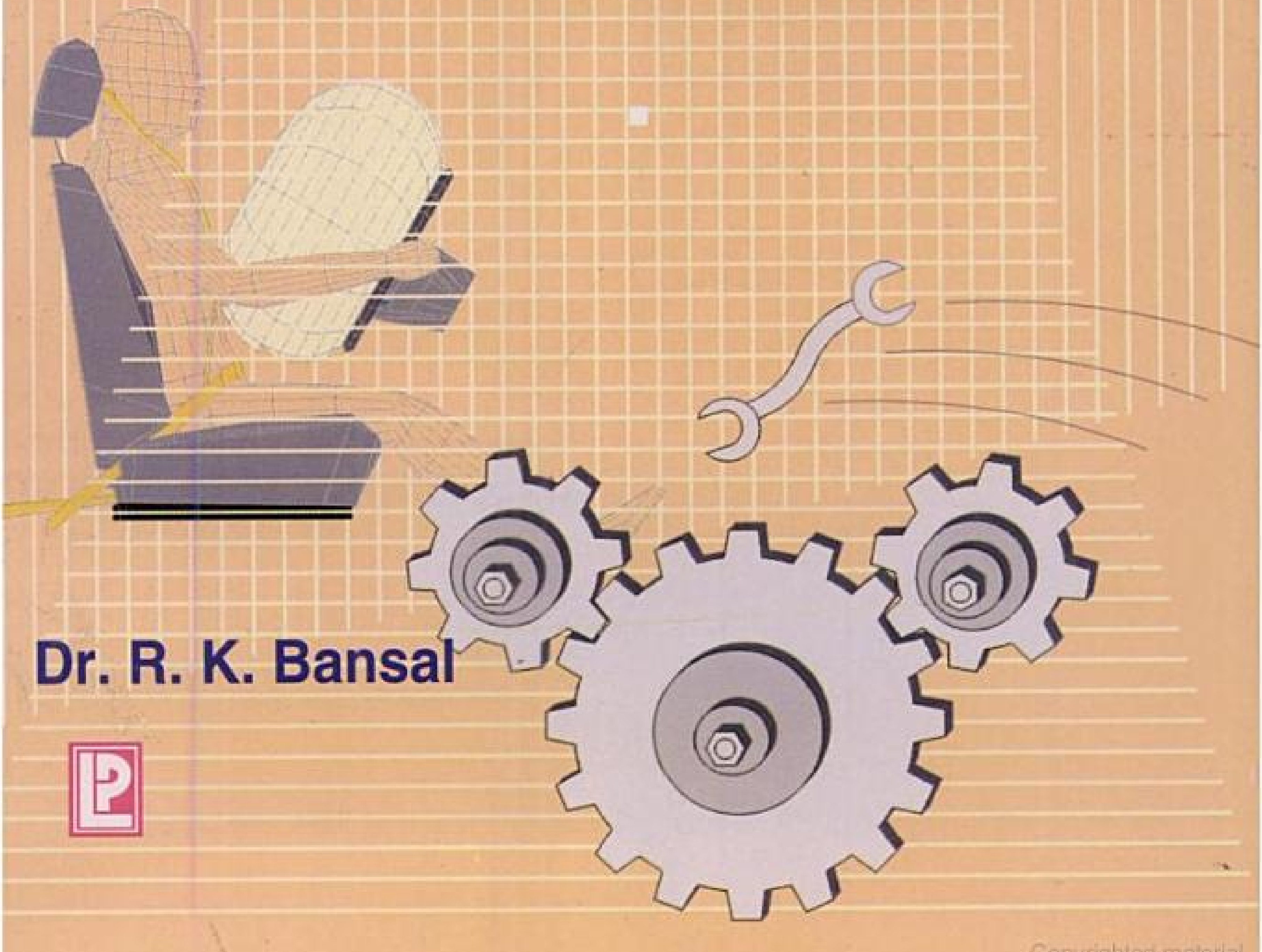
A Textbook of

Engineering Mechanics



Dr. R. K. Bansal

Published by :

LAXMI PUBLICATIONS (P) LTD
22, Golden House, Daryaganj,
New Delhi-110002.

Phones : 011-23 26 23 68
011-23 26 23 70

Faxes : 011-23 25 25 72
011-23 26 22 79

Branches :

- 129/1, IIIrd Main Road, IX Cross, Chamrajpet, **Bangalore** (*Phone* : 080-26 61 15 61)
- 26, Damodaran Street, T. Nagar, **Chennai** (*Phone* : 044-24 34 47 26)
- St. Benedict's Road, **Cochin** (*Phone* : 0484-239 70 04)
- Pan Bazar, Rani Bari, **Guwahati** (*Phones* : 0361-254 36 69, 251 38 81)
- 4-2-453, 1st Floor, Ramkote, **Hyderabad** (*Phone* : 040-24 75 02 47)
- Adda Tanda Chowk, N.D. 365, **Jalandhar City** (*Phone* : 0181-222 12 72)
- 106/A, Ist Floor, S.N. Banerjee Road, **Kolkata** (*Phones* : 033-22 27 37 73, 22 27 52 47)
- 18, Madan Mohan Malviya Marg, **Lucknow** (*Phone* : 0522-220 95 78)
- 128A, Block 3, First Floor, Noorani Building, L.J. Road, **Mumbai** (*Phone* : 022-24 46 39 98)
- Radha Govind Street, Tharpagna, **Ranchi** (*Phone* : 0651-230 77 64)

EMAIL : colaxmi@hotmail.com

WEBSITE : www.laxmipublications.com

EEM-0552-240-ENGG MECHANICS

Compiled by : **Smt. Nirmal Bansal**

First Edition	:	*1991*	*Reprint*	:	*1992, 1993*
Second Edition	:	*1994*	*Third Edition*	:	*1996*
Reprint	:	*1997, 1998, 1999, 2000, 2001, Jan. 2001*			
Fourth Edition	:	*2002*	*Reprint*	:	*April 2003, March 2004*
Reprint	:	*October 2004, June 2005*			

Price : **Rs. 240.00** *Only*

C—10642/05/06

Typesetting by : Goswami Printers, Delhi-110053. *Printed at* : Sanjeev Offset Printers, Delhi.

Contents

Chapter *Pages*

PART I—STATICS

1. Fundamentals of Engineering Mechanics **1—23**

1.1. Introduction ... 1
1.2. Definitions ... 1
1.2.1. Vector quantity ... 1
1.2.2. Scalar quantity ... 1
1.2.3. A particle ... 1
1.2.4. Law of parallelogram of forces ... 1
1.2.5. Law of triangle of forces ... 4
1.2.6. Lami's theorem ... 4
1.3. System of Units ... 5
1.3.1. C.G.S. system of units ... 5
1.3.2. M.K.S. system of units ... 5
1.3.3. S.I. system of units ... 5
1.4. Trigonometric Formulae and Expressions ... 6
1.5. Differentiation and Integration ... 6
1.5.1. Differentiations ... 6
1.5.2. Integrations ... 7
Solved Problems 1.1—1.9 ... 7
1.6. Resolution of a Force ... 14
1.7. Resolution of a Number of Coplanar Forces ... 14
Solved Problems 1.10—1.12 ... 14
1.8. Moment of a Force ... 16
1.8.1. Units of moment ... 17
1.8.2. Effect of Force and moment on a body ... 17
Solved Problem 1.13 ... 17
1.9. Laws of Mechanics ... 18
1.9.1. Newton's first and second laws of motion ... 19
1.9.2. Newton's third law ... 19
1.9.3. The Gravitional law of attraction ... 19
1.9.4. The parallelogram law ... 20
1.9.5. The Principle of transmissibility of forces ... 21
Highlights ... 21
Exercise 1 ... 22

2. Coplanar, Collinear and Concurrent Forces **24—42**

2.1. Introduction ... 24
2.2. Classification of a Force System ... 24
2.2.1. Coplanar collinear ... 24
2.2.2. Coplanar concurrent ... 25
2.2.3. Coplanar parallel ... 25
2.2.4. Coplanar non-concurrent, non-parallel ... 25
2.3. Resultant of Several Forces ... 25

Chap.				Pages
	2.4.	Resultant of Coplanar Forces	...	26
	2.5.	Resultant of Collinear Coplanar Forces	...	26
		2.5.1. Analytical method	...	26
		2.5.2. Graphical method	...	26
		Solved Problem 2.1	...	26
	2.6.	Resultant of Concurrent Coplanar Forces	...	27
		2.6.1. When two forces act at a point	...	27
		2.6.2. When more than two two forces act at a point	...	28
		Solved Problems 2.2—2.10	...	30
		Highlights	...	41
		Exercise 2	...	41
3.	**Coplanar Parallel Forces**			**43—60**
	3.1.	Introduction	...	43
	3.2.	Moment of a Force	...	43
		Solved Problem 3.1	...	44
	3.3.	Principle of Moments (or Varignon's Principle)	...	44
		Solved Problem 3.2	...	46
	3.4.	Types of Parallel Forces	...	47
		3.4.1. Like parallel forces	...	47
		3.4.2. Unlike parallel forces	...	47
	3.5.	Resultant of Two Parallel Forces	...	47
		3.5.1. Resultant of two like parallel forces	...	47
		3.5.2. Resultant of two unlike parallel forces (unequal in magnitude)	...	48
		3.5.3. Resultant of two unlike parallel forces which are equal in magnitude	...	49
		Solved Problems 3.3—3.5	...	49
	3.6.	Resolution of a Force into a Force and a Couple	...	52
		Solved Problem 3.6	...	52
	3.7.	General Case of Parallel Forces in a Plane	...	54
		Solved Problems 3.7—3.8	...	54
	3.8.	Equivalent System	...	55
		Solved Problems 3.9—3.10	...	55
		Highlights	...	57
		Exercise 3	...	58
4.	**Conditions of Equilibrium**			**61—91**
	4.1.	Introduction	...	61
	4.2.	Principle of Equilibrium	...	61
		4.2.1. Equations of equilibrium for non-current forces systems	...	61
		4.2.2. Equations of equilibrium for concurrent force system	...	61
	4.3.	Force Law of Equilibrium	...	62
		4.3.1. Two force system	...	62
		4.3.2. Three force system	...	62
		4.3.3. Four force system	...	63
		Solved Problems 4.1—4.7	...	63
	4.4.	Action and Reaction	...	71

Chap.			Pages
4.5.	Free Body Diagram	...	72
	Solved Problems 4.8—4.21	...	72
	Highlights	...	88
	Exercise 4	...	88
5.	**Support Reactions**		**92—113**
5.1.	Introduction	...	92
5.2.	Types of Supports	...	92
	5.2.1. Simple support or knife edge support	...	92
	5.2.2. Roller support	...	92
	5.2.3. Pin joint (or hinged) support	...	92
	5.2.4. Smooth surface support	...	93
	5.2.5. Fixed or built-in support	...	93
5.3.	Types of Loading	...	93
	5.3.1. Concentrated or point load	...	93
	5.3.2. Uniformly distributed load	...	94
	5.3.3. Uniformly varying load	...	94
5.4.	Method for Finding out the Reactions of a Beam	...	94
	5.4.1. Analytical method	...	94
	5.4.2. Graphical method for finding out the reactions of a beam	...	94
5.5.	Problems on Simple Supported Beams	...	95
	Solved Problems 5.1—5.5	...	95
5.6.	Problems on Overhanging Beams	...	100
	Solved Problems 5.6—5.7	...	100
5.7.	Problems on Roller and Hinged Supported Beams	...	101
	Solved Problems 5.8—5.12	...	101
5.8.	Problems When Beams are Subjected to Couples	...	110
	Solved Problem 5.13	...	110
	Highlights	...	111
	Exercise 5	...	111
6.	**Analysis of Perfect Frames**		**114—153**
6.1.	Introduction	...	114
6.2.	Types of Frames	...	114
	6.2.1. Perfect frame	...	114
	6.2.2. Imperfect frame	...	115
6.3.	Assumptions Made in Finding out the Forces in a Frame	...	115
6.4.	Reactions of Supports of a Frame	...	115
6.5.	Analysis of a Frame	...	116
	6.5.1. Method of joints	...	116
	Solved Problems 6.1—6.5	...	116
	6.5.2. Method of joints applied to cantilever trusses	...	127
	Solved Problems 6.6—6.7	...	127
	6.5.3. Method of joints applied to trusses carrying horizontal loads	...	129
	Solved Problems 6.8—6.9	...	130
	6.5.4. Method of joints applied to trusses carrying inclined loads	...	134
	Solved Problem 6.10	...	134

Chap.			*Pages*
6.6	Method of Sections	...	136
	Solved Problems 6.11—6.14	...	137
	6.6.1. Method of section, cutting more than three members	...	142
	Solved Problems 6.15—6.16	...	142
6.7.	Graphical Method	...	145
	Solved Problems 6.17—6.19	...	146
	Highlights	...	149
	Exercise 6	...	150
7.	**Centre of Gravity and Moment of Inertia**		**154—194**
7.1.	Centre of Gravity	...	154
7.2.	Centroid	...	154
7.3.	Centroid of Centre or Gravity of Simple Plane Figures	...	154
7.4.	Centre of Gravity of Plane Figures by the Method of Moments	...	154
	7.4.1. Centre of gravity of plane figures by integration method	...	155
	7.4.2. Centre of gravity of a line	...	155
7.5.	Important Points	...	156
	7.5.1. Centre of gravity of structural sections	...	156
	Solved Problems 7.1—7.5	...	156
	7.5.2. Problems of finding centre of gravity of area by integration method	...	162
	Solved Problems 7.6—7.7	...	162
	7.5.3. Problems of finding centre of gravity of line-segment by integration method	...	167
	Solved Problems 7.8—7.11	...	167
7.6.	Moment of Inertia	...	174
7.7.	Radius of Gyration	...	175
7.8.	Theorem of the Perpendicular Axis	...	175
7.9.	Theorem of Parallel Axis	...	176
7.10.	Determination of Moment of Inertia	...	177
	7.10.1. Moment of inertia of a rectangular section	...	177
	7.10.2. Moment of inertia of a circular section	...	180
	7.10.3. Moment of inertia of a triangular section	...	181
	7.10.4. Moment of inertia of a uniform thin rod	...	183
	7.10.5. Moment of inertia of area under a curve of given equation	...	183
	Solved Problems 7.12—7.14	...	185
	Highlights	...	189
	Exercise 7	...	190
8.	**Friction**		**195—238**
8.1.	Introduction	...	195
8.2.	Definitions	...	195
	8.2.1. Co-efficient of friction	...	195
	8.2.2. Angle of friction	...	195
	8.2.3. Cone of friction	...	196
8.3.	Types of Friction	...	197

Chap.			Pages
8.4.	Laws of Solid Friction	...	197
	Solved Problems 8.1—8.8	...	197
8.5.	Angle of Repose	...	204
8.6.	Equilibrium of a Body Lying on a Rough Inclined Plane	...	204
	Solved Problems 8.9—8.22	...	205
8.7.	Analysis of Ladder Friction	...	222
	Solved Problems 8.23—8.28	...	222
8.8.	Analysis of Wedge Friction	...	230
	Solved Problem 8.29	...	231
	Highlights	...	234
	Exercise 8	...	235
9.	**Lifting Machines**		**239—289**
9.1.	Introduction	...	239
9.2.	Definitions	...	239
9.3.	Reversible and Irreversible Machine	...	240
9.4.	Condition for Irreversibility of a Machine	...	240
	Solved Problems 9.1—9.4	...	241
9.5.	Law of a Machine	...	243
	9.5.1. Expression for maximum mechanical advantage	...	244
	9.5.2. Expression for maximum efficiency	...	245
	Solved Problems 9.5—9.6	...	245
9.6.	Friction of a Machine Expressed in Terms of Actual Effort (P)	...	247
9.7.	Friction of a Machine Expressed in Terms of Load	...	248
	Solved Problems 9.7—9.12	...	248
9.8	Important Lifting Machines	...	252
	9.8.1. Simple wheel and axle	...	253
	Solved Problem 9.13	...	253
	9.8.2. Differential wheel and axle	...	254
	Solved Problem 9.14	...	255
	9.8.3. Worm and worm wheel	...	255
	Solved Problem 9.15	...	257
	9.8.4. Single purchase crab winch	...	257
	Solved Problem 9.16	...	259
	9.8.5. Double purchase crab winch	...	259
	Solved Problem 9.17	...	260
	9.8.6. Pulleys	...	262
	Solved Problems 9.18—9.20	...	264
	9.8.7. Weston's differential pulley block	...	269
	Solved Problem 9.21	...	270
	9.8.8. Screw Jack	...	270
	Solved Problems 9.22—9.28	...	274
	Highlights	...	283
	Exercise 9	...	286
10.	**Virtual Work**		**290—331**
10.1.	Introduction	...	290

Chap.			Pages
	10.2. Work	...	290
	10.3. Virtual Work	...	291
	10.4. Principle of Virtual Work	...	291
	10.5. Units of Work	...	291
	10.6. Forces to be Omitted While Applying the Principle of Virtual Work	...	291
	10.7. Important Points to be Remembered while Applying the Principle of Virtual Work	...	291
	10.8. Uses of the Principle of Virtual Work	...	292
	10.8.1. The principle of virtual work for problems on beams	...	292
	Solved Problems 10.1—10.5	...	293
	10.8.2. The principle of virtual work for problems on Framed structure	...	302
	Solved Problems 10.6—10.17	...	302
	10.8.3. The principle of virtual work for problems on lifting machines	...	323
	Solved Problems 10.18—10.19	...	323
	10.9. Virtual Work Done by Moment (or Torque)	...	325
	Solved Problem 10.20	...	326
	Highlights	...	328
	Exercise 10	...	329
11.	**Belts, Ropes and Chain Drives**		**332—378**
	11.1. Introduction	...	332
	11.2. Open Flat Belt Drive	...	332
	11.2.1. Velocity ratio of open belt drive	...	333
	Solved Problems 11.1—11.2	...	334
	11.2.2. Slip of the belt	...	334
	11.2.3. Creep of the belt	...	336
	Solved Problems 11.3—11.5	...	336
	11.3. Cross Belt Drive	...	339
	11.4. Compound Belt Drive	...	339
	Solved Problem 11.6	...	340
	11.5. Length of Belt	...	341
	11.5.1. Length of an open belt drive	...	341
	11.5.2. Length of a cross-belt drive	...	343
	Solved Problems 11.7—11.8	...	345
	11.6. Ratio of Belt Tensions	...	348
	11.6.1. Angle of contact for open belt drive	...	350
	11.6.2. Angle of contact for crossed belt drive	...	350
	11.7. Power Transmitted by Belt	...	350
	Solved Problems 11.9—11.10	...	351
	11.8. Centrifugal tension	...	352
	11.9. Maximum Power Transmitted by a Belt	...	353
	Solved Problems 11.11—11.14	...	355
	11.10. Initial Tension in the Belt	...	359
	Solved Problems 11.15—11.17	...	360
	11.11. V-belt Drive and Rope Drive	...	365
	Solved Problems 11.18—11.23	...	366

Chap.			*Pages*
	11.12. Chain drive	...	373
	11.13. Relative Advantages and Disadvantages of Chain and Belt (or Rope) Drives	...	373
	Highlights	...	374
	Exercise 11	...	376

PART II—DYNAMICS

12.	**Linear Motion**		**381—413**
	12.1. Introduction	...	381
	12.2. Velocity	...	381
	12.3. Acceleration	...	381
	12.4. Equations of Motion in a Straight Line	...	382
	12.4.1. Equation for final velocity	...	382
	12.4.2. Equation of motion for distance covered (s)	...	382
	12.4.3. Derivation of $v^2 - u^2 = 2as$	...	383
	Solved Problems 12.1—12.6	...	385
	12.4.4. Distance travelled in the nth second	...	389
	Solved Problem 12.7	...	390
	12.4.5. Equation of motion due to gravity	...	390
	12.5. Points to be Remembered	...	391
	Solved Problems 12.8—12.18	...	391
	12.6. Velocity and Acceleration of a Body Moving in a Straight Line by Differentiation	...	401
	Solved Problems 12.19—12.20	...	401
	12.7. Velocity and Displacement of a Body Moving in a Straight Line by Integration	...	403
	Solved Problems 12.21—12.24	...	404
	Highlights	...	411
	Exercise 12	...	412
13.	**Curvilinear Motion, Circular Motion, Rotation and Translation**		**414—440**
	13.1. Introduction	...	414
	13.2. Angular Velocity	...	414
	13.2.1. Relation between linear velocity and angular velocity	...	414
	13.3. Angular Acceleration	...	415
	13.3.1. Relation between linear acceleration and angular acceleration	...	415
	13.4. Equation of Motion Along a Circular Path	...	415
	13.4.1. Equation for angular displacement (θ)	...	416
	13.4.2. Derivation of the angular displacement in terms of initial and final angular velocities	...	416
	13.4.3. Relationship between r.p.m. (N) and angular velocity (ω)	...	416
	Solved Problems 13.1—13.8	...	417
	13.5. Types of Motion	...	422
	13.5.1. Motion of translation	...	422
	13.5.2. Motion of rotation	...	423
	13.5.3. Combined motion of translation and rotation	...	423
	Solved Problem 13.9	...	424

Chap.			Pages
	13.6. Analysis of Reciprocating Engine Mechanism	...	425
	Solved Problems 13.10—13.11	...	429
	13.7. Analysis of Four Bar Mechanism	...	434
	Solved Problems 13.12—13.13	...	435
	Highlights	...	438
	Exercise 13	...	439
14.	**Projectiles**		**441—470**
	14.1. Introduction	...	441
	14.2. Terms Used with the Projectiles	...	441
	14.2.1. Velocity of projection	...	441
	14.2.2. Angle of projection	...	441
	14.2.3. Times of flight	...	441
	14.2.4. Horizontal range	...	441
	14.3. Equation For the Path of a Projectile	...	441
	14.3.1. Maximum height attained by projectile	...	442
	14.3.2. Time of flight	...	443
	14.3.3. Horizontal range of projectile (R)	...	443
	14.3.4. Value of angle of projection (∞) for maximum horizontal range	...	443
	14.3.5. Time to reach the highest point	...	443
	Solved Problems 14.1—14.11	...	444
	14.4. Motion of a Body Thrown Horizontally from a Given Height into the Air	...	454
	Solved Problems 14.12—14.17	...	455
	14.5. Projectile on an Inclined Plane	...	460
	14.5.1. Time of flight	...	461
	14.5.2. Range on inclined plane	...	461
	14.5.3. Projectile down the inclined plane	...	462
	Solved Problems 14.18—14.21	...	463
	Highlights	...	467
	Exercise 14	...	468
15.	**Laws of Motion**		**471—543**
	15.1. Introduction	...	471
	15.2. Newton'w Laws for Linear Motion	...	471
	15.3. Definitions	...	471
	15.3.1. Mass	...	471
	15.3.2. Weight	...	471
	15.3.3. Momentum	...	471
	15.4. Newton's First Law of Motion	...	471
	15.5. Newton's Second Law of Motion	...	472
	15.6. Units of Force	...	473
	Solved Problems 15.1—15.10	...	473
	15.7. Motion on an Inclined Smooth Surface	...	480
	15.8. Motion on an Inclined Rough Surface	...	480
	Solved Problems 15.11—15.15	...	481
	15.9. Analysis of Lift Motion	...	488
	Solved Problems 15.16—15.20	...	489

Chap.		Pages
15.10. Analysis of the Motion of Two Bodies Connected by a String	...	493
Solved Problems 15.21—15.25	...	494
15.11. Analysis of the Motion of Two Bodies Connected by a String When One Body is Lying on a Horizontal Surface and Other is Hanging Free	...	499
15.11.1. The horizontal surface is smooth and string is passing over a smooth pulley	...	499
15.11.2. The horizontal surface is rough and the string is passing over a smooth pulley	...	501
Solved Problems 15.26—15.29	...	502
15.11.3. The horizontal surface is rough and the string passes over a rough surface	...	504
Solved Problem 15.30	...	505
15.12. Analysis of the Motion of Two Bodies Connected by a String When One Body is Lying on Inclined Plane and the Other is Hanging Free in Air	...	506
15.12.1. First case when the inclined surface is smooth	...	506
15.12.2. Second case when the inclined surface is rough	...	508
Solved Problems 15.31—15.32	...	509
15.13. Momentum and Angular Momentum (or moment of momentum)	...	511
15.13.1. Momentum	...	511
15.13.2. Momentum of momentum or angular momentum	...	511
15.14. Moment of Inertia of a Circular Section	...	512
15.15. Radius of Gyration (K)	...	514
Solved Problems 15.33—15.34	...	515
15.16. Laws for Rotary Motion	...	516
15.17. Kinetic Energy Due to Rotation	...	517
15.18. Total Kinetic Energy of a Body	...	518
Solved Problems 15.35—15.38	...	518
15.19. Rotation Due to a Weight W Attached to One End of a String Passing Over a Pulley of Weight W_0	...	521
Solved Problem 15.39	...	522
15.20. Rotation due to Weights Attached to the Two Ends of a String, which Passes Over a Rough Pulley of Weight W_0	...	523
Solved Problems 15.40—15.42	...	525
15.21. D' Alembret's Principle Applicable to Plane Motion	...	530
Solved Problem 15.43	...	532
15.22. D' Alembret's Principle Applicable to Rotary Motion	...	533
Highlights	...	536
Exercise 15	...	539
16. Simple Harmonic Motion and Mechanical Vibrations		**544—579**
16.1. Introduction	...	544
16.2. Definitions of Some Terms Used with S.H.M.	...	547
Solved Problems 16.1—16.9	...	547
16.3. Oscillation of the Bodies Having Simple Harmonic Motion	...	556
16.3.1. Oscillation of a vertical elastic string or spring	...	556
Solved Problems 16.10—16.17	...	558

Chap.				Pages
		16.3.2. Oscillations of a simple pendulum	...	566
		Solved Problems 16.18—16.19	...	567
	16.4.	Gain or Loss of Oscillations Due to Change in g or 1 for a Pendulum	...	567
		Solved Problems 16.20—16.21	...	568
	16.5.	Free Vibration	...	569
		16.5.1. The longitudinal vibrations	...	569
		16.5.2. The transverse vibration	...	570
		16.5.3. The torsional vibrations	...	570
		Solved Problems 16.22—16.25	...	571
		Highlights	...	576
		Exercise 16	...	577
17.	**Collision of Elastic Bodies**			**580—605**
	17.1.	Introduction	...	580
	17.2.	Some Definition	...	580
		17.2.1. Time of compression	...	580
		17.2.2. Time of restitution	...	580
		17.2.3. Time of collision	...	580
		17.2.4. Law of conversion of momentum	...	580
	17.3.	Types of Impacts	...	580
		17.3.1. Direct impact of two bodies	...	580
		17.3.2. Indirect impact of two bodies	...	581
	17.4.	Co-efficient of restitution	...	583
		Solved Problems 17.1—17.7	...	584
	17.5.	Loss of Kinetic Energy During Impact	...	592
		Solved Problems 17.8—17.12	...	592
	17.6.	Impact of a Body on a Fixed Plane	...	597
	17.7.	Direct impact of a body on a fixed plane	...	597
		Solved Problems 17.13—17.15	...	598
	17.8.	Indirect Impact of a Body on a Fixed Plane	...	600
		Solved Problems 17.16—17.17	...	600
		Highlights	...	602
		Exercise 17	...	603
18.	**Work, Power and Energy**			**606—637**
	18.1.	Introduction	...	606
	18.2.	Work	...	606
	18.3.	Units of Work	...	606
		Solved Problems 18.1—18.7	...	607
	18.4.	Power	...	610
		Solved Problems 18.8—18.11	...	610
	18.5.	Work Done By a Torque	...	614
	18.6.	Power Developed by a Torque	...	614
		Solved Problems 18.12—18.13	...	614
	18.7.	Energy	...	615
	18.8.	Mechanical Energy	...	615
		18.8.1. Potential energy	...	615

Chap.			Pages
	18.8.2. Kinetic energy	...	615
	18.8.3. Work done is equal to change of kinetic energy	...	616
	Solved Problems 18.14—18.15	...	617
18.9.	Law of Conservation of Energy	...	618
	Solved Problems 18.16—18.21	...	619
18.10.	Energy Lost By a Body Falling on Another Body and to Calculate the Resistance Offered the Ground	...	626
	Solved Problems 18.22—18.28	...	627
	Highlights	...	635
	Exercise 18	...	635
19.	**Forces in Space**		**638—657**
19.1.	Introduction	...	638
19.2.	Notation of a Vector	...	638
	19.2.1. Magnitude of a vector	...	638
	19.2.2. Multiplication of a vector by a scalar	...	638
19.3.	Components of a Force	...	638
19.4.	Unit Vector	...	639
19.5.	Components of a Force When Two Points on its Line of Actions are Given	...	640
19.6.	Position Vector of a Given Point	...	640
	Solved Problems 19.1—19.5	...	640
19.7.	Important Vector Operations	...	646
	19.7.1. Vector addition	...	646
	19.7.2. Dot product	...	646
	19.7.3. Cross product or vector product	...	647
	Solved Problems 19.6—19.10	...	647
19.8	Moment of a Force	...	650
	Solved Problems 19.11—19.15	...	650
	Subject Index	...	**658**

PART I

STATICS

1

Fundamentals of Engineering Mechanics

1.1. INTRODUCTION

Engineering mechanics is that branch of science which deals with the behaviour of a body when the body is at rest or in motion. The engineering mechanics may be divided into Statics and Dynamics. The branch of science, which deals with the study of a body when the body is at rest, is known as *Statics* while the branch of science which deals with the study of a body when the body is in motion, is known as *Dynamics*. Dynamics is further divided into kinematics and kinetics. The study of a body in motion, when the forces which cause the motion are not considered, is called *kinematics* and if the forces are also considered for the body in motion, that branch of science is called *kinetics*. The classification of Engineering Mechanics are shown in Fig. 1.1 below.

ENGINEERING MECHANICS

1. Statics (Body is at rest)

2. Dynamics (Body is in motion)

   (*i*) Kinematics

   (*ii*) Kinetics

Fig. 1.1

Note. Statics deals with equilibrium of bodies at rest, whereas dynamics deals with the motion of bodies and the forces that cause them.

1.2. DEFINITIONS

1.2.1. Vector Quantity. A quantity which is completely specified by magnitude and direction, is known as a vector quantity. Some examples of vector quantities are : velocity, acceleration, force and momentum. A vector quantity is represented by means of a straight line with an arrow as shown in Fig. 1.2. The length of the straight line (*i.e.*, *AB*) represents the magnitude and arrow represents the direction of the vector. The symbol $\vec{AB}$ also represents this vector, which means it is acting from *A* to *B*.

Fig. 1.2. Vector Quantity.

1.2.2. Scalar Quantity. A quantity, which is completely specified by magnitude only, is known as a scalar quantity. Some examples of scalar quantity are : mass, length, time and temperature.

1.2.3. A Particle. A particle is a body of infinitely small volume (or a particle is a body of negligible dimensions) and the mass of the particle is considered to be concentrated at a point. Hence a particle is assumed to a point and the mass of the particle is concentrated at this point.

1.2.4. Law of Parallelogram of Forces. The law of parallelogram of forces is used to determine the resultant* of two forces acting at a point in a plane. It states, "If two forces, acting at a point be represented

*The resultant of a system of forces may be defined as a single force which has the same effect as system of forces acting on the body.

in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point".

Let two forces P and Q act at a point O as shown in Fig. 1.3. The force P is represented in magnitude and direction by OA whereas the force Q is presented in magnitude and direction by OB. Let the angle between the two forces be 'α'. The resultant of these two forces will be obtained in magnitude and direction by the diagonal (passing through O) of the parallelogram of which OA and OB are two adjacent sides. Hence draw the parallelogram with OA and OB as adjacent sides as shown in Fig. 1.4. The resultant R is represented by OC in magnitude and direction.

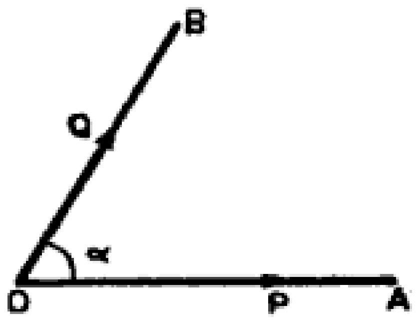


Fig. 1.3

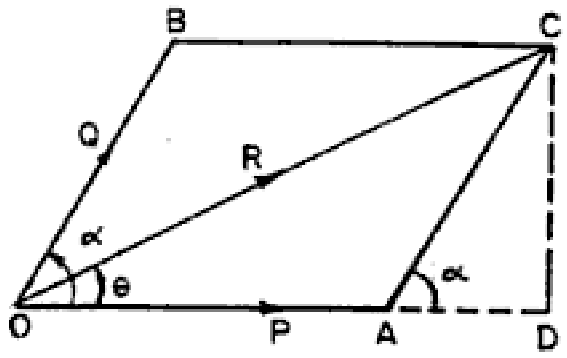


Fig. 1.4

Magnitude of Resultant (R)

From C draw CD perpendicular to OA produced.

Let α = Angle between two forces P and $Q = \angle AOB$

Now $\angle DAC = \angle AOB$ (Corresponding angles)

$= \alpha$

In parallelogram $OACB$, AC is parallel and equal to OB.

$\therefore$ $AC = Q$.

In triangle ACD,

$$AD = AC \cos \alpha = Q \cos \alpha$$

and $$CD = AC \sin \alpha = Q \sin \alpha.$$

In triangle OCD,

$$OC^2 = OD^2 + DC^2.$$

But $OC = R$, $OD = OA + AD = P + Q \cos \alpha$

and $DC = Q \sin \alpha$.

$$\therefore \quad R^2 = (P + Q \cos \alpha)^2 + (Q \sin \alpha)^2 = P^2 + Q^2 \cos^2 \alpha + 2PQ \cos \alpha + Q^2 \sin^2 \alpha$$

$$= P^2 + Q^2 (\cos^2 \alpha + \sin^2 \alpha) + 2PQ \cos \alpha$$

$$= P^2 + Q^2 + 2PQ \cos \alpha \qquad (\because \cos^2 \alpha + \sin^2 \alpha = 1)$$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha} \qquad ...(1.1)$$

Equation (1.1) gives the magnitude of resultant force R.

Direction of Resultant

Let θ = Angle made by resultant with OA.

Then from triangle OCD,

$$\tan \theta = \frac{CD}{OD} = \frac{Q \sin \alpha}{P + Q \cos \alpha}$$

$$\therefore \quad \theta = \tan^{-1} \left(\frac{Q \sin \alpha}{P + Q \cos \alpha} \right) \qquad ...(1.2)$$

Equation (1.2) gives the direction of resultant (R).

The direction of resultant can also be obtained by using sine rule [In triangle OAC, $OA = P$, $AC = Q$, $OC = R$, angle $OAC = (180 - \alpha)$, angle $ACO = 180 - [\theta + 180 - \alpha] = (\alpha - \theta)$]

$$\frac{\sin\theta}{AC} = \frac{\sin(180-\alpha)}{OC} = \frac{\sin(\alpha-\theta)}{OA}$$

$$\frac{\sin\theta}{Q} = \frac{\sin(180-\alpha)}{R} = \frac{\sin(\alpha-\theta)}{P}$$

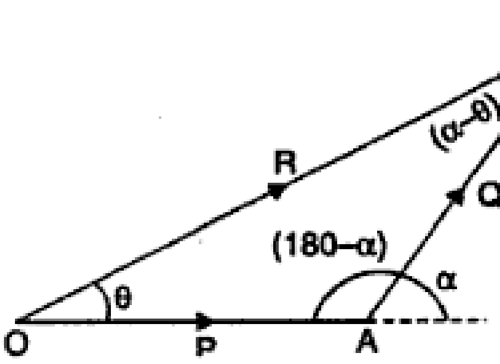


Fig. 1.4 (a)

Two cases are important.

1st Case. If the two forces P and Q act at right angles, then

$$\alpha = 90°$$

From equation (1.1), we get the magnitude of resultant as

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$$

$$= \sqrt{P^2 + Q^2} \qquad (\because \cos 90° = 0) \quad ...(1.2\,A)$$

From equation (1.2), the direction of resultant is obtained as

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

$$= \tan^{-1}\left(\frac{Q\sin 90°}{P + Q\cos 90°}\right) = \tan^{-1}\frac{Q}{P} \qquad (\because \sin 90° = 1 \text{ and } \cos 90° = 0)$$

2nd Case. The two forces P and Q are equal and are acting at an angle α between them. Then the magnitude and direction of resultant is given as

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{P^2 + P^2 + 2P \times P \times \cos\alpha} \qquad (\because P = Q)$$

$$= \sqrt{2P^2 + 2P^2\cos\alpha} = \sqrt{2P^2(1+\cos\alpha)}$$

$$= \sqrt{2P^2 \times 2\cos^2\frac{\alpha}{2}} \qquad \left(\because 1 + \cos\alpha = 2\cos^2\frac{\alpha}{2}\right)$$

$$= \sqrt{4P^2\cos^2\frac{\alpha}{2}} = 2P\cos\frac{\alpha}{2} \quad ...(1.3)$$

and

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right) = \tan^{-1}\frac{P\sin\alpha}{P + P\cos\alpha} \qquad (\because P = Q)$$

$$= \tan^{-1}\frac{P\sin\alpha}{P(1+\cos\alpha)} = \tan^{-1}\frac{\sin\alpha}{1+\cos\alpha}$$

$$= \tan^{-1}\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} \qquad \left(\because \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)$$

$$= \tan^{-1}\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \tan^{-1}\left(\tan\frac{\alpha}{2}\right) = \frac{\alpha}{2} \quad ...(1.4)$$

It is not necessary that one of two forces, should be along x-axis. The forces P and Q may be in any direction as shown in Fig. 1.5. If the angle between the two forces is 'α', then their resultant will be given by equation (1.1). The direction of the resultant would be obtained from equation (1.2). But angle θ will be the angle made by resultant with the direction of P.

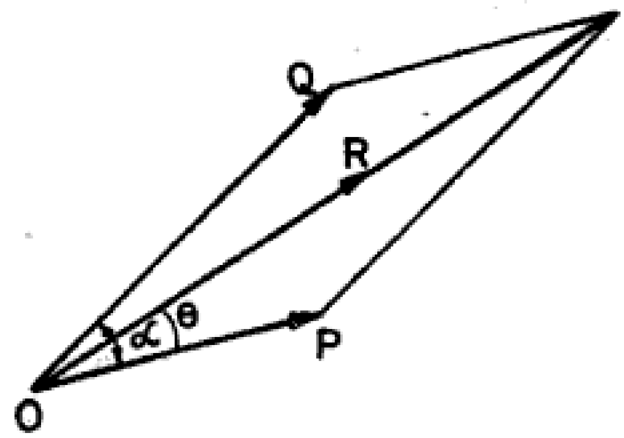


Fig. 1.5

1.2.5. Law of Triangle of Forces. It states that, "if three forces acting at a point be represented in magnitude and direction by the three sides of a triangle, taken in order, they will be in equilibrium."

1.2.6. Lami's Theorem. It states that, "If there forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces."

Suppose the three forces P, Q and R are acting at a point O and they are in equilibrium as shown in Fig. 1.6.

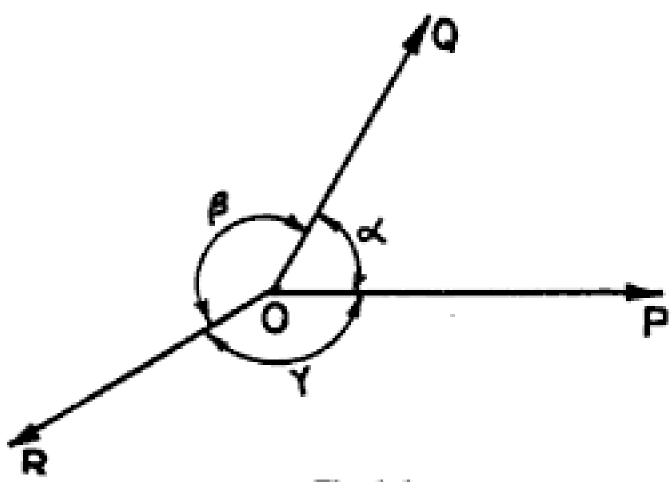


Fig. 1.6

Let α = Angle between force P and Q.

β = Angle between force Q and R.

γ = Angle between force R and P.

Then according to Lami's theorem,

$P \propto$ sine of angle between Q and $R \propto \sin \beta$.

$$\therefore \quad \frac{P}{\sin \beta} = \text{constant}$$

$$\text{Similarly} \quad \frac{Q}{\sin \gamma} = \text{constant and } \frac{R}{\sin \alpha} = \text{constant}$$

$$\text{or} \quad \frac{P}{\sin \beta} = \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha}$$

Proof of Lami's Theorem. The three forces acting on a point, are in equilibrium and hence they can be represented by the three sides of the triangle taken in the same order. Now draw the force triangle as shown in Fig. 1.6 (a).

Now applying sine rule, we get

$$\frac{P}{\sin(180 - \beta)} = \frac{Q}{\sin(180 - \gamma)} = \frac{R}{\sin(180 - \alpha)}$$

This can also be written

$$\frac{P}{\sin \beta} = \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha}$$

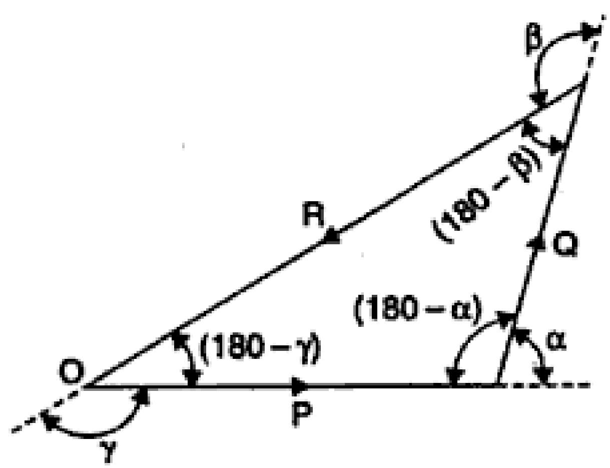


Fig. 1.6 (a)

This is same equation as equation (1.5).

Note. All the three forces should be acting either towards the point or away from the point.

1.3. SYSTEM OF UNITS

The following system of units are mostly used :

1. C.G.S. (*i.e.*, Centimetre-Gram Second) system of units.
2. M.K.S. (*i.e.*, Metre-Kilogram-Second) system of units.
3. S.I. (*i.e.*, International) system of units.

1.3.1 C.G.S. System of Units. In this system, length is expressed in centimetre, mass in gram and time in second. The unit of force in this system is *dyne*, which is defined as the force acting on a mass of one gram and producing an acceleration of one centimetre per second square.

1.3.2. M.K.S. System of Units. In this system, length is expressed in metre, mass in kilogram and time in second. The unit of force in this system is expressed as kilogram force and is represented as kgf.

1.3.3. S.I. System of Units. S.I. is abbreviation for 'The System International Units'. It is also called the International System of Units. In this system length is expressed in metre mass in kilogram and time in second. The unit of force in this system is Newton and is represented N. Newton is the force acting on a mass of one kilogram and producing an acceleration of one metre per second square. The relation between newton (N) and dyne is obtained as

$$\text{One Newton} = \text{One kilogram mass} \times \frac{\text{One metre}}{s^2}$$

$$= 1000 \text{ gm} \times \frac{100 \text{ cm}}{s^2} \qquad (\because \text{ one kg} = 1000 \text{ gm})$$

$$= 1000 \times 100 \times \frac{\text{gm} \times \text{cm}}{s^2}$$

$$= 10^5 \text{ dyne} \qquad \left\{\because \text{ dyne} = \frac{\text{gm} \times \text{cm}}{s^2}\right\}$$

When the magnitude of forces is very large, then the unit of force like kilo-newton and mega-newton is used. Kilo-newton is represented by kN.

One kilo-newton = 10^3 newton

or 1 kN = 10^3 N

and One mega newton = 10^6 Newton

The large quantities are represented by kilo, mega, giga and terra. They stand for :

Kilo = 10^3 and represented by ....k

Mega = 10^6 and represented by ....M

Giga = 10^9 and represented by ....G

Tera = 10^{12} and represented by ....T

Thus mega newton means 10^6 newton and is represented by MN. Similarly, giga newton means 10^9 N and is represented by GN. The symbol TN stands for 10^{12} N.

The small quantities are represented by milli, micro, nano and pico. They are equal to

Milli = 10^{-3} and represented by ....m

Micro = 10^{-6} and represented by ....μ

Nano = 10^{-9} and represented by ....n

Pico = 10^{-12} and represented by ....p.

Thus milli newton means 10^{-3} newton and is represented by mN. Micro newton means 10^{-6} N and is represented by μN.

The relation between kilogram force (kgf) and newton (N) is given by One kgf = 9.81 N

Weight of a body is the force with which the body is attracted towards earth. If W = weight of a body, m = mass in kg, then $W = m \times g$ Newtons

If mass, m of the body is 1 kg, then its weight will be,

$$W = 1\ (\text{kg}) \times 9.81\ \frac{\text{m}}{\text{s}^2} = 9.81\ \text{N}. \qquad \left(\because\ \text{N} = \text{kg}\ \frac{\text{m}}{\text{s}^2}\right)$$

1.4. TRIGONOMETRIC FORMULAE AND EXPRESSIONS

The following are the trigonometric formulae in a right-angled triangle ABC of Fig. 1.7.

(*i*) $\sin\theta = \dfrac{AC}{BC}$ (*ii*) $\cos\theta = \dfrac{AB}{BC}$

(*iii*) $\tan\theta = \dfrac{AC}{AB}$

(*iv*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(*v*) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(*vi*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(*vii*) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(*viii*) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(*ix*) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(*x*) $\sin 2A = 2 \sin A \cos A$

(*xi*) $\sin^2\theta + \cos^2\theta = 1.$

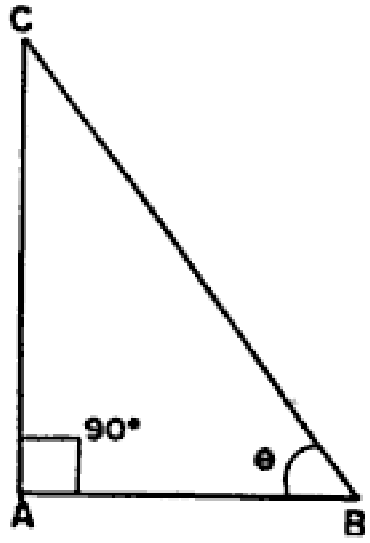


Fig. 1.7

1.5. DIFFERENTIATION AND INTEGRATION

1.5.1. Differentiations. (*i*) Differentiation of a quantity (say A) with respect to x is written as $\dfrac{d}{dx}(A)$ or $\dfrac{dA}{dx}$

(*ii*) $\dfrac{d}{dx}(x^4) = 4x^3, \dfrac{d}{dx}(x^n) = n\,x^{n-1}$ and $\dfrac{d}{dx}(x) = 1$

(*iii*) $\dfrac{d}{dx}(8x + 5)^4 = 4\,(8x + 5)^3 \times 8$

(*iv*) $\dfrac{d}{dx}(4) = 0$ as differentiation of constant is zero.

(*v*) $\dfrac{d}{dx}(u.v) = u \,.\, \dfrac{dv}{dx} + v \,.\, \dfrac{du}{dx}$ [when u and v are functions of x]

(*vi*) Differentiation of trigonometrical functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

1.5.2. Integrations. (*i*) Integration of a quantity (say A) with respect to x is written as $\int A dx$.

(*ii*) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$ (*iii*) $\int 4dx = 4x$

(*iv*) $\int (8x+5)^4 \, dx = \frac{(8x+5)^{4+1}}{(4+1) \times 8}$.

Problem 1.1. *Two forces of magnitude 10 N and 8 N are acting at a point. If the angle between the two forces is 60°, determine the magnitude of the resultant force.*

Sol. Given :

Force $P = 10$ N

Force $Q = 8$ N

Angle between the two forces, $\alpha = 60°$

The magnitude of the resultant force (R) is given by equation (1.1)

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha} = \sqrt{10^2 + 8^2 + 2 \times 10 \times 8 \times \cos 60°}$$

$$= \sqrt{100 + 64 + 2 \times 10 \times 8 \times \tfrac{1}{2}} \qquad (\because \cos 60° = \tfrac{1}{2})$$

$$= \sqrt{100 + 64 + 80} = \sqrt{244} = \mathbf{15.62 \text{ N. Ans.}}$$

Problem 1.2. *Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $20 \times \sqrt{3}$ N, find magnitude of each force.*

Sol. Given : Angle between the force, $\alpha = 60°$

Resultant, $R = 20 \times \sqrt{3}$

The forces are equal. Let P is the magnitude of each force.

Using equation (1.3), we have

$$R = 2P \cos \frac{\alpha}{2} \quad \text{or} \quad 20 \times \sqrt{3} = 2P \times \cos\left(\frac{60°}{2}\right) = 2P \cos 30°$$

$$= 2P \times \frac{\sqrt{3}}{2} = P \times \sqrt{3} \qquad \left(\because \cos 30° = \frac{\sqrt{3}}{2}\right)$$

$$\therefore \quad P = \frac{20 \times \sqrt{3}}{\sqrt{3}} = 20 \text{ N.}$$

$\therefore$ Magnitude of each force = **20 N. Ans.**

Problem 1.3. *The resultant of the two forces, when they act at an angle of 60° is 14 N. If the same forces are acting at right angles, their resultant is $\sqrt{136}$ N. Determine the magnitude of the two forces.*

Sol. Given : **Case I**

Resultant, $R_1 = 14$ N

Angle, $\alpha = 60°$

Case II

Resultant, $R_2 = \sqrt{136}$ N

Angle, $\alpha = 90°$

Let the magnitude of the two forces are P and Q.

Using equation (1.1) for case I.

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

or $$14 = \sqrt{P^2 + Q^2 + 2PQ \times \cos 60°} = \sqrt{P^2 + Q^2 + 2PQ \times \tfrac{1}{2}}$$

or $$14 = \sqrt{P^2 + Q^2 + PQ}$$

Squaring, $196 = P^2 + Q^2 + PQ$ ...(*i*)

Using equation (1.2 A) for case II,

$$R = \sqrt{P^2 + Q^2} \quad \text{or} \quad \sqrt{136} = \sqrt{P^2 + Q^2}$$

or $136 = P^2 + Q^2$ (Squaring both sides) ...(*ii*)

Subtracting equation (*ii*) from equation (*i*), we get

$$196 - 136 = P^2 + Q^2 + PQ - (P^2 + Q^2)$$

or $60 = PQ$ ...(*iii*)

Multiplying the above equation by two, we get $120 = 2PQ$ ...(*iv*)

Adding equation (*iv*) to equation (*ii*), we get $136 + 120 = P^2 + Q^2 + 2PQ$

or $256 = P^2 + Q^2 + 2PQ$ or $(16)^2 = (P + Q)^2$

$16 = P + Q$

$\therefore$ $P = (16 - Q)$ ...(*v*)

Substituting the value of P in equation (*iii*), we get

$$60 = (16 - Q) \times Q = 16Q - Q^2 \quad \text{or} \quad Q^2 - 16Q + 60 = 0$$

$\therefore$ This is a quadratic equation.

$$\therefore \quad Q = \frac{16 \pm \sqrt{(-16)^2 - 4 \times 60}}{2} = \frac{16 \pm \sqrt{256 - 240}}{2} = \frac{16 \pm 4}{4}$$

$$= \frac{16 + 4}{2} \quad \text{and} \quad \frac{16 - 4}{2} = 10 \text{ and } 6.$$

Substituting the value of Q in equation (*v*), we get

$$P = (16 - 10) \text{ or } (16 - 6) = 6 \text{ or } 10.$$

$\therefore$ Hence the two forces are 10 N and 6 N. **Ans.**

Problem 1.4. *Two forces are acting at a point O as shown in Fig. 1.8. Determine the resultant in magnitude and direction.*

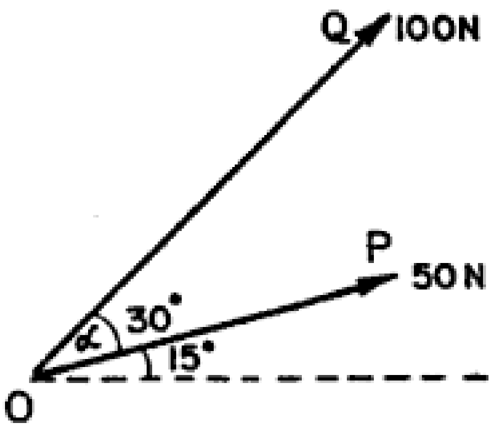


Fig. 1.8

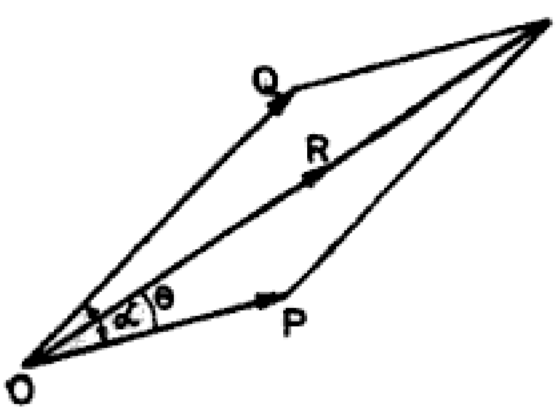


Fig. 1.9

Sol. Given :

Force $P = 50$ N, Force Q = 100 N

Angle between the two forces, $\alpha = 30°$

The magnitude of the resultant R is given by equation (1.1) as

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha} = \sqrt{50^2 + 100^2 + 2 \times 50 \times 100 \times \cos 30°}$$

$$= \sqrt{2500 + 10000 + 8660} = \sqrt{21160} = \mathbf{145.46 \text{ N. Ans.}}$$

The resultant R is shown in Fig. 1.9.

The angle made by the resultant with the direction of P is given by equation (1.2) as

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

or
$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right) = \tan^{-1}\left(\frac{100 \times \sin 30°}{50 + 100\cos 30°}\right)$$

$$= \tan^{-1} 0.366 = 20.10°$$

∴ Angle made by resultant with x-axis = $\theta + 15° = 20.10 + 15 =$ **35.10°. Ans.**

Problem 1.5. *The resultant of two concurrent forces is 1500 N and the angle between the forces is 90°. The resultant makes an angle of 36° with one of the force. Find the magnitude of each force.*

Sol. Given :

Resultant, $R = 1500$ N

Angle between the forces, $\alpha = 90°$

Angle made by resultant with one force, $\theta = 36°$

Let P and Q are two forces.

Using equation (1.2), $\tan\theta = \dfrac{Q\sin\alpha}{P + Q\cos\alpha}$

or
$$\tan 36° = \frac{Q\sin 90°}{P + Q\cos 90°} = \frac{Q \times 1}{P + Q \times 0} = \frac{Q}{P} \quad \text{or} \quad 0.726 = \frac{Q}{P}$$

or
$$Q = 0.726\,P \qquad \text{...}(i)$$

Using equation (1.1), $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

or
$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

or
$$1500^2 = P^2 + (0.726P)^2 + 2P(0.726P) \times \cos 90° \qquad (\because\ Q = 0.726P)$$

$$1500^2 = P^2 + 0.527P^2 + 0 \qquad (\because\ \cos 90° = 0)$$

$$= 1.527\,P^2$$

∴
$$P = \sqrt{\frac{1500^2}{1.527}} = \frac{1500}{1.2357} = 1213.86 \text{ N}$$

Substituting the value of P in equation (i), we get

$$Q = 0.726 \times 1213.86 = \mathbf{881.26\ N.\ Ans.}$$

Alternate Method. Refer to Fig. 1.9 (a). Consider triangle OAC.

