SELF-TESTING PROBLEMS

7.13 A 250 V moving-coil voltmeter, together with its multiplier, has an internal resistance of 500 kΩ. What value of multiplying resistor has to be added to enable the meter to have a full-scale deflection of 500 V? Find the current flowing through the meter if the indicated voltage is 415 V.

7.14 Find the resistance of a 25 A shunt if the voltage across it at full load is 0.05 V.

7.15 A Wheatstone bridge is balanced and the three known values of resistance are: R_1, 1 kΩ; R_2, 100 Ω; and R_3, 6.8 kΩ. Calculate the value of the resistance being measured.

7.16 A moving-coil meter has an internal resistance of 120 Ω. Calculate the shunt resistance value to extend the range ten times.

7.17 A permanent-magnet moving-coil meter has an internal resistance of 100 Ω and a full-scale deflection current of 1 mA. Calculate:

(a) the value of shunt resistor necessary to extend the range to 500 mA

(b) the value of the multiplying resistor required to extend the range to 250 V.

7.18 A moving-coil meter requires 10 mA to reach f.s.d. and has an internal resistance of 5 Ω. Determine the values of shunt resistors required to measure currents of 100 mA, 1 A, and 10 A.

7.19 A moving-coil meter of 1 mA f.s.d. and an internal resistance of 100 Ω is to be used as a voltmeter. What multiplier values of resistance are required if the ranges of the voltmeter are 10 V, 30 V, 100 V, 300 V, and 1000 V?

7.20 A permanent-magnet moving-coil meter of f.s.d. 50 μA and internal resistance 1000 Ω is to be used as a voltmeter with ranges of 2.5 V, 10 V, 25 V, 100 V, 250 V and 1000 V. Find the values of the necessary multiplying resistors.

7.21 A resistor was measured using the bridge network of Figure 7.22(a) and the following results obtained: $R_1 = 10\ \Omega$, $R_2 = 1000\ \Omega$, $R_3 = 987\ \Omega$. Find the value of the unknown resistor.

7.22 Using a Weston standard cell calibrated at 1.0183 V, a potentiometer was balanced at 564 mm. A cell to be tested was then substituted for the Weston cell and a balance was obtained at 852 mm. Find the test voltage of the unknown cell.

Chapter 8

Alternating current principles: single phase

8.1 INTRODUCTION

In Chapter 5 it was shown that a magnetic flux cutting across conductors produces an induced voltage. This is done by either moving a magnet in close proximity to a conductor or moving the conductor in close proximity to a magnet. For practical purposes this method of producing electrical energy is both inefficient and clumsy. Furthermore, when both magnet and conductors are stationary, no voltage is produced.

It is far easier to spin a magnet in close proximity to a group of conductors (or vice versa) than to plunge a magnet in and out of a coil continuously. It is also more efficient to create a strong electromagnet than to rely on a permanent magnet retaining all its magnetism over its life span. The mechanical method is almost universally used to produce what is known as alternating current.

8.2 ALTERNATING VOLTAGE WAVEFORMS

Almost all generators produce an alternating voltage. Even so-called direct current generators initially produce an alternating voltage. It is converted internally to direct current with the aid of a commutator before being available at the machine's terminals. Sinusoidal waveform voltages are used as a standard because of the many advantages offered.

8.2.1 Sinusoidal waveforms

The term *sinusoidal waveform* is derived from the fact that the waveform follows the strict mathematical function of the sine, as discussed separately in section 1.6.1.

When tables of sine values are plotted as a graph, the resultant characteristic curve is called a sine wave, that is, it is sinusoidal in shape. The first half of the curve is a mirror image of the second half and one half is always positive, while the second half is always negative. The advantages of a sine wave are:

1. It is an easy shape to reproduce because it is a naturally occurring waveform.
2. It is easily repeatable with other sources at other places.
3. It is the only waveform that produces a current flow with the same waveform as the voltage.
4. Its shape allows the maximum permissible power per unit size of machine.
5. Because of its alternating voltage, current and magnetic flux, it can be used to operate any equipment that relies on mutual induction for its operation.

8.2.2 Other waveforms

In the electrical power industry, any waveform other than sinusoidal is considered to be distorted. However, there are other waveforms in use. These are generated for specific purposes and not used for the transmission of electrical energy. Several of the more common waveforms are shown in Figure 8.1, where they can be compared.

