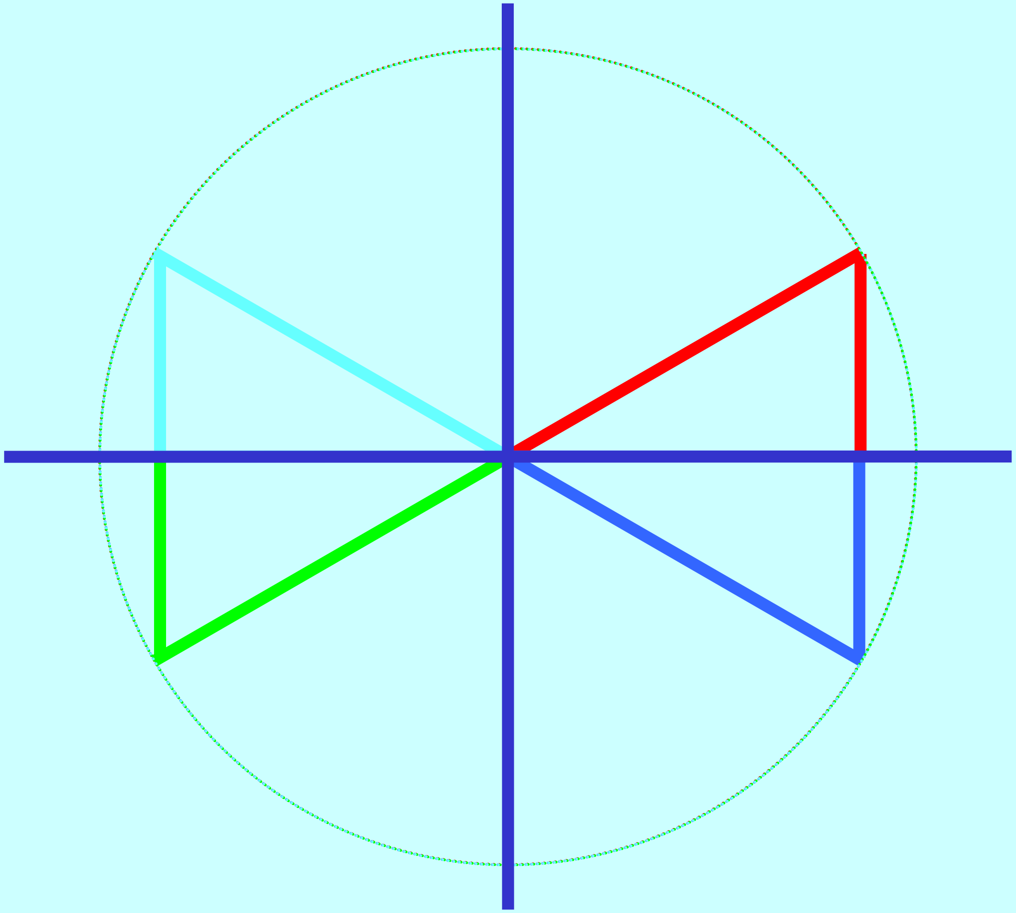
UEENEEG102A Solve problems in low voltage a.c. circuits

Trigonometry

Trigonometry

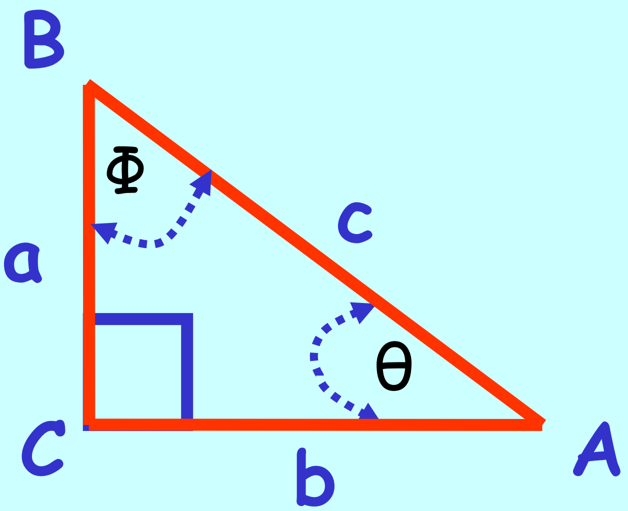


Objectives:

At the end of this lesson students should be able to:

1. State and apply the Sine, Cosine and Tangent ratios of a right-angle triangle.
2. Use a calculator to find the Sine, Cosine and Tangent of any angle.
3. Apply Pythagoras' Theorem to a right-angle triangle.

The Right Angle Triangle

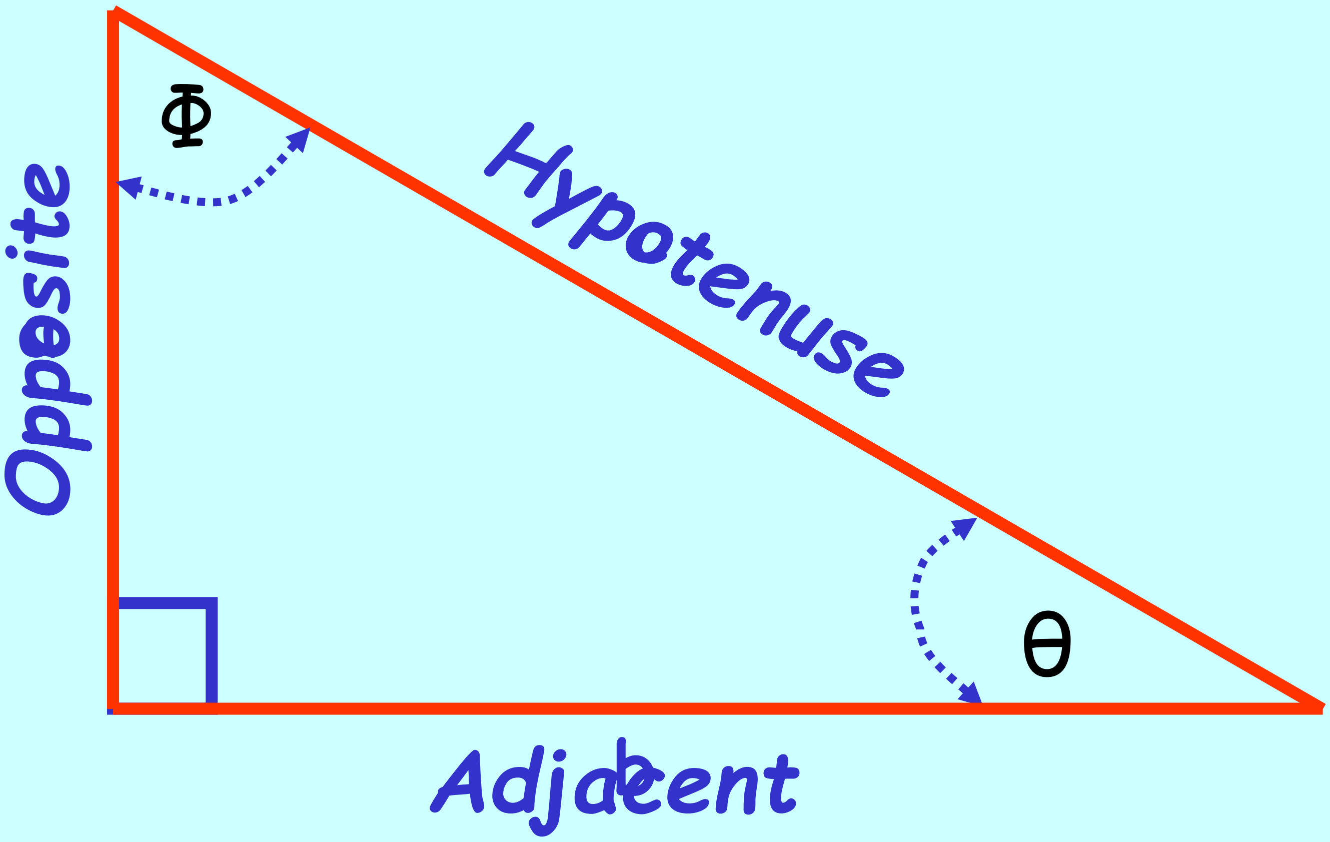


$\angle A = \angle CAB = \theta$ Side BC = a

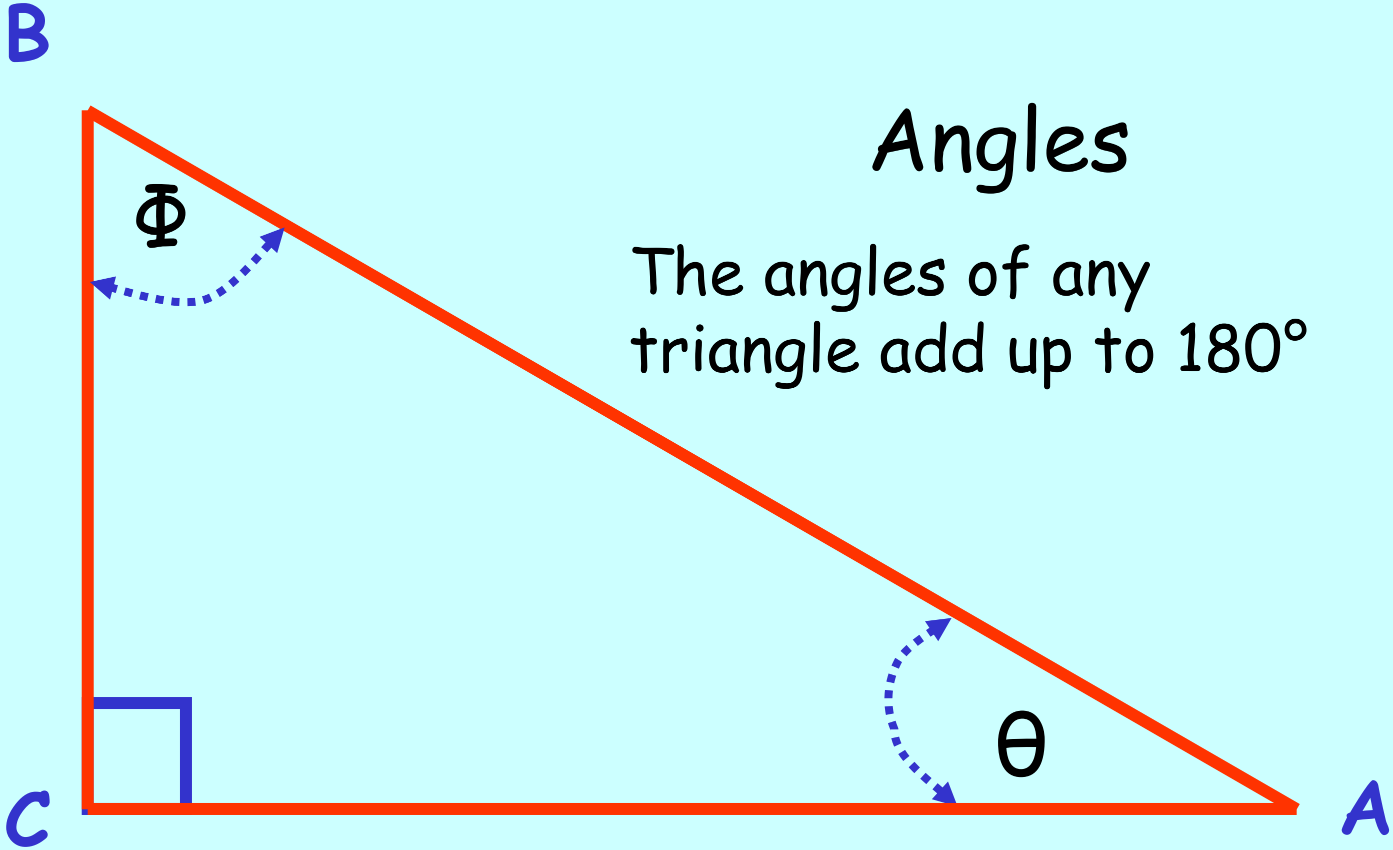
$\angle B = \angle ABC = \Phi$ Side CA = b

$\angle C = \angle BCA = 90^\circ$ Side AB = c

Names of Sides



Angles

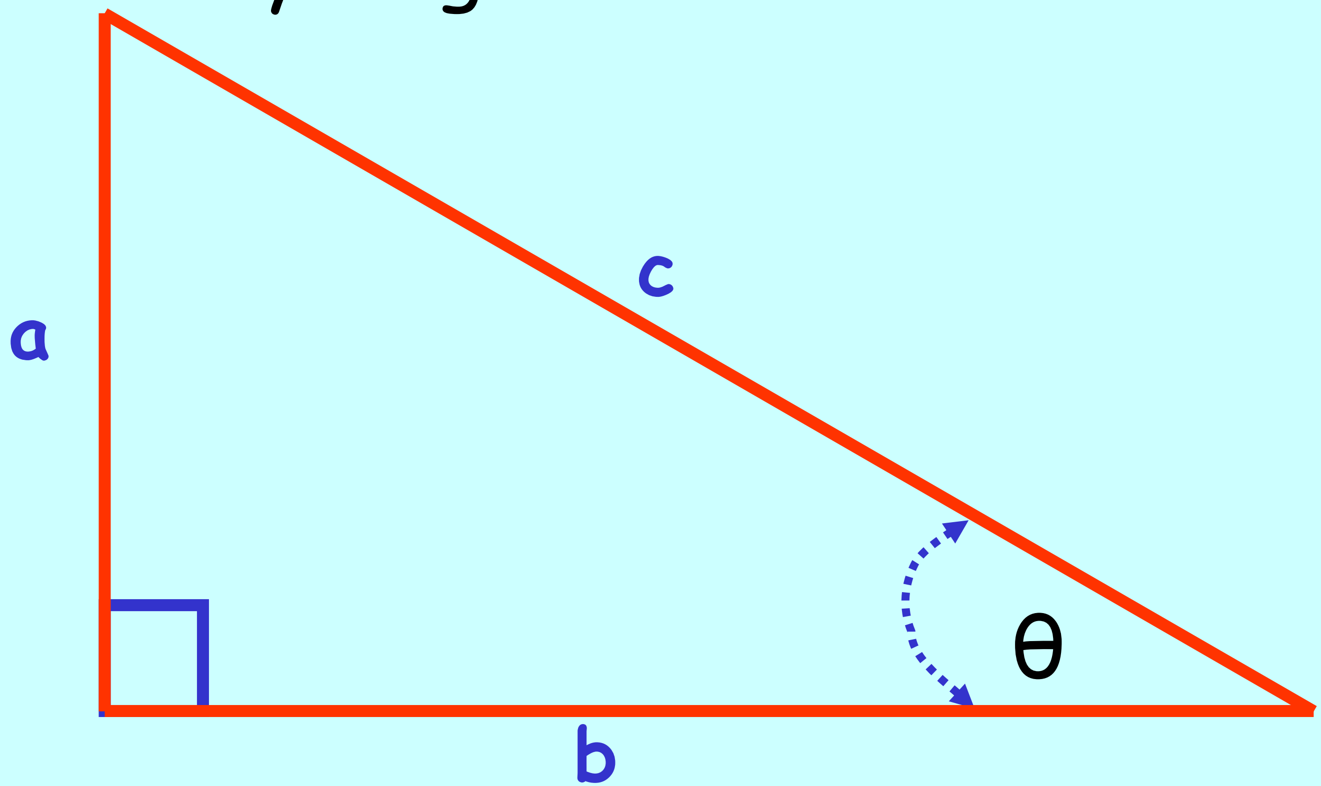


The angles of any triangle add up to 180°

$$\angle A + \angle B + \angle C = 180$$

$$\theta + \Phi = 90°$$

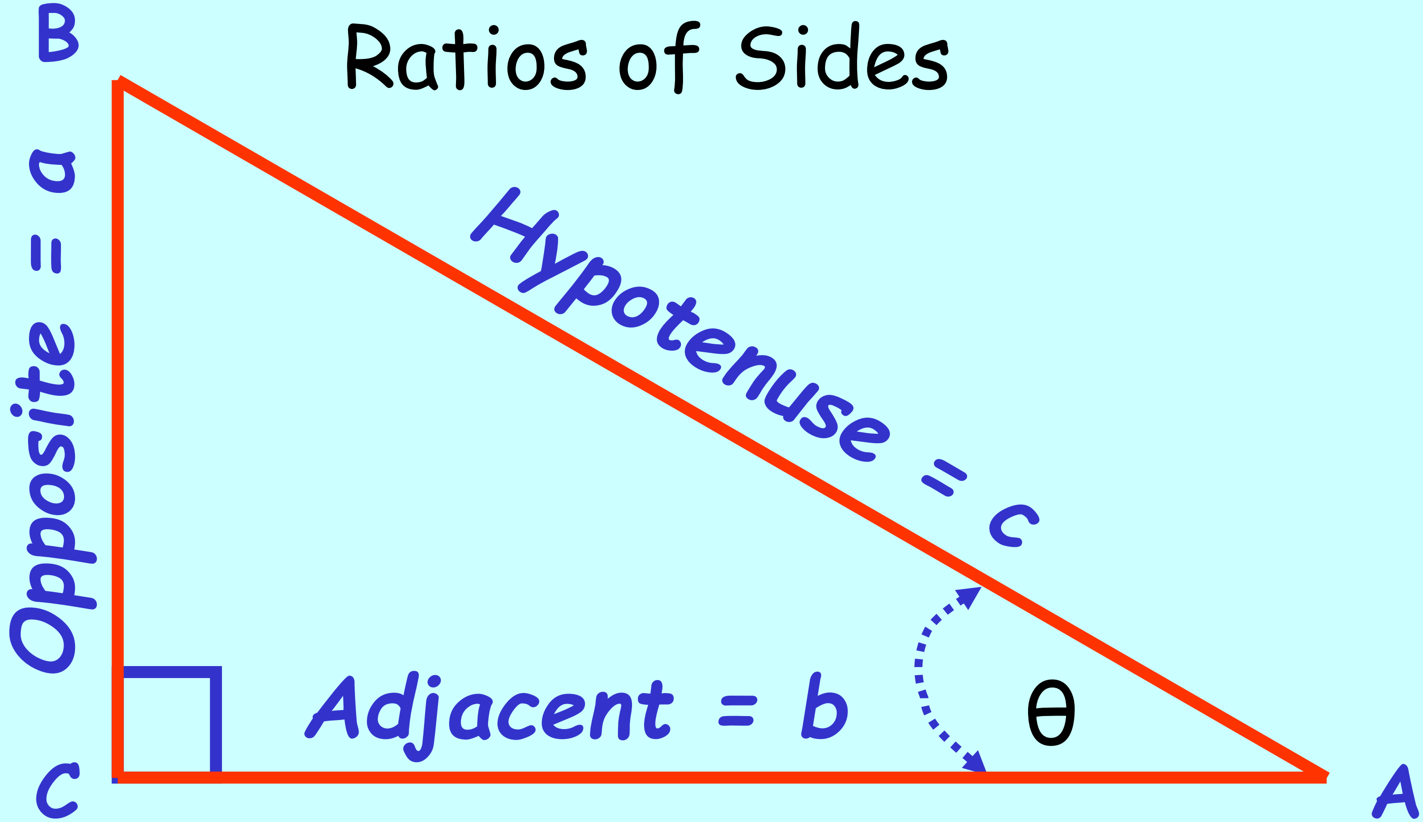
Pythagoras' Theorem



The square on the Hypotenuse is equal to the sum of the squares on the other two sides ….

$$c^2 = a^2 + b^2$$

Ratios of Sides



$$\sin\theta = \frac{Opp}{Hyp}$$

$$\cos\theta = \frac{Adj}{Hyp}$$

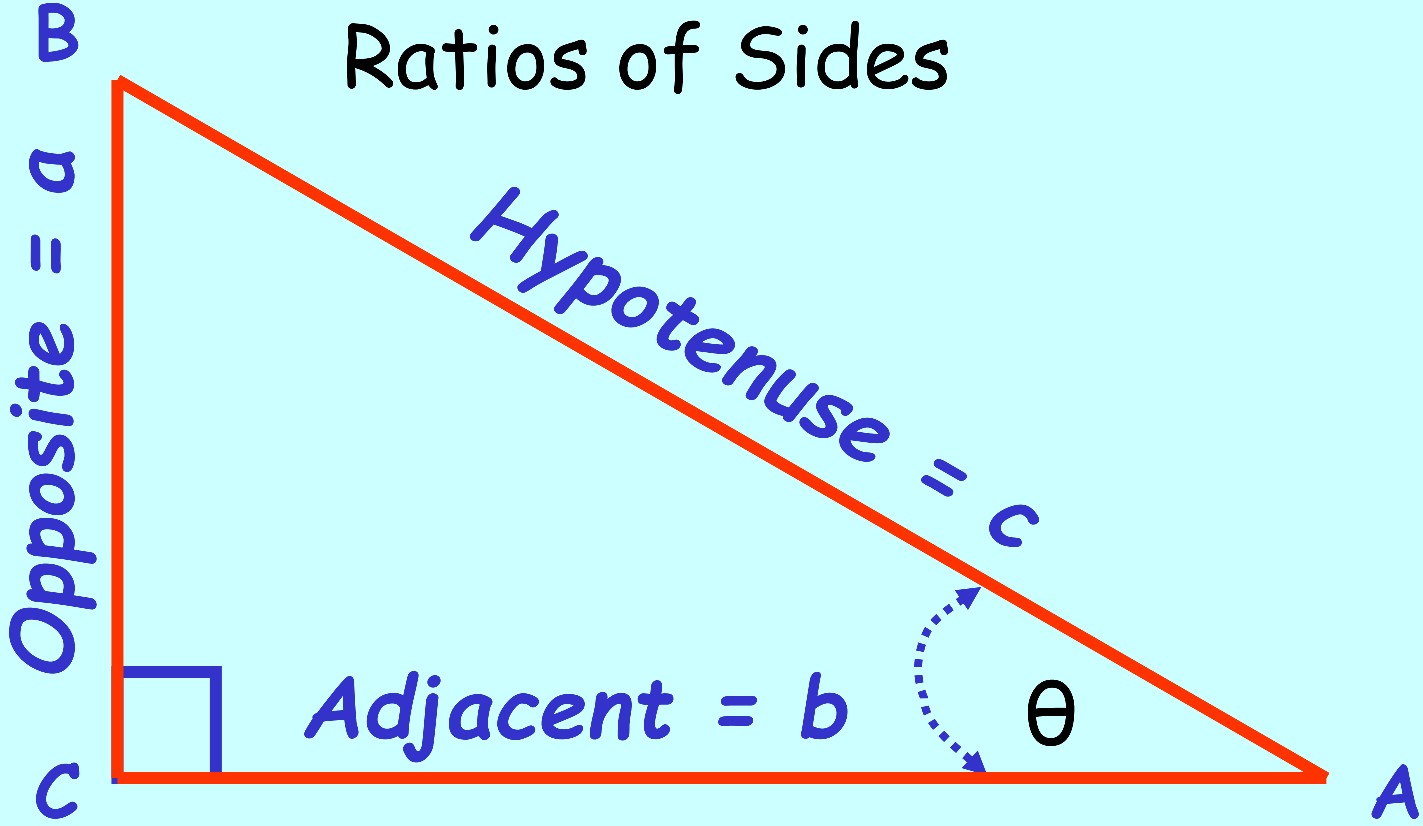
$$\tan\theta = \frac{Opp}{Adj}$$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$

Ratios of Sides



$$\sin\theta = \frac{Opp}{Hyp}$$

$$\cos\theta = \frac{Adj}{Hyp}$$

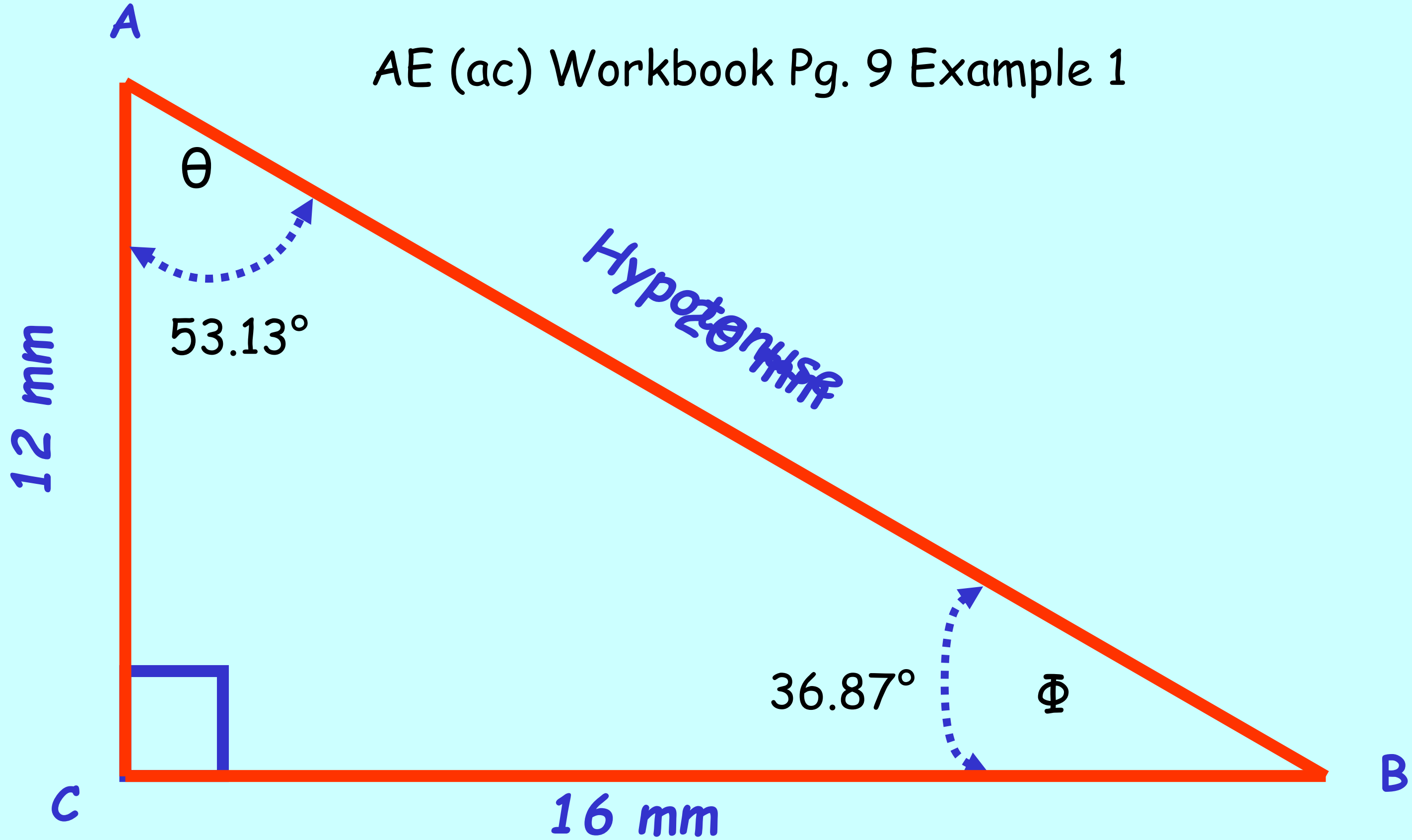
$$\tan\theta = \frac{Opp}{Adj}$$

SOH

SOH CAH TOA

TOA

AE (ac) Workbook Pg. 9 Example 1



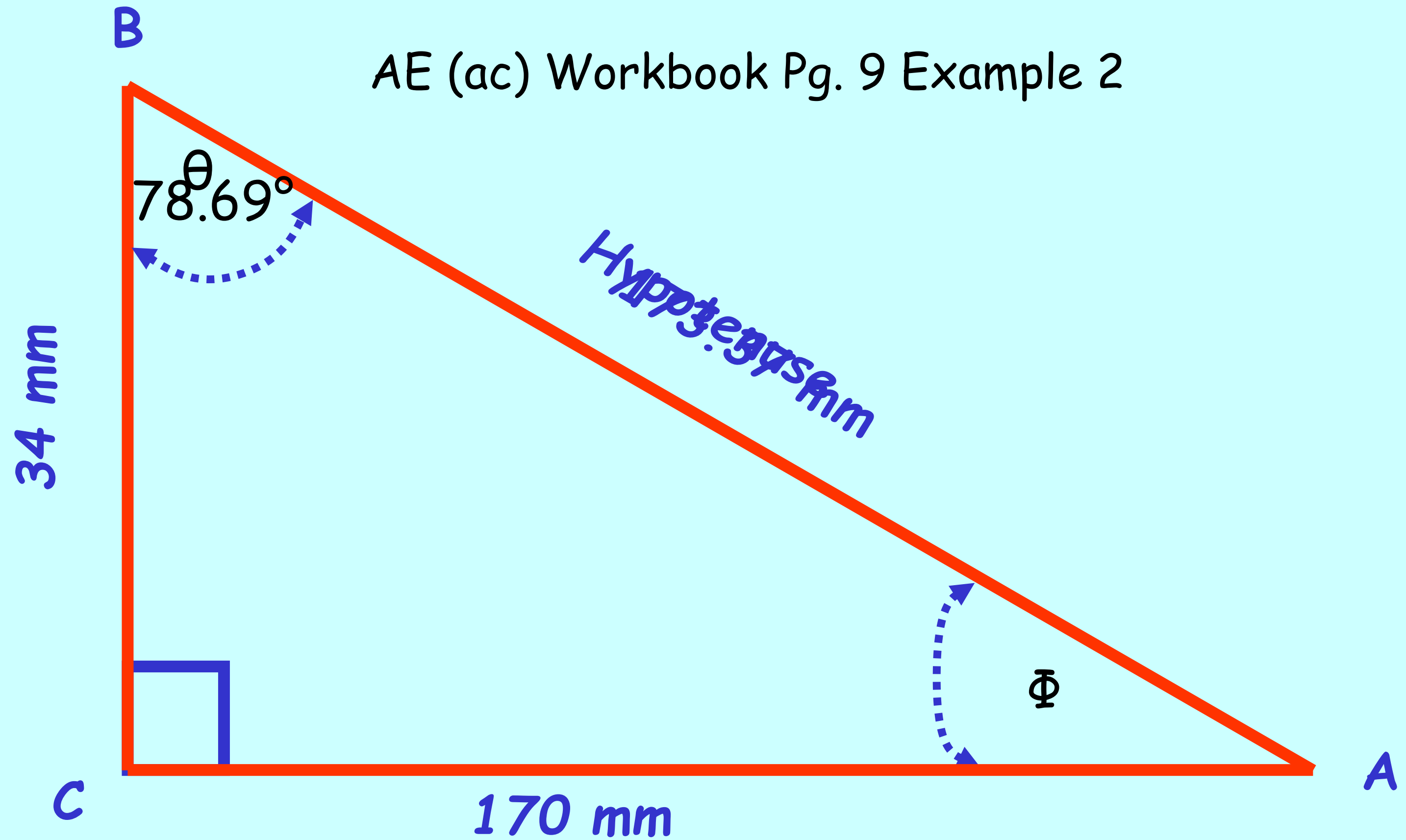
$$\theta = \cos^{-1}\frac{12}{20} = 53.13^\circ$$

$$\Phi = \sin^{-1}\frac{12}{20} = 36.87^\circ$$

$$\Phi = 90 - 53.13 = 36.87^\circ$$

$$BC = \sqrt{20^2 - 12^2} = 16\text{ mm}$$

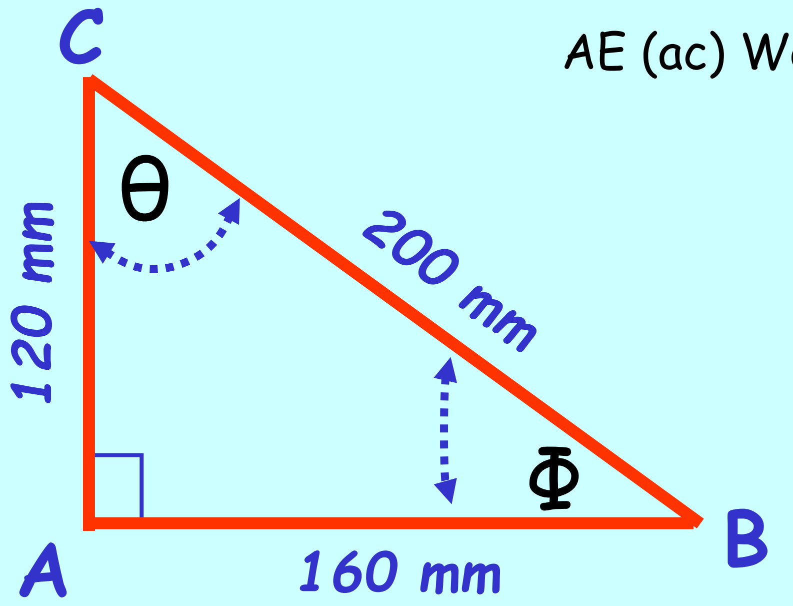
AE (ac) Workbook Pg. 9 Example 2



$$\theta = \tan^{-1}\frac{170}{34} = 78.69^{\circ}$$

$$AB = \sqrt{170^2 + 34^2} = 173.37\ \text{mm}$$

AE (ac) Workbook Pg. 10 Example 3



$$AC = \sqrt{200^2 - 160^2} = 120\,\text{mm}$$

(a)

$$\sin B = \frac{120}{200} = 0.6$$

$$\cos B = \frac{160}{200} = 0.8$$

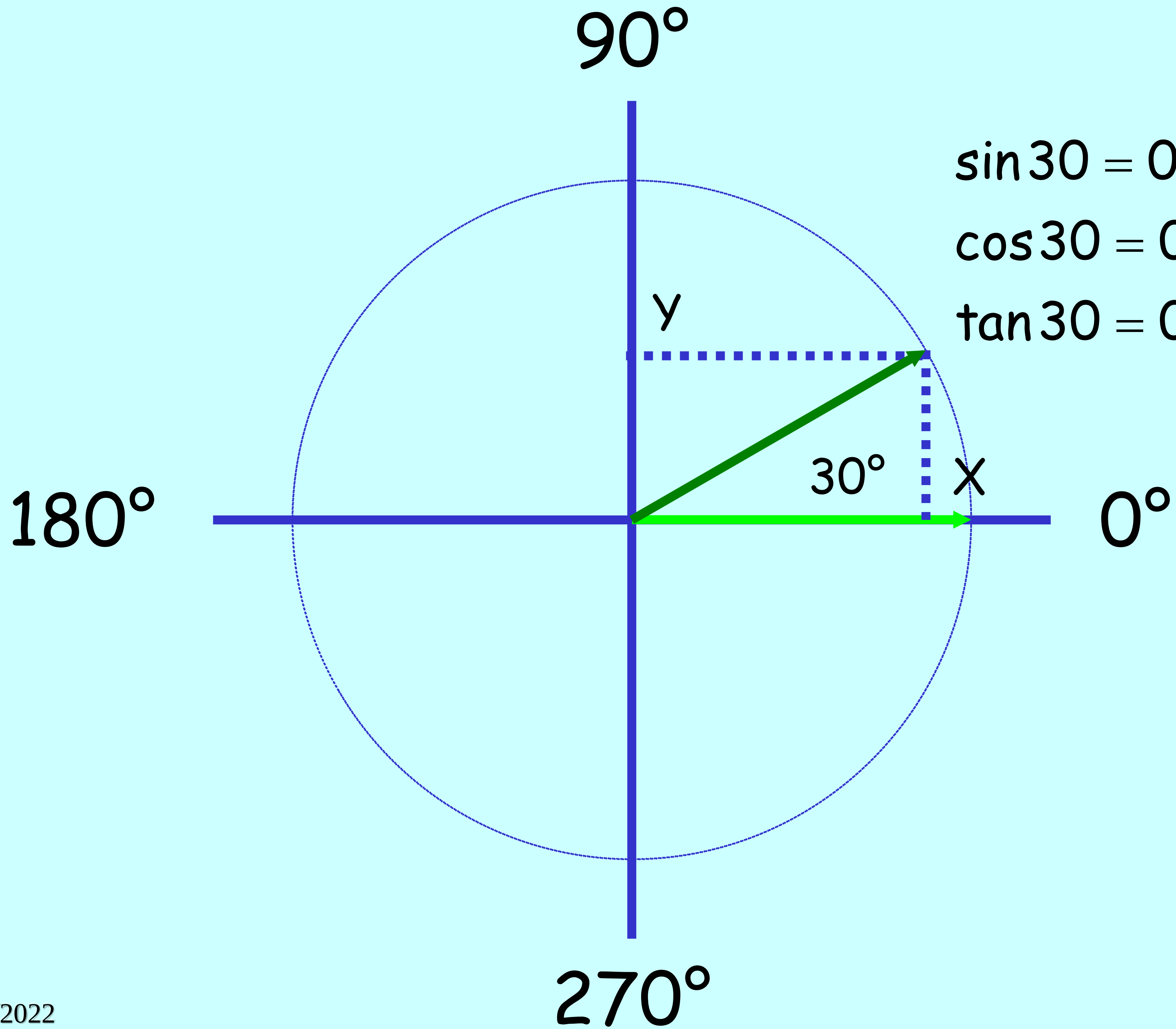
$$\tan B = \frac{120}{160} = 0.75$$

(b)

$$\sin C = \frac{160}{200} = 0.8$$

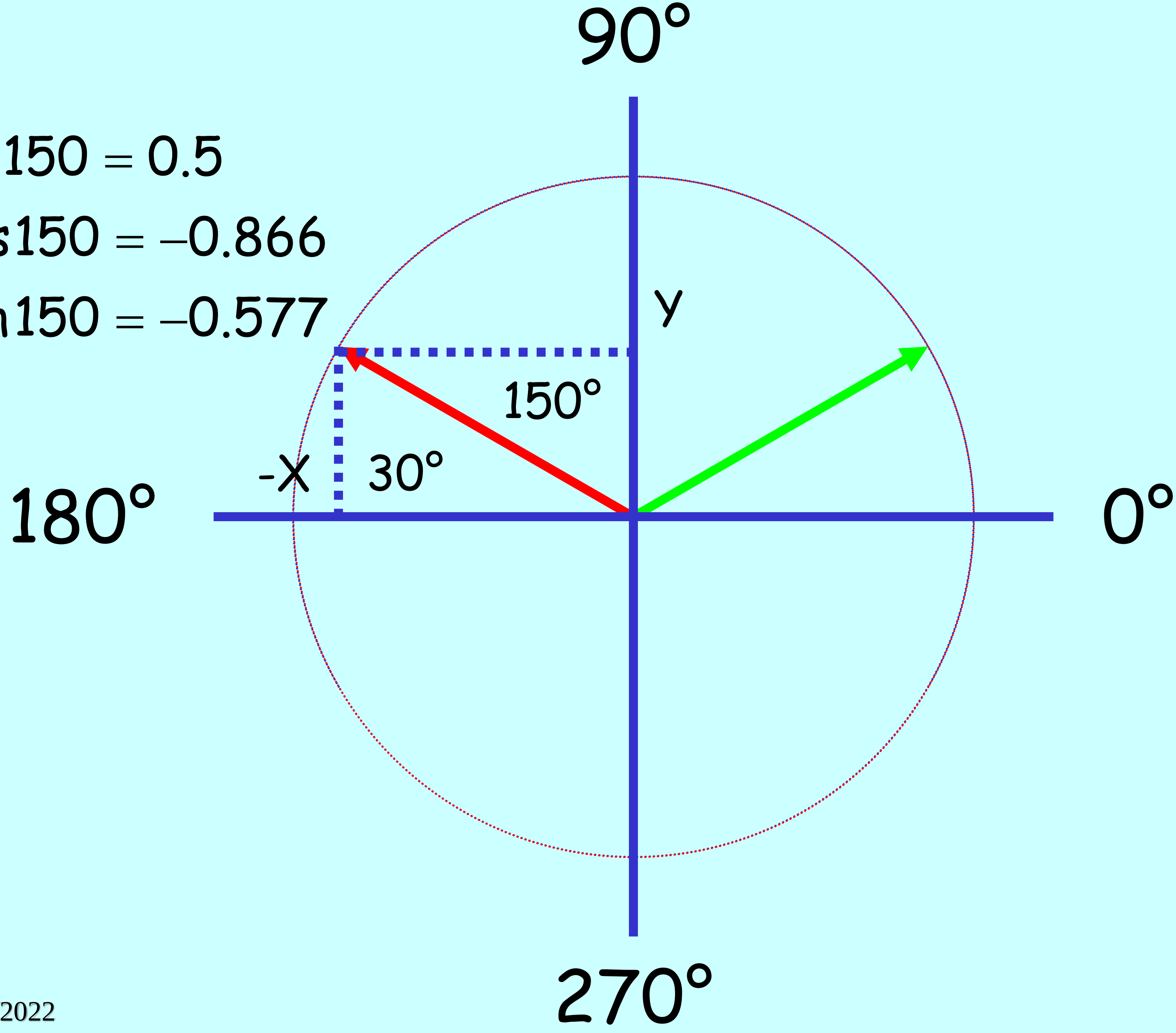
$$\cos C = \frac{120}{200} = 0.6$$

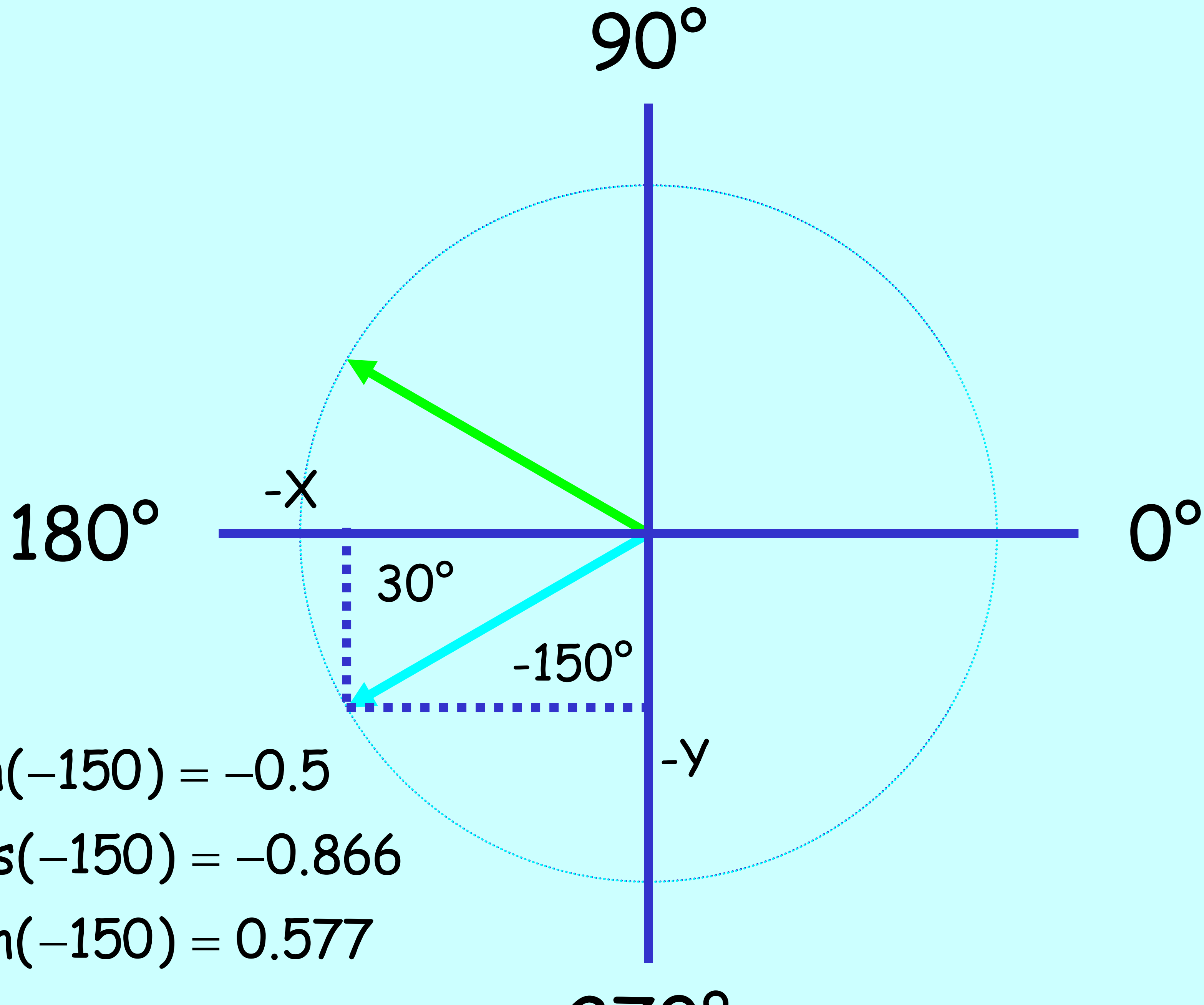
$$\tan C = \frac{160}{120} = 1.33$$



$$\sin 30 = 0.5$$
$$\cos 30 = 0.866$$
$$\tan 30 = 0.577$$

$$\sin 150 = 0.5$$
$$\cos 150 = -0.866$$
$$\tan 150 = -0.577$$

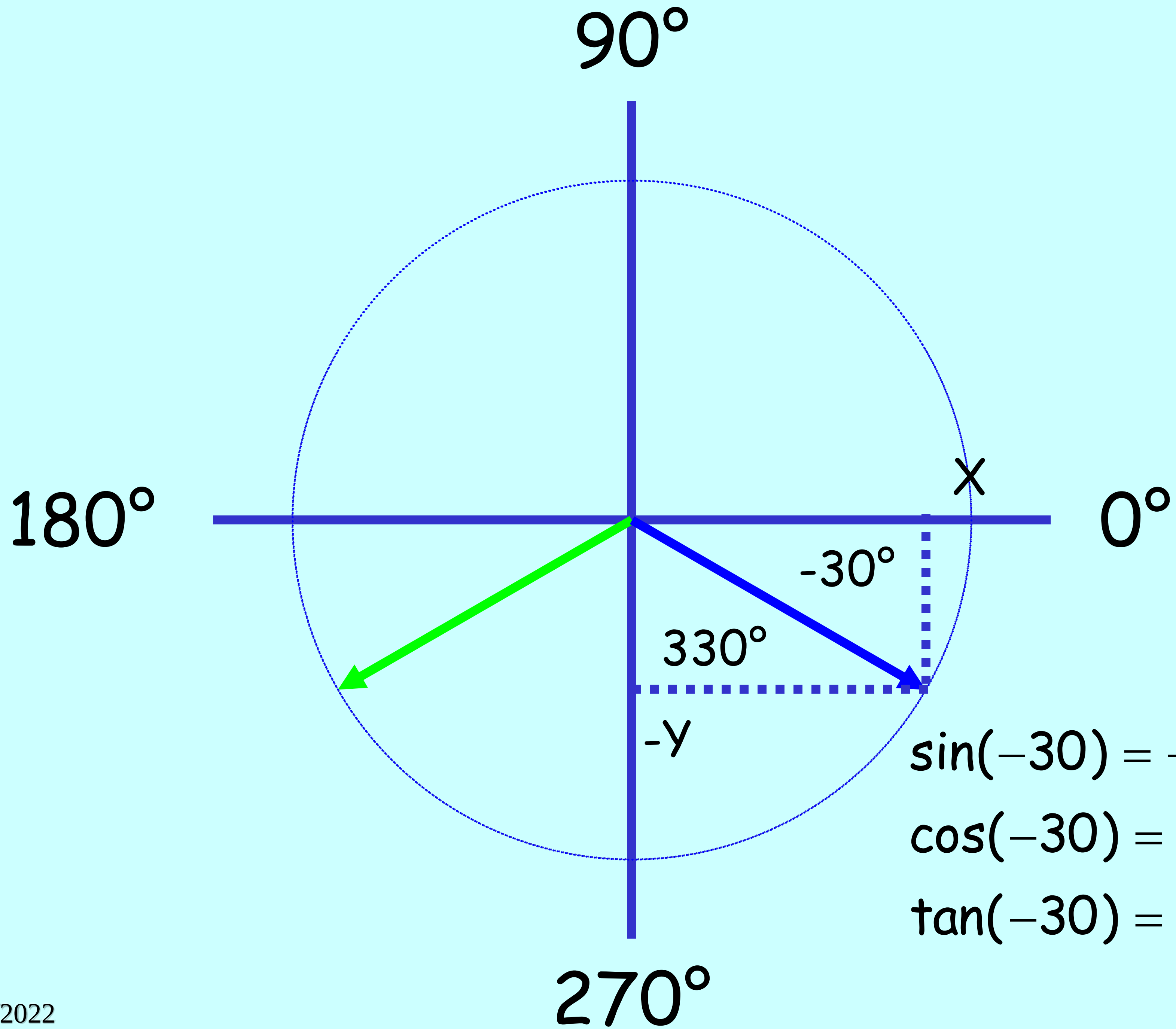




$\sin(-150) = -0.5$

$\cos(-150) = -0.866$

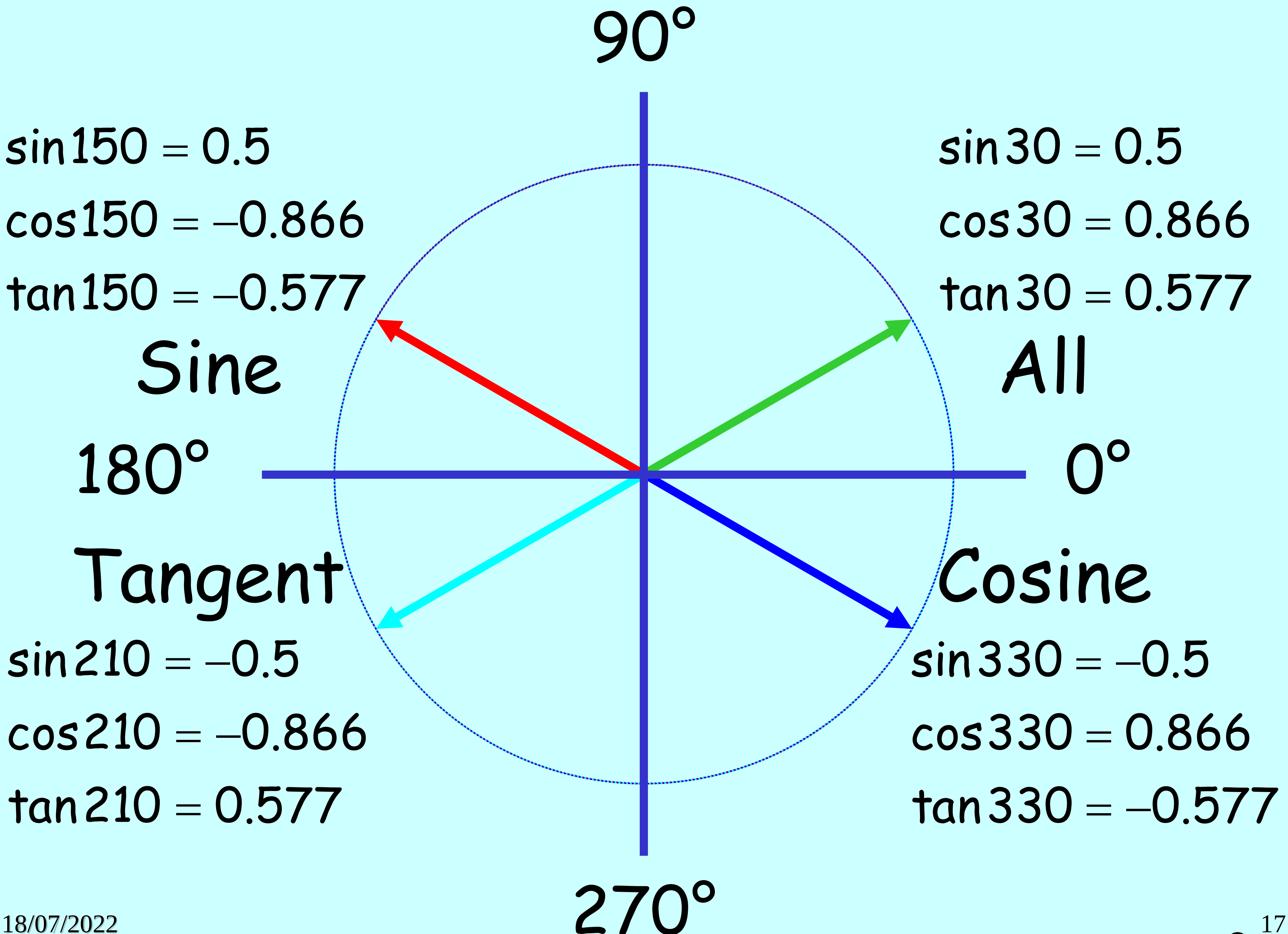
$\tan(-150) = 0.577$

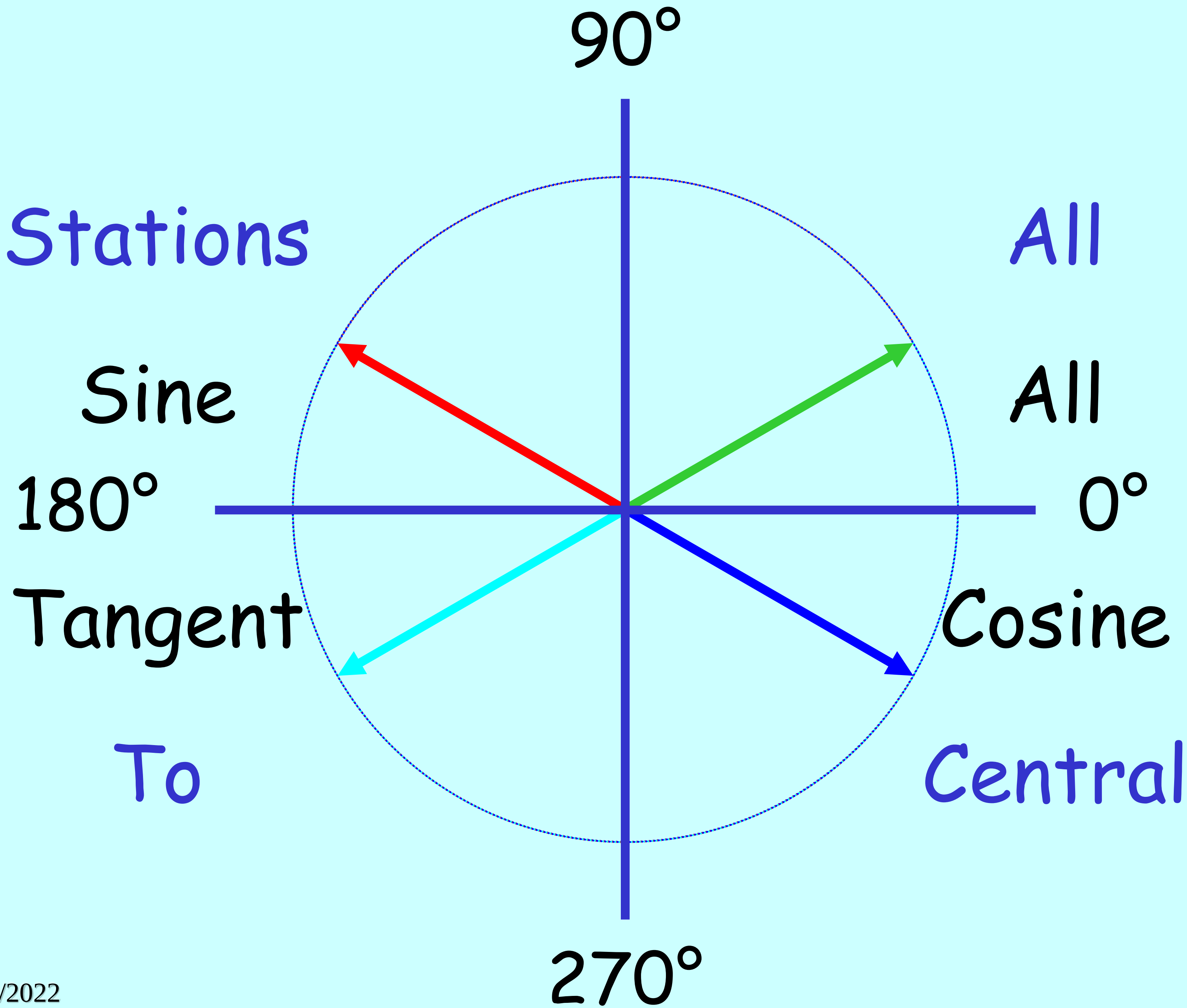


$$\sin(-30) = -0.5$$

$$\cos(-30) = 0.866$$

$$\tan(-30) = -0.577$$


90°
sin150 = 0.5
cos150 = –0.866
tan150 = –0.577
sin30 = 0.5
cos30 = 0.866
tan30 = 0.577
Sine
All
180°
0°
Tangent
Cosine
sin210 = –0.5
cos210 = –0.866
tan210 = 0.577
sin330 = –0.5
cos330 = 0.866
tan330 = –0.577
270°


90°
Stations
All
Sine
All
180°
0°
Tangent
Cosine
To
Central
270°

AE (ac) Workbook Pg. 10 Example 4

Angle θ	Sine θ	Cosine θ	Tangent θ
10°	0.1736	0.9848	0.1763
27°	0.4540	0.8910	0.5095
22.5°	0.3827	0.9239	0.4142
32°	0.5299	0.8480	0.6249
47°	0.7314	0.6820	1.0724
57°	0.8387	0.5446	1.5399
63°	0.8910	0.4540	1.9626
69°	0.9336	0.3584	2.6051
101°	0.9816	- 0.1908	- 5.1446
146°	0.5592	- 0.8290	- 0.6745
154°	0.4384	- 0.8988	- 0.4877
163.7°	0.2807	- 0.9598	- 0.2924

Problems

and

Exercises

UEENEEG102A Solve problems in low voltage a.c. circuits

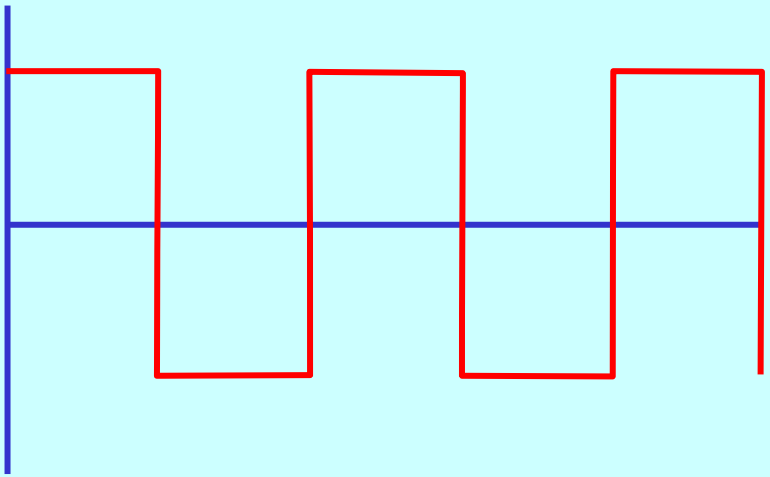
Alternating Quantities

Objectives:

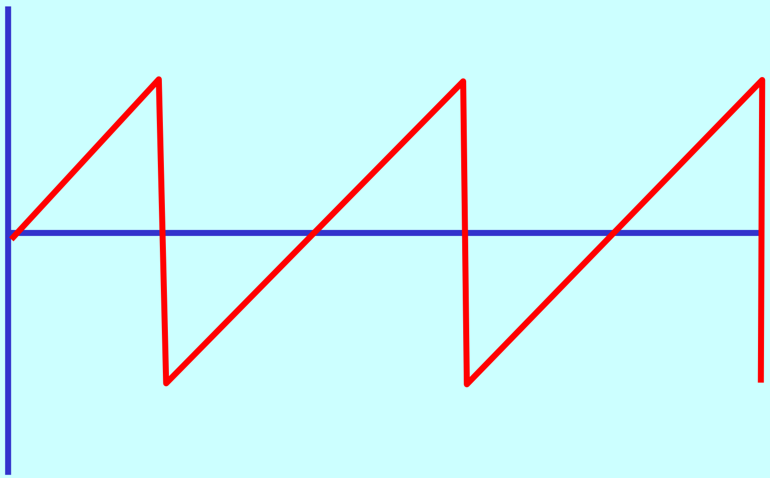
At the end of this lesson students should be able to:

1. Define the term *Alternating Current* and list advantages.
2. Identify basic waveshapes and list uses for each.
3. Sketch a simple SINE Wave and show Peak to Peak, Peak, and RMS values as well as the Period of the wave.
4. Calculate values associated with AC Waveforms.
5. Define Crest and Form Factors for an AC Waveform.

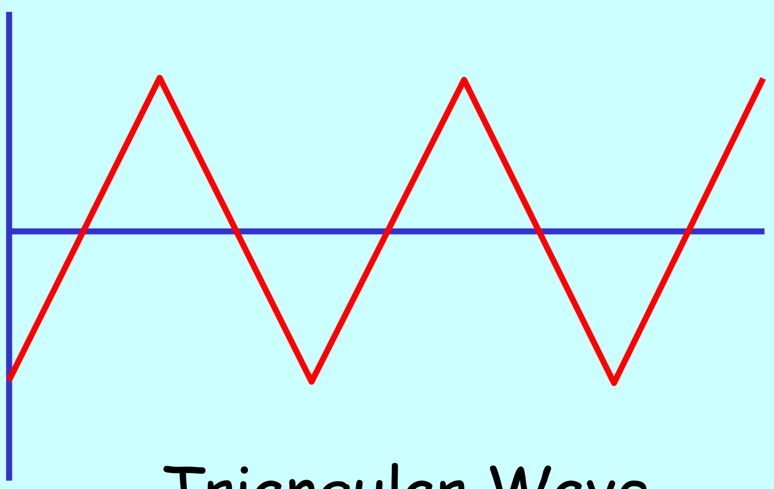
Common Wave Forms



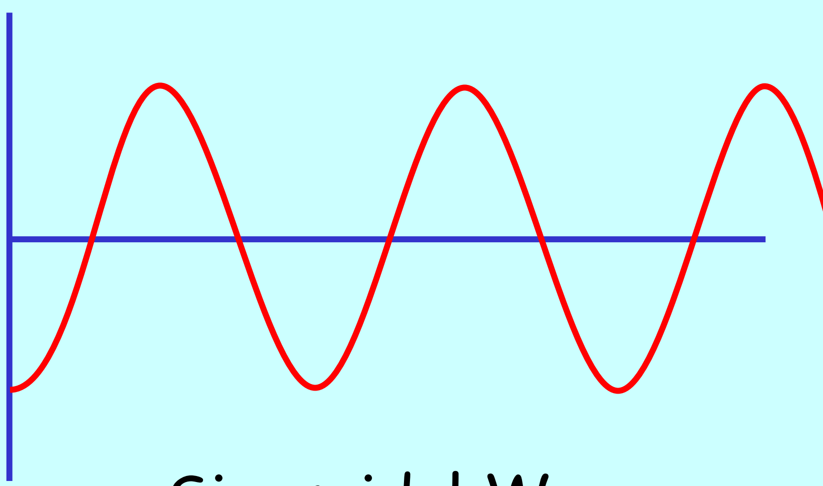
Square Wave



Sawtooth Wave

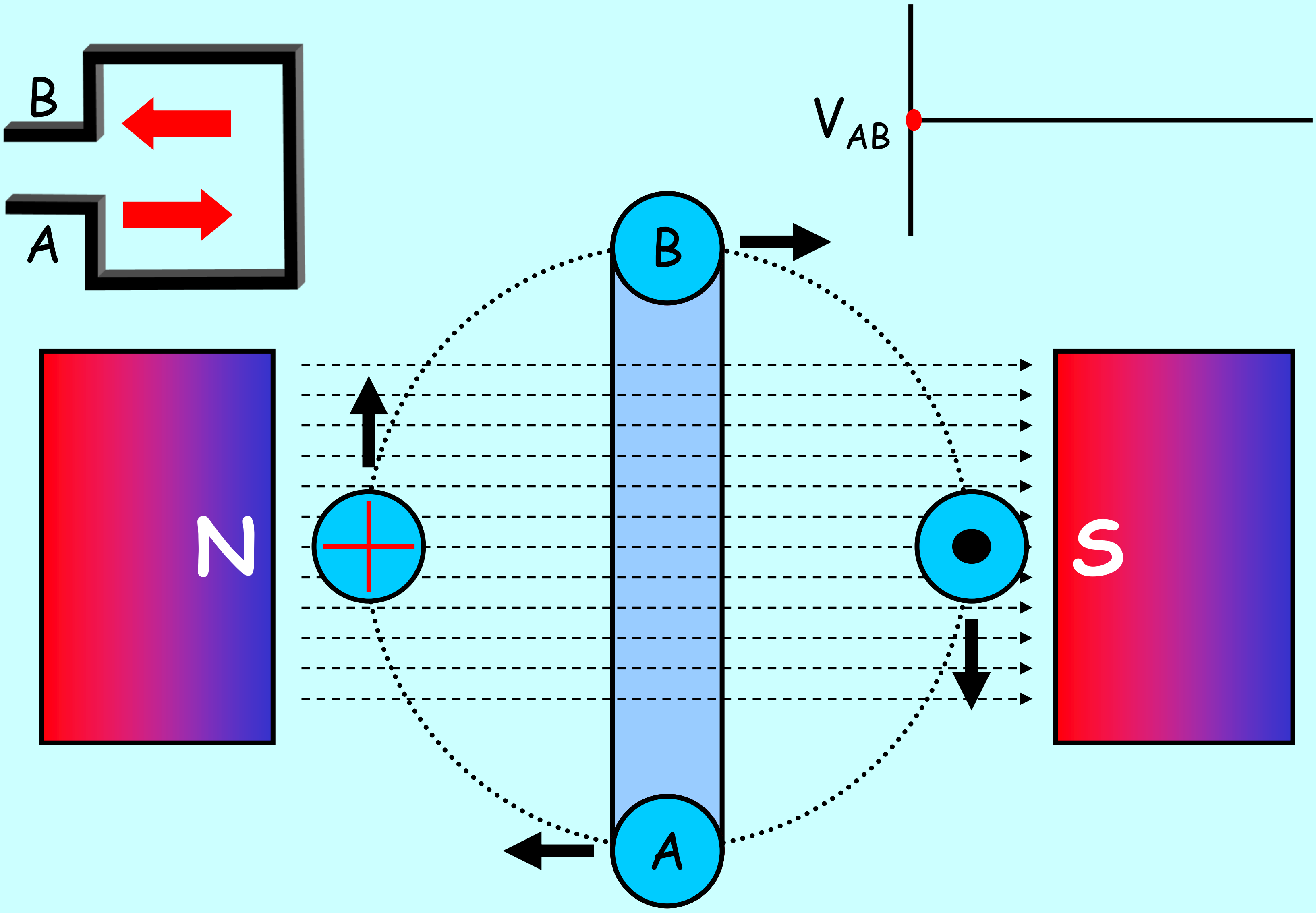


Triangular Wave

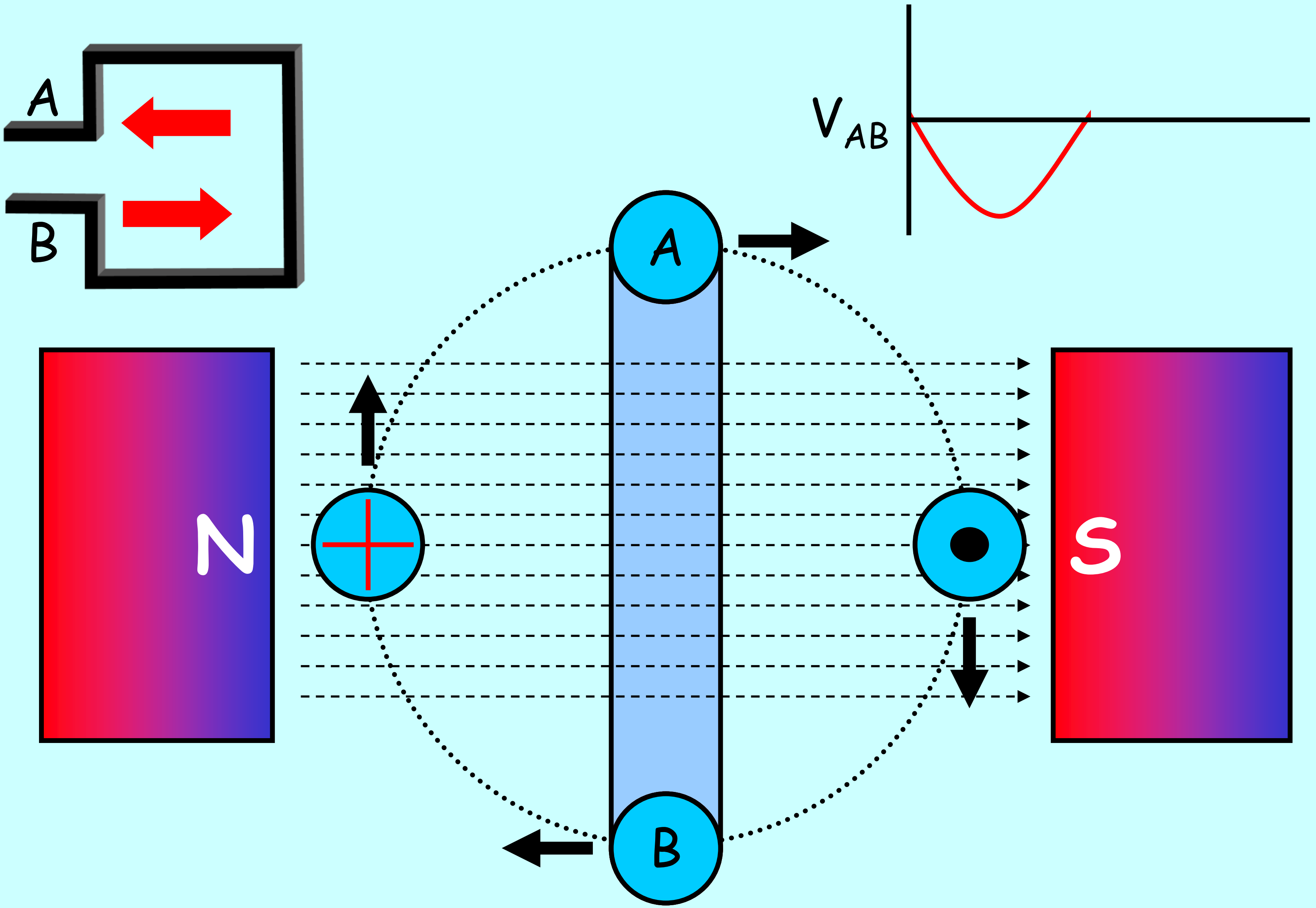


Sinusoidal Wave

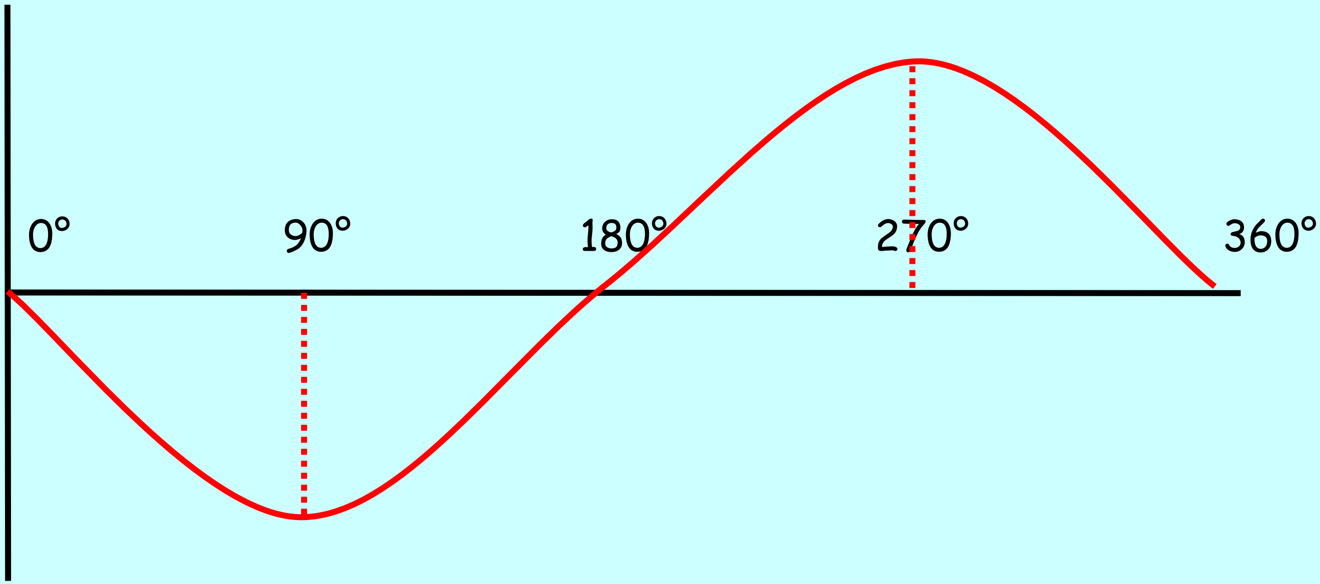
Generator Operation



Generator Operation



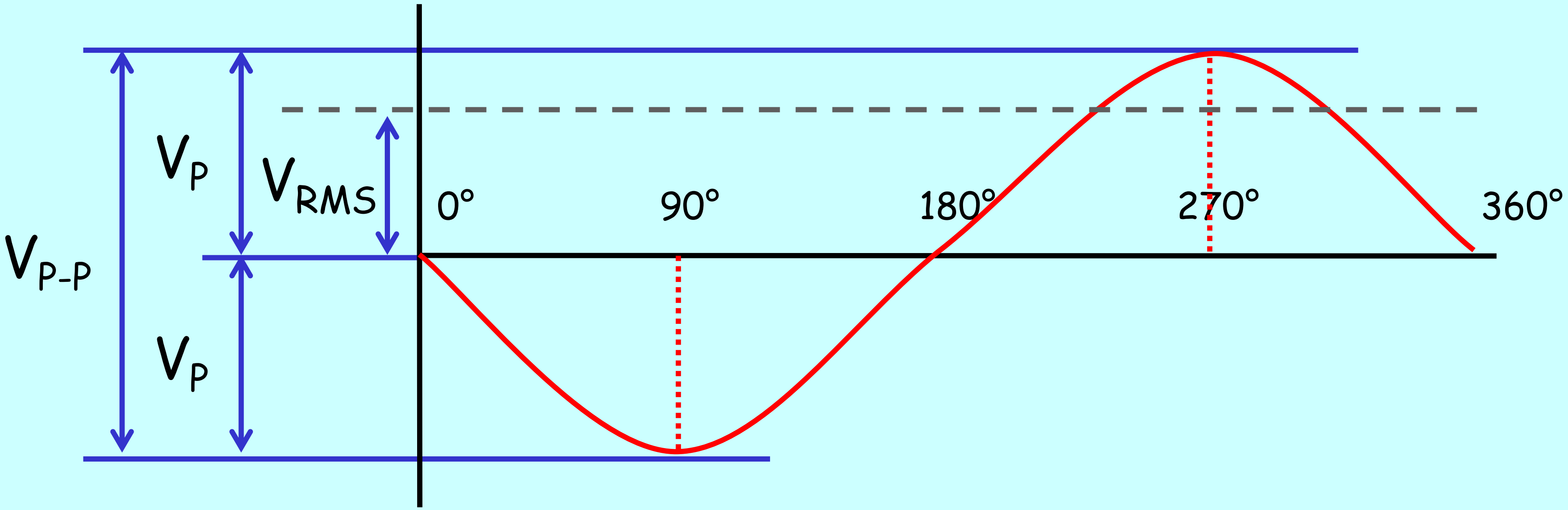
Alternating Quantities



The Output of a generator is ALTERNATING

The *Average* value of an AC Quantity is ZERO.

The Amplitude between the peaks of an AC Quantity is known as the Peak to Peak Value.

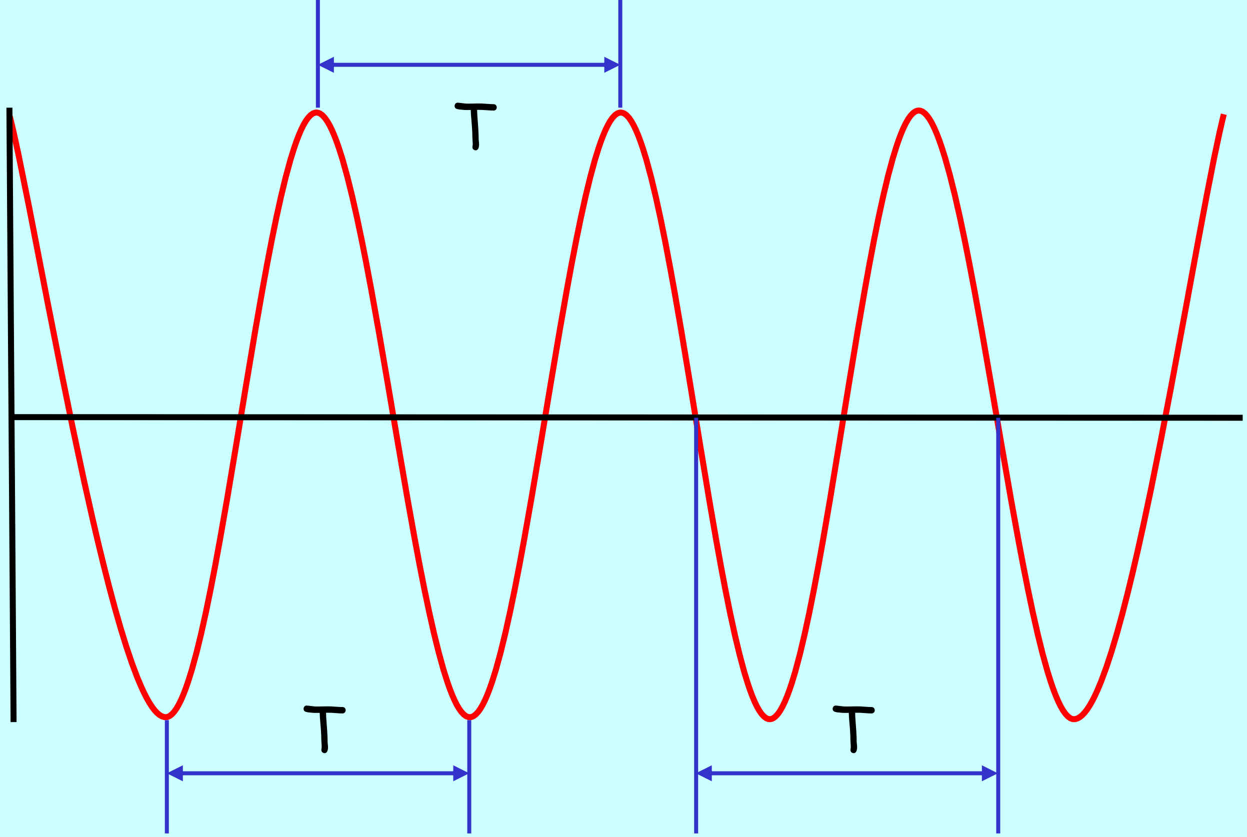


Half of the Peak to Peak Value is known as the Peak (or Maximum) Value.

$$V_{P-P} = 2V_P$$

The *Root Mean Square (RMS)* value of an AC Quantity is the equivalent DC Value that would do the same work.

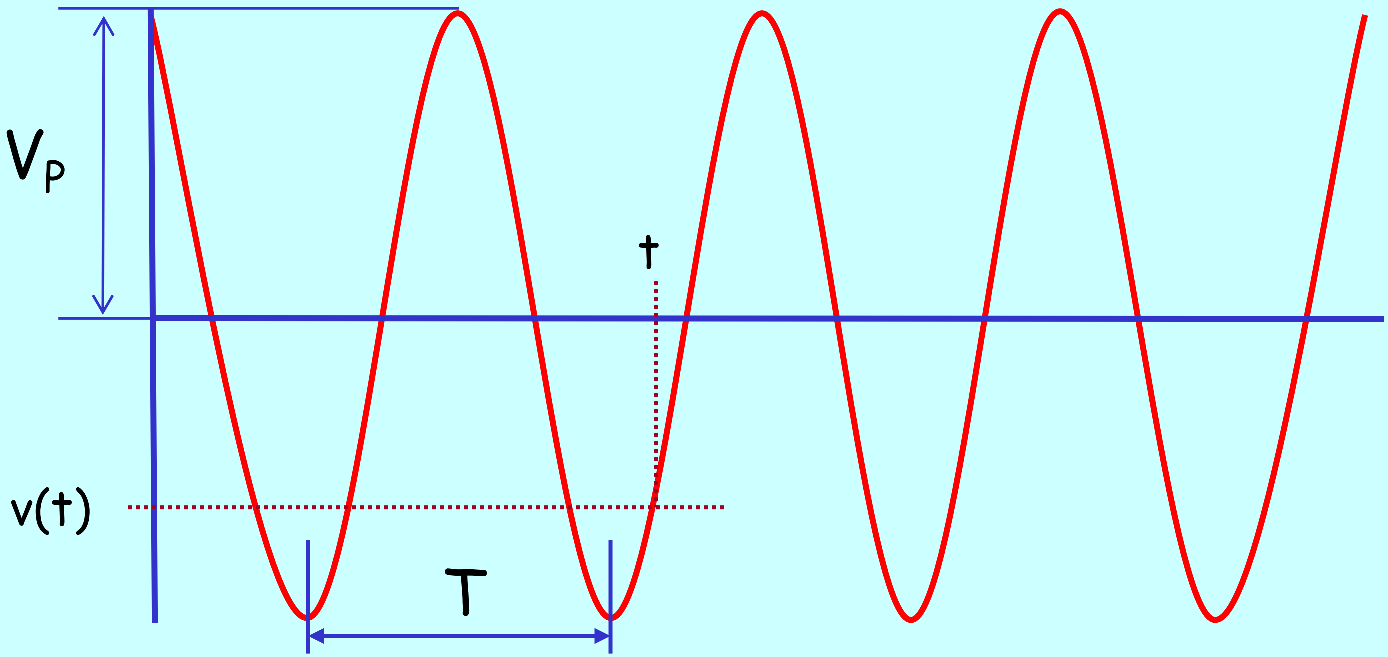
$$V_P = \sqrt{2}V_{RMS}$$



The time taken to complete one cycle (360°) of a waveform is known as the Period (T).

The Frequency (Hertz) of a waveform is the number of cycles per second (cps).

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$



Any Sinusoidal Waveform can be described mathematically by:

$$v(t) = V_P \sin(360ft) = V_P \sin\left(\frac{360t}{T}\right)$$

where: v(t) = The instantaneous value of the wave.

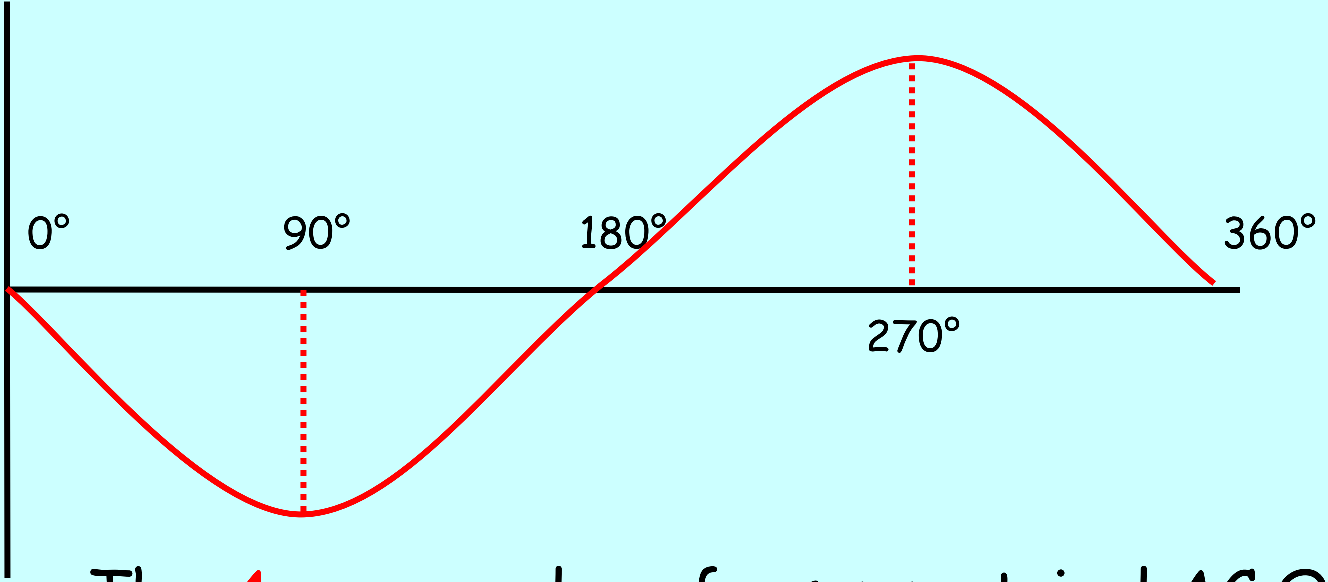
t = The instantaneous time.

V_P = The Peak value of the wave.

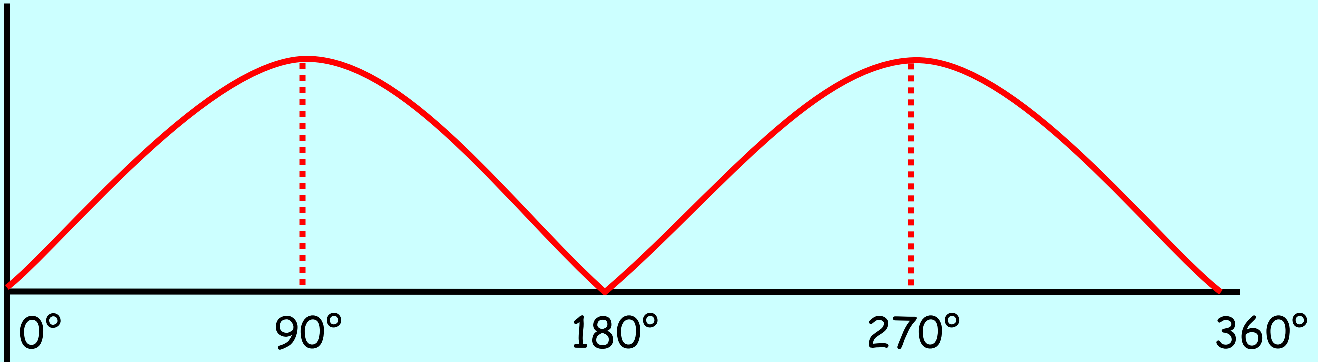
f = The frequency of the wave.

$$f = \frac{1}{T}$$

(360ft) = The angle at the instantaneous time t.

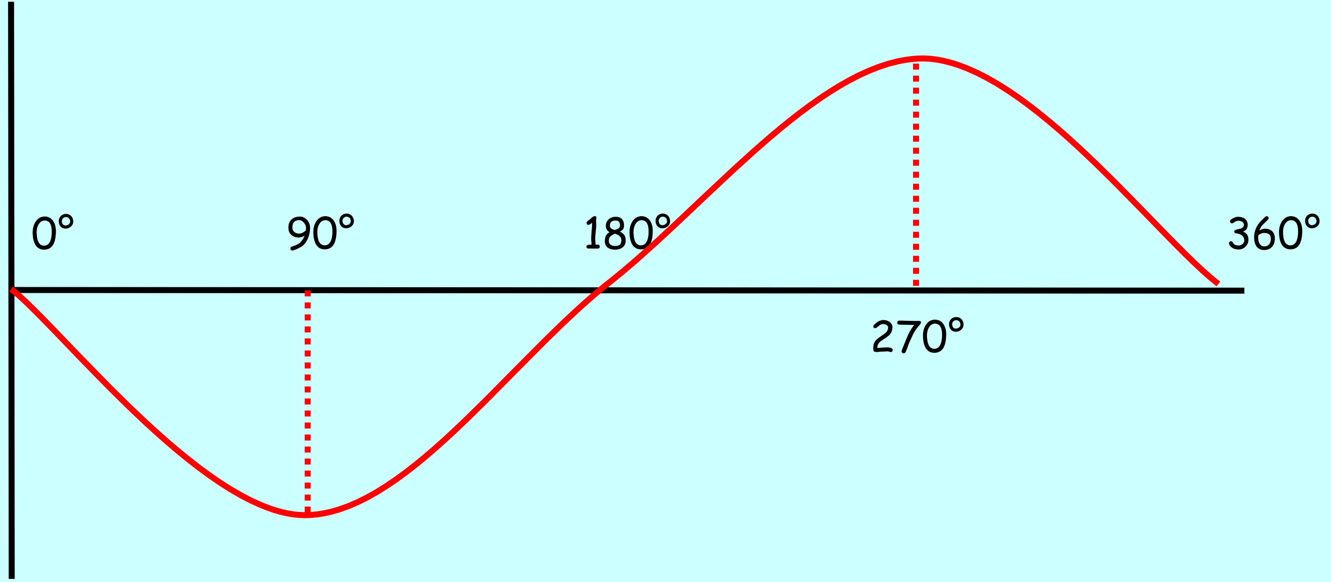


The *Average* value of a symmetrical AC Quantity is ZERO.



