NEW SOUTH WALES

DEPARTMENT OF TECHNICAL EDUCATION

ELECTRICAL ENGINEERING CERTIFICATE COURSE

STAGE III

ELECTRICAL MACHINES I

BOOK II

(UNITS 7 - 14)

INTRODUCTION

In the first book covering unit 1 to 6 inclusive the d.c. machine was discussed both from the point of view of its construction and also from its operation as a device to convert mechanical energy to electrical energy, that is to say, the operation of the machine in the generating mode.

It has been emphasized in the earlier units that basically there is no fundamental structural difference between d.c. motors and d.c. generators. Recent development in the increasing use of d.c. motors in conjunction with controlled rectifier power supplies have introduced restraints on motor design. As a consequence the tendency has been to incorporate design features in d.c. motors that are not essential in d.c. generators to give the necessary characteristics required of machines used on S.C.R. supplies.

These units (7-14) will also cover the starting of d.c. motors using series resistances as a means of limiting the current and the methods to be used in assessing the requirements of motors intended for particular drives.

Again the student's attention is drawn to the need for a systematic approach in his study techniques. This is of particular importance in correspondence courses. At this stage in the student's programme he should have received units back from his teacher. One further word of advice, in regard to returned units. If the student has not reached a minimum standard, units are sometimes returned to the student for resubmission. In other cases the student may not be satisfied with his own performance and it is then suggested that perhaps he could rework the problems taking note of any suggestions offered by the teacher.

UNIT NO. 7

This unit will cover the following topics:

7.1 Direction of Rotation.

7.2 Reversing Interpole Motors.

7.3 Speed Regulation of Motors.

7.4 Effect of Field Resistance on Speed and Torque.

7.5 Torque and Speed Relations in Series Motors.

7.6 Armature Speed Control by Series Resistance.

7.7 Shunt Field Resistance Control.

7.8 Armature Voltage Control.

7.9 Braking.

Review Questions.

Assignments.

7.1 DIRECTION OF ROTATION

The direction of rotation of a motor is dependent on the direction of the armature magnetic field, relative to the direction of the main pole magnetic field. Reversing the direction of either of these will reverse the armature rotation. In practice this means reversing either:

(a) the current flow in the armature, that is, reverse brush polarity, or

(b) reversing the direction of the shunt field current.

In the case of the compound wound motor if the shunt field current is reversed, the series field current also must be reversed. If the armature series field circuit current is reversed, the series field portion must also be reversed to maintain the same direction for the magnetic field and the shunt portion.

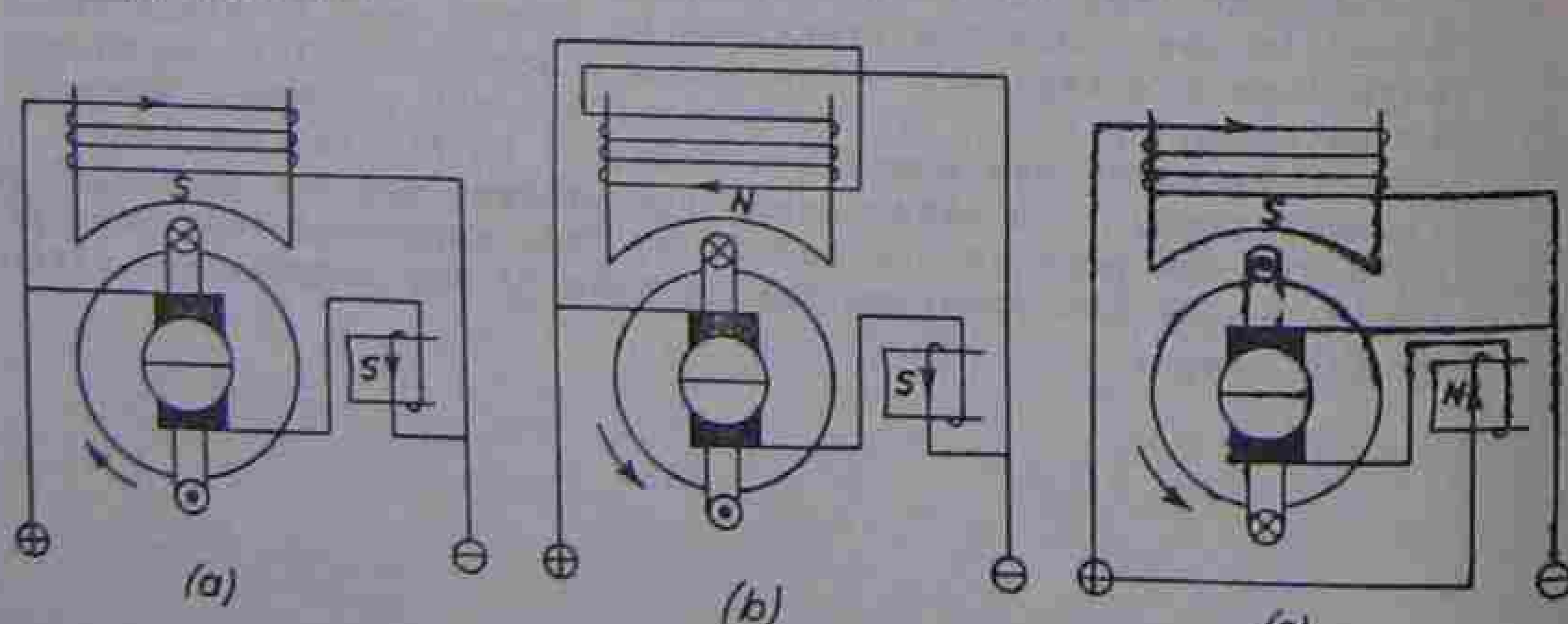


Figure 7.1 - Reversal of rotation of Shunt Motor.

Figure 7.1 shows how the direction of rotation of a shunt motor is reversed.

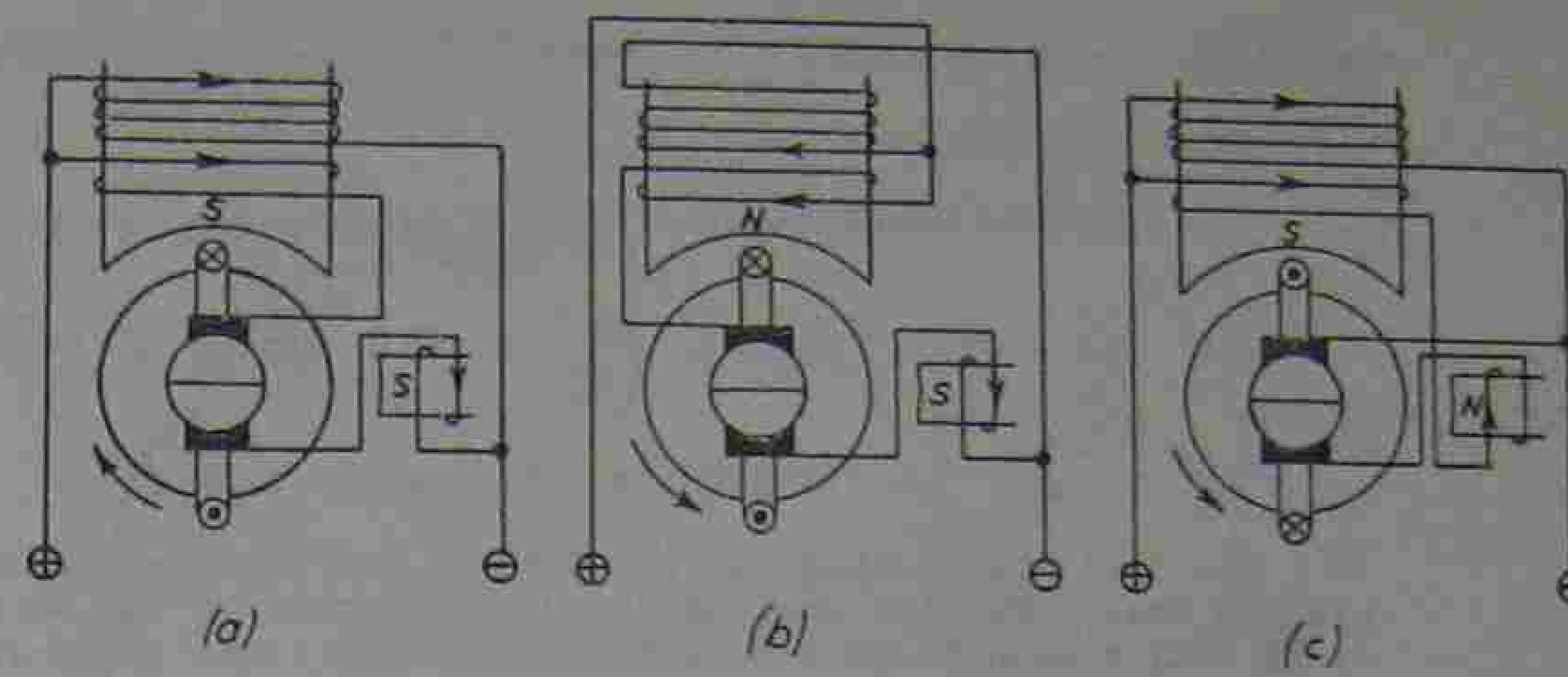


Figure 7.2 - Reversal of rotation of Compound Motor.

Figure 7.2 shows how the direction of rotation of a compound motor is reversed.

7.2 REVERSING INTERPOLE MOTORS

Where interpoles are fitted to a motor the interpole must be the same polarity as the preceding main pole considered from the standpoint of the direction of rotation. After interpoles have been correctly connected to an armature they are treated as part of the armature circuit. Fig. 7.3 shows that this rule holds good for either main field or armature current reversal. (See Fig. 7.3)

7.3 SPEED REGULATION OF MOTORS

The nameplate of a motor gives the rated power, volts, current and speed at which it will run for a given length of time without exceeding a certain temperature rise, the speed being taken at the end of a heat run.

In the case of compound and shunt wound motors, the no-load speed will be greater than the full-load speed for constant shunt field setting. The speed regulation is specified as the change in speed between rated load and no-load expressed as a per cent of the speed at rated load.

$$\text{per cent regulation} = \frac{\text{r p m N.L. - r p m.. rated}}{\text{r p m. rated}} \times 100 \quad - 7.1$$

Example 7.1

If the no-load r p m of a 20 kW 1150 r p m rated motor is 1210 r p m, calculate:

(a) the percent regulation!

(b) the motor speed when the output is 12 kW assuming a straight line speed-load variation.

Solution

From equation 7.1

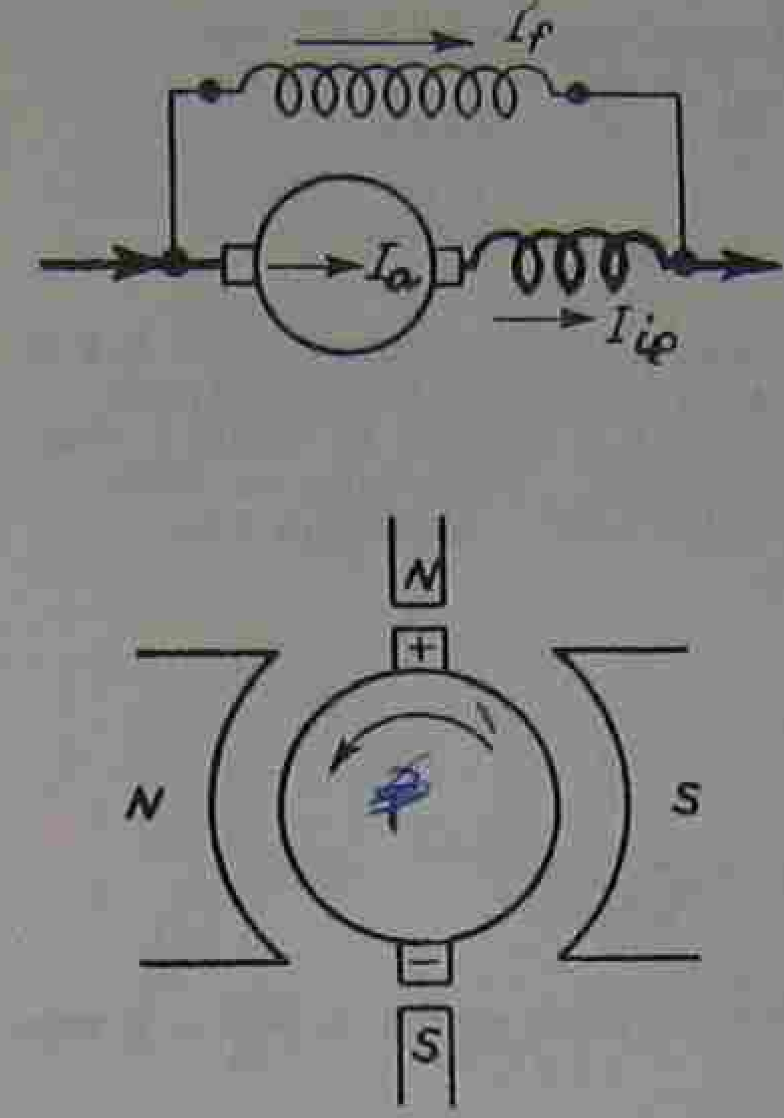
(a) percent regulation $= \dfrac{1210 - 1150}{1150} \times 100 = 5.22\%$

(b) $\text{r p m}_{12} = 1150 + \left(\dfrac{8}{20} \times 60\right) = 1150 + 24$

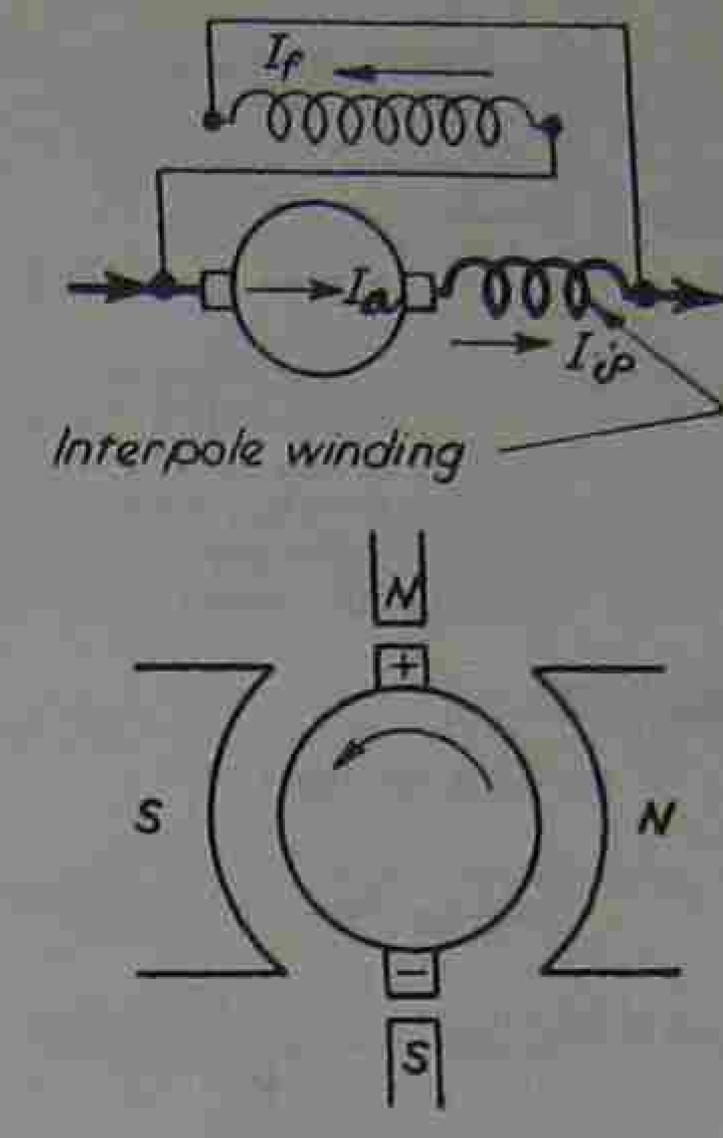
$= 1172$ r p m (by similar triangle).

The fact that a compound wound motor has a series field winding means that the no-load speed is governed by the shunt field flux strength and the load speed by the shunt field plus the series field strength. It, therefore, has a greater regulation than the shunt wound motor since, as it will be shown in a later section,

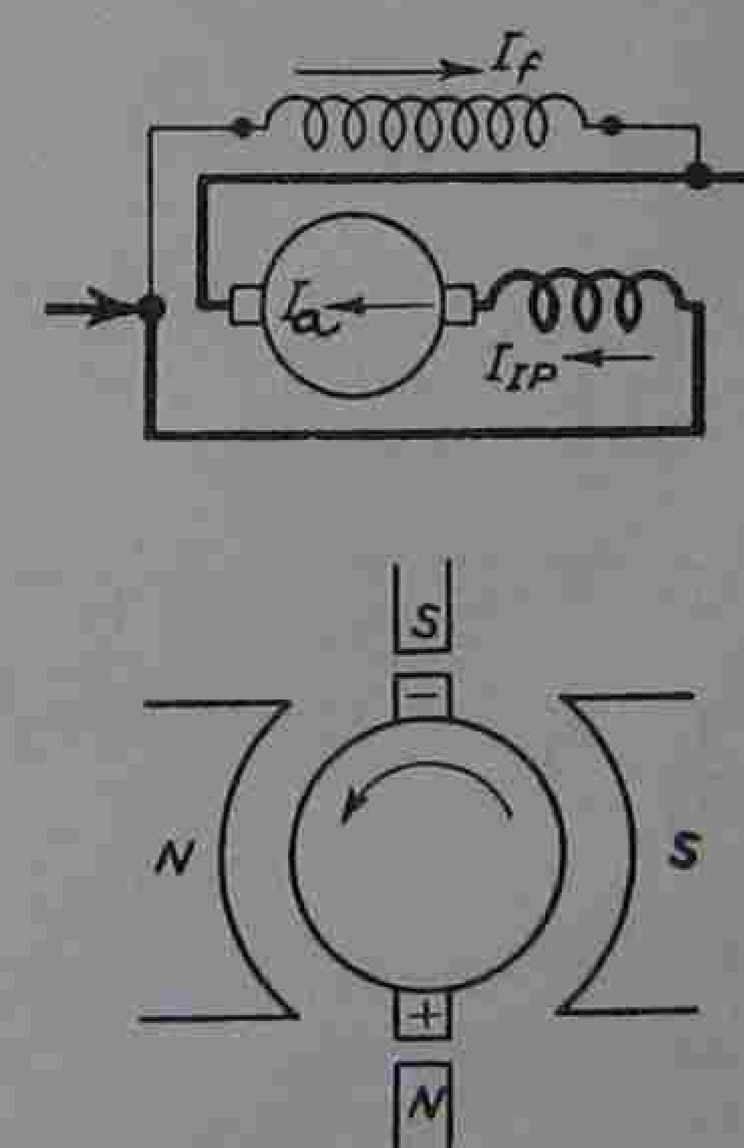
speed $\propto \dfrac{1}{\Phi}$



(a) Original rotation clockwise



(b) Field reversing, counter clockwise



(c) Armature-circuit reversing, counterclockwise

Figure 7.3 - Reversal of Rotation of Machines with Interpoles.

The following example illustrates the effect of the series field on regulation.

Example 7.2

The following are shunt motor data:

230 volts, full-load current 20 amps, speed 1200 r p m No-load current 5 amps, armature circuit resistance 0.3 ohms, brush drop full-load 2 volts, no load 1 volt shunt field resistance 115 ohms. Assuming no loss of flux from no-load to full-load, calculate the percent speed regulation.

Solution

As seen in Unit 4

$$N = \frac{V_t - I_a R_a}{K\varphi} \text{ r p m} \quad \text{where } K = \frac{Z\,P}{60a}$$

No-load; $\text{r p m} \quad \frac{(230-1) - (5-2)\,0.3}{K\varphi} = \frac{228.1}{K\varphi}$ (as I shunt $= \frac{230}{115} = 2$ amps.)

Full-load; $1200 = \frac{(230-2) - (20-2)\,0.3}{K\varphi} = \frac{222.6}{K\varphi}$

$$\therefore \; K\varphi = \frac{222.6}{1200}$$

thus $\text{r p m}_{N.L.} = 1200 \times \frac{228.1}{222.6} = 1230$ r p m

Alternatively,

since $\quad E_g \propto \Phi\, N$

$$\frac{E_{g1}}{\Phi_1\, N_1} = \frac{E_{g2}}{\Phi_2\, N_2}$$

or

$$N_2 = \frac{E_{g2}}{E_{g1}} \times \frac{\Phi_1}{\Phi_2}\, N_1$$

$$= \frac{228.1}{222.6} \times 1200$$

(Since $\Phi_1 = \Phi_2$)

$$= 1230 \text{ r p m}$$

Percent regulation $= \frac{1230 - 1200}{1200} \times 100 = 2.5\%$

If a series field 0.05 ohms is a connected long shunt compound and the torque adjusted to the full-load shunt value, the speed drops to 1075 r p m. and the line current to 18.4 Amperes $I_{SH} = 2$ Amperes. Calculations show a 10% increase in flux over the shunt motor. Determine the percent speed regulation.

$$\text{r p m}_{N.L.} = \frac{\overset{V_t - V_B}{(230-1)} - \overset{I_a R_a}{(5-2)\,0.3} - \overset{I_a R_{SF}}{(5-2)\,0.05}}{K\varphi}$$

$$1075 = \frac{230 - 2 - (0.05 \times 16.4) - (18.4 - 2)\,0.3}{1.1 \times K\varphi}$$

$$\therefore \; K = \frac{222.3}{1075 \times 1.1}$$

thus $\text{r p m}_{N.L.} = 1075 \times \frac{228.1}{222.3} \times 1.1 = 1210$ r p m.

% regulation $= \frac{1210 - 1075}{1075} \times 100 = 12.6\%$

In all the above cases it is assumed that no armature reaction effect is reducing the main field flux on load. In practice armature reaction causes the shunt wound motor to have an extremely flat characteristic and it is possible to obtain a negative regulation; that is, the speed rises with load, the machine being unstable. By fitting a very small series winding the near level characteristic can be obtained with stability. (Arm Resistance VERY LOW)

7.4 EFFECT OF FIELD RESISTANCE ON SPEED AND TORQUE

$$\text{r p m} = \frac{V_t - I_a R_a}{K\,\Phi} \qquad K = \frac{Z\,P}{60a}$$

Weakening the shunt field flux (Φ) by decreasing the shunt amperes can be seen to cause the motor speed to rise. When resistance is added to the shunt field circuit three changes take place, namely:

1. The field flux is reduced and from $E_g = \frac{\Phi\, ZNP}{60a}$, the back e.m.f. is reduced.
2. The armature current increases to balance the equation $E_g = V - I_a R_a$. The increase in armature current more than compensates for the drop in flux and increases the torque ($T = K\Phi\, I_a$) which accelerates the motor.

3. When the speed has increased sufficiently to compensate for the drop in flux, a state of equilibrium is reached with the same load torque as previously, increased power output due to higher speed and increased armature current.

(Since output $= E_g I_a$)

I_a

Example 7.3

Motor particulars 3.75 kW 230 volts, 18 amperes, 1750 r p m.

R_a = 0.3 ohms, brush drop 2 volts on load. Shunt

Calculate:

(a) Full-load torque.

(b) Initial rush of armature current and corresponding momentary maximum torque at the instant the field rheostat resistance is increased to reduce the field flux to 0.96 of the original value.

(c) The final armature current, speed and power assuming the torque is as in (a).

Solution

(a) $$T_a = \frac{60\ P}{2\pi\ N_{FL}} = \frac{60 \times 3750}{6.28 \times 1750} = 20.4 \text{ newton metres}$$

(b) $$\text{speed } N_{FL} = \frac{V_t - \text{brush drop} - I_a R_a}{K\ \Phi_a} = \frac{230 - 2 - 18 \times 0.3}{K\ \Phi_a} = 1750 \text{ r p m}$$

$$K\ \Phi_a = \frac{228 - 5.4}{1750} = 0.1273$$

$$\%\text{ flux reduction} \times K\ \varphi_a = \frac{V_t - \text{brush drop} - I_a R_a}{N}$$

$$0.96 \times 0.1273 = \frac{(230 - 2) - I_{ab} \times 0.3}{1750} = \frac{228 - 0.3\ I_{ab}}{1750}$$

$$228 - 0.3\ I_{ab} = 214\ ; \qquad 0.3\ I_{ab} = 228 - 214$$

$$\therefore I_{ab} = \frac{228 - 214}{0.3} = 46.7 \text{ amperes}$$

$$\frac{\text{Torque}_b}{\text{Torque}_a} = \frac{K'_T\ \Phi_b \times I_{ab}}{K'_T\ \Phi_a \times I} \qquad \boxed{\text{where } K'_T = \frac{.159 \times Z \times P}{a}}$$

$$\text{torque}_b = 20.4 \times \frac{0.96\ \Phi_a}{\Phi_a} \times \frac{46.7}{18}$$

= 52.9 newton metres.

= 50.56

$$\text{the \% current increase} = \frac{46.7 - 18}{18} \times 100 = 160\%$$

$$\text{the \% torque increase} = \frac{37.4 - 15}{15} \times 100 = 150\%$$?

(c) $I_{ac} \times (K\ \Phi)_c = I_{aa} \times K\ \varphi$ = torque

$$I_{ac} = 18 \times \frac{K_5\ \Phi_a}{0.96\ K\ \Phi_a} = 18.75 \text{ amperes}$$

$$\text{r p m}_c = \frac{V_t - \text{brush drop} - I_{ac} R_a}{K\ \Phi a \times 0.96} = \frac{(230 - 2) - 18.75 \times 0.3}{0.96 \times 0.1273}$$

$$= \frac{222.4}{0.1222} = 1820 \text{ r p m.}$$

$$\text{power} = \frac{2\ \pi\ NT}{60}$$

$$= \frac{2 \times 3.14 \times 1820 \times 20.4}{60}$$

= 3870 watts. ? 3888 W

When weakening the field of a motor it is, of course, necessary to go to the magnetisation curve to obtain the field ampere turns needed for the field weakened condition, and in the case of the compound wound motor one deals in the total number of ampere turns required to produce the correct flux for a desired speed, and from the total number of ampere turns the series ampere turns are deducted to give the shunt field ampere turns.

7.5 TORQUE AND SPEED RELATIONS IN SERIES MOTORS

Since torque $= K\ \Phi\ I_a$

and $\Phi \propto I_a$

for a series motor, until saturation of the magnetic circuit starts, then

$$T = K\ I_a^2$$

As saturation commences the flux increase becomes less until at saturation

torque $= K\ \Phi\ I_a$

Figure 7.4 shows a typical curve of torque against load current.

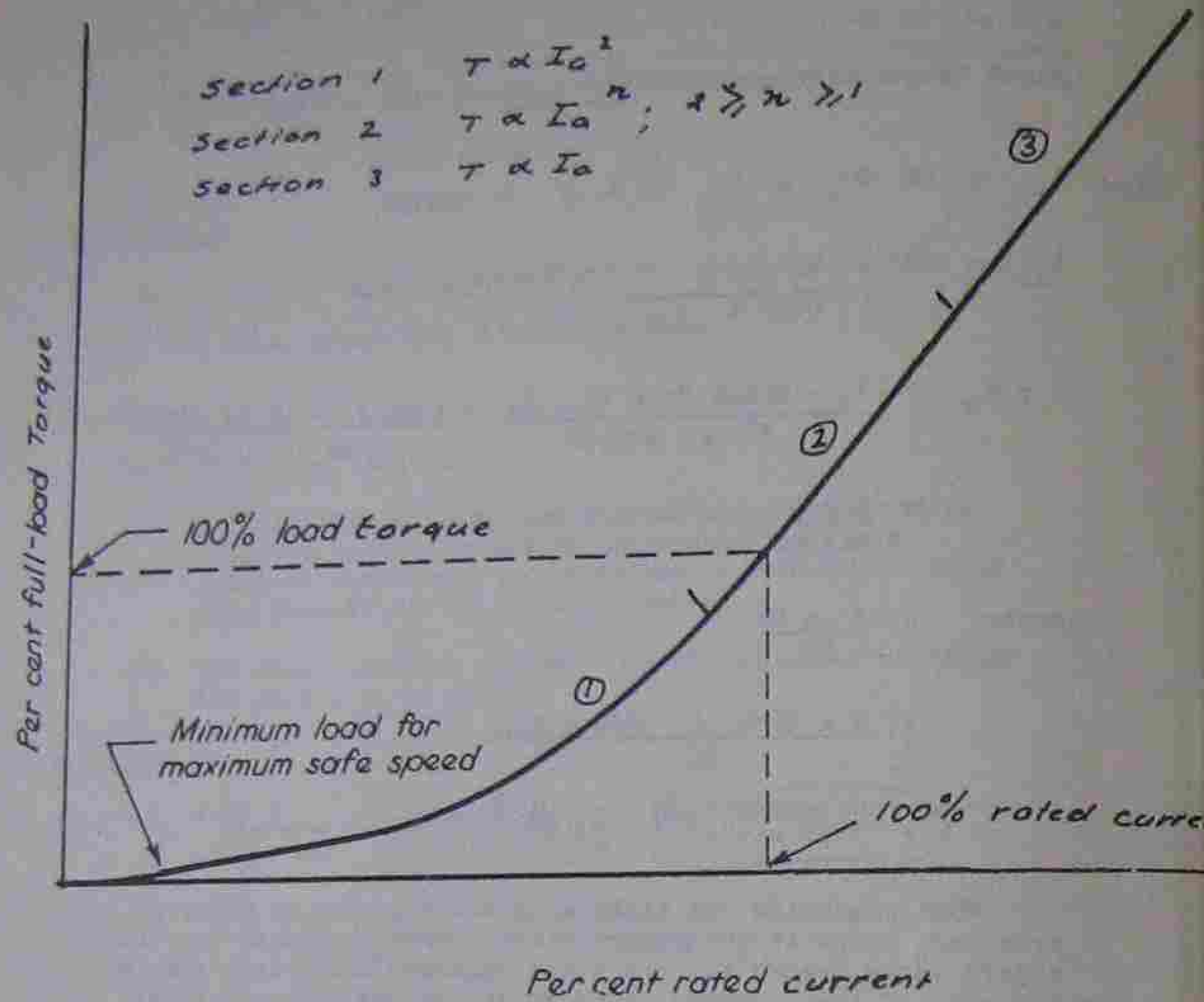


Figure 7.4 - Torque-Current Curve for Series Motor.

Example 7.4

A 15 kW 230 volt series motor takes 80 A. at full load at a speed of 1150 r p m. What torque is developed by the machine when the line current rises to 100 A. assuming at 100 amps the field flux increases 12 percent?

$$\text{torque}_{F.L.} = \frac{60P}{2\,\pi\,N} = \frac{60 \times 15000}{6.28 \times 1150}$$

$$= 125 \text{ newton metres}$$

$$\frac{\text{torque}_{100}}{\text{torque}_{F.L.}} = \frac{K\ \Phi_{100}\ I_{100}}{K\ \Phi_{F.L.} I_{F.L.}}$$

$$\text{torque}_{100} = 125 \times \frac{1.12}{1.0} \times \frac{100}{80}$$

$$= 175 \text{ newton metres.}$$

$$r\ p\ m = \frac{V_t - IR_a}{K\ \Phi} \quad R_a = \text{arm. resis. + series fld. resis. + interpole fld. resis.}$$

Until saturation $\Phi \propto I_a$

In the case of a series motor the effect of the $I_a R_a$ drop on the speed is small in comparison to change in Φ with a change in I_a.

Since Φ varies with I_a, if a very light load were put on the series motor the field flux would be very small and extremely high speeds could be reached. There is, therefore, a minimum load on which the motor can safely run. With heavy overloads the machine becomes saturated gradually and tends to go slower. These characteristics make it an extremely suitable motor for traction and similar duties. In order to determine the speed-load amp and torque-load amp curves, it is necessary to know the magnetisation curve of the machine. This is found by separately exciting the series field and driving it as a generator.

7.6 ARMATURE SPEED CONTROL BY SERIES RESISTANCE

The speed "N" of a d.c. motor is directly proportional to the back e.m.f., E_g and inversely proportional to the flux. For a given load since $E_g = V - I_a R_a$, if V is varied E_g can be taken as nearly proportional to V. Thus speed will vary approximately directly proportionally with supply volts. (Until $I_a R_a$ is large in respect to V.) The simplest manner in which the speed of a d.c. motor is varied is, therefore, to put a resistance in series with the armature circuit. Unfortunately, this can mean a considerable loss of power in the resistor as it has to carry the full armature load current. It is, therefore, only used where operating efficiency is not a major factor and only a small speed range is required. Fig. 7.5 illustrates the connections for this method of speed control.

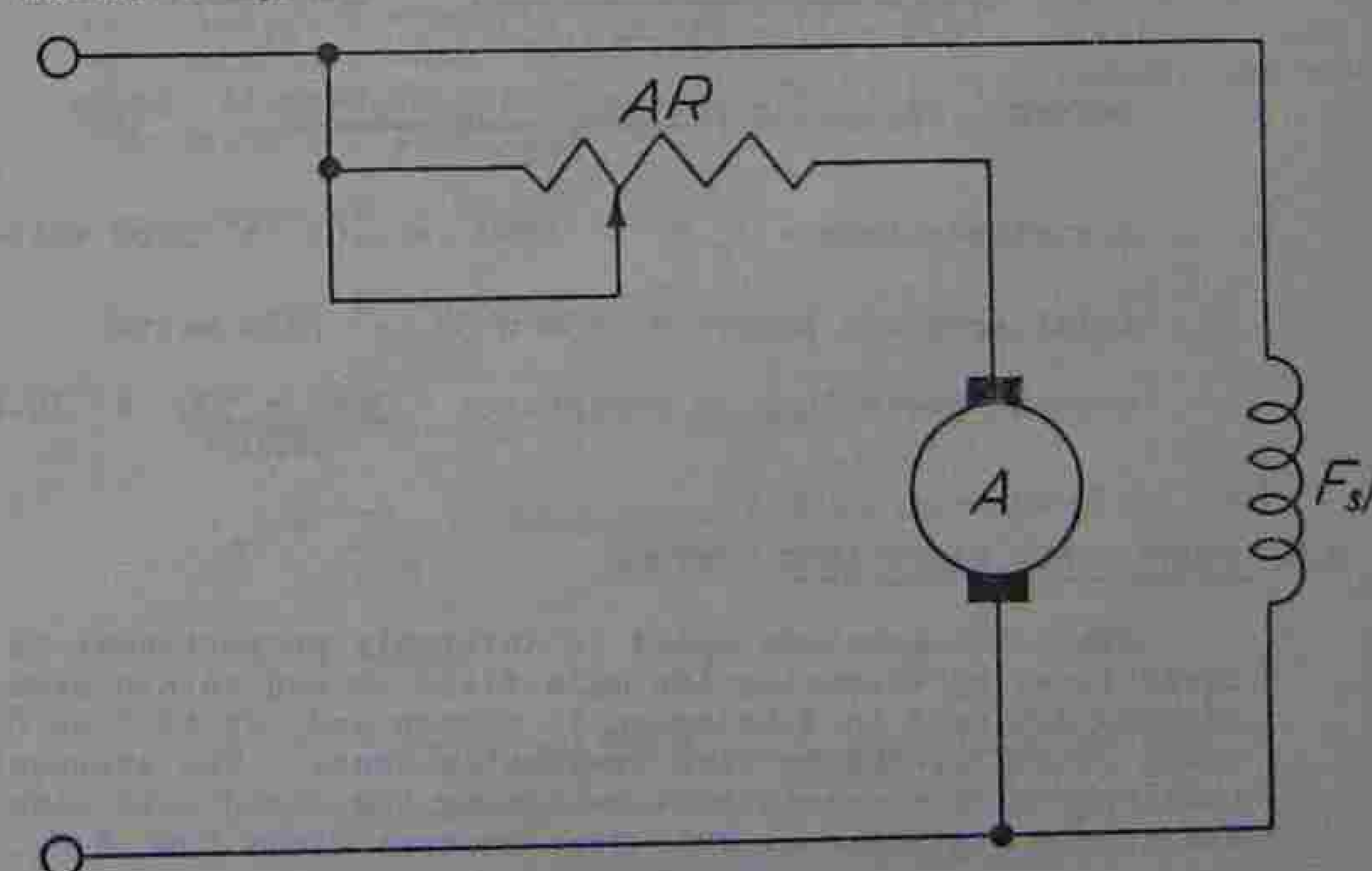


Fig. 7.5 - Armature speed control by series resistance.

