THE GOVERNMENT OF THE REPUBLIC OF THE UNION OF MYANMAR

MINISTRY OF EDUCATION

PHYSICS

GRADE 9

BASIC EDUCATION CURRICULUM, SYLLABUS AND

TEXTBOOK COMMITTEE

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အခြေခံပညာ သင်ရိုးညွှန်းတမ်း၊ သင်ရိုးမာတိကာနှင့် ကျောင်းသုံးစာအုပ်ကော်မတီ၏ မူပိုင်ဖြစ်သည်။

FOREWORD

This text book is prescribed for the tenth grade students. It is the first volume of the whole course for a student studying physics in the upper secondary level of basic education (i.e. the tenth grade and the eleventh grade).

The division and order of subject content in separate fields presented in the whole course of upper secondary level physics generally follow the sequence mentioned below:

(I) Mechanics

(2) Heat

(3) Waves and Sound

(4) Optics

(5) Electricity and Magnetism

(6) Modern Physics

The present text book covers introductory topics in the first five fields apart from Modern Physics which is dealt with in the eleventh grade text .

Physics is generally defined as the study of matter and motion. In fact, neither this nor any other one-sentence statement adequately covers the whole definition of physics. It is a unified structure of the following features:

(a) creativity,

(b) accumulation of knowledge,

(c) unification of concepts,

(d) mathematical equations and formulation,

(e) philosophical reasoning,

(f) practical applications.

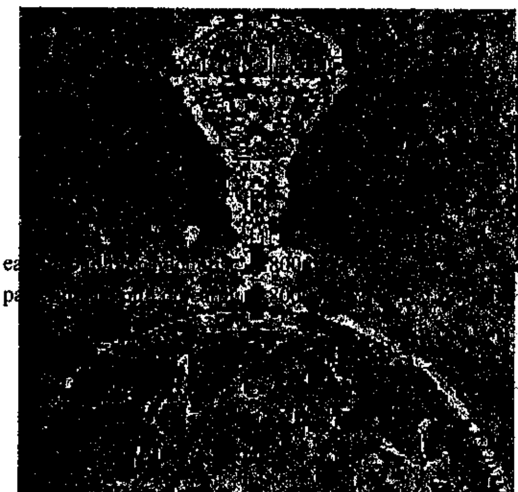
Both text books are designed to give students not only an understanding of the important facts, laws and basic concepts of physics, but the practical application of theoretical knowledge to solving problems also.

CONTENTS

MECHANICS

Sir Issac Newton MA DLit FRS (1642-1727)

MECHANICS



The gravity force acts on both objects equally

A see-saw uses the Principle of Moments for balancing

CHAPTER 1

PHYSICS AND MEASUREMENT

Physics, like other sciences, is rooted in observations and experiments, and for them to be meaningful, the data accumulated and the results must be based upon quantitative measurements.

Measurement essentially is a comparison process. Quantitative measurements must be expressed by numerical comparison to certain agreed upon set of standards. A standard quantity of some kind, referred to as a unit, is first established.

In expressing quantitative measurements, the following rules are generally followed:

(a) When measuring physical quantities such as length, mass and temperature we have to compare them with quantities of the same kind that are called units of measure.

(b) When we want to express exactly the magnitude of a physical quantity (Q) we use a dimensionless number (N) that represents the multiple of the unit (u) that represents the dimension.

Expressed in symbols

$$Q = Nu$$

For example, in expressing a mass Q = 15 kg, N=15 and u = kg (kilogram mass) and similarly for a velocity of $Q=100ms^{-1}$,N= 100 and $u = m\ s^{-1}$ (metre per second).

1.1 BASIC AND DERIVED UNITS

(a) Origin of Units

Units of measurement such as metre, kilogram, degree Celsius and so on are by no means fixed by nature. These units have been selected and prescribed by scientists at international conventions.

(b) Classification

As physical quantities are of the basic type (length, mass, time, temperature, etc.) and of the derived type (volume, velocity, work, etc.), their units are also called basic units and **derived units.(** a unit of measurement formed by combining the base units of a system)

(c) System of Units

In the present physics course for Basic Education we shall be using the following three systems of units.

- the British system
- the metric system
- the SI units

The British system is based on foot (ft), pound (lb) and second (s) and is therefore also called the FPS system. The metric system consists of (i) the CGS system which is based on centimetre (cm), gram (g) and second (s) and (ii) the MKS system which is based on metre (m), kilogram (kg)and second (s).The two systems are alike in the sense that units of length and mass of one system may be converted to those of the other by using powers of 10 (e.g. 1 m $=10^2$ cm; 1 kg = 10^3g). The units used in electricity and magnetism for the two systems are, however, quite different.

The SI units is just the modified form of the MKS system of units.

The British system of units is being used less and less, and is now almost obsolete. Only a few engineers use this system nowadays. The system is no longer used in physics research work. Even though the system is still being used, most physics textbooks nowadays are entirely in SI units.

Since the SI units is being used more and more in physics and in engineering subjects, we shall mainly employ this system of units in this book.

The SI units came into existence after slightly modifying the metric system of units. This was done at **the eleventh General Conference of Weights and Measures** held in 1960. "SI" is the abbreviation of the French words "Systeme International" which means international system. It was also agreed upon that the abbreviation of the system, namely SI, shall be used in all languages. The delegates to the above-mentioned conference also agreed to the use of SI units in science, technology, practical works and in teaching.

1.2 STANDARDS AND UNITS

(a) The Unit of Length

The concept of distance or the concept of change of position is indeed one of the earliest concepts studied by man. To define length we have to use the measurement or size of a standard object. For everyday use the standard may be a yard stick, ruler, metre stick and so on. At first, the metre was defined as the distance equal to one ten millionth ($1/10^7$) of the distance between the pole and the equator. According to this definition the circumference of the earth can be easily remembered. Its value is 4 x 10^7 m or 40 000 km. Later, a specified metal rod was used as a standard of length for quite a number of years. This rod was kept in a temperature controlled room near Paris, the capital of France. The length of the rod was defined as a metre. Nowadays the standard of length used is based on the wavelength of orange-red light emitted by a krypton 86 isotope. A metre is now defined as the length equivalent to 1 650 763.73 times the wavelength of this orange-red light (Fig. 1.1). (This is neither a round number nor can it be easily remembered. This particular number is chosen so that the metre defined in this manner is nearly equivalent to the metre defined earlier.)

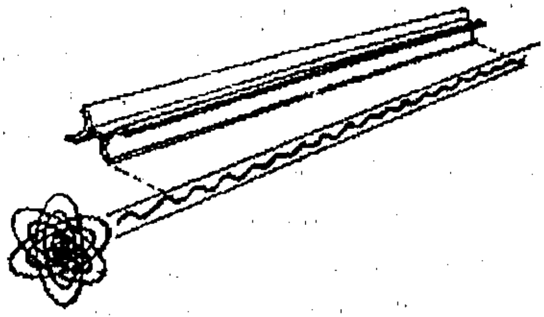


Fig. 1.1

The unit of length is the metre (m) in the MKS and SI systems. In the CGS system the unit of length is the centimetre (cm) and

$$1 \text{ cm} = \frac{1}{100} \text{m} = 10^{-2} \text{m}$$

In the FPS system the unit of length is the foot (ft) and

$$1 \text{ ft} = 0.3048 \text{ m} = 3.048 \times 10^{-1} \text{m}$$

The size of a physical system varies enormously. Elementary particles have the smallest size and the universe has the largest. The ratio of the largest size to the smallest size works out to be 10^{41} (forty-one zeroes after the number one). This means the size of the universe is 10^{41} times larger than the size of an elementary particle. Hence it is necessary that we choose the appropriate unit of length according to the field of study.

The unit of length used by particle physicists is the " fermi " or "femtometre" (fm) given by

$$1 \text{fm} = 10^{-15} \text{ m}$$

In the field of optics physicists use the unit: angstrom (Å), where

$$1 \text{Å} = 10^{-10} \text{ m}$$

Again, in astronomy, the most suitable units are the astronomical unit (AU) and the light-year unit which may be expressed respectively as follows

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

$$1 \text{ light-year} = 9.461 \times 10^{15} \text{ m}$$

The largest unit of length is the "parsec "where

$$1 \text{ parsec} = 3.084 \times 10^{16} \text{ m}$$

(b) The Unit of Time

The concept of time includes the idea of repetition. Human beings can feel and experience the change as well as the flow of time. The regular changes from day to night, then night to day together with the changes in the seasons are well known to man. Hunger and tiredness also seem to be periodic. A pendulum clock is just an instrument for counting a certain repetitive motion. The earth itself can be regarded as a clock. The earth rotating on its axis completes one revolution daily and it takes one year to complete its journey of one round about the sun. The rotation about its axis is called "spin" and the motion around the sun is called "rotation". At first a time of one second (1s) was defined as that time equal to $\frac{1}{86\,400}$ part of a day. Later it was found that it was more correct and reliable to calculate the rotation about the sun rather than the spin about the axis. Hence from the year 1900 onwards 1 s has been defined as that time which is equal to 1 /31 556 925.9747 part of the time required by the earth to go one round about the sun. Nowadays the standard time is measured by the **atomic clock** (see Fig 1.2a) (an extremely accurate timekeeping device regulated by the natural regular oscillations of an atom or molecule) which is based on the cesium atom. A particular frequency 9 192 631 770 s^{-1} (in SI units s^{-1} is replaced by **Hz**) emitted or absorbed by a cesium atom is used to define 1 s. In research works carried out in laboratories and in measuring very short times it is more appropriate and suitable to use the atomic standard instead of the earlier astronomical standards.

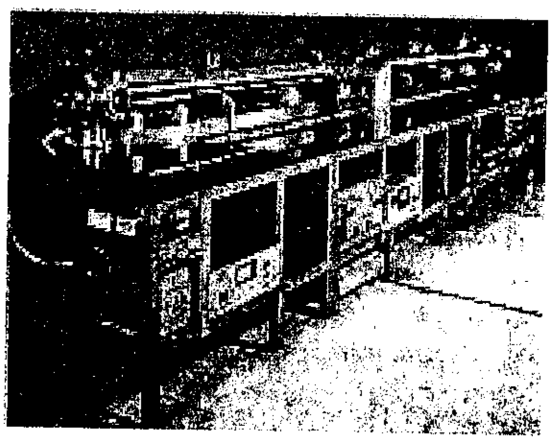


Fig 1.2a The atomic clock, (It is located at the National Bureau of Standards in Boulder, Colorado)

In measuring a time scale, it is quite difficult to specify what time interval is a short one or a long one. The time taken by light to traverse the diameter of an elementary particle is just 10^{-23} s. Compared to this one-millionth part of a second (10^{-6} s) is indeed a very long time interval. However, when we compare this with the average life-span of man we find that 10^{-6} s is in fact a very very short time.

Physicists carrying out research in various fields have to deal with very short time intervals such as the life-time of an elementary particle (10^{-23} s) and also very long time intervals such as the age of the universe (2×10^{18} s). The ratio of the two numbers, which gives the extent of the time scale, works out to about 10^{41}. That this figure happens to be the same as that for the extent of the length scale (discussed in Section 1.2) is by no means just a coincidence. Just as objects at the edge of the universe move with nearly the velocity of light, so do the elementary particles of the sub-atomic world. We can say therefore that it is actually the velocity of light which interconnects the distance and time measurements.

For measuring very short times the second can be sub-divided as shown.

Name of unit	*Symbol*	*Time interval*
picosecond	ps	10^{-12} s
nanosecond	ns	10^{-9} s
microsecond	μs	10^{-6} s
millisecond	ms	10^{-3} s

The unit of time is the second in all systems of measurement.

(c) The Unit of Mass

In defining the standards of length and time, objects of ordinary size were at first chosen as standard objects. Later these standard objects were changed to atom-sized particles: krypton atom for the standard of length and cesium atom for the standard of time. To define the standard of mass an object of ordinary size is still being used as the standard object to this day. This standard object is a cylinder of 1 kg mass made of platinum-iridium alloy. It serves as a standard of mass for international use and is kept at Sevres in France(Fig. 1.2b). Prototypes of the standard kilogram are distributed to research academies and laboratories situated in all parts of the world. (A prototype kilogram is an accurate copy of the original standard.)

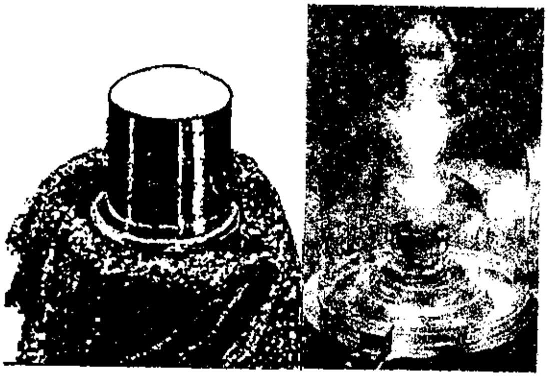


Fig. 1.2b

Before the modern standard kilogram was introduced, 1 kg mass was defined as the mass of one litre (10^3 cm^3 or 10^{-3} m^3) of water at 4 °C. It is interesting to note that the standards of length, time and mass used earlier were somehow connected to the earth. One-fourth the circumference of the earth is equivalent to ten million metres (10^7 m)

(Fig. 1.3). A day has (60 × 60 × 24 s) and the volume of water (whose mass was used to define the standard kilogram) can be expressed in terms of the standard metre, which is directly related to the circumference of the earth.

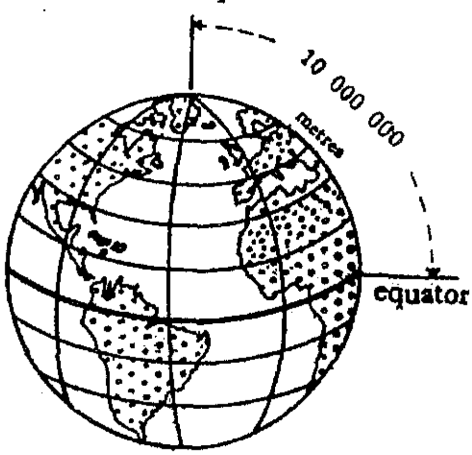


Fig. 1.3

The units of mass are

- kilogram (kg) for the SI unit,
- kilogram (kg) for the MKS system.
- gram (g) for the CGS system,
- pound (lb) and slug (sl) for the British system.

In one form of the British system the pound is used as the unit of mass. In another, the slug is used as the unit of mass and the pound is used as the unit of force. This could therefore lead to mistakes in the use of the British system. If we use the metric system instead of the British system such mistakes could not arise. In this textbook we shall use the slug for the unit of mass and the pound for the unit of force in the British system. The relation between kilogram, gram and slug is given by 1 kg = 10^3g = 6.852177×10^{-2} sl.

1.3 SYMBOLS FOR PHYSICAL QUANTITIES

It is said that "mathematics is the language of physics ". Physical laws and principles can be fully and effectively represented in mathematical forms. Since we have to express the relation between physical quantities in mathematical equations it is necessary that the symbols for the physical quantities be short and precise. The various symbols commonly used are: s for displacement, v for velocity, a for acceleration, p for momentum, F for force, E for energy, W for work, T for temperature, q for charge and so on. Depending on whether a physical quantity is a scalar or a vector, there is a way of writing the symbols such that the two may be differentiated. We shall see this in Chapter 2.

1.4 THE MEASUREMENT OF LENGTH

In length measurement, we must choose an instrument that is suitable for the length to be measured. Table 1.3 summarises the commonly used instruments and the lengths which they are suitable for measuring.

Table 1.3 Instruments used for measuring length

Length to be measured	Suitable instrument	Accuracy of instruments
Several metres(m)	Measuring tape	0.1 cm
Several centimetres (cm) to 1 m	Metre or half-metre rule	0.1 cm
between 1 cm and 10 cm	Vernier calipers	0.01 cm (usually)
less than 2 cm	Micrometer screw gauge	0.01 mm (or 0.001 cm)

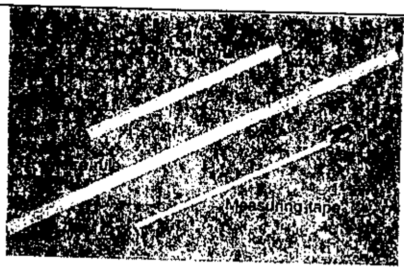


Fig 1.4 Meter rule, half-meter rule and measuring tape

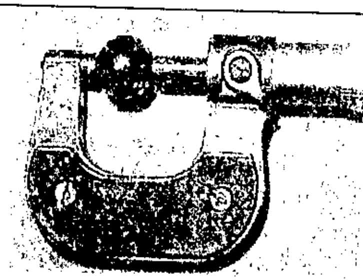


Fig 1.5 Micrometer screw gauge

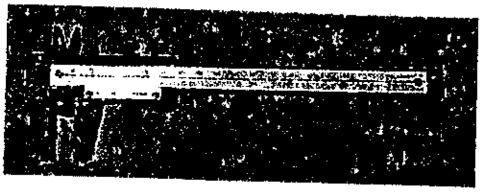


Fig 1.6 Vernier calipers

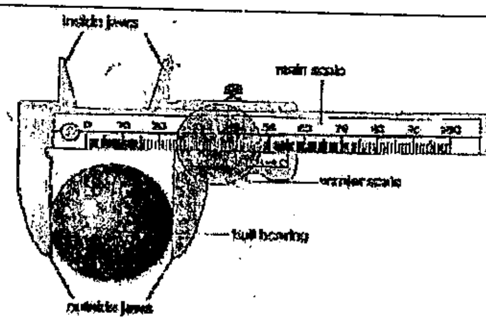


Fig 1.7 Using the vernier calipers

1. 5 THE MEASUREMENT OF TIME

Time

Time is measured in years, months, days, hours, minutes and seconds. The SI unit for time is second (s). Due to the wide range of time intervals that we want to measure, we need different kinds of clocks and watches. Table 1.4 shows useful clocks and watches that are currently in use.

Table 1.4 Some useful clocks and watches

Type of clock/watch	Use and accuracy
Atomic clock	Measures very short time intervals of about 10^{-10} seconds.
Digital stopwatch	Measures short time intervals (in minutes and seconds) to an accuracy to ± 0.01 s.
Analogue stopwatch	Measures short time intervals (in minutes and seconds) to an accuracy to ± 0.1 s.
Ticker-tape timer	Measures short time intervals of 0.02 seconds.
Watch	Measures longer time intervals in hours, minutes and seconds.
Pendulum clock	Measures longer time intervals in hours, minutes and seconds.
Radioactive decay clock	Measures (in years) the age of remains from thousands of years ago.

1. 6 THE MEASUREMENT OF MASS

Mass

The mass of an object is a measure of the amount of matter in it. It depends on the number of atoms it contains and the size of those atoms. It is a basic property of the body and cannot be changed by the location, shape and speed of the body (for speeds much less than the speed of light).

The SI unit for mass is the kilogram (kg). Large masses (e.g. mass of a car) are measured in tones (1 tonne = 1000 kg) while small masses (e.g. mass of a pencil) are measured in grams (1 gram (g) = 10^{-3} kg). Table 1.5 shows some masses in this universe.

Table 1.5 Some masses in this universe

Object	Mass in kilogram (kg)
electron	10^{-30}
a fine grain of sand	10^{-6} = 1 milligram
a pea	10^{-3} = 1 gram
an apple	10^{-1} = 0.1 kilogram
a medium-sized car	10^{3} = 1 tonne
Earth	10^{24}
Sun	10^{30}

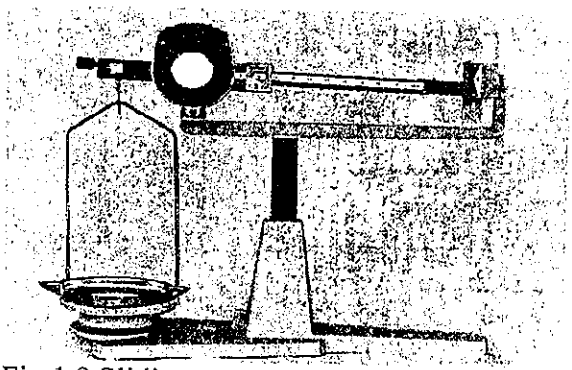


Fig 1.8 Sliding mass balance

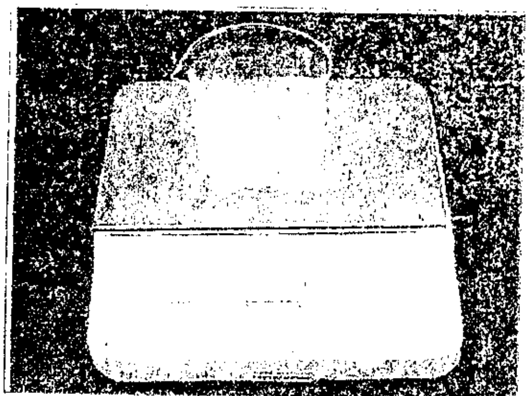


Fig 1.9 Electronic balance

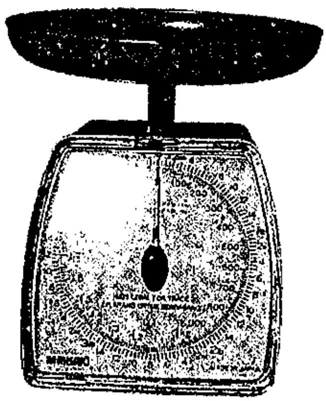
The Sliding Mass Balance and the Electronic Balance

Most masses used in laboratory work are measured either by the sliding mass balance or the electronic balance as shown in Figures 1.8 and 1.9 respectively.

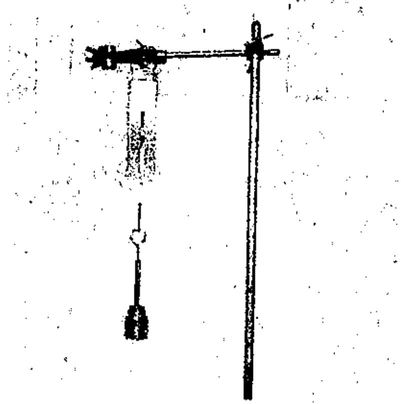
For the sliding mass balance, the unknown mass is placed onto the pan and its mass is obtained by sliding the movable masses on the beams until the beams are balanced. It is basically a beam balance.

The electronic balance is easier to use and also more accurate than the sliding mass balance. The unknown mass is simply placed on top of the pan and its mass read directly from a display placed on top of the pan and its mass read directly from a display screen.

Apparatus Used to Measure Weight

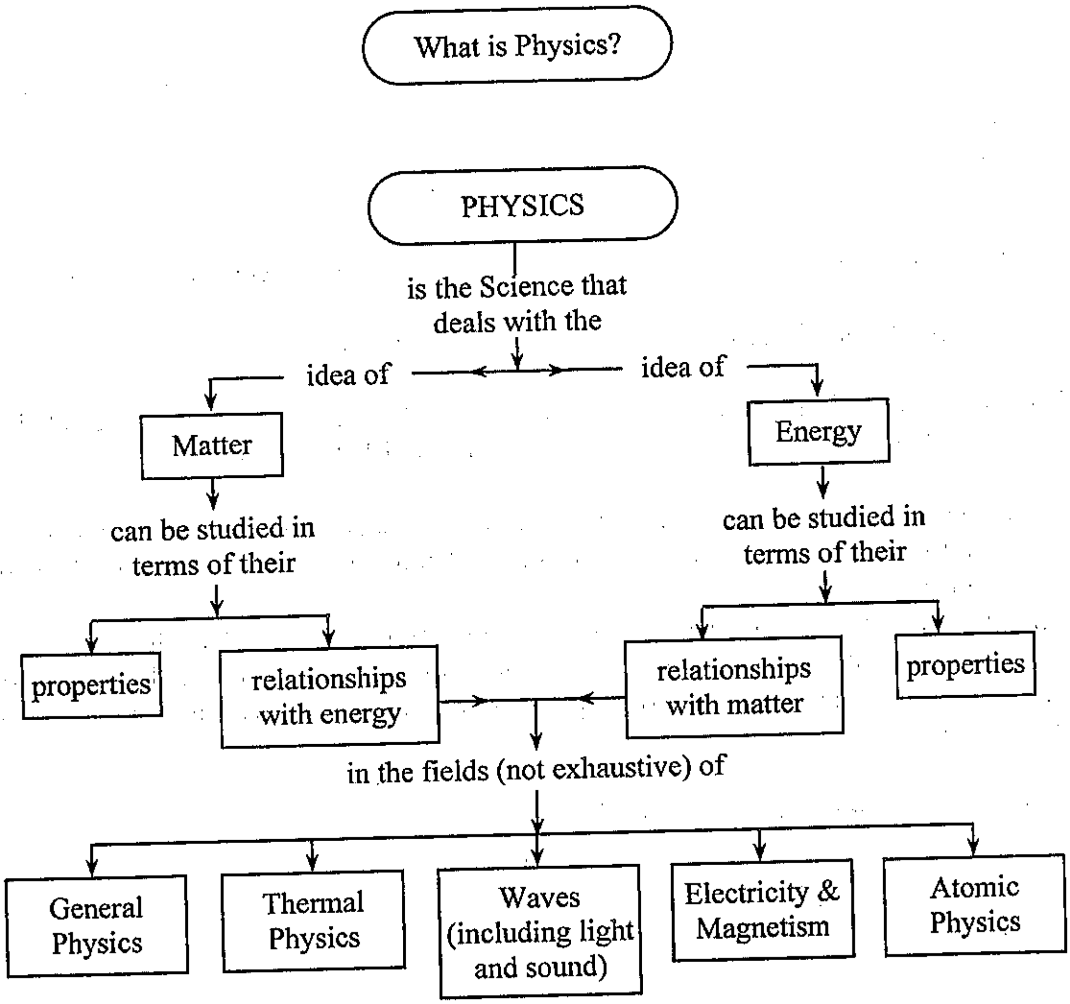


A compression balance

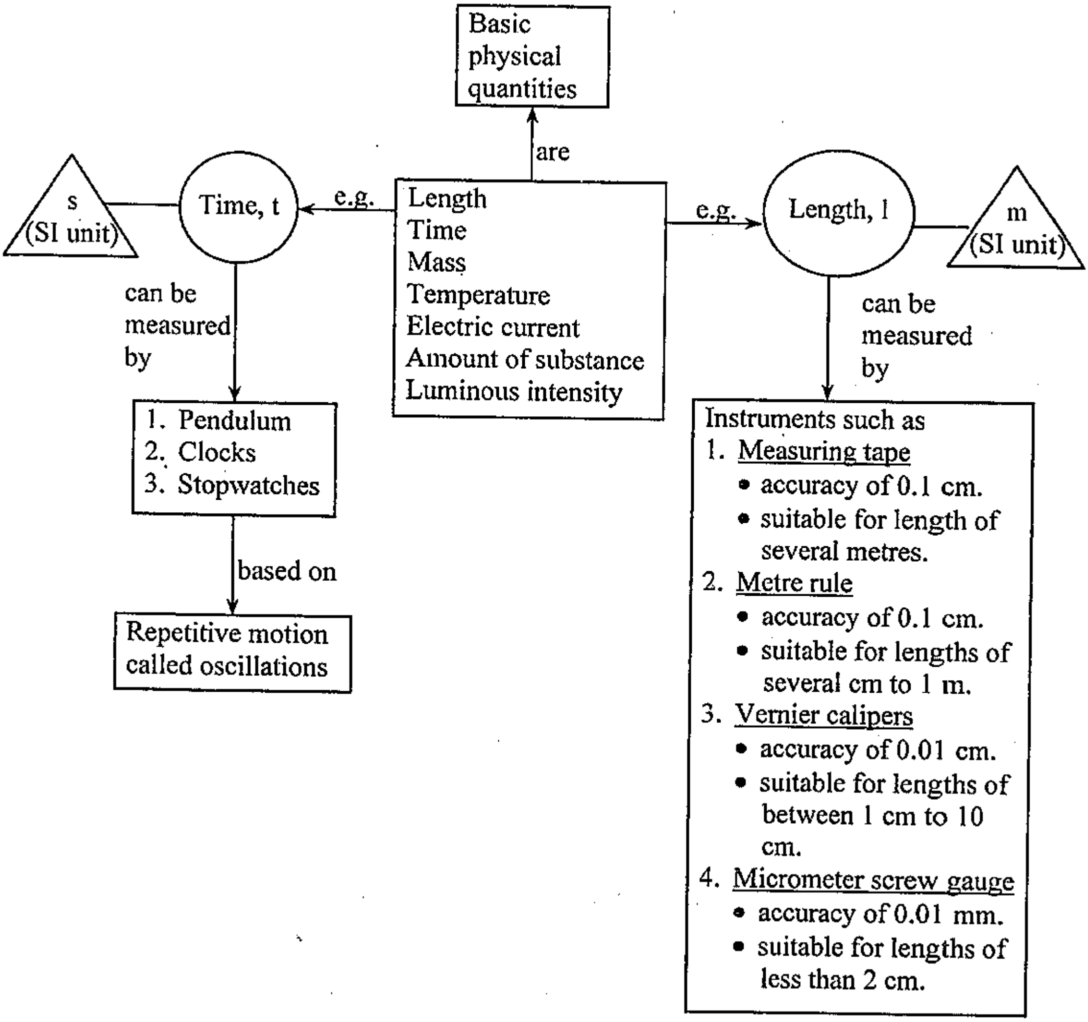


A spring balance

Concept map (Physics)



Concept Map (Basic Physical Quantities)



EXERCISES

1. Write a page on why physics ought to be studied.
2. Which of the following is/are a one-sentence definition of physics?

   (a) Physics is the study of energy and matter.

   (b) Physics is a subject which studies natural phenomena.

   (c) Physics is a collection of laws and principles which govern the behaviour of nature.

   (d) Physics is a subject which studies elements and compounds.

   (e) Physics is a subject which studies the structure of the animate world.
3. Define or explain the following:

   (a) Standards (b) Units (c) SI system (d) CGS system (e) British system.
4. The SI system has been accepted because it has many advantages compared to the previous systems used. Discuss these advantages.
5. Express the following measurements in decimal notation.

   (Example : $10^{-5} = 0.000\ 01$; $10^{-2} = 0.01$)

   Diameter of nucleus of an atom............. 10^{-15} m

   Diameter of sodium atom..................... 10^{-11} m

   Diameter of a virus ....2×10^{-8} m
6. Express the following in the **standard form** (numbers expressed using powers of 10)

   (Example : 1 000 000 = 10^6; 1000 = 10^3; 0.001 = 10^{-3})

   Length of a mouse .... 0.1 m

   A man's height... 1-2 m

   Diameter of the earth............................... 13 000 000 m

   Distance of the earth to the sun........ 130 000 000 000 m

   Distance of a distant galaxy ....100 000 000 000 000 000 000 000 000 m

7. The sun is a medium-sized star. In the Milky Way galaxy which includes the sun, there are one hundred billion stars. Write down this figure in the standard form
(1 billion = 10^9)

8. The tissue of a cell is 70 Å thick (Å = angstrom unit). If 1 Å $=10^{-10}$ m find the thickness of the tissue in terms of an inch.

9. The sizes of the atom, man, the sun and the universe are given as follows:
The atom............................... 4×10^{-10} m
Man .... 1-2 m
The sun........ .... 1.2×10^{9} m
The universe............................ 10^{26} m
Determine whether the following are correct.
Size of man $\sim \sqrt{\text{size of atom} \times \text{size of the sun}}$
Size of the sun $\sim \sqrt{\text{size of atom} \times \text{size of the universe}}$

10. The size of elementary particles (which are also the smallest particles) is of the order of $\sim 10^{-15}$ m and the size of the universe is of the order of $\sim 10^{26}$m. Determine the ratio of the size of universe to the size of an elementary particle.

11. One acre is equal to 43 560 ft^2. How much is it in m^2?

12. One litre is equivalent to 1000 cm^3. How many litres are there in 231 in^3?

13. The following time intervals are given in seconds. Convert them to years.
The existence of vertebrate animals on earth........................ 10^{16} s
The existence of life on earth... 10^{17} s
Age of the earth... 4×10^{17} s
Age of the universe.. 2×10^{18} s
The shortest life-time of an elementary particle.................. 10^{-23} s

14. The shortest life-time of an elementary particle is 10^{-23} s and the age of the universe is 10^{18} s. Find the ratio of the two time intervals.

15. In fixing the standard for time we can either choose the solar day or the atomic clock. Which do you think is the better of the two? Why?

16. The distance light travels in one year is defined as a light-year *(l. yr.)*. If the width of the Milky Way galaxy (which includes our solar system) is 10^5 *l.* yr., how long will a light signal take to travel that width?

17. According to observations and measurements the farthest galaxies are at a distance of 10^{26} m from the earth. When space travel becomes highly advanced do you think that man will be able to visit those galaxies? (Light travels 3×10^8 m in one second and there are approximately 10^8 s in a year.)

18. 40 mi h^{-1} is the quotient of 40 miles and 1 h and 1 m s^{-1} is the quotient of 1 m and 1 s . Convert 40 mi h^{-1} to m s^{-1} unit and 3 m s^{-1} to mi h^{-1} unit.

19. The mass of an electron is $m_e = 9.1 \times 10^{-31}$ kg. The mass of a muon is about 207 m_e and the mass of a proton is about 1 836 m_e. Find the masses of **muon** (an elementary particle with a mass about 200 times that of an electron) **and proton.**

CHAPTER 2

VECTORS

Motion is the cause of practically all changes which occur daily round us. Because the earth rotates on its axis we have day and night. Different seasons on earth are due to the earth moving around the sun along a definite path. Wind and its effects are caused by the motion of air. The growth of crops consumed by man and animals depends on the motion of water, air and nutrient absorbed by these crops. We come across different kinds of motion not only in transporting raw materials and finished products from one place to another but also in industrial production lines where raw materials are converted into finished goods.

Whatever kind of change we may study the key factor is motion itself.

When a car breaks down and stops we have to push in order to move it. A car is in motion because the engine is driving it forward. Whatever the case may be the thing that causes the car to move is called force. The force changes the state of motion of the car. When we want to slow down a moving car or stop it completely we have to apply the brakes. Here also the state of motion of the car is changed by the force.

So it is obvious that force not only controls motion, it also changes the state of motion. In order to study in detail how force and motion are connected we have to find out systematically the nature and important factors governing force and motion.

When a body is in motion there is a change in position involved. The change in position along a certain direction is called displacement. Suppose a body moves from point A to another point B ten miles away and situated north-west of A. How shall we describe the displacement? We cannot just say the body has moved ten miles from A. It will not be sufficient. This will mean that B can be situated at any point on the circumference of the circle whose radius is ten miles and whose centre is A. The position of B, therefore, cannot be exactly determined. It is necessary that we mention also the direction of the displacement. To describe the displacement more exactly, we should say that the displacement of the body from A to B is ten miles north-west. A quantity such as displacement is called a vector. A vector has both magnitude and direction. Vector is the abbreviation of vector quantity. Quantities that have only magnitude are called scalars, e.g. mass and length.

Other vectors which are important in the study of motion are velocity, acceleration, momentum and force.

If we just say that an aeroplane is flying at 500 miles per hour (500 mi h^{-1}) we are not giving a full description of the motion of the plane. We just mention the speed of the plane without referring to the flight direction. If we want to describe the motion of a plane at any instant we have to state its velocity. The velocity gives the speed as well as the direction of motion. Velocity is therefore a vector whose magnitude gives the speed and whose direction gives the direction along which the body is moving.

It is also obvious that force is a vector quantity. The effect of a force acting on a body depends not only on the magnitude of force but also on the direction of the force. That is why when we describe force we have to mention both its magnitude as well as its direction.

Taking force as an example let us now study vectors in general.

2.1 VECTOR SYMBOLS AND ADDITION OF VECTORS

Let us suppose that a number of forces are acting on a body simultaneously. How will we find the resultant force or the net force acting on the body? To do this we shall have to use the vector diagram drawn to a proper scale. The arrows drawn in a vector diagram represent the respective forces. The length of each arrow is proportional to the magnitude of the force it represents and the direction of the arrow represents the direction of the force concerned.

The symbols for vectors are usually indicated by placing arrows over their symbols, for instance $\vec{A}$, $\vec{B}$, $\vec{C}$ and so on. In this textbook the magnitude of vector $\vec{A}$ will be written A the magnitude of vector $\vec{B}$ will be B and so on. (See G.10 Teacher's manual Pg-18)

The method for adding vectors is illustrated below. To add $\vec{B}$ to $\vec{A}$ shift $\vec{B}$ parallel to itself until its tail is at the tip of $\vec{A}$. (In its new position $\vec{B}$ must still have its original length and direction.) Then draw a third vector $\vec{R}$ from the tail of $\vec{A}$ to the tip of $\vec{B}$ as shown in the figure. $\vec{R}$ is the vector sum $\vec{A} + \vec{B}$ and it is called the resultant vector. Note that the tail of $\vec{R}$ is at the tail of $\vec{A}$ and the tip of $\vec{R}$ is at the tip of $\vec{B}$.

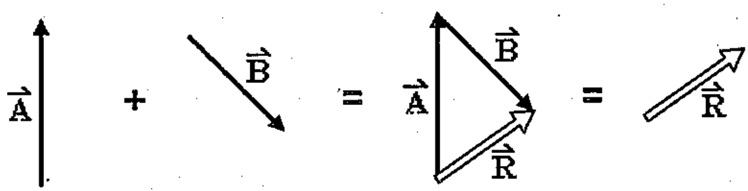


Fig.2.1

If $\vec{A}$ and $\vec{B}$ represent two forces acting on a body, then $\vec{R}$ represents the resultant or net force. In other words, the two forces represented by $\vec{A}$ and $\vec{B}$ can be replaced by a single force represented by $\vec{R}$. By so doing, the behaviour of the body will not be altered in any way.

The method of vector addition described above can be used for adding two or more vectors. The figure below shows how four vectors, whose magnitudes and directions differ, are added together.

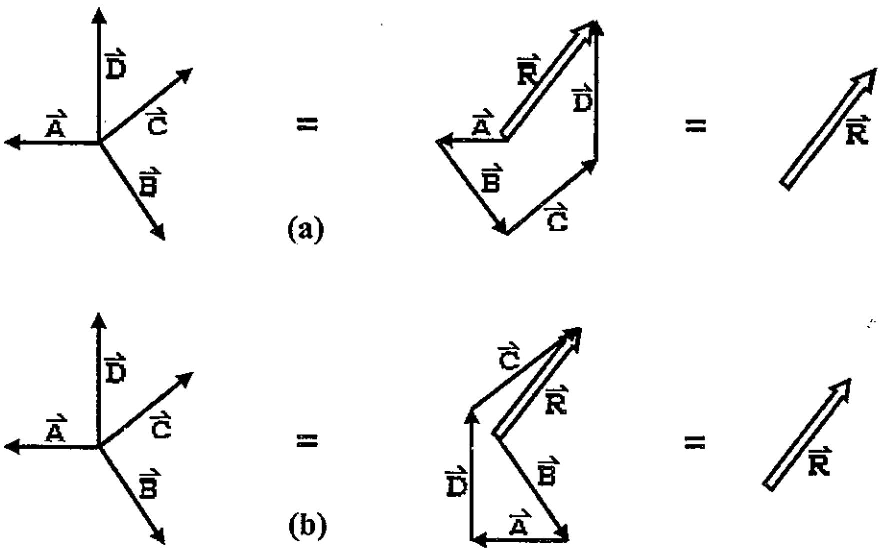


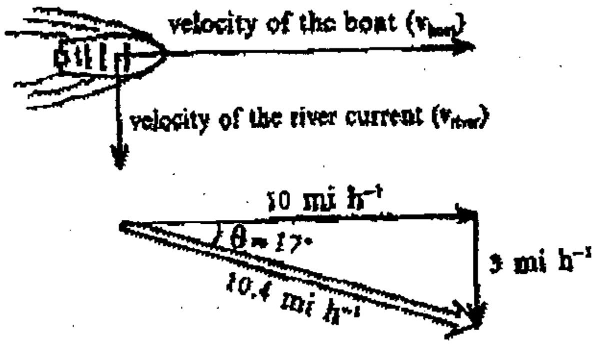
Fig.2.2

We can see from the figure that there are more than one method of adding the four vectors. The four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$ can be taken in any order and added, and still the resultant vector is the same. Fig. 2.2(b) illustrates a typical example. We may also note that the order in which the vectors are added does not affect the result; that is

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Vector diagrams can be used not only for forces but also for other vector quantities such as displacement, velocity....etc. We must be careful, however, that only vector quantities of the same kind may be added. Displacements may be added vectorially; so also velocities and forces. But different vector quantities like velocity and force cannot be added.

Let us study the following example of vector addition. A boat travels east at 10 mi h^{-1} in a river that flows south at 3 mi h^{-1}. We wish to find the boat's velocity relative to the river bank (the earth). Here we have to add the boat's velocity to the velocity of the river current. The two velocities can be represented by vectors and then they can be added by a vector diagram.



If v is the required velocity, then

$$v^2 = v^2_{\text{boat}} + v^2_{\text{river}} \quad (\because v_{\text{boat}} \perp v_{\text{river}})$$

$$v^2 = 10^2 + 3^2$$

$$= 109$$

$$v = 10.4 \text{ mi h}^{-1}$$

$$\tan\theta = \frac{v_{\text{river}}}{v_{\text{boat}}} = \frac{3}{10}$$

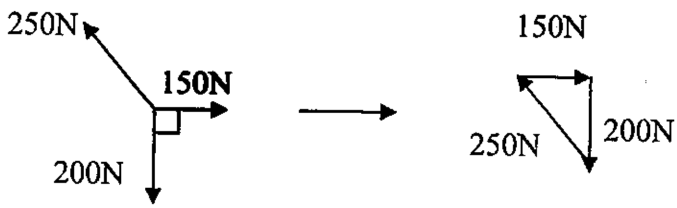
$$= 0.3$$

$$\therefore \theta = \tan^{-1} 0.3 = 17°$$

The addition of vectors is shown in the vector diagram and the determination of the magnitude and direction of the resultant vector is shown in the above calculation. In this example v_{boat} and v_{river} are perpendicular to each other. Hence we can use the Pythagoras theorem in finding the magnitude v of the resultant velocity $\vec{v}$.

Triangle of forces

If three forces acting on a body are in equilibrium, we can always represent these three forces by the sides of a triangle, with the direction of the forces taken in order.



Having learnt how vectors may be added let us now find out how to subtract one vector from another. Vector subtraction is also called vector difference.

Vector subtraction is, in effect, vector addition. If we subtract vector $\vec{A}$ from vector $\vec{B}$, we must first write vector $\vec{A}$ with a minus sign in front, such as $-\vec{A}$. This vector $-\vec{A}$ has the same magnitude as vector $\vec{A}$ but its direction is opposite to that of $\vec{A}$. Vector $\vec{B}$ can now be added to vector $-\vec{A}$ using the method described above. In what follows, the vector difference between $\vec{B}$ and $\vec{A}$ is found by means of algebraic notation as well as by drawing a vector diagram.

$$\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$$

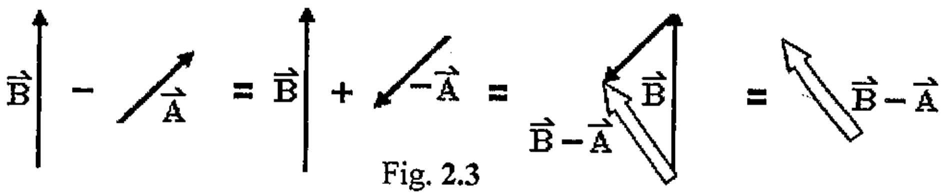


Fig. 2.3

Besides vector addition and subtraction we must also know how to resolve a vector. Resolution of a vector means that the given vector is resolved (subdivided) into vector components so that their sum total effect is the same as that of the original vector. A vector situated in a three dimensional space can be resolved into three vector components. If we consider only a two dimensional space, then a vector in such a space can be resolved into two vector components which are perpendicular to each other. "Vector component" is a new term introduced here. Let us find out what it really means.

2.2 RESOLUTION OF VECTORS

Just as a number of vectors can be added to obtain a resultant vector, it is also possible to sub-divide a given vector into a number of different vectors. As shown in Fig. 2.4, if vectors $\vec{A}$ and $\vec{B}$ combine to give a resultant vector $\vec{C}$, then it is equally true that vector $\vec{C}$ is equivalent to the sum of its vector components $\vec{A}$ and $\vec{B}$. The process of sub-dividing a vector into two or more vectors is called "resolution of a vector", and the new vectors obtained are called "vector components" of the original vector.

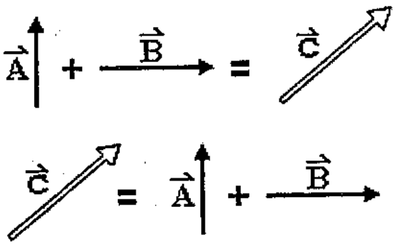


Fig. 2.4

Vector components as a rule need not be perpendicular to one another. But in most practical applications they are perpendicular to one another.

Let us look at an example of how resolution of a vector is put into practical use.

Fig. 2.5 shows a boy pulling a wagon with a rope which is at an angle α above the ground. Only part of the force he exerts affects the motion, since the wagon moves horizontally while the force $\vec{F}$ is not a horizontal one.

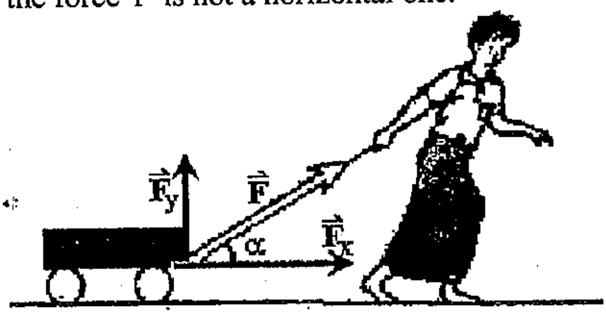


Fig.2.5

We can resolve $\vec{F}$ into two components, $\vec{F}_x$ and $\vec{F}_y$, which are the horizontal and vertical components of $\vec{F}$, respectively.

The horizontal component $\vec{F}_x$ is responsible for the wagon's motion, while the vertical component $\vec{F}_y$ merely pulls the wagon upwards. (Since $\vec{F}_y$ is counter-balanced by the weight of the wagon, the wagon does not really move upwards.) The magnitudes of $\vec{F}_x$ and $\vec{F}_y$ are

$$F_x = F \cos \alpha$$
$$F_y = F \sin \alpha$$

$\vec{F}_x$ is the projection of $\vec{F}$ in the horizontal direction, and $\vec{F}_y$ is the projection of $\vec{F}$ in the vertical direction.

We need just two quantities to specify $\vec{F}$. These two quantities can either be the magnitude of the force $\vec{F}$ and the angle α or $\vec{F}_x$ and $\vec{F}_y$. Generally, however, physical quantities which are represented by vectors are considered in three dimensional space rather than in two dimensional space. We therefore need three quantities to specify a vector. These three quantities are shown in Fig. 2.6.

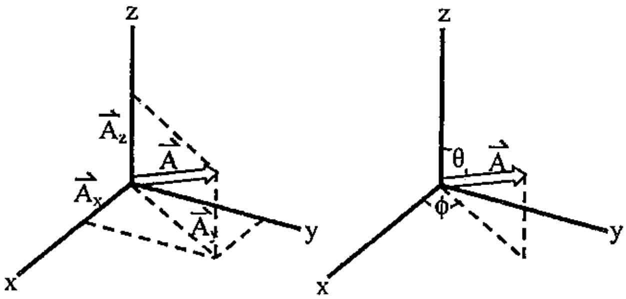
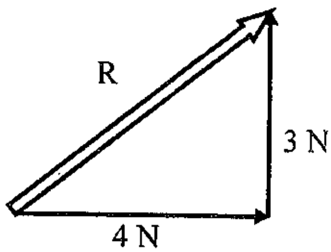


Fig.2.6

Two different sets of the three quantities are shown in Fig. 2.6. One set consists of the vector components $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ of vector $\vec{A}$ and the other consists of the magnitude A of vector $\vec{A}$ and the two angles φ and ϑ. It is found that the former is more convenient to use than the latter.

Example (1) A force of 3 N is perpendicular to a force of 4 N. Find the magnitude of the resultant of the two forces.

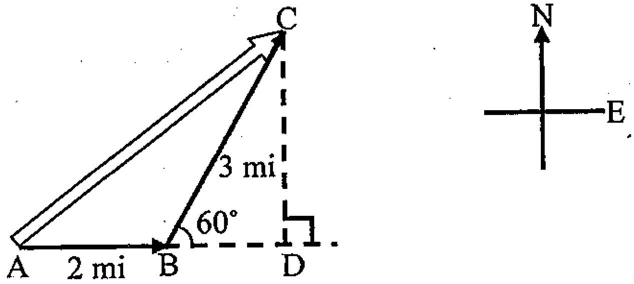
Let R be the magnitude of the resultant force.



$$R = \sqrt{(4N)^2 + (3N)^2} = \sqrt{25N^2} = 5N$$

(N is the abbreviation of the force unit: newton)

Example (2) A man walks 2 mi east and then 3 mi in a direction 60° north of east. Find the magnitude of his resultant displacement from his starting point.



