

# Learning Objectives

- General Layout of the System
- Power System and System Networks
- Systems of A.C. Distribution
- Effect of Voltage on Transmission Efficiency
- Comparison of Conductor Materials Required for Various Overhead Systems
- Reactance of an Isolated Single-Phase Transmission Line
- A.C. Distribution
   Calculations
- Load Division Between Parallel Lines
- Suspension Insulators
- Calculation of Voltage Distribution along Different Units
- Interconnectors
- Voltage Drop Over the Interconnectors
- Sag and Tension with Support at Unequal Levels
- Effect of Wind and Ice

# A.C. TRANSMISSION AND DISTRIBUTION



The above figure shows a mini AC power plant. AC has distinct advantages over DC in generation as well as transmission.

#### 41.1. General Layout of the System

The conductor system by means of which electric power is conveyed from a generating station to the consumer's premises may, in general, be divided into two distinct parts *i.e.* transmission system and distribution system. Each part can again be sub-divided into two—primary transmission and secondary transmission and similarly, primary distribution and secondary distribution and then finally the system of supply to individual consumers. A typical layout of a generating, transmission and distribution network of a large system would be made up of elements as shown by a single-line diagram of Fig. 41.1 although it has to be realized that one or more of these elements may be missing in any particular system. For example, in a certain system, there may be no secondary transmission and the distribution system proper may begin at the generator bus-bars.

Now-a-days, generation and transmission is almost exclusively three-phase. The secondary transmission is also 3-phase whereas the distribution to the ultimate customer may be 3-phase or singlephase depending upon the requirements of the customers.

In Fig. 41.1, C.S. represents the central station where power is generated by 3-phase alternators at 6.6 or 11 or 13.2 or even 32 kV. The voltage is then stepped up by suitable 3-phase transformers for transmission purposes. Taking the generated voltage as 11 kV, the 3-phase transformers step it up to 132 kV as shown. Primary or high-voltage transmission is carried out at 132 kV\*. The transmission voltage is, to a very large extent, determined by economic considerations. High voltage transmission requires conductors of smaller cross-section which results in economy of copper or aluminium. But at the same time cost of insulating the line and other expenses are increased. Hence, the economical voltage of transmission is that for which the saving in copper or aluminium is not offset (*i*) by the increased cost of insulating the line (*ii*) by the increased size of transmission-line structures and (*iii*) by the increased size of generating stations and sub-stations. A rough basis of determining the most economical transmission voltage is to use 650 volt per km of transmission line. For example, if transmission line is 200 km, then the most economical transmission voltage will be  $200 \times 650 \approx 132,000$  V or 132 kV.

The 3-phase, 3-wire overhead high-voltage transmission line next terminates in step-down transformers in a sub-station known as Receiving Station (R.S.) which usually lies at the outskirts of a city because it is not safe to bring high-voltage overhead transmission lines into thickly-populated areas. Here, the voltage is stepped down to 33 kV. It may be noted here that for ensuring continuity of service *transmission is always by duplicate lines*.

From the Receiving Station, power is next transmitted at 33 kV by underground cables (and occasionally by overhead lines) to various sub-stations (SS) located at various strategic points in the city. This is known as secondary or low-voltage transmission. From now onwards starts the primary and secondary distribution.

At the sub-station (SS) voltage is reduced from 33kV to 3.3kV 3-wire for primary distribution. Consumers whose demands exceeds 50 kVA are usually supplied from SS by special 3.3 kV feeders.

The secondary distribution is done at 400/230 V for which purpose voltage is reduced from 3.3 kV to 400 V at the distribution sub-stations. Feeders radiating from distribution sub-station supply power to distribution networks in their respective areas. If the distribution network happens to be at a great distance from sub-station, then they are supplied from the secondaries of distribution transformers which are either pole-mounted or else housed in kiosks at suitable points of the distribution networks. The most common system for secondary distribution is 400/230-V, 3-phase 4-wire system. The single-phase residential lighting load is connected between any one line and the neutral

High voltages like 750 kV are in use in USSR (Konakovo-Moscow line) and 735 kV in Canada (Montreal-Manicoagan Scheme).



whereas 3-phase, 400-V motor load is connected across 3-phase lines directly.

It should be noted that low-voltage distribution system is sub-divided into feeders, distributors and service mains. No consumer is given direct connection from the feeders, instead consumers are connected to distribution network through their service mains. The a.c. distributors are, in many ways, similar to the d.c. distributors as regards their constructional details and restrictions on drops in voltage.

Summarizing the above, we have

- 1. Generating voltage : 6.6, 11, 13.2 or 33 kV.
- 2. High voltage transmission : 220 kV, 132 kV, 66kV.
- **3.** High voltage or primary distribution : 3.3, 6.6 kV.
- Low-voltage distribution : A.C. 400/230, 3-phase, 4-wire D.C. 400/230 ; 3-wire system

The standard frequency for a.c. working is 50 Hz or 60 Hz (as in U.S.A.). For single-phase traction work, frequencies as low as 25 or 16 2/3Hz are also used.

### 41.2. Power Systems and System Networks

It is a common practice now-a-days to interconnect many types of generating stations (thermal and hydroelectric etc.) by means of a common electrical network and operate them all in parallel.

This combination of generating stations forms what is known as *power system*. The various elements of such a system like generating stations, transmission lines, the substations, feeders and distributors etc. become tied into a whole by the integrated process of continuous generation and consumption of electric energy.

A system network (or grid) is the name given to that part of power system which consists of the sub-stations and transmission lines of various voltage rating.

Fig. 41.2 shows single-line diagram representing the main connections of a power system consisting of a heating and power central station (HCS), a large-capacity hydro-electric station (HS) and



two regional thermal power stations (*RTS*-1 and *RTS*-2). The stations *HS*, *RTS*-1 and *RTS*-2 are situated at large distances from the consumers, hence voltage of the electric power generated by them has to be stepped up by suitable transformers before it is fed into the system transmission network.

As shown in Fig. 41.2, *HS* is connected with the main 110-kV network of the system with the help of (*i*) two-220 kV transmission lines *L*-1 and (*ii*) main (regional) sub-station *A* which houses two 220/110 kV, 2-winding transformers for interconnecting the two transmission lines.

Transmission lines L-2, L-3 and L-4 constitute a high-voltage loop or ring mains. As seen, disconnection of any one of the lines will not interrupt the connections between the elements of the system. Station *RTS*-1 feeds directly into the 110-kV line loop whereas *RTS*-2 is connected to the main network of the system by lines L-5 and L-6 through the buses of substations *B* and *C*. *HCS* is interconnected with 110-kV system through the buses of substation A by means of 10/110-kV transformers and line L-7.

It may be pointed out here that the main substations of the system are A and B. Substation B houses 3-winding transformers. The 35-kV supply is provided over quite large areas whereas 6-10 kV supply is supplied to consumers situated with a limited radius from the substations. Substations C and D are equipped with 2-winding transformers which operate at voltages indicated in the diagram. Substation C is known as a *through substation* whereas D is known as a *spur* or *terminal* substation.

Obviously, Fig. 41.2 shows only part of 220-kV and 110-kV lines and leaves out the 35, 10 and 6-kV circuits originating from the buses of the substations. Also, left out are low-voltage circuits for transmission and distribution (see Fig. 41.9).

## 41.3. Systems of A.C. Distribution

As mentioned earlier, a.c. power transmission is always at high voltage and mostly by 3-phase system. The use of single-phase system is limited to single-phase electric railways. Single-phase power transmission is used only for short distances and for relatively low voltages. As will be shown later, 3-phase power transmission requires less copper than either single-phase or 2-phase power transmission.

The distribution system begins either at the sub-station where power is delivered by overhead transmission lines and stepped down by transformers or in some cases at the generating station itself. Where a large area is involved, primary and secondary distributions may be used.

With respect to phases, the following systems are available for the distribution of a.c. power.

- 1. Single-phase, 2-wire system.
- 2. Single-phase, 3-wire system.
- 3. Two-phase, 3-wire system.
- 4. Two-phase, 4-wire system.
- 5. Three-phase, 3-wire system.
- **6.** Three-phase, 4-wire system.

# 41.4. Single-phase, 2-wire System



The above figure shows an electrical substation. A substation is a high-voltage electric system facility. It is used to switch generators, equipment, and circuits or lines in and out of a system. It is also used to change AC voltages from one level to another, and/or change alternating current to direct current or direct current to alternating current. Some substations are small with little more than a transformer and associated switches.

It is shown in Fig. 41.3 (a) and (b). In Fig. 41.3 (a), one of the two wires is earthed whereas in Fig. 41.3 (b) mid-point of the phase winding is earthed.

#### 41.5. Single-phase, 3-wire System

The 1-phase, 3-wire system is identical in principle with the 3-wire d.c. system. As shown in Fig. 41.4, the third wire or neutral is connected to the centre of the transformer secondary and earthed for protecting personnel from electric shock should the transformer insulation break down or the secondary main contact high voltage wire.



#### 41.6. Two-phase, 3-wire System

This system is still used at some places. The third wire is taken from the junction of the two-phase windings I and II, whose voltages are in quadrature with each other as shown in Fig. 41.5. If the voltage between the third or neutral wire and either of the two wires is V, then the voltage between the outer wires is V as shown. As compared to 2-phase, 4-wire system, the 3-wire system suffers from the defect that it produces voltage unbalance because of the unsymmetrical voltage drop in the neutral.



#### 41.7. Two-phase, 4-wire System

As shown in Fig. 41.6, the four wires are taken from the ends of the two-phase windings and the mid-points of the windings are connected together. As before, the voltage of the two windings are in quadrature with each other and the junction point may or may not be earthed. If voltage between the two wires of a phase winding be V, then the voltage between one wire of phase I and one wire of phase II is 0.707 V.

#### 41.8. Three-phase, 3-wire System

Three-phase systems are used extensively. The 3-wire system may be delta-connected or starconnected whose star point is usually earthed. The voltage between lines is V in delta-connection and  $\sqrt{3}$  V in case of star connection where V is the voltage of each phase as shown in Fig. 41.7 (*a*) and (*b*) respectively.



#### 41.9. Three-phase, 4-wire System

The 4th or neutral wire is taken from the star point of the star-connection as shown in Fig. 41.8 and is of half the cross-section of the outers or line conductors. If V is the voltage of each winding,

then line voltage is  $\sqrt{3}$  V. Usually, phase voltage *i.e.* voltage between any outer and the neutral for a symmetrical system is 230 V so that the voltage between any two lines or outers is  $\sqrt{3} \times 230 = 400$  V.

Single-phase residential lighting loads or single-phase motors which run on 230 V are connected between the neutral and any one of the line wires. These loads are connected symmetrically so that line wires are loaded equally. Hence, the resultant current in the neutral wire is zero or at least minimum. The three phase induction motors requiring higher voltages of 400 V or so are put across the lines directly.

#### 41.10. Distribution

The distribution system may be divided into feeders, distributors, sub-distributors and service mains. As already explained in Art. 41.1, feeders are the conductors which connect the sub-station (in some cases the generating station) to the distributors serving a certain allotted area. From distributors various tappings are taken. The connecting link between the distributors and the consumers' terminals are the service mains. The essential difference between a feeder and a distributor is that whereas the current loading of a feeder is the same throughout its length, the distributor has a distributed loading which results in variations of current along its entire length. In other words, no direct tappings are taken from a feeder to a consumer's premises.



In early days, *radial* distribution of tree-system type, as shown in Fig. 41.9, was used. In this system, a number of independent feeders branch out radially from a common source of supply *i.e.* a sub-station or generating station. The distribution transformers were connected to the taps along the length of the feeders. One of the main disadvantages of this system was that the consumer had to depend on one feeder only so that if a fault or breakdown occurred in his feeder, his supply of power was completely cut off till the fault was repaired. Hence, there was no absolute guarantee of continuous power supply.

For maintaining continuity of service, ring-main distributor (*R.M.D.*) system as shown in Fig. 41.10, is employed almost universally. *SS* represents the sub-station from which two feeders supply power to the ring-main distributor at feeding points  $F_1$  and  $F_2$ . The ring-main forms a complete loop and has *isolating* switches provided at the poles at strategic points for isolating a particular section in case of fault. In this way, continuity of service can be maintained to other consumers on

healthy sections of the ring-main. The number of feeders of the ring-main depends on (*i*) the nature of loading—heavy or light (*ii*) the total length of the R.M.D. and (*iii*) on the permissible/allowable drop of voltage. Service mains (S) are taken off at various points of the R.M.D. Sometimes sub-distributors are also used. Since a loop or closed ring-main can be assumed to be equivalent to a number of straight distributors fed at both ends, the voltage drop is small which results in economy of conductor material. The service mains are the connecting link between the consumer's terminals and the R.M.D. or sub-distributor.

# 41.11. Effect of Voltage on Transmission Efficiency

Let us suppose that a power of W watt is to be delivered by a 3-phase transmission line at a line voltage of *E* and power factor  $\cos \phi$ .

The line current 
$$I = \frac{W}{\sqrt{3} E \cos \phi}$$

Let

l = length of the line conductor;  $\sigma$  = current density  $\rho$  = specific resistance of conductor material, A = cross-section of conductor

then

 $A = \frac{I}{\sigma} = \frac{w}{\sqrt{3} E \cdot \sigma \cdot \cos \phi}$ 

=

R

Now

Line loss

$$= \frac{\rho l}{A} = \frac{\sqrt{3\sigma} \rho l E \cos \phi}{W}$$
  
= 3 × loss per conductor = 3  $I^2 R$   
=  $3 \frac{W^2}{3E^2 \cos^2 \phi} \times \frac{\sqrt{3\sigma} \rho l \cos \phi}{W} = \frac{\sqrt{3\sigma} \rho l W}{E \cos \phi}$  ...(1)

Line intake or input = output + losses = 
$$W + \frac{\sqrt{3}\rho\sigma lW}{E\cos\phi} = W \left(1 + \frac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right)$$

: efficiency of transmission

$$\frac{\text{output}}{\text{input}} = \frac{W}{W\left(1 + \frac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right)} = \left(1 - \frac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right) \text{approx} \quad \dots (2)$$

Voltage drop per line = 
$$IR = \frac{\sqrt{3} \sigma \rho l E \cos \phi}{W} \times \frac{W}{\sqrt{3} E \cos \phi} = \sigma \rho l$$
 ...(3)

Volume of copper = 
$$3 lA = \frac{3 W l}{\sqrt{3} \sigma E \cos \phi} = \frac{\sqrt{3} W l}{\sigma E \cos \phi}$$
 ...(4)

For a given value of transmitted power W, line length *l*, current density  $\sigma$  and specific resistance  $\rho$  of the given conductor material, the effect of supply voltage on transmission can be seen as follows :

- **1.** From equation (1), line loss is inversely proportional to *E*. It is also inversely proportional to power factor,  $\cos \phi$ .
- **2.** Transmission efficiency increases with voltage of transmission and power factor as seen from equation (2).
- **3.** As seen from equation (3) for a given current density, the resistance drop per line is constant (since ρ and *l* have been assumed fixed in the present case). Hence, percentage drop is decreased as *E* is increased.
- **4.** The volume of copper required for a transmission line is inversely proportional to the voltage and the power factor as seen from equation (4).

It is clear from the above that for long distance transmission of a.c. power, high voltage and high power factors are essential. But as pointed out earlier in Art. 41.1, economical upper limit of voltage is reached when the saving in cost of copper or aluminium is offset by the increased cost of insulation and increased cost of transformers and high-voltage switches etc. Usually, 650 volt per route km is taken as a rough guide.

# 41.12. Comparison of Conductor Materials Required for Various Overhead Systems

We will now calculate the amounts of conductor material required for various systems of a.c. power transmission. To do it without prejudice to any system, we will make the following assumptions :

- **1.** Amount of power transmitted by each system is the same.
- 2. Distance of transmission is the same in each case.
- 3. Transmission efficiency is the same *i.e.* the losses are the same in each case.
- 4. Loads are balanced in the case of 3-wire systems.
- 5. Cross-section of the neutral wire is half that of any outer.
- 6. Maximum voltage to earth is the same in all cases.

By way of illustration a few cases will be compared, although the reader is advised to attempt others as well.

#### (i) Two-wire d.c. System and Three-phase, 3-wire System

 $I_{2} = -$ 

Let in both cases, *E* be the maximum voltage between conductors and *W* the power transmitted. In d.c. system, let  $R_1$  be the resistance of each conductor and  $I_1$  the current, then  $I_1 = (W/E)$ , hence losses in the two conductors are  $= 2I_1^2R_1 = 2(W/E)^2R_1$ ...(*i*)

For a.c. system, let resistance per conductor be  $R_2$  and power factor  $\cos \phi$ , then since E is the maximum voltage, the *R.M.S.* voltage is  $E/\sqrt{2}$ .

Current in each line,

$$\sqrt{3} (E/\sqrt{2}) \cos \phi$$

Total losses in three conductors are,

$$3l_{2R_{2}}^{2} = \left[\frac{W}{\sqrt{3}(E/\sqrt{2})\cos\phi}\right]^{2}R_{2} = \frac{2W^{2}}{E^{2}} \times \frac{1}{\cos^{2}\phi}.R_{2} \qquad \dots (ii)$$

Since transmission efficiency and hence losses are the same, therefore equating (i) and (ii), we get

$$\frac{2W^2R_1}{E^2} = \frac{2W^2}{E^2} \cdot \frac{1}{\cos^2\phi} R_2 \text{ or } \frac{R_1}{R_2} = \frac{1}{\cos^2\phi}$$

Since area of cross-section is inversely proportional to the resistance.

$$\frac{\text{area of one a.c. conductor}}{\text{area of one d.c. conductor}} = \frac{1}{\cos^2 \phi}$$

Now, for a given length, volumes are directly proportional to the areas of cross-section.

$$\therefore \qquad \frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{1}{\cos^2 \phi}$$

Keeping in mind the fact, that there are two conductors in d.c. system and three in a.c. system under consideration, we have

$$\frac{\text{total volume in 3-wire a.c. system}}{\text{total volume in 2-wire d.c. system}} = \frac{1}{\cos^2 \phi} \cdot \frac{3}{2} = \frac{1.5}{\cos^2 \phi}$$

#### (ii) Three-phase, 4-wire and 3-wire d.c. Systems

The neutral conductor of each system is earthed. Let E be the maximum voltage to earth and W the power to be transmitted in both cases.

*.*..

For d.c. 3-wire system, voltage between outers is 2*E*. If  $I_1$  and  $R_1$  are the current in and resistance of each conductor, then  $I_1 = W/2E$ . Assuming a balanced load (in which case there would be no current flowing in the neutral conductor), the value of loss in two conductors is

$$2I_1^2 R_1 = 2(W/2E)^2 R_1 = W^2 R_1/2E$$

In the case of a.c. system, if *E* is the maximum voltage between any wire and neutral, then its R.M.S. value is  $E/\sqrt{2}$ . Hence, the line voltage =  $\sqrt{3} \cdot E/\sqrt{2}$ . The line current is

$$I_2 = \frac{W}{\sqrt{3} \times (\sqrt{3} E / \sqrt{2}) \cos \phi}$$

where  $\cos \phi$  is the power factor. If  $R_2$  is the resistance of each line, then total loss in 3 lines is

$$= 3I_{2R_2}^2 = 3 \times \left[\frac{W}{\sqrt{3}(\sqrt{3}E/\sqrt{2})\cos\phi}\right]^2 \times R_2 = \frac{2W^2R}{3E^2\cos^2\phi}$$

For equal transmission efficiencies, the two losses should be the same.

$$\frac{W^2 R_1}{2E^2} = \frac{2W^2 R_1}{3E^2 \cos^2 \phi} \text{ or } \frac{R_1}{R_2} = \frac{4}{3\cos^2 \phi}$$

Since volume is directly proportional to the area of cross-section which is itself inversely proportional to the resistance, hence

$$\frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{4}{3\cos^2 \phi}$$

Assuming the neutral wires of each system to have half the cross-section of the outers, there is 3.5 times the quantity of one conductor in a.c. and 2.5 times in the d.c. system. Hence

$$\frac{\text{total volume in the a.c. system}}{\text{total volume in the d.c. system}} = \frac{3.5}{2.5} \times \frac{4}{3\cos^2 \phi} = \frac{1.867}{\cos^2 \phi}$$

In Table 41.1 are given the ratios of copper in any system as compared with that in the corresponding d.c. 2-wire system which has been alloted a basic number of 100.

	<b>Table No. 41.1</b>	
System	With same maximum voltage to earth	With same maximum voltage between conductors
D.C. 2-wire	100	100
D.C. 2-wire Mid-point earthed	25	100
D.C. 3-wire Neutral = $\frac{1}{2}$ outer	31.25	125
D.C. 3-wire Neutral = $\frac{1}{2}$ outer	37.5	150
Single-phase, 2-wire	$200/\cos^2\phi$	$200/\cos^2\phi$
Single-phase, 2-wire Mid-point earthed	$50/\cos^2\phi$	$200/\cos^2\phi$
Single-phase, 3-wire Neutral = $\frac{1}{2}$ outer	$62.5/\cos^2\phi$	$250/\cos^2\phi$
Two-phase, 4-wire	$50/\cos^2\phi$	$200/\cos^2\phi$

	A.C. Italisiilis		1013
Two-phase, 3-wire	146/cos <sup>2</sup> φ	291/cos <sup>2</sup> ¢	
Three-phase, 3-wire	$50/\cos^2\phi$	$150/\cos^2\phi$	
Three-phase, 4-wire Neutral = outer	$67/\cos^2\phi$	$200/\cos^2\phi$	

iccion and Distribution

A comparison of costs for underground systems employing multicore cables can also be worked out on similar lines. In these case, however, maximum voltage between the conductors is taken as the criterion.

It would be clear from the above discussion, that for a.c. systems, the expression for volume contains  $\cos^2 \phi$  in denominator. Obviously, reasonable economy in copper can be achieved only when  $\cos \phi$  approaches unity as closely as possible. For equal line loss, the volume of copper required for  $\cos \phi = 0.8$  will be  $1/0.8^2 = 1.6$  times that required for  $\cos \phi = 1$ .

Since the cost of distribution system represents a large percentage of the total capital investment of power supply companies, it is quite reasonable that a consumer should be penalized for low power factor of his load. This point is fully dealt with in Chapt. 47.

**Example 41.1.** A 3-phase, 4-wire system is used for lighting. Compare the amount of copper required with that needed for a 2-wire D.C. system with same line voltage. Assume the same losses and balanced load. The neutral is one half the cross-section of one of the respective outers.

Solution. (a)	<b>Two-wire</b> <i>DC</i> Let <i>V</i>	= voltage between conductors
	Р	= power delivered, $R_1$ = resistance/conductor
Current	$I_1$	= P/V
	power loss	$= 2I_1^2 R_1 = 2 P^2 R_1 / V^2$

(b) Three-phase, 4-wire

...

...

Let V be the line-to-neutral voltage and  $R_2$  resistance of each phase wire.

$$P = 3VI_{2} \cos \phi = 3VI_{2} ---\text{if } \cos \phi = 1$$
  
Power loss =  $3I_{2}^{2}R_{2} = 3(P/3V)^{2}R_{2} = P^{2}R_{2}/3V^{2}$ 

Since power losses in both systems are equal

$$2P^2R_1/V^2 = P^2R_2/3V^2$$
 or  $R_1/R_2 = 1/6$ 

If  $A_1$  and  $A_2$  are the cross-sectional areas of conductors in the two systems, then  $A_1/A_2 = 6$ . Because  $R \propto l/A$ 

 $\therefore$  Cu reqd. for 2-wire system = 2A<sub>1</sub>l

Cu reqd. for 3- $\phi$ , 4-wire system =  $(3A_2l + A_2l/2)$ 

$$\frac{\text{Cu for } 3 - \phi \text{ system}}{\text{Cu for d.c. system}} = \frac{3.5 A_2 l}{2 A_1 l} = \frac{3.5}{2} \times \frac{1}{6} = 0.292$$

Cu for  $3-\phi$  system =  $0.292 \times Cu$  for d.c. system.

**Example 41.2.** Estimate the weight of copper required to supply a load of 100 MW at upf by a 3-phase, 380-kV system over a distance of 100 km. The neutral point is earthed. The resistance of the conductor is 0.045 ohm/cm<sup>2</sup>/km. The weight of copper is 0.01 kg/cm<sup>3</sup>. The efficiency of transmission can be assumed to be 90 percent. (Power Systems, AMIE, Sec. B, 1994)

**Solution.** Power loss in the line =  $(1 - 0.9) \times 100 = 10$  MW

Line current = 
$$100 \times 10^{6} / \sqrt{3} \times 380 \times 10^{3} \times 1 = 152$$
 A

Since  $I^2 R$  loss in 3-conductors is  $10 \times 106$  W, loss per conductor is  $= 10 \times 10^6/3 = 3.333 \times 10^6$  W

Resistance per conductor =  $10 \times 10^6/3 \times 152^2 = 144.3 \Omega$ 

Resistance per conductor per km = 144.3/100 = 1.443  $\Omega$ 

 $\therefore$  Conductor cross-section = 0.045/1.443 = 0.03 m<sup>3</sup>

Volume of copper per meter run =  $0.03 \times 100 = 3$  cm<sup>3</sup>

Weight of copper for 3-conductor for 100 km length =  $3 \times (3 \times 0.01) \times 100 \times 1000 = 9000$  kg

**Example 41.3.** A d.c. 2-wire distribution system is converted into a.c. 3-phase, 3-wire system by adding a third conductor of the same size as the two existing conductors. If voltage between conductors and percentage power loss remain the same, calculate the percentage additional balanced load which can now be carried by the conductors at 0.95 p.f.



#### Solution. (a) DC 2-wire system [Fig. 40.11 (a)]

If *R* is the resistance per conductor, then power transmitted is  $P = VI_1$  and power loss =  $2I_12R$ . Percentage power loss

$$= 2I_1^2 R \times 100/VI$$
$$= 2I_1 R \times 100/V$$

(b) as 3-phase, 3-wire system [Fig. 
$$40.11$$
 (b)

$$P_2 = \sqrt{3} V I_2 \cos \phi$$
, power loss =  $3I_2^2 R$ 

% power loss = 
$$3I_2^2 R/P_2 = \sqrt{3}I_2 R \times 100 V \cos \phi$$

Since losses in the two cases are the same

$$\therefore \qquad 2I_1 R \times 100/V = \sqrt{3}I_2 R \times 100/V \cos \phi \text{ or } I_2 = 2 \cos \phi \times I_1/\sqrt{3}$$

$$P_2 = \sqrt{3.V.2}\cos^2\phi \times I_1/\sqrt{3} = 2VI_1\cos^2\phi = 2VI_1(0.95)^2 = 1.8VI_1$$

Percentage additional power transmitted in a 3-phase, 3-wire system

$$= \frac{P_2 - P_1}{P_1} \times 100 = \frac{1.8 V I_1 - V I_1}{V I_1} \times 100 = 80\%$$

**Example 41.4.** A 2-phase, 3-wire a.c. system has a middle conductor of same cross-sectional area as the outer and supplies a load of 20 MW. The system is converted into 3-phase, 4-wire system by running a neutral wire. Calculate the new power which can be supplied if voltage across consumer terminal and percentage line losses remain the same. Assume balanced load.



$$= (3I_2^2 R/3VI_2 \cos \phi) \times 100$$
  
=  $I_2 R \times 100/V \cos \phi$ 

Since percentage line losses are the same in both cases

$$\therefore \qquad I_1 R \times 100/V \cos \phi = I_2 R \times 100/V \cos \phi \qquad \therefore \qquad I_1 = I_2$$
  
Now,  
$$P_1 = 2VI_1 \cos \phi = 20 \text{ MW}; \qquad \therefore \qquad VI_1 \cos \phi = 10 \text{ MW}$$
$$P_2 = 3VI_2 \cos \phi = 3VI_1 \cos \phi = 3 \times 10 = 30 \text{ MW}$$

#### 41.13. Constants of a Transmission Line

A transmission line not only has an ohmic resistance but also inductance and capacitance between its conductors. These are known as the constants of a transmission line. While calculating the drop in a.c. transmission and distribution circuits, we will have to consider (*i*) resistive or ohmic drop—in phase with the current (*ii*) inductive drop—leading the current by  $90^{\circ}$  and (*iii*) the capacitive drop and charging current taken by the capacitance of the line. The capacitance and hence the charging current is usually negligible for short transmission lines.

#### 41.14. Reactance of an Isolated Single-Phase Transmission Line

In Fig. 41.13 (a) are shown the cross-sections of two conductors of a single-phase line spaced D from centre to centre.

Since currents through the two conductors will, at any time, always be flowing in opposite directions, the fields surrounding them always reinforce each other in the space between them as shown in Fig. 41.13.





The two parallel conductors form a rectangular loop of one turn through which flux is produced by currents in the two conductors. Since this flux links the loop, the loop possesses inductance. It might be thought that this inductance is negligible because the loop has only one turn and the entire flux-path lies through air of high reluctance. But as the cross-sectional area of the loop is large, from 1 to 10 metre wide and several km long, even for a small flux density, the total flux linking the loop is large and so inductance is appreciable.

It can be proved that inductance per loop metre (when  $r \le D$ ) is

L

$$= \frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi}$$
 henry/metre

where

 $\mu$  = absolute permeability of the surrounding medium  $\mu_i$  = absolute permeability of the conductor material

...(i)

Now,  $\mu = \mu_0 \mu_r$  and  $\mu_i = \mu_0 \mu_r$  where  $\mu_r$  and  $\mu_r'$  are the relative permeabilities of the surrounding medium and the conductor material. If surrounding medium is air, then  $\mu_r = 1$ . Also, if conductor is made of copper, then  $\mu_r' = 1$ . Hence, the above expression becomes.

$$L = \left(\frac{\mu_{o} \mu_{r}}{\pi} \log h D/r + \frac{\mu_{o} \mu_{r}'}{4\pi}\right) H/m$$

 $= \frac{\mu_{o}}{4\pi} (\log h D/r + 1) H/m = 10^{-7} (1 + \log h Dr) H/m$ 

Loop reactance  $X = 2\pi f L$  ohm/metre

Obviously, the inductance of each single conductor is half the above value\*.

inductance/conductor = 
$$\frac{1}{2} (1 + \log h D/r) \times 10^{-7}$$
 H/m ...(*ii*)  
reactance/conductor =  $2 \pi f \times \frac{1}{2} (1 + \log h D/r) \times 10^{-7} \Omega/m$ 

#### 41.15. Reactance of 3-phase Transmission Line

In 3-phase transmission, it is more convenient to consider the reactance of each conductor instead of the looped line or of the entire circuit. Two cases will be considered for 3-phase lines.

