

...the expected to perform is to design, and operate a process that produces a commercially valuable product from the raw materials. In most cases, they are to ensure process fluids be transported from the storage tanks through the process equipment to the product storage in a controlled manner. For these tasks and many other, chemical engineers *must* have an understanding of Fluid Mechanics.

In this section, we briefly discuss the nature of fluids. Basic concepts such as density, viscosity, surface tension and pressure are introduced and discussed in detail. We will examine the cause of these properties using a description at molecular level and further investigate how they would behave at macroscopic scales.

## 1.1 Nature of fluids

The greatest scientist ever, Sir Isaac Newton, provided a definition for fluids based on the observation. In Book II, Section V of the Principia the definition is given as

*"A fluid is any body whose parts yield to any force impressed on it, by yielding, are easily moved among themselves."*

With a modest change to the above, describing the nature of the force, we still use this simple definition. A fluid can be defined as

*"a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be."*

From this definition, it is clear that two states of matter, Liquid and gas, are fluids. Even though solids yield under shear stress, the deformation it undergoes is finite and once the force is released, unlike fluids, it tends to assume its initial shape.

## 1.2 Fluid as a continuum

Fluids, like any other substance, are made of molecules. Weak cohesive forces keep molecules attracted to each other. However, the molecules are in constant motion. Distance a molecule travel before hitting another is called the mean free path  $\lambda$ . This mean free path is directly proportional to the temperature and inversely proportional to the pressure.

If we look at a liquid at microscopic length scale, we would be able to see molecules of the liquid moving in the space bouncing off each other and the container wall. At this length scale, fluids are discontinuous spatially. However, we very seldom work at this length scale when handling fluids. At a larger length scale, for instance when we consider a tiny liquid droplet of about 1 mm radius, it appears as a continuous phase. In this example, the diameter of the droplet is called the characteristic length: the length scale at which we observe the droplet. Assume the characteristic length scale to be  $L$ . The ratio between the mean free path and the characteristic length gives a nondimensional quantity called Knudsen number.

$$Kn = \frac{\lambda}{L} \quad (1.1)$$

Knudsen number gives a feeling about the continuity of a fluid at the length scale of observation.

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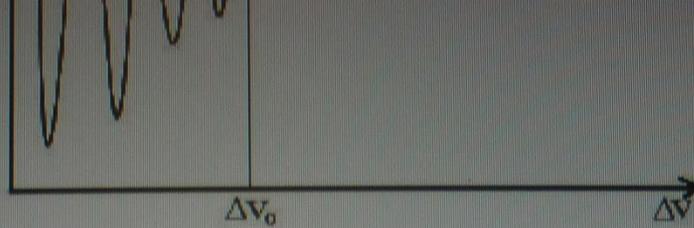
$Kn \leq 0.001 \Rightarrow L \geq 1000\lambda$ , fluid can be considered as a continuum.

$0.001 \leq Kn \leq 0.1 \Rightarrow 100\lambda \leq L \leq 10\lambda$ , rarefaction effects start to influences the properties.

Around  $Kn = 0.1$ , the assumption that a liquid is a continuum starts to break down.

$Kn > 10$  we are looking at molecules at a length scale smaller than their mean free path; the continuum approach breaks down completely.





**Figure 1.1** Variation of the ratio  $\Delta m/\Delta V$  against increasing length scale.

## 1.3 Properties of fluids

A bottle would weigh more when filled with water than olive oil. Again, you might have observed that honey flows slower than water. Fluids differ from one another due to the differences of the inherent properties. Important properties to consider when learning mechanics of fluids are