Let the resultant displacement be AC. We have,

$$BD = BC\cos 60^\circ$$
$$= 3\cos 60^\circ = 1.5\text{mi}$$
$$AD = AB + BD$$
$$= 2 + 1.5 = 3.5 \text{ mi}$$
$$CD = BC\sin 60^\circ$$
$$= 3\sin 60^\circ = 2.6 \text{ mi}$$
$$\therefore AC = \sqrt{(AD)^2 + (CD)^2}$$
$$= \sqrt{12.25 + 6.76}$$
$$= 4.4 \text{ mi}$$

EXERCISES

1. In the equation $s = v_o t + \frac{1}{2}at^2$, s,$v_o$and a are magnitudes of the respective vector quantities. Write down the equation in vector form.
2. The relation between displacement s, average velocity $\bar{v}$ and time t is given by $s = \bar{v}t$. Which of the quantity or quantities are vectors and which are scalars?
3. Fill in the blanks.

   Using graphical method a vector may be represented by an arrow. The (a) of the arrow is proportional to the magnitude of the vector and the direction of the arrow gives the (b) of the vector.

4. Fill in the blanks.
   A force directed to the east is represented by an arrow of length 3 cm pointing to the right. A force of equal magnitude directed to the west can be represented by an arrow of length (a) pointing to the (b).
5. Which of the following statements is definitely false?
   (a) A scalar is the magnitude of a vector.
   (b) A vector has both magnitude and direction, while a scalar has only magnitude.
   (c) A displacement is a scalar quantity since it has both magnitude and direction.
   (d) If the statement (b) is true, then (c) must be false.
6. Differentiate between vector and scalar quantities.
7. Can the magnitude of the resultant vector of two vectors having the same magnitude be
   (a) greater than, smaller than or equal to the magnitude of each vector?
   (b) greater than, smaller than or equal to the sum of the magnitudes of the two vectors? (Answer with diagrams.)
8. In ordinary arithmetic 2+2=4. Can we always use this kind of addition in vector summation? Explain.
9. If the sum of the two vectors is zero what can you say about these two vectors?
10. Can the sum of two vectors, having unequal magnitudes, be equal to zero?
11. Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the equation A + B - C = 0. Can you say that $\vec{C}$ is the resultant vector of $\vec{A}$ and $\vec{B}$?
12. If $\vec{A} = -2\vec{B}$ compare the magnitudes and directions of $\vec{A}$ and $\vec{B}$.
13. If $\vec{C} = \frac{3}{2}\vec{D}$ and C is represented by the arrow shown, draw an arrow that represents D.

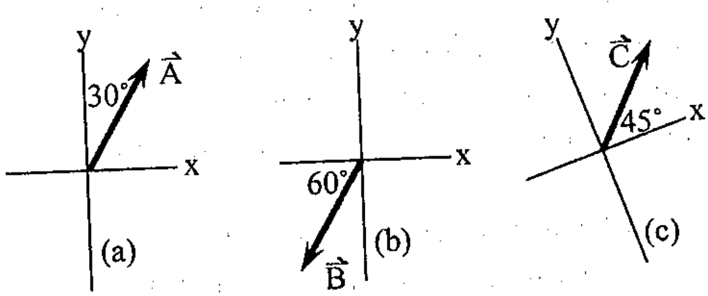
3 units

14. Add two vectors having 3 units each and both pointing north.
15. Find the sum of a vector having 4 units pointing north and a vector having 3 units pointing south.

16. A force 4 N, directed east, and a force 6 N, directed west, act on a particle. Find the magnitude and direction of the resultant force. (A particle is a very small object.)

17. Find the magnitude of the resultant force of a force 3 N pointing east and a force 6 N pointing north.

18. Draw the respective y components of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ shown in Figs. (a), (b) and (c). Also write down the values of these components .



19. A vector of magnitude 5 units is inclined at an angle 37° to the x axis. Find the magnitudes of the vector components along x and y axis.

20. A vector of magnitude 5 units is inclined at an angle 37° to the x axis and another vector of magnitude 10 units is inclined at an angle 53° to the x axis. What is the magnitude of the sum of the vector components along the x axis?

CHAPTER 3

DESCRIBING MOTION

We cannot say that motion is absolute and independent of other things. Motion actually is relative and we need a frame of reference to describe it. How fast are you moving as you sit and read this book? This may seem to be an absurd question but at this very moment you are moving along at a speed of 19 miles per second. This speed is equivalent to 70 000 mi h^{-1} and is many times the speed of a jet plane. You might be surprised but the question and the answer mentioned above are neither absurd nor surprising. That the answer is correct can be explained as follows. Since you are on the earth, it carries you along with it as it speeds around the sun in its orbit. Therefore, the earth's orbital speed, which is 19 miles per second, is also your speed. Ordinarily, you will say that you are at rest and that you do not think of yourself as being in motion. This is because, in everyday life, when you say that a body is moving, you mean that it is moving with respect to the surface of the earth.

You are either moving at 19 miles per second or you are at rest depending on the point of reference chosen. A point of reference in a given frame of reference is any body with respect to which the motion of another body is being described.

In the first instance the reference is the sun and in the second it is any object such as a tree or a building which is situated on the earth. A body may therefore be at rest in one frame of reference while it may be moving in another. Consider a passenger sitting in a bus that is travelling at 20 mi h^{-1}. The passenger will be moving at a speed of 20 mi h^{-1} with respect to the road, but he will be stationary with respect to the seats, floor, walls of the bus or the driver of the bus. If another bus, also travelling at 20 mi h^{-1}, should be coming toward the passenger, the speed of the passenger with respect to that bus would be 40 mi h^{-1}. This example illustrates that the speed changes as the frame of reference is varied. It is therefore necessary that we specify a frame of reference when we study motion, since motion, as we have seen, is not absolute but relative.

3.1 VELOCITY AND SPEED

In everyday life, we describe the motion of bodies by stating their speeds. The speed of a body tells us how far it travels during every unit of time. A typical automobile speed is 30 mi h^{-1} or 44 feet per second. This means that the automobile travels a distance of 44 ft each second. Some typical units of speed are feet per second (ft s^{-1}), centimetres per second (cm s^{-1}) and metres per second (m s^{-1}). An artificial earth satellite has a speed of about 5 miles per second (5 mi s^{-1}). We can compare this with the speed of light which is 186000 mi s^{-1}. This speed, so far, is the fastest known to man. Although there is a theoretical prediction that the speed of a certain particle exceeds the speed of light, we still have no concrete evidence that such a particle exists in nature.

In physics we give a more complete description of the motion of a body by stating its velocity instead of its speed. Velocity tells us two things about a moving body: its speed and its direction of motion. Thus, the velocity of an aeroplane should be stated as 300 kilometres per hour(km h^{-1}) westward, which then gives both the speed and the direction.

All the different kinds of motion we come across in the world can be classified as one of two main types, uniform motion and accelerated motion.
In uniform motion, both the speed and direction of the moving body remain the same. It is therefore motion at constant velocity. A car going at a steady speed of 30 mi h^{-1} on a straight road is travelling in uniform motion (Fig. 3.1).

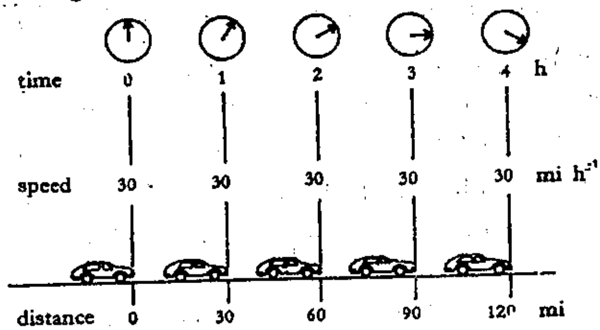


Fig.3.1

In practice, a car usually does not travel at constant velocity all the time. Changing road and traffic conditions make it necessary that the car changes its direction or speed, or both direction and speed. The velocity therefore keeps changing. Motion with changing velocity is called accelerated motion. In everyday usage, acceleration generally means speeding up. In physics, however, acceleration refers to any change of velocity. Velocity changes when there is change of direction, change of speed, or change of both direction and speed. Any change in velocity gives rise to acceleration. Since the change of speed can either be increasing or decreasing, the change of velocity in accelerated motion, therefore, can also be either increasing or decreasing.
Considering the points discussed in this chapter and in Chapter 2, we can now define exactly the physical quantities: displacement, velocity and acceleration which are necessary for describing motion.
Displacement is defined as the distance travelled along a particular direction.

Displacement has both magnitude and direction and therefore is a vector quantity. If a car travels 100 m towards the east, then its displacement is "100m, eastward".

Velocity is defined as the rate of change of distance along a particular direction. It can also be defined as the rate of change of displacement. Velocity is also a vector quantity. (Note that speed is just a scalar quantity.)

If a body travels a distance s along a particular direction in time t, then the magnitude of its velocity is

$$v = \frac{s}{t} \tag{3.1}$$

If the motion of the body is such that equal displacements take place in equal intervals of time, then the velocity is constant. If during equal time intervals the displacements are not the same, then the velocity is not constant and we use the term average velocity to describe the motion of the body.

Average velocity is defined as the ratio of the total distance travelled along a particular direction to the time taken to travel that distance. In symbols, magnitude of average velocity is written

$$\bar{v} = \frac{s}{t} \tag{3.2}$$

Since velocity is a vector quantity, velocity can be expressed in vector form as

$$\vec{v}_{av} = \frac{\vec{s}}{t} \tag{3.3}$$

If the velocity of a body changes, then the body is said to have acceleration. Acceleration is defined as the rate of change of velocity. It is therefore a vector quantity which can be shown in symbols as

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t} \tag{3.4}$$

where a represents the acceleration, v_0 the initial velocity, v the final velocity and t the time interval respectively. The magnitude of acceleration can be written

$$a = \frac{v - v_0}{t} \tag{3.5}$$

(Equation 3.5 holds true only for the special case of motion along a straight line.)

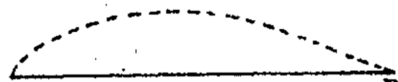
Even though the acceleration due to a decrease in velocity is termed deceleration in common usage, it is customary to use the technical terms "acceleration" and "negative acceleration" to refer to an increase or decrease in velocity respectively.

The units of displacement, velocity and acceleration in different systems are given in Table 3.1.

Table 3.1

Quantity	SI	MKS	CGS	FPS
Displacement	m	m	cm	ft
Velocity	m s^{-1}	m s^{-1}	cm s^{-1}	ft s^{-1}
Acceleration	m s^{-2}	m s^{-2}	cm s^{-2}	ft s^{-2}

Example (1) A body travels from A to B along a straight line and another body travels from A to B along a curve (shown by the dotted line).



If the straight-line distance between A and B is 3 km find the displacement of each body.

The displacement of each body is the same. (The displacement is the same although the paths of travel or the distances actually travelled are not the same.)

The displacement is 3 km from A to B.

Example(2) In the above example, the first body moves along the straight line from B back to A. The second body, however, moves along the curved-path back to the same starting point A. What are the displacements of the two bodies now?

Since the starting-point as well as the end-point for both the bodies is A, there is no net change in their position at all. Therefore the displacement of each body is zero although the distances travelled are not zero.

Example (3) If the first body takes 1.5 h to travel from A to B, what will be its velocity?

Since the displacement is 3 km and the time taken is 1.5 h,

$$\text{velocity} = \frac{3}{1.5} = 2\text{km h}^{-1}$$

$$\text{velocity} = \frac{3\times1000}{1.5\times60\times60} = 0.56 \text{ m s}^{-1}$$

(This result can also be shown in cm s^{-1}.)

NOTE: Since the motion of the body is along a straight line, we have not mentioned the direction of the velocity, which is toward B. A positive sign for the velocity is quite sufficient here. The velocity in this example is just the average velocity.

Example(4) A body moves from one point to another along a straight line with constant acceleration. If its initial velocity is 40 cm s^{-1} and final velocity is 80 cm s^{-1} find the average velocity of the body.

If v_o = the initial velocity and v = the final velocity, then

$$\text{average velocity } \bar{v} = \frac{v_0 + v}{2} = \frac{40+80}{2} = 60 \text{ cm s}^{-1}$$

Example (5) A body travels B a straight line for 5 s. The displacement of the body for each second is given in the table below.

Time	0 s	1 s	2 s	3 s	4 s	5 s
Displacement	0 cm	10 cm	25 cm	30 cm	48 cm	60 cm

Find the average velocity of the body. Is the body moving with uniform velocity?

$$\text{Average velocity } \bar{v} = \frac{s}{t}$$

$$= \frac{60}{5} = 12 \text{ cm s}^{-1}$$

OR

For each second, the velocity is

1st second	2nd second	3rd second	4th second	5th second
10cm s^{-1}	15 cm s^{-1}	5 cm s^{-1}	18 cm s^{-1}	12cm s^{-1}

$$\text{Average velocity } \bar{v} = \frac{10+15+5+18+12}{5} = \frac{60}{5} = 12 \text{ cm s}^{-1}$$

Since the velocity of the body is changing every second the body is not moving with uniform velocity.

Example(6) A car moving in a straight line with constant acceleration arrives at a certain point after travelling 5 s from the starting point. If the initial velocity is 44 ft s^{-1} and the final velocity is 66 ft s^{-1} find the acceleration of the car.

If v_o = the initial velocity and v = the final velocity, then

$$\text{acceleration } a = \frac{v - v_0}{t} = \frac{66-44}{5} = 4.4 \text{ ft s}^{-2}$$

3.2 LINEAR MOTION

Out of the many types of motions we shall first study the simplest type, which is motion in a straight line or linear motion. The study of linear motion of bodies is important for two reasons. First, many objects, such as freely falling bodies, actually move in straight-line paths. Second, many complicated motions of bodies can be considered as combinations of two or more straight-line motions and therefore can be analysed in terms of straight-line motions.

In discussing linear motion we shall use just v for the velocity symbol instead of the velocity vector symbol $\vec{v}$. The direction will be specified by using positive and negative signs. Since linear motion has only two directions, if one direction is taken positive then the other will be taken negative. It is important that we use the positive and negative signs correctly in our discussion. The symbol for acceleration will also be represented by a instead of $\vec{a}$. The acceleration due to an increase in velocity will be assigned a positive sign, while that due to a decrease in velocity will be assigned a negative sign.

We shall now derive the equations of motion for linear motion. Let us suppose that a body moving with velocity v_0 acquires an acceleration a. Therefore, for every second that the body is moving the increase in velocity is equal to a. In time t the increase in velocity will be " at" . Hence, the velocity after time t will be

$$v = v_0 + at \qquad (3.6)$$

If a body is moving with uniform acceleration, its average velocity is equal to half the sum of its initial velocity $v_0$and its final velocity v. Therefore,

$$\text{average velocity} \quad \bar{v} = \frac{v_0 + v}{2}$$

Since $v = v_0 + at$,

$$\text{average velocity} \quad \bar{v} = \frac{v_0 + v_0 + at}{2}$$

$$= v_0 + \tfrac{1}{2}\,at$$

Since the distance s travelled by the body (the displacement) is equal to the product of average velocity and time, we have

$$s = (v_0 + \frac{1}{2}a\,t)\,t \qquad \text{or} \qquad s = v_0 t + \frac{1}{2}at^2 \qquad (3.7)$$

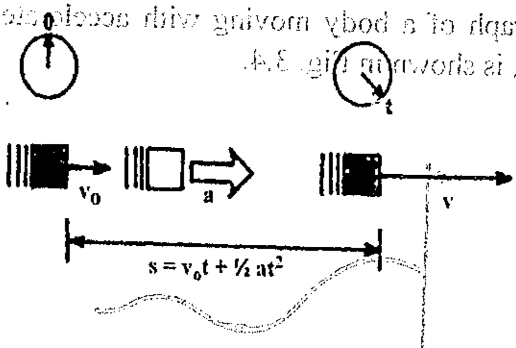


Fig. 3.2

Eliminating time t from the two equations obtained above we get another equation of motion as follows. Squaring both sides of the equation $v = v_0 + at$, we have

$$v^2 = v_0^2 + 2\, v_0\, at + a^2 t^2$$

This equation can be written in the form

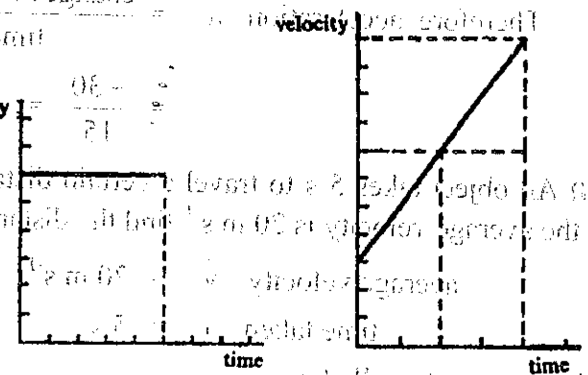
$$v^2 = v_0^2 + 2a\,(v_0 t + \tfrac{1}{2} at^2)$$

Since the terms shown in brackets correspond to s

$$v^2 = v_0^2 + 2as \qquad (3.8)$$

The above three equations of motion give the relation between displacement, velocity and acceleration of a body moving in a straight line with uniform acceleration. These equations are important and useful in analysing straight-line motion.

Straight-line motion may also be illustrated with the help of a velocity-time graph.



(a) (b)

Fig. 3.3

The velocity-time graph shown in Fig. 3.3(a) represents motion with uniform velocity and Fig. 3.3 (b) illustrates motion with uniform acceleration.

The velocity-time graph of a body moving with accelerated motion, but with non-uniform acceleration, is shown in Fig. 3.4.

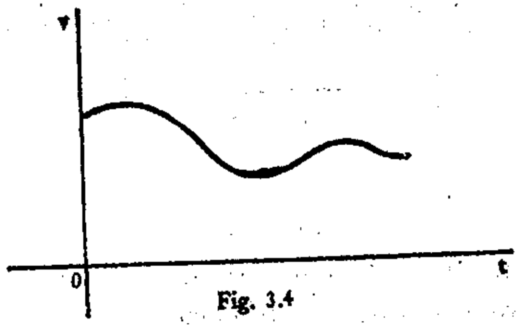


Fig. 3.4

Example (7) A car travelling with a speed 108 km h^{-1} stops in 15 s due to uniform acceleration. Find the value of the acceleration.

$$108 \text{ km h}^{-1} = \frac{108\times1000}{60\times60} = 30 \text{ ms}^{-1}$$

$$\text{initial velocity} = 30 \text{ m s}^{-1}$$

$$\text{final velocity} = 0 \text{ m s}^{-1}$$

$$\text{change of velocity} = \text{final velocity} - \text{initial velocity}$$

$$= (0\text{-}30) \text{ m s}^{-1} = -30 \text{ m s}^{-1}$$

$$\text{Therefore, acceleration } a = \frac{\text{change of velocity}}{\text{time}}$$

$$= \frac{-30}{15} = -2 \text{ m s}^{-2}$$

Example (8) An object takes 5 s to travel a certain distance. If the path of travel is straight and the average velocity is 20 m s^{-1} find the distance travelled by the object.

$$\text{average velocity } \bar{v} = 20 \text{ m s}^{-1}$$

$$\text{time taken } t = 5 \text{ s}$$

Therefore, the distance travelled is

$$s = \bar{v}\, t = 20 \times 5 = 100 \text{ m}$$

Example (9) A car starting from rest travels with uniform acceleration of 2m s^{-2} in the first 6 s. It then travels with a constant velocity for half an hour. Find the distance travelled in the first 6 s as well as the distance travelled in the following half an hour.

For the first part of the journey,

$$\text{initial velocity } v_0 = 0 \text{ m s}^{-1}$$
$$\text{acceleration } a = 2 \text{ m s}^{-2}$$
$$\text{time taken } t_1 = 6 \text{ s}$$

Therefore, the displacement $s_1 = \frac{1}{2}at_1^2 = \frac{1}{2} \times 2 \times 6^2 = 36 \text{ m}$

The velocity at the end of the first part of the journey is

$$v^2 = 2 a s_1$$
$$= 2 \times 2 \times 36$$
$$\therefore v = 12 \text{ m s}^{-1}$$

For the second part of the journey,

$$\text{constant velocity } v = 12 \text{ m s}^{-1}$$
$$\text{time taken } t_2 = 30 \text{ min} = 30 \times 60 \text{ s}$$

Therefore, the displacement $s_2 = v_{constant} t_2$

$$= 12 \times 30 \times 60$$

Therefore, $v = 21\,600 \text{ m}$

$$= 21.6 \text{ km}$$

3.3 MOTION GRAPHS AND THEIR INTERPRETATION

Motion can be described or analysed conveniently with the help of graphs. The usefulness of motion graphs is illustrated through examples in the following discussion.

Let us first look at the simplest type of motion-one in which a steel ball is moving with constant speed along a straight path. The speed of the ball is not changing. Let this speed be 5 cm s^{-1} ; then the ball travelled 5 cm in every second. On measuring the distances covered and the corresponding time elapsed, we get the data listed in Table 3.2.

Table 3.2

Time	Distance	Position
0 s	0 cm	A
1 s	5 cm	B
2 s	10 cm	C
3 s	15 cm	D
4 s	20 cm	E
5 s	25 cm	F

Table 3.2 lists the distances travelled by the ball at the end of each of five 1-second intervals. The respective positions of the ball are marked A, B, C, D, E and F, A being the starting position, B the position after 1 s and so on.

For each of the positions of the steel ball, we plot a point on the graph in Fig. 3.5. The distance travelled is shown on the vertical axis and the time elapsed is shown on the horizontal axis. On plotting the data in Table 3.2, we get the distance versus time graph shown in Fig. 3.5. This graph is called the distance-time graph. Point A of the graph represents the starting point and therefore lies at the origin. Point B represents the distance of 5 cm travelled by the ball after a time interval of 1 s. Its ordinate or vertical distance from the horizontal axis is 5 cm on the distance scale. Its abscissa or horizontal distance from the vertical axis is 1 s on the time scale. Similarly, point C represents the distance of 10 cm travelled at the end of 2 s of time. The remaining points D, E, F are also obtained in the same manner. On connecting these points, we find that the graph is a straight line.

Hence, a straight line distance-time graph represents a uniform motion; that is, motion with a constant speed along a straight line. And the slope or slant of the distance-time graph gives the speed of the moving object. The steeper or greater the slope of a straight line distance-time graph, the greater is the speed.

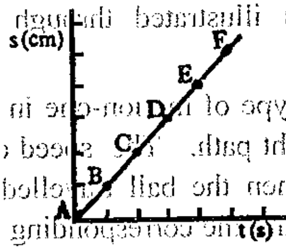


Fig. 3.5

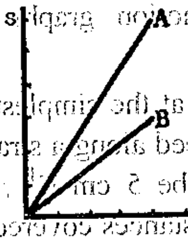


Fig. 3.6

The distance-time graphs of two moving objects are shown in Fig. 3.6. Since both the graphs are straight lines, they represent linear motions with constant speeds the graph with the steeper slope represents the motion with greater speed.

Another type of graph useful in the analysis of motion is the speed-time graph. This type of graph shows how the speed of a moving object varies with time. Let us again look at the motion (with constant speed along a straight path) described above. In Fig. 3.7 the constant speed of the steel ball moving at 5 cm s^{-1} is plotted against the time of travel. The speed is shown on the vertical axis and the time is shown on the horizontal axis. The speed-time graph is a straight line parallel to the horizontal axis. The fact that all the points on the graph are at equal distances from the horizontal axis means that the object is moving with the same speed throughout its motion.

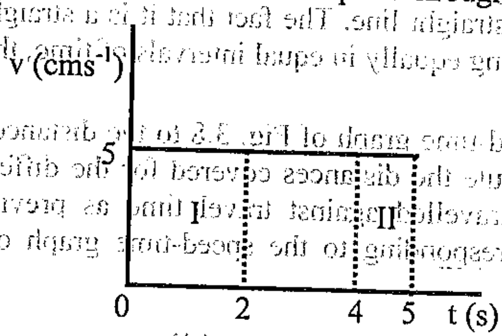


Fig. 3.7

We can find out the distance travelled by the steel ball between any two instants of time by calculating the area under the speed-time graph between those instants of time. Thus, area I of Fig. 3.7 gives the distance travelled by the ball between the time $t = 0$ s and $t = 2$ s, and similarly, area II gives the distance travelled in the time interval between $t = 4$ s and $t = 5$ s. Area I = $v \times t$, where $v = 5$ cm s^{-1} and $t = 2$ s, and therefore the distance travelled in that time interval of 2 s is 5 cm s^{-1} $\times$ 2 s or 10 cm. Similarly, the distance travelled between $t = 4$ s and $t = 5$ s is 5 cm s^{-1} $\times$ 1 s or 5 cm. The ability to find out the distance travelled by such means is a useful feature of the speed-time graph.

We will now go on to the next simplest type of linear motion. In this type of motion the object is moving along a straight path with uniform (constant) acceleration.

A car is travelling along a straight road and its speedometer readings are as listed in Table 3.3. The speedometer of the car indicates directly the speed of the car at the particular instant the reading is taken.

Table 3.3

Time	speed
0 s	0 m s^{-1}
2 s	12 m s^{-1}
4 s	24 m s^{-1}
6 s	36 m s^{-1}
8 s	48 m s^{-1}
10 s	60 m s^{-1}

On plotting the speed against time, we get the speed-time graph of Fig. 3.8. The nature of this graph is a straight line. The fact that it is a straight line shows that the speed of the car is changing equally in equal intervals of time; that is, the acceleration is constant.

We can convert the speed-time graph of Fig. 3.8 to the distance-time graph. In order to do that we first compute the distances covered for the different travel times, and then plot the distance travelled against travel time as previously described. The distance-time graph corresponding to the speed-time graph of Fig.3.8 is given in Fig.3.9

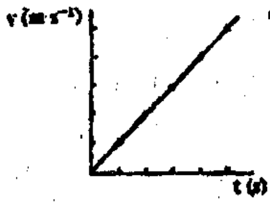


Fig. 3.8

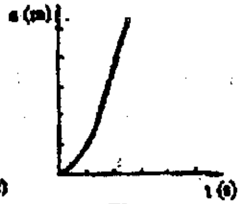


Fig. 3.9

Notice that the distance-time graph in the present case is not a straight line but a parabola.

We will continue to look at the motion of the car beyond the travel time of 10 s. The speedometer readings after the first 10 s give the speeds of the car as listed inTable3.4.

Table 3.4

Time	Speed
12 s	60 ms^{-1}
18 s	60 m s^{-1}
24 s	60 m s^{-1}
30 s	60 m s^{-1}
32 s	50 m s^{-1}
34 s	40 m s^{-1}
36 s	30 m s^{-1}
38 s	20 m s^{-1}
40 s	10 m s^{-1}
42 s	0 m s^{-1}

The speed-time graph of the data in Table 3.3 and 3.4 is shown in Fig. 3.10, and it is the graph for the entire time of travel. Note that the car stopped moving after 42 s of travel time. Fig. 3.10 shows that for the first 10 s the car was moving with increasing speed. It is, therefore, accelerating and since the increment of speed in equal time interval for this part of motion is the same, the acceleration is constant. From 10 s to 30 s time interval, the graph is a straight line parallel to the horizontal axis. This signifies that the speed of the car did not change. Then the last part of the graph, which is a straight line sloping down to the horizontal axis, indicates that at 30 s the car began to decrease its speed uniformly and finally at 42 s it stopped. In this last portion of motion, the car was moving with a constant negative acceleration; in layman terms, the car was decelerating after 30 s.

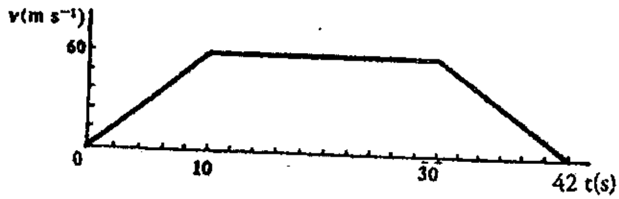


Fig.3.10

One can easily convert the speed-time graph of Fig. 3.10 into a distance-time graph.

Let us now try and describe the motions represented by the distance-time graphs of Fig. 3.6. The two graphs represent the motions, respectively, of two runners A and B who took part in a race. The race course is a straight path. The graphs show that runner A had run at a faster pace than B, in fact twice as fast. By the time A had reached the winning post, B had covered only half the distance and at that distance he stopped moving altogether.

Fig. 3.11 (a), (b), (c), show other speed-time graphs which are different from the ones we have discussed previously. Fig. 3.11 (a) is that of a lift whose speed increased uniformly along OA and then decreased uniformly in speed to rest along AB. The distance travelled is the area of the triangle OAB.

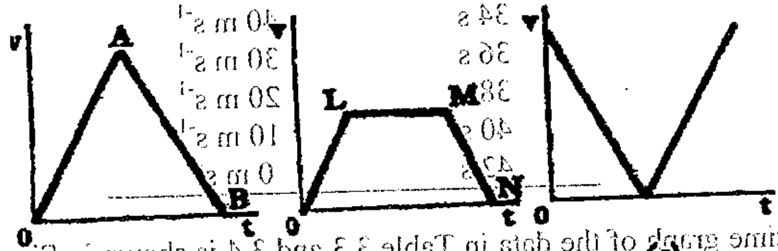


Fig. 3.11

Fig. 3.11 (b), shows the speed-time graph of a train between two stations. The train's speed increased along OL as it pulled away from the station; it then travelled with fairly uniform speed for a time along LM, finally decreased in speed and came to rest at the second station. The distance travelled is the area under the curve OLMN.

Fig. 3.11 (c) is the speed-time graph of a tennis ball thrown vertically upwards. The speed decreased uniformly as the ball rose upward till it came to a stop at the top of the flight. It then fell to the ground and on this downward flight the speed of the ball uniformly increased. When the thrower caught the ball its speed was the same as when it left the hand of the thrower.

The figure shows what happened when a ball was dropped whilst at the same instant a second ball was thrown sideways. When an object is projected horizontally, it travels in the horizontal direction with a constant velocity v_{x0} (as indicated by the constant horizontal vector lengths in the drawing). While traveling horizontally, the object is also falling under the influence of gravity, $a_y=g$. The combined motions produce a curved path. Note that the downward motion produce a curved path. Note that the downward motion of the projected ball is the same as that of a dropped ball.

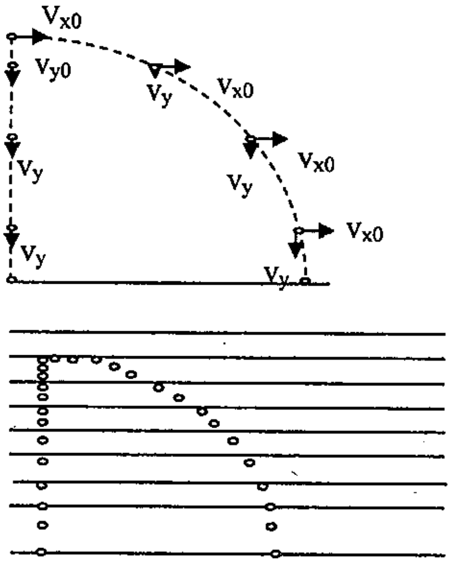


Fig. 3.12

The position marks on two edges of the figure 3.12 show that
1.Both balls hit the ground at the same time: the downward acceleration of both balls was exactly the same.
2. Horizontally, the second ball moves over the ground at a constant speed.
These results suggest that the **horizontal** and **vertical** movements of a falling object are quite independent of each other.

Motion graphs and their interpretation

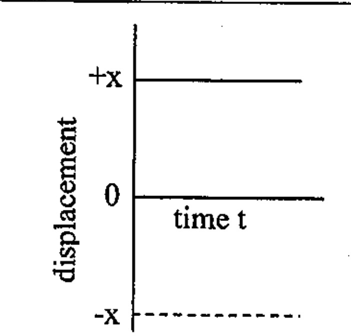
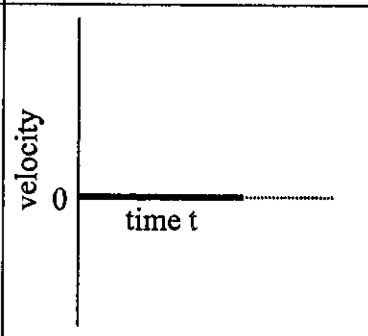
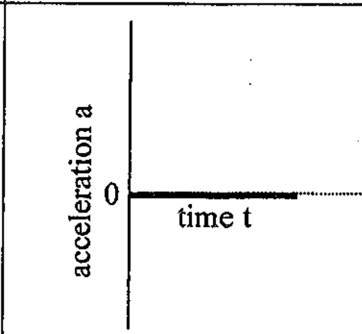
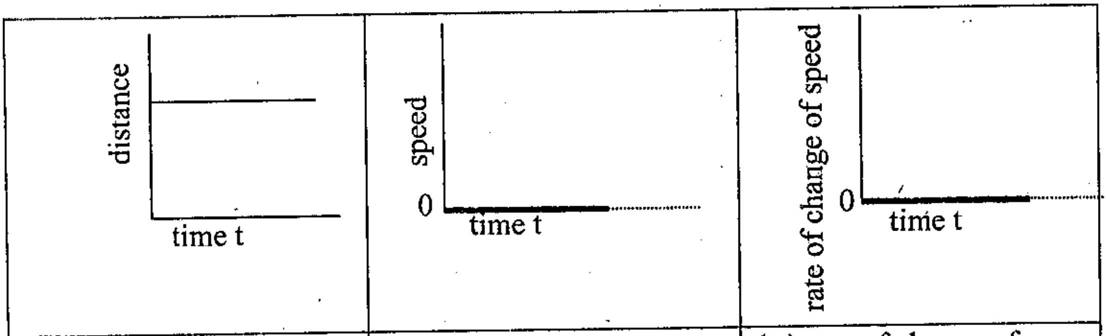
steady displacement at rest in fixed position	no velocity	no acceleration
 	 	
(a) displacement-time	(b) velocity-time	(c) acceleration-time

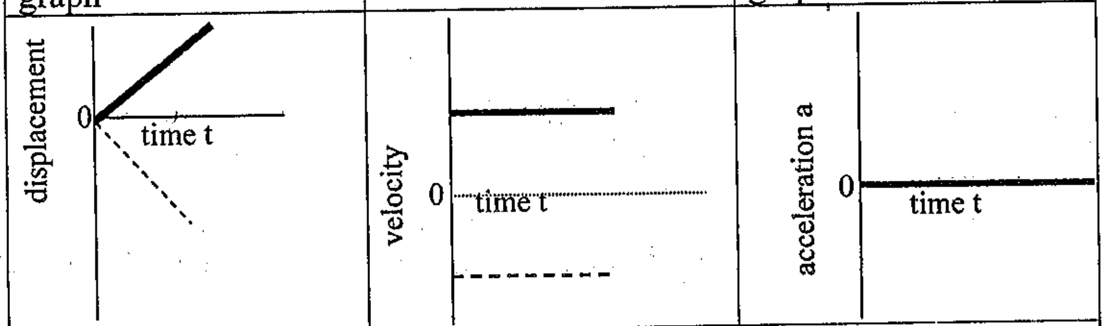
Fig. 3.13 **Motion** graphs for a stationary object at a distance x from the observer



(a)distance-time	(b) speed-time	(c) rate of change of speed-time

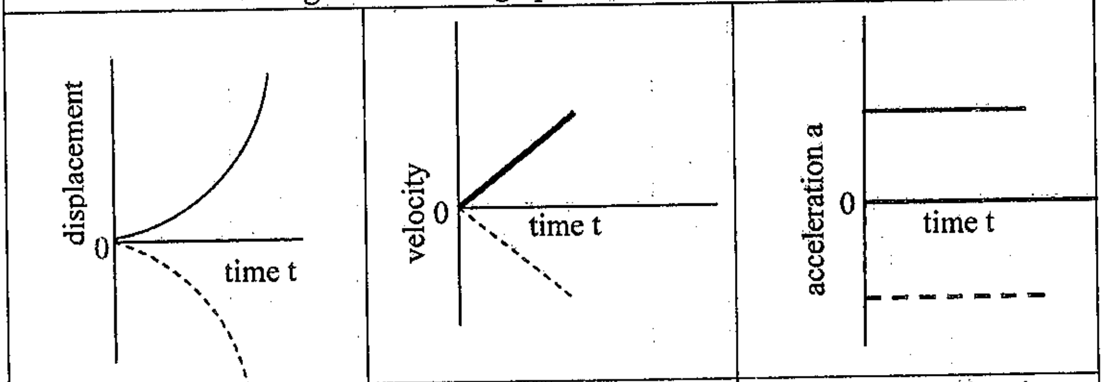
Fig. 3.14 **Scalar** graphs for a stationary object at a fixed distance from the observer

displacement-time graph	velocity-time graph	acceleration-time graph



(a) Regular increase in displacement	(b) Uniform velocity	(c) No acceleration

Fig 3.15 **Motion** graphs for uniform velocity



(a) Irregular increase in displacement with time	(b) Regular increase in velocity with time	(c) Uniform acceleration

Fig 3.16 **Motion** graphs for uniform acceleration

Example (10) A train starts from a station A, with an acceleration of 0.2 m s^{-2} and attains its maximum speed in 1.5 min. After continuing at this speed for 4 min it is uniformly retarded for 45 s before coming to rest in station B. Find ,by drawing a suitable graph,

(a) the distance between A and B in km,

(b) the maximum speed in km h^{-1} ,

(c) the average speed in m s^{-1}.

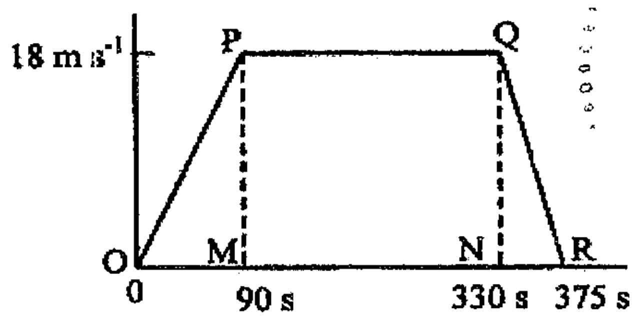
initial velocity $v_0 = 0$ m s^{-1}

acceleration $a = 0.2$ m s^{-2}

time taken $t = 1.5$min $= 1.5 \times 60$ s

$$v = v_0 + at$$
$$= 0 + 0.2 \times 1.5 \times 60$$
$$= 18 \text{ ms}^{-1}$$

Therefore, the maximum speed $= 18 \text{ ms}^{-1}$



Using the formula (Area of $\Delta = \frac{1}{2} base \times height$), one gets

Area OPQR = Area (ΔOMP+ ΔNQR) +Area of rectangular MNQP

$$= \frac{1}{2}(OM \times PM) + \frac{1}{2}(NR \times QN) + (PM \times MN)$$

Since PM=QN, we obtain

$$\text{Area OPQR} = PM\left[\frac{(OM + NR)}{2} + MN\right]$$

$$= 18\left[\frac{(90 + 45)}{2} + 240\right]$$

$$= 5535 \quad \text{m} = 5.535 \text{ km}$$

(a) The distance between A and B = 5.535 km

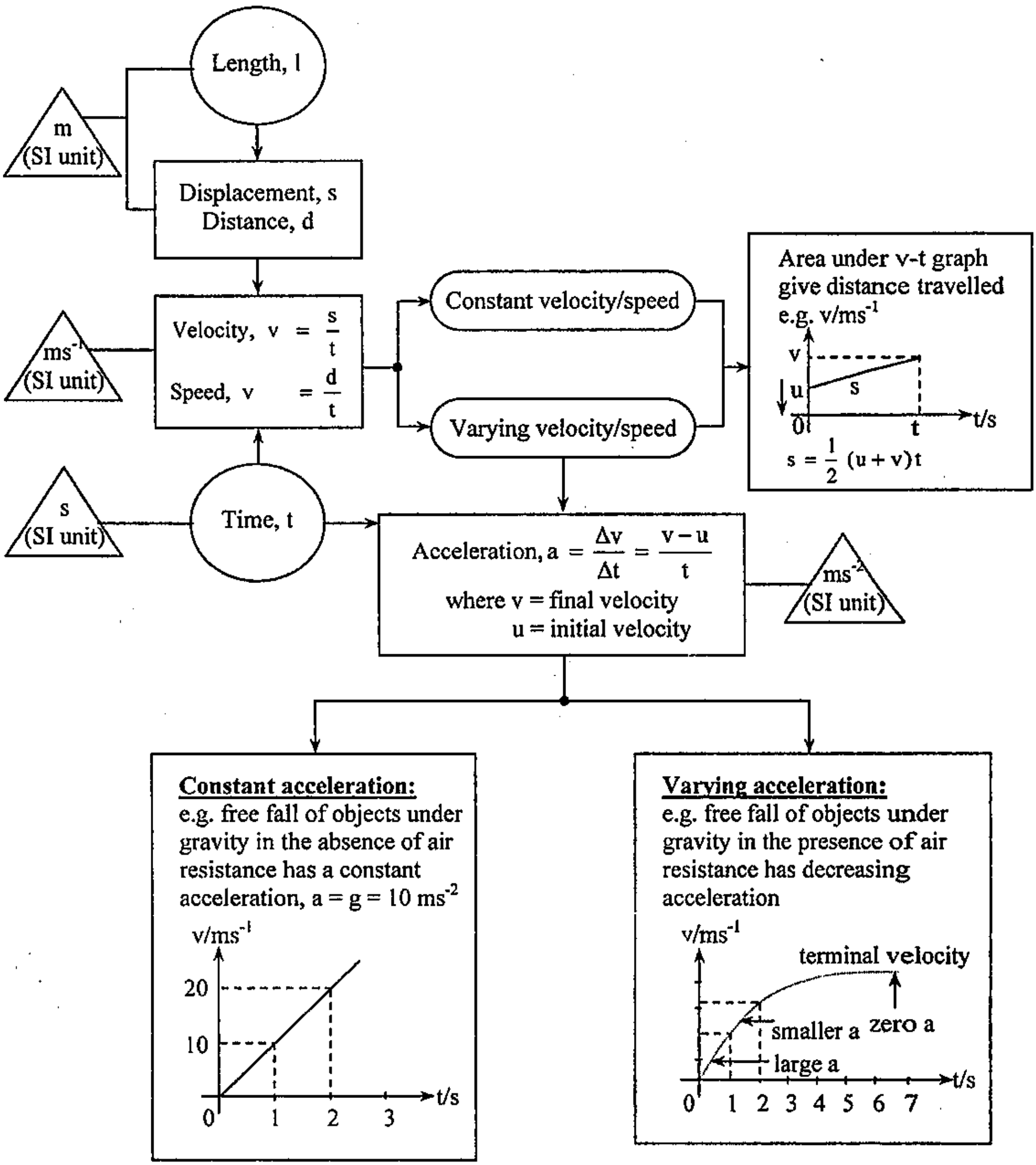
(b) The maximum speed = 18 m s^{-1}
= 64.8 km h^{-1}

(c) The average speed $\bar{v} = \frac{s}{t}$

$$= \frac{5535}{375}$$

$$= 14.76 \text{ m s}^{-1}$$

Concept Map (Length, *l*)



EXERCISES

1. Which of the following quantities are scalars and which are vectors?

   (a) speed (b) velocity (c) average velocity (d) acceleration (e) displacement.

2. The following equations are used to describe the motion of a body.

$$s = v_0 t + \frac{1}{2} at^2$$

$$v = v_0 + at$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$s = \bar{v}\, t$$

   Express them in vector form. (The symbols carry their usual meanings.) Are the above equations true for motion with non-uniform acceleration?

3. A person goes from his house to a nearby shop at the corner of the street and then returns home. Can you say that the distance travelled by him is equal to the magnitude of his displacement?

4. In a one-round-about-town walking race the starting point is the same as the finishing point. Whose magnitude of displacement is greater? The one who completes the race or the one who gives up half-way?

5. A body moves along a straight line. What can you say about its speed and its velocity?

6. Is the formula for average velocity $\bar{v} = \frac{v_0 + v}{2}$ always true?

7. For what particular case do the two equations $s = \bar{v}\, t$ and $s = vt$ become equivalent?

8. What form will the equations in question number 2 assume for motion with constant velocity?

9. Check whether the following statements are true or not.

   (a) "If the speed changes, the velocity also changes."

   (b) "Although speed changes, there is no acceleration."

   (c) "Even though velocity changes, the speed may or may not change."

(d)"If the speed does not change, but the direction changes, there will be acceleration."

10. Fill in the blanks.

Velocity is (a) position in a particular direction and (b) is the rate of change of velocity.

Since displacement has both magnitude and direction, displacement is a (c) quantity,

11. Choose the correct answer from (a), (b), (c) and (d), Displacement is

(a) rate of change of velocity,

(b) magnitude of acceleration,

(c) distance,

(d) change of position in a certain direction.

12. Define average velocity.

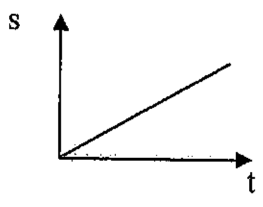
13. Define speed and velocity such that the two may be distinguished.,

14. What is the meaning of the unit cm s^{-2} ?

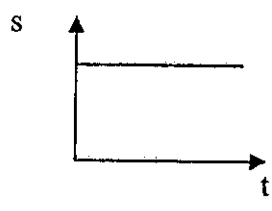
15. What component of a body's motion does the gradient (slope) of a displacement-time graph represent?

16. How can a body's acceleration be calculated from a velocity from a velocity time graph of its motion?

17. Which graph represents the motion of a body moving with constant velocity?

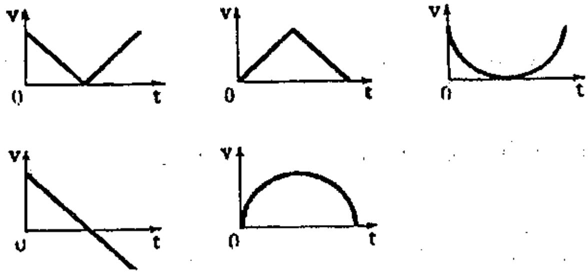


(a)

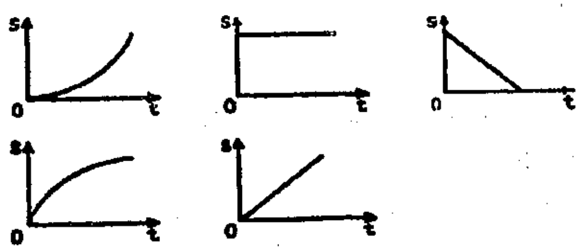


(b)

18. A ball is projected vertically upward. Which graph represents the velocity of the ball during its flight when air resistance is ignored?



19. A train decelerates at a constant rate during a period commencing at t = 0. Which graph represents the displacement of the train?



20. Find the average velocity of a sprinter who won the gold medal in a 100 m race with a time of 11.5 s.

21. In a 400 m race, the person running in the innermost lane clocked 50 s and won the gold medal. Find his average velocity. Is the magnitude of the average velocity the same as the value of average speed? (Hint: *For the innermost lane the starting point is the same as the finishing point.)*

22. An object moves with an initial velocity of 5 m s^{-1}. After 10 s its velocity is 10 m s^{-1}. If the object moves with constant acceleration in a straight line, find

    (a) its average velocity,

    (b) the distance travelled in 10 s and

    (c) its acceleration.

23. An object moves with an initial velocity of 4 m s^{-1}. If it moves with a uniform acceleration of 2 m s^{-2} find its displacement after 20 s.

24. A particle with initial velocity of 10 m s^{-1} travels in a straight line and stops completely after 12 s. Find the uniform acceleration of the particle.

25. A particle starting from rest moves along a straight line with a constant acceleration of 2 ms^{-2}.What is the velocity of the particle 9s after it started from rest?

26. A car is travelling with a constant velocity of 6 m s^{-1}. The driver applies the brakes as he sees a cow which is at a distance of 24 m from the car. Find the acceleration of the car if it stops just in front of the cow.

27. A body moving with a constant acceleration reaches the velocity 4 m s^{-1} after 10 s. If the body starts from rest, find the respective velocities after (a) 12 s (b) 14s (c) 16s and (d) 18s. Draw the velocity-time graph for the interval 10 s to 18 s. From the graph, find the velocities at 8 s and 20 s respectively.

28. Draw a graph of velocity against time for a body which starts with an initial velocity of 4 m s^{-1} and continues to move with an acceleration of 1.5 m s^{-2} for 6 s. Show how you would find from the graph:

(a) the average velocity,

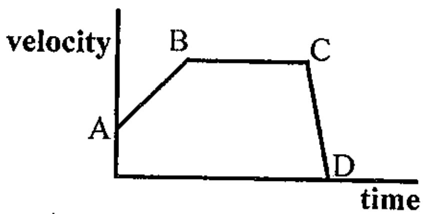
(b) the distance moved in those 6s.

29. A body starts from rest and accelerates at 3 m s^{-2}, for 4 s. Its velocity remains constant at the maximum value so reached for 7 s and it finally comes to rest with uniform negative acceleration after another 5 s. Find by the graphical method:

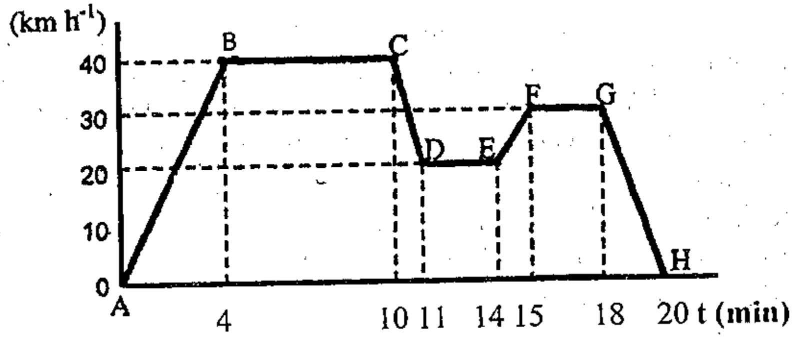
(a) the distance moved during each stage of motion,

(b) the average velocity over the whole period.