#### (i) Symmetrical Spacing

In Fig. 41.14 (a) are shown the three conductors spaced symmetrically i.e. each conductor is at

the apex of the same equilateral triangle ABC of side D. The inductance per conductor per metre is found by using equation (ii) in Art 41.14 because it can be shown that inductance per km of one conductor of 3-phase circuit is the same as the inductance per conductor per km of single-phase circuit with equivalent spacing.



#### (ii) Unsymmetrical Spacing

In Fig. 41.14 (b) is shown a 3-phase circuit with conductors placed unsymmetrically. In this case also, the inductance is given by equation (*ii*) of Art. 41.14 with the only modification that D is put equal to  $\sqrt[3]{(D_1 D_2 D_3)}$ .

#### 41.16. Capacitance of a Single-phase Transmission Line

We know that any two conductors which are separated by an insulating medium constitute a capacitor. When a potential difference is established across two such conductors, the current flows in at one conductor and out at the other so long as that p.d. is maintained. The conductors of an overhead transmission line fulfil these conditions, hence when an alternating potential difference is applied across a transmission line, it draws a leading current even when it is unloaded. This leading current is in quadrature with the applied voltage and is known as the *charging current*. Its value depends upon voltage, the capacitance of the line and the frequency of alternating current.

As shown in Art 5.10, the capacitance between conductors of a single-phase line is approximately given by

\* It may be noted that standard conductors have a slightly higher inductance.

$$C = \frac{\pi \epsilon}{\log h D/r} = \frac{\pi \epsilon_0 \epsilon_r}{2.3 \log_{10} D/r} \qquad F/m = \frac{0.0121 \epsilon_r}{\log_{10} D/r} \,\mu F/\mathrm{km}$$

Here, D is the distance between conductor centres and r the radius of each conductor, both expressed in the same units (Fig. 41.15).



As shown in Fig. 41.16, the capacitance to neutral  $C_n = 2C$  where point O is the neutral of the system. Obviously, the total capacitance between conductors A and B is given by finding the resultant of two capacitances each of value  $C_n$  joined in series, the resultant, obviously, being equal to C.

It is important to remember that if capacitance to neutral is used for calculating the charging current, then voltage to neutral must also be used.

$$\therefore \qquad C_n = 2C = \frac{2 \pi \epsilon}{\log_e D/r} \text{ F/m} \qquad \dots (ii)$$
  
$$\therefore \qquad \text{line charging current} = \frac{V}{X_C} = \frac{V}{1/(2\pi f C_n)} = 2\pi f C_n V \text{ A/m}$$

where V is the *voltage to neutral*.

However, it may be noted that ground effect has been neglected while deriving the above expression. This amounts to the tacit assumption that height h of the conductors is very large as compared to their spacing 'd'. In case ground effect is to be taken into account, the expression for capacitance becomes.

$$C_n = \frac{2\pi\epsilon}{\log h} \frac{d}{r\sqrt{1+d^2/4h^2}}$$
 F/m (Ex. 41.7)

**Example 41.5.** What is the inductance per loop metre of two parallel conductors of a single phase system if each has a diameter of 1 cm and their axes are 5 cm apart when conductors have a relative permeability of (a) unity and (b) 100. The relative permeability of the surrounding medium is unity in both cases. End effects may be neglected and the current may be assumed uniformly distributed over cross-section of the wires.

Solution. (a) Here,  

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}; \ \mu_i = 1$$

$$\therefore \qquad L = \frac{4\pi \times 10^{-7}}{\pi} \left( \log h^5 / 0.5 + \frac{1}{4} \right) = 1.02 \ \mu\text{H/m}$$
(b) Here,  

$$\mu = \mu_0 = 4\pi \times 10^{-7}; \quad \mu_i = 100 \ \mu_0$$

$$L = \frac{4\pi \times 10^{-7}}{\pi} \left( \log h^5 / 0.5 + \frac{100}{4} \right) = 10.9 \ \mu\text{H/m}$$

Example 41.6. A 20-km single-phase transmission line having 0.823 cm diameter has two line conductors separated by 1.5 metre. The conductor has a resistance of 0.311 ohm per kilometre. Find the loop impedance of this line at 50 Hz. (Gen. Trans. & Dist. Bombay Univ. 1992)

Loop length =  $20 \text{ km} = 2 \times 10^4 \text{ m}$ Solution.  $L = 2 \times 10^4 \left( \frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi} \right)$  henry Total loop inductance is D = 1.5 m;  $r = 0.823/2 = 0.412 \text{ cm} = 4.12 \times 10^{-3} \text{ m}$ Here. Assuming  $\mu_r = 1$  for copper conductors since they are non-magnetic and also taking  $\mu_r = 1$  for air,

we have

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$$L = 2 \times 10^{4} \left(\frac{\mu}{\pi} \log_{e} D/r + \frac{\mu_{i}}{4\pi}\right) = \frac{2 \times 10^{4} \times 4\pi \times 10^{-7}}{\pi} \left(\log_{e}^{1.5/4.12 \times 10^{-3}} + \frac{1}{4}\right) H$$
  
= 8 × 10<sup>-3</sup> (5.89 + 0.25) = 49.12 × 10<sup>-3</sup> H

Reactance  $X = 2\pi \times 50 \times 49.12 \times 10^{-3} = 15.44 \,\Omega$ ; Loop resistance  $= 2 \times 20 \times 0.311 = 12.44 \,\Omega$ 

Loop impedance =  $\sqrt{12.44^2 + 15.44^2} = 19.86 \,\Omega$ 

**Example 41.7.** The conductors in a single-phase transmission line are 6 m above ground. Each conductor has a diameter of 1.5 cm and the two conductors are spaced 3 m apart. Calculate the capacitance per km of the line (i) excluding ground effect and (ii) including the ground effect.

Solution. The line conductors are shown in Fig. 41.17.

$$C_n = \frac{2\pi\epsilon_0}{\log h \, d/r} = \frac{2\pi \times 8.854 \times 10^{-12}}{\log h \, 3/0.75 \times 10^{-2}} \, \text{F/m}$$

 $= 9.27 \times 10^{-12} \text{ F/m} = 9.27 \times 10^{-3} \, \mu\text{F/km}$ 

(*ii*) In this case,

*(i)* 

$$C_n = \frac{2 \pi \epsilon_0}{\log h \frac{d}{r \sqrt{1 + d^2/4h^2}}} F/m$$
  
=  $\frac{2\pi \times 8.854 \times 10^{-12}}{\log h \frac{3}{0.75 \times 10^{-12} \sqrt{1 + 3^2/4 \times 6^2}}}$   
=  $9.33 \times 10^{-12} F/m = 9.33 \times 10^{-3} mF/km$ 



It is seen that line capacitance when considering ground effect is more (by about 0.64% in the present case).

#### 41.17. Capacitance of a Three-phase Transmission Line

In Fig. 41.18 (a) are shown three conductors spaced symmetrically and having a capacitance of C between each pair. The value of C i.e. line-to-line capacitance is the same as given by equation (i) in Art. 41.16.

Considering the two conductors A and B, their line capacitance C can be regarded as consisting of two line-to-neutral capacitances connected in series, so that for the line-to-neutral capacitance of A with respect to neutral plane 1, as in Art. 41.16, we have



Fig. 41.18

Similarly, line-to-neutral capacitance of conductor A with respect to neutral plane 3 is

$$C_{n3} = \frac{2\pi \epsilon}{\log_e D/r} \,\mathrm{F/m}$$

The vector sum of these capacitances is equal to either because their phase angle is  $120^{\circ}$  *i.e.* the phase angle of the voltages which cut across them.

Hence, the capacitance to neutral per metre of one conductor of a 3-phase transmission line is

$$C_n = \frac{2\pi \epsilon}{\log_e D/r}$$
 F/m

In case the conductors are placed

asymmetrically, the distance used is  $D = \sqrt[3]{(D_1 D_2 D_3)}$ .

# 41.18. Short Single-phase Line Calculations

Single-phase circuits are always short and work at relatively low voltages. In such cases, the line constants can be regarded as 'lumped' instead of being distributed and capacitance can be neglected.

Let	$E_S =$ voltage at sending end;	$E_R$ = voltage at receiving end
	I = line current	$\cos \phi_R$ = power factor at receiving end
	R = resistance of both conductors:	$X$ = reactance of both conductors = $\omega L$

Then, the equivalent circuit of a short single-phase line is as shown in Fig. 41.19.

Resistive drop $= IR$	—in phase with
Reactive drop $= IX$	—in quadrature with

The vector diagram is shown in Fig. 41.20. From the right-angled triangle OMA, we get

$$OM^{2} = OA^{2} + AM^{2} = (OK + KA)^{2} + (AC + CM)^{2}$$
$$E_{S} = \sqrt{[(E_{R} \cos \phi_{R} + IR)^{2} + (E_{R} \sin \phi_{R} + IX)^{2}]}$$

An approximate expression for the voltage drop is as follows :



Draw the various dotted lines as in Fig. 41.20. Here, ML is  $\perp OF$ , CL is  $\parallel OF$  and CD is  $\perp OF$ . As seen

OM = OF (approximately). = OD + DF = OB + BD + DF $\begin{array}{rcl} E_S &=& E_R + BD + DF = E_R + BD + CL = E_R + IR\cos\varphi_R + IX\sin\varphi_R \\ E_S - E_R &=& IR\cos\varphi_R + IX\sin\varphi_R \ \therefore \ \mathrm{drop} = I(R\cos\varphi_R + X\sin\varphi_R) \ (\mathrm{approx.}) \end{array}$ or or Voltage regn. of line =  $[(E_S - E_R)/E_R] \times 100$ 

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**Solution in Complex Notation** Let us take  $E_R$  as reference vector as shown in Fig. 41.21 so that  $E_R = (E_R + j_0)$ As seen from  $\Delta OAB$ ,  $E_S$  is equal to the vector sum of  $E_R$  and IZ or  $\mathbf{E}_S = \mathbf{E}_R + \mathbf{IZ}$ φ<sub>R</sub>  $\mathbf{I} = I \angle -\phi_R = I (\cos \phi_R - j \sin \phi_R)$ Now.  $\mathbf{Z} = Z \angle \theta = (R + jX)$ Similarly, Fig. 41.21  $\mathbf{E}_{\mathbf{S}} = \mathbf{E}_{\mathbf{R}} + \mathbf{I}_{\mathbf{Z}} \angle \boldsymbol{\theta} - \boldsymbol{\phi}_{\mathbf{R}}$ *.*..  $\mathbf{E}_{S} = \mathbf{E}_{R} + \mathbf{I} \left( \cos \phi_{R} - j \sin \phi_{R} \right) \left( R + j X \right)$ or  $= \mathbf{E}_{R} + (IR\cos\phi_{R} + IX\sin\phi_{R}) + j(IX\cos\phi_{R} - I_{R}\sin\phi_{R})^{2}$  $E_{S} = \sqrt{\left(E_{R} + IR\cos\phi_{R} + IX\sin\phi_{R}\right)^{2} + \left(IX\sin\phi_{R} - IR\sin\phi_{R}\right)^{2}}$ .... If the p.f. is leading, then  $\mathbf{I} = I \angle \phi_R = I (\cos \phi_R + j \sin \phi_R)$  $\mathbf{E}_{S} = \mathbf{E}_{R} + \mathbf{I}\mathbf{Z} \angle \theta + \phi_{R} = \mathbf{E}_{R} + I \left(\cos \phi_{R} + j \sin \phi_{R}\right) \left(R + j X\right)$ .... It may be noted that the sending end power factor angle is  $\phi_s = (\phi_R + \alpha)$ .

**Example 41.8.** A single-phase line has an impedance of  $5 \angle 60^{\circ}$  and supplies a load of 120 A, 3,300 V at 0.8 p.f. lagging. Calculate the sending-end voltage and draw a vector diagram.

(City & Guides, London)

Solution. The vector diagram is similar to that shown in Fig. 41.21. Here				
	$\mathbf{E}_{\mathbf{R}} = 3,300 \angle 0^{\circ},  \mathbf{Z} = 5 \angle 60^{\circ}$			
Since	$\phi_R = \cos^{-1}(0.8) = 36^{\circ}52',  \therefore  \mathbf{I} = 120 \angle -36^{\circ}52'$			
Voltage drop	$=$ <b>IZ</b> = 120 × 5 $\angle$ 60° - 36°52′			
	$= 600 \angle 23^{\circ}8' = 600 (0.9196 + j0.3928) = 551.8 + j 235.7$			
.: <b>.</b>	$\mathbf{E}_{\mathbf{S}} = (3,300 + j_0) + (551.8 + j235.7) = 3,851.8 + j235.7$			
	$\mathbf{E}_{s} = \sqrt{3851.8^{2} + 235.7^{2}} = 3,860 \text{ V}$			

**Example 40.9.** An overhead, single-phase transmission line delivers 1100 kW at 33 kV at 0.8 p.f. lagging. The total resistance of the line is  $10 \Omega$  and total inductive reactance is  $15 \Omega$ . Determine (i) sending-end voltage (ii) sending-end p.f. and (iii) transmission efficiency.

(Electrical Technology-I, Bombay Univ.)

**Solution.** Full-load line current is  $I = 1100/33 \times 0.8 = 41.7$  A  $I^2 R = 41.7^2 \times 10 = 17,390 \text{ W} = 17.39 \text{ kW}$ Line loss \_  $\frac{\text{output}}{\text{output} + \text{losses}} = \frac{1100 \times 100}{1100 + 17.39} = 98.5\%$ (*iii*) Transmission efficiency = = **IZ** = 41.7(0.8 - *j* 0.6) (10 + *j* 15) = 709 + *j* 250 (*i*) Line voltage drop Sending-end voltage is  $\mathbf{E}_{\mathbf{S}} = \mathbf{E}_{\mathbf{R}} + \mathbf{I}_{\mathbf{Z}} = (33,000 + j0) + (709 + j250)$  $= 33,709 + j 250 = 33,710 \angle 25^{\circ}$ Hence, sending-end voltage is 33.71 kV (*ii*) As seen from Fig. 41.22,  $\alpha = 0^{\circ}25'$ Sending-end p.f. angle is  $\theta + \alpha = 36^{\circ}52' + 0^{\circ}25' = 37^{\circ}17'$ Fig. 41.22 p.f. =  $\cos 37^{\circ}17' = 0.795$  (lag). ...

Note. As seen from Fig. 41.22, approximate line drop is

 $= I(R \cos \phi_R + X \sin \phi_R) = 41.7 (10 \times 0.8 + 15 \times 0.6) = 709 V$  $\therefore E_S = 33,000 + 709 = 33,709 V - as above$ 

**Example 41.10.** What is the maximum length in km for a 1-phase transmission line having copper conductors of 0.775 cm<sup>2</sup> cross-section over which 200 kW at unity power factor and at 3300 V can be delivered? The efficiency of transmission is 90 per cent. Take specific resistance as  $(1.725 \times 10^{-8}) \Omega$ -m. (Electrical Technology, Bombay Univ.)

Solution. Since transmission efficiency is 90 per cent, sending-end power is 200/0.9 kW.

Line loss	= (200/0.9 - 200)	= 22.22  kW = 22,220  W	
Line current	= 200,000/3300 ×	1 = 60.6  A	
If $R$ is the resistance of	one conductor, then		
	$2I^2R$ = line loss or	$2 \times 60.62 \times R = 22,220$	or $R = 3.03$ W
Now, $R = \rho \frac{1}{A}$	$3.03 = 1.725 \times 10^{-8} \times 1/0.7$	$75 \times 10^{-4}$ : $l = 13,600,$	, <i>m</i> = <b>13.6 km</b>

**Example 41.11.** An industrial load consisting of a group of induction motors which aggregate 500 kW at 0.6 power factor lagging is supplied by a distribution feeder having an equivalent impedance of (0.15 + j0.6) ohm. The voltage at the load end of the feeder is 2300 volts.

- (a) Determine the load current.
- (b) Find the power, reactive power and voltampere supplied to the sending end of the feeder.
- (c) Find the voltage at the sending end of the feeder.

(•) • • • • • • • • • • • • • • • • • •		8
		(Electrical Technology, Vikram Univ., Ujjain)
Solution. (a)	I =	$500 \times 103/\sqrt{3} = 209 \text{ A}$
(c) Let V	$r_{R} =$	$(2300 + j0);  I = 209 \ (0.6 - j \ 0.8)$
Voltage drop	=	$\mathbb{Z} = 209 \ (0.6 - j \ 0.8) \ (0.15 + j \ 0.6) = 119 + j50$
V	$r_s =$	$\mathbf{V}_{R} + \mathbf{IZ} = 2300 + 119 + j\ 50 = 2419 + j\ 50 = 2420 \angle 1.2'$
( <i>b</i> ) Sending power	=	$\sqrt{3} \times 2420 \times 209 \times 0.5835 = 511.17 \text{ kW}$
Sending-end reactive power	=	$\sqrt{3} \times 2420 \times 209 \times 0.8121 = 711.42 \text{ kVAR}$
Sending-end volt ampere kVA	A =	$\sqrt{3} \times 2420 \times 209 = 876 \text{ kVA}$
or kVA	A =	$\sqrt{kW^2 + kVAR^2} = \sqrt{511.17^2 + 711.42^2} = 876$

#### 41.19. Short Three-phase Transmission Line Constants

Three-phase calculations are made in the same way as single-phase calculations because a 3-phase unit can be regarded as consisting of 3 single-phase units each transmitting one-third of the total power. In Fig. 41.23 is shown a 3-phase system in which each conductor has a resistance of  $R\Omega$  and inductance of  $X\Omega$ . In the case of a 3-phase system, it is advantageous to work in phase instead of in line-to-line values. In Fig. 41.24, one phase is shown separately. Since in a balanced load, there is no current in the neutral wire hence no current flows through the ground.

The resistive and reactive drops are calculated as before. By adding these drops *vectorially* to the receiving-end voltage, the sending-end voltage can be calculated. The line voltage at the sending-end can be found by multiplying this value by  $\sqrt{3}$ .



In the figure, a star-connected load is assumed. But if it is a delta-connected load then it can be replaced by an equivalent star-connected load.

**Example 41.12.** A 33-kV, 3-phase generating station is to supply 10 MW load at 31 kV and 0.9 power factor lagging over a 3-phase transmission line 3 km long. For the efficiency of the line to be 96%, what must be the resistance and reactance of the line?

			(Electrical Power-III, Bangalore Univ.)
Solution.	Power output	t =	10 MW ; $\eta = 0.96$ ; Power input = 10/0.96 = 10.417 MW
	Total loss	. =	0.417 MW
Now,	Ĺ	=	$10 \times 106/\sqrt{3} \times 31 \times 103 \times 0.9 = 207$ A
If $R$ is the rest	istance per phas	e, th	$\operatorname{hen} 3 \times 2072 \times R = 0.417 \times 106  \therefore  R = 3.24 \Omega$
Now, $V_S$ per p	hase	=	$33/\sqrt{3} = 19.052$ kV and $V_R$ per phase $= 31/\sqrt{3} = 17.898$ kV
Using the app	roximate relation	n of	Art. 36-18, we get
	$V_s$	; =	$V_R + I(R\cos\phi_R + X\sin\phi_R)$

19,052 = 17,898 + 207( $3.24 \times 0.9 + X \times 0.4368$ ) ∴ X = 6.1 Ω/phase.

**Example 41.13.** A balanced Y-connected load of  $(300 + j100) \Omega$  is supplied by a 3-phase line 40 km long with an impedance of  $(0.6 + j0.7) \Omega$  per km (line-to-neutral). Find the voltage at the receiving end when the voltage at the sending end is 66 kV. What is the phase angle between these voltages? Also, find the transmission efficiency of the line.

(Elect. Power Systems, Gujarat Univ.)

Solution. The circuit connections are shown in Fig. 41.25.		
Resistance for 40 km conductor length =	$= 40 \times 0.6 = 24 \ \Omega$	
Reactance for 40 km conductor length =	$= 40 \times 0.7 = 28 \ \Omega$	
Total resistance/phase =	$= 300 + 24 = 324 \Omega$	
Total reactance/phase =	$= 100 + 28 = 128 \Omega$	
Total impedance/phase =	$= \sqrt{324^2 + 128^2} = 348 \Omega$	
Line current =	$= \frac{66,000/\sqrt{3}}{348} = \frac{38,100}{348} = 110 \text{ A}$	
Now, $\tan \phi_R =$	= 00/300; $\phi_R = 18.4'$ ; cos $\phi_R = 0.949$ , sin $\phi_R = 0.316$	
Voltage drop in conductor resistance =	$= 110 \times 24 = 2640 \text{ V}$	
Voltage drop in conductor reactance =	$= 110 \times 28 = 3080 \text{ V}$	





Fig. 41.25

It is seen from Art. 41.18 that  $\mathbf{V}_{\mathbf{S}} = (V_R + IR\cos\phi_R + IX\sin\phi_R) + j(IX\cos\phi_R - IR\sin\phi_R)$  $= (V_R + 2640 \times 0.949 + 3080 \times 0.316) + j (3080 \times 0.949 - 2640 \times 0.316)$  $= (V_R + 3475) + j 2075$  $\mathbf{V}_{\mathbf{S}} = 66000 / \sqrt{3} = 38,100 \text{ V}$ Now.  $38,100^2 = (V_R + 3475)^2 + 2075^2, V_R = 34,585 \text{ V}$ *.*.. Line voltage across load =  $34,585 \times \sqrt{3} = 59.88 \text{ kV}$  $\mathbf{V}_{\mathbf{S}} = (34,585 + 3475) + j \ 2075 = 38,060 + j \ 2075 = 38,100 \ \angle \ 3.1^{\circ}$ *.*.. Obviously,  $V_s$  leads  $V_R$  by 3.1° as shown in Fig. 41.25 (b). Power delivered to the load/phase =  $110^2 \times 300$  W  $= 110^2 \times 324 \text{ W}$ Power transmitted/phase :. transmission efficiency  $\eta = (300/324) \times 100 = 92.6\%$ 

**Example 41.14.** *Define 'regulation' and 'efficiency' of a short transmission line.* 

A 3-phase, 50-Hz, transmission line having resistance of  $5\Omega$  per phase and inductance of 30 mH per phase supplies a load of 1000 kW at 0.8 lagging and 11 kV at the receiving end. Find.

(a) sending end voltage and power factor (b) transmission efficiency (c) regulation.

(Electrical Engineering-III. Poona Univ. 1990)

VR per phase =  $11,000/\sqrt{3} = 6,350$  V Solution. Line current, =  $1000 \times 10^3 / \sqrt{3} \times 11,000 \times 0.8 = 65.6 \text{ A}$  $X_L = 2\pi \times 50 \times 30 \times 10^{-3} = 9.4 \,\Omega, R = 5\Omega; Z_{ph} = (5 + j \,9.4) \,\Omega$ = 65.6 (0.8 - j 0.6) (5 + j 9.4) = 632 + j 296*.*.. drop per conductor  $V_{\rm s} = 6350 + (632 + j\,296) = 6,982 + j\,296 = 6,988 \angle 2.4^{\circ}$ (a)Sending-end line voltage  $= 6,988 \times \sqrt{3} = 12,100 \text{ V} = 12.1 \text{ kV}$ As seen from Fig. 41.25 (*b*),  $\alpha = 2.4^\circ$ , Now,  $\cos \phi_R = 0.8$ ,  $\phi_R = 36.9^\circ$  $\phi_s = 36.9^\circ + 2.4^\circ = 39.3^\circ, \cos \phi_s = 0.774$  (lag).  $= 3 I^2 R = 3 \times 65.62 \times 5 = 64.550 W = 64.55 kW$ (b) Total power loss Input power = 1000 + 64.55 = 1064.55 kW

....

: power transmission  $\eta = 1000/1064.55 = 0.9394$  or **93.94%** 

(c) Now, line regulation is defined as the rise in voltage when full-load is thrown off the line divided by voltage at the load end.

% regn. =  $\frac{(12.1 - 11)}{11} \times 100 = 10\%$ 

**Example 41.15.** A short 3- $\phi$  line with an impedance of  $(6 + j8) \Omega$  per line has sending and receiving end line voltages of 120 and 110 kV respectively for some receiving-end load at a p.f. of 0.9. Find the active power and the reactive power at the receiving end.

(Transmission and Distribution, Madras Univ.)

Solution. As seen from Art. 41.18, considering phase values, we have  $V_S = V_R + I(R \cos \phi R + X \sin \phi R)$ Now,  $V_S$  per phase =  $120/\sqrt{3} = 69,280 \text{ V}$ Similarly,  $V_R$  per phase =  $110/\sqrt{3} = 63,507 \text{ V}$   $\therefore$  69,280 =  $63,507 + I(6 \times 0.9 + 8 \times 0.435)$   $\therefore$  Line current I = 5,773/8.88 = 650 AActive power at receiving end =  $\sqrt{3}V_L I_L \cos \phi = \sqrt{3} \times 110 \times 650 \times 0.9 = 111,400 \text{ kW}$ Reactive power at receiving end =  $\sqrt{3}V_L I_L \sin \phi = \sqrt{3} \times 110 \times 650 \times 0.435 = 53,870 \text{ kVAR}$ 

**Example 41.16.** A 3-phase, 20 km line delivers a load of 10 MW at 11 kV having a lagging p.f. of 0.707 at the receiving end. The line has a resistance of 0.02  $\Omega$ /km phase and an inductive reactance of 0.07  $\Omega$ /km/phase. Calculate the regulation and efficiency of the line. If, now, the receivingend p.f. is raised to 0.9 by using static capacitors, calculate the new value of regulation and efficiency. (Electrical Engg. ; Bombay Univ.)

Solution. (*i*) When p.f. = 0.707 (lag) $= 10 \times 10^{6} / \sqrt{3} \times 11,000 \times 0.707 = 743$  A Line current  $= 11,000/\sqrt{3} = 6,352$  V  $V_{R}$  per phase Total resistance/phase for 20 km =  $20 \times 0.02 = 0.4$  W Total reactance/phase for 20 km  $= 20 \times 0.07 = 1.4$  W  $\therefore$  Total impedance/phase =  $(0.4 + j \ 1.4) \Omega$ If  $V_R$  is taken as the reference vector, then drop per phase = 743 (0.707 - j 0.707) (0.4 + j 1.4) = (945 + j 525) $V_{\rm s} = 6,352 + 945 + j\,525 = 7,297 + j\,525$ ...  $V_s = \sqrt{7297^2 + 525^2} = 7,315 \text{ V}$ or % regulation =  $\frac{7,315-6,352}{6,352} \times 100 = 15.1$  % *.*..  $= 3I^2R = 3 \times 7432 \times 0.4 = 662 \text{ kW}$ Total line loss = 10 + 0.662 = 10.662 MW  $\therefore$   $\eta = 10 \times 100/10.662 = 94\%$ Total output (ii) When p.f. = 0.9 (lag)  $= 10^{7}/\sqrt{3} \times 11,000 \times 0.9 = 583$  A Line current = 583 (0.9 - j 0.435) (0.4 + j 1.4) = 565 + j 633Drop/phase  $V_{\rm S} = 6,352 + (565 + j 633) = 6,917 + j 633$  $V_{\rm s} = \sqrt{6,917^2 + 633^2} = 6,947 \, {\rm V}$ ....

∴ % regulation	$= \frac{6,947 - 6,352}{6,352} \times 100 = 9.37\%$
Total line loss	$= 3 \times 5832 \times 0.4 = 408 \text{ kW}$ ; Total output = 10.408 MW
	$\eta = 10 \times 100/10.408 = 96.1\%$

**Example 41.17.** A load of 1,000 kW at 0.8 p.f. lagging is received at the end of a 3-phase line 10 km long. The resistance and inductance of each conductor per km are 0.531 W and 1.76 mH respectively. The voltage at the receiving end is 11 kV at 50 Hz. Find the sending-end voltage and the power loss in the line. What would be the reduction in the line loss if the p.f. of the load were improved to unity? (Elect. Power Systems, Gujarat Univ.)

Line current =  $1,000 \times 1,000/\sqrt{3} \times 11 \times 1,000 \times 0.8 = 65.6$  A Solution. Voltage/phase =  $11,000/\sqrt{3} = 6,352$  V  $\mathbf{X} = 2\pi \times 50 \times 176 \times 10\sqrt{3} \times 10 = 5.53 \,\Omega; \mathbf{R} = 0.531 \times 10 = 5.31 \,\Omega \quad \therefore \quad \mathbf{Z} = (5.31 + j \, 5.53)$ = 65.6 (0.8 - j 0.6) (5.31 + j 5.53) = 496.4 + j 81.2Voltage drop/phase :.  $V_s = 6,352 + 496.4 + j 81.2 = 6,848 + j 81 = 6,848 V$  numerically (approx.) :. line-to-line sending-end voltage =  $6,848 \times \sqrt{3} = 11,860 V = 11.86 \text{ kV}$  $= 3 \times 5.31 \times 65.62 = 68,550 \text{ W} = 68.55 \text{ kW}$ Total loss  $= 1.000/11 \times \sqrt{3} = 52.49$  A Line current for unity p.f.  $= 3 \times 5.31 \times 52.492 = 43.89 \text{ kW}$ : New losses ∴ reduction = 68.55 - 43.89 = 24.66 kW

**Example 41.18.** Estimate the distance over which a load of 15,000 kW at 0.85 p.f. can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0.905 W per kilometre. The voltage at the receiving end is to be 132 kV and the loss in transmission is to be 7.5% of the load. (Transmission and Distribution, Madras Univ.)

**Solution.** Line current = 
$$15,000/132 \times \sqrt{3} \times 0.85 = 77.2$$
 A  
Total loss =  $7.5\%$  of  $15,000 = 1,125$  kW  
If *R* is the resistance of one conductor, then  
 $3 I^2 R = 1,125,000$  or  $3 \times (77.2)^2 \times R = 1,125,000$ ;  $R = 62.94 \Omega$   
Length of the line =  $62.94/0.905 = 69.55$  km.