Example 7.5 7.46KW.

A ~~7.5~~ kW 230 volt 1750 r p m shunt motor armature resistance 0.35 ohm has a shunt field resistance of 62.2 ohms.

If no load current is 7.7 A., and full load efficiency 86%,

(a) brush drop 3 volts at full load, 1 volt at no-load, calculate the percent regulation.

(b) A 2.65 ohm resistance is placed in series with the armature circuit. Calculate the new speed, percent regulation and percent power loss in series resistance in respect of total power input at full load.

(i) $I_{FL} = \dfrac{\cancel{10} \times 746}{230 \times 0.86} = 37.7$ amperes

$I_{sh} = \dfrac{230}{62.2} = 3.7$ amperes

$I_{aFL} = 37.7 - 3.7 = 34$ amperes

$I_{aNL} = 7.7 - 3.7 = 4.0$ amperes

$N_{NL} = \dfrac{1750 \times (230 - 1) - (4 \times 0.35)}{(230 - 3) - (34 \times 0.35)}$

$= \dfrac{1750 \times 227.6}{215.1} = 1850$ r p m

percent regulation $= \dfrac{(1850 - 1750) \times 100}{1750}$

$= 5.7\%$

(ii) $N_{FL} = \dfrac{1750 \times (230 - 3) - (34 \times 3)}{(230 - 3) - (34 \times 0.35)} = \dfrac{1750 \times 125}{215.1} = 1020$ r p m

percent regulation $= \dfrac{(1850 - 1020) \times 100}{1020} = 81.5\%$

resistance loss $= I_a^2 R = (34)^2 \times 2.65 = 3060$ watts

total armature power $= 230 \times 34 = 7820$ watts

percent power loss in resistance $= \dfrac{3060 \times 100}{7820} = 39.2\%$

7.7 SHUNT FIELD RESISTANCE CONTROL

Since the armature speed is inversely proportional to the field flux, by weakening the main field we can obtain wide speed ranges, 2.5 to 3 to 1 being quite common and, up to 5 or 6 to 1 being possible, though less frequently used. The drawback to this type of control is that weakening the field will also reduce the torque available at full load current since $T \propto \Phi I_a$.

(Fig. 7.6) It is general with this type of control to fit a very light series field winding, commonly called a stabilising winding. This is meant to counteract the field demagnetising effect of the armature reaction which is more pronounced when the field is weak and thus ensure the speed drops as load is increased for a given field setting.

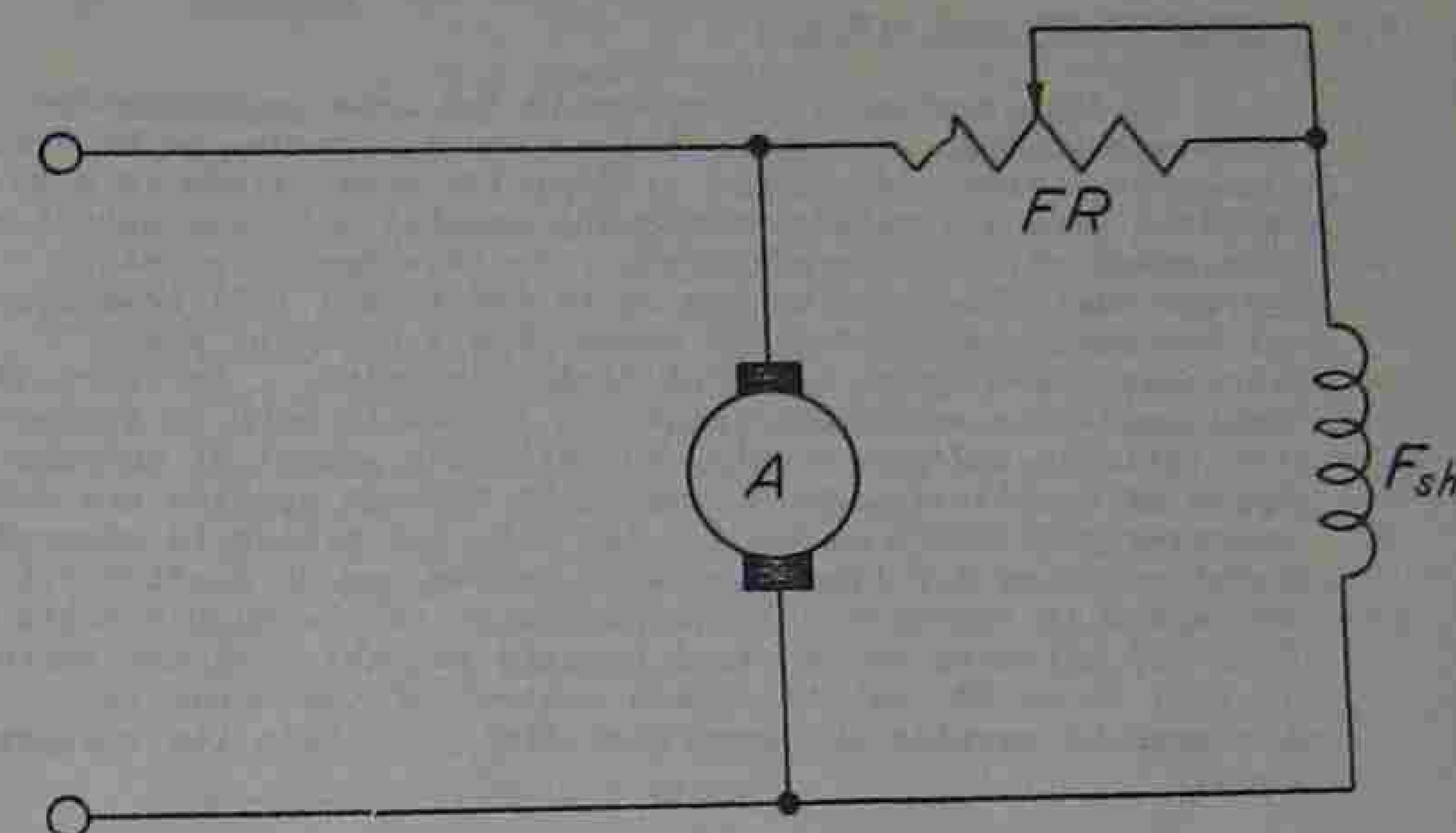


Fig. 7.6 - Field Control of Armature Speed.

Example 7.5

Using the data given in the example given in the previous Section calculate the speed if the field current is reduced to 2.7 amps. by means of a shunt field resistance. Assume this reduces the field flux by 20 percent. The motor torque remains the same. Determine the power loss in field resistance.

Since $T = K \Phi I_a$

$K = \dfrac{0.159 \times P \times Z}{a}$

I_a must increase for constant T

$I_a = \dfrac{34}{0.8} = 42.5$ amps. (flux is reduced to 80%)

$$\text{r p m} = \frac{1750 \times \left[\dfrac{-(230 - 3) - (42.5 \times 0.25)}{0.8 \; K \Phi}\right]}{\left[\dfrac{(230 - 3) - (34 \times 0.35)}{K \Phi}\right]}$$

$$= \frac{1750 \times 212.1}{215.1 \times 0.8} = 2160 \text{ r p m}$$

field resistance $R_{rh} = \frac{230}{2.7} - 62.2 = 23$ ohms

$$I_f^{\,2} R_{rh} = (2.7)^2 \times 23 = 167.5 \text{ watts.}$$

7.8 ARMATURE VOLTAGE CONTROL

In this system the armature is fed from an independent variable voltage supply. By this means, speeds can be varied to cover a very wide range. Since the shunt field is also excited from a separate supply the available torque output of the motor will not be affected. We thus have a constant torque variable speed system up to the normal full load speed of the motor and if desired above this a constant power variable speed range by shunt field weakening. For certain applications a motor generator set system is used to produce the variable voltage supply, but with the advent of various types of rectifiers, static variable voltage systems are used wherever possible nowadays. The M.G. set system is generally referred to as the "Ward Leonard" system, as in Section 7.6 the speed is approximately proportional to the supply volts. Fig. 7.7 illustrates the Ward Leonard variable voltage system. It will be noted that the speed control of the motor is achieved by varying the generator field and thus its voltage ouput.

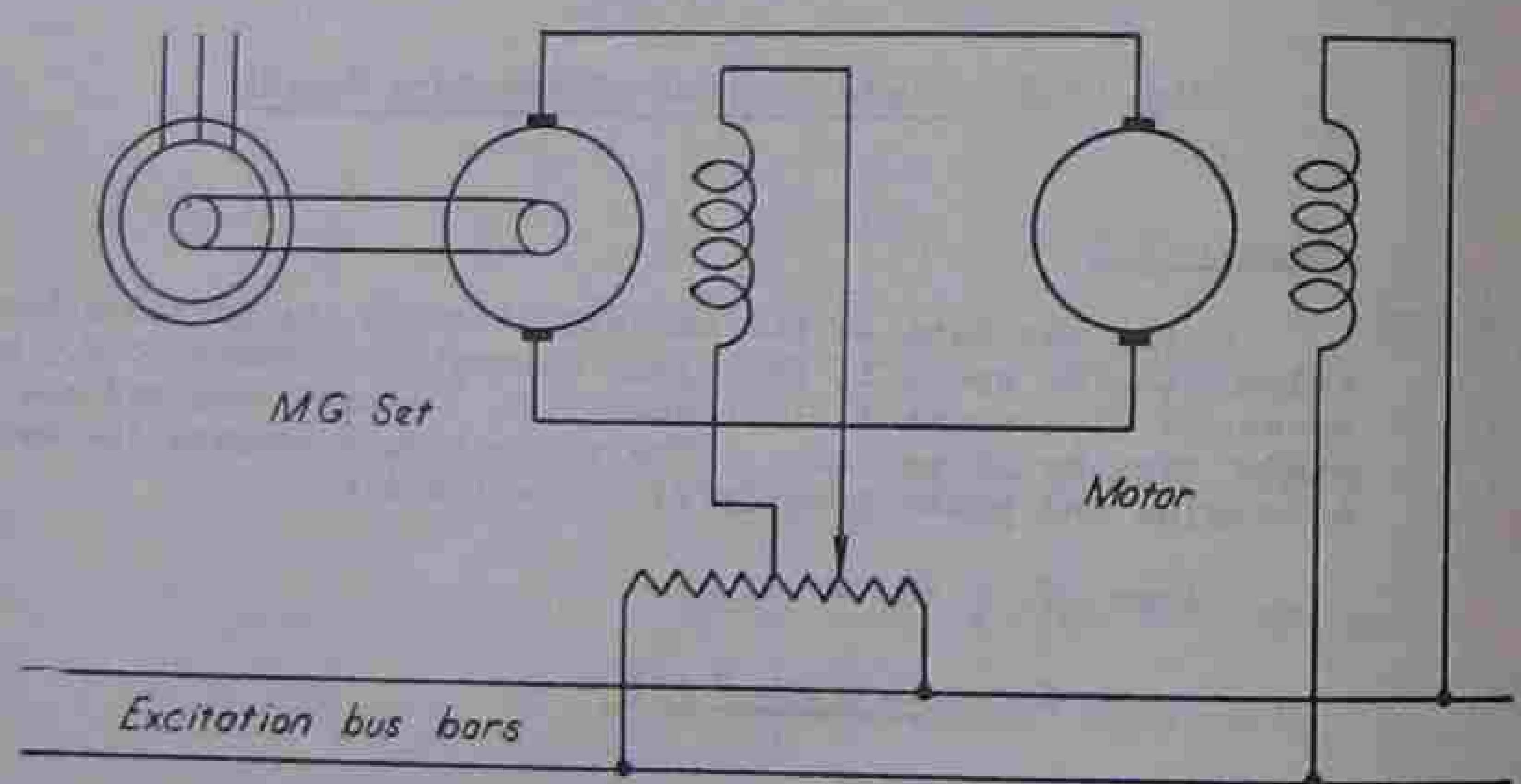


Fig. 7.7 - Ward Leonard Control.

7.9 BRAKING

The braking of an electric motor requires that the kinetic energy of the motor and its load be dissipated in order to bring the motor and load to rest when the electric supply is cut off. There are four common ways of doing this, the first of which is mechanical and the remaining three electrical:

(a) by means of some type of friction brake;
(b) plugging;
(c) dynamic braking;
(d) regenerative braking.

Examining these three last separately:

1. Plugging (See Fig. 7.8)

Using this system for a shunt motor the shunt field circuit is not disturbed and power is supplied to the motor armature in the opposite direction to normal. Thus the motor tries to run in the opposite direction, but just before it does this, that is, when the motor is at rest, the main switch is opened. At the instant the motor is plugged, the supply volts and the motor back e.m.f. are nearly equal (I_a is very small) and as the supply voltage has just been reversed they are additive, instead of being in opposition. The only factor limiting the initial rush of current due to this double voltage is therefore the armature resistance. It is necessary to insert a resistor in circuit to limit this current. The plugging resistance is about 85% greater than the starting resistance. Fig. 7.8 illustrates the plugging circuit for a shunt wound motor.

Ra

Where I_{BR} is the allowable braking current, the plugging resistance is given by:

$$R = \frac{V_t + E_g - V_b}{I_{BR}} - R_a \qquad - 7.2$$

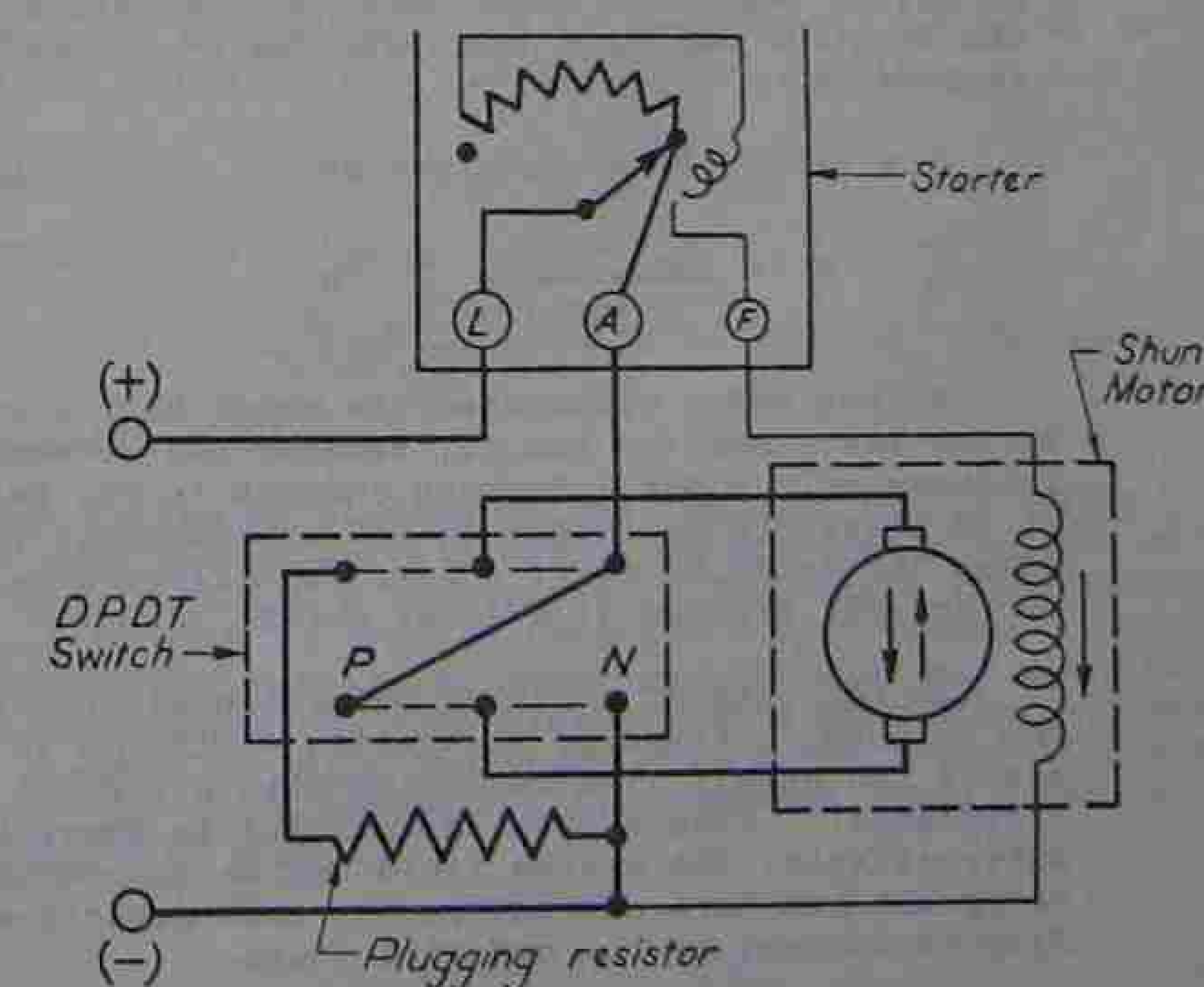


Figure 7.8 - Braking by Plugging.

2. Dynamic Braking

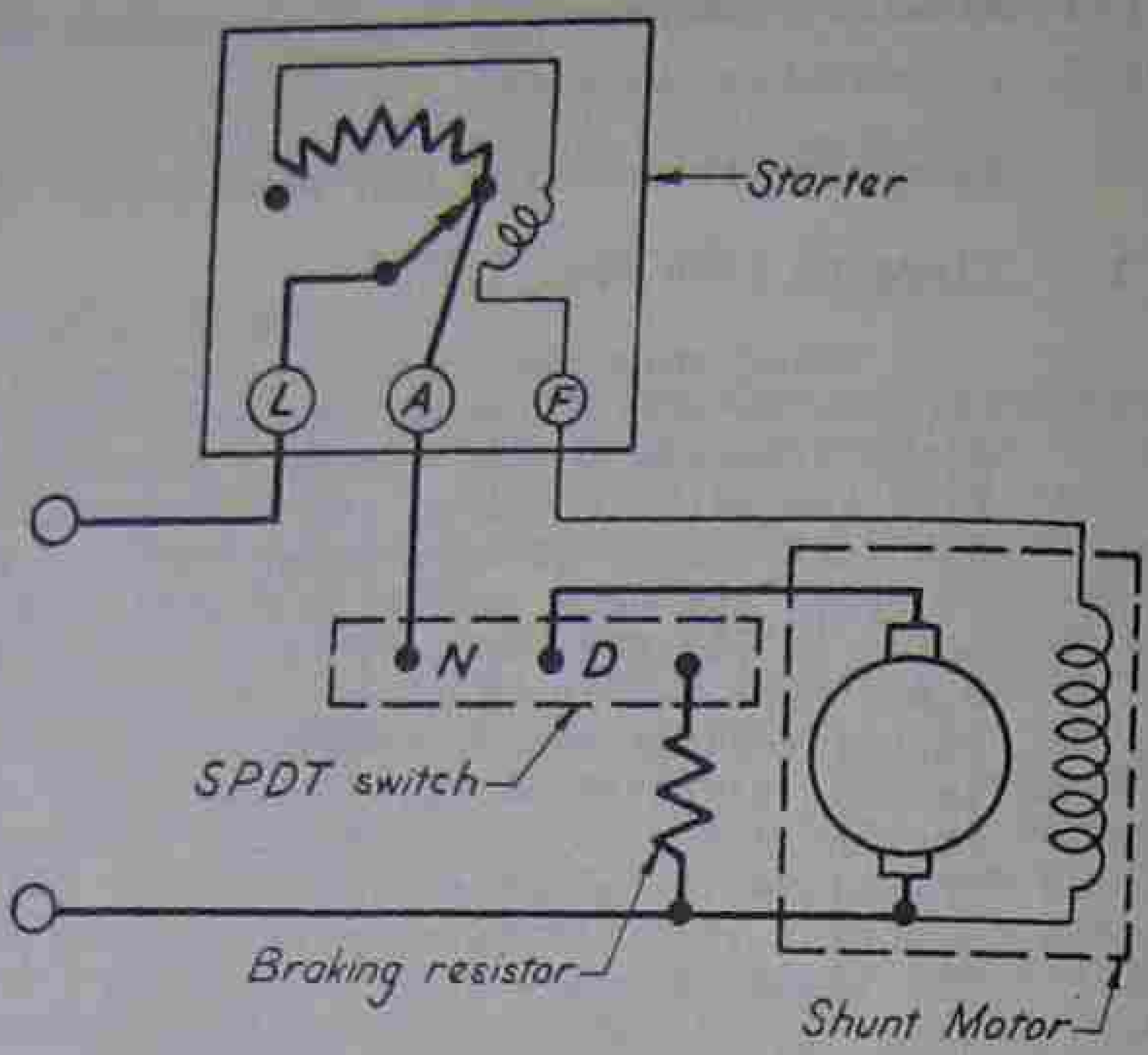


Fig. 7.9 - Dynamic Braking for Shunt Motor.

In this system the motor armature is disconnected from the supply and a resistance of comparatively low value is inserted across its terminals. The motor field is kept energised. The motor will now act as a generator supplying energy to the resistance, until all the kinetic energy in the armature has been dissipated as $I^2 R$ loss in the resistance.

The value of R is given by

$$R = \frac{E_g - V_b}{I_{BR}} - R_a \qquad - \quad 7.3$$

As the motor slows down the motor back e.m.f., which has become the generator e.m.f., reduces and at very low speeds produces only a small braking current in the resistance. It is therefore necessary to fit a mechanical brake to bring the motor to rest. Fig. 7.9 illustrates the circuit for dynamic braking of a shunt motor.

In the case of a series motor two alternative systems can be used. Firstly, the series field can be connected across the supply line in series with a current limiting resistance. This system is wasteful in energy. Alternatively, the series field can be connected to carry the armature current in the same direction as the flow of current in the motoring state.

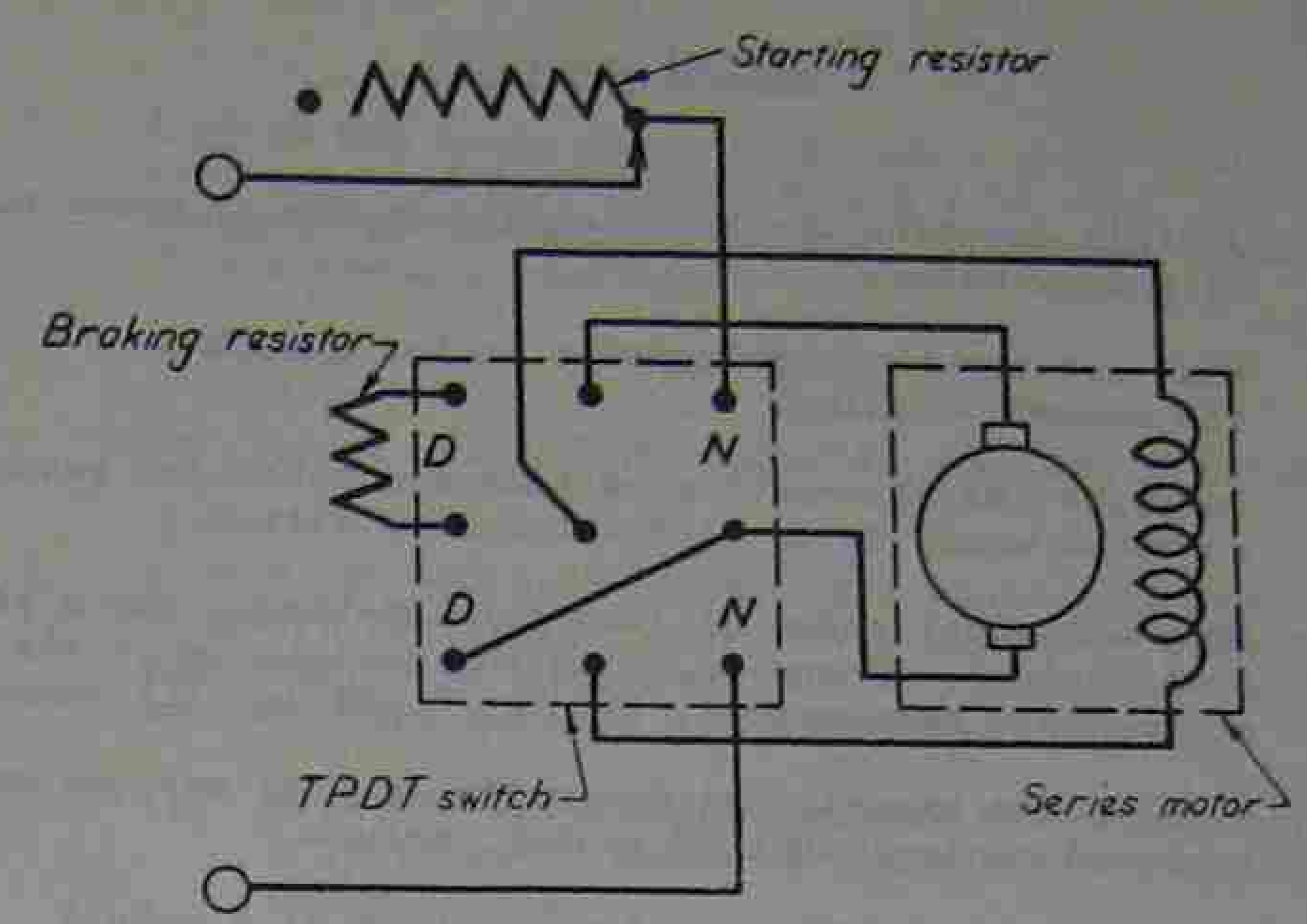


Fig. 7.10 - Dynamic Braking for Series Motor.

In both the above systems the armature power is dissipated in a resistance as for a shunt motor. Fig. 7.10 illustrates the dynamic braking circuit for a series motor.

Example 7.6

The braking current of a 230 volt 58 ampere motor, series wound, total armature and field resistance 0.28 ohms, is to be limited to 1.75 times the motor full load rating. Calculate the value of the dynamic braking resistor. Assume E_g is 94 percent of rated voltage and brush drop 3 volts.

Solution

Using equation 7.3

$$R = \frac{(0.94 \times 230) - 3}{1.75 \times 58} - 0.28 = 1.82 \text{ ohms.}$$

2. Regenerative Braking

This system is similar to the dynamic braking system except that the power is returned to the supply instead of being dissipated in a resistance. This means that the motor back e.m.f. must be greater than the supply volts. This can be achieved by running the motor above its normal speed or overstrengthening the main field. This system is used where it is desired to limit the speed of a motor rather than stop it, that is, for electric traction systems where there are long down hill runs.

REVIEW QUESTIONS

1. Briefly describe the Ward Leonard system of speed control of a d.c. motor.

2. What are the basic principles behind plugging, regenerative and dynamic braking?

3. Describe the alternative methods available for reversing a compound wound motor with interpoles fitted.

4. Draw a typical curve of load current-torque for a series wound motor. Describe, with the assistance of the appropriate expressions, why the curve is this shape.

5. Develop the expressions demonstrating the various methods of speed control applied to d.c. motors.

6. What is meant by the speed regulation of a motor?

7. List the advantages and disadvantages of the different methods of speed control.

8. Explain the various methods whereby d.c. motors can be braked electrically.

A S S I G N M E N T S

UNIT NO. 7

Marks

20 1. The following are the particulars of a motor 18 kW 550 volt, 900 r p m. long shunt compound wound:

R_a = 0.42 ohms, R_{sh} = 183 ohms, R_{se} = 0.06 ohms,

full loading efficiency 87%. Assume brush drop of 4 and 2 volts at full and no-load respectively. If the no-load line current is 5 amperes and the full-load flux is 10% greater than the no-load value, calculate the percent speed regulation.

20 2. A 37.5 kW 550 volt shunt motor has an armature resistance 0.088 ohms and takes an armature current of 74 amperes at full-load at 1350 r p m.

(a) Calculate full-load torque.

(b) Assume a brush drop of 5 volts and determine initial rush of armature current and corresponding momentary maximum torque at the instant the field rheostat resistance is increased in order to reduce the field flux to 97% of its original value.

Marks

15 3. A 15 kW 230 volts shunt motor has a shunt field resistance of 69.7 ohms. With a 1.41 ohm. resistor in the armature circuit the motor takes 74.3 amperes at full-load torque. Calculate:

(a) power taken by the field,

(b) power loss in the inserted armature resistor.

25 4. A 37.5 kW 550 volt 1400 r p m shunt motor has armature resistance 0.26 ohm. shunt field resistance 220 ohms. If full-load efficiency is 89.4% and no-load line current 4 amperes, calculate:

(a) percent regulation, assuming brush drops of 5 volts and 2 volts at full-load and no-load respectively,

(b) the total power loss in the armature circuit at full-load,

(c) the power taken by the field,

(d) power loss in rheostat. ?

20 5. A 45 kW 230 volts shunt motor has armature resistance 0.04 ohms. and field resistance 38.3 ohms. At the moment it is developing rated torque it is plugged. Determine the value of the plugging resistor if the latter limits the instantaneous armature current to 200% of its rated value. Assume brush drop 3 volts and motor efficiency 88.5%. If this motor were to bo dynamically braked, what would be the value of the braking resistor to keep the instantaneous current limit to 200%?

ELECTRICAL MACHINES I

UNIT NO. 8

8.1 Introduction to S.C.R. drives.

8.2 Circuit considerations.

8.3 Type of control.

8.4 Block diagram of system.

8.5 Heating effect of current ripple.

8.6 Derating of machines.

8.7 References.

Review questions.

Assignments.

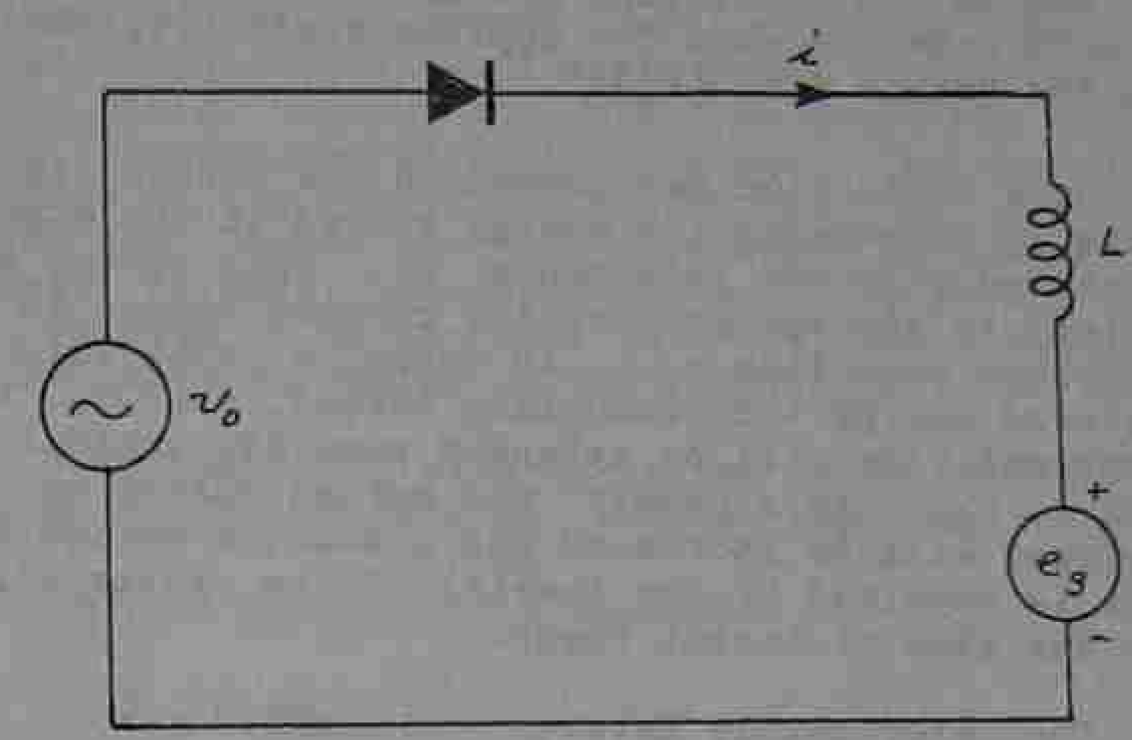
8.1 INTRODUCTION

The purpose of this unit is to cover the subject of S.C.R. control of d.c. motors from the point of view of the rectifier circuits and the effect of the different rectifier configurations on the behaviour of the motor.

Although it is possible to control the speed of a motor by means of a controlled output rectifier applied to the shunt field circuit the discussion in this unit will be limited to speed control by means of varying the average e.m.f. applied to the armature circuit. The method of controlling this e.m.f. will be by means of variation in the conduction period of each cycle as shown in Unit 2.

8.2 CIRCUIT CONSIDERATION

For the purpose of analysing the behaviour of the rectifier circuit it can be assumed that as the basic consideration is with variations in current, rather than with current magnitudes, that the resistance of the armature circuit can be neglected. As a consequence, if the rectifying device is assumed to be a half wave rectifier without any gating control, the equivalent circuit of the rectifier and armature is shown in Figure 8.1.



Note: i) "L" includes inductance of Armature and Transformer.

ii) Field may also be provided from separate variable d.c. source.

Figure 8.1 - Basic equivalent circuit for rectifier and motor.

In the diagram as well as neglecting the resistance of the armature circuit it has been assumed that:

(1) The diode is an ideal device having a negligible forward resistance and an infinite reverse resistance.

(2) The parameter L includes the inductance of the armature as well as the transformer leakage reactance.

(3) The instantaneous value of the back e.m.f., e_g , is due to the rotation of the armature in a constant flux field. Hence any variations in e_g are due only to variations in the speed, N.

