.1 Steady flow

Flow velocity at any given point does not change with time. However, velocity can vary in space. For example, consider a pipe with increasing diameter (expansion). Flow within the pipe can be steady with velocity constantly varying along the pipe. Figure 4.10 shows the velocity distribution at several cross sections and the velocity profile along the pipe. The velocity drops due to the increase of the flow area along the axis of the pipe but velocity at each point along the axis does not change with time.

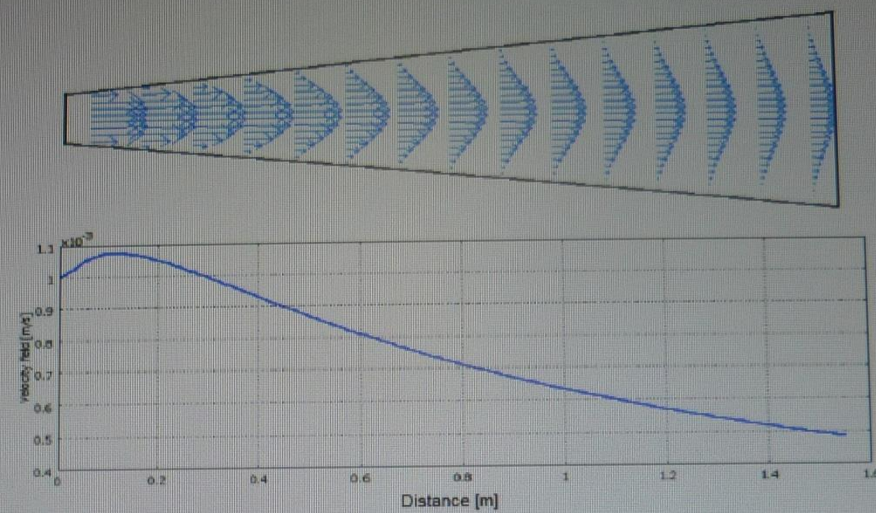

Figure 4.10. Velocity variation alone a diverging pipe.

4.4.2 Unsteady flow

Velocity of the fluid at any point within the flow domain changes with time. Example of this is the flow in a tea cup just after stirring to dissolve sugar or milk. The flow slows down and eventually come to rest if not disturbed.

4.4.3 Laminar flow

When fluid flows as if they move in parallel layers, it is called a laminar flow. For example, flow in viscous fluid like honey shows laminar flow behaviour. Laminar flow also takes place for less viscous liquids when flow through capillaries or flowing down an inclined plane as a thin film.

4.4.4 Turbulent flow

The most common form of flow is the turbulent flow. The fluid flows while mixing vigorously. Most of the open channel flows, rivers, large diameter pipe flows, blowing wind are common examples for turbulent flows.

4.5 Bernoulli equation

We have discussed streamlines in section 3.1.2. Any fluid particle on a streamline will move along the streamline as the velocity of that fluid particle is tangent to the streamline at all points. Fluid particle moving along the stream line may experience acceleration or deceleration due to the forces acting on the particle. If we consider an inviscid fluid at steady state (i.e. velocity at any point does not change with time), there are no shear stresses acting on the particle. Therefore, Newton's second law of motion can be applied to the fluid particle without much difficulty.

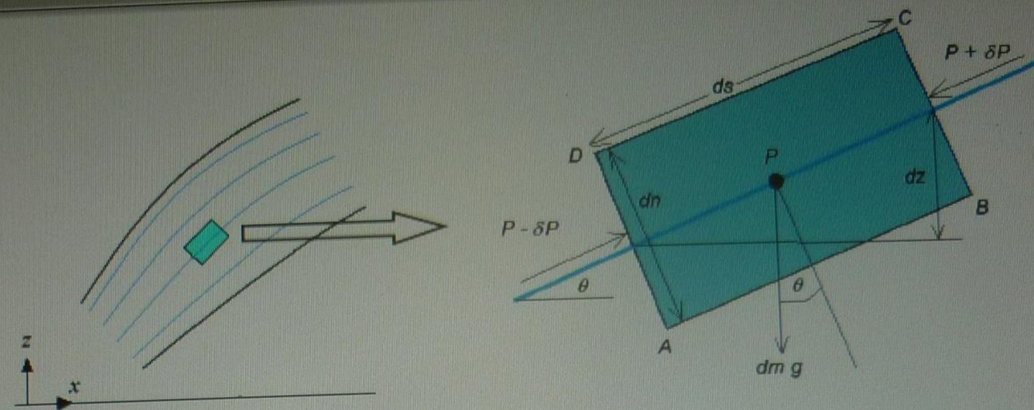

Figure 4.11. Fluid element in a flow

Consider the fluid particle shown in Figure 4.11. The streamline along which the fluid particle moves has an inclination angleθ. Length and the width of the fluid particle are ds and dn respectively. Pressure at the centre of the element is P.