**Example 41.19.** A  $3-\phi$  line has a resistance of  $5.31\Omega$  and inductance of 0.0176 H. Power is transmitted at 33 kV, 50-Hz from one end and the load at the receiving end is 3,600 kW at 0.8 p.f. lagging. Find the line current, receiving-end voltage, sending-end p.f. and efficiency of transmisson.

(Transmission and Distribution-I, Madras Univ.)

Solution.	$V_S$	=	$V_R + I_R \cos \phi_R + IX \sin \phi_R$ approximately
Now, power delivered	/phase	=	$V_R I \cos \phi_R$
∴ 1,	$200 \times 1,000$	=	$VR \cdot I \times 0.8$ $\therefore$ $I = 15 \times 105/V_R$
Also,	V <sub>s</sub> per phase	=	$33,000/\sqrt{3} = 19,050 \text{ V}$
	R	=	5.31 $\Omega$ ; <i>X</i> = 0.0176 × 314 = 5.54 $\Omega$
	19,050	=	$V_R + \frac{15 \times 10^5}{V_R} \times 5.31 \times 0.8 + \frac{15 \times 10^5}{V_R} \times 5.54 \times 0.6$
$\therefore V_R^2 - 19,050 V_R$	+ 11,358,000	=	0 or $V_R = \frac{19.050 \pm 17,810}{2} = \frac{36.860}{2} = 18,430 \text{ V}$
Line voltage at the re	eceiving end	=	$18,430 \times \sqrt{3} = 32 \text{ kV}$
	Ι	=	15 × 105/18,430 = <b>81.5</b> A

Now,  

$$V_S = V_R + I(\cos \phi_R - j \sin \phi_R) (R + jX)$$
  
 $= 18,430 + 81.5 (0.8 - j 0.6) (5.31 + j 5.54)$   
 $= 18,430 + 615 + j 100 = 19,050 \angle 0.3^{\circ}$   
 $\phi_R = \cos^{-1} (0.8) = 36^{\circ}52';$   
 $\phi_S = 36^{\circ}52' + 18' = 37^{\circ}10'$   
 $\therefore$  sending-end p.f.  
Power lost in line  
 $= 3 I^2 R = 3 \times 81.52 \times 5.31 = 106 \text{ kW}$   
 $= 3,600 + 106 = 3,706 \text{ kW}$   
 $\eta = 3,600 \times 100/3,706 = 97.2\%$ 

**Example 41.20.** A 3-phase short transmission line has resistance and reactance per phase of  $15\Omega$  and  $20 \Omega$  respectively. If the sending-end voltage is 33 kV and the regulation of the line is not to exceed 10%, find the maximum power in kW which can be transmitted over the line. Find also the k VAR supplied by the line when delivering the maximum power.

**Solution.** As seen from Art. 41.18  $V^2 = (V + I)$ 

$$V_{S}^{2} = (V_{R} + I_{R} \cos \phi_{R} + IX \sin \phi_{R})^{2} + (IX \cos \phi_{R} - I_{R} \phi_{R})^{2}$$
  
=  $V_{R}^{2} + 2IV_{R} (R \cos \phi_{R} + X \sin \phi_{R}) + I^{2} (R^{2} + X^{2})$   
real power/phase,  
$$P = V_{R} I \cos \phi_{R} ; \text{ reactive power/ phase, } Q = V_{R} I \sin \phi_{R}$$
  
$$\therefore \qquad P^{2} + Q^{2} = V_{R}^{2} I^{2} \text{ or } I^{2} = (P^{2} + Q^{2})/V_{R}^{2}$$

Substituting this value above, we get

or 
$$V_{S}^{2} = V_{R}^{2} + 2PR + 2QX + \frac{P^{2} + Q^{2}}{V_{R}^{2}} (R^{2} + X^{2})$$
$$V_{S}^{2} = V_{R}^{2} - 2PR - 2QX - \frac{P^{2} + Q^{2}}{V_{R}^{2}} (R^{2} + X^{2}) = 0 \qquad \dots (i)$$

To find the maximum power transmitted by the line, differentiate the above equation w.r.t. Q and put dP/dQ = 0 (treating  $V_s$  and  $V_R$  as constants).

$$\therefore -2R \frac{dP}{dQ} - 2X - 2P \cdot \frac{(R^2 + X^2)}{V_R} \cdot \frac{dP}{dP} - 2Q \frac{(R^2 + X^2)}{V_R^2} = 0$$
  
Since  $dP/dQ = 0 \therefore 2X + 2Q \frac{(R^2 + X^2)}{V_R^2} = 0$  or  $Q = \frac{-V_R^2 X}{R^2 + X^2} = \frac{-V_R^2 X}{Z^2}$ 

Putting this value of Q in Eq. (i) above we have

$$\frac{P^2 Z^2}{V_R^2} + 2 PR - V_S^2 + V_R^2 - \frac{V_R^2 X^2}{Z^2} = 0$$
$$\frac{P^2 Z^2}{V_R^2} + 2 PR - V_S^2 + V_R^2 \left(1 - \frac{X^2}{Z^2}\right) = 0$$

or

or 
$$\frac{P^2 Z^2}{V_R^2} + 2 PR + \frac{V_S^2 + V_R^2}{Z^2} - V_S = 0$$

Solving for *P*, we get  $P_{max} = \frac{V_R^2}{Z^2} \left( Z \cdot \frac{V_S}{V_R} - R \right)$  watts/phase In the present case,  $V_S = 33,000/\sqrt{3} = 19,050 \text{ V}$ ; Since regulation is limited to 10%,  $\therefore V_R + 10\% V_R = V_S$  or  $1.1 V_R = 19,050$ ;  $V_R = 17,320 \text{ V}$ ;  $Z = \sqrt{15^2 + 20^2} = 25 \Omega$ 

$$P_{\text{max}} \text{ per phase } = \left(\frac{17.320}{25}\right)^2 \left(25 \times \frac{19,050}{17,320} - 15\right) = 6 \times 10^6 \text{ W} = 6 \text{ MW}$$
  
Total maximum power =  $3 \times 6 = 18 \text{ MW}$   
kVAR supplied/phase  $Q = \frac{V_R^2 X}{Z^2} = -\frac{17,320^2 \times 20}{25^2} \times 10^{-3} = 9,598$   
Total kVAR supplie =  $3 \times 9,598 = 28,794$ .

**Example 41.21.** A 3-  $\phi$ , 50-Hz generating station supplies a load of 9,900 kW at 0.866 p.f. (lag) through a short overhead transmission line. Determine the sending-end voltage if the receiving-end voltage is 66 kV and also the efficiency of transmission. The resistance per km is  $4\Omega$  and inductance 40 mH. What is the maximum power in kVA that can be transmitted through the line if both the sending and receiving-end voltages are kept at 66 kV and resistance of the line is negligible.

Solution.  

$$R = 4 \Omega$$
;  $X = 40 \times 10^{-3} \times 314 = 12.56 \Omega$   
Line current  
 $I = 9,900/\sqrt{3} \times 66 \times 0.866 = 100 \text{ A}$   
 $V_R = 66,000/\sqrt{3} = 38,100 \text{ V}$ ;  $\cos \phi_R = 0.866$ ;  $\sin \phi_R = 0.5$   
 $V_S = V_R + I(R \cos \phi_R + X \sin \phi_R) = 38,100 + 100(4 \times 0.866 + 12.56 \times 0.5) = 39,075 \text{ V}$   
Line value of sending-end voltage  $= 39,075 \times \sqrt{3} = 67.5 \text{ kV}$   
Total line loss  $= 3I^2R = 3 \times 100^2 \times 4 = 120 \text{ kW}$   
 $\therefore$   
 $n = 9,900/(9,900 + 120) = 0.988 \text{ or } 98.8\%$ 

:. 
$$\eta = 9,900/(9,900 + 120) = 0.9$$
  
Max, value of Q for 3-phases  $= \frac{3 \cdot V_R^2}{Z^2} \cdot X$ 

Now.

*:*..

Now,  

$$V_S = V_R = 38,100 \text{ V}$$
 and resistance is negligible.  
 $\therefore$  Max. value of  
 $Q = \frac{3 \cdot V_R^2}{Z^2} \cdot X \cdot 10^{-3} = 3 \times 3,810 \times 10^{-3}/12.56 = 3,48,000 \text{ kVA}$ 

Example 41.22. A 3-phase load of 2,000 kVA, 0.8 p.f. is supplied at 6.6 kV, 50-Hz by means of a 33 kV transmission line 20 km long and a 5 : 1 transformer. The resistance per km of each conductor is 0.4  $\Omega$  and reactance 0.5 $\Omega$ . The resistance and reactance of the transformer primary are 7.5  $\Omega$  and 13.2  $\Omega$ , whilst the resistance of the secondary is  $0.35 \Omega$  and reactance  $0.65 \Omega$ . Find the voltage necessary at the sending end of transformission line when 6.6 kV is maintained at the load-end and find the sending-end power factor. Determine also the efficiency of transmission.



Solution. One phase of the system is shown in Fig. 41.26. Impedance per phase of high voltage line =  $(8 + j \ 10)$ 

Impedance of the transformer primary

$$= (7.5 + j 13.2)$$
 ohm

Total impedance on the high-tension side

$$= (8 + j 10) + (7.5 + j 13.2) = 15.5 + j 23.2$$

This impedance can be transferred to secondary side by using the relation given in Art. 30.12. Hence, impedance as referred to secondary side is

$$= (15.5 + j 23.2)/52 = 0.62 + j 0.928$$

Adding to them the impedance of the transformer secondary, we get the total impedance as referred to low-voltage side

	=	$(0.62 + j \ 0.928) + (0.35 + j \ 0.65) = 0.97 + j \ 1.578$
Now, kVA load per phase	=	2,000/3.0 = 667
Receiving-end voltage/phase	=	$6.6/\sqrt{3} = 3.81 \text{ kV}$
:. current in the line	=	667/3.81 = 175 A
Drop per conductor	=	$I(R\cos\phi + X\sin\phi) = 175(0.97 \times 0.8 + 1.578 \times 0.6) = 302 \text{ V}$
Now, $E_S$	=	$E_R + I(R\cos\phi + X\sin\phi)$
Hanna and in a and sealtane (a	1	$a_{1}$ to monotonal as metanomed to the large model $a_{2}$ and $a_{3}$ is 2.810 $\pm$

Hence, sending-end voltage (phase to neutral as referred to the lower voltage side) is 3,810 + 302 = 4,112 V. As referred to high-voltage side, its value =  $4,112 \times 5 = 20,560 V$ 

Line voltage =  $20,560 \times \sqrt{3}/1000 = 35.6 \text{ kV}$ 

If  $\phi_s$  is the power factor angle at the sending-end, then

$\tan \phi_S$	; =	$\frac{\sin\phi + (IX/E_B)}{\cos\phi + (IR/E_R)} = \frac{0.6 + (175 \times 1.578/3810)}{0.8 + (175 \times 0.97/3.10)} = 0.796$					
∴ ¢ <sub>5</sub>	, =	$\tan^{-1}(0.796) = 38^{\circ}31'$ $\therefore$ $\cos \phi_s = \cos 38^{\circ}31' = 0.782$					
Power loss/phase	=	$1752 \times 0.97/1000 = 29.7 \text{ kW}$					
Power at the receiving end/phase = $2000 \times 0.8/3 = 533.3$ kW							
: transmission efficiency	=	$\frac{533.3 \times 100}{533.3 + 29.7} = 94.7 \%$					

#### **Tutorial Problem No. 41.1**

- 500 kW at 11kV are received from 3-phase transmission line each wire of which has a resistance of 1.2 Ω and a reactance of 1 Ω. Calculate the supply pressure when the power factor of the load is (*i*) unity and (*ii*) 0.5 leading. [(*i*) 11,055 V (*ii*) 10,988 V]
- What load can be delivered by a 3-phase overhead line 5 km long with a pressure drop of 10%. Given that the station voltage is 11 kV, resistance per km of each line 0.09 Ω, reactance per km 0.08 Ω and the power factor of the load 0.8 lagging. [14,520 kW]
- 3. Estimate the distance over which a load of 15,000 kW at 0.85 power factor can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance  $0.56 \Omega$  per km. The p.d. at the receiving end is to be 132 kV and the loss in transmission is not to exceed 7.5%

#### [121.9 km] (I.E.E. London)

- 4. A d.c. 2-wire system is to be converted into 3-phase, 3-wire a.c. system by adding a third conductor of the same size as the two existing conductors. Calculate the percentage additional balanced load that can now be carried by the conductors at 0.96 p.f. lagging. Assume the same voltage between the conductors and the same percentage power loss. [84%]
- 5. A 3-phase short transmission line of resistance 8  $\Omega$  and reactance 11  $\Omega$  per phase is supplied with a voltage of 11 kV. At the end of the line is a balanced load of *P* kW per phase at a p.f. of 0.8 leading. For what value of *P* is the voltage regulation of the line zero?

#### [210 kW] (Electrical Technology, M.S. Univ. Baroda)

- A 3-ph, 50-Hz transmission line 10 km long delivers 2,500 kVA at 10 kV. The p.f. of the load is 0.8 (lag). The resistance of each conductor is 0.3 W/km and the inductance 1.82 mH/km. Find (*a*) the voltage and p.f. at the sending end (*b*) the efficiency of transmission and (*c*) the percentage regulation of the line. [(a) 11.48 kV; 0.763 (b) 91.4% (c) 14.8%]
- The conductors in a single-phase transmission line are 6 m above the ground. Each conductor has 1.5 cm diameter and the conductors are spaced 3 m apart. Starting from the fundamentals, determine the capacitance per kilometre of the line first including and then excluding the effect of ground. What do you conclude ? [4.60 × 10<sup>-3</sup> μF/km; 4.63 × 10<sup>-3</sup> μF/km] (*Ranchi Univ.*)

#### 41.20. Effect of Capacitance

So far we have neglected the effect of capacitance on the line regulation because the capacitances of short lines transmitting at relatively low voltages (up to 20 kV) are negligible. But as the voltage and length of the transmission line increase, the capacitance gradually becomes of greater importance. Similarly, the leakage across insulators also assumes greater importance. Hence, exact calculations of regulation for long lines take into consideration the capacitance and leakage reactance of the lines and are quite elaborate, the amount of elaboration depending on the transmitting voltage.

(*i*) In the case of short lines, ordinarily, the capacitance is negligible. But if in a problem, the line capacitance is given and if the line is less than 80 km, then the line capacitance can be lumped at the receiving or load end as shown in Fig. 41.27 (*a*) although this method of localizing the line capacitance at the load end over-estimates the effect of capacitance. In that case, the line current  $\mathbf{I}_{R}$  is the vector sum of the load current  $\mathbf{I}_{R}$  and the charging current  $\mathbf{I}_{C}$  of the capacitance. Hence,  $\mathbf{I}_{S} = \mathbf{I}_{R} + \mathbf{I}_{C}$ 



Fig. 41.27

Now, charging current 
$$\mathbf{I}_C = j\omega CE_R$$
  $\therefore$   $\mathbf{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$   
 $\therefore$   $\mathbf{I}_S = I_R \cos \phi_R - jI_R \sin \phi_R + j$   $CE_R = I_R \cos \phi_R + j(-I_R \sin \phi_R + \omega CE_R)$   
Line drop  $= I_S(R + jX)$   $\therefore$   $\mathbf{E}_S = \mathbf{E}_R + \mathbf{I}_S(R + \mathbf{J}_X)$ 

The vector diagram is shown in Fig. 41.27 (b).

(*ii*) In the case of lines with voltages up to 100 kV and 150 km in length, satisfactory solutions can be obtained by the so-called *T*-method and  $\pi$ -or pi method as described below.

**Example 41.23.** A (medium) single-phase transmission line 50 km long has the following constants :

resistance/km	$= 0.5 \ \Omega;$	reactance/km = 1.6 $\Omega$
susceptance/km	$= 28 \times 10^{-6} S;$	receiving-end line voltage = 66,000 V

Assuming that total capacitance of the line is located at receiving end alone, determine the sending-end voltage, the sending-end current and regulation. The line is delivering 15,000 kW at 0.8 p.f. lagging. Draw a vector diagram to illustrate your answer.

**Solution.** Let  $E_S$  and  $E_R$  be sending-end and receiving-end voltages respectively as shown in Fig. 41.28.

Load current at the receiving-end is  $I_R = 15 \times 10^4/66 \times 10^3 \times 0.8 = 284$  A Total resistance =  $0.5 \times 50 = 25 \Omega$ ; Total resistance =  $1.6 \times 50 = 80 \Omega$ Susceptance  $B = 28 \times 10^{-6} \times 50 = 14 \times 10^{-4}$  Siemens

Capacitive admittance  $Y = B = 14 \times 10^{-4}$  Siemens

As seen from vector diagram of Fig. 41.29, sending-end current  $I_S$  is the vector sum of load current  $I_R$  and capacitive current  $I_C$ .

Now, 
$$I_C = E_R Y = 66,000 \times 14 \times 10^{-4} = 92 \text{ A}$$
  
Let  $\mathbf{E}_R = (66,000 + j \, 0)$   
 $\mathbf{I}_R = 284 \, (0.8 - j \, 0.6) = 227 - j190; \, \mathbf{I}_C = j \, 92$   
 $\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C = (227 - j \, 190 + j \, 92) = 240 \, \angle -18^\circ 57'; \, Z = 25 + j \, 80 = 84 \, \angle 72^\circ 36'$   
Line drop  $= \mathbf{I}_S \mathbf{Z} = 240 \, \angle -18^\circ 57' \times 84 \, \angle 72^\circ 36' = 20,160 \, \angle 53^\circ 39' = 11,950 + j \, 16,240$   
 $\mathbf{E}_S = \mathbf{E}_R + \mathbf{I}_S \mathbf{Z} = 66,000 + (11,950 + j \, 16,240) = 79,500 \, \angle 11^\circ 44'$   
Regulation  $= \frac{E_S - E_R}{E_S} \times 100 = \frac{79,500 - 66,000}{66,000} \times 100 = 20.5\%$ 



# 41.21. "Nominal" T-method

In this *T*-method, also known as mid-capacitor method, the whole of the line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 41.30.

It is seen that

*.*..

$$\mathbf{E}_1 = \mathbf{E}_R + \mathbf{I}_R \mathbf{Z}_{BC}.$$
 Knowing  $E_1$ , we can find  $\mathbf{I}_C$  as under.  
$$\mathbf{I}_C = j\omega C_{F1} \quad \therefore \quad \mathbf{I}_S = \mathbf{I}_C + \mathbf{I}_R$$

Obviously, current through portion A B is  $I_S$ ; hence voltage drop =  $I_S Z_{AB}$ 

$$\mathbf{E}_{\mathbf{S}} = \mathbf{E}_1 + \mathbf{I}_S \, \mathbf{Z}_{AB}$$



Fig. 41.30



Fig. 41.31

The vector diagram is shown in Fig. 41.31. Receiving-end voltage  $E_R$  is taken as the reference vector. It may be pointed out here that all values are phase values *i.e.* line to neutral values.  $I_R$  is the load current lagging  $E_R$  by  $\phi_R$ ,  $CD = I_R \cdot R/2$  and parallel to  $I_R$ .  $BD = I_R \cdot X/2$  and perpendicular to  $I_R \cdot OB$  represents  $E_1$  *i.e.* the voltage across the middle capacitor. BE represents  $I_S \cdot R/2$  and is parallel to  $I_S$ . Similarly,  $EA = I_S \cdot X/2$  and perpendicular to  $I_S$ . OA represents the voltage at the sending end.

It may be noted that if leakage is appreciable, then leakage conductance G can be assumed to be concentrated at the middle point of the line and can be represented by non-inductive conductance G shunting the middle capacitor as shown in Fig. 41.32.





**Example 41.24.** A 3-phase, 50-Hz overhead transmission line 100 km long with 132 kV between lines at the receiving end has the following constants :

resistance/km/phase =  $0.15 \Omega$ ; inductance/km/phase = 1.20 mHcapacitance/km/phase = 0.01 mF

Determine, using an approximate method of allowing for capacitance, the voltage, current and p.f. at the sending end when the load at the receiving end is 72 MW at 0.8 p.f. lagging. Draw vector diagram for the circuit assumed. (Electrical Power System ; Gujarat Univ.)

Solution. For a 100-km length of the line,

$$R = 0.15 \times 100 = 15 \Omega; X_L = 314 \times 1.2 \times 10^{-3} \times 100 = 37.7 \Omega$$
$$X_C = 106/314 \times 0.01 \times 100 = 3187 \Omega$$

Using the nominal *T*-method, the equivalent circuit is shown in Fig. 41.33 (*a*)  $V_R = 132/\sqrt{3} = 76.23 \text{ kV} = 76,230 \text{ V}$ . Hence,  $\mathbf{V}_R = 76,230 + j 0$ Load current,  $\mathbf{I}_R = 72 \times 102/\sqrt{3} \times 132 \times 103 \times 0.8 = 394 \text{ A}$   $\therefore$   $\mathbf{I}_R = 394 (0.8 - j 0.6) = 315 - j 236 \text{ A}$ ;  $\mathbf{Z}_{BC} = (7.5 + j 18.85) \text{ W}$ Drop/phase over  $BC = \mathbf{I}_R \mathbf{Z}_{BC} = (315 - j 236) (7.5 + j 18.85) = 6802 + j 4180$   $\mathbf{V}_1 = \mathbf{VR} + \mathbf{I}_R \mathbf{Z}_{BC} = (76,230 + j0) + (6802 + j 4180) = 88,030 + j 4180$   $\mathbf{I}_C = \frac{\mathbf{V}_1}{\mathbf{X}_C} = \frac{83,030 + j 4180}{-j 3187} = -1.31 + j 26$   $\mathbf{I}_S = \mathbf{I}_C + \mathbf{I}_R = (-1.31 + j 26) + (315 - j 236) = (313.7 - j 210) = 377.3 \angle - 33.9^\circ$ Drop/phase over  $AB = \mathbf{I}_S \mathbf{Z}_{AB} = (313.7 - j 210) (7.5 + j 18.85) = 6320 + j 4345$  $\mathbf{V}_1 = \mathbf{V}_1 + \mathbf{I}_2 \mathbf{T}_2 = (22.020 + i 4180) + (6220 + i 4245) = 80.250 + i 8525 = 80.750 (550)$ 

 $\therefore \quad \mathbf{V}_{S} = \mathbf{V}_{1} + \mathbf{I}_{S} \mathbf{Z}_{AB} = (83,030 + j\,4180) + (6320 + j\,4345) = 89,350 + j\,8525 = 89,750 \angle 5.4^{\circ}$ Line value of sending-end voltage =  $\sqrt{3} \times 89,750 \times 10^{-3} = 155.7 \text{ kV}$ 



Fig. 41.33

Phase difference between  $V_s$  and  $I_s = 33.9^\circ + 5.4^\circ = 39.3^\circ$  with current lagging as shown in Fig. 41.33 (*b*)  $\cos \phi_s = \cos 39.3^\circ = 0.774$  (lag)

**Example 41.25.** A 3-phase, 50-Hz transmission line, 100 km long delivers 20 MW at 0.9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0.2  $\Omega$  and 0.4  $\Omega$  respectively while the capacitive admittance is  $2.5 \times 10^{-6}$  S per km. Calculate (a) the voltage and current at the sending end and (b) the efficiency of transmission. Use the nominal T-method.

(Electrical Power-I, M.S. Univ. Baroda)

**Solution.** Resistance for 100 km =  $0.2 \times 100 = 20 \Omega$ 

Reactance for 100 km =  $0.4 \times 100 = 40 \Omega$ 

Capacitive admittance for 100 km =  $2.5 \times 10^{-6} \times 100 = 2.5 \times 10^{-4}$  S

Let us take the receiving-end voltage  $E_R$  as reference vector.

$$E_R = 110/\sqrt{3} = 63.5 \text{ kV}$$
;  $I_R = 20 \times 106/\sqrt{3} \times 110 \times 103 \times 0.9 = 116.6 \text{ A}$ 

 $\cos \phi_R = 0.9$ ;  $\sin \phi_R = 0.435$  (from tables)

With reference to Fig. 41.34, we have

$$\mathbf{E}_{R} = (63.5 + j\ 0)\ \text{kV}$$
;  $\mathbf{I}_{R} = 116.6(0.9 - j\ 0.\ 435) = 105 - j\ 50.7$ ;  $\mathbf{Z}_{BC} = (10 + j\ 20)$ 





#### Fig. 41.34

Voltage drop between points *B* and *C* is

 $\mathbf{V}_{BC} = \mathbf{I}_{R} \mathbf{Z}_{BC} = (105 - j \ 50.7) \ (10 + j \ 20) = (2,064 + j \ 1,593) \ \mathbf{V}$   $\mathbf{E}_{1} = \mathbf{E}_{R} + \mathbf{I}_{R} \mathbf{Z}_{BC} = (63,500 + 2,064 + j \ 1,593) = 65,564 + j \ 1,593$   $\mathbf{I}_{C} = \mathbf{E}_{1} \mathbf{Y} = (65,564 + j \ 1,593) \times j \ 2.5 \times 10^{-4} = (-0.4 + j \ 16.4) \ \mathbf{A}$   $\mathbf{I}_{S} = \mathbf{I}_{R} + \mathbf{I}_{C} = (105 - j \ 50.7) + (-0.4 + j \ 16.4) = (104.6 - j \ 34.3) = 110.1 \ \angle 18^{\circ} \ 9$ 

Drop between points A and B is

 $\mathbf{V}_{AB} = \mathbf{I}_{S} \mathbf{Z}_{AB} = (104.6 - j \ 34.3) \ (10 + j \ 20) = 1,732 + j \ 1,749$  $\mathbf{E}_{S} = \mathbf{E}_{1} + \mathbf{V}_{AB} = (65,564 + j \ 1,593) + (1,732 + j \ 1,749) = 67,296 + j \ 3,342 = 67,380 \ \angle 2^{\circ} \ 51$ (*a*) Sending-end voltage (line value) is  $67,380 \times \sqrt{3} = 116,700 \ \text{V} = \mathbf{116.7 \ kV}$ Sending-end current = **110.1 A** 

Note. Phase difference between  $E_s$  and  $I_s$  is  $(-18^\circ9 - 2^\circ51) = -21^\circ$  with current lagging. Hence, p.f. at sending-end is  $\cos \phi_s = \cos 21^\circ = 0.934$  (lag).

(b) Copper loss for three phases between points B and C (Fig. 41.32) is

 $= 3 \times 116.6^2 \times 10 = 0.408$  MW

Copper loss for three phases between points A and B is  $= 3 \times 110.1^2 \times 10 = 0.363$  MW Total Cu loss for 100 km of line length = 0.408 + 0.363 = 0.771 MW Transmission  $\eta = 20 \times 100/20.771 = 96.27$  %

Note. Line losses could also be computed by finding the line input which =  $\sqrt{3} E_s I_s \cos \phi_s$ .

#### 41.22. "Nominal" π-method

In this method, the line-to-neutral capacitance is divided into two halves ; one half being concentrated or localized at the sending-end and the other half at the receiving-end as shown in Fig. 41.35 (*a*). The capacitance at the sending or generating end has no effect on line drop or line regulation but its charging current must be added to the line current in order to obtain the total sending-end current  $I_s$ .

It is obvious that  $I_S$  is the vector sum of  $I_{C2}$  and  $I_L$  where  $I_L$  is the vector sum of  $I_{C1}$  and  $I_R$ . The vector diagram is shown in Fig. 41.35 (b).  $E_R$  is taken as the reference vector. Current  $I_L$  is vector sum of  $I_R$  and  $I_{C1}$  (which is ahead of  $E_R$  by 90°). The drop  $A B = I_L R$  is in phase with vector for  $I_L$  and reactive drop  $BC = I_L X$  is in quadrature with  $I_L$ . OC represents the sending-end voltage. The sending-end current  $I_S$  is the vector sum of  $I_{C2}$  (which itself is ahead of  $E_S$  by 90°). It may be noted that



if leakage reactance is not negligible, then leakage conductance G can also be divided into two equal halves and put at both ends in parallel with the capacitors as shown in Fig. 41.36.



A long or medium transmission line has considerable capacitance and so draws leading charg-

ing current from the generating-end even when unloaded. Moreover, receiving-end voltage V<sub>R</sub> under no-load condition is found to be *greater* than sending-end voltage  $V_s$ . This phenomenon is known as Ferranti effect.





Fig. 41.37 (a) shows the distributed parameters of such a line. It may be replaced by the circuit of Fig, 41.37 (b) where these distributed parameters have been lumped. As shown in the phasor

Fig. 41.35

G/2

١ç

C/2

I<sub>C2</sub>

E<sub>s</sub>

 $Z_L$ 

I<sub>L</sub>

Neutral

Fig. 41.36

R

M X

G/2

M

 $I_R$ 

C/2

I<sub>C1</sub>

E<sub>R</sub>

I<sub>C</sub>X

I<sub>C</sub>R

diagram of Fig. 41.38, the charging current  $I_C$  leads  $V_R$ by 90° and produces a phase voltage drop =  $I_C Z = I_C (R + j X_L)$ . Obviously,  $V_S < V_R$  Now,  $I_C = V_R \omega C$ 

As seen from Fig. 41.38

$$W_S = \sqrt{(V_R - I_C X_L)^2 + (I_C R)^2}$$
  
gible then

 $\overline{2}$   $V_8$   $V_R$  Fig. 41.38

If R is negligible, then

 $V_S = (V_R - I_C X_L)$  or  $V_R = V_S + I_C X_L$ 

#### 41.24. Charging Current and Line Loss of an Unloaded Transmission Line

Fig. 41.39 shows the distribution of capacitance in a long transmission line of length l. Obviously, charging current  $I_C$  has maximum value at the sending end and linearly falls to zero at the receiving end. Accordingly, value of charging current at distance x from the sending end is proportionally equal to  $I_C (l-x)/l$ . The *RMS* value I of this current is given by

$$I^{2} = \frac{1}{l} \int_{0}^{l} \frac{I_{C}^{2} (l-x)^{2}}{l^{2}} dx = \frac{I_{C}^{2}}{l^{3}} \int_{0}^{l} (l^{2} + x^{2} - 2lx) dx$$
$$= \frac{I_{C}^{2}}{l^{3}} \left[ l^{2}x + \frac{x^{3}}{3} - lx^{2} \right]_{0}^{l} = \frac{I_{C}^{2}}{3}$$
$$I = I_{C} / \sqrt{3}$$

*.*.