30. The graph in figure shows the relationship between velocity and time for a moving body. What kind of motion is represented by (a) AB, (b) BC, (c) CD ?



31. The figure represents graphically the velocity of a car moving along a straight level road over a period of twenty minutes.



(a) Describe the motion of the car between A and B.
(b) Describe the motion of the car between D and E.
(c) How far has the car travelled between B and D?
(d) Calculate the acceleration of the car between B and D.

CHAPTER 4

FORCES

The portion describing motion which is called kinematics was discussed in Chapter 3. The concepts such as displacement, velocity and acceleration, which are required for the discussion of kinematics, have also been defined precisely.

Besides kinematics, which describes motion, dynamics, which explains motion, is also important in the study of motion. A fundamental concept in dynamics is force. In this chapter, together with force concept, mass, which is another concept required for the explanation of motion, will be defined; and the relation between force and mass will also be presented and discussed.

The use of the word "force" is quite common in everyday life. In ordinary usage one comes across such statements as: "his writing consists of many forceful words", "the atom bomb explodes with tremendous force", "collision with great force", "the earthquake of great intensity and force" in which the meaning of the word force is not exact or well defined. From the point of view of physics such general usages have mixed up the meanings of force, energy and intensity. However, force is defined precisely and explicitly in physics.

Although a force is commonly understood as a push or a pull, it cannot be said that this definition is sufficient and complete. In order that the meaning of force be more complete and exact, the definition must be modified. Force is defined precisely by Newton's laws of motion. The exact relation of force and mass is also derived from one of these laws.

4.1 NEWTON'S LAWS OF MOTION

Firstly, Newton's three laws of motion will be stated in words and then expressed in mathematical forms.

First Law

When no net external force acts upon it, a particle at rest will remain at rest and a particle in motion at a constant velocity will continue to move with the same constant velocity.

In mathematical form

$$\text{If } \vec{F} = 0 \text{ then } \vec{a} = 0 \qquad (4.1)$$

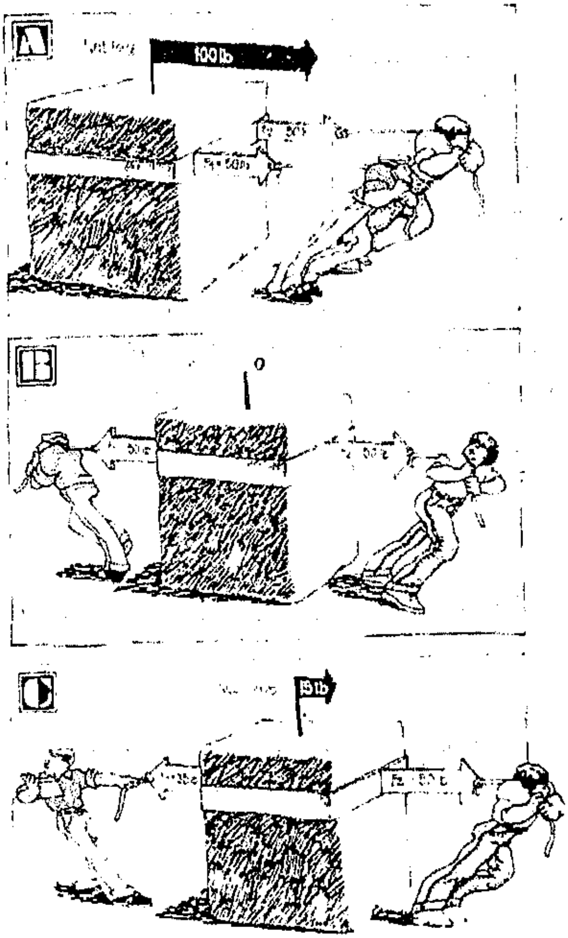
Second Law

The net external force acting upon a particle is equal to the product of the mass and the acceleration of the particle.

$$\vec{F} = m\vec{a} \qquad (4.2)$$

In the above equations $\vec{F}$ is the net external force.

The following figures A , B ,C show to get the clear understanding of the term the net force.



Third Law

Whenever two particles interact, the force exerted by the second on the first is equal in magnitude and opposite in direction to the force exerted by the first on the second.

$$\vec{F}_{\text{Second on first}} = -\vec{F}_{\text{First on second}} \qquad (4.3)$$

Let us begin with the discussion of the first law. This law means that if a net external force acts on a particle, the initial state of the motion of the particle will be changed. Although two or more forces act simultaneously on the particle, its state may not be changed. For example, if two equal and opposite forces act simultaneously on a particle at rest, it will remain at rest. In this case, the net force acting on the particle is zero since the two forces cancel out. Therefore, the initial state of the particle is totally unchanged.

If there is no external force a particle at rest will remain at rest. Even a person with no knowledge of physics will readily accept this fact. A lay person will think that every motion is caused by force or a body will be in motion only when force acts upon it. Even in the absence of an acting force a body can still be in motion at constant velocity; but it would be difficult to relate this fact with one's own common sense. According to the statement of the first law, if there is no net external force of any kind, a particle initially in motion at a constant velocity will continue to remain in the same state of motion. Again, although external forces are simultaneously acting on a particle, if the resultant of the applied forces is zero, the initial state of the particle will not be changed. It is more correct to say "force changes the state of motion" rather than to say "force causes motion." This is one property of force.

Newton's first law expresses the idea of inertia. The inertia of a body is its reluctance to start moving, and its reluctance to stop once it has begun moving. (e.g. It is much easier to push a 5 kg mass than a 500 kg mass because a 500 kg mass has a greater inertia than 5 kg mass. It is often noted that a running boy of mass 50 kg is easier to stop than a running footballer of mass 90 kg) This property of the body is also demonstrated by a driver in a car who is jerked forward when the vehicle stops suddenly (Fig. 4.1).

moving with constant velocity sudden stop

Fig. 4.1

It would be difficult, at the first encounter, to understand Newton's second law of motion. This is because two concepts, namely force and mass, which had not yet been precisely defined earlier, are included in this law. However, by combining the second and the third laws, force and mass can be precisely defined.

The mathematical equation: $\vec{F} = m\,\vec{a}$, which describes the second law is a vector equation; and according to this equation the direction of the acceleration is the same as that of the force. Moreover, this equation will be a vector equation only if the mass of the particle is a scalar.

The second law may also be viewed as follows.

If a net external force acts upon a particle, the force produces acceleration, and the ratio of the force to the acceleration is the mass of the particle.

Let us consider a particle. Assume that a force $\vec{F}_1$ produces an acceleration $\vec{a}_1$ when applied to the particle, and a force $\vec{F}_2$ applied to the same particle produces an acceleration $\vec{a}_2$. Hence, according to Newton's second law we have

$$\frac{\vec{F}_1}{\vec{a}_1} = \frac{\vec{F}_2}{\vec{a}_2} = m = \text{constant} \qquad (4.4)$$

where the constant m is the mass of the particle. If $F_1 > F_2$ then $a_1 > a_2$. It means that as the magnitude of the force acting on a particle increases, the acceleration of the particle will increase accordingly. It is equivalent to saying that acceleration is directly proportional to force. In symbols, $\vec{a} \propto \vec{F}$.

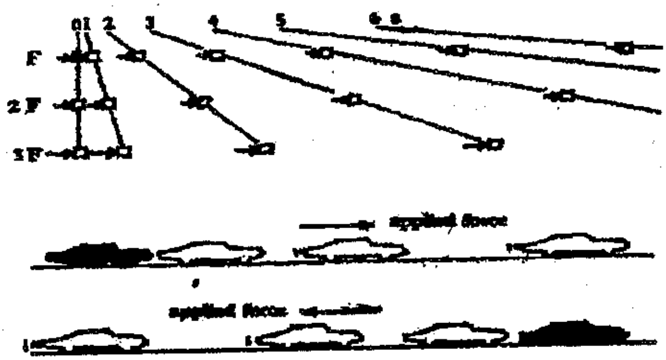


Fig. 4.2.

In order to discuss and explain Newton's third law the following cases will be considered. Consider a man sitting on a chair. The man exerts a force which is equal to his bodyweight on the chair. At the same time the chair exerts a reaction force, which is equal in magnitude and opposite in direction, on the man. In this case, if one of the said forces is termed "action" the other is called "reaction". If the force exerted by the man is called "action", the force exerted by the chair should be called "reaction", or it can be said that the force exerted by the chair is "action" when the force exerted by the man is taken as "reaction". There is no strict rule as to which one should be taken as "action", and which one as "reaction".

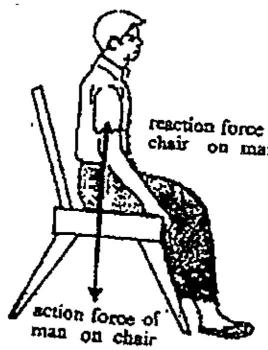


Fig. 4.3

As another case, let us look at a man firing a gun at a target. The gun exerts a force on the bullet, and the bullet exerts an equal reaction force on the gun. This gives rise to a recoil force to the shoulder. The two forces are equal in magnitude but opposite in direction.

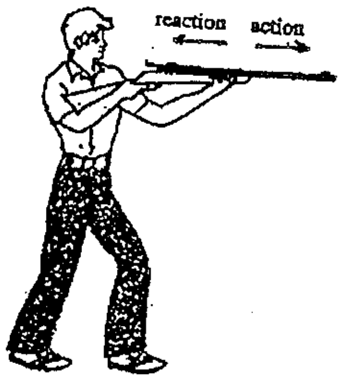


Fig . 4.4

In each of the above cases action and reaction act as a pair at the same time but the pair of forces acts on two separate objects. Important facts relating to force which arise from Newton's third law are as follows:

- It is not a single force acting by itself but a pair of forces acting simultaneously. This pair of forces is action-reaction pair.
- Action-reaction pair does not act on a single object but acts on two separate objects.
- Action force and reaction force cannot cancel out each other.

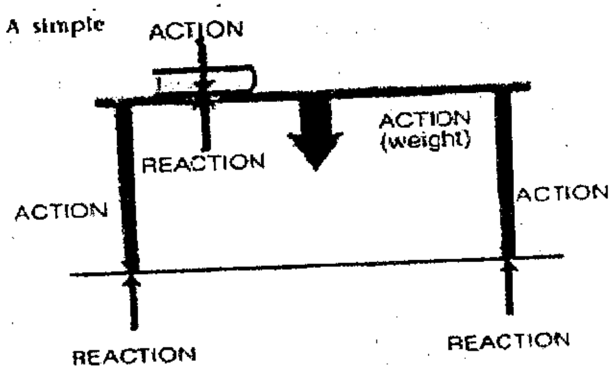


Fig. 4.5

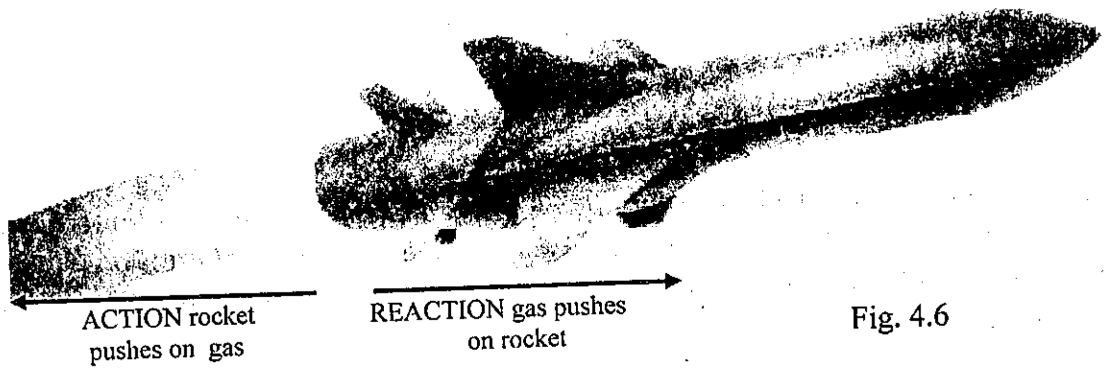


Fig. 4.6

According to the above discussions the three laws of Newton can be designated, as follows:

- First law as law of inertia;
- Second law as law of force and acceleration;
- Third law as law of action and reaction.

Usefulness of the second law of motion will be shown through derivation of units of force.

Units of Force

Newton (N) and dyne which are units of force can now be defined explicitly from $\vec{F} = m\vec{a}$. A body of mass 1 kg resting on a frictionless horizontal plane will be considered.A force that is acting on this 1 kg mass to give it an acceleration of 1 m s^{-2} is called 1newton. (1N= 1 kg m s^{-2})

Similarly, a force that is acting on 1 g mass to give it an acceleration of 1 cm s^{-2} is called 1 dyne. (1 dyne = 1 g cm s^{-2})

Since $1\ g = 10^{-3}$ kg and 1 cm= 10^{-2} m,

$$1\ \text{dyne} = 10^{-3} \times 10^{-2} = 10^{-5}\ \text{newton or 1 newton} = 10^{5}\text{dynes}$$

The newton and the dyne are particularly useful units of force because acceleration of the bodies on which the forces act can be obtained directly from their definitions.

Not only units of force but the definition of the slug, which is the unit of mass, can also be derived from Newton's second law. The slug is the unit of mass in British engineering system. It is defined as follows: when 1 pound force acts on a body and if the acceleration of the body is 1 ft s^{-2}, then the mass of the body is called 1 slug. Therefore, if a force of 2 pounds acts on a mass of 1 slug the acceleration thus produced will be 2 ft s^{-2}; a force of 3 pounds will give a mass of 1 slug an acceleration of 3 ft s^{-2}; a force of 4 pounds will give a mass of 1 slug an acceleration of 4 ft s^{-2} and so on. A mass of 1 slug is approximately equal to a mass of 14.6 kg.

Units of force, mass, length and time which are all useful to the application of the equation F = ma are shown in the following table in SI, MKS, CGS and British engineering systems.

Table 4.1

Quantity	SI	MKS	CGS	British
Force	newton	newton	dyne	pound
Mass	kilogram	kilogram	gram	slug
Length	metre	metre	centimetre	foot
Time	second	second	second	second

4.2 GRAVITATIONAL FORCE AND NEWTON'S LAW OF GRAVITATION

Newton was able to point out and express precisely that all bodies in the universe are attracting one another. Gravitational force causes bodies which are above the earth's surface to fall onto the earth's surface. The gravitational force enables the moon to go round the earth and the earth to go round the sun. These are some examples of the effects of gravitational force.

Newton stated the gravitational law as follows:

Everybody attracts every other body in the universe. The gravitational force between the two bodies is directly proportional to the product of the masses and inversely proportional to the square of the distance between them. In symbols

$$F \propto \frac{m_1 m_2}{r^2} \qquad (4.5)$$

where F is the gravitational force between the masses m_1 and m_2 whose distance apart is r. If it is expressed as an equation in vector notation:

$$\vec{F} = -G\frac{m_1 m_2}{r^2}\hat{r} \qquad (4.6)$$

where G is a constant which is the same for all bodies in the universe. According to experimental measurements the value of G in MKS system is found to be $6.67\times10^{-11} m^3\, kg^{-1}\, s^{-2}$.

$\hat{r} = \frac{\vec{r}}{r}$; $|\hat{r}|=1$, $\hat{r}$ = **unit vector** (A vector that has the magnitude 1).

Applications of Newton's law of gravitation

(a) Tides

The attraction of the moon and the sun upon water of the earth cause tides.

(b) Orbits of satellites round the earth

Satellites can be launched from the earth's surface to circle the earth.
They are kept in their orbit by the gravitational attraction of the earth.

Example (1) If 10 N force acts upon a 2 kg mass, find the acceleration produced.

Since F= 10 N and m = 2kg, we have

F = ma

10 N = 2 kg× a

$a = 5N\, kg^{-1} = 5ms^{-2}$

(Since MKS system is used the unit of acceleration is $m\, s^{-2}$)

Example(2) Find the magnitude of a force needed to accelerate an electron from rest to a velocity of 10^9 cm s^{-1} in 10 s. (An electron mass = 9.1×10^{-28} g)

The electron starts from rest and is uniformly accelerated to velocity v in time t, and

acceleration $$a = \frac{v}{t}$$

Substituting the values $v = 10^9$ cm s^{-1} and t = 10 s in the equation, we have

$$a = \frac{10^9}{10} = 10^8 \text{ cm s}^{-2}$$

Using this value of a and the value $m = 9.1 \times 10^{-28}$ g, we have

$$F = ma$$
$$= 9.1 \times 10^{-28} \times 10^8$$
$$= 9.1 \times 10^{-20} \text{ dynes}$$

(Since CGS system is used, the force is in dynes.)

Example (3) A 12 lb force gives a body an acceleration of 4 ft s^{-2}. Find the mass of the body.

Since F = 12 lb and a = 4 ft s^{-2}, we have $F = ma$

$$12 = m \times 4$$
$$m = 3 \text{ sl}$$

(As the British engineering system is used, the mass is expressed in slug.)

Example (4) A 4 kg ball is at rest on a perfectly smooth plane which is in a horizontal position. A force of 10 N is applied horizontally to the ball. Find the speed of the ball and the distance travelled after 6s.

From F = ma, we have

$$a = \frac{F}{m}$$

$$a = \frac{10}{4}$$

$= 2.5$ m s^{-2} is obtained.

According to Newton's second law, the direction of a and F are the same. The speed of the ball 6 s after the force started to act is

$$v = at$$
$$= 2.5 \times 6 = 15 \text{m s}^{-1}$$

and the distance travelled is

$s = \frac{1}{2} at^2 = \frac{1}{2} \times 2.5 \times 6^2$

$= \frac{1}{2}\ 2.5 \times 36 = 45\text{m}$

(Since the ball started from rest the initial speed of the body is assumed to be $v_0=0$.)

Example (5) A 2 kg ball is moving with an initial speed of 15 m s^{-1} on a rough plane which is in a horizontal position, and gradually slows down and stops after travelling 20 m. Find the magnitude of the force which resists the motion of the ball.

The speed of the ball changes from 15 m s^{-1} to 0 ms^{-1} after travelling 20 m. Thus, we have

$$v^2 = v_0^2 + 2as$$

$$0 = (15)^2 + 2\,a \times 20$$

$$0 = 225 + 40\,a$$

$$a = \frac{-225}{40} = -5.6\text{ m s}^{-2}$$

As indicated by the minus sign it is found to be a negative acceleration. The force resisting the motion of the ball is

$$\vec{F} = m\vec{a}$$

$$= 2 \times (-5.6) = -11.2\text{ N}$$

The minus sign indicates that the direction of the force is opposite to that of the motion of the ball. The magnitude of the force is 11.2 N.

Example (6) Find the gravitational force between two 1 kg masses held 1 m apart.

Since $m_1 = m_2 = 1$ kg, $r = 1$m and $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$, the force acting between the two masses is

$$F = G\frac{m_1 m_2}{r^2}$$

$$= 6.67 \times 10^{-11} \times \frac{1\times1}{(1)^2}$$

$$= 6.67 \times 10^{-11}\text{ N}$$

Example (7) Express the value of G in the CGS units.

$$G = 6.67 \times 10^{-11} \frac{m^3}{kg\,s^2}$$

$$= 6.67 \times 10^{-11} \times \frac{(10^2\ cm)^3}{(10^3\ g)\times(1\,s)^2}$$

$$= \frac{6.67\times10^{-11}\times10^6}{10^3} \frac{cm^3}{g\,s^2} \qquad = 6.67 \times 10^{-8}\,cm^3\,g^{-1}s^{-2}$$

4.3 DIFFERENT KINDS OF FORCES

So far, only four fundamental forces are known. They are the gravitational force, weak interaction, electromagnetic force and nuclear force. Nuclear force is the strongest and the gravitational force is the weakest of these forces. The electromagnetic force is the second strongest force. Amongst the four forces the gravitational and the electromagnetic forces are long-range forces and the remaining two forces are short-range forces.

The gravitational force acts between particles having mass, the electromagnetic force acts between charged particles, the weak interaction between all elementary particles (sub-atomic particles), and the nuclear force acts between some elementary particles such as proton, neutron, pion and strange particles.

The fundamental force encountered in the study of mechanics is the gravitational force. The electromagnetic force will be encountered in the study of electromagnetics. Only these two fundamental forces will be studied in elementary physics.

In the study of mechanics, apart from gravitational force, frictional force and elastic force will also be encountered. However, unlike gravitational force, these two mechanical forces are not fundamental forces.

When a body is placed on a floor, the bottom part of the body and the surface of the floor are in contact, and there is a force, between the two surfaces which resists the motion of the body. The force that acts to resist the motion of the body is frictional force. The frictional force depends, in a complicated manner, on the smoothness and cleanliness of the surfaces, the force pressing the two surfaces together and the speed of the body.

Although frictional force is not a fundamental type of force, it is very important in mechanics. Since the effects of frictional force play a major role in the effective utilization of machineries, prior consideration should be given to frictional force in the construction of machineries. Frictional force causes undesired effects in many cases. Frictional force reduces efficiency of machines. Therefore, great efforts should be made to minimize frictional force as much as possible.

Advantages as well as disadvantages are associated with frictional force. Ability of human beings to walk on the earth's surface is due to friction. The possibility of putting nails in wood and the ability of belts to rotate pulleys and wheels is also due to friction.

4.4 MASS AND WEIGHT

It is known that a body dropped from a height above the surface of the earth will fall (towards the centre of the earth) onto the ground. If the height is not too far from the ground the body will fall at a constant acceleration. The acceleration due to the gravitational force is called acceleration due to gravity and it is represented by the symbol g. The attracting force of the earth acting on a body is defined as the weight of the body. Let the mass of the body be m ; and if a = g is substituted in Newton's second law: F = ma, the gravitational force acting on the body or the weight of the body is found to be

$$w = mg \qquad (4.7)$$

This relation is true not only for freely falling bodies but also for bodies on the ground.

According to the relation w = mg, it is clear that mass and weight are different quantities. Weight is force and its units are newton, dyne and pound while the units of mass are kilogram, gram and slug. Those who begin to study elementary physics usually confuse mass and weight. There is no reason to make such a mistake once w = mg is learnt.

Mass is the quantity of matter in a body. Mass is also a measure of inertia. The mass of this type defined from this point of view is called inertial mass while the mass of the type defined by m = w /g is called gravitational mass. It is known from exact measurements that gravitational and inertial masses are equal. Newton believed that this equality was a mere coincidence. However, Einstein had assumed that it was not a coincidence but an absolute equality, and that this was a fundamental fact. Based on this fact Einstein was able to formulate his famous general theory of relativity.

Without differentiating between inertial mass and gravitational mass, only the word "mass" will be used in this book.

Mass is always a constant. Wherever a body may be, there is no change in the value of the mass of the body. But the weight of the body can change according to its position. For example, if a body be carried from the equator($g = 9.78\ m\ s^{-2}$) to the pole($g = 9.83\ m\ s^{-2}$), the weight of the body will vary slightly. Similarly, if it be carried from a valley to a height on a mountain, the weight of the body will also change slightly. This variation of weight is due to the change in the value of the gravitational force as the distance from the body to the centre of the earth changes. (It should be noticed here that the earth is not a perfect sphere but slightly flattened at the poles).

Example (8) Find the value of the acceleration due to gravity g.

Let the mass of a body at the earth's surface be m and that of the earth be M. The distance from the body to the centre of the earth is just the radius of the earth; and if that distance is denoted by r, the gravitational force acting on the body is

$$F = G\frac{mM}{r^2}$$

According to definition this gravitational force is the weight of the body and since the weight of the body is mg, we have

$$mg = G\frac{mM}{r^2}$$

$$g = \frac{GM}{r^2}$$

(Note that the properties of the body are totally excluded in the above relation.)

If $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$, $M = 5.97 \times 10^{24}$ kg and $r = 6.37 \times 10^6$ m are used, we have

$$g = 6.67 \times 10^{-11} \times \frac{5.97\times10^{24}}{(6.37\times10^6)^2}$$

$$= \frac{3.98\times10^{14}}{(6.37\times10^6)^2}$$

$$= 9.8\ m\ s^{-2}$$

In the CGS system the value of g is

$$g = 9.8 \times 10^2\ cm\ s^{-2} = 980\ cm\ s^{-2}$$

But the value of g is often taken approximately as 10 m s^{-2} or 10^3 cm s^{-2} in order to simplify calculations. The value of g in the FPS system is 32 ft s^{-2}.

Example (9) 5 kg and 10 kg masses are at a place on the top of a hill where g has the value 9.75 ms^{-2}. Find the weights of the masses.

Since weight of a body is the gravitational force acting on the body, we have

$$\omega = mg$$

Hence, the weight of 5 kg mass is,

$$\omega_{5kg} = 5 \times 9.75$$
$$= 48.75 \text{ N}$$

and the weight of 10 kg mass is

$$\omega_{10kg} = 10 \times 9.75$$
$$= 97.5 \text{ N}$$

Example (10) Find the mass of a body weighing 3232 lb.

$$\omega = mg$$
$$m = \frac{\omega}{g}$$
$$= \frac{3232}{32} = 101 \text{ sl}$$

Example (11) If a body weighing 320 lb is moving at an acceleration of 10 ft s^{-2}, find the net force acting on the body.

The mass of the body is $m = \frac{\omega}{g}$

$$= \frac{320}{32} = 10 \text{ sl}$$

and the net force acting on the body is

$$F = ma = 10 \times 10 = 100 \text{ lb}$$

4.5 DENSITY, RELATIVE DENSITY AND PRESSURE

Mechanics of particles was treated in previous chapters. In this chapter mechanics of liquids will be discussed. In the study of mechanics of particles, the type of motion and the state of equilibrium can be predicted exactly if the forces acting on the particles are known. Similar methods of approach will be used in the study of mechanics of liquids. However, there is one thing different in the study of mechanics of liquids. Unlike solids a liquid has no regular shape. The shape of a liquid changes according to the shape of its container. Therefore, in the detailed study of mechanics of liquids the geometrical problems which arise from the variation of shape will become more and more complicated. The concepts of density and pressure, instead of mass and force, will be used to avoid these problems and to understand mechanics of liquids more fully.

After defining density and pressure, liquids at rest as well as non-viscous liquids which are in motion can be studied. In studying liquids at rest one must first try and understand why a body remains submerged or floats in a liquid. For liquids which are in motion Bernoulli's equation can be used. Work and energy concepts are included in this equation and it is stated in such a way that it can be applied conveniently to liquids.

The study of liquids at rest is called hydrostatics and the study of liquids in motion is called hydrodynamics. Only hydrostatics will be discussed in this chapter.

Since direct use of mass and force concepts in mechanics of liquids is inconvenient, they must be replaced by density and pressure. Density (ρ) and pressure (p) are not only non-vectors, they are also free from geometrical restrictions. Before studying liquids at rest let us first define the concepts involved.

Density

For any shape of liquid which has a known mass the volume is fixed. Therefore, the density of a liquid is defined as the ratio of its mass to volume. In symbols

$$\rho = \frac{m}{V} \qquad (4.8)$$

where ρ, m and V represent the density, the mass and the volume of the liquid respectively. In SI units density is expressed in kilograms per cubic metre (kg m^{-3}). The densities of some liquids are shown in Table 4.2.

Table 4.2

Liquid	Density ($kg\ m^{-3}$)	Temperature (°C)
Water, pure	1000	4
	958	100
Sea water	1025	15
Alcohol, ethyl	791	20
Chloroform	1490	20
Mercury	13600	0
Whole blood	1059 .5	25
Blood plasma	1026 .9	25
Milk	1030	
ice	920	

Relative Density

We have to measure mass and volume in the determination of density. Mass can be measured accurately with a laboratory balance and it can be measured with greater accuracy than volume. Therefore, the value of relative density can be measured more accurately than that of the density because it is necessary to measure only the mass and not the volume.

The ratio of the density of a body to the density of water at 4 °C is called relative density or specific gravity.

The values of relative densities can be obtained readily from Table 4.2 because the density of water at 4 °C is 1000 kg m^{-3}. For example, the relative density of mercury is just a number without units, $\frac{13\,600}{1\,000} = 13.6$.

The mass or the weight of a body of known volume can be obtained if the relative density is given. At 4 °C the mass of 1 cm^3 of water is 1 g. The weight of 1 ft^3 of water is 62.5 lb. Therefore, the mass of 1 cm^3 of mercury is 13.6×1 g = 13.6 g and the weight of 1 ft^3 of mercury will be $13.6 \times 62.5 = 850$ lb.

Relative density can also be defined by the following relation:

$$\text{relative density} = \frac{\text{mass of certain volume of body}}{\text{mass of equal volume of water}}$$

Since the mass of an object is directly proportional to its weight the relation

$$\text{Relative density} = \frac{\text{weight of body having a certain volume}}{\text{weight of equal volume of water}}$$

is also correct.

The density and relative density concepts are not only important in mechanics of liquids but also very useful in practice. When drawing the designs of bridges, flyovers and buildings, architects and engineers have to know the densities of materials which are to be used for the construction. The volume of each part of the material is calculated from the designed models. If the volume is known, the mass is obtained by multiplying the volume by the density. The weight is known when the mass is known. Only when the weight is known can one calculate how strong the foundation should be and how strong the posts (pillars) should be.

Chemists can check roughly the purity of a substance by measuring the densities.

Geologists can sometimes identify gems by measuring their densities.

Example (12) In the British engineering system the density of water is 1.94 sl ft^{-3}. Find the weight of 1 ft^3 of water.

Since the mass of 1 ft^3 of water is

$$m = 1.94 \text{ sl},$$

the weight is

$$\begin{aligned} \omega &= mg \\ &= 1.94 \text{ sl} \times 32.2 \text{ ft s}^{-2} \\ &= 62.5 \text{ lb} \end{aligned}$$

Example (13) If the volume of a metal sphere of 210 g mass is 20 cm^3 what is the density of the metal ?

The density is

$$\begin{aligned} \rho &= \frac{m}{V} \\ &= \frac{210}{20} = 10.5 \text{ g cm}^{-3} \end{aligned}$$

Example (14) Express the value of the density from example (13) in SI units.

$$\begin{aligned} \rho &= 10.5 \text{ g cm}^{-3} \\ &= 10.5 \times \frac{10^{-3}}{(10^{-2})^3} \\ &= 1.05 \times 10^4 \text{ kg m}^{-3} \end{aligned}$$

Example (15) The density of helium at 0° C is 0.178 kg m^{-3}. Find the mass of the helium gas which is in a balloon of volume 1000 m^3.

The mass of helium is $m = \rho V$

$$= 0.178 \times 10^3 = 178 \text{ kg}$$

Example (16) Find the relative density of helium at 0°C.

The relative density of helium $= \dfrac{\rho_{He}}{\rho_{water}}$

$$= \frac{0.178}{1000}$$

$$= 1.78 \times 10^{-4}$$

Example (17) Find the relative density of glycerine at 0°C (densityof glycerine = 1260 kg m^{-3}).

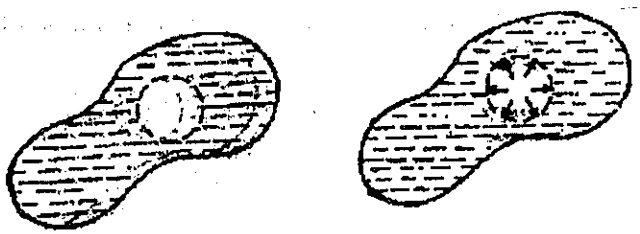
The relative density of glycerine $= \dfrac{\rho_G}{\rho_{water}}$

$$= \frac{1260}{1000}$$

$$= 1.26$$

4.6 PRESSURE

The concept of pressure will now be explained and defined. Consider a liquid which is in static equilibrium as shown in Fig.4.7 (a). Suppose the volume of liquid in the sphere is to be removed without disturbing the liquid. In order to do this, a group of forces must be .acting in some way inside the cavity of the sphere as shown in Fig. 4.7(b).



(a) (b)

Fig.4.7

The effect of this group of forces must be the same as the original effect of the liquid which has been removed. Also, each of the forces will be normal at every corresponding point on the inner surface of the sphere.

The ratio obtained by dividing the total magnitude of the normal forces by the surface area is called average pressure. Therefore,

$$\text{Average pressure} = \frac{\text{magnitude of the normal forces}}{\text{surface area}}$$

$$p = \frac{F}{A} \qquad (4.9)$$

In order to obtain the pressure at a point the radius and the area of the imaginary sphere in Fig.4.7 (b) may be compressed so that they become extremely small or the sphere may be compressed till it nearly becomes a point.

We have defined pressure and density only with reference to liquids. In reality, these concepts are applicable not only to liquids but also to solids and gases. Generally, gases and liquids are together called "fluids". Although it has been said in the beginning of this chapter that mechanics of liquids will be discussed, actually it will be more appropriate to refer to it as fluid mechanics.

The unit of pressure is the ratio of the unit of force to the unit of area. The unit of pressure in SI units is pascal (Pa)and 1 Pa =1 Nm^{-2}.The units of pressure widely used in meteorology are bar(b)and millibar (mb)and the unit of pressure used in medicine and physiology is torr or millimetre of mercury (mm Hg). (At present, Pa is frequently use in meteorology.)

The unit of pressure in the British engineering system is pounds per square inch (lb in^{-2})

The relation between the units of pressure are

$$1 \text{ lb in}^2 = 6.895 \times 10^3 \text{ Pa}$$

$$1 \text{ b} = 10^5 \text{Pa}$$

$$1 \text{ mb} = 10^{-3} \text{ b}$$

$$1 \text{ Pa} = 1.45 \times 10^{-4} \text{ lb in}^{-2} = 10^{-5} \text{ b}$$

Example (18) A 25 N force acts normally on a surface whose area is 5×10^{-4} m^2. What is the pressure?

$$\text{The pressure} \quad p = \frac{F}{A}$$

$$= \frac{25}{5\times10^{-4}}$$

$$= 5 \times 10^4 \text{ N m}^{-2}$$

$$= 5 \times 10^4 \text{ Pa}$$

Example (19) A 25 N force is applied to the piston of a syringe. The area of the piston is 10^{-4} m^2. If no liquid flows out of the syringe find the increase in pressure on the liquid.

$$\text{Increase in pressure} \quad p = \frac{F}{A}$$

$$p = \frac{25}{10^{-4}}$$

$$= 2.5 \times 10^5 \text{ Pa}$$

(There is only atmospheric pressure before the 25 N force is applied. Atmospheric pressure is explained in the next section.)

Example (20) Express 2×10^5 Pa pressure in lb in^{-2} and in bars.

$$2 \times 10^5 \text{ Pa} = 2 \times 10^5 \times (1.45 \times 10^{-4} \text{ lb in}^{-2})$$

$$= 2.9 \times 10$$

$$= 29 \text{ lb in}^{-2}$$

$$2 \times 10^5 \text{ Pa} = \frac{2\times10^5}{10^5}$$

$$= 2 \text{ b}$$

Example(21) Which of the pressures 2 x 10^5 Pa, 1 b, 10^4 m b is the greatest?

$$2 \times 10^5 \text{ Pa} = 2\text{b} > 1 \text{ b}$$

$$2 \times 10^5 \text{ Pa} = 2\text{b}$$

$$10^5 = 2 \times 10^3 \text{ m b} < 10^4 \text{ m b}$$

Therefore, the 10^4 mb pressure is the greatest.

4.7 MOMENTUM AND LAW OF CONSERVATION OF MOMENTUM

Another important concept in mechanics is momentum. Momentum (p) of a body is defined as the product of the mass of the body and its velocity.

It is written as $\vec{p} = m\vec{v}$

One fundamental law of physics is the law of conservation of momentum.

The law states:

If there is no net external force acting on a system consisting of two bodies, the sum of the momentum of the two bodies will remain constant.

When two bodies of masses m_A and m_B collide, we have

$$\vec{p}_A+\vec{p}_B = \vec{p}'_A \ \vec{p}'_B$$

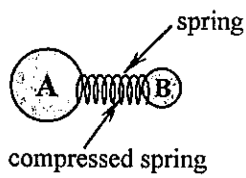
$$m_A\vec{v}_A+m_B\vec{v}_B = m'_A\vec{v}'_A + m'_B\vec{v}'_B \qquad (4.10)$$

$\vec{v}_A$ and $\vec{v}_B$ are velocities of the masses before collision; and $\vec{v}'_A$ and $\vec{v}'_B$ are their velocities after collision.

The law of conservation of momentum is a general law and is true not only for ordinary-sized objects but also for very small elementary particles such as protons and electrons.

Let us apply the law of conservation of momentum to a very simple and easy case. A compressed spring is placed between two wooden balls of different sizes as shown in the following figure. Both balls are initially at rest. Hence

$$\vec{v}_A = \vec{v}_B = 0$$



When the spring is released there will be interaction between the two balls. (It is the recoil force of the spring). If the mass of the spring is so small that it can be neglected, we have

$$0 = m_A\vec{v}'_A + m_B\vec{v}'_B$$

Since the total initial momentum is zero, the left-hand side of the equation is zero. From the above equation we have

$$m_B \vec{v}'_B = -m_A \vec{v}'_A$$

The minus sign indicates that the two velocity vectors are parallel but opposite in direction. Taking only the magnitude, we have

$$m_B v'_B = m_A v'_A$$

or

$$m_B = m_A \frac{v'_A}{v'_B}$$

where v'_A and v'_B are the magnitudes of the velocity vectors. By using this relation inertial mass can be measured. In such measurement m_A, should be chosen as a standard mass.

Example (22) A bullet of mass 50 g leaves the muzzle of a gun with a velocity of 20000 cm s^{-1}. Find the momentum of the bullet.

Since m = 50 g and $v = 20\,000$ cm s^{-1}, we have

$$p = mv$$
$$= 50 \times 20\,000$$
$$= 10^6 \text{ g cm s}^{-1}$$

Example (23) Find the total momentum of the following system, (a) Two electrons each having mass m are moving towards each other with the speed of 0.01 c. (b) One of the two electrons having mass m is moving at a velocity of 0.01c while the other is moving at a velocity of 0.02 c in the same direction.(c is the velocity of light and its value is 3×10^8 ms^{-1}.)

(a) The momentum of the first electron $= mv_1$
$= m \times 0.01\ c$
$= 0.01\ mc$

The momentum of the second electron $= -mv_2$
$= -m \times 0.01c = -0.01\ mc$

The total momentum $p = mv_1 + (-mv_2)$
$= 0.01\ mc + (-0.01\ mc) = 0$

(b)The momentum of the first electron $= mv_1$
$= m \times 0.01\ c = 0.01\ mc$

The momentum of the second electron $= mv_2$
$= m \times 0.02c = 0.02\ mc$

The total momentum $p = mv_1 + mv_2$

$= 0.01\ mc + 0.02\ mc$

$= 0.03\ mc$

Example (24) A bullet of mass 16 g (0.016 kg) is fired from a 4 kg gun with a velocity of 600 m s^{-1}. What is the recoil velocity of the gun ?

(For a system consisting of a gun and a bullet, the total momentum before firing = 0. Therefore, according to the law of conservation of momentum, the total momentum after firing =0. Hence, the magnitudes of the momentum of the bullet and the recoil momentum of the gun are the same.)

The momentum of the bullet after firing $p_b = m_b v_b$

$= 0.016 \times 600 = 9.6\ kg\ m\ s^{-1}$

The momentum of the gun $p_g = m_g v_g$

$= 4 \times v_g\ kg\ m\ s^{-1}$

By the law of conservation of momentum, we can write

$4v_g = 9.6$

Therefore $v_g = 2.4\ m\ s^{-1}$

Example (25) A man dived horizontally with a velocity of 1.5 m s^{-1} from a 100 kg boat. If the recoil velocity of the boat is 0.9 m s^{-1} what is the mass of the man?

The momentum of the man $p = mv$

$= m \times 1.5 = 1.5\ m\ kg\ ms^{-1}$

The momentum of the boat $p_{bt} = m_{bt} v_{bt}$

$= 100 \times 0.9$

$= 90\ kg\ ms^{-1}$

The magnitudes of the momentum of the man and that of the boat are the same.

Hence,

$$1.5m = 90$$

$$m = \frac{90}{1.5} = 60\ kg$$

4.8 FREELY FALLING BODIES

If a body is dropped from a height near the earth's surface, the body will fall onto the ground with a constant acceleration g. The air resistance is totally neglected for the fall. Only then can the fall of the body be defined as free fall.

Equations of motion under constant acceleration derived in Chapter 3 can be used for free fall. In free fall, the acceleration due to gravity, which is a constant, is a vector and its direction is always downwards (towards the centre of the earth).

Since the initial velocity of a body dropped from a height is zero and its acceleration is $a = g$, we have

$$s = \frac{1}{2} gt^2 \qquad (4.11)$$

Since the displacement is measured from the starting point, the directions of the displacement and acceleration are the same for the falling object. The velocity of the body at time t after it has started to fall is

$$v = gt \qquad (4.12)$$

The relation between v and s can be derived as follows.

$$s = \frac{1}{2g}(gt)^2$$

$$= \frac{1}{2g} v^2$$

from which

$$v^2 = 2gs \qquad (4.13)$$

is obtained.

(If $v_o = 0$, $a = g$ are used in $v^2 = v_o^2 + 2as$ the same relation will be obtained.)

When a body is thrown upwards with an initial velocity v_o, g must be given a minus sign while s and v are given positive signs since the displacement and the velocity are opposite in direction to the acceleration due to gravity. Therefore the equations to be used for describing the motion of a body thrown upwards (while going upwards) are

$$s = v_o t - \frac{1}{2} gt^2 \qquad (4.14)$$

$$v = v_o - gt \qquad (4.15)$$

$$v^2 = v_o^2 - 2gs \qquad (4.16)$$

The maximum height h the body will reach can be obtained as follows.

Since at the highest point $v = 0$ and $s = h$, we have

$$0 = v_o^2 - 2gh$$

$$h = \frac{v_o^2}{2g}$$

From equations (4.12) and (4.13)the time taken to reach the highest point is found to be

$$t = \sqrt{\frac{2h}{g}} \qquad (4.17)$$

The time taken to reach the highest point is the same as the time taken to reach the ground from the highest point.

Example (26) A ball is thrown upwards with a velocity of 40 ms^{-1}. How long does the ball stay in the air ? What height does the ball reach ? (Assume that $g = 10 m\ s^{-2}$.)

$$v_0 = 40 m\ s^{-1} \quad g = 10 m\ s^{-2}, \quad v = 0 \text{ and}$$

$$v = v_o - gt$$

$$0 = 40 - 10\ t$$

$$t = 4\ s$$

The total time the ball stays in the air = $2 \times 4 = 8s$, and

$$v^2 = v_0^2 - 2gh$$

$$0 = (40)^2 - 2 \times 10 \times h$$

$$20\ h = 1600$$

$$h = 80\ m$$

The ball reached a height of 80 m.

Example (27) What is the velocity of a stone freely falling from a height of 2000 cm when it strikes the ground? How long does the stone take to reach the ground?

(Assume that $g = 1000\ cm\ s^{-2}$.)

$h = 2000$ cm, $g = 1000\ cm\ s^{-2}$ and $v_o = 0$, and

$$v^2 = v_0^2 + 2gh$$

$$= 0 + 2 \times 1000 \times 2000$$

$$= 4 \times 10^6$$

$$v = 2 \times 10^3\ cm\ s^{-1}$$

Next. we have

$$h = v_0 t + \frac{1}{2} g t^2$$

$$2000 = 0 + \frac{1}{2} \times 1000\, t^2$$

$$500\, t^2 = 2000$$

$$t^2 = 4$$

$$t = 2s$$

Example (28) An object is hurled vertically upwards with a speed of 50 m s^{-1}. How long does it take the object to be caught again? What height does the object reach? What is the average velocity for the whole distance travelled?

(Assume that g =10 ms^{-2})

$v_0 = 50$ m s^{-1}, g = 10 m s^{-2}, v= 0 and

$$v = v_0 - gt$$

$$o = 50\text{-}10\, t$$

$$t = 5\ s$$

The total time the object is in the air = 2 × 5= 10 s. Therefore, the object will be caught again 10 s after being hurled.

Let the maximum height the object will reach = h. Then

$$h = v_0 t - \frac{1}{2} gt^2$$

$$= 50 \times 5 - \frac{1}{2} \times 10 \times 25$$

$$= 250\text{-}125$$

$$= 125m$$

Let the velocity when the object is caught = v_1,

Since $v_o = 50m\ s^{-1}$, t = 10s and g = $10ms^{-2}$, we have

$$v_1 = v_0 - g\,t$$

$$= 50 - 10 \times 10$$

$$= -50\ m\ s^{-1}$$

The negative sign appears because the body is descending.

$$\therefore \text{ Average velocity} = \frac{v_0 + v_1}{2}$$

$$= \frac{(50)+(-50)}{2}$$

$$= \frac{0}{2}$$

$$= 0 \text{ m s}^{-1}$$

Example (29) A projectile is shot up vertically with a velocity of 100 m s^{-1}. How long does it take the projectile to reach a height of 375 m? (Assume that g = 10 ms^{-2}.)

v_o = 100 m s^{-1}, h = 375 m, g = 10 m s^{-2}, and

$$h = v_0 t - \tfrac{1}{2} gt^2$$

$$375 = 100\,t - 5\,t^2$$

$$t^2 - 20\,t + 75 = 0$$

$$(t - 15)(t - 5) = 0$$

$$t = 5 \text{ s or } 15 \text{ s}$$

t = 5 s is the time taken to reach the height of 375 m while travelling upwards.

t = 15 s is the time taken to go up to the highest point and fall back again to 375 m height.

(Two values are obtained since the projectile passes a point twice: once while rising and once while descending.)

4.9 THE TURNING EFFECT OF A FORCE

We encounter turning effects of forces in everyday life. Swinging open the door of a room or the garden gate about the hinges, tightening a nut by turning a spanner and kids playing seesaw are familiar to us. In all these cases, the objects experiencing the turning effects are pivoted either at the hinges or fulcrums. A force which acts on a pivoted body at a distance from the fulcrum tends to make that body rotate. The turning effect of a force about a particular fulcrum is measured by the moment of that force.

The moment of a force is the product of the magnitude of that force (in newtons) and the perpendicular distance (in metres) of its line of action from the fulcrum. Its unit, in the SI system, is N m, which is never shortened to J (joules).

Fig. 4.8 shows the left-hand half of a wooden bar pivoted at the centre. The two forces, 15 N and 25 N are acting at the points which are respectively 10 cm and 20 cm away from the pivot. Both these forces are acting downwards and tending to rotate the bar the same way in the anticlockwise sense. To work out the total anticlockwise moment caused by the two forces, we first find the moments due to each separate force and then add the two up (since the moments tend to rotate the bar in the same sense or direction).

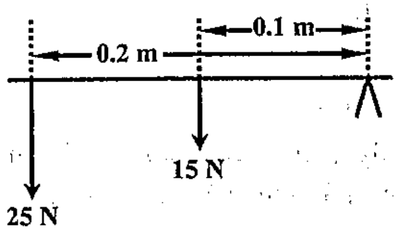


Fig. 4.8

Moment due to the 15 N force is

$$L_1 = 15\ \text{N} \times 0.1\ \text{m}$$
$$= 1.5\ \text{N m}$$

and the moment due to the 25 N force is

$$L_2 = 25\ \text{N} \times 0.2\ \text{m}$$
$$= 5\ \text{N m}$$

Thus, the total moment is

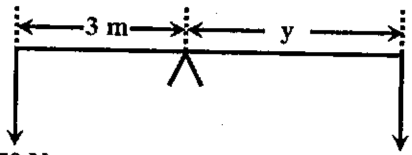
$$L_1 + L_2 = 1.5\ \text{N m} + 5\ \text{N m}$$
$$= 6.5\ \text{N m}$$

The Principle of Moments

The condition necessary for a pivoted object to be in balance is given by the principle of moments. This principle states that if an object such as a bar or a plank is to be in balance, the total clockwise moment about the fulcrum must equal the total anticlockwise moment. As an application of this principle, let us look at the following example.

Example (30) A seesaw is pivoted at its centre. One boy weighing 250 N is sitting at the left-hand side, 3 m from the fulcrum. Another boy weighing 225 N is at the other side. If the two boys are in balance, find the distance of the second boy from the fulcrum.

First draw a diagram showing the forces that act on the seesaw.



Since the boys are in balance,

clockwise moment of the 225 N boy	=	anticlockwise moment of the 250 N boy

Thus,

$$225 \times y = 250 \times 3$$

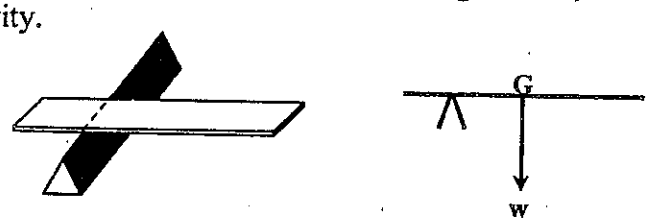
$$y = \frac{250\times3}{225}$$

$$= 3.33 \text{ m}$$

That is, the second boy is sitting on the other side 3.33 m away from the pivot.

Another concept, needed for the study of pivoted bars or planks is the centre of gravity. The centre of gravity of a particular object is the point at which all its weight may be considered to act. Thus, for example, the weight of a metre stick of uniform density is considered to be acting at the 50 cm mark-its mid-point.

Let us now look at situations where bars or planks are pivoted off centre. One such situation is shown in Fig. 4.9 (a). Here a ruler is placed off centre. It is pivoted near the left -hand end. Fig. 4.9 (b) shows that a single force, its weight is acting at the centre of gravity.