1. Density
2. Viscosity and
3. Surface tension

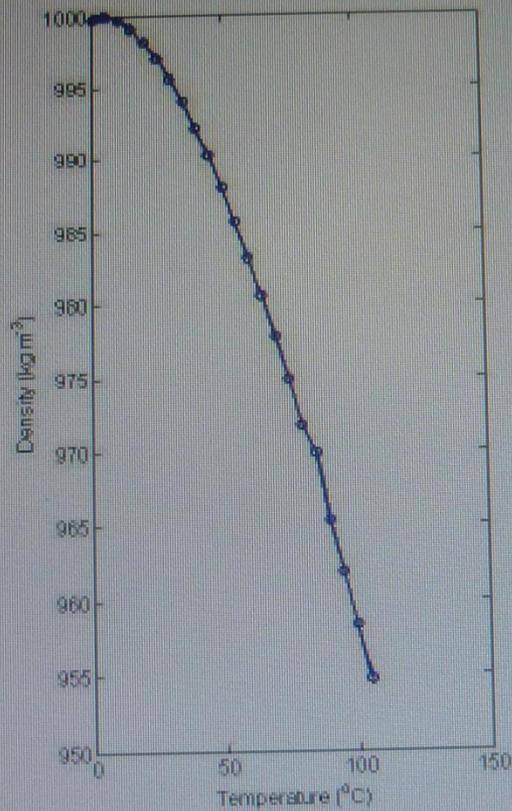
There are other properties such as boiling point, freezing point that are not considered here.

### 1.3.1 Density

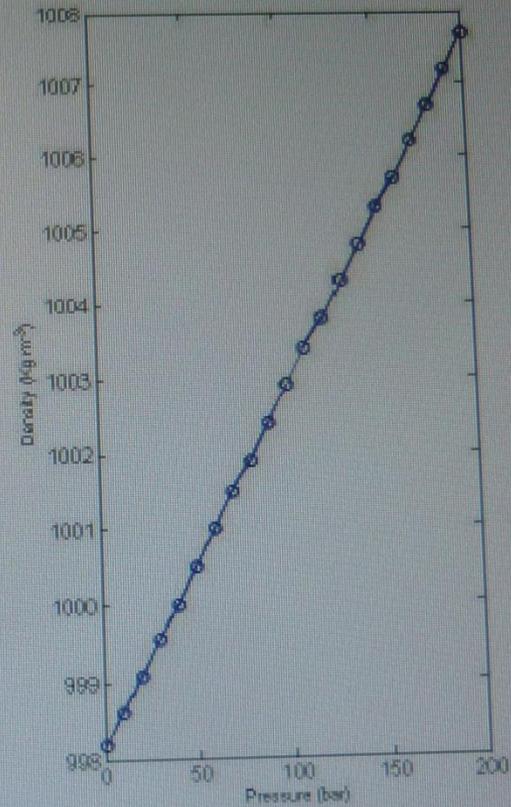
Density of any substance (i.e. fluids and solids) is defined as the mass of a unit volume of that substance. It is often expressed in  $\text{kg}/\text{m}^3$  and usually designated by the Greek symbol  $\rho$  (rho). Therefore, the density,

$$\rho = \frac{m}{V} \quad (1.2)$$

where  $m$  and  $V$  represent the mass and the volume respectively. On the other hand, the specific volume is the volume per unit mass. It is given by the reciprocal of the density –that is



(a)



(b)

Figure 1.2. Variation of density with temperature and pressure

1.1. Density of some common fluids are given in Table 1.1. Usually, densit

Fluid	Density/(Kg m <sup>-3</sup> )	Viscosity/(Pa s)
<b>Gases</b>		
Air	1.205	$1.8 \times 10^{-5}$
Ammonia	0.717	$9.8 \times 10^{-6}$
Carbon dioxide	1.842	$1.4 \times 10^{-5}$
Chlorine	2.994	$1.29 \times 10^{-5}$
Oxygen	1.331	$1.92 \times 10^{-5}$
<b>Liquids</b>		
Water	998	0.001
Olive oil	800	0.081
Castor oil	955	0.985
Glycerol	1260	1.495
Kerosene	820	0.0025

**Table 1.1** Properties of common gasses and liquids at 20 C and 1 atm pressure.

### 1.3.2 Viscosity

As already pointed out, different liquids flow at different rates given all other conditions remains same. This means there is some property that affects the way fluids flow. This property is called *viscosity*.