Using sine rule, we get

$$\frac{\sin 90°}{R} = \frac{\sin 36°}{Q} = \frac{\sin 54°}{P}$$

or
$$\frac{\sin 90°}{R} = \frac{\sin 36°}{Q}$$

or
$$Q = \frac{R\sin 36°}{\sin 90°} \qquad (\text{where } R = 1500 \text{ N})$$

$$= \frac{1500 \times 0.5877}{1} = \mathbf{881.67\ N.\ Ans.}$$

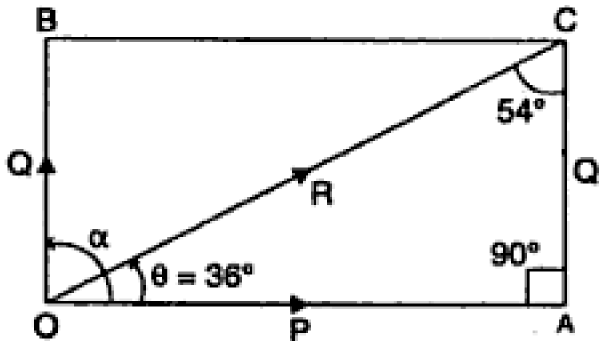


Fig. 1.9 (a)

Also, we have $\dfrac{\sin 90°}{R} = \dfrac{\sin 54°}{P}$

∴
$$P = \frac{R\sin 54°}{\sin 90°} = \frac{1500 \times 0.8090}{1}$$

$$= \mathbf{1213.52\ N.\ Ans.}$$

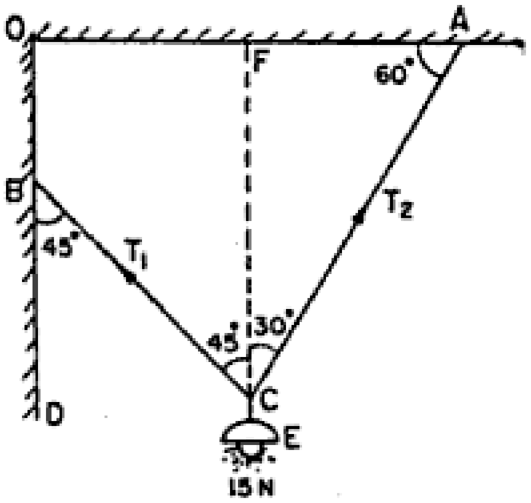


Fig. 1.13

Sol. Given : Weight at $C = 15$ N

$\angle OAC = 60°$

$\angle CBD = 45°$

Let T_1 = Force in string BC

T_2 = Force in string AC

1st Method

Using Lami's theorem at C

$$\frac{15}{\sin \text{ of } \angle BCA} = \frac{T_1}{\sin \text{ of } \angle ACE} = \frac{T_2}{\sin \text{ of } \angle ACE}$$

But $\angle BCA = 45° + 30° = 75°$

$\angle ACE = 180° - 30° = 150°$

$\angle BCE = 180° - 45° = 135°$

$$\therefore \quad \frac{15}{\sin 75°} = \frac{T_1}{\sin 150°} = \frac{T_2}{\sin 135°}$$

$$\therefore \quad T_1 = \frac{15 \times \sin 150°}{\sin 75°} = \mathbf{7.76\ N.\ Ans.}$$

and

$$T_2 = \frac{15 \times \sin 135°}{\sin 75°} = \mathbf{10.98\ N.\ Ans.}$$

2nd Method

The point C is in the equilibrium. The forces acting at C are 15 N, T_1 and T_2.

Resolving all forces at C in the horizontal direction

$$T_1 \sin 45° = T_2 \sin 30° \quad \text{or} \quad T_1 \times \frac{1}{\sqrt{2}} = T_2 \times \frac{1}{2}$$

$$\therefore \quad T_2 = T_1 \times \frac{2}{\sqrt{2}} = \sqrt{2} \times T_1 \qquad \text{...}(i)$$

Resolving all forces at C in the vertical direction,

$$T_1 \cos 45° + T_2 \cos 30° = 15$$

or

$$T_1 \times \frac{1}{\sqrt{2}} + T_2 \times \frac{\sqrt{3}}{2} = 15 \qquad \text{...}(ii)$$

Substituting the value of T_2 from equation (i) into equation (ii),

$$T_1 \times \frac{1}{\sqrt{2}} + \sqrt{2} \times T_1 \times \frac{\sqrt{3}}{2} = 15$$

or

$$\frac{T_1}{\sqrt{2}} + \frac{\sqrt{3}T_1}{\sqrt{2}} = 15 \qquad \left(\because \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{1}{\sqrt{2}}\right)$$

or

$$T_1 + \sqrt{3}T_1 = 15 \times \sqrt{2} \quad \text{or} \quad T_1(1 + \sqrt{3}) = 15 \times \sqrt{2}$$

$$\therefore \quad T_1 = \frac{15 \times \sqrt{2}}{1 \times \sqrt{3}} = \mathbf{7.76\ N.\ Ans.}$$

Substituting this value of T_1 in equation (i), we get

$$T_2 = \sqrt{2} \times T_1 = \sqrt{2} \times 7.76 = \mathbf{10.98\ N.\ Ans.}$$

1.6. RESOLUTION OF A FORCE

Resolution of a force means "finding the components of a given force in two given directions."

Let a given force be R which makes an angle θ with X-axis as shown in Fig. 1.14. It is required to find the components of the force R along X-axis and Y-axis.

Components of R along X-axis $= R \cos \theta$.

Component of R along Y-axis $= R \sin \theta$.

Hence, the resolution of forces is the process of finding components of forces in specified directions.

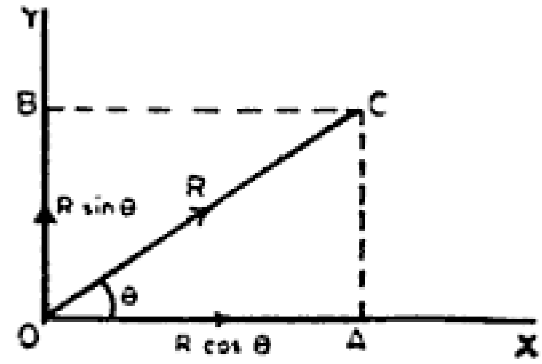


Fig. 1.14

1.7. RESOLUTION OF A NUMBER OF COPLANAR FORCES

Let a number of coplanar forces (forces acting in one plane are called co-planar forces) $R_1, R_2, R_3, ...$ are acting at a point as shown in Fig. 1.15.

Let θ_1 = Angle made by R_1 with X-axis
θ_2 = Angle made by R_2 with X-axis
θ_3 = Angle made by R_3 with X-axis
H = Resultant component of all forces along X-axis
V = Resultant component of all forces along Y-axis
R = Resultant of all forces
θ = Angle made by resultant with X-axis.

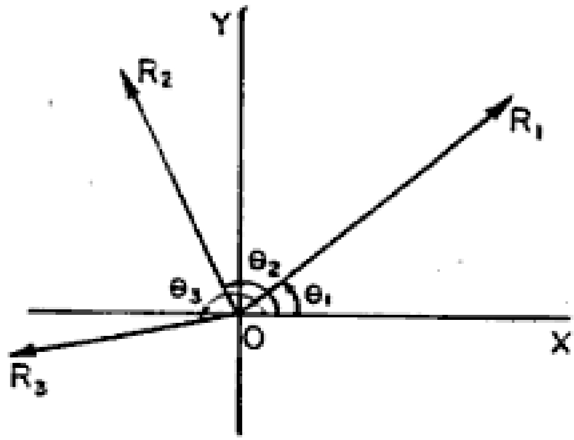


Fig. 1.15

Each force can be resolved into two components, one along X-axis and other along Y-axis.

Component of R_1 along X-axis $= R_1 \cos \theta_1$

Component of R_1 along Y-axis $= R_1 \sin \theta_1$.

Similarly, the components of R_2 and R_3 along X-axis and Y-axis are $(R_1 \cos \theta_2, R_2 \sin \theta_2)$ and $(R_3 \cos \theta_3, R_3 \sin \theta_3)$ respectively.

Resultant components along X-axis

= Sum of components of all forces along X-axis.

$$\therefore \quad H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3 + ... \quad ...(1.6)$$

Resultant component along Y-axis.

= Sum of components of all forces along Y-axis.

$$\therefore \quad V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3 + ... \quad ...(1.7)$$

Then resultant of all the forces, $R = \sqrt{H^2 + V^2}$ ...(1.8)

The angle made by R with X-axis is given by, $\tan \theta = \dfrac{V}{H}$ ...(1.9)

Problem 1.10. *Two forces are acting at a point O as shown in Fig. 1.16. Determine the resultant in magnitude and direction.*

Sol. The above problem has been solved earlier. Hence it will be solved by resolution of forces.

Force $P = 50$ N and force $Q = 100$ N.

Let us first find the angles made by each force with X-axis.

Angle made by P with x-axis = 15°

Angle made by Q with x-axis = 15 + 30 = 45°

Let H = Sum of components of all forces along X-axis.

V = Sum of components of all forces along Y-axis.

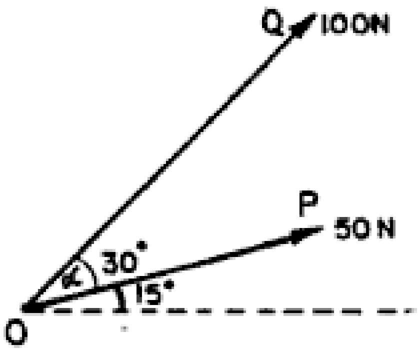


Fig. 1.16

The sum of components of all forces along X-axis is given by,

$$H = P\cos 15° + Q\cos 45°$$
$$= 50 \times \cos 15° + 100\cos 45° = 119 \text{ N}$$

The sum of components of all forces along Y-axis is given by,

$$V = P\sin 15° + Q\sin 45°$$
$$= 50\sin 15° + 100\sin 45° = 83.64 \text{ N}$$

The magnitude of the resultant force is given by equation (1.8),

$$R = \sqrt{H^2 + V^2} = \sqrt{119^2 + 83.64^2} = \mathbf{145.46\ N.\ Ans.}$$

The direction of the resultant force is given by equation (1.9), $\tan\theta = \frac{V}{H} = \frac{83.64}{119}$

$$\therefore \quad \theta = \tan^{-1}\frac{83.64}{119} = \mathbf{35.10°.\ Ans.}$$

Here θ is the angle made by resultant R with x-axis.

Problem 1.11. *Three forces of magnitude 40 kN, 15 kN and 20 kN are acting at a point O as shown in Fig. 1.17. The angles made by 40 kN, 15 kN and 20 kN forces with X-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force.*

Sol. Given :

$R_1 = 10$ kN, $\theta_1 = 60°$

$R_2 = 15$ kN, $\theta_2 = 120°$

$R_3 = 20$ kN, $\theta_3 = 240°$

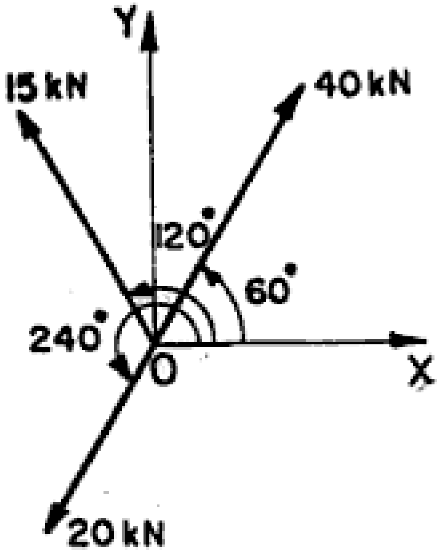


Fig. 1.17

The sum of components of all forces along X-axis is given by equation (1.6) as

$$H = R_1\cos\theta_1 + R_2\cos\theta_2 + R_3\cos\theta_3$$
$$= 40 \times \cos 60° + 15 \times \cos 120° + 20 \times \cos 240°$$
$$= 40 \times \tfrac{1}{2} + 15 \times (-\tfrac{1}{2}) + 20 \times (-\tfrac{1}{2})$$
$$= 20 - 7.5 - 10 = 2.5 \text{ kN.}$$

The resultant component along Y-axis is given by equation (1.7) as

$$V = R_1\sin\theta_1 + R_2\sin\theta_2 + R_3\sin\theta_3$$
$$= 40 \times \sin(60°) + 15 \times \sin(120°) + 20 \times \sin(240°)$$
$$= 40 + \frac{\sqrt{3}}{3} + 15 \times \frac{\sqrt{3}}{2} + 20 \times \left(\frac{-\sqrt{3}}{2}\right)$$
$$= 20 \times \sqrt{3} + 7.5 \times \sqrt{3} - 10 \times \sqrt{3} = 17.5 \times \sqrt{3} \text{ kN} = 30.31 \text{ kN.}$$

The magnitude of the resultant force is given by equation (1.8)

$$R = \sqrt{H^2 + V^2} = \sqrt{2.5^2 + 30.31^2} = \mathbf{30.41\ kN.\ Ans.}$$

The direction of the resultant force is given by equation (1.9)

$$\tan\theta = \frac{V}{H} = \frac{30.31}{2.5} = 12.124 = \tan 85.28°$$

$$\therefore \quad \theta = \mathbf{85.28°\ or\ 85°\ 16.8'.\ Ans.}$$

7. According to Lami's theorem, "If three forces acting at a point are equilibrium, each force will be proportional to the sine of the angle between the other two forces."
8. The relation between newton and dyne is given by One newton = 10^5 dyne.
9. Moment of a force about a point = Force × perpendicular distance of the line of action of the force from that point.
10. The force causes linear displacement while moment causes angular displacement. A body will be in equilibrium if (*i*) resultant force in any direction is zero and (*ii*) the net moment of the forces about any point is zero.
11. Gravitational law of attraction is given by,

$$F = G\frac{m_1 \times m_2}{r^2}$$

where G = Universal gravitational constant
m_1, m_2 = mass of bodies
r = Distance between the bodies
F = Force of attraction between the bodies.

EXERCISE 1

(A) Theoretical Problems

1. What do you mean by scalar and vector quantities ?
2. Define the law of parallelogram of forces. What is the use of this law ?
3. State triangle law of forces and Lami's theorem.
4. Two forces P and Q are acting at a point in a plane. The angle between the forces is 'α'. Prove that the resultant (R) of the two forces is given by $R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$.
5. Define the following terms : dyne, newton, meganewton and moment of a force.
6. Prove that one newton is equal to 10^5 dyne.
7. Explain the terms : clockwise moments and anti-clockwise moments.
8. What is the effect of force and moment on a body ?
9. Indicate whether the following statement is true or false.
"The resultant components of the forces acting on a body along any direction is zero but the net moment of the forces about any point is not zero, the body will be in equilibrium'. [Ans. False]
10. Write the S.I. units of : Force, moment and velocity.
11. What do you mean by resolution of a force ?
12. A number of coplanar forces are acting at a point making different angles with x-axis. Find an expression for the resultant force. Find also the angle made by the resultant force with x-axis.
13. State and explain the pinciple of transmissibility of forces.
14. State and explain the following laws :
(*i*) Newton's laws of motion.
(*ii*) The gravitational law of attraction.
15. Using gravitation law of attraction, prove that $W = m \times g$.
16. Explain fully the following terms :
(*i*) Resolved part of a given force in a given direction.
(*ii*) Lami's theorem.

(B) Numerical Problems

1. Determine the magnitude of the resultant of the two forces of magnitude 12 N and 9 N acting at a point if the angle between the two forces is 30°. [Ans. 20.3 N]

2. Find the magnitude of two equal forces acting at a point with an angle of 60° between them, if the resultant is equal to $30 \times \sqrt{3}$ N. [Ans. 30 N]

3. The resultant of two forces when they act at right angles is 10 N, whereas when they act at an angle of 60° the resultants is $\sqrt{148}$. Determine the magnitude of the two forces. [Ans. 8 N and 6 N]

4. Three forces of magnitude 30 kN, 10 kN and 15 kN are acting at a point *O*. The angles made by 30 kN force, 10 kN force and 15 kN force with *x*-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force. [Ans. 21.79 kN, 83° 24]

5. A weight of 800 N is supported by two chains as shown in Fig. 1.26. Determine the tension in each chain. [Ans. 273.5 N, 751.7 N]

6. An electric light fixture weighing 20 N hangs from a point *C*, by two strings *AC* and *BC*. *AC* is inclined at 60° to the horizontal and *BC* at 30° to the vertical as shown in Fig. 1.27. Using Lami's theorem or otherwise determine the forces in the strings *AC* and *BC* [Ans. 8.929 N, 13.05 N]

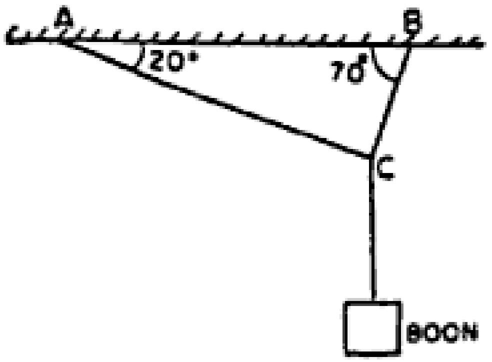


Fig. 1.26

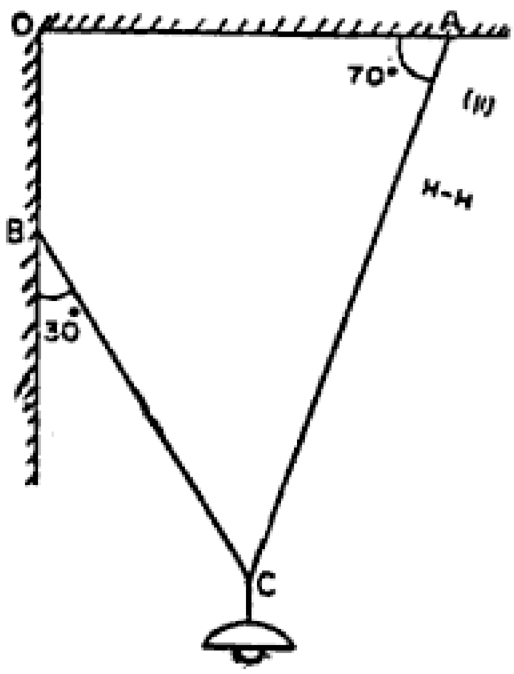


Fig. 1.27

7. A beam *AB* of span 6 m carries a point load of 100 N at a distance 2 m from *A*. Determine the beam reaction. [Ans. R_A = 66.67 N ; R_B = 33.33 N]

8. Four forces of magnitudes 20 N, 30 N, 40 N and 50 N are acting respectively along the four sides of a square taken in order. Determine the magnitude, direction and position of the resultant force.

$$\left[\text{Ans. } 20 \times \sqrt{2} \text{ N}, 225°, \frac{7a}{2 \times \sqrt{2}}\right]$$

9. Two forces magnitude 15 N and 12 N are acting at a point. If the angle between the two forces is 60°, determine the resultant of the forces in magnitude and direction. [Ans. 23.43 N, 26.3°]

10. Four forces of magnitude *P*, 2*P*, $3 \times \sqrt{3}P$ and 4*P* are acting at a point *O*. The angles made by these forces with *x*-axis are 0°, 60°, 150° and 300° respectively. Find the magnitude and direction of the resultant force. [Ans. *P*, 1200]

2

Coplanar Collinear and Concurrent Forces

2.1. INTRODUCTION

Coplanar forces means the forces in a plane. The word collinear stands for the forces which are having common lines of action whereas the word concurrent stands for the forces which intersect at a common point. When several forces act on a body, then they are called *a force system* or *a system of forces*. In a system in which all the forces lie in the same plane, it is known as *coplanar force system*. Hence this chapter deals with a system of forces which are acting in the same plane and the forces are either having a common line of action or intersecting at a common point.

2.2. CLASSIFICATION OF A FORCE SYSTEM

A force system may be coplanar or non-coplanar. If in a system all the forces lie in the same plane then the force system is known as coplanar. But if in a system all the forces lie in different planes, then the force system is known as non-coplanar. Hence a force system is classified as shown in Fig. 2.1.

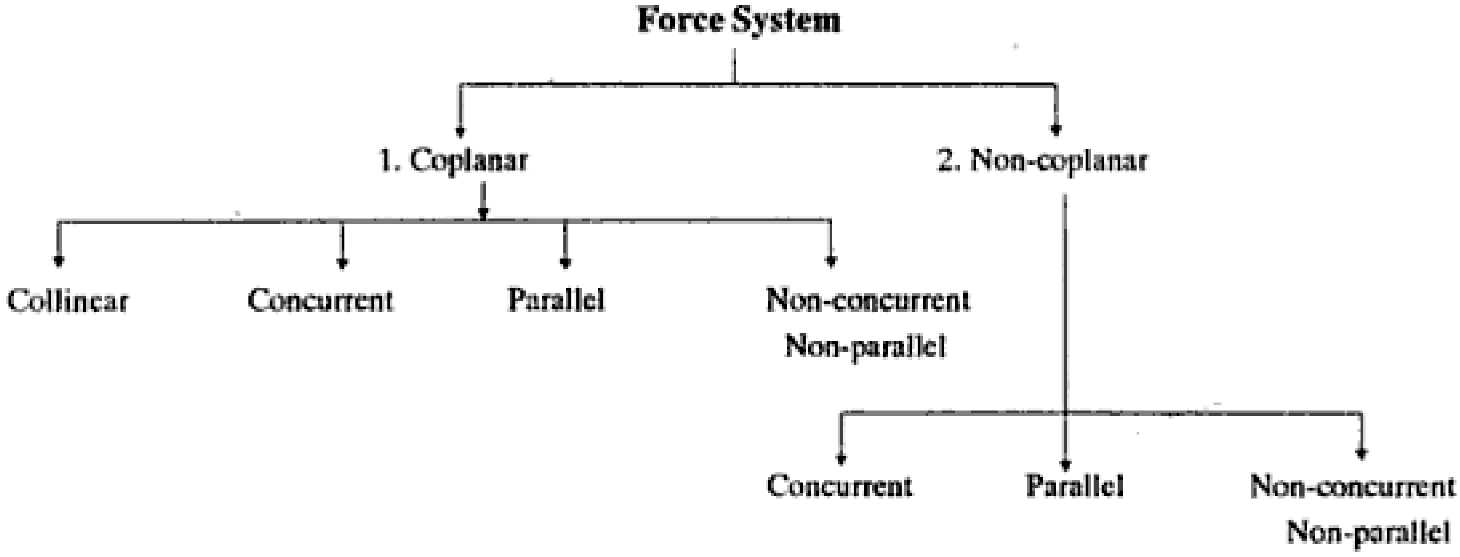


Fig. 2.1

In this chapter, we shall discuss only coplanar force system, in which the forces may be :

(*i*) Collinear

(*ii*) Concurrent

(*iii*) Parallel

(*iv*) Non-concurrent, non-parallel (or General system of forces).

2.2.1. Coplanar Collinear. Fig. 2.2 shows three forces F_1, F_2 and F_3 acting in a plane. These three forces are in the same line *i.e.*, these three forces are having a common line of action. This system of forces is known as coplanar collinear force

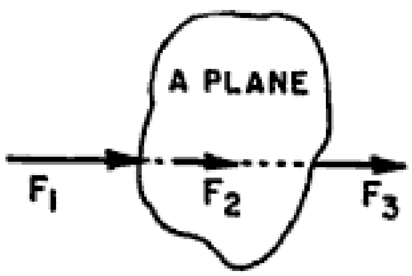


Fig. 2.2. Coplanar Collinear Forces.

system. Hence in coplanar collinear system of forces, all the forces act in the same plane and have a common line of action.

2.2.2. Coplanar Concurrent. Fig. 2.3 shows three forces F_1, F_2 and F_3 acting in a plane and these forces intersect or meet at a common point O. This system of forces is known as coplanar concurrent force system. Hence in coplanar concurrent system of forces, all the forces act in the same plane and they intersect at a common point.

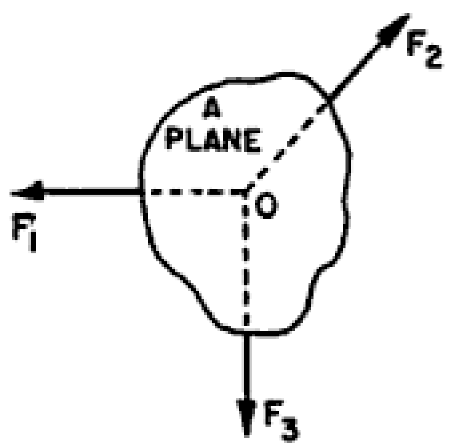


Fig. 2.3. Concurrent Coplanar Forces.

2.2.3. Coplanar Parallel. Fig. 2.4 shows three forces F_1, F_2 and F_3 acting in a plane and these forces are parallel. This system of forces is known as coplanar parallel force system. Hence in coplanar parallel system of forces, all the forces act in the same plane and are parallel.

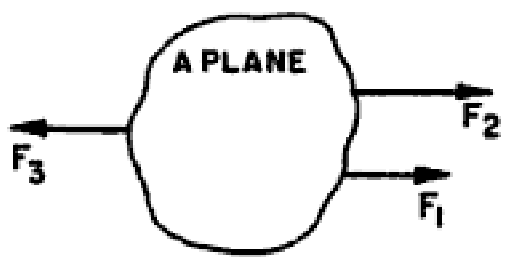


Fig. 2.4. Coplanar Parallel Forces.

2.2.4. Coplanar Non-concurrent Non-parallel. Fig. 2.5 shows four forces F_1, F_2, F_3 and F_4 acting in a plane. The lines of action of these forces lie in the same plane but they are neither parallel nor meet or intersect at a common point. This system of forces is known as coplanar non-concurrent non-parallel force system. Hence in coplanar non-concurrent non-parallel system of forces, all the forces act in the same plane but the forces are neither parallel nor meet at a common point. This force system is also known as *general system of forces*.

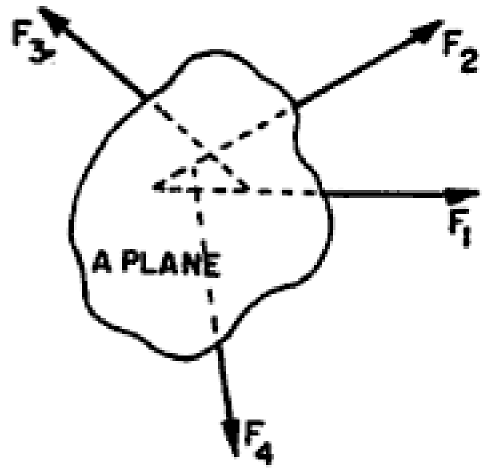


Fig. 2.5. Non-concurrent Non-parallel.

2.3. RESULTANT OF SEVERAL FORCES

When a number of coplanar forces are acting on a rigid* body, then these forces can be replaced by a single force which has the same effect on the rigid body as that of all the forces acting together, then this single force is known as the *resultant* of several forces. Hence a single force which can replace a number of forces acting on a rigid body, without causing any change in the external effects on the body, is known as the *resultant force*.

2.4. RESULTANT OF COPLANAR FORCES

The resultant of coplanar forces may be determined by following two methods :

1. Graphical method
2. Analytical method.

*Rigid body is a body which does not deform under the action of loads or external forces. In case of rigid body, the distance between any two points of the body remains constant, when this body is subjected to loads. Though all the bodies do deform to same extent under the action of loads, but in many situation, this deformation is negligible small.

The resultant of the following coplanar forces will be determined by the above two methods :

(*i*) Resultant of collinear coplanar forces

(*ii*) Resultant of concurrent coplanar forces.

2.5. RESULTANT OF COLLINEAR COPLANAR FORCES

As defined in Art. 2.2.1, collinear coplanar forces are those forces which act in the same plane and have a common line of action. The resultant of these forces are obtained by analytical method or graphical method.

2.5.1. Analytical method. The resultant is obtained by adding all the forces if they are acting in the same direction. If any one of the forces is acting in the opposite direction, then resultant is obtained by subtracting that force.

Fig. 2.6 shows three collinear coplanar forces F_1, F_2 and F_3 acting on a rigid body in the same direction. Their resultant R will be sum of these forces.

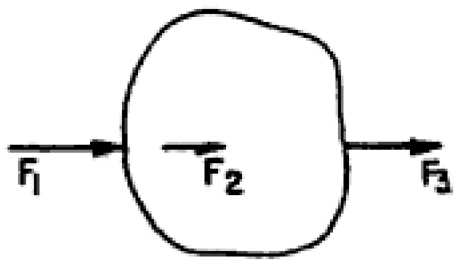


Fig. 2.6

$$\therefore \quad R = F_1 + F_2 + F_3 \quad ...(2.1)$$

If any one of these forces (say force F_2) is acting in the opposite direction, as shown in Fig. 2.7, then their resultant will be given by

$$R = F_1 - F_2 + F_3 \quad ...(2.2)$$

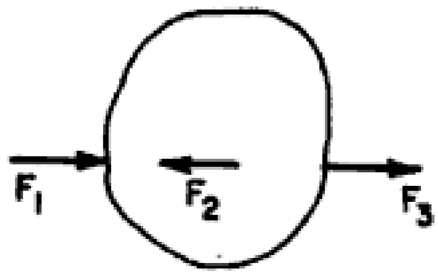


Fig. 2.7

2.5.2. Graphical Method. Some suitable scale is chosen and vectors are drawn to the chosen scale. These vectors are added/or subtracted to find the resultant. The resultant of the three collinear forces F_1, F_2 and F_3 acting in the same direction will be obtained by adding all the vectors. In Fig. 2.8, the force $F_1 = ab$ to some scale, force $F_2 = bc$ and force $F_3 = cd$. Then the length ad represents the magnitude of the resultant on the scale chosen.

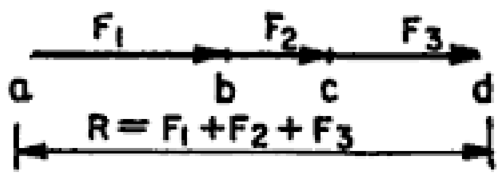


Fig. 2.8

The resultant of the forces F_1, F_2 and F_3 acting on a body shown in Fig. 2.7 will be obtained by subtracting the vector F_2. This resultant is shown in Fig. 2.9, in which the force $F_1 = ab$ to some suitable scale. This force is acting from a to b. The force F_2 is taken equal to bc on the same scale in opposite direction. This force is acting from b to c. The force F_3 is taken equal to cd. This force is acting from c to d. The resultant force is represented in magnitude by ad on the chosen scale.

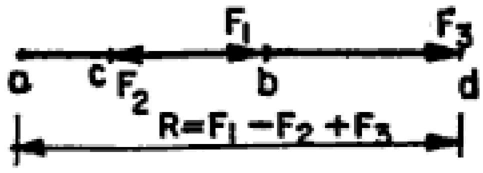


Fig. 2.9

Problem 2.1. *Three collinear horizontal forces of magnitude 200 N, 100 N and 300 N are acting on a rigid body. Determine the resultant of the forces analytically and graphically when*

(*i*) *all the forces are acting in the same direction,*

(*ii*) *the force 100 N acts in the opposite direction.*

Sol. Given : $F_1 = 200$ N, $F_2 = 100$ N and $F_3 = 300$ N

(*a*) *Analytical Method*

(*i*) When all the forces are acting in the same direction, then resultant is given by equation (2.1) as

$$R = F_1 + F_2 + F_3 = 200 + 100 + 300 = \mathbf{600\ N.} \quad \textbf{Ans.}$$

(*ii*) When the force 100 N acts in the opposite direction, then resultant is given by equation (2.2) as

$$R = F_1 + F_2 + F_3 = 200 - 100 + 300 = \mathbf{400\ N.}\quad \textbf{Ans.}$$

(*b*) *Graphical Method*

Select a suitable scale. Suppose 100 N = 1 cm. Then to this scale, we have

$$F_1 = \frac{200}{100} = 2 \text{ cm},$$

$$F_2 = \frac{100}{100} = 1 \text{ cm},$$

and $$F_3 = \frac{300}{100} = 3 \text{ cm}.$$

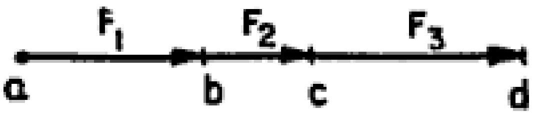


Fig. 2.10

(*i*) *When all the forces act in the same direction.*

Draw vectors ab = 2 cm to represent F_1,

vector bc = 1 cm to represent F_2 and vector

cd = 3 cm to represent F_3 as shown in Fig. 2.10.

Measure vector ad which represents the resultant.

By measurement length ad = 6 cm

∴ Resultant = Length ad × chosen scale (∵ Chosen scale is 1 cm = 100 N)

= 6 × 100 = **600 N. Ans.**

(*ii*) When force 100 N = F_2, acts in the opposite direction

Draw length ab = 2 cm to represent force F_1.

From b, draw bc = 1 cm in the opposite direction to represent F_2. From c draw cd = 3 cm to represent F_3 as shown in Fig. 2.10(*a*).

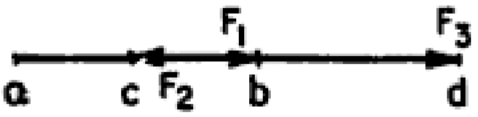


Fig. 2.10 (*a*)

Measure length ad. This gives the resultant.

By measurement, length ad = 4 cm

∴ Resultant = Length ad × chosen scale

= 4 × 100 = **400 N. Ans.**

2.6. RESULTANT OF CONCURRENT COPLANAR FORCES

As defined in Art. 2.2.2, concurrent coplanar forces are those forces which act in the same plane and they intersect or meet at a common point. We will consider the following two cases :

(*i*) When two forces act at a point

(*ii*) When more than two forces act at a point.

2.6.1. When two forces act at a point

(*a*) *Analytical Method*

In Art. 1.2.4, we have mentioned that when two forces act at a point, their resultant is found by the law of parallelogram of forces. The magnitude of resultant is obtained from equation (1.1) and the direction of resultant with one of the forces is obtained from equation (1.2).

Suppose two forces P and Q act at point O as shown in Fig. 2.11 and α is the angle between them. Let θ is the angle made by the resultant R with the direction of force P.

Forces P and Q form two sides of a parallelogram and according to the law, the diagonal through the point O gives the resultant R as shown.

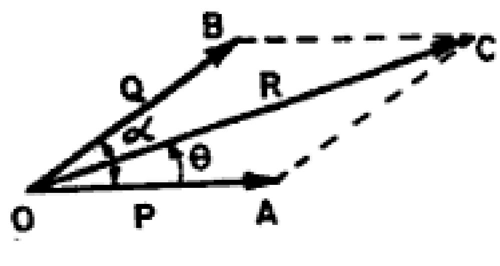


Fig. 2.11

The magnitude* of resultant is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

The above method of determining the resultant is also known as the *cosine law method.*

The direction* of the resultant with the force P is given by

$$\theta = \tan^{-1}\left(\frac{Q \sin \alpha}{P + Q \cos \alpha}\right)$$

(*b*) *Graphical Method*

(*i*) Choose a convenient scale to represent the forces P and Q.

(*ii*) From point O, draw a vector $Oa = P$.

(*iii*) Now from point O, draw another vector $Ob = Q$ and at an angle of α as shown in Fig. 2.12.

(*iv*) Complete the parallelogram by drawing lines $ac \parallel$ to Ob and $bc \parallel$ to Oa.

(*v*) Measure the length OC.

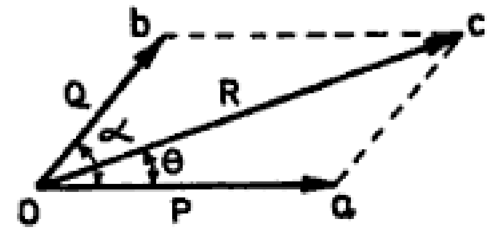


Fig. 2.12

Then resultant R will be equal to length $OC \times$ chosen scale.

(*vi*) Also measure the angle θ, which will give the direction of resultant.

The resultant can also be determine graphically by drawing a triangle *oac* as explained below and shown in Fig. 2.13.

(*i*) Draw a line *oa* parallel to P and equal to P.

(*ii*) From a, draw a vector ac at an angle α with the horizontal and cut ac equal to Q.

(*iii*) Join *oc*. Then *oc* represents the magnitude and direction of resultant R.

Magnitude of resultant R = Length $OC \times$ chosen scale. The direction of resultant is given by angle θ. Hence measure the angle θ.

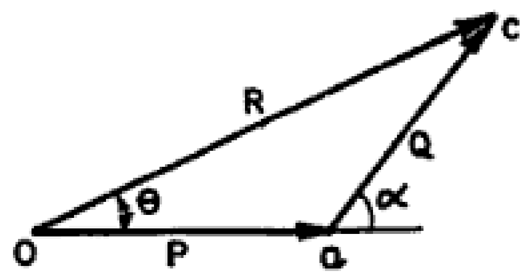


Fig. 2.13

2.6.2. When more than two forces act at a point

(*a*) *Analytical Method*

The resultant of three or more forces acting at a point is found analytically by a method which is known as rectangular components methods (Refer to Art. 1.7). According to this method all the forces acting at a point are resolved into horizontal and vertical components and then *algebraic summation*** of horizontal and vertical components is done separately. The summation of horizontal component is written as ΣH and that of vertical as ΣV. Then resultant R is given by

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}\ .$$

The angle made by the resultant with horizontal is given by

$$\tan \theta = \frac{(\Sigma V)}{(\Sigma H)}$$

∴ Let four forces F_1, F_2, F_3 and F_4 act at a point O as shown in Fig. 2.14.

*Refer Art. 1.2.4, for the derivation of magnitude and direction of resultant on page 1.

**Summation means addition. Algebraic summation of horizontal components means that if all the horizontal components are in the same direction then they are added. But if one horizontal component is in opposite direction then it is subtracted.

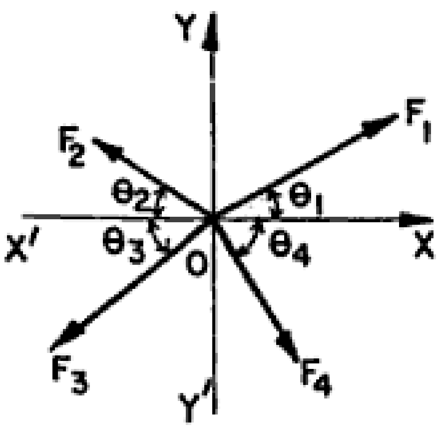


Fig. 2.14

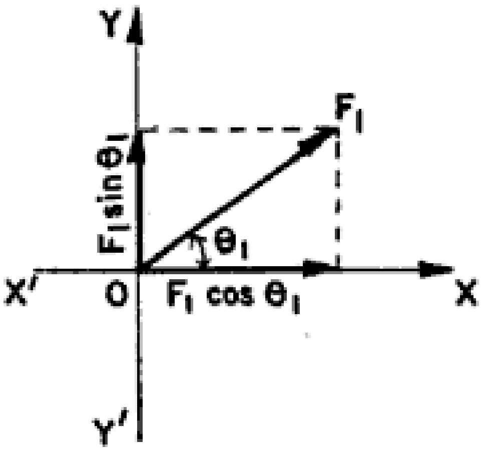


Fig. 2.14 (a)

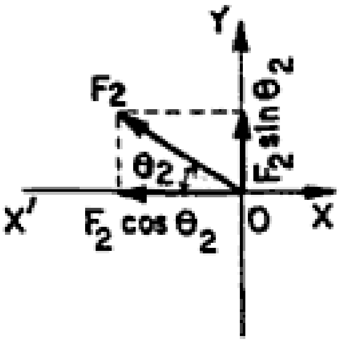


Fig. 2.14 (b)

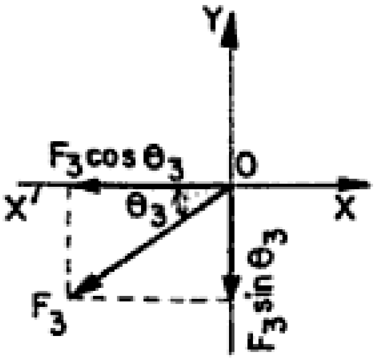


Fig. 2.14 (c)

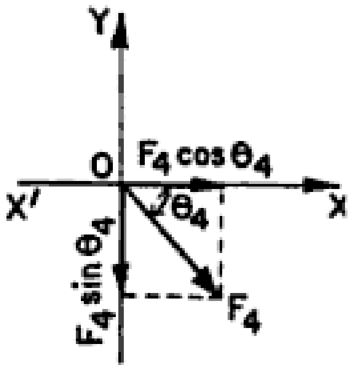


Fig. 2.14 (d)

The inclination of the forces is indicated with respect to horizontal direction. Let

θ_1 = Inclination of force F_1 with OX

θ_2 = Inclination of force F_2 with OX'

θ_3 = Inclination of force F_3 with OX'

θ_4 = Inclination of force F_4 with OX.

The force F_1 is resolved into horizontal and vertical components and these components are shown in Fig. 2.14 (*a*). Similarly, Fig. 2.14 (*b*), (*c*) and (*d*) shows the horizontal and vertical components of forces F_2, F_3 and F_4 respectively. The various horizontal components are :

$F_1 \cos \theta_1 \rightarrow (+)$

$F_2 \cos \theta_2 \leftarrow (-)$

$F_3 \cos \theta_3 \leftarrow (-)$

$F_4 \cos \theta_4 \rightarrow (+)$

$\therefore$ Summation or algebraic sum of horizontal components :

$$\Sigma H = F_1 \cos \theta_1 - F_2 \cos \theta_2 - F_3 \cos \theta_3 + F_4 \cos \theta_4$$

Similarly, various vertical components of all forces are :

$F_1 \sin \theta_1 \uparrow (+)$

$F_2 \sin \theta_2 \uparrow (+)$

$F_3 \sin \theta_3 \downarrow (-)$

$F_4 \sin \theta_4 \downarrow (-)$

$\therefore$ Summation or algebraic sum of vertical components :

$$\Sigma V = F_1 \sin \theta_1 + F_2 \sin \theta_2 - F_3 \sin \theta_3 - F_4 \sin \theta_4$$

Then the resultant will be given by $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$ ...(2.1)

And the angle (θ) made by resultant with x-axis is given by $\tan\theta = \dfrac{(\Sigma V)}{(\Sigma H)}$ ...(2.2)

(*b*) *Graphical method*

The resultant of several forces acting at a point is found graphically with the help of the *polygon law of forces*, which may be stated as

"If a number of coplanar forces are acting at a point such that they can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in the opposite order.

Let the four forces F_1, F_2, F_3 and F_4 act at a point O as shown in Fig. 2.15. The resultant is obtained graphically by drawing polygon of forces as explained below and shown in Fig. 2.15 (*a*).

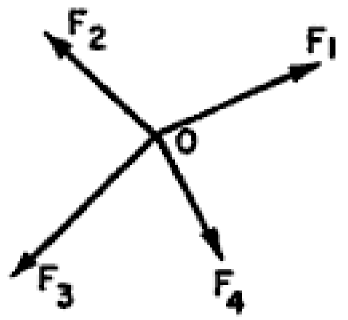


Fig. 2.15

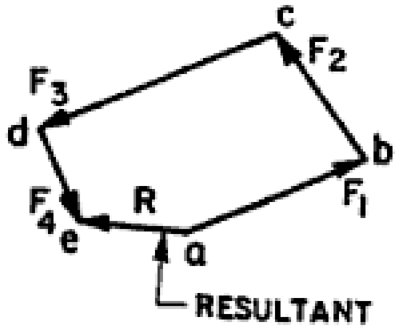


Fig. 2.15 (*a*)

(*i*) Choose a suitable scale to represent the given forces.

(*ii*) Take any point *a*. From *a*, draw vector *ab* parallel to OF_1. Cut *ab* = force F_1 to the scale.

(*iii*) From point *b*, draw *bc* parallel to OF_2. Cut *bc* = force F_2.

(*iv*) From point *C*, draw *cd* parallel to OF_3. Cut *cd* = force F_3.

(*v*) From point *d*, draw *de* parallel to OF_4. Cut *de* = force F_4.

(*vi*) Join point *a* to *e*. This is the closing side of the polygon. Hence *ae* represents the resultant in magnitude and direction.

Magnitude of resultant R = Length *ae* × scale.

The resultant is acting from *a* to *e*.

Problem 2.2. *Two forces of magnitude 240 N and 200 N are acting at a point O as shown in Fig. 2.16. If the angle between the forces is 60°, determine the magnitude of the resultant force. Also determine the angle β and γ as shown in the figure.*

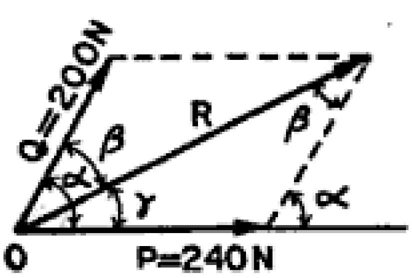


Fig. 2.16

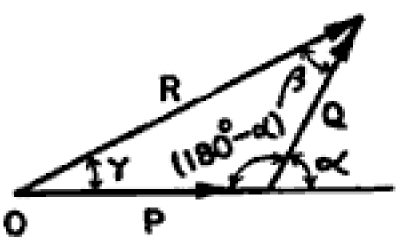


Fig. 2.16 (*a*)

Sol. Given :

Force $P = 240$ N, $Q = 200$ N

Angle between the forces, $\alpha = 60°$

The magnitude of resultant R is given by,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{240^2 + 200^2 + 2 \times 240 \times 200 \times \cos 60°}$$
$$= \sqrt{57600 + 40000 + 48000} = \mathbf{381.57\ N.\ Ans.}$$

Now refer to Fig. 2.16 (*a*). Using sine formula, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)} \quad ...(i)$$

or

$$\frac{P}{\sin\beta} = \frac{R}{\sin(180° - \alpha)}$$

$\therefore$
$$\sin\beta = \frac{P\sin(180° - \alpha)}{R} = \frac{240\sin(180 - 60)}{381.57} \quad (\because\ P = 240\text{ N}, \alpha = 60°, R = 381.57\text{ N})$$
$$= \frac{240 \times \sin 120°}{381.57} = 0.5447$$

$\therefore$
$$\beta = \sin^{-1} 0.5447 = \mathbf{33°.}\quad \textbf{Ans.}$$

From equation (*i*), also we have $\dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin(180 - \alpha)}$

$\therefore$
$$\sin\gamma = \frac{Q\sin(180 - \alpha)}{R}$$
$$= \frac{200\sin(180 - 60)}{381.57} = \frac{200\sin 120°}{381.57} = 0.4539$$

$\therefore$
$$\gamma = \sin^{-1} 0.4539 = \mathbf{26.966°.}\quad \textbf{Ans.}$$

Problem 2.3. *Two forces P and Q are acting at a point Q as shown in Fig. 2.17. The resultant force is 400 N and angles β and γ are 35° and 25° respectively. Find the two forces P and Q.*

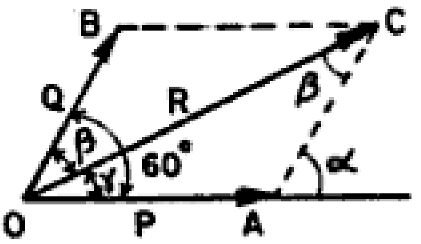


Fig. 2.17

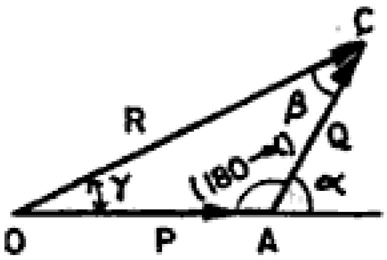


Fig. 2.17 (*a*)

Sol. Given :

Resultant, $R = 400$ N

Angles, $\beta = 35°, \gamma = 25°$

$\therefore$ Angle between the two forces, $\alpha = \beta + \gamma = 35° + 25° = 60°$

Refer to Fig. 2.17 (*a*). Using sine formula for ΔOAC, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180 - \alpha)} \quad ...(i)$$

$$\frac{P}{\sin\beta} = \frac{R}{\sin(180 - \alpha)}$$

$\therefore$
$$P = \frac{R\sin\beta}{\sin(180 - \alpha)} = \frac{400 \times \sin 35°}{\sin(180 - 60)} \quad (\because\ R = 400, \beta = 35, \alpha = 60°)$$
$$= \frac{400 \times 0.5736}{0.866} = \mathbf{264.93\ N.}\quad \textbf{Ans.}$$

Also from equation (*i*), we have $\frac{Q}{\sin\gamma} = \frac{R}{\sin(180-\alpha)}$

$$\therefore \qquad Q = \frac{R\sin\gamma}{\sin(180-\alpha)} = \frac{400 \times \sin 25°}{\sin(180° - 60°)} = \frac{400 \times 0.4226}{0.866} = \mathbf{195.19\ N.} \quad \textbf{Ans.}$$

Problem 2.4. *Two forces P and Q are acting at a point O as shown in Fig. 2.18. The force P = 240 N and force Q = 200 N. If the resultant of the forces is equal to 400 N, then find the values of angles β, γ and α.*

Sol. Given :

Forces, $P = 240$ N, $Q = 200$ N

Resultant, $R = 400$ N

Let β = Angle between R and Q,

γ = Angle between R and P.

From Fig. 2.18, it is clear that, $\alpha = \beta + \gamma$.

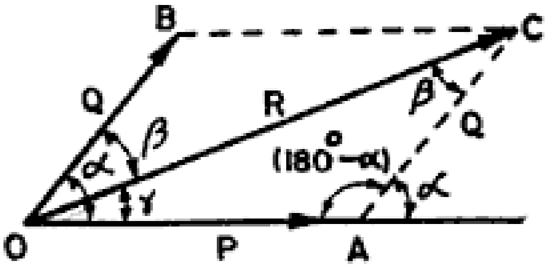


Fig. 2.18

Let us first calculate the angle α (*i.e.*, angle between the two forces).

Using the relation,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \text{or } R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

or $$400^2 = 240^2 + 200^2 + 2 \times 240 \times 200 \times \cos\alpha$$

or $$16000 = 57600 + 40000 + 96000 \times \cos\alpha$$

$$\therefore \qquad \cos\alpha = \frac{16000 - 57600 - 40000}{96000} = 0.65$$

$$\therefore \qquad \alpha = \cos^{-1} 0.65 = 49.458° = 49° (0.458 \times 60') = 49° 27.5'$$

Now using sine formula for ΔOAC of Fig. 2.18, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180-\alpha)} \qquad ...(i)$$

or $$\frac{P}{\sin\beta} = \frac{R}{\sin(180-\alpha)}$$

$$\therefore \qquad \sin\beta = \frac{P\sin(180-\alpha)}{R} = \frac{240\sin(180-49.458)}{400} \qquad (\because \; P = 240, \alpha = 49.458°)$$

$$= \frac{240\sin(130.542°)}{400} = 0.4559$$

$$\therefore \qquad \beta = \sin^{-1} 0.4559 = \mathbf{27.12°.}\ \textbf{Ans.}$$

Also from equation (*i*), we have $\frac{Q}{\sin\gamma} = \frac{R}{\sin(180-\alpha)}$

$$\therefore \qquad \sin\gamma = \frac{Q\sin(180-\alpha)}{R} = \frac{200 \times \sin(180-49.458)}{400}$$

$$= \frac{200 \times \sin(130.542°)}{400} = 0.3799$$

$$\therefore \qquad \gamma = \sin^{-1} 0.3799 = \mathbf{22.33°.}\ \textbf{Ans.}$$

Problem 2.5. *A force of 100 N is acting at a point making an angle of 30° with the horizontal. Determine the components of this force along X and Y directions.*

Sol. Given :

Force, $F = 100$ N

Angle made by F with horizontal, $\theta = 30°$

Let F_x = Component along x-axis

F_y = Component along y-axis

Then $F_x = F \cos\theta = 100 \cos 30°$

$= 100 \times 0.866$

$=$ **86.6 N. Ans.**

and $F_y = F \sin\theta = 100 \sin 30°$

$= 100 \times 0.5 =$ **50 N. Ans.**

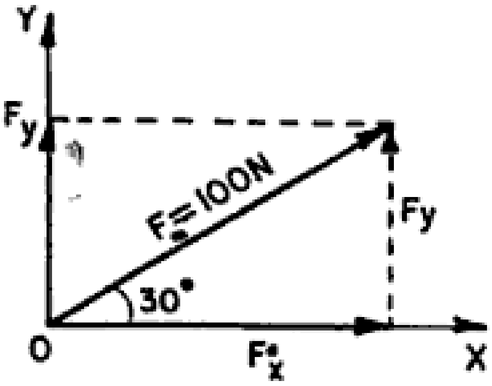


Fig. 2.19

Problem 2.6. *A small block of weight 100 N is placed on an inclined plane which makes an angle $\theta = 30°$ with the horizontal. What is the component of this weight ; (ii) parallel to the inclined plane and (iii) perpendicular to the inclined plane ?*

Sol. Given :

Weight of block, $W = 100$ N

Inclination of plane, $\theta = 30°$

The weight of block $W = 100$ N is acting vertically downwards through the C.G. of the block. Resolve this weight into two components *i.e.*, one perpendicular to the inclined plane and other parallel to the inclined plane as shown in Fig. 2.20. The perpendicular (normal) component makes an angle of 30° with the direction of W.

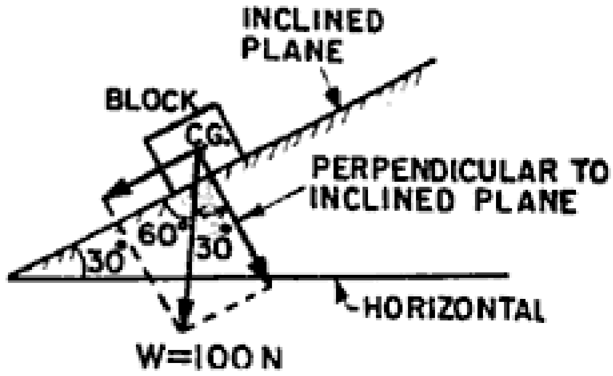


Fig. 2.20

Hence component of the weight perpendicular to the inclined plane

$= W \cos 30° = 100 \times 0.866 =$ **8.66 N. Ans.**

Component of the weight (W) parallel to the inclined plane

$= W \sin 30° = 100 \times 0.5 =$ **50 N. Ans.**

Problem 2.7. *Fig. 2.21 shows a particular position of the connecting rod BA and crank AO. At this position, the connecting rod of the engine exerts a force 2500 N on the crank pin at A. Resolve this force into horizontal and vertical components at A. Also resolve the given force at A along AO and along a direction perpendicular to AO.*

Sol. Given :

Length $BA = 50$ cm

Length $AO = 25$ cm.

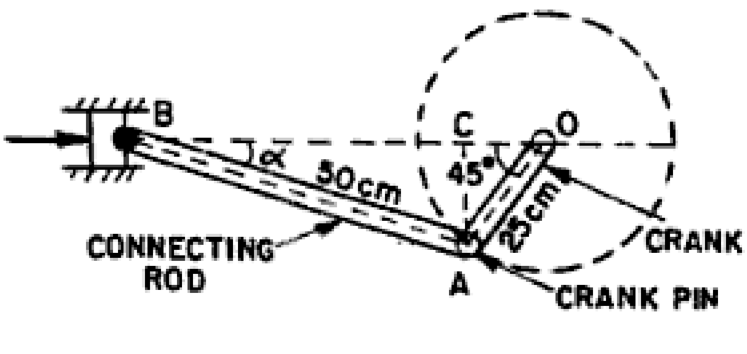


Fig. 2.21

Force exerted by connecting rod BA at A = 2500 N. This force is acting along BA at point A.

Let α = Angle made by BA with horizontal.

This angle can be calculated by drawing a perpendicular AC from point A on the horizontal axis. Now the side AC is common in triangles ABC and AOC.