If *R* is the resistance of the line per phase, then total power loss in the line is

$$= 3 I^2 R = 3 \left(\frac{I_C}{\sqrt{3}}\right)^2 R = I_C^2 R.$$

**Example 41.26.** A 3-phase transmission line, 100 km long has following constants: resistance per km per phase = 0.28  $\Omega$ ; inductive reactance per km per phase = 0.63  $\Omega$ . Capacitive susceptance per km per phase =  $4 \times 10^{-6}$  siemens. If the load at the receiving end is 75 MVA at 0.8 p.f. lagging with 132 kV between lines calculate sending-end voltage, current and p.f. Use nominal- $\pi$ -method.

(Power System-I, AMIE, Sec. B. 1994)

**Solution.** For a line of length 100 km, resistance/phase =  $0.28 \times 100 = 28 \Omega$ ; inductive reactance/phase =  $0.63 \times 100 = 63 \Omega$ Capacitive susceptance/phase =  $4 \times 10^{-6} \times 100$ =  $4 \times 10^{-4}$  S

Capacitive susceptance at each end  $= 2 \times 10^{-4}$  S  $\mathbf{V}_R = 132 \times 10^3 / \sqrt{3} = 76,230 \text{ V}; \mathbf{V}_R = (76,230 \text{ } + j \text{ } 0); \mathbf{I}_R = 75 \times 10^6 / \sqrt{3} \times 132 \times 10^3 \times 0.8 = 410$   $\mathbf{I}_R = 410 (0.8 - j \text{ } 0.6) = 328 - j \text{ } 246$  $\mathbf{Y}_{CL} = j 2 \times 10^{-4}$  S;  $\mathbf{I}_{CL} = \mathbf{V}_R$ ,  $\mathbf{Y}_{CL}$ 

$$C_1 = j 2 \times 10^{-5} \text{ s}, r_{C_1} = \mathbf{v}_R \cdot \mathbf{r}_{C_1}$$
  
= 76,230 × j 2 × 10<sup>-4</sup> = j 15.25 A



Fig. 41.39

 $I_{L} = I_{R} + I_{C1} = 328 - j 231 \text{ A}$ Drop per conductor =  $I_{L}$ .  $Z_{L} = (328 - j 231) (28 + j 63) = 23,737 + j 14,196$  $V_{S} = V_{R} + I_{L} Z_{L} = 76,230 + 23,737 + j 14,196 = 99,967 + j 14,196 = 100,970 \angle 8.1^{\circ}$ Line value of sending-end voltage =  $100,970 \times 10^{-3} \times \sqrt{3} = 174.9 \text{ kW}$  $I_{C} = Y_{S} Y_{C2} = 100,970 \angle 8.1^{\circ} \times 2 \times 10^{-4} \angle 90^{\circ} = 20.19 \angle 98.1^{\circ} = -2.84 + j 20$  $I_{S} = I_{C}^{2} + I_{L} = (-2.84 + j 20) + (328 - j 231) = 325.2 - j 211 = 388 \angle -33^{\circ}$ Angle between  $V_{S}$  and  $I_{S} = 8.1^{\circ} + 33^{\circ} = 41.4^{\circ}$ ; cos  $\phi = 0.877$  (lag)





**Example 41.27.** A 100-km long, three-phase, 50-Hz transmission line has resistance/phase/ km = 0.1  $\Omega$ ; reactance/phase/km = 0.5  $\Omega$ ; susceptance/phase/km = 10 × 10<sup>-6</sup> siemens.

If the line supplies a load of 20 MW at 0.9 p.f. lagging at 66 kV at the receiving end, calculate by nominal 'p' method, the regulation and efficiency of the line. Neglect leakage.

(Electrical Power-I, Bombay Univ. 1991)

Solution. For a 100 km line resistance/phase =  $0.1 \times 100 = 10 \ \Omega$ ; inductive reactance/phase =  $0.5 \times 100 = 50 \ \Omega$ Capacitive susceptance/phase =  $10 \times 10^{-4} \times 100 = 10 \times 10^{-4}$  siemens Susceptance at each end =  $5 \times 10^{-4}$  siemens :.  $\mathbf{V}_{R} = (38,100 + j \, 0)$  $V_R = 66 \times 10^3 / \sqrt{3} = 38,100 \text{ V}$  $I_R = 20 \times 10^6 / \sqrt{3} \times 66 \times 10^3 \times 0.9 = 195 \text{ A}$ ;  $I_R = 195(0.9 - j\ 0.435) = 176 - j\ 85$  $\mathbf{I}_{C1} = 38100 \times j 5 \times 10^{-4} = j 19.1 \text{ A}; \mathbf{I}_{L} = \mathbf{I}_{R} + \mathbf{I}_{C1} = 176 - j 66$ drop/conductor =  $\mathbf{I}_{L} \mathbf{Z}_{L} = (176 - j \ 66) \ (10 + j \ 50) = 5060 + j \ 8140$  $\mathbf{V}_{S} = \mathbf{V}_{R} + \mathbf{I}_{L} \mathbf{Z}_{L} = 38,100 + 5060 + j 8140 = 43,160 + j 8140 = 43,920 \angle 10.7^{\circ}$ Line value =  $43,920 \times 10^{-3} \times \sqrt{3} = 76 \text{ kV}$  $\mathbf{I}_{C2} = \mathbf{V}_{S} \cdot \mathbf{Y}_{C2} = 43,920 \angle 10.7^{\circ} \times 5 \times 10^{-4} \angle 90^{\circ} = 21.96 \angle 100.7^{\circ} = -4.1 + j \, 21.6$  $\mathbf{I}_{S} = \mathbf{I}_{C2} + \mathbf{I}_{L} = 172 - j \, 44.4 = 177.6 \, \angle -14.5^{\circ}; \, \phi_{S} = 14.5^{\circ} + 10.7^{\circ} = 25.2^{\circ}$  $\cos \phi_s = 0.905 (\text{lag})$ % regulation =  $\frac{76-66}{66} \times 100 = 15.15\%$ *(i)* 

line input  $= \sqrt{3} \times 76 \times 103 \times 177.6 \times 0.905 = 21.5 \times 10^{6} \Omega = 21.15 \text{ MW}$ (ii)

: transmission  $\eta = 20 \times 100/21.15 = 94.56\%$ 

Example 41.28. (a) A 50-Hz, 3-phase, 100-km long line delivers a load of 40 MVA at 110 kV and 0.7 p.f. lag. The line constants (line to neutral) are : resistance of 11 ohms, inductive reactance of 38 ohms and capacitive susceptance of  $3 \times 10^{-4}$  siemens. Find the sending-end voltage, current, power factor and efficiency of power transmission.

(b) draw the vector diagram.

(c) If the sending-end voltage is held constant and load is removed, calculate the receiving-end voltage and current. (Electrical Power-II, Bangalore Univ.)

**Solution.** (a) Nominal  $\pi$ -method will be used to solve this problem. Capacitive susceptance (or admittance) at each end =  $Y/2 = 3 \times 10^{-4}/2 = 1.5 \times 10^{-4}$  siemens as shown in Fig. 41.41.



- :. power transmission  $\eta = 28/29.41 = 0.952$  or **95.2%**
- (b) vector diagram is similar to that shown in Fig. 41.35 (b).
- (c) As seen from Fig. 41.41 under no-load condition, current in the conductor is

= **I**<sub>C1</sub> = *j* 9.5 A Drop/phase =  $\mathbf{I}_{C_1} \mathbf{Z}_L = j 9.5(11 + j 38) = -361 + j 105$  $\mathbf{V}_{\rm c} = 63,500 + (-361 + j\,105) = 63,140 + j\,105 = 63,140 \ \angle 0.1^{\circ}$  $\mathbf{I}_{C2} = (63,140 + j\ 105) \times j\ 1.5 \times 10^{-4} = (-\ 0.016 + j\ 9.47) \text{ A}$  $\mathbf{I}_{S} = \mathbf{I}_{C_{1}} + \mathbf{I}_{C_{2}} = j \, 9.5 - 0.016 + j \, 9.47 = (-0.016 + j \, 18.97) \, \mathrm{A}$ 

# **Tutorial Problem No. 41.2**

A 50-Hz, 3-phase line 100 km long delivers a load of 40,000 kVA at 110 kV and a lagging power 1. factor of 0.7. The line constants (line-to-neutral) are : resistance 11  $\Omega$ ; inductive reactance 38  $\Omega$ ; capacitive susceptance  $3 \times 10^{-4}$  S (half at each end), leakage negligible. Find the sending-end voltage, current, power factor and power input. [122 kV ; 195.8 A ; 0.715 ; 29.675 kW]

- 2. A 50-Hz, 3-phase transmission line has the following constants (line-to-neutral). Resistance 11  $\Omega$ , reactance 38  $\Omega$ ; susceptance  $3 \times 10^{-4}$  S, leakage negligible. The capacitance can be assumed located half at each end of the line. Calculate the sending-end voltage, the line current and the efficiency of transmission when the load at the end of the line is 40,000 kVA at 110 kV power factor 0.7 lagging. [122 kV ; 203.35 A ; 95.5%]
- 3. A 3-phase transmission line has the following constants (line-to-neutral);  $R = 10 \Omega$ ; inductive reactance =  $20 \Omega$ , capacitive reactance =  $2.5 k \Omega$ . Using the nominal *T*-method, calculate the voltage, line current and power factor at sending-end and the efficiency of transmission when the transmission line supplies a balanced load of 10 MW at 66 kV and power factor 0.8 lagging.

[69.5 kV; 100 A; 0.85 lag; 96.8 %] (Electrical Technology; M.S. Univ. Baroda)

- A 3-phase line has a resistance of 5.31 ohms and inductance of 0.0176 henry. Power is transmitted at 33 kV, 50 Hz from one end and the load at the receiving end is 3600 kW at 80 per cent power factor. Find the line current, receiving-end voltage, sending-end power factor and the efficiency of transmission.
   [81.5 A, 31.9 kV, 0.796, 97.2%] (U.P.S.C.)
- Calculate the % regulation of 6.6 kV single-phase A.C. transmission line delivering 40 amps current at 0.8 lagging power factor. The total resistance and reactance of the line are 4.0 ohm and 5.0 ohm per phase respectively.
   [3.76%]
- 6. A-3 phase load of 2,000 kVA at 0.8 power factor is supplied at 6.6 kV, 50 Hz by means of a 33 kV transmission line, 32 km long in association with a 33/6.6 kV step-down transformer. The resistance and reactance of each conductor per km are 0.4 ohm and 0.5 ohm respectively. The resistance and reactance of the transformer primary are 7.5 ohm and 13.2 ohm while those of secondary are 0.35 ohm and 0.65  $\Omega$  respectively. Find the voltage necessary at the sending end of the transmission line when the voltage is maintained at 6.6 kV at the receiving end. Determine also the sending-end power and efficiency of transmission. [35.4 kV, 1,689.5 kW, 94.7%]
- The generalised A and B constants of a transmission line are 0.96 ∠1° and 120 ∠80° respectively. If the line to line voltages at the sending and receiving ends are both 110 kV and the phase angle between them is 30°, find the receiving-end power factor and the current. With the sending-end voltage maintained at 110 kV, if the load is suddenly thrown off, find the corresponding receiving-end voltage. [0.868 (lead), 326 A,114.5 kV]
- corresponding receiving-end voltage. [0.868 (lead), 326 A,114.5 kV]
   A 3-phase, 50-Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0.1 Ω, 0.5 Ω and 3 × 10<sup>-6</sup> siemens/km/phase. If the line delivers 50 MW at 110 kV and 0.8 p.f. lag, determine the sending-end voltage and current. Assume a nominal π-circuit for the line.

[143.7 kV, 306.3 A]

## 41.25. Generalised Circuit Constants of a Transmission Line

For any 4-terminal network *i.e.* one having two input and two output terminals (like a transmission line) the sending-end voltage per phase and the currents at the receiving and sending-end can be expressed by the following two equations :

$$\mathbf{V}_{S} = \mathbf{A}\mathbf{V}_{R} + \mathbf{B}\mathbf{I}_{R} \qquad \dots(i)$$
$$\mathbf{I}_{S} = \mathbf{C}\mathbf{V}_{R} + \mathbf{D}\mathbf{I}_{R} \qquad \dots(ii)$$

where **A**, **B**, **C** and **D** are the constants known as '*generalized circuit constants*' of the transmission line. Their values depend on the particular method adopted for solving the transmission network. Let us consider the following cases.

(*i*) Short Line. In the case of lines up to 50 km, the effect of capacitance on the line performance is negligible. Hence, such a line can be represented as shown in Fig. 41.42 (*a*).

Here, 
$$\mathbf{I}_{S} = \mathbf{I}_{R}$$
 and  $\mathbf{V}_{S} = \mathbf{V}_{R} + \mathbf{I}_{R}\mathbf{Z}$ 

Comparing these with Eq. (i) and (ii) above, we get that

**A** = 1; **B** = **Z**, **C** = 0 and **D** = 1
(ii) Medium Line-Nominal-T Method

The circuit is shown in Fig. 41.42 (b).

It is seen that, Also Now,

*:*.





Fig. 41.42

Eliminating  $\mathbf{V}_1$  from Eq. (*iii*) and (*iv*) we get

$$\mathbf{Y}_{S} = \mathbf{V}_{R} + \frac{\mathbf{I}_{S}\mathbf{Z}}{2} + \frac{\mathbf{I}_{R}\mathbf{Z}}{2} \qquad \dots (vi)$$

Substituting the value of  $I_s$ , we get

v

$$\mathbf{V}_{S} = \left(1 + \frac{\mathbf{Y}\mathbf{Z}}{2}\right)\mathbf{V}_{R} + \left(\mathbf{Z} + \frac{\mathbf{Y}\mathbf{Z}^{2}}{4}\right)\mathbf{I}_{R} \qquad \dots (\nu ii)$$

Comparing Eq. (vii) and (v) with Eq. (i) and (ii) respectively, it is found that

$$\mathbf{A} = \mathbf{D} = 1 + \frac{\mathbf{Y}\mathbf{Z}}{2}$$
;  $\mathbf{B} = \left(1 + \frac{\mathbf{Y}\mathbf{Z}}{4}\right)$  and  $\mathbf{C} = \mathbf{Y}$ 

It can again be proved that AD - BC = 1.

## (*iii*) Medium Line–Nominal- $\pi$ Method

The circuit is shown in Fig. 41.42 (c). Here, series impedance per phase = (R + j X) and admittance is  $\mathbf{Y} = j\omega C$ .

As seen,  

$$\mathbf{I}_{S} = \mathbf{I} + \mathbf{I}_{C2} = \mathbf{I} + \mathbf{V}_{S} \mathbf{Y}/2 \qquad \dots (viii)$$
Also  

$$\mathbf{I} = \mathbf{I}_{C1} + \mathbf{I}_{R} = \mathbf{I}_{R} + \mathbf{V}_{R} \mathbf{Y}/2 \qquad \dots (ix)$$
Now,  

$$\mathbf{V}_{S} = \mathbf{V}_{R} + \mathbf{I}_{Z} = \mathbf{V}_{R} + \mathbf{Z} \left(\mathbf{I}_{R} + \frac{\mathbf{V}_{R} \mathbf{Y}}{2}\right)$$

Now,

$$= \left(1 + \frac{\mathbf{Y}\mathbf{Z}}{2}\right)\mathbf{V}_R + \mathbf{Z}\mathbf{I}_R \qquad \dots (\mathbf{x})$$

Eliminating I from Eq. (*viii*) and (*ix*), we get

$$\mathbf{I}_{S} = \mathbf{I}_{R} + \frac{\mathbf{V}_{R}\,\mathbf{Y}}{2} + \frac{\mathbf{V}_{S}\,\mathbf{Y}}{2}$$

Now, substituting the value of  $V_s$ , we have

$$\mathbf{I}_{S} = \mathbf{I}_{R} + \frac{\mathbf{V}_{R}\mathbf{Y}}{2} + \frac{\mathbf{Y}}{2} \left[ \left( 1 + \frac{\mathbf{Y}\mathbf{Z}}{2} \right) \mathbf{V}_{R} + \mathbf{Z}\mathbf{I}_{R} \right]$$
$$\mathbf{I}_{S} = \mathbf{Y} \left( 1 + \frac{\mathbf{Y}\mathbf{Z}}{4} \right) \mathbf{V}_{R} + \left( 1 + \frac{\mathbf{Y}\mathbf{Z}}{2} \right) \mathbf{I}_{R} \qquad \dots (xi)$$

or

Comparing Eq. (x) and (ii) with Eq. (i) and (xi) above, we get,

$$\mathbf{A} = \mathbf{D} = \left(1 + \frac{\mathbf{Y}\mathbf{Z}}{2}\right); \mathbf{B} = \mathbf{Z}, \mathbf{C} = \mathbf{Y}\left(1 + \frac{\mathbf{Y}\mathbf{Z}}{4}\right)$$

Again, it can be shown that AD - BC = 1

**Example 41.29.** Find the following for a single-circuit transmission line delivering a load of 50 MVA at 110 kV and p.f. 0.8 lagging :

(i) sending-end voltage, (ii) sending-end current, (iii) sending-end power, (iv) efficiency of transmission. (Given  $A = D = 0.98 \angle 3^{\circ}$ ,  $B = 110 \angle 75^{\circ}$  ohm,  $C = 0.0005 \angle 80^{\circ}$  ohm)

(Power Systems, AMIE, Sec. B, 1993)

**Solution.** Receiving-end voltage,  $V_R = 110/\sqrt{3} = 63.5$  kV

Taking this voltage as reference voltage, we have

 $V_{R} = (63,500 + j 0)$   $I_{R} = 50 \times 10^{6} / \sqrt{3} \times 110 \times 10^{3} = 262.4 \text{ A}$   $I_{R} = 262.4 \angle \cos^{-1} 0.8 = 262.4 (0.8 - j 0.6) = (210 - j 157.5) \text{ A}$   $AV_{R} = 0.98 \angle 3^{\circ} \times 63,500 \angle 0^{\circ} = 62,230 \angle 3^{\circ} = (62145 + j 3260) \text{ V}$   $BI_{R} = 110 \angle 75^{\circ} \times 262.4 \angle 36.86^{\circ} = 28,865 \angle 111.8^{\circ} = (-10,720 + j 26,800) \text{ V}$   $V_{S} = AV_{R} + BI_{R} = 51,425 + j 30,060 = 59,565, \angle 30.3^{\circ} \text{ V}$   $CV_{R} = 0.0005 \angle 80^{\circ} \times 63,500 \angle 0^{\circ} = 31.75 \angle 80^{\circ} = (5.5 + j 31.3) \text{ A}$ 







 $I_{S} = CV_{R} + DI_{R} = 203 + j \ 196 = 282 \ \angle 44^{\circ}A$ 

 $\mathbf{DI}_{R} = 0.98 \angle 3^{\circ} \times 262.4 \angle 36.86^{\circ} = 257.75 \angle 39.86^{\circ} = 197.4 + j \ 164.8$ 

*(ii)* 

(*iii*) Sending-end power =  $3V_S I_S \cos \phi_S = 3 \times 59.565 \times 282 \times \cos \angle 44.30^\circ - 30.3^\circ = 48.96$  MW Receiving end power =  $50 \times 0.8 = 40$  MW

(*iv*) Transmission  $\eta = 40 \times 100/48.96 = 81.7\%$ 

**Example 41.30.** A 150 km,  $3-\phi$ , 110-V, 50-Hz transmission line transmits a load of 40,000 kW at 0.8 p.f. lag at receiving end.

resistance/km/phase = 0.15  $\Omega$ , reactance/km/phase = 0.6  $\Omega$ ; susceptance/km/phase =  $10^{-5}$  S (a) determine the A, B, C and D constants of the line (b) find regulation of the line.

(Power System-I, AMIE, 1993)

**Solution.** We will use the nominal- $\pi$  method to solve the problem. For a length of 150 km :

**R** =  $0.15 \times 150 = 22.5 \ \Omega$ ; **X** =  $0.6 \times 150 = 90 \ \Omega$ ; **Y** =  $150 \times 10^{-5} \ \text{S} = 15 \times 10^{-4} \ \text{S}$ 

**Z** =  $(\mathbf{R} + j \mathbf{X}) = (22.5 + j 90) = 92.8 \angle 75^{\circ} \text{ ohm}; \mathbf{Y} = 15 \times 10^{-4} \angle 90^{\circ} \text{ S}$ 

(a) 
$$\mathbf{A} = \mathbf{D} = \left(1 + \frac{\mathbf{YZ}}{2}\right) = 1 + \frac{j \cdot 15 \times 10^{-4}}{2} (22.5 + j \cdot 90) = (0.9675 + j \cdot 0.01688)$$
  
= 0.968  $\angle 1.0^{\circ}$   
 $\mathbf{B} = \mathbf{Z} = 92.8 \angle 7.5^{\circ}; \mathbf{C} = \mathbf{Y} \left(1 + \frac{\mathbf{YZ}}{4}\right) = j \cdot 15 \times 10^{-4} \left[1 + \frac{1}{4} \cdot j \cdot 15 \times 10^{-4} (22.5 + j \cdot 90)\right]$   
= - 0.00001266 + j 0.00145 = 0.00145  $\angle 90.5^{\circ}$ 

.

(b) Now, the regulation at the receiving-end is defined as the change in voltage when full-load is thrown off, the sending-end voltage being held constant.

Now, when load is thrown off,  $I_R = 0$ . Hence, putting this value in  $V_S = AV_R + BI_R$ , we get

$$\mathbf{V}_{S} = \mathbf{A}\mathbf{V}_{RO} \text{ or } \mathbf{V}_{RO} = \frac{\mathbf{V}_{S}}{\mathbf{A}} \quad \therefore \quad \% \text{ regn.} = \frac{V_{RO} - V_{R}}{V_{R}} \times 100 = \frac{(V_{S}/A - V_{R})}{V_{R}} \times 100$$

Now,  $V_R = 100/\sqrt{3}$  kV = 63,520 V—at load  $\mathbf{I}_R = 40 \times 10^6/\sqrt{3} \times 110 \times 10^3 \times 0.8 = 263$  A;  $\mathbf{I}_R = 263 (0.8 - j 0.6) = (210 - j 158)$   $\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R = 63,520 (0.9675 + j 0.01688) + (22.5 + j 90) (210 - j 158)$  $= 80,450 + j 16,410 = 82,110 \angle 11.5^\circ$ 

$$V_S/A = 82,110/0.968 = 84,800$$
; regn. =  $\frac{84,800 - 63,520}{63,520} \times 100 = 33.5\%$ 

**Example 41.31.** A 132-kV, 50-Hz, 3-phase transmission line delivers a load of 50 MW at 0.8 p.f. lagging at receiving-end.

The generalised constants of the transmission line are

 $A = D = 0.95 \angle 1.4^{\circ}$ ;  $B = 96 \angle 7.8^{\circ}$ ;  $C = 0.0015 \angle 90^{\circ}$ 

Find the regulation of the line and the charging current. Use nominal T-method.

(Electrical Power-I, Bombay Univ.)

Solution.  

$$I_{R} = 50 \times 10^{\circ} / \sqrt{3} \times 132 \times 10^{3} \times 0.8 = 273 \text{ A}$$

$$I_{R} = 273 (0.8 - j 0.6) = 218 - j 164 = 273 \angle -36.9^{\circ}$$

$$V_{R} = 132,000 / \sqrt{3} = 76,230 ; V_{R} = 76,230 + j 0$$

$$V_{S} = AV_{R} + BI_{R} = 76,230 \angle 0^{\circ} \times 0.95 \angle 1.4^{\circ} + 96 \angle 78^{\circ}.273 \angle -36.9^{\circ}$$

$$= 72,418 \angle 1.4 + 26,208 \angle 41.1^{\circ} = 89,000 + j 21,510 = 92,150 \angle 13^{\circ}$$

$$I_{S} = CV_{R} + DI_{R} = 76,230 \times 0.0015 \angle 90^{\circ} + 0.95 \angle 1.4^{\circ}.273 \angle -36.9^{\circ}$$

$$= j 114 + 259.3 \angle -35.5^{\circ} = 211 - j 36$$

$$V_{RO} = V_{S}/A = 92,150/0.95 = 97,000 ; \% \text{ regn} = \frac{97,000 - 76,230}{76,230} = 27.1$$

$$I_{C} = I_{S} - I_{R} = (211 - j 36) - (218 - j 164) = -7 + j 128 = 128.2 \angle 93.1^{\circ}$$

**Example 41.32.** A 3-phase transmission line consists of two lines 1 and 2 connected in series, line 1 being at the sending end and 2 at the receiving end. The respective auxiliary constants of the two lines are :  $A_1$ ,  $B_1$ ,  $C_1$ ,  $D_1$  and  $A_2$ ,  $B_2$ ,  $C_2$ ,  $D_2$ . Find the A, B, C, D constants of the whole line which is equivalent to two series-connected lines.

**Solution.** The two series-connected lines along with their constants as shown in Fig. 41.43. For line No. 1

$\mathbf{V}_S = \mathbf{A}_1 \mathbf{V} + \mathbf{B}_1 \mathbf{I}$	;	$\mathbf{I}_{S} = \mathbf{C}_{1}\mathbf{V} + \mathbf{D}_{1}\mathbf{I}$	( <b>i</b> )
For line No. 2			

 $\mathbf{V} = \mathbf{A}_2 \mathbf{V}_R + \mathbf{B}_2 \mathbf{I}_R \quad ; \qquad \mathbf{I} = \mathbf{C}_2 \mathbf{V}_R + \mathbf{D}_2 \mathbf{I}_2 \qquad \dots (\mathbf{i})$ 

Substituting the values of  $\mathbf{V}$  and  $\mathbf{I}$  from Eq. (*ii*) into Eq. (*i*), we get

$$V_{S} = A_{1}(A_{2}V_{R} + B_{2}I_{R}) + B_{1}(C_{2}V_{R} + D_{2}I_{R}) = (A_{1}A_{2} + B_{1}C_{2})V_{R} + (A_{1}B_{2} + B_{1}D_{2})I_{R}$$
  
and  $I_{S} = C_{1}(A_{2}V_{R} + B_{2}I_{R}) + D_{1}(C_{2}V_{R} + D_{2}I_{R}) = (C_{1}A_{2} + D_{1}C_{2})V_{R} + (C_{1}B_{2} + D_{1}D_{2})I_{R}$   
Hence, the two lines connected in series have equivalent auxiliary constants of

#### 41.26. Corona

where

When an alternating potential difference is applied across two conductors whose spacing is large as compared to their diameter, there is no apparent change in the condition of the atmospheric air surrounding the wires if voltage is low. However, when the p.d. is increased, then a point is reached when a faint luminous glow of bluish colour appears along the lengths of conductors and at the same time a hissing sound is heard. This bluish discharge is known as *corona*. Corona is always accompanied by the production of ozone which is readily detected because of its characteristic odour. If the p.d. is further increased, then the glow and hissing both increase in intensity till a spark-over between the conductors takes place due to the break-down of air insulation. If the conductors are smooth and polished, the corona glow is uniform along their length but if there is any roughness, they will be picked up by relatively brighter illumination. In the case of conductors with a spacing shorter as compared to their diameters, sparking may take place without any visible glow. If the p.d. between wires is direct instead of alternating, there is a difference in the appearance of the two wires. The positive wire has a smooth glow about it whereas the glow about the negative wire is spotty.

Corona occurs when the electrostatic stress in the air around the conductors exceeds 30 kVA (maximum)/cm or 21.1 kV (r.m.s.)/cm. The effective disruptive critical voltage to neutral is given by the relation.

 $V_C = m_0 g_0 \,\delta r \log h D/r \,kV/\text{phase} = 2.3 \,m_0 g_0 \,\delta r \log_{10} D/r \,kV/\text{phase}$ 

 $m_0$  = irregularity factor which takes into account the surface conditions of the conductor.

- D = distance between conductors in cm.
- r = radius of the conductor in cm or the radius of the circumscribing circle in a stranded cable.
- $g_0$  = breakdown strength or disruptive gradient of air at 76 cm of mercury and 25°C
  - = 30 kV (max.)/cm = 21.1 kV (r.m.s.)/cm
- $\delta$  = air density factor

Substituting the value of  $g_0$ , we have

$$m_0 = 21.1 m_0 \,\delta r \log h D/r \,kV/\text{phase} = 21.1 m_0 \,\delta r \times 2.3 \log_{10} D/r \,kV/\text{phase}$$

 $= 48.8 m_0 \delta r \log_{10} D/r \text{ kV/phase}$ 

The value of  $\delta$  is given by  $\delta = \frac{3.92 b}{273 + t}$ 

b = barometric pressure in cm of Hg and t = temp. in degrees centigrade. where

when b = 76 cm and  $t = 25^{\circ}$ , C  $\delta = 1$ 

The irregularity factor  $m_0$  depends on the shape of cross-section of the wire and the state of its surface. Its value is unity for an absolutely smooth wire of one strand of circular section and less than unity for wires roughened due to weathering as shown below :

Iri	regular	ity Factor
Polished wires		1.0
Weathered wires		0.93 to 0.98
7-strand cables, concentric lay		0.83 to 0.87
Cables with more than 7-strands		0.80 to 0.85

#### 41.27. Visual Critical Voltage

This voltage is higher than the disruptive critical voltage  $V_c$ . It is the voltage at which corona appears all along the line. The *effective* or r.m.s. value of visual critical voltage is given by the following empirical relation.

$$V_{v} = m_{v} g_{0} \delta r \left( 1 + \frac{0.3}{\sqrt{\delta r}} \right) \operatorname{logh} D/r \, \mathrm{kV/phase}$$
  
= 21.1  $m_{v} \delta r \left( 1 + \frac{0.3}{\sqrt{\delta r}} \right) \times 2.3 \, \operatorname{log}_{10} D/r \, \mathrm{kV/phase}$   
= 48.8  $m_{v} \delta r \left( 1 + \frac{0.3}{\sqrt{\delta r}} \right) \operatorname{log}_{10} D/r \, \mathrm{kV/phase}$ 

where  $m_v$  is another irregularity factor having a value of 1.0 for smooth conductors and 0.72 to 0.82 for rough conductors.