Viscosity of a fluid originates from the nature of molecular interactions. Liquids, unlike gasses, have restricted molecular

respect to other molecules. To dislocate a molecule, a certain amount of energy is required. Viscosity is the energy that needs to dislocate a mole of a fluid<sup>1</sup>.

$$\mu = \frac{\bar{N}h}{\bar{V}} e^{(3.8T_b/T)} \quad (1.5)$$

- $\mu$  = Viscosity
- $\bar{N}$  = Avogadro number
- $h$  = Plank's constant
- $\bar{V}$  = Volume of a mole of liquid
- $T_b$  = Boiling point of the liquid
- $T$  = Temperature

Viscosity characterises the flow of fluids. Newton, studying the flow realised that the applied shear force and the amount of deformation relate to one another. For example consider a rectangular fluid packet as shown in figure 1.3. A shear force  $F$  is applied to the upper surface at time  $t=0$ . During a small period of  $\delta t$ , upper surface moves a small distance  $\delta x$  deforming the rectangle to its new position shown in (b).

For the proof and an informative discussion see Bird, R.B., Stewart, W.E. and Lightfoot, E.N., Transport Phenomena, 2 Edition, John Wiley, 2002

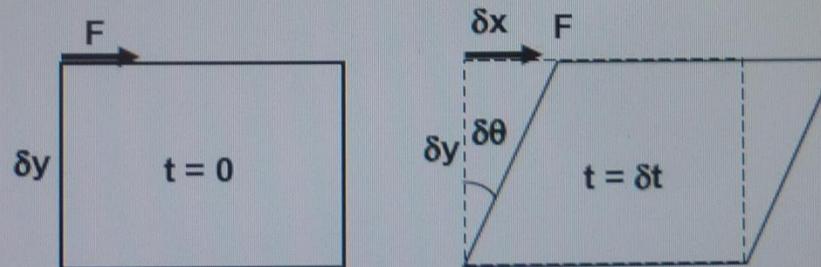


Figure 1.3. Deformation of a rectangular fluid element under applied shear stress

As long as the force  $F$  is applied, the fluid element will continue to deform. The rate of deformation is given by the rate at which the angle  $\delta \theta$  changes. The rate of deformation is proportional to the shear stress applied. Shear stress is normally

$$\tau \propto \frac{\delta\theta}{\delta t}$$

The angle  $\delta\theta$  is given by

$$\tan\delta\theta = \frac{\delta x}{\delta y}$$

For small angles  $\tan\delta\theta \approx \delta\theta$

Therefore,

$$\tau \propto \frac{\delta x}{\delta y \delta t}$$

Since  $\lim_{\delta t \rightarrow 0} \frac{\delta x}{\delta t} \rightarrow du$ , where  $du$  is the velocity induced by the applied force.

$$\tau \propto \frac{du}{dy}$$

Newton postulated that proportionality constant is the viscosity. This gives



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Newton postulated that proportionality constant is the viscosity. This gives

$$\tau = \mu \frac{du}{dy} \tag{1.7}$$

This equation achieves dimensional homogeneity only if  $\mu$  has units Pa s (Pascal seconds). However, it is common practice to give the viscosity in *Poise* (P) or *centipoises* (cP), a unit named after French physicist Jean Marie Poiseuille.

$$1P = 1 \text{ g cm}^{-1} \text{ s}^{-1}$$

The term  $\frac{du}{dy}$  is called the *velocity gradient*.