A method to completely analyse the circuit should be cognizant of the mechanical aspects of the motor operation such as the inertial energy of the system and the damping coefficient but as this is essentially a qualitative rather than a quantitative analysis this aspect of the machine operation will be neglected apart from the comment that during periods of zero current the output of the machine comes solely from the inertial energy of the rotating members. As a consequence of this there will be a small drop in speed (ΔN) during such periods.

The basic loop law for the circuit of Figure 8.1 can be written as:

$$v_o - L\frac{di}{dt} - e_g = 0 \qquad - \qquad 8.1$$

Due to the rectifying action of the diode current flow can only commence when the applied e.m.f. (v_o) is greater than the back e.m.f. (e_g).

Once conduction has commenced the current increases gradually increasing the energy stored in the magnetic field associated with the inductance L. This energy must be returned to the system when the current begins to fall as v_o becomes less than e_g. In order to allow this discharge of stored energy the conduction period, that is, the duration of current flow will be extended past the instant that v_o is equal to e_g. As a result, the period during which torque is developed will be increased and a shorter period during which power is supplied by the inertia of the system will mean a smoother flow of output power.

The relationship between the applied e.m.f. and the back e.m.f. is shown in Figure 8.2 where, for easier explanation, the generated e.m.f. curve is shown to be constant.

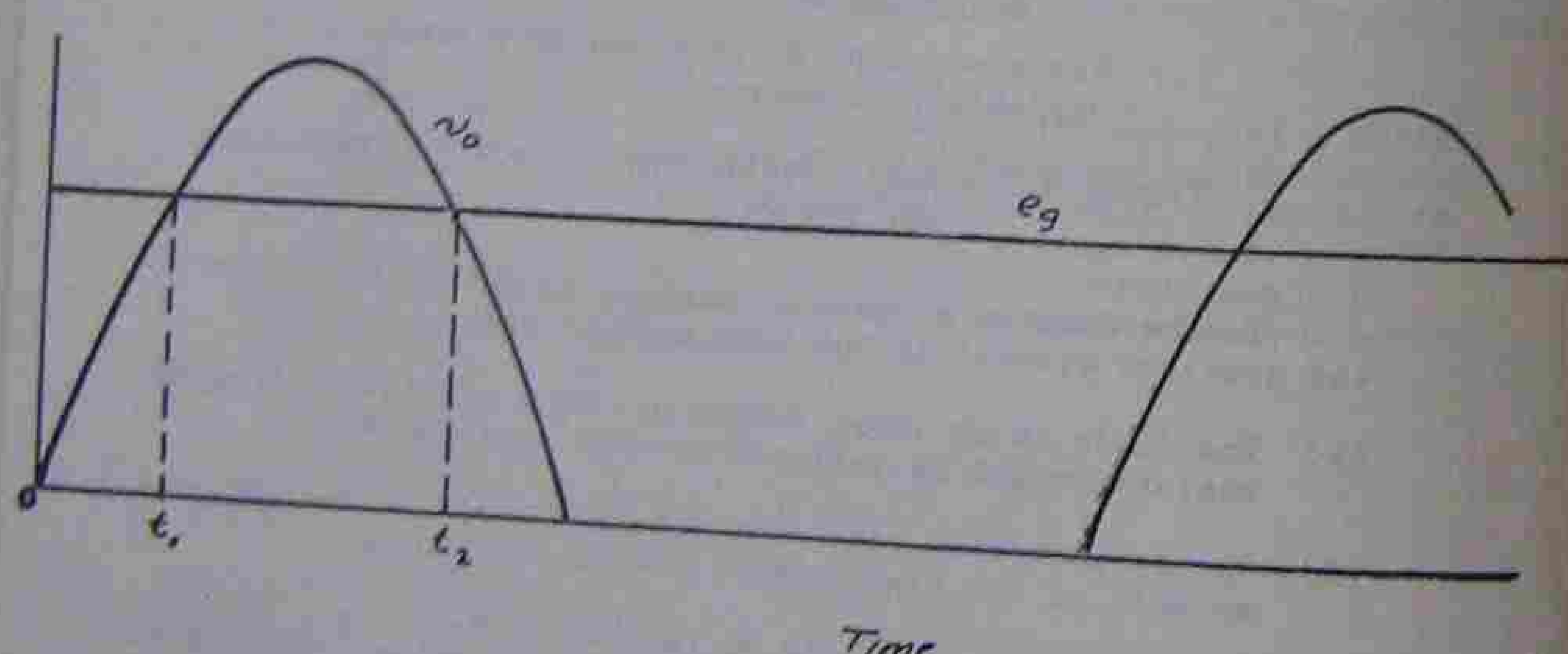


Figure 8.2 - Applied e.m.f. and back e.m.f. curves for circuit of Figure 8.1.

As soon as the point on the cycle is reached where v_o is greater than e_g the conditions of equation 8.1 apply and current will flow in the armature circuit. Due, however, to the inductive nature of the circuit the current will not rise instantaneously but will rise following a complex expression including an exponential and a sinusoidal component. As the current rises the energy stored in the armature field will increase due to the inductance of the armature. The energy thus stored will reach a maximum at the instant the current reaches its maximum value which, due to the delay in the current rise, will occur at approximately the instant v_o falls to a value equal to e_g. If the armature circuit contained resistance only the current would again be zero at this instant (i.e., when v_o is again equal to e_g) but due to the stored energy the current must flow until the collapsing field restores energy to the system. The energy is not restored to the electrical system but, due to the fact that torque can only be produced when there is current in the armature circuit, this prolonging of the period of current flow results in an increase in N with a consequent increase in the inertial energy of the rotating system.

This condition also satisfies equation 8.1 for as the current begins to fall $\frac{di}{dt}$ becomes negative and this accounts for the fact that v_o is less than e_g.

$$v_o - L\frac{di}{dt} - e_g = 0$$

from which

$$v_o - e_g = L\frac{di}{dt}$$

for falling current $\frac{di}{dt}$ is negative

hence $$v_o - e_g = -L\frac{di}{dt}$$

which implies that

$$e_g > v_o$$

When the energy stored in the field is released current will cease to flow and in the absence of current no torque will be developed and energy will be passed on to the load by the system giving up inertial energy. As a consequence the speed of the system will fall. This rise and fall in speed means that the speed curve can no longer be represented by a straight line relationship.

The foregoing discussion illustrates that inductance in the armature circuit can be beneficial in that the presence of inductance lengthens the conduction period and reduces fluctuation both in armature speed and developed torque. It should also be recalled that the presence of this inductance in the actual armature makes good commutation harder to achieve and the inductance should be either due to a line inductor or the leakage reactance of the armature. The relationship between the applied e.m.f., the generated e.m.f., the speed, the current in the armature and time are shown in Figure 8.3.

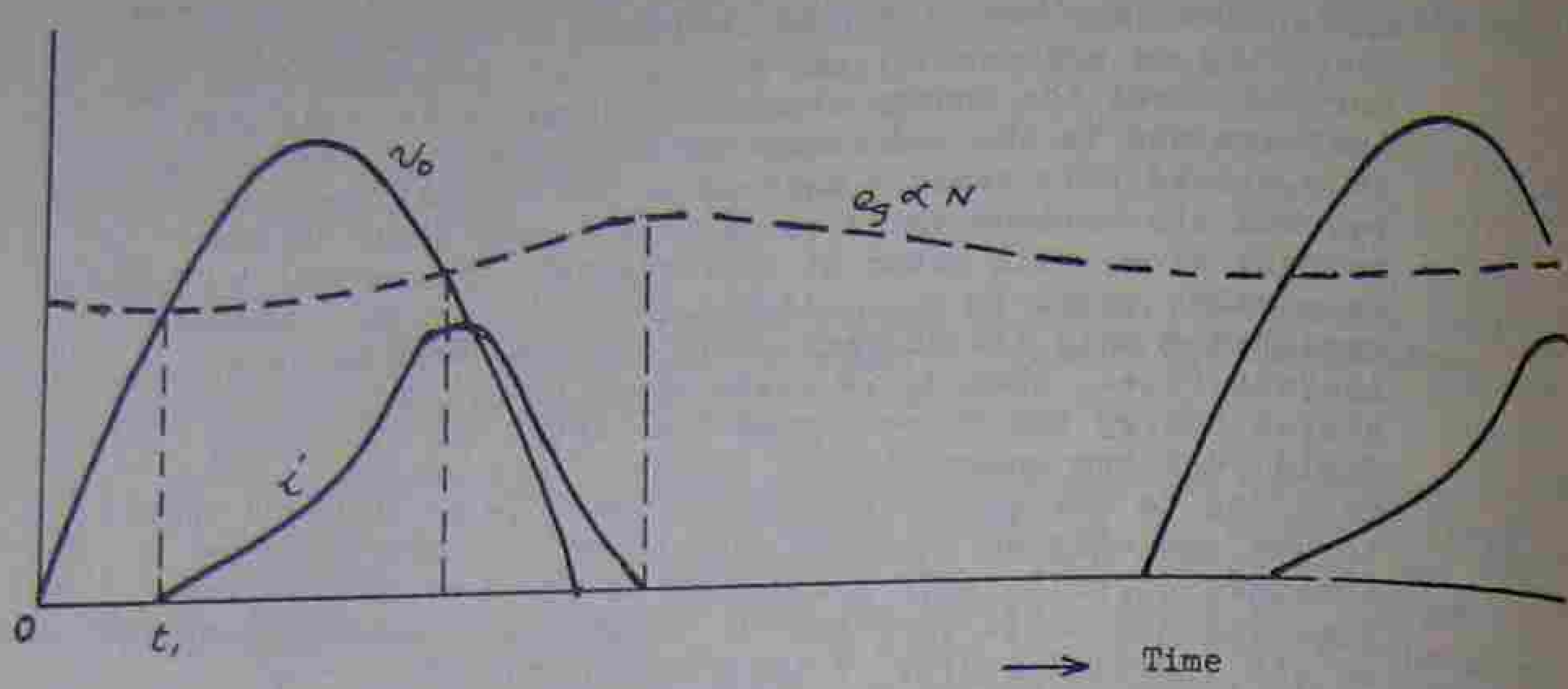


Figure 8.3 - Current voltage wave for half wave rectifier.

This analysis applies to a circuit in which no control is possible except by variation in the maximum value of v_o. This would involve a variable transformer with a large number of tappings to obtain a smooth variation. As stated earlier, the control of the average e.m.f. is increased by varying the point in the conduction cycle that the rectifier begins to conduct.

8.3 TYPE OF CONTROL

The rectifying device used in d.c. controlled rectifier drives is termed the silicon controlled rectifier (S.C.R.) or thyristor which is a three terminal device, the solid state equivalent of the gas triode. The idealized voltage current relationship for such a device is shown in Figure 8.4

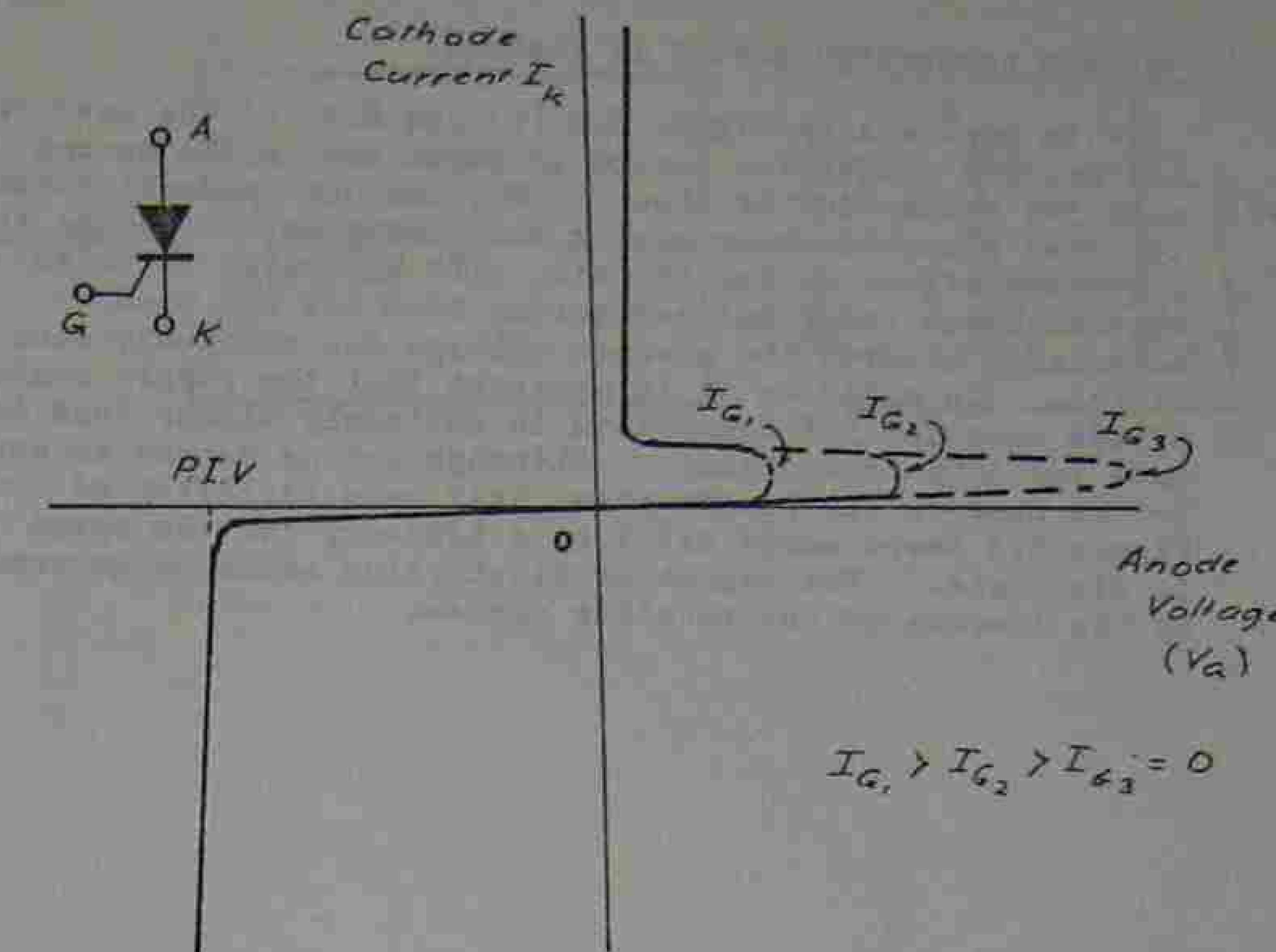


Figure 8.4 - Idealized S.C.R. characteristics.

As shown in the diagram the S.C.R. will not begin to conduct until a very high e.m.f. is applied between the anode and the cathode. Under these conditions the current in the gate circuit is zero. If the current in the gate circuit is increased the difference in potential across the device necessary to initiate conduction is reduced. Hence, if the gate current is of sufficient magnitude the S.C.R. will behave in the same manner as the conventional two terminal diode. The control problem is then one of adjusting the gate current so that conduction commences at the appropriate point in the cycle. The methods of achieving this control such as phase shift control and relaxation oscillators together with their practical refinements will not be covered in this course, the consideration of gating methods being confined to the statement that such methods do exist and are used in commercial control units.

8.4 BLOCK DIAGRAM OF SYSTEM

It is obvious that a single-phase half wave rectifier as shown in Figure 8.1 even under the most favourable operating and circuit conditions would produce distinct fluctuations both in the speed and armature current of a motor supplied from the rectifier. As a consequence a more practical control system would either be a single-phase full wave, a three-phase semi-converter or ideally a three-phase full converter. The latter type of rectifier gives 6 pulses per cycle and, as a consequence, produces an output with an inherently lower ripple voltage. The basic circuit diagram and the resultant output voltage waveform for maximum conduction conditions are shown in Figure 8.5 (a). Note that no gating circuit is shown and that the maximum conduction period is assumed.

In one case in Figure 8.5 (b) and 8.5 (c) the wave for the maximum conduction period is shown and in the second case 50% conduction is shown. For the 50% conduction case the rise trace is shown rising vertically which ignores the inductive effect of the circuit. It will also be noted that although the total conduction time has not been materially altered the average voltage has obviously been varied. In addition it is apparent that the ripple content of the wave for 50% conduction is obviously higher than the wave for 100% conduction. Although not as marked as would be the case in the single-phase half wave rectifier of Figure 8.3 there would still be a tendency for the speed to fluctuate. The degree of fluctuation would be governed by the inertia of the rotating system.

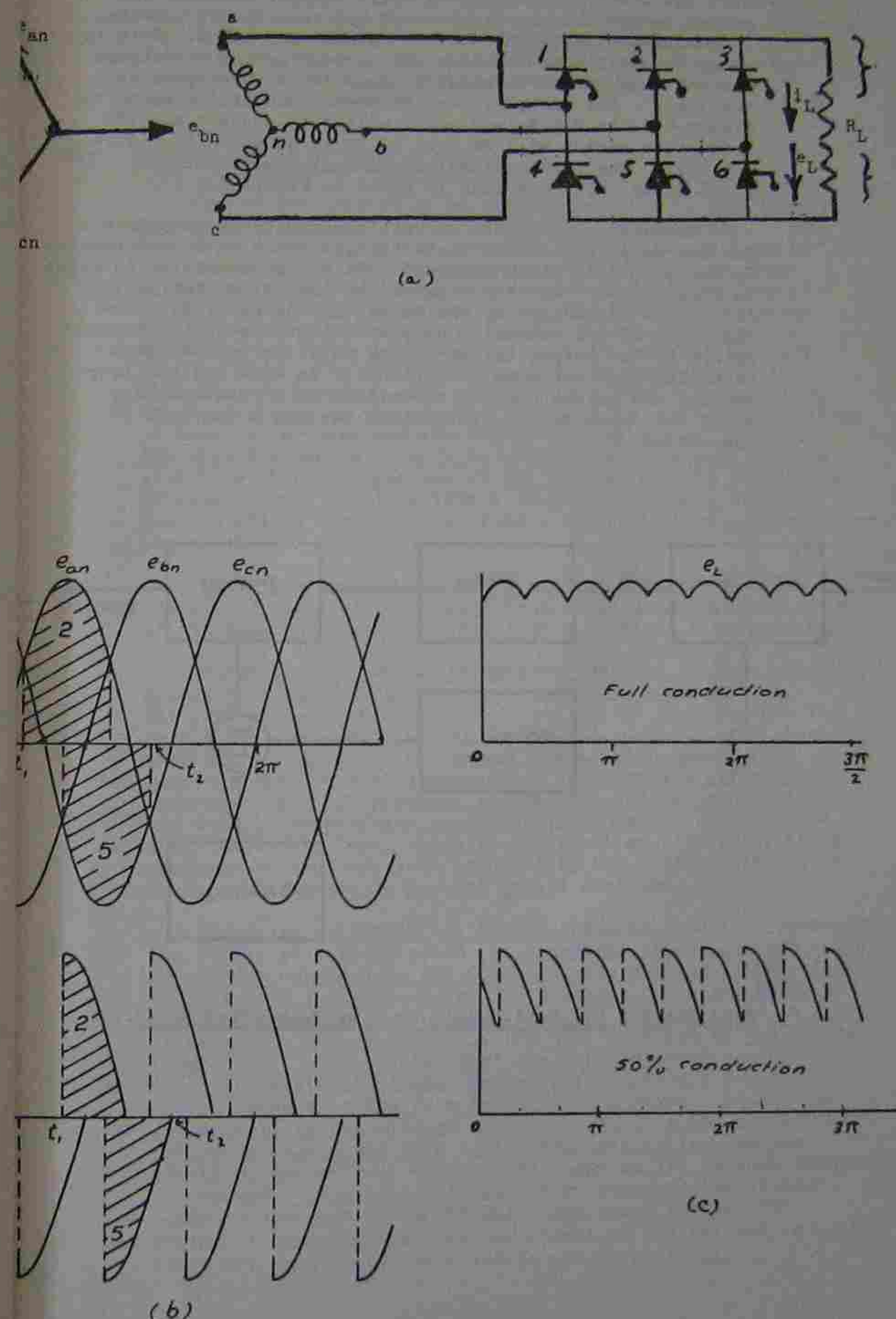


Figure 8.5 - Three-Phase Full Converter.

The S.C.R.'s in group "a" will conduct whenever the gating conditions are correct and the transformer terminal is positive with respect to the cathode and the S.C.R.'s in group "b" will conduct under the correct gating conditions and the transformer terminal is negative with respect to the load. For example, for the phasor conditions shown in Figure 8.5 the S.C.R.'s 2 and 5 will both conduct and the conduction period will be from t_1 to t_2 as shown in Figure 8.5 (b). The resultant voltage output wave for the three-phase full converter is shown in Figure 8.5 (c).

Without resorting to circuit components it is possible for the behaviour of the converter and motor to be represented in the form of a block diagram. This representation is shown in Figure 8.6. In this diagram the control of the gating circuit is influenced by the output of the machine. The magnitude of the converter output voltage can be influenced either by the output torque of the motor or by the speed depending upon the mode of control. In this way the loop through the gating control circuit forms a feedback loop since the input to the system then becomes a function of the output.

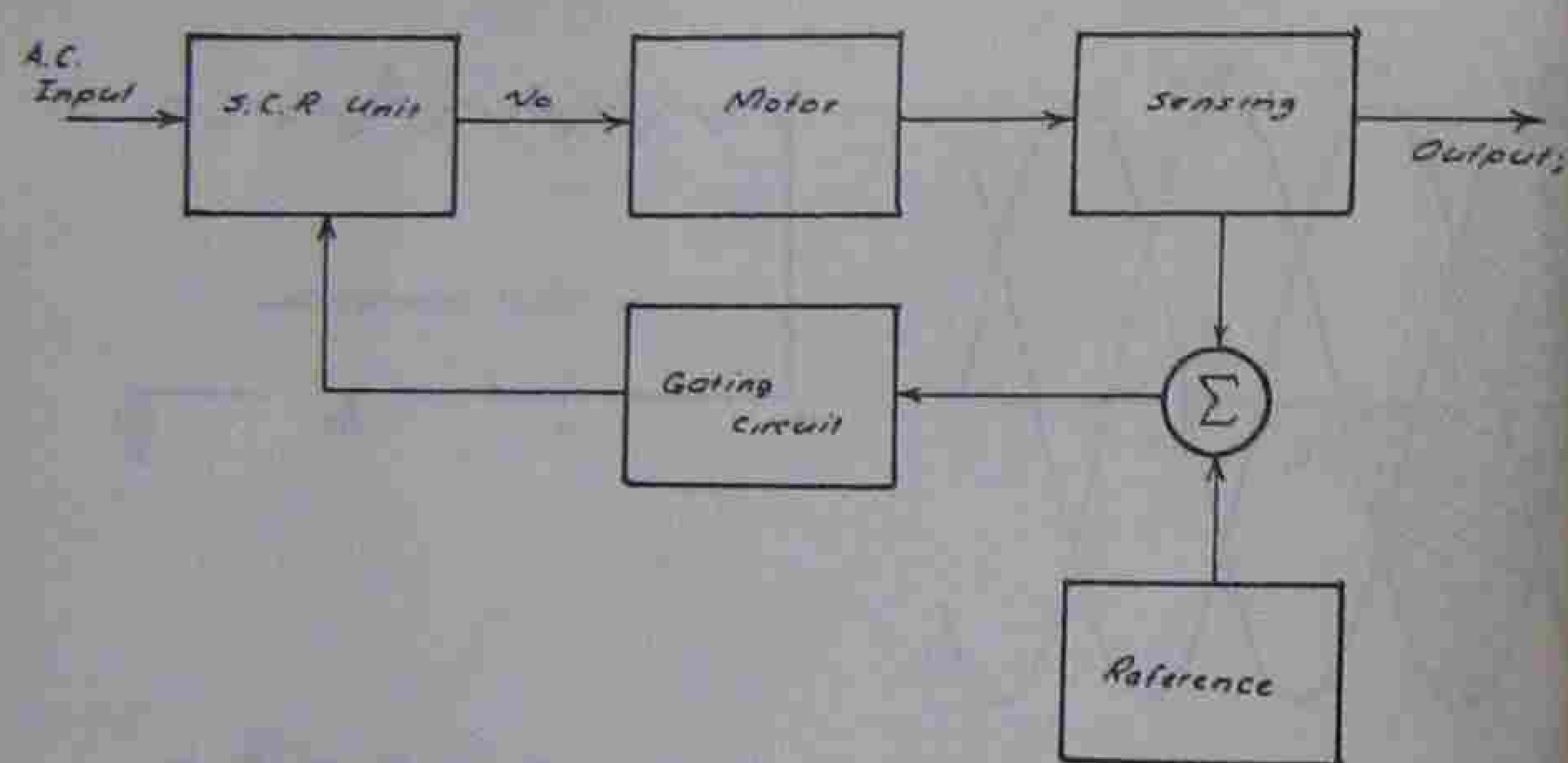


Figure 8.6 - Block Diagram of S.C.R. Controlled Motor.

The circuitry of the block shown for sensing, reference and control will depend on the degree of sophistication required and the tolerance permitted in the output parameter. Such circuitry belongs to a Motor Control oriented subject rather than a subject where the primary emphasis is on machine construction and behaviour.

8.5 HEATING EFFECT OF CURRENT RIPPLE

One of the effects of an electric current is that heat is produced whenever current is present in the conductor. This heat is a problem from the point of view of contributing to the losses of the machine and, also, the temperature rise associated with this heat can cause damage to insulation if permitted to become excessive. As this heat is a function of the load on the machine it is the heating effect of the current which helps to place limits on the effective output of a machine. In practice, a machine is generally fitted with some device that will enhance the normal cooling action of the rotation of the armature and in this way the dissipation of heat is improved and the permitted load can be increased. For this discussion it will be assumed that the only concern will be with the comparison of the heating effects of the same average current supplied from a steady d.c. source and the heating effect when supplied from a silicon controlled rectifier power unit.

There are two reasons why a machine supplied from a controlled rectifier unit will tend to produce more heat than if the same machine was driven from a straight d.c. supply.

Firstly, the current variations inherent in this type of supply produce fluctuations in the flux in the armature. The flux variations so produced fulfill the requirements for an e.m.f. to be produced and this e.m.f. causes circulating currents in the tooth section of the armature where the fluctuations are most pronounced. The overall effect is to increase the heat produced in the iron and to contribute to the overall temperature rise in the machine. It should be noted that this loss, which is in effect an iron loss, is in addition to the iron loss produced by the normal rotation of the armature which is present in all machines.

Secondly, although the torque produced, and as a consequence the output at a given speed, is dependent on the average value of current supplied to the armature, the heating effect of this current is a function of the R.M.S. value. As the R.M.S. value of the current is higher than the average value it follows as a consequence that the heat produced by the current is greater. This latter fact leads to the conclusion that the heating effect of the current derived from a S.C.R. source depends on the form factor of the wave. In general, as controlling the average current by varying the conduction period increases the form factor the comparitive heating effect of the current tends to increase as the average current is reduced. The ratio of the R.M.S. value to the average tends to decrease below the half load point but in this region of operation the heating effect of the current is well below the full load effect and is not so critical.

8.6 DERATING OF MACHINES

As a result of the increased heating effect in an armature that is supplied from a silicon controlled rectifier power supply some method must be found to compensate for the increased heat involved. The method may be either one of the following alternatives or a combination of both depending on the load cycle and the type of insulation on the machine.

The two alternatives are:

Firstly, to ignore the effects of the increased losses and to operate the machine at its rated load regardless of the type of supply. This alternative is only viable if the designer or user has ascertained the exact extent of the temperature rise caused by the increase in the heating effect of the current due to the S.C.R. drive and determined that the rise so caused will not exceed the safe operating limits of the insulation. The dangers that are associated with excessive temperature rise in insulation may be temporary or transient, such as decrease in insulation resistance, or the degradation of the insulation due to carbonisation or oxidation with some materials or deformation in plastic based materials.

Secondly, if an evaluation of the effects of the rise in temperature indicates that it would be unwise to retain the original rating of the machine the rating of the machine must be reduced to keep the actual temperature rise within the accepted limits for the insulation material concerned.

The extent of the actual derating will depend on both the type of rectifier unit and the nature of the load, in this latter case with particular emphasis on the variations in speed.

As stated in Section 8.3, where the heating effect of a current derived from a silicon controlled rectifier power supply was discussed, the extra heat produced in the machine winding by such a supply is dependent on the form factor of the current. It can be shown that the form factor tends to increase as the wave moves further from the d.c. and, hence, in rectifiers where the ripple is pronounced, the form factor increases with an increase in the heat produced by the winding. Therefore, the closer the output of a power supply approximates d.c. conditions the lesser will be the degree of derating. Consequently, when a machine is operated from a 3-phase semi-converter the degree of derating would be less marked than if the machine was operated in a single-phase half wave rectifier unit.

With regards to the operating speed of the machine at maximum speed the cooling system is operating at the peak of its effectiveness. As the speed of the armature rotation is reduced the output of the cooling system drops rapidly and tends to vary at the cube of the speed. Hence, as the heating effect varies at the square of the current the relative cooling effect of the fan tends to fall as the speed decreases. As a result, if the machine operates for some time at a reduced speed the degree of derating will be more marked. At speeds below about one half of the rated speed the decrease of the effectiveness of the cooling system is less marked due to the fact that operation at lower speeds is generally associated with a reduced current.

As a consequence, the degree of derating must be related to both the type of service and the heat resistance capabilities of the insulation.

8.7 REFERENCES

The foregoing discussion has only been a resume of some of the factors that must be considered when the operation of machines on S.C.R. drives is being discussed. For further reading on this subject the student is referred to a paper by V.E. Vrana entitled "D.C. Motor Considerations" presented to the "Seminar on Feedback Control and Thyristor Drives" and which was published in the Westinghouse Engineer in July, 1967.

In this paper the emphasis is towards the motor design and considerations. For a more detailed analysis of the mathematics of the rectifier output the student is referred to the text "Electrical Machinery" by Fitzgerald, Kingsley and Kusko, and particularly to the third edition published in 1971 by McGraw-Hill. The mathematical treatment is advanced but not beyond the comprehension of certificate students.

REVIEW QUESTIONS

1. Write down the loop law equation for the circuit of Figure 8.1 and give the significance of the various terms.

2. In the expression of Question 1 which terms can be neglected when considering:

   (i) current waves with a high ripple content,

   (ii) current waves with negligible ripple?

3. Explain why conduction can continue when e_g is greater than v_o.

4. The presence of inductance in the circuit of Figure 8.1 improves the character of the wave form. Why should this inductance be external to the armature windings? What are the various methods of introducing this inductance in the circuit?

5. Why is the torque produced by the conductors pulsating in nature and why is the net output torque relatively constant? What part does the inertia of the system play in this latter respect?

6. What is the most common rectifying device now used in controlled rectifier drives?

7. Name the various types of rectifier configurations.

8. What factors increase the heating effect of the current in machines driven from S.C.R. power supplies?

A S S I G N M E N T S

As the treatment of the material in this unit has been essentially qualitative in nature the work to be forwarded will be in the form of a revision of the work done in earlier assignments.

Marks

15 1. A 4-pole simplex wave wound armature has 47 slots and each coil is made up of four elements.

Determine:

(a) The pole pitch.

(b) A suitable coil span.

(c) The commutator pitch stating whether the winding is progressive or retrogressive.

20 2. A shunt generator has an external characteristic defined by the following relationship:

V_t	280	274	268	262	254	242	228	212	196
I_L	0	5	10	15	20	25	30	35	40

For the generator determine:

(a) The terminal voltage when supplying a load with a total resistance of 9.6 ohms.

(b) The regulation assuming that condition (a) is for full load.

(c) Load resistance when the current is one half of the full load current.

(d) Explain why one half of the full load current does not give one half of the full load power.

20 3. A d.c. generator has an armature wound with 478 conductors and the winding is 4-pole lap. If the machine is considered to be separately excited and when running at speed of 1000 r.p.m. in a field of 0.04 webers produces a current of 100 amperes in the load. Assuming the rotational losses to be 600 watts and the armature resistance to be 0.1 ohms, determine:

(a) e.m.f. generated in armature.

(b) Terminal voltage.

(c) Load resistance.

(d) Power input to generator.

(e) Torque applied by prime mover.

Marks

30 4. The open circuit curve of a self excited shunt generator is defined by:

E_g	10	50	91	137	186	218	235	257	263
I_f	0	.4	.8	1.2	1.6	2.0	2.4	2.8	3.0

(a) If the effects of armature reaction are neglected, determine the value of the field rheostat resistance to give a terminal voltage of 200 volts at full load if the full load armature drop is 30 volts and the field winding resistance is ~~100~~ 80 ohms.

(b) If, for the field resistance chosen, the actual terminal voltage due to armature reaction was 180 volts, determine the new value of field rheostat resistance to compensate for the decrease in flux produced by the armature reaction.

15 5. The input to a d.c. motor on no-load was 75 amperes at 240 volts and the shunt field circuit resistance was measured as being 96 ohms. Neglecting the resistance of the armature determine the rotational losses.

ELECTRICAL MACHINES I

UNIT NO. 9

9.1 Introduction.

9.2 Torque speed relationship.

9.3 Torque speed relationship for various loads.

9.4 Matching motors to drives.

Review Questions.

Assignments.

9.1 INTRODUCTION

In Unit 7 the relationships between torque and armature current and speed and armature current for various machines were developed for series shunt and compound machines. These curves provide useful means of comparing the operation of different motors for the same armature current but are of little use when comparing the performance of a machine against a given load. The purpose of this unit is to develop a relationship between developed torque and armature speed for the shunt, series and compound machines as a means of selecting a motor for a given task.

The relationship between torque and speed is chosen as the most workable criteria for choosing motors for specific loads since as shown in Unit 1, the relationship between the same two quantities can be used to define the behaviour of a load.

The characteristics of both the motor and the load can be plotted and compared and using the techniques discussed in Unit 1, the accelerating time for a given set of conditions can be determined.

9.2 TORQUE SPEED RELATIONSHIPS

The torque speed curves for a motor in a practical situation would be best obtained by test as this would be the best indicator of the motor performance and behaviour. In the following discussion, as a guide to various techniques, a mathematical relationship between the torque and load for the various types of machines will be derived by making the following assumptions:

(1) In shunt machines the field current will be neglected. Thus the following relationship will hold:

9.1

(2) The effects of saturation will be neglected. As a consequence of this assumption it can be said that

(a) for shunt machines

$$\Phi \propto V_t \qquad - \quad 9.2$$

(b) for series machines

$$\Phi \propto I_a \qquad - \quad 9.3$$

(3) The frictional torque due to windage and friction, which is a function of the speed, will be neglected.