Newton's second law of motion states that the force is equal to the rate of change of momentum.

$$\boldsymbol{F} = \frac{d(mv_2 - mv_1)}{dt} = ma$$

Where a is the acceleration. Consider the forces acting along the streamline on the fluid element. Force due to the pressure F_p

$$\boldsymbol{F_P} = (P - \delta P)dndy - (P + \delta P)dndy \tag{4.3}$$

$$\delta P = \frac{dP}{ds}\frac{ds}{2} \tag{4.4}$$

Therefore, $\boldsymbol{F_P} = -\frac{dP}{ds}dndsdy$ (4.5)

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = E \tag{4.13b}$$

4.6 Physical meaning of the Bernoulli equation

Each term in the l.h.s of the equation (4.13b) is called a "head".

$\frac{P}{\rho g}$ = Pressure head

$\frac{v^2}{2g}$ = Velocity head

z = Elevation head

Consider the elevation head. It has the dimensions of length: meters. Since the equation (4.13b) obeys dimensional homogeneity, other two terms too have dimensions of length.

Head is a measure of energy per unit weight of the fluid. For example, a fluid volume with mass m elevated to a height of h above the ground has a potential energy mgh (units: Joules). In this case, ground is considered as the reference level with zero potential energy. If we compute the energy per unit weight , it gives h. The elevation head in equation (4.13b) can be considered as the height above some reference level which is often called the "Datum level".

that the total head of a fluid element remains a constant as it flows along the streamline.

Consider a fluid element initially at location 1, z_1 above the datum flow along the streamline to location 2 gaining a height z_2 above the datum as shown in Figure 4.12. Consider the total head at 1 to be E_1 and at 2 to be E_2.

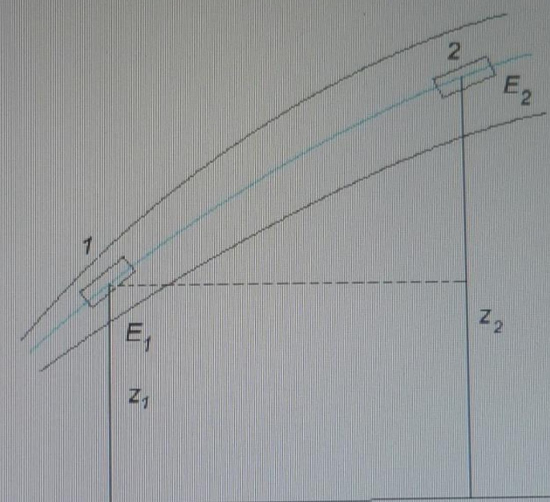

Figure 4.12. Application of Bernoulli equation

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = E \qquad (4.14)$$

4.7 Applications of Bernoulli equation

Even though Bernoulli equation is derived for inviscid fluids, it provides a good estimation for real fluids when the viscosity is small. This flexibility gives a wide variety of applications especially in flow velocity measurements. Equation (4.14) can be applied to two locations within the flow. If five of the six variables are known, then the unknown could easily be determined.

Here we consider several applications of the Bernoulli equation.

4.7.1 Free jets

A free jet forms when a liquid accelerate through a nozzle. Consider a nozzle at the bottom of a reservoir as shown in Figure 4.13. The smooth and well contoured nozzle has a diameter d.

Consider a fluid element moving from (1) at the surface to (2) at the nozzle exit along the streamline. The free surface of the liquid is h distance above the nozzle.

Assuming the datum is at the nozzle, applying Bernoulli equation between (1) and (2) gives

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

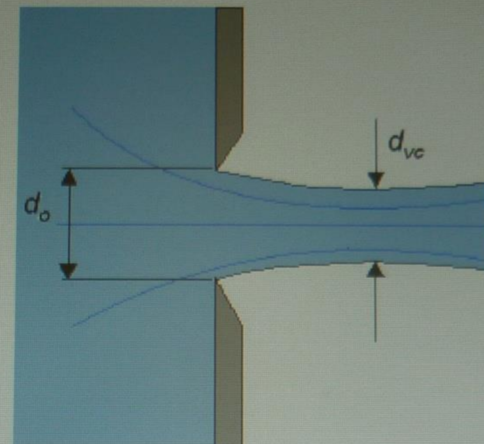

Figure 4.14. Vena Contracta

Fluid streamlines cannot change direction abruptly. When the fluid elements takes the 90 degree angle passing through the orifice follows a path that creates a jet diameter less than the orifice diameter. This thinning that occurs downstream just outside the orifice is called *Vena Contracta.*

Applying Bernoulli equation between two points at the surface (p_1, v_1, h_1) and the *vena contracta* (p_2, v_2, h_2)

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$