Above equation shows that the shear stress is linearly proportional to the velocity gradient. Fluids that show this linear relationship is called *Newtonian fluids*. Water, air, and crude oil are some examples of Newtonian fluids. However, there are fluids that do not show the linear relationship. They are called non-Newtonian fluids. Polymer melts, xanthan gum

Viscosity is measured using a wide range of viscometers that measures the time taken to flow a known amount of the liquid or measuring the shear rate indirectly measuring the torque of a shaft rotating in the liquid. Ostwalt, Cannon-Fenske and Saybolt viscometers measures the flow time and cone and plate type viscometers use the torque measurements.

### 1.3.3 Surface Tension

Consider a liquid at rest in an open vessel. The liquid surface is in contact with the air at the room temperature. Consider a molecule of the liquid in the bulk surrounded by other molecules as shown by A in Figure 1.5. As we have discussed in section 1.2, this molecule is attracted to the neighbouring molecules making it to move. If the time averaged distance is considered, the molecule will be in the close vicinity of its initial location as the force exerted by the neighbouring molecules acts on all directions.

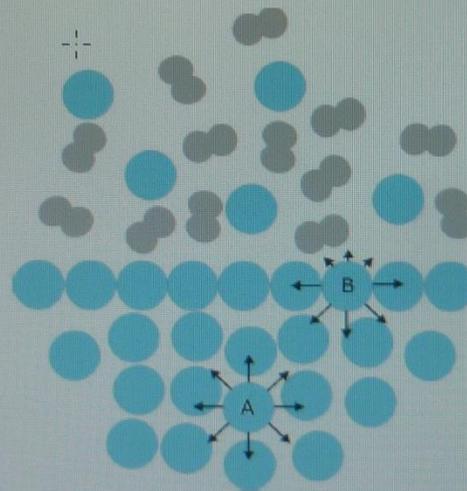


Figure 1.5. Intermolecular forces acting on liquid molecules.



Consider a spherical soap bubble with radius  $r$ . It has two surfaces, one inside and the other outside as shown in Figure 1.6. Assume the surface tension of the soap solution is  $\sigma$ . Pressure inside has to be higher than the outside. The force applied on the projected area is  $\Delta p \times \pi r^2$ . This force is balanced by the surface tension. The force exerted by the surface tension is  $2\pi r\sigma$ .

Considering the force balance,

$$\Delta p \times \pi r^2 = 2 \times 2\pi r\sigma$$

$$\Delta p = 4\sigma \left( \frac{1}{r} \right)$$

For a surface with difference curvatures in the two mutually perpendicular directions (like that of an ellipsoid) the above equation can be written as

$$\Delta p = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \quad (1.9)$$

This equation is known as the Young-Laplace equation. 

Another interesting concept associated with surface tension is the wetting property. A liquid drop, when deposited on a solid substrate, will spread until it reaches the equilibrium. The line at which the liquid, air and the solid substrate meet is known as the contact line. Equilibrium is achieved when the forces acting at the contact line balances each other. A liquid



**Figure 1.7.** Contact angle and the forces acting at the contact line of a liquid drop sitting on a horizontal plate.

Figure 1.7 shows the forces acting at the contact line.

$\sigma_{l,a}$  : liquid-air interfacial energy (surface tension)

$\sigma_{l,s}$  : liquid-solid interfacial energy

$\sigma_{s,a}$  : solid-air interfacial energy

Like surface tension the other two are also defined as energies per unit area or forces acting on a unit length of the contact line. The force balance gives

$$\sigma_{a,s} = \sigma_{l,s} + \sigma_{a,l} \cos \theta.$$



A liquid will spread on any surface until the free energy assumes the minimum possible value. Liquids rise up in capillaries against gravity due to the same reason.