## 41.28. Corona Power

Formation of corona is always accompanied by dissipation of energy. This loss will have some effect on efficiency of the line but will not have any appreciable effect on the line regulation. This loss is affected both by atmospheric and line conditions. Soon after the critical voltage is reached, the corona loss increases as the square of the excess voltage. The loss for voltage of *V* kilovolt to neutral is given by the following empirical relation.

$$P = 241 \frac{(f+25)}{\delta} \sqrt{\left(\frac{r}{D}\right)(V - V_c)^2 \times 10^{-5}} \text{ kW/km/phase}$$

where f is the frequency of the a.c. in Hz. Obviously, total loss equals 3 times the above.

#### 41.29. Disadvantages of Corona

- 1. There is a definite dissipation of power although it is not so important except under abnormal weather conditions like storms etc.
- 2. Corrosion due to ozone formation.
- 3. The current drawn by the line due to corona losses is non-sinusoidal in character, hence it causes non-sinusoidal drop in the line which may cause some interference with neighbouring communication circuits due to electromagnetic and electrostatic induction. Such a shape of corona current tends to introduce a large third harmonic component.

However, it has been found that corona works as a safety valve for surges.

4. Particularly intense corona effects are observed at a working voltage of 35 kV or higher. Hence, designs have to be made to avoid any corona on the bus-bars of substations rated for 35 kV and higher voltages during their normal operation. Corona discharge round bus-bars is extremely undesirable because the intense ionization of the air reduces its dielectric strength, makes it easier for the flashover to occur in the insulators and between phases particularly when the surfaces concerned are dirty or soiled with other deposits. The ozone produced due to corona discharge aggressively attacks the metallic components in the substations and switchgear, covering them with oxides. Moreover, the crackling sound of the corona discharge in a substation masks other sounds like light crackling noise due to arcing in a loose contact, the sound of an impending breakdown or creepage discharge in the equipment, the rattling noise due to the loosening of steel in a transformer core etc. The timely detection of such sounds is very important if any serious breakdown is to be avoided.

**Example 41.33.** Find the disruptive critical voltage for a transmission line having : conductor spacing = 1 m; conductor (stranded) radius = 1 cmbarometric pressure = 76 cm of Hg;  $temperature = 40^{\circ}C$ Air break-down potential gradient (at 76 cm of Hg and at  $25^{\circ}C$ ) = 21.1 kV (r.m.s.)/cm. (Electric Power Systems-I, Gujarat Univ.)  $V_C = 2.3 m_0 g_0 r \log_{10} D/r \, \text{kV/phase}$ Solution. Here.  $g_0 = 21.1 \text{ kV} (\text{r.m.s.})/\text{cm}$ ;  $m_0 = 0.85$  (assumed)  $\delta = 3.92 \times 76/(273 + 40) = 0.952$ ;  $\log_{10} D/r = \log_{10} 100/1 = 2$  $V_c = 2.3 \times 0.85 \times 21.1 \times 0.952 \times 1 \times 2 = 78.54$  kV (r.m.s.)/phase *.*.. =  $78.54 \times \sqrt{3} = 136$  kV (r.m.s.)/phase Line value **Example 41.34.** Find the disruptive critical and visual corona voltage of a grid-line operating at 132 kV. conductor dia  $= 1.9 \, cm$ ; conductor spacing = 3.81 m $= 44^{\circ}C$ temperature barometric pressure  $= 73.7 \ cm$ ÷ conductor surface factor : = 0.8 = 0.66. fine weather ; rough weather (Electrical Power Systems-III, Gujarat Univ.)  $V_c = 48.8 m_0 \,\delta r \log_{10} D/r \,\mathrm{kV/phase}$ Solution.  $m_0 = 0.8; \delta = 3.92 \times 73.7/(273 + 44) = 0.91$ Here,  $\log_{10} 381/1.9 = \log_{10} 200.4 = 2.302$  $V_c = 48.8 \times 0.8 \times 0.91 \times 1.9 \times 2.302 = 155.3 \text{ kV/phase}$ ....  $V_v = 48.8 m_v \,\delta r \left(1 + \frac{0.3}{\sqrt{\delta r}}\right) \log_{10} D/r \,\mathrm{kV/phase}$  $m_{\nu} = 0.66; \delta = 0.91; \sqrt{\delta r} = \sqrt{0.91 \times 1.9} = 1.314$ Here.  $V_{\nu} = 48.8 \times 0.66 \times 0.91 \times 1.9 \left(1 + \frac{0.3}{1.314}\right) 2.302 = 157.5 \text{ kV/phase}$ *.*..

**Example 41.35.** A certain 3-phase equilateral transmission line has a total corona loss of 53 kW at 106 kV and a loss of 98 kW at 110.9 kV. What is the disruptive critical voltage between lines? What is the corona loss at 113 kV? (Electrical Power Systems-II, Gujarat Univ.)

Solution. As seen from Art. 41.28, the total corona loss for three phases is given by

$$P = 3 \times \frac{241(f+25)}{\delta} \cdot \sqrt{\frac{r}{D}(V-V_c)^2 \times 10^{-5}} \text{ kW/km}$$

Other things being equal,  $P \propto (V - V_c)$ 

 $53 \propto \left(\frac{106}{\sqrt{3}} - V_c\right)^2 \propto (61.2 - V_c)^2 \qquad -1\text{st case}$   $98 \propto \left(\frac{110.9}{\sqrt{3}} - V_c\right)^2 \propto (64 - V_c)^2 \qquad -2\text{nd case}$   $\frac{98}{53} = \frac{(64 - V_c)^2}{(61.2 - V_c)^2} \quad \text{or} \quad V_c = 54.2 \text{ kV/km}$ 

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...

$$\frac{98}{53} = \frac{(64 - V_c)^2}{(61.2 - V_c)^2} \text{ or } V_c = 54.2 \text{ I}$$
$$W \propto \left(\frac{113}{\sqrt{3}} - V_c\right)^2 \propto (65.2 - V_c)^2$$

Similarly,

$$\frac{W}{98} = \frac{(65.2 - V_c)^2}{(64 - V_c)^2} = \frac{(65.2 - 54.2)^2}{(64 - 54.2)^2} \quad \therefore \quad W = 123.4 \text{ kW}$$

**Example 41.36.** A 3-phase, 50-Hz, 220-kV transmission line consists of conductors of 1.2 cm radius spaced 2 metres at the corners of an equilateral triangle. Calculate the corona power loss per km of the line at a temperature of 20°C and barometric pressure of 72.2 cm. Take the surface factors of the conductor as 0.96. (Electrical Power-II, Bangalore Univ.)

Solution. As seen from Art. 41.28, corona loss per phase is

Here,

*.*..

$$\delta = \frac{3.92 \, b}{273 + t} = \frac{3.92 \times 72.2}{273 + 20} = 0.966$$

$$V_c = 48.8 \, m_0 \, \delta \, r \log_{10} D/r = 48.8 \times 0.96 \times 0.966 \times 1.2 \times \log_{10} \frac{200}{1.2}$$

$$= 120.66 \, \text{kV/phase}$$

$$V = 220/\sqrt{3} = 127 \, \text{kV/phase}$$

$$P = 241 \times \frac{75}{0.966} \times \sqrt{\left(\frac{1.2}{200}\right)} \times (127 - 120.66)^2 \times 10^{-5} = 0.579 \, \text{kW/km/phase}$$

 $P = 241 \frac{(f+25)}{2} \sqrt{(r/D)} \cdot (V-V_{\star})^2 \times 10^{-5} \text{ kW/km/phase}$ 

Total loss for 3 phase =  $3 \times 0.579 = 1.737$  kW/km

## 41.30. Underground Cables

Underground cables are used where overhead lines are not possible as in large cities despite the fact that in their case, cost per kW per km is much more as compared to overhead transmission lines. Another advantage of overhead system for distributors is that tapping can be made at any time without any disturbance, which is of great importance in rapidly developing areas. However, underground cables are more advanta-



Underground cables are used where overhead transmission is difficult

geous for feeders which are not likely to be disturbed for tapping purposes because, being less liable to damage through storms or lighting or even wilful damage, they offer a safer guarantee of supply. But this advantage may be offset by the cost of trenching and expensive jointing necessary in case of repairs.

However, cables score over overhead lines in cases where voltage regulation is more important, because, due to very small spacing of their conductors, they have a very low inductance and hence low inductive drops.

Cables may be classified in two ways according to (*i*) the type of insulating material used in their manufacture or (*ii*) the voltage at which they transmit power. The latter method of classification is, however, more generally used according to which cables are divided into three groups :

- 1. Low-tension cables–up to 1000 V
- 2. High-tension cables–up to 23,000 V
- 3. Super-tension cables-from 66 kV to 132 kV

For all cables, the conductor is tinned stranded copper of high conductivity. Stranding is done to secure flexibility and the number of conductors in a core is generally 3, 7, 19 and 37 etc. Except for 3-strand, all



numbers have a centrally-disposed conductor with all others surrounding it. A cable may have one or more than one core depending on the type of service for which it is intended. It may be (i) single-core (ii) two-core (iii) three-core and (iv) four-core etc.

In Fig. 41.44 is shown a section through a twin-cored, high-tension lead-covered underground cable sheathed with a continuous tube of pure lead whereas Fig. 41.45 shows a section of a typical concentric type 2-core cable used for single-phase distribution. The cores are arranged concentrically. The outer core is arranged in the form of hollow tubing. Both cores are of stranded copper and paper-insulated and are protected by a lead-sheath.

In Fig. 41.46 is shown a section through a 3-core extra-high-tension paper-insulated lead-covered and steel-wire armoured cable.

The cores are surrounded by insulation or impregnated paper, varnished cambric or vulcanised bitumen.

The insulation is, in turn, surrounded by a metal sheath made of lead or a lead alloy and prevents the entry of moisture into the inner parts. On the sheath is applied the bedding which consists of two compounded paper tapes along with suitable compounded fibrous materials.

Next comes 'armouring' which is placed over the bedding and consists of either galvanized steel wires or two layers of steel tape. Armouring may not be done in the case of some cables.

Next comes 'serving' which consists of compounded fibrous material (jute etc.) placed over the armouring in the case of armoured cables or over the metal sheath in the case of unarmoured cablesin which case serving consists of two layers of compounded paper tapes and a final covering of compounded fibrous material.



## 41.31. Insulation Resistance of a Single-core Cable

The expression for the insulation resistance of such a cable, as derived in Art. 5.14.

$$= \frac{2.3 \rho}{2 \pi l} \log_{10} \frac{r_2}{r_1} \text{ ohm}$$

The value of specific resistance for paper is approximately 500  $\Omega$ -m.

## 41.32. Capacitance and Dielectric Stress

This has already been dealt with in Art. 5.9 and 5.13.

### 41.33. Capacitance of 3-core Belted Cables

The capacitance of a cable is much more important than of the overhead wires of the same length because in cables (i) conductors are nearer to each other and the sheath and (ii) they are separated by a dielectric medium of higher permittivity as compared to air. Fig. 41.47 shows a system of capacitances





in a belted 3-core cable used for 3-phase system. It can be regarded as equivalent to three-phase cables having a common sheath. Since there is a p.d. between pairs of conductors and between each conductor and sheath, there exist electrostatic fields as shown in Fig. 41.47 (a) which shows average distribution of electrostatic flux, though, actually, the distribution would be continually changing

because of changing potential difference between conductors themselves and between conductors and sheath. Because of the existence of this electrostatic coupling, there exist six capacitances as shown in Fig. 41.47 (b). The three capacitances between three cores are delta-connected whereas the other three between each core and the sheath are star-connected, the sheath forming the star-point [Fig. 41.47 (c)].

 $C_{2}=3C_{1}$  $C_1$ C  $C_{2}=3C$  $C_1$ *(b)* (a)Fig. 41.48

The three delta-connected capacitances each of value  $C_1$  can be converted into equivalent star-capacitance  $C_2$  which

will be three times the delta-capacitances  $C_1$  as shown in Fig. 41.48.

The two star capacitances can now be combined as shown in Fig. 41.49 (a). In this way, the whole cable is equivalent to three star-connected capacitors each of capacitance  $C_n = 3C_1 + C_S$  as shown in Fig. 41.49 (*b*).

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## Fig. 41.49

If Vp is the phase voltage, then the charging current is given by

 $I_C = V_P (C_S + 3C_1) \omega = V_P \omega C_n$ 

## 41.34. Tests for Three-phase Cable Capacitance

The following three tests may be used for the measurement of  $C_1$  and  $C_5$ :

(*i*) In the first test, all the three cores are bunched together and then capacitance is measured by usual methods between these bunched cores and the earthed sheath. This gives  $3 C_s$  because the three capacitances are in parallel.

(*ii*) In the second method, two cores are bunched with the sheath and capacitance is measured between these and the third core. It gives  $2C_1 + C_s$ . From this value,  $C_1$  and  $C_s$  can be found.

(iii) In the third method, the capacitance  $C_L$  between two cores or lines is measured with the third core free or shorted to earth. This gives

$$\left(C + \frac{C_1}{2} + \frac{C_s}{2}\right) = \frac{1}{2}(3C_1 + C_s) = \frac{C_n}{2}$$

Hence,  $C_n$  is twice the measured value *i.e.*  $C_n = 2C_L$ 

Therefore, charging current  $I_C = \omega V_P C_n = \frac{2}{\sqrt{3}} \cdot \omega V_L C_L$ 

where  $V_L$  is the line voltage (not phase voltage).

**Example 41.37.** A single-core lead-covered cable is to be designed for 66-kV to earth. Its conductor radius is 1.0 cm and its three insulating materials A, B, C have permittivities of 5.4 and 3 respectively with corresponding maximum safe working stress of 38 kV per cm (r.m.s. value), 26-kV per cm and 20-kV per cm respectively. Find the minimum diameter of the lead sheath.

Solution.

 $g_{1\text{max}} = \frac{Q}{2\pi\epsilon_0\epsilon_{r1}r} \text{ or } 38 = \frac{Q}{2\pi\epsilon_0\times5\times1} \qquad \dots (i)$ 

$$g_{2\text{max}} = \frac{Q}{2\pi\epsilon_0\epsilon_{r2} r_1} \text{ or } 26 = \frac{Q}{2\pi\epsilon_0 \times 4 \times r_1} \qquad \dots (ii)$$

$$g_{3\max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r3} r_2} \text{ or } 20 = \frac{Q}{2\pi\epsilon_0 \times 3 \times r_2} \qquad \dots (iii)$$

From (*i*) and (*ii*), we get,  $38/26 = 4r_1/5$ ,  $r_1 = 1.83$  cm Similarly, from (*i*) and (*ii*),

## A.C. Transmission and Distribution 1649 $38/20 = 3r_2/5$ ; $r_2 = 3.17$ cm r<sub>3</sub> $V_1 = g_{1\max} \times r \times 2.3 \log_{10} r_1 / r_1$ $= 38 \times 1 \times 2.3 \log_{10} \frac{1.83}{1} = 22.9 \text{ kV}$ $r_2$ $V_2 = g_{2\max} \times r_1 \times 2.3 \log_{10} \frac{r_2}{r_1}$ $= 26 \times 1.83 \times 2.3 \log_{10} \frac{3.17}{1.83}$ С В = 26.1 kV $V_3 = g_{3\max} \times r_2 \times 2.3 \log_{10} \frac{r_3}{r_2}$ $\in$ $\in {}_2$ $= 20 \times 3.17 \times 2.3 \log_{10} \frac{r_3}{3.17}$ € 3 $= 145.82 \log_{10} \frac{r_3}{3.17}$ $V = V_1 + V_2 + V_3$ Now, Fig. 41.50 $66 = 22.9 + 26.1 + 145.82 \log_{10} \frac{r_3}{3.17}$

*.*..

$$r_3 = 4.15 \,\mathrm{cm}$$

$$\therefore$$
 diameter of the sheath =  $2r_3 = 2 \times 4.15 = 8.3$  cm

**Example 41.38.** The capacitances per kilometer of a 3-phase cable are 0.63  $\mu$ F between the three cores bunched and the sheath and  $0.37 \,\mu F$  between one core and the other two connected to sheath. Calculate the charging current taken by eight kilometres of this cable when connected to a 3-phase, 50-Hz, 6,600-V supply.

Solution. As shown in Art. 40.33, 
$$0.63 = 3C_s$$
  $\therefore$   $C_s = 0.21 \,\mu$  F/km  
From the second test,  
 $0.37 = 2C_1 + C_2 = 2C_1 + 0.21$   $\therefore$   $C_1 = 0.08 \,\mu$  F/km  
 $\therefore$   $C_s$  for 8 km = 0.21 × 8 = 1.68  $\mu$  F;  $C_1$  for 8 km = 0.08 × 8 = 0.64  $\mu$  F  
 $C_n = C_s + 3C_1 = 1.68 + (3 \times 0.64) = 3.6 \,\mu$  F  
Now,  $V_p = (6,600/\sqrt{3}); \quad \omega = 314 \, \text{rad/s}$   
 $I_C = \omega V_p C_n = (6,600/\sqrt{3}) \times 3.6 \times 10^{-6} \times 314 = 4.31 \,\text{ampere}$ 

**Example 41.39.** A 3-core, 3-phase belted cable tested for capacitance between a pair of cores on single phase with the third core earthed, gave a capacitance of 0.4 mF per km. Calculate the charging current for 1.5 km length of this cable when connected to 22 kV, 3-phase, 50-Hz supply.

Solution.

$$C_L = 0.4 \,\mu\text{F}, V_L = 22,000 \,\text{V}, \quad \omega = 314 \,\text{rad/s}$$
  
 $I_C = \frac{2}{\sqrt{3}} \quad \omega V_L C_L = \frac{2}{\sqrt{3}} \times 22,000 \times 0.4 \times 10^{-6} \times 314 = 3.2 \,\text{A}$   
or 15 km = 3.2 × 15 = **48** A

Charging current for 15 km

**Example 41.40.** A 3-core, 3-phase metal-sheathed cable has (i) capacitance of 1  $\mu$ F between shorted conductors and sheath and (ii) capacitance between two conductors shorted with the sheath and the third conductor 0.6  $\mu$ F. Find the capacitance (a) between any two conductors (b) between any two shorted conductors and the third conductor. (Power Systems-I, AMIE, Sec. B, 1993)

Solution. (a) The capacitance between two cores or lines when the third core is free or shorted to earth is given by  $1/2 (3C_1 + C_s)$ 

we have  $3C_s = 1\mu$  F or  $C_s = 1/3 \mu$  F = 0.333  $\mu$  F Now, (*i*)

From (*ii*) we get,  $2C_1 + C_s = 0.6 \,\mu$  F,  $2C_1 = 0.6 - 0.333 = 0.267 \,\mu$  F,  $\therefore \qquad C_1 = 0.133 \,\mu$  F

(b) The capacitance between two shorted conductors and the other is given by

$$2C_1 + \frac{2C_s \times C_s}{3C_s} = 2C_1 + \frac{2}{3}C_s = 2 \times 0.133 + \frac{2}{3} \times 0.333 = 0.488 \text{ mF}$$

## 41.35. A.C. Distributor Calculations

These calculations are similar to those for d.c. distributor but with the following differences :

- **1.** The loads tapped off will be at different power factors. Each power factor is taken with respect to the voltage at the feeding point which is regarded as a reference vector.
- 2. The currents in the sections of the distributor will be given by the vector sum of load currents and not by their arithmetic sum as in a d.c. distributor. Currents can be added algebraically only when they are expressed in the complex notation.
- **3.** The voltage drop, in the case of a.c. circuits, is not only due to ohmic resistance but due to inductive reactance as well (neglecting capacitive reactance if any).

It has already been shown that voltage drop in an inductive circuit is given by

 $I(R\cos\phi + X\sin\phi)$ 

The total drop will be given by  $\sum I (R \cos \phi + X \sin \phi)$ .

Questions on a.c. distributors may be solved in the following three ways :

- 1. Express voltages, currents and impedances in complex notation and then proceed exactly as in d.c. distributors.
- 2. Split the various currents into their active and reactive components. Now, the drop in the case of active components will be due to resistance only and in the case of reactive components due to reactance only. Find out these two drops and then add the two to find the total drop.
- 3. In cases where approximate solutions are sufficient, quick results can be obtained by finding the "distribution centre" or centre of gravity of the load.

All these three methods are illustrated by Ex. 41.42 given on the next page :

**Example 41.41.** A 2-wire a.c. feeder 1 km long supplies a load of 100 A at 0.8 p.f. lag 200 volts at its far end and a load of 60 A at 0.9 p.f. lag at its mid-point. The resistance and reactance per km (lead and return) are 0.06 ohm and 0.08 ohm respectively. Calculate the voltage drop along the distributor from sending end to mid-point and from mid-point to far end.

#### (Power Systems-I, AMIE, Sec. B, 1993)

Solution. Fig. 41.51 shows the feeder AC 1 km long having B as its mid-point and A as its sending-end point.



Fig. 41.51

Let the voltage of point *C* be taken as reference voltage.

 $V_{C} = 200 + j_{0}, I_{C} = 100(0.8 - j\ 0.6) = (80 - j\ 60) \text{ A}$ Loop impedance of feeder *BC* (lead and return) =  $(0.06 + j\ 0.08)/2 = (0.03 + j\ 0.04)$  ohm Voltage drop in  $BC = (80 - j\ 60)\ (0.03 + j\ 0.04) = (\mathbf{4.8} + j\mathbf{1.4}) \text{ V}$   $\therefore \qquad V_{B} = 200 + 4.8 + j\ 1.4 = (204.8 + j\ 1.4) \text{ V}$ Current  $I_{B} = 60\ (0.9 - j\ 0.4357) = (54 - j\ 26.14) \text{ A}$ Current in feeder *AB i.e.*   $I_{AB} = I_{C} + I_{B} = (80 - j\ 60) + (54 - j\ 26.14) = (134 - j\ 86.14) \text{ A}$ Drop in section  $AB = (134 - j\ 86.14)\ (0.03 + j\ 0.04) = (7.46 + j\ 2.78) \text{ V}$ Voltage drop from point *A* to point *B* = (7.46 + j\ 2.78) \text{ V}

**Example 41.42.** A single-phase a.c. distributor 500 m long has a total impedance of  $(0.02 + j0.04) \Omega$  and is fed from one end at 250V. It is loaded as under :

- (*i*) 50 A at unity power factor 200 m from feeding point.
- (ii) 100 A at 0.8 p.f. lagging 300 m from feeding point.
- (iii) 50 A at 0.6 p.f. lagging at the far end.

Calculate the total voltage drop and voltage at the far end. (Power System-I, AMIE, 1994) Solution. First Method

Current in section AD (Fig. 41.52) is the vector sum of the three load currents.

 $\therefore$  current in AD

= 50 + 100 (0.8 - j 0.6) + 50 (0.6 - j 0.8)= 160 - i100

$$= 100 - j100$$

Impedance of section AD

= (200/500) (0.02 + j 0.04)

$$= (0.008 + j \ 0.016)$$
 W





Voltage drop in section	$AD = (160 - j \ 100) \times (0.008 + j \ 0.016) = (2.88 + j \ 1.76) V$
Current in section	DC = (160 - j100) - 50 = (110 - j100)  A
Impedance of	$DC = (0.004 + j \ 0.008) \ \Omega$
Drop in	CD = (100 - j100) (0.004 + j 0.008) = (1.24 + j 0.48) V
Current in	CB = 50 (0.6 - j0.8) = (30 - j40)  A
Impedence of	$CB = (0.008 + j \ 0.016) \ \Omega$
∴ drop in	CB = (30 - j40) (0.008 + j0.016) = (0.88 + j0.16) V
Total drop	= (2.88 + j 1.76) + (1.24 + j 0.48) + (0.88 + j 0.16) = (5 + j 2.4) V
Voltage at far end	= (250 + j0) - (5 + j2.4) = 245 - j2.4 volt
Its magnitude is	$= \sqrt{245^2 + 2.4^2} = 245 \text{ V (approx)}$

#### Second Method

We will split the currents into their active and reactive components as under :

 $50 \times 1 = 50 \text{ A}; \quad 100 \times 0.8 = 80 \text{ A}; \quad 50 \times 0.6 = 30 \text{ A}$ 

These are shown in Fig. 41.51 (a). The reactive or wattless components are  $50 \times 0^{-1} = 0^{-1} = 100 \times 0.6 = 60^{-1} = 50 \times 0.8 = 40^{-1} = 40^{-1} = 50^$ 

$$50 \times 0 = 0$$
;  $100 \times 0.6 = 60$  A;  $50 \times 0.8 = 40$  A

These are shown in Fig. 41.53 (*b*). The resistances and reactances are shown in their respective figures.





Drops due to active components of currents are given by taking moments

 $= 50 \times 0.008 + 80 \times 0.012 + 30 \times 0.02 = 1.96 \text{ V}$ 

Drops due to reactive components  $= 60 \times 0.024 + 40 \times 0.04 = 3.04$ 

Total drop = 1.96 + 3.04 = 5 V

This is approximately the same as before.

#### **Third Method**

The centre of gravity (C.G.) of the load is at the following distance from the feeding end  $50 \times 200 + 100 \times 300 + 50 \times 500$ 

200	= 325  m
Value of resistance upto C.G.	$= 325 \times 0.02/500 = 0.013 \ \Omega.$
Value of reactance upto C.G.	$= 325 \times 0.04/500 = 0.026 \ \Omega.$
Average p.f.	$= \frac{50 \times 1 + 100 \times 0.8 + 50 \times 0.6}{200} = 0.8$
$\cos \phi_{av}$	$= 0.8$ ; sin $\phi_{av} = 0.6$
Drop	$= 200(0.013 \times 0.8 + 0.026 \times 0.6) = 5.2 \text{ V}$

This is approximately the same as before.

**Example 41.43.** A single-phase distributor, one km long has resistance and reactance per conductor of 0.2  $\Omega$  and 0.3  $\Omega$  respectively. At the far end, the voltage  $V_B = 240$  V and the current is 100 A at a power factor of 0.8 lag. At the mid-point A of the distributor current of 100 A is tapped at a power factor of 0.6 lag with reference to the voltage  $V_A$  at the mid-point. Calculate the supply voltage  $V_S$  for the distributor and the phase angle between  $V_S$  and  $V_B$ .

**Solution.** As shown in Fig. 41.54 (*a*), let *SB* be the distributor with *A* as the mid point. Total impedance of the distributor is =  $(0.4 + j0.6) \Omega$ .

Let the voltage  $V_B$  at point B be taken as the reference voltage.

 $\therefore \qquad \mathbf{V}_{B} = (240 + j_{0})\mathbf{V}; \mathbf{I}_{B} = 100 (0.8 - j \ 0.6) = 80 - j \ 60 \ A$ Drop in section  $AB = (80 - j \ 60) (0.2 + j \ 0.3) = (34 + j \ 12) \ \mathbf{V}$   $\mathbf{V}_{A} = \mathbf{V}_{B} + \text{drop over } \mathbf{AB} = (240 + j \ 0) + (34 + j \ 12) = (274 + j \ 12) \ \mathbf{V}$ 

The phase difference between  $V_A$  and  $V_B$  is  $= \tan^{-1} (12/274) = 2^{\circ}30'$ 

The load current  $I_A$  has lagging power factor of 0.6 with respect to  $V_A$ . It lags  $V_A$  by an angle  $\phi = \cos^{-1}(0.6) = 53^{\circ}8'$ .

Hence, it lags behind  $V_B$  by an angle of  $(53^{\circ}8' - 2^{\circ}30') = 50^{\circ}38'$  as shown in the vector diagram of Fig. 41.54 (*b*).





#### Fig. 41.54

 $\mathbf{I}_{A} = 100 (\cos 50^{\circ}38' - j \sin 50^{\circ}38') = (63.4 - j77.3) \text{ A}$   $\mathbf{I} = \mathbf{I}_{A} + \mathbf{I}_{B} = (80 - j 60) + (63.4 - j77.3) = (143.4 - j137.3) \text{ A}$ Drop in section  $S_{A} = (143.4 - j137.3) (0.2 + j 0.3) = (69.87 + j 15.56)$   $\mathbf{V}_{S} = \mathbf{V}_{A} + \text{drop in section } S_{A}$   $= (274 + j12) + (69.87 + j15.56) = 343.9 + j 27.6 = 345 \angle 5^{\circ}28' \text{ V}$ 

Hence, supply voltage is 345 V and lead  $V_B$  by 5°28'

**Example 41.44.** A 1-phase ring distributor ABC is fed at A. The loads at B and C are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at A. The total impedances of the sections AB, BC and CA are (1 + j1), (1 + j2) and (1 + j3) ohm respectively. Find the total current fed at A and the current in each section.

#### (Transmission and Distribution-II, Madras Univ.)

**Solution.** Thevenin's theorem will be used to solve this problem. The ring distributor is shown in Fig. 41.55 (*a*). Imagine feeder *BC* to be removed [Fig. 41.55 (*b*)].

Current in AB = 20(0.8 - j0.6) = (16 - j12) A Current in section AC = 15(0.6 - j0.8) = (9 - j12) A Drop over AB = (16 - j12)(1 + j1) = (28 + j4) V Drop over AC = (9 - j12)(1 + j3) = (45 + j15) V Obviously, point C is at a larger stantial as compared to point P.

Obviously, point C is at a lower potential as compared to point B.

p.d. between *B* and C = (45 + j15) - (28 + j4) = (17 + j11) V

Impedance of the network as looked into from points *B* and *C* is  $= (1+j1) + (1+j3) = (2+j4)\Omega$ . The equivalent Thevenin's source is shown in Fig. 41.55 (*c*) with feeder *BC* connected across it.