In triangle ABC, $\sin\alpha = \dfrac{AC}{AB}$ or $AC = AB\sin\alpha = 50\sin\alpha$ $(\because AB = 50)$

In triangle AOC, $\sin 45° = \dfrac{AC}{AO}$ or $AC = AO\sin 45° = 25\sin 45°$ $(\because AO = 25)$

Equating the two values of AC, we get $50\sin\alpha = 25\sin 45°$

$$\therefore \quad \sin\alpha = \frac{25\sin 45°}{50} = 0.3535 \quad \text{or} \quad \alpha = \sin^{-1} 0.3535 = 20.7°$$

Now the force 2500 N is acting along BA at point A as shown in Fig. 2.21 (a).

$\therefore$ Horizontal component of this force at

$A = 2500\cos\alpha$
$= 2500\cos 20.7°$
$=$ **2338.61 N. Ans.**

Vertical component $= 2500\sin\alpha$
$= 2500 \times \sin 20.7$
$=$ **883.75 N. Ans.**

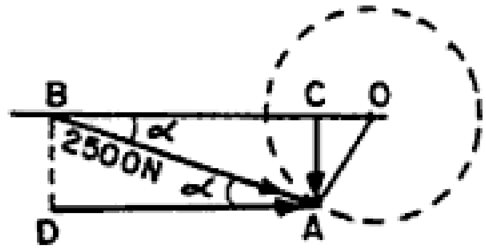


Fig. 2.21 (a)

IInd Part

The force of 2500 N is acting along BA at point A as shown in Fig. 2.21 (b). This force at point A is represented by AD. Hence BAD is a straight line. Resolve the force AD along AO and perpendicular to AO *i.e.*, in the direction AE where AE is perpendicular to AO at point A.

Now angle OAD $= 45 + 20.7 = 65.7°$

Component of force AD along

$AO = AD\cos 65.7°$
$= 2500\cos 65.7°$
$=$ **1028.78 N. Ans.**

Component of force AD along $AE = AD\sin 65.7°$
$= 2500 \times \sin 65.7$
$=$ **2278.5 N. Ans.**

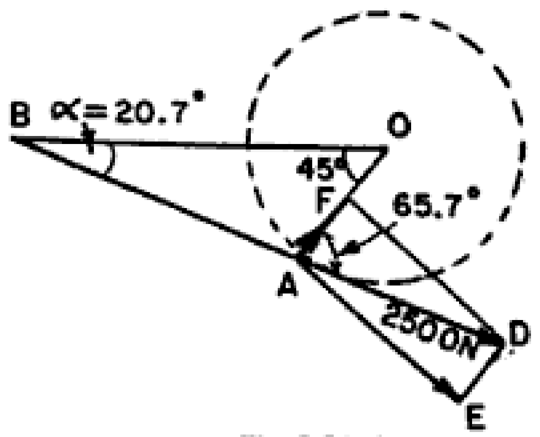


Fig. 2.21 (b)

Problem 2.8. *The four coplanar forces are acting at a point as shown in Fig. 2.22. Determine the resultant in magnitude and direction analytically and graphically.*

Sol. Given :

Forces, $F_1 = 104$ N,
$F_2 = 156$ N,
$F_3 = 252$ N, and
$F_4 = 228$ N.

(a) *Analytical Method.* Resolve each force along horizontal and vertical axes. The horizontal components along OX will be considered as +ve whereas along OX' as –ve. Similarly, vertical components in upward direction will be +ve whereas in downward direction as –ve.

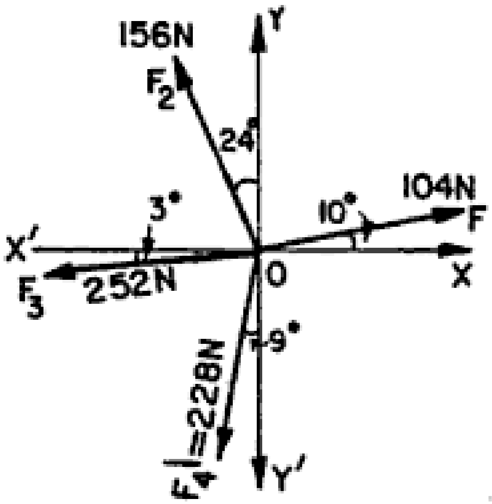


Fig. 2.22

(*i*) *Consider force F_1 = 104 N.* Horizontal and vertical components are shown in Fig. 2.22 (*a*).

Horizontal component,

$$Fx_1 = F_1 \cos 10° = 104 \times 0.9848$$
$$= 102.42 \text{ N}$$

Vertical component,

$$Fy_1 = F_1 \sin 10° = 104 \times 0.1736$$
$$= 18.06 \text{ N.}$$

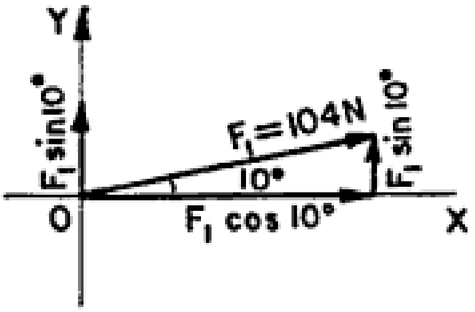


Fig. 2.22 (*a*)

(*ii*) *Consider force F_2 = 156 N.* Horizontal and vertical components are shown in Fig. 2.22 (*b*).

Angle made by F_2 with horizontal axis

$$OX' = 90 - 24 = 66°$$

∴ Horizontal components,

$$Fx_2 = F_2 \cos 66° = 156 \times 0.4067$$
$$= 63.44 \text{ N.}$$

It is negative as it is acting along OX'.

Vertical component,

$$Fy_2 = F_2 \sin 66° = 156 \times 0.9135$$
$$= 142.50 \text{ N.} \quad (+\text{ve})$$

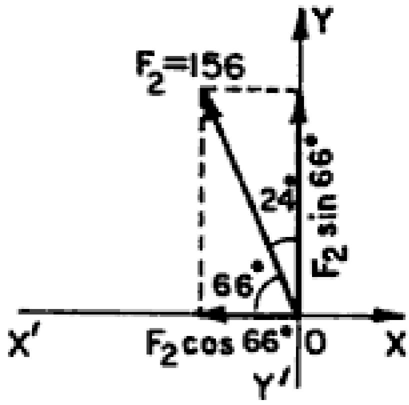


Fig. 2.22 (*b*)

(*iii*) *Consider force F_3 = 252 N.* Horizontal and vertical components are shown in Fig. 2.22 (*c*).

Horizontal component,

$$Fx_3 = F_3 \cos 3° = 252 \times 0.9986$$
$$= 251.64 \text{ N.} \quad (-\text{ve})$$

Vertical component,

$$Fy_3 = F_3 \sin 3° = 252 \times 0.0523$$
$$= 13.18 \text{ N.} \quad (-\text{ve})$$

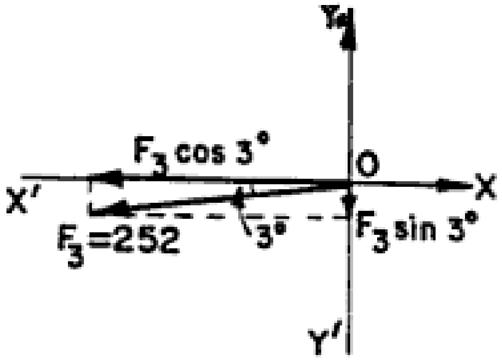


Fig. 2.22 (*c*)

(*iv*) *Consider force F_4 = 228 N.* Horizontal and vertical components are shown in Fig. 2.22 (*d*).

Angle made by F_4 with horizontal axis

$$OX' = 90 - 9 = 81°.$$

∴ Horizontal component,

$$Fx_4 = F_4 \cos 81° = 228 \times 0.1564$$
$$= 35.66 \text{ N} \quad (-\text{ve})$$

Vertical component,

$$Fy_4 = F_4 \sin 81° = 228 \times 0.9877$$
$$= 225.2 \text{ N.} \quad (-\text{ve})$$

Now algebraic sum of horizontal components is given by,

$$\Sigma H = Fx_1 - Fx_2 - Fx_3 - Fx_4$$
$$= 102.4 - 63.44 - 251.64 - 35.66$$
$$= -248.32 \text{ N.}$$

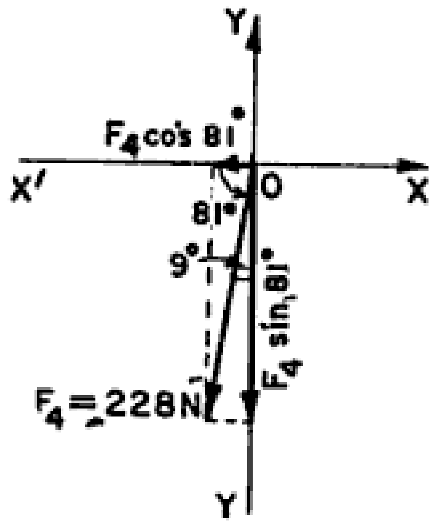


Fig. 2.22 (*d*)

– ve sign means that ΣH is acting along OX' as shown in Fig. 2.22 (*e*).

Similarly, the algebraic sum of vertical components is given by,

$$\Sigma V = 18.06 = 142.50 + 13.18 - 225.2$$
$$= -77.82 \text{ N.}$$

–ve sign means that ΣV is acting along OY' as shown in Fig. 2.22 (*e*).

The magnitude of resultant (*i.e.*, R) is obtained by using equation (2.1).

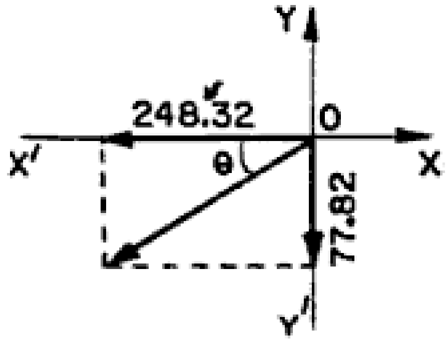


Fig. 2.22 (*e*)

$$\therefore \quad R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(248.32)^2 + (77.82)^2}$$
$$= \mathbf{260.2 \text{ N.}} \quad \textbf{Ans.}$$

The direction of resultant is given by equation (2.2).

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{77.82}{248.32} = 0.3134$$

$$\therefore \quad \theta = \tan^{-1} 0.3134 = 17.4°. \quad \textbf{Ans.}$$

(*b*) *Graphical method.* Fig. 2.23 (*a*), shows the point at which four forces 104 N, 156 N, 252 N and 228 N are acting. The resultant force is obtained graphically by drawing polygon of forces as explained below and shown in Fig. 2.23 (*b*) :

(*i*) Choose a suitable scale to represent the given forces. Let the scale is 25 N = 1 cm. Hence the force 104 N will be represented by $\frac{104}{25} = 4.16$ cm, force 156 N will be represented by $\frac{156}{25} = 6.24$ cm force 252 N will be represented by $\frac{252}{25} = 10.08$ cm and the force 228 N will be represented by $\frac{228}{25} = 9.12$ cm.

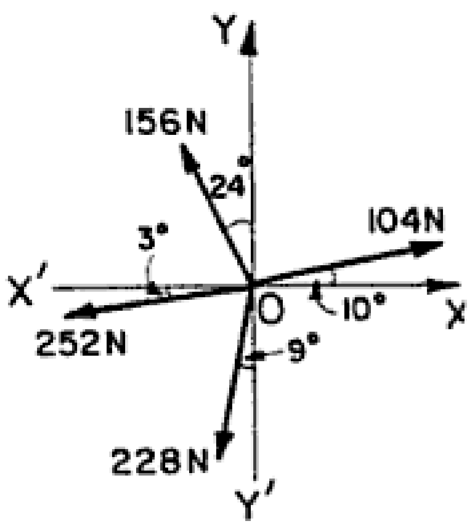


Fig. 2.23 (*a*)

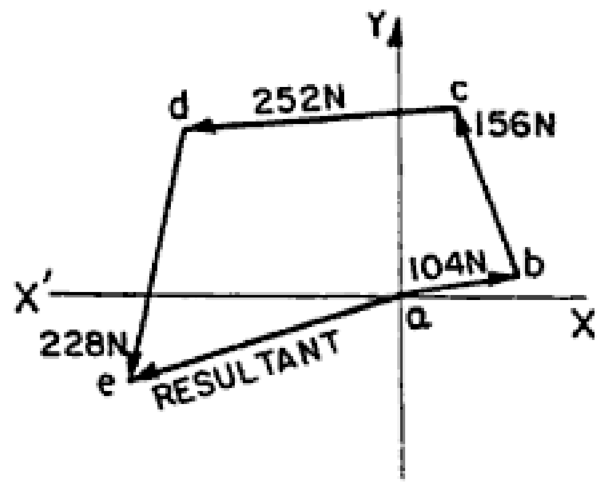


Fig. 2.23 (*b*)

(*ii*) Take any point *a*. From point *a*, draw vector *ab* parallel to line of action of force 104 N. Cut *ab* = 4.16 cm. Then *ab* represents the force 104 N in magnitude and direction.

(*iii*) From point *b*, draw vector *bc* parallel to force 156 N and cut *bc* = 6.24 cm. Then vector *cd* represents the force 156 N in magnitude and direction.

(*iv*) From point *c*, draw a vector *cd* parallel 252 N force and cut *cd* = 10.08 cm. Then vector *cd* represents the force 252 N in magnitude and direction.

(*v*) Now from point *d*, draw the vector *de* parallel to 228 N force and cut *de* = 9.12 cm. Then vector *de* represents the force 228 N in magnitude and direction.

(*vi*) Join point *a* to *e*. The line *ae* is the closing side of the polygon. Hence the side *ae* represents the resultant in magnitude and direction. Measure the length of *ae*.

By measurement, length *ae* = 10.4 cm

$\therefore$ Resultant, R = Length ae × Scale = 10.4 × 25 $\quad$ ($\because$ 1 cm = 25 N)

= **260 N. Ans.**

Now measure angle made by *ae* with horizontal. This angle is 17.4° with axis OX'. **Ans.**

Problem 2.9. *The resultant of four forces which are acting at a point O as shown in Fig. 2.24, is along Y-axis. The magnitude of forces F_1, F_3 and F_4 are 10 kN, 20 kN and 40 kN respectively. The angles made by 10 kN, 20 kN and 40 kN with X-axis are 30°, 90° and 120° respectively. Find the magnitude and direction of force F_2 if resultant is 72 kN.*

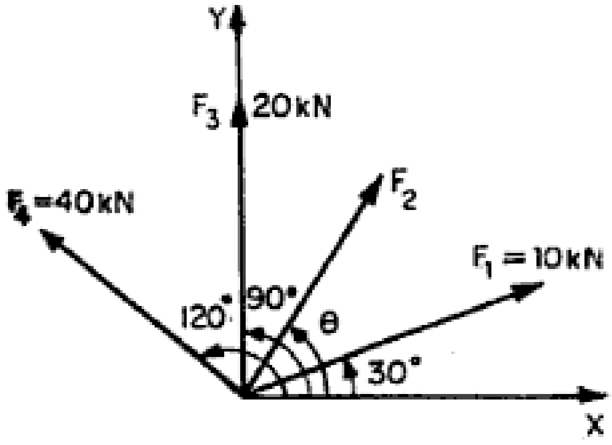


Fig. 2.24

Sol. Given :

$$F_1 = 10 \text{ kN}, \theta_1 = 30°$$
$$F_2 = ?, \qquad \theta_2 = \theta$$
$$F_3 = 20 \text{ kN}, \theta_3 = 90°$$
$$F_4 = 40 \text{ kN}, \theta_4 = 120°$$

Resultant, $R = 72$ kN

Resultant is along Y-axis.

Hence the algebraic sum of horizontal component should be zero and algebraic sum of vertical components should be equal to the resultant.

$\therefore$ $\quad \Sigma H = 0$ and $\Sigma V = R = 72$ kN

But
$$\Sigma H = F_1 \cos 30° + F_2 \cos \theta + F_3 \cos 90° + F_4 \cos 120°$$
$$= 10 \times 0.866 + F_2 \cos \theta + 20 \times 0 + 40 \times (-\tfrac{1}{2})$$
$$= 8.66 + F_2 \cos \theta + 0 - 20$$
$$= F_2 \cos \theta - 11.34$$

$\therefore$ $\quad \Sigma H = 0$ or $F_2 \cos \theta - 11.34 = 0$

or
$$F_2 \cos \theta = 11.34 \qquad \text{...}(i)$$

Now
$$\Sigma V = F_1 \sin 30° + F_2 \sin \theta + F_3 \sin 90° + F_4 \sin 120°$$
$$= 10 \times \frac{1}{2} + F_2 \sin \theta + 20 \times 1 + 40 \times 0.866$$
$$= 5 + F_2 \sin \theta + 20 + 34.64$$
$$= F_2 \sin \theta + 59.64$$

But
$$\Sigma V = R$$

$\therefore$ $\quad F_2 \sin \theta + 59.64 = 72$

$\therefore$
$$F_2 \sin \theta = 72 - 59.64 = 12.36 \qquad \text{...}(ii)$$

Dividing equation (*ii*) and (*i*),

$$\frac{F_2 \sin \theta}{F_2 \cos \theta} = \frac{12.36}{11.34} \quad \text{or} \quad \tan \theta = 1.0899$$

$\therefore$
$$\theta = \tan^{-1} 1.0899 = \mathbf{47.46°}. \quad \textbf{Ans.}$$

Substituding the value of θ in equation (*ii*), we get $F_2 \sin(47.46°) = 12.36$

or $$F_2 = \frac{12.36}{\sin(47.46°)} = \frac{12.36}{0.7368} = 16.77 \text{ kN.} \quad \textbf{Ans.}$$

Problem 2.10. *Determine the magnitude, direction and position of a single force P, which keeps in equilibrium the system of forces acting at the corners of a rectangular block as shown in Fig. 2.25. The position of the force P may be stated by reference to axes with origin O and coinciding with the edges of the block.*

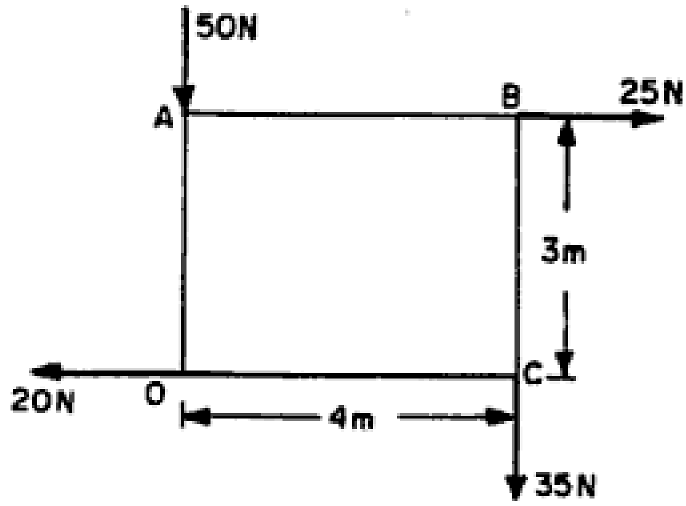


Fig. 2.25

Sol. Given :

Length $OC = 4$ m,

Length $BC = 3$ m

Force at $O = 20$ N (←)

Force at $C = 35$ N (↓)

Force at $B = 25$ N (→)

Force at $A = 50$ N (↓)

Let O be the origin and OX and OY be the reference axes as shown in Fig. 2.26.

Forces 50 N and 20 N form a concurrent system and their line of action intersect at O.

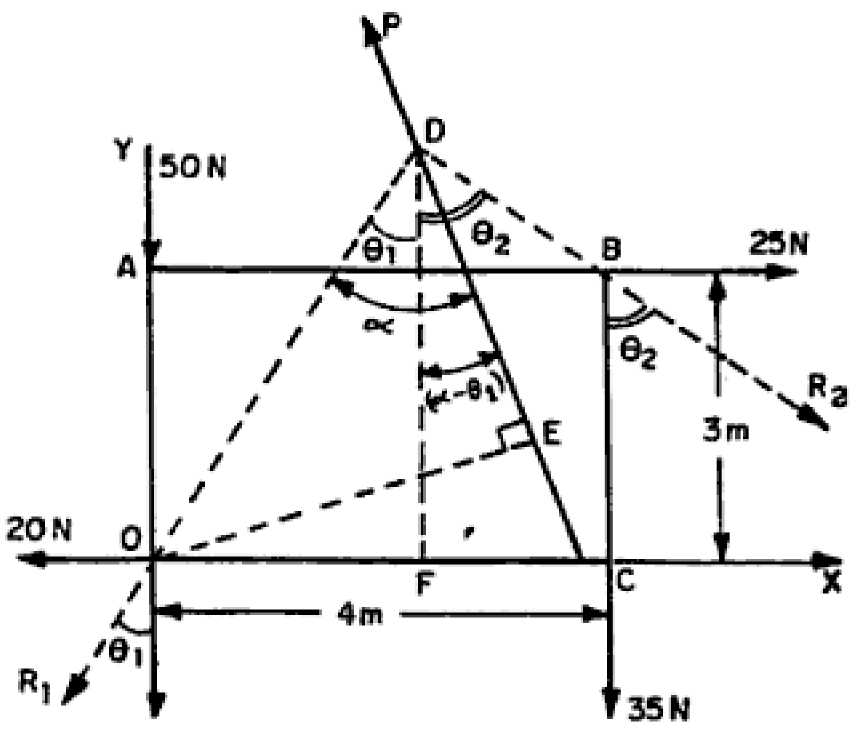


Fig. 2.26

The resultant of these forces

$$R_1 = \sqrt{50^2 + 20^2} = \sqrt{2900} = 53.85 \text{ N}$$

and $$\theta_1 = \tan^{-1}\left(\frac{20}{50}\right) = 21.8° \text{ with vertical axis.}$$

Similarly the forces 35 N and 25 N form a concurrent system and their line of action intersect at B.

The resultant of these forces

$$R_2 = \sqrt{25^2 + 35^2} = \sqrt{1850} = 43.01 \text{ N}$$

and $$\theta_2 = \tan^{-1}\left(\frac{25}{35}\right) = 35.53° \text{ with } BC \textit{ i.e.}, \text{ with vertical line.}$$

These two forces R_1 and R_2 intersect at D. The angle between these forces is $\theta_1 + \theta_2$ *i.e.*, angle $R_1DR_2 = \theta_1 + \theta_2 = 21.8° + 35.53° = 57.33°$.

Let P be the resultant of the forces R_1 and R_2.

$$\therefore \quad P = \sqrt{R_1^2 + R_2^2 + 2R_1 \times R_2 \times \cos(57.33°)}$$

$$= \sqrt{53.85^2 + 43.01^2 + 2 \times 53.85 \times 43.01 \times \cos 57.33°}$$

$$= \sqrt{2900 + 1850 + 4632.17 \times 0.5398} = \mathbf{85.15\ N.} \quad \textbf{Ans.}$$

The angle made by the resultant P with R_1 is given by

$$\tan\alpha = \frac{R_2 \sin 57.33°}{R_1 + R_2 \cos 57.33°} = \frac{43.01 \sin 57.33°}{53.85 + 43.01 \times \cos 57.33°}$$

$$= \frac{43.01 \times 0.8418}{53.85 + 23.21} = \frac{36.2058}{77.06} = 0.4698$$

$$\therefore \quad \alpha = \tan^{-1} 0.4698 = 25.16°$$

Hence the resultant P makes $(\alpha - \theta_1)$ angle with vertical in anti-clockwise direction *i.e.*, P makes (25.16 − 21.8 = 3.36°). **Ans.**

Position of the force P

The position of the force P is obtained by equating the clockwise moments and anti-clockwise moments about O (Refer Fig. 2.26).

Let OE = Perpendicular distance between O and line of action of the force P.

Taking moments of all forces about O,

$$20 \times 0 + 50 \times 0 + 35 \times 4 + 25 \times 3 = P \times OE$$

or $$0 + 0 + 140 + 75 = 85.147 \times OE \quad \text{or} \quad OE = \frac{215}{85.147} = 2.525 \text{ m}$$

From right angled triangle OED, $\sin\alpha = \frac{OE}{OD}$

$$\therefore \quad OD = \frac{OE}{\sin\alpha} = \frac{OE}{\sin 25.16°} = \frac{2.525}{0.4241} = 5.939$$

Let x and y are the co-ordinates of the force P with reference to the axes with origin O.

Then x = OF and y = DF

In right angled triangle OFD,

$$OF = OD \times \sin\theta_1 = 5.939 \times \sin 21.8° \qquad (\because \theta_1 = 21.8°)$$

$$= 2.20 \text{ m}$$

Also $$FD = OD \times \cos\theta_1 = 5.939 \times \cos 21.8° = 5.514 \text{ m}$$

$$\therefore \quad x = OF = \mathbf{2.20\ m.} \quad \textbf{Ans.}$$

and $$y = FD = \mathbf{5.514\ m.} \quad \textbf{Ans.}$$

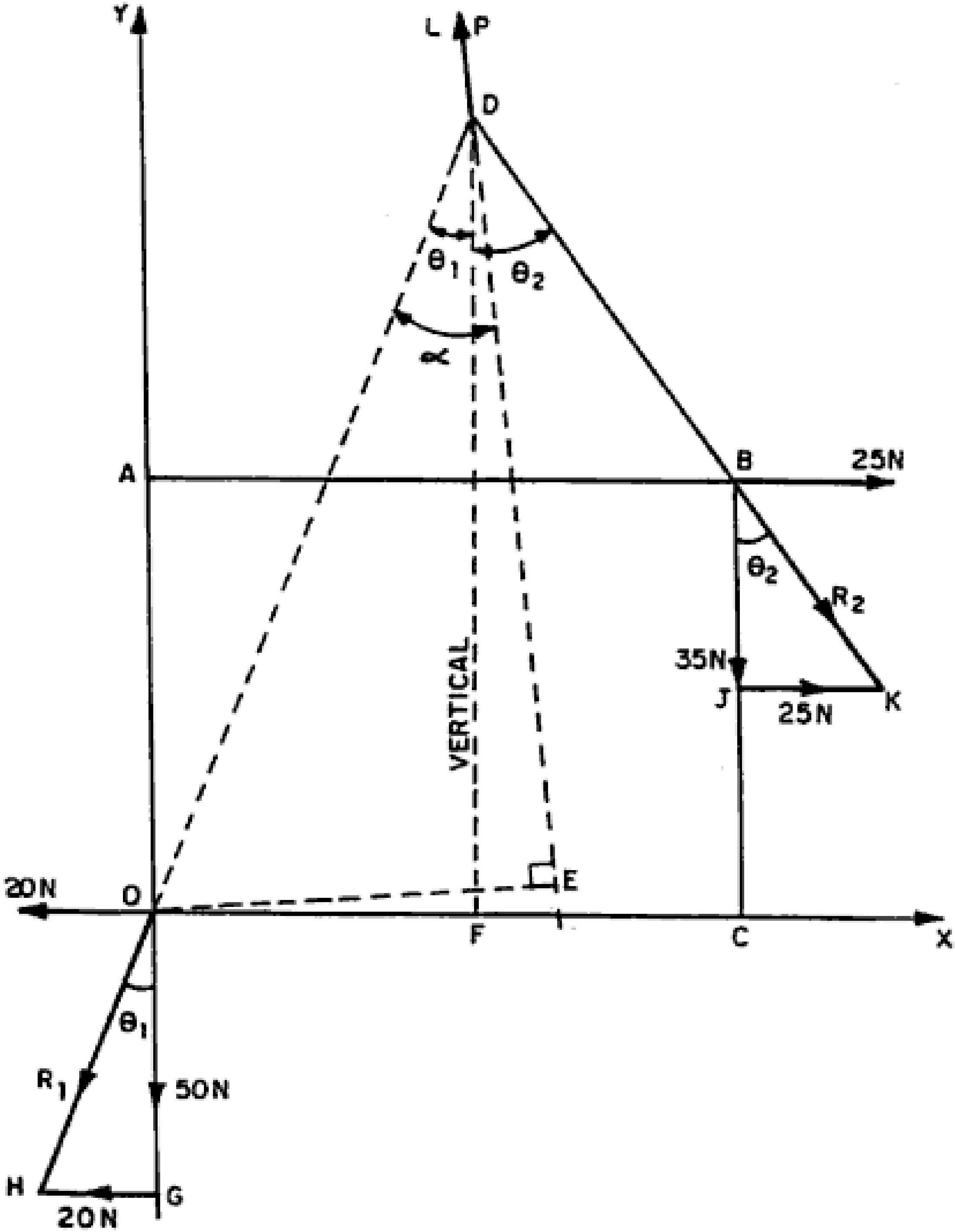


Fig. 2.27 (*a*)

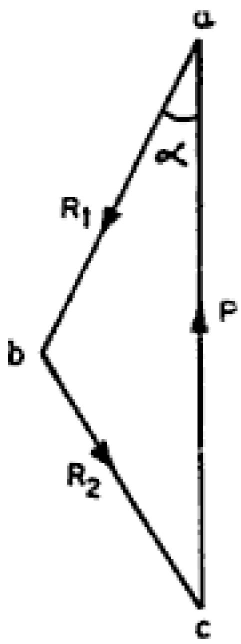


Fig. 2.27 (*b*)

Graphical Method [Refer to Fig. 2.27 (*a*)]

(*i*) To a suitable scale, take $OG = 50$ N and $GH = 20$ N. Join OH. Then OH represents the resultant R_1 in magnitude and direction. Produce the line HO backward.

(*ii*) From point B, Take $BJ = 35$ N and $JK = 25$ N. Join BK, which represents the resultant R_2 in magnitude and direction. Produce KB in the backward direction to interest the line of action of R_1 at point D.

(*iii*) To find the resultant of R_1 and R_2 (*i.e.*, force P) refer to Fig. 2.27 (*b*).

(*iv*) Take any point '*a*'. From this point draw line ab parallel to R_1 and equal to R_1. From point '*b*', draw line bc parallel to R_2 and equal to R_2. Join the point c to a.

(*v*) Then ca represents in magnitude and direction the force P. Hence measure ca. Then

$$P = ca = 85.15 \text{ N.} \quad \textbf{Ans.}$$

(*vi*) From point D, draw the line DL parallel to ca. Hence DL represents the direction of the force P.

(*vii*) To find the position of the force P which is acting at point D, draw DF parallel to axes OY. Then OF represents the x-coordinate and FD represents the y-coordinate of the force P. Measure OF and FD. Then by measurement,

$OF = x = 2.20$ m. **Ans.**

and $FD = y = 5.514$. **Ans.**

HIGHLIGHTS

1. Coplanar forces means the forces are acting in one plane.
2. Concurrent forces means the forces are intersecting at a common point.
3. Collinear forces means the forces are having same line of action.
4. The resultant of coplanar forces are determined by analytical and graphical methods.
5. The resultant (R) of three collinear forces F_1, F_2 and F_3 acting in the same direction, is given by $R = F_1 + F_2 + F_3$. If the force F_2 is acting in opposite direction then their resultant will be, $R = F_1 - F_2 + F_3$.
6. The resultant of the two forces P and Q having an angle α between them and acting at a point, is given by consine law method as $R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$. And the direction of the resultant with the force P is given by,
$$\tan \theta = \frac{\theta \sin \alpha}{P + \theta \cos \alpha}.$$
7. The resultant of three or more forces acting at a point is given by, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$, where ΣH = Algebraic sum of horizontal components of all forces, ΣV = Algebraic sum of vertical components of all forces. The angle made by the resultant with horizontal is given by, $\tan \theta = \frac{(\Sigma V)}{(\Sigma H)}$.
8. The resultant of several forces acting at a point is found graphically by using polygon law of forces.
9. Polygon law of forces states that if a number of coplanar forces are acting at a point such that they can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in the opposite order.

EXERCISE 2

(A) Theoretical Questions

1. Define and explain the following terms :
   (*i*) Coplanar and non-coplanar forces (*ii*) Collinear and concurrent forces
   (*iii*) Parallel and non-parallel forces.
2. What is the difference between collinear and concurrent forces ?
3. State and explain the following laws of forces :
   (*i*) Law of parallelogram of forces (*ii*) Law of triangle of forces
   (*iii*) Law of polygon of forces.
4. Derive an expression for the resultant in magnitude and direction of two coplanar concurrent forces using cosine law method.
5. Explain in detail the method of finding resultant in magnitude and direction of three or more forces acting at a point by analytical and graphical method.
6. Explain the procedure of resolving a given force into two components at right angles to each other.
7. Three collinear forces F_1, F_2 and F_3 are acting on a body. What will be the resultant of these forces, if
   (*a*) all are acting in the same direction (*b*) force F_3 is acting in opposite direction.
8. State the law of parallelogram of forces and show that the resultant $R = \sqrt{P^2 + Q^2}$ when the two forces P and Q are acting at right angles to each other. Find the value of R if the angle between the forces is zero.

(B) Numerical Problems

1. Three collinear horizontal forces of magnitude 300 N, 100 N and 250 N are acting on rigid body. Determine the resultant of the forces analytically and graphically when : (*i*) all the forces are acting in the same direction ; (*ii*) the force 100 N acts in the opposite direction. **[Ans.** (*i*) 650 N, (*ii*) 450 N**]**
2. Two forces of magnitude 15 N and 12 N are acting at a point. The angle between the forces is 60°. Find the resultant is magnitude. **[Ans.** 20.43 N**]**
3. A force of 1000 N is acting at a point, making an angle of 60° with the horizontal. Determine the components of this force along horizontal and vertical directions. **[Ans.** 500 N, 866 N**]**
4. A small block of weight 100 N is placed on an inclined plane which makes an angle of 60° with the horizontal. Find the components of this weight (*i*) perpendicular to the inclined plane and (*ii*) parallel to the inclined plane. **[Ans.** 50 N, 86.6 N**]**
5. Two forces P and Q are acting at a point O as shown in Fig. 2.28. The force P = 264.9 N and force Q = 195.2 N. If the resultant of the forces is equal to 400 N then find the values of angles β, γ, α. **[Ans.** β = 35°, γ = 25°, α = 60°**]**
6. A small block of unknown weight is placed on an inclined plane which makes an angle of 30° with horizontal plane. The component of this weight parallel to the inclined plane is 100 N. Find the weight of the block. **[Ans.** 200 N**]**
7. In question 6, find the component of the weight perpendicular to the inclined plane. **[Ans.** 173.2 N**]**
8. The four coplanar forces are acting at a point as shown in Fig. 2.29. Determine the resultant in magnitude and direction analytically and graphically. **[Ans.** 1000 N, θ = 60° with OX**]**

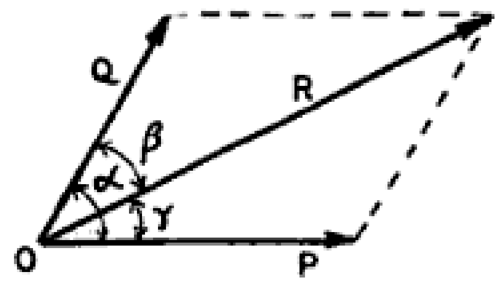


Fig. 2.28

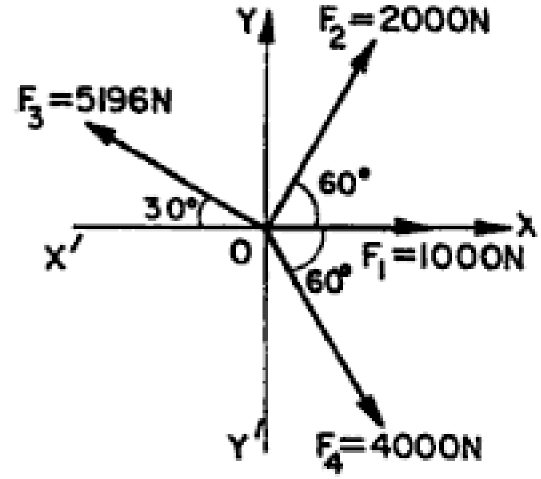


Fig. 2.29

9. The four coplanar forces are acting at a point as shown in Fig. 2.30. One of the forces is unknown and its magnitude is shown by P. The resultant is having a magnitude 500 N and is acting along x-axis. Determine the unknown force P and its inclination with x-axis. **[Ans.** P = 286.5 N and θ = 53° 15′**]**

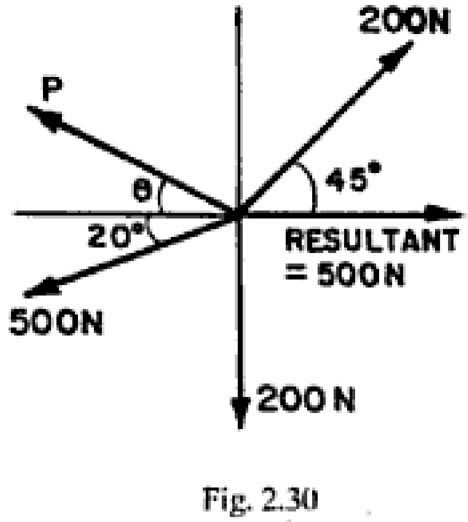


Fig. 2.30

3

Coplanar Parallel Forces

3.1. INTRODUCTION

The forces, which are having their line of actions parallel to each other, are known parallel forces. The two parallel forces will not intersect at a point. The resultant of two coplanar concurrent forces (*i.e.*, forces intersecting at the same point) can be directly determined by the method of parallelogram of forces. This method along with other methods for finding resultant of collinear and concurrent coplanar forces, were discussed in earlier chapters.

The parallel forces are having their lines of action parallel to each other. Hence, for finding the resultant of two parallel forces, (two parallel forces do not intersect at a point) the parallelogram cannot be drawn. The resultant of such forces can be determined by applying the *principle of moments.* Hence in this chapter first the concepts of moment and principle of moments will be dealt with. Thereafter the methods of finding resultant of parallel and even non-parallel forces will be explained.

3.2. MOMENT OF A FORCE

The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

Let F = A force acting on a body as shown in Fig. 3.1.

r = Perpendicular distance from the point O on the line of action of force F.

Then moment (M) of the force F about O is given by, $M = F \times r$

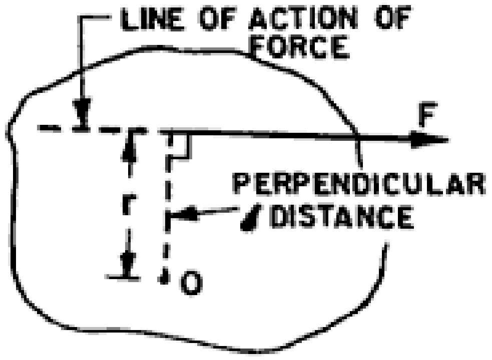


Fig. 3.1

The tendency of this moment is to rotate the body in the clockwise direction about O. Hence this moment is called *clockwise moment.* If the tendency of a moment is to rotate the body in anti-clockwise direction, then that moment is known as *anti-clockwise moment.* If clockwise moment is taken – ve then anti-clockwise moment will be + ve.

In S.I. system, moment is expressed in N m (Newton metre).

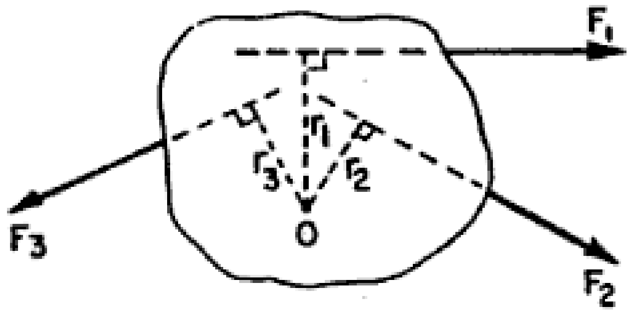


Fig. 3.2

Fig. 3.2 shows a body on which three forces F_1, F_2 and F_3 are acting. Suppose it is required to find the resultant moments if these forces about point O.

Let r_1 = Perpendicular distance from O on the line of action of force F_1.

r_2 and r_3 = Perpendicular distances from O on the lines of action of force F_2 and F_3 respectively.

Moment of F_1 about $O = F_1 \times r_1$ (clockwise) (–)

Moment of F_2 about $O = F_2 \times r_2$ (clockwise) (–)

Moment of F_3 about $O = F_3 \times r_3$ (anti-clockwise) (+)

The resultant moment will be *algebraic sum* of all the moments.

∴ The resultant moment of F_1, F_2 and F_3 about O

$$= - F_1 \times r_1 - F_2 \times r_2 + F_3 \times r_3.$$

Problem 3.1. *Four forces of magnitude 10 N, 20 N, 30 N and 40 N are acting respectively along the four sides of a square ABCD as shown in Fig. 3.4. Determine the resultant moment about the point A. Each side of the square is given 2 m.*

Sol. Given :

Length $AB = BC = CD = DA = 2$ m

Force at B = 10 N,

Force at C = 20 N,

Force at D = 30 N,

Force at A = 40 N,

The resultant moment about point A is to be determined.

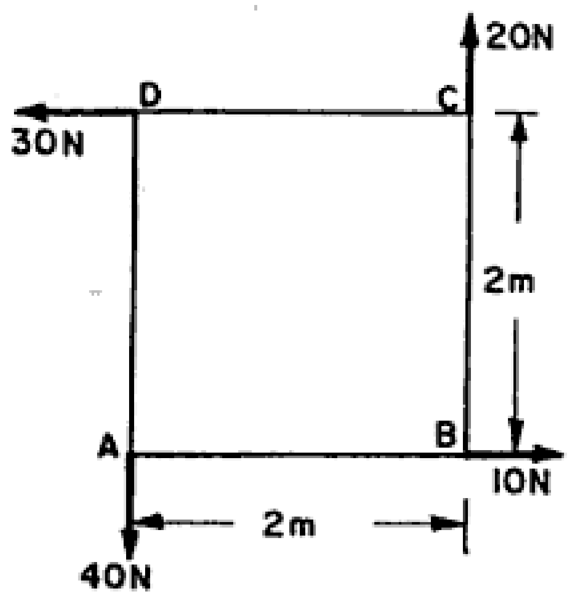


Fig. 3.3

The forces at A and B passes through point A. Hence perpendicular distance from A on the lines of action of these forces will be zero.

Hence their moments about A will be zero. The moment of the force at C about point A.

= Force at $C \times \perp$ distance from A on the line of action of force at C.

= (20 N) × (Length AB).

= 20 × 2 N m = 40 N m (anti-clockwise).

The moment of force at D about point A.

= Force at $D \times \perp$ distance from A on the line of action of force at D.

= (30 N) × (Length AD).

= 30 × 2 Nm = 60 Nm (anti-clockwise).

∴ Resultant moment of all forces about A.

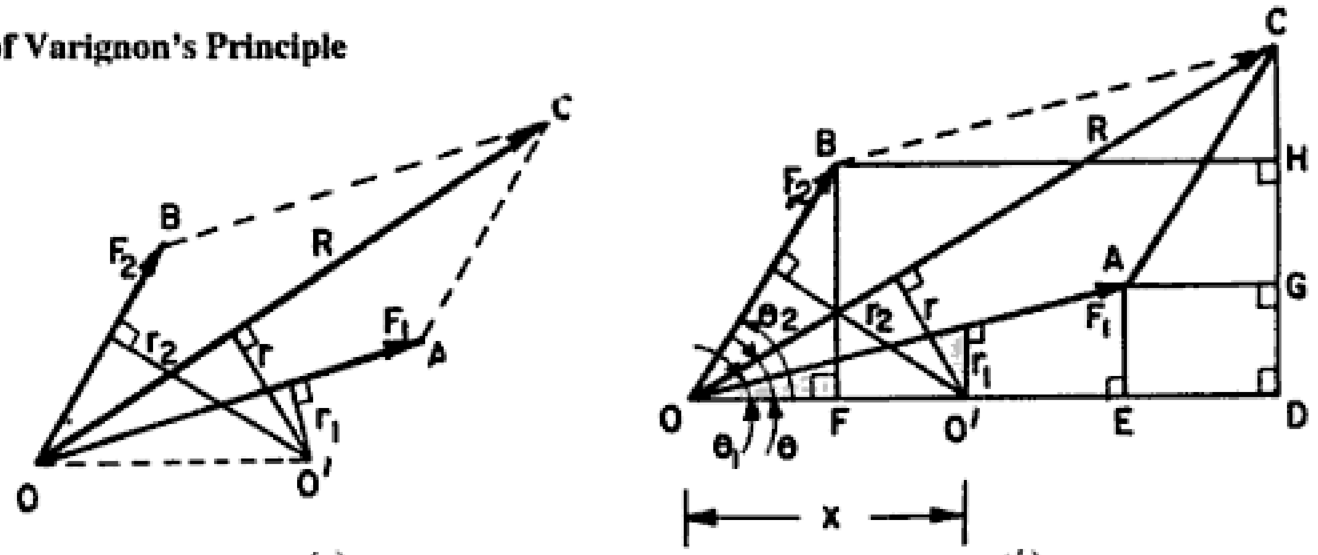
= 40 + 60 = 100 Nm (anti-clockwise). **Ans.**

3.3. PRINCIPLE OF MOMENTS (OR VARIGNON'S PRINCIPLE)

Principle of moments states that the moment of the resultant of a number of forces about any point is equal to the *algebraic sum* of the moments of all the forces of the system about the same point.

And according to Varignon's principle, the moment of a force about any point is equal to the *algebraic sum* of the moments of its components about *that point.*

Proof of Varignon's Principle



(a) (b)

Fig. 3.4

Fig. 3.4 (*a*) shows two forces F_1 and F_2 acting at point O. These forces are represented in magnitude and direction by OA and OB. Their resultant R is represented in magnitude and direction by OC which is the diagonal of parallelogram $OACB$. Let O' is the point in the plane about which moments of F_1, F_2 and R are to be determined. From point O', draw perpendiculars on OA, OC and OB.

Let r_1 = Perpendicular distance between F_1 and O'.

r = Perpendicular distance between R and O'.

r_2 = Perpendicular distance between F_2 and O'.

Then according to Varignon's principle ;

Moment of R about O' must be equal to algebraic sum of moments of F_1 and F_2 about O'.

or $R \times r = F_1 \times r_1 + F_2 \times r_2$

Now refer to Fig. 3.4 (*b*). Join OO' and produce it to D. From points C, A and B draw perpendiculars on OD meeting at D, E and F respectively. From A and B also draw perpendiculars on CD meeting the line CD at G and H respectively.

Let θ_1 = Angle made by F_1 with OD,

θ = Angle made by R with OD, and

θ_2 = Angle made by F_2 with OD.

In Fig. 3.4 (*b*), $OA = BC$ and also OA parallel to BC, hence the projection of OA and BC on the same vertical line CD will be equal *i.e.*, $GD = CH$ as GD is the projection of OA on CD and CH is the projection of BC on CD.

Then from Fig. 3.4 (*b*), we have

$P_1 \sin \theta_1 = AE = GD = CH$

$F_1 \cos \theta_1 = OE$

$F_2 \sin \theta_1 = BF = HD$

$F_2 \cos \theta_2 = OF = ED$ $\because$ ($OB = AC$ and also $OB \parallel AC$. Hence projections of OB and AC on the same horizontal line OD will be equal *i.e.*, $OF = ED$)

$R \sin \theta = CD$

$R \cos \theta = OD$

Let the length $OO' = x$.

Then $x \sin \theta_1 = r_1$, $x \sin \theta = r$ and $x \sin \theta_2 = r_2$

Now moment of R about O'

$= R \times (\perp$ distance between O' and $R) = R \times r$

$= R \times x \sin \theta$ $(\because \; r = x \sin \theta)$

$= (R \sin \theta) \times x$

$= CD \times x$ $(\because\ R \sin \theta = CD)$

$= (CH + HD) \times x$

$= (F_1 \sin \theta_1 + F_2 \sin \theta_2) \times x$ $(\because\ CH = F_1 \sin \theta_1 \text{ and } HD = F_2 \sin \theta_2)$

$= F_1 \times x \sin \theta_1 + F_2 \times x \sin \theta_2$

$= F_1 \times r_1 + F_2 \times r_2$ $(\because\ x \sin \theta_1 = r_1 \text{ and } x \sin \theta_2 = r_2)$

= Moment of F_1 about O' + Moment of F_2 about O'.

Hence moment of R about any point in the algebraic sum of moments of its components (*i.e.*, F_1 and F_2) about the same point. Hence Varignon's principle is proved.

The principle of moments (or Varignon's principle) is not restricted to only two concurrent forces but is also applicable to any coplanar force system, *i.e.*, concurrent or non-concurrent or parallel force system.

Problem 3.2. *A force of 100 N is acting at a point A as shown in Fig. 3.5. Determine the moments of this force about O.*

Sol. Given :

Force at $A = 100$ N

Draw a perpendicular from O on the line of action of force 100 N. Hence OB is the perpendicular on the line of action of 100 N as shown in Fig. 3.5.

1st Method

Triangle OBC is a right-angled triangle. And angle

$$OCB = 60°.$$

$$\therefore \quad \sin 60° = \frac{OB}{OC}$$

$$\therefore \quad OB = OC \sin 60°$$

$$= 3 \times 0.866$$

$$= 2.598 \text{ m}$$

Moment of the force 100 N about O

$= 100 \times OB = 100 \times 2.598$

$= 259.8$ Nm (clockwise). **Ans.**

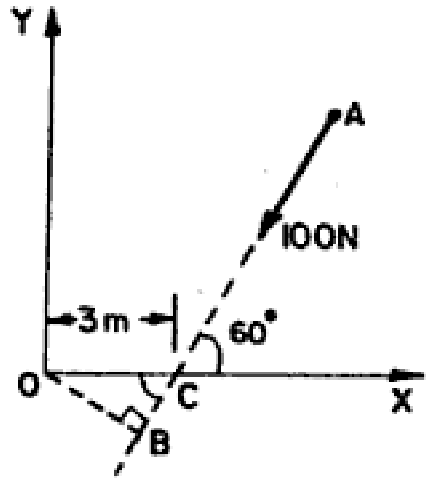


Fig. 3.5

2nd Method

The moment of force 100 N about O, can also be determined by using Varignon's principle. The force 100 N is replaced by its two rectangular components at any convenient point. Here the convenient point is chosen as C. The horizontal and vertical components of force 100 N acting at C are shown in Fig. 3.6.

(*i*) The horizontal component

$= 100 \times \cos 60° = 50$ N

But this force is passing through O and hence has no moment about O.

The vertical component

$= 100 \times \sin 60° = 100 \times 0.866 = 86.6$ N

This force is acting vertical downwards at C. Moment of this force about O.

$= 86.6 \times OC = 86.6 \times 3$ $(\because\ OC = 3 \text{ m})$

$= 259.8$ N (clockwise). **Ans.**

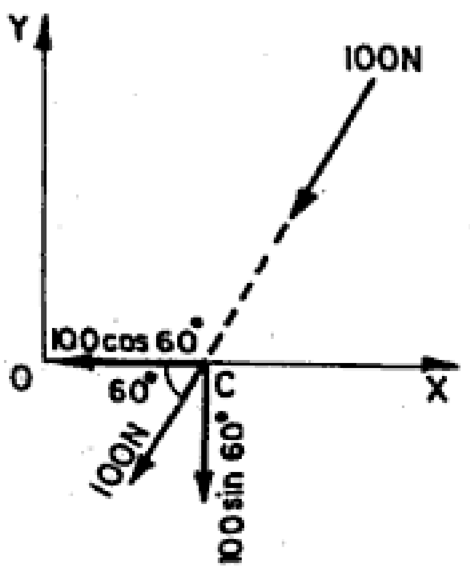


Fig. 3.6

3.4. TYPES OF PARALLEL FORCES

The following are the important types of parallel forces :

1. Like parallel forces,
2. Unlike parallel forces.

3.4.1. Like parallel forces. The parallel forces which are acting in the same direction, are known as like parallel forces. In Fig. 3.7, two parallel forces F_1 and F_2 are shown. They are acting in the same direction. Hence they are called as like parallel forces. These forces may be equal or unequal in magnitude.

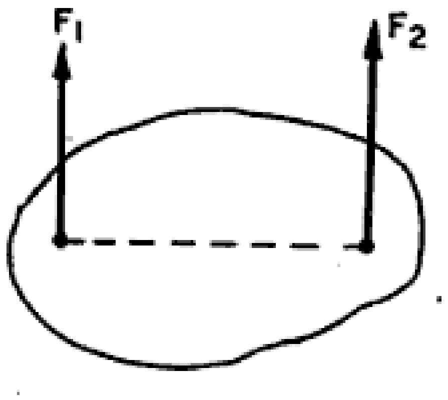


Fig. 3.7

3.4.2. Unlike Parallel Forces. The parallel forces which are acting in the opposite direction, are known as unlike parallel forces. In Fig. 3.8, two parallel forces F_1, F_2 are acting in opposite direction. Hence they are called as unlike parallel forces. These forces may be equal or unequal in magnitude.

The unlike parallel forces may be divided into : (*i*) unlike equal parallel forces, and (*ii*) unlike unequal parallel forces.

Unlike equal parallel forces are those which are acting in opposite direction and are equal in magnitude.

Unlike unequal parallel forces are those which are acting in opposite direction and are unequal in magnitude.

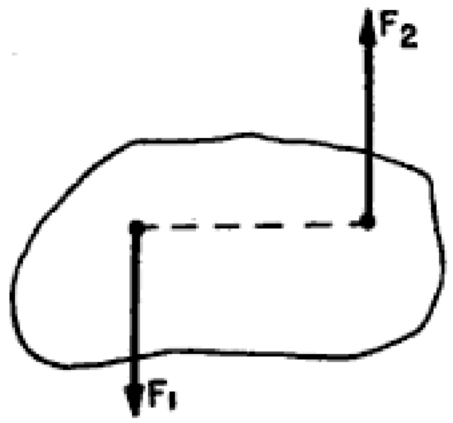


Fig. 3.8

3.5. RESULTANT OF TWO PARALLEL FORCES

The resultant of following two parallel forces will be considered :

1. Two parallel forces are like.
2. Two parallel forces are unlike and are unequal in magnitude.
3. Two parallel forces are unlike but equal in magnitude.

3.5.1. Resultant of two like parallel forces. Fig. 3.9 shows a body on which two like parallel forces F_1 and F_2 are acting. It is required to determine the resultant (R) and also the point at which the resultant R is acting. For the two parallel forces which are acting in the same direction, obviously the resultant R is given by,

$$R = F_1 + F_2$$

In order to find the point at which the resultant is acting, Varignon's principle (or method of moments) is used. According to this, the algebraic sum of moments of F_1 and F_2 about any point should be equal to the moment of the resultant (R) about that point. Now arbitrarily choose any point O along line AB and take moments of all forces about this point.

Moment of F_1 about $O = F_1 \times AO$ (clockwise) (–)

Moment of F_2 about $O = F_2 \times BO$ (anti-clockwise) (+ ve)

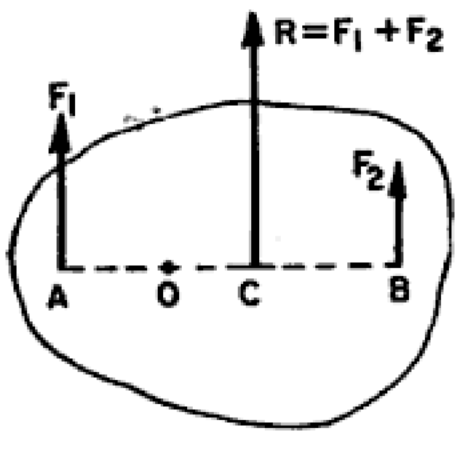


Fig. 3.9

∴ Algebraic sum of moments of F_1 and F_2 about O

$$= -F_1 \times AO + F_2 \times BO$$

Moment of resultant about $O = R \times OC$ (anti-clockwise)(+)

But according to principle of moments the algebraic sum of moments of F_1 and F_2 about O should be equal to the moment of resultant about the same point O.

$$\therefore \quad -F_1 \times AO + F_2 \times BO = +R \times CO = (F_1 + F_2) \times CO \qquad (\because R = F_1 + F_2)$$

or $$F_1(AO + CO) = F_2(BO - CO)$$

or $$F_1 \times AC = F_2 \times BC \qquad (\because AO + CO = AC \text{ and } BO - CO = BC)$$

or $$\frac{F_1}{F_2} = \frac{BC}{AC}$$

The above relation shows that the resultant R acts at the point C, parallel to the lines of action of the given forces F_1 and F_2 in such a way that the resultant divides the distance AB in the ratio inversely proportional to the magnitudes of F_1 and F_2. Also the point C lies in line AB *i.e.*, point C is not outside AB.