### 1.3.4 Compressibility

Seventeenth century British philosopher/physicist Robert Boyle published his observations on the influence of pressure on a fixed volume of gas in the second edition of his book<sup>2</sup> “*New Experiments Physico-Mechanicall, Touching the Spring of the Air,.....*” published in 1662. He observed that for a fixed amount of an ideal gas maintained at a constant temperature, the volume (V) is inversely proportional to the pressure (P).

$$V \propto \frac{1}{P}$$

Change of volume in a unit volume per unit change of pressure is defined as the compressibility. If the change of a unit volume is  $\delta v$  for an increase of pressure by a  $\delta p$  amount, the compressibility can be defined as

$$K = \frac{\delta V}{V \delta P} \quad (1.11)$$

1/K is called the bulk modulus and is a measure of resistance to the change of volume under pressure. K itself is a function of pressure.

2 Boyle, R., 1662, “*New Experiments Physico-Mechanicall, Touching the Spring of the Air and its Effects (Made, for the Most Part, in a New Pneumatical Engine) Written by Way of Letter to the Right Honorable Charles Lord Vicount of Dungarvan, Eldest Son to the Earl of Corke*”. 2nd Ed., Oxford

This book considers incompressible fluids unless stated otherwise.

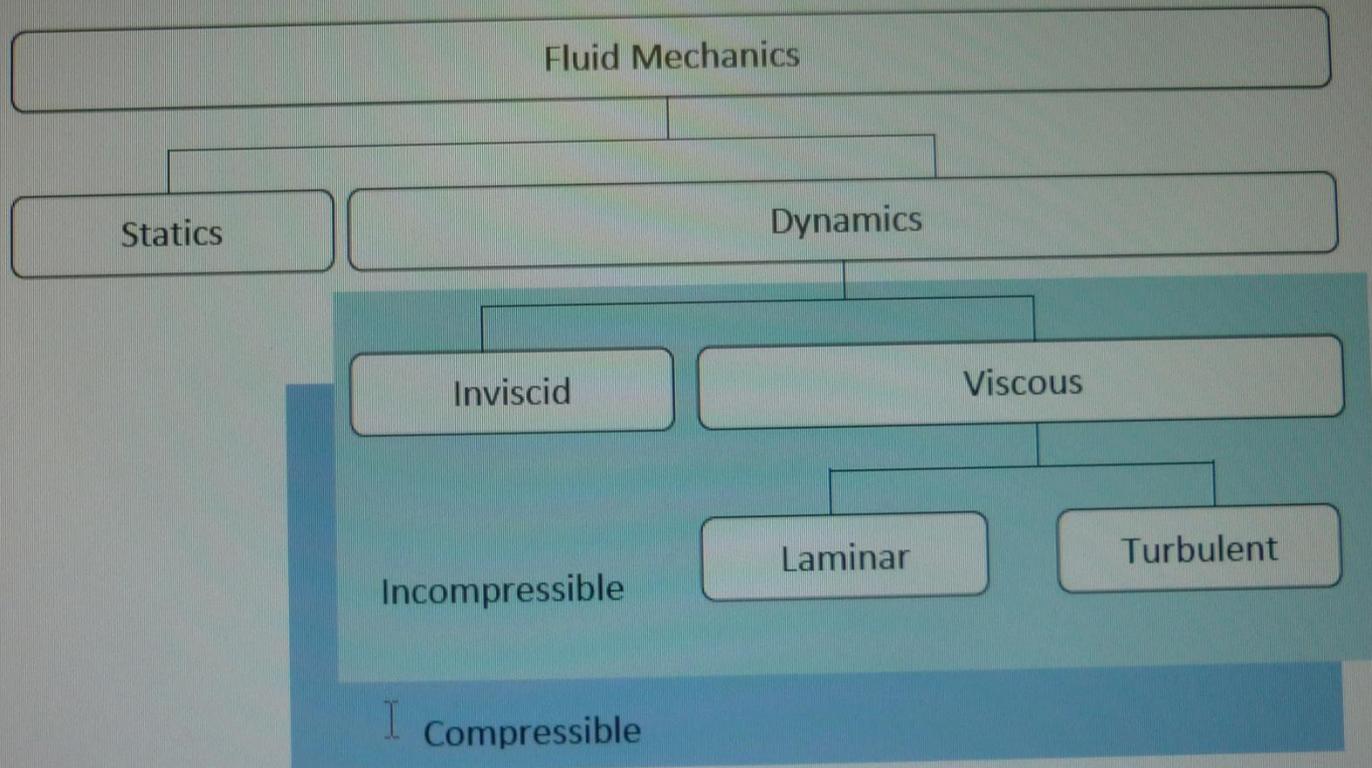


Figure 1.9. A broad classification of fluid mechanics



# 2 Fluid Statics

## Introduction

Statics is the area of fluid mechanics that studies fluids at rest. It also extends to fluids in motion when there is no relative motion between adjacent fluid particles (e.g. rigid body motion). In Chemical engineering context, fluid statics provide an essential body of knowledge to design liquid storage tanks considering forces acting on viewing glasses, sluice gates, required wall thicknesses etc.