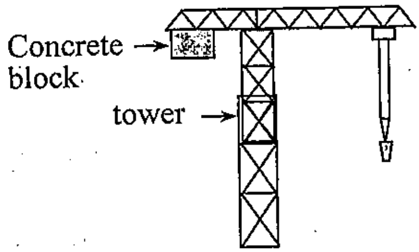


(*a*) (*b*)

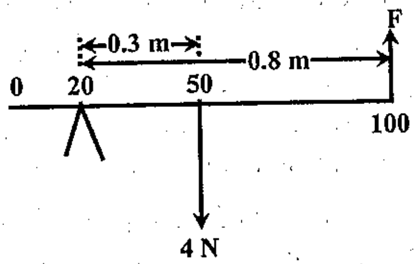
Fig. 4.9

It is evident that the ruler is not balanced. It will rotate and slide off. In this case the clockwise moment of the weight about the fulcrum has no counter balancing anticlockwise moment. Thus, according to the principle of moments it cannot be in balance.

There is one way by which we can place the ruler of Fig. 4.9 in balance. This is done by applying a force of the right size at the right-hand end of the ruler .The direction of this force must be such that the moment due to it causes a moment in the anticlockwise sense. That is, the applied force at the right-hand end must act upwards, opposite to the direction in which the weight is acting.
The following figure shows the application of moment in building site crane.



Example (31) A uniform metre rule weighing 4 N, pivoted at the 20 cm mark, is supported at the right-hand end at the 100 cm mark, by a vertical thread. Find the tension in the thread.
First, draw a diagram showing all the forces acting on the metre rule.



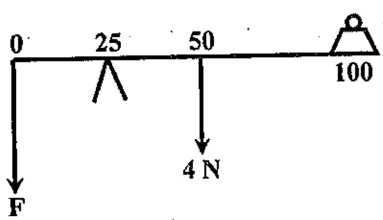
The diagram shows the distances of the respective forces also . For balance,

$$\text{total clockwise moment} = \text{total anticlockwise moment}$$

$$4 \times 0.3 = F \times 0.8$$

$$F = \frac{4\times0.3}{0.8} = 1.5 \text{ N}$$

Example (32) A uniform metre rule weighing 4 N is pivoted at the 25 cm mark. A load of 100 g is placed at the 100 cm mark. If the metre rule, along with the load, is to be in balance, what force must be applied at other end: 0 cm mark. (acceleration due to gravity g =10 N kg^{-1})
As usual, we first draw the diagram showing all the forces acting on the metre rule.



For balance,

$$\text{total anticlockwise moment} = \text{total clockwise moment}$$

$$F \times 0.25 = 4 \times 0.25 + 0.1 \times 10 \times 0.75$$

$$0.25\,F = 1.75$$

$$F = 7\text{ N}$$

A force of 7 N must act downward at the 0 cm mark to keep the metre rule in balance. Finally, we will look at the case where the force acting on a pivoted rule is not perpendicular to the rule. Such a situation is illustrated in Fig. 4.10.

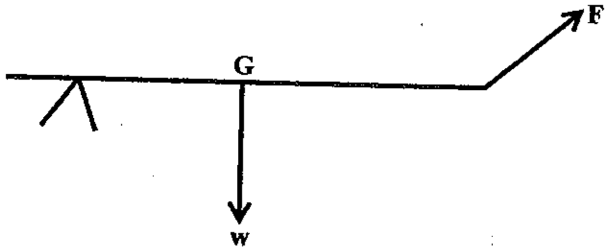


Fig.4.10

The moment due to the weight of the ruler is the product of the weight and the distance of the centre of gravity, G, from the pivoted point (fulcrum). But the moment due to the force F is not equal to F times the distance, along the ruler, of F from the fulcrum. Instead it should be F times the perpendicular distance of the fulcrum from the line of action of F,

(Fig.4.11)The moment of the weight w is clockwise, while that of F is anticlockwise. For balance, the two moments must be equal.

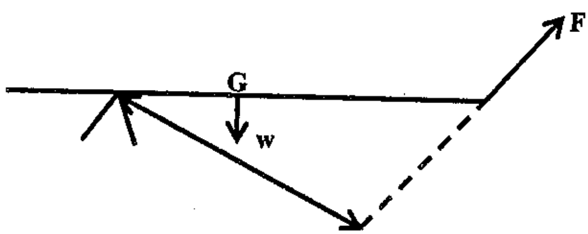
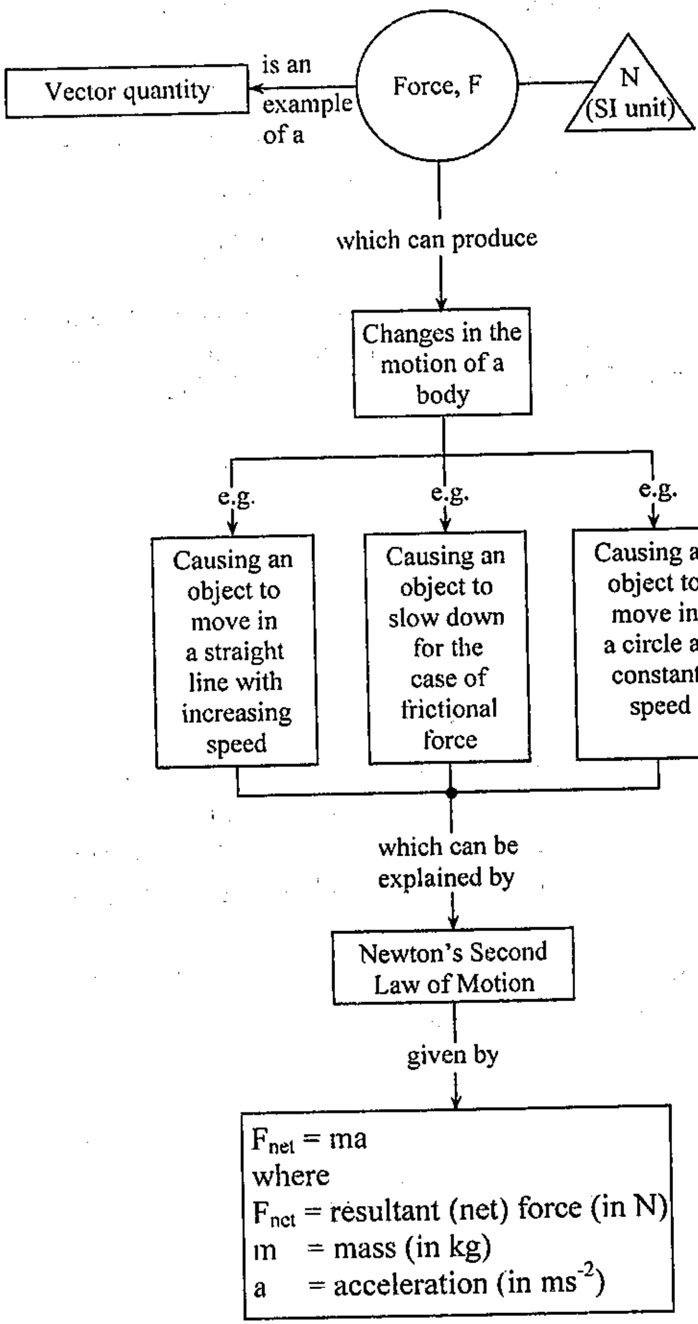
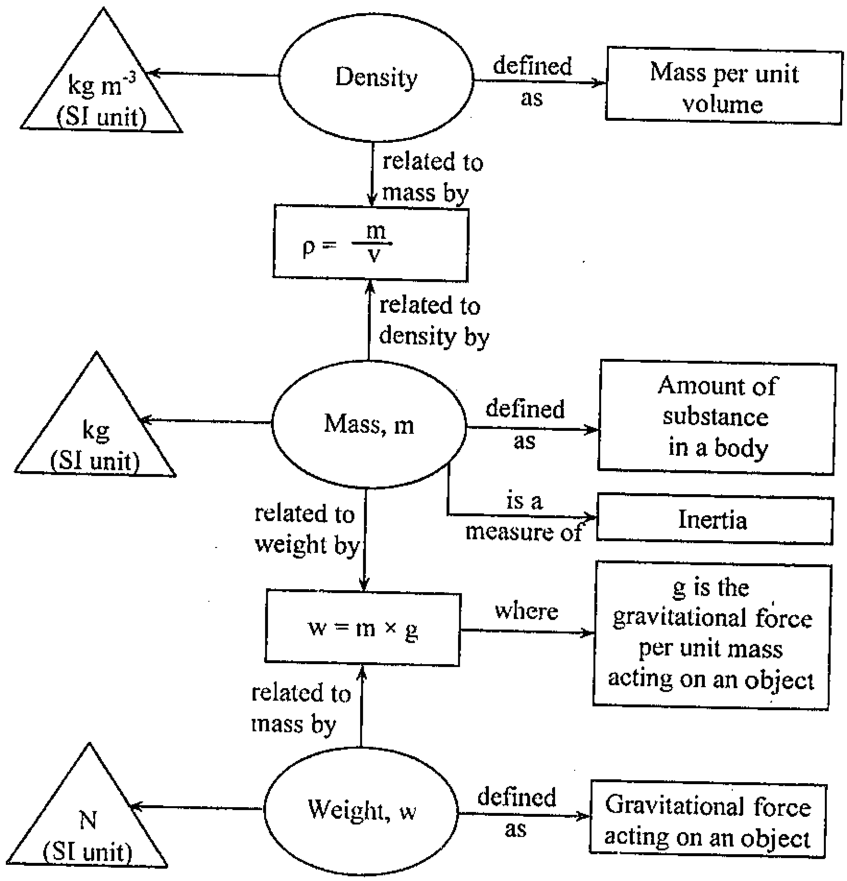


Fig.4.11

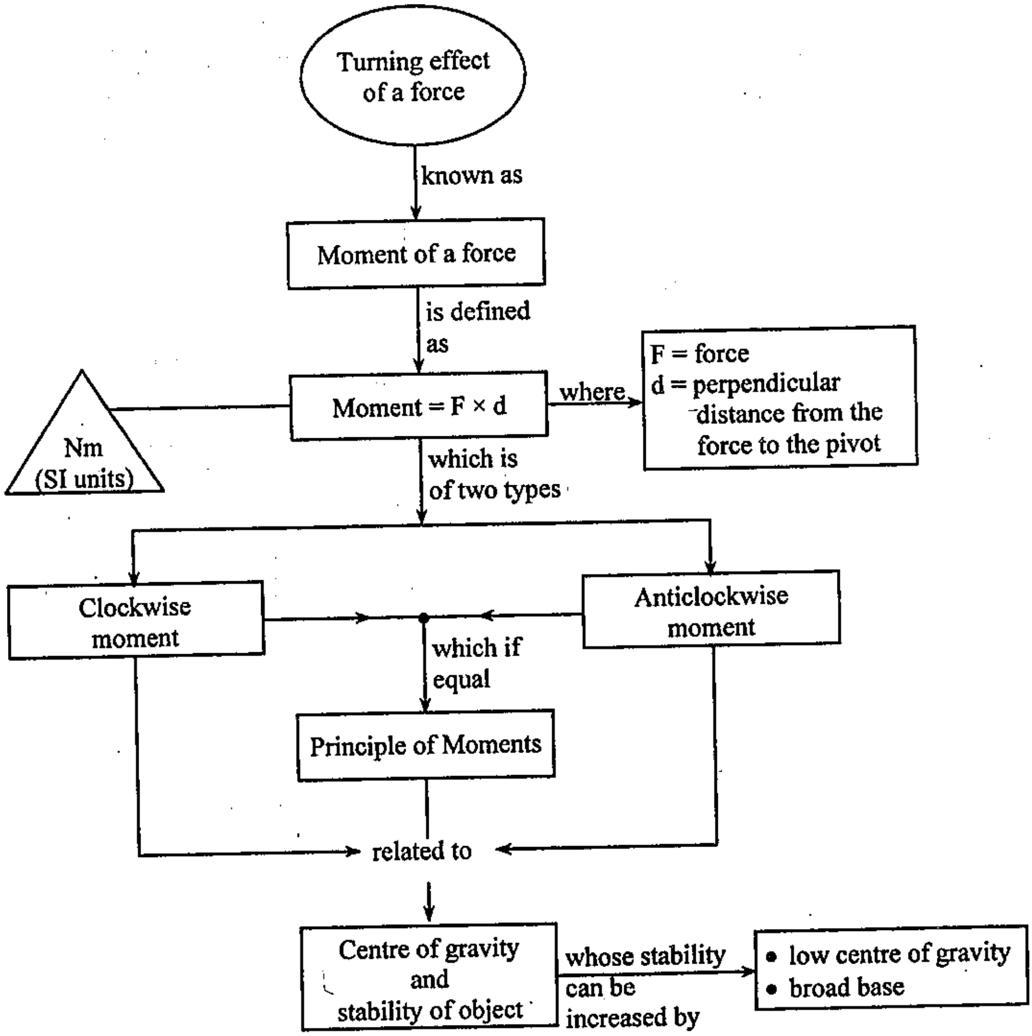
Concept Map (Force)



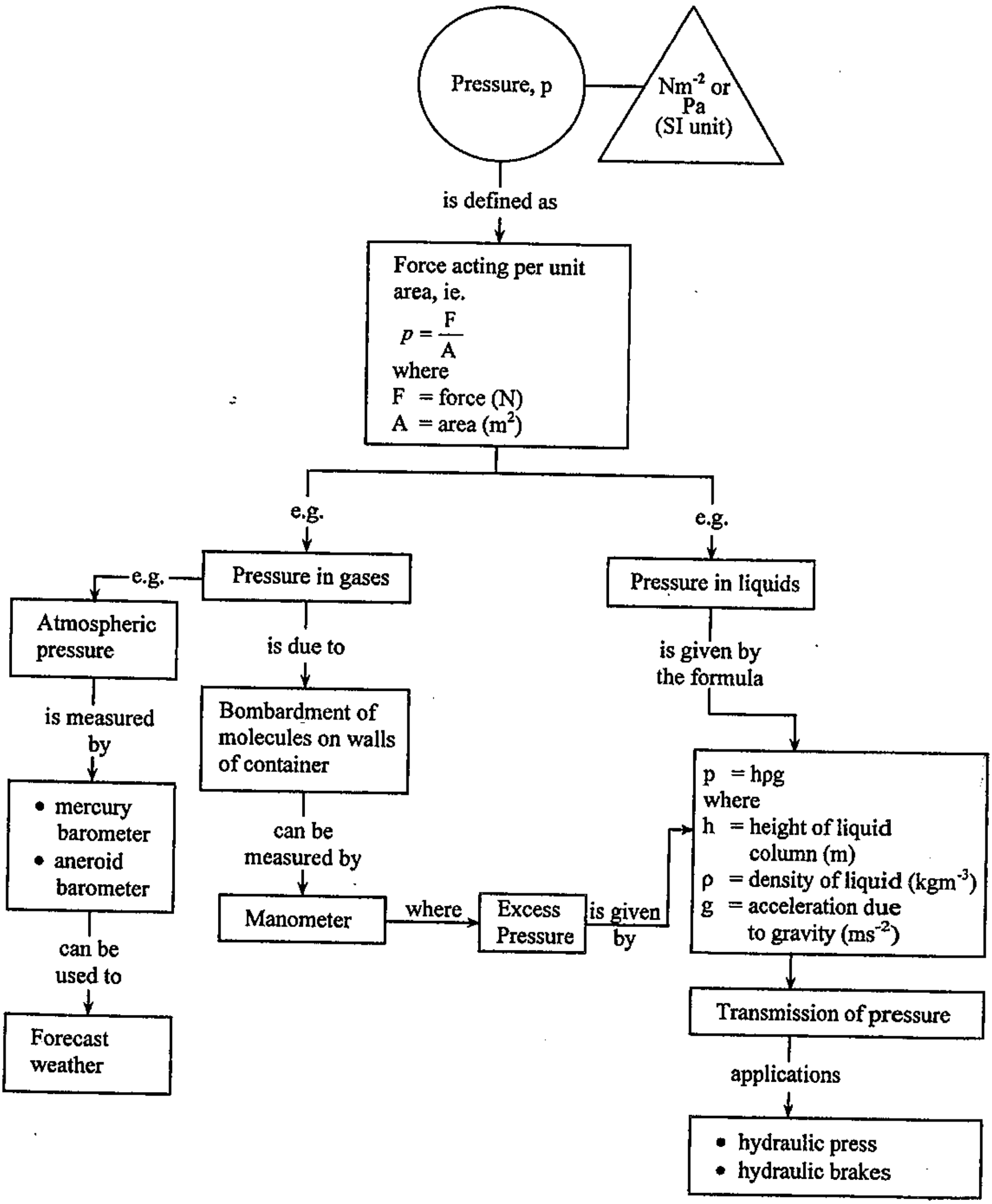
Concept Map (Density, Mass, Weight)



Concept Map (Turning effect of a force)



Concept Map (Pressure)



EXERCISES

1. Which is more difficult? To move a wooden box or a big stone. Why?
2. Which is easier? To stop a train with few carriages or a train with many carriages. Why?
3. Explain why passengers standing in a bus are thrown forward when the bus suddenly stops.
4. It is dangerous for a person to jump from a moving car. Why?
5. Will a box at rest on a floor gain an acceleration when a force is applied to it? (Hint: Frictional force must be considered here.)
6. When two forces of equal magnitude are each applied to two masses separately, the acceleration of one of the masses is twice that of the other. Compare the two masses.
7. Rewrite the relation $m \frac{v - v_0}{t} \ \alpha \ F$ in vector notation.
8. Is it correct to describe Newton's second law in symbols as $F \ \alpha \ a$?
9. Although two forces act simultaneously on a body, it continues to move with a constant velocity. What can be said about the two forces?
10. A 4.0 kg object is moving across a friction-free surface with a constant velocity of 2m/s. Which one of the following horizontal forces is necessary to maintain this state of motion?
    a. 0 N b. 0.5 N c. 2.0 N d. 8.0 N e. depends on the speed.
11. If the forces acting upon an object are balanced, then the object
    a. must not be moving
    b. must be moving with a constant velocity
    c. must not be accelerating
    d. none of these
12. The gravitational force due to the Earth on a 1kg mass at one Earth radius above the surface of the Earth is
    a. equal to b. 1/2 of c. 1/4 of d. 1/8 of
    e. 1/16 of the force on the same mass on the surface of the Earth.

    Answer-The answer is 3. The radius is doubled, (remember to measure from the center of the Earth), so the square radius is quadrupled. The Force depends on the inverse of the square radius.
13. The mass of Mars is about 1/10 of the mass of the Earth and the radius of Mars is approximately 1/2 of the Earth's radius. Approximately what is the acceleration due to gravity on the surface of Mars?
    a. $4 m/s^2$ b. $2 m/s^2$ c. $1/2\ m/s^2$ d. $1/4\ m/s^2$ 5. $1/10\ m/s^2$

Answer- The answer is 1. The acceleration is proportional to the mass and inversely proportional to the radius. Therefore $(1/10)(1/2)^2 = 4/10$. The acceleration due to gravity on the surface of the Earth is approx. $10 m/s^2$ is the acceleration due to gravity on Mars is about $(4/10)*(10) = 4 m/s^2$.

14. Can action and reaction cancel each other? Why?

15. There are only object A and object B in an isolated system. If the magnitude of a force exerted by A on B is F_A and that of the reaction exerted by B on A is F_B, express the relation between the two forces in symbols. (Both objects A and B are moving.)

16. A body weighing 'ω' is moving with an acceleration 'a' along a horizontal straight line. What is the force acting on the body?

17. The weight of a body may change when its position is changed, but mass does not. Why?

18. Is there any consistency between the two statements "There is gravitational force acting on an astronaut" and " An astronaut is in a weightless state"?

19. Fill in the blanks.

(a) 1 N force acting horizontally on a body ---------- gives it an acceleration of $1 ms^{-2}$.

(b) ------force acting horizontally on a body of mass 1 sl gives it an acceleration of $1 ft\ s^{-2}$.

(c) 1 dyne force acting horizontally on a body of mass 1 g gives it an acceleration of……

20. What do you understand by the "moment of a force" about a point?

21 State the conditions of equilibrium when a body is acted upon by a number of parallel forces.

22 What is meant by the centre of gravity of a body ?

23. A uniform metal tube of length 5 m and mass 9 kg is suspended horizontally by two vertical wires attached at 50 cm and 150 cm, respectively, from the ends of the tube. Find the tension in each wire.

24. (a) Find the magnitude of a force that must act on a body of 10 kg mass to give it an acceleration of $5 m\ s^{-2}$.

25. Find the maximum and the minimum accelerations, along the horizontal direction, of a body of 5 kg mass due to a 100 N force acting upon it.

26. If a proton of mass 1.675 x 10^{-24} g is accelerated by an accelerator to an acceleration of 10^6 ms^{-2}, find the net force acting on the proton.

27. If the velocity of a car of 1200 kg mass increases from 60 m s^{-1} to 120 m s^{-1} in 10s, what is the net force acting on the car?

28. A truck of 2000 kg mass moving at a velocity of 12m s^{-1} slides 15 m before it comes to a stop after applying the brakes. What is the resisting force of the brakes?

29. What is the acceleration of a body weighing 20 N due to the applied force F_{net}= 20 N?

30. Find the force required to move a body of 2 kg mass upward with an acceleration of $5ms^{-2}$.

31. 10 N force is applied horizontally in turn to a body of 1 kg mass and a body weighing 1 N. Which of the bodies will have greater acceleration?

32. Compare the accelerations of the two bodies of masses M and 3 M when the same net force of 20 N is applied to each of them.

33. A 100 lb force acts horizontally on a body of mass 1.5 sl. Find the acceleration and the weight of the body.

34. A 3 ton car moving with the velocity of 30 mi h^{-1} is brought to a stop in 2 s. Find the resisting force of the brakes acting on the car.

35. A lift weighing 2000 lb is pulled up by a cable of tension 5000 lb. Find the mass of the lift and its upward acceleration.

36. A 60 kg swimmer dived from a 5 m high diving board into a pool .What is the acceleration of the earth while the swimmer is falling? Take the mass of the earth as 6×10^{24} kg and the acceleration due to gravity as 10 m s^{-2}.

37. If a body weighing 100 N is carried to the moon and put on the moon's surface, what will happen to the weight of the body ?Will the mass of the body change?

38. Two bodies of masses 2 kg and 5 kg are at rest 2 m apart. Find the gravitational force interacting between the two bodies. (G= 6.67×10^{-11} N m^2 kg^{-2})

39. Compare the moon's gravitational forces acting on the two bodies of masses M and 3 M which are falling simultaneously onto the moon's surface from a height near the surface. If M = 0.2 kg and the acceleration due to the gravity of the moon is 1.6 m s^{-2}, find the moon's gravitational forces acting on each of the bodies.

40. If the velocity of a 0.02 kg bullet is 500 m s^{-1}, find the magnitude of the momentum of the bullet. If the bullet is fired towards the north, what is the direction of its momentum?

41. A body of 10 sl is moving towards the north at a constant velocity of 60 ft s^{-1}. What is the momentum of the body?

42. If the muzzle velocity of a 0.005 kg bullet is 500 m s^{-1}, find the magnitude of the muzzle momentum of the bullet. The gun is aimed and fired at a target in the north and then at a target in the east. Will the muzzle momenta of the bullets be equal?

43. What is the total momentum of two neutrons each having a mass of 1.67×10^{-27} kg

    (a) moving toward each other with equal speeds ?

    (b) moving towards the east at speeds of 10^5 m s^{-1} and 10^6 ms^{-1} respectively?

44. A basket-ball is thrown with a velocity of 20 m s^{-1} towards the wall. With what velocity will the ball bounce back to the thrower?

45. A tiger of mass 400 kg is running horizontally, at 50 m s^{-1} towards a hunter. The hunter fires a gun from a place straight in front of the tiger and the tiger falls and dies on the spot. If the mass of the bullet is 0.002 kg, find the velocity of the bullet. Give a comment on your answer.

46. A30 g bullet moving at a constant velocity hits a wooden block of mass 3 kg and continues to travel at the velocity of 2 m s^{-1} together with the block. What is the velocity of the bullet when it hits the block?

47. A 60 kg man dived into the water with the velocity 20 m s^{-1} from a 120 kg boat. Find the recoil velocity of the boat.

48. A 0.2 kg marble is at rest on a smooth table. Another marble of mass 0.1 kg moving at a constant velocity of 10 m s^{-1} towards the east hits this marble and it recoils to the west with the velocity of 5 m s^{-1}. Find the velocity of the marble which was at rest?

49. What is the time taken to reach the ground by a stone falling from the edge of a roof which is 64 ft high? What is the velocity of the stone when it strikes the ground ?

50. A ball thrown vertically upward reaches a maximum height of 80 ft. With what velocity has the ball been thrown? What is the time taken to reach the maximum height?

51. A stone is thrown vertically upward with 40 m s^{-1}. What will be its respective velocities 3 s, 4 s and 5 s after it has been thrown?

52. A ball is thrown vertically upward and it is caught again after 6 s.

(a) Find the velocity with which it is thrown.

(b) Find the maximum height reached.

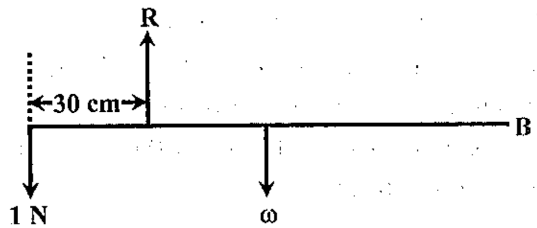
(c) Find the total displacement for the whole distance travelled.

(d) Find the average velocity for the whole distance travelled.

53. A man throws a stone vertically upward at 30 m s^{-1}. How long does it take the stone to reach the height of 40 m?

54. A stone, thrown by a girl, reaches a height of 20 m. How long does it take the stone to be caught back? With what velocity does she throw the stone?

55. If AB is a uniform metre rule which is balanced as shown in the diagram, (a) what is the weight ω of the rule? (b) what is the reaction R?



56. A pole AB of length 10 m and weight 500 N has its centre of gravity 4 m from the end A, and lies on horizontal ground. The end B is to be lifted by a vertical force applied at B. Calculate the least force required to do this. Why would this force, applied at the end A, not be sufficient to lift the end A?

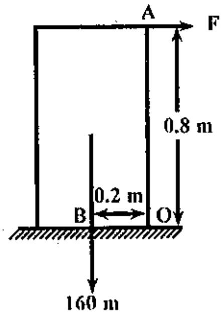
57. In order to "weigh" a boy in the laboratory, a uniform plank of wood AB 3 m long and having a mass of 8 kg is pivoted about a point 0.5 m from A. The boy stands 0.3 m from A and a mass of 2 kg is placed 0.5 m from B in order to balance the plank horizontally. Calculate the mass of the boy.

58. A bridge over a stream is made from a uniform wooden beam which weighs 4500 N and is 16 m long. Its ends A and B are supported on boulders. If a man weighing 800 N is standing on the bridge 4 m from A, what is the reaction at the boulder

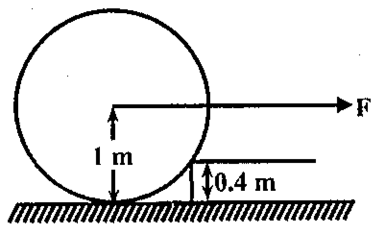
(a) under A?

(b) under B?

59. A heavy solid block is tied with a rope to is upper edge A. The weight of the block is 160 N acting vertically, OB is 0.2 m and OA is 0.8 m. Find the force F which will just tilt the block about O.



60. The diagram shows a wheel of mass 15 kg and radius 1m being pulled by a horizontal force F against a step 0.4 m high. What initial force is just sufficient to turn the wheel so that it will rise over the step? What happens to the size of this horizontal force as the wheel rises?



61. Define density and relative density.

    What is the unit for density in SI units?

    Why is relative density unitless?

62. Having density concept why do we still need to use relative density?

63. A wax block floats in water and an iron block sinks in water; compare the densities of wax, water and iron.

64. A small iron alloy is inserted in a block of wax. Explain whether the block of wax would float or sink in water.

65. What experiment would you do to find out whether the relative density of kerosene is less than or greater than 1?

66. Find the mass of water required to fill the aquarium of length 100 cm, breadth 40 cm and depth 30 cm.

67. "Since gold is denser than aluminum, gold is always heavier than aluminum." Why is this statement wrong? Write down the correct statement.

68. Which is more appropriate to express the amount of matter contained in a body: in volume or in density?

69. Define pressure. Is pressure a scalar or a vector? Express the unit of pressure in SI units.

70. It is more effective to use a sharper knife than a blunt one. Why?

71. Explain why the tip of a pin feels sharper than the tip of an iron nail.

72. The density of aluminum is 2700 kg m^{-3}. Find the mass of a sheet of aluminum having a length of 1 m, a breadth of 0.5 m and a thickness of 0.001m.

73. If the mass of 50 cm^3 sulphuric acid solution is 65 g, find the density of the solution.

74. The relative density of sulphur is 2. Find the volume of 1 kg of sulphur.

75. The mass of a statue which is made of silver is 120 g. If the density of silver is10.5 g cm^{-3}, find the volume of the statue.

76. The mass of hydrogen atom is 1.7×10^{-27} kg. Assuming the normal state of hydrogen atom as a sphere of radius 0.5×10^{-10} m, find the average density of the atom. If the radius of hydrogen atom in its first excited state is about 2×10^{-10} m would the density of hydrogen atom increase or decrease?

77. (a) What is the mass of 1 m^3 of water?

    (b) What is the mass of 12 m^3 of methylated spirit whose relative density is 0.8?

78. The diameter of the contact area of the stylus and the record is 0.2 mm. Find the pressure if the force exerted by the stylus is 5 N. Express the pressure in N m^{-2} and Pa.

CHAPTER 5

WORK AND ENERGY

In studying a branch of physics one must be well-versed in the concepts which are involved in that branch. This means that one must have not only a mere understanding of the concepts but must also know their importance and usefulness and how they are related to one another if there are inter-relationships among them.

The concepts which are involved in mechanics are not very numerous. The concepts, including derived concepts such as velocity and acceleration, number no more than a dozen. These concepts are subdivided into fundamental concepts and ordinary concepts. The momentum concept, which has been described in the previous chapter, is a concept in mechanics. Energy which is another fundamental concept will be discussed in this chapter.

In the beginning, energy was just an ordinary concept but later it was transformed into a fundamental one. At present, energy is a fundamental concept not only for physics but also for science as a whole.

There are different forms of energy. They are mechanical energy, heat energy, light energy, electrical energy, nuclear energy and so on.

In this chapter only mechanical energy will be discussed. Before discussing mechanical energy it is necessary to define a concept called "work" which is related to energy.

5.1 WORK AND ITS UNITS

The work done on a particle is equal to the product of the force acting on the particle and the distance travelled in the direction of the force. In symbols,

$$W = Fs \qquad (5.1)$$

where W is the work done, F is the force acting on the particle and s is the distance travelled, respectively. There is a relationship between this technical definition and the common notion of work. Normally we would say that more work has to be done to push a heavy load than to push a lighter one, and the work done has to be doubled to push a load twice the distance. Here, the technical definition is equivalent to the ordinary meaning. Although a man standing up and holding a heavy load is said to be doing work in everyday usage, according to the technical definition he is not doing any work at all. This is because the distance travelled by the man is zero. Hence, just as there are cases where the common notion of work and its technical definition coincide, there are also cases where they differ. This shows how the meaning of a word may become distorted when everyday usage of a word is applied in a certain subject. It would of course be possible, as an alternative, to use a different word. However, if we label such concepts with entirely different words, there is the risk that their origins and their connections with common usage would become lost.

Concerning work, the technical meaning of the word as defined above is used in physics. This definition is the simplest and it is for a particular condition where the directions of the applied constant force and the motion are the same (Fig. 5.1).

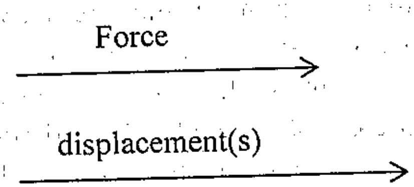


Fig. 5.1

(It can also be stated that the force vector and the displacement vector are parallel.)

When the unit of force is in pounds (lb) and if the distance is in feet (ft), the unit of work is given in ft-lb. When the unit of force is in $\binom{\textit{dynes}}{\textit{newtons}}$ and if the distance is in $\binom{\textit{centimetres}}{\textit{metres}}$ the unit of work is then given in $\binom{\textit{ergs}}{\textit{joules}}$.

When a force is constant and if the directions of the force and the motion are not the same, work is defined as follows.

Work is the product of the component of the force in the direction of motion and the distance moved.

The following illustrates this definition.

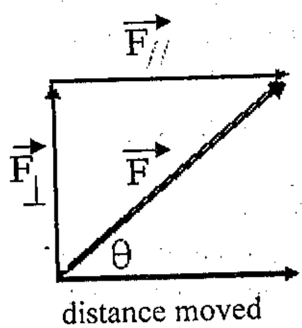


Fig.5.2

In this example the magnitude of $F_{//}$ is F cos θ and that of $F_{\perp}$ is F sin θ. $F_{//}$ and $F_{\perp}$ are the components of the force F.

$F_\perp$ is the component which is perpendicular to the direction of the motion and $F_{//}$ is the component which is parallel to the direction of motion and thus,

$$F_\perp + F_{//} = F$$

The definition of work done for this example is then,

$$\text{work} = F_{//} \times \text{the distance moved.}$$

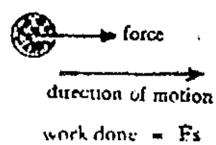
We can then generalize our original definition as follows:

If a constant force is acting on a body the work done is the product of the component of force in the direction of the motion and the distance moved.

According to this definition, for a particular value of force acting on a body, the magnitude of work will have its maximum value when the directions of the force and the motion are the same.

The three specific examples encountered in the consideration of the work done by constant forces are:

(1) when the directions of the force and the motion are the same



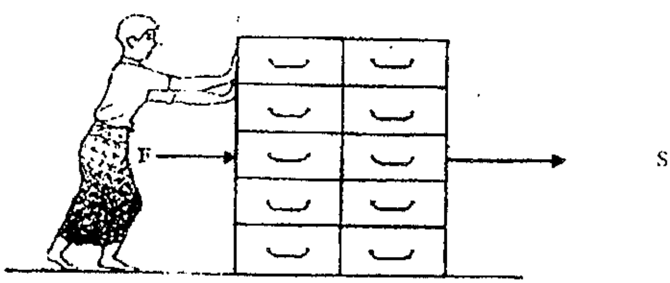
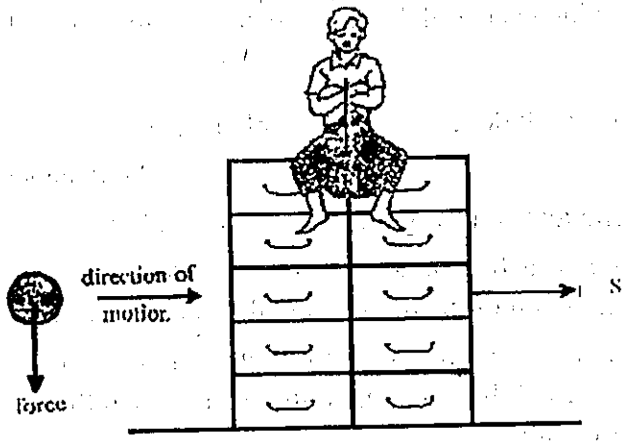


Fig. 5.3

(2) When the force and the motion are perpendicular to each other

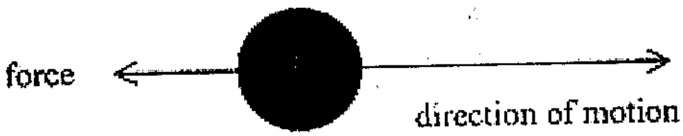


work done = $F_{//} s = 0$

$(F_{//} = F \cos 90^\circ = 0)$

Fig.5.4

(3) When the force and the motion are in opposite directions



work done = $F_{//} s = -Fs$

$(F_{//} = F \cos 180^\circ = -F)$

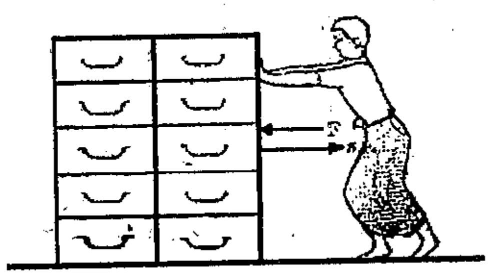


Fig. 5.5

We can now consider the case where no work is done although there is applied force and the body is in motion.

From the point of view of physics a labourer who carries a load on his back and walks along a horizontal path with a uniform velocity is not doing any work at all.

The concept called work is now simple and clear. Next, let us consider energy concept which is broader and deeper. Energy is the capacity to do work. When we say that a body possesses energy, it means that the body has ability to exert a force on another body and does work. Again, when it is said that work is done on a body, it means that the energy which is equal to the magnitude of the work is given to the body. Now energy can be defined numerically as follows.

If a body is displaced $1 \left(\begin{matrix} \text{ft} \\ \text{cm} \\ \text{m} \end{matrix}\right)$ by applying $1 \left(\begin{matrix} \text{lb} \\ \text{dyne} \\ \text{newton} \end{matrix}\right)$ force, it is said that one unit of energy is given to the body. That one unit of energy is $1 \left(\begin{matrix} \text{ft-lb} \\ \text{erg} \\ \text{joule} \end{matrix}\right)$ The joule (J) is the unit of energy in SI system of units. The units of energy are the same as the units of work.

Example (1) How many ergs are equivalent to 1 J?

$$\begin{aligned} 1\,\text{J} &= 1\,\text{N} \times 1\,\text{m} \\ &= 10^5\,\text{dynes} \times 10^2\,\text{cm} \\ &= 10^7\,\text{ergs} \end{aligned}$$

Among the different forms of energy which have been described previously, only mechanical energy will be discussed in this chapter. The mechanical energy is divided into two types: kinetic energy which is energy due to motion and potential energy which is energy due to the position and configuration of the body.

5.2 KINETIC ENERGY AND POTENTIAL ENERGY

Kinetic Energy

Let us consider a body of mass m which is at rest. Let an external force $F_{external}$ be applied to the body. Then, according to Newton's second law, the acceleration of the body must be $a = F_{external}/m$.

Due to the applied force the body will be in motion and if its velocity increases to v after travelling the distance s, we have

$$v^2 = 2as$$

or

$$v^2 = 2\left(\frac{F_{external}}{m}\right)s$$

from which

$$\frac{1}{2} m v^2 = F_{external}\, s \text{ is obtained.}$$

$F_{external}s$ is the work done on the body and is also the amount of energy given to the body. Therefore, $\frac{1}{2}mv^2$ which is on the left-hand side of the equation is the energy received by the body.

Half of the product of the mass and the velocity squared of the body is defined as the kinetic energy of the body.

The kinetic energy of the body is

$$KE = \frac{1}{2}mv^2 \qquad (5.2)$$

If the body is not initially at rest and is moving with an initial velocity v_0 we have

$$v^2 - v_0^2 = 2as = 2\left(\frac{F_{external}}{m}\right)s$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_{external}\, s$$

According to this, the work done on the body of mass m appears as the increase in the kinetic energy of the body. Here, it is obvious that 'work' is just the term for the energy which is given to the body by the external force.

Example (2) A ball of 1 kg mass is thrown with a velocity of 5 m s^{-1}. Find the kinetic energy of the ball.

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 1 \times (5^2) = 12.5\ J$$

Example (3) A car weighing 3200 lb is driven at a velocity of 15 mi h^{-1} (22 ft s^{-1}). Find the kinetic energy of the car.

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{\omega}{g}v^2$$

$$= \frac{1}{2} \times \frac{3200}{32} \times (22)^2$$

$$= 24\,200 \text{ ft-lb}$$

$$= 2.42 \times 10^4 \text{ ft-lb}$$

Example (4) What will be the kinetic energy of the car in example (3) if it is driven at a velocity of 60 mi h^{-1} (88 ft s^{-1})?

$$KE = \frac{1}{2} \quad \frac{\omega}{g} v^2$$
$$= \frac{1}{2} \times \frac{3200}{32} \times (88)^2$$
$$= 387\,200 \text{ ft-lb} = 3.9 \times 10^5 \text{ ft-lb}$$

The kinetic energy of the car when it is travelling at 60 mi h^{-1} is 16 times greater than that of the same car travelling at 15 mi h^{-1}. Since the kinetic energy is directly proportional to the square of the velocity, an accident which occurs during high speed driving is more severe.

Example (5) Electrons are the smallest particles present in ordinary matter. Its mass is only 9.1×10^{-31} kg. If the velocity of an electron is 3×10^7 m s^{-1}, find its kinetic energy.

$$KE = \frac{1}{2} mv^2$$
$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (3 \times 10^7)^2$$
$$= 4.1 \times 10^{-16} J$$

Potential Energy

Let us consider a body of mass m which is on the ground. When the body is raised to a height h above the ground, the amount of work done against the gravitational force is $$W = mgh \qquad (5.3)$$

In this case, as the increase in the kinetic energy is zero where has the energy gone? The answer is that the energy is stored in the body which is at the height h, as its potential energy. In order to get back the kinetic energy, the body has to be simply dropped. After falling through the height h, the velocity of the body of mass m can be calculated from the formula $v^2 = 2gh$. If the kinetic energy is calculated, we will find that

$$KE = \frac{1}{2} mv^2$$
$$= \frac{1}{2} m(2gh)$$
$$= mgh$$

Therefore, it is found that the amount of work done (mgh) to raise the body to the height h can be totally transformed into the kinetic energy.

The energy stored in a body due to its position is called the potential energy.
In the above example, the potential energy is

$$PE = mgh \tag{5.4}$$

In raising a body to the height h the potential energy (mgh) remains the same regardless of the path taken by the body (Fig. 5.6).

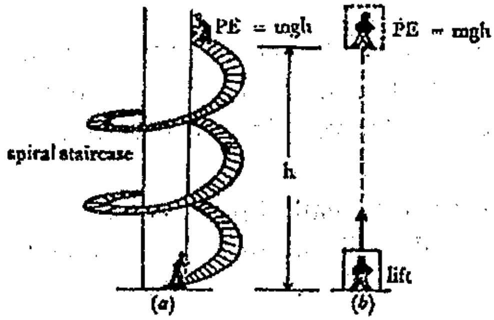


Fig. 5.6

The potential energy is not only the energy due to the position of a body but also the energy due to the configuration of the body. For example a wound clock spring, a coil of compressed spring, a stretched bow with an arrow, all have potential energy. Therefore, the potential energy of a body may be defined generally as the energy stored in a body by virtue of its position or configuration. It was shown above that the velocity of a body of mass m when it strikes the ground is $v = \sqrt{2gh}$ after falling from the height h.

In this example the body has fallen vertically downward. When a body is sliding down along a smooth and curved sliding board instead of falling vertically, the final velocity of the body when it strikes the ground can be obtained by letting the initial potential energy equal to the final kinetic energy of the body. If the starting point is at a height h, the velocity when it strikes the ground is still $v = \sqrt{2gh}$ (Fig.5.7).

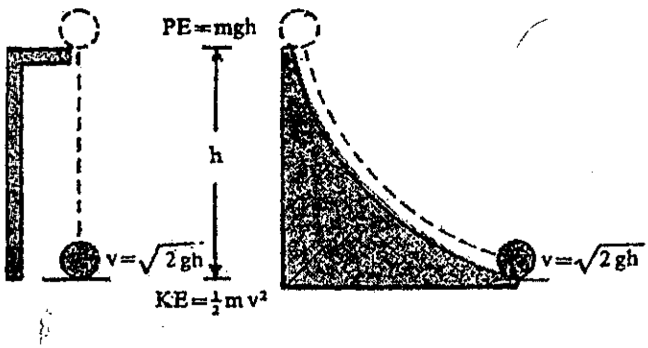


Fig. 5.7

The choice of the reference point (h = 0 point) is very important in the calculation of the potential energy of a body.

Referring to Fig. 5.8, let us find out the potential energy of a book of 1 lb which is 1 ft above the table at the top-floor of a building, for different points of reference.

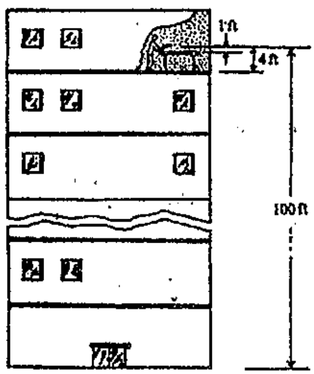


Fig. 5.8

If the table-top is taken as the reference point the potential energy of the book is 1 ft-lb ; if the floor of the top-floor is chosen as the reference point the potential energy is 4 ft-lb ; and if the ground is taken as yet another reference point the potential energy of the book is 100 ft-lb. Therefore, the value of h must be chosen carefully in the calculation of the potential energy.

Example (6) A guava of 0.3 kg is at the height of 3 m above the ground. With reference to the ground what is the potential energy of the guava?

$$PE = mgh$$
$$= 0.3 \times 9.8 \times 3$$
$$= 8.82 \text{ J}$$

Example (7) A 3200 lb car is lifted to the height of 100 ft by a crane. Find the potential energy of the car.

$$PE = mgh$$
$$= 3200 \times 100 = 320\ 000 \text{ ft-lb}$$

This energy value is smaller than the kinetic energy of a 3200 lb car which is travelling at the velocity of 60 mi h^{-1} (see example 4).Thus, a stationary object hit by a car travelling at 60 mi h^{-1} suffers greater damage than when a car of the same mass falls on it from a height of 100 ft.

Example (8) What is the potential energy of an electron which is at the height of 0.lm above the ground?

$$PE = mgh$$
$$= 9.1 \times 10^{-31} \times 9.8 \times 0.1$$
$$= 8.9 \times 10^{-31} \text{ J}$$

This value is 10^{15} times smaller than the kinetic energy of an electron moving at normal rate (3×10^7 m s^{-1})

Mechanical energy

The two types of mechanical energy that a body may possess are kinetic energy and potential energy.

(a) Kinetic energy (KE)

(b) Potential energy (PE)
- (i) gravitational PE
- (ii) elastic PE

(a) Kinetic energy
all objects in motion

(b) Potential energy
- (i) a waterfall, raised objects.
- (ii) composed or stretched springs, the bent condition of a diving board, the stretched elastic band of a catapult.

In mechanical processes the potential energy and the kinetic energy can change from one form to another .The interchange of potential energy and kinetic energy will be illustrated by an example. A weight w is dropped from a height onto a wedge which is placed on the ground as shown in Fig. 5.9. The weight has only potential energy when it is at that height.

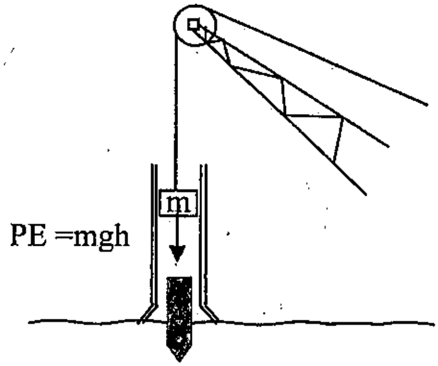


Fig 5.9

When it is dropped, potential energy is changed to kinetic energy and when it strikes the wedge kinetic energy is changed to work and hence pushes the wedge into the ground. There are many other mechanical processes where potential energy is changed to kinetic energy and kinetic energy to work, one step after another, as in the above process.

Example(9) A mechanical pile driver with a diver weight of 480lb and a vertical length of 20ft is used to drive a small pile into the ground. (Fig 5.9)

(a) How much energy is delivered to the pile on the initial strike?

(b) With what velocity does the driver strike the pile? (Neglect friction)

Solution (a) With the driver at the top position (h_0=20 ft), it has a total energy of

$$E_0 = PE_0 = mgh_0 = wh_0 = (480\text{lb})(20\text{ft})$$
$$= 9600 \text{ ft-lb}$$

The energy is converted to kinetic energy as the weight falls and is delivered to the pile when the driver strikes the pile.

(b) Just before the driving weight strikes the pile, it has a velocity of

$$v = \sqrt{2g\Delta h} = \sqrt{2(32\text{ft/s}^2)(20\text{ft})} = 36\text{ft/s}$$

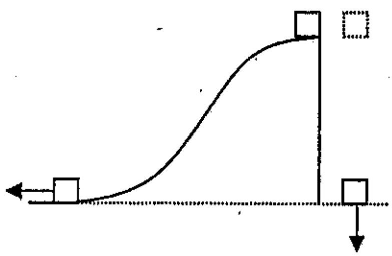
(As the pile is driven into the ground, the height of fall of the driving weight increases, and the striking velocity increases with each successive strikes. The energy of the driver goes into the work of driving the pile into the ground. Once the driver strikes the pile, the mechanical energy of the system is no longer conserved. Why?)

Energy transfer – true or false?

Look at the diagrams in figure before you read the explanations below.

(In each example, the effects of air resistance and friction are neglected).

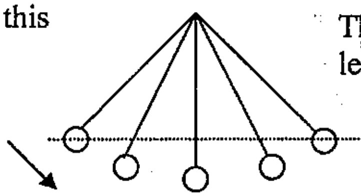
The block which slides down the smooth slope gains the same velocity



As the block which drops straight down

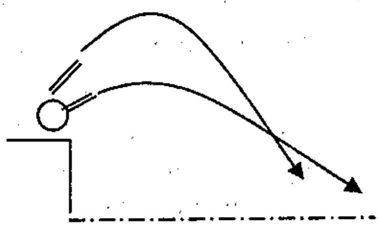
true

Both blocks start with the same amount of potential energy, so they have the same amount of kinetic energy when they reach ground level.



true

The pendulum gains the same amount of potential energy during the upward part of its swing as it loses during the downward part – despite the presence of the bench.



true

Whichever way the gun points, the shell leaves the barrel with the same amount of kinetic and potential energy. All of this energy is in the form of kinetic energy as the shell strikes the water.

