Intermediate Thermodynamics

Cycles

Heat-Engine Cycle

A heat engine produces work receiving heat from a reservoir at T_H and rejecting heat to a reservoir at T_C. The first law requires that:

$$Q_H + Q_C + W = 0 \tag{1}$$

since the change in energy within a cycle is zero. The second states that:

$$\sigma_{eng} = \Delta S - \sum_i \frac{Q_i}{T_i} = -\frac{Q_H}{T_h} - \frac{Q_C}{T_c} \geq 0 \tag{2}$$

where the change in entropy is zero in a cycle and T_h and T_c are the engine surface temperatures at which heat is received and rejected, respectively. The irreversibility can be calculated from Eq. (3:23) for a control mass:

$$I_{eng} = W_{act} + Q_H\left(1 - \frac{T_0}{T_H}\right) + Q_C\left(1 - \frac{T_0}{T_C}\right) \tag{3}$$

The effectiveness of a heat engine is defined as:

$$\varepsilon = \left\| \frac{W_{act}}{\Phi_{Q,H}} \right\| = \frac{\eta_{act}}{\eta_{Carnot}} \tag{4}$$

where the last equality can be proved using Eqs. (1-3) for the reversible case.

In the case of irreversible heat transfer with the heat reservoirs, the irreversibility within the engine is:

$$I_{eng} = W_{act} + Q_H\left(1 - \frac{T_0}{T_h}\right) + Q_C\left(1 - \frac{T_0}{T_c}\right) \tag{5}$$

and the irreversibilities associated to the heat transfers from the hot and to the cold reservoir are:

$$I_{Q,H} = T_0\, Q_H \left(\frac{1}{T_h} - \frac{1}{T_H} \right), \qquad I_{Q,C} = T_0\, Q_C \left(\frac{1}{T_C} - \frac{1}{T_c} \right) \tag{6}$$

Refrigeration and Heat Pump Cycles

A refrigeration cycle uses work to take heat from a reservoir at T_C while rejecting heat to a reservoir at T_H.. First and second law requirements are the same as Eqs. (1) and (2). The coefficient of performance (COP) is defined as:

$$COP_{ref} = \frac{Q_C}{W} \tag{7}$$

and the effectiveness is defined as:

$$\varepsilon_{ref} = \left\| \frac{\Phi_{Q,C}}{W_{act}} \right\| = \frac{COP_{act}}{COP_{Carnot}} \tag{8}$$

A heat pump cycle works like a refrigeration cycle but the objective is to reject heat to be used in a process or in building heating. The COP and the effectiveness are thus defined as:

$$COP_{ref} = \frac{Q_H}{W}, \qquad \varepsilon_{hp} = \left\| \frac{\Phi_{Q,H}}{W_{act}} \right\| = \frac{COP_{act}}{COP_{Carnot}} \tag{9}$$

Gas-Turbine Cycle

A schematic of a gas-turbine is shown in Fig. 1. Notice that the cycle is open. Assuming adiabatic compression and expansion, the first law states:

$$w_c = h_{2a} - h_1, \qquad q_{comb} = h_3 - h_{2a}, \qquad w_c = h_{4a} - h_3 \tag{10}$$

The efficiency of the compression and turbine processes are:

$$\eta_c = \frac{h_{2s} - h_1}{h_{2a} - h_1}, \qquad \eta_t = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \tag{11}$$

Availability balances are written as:

$$w_c = \left(b_{2a} - b_1 \right) + i_c \tag{12}$$

$$\sum_i q_i \left(1 - \frac{T_0}{T_i}\right) = b_3 - b_{2a} + i_{comb} \tag{13}$$

$$w_t = \left(b_{4a} - b_3\right) + i_t \tag{14}$$

and the cycle effectiveness is:

$$\varepsilon_{cycle} = \frac{work\ output}{exergy\ input} = \frac{w_{net}}{\Delta\Psi_{comb}} \tag{15}$$

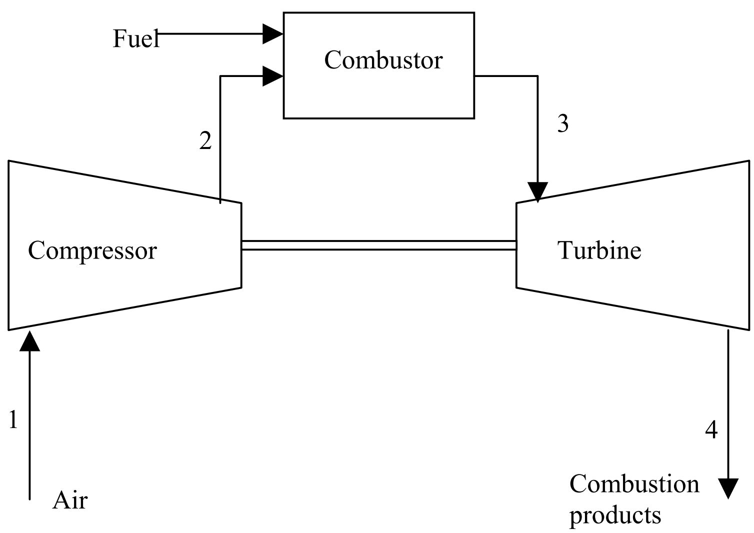

Figure 1: Gas-turbine open cycle.