The diagram shows several waveforms along a broken line, indicating a central point in each case. Figure 8.1(a)

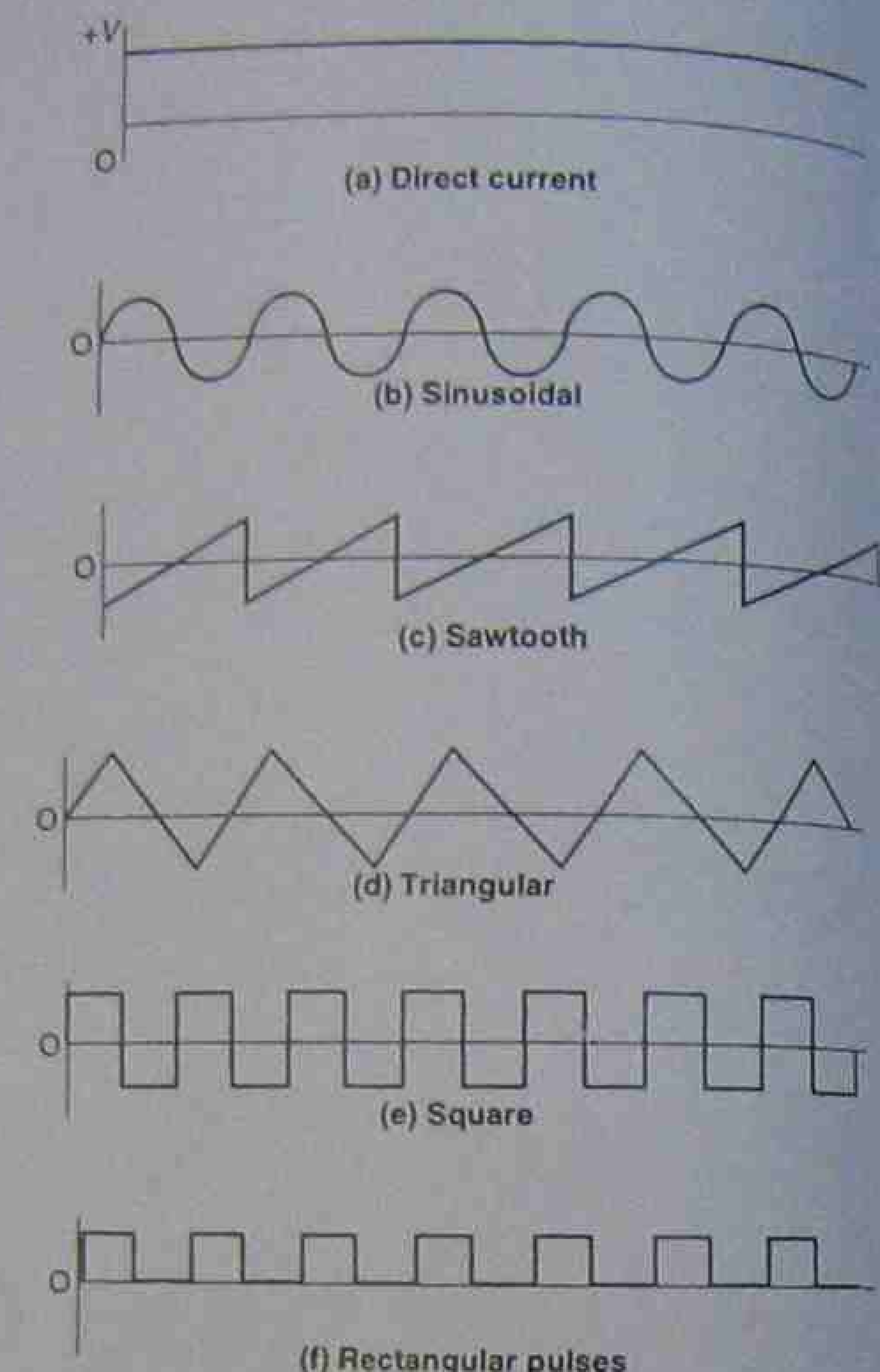


Figure 8.1 • Typical waveforms

depicts a d.c. voltage. Note that it attains a steady value and remains constant at that value. It is the waveform used where steady and unidirectional currents are desired.

The sinusoidal waveform is drawn in Figure 8.1(b). For a pure sine wave, it is continually changing at various rates. Its construction, shape and uses are discussed in greater depth later in this chapter. It is important to note that its shape is symmetrical about the broken line and it is not a series of semicircles as so often depicted.