Imagine a small fluid element surrounded by the rest of the fluid. The boundary of this fluid particle experience shear stress due to intermolecular forces. The fluid element experiences the gravitational force irrespective to its motion or the position. The force acting on the fluid element due to the gravity is given by the product  $mg$  where  $m$  and  $g$  are the mass of the element and the acceleration due to gravity respectively. Such forces appear in fluids due to external fields such as gravity or electromagnetic fields are called the body forces. Body forces acts on the whole volume of the fluid particle: hence the name.

The stresses appearing at the boundary could be divided into two categories; (1) pressure and (2) viscous stresses. The viscous stresses arise due to the relative motion of neighbouring fluid molecules. Viscous stresses on this fluid particle change locally depending on the relative velocity of the surrounding fluid. Therefore, for fluids at rest it is important to understand the role of the pressure.



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## 2.1 Pressure

In weather reports, you might have heard of “high pressure” or “low pressure” regions that make clouds to move. This refers to the force air mass above the ground applying on a unit area on the ground.

Pressure is defined as the total force applied normal (perpendicular) to a unit surface area.

$$\text{Pressure} = \frac{\text{Total static force exerted normal to the area}}{\text{Area on which the force is applied}}$$

Consider a force  $F$  applied on an area  $A$  as shown in Figure 2.1.