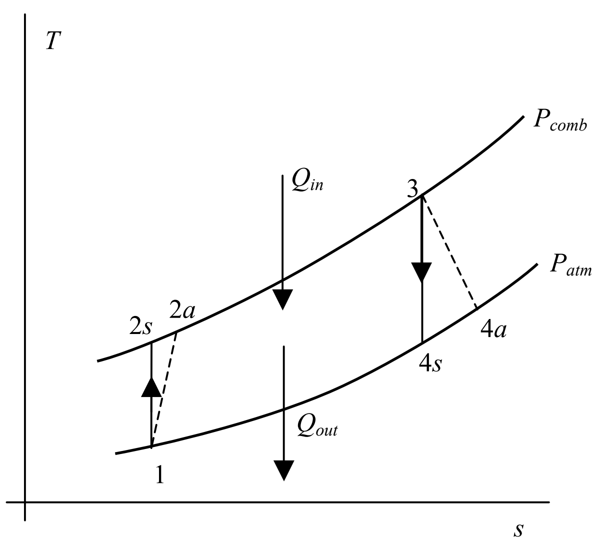

Figure 2: T-s diagram.

It is frequent to use a regenerator to take advantage of the high temperature gases on the output of the turbine to preheat the air at the entrance of the combustor. This is called a regenerative turbine cycle. The schematics are shown in Fig. 3.

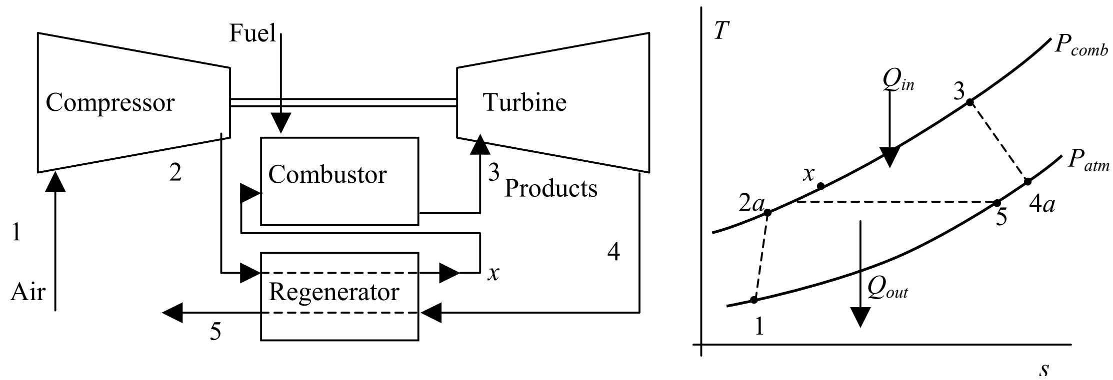

Figure 3: Regenerative gas-turbine open cycle.

The energy balance on the regenerator, assuming adiabatic behavior is:

$$h_x - h_2 = h_5 - h_4 \tag{16}$$

and the effectiveness is defined as:

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \tag{17}$$

The availability balance for the regenerative cycle can be written in *input=output* format as:

$$\sum_i q_i\left(1 - \frac{T_0}{T_i}\right) + b_1 = b_5 - (w_t - w_c) + i_c + i_t + i_{reg} \tag{18}$$

where all the irreversibilities have been neglected with the exception of that of the compressor, turbine and regenerator units.

Steam Power Cycle

A simple Rankine cycle is shown in Fig. 4. The Rankine cycle is extensively used in steam power stations, typically nuclear and coal fueled.

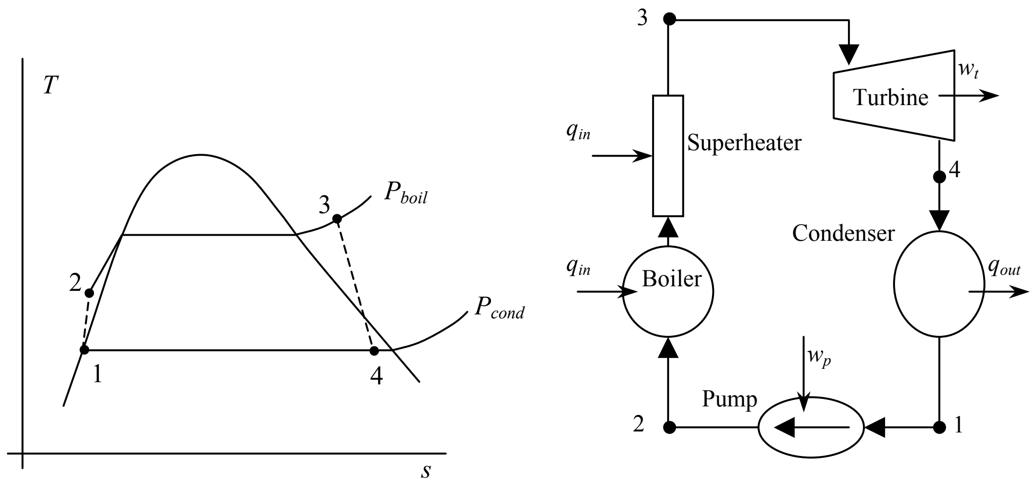

Figure 4: Rankine cycle.