5.3 LAW OF CONSERVATION OF ENERGY

Laws of physics play an important and useful role in the investigation of nature. The regularity and the systematic behaviour of the natural processes are usually revealed by these laws. This revelation has given a guide-line to physicists for the systematic study of nature. Among the laws which are obeyed by the observable physical quantities, the most fundamental and important laws are the conservation laws. The law of conservation of momentum has been described and explained in the previous chapter. In this section the law of conservation of energy will be discussed.

Before the law of conservation of energy is discussed, it may be necessary to explain the statement: "An observable quantity obeys a conservation law". For an isolated system, although time changes, if there is no change in the total magnitude of an observable quantity, the observable quantity is said to obey the conservation law. If we review the law of conservation of momentum it can be seen that the law has been stated such that the total momentum of an isolated system must be a constant. Let us take another example in order to have a better understanding of the idea of "conservation". Consider a group consisting of five boys playing a game of marbles. Let us assume that the total number of marbles they have is exactly 100. During the game, if there is no one in the surrounding to give them some more marbles and also no player has given away any marble to any person in the surrounding, then the total number of marbles among them will still be 100. However, there can be individual loss and gain; and thus the number of marbles can increase or decrease for each of the players, but the total number of marbles, circulating among the players is still 100. In this example, a group formed by five boys is a system. Since there is no interaction with the surrounding (no marble has been taken in or out), the system is an isolated system. The players are said to interact with one another as they are interchanging marbles. In this example, the thing that is conserved is the total number of marbles. Now replace the players with particles and replace the marbles with energy. An isolated system of particles is then obtained. The particles interact with one another. They are interchanging energy. The total energy of the system must be constant. Therefore, this isolated system obeys the law of conservation of energy.

The law of conservation of energy is stated briefly as follows:

The total energy of an isolated system is constant.

This law is often written as:

Energy cannot be created or destroyed. The total energy of the universe is constant.

These two statements are equivalent. In the second statement the whole universe is taken as an isolated system.

Energy cannot be created or destroyed but energy can be changed from one form to another.Therefore, for an isolated system the sum of the different forms of energy must be constant.

Let us verify the conservation of energy with a particular example.

Let us consider a two-particle system which consists of only a stone and the earth. Let the mass of the stone be m. The stone is dropped from a height h_o above the ground. The freely falling stone and the earth are attracting each other with equal forces. But only the motion of the stone is noticeable and the motion of the earth can be neglected since the mass of the earth when compared with the stone is many times larger.

Due to the gravitational force acting on the stone its acceleration will be $g = 9.8\ m\ s^{-2}$. Let us assume that the stone has fallen from the height h_o to the height h and its velocity changes from v_o to v during the period of time t. The kinetic energy will change because the velocity of the stone changes. The relationship between the energy change and work is

$$W = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$$

But $$W = Fd$$

For this example, since F, the weight of the stone, is mg, the distance d is

$$d = h_0 - h$$

we get $$mg(h_0 - h) = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$$

$$\frac{1}{2} mv^2 + mgh = \frac{1}{2} mv_0^2 + mgh_0$$

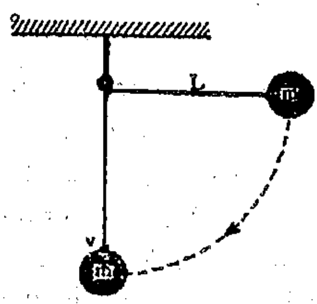
The quantity at the left is sum of the potential energy and the kinetic energy or the total mechanical energy at time t after the stone has started to fall; and the quantity at the right is the initial total mechanical energy of the stone. The value of this quantity (the total mechanical energy) is conserved throughout the distance travelled by the falling stone. It can be easily remembered by writing it as

kinetic energy + potential energy = total energy = constant

If the symbol T is used for kinetic energy, U for potential energy and E for total energy the above relation can be represented as

$$E = T + U = \text{constant} \qquad (5.5)$$

Example (10) A pendulum of mass m is held in a horizontal position as shown in the figure. If the length of the thread holding the pendulum is L and the pendulum is released, what is its velocity when it passes the lowest point?



Initially $v_0 = 0$ and $U_0 = mgL$,

and at the lowest point $v = v$ and $U = 0$.

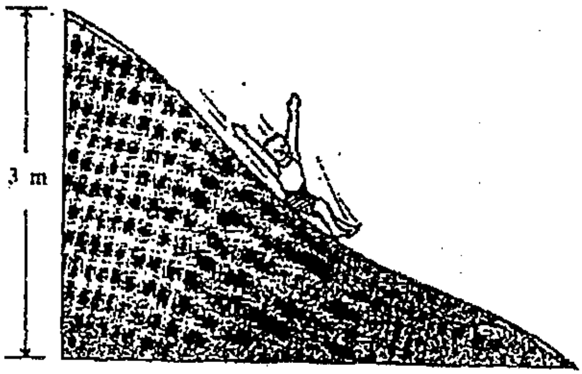
According to the law of conservation of energy

$$\frac{1}{2} mv^2 + 0 = 0 + mgL$$

$$v^2 = 2gL$$

$$v = \sqrt{2gL}$$

Example (11) A boy slides down a sliding board as shown in the figure. What is his velocity when he reaches the bottom of the board?



Let the mass of the boy be m.

Since his velocity is zero at the starting point there is no kinetic energy at all. Because the total energy is just the potential energy, we have

$$E = 0 + U$$
$$= 0 + mgh$$
$$= m \times 9.8 \times 3 \text{ J}$$

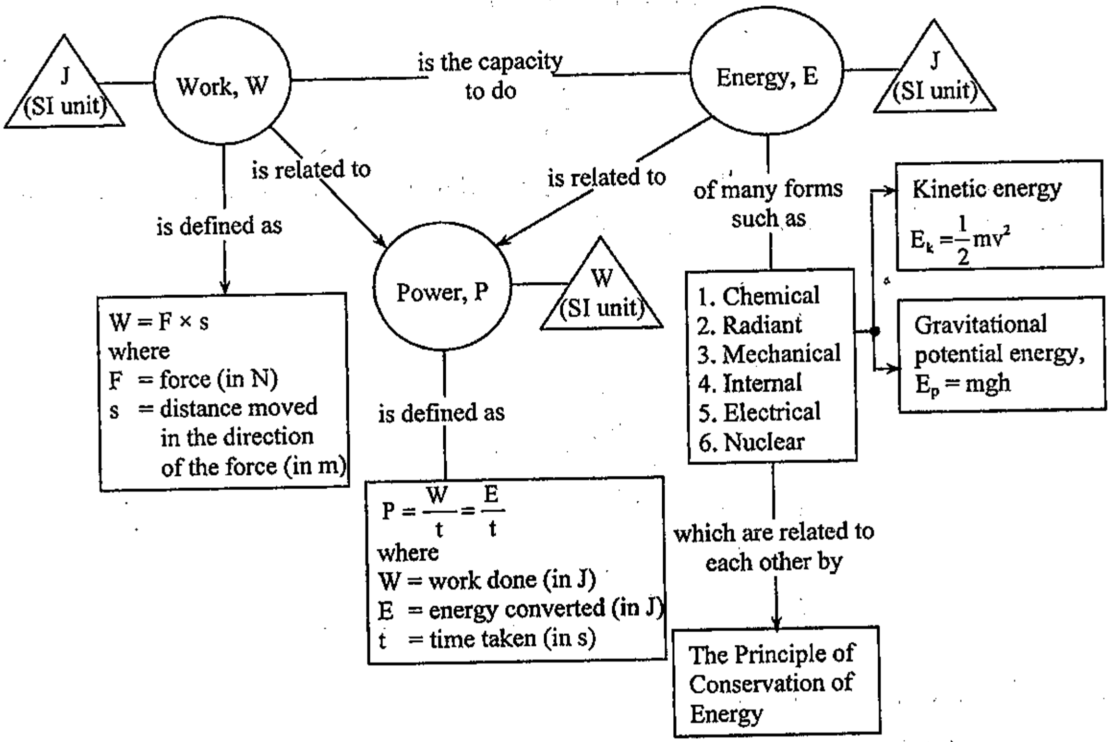
At the bottom of the board the potential energy is zero and the total energy is just the kinetic energy. Let his velocity at the bottom be v, then

$$E = \frac{1}{2} mv^2 + 0$$
$$= \frac{1}{2} mv^2$$

According to the law of conservation of energy

$$\frac{1}{2} mv^2 = m \times 9.8 \times 3$$
$$v^2 = 2 \times 9.8 \times 3$$
$$v = \sqrt{58.8} = 7.67 \text{ m s}^{-1}$$

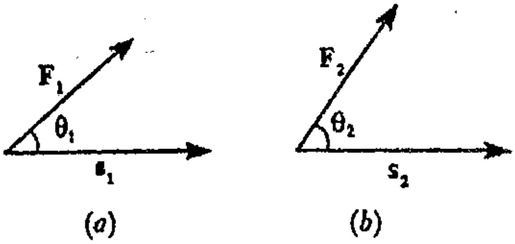
Concept Map (Work, Energy, Power)



EXERCISES

1. Define "work". Write down the unit or work in SI system.
2. Is work a vector or a scalar?
3. A labourer who supports a load on his back walks along a horizontal straight path. Is there any work done if the speed of the labourer is uniform? Why?
4. A vegetable-seller puts down the vegetable basket which she is carrying onto the ground. Is there any work done or not? Give reason for your answer.

5. For which of the following two cases is the quantity of work done greater?

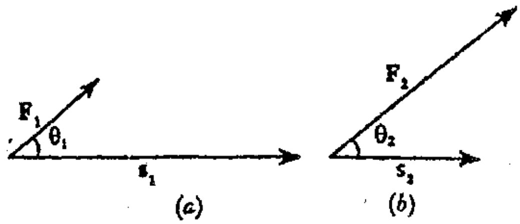


$F_1 = F_2$

$s_1 = s_2$

$\theta_1 < \theta_2$

6. Are the quantities of work done for the following two cases equal ?



$F_1 = ½ F_2$

$s_1 = 2 s_2$

$\theta_1 = \theta_2$

7. Why are the unit of energy and that of work the same?
8. Among the different forms of energy which of them has a special importance for industrial production of goods and for upgrading the living conditions of the people ? Discuss briefly.
9. Discuss the correctness of the statement, "Energy is needed to produce energy," giving an example for it.
10. Discuss the role of energy for economic development and industrialization.
11. Write down the law of conservation of energy. Is this a fundamental law?
12. Explain the conservation of mechanical energy.
13. Are the following right or wrong?

    (a) In nature it is observed that although there are conversions or interchanges of energy form one form to another, the total energy is constant.

    (b) The law of conservation of energy was understood fully only when Einstein showed that matter and energy are two different forms of the same quantity.

    (c) Energy can be changed from one form to another and it can also be converted to matter.
14. Define "energy".
15. A woman pushes a child, who is riding a tricycle, with a 200 N force. The tricycle moves a distance of 2m and the work done by the woman is 100 J. Find the angle between the force and the displacement.

16. A child is pulling a toy car with a 10 N force. The direction of the force makes an angle of 20° with the horizontal plane. If the car moves 6 m, how much work does the child do?

17. A woman is pushing a chair along a horizontal plane with a 300 N force. Find the work done for the following cases :

    (a) The chair moves 2 m parallel to the force.

    (b) The chair does not move at all.

18. If the velocity of a 1000 kg car is 40 km h^{-1}, find the kinetic energy of the car.

19. A wooden sphere of mass 0.15 kg is thrown with a velocity of 30 m s^{-1}.

    (a) Find the kinetic energy of the sphere.

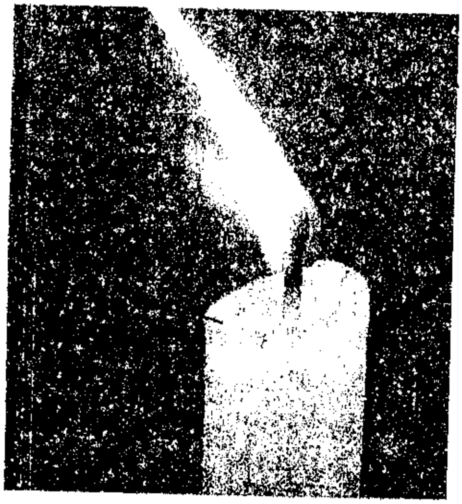
    (b) If the man who throws the wooden sphere has exerted a constant force on the sphere for 1.5 m, what is the force exerted by the man on the sphere?

20. An empty can is dropped from a window which is 100 ft high from the ground. What is the velocity of the can when it strikes the ground? (Neglect air resistance.)

21. When riding a swing, the highest point a child can reach is 6 ft above the lowest point. What is the velocity when the swing reaches the lowest point? (Neglect frictional forces.)

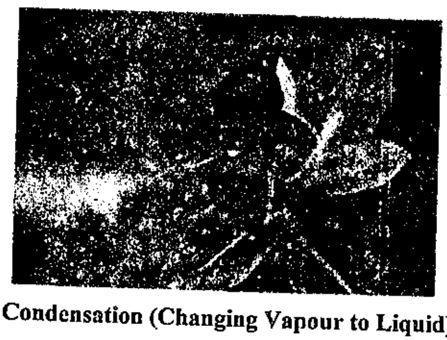
22. A tennis ball which is thrown vertically upward reaches the height of 50 m. Find the initial velocity of the ball. (Neglect air resistance.)

23. Water which flows over a dam falls into a canal which is at a depth of h m from the top of the dam. What is the velocity of water when it reaches the canal if the velocity when it starts to fall from the dam is zero?

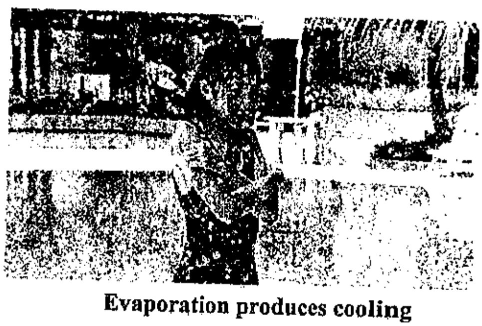


Melting (Changing Solid to Liquid)

HEAT



Condensation (Changing Vapour to Liquid)



Evaporation produces cooling

CHAPTER 6

HEAT AND TEMPERATURE

The study of heat and thermal properties of matter is actually a study of energy and energy transfer.

It has long been recognized that heat is a form of energy. When heat is applied to various kinds of engines, it can supply forces that do work. The internal structure of matter is examined by the study of the effects of heat energy upon matter. It suggests that all matter consists of tiny particles such as atoms and molecules. Heat is closely associated with the motions of such tiny particles. Therefore heat must be studied on molecular basis.

6.1 HEAT AND TEMPERATURE

The motions and positions of molecules in matter result in the kinetic energy and potential energy. The total energy, that is, the sum of the potential energy and the kinetic energy, of molecules in matter is in fact the internal energy of that matter. Temperature is related to that internal energy. Temperature is a measure of the internal energy of molecules.

The concept of temperature is very important for the physical and biological sciences. This is because the temperature of an object is directly related to the energies of molecules composing the object. Natural processes often involve energy changes and the temperature is an indicator for these changes.

The sensations of hotness, warmness and coldness can be experienced by touching the objects. Temperature is the quantity that determines how cold or how hot the object is. The temperature of a hot body is higher than that of a cold body. The temperature of the object cannot be known accurately by experiencing the sensations of hotness or coldness. For example, if one touch the metal knob of a wooden door with one hand and touch the wood of the door with the other hand, he will feel that the metal knob is colder than the wood. This is so even though both the metal knob and the wood of the door are at the same temperature. This is just like saying that the mass of an object cannot be known accurately by lifting that object. Just as there are balances to measure the mass of the object, there are thermometers to measure temperatures accurately.

There is a relation between heat and temperature. The energy exchanged between an object and its surrounding due to different temperatures is defined as heat.

Heat and temperature are different quantities. When a body at a higher temperature is in contact with a body at a lower temperature heat flows from the first to the second body. Although the temperature of the first body is known, it is impossible to know how much heat has been transferred to the second body which is not as hot as the first one. Two bodies may have the same temperature, but may not be able to supply the same quantities of heat when put into contact with colder bodies under the same condition. Generally, the quantities of heat that can be supplied by different bodies at the same temperature are not the same. For example, a gallon of boiling water and half a gallon of boiling water may have the same temperature. However, under the same condition, the heat which can be supplied to a large block of ice by a gallon of boiling water is twice the amount supplied by half a gallon of boiling water.

The units used to measure temperature will be discussed before describing the units used to measure heat.

6.2 UNITS OF TEMPERATURE OR TEMPERATURE SCALES

The thermometers marked with Celsius (Centigrade) scale and Fahrenheit scale are used for ordinary cases such as measuring room-temperature, measuring body-temperature and measuring the temperature of hot water. Temperature is usually expressed by writing $^{\circ}$**C** and $^{\circ}$**F** just after the number of degrees. C stands for Celsius and F for Fahrenheit. The third temperature scale, the Kelvin or the absolute temperature is used in scientific work. Temperature is expressed by writing K just after the number. K stands for Kelvin.

The values of physical quantities remain the same at a given temperature. For example, the density of water at 4 $^{\circ}$**C** is 1000 kg m^{-3}. The values of many physical quantities vary with temperature. The length of an iron bar varies with temperature. But the length of that iron bar is the same whenever it is put into a container having a mixture of ice and water. A temperature scale can thus be defined by using such properties. A liquid such as mercury or alcohol is used in home thermometers to indicate temperature. The thermometer consists of a glass tube attached to a bulb. The bulb and part of the glass tube are filled with mercury. As the temperature increases the volume of mercury increases faster than that of the bulb. And the mercury rises in the glass tube.

To calibrate a thermometer, two reference points are chosen and the interval between these points is subdivided into a number of equal parts. The freezing point and boiling point of water under normal atmospheric pressure are chosen as reference points which are marked on the thermometer. The interval between these two points is divided into one hundred equal parts for the Celsius scale. If the freezing point of water or ice point is marked **0**$^{\circ}$ and the boiling point of water or steam point is marked **100**$^{\circ}$, the thermometer scale is the Celsius scale. On the Celsius scale, the ice point is 0 °C and the steam point is 100 °C. On the Fahrenheit scale the ice point is 32 °F and the steam point 212 °F.

The relationship between the Celsius temperature T_c and the Fahrenheit temperature T_F is given by the equation

$$T_c = \frac{5}{9}(T_F - 32) \qquad (6.1)$$

For example, normal body temperature is 98.6 °F. On the Celsius scale, this is

$$\begin{aligned} T_c &= \frac{5}{9}(T_F - 32) \\ &= \frac{5}{9}(98.6 - 32) \\ &= 37.0\ ^\circ C \end{aligned}$$

The relationship between the Celsius temperature T_c and Kelvin temperature T_K is given by

$$T_c + 273 = T_K \qquad (6.2)$$

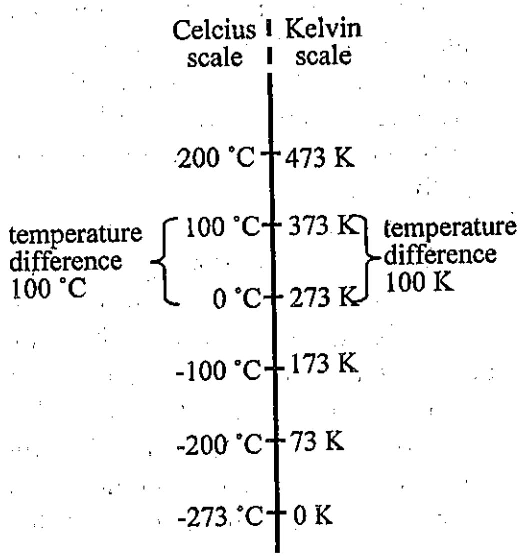


Fig.(6.1) Celsius scale and the corresponding Kelvin scale

In line with the decisions of the General Conference of the International Committee of Weights and Measures, the symbols °C, °*F* and K are used throughout this book to represent both the temperature difference and the temperature.

6.3 THERMAL EXPANSION OF SUBSTANCES

When a substance is heated, its volume usually increases. The dimensions of the substance increase correspondingly. This increase in size can be explained in terms of the increased kinetic energy of the molecules. The additional kinetic energy results in each molecule colliding more forcefully with its neighbours. Therefore, the molecules push each other further apart and the substance which is heated increases in size.

With few exceptions, substances expand when heated and if there is an obstruction to the expanding bodies, very large forces may be exerted on that obstruction by the expanding bodies.

If concrete road surfaces were laid down in one continuous piece, cracks would appear due to the difference between summer and winter temperatures. To avoid this, the concrete road surface is laid in small sections, each one being separated from the next by a small gap. The gap must be filled with a compound of pitch. On hot days, expansion of concrete squeezes the compound of pitch out of the gap. It goes back into the gap when the concrete contracts.

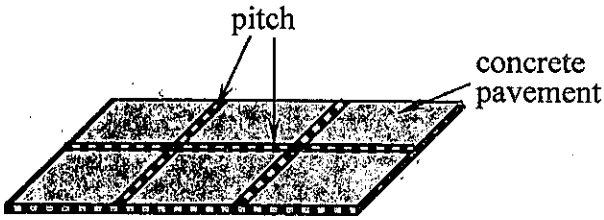
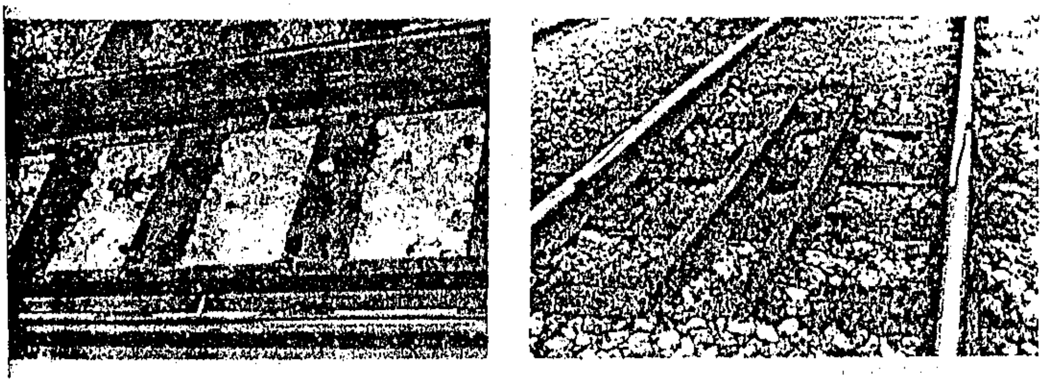


Fig.6.2 Making allowance to the expansion of concrete pavements and road surface

In laying railway tracks, gaps must be left between successive lengths of rail. The buckling of the track as the rails expand due to temperature rise may thereby be prevented. This method of laying railway tracks is an old one. In a new method, railway lines are welded together as shown in Fig. 6.3. (Note that the welded ends are cut into wedge shapes.) The last fifty to one hundred metres of both ends of such welded rail show expansion which is of a few centimetres. Concrete sleepers are used for these railway tracks.



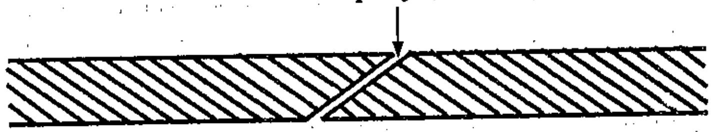


Fig. 6.3 Taper joint on a railway line

Although expansion can be troublesome for some practical work, it has many beneficial effects.

The steel tyres of locomotive wheels have to be renewed from time to time. To ensure a tight fit the tyre is made slightly smaller in diameter than the wheel. Before being fitted, the tyre is heated uniformly. The tyre expands and slips over the wheel. When the tyre cools it contracts and makes a tight fit. This idea is also used in fitting the wheel of a bullock-cart with an iron tyre. The thermal expansion property is utilized in riveting together steel plates and girders used in the dockyards and other constructional works.

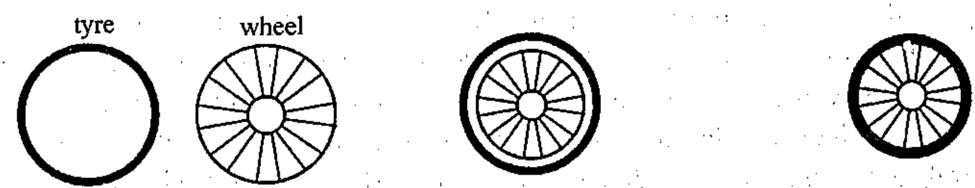


Fig. 6.4 Fixing a steel tyre onto the wheel of a locomotive wheel

6.4 LINEAR EXPANSION

When two different metal bars of the same length are heated so that the increase in temperature is the same, the magnitudes of their expansion may not be the same. For example, the expansion of copper is one and a half times that of steel. Aluminium expands twice as much as steel does.

A relation between the change in length of an object and the temperature change can easily be obtained. Let the original length of the object in Fig. 6.2. be l. Suppose the increase in length is Δl when the temperature increases by an amount ΔT . (The symbol ΔT represents a small change. If temperature T_2 is slightly higher than T_1, then $T_2 - T_1 = \Delta T$. So, ΔT is a small increase in temperature. Similarly, Δl is a small increase in length.)

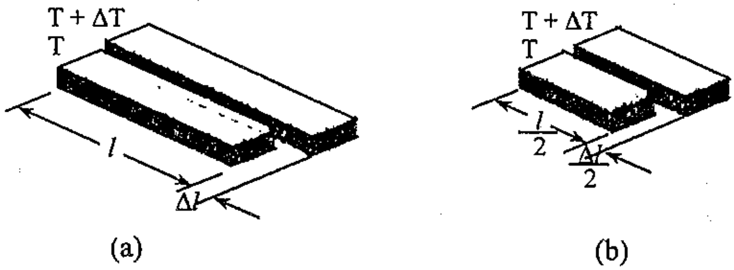


Fig. 6.5 Linear expansion of an object

If the object in Fig. 6.5 (a) is exactly halved, two equal parts will be obtained. Each part will have a length $l/2$ and the increase in length will be $\Delta l / 2$. Therefore, the increase in length Δl is directly proportional to the length l. In addition, it is known from experiments that if the increase in temperature is $2\ \Delta T$ instead of ΔT , the increase in length also doubles. Therefore, Δl is directly proportional to both l and ΔT . Both proportionalities may be combined into one expression:

$$\Delta l \;\alpha\; l \Delta T$$

Therefore,

$$\Delta l = \alpha l \Delta T \qquad (6.3)$$

In the above relation, α is the coefficient of linear expansion. This coefficient is a property of a given material and depends somewhat on the temperature. The values of α for some materials are given in Table 6.1.

Table 6.1

Material	Temperature (°C)	α (K^{-1})
Aluminium	-23	2.21×10^{-5}
	20	2.30×10^{-5}
	77	2.41×10^{-5}
	527	2.35×10^{-5}
Diamond	20	1.00×10^{-6}
Celluloid	50	1.09×10^{-4}
Glass (most types)	50	8.30×10^{-6}
Glass (Pyrex)	50	3.20×10^{-6}
Ice	-5	5.07×10^{-5}
Steel	20	1.27×10^{-5}
Platinum	20	8.90×10^{-6}

The unit of α is per K. Per K can be written as K^{-1}. The unit of α was expressed in per °C in the old physics textbooks. Now, the modern SI units are being used more and per °C is replaced by per K. Although there is a change in unit, the value of α remains the same.

Although ΔT in the above equation ($\Delta l = \alpha l \Delta T$) is a small increase in temperature, the equation is still correct for fairly large values of ΔT. It will still be correct even for $\Delta T = 100\text{ K} (= 100\ ^\circ\text{C})$.

The following example (1) illustrates the importance of linear expansion.

Example (1) The roadbed of a steel bridge is 1280 m long. If the temperature varies from 10 °C to 35 °C during a certain year, what is the difference in lengths at those temperatures? The road is supported by steel girders. For steel, $\alpha = 1.27 \times 10^{-5} K^{-1}$

$$\Delta T = 35^\circ C - 10^\circ C$$

$$= 25^\circ C = 25 K$$ Here we write $\Delta T = 25\ ^\circ C = 25$ K, since $\Delta T = (35 + 273) - (10 + 273)\text{ K} = 25\text{ K}$

Thus, $$\Delta l = \alpha l \Delta T$$

$$= 1.27 \times 10^{-5} K^{-1} \times 1280\text{ m} \times 25\text{ K}$$

$$= 0.406\text{ m or } 40.6\text{ cm}$$

Hence, the change in the roadbed length due to linear expansion must be allowed for in the design of the bridge so as not to damage the bridge.

Example (2) The length of a metal bar having coefficient of linear expansion α is l at the temperature T. What is the length of that metal bar at the temperature $T + \Delta T$?

The change in length due to the temperature change ΔT is

$$\Delta l = \alpha l \Delta T$$

Therefore, the length of the metal bar at $T + \Delta T$ is

$$l' = l + \Delta l$$

$$= l + \alpha l \Delta T$$

$$= l(1 + \alpha \Delta T)$$

Example (3) Define the coefficient of linear expansion.

$$\Delta l = \alpha l \Delta T$$

$$\alpha = \frac{\Delta l}{l} \frac{1}{\Delta T}$$

Therefore, the coefficient of linear expansion is the change in length per unit length for one degree change in temperature.

It can be expressed as

$$\alpha = \frac{N}{K} = \frac{N}{°C} = \frac{\frac{5}{9}N}{°F}$$

In this equation, N is the number which depends upon the type of substance, N does not depend upon the temperature change.

6.5 AREA EXPANSION AND VOLUME EXPANSION

The relations analogous to the one which gives the increase in length Δl for the increase in temperature ΔT can be derived for the area expansion and volume expansion. The relations obtained are

$$\Delta A = \beta A \Delta T \text{ (for area expansion)} \qquad (6.4)$$

and

$$\Delta V = \gamma V \Delta T \text{ (for volume expansion)} \qquad (6.5)$$

In these equations, β is the coefficient of superficial or area expansion and γ is the coefficient of cubical or volume expansion.

The derivation of the equation for the volume change only will be expressed here. The derivation of this equation is as follows.

Consider a cube of volume V. When the temperature of the cube changes from T to T + ΔT the length of each side will change from l to $l + \Delta l$. So, the change in volume is $\Delta V = (l+\Delta l)^3 - l^3$. $(l+\Delta l)^3$ can be approximated by $l^3 + 3l^2\Delta l$. Then the change in volume is $\Delta V = 3l^2\Delta l = 3V\Delta l/l$. If $\Delta l = \alpha l \Delta T$ is used

$$\Delta V = 3\alpha V\Delta T = \gamma V \Delta T$$

where

$$\gamma = 3\alpha$$

Thermal expansion occurs not only in solids but also in liquids and gases. Unlike solids, liquids and gases do not have well defined shapes. As the shape is just the shape of the container, only the volume expansion needs to be considered for liquids and gases. The previous equation $\Delta V = \gamma V \Delta T$ can be used to calculate the volume expansion of liquids as well as gases.

In the expansion of liquids, the volume expansion of water is quite interesting.

The anomalous behaviour of water is that it has a negative coefficient of volume expansion at some temperatures. Fig. 6.7 shows the change in γ and density of water with the temperature, γ varies as the temperature changes and the sign of γ changes at 3.98 °C. As the temperature T rises from 0 °C, water contracts up to 3.98 °C and then expands as the temperature increases further. The density of water is greatest at 3.98 °C.

The anomalous expansion of water is very important for aquatic animals in very cold regions. As the temperature of air decreases in early winter, the surface water of the lake cools. When this surface water reaches 3.98 °C, it sinks to the bottom of the lake. The water from beneath is less dense and it floats to the surface. The cool descending water carries fresh oxygen with it. Thus, the cold water mixes with the warm water and then the temperature of the lake becomes uniform. The entire lake cools until it reaches 3.98 °C. The temperature of surface water decreases further and finally ice is formed. The water freezes from the surface downward. Fishes and other aquatic animals survive the winter in the water beneath the ice.

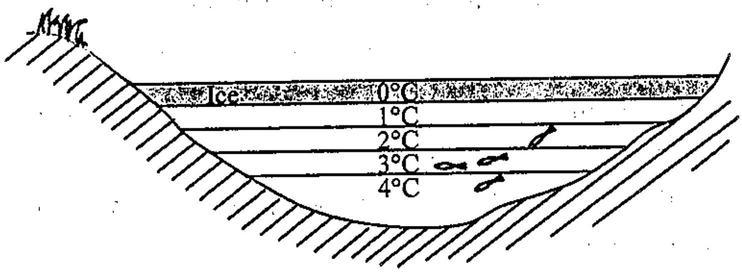


Fig. 6.6 Temperature in an ice covered lake

As stated above, the equation

$$\Delta V = \gamma V \Delta T$$

can be used for the expansion of gases. But there is a difference for solids and liquids. In defining γ in the above equation there is no restriction on the temperature for the initial volumes of solids and liquids.

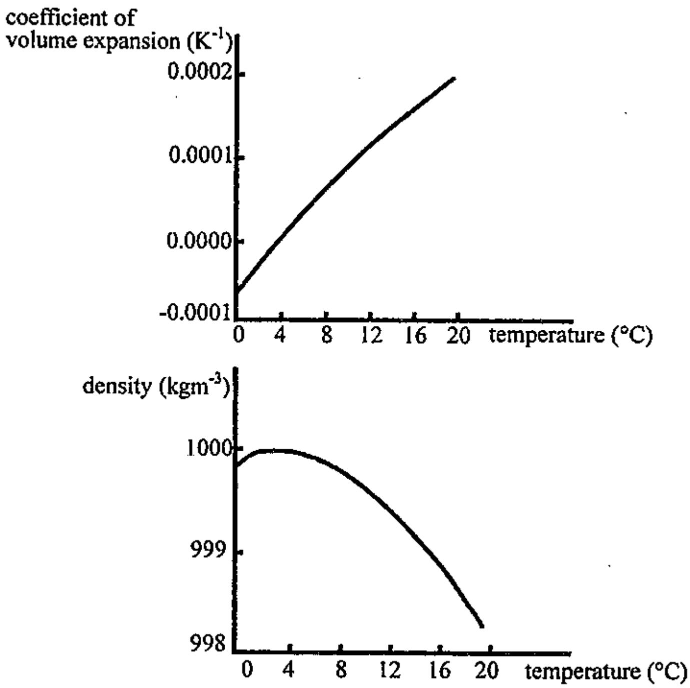


Fig 6.7 The change in coefficient of volume and density with temperature

But there is a restriction for gases that the initial volume must be at 0 °C and the pressure must be kept constant. Let the volume of the gas be V_o at 0 °C. If the volume becomes V when the temperature increases to T,

$$\Delta V = V - V_o$$

and $$\Delta T = T - 0 = T$$

Thus, $$V - V_o = \gamma V_o T$$

or $$\gamma = \frac{V - V_0}{V_0}\frac{1}{T} \text{ per } ^\circ C \text{ or per K.}$$

To determine γ experimentally, the increases in volume with temperature are measured and plotted on a graph. By reading V_o and the volume at 100 °C, V_{100}, from the graph, γ can be evaluated from the equation:

$$\gamma = \frac{V_{100} - V_0}{V_0 \times 100}$$

The result gives the coefficient of volume expansion of a gas. From experiments, the coefficient of volume expansion of gases is found to be 1/273.

Example (4) The area of a metal plate is A_1 at the temperature T_1 and A_2 at T_2. If $T_2 > T_1$, obtain the relation between A_1 and A_2. The coefficient of area expansion of metal is β.

$$\Delta A = A_2 - A_1 \text{ and}$$
$$\Delta T = T_2 - T_1$$

Thus,
$$A_2 - A_1 = \beta A_1 (T_2 - T_1)$$
$$A_2 = A_1 + \beta A_1 (T_2 - T_1)$$
$$= A_1[1 + \beta (T_2 - T_1)]$$

Example (5) Define the coefficient of volume expansion of a liquid.

From
$$\Delta V = \gamma V \Delta T ,$$
$$\gamma \quad \frac{\Delta V}{V} \cdot \frac{1}{\Delta T}$$

Therefore, the coefficient of volume expansion of a liquid is the change in volume per unit volume for one degree change in temperature.

In symbols.
$$\gamma = \frac{N}{K} = \frac{N}{{}^{\circ}C} = \frac{5/9\, N}{{}^{\circ}F}$$

where N is the number which depends upon the type of substance; N does not depend upon the temperature change.

Concept Map (Temperature)

Temperature

↓

Scale of temperature requires:

- a thermometric substance that has a physical property which varies continuously with temperature.
- two fixed points and a scale between them.

↓

Centigrade (Celsius) scale

- two fixed points:
  - (i) ice point (0°C)
  - (ii) steam point (100°C)
- defining equation:

$$T/^\circ C = \frac{X_T - X_0}{X_{100} - X_0} \times 100$$

Converting Temperatures

$$T/^\circ C = T/K - 273$$

Kelvin (Absolute) scale

- one fixed point: the triple point of water.
- The Kelvin (K) is the SI unit for temperature.

SUMMARY

- ***Temperature*** is a measure of "hotness" or "coldness" of a body.
- ***Heat*** is a form of energy. It is the energy exchanged between an object and it surrounding due to different temperatures.
- Solids, liquid and gases expand when heated.
- Coefficient of expansion is a property of a given material and depends somewhat on the temperature.
- ***Coefficient of linear expansion*** is defined as the change in length (of a substance) per unit length for one degree change in temperature.
- ***Coefficient of area expansion*** is defined as the change in surface area (of a substance) per unit area for one degree change in temperature.
- ***Coefficient of volume expansion*** is defined as the change in volume (of a substance) per unit volume for one degree change in temperature.
- The anomalous behaviour of water is that it contracts as the temperature rises from 0 °C to 4 °C and it expands as the temperature increases from 4 °C to 100 °C

  The density of water is greatest at 4 °C.

EXERCISES

1. Nowadays, the Celsius scale rather than the Fahrenheit scale is widely used. Discuss why this is so.
2. Choose the best answer from the following. For scientific work
   (a) only the Kelvin temperature scale is used.
   (b) only the Celsius scale is used.
   (c) only the Fahrenheit scale is used.
3. Fill in the blanks.
   The temperature 250 K is <u>*(a)*</u> than the ice point and the temperature 400 K is <u>*(b)*</u> than the ice point.
4. Define the coefficient of linear expansion.

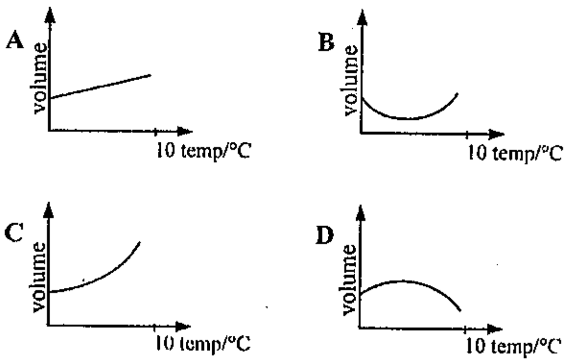
   Express the unit of that coefficient in SI units.

   "The coefficient of linear expansion of steel is 1.27×10^{-5} K^{-1}."

   Explain the meaning of this statement.
5. Define the coefficient of area expansion and the coefficient of volume expansion.

   Express the units of those coefficient in SI units.

6. Choose the correct answer. The density of a solid decreases when it is heated because (a) its mass decreases. (b) its mass increases. (c) its volumes decreases. (d) its volumes increases.
7. A solid expands when heated. What happens to its (a) mass; (b) volume; (c) density?
8. Should telephone wires be fixed to their supporting poles on a hot day or on a cold day? Explain your answer.
9. Which of the following graphs shows how the volume of water changes as it is heated from 0 °C to 10 °C?



10. Discuss the problems which may arise in construction works if the effect of linear expansion is not taken into account.
11. If the unit of the coefficient of linear expansion is changed from per K to per °F, does the numerical value of that coefficient change?
12. Why is it possible to open a jar of jam when its tight lid is heated?
13. The coefficient of volume expansion of pyrex glass is one-third that of ordinary glass. Which glass can stand more thermal strain?
14. What temperature on the Celsius scale corresponds to 100° on the Fahrenheit scale?
15. The decline of average temperature of the universe with its age is given below:

$t \sim 10^{-23}$ s .... $T \sim 10^{12}$ K

$t \sim 10^{-4}$ s .... $T \sim 10^{10}$ K

$t \sim 10^{6}$ yrs .... $T \sim 3000$ K

$t \sim 2 \times 10^{10}$ yrs (today) .... $T \sim 3$ K

Convert the temperatures from K to °C.

16. What temperature on the Celsius scale corresponds to 104 °F, the body temperature of a person who is gravely ill?

17. At what temperatures are the readings on a Fahrenheit and Celsius scales the same?

18. A steel railroad track is 20 m long at 20 °C. How much longer is it at 40 °C?

α steel = $1.27 \times 10^{-5} K^{-1}$

19. A steel railroad track is 30 m long at 0 °C. How much shorter is it at -20 °C ?

α steel = $1.27 \times 10^{-5} K^{-1}$

20. An aluminium metre stick is exactly 1 m long at 20 °C. How much shorter is it at 0 °C? Use $\alpha = 2.30 \times 10^{-5} K^{-1}$.

21. The surface area of a solid increases with temperature according to the formula $\Delta A = 2\alpha A \Delta T$. How much does the area of a rectangular steel plate 0.5 m by 2.5 m increase when it is heated from 0 °C to 40 °C?

22. A heat-resistant glass at 15 °C is fully filled with 250 cm^3 of glycerine. If the temperature increases to 25 °C how much glycerine overflows? The coefficient of volume expansion of glycerine is $5.1 \times 10^{-4} K^{-1}$ and that of heat-resistant glass is $0.09 \times 10^{-4} K^{-1}$.

CHAPTER 7

MEASUREMENT OF HEAT

Heat is a form of energy and therefore it may be expressed in energy units: joules, ergs or foot-pounds. Special units for heat, however, were introduced into physics during the later part of the eighteenth century when the relation of heat to energy was not quite understood. These special units, the calorie (cal) and the British thermal unit (Btu) are still used, although heat is no longer viewed as a substance. In this chapter the relations between different units of heat as well as the methods of measuring heat will be discussed.

7.1 MEASUREMENT OF HEAT

Heat has been defined as the amount of energy transferred from one object to another because of a difference in temperature. The unit used to measure how much heat the object attains is the kilocalorie.

Heat required to change the temperature of 1 kg mass of water by 1 K is called 1 kilocalorie.

This definition was given before the relation between heat and energy was known.

As heat is a form of energy, heat can be expressed in terms of the work unit. The SI unit of work is the joule (J) and the relation between kilocalorie (kcal) and joule is

$$1 \text{ kcal} = 4184 \text{ J}$$

The thermal unit of the British system is the British thermal unit (Btu) and

$$1 \text{ Btu} = 1055 \text{ J}$$

The thermal unit used in electrical engineering is the kilowatt hour (kWh) and

$$1 \text{ kWh} = 860 \text{ kcal}$$

$$= 3413 \text{ Btu}$$

$$1 \text{ kcal} = 4184 \text{ J or } 1 \text{ cal} = 4.184 \text{ J}$$

The German doctor, Robert Mayer, found first that the energy which represents 1 cal is 4.184 J. He was also the first to suggest the conservation of energy.

Although it was known about one hundred and fifty years ago that heat is a form of energy and 1 cal = 4.184 J, the unit "joule" has not been widely used in measuring heat yet. Calorie and kilocalorie units are still in use. Joule, being the SI unit, is the more modem unit for all types of energy including heat. The confusion relating to measurement of heat arises due to the use of the British thermal unit (Btu) as well as kWh in some countries. In addition, the calorie unit used by the nutritionists in measuring the energy values of food is in fact the kilocalorie used in the physical science, and this adds to the confusion.

No confusion will arise, if all the scientists use the internationally accepted SI unit, the joule, uniformly.

Example (1) Express the relations between the units used in measuring heat.

1 joule	$= 10^7$ ergs	$= 0.2390$ calories
1 calorie	$= 4.184$ joules	
1 kcal	$= 10^3$ calories	
1 kWh	$= 3.6 \times 10^6$ joules	
1 Btu	$= 1.054 \times 10^3$ joules	

Example (2) How many kilocalories are equal to the work unit, 1 ft-lb?

$$\begin{aligned} 1 \text{ ft - lb} &= 1.356 \text{ J} \\ &= 1.356 \times 0.2390 \text{ cal} \\ &= \frac{1.356 \times 0.2390}{1000} \\ &= 3.241 \times 10^{-4} \text{ kcal} \end{aligned}$$

Example (3) An egg contains 8 x 10^4 cal of the average energy value of food. If that cal unit is the thermal unit used in the physical science, express the energy value of food of the egg in joules.

$$1 \text{ cal} = 4.184 \text{ J}$$

The average energy value of food of the egg is

$$\begin{aligned} 8 \times 10^4 \text{ cal} &= 8 \times 10^4 \times 4.184 \\ &= 334\ \ 720 \text{ J} \end{aligned}$$

334 720 J of energy is obtained from an egg.

7.2 THERMAL CAPACITY

When an object at a certain temperature is placed in contact with another object at a higher temperature, the heat energy is transferred from the object at a higher temperature to one at a lower temperature. Since the object at the lower temperature receives additional energy, its temperature increases. The ratio of the amount of energy transferred to the temperature change is called the thermal capacity. If the energy transferred to the object is ΔU and the temperature change is ΔT, the thermal capacity of that body is

$$C = \frac{\Delta U}{\Delta T} \qquad (7.1)$$

The energy may be transferred from one object to another because of the temperature difference and the energy may be transferred by doing work on the object as well. For example, since work is done by stirring a liquid or by compressing a gas, the energy transfer occurs and temperature rises. Therefore, when work is done on the substance in the above manner or heat is added to the substance, the internal energy U of that substance increases. If the volume of the substance does not change, the heat added is equal to the increase in internal energy.

Suppose the internal energy of n moles of a substance changes by ΔU due to the temperature change ΔT. Then, the thermal capacity for 1 mole is

$$C = \frac{1}{n}\frac{\Delta U}{\Delta T} \qquad (7.2)$$

The unit of C is joules per mole per kelvin ($J\ mol^{-1}\ K^{-1}$).

The thermal capacity is an important property of materials. The best example of this is water. It has a relatively high thermal capacity. Because of high thermal capacity, water is used to cool engines. In cold countries, water is used to store heat in solar heating system of the houses. Another common application is the use of hot water bags (Fig. 7.1) to keep warm. This relies on the ability of hot water to store a large amount of energy.

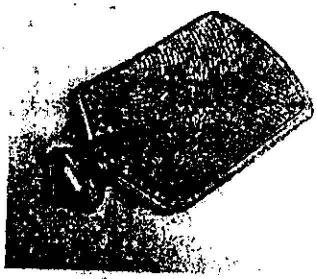


Figure (7.1) Hot water bag

The high thermal capacity of water also affects the climate. The climate of regions near large bodies of water such as lakes is found to be milder. This is because the thermal capacity of water is very much higher than those of the earth and air. For one degree rise in temperature, water absorbs much more heat than either land or air. The lake is a reservoir of heat for the surrounding regions. In summer, as the lake absorbs much more heat than the surrounding land and air, the surrounding region remains cool. In winter, when the surrounding land air become cold, the lake gives off the heat it has stored as its temperature drops. Thus, the surrounding region remains warm.

7.3 SPECIFIC HEAT CAPACITY

The physical quantity closely related to the thermal capacity is the specific heat capacity. The specific heat capacity of a substance is the heat needed to change the temperature of a unit mass of that substance by one degree. The specific thermal capacity is represented by the symbol c. The relation between c and thermal capacity C for 1 mole is

$$c = \frac{C}{M} \qquad (7.3)$$

where M is the mass of 1 mole of the substance.

Suppose the specific heat capacity of the object is c and the mass of that object is m. Then, heat required to change the temperature of the object by ΔT will be

$$\Delta Q = mc \ \Delta T \qquad \text{(at constant volume)} \qquad (7.4)$$

ΔQ is the additional heat gained by the object. In the SI system, the unit of the specific heat capacity is joules per kilogram per kelvin ($J\ kg^{-1}\ K^{-1}$).The old units used previously are cal $g^{-1}\ {}^{\circ}C^{-1}$and kcal $kg^{-1}\ {}^{\circ}C^{-1}$.

The specific heat capacities of various substances are given in Table 7.1.

Table 7.1

Substance	Specific Heat Capacity c kcal $kg^{-1}\ K^{-1}$	 $J\ kg^{-1}\ K^{-1}$
Aluminum	0.215	898
Steel	0.107	447
Diamond	0.124	518
Lead	0.031	130
Copper	0.092	385
Helium (gas)	1.240	5180
Hydrogen (gas)	3.410	14250
Iron	0.105	443
Nitrogen (gas)	0.249	1040
Oxygen (gas)	0.219	915
Water	0.996	4169
Ice(-10°C to 0°C)	0.500	2089
Steam (100 C to 200 °C)	0.470	1963
Glass	0.200	837

Example (4) When 3 x 10^5 J of heat is added to 10 kg of an object, the temperature of that object increases by 10°C. Find the thermal capacity of that object.

$$\Delta T = 10^{\circ}C = 10\ K$$

and $$\Delta Q = 3 \times 10^5\ J$$

Thermal capacity of the object $= \dfrac{\Delta Q}{\Delta T}$

$$= \frac{3\times10^5}{10}$$

$$= 3 \times 10^4\ \ J\ K^{-1}$$

Example (5) Find the specific heat capacity of the object from example (4).