The next waveform is the saw-tooth wave shown in Figure 8.1(c), and it is immediately above the triangular wave Figure 8.1(d) with which it is often confused. Both rise symmetrically, but while one decreases at the same rate as it increases, the other changes abruptly to its most negative value. Both are used in timing circuits because of the positive voltage peak that occurs periodically.

The square or rectangular waveform is shown in Figure 8.1(e). Whether it is considered square or rectangular depends on the periodic time constants of the wave. The effect is to switch a voltage from positive to negative at a steady rate. There is a positive voltage for a period and then a negative value for a further period. The transition from positive to negative is abrupt. The positive period of voltage may or may not be equal to the negative voltage period. To ensure that an electronic circuit is switched off completely, it is sometimes necessary to make sure there is a negative voltage at some point. The wave is also used in logic circuit switching. Electronic components sometimes

use this waveform in comparison-type circuits as an indication that some event has occurred, for example, one voltage has exceeded another voltage.

The waveform shown in Figure 8.1(f) is an adaptation of the square wave. The negative part of the wave is missing, leaving only positive pulses. Sometimes called short rectangular pulses, they are the basis of information transmission in digital electronics. There are two ways of varying the information in this wave. One is by varying the height, and the other is by varying either the width of the positive pulses or their spacing.

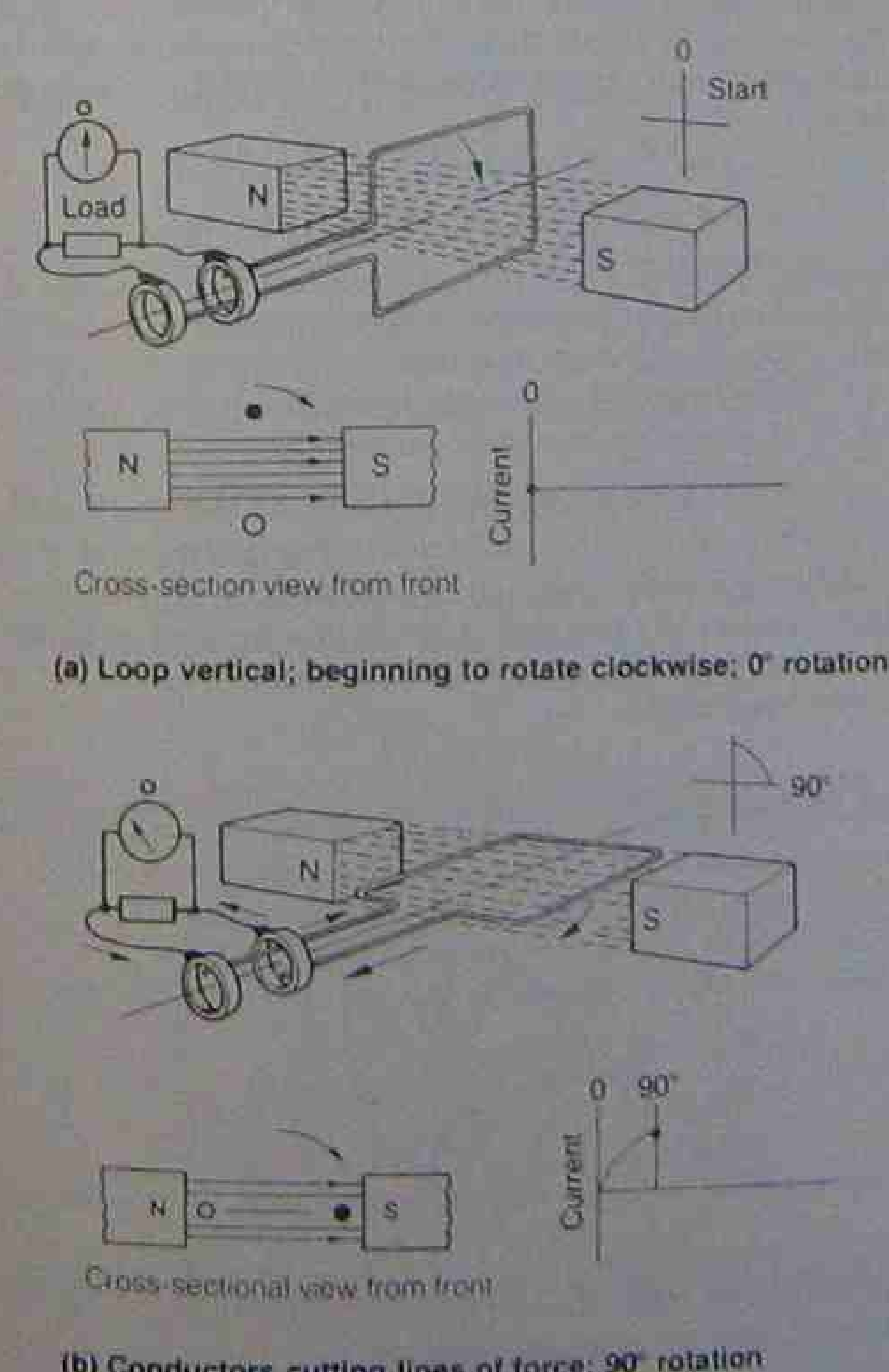
8.3 ALTERNATORS

A machine producing a voltage with an alternating waveform at its terminals is called an *alternator*. On occasion it can be called an alternating current generator. Both names are correct, although usually the term *generator* is reserved for direct current machines. The modern generator and alternator both consist of electromagnets rotating within a frame containing many coils, each producing an e.m.f. These coils may be connected in series or parallel to produce a desired voltage at the terminals. Some alternators use the principle of rotating the coils between stationary magnets with the same result. Larger machines use rotating electromagnets, while smaller machines use stationary electromagnets.

8.3.1 Loop rotating in a magnetic field

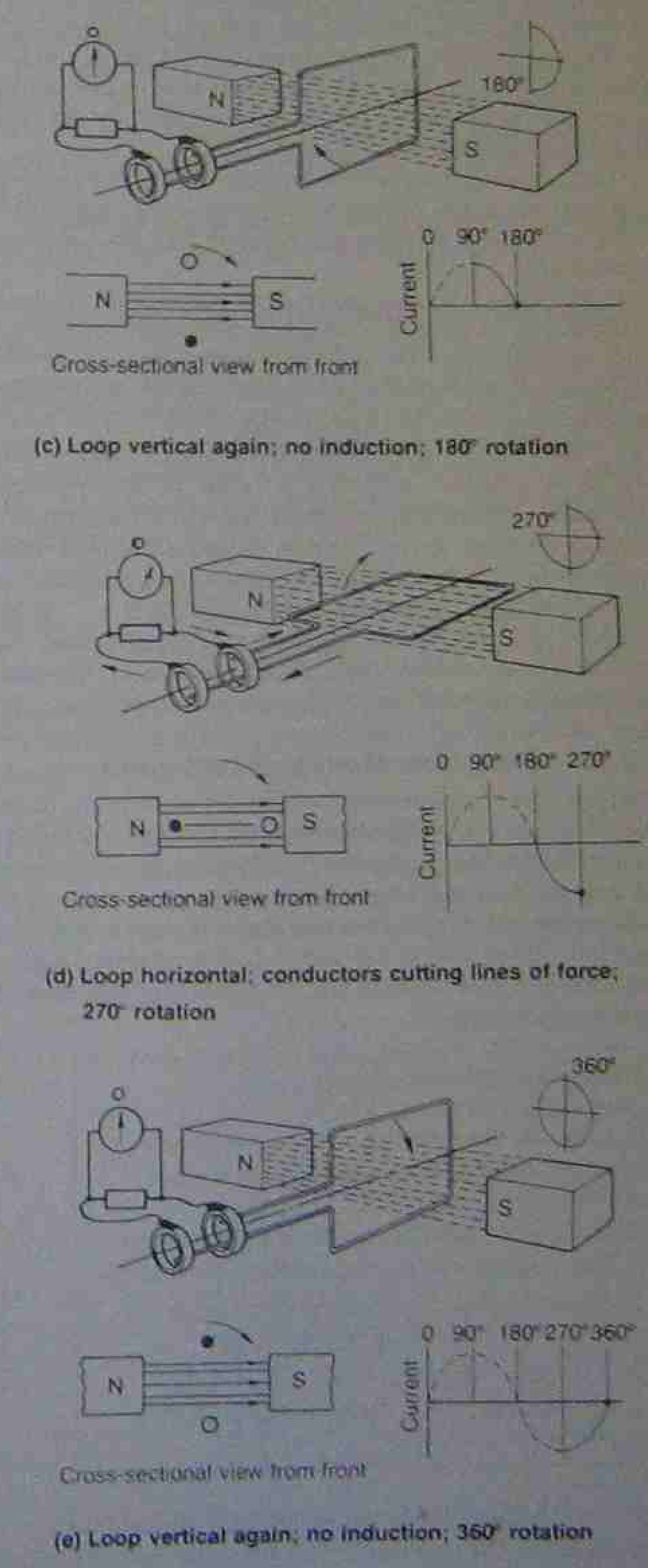
Figure 8.2(a) shows a single-loop generator in which the loop may be rotated in the field of two fixed, permanent magnets. As the loop is rotated, an e.m.f. will be generated in it.