A. <u>Shunt Machines</u>

For a given machine, in terms of the above assumptions

$$T = K_t \ \Phi \ I_a \qquad - \quad 9.4$$

where $$K_t = \frac{PZ}{2\pi a} \qquad - \quad 9.5$$

also $$E_g = V_t - I_a R_a$$

from which

$$I_a = \frac{V_t - E_g}{R_a} \qquad - \quad 9.6$$

but $$E_g = K_e \ \Phi \ N \qquad - \quad 9.7$$

where $$K_e = \frac{PZ}{60_a} \qquad - \quad 9.8$$

$$\therefore \quad I_a = \frac{V_t - K_e \ \Phi \ N}{R_a} \qquad - \quad 9.9$$

Substituting this value (equation 9.9) for I_a in equation 9.4 will give:

$$T = K_t \ \Phi \left[\frac{V_t - K_e \ \Phi \ N}{R_a} \right]$$

$$= \frac{K_t \ \Phi \ V_t}{R_a} - \frac{K_e \ K_t \ \Phi^2}{R_a} \cdot N \qquad - \quad 9.10$$

Thus the torque of a shunt machine varies inversely as the speed. The relationship between the torque and the speed is given in Figure 9.1

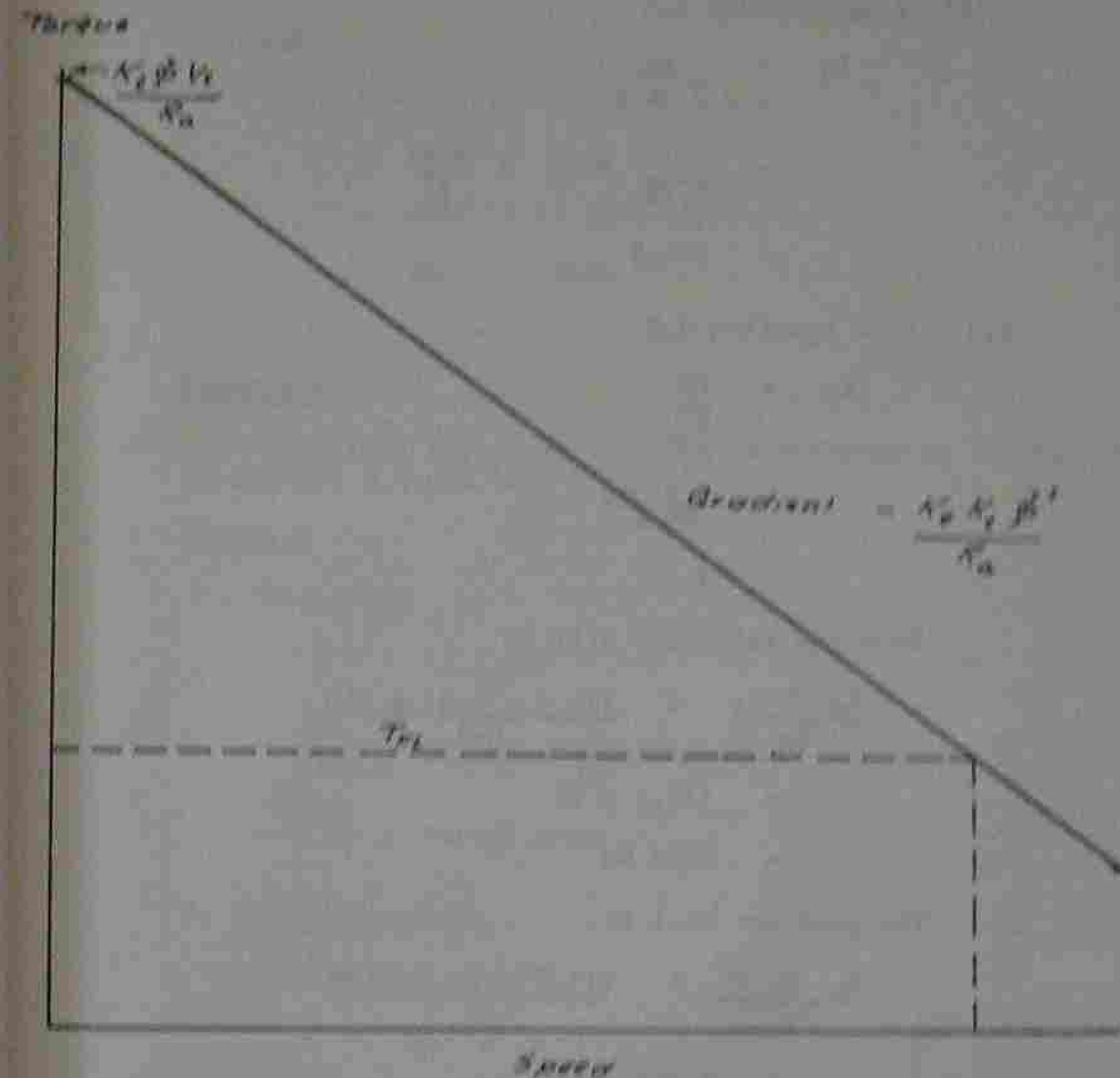


Figure 9.1 - <u>Torque Speed Curve for Shunt Motor.</u>

Additional information that can be obtained from the relationship given in equation 9.10 is:

(i) Since flux is proportional to supply voltage the torque is proportional to the square of the supply voltage.

(ii) Torque is inversely proportional to the resistance of the armature circuit.

Thus, if the parameters of a given machine are known equation 9.10 can be evaluated.

<u>Example 9.1</u>

A 4-pole wave wound armature operating in a field of flux 0.04 webers is wound with 360 armature conductors. Determine the expression for torque as a function of speed if V_t = 250 volts and R_a = 0.1 Ω .

Solution

(i) From equation 9.5

$$K_t = \frac{PZ}{2\pi a}$$

$$= \frac{1}{6.28} \times 4 \times \frac{360}{2}$$

$$= 114.5$$

(ii) From equation 9.8

$$K_e = \frac{PZ}{60_a}$$

$$= \frac{4 \times 360}{60 \times 2}$$

$$= 12$$

Thus the intercept will be:

$$\frac{K_t \ \Phi \ V_t}{R_a} = \frac{114.5 \times 0.01 \times 250}{0.1}$$

$$= 114.5 \times 25$$

$$= 2860 \text{ Nm}$$

The gradient will be:

$$\frac{K_e \ K_t \ \phi^2}{R_a} = \frac{114.5 \times 12 \times (.01)^2}{.1}$$

$$= 1.145 \times 1.2$$

$$= 1.38$$

The expression for torque as a function of speed would be:

$T = 2860 - 1.38N$ newton metres.

From this expression, the machine would speed up until the value of T is zero. That is,

$$2860 - 1.38\ N = 0$$

$$\text{or} \quad N = \frac{2860}{1.38}$$

$$= 2070 \text{ r.p.m.}$$

In practice, the speed would increase until the developed torque balanced out the frictional torque at which point the accelerating torque would be zero and the equilibrium no-load speed would be reached.

Example 9.2

Assuming that the no-load speed of the motor in example 9.1 is 2065 r.p.m. calculate the frictional torque.

Solution

Using the results of example 9.1

$$T = 2860 - 1.38\ N$$

$$= 2860 - 1.38 \times 2065$$

$$= 2860 - 2850$$

$$= 10 \text{ Nm}$$

Example 9.3

Calculate the power loss due to friction in the machine of example 9.1 and 9.2.

Solution

Recalling

$$P = \frac{2\pi NT}{60}$$

$$P = \frac{2 \times 3.142 \times 10 \times 2065}{60}$$

$$= 2160 \text{ watts}$$

B. Series Machines

For a series machine

$$T = K_{ts}\ I_a^{\ 2} \qquad - 9.11$$

$$\text{and } E_g = V_t - I_a R_a$$

$$\text{or} \quad K_{es}\ I_a N = V_t - I_a R_a \qquad - 9.12$$

Note:

K_{ts} and K_{es} are given on the basis of the assumption that

$$\Phi \propto I_a$$

and as a consequence are difficult to evaluate since the relationship between Φ and I_a is not known explicitly.

From 9.12

$$\left(\frac{K_{es}\ N}{R_a}\right) I_a + I_a = \frac{V_t}{R_a}$$

$$I_a \left(\frac{K_{es}N + R_a}{R_a}\right) = \frac{V_t}{R_a}$$

$$I = \frac{V_t}{K_{es}N + R_a} \qquad - 9.13$$

Substituting this value for I_a in equation 9.11 will give

$$T = K_{ts}\left(\frac{V_t}{K_{es}N + R_a}\right)^2 \qquad - 9.14$$

Due to the problems associated with the evaluation of K_{ts} and K_{es} this is as far as this expression can be taken to obtain meaningful results. However, the expression in this form can be used to deduce several characteristics of the torque speed curve. These may be listed as:

(i) Torque is proportional to the square of the terminal voltage.

(ii) Torque is inversely proportional to the square of the armature speed.

(iii) Torque varies inversely as the square of the armature circuit resistance.

Thus, while the general form of the torque speed relationship for the series motor can now be sketched the final form would depend on a knowledge of the relationship between the flux and the armature current and, in addition, a knowledge of the frictional torque speed relationship for the machine running free.

In addition to the information deduced from equation 9.14 it can be seen that if the frictional torque was zero the armature speed would tend to infinity. As a consequence, as stated previously when dealing with the torque-armature current characteristic, there is a lower limit to the load applied to a series motor to ensure that the speed of the machine does not become excessive.

C. Compound Machines

The difficulty in obtaining an explicit relationship between torque and speed for a compound machine is made more difficult by the fact that the flux has two components:

(i) That due to the shunt field which is a function of the terminal voltage and can be considered a constant.

(ii) That due to the series field which is a function of the armature current and is therefore load dependent.

In addition, the relative strength of the fields is unknown and it is also possible that the fields may be either opposing or assisting. As a consequence it will only be stated that, depending on the above factors and their relationship, the torque speed characteristic of a compound motor is somewhere between that of a shunt and that of a series motor.

9.3 TORQUE SPEED RELATIONSHIP FOR VARIOUS LOADS

Due to the varying nature of the loads that d.c. motors may be required to drive before a motor can be properly matched to a load it is first necessary to compare the torque speed characteristic of both the motor and the load. The characteristics of different types of loads will be covered in more detail in a later unit. The subject will be introduced at this stage in a general form by specifying a torque speed relationship mathematically as shown in the following:

A. Constant Torque

A constant torque load is one in which the torque is independent of speed. A typical torque speed curve for such a load is given in Figure 9.2.

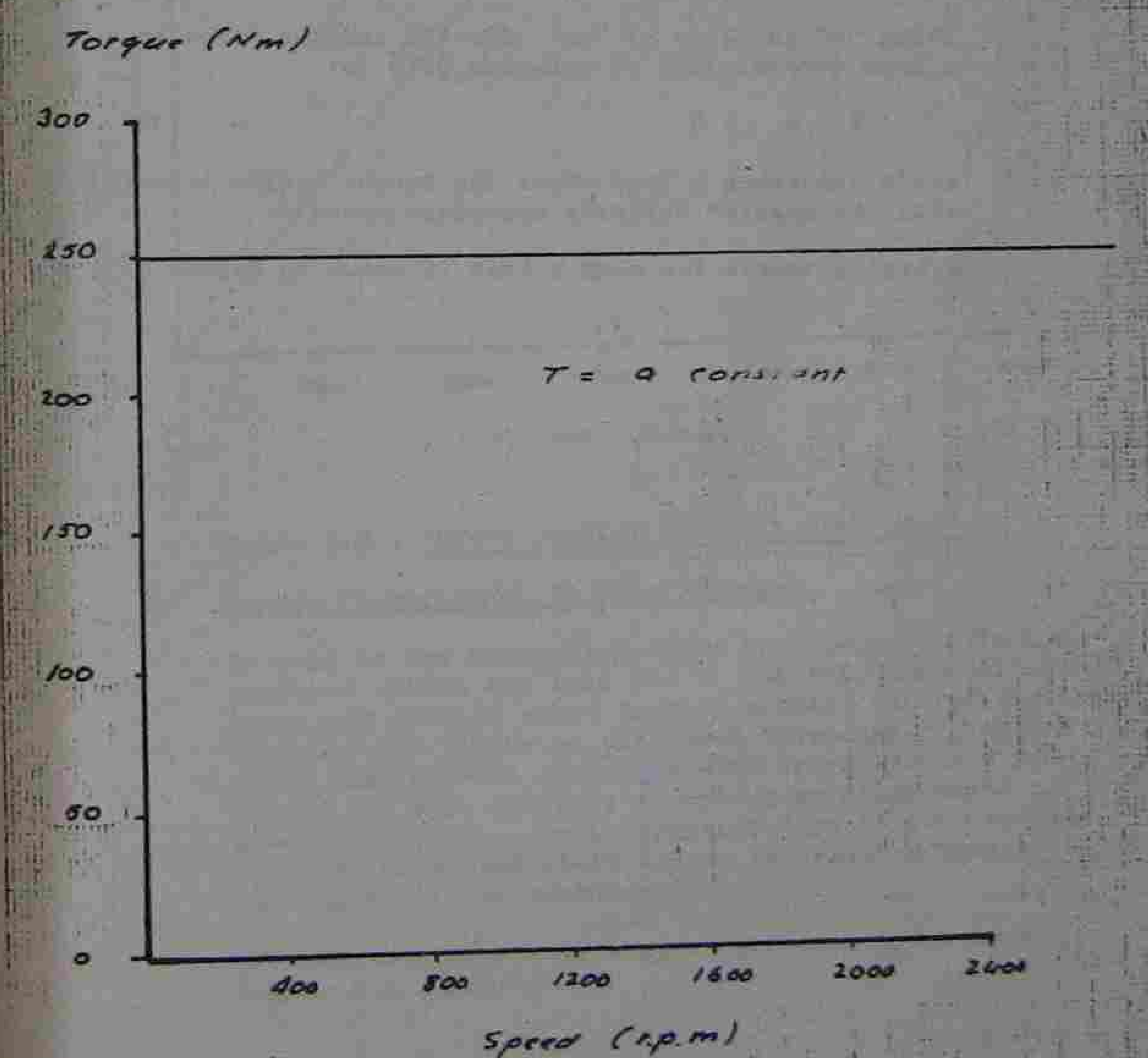


Figure 9.2 - Constant Torque Characteristic.

B. Torque Proportional to Speed

With some classes of loads the internal torque developed by the load may be a function of the angular velocity of the load. It will be recalled that the torque expression for a rotating system can be written as:

$$T = \beta\omega + I\alpha \qquad - \quad 9.15$$

where β is the damping or friction coefficient

ω the angular velocity

I the moment of inertia

and α the angular acceleration

For constant speed loads the acceleration term can be ignored and equation 9.15 becomes

$$T = \beta\omega \qquad - \quad 9.16$$

Using "N" in place of "ω" for the angular velocity, a more general form of equation 9.16 is

$$T = k\,N \qquad - \quad 9.17$$

which describes a load where the torque varies linearly with the angular velocity (armature speed).

A typical curve for such a load is shown in Figure 9.3.

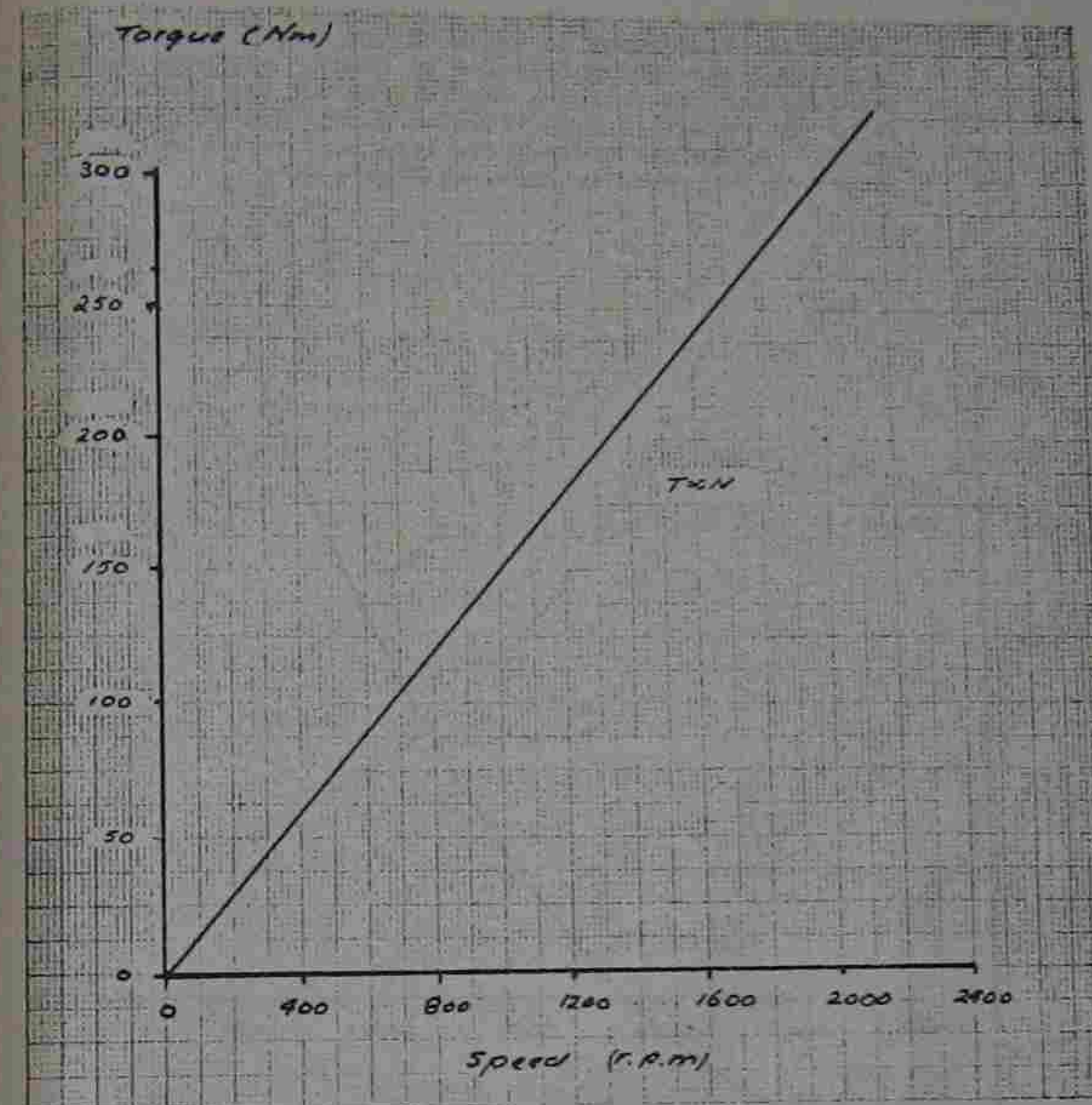


Figure 9.3 - Torque varying linearly with Speed.

C. Torque Proportional to Speed Squared

As well as the increasing speed increasing the torque produced within the load due to the friction coefficient there may also be other factors present that will act to increase the torque as the speed increases. In such cases the increase of torque with speed will not be linear but will increase according to a law based on some power of N. As a representative of such systems, the case of a load where torque varies as a square of the speed will be considered.

The relationship between speed and torque for such a load can be written as:

$$T = K\,N^2 \qquad - \quad 9.18$$

In practice the power to which N is raised is seldom an integer but varies from one to three. In some cases the load may not follow a regular law but the power to which N is raised may itself be a function of the speed. A typical curve for a load that varies as the square of the speed is shown in Figure 9.4.

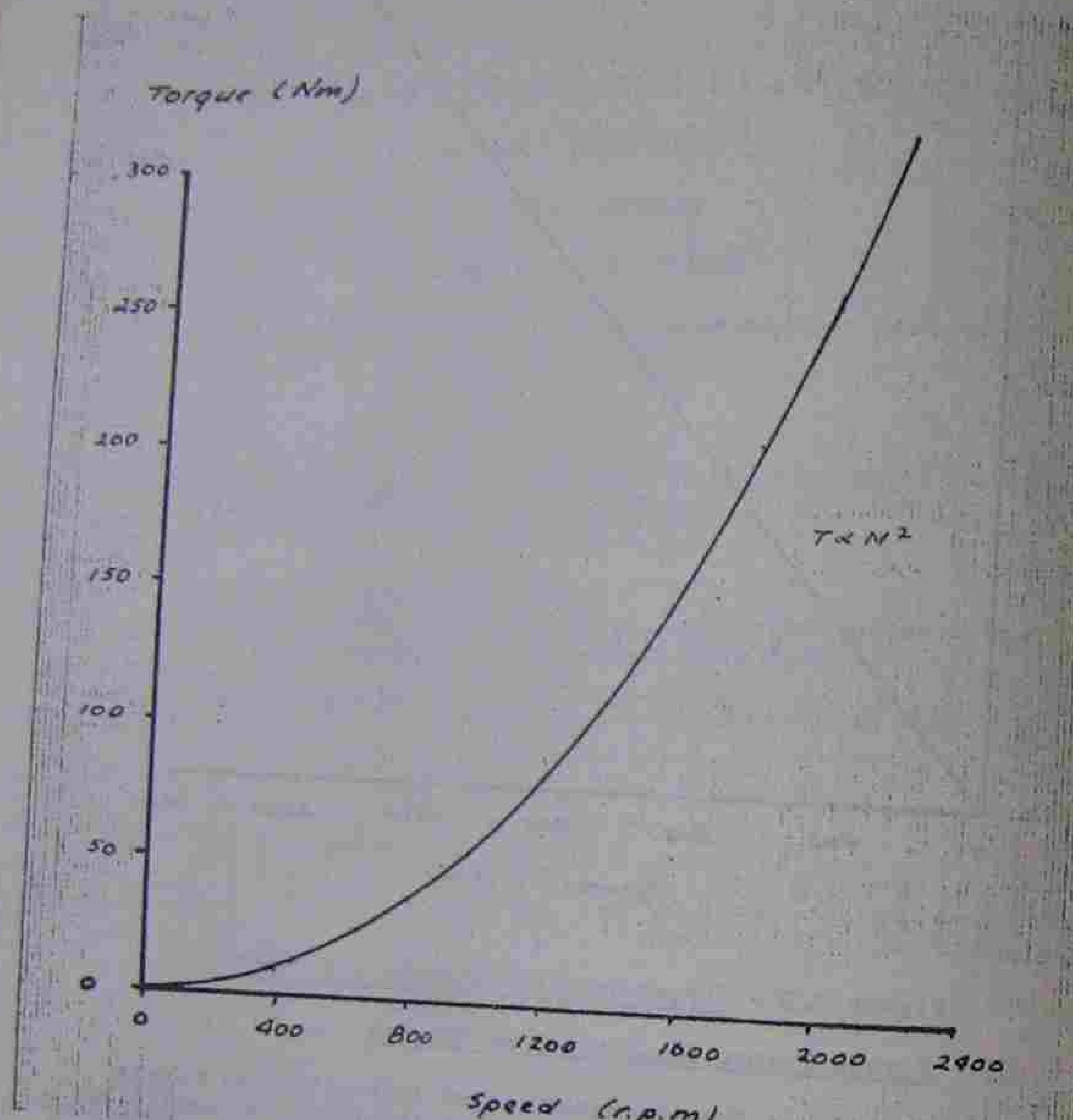


Figure 9.4 - Torque varying as the Square of the Speed.

D. Combined Loads

An example of a load that does not obey a simple law is one that has a fixed component and a component that varies either linearly with N or to some power of N. Two such loads are shown in Figure 9.5

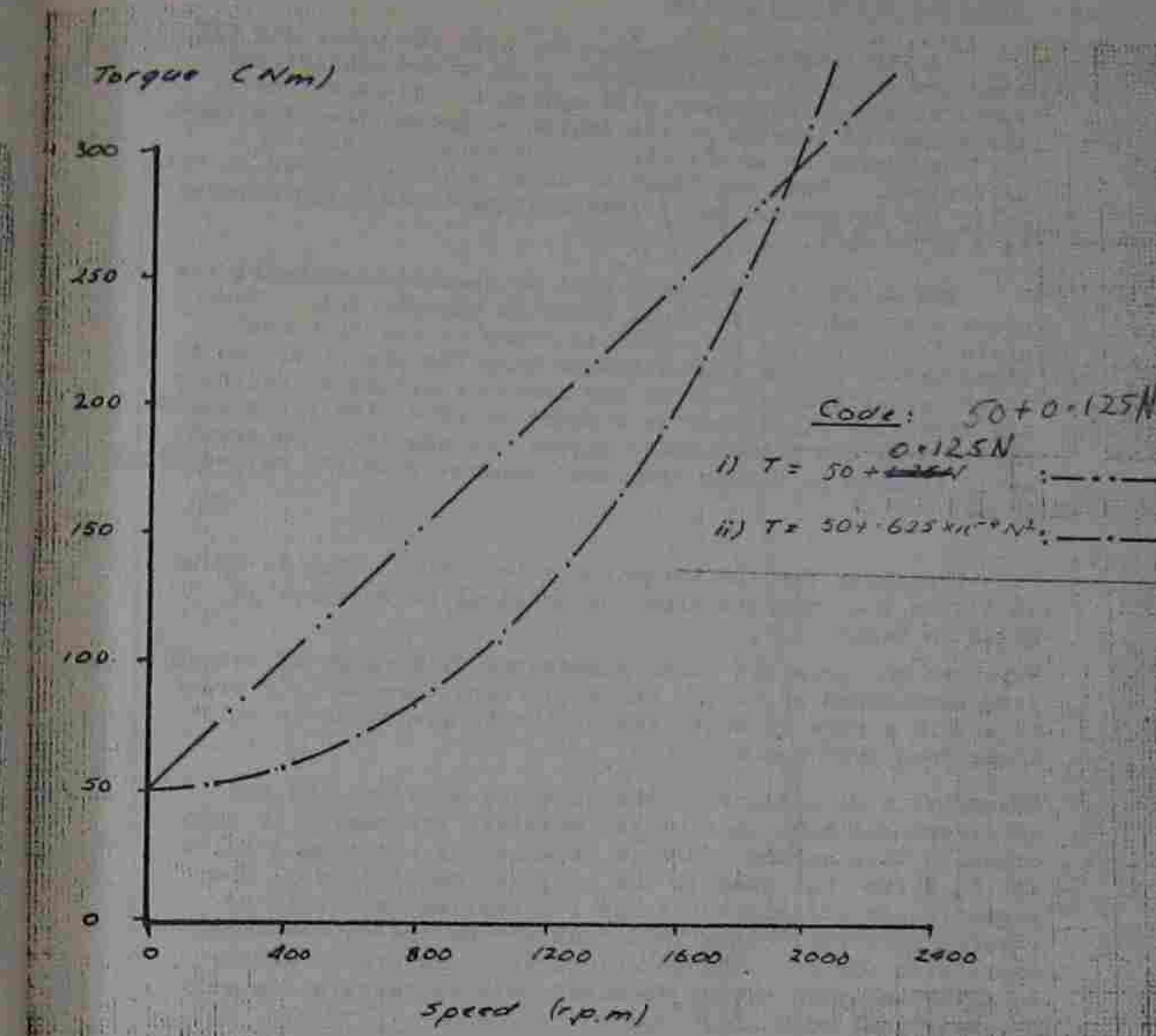


Figure 9.5 - Composite Load Examples.

These would be more practical examples of load than the simple one term loads of parts A, B and C. The curves shown would represent loads that had a constant torque factor and a factor with a frictional content that varied with motor or load speed. In fact, part of the speed dependent load could be contributed by the windage and friction torque of the motor itself.

It should be appreciated that in practice it is seldom possible to have loads as clearly defined mathematically as those shown in the preceding examples. Where industrial loads are concerned such factors as the moment of inertia of the load must be considered together with aspects such as static friction as opposed to dynamic friction all of which tend to make the mathematical prediction of load characteristics a complex task.

9.4 MATCHING MOTORS TO DRIVES

If the torque speed curves for both the motor and the load are known it is possible to predict the equilibrium speed at which the system will operate. In addition, if the moment of inertia of the system is known, the time taken for the system to accelerate to this speed can also be determined. Only the first of these will be covered in this unit, the determination of accelerating times being covered in a later unit.

The matching techniques will be demonstrated using the curve obtained for the shunt motor in example 9.2. Once again the student's attention is drawn to the fact that these curves are an approximation only for practice, due to the demagnetizing effect of the armature acting to decrease the effective field flux in a shunt machine, the curve would depart from the linear model given. In practice the speed would tend to be higher than that indicated in the mathematical model.

The curve for the motor characteristic, which is drawn in Figure 9.6, differs from the original curve which is given in Figure 9.1.

Firstly, the original curve showed the full range of torque from zero speed up to the speed for zero torque. In order to obtain a more accurate result the torque range is only shown from 1900 r.p.m.

Secondly, a second curve with both the gradient and the intercept one half that of the original expression is also drawn. This second curve is obtained by assuming a value for R_a twice that used in the original expression. The significance of this curve and its implications will be covered more fully in a later unit. As the mathematical expression for both the motor and the load are known the equilibrium speed can be obtained both by calculation and by analytical solution.

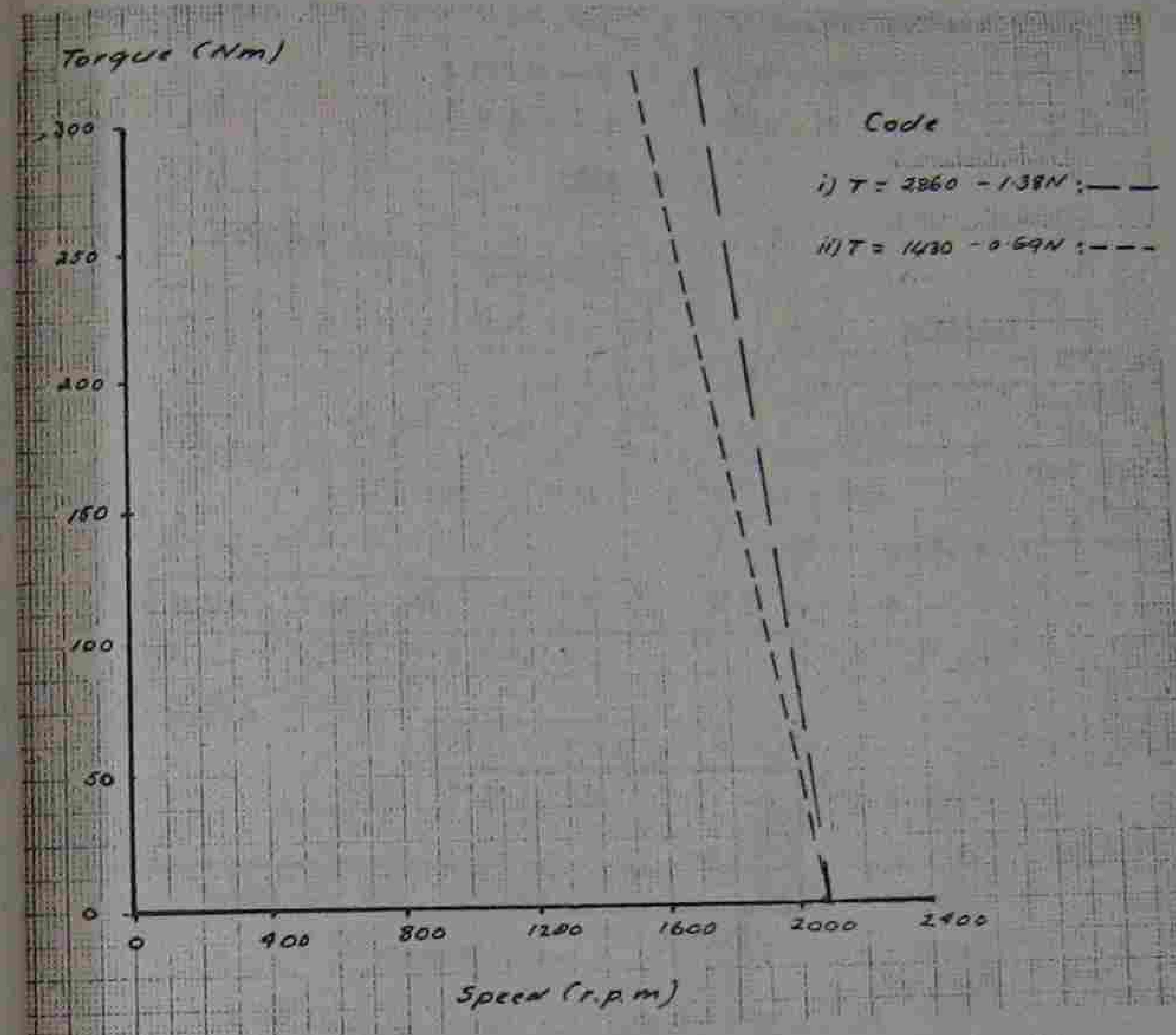


Figure 9.6 - Modified load characteristics for Motor of Example 9.2.

Example 9.4

For this example the original motor expression will be used and matched against both the loads shown in Figure 9.5 in order to obtain the equilibrium speed in each instance.