The specific heat capacity of the object is

$$c = \frac{1}{m}\frac{\Delta Q}{\Delta T} \quad (m = \text{mass of the object})$$

and

$m = 10$ kg

Hence, $$c = \frac{1}{10} \times 3 \times 10^4$$

$$J = 3 \times 10^3\ J\ kg^{-1}K^{-1}$$

If c is to be expressed in kcal $kg^{-1}K^{-1}$ unit

$$c = \frac{3\times10^3}{4184}$$

$$= 0.7170\ \ kcal\ \ kg^{-1}K^{-1}$$

Example (6) The thermal capacity of 1 mole of helium (He) gas is 12.47 J mol^{-1} K^{-1}. If the mass of 1 mole of He is 4 x 10^{-3} kg, find the specific heat capacity of He.

$$C = 12.47\ J\ mol^{-1}K^{-1}$$

and $$M = 4\times10^{-3} kg$$

The specific heat capacity of He is

$$c = \frac{C}{M}$$

$$= \frac{12.47}{4\times10^{-3}}$$

$$= 3.12 \times 10^3\ J\ kg^{-1}K^{-1}$$

7.4 LAW OF HEAT EXCHANGE

When the specific heat capacity of a substance is known, the heat gained by a given mass of that substance as its temperature increases can be determined. If m is the mass and c is the specific heat capacity, heat gained by the object as its temperature increases by one degree is

$$\Delta Q_{gained} = mc$$

The heat gained when the temperature increases by ΔT is

$$\Delta Q_{gained} = mc\ \Delta T$$

The rise in temperature by ΔT is the difference between the final temperature and the initial temperature.

Similarly, the heat lost by the object when its temperature decreases by ΔT is

$$\Delta Q_{lost} = mc\ \Delta T$$

Using the above equations, the heat gained or lost by the object due to the heat transfer can be calculated.

When heat is transferred from one object to another object the total heat lost by one object is equal to the total heat gained by the other object.

This statement is the law of heat exchange. Since heat is energy, the law of heat exchange is one particular statement of the law of conservation of energy. In the above statement, the system consisting of two objects must be regarded as an isolated system.

The specific heat capacity can be determined using the law of heat exchange. The method of determination is as follows.

The sample whose specific heat capacity is to be determined is placed in a calorimeter which is well insulated. When the calorimeter and sample are heated causing a temperature change of ΔT, the heat absorbed by the sample $= mc\ \Delta T$

where m is the mass of the sample and c is the specific heat capacity of the sample.

Heat absorbed by the calorimeter $= m_c c_c\ \Delta T$

where m_c is the mass of calorimeter and c_c is the specific heat capacity of calorimeter.

Therefore, the heat gained by the sample and calorimeter is

$$\Delta Q_{gained} = mc\ \Delta T + m_c\ c_c \Delta T$$

If the heat lost by the electric heater is

$$\Delta Q_{lost} = \Delta Q$$

and since

$$\Delta Q_{lost} = \Delta Q_{gained}$$

$$\Delta Q = mc\,\Delta T + m_c c_c \Delta T$$

$$mc\,\Delta T = \Delta Q - m_c c_c\,\Delta T$$

$$c = \frac{\Delta Q - m_c c_c \Delta T}{m\,\Delta T}$$ is obtained.

Example (7) A calorimeter at 15 °C contains 0.1 kg of carbon. The calorimeter is made of aluminum and its mass is 0.02 kg. When 892 J of heat is added to the calorimeter and carbon, the temperature increases to 28 °C. Find the specific heat capacity of carbon. Assume that the specific heat capacity for aluminum at the said temperature range is 900 J kg^{-1} K^{-1}.

$$c = \frac{\Delta Q - m_c c_c \Delta T}{m\,\Delta T}$$

$$= \frac{892 - 0.02 \times 900 \times 13}{0.1 \times 13}$$

$$= 506 \text{ J kg}^{-1}\text{ K}^{-1}$$

Example (8) 0.20 kg coffee at 90 °C is poured into 0.5 kg glass at 20 °C. If no heat is lost and gained from outside, what is the final temperature of coffee?

Assume that the specific heat capacity of coffee is the same as that of water.

The heat gained by the glass $= m_{cup} c_{cup} \Delta T$

$= 0.50 \times 837 \times (T-20)$

$= (418.5\,T - 8370)$ J

The heat lost by coffee $= m_{coffee}\, c_{coffee}\, \Delta T$

$= 0.20 \times 4169 \times (90-T)$

$= (75\,042 - 833.8\,T)$ J

Heat gained = Heat lost

$418.5T - 8370 = 75042 - 833.8T$

$1252.3\,T = 83\,412$

$T = 67$ °C

SUMMARY

- *Heat* is a form of energy and therefore it may be expressed in energy units: joules,
  ergs or foot-pounds.
- The special units, the calorie (cal) and the British thermal unit (Btu) are still used.
- The **SI** unit of heat is the joule (J).

  1 joule = 10^7 ergs = 0.2390 calories

  lcal = 4.184J or 1 kcal = 4184J (1 kcal = 10^3 calories)
- The thermal unit of the British system is the British thermal unit (Btu).

  1 Btu = 1055 J
- The thermal unit used in electrical engineering is the kilowatt hour (kWh).

  1 kWh = 3.6 ×10 × joules
- The ratio of the amount of energy transferred to the temperature change is called *the thermal capacity.* $C = \frac{\Delta U}{\Delta T}$
- A mole of any substance is the amount of that substance that contains Avogadro's number of molecules, where Avogadro's number $N_A = 6.022 \times 10^{23}$. If we have n mole of a substance, the number of molecules is $N = n\, N_A$.
- The thermal capacity for 1 mole is $C = \frac{1}{n}\frac{\Delta U}{\Delta T}$
- The unit of C is joules per mole per kelvin ($J\ mol^{-1}\ K^{-1}$).
- The specific heat capacity (c) of a substance is the heat needed to
- change the temperature of a unit mass of that substance by one degree.

  $c = \frac{1}{m}\frac{\Delta Q}{\Delta T}$ (m = mass of the object)
- The S1 unit of the specific heat capacity is joules per kilogram per kelvin ($J\ kg^{-1}K^{-1}$).
- The relation between c and thermal capacity C for 1 mole is $c = \frac{C}{M}$

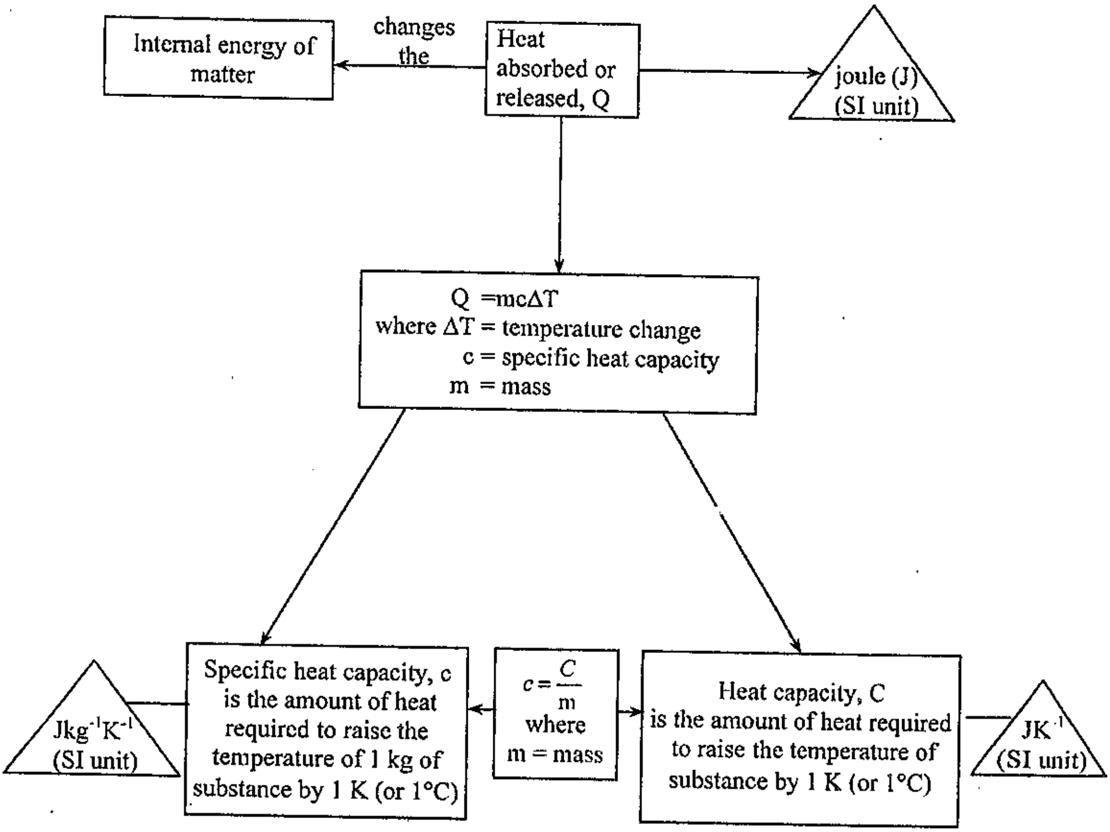
  where M is the mass of 1 mole of the substance.
- Heat required to change the temperature of an object of mass m by ΔT (at constant volume) is $\Delta Q = mc\Delta T$.
- The heat gained when the temperature increases by ΔT is

  $$\Delta Q_{gained} = mc\,\Delta T$$
- The heat lost by the object when its temperature decreases by ΔT is

  $$\Delta Q_{lost} = mc\,\Delta T$$

- ***The law of heat exchange.*** When heat is transferred from one object to another object the total heat lost by one object is equal to the total heat gained by the other object. ($\Delta Q_{lost} = \Delta Q_{gained}$)

Concept Map (Heat absorbed or released, Q)



EXERCISES

1. What is "thermal capacity"?
2. Define "specific heat capacity". Does it have any unit? If it has, write down the unit in SI system.
3. State the law of heat exchange.
4. How does the specific heat capacity of water moderate the climate in a region near a lake?

5. When a piece of iron is cooled from 70 °C to 40 °C, the heat given out is 690 J. What is the heat capacity of the piece of iron?

6. The heat capacity of a piece of copper is 200 J $°C^{-1}$. What is the amount of heat required to raise its temperature from 30 °C to 100 °C?

7. What is the amount of heat required to raise the temperature of 2 kg of copper from 30 °C to 80 °C? Assume that the specific heat capacity of copper is 400 J kg^{-1} $°C^{-1}$.

8. 600 000 J of heat energy is supplied to a kettle with 2.0 kg of water to raise the temperature of the water from 30 °C to 100 °C. Assuming no heat lost to the surroundings, find the specific heat capacity of water?

9. 0.5 kg of orange squash at 30 °C is placed in a refrigerator which can remove heat at an average rate of 25 Js^{-1}. How long will it take to cool the orange squash to 5 °C? The specific heat capacity of orange squash is 4200 J kg^{-1} $°C^{-1}$.

10. 1 litre of water at 100 °C is added to 4 litre of water at 30 °C. What will be the final temperature of the water?

11. 0.2 kg of water at 90 °C is poured into a steel cup of mass 0.1 kg at 30 °C. What is the final temperature of water assuming there is no heat lost to the surroundings? (Specific heat capacity of steel is 450 J kg^{-1} K^{-1} and that of water is 4200 J kg^{-1} $°C^{-1}$.)

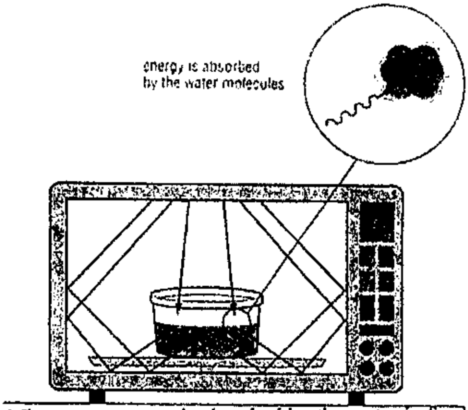
12. Mercury has a much lower specific heat capacity than water. If 1 kg of mercury at 100 °C is mixed with 1 kg of water at 0 °C, will the temperature of the mixture be 50 °C, above 50 °C, or below *50* °C? What will happen to the heat lost by the mercury during the mixing of mercury and water?

13. A 0.6 kg copper container holds 1.5 kg of water at 20 °C. If 0.1 kg iron ball at 120 °C is dropped into the water, what is the final temperature of the water?

14. How much heat must be added to change the temperature of 0.15kg helium (gas) from 20°C to 80 °C without changing the volume? The specific heat capacity of helium is 5.18×10^3 Jkg^{-1} K^{-1}.

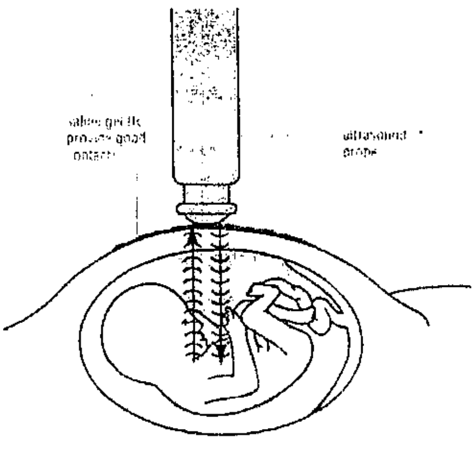
15. The specific heat capacity of 0.4 kg mass of a calorimeter is 627.6 J kg^{-1} K^{-1}. A 0.55 kg substance is in that calorimeter. The temperature of the calorimeter increases by 4 K when 2450 J of energy is added to it. Find the specific heat capacity of the substance in the calorimeter.

16. A 60 kg woman is on a diet that provides her with 1.046×10^7 J daily. If this amount of heat is added to 60 kg of water at 37 °C what will be the final temperature of the water?

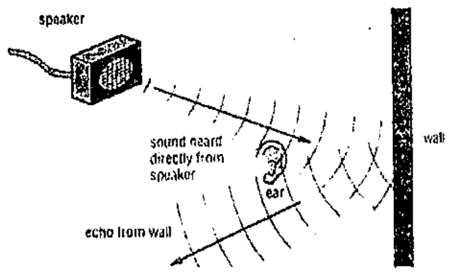


Microwave energy is absorbed by the water in food raising the temperature and cooking the food

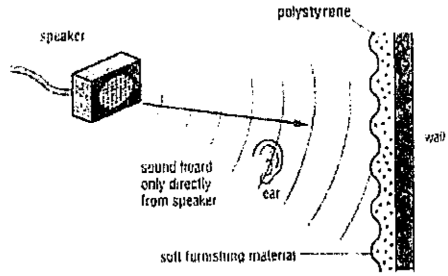
WAVES AND SOUND



An ultrasound scan of the womb



Most of the energy of a sound wave is reflected from hard smooth surfaces



Soft, irregular surfaces reflect very little sound in any one direction – most of the energy is absorbed

CHAPTER 8

WAVE CONCEPT AND SOUND WAVE

Most of the wave motions with which we are familiar involve large scale coordinated disturbances of many particles. Although the individual particles in those disturbances do not move far, the disturbance itself may travel great distances. Energy and momentum are carried along with the disturbance. The motions of the particles vary with the type of waves. For example, in a water wave, the water molecules move in almost circular paths. In a sound wave molecules vibrate back and forth. In a wave along a string parts of the string move up and down.

Light waves are also coordinated disturbances involving changes in electric and magnetic fields. For such waves the particles do not move, but the waves nevertheless carry energy and momentum. The mathematical description of these waves is nearly the same as that of mechanical waves such as water wave, sound wave and wave in a string.

All waves have a number of common characteristics. These are mentioned in the next section where waves in strings and springs are mainly discussed. The discussions on sound waves are described in the last section of this chapter.

Wave is a basic concept of physics. Waves are important because they carry energy and momentum from one place to another. The concepts of mechanics and electromagnetic theory are used as a basis in describing wave motion.

8.1 TRANSVERSE AND LONGITUDINAL WAVES

Waves are classified as transverse and longitudinal waves. If the displacements of particles of the medium are perpendicular to the direction of the wave. such a wave is called a transverse wave. Light waves are transverse waves. Waves in a vibrating string are also transverse waves. If the displacements of particles of the medium are parallel to the direction of the wave, such a wave is called a longitudinal wave. Compressional waves in a coiled spring as well as sound waves are longitudinal waves.

Waves have linearity property. Linearity property here means that when two or more waves pass the same point, the resultant wave at that point is the sum of the individual waves. After passing that point the waves continue along their original paths without any change whatsoever. The description of such linearity property is called the principle of superposition.

8.2 WAVE FREQUENCY, WAVE VELOCITY AND WAVELENGTH

As described above, there are different types of waves in nature. Light wave is a transverse electromagnetic wave, with changing electric and magnetic fields at right angles to each other and both being perpendicular to the direction of the light wave. The path of a sound wave consists of alternate compressions and rarefactions of the medium and hence it is a longitudinal wave. Water waves are a mixture of transverse and longitudinal waves.

Waves can be represented by graphs. The graphs representing transverse and longitudinal waves are similar. In Fig.8.1 *(a)* and *(b)*. a pulse of wave produced in a string and in a spring due to disturbances at their left ends and travelling toward the right are shown.

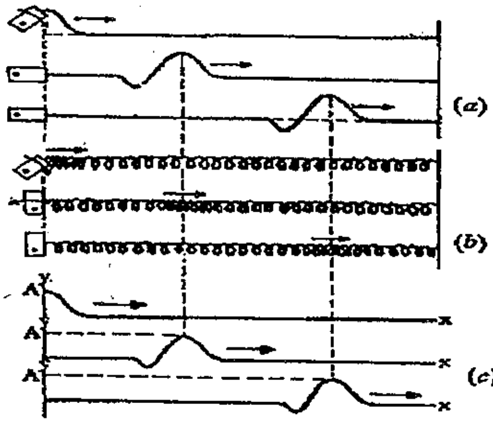


Fig.8.1

In Fig.8.1 *(a)* the motion of the particles is transverse to the motion of the wave. In Fig.8.1 *(b)* the motion of the particles is along the direction of the wave. Both types of waves are represented by graphs in Fig.8.1 *(c)*. For the string y is the displacement of the string from its undisturbed position and for the spring y is a measure of the compression or extension of the spring. For the string the displacements above the equilibrium position are positive and for the spring a compression is regarded as a positive displacement.

Fig. 8.2. (a) and (b) show the periodic wave trains produced by transverse up-and-down motion of the left end of the string and longitudinal to-and-fro motion of the left end of the spring respectively.

These periodic wave trains are shown graphically in Fig. 8.2(c)

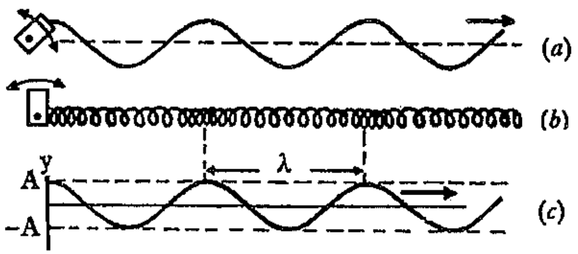


Fig.8.2

The broken line in Fig. 8.2 (*a*) indicates the undisturbed position of the string.Fig.8.2 *(b)* the alternate compressed and extended portions of the spring are shown. Let us now discuss some quantities which characterize the common properties of periodic waves. Let us start with frequency.

The *frequency* f is the number of waves (wavelengths) passing a point per second and it depends on the vibrating source. For example, in the string and spring of Fig. 8.2 the frequency, is the rate at which the oscillations occur at the left end.

The *period* T is the time taken by the wave to travel the distance between any two consecutive wave crests, and it is the reciprocal or inverse of the frequency:

$$T = \frac{1}{f}$$

The *velocity* v of a wave is the speed with which a wave crest travels.

The *wavelength* λ of a periodic wave is the distance between any two consecutive wave crests.

The *amplitude* A of a wave is the maximum value of the displacement. The displacement of a periodic wave varies back and forth between A and -A.

Most of the periodic waves are represented by sine or cosine graphs.

The relationship between the frequency, wavelength, and velocity of a periodic wave can be obtained from Fig.8.3.

In Fig. 8.3 it is shown that the wave travels one wavelength λ during the period T. (Period T can also be defined as the time required for one complete oscillation.) Since the wave travels the distance λ in the period T the wave velocity v is

$$v = \frac{\lambda}{T}$$

Again, since $T = \frac{1}{f}$

$$v = f\lambda$$

is obtained from the above two equations.

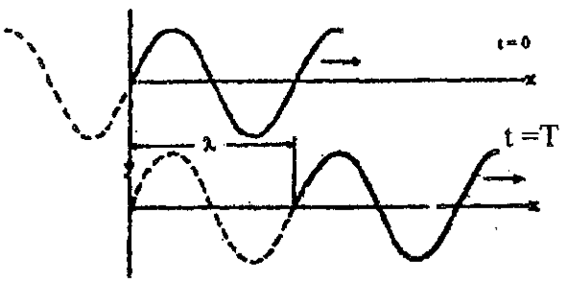


Fig 8.3

Example (1) A wave pulse in a string moves a distance of 10 m in 0.05 s. (a) Find the velocity of the wave pulse. (b) Find the frequency of a periodic wave in the same string if its wavelength is 0.8 m.

(a) The velocity of the wave pulse is

$$v = \frac{d}{t}$$

$$= \frac{10\text{m}}{0.05\text{s}} = 200\text{ms}^{-1}$$

(b) The periodic wave has the same velocity of 200 m s^{-1}. Using $f\lambda = v$, the frequency of the 0.8 m wave is

$$f = \frac{v}{\lambda}$$

$$= \frac{200\text{ms}^{-1}}{0.8\text{ m}} = 250\text{ s}^{-1} \text{ or } 250\text{ Hz}$$

Example(2) A typical sound wave associated with human speech has a frequency of 500 Hz and the frequency of the yellow light is about 5×10^{14} Hz . The velocity of sound in air is 344 m s^{-1} and the velocity of light is 3×10^8 m s^{-1}. Find the wavelengths of the two waves.

For both waves $f\lambda = v$, so for the sound wave

$$\lambda = \frac{v}{f}$$

$$= \frac{344}{500} = 0.688\text{ m}$$

For the light wave

$$\lambda = \frac{c}{f}$$

$$= \frac{3\times10^8}{5\times10^{14}} = 6 \times 10^{-7}$$

$$= 6000\text{ Å}$$

The ripple tank

The ripple tank is a convenient piece of apparatus for demonstrating the properties of wave pulses and waves. It consists of a sheet of glass in a frame about 5 cm deep. Sheets of sponge lime the frame to absorb the ripples and prevent reflection by the sides. The tank stands on legs above a large sheet of white paper or painted hardboard. A lamp above the tank throws the shadows of the wave or ripples onto the white screen. These shadows are seen most clearly if they are *not* viewed through the water. Before use the tank is leveled and water is poured into a depth of 5-10 mm.

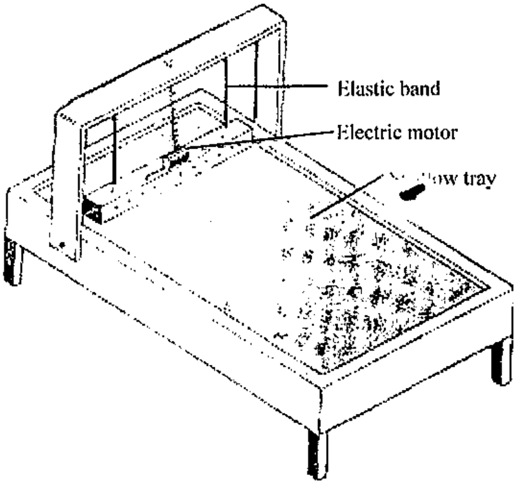


Fig. 8.4

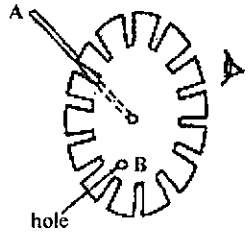


Fig.8.5

Plane wave pulses are produced by dipping a rule into the water, circular wave pulses are produced by any pointed object. When continuous waves are required a wooden beam is hung above the water by elastic bands.(Fig. 8.4) A small electric motor with an eccentric shaft is fitted to the beam which, when switched on, makes both motor and beam vibrate. A wooden strip fitted to the bottom of the beam produces a plane wave, circular waves are formed by a small wooden sphere.

Continuous waves are viewed using a stroboscope. One type of stroboscope is simply a flashing light. When this flashes with the same frequency as the frequency of the waves, the waves appear to be stationary. Another type is a hand stroboscope, a disc with a number of radial slits equally spaced around its edge. (Fig. 8.5) The disc is attached to an axle passing through its center and fixed to the handle A. It is rotated by a finger placed in the hole B. The waves are viewed through the slits, and when the frequency with which the slits pass the eye is the same as the frequency of the waves, the waves appear to be stationary.

8.3 SOUND WAVE FORMS AND VELOCITY OF SOUND

Sound wave forms

The wave forms in Fig.8.6 may be regarded as longitudinal sound waves. The wave forms in parts (a), (b) and (e) have the same frequency, as do those in parts (c) and (d).

Pitch The pitch of a note (how 'high' it is) is determined by its frequency, so the notes produced by the waves in (a), (b) and (e) in Fig.8.6 all have the same pitch. The wavelength of the waves in parts (c) and (d) is half of the wavelength in the others so the frequency and hence the pitch are doubled. These notes are said to be **one octave** higher than the others.

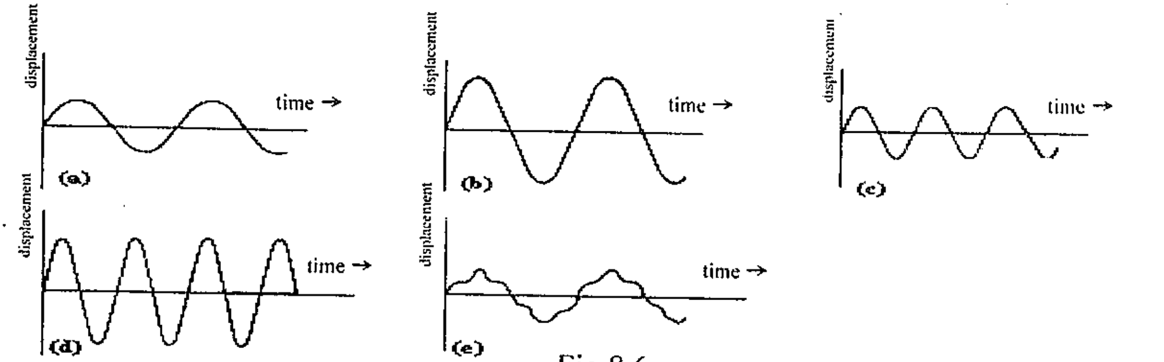


Fig.8.6

Loudness The loudness of a note depends upon the amplitude of the wave that produces it. The greater the amplitude the louder the note, because more energy is used to produce a larger amplitude.

Note The energy transmitted by a wave depends upon the frequency as well as the amplitude. If the frequency of a note is doubled, twice as many compressions and rarefactions strike the ear each second and more energy is received. In fact the energy in a wave is proportional to both (frequency $)^2$ and (amplitude $)^2$.

Quality or timbre The note in Fig.8.6 (e) has the same frequency and hence the same pitch as the notes in (a) and (b). It has the same amplitude as (a) and will therefore be just as loud. However, (e) sounds different from (a) because it has a different wave form. It is said to differ in quality or timber. It sounds richer than the other notes because it is not a simple note, but contains **overtones.** Notes of the same pitch played upon different musical instruments are distinguished from each other by their quality.

Velocity of sound

The velocity of sound in air, at 0 $^{\circ}$C is 332 ms^{-1} or 1090 fts^{-1}. Whenever the air temperature increases by 1 $^{\circ}$C, the velocity of sound will increase by 0.2 %. The velocity of sound in air can be expressed as

$$v = 332\sqrt{\frac{T}{273}}$$

Here T is given in terms of K and v in m s^{-1}. The above relation can be approximated by the following relation

$$v \simeq 332 + 0.6\,(T - 273)$$

Sound can travel not only through air but also through solids, liquids and gases. Generally, the denser the medium, the greater will be the velocity of sound. This is natural because the denser the medium is the more will the particles of the medium tightly bind themselves together. This means the disturbance can be transferred more quickly from one particle to the next. The velocity of sound in some of the solids and liquids are given in Table 8.1.

Table 8.1

Medium	Velocity		Temperature
	ms^{-1}	fts^{-1}	$^{\circ}C$
Air	332	1090	0
$C0_2$	259	850	0
Cl_2	206	676	0
Water, pure	1404	4605	0
Copper	3560	11680	20
Iron	5130	16830	20

Example (3) The frequency of a musical note is 440 Hz. Find the wavelengths of that sound in air and water.

Since the velocity of sound in air is 332 m s^{-1}, the wavelength of musical note in air is

$$\lambda = \frac{v}{f}$$

$$= \frac{332}{440} = 0.7544 \text{ m}$$

$$= 75.44 \text{ cm}$$

Since the velocity of sound in water is 1404 m s^{-1}, the wavelength of musical note in water is

$$\lambda = \frac{v}{f}$$

$$= \frac{1404}{440} = 3.19 \text{ m}$$

Therefore, the wavelength of the musical note in water is more than four times that of the musical note in air. When a sound wave travels from one medium to another its velocity and wavelength change but the frequency remains the same.

Example (4) What is the velocity of sound in air at 40 °C?

The velocity of sound in air at 40 °C is

$$v = 332\sqrt{\frac{T}{273}}$$

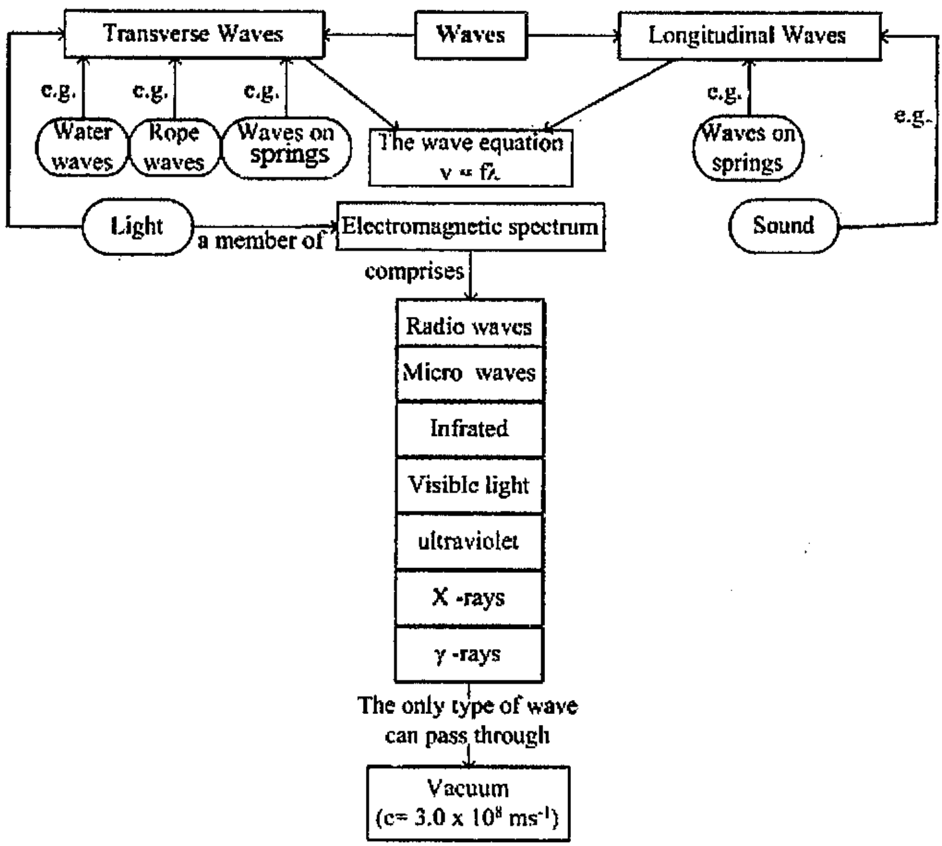
$$= 332\sqrt{\frac{273+40}{273}} = 355.5 \text{ m s}^{-1}$$

If the approximate formula is used

$$v \simeq 332 + 0.6\,(T - 273)$$

$$\simeq 332 + 0.6(313 - 273) \simeq 356 \text{ m s}^{-1}$$

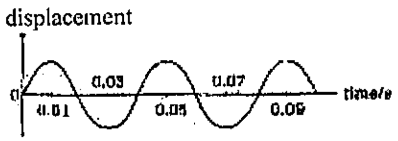
Concept Map (Waves)



EXERCISES

1. Sound waves can travel in all of the following except
   A. solids. C. air
   B. liquids D. vacuum
2. How does the speed of sound vary in the following media: water, air and wood?

	Highest speed	Lowest speed
A.	Air	water
B.	Water	wood
C.	Wood	water
D.	Wood	air

3. Which one of the following statements is true for both sound and light waves?
   A. They are transverse waves.
   B. They are reflected from a glass surface.
   C. They travel faster in air than in water.
   D. They are electromagnetic waves.
4. 

   The displacement of an air particle with time as a sound wave travels through the air is as shown above.
   What is the frequency of the sound wave?
   A. 10 Hz C. 50 Hz
   B. 25 Hz D. 100 Hz
5. A sound of frequency 400 Hz has a wavelength of 4.0 m in a medium. What is the speed of sound in the medium?
   A. 10^{-2} m/s C. 1600 m/s
   B. 100 m/s D. 8000 m/s
6. A boy hears the thunder 2.0 s after seeing lighting flash. How far is the lightning flash from the boy? (Speed of sound = 330 m/s)
   A. 165 m C. 660m
   B. 330 m D. 1320 m
7. Which of the following describes correctly the changes, if any, to the frequency, wavelength and speed of sound as it travels from air into water?

	Frequency	Wavelength	Speed
A.	Remains unchanged	Decreases	Decreases
B.	Remains unchanged	Increases	Increases
C.	Increases	Increases	Increases
D.	Increases	Remains unchanged	Decreases

8. Define wavelength, frequency and velocity of a sound wave. Write down the relationship between them. Can this relationship be used for other waves (such as light waves)?

9. Are the following statements true or false? Correct the statements which are wrong.

(a) The frequency of a wave is directly proportional to its wavelength.

(b) Sound wave is transverse wave and water wave is longitudinal wave.

(c) The velocity of sound is the same in water, air and helium gas.

(d) Sound waves cannot travel through vacuum.

10. Write down the relation between period and frequency.

11. How does the velocity of sound depend on the temperature of the medium through which it travels?

12. Fill in the blanks.

The velocity of sound in air at 0 °C is *(a)* m s^{-1} and the velocity of sound will increase by *(b)* % whenever the air temperature rises 1 °C.

13. Fill in the blank.

The relation $v \simeq 332 + 0.6\,(T - 273)$ can be derived from the relation ___.

14. "Generally, the denser the medium the greater will be the velocity of sound." Explain this statement.

15. Find the wavelength of a wave with frequency 1000 Hz and velocity 344 ms^{-1}.

16. Find the frequency of a wave of velocity 200 m s^{-1} and wavelength 0.5 m.

17. A radar antenna emits electromagnetic radiation ($c = 3 \times 10^8\ ms^{-1}$) of wavelength 0.03 m for 0.5 s. *(a)* Find the frequency of radiation. *(b)*How many complete waves are emitted in 0.5 s?

18. Find the frequency, of a wave of 29 m wavelength telecast by a TV station. The velocity of that wave is the same as that of other electromagnetic waves and is 3×10^8 m s^{-1}.

19. The shortest wavelength of an ultrasonic wave emitted by a bat (in air at 0 °C) is 3.3 mm. What is the frequency of this wave? Is this frequency the largest or the smallest?

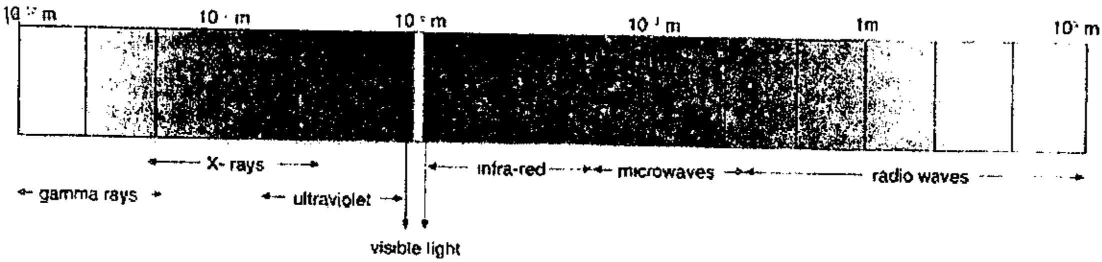
20. A stone is dropped in a well whose water level is 20 m down. How much time elapses until the sound of the splash is heard? Assume the speed of sound to be 340 ms^{-1}.

21. The frequency of a musical note in air is 440 Hz. What is the wavelength of that sound in sea water and in CO_2 gas?

22. What is the velocity of sound in air at 20 $^{\circ}C$?

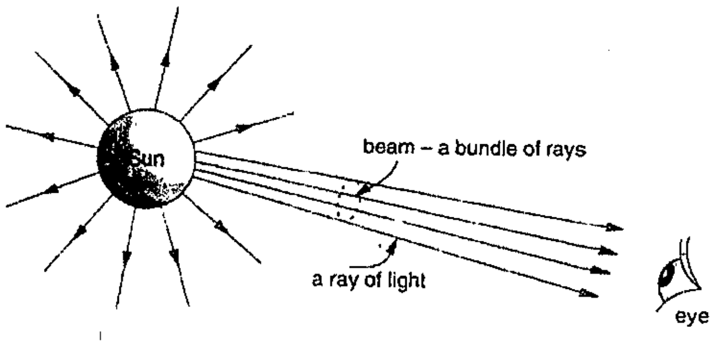
23. If the temperature of the medium is increased from 0 $^{\circ}C$ to 40 $^{\circ}C$, by what percentage has the velocity of sound increased?

24. Draw the v^2 - T graph. (v = velocity of sound, T = temperature of medium and $v \propto \sqrt{T}$.)

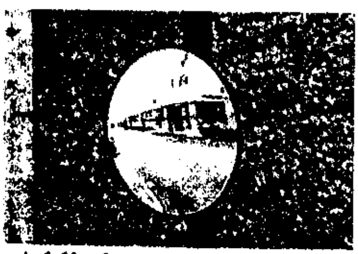


The electromagnetic spectrum

LIGHT



Light travels in straight line called rays



A blind corner mirror

CHAPTER 9
REFLECTION OF LIGHT

When light is incident on the surface of an object some of the light is sent back and this phenomenon is called reflection of light. Generally, light incident on the surface of an object is always partially reflected. When light is incident on a transparent substance such as glass, some of the light passes through the glass, some reflected and some absorbed. Highly-polished surfaces reflect much more light than rough surfaces. Highly-polished metal surfaces reflect about 80 to 90 per cent of the light incident on them. Objects are visible to us because the reflected light rays from those objects enter the eye.

Properties of light may be studied by subdividing optics into geometrical optics and physical optics. Geometrical optics is based upon the fact that light travels in a straight line. Ray diagrams are used in explaining the optical phenomena. On the other hand, physical optics is based upon the fact that light propagates by means of a wave-motion. Of these two, only geometrical optics will be studied here.

9.1 SOURCES OF LIGHT

The sun, the stars, the candle flame, the fluorescent lamp, etc., are all sources of light. Such sources of light are called self-luminous bodies. Human beings, trees, books, etc., are non-luminous bodies. These non-luminous bodies are visible when the reflected rays of light from them enter the eye.

The sun is the chief source of light. The fact that light coming from the sun passes through the empty space on its way to the earth shows that light can travel through vacuum. It also shows that light is an electromagnetic wave. The speed of light in vacuum is 3×10^8 m s^{-1}.

9.2 REFLECTION AND LAWS OF REFLECTION

A ray of light is a path along which the light energy travels. A ray is represented by a straight line with an arrow-head. The arrow-head points in the direction of light propagation.

A beam of light is a collection of rays of light. If the rays are parallel to one another, the beam of light is called a parallel beam of light (Fig.9.1). Searchlights, used in trains and lighthouses, emit parallel beams of light. A beam of light received from a distant source can also be considered as a parallel beam.

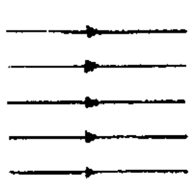
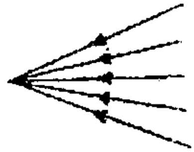
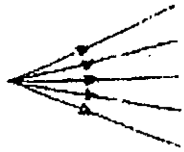




Fig.9.1. Parallel rays   Fig. 9.2 Convergent rays   Fig.9.3Divergent rays

If the rays of light are directed towards a point or if the rays of light converge to a point, the beam of light is called a convergent beam (Fig. 9.2). A parallel beam of light becomes a convergent beam after passing through a convex lens. If the rays of light diverge from a point or if they appear to come from a point, the beam of light is called a divergent beam (Fig. 9.3). A beam emitted by a light bulb is a divergent beam.

Rays of light are used in studying the reflection of light from a surface. The light incident on the surface consists of several rays. But the incident light is represented by only one of those rays. Similarly, the reflected light from a surface is represented by one of the rays of reflected light.

A ray which represents the incident light is an incident ray. A line perpendicular to the surface at the point of incidence is called a normal. A ray which represents the reflected light is a reflected ray. An angle between the incident ray and the normal is an angle of incidence and an angle between the reflected ray and the normal is an angle of reflection.

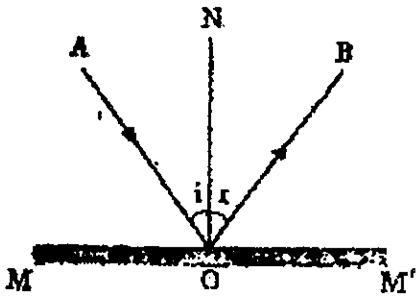


Fig. 9.4 Reflection at a plane surface

The reflection of light from a plane surface MM' is shown in Fig 9.4 This figure illustrates the terms used in connection with the reflection of light.

AO = incident ray OB = reflected ray ON = normal

∠ AON = i = angle of incidence   ∠ BON = r = angle of reflection

When the angle of incidence, i, is varied the angle of reflection, r, also varies and experiments show that i is always equal to r. In addition, it is found that the incident ray, the reflected ray and the normal all lie in the same plane. These findings are stated in the laws of reflection.

Laws of Reflection

(1) The incident ray, the reflected ray and the normal all lie in the same plane.

(2) The angle of incidence is equal to the angle of reflection.

Regular and Diffuse Reflection

Plane mirrors and glass slabs have smooth surfaces. When a ray of light is incident on such surfaces it is reflected in a definite direction in accordance with the laws of reflection. When a parallel beam of light is incident on a plane mirror the angles of incidence of all the rays are equal. Thus all the rays are reflected in one direction. Such reflection of light is called regular reflection.

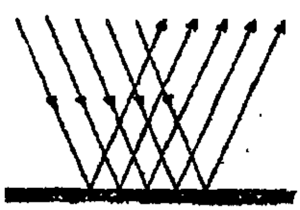


Fig. 9.5 Regular reflection

Fig. 9.6 Diffuse reflection

When a parallel beam of light is incident on a rough surface the rays are reflected in different directions (Fig.9.6). Such reflection of light is called diffuse reflection. The surface of paper is not smooth. The roughness of the paper surface can be seen with a microscope. Thus, reflection from the surface of the paper is diffuse reflection. All of the reflected rays from the paper surface obey the laws of reflection. However, the incident rays have different angles of incidence due to the roughness of the paper surface. Therefore, the rays are reflected in different directions. Reflections from objects in everyday life such as flowers, books, people and brick walls, are diffuse reflections.

9.3 REFLECTION AT PLANE SURFACES

An object having a smooth reflecting surface is called a mirror. If the reflecting surface is plane, the mirror is called a plane mirror. Looking glass, in everyday use, is one kind of plane mirror.

We shall first of all study the formation of image of a point object due to a plane mirror. In Fig. 9.7 a point object O is placed in front of the plane mirror M. A ray OB from O is incident on the plane mirror at B and reflected along BC. BN is the normal. A ray OA from O is incident normally on the plane mirror and reflected along AO. When OA and CB are produced they meet at a point I behind the mirror. I is the image of O. We are assuming here that all the rays lie in the plane which is perpendicular to the surface of the plane mirror.

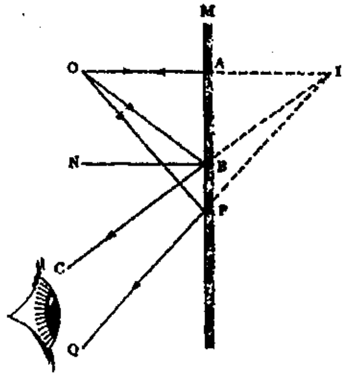


Fig. 9.7 Formation of image of a point object

In Fig. 9.7 OA is parallel to NB, so that

$\angle$ OBN $= \angle$ AOB (alternate angles)

$\angle$ CBN $= \angle$ AIB (corresponding angles)

By the laws of reflection $\angle$ OBN $= \angle$ CBN

Therefore $\angle$AOB $= \angle$AIB

In the triangles OAB and IAB

$\angle$ AOB $= \angle$ AIB

$\angle$ OAB $= \angle$ IAB (right angles)

AB = AB (common side)

Therefore Δ OAB $\cong \Delta$ AIB

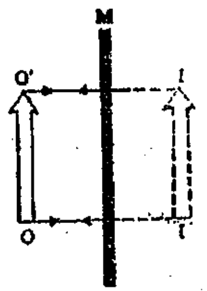
and AO = AI

AO is the perpendicular distance of the object from the plane mirror and AI is that of the image from the plane mirror. The object and image are at equal perpendicular distances from the mirror. In other words, the image is as far behind the mirror as the object is in front.

In Fig. 9.7 the incident ray OP is reflected along PQ and the reflected ray PQ produced backwards passes through I. Similarly, other reflected rays produced backwards will pass through I. Thus, we can say that the point image I is formed on the line passing through O and perpendicular to the plane mirror.

The image I is observed as the reflected rays enter the eye. The reflected rays appear to diverge from I. The reflected rays do not actually pass through I; only the reflected rays produced backwards pass through I. Thus, I is a virtual image. The virtual image cannot be formed on a screen. The image formed by the actual intersection of the reflected rays is a real image. A real image can therefore be focused on a screen.

The formation of image of an extended object due to a plane mirror will be studied next. In Fig. 9.8, II' is the image of an object OO'. The object OO' can be considered as an object formed by several point objects. I is the image of a point O and I' is that of a point O'. The points between O and O' have corresponding images. When all of the images are connected the image of the whole object is obtained.



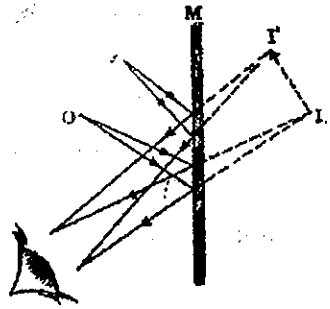


Fig.9.8 Image of an extended object Fig.9. 9 Ray diagram for an extended object

From Fig.9.8 we can see that OO' = II'. This means that the image and the object are of the same size. The image II' is virtual and erect. Fig.9. 9 shows the image II' of a slightly inclined object OO' formed in the plane mirror.

Lateral inversion

Suppose that a man is looking at himself (at his image) in a looking glass. When he tilts his head to the right, the head of the image in the mirror is found to tilt to the left with respect to the image. When he tilts his head to the left, the head of the image is found to tilt to the right with respect to the image. This effect is called lateral inversion. You can do this experiment yourself.

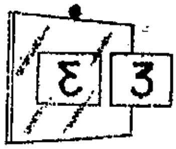


Fig. 9.10 Lateral inversion of a number

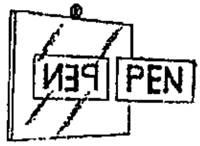


Fig 9.11 Lateral inversion of a word

When a paper on which a number 3 is written is placed in front of a looking glass, its image in the looking glass is laterally inverted as shown in Fig. 9.10.When a paper on which the word PEN is written is placed in front of a looking glass the lateral inversion is as shown in Fig. 9.11.

Properties of an Image in a Plane Mirror

The properties of an image formed in a plane mirror are as follows:—

1. The image is of the same size as the object.
2. The image is virtual.
3. The image is erect.
4. The image is laterally inverted.
5. The image is situated on the line passing through the object and perpendicular to the plane mirror.
6. The image is as far behind the mirror as the object is in front.

Principle of Reversibility of Light

In Fig. 9.12 a ray AO is incident on the plane mirror at a point O and is reflected along OB. If a ray were incident on the mirror along BO, it would be reflected along OA in accordance with the laws of reflection. If the direction of a ray of light is reversed, the light ray will travel along its original path. This is known as the principle of reversibility of light. Any ray of light obeys this principle.

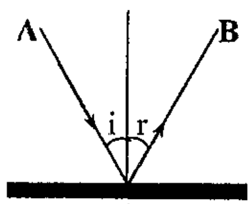


Fig. 9.12 illustration of reversibility of light

The images studied so far are formed in the plane mirror when both the object and the mirror are stationary. Now, the images formed in the mirror will be studied when either the object or the mirror is in motion.