Some means must be used to connect the loop to an external circuit, and for this reason two metallic slip-rings are connected to the respective loop ends. Carbon brushes, bearing on the slip-rings, complete the connection.



(a) Loop vertical; beginning to rotate clockwise; 0° rotation

(b) Conductors cutting lines of force; 90° rotation



(c) Loop vertical again; no induction; 180° rotation

(d) Loop horizontal; conductors cutting lines of force; 270° rotation

(e) Loop vertical again; no induction; 360° rotation

Figure 8.2 • Production of one cycle of a.c. by a simple a.c. generator

Effect on e.m.f. as the loop rotates

[illegible]

As the voltage generated alternates in direction, the [illegible] generator described above is known as an alternating-current generator, or simply as an alternator.

8.3.3 Direction of an induced e.m.f.

The direction of a generated voltage depends on the direction of relative movement between a magnetic flux and the conductors that link with it. A detailed explanation of the term relative movement has been given in section 5.9.

A convenient rule, known as Fleming's right-hand rule, may be applied to find the direction of an induced e.m.f., provided the direction of flux and relative direction of motion are known.

> **Fleming's right-hand rule (use for generators)**
> Arrange the thumb, first, and centre fingers of the right hand at right angles (90°) to each other.
> 1. Point the thumb in the direction in which the conductor is moving through the flux.
> 2. Point the first finger in the direction in which the lines of force are acting.
> 3. The centre finger will then point in the direction in which the induced e.m.f. acts along the conductor.

8.3.4 Magnitude of a generated e.m.f.

In section 8.3.1 it was stated that the value of an induced e.m.f. depends on three factors. These are the strength of