	Current in <i>BC</i>	=	$\frac{17+j11}{(2+j4)+(1+j2)} = (2.6-j1.53) \text{ A}$
Current in A B		=	$(16 - j12) + (2.6 - j1.53) = 18.6 - j13.53 = 23 \angle -36^{\circ} \text{ A}$
Current in BC		=	$(9-j12) - (2.6-j1.53) = 6.4 - j11.5 = 13.2 \angle -60.9^{\circ} \text{ A}$
Total current fe	d at point A	=	$(16 - j12) + (9 - j12) = 25 - j24 = 34.6 \angle -43.8^{\circ} \text{ A}.$





Example 41.45. A 2-wire ring distributor ABC is supplied at A at 400 V. Point loads of 20 A at a p.f. of 0.8 lagging and 30 A at a p.f. 0.6 lagging are tapped off at B and C respectively. Both the power factors refer to the voltage at A. The respective go-and-return impedances of sections AB,BC and CA are (1 + j2) ohm, (2 + j3) ohm and (1 + j3) ohm. Calculate the current flowing through each section and the potentials at B and C. Use Superposition theorem.

(Power Systems-I, M.S. Univ. Baroda 1992)

Solution. The distributor circuit is shown in Fig. 41.56 (a). Currents in various sections are as shown. First, consider the load at point B acting alone as in Fig. 41.56(b). The input current at point A divides in the inverse ratio of the impedances of the two paths A B and A CB.

Let the currents be  $\mathbf{I}_1'$  and  $\mathbf{I}_2'$ 

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$$\mathbf{I_1'} = (16 - j12) \times \frac{(1 + j3) + (2 + j3)}{(1 + j3)(2 + j3) + (1 + j3)} = (16 - j12) \times \frac{(3 + j6)}{4 + j8} = (12 - j9) A$$
$$\mathbf{I_2'} = (16 - j12) - (12 - j9) = (4 - j3) A$$

Now, consider the load at point C to act alone as shown in Fig. 40.56 (c). Let the currents now be  $\mathbf{I}_1''$  and  $\mathbf{I}_2''$ .



Fig. 41.56

$$\mathbf{I}_{1}'' = (18 - j24) \times \frac{1 + j3}{4 + j8} = 7.5 - j7.5; \ \mathbf{I}_{2}'' = (18 - j24) - (7.5 - j7.5) = (10.5 - j16.5) \text{ A}$$

As per Superposition theorem, the current  $\mathbf{I}_1$  in section *A B* is the vector sum of  $\mathbf{I}_1'$  and  $\mathbf{I}_1''$ .

 $\therefore \quad \mathbf{I}_{1} = \mathbf{I}_{1}' + \mathbf{I}_{1}'' = (12 - j9) + (7.5 - j7.5) = (19.5 - j16.5) \text{ A}$   $\mathbf{I}_{2} = \mathbf{I}_{2}' + \mathbf{I}_{2}'' = (4 - j3) + (10.5 - j16.5) = (14.5 - j19.5) \text{ A}$   $\mathbf{I}_{3} = \mathbf{I}_{1}'' - \mathbf{I}_{2}' = (7.5 - j7.5) - (4 - j3) = (3.5 - j4.5) \text{ A}$ Potential of B = 400 - drop over AB  $= 400 - (19.5 - j16.5)(1 + j2) = 347.5 - j22.5 = 348 \angle -3.7^{\circ}\text{V}$ Potential of  $C = 400 - (14.5 - j19.5)(1 + j3) = 327 - j24 = 328 \angle -4.2^{\circ}\text{V}$ 

**Example 41.46.** A 3-phase ring main ABCD, fed from end A, supplies balanced loads of 50 A at 0.8 p.f. lagging at B, 120 A at u.p.f. at C and 70 A at 0.866 p.f. lagging at D, the load currents being referred to the voltage at point A.

The impedance per phase of the various line sections are :

section AB =  $(1 + j0.6) \Omega$ ; section BC =  $(1.2 + j0.9) \Omega$ 

section CD =  $(0.8 + j0.5) \Omega$ ; section DA =  $(3 + j2) \Omega$ 

Determine the currents in the various sections.(Electrical Power-I, Bombay Univ.)Solution. One phase of the ring main is shown in Fig. 41.57. Let the current in sectionAB = (x + jy) A.





Current in BC = (x + jy) - 50(0.8 - j0.6) = (x - 40) + j(y + 30)Current in CD = (x - 40) + j(y + 30) - (120 + j0) = (x - 160) + j(y + 30)Current in DA = (x - 160) + j(y + 30) - 70(0.866 - j0.5) = (x - 220.6) + j(y + 65)Applying Kirchhoff's voltage law to the closed loop ABCDA, we have (1 + j0.6)(x + jy) + (1.2 + j0.9)[(x - 40) + j(y + 30)] + (0.8 + j0.5)[(x - 160) + j(y + 30)] + (3 + j2)[(x - 220.6) + j(y + 65)] = 0or (6x - 4y + 1009.8) + j(4x + 6y - 302.2) = 0Since the real (or active) and imaginary (or reactive) parts have to be separately zero. 6x - 4y + 1009.8 = 0 and 4x + 6y - 302.2 = 0... Solving for x and y, we get x = 139.7 and y = -42.8AB = (139.7 - j42.8) A:. Current in section Current in section BC = (139.7 - 40) + j(-42.8 + 30) = (99.7 - j12.8) A

Current in section CD = (139.7 - 160) + j(-42.8 + 30) = (-20.3 - j12.8) A Current in section DA = (139.7 - 220.6) + j(-42.8 + 65) = (-80.9 + j22.2) A

#### 41.36. Load Division Between Parallel Lines

It is common practice to work two or more cables or overhead lines in parallel when continuity of supply is essential. In the case of a fault developing in one line of cable, the other lines or cables carry the total load till the fault is rectified."

Let us take the case of two lines in parallel and having impedances of  $\mathbb{Z}_1$  and  $\mathbb{Z}_2$ . Their combined impedance is

$$\mathbf{Z} = \frac{\mathbf{Z}_1 \times \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

If **I** is the current delivered to both lines, then total drop =  $\mathbf{IZ} = \mathbf{I} \times \frac{\mathbf{Z}_1 \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$ 

If  $\mathbf{I}_1$  and  $\mathbf{I}_2$  are the respective currents flowing in the two lines, then

$$\mathbf{I}_1 = \frac{\text{voltage drop}}{\mathbf{Z}_1} = \frac{\mathbf{I}\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}; \text{ Similarly, } \mathbf{I}_2 = \frac{\mathbf{I}\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

It may be noted that in the case of two impedances in parallel, it is convenient to take voltage vector as the reference vector.

**Example 41.47.** A total load of 12,000 kW at a power factor of 0.8 lagging is transmitted to a substation by two overhead three-phase lines connected in parallel. One line has a conductor resistance of 2  $\Omega$  per conductor and reactance (line to neutral) of 1.5  $\Omega$ , the corresponding values for the other line being 1.5 and 1.2  $\Omega$  respectively. Calculate the power transmitted by each overhead line.

(London Univ.)

Solution. Let us assume a line voltage of 1000 kV for convenience.

Total load current  $\begin{aligned} \mathbf{Z}_{1} &= (2+j1.5); \, \mathbf{Z}_{2} = (1.5+j1.2) \, \Omega \\ \mathbf{I} &= 12,000/\sqrt{3} \times 1000 \times 0.8 = 8.66 \, \mathrm{A} \\ \text{Taking voltage along reference vector, we have } \mathbf{I} = 8.66 \, (0.8-j0.6) \\ \mathbf{I}_{1} &= \frac{8.66 \, (0.8-j\, 0.6) \, (1.5+j\, 1.2)}{(2+j\, 1.5) \, (1.5+j\, 1.2)} = 4.437(6.882-j5.39) \, \mathrm{A} \end{aligned}$ 

: power transmitted by 1st line is

$$W_{1} = 10^{6} \times \frac{\sqrt{3}}{1000} \times 4.437 \times 6.882 = 5,280 \text{ kW}$$
$$I_{2} = \frac{8.66 (0.8 - j \ 0.6) (2 + j \ 1.5)}{(2 + j \ 1.5) (1.5 + j \ 1.2)} = 4.437(8.75 - j6.75) \text{ A}$$

Similarly

$$W_2 = 10^6 \times \frac{\sqrt{3}}{1000} \times 4.437 \times 8.75 = 6,720 \text{ kW}$$
  
= 5,280 + 6,720 = 12,000 kW

As a check, total power

41.37. Suspension Insulators

Suspension insulators are used when transmission voltage is high. A number of them are connected in series to form a chain and the line conductor is carried by the bottom most insulator.

As shown in Fig. 41.58 a 'cap' type suspension insulator consists of a single disc-shaped piece of porcelain grooved on the under surface to increase the surface leakage path. A galvanized cast iron cap is cemented at the top of the insulator. In the hollow cavity of the insulator is cemented a galvanized forged steel pin, the lower enlarged end of which fits into the cavity of the steel cap of the lower suspension insulator and forms a ball and socket connection.

A string of suspension insulators consists of many units, the number of units depending on the value of the transmission voltage. There exists mutual capacitance between different units and, in addition, there is capacitance to ground of each unit because of the nearness of the tower, the cross-arm and the line. Due to this capacitance to ground, the total system voltage is not equally distributed over the different units of the string. The unit nearest to the line conductor carries the maximum percentage of voltage, the figure progressively decreasing as the unit nearest to the tower is approached. The inequality of voltage distribution between individual units becomes more pronounced as the number of insulators increases and it also depends on the ratio (capacitance of insulator/capacitance of earth).



#### String Efficiency

If there are *n* units in the string, then its efficiency is given by

% string 
$$\eta = \frac{\text{total voltage across the string}}{n \times \text{voltage across unit adjacent the line}} \times 100$$

## 41.38. Calculation of Voltage Distribution along Different Units

Let, C = capacitance to ground

kC = mutual capacitance between units

The current and voltage distribution is as shown in Fig. 41.59. It is seen that

$$I_1 = \frac{V_1}{1/\omega kC} = \omega k C V_1; \text{ Similarly } i_1 = \frac{V_1}{1/\omega C} = \omega C V_1$$

Now,  $I_2 = I_1 + i_1 = \omega C V_1 (1+k)$  and  $V_2 = \frac{I_2}{\omega k C} = \frac{V_1 (1+\kappa)}{k}$ 

The current  $i_2$  is produced by the voltage combination of  $(V_1 + V_2)$ 

Now, 
$$V_1 + V_2 = V_1 \left( 1 + \frac{1+k}{k} \right) = V_1 \left( 1 + \frac{1+2k}{k} \right); \quad \therefore \quad i_2 = \omega C V_1 \left( 1 + \frac{1+2k}{k} \right)$$

At junction *B*, we have

$$I_{3} = I_{2} + i_{2} = \omega C V_{1} \left\{ 1 + k \frac{(1+2k)}{k} \right\}$$
$$= \omega C V_{1} \frac{(1+3k+k^{2})}{k}$$

However,  $I_3 = \omega k C V_3$ ;  $\therefore V_3 = \frac{V_1 (1 + 3k + k^2)}{k^2} = V_1 \left( 1 + \frac{3}{k} + \frac{1}{k^2} \right)$ The current  $i_3$  is produced by the voltage combination of  $(V_1 + V_2 + V_3)$ 

Now, 
$$V_1 + V_2 + V_3$$
  

$$= V_1 \left[ 1 + \frac{(1+k)}{k} + \frac{(1+3k+k^2)}{k^2} \right]$$

$$\therefore \quad i_3 = \omega C V_1 \left( \frac{1+4k+3k^2}{k^2} \right)$$
At junction *C*, we have  
 $I_4 = I_3 + i_3 = \omega C V_1$   
 $\left[ \frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2} \right]$   
Now,  $I_4 = \omega k C V_4$ ,  
 $\therefore \quad V_4 = V_1 \left[ \frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2} \right]$   
 $= V_1 \left( 1 + \frac{6}{k} + \frac{5}{k^2} + \frac{1}{k^2} \right)$   
For the fifth insulator from the top, we have



and so on.

If the string has *n* units, the total voltage is given by

 $V_5 = \left(1 + \frac{10}{k} + \frac{15}{k^2} + \frac{7}{k^3} + \frac{1}{k^4}\right)$ 

$$V = (V_1 + V_2 + V_3 + V_4 + \dots + V_n)$$

**Example 41.48.** For a string insulator with four discs, the capacitance of the disc is ten times the capacitance between the pin and earth. Calculate the voltage across each disc when used on a 66-kV line. Also, calculate the string efficiency. (Power Systems-I, AMIE Sec. B, 1994)

**Solution.** Let *C* be the self-capacitance of each disc and kC the capacitance between each link pin and earth. We are given that k = 10

As seen from Art. 41.38,

$$V_{2} = \frac{(1+k)}{k} V_{1} = \frac{11}{10} V_{1}$$

$$V_{3} = V_{1} \left( 1 + \frac{3}{k} + \frac{1}{k^{2}} \right) = \frac{131}{100} V_{1}$$

$$V_{4} = V_{1} \left( 1 + \frac{6}{k} + \frac{5}{k^{2}} + \frac{1}{k^{3}} \right) = \frac{1561}{100} V_{1}$$

$$V = V_{1} + V_{2} + V_{3} + V_{4} = V_{1} + \frac{11}{10} V_{1} + \frac{131}{100} V_{1} + \frac{1561}{1000} V_{1} = \frac{4971}{1000} V_{1}$$

$$V_{1} = V \frac{1000}{4971} = \frac{66}{\sqrt{3}} \times \frac{1000}{4971} = 7.66 \text{ kV}$$

$$V_{2} = \frac{11}{10} V_{1} = \frac{11}{10} \times 7.66 = 8.426 \text{ kV}$$

$$V_{3} = \frac{131}{100} V_{1} = \frac{131}{100} \times 7.66 = 10.03 \text{ kV}$$

*.*..

$$V_4 = \frac{1561}{1000} V_1 = \frac{1561}{1000} \times 7.66 = 11.96 \text{ kV}$$
  
String  $\eta = \frac{V}{4 \times V_4} \times 100 = \frac{66 \times 100}{\sqrt{3} \times 4 \times 11.96} = 79.65 \%$ 

**Example 41.49.** Explain what is meant by string efficiency and how it can be improved. Each line of a 3-phase, 33-kV system is suspended by a string of 3 identical insulator discs. The capacitance of each disc is 9 times the capacitance to ground. Find voltage distribution across each insulator and the string efficiency. Suggest a method for improving the string efficiency.

(Power Systems-I, AMIE, Sec. B, 1993)

**Solution.** Let *C* be the self-capacitance of each disc and kC the mutual capacitance between the units. We are given that k = 9.

$$V_{2} = V_{1} \frac{(1+k)}{k} = \frac{10}{9} V_{1};$$
  
$$V_{3} = V_{1} \frac{(1+3k+k^{2})}{k^{2}} = \frac{109}{81} V_{1}$$

 $(1 \pm k) = 10$ 

Total voltage,

$$\frac{33}{\sqrt{3}} = \frac{280}{81}V_1; \quad \therefore \quad V_1 = \frac{33 \times 81}{\sqrt{3} \times 280} = 5.51 \text{ kV}$$
$$V_2 = (10/9) V_1 = 6.12 \text{ kV}; \quad V_3 = (109/81)V_1 = 7.41 \text{ kV}$$
$$= \frac{V}{3 \times V_3} \times 100 = \frac{33/\sqrt{3}}{3 \times 7.41} \times 100 = 85\%$$

 $V = V_1 + V_2 + V_3 = V_1 + \frac{10}{9}V_1 + \frac{109}{81}V_1 = \frac{280}{81}V_1$ 

String efficiency  $= \frac{1}{3}$ 

The string efficiency can be improved by providing a guard ring surrounding the lower-most unit which is connected to the metal work at the bottom. This ring increases the capacitance between the metal work and the line and helps in equalising the voltage distribution along the different units. The efficiency can also be improved by grading the insulators and by making the ratio (capacitance to earth/capacitance per insulator) as small as possible.

#### 41.39. Interconnectors

An interconnector is a tieline which enables two generating stations to operate in parallel. It facilitates the flow of electric power in either direction between the two stations.

## 41.40. Voltage Drop Over the Interconnector

Let station 1 supply a current of 1 to station 2 along an interconnector having a resistance of RW and reactance of XTW per phase as shown in Fig. 41.60 (*a*). If the receiving end p.f. is  $\cos \phi$  lagging, then the vector diagram will be as shown in Fig. 41.60 (*b*).

Voltage drop over the interconnector

$$= I(\cos \phi - j\sin \phi) (R + jX)$$
  
=  $I(R\cos \phi + X \sin \phi) + jI(X\cos \phi - R \sin \phi)$   
=  $\left\{ \frac{I(R\cos \phi + X \sin \phi) + jI(X \cos \phi - R \sin \phi)}{E_2} \right\} \times 100$ 

% voltage drop

Let  $I \cos \phi = I_d$  and  $I \sin \phi = I_q$ , where  $I_d$  and  $I_q$  are the in-phase and quadrature components.

 $\therefore \text{ voltage drop} = \left\{ \frac{(I_d R + I_q X) + j (I_d X - I_q R)}{E_2} \right\} \times 100$ 

 $\therefore \text{ in-phase voltage drop} = \frac{(I_d R + I_q X)}{E_2} \times 100$ and quadrature voltage drop =  $\frac{(I_d X^2 - I_q R)}{E_2} \times 100$ 





**Example 41.50.** The bus-bar voltages of two stations A and B are 33 kV and are in phase. If station A sends 8.5 MW power at u.p.f. to station B through an interconnector having an impedance of  $(3 + j4)\Omega$ , determine the bus-bar voltage of station A and the phase angle shift between the bus-bar voltages.

А

Stn

В

Stn

Fig. 41.61

Solution. With reference to Fig. 41.61,

Voltage of station *B*,  $V_B = 33,000/\sqrt{3} = 19,050$  V/phase Since power transferred to station *B* is 8.5 *M* W, current in the interconnector is

 $I = 8.5 \times 10^3 / \sqrt{3} = 148.7 \text{ A}$ 

Taking  $V_B$  as the reference vector,  $V_A = V_B + I(\cos \phi - j \sin \phi) (R + jX)$  = 19,050 + 148.7 (1 - j0) (3 + j4)  $= 19,496 + j595 = 19,502 \angle 1.75^{\circ}$ The line-to-line bus-bar voltage of station

A is

 $= 19,502 \times \sqrt{3} = 33.78 \text{ kV}$ 

The phase-shift angle between bus-bar voltages is 1.75°

### 41.41. Sag and Stress Analysis

The conductors of a transmission line are attached to suitable insulators carried on supports of wood, iron, steel or reinforced concrete. Obviously, the supports must be strong enough to withstand not only the dead weight of the conductors themselves but also the loads due to ice and sleet that may adhere to them and to wind pressure. Moreover, the minimum factor of safety for the conductors should be 2.0 based on ultimate strength.

Sag and stresses vary with temperature on account of thermal expansion and contraction of the line conductors. The value of sag as well as tension of a conductor would now be calculated when (i) supports are at equal levels and (ii) supports are at unequal levels.

## 41.42. Sag and Tension with Supports at Equal Levels

Fig. 41.62 shows a span of a wire with the two supports at the same elevation and separated by a horizontal distance 2l. It can be proved that the conductor AB forms a catenary with the lowest point O forming the mid-point (where the curve is straight).



Let *W* be the weight of the wire per unit length and let point O be chosen as the reference point for measuring the co-ordinates of different points on the wire. Consider a point P having co-ordinates of x and y. The tension T at point P can be resolved into two rectangular components.  $T_x$ and  $T_y$  so that  $T = \sqrt{T_x^2 + T_y^2}$ . If *S* is the length of the arc *OP*, then its weight is WS which acts vertically downward through the centre of gravity of OP. There are four forces acting on OP-two vertical and two horizontal. Since OP is in



equilibrium, the net force is zero. Equating the horizontal and vertical components, we have,

$$T_0 = T_x \text{ and } T_y = W_s.$$

It may be noted that the horizontal component of tension is constant throughout the length of the wire :

Since line *PT* is tangential to the curve *OB* at point *P*,  $\tan \theta = \frac{T_y}{T_x}$ 

It is also seen from the elementary piece *PP'* of the line that  $\tan \theta = dy/dx$ 

$$\therefore \qquad dy/dx = \tan \theta = \frac{T_y}{T_x} \text{ or } dy/dx = WS/T_0 \qquad \dots (i)$$

If  

$$PP' = dS, \text{ then } dS = \sqrt{(dx)^2 + (dy)^2} = dx \sqrt{1 + (dy/dx)^2}$$
or  

$$\frac{dS}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \left(\frac{WS}{T_0}\right)^2} \text{ or } dx = \frac{dS}{\sqrt{1 + \left(\frac{W^2}{dx}\right)^2}}$$

Integrating both sides, we have  $x = \left(\frac{T_0}{W}\right) \sinh^{-1} \left(\frac{WS}{T_0}\right) + C$ where *C* is the constant of integration.

Now, when x = 0, S = 0. Putting these values above, we find that C = 0.  $\therefore \qquad x = \left(\frac{T_0}{W}\right) \sinh^{-1}\left(\frac{WS}{T_0}\right) \text{ or } S = \frac{T_0}{W} \sinh\left(\frac{Wx}{T_0}\right) \qquad \dots (ii)$ 

Substituting this value of S in Eq. (i), we get

$$\frac{dy}{dx} = \sinh\left(\frac{Wx}{T_0}\right)$$
 or  $dy = \sinh\left(\frac{Wx}{T_0}\right)dx$ 

....

$$y = \int \sinh\left(\frac{Ex}{T_0}\right) dx = \left(\frac{T_0}{W}\right) \cosh\left(\frac{Wx}{T_0}\right) + D$$

where *D* is also the constant of integration. At the origin point *O*, x = 0 and y = 0. Hence, the above equation becomes

$$0 = \left(\frac{T_0}{W}\right)\cosh 0 + D + \left(\frac{T_0}{W}\right) + D \qquad \therefore \quad D = -\frac{T_0}{W}$$

Substituting this value of D in the above equation, we get

$$y = \left(\frac{T_0}{W}\right) \cosh\left(\frac{W_X}{T_0}\right) - \frac{T_0}{W} = \frac{T_0}{W} \left[\cosh\left(\frac{W_X}{T_0}\right) - 1\right] \qquad \dots (iii)$$

This is the equation of the curve known as catenary. Hence, when a wire is hung between two supports, it forms a catenary

(a) The tension at point P(x, y) is given by

$$T^{2} = T_{x2} + T_{y2} = T_{02} + W^{2}S^{2} = T_{02} + T_{02}\sinh^{2}\left(\frac{Wx}{T_{0}}\right) - \text{from Eq. (iii)}$$
$$T_{0} = \left[1 + \operatorname{sigh}^{2}\left(\frac{Wx}{T_{0}}\right)\right] = T_{0} - 2\operatorname{sigh}^{2}\left(\frac{Wx}{T_{0}}\right) - \operatorname{from Eq. (iii)}$$

$$= T_{02} \left[ 1 + \sinh^2 \left( \frac{Wx}{T_0} \right) \right] = T - 2 \cosh^2 \left( \frac{Wx}{T_0} \right) \quad \therefore \quad T = T_0 \cosh \left( \frac{Wx}{T_0} \right) \quad \dots (i\nu)$$

(b) Tension at points A and B where  $x = \pm l$  is given by  $T = T_0 \cosh(W_1/T_0)$  ...(v)

(c) The maximum sag is represented by the value of y at either of the two points A and B for which x = +l and x = -l respectively. Writing  $y = d_{max}$  and putting  $x = \pm l$  in Eq. (*iii*), we get,

$$d_{\max} = \frac{T_0}{W} \left[ \cosh\left(\frac{Wl}{T_0}\right) - 1 \right] \qquad \dots (vi)$$

(d) The length of the wire or conductor in a half span is as seen from Eq. (ii) above,

$$S = \frac{T_0}{W} \sinh\left(\frac{Wl}{T_0}\right)$$

## Approximate Formulae

The hyperbolic sine and cosine functions can be expanded into the following series

$$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \frac{z^7}{7!} + \dots \text{ and } \cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \frac{z^6}{6!} + \dots$$

Using the above, the approximate values of T d and S points at A and B may be found as follows :

(i) 
$$T = T_0 \cosh\left(\frac{Wl}{T_0}\right) = T_0 \left(1 + \frac{W^2 l^2}{2T_0^2} + \dots\right) - \text{neglecting higher powers}$$
$$= T_0 + \frac{W^2 l^2}{2T_0} \cong T_0 \qquad -\text{if } \frac{W^2 l^2}{2T_0} \text{ is negligible}$$

 $\therefore$   $T = T_0 - i.e.$  tension at the supports is very approximately equal to the horizontal tension acting at any point on the wire.

(ii)  
$$d = \frac{T_0}{W} \left[ \cosh\left(\frac{Wl}{T_0}\right) - 1 \right] = \frac{T_0}{W} \left[ \left( 1 + \frac{W^2 l^2}{2T_0^2} + \dots \right) - 1 \right]$$
$$= \frac{Wl^2}{2T_0} = \frac{Wl^2}{2T} \text{ (approx)} \quad \therefore \quad d = \frac{Wl^2}{2T}$$

It should be noted that W and T should be in the same units *i.e.* kg-wt or newton.

(iii) 
$$S = \frac{T_0}{W}\sinh + \left(\frac{Wl}{T_0}\right) = \frac{T_0}{W}\left(\frac{Wl}{T_0} + \frac{W^3l^3}{6T_0^3} + \dots\right) = l + \frac{W^2l^3}{6T_0^2}$$
 —negle  
The total length of the wire along the curve is  $L = 2S = 2l + \frac{W^2l^2}{3T^2}$ 

-neglecting higher terms.

This length consists of the unstertched length  $L_u$  and the stretch or extension.

:. unstretched length  $L_u = L$  —extension  $\Delta L$ 

Now, 
$$E = \frac{T/A}{\Delta l/2l}$$
 or  $\Delta l = \frac{2lT}{EA}$ 

Substituting the value of T from the relation  $d = W l^2 / 2T$ , we have

$$\Delta l = \frac{2l}{EA} \cdot \frac{Wl^2}{2d} = \frac{Wl^3}{EAd} \quad \therefore \quad L_u = L - \frac{Wl^3}{EAd}$$

$$(iv) \qquad \qquad y = \frac{T_0}{W} \left[ \cosh\left(\frac{Wx}{T_0}\right) - 1 \right] = \frac{T_0}{W} \left[ \left(1 + \frac{W^2 x^2}{2T_0^2} + \dots\right) - 1 \right]$$
or
$$y = \frac{Wx^2}{2T} \qquad \qquad -\text{the equation of a parabola}$$

It shows that the catenary curve formed by the sagging is very approximately like a parabola in shape.

The above formulae are sufficiently accurate for all practical purposes provided sag is less than 10% of the span.



Overhead cables are economical and easy to maintain, which are usually made of bare copper or aluminum wires

# 41.43. Sag and Tension with Supports at Unequal Levels

Fig. 41.63 shows a span between two supports *A* and *B* whose elevations differ by  $\eta$ , their horizontal spacing being 2*l* as before. Such spans are generally met with in a hilly country. Let *O* be the lowest point of the catenary *AOB*. Obviously, *OA* is a catenary of half-span  $x_1$  and *OB* of half-span  $x_2$ .



## Fig. 41.63

The equations derived in Art. 41.42 also apply to this case.

(*i*) As seen from Eq. (vi) of Art. 41.42.

$$d_{1} = \frac{T_{0}}{W} \left[ \cosh\left(\frac{Wx_{1}}{T_{0}}\right) - 1 \right] \qquad --\text{putting } l = x_{1}$$

$$d_{2} = \frac{T_{0}}{W} \left[ \cosh\left(\frac{Wx_{2}}{T_{0}}\right) - 1 \right] \qquad --\text{putting } l = x_{2}$$

$$T_{1} = T_{0} \cosh\left(\frac{Wx_{1}}{T_{0}}\right) \text{ and } T_{2} = T_{0} \cosh\left(\frac{Wx_{2}}{T_{0}}\right)$$

*(ii)* 

It is obvious that maximum tension occurs at the higher support B.

$$\therefore$$
 max. permissible tension =  $T_0 \cosh\left(\frac{Wx_2}{T_0}\right)$ 

## **Approximate Formulae**

Is is obvious that

$$d_2 - d_1 = \frac{T_0}{W} \left[ \cosh\left(\frac{Wx_2}{T_0}\right) - \cosh\left(\frac{Wx_2}{T_0}\right) \right] = h \text{ and } x_1 + x_2 = 2l$$

Using approximations similar to those in Art. 41.42, we get

$$d_1 = \frac{Wx_1^2}{2T_0} = \frac{Wx_1^2}{2T}$$
 and  $d_2 = \frac{Wx_2^2}{2T_0} = \frac{Wx_2^2}{2T}$ 

(it has been assumed, as before, that  $T = T_0$ )

$$\therefore \qquad d_2 - d_1 = h = \frac{W}{2T} (x_2^2 - x_1^2) = \frac{W}{2T} (x_2 + x_1) (x_2 - x_1) = \frac{Wl}{T} (x_2 - x_1)$$

$$\therefore \qquad x_2 - x_1 = \frac{hT}{Wl}; \quad \text{Now, } x_2 - x_1 = hT/Wl \text{ and } x_2 + x_1 = 2l$$
$$\therefore \qquad x_1 = 1 - \frac{hT}{2Wl} \quad \text{and} \quad x_2 = 1 + \frac{hT}{2Wl} *$$

Having found  $x_1$  and  $x_2$ , values of  $d_1$  and  $d_2$  can be easily calculated. It is worth noting that in some cases,  $x_1$  may be negative which means that there may be no horizontal point (like point O) in the span. Such a thing is very likely to happen when the line runs up a steep mountain side.