The location of the point C, at which the resultant R is acting, can also be determined by taking moments about points A of Fig. 3.9. As the force F_1 is passing through A, the moment of F_1 about A will be zero.

The moment of F_2 about $A = F_2 \times AB$ (anti-clockwise) (+)

Algebraic sum of moments of F_1 and F_2 about O

$$= O + F_2 \times AB = F_2 \times AB \text{ (anti-clockwise) (+)} \qquad ...(i)$$

The moment of resultant R about A

$$= R \times AC \text{ (anti-clockwise)(+)} \qquad ...(ii)$$

But according to the principle of moments, the algebraic sum of moments of F_1 and F_2 about A should be equal to the moment of resultant about the same point A. Hence equating equations (*i*) and (*ii*),

$$F_2 \times AB = R \times AC$$

But $R = (F_1 + F_2)$ hence the distance AC should be less than AB. Or in other words, the point C will lie inside AB.

3.5.2. Resultant of Two Unlike Parallel Forces (Unequal in magnitude). Fig 3.10 shows a body on which two unlike parallel forces F_1 and F_2 are acting which are unequal in magnitude. Let us assume that force F_1 is more than F_2. It is required to determined the resultant R and also the point at which the resultant R is acting. For the two parallel forces, which are acting in opposite direction, obviously the resultant is given by,

$$R = F_1 - F_2$$

Let the resultant R is acting at C as shown in Fig. 3.10.

In order to find the point C, at which the resultant is acting, principle of moments is used.

Choose arbitrarily any point O in line AB. Take the moments of all forces (*i.e.*, F_1, F_2 and R) about this point.

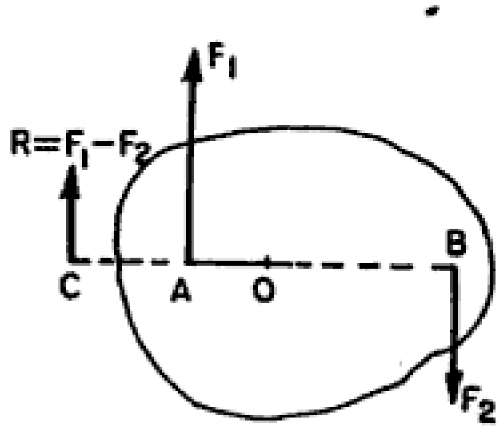


Fig. 3.10

Moment of F_1 about $O = F_1 \times AO$ (clockwise)

Moment of F_2 about $O = F_2 \times BO$ (clockwise)

Algebraic sum of moments of F_1 and F_2 about O

$$= F_1 \times AO + F_2 \times BO \qquad ...(i)$$

Moment of resultant force R about O

$$= R \times CO \text{ (clockwise)}$$

$$= (F_1 - F_2) \times CO \qquad (\because R = F_1 - F_2)$$

$$= F_1 \times CO - F_2 \times CO \qquad ...(ii)$$

But according to the principle of moments, the algebraic sum of moments of all forces about any point should be equal to the moment of resultant about that point. Hence equating equations (*i*) and (*ii*), we get

$$F_1 \times AO + F_2 \times BO = F_1 \times CO - F_2 \times CO$$

or $$F_2(BO + CO) = F_1(CO - AO)$$

$$F_2 \times BC = F_1 \times AC \qquad (\because \ BO + CO = BC \text{ and } CO - AO = AC)$$

or $$\frac{BC}{AC} = \frac{F_1}{F_2} \quad \text{or} \quad \frac{F_1}{F_2} = \frac{BC}{AC}$$

But $F_1 > F_2$, hence BC will be more than AC. Hence point O lies outside of AB and on the same side as the larger force F_1. *Thus in case of two unlike parallel forces the resultant lies outside the line joining the points of action of the two forces and on the same side as the larger force.*

The location of the point C, at which the resultant R is acting, can also be determined by taking moments about point A, of Fig. 3.10. As the force F_1 is passing through A, the moment of F_1 about A will be zero.

The moment of F_2 about $A = F_2 \times AB$ (clockwise) (–)

Algebraic sum of moments of F_1 and F_2 about A

$$= O + F_2 \times AB = F_2 \times AB \text{ (clockwise) } (-) \qquad ...(i)$$

The moment of resultant R about A should be equal to the algebraic sum of moments of F_1 and F_2 (*i.e.*, $= F_2 \times AB$) according to the principle of moments. Also the moment of resultant R about A should be clockwise. As R is acting upwards [$\because$ $F_1 > F_2$ and $R = (F_1 - F_2)$ so R is acting in the direction of F_1], the moment of resultant R about A would be clockwise only if the points C is towards the left of point A. Hence the point C will be outside the line AB and on the side of F_1 (*i.e.*, larger force).

Now the moment of resultant R about A

$$= R \times AC \text{ (clockwise) } (-) \qquad ...(ii)$$

Equating equations (*i*) and (*ii*),

$$F_2 \times AB = R \times AC$$

$$= (F_1 - F_2) \times AC \qquad (\because \ R = F_1 - F_2)$$

As F_1, F_2 and AB are known, hence AC can be calculated. Or in other words, the location of point C is known.

3.5.3. Resultant of two unlike parallel forces which are equal in magnitude. When two equal and opposite parallel forces act on a body, at some distance apart, the two forces from a *couple* which has a tendency to rotate the body. The perpendicular distance between the parallel forces is known as *arm of the couple*.

Fig. 3.11 shows a body on which two parallel forces, which are acting in opposite direction but equal in magnitude are acting. These two forces will form a couple which will have a tendency to rotate the body in clockwise direction. The moment of the couple is the product of either one of the forces and perpendicular distance between the forces.

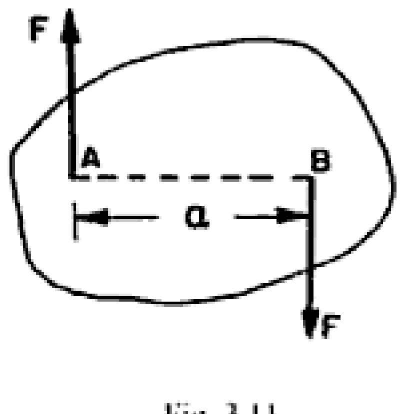


Fig. 3.11

Let F = Force at A or at B

a = Perpendicular distance (or arm of the couple)

The moment (M) of the couple is given by, $M = F \times a$.

The units of moment will be Nm.

Problem 3.3. *Three like parallel forces 100 N, 200 N and 300 N are acting at points A, B and C respectively on a straight line ABC as shown in Fig. 3.12. The distances are AB = 30 cm and BC = 40 cm. Find the resultant and also the distance of the resultant from point A on line ABC.*

Sol. Given :

Force at $A = 100$ N

Force at $B = 200$ N

Force at $C = 300$ N

Distance $AB = 30$ cm, $BC = 40$ cm. As all the forces are parallel and acting in the same direction, their resultant R is given by

$$R = 100 + 200 + 300 = 600 \text{ N}$$

Let the resultant is acting at a distance of x cm from the point A as shown in Fig. 3.12.

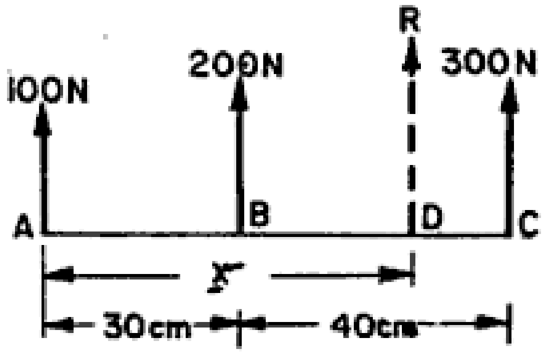


Fig. 3.12

Now take the moments of all forces about point A. The force 100 N is passing A, hence its moment about A will be zero.

$\therefore$ Moment of 100 N force about $A = 0$

Moment of 200 N force about $A = 200 \times 30 = 6000$ N cm (anti-clockwise)

Moment of 300 N force about $A = 300 \times AC$

$= 300 \times 70 = 21000$ N cm (anti-clockwise)

Algebraic sum of moments of all forces about A

$= 0 + 6000 + 21000 = 27000$ N cm (anti-clockwise)

Moment of resultant R about $A = R \times x$

$= 600 \times x$ N cm ($\because$ $R = 600$)

But algebraic sum of moments of all forces about A

= Moment of resultant about A

or $27000 = 600 \times x$ or $x = \dfrac{27000}{600} = 45$ **cm. Ans.**

Problem 3.4. *The three like parallel forces of magnitude 50 N, F and 100 N are shown in Fig. 3.13. If the resultant R = 250 N and is acting at a distance of 4 m from A, then find*

(*i*) *Magnitude of force F.*

(*ii*) *Distance of F from A.*

Sol. Given :

Forces at $A = 50$ N, at $B = F$ and $D = 100$ N

$R = 250$ N, Distance $AC = 4$ m, $CD = 3$ m.

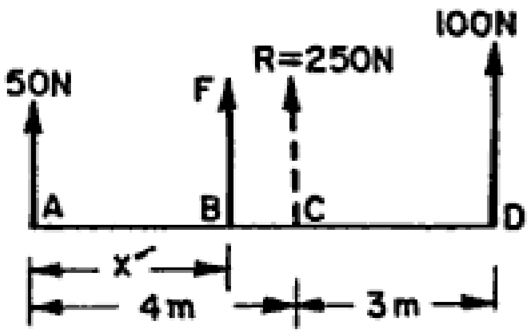


Fig. 3.13

(*i*) *Magnitude of force F*

The resultant R of three like forces is given by,

$$R = 50 + F + 100$$

or $250 = 50 + F + 100$ ($\because$ $R = 250$)

$\therefore$ $F = 250 - 50 - 100 = 100$ **N. Ans.**

(*ii*) *Distance of F from A*

Take the moments of all forces about point A.

Moment of force 50 N about $A = 0$ ($\because$ force 50 N is passing through)

Moment of force F about $A = F \times x$ (anti-clockwise)

Moment of force 100 N about $A = 100 \times AD = 100 \times 7 = 700$ N m (anti-clockwise)

$\therefore$ Algebraic sum of moments of all forces about A

$= 0 + F \times x + 700$ N m

$= F \times x + 700$ N m (anti-clockwise)

Moment of resultant R about A

$= R \times 4 = 250 \times 4 = 1000$ N m (anti-clockwise)

But algebraic sum of moments of all forces about A must be equal to the moment of resultant R about A.

$\therefore$ $F \times x + 700 = 1000$ or $F \times x = 1000 - 700 = 300$

or $x = \dfrac{300}{F} = \dfrac{300}{100}$ ($\because$ $F = 100$ N)

$= 3$ m. **Ans.**

Problem 3.5. *Four parallel forces of magnitudes 100 N, 150 N, 25 N and 200 N are shown in Fig. 3.14. Determine the magnitude of the resultant and also the distance of the resultant from point A.*

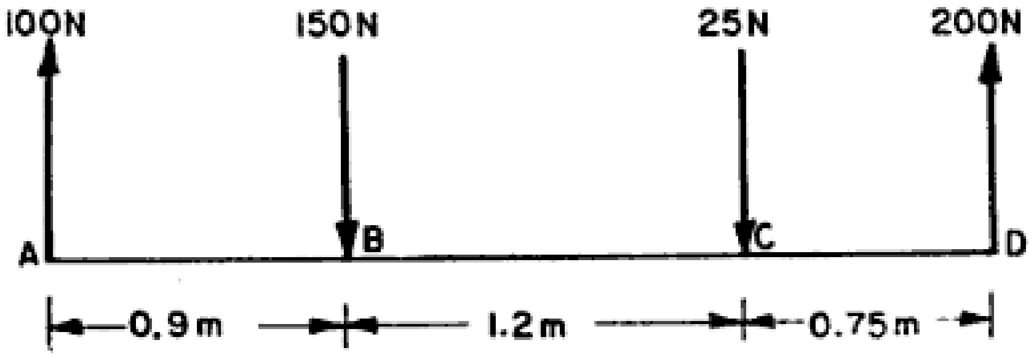


Fig. 3.14

Sol. Given :

Forces are 100 N, 150 N, 25 N and 200 N.

Distances $AB = 0.9$ m, $BC = 1.2$ m, $CD = 0.75$ m.

As all the forces are acting vertically, hence their resultant R is given by.

$R = 100 - 150 - 25 + 200$ (Taking upward force + ve and downward as – ve)

$= 300 - 175 = 125$ N

+ve sign shows that R is acting vertically upwards. To find the distance of R from point A, take the moments of all forces about point A.

Let x = Distance of R from A in metre.

As the force 100 N is passing through A, its moment about A will be zero.

Moment of 150 N force about $A = 150 \times AB$

$= 150 \times 0.9$ (clockwise) (–) $= -135$ Nm

Moment of 25 N force about $A = 25 \times AC = 25 \times (0.9 + 1.2)$

$= 25 \times 2.1$ (clockwise) (–) $= -52.5$ Nm.

Moment of 200 N force about $A = 200 \times AD$

$= 200 \times (0.9 + 1.2 + 0.75)$

$= 200 \times 2.85$ (anti-clockwise) (+) $= 570$ Nm

Algebraic sum of moments of all forces about A

$= -135 - 52.5 + 570 = 382.5$ Nm ...(*i*)

+ ve sign shows that this moment is anti-clockwise. Hence the moment of resultant R about A must be 382.5 Nm, *i.e.*, moment of R should be anti-clockwise about A. The moment of R about A will be anti-clockwise if R is acting upwards and towards the right of A.

Now moment of R about $A = R \times x$. But $R = 125$

$= 125 \times x$ (anti-clockwise) (+)

$= +125 \times x$ ...(*ii*)

Equating (*i*) and (*ii*),

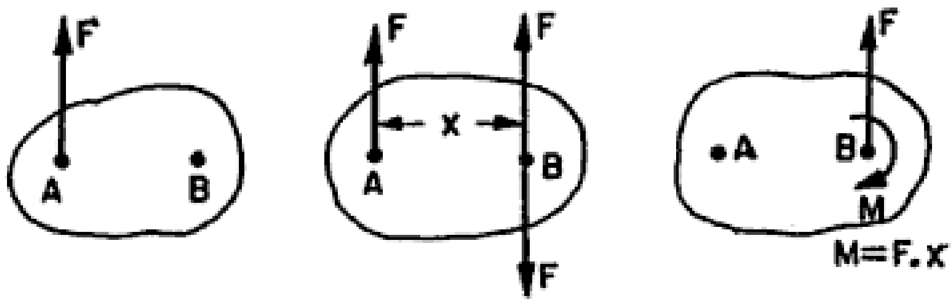
$$382.5 = 125 \times x \quad \text{or} \quad x = \frac{382.5}{125} = \mathbf{3.06\ m.} \quad \textbf{Ans.}$$

∴ Resultant (R = 125 N) will be 125 N upwards and is acting at a distance of 3.06 m to the right of point A.

3.6. RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE

A given force F applied to a body at any point A can always be replaced by an equal force applied at another point B together with a couple which will be equivalent to the original force. This is proved as given below :

Let the given force F is acting at point A as shown in Fig. 3.15 (*a*).



(*a*) (*b*) (*c*)

Fig. 3.15

This force is to be replaced at the point B. Introduce two equal and opposite forces at B, each of magnitude F and acting parallel to the force at A as shown in Fig. 3.15 (*b*). The force system of Fig. 3.15 (*b*) is equivalent to the single force acting at A of Fig. 3.15 (*a*). In Fig. 3.15 (*b*) three equal forces are acting. The two forces *i.e.*, force F at A and the oppositely directed force F at B (*i.e.*, vertically downward force at B) from a couple. The moment of this couple is $F \times x$ clockwise where x is the perpendicular distance between the lines of action of forces at A and B. The third force is acting at B in the same direction in which the force at A is acting. In Fig. 3.15 (*c*), the couple is shown by curved arrow with symbol M. The force system of Fig. 3.15 (*c*) is equivalent to Fig. 3.15 (*b*). Or in other words the Fig. 3.15 (*c*) is equivalent to Fig. 3.15 (*a*). Hence the given force F acting at A has been replaced by an equal and parallel force applied at point B in the same direction together with a couple of moment $F \times x$.

Thus a force acting at a point in a rigid body can be replaced by an equal and parallel force at any other point in the body, and a couple.

Problem 3.6. *A system of parallel forces are acting on a rigid bar as shown in Fig. 3.16. Reduce this system to :*

(*i*) *a single force*

(*ii*) *a single force and a couple at A*

(*iii*) *a single force and a couple at B.*

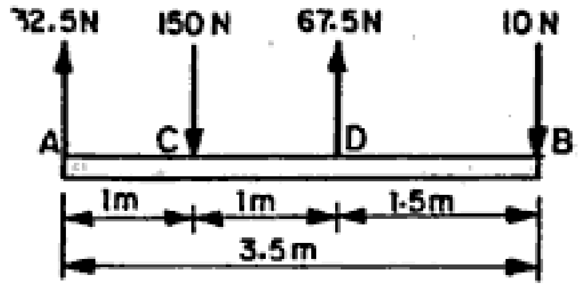


Fig. 3.16

Sol. Given :

Forces at A, C, D and B are 32.5 N, 150 N, 67.5 N and 10 N respectively.

Distances AC = 1 m, CD = 1 m and BD = 1.5 m.

(*i*) *Single force system.* The single force system will consist only resultant force in magnitude and location. All the forces are acting in the vertical direction and hence their resultant (R) in magnitude is given by

$$R = 32.5 - 150 + 67.5 - 10 = \mathbf{-60\ N.} \quad \textbf{Ans.}$$

Negative sign shows that resultant is acting vertically downwards.

Let x = Distance of resultant from A towards right. To find the location of the resultant take the moments of all forces about A, we get Moment of resultant about A.

= Algebraic sum of moments of all forces about A

or $$R \times x = -50 \times AC + 67.5 \times AD - 10 \times AB$$

(Taking clockwise moment –ve and anticlockwise moment +ve)

or $$(-60) \times x = 50 \times 1 + 67.5 \times 2 - 10 \times 3.5$$

$$(\because \; R = -60)$$

or $$-60x = -150 + 135 - 35 = -50$$

$$\therefore \quad x = \frac{-50}{-60} = \mathbf{0.833 \text{ m.}} \quad \textbf{Ans.}$$

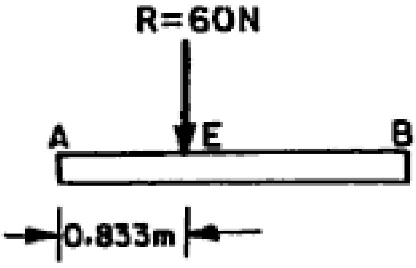


Fig. 3.16 (*a*)

Hence the given system of parallel forces is equivalent to a single force 60 N acting vertically downwards at point E at a distance of 0.833 m from A shown in Fig. 3.16 (*a*).

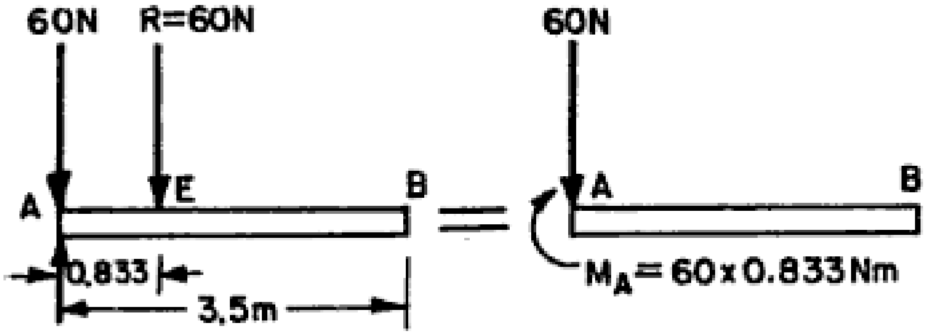
(*ii*) *A single force and a couple at A.* The resultant force R acting at point E as shown in Fig. 3.16 (*a*) can be replaced by an equal force applied at point A in the same direction together with a couple. This is shown in Fig. 3.16 (*c*).

The moment of the couple = 60 × 0.833 Nm (clockwise)

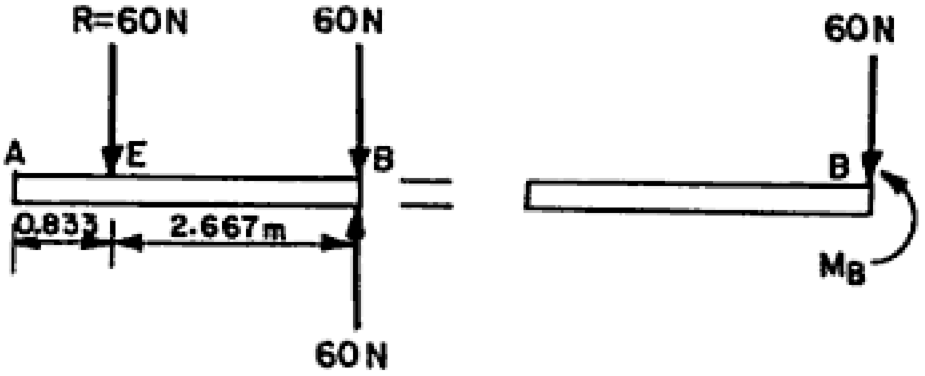
= – **49.98 Nm. Ans.** (– ve sign is due to clockwise)

(*iii*) *A single force and a couple at B.* First find distance BE. But from Fig. 3.16 (*b*), the distance

$$BE = AB - AE = 3.5 - 0.833 = 2.667 \text{ m.}$$



(*b*) (*c*)



(*d*) (*e*)

Fig. 3.16

Now if the force R = 60 N is moved to the point B, it will be accompanied by a couple of moment 60 × BE or 60 × 2.667 Nm. This is shown in Fig. 3.16 (*e*).

The moment of the couple = 60 × 2.667 Nm (anti-clockwise)

= **160 Nm. Ans.**

3.7. GENERAL CASE OF PARALLEL FORCES IN A PLANE

Fig. 3.17 shows a number of parallel forces acting on a body in one plane. The forces F_1, F_2 and F_4 are acting in one direction, whereas the forces F_3 and F_5 are acting in the opposite direction. Let R_1 = Resultant

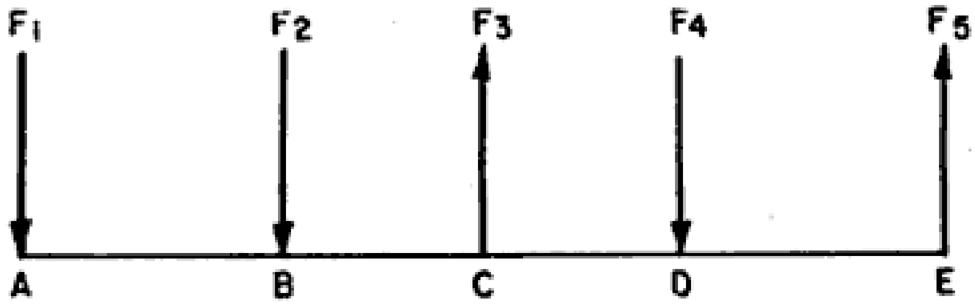


Fig. 3.17

of forces F_1, F_2 and F_5 and R_2 = Resultant of forces F_3 and F_5. The resultants R_1 and R_2 are acting in opposite direction and are parallel to each other. Now three important cases are possible.

1. R_1 may not be equal to R_2. Then we shall have two unequal parallel forces (R_1 and R_2) acting in the opposite direction. The resultant R of these two forces (R_1 and R_2) can be easily obtained. The point of application of resultant R can be obtained by equating the moment of R about any point to the algebraic sum of the moments of individual forces about the same point.

2. R_1 is equal to R_2. Then we shall have two equal parallel forces (R_1 and R_2) acting in the opposite direction. The resultant R of these two forces will be zero. Now the system may reduce to a couple or the system is in equilibrium. To distinguish between these two cases, the algebraic sum of moments of all forces (F_1, F_2, ...F_5) about any point is taken. If the sum of moments is not zero, the system reduces a resultant couple. The calculated moment gives the moment of this couple.

3. R_1 is equal to R_2 and sum of moments of all forces (F_1, F_2, F_3, F_4, F_5, ...) about any point is zero, then the system will not be subjected to any resultant couple but the system will be in equilibrium.

Problem 3.7. *Determine the resultant of the parallel force system shown in Fig. 3.18.*

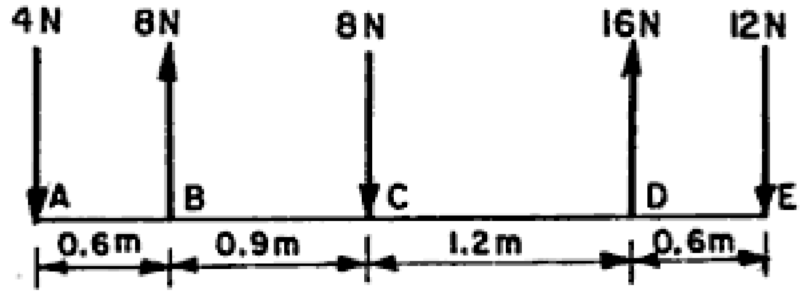


Fig. 3.18

Sol. Given :

Forces at A, B, C, D and E and 4 N, 8 N, 8 N, 16 N and 12 N respectively.

Distances $AB = 0.6$ m, $BC = 0.9$ m,

$CD = 1.2$ m, and $DE = 0.6$ m.

Since all the forces are vertical and parallel, hence their resultant is given by

$$R = -4 + 8 - 8 + 16 - 12 = 0$$

As the resultant force on the system is zero, there will be two possibilities. The system has a resultant couple or the system is in equilibrium. To distinguish between these two possibilities, take the sum of moments of all forces about any point. Let us take the moments about point A.

∴ Algebraic sum of moments of all forces about A

$$= 4 \times 0 + 8 \times AB - 8 \times AC + 16 \times AD - 12 \times AE$$
$$= 0 + 8 \times 0.6 - 8 \times (0.6 + 0.9) + 16 \times (0.6 + 0.9 + 1.2) - 12 \times (0.6 + 0.9 + 1.2 + 0.6)$$
$$= 0 + 4.8 - 12 + 16 \times 2.7 - 12 \times 3.3 \text{ Nm}$$
$$= 4.8 - 12 + 43.2 - 39.6 = 48 - 51.6$$
$$= -3.6 \text{ Nm}$$

As the algebraic sum of moments of all forces about any point is not zero, the system will have a resultant couple of magnitude – 3.6 Nm *i.e.*, a clockwise couple. **Ans.**

Problem 3.8. *Determine the resultant of the parallel forces acting on a body as shown in Fig. 3.19.*

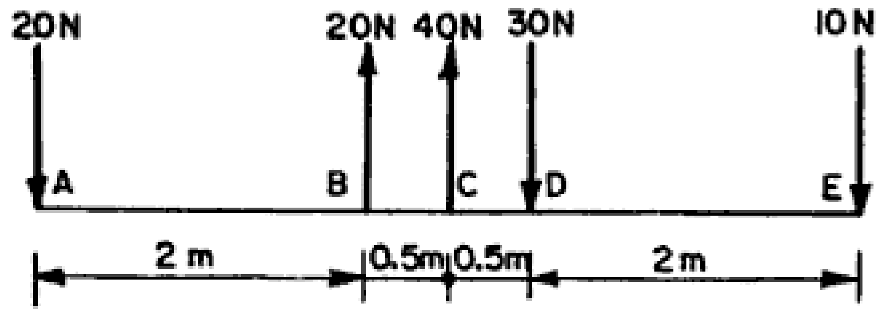


Fig. 3.19

Sol. Since all the forces are vertical and parallel, hence their resultant is given by

$$R = -20 + 20 + 40 - 30 - 10 = 0$$

Taking moments of all forces about the point A, we get

Resultant moment $= 20 \times 0 + 20 \times 2 + 40 \times 2.5 - 30 \times 3 - 10 \times 5$

$$= 0 + 40 + 100 - 90 - 50 = 140 - 140 = 0$$

As the resultant moment is zero and also the resultant force on the body is zero, the body will be in equilibrium. **Ans.**

3.8. EQUIVALENT SYSTEM

An equivalent system for a given system of coplanar forces, is a combination of a force passing through a given point and a moment about that point. The force is the resultant of all forces acting on the body. And the moment is the sum of all the moments about that point.

Hence equivalent system consists of :

(*i*) a single force R passing through the given point P and

(*ii*) a single moment M_R

where R = the resultant of all force acting on the body.

M_R = sum of all moments of all the forces about point P.

Problem 3.9. *Three external forces are acting on a L-shaped body as shown in Fig. 3.20. Determine the equivalent system through point O.*

Sol. Given :

Force at $A = 2000$ N, Angle = 30°

Force at $B = 1500$ N

Force at $C = 1000$ N

Distance $OA = 200$ mm, $OB = 100$ mm and $BC = 200$ mm

Angle $COA = 90°$

Determine the equivalent system through O. This means find

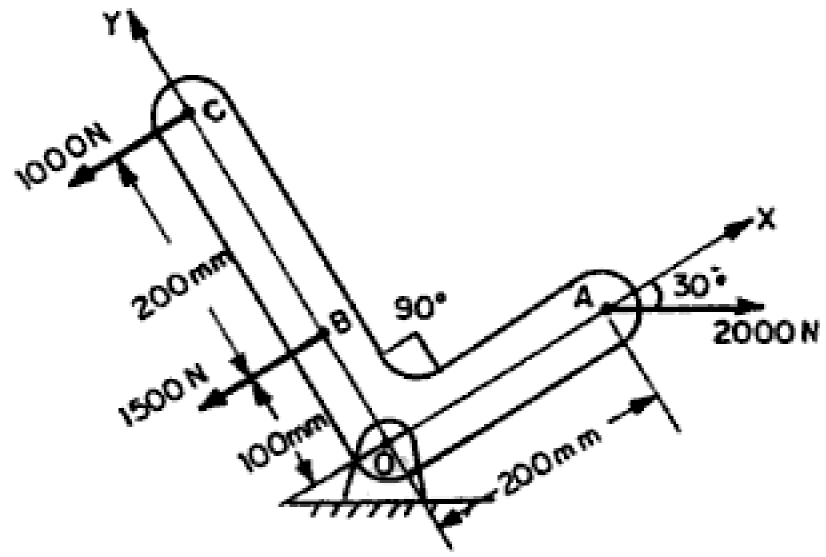


Fig. 3.20

(*i*) single resultant force, R

(*ii*) single moment through O.

Taking x-axis along OA and y-axis along OC.

The force at A is resolved into two components.

Component along x-axis = $2000 \times \cos 30° = 1732$ N

Component along y-axis = $2000 \times \sin 30° = 1000$ N

Resolving all forces along X-axis *i.e.*,

$$\Sigma F_x = 2000 \cos 30° - 1500 - 1000 = -768 \text{ N}$$

Similarly $\Sigma F_y = -2000 \times \sin 30° = -1000$ (– ve sign is due to downward)

$\therefore$ Resultant, $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(-768)^2 + (-1000)^2}$

$$= \sqrt{589824 + 1000000} = 1260.88 \text{ N}$$

Taking moments of all forces about point O,

$$M_0 = (-2000 \sin 30) \times 200 + 1500 \times 100 + 1000 \times 300$$

$$= -200000 + 150000 + 300000$$

$$= 250000 \text{ Nmm} = 250 \text{ Nm}$$

$\therefore$ Equivalent system through point O is

$$R = 1260.88 \text{ N}$$

$$M = 250 \text{ Nm}$$

Problem 3.10. *Fig. 3.21 shows two vertical forces and a couple of moment 2000 Nm acting on a horizontal rod which is fixed at end A.*

(*i*) *Determine the resultant of the system.*

(*ii*) *Determine an equivalent system through A.*

Sol. Given :

Force at C = 4000 N, Force at B = 2500 N

Moment at D = 2000 Nm

Distance AC = 1 m, BC = 1.5 m

CD = 0.8 m, BD = 0.7 m

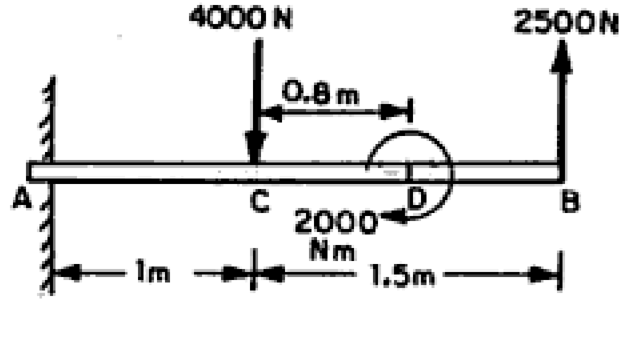


Fig. 3.21

(*i*) *Resultant of the system*

This means to find the resultant of all the forces and also the point at which the resultant is acting. There are two vertical forces only.

Hence resultant, $R = 4000 - 2500 = 1500$ N acting downward

The point at which the resultant is acting is obtained by taking moments about point *A*. For moments there are two forces (4000 N at *C* and 2000 N at *B*) and also a moment at *D*.

Moment of force 4000 N about point $A = 4000 \times 1$

$= 4000$ Nm (clockwise)

Moment of force 2500 N about point *A*

$= 2500 \times (1 + 1.5) = 2500 \times 2.5$

$= 6250$ Nm (anti-clockwise)

Moment at $D = 2000$ Nm (clockwise)

$\therefore$ Sum of all moments about *A*

= 4000 (clockwise) – 6250 (anti-clockwise) + 2000 (clockwise)

= – 250 (anti-clockwise)

The resultant is acting vertically downward. If it is acting towards right of *A*, then it will give clockwise moment. But we want anti-clockwise moment. Hence the resultant must act towards the left of *A*.

Let x = Distance of resultant force (1500 N) from *A*

$\therefore$ Moment of resultant force (*R*) about *A*

$= 1500 \times x$

$$\therefore \quad 1500 \times x = 250$$

$$\therefore \quad x = \frac{250}{1500} = 0.166 \text{ m.}$$ **Ans.**

Hence resultant of the system is 1500 N ↓ acting at a distance of 0.166 m left to *A*. **Ans.**

(*ii*) *Equivalent system through A*

This means to find a single resultant force and a single moment through *A*.

Single resultant force, $R = 1500$ N

Single moment through, $A = 250$ Nm. **Ans.**

HIGHLIGHTS

1. Parallel forces are having their lines of action parallel to each other.
2. The moment of a force about any point is the product of force and perpendicular distance between the point and line of action of force.
3. Anti-clockwise moment is taken +ve whereas clockwise moment is taken – ve.
4. Varignon's principle states that the moment of a force about any point is equal to the algebraic sum of moments of its components about that point.
5. Like parallel forces are parallel to each other and are acting in the same direction, whereas the unlike parallel forces are acting in opposite direction.
6. The resultant of two like parallel forces is the sum of the two forces and acts at a point between the line in such a way that the resultant divides the distance in the ratio inversely proportional to the magnitudes of the forces.
7. When two equal and opposite parallel forces act on a body at some distance apart, the two forces form a couple which has a tendency to rotate the body. The moment of this couple is the product of either one of the forces and perpendicular distance between the forces.

8. A given force F applied to a body at any point A can always be replaced by an equal force applied at another point B in the same direction together with a couple.
9. If the resultant of a number of parallel forces is not zero, the system can be reduced to a single force, whose magnitude is equal to the algebraic sum of all forces. The point of application of this single force is obtained by equating the moment of this single force about any point to the algebraic sum of moments of all forces acting on the system about the same point.
10. If the resultant of a number of parallel forces is zero, then the system may have a resultant couple or may be in equilibrium. If the algebraic sum of moments of all forces about any point is not zero, then system will have a resultant couple. But if the algebraic sum of moments of all forces about any point is zero, the system will be in equilibrium.

EXERCISE 3

A. Theoretical Questions

1. Define the terms : Coplanar parallel forces, like parallel forces and unlike parallel forces.
2. Define and explain the moment of a force. Differentiate between clockwise moment and anti-clockwise moment.
3. (*a*) State the Varignon's principle. Also give the proof of Varignon's principle.
   (*b*) Differentiate between :
   (*i*) Concurrent and non-concurrent forces,
   (*ii*) Coplanar and non-coplanar forces,
   (*iii*) Moment of a force and couple.
4. Define moment of a force about a point and show that the algebraic sum of the moments of two coplanar forces about a point is equal to the moment of their resultant about that point.
5. What are the different types of parallel forces ? Distinguish between like and unlike parallel forces.
6. Prove that the resultant of two like parallel forces F_1 and F_2 is $F_1 + F_2$. Also prove that the resultant divides the line of joining the points of action of F_1 and F_2 internally in the inverse ratio of the forces.
7. Prove that in case of two unlike parallel forces the resultant lies outside the line joining the points of action of the two forces and on the same side as the larger force.
8. Describe the method of finding the line of action of the resultant of a system of parallel forces.
9. The resultant of a system of parallel forces is zero, what does it signify ?
10. Describe the method of finding the resultant of two unlike parallel forces which are equal in magnitude.
11. Prove that a given force F applied to a body at any point A can always be replaced by an equal force applied at another point B together with a couple.
12. State the principle of moment.
13. Indicate whether the following statements are True or False.
   (*i*) Force is an agency which tends to cause motion.
   (*ii*) The tension member of a frame work is called a street.
   (*iii*) The value of g reduces slightly as we move from poles towards the equator.
   (*iv*) Coplanar forces are those which have the same magnitude and direction.
   (*v*) A couple consists of two unequal and parallel forces acting on a body, having the same line of action.
   (*vi*) A vector diagram of a force represents its magnitude, direction, sense and point of application.
   (*vii*) The force of gravitation on a body is called its weight.
   (*viii*) The centre of gravity of a body is the point, through which the resultant of parallel forces passes in whatever position may the body be placed.

[**Ans.** (*i*) True (*ii*) False (*iii*) True (*iv*) False (*v*) False (*vi*) False (*vii*) True (*viii*) True.]

B. Numerical Problems

1. Four forces of magnitudes 20 N, 40 N, 60 N and 80 N are acting respectively along the four sides of a square *ABCD* as shown in Fig. 3.22. Determine the resultant moment about point *A*.
Each side of square is 2 m.
[Ans. 200 Nm anti-clockwise]

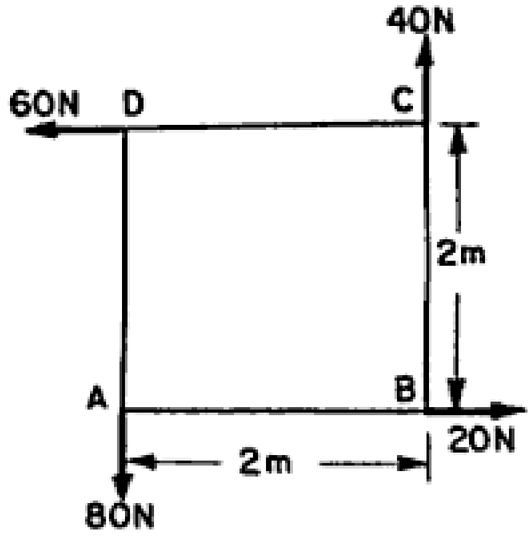


Fig. 3.22

2. A force of 50 N is acting at a point *A* as shown in Fig. 3.23. Determine the moment of this force about *O*.
[Ans. 100 Nm clockwise]

3. Three like parallel forces 20 N, 40 N and 60 N are acting at points, *A*, *B* and *C* respectively on a straight line *ABC*. The distances are *AB* = 3 m and *BC* = 4 m. Find the resultant and also the distance of the resultant from point *A* on line *ABC*. **[Ans.** 120 N, 4.5 m]

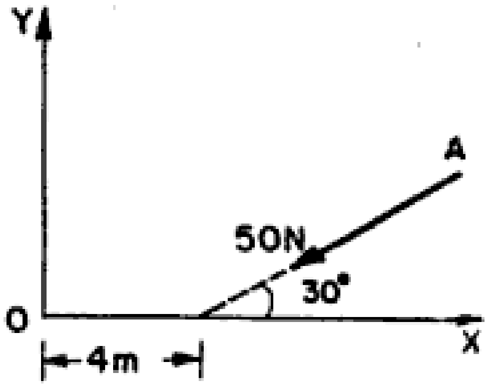


Fig. 3.23

4. The three like parallel forces 101 N, *F* and 300 N are acting as shown in Fig. 3.24. If the resultant *R* = 600 N and is acting at a distance of 45 cm from *A*, then find the magnitude of force *F* and distance of *F* and *A*.
[Ans. 200 N, 30 cm]

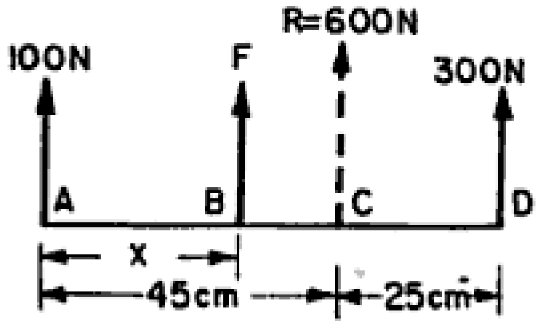


Fig. 3.24

5. Four parallel forces of magnitudes 100 N, 200 N, 50 N and 400 N are shown in Fig. 3.25. Determine the magnitude of the resultant and also the distance of the resultant from point *A*. **[Ans.** *R* = 350 N, 3.07 m]

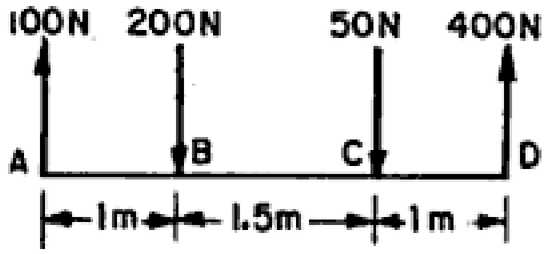


Fig. 3.25

6. A system of parallel forces are acting on a rigid bar as shown in Fig. 3.26. Reduce this system to :

(*i*) a single force, [**Ans.** (*i*) R = 120 N at 2.83 m from A

(*ii*) a single force and a couple at A (*ii*) R = 120 N and $M_A = -340$ Nm

(*iii*) a single force and a couple at B. (*iii*) R = 120 N and M_B = 120 Nm]

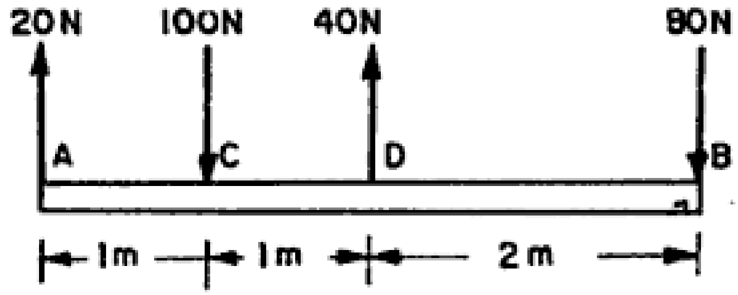


Fig. 3.26

7. Five forces are acting on a body as shown in Fig. 3.27. Determine the resultant.

[**Ans.** R = 0, Resultant couple = 10 Nm]

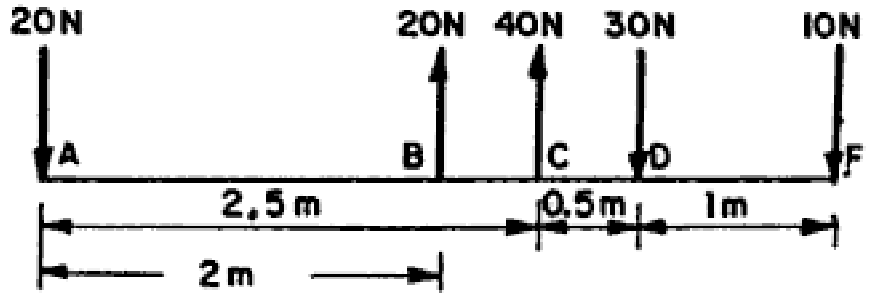


Fig. 3.27

8. Determine the resultant of the parallel forces shown in Fig. 3.28. [**Ans.** Body is in equilibrium]

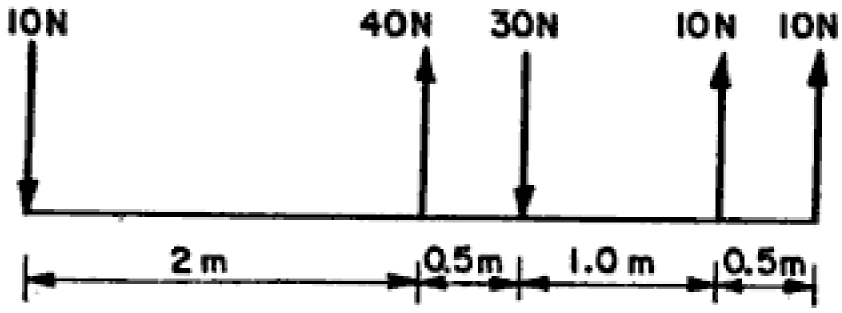


Fig. 3.28

4

Conditions of Equilibrium

4.1. INTRODUCTION

When some external forces (which may be concurrent or parallel) are acting on a stationary body, the body may start moving or may start rotating about any point. But if the body does not start moving *and also* does not start rotating about any point, then the body* is said to be in equilibrium. In this chapter, the conditions of equilibrium for concurrent forces (*i.e.*, forces meeting at a point) and for parallel forces will be described. Also the concept of free body diagram, different types of support reactions and determination of reactions will be explained.

4.2. PRINCIPLE OF EQUILIBRIUM

The principle of equilibrium states that, a stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces is zero *and also* the algebraic sum of moments of all the external forces about any point in their plane is zero. Mathematically, it is expressed by the equations :

$$\Sigma F = 0 \quad \text{...(4.1)}$$

$$\Sigma M = 0 \quad \text{...(4.2)}$$

The sign Σ is known as sigma which is a Greek letter. This sign represents the *algebraic sum* of forces or moments.

The equation (4.1) is also known as force law of equilibrium whereas the equation (4.2) is known as moment law of equilibrium.

The forces are generally resolved into horizontal and vertical components. Hence equation (4.1) is written as

$$\Sigma F_x = 0 \quad \text{...(4.3)}$$

and

$$\Sigma F_y = 0 \quad \text{...(4.4)}$$

where ΣF_x = Algebraic sum of all horizontal components

and ΣF_y = Algebraic sum of all vertical components.

4.2.1. Equations of equilibrium for Non-concurrent forces systems. A non-concurrent force systems will be in equilibrium if the resultant of all forces and moment is zero.

Hence the equations of equilibrium are

$$\Sigma F_x = 0, \Sigma F_y = 0 \quad \text{and} \quad \Sigma M = 0.$$

4.2.2. Equations of equilibrium for concurrent force system. For the concurrent forces, the lines of action of all forces meet at a point, and hence the moment of those force about that very point will be zero or $\Sigma M = 0$ automatically.

Thus for concurrent force system, the condition $\Sigma M = 0$ becomes redundant and only two conditions, *i.e.*, $\Sigma F_x = 0$ and $\Sigma F_y = 0$ are required.

*A body will be in equilibrium if both the resultant of the forces and resultant moment are zero, whereas a paraticle will be in equilibrium if the resultant force acting on it is zero.

Now let us apply the three conditions of equilibrium :

(i) $\Sigma F_x = 0$ as there is no horizontal force acting on the body

(ii) $\Sigma F_y = 0$ i.e., $F_1 + F_3 = F_2$

(iii) $\Sigma M = 0$ about any point.

Taking the moments of F_1, F_2 and F_3 about point A,

$$\Sigma M_A = -F_2 \times AB + F_3 \times AC$$

(Moment of F_3 is anti-clockwise whereas moment of F_2 is clockwise)

For equilibrium, ΣM_A should be zero

i.e., $$-F_2 \times AB + F_3 \times AC = 0$$

If the distances AB and AC are such that the above equation is satisfied, then the body will be in equilibrium under the action of three parallel forces.

4.3.3. Four force system. The body will be in equilibrium if the resultant force in horizontal direction is zero (i.e., $\Sigma F_x = 0$), resultant force in vertical direction is zero (i.e., $\Sigma F_y = 0$) and moment of all forces about any point in the plane of forces is zero (i.e., $\Sigma M = 0$).

Problem 4.1. *Two forces F_1 and F_2 are acting on a body and the body is in equilibrium. If the magnitude of the force F_1 is 100 N and its acting at O long x-axis as shown in Fig. 4.4, then determine the magnitude and direction of force F_2.*

Sol. Given :

Force, $F_1 = 100$ N

The body is in equilibrium under the action of two forces F_1 and F_2.

When two forces are acting on a body and the body is in equilibrium, then the two forces should be collinear, equal and opposite.

$\therefore$ $F_2 = F_1 = 100$ N

The force F_2 should pass through O, and would be acting in the opposite direction of F_1.

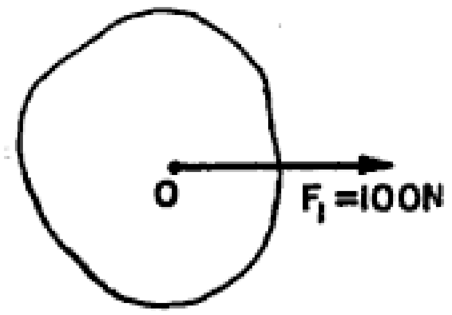


Fig. 4.4

Problem 4.2. *Three forces F_1, F_2 and F_3 are acting on a body as shown in Fig. 4.5 and the body is in equilibrium. If the magnitude of force F_3 is 400, N, find the magnitudes of force F_1 and F_2.*

Sol. Given :

Force, $F_3 = 400$ N.

As the body is in equilibrium, the resultant force is x-direction should be zero and also the resultant force in y-direction should be zero.

(i) For $\Sigma F_x = 0$, we get

$F_1 \cos 30° - F_2 \cos 30° = 0$

or $F_1 - F_2 = 0$

or $F_1 = F_2$ ...(i)

(ii) For $\Sigma F_y = 0$, we get

$F_1 \sin 30° + F_2 \sin 30° - 400 = 0$

or $F_1 \times 0.5 + F_2 \times 0.5 = 400$

or $F_1 \times 0.5 + F_2 \times 0.5 = 400$ $(\because\ F_2 = F_1)$

or $F_1 =$ **400 N. Ans.**

Also $F_2 = F_1 =$ **400 N. Ans.**

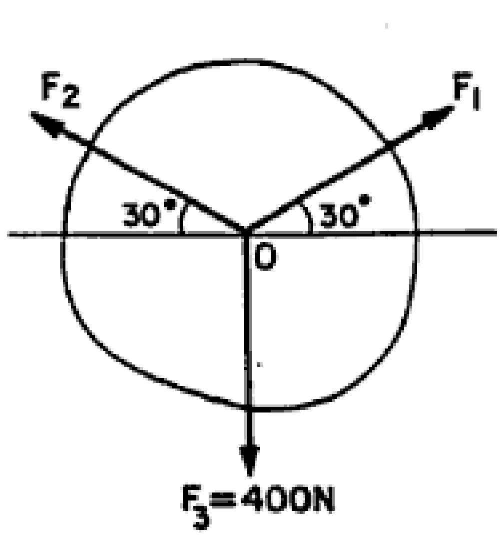


Fig. 4.5

2nd Method

If three forces are acting on a body at a point and the body is in equilibrium, Lami's Theorem can be applied.

Using Lami's theorem,

$$\frac{F_1}{\sin 120°} = \frac{F_2}{\sin 120°} = \frac{400}{\sin 120°}$$

or $$F_1 = F_2 = 400 \text{ N.} \quad \textbf{Ans.}$$

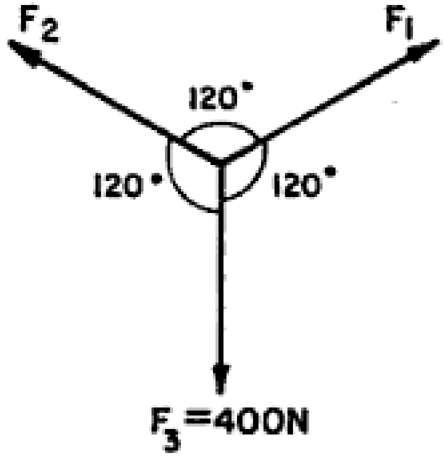


Fig. 4.5 (*a*)

Problem 4.3. *Three parallel forces F_1, F_2 and F_3 are acting on a body as shown in Fig. 4.6 and the body is in equilibrium. If force F_1 = 250 N and F_3 = 1000 N and the distance between F_1 and F_2 = 1.0 m, then determine the magnitude of force F_2 and the distance of F_2 from force F_3.*

Sol. Given :

Force, $F_1 = 250$ N

Force, $F_3 = 1000$ N

Distance $AB = 1.0$ m

The body is in equilibrium.

Find F_2 and distance BC.

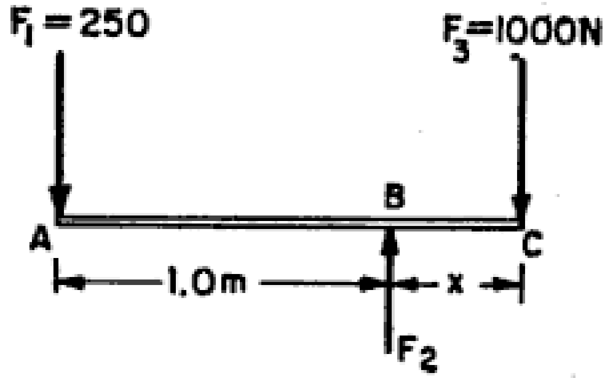


Fig. 4.6

For the equilibrium of the body, the resultant force in the vertical direction should be zero (here there is no force in horizontal direction).

∴ For $\Sigma F_y = 0$, we get

$$F_1 + F_3 - F_2 = 0$$

or $$250 + 1000 - F_2 = 0$$

or $$F_2 = 250 + 1000 = 1250 \text{ N.} \quad \textbf{Ans.}$$

For the equilibrium of the body, the moment of all forces about any point must be zero.

Taking moments of all forces about point A and considering distance $BC = x$, we get

$$F_2 \times AB - AC \times F_3 = 0$$

or $$1250 \times 1.0 - (1 + x) \times 1000 = 0 \qquad (\because \; AC = AB + BC = 1 + x)$$

or $$1250 - 1000 - 1000x = 0$$

or $$250 = 1000x$$

or $$x = \frac{250}{1000} = 0.25 \text{ m.} \quad \textbf{Ans.}$$

Problem 4.4. *The five forces F_1, F_2, F_3, F_4 and F_5 are acting at a point on a body as shown in Fig. 4.7 and the body is in equilibrium. If F_1 = 18 N, F_2 = 22.5 N, F_3 = 15 N and F_4 = 30 N, find the force F_5 in magnitude and direction.*

Sol. Given :

Forces, $F_1 = 18$ N, $F_2 = 2.25$ N,

$F_3 = 15$ N and $F_4 = 30$ N.

The body is in equilibrium. Find force F_5 in magnitude and direction. This problem can be solved analytically and graphically.