The *Average* value of a Rectified AC Quantity is

$$\frac{2}{\pi} = 0.637$$



The *form factor* of an AC Waveform is:

$$\text{form factor} = \frac{V_{RMS}}{V_{Avg}} = \frac{\pi}{2\sqrt{2}} = \frac{0.707}{0.637} = 1.11$$

The *crest factor* of an AC Waveform is:

$$\text{crest factor} = \frac{V_{Max}}{V_{RMS}} = \sqrt{2} = 1.414$$

Problems

and

Exercises

End of Lesson

Practical Exercises

Sinusoidal Waveforms
Pp. 53 - 57

L

Pg. 50 Ex. 1

$$v(t) = V_P \sin(360ft)$$

V_P = 340 V

A. (360ft) = 45°

$$v(t) = 340\sin(45)$$

v(t) = 240.42 V

B. (360ft) = 120°

$$v(t) = 340\sin(120)$$

v(t) = 294.45 V

C. (360ft) = 270°

$$v(t) = 340\sin(270)$$

v(t) = -340 V

Pg. 51 Ex. 2

$$i(t) = I_P \sin(360ft)$$

I_P = 20 A   f = 50 Hz

A. (360ft) = 360x50x0.006 = 108°

$$i(t) = 20\sin(108)$$

i(t) = 19.02 A

B. (360ft) = 360x50x0.009 = 162°

$$i(t) = 20\sin(162)$$

i(t) = 6.18 A

C. (360ft) = 360x50x0.015 = 270°

$$i(t) = 20\sin(270)$$

i(t) = -20 A

Pg. 52 Ex. 3

A. $V_{PP} = 400\ V$

B. $T = 0.010\ S = 10\ mS$

C. $f = 1/0.010 = 100\ Hz$

Pg. 51 Ex. 4

$$V_{RMS} = \frac{V_P}{\sqrt{2}} = \frac{340}{\sqrt{2}}$$

$V_{RMS} = 240\ V$

Pg. 51 Ex. 5

$$I_P = \sqrt{2} I_{RMS} = 10\sqrt{2}$$

$I_P = 14.14\ A$

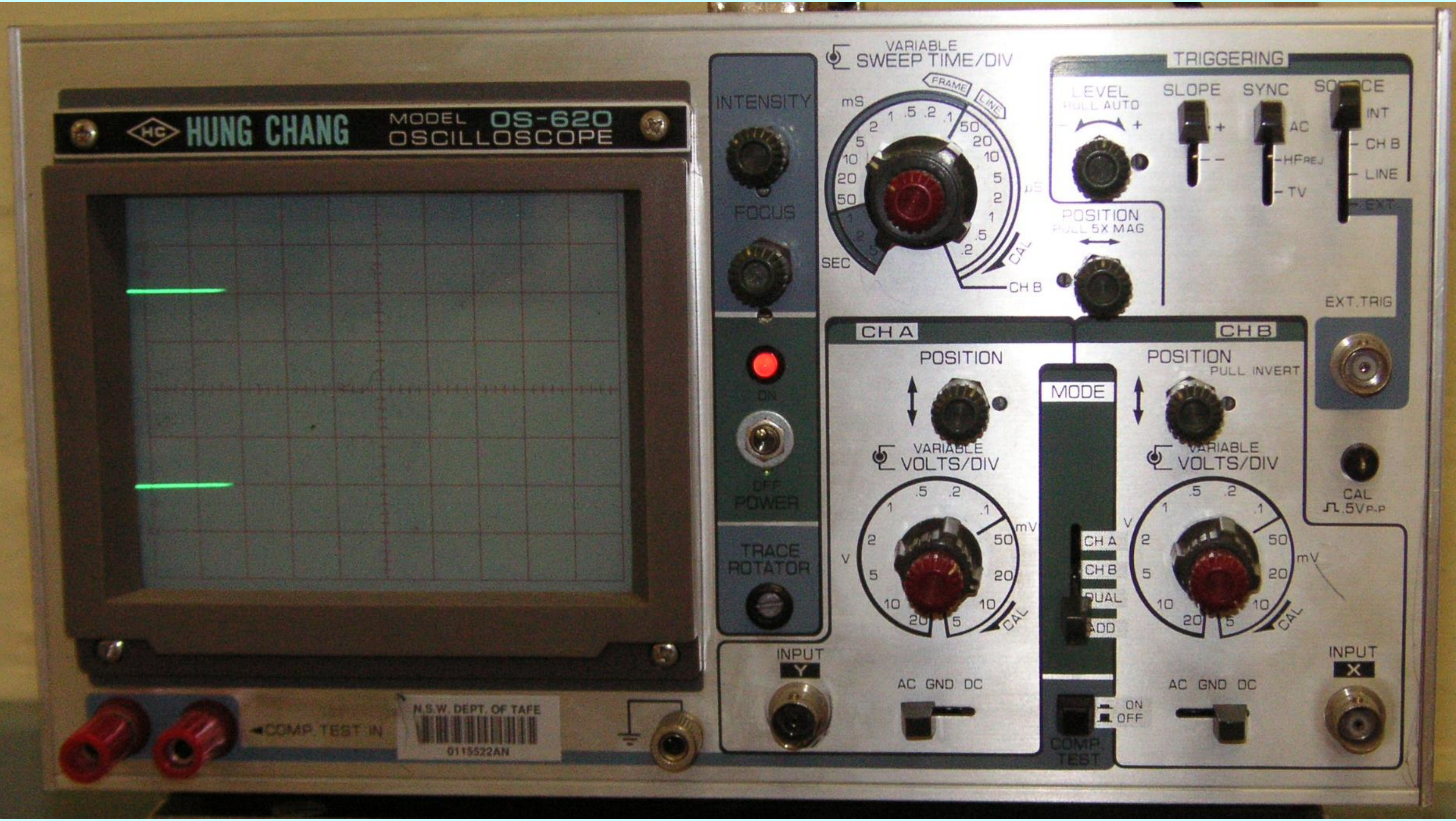
UEENEEG102A
Solve problems in low voltage a.c. circuits

The CRO

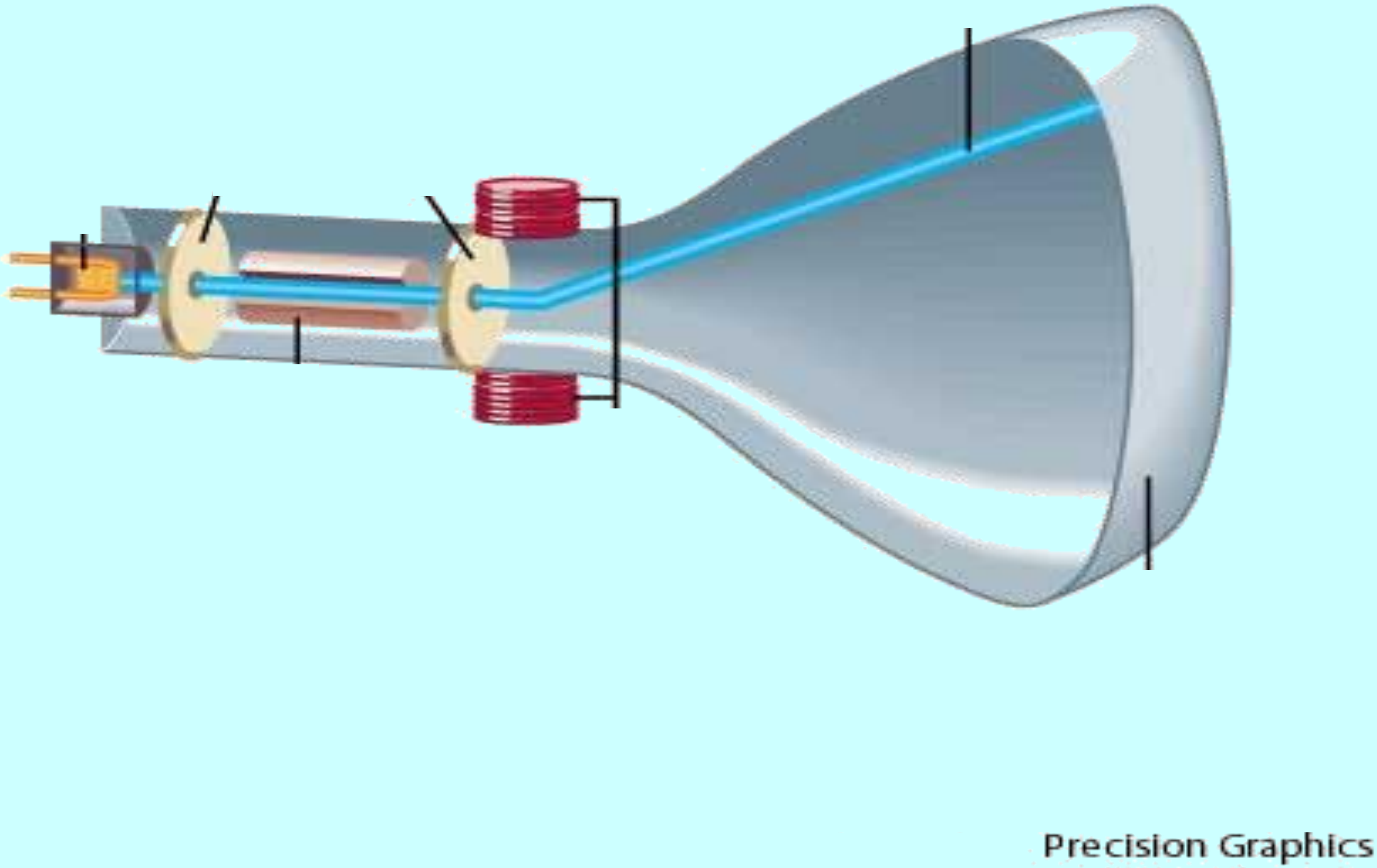
Objectives:

At the end of this lesson students should be able to:

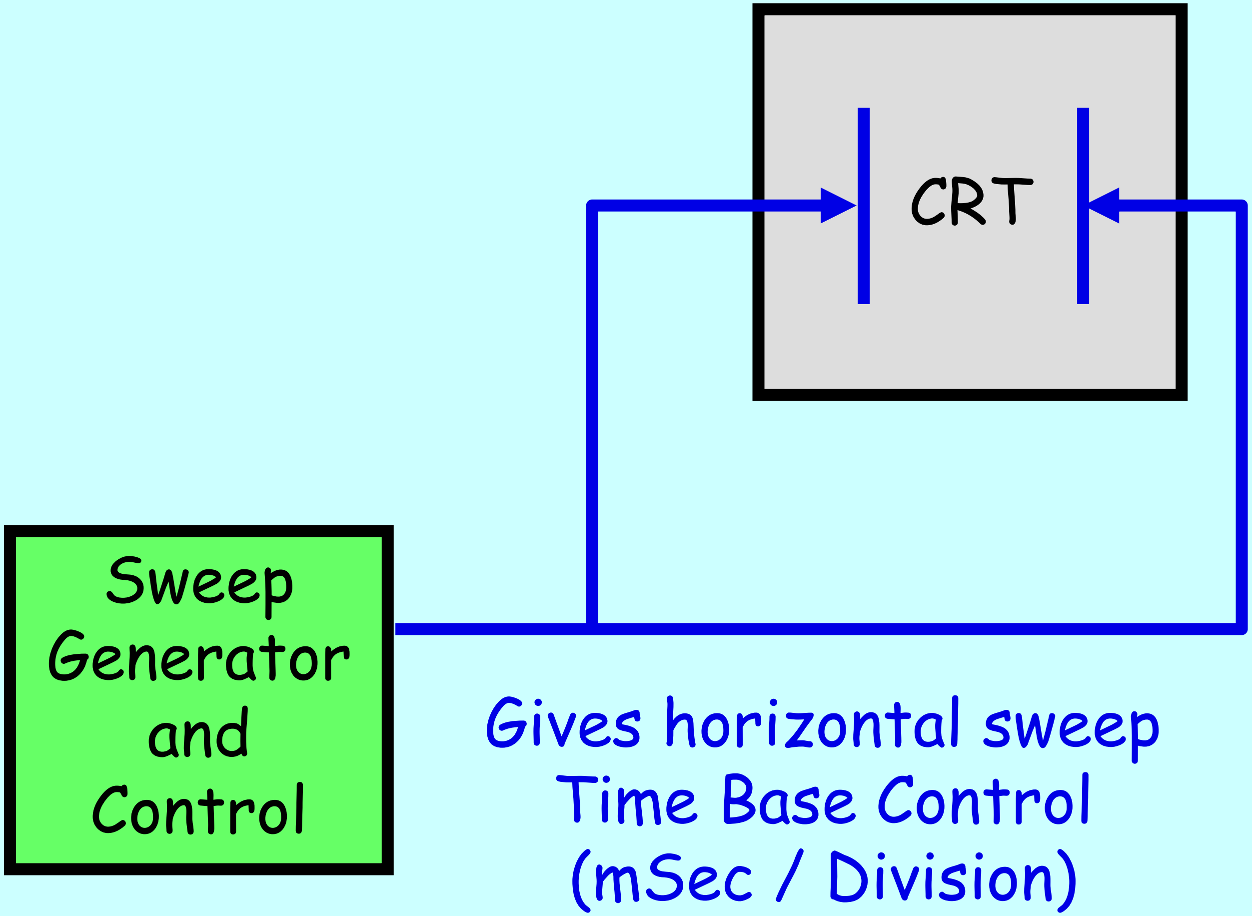
1. Correctly adjust the controls of a basic CRO.
2. Check the calibration of a basic CRO.
3. Measure DC and AC Voltages on a basic CRO.
4. Use a CRO to determine the phase difference between two sinewaves.


HUNG CHANG
MODEL OS-620
OSCILLOSCOPE
INTENSITY
FOCUS
ON
OFF
POWER
TRACE ROTATOR
VARIABLE
SWEEP TIME/DIV
FRAME
LINE
mS
SEC
μS
CAL
CH B
TRIGGERING
LEVEL
PULL AUTO
SLOPE
SYNC
AC
HF REJ
TV
INT
CH B
LINE
EXT
POSITION
PULL 5X MAG
EXT. TRIG
CH A
POSITION
VARIABLE
VOLTS/DIV
V
mV
MODE
CH A
CH B
DUAL
ADD
CH B
POSITION
PULL INVERT
VARIABLE
VOLTS/DIV
CAL
.5V P-P
INPUT
Y
AC GND DC
ON
OFF
COMP. TEST
AC GND DC
INPUT
X
COMP. TEST IN
N.S.W. DEPT. OF TAFE
0115522AN

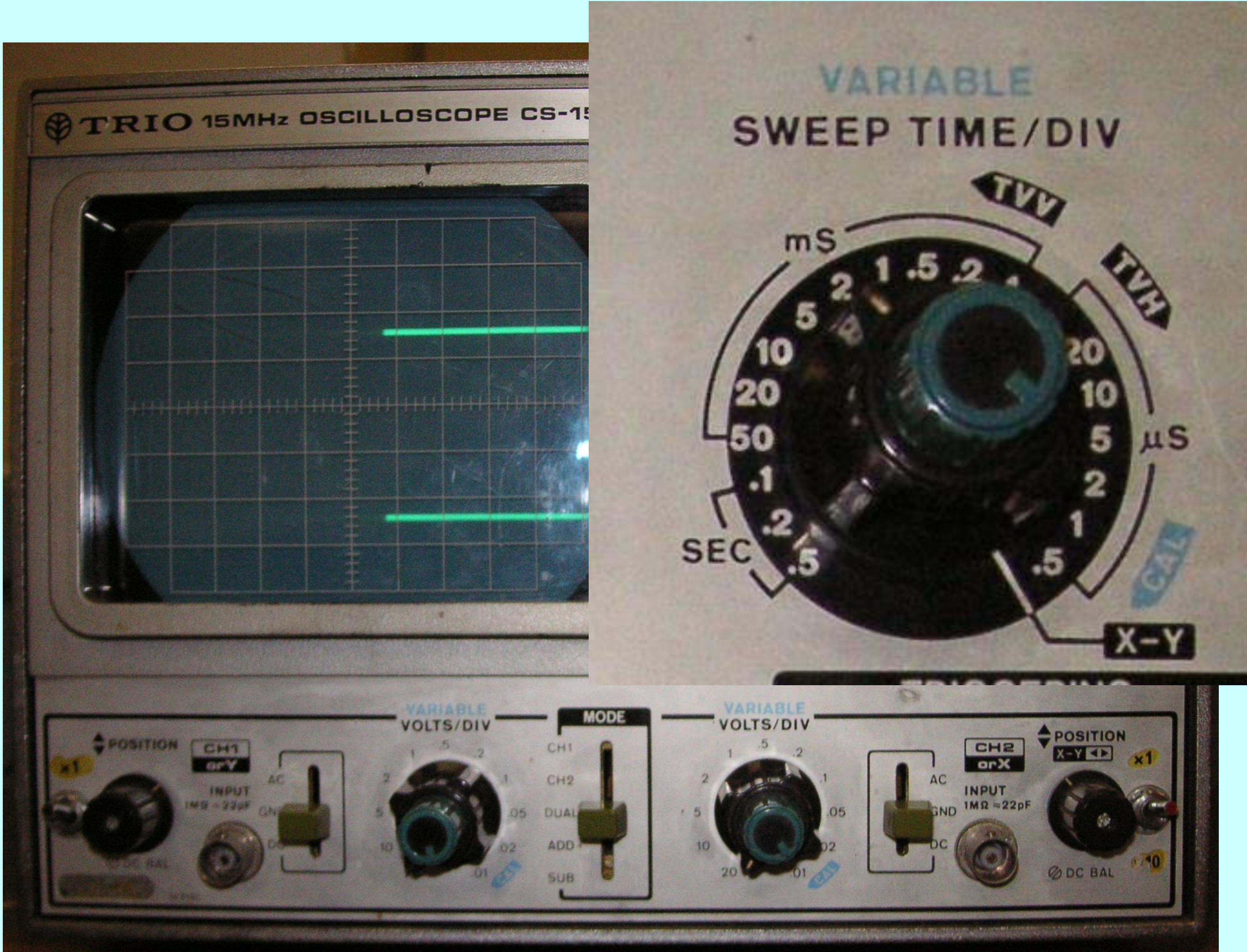

TRIO 15MHz OSCILLOSCOPE CS-1560A
POWER
ILLUM
OFF
VARIABLE
SWEEP TIME/DIV
CAL
1Vp-p
POSITION
PULL X5 MAG
INTENSITY
TRIGGERING
LEVEL
PULL AUTO
ASTIG
SYNC
NOR
TV
FOCUS
SOURCE
CH1 CH2 EXT
EXT. TRIG
VARIABLE
VOLTS/DIV
MODE
CH1
CH2
DUAL
ADD
SUB
CH1
or Y
CH2
or X
INPUT
1MΩ ≈22pF
AC
GND
DC
POSITION
X-Y
DC BAL

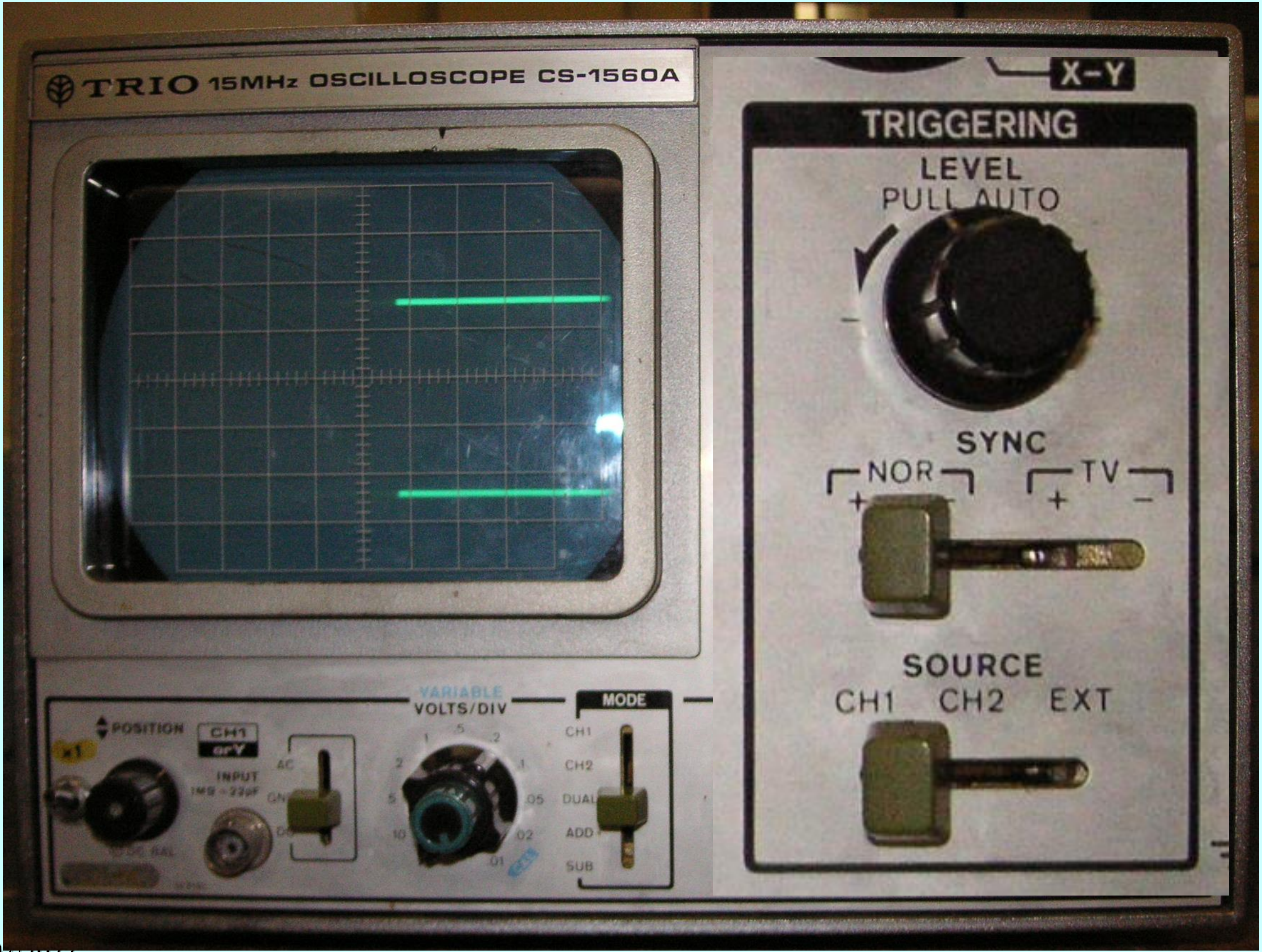

Precision Graphics

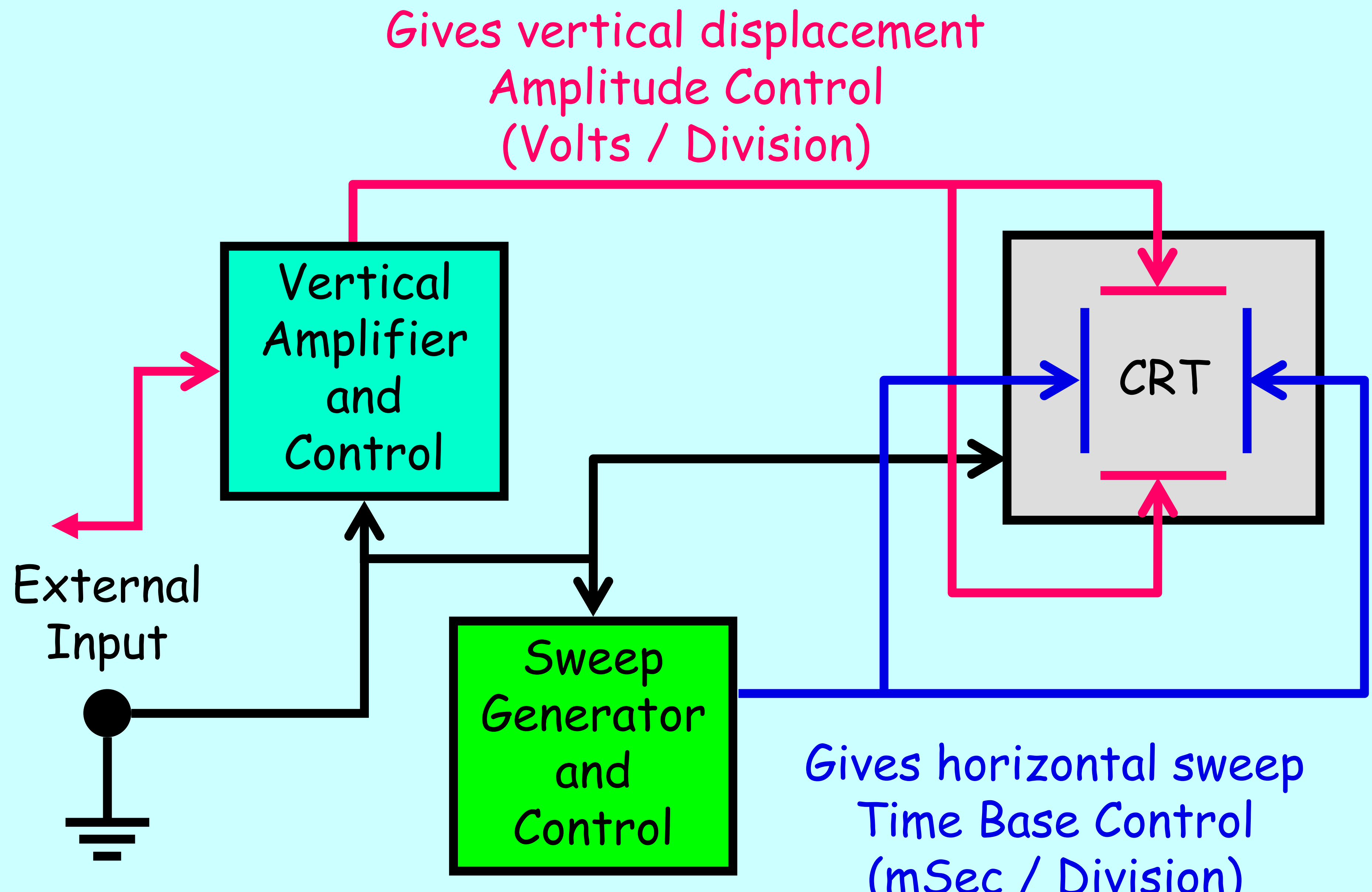

TRIO 15MHz OSCILLOSCOPE CS-1560A
POWER
ILLUM
OFF
INTENSITY
FOCUS



Typical Block Diagram of a Oscilloscope

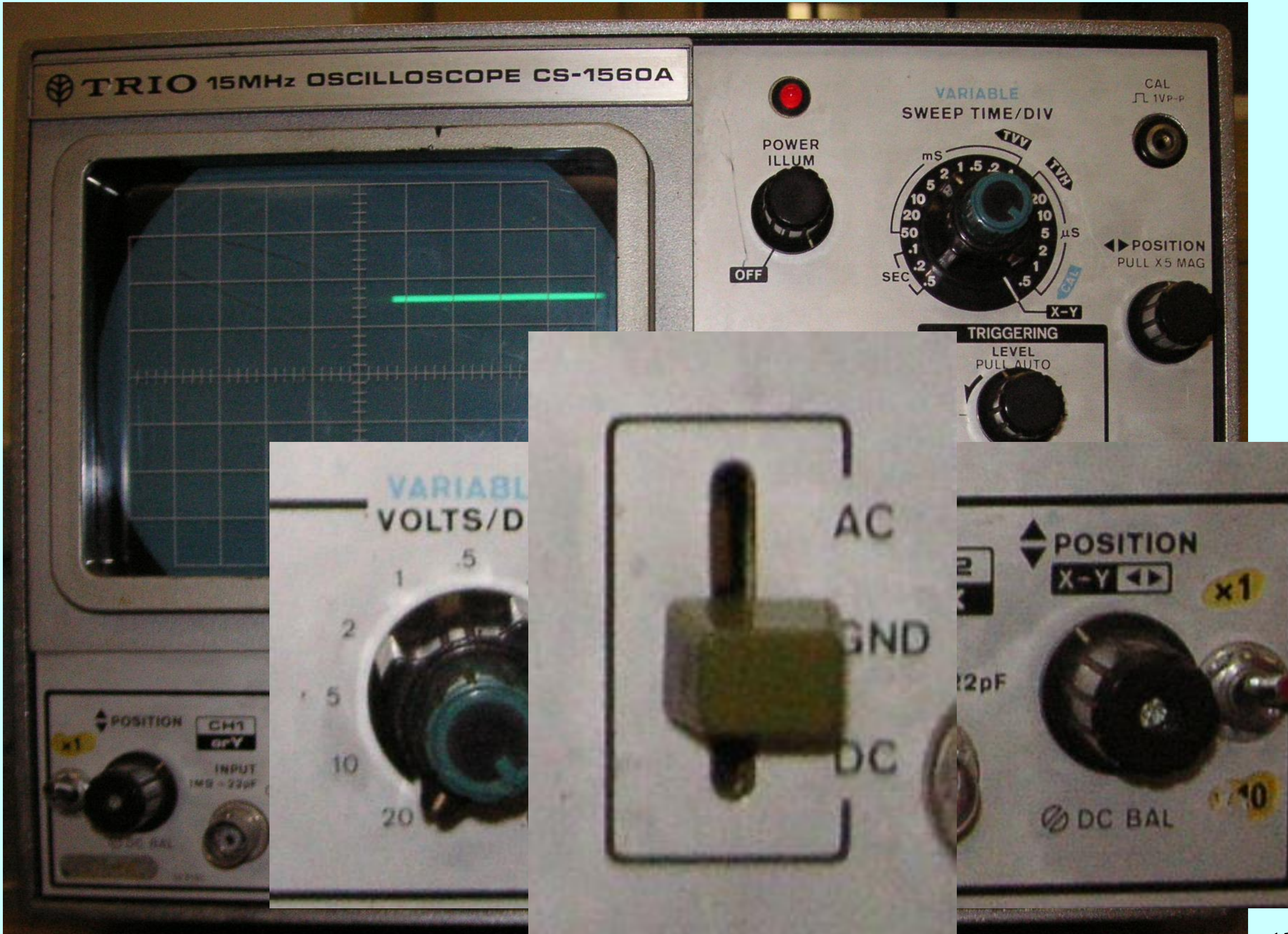

TRIO 15MHz OSCILLOSCOPE CS-15
VARIABLE
SWEEP TIME/DIV
TVV
TVH
mS
µS
SEC
CAL
X-Y
POSITION
CH1 or Y
INPUT
VARIABLE
VOLTS/DIV
MODE
CH1
CH2
DUAL
ADD
SUB
VARIABLE
VOLTS/DIV
AC
GND
DC
CH2 or X
INPUT
1MΩ ≈22pF
POSITION
X-Y
×1
×10
DC BAL


TRIO 15MHz OSCILLOSCOPE CS-1560A
X-Y
TRIGGERING
LEVEL
PULL AUTO
SYNC
NOR
TV
SOURCE
CH1 CH2 EXT
VOLTS/DIV
MODE
CH1
CH2
DUAL
ADD
SUB
POSITION
INPUT



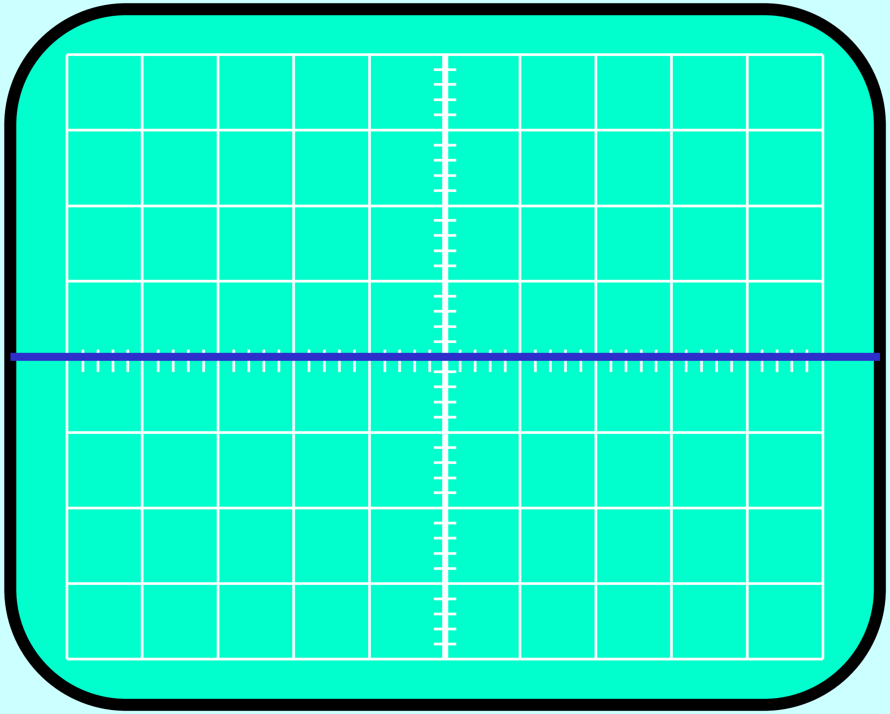
Typical Block Diagram of a Oscilloscope


TRIO 15MHz OSCILLOSCOPE CS-1560A
POWER ILLUM
VARIABLE
SWEEP TIME/DIV
CAL
1Vp-p
POSITION
PULL X5 MAG
X-Y
TRIGGERING
LEVEL
PULL AUTO
SYNC
NOR
TV
ASTIG
EXT. TRIG
VARIABLE
VOLTS/DIV
CAL
CH2
or X
POSITION
X-Y
x1
INPUT
1MΩ ≈22pF
AC
GND
DC
DC BAL


TRIO 15MHz OSCILLOSCOPE CS-1560A
POWER ILLUM
OFF
VARIABLE
SWEEP TIME/DIV
mS
TVV
TVH
SEC
µS
CAL
X-Y
CAL
1Vp-p
POSITION
PULL X5 MAG
TRIGGERING
LEVEL
PULL AUTO
VARIABL
VOLTS/D
AC
GND
DC
POSITION
X-Y
×1
DC BAL
CH1
INPUT

TRIO 15MHz OS
MODE
CH1
CH2
DUAL
ADD
SUB
VARIABLE
SWEEP TIME/DIV
CAL
1Vp-p
mS
µS
X-Y
POSITION
PULL X5 MAG
TRIGGERING
LEVEL
PULL AUTO
SYNC
NOR
TV
SOURCE
CH1 CH2 EXT
EXT. TRIG
CH1 or Y
CH2 or X
INPUT
1MΩ ≈22pF
AC
GND
DC
DC BAL
×1

Exercise 1 Pg. 24



Vertical Scale
10 V/div

Horizontal Scale
2 mS/div

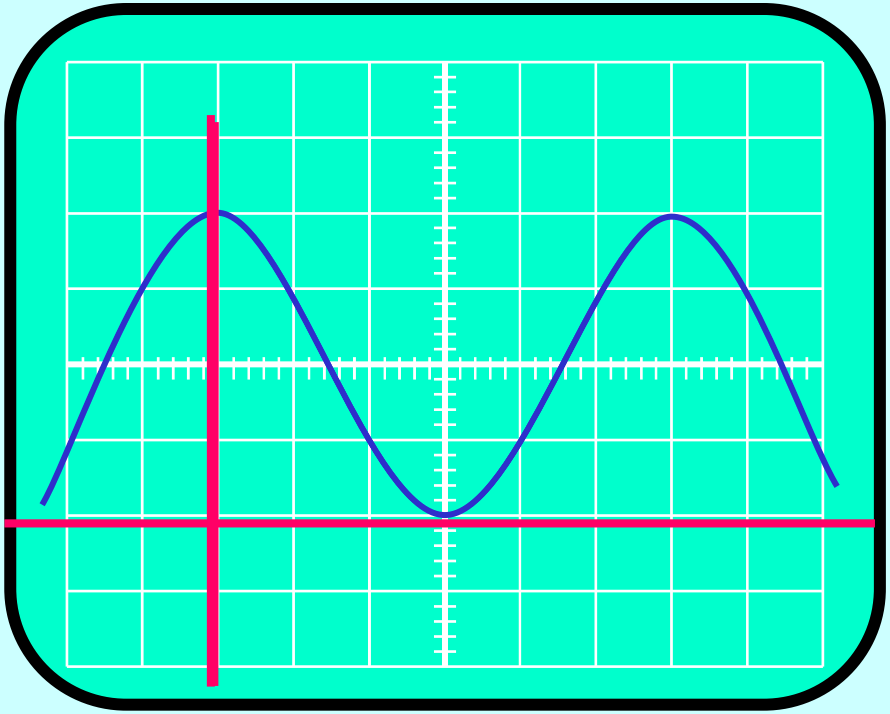
Trace centred at
ZERO when Earthed.

A. The signal is DC

B. Change the Vertical Scale to 20 V/div and leave the Time Base at 2 mS/div.

C. Change the Vertical Scale back to 10 V/div and change the Time Base to 0.5 mS/div.

Exercise 2 Pg. 25



Vertical Scale
1 V/div

Horizontal Scale
0.1 mS/div

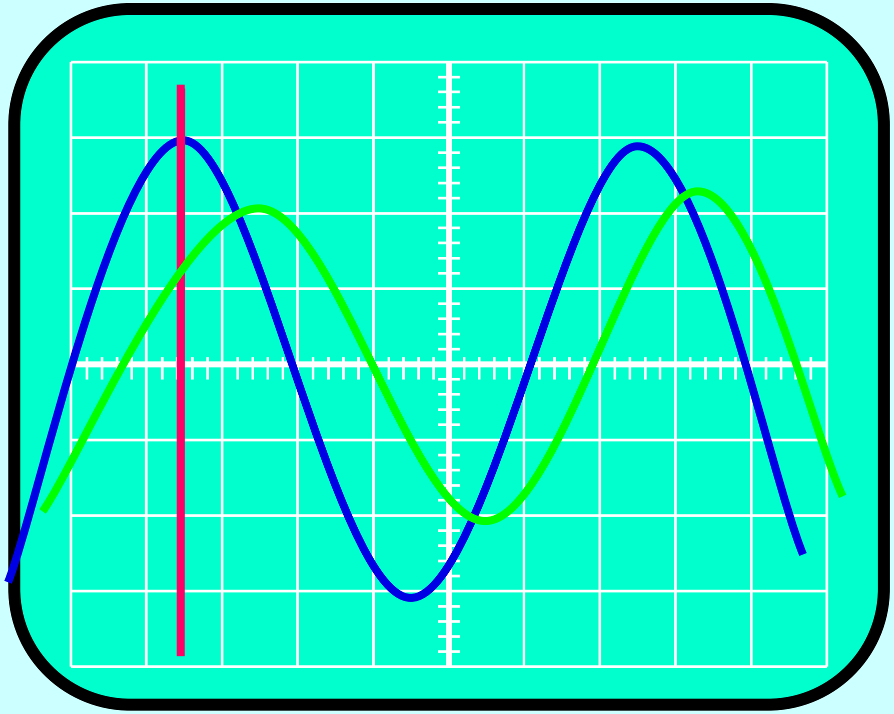
A. $V_{Peak\ to\ Peak}$ = 4 x 1 = 4 V

B. V_{Peak} = 4/2 = 2 V

C. T = 6 x 0.1 = 0.6 mS

D. f = 1 / T = 1 / 0.6 mS = 1,666 Hz

Exercise 3 Pg. 26



Vertical Scale
Ch A = 1 V/div

Ch B = 0.5 V/div

Horizontal Scale
0.1 mS/div

A. $V_{A(Peak)}$ = (6.2 x 1)/2 = 3.1 V

B. $V_{B(Peak)}$ = (4.2 x 0.5)/2 = 1.05 V

C. T = 6 x 0.1 = 0.6 mS

D. f = 1 / T = 1 / 0.6 mS = 1,666 Hz

E. Φ_T = 1 x 0.1 mS = 0.1 mS

F. $\Phi°$ = 360 x (0.1/0.6) = 60°

End of Lesson

Practical Exercises

Pp. 27 - 37

2.1, 2.2, 2.3 & 2.4

UEENEEG102A
Solve problems in low voltage a.c. circuits

Vectors and Phasors

Objectives:

At the end of this lesson students should be able to:

1. Define the term *Vector*.
2. Define the term *Phasor*.
3. Draw the phasor representation of AC Sinusoidal waveforms.
4. Identify leading and lagging phase angles.
5. Draw and read phasor diagrams.

What is a ***Vector***?

A quantity that has MAGNITUDE and DIRECTION.

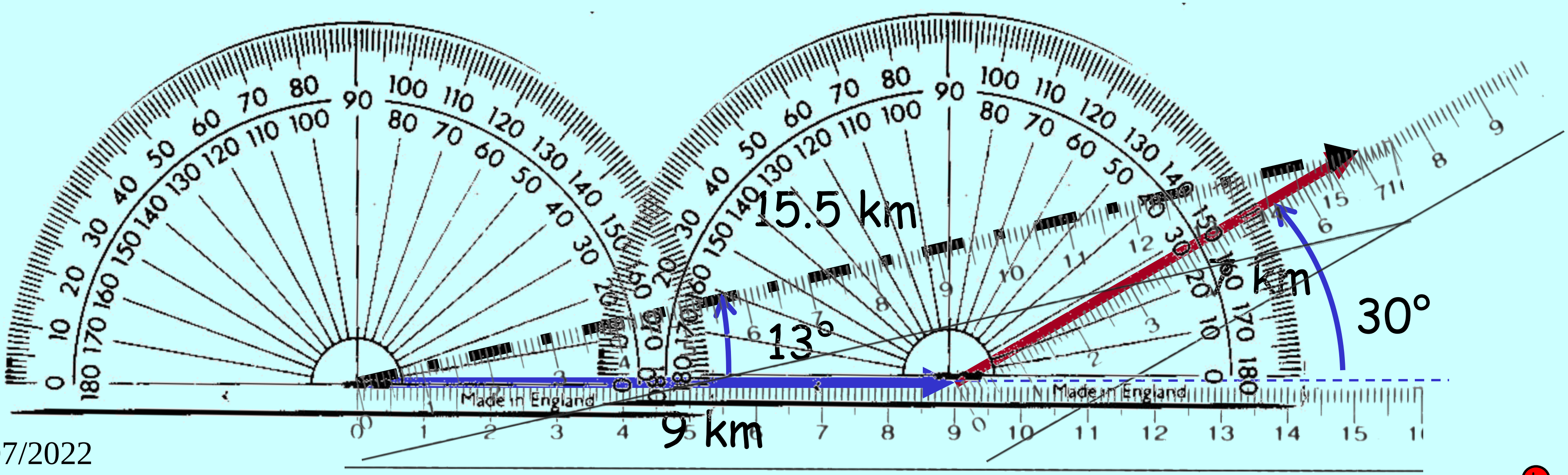
Examples: Displacement, Velocity, Acceleration, Force

A person travels due East for 9 km

and then travels 30° North of East for 7 km

What is their displacement from the original starting position?

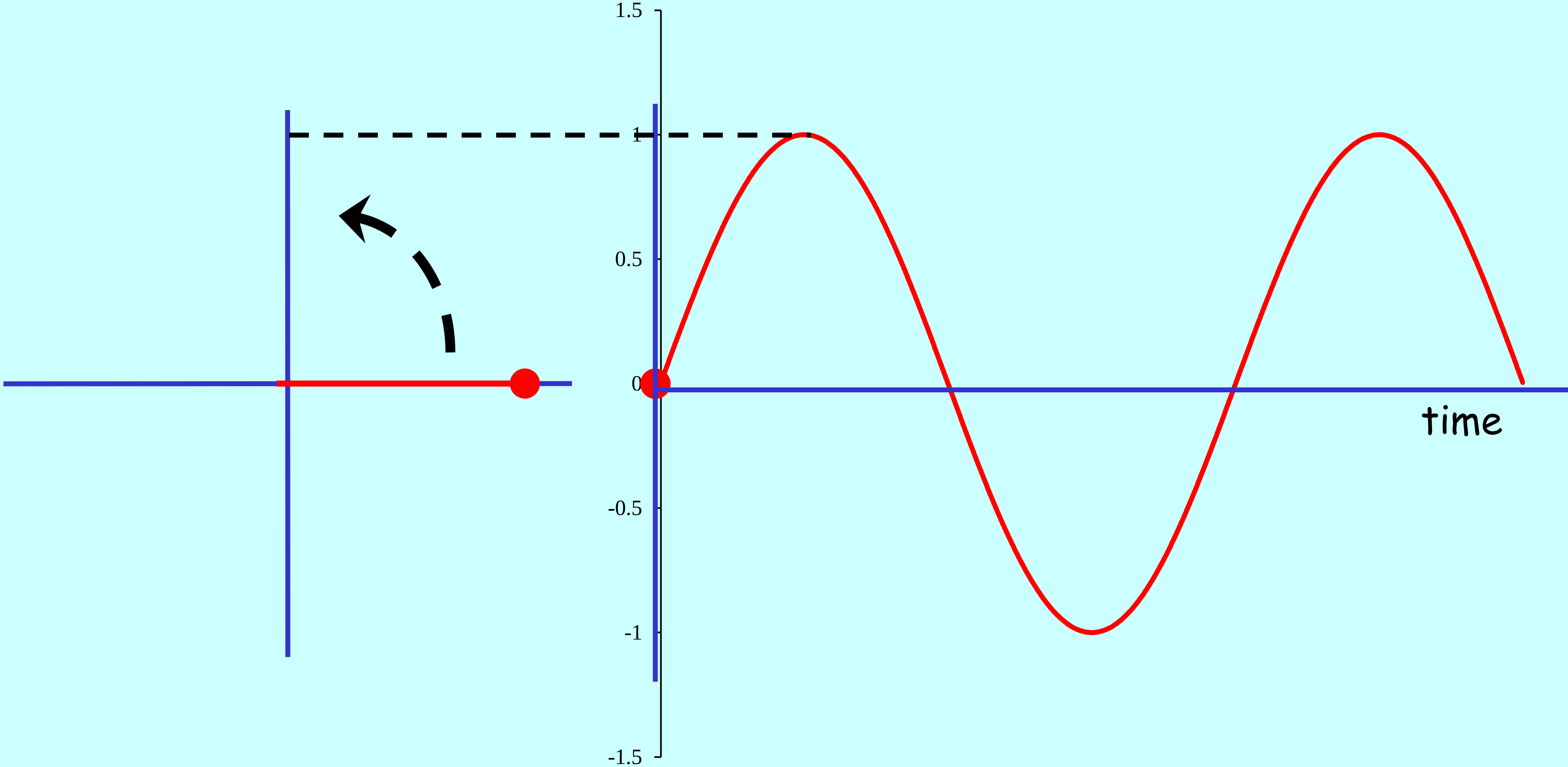
15.5 km ∠ 13°



What is a ***Phasor***?

A Vector representation of an electrical quantity.

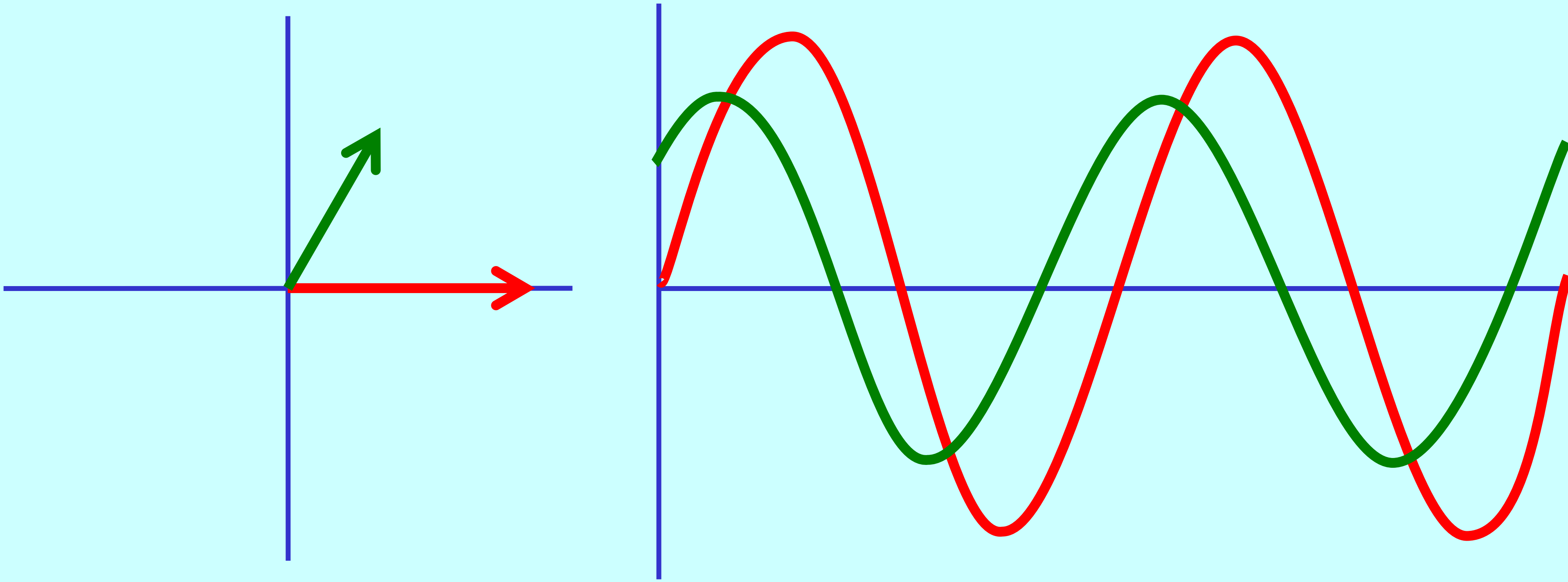
Specifically Sinusoidal quantities.



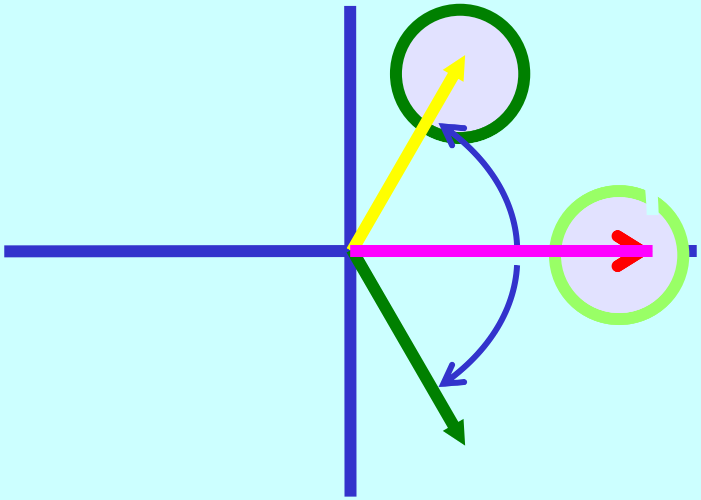
And if we have more than one quantity?

Simply add more Vectors with appropriate phase angles.

The only stipulations are that the quantities MUST be the same units (ie. V, A, Ω, etc.) and have the same frequency.



Conventions



The Reference Phasor is always at ZERO degrees.

Voltage Phasors are shown by an open arrow head.

Current Phasors are shown by a closed arrow head.

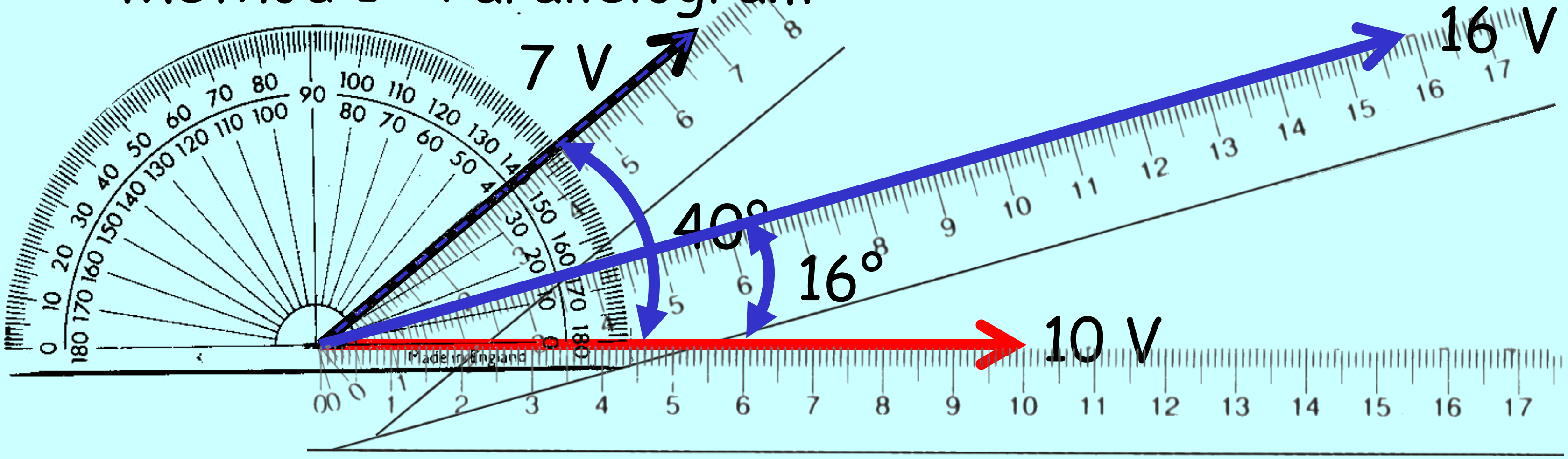
Phasors rotate in an Anti-Clockwise direction.

Phasor Angles are specified as either Leading or Lagging.

Phasor Magnitudes are generally specified in RMS values.

Phasor Addition

Method 1 - Parallelogram



Always start by drawing the Reference Phasor which will be at ZERO degrees.

Measure the Phase Angle of the Phasor to be added to the reference.

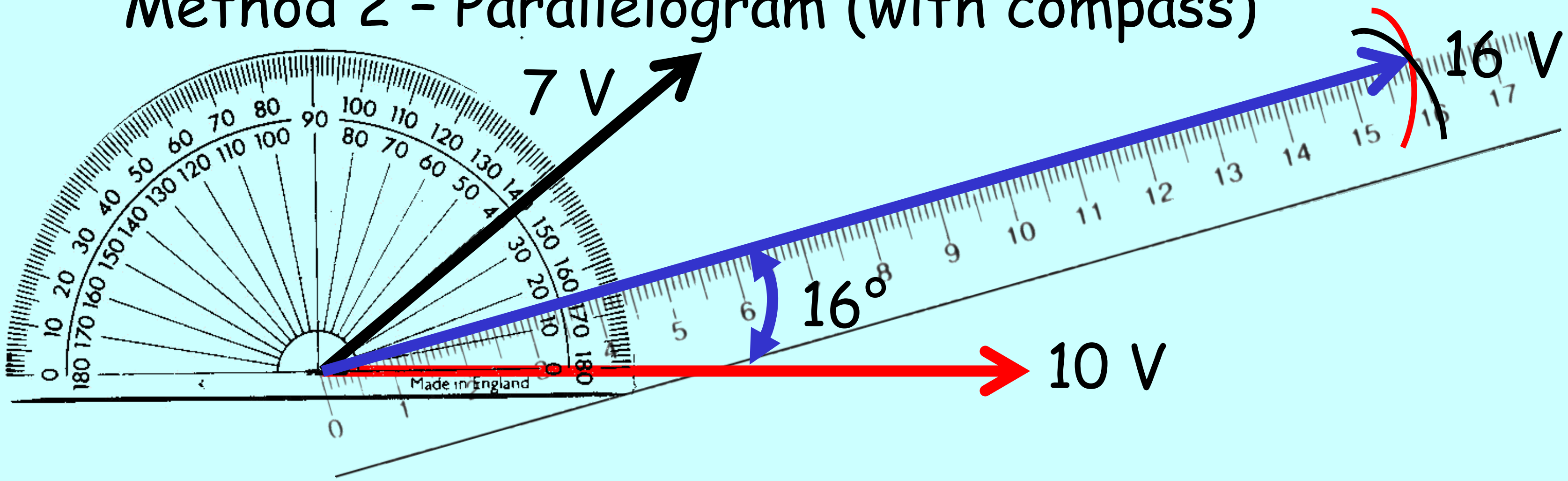
Draw in the second Phasor and complete the parallelogram.

Draw in the diagonal of the parallelogram. This is the resultant Phasor.

Measure the Magnitude and Phase Angle of the Resultant Phasor.

Phasor Addition

Method 2 – Parallelogram (with compass)



Always start by drawing the Reference Phasor which will be at ZERO degrees.

Measure the Phase Angle of the Phasor to be added to the reference.

Draw in the second Phasor.

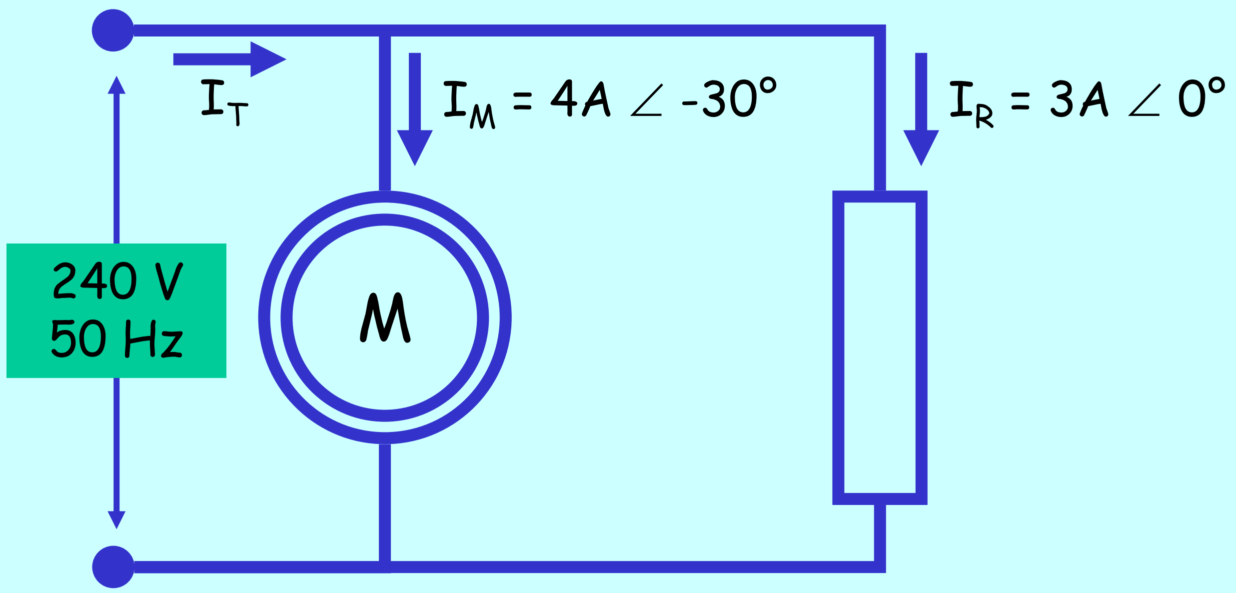
Measure the length of the reference phasor with a compass and draw an arc from the tip of the second phasor.

Measure the length of the second phasor with a compass and draw an arc from the tip of the reference phasor.

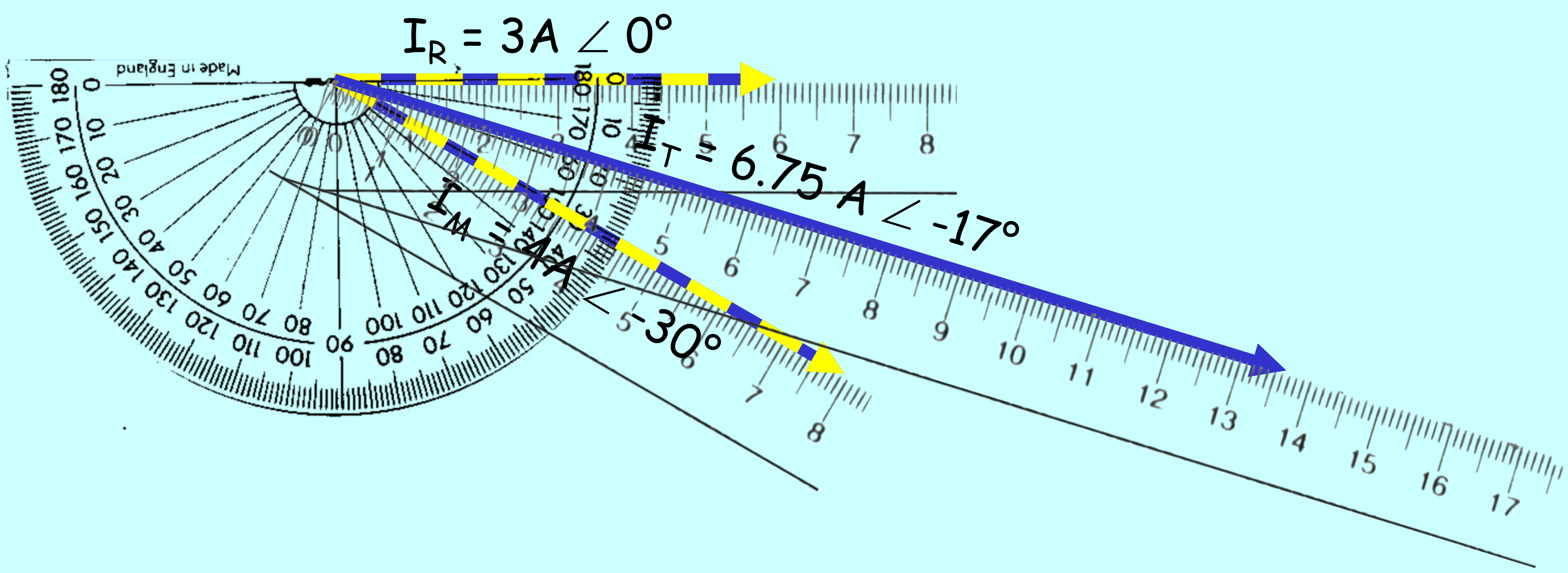
Draw in the resultant Phasor.

Measure the Magnitude and Phase Angle of the Resultant Phasor.

Pg. 67 Ex. 1

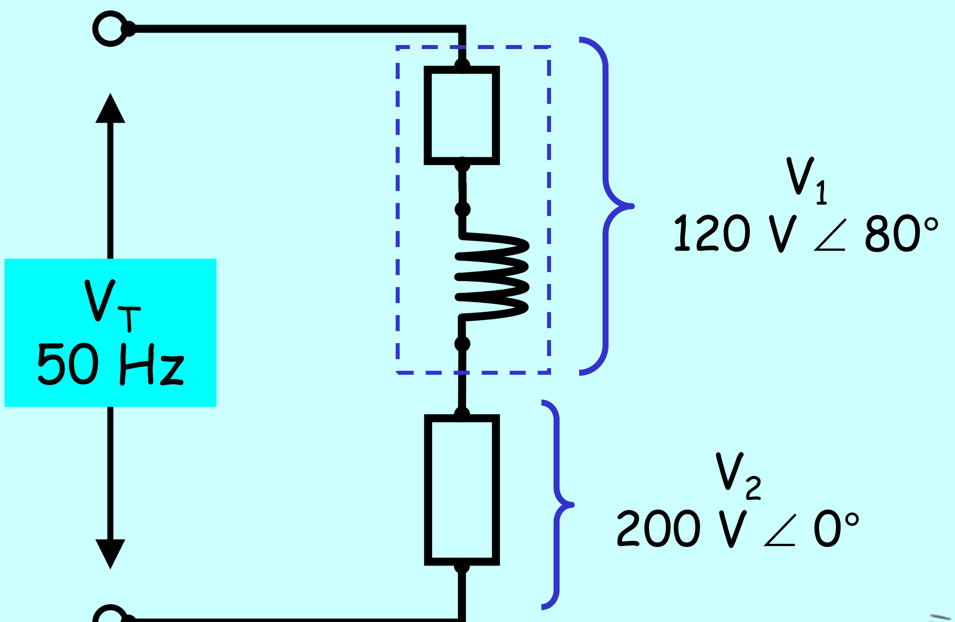


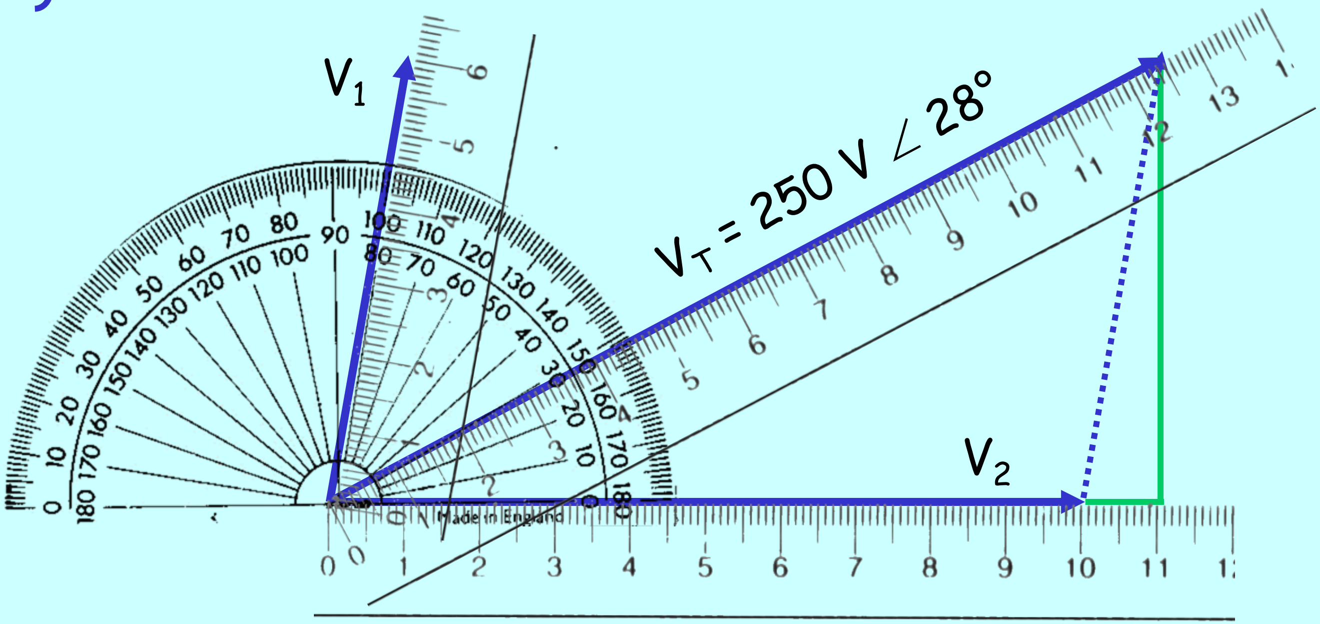
I_T
I_M = 4A ∠ -30°
I_R = 3A ∠ 0°
240 V
50 Hz
M



I_R = 3A ∠ 0°
I_T = 6.75 A ∠ -17°
I_M = 4A ∠ -30°
Made in England

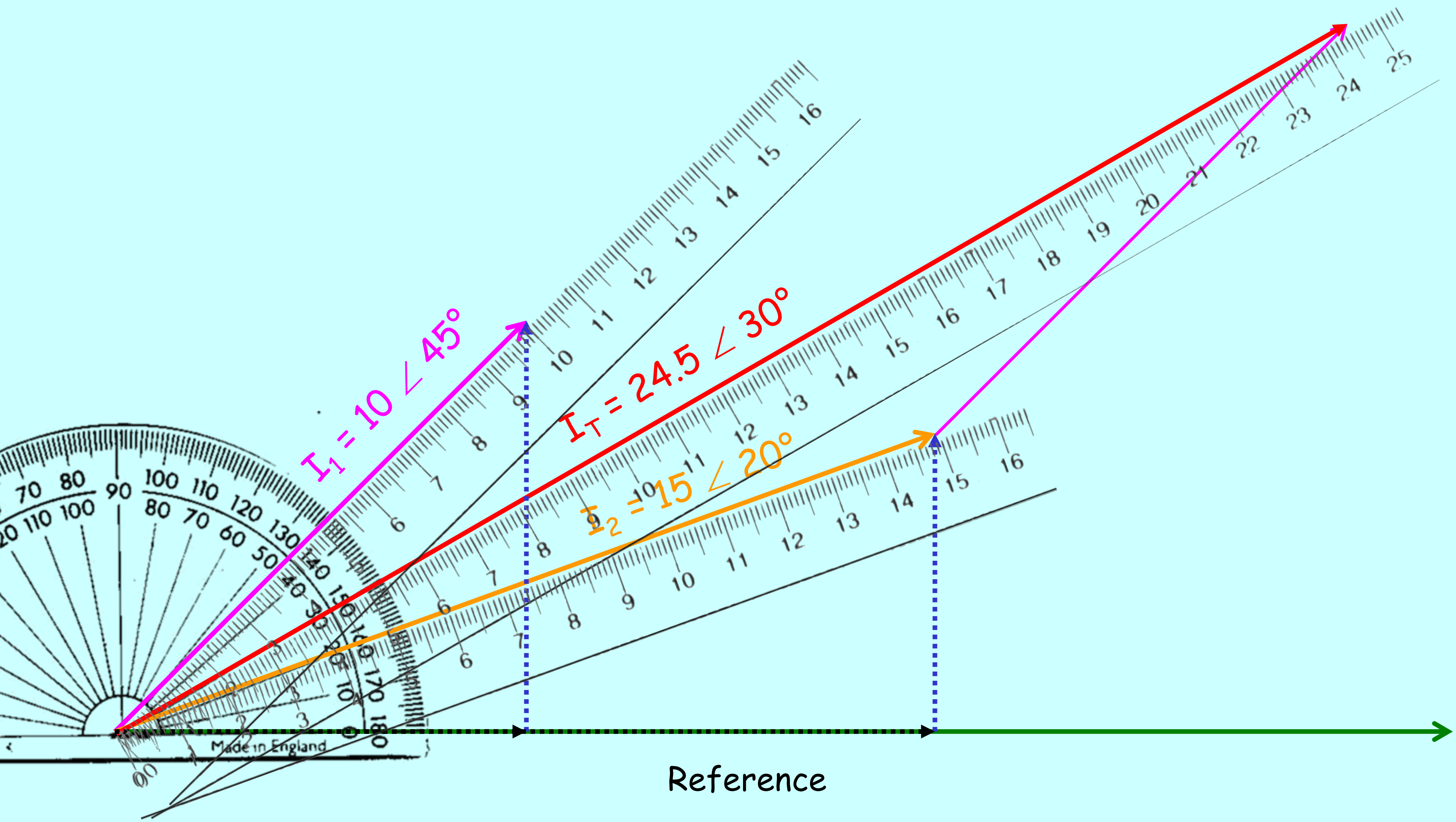
Pg. 68 Ex. 2



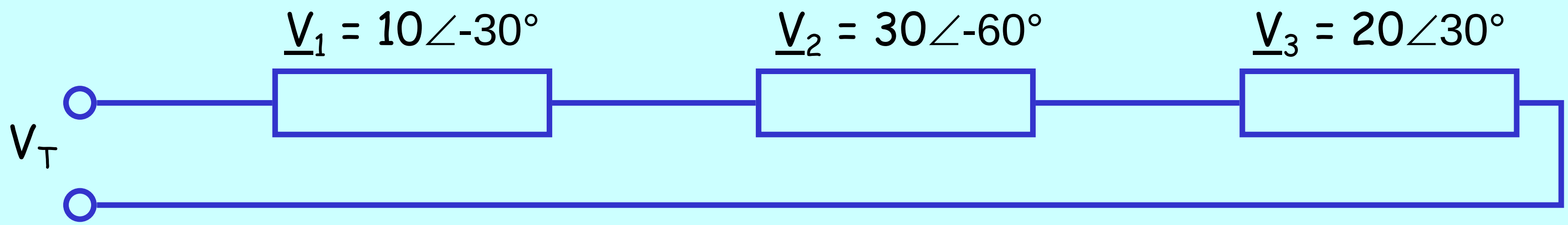


Pg. 72 Ex. 2

$I_1 = 10 \angle 45°$ $I_2 = 15 \angle 20°$

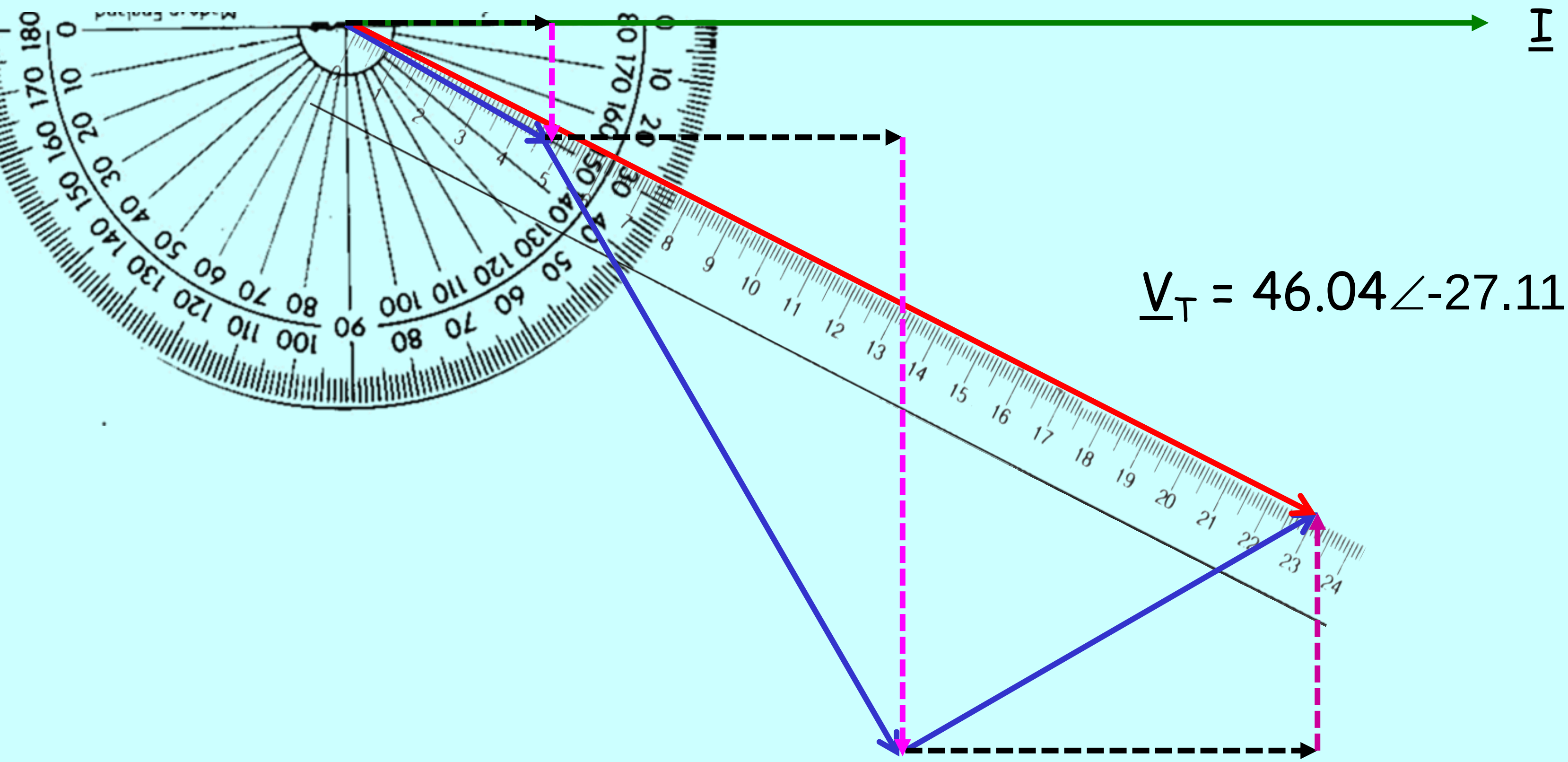


Pg. 73 Ex. 3



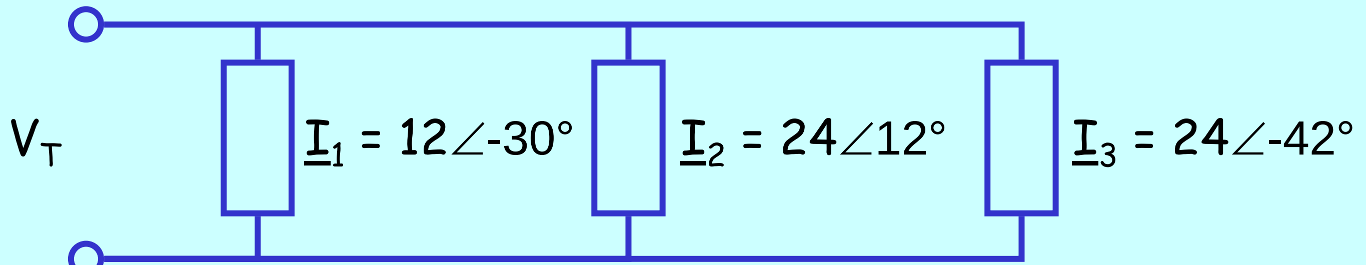
Series Circuit

∴ the Reference is Current AND $\underline{V}_T = \underline{V}_1 + \underline{V}_2 + \underline{V}_3$



$\underline{V}_T = 46.04\angle -27.11°$

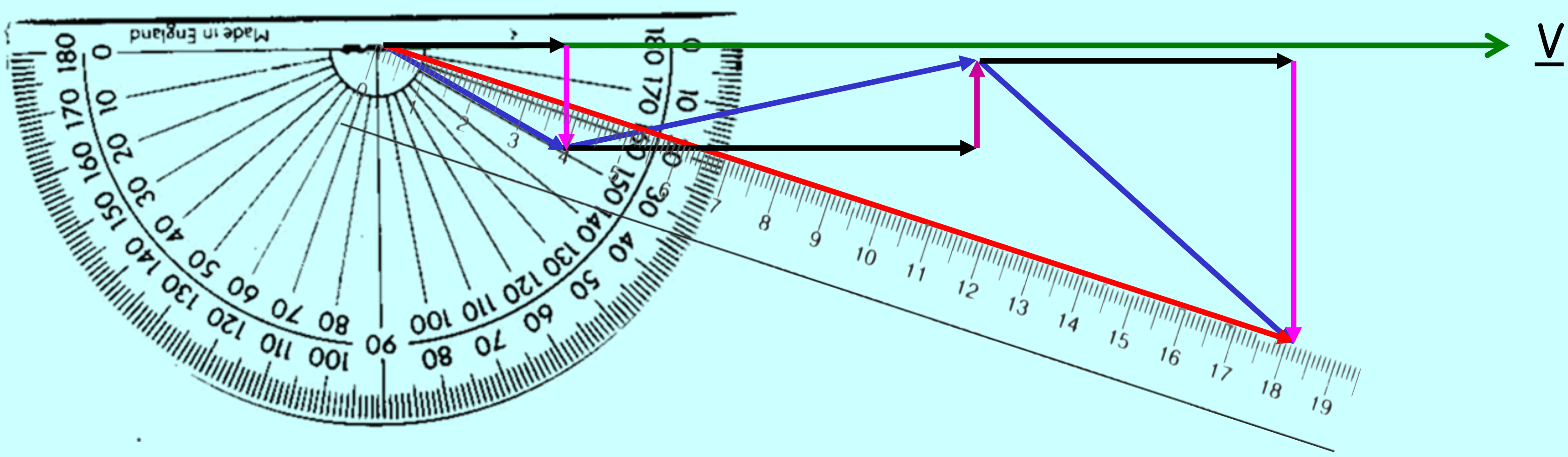
Pg. 74 Ex. 4



Parallel Circuit

∴ the Reference is Voltage

AND $I_T = I_1 + I_2 + I_3$

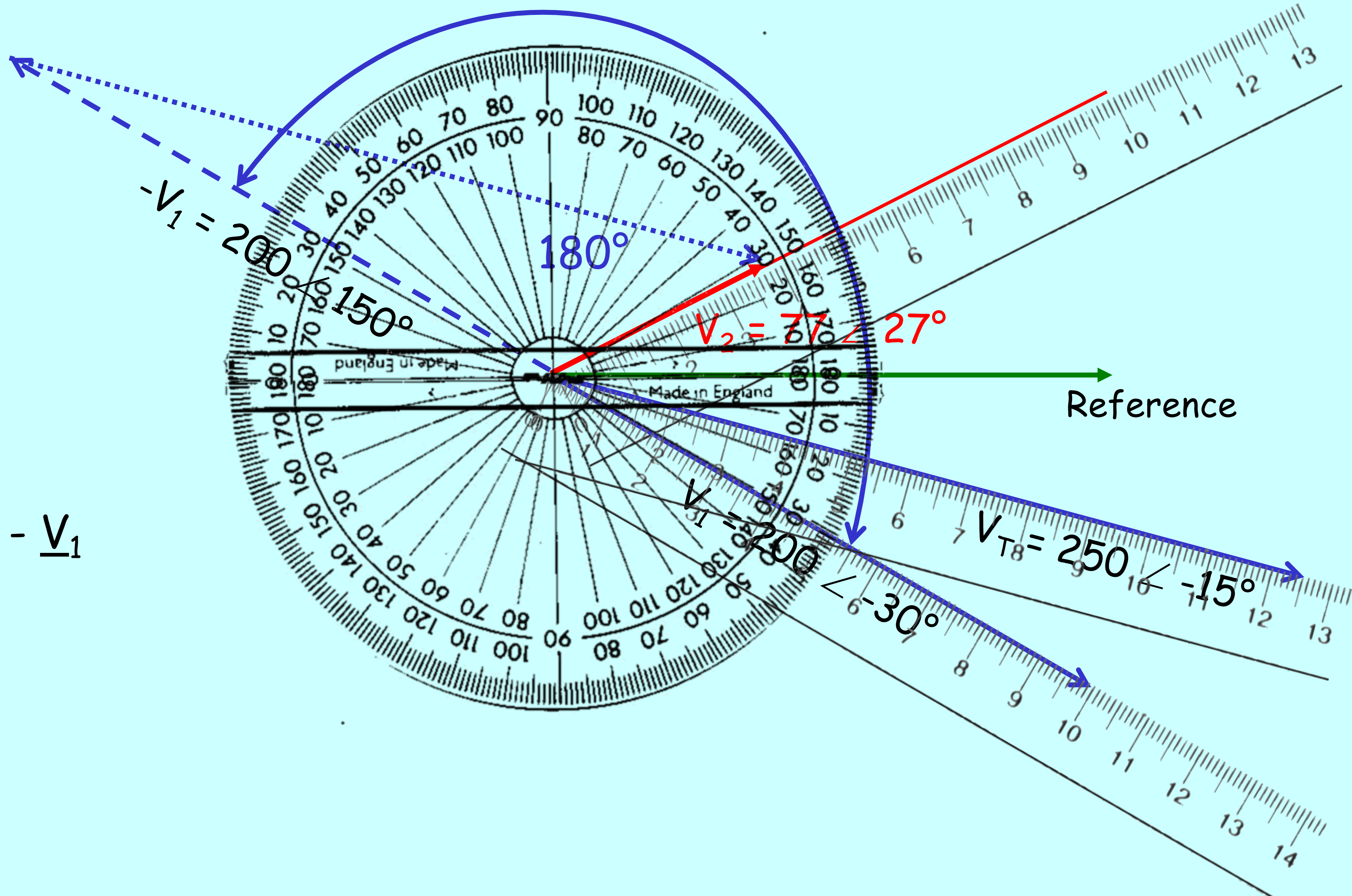


$I_T = 54.45\angle -18.27°$

Pg. 71 Ex. 1

$V_1 = 200 \angle -30°$

$\underline{V}_T = 250 \angle -15°$



$\underline{V}_2 = \underline{V}_T - \underline{V}_1$

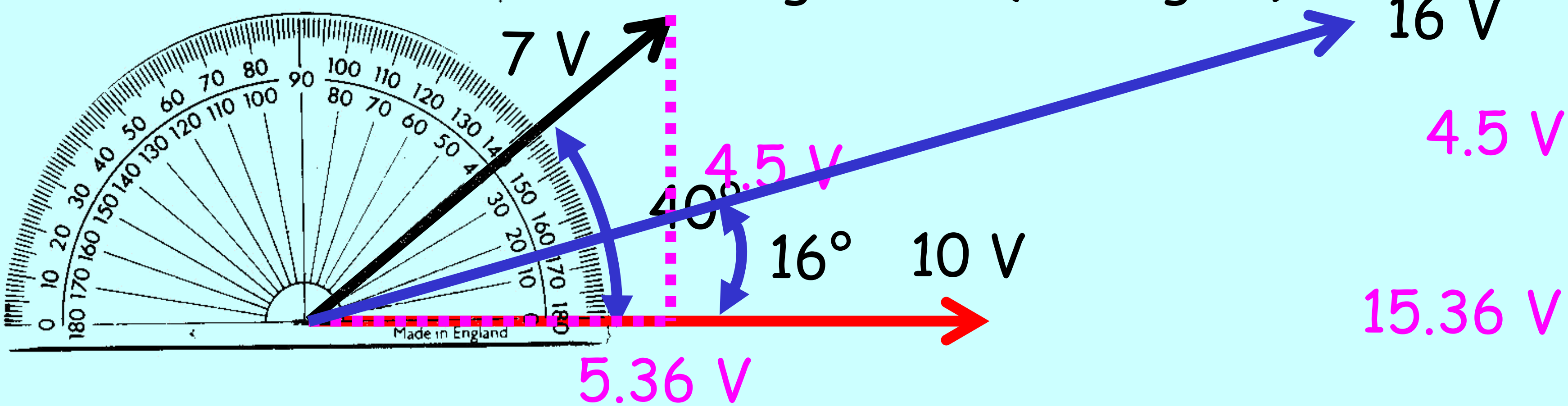
End of Lesson

Practical Exercises

Phasor Addition
Pp. 75 - 80

Phasor Addition

Method 2 – Algebraic (Triangles)



Start with the Reference Phasor which will be at ZERO degrees.

All other Phasors can be considered to be the hypotenuse of a Right Angle Triangle whose base is the Reference.

Calculate the lengths of the Opposite and Adjacent Sides.

$X = r \cos \theta$ $\qquad$ $Y = r \sin \theta$

Add the Horizontal and Vertical Components to get the Resultant Phasor.

Calculate the Magnitude and Phase Angle of the Resultant Phasor.

$$r = \sqrt{X^2 + Y^2} \qquad \theta = \tan^{-1}\left(\frac{Y}{X}\right)$$

UEENEEG102A
Solve problems in low voltage a.c. circuits

Resistive AC Circuits

Objectives:

At the end of this lesson students should be able to:

1. Apply Ohm's Law in a Resistive ac circuit.

2. State the phase relationship between Voltage and Current in a Resistive ac circuit.

3. Draw the phasor diagram for a Resistive ac circuit.

4. Calculate the Power consumed by a Resistive ac circuit.

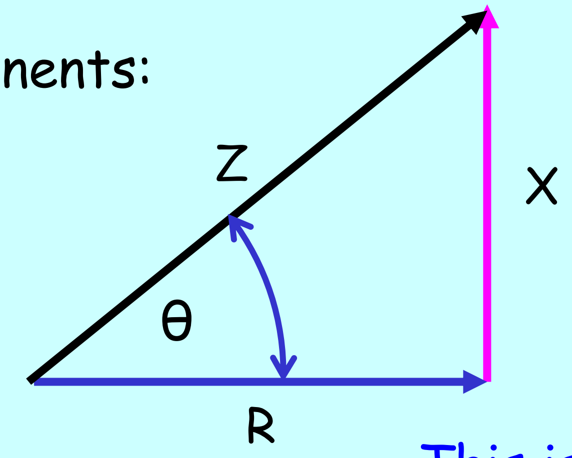
Terminology and Relationships

In an ac circuit opposition to current flow is called:

Impedance (Z). $$Z = \frac{V}{I}$$

Ohm's Law for ac circuits

Impedance (Z) has three (3) components:



R Resistance

X Reactance

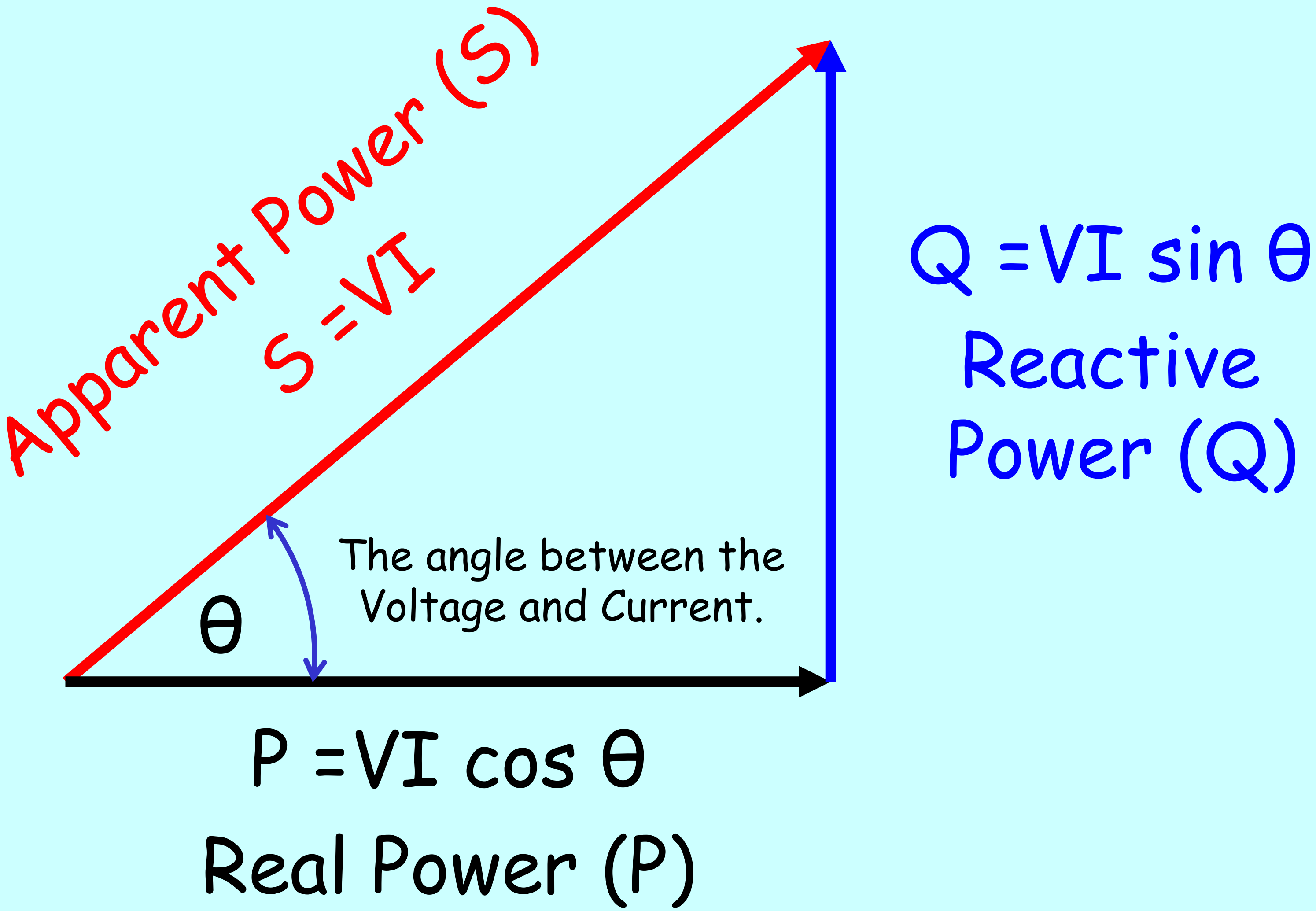
X_L Inductive Reactance

X_C Capacitive Reactance

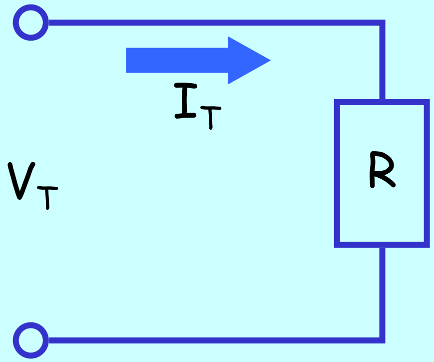
This is called the Impedance Triangle

θ The angle between the Voltage and Current.