# 41.44. Effect of Wind and Ice

In the formulae derived so far, effect of ice and wind loading has not been taken into account. It is found that under favourable atmospheric conditions, quite an appreciable thickness of ice is formed on transmission lines. The weight of ice acts vertically downwards i.e. in the same direction as the weight of the conductor itself as shown in Fig. 41.64. In addition, there may be high wind which exerts considerable force on the conductor. This force is supposed to act in a horizontal direction. If  $W_i$  is the weight of ice per unit length of the



designing electric cables

conductor and  $W_{\mu}$  the force per unit length exerted by the wind, then total weight of the conductor per



 $Wt = \sqrt{\left(W + W_i\right)^2 + W_w^2}$ 

It is obvious that in all equations derived in Art. 41.42 and 41.43, W should be replaced by Wt. As seen from Fig. 41.64(b), the conductor sets itself in a plane at an angle of  $\theta = \tan^{-1}$  $[W_{\omega}/(W + W_i)]$  to the vertical but keeps the shape of a catenary (or a parabola approximately).

**Note.** (*i*) If *P* is the wind pressure per unit projected area of the conductor, then wind load or force per unit length of the ice-covered conductor is

 $W_{\omega} = P \times (D + 2R) \times l = P \times D$ 

—if conductor is without ice

If we put h = 0 *i.e.* assume supports at the same level, then as expected  $x_1 = x_2 = l$ 

(ii) Ice is in the form of a hollow cylinder of inner diameter = D and outer diameter = (D + 2R). Hence, volume per unit length of such a cylinder is

$$= \frac{\pi}{4} \left[ (D+2R)^2 - D^2 \right] \times 1 = \frac{\pi}{4} (4DR + 4R^2) = \pi R(D+R)$$

If  $\rho$  is the ice density, then weight of ice per unit length of the conductor is

$$W_i = \rho \times \text{volume} = \pi \rho R(D + R)$$

When ice and wind loads are taken into account, then the approximate formulae derived in Art. 41.42 become

$$d = \frac{W_t l^2}{2T}; \quad S = 1 + \frac{W_t^2 l^3}{6T^2} \quad \text{and} \quad y = \frac{W_t x^2}{2T}$$

Here 'd' represents the slant sag in a direction making an angle of  $\theta$  with the vertical. The vertical sag would be =  $d \cos \theta$ .

Similarly, formulae given in Art. 41.43 become

$$d_1 = \frac{W_t \cdot x_1^2}{2T}; \quad d_2 = \frac{W_t \cdot x_2^2}{2T}; \quad x_1 = 1 - \frac{hT}{2W_t \cdot l} \text{ and } x_2 = 1 + \frac{hT}{2W_t \cdot l}$$

**Example 41.51.** A transmission line conductor at a river crossing is supported from two towers at heights of 70 m above water level. The horizontal distance between towers is 300 m. If the tension in conductor is 1,500 kg, find the clearance at a point midway between the towers. The size of conductor is  $0.9 \text{ cm}^2$ . Density of conductor material is  $8.9 \text{ g/cm}^3$  and suspension length of the string is 2 metres.

Solution.

Sag.  $d = Wl^2/2T$  $l = 150 \text{ m}, W = lA\rho = 2 \times (0.9 \times 10^{-4}) \times (8.9 \times 10^{3}) = 0.8 \text{ kg wt}; T = 1500 \text{ kg wt}$  $d = W l^2 / 2T = 0.8 \times 150^2 / 2 \times 1500 = 6 \text{ m}$ 

Clearance between conductor and water at mid-way between the towers

$$= 70 - 6 - 2 = 62 \text{ m}$$

**Example 41.52.** The effective diameter of a line is 1.96 cm and it weighs 90 kg per 100 metre length. What would be the additional loading due to ice of radial thickness 1.25 cm and a horizontal wind pressure of 30  $kg/m^2$  of projected area? Also, find the total weight per metre run of the line. Density of ice is  $920 \text{ kg/m}^3$ .

Solution. It should be noted that weights of the conductor and ice act vertically downwards whereas wind pressure is supposed to act horizontally. Hence, the total force on one metre length of the conductor is found by adding the horizontal and vertical forces vectorially.

∴ total weight	$W_t = \sqrt{(W + W_i)^2 + W_w^2},  W_i = \pi \rho R (D + R)$
Here	$\rho = 920 \text{ kg/m}^3$ , $R = 0.0125 \text{ m}$ ; $D = 0.0196 \text{ m}$
$W_w = P(D+2R) =$	$30(0.0196 + 2 \times 0.0125) = 1.34$ kg wt/m; $W = 90/100 = 0.9$ kg wt/m
<i>.</i>	$W_i = \pi \times 920 \times 0.0125(0.0196 + 0.0125) = 1.16 \text{ kg wt/m}$
<i>:</i>	$W_t = \sqrt{(W + W_i)^2 + W_w^2} = \sqrt{(0.9 + 1.16)^2 + 1.34^2} = 2.46 \text{ kg wt/m}$

**Example 41.53.** A transmission line has a span of 150 metres between supports, the supports being at the same level. The conductor has a cross-sectional area of  $2 \text{ cm}^2$ . The ultimate strength is  $5,000 \text{ kg/cm}^2$ . The specific gravity of the material is 8.9. If the wind pressure is 1.5 kg/m length of the conductor, calculate the sag at the centre of the conductor if factor of safety is 5.

(Electrical Technology-I, Bombay Univ.)

Solution Sofety for	Safety facto	tor -	breaking or ultimate stress
Solution. Salety lact		<i>n</i> –	working stress
∴ workir	ng stress = $5,000/$	5 =	$10^3$ kg/cm <sup>2</sup> ; Working tension, $T = 10^2 \times 2 = 2,000$ kg wt.
Vol. of	one metre length	of co	nductor = $2 \times 100 = 200 \text{ cm}^3$
Wt.	of 1 m of materia	al =	$8.9 \times 200 \ g$ or $W = 8.9 \times 200/1,000 = 1.98 \ \text{kg-wt}.$
Total Wt. pe	er metre	=	$\sqrt{W^2 + W_w^2} = \sqrt{1.98^2 + 1.5^2} = 2.48$ kg-wt
Now		<i>d</i> =	$Wl^2/2T = 2.48 \times (150/2)^2/2 \times 2,000 = 3.5$ metre

**Example 41.54.** A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of  $1.29 \text{ cm}^2$ , weighs 1,170 kg/km and has a breaking stress of  $4,218 \text{ kg/cm}^2$ . Calculate the sag for a factor of safety of 5 allowing a wind pressure of 122 kg per  $m^2$  of projected area. What is the vertical sag?

(Transmission and Distribution-II, Kerala Univ.)

Calada a	C. C. C. C. Mark	breaking or ultimate stress
Solution.	Safety factor =	working stress
	working stress =	$4,218/5 = 843.6 \text{ kg/cm}^2$
Working tension	on $T =$	$843.6 \times 1.29 = 1,088$ kg.wt
<i>:</i> .	W =	1,170 kg/km = 1.17 kg.wt/metre
Let us now fin	d diameter of the co	onductor from the equation
	$\pi d^2/4 =$	1.29 or $d = 1.28$ cm
.: Projected	area of the conduct	tor per metre length is $= 1.28 \times 10^{-2} \times 1 = 1.28 \times 10^{-2} \text{ m}^2$
	$W_{_{W}}$ =	$122 \times 1.28 \times 10^{-2} = 1.56 \text{ kg-wt/m}$
<i>.</i>	$W_t =$	$\sqrt{W^2 + W_w^2} = \sqrt{1.17^2 + 1.56^2} = 1.95 \text{ kg-wt/m}$
Slant sag	d =	$W_t l^2 / 2T = 1.95 \times 100^2 / 2 \times 1088 = 8.96 \text{ m}$
Now,	$\tan \theta =$	$W_w/W = 1.56/1.17 = 1.333; \theta = 53.2^{\circ}$
<i>:</i> .	vertical sag =	$d\cos 53.2^\circ = 8.96 \times 0.599 = 5.3 \mathrm{m}$

**Example 41.55.** A transmission line has a span of 214 metres. The line conductor has a crosssection of 3.225 cm<sup>2</sup> and has an ultimate breaking strengh of 2,540 kg/cm<sup>2</sup>. Assuming that the line is covered with ice and provides a combined copper and ice load of 1.125 kg/m while the wind pressure is 1.5 kg/m run (i) calculate the maximum sag produced. Take a factor of safety of 3 (ii) also determine the vertical sag. (Electrical Power Systems, Gujarat Univ.)

<b>Solution.</b> ( <i>i</i> ) Maximum sag in a direction making an angle of $\theta$ (Fig. 41.64) is $d = W_t l^2 / 2T$			
Here,	W + Wi	=	1.125  kg-wt/m; $Ww = 1.5  kg-wt/m$
	$W_t$	=	$\sqrt{1.125^2 + 1.5^2} = 1.874$ kg-wt/m; $l = 214/2 = 107$ m
Safety factor		=	ultimate breaking stress working stress
working stress	8	=	$2,540/3 = 847 \text{ kg-wt/cm}^2$
Permissible tensio	n T	=	$847 \times 3.225 = 2,720$ kg-wt
.:.	d	=	$1.874 \times 107^2 / 2 \times 2,720 = 3.95 \text{ m}$
(ii) Now	tan θ	=	$W_w/(W_i + W) = 1.5/1.125 = 0.126$ ; $\theta = 53.2^\circ$ ; $\cos \theta = 0.599$
.:.	vertical sag	=	$d\cos\theta = 3.95 \times 0.599 = 2.35 \text{ m}$

**Example 41.56.** Two towers of height 30 and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance between the towers is 500 m. If the tension in the conductor is 1,600 kg, find the minimum clearance of the conductor and the clearance of the conductor mid-way between the supports. Weight of the conductor is 1.5 kg/m. Bases of the towers can be considered to be at the water level. (Electrical Power-I, Bombay Univ.)



As seen from Fig. 41.65, clearance of the lowest point *O* from the water level

$$= 30 - 7 = 23 \text{ m}$$

The horizontal distance of mid-point *P* from the reference point *O* is x = (250 - 122) = 128m\*. The height of the point *P* above *O* is

$$d_{mid} = \frac{Wx_1^2}{2T} = \frac{1.5 \times 128^2}{2 \times 1,600} = 7.68 \text{ m}$$

Hence, clearance of mid-point above water level is 23 + 7.68 = 30.68 m

**Example 41.57.** An overhead transmission line at a river crossing is supported from two towers at heights of 50 m and 100 m above the water level, the horizontal distance between the towers being 400 m. If the maximum allowable tension is 1,800 kg and the conductor weighs 1 kg/m, find the clearance between the conductor and water at a point mid-way between the towers.

**Solution.** Here, h = 100 - 50 = 50 m; В l = 400/2 = 200 m $T = 1800 \, \text{kg-wt}$ W = 1 kg-wt/m $d_2$  $x_1 = l - \frac{hT}{2Wl}$ d'mid  $= 200 - \frac{50 \times 1800}{2 \times 1 \times 200}$ 25 m= -25 m $x_2 = 425 \text{ m}$  $x_2 = l + \frac{hT}{2Wl} 200 + 225$ = 425 mFig. 41.66 Or  $x = \frac{1}{2} (x_2 - x_1)$ . In this case,  $x = \frac{1}{2} (378 - 122) = 128$  m

(Power System-I, AMIE, Sec. B, 1994)

Since  $x_1$  turns out to be negative, point A lies on the same side of O as B (Fig. 41.66). Distance of mid-point P from O is x = (425 + 25) = 225 m. Hence, height of P above O is

$$d_{mid} = \frac{Wx^2}{T} = \frac{1 \times 225^2}{2 \times 1,800} = 14 \text{ m}$$
$$d_2 = \frac{Wx_2^2}{2T} = \frac{1 \times 425^2}{2 \times 1,800} = 50.2 \text{ m}$$

Now,

Hence, P is (50.2-14) = 36.2 m below point B. It means that mid-point P is (100 - 36.2) = 63.8 m above water level.

**Example 41.58.** A conductor is strung across a river, being supported at the two ends at heights of 20 m and 16 m respectively, from the bed of the river. The distance between the supports is 375 m and the weight of the conductor = 1.2 kg/m.

If the clearance of the conductor from the river bed be 9 m, find the horizontal tension in the conductor. Assume a parabolic configuration and that there is no wind or ice loading.

#### (Electrical Power-I, Bombay Univ.)

 $= 2m \times 112.5 = 225$  m)

9

Solution. Here,	l = 375/2 = 187.5  m; h = 20 - 16 = 4  m; W = 1.2  kg-wt/m; T =
	$x_1 = l - \frac{hT}{2Wl} = 187.5 - \frac{4 \times T}{2 \times 1.2 \times 187.5} = 187.5 - \frac{T}{112.5}$
or	$x_1 = (187.5 - m)$ where $m = T/112.5$
A.1	$d \rightarrow 0$ 16 and 7 $Wx_1^2$

Also,

*.*..

$$d_1 + 9 = 16$$
 or  $d_1 = 7 = \frac{Wx_1^2}{2T}$   
 $7 = \frac{1.2(187.5 - m)^2}{225 m}$  (:: 2T

or  $1.2 m^2 - 2,025 m + 42,180 = 0$  or m = 1,677 or 10.83

Rejecting the bigger value which is absurd, we have

m = 10.83 or T/112.5 = 10.83 or T = 1,215 kg-wt/m

#### **Tutorial Problem No. 41.3**

1. Show diagrammatically the distribution of electrostatic capacitance in a 3-core, 3-phase lead-sheathed cable.

The capacitance of such a cable measured between any two of the conductors, the sheathing being earthed, is 0.3  $\mu$ F per km. Find the equivalent star-connected capacitance and the kVA required to keep 10 km of the cable charged when connected to 20,000-V, 50 Hz bus-bars. **[0-6 \mu F; 754 kVA]** 

2. The 3-phase output from a hydro-electric station is transmitted to a distributing centre by two overhead lines connected in parallel but following different routes. Find how a total load of 5,000 kW at a p.f. of 0.8 lagging would divide between the two routes if the respective line resistances are 1.5 and 1.0  $\Omega$  and their reactances at 25 Hz are 1.25 and 1.2  $\Omega$ .

#### [2,612 kW ; 2,388 kW](City & Guilds, London)

3. Two 3-phase cables connected in parallel supply a 6,600-V, 1,000-kW load at a lagging power factor of 0.8. The current in one of the cables is 70 A and it delivers 600 kW. Calculate its reactance and resistance, given that the other cable has reactance of 2.6  $\Omega$  and a resistance of 2  $\Omega$ .

 $[\mathbf{R} = .795 \,\Omega; \mathbf{X} = 1.7 \,\Omega]$  (London Univ.)

4. A concentric cable has a conductor diameter of 1 cm and an insulation thickness of 1.5 cm. Find the maximum field strength when the cable is subjected to a test pressure of 33 kV.

[47.6 kV/cm (r.m.s.) or 67.2 kV/cm (peak)] (London Univ.)

5. A single-phase ring distributor XYZ is fed at X. The loads at Y and Z are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at X. The total impedances of the three sections X Y, YZ and ZX are (1 + j1), (1 + j2) and (1 + j3) ohms respectively. Find the total current fed at X and the current in each section with respect to suply voltage at X.

#### (Allahabad Univ.)

#### [34.6 ∠ - 43.8° A ; AXY = 23.1 ∠- 32° A, AXZ = 13.1 ∠- 60.8°. YZ = 3.01 ∠- 30.5°A]

- 6. A single-phase distributor has a resistance of 0.2 ohm and reactance 0.3 ohm. At the far end, the voltage  $V_B = 240$  volts, the current is 100 A and the power factor is 0.8. At the mid point A, current of 100 A is supplied at a power factor 0.6 with reference to voltage  $V_A$  at point A. Find supply voltage  $V_S$  and the phase angle between  $V_S$  and  $V_B$ . [292 V, 2.6°]
- Estimate the corona loss for a 3-phase 110-kV, 50-Hz, 150-km long transmission line consisting of three conductors, each of 10 mm diameter and spaced 2.5 metre apart in an equilateral triangle formation. The temperature of air is 30°C and the atmospheric pressure is 750 mm of mercury. Take the irregularity factor as 0.85. [385 kW](AMIE)
- **8.** A transmission line conductor at a river crossing is supported from two towers at heights of 20 m and 60 m above water level. The horizontal distance between the towers is 300 m. If the tension in the conductor is 1800 kg and the conductor weighs 1.0 kg per metre, find the clearance between the conductor and the water level at a point mid-way between the towers. Use approximate method.

#### (18.75 m) (AMIE)

**9.** Show that the positive and negative sequence impedances of transmission lines are same where as its zero sequence impedance is higher than positive sequence impedance.

(Nagpur University, Summer 2004)

10. A 132 kV, 3 phase, 50 Hz overhead line of 100 km length has a capacitance to earth of each line of 0.01 μF/km. Determine inductance and kVA rating of the arc suppression coil suitable for this system. (Nagpur University, Summer 2004)

# **OBJECTIVE TESTS – 41**

1. With same maximum voltage between conductors, the ratio of copper volumes in 3-phase, 3wire system and 1-phase, 2-wire system is

( <i>a</i> )	4/3	<i>(b)</i>	3/4
	= 10	( <b>D</b>	a ( =

(c) $5/3$ (d)	3/5
---------------	-----

- 2. The volume of copper required for an a.c. transmission line is inversely proportional to
  - (a) current (b) voltage
  - (c) power factor (d) both (b) and (c)
  - (e) both (a) and (c).
- **3.** For a.c. transmission lines less than 80 km in length, it is usual to lump the line capacitance at
  - (a) the receiving end
  - (*b*) the sending end
  - (c) the mid-point
  - (d) any convenient point.
- Corona occurs between two transmission wires when they
  - (a) are closely-spaced
  - (b) are widely-spaced
  - (c) have high potential difference

(d) carry d.c. power.

- 5. The only advantage of corona is that it (*a*) makes line current non-sinusoidal
  - (b) works as a safety-valve for surges
  - (c) betrays its presence by hissing sound
  - (d) produces a pleasing luminous glow.
- 6. The sag produced in the conductor of a transmission wire depends on
  - (a) weight of the conductor per unit length
  - (b) tension in the conductor
  - (*c*) length of the conductor
  - (*d*) all of the above
  - (*e*) none of the above.
- Suspension insulators are used when transmission voltage is

(b) low

- (a) high
- (c) fluctuating (d) steady
- **8.** The string efficiency of suspension insulators can be increased by
  - (a) providing a guard ring
  - (b) grading the insulators

- (c) using identical insulator disc
- (*d*) both (*a*) & (*b*).
- **9.** An interconnector between two generating stations facilitates to
  - (a) keep their voltage constant
  - (b) run them in parallel
  - (c) transfer power in either direction
  - (*d*) both (*b*) & (*c*)
- **10.** The effective disruptive critical voltage of a transmission line does NOT depend on
  - (a) irregularity factor
  - (b) conductor radius
  - (c) distance between conductors
  - (d) material of the conductors.
- **11.** By which of the following systems electric power may be transmitted?
  - (a) Overhead system
  - (b) Underground system
  - (c) Both (a) and (b)
  - (d) None of the above
- **12.** ..... are the conductors, which cannect the consumer's terminals to the distribution
  - (a) Distributors
  - (b) Service mains
  - (c) Feeders
  - (d) None of the above
- **13.** The underground system cannot be operated above

( <i>a</i> )	440 V	(b)	11 kV
( <i>c</i> )	33 kV	(d)	66 kV

- 14. Overhead system can be designed for operation upto
  - (a) 11 kV (b) 33 kV
  - (c) 66 kV (d) 400 kV
- **15.** If variable part of annual cost on account of interest and depreciation on the capital outlay is equal to the annual cost of electrical energy wasted in the conductors, the total annual cost will be minimum and the corresponding size of conductor will be most economical. This statement is known as
  - (a) Kelvin's law (b) Ohm's law
  - (c) Kirchhoff's law (d) Faraday's law
  - (e) none of the above
- **16.** The wooden poles well impregnated with creosite oil or any preservative compound have life
  - (a) from 2 to 5 years
  - (b) 10 to 15 years
  - (c) 25 to 30 years
  - (*d*) 60 to 70 years
- 17. Which of the following materials is not used

for transmission and distribution of electrical

- power?
- (a) Copper(b) Aluminium
- (c) Steel
- (d) Tungsten
- 18. Galvanised steel wire is generally used as
  - (a) stay wire
  - (b) earth wire
  - (c) structural components
  - (*d*) all of the above
- 19. The usual spans with R.C.C. poles are
  - (a) 40-50 metres
  - (b) 60-100 metres
  - (c) 80–100 metres
  - (d) 300-500 metres
- **20.** The corona is considerably affected by which of the following?
  - (a) Size of the conductor
  - (*b*) Shape of the conductor
  - (c) Surface condition of the conductor
  - (d) All of the above
- **21.** Which of the following are the constants of the transmission lines?
  - (a) Resistance
  - (b) Inductance
  - (c) Capacitance
  - (*d*) All of the above
- **22.** % age regulation of a transmission line is given by

(a) 
$$\frac{V_R - V_S}{V_R^2} \times 100$$
  
(b) 
$$\frac{V_S - V_R}{V_R} \times 100$$

(c) 
$$\frac{V_S - V_R}{V_S} \times 100$$
  
(d) 
$$\frac{V_S - V_R}{V_{P^2}} \times 100$$

where  $V_S$  and  $V_R$  are the voltages at the sending end and receiving and respectively.

- 23. The phenomenon of rise in voltage at the receiving end of the open-circuited or lightly loaded line is called the
  - (a) Seeback effect
  - (b) Ferranti effect
  - (c) Raman effect
  - (d) none of the above

- **24.** The square root of the ratio of line impedance and shunt admittance is called the
  - (a) surge impedance of the line
  - (b) conductance of the line
  - (c) regulation of the line
  - (d) none of the above
- **25.** Which of the following is the demerit of a 'constant voltage transmission system'?
  - (a) Increase of short-circuit current of the system
  - (*b*) Availability of steady voltage at all loads at the line terminals
  - (c) Possibility of better protection for the line due to possible use of higher terminal reactances
  - (d) Improvement of power factor at times of moderate and heavy loads
  - (e) Possibility of carrying increased power for a given conductor size in case of longdistance heavy power transmission
- 26. Low voltage cables are meant for use up to
  - (a) 1.1 kV
  - (b) 3.3 kV
  - (c) 6.6 kV
  - (*d*) 11 kV
- 27. The operating voltage of high voltage cables is upto
  - (a) 1.1 kV
  - (b) 3.3 kV
  - (c) 6.6 kV
  - (*d*) 11 kV
- **28.** The operating voltage of supertension cables is upto
  - (a) 3.3 kV
  - (b) 6.6 kV
  - (c) 11 kV
  - (d) 33 kV
- **29.** The operating voltage of extra high tension cables is upto
  - (a) 6.6 kV
  - (b) 11 kV
  - (c) 33 kV
  - (*d*) 66 kV
  - (e) 132 kV
- **30.** Which of the following methods is used for laying of underground cables?
  - (a) Direct laying
  - (b) Draw-in-system
  - (c) Solid system
  - (*d*) All of the above
- **31.** Which of the following is the source of heat generation in the cables?
  - (a) Dielectric losses in cable insulation
  - (b)  $I^2R$  losses in the conductor
  - (c) Losses in the metallic sheathings and armourings

(*d*) All of the above

- **32.** Due to which of the following reasons the cables should not be operated too hot?
  - (*a*) The oil may loose its viscosity and it may start drawing off from higher levels
  - (b) Expansion of the oil may cause the sheath to burst
  - (c) Unequal expansion may create voids in the insulation which will lead to ionization
  - (d) The thermal instability may rise due to the rapid increase of dielectric losses with temperature
- **33.** Which of the following D.C. distribution system is the simplest and lowest in first cost?
  - (a) Radial system
  - (b) Ring system
  - (c) Inter-connected system
  - (d) Non of the above
- 34. A booster is a
  - (a) series wound generator
  - (b) shunt wound generator
  - (c) synchronous generator
  - (*d*) none of the above
- **35.** Besides a method of trial and error, which of the following methods is employed for solution of network problems in interconnected system?
  - (a) Circulating current method
  - (b) Thevenin's theorem
  - (c) Superposition of currents
  - (d) direct application of Kirchhoff's laws
  - (e) All of the above
- **36.** Which of the following faults is most likely to occur in cables?
  - (a) Cross or short-circuit fault
  - (b) Open circuit fault
  - (c) Breakdown of cable insulation
  - (d) all of the above
- **37.** The cause of damage to the lead sheath of a cable is
  - (a) crystallisation of the lead through vibration
  - (b) chemical action on the lead when
  - (c) mechanical damage
  - (*d*) all of the above
- **38.** The voltage of the single phase supply to residential consumers is
  - (a) 110 V
  - (b) 210 V
  - (c) 230 V
  - (d) 400 V
- **39.** Most of the high voltage transmission lines in India are
  - (a) underground
  - (b) overhead

- (c) either of the above
- (d) none of the above
- 40. The distributors for residential areas are
  - (a) single phase
  - (b) three-phase three wire
  - (c) three-phase four wire
  - (d) non of the above
- **41.** The conductors of the overhead lines are
  - (a) solid
  - (b) stranded
  - (c) both solid and stranded
  - (d) none of the above
- 42. High voltage transmission lines use
  - (a) suspension insulators
  - (b) pin insulators
  - (c) both (a) and (b)
  - (d) none of the above
- 43. Multicore cables generally use
  - (a) square conductors
  - (b) circular conductors
  - (c) rectangular conductors
  - (d) sector-shaped conductors
  - (e) none of the above
- 44. Distributio lines in India generally use
  - (a) wooden poles
  - (b) R.C.C. poles
  - (c) steel towers
  - (d) none of the above
- **45.** The material commonly used for insulation in high voltage cables is
  - (a) lead
  - (b) paper
  - (c) rubber
  - (d) none of the above
- **46.** The loads on distributors systems are generally
  - (a) balanced
  - (b) unbalanced
  - (c) either of the above
  - (d) none of the above
- **47.** The power factor of industrial loads is generally
  - (a) unity
  - (b) lagging
  - (c) leading
  - (d) zero
- 48. Overhead lines generally use
  - (a) copper conductors
  - (b) all aluminium conductors
  - (c) A.C.S.R. conductors
  - (d) none of these
- **49.** In transmission lines the cross-arms are made of
  - (a) copper
  - (b) wood
  - (c) R.C.C.