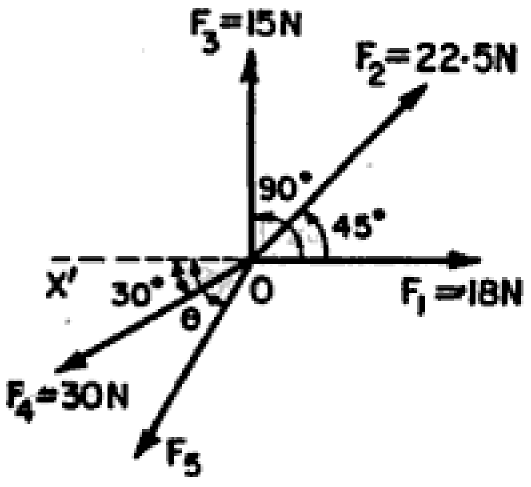


Fig.4.7

1. Analytical Method

Let θ = Angle made by force F_5 with horizontal axis O-X'.

As the body is in equilibrium, the resultant force in x-direction and y-direction should be zero.

(*i*) For $\Sigma F_x = 0$, we get

$$F_1 + F_2 \cos 45° - F_4 \cos 30° - F_5 \cos \theta = 0$$

or $$18 + 22.5 \times 0.707 - 30 \times 0.866 - F_5 \cos \theta = 0$$

or $$18 + 15.9 - 25.98 - F_5 \cos \theta = 0$$

or $$F_5 \cos \theta = 18 + 15.9 - 25.98$$

or $$F_5 \cos \theta = 7.92 \quad \text{...}(i)$$

(*ii*) For $\Sigma F_y = 0$, we get

$$F_2 \sin 45° + F_3 - F_4 \sin 30° - F_5 \sin \theta = 0$$

or $$22.5 \times 0.707 + 15 - 30 \times 0.5 - F_5 \sin \theta = 0$$

or $$15.9 + 15 - 15 - F_5 \sin \theta = 0$$

or $$F_5 \sin \theta = 15.9 \quad \text{...}(ii)$$

Dividing equation (*ii*) by equation (*i*), we get $\dfrac{F_5 \sin \theta}{F_5 \cos \theta} = \dfrac{15.9}{7.92}$ or $\tan \theta = 2.0075$

$$\therefore \quad \theta = \tan^{-1} 2.0075 = \mathbf{63.52°}. \quad \textbf{Ans.}$$

Substituting the value of θ in equation (*i*), we get

$$F_5 \cos 63.52° = 7.92$$

$$\therefore \quad F_5 = \frac{7.92}{\cos 63.52} = \mathbf{17.76\ N}. \quad \textbf{Ans.}$$

2. Graphical Method

(*i*) First draw a space diagram with given four forces F_1, F_2, F_3 and F_4 at correct angles as shown in Fig. 4.8 (*a*).

(*ii*) Now choose a suitable scale, say 1 cm = 5 N for drawing a force diagram. Take any point O in the force diagram as shown in Fig. 4.8 (*b*).

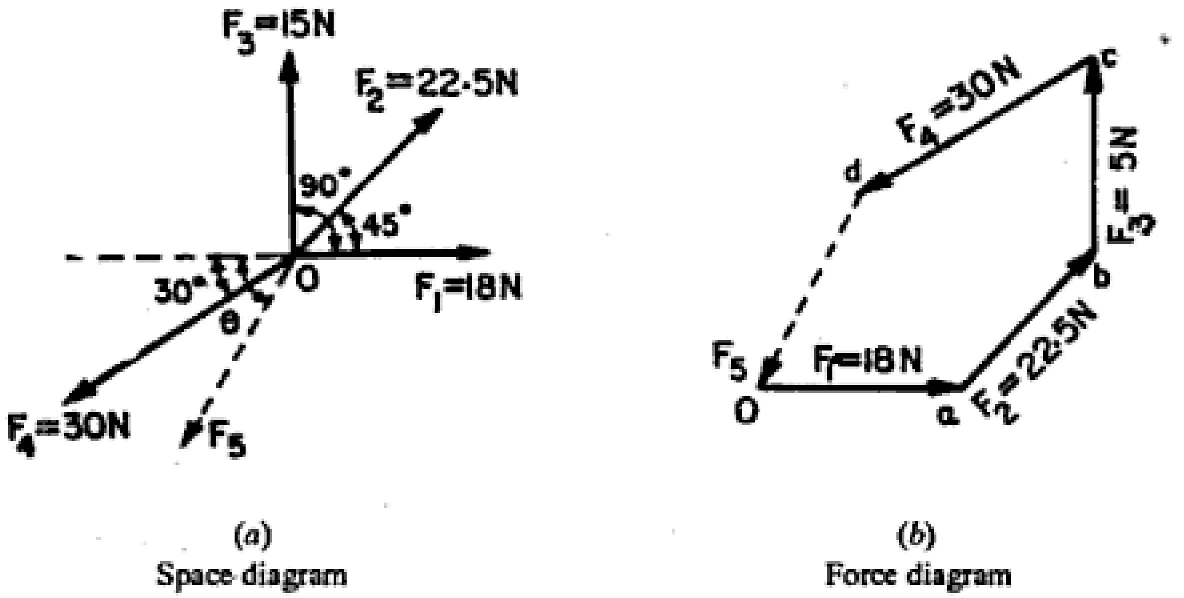


Fig. 4.8

(*iii*) Draw line Oa parallel to force F_1 and cut $Oa = F_1 = 18$ N to the same scale.

(*iv*) From a, draw the line ab parallel to F_2 and cut $ab = F_2 = 22.5$ N

(*v*) From b, draw the line bc parallel to F_3 and cut $bc = F_3 = 15$ N

(*vi*) From c, draw the line cd parallel to F_4 and cut $cd = F_4 = 30$ N

(*vii*) Now join d to O. Then the closing side dO represents the force F_5 in magnitude and direction. Now measure the length dO.

By measurement, length $dO = 3.55$ cm.

$\therefore$ Force F_5 = Length dO × Scale = 3.55 × 5 = **17.75 N. Ans.**

The direction is obtained in the space diagram by drawing the force F_5 parallel to line dO.

Measure the angle θ, which is equal to 63.5°. Or the force F_5 is making an angle of 180 + 63.5 = 243.5° with the force F_1.

Problem 4.5. *Fig. 4.8 (c) shows the coplanar system of forces acting on a flat plate. Determine : (i) the resultant and (ii) x and y intercepts of the resultant.*

Sol. Given :

Force at $A = 2240$ N.

Angle with x-axis = 63.43°

Force at $B = 1805$ N.

Angle with x-axis = 33.67°

Force at $C = 1500$ N.

Angle with x-axis = 60°

Lengths $OA = 4$ m,

$DB = 3$ m,

$DC = 2$ m

and $OD = 3$ m

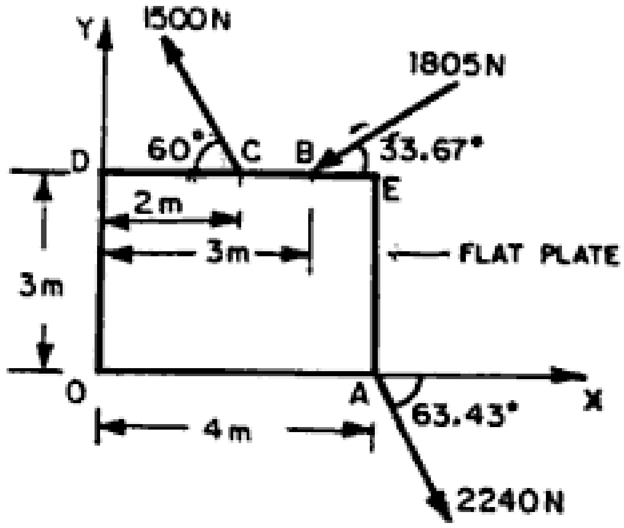


Fig. 4.8 (*c*)

Each force is resolved into X and Y components as shown in Fig. 4.8 (*d*).

(*i*) Force at $A = 2240$ N.

Its X-component = 2240 × cos 63.43° = 1001.9 N,

Its Y-component = 2240 × sin 63.43° = 2003.4 N

(*ii*) Force at $B = 1805$ N.

X-component = 1805 × cos 33.67° = 1502.2 N

Y-component = 1805 × sin 33.67° = 1000.7 N

(*iii*) Force at $C = 1500$ N.

X-component = 1500 × cos 60° = 750 N

Y-component = 1500 × sin 60° = 1299 N

The net force along X-axis,

$$R_x = \Sigma F_x = 1001.9 - 1502.2 - 750 = -1250.3 \text{ N}$$

The net force along Y-axis,

$$R_y = \Sigma F_y = -2003.4 - 1000.7 + 1299 = -1705.1 \text{ N}$$

(*i*) *The resultant force is given by,*

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-1250.3)^2 + (-1705.1)^2}$$

$$= \sqrt{1563250 + 2907366} = \mathbf{2114.4\ N.}\quad \textbf{Ans.}$$

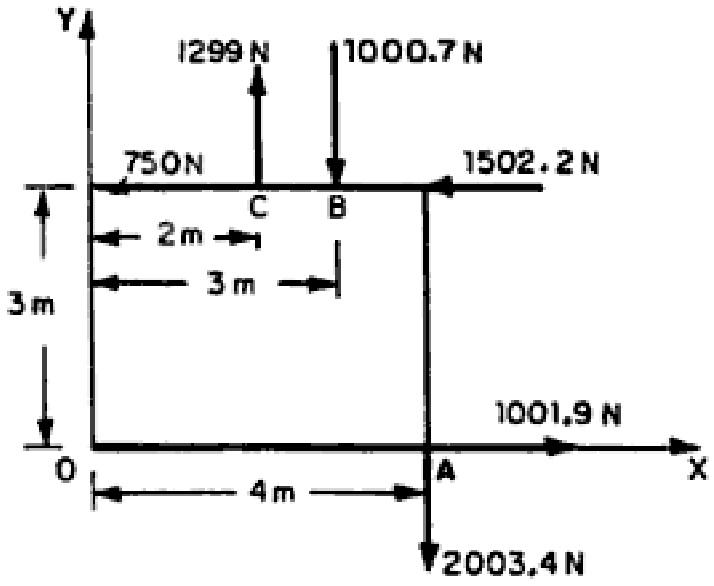


Fig. 4.8 (*d*)

The angle made by the resultant with x-axis is given by

$$\tan\theta = \frac{R_y}{R_x} = \frac{-1705.1}{-1250.3} = 1.363$$

$\therefore$ $\theta = \tan^{-1} 1.363 = 53.70$

The net moment* about point O,

$$M_0 = 2003.4 \times 4 + 1000.7 \times 3 - 1299 \times 2 - 1502.2 \times 3 - 750 \times 3$$
$$= 8012.16 + 3002.1 - 2598 - 4506.6 - 2250$$
$$= 11014.26 - 9354.6 = 1659.55 \text{ Nm}$$

As the net moment about O is clockwise, hence the resultant must act towards right of origin O, making an angle = 53.7° with x-axis as shown in Fig. 4.8 (*e*). The components R_x and R_y are also negative. Hence this condition is also satisfied.

(*ii*) *Intercepts of resultant on x-axis and y-axis* [Refer to Fig. 4.8 (*e*)].

Let x = Intercept of resultant along x-axis.

y = Intercept of resultant along y-axis.

The moment of a force about a point is equal to the sum of the moments of the components of the force about the same point. Resolving the resultant (R) into its component R_x and R_y at F.

Moment of R about O = Sum of moments of R_x and R_y at O

But moment of R about O = 1659.66

(M_0 = 1659.66)

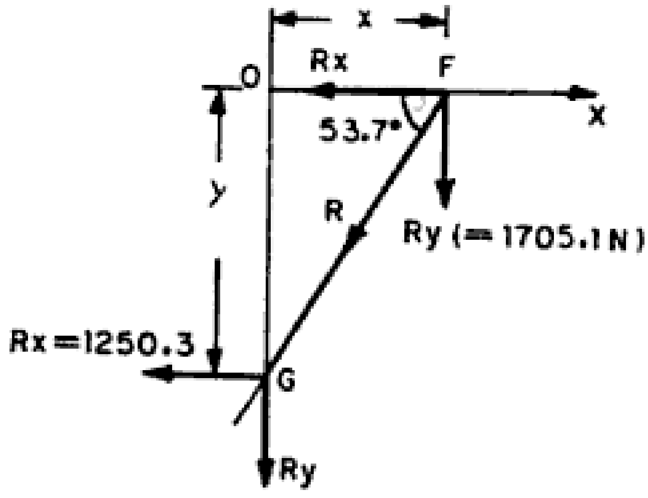


Fig. 4.8 (*e*)

$\therefore$ $1659.66 = R_x \times O + R_y \times x$

(as R_x at F passes through O hence it has no moment)

*Considering clockwise moment positive and anti-clockwise moment as negative. At A, the X component of 1001.9 N passes through O and hence has no moment.

$\therefore \quad 1659.66 = 1705.1 \times x \qquad (\because R_y = 1705.1)$

$\therefore \quad x = \frac{1659.66}{1705.10} = 0.97$ m right of O. **Ans**

To find y-intercept, resolve the resultant R at G into its component R_x and R_y.

$\therefore$ Moment of R about O = Sum of moments of R_x and R_y at O

or $\quad 1659.66 = R_x \times y + R_y \times O.$

(At G, R_y passes through O and hence has no moment)

$\therefore \quad 1659.66 = 1250.3 \times y$

$\therefore \quad y = \frac{1659.66}{1250.30} = 1.32$ m below O. **Ans.**

Problem 4.6. *A lamp weighing 5 N is suspended from the ceiling by a chain. It is pulled aside by a horizontal cord until the chain makes an angle of 60° with the ceiling as shown in Fig. 4.9. Find the tensions in the chain and the cord by applying Lami's theorem and also by graphical method.*

Sol. Given :

Weight of lamp = 5 N

Angle made by chain with ceiling = 60°

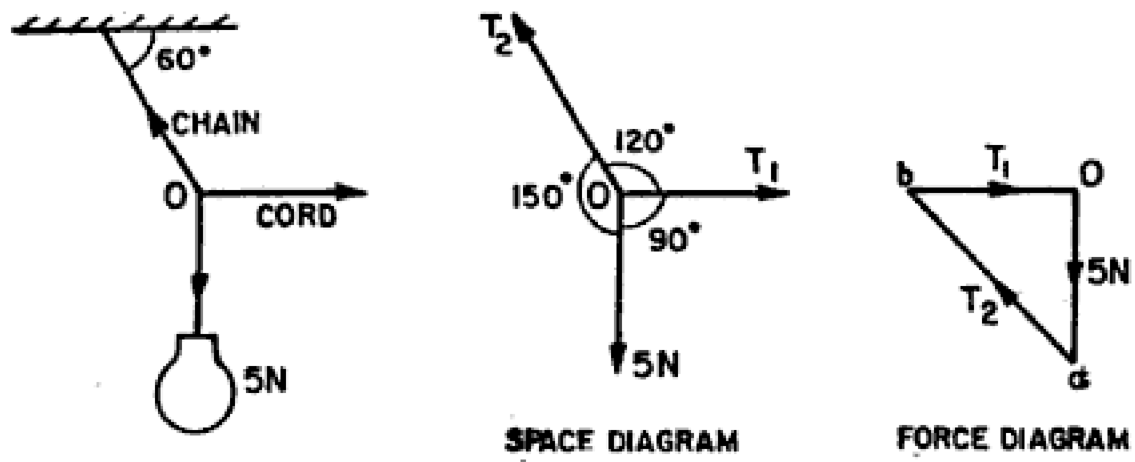
Cord is horizontal as shown in Fig. 4.9.

(*i*) *By Lami's Theorem*

Let $\quad T_1$ = Tension (or pull) in the cord

T_2 = Tension (or pull) in the chain.

Now from the geometry, it is obvious that angles between T_1 and lamp will be 90°, between lamp and T_2 150° and between T_2 and T_1 120° [Refer to Fig. 4.9 (*b*)].



(*a*) (*b*) (*c*)

Fig. 4.9

Applying Lami's theorem, we get

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 90°} = \frac{5}{\sin 120°}$$

$\therefore \quad T_1 = 5 \times \frac{\sin 150°}{\sin 120°} = $ **2.887 N. Ans.**

and $\quad T_2 = 5 \times \frac{\sin 90°}{\sin 120°} = $ **5.774 N. Ans.**

(*ii*) *By Graphical Method*

(1) First draw the space diagram at correct angles as shown in Fig. 4.9 (*b*). Now choose a suitable scale say 1 cm = 1 N for drawing a force diagram as shown in Fig. 4.9 (*c*). Take any point *O* in the force diagram.

(2) From *O*, draw the line *Oa* vertically downward to represent the weight of the lamp. Cut *Oa* = 5 N.

(3) From *a*, draw the line *ab* parallel to T_2. The magnitude of T_2 is unknown. Now from *O*, draw the line *Ob* horizontally (*i.e.*, parallel to T_1) cutting the line *ab* at point *b*.

(4) Now measure the lengths *ab* and *bO*.

Then *ab* represents T_2 and *bO* represents T_1. By measurements, *ab* = 5.77 cm and *bO* = 2.9 cm.

∴ Pull in the cord = *bO* = 2.9 cm × scale = 2.9 × 1

= **2.9 N. Ans.**

Pull in the chain = *ab* = 5.77 cm × scale = 5.77 × 1

= **5.77 N. Ans.**

Problem 4.7. *On a horizontal line PQRS 12 cm long, where PQ = QR = RS = 4 cm, forces of 1000, 1500, 1000 and 500 N are acting at P, Q, R and S respectively, all downwards, their lines of action making angles of 90, 60, 45 and 30 degrees respectively with PS. Obtain the resultant of the system completely in magnitude, direction and position graphically and check the answer analytically.*

Sol. Given :

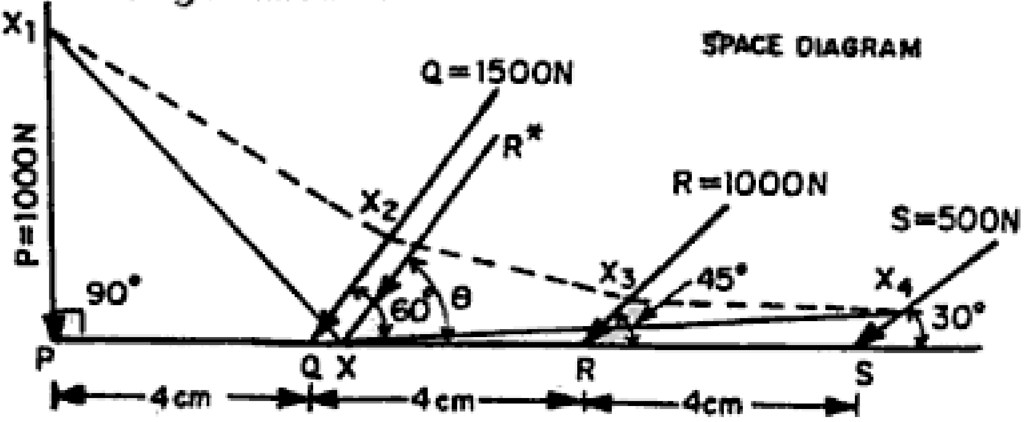
$PQ = QR = RS = 4$ cm

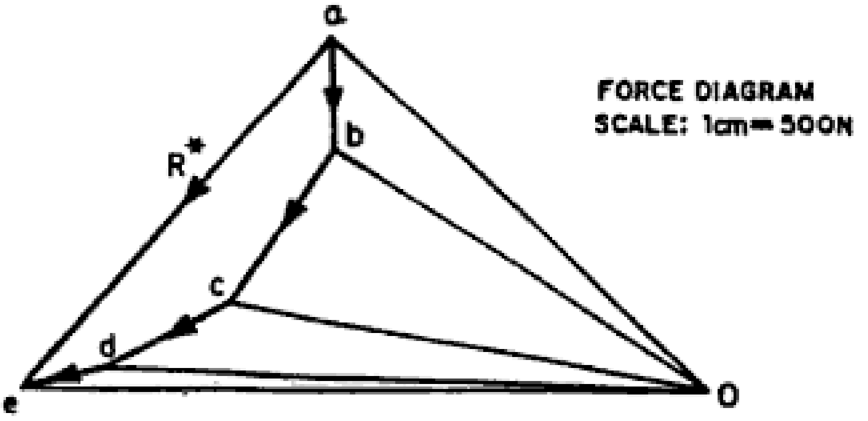
Force at P = 1000 N. Angle with $PS = 90°$

Force at Q = 1500 N. Angle with $QS = 60°$

Force at R = 1000 N. Angle with $RS = 45°$

Force at S = 500 N Angle with $PS = 30°$





(*c*)

Fig. 4.9

Graphical Method

Draw the space diagram of the forces as shown in Fig. 4.9 (*d*). The procedure is as follows :

(*i*) Draw a horizontal line *PQRS* = 12 cm in which take *PQ* = *QR* = *RS* = 4 cm

(*ii*) Draw the line of action of forces *P, Q, R, S* of magnitude 1000 N, 1500 N, 1000 N and 500 N respectively at an angle of 90°, 60°, 45° and 30° respectively with line *PS* as shown in Fig. 4.9.

Magnitude and direction of Resultant force (R*)

To find the magnitude and direction of the resultant force, the force diagram is drawn as shown in Fig. 4.9 (*e*) as given below :

(*i*) Draw the vector *ab* to represent the force 1000 N to a scale of 1 cm = 500 N. The vector *ab* is parallel to the line of action of force *P*.

(*ii*) From point *b*, draw vector *bc* = 1500 N and parallel to the line of action of force *Q*. Similarly the vectors, *cd* = 1000 N and parallel to line of action of force *R* and *de* = 500 N and parallel to the line of action of force *S*, are drawn.

(*iii*) Join *ae* which gives the magnitude of the resultant. Measuring *ae*, the resultant force is equal to 3770 N.

(*iv*) To get the line of action of the resultant, choose any point *O* on force diagram (called the pole) and join *Oa, Ob, Oc, Od* and *Oe*.

(*v*) Now choose any point X_1 on the line of action of force *P* and draw a line parallel to *Oa*.

(*vi*) Also from the point X_1, draw another line parallel to *Ob*, which cuts the line of action of force *Q* at X_2. Similarly from point X_2, draw a line parallel to *Oc* to cut the line of action of force *R* at X_3. From point X_3, draw a line parallel to *Od* to cut the line of action of force *S* at X_4.

(*vii*) From point X_4, draw a line parallel *Oe*.

(*viii*) Produce the first line (*i.e.*, the line from X_1 and parallel to *Oa*) and the last line (*i.e.*, the line from X_4 and parallel to *Oe*) to interest at *X*. Then the resultant must pass through this point.

(*ix*) From point *X*, draw a line parallel to *ae* which determines the line of action of resultant force. Measure *PX*. By measurements :

Resultant force, R^* = 3770 N

Point of action, *PX* = 4.20 cm

Direction, $\theta = 60° \ 30'$ with *PS*

Analytical Method

In analytical method, all the forces acting can be resolved horizontally and vertically. Resultant of all vertical and horizontal forces can be calculated separately and then the final resultant can be obtained.

Resolving all forces and considering the system for vertical forces only.

Vertical force at $P = 1000$ N

Vertical force at $Q = 1500 \sin 60° = 1299$ N

Vertical force at $R = 1000 \sin 45° = 707$ N

Vertical force at $S = 500 \sin 30° = 250$ N

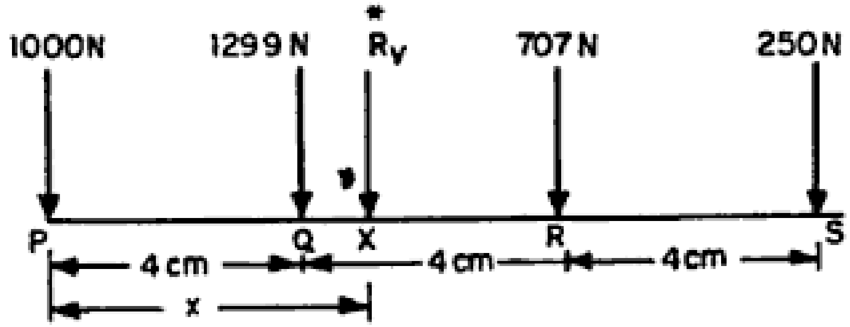


Fig. 4.9 (*f*)

Let R_V^* = the resultant of all vertical forces and acting at a distance x cm from P.

$$= 1000 + 1299 + 707 + 250 = 3256 \text{ N}$$

Taking moments of all vertical forces about point P,

$$R_V^* \times x = 1299 \times 4 + 707 \times 8 + 250 \times 12 = 13852$$

$$\therefore \qquad x = \frac{13852}{R_V^*} = \frac{13852}{3256} = 4.25 \text{ cm}$$

Now consider the system for horizontal forces only Horizontal force at $P = 0$

Horizontal force at $Q = 1500 \times \cos 60° = 750$ N

Horizontal force at $R = 1000 \times \cos 45° = 707$ N

Horizontal force at $S = 500 \times \cos 30° = 433$ N

Resultant of all horizontal forces will be,

$$R_H^* = 0 + 750 + 707 + 433 = 1890 \text{ N}$$

The resultant R^* of R_V^* and R_H^* will also pass through point X which is at a distance of 4.25 cm from P.

$$\therefore \qquad R^* = \sqrt{R_V^{*2} + R_H^{*2}} = \sqrt{3256^2 + 1890^2} = 3764 \text{ N.} \quad \textbf{Ans.}$$

The resultant will make an angle θ with PS and is given by

$$\tan\theta = \frac{R_V^*}{R_H^*} = \frac{3256}{1890} = 1.723$$

$$\therefore \qquad \theta = \tan^{-1} 1.723 = 59.9°$$

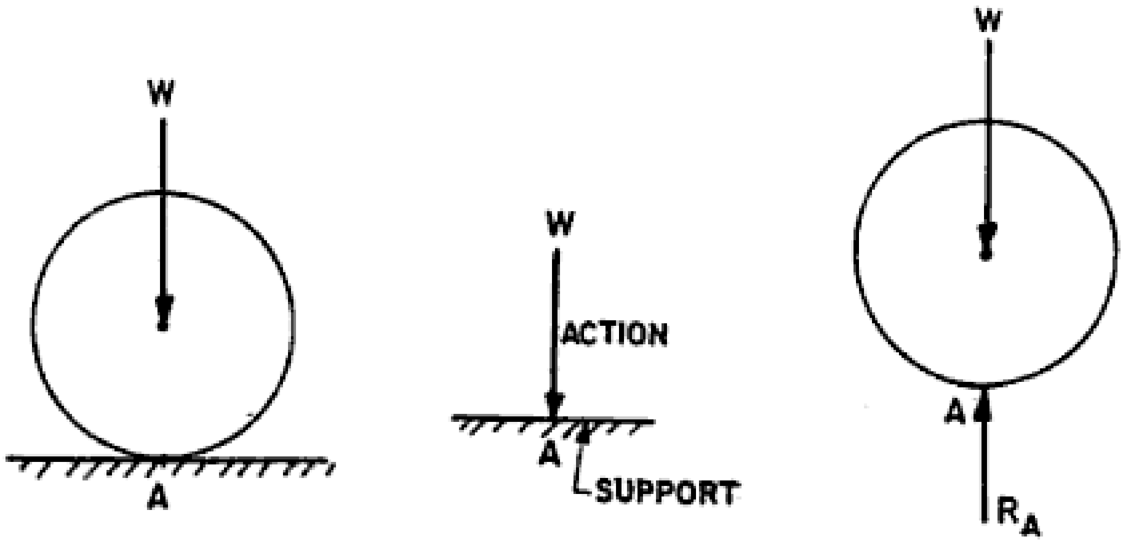
Thus the resultant of 3764 N makes an angle 59.9° with PS and passing through point X which is at a distance of 4.25 cm from point P.

This result confirms closely with the values obtained by graphical method.

4.4. ACTION AND REACTION

From the Newton's third law of motion, we know that to every action there is equal and opposite reaction. Hence reaction if always equal and opposite to the action.

Fig. 4.10 (*a*) shows a ball placed on a horizontal surface (or horizontal plane) such that it is free to move along the plane but cannot move vertically downward. Hence the ball will exert a force vertically



(*a*) (*b*) (*c*)

Fig. 4.10

downwards at the support as shown in Fig. 4.10 (*b*). This force is known as *action*. The support will exert an equal force vertically upwards on the ball at the point of contact as shown in Fig. 4.10 (*c*).

The force, exerted by the support on the ball, is known as *reaction*. Hence '*any force on a support causes an equal and opposite force from the support so that action and reaction are two equal and opposite forces*'.

4.5. FREE BODY DIAGRAM

The equilibrium of the bodies which are placed on the supports can be considered if we remove the supports and replace them by the reactions which they exert on the body. In Fig. 4.10 (*a*), if we remove the supporting surface and replace it by the reaction R_A that the surface exerts on the balls as shown in Fig. 4.10 (*c*), we shall get *free-body diagram.*

The point of application of the reaction R_A will be the point of contact A, and from the law of equilibrium of two forces, we conclude that the reaction R_A must be vertical and equal to the weight W.

Hence Fig. 4.10 (*c*), in which the ball is *completely isolated* from its support and in which all forces acting on the ball are shown by vectors, is known a free-body diagram. Hence to draw the free-body diagram of a body we remove all the supports (like wall, floor, hinge or any other body) and replace them by the reactions which these support exert on the body. Also the body should be completely isolated.

Problem 4.8. *Draw the free body diagram of ball of weight W supported by a string AB and resting against a smooth vertical wall at C as shown in Fig. 4.11 (a).*

Sol. Given :

Weight of ball = W

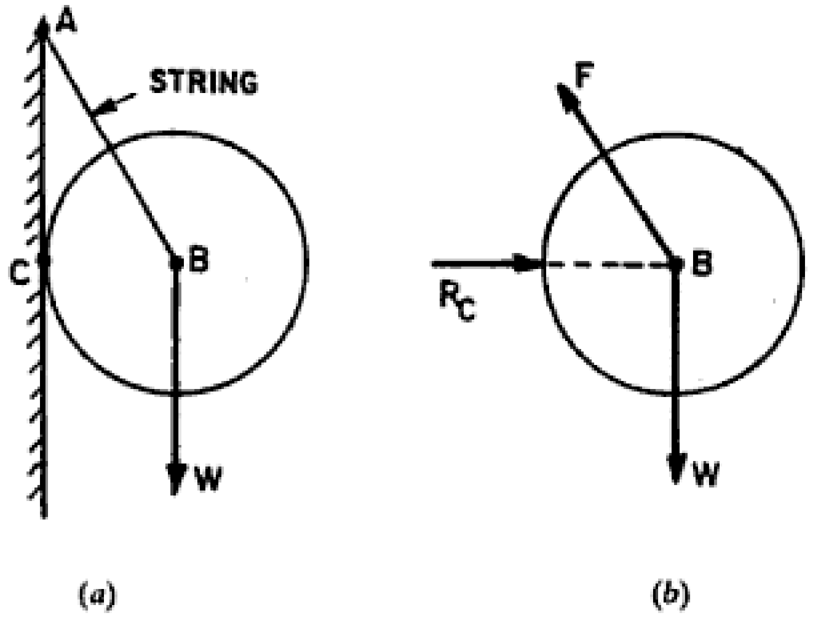


Fig. 4.11

The ball is supported by a string AB and is resting against a vertical wall at C.

To draw the free-body diagram of the ball, isolate the ball completely (*i.e.*, isolate the ball from the support and string). Then besides the weight W acting at B, we have two reactive forces to apply one replacing the string AB and another replacing the vertical wall AC. Since the string is attached to the ball at B and since a string can pull only along its length, we have the reactive force F applied at B and parallel to BA. The magnitude of F is unknown.

The reaction R_C will be acting at the point of contact of the ball with vertical wall *i.e.*, at point C. As the surface of the wall is *perfectly smooth**, the reaction R_C will be normal to the vertical wall (*i.e.*, reaction R_C will be horizontal in this case) and will pass through the point B. The magnitude of R_C is also unknown. The complete free-body diagram is shown in Fig. 4.11 (*b*).

*The reaction at a perfectly smooth surface is always normal to the surface.

Problem 4.9. *A circular roller of weight 100 N and radius 10 cm hangs by a tie rod AB = 20 cm and rests against a smooth vertical wall at C as shown in Fig. 4.12 (a). Determine : (i) the force F in the tie rod, and (ii) the reaction R_C at point C.*

Sol. Given :

Weight of roller, $W = 100$ N

Radius of roller, $BC = 10$ cm

Length of tie rod, $AB = 20$ cm

From ΔABC, we get $\sin\theta = \frac{BC}{AB} = \frac{10}{20} = 0.5$

$\therefore \quad \theta = \sin^{-1} 0.5 = 30°$

The free-body diagram of the roller is shown in Fig. 4.12 (*b*) in which

R_C = Reaction at C

F = Force in the tie rod AB

Free-body diagram shows the equilibrium of the roller. Hence the resultant force in x-direction and y-direction should be zero.

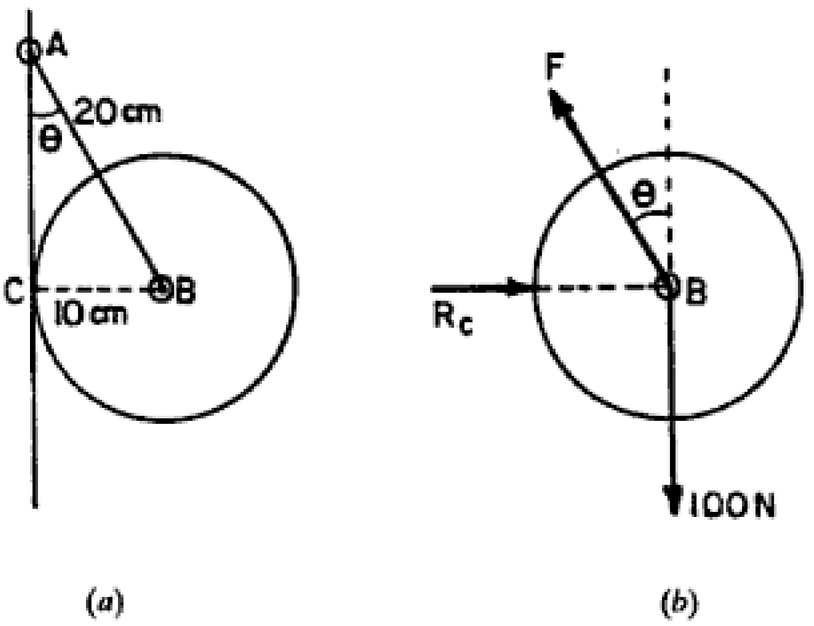


Fig. 4.12

For $\Sigma F_x = 0$, we get $R_C - F\sin\theta = 0$

or $R_C = F\sin\theta$ ...(*i*)

For $\Sigma F_y = 0$, we get $100 - F\cos\theta = 0$

or $100 = F\cos\theta$

or $F = \frac{100}{\cos\theta} = \frac{100}{\cos 30°}$ $(\because \theta = 30°)$

= 115.47 N. Ans.

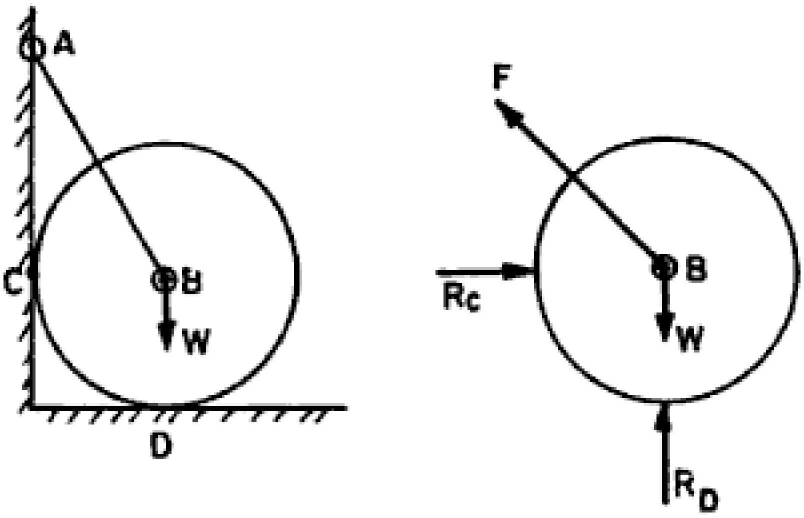
Substituting the value of F in equation (*i*),

$R_C = 115.47 \times \sin 30° =$ **57.73 N. Ans.**

Problem 4.10. *Draw the free-body diagram of a ball of weight W, supported by a string AB and resting against a smooth vertical wall at C and also resting against a smooth horizontal floor at D as shown in Fig. 4.13 (a).*

Sol. Given :

To draw the free-body diagram of the ball, the ball should be isolated completely from the vertical support, horizontal support and string AB. Then the forces acting on the isolated ball as shown in Fig. 4.13 (*b*), will be :



(*a*) (*b*)

Fig. 4.13

(*i*) Reaction R_C at point C, normal to AC.

(*ii*) Force F in the direction of string.

(*iii*) Weight W of the ball.

(*iv*) Reaction R_D at point D, normal to horizontal surface.

The reactions R_C and R_D will pass through the centre of the ball *i.e.*, through point B.

Problem 4.11. *A ball of weight 120 N rests in a right-angled groove, as shown in Fig. 4.14 (a). The sides of the groove are inclined to an angle of 30° and 60° to the horizontal. If all the surfaces are smooth, then determine the reactions R_A and R_C at the points of contact.*

Sol. Given :

Weight of ball, $W = 120$ N

Angle of groove $= 90°$

Angle made by side FD with horizontal = 30°

Angle made by side ED with horizontal = 60°

$\therefore$ Angle $FDH = 30°$ and angle $EDG = 60°$

Consider the equilibrium of the ball. For this draw the free body diagram of the ball as shown in Fig. 4.14 (*b*).

The forces acting on the isolated ball will be :

(*i*) Weight of the ball = 120 N and acting vertically downwards.

(*ii*) Reaction R_C acting at C and normal to FD.

(*iii*) Reaction R_A acting at A and normal to DE.

The reactions R_A and R_C will pass through B, *i.e.*, centre of the ball. The angles made by R_A and R_C at point B will be obtained as shown in Fig. 4.14 (*c*).

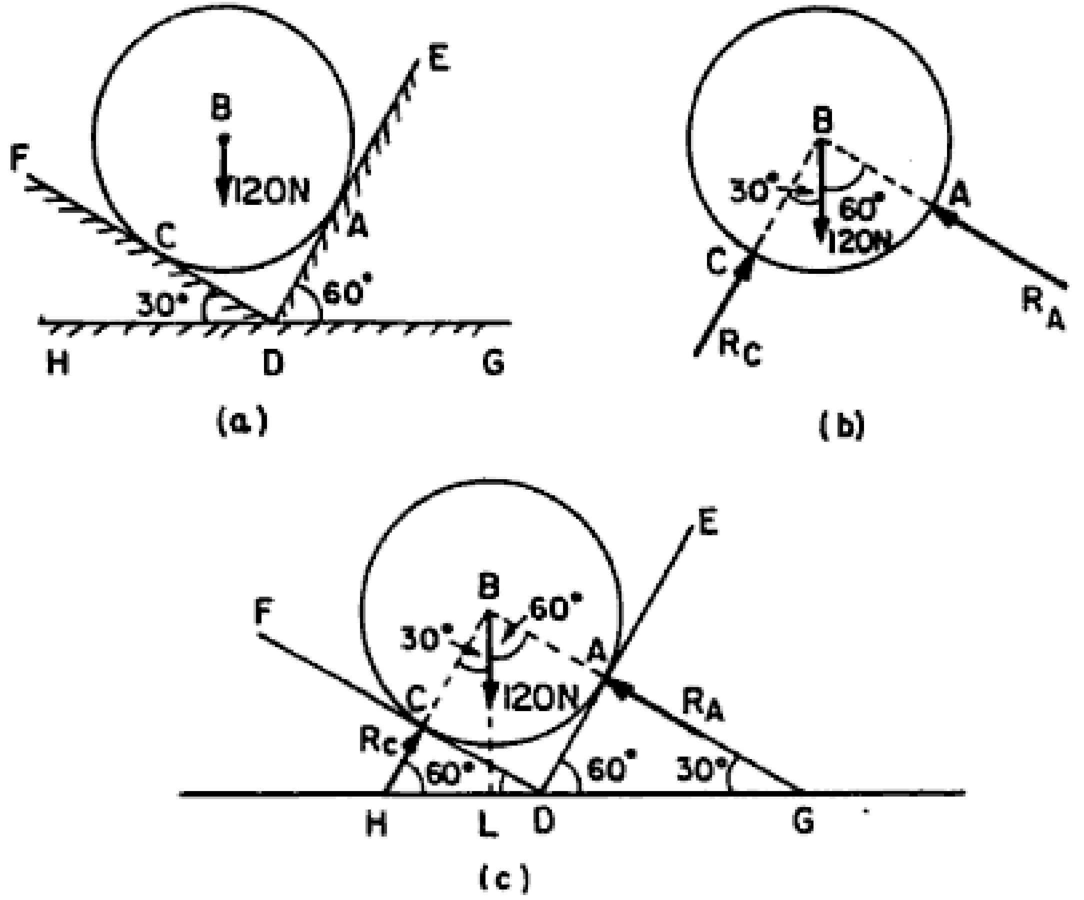


Fig. 4.14

In ΔHDC, $\angle CDH = 30°$ and $\angle DCH = 90°$. Hence $\angle DHC$ will be 60°. Now in ΔHBL, $\angle BLH = 90°$ and angle $LHB = 60°$. Hence $\angle HBL$ will be 30°.

Similarly, $\angle GBL$ may be calculated. This will be equal to 60°.

For the equilibrium of the ball,

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

For $\Sigma F_x = 0$, we have $R_C \sin 30° - R_A \sin 60° = 0$

or $$R_C \sin 30° = R_A \sin 60°$$

or $$R_C = R_A \times \frac{0.866}{\sin 30°} = 1.732\, R_A \quad \text{...}(i)$$

For $\Sigma F_y = 0$, we have $120 - R_A \cos 60° - R_C \cos 30° = 0$

or $$120 = R_A \cos 60° + R_C \cos 30°$$

$$= R_A \times 0.5 + (1.732\, R_A) \times 0.866 \quad (\because \; R_C = 1.732\, R_A)$$

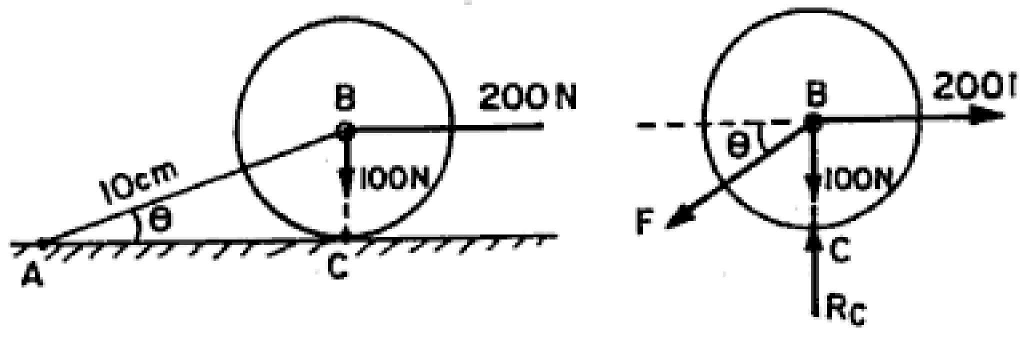
$$= 0.5\, R_A + 1.5\, R_A = 2R_A$$

$\therefore$ $$R_A = \frac{120}{2} = \mathbf{60\ N.} \quad \textbf{Ans.}$$

Substituting this value in equation (*i*), we get

$$R_C = 1.732 \times 60 = \mathbf{103.92\ N.} \quad \textbf{Ans.}$$

Problem 4.12. *A circular roller of radius 5 cm and of weight 100 N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 10 cm as shown in Fig. 4.15. A horizontal force of 200 N is acting at B. Find the tension (or Force) in the bar AB and the vertical reaction at C.*



(a) (b)

Fig. 4.15

Sol. Given :

Weight, $W = 100$ N

Radius *i.e.*, $BC = 5$ cm

Length of bar, $AB = 10$ cm

Horizontal force at $B = 200$ N

In ΔABC, $\sin\theta = \frac{BC}{AB} = \frac{5}{10} = 0.5$

$\therefore$ $\theta = \sin^{-1} 0.5 = 30°$

Let F = Tension in the string AB.

Consider the equilibrium of the roller. For this draw the free body diagram of the roller as shown in Fig. 4.15 (*b*).

The reaction R_C at point C will pass through point B.

The tension (or force F) will be acting along the length of the string.

As the roller is in equilibrium in Fig. 4.15 (*b*), the resultant force in x-direction and y-direction should be zero.

For $\Sigma F_x = 0$, we have $F\cos\theta - 200 = 0$

$\therefore$ $F = \frac{200}{\cos\theta} = \frac{200}{\cos 30°}$ $(\because \theta = 30°)$

$= 230.94$ **N. Ans.**

For $\Sigma F_y = 0$, we have $R_C - W - F\sin\theta = 0$

or $R_C = W + F\sin\theta = 100 + 230.94 \times \sin 30$

$= 215.47$ **N. Ans.**

Problem 4.13. *Two identical rollers P and Q, each of weight W, are supported by an inclined plane and a vertical wall as shown in Fig. 4.16 (a). Assume all the surfaces to be smooth. Draw the free body diagrams of :*

(*i*) *roller Q*, (*ii*) *roller P and* (*iii*) *rollers P and Q taken together.*

Sol. Given :

Weight of each roller $= W$

Radius of each roller $= R$

Identical rollers means the radius of each roller is same.

Hence the line EF in Fig. 4.16 (*a*) will be parallel to surface AB.

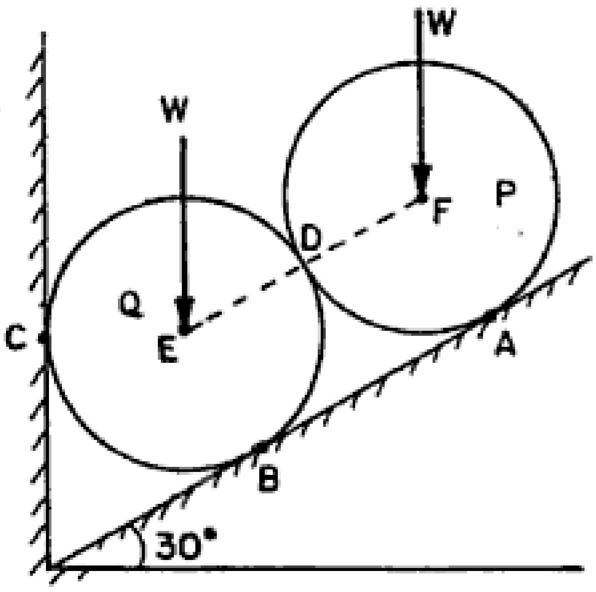


Fig. 4.16 (*a*)

Each surface is smooth, hence reaction at the point of contact will be normal to the surface.

Let R_A = Reaction at point A

R_B = Reaction at point B

R_C = Reaction at point C

The two rollers are also in contact at point D. Hence there will be a reaction R_D at the point D.

(*i*) **Free-body diagram of roller Q.** To draw the free-body diagram of roller Q, isolate the roller Q completely and find the forces acting on the roller Q. The roller Q has points of contact at B, C and D. The forces acting on the roller Q will be :

(*i*) Weight of roller W.

(*ii*) Reaction R_B at point B.

This will be normal to the surface BA at point B.

(*iii*) Reaction R_C at point C. This will be normal to the vertical surface at point C.

(*iv*) Reaction R_D at point D. This will be normal to the tangent at point D.

The reactions R_B, R_C and R_D will pass through the centre E of the roller Q. These three reactions are unknown.

(*ii*) **Free-body diagram of roller P.** Free-body diagram of roller P is shown in Fig. 4.16 (*c*). The roller P has points of contact at A and D. The forces acting on the roller P are :

(*i*) Weight W

(*ii*) Reaction R_A at point A

(*iii*) Reaction R_D at point D.

The reactions R_A and R_D will pass through point F, *i.e.*, centre of roller P. These two reactions are unknown. If W is given, then these reactions can be calculated.

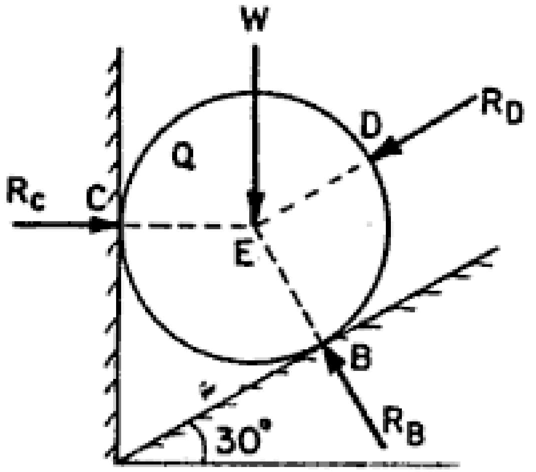


Fig. 4.16 (*b*)

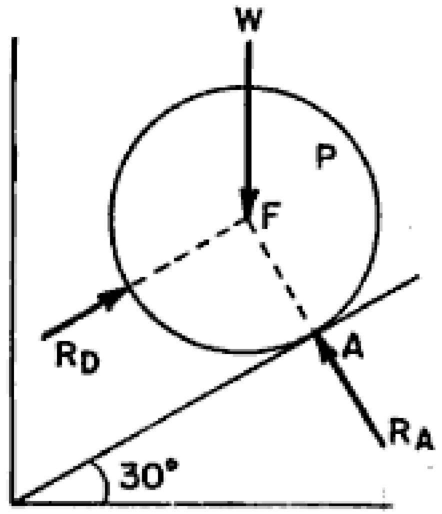


Fig. 4.16 (*c*)

(*iii*) **Free-body diagram of rollers P and Q taken together.** When the rollers *P* and *Q* are taken together, then points of contacts are *A*, *B* and *C*. The free-body diagram of this case is shown in Fig. 4.16 (*d*). The forces acting are :

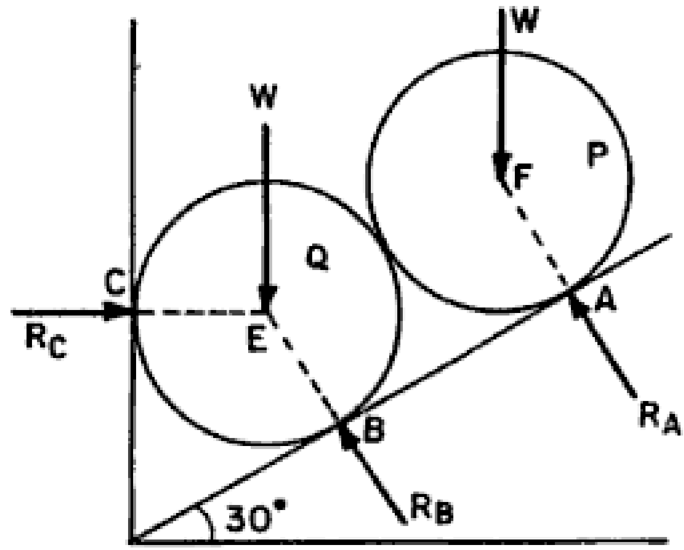


Fig. 4.16 (*d*)

(*i*) Weight *W* on each roller

(*ii*) Reaction R_A at point *A*

(*iii*) Reaction R_B at point *B*

(*iv*) Reaction R_C at point *C*.

In this case there will be no reaction at point *D*.

Problem 4.14. *Two identical rollers, each of weight W = 1000 N, are supported by an inclined plane and a vertical wall as shown in Fig. 4.17 (a). Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth.*

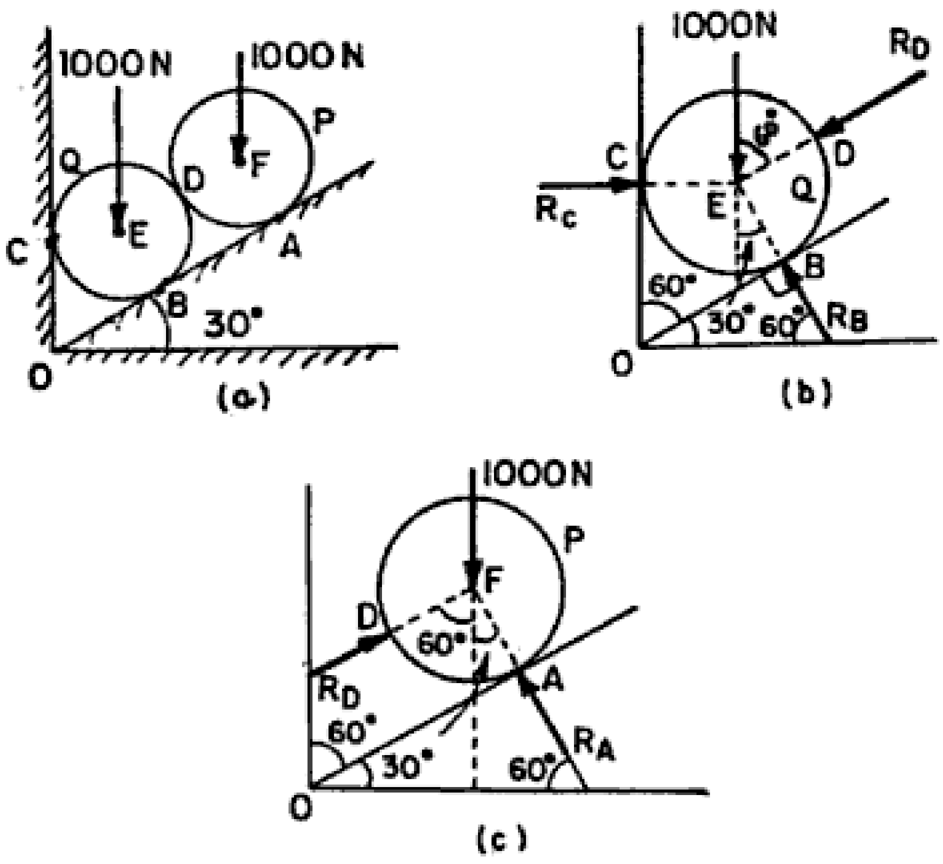


Fig. 4.17

Sol. Given :

Weight of each roller = 1000 N

Radius of each roller is same. Hence line *EF* will be parallel to *AB*.

Equilibrium of Roller P

First draw the free-body diagram of roller *P* as shown in Fig. 4.17 (*c*). The roller *P* has points of contact at *A* and *D*. Hence the forces acting on the roller *P* are :

(*i*) Weight 1000 N acting vertically downward.

(*ii*) Reaction R_A at point *A*. This is normal to *OA*.

(*iii*) Reaction R_D at point *D*. This is parallel to line *OA*.

The resultant force in *x* and *y* directions on roller *P* should be zero.

For $\Sigma F_x = 0$, we have

$$R_D \sin 60° - R_A \sin 30° = 0 \quad \text{or} \quad R_D \sin 60° = R_A \sin 30°$$

$$\therefore \qquad R_D = R_A \frac{\sin 30°}{\sin 60°} = 0.577\, R_A \qquad \text{...}(i)$$

For $\Sigma F_y = 0$, we have

$$R_D \cos 60° + R_A \cos 30° - 1000 = 0$$

$$(0.577\, R_A) \cos 60° + R_A \cos 30° = 1000 \qquad (\because \; R_D = 0.577\, R_A)$$

or $$0.577 \times 0.5\, R_A + R_A \times 0.866 = 1000$$

$$1.1545\, R_A = 1000 \quad \text{or} \quad R_A = \frac{1000}{1.1545} = \mathbf{866.17\ N.} \quad \textbf{Ans.}$$

Substituting this value in equation (*i*), we get

$$R_D = 0.577 \times 866.17 = 499.78$$

Equilibrium of Roller Q

The free-body diagram of roller *Q* is shown in Fig. 4.17 (*b*). The roller *Q* has points of contact at *B*, *C* and *D*.