Neglecting kinetic and potential energy, the steady-state energy balances on the components of the cycle are (assuming adiabatic turbine and pump):

$$q_{in} = h_3 - h_2, \quad w_t = h_4 - h_3, \quad q_{cond} = h_1 - h_4, \quad w_p = h_2 - h_1 \tag{19}$$

and the turbine and pump efficiencies are:

$$\eta_t = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}, \qquad \eta_p = \frac{h_{2s} - h_1}{h_{2a} - h_1} \tag{20}$$

The exergy balances are:

$$\sum_i q_i \left(1 - \frac{T_0}{T_i}\right) + b_2 = b_3 + i_{boiler} \tag{21}$$

$$b_3 = b_{4a} - w_t + i_t \tag{22}$$

$$b_{4a} = b_1 - \sum_i q_i \left(1 - \frac{T_0}{T_i}\right) + i_{cond} \tag{23}$$

$$w_p + b_1 = b_{2a} + i_p \tag{24}$$

and the effectiveness is given by Eq. (15).

Refrigeration Cycle

A vapor-compression refrigeration cycle is shown in Fig. 5. Under the same assumptions as in the previous section we can write:

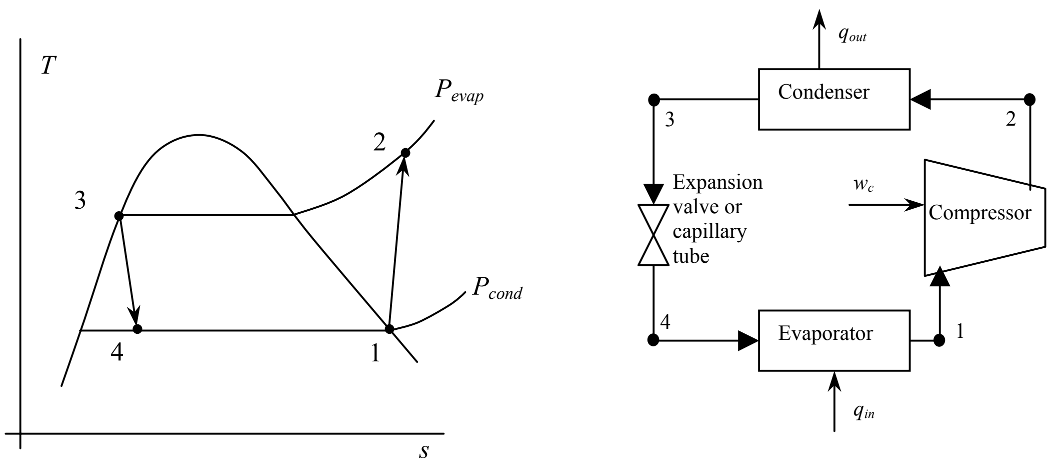

Figure 5: Vapor compression refrigeration cycle.

Neglecting kinetic and potential energy, the steady-state energy balances on the

$$q_c + w_c = h_{2a} - h_1, \quad q_{cond} = h_3 - h_{2a}, \quad h_3 - h_4, \quad q_{evap} = h_1 - h_4 \tag{25}$$

where heat losses on the compressor have been included. Availability balances result in:

$$b_1 + w_c = b_2 - \sum_i q_i \left(1 - \frac{T_0}{T_i}\right) + i_c \tag{26}$$

$$b_2 = b_3 - \sum_i q_i \left(1 - \frac{T_0}{T_i}\right) + i_{cond} \tag{27}$$

$$b_3 = b_4 + i_{valve} \tag{28}$$

$$b_4 = b_1 - \sum_i q_i \left(1 - \frac{T_0}{T_i}\right) + i_{evap} \tag{29}$$

and the effectiveness is defined in this case as:

$$\varepsilon_{cycle} = \frac{refrigeration}{work\ input} = -\frac{\phi_{Q,evap}}{w_c} \tag{30}$$

Homework

1.- An irreversible gas turbine operates between pressures of 1.0 and 6.4 bar with compressor and turbine inlet temperatures of 22 and 807 ^oC. Determine the compressor and turbine work, the change in availability across each device, and the irreversibility of all three devices, in *kJ/kg* of air. Consider then the case in which a regenerator is added to preheat the inlet of the compressor with the output of the turbine. Assume a 75 % effectiveness for the regenerator and repeat the previous analysis.

2.- A steam power cycle operates with superheated vapor at 140 bar, 560 ^oC, and condenses steam at 0.06 bar. The cooling water on the cold side of the condenser rises from 18 to 28 ^oC. The first-law efficiencies of the turbine and the pump are 85 % and 70 % respectively. Perform an energy and exergy analysis of the cycle.