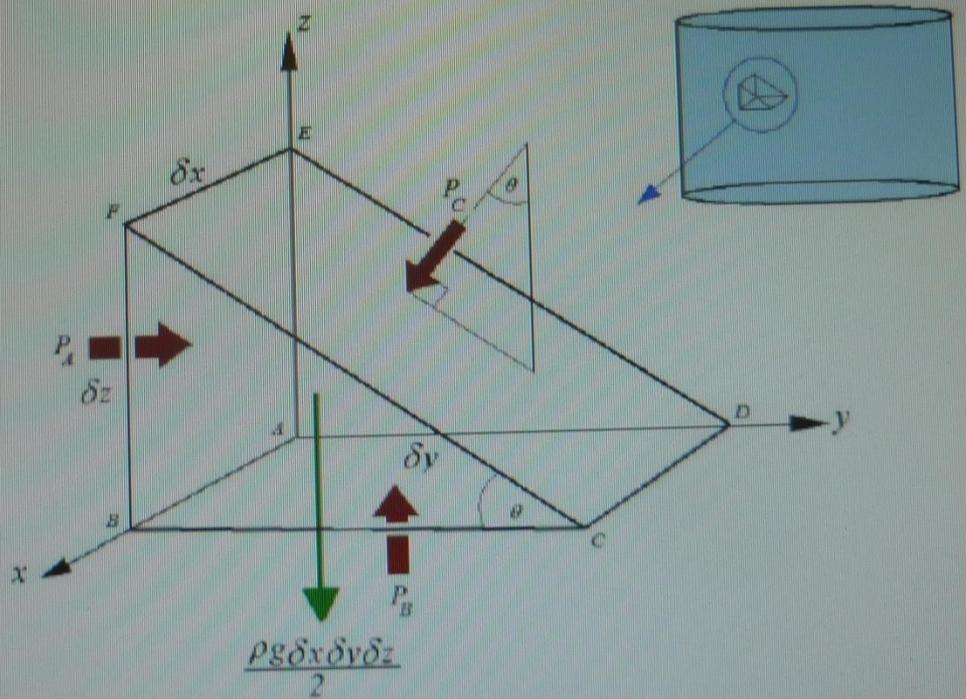


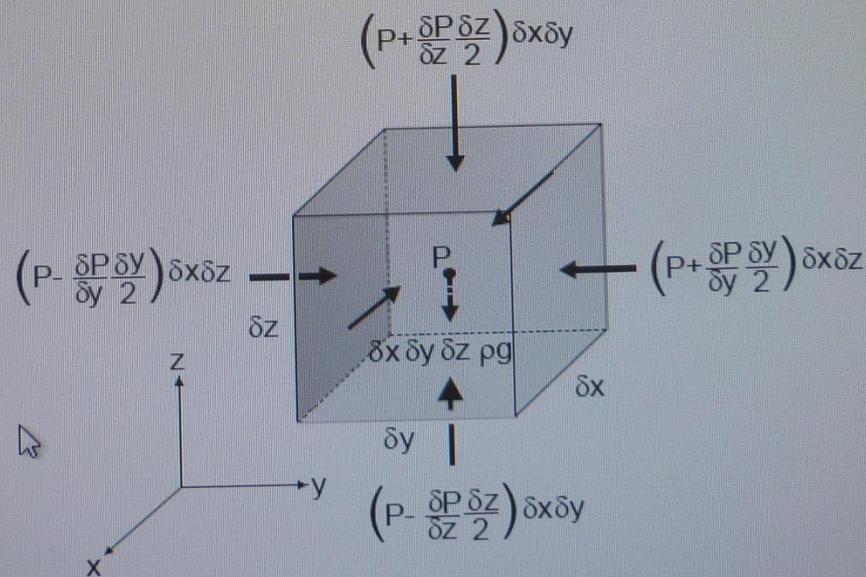
Figure 2.2. Pressure acting on a fluid prism at rest.



## 2.3 Pressure variation in a static fluid

Consider a static fluid at equilibrium. Pressure at any arbitrary point is indicated by  $P$ . Assume an infinitesimal fluid element with sides  $\delta x$ ,  $\delta y$ , and  $\delta z$  having the arbitrary point at the centre. The weight of the fluid element acts in the direction of gravity only. This is shown in the Figure 2.3.

Pressure at a point  $\delta y$  distance to the right of the initially selected point is  $P + \delta p$ . Therefore, the variation of pressure in  $y$  direction per unit distance can be defined as  $\delta P / \delta y$ . Therefore, pressure at the centre of the surface  $\frac{\delta y}{2}$  distance to the right of the selected point can be written as  $P + \frac{\delta P}{\delta y} \frac{\delta y}{2}$ . Similarly, pressure at a point  $\frac{\delta y}{2}$  to the left of the selected point will be  $P - \frac{\delta P}{\delta y} \frac{\delta y}{2}$ . The sign convention assumed that the pressure increases in the positive directions of the Cartesian coordinates. Figure 2.3 shows the pressures at the surfaces of the fluid element (values for the  $x$ -direction is not shown).



Taking the limit  $\delta z \rightarrow 0$  suggests that we have shrunk the volume to an infinitesimally small region. This emphasizes the spatial continuity of the property.