The forces acting on the roller *Q* are :

(*i*) Weight $W = 1000$ N ;

(*ii*) Reaction R_B at point *B* and normal to *BO* ;

(*iii*) Reaction R_C at point *C* and normal to *CO* ; and

(*iv*) Reaction R_D at point *D* and parallel to *BO*.

For $\Sigma F_x = 0$, we have

$$R_B \sin 30° + R_D \sin 60° - R_C = 0$$

or $$R_B \times 0.5 + 449.78 \times 0.866 - R_C = 0$$

or $$R_C = 0.5\, R_B + 432.8 \qquad \text{...}(ii)$$

For $\Sigma F_y = 0$, we have

$$R_B \times \cos 30° - 1000 - R_D \times \cos 60° = 0$$

or $$R_B \times 0.866 - 1000 - 499.78 \times 0.5 = 0 \qquad (\because \; R_D = 499.78)$$

or $$0.866\, R_B - 1249.89 = 0 \quad \text{or} \quad R_B = \frac{1249.89}{0.866} = \mathbf{1443.3\ N.} \quad \textbf{Ans.}$$

Substituting this value in equation (*ii*), we get

$$R_C = 0.5 \times 1443.3 + 432.8 = \mathbf{1154.45\ N.} \quad \textbf{Ans.}$$

Problem 4.15. *Two spheres, each of weight 1000 N and of radius 25 cm rest in a horizontal channel of width 90 cm as shown in Fig. 4.18. Find the reactions on the points of contact A, B and C.*

Sol. Given :

Weight of each sphere, $W = 1000$ N

Radius of each sphere, $R = 25$ cm

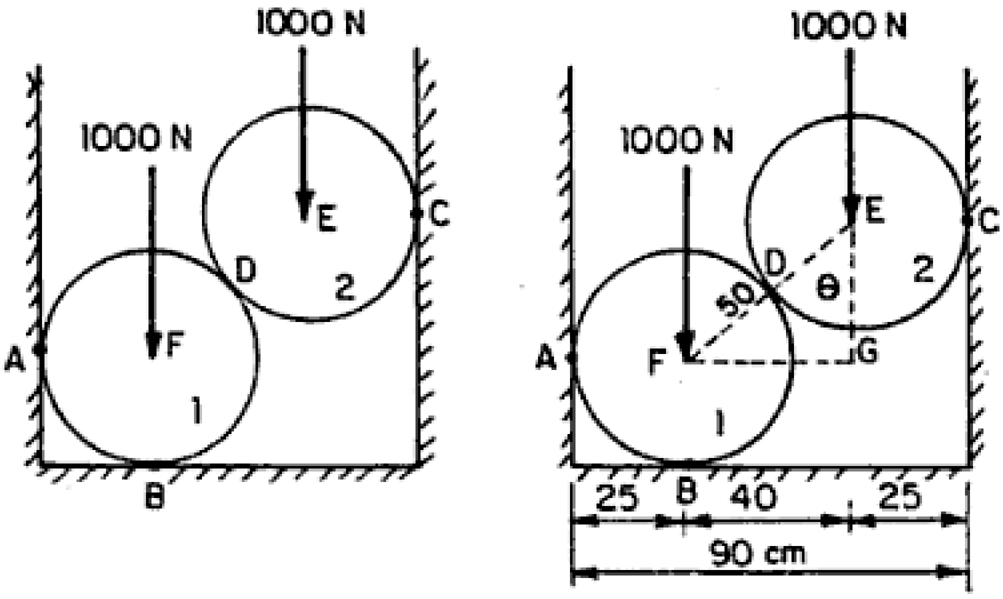
$\therefore$ $AF = BF = FD = DE = CE = 25$ cm

Width of horizontal channel = 90 cm

Join the centre E to centre F as shown in Fig. 4.18 (b).

Now $EF = 25 + 25 = 50$ cm, $FG = 40$ cm

In $\Delta EFG, EG = \sqrt{EF^2 - FG^2} = \sqrt{50^2 - 40^2} = \sqrt{2500 - 1600} = 30$



(a) (b)

Fig. 4.18

$$\therefore \quad \cos\theta = \frac{EG}{EF} = \frac{30}{50} = \frac{3}{5} \quad \text{and} \quad \sin\theta = \frac{FG}{EF} = \frac{40}{50} = \frac{4}{5}.$$

Equilibrium of Sphere No. 2

The sphere 2 has points of contact at C and D.

Let R_C = Reaction at C

and R_D = Reaction at D

The free-body diagram of sphere No. 2 is shown in Fig. 4.18 (c).

The reaction R_D at point D, will pass through the centre E of the sphere No. 2, as any line normal to any point on the circumference of the circle will pass through the centre of circle. For the equilibrium of the sphere No. 2, the resultant force in x and y directions should be zero.

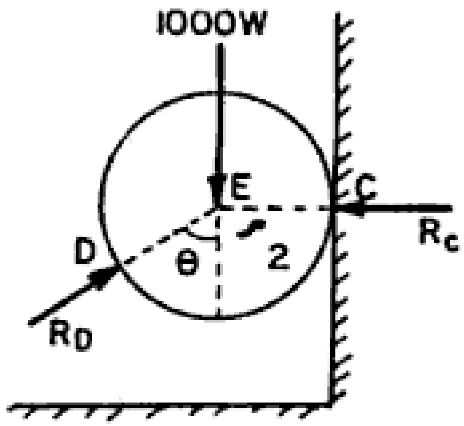


Fig. 4.18 (c)

For $\Sigma F_x = 0$, we have $R_D \sin\theta = R_C$ ...(i)

For $\Sigma F_y = 0$, we have $R_D \cos\theta = 1000$

or $$R_D = \frac{1000}{\cos\theta} = \frac{1000}{\left(\frac{3}{5}\right)} \quad \left(\because \cos\theta = \frac{3}{5}\right)$$

$$= 1000 \times \frac{5}{3} = \frac{5000}{3} \text{ N} \qquad ...(ii)$$

Substituting the value of R_D in equation (*i*),

$$\frac{5000}{3} \times \sin\theta = R_C \quad \text{or} \quad \frac{5000}{3} \times \frac{4}{5} = R_C \qquad \left(\because \sin\theta = \frac{4}{5}\right)$$

or $1333.33 = R_C$

$\therefore$ $R_C =$ **1333.33 N. Ans.**

Equilibrium of sphere No. 1. The sphere 1 has points of contact at *A*, *B* and *D*.

Let R_A = Reaction at point *A*

R_B = Reaction at point *B*

The free-body diagram of sphere No. 1 is shown in Fig. 4.18 (*d*). The reactions R_A, R_B and R_D will pass through the centre *F* of the sphere No. 1.

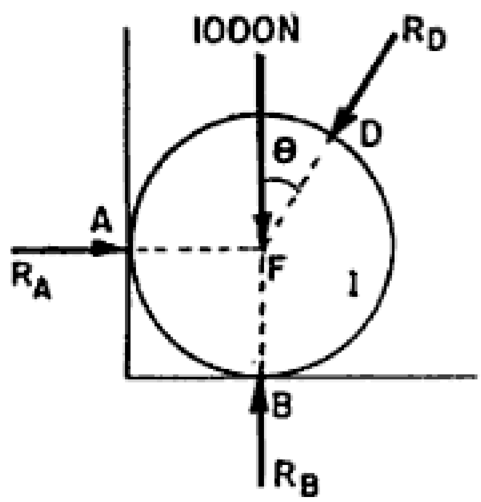


Fig. 4.18 (*d*)

For $\Sigma F_x = 0$, we have

$$R_A - R_D \sin\theta = 0$$

or $R_A = R_D \sin\theta$

$$= \frac{5000}{3} \times \frac{4}{5} \qquad \left(\because R_D = \frac{5000}{3} \text{ and } \sin\theta = \frac{4}{5}\right)$$

= **1333.33 N. Ans.**

For $\Sigma F_y = 0$, we have

$$R_B - 1000 - R_D \cos\theta = 0$$

$\therefore$ $R_B = 1000 + R_D \cos\theta$

$$= 1000 + \frac{5000}{3} \times \frac{3}{5} \qquad \left(\because \cos\theta = \frac{3}{5}\right)$$

= **2000 N. Ans.**

Problem 4.16. *Two smooth circular cylinders, each of weight W = 1000 N and radius 15 cm, are connected at their centres by a string AB of length = 40 cm and rest upon a horizontal plane, supporting above them a third cylinder of weight = 2000 N and radius 15 cm as shown in Fig. 4.19. Find the force S in the string AB and the pressure produced on the floor at the points of contact D and E.*

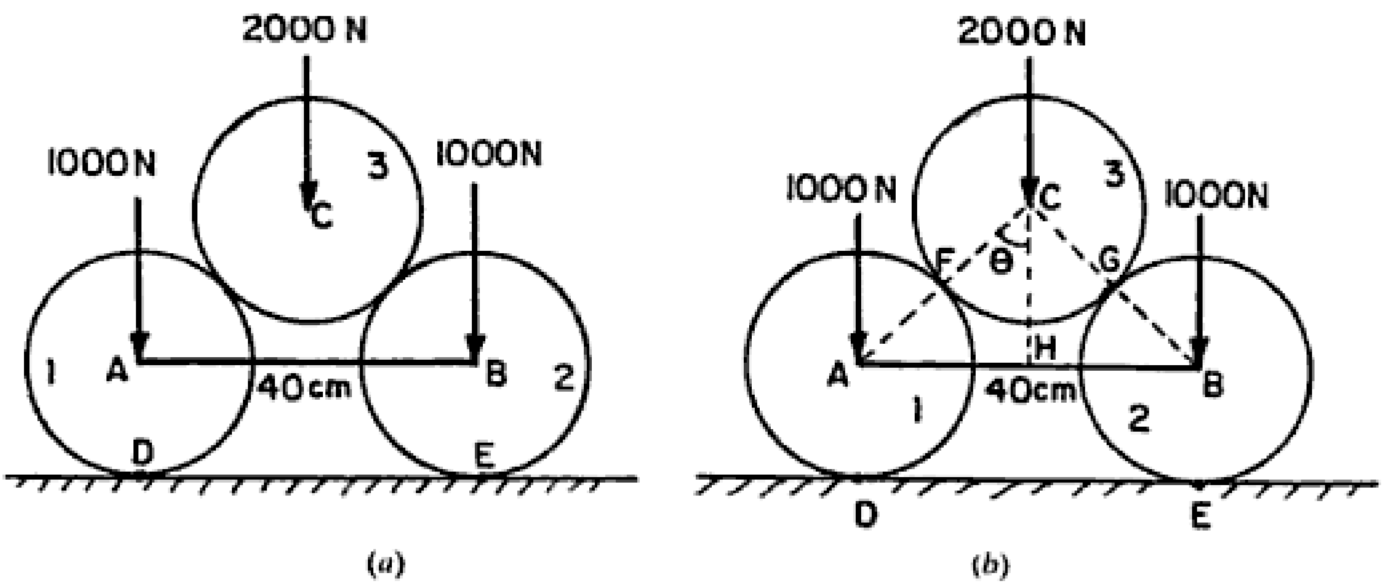


Fig. 4.19

Sol. Given :

Weight of cylinders 1 and 2 = 1000 N

Weight of cylinder 3 = 2000 N

Radius of each cylinder = 15 cm

Length of string $AB = 40$ cm

From Fig. 4.19 (*b*), $AC = AF + FC = 15 + 15 = 30$ cm

$$AH = \frac{1}{2} \times AB = \frac{1}{2} \times 40 = 20 \text{ cm.}$$

From ΔACH, $\sin \theta = \frac{AH}{AC} = \frac{20}{30} = 0.667$

$\therefore$ $\theta = \sin^{-1} 0.667 = 41.836°$.

Equilibrium of cylinder 3. The cylinder 3 has points of contact at F and G. The reactions R_F and R_G will pass through the centre of sphere 3. The free-body diagram is shown in Fig. 4.19 (*c*). Resolving forces horizontally,

$$R_F \sin \theta - R_G \sin \theta = 0$$

or $$R_F = R_G \quad \text{...(i)}$$

Resolving forces vertically,

$$R_F \cos \theta + R_G \cos \theta = 2000$$

or $$R_F \cos \theta + R_F \cos \theta = 2000 \quad (\because R_F = R_G)$$

$\therefore$ $$R_F = \frac{2000}{2 \times \cos \theta} = \frac{1000}{\cos 41.836°} = 1342.179 \text{ N} \quad \text{...(ii)}$$

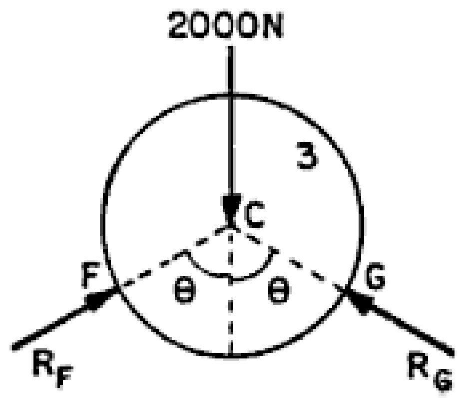


Fig. 4.19 (*c*)

Equilibrium of cylinder 1

The cylinder 1 has points of contact at D and F. Also the cylinder 1 is connected to cylinder 2 by a string AB. To draw the free-body diagram of cylinder 1, there will be reactions R_F and R_D at points F and D as shown in Fig. 4.19 (*d*). Also there will be a force S in the direction of the string AB.

For $\Sigma F_x = 0$, we have

$$S - R_F \sin \theta = 0$$

$\therefore$ $$S = R_F \sin \theta$$

$$= 1342.179 \times \sin 41.836°$$

[$\because$ From equation (*ii*), $R_F = 1342.179$]

= **895.2 N. Ans.**

For $\Sigma F_y = 0$, we have

$$R_D - 1000 - R_F \cos \theta = 0$$

or $$R_D = 1000 + R_F \cos \theta$$

$$= 1000 + 1342.179 \times \cos 41.836$$

$= 1999.99 \simeq$ **2000 N. Ans.**

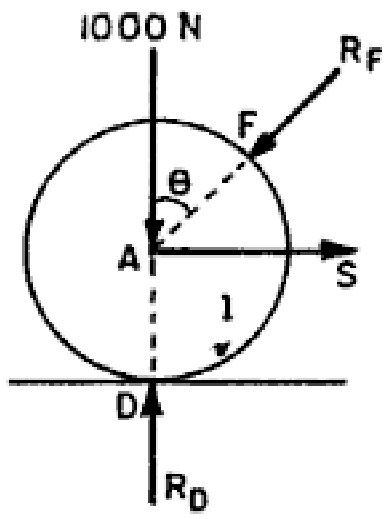


Fig. 4.19 (*d*)

The equilibrium of cylinders 1, 2 and 3 taken together

The three cylinders taken together have points of contact at D and E. The free-body diagram is shown in Fig. 4.19 (e). In this case only vertical forces exist. Hence resultant force in y-direction should be zero.

$$\therefore \quad R_D + R_E - 1000 - 2000 - 10000 = 0$$

or $$R_E = 1000 + 2000 + 1000 - R_D$$

or $$R_E = 4000 - R_D$$

$$= 4000 - 2000 \qquad (\because \quad R_D = 2000)$$

$$= \mathbf{2000\ N.} \quad \textbf{Ans.}$$

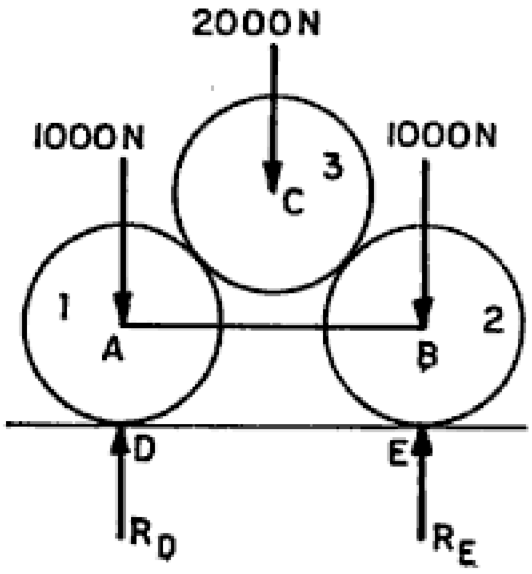


Fig. 4.19 (e)

Problem 4.17. *A roller of radius 40 cm, weighing 3000 N is to be pulled over a rectangular block of height 20 cm as shown in Fig. 4.20, by a horizontal force applied at the end of a string wound round the circumference of the roller. Find the magnitude of the horizontal force which will just turn the roller over the corner of the rectangular block. Also determine the magnitude and direction of reactions at A and B. All surfaces may be taken as smooth.*

Sol. Given :

Radius of roller = 40 cm

Weight, $W = 3000$ N

Height of block = 20 cm

Find horizontal force P, reaction R_A and reaction R_B when the roller just turns over the block.

When the roller is about to turn over the corner of the rectangular block, the roller lifts at the point A and then there will be no contact between the roller and the point A. Hence reaction R_A at point A will become zero.

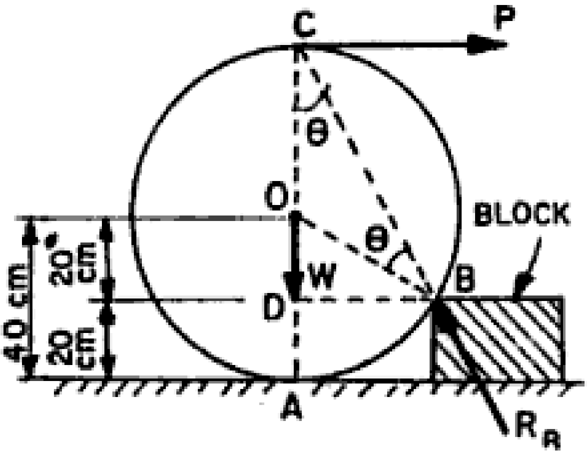


Fig. 4.20

Now the roller will be in equilibrium under the action of the following three forces :

(i) its weight W acting vertically downward

(ii) horizontal force P

(iii) reaction R_B at point B. The direction of R_B is unknown.

For the equilibrium, these three forces should pass through a common point. As the force P and weight W is passing through point C, hence the reaction R_B must also pass through the point C. Therefore, the line BC gives the direction of the reaction R_B.

In ΔBOD, $\quad BO$ = Radius = 40 cm,

$$OD = OA - AD = 40 - 20 = 20 \text{ cm}$$

$$\therefore \quad BD = \sqrt{BO^2 - OD^2} = \sqrt{40^2 - 20^2} = \sqrt{1200} = 34.64$$

Now in ΔBCD, $$\tan\theta = \frac{BD}{CD} = \frac{34.64}{CO + OD} = \frac{34.64}{(40 + 20)} = 0.5773$$

$$\therefore \quad \theta = \tan^{-1} 0.5773 = 29.999° \simeq 30°$$

11. Two identical rollers, each of weight 50 N, are supported by an inclined plane and a vertical wall as shown in Fig. 4.33. Find the reactions at the points of supports *A*, *B* and *C*. Assume all the surfaces to be smooth.

[**Ans.** $R_A = 43.3$ N, $R_B = 72$ N, $R_C = 57.5$ N]

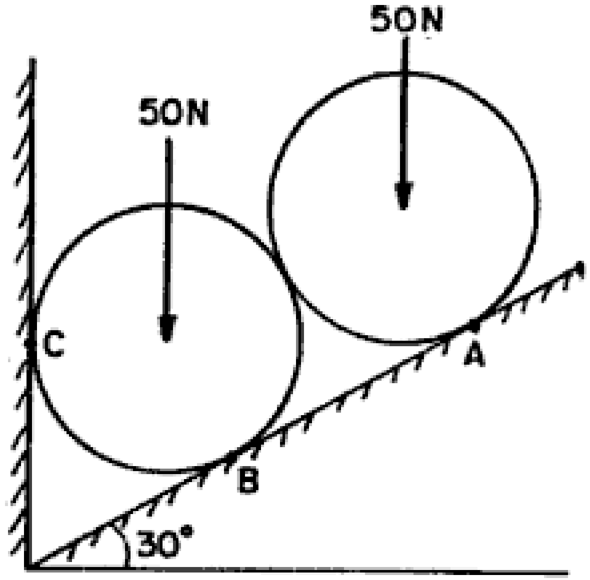


Fig. 4.33

12. Two spheres, each of weight 50 N and of radius 10 cm rest in a horizontal channel of width 36 cm as shown in Fig. 4.34. Find the reactions on the points of contact *A*, *B* and *C*.

[**Ans.** $R_A = R_C = 66.67$ N, $R_B = 100$ N]

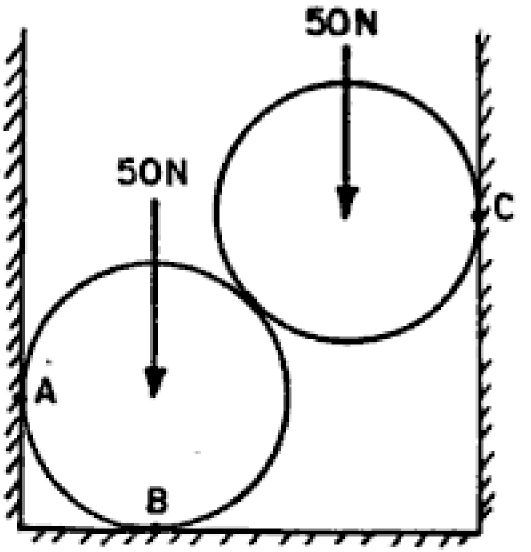


Fig. 4.34

5

Support Reactions

5.1. INTRODUCTION

When a number of forces are acting on a body, and the body is supported on another body, then the second body exerts a force known as reactions on the first body at the points of contact so that the first body is in equilibrium. The second body is known as support and the force, exerted by the second body on the first body, is known as support reactions.

5.2. TYPES OF SUPPORTS

Though there are many types of supports, yet the following are important from the subject point of view :

(*a*) Simple supports or knife edge supports
(*b*) Roller support
(*c*) Pin-joint (or hinged) support
(*d*) Smooth surface support
(*e*) Fixed or built-in support.

5.2.1. Simple support or knife edge support. A beam supported on the knife edges A and B is shown in Fig. 5.1 (*a*). The reactions at A and B in case of knife edge support will be normal to the surface of the beam. The reactions R_A and R_B with free-body diagram of the beam is shown in Fig. 5.1. (*b*).

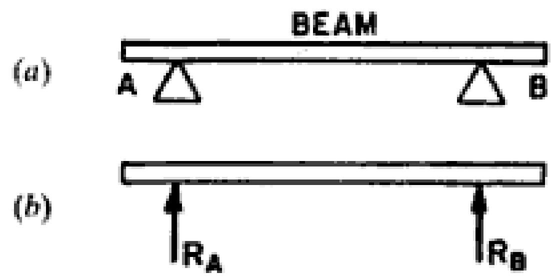


Fig. 5.1

5.2.2. Roller Support. A beam supported on the rollers at points A and B is shown in Fig. 5.2 (*a*). The reactions in case of roller supports will be normal to the surface on which rollar are placed as shown in Fig. 5.2 (*b*).

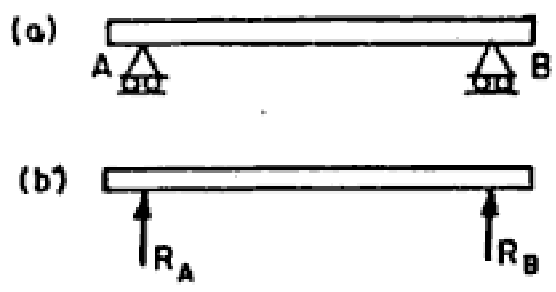


Fig. 5.2

5.2.3. Pin joint (or hinged) support. A beam, which is hinged (or pin-joint) at point A, is shown in Fig. 5.3. The reaction at the hinged end may be *either vertical or inclined* depending upon the type of loading. If the load is vertical, then the reaction will also be vertical. But if the load is inclined, then the reaction at the hinged end will also be inclined.

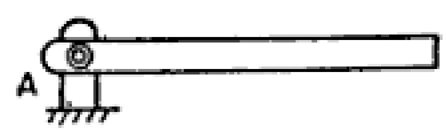
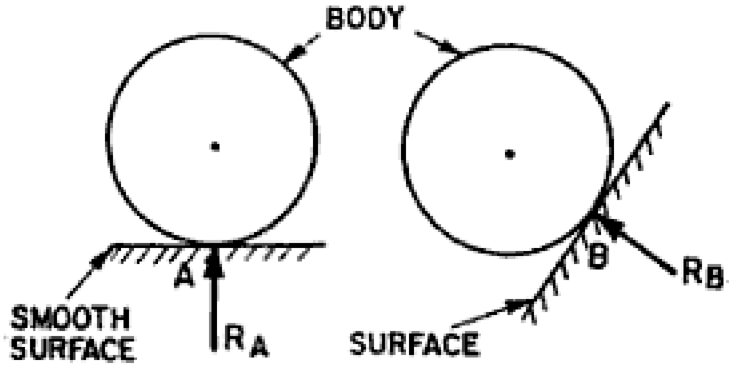


Fig. 5.3

5.2.4. Smooth Surface Support. Fig. 5.4 shows a body in contact with a smooth surface. The reaction will always act normal to the support as shown in Fig. 5.4 (*a*) and 5.4 (*b*).



(*a*) (*b*)

Fig. 5.4

Fig. 5.5 shows a rod *AB* resting inside a sphere, whose surface are smooth. Here the rod becomes body and sphere becomes surface. The reactions on the ends of the rod (*i.e.*, at point *A* and *B*) will be normal to the sphere surface at *A* and *B*. The normal at any point on the surface of the sphere will always pass through the centre of the sphere. Hence reactions R_A and R_B will have directions *AO* and *BO* respectively as shown in Fig. 5.5.

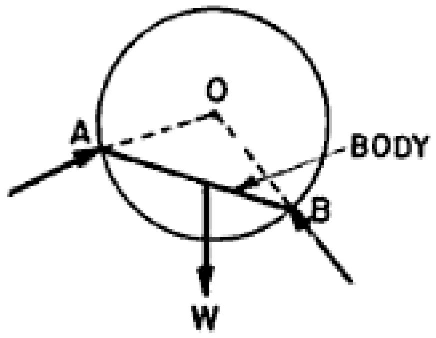


Fig. 5.5

5.2.5. Fixed or built-in Support. Fig. 5.6 shows the end *A* of a beam, which is fixed. Hence the support at *A* is known as a fixed support. In case of fixed support, the reaction will be inclined. Also the fixed support will provide a couple.

Fig. 5.6

5.3. TYPES OF LOADING

The following are the important types of loading :

(*a*) Concentrated or point load,

(*b*) Uniformly distributed load, and

(*c*) Uniformly varying load.

5.3.1. Concentrated or point load. Fig. 5.7 shows a beam *AB*, which is simply supported at the ends *A* and *B*. A load *W* is acting at the point *C*. This load is known as point load (or concentrated load). Hence any load acting at a point on a beam, is known as point load.

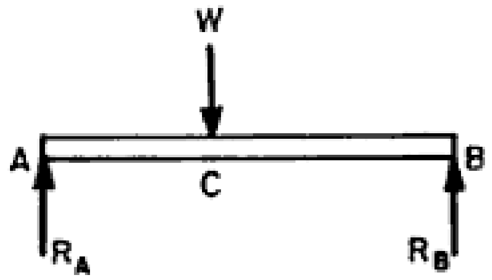


Fig. 5.7

The given beam is drawn to a suitable scale along with the loads and the reactions R_A and R_B. This step is known as construction of space diagram.

The different loads and forces (*i.e.*, reactions R_A and R_B) are named by two capital letters, placed on their either side of the space diagram as shown in Fig. 5.11. This step is known as *Bow's notation*. The load W_1 is named by *PQ*, W_2 by *QR*, reaction R_B by *SR* and reaction R_A by *SP*.

Now the vector diagram is drawn according to the following steps :

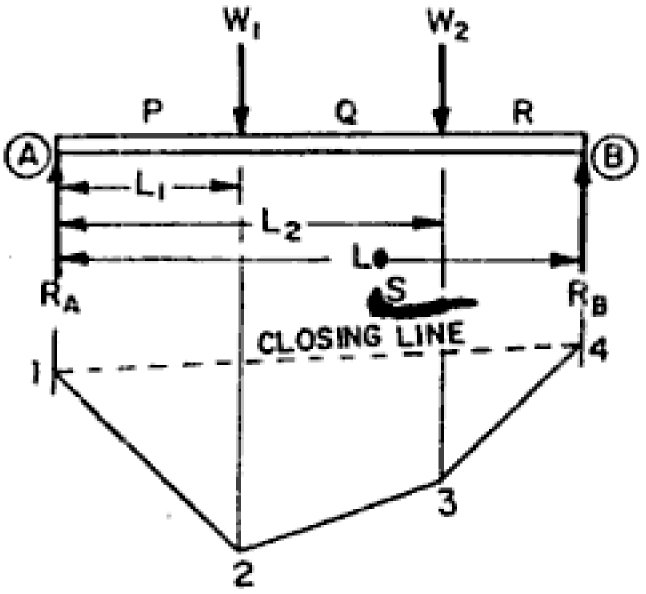
(*i*) Choose a suitable scale to represent the various loads. Now take any point *p* and draw *pq* parallel and equal to the load *PQ* (*i.e.*, W_1) vertically downward to the same scale.

(*ii*) Now through *q*, draw *qr* parallel and equal to *QR* vertically downward to the same scale.

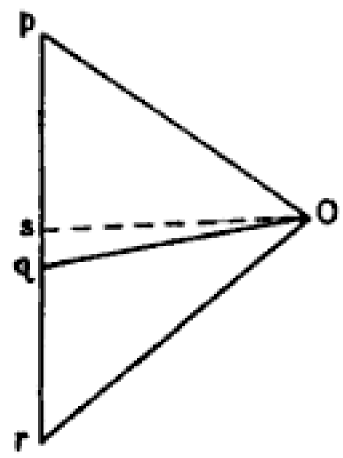
(*iii*) Select any suitable point *O*. Now join the point *O* to points *p*, *q* are *r* as shown in Fig. 5.11 (*b*).

(*iv*) Now in Fig. 5.11 (*a*), extend the lines of action of the loads and the two reactions. Take any point 1, on the line of action of the reaction R_A. Through 1, draw the line 1-2 parallel to *pO*, intersecting the line of action of load W_1 at 2.

(*v*) Now from point 2, draw line 2-3 parallel to *qO*, intersecting the line of action of load W_2 at 3. Similarly, from point 3, draw the line 3-4 parallel to *rO*, intersecting the line of action of reaction R_B at point 4.



(*a*) Space diagram



(*b*) Vector diagram

Fig. 5.11

(*vi*) Now join the point 1 to point 4. The line 1-4 is known as closing line. Now from point *O* (*i.e.*, from vector diagram) draw line *Os* parallel to line 1-4.

(*vii*) Now in the vector diagram the length *sp* represents the magnitude of reaction R_A to the same scale. Similarly, the length *rs* represents the magnitude of reaction R_B to the same scale.

5.5. PROBLEMS ON SIMPLE SUPPORTED BEAMS

Problem 5.1. *A simply supported beam AB of span 6 m carries point loads of 3 kN and 6 kN at a distance of 2 m and 4 m from the left end A as shown in Fig. 5.12. Find the reactions at A and B analytically and graphically.*

Sol. Given :

Span of beam = 6 m

Let R_A = Reaction at *A*

R_B = Reaction at *B*

(*a*) **Analytical Method.** As the beam is in equilibrium, the moments of all the forces about any point should be zero.

Now taking the moment of all forces about A, and equating the resultant moment to zero, we get

$$R_B \times 6 - 3 \times 2 - 6 \times 4 = 0$$

or $$6R_B = 6 + 24 = 30$$

or $$R_B = \frac{30}{6} = 5 \text{ kN. } \textbf{ Ans.}$$

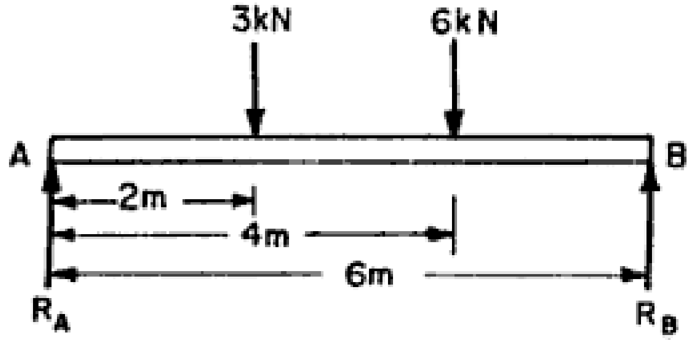


Fig. 5.12

Also for equilibrium, $\Sigma F_y = 0$

$\therefore$ $$R_A + R_B = 3 + 6 = 9$$

$\therefore$ $$R_A = 9 - R_B = 9 - 5 = 4 \text{ kN. } \textbf{ Ans.}$$

(*b*) **Graphical Method.** First of all draw the space diagram of the beam to a suitable scale. Let 1 cm length in space diagram represents 1 m length of beam. Hence take AB = 6 cm, distance of load 3 kN from A = 2 cm and distance of 6 kN from A = 4 cm as shown in Fig. 5.13 (*a*).

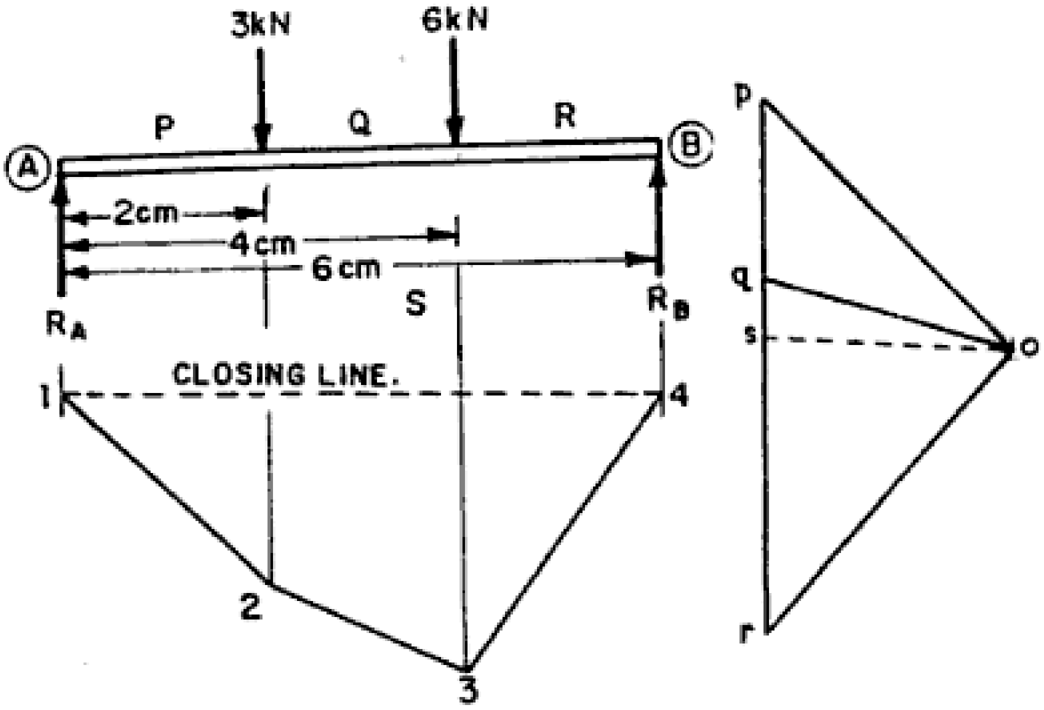
Now name all the loads and reactions according to Bow's notation *i.e.*, load 3 kN is named by PQ, load 6 kN by QR, reaction R_B by SR and reaction R_A by SP.

Now the vector diagram is drawn according to the following steps : [Refer to Fig. 5.13 (*b*)].

1. Choose a suitable scale to represent various loads. Let 1 cm represents 1 kN load. Hence load PQ (*i.e.*, 3 kN) will be equal to 3 cm and load QR (*i.e.*, 6 kN) 6 cm.

2. Now take any point p and draw line pq parallel to load PQ (*i.e.*, 3 kN). Take pq = 3 cm to represent the load of 3 kN.

3. Through q, draw line qr parallel to load QR (*i.e.*, 6 kN). Cut qr equal to 6 cm to represent the load of 6 kN.



(*a*) Space diagram (*b*) Vector diagram

Fig 5.13

4. Now take any point O. Join the point O to the points p, q and r as shown in Fig. 5.13 (*b*).

5. Now in Fig. 5.13 (*a*), extend the lines of action of the loads (3 kN and 6 kN), and the two reactions. Take any point 1, on the line of action of the reaction R_A. Through 1, draw the line 1-2 parallel to *pO*, intersecting the line of action of load 3 kN at point 2.

6. From point 2, draw line 2-3 parallel to *qO*, intersecting the line of action of load 6 kN at 3. Similarly, from point 3, draw a line 3-4 parallel to *rO*, intersecting the line of action of reaction R_B at point 4.

7. Join 1 to 4. The line 1-4 is known as closing line. From the vector diagram, from point *O*, draw line *Os* parallel to line 1-4.

8. Measure the length *sp* and *rs*. The length *sp* represents the reaction R_A and length *rs* represents the reaction R_B.

By measurement, $sp = 4$ cm and $rs = 5$ cm

$\therefore$ R_A = Length $sp \times$ scale = 4×1 kN = **4 kN. Ans.**

R_B = Length $rs \times$ scale = 5×1 kN = **5 kN. Ans.**

Problem 5.2. *A simply supported beam AB of length 9 m, carries a uniformly distributed load of 10 kN/m for a distance of 6 m from the left end. Calculate the reactions at A and B.*

Sol. Given :

Length of beam = 9 m

Rate of U.D.L. = 10 kN/m

Length of U.D.L. = 6 m

Total load due to U.D.L. = (Length of U.D.L.) × Rate of U.D.L.

= 6 × 10 = 60 kN

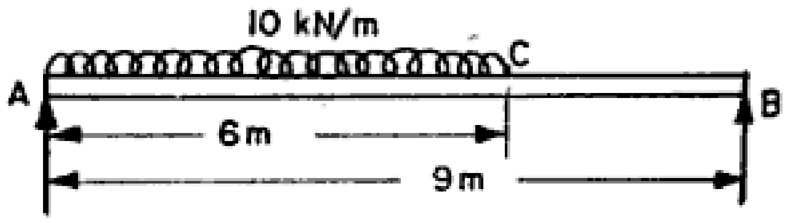


Fig. 5.14

This load of 60 kN will be acting at the middle point of *AC* *i.e.*, at a distance of $\frac{6}{2} = 3$ m from *A*.

Let R_A = Reaction at *A* and R_B = Reaction at *B*

Taking the moments of all forces about point *A*, and equating the resultant moment to zero, we get

$$R_B \times 9 - (6 \times 10) \times 3 = 0 \quad \text{or} \quad 9R_B - 180 = 0$$

$\therefore$ $R_B = \frac{180}{9}$ = **20 kN. Ans.**

Also for equilibrium, $\Sigma F_y = 0$

or $R_A + R_B = 6 \times 10 = 60$

$\therefore$ $R_A = 60 - R_B = 62 - 20$ = **40 kN. Ans.**

Problem 5.3. *A simply supported beam of length 10 m, carries the uniformly distributed load and two point loads as shown in Fig. 5.15. Calculate the reactions R_A and R_B.*

Sol. Given :

Length of beam = 10 m

Length of U.D.L. = 4 m

Rate of U.D.L. = 10 kN/m

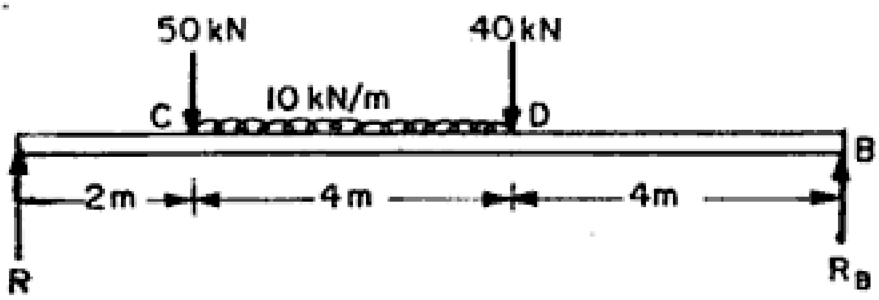


Fig. 5.15

$\therefore$ Total load due to U.D.L. = 4 × 10 = 40 kN

This load of 40 kN due to U.D.L. will be acting at the middle point of *CD, i.e.*, at a distance of $\frac{4}{2} = 2$ m from *C* (or at a distance of 2 + 2 = 4 m from point *A*).

Let R_A = Reaction at *A*

and R_B = Reaction at *B*.

Taking the moments of all forces about point *A* and equating the resultant moment to zero, we get

$$R_B \times 10 - 50 \times 2 - 40 \times (2 + 4) - (10 \times 4)\left(2 + \frac{4}{2}\right) = 0$$

or $$10R_B - 100 - 240 - 160 = 0$$

or $$10R_B = 100 + 240 + 160 = 500$$

$\therefore$ $$R_B = \frac{500}{10} = \mathbf{50\ kN.}$$ **Ans.**

Also for equilibrium of the beam, $\Sigma F_y = 0$

$\therefore$ $$R_A + R_B = \text{Total load on the beam} = 50 + 10 \times 4 + 40 = 130$$

$\therefore$ $$R_A = 130 - R_B = 130 - 50 = \mathbf{80\ kN.}$$ **Ans.**

Problem 5.4. *A simply supported beam of span 9 m carries a uniformly varying load from zero at end A to 900 N/m at end B. Calculate the reactions at the two ends of the support.*

Sol. Given :

Span of beam = 9 m

Load at end *A* = 0

Load at end *B* = 900 N/m

Total load on the beam = Area of $ABC = \frac{AB \times BC}{2} = \frac{9 \times 900}{2}$

= 4050 N

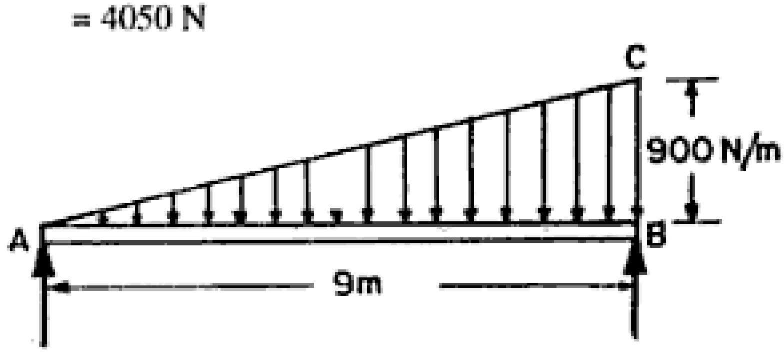


Fig. 5.16

or $$5R_B - (5 \times 800) \times 2.5 - \left(\frac{1}{2} \times 5 \times 800\right) \times \left(\frac{2}{3} \times 5\right) = 0$$

or $$5R_B - 1000 - 6666.66 = 0$$

or $$5R_B = 1000 + 6666.66 = 16666.66$$

or $$R_B = \frac{16666.66}{5} = \mathbf{3333.33\ N.}\quad \textbf{Ans.}$$

Also for the equilibrium of the beam, $\Sigma F_y = 0$

$\therefore$ $R_A + R_B$ = Total load on the beam

= 6000 ($\because$ Total load on beam = 6000 N)

$\therefore$ $R_A = 6000 - R_B = 6000 - 3333.33 =$ **2666.67 N. Ans.**

5.6. PROBLEMS ON OVERHANGING BEAMS

If the end portion of a beam is extended beyond the support, then the beam is known as over hanging beam. Over hanging portion may be at one end of the beam or at both ends of the beam as shown in Fig. 5.18.

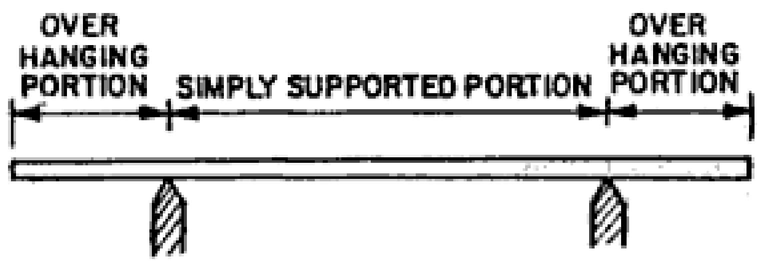


Fig. 5.18

Problem 5.6. *A beam AB of span 8 m, overhanging on both sides, is loaded as shown in Fig. 5.19. Calculate the reactions at both ends.*

Sol. Given :

Span of beam = 8 m

Let R_A = Reaction at A

and R_B = Reaction at B.

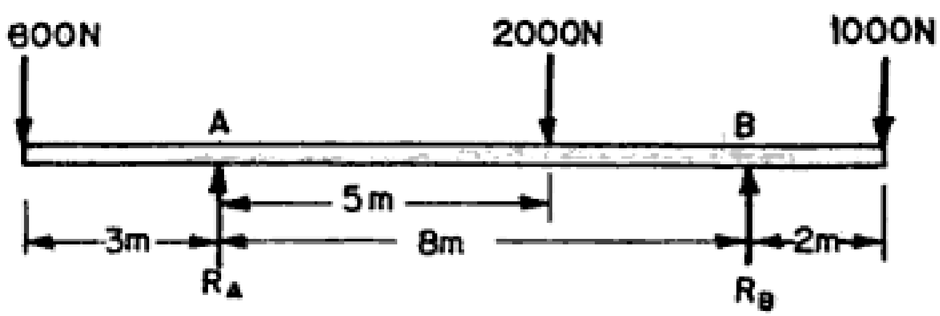


Fig. 5.19

Taking the moments of all forces about point A and equating the resultant moment to zero, we get

$$R_B \times 8 + 800 \times 3 - 2000 \times 5 - 1000 \times (8 + 2) = 0$$

or $$8R_B + 2400 - 10000 - 10000 = 0$$

or $$8R_B = 20000 - 2400 = 17600$$

$\therefore$ $$R_B = \frac{17600}{8} = \mathbf{2200\ N.}\quad \textbf{Ans.}$$

Also for the equilibrium of the beam, we have

$$R_A + R_B = 800 + 2000 + 1000 = 3800$$

$\therefore$ $R_A = 3800 - R_B = 3800 - 2200 =$ **1600 N. Ans.**

Problem 5.7. *A beam AB of span 4 m, overhanging on one side upto a length of 2 m, carries a uniformly distributed load of 2 kN/m over the entire length of 6 m and a point load of 2 kN/m as shown in Fig. 5.20. Calculate the reactions at A and B.*

Sol. Given :

Span of beam = 4 m

Total length = 6 m

Rate of U.D.L. = 2 kN/m

Total load due to U.D.L. = 2 × 6 = 12 kN

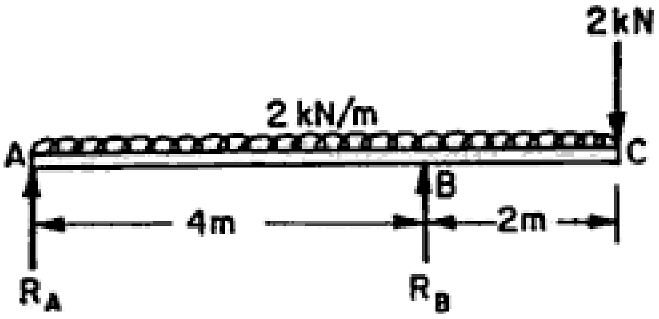


Fig. 5.20

The load of 12 kN (*i.e.*, due to U.D.L.) will act at the middle point of *AC*, *i.e.*, at a distance of 3 m from *A*.

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking the moments of all forces about point *A* and equating the resultant moment to zero, we get

$$R_B \times 4 - (2 \times 6) \times 3 - 2 \times (4 + 2) = 0$$

or $$4R_B - 36 - 12 = 0$$

or $$4R_B = 36 + 12 = 48$$

$\therefore$ $R_B = \frac{48}{4} =$ **12 kN. Ans.**

Also for equilibrium, $\Sigma F_y = 0$ or $R_A + R_B = 12 + 2 = 14$

$\therefore$ $R_A = 14 - R_B = 14 - 12 =$ **2 kN. Ans.**

5.7. PROBLEMS ON ROLLER AND HINGED SUPPORTED BEAMS

In case of roller supported beams, the reaction on the roller end is always *normal* to the support. All the steel trusses of the bridges is generally having one of their ends supported on rollers. The main advantage of such a support is that beam, due to change in temperature, can move easily towards left or right, on account of expansion or contraction.

In case of a hinged supported beam, the reaction on the hinged end may be either vertical or inclined, depending upon the type of loading. The main advantage of a hinged end is that the beam remains stable. Hence all the steel trusses of the bridges, have one of their end on rollers and the other end as hinged.

Problem 5.8. *A beam AB 1.7 m long is loaded as shown in Fig. 5.21. Determine the reactions at A and B.*

Sol. Given :

Length of beam = 1.7 m

Let R_A = Reaction at A

and R_B = Reaction at B.

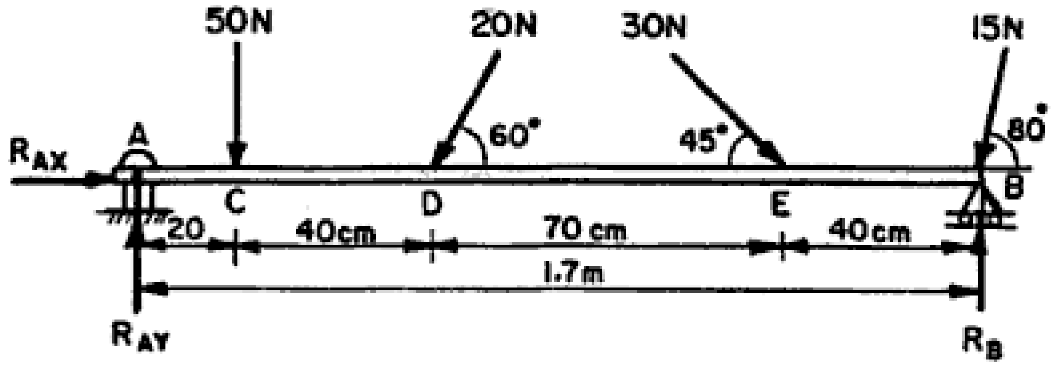


Fig. 5.21

Since the beam is supported on rollers at B, therefore the reaction R_B will be vertical.

The beam is hinged at A, and is carrying inclined load, therefore the reaction R_A will be inclined. This means reaction R_A will have two components, *i.e.*, vertical component and horizontal component.

Let R_{AX} = Horizontal component of reaction R_A

R_{AY} = Vertical component of reaction R_A.

First resolve all the inclined loads into their vertical and horizontal components.

(*i*) Vertical component of load at D

$$= 20 \sin 60° = 20 \times 0.866 = 17.32 \text{ N}$$

and its horizontal component

$$= 20 \cos 60° = 10 \text{ N} \leftarrow$$

(*ii*) Vertical component of load at E

$$= 30 \sin 45° = 21.21 \text{ N}$$

and its horizontal component

$$= 30 \cos 45° = 21.21 \text{ N} \rightarrow$$

(*iii*) Vertical component of load at B

$$= 15 \sin 80° = 14.77 \text{ N}$$

and its horizontal component

$$= 15 \cos 80° = 2.6 \text{ N} \leftarrow$$

From condition of equilibrium, $\Sigma F_x = 0$

or $R_{AX} - 10 + 21.21 - 2.6 = 0$

or $R_{AX} = 10 - 21.21 + 2.6 = -8.61 \text{ N}$

– ve sign shows that the assumed direction of R_{AX} (*i.e.*, horizontal component of R_A) is wrong. Correct direction will be opposite to the assumed direction. Assumed direction of R_{AX} is towards right. Hence correct direction of R_{AX} will be towards left at A.

$\therefore$ $R_{AX} = 8.61 \text{ N} \leftarrow$

To find R_B, take moments* of all forces about A.

For equilibrium, $\Sigma M_A = 0$

$$\therefore \quad 50 \times 20 + (20 \sin 60°) \times (20 + 40) + (30 \times \sin 45°) \times (20 + 40 + 70) + (15 \sin 80°) \times (170) - 170\, R_B = 0$$

*The moment of all horizontal components about point A, will be zero.

or $\quad 1000 + 1039.2 + 2757.7 + 2511 - 170\,R_B = 0$

or $\quad 7307.9 - 170\,R_B = 0$

$\therefore \quad R_B = \dfrac{7307.9}{170} = \mathbf{42.98\ N.}$ **Ans.**

To find R_{AY}, apply condition of equilibrium, $\Sigma F_y = 0$

or $\quad R_{AY} + R_B = 50 + 20 \sin 60° + 30 \sin 45° + 15 \sin 80°$

or $\quad R_{AY} + 42.98 = 50 + 17.32 + 21.21 + 14.77 = 103.3$

$\therefore \quad R_{AY} = 103.3 - 42.98 = 60.32\ N \uparrow$

$\therefore$ Reaction at A, $\quad R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$

$= \sqrt{8.61^2 + 60.32^2}$

$= 60.92\ N$

The angle made by R_A with x-direction is given by

$$\tan\theta = \frac{R_{AY}}{R_{AX}} = \frac{60.32}{8.61} = 7.006$$

$\therefore \quad \theta = \tan^{-1} 7.006 = \mathbf{81.87°}$. **Ans.**

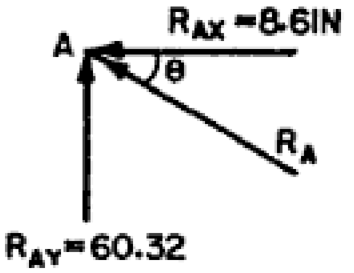


Fig. 5.21 (*a*)

Problem 5.9. *A beam AB 6 m long is loaded as shown in Fig. 5.22. Determine the reactions at A and B by (a) analytical method, and (b) graphical method.*

Sol. Given :

Length of beam $= 6$ m

Let R_A = Reaction at A

R_B = Reaction at B.

The reaction R_B will be vertical as the beam is supported on rollers at end B.

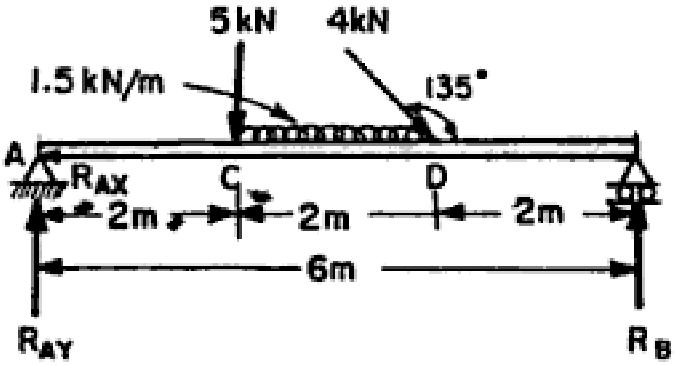


Fig. 5.22

The reaction R_A will be inclined, as the beam is hinged at A and carries inclined load.

Let R_{AX} = Horizontal component of reaction R_A

R_{AY} = Vertical component of reaction R_A.

(*a*) **Analytical Method.** First resolve the inclined load of 4 kN into horizontal and vertical components.

Horizontal component of 4 kN at D

$= 4 \cos 45° = 2.828\ kN \rightarrow$

and its vertical component

$= 4 \sin 45° = 2.828\ kN \downarrow$

For equilibrium, $\Sigma F_x = 0$

$\therefore$ $-R_{AX} + 2.828 = 0$

or $R_{AX} = 2.828$ N

To find R_B, take the moments* of all forms about point A.