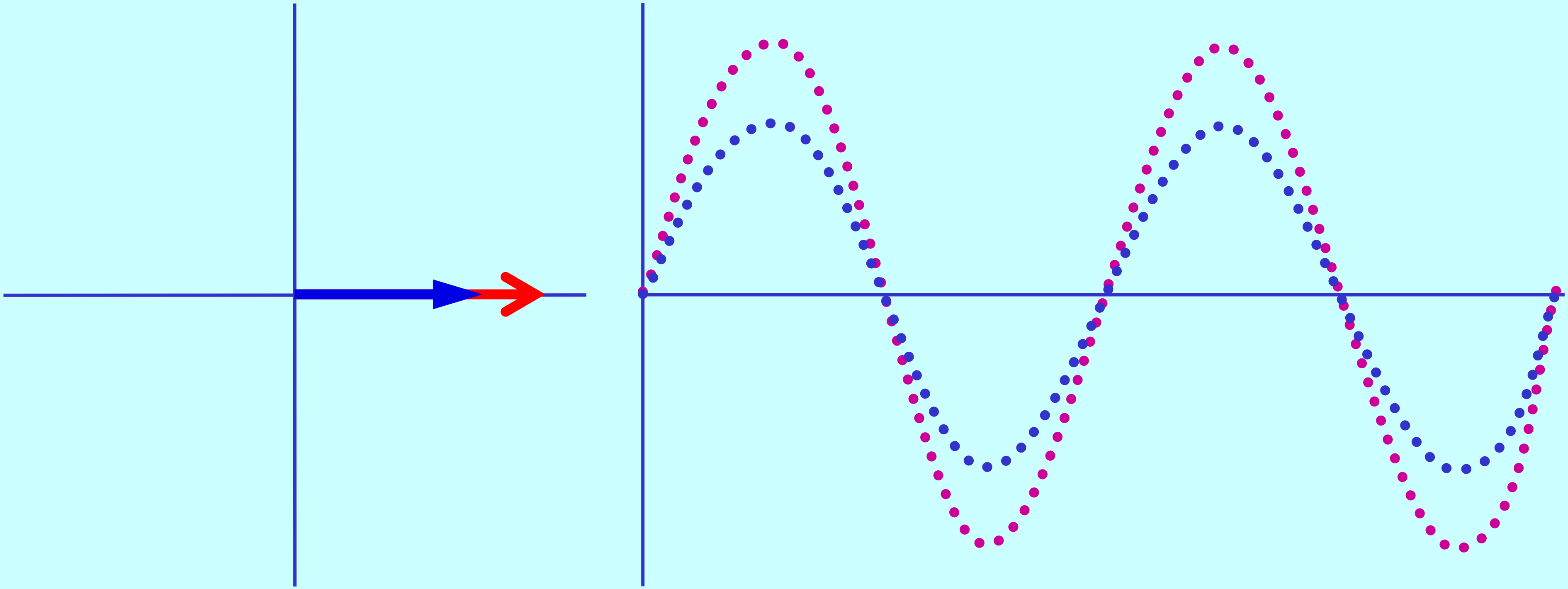
Power Triangle



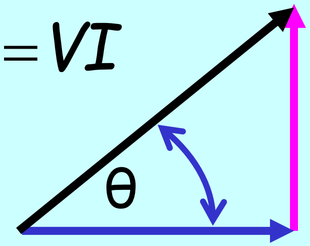
cos θ is called the Power Factor (λ) of the circuit because it relates the Real Power to the Apparent Power.



In a purely RESISTIVE ac circuit the Current is IN PHASE with the Voltage.



From the Power Triangle:



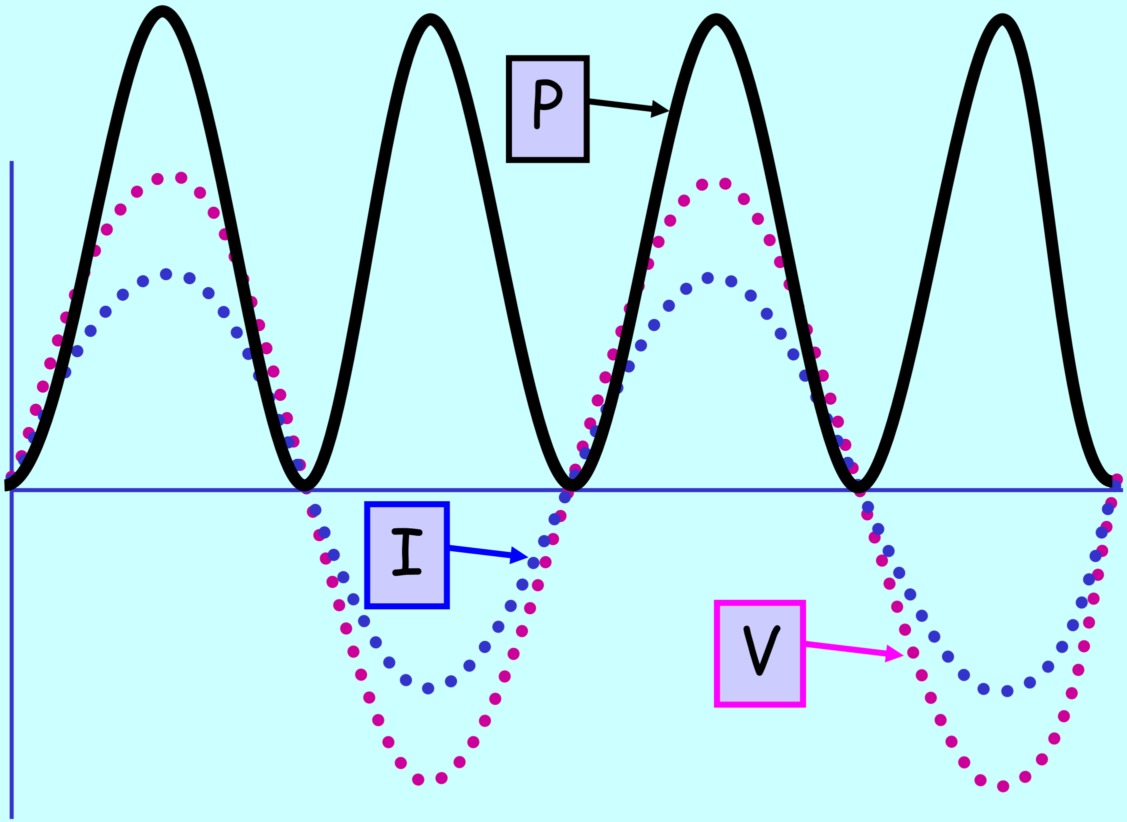
$S = VI$

$Q = VI \sin\theta$

$P = VI \cos\theta$

In a purely resistive circuit Current is In Phase with Voltage ∴ θ = 0°
(cos 0 = 1 & sin 0 = 0)

$P = VI$ $Q = 0$



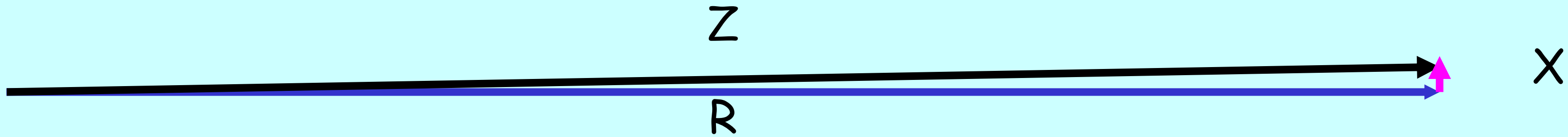
All practical components have some resistance and therefore use some Real Power.

Resistance is that property of a circuit that opposes current flow.

In any circuit: $$Z = \frac{V}{I}$$

So in a purely Resistive circuit: $Z \approx R$

In an ac circuit the Voltage is continually changing,
but the Resistance is constant.



With a phase angle of -θ (≈ 0°).

Example Calculations

$$Z = \frac{V}{I}$$

$$Z \approx R$$

$$S = VI$$

$$P = VI\cos\theta$$

$$Q = VI\sin\theta$$

Ex. 1

The element of a toaster has a resistance of 60 Ω. Determine the circuit current if the toaster is connected to a 240 V 50 Hz supply.

What do we know?

$\mathrm{R} = 60\ \Omega$

$\mathrm{V} = 240\ \mathrm{V}$

What do we want to know?

$$I = \frac{V}{Z}$$

$$I = \frac{240}{60}$$

$$I = 4A$$

Ex. 2

A purely resistive lamp is connected to a 24 V 50 Hz supply. If the lamp draws 1.25 A determine the circuit resistance, circuit impedance and the power consumption.

What do we know?

What do we want to know?

$$V = 24\ \mathrm{V}$$

$$I = 1.25\ \mathrm{A}$$

$$Z = R$$

$$Z = \frac{V}{I}$$

$$Z = \frac{24}{1.25}$$

$$Z = 19.2\Omega$$

$$P = VIcos\theta$$

$$P = 24 \times 1.25 \times 1$$

$$P = 30W$$

Ex. 3

A 2k2 Ω resistor is connected to a sinewave generator. If the frequency of the signal is set to 2 kHz and the circuit draws 4 μA determine the output voltage of the generator.

What do we know?

$Z = R = 2k2\ \Omega$

$f = 2\ kHz$

$I = 4\ \mu A$

What do we want to know?

$V = IZ$

$V = 2200 \times 4\mu$

$V = 8.8mV$

Ex. 4

A circuit has a resistance of 20 Ω and draws a current of 16 A. Determine:

a. The applied voltage,
b. The circuit impedance, and
c. The power drawn by the circuit.

What do we know?

$$R = 20\ \Omega$$

$$I = 16\ A$$

What do we want to know?

(a) $V = ZI$

$V = 20 \times 16$

$V = 320V$

(b) $Z = R = 20\Omega$

(c) $P = VI$

$P = 320 \times 16$

$P = 5.12\ kW$

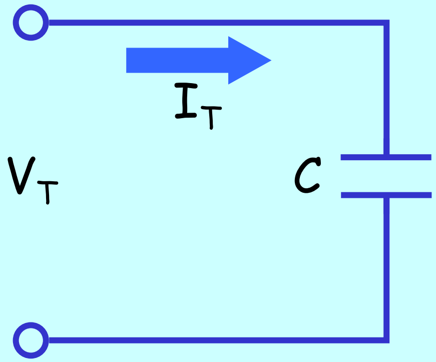
UEENEEG102A
Solve problems in low voltage a.c. circuits

Capacitive AC Circuits

Objectives:

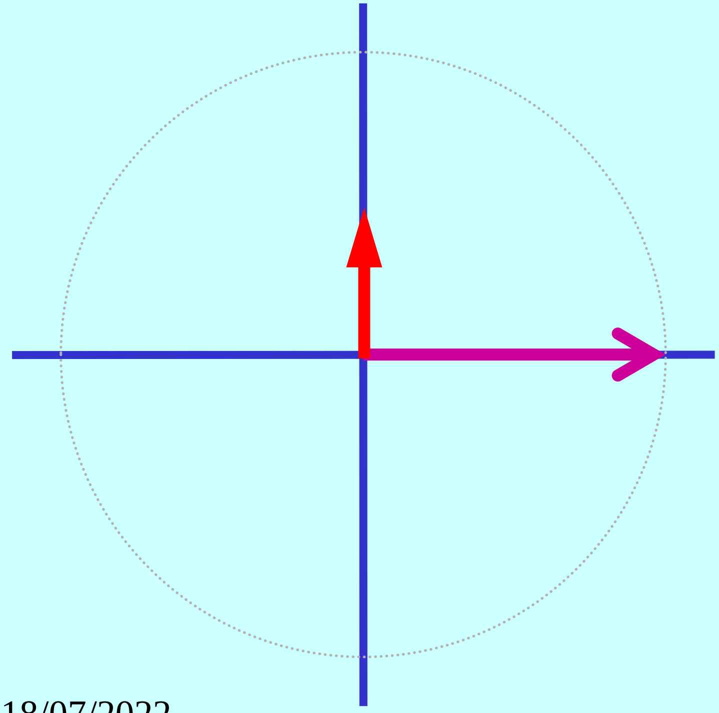
At the end of this lesson students should be able to:

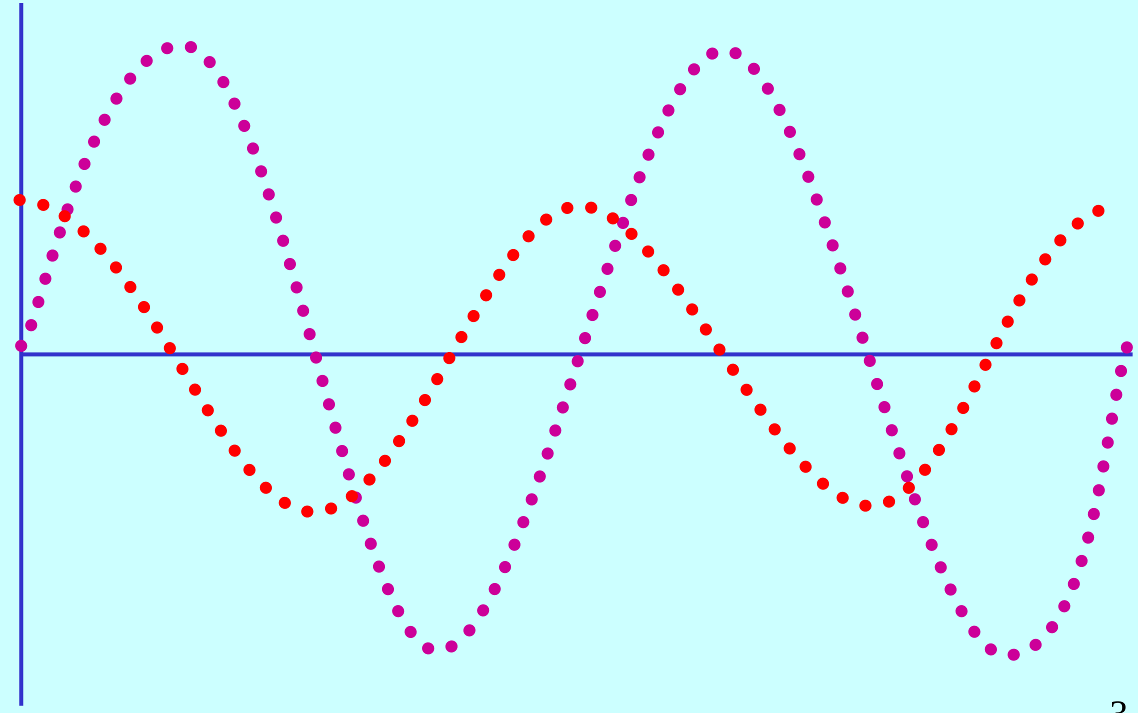
1. List the effects and applications of Capacitance in an ac circuit.
2. Define the term Capacitive Reactance.
3. Draw Impedance, Current and Voltage phasors in an Ideal Capacitive circuit.
4. Calculate Impedance, Currents and Voltages in an Ideal Capacitive circuit given certain characteristics.
5. Calculate the Power consumed by a Capacitive ac circuit.



Capacitance is that property of a circuit that opposes changes in voltage.

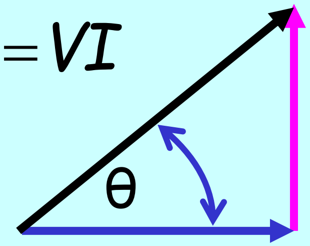
In a purely CAPACITIVE ac circuit the Current Leads the Voltage by 90°.





From the Power Triangle:

$S = VI$

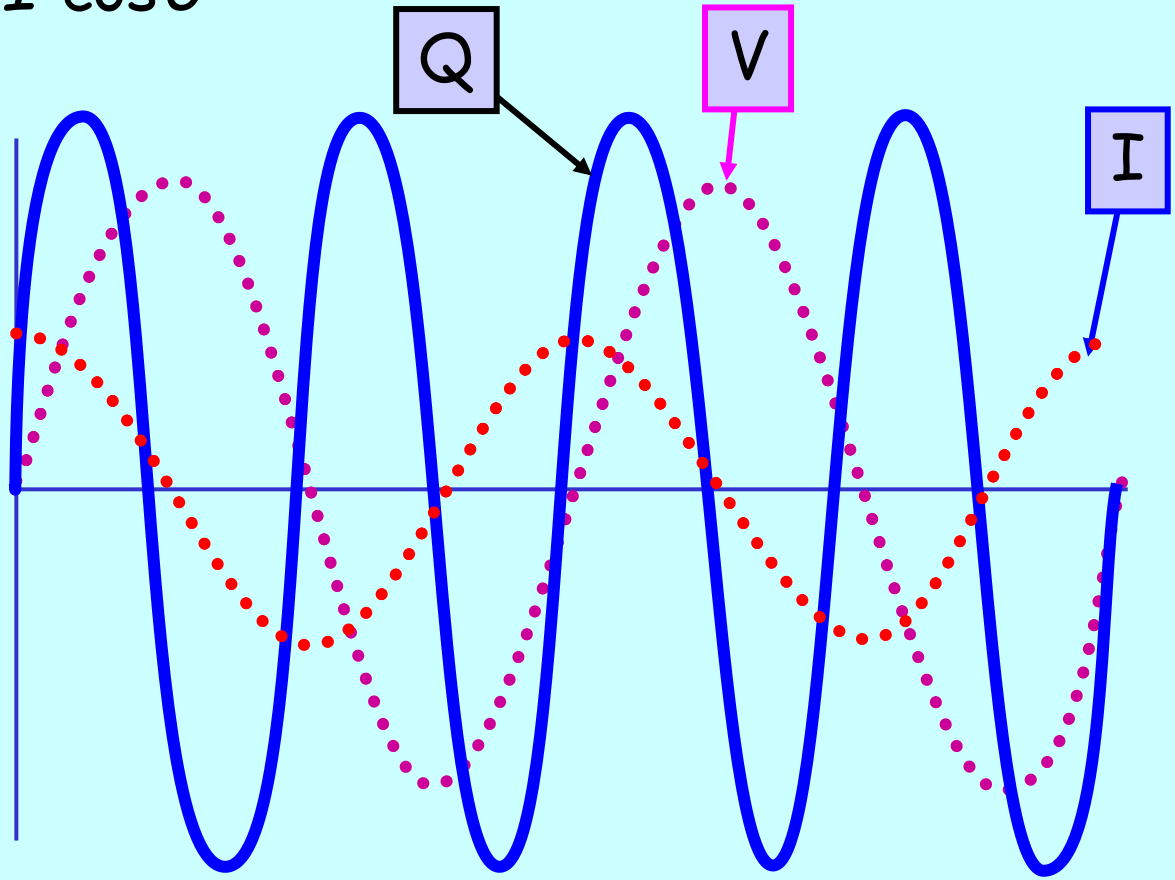


$Q = VI \sin\theta$

$P = VI \cos\theta$

In a purely Capacitive circuit Current Leads Voltage ∴ θ = 90°
(cos 90 = 0 & sin 90 = 1)

$P = 0$ $Q = VI$

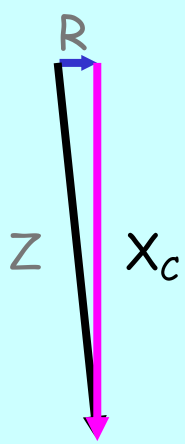


Capacitance is that property of a circuit that opposes changes in voltage.

In any circuit: $$Z = \frac{V}{I}$$

So in a purely Capacitive circuit $Z \approx X_C$ (X_C = Capacitive Reactance)

In an ac circuit the Voltage is continually changing, and the Capacitive Reactance is frequency dependant.



$$|X_C| = \frac{1}{2\pi f C}$$

With a phase angle of -θ (≈ -90).

Example Calculations

C I V I L

$$Z = \frac{V}{I}$$

$$Z \approx X_C$$

$$S = VI$$

$$|X_C| = \frac{1}{2\pi f C}$$

$$P = VI\cos\theta$$

$$|X_L| = 2\pi f L$$

$$Q = VI\sin\theta$$

Ex. 1

Determine the impedance of a purely capacitive circuit which draws 4.7 A from a 240 V, 50 Hz supply.

What do we know?

$$I = 4.7\ A$$

$$V = 240\ V$$

What do we want to know?

$$Z\ = ?$$

$$Z\ \approx X_C$$

$$X_C = \frac{V}{I}$$

$$X_C = \frac{240}{4.7}$$

$$X_C = 51\ \Omega$$

Ex. 2

Determine the reactance and circuit current drawn by an 8 μF Capacitor connected to a 240 V, 50 Hz supply. Draw the Phasor diagram for this component.

What do we know?

$$C = 8\ \mu F$$

$$V = 240\ V$$

$$f = 50\ hZ$$

What do we want to know?

$$X_c = \frac{1}{2\pi f C}$$

CIVIL

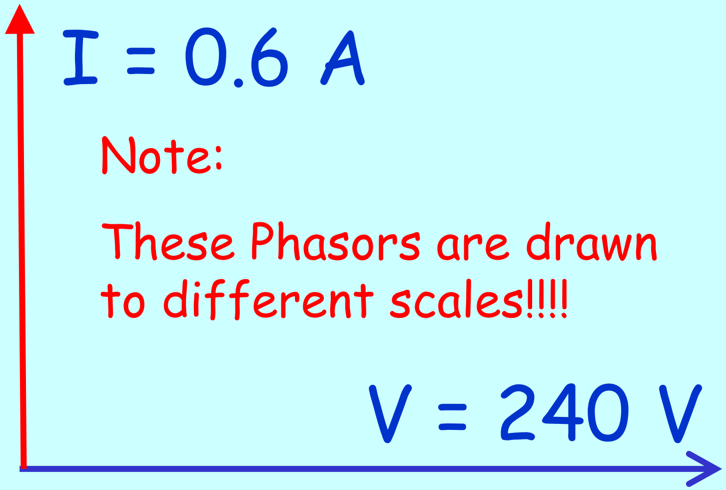
$$X_c = \frac{1}{2\pi \times 50 \times 8\mu}$$

$$X_c = 397.89\Omega$$

$$I = \frac{V}{Z}$$

$$I = \frac{240}{397.89}$$

$$I = 0.603\ A$$



Note:

These Phasors are drawn to different scales!!!!

Ex. 2

A single phase synchronous motor draws 40 A with a leading power factor of 0.22 from a 240 V, 50 Hz supply. Determine the Apparent and True Powers.

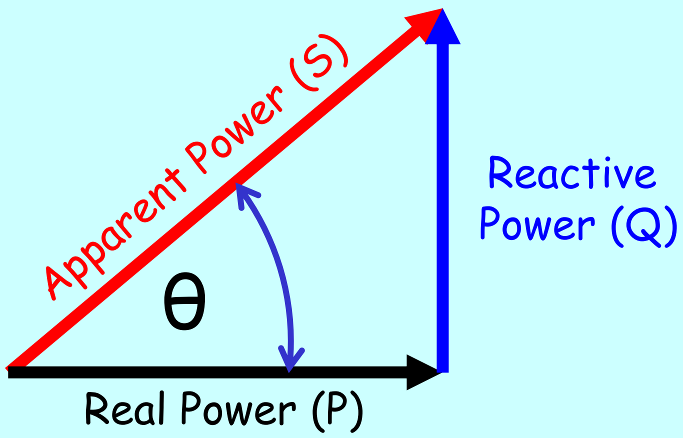
What do we know?

I = 40 A

ʎ = 0.22 lead

V = 240 V

f = 50 Hz



What do we want to know?

$S = VI$

$S = 240 \times 40$

$S = 9.6$ kVA

$P = VI \cos \theta$

$ʎ = \cos \theta$

$P = 9.6\ k \times 0.22$

$P = 2.1$ kW

Ex. 3

Calculate the phase angle for a circuit which has a Power Factor (λ) of:

a. 0.1 lead, and

b. 0.33 lead.

What do we know?

$\lambda_a = 0.10$ lead

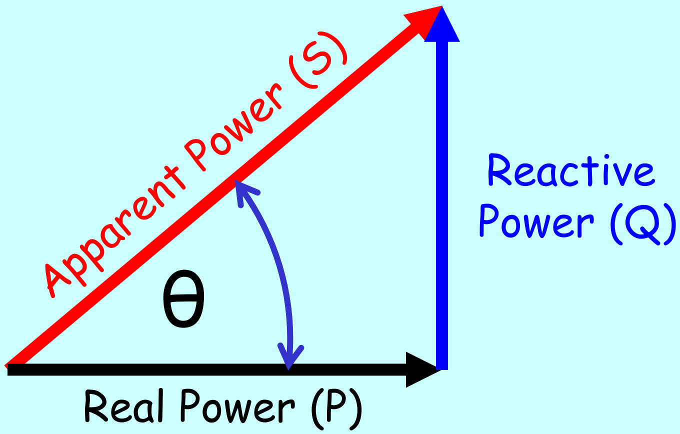
$\lambda_b = 0.33$ lead

What do we want to know?

$\theta_a = \cos^{-1}(0.10)$ $\quad\theta_a = 84.26°$

$\theta_b = \cos^{-1}(0.33)$ $\quad\theta_b = 70.73°$

Circuits are Capacitive.



Ex. 3

Calculate the phase angle for a circuit which has a Power Factor (λ) of:

a. 0.1 lead, and

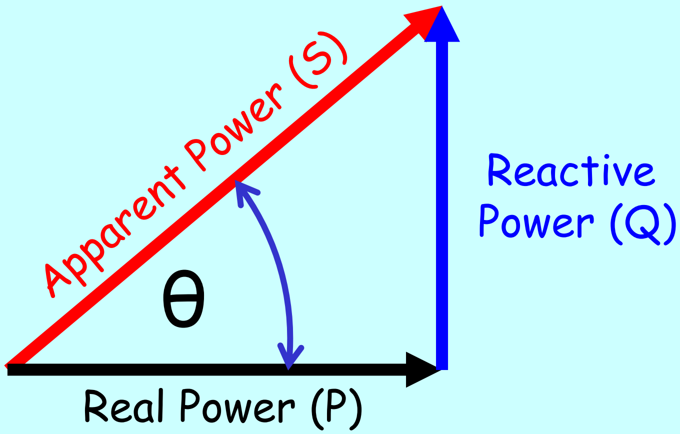
b. 0.33 lead.

What do we know? What do we want to know?

$\lambda_a = 0.10$ lead $\quad \theta_a = \cos^{-1}(0.10) \quad \theta_a = 84.26°$

$\lambda_b = 0.33$ lead $\quad \theta_b = \cos^{-1}(0.33) \quad \theta_b = 70.73°$

Circuits are Capacitive.



Ex. 3

Calculate the capacitance of a capacitor if its capacitive reactance is 100 Ω at a frequency of 50 Hz.

What do we know?

$X_C = 100\ \Omega$

$f = 50\ \text{Hz}$

$$X_C = \frac{1}{2\pi f C}$$

What do we want to know?

$C = ?$

$$C = \frac{1}{2\pi f X_C}$$

$$C = \frac{1}{100\pi \times 100} = 31.8\ \mu\text{F}$$

Ex. 4

Determine the current taken by a 100 μF capacitor which is connected to a 230 V 50 Hz supply.

What do we know?

$C = 100\ \mu F$

$V = 230\ V$

$f = 50\ Hz$

What do we want to know?

$I = ?$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{100\pi \times 100\mu} = 31.8\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{31.8} = 7.23\ A$$

End of Lesson

Practical Exercises

Ohm's Law in ac & dc Circuits

Inductive Reactance

Capacitive Reactance

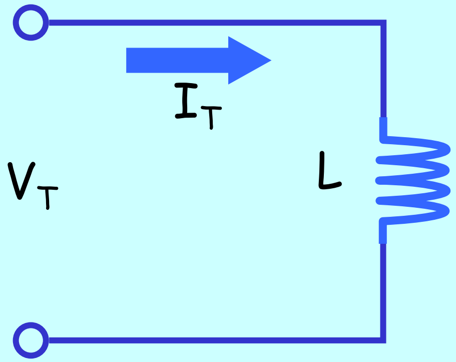
UEENEEG102A
Solve problems in low voltage a.c. circuits

Inductive AC Circuits

Objectives:

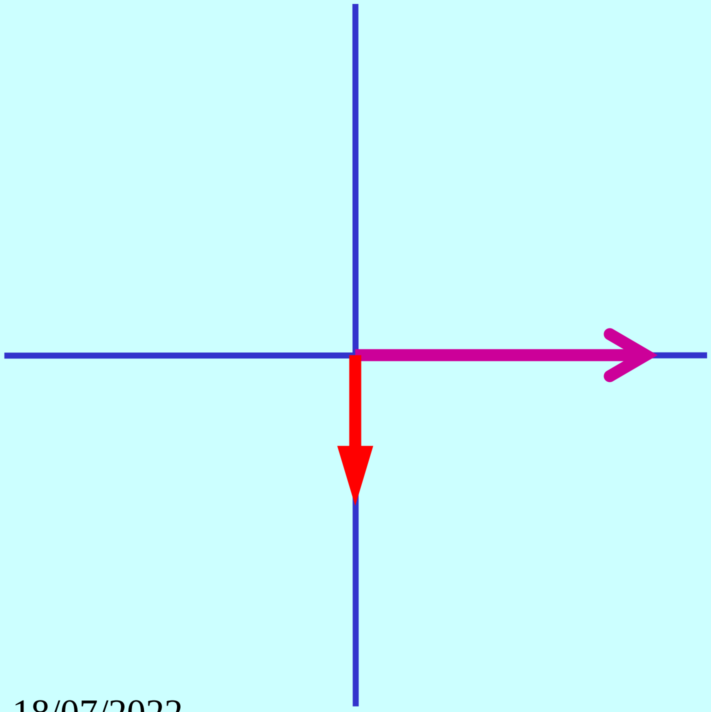
At the end of this lesson students should be able to:

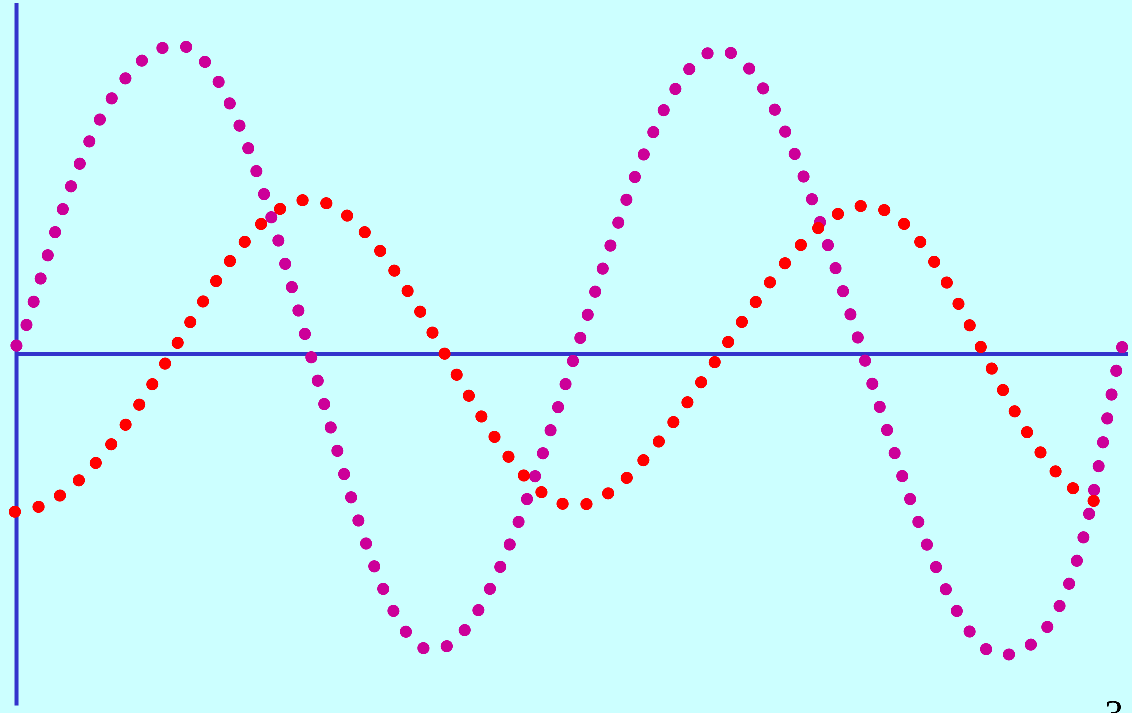
1. List the effects and applications of Inductance in an ac circuit.
2. Define the term Inductive Reactance.
3. Draw Impedance, Current and Voltage phasors in an Ideal Inductive circuit.
4. Calculate Impedance, Currents and Voltages in an Ideal Inductive circuit given certain characteristics.
5. Calculate the Power consumed by an Inductive ac circuit.



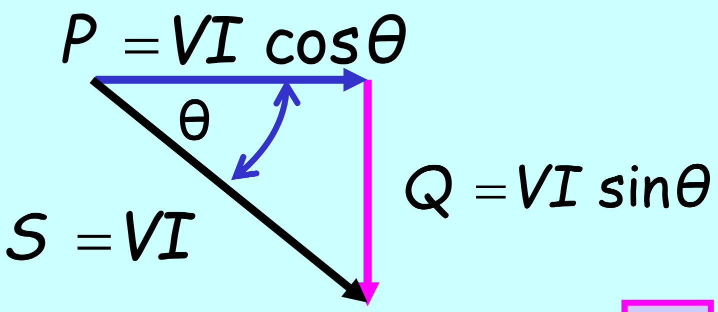
Inductance is that property of a circuit that opposes changes in current.

In a purely INDUCTIVE ac circuit the Current Lags the Voltage by 90°.





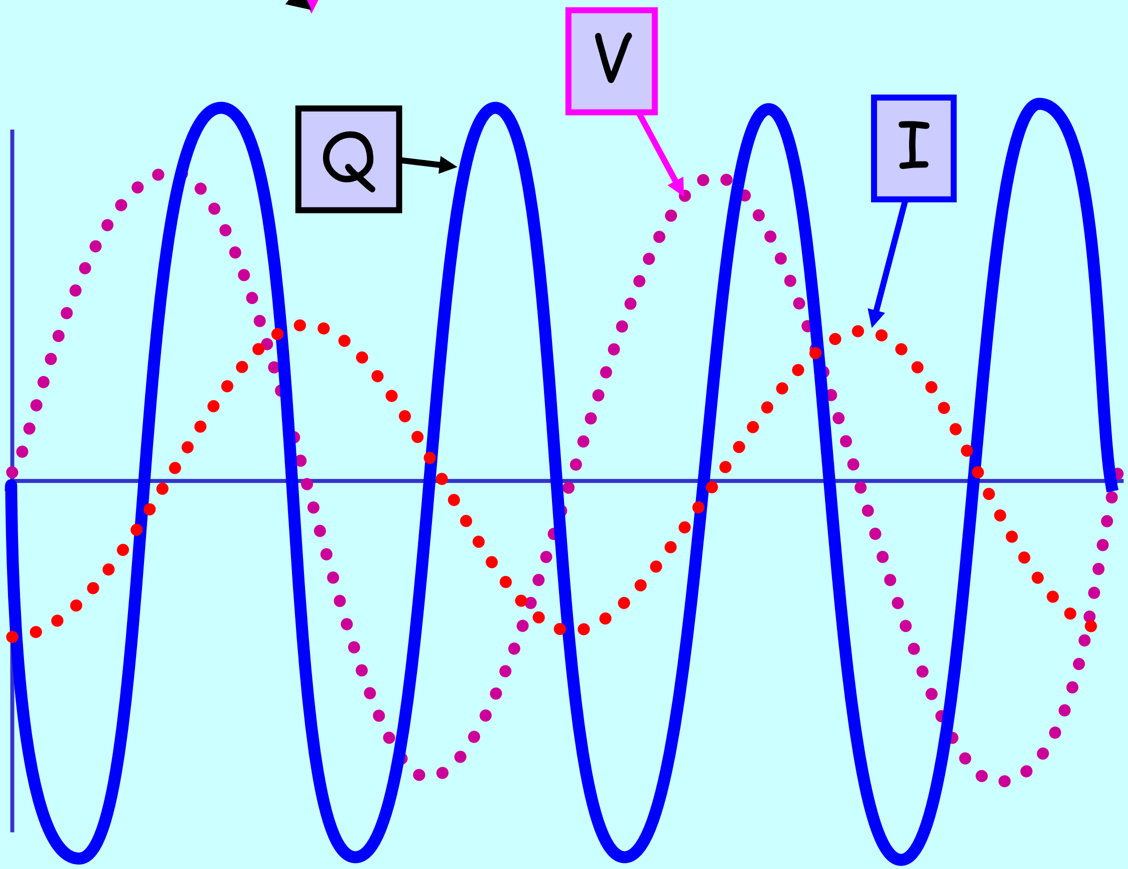
From the Power Triangle:



In a purely Inductive circuit Current Lags Voltage ∴ θ = -90°
(cos -90 = 0 & sin -90 = -1)

$$P = 0$$

$$Q = -VI$$



Practical Inductors have some resistance and therefore use some Real Power (I^2R).

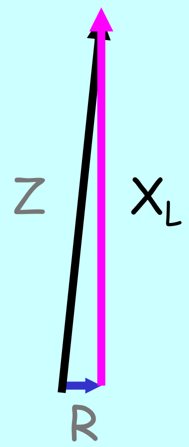
Inductors are used in AC Circuits because they use very little Real Power.

Inductance is that property of a circuit that opposes changes in current.

In any circuit: $$Z = \frac{V}{I}$$

So in a purely Inductive circuit $Z \approx X_L$ (X_L = Inductive Reactance)

In an ac circuit the Voltage is continually changing, and the Inductive Reactance is frequency dependant.



$$|X_L| = 2\pi f L$$

With a phase angle of -θ (≈ 90°).

Example Calculations

$$Z = \frac{V}{I}$$

$$S = VI$$

$$P = VI\cos\theta$$

$$Q = VI\sin\theta$$

$$Z \approx X_L$$

$$|X_L| = 2\pi f L$$

Ex. 1

Determine the impedance of an inductor if it draws 1.8 A from a 230 V 50 Hz supply.

What do we know?

$$I = 1.8\ A$$

$$V = 240\ V$$

What do we want to know?

$$Z\ = ?$$

$$Z\ \approx X_L = \frac{V}{I}$$

$$Z\ \approx X_L = \frac{V}{I} = \frac{240}{1.8} = 133.3\ \Omega$$

Ex. 2

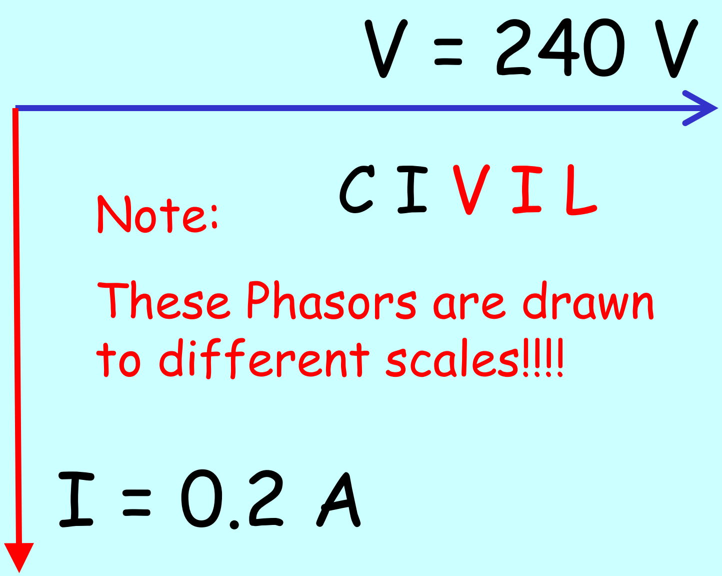
Determine the impedance and inductance of a coil with negligible resistance which draws 0.2 A when connected to a 240 V, 50 Hz supply.
Draw the Phasor diagram for this component.

What do we know?

$$I = 0.2\ A$$

$$V = 240\ V$$

$$f = 50\ hZ$$



C I V I L

Note:

These Phasors are drawn to different scales!!!!

What do we want to know?

$$Z \approx X_L = \frac{V}{I}$$

$$X_L = \frac{240}{0.2}$$

$$X_L = 1200\ \Omega$$

$$L = \frac{X_L}{2\pi f}$$

$$X_L = 2\pi f L$$

$$L = \frac{1200}{2\pi \times 50}$$

$$L = 3.82\ H$$

Ex. 3

Draw the V-I Phasor diagram (with values) for a 0.2 H coil with negligible resistance when it is connected to a 240 V, 50 Hz supply.

What do we know?

L = 0.2 H

V = 240 V

f = 50 Hz

C I V I L

What do we want to know?

$$Z \approx X_L = 2\pi f L$$

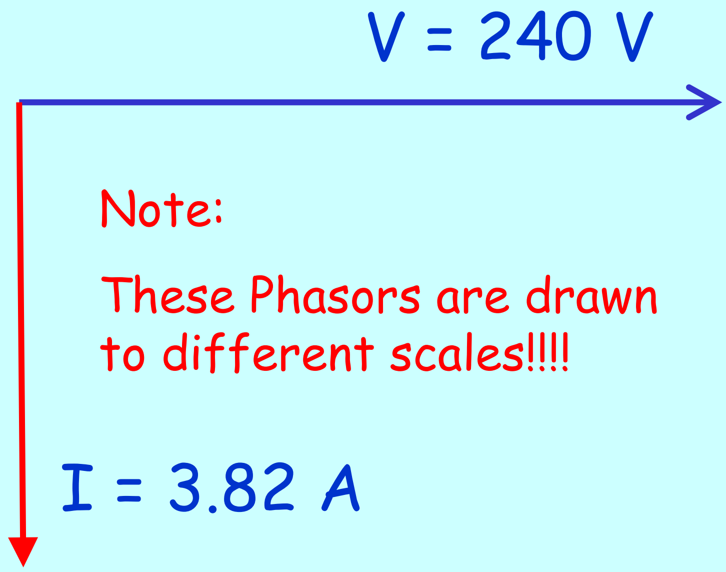
$$X_L = 2\pi \times 50 \times 0.2$$

$$X_L = 62.8\,\Omega$$

$$I = \frac{V}{Z}$$

$$I = \frac{240}{62.8}$$

$$I = 3.82\,A$$



Note:

These Phasors are drawn to different scales!!!!

Ex. 4

A welder draws 40 A with a lagging power factor of 0.22 from a 240 V, 50 Hz supply. Determine the Apparent and True Powers for the welder.

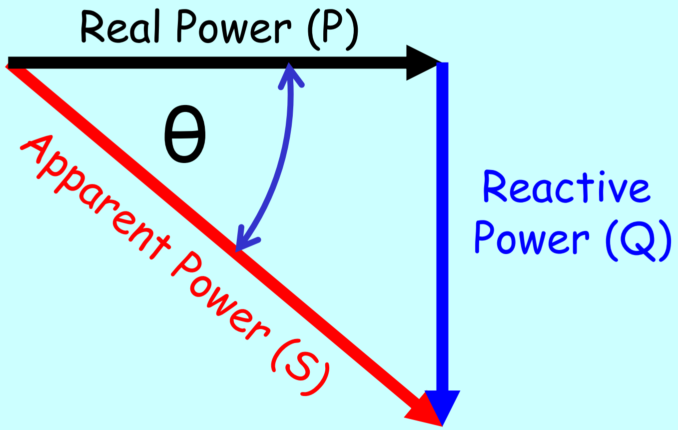
What do we know?

I = 40 A

λ = 0.22 lead

V = 240 V

f = 50 Hz



What do we want to know?

S = VI

S = 240 x 40

S = 9.6 kVA

P = VI cos θ

λ = cos θ

P = 9.6 k x 0.22

P = 2.1 kW

Ex. 5

Calculate the phase angle for a circuit which has a Power Factor (λ) of:

a. 0.76 lag, and

b. 0.96 lag.

What do we know?

What do we want to know?

$\lambda_a = 0.76$ lag

$\theta_a = \cos^{-1}(0.76)$

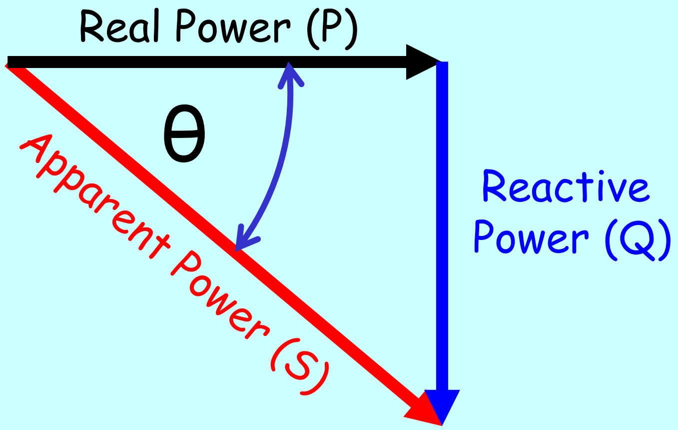
$\theta_a = -40.5°$

$\lambda_b = 0.96$ lag

$\theta_b = \cos^{-1}(0.96)$

$\theta_b = -16.3°$

Circuits are Inductive.



Ex. 6

An ideal inductor draws 10 A from a 240 V, 50 Hz supply.

How much current would it draw from a 120 V, 100 Hz supply?

What do we know?

$I_1 = 10\ A$

$V_1 = 240\ V$

$f_1 = 50\ Hz$

What do we want to know?

$I_2 = ?$

$V_2 = 120\ V$

$f_2 = 100\ Hz$

$$I_2 = \frac{120}{48}$$

$$I_2 = 2.5\ A$$

$$I = \frac{V}{Z}$$

$$\boldsymbol{Z = X_L = 2\pi f L}$$

$$X_{L1} = \frac{V}{I} = \frac{240}{10} = \mathbf{24\ \Omega}$$

$$L = \frac{X_{L1}}{2\pi f} = \frac{24}{2\pi \times 50} = \mathbf{76.4\ mH}$$

$$X_{L2} = 2\pi f L = 2\pi \times 100 \times 76.4m$$

$$\boldsymbol{X_{L2} = 48\ \Omega}$$

$$f_2 = 2f_1 \Rightarrow X_{L2} = 2X_{L1}$$

$$\boldsymbol{X_{L2} = 48\ \Omega}$$

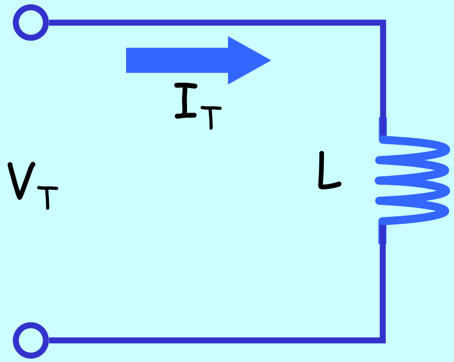
UEENEEG102A
Solve problems in low voltage a.c. circuits

Series RL AC Circuits

Objectives:

At the end of this lesson students should be able to:

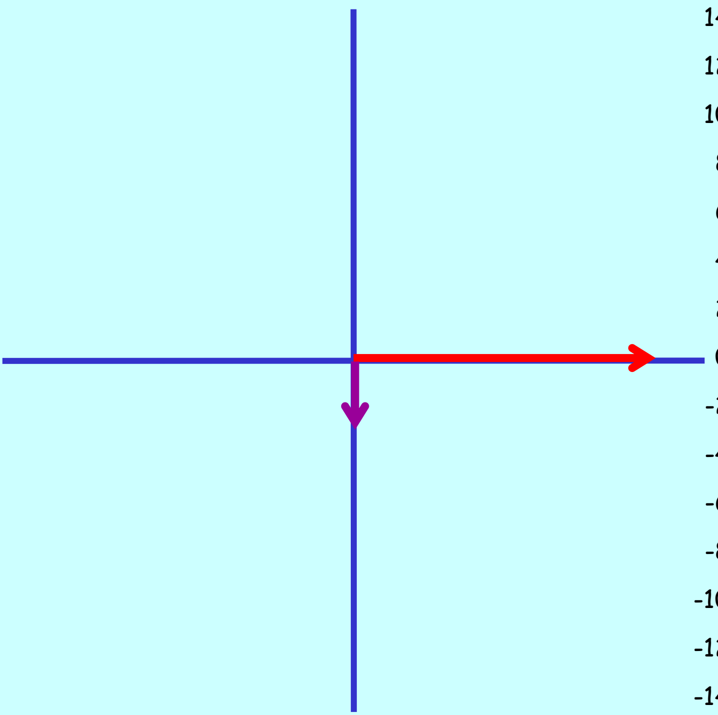
1. Determine circuit quantities and characteristics of Series RL ac circuits.
2. Draw the equivalent circuit for a practical Inductor.
3. Draw and label the Phasor Diagram for Series RL ac circuits.
4. Draw and label Impedance and Power Triangles for Series RL ac circuits.
5. List a number of practical applications for RL circuits.

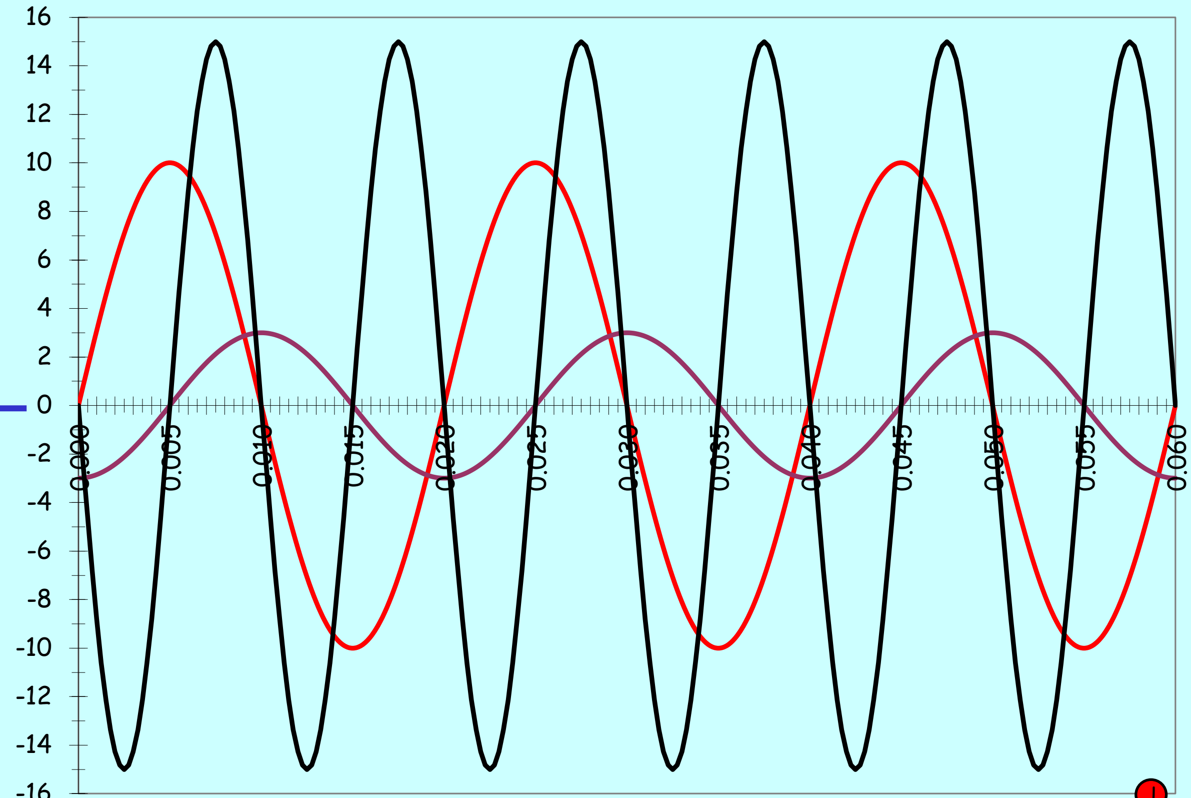


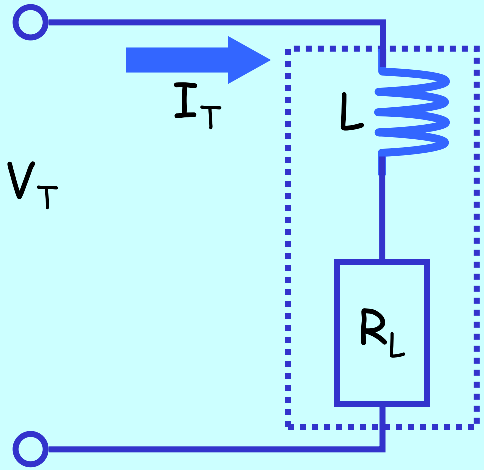
Inductance is that property of a circuit that opposes changes in current.

In a purely Inductive ac circuit the Current Lags the Voltage by 90°.

Average Real Power is ZERO Watts.

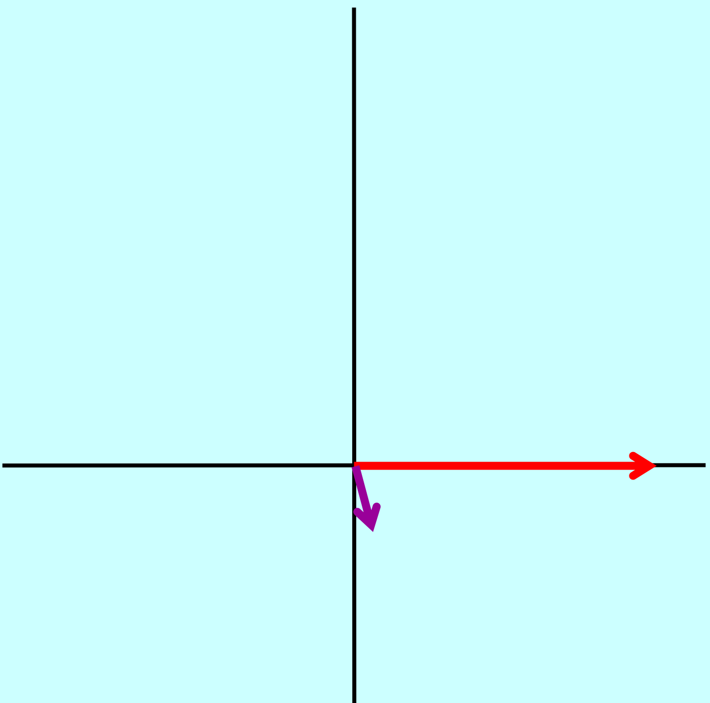


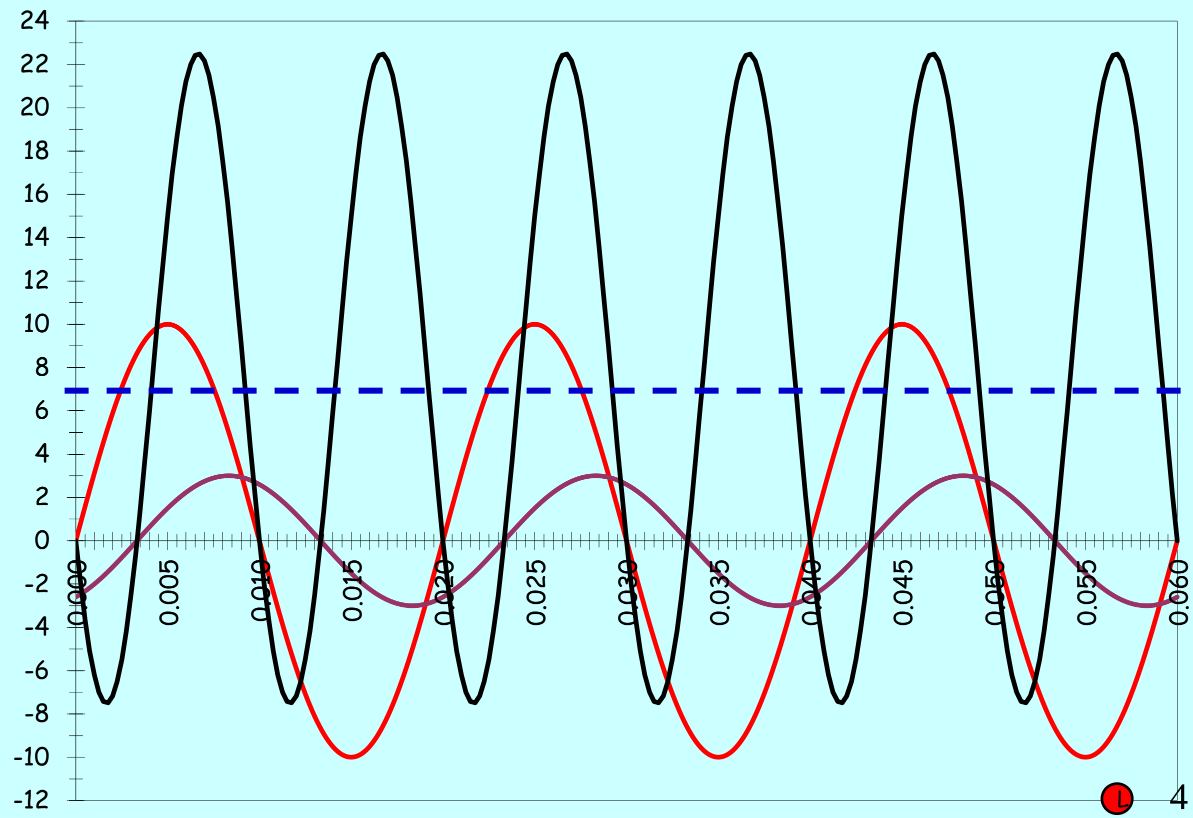




In a practical Inductive ac circuit there is some Resistance and therefore the Current Lags the Voltage by some angle less than 90°.

Average Real Power is NOT ZERO.





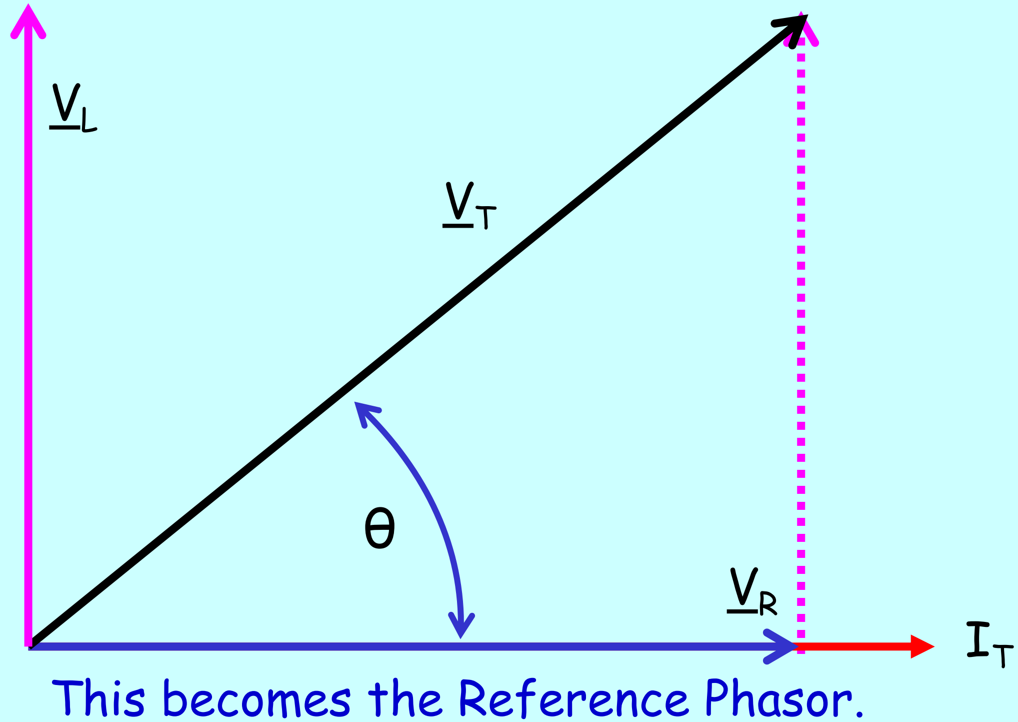
In a SERIES Circuit the common factor is current.

The Voltage across the Resistor ($\underline{V}_R$) is in phase with the current.

The Voltage across the Inductor ($\underline{V}_L$) Leads the current, and therefore $\underline{V}_R$, by 90°.

The Algebraic Addition: $\underline{V}_R + \underline{V}_L$ *gives* $\underline{V}_T$.

Which then gives us the circuit's phase angle.



In a Series Circuit (where I is common to ALL elements) we can develop the Voltage Triangle.

And then use Ohm's Law to find

The Impedance Triangle.

The Impedance (Z_T) can be found from:

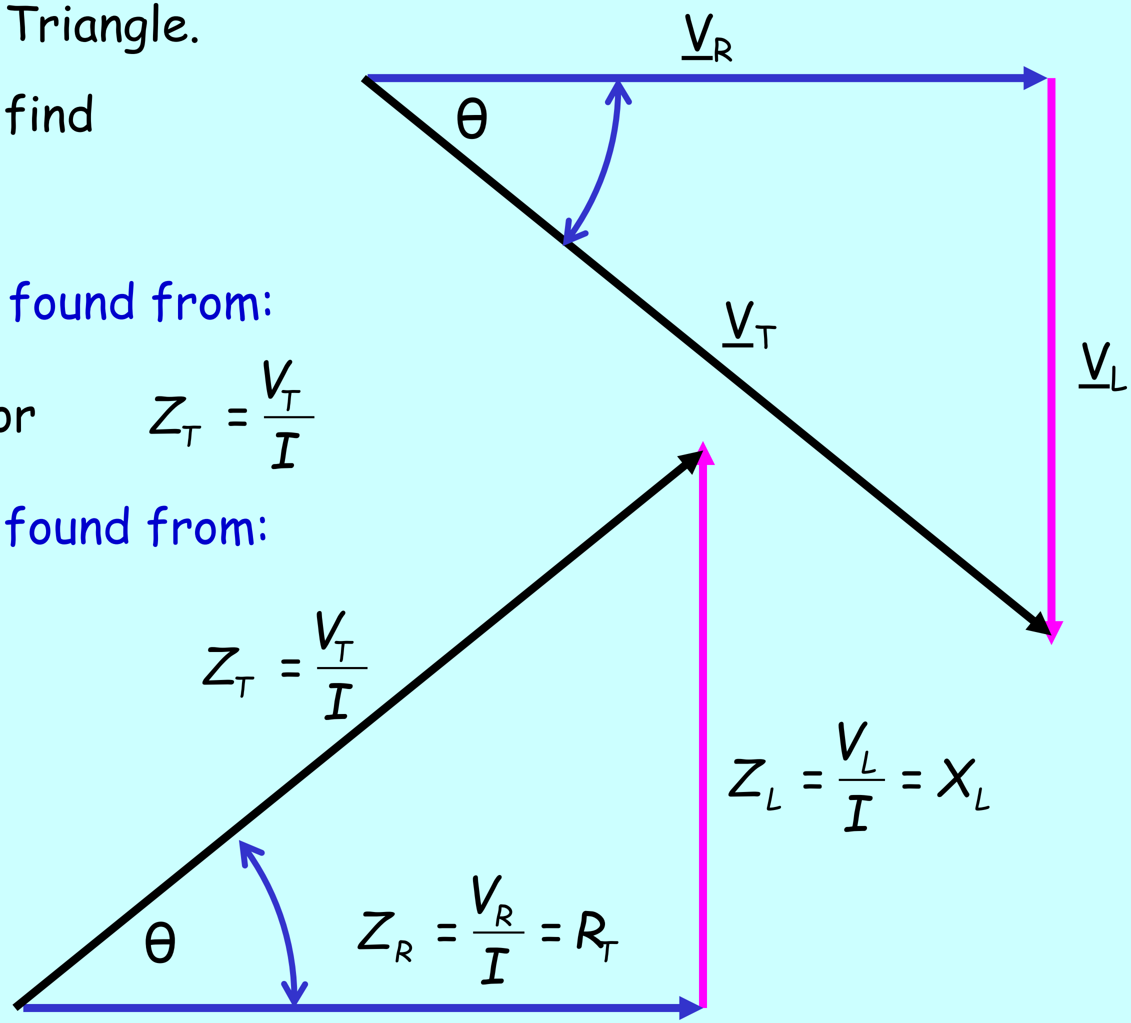
$$Z_T = \sqrt{R_T^2 + X_L^2} \quad \text{or} \quad Z_T = \frac{V_T}{I}$$

The Phase Angle (θ) can be found from:

$$\sin\theta = \frac{X_L}{Z_T} = \frac{V_L}{V_T}$$

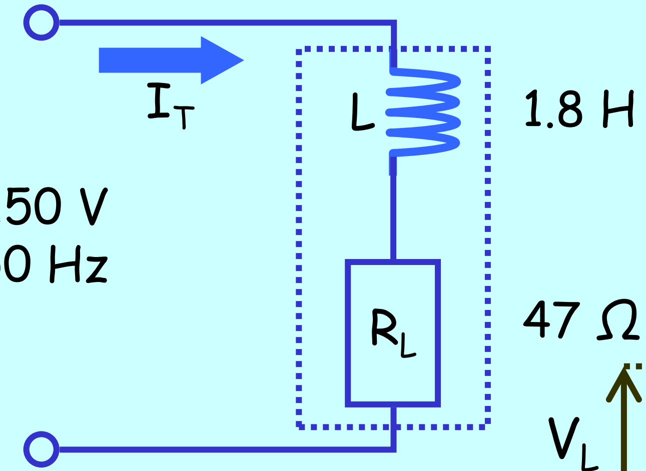
or $$\cos\theta = \frac{R_T}{Z_T} = \frac{V_R}{V_T}$$

or $$\tan\theta = \frac{X_L}{R_T} = \frac{V_L}{V_R}$$



These are Similar Triangles.
They have the same angles,
and their sides are proportional.

Example 1



A. Find Inductive Reactance (X_L)

B. Find Circuit Impedance (Z)

C. Draw the Phasor Diagram for the circuit.

A. $X_L = 2\pi f L$

$X_L = 2\pi \times 50 \times 1.8$

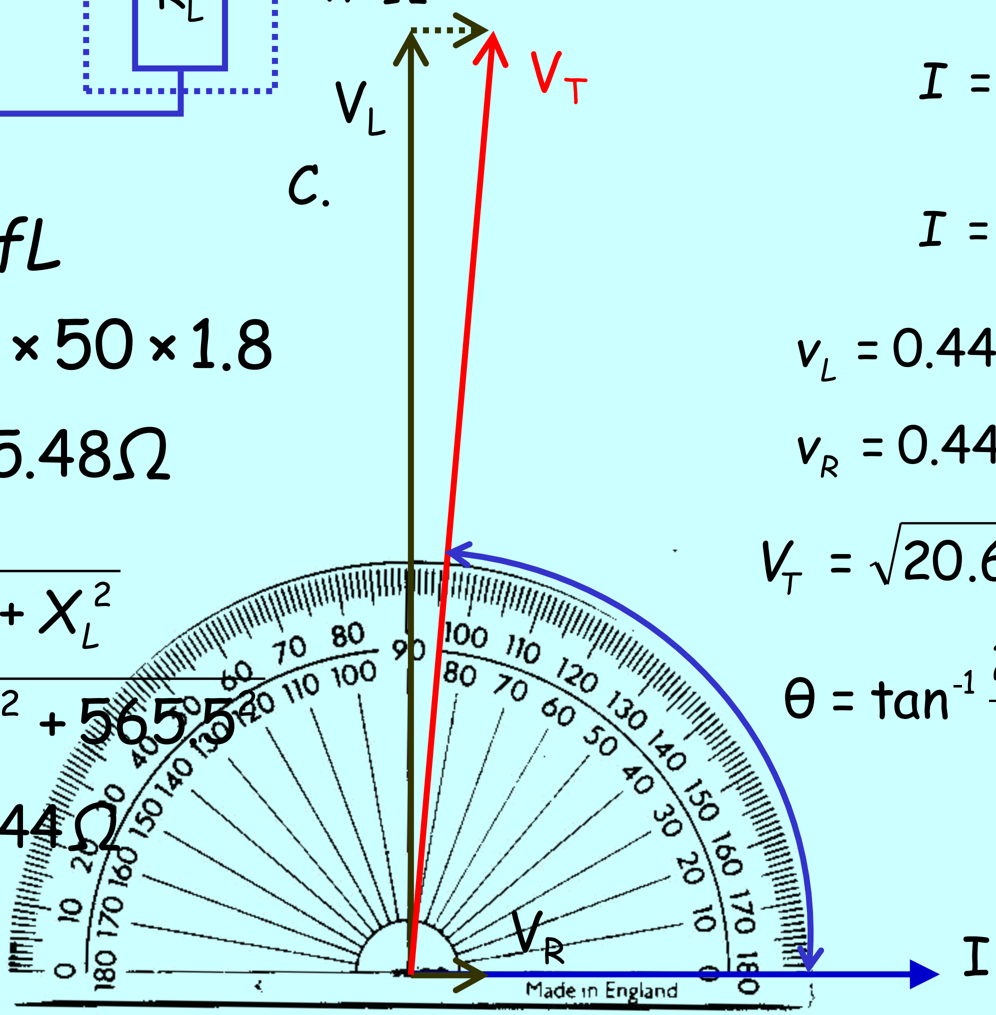
$X_L = 565.48\Omega$

B. $Z = \sqrt{R^2 + X_L^2}$

$Z = \sqrt{47^2 + 565.5^2}$

$Z = 567.44\Omega$

C.



$I = \frac{V}{Z}$

$I = \frac{250}{567.4} = 440\,\text{mA}$

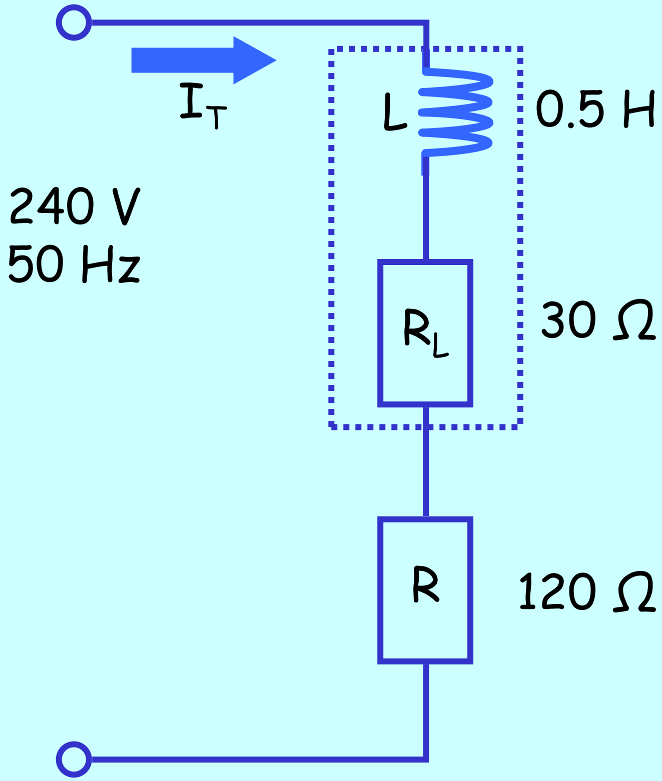
$v_L = 0.44 \times 567 = 249.48\,V$

$v_R = 0.44 \times 47 = 20.68\,V$

$V_T = \sqrt{20.68^2 + 249.48^2} = 250\,V$

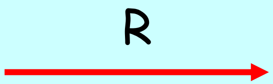
$\theta = \tan^{-1}\frac{249.48}{20.68} = 85.26^\circ$

Example 2



a. $R_T = R + R_L$

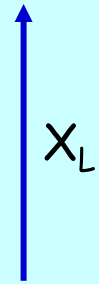
$R_T = 120 + 30 = 150\,\Omega$



b. $X_L = 2\pi f L$

$X_L = 2\pi \times 50 \times 0.5$

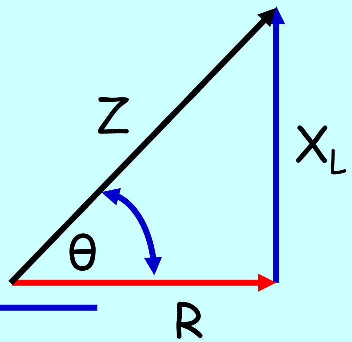
$X_L = 157\,\Omega$



c. $Z = \sqrt{R_T^2 + X_L^2}$

$Z = \sqrt{150^2 + 157^2}$

$Z = 217\,\Omega$



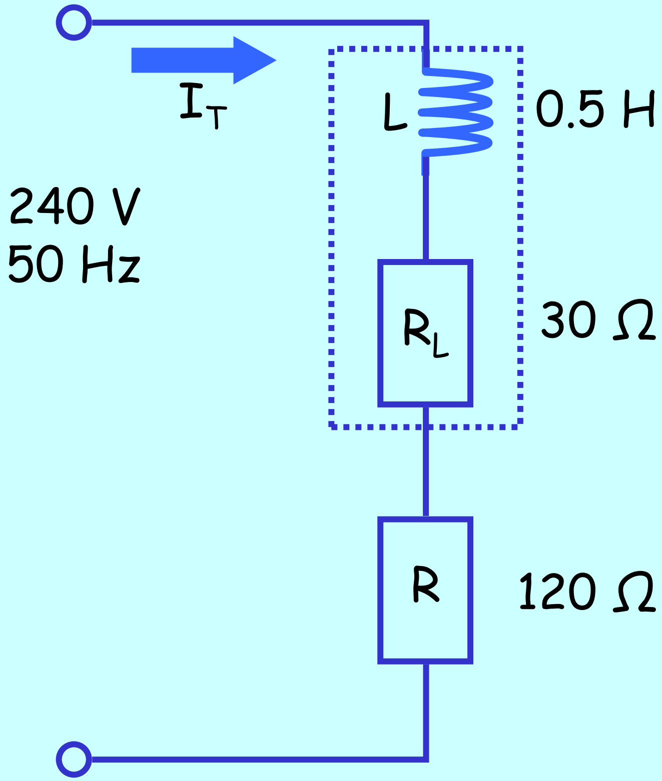
d. $I = \dfrac{V}{Z}$

$I = \dfrac{240}{217} = 1.1\,A$

e. $\theta = \tan^{-1}\dfrac{X}{R}$

$\theta = \tan^{-1}\dfrac{157}{150} = 46.3^{\circ}$

Example 2 (cont)



f. $\lambda = \cos\theta \qquad \lambda = \dfrac{P}{S}$

$\lambda = \cos 46.3 = 0.691$

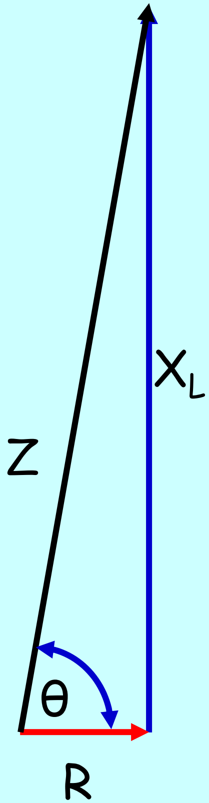
g. $Z_L = \sqrt{R_L^2 + X_L^2}$

$Z_L = \sqrt{30^2 + 157^2}$

$Z_L = 159.8\Omega$

h. $\theta = \tan^{-1}\dfrac{X}{R}$

$\theta = \tan^{-1}\dfrac{157}{30} = 79.2^\circ$



i. $V_R = IR$

$V_R = 1.1 \times 120$

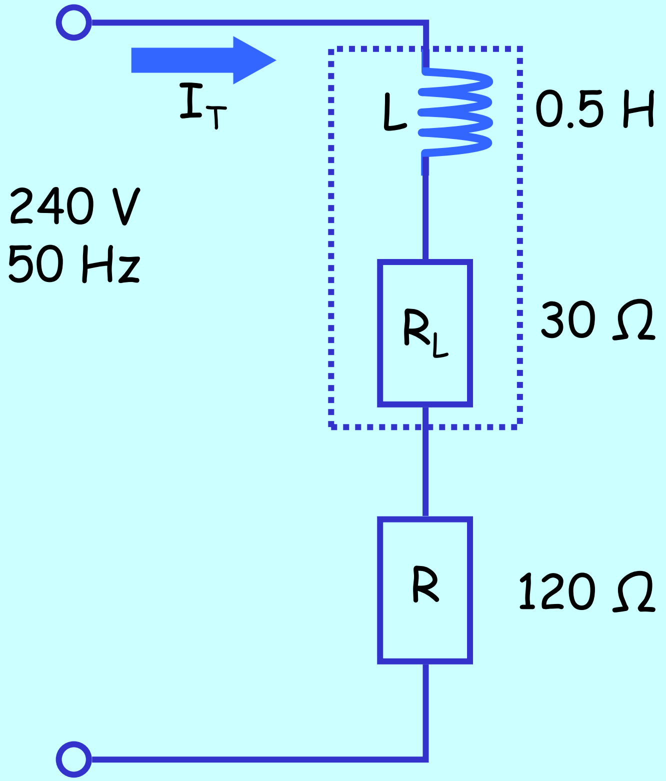
$V_R = 132\ \text{V}$

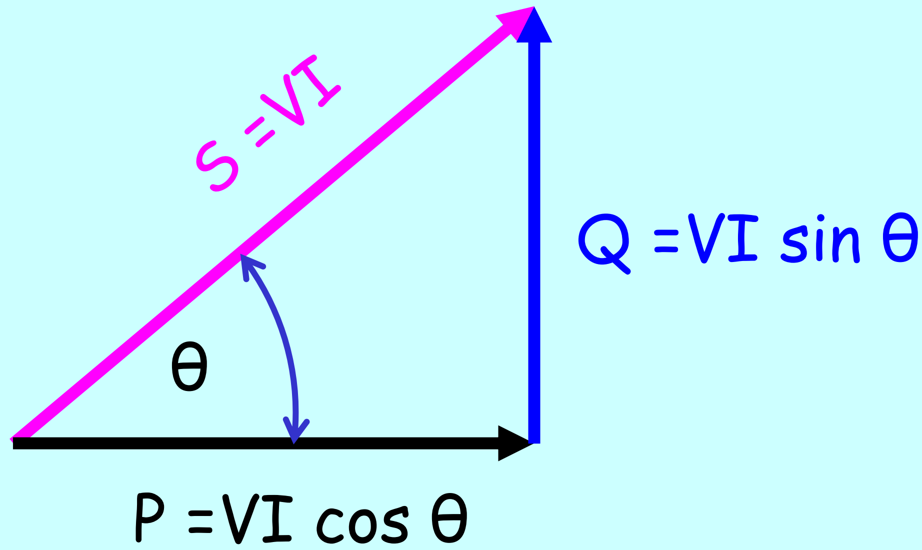
j. $V_L = IZ_L$

$V_L = 1.1 \times 159.8$

$V_L = 175.8\ \text{V}$

Example 2 (cont)





k. $S = VI$

$S = 240 \times 1.1$

$S = 264 \text{ VA}$

l. $P = VI \cos\theta$

$P = 240 \times 1.1 \times \cos 46.3$

$P = 182 \text{ W}$

m. $Q = VI \sin\theta$

$Q = 240 \times 1.1 \times \sin 46.3$

$Q = 191 \text{ Var}$

Example Calculations

CIVIL

$$Z = \frac{V}{I} \qquad Z = \sqrt{R^2 + X^2} \qquad |X_C| = \frac{1}{2\pi f C} \qquad |X_L| = 2\pi f L$$

$$\sin\theta = \frac{X_L}{Z_T} = \frac{V_L}{V_T} = \frac{Q}{S}$$

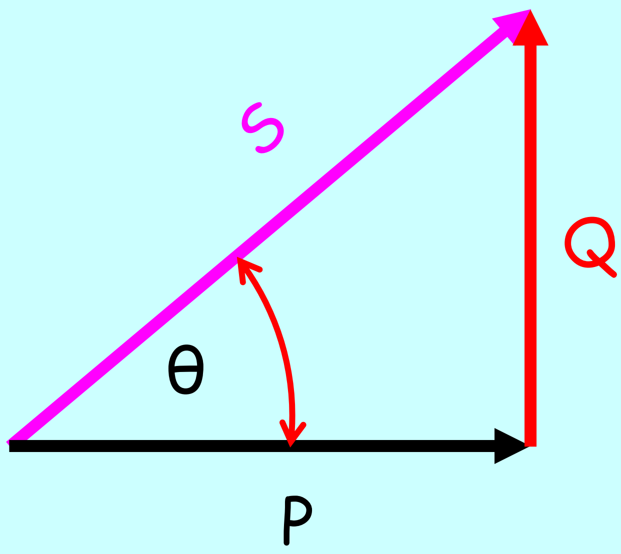
$$\cos\theta = \frac{R_T}{Z_T} = \frac{V_R}{V_T} = \frac{P}{S}$$

$$\tan\theta = \frac{X_L}{R_T} = \frac{V_L}{V_R} = \frac{Q}{P}$$

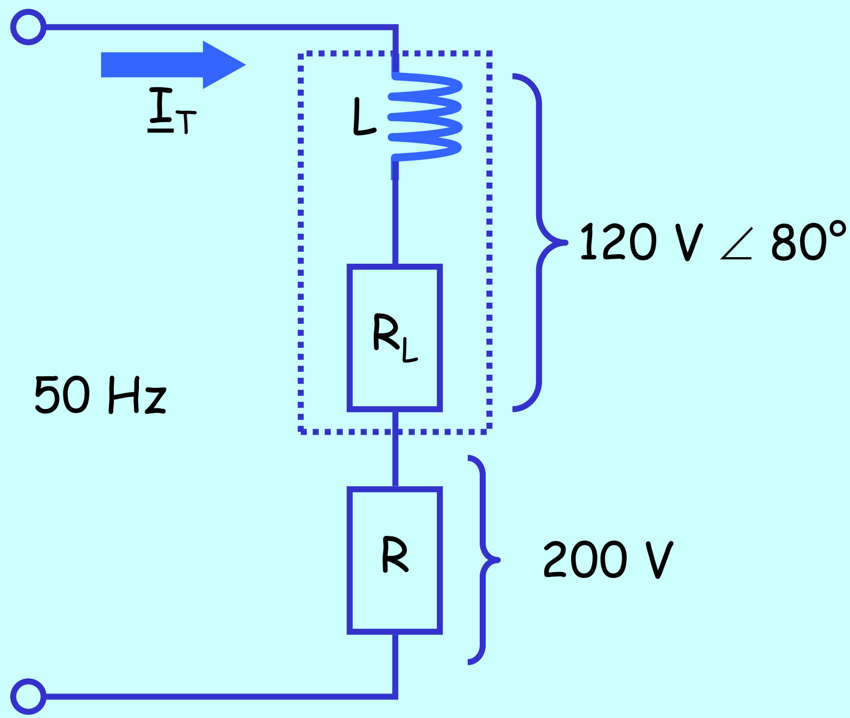
$$S = VI$$

$$P = VI\cos\theta$$

$$Q = VI\sin\theta$$



Exercise 1



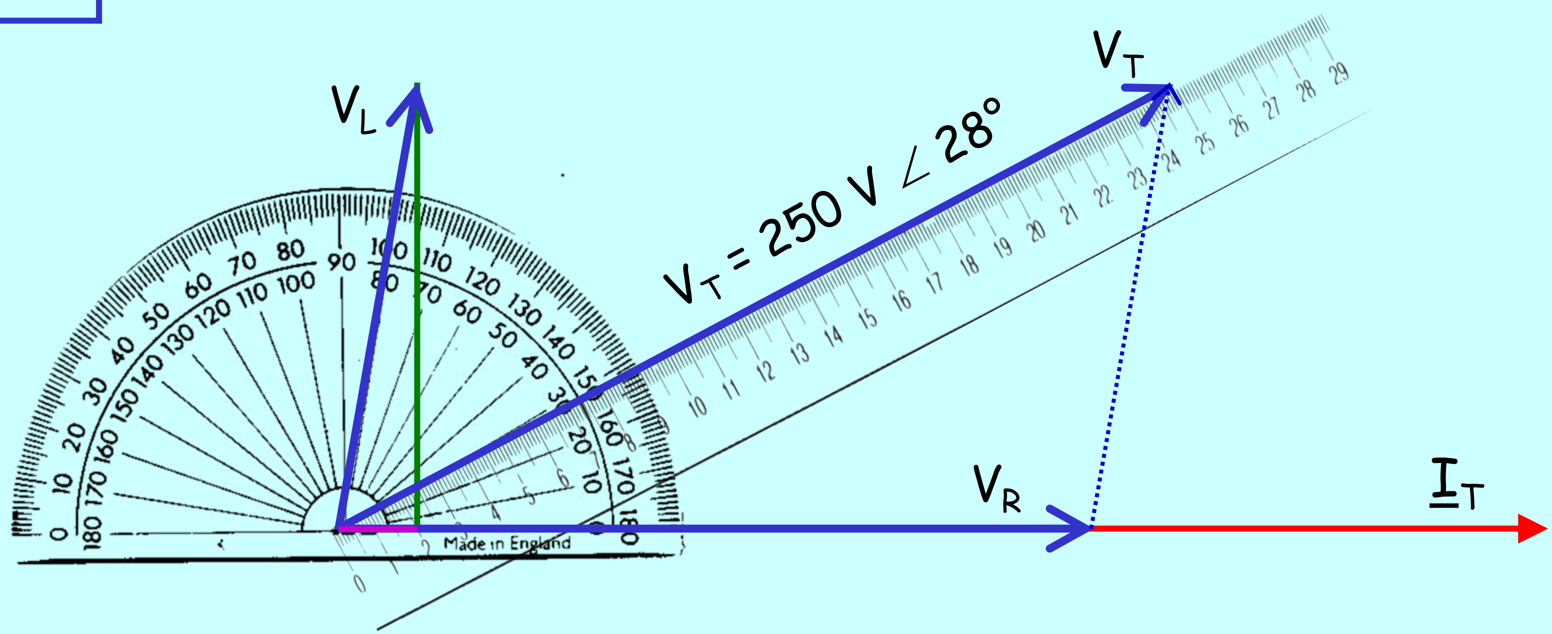
Draw the Phasor diagram for the circuit and find V_T.