Solution

(a) For motor

$$T = 2860 - 1.38\,N \qquad - (1)$$

(b) For loads

(i) $$T = 50 + 0.125\,N \qquad - (2)$$

(ii) $$T = 50 + .625 \times 10^{-4} N^2 \qquad - (3)$$

Firstly

equating (1) and (2)

$$2860 - 1.38\ N = 50 + 0.125\ N$$

$$2860 = 1.505\ N$$

$$N = \frac{2860}{1.505}$$

$$= \underline{1900\ \text{r.p.m.}} \quad 1867\,RPM$$

Secondly

equating (1) and (3)

$$2860 - 1.38\ N = 50 + .625 \times 10^{-4}\ N^2$$

$$.625 \times 10^{-4}\ N^2 + 1.38\ N - 2860 = 0$$

From which

$$N = \frac{-1.38 \pm \sqrt{1.38^2 + (4 \times .625 \times 10^{-4} \times 2860)}}{2 \times 0.625 \times 10^{-4}} \quad 2810$$

$$= \frac{-1.38 \pm \sqrt{1.904 + .715}}{1.25 \times 10^{-4}}$$

$$= \frac{-1.38 \pm \sqrt{2.629}}{1.25 \times 10^{-4}} \quad \sqrt{2.6069}$$

$$= \frac{-1.38 \pm 1.622}{1.25 \times 10^{-4}} \quad 1.615$$

$$= \frac{.2420}{1.25 \times 10^{-4}} \quad \frac{0.235}{1.25 \times 10^{-4}}$$

$$= 1930\ \text{r.p.m.}$$

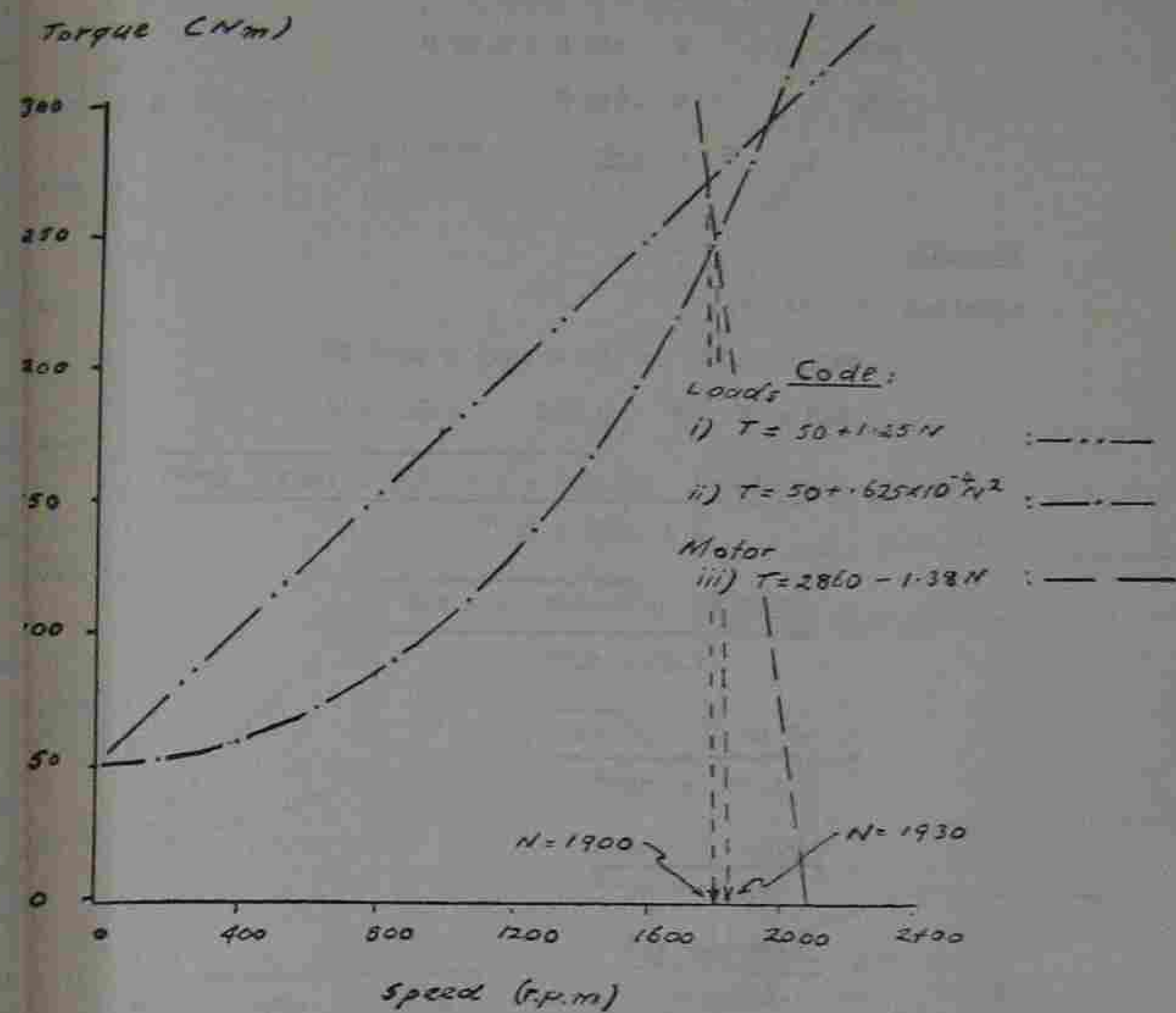


Figure 9.7 - Solution to Example 9.4

Example 9.5

Repeat example 9.3 for the motor characteristic obtained by doubling the armature circuit resistance.

Solution

(a) Motor expression

$$T = 1430 - 0.69\ N \qquad - \quad (1)$$

(b) Load expression

(i) $$T = 50 + 0.125\ N \qquad - \quad (2)$$

(ii) $$T = 50 + .625 \times 10^{-4}\ N^2 \qquad - \quad (3)$$

Firstly

equating (1) and (2)

$$1430 - 0.69\,N = 50 + 0.125\,N$$

$$1430 - 50 = .125\,N + 0.69\,N$$

$$1380 = .815\,N$$

$$N = \frac{1380}{.815} = 1695 \text{ r.p.m.}$$

Secondly

equating (1) and (3)

$$1430 - 0.69N = 50 + .625 \times 10^{-4}\,N^2$$

$$.625 \times 10^{-4}\,N^2 + 0.69N - 1380 = 0$$

$$N = \frac{-0.69 \pm \sqrt{0.69^2 + (4 \times .625 \times 1380 \times 10^{-4})}}{2 \times .625 \times 10^{-4}}$$

$$= \frac{-0.69 \pm \sqrt{0.4761 + .345}}{1.25 \times 10^{-4}}$$

$$= \frac{-0.69 \pm \sqrt{.821}}{1.25 \times 10^{-4}}$$

$$= \frac{-0.69 \pm .9061}{1.25 \times 10^{-4}}$$

$$= \frac{.2161}{1.25} \times 10^3$$

$$= 1735 \text{ r.p.m.}$$

The graphical solution to this problem is given in Figure 9.8

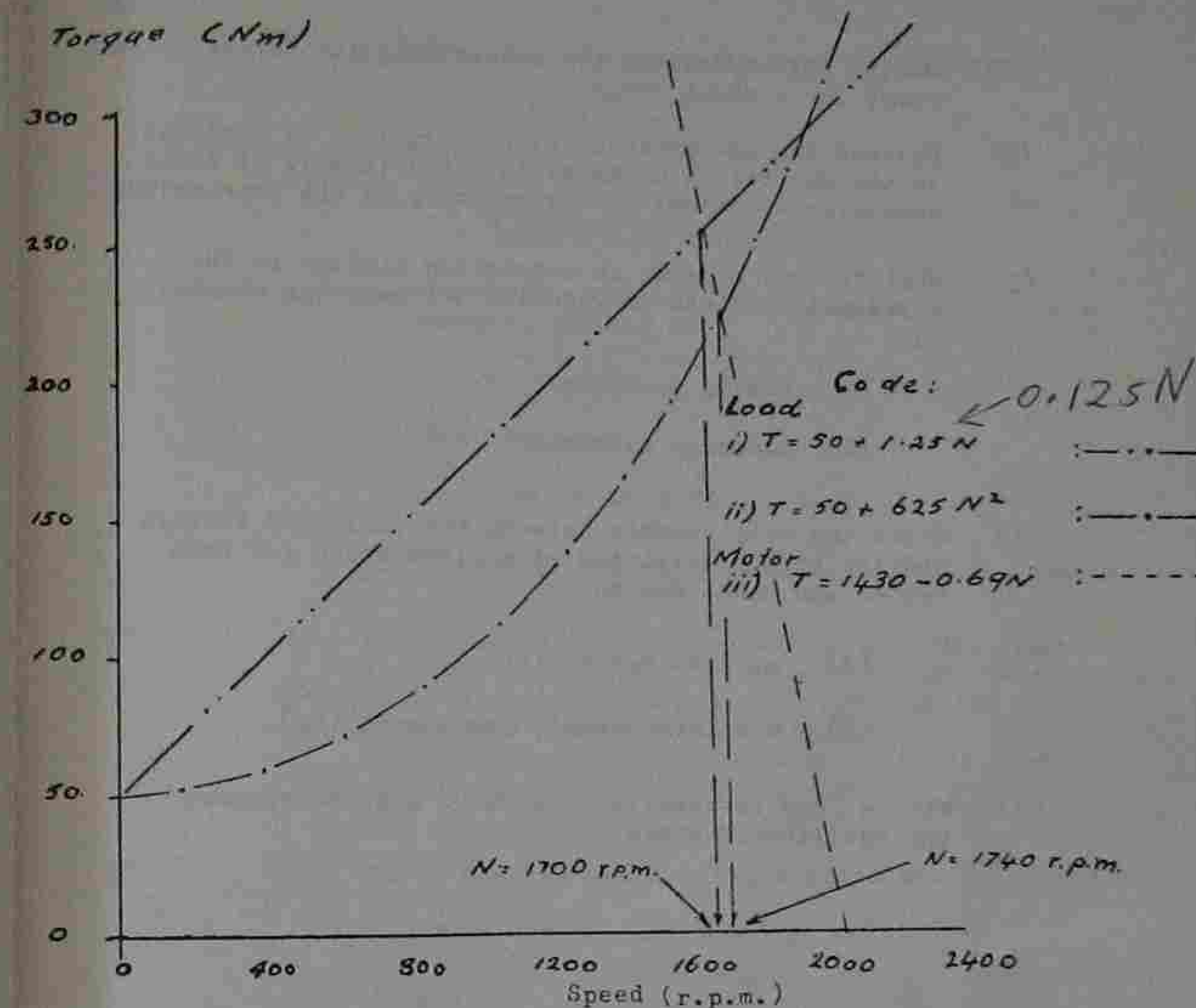


Figure 9.8 - Solution to Example 9 5.

REVIEW QUESTIONS

(1) Derive the expression for determining the torque speed curve for a shunt motor.

(2) Discuss the assumptions, either explicit or implicit, in the derivation of Question 1 and in view of these assumptions comment on the validity of the expression.

(3) What factors prevent an expression similar to the expression derived in Question 1 from being obtained for:

- (a) series machines,
- (b) compound machines?

(4) State the relationship between the following factors and the torque developed at a given speed for both series and shunt motors:

- (1) applied e.m.f. (V_t),
- (2) armature circuit resistance (R_a).

(5) When a load is applied to a motor what determines the equilibrium speed?

A S S I G N M E N T S

Marks

20 (1) a. Draw the curve for the load defined by:

$$T = 1 \times 10^{-4} N^2$$

for the range

$$0 \leq N \leq 2500 \text{ r.p.m.}$$

b. On the same set of axes draw the curve for a machine defined by:

$$T = 5000 - 2.0 N$$

for the range

$$2300 \leq N \leq 2500 \text{ r.p.m.}$$

c. Determine the equilibrium speed of the system.

20 (2) A 500 volt d.c. shunt motor has a 4-pole lap wound armature comprising 580 conductors and the armature resistance is 0.2 ohms. If the flux per pole is 0.02 weber determine the expression for the developed torque as a function of speed (neglect the demagnetizing effect of armature reaction).

30 (3) For a machine with a torque speed curve defined by

$$T = 2000 - 0.2 N - 4 \times 10^{-4} N^2$$

determine:

a. the no load speed if the frictional torque is 5 newton metres,

b. the available load torque if the speed regulation on full load is 10%.

c. the full load output power.

Marks

15 (4) Using the torque speed relationship given in Question 3, determine the operating speed and developed torque if the load supplied requires a constant torque of 100 newton metres and the frictional torque of the motor can be represented by:

$$T = 1.25 \times 10^{-6} N^2$$

15 (5) Redraw the curve for the loads given in Figure 9.5 and, from this curve, determine the resistance of the armature as a ratio of the original resistance for the motor of Example 9.2 to give an equilibrium speed of 1750 r.p.m. on the linear curve.

ELECTRICAL MACHINES I

UNIT NO. 10

10.1 Introduction.

10.2 Graphical determination of R.M.S.

10.3 Duty cycle calculations.

10.4 Manual starter for d.c. motor.

10.5 Series motor controllers.

10.6 Acceleration of d.c. motors.

Review Questions.

Assignments.

10.1 INTRODUCTION

As well as selecting a motor that is capable of developing sufficient torque to drive a load at the required speed, the motor so chosen must be able to provide this torque and not suffer any undue damage as a consequence to the heat produced within the machine by the currents in the various windings. Where the load is constant this represents no real problem as it becomes a relatively simple matter to then select a machine of the appropriate power rating. Where the load is cyclic in nature the average load cannot be arrived at by simply considering the loads as such and the duration of the loads, but as the heating effect is a function of the R.M.S. current the R.M.S. value of the load must be obtained.

Again, if a mathematical expression for the load cycle is available this R.M.S. value can be obtained analytically but as this is seldom the case the only alternative is to resort to graphical means.

10.2 GRAPHICAL DETERMINATION OF R.M.S.

The student will probably recall the mid-ordinate method of obtaining the area under an irregular curve used in earlier subjects. In the general mid-ordinate method the base line is divided into sections of equal length and the height of the curve at the mid-point of each of the sections recorded. The total area is then the average of all the mid-ordinate values of the function multiplied by the length of the base. If the average value of the function is required over the range of the function the average value is the total area divided by the base, and this is simply the average of all the mid-ordinate values with the qualification that where the function is zero over one of the base increments the mid-ordinate value must be recorded as zero.

This can be written in calculus notation as

$$x_{ave} = \frac{1}{y} \int_0^y x \, dx \qquad - \qquad 10.1$$

where x is a function of time.

As an introduction to this present topic this method will be used to determine the area of a triangle.

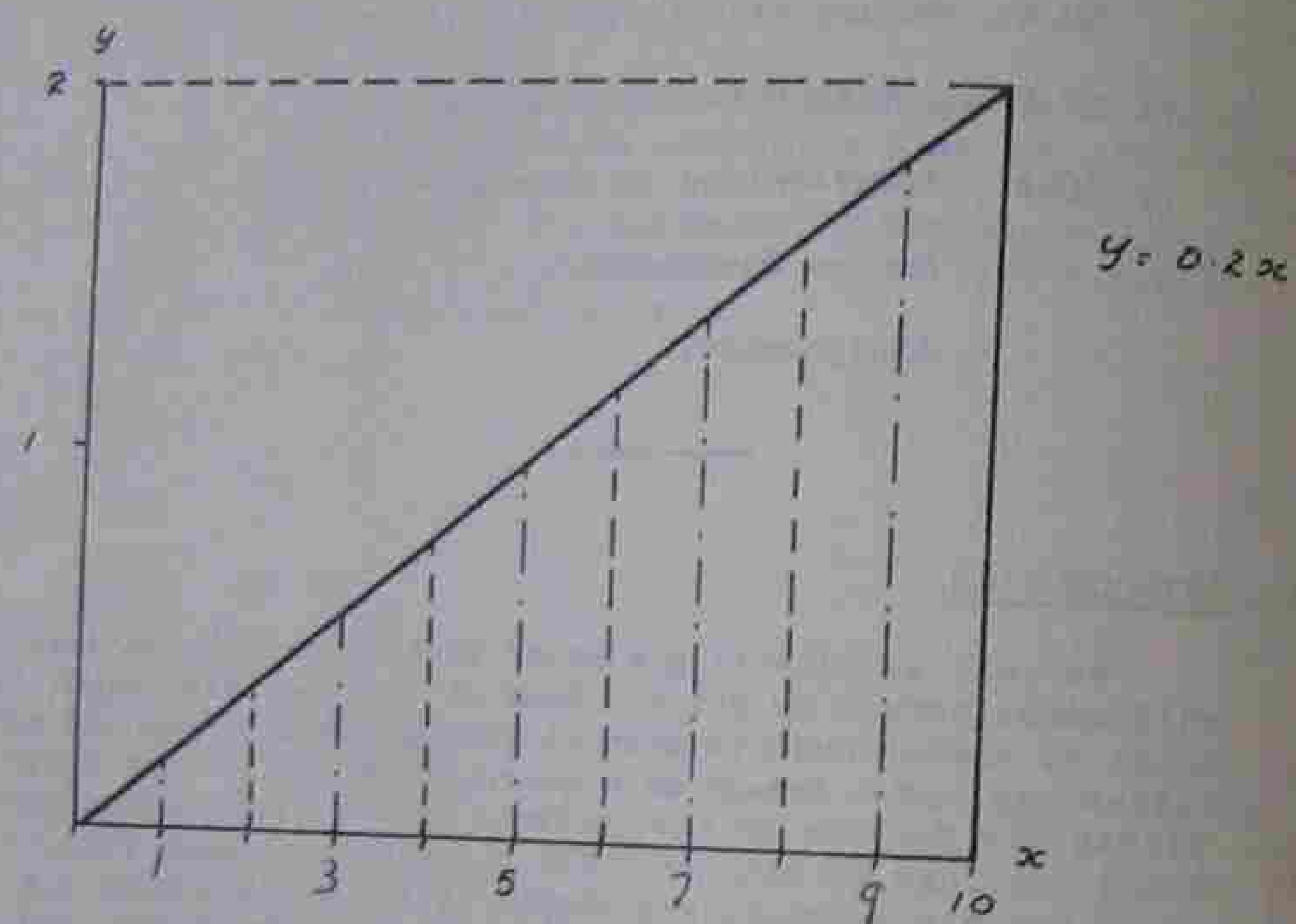


Figure 10.1 - Diagram for Example 10.1.

Example 10.1

Use the mid-ordinate method to determine the area of the triangle of Figure 10.1.

Solution

As shown in the diagram, the base line is divided into five equal sections the mid-points of which are 1, 3, 5, 7 and 9. Since the curve is linear an equation for the curve can be obtained and used to calculate the value of y for each mid-ordinate value of x. The area of each section is then the height of the mid-ordinate multiplied by the base increment. The results are tabulated in Table 10.1.

Section	Mid-point	Mid-ordinate	Area
1	1	0.2	.4
2	3	0.6	1.2
3	5	1.0	2.0
4	7	1.4	2.8
5	9	1.8	3.6
		Total	10.0

Table 10.1

Alternatively, using equation 10.1.

$$y = \int_0^{10} 0.2x \, dx$$

$$= \left[0.1\, x^2\right]_0^{10}$$

$$= 0.1 \left[10^2 - 0^2\right]$$

$$= 10$$

It will be observed that the same results would have been obtained by using the geometric expression for the area of the triangle, namely:

$$\text{area} = \frac{\text{base x height}}{2}$$

$$= \frac{10 \times 2}{2}$$

$$= 10$$

As this is the equivalent of using the mid-ordinate method for only one section in the base which would make the mid-ordinate 5 and the value of y corresponding to $x = 5$ is 1. This leads to the conclusion that for average values of linear relationships the accuracy of the result is not affected by the magnitude of the increments.

The R.M.S. value of a function is defined mathematically as:

$$x_{R.M.S.} = \sqrt{\frac{1}{y} \int_0^y x^2 \, dx} \qquad - \qquad 10.2$$

Referring again to the diagram of Figure 10.1 the R.M.S. value of the function over the given interval is:

$$x_{R.M.S.} = \left[\frac{1}{10}\int_{0}^{10} (0.2\ x)^2\ dx\right]^{1/2}$$

$$= \left[\frac{1}{10}\int_{0}^{10} 0.04\ x^2\ dx\right]^{1/2}$$

$$= \left[\frac{1}{10}\left(\frac{0.04}{3}x^3\ \Big|_{0}^{10}\right)\right]^{1/2}$$

$$= \left[\frac{1}{10}\left(\frac{0.04 \times 1000}{3}\right)\right]^{1/2}$$

$$= (1.33)^{1/2}$$

$$= 1.153$$

The procedure used to determine the R.M.S. value of the function repreeented by the figure in diagram in Figure 10.1 may be summarized as follows:

(1) Divide the base into an equal number of increments.

(2) Determine the value of the mid-ordinate for each section either by measurement or by calculation if the equation of the expression is known.

(3) Calculate the square of each mid-ordinate value.

(4) Sum the squares of the values.

(5) Determine the average of the squares.

(6) Extract the square root of the average.

Applying the technique to the Figure 10.1 the results are as tabulated in Table 10.2.

Section	Mid-point	Mid-ordinate	Mid-ordinate Squared
1	1	0.2	0.04
2	3	0.6	0.36
3	5	1.0	1.0
4	7	1.4	1.96
5	9	1.8	3.24
		Total	6.60

Average $6.6 \div 5 = 1.32$

R.M.S. value $= \sqrt{1.32}$

$= 1.149$

Table 10.2

That is, the R.M.S. value of the function is 1.149. As the average value of the function is 1.0 this shows that as stated in the section of the heating effect of the ripple current in S.C.R. drives that the R.M.S. value of a function is greater than the average value. This last result also shows that the accuracy of the result is dependent on the number of divisions of the base line.

As the divisions of the base were of equal length in Example 10.2 it was not necessary to first compute the area in order to determine the R.M.S. value. In practice the duty cycle of a machine will generally have periods of different loads of unequal duration and the area under the squared curve must be determined to find the average ordinate prior to extracting the square root.

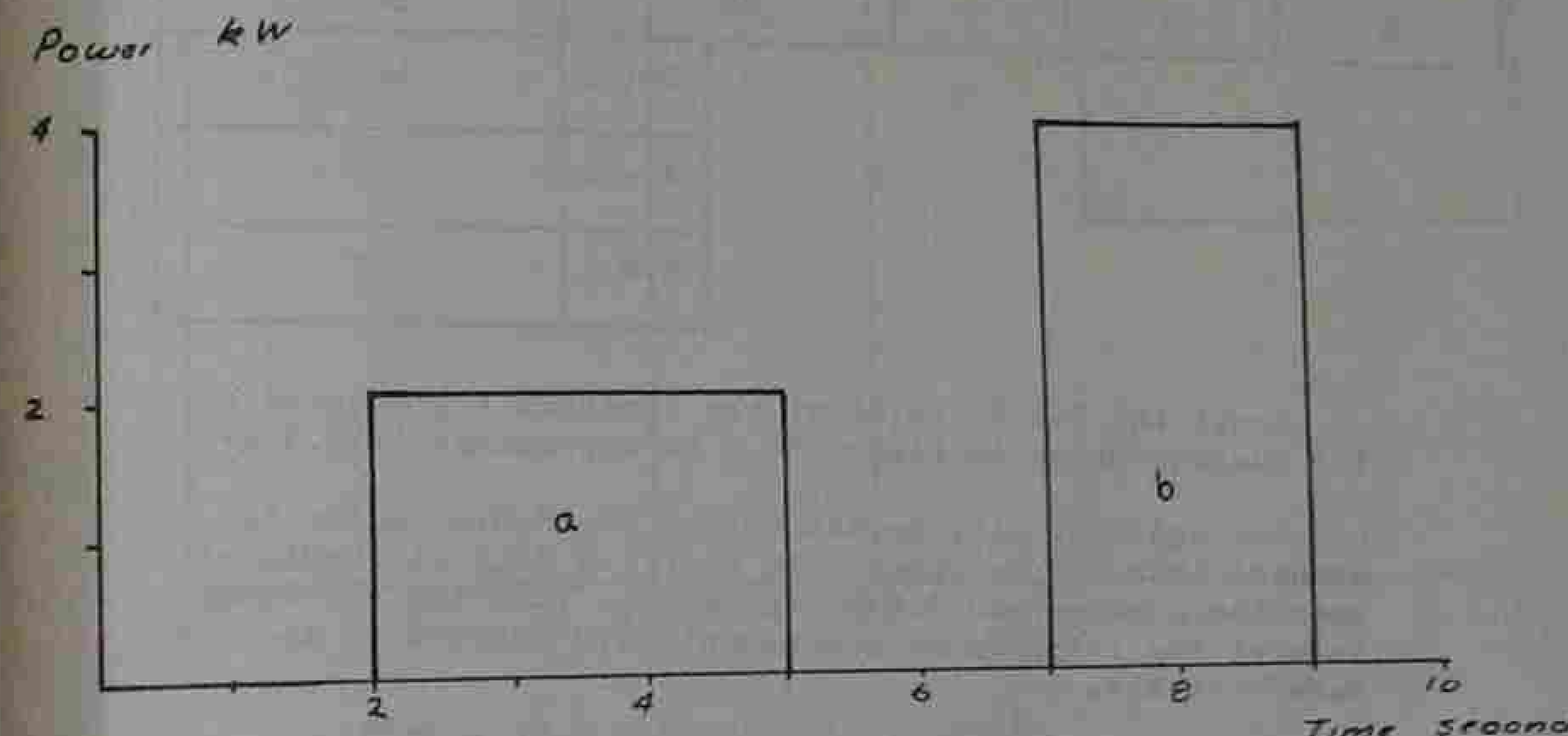


Figure 10.2 - Diagram for Examples 10.3 and 10.4.

As a further comparison of the average and R.M.S. value of a function and as an example of this technique the following examples will find the average and then the R.M.S. value of the function shown in Figure 10.2.

Example 10.3

Determine the average value of the function shown in Figure 10.2.

Solution

Section	Duration	Value	Area
a	3	2	6
b	2	4	8
		Total =	14

Average $= 14 \div 10$

$= 1.4$ kW

Thus the average value of the function of the interval 0 to 10 is 1.4 kW.

Example 10.4

Determine the R.M.S. value of the function shown in Figure 10.2.

Solution

Section	Duration	Value	Value Squared	Area of Function Squared
a	3	2	4	12
b	2	4	16	32
			Total	44
			Average	4.4
			R.M.S.	2.1

Hence the R.M.S. value of the function represented by the diagram shown in Figure 10.2 is approximately 2.1 kW.

One again it will be noted that the R.M.S. value is greater than the average. In fact the only circumstance when the average and R.M.S. value of a function are equal is when the function is a constant with respect to the base variable.

10.3 DUTY CYCLE CALCULATIONS

The techniques demonstrated in Section 10.2 can now be used to determine the R.M.S. power of a motor to provide the power adequate for a given load cycle. Although practical duty cycles are rarely simple functions and generally are not constant for any period it will be assumed that the duty cycles will be constant with respect to time over the duration of their application.

Example 10.5

A certain motor has to provide the power to drive a load that requires an input power of 100 kW for 20 seconds, running free (no-load) for 10 seconds; for the next 10 seconds the required input is 120 kW followed by a 20 second period when the power required is 40 kW. After this the cycle is repeated. Determine the R.M.S. value of the power required from the motor.

Solution

The duty cycle for the motor over the cycle duration, (60 seconds) is shown in Figure 10.3.

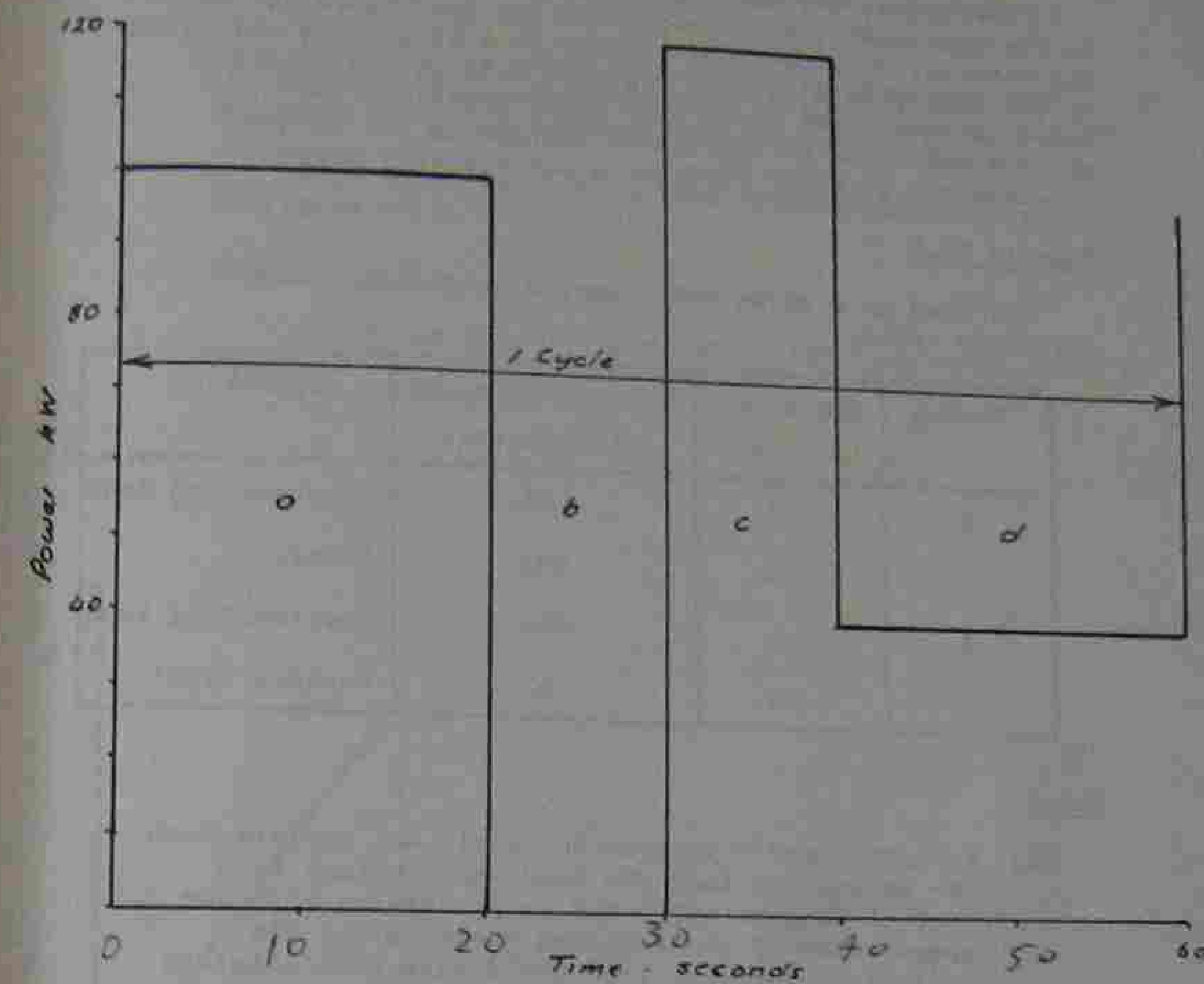


Figure 10.3 - Diagram for Example 10.5

The tabulated results are shown in Table 10.3.

Section	Duration (Seconds)	Power Required	Power Squared	Power^2 time product
a	20	100	10000	200000
b	10	0	0	0
c	10	120	14400	~~172800~~ 144000
d	20	40	1600	~~64000~~ 32000
Total	60		Total	~~336800~~ 376000
			Average =	~~8943~~ 6267
			R.M.S. value =	~~94.6~~ 79.16

Table 10.3

Therefore, the load cycle illustrated in Figure 10.3 is the equivalent, with respect to the heating effect on the machine, of a constant load of 94.6 kW and the selection of the appropriate machine would be made on that basis. Where extreme loads occur for small durations the machine must also be capable of sustaining the torque requirements of these loads at a satisfactory speed.

Example 10.6

The load on a given motor varies according to the following table.

Period	Duration (Seconds)	Power Requirement (kilowatts)	Operation
1	10	60	accelerating load
2	20	100	process
3	5	120	decelerating load
4	15	20	running light

Notes

(i) Periods 2 and 4 in practice would not conform with the assumption that the load was constant with respect to time but prior to proceeding with the computation of the required motor rating the equivalent constant load could have been obtained using the method demonstrated in Example 10.2.

(ii) In Period 3 the motor would have been operating in the braking mode but would still be required to provide the power tabulated with the consequent heating effect involved.

The solution to the problem of Example 10.6 can now be tabulated as shown in Table 10.4.

Solution

Period	Duration	Power	$Power^2$	$Power^2$ x Duration
1	10	60	3600	36000
2	20	100	10000	200000
3	5	120	14400	72000
4	15	20	400	6000
Totals	50			314000
				mean = 6280
				R.M.S. power = 79.25

Table 10.4

Thus, the power requirement of the motor is 79.25 kW and the next standard size above this rating would be chosen.

It should be noted that where the power requirement of the motor is fluctuating rapidly it is necessary to adopt the more classical approach demonstrated in Example 10.2.

This would apply to a load cycle similar to that shown in Figure 10.4.

Example 10.7

Determine the R.M.S. value of the load cycle shown in Figure 10.4.

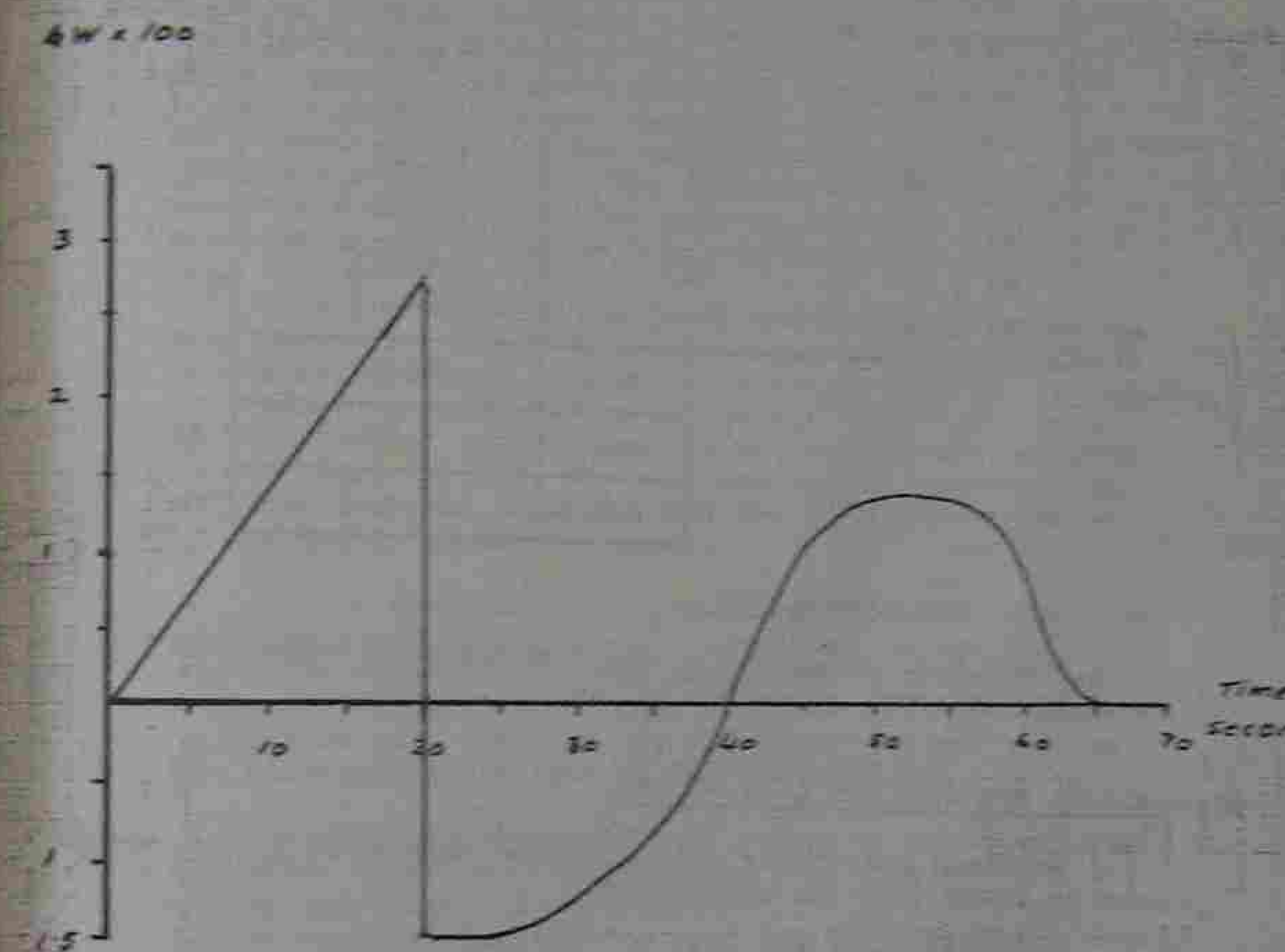


Figure 10.4 - Diagram for Example 10.7.