$\therefore$ For equilibrium, $\Sigma M_A = 0$

$$R_B \times 6 - 5 \times 2 - (2 \times 1.5)\left(2 + \frac{2}{2}\right) - (4 \sin 45°)(2 + 2) = 0$$

or $6R_B - 10 - 9 - 11.312 = 0$

or $6R_B = 10 + 9 + 11.312 = 30.312$

$\therefore$ $R_B = \dfrac{30.312}{6} = $ **5.052 kN. Ans.**

To find R_{AY}, apply the condition of equilibrium, $\Sigma F_y = 0$

$\therefore$ $R_{AY} + R_B - 5 - (1.5 \times 2) - 4 \sin 45° = 0$

or $R_{AY} + 5.052 - 5 - 3 - 2.828 = 0$

$\therefore$ $R_{AY} = -5.052 + 5 + 3 + 2.828 = 5.776$ kN

$\therefore$ Reaction at A is given by

$$R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$$

$$= \sqrt{2.828^2 + 5.776^2}$$

$$= \sqrt{7.997 + 33.362}$$

$= \sqrt{41.359} =$ **6.43 kN. Ans.**

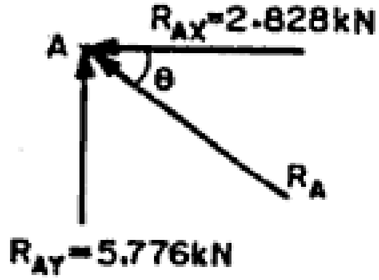


Fig. 5.22 (*a*)

Let θ = Angle made by R_A with x-direction.

$\therefore$ $\tan\theta = \dfrac{R_{AY}}{R_{AX}} = \dfrac{5.775}{2.828} = 2.0424$

$\therefore$ $\theta = \tan^{-1} 2.0424 =$ **63.9°. Ans.**

(*b*) **Graphical Method.** First of all convert the uniformly distributed load (U.D.L.) into its equivalent point load acting at the C.G. of the portion on which U.D.L. is acting. Hence total load due to U.D.L. will be $1.5 \times 2 = 3$ kN acting at a distance of 3 m from point A :

(*i*) Now draw the space diagram of the beam according to some suitable scale, as shown in Fig. 5.23 (*a*).

(*ii*) Name all the loads and reactions according to Bow's notation. Now draw the vector diagram as shown in Fig. 5.23 (*b*). Choose any suitable scale for vector diagram.

(*iii*) Take any point p for drawing vector diagram. From p, draw line pq parallel and equal to load 5 kN (*i.e.*, load PQ). From q, draw qr parallel and equal to 3 kN. From r, draw rs parallel and equal to 4 kN load.

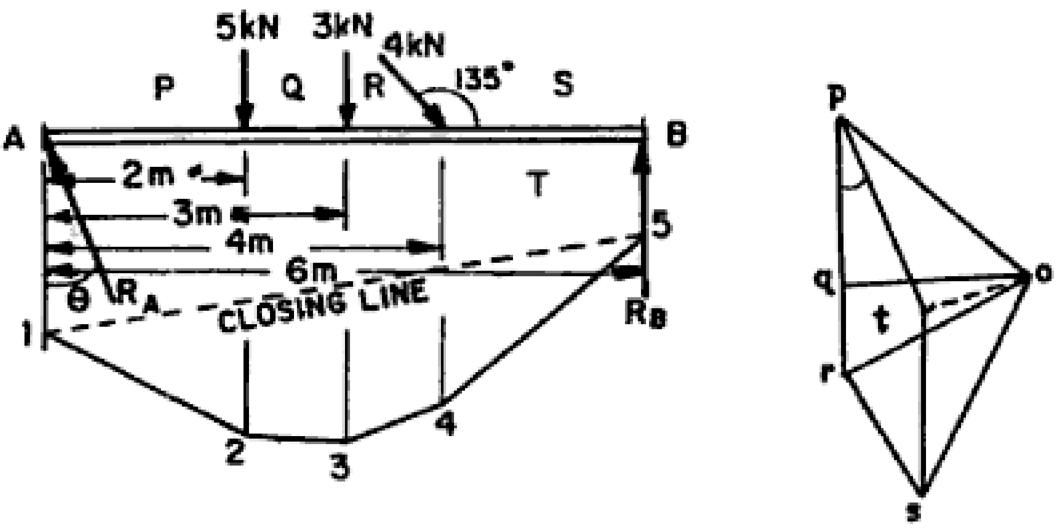
(*iv*) Now take any point O, and join Op, Oq, Or and Os.

(*v*) Now in space diagram [*i.e.*, Fig. 5.23 (*a*)], extend the lines of actions of loads PQ, QR, RS and reaction R_B.

(*vi*) Take any point 1, vertically below the point A as shown in Fig. 5.23 (*a*). From point 1, draw line 1-2 parallel to line pO, intersecting the line of action of 5 kN at point 2.

(*vii*) Similarly, draw lines 2-3, 3-4 and 4-5 parallel to qO, rO and sO respectively. Join point 1 to 5. Line 1-5 is the closing line in space diagram.

*The moment of horizontal component about A, will be zero.



(*a*) Space diagram (*b*) Vector diagram

Fig. 5.23

(*viii*) From *O* in vector diagram, draw a line parallel to closing line 1-5. Now through *s*, draw a line *st* vertical (as the reaction R_B is vertical), intersecting the line through *O* at *t*. Join *t* to *p*.

(*ix*) The length *st* represents the reaction R_B in magnitude and direction whereas the length *tp* gives the magnitude and direction of reaction R_A. At point *A*, draw a line parallel to *tp* as shown in Fig. 5.23 (*a*). By measurement, we get

R_A = length *tp* = 6.43 kN

R_B = length *st* = 5.052 kN

and $\theta = 26.1°$.

Problem 5.10. *A beam AB 10 m long is hinged at A and supported on rollers over a smooth surface inclined at 30° to the horizontal at B. The beam is loaded as shown in Fig. 5.24. Determine reactions at A and B.*

Sol. Given :

Length of beam = 10 m

Let R_A = Reaction at *A*

and R_B = Reaction at *B*

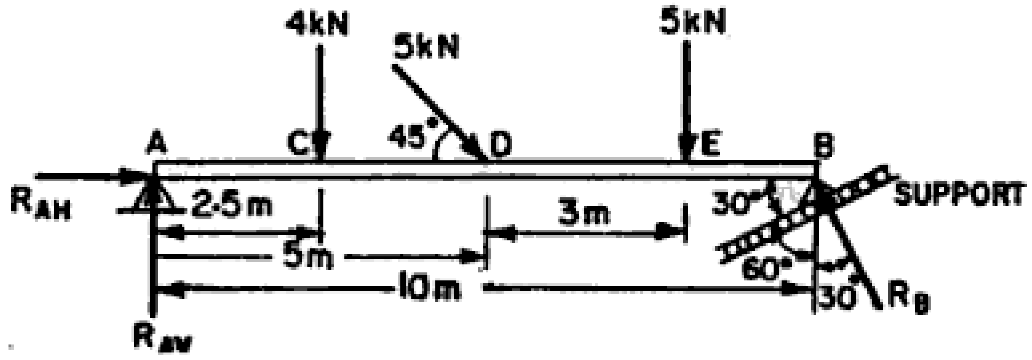


Fig. 5.24

The reaction R_B will be normal to the support as the beam at *B* is supported on the rollers. But the support at *B* is making an angle 30° with the horizontal or 60° with the vertical as shown in Fig. 5.24. Hence the reaction R_B is making an angle of 30° with the vertical.

The vertical component of R_B

$$= R_B \cos 30°$$

and horizontal component of R_B

$$= R_B \sin 30°$$

These components are shown in Fig. 5.24 (*a*).

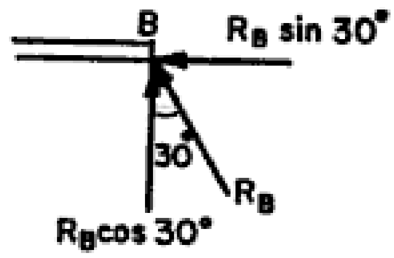


Fig. 5.24 (*a*)

Resolving the load of 5 kN acting at D into horizontal and vertical components, we get

Vertical component of 5 kN

$$= 5 \sin 45° = 5 \times 0.707 = 3.535 \text{ kN}$$

Horizontal component of 5 kN

$$= 5 \cos 45° = 5 \times 0.707 = 3.535 \text{ kN}$$

The reaction at A will be inclined, as the end A is hinged and beam carries inclined load.

Let R_{AX} = Horizontal component of reaction R_A

R_{AY} = Vertical component of reaction R_A

For equilibrium of the beam, the moments of all forces about any point should be zero.

Taking the moments* about point A,

$$(R_B \cos 30°) \times 10 - 4 \times 2.5 - (5 \sin 45°) \times 5 - 5 \times 8 = 0.$$

$$8.66\, R_B - 10 - 17.675 - 40 = 0$$

or $$R_B = \frac{10 + 17.675 + 40}{8.66} = \mathbf{7.81 \text{ kN}} \quad \textbf{Ans.}$$

For equilibrium, $\Sigma F_x = 0$

or $$R_{AH} + 5 \cos 45° - R_B \sin 30° = 0$$

or $$R_{AH} + 3.535 - 7.81 \times 0.5 = 0$$

$\therefore$ $$R_{AH} = 7.81 \times 0.5 - 3.535 = 0.37 \text{ kN}$$

For equilibrium, $\Sigma F_y = 0$

$\therefore$ $$R_{AV} + R_B \cos 30° - 4 - 5 \sin 45° - 5 = 0$$

or $$R_{AV} + 7.81 \times 0.866 - 4 - 3.535 - 5 = 0$$

or $$R_{AV} + 6.763 - 12.535 = 0$$

or $$R_{AV} = 12.535 - 6.763 = 5.77 \text{ kN}$$

$\therefore$ Reaction at A, $$R_A = \sqrt{R_{AH}^2 + R_{AV}^2} = \sqrt{0.37^2 + 5.77^2}$$

$$= \mathbf{5.78 \text{ kN.}} \quad \textbf{Ans.}$$

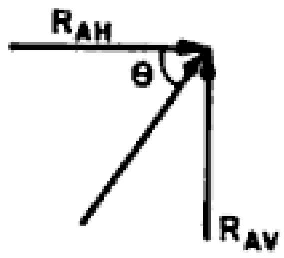


Fig. 5.24 (*b*)

The angle made by R_A with x-direction is given by

$$\tan \theta = \frac{R_{AV}}{R_{AH}} = \frac{5.77}{0.37} = 15.59$$

$\therefore$ $$\theta = \tan^{-1} 15.59 = \mathbf{86.33°.} \quad \textbf{Ans.}$$

Problem 5.11. *Determine the reactions at the hinged support A and the roller support B as shown in Fig. 5.25 (a).*

Sol. Given :

The support at A is hinged whereas the support at B is placed on the roller. Hence the reaction at the roller support will be perpendicular to the inclined surface.

*The moments of horizontal components of 5 kN at D and of reaction R_B will be zero about the point A.

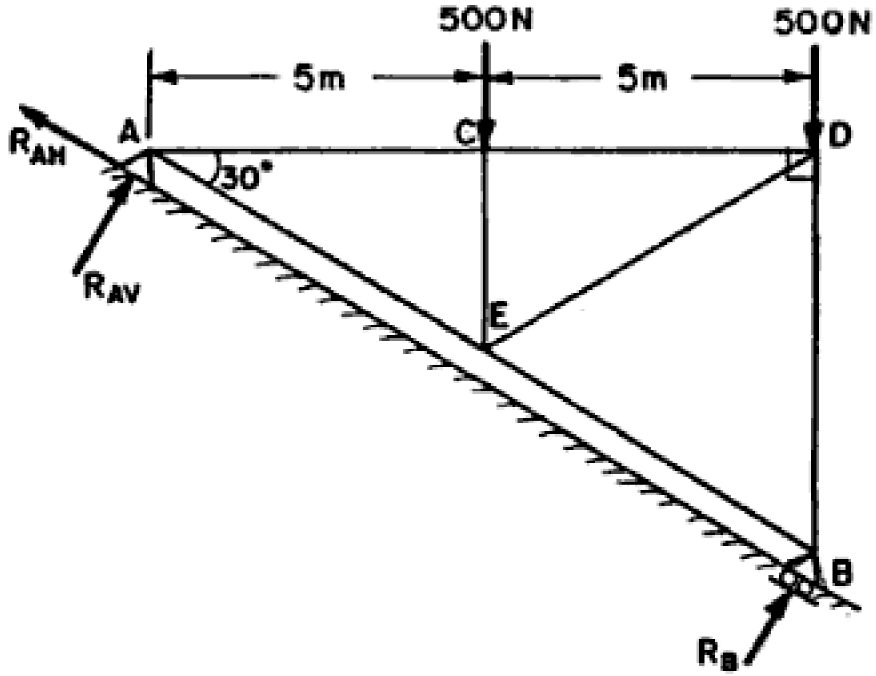


Fig. 5.25 (*a*)

The reaction at the hinged support *A* (*i.e.*, reaction R_A) will be inclined at some angle to the inclined surface *AB*.

Let R_{AV} = Component of reaction R_A normal to inclined surface *AB*

R_{AH} = Component of reaction R_A along the inclined surface *AB*.

The given vertical force of 500 N at *C* is resolved parallel and perpendicular to the inclined surface *AB*. The value of its component parallel to the inclined surface is equal to 500 sin 30° or 250 N and its value perpendicular to the inclined surface is equal to 500 cos 30° or $250 \times \sqrt{3}$ N. The directions of these components are shown in Fig. 5.25 (*b*).

Similarly, the vertical force of 500 N at *D* is resolved as shown in Fig. 5.25 (*b*).

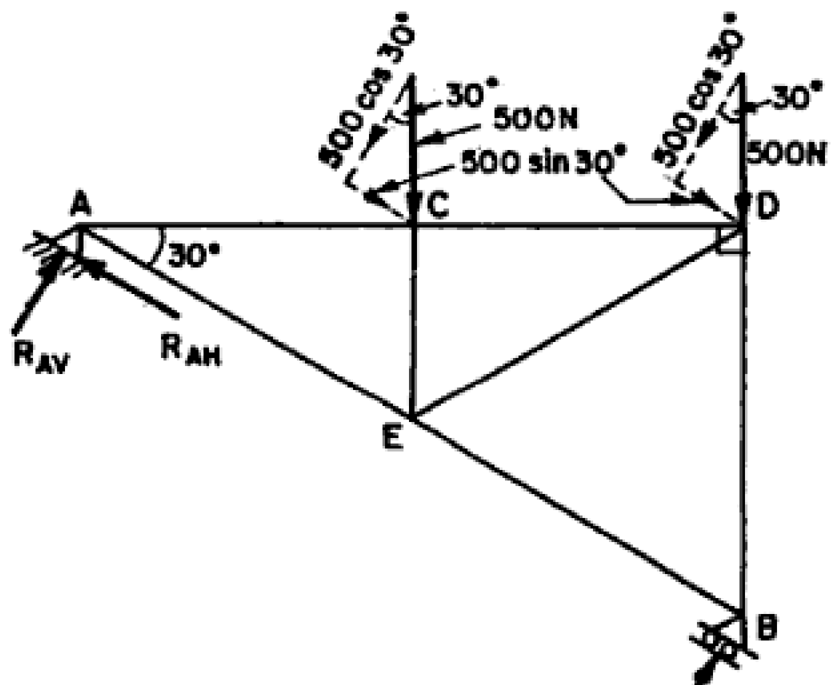


Fig. 5.25 (*b*)

Now sum of the components parallel to the inclined surface $AB = 500 \sin 30° + 500 \sin 30° = 250 + 250 = 500$ N acting from left to right.

Hence the value of the reaction component R_{AH} would be equal to 500 N and should act from right to left as shown.

To find R_B, take the moments of all forces about point A shown in Fig. 5.25 (*a*).

Hence applying $\Sigma M_A = 0$, we get

$$500 \times AC + 500 \times AD = R_B \times AB$$

But $AC = 5 \text{ cm}, AD = 10 \text{ cm}$

From ΔABD, $\cos 30° = \frac{AD}{AB}$.

Hence $AB = \frac{AD}{\cos 30°} = \frac{10 \times 2}{\sqrt{3}} = \frac{20}{\sqrt{3}}$

Hence above equation becomes,

$$500 \times 5 + 500 \times 10 = R_B \times \frac{20}{\sqrt{3}} \quad \text{or} \quad 2500 + 5000 = R_B \times \frac{20}{\sqrt{3}}$$

$\therefore$ $R_B = \frac{(2500 + 5000) \times \sqrt{3}}{20} = \mathbf{649.5 \text{ N.}}$ **Ans.**

Now from Fig. 5.25 (*b*), equating all the forces perpendicular to the inclined surface AB, we get

$$R_{AV} + R_B = 500 \times \cos 30° + 500 \cos 30°$$

or $R_{AV} + 649.5 = 1000 \cos 30° = 1000 \times \frac{\sqrt{3}}{2}$

$\therefore$ $R_{AV} = 1000 \times \frac{\sqrt{3}}{2} - 649.5 = 216.5$ N

But $R_{AH} = 500$ N

$\therefore$ $R_A = \sqrt{R_{AH}^2 + R_{AV}^2} = \sqrt{500^2 + 216.5^2} = \mathbf{545 \text{ N.}}$ **Ans.**

Problem 5.12. *Find reactions at supports of an L-bent shown in Fig. 5.26.*

Sol. Given :

Force at point D = 100 N at an angle of 30° with horizontal

Force at point C = 70 N at an angle of 45° with vertical

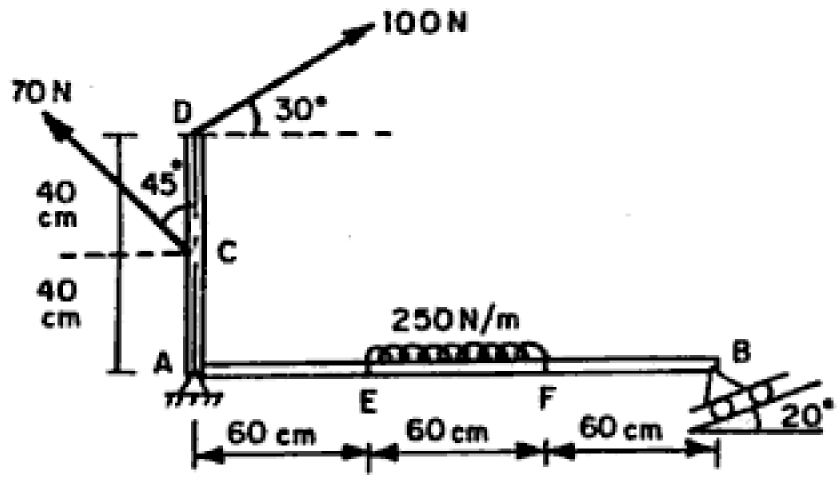


Fig. 5.26

Load on $EF = 250$ N/m $= 250 \times$ Length EF in metre

$= 250 \times 0.6 = 150$ N

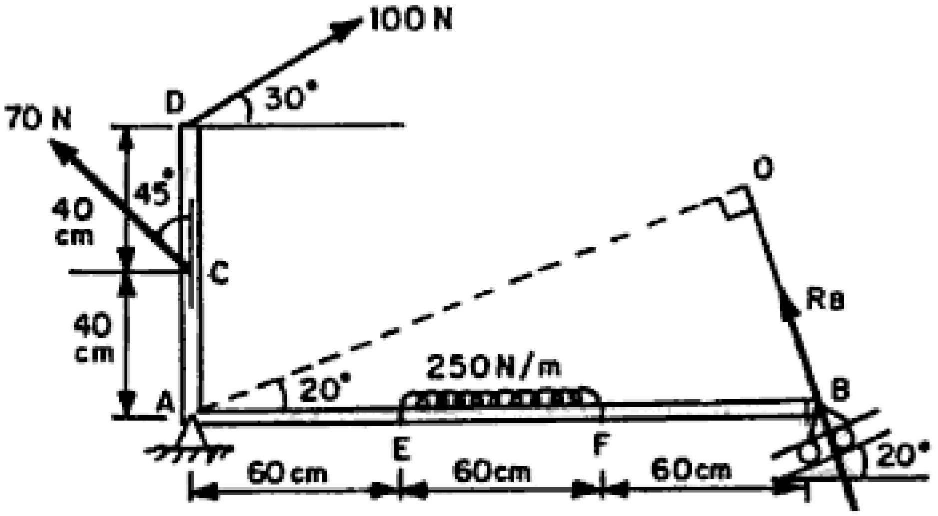


Fig. 5.27 (a)

The load on EF will be acting at the middle point of EF *i.e.*, at a distance of $0.6/2 = 0.3$ m from E or at a distance of $0.6 + 0.3 = 0.9$ m from A. The point B is placed on roller at an angle of 20° with the horizontal. Hence reaction at B will be normal to the surface of the roller.

The perpendicular distance from A on the line of action of $R_B = AO = AB \cos 20° = 180 \times \cos 20°$ cm $= 1.8 \cos 20°$ m as shown in Fig. 5.27 (a). For equilibrium of the beam, the moments of all forces about any point should be zero. Taking moments of all forces about point A, we get

[Horizontal component at D] $\times AD$ – [Horizontal component at C] $\times AC$ + Load on $EF \times 90 - R_B \times AO = 0$

or $(100 \cos 30) \times 80 - (70 \times \sin 45) \times 40 + 150 \times 90 - R_B \times 180 \cos 20 = 0$

(**Note.** The vertical components at D and C, pass through the point A. Hence moments of these vertical components about A are zero)

or $6928 - 1979.6 + 13500 - 169.14\, R_B = 0$

or $18448.4 = 169.14\, R_B$

$\therefore \quad R_B = \dfrac{18448.4}{169.14} = 109.07$

Let R_A = Reaction at the point A

The reaction at A can be resolved in two components *i.e.*, R_{Ax} and R_{Ay}.

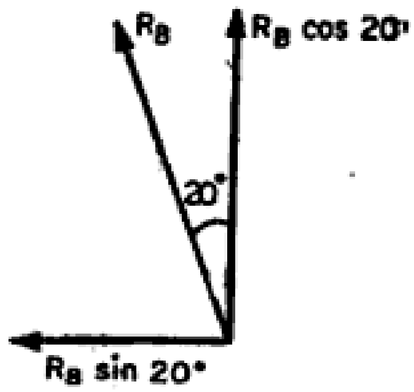


Fig. 5.27 (b)

For equilibrium, $\Sigma F_x = 0$

or $R_{Ax} + 100 \cos 30° - 70 \sin 45° - R_B \sin 20 = 0$

or $R_{Ax} = R_B \sin 20° + 70 \sin 45° - 100 \cos 30°$

$= 109.07 \times 0.342 + 70 \times 0.707 - 100 \times 0.866$

$= 37.3 + 49.49 - 86.6 = 0.19$ N

For equilibrium, $\Sigma F_y = 0$

or $R_{Ay} + 100 \sin 30° + 70 \cos 45° + R_B \cos 20° = 150$

or $R_{Ay} = 150 - 100 \sin 30° - 70 \cos 45° - R_B \cos 20°$

$= 150 - 50 - 49.49 - 109.07 \times 0.9396 = -51.98$ N

(– ve sign means, R_{Ay} will be acting vertically downward)

Refer to Fig. 5.27 (c)

$$\therefore \qquad R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2}$$

or
$$R_A = \sqrt{0.19^2 + (-51.98)^2}$$
$$= \sqrt{0.0361 + 2701.92}$$
$$= \textbf{51.9803 kN.} \quad \textbf{Ans.}$$

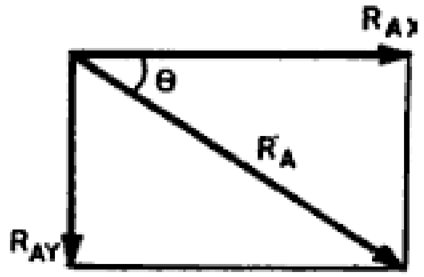


Fig. 5.27 (c)

The angle made by R_A with x-axis is given by

$$\tan\theta = \frac{R_{Ay}}{R_{Ax}} = \frac{51.98}{0.19} = 273.57$$

$$\therefore \qquad \theta = \tan^{-1} 273.57 = \mathbf{89.79°} \quad \textbf{Ans.}$$

5.8. PROBLEMS WHEN BEAMS ARE SUBJECTED TO COUPLES

In this section, the reactions of the beam will be calculated when beams are subjected to clockwise or anti-clockwise couple along with the other loads. While taking the moments about any point, the magnitude and sense of the couple is taken into consideration. But when the total load on the beam is calculated the magnitude and sense of the couple is not considered.

Problem 5.13. *A simply supported beam AB of 7 m span is subjected to : (i) 4 kN m clockwise couple at 2 m from A, (ii) 8 kN m anti-clockwise couple at 5 m from A and (iii) a triangular load with zero intensity at 2 m from A increasing to 4 kN per m at a point 5 m from A. Determine reactions at A and B.*

Sol. Given :

Span of beam = 7 m

Couple at C (*i.e.*, at 2 m from A) = 4 kN m (clockwise)

Couple at D (*i.e.*, at 5 m from A) = 8 kN m anti-clockwise)

Triangular load from C to D with :

Vertical Load at C = 0

Vertical Load at D = 4 kN/m

$\therefore$ Total load on beam = Area of triangle $CDE = \dfrac{CD \times DE}{2} = \dfrac{3 \times 4}{2} = 6$ kN

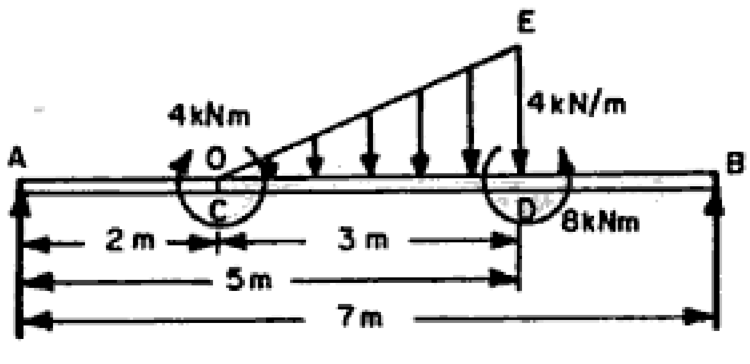


Fig. 5.28

This load will be acting at the C.G. of the ΔCDE *i.e.*, at a distance of $\dfrac{2}{3} \times CD = \dfrac{2}{3} \times 3 = 2$ m from C or 2 + 2 + 4 m from end A.

Let R_A = Reaction at A.

R_B = Reaction at B.

Taking the moments of all forces about point A and equating the resultant moment to zero (*i.e.*, $\Sigma M_A = 0$ and considering clockwise moment positive and anti-clockwise moment negative), we get

$$-R_B \times 7 + 4^* - 8^{**} + (\text{Total load on beam}) \times \text{Distance of total load from } A) = 0$$

or $$-7R_B + 4 - 8 + 6 \times 4 = 0$$

$$-7R_B + 4 - 8 + 24 = 0$$

or $$20 = 7R_B$$

or $$R_B = \frac{20}{7} \text{ kN.} \quad \textbf{Ans.}$$

Also for the equilibrium of the beam $\Sigma F_y = 0$

or $$R_A + R_B = \text{Total load on the beam} = 6 \text{ kN}$$

$\therefore$ $$R_A = 6 - R_B = 6 - \frac{20}{7} = \frac{22}{7} \text{ kN.} \quad \textbf{Ans.}$$

HIGHLIGHTS

1. The reaction at the knife edge support will be normal to the surface of the beam.
2. The reaction in case of roller support will be normal to the surface of roller base.
3. The reaction at the hinged end (or pinned end) will be either vertical or inclined depending upon the type of loading. If the load is vertical, then reaction will be vertical. But if the load is inclined, then the reaction will also be inclined.
4. For a smooth surface, the reaction is always normal to the support.
5. A load, acting at a point on a beam, is known as point load or concentrated load.
6. If each unit length of the beam carries same intensity of load, then that type of load is known as uniformly distributed load which is written as U.D.L.
7. The reactions of a beam can be determined by analytical method and graphical method.
8. The reactions by analytical method are obtained by using equations of equilibrium, *i.e.*, $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$.
9. The reactions by graphical method are obtained by drawing a space diagram and a vector diagram.
10. If a beam is loaded with inclined loads, then the inclined loads are resolved normal to the beam and along the beam. Now the equations of equilibrium are used for finding reactions.

EXERCISE 5

A. Theoretical Questions

1. Explain the term 'support reactions.' What are the different types of support ?
2. What is the difference between a roller support and a hinged support ?
3. What are the important types of loading on a beam ? Differentiate between uniformly distributed load and uniformly varying load on a beam.
4. Name the different methods of finding the reactions at the two supports of a beam.
5. A beam AB of length L is simply supported at the ends A and B. It carries two point loads W_1 and W_2 at a distance L_1 and L_2 from the end A respectively. How will you find the reactions R_A and R_B by analytical method.
6. Describe in details the different steps involved in finding the reactions of a beam by graphical method.
7. Define and explain an overhanging beam.
8. What is the main advantage of roller support in case of the steel trusses of the bridges ?

---

*The couple at C is 4 kN m clockwise. Hence its sense is positive.

**The couple at D is 8 kN m anti-clockwise. Hence its sense is negative. The couple is also moment.

(B) Numerical Problems

1. A simply supported beam of length 8 m carries point loads of 4 kN and 6 kN at a distance of 2 m and 4 m from the left end. Find the reactions at both ends analytically and graphically. [Ans. 6 kN, 4 kN]
2. A simply supported beam of length 8 m carries a uniformly distributed load of 10 kN/m for a distance of 4 m, starting from a point which is at a distance of 1 m from the left end. Calculate the reactions at both ends. [Ans. 25 kN, 15 kN]
3. A beam 6 m long is simply supported at the ends and carries a uniformly distributed load of 1.5 kN/m and three concentrated loads 1 kN, 2 kN and 3 kN acting respectively at a distance of 1.5 m, 3 m and 4.5 m from the left end. Calculate the reactions at both ends. [Ans. 7 kN, 8 kN]
4. A simply supported beam of span 10 m carries a uniformly varying load from zero at the left end to 1200 N/m at the right end. Calculate the reactions at both ends of the beam. [Ans. 2000 N and 4000 N]
5. A simply supported beam *AB* is subjected to a distributed load increasing from 1500 N/m to 4500 N/m from end *A* to end *B* respectively. The span *AB* = 6 m. Determine the reactions at the supports. [Ans. R_A = 7500 N, R_B = 10500 N]
6. An overhanging beam carries the loads as shown in Fig. 5.29. Calculate the reactions at both ends. [Ans. R_A = 1 kN, R_B = 6 kN]

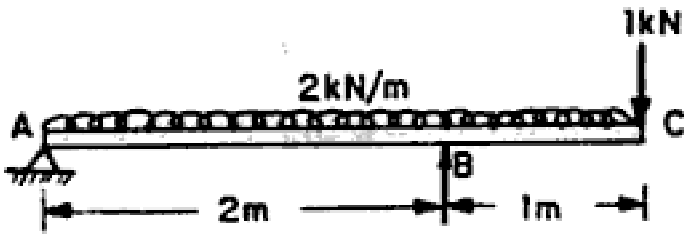


Fig. 5.29

7. An overhanging beam carries the loads as shown in Fig. 5.30. Calculate the reactions at both ends. [Ans. R_A = 10 kN, R_B = 11 kN]

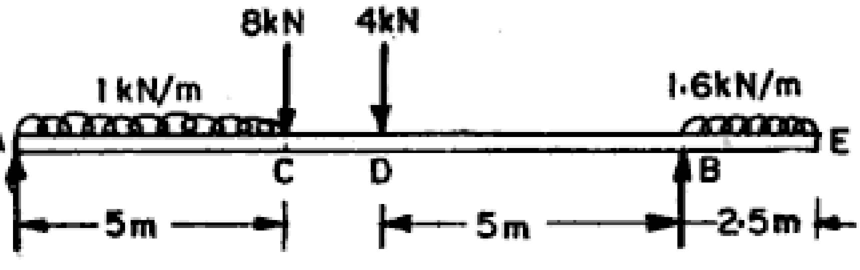


Fig. 5.30

8. A beam is loaded as shown in Fig. 5.31. Determine the reactions at both ends. [Ans. R_{AV} = 2.875 kN, R_{AH} = 5.196 kN ←, R_B = 7.125 kN]

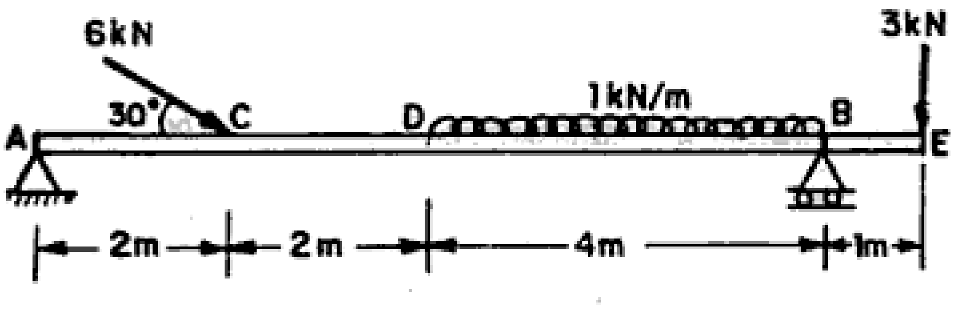


Fig. 5.31

9. A beam AB of span 6 m is hinged at A and supported on rollers at end B and carries load as shown in Fig. 5.32. Determine the reactions at A and B. [**Ans.** R_{AV} = 5.87 kN, R_{AH} = 3.222 kN ←, R_B = 7.3 kN]

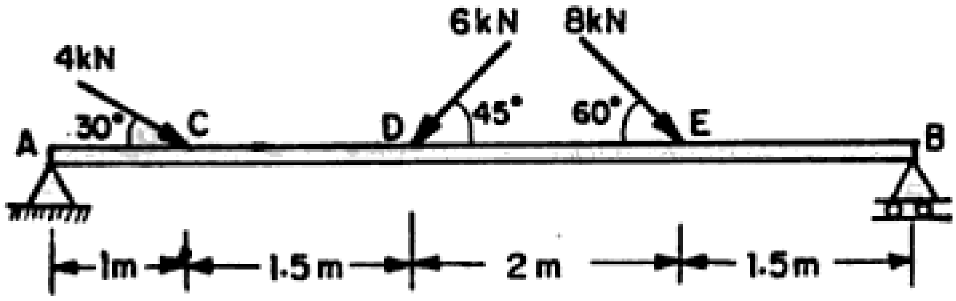


Fig. 5.32

10. A beam AB of span 8 m is subjected to the uniformly distributed load of 1 kN/m over the entire length and the moment 32 kN/m at C as shown in Fig. 5.33. Determine the reactions at the both ends. [**Ans.** R_A = 0, R_B = 8 kN]

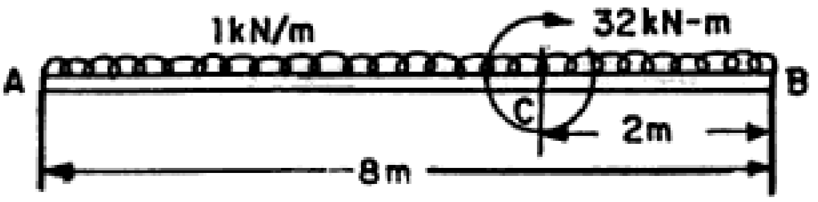


Fig. 5.33

11. A simply supported beam AB is subjected to a distributed load increasing from 1500 N/m to 4500 N/m from end A to end B. The span AB = 6 m. Determine the reactions at the supports.

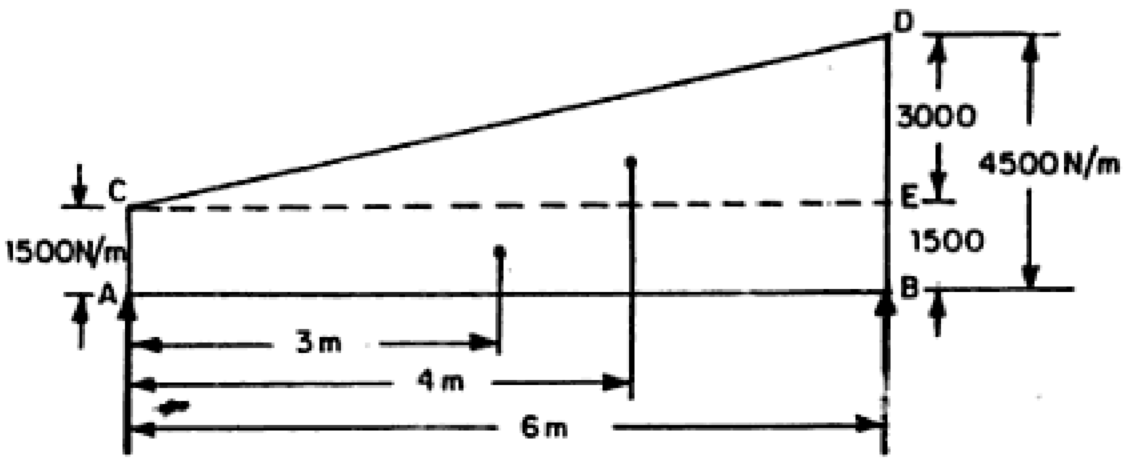


Fig. 5.34

[**Hint.** Area of rectangle = 1500 × 6 = 9000 N, Area of triangle = $\frac{3000 \times 6}{2}$ = 9000 N. C.G. of rectangle from

A = 3 m, C.G. of triangle from $A = \frac{2}{3} \times 6 = 4$ m.

$\Sigma M_A = 0$, $6\,R_B = 9000 \times 3 + 9000 \times 4 = 63000$

∴ R_B = 10500 N. R_A = (9000 + 9000) – 10500 = 7500 N]

Suppose we add a set of two members and a joint again, we get a perfect frame as shown in Fig. 6.2 (*b*). Hence for a perfect frame, the number of joints and number of members are given by,

$$n = 2j - 3$$

where n = Number of members, and

j = Number of joints.

6.2.2. Imperfect Frame. A frame in which number of members and number of joints are not given by

$$n = 2j - 3$$

is known, an imperfect frame. This means that number of members in an imperfect frame will be either more or less than $(2j - 3)$.

(*i*) If the number of members in a frame are less than $(2j - 3)$, then the frame is known as *deficient frame.*

(*ii*) If the number of members in a frame are more than $(2j - 3)$, then the frame is known as *redundant frame.*

6.3. ASSUMPTIONS MADE IN FINDING OUT THE FORCES IN A FRAME

The assumptions made in finding out the forces in a frame are :

(*i*) The frame is a perfect frame

(*ii*) The frame carries load at the joints

(*iii*) All the members are pin-joined.

6.4. REACTIONS OF SUPPORTS OF A FRAME

The frames are generally supported

(*i*) on a roller support or

(*ii*) on a hinged support.

If the frame is supported on a roller support, then the line of action of the reaction will be at right angles to the roller base as shown in Figs. 6.3 and 6.4.

If the frame is supported on a hinged support, then the line of action of the reaction will depend upon the load system on the frame.

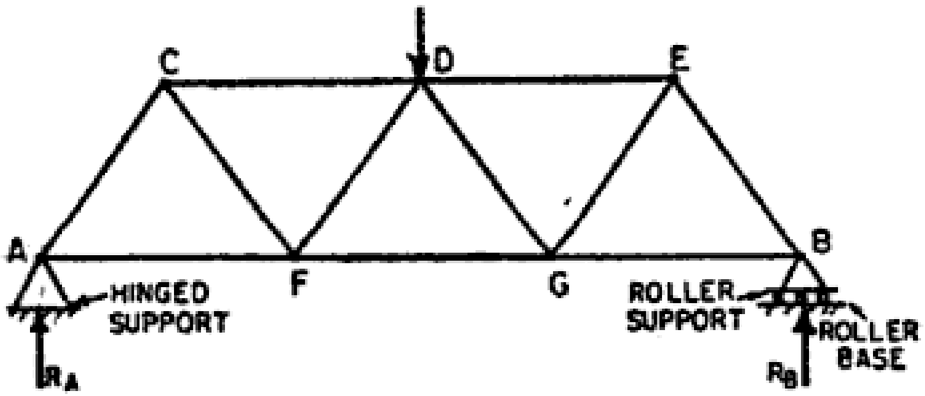


Fig. 6.3

The reactions at the supports of a frame are determined by the conditions of equilibrium. The external load on the frame and the reactions at the supports must form a system of equilibrium.

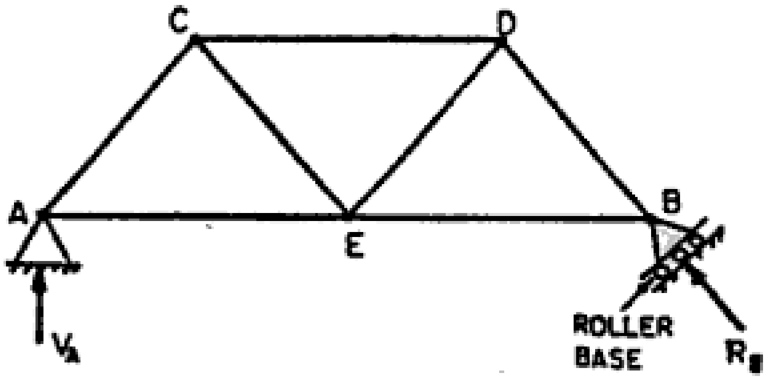


Fig. 6.4

6.5. ANALYSIS OF A FRAME

Analysis of a frame consists of :

(*i*) Determinations of the reactions at the supports and

(*ii*) Determination of the forces in the members of the frame.

The reactions are determined by the condition that the applied load system and the induced reactions at the supports form a system in equilibrium.

The forces in the members of the frame are determined by the condition that every joint should be in equilibrium and so, the forces acting at every joint should form a system in equilibrium.

A frame is analysed by the following methods :

(*i*) Method of joints,

(*ii*) Method of sections and

(*iii*) Graphical method.

6.5.1. Method of Joints. In this method, after determining the reactions at the supports, the equilibrium of every joint is considered. This means the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown. The force in the member will be compressive if the member *pushes the joint* to which it is connected whereas the force in the member will be tensile if the member *pulls the joint* to which it is connected.

Problem 6.1. *Find the forces in the members AB, AC and BC of the truss shown in Fig. 6.5.*

Sol. First determine the reactions R_B and R_C. The line of action of load of 20 kN acting at A is vertical. This load is at a distance of $AB \times \cos 60°$ from the point B. Now let us find the distance AB.

The triangle ABC is a right-angled triangle with angle $BAC = 90°$. Hence AB will be equal to $BC \times \cos 60°$.

$$\therefore \quad AB = 5 \times \cos 60° = 5 \times \frac{1}{2} = 2.5 \text{ m}$$

Now the distance of line of action of 20 kN from B is $AB \times \cos 60°$ or $2.5 \times \frac{1}{2} = 1.25$ m.

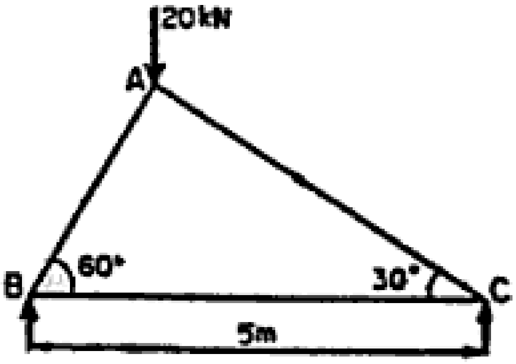


Fig. 6.5

Taking the moments about B, we get

$$R_C \times 5 = 20 \times 1.25 = 25$$

$$\therefore \quad R_C = \frac{25}{5} = 5 \text{ kN}$$

and $$R_B = \text{Total load} - R_C = 20 - 5 = 15 \text{ kN}$$

Now let us consider the equilibrium of the various joints.

Joint B

Let F_1 = Force in member AB

F_2 = Force in member BC

Let the force F_1 is acting towards the joint B and the force F_2 is acting away* from the joint B as shown in Fig. 6.6. (The reaction R_B is acting vertically up. The force F_2 is horizontal. The reaction R_B will be balanced by the vertical component of F_1. The vertical component of F_1 must act downwards to balance R_B. Hence F_1 must act towards the joint B so that its vertical component is downward. Now the horizontal component of F_1 is towards the joint B. Hence force F_2 must act away from the joint to balance the horizontal component of F_1).

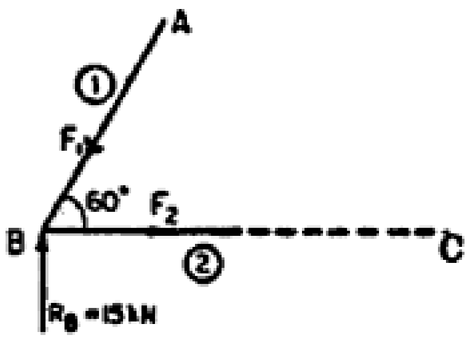


Fig. 6.6

Resolving the forces acting on the joint B, vertically

$$F_1 \sin 60° = 15$$

$$\therefore \quad F_1 = \frac{15}{\sin 60°} = \frac{15}{0.866} = 17.32 \text{ kN (Compressive)}$$

As F_1 is pushing the joint B, hence this force will be compressive. Now resolving the forces horizontally, we get

$$F_2 = F_1 \cos 60° = 17.32 \times \frac{1}{2} = 8.66 \text{ kN (tensile)}$$

As F_2 is pulling the joint B, hence this force will be tensile.

Joint C

Let F_3 = Force in the member AC

F_2 = Force in the member BC

The force F_2 has already been calculated in magnitude and direction. We have seen that force F_2 is tensile and hence it will pull the joint C. Hence it must act away from the joint C as shown in Fig. 6.7.

Resolving forces vertically, we get

$$F_3 \sin 30° = 5 \text{ kN}$$

$$\therefore \quad F_3 = \frac{5}{\sin 30°} = 10 \text{ kN} \quad \text{(Compressive)}$$

As the force F_3 is pushing the joint C, hence it will be compressive.

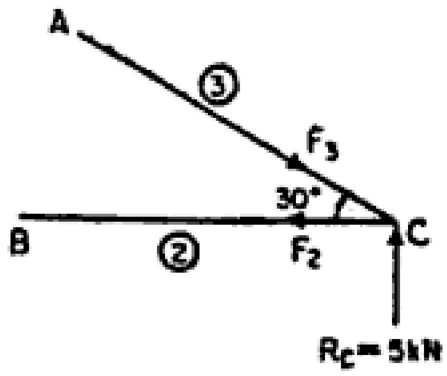


Fig. 6.7

Problem 6.2. *A truss of span 7.5 m carries a point load of 1 kN at joint D as shown in Fig. 6.8. Find the reactions and forces in the members of the truss.*

Sol. Let us first determine the reactions R_A and R_B

Taking moments about A, we get $R_B \times 7.5 = 5 \times 1$

$$\therefore \quad R_B = \frac{5}{7.5} = \frac{2}{3} = 0.667 \text{ kN}$$

*The direction of F_2 can also be taken towards the joint B. Actually when we consider the equilibrium of the joint B, if the magnitude of F_1 and F_2 comes out to be positive then the assumed direction of F_1 and F_2 are correct. But if any one of them is having a negative magnitude then the assumed direction of that force is wrong. Correct direction then will be the reverse of the assumed direction.

Joint F

The forces F_{FA} and F_{FC} are known in magnitude and directions. The assumed directions of the forces F_{DF} and F_{GF} are shown in Fig. 6.23 (*d*).

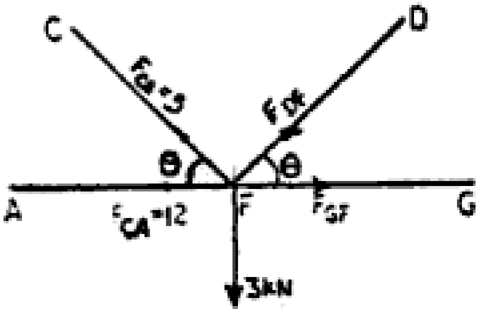


Fig. 6.23 (*d*)

Resolving the forces vertically, we get

$$5 \times \sin\theta + F_{DF}\sin\theta = 3$$

or
$$F_{DF} = -\frac{5\sin\theta + 3}{\sin\theta}$$

$$= -5 + \frac{3}{\sin\theta} = -5 + \frac{3}{0.6} = -5 + 5 = 0$$

Resolving the forces horizontally, we get

$$12 + 5\cos\theta = F_{GF} + F_{DF}\cos\theta$$

or
$$12 + 5 \times 0.8 = F_{GF} + 0 \quad \text{or} \quad 12 + 4 = F_{GF}$$

$\therefore$
$$F_{GF} = 12 + 4 = 16 \text{ kN (Tensile)}$$

Now consider the joint *D*.

Joint D

The forces F_{DC} and F_{FD} are known in magnitude and direction. The assumed directions of F_{DG} and F_{DE} are shown in Fig. 6.23 (*e*).

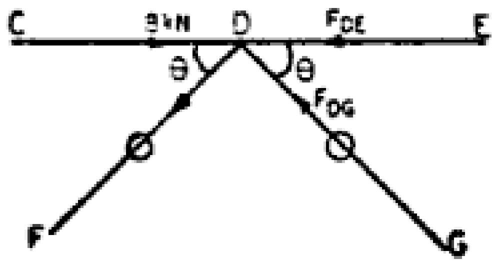


Fig. 6.23 (*e*)

Resolving vertically, we get

$$F_{DG}\sin\theta = F_{DF} \times \sin\theta = 0$$

$\therefore$
$$F_{DG} = 0$$

Resolving forces horizontally, we get

$$F_{DE} = F_{CD} = 8 \text{ kN}$$

$\therefore$
$$F_{DE} = 8 \text{ kN (Compressive)}$$

Now consider the joint *G*.

Joint G

The forces F_{DG} and F_{FG} are known in magnitude and direction. The assumed directions of F_{GE} and F_{GB} are shown in Fig. 6.23 (*f*).

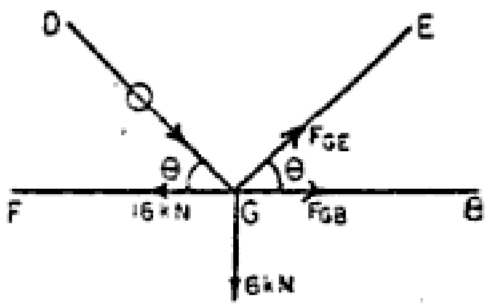


Fig. 6.23 (*f*)

Resolving the forces vertically, we get

$$F_{GE}\sin\theta = F_{DG}\sin\theta + 6 = 6$$

or
$$F_{GE} = \frac{6}{\sin\theta} = \frac{6}{0.6}$$

$$= 10 \text{ kN (Tensile)}$$

Resolving forces horizontally,

$$F_{GB} = 16 - F_{GE}\cos\theta$$

$$= 16 - 10 \times 0.8 = 8 \text{ kN} \qquad \text{(Tensile)}$$

The truss is hinged at A and hence the support reactions at A will consists of a horizontal reaction H_A and a vertical reaction R_A.

Now length $AC = 4 \times \cos 30 = 4 \times 0.866 = 3.464$ m

and length $AD = 2 \times AC = 2 \times 3.464 = 6.928$ m

Now taking moments about A, we get

$$R_B \times 12 = 2 \times AC + 1 \times AD + 1 \times AE$$
$$= 2 \times 3.464 + 1 \times 6.928 + 1 \times 4 = 17.856$$

$$\therefore \quad R_B = \frac{17.856}{12} = 1.49 \text{ kN}$$

Total vertical components of inclined loads

$$= (1 + 2 + 1) \times \sin 60° + 1.0$$
$$= 4 \times 0.866 + 1.0 = 4.464 \text{ kN}$$

Total horizontal components of inclined loads

$$= (1 + 2 + 1) \cos 60° = 4 \times 0.5 = 2 \text{ kN}$$

Now R_A = Vertical components of inclined loads $- R_B$

$$= 4.464 - 1.49 = 2.974 \text{ kN} (\uparrow)$$

and H_A = Sum of all horizontal components = 2 kN

Now the forces in the members can be calculated.

Consider the equilibrium of joint A.

Joint A

Let F_{AE} = Force in member AE

and F_{AC} = Force in member AC

Their directions are assumed as shown in Fig. 6.24 (*a*).

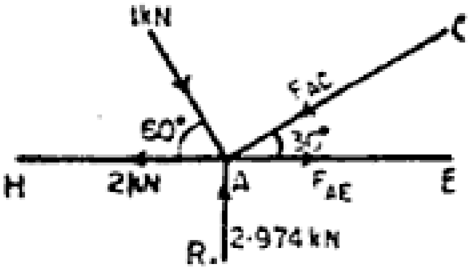


Fig. 6.24 (*a*)

Resolving the forces vertically, we get

$$F_{AC} \times \sin 30° + 1 \times \sin 60° = 2.974$$

or $$F_{AC} \times 0.5 + 0.866 = 2.974$$

$$\therefore \quad F_{AC} = \frac{2.974 - 0.866}{0.5}$$
$$= 4.216 \text{ kN (Compressive)}$$

Resolving the forces horizontally, we get

$$F_{AE} = 2 + F_{AC} \cos 30° - 1 \times \cos 60°$$
$$= 2 + 4.216 \times 0.866 - 0.5 = 5.15 \text{ kN (Tensile)}$$

Now consider the joint C.

Joint C

From Fig. 6.24 (*b*), we have

$$F_{CD} = F_{AC} = 4.216 \quad \text{(Compressive)}$$

and $$F_{CE} = 2 \text{ kN} \quad \text{(Compressive)}$$

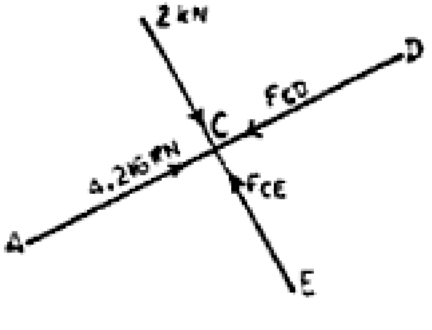


Fig. 6.24 (*b*)

Now consider joint E.

Joint E [See Fig. 6.24 (*c*)]

Resolving forces vertically, we get

$$1 + 2 \times \sin 60° = F_{ED} \times \sin 60°$$

or $$F_{ED} = 2 + \frac{1}{\sin 60} = 3.155 \quad \text{(Tensile)}$$

Resolving forces horizontally, we get

$$5.15 - 2 \times \cos 60° - F_{ED} \cos 60° - F_{EF} = 0$$

or
$$5.15 - 2 \times \frac{1}{2} - 3.15 \times \frac{1}{2} - F_{EF} = 0$$

$$F_{EF} = 5.15 - 1 - 1.57 = 2.58 \text{ kN}$$

(Tensile)

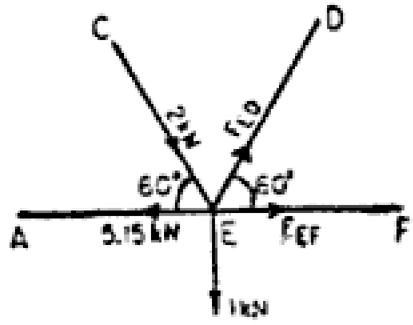


Fig. 6.24 (*c*)

At the joint *G*, two forces, *i.e.*, F_{BG} and F_{DG} are in the same straight line and hence the third force, *i.e.*, F_{GF} should be zero.

$\therefore \quad F_{GF} = 0$

Now consider the joint *F*.

Joint F [See Fig. 6.24 (*d*)]

Resolving forces vertically, we get

$$F_{DF} \times \sin 60° = 0$$

$$\therefore \quad F_{DF} = 0$$

Resolving horizontally, we get

$$F_{FB} = F_{EF} = 2.58 \text{ kN}$$

$$\therefore \quad F_{FB} = 2.58 \text{ kN} \quad \text{(Compressive)}$$

Now consider the joint *B*.