$\underline{V}_R = 200\angle 0°$

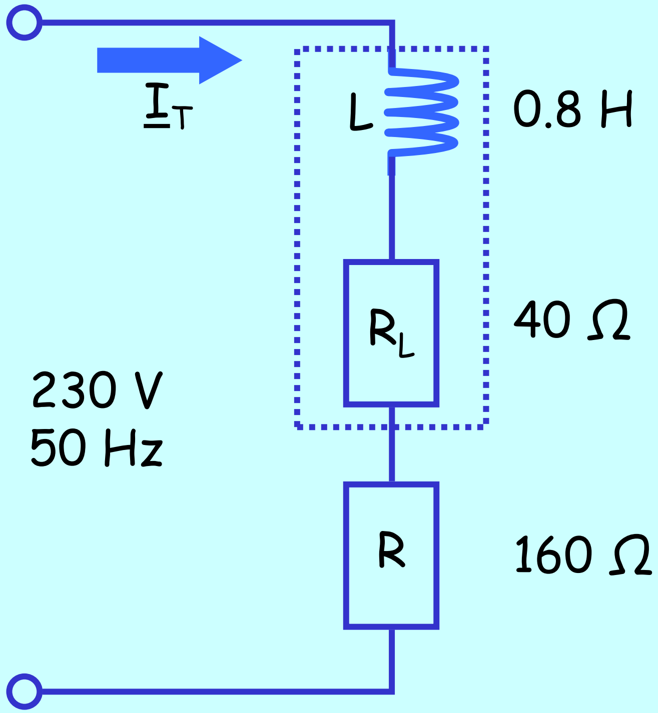
$\underline{V}_L = 120\angle 80°$

$|V_T| = 250.5$ V

$\underline{\theta} = 28°$



Exercise 2



a. R_T

b. X_L

c. Z_T

d. I_T

e. Θ_T

f. Z_L

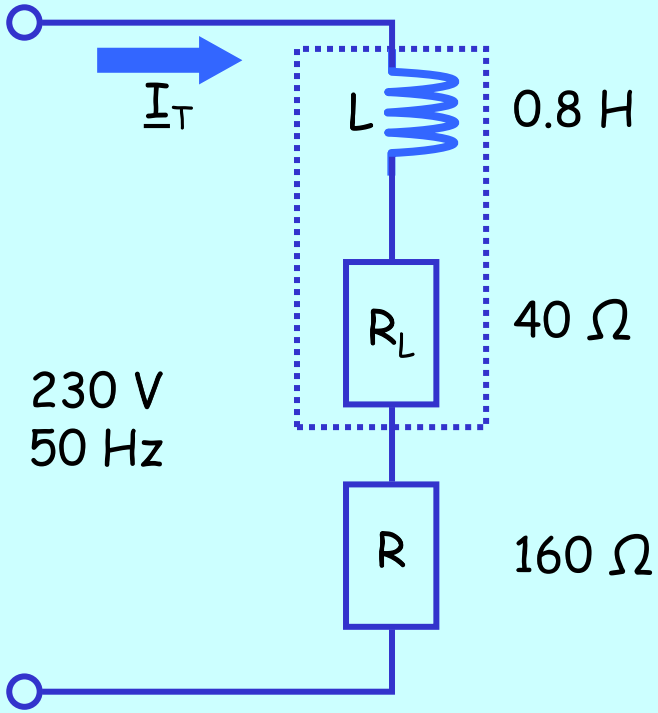
g. Θ_L

h. V_R

i. V_L

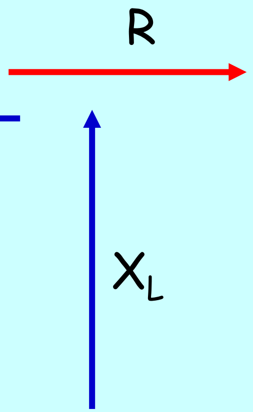
j. *Draw the Phasor diagram for the circuit.*

Exercise 2



a. $R_T = R + R_L$

$R_T = 160 + 40 = 200\Omega$



b. $X_L = 2\pi f L$

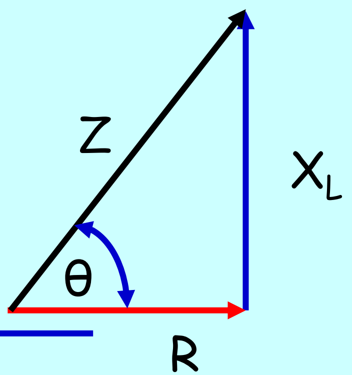
$X_L = 2\pi \times 50 \times 0.8$

$X_L = 251.3\ \Omega$

c. $Z = \sqrt{R_T^2 + X_L^2}$

$Z = \sqrt{200^2 + 251^2}$

$Z = 321\ \Omega$



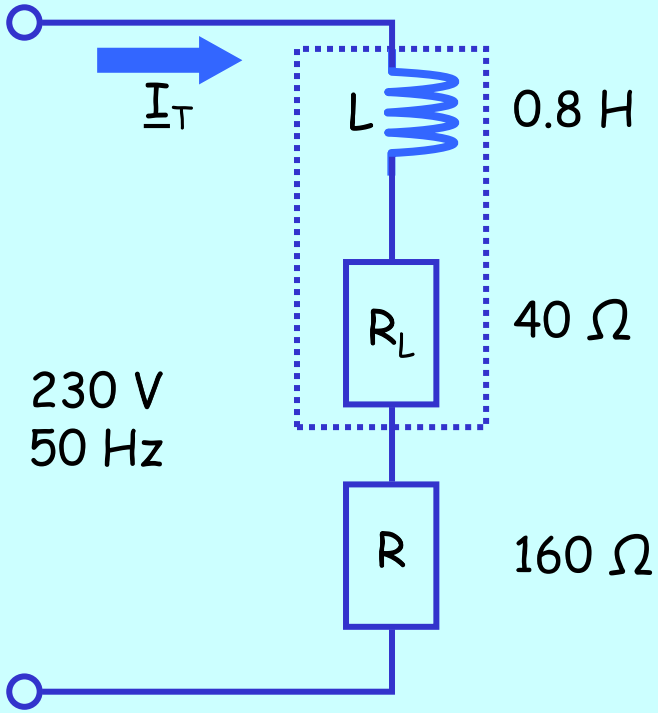
d. $I = \frac{V}{Z}$

$I = \frac{230}{321} = 0.717\ A$

e. $\theta = \tan^{-1}\frac{X}{R}$

$\theta = \tan^{-1}\frac{251}{200} = 51.45^\circ$

Exercise 2 (cont)



$R_T = 200\ \Omega$

$X_L = 251\ \Omega$

$Z = 321\ \Omega$

$I = 0.717\ A$

$\theta = 51.45°$

$$\lambda = \cos 51.45 = 0.623$$

f. $$Z_L = \sqrt{R_L^2 + X_L^2}$$

$$Z_L = \sqrt{40^2 + 251^2}$$

$$Z_L = 254\ \Omega$$

g. $$\theta = \tan^{-1}\frac{X}{R}$$

$$\theta_L = \tan^{-1}\frac{251}{40} = 80.9^\circ$$

h. $$V_R = IR$$

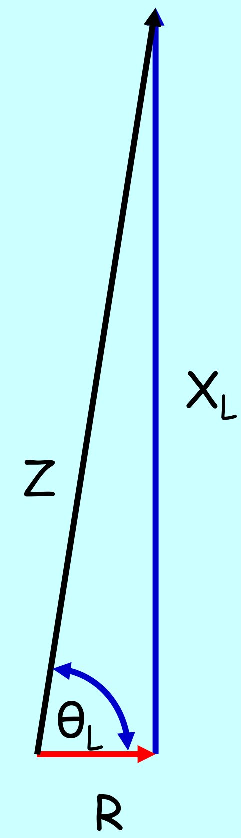
$$V_R = 0.717 \times 160$$

$$V_R = 114.72\ V$$

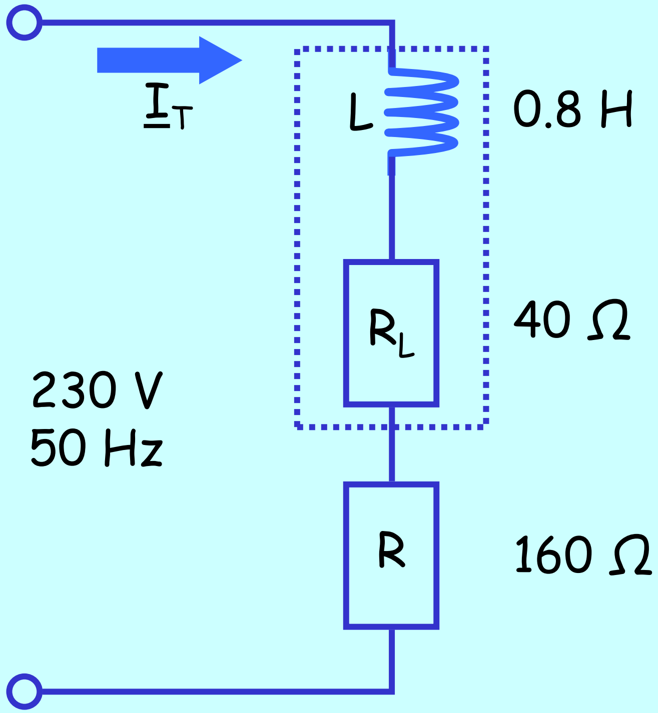
i. $$V_L = IZ_L$$

$$V_L = 0.717 \times 254$$

$$V_L = 182.12\ V$$

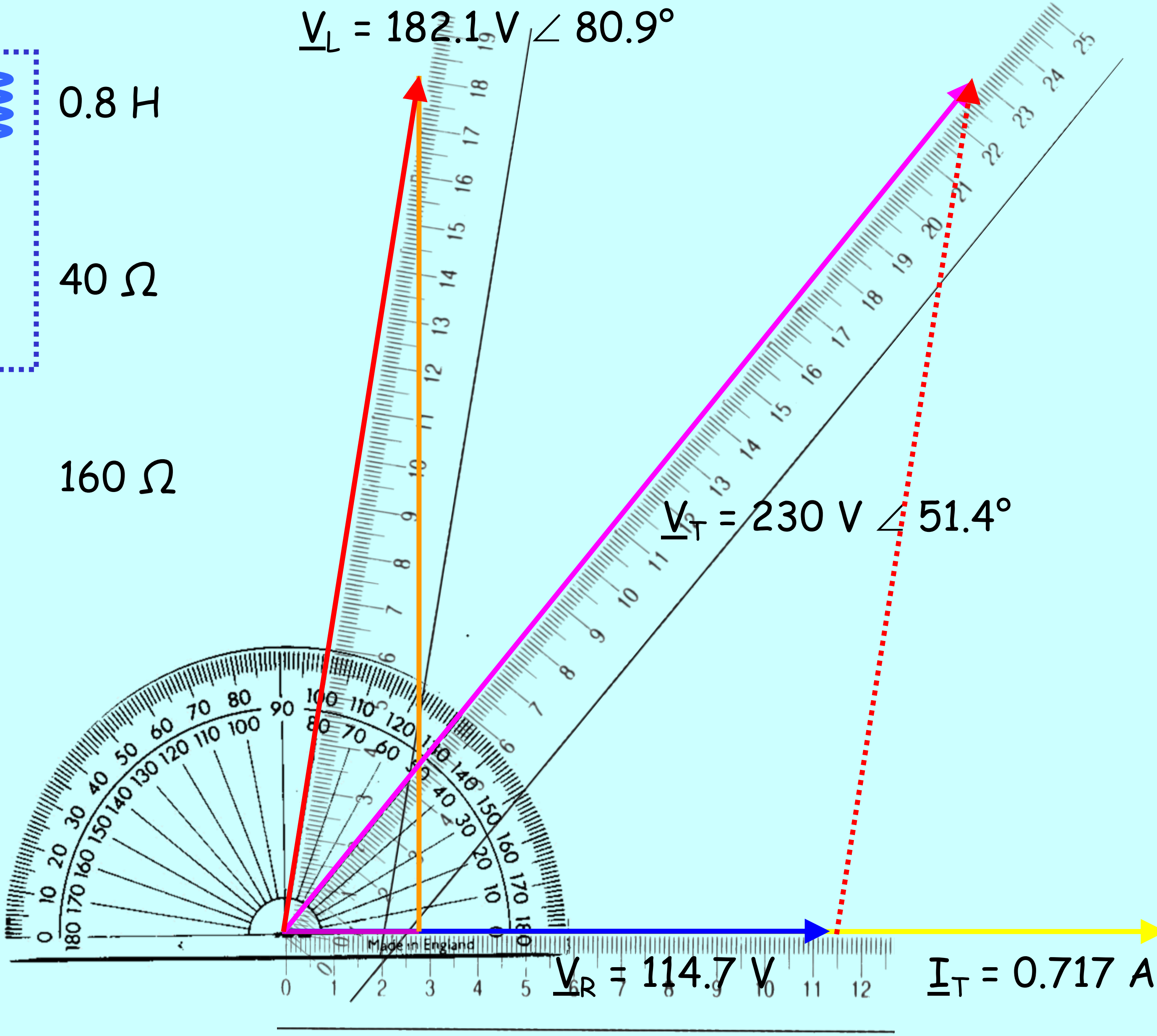


Exercise 2



$|V_T|$ = 230 V

$\underline{\theta}$ = 51°



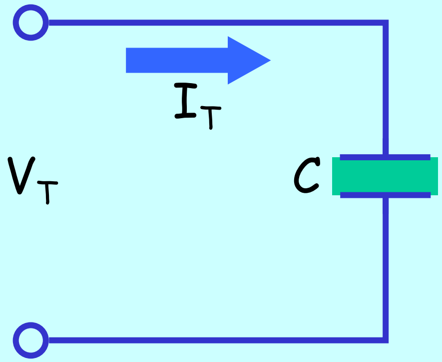
UEENEEG102A
Solve problems in low voltage a.c. circuits

Series RC AC Circuits

Objectives:

At the end of this lesson students should be able to:

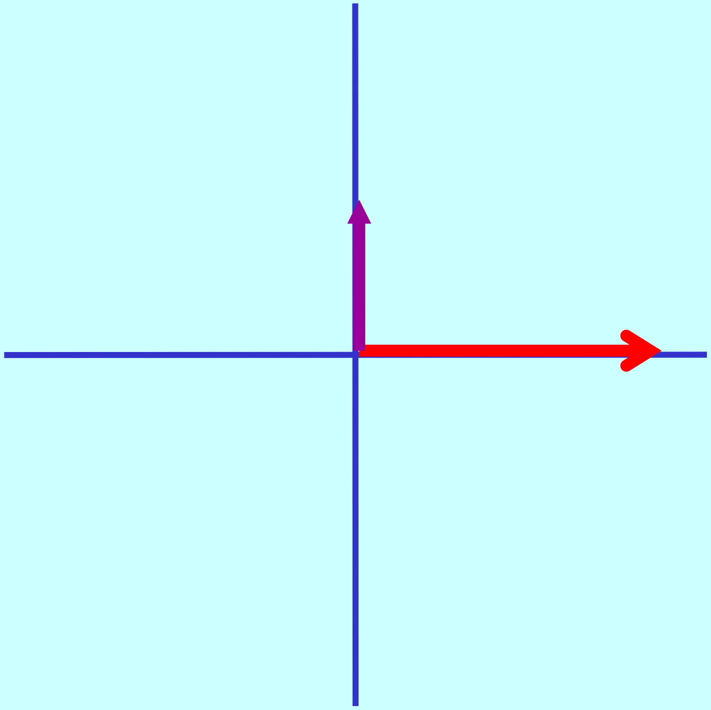
1. Determine circuit quantities and characteristics of Series RC ac circuits.
2. Draw and label Impedance and Power Triangles for Series RC ac circuits.
3. Draw and label the Phasor Diagram for Series RC ac circuits.
4. List a number of practical applications for RC circuits.

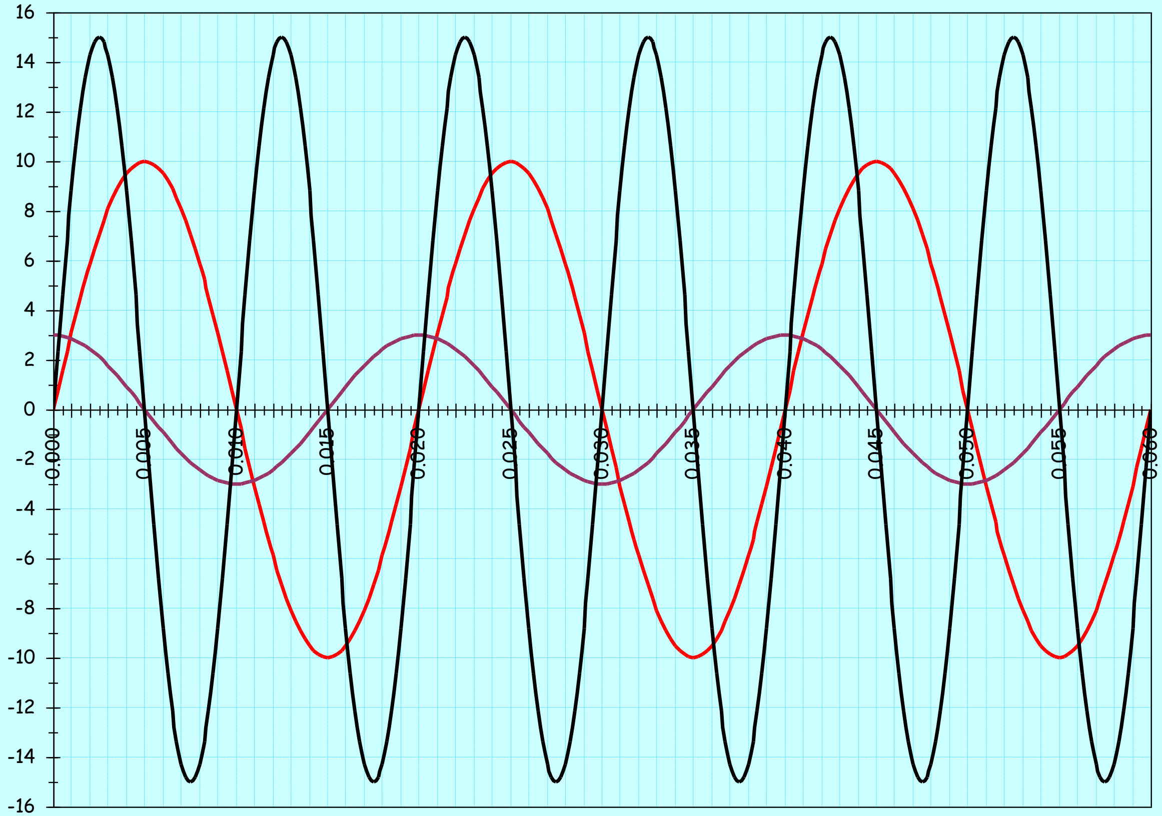


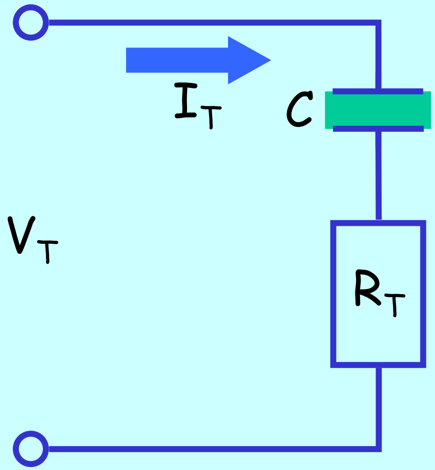
Capacitance is that property of a circuit that opposes changes in voltage.

In a purely CAPACITIVE ac circuit the Current Leads the Voltage by 90°.

Average Real Power is ZERO.

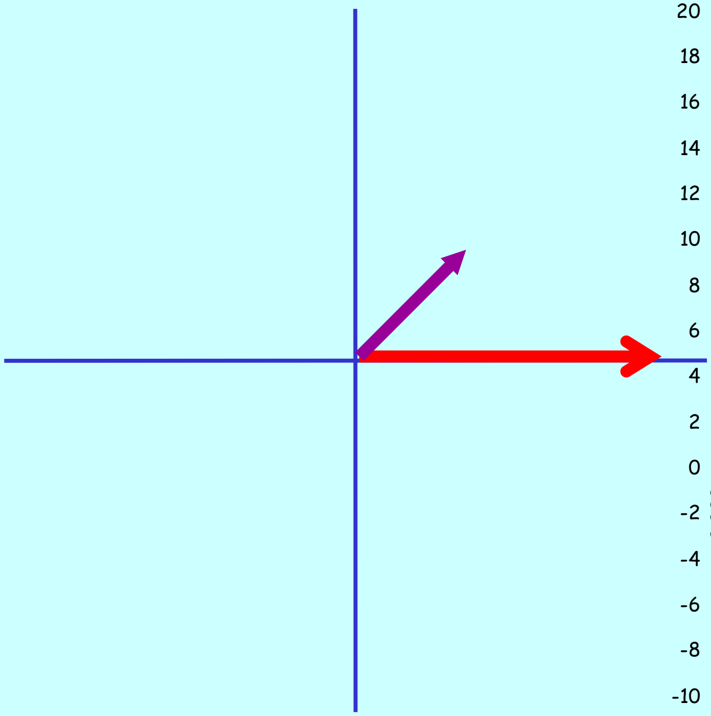


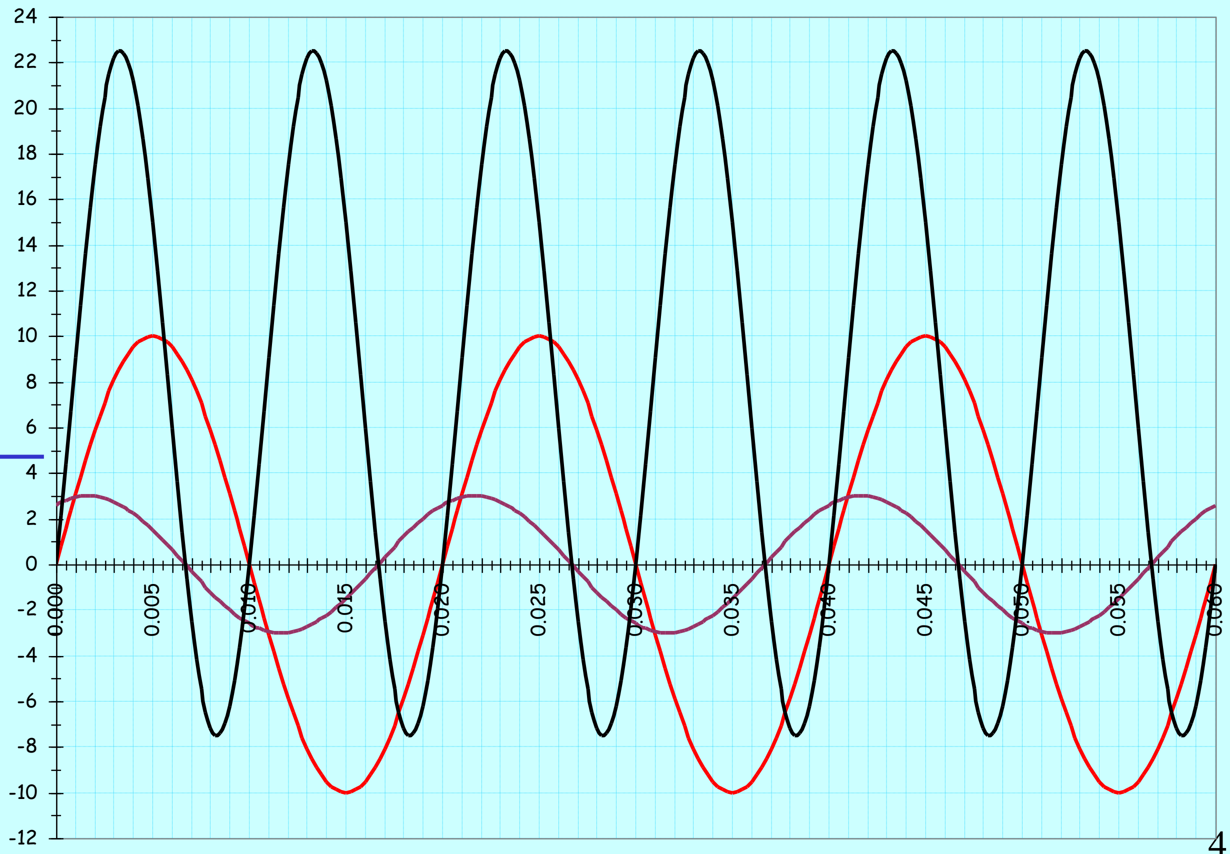




In a practical Capacitive ac circuit there is some Resistance and therefore the Current Leads the Voltage by some angle less than 90°.

Average Real Power is NOT ZERO.





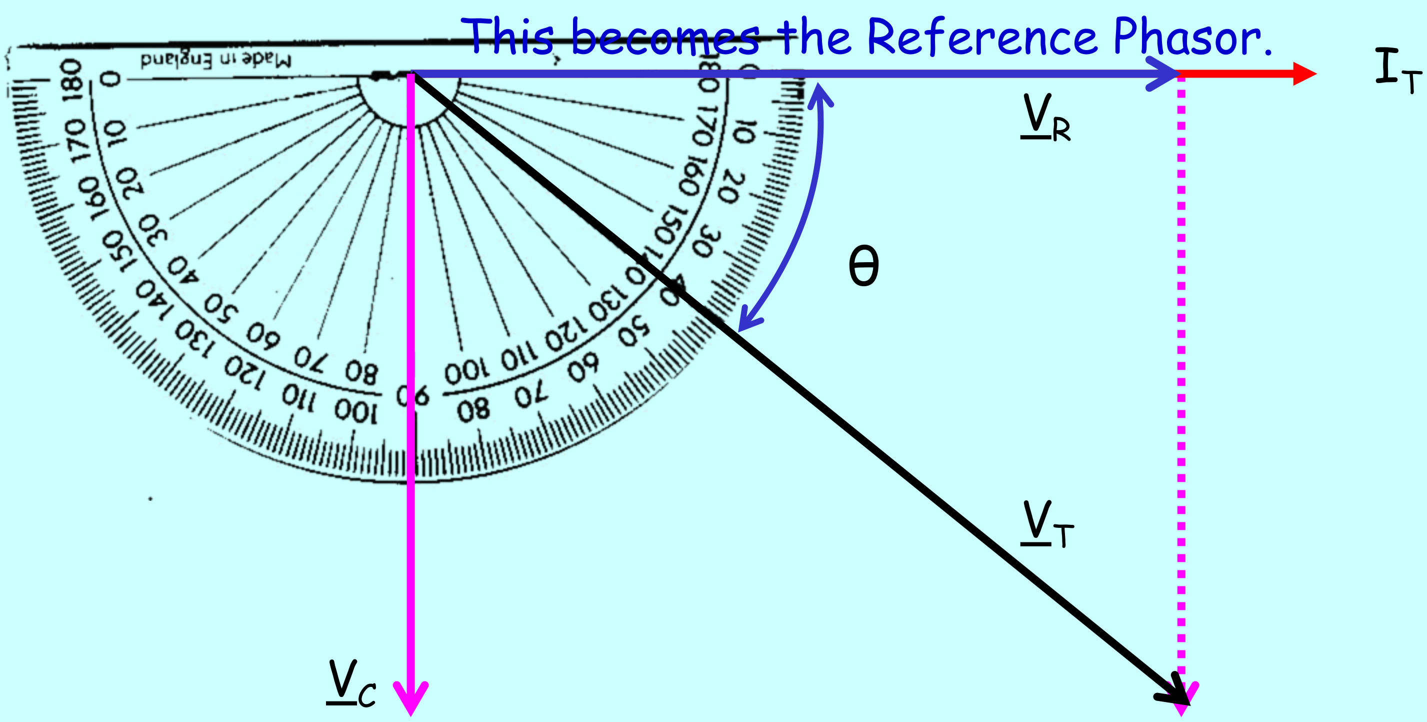
In a SERIES Circuit the common factor is current.

The Voltage across the Resistor ($\underline{V}_R$) is in phase with the current.

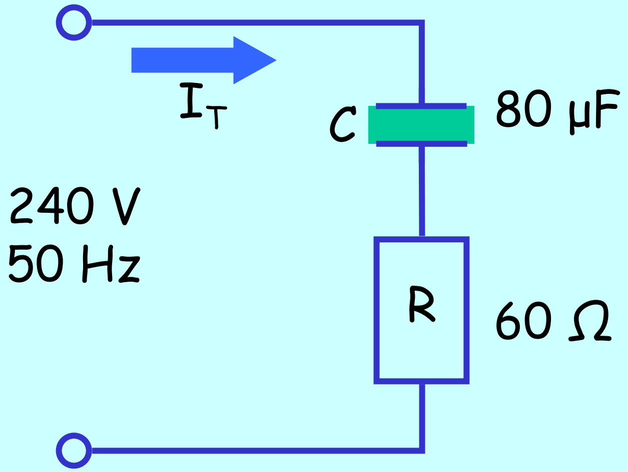
The Voltage across the Capacitor ($\underline{V}_C$) Lags the current, and therefore $\underline{V}_R$, by 90°.

The Algebraic Addition: $\underline{V}_R + \underline{V}_C$ *gives* $\underline{V}_T$.

Which then gives us the circuit's phase angle.



Example



a. $X_C = \dfrac{1}{2\pi f C}$

$X_C = \dfrac{1}{2\pi \times 50 \times 80\mu}$

$X_C = 39.8\Omega$

b. $Z = \sqrt{R_T^2 + X_C^2}$

$Z = \sqrt{60^2 + 39.8^2}$

$Z = 72\Omega$

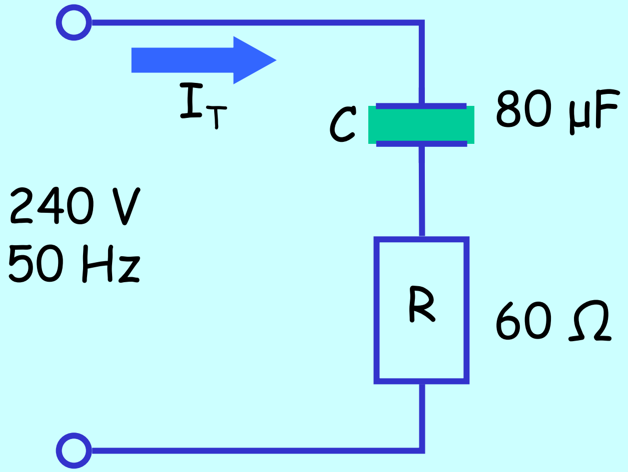
c. $I = \dfrac{V}{Z}$

$I = \dfrac{240}{72} = 3.33\ A$

d. $\theta = \tan^{-1}\dfrac{X}{R}$

$\theta = \tan^{-1}\dfrac{39.8}{60} = 33.56^{\circ}$

Example (cont)



$X_C = 39.8\Omega$

$Z = 72\Omega$

$I = 3.33\ A$

$\theta = 33.56^\circ$

e. $\lambda \equiv \cos\theta$ $\qquad \lambda = \dfrac{P}{S}$

$\lambda = \cos 33.56 = 0.833$

---

f. $V_R \equiv IR$

$V_R = 3.33 \times 60$

$V_R = 199.8\ V$

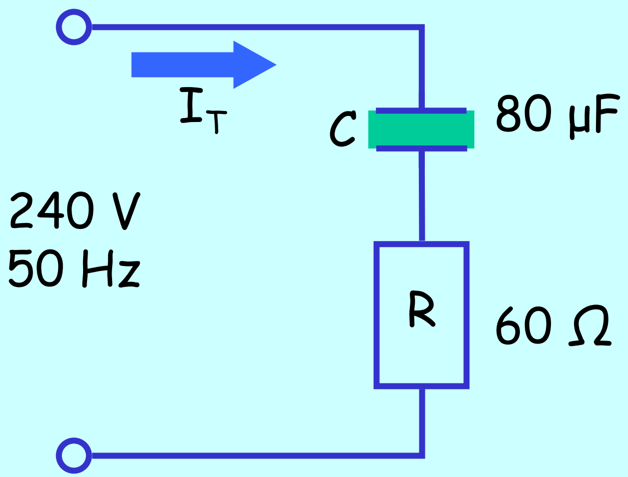
---

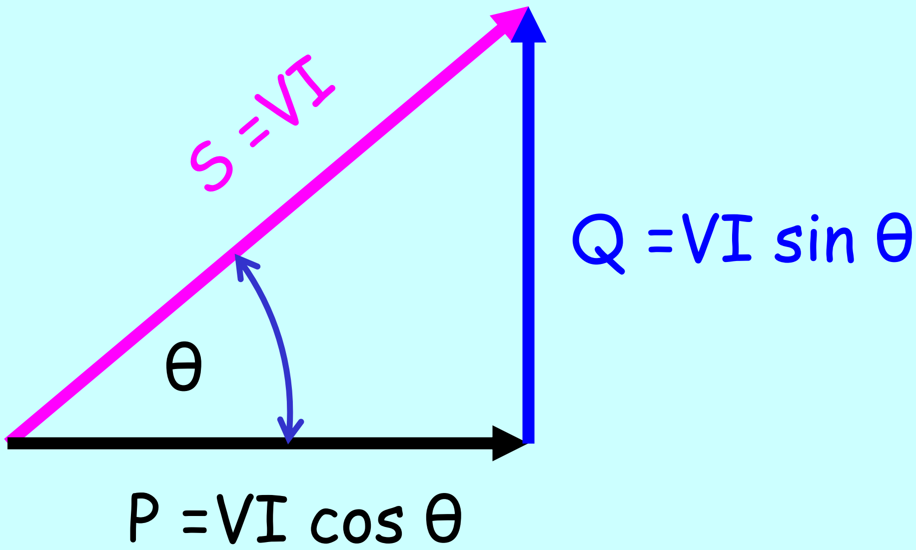
g. $V_C \equiv IX_C$

$V_C = 3.33 \times 39.8$

$V_C = 131.87\ V$

Example (cont)





$$X_C = 39.8\Omega$$
$$Z = 72\Omega$$
$$I = 3.33\,A$$
$$\theta = 33.56^\circ$$

h. $S = VI$

$$S = 240 \times 3.33$$
$$S = 799.2\,VA$$

i. $P = VI\cos\theta$

$$P = 240 \times 3.33 \times \cos 33.56$$
$$P = 665.98\,W$$

j. $Q = VI\sin\theta$

$$Q = 240 \times 3.33 \times \sin 33.56$$
$$Q = 441.81\,Var$$

Example Calculations

C I V I L

$$Z = \frac{V}{I} \qquad Z = \sqrt{R^2 + X^2} \qquad |X_C| = \frac{1}{2\pi f C} \qquad |X_L| = 2\pi f L$$

$$\sin\theta = \frac{X_L}{Z_T} = \frac{V_L}{V_T} = \frac{Q}{S}$$

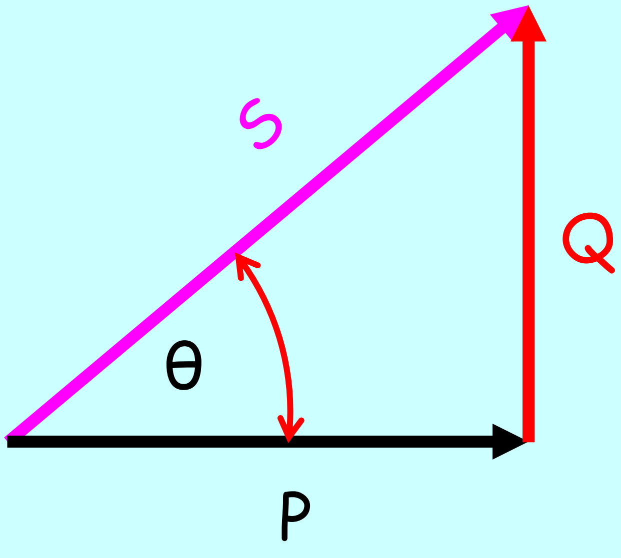
$$\cos\theta = \frac{R_T}{Z_T} = \frac{V_R}{V_T} = \frac{P}{S}$$

$$\tan\theta = \frac{X_L}{R_T} = \frac{V_L}{V_R} = \frac{Q}{P}$$

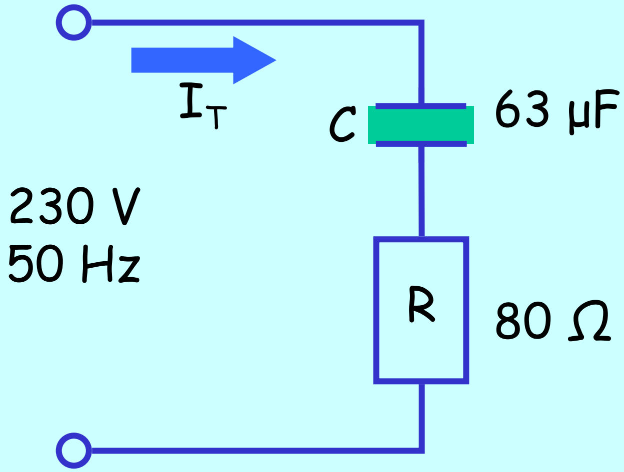
$$S = VI$$

$$P = VI\cos\theta$$

$$Q = VI\sin\theta$$



Example 1



a. R_T

b. X_C

c. Z_T

d. I_T

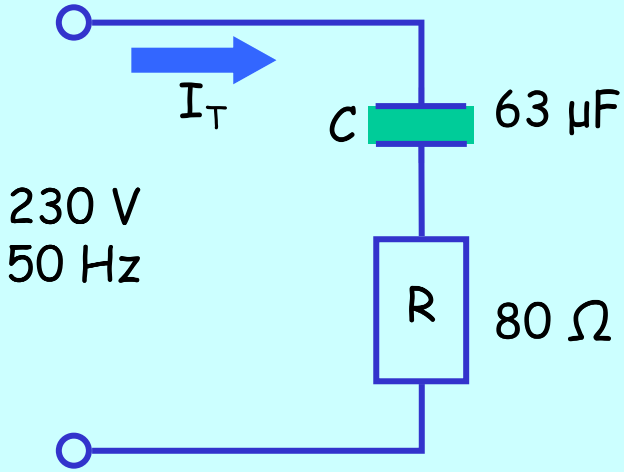
e. Θ_T

f. V_C

g. V_R

h. *Draw the Phasor diagram for the circuit.*

Example 1



a. $R_T = 80\Omega$

b. $X_C = \dfrac{1}{2\pi f C}$

$X_C = \dfrac{1}{2\pi \times 50 \times 63\mu}$

$X_C = 50.5\ \Omega$

c. $Z = \sqrt{R_T^2 + X_C^2}$

$Z = \sqrt{80^2 + 50.5^2}$

$Z = 94.6\ \Omega$

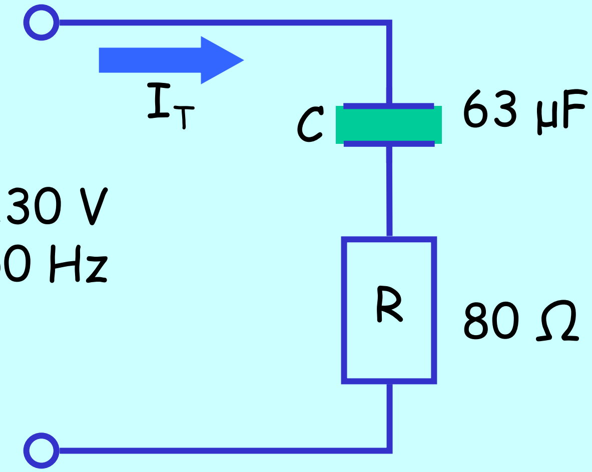
d. $I = \dfrac{V}{Z}$

$I = \dfrac{230}{94.6} = 2.43\ A$

e. $\theta = \tan^{-1}\dfrac{X}{R}$

$\theta = \tan^{-1}\dfrac{50.5}{80} = 32.26^\circ$

Example 1 (cont)



f.

$$V_C = IX_C$$
$$V_C = 2.43 \times 50.5$$
$$V_C = 122.72\,V$$

g.

$$V_R = IR$$
$$V_R = 2.43 \times 80$$
$$V_R = 194.4\,V$$

$X_C = 50.5\,\Omega$

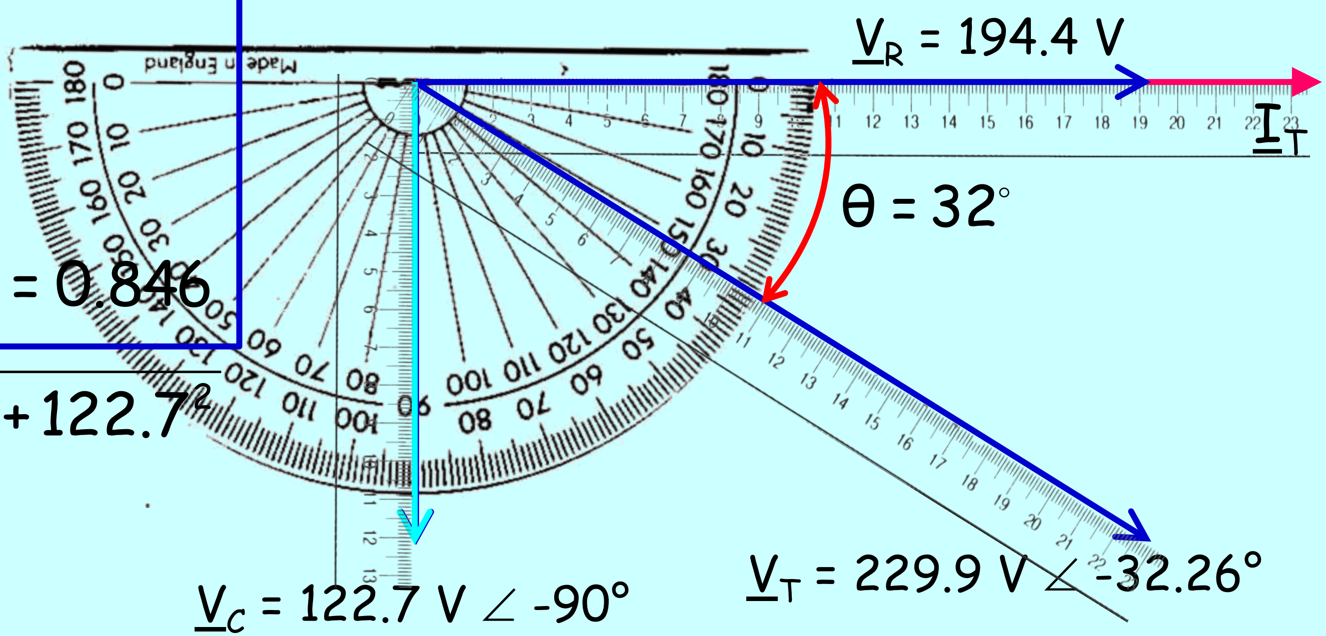
$Z = 94.6\,\Omega$

$I = 2.43\,A$

$\theta = 32.26^\circ$

$\lambda = \cos 32.26 = 0.846$

$$|V_T| = \sqrt{194.4^2 + 122.7^2}$$
$$|V_T| = 229.9\,V$$



End of Lesson

Practical Exercises

Series RL ac circuits

Series RC ac circuits

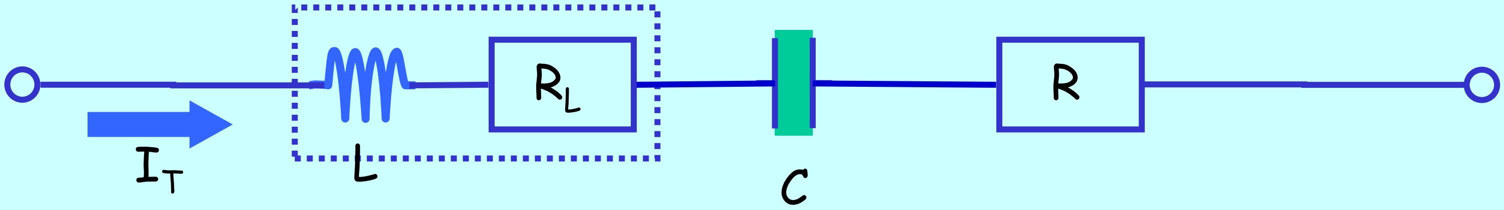
UEENEEG102A
Solve problems in low voltage a.c. circuits

Series RLC AC Circuits

Objectives:

At the end of this lesson students should be able to:

1. Determine circuit quantities and characteristics of RLC Series Circuits.
2. Draw and label Impedance and Power Triangles for RLC Series Circuits.
3. Draw and label the Phasor Diagram for RLC Series Circuits.
4. State the effect of, and calculate the frequency of resonance in an RLC Series Circuits.
5. List a number of practical applications for RLC Series Circuits.



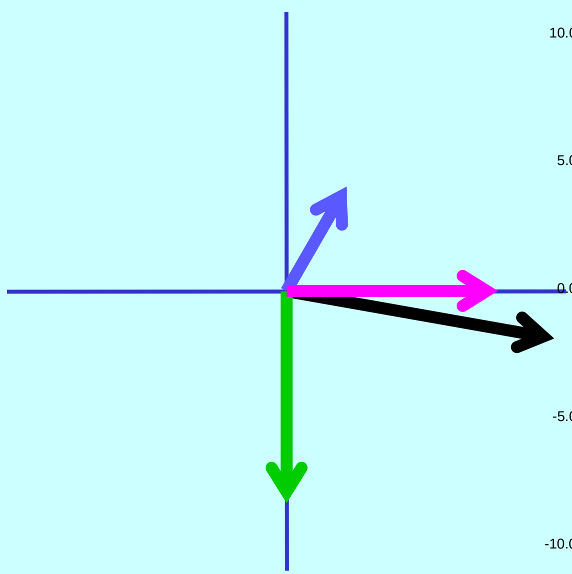
In a practical Series ac Circuit the Current is Common.

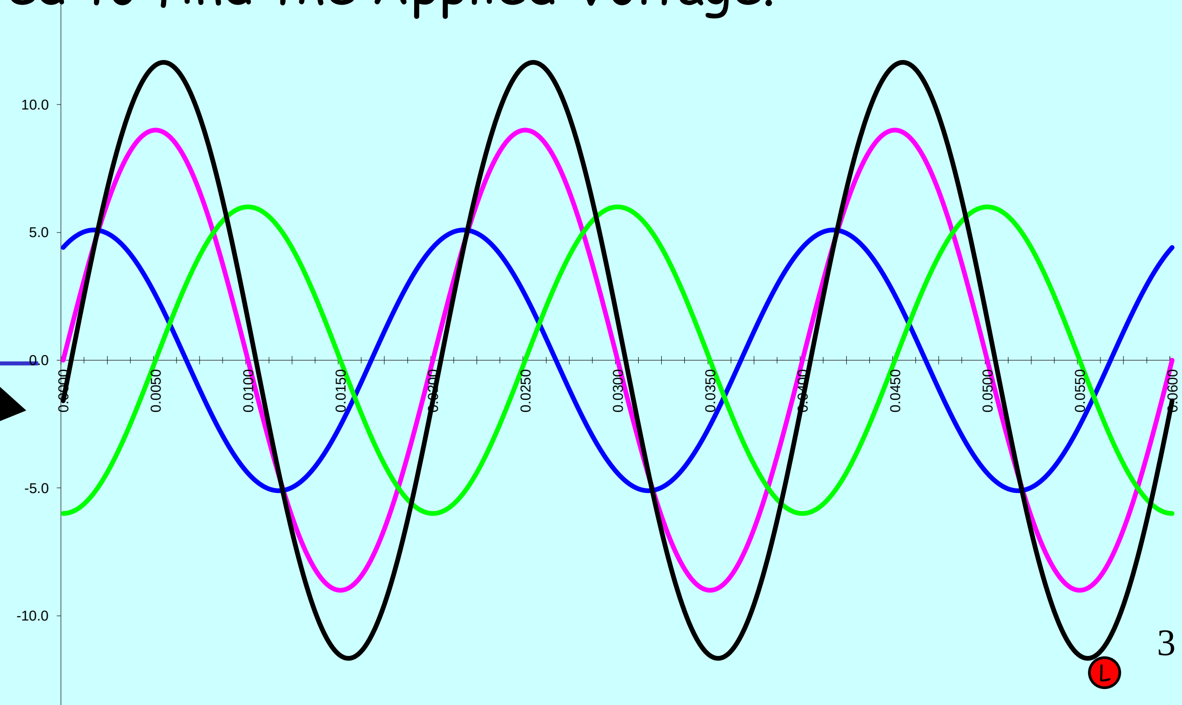
The voltage across the Resistor is in phase with the current,

The voltage across the Inductor leads the current by some angle less than 90°,

The voltage across the Capacitor lags the current by about 90°.

and these can be resolved to find the Applied Voltage.

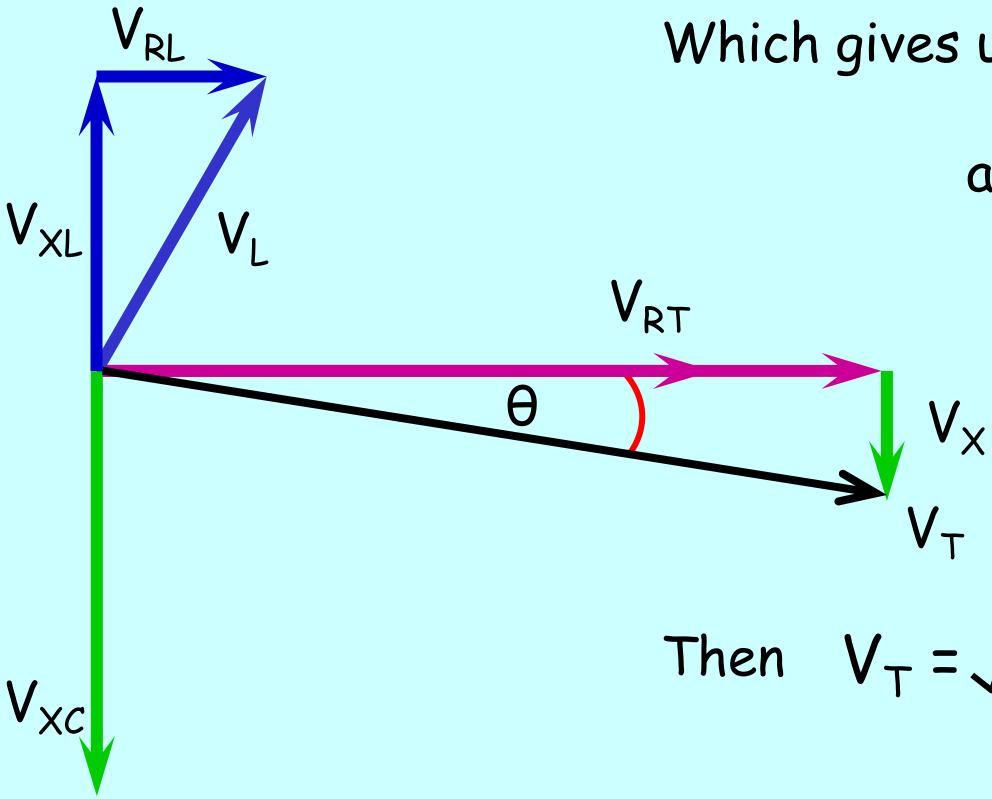




The Voltage Phasor diagram

can be resolved thus:

V_L has components V_{XL} and V_{RL}.



Which gives us: $V_{RT} = V_R + V_{RL}$

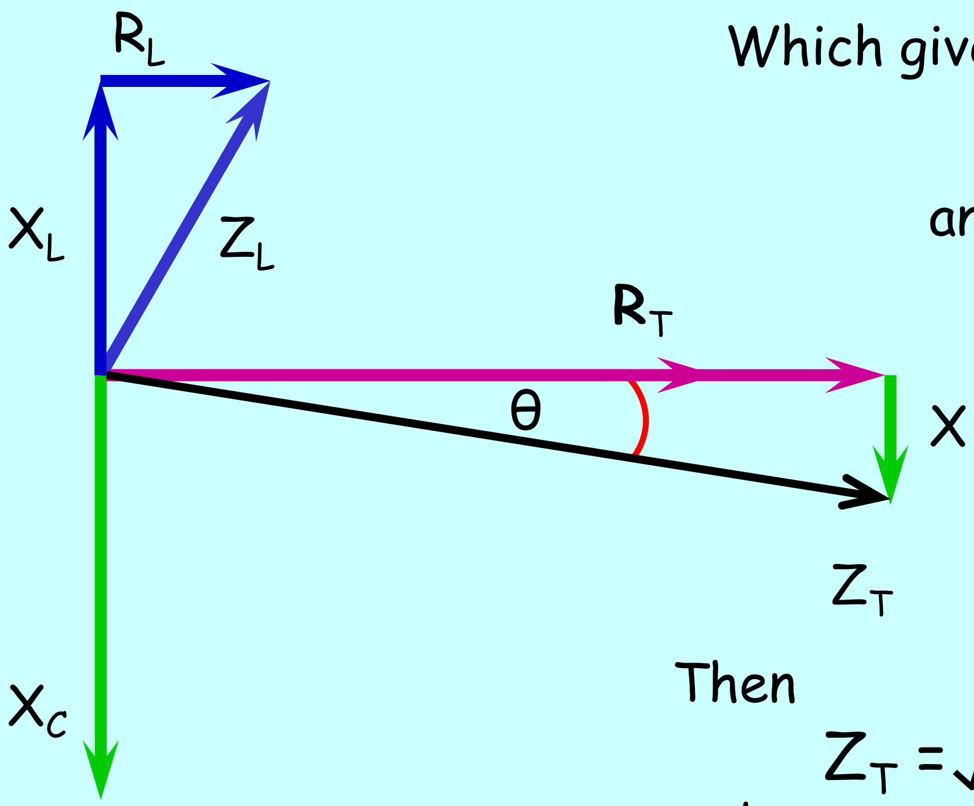
and $V_X = V_{XL} - V_{XC}$

Then $V_T = \sqrt{\{(V_R + V_{RL})^2 + (V_{XL} - V_{XC})^2\}}$

and $\theta = \tan^{-1}\frac{V_X}{V_{RT}}$

The same thing happens with the Impedance Triangle.

Z_L has components X_L and R_L.



Which gives us:

$$R_T = R + R_L$$

and

$$X = X_L - X_C$$

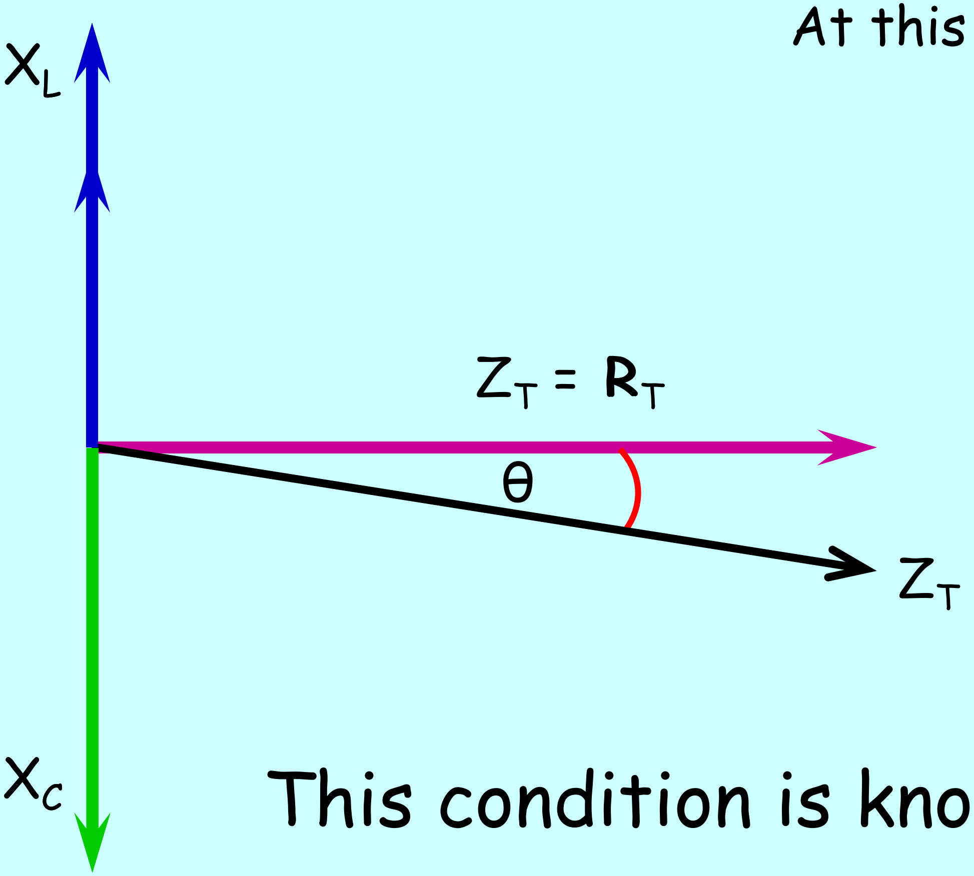
Then

$$Z_T = \sqrt{\{(R + R_L)^2 + (X_L - X_C)^2\}}$$

and

$$\theta = \tan^{-1} \frac{X}{R_T}$$

A special condition occurs when $X_L = X_C$.



At this point: $X = X_L - X_C = 0$

$R_T = R$

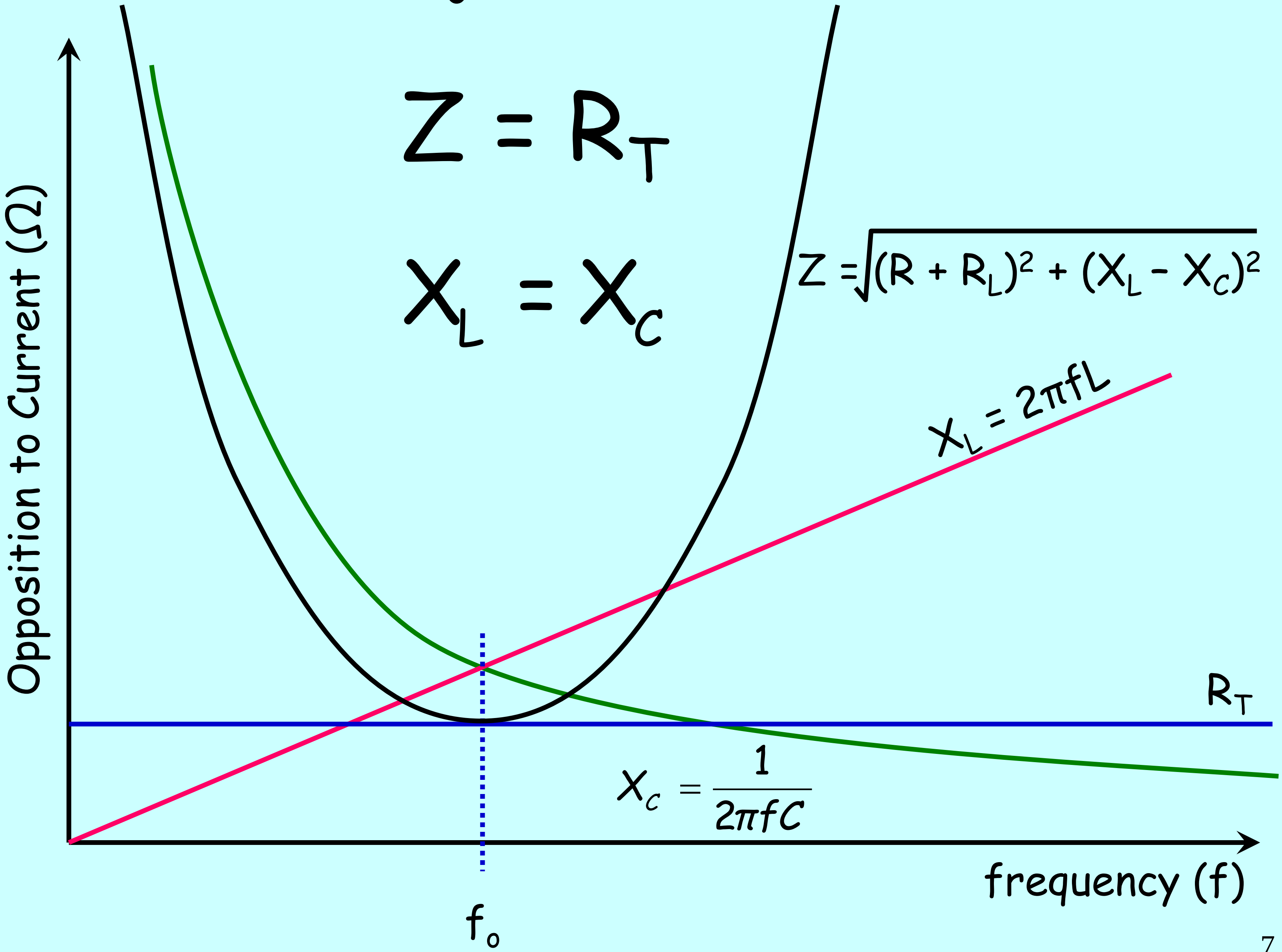
$Z_T = R$

$\theta = 0$

and $\lambda = 1$

This condition is known as Resonance.

At Resonance (f_o):



At Resonance (f_o):

$$Z = R_T$$

Circuit Impedance is a MINIMUM at Resonance (f_o).

$$X_L = X_C$$

$$2\pi f_o L = \frac{1}{2\pi f_o C}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Example Calculations

C I V I L

$$Z = \frac{V}{I}$$

$$Z = \sqrt{R^2 + X^2}$$

$$|X_C| = \frac{1}{2\pi f C}$$

$$|X_L| = 2\pi f L$$

$$\sin\theta = \frac{X_L}{Z_T} = \frac{V_L}{V_T} = \frac{Q}{S}$$

$$\cos\theta = \frac{R_T}{Z_T} = \frac{V_R}{V_T} = \frac{P}{S}$$

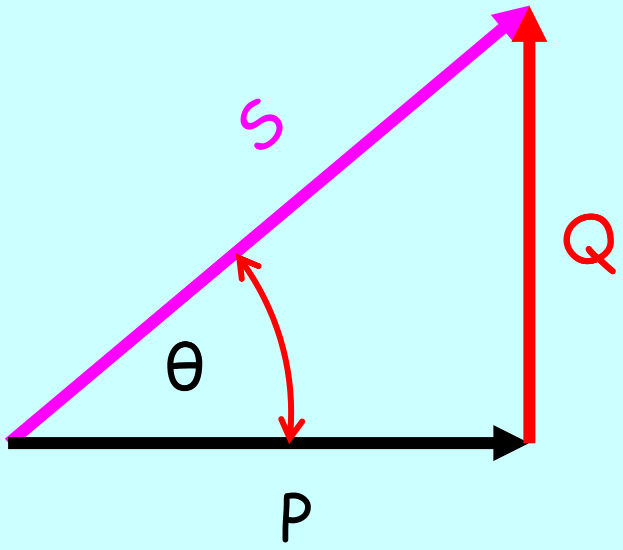
$$\tan\theta = \frac{X_L}{R_T} = \frac{V_L}{V_R} = \frac{Q}{P}$$

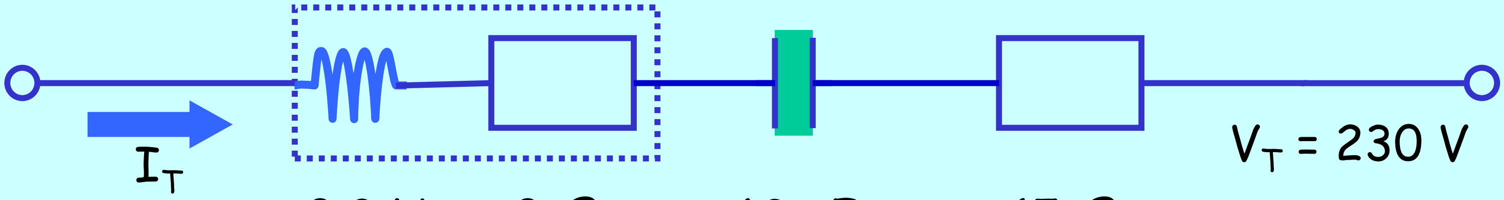
$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$S = VI$$

$$P = VI\cos\theta$$

$$Q = VI\sin\theta$$





X_L = ?

X_C = ?

Z = ?

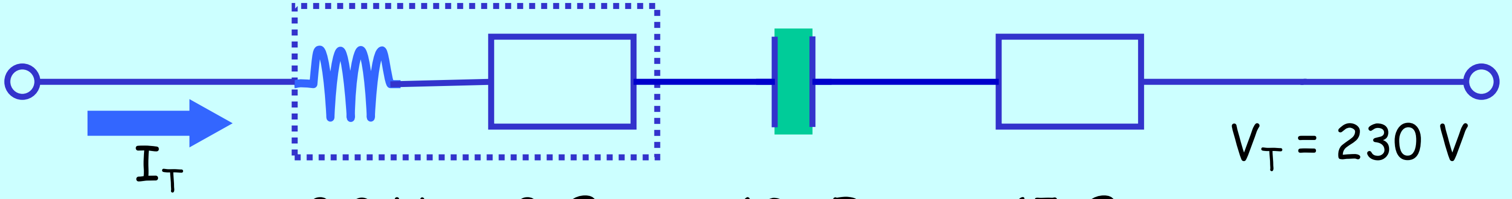
I = ?

θ = ?

V = ?

V = ?

V = ?



$$X_L = 2\pi f L = 100\pi \times 0.2 = 62.83\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{100\pi \times 63\mu} = 50.53\ \Omega$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{15^2 + 12.3^2} = 19.4\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{19.4} = 11.86\ A$$

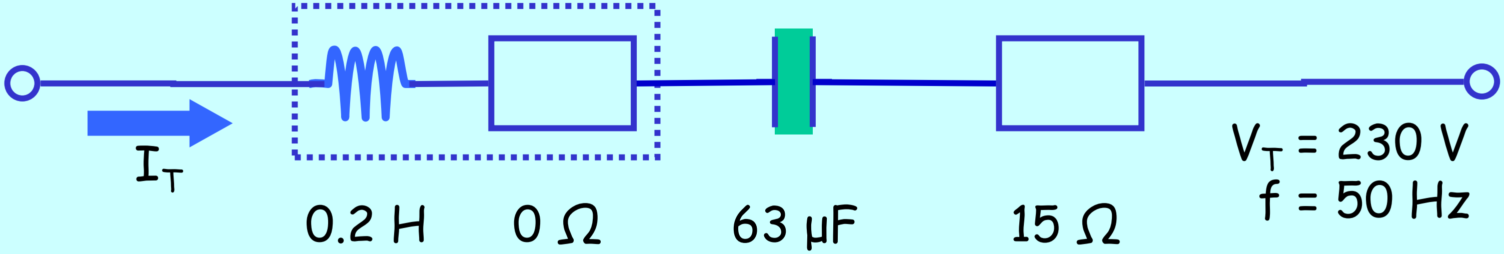
$$\theta = \cos^{-1}\frac{R}{Z} = \cos^{-1}\frac{15}{19.4} = 39.36^{o}$$

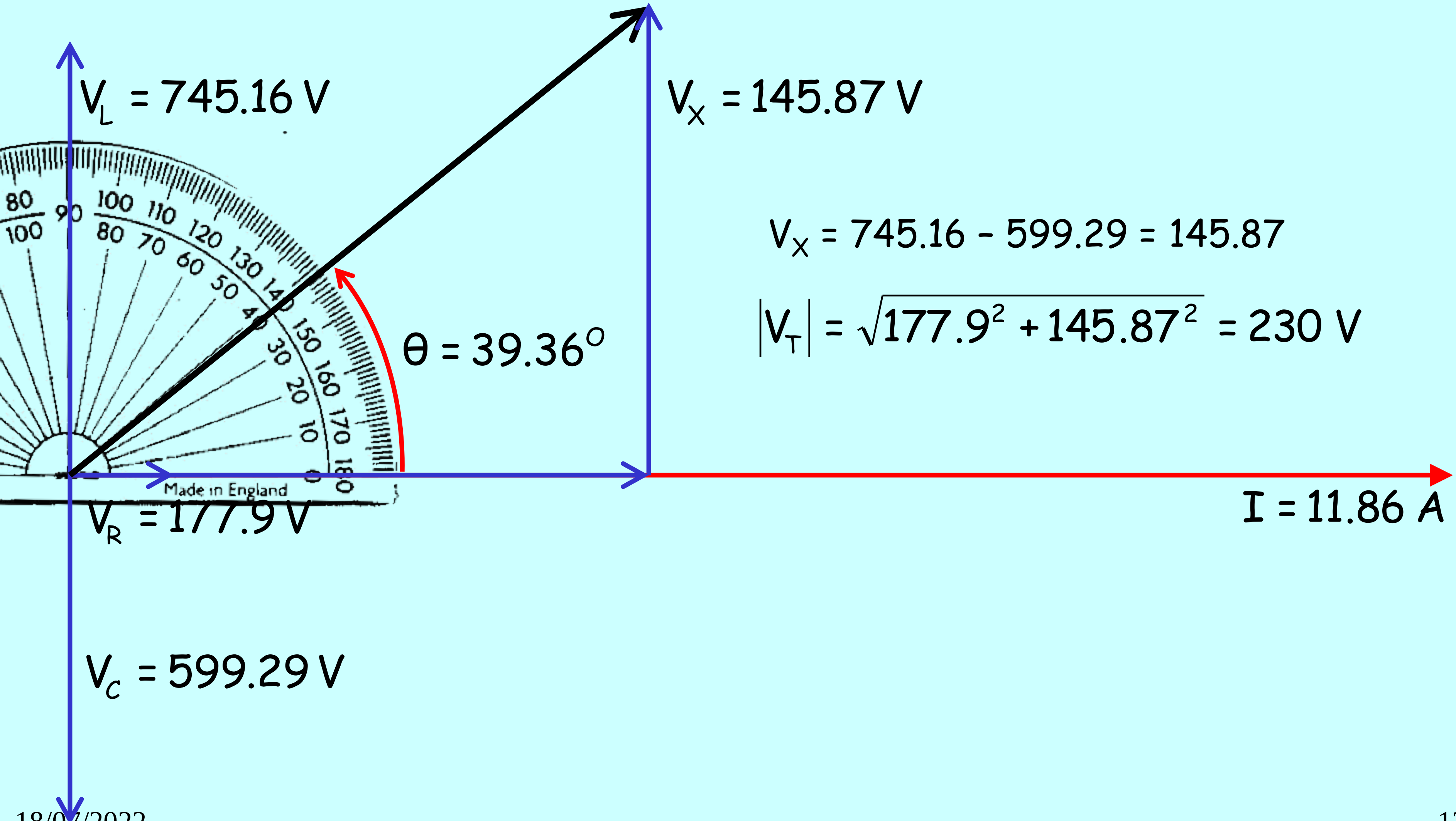
$$V = IR = 11.86 \times 15 = 177.9\ V$$

$$V = IX_L = 11.86 \times 62.83 = 745.16\ V$$

$$V = IX_C = 11.86 \times 50.53 = 599.29\ V$$

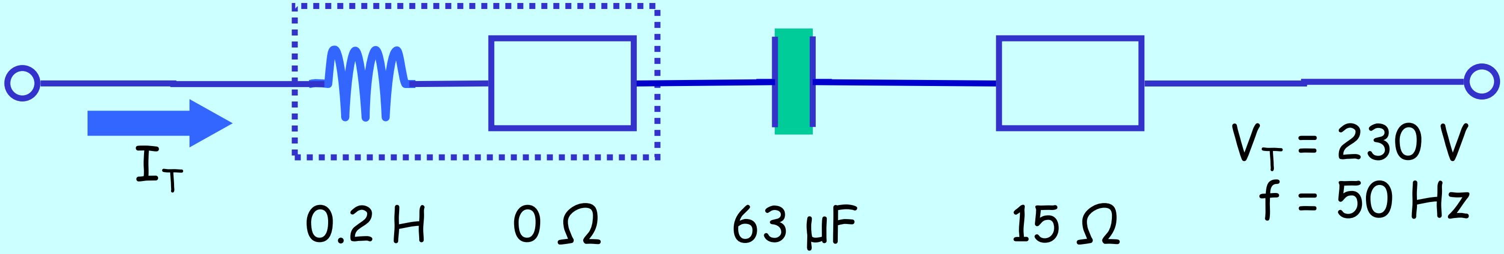
Pg. 160





$$V_X = 745.16 - 599.29 = 145.87$$

$$|V_T| = \sqrt{177.9^2 + 145.87^2} = 230\ V$$



$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$Z = 15\ \Omega$$

$$f_o = \frac{1}{2\pi\sqrt{0.2 \times 63\mu}}$$

$$\theta = 0^\circ$$

$$f_o = 44.84\ Hz$$

$$\lambda = 1$$

End of Lesson

Practical Exercises

Series RLC Circuits.

UEENEEG102A
Solve problems in low voltage a.c. circuits

Parallel RLC AC Circuits

Objectives:

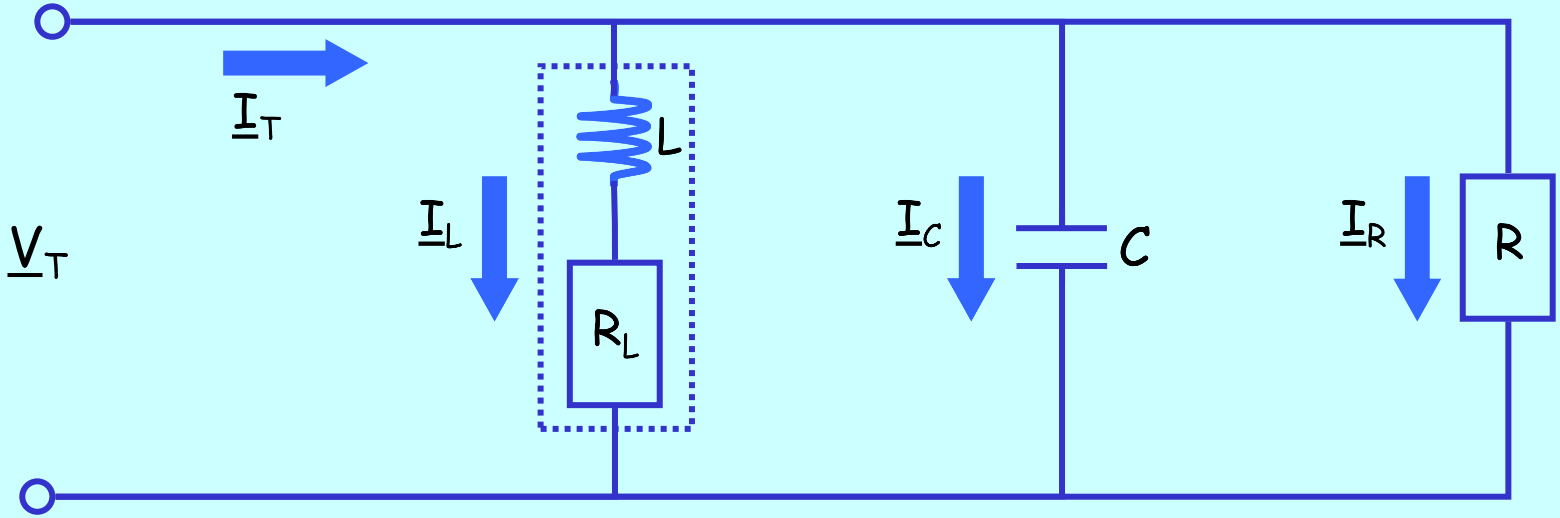
At the end of this lesson students should be able to:

1. Determine circuit quantities and characteristics of ac Parallel Circuits.

2. Draw and label Impedance and Power Triangles for ac Parallel Circuits.

3. Draw and label the Phasor Diagram for ac Parallel Circuits.

4. List a number of practical applications for ac Parallel Circuits.



In a parallel circuit Voltage is common. $\underline{V}_T = \underline{V}_L = \underline{V}_C = \underline{V}_R$

C I V I L

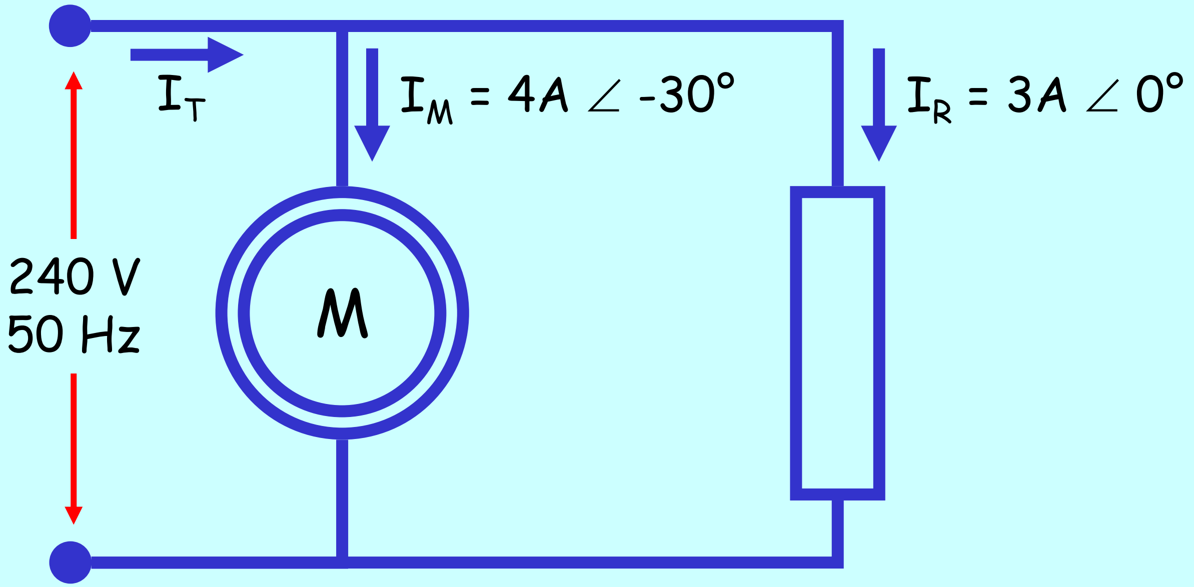
The current through the Inductor lags the applied voltage by some angle less than 90°.

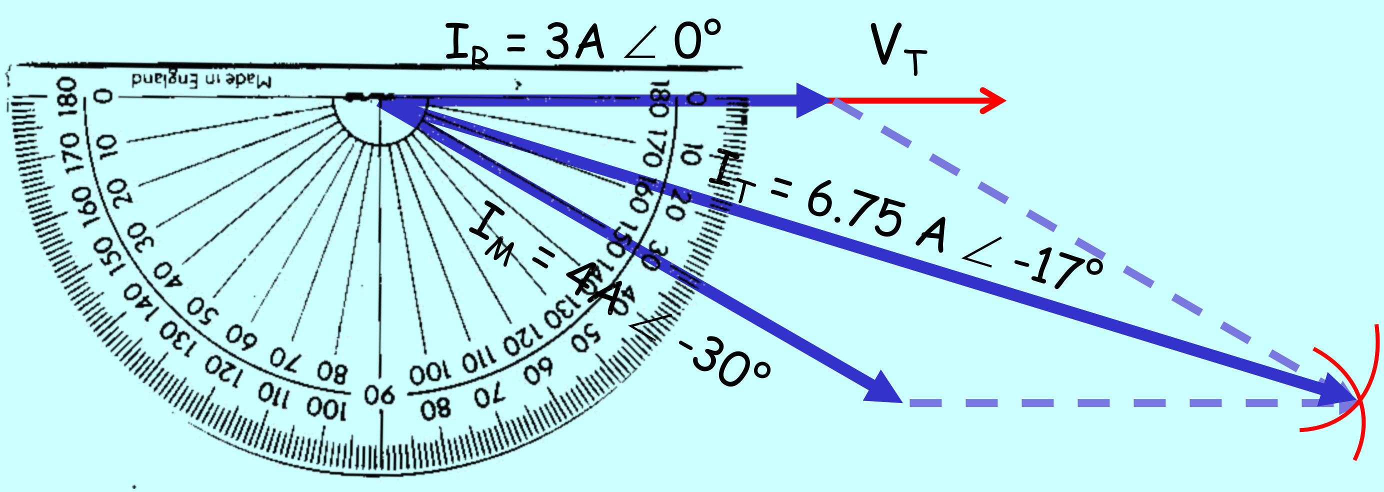
The current through the Capacitor leads the applied voltage by about 90°.

The current through the Resistor is in phase with the applied voltage.