Solution

Due to the irregular nature of the curve from 20 seconds to 65 seconds the increments selected should be small since, in an example like this, the accuracy of the result will be affected by the size of the base division. As a compromise value the base divisions will be 5 seconds intervals. As the divisions are of equal length in this case it will not be necessary to calculate the area of the squared curve as shown in Table 10.5

Section	Mid-ordinate	Mid-ordinate^2
0 - 5	0.32	0.091
5 - 10	1	1
10 - 15	1.75	3.06
15 - 20	2.4	5.76
20 - 25	-1.5	2.25
25 - 30	-1.4	1.96
30 - 35	-1.1	1.21
35 - 40	-.5	0.25
40 - 45	0.5	0.25
45 - 50	1.25	1.56
50 - 55	1.4	1.96
55 - 60	1.25	1.56
60 - 65	.25	0.063
65 - 70	0	0
	Total	20.974
	mean	1.495
	R.M.S.	1.23

Table 10.5

The R.M.S. value of the function shown in Figure 10.3 over the interval given is 123 kW.

10.4 MANUAL STARTERS FOR D.C. MOTORS

As shown previously, the armature current in a d.c. motor can be found from the expression:

$$I_a = \frac{V_t - E_g}{R_a}$$

As the back e.m.f., E_g, is proportional to the speed, at standstill the armature current will be given by the relationship.

$$I_a = \frac{V_t}{R_a}$$

In most machines if this current is permitted to flow in the armature circuit the resultant forces can cause distortion of the armature windings as well as seriously disturbing the supply system. Also the very high starting torques produced can cause mechanical damage to the drive system. Consequently, as V_t is generally taken to be constant the only way in which the armature current can be limited is to add extra resistance to the armature circuit

by means of a variable resistor. As the speed of the armature then increases, resistance can be successively reduced in order to ensure that sufficient current to produce the required torque is permitted to flow. The resistance of the added resistor is such that the standstill current is limited to between 125% and 150% of the full load current. Thus for shunt machines this implies that the starting torque is limited in the same ratio. The starting resistance is generally only intended for intermittent use and consequently is short time rated.

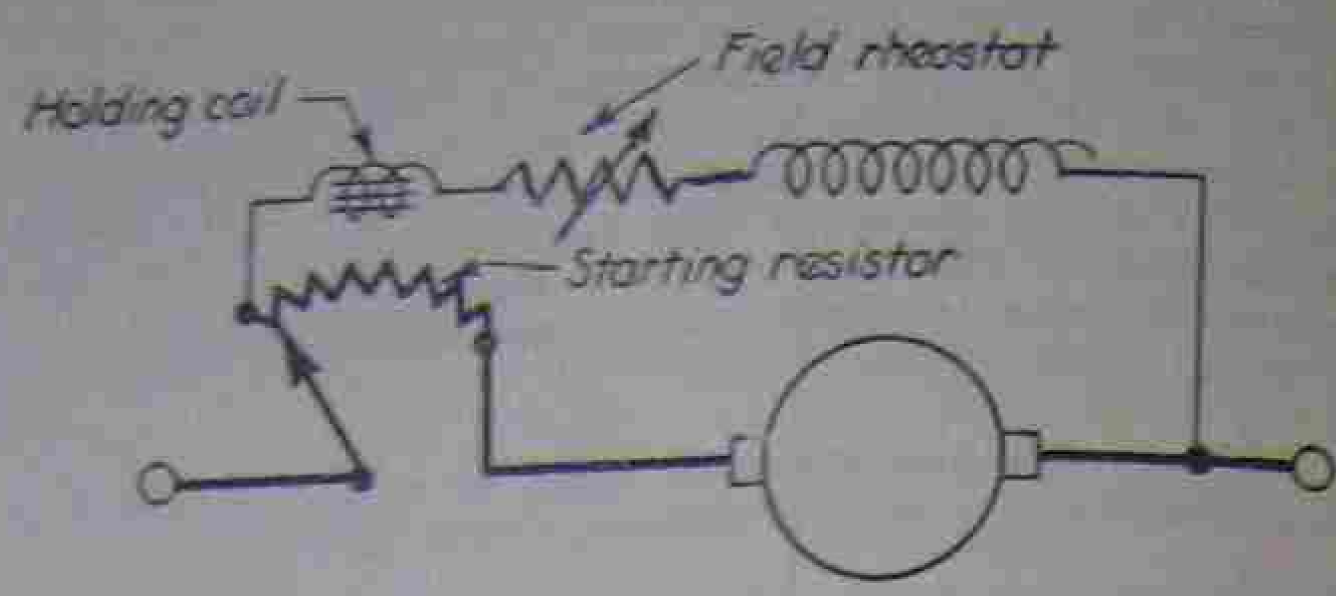
There are two standard types of manual starters for d.c. shunt and compound motors. (See page 66)

Figure 10.5 shows the schematic and actual wiring diagram for a three-point starter for a shunt wound motor. The starter has three terminals labelled L (line terminal), F (field or shunt terminal) and A (armature terminal). The starter arm is held in the off position by a strong spiral spring. To start the motor the starter arm is moved to notch one and the main switch closed. Using this procedure the circuit is made on the master switch and not on the first resistance stud. The armature should now begin to accelerate and after it has done so for a few seconds, the starting handle should be moved to the second notch.

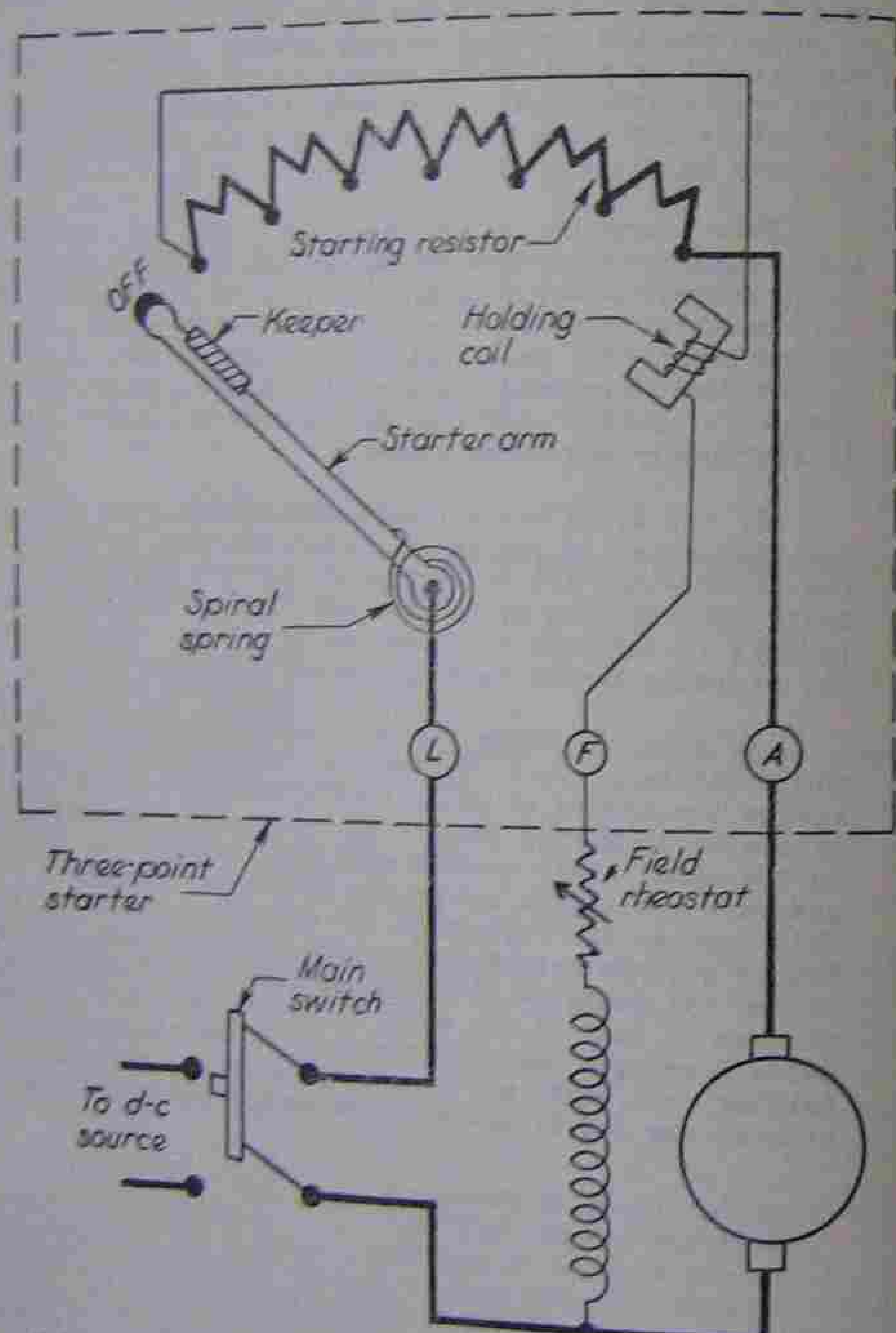
This procedure is continued until the last notch has been reached. The iron keeper on the starting handle will be attracted to and held by the magnetic poles of the holding coil which is connected in series with the shunt field. In the event of a power failure or open circuit occurring in the field circuit, the starting handle returns to the off position thus ensuring that the motor cannot restart if the supply should be remade.

If the field circuit were suddenly broken, since the armature speed is inversely proportional to the field flux the motor would try to overspeed, but the break in the field current will also cause the flux in the holding coil to fall to zero and so allow the spiral spring to bring the starting handle back to the off position.

To stop the motor the main switch is opened and since the motor back e.m.f. opposes the line voltage the e.m.f. across the blades of the switch will not be very high; thus, little or no arcing will occur. The electromagnetic energy in the shunt field circuit will be dissipated in the armature resistance and as the armature slows down. This reduces the voltage in the circuit. The starter handle will be released by the holding coil and returns to the off position. The drawback to the three-point starter is that if the shunt field current were reduced below a certain value by the shunt field rheostat the holding coil would not be strong enough to hold the starter arm and it would return to the off position.

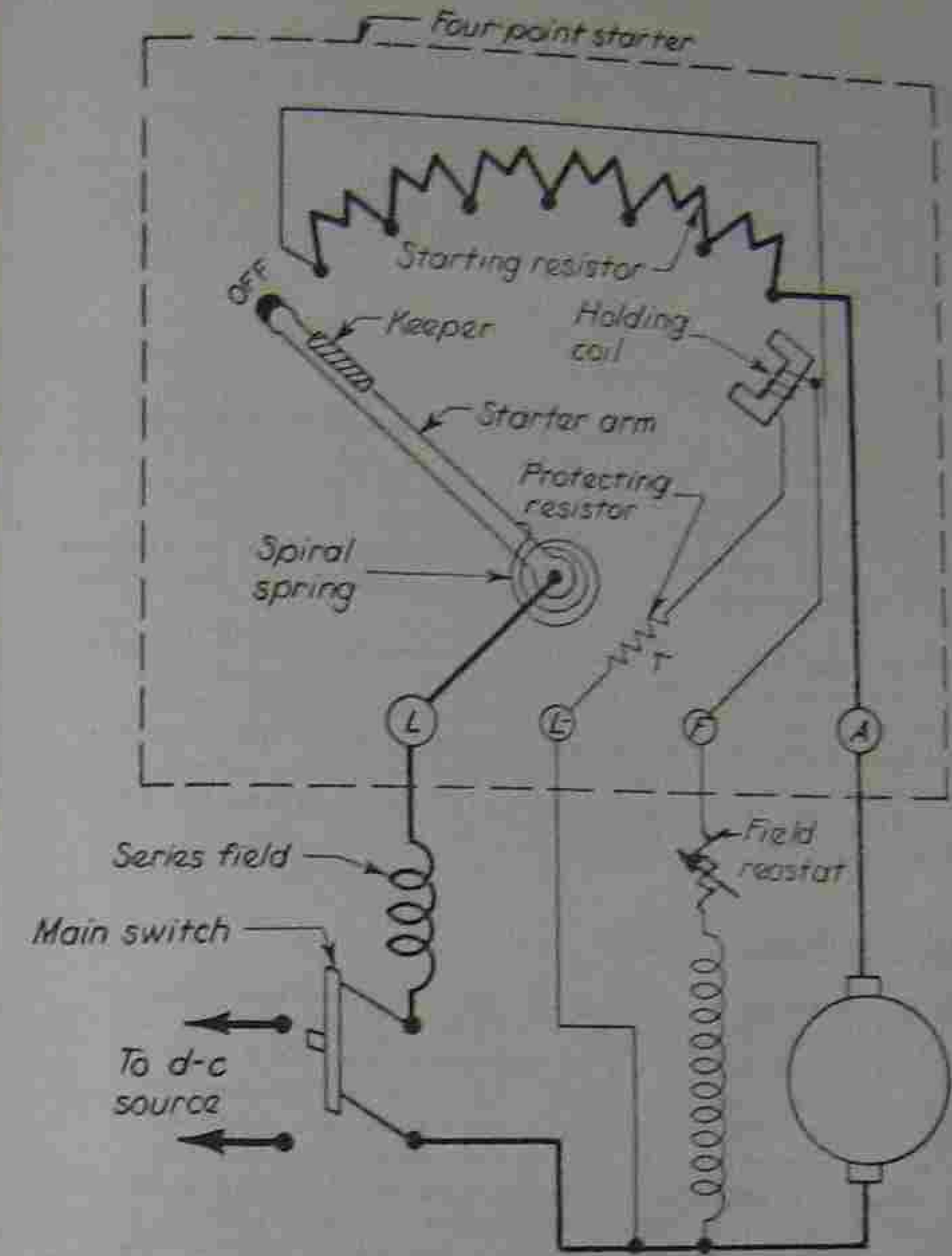


(a) Schematic diagram of 3-point starter.

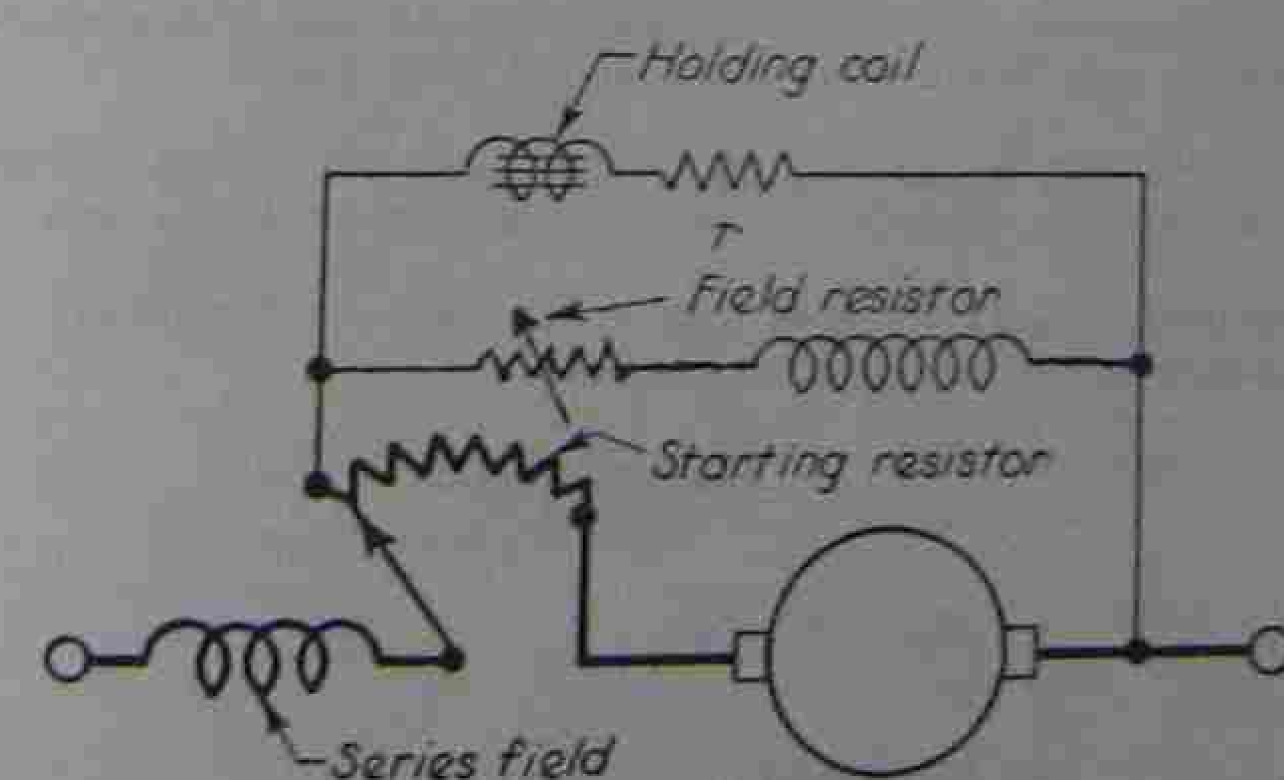


(b) Wiring diagram of 3-point starter.

Figure 10.5 - Schematic and Wiring Diagram for Three-point Starter.



(1) Wiring diagram of 4-point starter connected to short shunt compound motor.



(2) Schematic diagram of 4-point starter connected to short shunt compound motor.

Figure 10.6

To overcome this objection the holding coil is connected in parallel with the shunt field circuit and in series with a current limiting resistor. This necessitates bringing out another terminal (L_-).

The wiring diagram and schematic diagram for a four-point starter for a short shunt compound wound motor are shown in Figure 10.6.

10.5 SERIES MOTOR CONTROLLERS

When a starter has incorporated in it some means of controlling the speed of the motor it is called an electric controller. The main basic functions of the controller are control of acceleration, deceleration, speed and reversing. The controller is continuously rated as against the short time rating of the three and four point starters.

The series motor drum controller consists of a series of cams fixed to a central shaft, which can be moved in either direction from central off position. The cams make contact with spring loaded copper fingers or contacts as the drum revolves. Either baffles between contacts or magnetic blow out coils are used to control arcing at the contacts.

10.6 ACCELERATION OF D.C. MOTORS

When operating at full load, speed and output and assuming constant flux, $I_a R_a = V_t - E_g$ where E_g = armature back e.m.f. and I_a is dependent on the load. As stated earlier, at standstill E_g is zero because the armature is stationary and resistance must be inserted to limit the current. As the motor speeds up its back e.m.f. builds up and its current drops maintaining a value sufficient to overcome the load torque. In practice the armature resistance is cut out in a series of steps, thus obtaining a smooth start.

Example

The armature of a 230 volt shunt motor has a resistance of 0.82 ohm. and takes 28.2 amperes at full load:

(a) if I_a is not to exceed 150% of normal full load current at starting, calculate the starting resistance, and

(b) determine I_a if no starting resistance is inserted (assume brush drop three volts).

a. $$1.5 \times 28.2 = \frac{(230 - 3) - 0}{.82 + R_2}$$

$$R_2 = \frac{227}{42.3} - .82$$

$$= 4.55 \text{ ohms.}$$

b. $$I_a = \frac{230 - 3}{.82} = 277 \text{ amperes.}$$

The number of steps in a starting resistance and the resistance between each step may be determined as follows:

Assume that the initial value of the starting current is denoted by I_a and the current at full load is I_r. As the final resistance in the circuit will be that of the armature only the resistance on the final or $(n + 1)^{th}$ stud will be R_a.

At starting, the current in the armature circuit is limited to I_a by the total resistance of the armature circuit and the added starting resistor. If this total resistance is denoted by R_1 then

$$I_a = \frac{V_t}{R_1} \qquad (1)$$

Once current appears in the armature circuit the motor will commence to rotate and the generated e.m.f. will rise until

$$I_r = \frac{V_t - E_{g1}}{R_1} \qquad (2)$$

At this point the armature circuit resistance is reduced to R_2 such that

$$I_a = \frac{V_t - E_{g1}}{R_2} \qquad (3)$$

But from equation (2)

$$E_{g1} = V_t - I_r R_1 \qquad (4)$$

Substituting this value of E_{g1} in equation (3) will give:

$$I_a = \frac{V_t - V_t + I_r R_1}{R_2}$$

from which

$$\frac{I_a}{I_r} = \frac{R_1}{R_2} \qquad (5)$$

Similarly

$$\frac{I_a}{I_r} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \ldots = \frac{R_n}{R_{n+1}} \qquad (6)$$

but as $R_{n+1} = R_a$, from equation (6)

$$\left(\frac{I_a}{I_r}\right)^n = \frac{R_1}{R_2} \times \frac{R_2}{R_3} \times \ldots \times \frac{R_{n-1}}{R_a} \times \frac{R_n}{R_{n+1}}$$

(R_n) (R_a)

which gives

$$\left(\frac{I_{max}}{I_{min}}\right)^n = \left(\frac{I_s}{I_r}\right)^n = \frac{R_1}{R_a} \qquad - 10.3$$

and the general expression

$$\frac{I_s}{I_r} = \frac{R_k}{R_{k+1}} \qquad - 10.4$$

The resistance between any two studs is given by

$$R_{k-(k+1)} = R_k - R_{k+1} \qquad - 10.5$$

If both sides of equation 10.3 are multiplied by the term in the brackets on the left hand side of the expression the result is:

$$\left(\frac{I_{max}}{I_{min}}\right)^n = \left(\frac{I_s}{I_r}\right)^{n+1} = \frac{I_s}{I_r}\frac{R_1}{R_a}$$

but as $I_s R_1 = V_t$ as may be seen by referring to equation (1)

Then

$$\left(\frac{I_{max}}{I_{min}}\right)^{n+1} = \left(\frac{I_s}{I_r}\right)^{n+1} = \frac{V_t}{I_r R_a}$$

This eliminates the need to specifically determine R_1 prior to calculating the number of steps. The minimum current I_r is dictated by the torque necessary to accelerate the load.

It may be preferable in some circumstances to refer to the currents involved as I_{max} and I_{min} in preference to I_r and I_s.

Example 10.8

The resistance an armature of a 240 volt d.c. shunt motor is 0.5 ohms. Assuming that it is required that the current at starting be limited to 200% of the full load current and the full load current is 15 amperes determine:

(a) the total resistance of the armature circuit at starting;

(b) the number of studs on the starter;

(c) the resistance between each stud.

Solution

(a) $I_s = 2 \times I_R = 2 \times 15$

$= 30$ amperes

$R_1 = \frac{V_t}{I_s} = \frac{240}{30}$

$= 8$ ohms

(b) From equation 10.3

$$\left(\frac{I_s}{I_R}\right)^n = \frac{R_1}{R_a}$$

$$\left(\frac{30}{15}\right)^n = \frac{8}{0.5}$$

$$2^n = 16$$

Resistor Div from which $n = 4$.

$$\left(\frac{I_s}{I_r}\right)^{n+1} = \frac{V_t}{I_r R_a}$$

$$\left(\frac{30}{15}\right)^{n+1} = \frac{240}{15 \times 0.5}$$

$$2^{n+1} = 32$$

$n+1 = 5$ = (Contactors Studs)

$n = 4$ = Res. Div

Hence the number of studs will be 5.

(c) From equation 10.4

$$R_2 = \frac{R_1 I_r}{I_s} = \frac{8 \times 15}{30} = 4\,\Omega$$

The resistance between studs 1 and 2 will be: (using equation 10.5)

$$R_{1-2} = R_1 - R_2 = 8 - 4$$

$$= 4\,\Omega$$

Similarly

(i) $R_3 = \frac{R_2 I_r}{I_s} = \frac{4 \times 15}{30} = 2\,\Omega$

and $R_{2-3} = R_2 - R_3 = 2\,\Omega$

(ii) $R_4 = \frac{R_3 I_r}{I_s} = \frac{2 \times 15}{30} = 1\,\Omega$

and $R_{3-4} = R_3 - R_2 = 2 - 1 = 1\,\Omega$

(iii) $R_5 = \frac{R_4 I_r}{I_s} = \frac{1 \times 15}{30} = 0.5\,\Omega$

and $R_{4-5} = R_4 - R_5 = 1 - 0.5 = 0.5\,\Omega$

The arrangement of the circuit is shown in Figure 10.7.

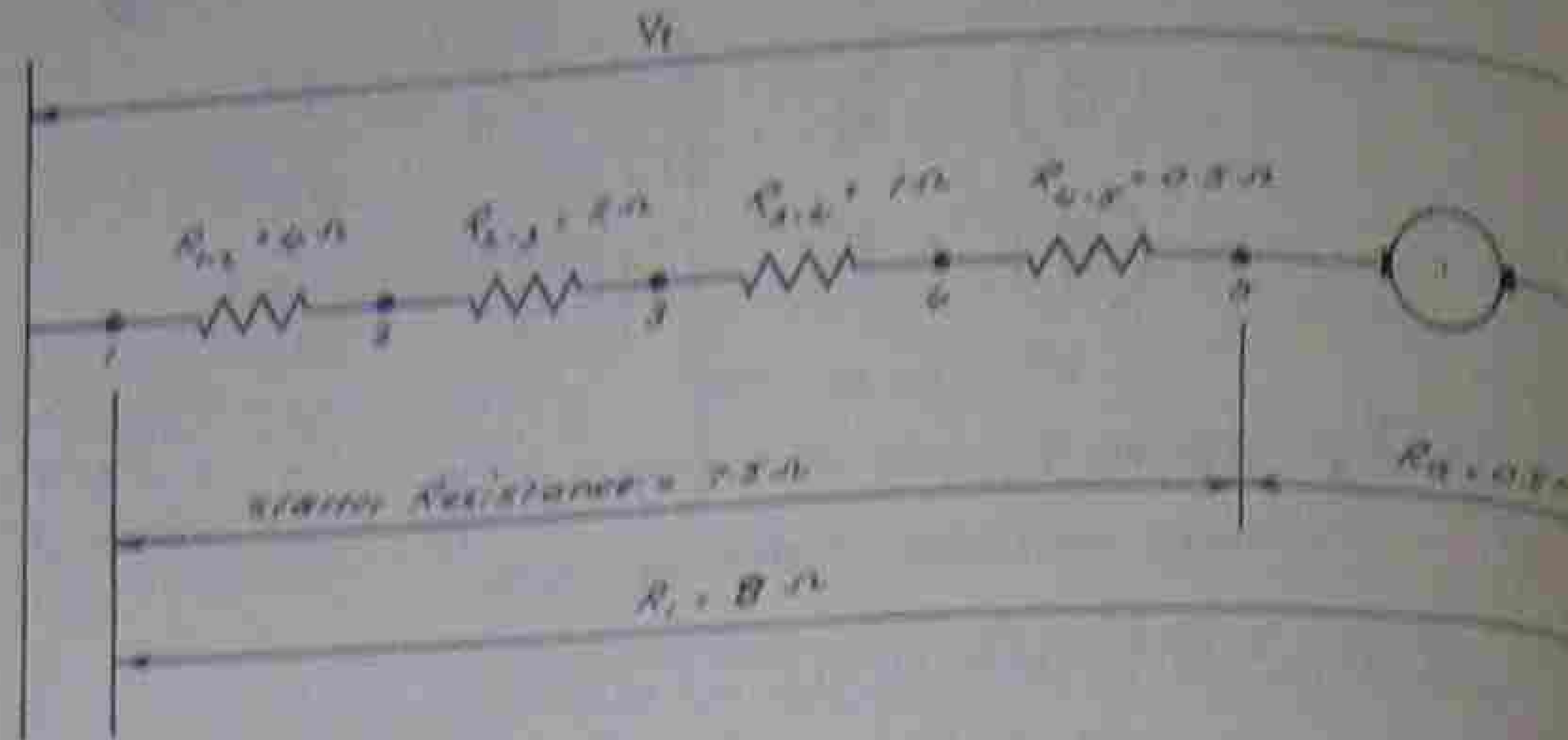


Figure 10.7 - Diagram for Example 10.8

As an alternative to the calculation of the number of steps and the resistance between each stud the information can be obtained by graphical construction.

The procedure may be summarised as follows, the actual construction appearing in Figure 10.8 where the example used is the motor given in Example 10.8.

Step 1 - Determine the value of the resistance of the armature circuit at starting and draw this to scale as the base axis. In this case the resistance is 8 ohms. Note that the resistance axis is calibrated from right to left along the axis.

Step 2 - Along the left hand vertical axis mark off to scale the starting and running current values, drawing lines through these points parallel to the base line.

Step 3 - Draw a diagonal from the origin of the resistance scale to the I_R point on the current scale. The intercept point when this diagonal cuts the line drawn through the I_r point will give the value of the resistance between the first and second studs.

Step 4 - From the intercept on the I_R line obtained in Step 3 draw a vertical line to cut the I_R line and draw a line joining this point of intersection with the origin of the resistance scale.

Step 5 - The second line drawn in Step 4 will cut the I_r line, the distance between this intercept and that obtained previously representing the resistance between the second and third studs.

The process can now be repeated until the line drawn from the intercept with the I_R line and the resistance origin either passes through the point representing the armature resistance or intersects the I_r line at a value less than R_a.

Figure 10.8 - Graphical Determination at Starting Resistance Details.

REVIEW QUESTIONS

(1) What factors are to be considered when selecting a motor for a given task?

(2) Describe how the mid-ordinate method can be used to determine:

(a) the average value,

(b) the R.M.S. value,

of a non-linear function.

(3) What effect does the operation of a motor in the driving mode or the braking mode have on:

(a) the average value,

(b) the R.M.S. value,

of the power rating of a motor?

(4) Why is it necessary for added resistance to be included in the armature circuit of a d.c. motor when the motor is started? How does the inclusion of this added resistance affect the motor starting torque?

(5) What is the essential difference between three-point and four-point manual starters?

(6) Sketch the schematic diagram for a three-point starter.

(7) Derive equation 10.3.

(8) In what important output do the resistors used in starters and controllers vary?

ASSIGNMENTS

Marks

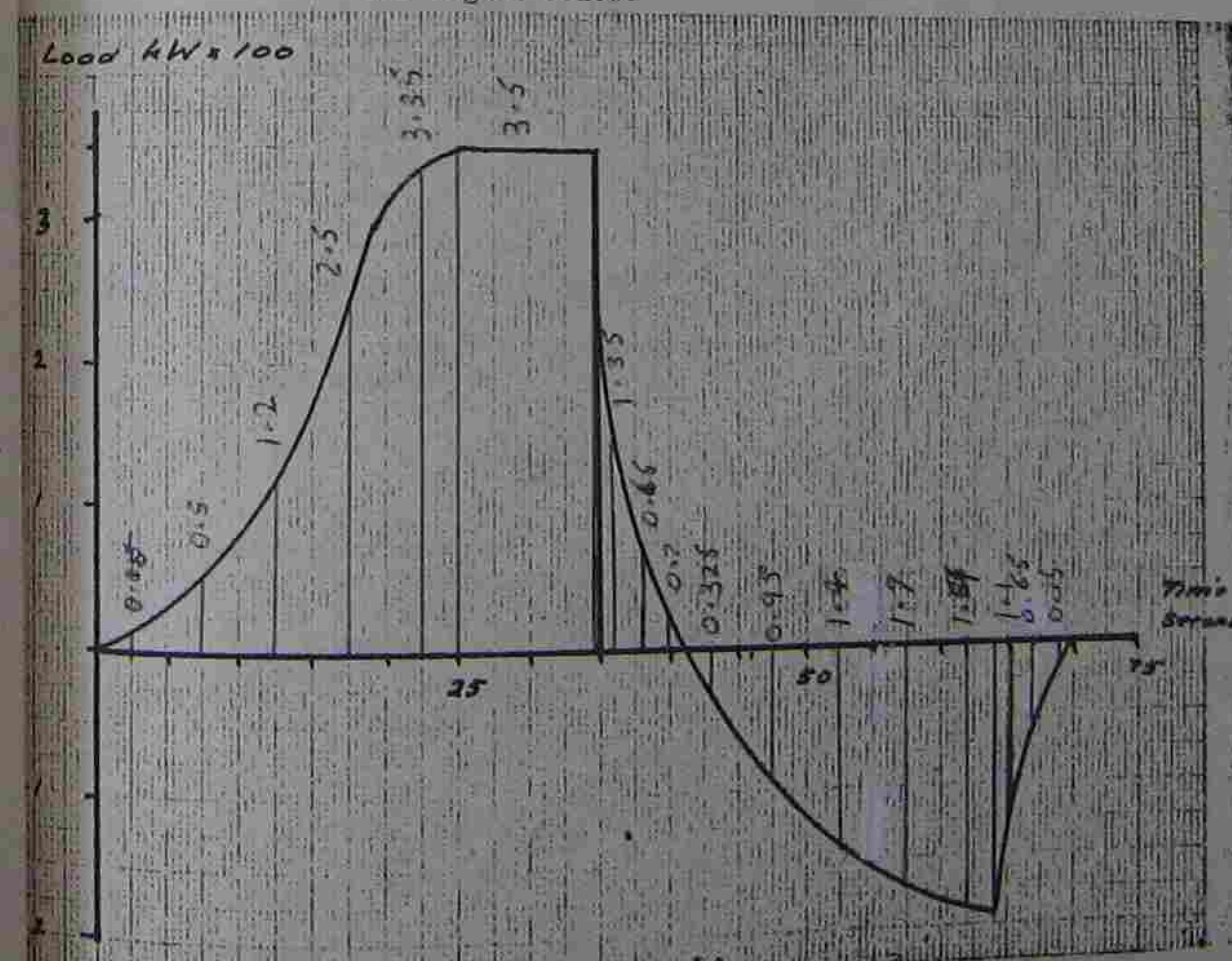
15 1. Use the methods given in this unit to obtain the average and the R.M.S. value of a sinusoidal wave having a maximum value of 100 kilowatts.

25 2. (a) Determine the R.M.S. power rating of a motor to perform the following duty cycle:

Time	Load (kW)
0 - 5	5
5 - 15	100
15 - 20	40
20 - 25	-60 (braking)
25 - 50	-30 (braking)
50 - 60	-20 (braking)

(b) Compare the R.M.S. value of power with the average power output of the motor.

20 3. Determine the R.M.S. value of the power shown in the figure below:



Marks

15 4. The armature and shunt field resistance of a 230 volt shunt motor are 0.28 and 153.5 ohm, respectively. If the starting resistance is 3.22 ohm, calculate:

(a) The total line current at instant of starting, assuming brush drop to be three volts.

(b) The total line current if no starting resistance is used.

45.6 (60HP)

25 5. The armature resistance of a 456 kW 240 volt shunt motor is 0.067 ohm and the field resistance is 40 ohms. It is desired that the motor develop 150% full load torque at starting.

(a) Calculate the total starter resistance, assuming full load motor efficiency of 83.8% and a brush drop of three volts.

(b) Determine the number of resistance steps in the starter and the resistance between each step if the lower current limit is 100% of full load current.

ELECTRICAL MACHINES I

UNIT NO. 11

11.1 Introduction.

11.2 Acceleration of d.c. motors.

11.3 Automatic starters.

11.4 Back e.m.f. starter.

11.5 Series lockout (current dependent) starter.

11.6 Time limit starters.

11.7 Braking.

Review questions.

Assignments.

11.1 INTRODUCTION

As the power is supplied to a motor the armature will accelerate up to the equilibrium speed where the developed torque is equal to the load torque. The way in which the speed builds up will depend on the method of starting and the reduction in torque as a consequence of the reduction in starting current. The methods of producing a deceleration in motor speed electrically will also be discussed and the considerations of current limiting with the various methods of braking.