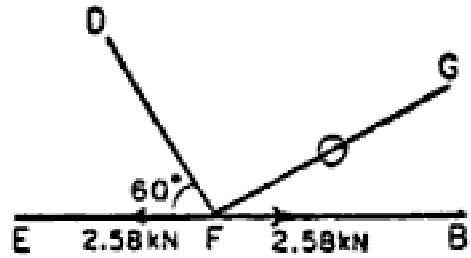


Fig. 6.24 (*d*)

Joint B

Resolving vertically, we get

$$F_{BG} \times \sin 30° = 1.49$$

$$\therefore \quad F_{BG} = \frac{1.49}{0.5} = 2.98 \text{ kN}$$

(Compressive)

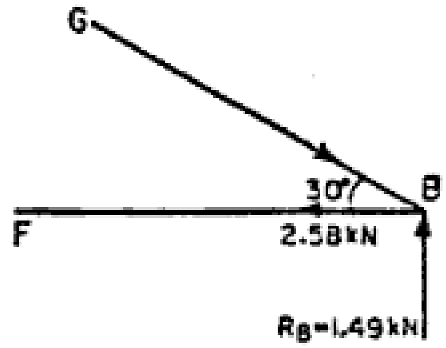


Fig. 6.24 (*e*)

Joint G

$$F_{GD} = F_{BG} = 2.98 \text{ kN (Compressive)}$$

The forces are shown in a tabular form as

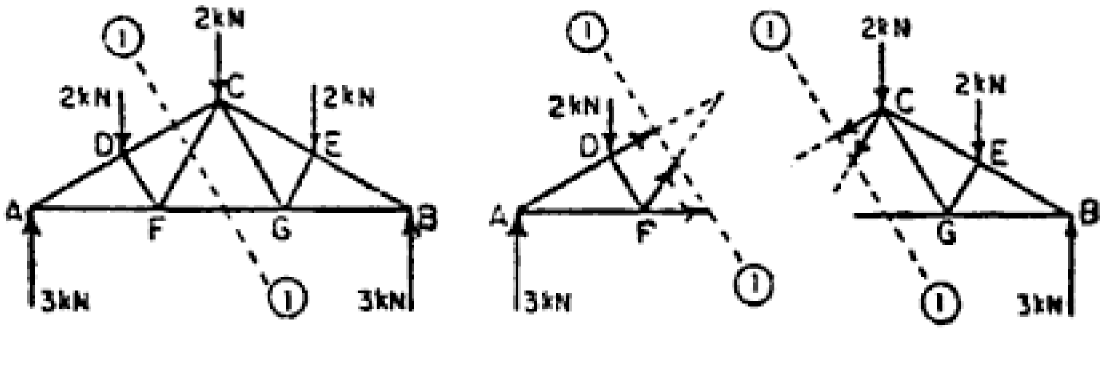
Member	*Force in the member*	*Nature of force*
AC	4.216 kN	Compressive
AE	5.15 kN	Tensile
CE	2 kN	Compressive
CD	4.216 kN	Compressive
ED	3.155 kN	Tensile
EF	2.58 kN	Tensile
DF	0	Nil
DG	2.98 kN	Compressive
GB	2.98 kN	Compressive
FB	2.58 kN	Compressive
FG	0	Nil

6.6. METHOD OF SECTIONS

When the forces in a few members of a truss are to be determined, then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

In this method, a section line is passed through the members, in which forces are to be determined as shown in Fig. 6.25. The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown. The part of the truss, on any one side of the section line, is treated as a free body in equilibrium under the action of external forces on that part and forces in the members cut by the section line. The unknown forces in the members are then determined by using equations of equilibrium as

$$\Sigma F_x = 0, \quad \Sigma F_y = 0 \quad \text{and} \quad \Sigma M = 0.$$



(*a*) Given Truss (*b*) Left Part (*c*) Right Part

Fig. 6.25

If the magnitude of the forces, in the members cut by a section line, is positive then the assumed direction is correct. If magnitude of a force is negative, then reverse the direction of that force.

Problem 6.11. *Find the forces in the members AB and AC of the truss shown in Fig. 6.26 using method of section.*

Sol. First determine the reaction R_B and R_C.

The distance of line of action of 20 kN from point B is $AB \times \cos 60°$ or $2.5 \times \frac{1}{2} = 1.25$ m

Taking moments about point B, we get

$$R_C \times 5 = 20 \times 1.25$$

$$\therefore \quad R_C = \frac{20 \times 1.25}{5} = 5 \text{ kN}$$

and $$R_B = 20 - 5 = 15 \text{ kN}$$

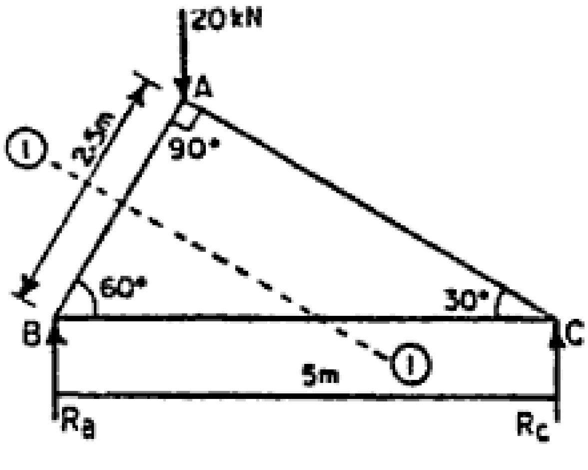


Fig. 6.26

Now draw a section line (1.1), cutting the members AB and BC in which forces are to be determined. Now consider the equilibrium of the left part of the truss. This part is shown in Fig. 6.27.

Let the directions of F_{BA} and F_{BC} are assumed as shown in Fig. 6.27.

Now taking the moments of all the forces acting on the left part about point C, we get

$$15 \times 5 + (F_{BA} \times AC)^* = 0$$

($\because$ The perpendicular distance between the line of action of F_{BA} and point C is equal to AC)

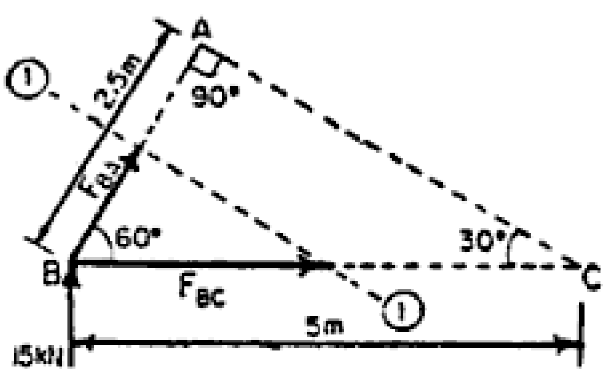


Fig. 6.27

*The moment of the force F_{BA} about point C, is also taken by resolving the force F_{BA} into vertical and horizontal components at point B. The moment of the horizontal component about C is zero, whereas the moment of vertical component will be $(F_{BA} \times \sin 60°) \times 5 = F_{BA} \times 5 \times \sin 60°$ or $F_{BA} \times 5 \times \cos 30°$. ($\because$ $\sin 60° = \cos 30°$)

or $$75 + F_{BA} \times 5 \times \cos 30° = 0 \qquad (\because \; AC = BC \times \cos 30°)$$

or $$F_{BA} = \frac{-75}{5 \times \cos 30°} = -17.32 \text{ kN}$$

The negative sign shown that F_{BA} is acting in the opposite direction (*i.e.*, towards point B). Hence force F_{BA} will be a compressive force.

$$\therefore \qquad F_{BA} = 17.32 \text{ kN (Compressive).} \quad \textbf{Ans.}$$

Again taking the moments of all the forces acting on the left part about point A, we get

15 × Perpendicular distance between the line of action of

15 kN and point C = F_{BC} × Perpendicular distance between F_{BC} and point A

$$15 \times 2.5 \times \cos 60° = F_{BC} \times 2.5 \times \sin 60°$$

$$\therefore \qquad F_{BC} = \frac{15 \times 2.5 \times \cos 60°}{2.5 \times \sin 60°} = \frac{15 \times 0.5}{0.866}$$

$$= 8.66 \text{ kN (Tensile).} \quad \textbf{Ans.}$$

These forces are same as obtained in Problem 6.1.

Problem 6.12. *A truss of span 5 m is loaded as shown in Fig. 6.28. Find the reactions and forces in the members marked 4, 5 and 7 using method of section.*

Sol. Let us first determine the reactions R_A and R_B.

Triangle ABD is a right-angled triangle having angle

$$ADB = 90°$$

$$\therefore \qquad AD = AB \cos 60° = 5 \times 0.5 = 2.5 \text{ m}$$

The distance of line of action the vertical load 10 kN from point A will be $AD \cos 60°$ or $2.5 \times 0.5 = 1.25$ m.

From triangle ACD, we have

$$AC = AD = 2.5 \text{ m}$$

$$\therefore \qquad BC = 5 - 2.5 = 2.5 \text{ m}$$

In right-angled triangle CEB, we have

$$BE = BC \cos 30° = 2.5 \times \frac{\sqrt{3}}{2}$$

∴ The distance of line of action of vertical load 12 kN from point B will be $BE \cos 30°$ or $BE \times \frac{\sqrt{3}}{2}$

$$= \left(2.5 \times \frac{\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{2}$$

$$= 1.875 \text{ m}$$

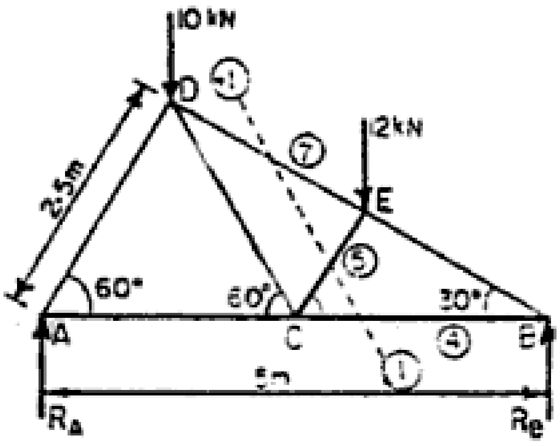


Fig. 6.28

∴ The distance of the line of action of the load of 12 kN from point A will be

$$(5 - 1.875) = 3.125 \text{ m}$$

Now taking the moments about A, we get

$$R_B \times 5 = 10 \times 1.25 + 12 \times 3.125 = 50$$

$$\therefore \qquad R_B = \frac{50}{5} = 10 \text{ kN} \quad \text{and} \quad R_A = (10 + 12) - 10 = 12 \text{ kN}$$

Now draw a section line (1.1), cutting the members 4, 5 and 7 in which forces are to be determined. Consider the equilibrium of the right part of the truss (because it is smaller than the left part).

This part is shown in Fig. 6.29. Let F_4, F_5 and F_7 are the forces in members 4, 5 and 7. Let their directions are assumed as shown in Fig. 6.29.

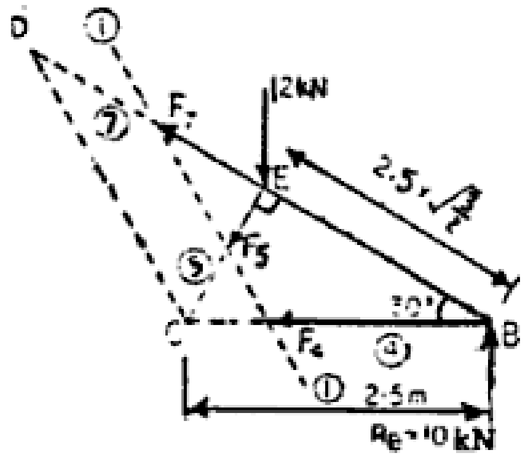


Fig. 6.29

Now taking the moments of all the forces acting on the right part about point E, we get

$$R_B \times BE \cos 30° = F_4 \times (BE \times \sin 30°)$$

or $$10 \times \left(2.5 \times \frac{\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{2} = F_4 \times 2.5 \times \frac{\sqrt{3}}{2} \times 0.5$$

or $$10 \times \frac{\sqrt{3}}{2} = F_4 \times 0.5$$

$$\therefore \quad F_4 = 10 \times \frac{\sqrt{3}}{2} \times \frac{1}{0.5} = 17.32 \text{ kN (Tensile).}$$

Now taking the moments of all the forces about point B acting on the right part, we get

$$12 \times BE \cos 30° + F_5 \times BE = 0$$

or $$12 \times \cos 30° + F_5 = 0$$

$$\therefore \quad F_5 = -12 \times \cos 30° = -10.392 \text{ kN}$$

– ve sign indicates that F_5 is compressive.

$$\therefore \quad F_5 = 10.392 \text{ kN (Compressive).} \quad \textbf{Ans.}$$

Now taking the moments about point C of all the forces acting on the right parts, we get

$$12 \times (2.5 - BE \cos 30°) = F_7 \times CE + R_B \times BC$$

or $$12 \times \left(2.5 - 2.5 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) = F_7 \times 2.5 \times \sin 30° + 10 \times 2.5$$

or $$12 \times (2.5 - 1.875) = F_7 \times 1.25 + 25 \quad \text{or} \quad 7.5 = 1.25F_7 + 25$$

or $$F_7 = \frac{7.5 - 25}{1.25} = -14 \text{ kN}$$

Negative sign shows that F_7 is compressive.

$$\therefore \quad F_7 = 14 \text{ kN (Compressive).} \quad \textbf{Ans.}$$

These forces are same as obtained in Problem 6.3.

Problem 6.13. *A truss of span 9 m is loaded as shown in Fig. 6.30. Find the reactions and forces in the members marked 1, 2 and 3.*

Sol. Let us first calculate the reactions R_A and R_B.

Taking moments about A, we get

$$R_B \times 9 = 9 \times 3 + 12 \times 6$$
$$= 27 + 72 = 99$$

$$\therefore \quad R_B = \frac{99}{9} = 11 \text{ kN}$$

and $$R_A = (9 + 12) - 11 = 10 \text{ kN}$$

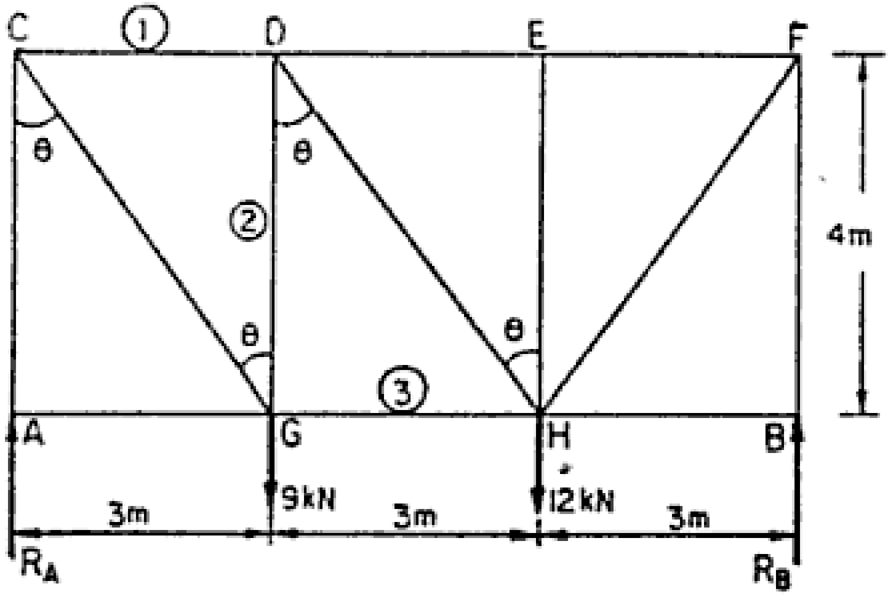


Fig. 6.30

Now draw a section line (1-1), cutting the members 1, 2 and 3 in which forces are to be determined. Consider the equilibrium of the left part of the truss (because it is smaller than the right part). This part is shown in Fig. 6.30 (*a*). Let F_1, F_2 and F_3 are the forces members 1, 2 and 3 respectively. Let their directions are assumed as shown in Fig. 6.30 (*a*).

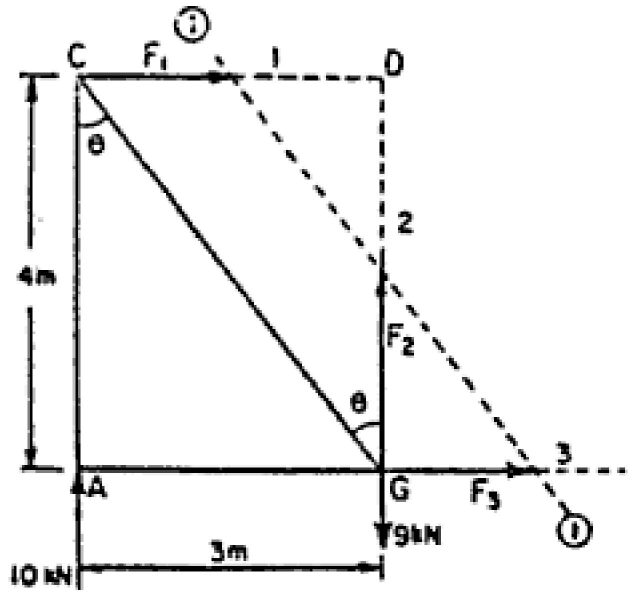


Fig. 6.30 (*a*)

Taking moments of all the forces acting on the left part about point *D*, we get

$$10 \times 3 = F_2 \times 4$$

$$\therefore \qquad F_3 = \frac{10 \times 3}{4}$$

$$= 7.5 \text{ kN (Tensile)}. \quad \textbf{Ans.}$$

Now taking the moments of all the forces acting on the left part about point *G*, we get

$$10 \times 3 + F_1 \times 4 = 0$$

$$\therefore \qquad F_1 = \frac{-30}{4} = -7.5 \text{ kN}$$

Negative sign shows that force F_1 is compressive.

$$\therefore \qquad F_1 = 7.5 \text{ kN (Compressive)}. \quad \textbf{Ans.}$$

Now taking the moments about the point *C*, we get

$$F_2 \times 3 - 9 \times 3 + F_3 \times 4 = 0$$

or $$F_2 \times 3 - 27 + 7.5 \times 4 = 0 \qquad (\because \; F_3 = 7.5)$$

or $$F_2 = \frac{27 - 7.5 \times 4}{3} = \frac{-3}{3} = -1.0 \text{ kN}$$

Negative sign shows that force F_2 is compressive.

$$\therefore \qquad F_2 = 1.0 \text{ kN (Compressive)}. \quad \textbf{Ans.}$$

Problem 6.14. *For the pin-joined truss shown in Fig. 6.31, find the forces in the members marked 1, 2 and 3 with the single load of 80 kN as shown.*

Now taking the moments about A, we get

$$R_B \times 12 = 2 \times AC + 1 \times AD + 1 \times AE$$

$$= 2 \times 3.464 + 1 \times 6.928 + 1 \times 4 = 17.856$$

$$\therefore \quad R_B = \frac{17.856}{12} = 1.49 \text{ kN}$$

Now draw the section line (1-1), passing through members DG, DF and EF in which the forces are to be determined. Consider the equilibrium of the right part of the truss. This part is shown in Fig. 6.33 (*a*). Let F_{DG}, F_{FD} and F_{EF} are the forces in members DG, FD and EF respectively. Let their directions are assumed a shown in Fig. 6.33 (*a*). Taking moments of all forces acting on right part about point F, we get

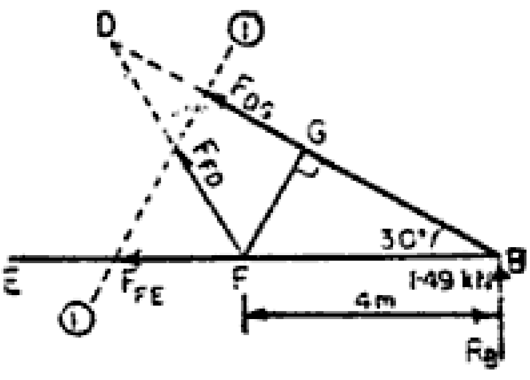


Fig. 6.33 (*a*)

$$R_B \times 4 + F_{DG} \times FG = 0$$

or $$1.49 \times 4 + F_{DG} \times (4 \times \sin 30°) = 0$$

$$(\because \quad FG = 4 \times \sin 30°)$$

or $$F_{DG} = \frac{-1.49 \times 4}{4 \times \sin 30°} = -2.98 \text{ kN}$$

– ve sign shows that the force F_{DG} is compressive.

$$\therefore \quad F_{DG} = 2.98 \text{ kN (Compressive).} \quad \textbf{Ans.}$$

Now taking the moments about point D, we get

$$R_B \times BD \cos 30 = F_{FE} \times BD \times \sin 30$$

or $$R_B \times \cos 30 = F_{FE} \times \sin 30$$

$$\therefore \quad F_{FE} = \frac{1.49 \times \cos 30}{\sin 30} = \frac{1.49 \times 0.866}{0.5}$$

$$= 2.58 \text{ kN} \quad \text{(Tensile).} \quad \textbf{Ans.}$$

Now taking the moments of all forces acting on the right part about B, we get

$F_{FD} \times \perp$ distance between F_{FD} and $B = 0$

$\therefore \quad F_{FD} = 0.$ **Ans.** $(\because \perp$ distance between F_{FD} and B is not zero)

6.7. GRAPHICAL METHOD

The force in a perfect frame can also be determined by a graphical method. The analytical methods (such as method of joints and method of sections) give absolutely correct results, but sometimes it is not possible to get the results from analytical methods. Then a graphical method can be used conveniently to get the results. The graphical method also provides reasonable accurate results.

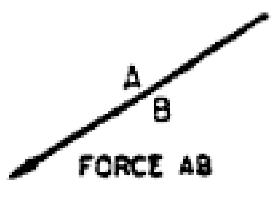


Fig. 6.34

The naming of the various members of a frame are done according to *Bow's notations*. According to this notation of force is designated by two capital letters which are written on either side of the line of action of the force. A force with letters A and B on either side of the line of action is shown in Fig. 6.34. This force will be called AB.

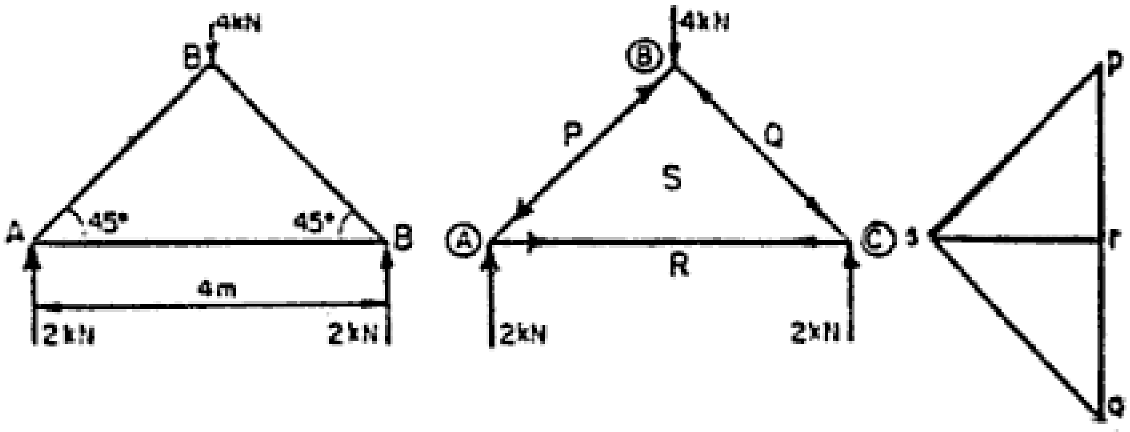
The following steps are necessary for obtaining a graphical solution of a frame.

(*i*) Making a space diagram

(*ii*) Constructing a vector diagram

(*iii*) Preparing a force table.

1. Making a space diagram. The given truss or frame is drawn accurately according to some linear scale. The loads and support reactions in magnitude and directions are also shown on the frame. Then the various members of the frame are named according to Bow's notation. Fig. 6.35 (*a*) shows a given truss and



(*a*) Given Diagram (*b*) Space Diagram (*c*) Vector Diagram

Fig. 6.35

the forces in the members *AB, BC* and *AC* are to be determined. Fig. 6.35 (*b*) shows the space diagram to same linear scale. The member *AB* is named as *PS* and so on.

2. Constructing a vector diagram. Fig. 6.35 (*c*) shows a vector diagram, which is drawn as given below :

(*i*) Take any point *p* and draw *pq* parallel to *PQ* vertically downwards. Cut *pq* = 4 kN to same scale.

(*ii*) Now from *q* draw *qr* parallel to *QR* vertically upwards and cut *qr* = 2 kN to the same scale.

(*iii*) From *r* draw *rp* parallel to *RP* vertically upwards and cut *rp* = 2 kN to the same scale.

(*iv*) Now from *p*, draw a line *ps* parallel to *PS* and from *r*, draw a line *rs* parallel to *RS*, meeting the first line at *s*. This is vector diagram for joint (*A*). Similarly the vector diagrams for joint (*B*) and (*C*) can be drawn.

3. Preparing a force table. The magnitude of a force in a member is known by the length of the vector diagram for the corresponding member, *i.e.*, the length *ps* of the vector diagram will give the magnitude of force in the member *PS* of the frame.

Nature of the force (*i.e.*, tensile or compressive) is determined according to the following procedure :

(*i*) In the space diagram, consider any joint. Move round that joint in a *clockwise direction.* Note the order of two capital letters by which the members are named. For example, the members at the joint (*A*) in space diagram Fig. 6.35 (*b*) are named as *PS, SR* and *RP*.

(*ii*) Now consider the vector diagram. Move on the vector diagram in the order of the letters (*i.e., ps, sr* and *rp*).

(*iii*) Now mark the arrows on the members of the space diagram of that joint (here joint *A*).

(*iv*) Similarly, all the joints can be considered and arrows can be marked.

(*v*) If the arrow is pointing towards the joint, then the force in the member will be compressive whereas if the arrow is away from the joint, then the force in the member will be tensile.

Problem 6.17. *Find the forces in the members AB, AC and BC of the truss shown in Fig. 6.36.*

Sol. First determine the reactions R_B and R_C.

From Fig. 6.36 (*a*), $AB = BC \times \cos 60° = 5 \times \frac{1}{2} = 2.5$ m

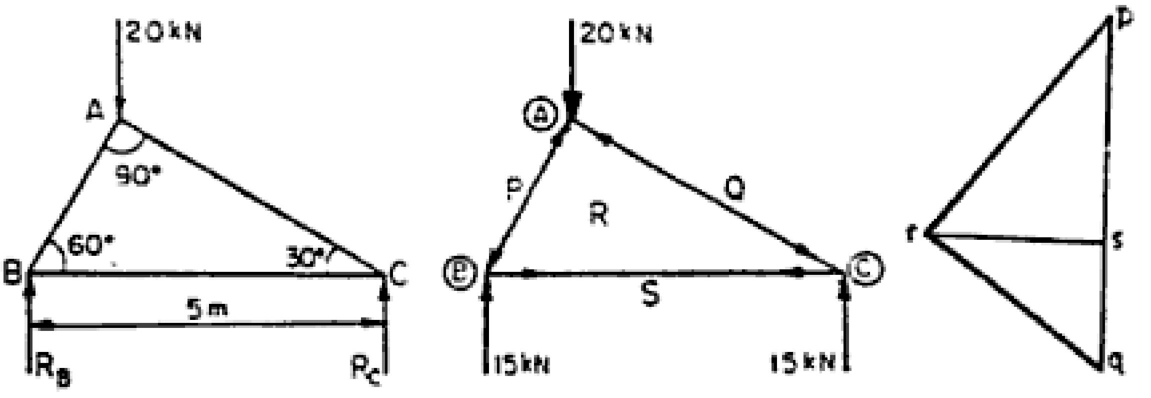
Distance of line of action of 20 kN from point B

$$= AB \cos 60^\circ = 2.5 \times \frac{1}{2} = 1.25 \text{ m}$$

Now taking moments about B, we get

$$R_C \times 5 = 20 \times 1.25 = 25$$

$\therefore$ $$R_C = \frac{25}{5} = 5 \text{ kN} \quad \text{and} \quad R_B = 20 - 5 = 15 \text{ kN}$$



(a) Given Diagram (b) Space Diagram (c) Vector Diagram

Fig. 6.36

Now draw the space diagram for the truss alongwith load of 20 kN and the reactions R_B and R_C equal to 15 kN and 5 kN respectively as shown in Fig. 6.36 (b). Name the members AB, AC and BC according to Bow's notations as PR, QR and RS respectively. Now construct the vector diagram as shown in Fig. 6.36 (c) and as explained below :

(i) Take any point p and draw a vertical line pq downward equal to 20 kN to some suitable scale. From q draw a vertical line qs upward equal to 5 kN to the same scale to represent the reaction at C. Then sp will represent the reaction R_B to the scale.

(ii) Now draw the vector diagram for the joint (B). From p, draw a line pr parallel to PR and from s draw a line sr parallel to SR, meeting the first line at r. Now prs is the vector diagram for the joint (B). Now mark the arrows on the joint B. The arrow in member PR will be towards to joint B, whereas the arrow in the member RS will be away from the joint B as shown in Fig. 6.36 (b).

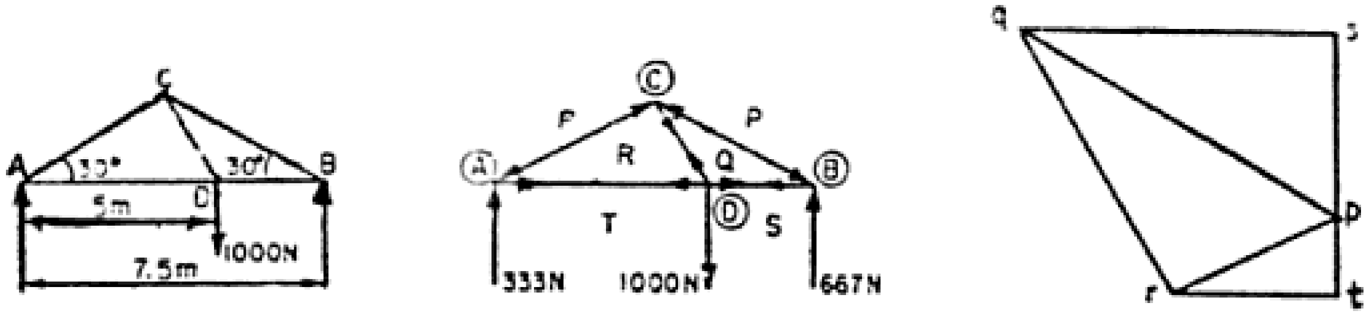
(iii) Similarly draw the vector diagrams for joint A and C. Mark the arrows on these joints in space diagram.

Now measure the various sides of the vector diagram. The forces are obtained by multiplying the scale factor. The forces in the members are given in a tabular form as :

Member		*Force in member*	*Nature of force*
According to given truss	*According to Bow's notation*		
AB	*PR*	17.3 kN	Compressive
AC	*QR*	10.0 kN	Compressive
BC	*RS*	8.7 kN	Tensile

Problem 6.18. *A truss of span 7.5 m carries a point load of 1000 N at joint D as shown in Fig. 6.37. Find the reactions and forces in the member of the truss.*

Sol. First determine the reactions R_A and R_B.



(*a*) Given Diagram (*b*) Space Diagram (*c*) Vector Diagram

Fig. 6.37

Taking moments about A, we get

$$R_B \times 7.5 = 5 \times 1000$$

$$\therefore \qquad R_A = \frac{5000}{7.5} = 667 \text{ N} \quad \text{and} \quad R_A = 1000 - 667 = 333 \text{ N}.$$

Now draw the space diagram for the truss alongwith load of 1000 N and reactions R_A and R_B equal to 333 N and 667 N respectively as shown in Fig. 6.37 (*b*). Name the members *AC, CB, AD, CD* and *DB* according to Bow's notations as *PR, PQ, RT, QR* and *QS* respectively. Now construct the vector diagram as shown in Fig. 6.37 (*c*) and as explained below :

(*i*) Take any point *s* and draw a vertical line *st* downward equal to load 1000 N to some suitable scale. From *t* draw a vertical line *tp* upward equal to 333 N to the same scale to represent the reaction at *A*. The *ps* will represent the reaction R_B to the scale.

(*ii*) Now draw the vector diagram for the joint *A*. From *p*, draw a line *pr* parallel to *PR* and from *t* draw a line *tr* parallel to *RT*, meeting the first line at *r*. Now *prt* is the vector diagram for the joint *A*. Now mark the arrows on the joint *A*. The arrow in the member *PR* will be towards the joint *A*, whereas the arrow in the member *RT* will be away from the joint *A* as shown in Fig. 6.37 (*b*).

(*iii*) Similarly draw the vector diagrams for the joint *C, B* and *D*. Mark the arrows on these joints as shown in Fig. 6.37 (*b*).

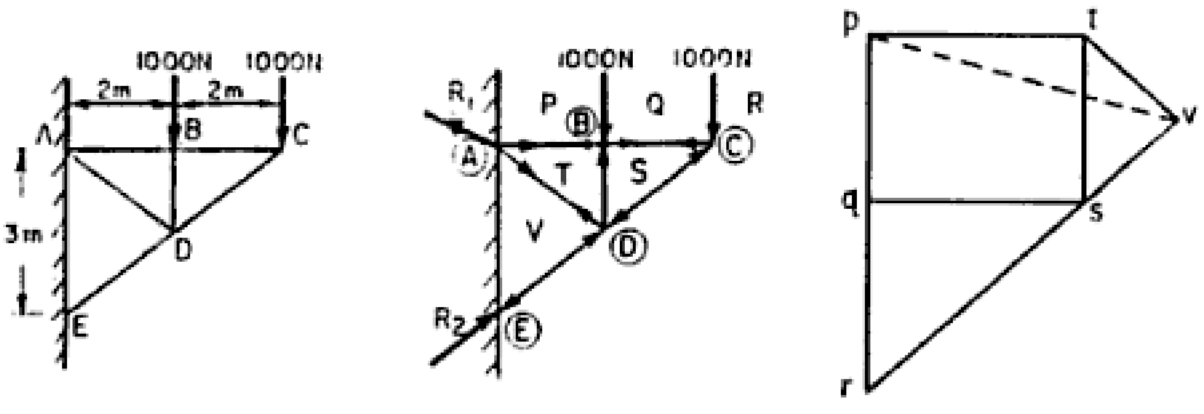
Now measure the various sides of the vector diagrams. The forces in the members are obtained by multiplying the scale factor to the corresponding sides of the vector diagram. The forces in members are given in a tabular form as :

Member		*Force in member*	*Nature of force*
According to given truss	*According to Bow's notation*		
AC	*PR*	666 N	Compressive
AD	*RT*	576.7 N	Tensile
CB	*PQ*	1333 N	Compressive
CD	*QR*	1155 N	Tensile
DB	*QS*	1555 N	Tensile

Problem 6.19. *Determine the forces in all the members of a cantilever truss shown in Fig. 6.38.*

Sol. In this case the vector diagram can be drawn without knowing the reactions. First of all draw the space diagram for the truss along with loads of 1000 N of joints *B* and *C*. Name the members *AB, BC, CD, DE, AD* and *BD* according to Bow's notation as *PT, QS, SR, RV, VT* and *ST* respectively. Now construct the vector diagram as shown in Fig. 6.38 (*c*) and as explained below :

(*i*) The vector diagram will be started from joint *C* where forces in two members are unknown. Take any point *q* and draw a vertical line *qr* downward equal to load 1000 N to some suitable scale. From *r*, draw a line *rs* parallel to *RS* and from *q* draw a line *qs* parallel to *QS*, meeting the first line at *s*. Now *qrS* is the vector diagram for the joint *C*. Now mark the arrows on the joint *C*. The arrow in the member *RS* will be towards the joint *C*, whereas the arrow in the member *SQ* will be away from the joint *C* as shown in Fig. 6.38 (*b*).



(*a*) Given Figure (*b*) Space Diagram (*c*) Vector Diagram

Fig. 6.38

(*ii*) Now draw the vector diagram for the joints *B* and *D* similarly.

Mark the arrows on these joints as shown in Fig. 6.38 (*b*).

Now measure the various sides of the vector diagram. The forces in the members are given in a tabular form as :

Member		*Force in member*	*Nature of force*
According to given truss	*According to Bow's notation*		
AB	*PT*	1333 N	Tensile
BC	*QS*	1333 N	Tensile
CD	*SR*	1666 N	Compressive
DE	*RV*	2500 N	Compressive
AD	*VT*	833 N	Tensile
BD	*ST*	1000 N	Compressive

From the vector diagram, the reactions R_1 and R_2 at *A* and *E* can be determined in magnitude and directions.

Reaction $R_2 = rv = 2500$ N. This will be towards point *E*.

Reaction $R_1 = vp = 2000$ N. This will be away from the point *A* as shown in Fig. 6.38 (*b*). The reaction R_1 is parallel to *vp*.

HIGHLIGHTS

1. The relation between number of joints (*j*) and number of members (*n*) in a perfect frame is given by $n = 2j - 3$.
2. Different frame is a frame is which number of numbers are less than $(2j - 3)$ whereas a redundant frame is a frame in which number of members are more than $(2j - 3)$.

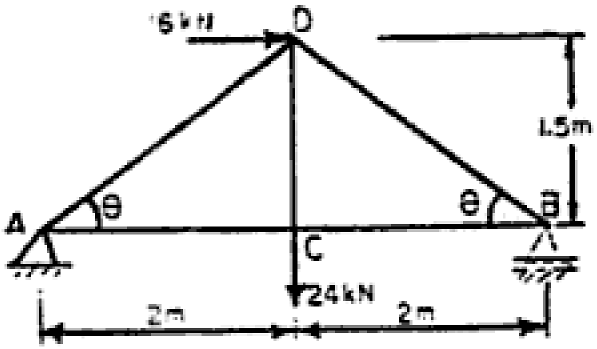
3. The reaction on a roller support is at right angles to the roller base :
4. The forces in the members of a frame are determined by :
   (*i*) Method of joints (*ii*) Method of sections and
   (*iii*) Graphical method.
5. The force in a member will be compressive if the member pushes the joint the which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.
6. While determining forces in a member by method of joints, the joint should be selected in such a way that at any time there are only two members, in which the forces are unknown.
7. If three forces act at a joint and two of them are along the same straight line then third force would be zero.
8. If a truss (or frame) carries horizontal loads, then the support reaction at the hinged end will consists of (*i*) horizontal reaction and (*ii*) vertical reaction.
9. If a truss carries inclined loads, then the support reaction at the hinged end will consists of : (*i*) horizontal reaction and (*ii*) vertical reaction. They will be given as :
   Horizontal reaction = Horizontal components of inclined loads
   Vertical reaction = Total vertical components of inclined loads – Roller support reaction.
10. Method of section is mostly used, when the forces in a few members of a truss are to be determined.
11. The following steps are necessary for obtaining a graphical solution of a frame :
    (*i*) Making a space diagram,
    (*ii*) Constructing a vector diagram, and
    (*iii*) Preparing a force table.
12. The various members of a frame are named according to Bow's notation.

EXERCISE 6

A. Theoretical Questions

1. Define and explain the terms : Perfect frame, imperfect frame, deficient frame and a redundant frame.
2. (*a*) What is a frame ? State the difference between a perfect frame and an imperfect frame.
   (*b*) What are the assumptions made in finding out the forces in a frame ?
3. What are the different methods of analysing (or finding out the forces) a perfect frame ? Which one is used where and why ?
4. How will you find the forces in the members of a truss by method of joints when
   (*i*) the truss is supported on rollers at one end and hinged at other end and carries vertical loads.
   (*ii*) the truss is acting as a cantilever and carries vertical loads.
   (*iii*) the truss is supported on rollers at one end and hinged at other end and carries horizontal and vertical loads.
   (*iv*) the truss is supported on rollers at one end and hinged at other end and carries inclined loads.
5. (*a*) What is the advantage of method of section over method of joints ? How will you use method of section in finding forces in the members of a truss ?
   (*b*) Explain with simple sketches the terms (*i*) method of sections and (*ii*) method of joints, as applied to trusses.
6. How will you find the forces in the members of a joint by graphical method ? What are the advantages or disadvantages of graphical method over method of joints and method of section ?
7. What is the procedure of drawing a vector diagram for a frame ? How will you find out (*i*) magnitude of a force. and (*ii*) nature of a force from the vector diagram ?
8. How will you find the reactions of a cantilever by graphical method ?
9. What are the assumptions made in the analysis of a simple truss.

8. Determine the forces in the truss shown in Fig. 6.46 which carries a horizontal load of 16 kN and a vertical load of 24 kN.



[**Ans.** AC = 24 kN (Tens.)
AD = 10 kN (Comp.)
CD = 24 kN (Tens.)
CB = 24 kN (Tens.)
BD = 30 kN (Comp.)]

Fig. 6.46

9. Find the forces in the member AB and AC of the truss shown in Fig. 6.39 of question 1, using method of sections.
[**Ans.** AB = 4.33 kN (Comp.)
AC = 2.5 kN (Comp.)]

10. Find the forces in the members marked 1, 3, 5 of truss shown in Fig. 6.40 of question 2, using method of sections.
[**Ans.** F_1 = 333 N (Comp.)
F_3 = 577.5 N (Tens.)
F_5 = 577.5 N Tens.)]

11. Find the forces in the members DE, CE and CB of the truss, shown in Fig. 6.41 of equation 3, using method of sections.
[**Ans.** DE = 3.5 kN (Comp.)
CE = 2.598 kN (Comp.)
BC = 4.33 kN (Tens.)]

12. Using method of section, determine the forces in the members CD, FD and FE of the truss shown in Fig. 6.42 of question 4.
[**Ans.** CD = 800 N (Comp.)
FD = 400 N (Comp.)
FE = 600 N Tens.)]

13. Using method of section, determine the forces in the members CD, ED and EF of the truss shown in Fig. 6.47.
[**Ans.** CD = 4.216 kN (Comp.)
ED = 3.155 kN (Tens.)
EF = 2.58 kN (Tens.)]

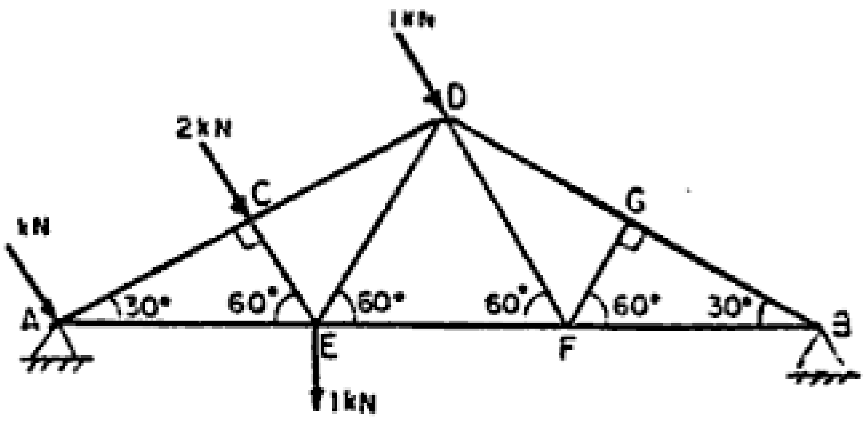


Fig. 6.47

14. Find the forces in the members AB, AC and BC of the truss shown in Fig. 6.39 of question 1, using graphical method.

15. Using graphical method, determine the magnitude and nature of the forces in the members of the truss shown in Fig. 6.40 of question 2.

16. Determine the forces in all the members of a cantilever truss shown in Fig. 6.44 of question 6, using graphical method. Also determine the sections of the cantilever.

7

Centre of Gravity and Moment of Inertia

7.1. CENTRE OF GRAVITY

Centre of gravity of a body is the point through which the whole weight of the body acts. A body is having only one centre of gravity for all positions of the body. It is represented by C.G. or simply G.

7.2. CENTROID

The point at which the total area of a plane figure (like rectangle, square, triangle, quadrilateral, circle etc.) is assumed to be concentrated, is known as the centroid of that area. The centroid is also represented by C.G. or simply G. The centroid and centre of gravity are at the same point.

7.3. CENTROID OF CENTRE OR GRAVITY OF SIMPLE PLANE FIGURES

(*i*) The centre of gravity (C.G.) of a uniform rod lies at its middle point.

(*ii*) The centre of gravity of a triangle lies at the point where the three medians* of the triangle meet.

(*iii*) The centre of gravity of a rectangle or of a parallelogram is at the point, where its diagonal meet each other. It is also the point of intersection of the lines joining the middle points of the opposite sides.

(*iv*) The centre of gravity of a circle is at its centre.

7.4. CENTRE OF GRAVITY OF PLANE FIGURES BY THE METHOD OF MOMENTS

Fig. 7.1 shows a plane figure of total area A whose centre of gravity is to be determined. Let this area A is composed of a number of small areas a_1, a_2, a_3, a_4, ... etc.

$\therefore \quad A = a_1 + a_2 + a_3 + a_4 + \ldots$

Let x_1 = The distance of the C.G. of the area a_1 from axis OY

x_2 = The distance of the C.G. of the area a_2 from axis OY

x_3 = The distance of the C.G. of the area a_3 from axis OY

x_4 = The distance of the C.G. of the area a_4 from axis OY and so on.

The moments of all small areas about the axis OY

$$= a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \ldots \qquad \ldots(i)$$

Let G is the centre of gravity of the total area A whose distance from the axis OY is $\bar{x}$.

Then moment of total area about $OY = A\bar{x}$ ...(*ii*)

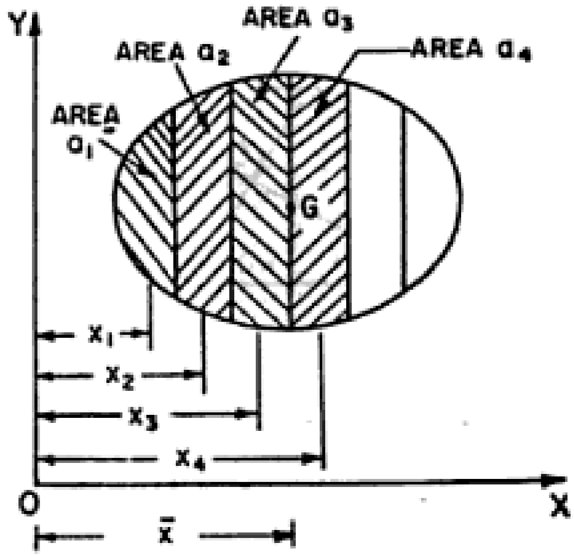


Fig. 7.1

*The line connecting the vertex and the middle point of the opposite side of a triangle is known as median of the triangle.

The moments of all small areas about the axis OY must be equal to the moment of total area about the same axis. Hence equating equations (*i*) and (*ii*), we get

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \ldots = A\bar{x}$$

or
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \ldots}{A} \quad \ldots(7.1)$$

where $A = a_1 + a_2 + a_3 + a_4 \ldots$

If we take the moments of the small areas about the axis OX and also the moment of total area about the axis OX, we will get

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4 + \ldots}{A} \quad \ldots(7.2)$$

where $\bar{y}$ = The distance of G from axis OX

y_1 = The distance of C.G. of the area a_1 from axis OX

y_2, y_3, y_4 = The distance of C.G. of area a_2, a_3, a_4 from axis OX respectively.

7.4.1. Centre of Gravity of Plane Figures by Integration Method. The equations (7.1) and (7.2) can be written as

$$\bar{x} = \frac{\Sigma\, a_i\, x_i}{\Sigma a_i} \quad \text{and} \quad \bar{y} = \frac{\Sigma\, a_i\, y_i}{\Sigma a_i}$$

where $i = 1, 2, 3, 4, \ldots..$

x_i = Distance of C.G. of area a_i from axis OY and

y_i = Distance of C.G. of area a_i from axis OX.

The value of i depends upon the number of small areas. If the small areas are large in number (mathematically speaking infinite in number), then the summations in the above equations can be replaced by integration. Let the small areas are represented by dA instead of 'a', then the above equations are written as :

$$\bar{x} = \frac{\int x^* dA}{\int dA} \quad \ldots(7.2\text{ A})$$

and
$$\bar{y} = \frac{\int y^* dA}{\int dA} \quad \ldots(7.2\text{ B})$$

where $\int x^* dA = \Sigma x_i a_i$

$\int dA = \Sigma a_i$

$\int y^* dA = \Sigma y_i a_i$

Also x^* = Distance of C.G. of area dA from axis OY

y^* = Distance of C.G. of area dA from axis OX.

7.4.2. Centre of Gravity of a Line. The centre of gravity of a line which may be straight or curve, is obtained by dividing the given line, into a large number of small lengths as shown in Fig. 7.1 (*a*).

The centre of gravity is obtained by replacing dA by dL in equations (7.2 A) and (7.2 B).

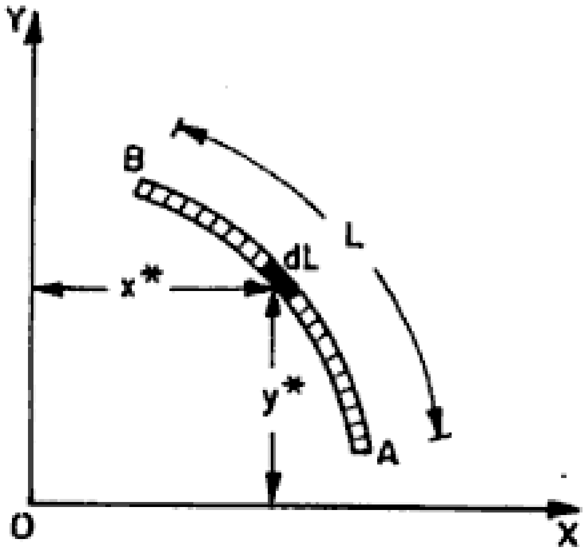


Fig. 7.1 (*a*)

Then these equations become $\bar{x} = \frac{\int x^* \, dL}{\int dL}$ ...(7.2 C)

and $\bar{y} = \frac{\int y^* \, dL}{\int dL}$ ...(7.2 D)

where x^* = Distance of C.G. of length dL from y-axis, and

y^* = Distance of C.G. of length dL from x-axis.

If the lines are straight, then the above equations are written as :

$$\bar{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3 + \ldots\ldots}{L_1 + L_2 + L_3 + \ldots\ldots} \qquad \text{...(7.2 E)}$$

and

$$\bar{y} = \frac{L_1y_1 + L_2y_2 + L_3y_3 + \ldots\ldots}{L_1 + L_2 + L_3 + \ldots\ldots} \qquad \text{...(7.2 F)}$$

7.5. IMPORTANT POINTS

(*i*) The axis, about which moments of areas are taken, is known as axis of reference. In the above article, axis *OX* and *OY* are called axis of reference.

(*ii*) The axis of reference, of plane figures, is generally taken as the lowest line of the figure for determining $\bar{y}$, and left line of the figure for calculating $\bar{x}$.

(*iii*) If the given section is symmetrical about *X-X* axis or *Y-Y* axis, then the C.G. of the section will lie on the axis is symmetry.

7.5.1. Centre of Gravity of Structural Sections. The centre of gravity of structural sections like *T*-section, *I*-section, *L*-sections etc. are obtained by splitting them into rectangular components. Then equations (7.1) and (7.2) are used.

Problem 7.1. *Find the centre of gravity of the T-section shown in Fig. 7.2 (a).*

Sol. The given *T*-section is split up into two rectangles *ABCD* and *EFGH* as shown in Fig. 7.2 (*b*). The given *T*-section is symmetrical about *Y-Y* axis. Hence the C.G. of the section will lie on this axis. The lowest line of the figure is line *GF*. Hence the moments of the areas are taken about this line *GF*, which is the axis of reference in this case.

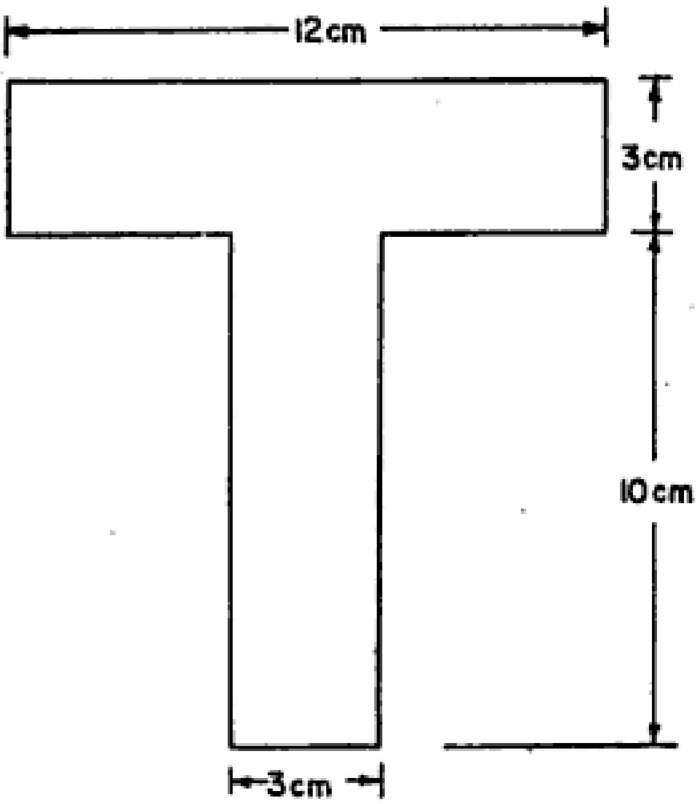


Fig. 7.2 (*a*)

Let $\bar{y}$ = The distance of the C.G. of the *T*-section from the bottom line *GF* (which is axis of reference)

a_1 = Area of rectangle *ABCD* = $12 \times 3 = 36\ \text{cm}^2$

y_1 = Distance of C.G. of area a_1 from bottom line *GF* = $10 + \frac{3}{2} = 11.5$ cm

a_2 = Area of rectangle *EFGH* = $10 \times 3 = 30\ \text{cm}^2$

y_2 = Distance of C.G. of area a_2 from bottom line *GF* = $\frac{10}{2} = 5$ cm.

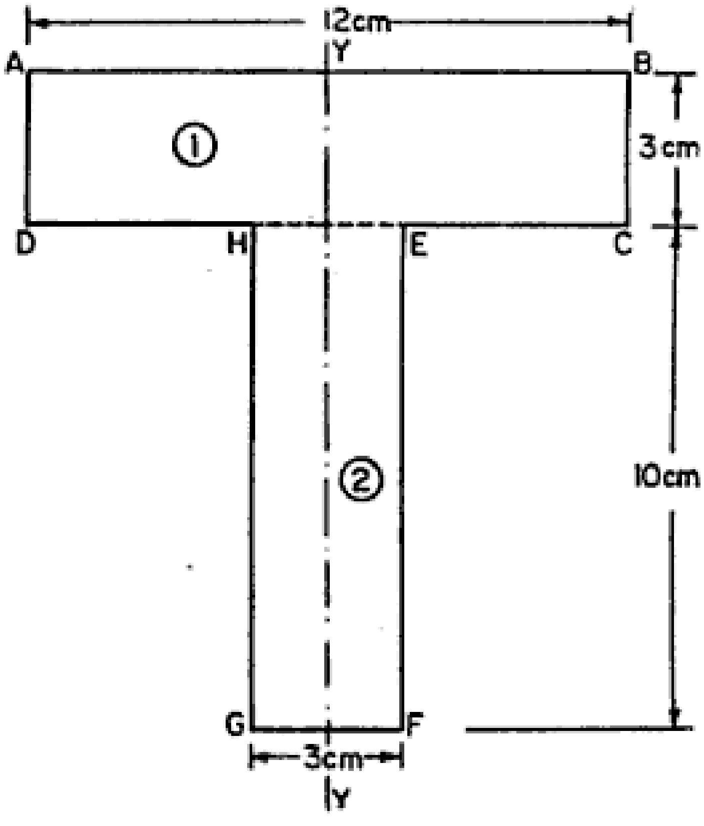


Fig. 7.2 (*b*)

ISBN 81-7008-305-2
9788170083054

LAXMI PUBLICATIONS (P) LTD