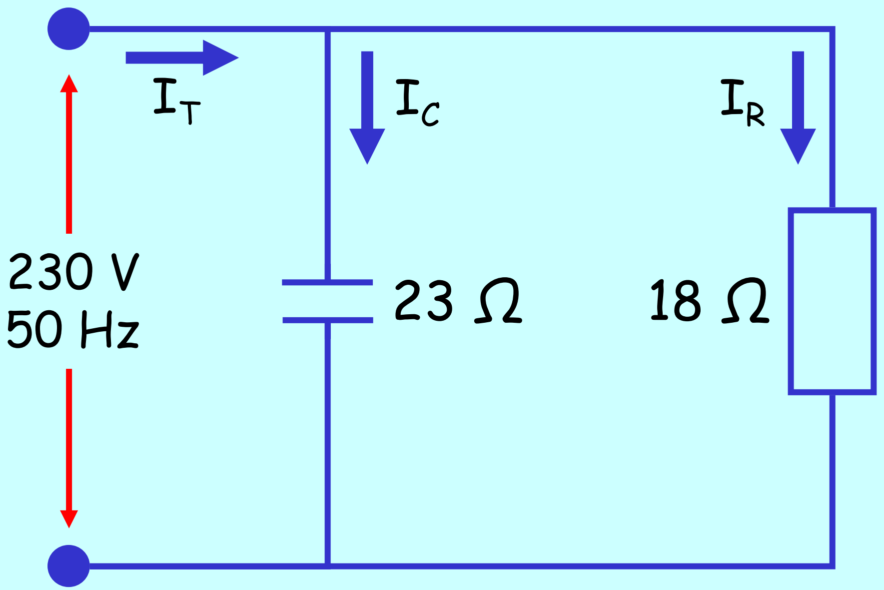
The total current is the algebraic sum of the branch currents.

$$\underline{I}_T = \underline{I}_L + \underline{I}_C + \underline{I}_R$$


I_T
$I_M = 4A \angle -30°$
$I_R = 3A \angle 0°$
240 V
50 Hz
M


$I_R = 3A \angle 0°$
V_T
$I_T = 6.75\ A \angle -17°$
$I_M = 4A \angle -30°$
Made in England

Pg. 175 Ex. 2

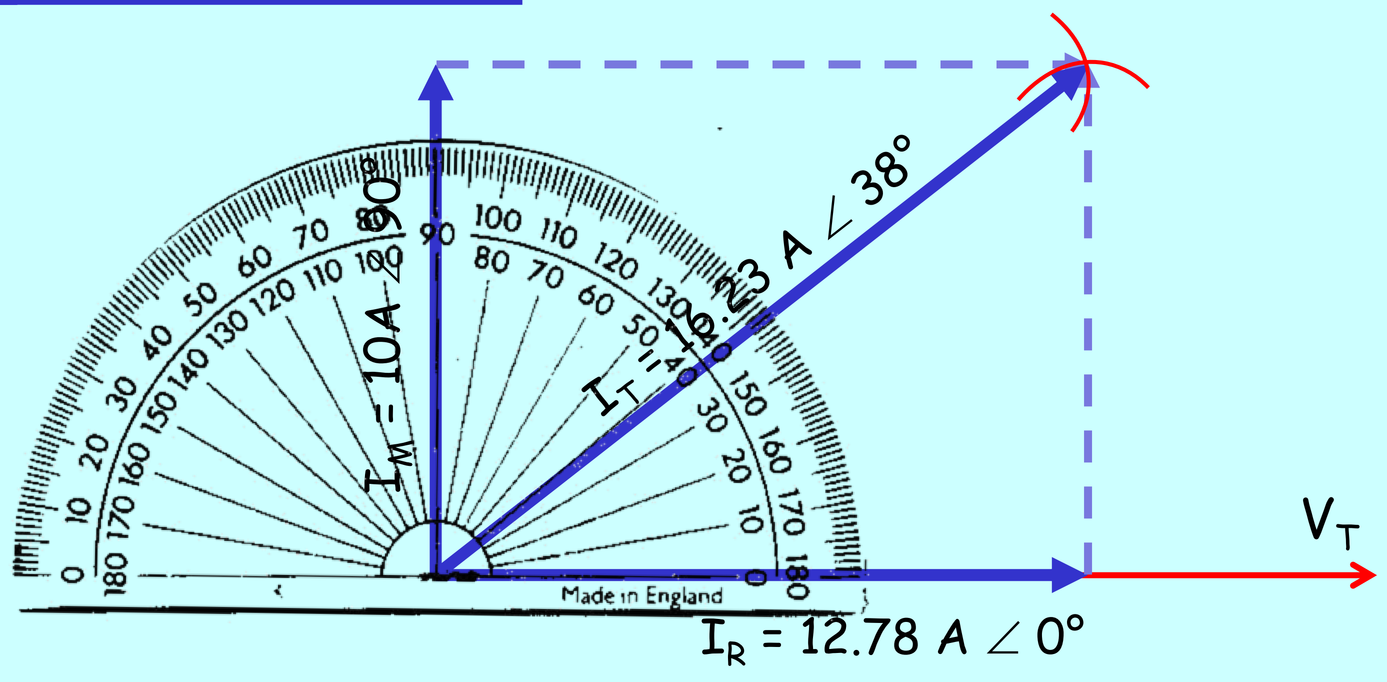


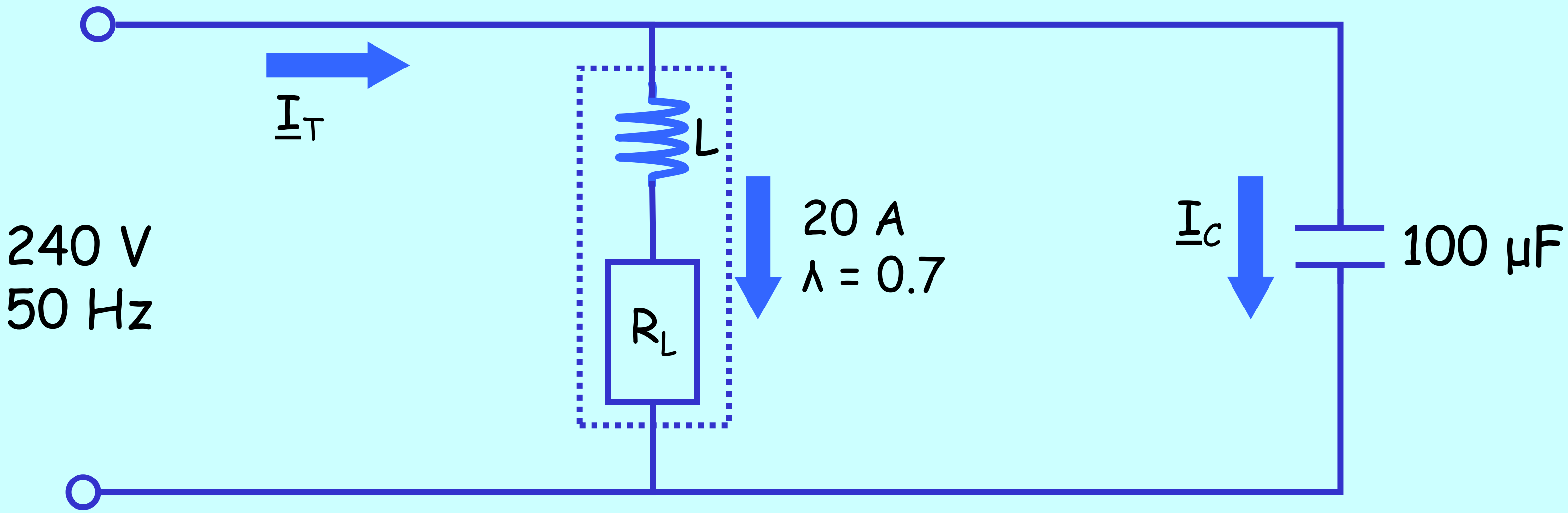
$$I_R = \frac{230}{18} = 12.78\ A$$

$$I_R = (12.78 \angle 0)$$

$$I_C = \frac{230}{23} = 10\ A$$

$$I_C = (10 \angle 90)$$





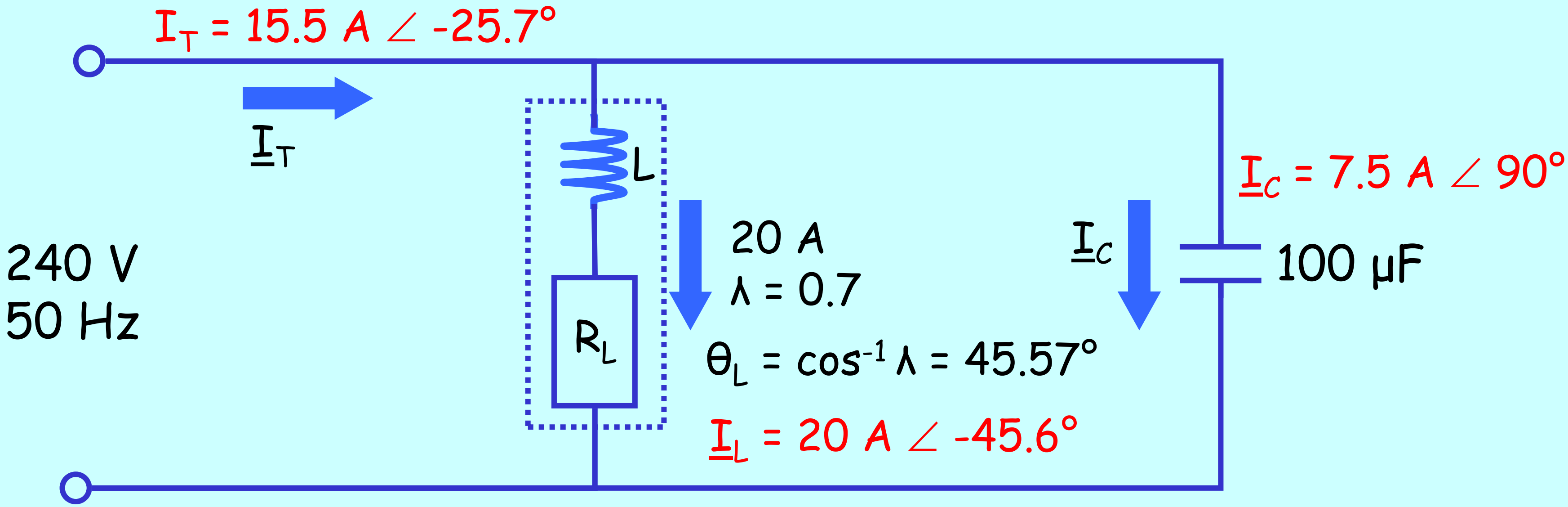
Find:

(a) Total Current ($\underline{I}_T$)

(b) Total Impedance ($\underline{Z}_T$)

(c) Power Factor (λ)

(d) Real Power (P)



(a) $\underline{I}_T = \underline{I}_L + \underline{I}_C$

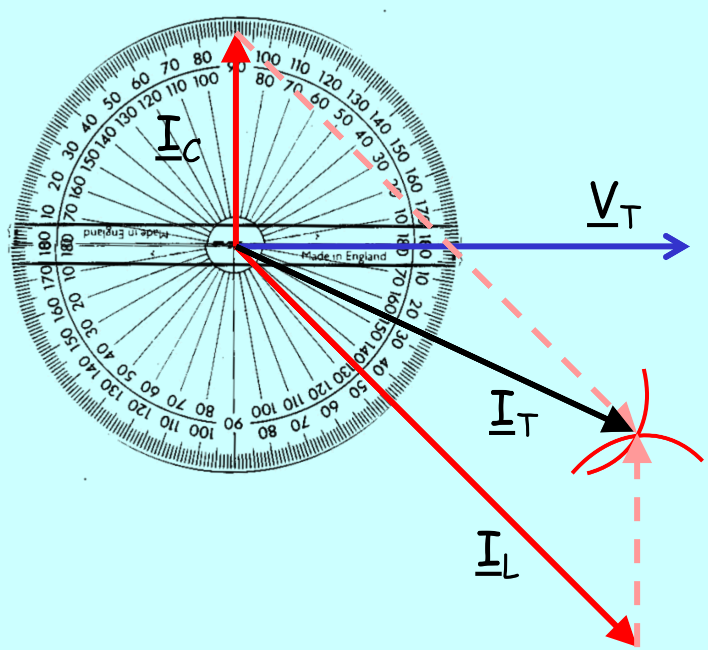
$$I_C = \frac{V}{X_C}$$

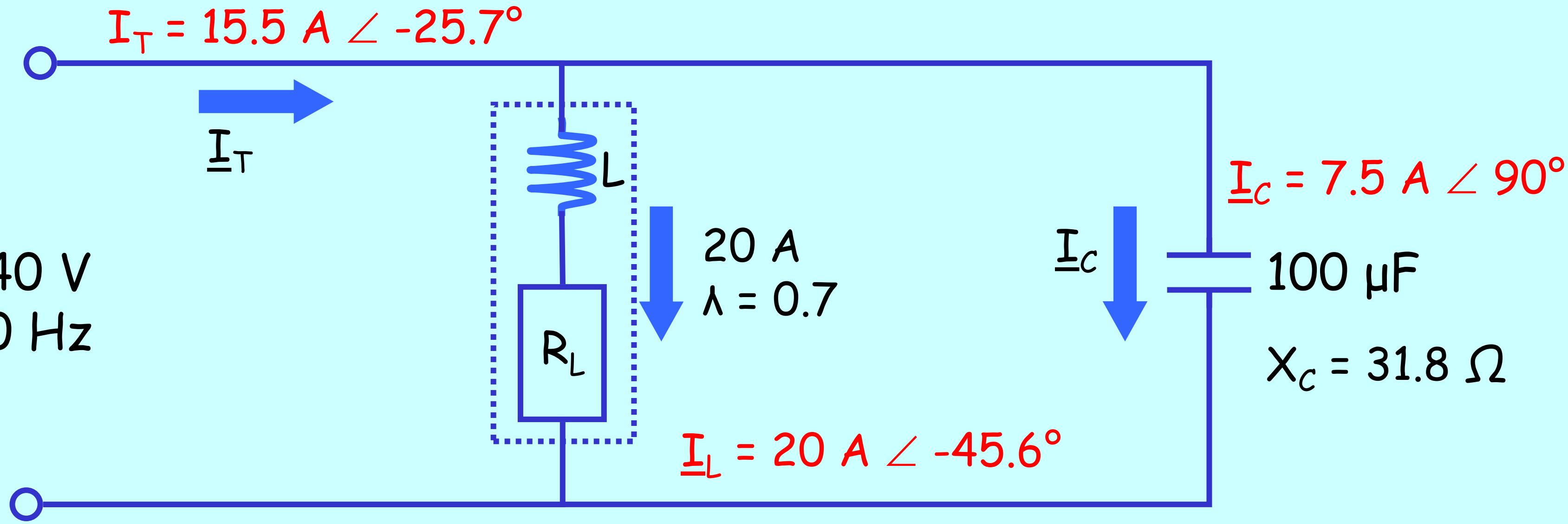
$$|X_C| = \frac{1}{2\pi f C}$$

$$|X_C| = \frac{1}{100\pi \times 100\mu}$$

$$|X_C| = 31.83\ \Omega$$

$$I_C = \frac{240}{31.83} = 7.54\ A$$





(b) $$Z_T = \frac{V}{I_T}$$

$$|Z_T| = \frac{240}{15.54} = 15.44\ \Omega$$

$$\theta_Z = 25.7°$$

$$Z_T = 15.4\ \Omega \angle 25.7°$$

(c) $$\lambda = \cos\theta$$

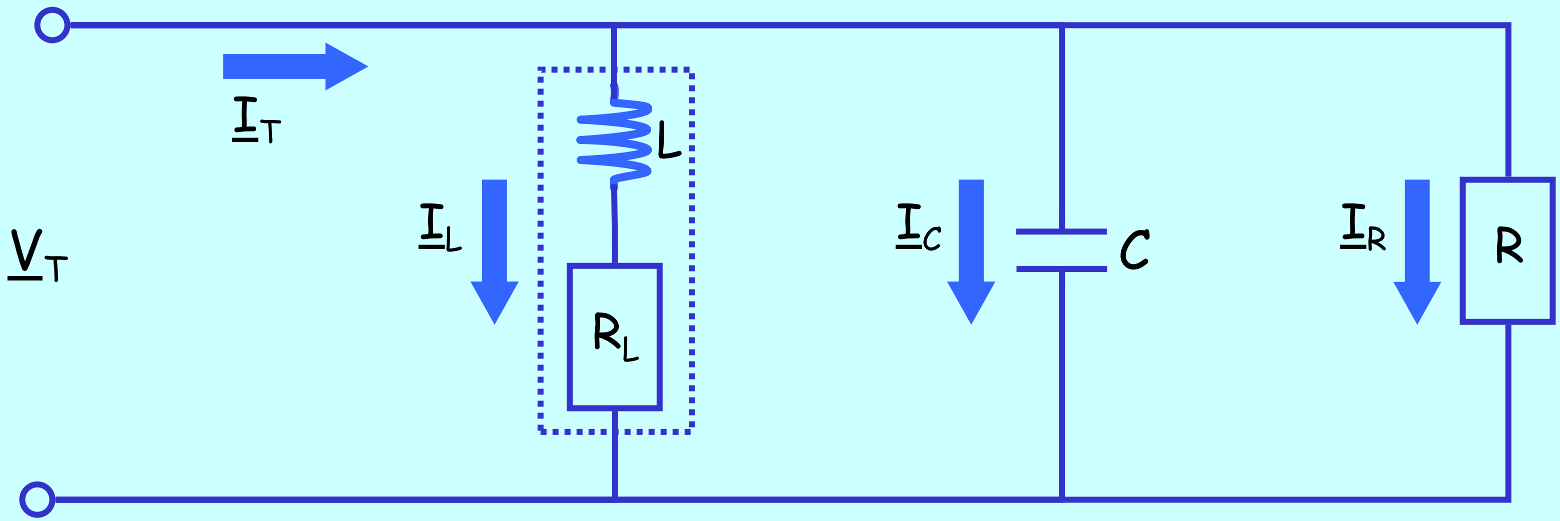
$$\lambda = \cos(-25.7)$$

$$\lambda = 0.9$$

(d) $$P = VI\cos\theta = VI\lambda$$

$$P = 240 \times 15.5 \times 0.9$$

$$P = 3{,}348\ W = 3.35\ kW$$



Resonance occurs in a parallel circuits.
This happens when:

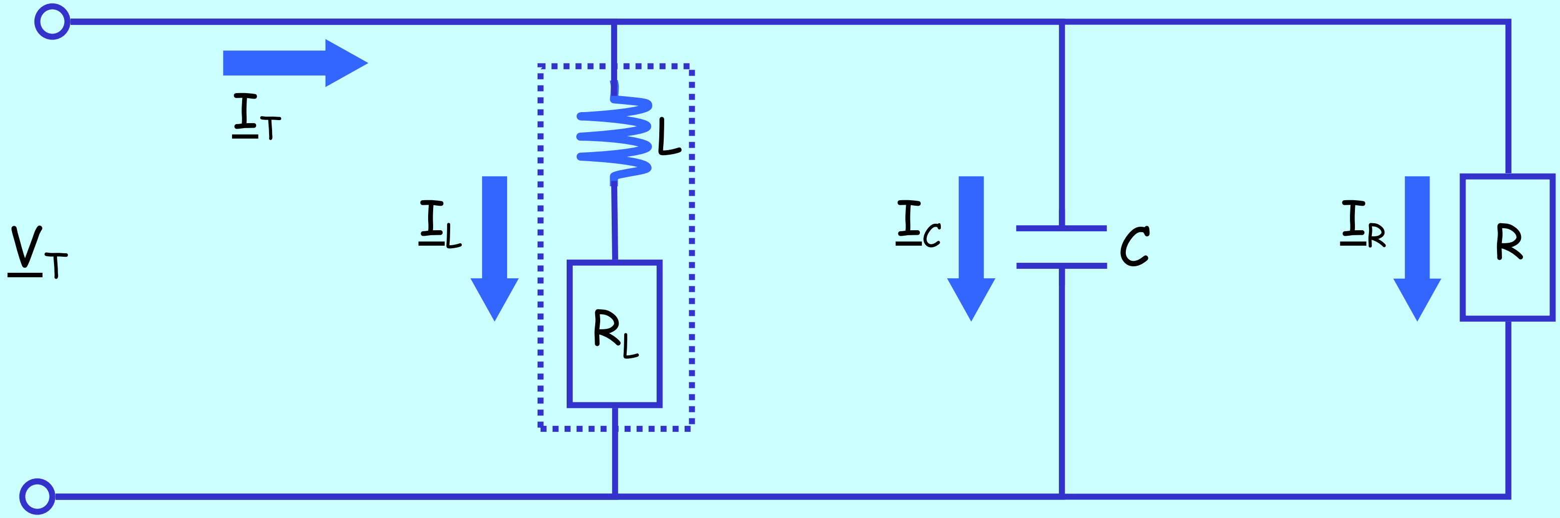
$$X_L = X_C$$

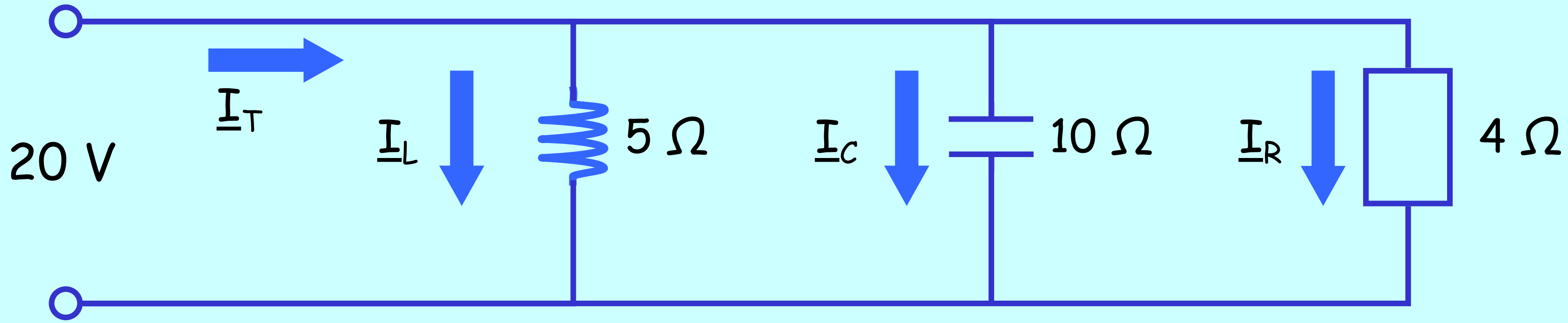
As X_L and X_C cancel each other they appear to be an open circuit to the source.
This means that:

$$Z = R_T$$

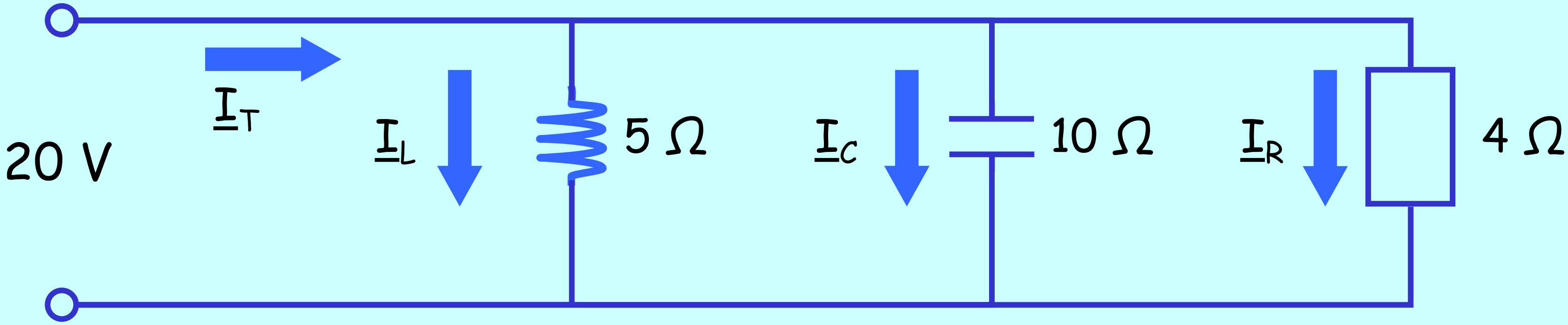
Circuit Impedance is a MAXIMUM at Resonance (f_o).

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$





No R in series with L ∴ assume ideal components!



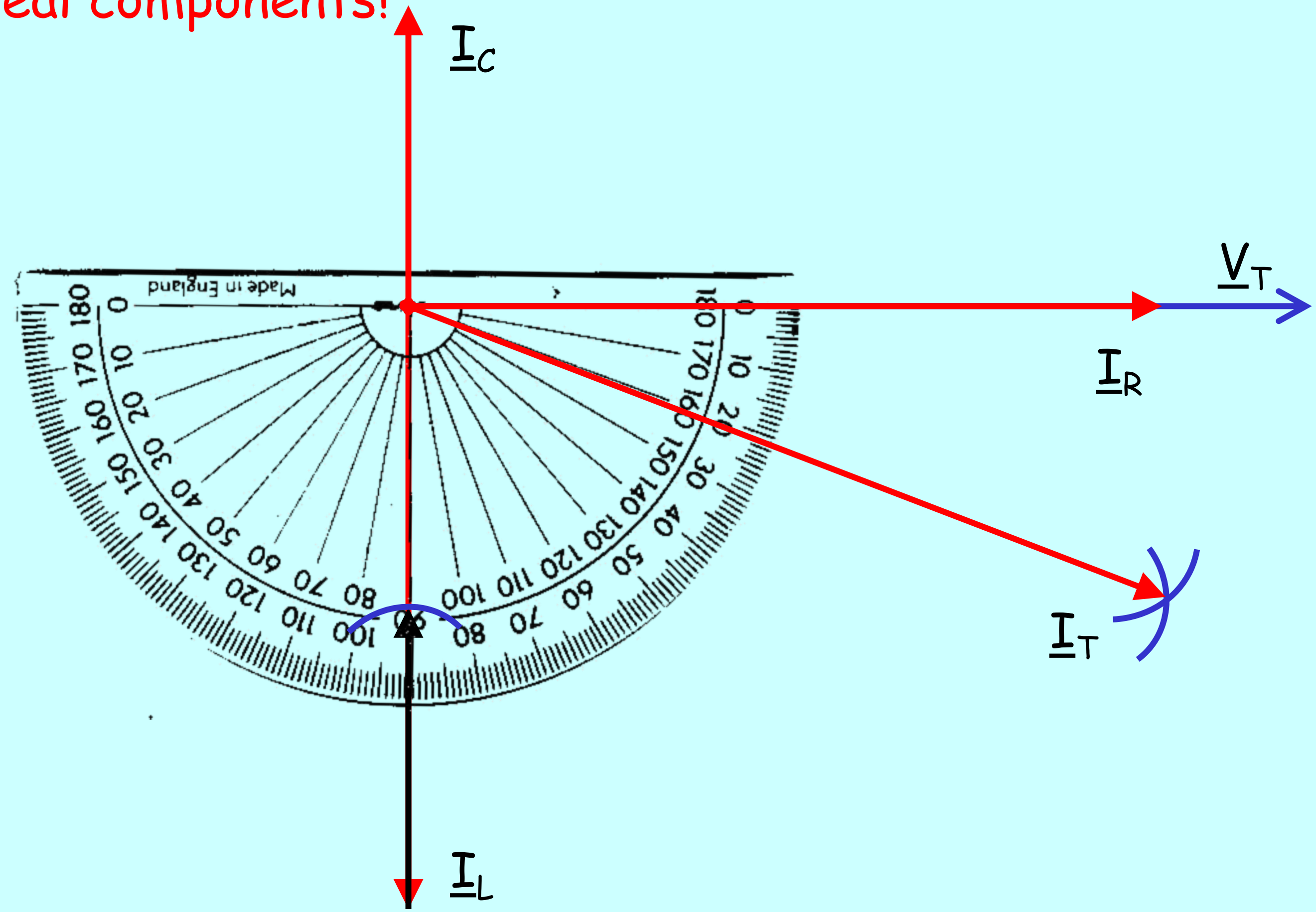
Assume ideal components!

$$|I_R| = \frac{20}{4} = 5\ A$$

$$|I_L| = \frac{20}{5} = 4\ A$$

$$|I_C| = \frac{20}{10} = 2\ A$$

$$I_T = 5.39 \angle -21.8°$$



End of Lesson

Practical Exercises

Parallel ac Circuits

Parallel RLC ac Circuits

UEENEEG102A
Solve problems in low voltage a.c. circuits

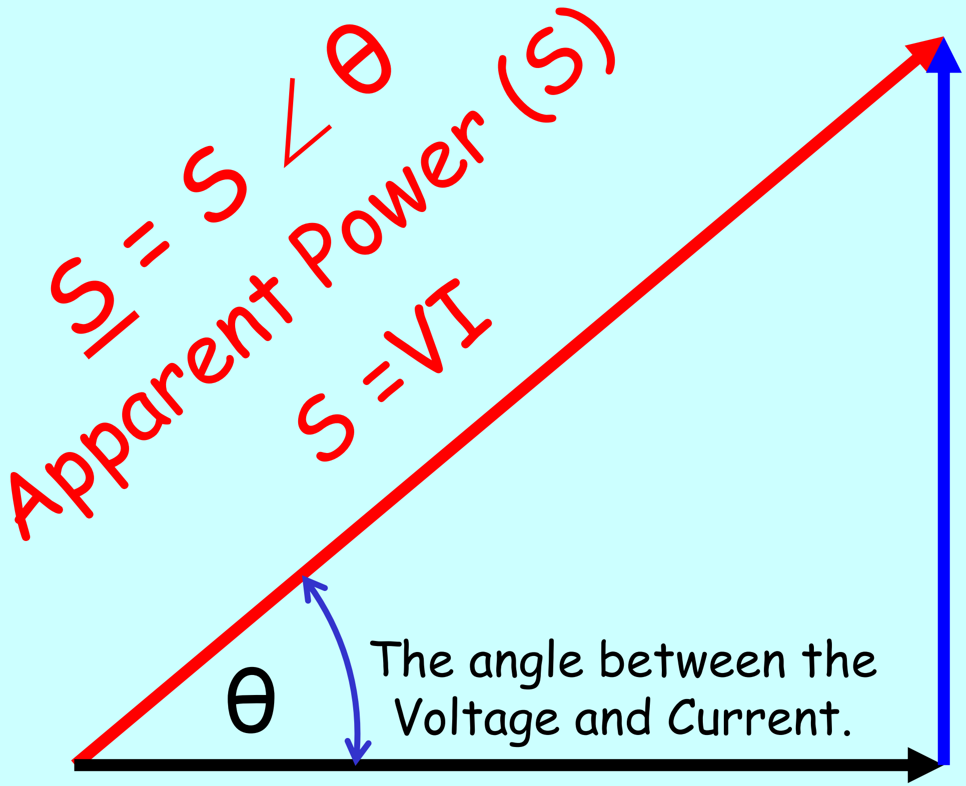
Power in AC Circuits

Objectives:

At the end of this lesson students should be able to:

1. Calculate values of the Power Triangle.
2. Draw and label Power Triangles for AC Circuits.
3. Draw Circuit diagrams showing Wattmeter connections.
4. Measure True and Apparent power.

Power Triangle



Reactive Power (Q)

$Q = VI \sin \theta$

$Q = S \sin \theta$

Real Power (P)

$P = VI \cos \theta$

$P = S \cos \theta$

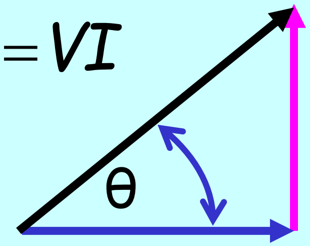
$$Power\ Factor\ (\lambda) \triangleq \frac{P}{S}$$

cos θ is often called the Power Factor (λ) because it relates the Real Power to the Apparent Power.

From the Power Triangle:

$S = VI$

$Q = VI \sin\theta$

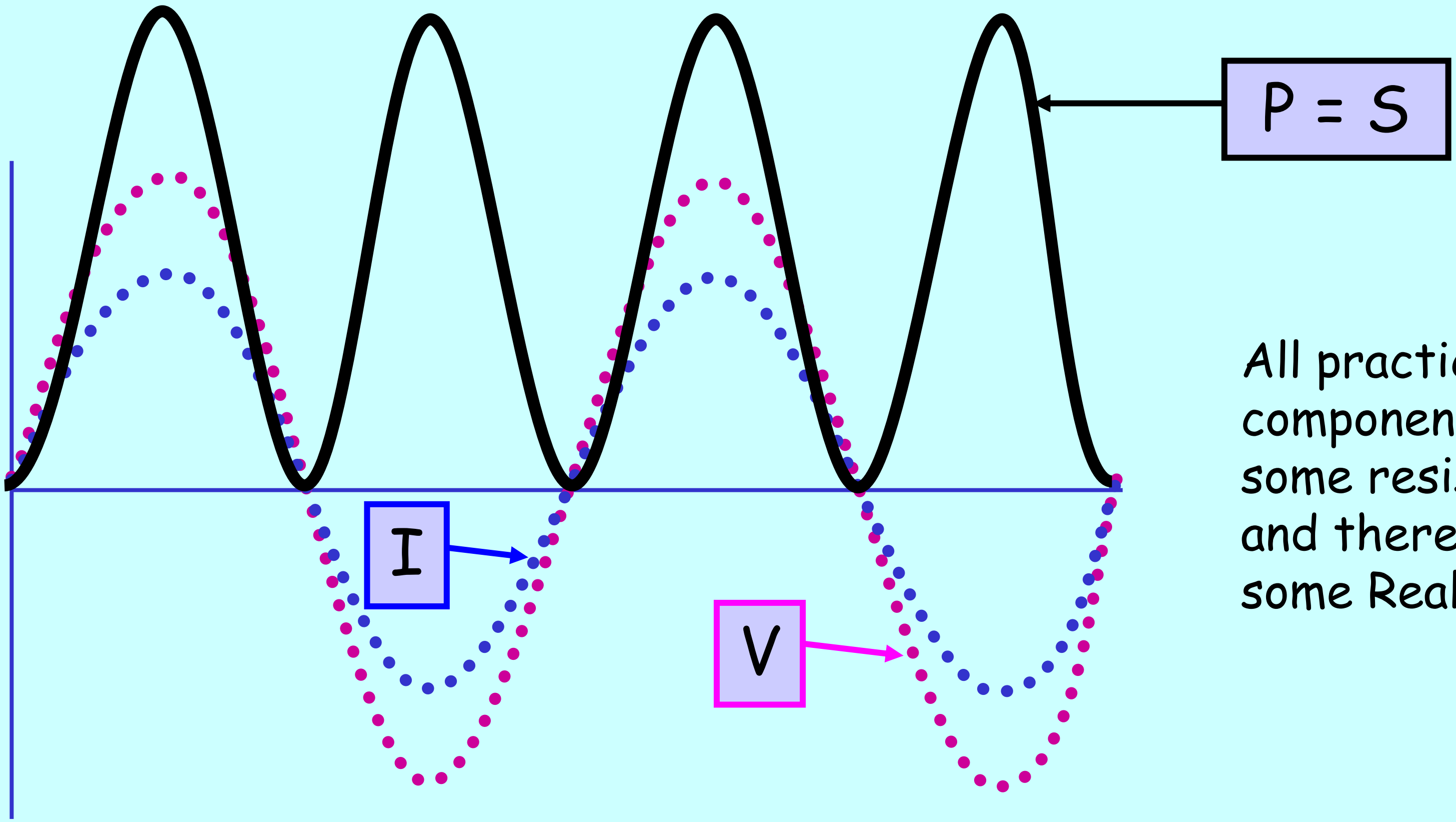


$P = VI \cos\theta$

In a purely resistive circuit Current is In Phase with Voltage ∴ θ = 0°

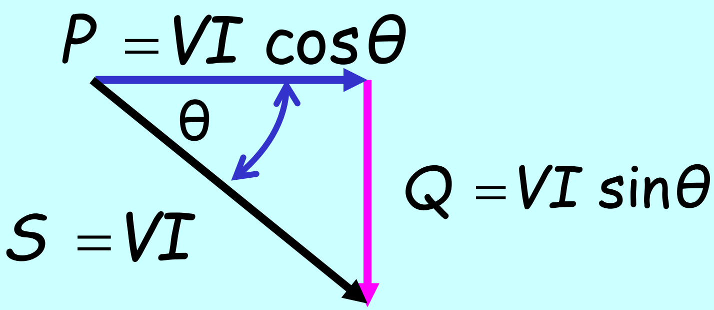
(cos 0 = 1 & sin 0 = 0)

$P = VI$ $Q = 0$



All practical components have some resistance and therefore use some Real Power.

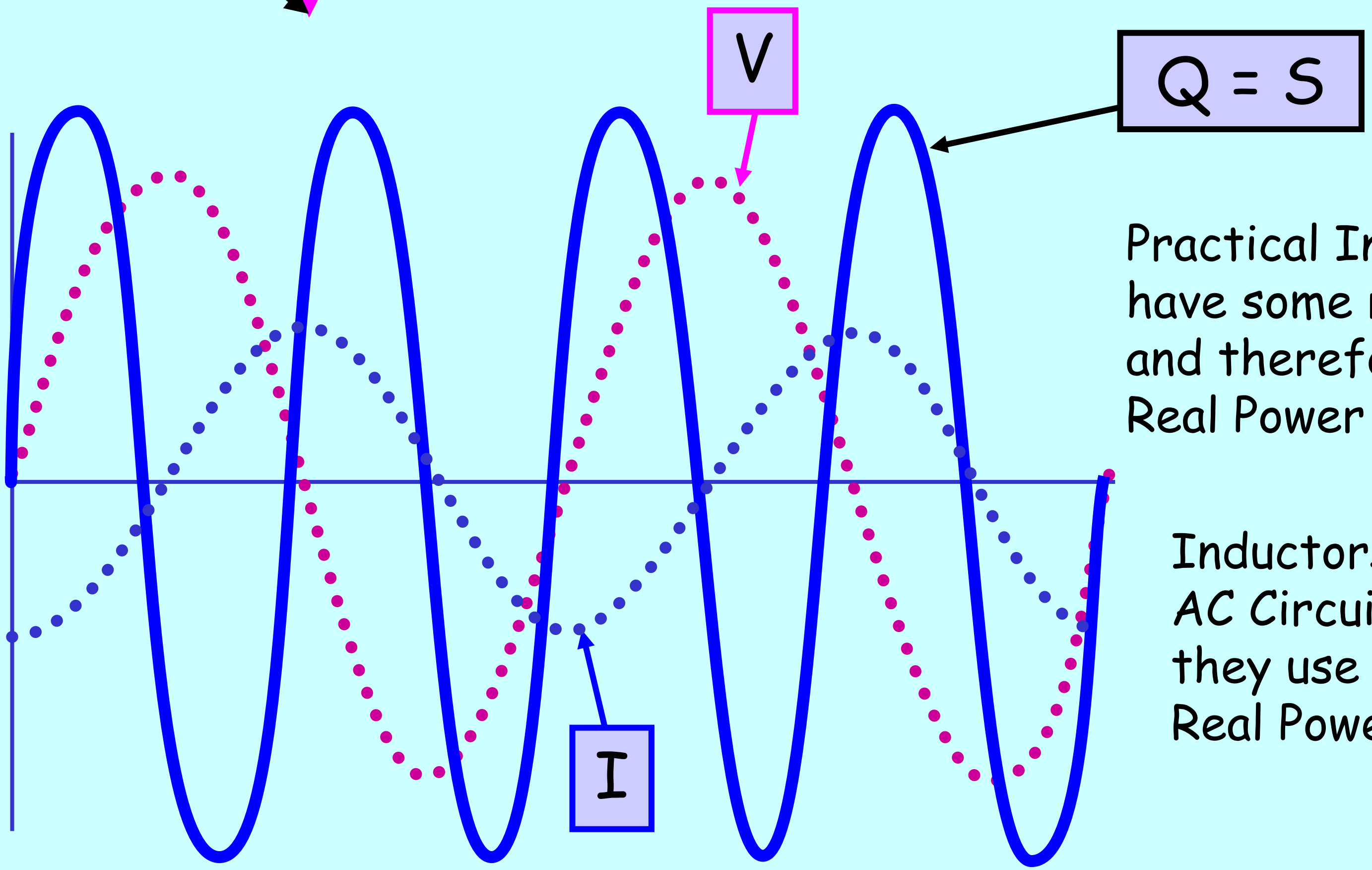
From the Power Triangle:



In a purely Inductive circuit Current Lags Voltage ∴ θ = -90°

(cos -90 = 0 & sin -90 = -1)

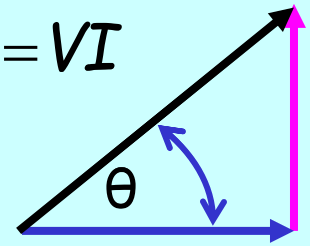
$$P = 0 \qquad Q = -VI$$



Practical Inductors have some resistance and therefore use some Real Power (I^2R).

Inductors are used in AC Circuits because they use very little Real Power.

From the Power Triangle:



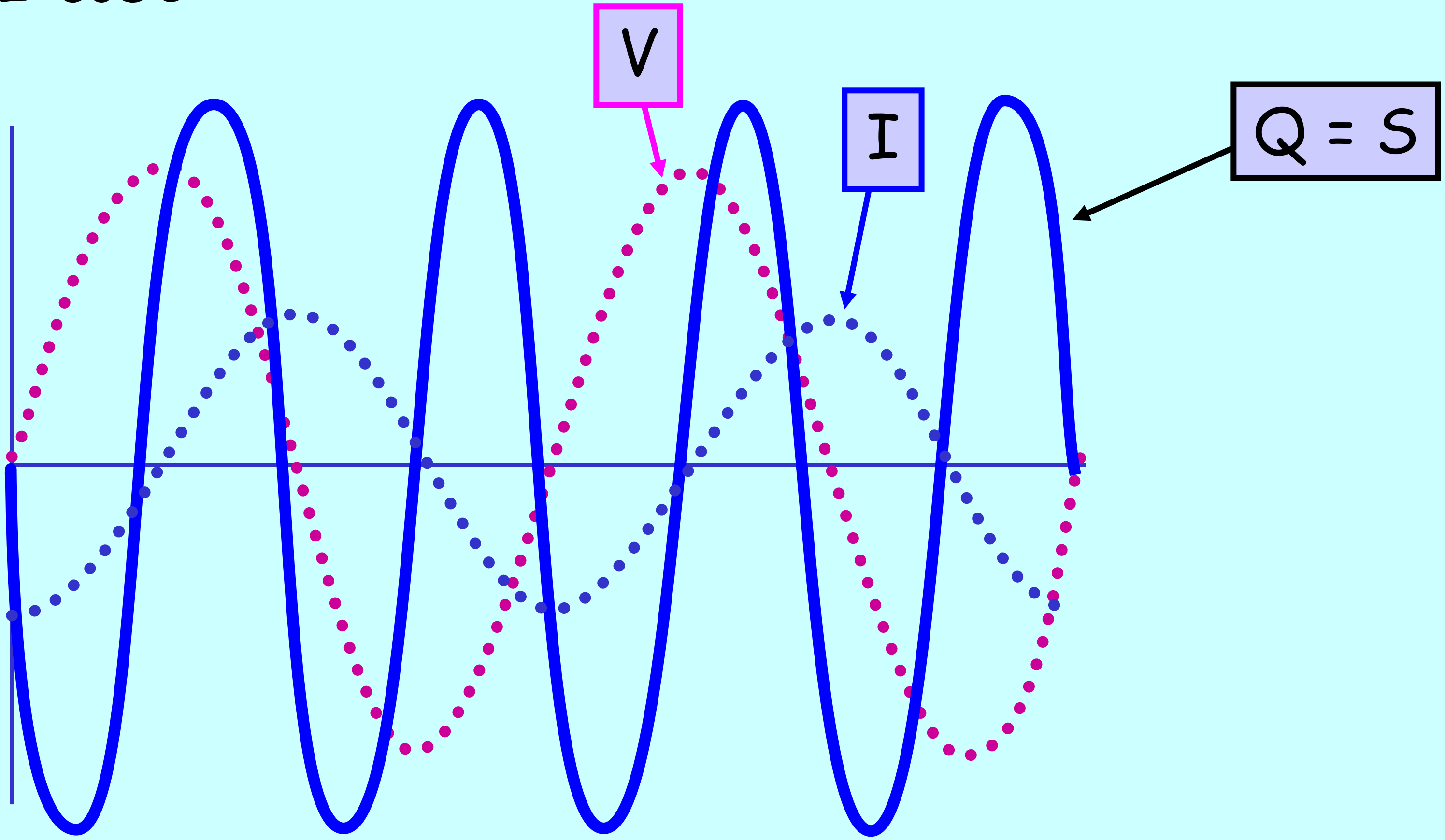
$S = VI$

$Q = VI \sin\theta$

$P = VI \cos\theta$

In a purely Capacitive circuit Current Leads Voltage ∴ θ = 90°
(cos 90 = 0 & sin 90 = 1)

$P = 0$ $Q = VI$



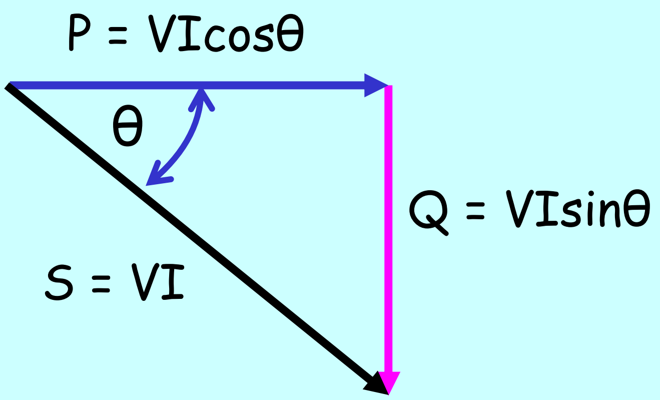
A single phase 20 Amp load has a Power Factor of 0.866 when connected to a 240 Volt 50 Hz supply. Determine the Circuit Phase Angle, Apparent, Real and Reactive Power of the load.

What do we know?

$I = 20\ A$ $\lambda = 0.866$ lagging

$V = 240\ V$

$f = 50\ Hz$



What do we want to know?

Find θ, S, P & Q $-\theta = \cos^{-1} \lambda$

$\theta = -30°$

$Q = \sqrt{(S^2 - P^2)}$

$S = VI$	$P = VI \cos \theta$	$Q = VI \sin \theta$
$S = 20 \times 240$	$P = 4.8k \times 0.866$	$Q = 4.8k \times -0.5$
$S = 4.8\ kVA$	$P = 4.16\ kW$	$Q = -2.4\ kVar$

A single phase 240 V 50 Hz supply is connected to a small factory. If the total current drawn from the supply is 30 Amps with a lagging power factor of 0.7, determine the power dissipated by the load, the reactive and apparent powers and draw the Power Triangle.

What do we know? $I = 30\ A$ $\lambda = 0.7$ lagging $V = 240\ V$ $f = 50\ Hz$

What do we want to know? $-\theta = \cos^{-1} \lambda$ $\theta = -45.57°$

$P = VI \cos \theta$

$P = 7.2k \times 0.7$

$P = 5.04\ kW$

$Q = VI \sin \theta$

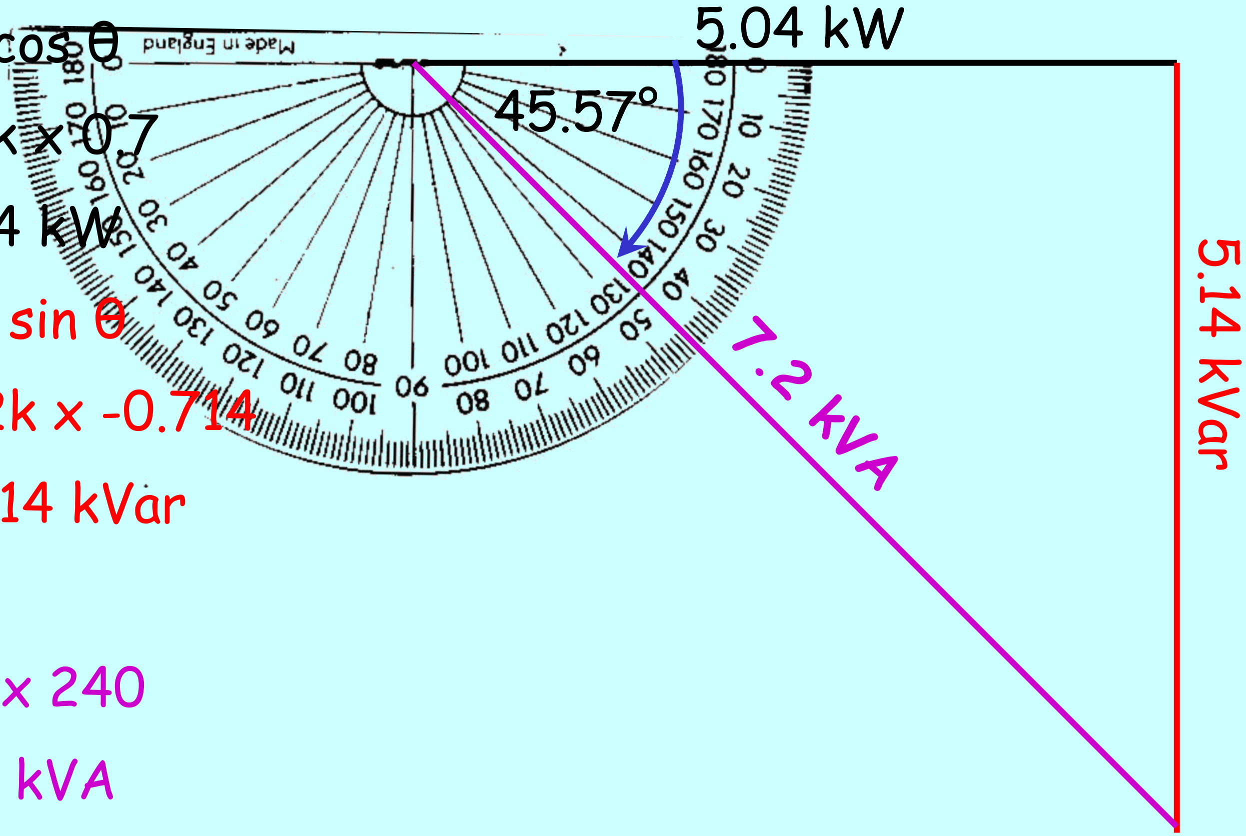
$Q = 7.2k \times -0.714$

$Q = -5.14\ kVar$

$S = VI$

$S = 30 \times 240$

$S = 7.2\ kVA$



Load #1 = 5.04 kW @ 0.7 lag

Load #2 = Capacitive @ 1.92 kVar

Load #3 = Purely resistive @ 2.4 kW

Source 240 V @ 50 Hz

Draw the circuit Power Triangle and find I_T.

P_L = 5.04 kW

$$S_L = \frac{P}{\lambda} = \frac{5.04}{0.7}$$

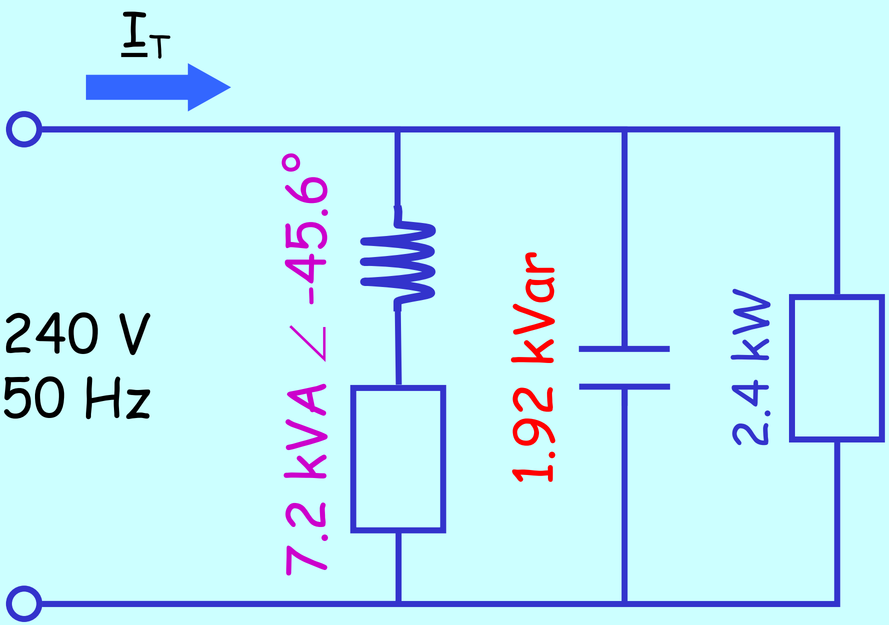
λ_L = 0.7 lagging

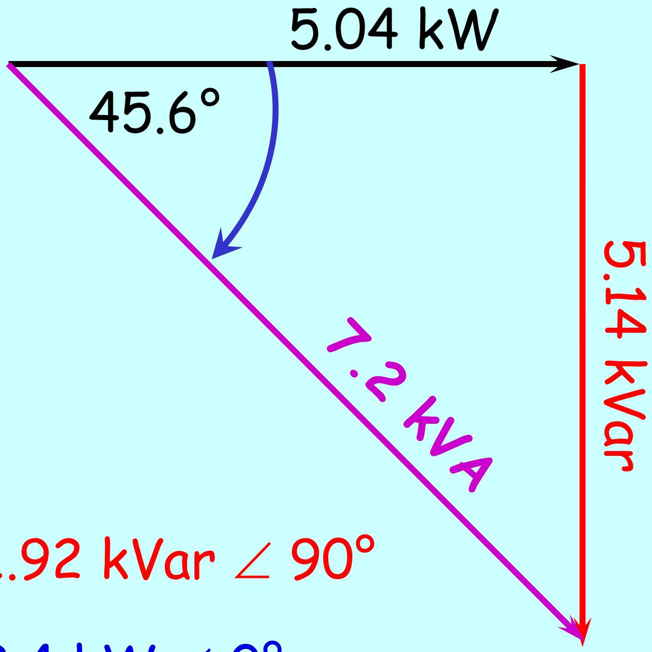
S_L = 7.2 kVA

$-\theta_L = \cos^{-1} 0.7 = -45.6°$

$\underline{S}_L = 7.2 \text{ kVA} \angle -45.6°$

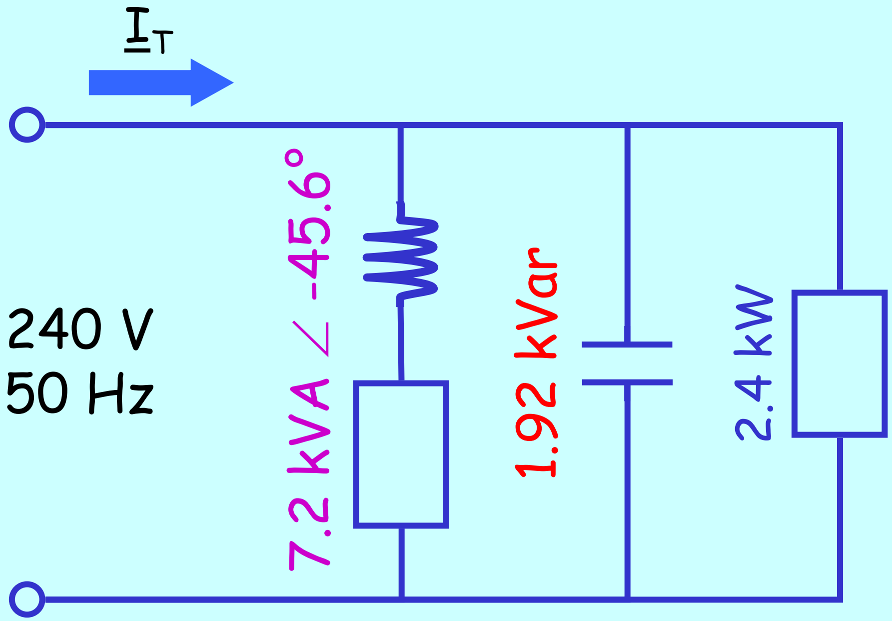
$Q_L = 7.2\text{k} \sin(-45.6°) = 5.14 \text{ kVar}$





$\underline{Q}_C = 1.92 \text{ kVar} \angle 90°$

$\underline{P}_R = 2.4 \text{ kW} \angle 0°$

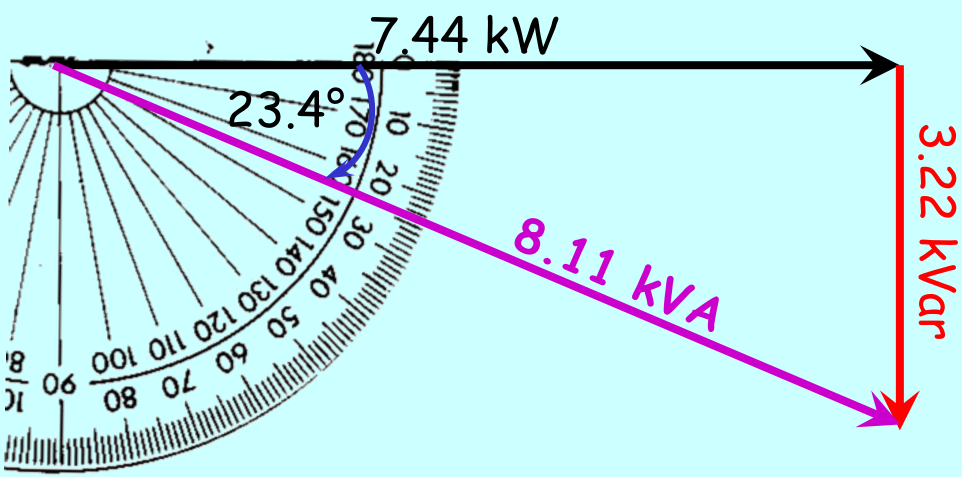


$$P_T = (5.04 + 2.4) = 7.44 \text{ kW}$$

$$Q_T = (1.92 - 5.14) = -3.22 \text{ kVar}$$

$$S_T = \sqrt{(7.44^2 + 3.22^2)} = 8.11 \text{ kVA}$$

$$\theta_T = \tan^{-1}(^{-3.22}/_{7.44}) = -23.4°$$



$$\lambda = \cos \theta$$

$$\lambda = \cos -23.4$$

$$\lambda = 0.918$$

$$I = \frac{S}{V}$$

$$I = \frac{8.11k\angle -23.4}{240\angle 0}$$

$$I = 33.8 \text{ A} \angle -23.4°$$

End of Lesson

Practical Exercises

Single Phase Power Measurement

Need to add pix of Power Meter and how to connect it.

Circuit diagrams as well.

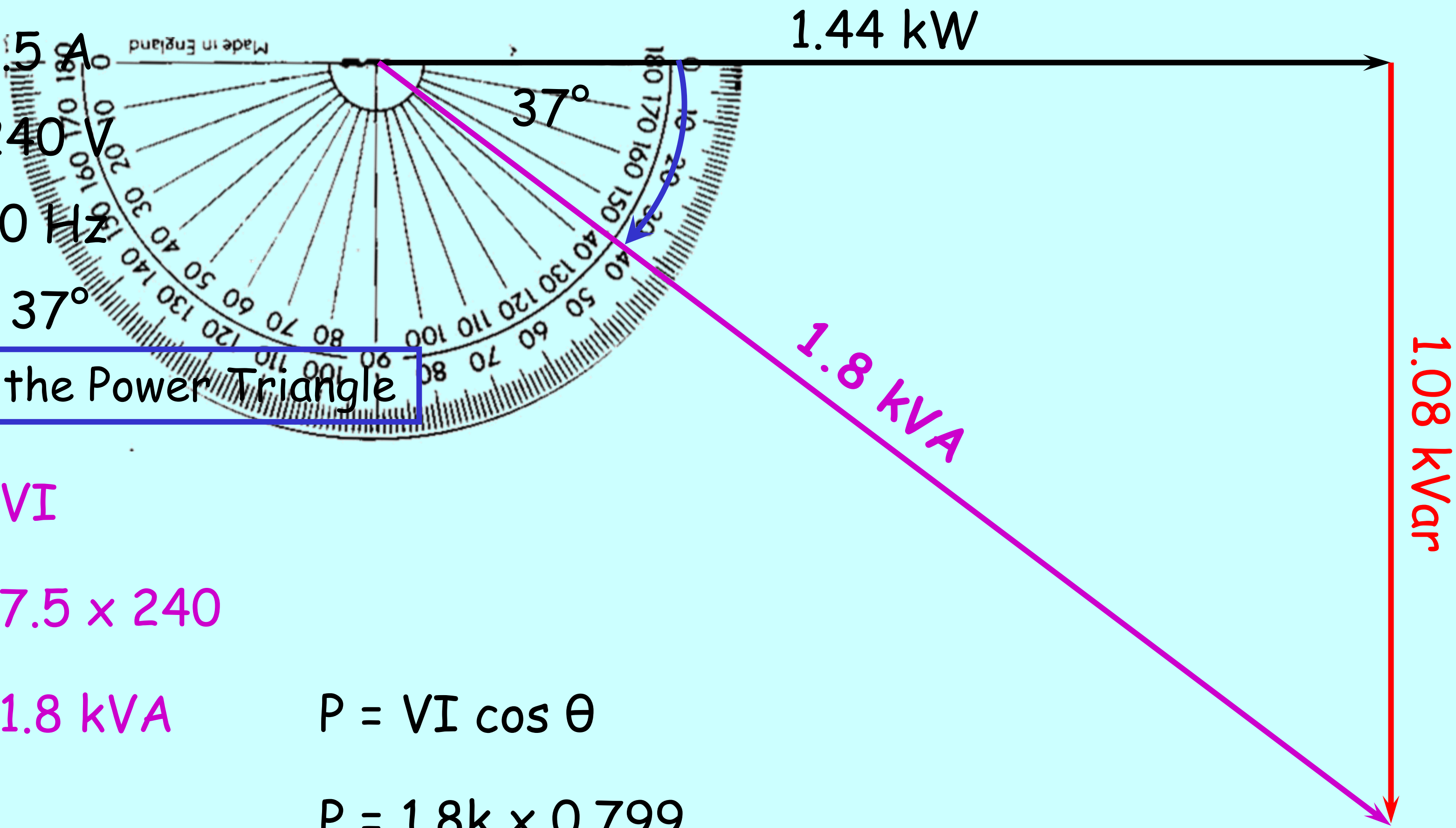
I = 7.5 A

V = 240 V

f = 50 Hz

θ = - 37°

Draw the Power Triangle



S = VI

S = 7.5 x 240

S = 1.8 kVA

P = VI cos θ

P = 1.8k x 0.799

P = 1.44 kW

Q = VI sin θ

Q = 1.8k x -0.602

Q = -1.08 kVar

$P = 10\ kW$

$V = 240\ V$

$$\underline{S} = \frac{P}{\lambda}$$

$$I = \frac{S}{V}$$

(a) $\lambda = 1$

$S = 10\ kVA$

$$I = \frac{10k}{240}$$

$I = 41.7\ A$

(b) $\lambda = 0.8$ lag

$$\underline{S} = \frac{10k}{0.8}$$

$S = 12.5\ kVA$

$$I = \frac{12.5k}{240}$$

$I = 52.1\ A$

(c) $\lambda = 0.6$ lag

$$\underline{S} = \frac{10k}{0.6}$$

$S = 16.7\ kVA$

$$I = \frac{16.7k}{240}$$

$I = 69.6\ A$

Note that as Power Factor decreases Current increases

Find the Apparent Power, the Reactive Power & the Power Factor

$V = 240\ V$

$I = 9\ A$

$P = 2\ kW$

$S = VI$

$S = 240 \times 9$

$S = 2.16\ kVA$

$Q = \sqrt{(2.16^2 - 2^2)}\ kVar$

$Q = 0.815\ kVar$

$\lambda = \dfrac{P}{S} = \dfrac{2k}{2.16k}$

$\lambda = 0.926$

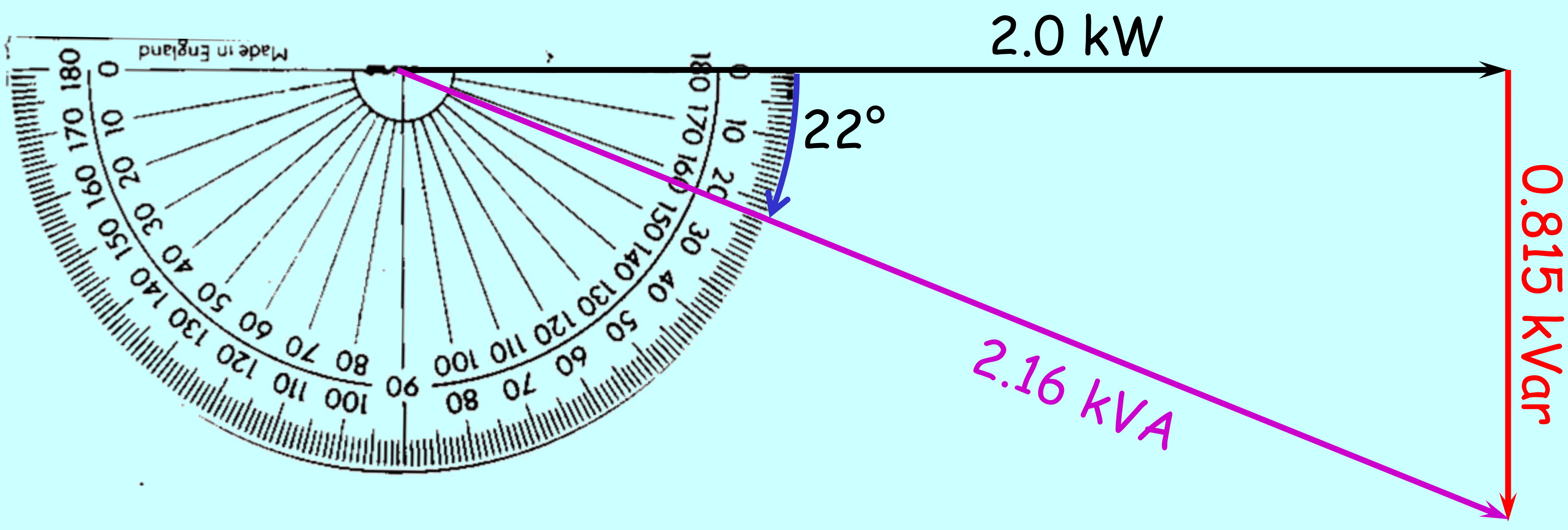
$\theta = \cos^{-1} \lambda$

$\theta = 22.2°$

$Q = S \sin \theta$

$Q = 2.16k \sin 22.2$

$Q = 0.816\ kVar$



UEENEEG102A
Solve problems in low voltage a.c. circuits

Power Factor Improvement

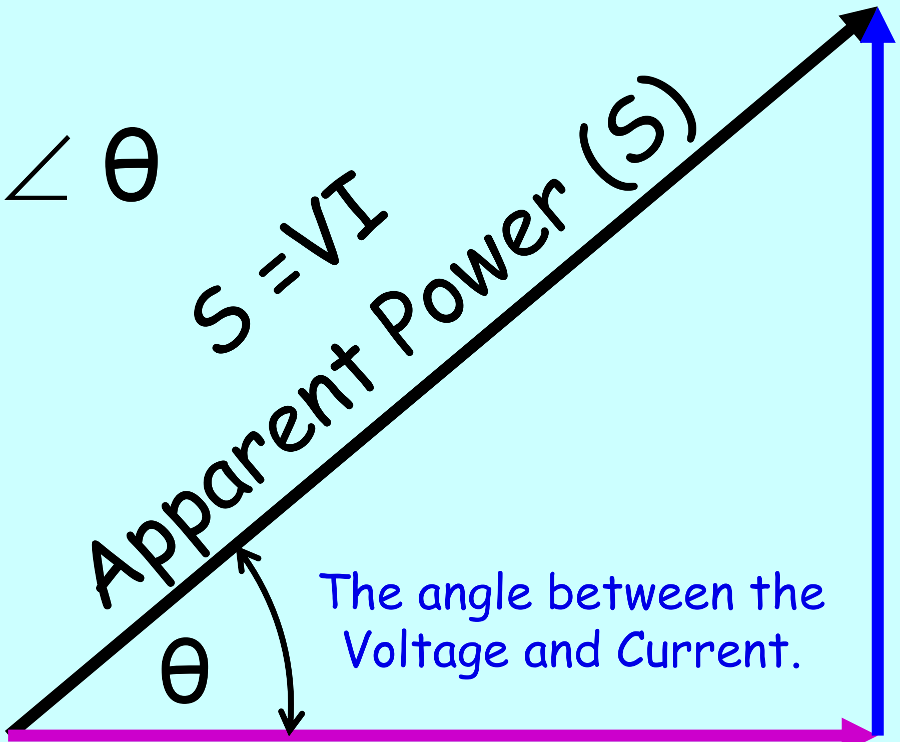
Objectives:

At the end of this lesson students should be able to:

1. Determine the Power Factor of a Single Phase Circuit.
2. State the effects of low power factor in a single phase circuit.
3. Determine the value of capacitance or reactive power required to improve power factor.
4. State the requirements concerning installation power factor.

Power Triangle

$\underline{S} = S \angle \theta$



Reactive Power (Q)

$Q = VI \sin\theta$

Real Power (P)

$P = VI \cos\theta$

$\cos\theta$ is called the Power Factor (λ) of the circuit because it relates the Real Power to the Apparent Power.

Power Factor (λ) is a measure of an installations efficiency (η)

$$\lambda = \cos\theta$$

θ = The Phase angle of the circuit

Low Power Factor (λ) can be caused by:

- Inductive loads (lagging power factor).
  - Fluorescent light Ballasts.
  - Lightly loaded Motors and Transformers
- Capacitive Loads (leading power factor).
  - Long cable runs.

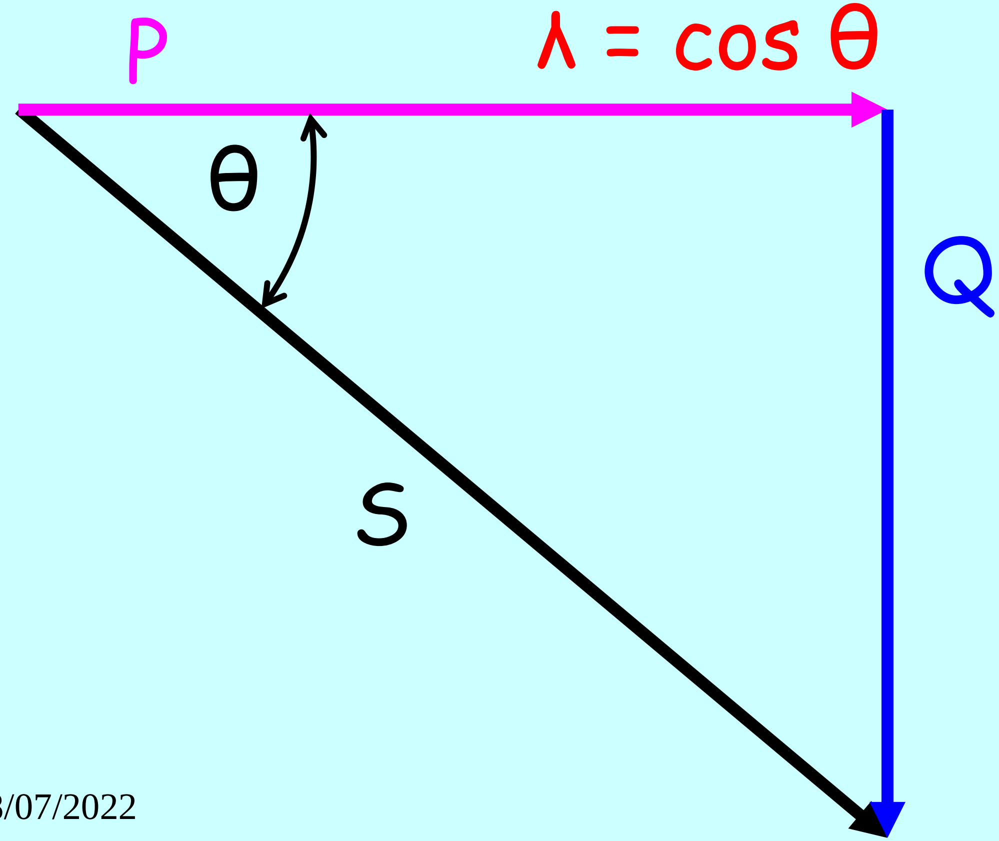
Low Power Factor (λ) causes:

- Increased Losses (I^2R).
- Increased Cable Sizes.
- Increased Equipment Costs (require higher ratings).
- Increased Generation Costs.
- Operational problems.

Power Factor Improvement

The ***NSW Service and Installation Rules*** requires that *a consumers installation* should have a power factor of not less than 0.9 lagging and that the power factor of any installation MUST NOT become leading at any time.

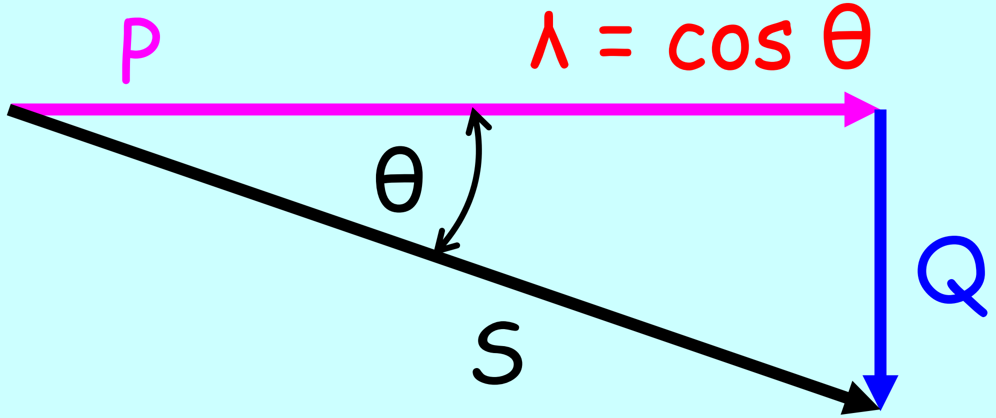
How can we improve power factor?



Power Factor Improvement

The ***NSW Service and Installation Rules*** requires that *a consumers installation* should have a power factor of not less than 0.9 lagging and that the power factor of any installation MUST NOT become leading at any time.

How can we improve power factor?



Reduce 'Q'

This reduces 'S' and θ

But does not change 'P'

Most reactive loads are Inductive and hence have a lagging power factor.

Counteract Inductive loads by placing a Capacitor in parallel.

Tutorial pg. 51 Example:

A 240 V, 50 Hz, single phase installation draws 40 A from a supply at a power factor of 0.4 lag. Determine the Farads rating of a capacitor bank to be connected in parallel with the load to achieve an installation power factor of:

a) 0.866 lag

b) Unity

What do we know?

V = 240 V
f = 50 Hz
I = 40 A
ʎ = 0.4 lag

What do we want to know? Capacitance

What can we calculate?

$V = 240$ V

$f = 50$ Hz

$I = 40$ A

$\lambda = 0.4$ lag

$$\theta = \cos^{-1} \lambda$$

$$\theta = \cos^{-1} 0.4$$

$$\theta = -66.4°$$

$$S = VI$$

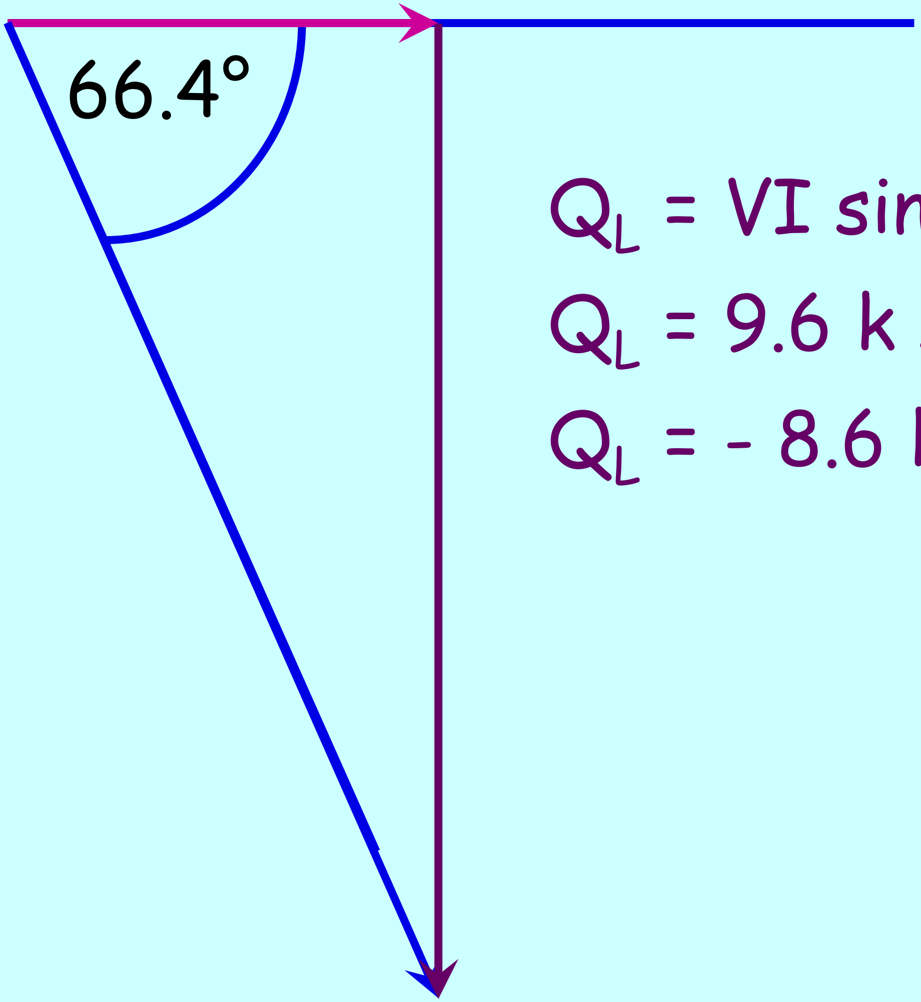
$$S = 40 \times 240$$

$$S = 9.6 \text{ kVA}$$

$$P = VI \cos \theta$$

$$P = 9.6 \text{ k} \times 0.4$$

$$P = 3.8 \text{ kW}$$



$$Q_L = VI \sin \theta$$

$$Q_L = 9.6 \text{ k} \times (-0.9)$$

$$Q_L = -8.6 \text{ kVar}$$

a) 0.866 lag

V = 240 V
f = 50 Hz

λ = 0.866 lag

$\theta = \cos^{-1}\lambda$

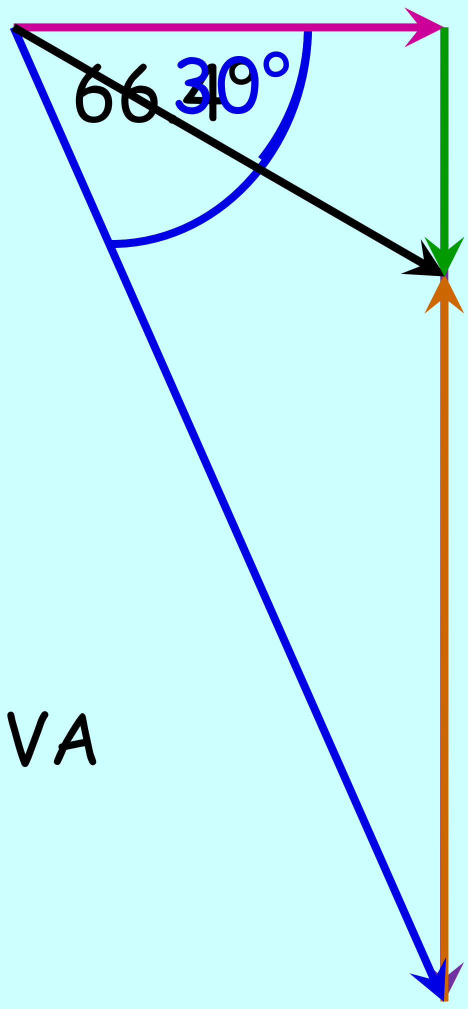
$\theta_a = \cos^{-1} 0.866$

$\theta_a = -30°$

$S = \dfrac{P}{\lambda}$

$S = \dfrac{3.8}{0.866} = 4.4\,kVA$

P = 3.8 kW



$Q_L = -8.6$ kVar

$Q_T = S \sin\theta$

$Q_T = 4.4\,k \times (-0.5)$

$Q_T = -2.2$ kVar

$Q_T = Q_L + Q_C$

$Q_C = Q_T - Q_L$

$Q_C = -2.2 + 8.6$

$Q_C = 6.4$ kVar

a) 0.866 lag

$$Q_C = 6.4\ kVar$$

V = 240 V
f = 50 Hz

$$Q_C = V_C I_C$$

$$I_C = \frac{V_C}{X_C}$$

$$Q_C = \frac{V_C^{\ 2}}{X_C}$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{V^2}{Q_C}$$

$$C = \frac{1}{2\pi f X_C}$$

$$X_C = \frac{240^2}{6.4k}$$

$$C = \frac{1}{2\pi \times 50 \times 9}$$

$$X_C = 9\ \Omega$$

$$C = 353.7\ \mu F$$

b) Unity

V = 240 V

f = 50 Hz

$\lambda = 1.0$

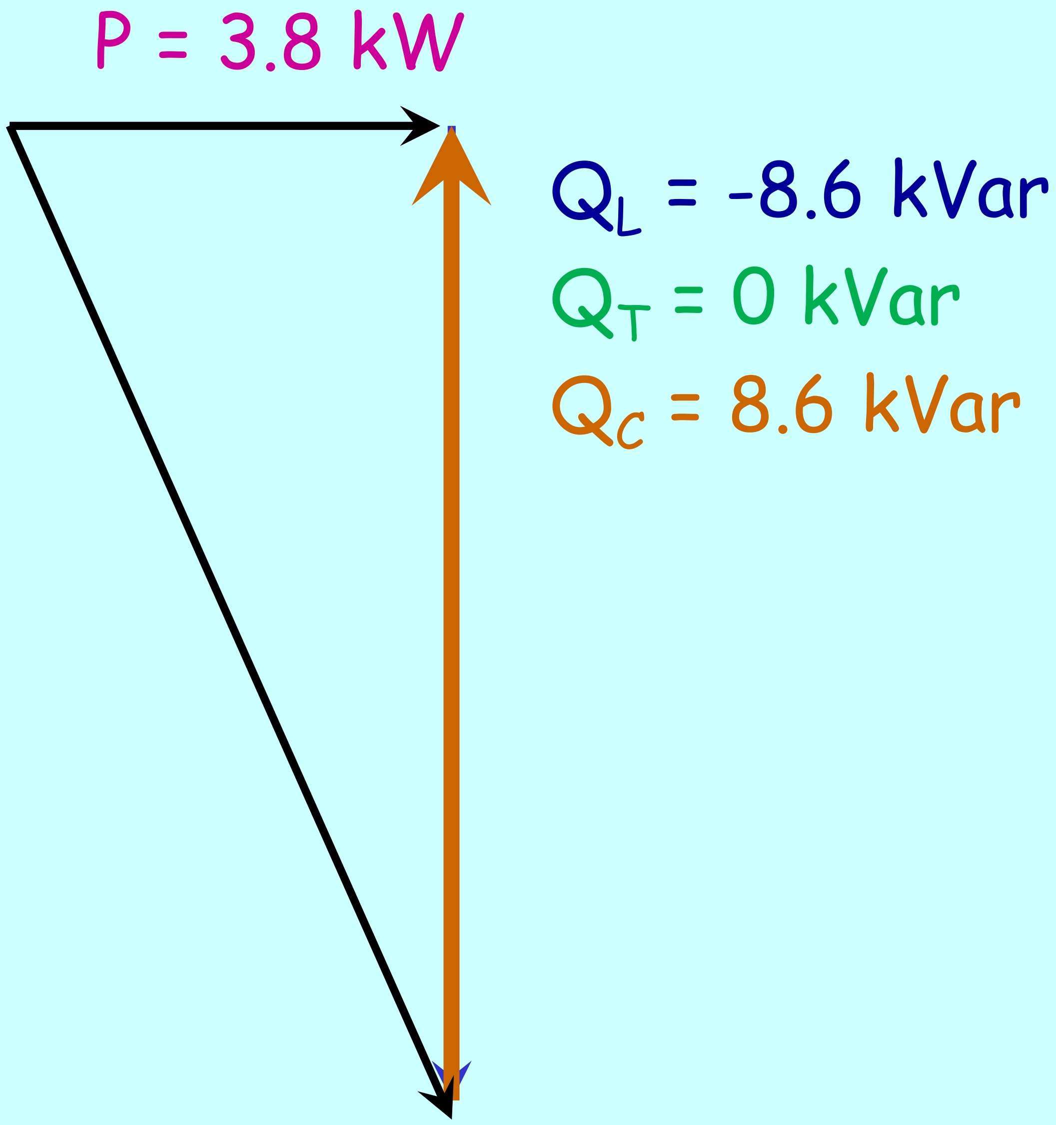
$\theta = \cos^{-1}\lambda$

$\theta_a = \cos^{-1} 1.0$

$\theta_a = 0°$

$$S = \frac{P}{\lambda}$$

$$S = \frac{3.8}{1.0} = 3.8\,kVA$$



b) Unity

$$Q_C = 8.6\ kVar$$

V = 240 V
f = 50 Hz

$$Q_C = V_C I_C$$

$$I_C = \frac{V_C}{X_C}$$

$$Q_C = \frac{V_C}{X_C}^2$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{V^2}{Q_C}$$

$$C = \frac{1}{2\pi f X_C}$$

$$X_C = \frac{240^2}{8.6k}$$

$$C = \frac{1}{2\pi \times 50 \times 6.7}$$

$$X_C = 6.7\ \Omega$$

$$C = 475.3\ \mu F$$

A 240 V, 50 Hz, single phase installation draws 10 A from a supply at a power factor of 0.6 lag. Determine the Farads rating of a capacitor bank to be connected in parallel with the load to make it comply with the requirements of the ***NSW Service and Installation Rules.***

What do we know?	V = 240 V f = 50 Hz I = 10 A λ = 0.6 lag
What do we want?	λ = 0.9 lag
What do we need to know?	Capacitance
What can we calculate?	

$P = VI \cos\theta$

$P = 2.4\text{ k} \times 0.6$

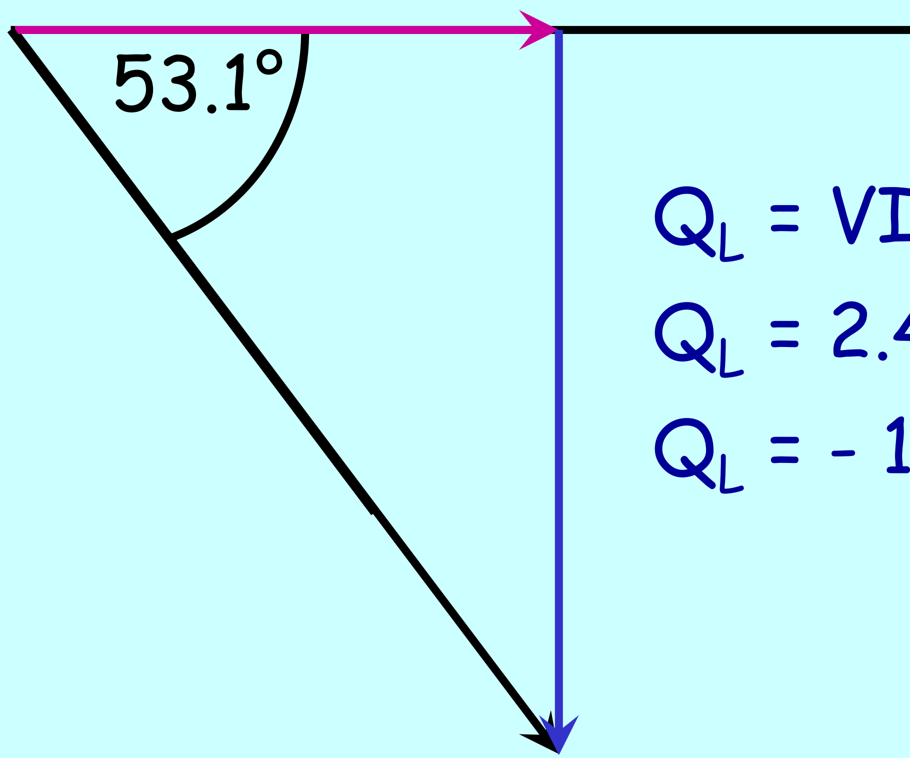
$P = 1.44\text{ kW}$

$V = 240\text{ V}$

$f = 50\text{ Hz}$

$I = 10\text{ A}$

$\lambda = 0.6\text{ lag}$



$\theta = \cos^{-1}\lambda$

$\theta = \cos^{-1} 0.6$

$\theta = -53.1°$

$S = VI$

$S = 10 \times 240$

$S = 2.4\text{ kVA}$

$Q_L = VI \sin\theta$

$Q_L = 2.4\text{ k} \times (-0.8)$

$Q_L = -1.92\text{ kVar}$

V = 240 V

f = 50 Hz

I = 10 A

λ = 0.6 lag

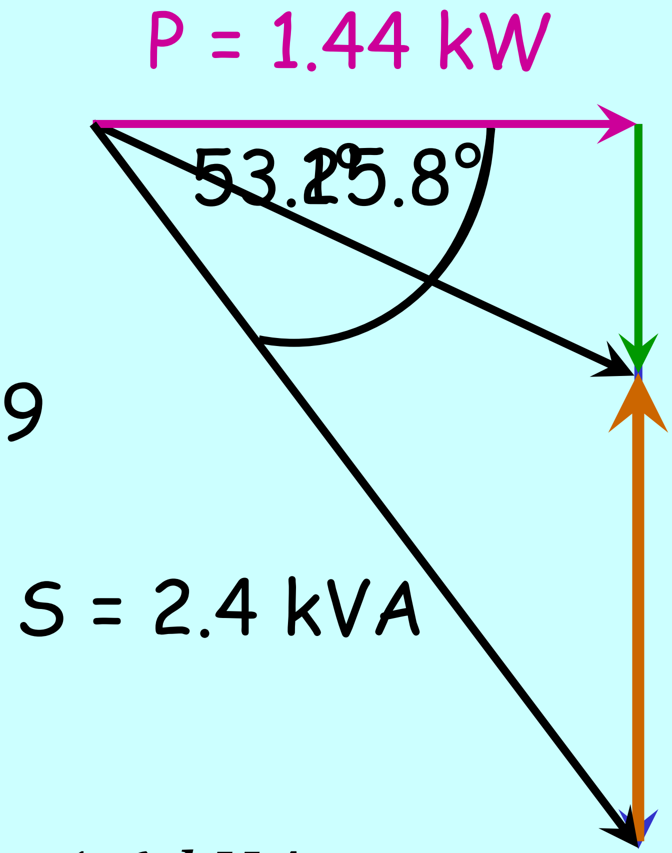
$\theta = \cos^{-1} \lambda$

$\theta_a = \cos^{-1} 0.9$

$\theta_a = -25.8°$

$$S = \frac{P}{\lambda}$$

$$S = \frac{1.44k}{0.9} = 1.6\ kVA$$



$Q_L = -1.92$ kVar

$Q_T = S \sin \theta$

$Q_T = 1.6\ k \times (-0.44)$

$Q_T = -0.7$ kVar

$Q_T = Q_L + Q_C$

$Q_C = Q_T - Q_L$

$Q_C = -0.7 + 1.92$

$Q_C = 1.22$ kVar

$$Q_C = 1.22\ kVar$$

V = 240 V
f = 50 Hz

$$Q_C = V_C I_C$$

$$I_C = \frac{V_C}{X_C}$$

$$Q_C = \frac{V_C}{X_C}^2$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{V^2}{Q_C}$$

$$C = \frac{1}{2\pi f X_C}$$

$$X_C = \frac{240^2}{1.22k}$$

$$C = \frac{1}{2\pi \times 50 \times 47.2}$$

$$X_C = 47.2\ \Omega$$

$$C = 67.4\ \mu F$$

Tutorial pg. 52 Exercise 1

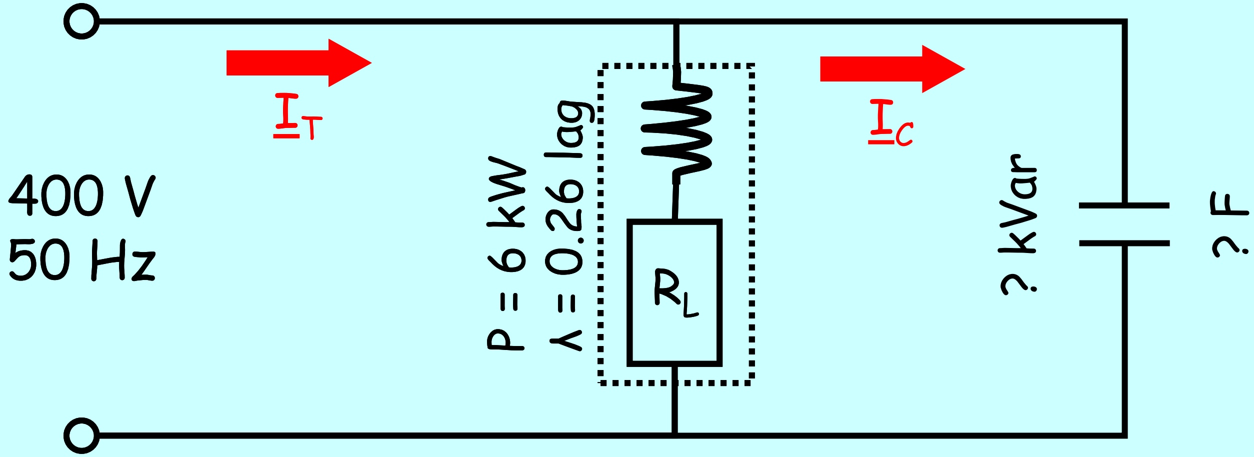
A 400V 50Hz, fluorescent lighting load in a shopping centre is measured at 6000 Watts at a power factor of 0.26 lagging. Determine:

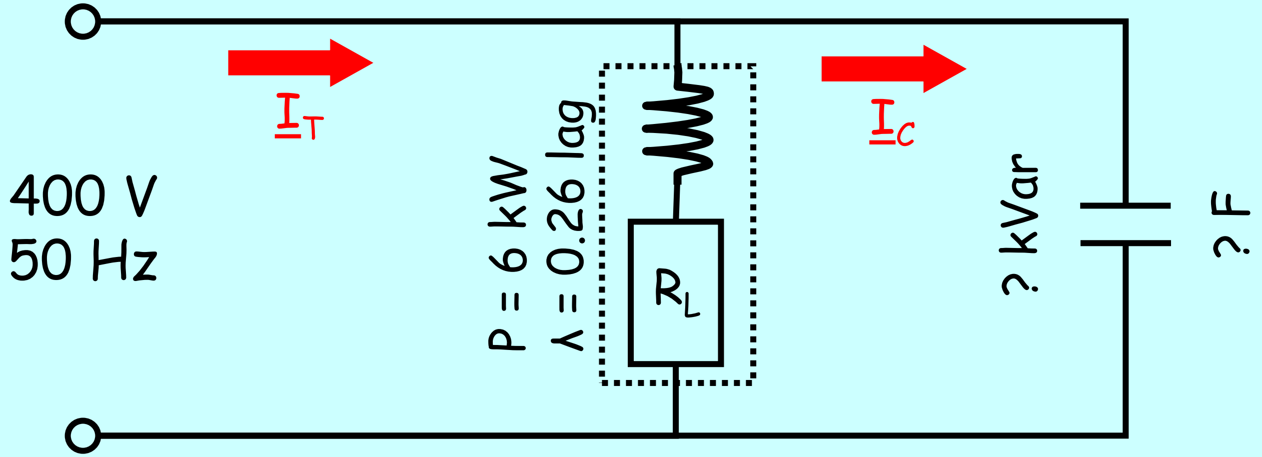
a. The KVar Rating of a Capacitor Bank to improve the power factor to 0.8. Use both the Measurement & Calculation methods.

b. Determine the current through the capacitor bank

c. Determine the capacitance value of the capacitor required.

Where could we start?