Stationary Object and Moving Plane Mirror

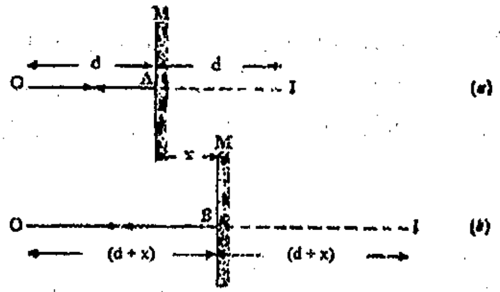


Fig.9.13. Image formation in a moving plane mirror

In Fig. 9.13 (a) M is a plane mirror and the image of an object O is I. Therefore OA = AI = d.

In this position, the distance between O and I = d + d = 2d.

In Fig. 9.13 (b) M is moved a distance x away from O. In this position, the distance between O and I = (d + x) + (d + x)

= 2d+2x

The position of O remains the same but the position of I is changed.

The displacement of I from its original position = (2d + 2x) - 2d

= 2x

Similarly, if M is moved a distance x towards the object, the image will be moved a distance 2x from its original position.

Generally, therefore, if a mirror is moved a distance x away from or towards the object, the image will move through a distance 2x.

Stationary Plane Mirror and a Moving Object

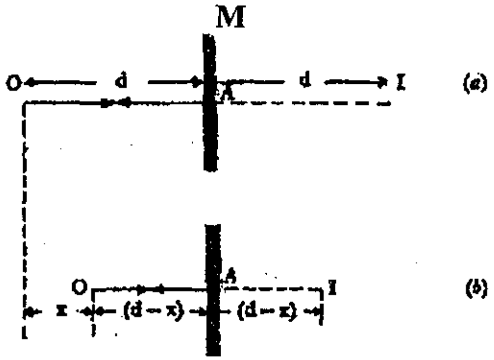


Fig. 9.14 Image of a moving object

In Fig. 9.14 (*a*) the distance between the object and the mirror is d. Therefore OA=AI=d.

In Fig. 9.14 (*b*) O is moved a distance x towards the mirror. In this position OA = AI = d-x.

I moves towards M from its original position.

The displacement of I from its original position $= d - (d - x) = x$

Similarly, if O is moved a distance x away from M, I will also move a distance x away from M.

Therefore, if an object is moved a distance x away from or towards the mirror, its image will move through a distance x from its original position.

Deviation of Light by a Plane Mirror

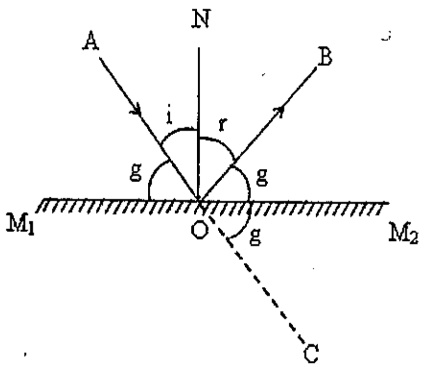


Fig. 9.15 The glancing angle

In Fig 9.15,
AO = Incident ray
ON = Normal
OB = Reflected ray
OC = Original path of the incident ray
$\angle AON = i$ = angle of incidence
$\angle BON = r$ = angle of reflection
$\angle AOM_1 = g$ = glancing angle

Fig. 9.15 shows the reflection of light from a plane mirror M_1M_2. The angle between the incident ray AO and the plane mirror is called the glancing angle g.

By the laws of reflection $i = r$ and hence $\angle AOM_1 = \angle BOM_2 = g$.

In addition, $\angle AOM_1 = \angle COM_2 = g$.

Although the original direction of the incident ray AO is along OC it is reflected along OB by the mirror.

The angle of deviation of AO $= \angle COB$

$= g + g = 2g$

Therefore, the angle of deviation of a ray by a plane mirror is twice the glancing angle.

Deviation of Reflected Ray by Rotating a Plane Mirror

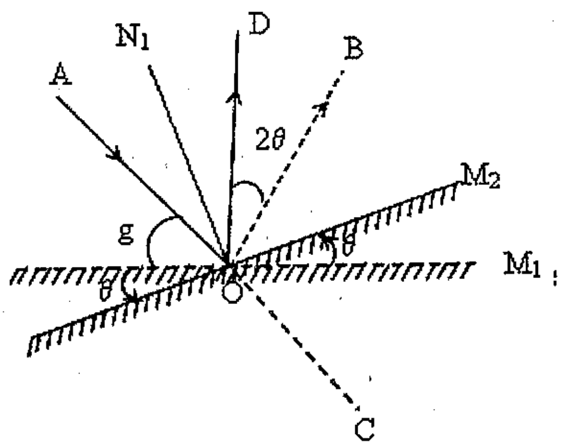


Fig. 9.16 (a) Deviation of reflected ray (Mirror after rotating anti- clockwise direction)

In Fig. 9.16 (a)

AO = Original incident ray

ON_1 = New normal obtained due to rotating by the mirror

OD = New reflected ray

OB = Original reflected ray

OC = Original path of the incident ray

θ = Angle of rotation by the mirror

$\angle COD$ = New angle of deviation

In Fig. 9.16 (a), the incident ray AO is reflected along OB by a plane mirror M_1. The angle of deviation $\angle COB = 2g$, where g is the glancing angle.M_1 can rotate about an axis O. The axis O is perpendicular to the plane containing the incident and reflected rays and passes through O. In Fig. 9.16 (a) the incident and reflected rays lie on the plane of the paper so that the axis O is perpendicular to the plane of the paper. As shown in Fig. 9.16(a), M_1 is rotated anticlockwise through an angle θ to the position M_2. The incident ray AO is still in its original position.

But the reflected ray OB moves anticlockwise and lies along OD which is a new reflected ray.

When the plane mirror is in the position M_2, the new glancing angle is $g + \theta$. Since OD is the new reflected ray of the incident ray AO, the angle of deviation = $\angle COD$ = twice the glancing angle = $2(g+\theta)$

The angle between OB and OD (two reflected rays) = $\angle BOD$

$$= \angle COD - \angle COB$$
$$= 2(g+\theta) - 2g$$
$$= 2\theta$$

where θ is the angle of rotation by the mirror.

Thus if the plane mirror rotates about an axis through an angle θ in the direction of anti-clockwise, the reflected ray is rotated through an angle 2θ.

Similarly, one can find that if the plane mirror rotates about an axis through an angle θ in the clockwise direction (Fig 9.16 (b)), the reflected ray is also rotated through an angle 2θ.

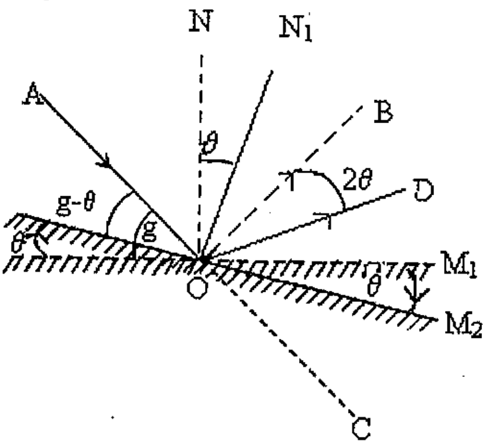


Fig. 9.16 (b) Deviation of reflected ray (Mirror after rotating clockwise direction)

New glancing angle = $g - \theta$

The angle of deviation = $\angle COD$

= twice the glancing angle

$$= 2(g - \theta)$$
$$= 2g - 2\theta$$

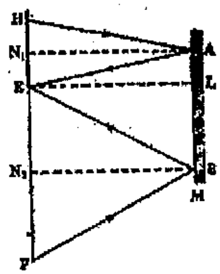
One has known that

$\angle COB = 2g$

The angle between OB and OD (two reflected rays)

$$\angle BOD = \angle COB - \angle COD$$
$$= 2g - (2g - 2\theta)$$
$$= 2\theta$$

Example (1) A man 5 ft 6 in tall and whose eye level is 5 ft 2 in above the ground, looks at his image in a looking glass. What is the minimum vertical length of the looking glass if the man is to be able to see the whole of himself?



In the above figure M is the looking glass. H represents the man's head, E his eyes and F his feet, respectively.

Therefore, HF = 66 in, EF = 62 in and HE = 66 - 62 = 4 in.

For the man to be able to see his head, an incident ray from H to the top A of M must be reflected to his eyes E.

Since the normal AN_1 bisects HE

$$AL = EN_1 = \frac{1}{2} HE = \frac{1}{2} \times 4 \text{ in} = 2 \text{ in}$$

For the man to be able to see his feet F, an incident ray from F to the bottom B of M must be reflected to his eyes E.

Since the normal BN_2 bisects EF,

$$LB = EN_2 = \frac{1}{2} EF = \frac{1}{2} \times 62 \text{ in} = 31 \text{ in}$$

Therefore, the vertical length of M $= AL + LB$
$= 2 \text{ in} + 31 \text{ in}$
$= 33 \text{ in}$
$= 2 \text{ ft } 9 \text{ in}$

The looking glass must have a minimum vertical length of 2 ft 9 in.

Example (2) A pin, 2 cm high, is placed 6 cm in front of a plane mirror. If the pin is moved 1.5 cm closer to the mirror, by how much is the distance between the pin and the image reduced?

The distance between the pin and the plane mirror is equal to the distance between the pin's image and the mirror.

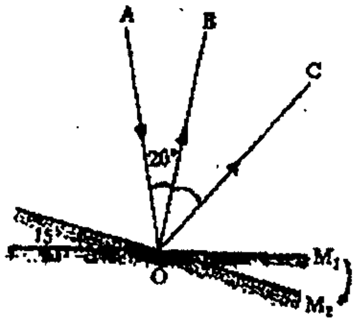
In the first position, the distance between the pin and its image = 6 + 6 = 12 cm.

In the second position, the distance between the pin and the mirror = 6 -1.5 = 4.5 cm.

Therefore, the distance between the pin and its image = 4.5 + 4.5 = 9 cm.

The distance reduced between the pin and its image =12- 9=3 cm.

Example (3) The reflected ray makes an angle of 20° with a ray incident on a plane mirror. The mirror is rotated through 15° about an axis perpendicular to the plane containing the incident and reflected rays. Find the angles between the incident ray and the new reflected ray.



In the above figure, M_1 is the original position of a plane mirror. For that arrangement the angle between the incident ray AO and the reflected ray OB= $\angle AOB=20^\circ$,

When the mirror is rotated clockwise through 15° from position M_1 to M_2, the reflected ray is rotated to the right along OC.

When the mirror rotates about an axis through an angle θ the reflected ray is rotated through an angle 2θ.

Thus, the angle between the first reflected ray OB and the new reflected ray OC $=2\times 15^\circ=30^\circ$

And, the angle between the incident ray and the new reflected ray $=\angle AOC=20^\circ+30^\circ = 50^\circ$

When the mirror in the position M_1 is rotated anticlockwise through an angle 15°, the reflected ray is rotated to the left.

For that arrangement, the angle between the reflected ray OB and the new reflected ray $= 2 \times 15^\circ = 30^\circ$

The new reflected ray is to the left of the incident ray AO.

Therefore, the angle between the incident ray and the new reflected ray $=30^\circ-20^\circ =10^\circ$

And the possible angles between the incident ray and the new reflected rays are 10° and 50°.

The images studied so far are formed in the plane mirror when both the object and the mirror are stationary.

Some Important Applications of Plane Mirrors

(i) Optical testing

The illuminated letters are laterally inverted so that the patient can see the letters correctly in the mirror. At the same time, the letters appear further than they actually are, so the room need not be that long.

(ii) Periscope

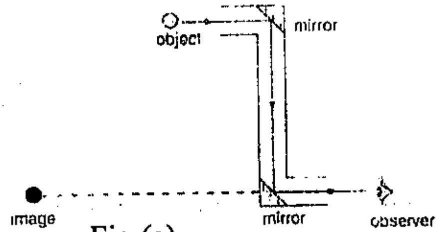


Fig (a)

When the view of an object is obstructed by an obstacle, the periscope can be used to see the object clearly. For example, when the persons in front obstruct the view of a singer on the stage, the singer can be viewed by the use of a periscope.

As shown in Fig (a) a simple periscope consists, of two plane mirrors facing one another. They are parallel and fixed at an angle of 45° to the line joining them.

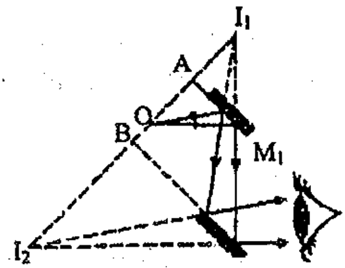
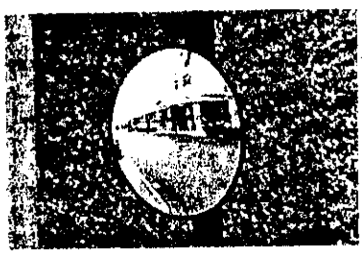


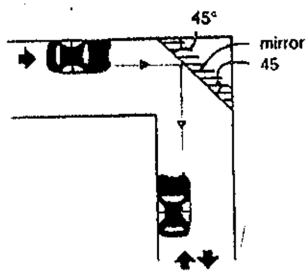
Fig (b)

The formation of successive images in the periscope is shown in Fig (b).

The image I_1 formed in the mirror M_1. I_1 becomes an object for the mirror M_2. I_2 is the final image of the object seen in M_2. When constructing a ray diagram it should be noticed that $OA = I_1A$ and $BI_1 = BI_2$. In addition, the line I_1I_2 is perpendicular to the mirrors.



A blind corner mirror



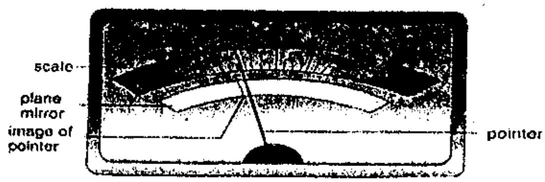
In the periscopes used in submarines, prisms are used instead of mirrors. In addition, periscopes which include telescopes are used in the observation of very distant objects.

(iii) Blind corners

Fitting a plane mirror at a corner allows drivers to see around blind turns (the diagram is not drawn to scale).

(iv) Instrument scales

By forming an image of the pointer, the plane mirror eliminates parallax errors in the reading of instrument scales.



(v) Other uses

Plane mirrors are also used in many optical instruments such as telescopes. Overhead projectors as well as lasers. Another common use of the plane mirror is in the construction of a kaleidoscope which gives colourful multiple images of small pieces of coloured glass.

9.4 REFLECTION AT CURVED MIRRORS

Now that we have studied the reflection at the plane mirror, let us proceed to the study of reflection at curved mirrors. In the case of reflection at curved mirrors also, the rays of light obey the laws of reflection stated previously. The reflecting surfaces used in the motorcar headlamps, torchlights, searchlights and so on are curved mirrors. The Hale telescope at Mt. Palomar, California, in the United States of America is one of the largest telescopes in the world, it uses a huge concave mirror having a diameter of 5 m.

If only a small part of the surface of a curved mirror is used for reflection, it can be considered as an outer or an inner surface of a hollow sphere. Only concave and convex mirrors having spherical surfaces are used in most experiments. We shall now discuss the reflections at such mirrors. Definitions concerning curved mirrors are given below.

(a) Concave Mirror

If the reflecting surface of a mirror forms part of the inner surface of a hollow sphere, the mirror is called a concave mirror.

(b) Convex Mirror

If the reflecting surface of a mirror forms part of the outer surface of a hollow sphere, the mirror is called a convex mirror.

(c) Pole of a Concave or Convex Mirror

The centre of the surface of a concave or convex mirror is called its pole.

(d) Centre of Curvature of a Concave or Convex Mirror

The centre of a sphere, part of whose surface is the concave or convex mirror, is called the centre of curvature of that mirror.

(The centre of curvature of a concave mirror is in front of the reflecting surface and that of a convex mirror is behind the reflecting surface.)

(e) Principal Axis

The line passing through the centre of curvature and the pole of a concave or convex mirror is called the principal axis.

(f) Radius of Curvature of a Concave or Convex Mirror

The radius of a sphere, part of whose surface is the concave or convex mirror, is called the radius of curvature of that mirror.

(g) Principal Focus

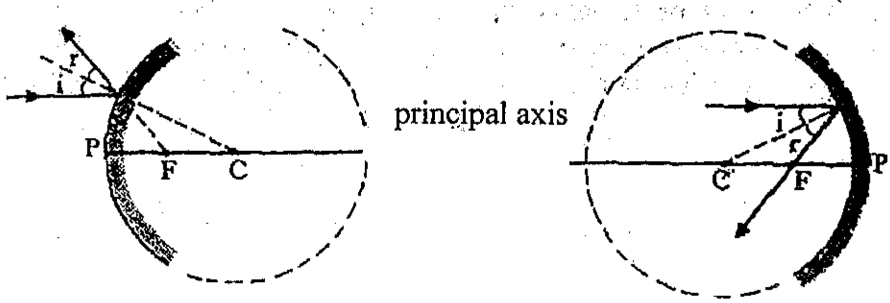
When the rays parallel and close to the principal axis are incident on a concave mirror the reflected rays pass through a point on the principal axis. That point is called the principal focus of the concave mirror. Since the reflected rays actually intersect at that point, the focus of a concave mirror is a real focus.

When the rays parallel to the principal axis are incident on a convex mirror the reflected rays appear to come from a point on the principal axis. That point is called the principal focus of the convex mirror. Since the reflected rays do not actually pass through that point, the principal focus of a convex mirror is a virtual focus.

(h) Focal Length

The distance between the pole and the focus of a concave or convex mirror is called the focal length of the concave or convex mirror.

Fig. 9.17 illustrates the stated definitions and the corresponding symbols.



P = pole, C = centre of curvature, F = focus
PF = f = focal length, PC = R = radius of curvature

Fig. 9.17 Convex and concave mirrors

Rays close to the Principal Axis (Paraxial Rays)

In order to simplify the study of reflections at mirrors and the refraction through lenses, only paraxial rays will be considered. Paraxial rays are rays parallel and close to the principal axis or rays which make very small angles with the principal axis.

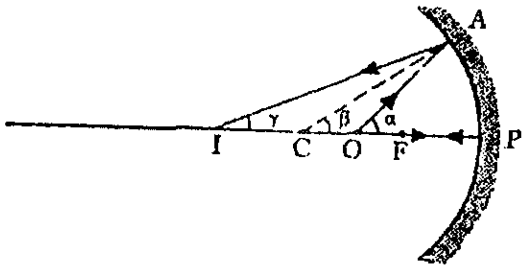


Fig. 9.18 Paraxial rays

In Fig. 9.18 the incident ray OA and the reflected ray AI are the rays close to the principal axis. This figure is exaggerated for the purpose of clarity. In practice, A must be very close to P since the angles α, β and γ are very small. For large objects which lie close to the mirror, paraxial rays do not give exact results, but only approximate results. The image aberration can be neglected when using the rays close to the principal axis.

Since the angles corresponding to the rays close to the principal axis are very small, the following mathematical assumptions are used. If θ in radian is very small, then

$$\text{Sin } \theta \approx \tan \theta \approx \theta$$

It then follows that the distance between any point on the axis and any point on the reflecting surface is equal to the distance between that point on the axis and the pole.

In Fig. 9.18 A is very close to P so that the curve AP can be taken as a straight line. Besides, it can be assumed that

$$OA = OP, \; CA = CP \text{ and } IA = IP.$$

The angle θ in radian is defined as follows.

$$\theta(\text{rad}) = \frac{\text{arc}}{\text{radius}}$$

The curve AP is the arc subtended by the angles α , β and γ.

Therefore
$$\alpha(\text{rad}) = \frac{PA}{OP} = \frac{PA}{OA} = \sin\alpha = \tan\alpha$$

$$\beta(\text{rad}) = \frac{PA}{PC} = \frac{PA}{CA} = \sin\beta = \tan\beta$$

$$\gamma(\text{rad}) = \frac{PA}{PI} = \frac{PA}{IA} = \sin\gamma = \tan\gamma$$

Relation between Focal Length f and Radius of Curvature R

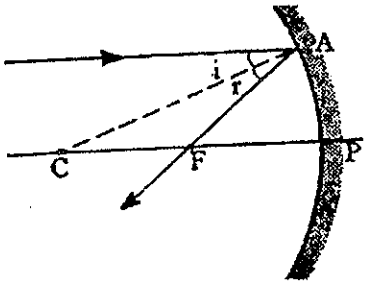


Fig. 9.19 Focal length and radius of curvature of a concave mirror

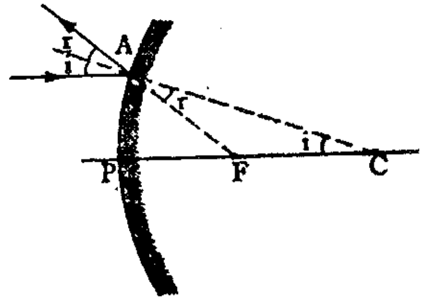


Fig. 9.20 Focal length and radius of curvature of a convex mirror

In Fig. 9.19 the ray parallel to the principal axis is incident on the concave mirror the point A and reflected through the focus F. In Fig. 9.20 the ray parallel to the principal axis is incident on the convex mirror at the point A and the reflected ray produced backward passes through F. The line passing through A and C is the normal.

By the laws of reflection $i = r$

Since the incident ray is parallel to the principal axis,

$$i = \angle ACP$$

Therefore, in the triangle ACF, $AF = FC$

For the rays close to the principal axis $AF = PF$

Therefore, $PF = FC$ or $PF = PC/2$

Since $PF = f$ and $PC = R$,

$$f = R/2$$

The focal length f of the curved mirror is one half its radius of curvature R.

Principal Rays and their Properties

The formation of images of an object in curved mirrors can be shown by constructing a ray diagram as in the case of formation of images in the plane mirror. The reflection at curved mirrors is also in accordance with the laws of reflection. The line passing through the point of incidence and the centre of curvature of the concave or convex mirror is the normal.

We have seen that for a plane mirror, the position of image of a point object is the point through which the reflected rays pass, when produced backwards. In order to show the size of the image of an extended object in a plane mirror, it is necessary to find only the positions of the two ends of that object. Similarly, only the rays coming from the two ends of an object need to be considered in order to show the formation of an image in a curved mirror. Furthermore, only two rays are necessary for drawing ray diagrams. Principal rays are used to draw ray diagrams. The principal rays and their properties are stated below.

1. When a ray parallel to the principal axis is incident on the concave mirror, the reflected ray passes through the focus; and when this ray is incident on the convex mirror, the reflected ray produced backward passes through the focus.
2. When a ray directed towards the principal focus is incident on the concave or convex mirror, the reflected ray travels parallel to the principal axis.
3. When a ray directed towards the centre of curvature is incident on the concave or convex mirror it is reflected along its original path.
4. When a ray directed towards the pole is incident on the pole of concave or convex mirror, the reflected ray is on the other side of the principal axis; the principal axis is the normal in this case.

Formation of Images in a Concave Mirror

The formation of images in the concave mirror for various positions of the object are shown in Figs.9.21 -9.26. In these figures, the object OO' is situated vertically on the principal axis. Since the ray from O is incident at P and reflected along its original path, the position of image I of O is somewhere on the axis. Thus, it is necessary to find the position of the image I' of the point O' at the top of the object by drawing the ray diagram. In doing so, only two convenient rays are used. The perpendicular distance from I' to the principal axis is the image II' of the object OO'. According to the principle of reversibility of light, if II' is the object, 00' is its image.

Figs.9.21 - 9.26 show that when the object OO' moves closer to the concave mirror, the image II' moves farther away from the mirror.

In Fig. 9.21 the object is at infinity and its image is

1. at F,
2. real,
3. inverted, and
4. smaller than the object.

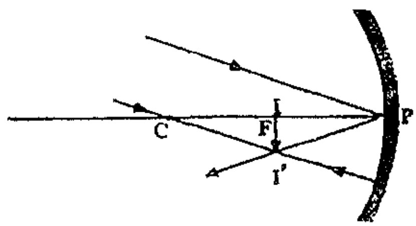


Fig. 9.21 the object at infinity

In Fig. 9.22 the object is beyond C and its image is

1. between C and F,
2. real,
3. inverted, and
4. smaller than the object.

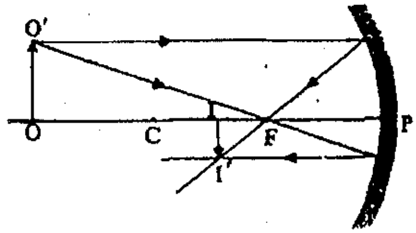


Fig. 9.22 object beyond C

In Fig. 9.23 the object at C and its image is

1. at C,
2. real,
3. inverted, and
4. of the same size as the object.

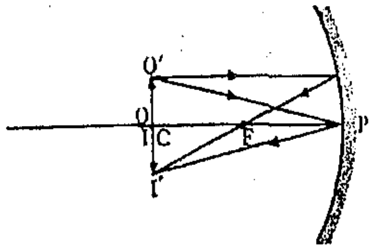


Fig. 9.23 the object at C

In Fig.9.24 the object is between C and F and its image is

1. beyond C,
2. real,
3. inverted, and
4. larger than the object.

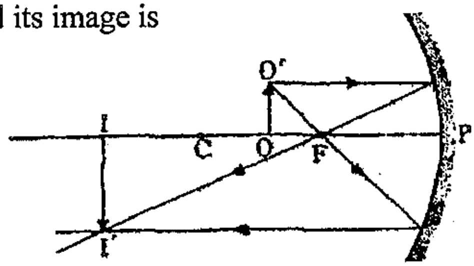


Fig 9.24 Object between C and F

In Fig. 9.25 the object is at F and its image is at infinity

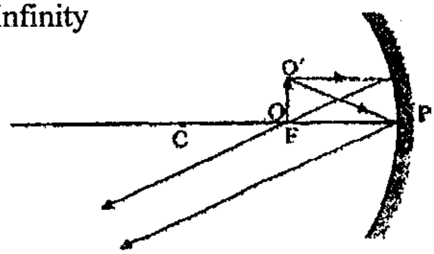


Fig. 9.25 Object at F

In Fig. 9.26 the object is between F and P and its image is

1. behind the mirror,
2. virtual,
3. erect, and
4. larger than the object.

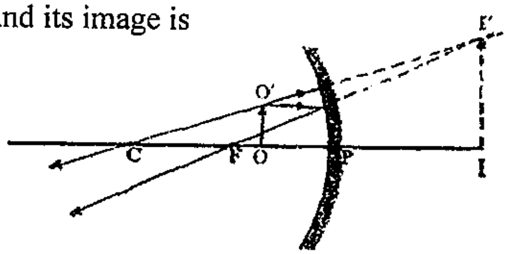


Fig. 9.26 Object between F and P

Formation of Images in a Convex Mirror

The image formed in the concave mirror may be real or virtual depending upon the position of the object. In addition, it may be erect or inverted. But the image formed in the convex mirror is always virtual, erect and smaller than the object. It is formed between P and F no matter where the object is situated.

The formation of an image in the convex mirror is illustrated with a ray diagram in Fig.9.27.

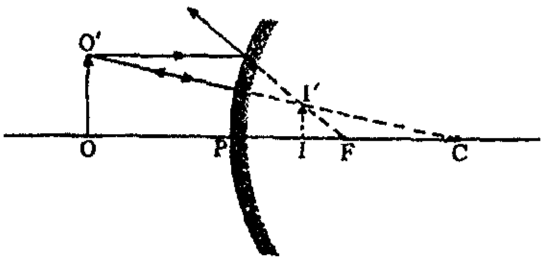


Fig. 9.27 Image formed by a convex mirror

A convex mirror has a wider field of view than a plane mirror of the same size (Fig.9.28). Convex mirrors are used as rear view mirrors of motor cars since they always give an erect image and a wide field of view.

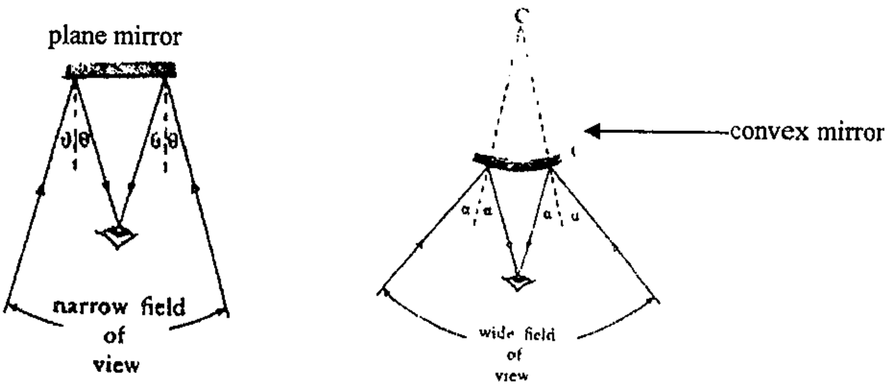


Fig. 9.28 Advantage of a rear view mirror

Sign Conventions

The facts obtained by studying the images formed in a concave mirror by constructing ray diagrams are summarized as follows:

1. The image is formed sometimes in front of a concave mirror and sometimes behind it.
2. The image may be either real or virtual.
3. The image may be either erect or inverted.

These facts make it necessary to establish sign conventions. The position of the image (in front of or behind the mirror), the nature of the image (real or virtual) and the configuration of the image (erect or inverted) are specified by the use of sign conventions.

All distances are measured from the pole of the mirror and the sign conventions in common use are given below.

1. Distances of real object, real image and real focus are positive. Distances of virtual object, virtual image and virtual focus are negative.
2. The focal length of a concave mirror is positive, and that of a convex mirror is negative. Since $f = \frac{R}{2}$ it implies that the radius of curvature is positive for a concave mirror and negative for convex one.
3. The perpendicular distance measured above the principal axis is positive and that below the principal axis is negative.

Mirror Formula

For both concave and convex mirrors

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad (9.1)$$

where u = object distance from the mirror
v = image distance from the mirror
f = focal length

Equation (9.1) is known as the mirror formula and it can be derived as follows.

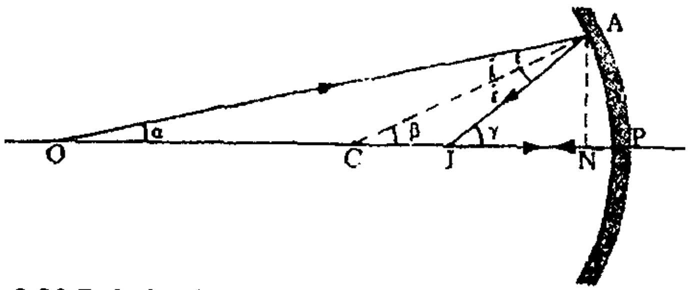


Fig. 9.29 Relation between u, v and f in a concave mirror

In Fig. 9.29 a point object O is situated on the principal axis of a concave mirror. The incident ray OA is reflected by the concave mirror along AI. The incident ray OP is reflected back along its original path. The point of intersection of these reflected rays I is the real image of O. CA is the normal and by the laws of reflection

$$\angle OAC = \angle IAC = i$$

OA, CA and IA make the angles α, β and γ respectively, with the principal axis. These angles are very small since only the rays close to the principal axis are considered here. (Fig. 9.29) is exaggerated for the purpose of clarity. AN is the normal to OP. In practice, N is almost coincident with P.

As β is the exterior angle of triangle OAC,

$$\beta = \alpha + i$$

or

$$i = \beta - \alpha \qquad (1)$$

Since γ is the exterior angle of triangle CIA.

$$\gamma = \beta + i$$

$$i = \gamma - \beta \qquad (2)$$

From equations (1) and (2), it follows that

$$\gamma - \beta = \beta - \alpha$$

Therefore

$$\alpha + \gamma = 2\beta \qquad (3)$$

In Fig 9.29

$$\alpha = \tan\alpha = \frac{AN}{ON} = \frac{AN}{OP}$$

$$\beta = \tan\beta = \frac{AN}{CN} = \frac{AN}{CP}$$

$$\gamma = \tan\gamma = \frac{AN}{IN} = \frac{AN}{IP}$$

Substituting these values of α, β and γ in equation (3)

$$\frac{AN}{OP} + \frac{AN}{IP} = 2\frac{AN}{CP}$$

Dividing by AN, we get

$$\frac{1}{OP} + \frac{1}{IP} = \frac{2}{CP}$$

Since OP = u, IP = v and CP = R = 2f, the above equation becomes

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

When the sign conventions are used

$$\frac{1}{+u} + \frac{1}{+v} = \frac{1}{+f}$$

or
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad (4)$$

Equation (4) has been derived for the concave mirror when the point object is beyond C on the principal axis. This equation can be derived also for the point object or an extended object situated somewhere on the principal axis in front of the concave mirror. The corresponding sign conventions for u, v and f must again be used.

Magnification

The images formed by the concave and convex mirrors have various sizes depending upon the position of the object. Thus, the lateral magnification produced by a mirror is defined by

$$\text{Magnification} = \frac{\text{height of image}}{\text{height of object}}$$
$$= \frac{\text{size of image}}{\text{size of object}}$$

If m = magnification, II' = size of image and OO' = size of object

then
$$m = \frac{II'}{OO'} \qquad (9.2)$$

The magnification m can also be expressed in terms of the object distance u, and the image distance v. In Fig. 9.26 the image II' of the object OO' beyond C is between F and C.

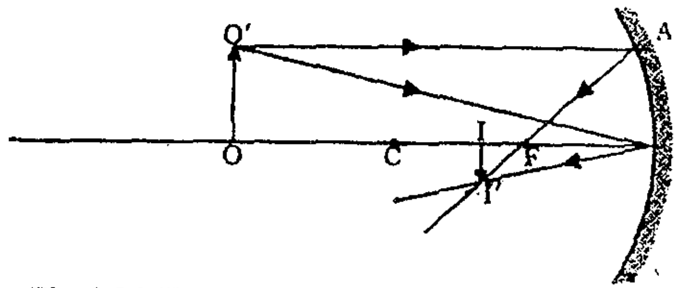


Fig. 9.26 Illustration of magnification

In Fig. 9.26 Δ OO'P and Δ II'P are similar

Therefore, $$\frac{II'}{OO'} = \frac{IP}{OP}$$

Since OP = u and IP = v, we get

$$\frac{II'}{OO'} = \frac{v}{u}$$

When the sign conventions are used,

$$\frac{-II'}{+OO'} = \frac{+v}{+u}$$

Here a minus sign is used for II' since it is below the principal axis. Thus, the formula for the magnification m can be expressed as

$$m = \frac{II'}{OO'} = -\frac{v}{u} \qquad (9.3)$$

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}}$$

$$= -\frac{\text{image distance}}{\text{object distance}}$$

Equation (9.3) can be derived for various positions of the object situated in 'front of the concave mirror. This equation can be used for the convex mirror as well. The minus sign determines both the nature and configuration of the image.

Some Applications of Mirror Formula

To understand how to apply the formulae $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ and

$m = \frac{II'}{OO'} = -\frac{v}{u}$ correctly, study the following examples carefully.

Example (4) An object is placed (a) 20 cm (b) 4 cm in front of a concave mirror of focal length 12 cm. Find the nature and position of the image in each case.

(a)

$$u = +20\text{ cm}$$
$$f = +12\text{ cm}$$
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
$$\frac{1}{+20} + \frac{1}{v} = \frac{1}{+12}$$
$$\frac{1}{v} = \frac{1}{12} - \frac{1}{20}$$
$$v = 30\text{ cm}$$

Since v is positive the image is real. It is formed 30 cm from the concave mirror on the same side as the object.

(b)

$$u = +4\text{ cm}$$
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
$$\frac{1}{+4} + \frac{1}{v} = \frac{1}{+12}$$
$$\frac{1}{v} = \frac{1}{12} - \frac{1}{4}$$
$$v = -6\text{cm}$$

Since v is negative the image is virtual. It is formed 6 cm behind the concave mirror.

Example (5) An object is placed 10 cm in front of a concave mirror of focal length 15 cm. Find the image position and the magnification.

$$u = +10\text{ cm}$$
$$f = +15\text{ cm}$$
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
$$\frac{1}{+10} + \frac{1}{v} = \frac{1}{+15}$$
$$\frac{1}{v} = \frac{1}{15} - \frac{1}{10}$$
$$v = -30\text{ cm}$$

Since v is negative the image is virtual. It is formed 30 cm behind the concave mirror.

$$\text{Magnification} \quad m = -\frac{v}{u}$$

$$m = -\frac{(-30)}{+10} = 3$$

Since $m = \frac{II'}{OO'} = 3$ or $II' = 3 \times OO'$, it can be said that the image is 3 times the size of the object and it is erect.

Example (6) The image of an object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the object position and the magnification.

The image in a convex mirror is always virtual.

$$v = -4 \text{ cm}$$

$$R = -24 \text{ cm}$$

$$R = 2f,\ f = R/2 = \left(\frac{-24}{2}\right) = -12 \text{ cm}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{u} + \frac{1}{-4} = \frac{1}{-12}$$

$$\frac{1}{u} = -\frac{1}{12} + \frac{1}{4}$$

$$u = 6 \text{ cm}$$

Since u is positive the object is real. It is 6 cm from the convex mirror.

$$\text{Magnification} \quad m = -\frac{v}{u}$$

$$m = -\frac{(-4)}{6}$$

$$= \frac{2}{3}$$

Since $m = \frac{II'}{OO'} = \frac{2}{3}$ or $II' = \frac{2}{3} \times OO'$, the size of the image is $\frac{2}{3}$ times the size of the object and the image is erect.

Example (7) The image of an object in a concave mirror is erect and three times the size of the object. If the mirror has a radius of curvature of 36 cm, find the position of the object.

Size of the image = 3 × size of object

$$II' = 3 \times OO'$$

$$\frac{II'}{OO'} = 3$$

$$m = +3$$

$$m = -\frac{v}{u}$$

$$+3 = -\frac{v}{u}$$

$$v = -3u$$

$$R = 36 \text{ cm}, \quad R = 2f, \quad f = \frac{R}{2} = \frac{36}{2} = 18 \text{ cm}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

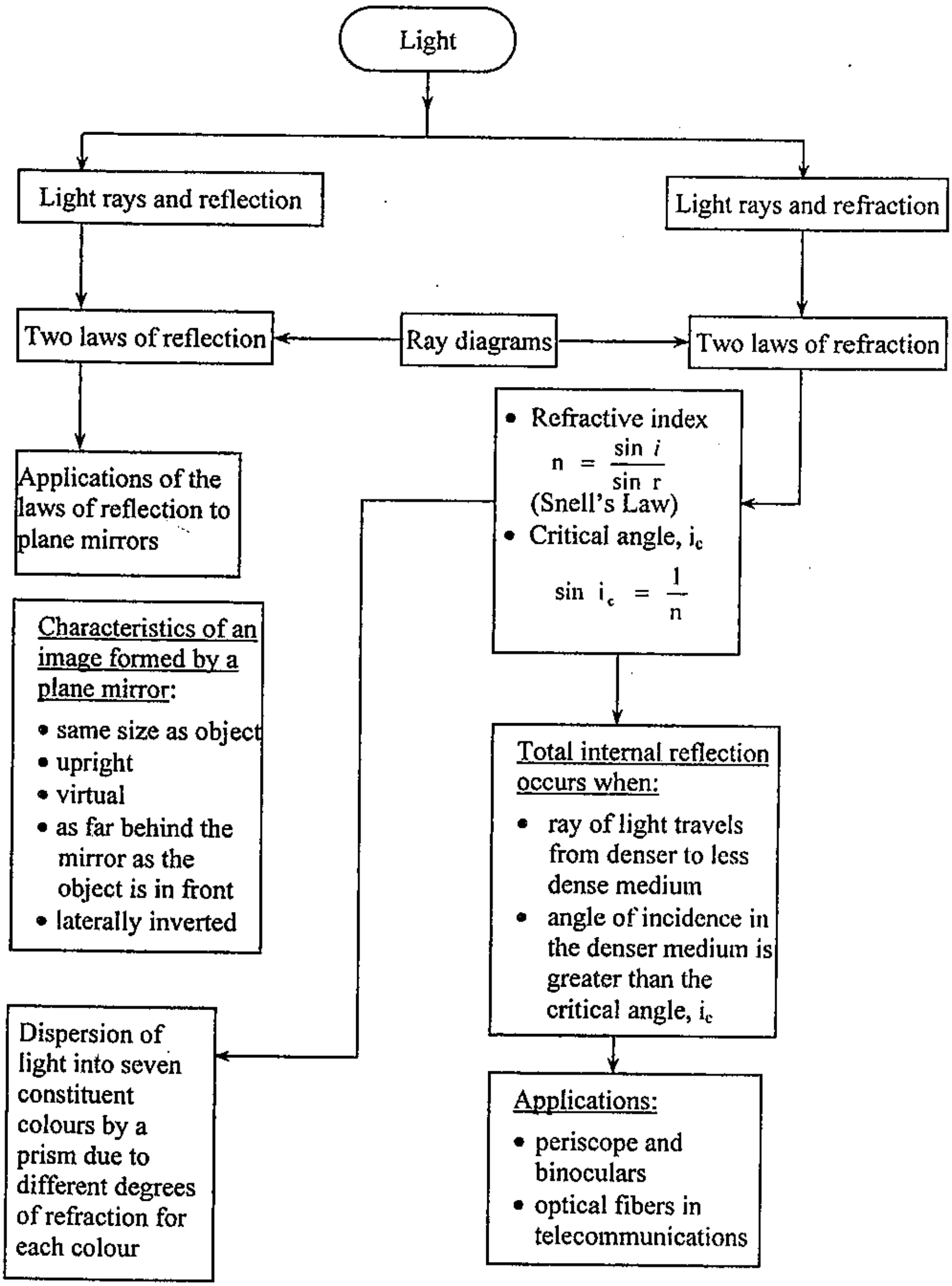
$$\frac{1}{u} + \frac{1}{-3u} = \frac{1}{18}$$

$$\frac{2}{3u} = \frac{1}{18}$$

$$u = 12 \text{ cm}$$

The object is 12 cm from the concave mirror.

Concept Map (Light)



EXERCISES

1. State the laws of reflection of light.
2. What is the difference between real and virtual images?
3. A man is looking into a plane mirror on the wall which is 6 ft away from him. He views the image of a chart which faces the mirror and is 2 ft behind him. Find the distance between his eyes and the image of the chart.
4. Draw a ray diagram to show that a vertical plane mirror need not be 5 ft long in order that a boy 5ft tall may see a full-length image of himself in it.
5. In the above problem, if the boy's eyes are 4 in below the top of his head find the height of the base of the mirror above floor level.
6. Show that a point object and its image are at equal distances from any point on the plane mirror.
7. What is meant by lateral inversion? The letter R is 5 cm in front of a plane mirror. Draw accurately the image of R in the mirror.
8. An object is in front of a plane mirror. If the object and the mirror each recede x from their original positions, by how much is the distance between the object and its image changed?
9. State the similarity and differences between the virtual images formed by the concave and convex mirrors.
10. By using the laws of reflection, prove that when a ray parallel to the principal axis is incident on a concave mirror, it passes through the focus after reflection.
11. Choose the correct answer from the following.

    (a) Only a virtual image smaller than the object is formed by a concave mirror.

    (b) Only a virtual image larger than the object is formed by a convex mirror.

    (c) The statements given in (a) and (b) are correct.

    (d) The statements given in (a) and (b) are wrong.

12. Choose the correct answer from the following.

(a) Only real images are formed by a concave mirror. (b) Real and virtual images can be formed by a concave mirror. (c) Real and virtual images can be formed by a convex mirror.

13. Choose the correct answer from the following.

When an object is at the centre of curvature of a concave mirror the magnification is (a) 0.5 (b) -1.0 (c) 1.5.

14. A concave mirror can produce an image which is twice the size of the object. Draw a ray diagram to show this.

15. An image is 6 cm from a convex mirror which has a radius of curvature of 36 cm. Find the object position and the magnification.

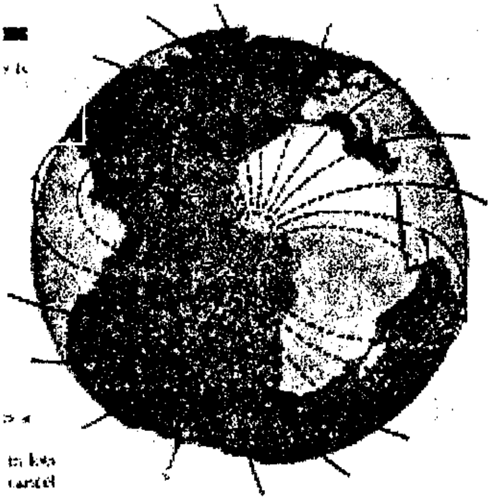
16. An image one-third the size of an object is formed by a convex mirror of focal length 15 cm. How far is the object from the convex mirror?

17. An object is 20 cm in front of a concave mirror of focal length 15 cm. How far must the screen be placed from the centre of curvature of the concave mirror to receive the image of the object? If the object is 2 cm tall, find the size of the image.

18. An object 5 cm tall is 15 cm in front of a concave mirror of focal length 10cm. Can its image be received on a screen each side 9 cm long? If so, in which position should the screen be placed?

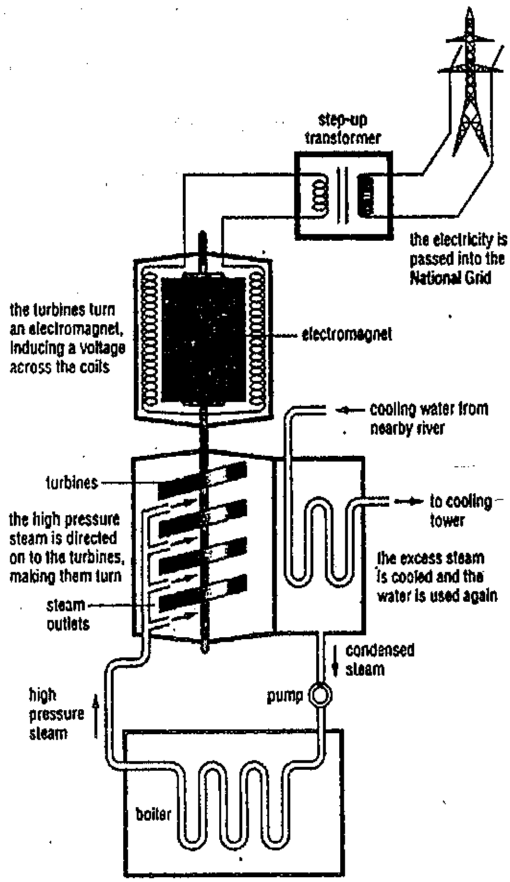
19. An object is 20 cm from a mirror. If the virtual image is half the size of the object, find the radius of curvature of the mirror.

20. The glancing angle of a ray incident on a plane mirror is 60°. Find the angle between the incident ray and the new reflected ray when the plane mirror is rotated (a) through 15° in the clockwise direction, (b) through 30° in the anticlockwise direction.



The Earth's magnetic field. The Earth behaves like a giant bar magnet, with a south-seeking pole at the North Pole, and a north-seeking pole at the South Pole.

ELECTRICITY



Generating electricity at a thermal power plant

CHAPTER 10
ELECTRICITY AND MAGNETISM

STATIC ELECTRICITY

Electricity is a form of energy. Electrical energy can be transformed into other forms of energy, such as heat energy, mechanical energy, light energy and sound energy. It is used in domestic electrical appliances, in industries, transportation and communication works. It is obvious that the electrical energy plays an important role in the development of a country. In technologically advanced countries scientists are trying to generate considerable amounts of electrical energies from the wind, from the sea and from the sun.

10.1 ELECTRIC CHARGES

In this chapter we shall discuss electric charges at rest and forces between them. Electric charges may be divided into static charges (charges at rest) and moving charges or flow of charges (the current). In this chapter, we shall study only static charges.

Positive Charge and Negative Charge

The French scientist, Du Fay, studied the nature of electric charges possessed by the substances and found that there were only two kinds of charges. Benjamin Franklin named them positive charge and negative charge. The positive charge is represented by a plus sign and the negative charge by a minus sign.

Experiments show that the charge formed on the glass rod when stroked with a silk cloth and the charge formed on the plastic rod when stroked with fur are different in nature. The charge possessed by the glass rod is called a positive charge and the charge possessed by the plastic rod is called a negative charge.

10.2 MATTER AND ELECTRICITY

Matter is composed of atoms which are very small in size. An atom consists of a core called the nucleus around which the particles called electrons are moving.

An electron is a negatively charged particle. The nucleus consists of two kinds of particles called proton and neutron. A proton is a positively charged particle and a neutron is an uncharged particle. Therefore, the nucleus has net positive charge. The magnitude of positive charge of the nucleus is equal to the sum of the positive charges of all the protons present in the nucleus.

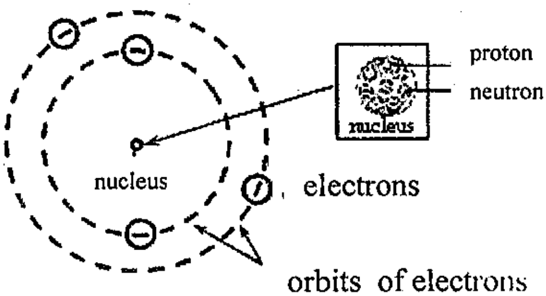


Fig. 10.1 A neutral beryllium atom with 4 electrons, 4 protons and 4 neutrons

In a normal atom the number of electrons is always equal to the number of protons. An electron and a proton have the same magnitude of electric charge. Therefore, since the magnitude of positive charge of the nucleus is equal to that of the total negative charge of electrons, a normal atom has no net charge. We say that a normal atom is electrically neutral.