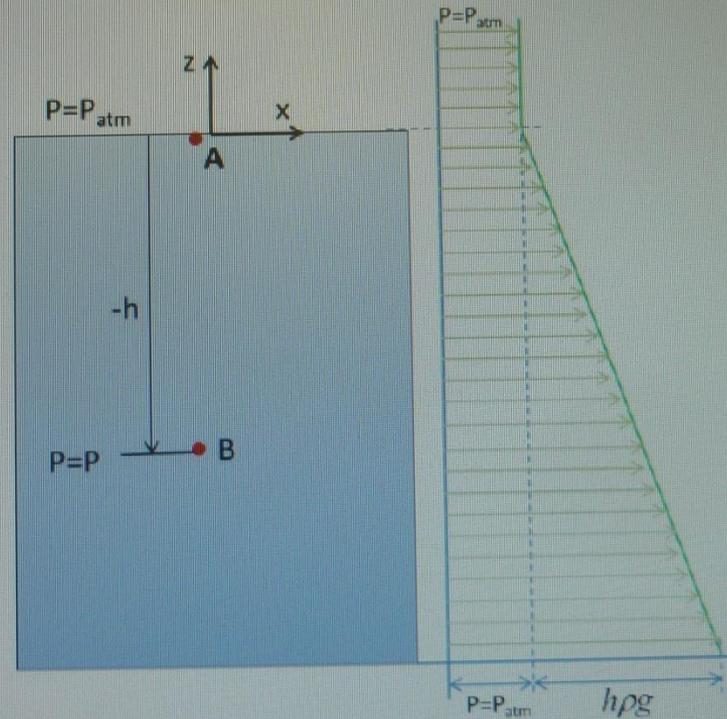


Figure 2.4. Pressure variation in the direction of gravity



## 2.4 Pressure and head

Pressure has the units  $\text{N/m}^2$ . This could be written as  $\text{J/m}^3$  and pressure could be defined as the energy per unit volume of the fluid. The  $h\rho g$  in equation 2.11 gives the same units indicating that it is a form of energy. In fluid mechanics a common term used to indicate pressure is the *head*. *Head is defined as the energy per unit weight of fluid.*

Dividing equation 2.11 by  $\rho g$ , we get the head due to the height of a liquid column as

$$\frac{P - P_{atm}}{\rho g} = \frac{h\rho g}{\rho g} = h \quad (2.12)$$

Energy per unit weight or the head at a point within a static fluid is simply given by the height of the fluid above the point. Head is a concept we will encounter again when we discuss dynamics in chapter 4 and pumping of fluids in chapter 7.

## 2.5 Measurement of pressure

Pressure is a very important characteristic of fluids. As a result there are many measuring techniques used to measure pressure. As shown in equation 2.11, height of a liquid column could be used to measure pressure.

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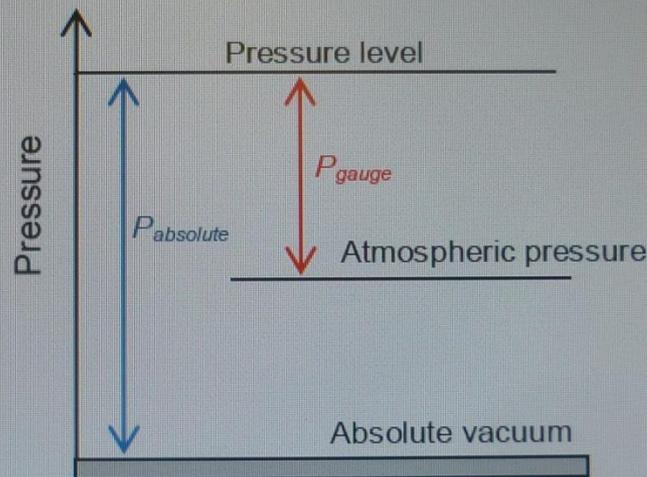


Figure 2.5. Relationship between gauge, absolute and atmospheric pressures.

It is essential to understand that the pressure is measured relative to the atmospheric pressure. In other words, some measuring techniques measure the pressure difference between the fluid and the atmosphere. This is called the *gauge pressure*. For instance,  $P - P_{atm}$  gives the gauge pressure. To obtain the *absolute pressure* one has to add the atmospheric