$Q_C = Q_T - Q_L$

$Q_C = -4.5 + 22.2$

$Q_C = 17.7$ kVar

$\theta = \cos^{-1}\lambda$

$\theta_L = \cos^{-1} 0.26$

$\theta_L = -74.9°$

$$\tan\theta = \frac{Q}{P}$$

$Q_L = P_L \times \tan\theta$

$Q_L = 6k \times \tan -74.9$

$Q_L = 22.2\ kVar$

$\theta = \cos^{-1}\lambda$

$\theta_T = \cos^{-1} 0.8$

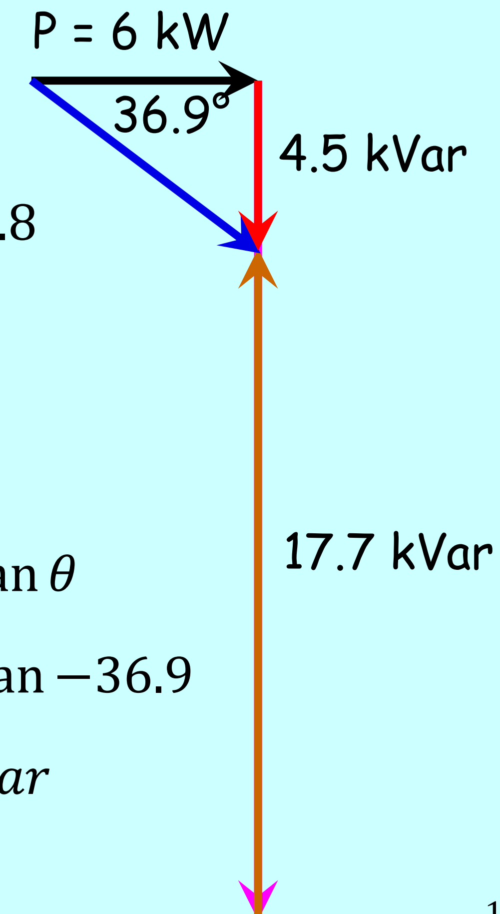
$\theta_T = -36.9°$

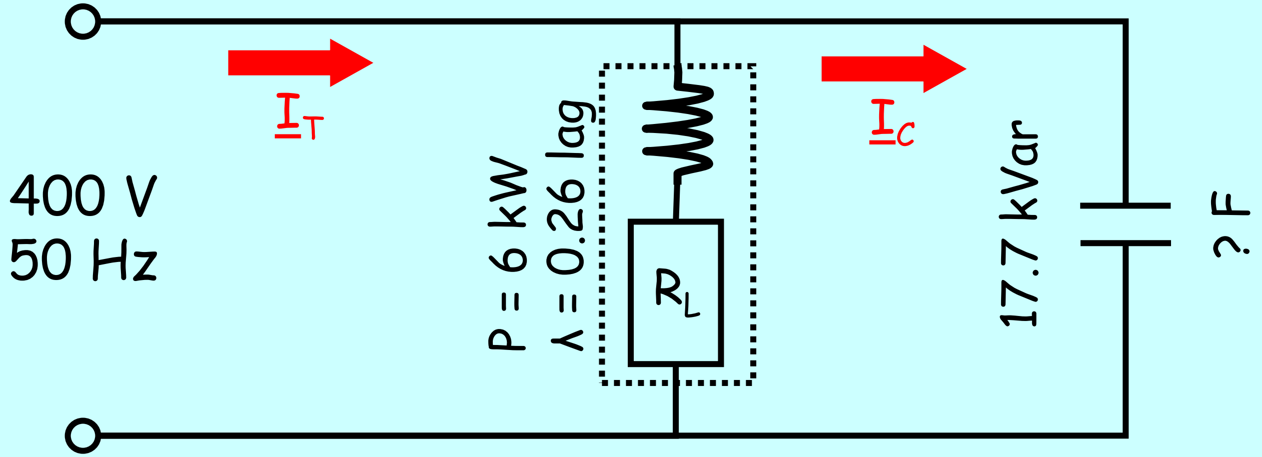
$$\tan\theta = \frac{Q}{P}$$

$Q_T = P_L \times \tan\theta$

$Q_T = 6k \times \tan -36.9$

$Q_T = 4.5\ kVar$





$$Q_C = V_C I_C$$

$$X_C = \frac{V_C}{I_C}$$

$$X_C = \frac{1}{2\pi f C}$$

$$I_C = \frac{Q_C}{V_C}$$

$$X_C = \frac{400}{44.25}$$

$$C = \frac{1}{2\pi f X_C}$$

$$I_C = \frac{17.7k}{400}$$

$$X_C = 9\ \Omega$$

$$C = \frac{1}{2\pi \times 50 \times 9}$$

$$I_C = 44.25\ A$$

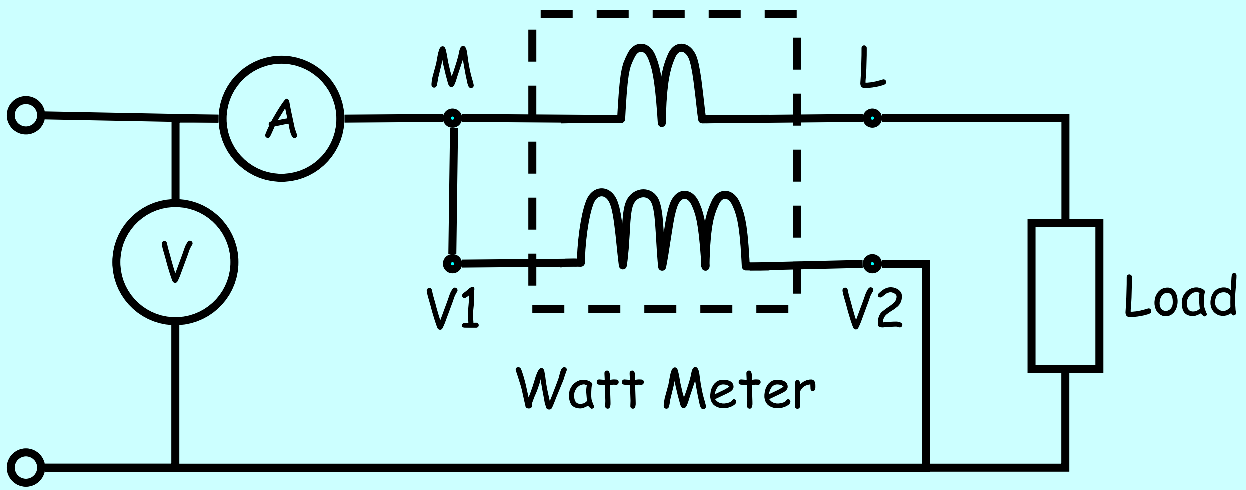
$$C = 353.7\ \mu F$$

Tutorial pg. 52 Exercise 4

The following meter readings were taken on a single-phase circuit. Use these to determine the power factor and the phase angle of the circuit.

Voltmeter = 240V   Ammeter = 20A   Wattmeter = 4.2kW

What do we know?



Tutorial pg. 52 Exercise 4

The following meter readings were taken on a single-phase circuit. Use these to determine the power factor and the phase angle of the circuit.

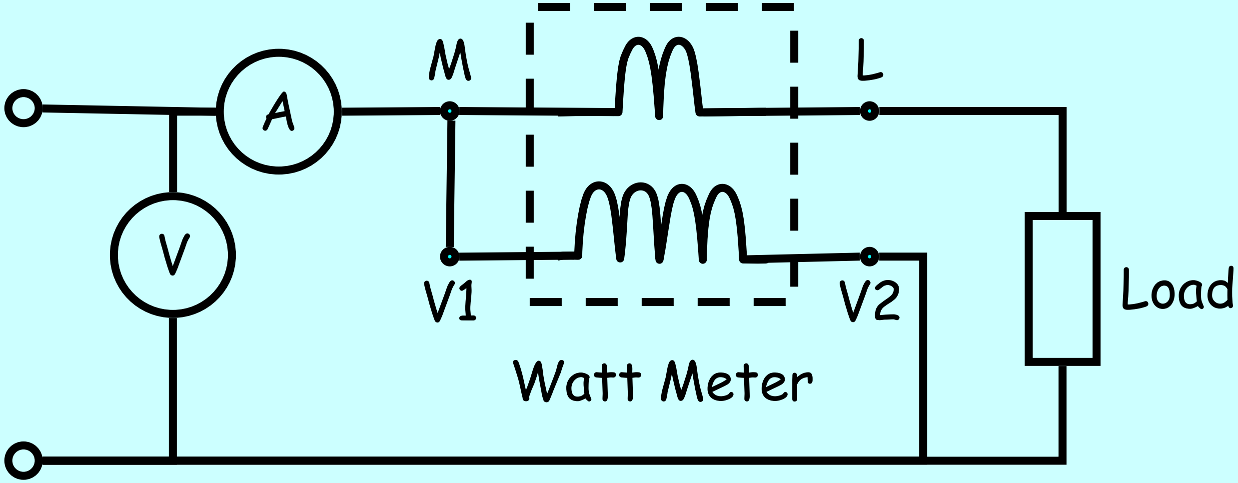
Voltmeter = 240V   Ammeter = 20A   Wattmeter = 4.2kW

What do we know?

V = 240 V
I = 20 A
P = 4.2 kW

What do we want to know? ʎ & θ

How can we calculate? $\lambda = \frac{P}{S}$ and $\theta = \cos^{-1}\lambda$



V = 240 V

I = 20 A

P = 4.2 kW

Apparent Power

S = VI

S = 240 x 20

S = 4,800 VA

Power Factor

$$\lambda = \frac{P}{S}$$

$$\lambda = \frac{4.2k}{4.8k} = 0.875$$

Phase Angle

$\theta = \cos^{-1} \lambda$

$\theta = \cos^{-1} 0.875$

$\theta = 29.0°$

End of Lesson

Practical Exercise

Power Factor Improvement

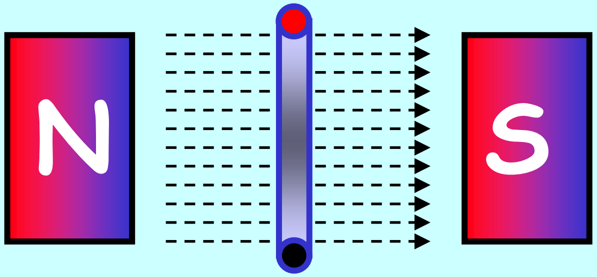
UEENEEG102A
Solve problems in low voltage a.c. circuits

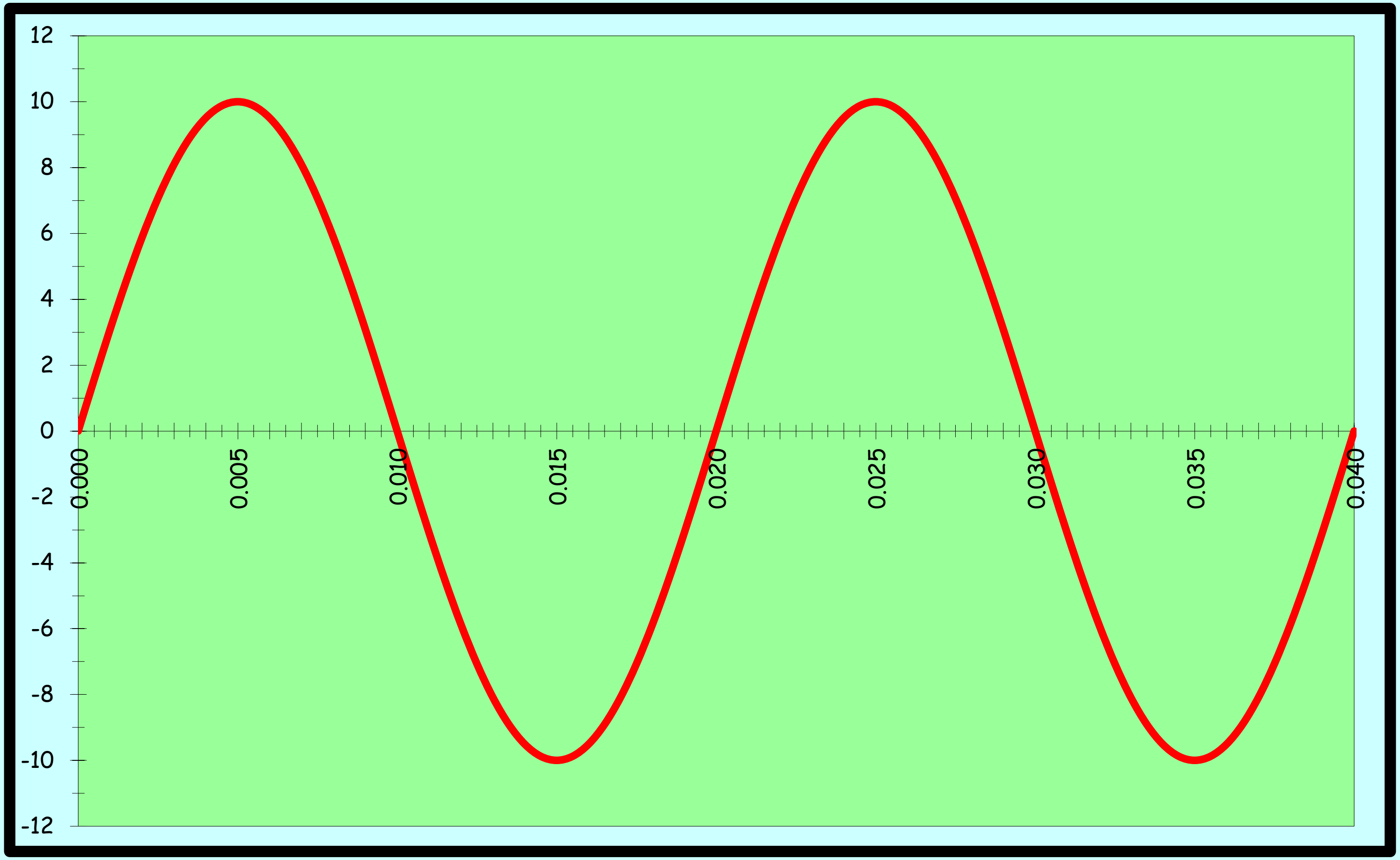
Three Phase Systems & Harmonics

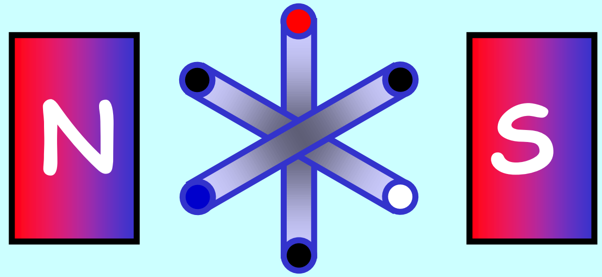
Objectives:

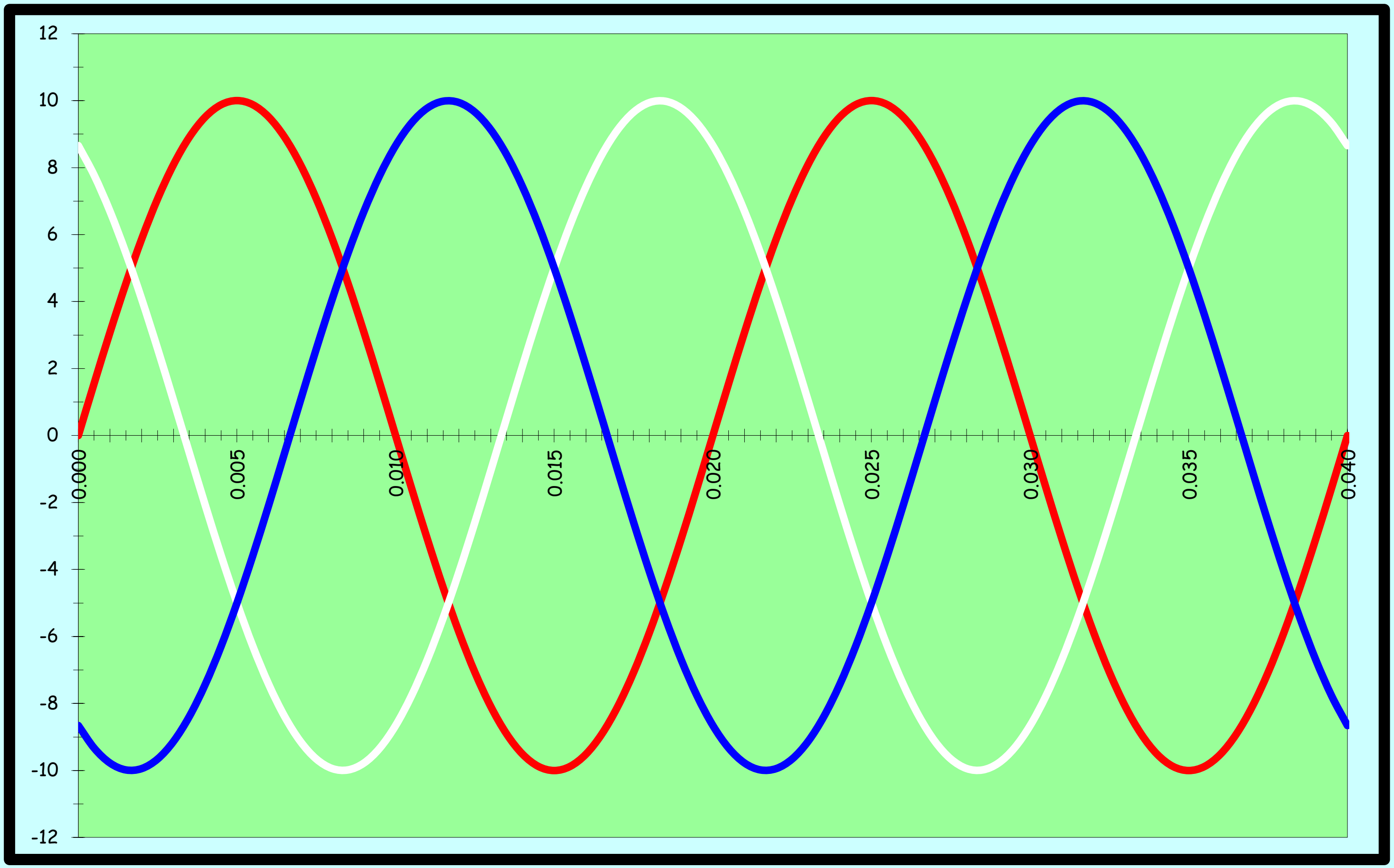
At the end of this lesson students should be able to:

1. Draw the voltage waveforms and Phasor diagram for a three phase system.
2. Briefly describe the principle of three phase generation.
3. State at least four advantages of three phase systems.
4. Determine the phase sequence of a three phase supply.

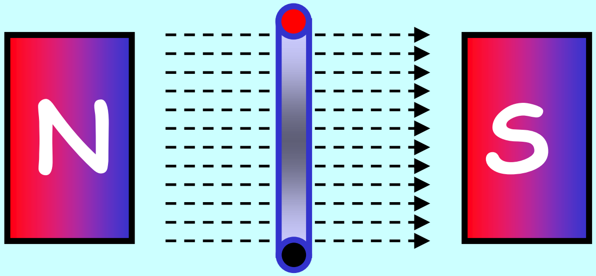

N
S


12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040

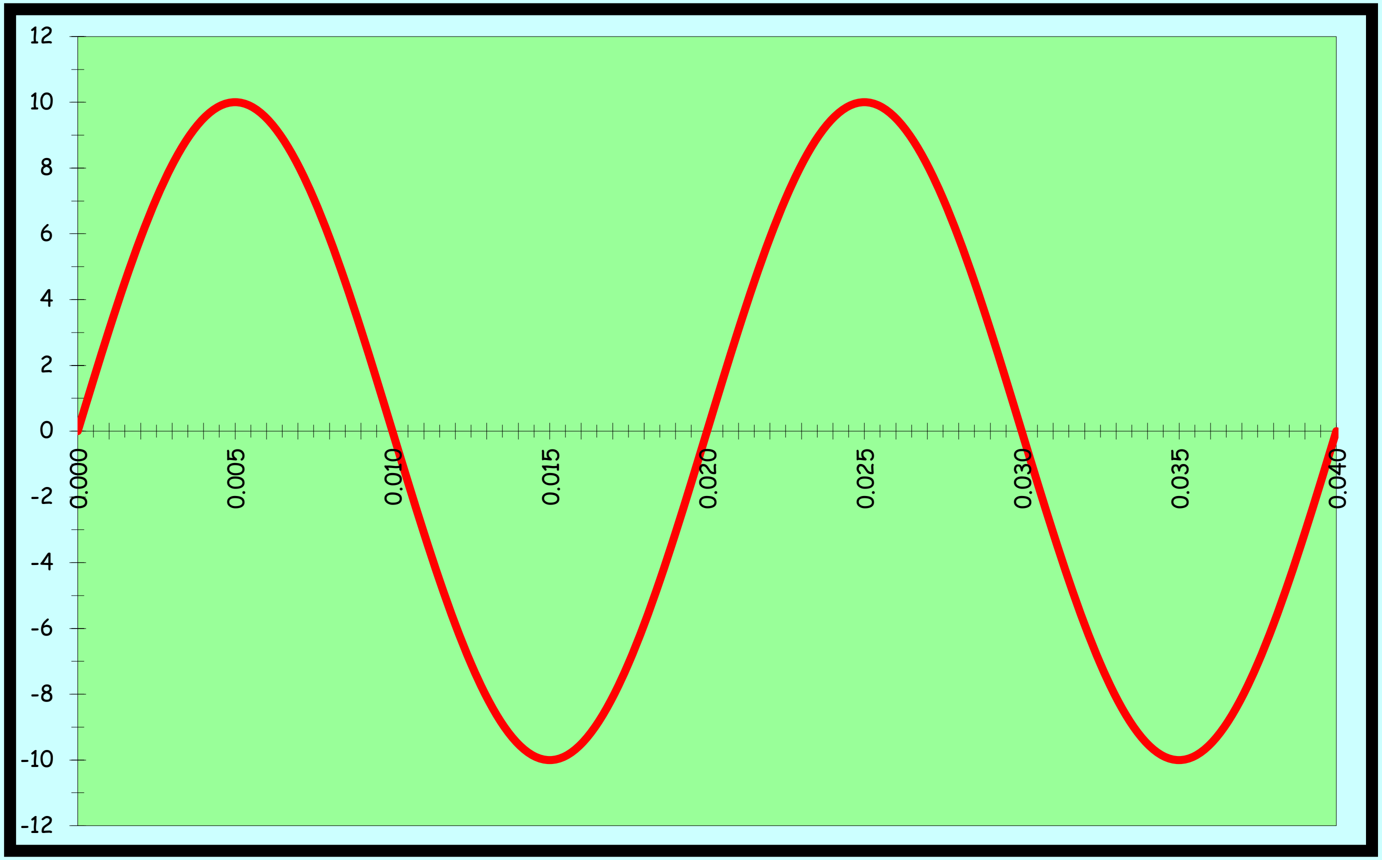

N
S


12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040

Generation Principles



$$f = \frac{N}{60} \times \frac{P}{2} = \frac{NP}{120}$$



$$f = \frac{N}{60} \times \frac{P}{2} = \frac{NP}{120}$$

$P = 2$

$N = 3{,}000$ rpm

$$f = \frac{2 \times 3000}{120}$$

$f = 50$ hz

$P = 6$

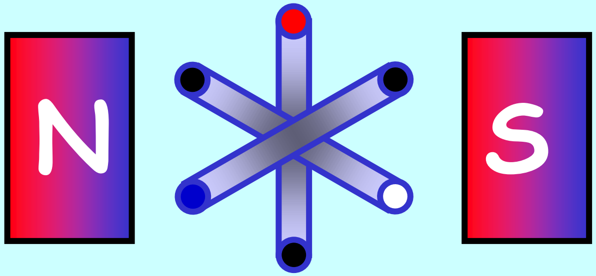
$f = 50$ Hz

$$N = \frac{120f}{P}$$

$$N = \frac{120 \times 50}{6}$$

$N = 1{,}000$ rpm

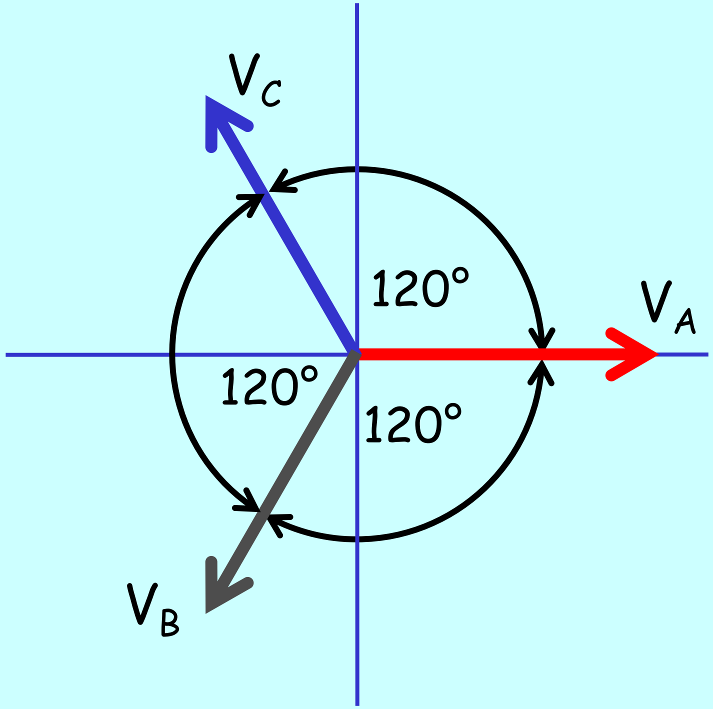
Three Phase Phasors



$$V_A = V_M \sin \theta$$

$$V_B = V_M \sin (\theta - 120)$$

$$V_C = V_M \sin (\theta + 120)$$



Advantages

- More Available Power
- Smoother Power delivery
- Varying Voltages
  - Phase to Phase (400) and
  - Phase to Neutral (230)
- Lower Current per phase
- Smaller Conductors

Find the line voltages of a three phase ac system at $\theta = 60°$ if the maximum voltage in the system is 110 V.

What do we know?

$$V_M = 110 \text{ V}$$

$$\theta = 60°$$

What do we want to know?

$$V_A = V_M \sin \theta$$

$$V_A = V_M \sin 60°$$

$$V_A = 95.3 \text{ V}$$

$$V_B = V_M \sin(\theta - 120)$$

$$V_B = V_M \sin(60 - 120)$$

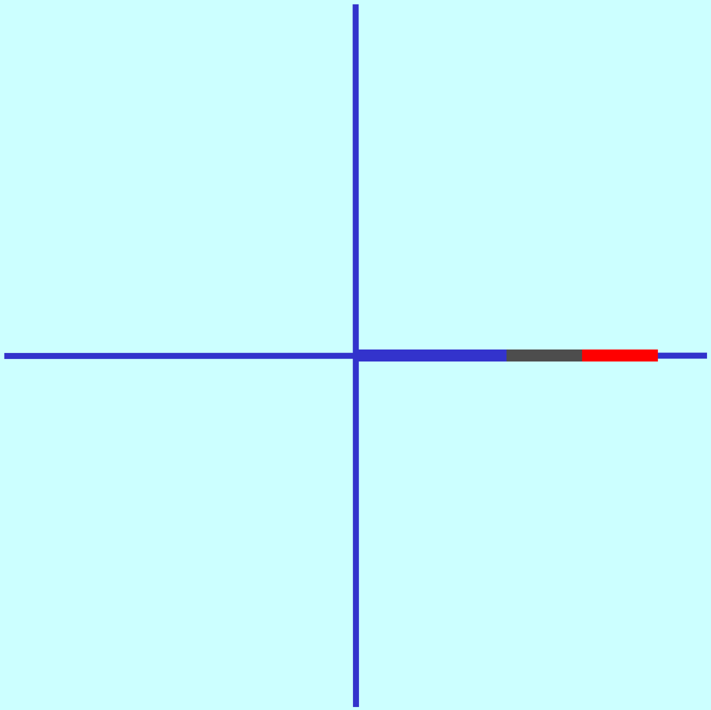
$$V_B = -95.3 \text{ V}$$

$$V_C = V_M \sin(\theta - 240)$$

$$V_C = V_M \sin(60 - 240)$$

$$V_C = 0 \text{ V}$$

Harmonics



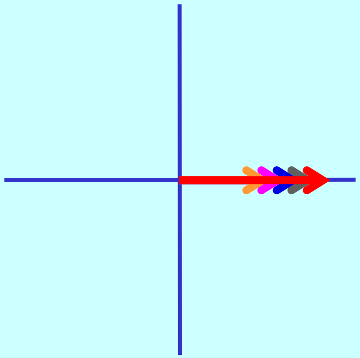
Objectives:

At the end of this lesson students should be able to:

1. Define what a Harmonic Waveform is.
2. Define the term selective resonance.
3. List the types and classifications of harmonic waveforms.
4. Explain the effects of harmonics.
5. Describe how harmonics are generated, produced, found and reduced.

Harmonics are whole number multiples of a Fundamental Frequency (1st Harmonic)

Eg. Fundamental frequency = 50 Hz



1st Harmonic = (50 x 1) = 50 Hz

2nd Harmonic = (50 x 2) = 100 Hz

3rd Harmonic = (50 x 3) = 150 Hz

4th Harmonic = (50 x 4) = 200 Hz

5th Harmonic = (50 x 5) = 250 Hz

Harmonics are whole number multiples of a Fundamental Frequency (1st Harmonic)

Eg. Fundamental frequency = 50 Hz

1st Harmonic = (50 x 1) = 50 Hz

2nd Harmonic = (50 x 2) = 100 Hz

3rd Harmonic = (50 x 3) = 150 Hz

4th Harmonic = (50 x 4) = 200 Hz

5th Harmonic = (50 x 5) = 250 Hz

ODD Harmonics = Fundamental frequency x ODD Number

Harmonics are whole number multiples of a Fundamental Frequency (1st Harmonic)

Eg. Fundamental frequency = 50 Hz

1st Harmonic = (50 x 1) = 50 Hz

2nd Harmonic = (50 x 2) = 100 Hz

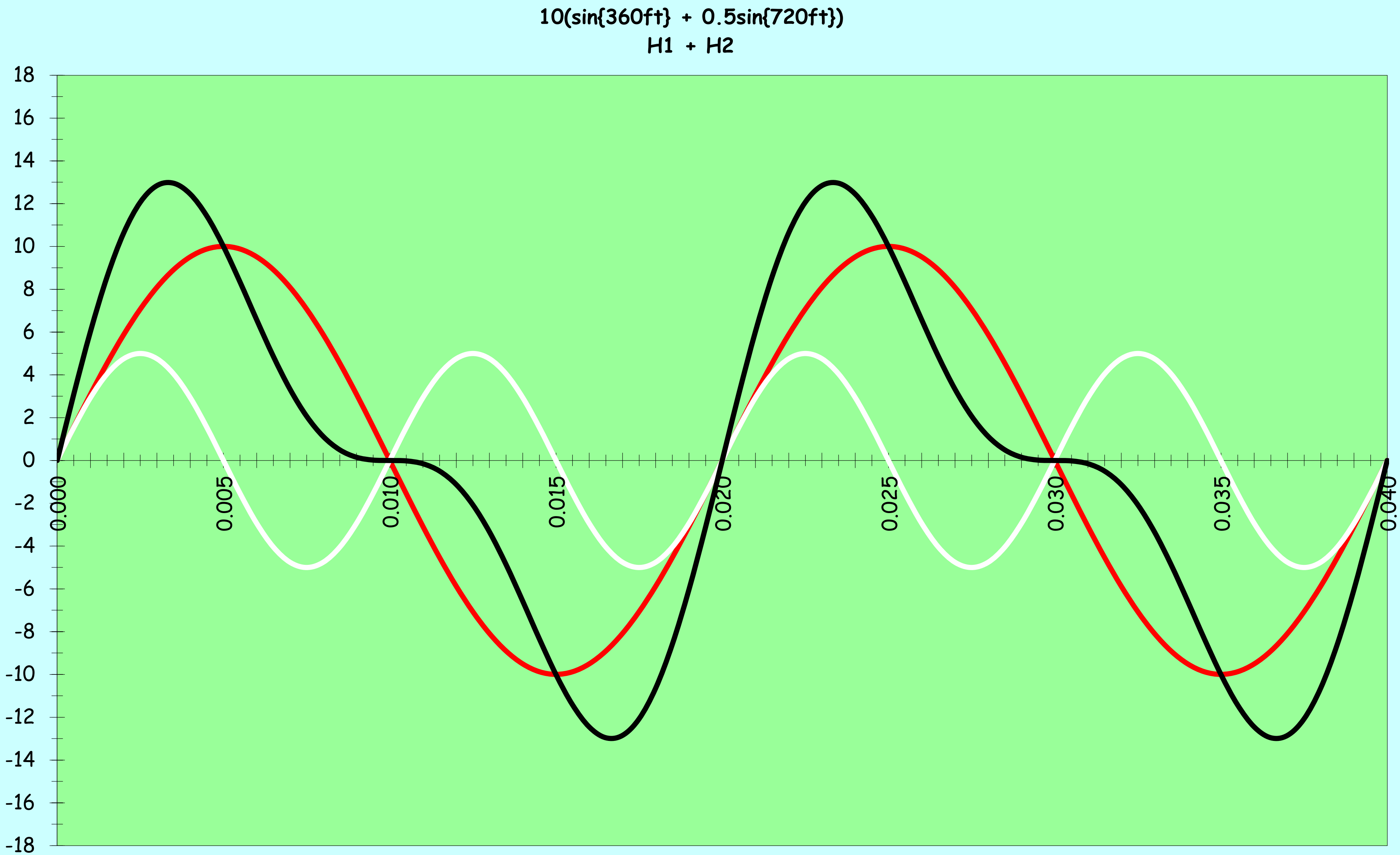
3rd Harmonic = (50 x 3) = 150 Hz

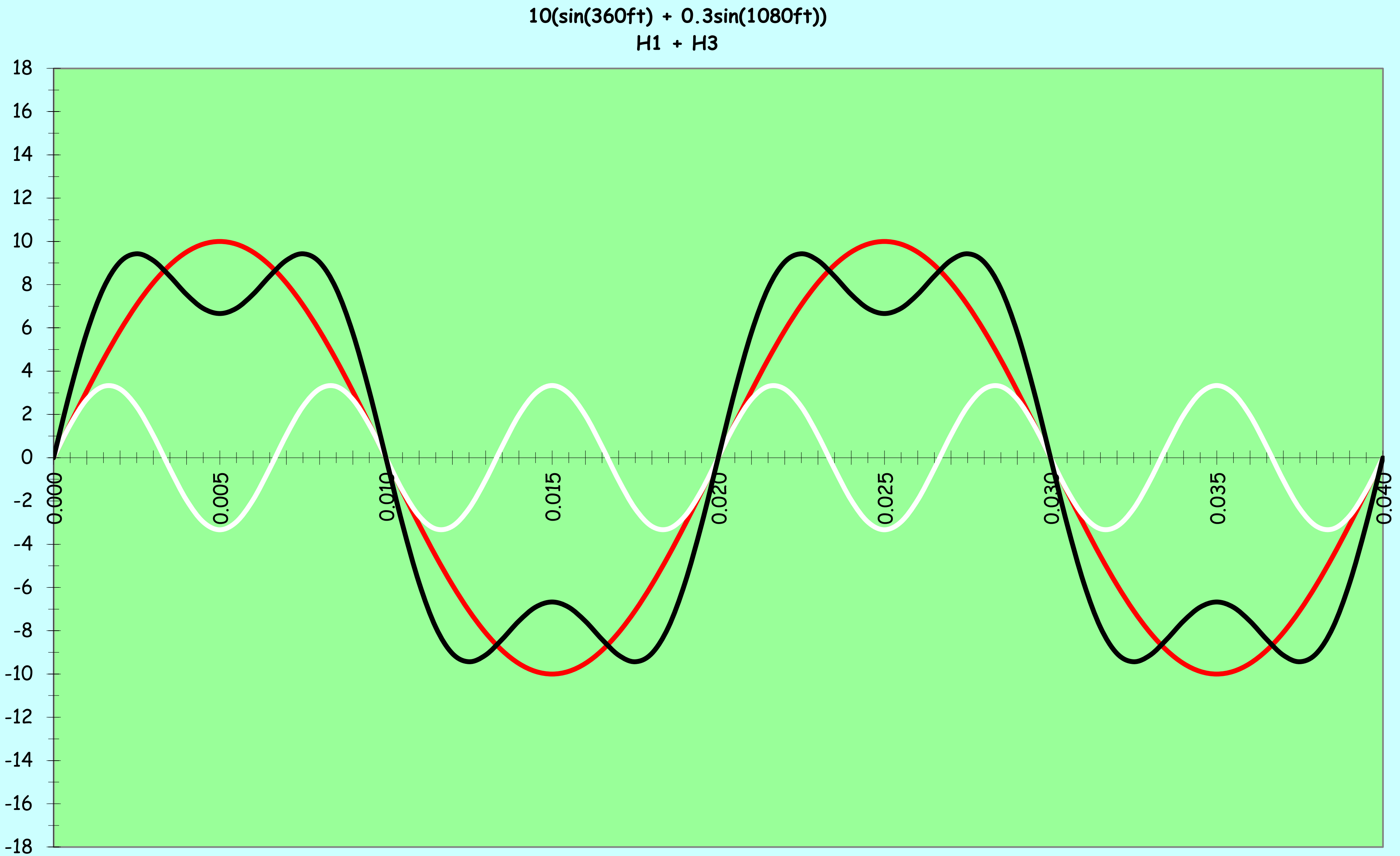
4th Harmonic = (50 x 4) = 200 Hz

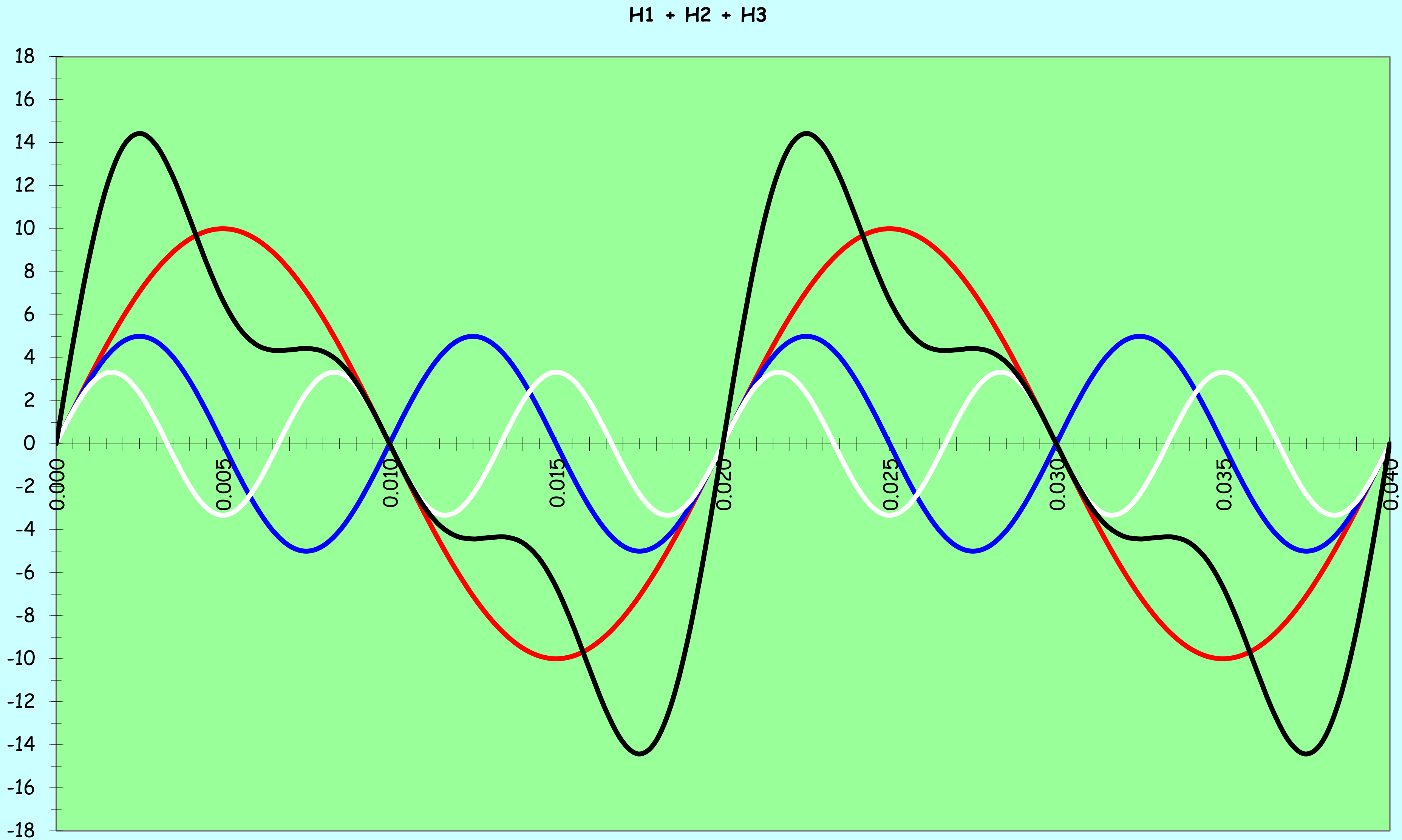
5th Harmonic = (50 x 5) = 250 Hz

ODD Harmonics = Fundamental frequency x ODD Number

EVEN Harmonics = Fundamental frequency x EVEN Number


10(sin{360ft} + 0.5sin{720ft})
H1 + H2
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040

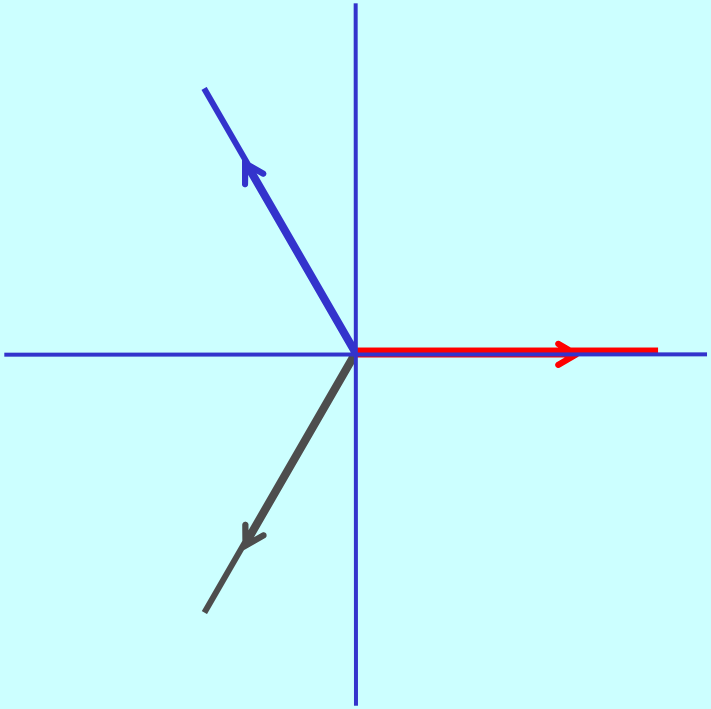

10(sin(360ft) + 0.3sin(1080ft))
H1 + H3
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040

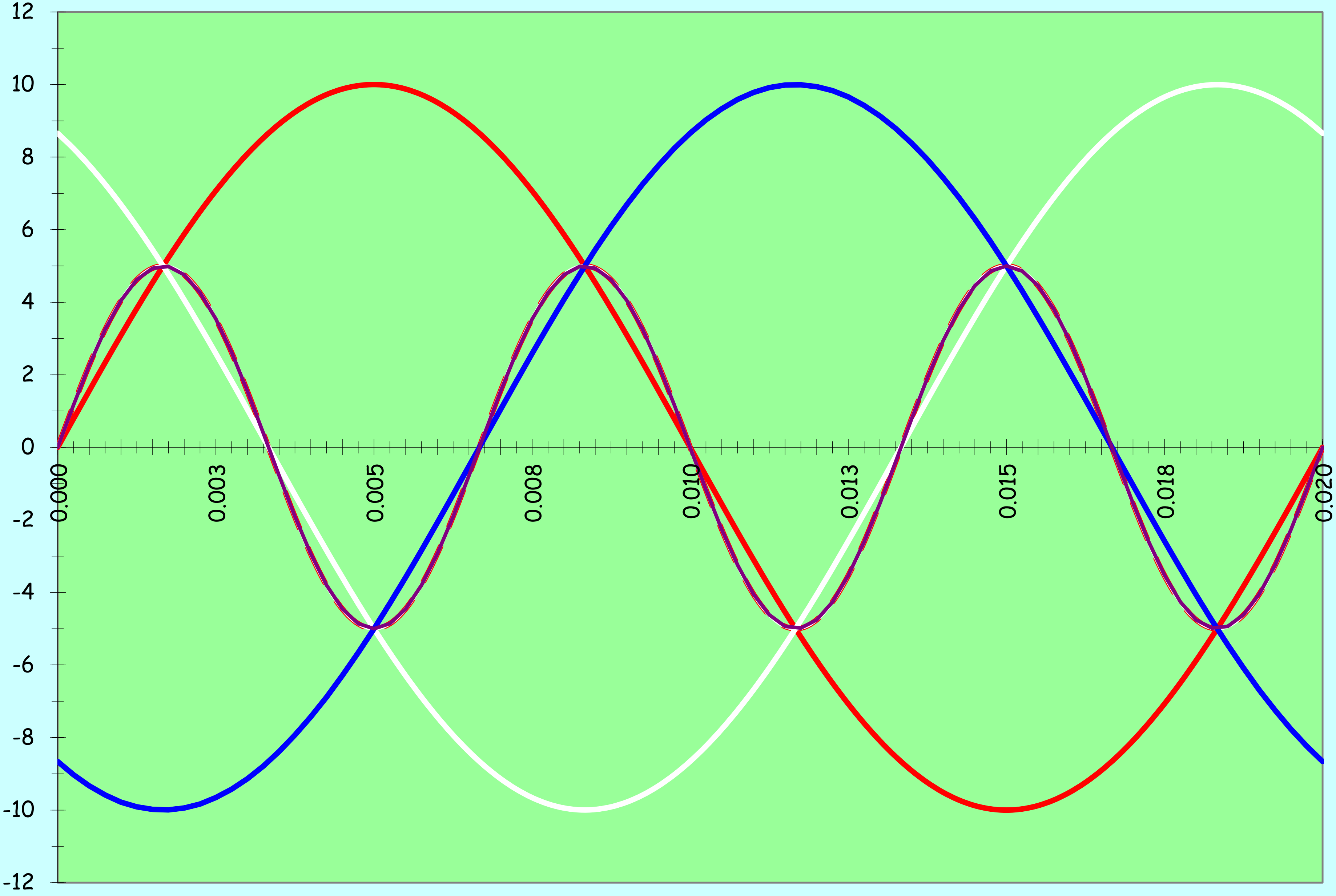

H1 + H2 + H3
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040

Harmonics which "rotate" with the same sequence as the fundamental are called *positive sequence*, eg. 7th Harmonic.

Harmonics which "rotate" in the opposite sequence to the fundamental are called *negative sequence*, eg. 5th Harmonic.

Harmonics which don't "rotate" at all because they're in phase with each other are called *zero sequence*, Eg. 3rd Harmonic.





12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0.000
0.003
0.005
0.008
0.010
0.013
0.015
0.018
0.020

Effects for Harmonics

Sequence	Effects on a Motor	Effects on a Distribution System
Positive	Forward rotating magnetic field Assists Torque	Heating
Negative	Reverse rotating magnetic field Reduces Torque	Heating and Motor Problems
Zero	Little or None	Heating and Excessive Neutral Currents

Testing for Harmonics

Display the waveform

Use a CRO to look at the shape of the waveform.

Measure the frequency components

Use a Spectrum Analyser to look at the frequency components of the waveform.

Measure the current components

Use a True RMS or Average Responding Ammeter to measure the currents.

Other Indicators

Abnormally Hot Transformers and other components

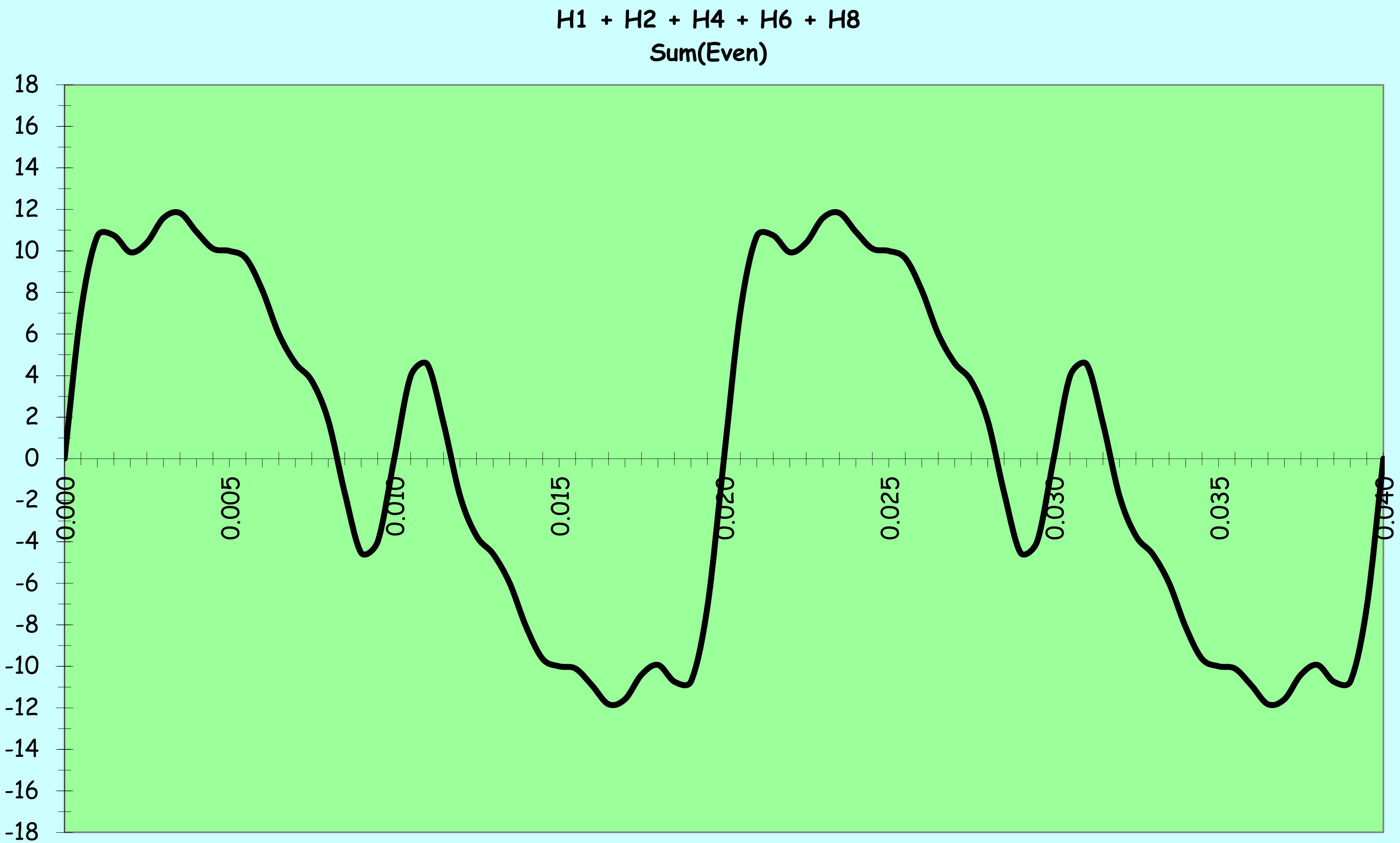
Abnormal vibrations

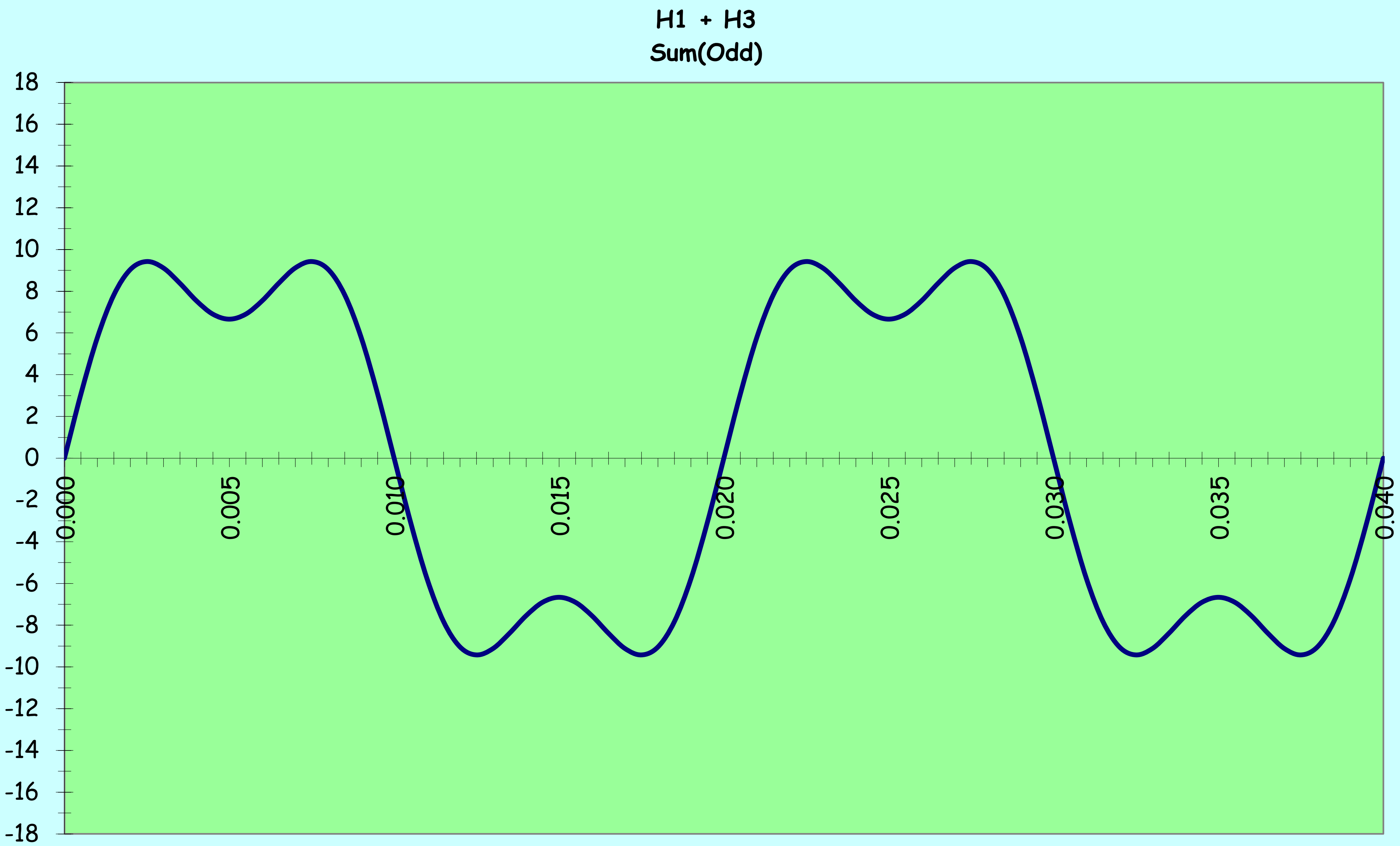
Erratic motor behaviour

Methods of overcoming Harmonic effects

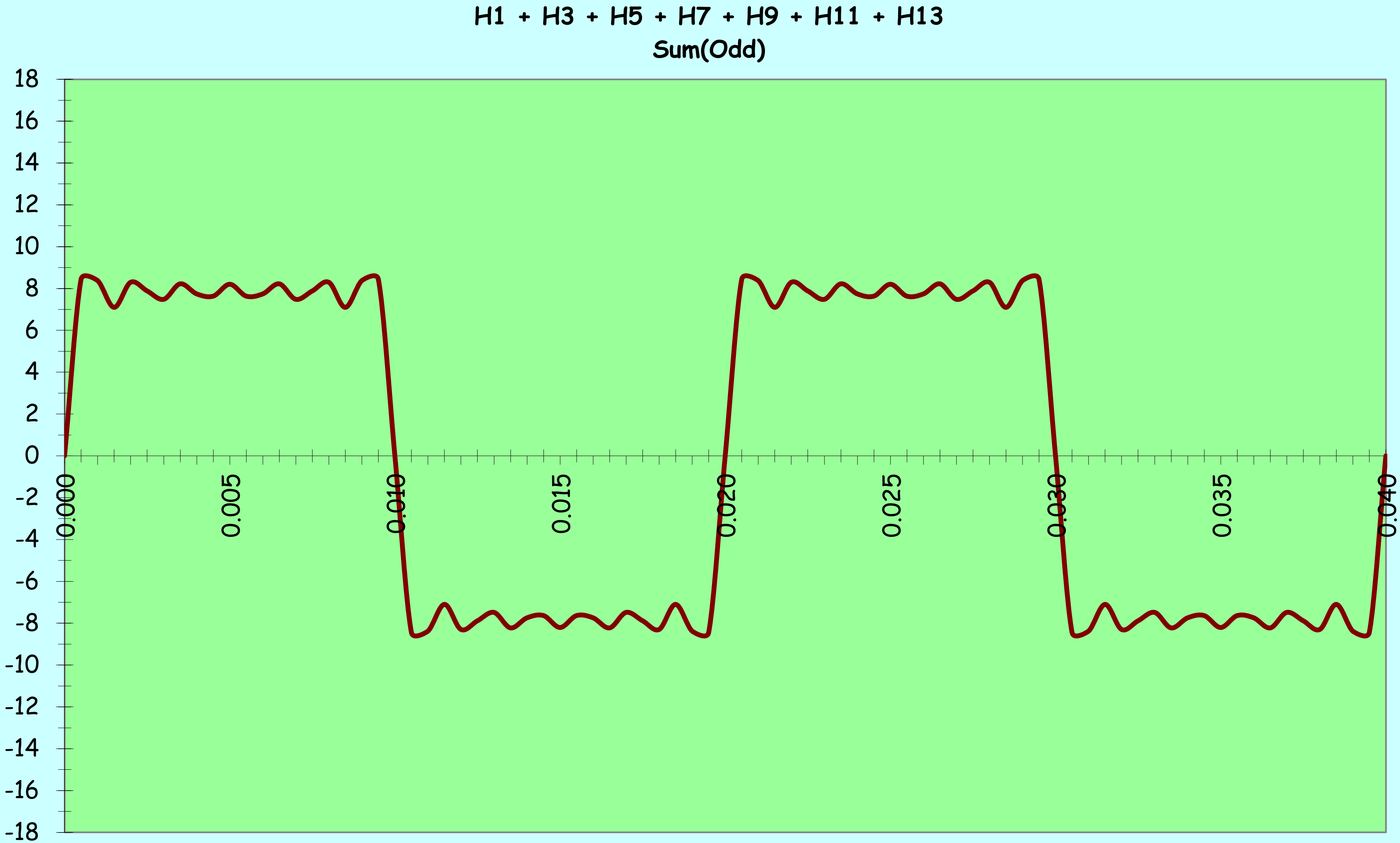
Good design practices

Filters


H1 + H2 + H4 + H6 + H8
Sum(Even)
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040



H1 + H3
Sum(Odd)
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040



H1 + H3 + H5 + H7 + H9 + H11 + H13
Sum(Odd)
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040

End of Lesson

Practical Exercises

Three Phase Waveforms

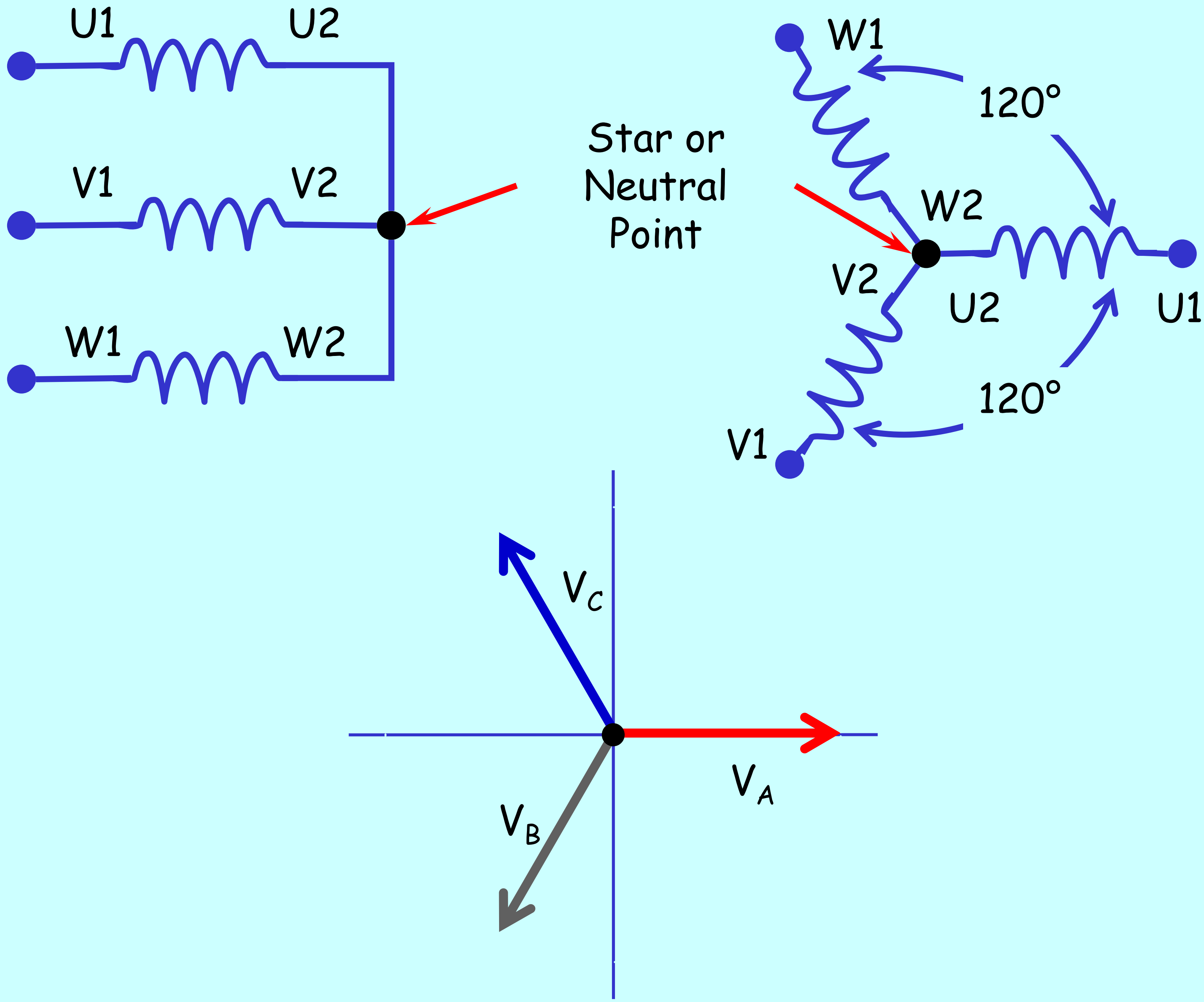
UEENEEG102A
Solve problems in low voltage a.c. circuits

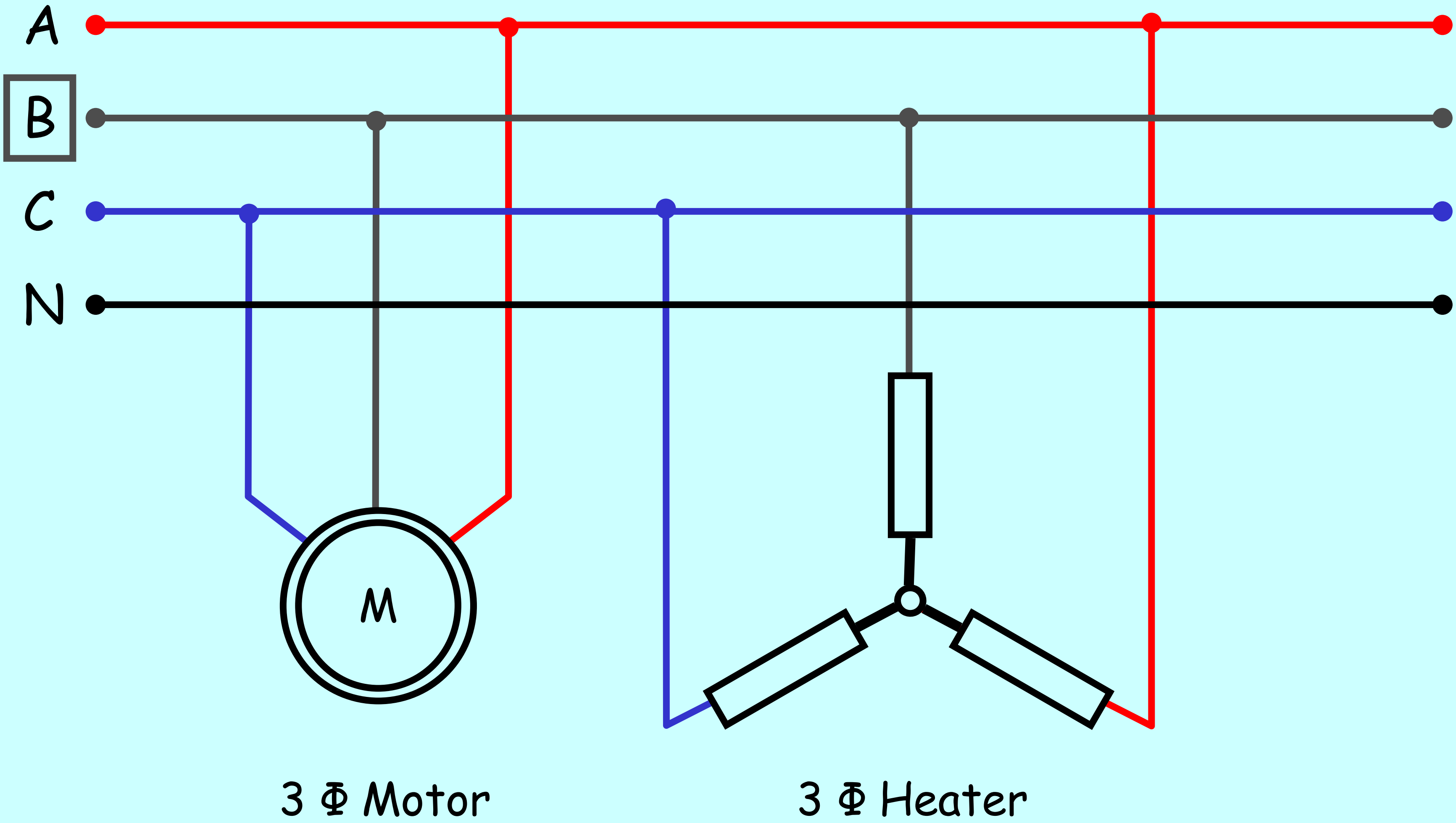
Three Phase Four Wire Systems

Objectives:

At the end of this lesson students should be able to:

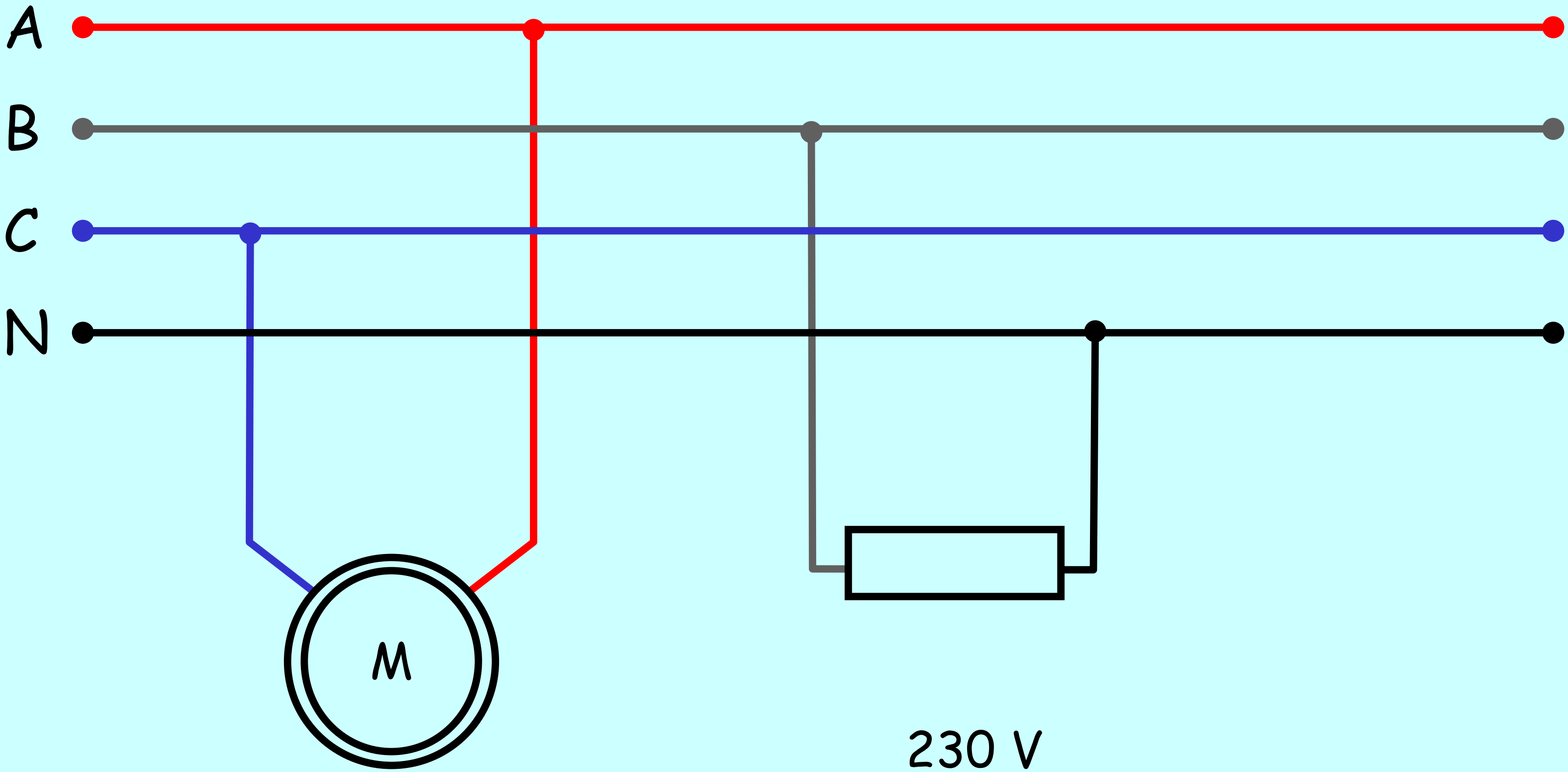
1. Draw the circuit connections for a Star Connected System.
2. Calculate Line & Phase Voltages and Currents for a three phase star connected system.
3. Develop Phasor Diagrams for a star connected system.
4. Connect a load in star.


U1
U2
V1
V2
W1
W2
Star or Neutral Point
W1
120°
W2
V2
U2
U1
120°
V1
V_C
V_A
V_B



3 Φ Balanced Loads

3 Φ Unbalanced Loads

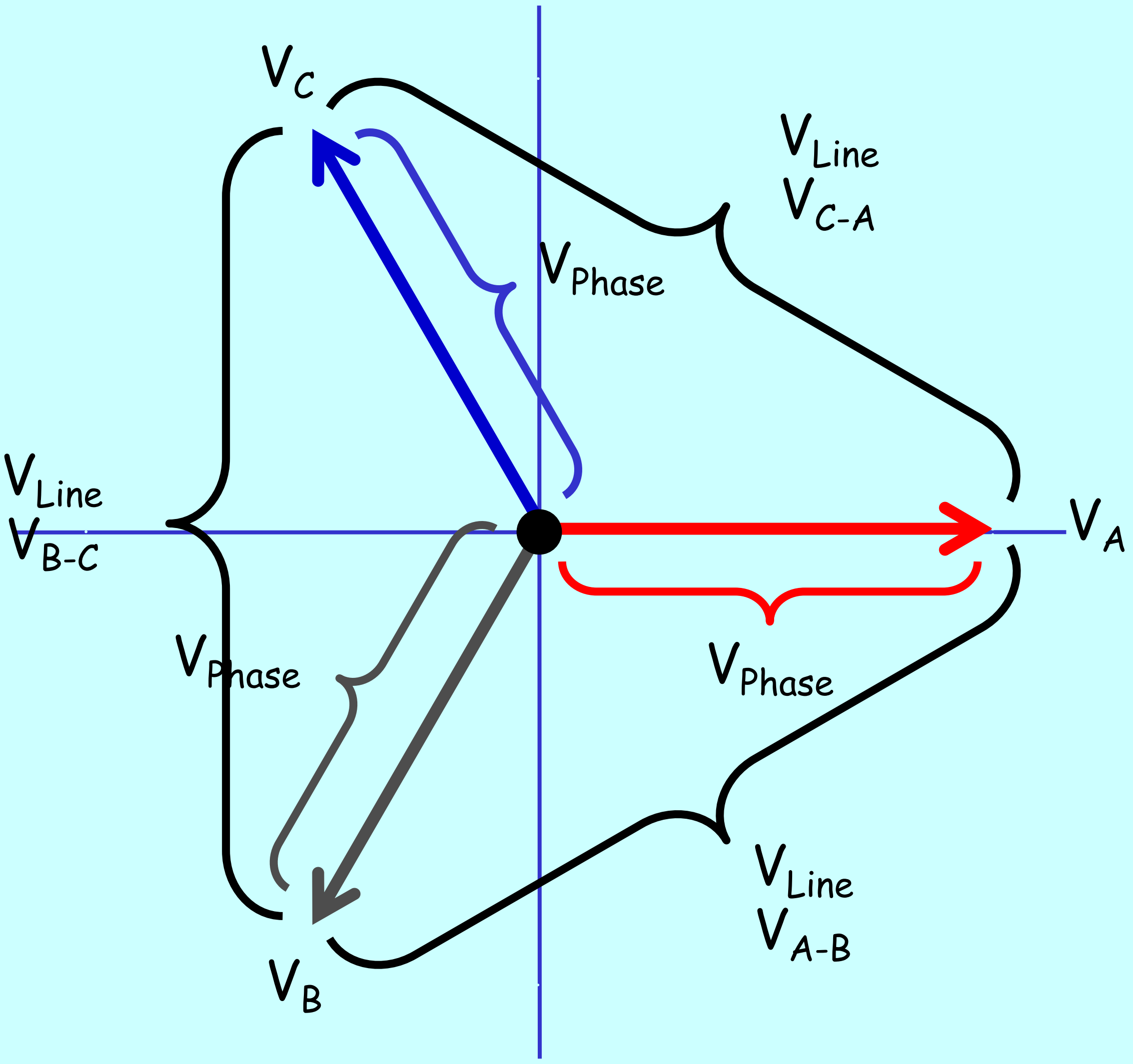


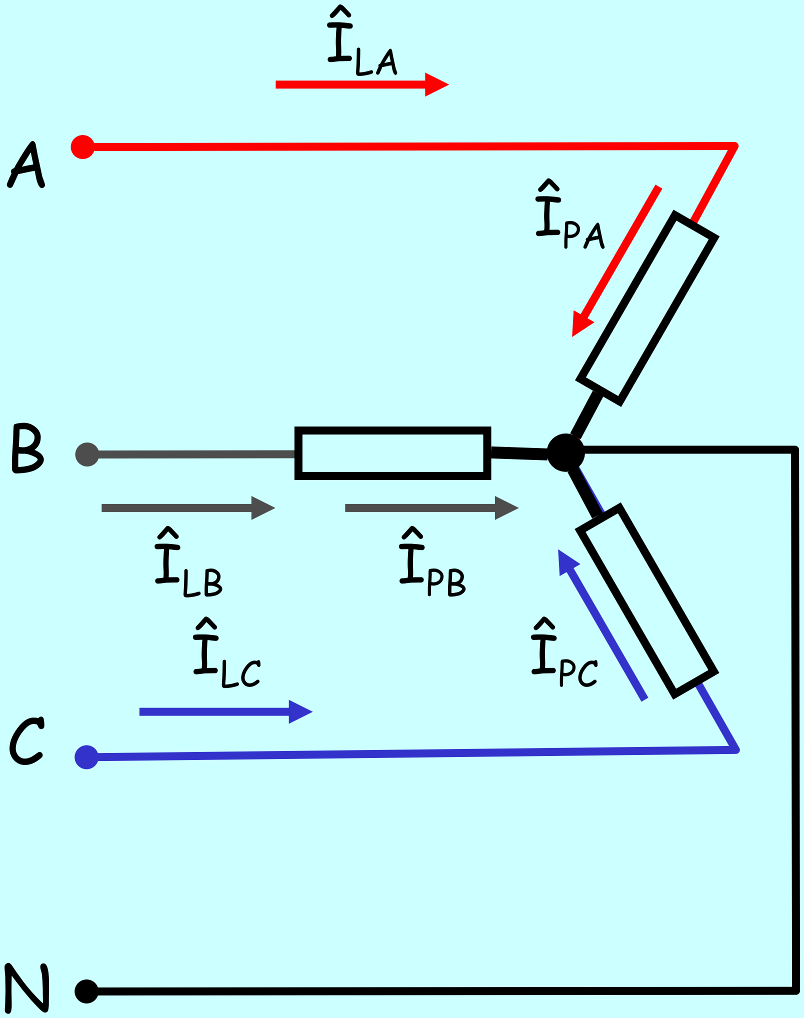
Single Φ Loads

Always Unbalanced

but we try to share the load evenly between phases

Phase & Line Voltages

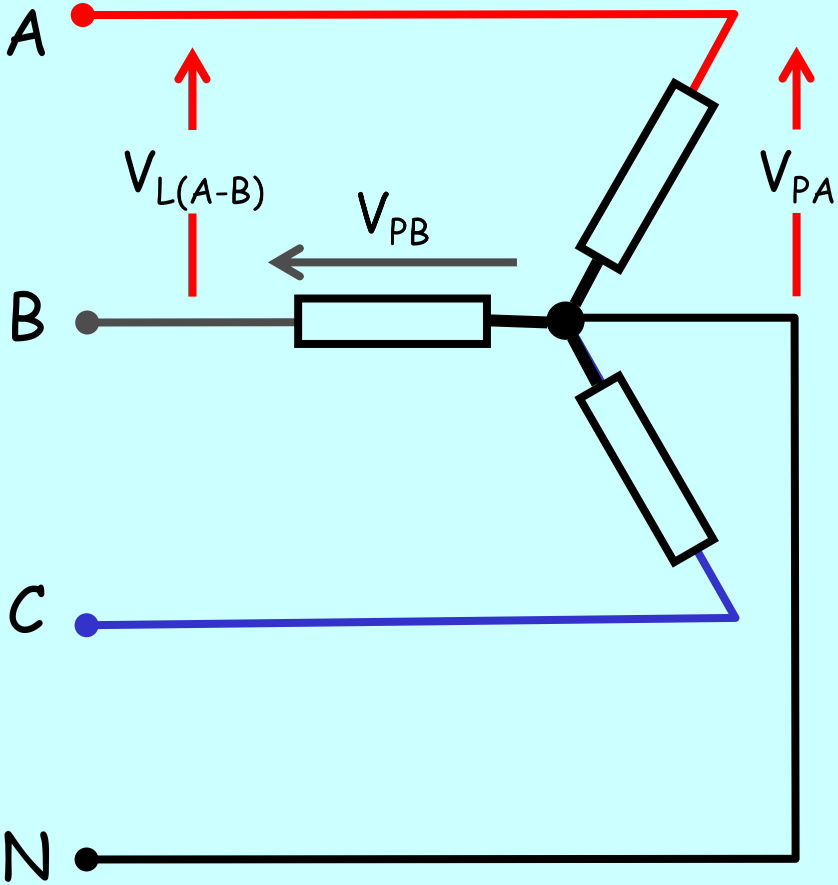




Voltage & Current Relationships

Line Current   Phase Current

$$\hat{I}_L = \hat{I}_P$$



Voltage & Current Relationships

Line Current — Phase Current

$$\hat{I}_L = \hat{I}_P$$

Line Voltage — Difference of two Phase Voltages

$$\underline{V}_L = \underline{V}_{PA} - \underline{V}_{PB}$$

Phase Voltages

$\underline{V}_A = V_M \sin\theta$

$\underline{V}_B = V_M \sin(\theta - 120)$

Line Voltage

$\underline{V}_{AB} = \underline{V}_A - \underline{V}_B$

$$V_{AB}^2 = V_A^2 + V_B^2 - 2V_A V_B \cos(120)$$

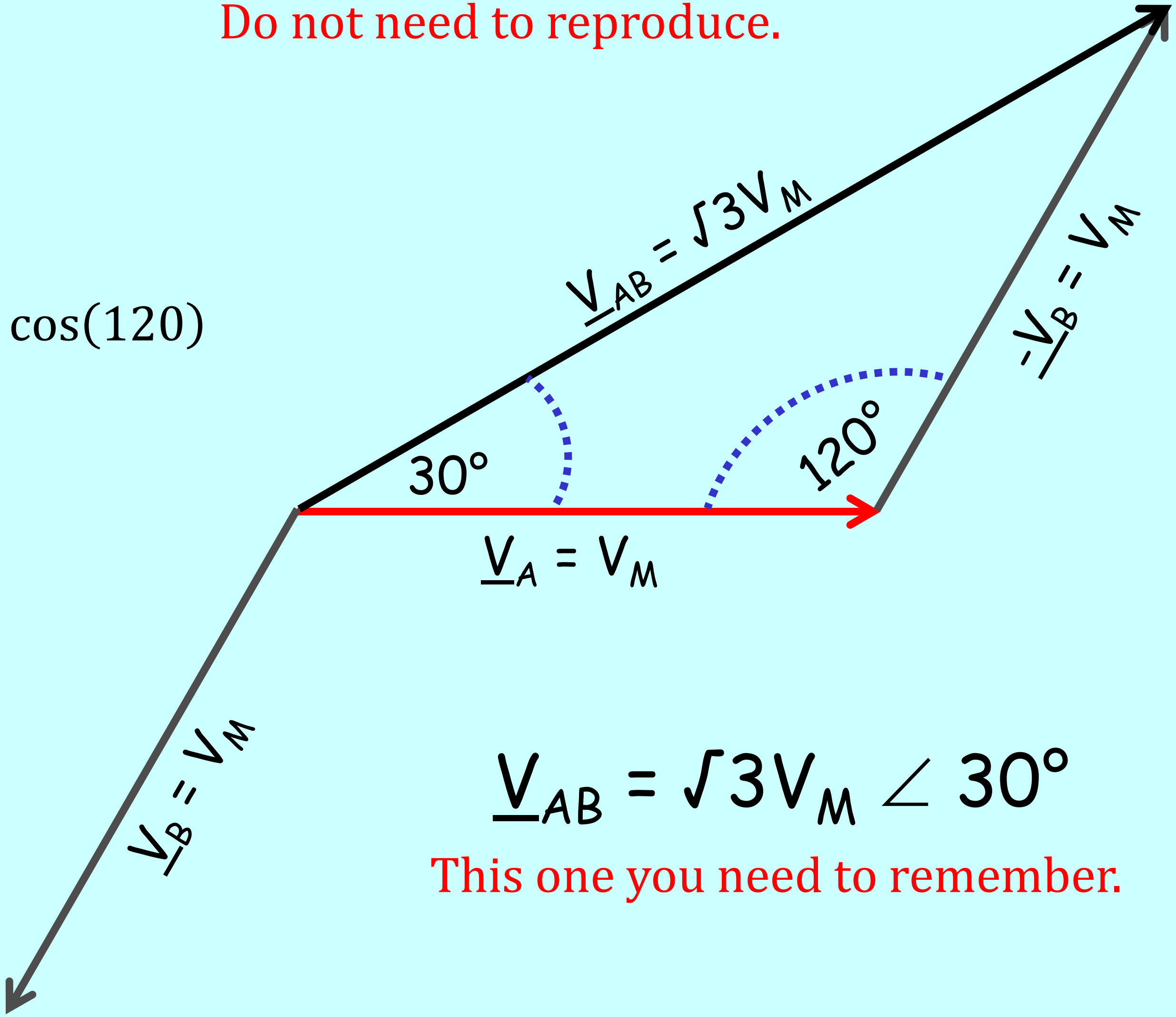
$$V_{AB}^2 = 2(1 - \cos(120))$$

$$V_{AB}^2 = 3$$

Cosine Rule

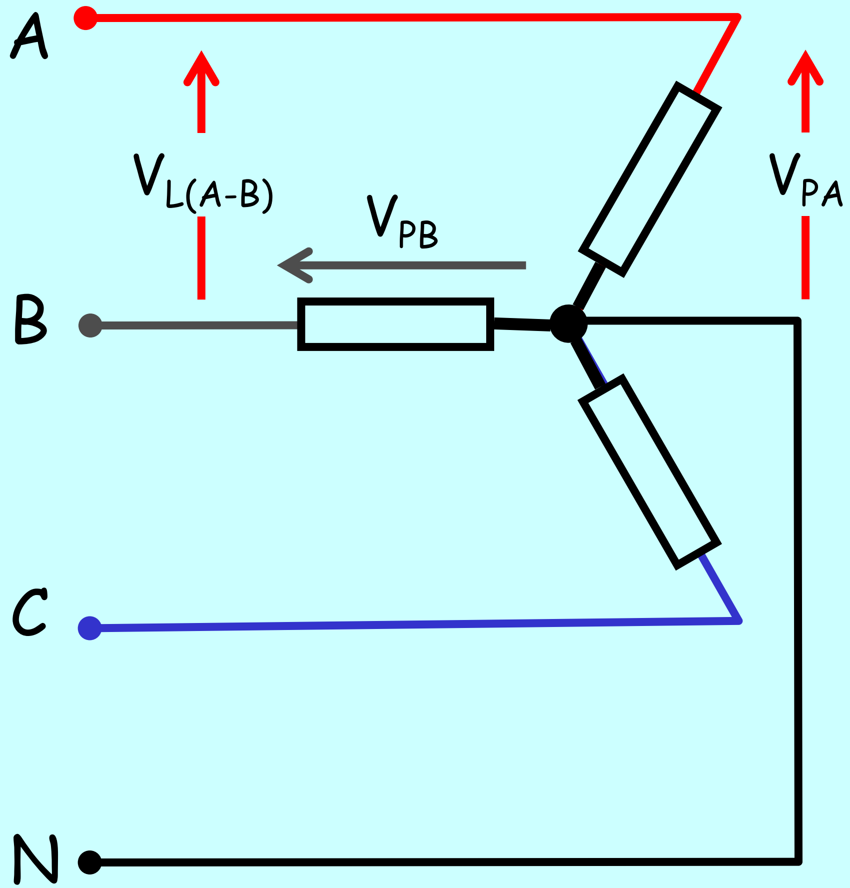
$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

For proof only!!!
Do not need to reproduce.



$\underline{V}_{AB} = \sqrt{3}V_M \angle 30°$

This one you need to remember.



Voltage & Current Relationships

Line Current — Phase Current

$$\hat{I}_L = \hat{I}_P$$

Line Voltage — Difference of two Phase Voltages

$$\underline{V}_L = \underline{V}_{PA} - \underline{V}_{PB}$$

$$|V_L| = \sqrt{3}|V_P|$$

A Star connected alternator produces 6.35 kV in each phase winding. What is the Line Voltage?

What do we know?

$V_P = 6.35$ kV

What do we want to know?

$V_L = \sqrt{3} \times V_P$

$V_L = \sqrt{3} \times 6.35$ kV

$V_L = 11$ kV

A Star connected transformer has a line voltage of 400 V.
What is the Phase Voltage?

What do we know?

$V_L = 400\ V$

What do we want to know?

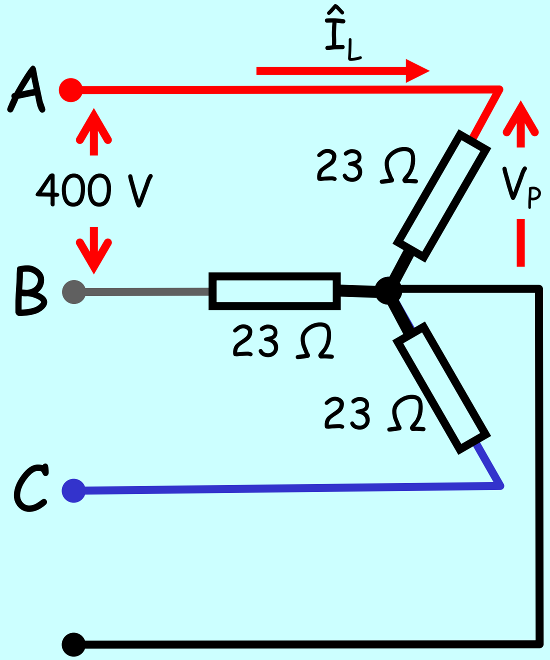
$$V_P = \frac{V_L}{\sqrt{3}}$$

$$V_P = \frac{400}{\sqrt{3}}$$

$V_P = 230.9\ V$

A 3 Phase Star connected heater has a line voltage of 400 V.
Determine the Line Current if each element has an impedance of 23 Ω.

What do we know?



$I_L = I_P$

What do we want to know?

$$V_P = \frac{V_L}{\sqrt{3}}$$

$$V_P = \frac{400}{\sqrt{3}} = 230\,V$$

$$I_P = \frac{V_P}{Z_P} = \frac{230}{23} = 10\ A$$

$$I_L = I_P = 10\ A$$

A three phase star connected four wire supply has a phase voltage of 220 V. What is the Line Voltage?

What do we know?

$$V_P = 220\ V$$

What do we want to know?

$$V_L = \sqrt{3} \times V_P$$

$$V_L = \sqrt{3} \times 220$$

$$V_L = 381\ V$$

A star connected induction motor draws 48 A from a system with a line voltage of 400 V. What is the Impedance of the Phase Windings?

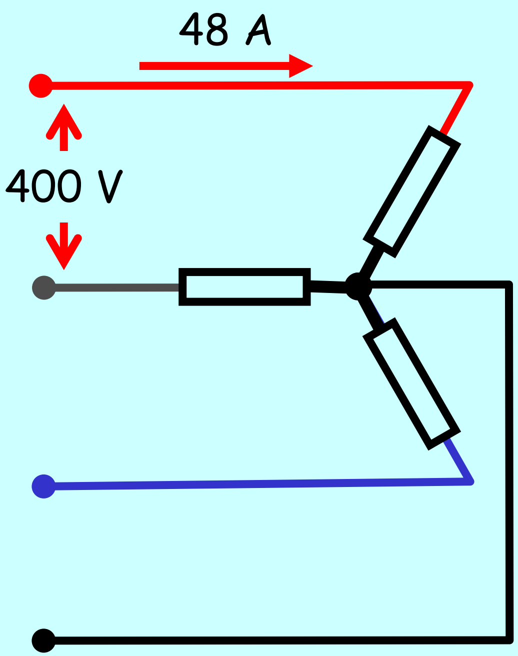
What do we know?