- (d) steel
- **50.** The material generally used for armour of high voltage cables is
  - (a) aluminium
  - (b) steel
  - (c) brass
  - (d) copper
- **51.** Transmission line insulators are made of *(a)* glass
  - (u) glass
  - (b) porcelain(c) iron
  - (d) P.V.C.
  - (a) 1.v.C.
- **52.** The material commonly used for sheaths of underground cables is
  - (a) lead
  - (b) rubber
  - (c) copper
  - (d) iron
- **53.** The minimum clearance between the ground and a 220 kV line is about
  - (*a*) 4.3 m
  - (b) 5.5 m
  - (c) 7.0 m
  - (d) 10.5 m
- The spacing between phase conductors of a 220 kV line is approximately equal to
  - (*a*) 2 m
  - (b) 3.5 m
  - (c) 6 m
  - (d) 8.5 m
- **55.** Large industrial consumers are supplied electrical energy at
  - (*a*) 400 V
  - (b) 11 kV
  - (c) 66 kV
  - (*d*) 400 kV
- **56.** In a D.C. 3-wire distribution system, balancer fields are cross-connected in order to
  - (a) boost the generated voltage
  - (b) balance loads on both sides of the neutral
  - (c) make both machines run as unloaded motors
  - (d) equalize voltages on the positive and negative outers
- **57.** In a D.C. 3-wire distributor using balancers and having unequal loads on the two sides
  - (*a*) both balancers run as generators
  - (b) both balancers run as motors
  - (c) balancer connected to lightly-loaded side runs as a motor
  - (*d*) balancer connected to heavily-loaded side runs as a motor
- **58.** Transmitted power remaining the same, if supply voltae of a D.C. 2-wire feeder is increased 100 percent, saving in copper is

- (a) 25 percent
- (b) 50 percent
- (c) 75 percent
- (d) 100 percent
- **59.** A uniformly-loaded D.C. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at a middle point is
  - (a) one-fourth
  - (b) one-third
  - (c) one-half
  - (d) twice
  - (e) none of the above
- **60.** As compared to a 2-wire D.C. distributor, a 3-wire distributor with same maximum voltage to earth uses only
  - (*a*) 31.25 percent of copper
  - (b) 33.3 percent of copper
  - (c) 66.7 percent of copper
  - (d) 125 percent of copper
- **61.** Which of the following is usually not the generating voltage?
  - (a) 6.6 kV
  - (b) 8.8 kV
  - (c) 11 kV
  - (*d*) 13.2 kV
- **62.** For an overhead line, the surge impedance is taken as
  - (a) 20-40 ohms
  - (b) 70-80 ohms
  - (c) 100-200 ohms
  - (d) 500–1000 ohms
  - (e) none of the above
- 63. The presence of ozone due to corona is harmful because it
  - (a) reduces power factor
  - (b) corrodes the material
  - (c) gives odour
  - (d) transfer energy to the ground
  - (e) none of the above
- **64.** A feeder, in a transmission system, feeds power to
  - (a) distributors
  - (b) generating stations
  - (c) service mains
  - (d) all of the above
- 65. The power transmitted will be maximum when
  - (a) corona losses are minimum
  - (b) reactance is high
  - (c) sending end voltage is more
  - (d) receiving end voltage is more
- 66. A 3-phase 4 wire system is commonly used on
  - (a) primary transmission
  - (b) secondary transmission
  - (c) primary distribution

- (d) secondary distribution
- **67.** Which of the following materials is used for overhead transmission lines?
  - (*a*) Steel cored aluminium
  - (b) Galvanised steel
  - (c) Cadmium copper
  - (d) Any of the above
- **68.** Which of the following is not a constituent for making porcelain insulators?
  - (a) Quartz
  - (b) Kaolin
  - (c) Felspar
  - (d) Silica
- **69.** There is a greater possibility of occurence of corona during
  - (a) dry weather
  - (b) winter
  - (c) summer heat
  - (d) humid weather
  - (e) none of the above
- **70.** Which of the following relays is used on long transmission lines?
  - (a) Impedance realy
  - (b) Mho's relay
  - (c) Reactance relay
  - (d) None of the above
- **71.** The steel used in steel cored conductors is usually
  - (a) alloy steel
  - (b) stainless steel
  - (c) mild steel
  - (d) high speed steel
  - (e) all of the above
- **72.** Which of the following distribution system is more reliable?
  - (a) Radial system
  - (b) Tree system
  - (c) Ring main system
  - (d) All are equally reliable
- **73.** Which of the followin characteristics should the line supports for transmission lines possess?
  - (a) Low cost
  - (b) High mechanical strength
  - (c) Longer life
  - (d) All of the above
- 74. Transmission voltage of 11 kV is normally used for distance upto
  - (a) 20–25 km
  - (*b*) 40–50 km
  - (c) 60–70 km
  - (d) 80–100 km
- **75.** which of the following regulations is considered best?
  - (*a*) 50%
  - (*b*) 20%
- (c) 10%
- (*d*) 2%
- 76. Skin effect is proportional to
  - (a) (conductor diameter)<sup>4</sup>
  - (b) (conductor diameter)<sup>3</sup>
  - (c) (conductor diameter)<sup>2</sup>
  - (d) (conductor diameter) $^{1/2}$
  - (e) none of the above
- 77. A conductor, due to sag between two supports, takes the form of
  - (a) semi-circle
  - (b) triangle
  - (c) ellipse
  - (d) catenary
- 78. In A.C.S.R. conductors, the insulation between aluminium and steel conductors is
  - (a) insulin
  - (b) bitumen
  - (c) varnish
  - (d) no insulation is required
- 79. Which of the following bus-bar schemes has the lowest cost?
  - (a) Ring bus-bar scheme
  - (b) Single bus-bar scheme
  - (c) Breaker and a hlaf scheme
  - (d) Main and transfer scheme
- 80. Owing to skin effect
  - (a) current flows through the half crosssection of the conductor
  - (b) portion of the conductor near the surface carries more current and core of the conductor carries less current
  - (c) portion of the conductor near the surface carries less current and core of the conductor carries more current
  - (d) none of the above
- 81. By which of the following methods string efficiency can be improved?
  - (a) Using a guard ring
  - (b) Grading the insulator
  - (c) Using long cross arm
  - (d) Any of the above
  - (e) None of the above
- 82. In aluminium conductors, steel core is provided to
  - (a) compensate for skin effect
  - (b) neutralise proximity effect
  - (c) reduce line inductance
  - (d) increase the tensile strength
- 83. By which of the following a bus-bar is rated?
  - (a) Current only
  - (b) Current and voltage
  - (c) Current, voltage and frequency
  - (d) Current, voltage, frequency and short time current

- 84. A circuit is disconnected by isolators when
  - (a) line is energize
  - (b) there is no current in the line
  - (c) line is on full load
  - (d) circuit breaker is not open
- 85. For which of the following equipment current rating is not necessary? (a) Circuit breakers
  - (b) Isolators

  - (c) Load break switch
  - (d) Circuit breakers and load break switches
- **86.** In a substation the following equipment is not installed
  - (a) exciters
  - (b) series capacitors
  - (c) shunt reactors
  - (d) voltage transformers
- 87. Corona usuall occurs when the electrostatic stress in air around the conductor exceeds
  - (a) 6.6 kV (r.m.s. value)/cm
  - (b) 11 kV (r.m.s. value)/cm
  - (c) 22 kV (maximum value)/cm
  - (d) 30 kV (maximum value)/cm
- 88. The voltage drop, for constant voltage transmission is compensated by installing (a) inductors
  - (b) capacitors
  - (c) synchronous motors
  - (d) all of above
  - (e) none of the above
- **89.** The use of strain type insulators is made where the conductors are
  - (a) dead ended
  - (b) at intermediate anchor towers
  - (c) any of the above
  - (d) none of the above
- 90. The current drawn by the line due to corona losses is
  - (a) non-sinusoidal
  - (b) sinusoidal
  - (c) triangular
  - (d) square
- 91. Pin type insulators are generally not used for voltages beyond
  - (a) 1 kV
  - (b) 11 kV
  - (c) 22 kV
  - (d) 33 kV
- 92. Aluminium has a specific gravity of
  - (*a*) 1.5
  - (*b*) 2.7
  - (c) 4.2
  - (*d*) 7.8
- **93.** For transmission of power over a distance of 200 km, the transmission voltage should be

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- (a) 132 kV
- (b) 66 kV
- (c) 33 kV
- (*d*) 11 kV
- **94.** For aluminium, as compared to copper, all the following factors have higher values except
  - (a) specific volume
  - (b) electrical conductivity
  - (c) co-efficient of linear expansion
  - (*d*) resistance per unit length for same cross-section
- **95.** Which of the following equipment, for regulating the voltage in distribution feeder, will be most economical?
  - (a) Static condenser
  - (b) Synchronous condenser
  - (c) The changing transformer
  - (d) Booster transformer
- **96.** In a tap changing transformer, the tappings are provided on
  - (a) primary winding
  - (b) secondary winding
  - (c) high voltage winding
  - (*d*) any of the above
- **97.** Constant voltage transmission entails the following disadvantage
  - (*a*) large conductor area is required for same power transmission
  - (b) short-circuit current of the system is increased
  - (c) either of the above
  - (d) none of the above
- **98.** On which of the following factors skin effect depends?
  - (a) Frequency of the current
  - (b) Size of the conductor
  - (c) Resistivity of the conductor material
  - (d) All of the above
- 99. The effect of corona can be detected by
  - (a) presence of zone detected by odour
  - (b) hissing sound
  - (c) faint luminous glow of bluish colour
  - (d) all of the above
- **100.** for transmission of power over a distance of 500 km, the transmission voltage should be in the range
  - (a) 150 to 220 kV
  - (b) 100 to 120 kV
  - (c) 60 to 100 kV
  - (d) 20 to 50 kV
- **101.** In the analysis of which of the following lines shunt capacitance is neglected?
  - (*a*) Short transmission lines
  - (b) Medium transmission lines
  - (c) Long transmission lines

- (d) Modium as well as long transmission lines
- **102.** When the interconnector between two stations has large reactance
  - (*a*) the transfer of power will take place with voltage fluctuation and noise
  - (b) the transfer of power will take place with least loss
  - (c) the stations will fall out of step because of large angular displacement between the stations
  - (*d*) none of the above
- **103.** The frequency of voltage generated, in case of generators, can be increased by
  - (a) using reactors
  - (b) increasing the load
  - (c) adjusting the governor
  - (d) reducing the terminal voltage
  - (e) none of the above
- **104.** When an alternator connected to the bus-bar is shut down the bus-bar voltage will
  - (a) fall
  - (b) rise
  - (c) remain uncharged
  - (d) none of the above
- **105.** The angular displacement between two interconnected stations is mainly due to
  - (a) armature reactance of both alternators
  - (b) reactance of the interconnector
  - (c) synchronous reactance of both the alternators
  - (*d*) all of the above
- **106.** Electro-mechanical voltage regulators are generally used in
  - (a) reactors
  - (b) generators
  - (c) transformers
  - (d) all of the above
- 107. Series capacitors on transmission lines are of little use when the load VAR requirement is (*a*) large
  - (b) small
  - (*c*) fluctuating
  - (*d*) any of the above
  - (*a*) any of the above
- **108.** The voltage regulation in magnetic amplifier type voltage regulator is effected by
  - (a) electromagnetic induction
  - (*b*) varying the resistance
  - (c) varying the reactance
  - (*d*) variable transformer
- 109. when a conductor carries more current on the surface as compared to core, it is due to (a) permeability variation
  - (*b*) corona

  - (c) skin effect
  - (d) unsymmetrical fault

- (e) none of the above
- 110. The following system is not generally used
  - (a) 1-phase 3 wire
  - (b) 1-phase 4 wire
  - (c) 3-phase 3 wire
  - (d) 3-phase 4 wire
- **111.** The skin effect of a conductor will reduce as the
  - (a) resistivity of conductor material increases
  - (b) permeability of conductor material increases
  - (*c*) diameter increases
  - (d) frequency increases
- **112.** When a live conductor of public electric wupply breaks down and touches the earth which of the following will happen?
  - (a) Current will flow to earth
  - (b) Supply voltage will drop
  - (c) Supply voltage will increase
  - (d) No current will flow in the conductor
- **113.** 310 km line is considered as
  - (a) a long line
  - (b) a medium line
  - (c) a short line
  - (d) any of the above
- 114. The conductors are bundled primarily to
  - (a) increase reactance
  - (b) reduce reactance
  - (c) reduce ratio interference
  - (d) none of the above
- **115.** The surge impedance in a transmission line having negligible resistance is given as
  - (a)  $\sqrt{\text{LC}}$
  - (b)  $\sqrt{L/C}$
  - (c)  $\sqrt{1/LC}$
  - (d)  $\sqrt{L+C}$
  - (e) none of the above
- **116.** The top most conductor in a high transmission line is
  - (a) earth conductor
  - (b) R-phase conductor
  - (c) Y-phase conductor
  - (d) B-phase conductor
- **117.** In A.C.S.R. conductor the function of steel is to
  - (a) provide additional mechanical strength
  - (b) prevent corona
  - (c) take care of surges
  - (*d*) reduce inductance and subsequently improve power factor
- 118. In transmission and distribution system the

permissible voltage variation is

- (a)  $\pm 1$  percent
- (b)  $\pm 10$  percent
- (c)  $\pm 20$  percent
- (d)  $\pm$  30 percent
- (e) none of the above
- **119.** By which of the following methods voltage of transmission can be regulated?
  - (*a*) use of series capacitors to neutralies the effect of series reactance
  - (b) Switching in shunt capacitors at the receiving end during have loads
  - (c) Use of tap changing transformers
  - (d) Any of the above methods
- **120.** Which of the following distribution systems is the most economical?
  - (a) A.C. 1-phase system
  - (b) A.C. 3-phase 3 wire system
  - (c) A.C. 3-phase 4 wire system
  - (d) Direct current system
- **121.** Which of the following is the main advantage of A.C. transmission system over D.C. transmission system?
  - (a) Less instability problem
  - (b) Less insulation problems
  - (c) Easy transformation
  - (d) Less losses in transmission over long distances
- 122. A tap changing transformer is used to
  - (a) supply low voltage current for instruments
  - (*b*) step up the voltage
  - (c) step down the voltage
- (*d*) step up as well as step down the voltage **123.** Which of the following bar schemes is the
  - most expensive?
    - (a) Double bus-bar double breaker
    - (b) Ringbus-bar scheme
    - (c) Single bus-bar scheme
    - (d) Main and transfer scheme
- **124.** By which of the following methods the protection agains direct lightning strokes and high voltage sweep waves is provided?
  - (*a*) Lightening arresters
  - (b) Ground wire
  - (c) Lightening arresters and ground wires
  - (d) Earthing of neutral
  - (d) None of the above
- **125.** In which of the following voltage regulators the effect of dead zero is found?
  - (*a*) Electromagnetic type
  - (b) Magnetic amplifier
  - (c) Electronic type using integrated circuits
  - (e) all of the above
- **126.** Corona results in

- (a) radio interference
- (b) power factor improvement
- (c) better regulation
- (d) none of the above
- **127.** Which of the following has least effect on corona?
  - (a) Atmospheric temperature
  - (b) Number of ions
  - (c) Size and charge per ion
  - (d) Mean free path
- **128.** In context of corona, if the conductors are polished and smooth, which of the following statements is correct?
  - (a) Hissing sound will be more intense
  - (b) Power loss will be least
  - (c) Corona glow will be uniform along the length of the conductor
  - (d) Corona glow will not occur
- **129.** Power loss due to corona is not directly proportional to
  - (a) spacing between conductors
  - (b) supply voltage frequency
  - (c) phase-neutral voltage
  - (*d*) all of the above
- **130.** Poles which carry transformers are usually (*a*) circular
  - (a) circuit
  - (b) I-type
  - (*c*) A-type(*d*) H-type
  - (*e*) none of the above
- **131.** Out of the following which type of poles are bulky?
  - (a) Transmission towers
  - (b) Concrete poles
  - (c) Tubular steel poles
  - (d) Wooden poles
- **132.** The effect of ice on transmission line conductors is to increase the
  - (a) transmission losses
  - (b) weight of the conductor
  - (c) tendency for corona
  - (d) resistance to flow of current
- **133.** If the height of transmission tower is increased (*a*) the line capacitance will decrease but line
  - inductance will remain uncharged
  - (b) the line capacitance and inductance will not change
  - (c) the line capacitance will increase but line inductance will decrease
  - (*d*) the line capacitance will decrease and line inductance will increase
- **134.** If staring efficiency is 100 percent it means that
  - (a) potential across each disc is zero
  - (b) potential across each disc is same

- (c) one of the insulator discs is shorted
- (*d*) none of the above **135.** In a 70/6 A.C.S.R. conductor there are
  - (a) 35 aluminium conductors and 3 steel conductors
    - (b) 70 aluminium conductors and 6 steel conductors
  - (c) 70 steel conductors and 6 aluminium conductors
  - (d) none of the above
- **136.** On which of the following does the size of a feeder depend?
  - (a) Voltage drop
  - (b) Voltage
  - (c) Frequency
  - (d) Current carrying capacity
- **137.** Which of the following are connected by the service mains?
  - (a) Transformer and earth
  - (b) Distributor and relay system
  - (c) Distributor and consumer terminals
  - (d) Distributor and transformer
- **138.** In the design of a distributor which of the following is the major consideration?
  - (a) Voltage drop
  - (b) Current carrying capacity
  - (c) Frequency
  - (d) kVA of system
  - (e) None of the above
- 139. In a distribution system major cost is that of
  - (*a*) earthing system
  - (b) distribution transformer
  - (c) conductors
- (d) meters
- **140.** A booster is connected in
  - (a) parallel with earth connection
  - (b) parallel with the feeder
  - (c) series with the feeder
  - (d) series with earth connection
- **141.** With which of the following are step-up substations associated?
  - (a) Concentrated load
  - (b) Consumer location
  - (c) Distributors
  - (d) Generating stations
  - (e) None of the above
- **142.** Which of the following equipment should be installed by the consumers having low power factor?
  - (a) Synchronous condensers
  - (b) Capacitor bank
  - (c) Tap changing transformer
  - (*d*) Any of the above
  - (e) None of the above
- 143. Which of the following equipment is used to

limit short-circuit current level in a substation?

- (a) Isolator
- (b) Lightning switch
- (c) Coupling capacitor
- (d) Series reactor
- **144.** Steepness of the travelling waves is alternated by ...... of the line
  - (*a*) capacitance
  - (*a*) capacitance (*b*) inductance
  - (b) inductance
  - (c) resistance
  - (d) all of the above
- **145.** The limit of distance of transmission line may be increased by the use of
  - (a) series resistances
  - (b) shunt capacitors and series reactors
  - (c) series capacitors and shunt reactors
  - (d) synchronous condensers
  - (e) none of the above
- **146.** By which of the following feactors is the sag of a transmission line least affected?
  - (*a*) Current through the conductor
  - (b) Ice deposited on the conductor
  - (c) Self weight of conductor
  - (d) Temperature of surrounding air
  - (e) None of the above
- **147.** Which of the following cause transient disturbances?
  - (a) Faults
  - (b) Load variations
  - (c) Switching operations
  - (d) Any of the above
- 148. A gay wire
  - (a) protects conductors against shortcircuiting
  - (b) provides emergency earth route
  - (c) provides protection against surges
  - (d) supports the pole
- **149.** Which of the following is neglected in the analysis of short transmission lines?
  - (a) Series impedance
  - (b) Shunt admittance
  - (c)  $I^2 R$  loss
  - (d) None of the above
  - (e) All of the above
- **150.** Basically the boosters are
  - (a) synchronous motors
  - (b) capacitors
  - (c) inductors
  - (d) transformers
- **151.** Which of the following is a static exciter? (*a*) Rectifier
  - (b) Rotorol
  - (c) Amplidyne
  - (d) D.C. separately excited generator
- 152. For exact compensation of voltage drop in the

- feeder the booster
- (a) must be earthed
- (b) must work on line voltage
- (c) must work on linear portion of its V-I characteristics
- (*d*) must work on non-linear portion of its V-I characteristics
- 153. The purpose of using a booster is to
  - (a) increase current
  - (b) reduce current
  - (c) reduce voltage drop
  - (d) compensate for voltage drop
  - (e) none of the above
- **154.** Induction regulators are used for voltage control in
  - (a) alternators
  - (b) primary distribution
  - (c) secondary distribution
  - (d) none of the above
- **155.** A synchronous condenser is generally installed
  - at the ..... of the transmission line
  - (a) receiving end
  - (b) sending end
  - (c) middle
  - (d) none of the above
- **156.** The area of cross-section of the neutral in a 3-wire D.C. system is generally ...... the area of cross-section of main conductor
  - (a) same as
  - (b) one-fourth
  - (c) one half
  - (d) double
- **157.** For which of the following, the excitation control method is satisfactory?
  - (a) Low voltage lines
  - (*b*) High voltage lines
  - (c) Short lines
  - (d) Long lines
- **158.** In which of the following cases shunt capacitance is negligible?
  - (a) Short transmission lines
  - (b) Medium transmission lines
  - (c) Long transmission lines
  - (d) All transmission lines
- **159.** A lightning arrester is usually located nearer to
  - (a) transformer
  - (b) isolator
  - (c) busbar
  - (d) circuit breaker
  - (e) none of the above
- **160.** The material used for the manufacture of grounding wires is
  - (a) cast iron
  - (b) aluminium

- (c) stainlesssteel
- (d) galvanised steel
- 161. Surge absorbers protect against ......
  - (a) high voltage high frequency
  - (b) high voltage low frequency
  - (c) low voltage high frequency
  - (d) low voltage low frequency
- **162.** Skin effect is noticeable only at ...... frequencies
  - (a) audio
  - (*b*) low
  - (c) high
  - (d) all
- 163. Per system stability is least affected by
  - (a) reactance of generator
  - (b) input torque
  - (c) losses
  - (d) reactance of transmission line
- **164.** When the load at the receiving end of a long transmission line is removed, the sending end voltage is less than the receiving end voltage. This effect is known as
  - (a) Ferranti effect
  - (b) Proximity effect
  - (c) Kelvin effect
  - (d) Faraday effect
  - (e) Skin effect
- **165.** In medium transmission lines the shunt capacitance is taken into account in
  - (a) T-method
  - (b)  $\pi$ -method
  - (c) steinmetz method
  - (d) all of the above
- **166.** System grounding is done so that
  - (*a*) inductive interference between power and communication circuits can be controlled
  - (*b*) the floating potential on the lower voltage winding for a transformer is brought down to an insignificant value
  - (c) the arcing faults to earth would not set up dangerously high voltage on healthy phases(d) for all above reasons
  - (a) for all above reasons
- **167.** Which of the following can be used for bus-bars?
  - (a) Tubes
  - (b) Rods
  - (c) Bars
  - (d) Any of the above
- **168.** If the height of transmission tower is increased, which of the following parameters is likely to change?
  - (a) Capacitance
  - (b) Inductance
  - (c) Resistance

- (d) All of the above
- (e) None of the above
- **169.** A.C.S.R. conductor having 7 steel stands surrounded by 25 aluminium conductors will be specified as
  - (*a*) 25/7
  - (*b*) 50/15
  - (c) 7/25
  - (*d*) 15/50
- **170.** Impedance relay is used on ...... transmission lines
  - (a) short
  - (b) medium
  - (c) long
  - (d) all
- 171. Corona is likely to occur maximum in
  - (a) transmission lines
  - (b) distribution lines
  - (c) domestic wiring
  - (d) all of the above
- **172.** The effect of wind pressure is more predominant on
  - (a) supporting towers
  - (b) neutral wires
  - (c) transmission lines
  - (d) insulators
- **173.** As compared to cables, the disadvantages of transmission lines is
  - (*a*) inductive interference between power and communication circuits
  - (b) exposure to lightning
  - (c) exposure to atmospheric hazards like smoke, ice, etc.
- (d) all of the above
- **174.** In overhead transmission lines the effect of capacitance cand be neglected when the length of line is less than
  - (a) 80 km
  - (b) 110km
  - (c) 150 km
  - (*d*) 210 km
- **175.** The effective resistance of a conductor will be the same as 'ohmic resistance' when
  - (a) power factor is unity
  - (b) current is uniformly distributed in the conductor cross-section
  - (c) voltage is low
  - (d) current is in true sine wave from
- **176.** Conductors for high voltage transmission lines are suspended from towers to
  - (a) increase clearance from ground
  - (b) reduce clearance from ground
  - (c) take care of extension in length during summer
  - (d) reduce wind and snow loads

- (e) none of the above
- **177.** To increase the capacity of a transmission line for transmitting power which of the following must be decreased?
  - (a) Capacitance
  - (b) Line inductance
  - (c) Voltage
  - (d) All of the above
- **178.** By using bundled conductors which of the following is reduced?
  - (a) Power loss due to corona
  - (b) Capacitance of the circuit
  - (c) Inductance of the circuit
  - (*d*) None of the above
  - (e) All of the above
- **179.** Which of the following short-circuits is most dangerous?
  - (a) Dead short-circuit
  - (b) Line to ground short-circuit
  - (c) Line to line short-circuit
  - (d) Line to line and ground short-circuit
  - (e) all of the above
- **180.** Due to which of the following reasons aluminium is being favoured as busbar material?
  - (a) Low density
  - (b) Low cost
  - (c) Ease of fabrication
  - (*d*) None of the above
- **181.** In case of transmission line conductors with the increase in atmospheric temperature
  - (a) length decreases but stress increases
  - (b) length increases but stress decreases
  - (c) both the length and stress increases
  - (d) both the length and stress decrease
- 182. Skin effect exists only in
  - (a) a.c. transmission
  - (b) high voltage d.c. overhead transmission
  - (c) low voltage d.c. overhead transmission
  - (d) cables carrying d.c. current
- **183.** Floating neutral, in 3-phase supply, is undesirable because it causes
  - (a) low voltage across the load
  - (b) high voltage across the load
  - (c) unequal line voltages across the load
  - (d) none of the above
- 184. The surge resistance of cables is
  - (a) 20 ohms
  - (*b*) 50 ohms
  - (c) 200 ohms
  - (d) 300 ohms
- **185.** The electrostatic stress in underground cables is
  - (a) zero at the conductor as well as on the

#### sheath

- (b) same at the conductor and sheath
- (c) minimum at the conductor and minimum at the sheath
- (*d*) maximum at the conductor and minimum at the sheath
- **186.** The ground ring transmission lines are used to *(a)* reduce the transmission losses
  - (b) reduce the earth capacitance of the lowest unit
  - (c) increase the earth capacitance of the lowest unit
  - (d) none of the above
- **187.** The string efficiency of an insulator can be increased by
  - (a) correct grading of insulators of various capacitances
  - (b) reducing the number of strings
  - (c) increasing the number of strings in the insulator
  - (d) none of the above
- 188. High voltages for transmitting power is economically available from(a) d.c. currents
  - (b) a.c. currents
  - (c) carrier currents
  - (*d*) none of the above
- 189. High voltage is primarily used, for long
  - distance power transmission, to
  - (a) reduce the time of transmission
  - (b) reduce the transmission losses
  - (c) make the system reliable
  - (d) none of the above
- **190.** By using bundle conductors, the critical voltage for the formation of corona will
  - (a) remain same
  - (b) decrease
  - (c) increase
  - (d) not occur
- **191.** If the voltage is increased x times, the size of the conductor would be
  - (a) reduced to  $1/x^2$  times
  - (b) reduced to 1/x times
  - (c) increased x times
  - (d) increased to  $x^2$  times
  - (e) none of the above
- **192.** The colour of the neutral of three-core flexible cable is
  - (a) blue
  - (b) brown
  - (c) red
  - (d) black
- **193.** In the cablessheaths are used to
  - (*a*) prevent the moisture from entering the cable

- (*b*) provide strength to the cable
- (c) provide proper insulation
- (d) none of the above
- **194.** The charging current in the cables
  - (a) leads the voltage by  $180^{\circ}$
  - (b) leads the voltageby  $90^{\circ}$
  - (c) lags the voltage by  $90^{\circ}$
  - (d) lags the voltage by  $180^{\circ}$
- 195. Ground wire is used to
  - (a) avoid overloading
  - (b) give the support to the tower
  - (c) give good regulation
  - (*d*) connect a circuit conductor or other device to an earth-plate
- **196.** Earthing is necessary to give protection against
  - (a) danger of electric shock
  - (b) voltage fluctuation
  - (c) overloading
  - (d) high temperature of the conductors
- **197.** Resistance grounding isused for voltage between
  - (a) 3.3kV to 11 kV
  - (b) 11 kV to 33 kV

- (c) 33 kV to 66 kV
- (d) none of the above
- **198.** Solid grounding is adopted for voltages below (*a*) 100 V
  - (*a*) 100 V(*b*) 200 V
  - (c) 400 V
  - (L) 400 V
  - (*d*) 660 V
- **199.** The size of the earth wire is determined by (*a*) the atmospheric conditions
  - (b) the voltage of the service wires
  - (c) the ampere capacity of the service wires
  - (d) none of the above
- 200. Transmission lines link
  - (a) generating station to receiving and station
    - (b) receiving and station to distribution transformer
    - (c) distribution transformer to consumer premises
    - (d) service points to consumer premises
  - (e) none of the above

<b>1.</b> (b) <b>2.</b> (d) <b>3.</b> (a) <b>4.</b> (c) <b>5.</b> (b) <b>6.</b> (d) <b>7.</b> (a) <b>8.</b> (d) <b>9.</b> (d)	<b>10.</b> ( <i>d</i> )
<b>11.</b> (c) <b>12.</b> (b) <b>13.</b> (d) <b>14.</b> (d) <b>15.</b> (a) <b>16.</b> (c) <b>17.</b> (d) <b>18.</b> (d) <b>19.</b> (c)	<b>20.</b> ( <i>d</i> )
<b>21.</b> (d) <b>22.</b> (b) <b>23.</b> (b) <b>24.</b> (a) <b>25.</b> (a) <b>26.</b> (c) <b>27.</b> (d) <b>28.</b> (d) <b>29.</b> (d)	<b>30.</b> ( <i>d</i> )
<b>31.</b> (d) <b>32.</b> (e) <b>33.</b> (a) <b>34.</b> (a) <b>35.</b> (e) <b>36.</b> (d) <b>37.</b> (d) <b>38.</b> (c) <b>39.</b> (b)	<b>40.</b> ( <i>c</i> )
<b>41.</b> (b) <b>42.</b> (a) <b>43.</b> (d) <b>44.</b> (b) <b>45.</b> (b) <b>46.</b> (b) <b>47.</b> (b) <b>48.</b> (c) <b>49.</b> (d)	<b>50.</b> ( <i>b</i> )
<b>51.</b> (b) <b>52.</b> (a) <b>53.</b> (c) <b>54.</b> (c) <b>55.</b> (c) <b>56.</b> (d) <b>57.</b> (c) <b>58.</b> (b) <b>59.</b> (a)	<b>60.</b> ( <i>a</i> )
<b>61.</b> (b) <b>62.</b> (c) <b>63.</b> (b) <b>64.</b> (a) <b>65.</b> (c) <b>66.</b> (d) <b>67.</b> (d) <b>68.</b> (d) <b>69.</b> (d)	<b>70.</b> ( <i>b</i> )
<b>71.</b> (c) <b>72.</b> (c) <b>73.</b> (d) <b>74.</b> (a) <b>75.</b> (b) <b>76.</b> (c) <b>77.</b> (d) <b>78.</b> (d) <b>79.</b> (b)	<b>80.</b> ( <i>b</i> )
<b>81.</b> (d) <b>82.</b> (d) <b>83.</b> (d) <b>84.</b> (b) <b>85.</b> (b) <b>86.</b> (a) <b>87.</b> (d) <b>88.</b> (c) <b>89.</b> (c)	<b>90.</b> ( <i>a</i> )
<b>91.</b> (d) <b>92.</b> (b) <b>93.</b> (a) <b>94.</b> (b) <b>95.</b> (d) <b>96.</b> (b) <b>97.</b> (b) <b>98.</b> (d) <b>99.</b> (d)	<b>100.</b> ( <i>a</i> )
<b>101.</b> (a) <b>102.</b> (c) <b>103.</b> (c) <b>104.</b> (c) <b>105.</b> (b) <b>106.</b> (b) <b>107.</b> (b) <b>108.</b> (c) <b>109.</b> (c)	<b>110.</b> ( <i>b</i> )
111. (a) 112. (a) 113. (a) 114. (b) 115. (b) 116. (a) 117. (a) 118. (b) 119. (d)	<b>120.</b> ( <i>d</i> )
121. (d) 122. (d) 123. (a) 124. (c) 125. (a) 126. (a) 127. (a) 128. (c) 129. (a)	<b>130.</b> ( <i>d</i> )
131. (b) 132. (b) 133. (a) 134. (b) 135. (b) 136. (d) 137. (c) 138. (a) 139. (b)	<b>140.</b> ( <i>c</i> )
141. (d) 142. (b) 143. (d) 144. (c) 145. (c) 146. (a) 147. (d) 148. (d) 149. (b)	<b>150.</b> ( <i>d</i> )
151. (a) 152. (c) 153. (d) 154. (b) 155. (a) 156. (c) 157. (c) 158. (a) 159. (a)	<b>160.</b> ( <i>d</i> )
<b>161.</b> (c) <b>162.</b> (c) <b>163.</b> (c) <b>164.</b> (a) <b>165.</b> (d) <b>166.</b> (d) <b>167.</b> (d) <b>168.</b> (a) <b>169.</b> (a)	<b>170.</b> ( <i>b</i> )
171. (a) 172. (a) 173. (d) 174. (a) 175. (b) 176. (a) 177. (b) 178. (a) 179. (a)	<b>180.</b> ( <i>b</i> )
<b>181.</b> (b) <b>182.</b> (a) <b>183.</b> (c) <b>184.</b> (b) <b>185.</b> (d) <b>186.</b> (b) <b>187.</b> (a) <b>188.</b> (b) <b>189.</b> (b)	<b>190.</b> ( <i>c</i> )
<b>191.</b> (a) <b>192.</b> (a) <b>193.</b> (a) <b>194.</b> (b) <b>195.</b> (d) <b>196.</b> (a) <b>197.</b> (a) <b>198.</b> (d) <b>199.</b> (c)	<b>200.</b> ( <i>a</i> )