When one or more electrons are removed from an atom, the atom has more protons than electrons and hence, it carries a net positive charge. When one or more electrons are added to an atom, the atom has more electrons than protons and hence, it carries a net negative charge

Principle of Conservation of Electric Charge

The laws of conservation of momentum and energy have been studied in mechanics. Like momentum and energy, electric changes are also conserved.

The principle of conservation of electric charge states that the net electric charge in an isolated system remains constant.

The net charge is the algebraic sum of the charges present in an isolated system. This means that the signs of the charges must be included in summing them up. Net charge can be positive, negative or zero.

For example, in the experiment on electrification by stroking the glass rod with a silk cloth, the glass rod and the silk cloth together form an isolated system. No charge flows to the surrounding from the glass rod and the silk cloth, and no charge flows into the glass rod and the silk cloth from the surrounding either. And the net charge of the isolated system - the glass rod and the silk cloth - remained constant. The glass rod and the silk cloth are uncharged bodies at first; the net charge of that system is initially zero. After stroking the glass rod with the silk cloth the algebraic sum of the total charge on the glass rod and that on the silk cloth must again be zero.

Separating or bringing together charges does not affect their magnitudes so that the net charge is unaffected. Every known physical process is found to conserve electric charge.

10.3 CONDUCTORS AND INSULATORS

As already mentioned (sec 10.2), in an atom the negatively charged electrons are moving round a positively charged nucleus. Some of these electrons are near the nucleus while other electrons are further away from the nucleus. Since positive and negative charges attract each other the electrons experience an attractive force of the nucleus. As the attractive force is greater for the electrons closer to the nucleus, the electrons closer to the nucleus or the inner electrons cannot move freely. This means that the inner electrons are tightly bound by the nucleus. These electrons closer to the nucleus are called bound electrons.

The electrons far away from the nucleus or the outer electrons experience less attractive force of the nucleus. This means that the outer electrons are loosely bound and are called free electrons. They can easily move from one atom to another.

The number of free electrons in a substance depends only upon the nature of that substance. The substance which has plenty of free electrons is called a conductor and the substance which has very few or no free electrons is called an insulator.

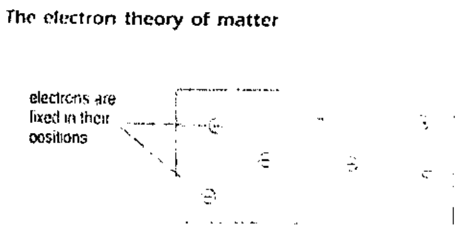


Fig 10.2(a) Insulator

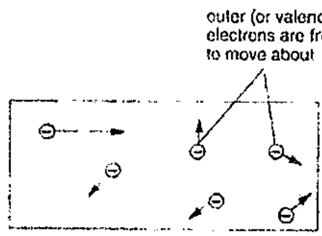
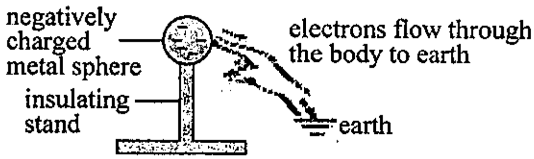


Fig 10.2(b) Conductor

Metals such as copper, brass, aluminum and silver are conductors and substances such as glass, wax, quartz, and plastic are insulators.

Insulators are materials that do not have free electrons, and thus cannot conduct electricity. Conductors are materials that have free electrons, and are able to conduct electricity.

Some substances contain a moderate amount of free electrons. Such substances are neither conductors nor insulators. They are called semiconductors. Silicon and germanium are widely used semiconductors. Transistors are made from semiconductors.



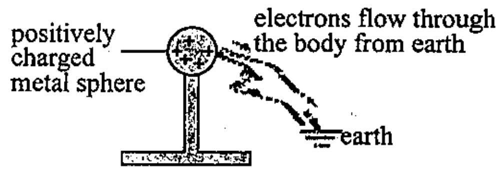


Fig 10.3(a) Earthing a negatively-charged metal sphere

Fig 10.3(b) Earthing a positively-charged metal sphere

10.4 CHARGING BY INDUCTION

Induction is the process of charging a conductor with any contact with the charging body.

(i) To charge two conductors with equal and opposite charges

In Fig. 10.4 *(a)* two metal spheres A and B supported on insulating stands are in contact. These spheres can be considered as a single conductor. They are uncharged spheres.

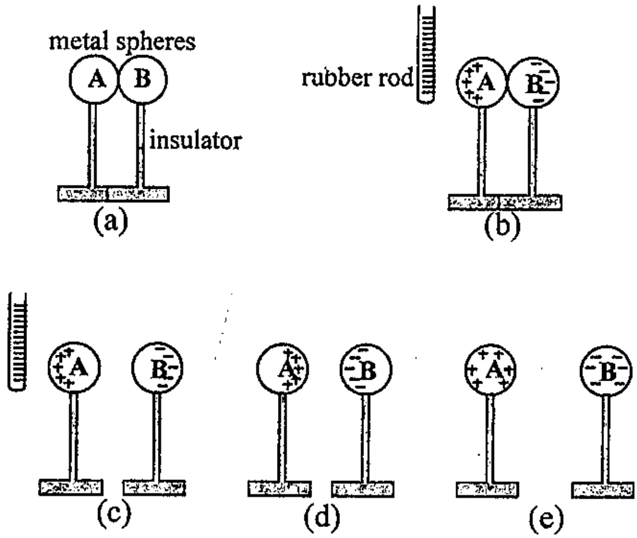


Fig.10.4 Electrostatic induction

(a) **insulated, uncharged conductors (two metal spheres: A and B) are in contact**

(b) **charging by electrostatic induction process on A and B using a negatively charged rubber rod:** +ve and − ve charges are induced on A and B respectively

(c) **A and B separated in the presence of inducing charge**

(d) **inducing charged removed**: A and B have opposite charges (But the charges on A and B are as close together as possible since unlike charges attract each other.)

(e) **inducing charged removed**: uniformly distributed charges on A and B

A negatively charged rubber rod is brought near the sphere A as shown in Fig. 10.4 (b). Since like charges repel the free electrons in the spheres A and B move away from the rod and they collect at the right surface of B. Thus an excess negative charge accumulates at the right surface of B. Since A is deficient in electrons an excess positive charge accumulates at the left surface of A. These excess charges on the surfaces of A and B are called induced charges. Spheres A and B, which were originally uncharged, become charged bodies when a charged rubber rod is placed near A. This process is called ***charging by induction.***

Keeping the rubber rod in position, B is moved slightly from A as shown in Fig. 10.4 (c).

Then the rubber rod is removed. A becomes a positively charged sphere and B becomes a negatively charged sphere as shown in Fig. 10.4 (d). Since unlike charges attract each other the charges on A and B are as close together as possible.
When A and B are separated by a distance as shown in Fig 10.4 (e) the charges are uniformly distributed on the surfaces of A and B. Although opposite charges have been induced on A and B by the rubber rod, the magnitude of the charge on the rubber rod remains unchanged.

(ii) To charge a single conductor by induction

If a single uncharged metal sphere is to be charged by induction the steps *(a)* to *(d)*, illustrated in Fig 10.5 must be carried out.

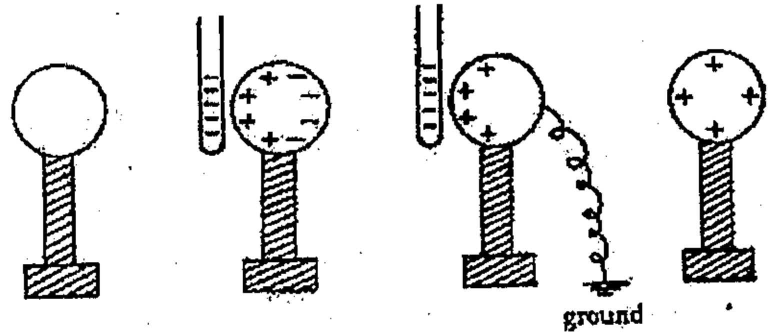


Fig. 10.5 Charging positively by induction

(a) A conductor is held by an insulating stand.

(b) Bring a charged rod (say negatively charged) to the vicinity of the conductor. Since the electrons in the metal sphere are repelled by the negative charge on the rubber rod, they move towards the right surface of the sphere. An excess negative charge accumulates at that surface.

(c) The surface is then touched momentarily by a conducting wire which is connected to the earth. The electrons are expelled to the earth through the wire and only positive charges remain on the sphere.

(d) When the rubber rod is removed, positive charges are uniformly distributed on the surface of the sphere. If that sphere is touched with another sphere of the same size and same material, charges will flow until both spheres have equal charges.

Note also that charging a single conductor by induction will always result in a charge that has the opposite sign to that of the charging rod.

Thus, bringing a positively charged rod to the vicinity of a single uncharged metal sphere (a conductor) and carrying out the same steps *(a)* to *(d)* of induction, the sphere becomes a negatively charged sphere.

EXERCISES

1. When a negatively charged sphere is brought near a suspended body, the suspended body is attracted to it. Is it correct to assume that the body is positively charged?
2. (a) When two bodies attract each other electrically, must both of them be charged?

   (b) When two bodies repel each other electrically, must both of them be charged?
3. Choose the correct answer from the following.

   (a) When an object contains an excess of electrons it has a positive electric charge.

   (b) When an object contains a deficiency of electrons it has a positive electric charge.

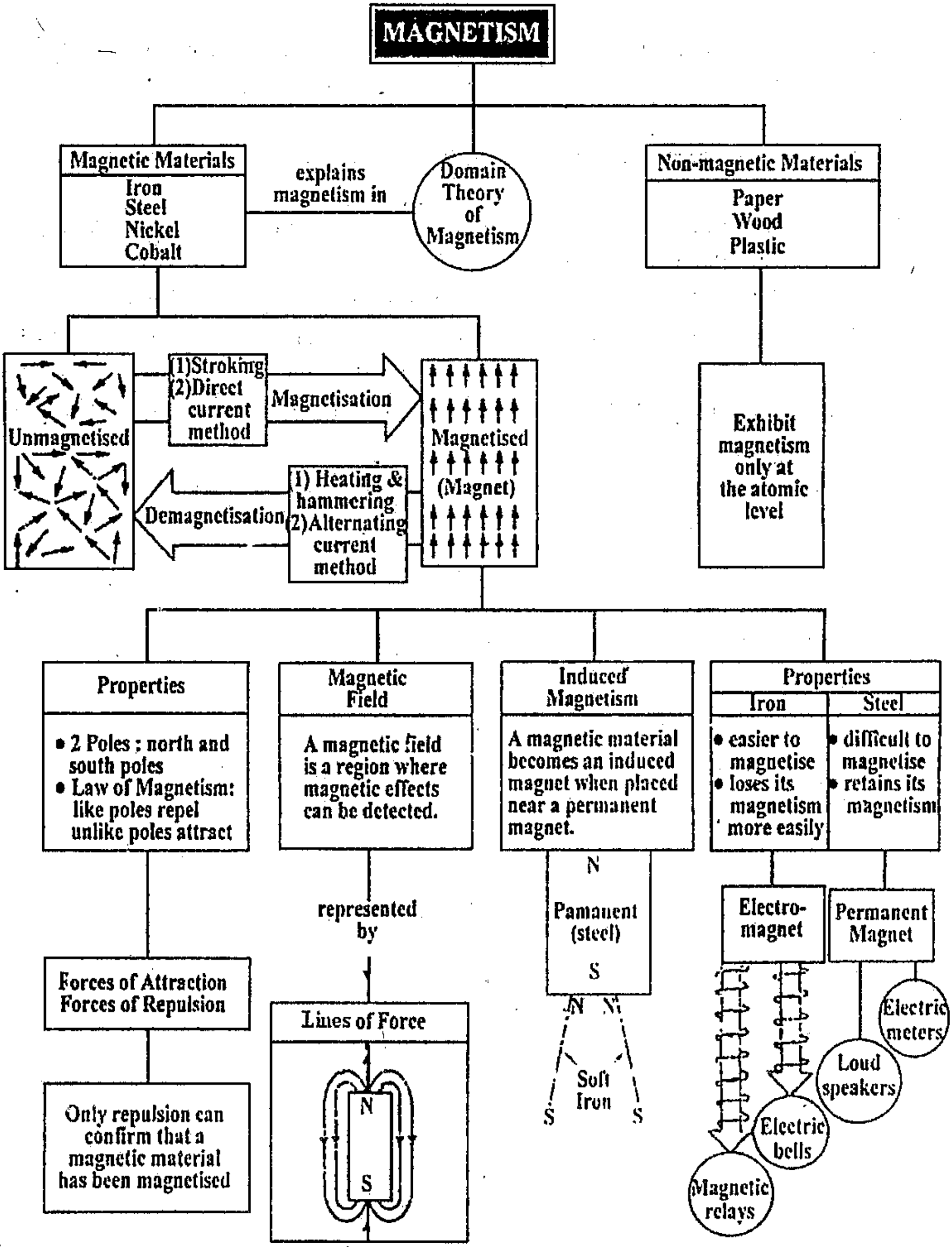
   (c) Since the nuclei of the atoms in an object are positively charged it has a positive electric charge.

   (d) When the electrons of the atoms in an object are positively charged it has a positive electric charge.
4. Choose the correct answer from the following.

   The magnitude of the charge of an electron is 1.6×10^{-19} C. A total of 10^4 electrons have been removed from an uncharged pith ball. Its charge now is

   (a) $+1.6 \times 10^{-15}$ C *(b)* $+1.6 \times 10^{-23}$C.

   (c) -1.6×10^{-15}C. *(d)* -1.6×10^{-23} C
5. In question number (5), what will be the answer if 10^4 electrons are added to the uncharged pith ball?
6. State the principle of conservation of electric charge.
7. *(a)* What do you understand by a bound electron and a free electron ? *(b)* Is your body a conductor or an insulator? *(c)* Mention five insulators and five conductors.


MAGNETISM
Magnetic Materials
Iron
Steel
Nickel
Cobalt
explains magnetism in
Domain Theory of Magnetism
Non-magnetic Materials
Paper
Wood
Plastic
Exhibit magnetism only at the atomic level
Unmagnetised
(1)Stroking
(2)Direct current method
Magnetisation
Magnetised (Magnet)
1) Heating & hammering
2)Alternating current method
Demagnetisation
Properties
• 2 Poles ; north and south poles
• Law of Magnetism: like poles repel unlike poles attract
Forces of Attraction
Forces of Repulsion
Only repulsion can confirm that a magnetic material has been magnetised
Magnetic Field
A magnetic field is a region where magnetic effects can be detected.
represented by
Lines of Force
N
S
Induced Magnetism
A magnetic material becomes an induced magnet when placed near a permanent magnet.
N
Pamanent (steel)
S
N
N
Soft Iron
S
S
Properties
Iron
Steel
• easier to magnetise
• loses its magnetism more easily
• difficult to magnetise
• retains its magnetism
Electro-magnet
Permanent Magnet
Magnetic relays
Electric bells
Loud speakers
Electric meters

10.5 MATERIALS AND MAGNETS

The discovery of magnetism

About 900 years ago, the Chinese first discovered that a certain type of rock called magnetite (or lodestone) possessed a unique property.
They found that a dish carrying a piece of lodestone would float in water in such a way that the lodestone always settled in a North-South direction. This unique property of the lodestone (literally means 'leading stone') forms the basis for the compass which is a very important piece of equipment for navigation and exploration both on land and at sea. Fig. 10.6 shows a lodestone or magnetite compass used by the Chinese while Fig. 10.7 shows a modern-day compass.

Fig. 10.6 A lodestone or magnetite compass used by the Chinese about 900 years ago

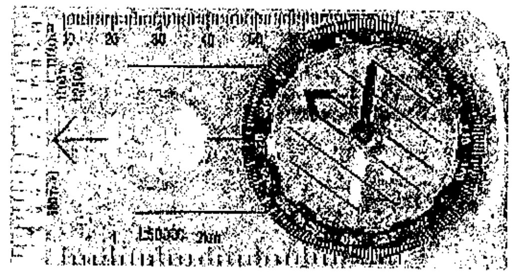


Fig. 10.7 A modern-day navigation compass

Magnetic and non-magnetic materials

Magnetite consists of an oxide of iron. This natural magnet attracts certain materials such as Cobalt, nickel and some alloys such as steel. We call these materials **magnetic materials.** Materials such as brass, copper, wood and plastics that are not attracted by a magnet are called **non-magnetic** materials.

Any material (such as magnetite) that is able to keep its magnetism for a long time is called a **permanent magnet.** Modern-day permanent magnets are usually made of steel (an alloy of iron) and special alloys such as alcomax and alnicol which contain metals such as iron, nickel, copper, cobalt and aluminum. Another type of permanent magnet is the ceramic magnet which is made from powders called ferrites (compounds of iron oxide with other metal oxides). These ceramic magnets are brittle, however.

Properties of magnets

Besides exhibiting the property of attracting magnetic materials, all magnets also exhibit the following properties:

1. Magnetic poles

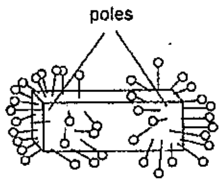


Fig.10.8 The pins show the positions of the poles of the magnet.

Most of the pins are attracted to the two ends of the bar magnet. We call these two ends the **poles** of the magnet.

2. North and South poles

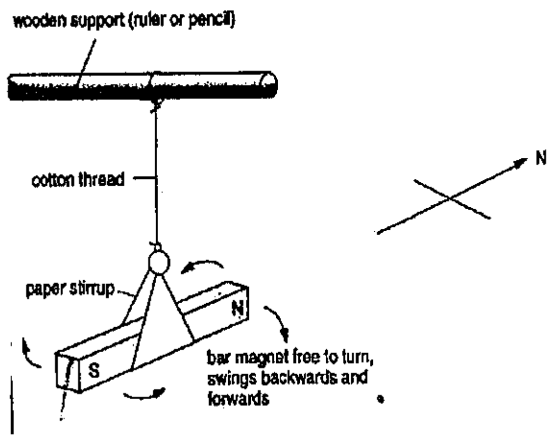


Fig.10.9 A suspended magnet always points exactly the same way

When the bar magnet comes to rest, one end always points towards the northern end of the Earth. This end of the magnet is thus called the North-seeking pole. Similarly, the other end of the magnet is called the south-seeking pole. The North-seeking pole and South-seeking pole of the magnet are usually referred to as simply the North pole (N-pole) and the South pole (S-pole) of the magnet. A magnet can therefore be used as a compass for navigation.

3. Laws of magnetic poles

Like poles repel, unlike poles attract.

Electromagnet

Similarly, if a soft iron bar is placed inside the solenoid of insulated wire and a current flow through it, the bar becomes magnetized. It is demagnetized when the current stops. As the soft iron bar is magnetized only when the current is flowing such a magnet is called a temporary magnet or an electromagnet.

10.6 THE THEORY OF MAGNETISM

The mechanism of magnetism is still not fully understood. We know that substances are made up of a large number of atoms. There are electrons which move in orbits about the nucleus of the atom. It is believed that the movement of the electrons in the atoms of a magnetic material make each atom a magnet. This atomic magnet is very small and weak. However, in a magnetic material, the millions of atoms arrange themselves in groups each with all its atomic magnets pointing a certain direction. These groups are known as magnetic domains and each single domain is a strong magnet.

- A flow of electrons from atom to atom constitutes a current (electricity).
- Electricity can be used to make a magnetic field.
- A wire carrying an electric current has a magnetic field around it.
- If the wire is twisted into a coil, the magnetic field inside is made stronger.
- The field becomes many times stronger if a soft iron core is put inside the coil. The coil and core together are called an electromagnet.
- An electromagnet can be turned on and off. This is done by controlling the current in the coil.
- This makes the electromagnet more useful than other magnets.
- Super electromagnets are promising as a way to make electrical power. They could be 40.000 times more powerful than ordinary magnets.
- Their coils would be made of special wires that are able to carry huge electric currents. These wires are called **"superconductors"**. They work only at very low temperatures.

In the unmagnetized state [Fig.10.10 (*a*)] the domains all point in different directions and their magnetic effects cancel one another. When an iron bar is placed in a current-carrying solenoid [Fig.10.10(b)], the domains tend to align themselves until the magnet is at its greatest possible strength [Fig.10.10 (c)]. A magnet is thus produced.

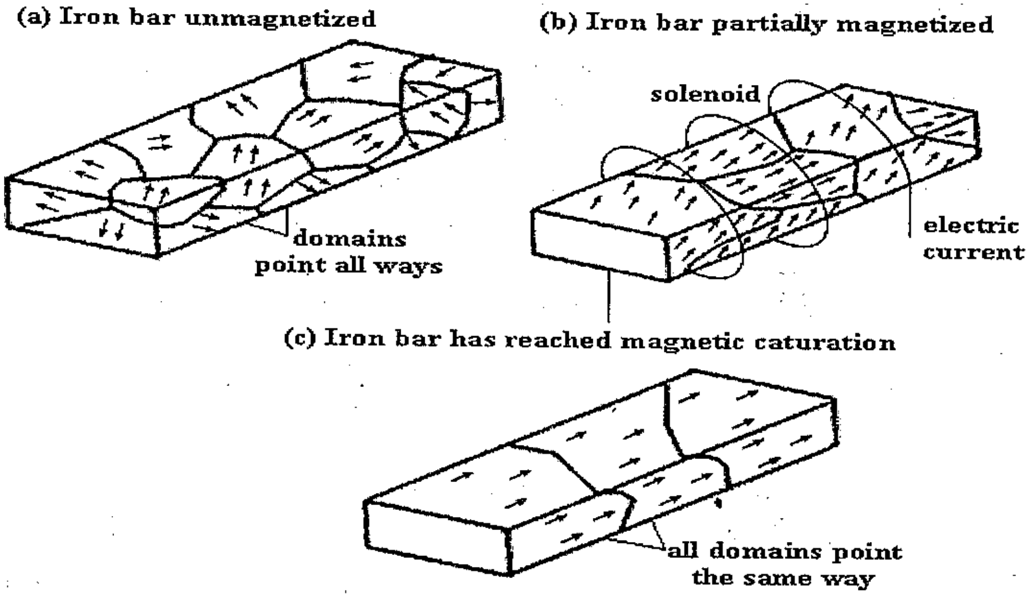


Fig. 10.10 The domain theory of magnetism. The tiny arrows show the direction of magnetisation. The domains are actually much smaller than the ones shown here.

A magnetised bar and an unmagnetised bar

If we take a thin piece of magnetized steel bar and cut it into three smaller pieces, we will notice that every piece is a magnet with an N-and S-pole (Fig. 10.11).

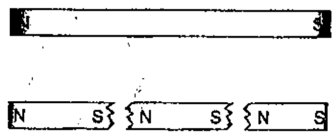


Fig.10.11 Each piece of the magnetised steel bar is a magnet

Therefore, it would be reasonable to imagine that if we keep on cutting each piece of the magnet into even smaller pieces, they would still be magnetized. In other words, we can suppose that the original magnet was made up of lots of 'tiny' magnets all lined up with their N-poles pointing in the same direction (Fig. 10.12).

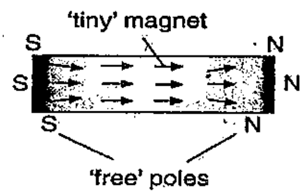


Fig.10.12 A magnetised bar

In Fig. 10.12, we note that the tiny magnets at the ends of the bar magnet splay out due to the mutual repulsion between like poles. This explains why the poles of the magnet are around the ends. In the case of an unmagnetised bar, we can imagine the tiny magnets pointing in random directions as shown in Fig.10.13. The resulting magnetic effect of all the tiny magnets are then cancelled out and thus the steel bar is said to be unmagnetised.

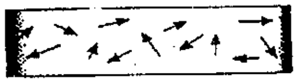
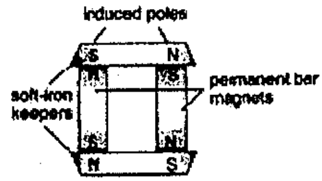


Fig.10.13 An unmagnetised bar

Based on this theory, we can account for the following

1. Storage of magnets using keepers
2. Magnetic saturation
3. Demagnetisation of magnets



Soft-iron keepers help the permanent bar magnet to stay strongly magnetised

10.7 MAGNETIC FIELDS

We have seen that a magnet affects magnetic substances placed near it. The region around a magnet in which this magnetic effect can be detected is called a magnetic field.

10.8 MAGNETIC PROPERTIES OF IRON AND STEEL

The magnetic properties of iron:

- easily magnetized and demagnetized.
- Can be magnetized by a weak magnetic field.

Iron is used in electromagnetic, transformer cores and magnetic shields.

The magnetic properties of steel:

- hard to magnetise and demagnetize than iron.
- requires a strong magnetic field to magnetise.

Steel is very good for making permanent magnets and it is used to make bar magnets.

Fig. 10.14 shows two chains of small iron paper clips and steel pen nibs hanging from a magnet. Each clip or nib induces magnetism in the one below it and the unlike poles so formed attract each other. The clips or nibs are added one by one to form a chain. The adding of the clip or nib only stops when no more clip or nib stays attached by induced magnetism.

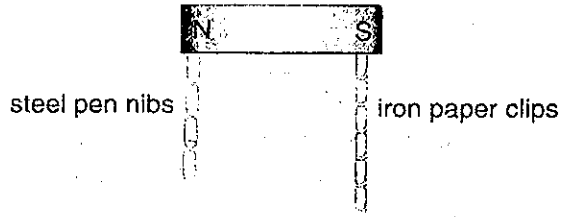


Fig. 10.14 The two chains of iron clips and steel nibs

From Fig. 10.14, it can be observed that the chain formed by iron paper clips is longer than that formed by the steel pen nibs. This shows that iron is more easily magnetised than steel.

If the chain formed by the iron paper clips is removed by slowly pulling the topmost clip away from the magnet, the entire chain collapses. This shows that the magnetism induced in iron is temporary.

However, when the same is done to the chain formed by the steel pen nibs, the chain does not collapse but the nibs remain attracted to each other. This shows that the magnetism induced in steel is permanent.

Magnetic materials such as steel which are harder to magnetise but retain their magnetism longer are called **hard magnetic materials.**

Magnetic materials such as iron or special alloys like mumetal and stalloy which are easier to magnetise but do not retain their magnetism are called **soft magnetic materials.**

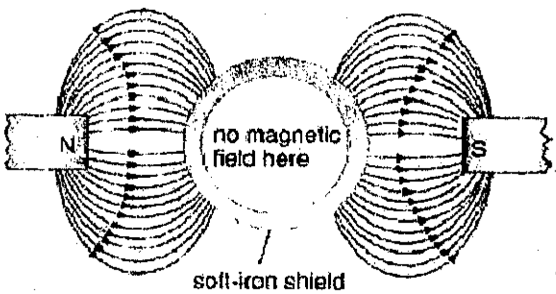


Fig. 10.15 Magnetic shielding to store magnetically sensitive instruments such as watches

Both types of magnetic materials have their own useful applications. For example, the hard magnetic materials such as steel are used in the making of permanent magnets while soft magnetic materials (such as iron) are used in the cores of transformers, electromagnets, and magnetic shielding (see Fig. 10.15).

EXERCISES

Multiple Choice Questions

1. It can be confirmed that a metal bar is already magnetized if

   A. a magnet is attracted to it.

   B. an aluminium bar is attracted to it.

   C. both ends of a compass needle are attracted to the same end of the bar

   D. one end of a compass needle is repelled by one end of the bar.

2. A small compass is placed in the uniform magnetic field as shown in Fig. 10.16.

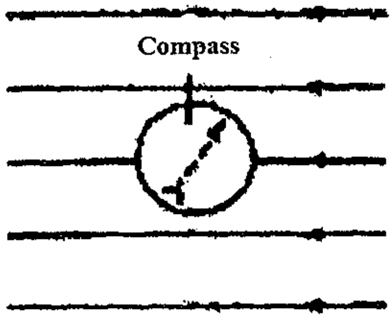


Fig.10.16

To which of the following directions will the compass needle point finally?

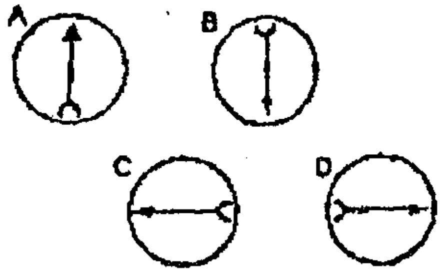


Fig. 10.17

3. A metal bar PQ hung by a thin thread always comes to rest with end Q pointing North. Another bar XY of the same metal settles in no definite direction. Which of the following is true?

   A. End Q attracts end X but repels end Y.

   B. End Q repels end X but attracts end Y.

   C. End Q attracts both end X and end Y.

   D. End Q neither attracts nor repels end X and end Y

4. Figure 10.18 shows a strong magnet holding six paperclips. If a weaker magnet brought close to the end of the last clip as shown it will
   A. bend away from the magnet.
   B. bend towards the magnet.
   C. fall to the ground.
   D. stay still.

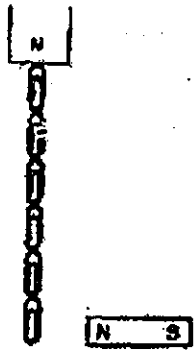


Fig. 10.18.

5. Which one of the following materials is most suitable for the core of an electromagnet?
   A. Steel
   B. Brass
   C. Iron
   D. Aluminum

6. As shown in Fig. 10.19, when the switch is closed, which of the following pairs of poles is correct?
   A. P is north and X is south.
   B. P is south and X is south.
   C. P is north and X is north.
   D. P is south and X is north.

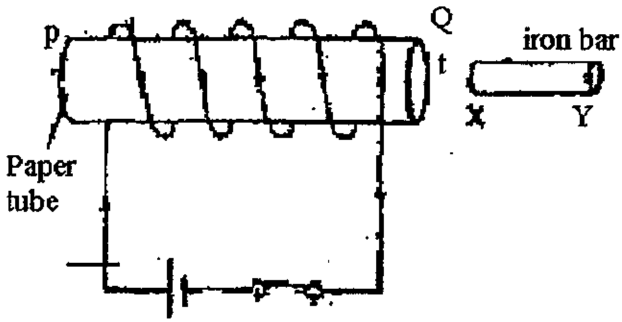


Fig. 10.19

7. Which of the following materials is correctly described?

material	property	use
A. iron	not easily demagnetised	permanent magnet
B. iron	easily demagnetised	electro-magnet
C. steel.	not easily demagnetised	electro-magnet
D. steel	easily demagnetised	permanent magnet

8. In which device is a permanent magnet used?
   A. An electric bell
   B. An electromagnet
   C. A plotting compass
   D. A relay

Structured Questions

1. Describe an experiment to determine the positions of the poles of a bar magnet.
2. What are the main differences in the magnetic properties of soft iron and steel ? How would you demonstrate them, experimentally? For each substance, name an instrument or piece of apparatus in which it is used because of its magnetic properties.
3. Describe briefly, with the help of simple diagrams if necessary,
   (a) how you would magnetize a steel rod PQ using a bar magnet so that P is a S pole;
   (b) how an electric current can be used to make P a S pole;
   (c) how you would check that the end P was a S pole after operations (a) and (b);
   (d) an electrical method to demagnetize P Q.
4. Experiments were conducted to test the ability of a vertically held bar magnet to attract soft iron hails. The results are shown in Fig. 10.20.

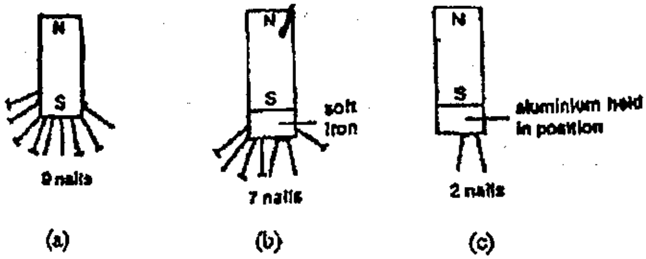


Fig.10.20

(a) What happened to the soft iron nails when they were placed in contact with the magnet?

(b) Suggest why the soft iron in Fig. 10.20 (b) picked up almost as many nails as the magnet alone.

(c) State and explain what would happen if the magnet was gently removed whilst the soft iron is still holding the 7 nails.

(d) Although aluminium is a non-magentic material, a few nails were attracted to it when it was placed at the end of the magnet. Suggest a reason for this.

5. Give brief explanations of the following,

(a) A piece of soft iron is attracted by a magnet.

(b) A small bar manget placed on top a cork floating on water, does not move towards the north.

(c) Two steel needles hanging from the lower end of a vertical bar magnet do not hang vertically (Fig. 10.21).

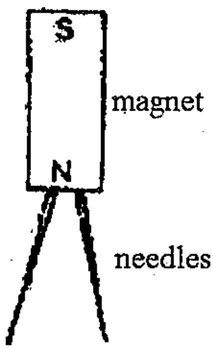


Fig.10.21

Glossary

Motion, Work and Energy

Acceleration The rate of change of velocity of a body.

Acceleration due to gravity The acceleration of a freely falling body within a gravitational field. Close to the surface of the earth its value is 10 m/s^2.

Centre of gravity The single point on a body at which the weight appears to act.

Gravitational potential energy The energy stored in a body that has been raised within the earth's gravitational field.

Gravity An 'action at a distance' force of attraction between two bodies.

Joule The unit of energy. 1 joule is the work done when a force of 1 newton moves its point of application through a distance of 1 metre in the direction of the force.

kilowatt A unit of power equal to 1000 watts, or a rate of energy transfer of 1000 joules per second.

Kinetic energy Energy associated with the movement of a body.

Machine An appliance that enables work to be done.

Potential energy The energy stored in a body due to its position and configuration.

Tension The state of anything that is subject to outward acting (stretching) forces.

Velocity The rate of change of displacement of a moving body (in terms of both distance and direction) with time.

Watt The unit of power, equal to a rate of energy transfer (or work done) of 1 joule per second.

Weight The gravitational force acting on a body.

Work The energy transferred in any system where a force causes movement. The work done is the product of the force and the distance moved by its point of application along the line in which the force acts.

Force

Centripetal force The force that acts towards the centre of the circle in which a body is moving, which keeps the body in circular motion.

Equilibrium The situation in which the effects of several forces cancel one another in terms of both magnitude and direction, producing zero resultant force. The state of a body at rest, or moving with constant velocity.

Friction A force caused by contact between two uneven moving past one another, or between the surface of a body and the fluid in which it moves, which resists the motion of a body.

Gravitational field strength The gravitational force exerted on a 1 kg mass placed within any gravitational field.

Moment The turning effect created by a force acting on a lever.

Momentum A property associated with the mass and velocity of a body.

Newton The unit of force. 1 newton is the force that gives a 1 kg mass an acceleration of 1 m/s^2.

Resultant force The effective force on a body on which several forces, of different magnitudes, may be acting in different direction.

Pressure

Atmospheric pressure The pressure exerted on a body by the atmosphere, due to the weight of the atmosphere. At the surface of the earth atmospheric pressure is 100 kNm^{-2} (100 kPa)

Pascal A unit of pressure equivalent to a force of 1 newton acting on 1 m^2.

Pressure The force per unit area acting on a surface in such a way that it is tending to change the dimensions of the surface.

Heat

Absolute zero The lowest temperature (**-273**°C) that any substance can reach. At this temperature the molecules or atoms of the substance have no heat energy.

Boiling point The temperature at which a substance undergoes a change of state between liquid and gas (vapour).

Conduction (thermal) The process of transferring heat through a material without any visible change in the motion of the particles of the material.

Convection The process of transferring heat by the movement of the fluid (liquid or gas) through which heat is being transferred.

Convection current The continuous transfer of heat by circulation in a fluid. Convection currents are created by changes in the density of the fluid - the warmed fluid expands, and therefore has a lower density than the surrounding fluid. The less dense fluid rises through the denser, cooler fluid, carrying heat with it.

Evaporation A process by which liquid molecules may become vapour molecules at a temperature other than the boiling point.

Heat Thermal energy in the process of being transferred in, some way.

Latent heat The heat exchanged when a substance undergoes some change of state (with no accompanying change in temperature).

Melting point The temperature at which a substance undergoes a change of state between solid and liquid.

Specific heat capacity The heat exchanged when 1 kg of a substance undergoes a change of temperature of $1^{\circ}C$.

Specific latent heat of fusion The heat exchanged when 1 kg of a substance undergoes a change of state between solid and liquid.

Specific latent heat of vaporisation The heat exchanged when 1 kg of a substance undergoes a change of state between liquid and vapour.

Temperature A measure of the relative 'hotness' of bodies; it depends on the average kinetic energy of the particles of a body.

Thermal energy Energy associated with heating effects.

Diffraction The spreading of waves as they pass by the edge of an obstacle or through a narrow slit.

Frequency The rate at which some regular disturbance takes place. For a wave this represents the number of complete oscillations per second.

Hertz A unit of frequency of vibrations. 1 hertz is equivalent to one oscillation per second.

Longitudinal wave An energy-carrying wave in which the movement of the particles is in line with the direction in which the energy is being transferred.

Oscillation One complete to-and-fro motion of a vibrating object.

Transverse wave A wave in which the oscillations are at right angles to the direction in which the wave transfers energy.

Wave equation The relation ***speed = frequency × wavelength*** which applies to all forms of wave motion.

Wavelength The distance between two successive points on a wave that are at the same stage of oscillation, i.e. in terms of their direction and displacement from their mean position.

Sound

Compression The state of any object that is subject to inward-acting (squashing) forces. Also, a region of a medium within which the particles are at above mean pressure, due to the passing of a sound wave.

Echo A sound that has undergone reflection.

Pitch The property of a note that determines how 'high' or 'low' it sounds to the listener.

Rarefaction A region of a medium within which the particles are at below mean pressure, due to the passing of a sound wave.

Sound energy Energy that has been transformed into an audible form.

Sound wave A longitudinal wave that transfers sound vibrations from place to place.

Ultrasound Sound waves with frequencies beyond the upper limit of human hearing.

Light

Electromagnetic spectrum A continuous arrangement that displays electromagnetic waves in order of increasing frequency or wavelength.

Electromagnetic waves Transverse waves that consist of electric and magnetic oscillations at right angles to one another and to the direction of travel.

Internal reflection The phenomenon in which light undergoes reflection at the boundary between two surfaces which have different optical densities.

Optical fibres Very thin strands of pure optical glass through which light undergoes total internal reflection.

Primary light colours Red, blue, green. By mixing these colours of light any other colour can be produced. Combining them equally produces white light.

Real image An image formed by the convergence of real rays of light and which can be displayed on a screen.

Refraction The phenomenon that occurs when a wave passes from one medium into another, causing a change in speed, and, possibly, direction.

Secondary light colours Magenta, yellow, cyan. Produced by mixing two primary colours.

Virtual image An image formed by the apparent convergence of virtual (non-real) rays of light and which cannot be displayed on a screen.

Electricity

Alternating current (a.c.) Electric current whose direction alternates (changes) at regular intervals.

Ammeter An instrument used to measure electric current.

ampere The unit used to measure electric current.

Capacitor A component of electronic systems which can be charged and discharged, and which may be used to create time delays.

Conductors Materials that allow the ready transfer of heat by conduction, or of electricity by current flow.

coulomb The unit representing the amount of charge passing any point in a circuit when a current of 1 ampere flows past that point for 1 second.

Direct current (d.c.) The flow of charge through a circuit in one direction only.

Electric charge A quantity of unbalanced (positive or negative) electricity.

Electric current The rate at which charge flows through a conductor.

Electrical energy Energy associated with the flow of charge through any part of a conducting circuit.

Electrons Negatively charged particles found orbiting the nucleus of an atom. Also emitted from a radioactive nucleus in beta decay.

Free electrons Electrons that are able to move freely from atom to atom within a material.

Ions Particles that have excess negative or positive charge.

Isotopes Forms of the same element with the same atomic number but different mass numbers. Some elements have only one natural isotope but all have artificially created isotopes.

Kilowatt-hour A unit used by electricity supply companies, representing the energy dissipated in one hour by a device with a power of 1 kilowatt.

Mass number The number of particles (protons and neutrons) inside the nucleus of an atom.

Neutrons Uncharged particles found in the nuclei of atoms.

ohm The unit of electrical resistance. 1 ohm is the resistance of a sample of conducting material across which a potential difference of 1 volt causes a current of 1 ampere to flow.

Ohm's Law A relationship between the current flowing through a conductor and the potential difference across the ends of the conductor: ***the current through a conductor is proportional to the potential difference across the ends of that conductor, provided the temperature of the conductor is constant.***

Parallel circuit A circuit in which the current passes through two or more paths, or loops, before rejoining.

Protons Positively charged particles found in the nuclei of all atoms.

Resistance A property of materials which resist the flow of electric current through them to some greater or lesser degree.

Resistivity The resistance per unit length of unit cross-section of a material.

Series circuit A circuit in which all devices are connected in one continuous loop through which a common current flows.

Volt The unit of potential difference. A potential difference of 1 volt exists between two points when 1 joule of work is done in transferring 1 coulomb of charge between the two points. Alternatively, a potential difference of 1 volt exists between two points when 1 ampere of current dissipates 1 watt of power on passing between the two points.

Voltage The value of the potential difference between two points (e.g., the terminals of a cell).

Voltmeter An instrument used to measure potential difference (voltage).

Magnetism

Electromagnet A soft iron core surrounded by a coil of wire, which acts as a magnet when current flows through the coil.

Electromagnetic induction The generation of an induced electric current when a conductor is moved through a magnetic field. The transfer of electrical power from one circuit to another (as in the case of transformers).

Induced current A current that is induced in a conductor due to the relative motion of the conductor and a magnetic field.

Magnetic field A space in which forces would act on magnetic poles placed within it.

ANSWERS TO ODD-NUMBERED PROBLEMS

Chapter 1

13. 4047 m^2
15. 3.17×10^8yr
    3.17×10^9yr
    1.27×10^{10}yr
    6.34×10^{10}yr
    3.17×10^{-31}yr
19. No
21. 1.88×10^{-28} kg
    1.67×10^{-27} kg

Chapter 2

15. 1 unit (north)
17. 6.71 N
19. 3.99 units
    3.01 units

Chapter 3

21. Zero.
23. 480 m
25. 18 ms^{-1}
27. (a)4.8 ms^{-1}
    (b)5.6 ms^{-1}
    (c)6.4ms^{-1}
    (d) 7.2ms^{-1}
    3.2ms^{-1}
29. (a) 24m, 84m, 30m
    (b) 8.625ms^{-1}
31. (c)4.5 km
    (d) 15kmh^{-1} min^{-2}

Chapter 4

23. 60 N
    30 N
25. 20 ms^{-2}
    -20ms^{-2}
27. 7200 N
29. 10 ms^{-2}
31. The body weighting 1 N
33. 66.67 ft s^{-2}
    48 lb
35. 62.5 sl , 48 ft s^{-2}
37. The weight of the body will decrease.
39. The force acting on 3 M is 3 times. greater
    0.32 N, 0.96 N
41. 600 sl fts^{-1}(north)
43. (a) 0
    (b) 18.37×10^{-22} kg ms^{-1}(east)
45. 10^7 ms^{-1} The answer is absurd.
    -10 ms^{-1}
49. 2 s, 64 ft s^{-1}
51. 10 m s^{-1}
    0 m s^{-1}
    -10 m s^{-1}
53. 2 s (or) 4 s
55. (a) 1.5 N
    (b) 2.5 N
57. 60 kg
59. 40 N
73. 1.3 g cm^{-3}
75. 11.43 cm^{-3}
77. (a) 1000 kg
    (b) 9600 kg

Chapter 5

15. $75^{\circ} 31'$
17. (a)600 J
    (b) 0
19. (a)67.5J
    (b)45N
21. 19.6 ft s^{-1}

Chapter 6

15. $(10^{12} - 273)\,^{\circ}C \simeq 10^{12}\,^{\circ}C$
    $(10^{10} - 273)\,^{\circ}C \simeq 10^{10}\,^{\circ}C$
    2727°C
    - 270 °C
17. - 40°
19. 7.62×10^{-3} m
21. $1.27 \times 10^{-3} m^2$

Chapter 7

13. 20.7 °C
15. 657.2 $J^{-1}\ kg^{-1}\ K^{-1}$

Chapter 8

23. 400Hz
25. 10.3 MHz
27. 2.06 s
29. 344 ms^{-1}

Chapter 9

3. 14 ft
5 2 ft 4 in
15. 9 cm, 2/3
17. 30 cm, 6 cm
19. 40 cm

Chapter 10

Answers to Multiple Choice Questions

1. D
3. C
5. C
7. B

APPENDIX
Common SI base units and derived units

Quantity	Base Units	Symbols
Length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
luminous intensity	candela	cd
substance	mole	mol

Quantity	Derived Units (Selected)	Formula
acceleration	metre per second squared	$m\ s^{-2}$
area	square metre	m^2
density	kilogram per cubic metre	$kg\ m^{-3}$
electric capacitance	farad(F)	$A\ s\ V^{-1}$
electric charge (quantity of electricity)	coulomb (C)	A s
energy	joule (J)	N m
force	newton (N)	$kg.\ m\ s^{-2}$
frequency	hertz(Hz)	cycle s^{-1}
magnetic flux density	tesla(T)	$W\ b\ m^{-2}$
power	watt(W)	$J\ s^{-1}$
pressure	pascal (Pa)	$N\ m^{-2}$
thermal conductivity	watt per metre per kelvin	$W\ m^{-1}K^{-1}$
velocity	metre per second	$m\ s^{-1}$
work	joule (J)	N m

SI unit prefixes, symbols and power of ten multiple and submultiple values

Prefix	Symbol	Value as Power of Ten	Multiplication Factor
deka	da	10	10
hecto	h	10^2	100
kilo	k	10^3	1000
mega	M	10^6	1 000 000
giga	G	10^9	1 000 000 000
tera	T	10^{12}	1 000 000 000 000

Prefix	Symbol	Value as Power of Ten	Multiplication Factor
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001
femto	f	10^{-15}	0.000 000 000 000 001
atto	a	10^{-18}	0.000 000 000 000 000 001

Greek Alphabets

a	alpha	λ	lambda
β	beta	μ	mu
γ	gamma	ρ	rho
δ	delta	σ	sigma
ε	epsilon	τ	tau
η	eta	φ	phi
θ	theta	Ω	omega
κ	kappa		

CONVERSION FACTORS

Length

1 metre (m)	= 39.4 in	1 foot (ft)	= 0.305 m
	= 3.28 ft	1 inch(in)	= 0.0833 ft
1 centimetre(cm)	= 0.394 in		= 2.54 cm
1 kilometre(km)	= 0.621 mi	1 mile(mi)	= 1.61 km

Area

1 m^2	= 10^4 cm^2	1 ft^2	= 9.29 × $10^{-2} m^2$
	= 1.55 × 10^3 in^2		= 929 cm^2
	= 10.76 ft^2		
1 cm^2	= 10^{-4} m^2		
	= 0.155 in^2		

Volume

1 m^3	= 10^6 cm^3	1 ft^3	= 2.83 × $10^{-2} m^3$
	= 35.3 ft^3		= 28.3 litres
	= 6.10 × 10^4 in^3		= 7.48 gal
		1 US gal	= 0.134 ft^3
1 Imperial gal	= 1.2 US gal		= 3.79 × 10^{-3} m^3

Mass

1 kilogram(kg)	= 0.0685 slug	1 slug (sl)	= 14.57 kg
		1 lb mass	= 454 g
			= 0.454 kg

Velocity

1 m s^{-1}	= 3.28 ft s^{-1}	1 ft s^{-1}	= 0.305 m s^{-1}
	= 3.60 km h^{-1}		= 0.682 mi h^{-1}
	= 2.24 mi h^{-1}		= 1.10 km h^{-1}
1 km h^{-1}	= 0.278 ms^{-1}	1 mi h^{-1}	= 1.47 fts^{-1}
	= 0.913 ft s^{-1}		= 0.447 ms^{-1}
	= 0.621 mih^{-1}		= 1.61 kmh^{-1}
		60 mi h^{-1}	= 88 ft s^{-1}

Force

1 newton(N)	= 0.225 lb	1 lb	= 4.45 N
	= 3.60 oz		= 4.45×10^5 dynes
	= 10^5 dynes		

Pressure

1 pascal(Pa)	= 1 Nm^{-2}	1 lb in^{-2}	= 6.90×10^3 Pa
	= 1.45×10^{-4} lb in^{-2}		
1 atm	= 1.013×10^5 Nm^{-2}		
	= 14.7 lb in^{-2}		

Energy

1 joule(J)	= 0.738 ft-lb	1 ft-lb	= 1.36 J
	= 2.39×10^{-4} kcal		= 1.29×10^{-3} Btu
	= 6.24×10^{18} eV		= 3.25×10^{-4} kcal
1 kilocalorie(kcal)	= 4184 J	1 Btu	= 778 ft-lb
	= 3.97 Btu		= 0.252 k cal
	= 3077 ft-lb		
1 electron volt (eV)	= 1.60×10^{-19} J		

Power

1 watt (W)	= 1 Js^{-1}	= 0.738 ft-lb s^{-1}
1 kilowatt (kW)	= 1.34 hp	
1 horse power (hp)	= 746 W	= 550 ft-lb s^{-1}

Temperature

$T_k = T_c + 273°$

$T_c = 5/9\,(T_F - 32°)$

$T_F = 9/5\,T_c + 32°$

Time

1 day	= 1.44×10^3 min	= 8.64×10^4 s	
1 year	= 8.76×10^3 h	= 5.26×10^6 min	= 3.15×10^7 s

Angle

1 radian (rad)	= 57° .18′ = 57.30°	1°	= 0.01745 rad
1 rad s^{-1}	= 9.55 rev min^{-1}	1 rev min-1 (rpm)	= 0.1047rad s^{-1}