$V_L = 400\ V$

$I_L = I_P = 48\ A$

What do we want to know?

$$Z = \frac{V}{I}$$



$$V_P = \frac{400}{\sqrt{3}}$$

$$Z = \frac{230}{48}$$

$V_P = 230\ V$

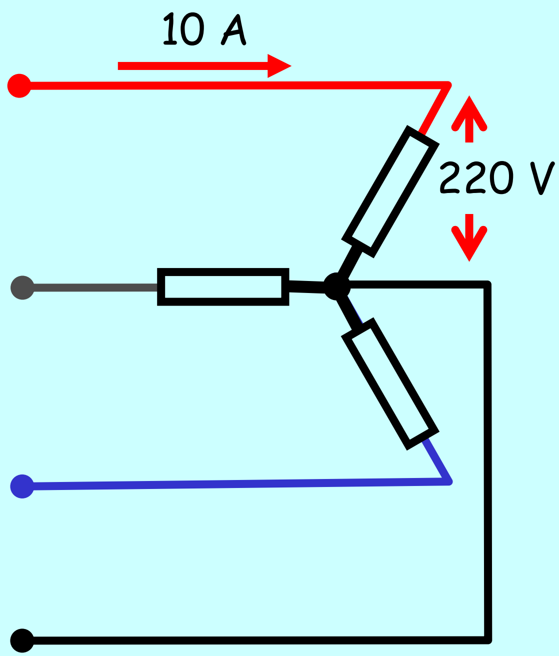
$Z = 4.79\ \Omega$

A star connected symmetrical load draws 10 A from a system with a phase voltage of 220 V. What is the Impedance of each load and the Line Voltage?

What do we know?

$V_P = 220\ V$

$I_L = I_P = 10\ A$



What do we want to know?

$$Z = \frac{V}{I}$$

$$Z = \frac{220}{10}$$

$$Z = 22\ \Omega$$

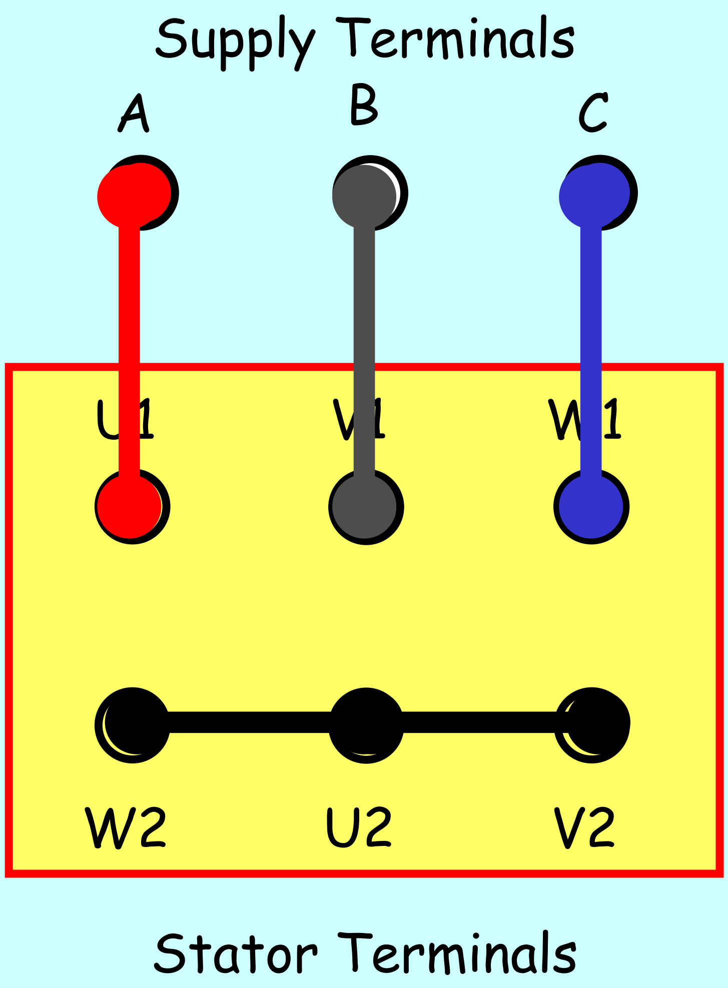
$$V_L = \sqrt{3} \times V_P$$

$$V_L = \sqrt{3} \times 220$$

$$V_L = 381\ V$$

The adjacent diagram represents the terminal connections for a 3 Phase Motor (U, V & W) and the terminals for a 3 Phase Supply (A, B & C).

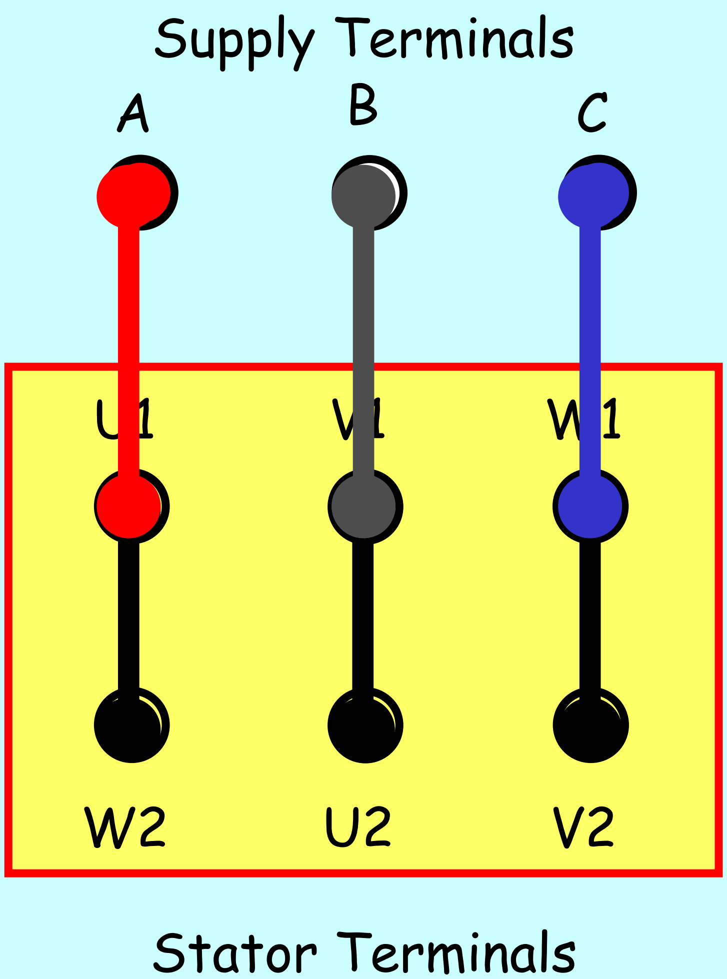
What connections could be made to have the motor to operate in **STAR configuration**?



Star Connection

The adjacent diagram represents the terminal connections for a 3 Phase Motor (U, V & W) and the terminals for a 3 Phase Supply (A, B & C).

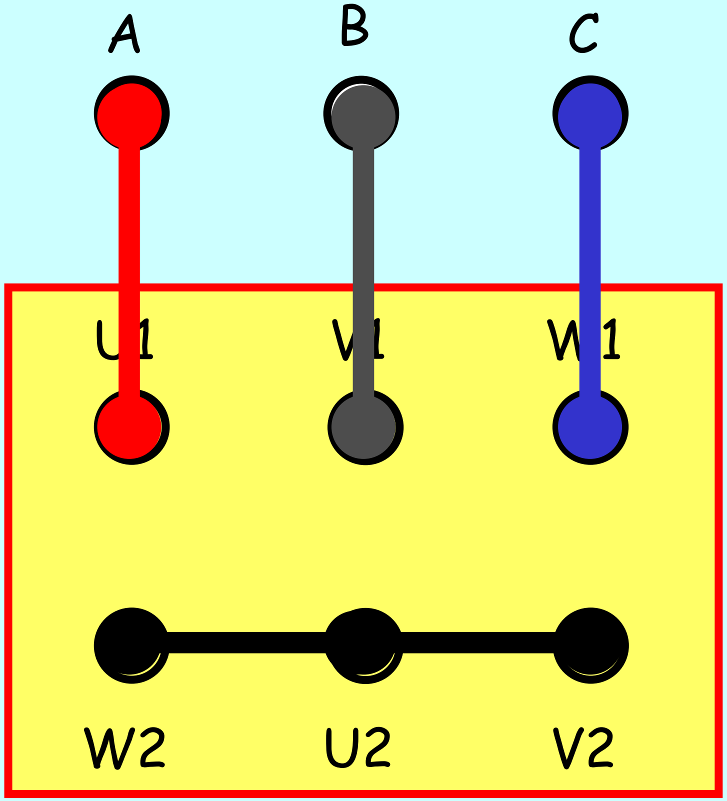
What connections could be made to have the motor to operate in **Delta configuration**?



Delta Connection

Star Connection

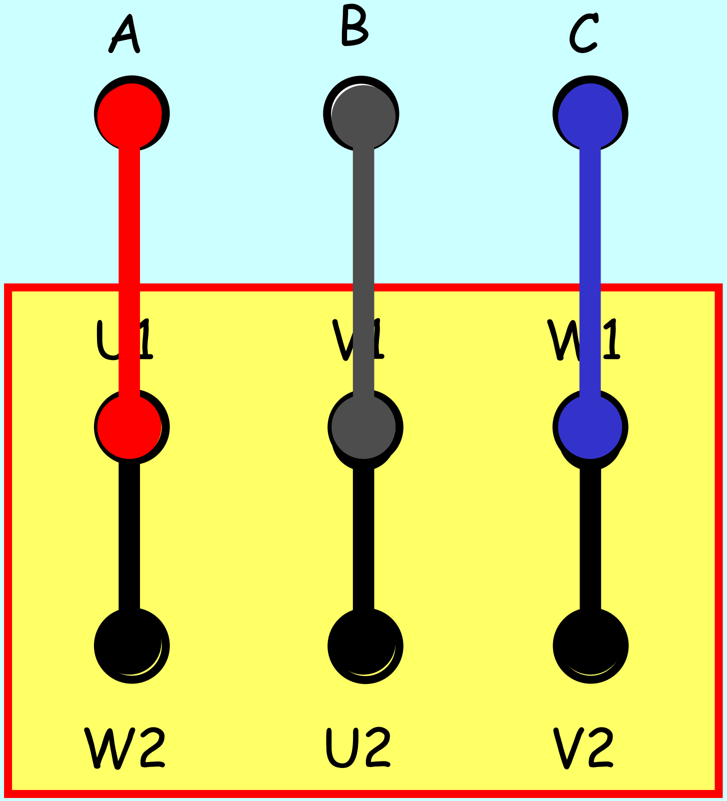
Supply Terminals



Stator Terminals

Delta Connection

Supply Terminals



Stator Terminals

UEENEEG102A
Solve problems in low voltage a.c. circuits

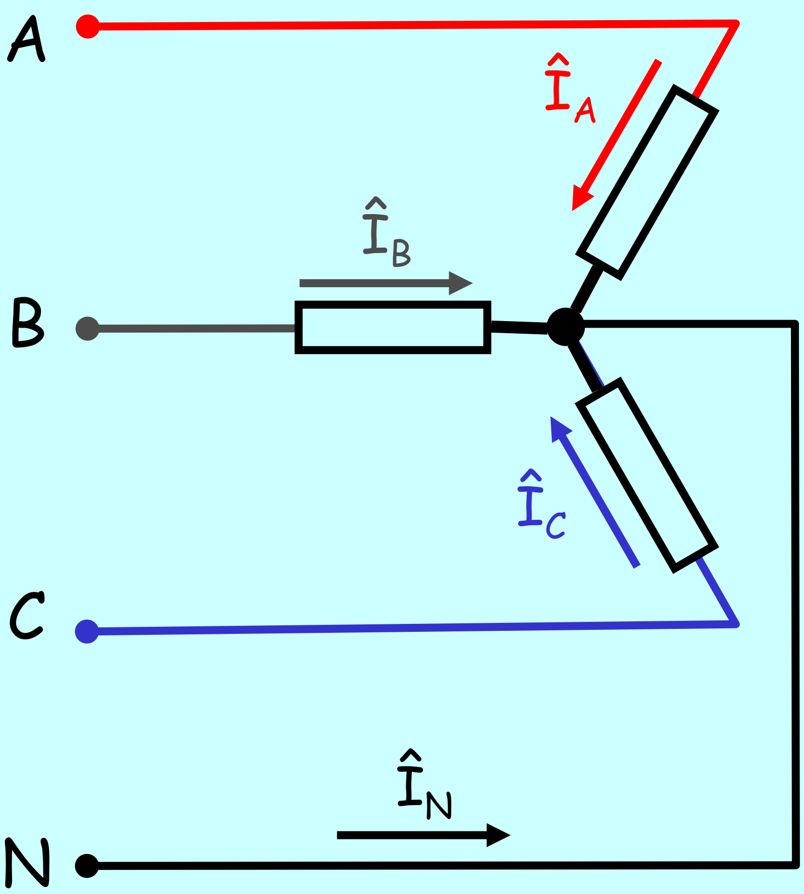
Three Phase Four Wire Systems

Objectives:

At the end of this lesson students should be able to:

1. Determine Phase and Neutral Currents for Balanced and Unbalanced four wire systems.
2. State the purpose of the Neutral conductor in a three phase system.
3. Develop Phasor Diagrams for a star connected system.
4. State the requirements regarding the size of Neutral Conductors.

Neutral Current



By Kirchoff's Current Law (KCL):

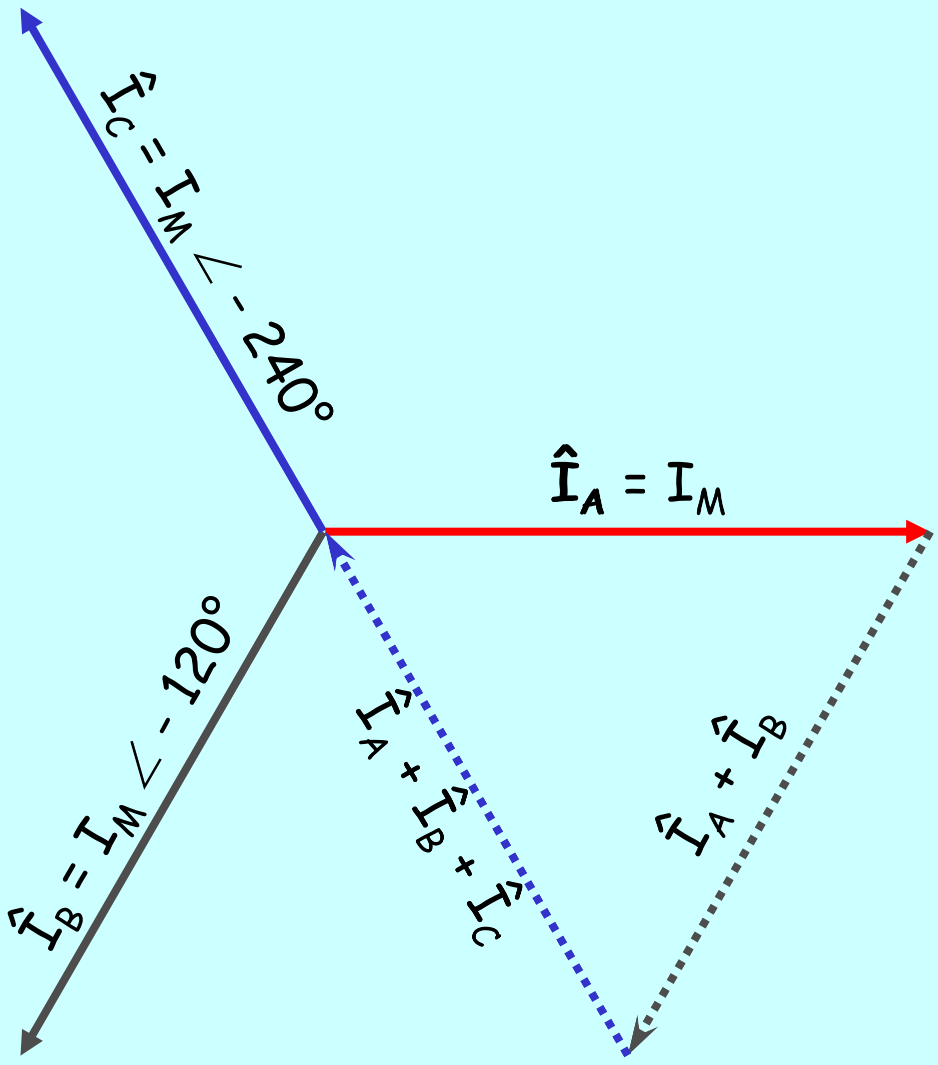
$$\hat{I}_N + \hat{I}_A + \hat{I}_B + \hat{I}_C = 0$$

For a balanced load

$$I_A = I_M \angle 0°$$

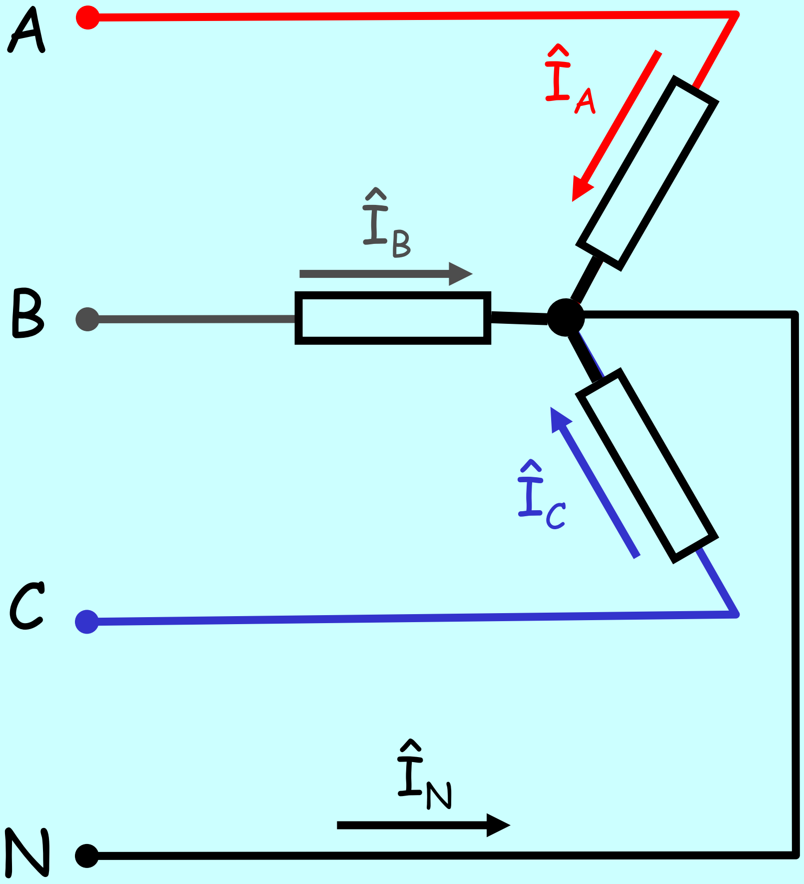
$$I_B = I_M \angle -120°$$

$$I_C = I_M \angle -240°$$



$\hat{I}_C = I_M \angle -240°$
$\hat{I}_A = I_M$
$\hat{I}_B = I_M \angle -120°$
$\hat{I}_A + \hat{I}_B + \hat{I}_C$
$\hat{I}_A + \hat{I}_B$

Neutral Current



By Kirchoff's Current Law (KCL):

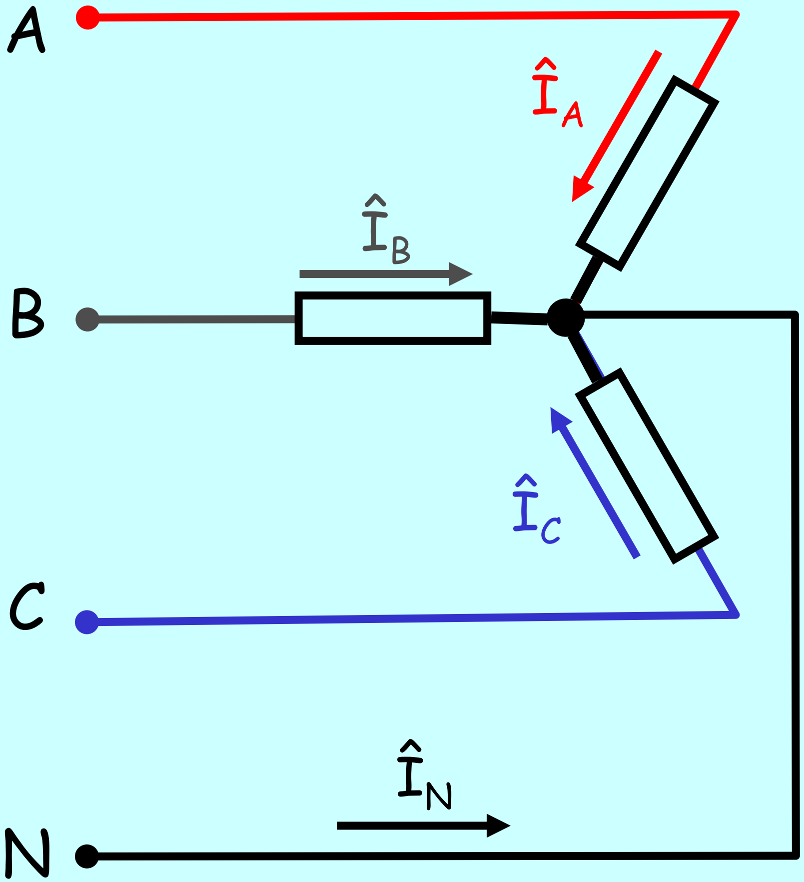
$$\hat{I}_N + \hat{I}_A + \hat{I}_B + \hat{I}_C = 0$$

$$\hat{I}_N = -(\hat{I}_A + \hat{I}_B + \hat{I}_C)$$

$$\hat{I}_N = 0\ A$$

What would happen if the loads are NOT Balanced?

Neutral Current



By Kirchoff's Current Law (KCL):

$$\hat{I}_N + \hat{I}_A + \hat{I}_B + \hat{I}_C = 0$$

$$\hat{I}_N = -(\hat{I}_A + \hat{I}_B + \hat{I}_C)$$

$$\hat{I}_N = ?\ A$$

$\hat{I}_A = 25$ A @ $\lambda = 0.707$ lag

$\hat{I}_A = 25 \angle -45°$

$\hat{I}_B = 10$ A @ $\lambda = 0.866$ lag

$\hat{I}_B = 10 \angle -30°$

$\hat{I}_C = 15$ A @ $\lambda = 1.0$

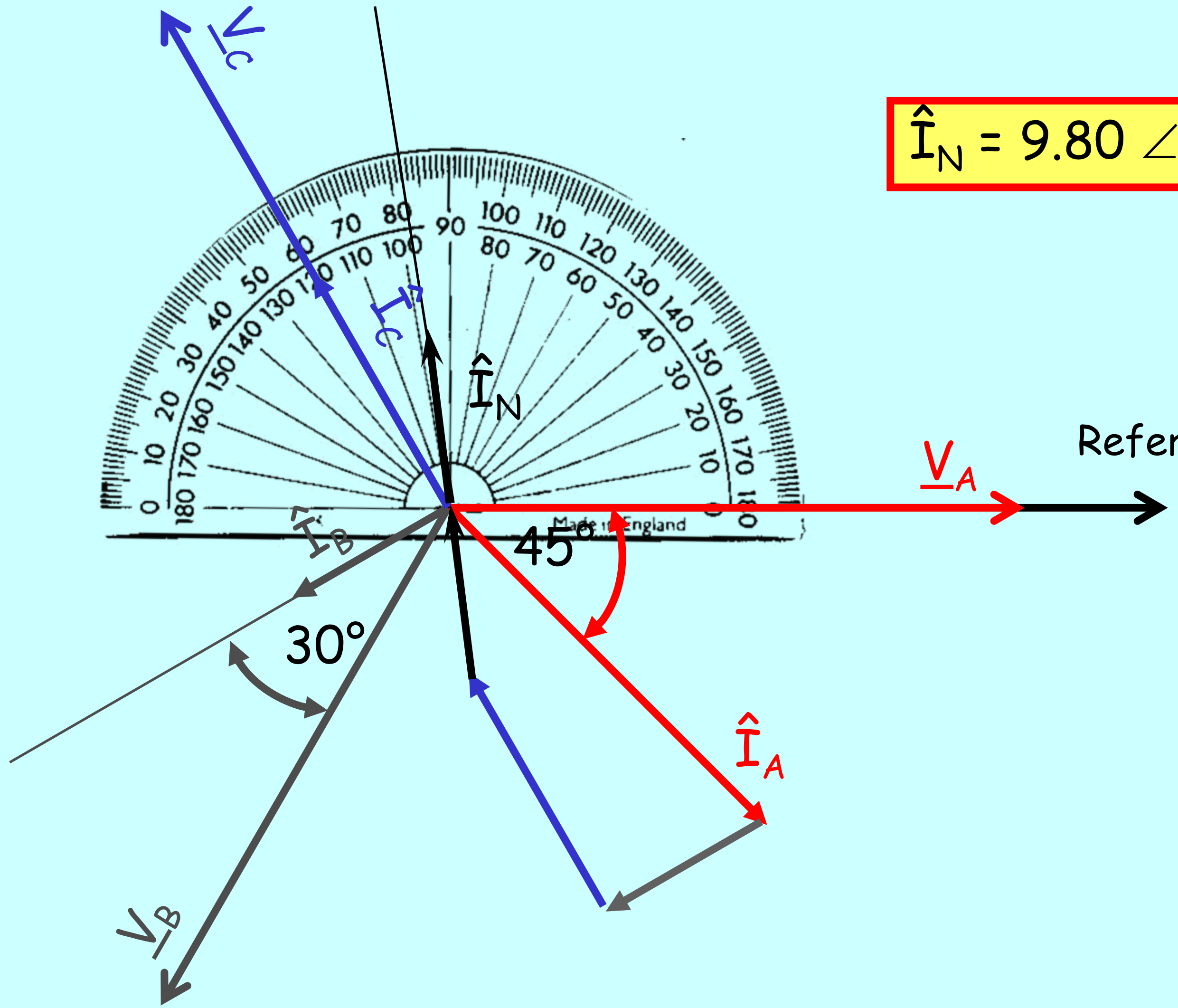
$\hat{I}_C = 15 \angle 0°$

$\hat{I}_A = 25 \angle -45°$

$\hat{I}_B = 10 \angle -30°$

$\hat{I}_C = 15 \angle 0°$

$\hat{I}_N = 9.80 \angle 99°$ A



Three single phase 240 V loads are connected to different phases of a 415 V 3-Phase 4-Wire Supply. Determine the Neutral Current if the loads are:

Phase A — A Capacitor Run Motor drawing 10 A @ a power factor of 0.9 lead

Phase B — A Split Phase A/C Motor taking 15 A at a power factor of 0.65 lag

Phase C — A 2.4 kW Radiator.

What do we know?

$\hat{I}_A = 10\ A\ @\ \lambda = 0.9$ lead → $\hat{I}_A = 10 \angle 25.8°$

$\hat{I}_B = 15\ A\ @\ \lambda = 0.65$ lag → $\hat{I}_B = 15 \angle -49.5°$

Radiators are Purely Resistive ∴ power factor is 1

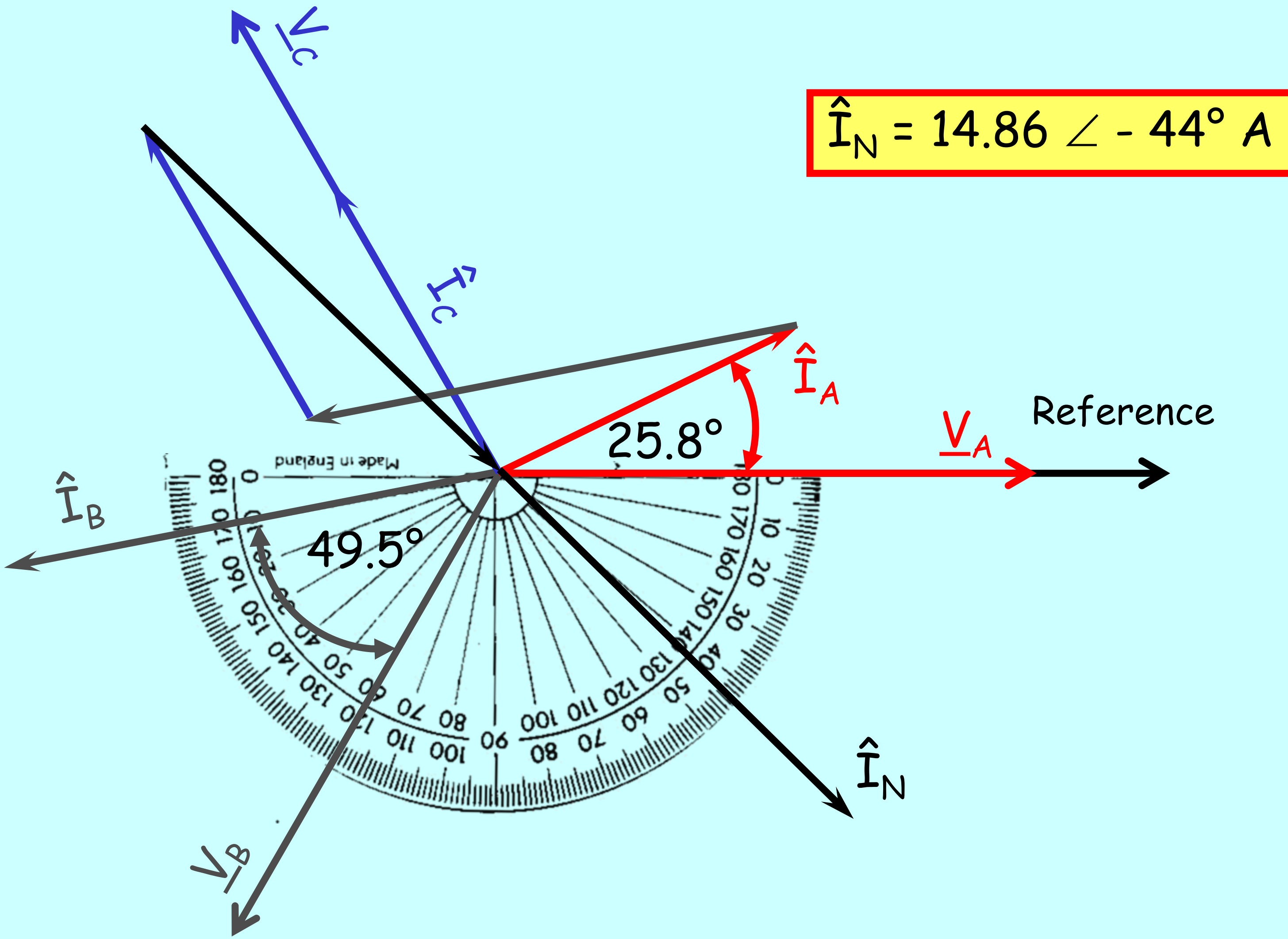
$$I = \frac{P}{V\cos\theta} = \frac{2400}{240 \times 1} = 10\ A$$

$\hat{I}_C = 10\ A\ @\ \lambda = 1.0$ → $\hat{I}_C = 10 \angle 0°$

$\hat{I}_A = 10 \angle 25.8°$

$\hat{I}_B = 15 \angle -49.5°$

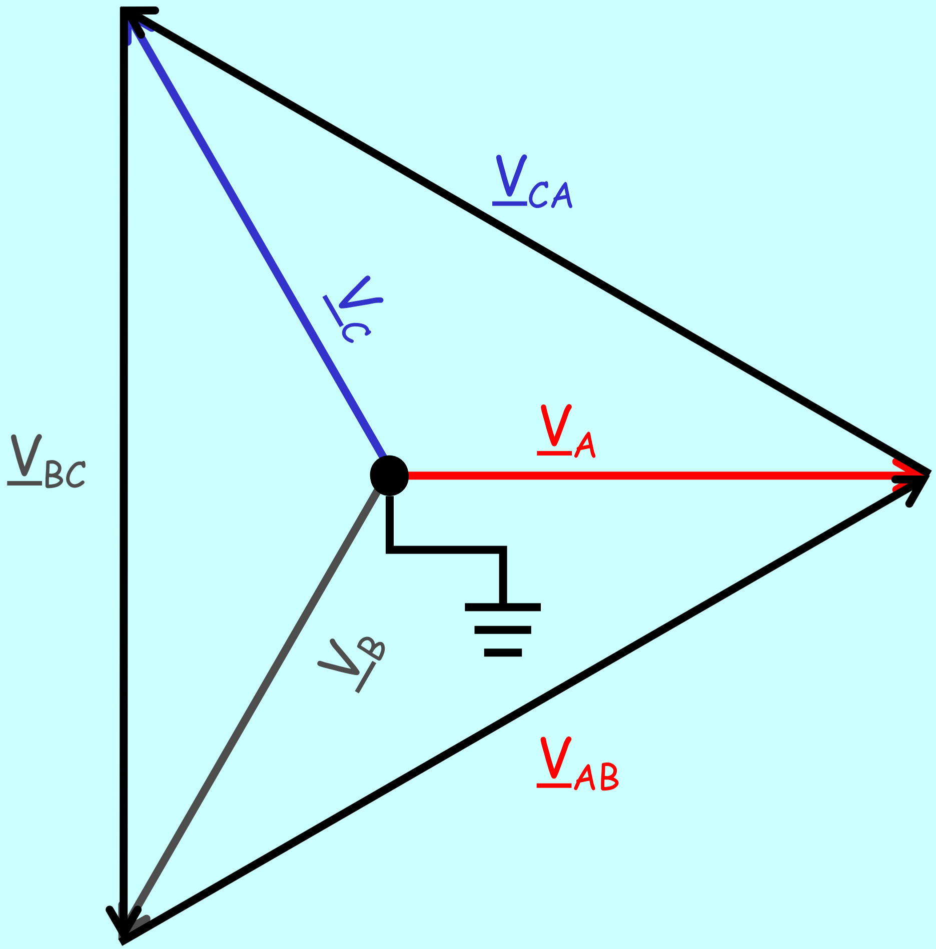
$\hat{I}_C = 10 \angle 0°$



Neutral Conductor Functions

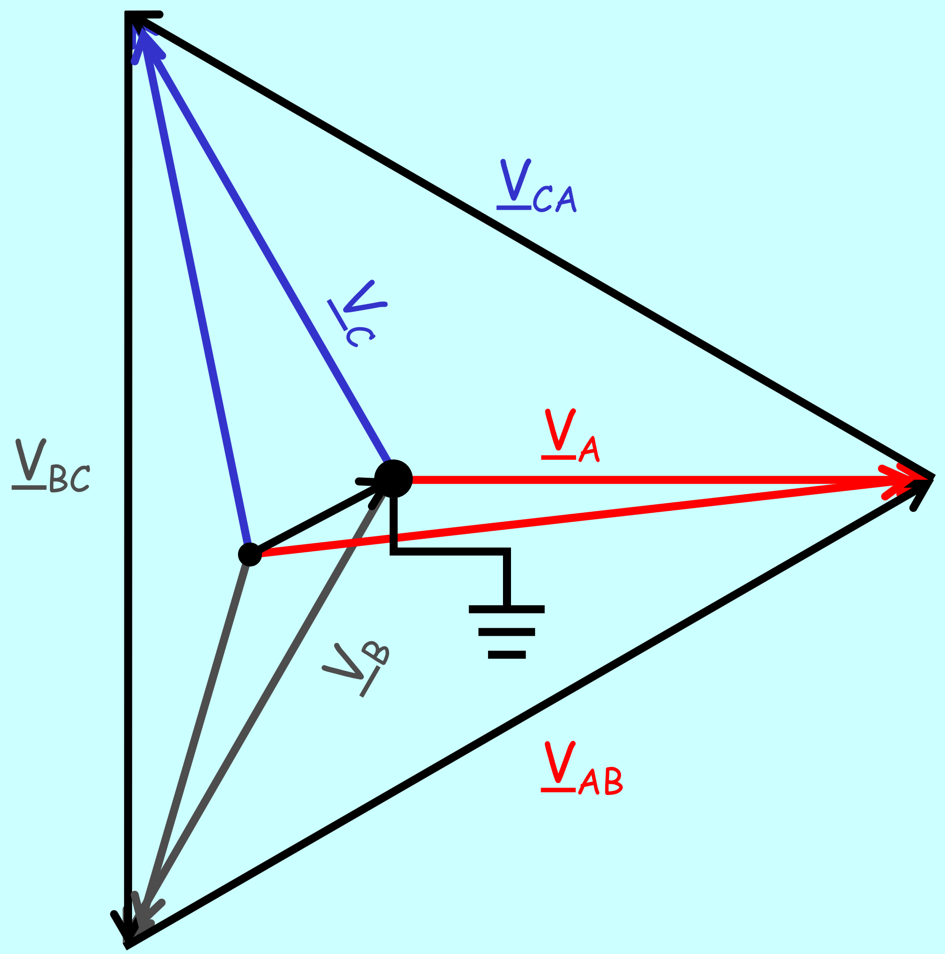
- Allow Single Phase Loads
- Maintain Phase Voltage equality
- Ensure correct operation of Protective Devices
- Carry out of balance currents
- Carry 3rd Harmonic Currents

The Neutral point forces the Line and Phase Voltages to remain even.



If the Neutral point becomes Open Circuit the Line Voltages remain the same, but the Phase Voltages change.

And now there is a Potential Difference from the common point to Earth.



Neutral Conductor Size

Check this reference

IAW AS3000 Clause 3.5.2

Factors

- Current Carrying Capacity
- Active Conductor Size
- Presence of Harmonics
- Detection Devices

A three phase star connected load is supplied by a 400 V three phase four wire supply. If each element of the heater has a resistance of 12 Ω, determine the magnitude of the Neutral Current.

What do we know?

This is a BALANCED Load!!!!!

$\hat{I}_N = 0\ A$

A 3-Phase 4-Wire Supply has the phase currents:

Phase A 3 A in a purely resistive load

Phase B 2 A lagging by 60°

Phase C 2 A leading by 50°

Determine the current in the Neutral Conductor

What do we know?

This is an unbalanced load

Purely Resistive loads have a power factor of 1 and ∠ 0°

$\hat{I}_A = 3 \angle 0°$

$\hat{I}_B = 2 \angle -60°$

$\hat{I}_C = 2 \angle +50°$

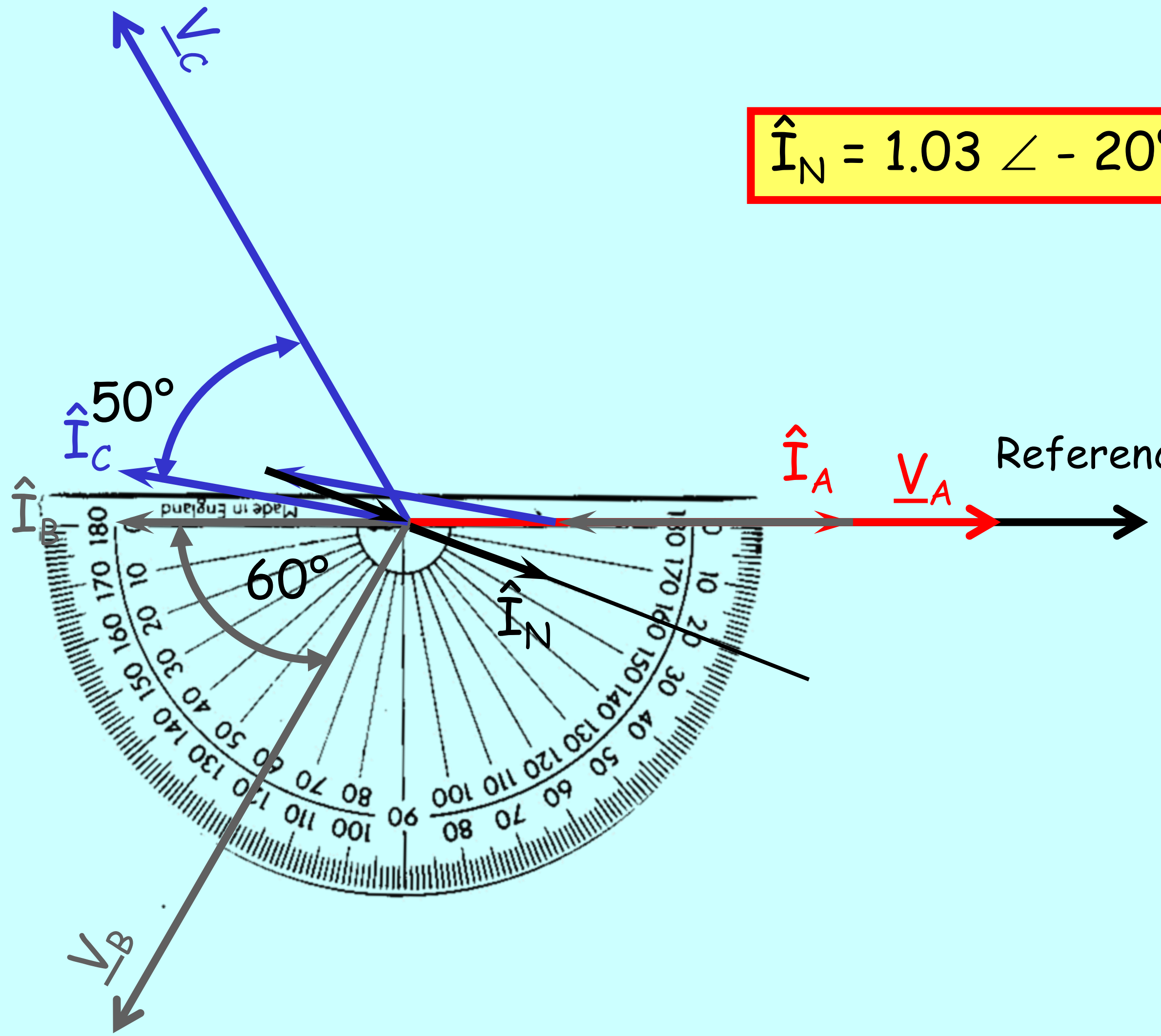
Do this on the board

$\hat{I}_A = 3 \angle 0°$

$\hat{I}_B = 2 \angle -60°$

$\hat{I}_C = 2 \angle 50°$

$\hat{I}_N = 1.03 \angle -20°$ A



Determine the Phase C to Star Point and Star Point to Earth Voltages of a 3_Phase, 4-Wire, 400 Volt system is the Phase A to Star Point voltage is measured at 270 V and the Phase B to Star Point voltage is 180 V.

What do we know?

$V_{AB} = V_{BC} = V_{CA} = 400\ V$

$V_{AN} = 270\ V$

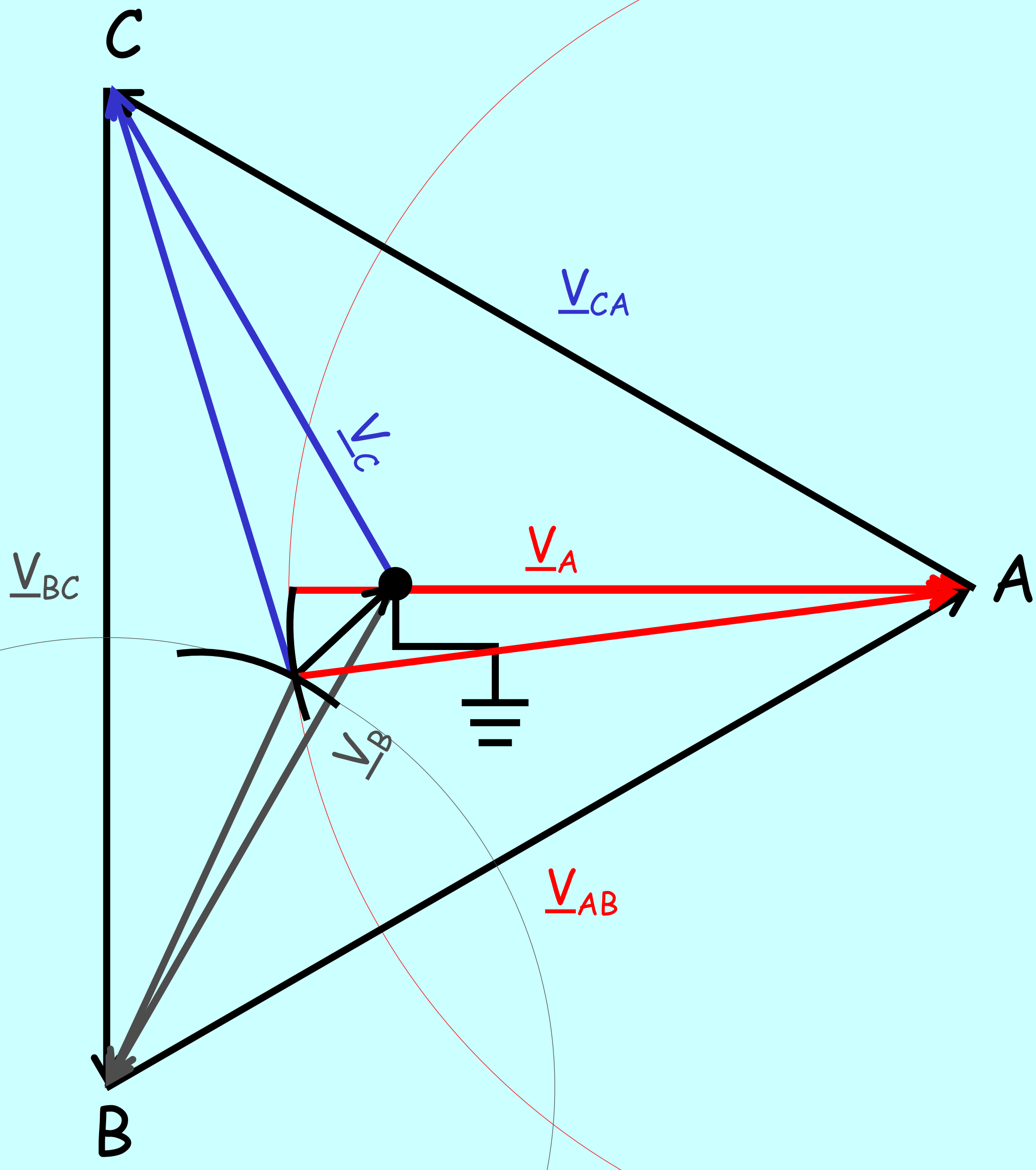
$V_{BN} = 180\ V$

Do this on the board

Pg. 289 Ex. 4

V_{AN} = 270 V
V_{BN} = 180 V

V_{CN} = 248 V
V_{N0} = 54.7 V



End of Lesson

Practical Exercises

Three Phase Star Connected Systems.

Neutral Conductor Current.

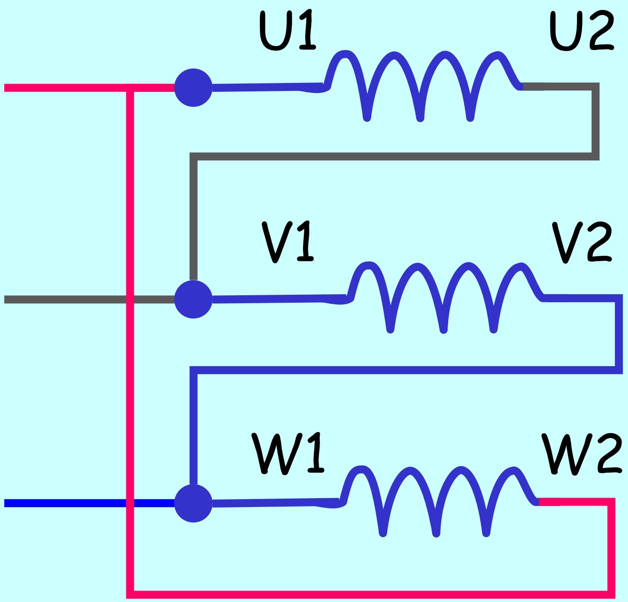
UEENEEG102A Solve problems in low voltage a.c. circuits

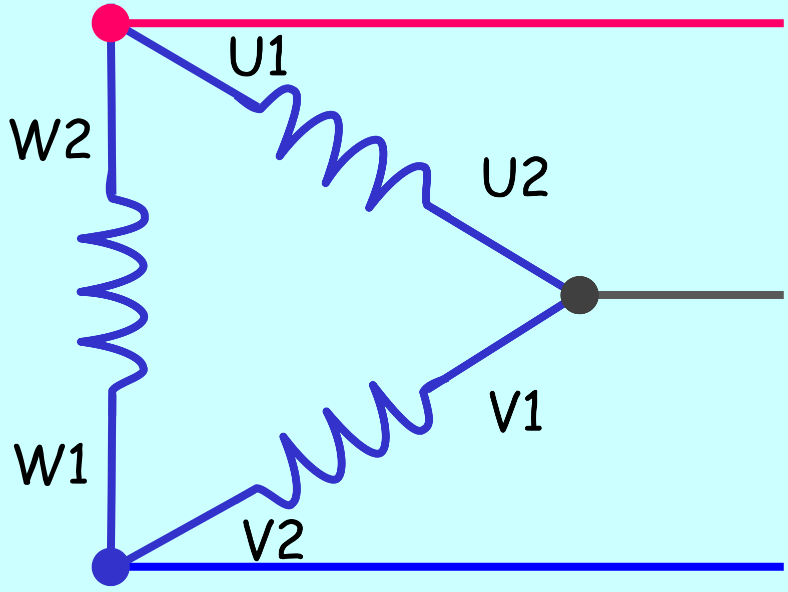
Three Phase Delta

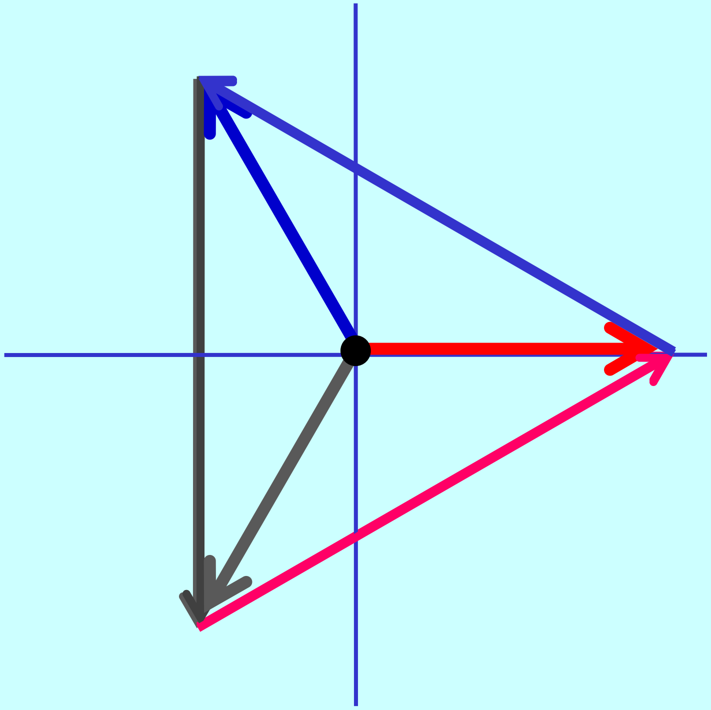
Objectives:

At the end of this lesson students should be able to:

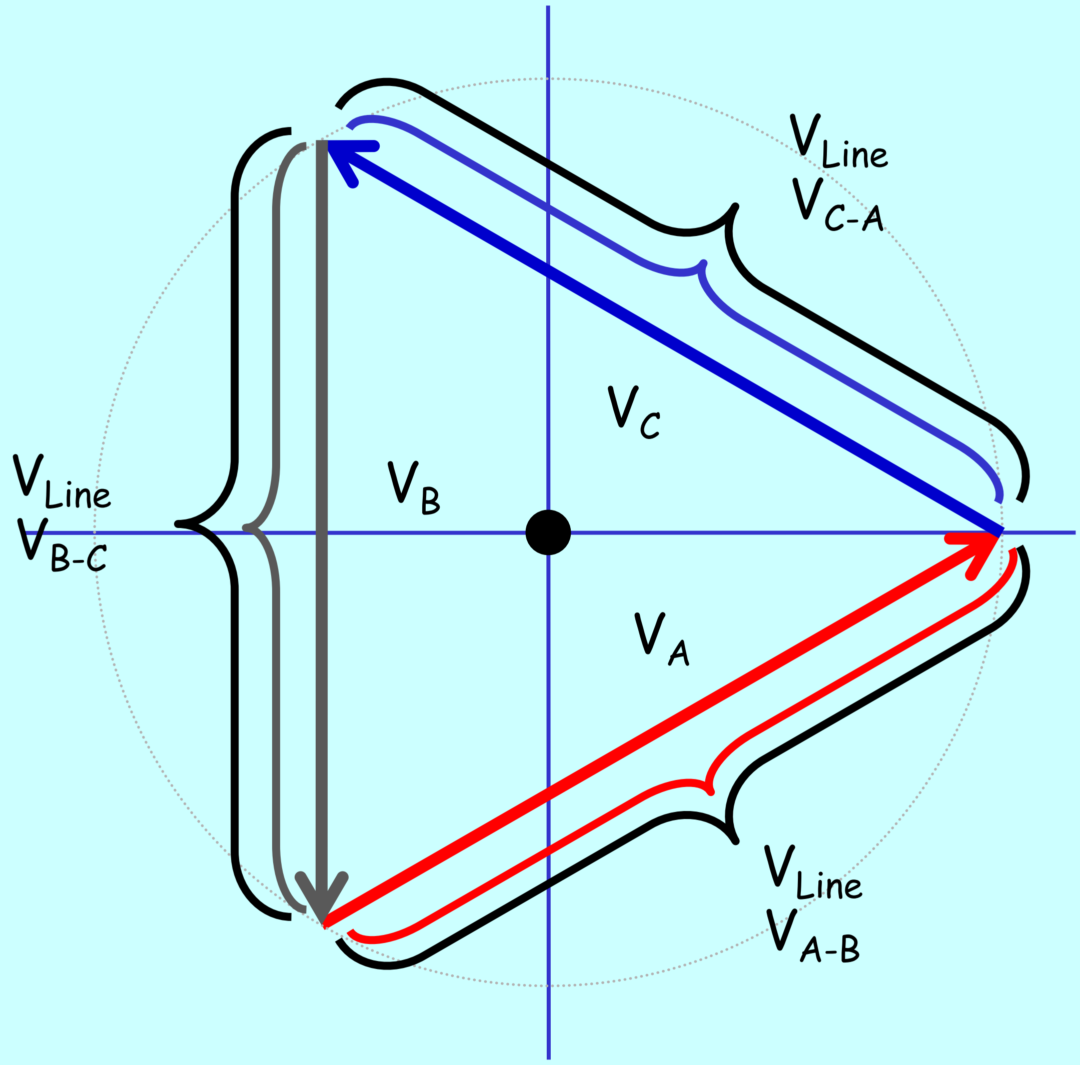
1. Draw the circuit for a three phase Delta Connection.
2. Determine Line & Phase Voltages for a Delta Connected System.
3. Determine Line & Phase Currents for a Delta Connected System.
4. Draw and label the Phasor Diagrams for a Delta connected system.
5. State the limitations of Delta connected systems.
6. Connect a Delta connected load.

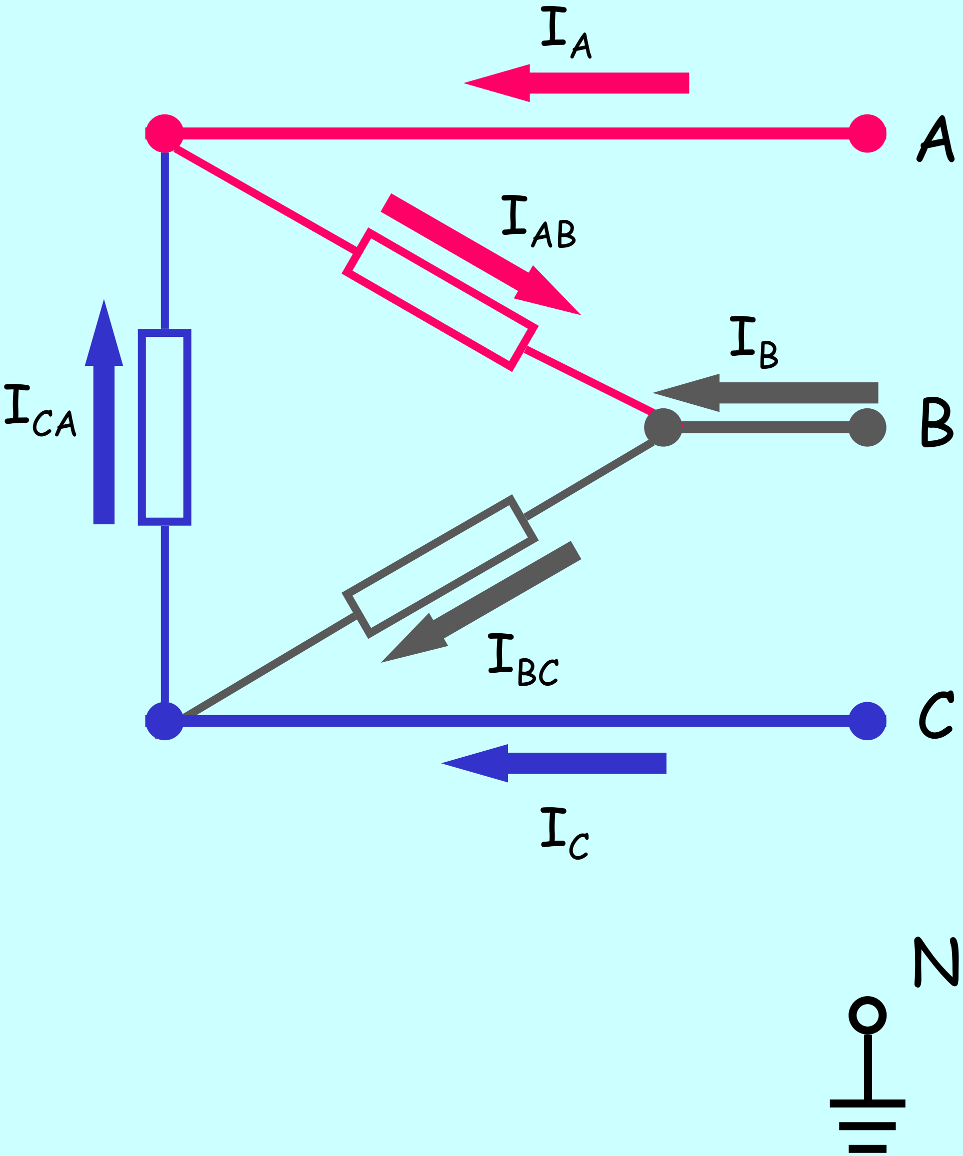

U1
U2
V1
V2
W1
W2


U1
W2
U2
V1
W1
V2



Phase & Line Voltages





Delta

Voltage & Current Relationships

Line Voltage		Phase Voltage
V_L	=	V_P

Line Current		Difference of two Phase Currents
$\hat{I}_A$	=	$\hat{I}_{AB} - \hat{I}_{CA}$

Phase Currents

$\underline{I}_{AB} = I_M \sin\theta$ $\underline{I}_{AB} = I_M \angle 0°$

$\underline{I}_{BC} = I_M \sin(\theta - 120)$ $\underline{I}_{BC} = I_M \angle -120°$

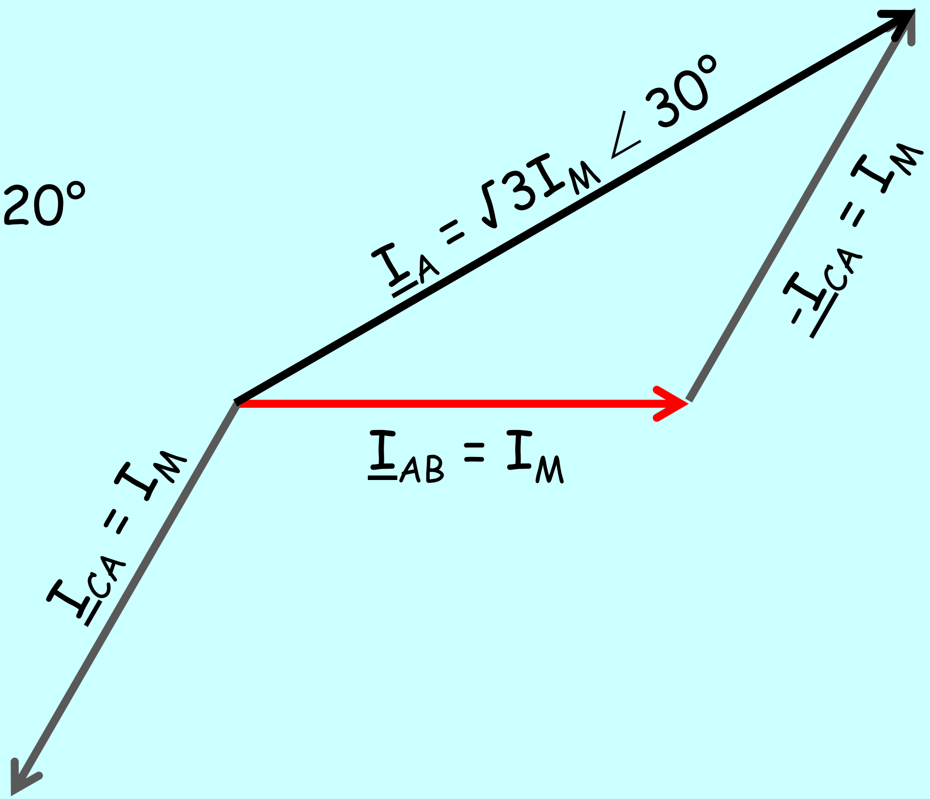
$\underline{I}_{CA} = I_M \sin(\theta - 240)$ $\underline{I}_{CA} = I_M \angle -240°$

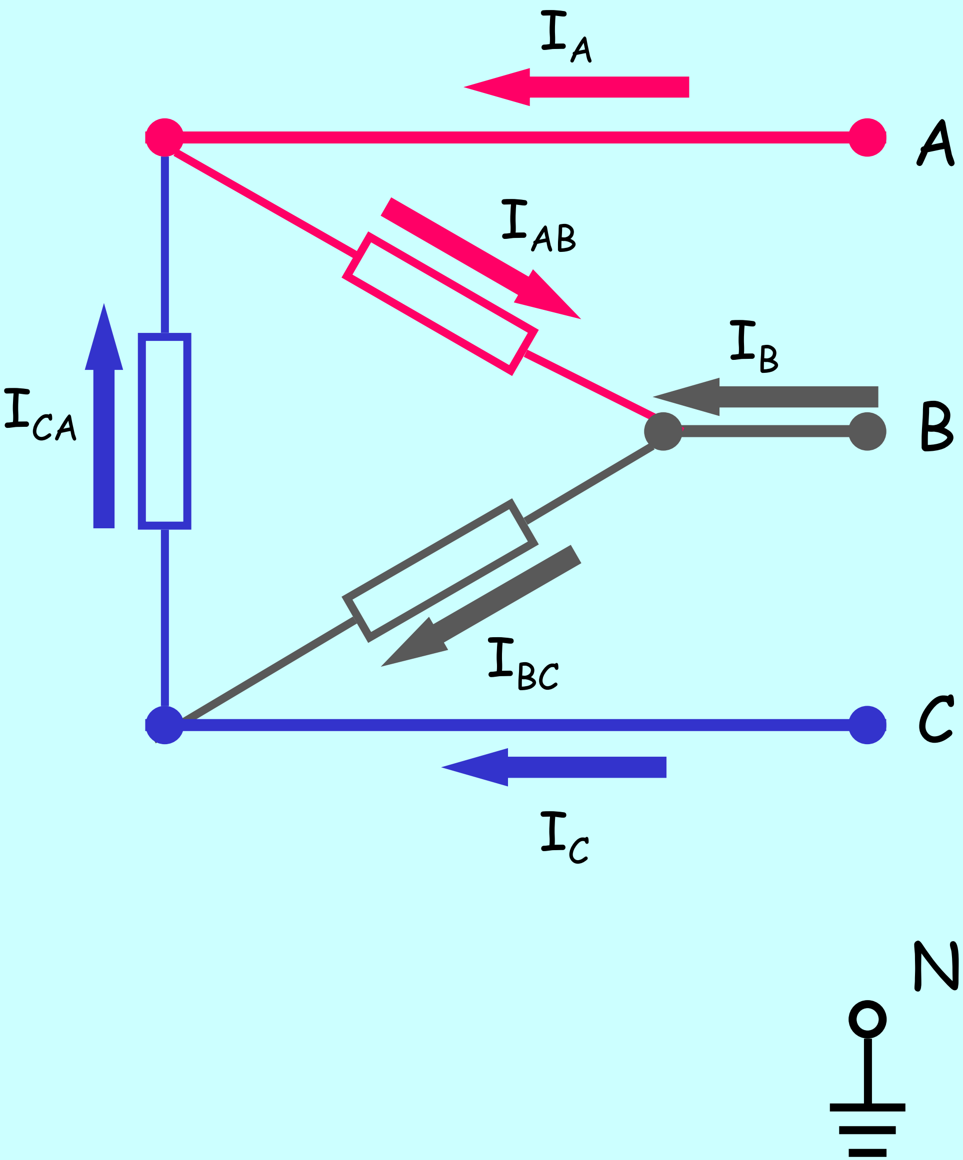
Line Currents

$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA}$

$\underline{I}_A = I_M - I_M \angle -120°$

$\underline{I}_A = \sqrt{3} I_M \angle 30°$





Delta

Voltage & Current Relationships

Line Voltage		Phase Voltage

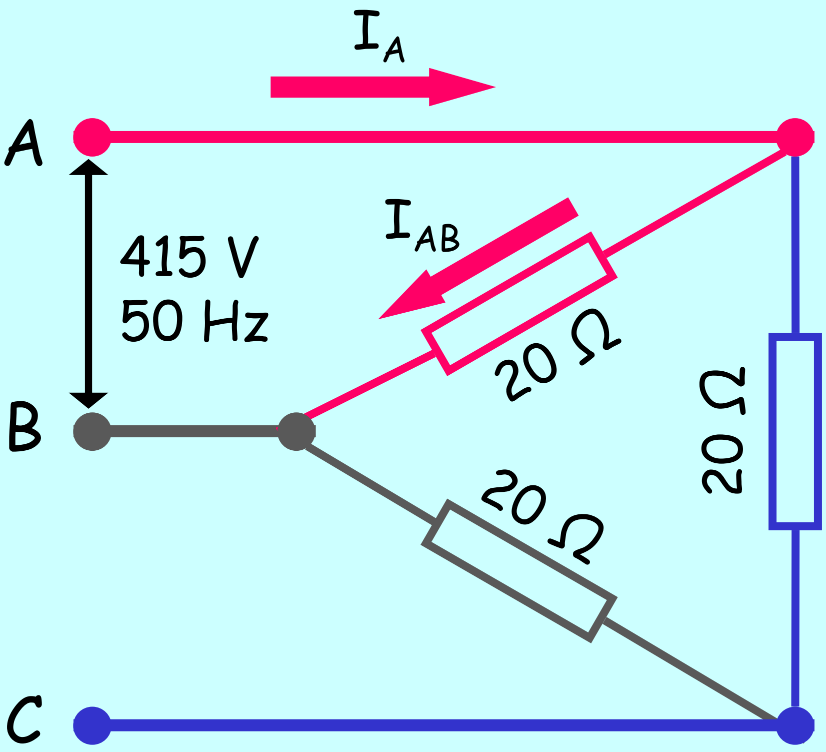
$$\underline{V}_L = \underline{V}_P$$

Line Current		Difference of two Phase Currents

$$\hat{I}_L = \hat{I}_{P1} - \hat{I}_{P2}$$

$$I_L = \sqrt{3} I_P$$

Determine the line current for a 3-phase motor stator winding which has an impedance of 20 Ω per phase when it is connected to a 415 V 50 Hz supply.



$$V_P = V_L = 415\text{ V}$$

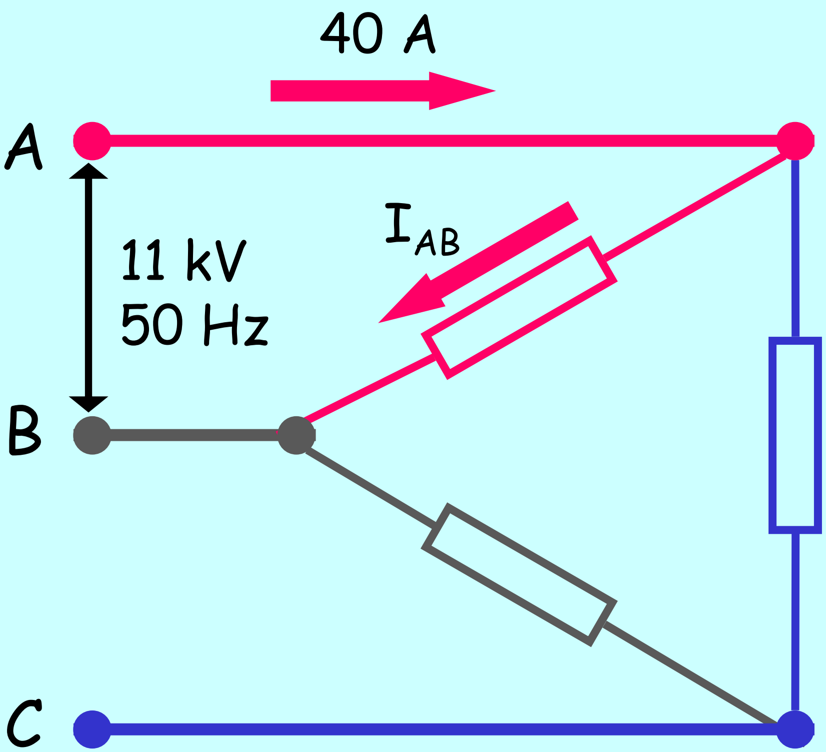
$$I_P = \frac{V_P}{Z_P}$$

$$I_P = \frac{415}{20} = 20.75\text{ A}$$

$$I_L = \sqrt{3}I_P = \sqrt{3} \times 20.75$$

$$I_L = 35.94\text{ A}$$

Determine the phase current for a 3-phase Delta connected transformer which delivers 40 A when it is connected to an 11 kV 50 Hz supply.

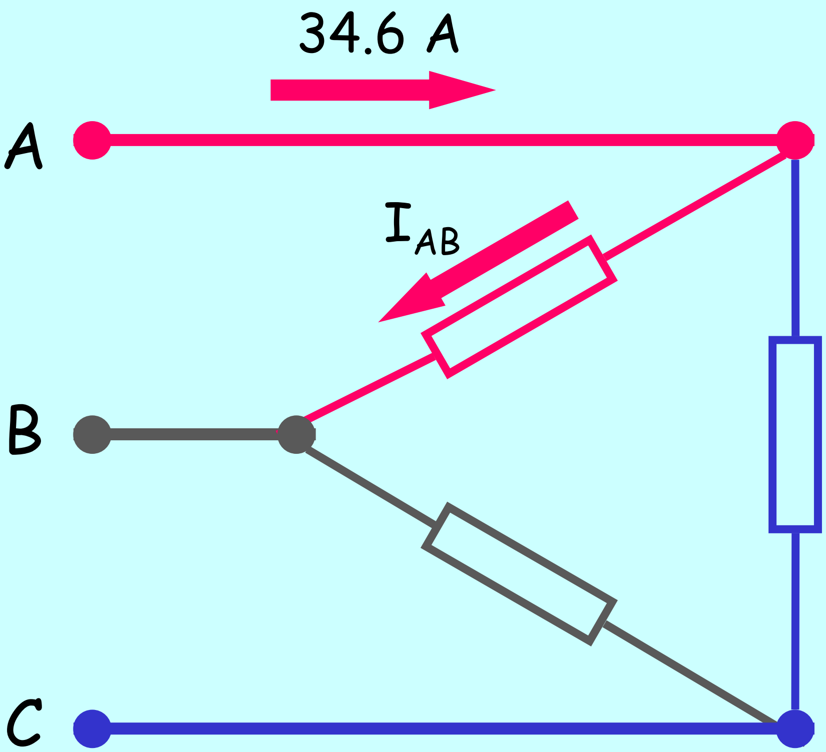


$$I_P = \frac{I_L}{\sqrt{3}}$$

$$I_P = \frac{40}{\sqrt{3}}$$

$$I_P = 23.1\ A$$

Determine the phase current for a 3-phase Delta connected resistive load which has a line current of 34.6 Amps.

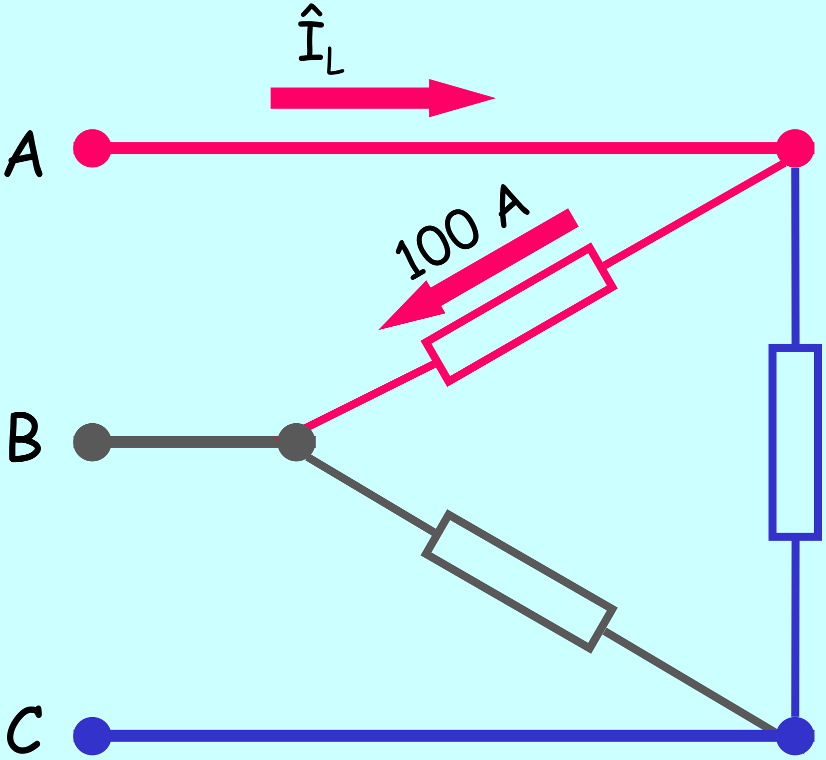


$$I_P = \frac{I_L}{\sqrt{3}}$$

$$I_P = \frac{34.6}{\sqrt{3}}$$

$$I_P = 19.98\ A$$

Determine the line current for a 3-phase Delta connected resistive load which has a phase current of 100 Amps.

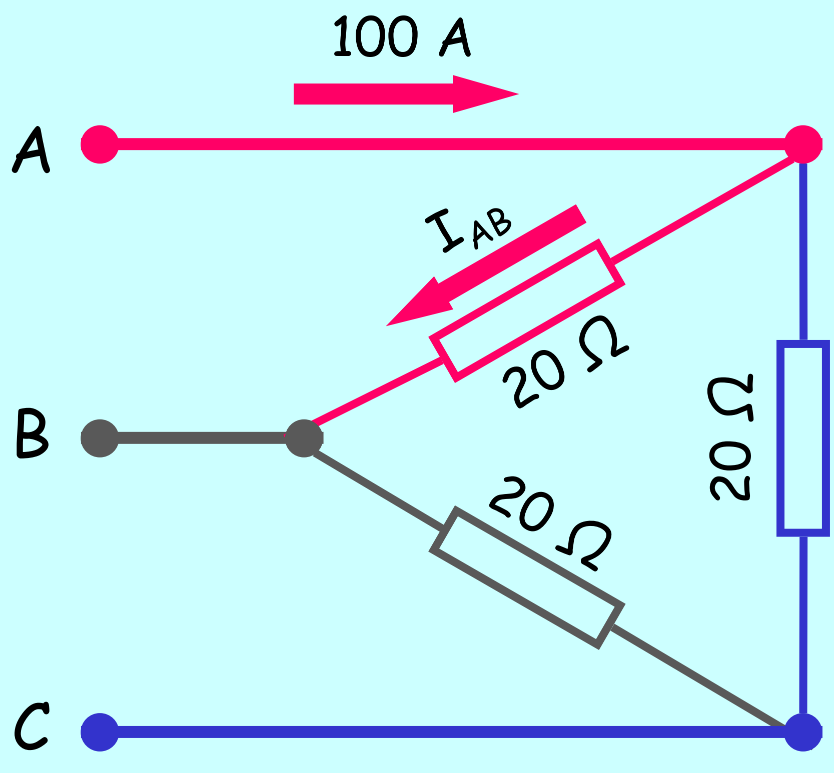


$$I_L = \sqrt{3} I_P$$

$$I_L = 100 \times \sqrt{3}$$

$$\boxed{I_L = 173.2\ A}$$

Determine the phase voltages and currents for a balanced 3-phase Delta connected resistive load which has a line current of 100 Amps and phase impedance of 20 Ω.



$$I_P = \frac{I_L}{\sqrt{3}}$$

$$I_P = \frac{100}{\sqrt{3}}$$

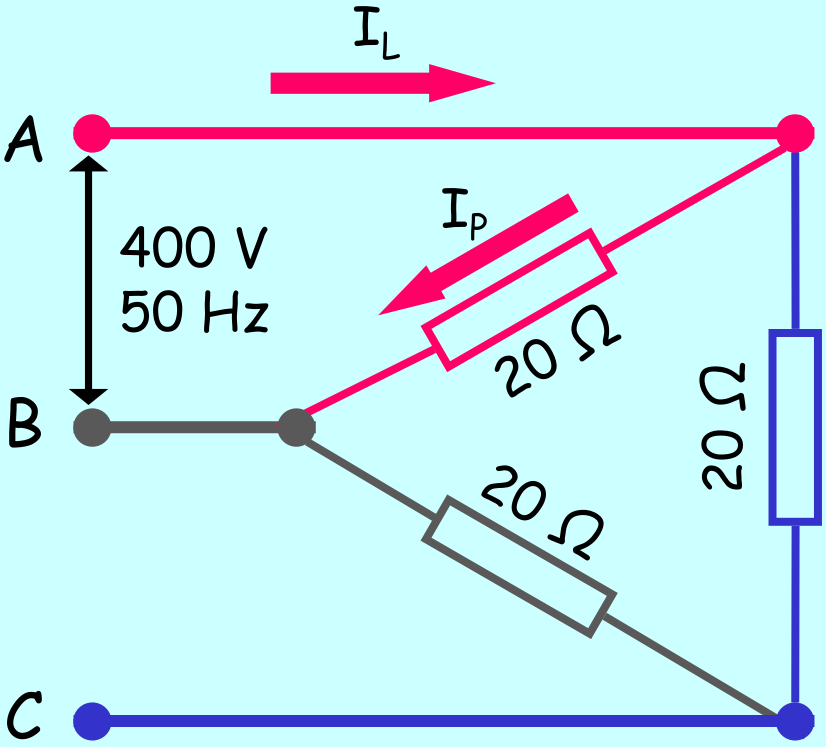
$$I_P = 57.74\ A$$

$$V_L = V_P = I_P Z$$

$$V_L = 57.74 \times 20$$

$$V_L = 1{,}154.8\ V$$

Determine the line current for a 3-phase motor which has a balanced impedance of 20 Ω per phase when it is connected to a three wire 400V 50Hz supply.



$$V_P = V_L = 400\ V$$

$$I_P = \frac{V_P}{Z_P}$$

$$I_P = \frac{400}{20} = 20\ A$$

$$I_L = \sqrt{3}I_P = \sqrt{3} \times 20$$

$$\boxed{I_L = 34.64\ A}$$

UEENEEG102A
Solve problems in low voltage a.c. circuits

Star-Delta Systems

Objectives:

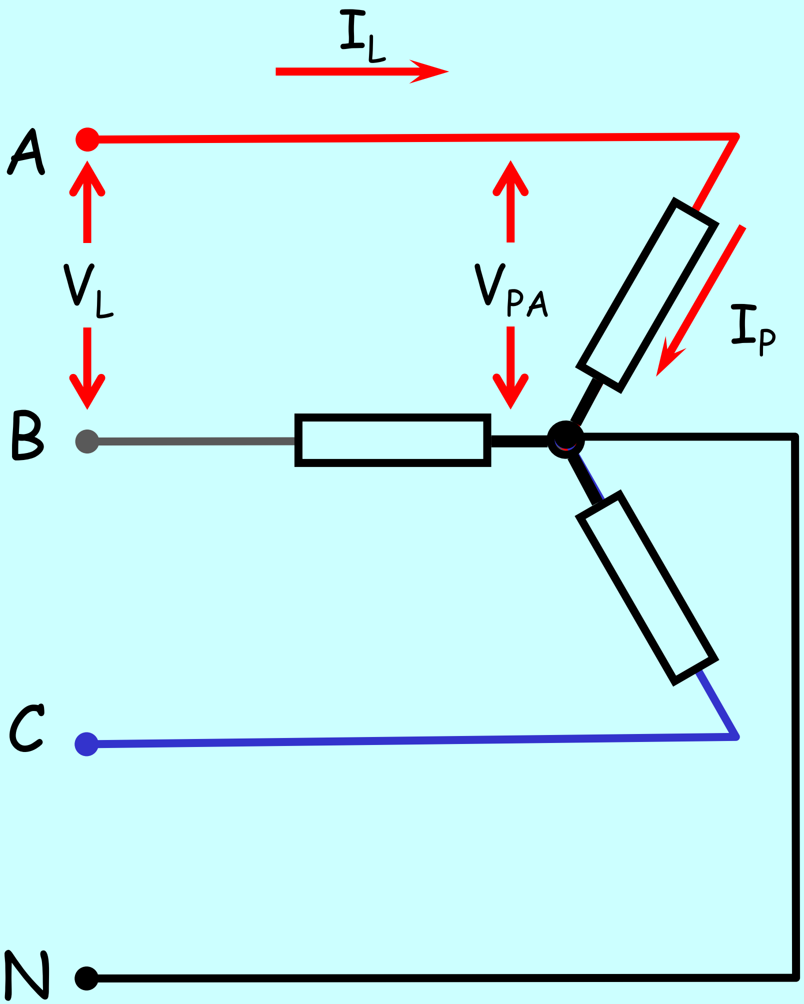
At the end of this lesson students should be able to:

1. Identify interconnected Star – Delta Systems.

2. Show the relationships between Line & Phase Voltages and Currents for a three phase Systems.

3. State the effect of reversing a phase winding in a three phase system.

4. Connect Star and Delta Systems.



Star

Voltage & Current Relationships

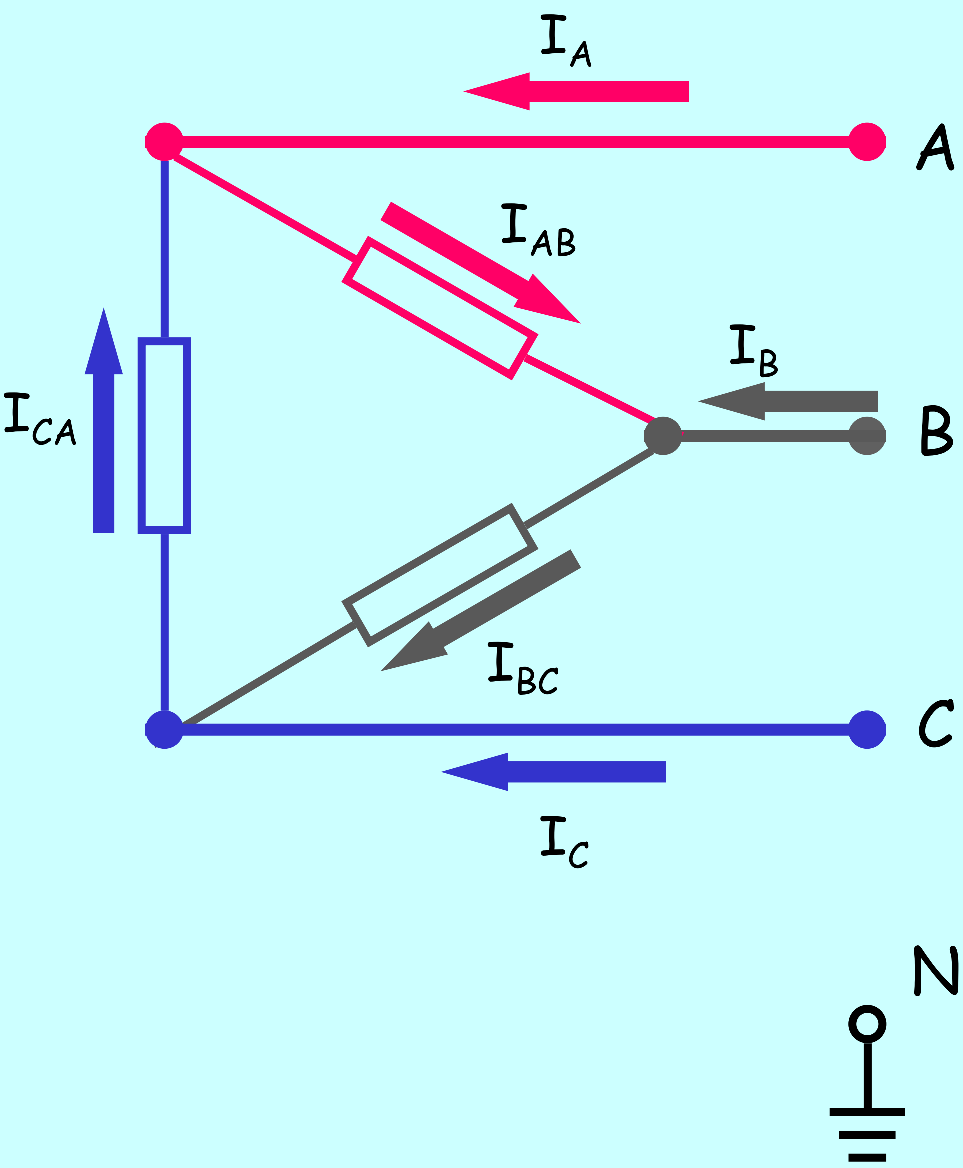
Line Current = Phase Current

$$I_L = I_P$$

$$I_N + I_A + I_B + I_C = 0$$

Line Voltage = Difference of two Phase Voltages

$$V_L = \sqrt{3}V_P$$



Delta

Voltage & Current Relationships

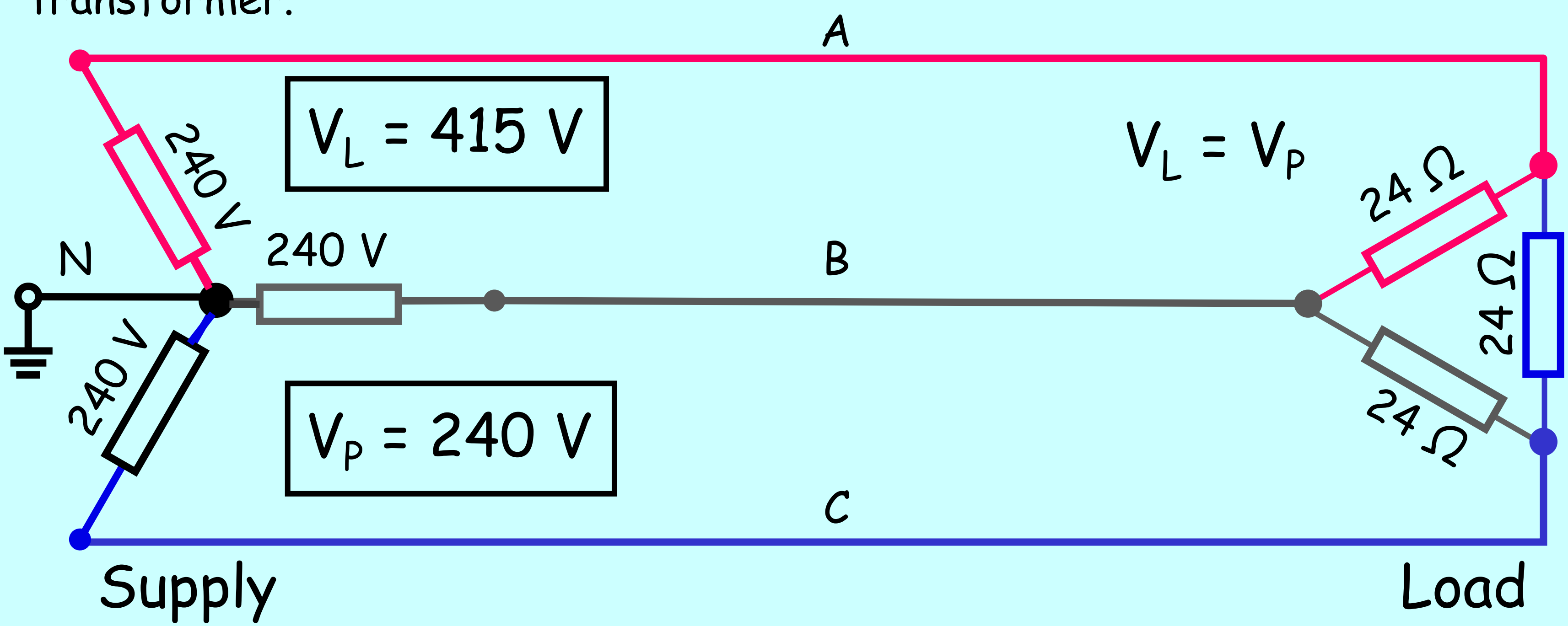
Line Voltage = Phase Voltage

$$V_L = V_P$$

Line Current = Difference of two Phase Currents

$$I_L = \sqrt{3} I_P$$

Determine the line current of a 3 phase Delta connected motor which has an impedance of 24 Ω per phase when it is connected to a 240V Star connected transformer.