11.2 ACCELERATION OF D.C. MOTORS

The current in the armature of a motor circuit is given by the expression.

$$I_a = \frac{V_t - E_g}{R_a} \qquad - 11.1$$

where in this case, for a motor where a starter is used to limit the starting current, the armature circuit resistance is the resistance of the armature and the added resistance. Thus equation 11.1, for the case where a starter is used becomes

$$I = \frac{V_t}{R_1} - \frac{E_g}{R_1} \qquad - 11.2$$

as $$\frac{V_t}{R_1} = I_s \qquad - 11.3$$

The general expression for the armature current becomes

$$I_a = I_s - \frac{1}{R_1} \times E_g \qquad - 11.4$$

Equation 11.4 is a linear expression and can be plotted as shown in Figure 11.1.

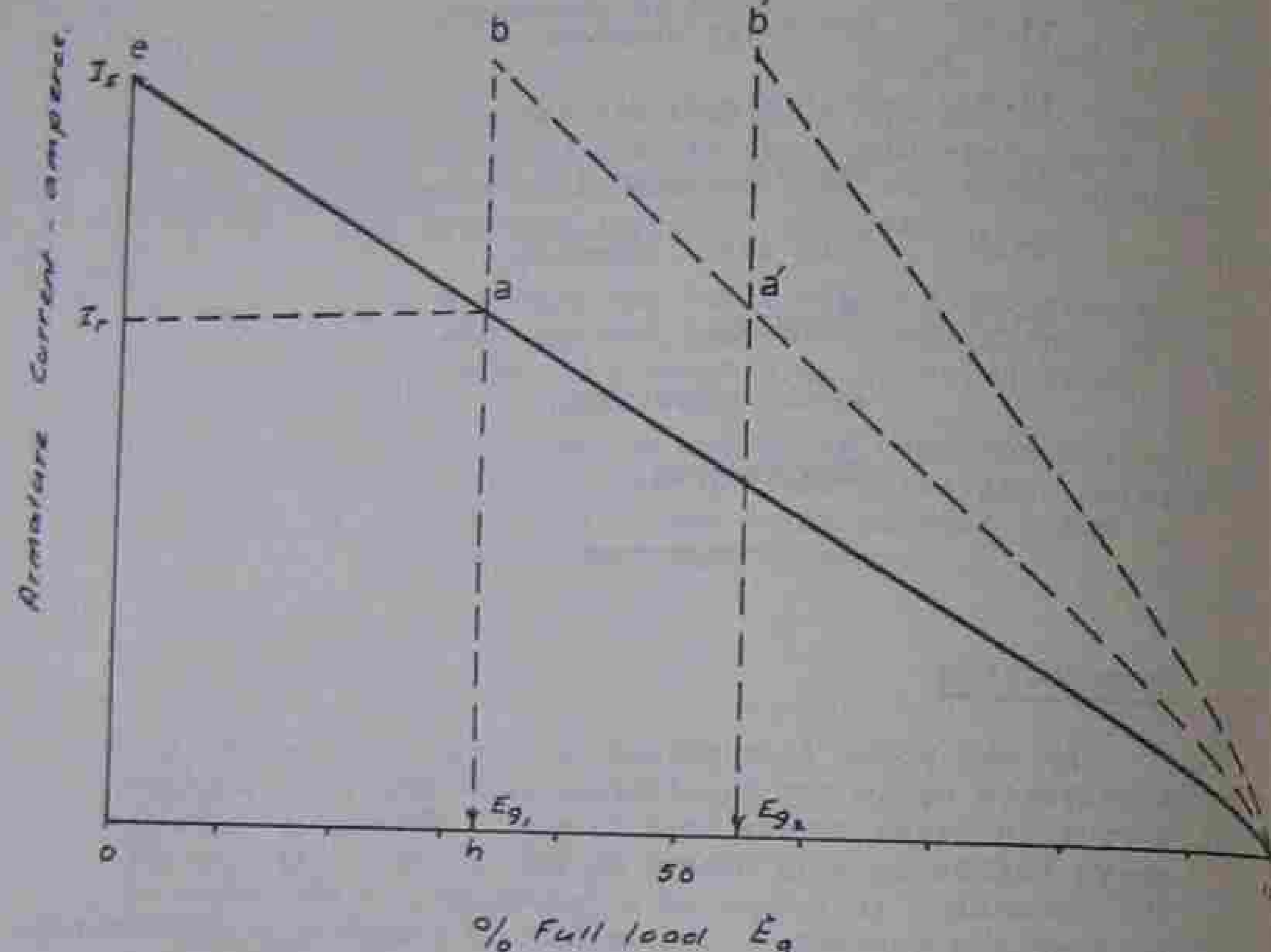


Figure 11.1 - Variation of Armature Current with Generated e.m.f.

The curve representing equation 11.4 will reach a point where I_a is equal to the full load current, I_r; at the point

Putting I_a equal to I_r in equation 11.4 will give

$$I_r = I_s - \frac{1}{R_1} E_{g1}$$

from which

$$E_{g1} = R_1 (I_s - I_r)$$

If, at this point, the resistance of the armature circuit is reduced to R_2 such that

$$I_s = \frac{V_t - E_{g1}}{R_2}$$

the armature current will rise to the initial starting current. The equation for the new expression for armature current will be:

$$I_a = \frac{V_t}{R_2} - \frac{1}{R_2} E_g \qquad - 11.5$$

At point "b" on the diagram of Figure 11.1, the new value of armature current will be I_s and the value of E_g will be E_{g1}. Substituting these values in equation 11.5 will give:

$$I_s = \frac{V_t}{R_2} - \frac{R_1}{R_2} (I_s - I_r)$$

since $\quad E_{g1} = R_1 (I_s - I_r)$

but $\quad V_t = I_s R_1$ (from 11.3)

therefore $\quad I_s = \frac{I_s R_1}{R_2} - \frac{I_s R_1}{R_2} + \frac{I_r R_1}{R_2}$

$$= \frac{I_r R_1}{R_2}$$

from which $\quad \frac{I_s}{I_r} = \frac{R_1}{R_2}$

This is a result obtained previously in Unit 10. Referring this result to the triangle Oef in Figure 11.1 shows that as

$$\frac{Of}{hf} = \frac{I_s}{I_r}$$

then if the length of the side Of of the triangle is calibrated to correspond to the resistance of R_1 the length hf will correspond to the resistance of R_2. This is the justification of the graphical method for the determination of starting resistance given in Unit 10.

The procedure can be continued to obtain the values of R_3 and R_4 and any further values of resistance required and, at the same time, the value of the generated e.m.f. at any particular stud can be obtained directly from the base calibrations.

Since in shunt motors where the flux is assumed to be constant the base line of Figure 11.1 can be calibrated in terms of the no-load speed. Figure 11.2 (a) shows the arrangement of a typical starting circuit where a five stud starter is used. The diagram of Figure 11.2 (b) shows the modification of Figure 11.1 for the starter of 11.2 (a) where the base line has been calibrated in terms of the no-load speed.

As the motor accelerates the armature current will decrease due to the increase in the generated e.m.f. As a consequence of the decreasing current the accelerating torque, the difference between the developed torque and the load torque will decrease. If the moment of inertia of the rotating system is assumed to be constant the increase in speed with respect to time will be exponential. The exact relationship will depend on the torque speed load curve of the combined load and motor frictional torque. As a consequence, the armature current versus time and the armature speed versus time curves will have the form shown in Figure 11.2 (c) and (d). The actual values of speed reached at the end of any given interval is given by the expression:

$$N = \frac{V_t - I_r R_n}{V_t - I_r R_a} \times N_{FL} \qquad - 11.6$$

where I_r is the full load armature current

R_a the resistance of the armature circuit alone

R_n the resistance of the armature circuit and starting resistance at the specified stud.

and N_{FL} the full load speed (rpm)

Example 11.1

The full load speed of the armature of a 240 volt motor is 1000 rpm and the resistance of the armature circuit is 0.25 ohms. If the full load current is 40 amperes calculate the speed for $I_a = I_{FL}$ when the total armature circuit resistance is 3.125 ohms.

Solution

From equation 11.6

$$N = \frac{240 - 40 \times 0.25}{240 - 40 \times 3.125} \times 1000$$

$$= \frac{240 - 10}{240 - 125} \times 1000$$

$$= 500 \text{ r.p.m.}$$

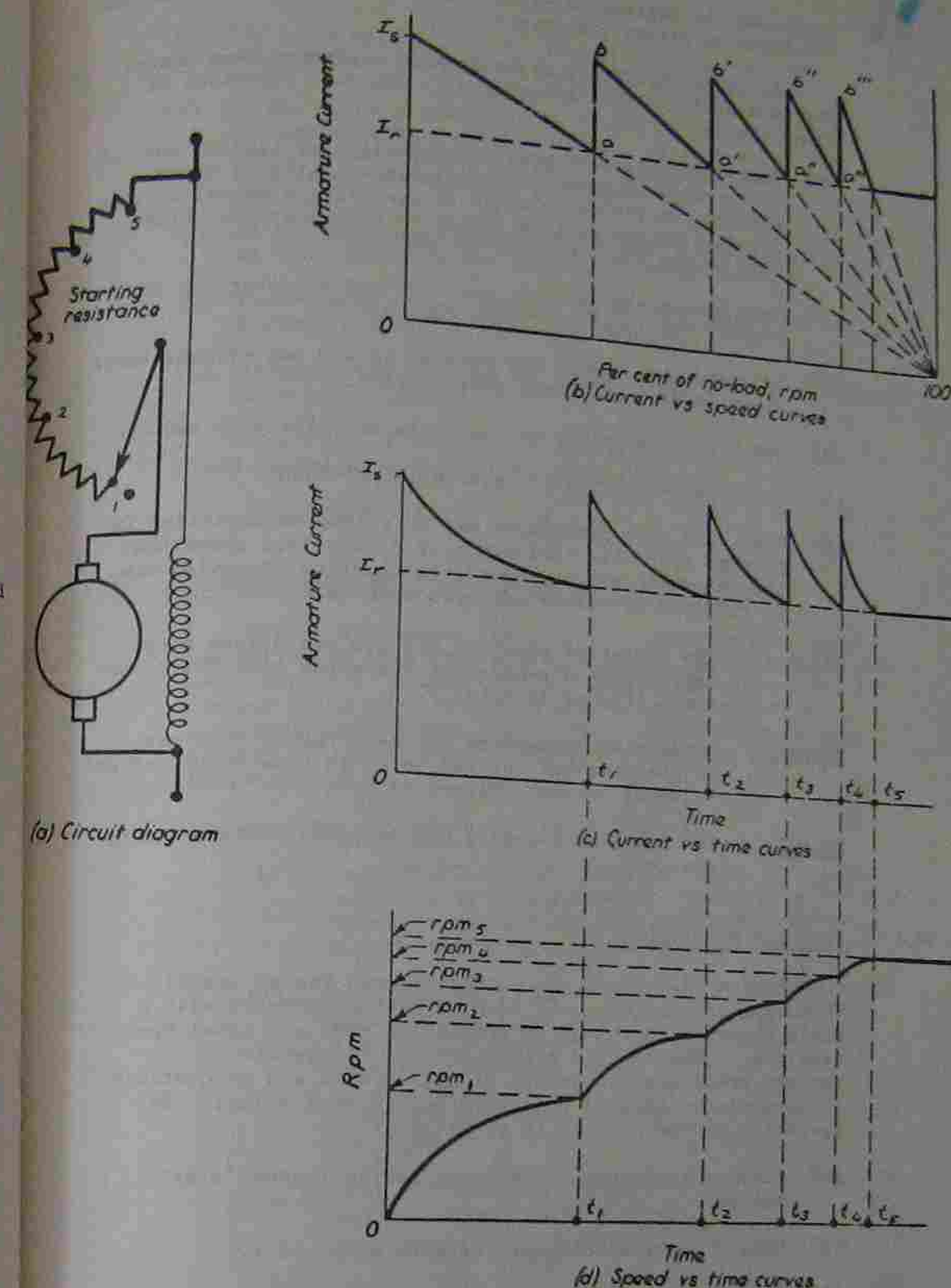


Figure 11.2 - Curves for Motor Starting Data.

11.3 AUTOMATIC STARTERS

An automatic starter is one where the starting sequence is not controlled by the operator but once the sequence has commenced the control is exercised by conditions either within the motor or within the starter.

Automatic starters, although more expensive than manual starters offer several advantages. The most important of these are:

(1) They ensure correct start irrespective of loading and can be operated by non-technical personnel after a short instruction period, whereas if operating a manual starter, the main switch must be closed or opened at the correct time or the starter could be damaged.

(2) Push button stations for starting and stopping can be located for local or remote control.

(3) Adjustment of timing for cutting in and out of resistance steps ensures smoothest acceleration.

(4) Electric braking may be incorporated relatively easily.

(5) Overload protection is invariably provided.

(6) Start and stop operations may be automatic assisting in process control. Reference to Figure 11.2 shows that the automatic sequencing of the starting operation can be effected by either of three methods.

<u>Firstly</u> the contactor shorting out the resistance steps can be made dependent on the motor back e.m.f. or speed (Figure 11.2 (a)).

<u>Secondly</u> successive contactors can be arranged to close as the armature current falls to some predetermined value of current (Figure 11.2 (b)).

<u>Thirdly</u> the starting sequence can be operated by a timing device.

11.4 BACK E.M.F. STARTER

Figure 11.3 shows the wiring diagram for an example of this type of starter. It is push button operated with a field rheostat which may be present to give any final speed required. The starter relays are set to operate at various predetermined values of back e.m.f. and an overload relay operates above a predetermined current value. The operation of the starter is as follows:

(1) Close main switch and depress spring loaded 'start' button.

(2) Coil M picks up closing contacts s, m and r.

(3) Closing 's', coil M maintains itself and the start button can be released.

(4) The armature and shunt field circuits are energised.

(5) The volts drop across R_C, R_B and R_A actuates field relay F short circuiting the rheostat.

(6) As the motor accelerates E_g builds up and when the voltage drop across R_A falls to a certain value coil A operates and relay contacts 'a' short circuits the section of the starting resistance R_A.

(7) As the motor accelerates the relays b and c operate in turn until all the resistance is shorted out.

(8) The voltage across relay F is now negligible and relay F opens putting in the field rheostat.

(9) Pressing the stop button demagnetises M causing contacts s, m and r to open.

This type of starter is used with small and medium sized motors having light starting torques.

(See Figure 11.3)

11.5 CURRENT DEPENDENT STARTER (SERIES "LOCKOUT")

Figure 11.4 shows the wiring diagram for a typical controller of this type where the shunt contactors are held open, that is, "locked out" by the series relays until the current falls to a predetermined value. The shunt contactor then closes, short circuiting the appropriate section of the starting resistances together with the series relay.

The sequence of operations is as follows:

(i) Press START button. Coil "a" picks up closing contactor 1, auxiliary contacts X and A. (Local circuit through X and A provides maintaining circuit short circuiting start button.)

(ii) As contactor 1 closes armature picks up through series relay 1 and resistance R_1, R_2 and R_3. High initial current in series relay 1 opens auxiliary contacts (shown "normally closed"). As armature accelerates, current falls to predetermined level allowing auxiliary contacts to close.

(iii) Coil "b" picks up through auxiliary contacts on series relay 1 closing shunt contactor 2. R_1 and series relay 1 short circuited.

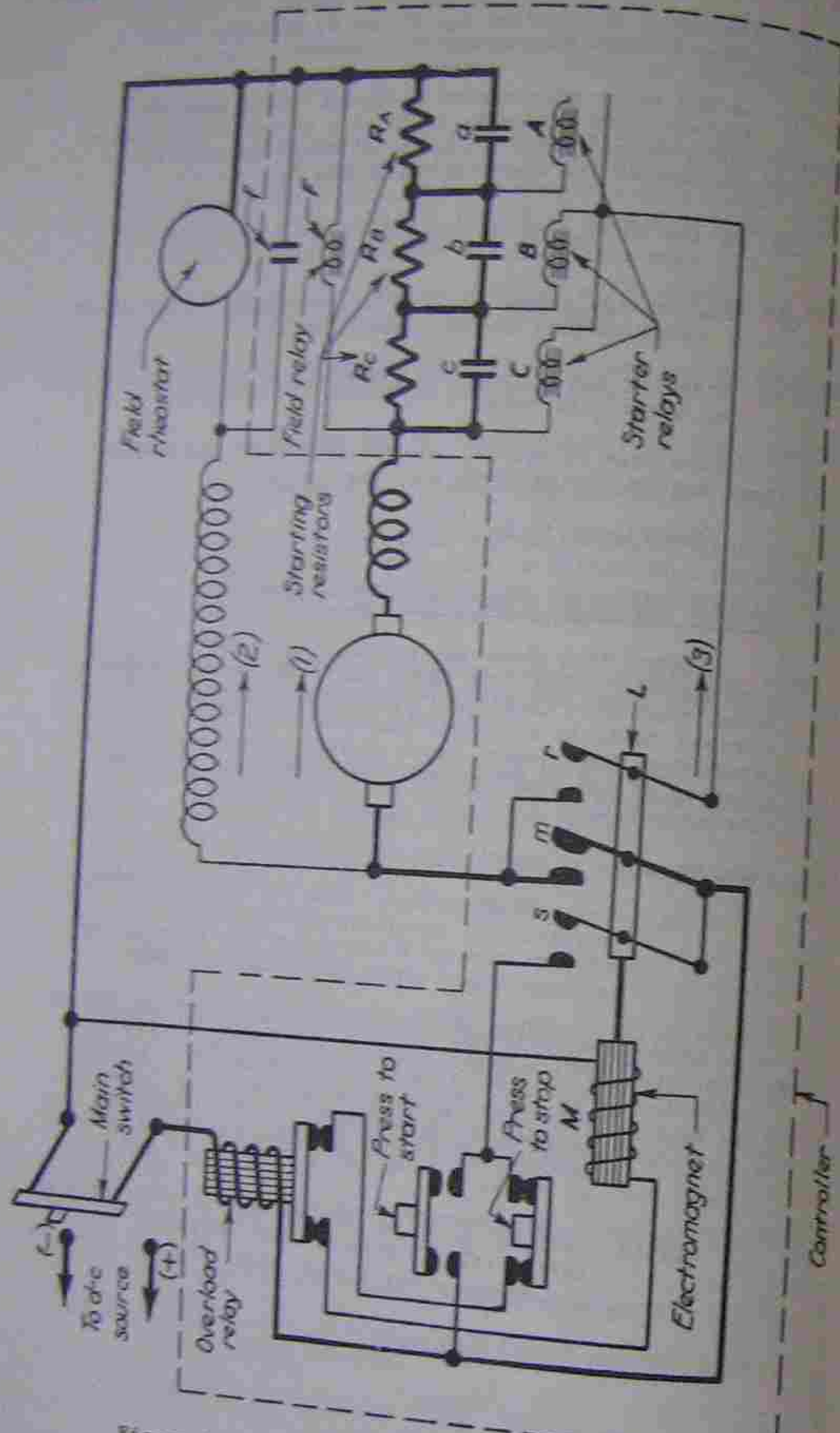


Figure 11.3 - Wiring diagram showing counter e.m.f. type automatic starter connected to a long shunt compound motor.

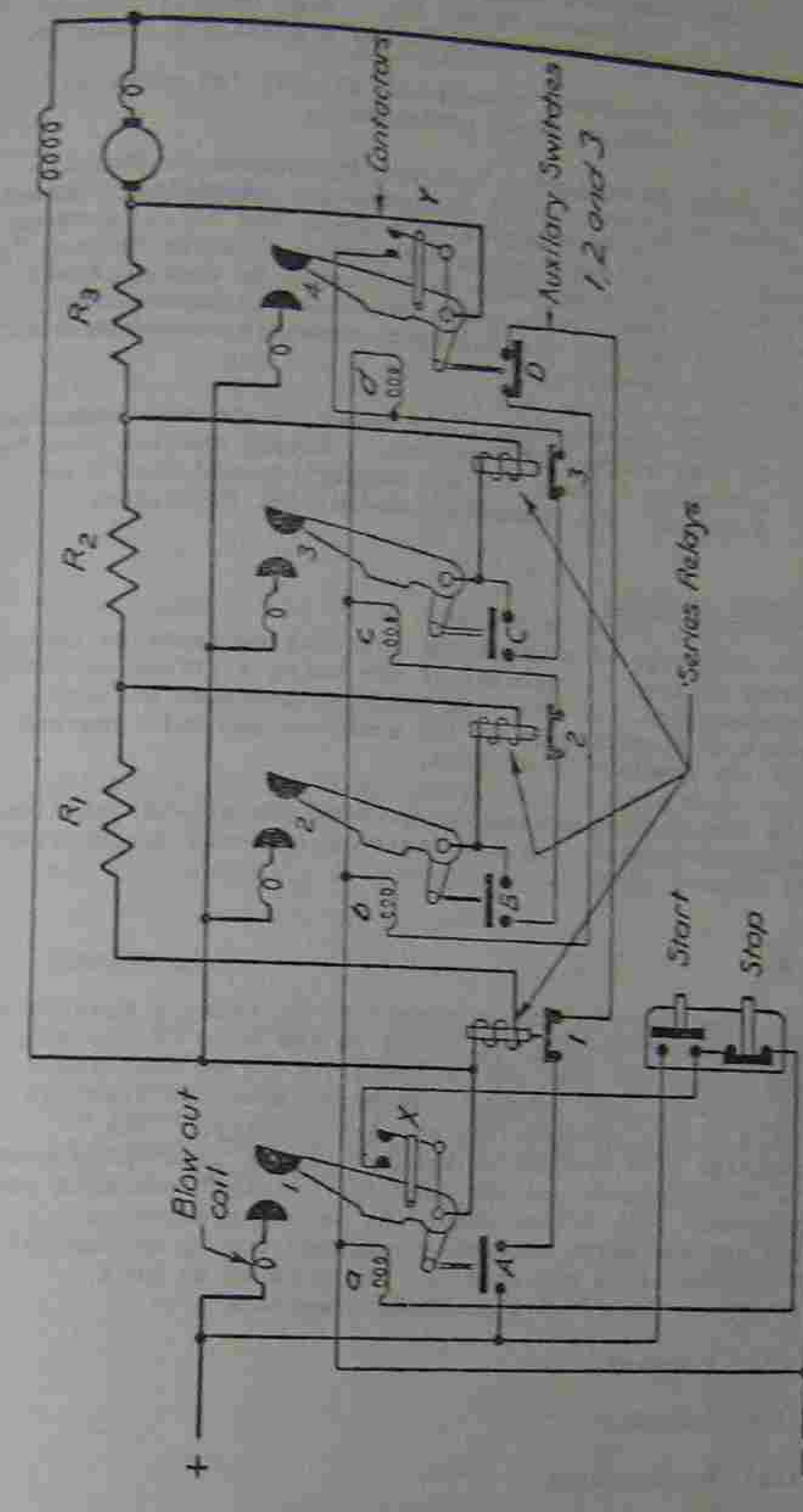


Figure 11.4 - Series Lockout Starter.

(iv) Armature picks up through series relay 2 and resistance R_2 and R_3. High initial current holds open series relay 2 auxiliary contacts.

(v) Procedure repeated until coil "d" picks up closing shunt contactor 4.

(vi) As contactor 4 closes R_3 shorted out supply is now direct to armature. Auxiliary contact Y closes providing hold-in for "d". D opens cutting supply from potential coils "b" and "c" allowing contactors 2 and 3 to open. Final conditions, contactors 1 and 4 maintained closed, 2 and 3 open, armature direct-on-line (d-o-l).

Note: Circuit provides for field d-o-l on closing contactor 1. To stop - Press STOP button. Supply removed from "a" contactor 1 drops out and supply removed from d as A opens. All contactors in initial condition.

11.6 TIME LIMIT STARTERS

In this type of starter the starting sequence is controlled by timing devices independent of the motor starting conditions. As a consequence, the starting sequence proceeds without reference to acceleration of the armature and will proceed even if the armature is stalled.

As this could cause damage to both the starter and the machine, some overcurrent sensing device should be incorporated in the control circuit.

11.7 BRAKING

In some cases it may be necessary to cause a machine to decelerate more rapidly than would be the case if the only braking torque was that due to the friction of the system or the ordinary load torque. In such cases the load can be braked either mechanically or electrically. The disadvantage with mechanical braking is the inevitable wear of the friction pads and the need for constant checking and maintenance. The better method is to use electrical braking systems where necessary supplemented by mechanical braking where it is required for the load to be held stationary. The electrical braking methods are:

(i) Plugging.

(ii) Dynamic.

(iii) Regenerative.

(a) Plugging

With plugging, the supply to the armature is reversed so that the expression for the armature current becomes:

$$I_a = \frac{-V_t - E_g}{R_a}$$

from which

$$I_a = \frac{-(V_t + E_g)}{R_a} \qquad - \quad 11.7$$

The torque thus produced opposes the inertial torque of the load and the rotating system. However, since the reversal of the supply to the armature results in the e.m.f. acting on the armature circuit being the sum of terminal voltage and the back e.m.f. the current is usually limited during braking operations by a series resistor. The resistance of the resistor is such that the initial braking current is limited to about 1.5 to 1.8 times that of the full load current. If the brush drop is considered, expression 11.7 becomes:

$$I_a = \frac{-(V_t + E_g - V_b)}{R_a} \qquad - \quad 11.8$$

At standstill the motor develops a torque due to the current,

$$I = \frac{-V_t - V_b}{(R_a + R_{br})} \qquad - \quad 11.9$$

and would, if not immediately disconnected from the supply, start to accelerate in the opposite direction. To prevent this the machine can be mechanically braked at standstill and automatically de-energised.

Example 11.2

A 240 V motor is required to be braked by plugging. If the armature current at full load is 100 amperes and the armature resistance is 0.1 ohms calculate the resistance of the braking resistor if the initial braking current is to be limited to 150% of the full load current and the brush drop is taken to be 5 volts.

Solution

At full load $E_g = V_t - V_b - I_a R_a$

$= 240 - 5 - 100 \times 0.1$

$= 225$ volts.

For braking the current must be limited to:

$I_{br} = I_a \times 1.5$

$= 100 \times 1.5$

$= 150$ amperes

Total resistance in armature circuit is then

$$R = \frac{V_t - V_b + E_g}{I_{br}}$$

$$= \frac{240 - 5 + 225}{150}$$

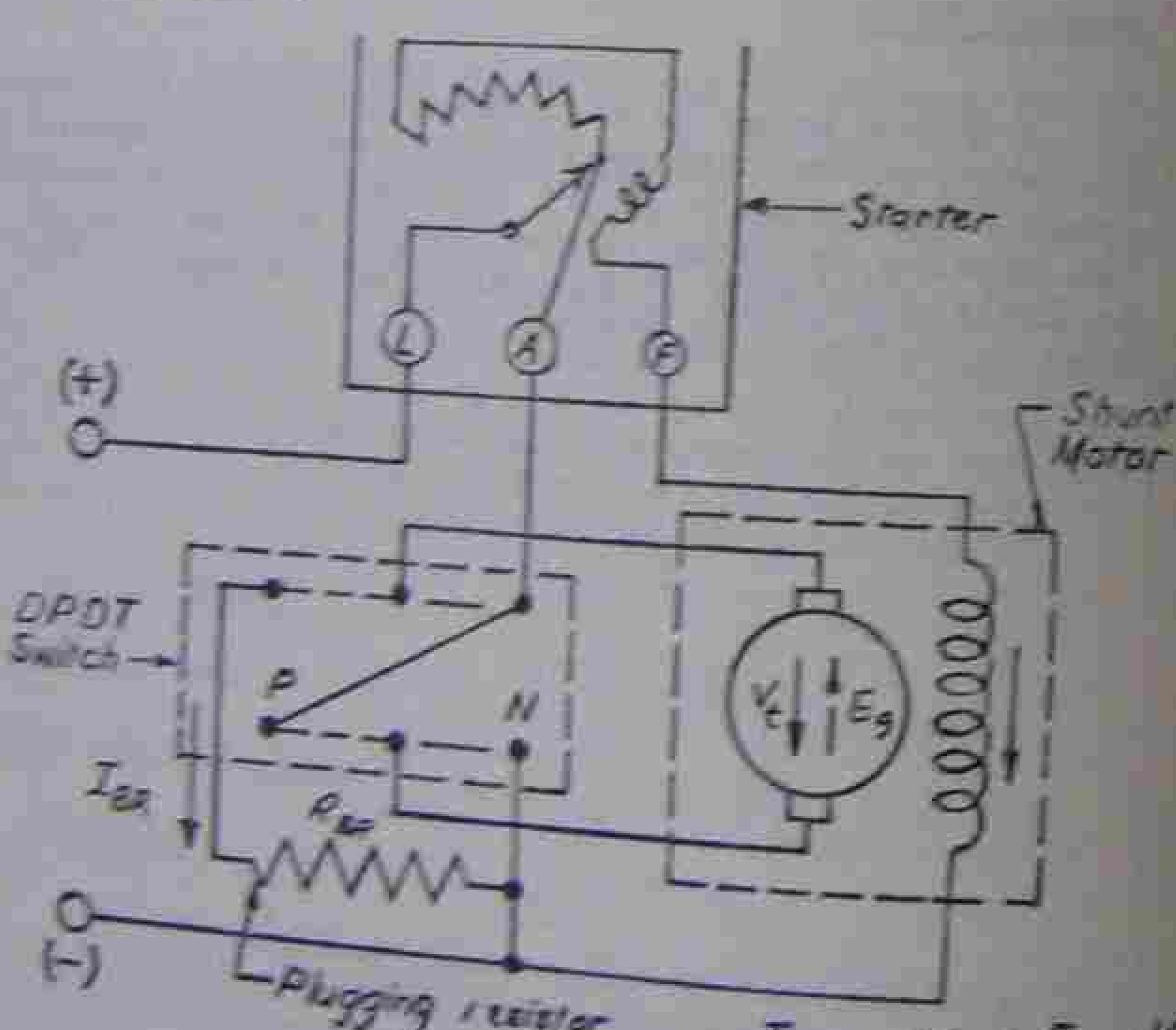
$$= \frac{460}{150}$$

$= 3.05$ ohms

$\therefore$ Added Resistance $= 3.05 - 0.1$

$= 2.95$ ohms

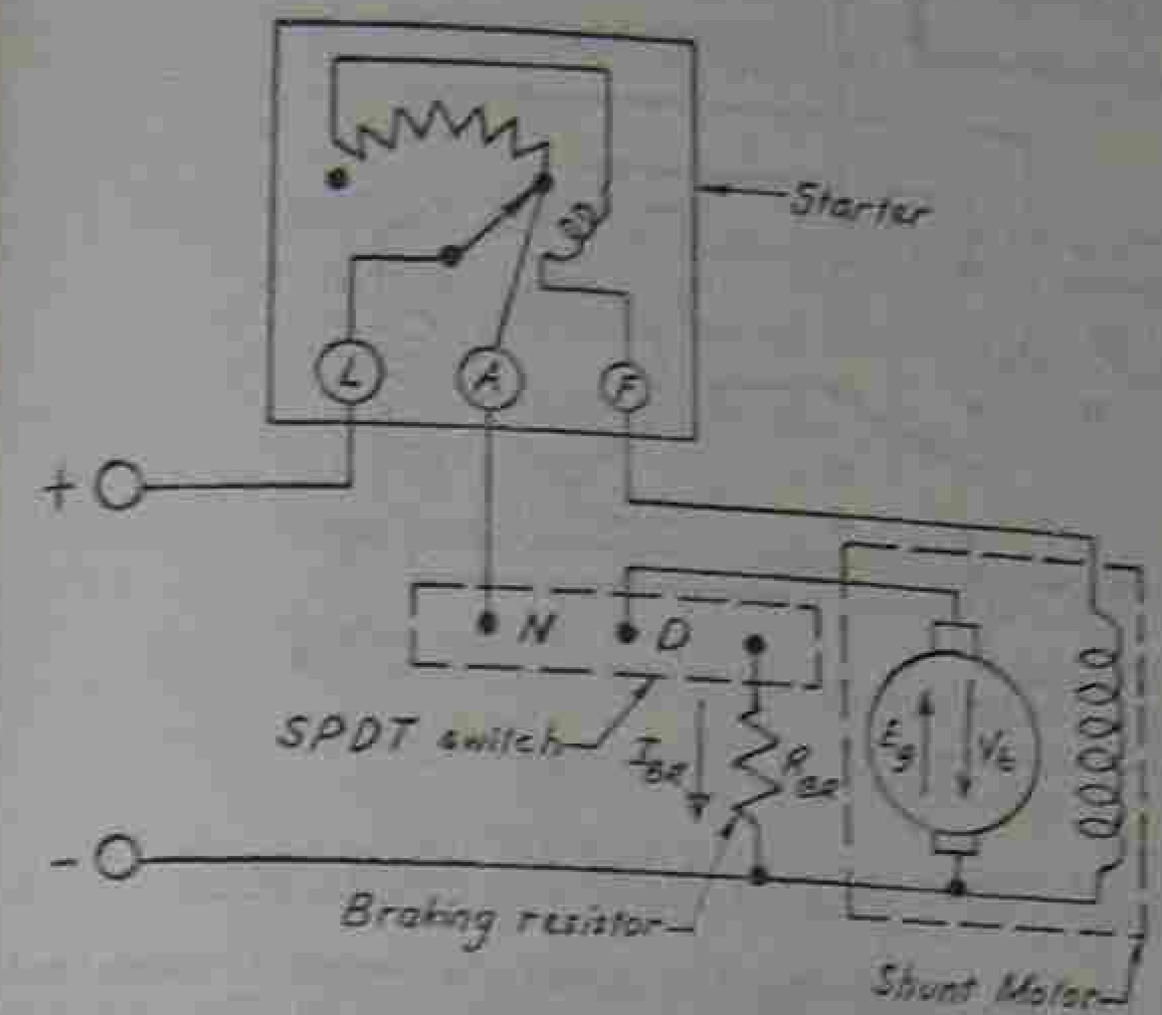
Figure 11.5 shows a typical circuit for braking a motor by plugging.



$$J_{BR} = \frac{V_t + E_g - V_b}{R_{BR} + R_a}$$

Figure 11.5 - Braking Circuit - Plugging.

(b) Dynamic Braking



$$I_{BR} = \frac{E_g - V_b}{R_{BR} + R_a}$$

Figure 11.6 - Dynamic Braking of Shunt Motor.

In this system the motor armature is disconnected from the supply and a resistance of comparatively low value is inserted across its terminals. The motor field is kept energised. The motor will now act as a generator supplying energy to the resistance, until all the kinetic energy in the armature has been dissipated as $I^2 R$ loss in the resistance.

As the motor slows down the motor back e.m.f. which has become the generator e.m.f., reduces and, at very low speeds produces only a small braking current in the resistance. It is therefore necessary to fit a mechanical brake to bring the motor to rest. Figure 11.5 illustrates the circuit for dynamic braking of a shunt motor.

In the case of a series motor two alternative systems can be used. Firstly the series field can be connected across the supply line in series with a current limiting resistance. This system is wasteful in energy. Alternatively the series field can be connected to carry the armature current in the same direction as the flow of current in the motoring state. Reversed.

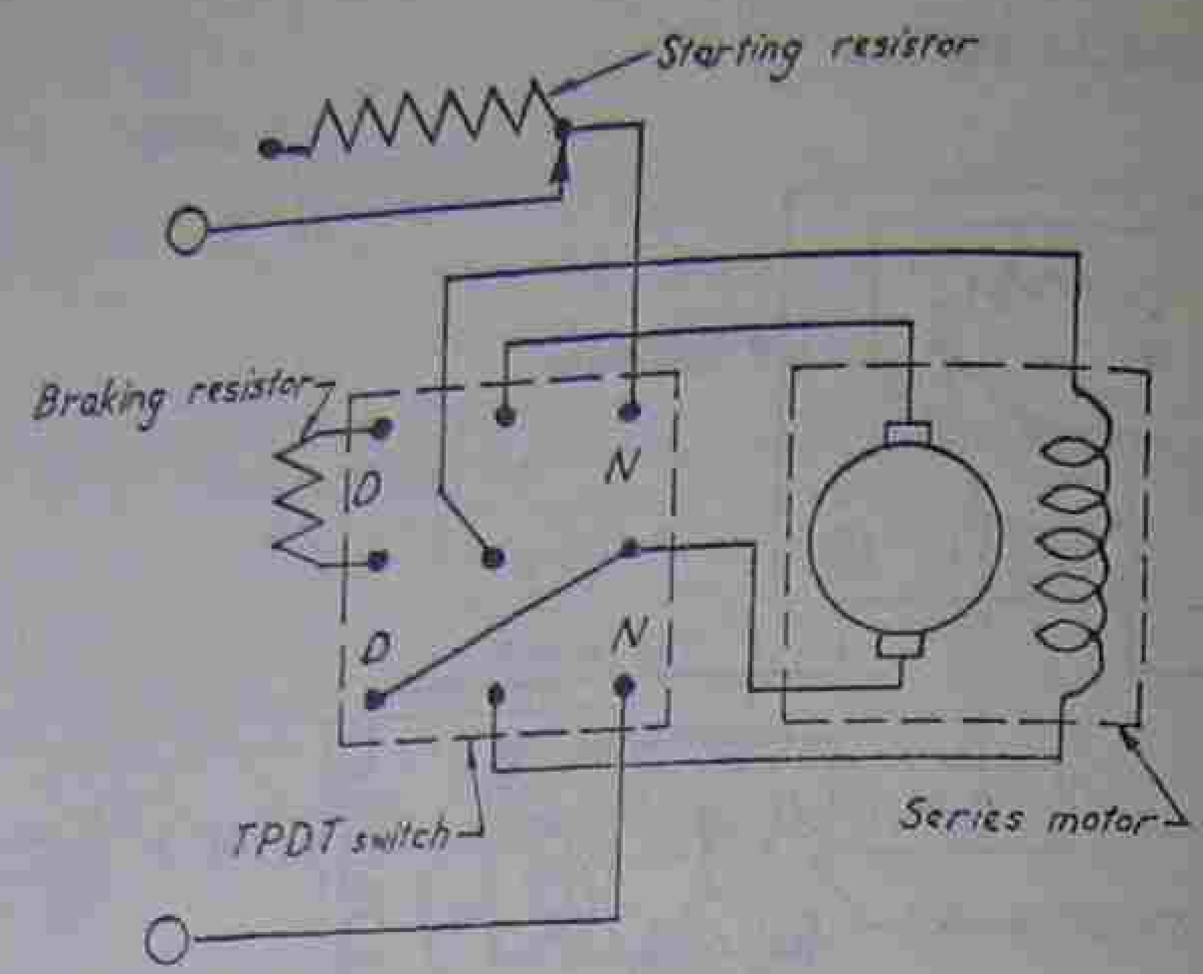


Figure 11.7 - Dynamic Braking - Series Motor

In both the above systems the armature power is dissipated in a resistance as for a shunt motor. Figure 11.7 illustrates the dynamic braking circuit for a series motor.

Example 11.3

The braking current of a 12.5 kW 230 volt 58 amperes motor, series wound, total armature and field resistance 0.28 ohms, is to be limited to 1.75 times the motor full load rating. Calculate the value of the dynamic braking resistor. Assume E_g is 94% of rated voltage and brush drop 3 volts.

Solution

$$R = \frac{(0.94 \times 230) - 3}{1.75 \times 58} - 0.28 = 1.82 \text{ ohms}$$

(c) Regenerative Braking

This system is similar to the dynamic braking system except that the power is returned to the supply instead of being dissipated in a resistance. This means that the motor back e.m.f. must be greater than the supply volts. This can be achieved by running the motor above its normal speed or overstrengthening the main field. This system is used where it is desired to limit the speed of a motor rather than stop it, that is, for electric traction systems where there are long down hill runs.

REVIEW QUESTIONS

(1) Explain why the following curves are not linear:

(a) Speed versus time during starting.

(b) Current versus time during starting.

(2) What are the advantages of automatic starters?

(3) Explain the principle of operation of:

(a) Back e.m.f. starters for shunt d.c. motors.

(b) Series lockout starters for d.c. motors.

(4) What is the main disadvantage of definite time starters?

(5) What are the basic principles behind plugging, regenerative and dynamic braking?

(6) Explain the difference in braking torque developed by the three methods of electrical braking.

(7) Which of the methods mentioned in Question 5 is best suited for traction work?

A S S I G N M E N T S

MARKS

20 1. A 45 kW 230 volt shunt motor has armature resistance 0.04 ohms and field resistance 38.3 ohms. At the moment it is developing rated torque it is plugged. Determine the resistance of the plugging resistor if the latter limits the instantaneous armature current to 200% of the rated full load value. Assume a brush drop of three volts and motor efficiency 88.5%. If this motor were to be dynamically braked, what would be the value of the braking resistor to keep the instantaneous current limit to 200% of the full load current?

20 2. A 240 volt d.c. motor has an armature resistance of 0.15 ohms and when delivering full load the speed is 1000 r.p.m. for an armature current of 100 amperes. Assuming that the motor is to be braked dynamically calculate the value of the added resistor if the current is to be limited to 175 amperes. What would be the initial braking torque developed by the motor and the braking torque when N = 500 rpm? Assume the brush voltage drop to be three volts. At what speed would the torque be zero?

25 3. Repeat Problem 2 for the case where the machine is to be braked by plugging. Also, determine the torque when the motor armature is stationary.

20 4. Assuming that the motor of Problem 2 was to be braked regeneratively and to accomplish this the field flux was increased by 10%, determine

(a) initial braking torque;

(b) speed for zero torque.

15 5. For the conditions given for the motor in Problem 3 draw a curve showing the relationship between developed power and speed from zero to 1000 rpm.

ELECTRICAL MACHINES I

UNIT NO. 12

12.1 Introduction.

12.2 Accelerating time considerations.

12.3 Determination of accelerating times.

12.4 Motor selection.

Review questions.

Assignments.

12.1 INTRODUCTION

The work done in previous units has covered aspects of motor selection and acceleration and has looked at the suitability of motors for a given load cycle. The purpose of this unit is to relate the various aspects of motor performance at starting to ensure that a motor chosen to produce a given torque is capable of accelerating the associated system, under the restrictions placed on the developed torque by the use of current limiting resistors for starting, in a sufficiently short period to prevent the short time rated resistors from sustaining damage.

12.2 ACCELERATING TIME CONSIDERATIONS

In Unit 1 it was shown that if the torque speed curve of both the load and the motor were known and, also, if the moment of inertia of the system could be determined it was possible to calculate the time taken for the system to accelerate between two values of speed.

In this unit the following assumptions will be made for shunt connected machines:

(1) The effect of demagnetization due to armature reaction can be neglected and the torque speed curve, as a consequence is linear.

(2) The shunt field current can be ignored and hence

$$I_a = I_L$$

(3) The moment of inertia of the system remains constant and that all values are referred to the motor shaft speed.

As the emphasis in this particular unit and the course itself is primarily concerned with the electrical aspects of motor design and application, it is not the purpose of the unit to carry out an exhaustive study of the subject of rotational motion or to treat the topic of the moment of inertia of a system any more fully than has already been done previously in Unit 1. Where required, the moment of inertia of a system can be determined from a knowledge of the geometry of the rotating members and the mass of the members involved and by reference to the appropriate tables. In this treatment the moment of inertia will be given both for the load and for the motor.

Due to the practical and electrical factors involved with the starting time calculations of d.c. motors the following factors must be considered:

<u>Firstly</u> In the expressions derived in Unit 1 for the accelerating time the angular velocity (ω) was given in radians per second. A more practical method is to express the angular velocity in terms of revolutions per minute (N). As a speed of N revolutions per minute is equivalent to $2\pi N/60$ radians per second the expression for accelerating time can be written as

$$t = \Delta N \left(\frac{2\pi I}{60 \; T} \right) \text{ seconds} \qquad - \; 12.1$$

where t = time in seconds

ΔN = change in speed (r.p.m.)

I = moment of inertia (Nms^2)

and T = torque (Nm)

Note: In the graphical determination of accelerating time the torque value in equation 12.1 is the mid-ordinate value of torque.

<u>Secondly</u> Where the starting of the motor and the associated load is effected by means of equipment that either reduces the supply voltage or limits the armature current by means of series resistors the torque speed curve of the motor must be modified accordingly as shown in the discussion of torque speed curves for shunt motor in Unit 9.

<u>Thirdly</u> The torque speed curve for the load is independent of motor conditions and the torque of the load is a function of the load speed only.

12.3 <u>DETERMINATION OF ACCELERATING TIMES</u>

The steps in determining the accelerating time for a given motor when coupled to a given load may be summarized as follows

<u>Step 1</u> - Determine the maximum and minimum values of armature current during the starting sequence. For example, it may be decided that for reasons imposed by the supply source that the maximum permitted value of current is 150% at the full load armature current.

<u>Step 2</u> - Using either the analytical or the graphical method determine the number of steps and the resistance of each step in the starter.

Note: The method here outlined is based on the assumption that the motor is started by means of an automatic starter in conjunction with a stepped starting resistor. With S.C.R. driven current limiting during starting could be a function of the control circuitry.

<u>Step 3</u> - From the information obtained in Step 2 determine the speed of the machine at each starting period.

<u>Step 4</u> - Assuming linearity for the torque speed curve of the motor determine the actual equation for the torque speed curve between the speed values calculated in Step 3.

<u>Step 5</u> - Plot the speed torque curve for the load and the motor torque speed curves obtained in Step 4 on the same set of axes.

<u>Step 6</u> - Using the techniques of Unit 1 determine the average accelerating torque values and hence obtain the required accelerating time.

Note: Due to the fact that the demagnetizing effects of armature reaction have been neglected and the armature torque speed curve is assumed to be linear the starting time determined will be a pessimistic estimate and the actual starting time should be less than the calculated value.

<u>Example 12.1</u>

A 100 kW 500V d.c. shunt motor has an armature resistance of 0.204 ohms and a full load speed of 920 r.p.m. If the load has a torque speed curve which may be defined by the expression:

$$T = 100 + 9.2 \times 10^{-4} N^2$$

and the motor armature particulars are:

Z = 756 conductors

P = 4 pole pairs

a = 8 parallel paths

ϕ = 0.04 webers

and if the combined moment of inertia of the load and the motor is 120 Nms^2 and the starting current is limited to 150% of the full load current at the upper limit and 90% of the full load current at the lower limit, determine the time taken to accelerate the motor to the full operating speed if any time delay in the operation of the starter contacts are neglected. Assume an efficiency of 90%.

Solution

Step 1 - $I_{FL} = \dfrac{100 \times 1000 \times 100}{500 \times 90}$

$\approx$ 220 amperes

$\therefore I_s = 220 \times \dfrac{150}{100}$

$= 330$ amperes

and $I_r = 220 \times \dfrac{90}{100}$

$\approx$ 200 amperes

from which

$$R_1 = \frac{V_t}{I_s} = \frac{500}{330}$$

$$= 1.51\ \Omega$$

Step 2 - $\left(\dfrac{I_s}{I_r}\right)^n = \dfrac{R_1}{R_a}$

$$\left(\frac{330}{200}\right)^n = \frac{1.51}{0.204}$$

$$1.65^n = 7.4$$

$$n = 4$$

(i) $R_2 = R_1 \times \dfrac{I_r}{I_s} = 1.51 \times \dfrac{200}{330} = 0.92$ ohms

(ii) $R_3 = R_2 \times \dfrac{I_r}{I_s} = 0.92 \times \dfrac{200}{330} = 0.555$ ohms

(iii) $R_4 = R_3 \times \dfrac{I_r}{I_s} = 0.555 \times \dfrac{200}{330} = 0.336$ ohms

(iv) $R_5 = R_4 \times \dfrac{I_r}{I_s} = 0.336 \times \dfrac{200}{330} = 0.204$ ohms

This is the final stud and the remainder of the circuit resistance is due to the armature.

Step 3

At full load

$I_a = 220$ amperes

$\therefore E_g = V_t - I_a R_a$

$= 500 - 220 \times 0.204$

$= 500 - 44.6$

$= 455.4$ volts

(i) At the end of the first accelerating period

$I_a = I_r = 200$ amperes

$R_a = R_1 = 1.51$ ohms

hence $E_{g1} = V_t - I_a R_a$

$= 500 - 200 \times 1.51$

$= 198$ volts

The speed at the end of the first accelerating period will be

$N_1 = N_{FL} \times \dfrac{198}{456}$

$= \dfrac{920}{456} \times 198$

$= 400$ r.p.m.

Note:

The term $\dfrac{N_{FL}}{E_{gFL}} = \dfrac{920}{456} = 2.02$

is a constant for this problem and will be used in all further calculations.

(ii) At the end of the second accelerating period

$I_a = 200$ amperes

$R_a = R_2 = 0.92$ ohms

hence $E_{g2} = 500 - 200 \times .92$

$= 500 - 184 \qquad = 316$ volts

The speed will then be

$$N_2 = 2.02 \times E_{g2}$$

$$= 2.02 \times 316$$

$$= 625 \text{ r.p.m.}$$

(iii) At the end of the third accelerating period

$$I_a = 200 \text{ amperes}$$

$$R_a = R_3 = 0.555 \text{ ohms}$$

hence $$E_{g3} = 500 - 200 \times .555$$

$$= 389 \text{ volts}$$

The corresponding speed will be

$$N_3 = 2.02 \times 389$$

$$= 780 \text{ r.p.m.}$$

(iv) At the end of the fourth period

$$I_a = 200 \text{ amperes}$$

$$R_a = R_4 = 0.336 \text{ ohms}$$

hence $$E_{g4} = 500 - 200 \times .336$$

$$= 500 - 67.2$$

$$= 433 \text{ volts}$$

The speed will then be

$$N_4 = 2.02 \times 433$$

$$= 860 \text{ r.p.m.}$$

(v) For the last accelerating period the remaining resistance is due only to the resistance of the armature. As a consequence the speed for a current of 220 amperes would be 920 r.p.m. However, it will be assumed that the current falls to 200 amperes.

$$E_g = V_t - I_a R_a$$

$$= 500 - 200 \times .204$$

$$= 500 - 42$$

$$= 458 \text{ volts}$$

Finally

$$N_5 = 458 \times 2.02$$

$$= 930 \text{ r.p.m.}$$

The actual final speed will be determined from the intersection of the load torque line and the motor developed torque line.

Step 4

From Unit 9

$$T = \frac{K_t \phi V_t}{R_a} - \frac{K_e K_t \phi^2}{R_a} N \qquad - 9.10$$

where

$$K_e = \frac{PZ}{60_a} \qquad - 9.8$$

and $$K_t = \frac{PZ}{2 \pi a} \qquad - 9.5$$

From the motor data

$$K_e = \frac{PZ}{60_a} = \frac{8 \times 756}{60 \times 8}$$

$$= 12.6$$

and

$$K_t = \frac{PZ}{2 \pi a} = \frac{8 \times 746}{16 \pi}$$

$$= 121$$

For the motor under discussion equation 9.10 can now be written as

$$T = \frac{121 \times 500 \times 0.04}{R_a} - \frac{121 \times 12.6 \times (0.04)^2}{R_a} N$$

$$= \frac{2420}{R_a} - \frac{2.44}{R_a} N$$

(i) In the first interval, the torque expression becomes:

$$T = \frac{2420}{1.51} - \frac{2.44}{1.51} N$$

$$= 1600 - 1.62 N$$

At the commencement of the interval when the speed is zero the torque will be:

$$T_i = 1600 - 1.62 \times 0$$

$$= 1600 \text{ Nm}$$

When the current falls to 200 amperes the torque will be:

$$T_f = 1600 - 1.62 \times 400$$

$$= 1600 - 650$$

$$= 950 \text{ Nm}$$

(ii) For the second interval equation 9.10 becomes:

$$T = \frac{2420}{.92} - \frac{2.44}{.92} N$$

$$= 2620 - 2.64 N$$

$$= 2620 - 2.64 N$$

and $T_i = 2.620 - 2.64 \times 400$

$$= 1620 \text{ Nm}$$

$$T_f = 2620 - 2.64 \times 625$$

$$= 970 \text{ Nm}$$

(iii) For the third interval the torque expression is:

$$T = \frac{2420}{.555} - \frac{2.44}{.555} N$$

$$= 4370 - 4.39 N$$

The torque values are then:

$$T_i = 4370 - 4.39 \times 625$$

$$= 1620 \text{ Nm}$$

$$T_f = 4370 - 4.39 \times 785$$

$$= 950 \text{ Nm}$$

(iv) For the fourth interval

$$T = \frac{2420}{.336} - \frac{2.44}{.336} N$$

$$= 7150 - 7.24 N$$

$$T_i = 7150 - 7.24 \times 785$$

$$= 1470 \text{ Nm}$$

$$T_f = 7150 - 7.24 \times 870$$

$$= 900 \text{ Nm}$$

$N_{NL} = 996$ RPM
When $T_m = 0$.

(v) For the final interval

$$T = \frac{2420}{.204} - \frac{2.44}{.204} N$$

$$= 11900 - 11.95 N$$

$$T_i = 11900 - 11.95 \times 870$$

$$= 1600 \text{ Nm}$$

$$T_f = 11900 - 11.95 \times 930$$

$$= 800 \text{ Nm}$$

The complete results to this stage may now be tabulated as shown in Table 12.1

Period	R_a	E_g (Final) (volts)	N_{Final} (r.p.m.)	Torque Expression	T_i	T_f
1	1.51	198	400	1600 - 1.62 N	1600	950
2	.92	316	625	2620 - 2.64 N	1620	970
3	.555	389	780	4370 - 4.39 N	1620	950
4	.336	433	870	7150 - 7.24 N	1470	900
5	.204	458	930	11900 - 11.95 N	1600	800

Table 12.1 - Results for example 12.2

Step 5

The values for the load speed torque curves may be calculated from the given expression and are tabulated in Table 12.2

Speed r.p.m.	0	200	400	600	800	1000
Torque Nm	100	137	147 247	431	688	1020

Table 12.2 - Torque speed curve for load example 12.1

The torque speed curves for both the motor and the load can now be plotted using the results listed in Tables 12.1 and 12.2.

In theory, as the starting and finishing current for each phase of the starting period are the same and, as the flux is constant, the initial and final values of torque should be the same. As the calculations were limited to an accuracy of three significant figures the resultant values obtained do not agree with the theoretical ideal, the deviation from the expected values increasing as the speed increases. A method of eliminating the error and also of speeding up the construction of the motor curve is to calculate the initial and final values of torque on the first step and use these as a basis for determining the total characteristic. As this method only requires the speed at the end of the first step and the associated torque expression this will effect a considerable saving in calculation time. The complete alternate curve is shown in Figure 12.2.

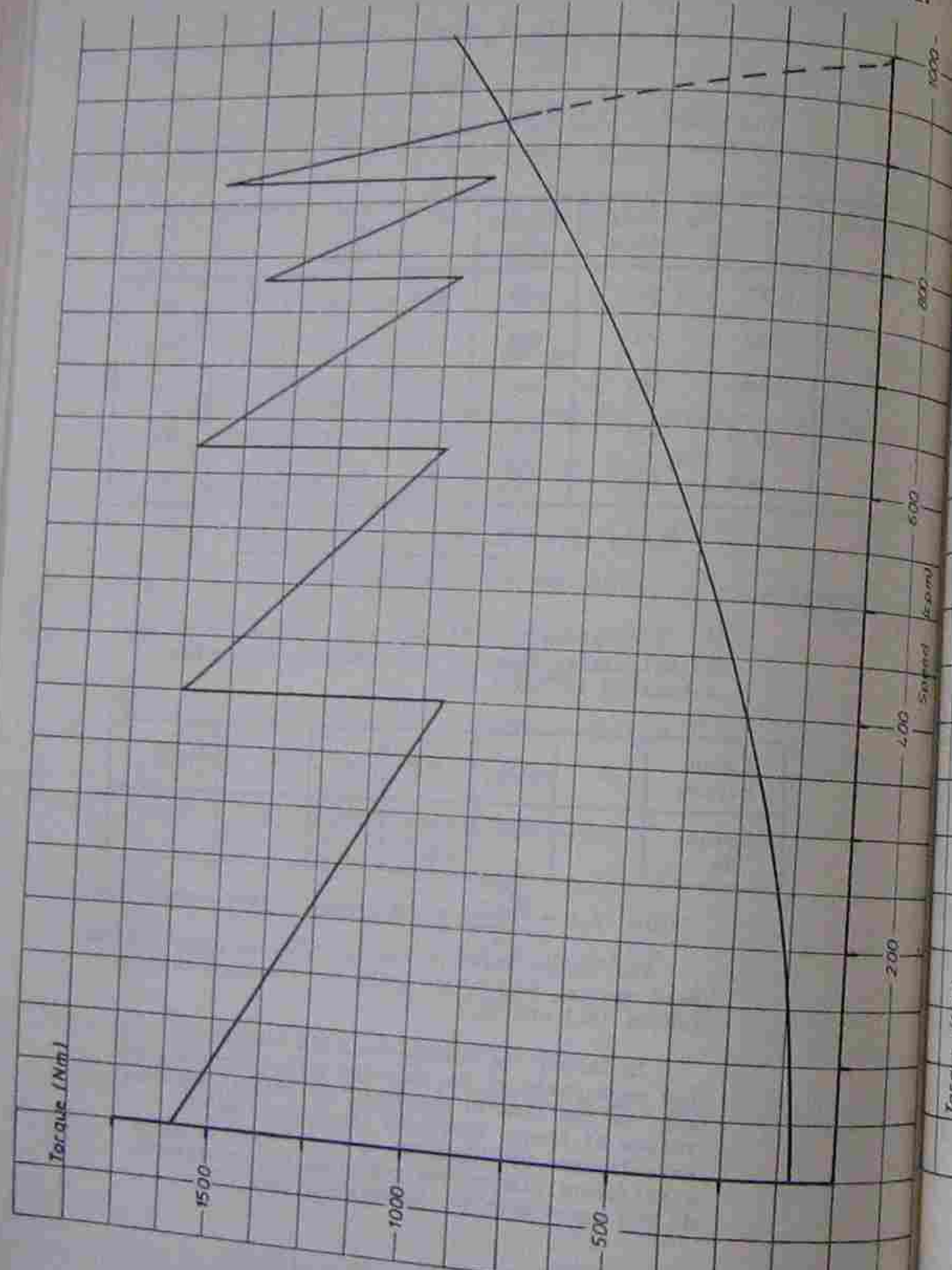


Figure 12.1 - Motor and Load Torque Speed Curves.

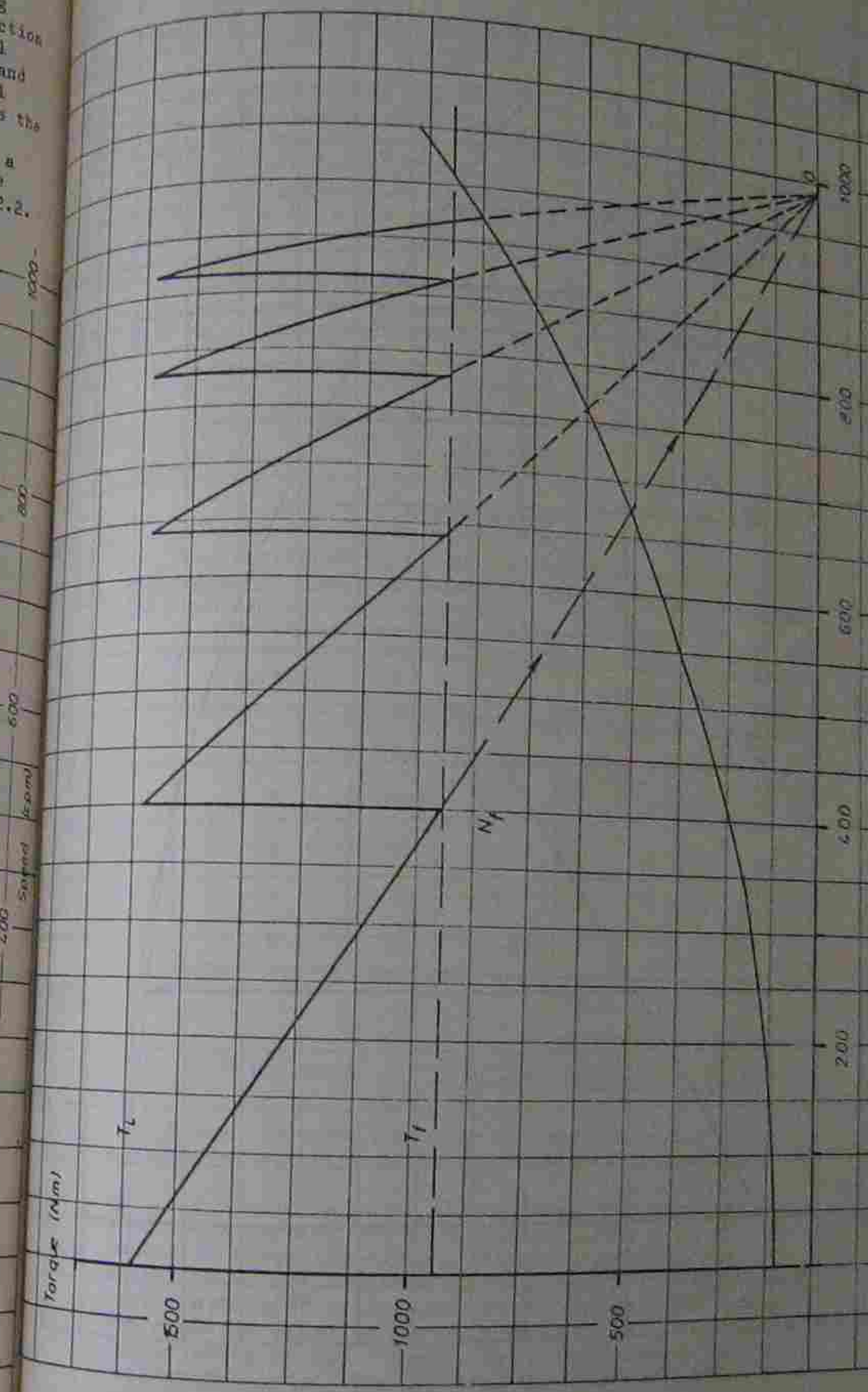


Figure 12.2 - Construction to obtain Motor Characteristic.

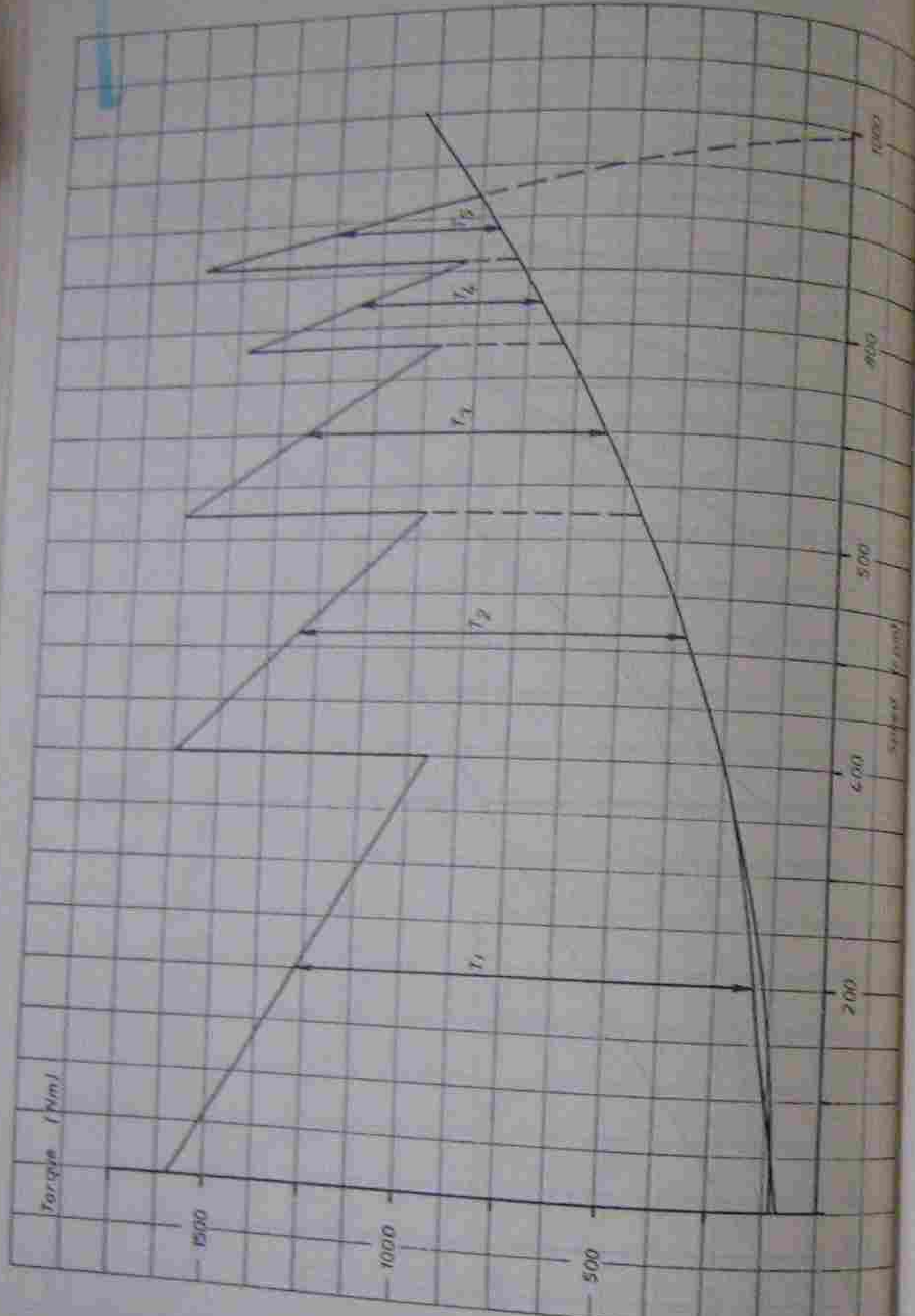


Figure 12.3 - Accelerating torque construction.

In Figure 12.2 the point O is obtained by projecting the torque characteristic to cut the speed axis. The remainder of the construction is similar to the graphical method for obtaining the number of steps in a motor starter.

The construction and the final curve used to obtain the starting time will be that given in Figure 12.1. Figure 12.3 shows the curves with the motor curve approximated by linear relationships with the average torque over the various intervals drawn as the appropriate mid-ordinate. The starting time can now be determined using the method of Unit 1.

From Figure 12.3 the first speed interval is from 0 to 400 r.p.m. and the average torque over this interval is 1100 Nm. Using equation 12.1 the accelerating time is

$$t = \Delta N \frac{2\pi}{60T} I$$

$$= \frac{400 \times 6.28 \times 120}{60 \times 1100}$$

$$= 4.55 \text{ seconds}$$

The accelerating times for the other intervals may be calculated in the same manner. Table 12 gives the times for each of the other intervals.

Section	N	T_{ave}	Accelerating Time
1	400	1100	4.55
2	225	980	3.12
3	165	688	3.01
4	80	413	2.42
5	60	410	1.83
		Total	14.93

Table 12.3 - Calculated Accelerating Times.

Hence the time taken for the motor to reach the equilibrium speed would be approximately 15 seconds.

(It should be noted that although the calculations which were carried out by means of a slide rule, give an answer to three significant figures implying an accuracy in the order of less than 1% due to the nature of the data used, particularly the value of the accelerating torque, the expected order of accuracy would be more likely to be in the order of 5%. Hence, the significance of the term "approximately". The accelerating time would probably be somewhere between 14 and 16 seconds.)

12.4 MOTOR SELECTION

The times calculated in Example 12.1 were for a shunt motor assuming that the demagnetizing effects of armature reaction were disregarded. For a different load or for a system having a higher moment of inertia the starting times could be much higher than those of the example with the consequent increase in the probability of overheating in the starting resistances. Where high inertia loads are to be driven a significant decrease in the accelerating time can be achieved by means of an added series winding to give a compound machine. The additional series winding will produce a higher starting torque and consequent reduction is accelerating time but the speed regulation of the compound machine will be greater than that of the shunt machine. The extent of the increase will depend on the degree of compounding. Where a load has a very high moment of inertia a series motor may be used. The series motor, however, has a very poor speed regulation characteristic. To select a motor for a particular load, once the considerations regarding the rating of the machine have been taken into account, the further factors to be considered are the speed regulation, the inertia of the system and the accelerating time.

In general, as shown previously, there are several ways in which the speed of d.c. motors can be controlled and, consequently, by using suitable control methods the inherently poor speed regulation of compound motors can be improved and this type of machine is generally used except in traction work where the inertia of the system is high.

The consideration of speed regulation is only valid where the machine is not intended for a controlled speed situations. The trend in current practice is to use d.c. motors in situations where their speed control capabilities can be more fully utilized. As a consequence the over-riding consideration is that of providing the maximum starting torque consistent with the limitations of the supply system in order to reduce the starting period duration.

Note: In this unit the symbol used for moment of inertia is I as per the Australian standard AS 1046 - 1972. The standard also gives J as an alternative, preferred symbol and in some respects the use of J could be preferred to the use of the sumbol I which is commonly associated with current. However, as current and moment of inertia seldom appear in the same expression there appears little possibility of the two being confused as the quantity the symbol represents should be apparent from the context if not explicibly stated. As the alternate symbol J is also associated with current density there seemed no valid reason to use J in preference to I. The student should be aware that the two alternative symbol are allowed and be prepared to recognize this possibility if it is encountered.

REVIEW QUESTIONS

(1) What effect has demagnetization on the linearity of the torque speed curve of a shunt motor?

(2) What factors affect the torque speed relationships of loads?

(3) What effect has the resistance of the armature circuit on the torque speed relationship of a shunt motor?

(4) Derive Equation 9.10.

(5) Discuss the factors which must be considered when determining the suitability of a motor for a given task.