$V_L = \sqrt{3}V_P$

$V_L = \sqrt{3} \times 240$

$V_L = 415$ V

$I_L = \sqrt{3}I_P$

$I_L = \sqrt{3} \times 17.32$

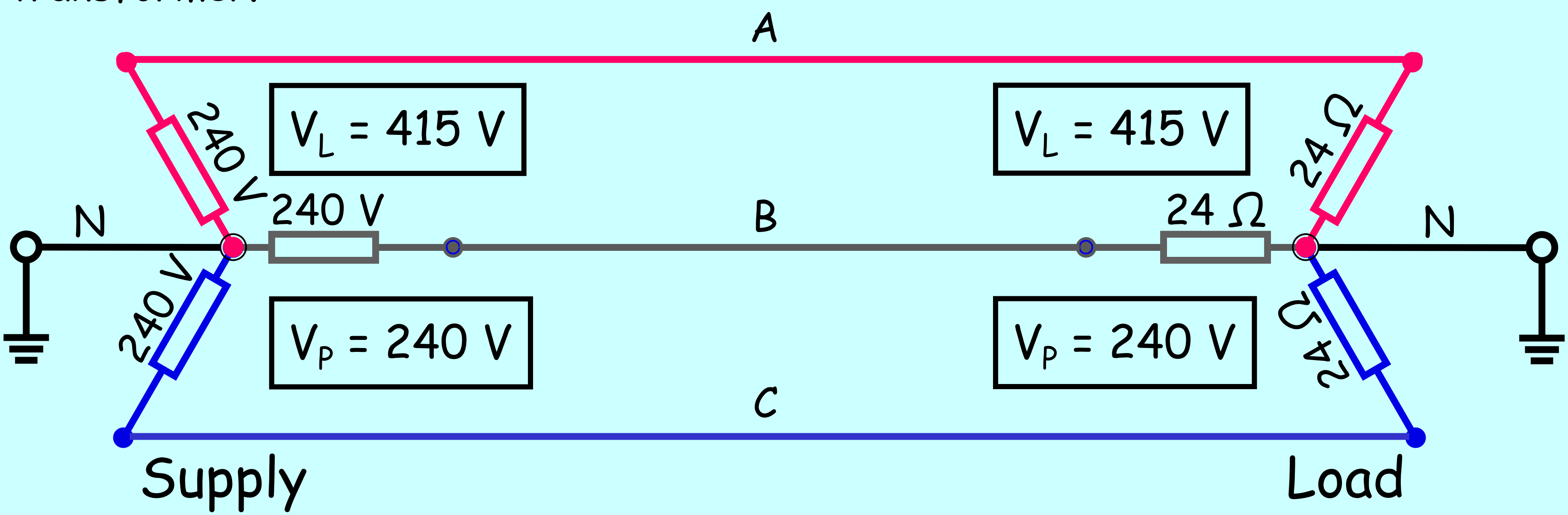
$I_L = 30$ A

$$I_P = \frac{V_P}{Z_P}$$

$$I_P = \frac{415}{24}$$

$I_P = 17.32$ A

Determine the line current of a 3 phase Star connected motor which has an impedance of 24 Ω per phase when it is connected to a 240V Star connected transformer.



$V_L = \sqrt{3}V_P$

$I_L = I_P$

$$I_P = \frac{V_P}{Z_P}$$

$$V_P = \frac{V_L}{\sqrt{3}}$$

$$I_P = \frac{240}{24}$$

Delta

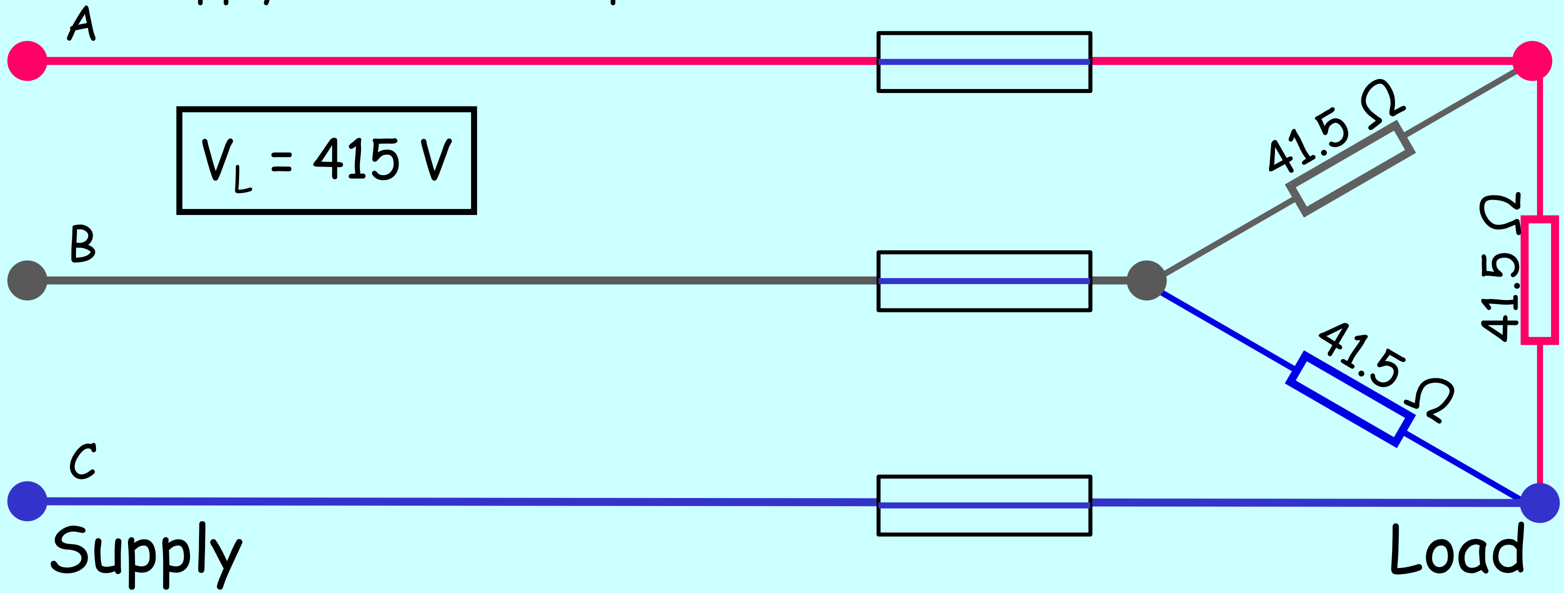
$I_L = 30\ A$

STAR

$I_L = 10\ A$

$I_P = 10\ A$

Determine the current in the elements of a 3 phase Delta connected heater which has an impedance of 41.5 Ω per phase when it is connected to a 415 V three wire supply if the fuse in phase C has blown.



Normal Operation

$$I_P = \frac{V_P}{Z_P}$$

$$I_L = \sqrt{3} I_P$$

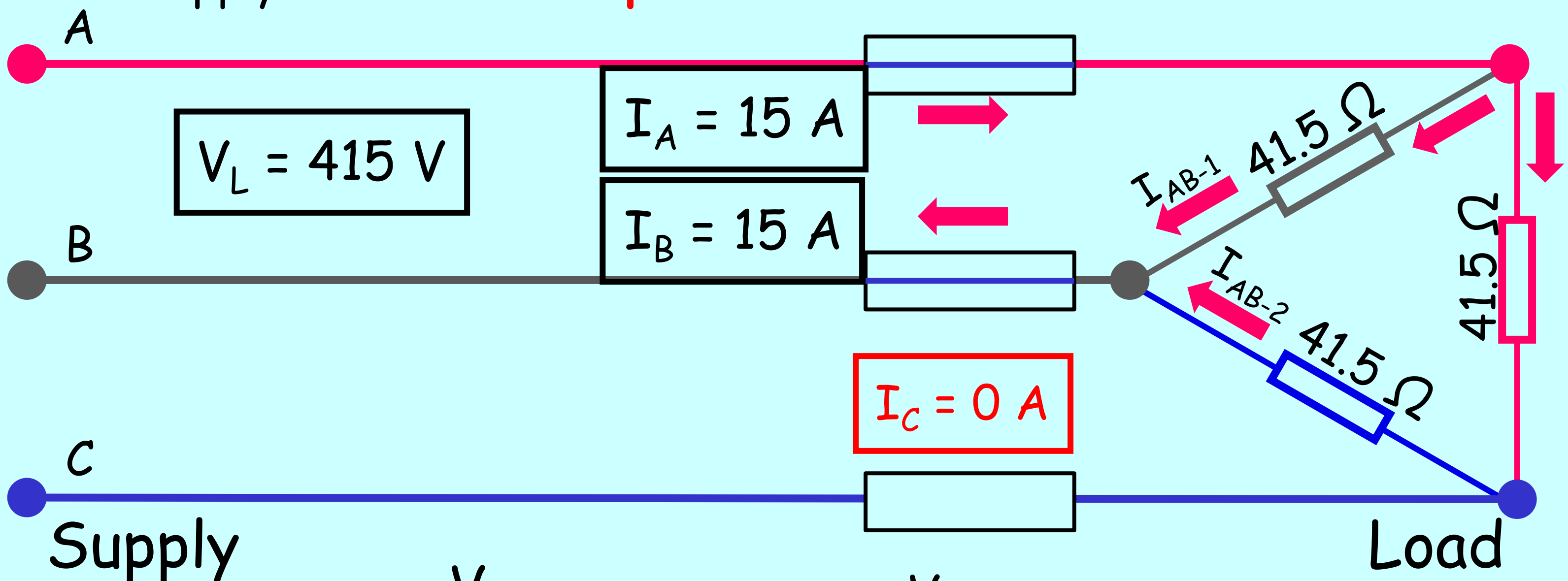
$$I_P = \frac{415}{41.5}$$

$$I_L = \sqrt{3} \times 10$$

$$I_P = 10\ A$$

$$I_L = 17.32\ A$$

Determine the current in the elements of a 3 phase Delta connected heater which has an impedance of 41.5 Ω per phase when it is connected to a 415 V three wire supply **if the fuse in phase C has blown.**



Fuse blown Operation

$$I_P = \frac{V_P}{Z_P}$$

$$I_{AB-1} = \frac{V_{AB}}{Z_{AB-1}}$$

$$I_{AB-1} = \frac{415}{41.5}$$

$$I_{AB-2} = \frac{V_{AB}}{Z_{AB-2}}$$

$$I_{AB-2} = \frac{415}{41.5 + 41.5}$$

$$I_A = I_B = I_{AB-1} + I_{AB-2}$$

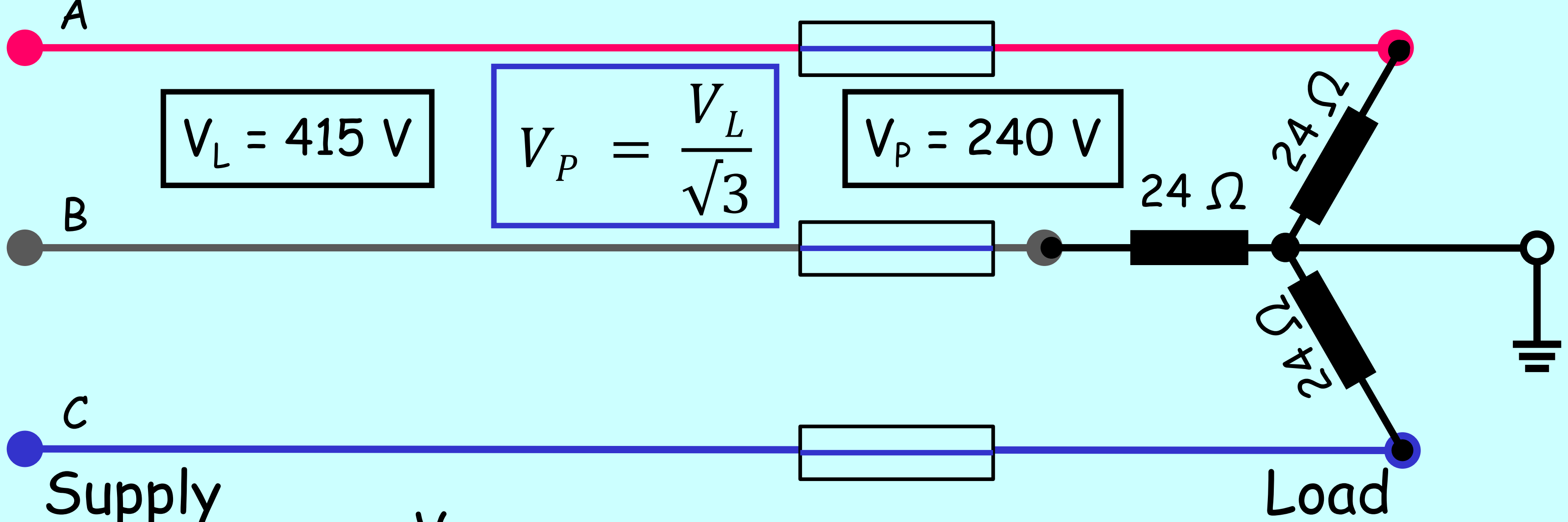
$$I_A = 10 + 5$$

I_{AB-1} = 10 A

I_{AB-2} = 5 A

$I_A = I_B$ = 15 A

Determine the phase voltages and line currents for the elements of a 3 phase Star connected heater which has an impedance of 24 Ω per phase when it is connected to a 415 V four wire supply if the fuse in phase C has blown.



$V_L = 415$ V

$$V_P = \frac{V_L}{\sqrt{3}}$$

$V_P = 240$ V

Normal Operation

$$I_P = \frac{V_P}{Z_P}$$

$$I_P = \frac{240}{24}$$

$I_L = I_P$

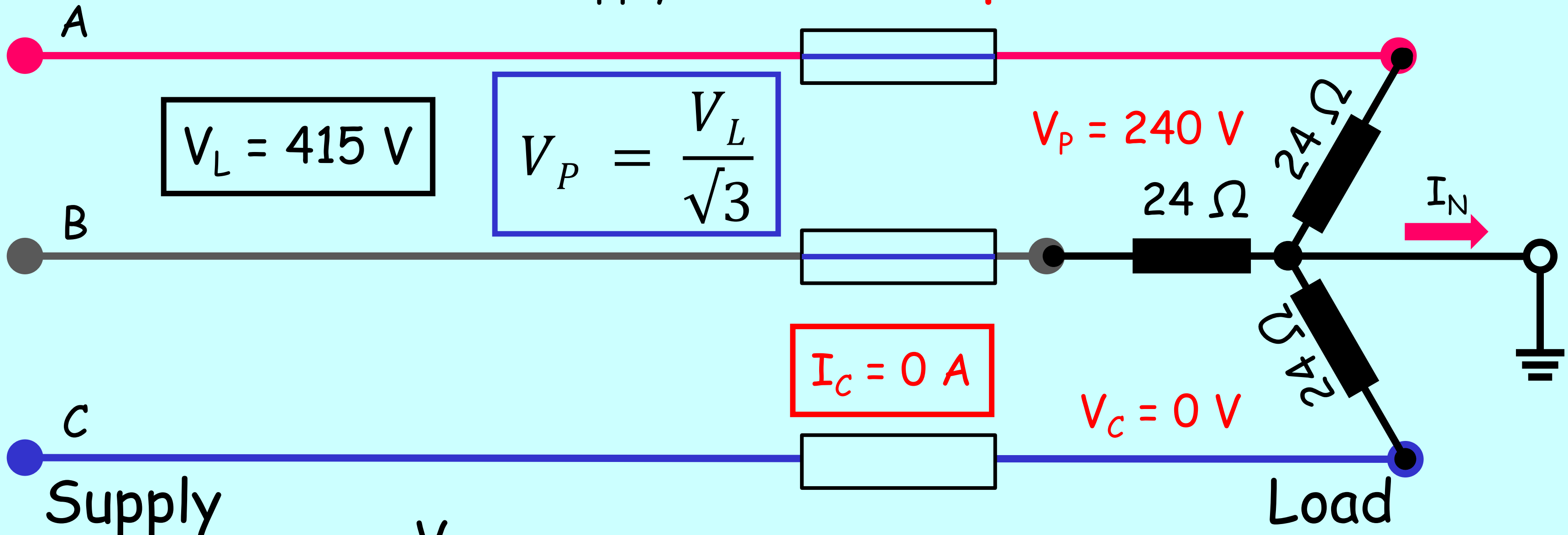
$\hat{I}_N + \hat{I}_A + \hat{I}_B + \hat{I}_C = 0$

$I_P = 10$ A

$I_L = 10$ A

$I_N = 0$ A

Determine the phase voltages and line currents for the elements of a 3 phase Star connected heater which has an impedance of 24 Ω per phase when it is connected to a 415 V four wire supply if the **fuse in phase C has blown**.



Fuse blown Operation

$$I_P = \frac{V_P}{Z_P}$$

$$I_P = \frac{240}{24}$$

$I_L = I_P$

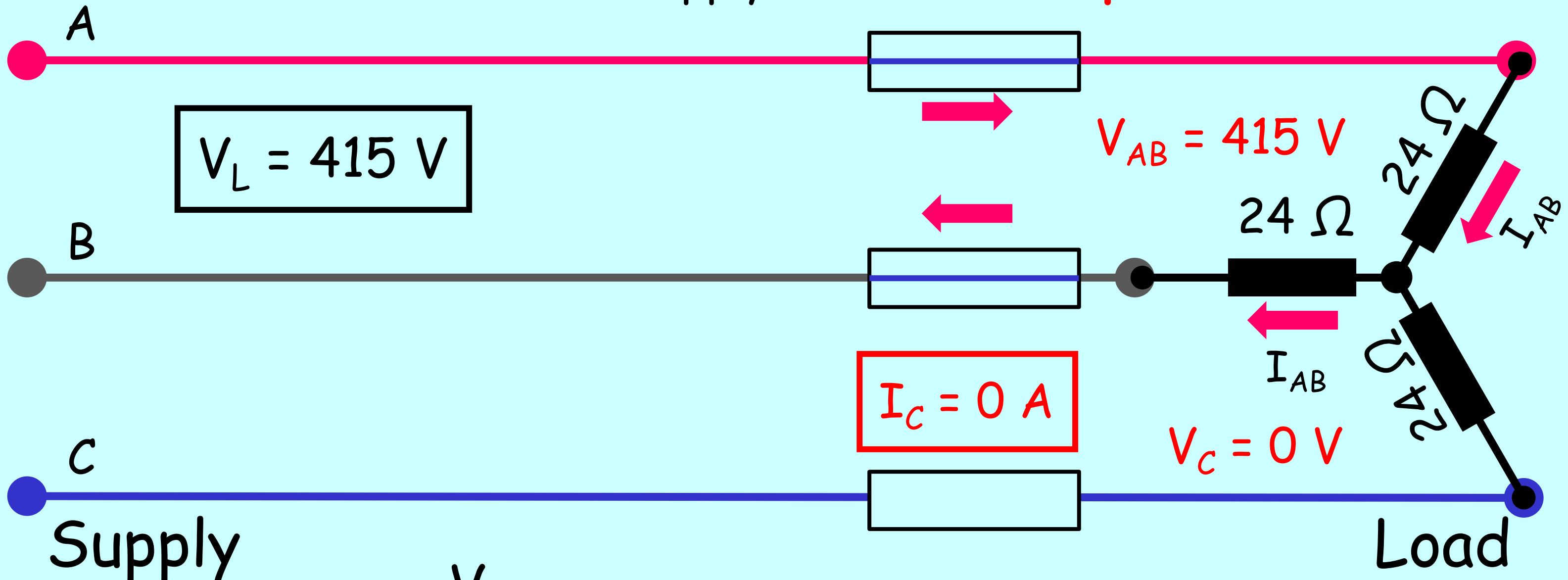
$\hat{I}_N + \hat{I}_A + \hat{I}_B + \hat{I}_C = 0$ (with $\hat{I}_C$ struck through)

$I_P = 10\ A$

$I_L = 10\ A$

$I_N = I_L = 10\ A$

Determine the phase voltages and line currents for the elements of a 3 phase Star connected heater which has an impedance of 24 Ω per phase when it is connected to a 415 V **three wire** supply if the **fuse in phase C has blown**.



Fuse blown Operation

$$I_P = \frac{V_P}{Z_P}$$

$$I_A = -I_B$$

$$I_P = \frac{415}{24 + 24}$$

$$I_P = 8.65\ A$$

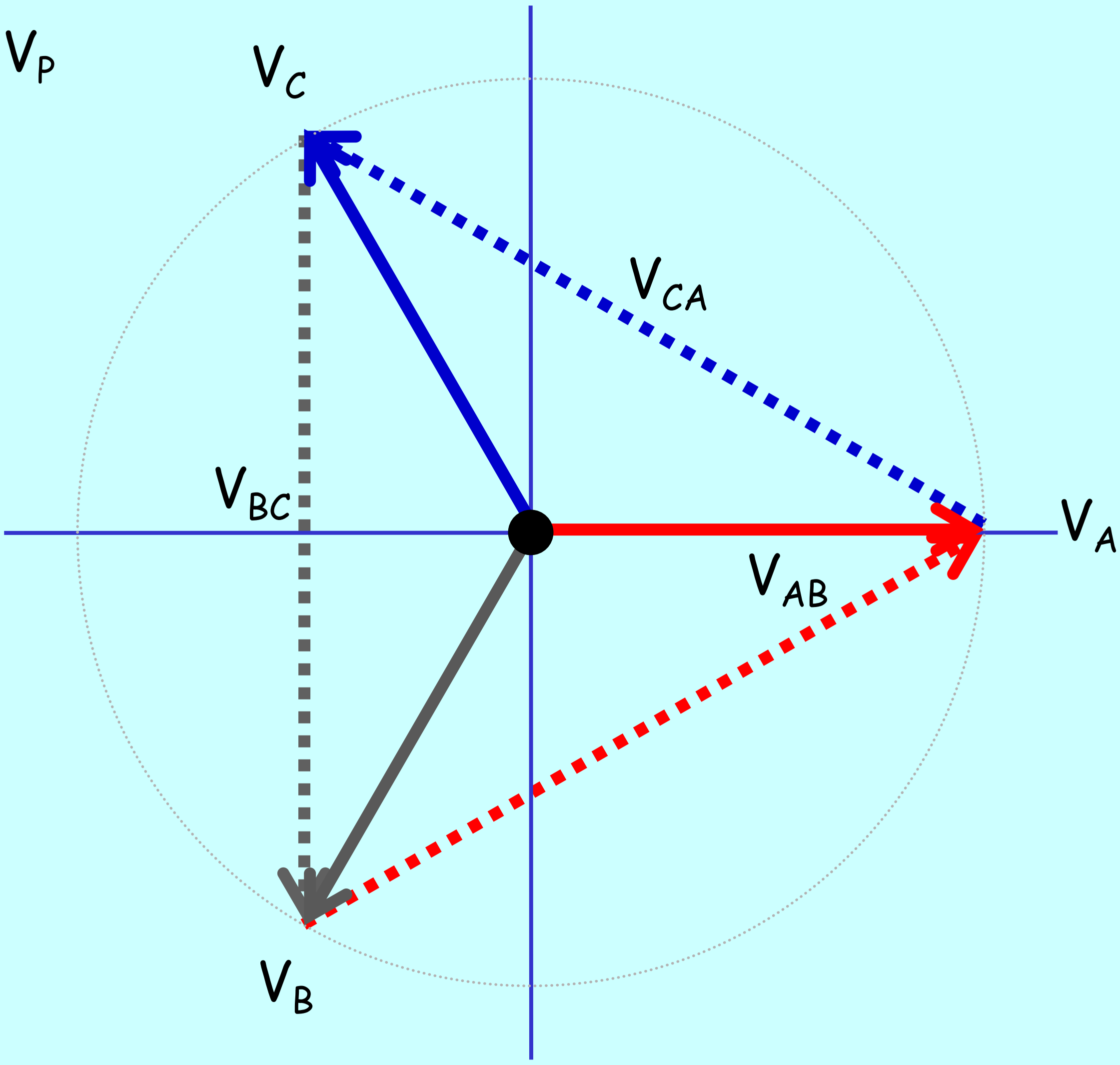
Star Voltages

$V_A = V_B = V_C = V_P$

$V_{AB} = \sqrt{3}V_P$

$V_{BC} = \sqrt{3}V_P$

$V_{CA} = \sqrt{3}V_P$



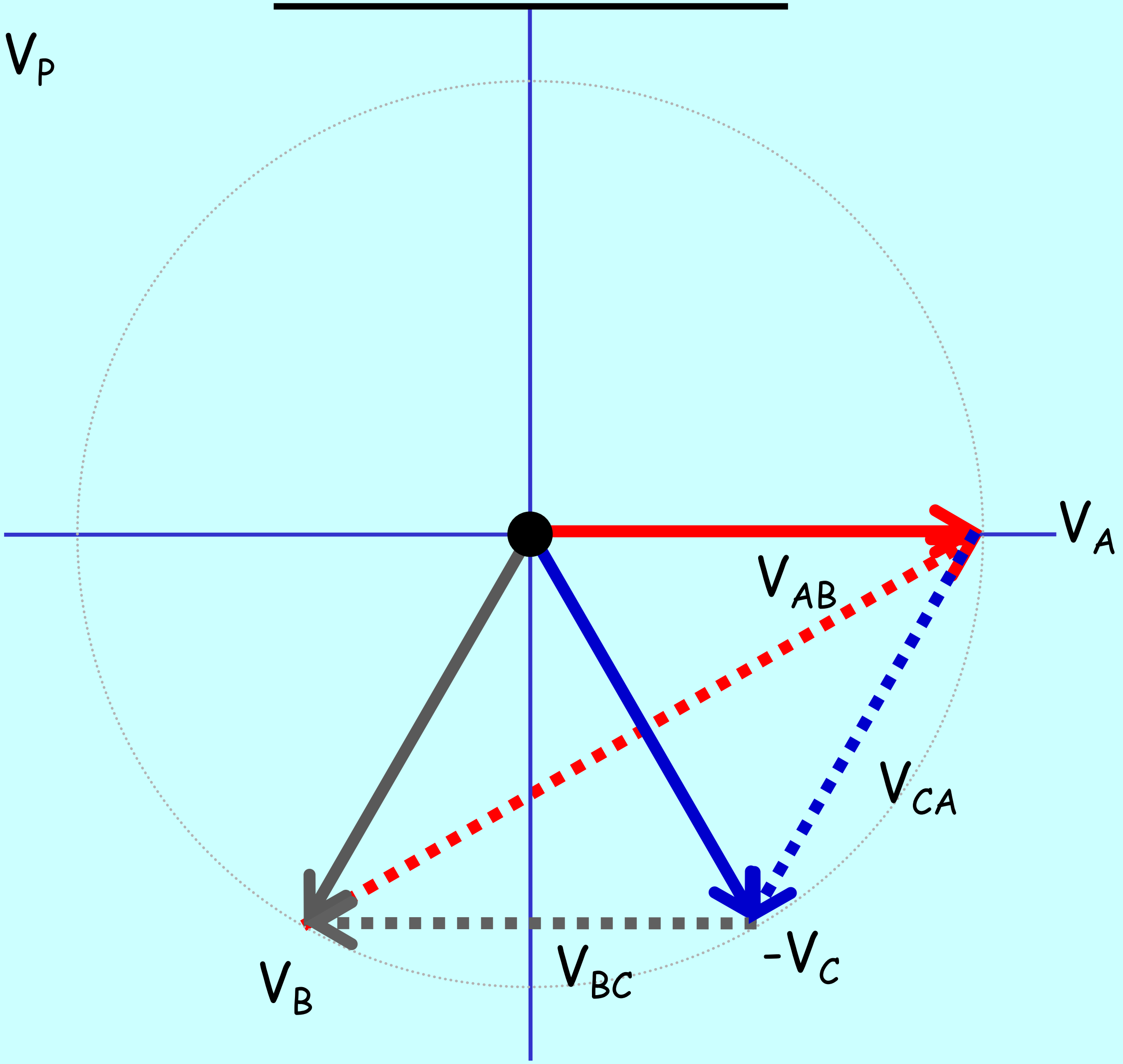
Star Voltages
Phase Reversal

$V_A = V_B = V_C = V_P$

$V_{AB} = \sqrt{3}V_P$

$V_{BC} \neq \sqrt{3}V_P$

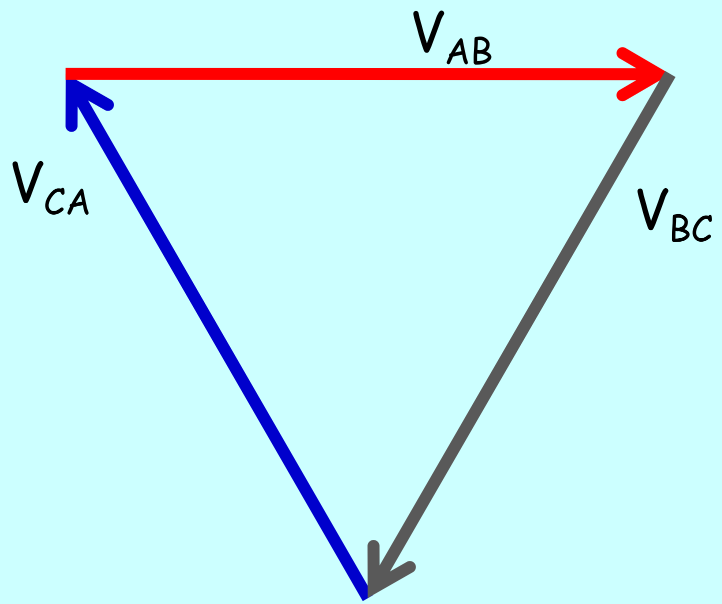
$V_{CA} \neq \sqrt{3}V_P$



Delta Voltages

$V_{AB} = V_{BC} = V_{CA} = V_P$

$V_{AB} + V_{BC} + V_{CA} = 0$

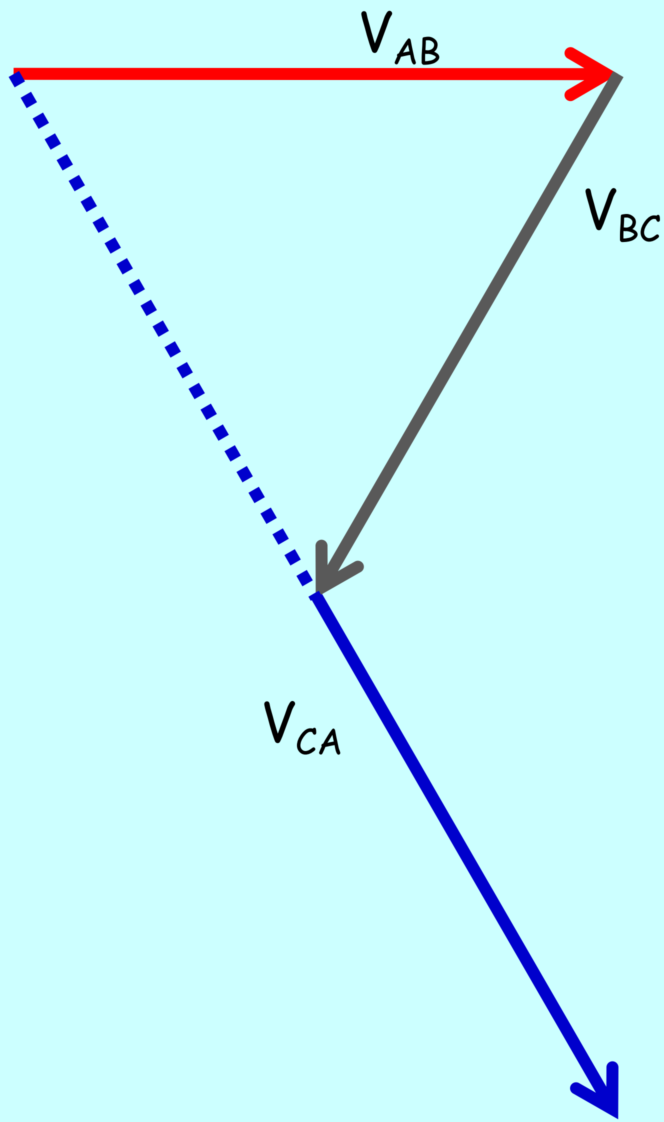


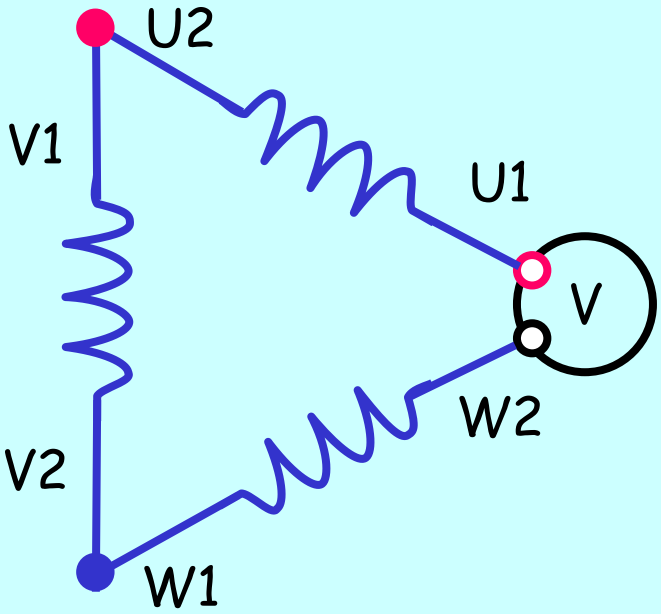
Delta Voltages
Phase Reversal

$V_{AB} + V_{BC} + V_{AC} \neq 0$

$V_{AB} = V_{BC}$

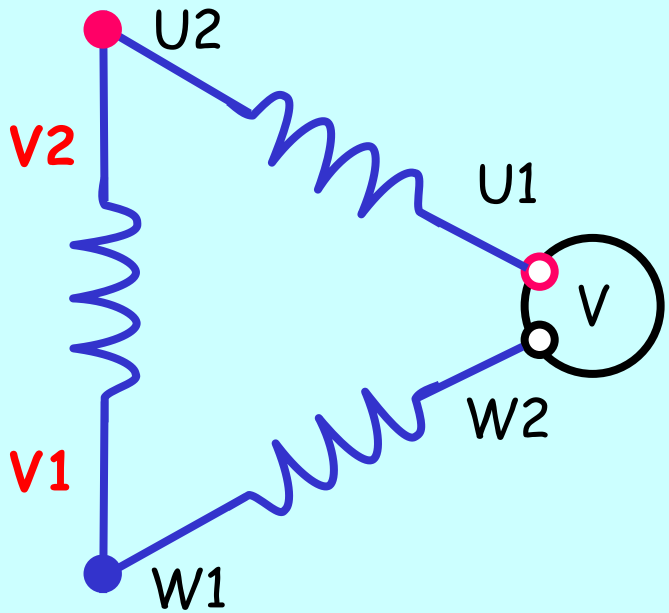
$V_{CA} = 2V_{AB}$





If rotation is correct

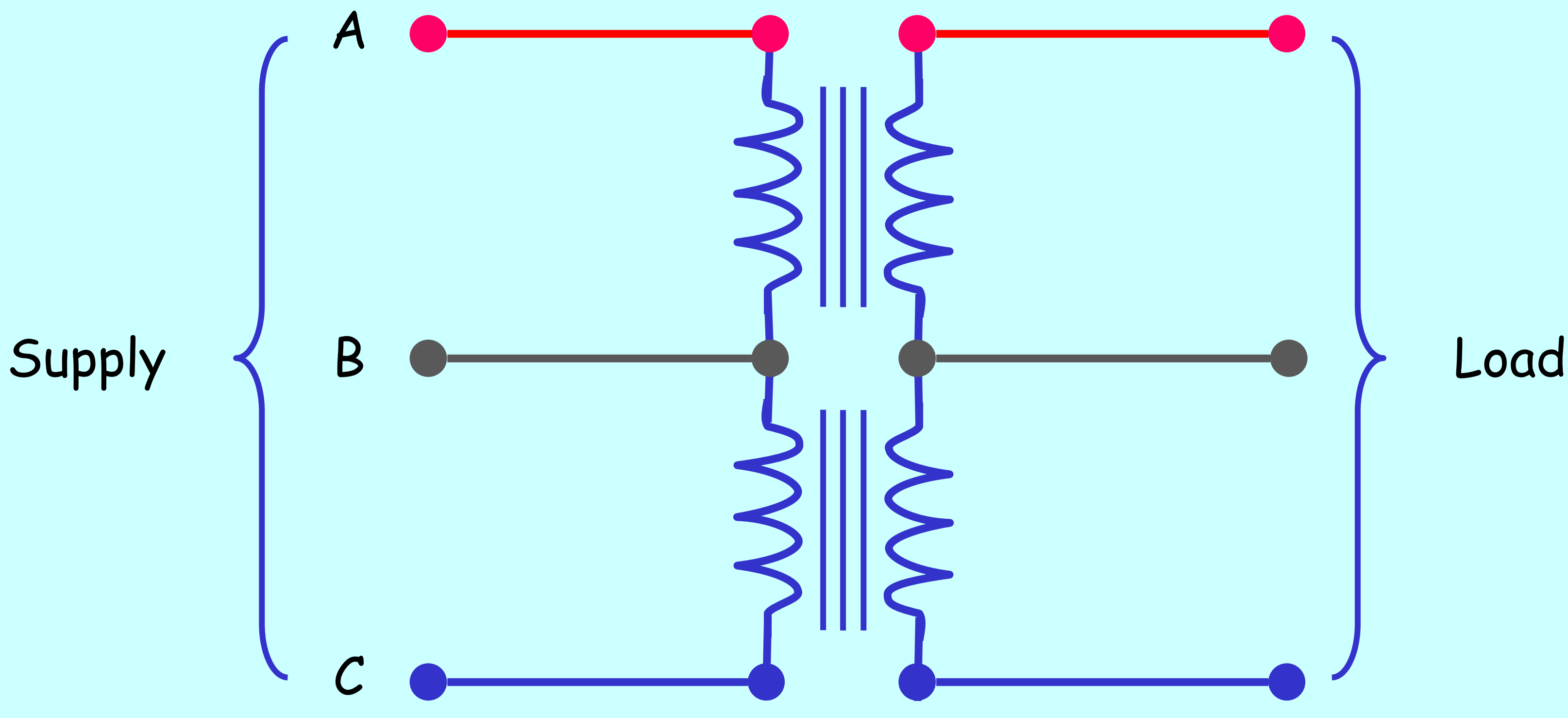
Voltage reading is Zero



If any phase is reversed

Voltage reading is $2V_P$

Open Delta Connected System



Three Phase Input

Two Single Phase Transformers

Three Phase Output

Simple and Easy to construct
Produces same Voltages

$I_L = I_P$

Does NOT have the same POWER Capabilities
Only 57.7%

End of Lesson

Practical Exercises

Three Phase Delta Connected System.

Star - Delta Connected System.

UEENEEG102A Solve problems in low voltage a.c. circuits

Energy & Power in 3 Phase Systems

Objectives:

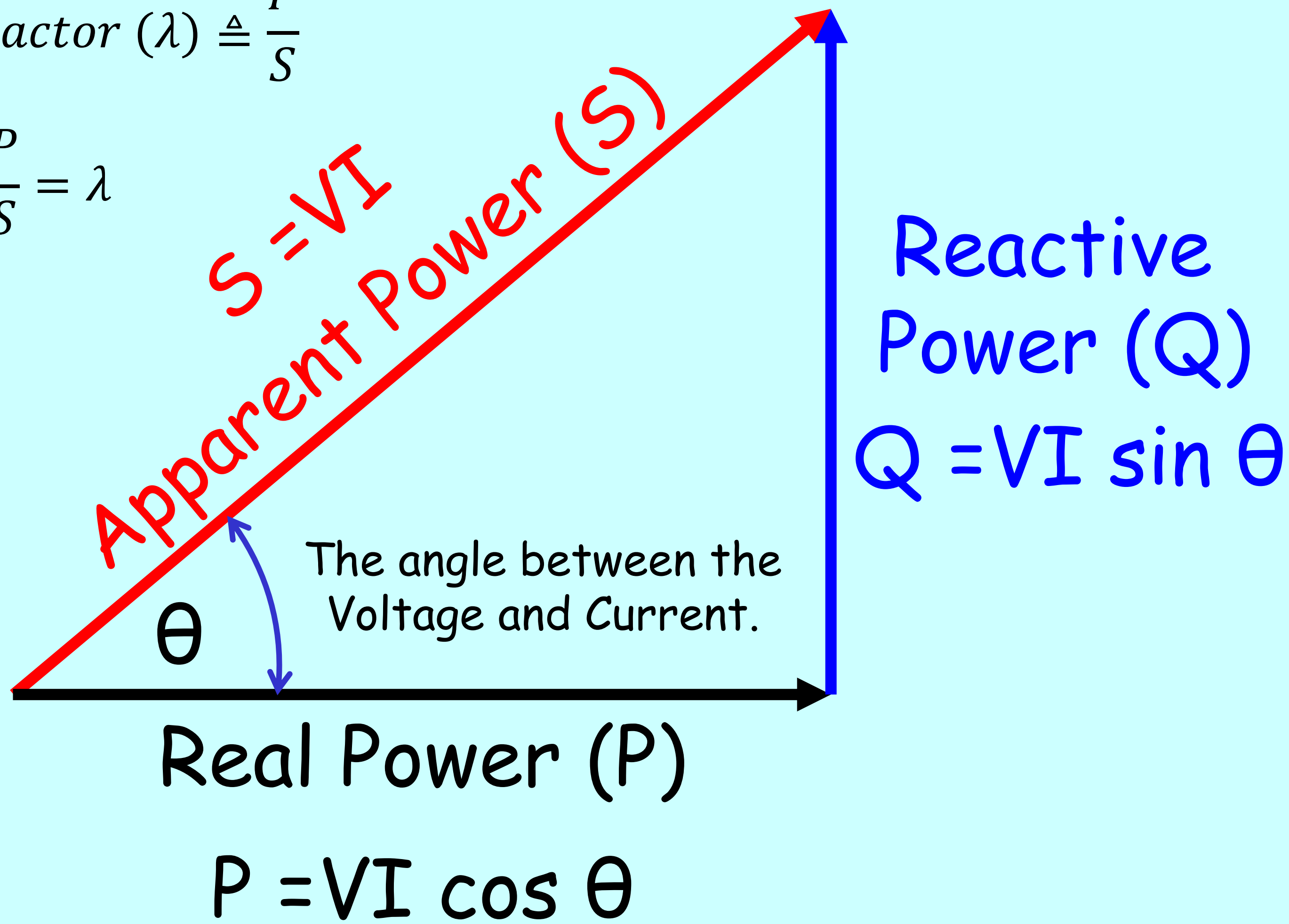
At the end of this lesson students should be able to:

1. Calculate True, Apparent and Reactive Power in a Three Phase System.
2. Measure True Power in a Three Phase System.

Power Triangle

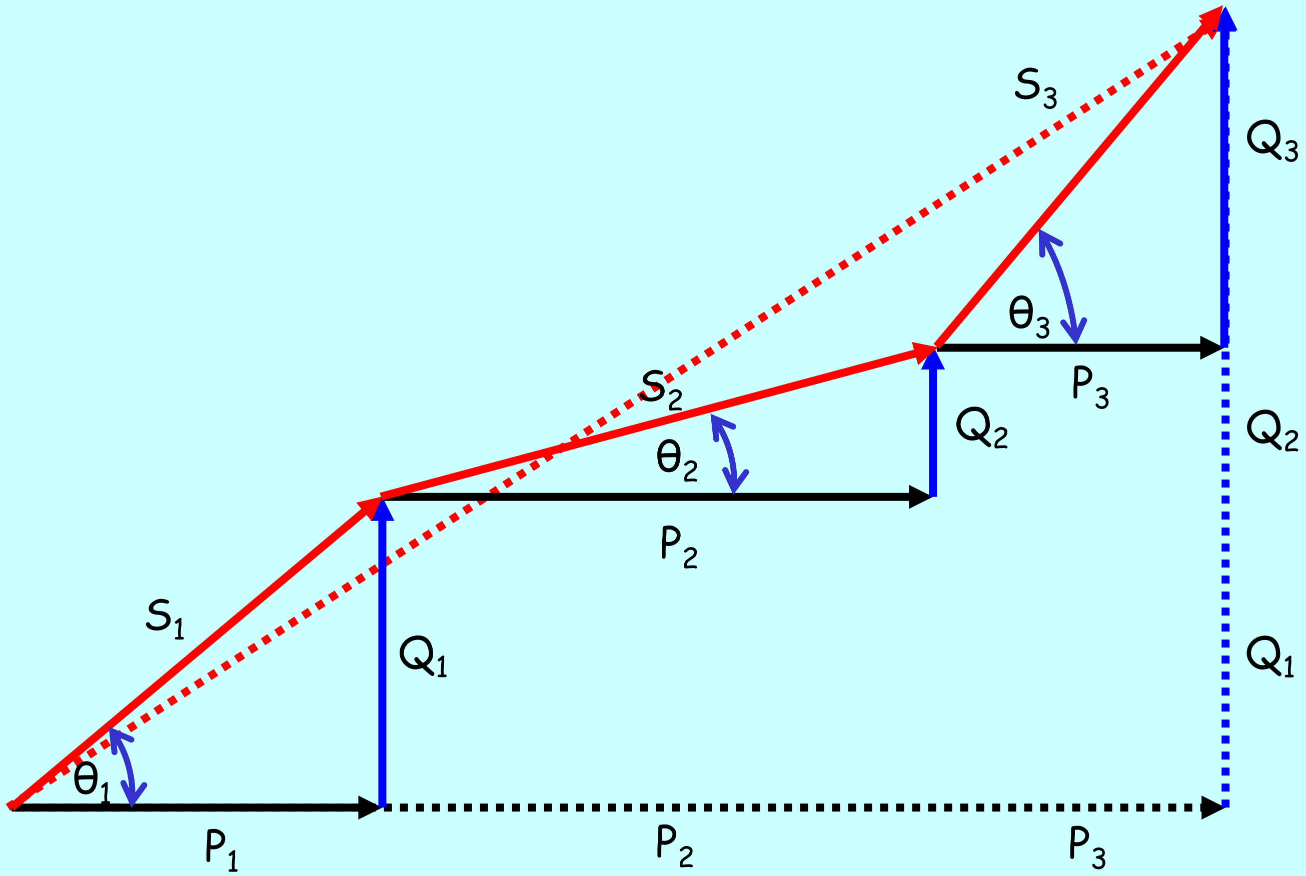
$$Power\ Factor\ (\lambda) \triangleq \frac{P}{S}$$

$$\cos\theta = \frac{P}{S} = \lambda$$



The Power Factor (λ) of the circuit relates the Real Power to the Apparent Power.

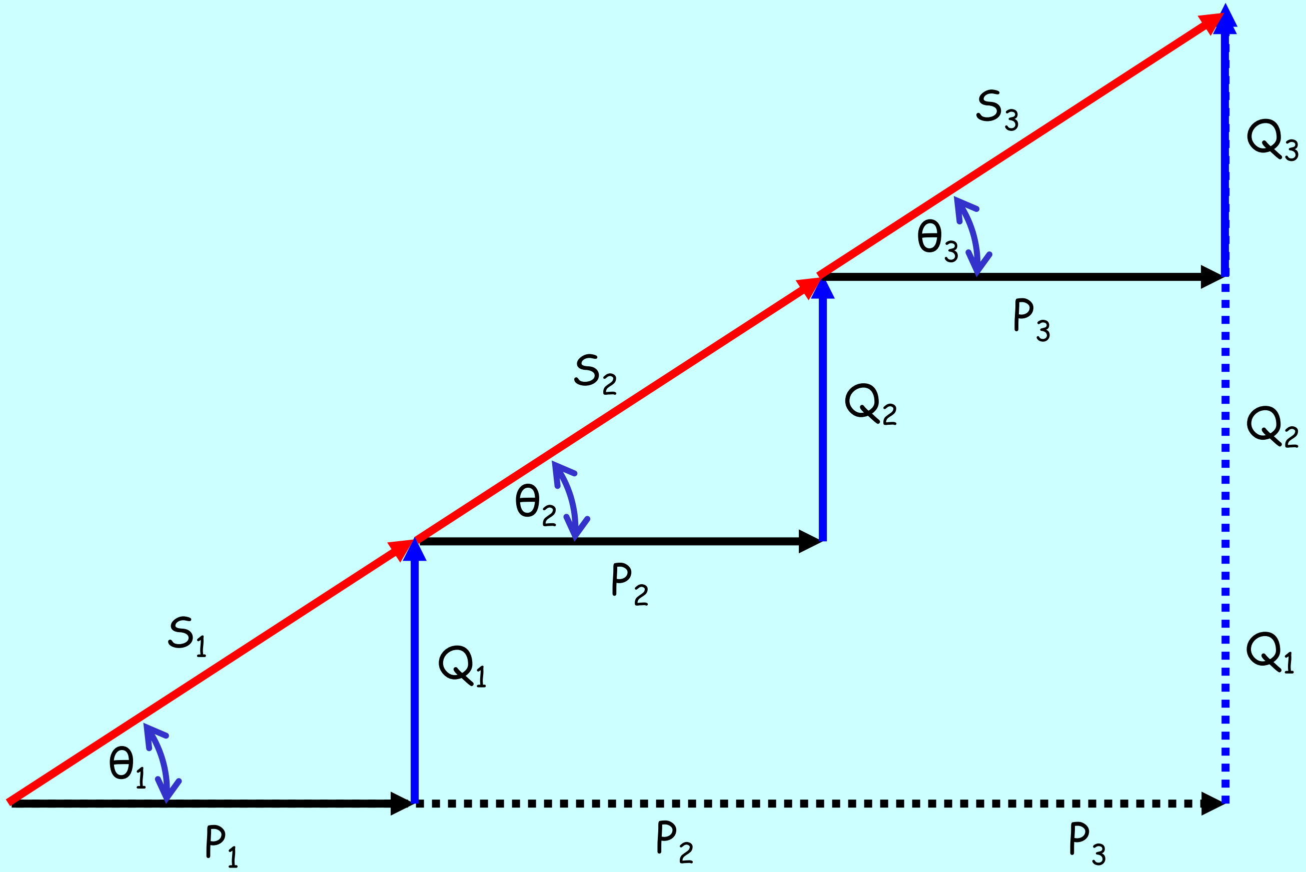
In an Unbalanced Three Phase System



Real Power in the Three Phase System

$$\underline{P}_T = \underline{P}_A + \underline{P}_B + \underline{P}_C$$

In a Balanced Three Phase System



Real Power in the Three Phase System

$$\underline{P}_T = \underline{P}_A + \underline{P}_B + \underline{P}_C$$

Real Power in any Three Phase System

$$\underline{P}_T = \underline{P}_A + \underline{P}_B + \underline{P}_C$$

In a Balanced Three Phase System

$$P_T = 3P_P = 3V_P I_P \cos\theta$$

In a Star Connected System

$$V_P \, x \, I_P = I_L \, x \, \frac{V_L}{\sqrt{3}}$$

In a Delta Connected System

$$V_P \, x I_P = V_L \, x \, \frac{I_L}{\sqrt{3}}$$

$$P_T = 3 \times \frac{V_L I_L}{\sqrt{3}} \times \cos\theta$$

$$P_T = 3 \times V_P I_P \times \cos\theta$$

$$P_T = \sqrt{3} \times V_L I_L \times \cos\theta$$

Determine the true power delivered to a 3-phase Delta connected induction motor which draws a balanced **line** current of 20 Amps at a lagging power factor of 0.866 from a 415 V, 50 Hz supply.

$V_L = 415\ V$

$I_L = 20\ A$

$\lambda = 0.866$

$P_T = \sqrt{3} V_L I_L \cos \theta$

$P_T = \sqrt{3} \times 415 \times 20 \times 0.866$

$P_T = 12{,}449\ W$

$P_T = 12.5\ kW$

Determine the line current delivered by an 11 kV transformer to a 3-phase balanced load which uses 300 kW at a lagging power factor of 0.9.

V_L = 11 kV

P_T = 300 kW

λ = 0.9

$$I_L = \frac{P_T}{\sqrt{3}V_L\cos\theta}$$

$$I_L = \frac{300k}{\sqrt{3}\times 11k \times 0.9}$$

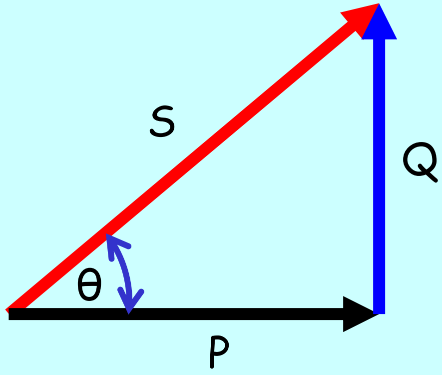
$$\boxed{I_L = 17.5\ A}$$

Determine the true and reactive power that can be delivered by a 3-phase transformer rated at 50 kVA if the load has a 0.5 lagging power factor.

$S_T = 50 \text{ kVA}$

$\lambda = 0.5 \text{ lag}$

$\lambda = \cos\theta$



$\theta = \cos^{-1}\lambda$

$\theta = \cos^{-1} 0.5$

$\theta = 60°$

$P_T = S\cos\theta$

$P_T = S\lambda$

$P_T = 50\text{ k} \times 0.5$

$P_T = 25 \text{ kW}$

$Q_T = \sqrt{(S^2 - P^2)}$

$Q_T = \sqrt{(50^2 - 25^2)}$

$Q_T = 43.3 \text{ kVar}$

$Q_T = S\sin\theta$

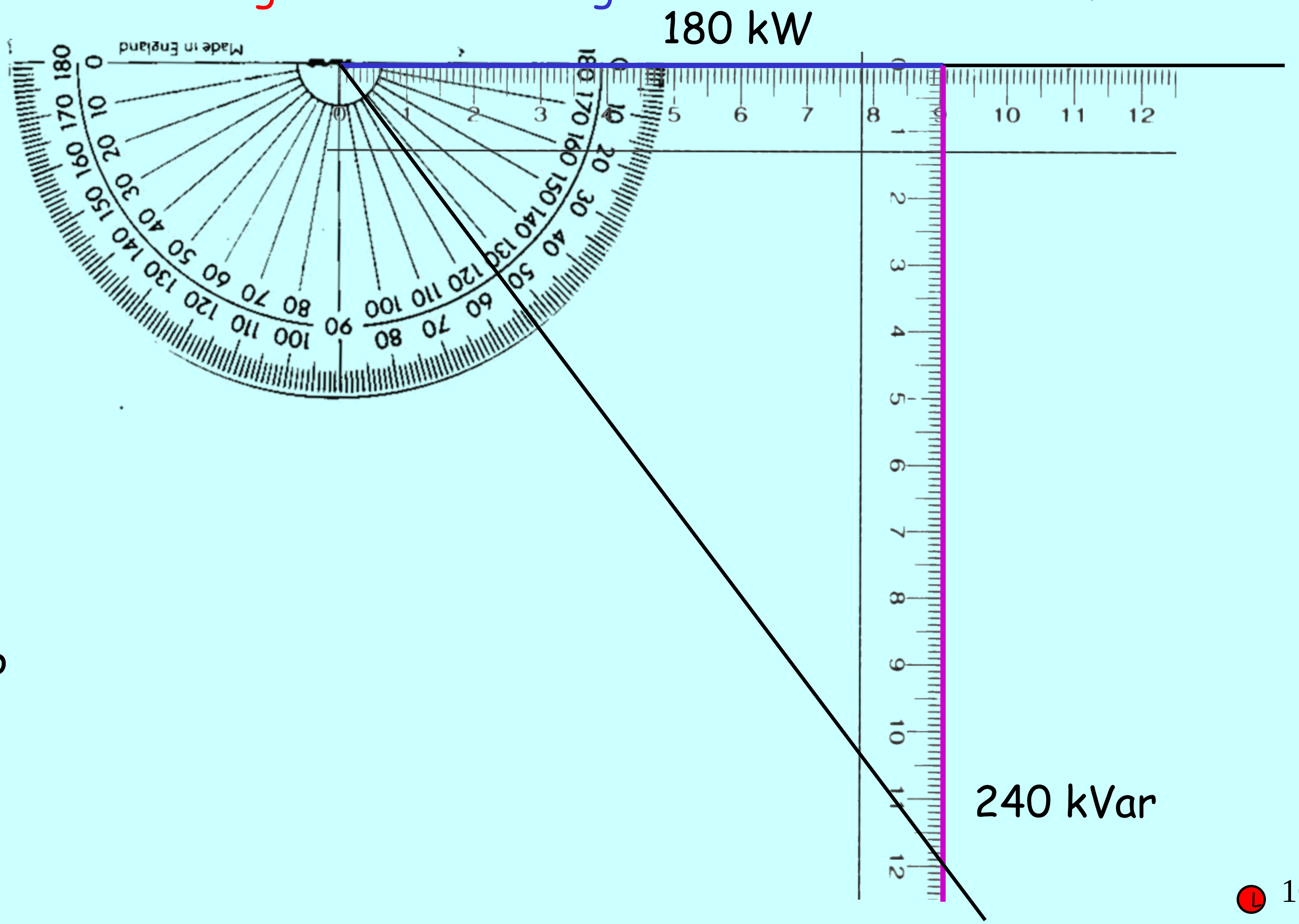
$Q_T = 50\text{ k} \times 0.866$

$Q_T = 43.3 \text{ kVar}$

A 3-phase balanced load requires 180kW of power when operating at 0.6 lag pf. and connected to a 415V 50Hz 3-phase supply. Determine the kvar rating and capacitance of a star connected capacitor bank that would improve the pf. to 0.9 lag.

Step 1: Construct the original Power Triangle

Scale: 1 cm = 20 k



$P_T = 180$ kW

$\lambda_1 = 0.6$

$\theta_1 = \cos^{-1} \lambda$

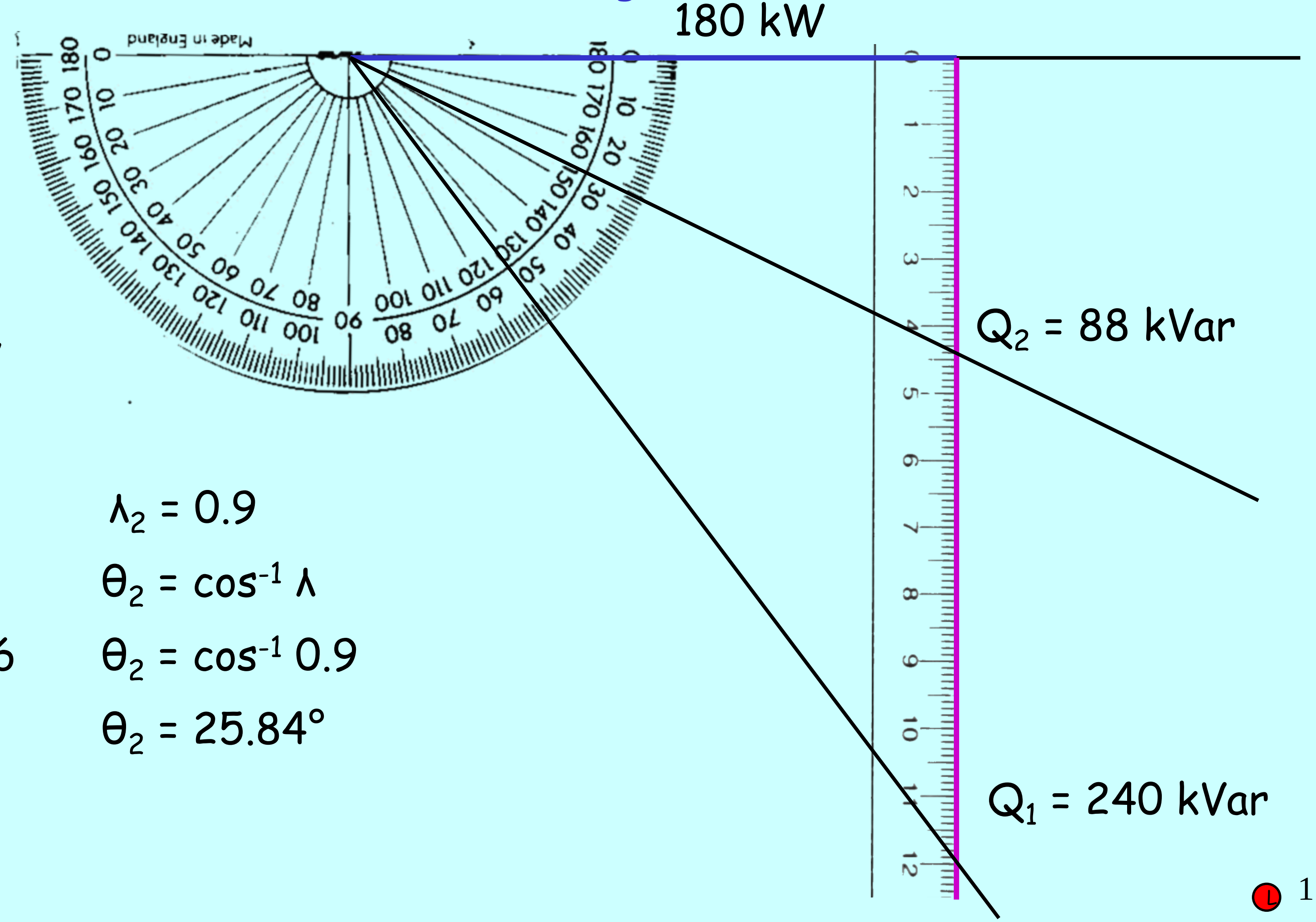
$\theta_1 = \cos^{-1} 0.6$

$\theta_1 = 53.13°$

A 3-phase balanced load requires 180kW of power when operating at 0.6 lag pf. and connected to a 415V 50Hz 3-phase supply. Determine the kvar rating and capacitance of a star connected capacitor bank that would improve the pf. to 0.9 lag.

Step 2: Construct the desired Power Triangle

Scale: 1 cm = 20 k



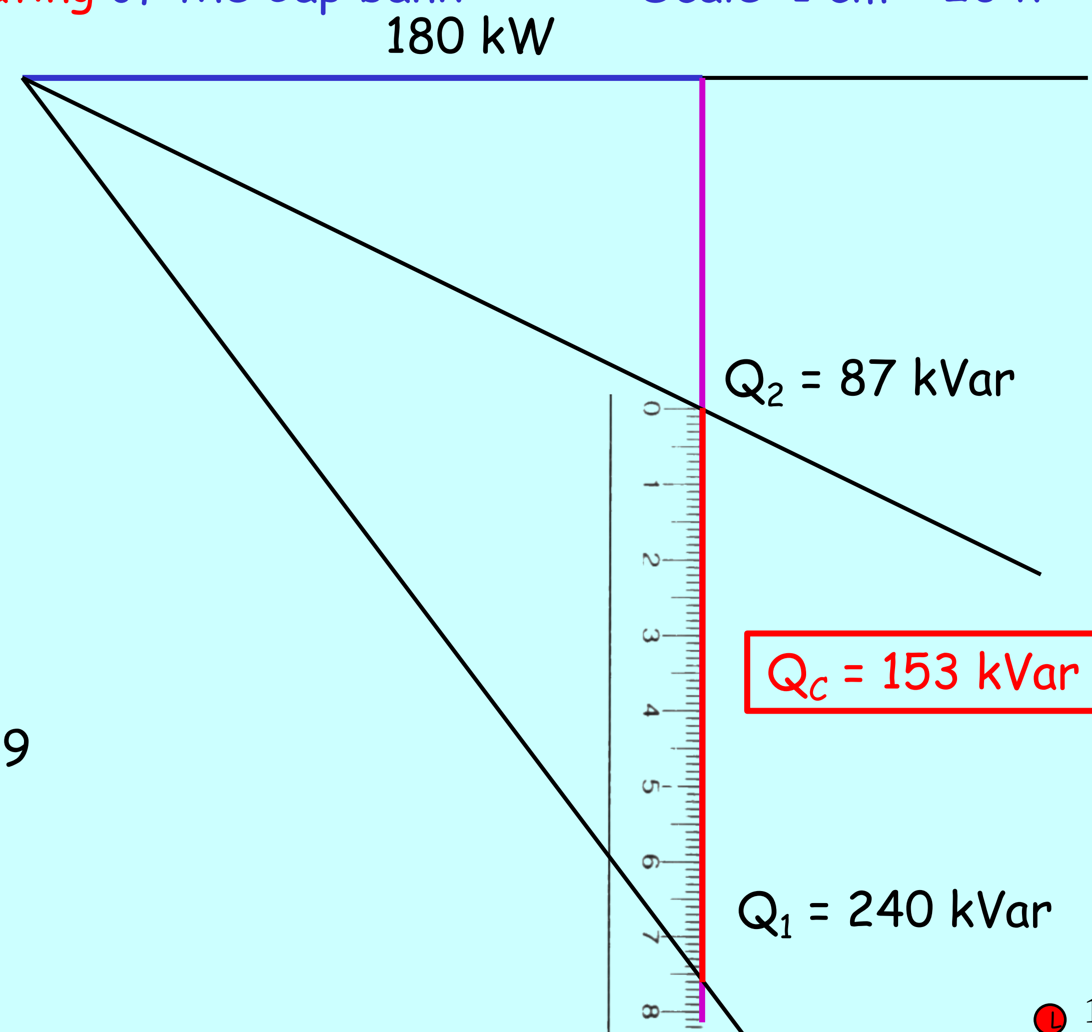
P_T = 180 kW

$\lambda_1 = 0.6$	$\lambda_2 = 0.9$
$\theta_1 = \cos^{-1} \lambda$	$\theta_2 = \cos^{-1} \lambda$
$\theta_1 = \cos^{-1} 0.6$	$\theta_2 = \cos^{-1} 0.9$
$\theta_1 = 53.13°$	$\theta_2 = 25.84°$

A 3-phase balanced load requires 180kW of power when operating at 0.6 lag pf. and connected to a 415V 50Hz 3-phase supply. Determine the kvar rating and capacitance of a star connected capacitor bank that would improve the pf. to 0.9 lag.

Step 3: Measure the kVar rating of the Cap bank

Scale: 1 cm = 20 k



$P_T = 180$ kW

$\lambda_1 = 0.6$	$\lambda_2 = 0.9$
$\theta_1 = \cos^{-1} \lambda$	$\theta_2 = \cos^{-1} \lambda$
$\theta_1 = \cos^{-1} 0.6$	$\theta_2 = \cos^{-1} 0.9$
$\theta_1 = 53.13°$	$\theta_2 = 25.84°$

A 3-phase balanced load requires 180kW of power when operating at 0.6 lag pf. and connected to a 415V 50Hz 3-phase supply. Determine the kvar rating and capacitance of a star connected capacitor bank that would improve the pf. to 0.9 lag.

Step 4: Calculate the CAPACITANCE of the Cap bank

Q_C = 153 kVar

$$Q_P = \frac{Q_C}{3}$$

$$Q_P = \frac{153}{3}$$

$$Q_P = 51 \text{ kVar}$$

$$X_P = \frac{V_P^2}{Q_P}$$

$$V_P = \frac{V_L}{\sqrt{3}}$$

$$X_P = \frac{240^2}{51\,000}$$

$$X_P = 1.13\ \Omega$$

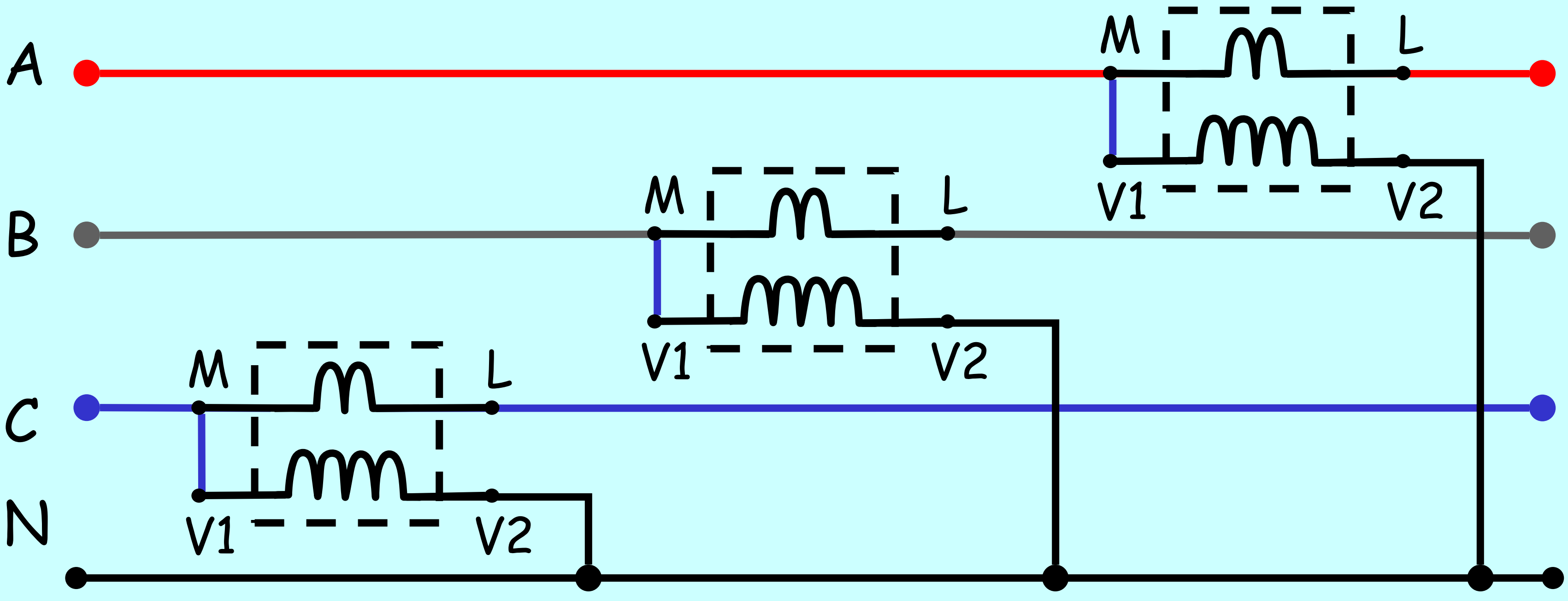
$$X_P = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_P}$$

$$C = \frac{1}{2\pi \times 50 \times 1.13}$$

$$C = 2.82\ mF$$

Power Measurement in 3 Φ Systems



Three Watt Meters – Four Wire method

$$P_T = P_A + P_B + P_C$$

Advantages

Can be used on Balanced OR Unbalanced Loads

Total Power easily monitored

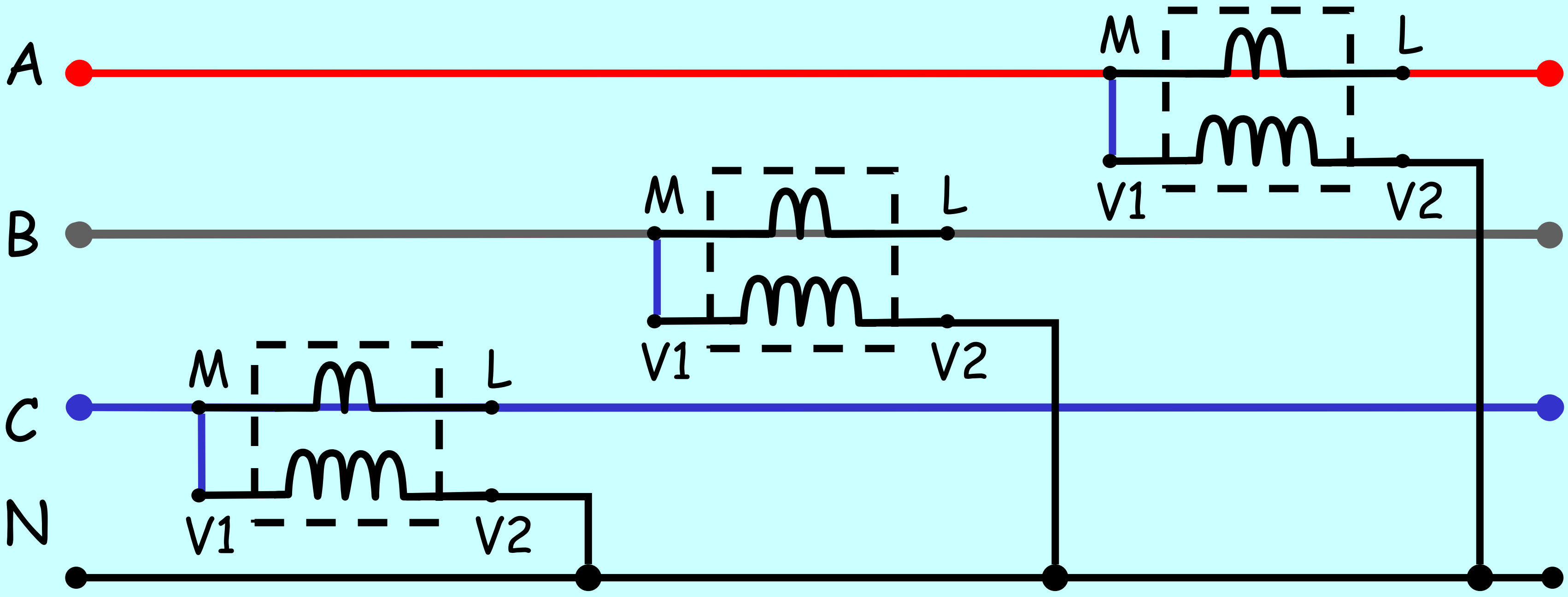
Reasonably accurate

Disadvantages

Three Wattmeters required

Requires a Neutral (Star System)

Power Measurement in 3 Φ Systems



One Watt Meter (move from phase to get each reading)

$$\underline{P}_T = \underline{P}_A + \underline{P}_B + \underline{P}_C$$

Advantages

Only ONE Meter used

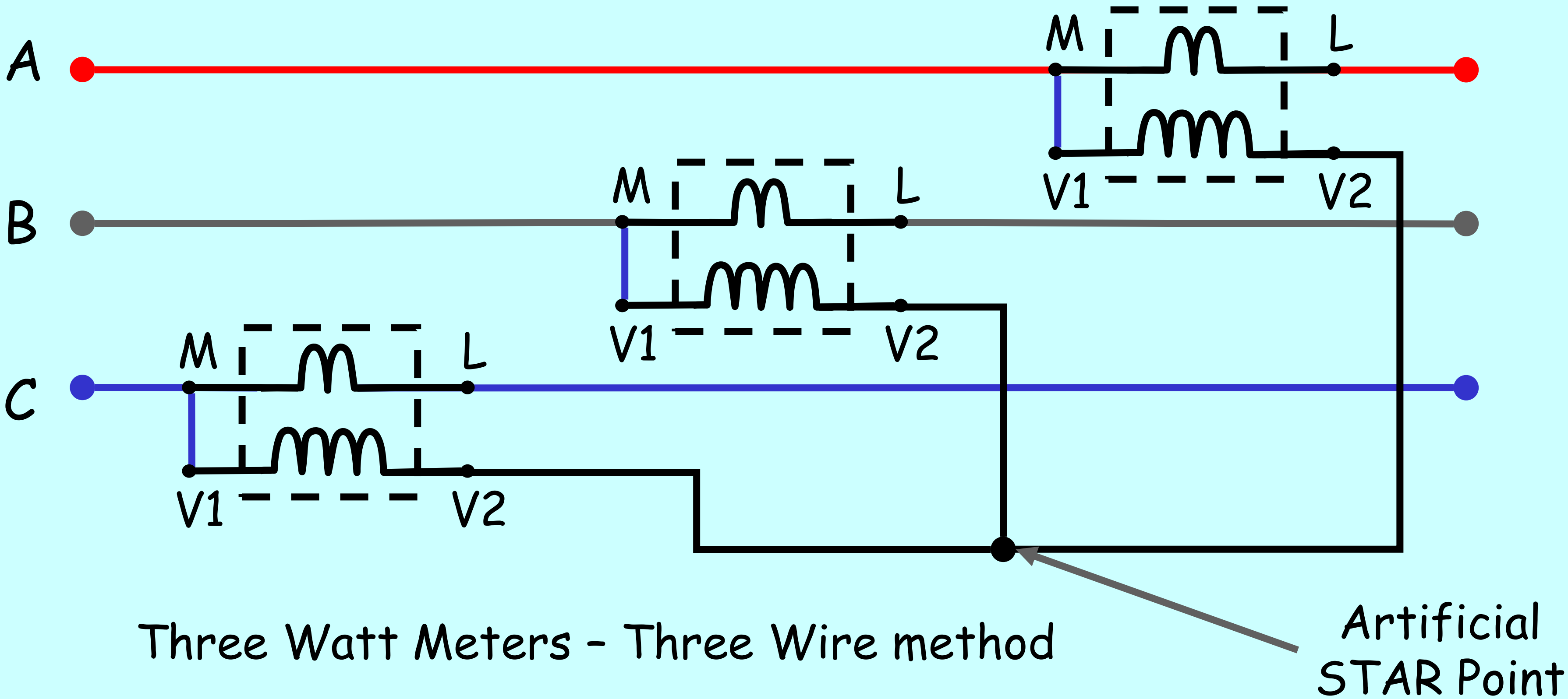
Simple & cheap

Can be used on Balanced OR Unbalanced Loads

Disadvantages

Not very accurate on Unbalanced Loads

Power Measurement in 3 Φ Systems



Three Watt Meters – Three Wire method

Advantages

Can be used on Balanced OR Unbalanced Loads

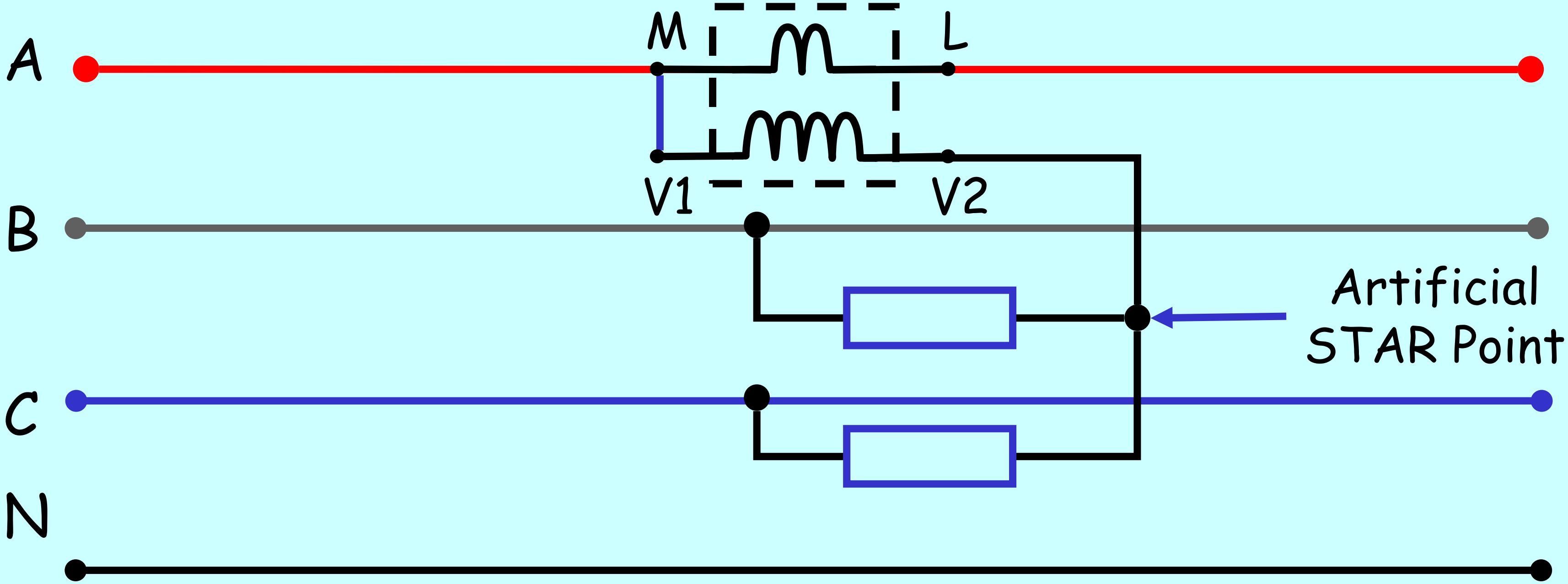
Total Power easily monitored

Reasonably accurate

Disadvantages

Requires Three identical Wattmeters

Power Measurement in 3 Φ Systems



One Watt Meter – Three Wire Method

Advantages

Only ONE Meter used

Simple & cheap

Can be used on Balanced OR Unbalanced Loads

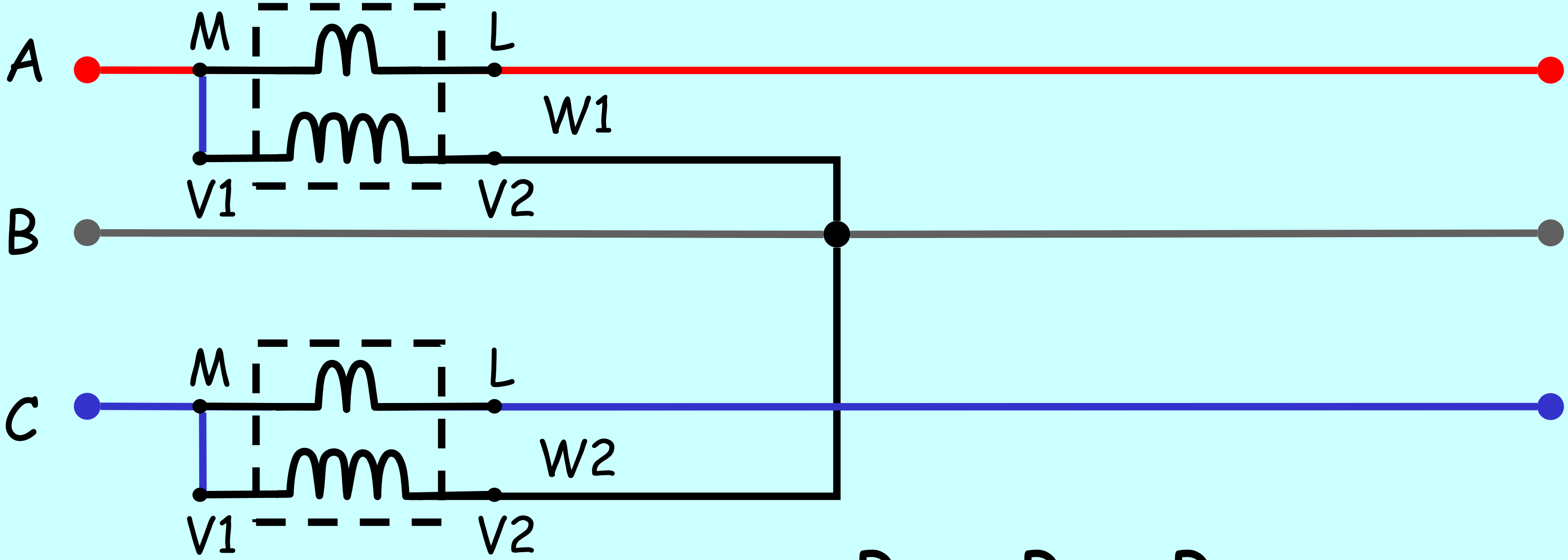
Disadvantages

Must construct the Artificial STAR Point

Not accurate on varying Unbalanced Loads

Need to reconnect to each phase for Unbalanced Loads

Power Measurement in 3 Φ Systems



$$\underline{P}_T = \underline{P}_1 + \underline{P}_2$$

Two Watt Meters

Advantages

Can be used on Balanced OR Unbalanced Loads

Total Power easily monitored

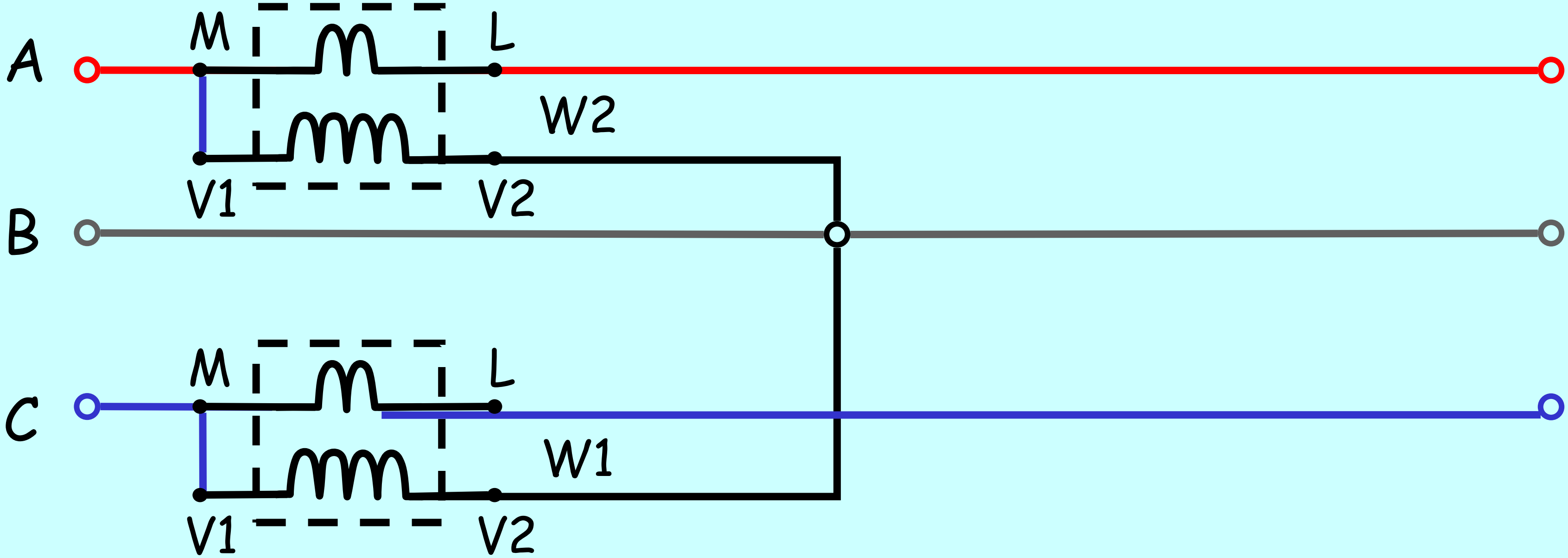
Only two Wattmeters required

Disadvantages

Can be used with Three Wire systems only

Not accurate for loads with low power factor

Power Measurement in 3 Φ Systems



Two Watt Meter Procedure

$$P_T = P_1 + P_2$$

For balanced loads

Locate W2 in the phase immediately following W1 in the phase sequence.

$$\tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)$$

Two Element Watt meters

Exercises

Page 73

1. A star connected A.C. generator develops 11000 volts per phase. Determine the MVA rating of the machine if the current per phase is 50 amperes. (1.65 MVA)

$V_P = 11\ kV$

$I_P = 50\ A$

$S_T = 3V_PI_P$

$S_T = 3 \times 11\ k \times 50$

$S_T = 1.65\ MVA$

2. A 3 phase alternator delivers a full load of 140 amperes at a power factor of 0.9 lagging. If the terminal voltage is 400 volts, calculate the:

a. Kilovolt ampere rating, (97 kVA)

b. Kilowatt output.(87.3 kW).

$V_L = 400\ V$

$I_L = 140\ A$

$\lambda = 0.9$

$S_T = \sqrt{3} V_L I_L$

$S_T = \sqrt{3} \times 400 \times 140$

$S_T = 96.99\ kVA$

$P_T = S_L \times \lambda$

$P_T = 97\ k \times 0.9$

$P_T = 87.3\ kW$

3. The power in a 3 phase 400 volt system is measured by the two wattmeter method where W1 indicates 30 kW and W2 indicates 20 kW. Calculate the:

a. Total power (50 kW),

b. Power factor (0.94),

c. Line current (76.78 A).

$$\tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)$$

V_L = 400 V

W_1 = 30 kW

W_2 = 20 kW

$$\theta = \tan^{-1}\left\{\sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)\right\}$$

$$\theta = \tan^{-1}\left\{\sqrt{3}\left(\frac{20 - 30}{20 + 30}\right)\right\}$$

θ = -19.1°

P_T = W_1 + W_2

P_T = (30 + 20) kW

P_T = 50 kW

$\lambda = \cos\theta$

$\lambda = \cos -19.1$

λ = 0.94

$P_T = \sqrt{3} V_L I_L \cos\theta$

$$I_L = \frac{P_T}{\sqrt{3} V_L \cos\theta}$$

$$I_L = \frac{50k}{\sqrt{3} \times 400 \times 0.94}$$

I_L = 76.78 A

4. The power input to a 3 phase, 400 volt induction motor is measured by the two wattmeter method where W1 indicates 10 kW and W2 indicates -6 kW (note the minus sign). Calculate the:

a. power input (4 kW),

b. power factor of the motor (0.14),

$$\tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)$$

$V_L = 400$ V

$W_1 = 10$ kW

$W_2 = -6$ kW

$$\theta = \tan^{-1}\left\{\sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)\right\}$$

$$\theta = \tan^{-1}\left\{\sqrt{3}\left(\frac{-6 - 10}{-6 + 10}\right)\right\}$$

$\theta = -81.79^\circ$

$P_T = W_1 + W_2$

$P_T = (10 - 6)$ kW

$P_T = 4$ kW

$\lambda = \cos\theta$

$\lambda = \cos -81.79$

$\lambda = 0.14$

End of Theory

Next Lesson

Final Theory Test