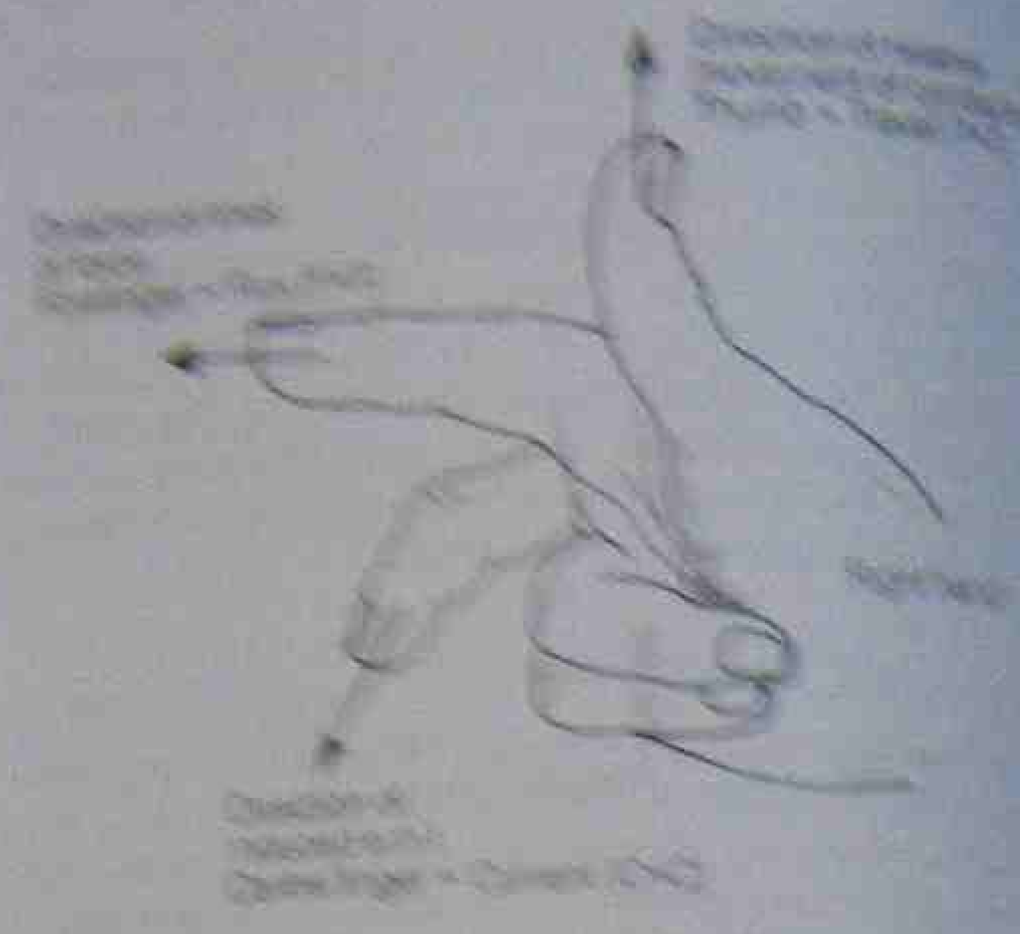


Figure 8.3 • Illustrating Fleming's right-hand rule

the magnetic field, the number of conductors in series and the relative rate of motion between the first two factors.

In practical machines it is easy to determine the speed of rotation of the machine and then take into account the comparative direction of the conductors in relation to the magnetic field. By combining these last two, the relative rate of motion can be found from the expression $v \sin \theta$. By combining all of these factors an equation can be developed; that is, the generated voltage can be found from:

$$\boxed{V = Blv \sin \theta}$$

where V = value of induced voltage
B = flux density in webers
l = length of conductor in metres
v = velocity in metres per second

This also agrees with the concept of Fleming's right-hand rule, where the movement of the conductor is at right angles to the magnetic flux. The formula also takes into account the fact that a conductor might not always be travelling at right angles to the magnetic flux, but at some other angle.

Example 8.1

A conductor 0.3 m long is rotating on the periphery of an armature at 24 m/s. If the flux density is 0.4 T, calculate the maximum voltage induced in the conductor when it is cutting the field at an angle of 90°.

$V = Blv \sin \theta$
$= 0.4 \times 0.3 \times 24 \times 1$
$= 2.88$ V

Example 8.2

If the conductor in example 8.1 had passed the centre of the magnetic flux and was then cutting it at an angle of 45°, find the new voltage generated.

$V = Blv \sin \theta$ (sin 45° = 0.707)
$= 0.4 \times 0.3 \times 24 \times 0.707$
$= 2.04$ V

8.3.5 Alternative alternator construction

The basic construction of an alternator has been shown to be a loop rotating in a magnetic field as illustrated in Figure 8.4(a). The alternating voltage produced when the ends of the loop were connected to slip-rings caused current to flow alternately first in one direction and then in the other.

This result can also be achieved by keeping the loop stationary and rotating the magnetic field instead, to give the relative motion. From a constructional viewpoint it is easier to have the loop on the outside of the magnetic field, as shown in Figure 8.4(b).

There are several advantages of this type of construction:

- with smaller alternators, the rotating magnetic field can be obtained from a permanent magnet
- slip-rings are not necessary for connection to the loop—enabling solid connections to be made.

In practical machines, the more solid connections allow higher voltages and currents to be handled and to be connected solely to the external circuit. Sliding connections to the slip-rings (brush gear and brushes) are kept to a minimum, so reducing maintenance problems.

8.4 METHODS BY WHICH THE GENERATED E.M.F. MAY BE INCREASED

It has been shown above that an e.m.f. may be produced by rotating a single-loop conductor in a magnetic field. This e.m.f. is too small to be of any practical use and methods of increasing its value must be considered.

8.4.1 Increase in number of effective conductors

Figure 8.5(a) illustrates the use of a coil rather than a single-turn loop in order to increase the number of flux linkages in a given time. Since the turns of the coil are equivalent to a number of separate conductors connected in series, the overall e.m.f. produced is the sum of the e.m.f. produced in each turn.

Generated e.m.f. may be further increased by adding coils and connecting them in series. This is shown in Figure 8.5(b), in which the coils are joined so that the starting lead of one is connected to the finishing lead of the other. Connecting the coils in this way ensures that the e.m.f. generated in them will be acting in the same direction and that the voltage will be additive.

Since e.m.f. is now generated in two coils, the overall e.m.f. produced will be greater.

The maximum e.m.f. produced by the two coils in Figure 8.5(b) will not be twice that of a single coil because the two coils are placed at an angle to each other and both cannot generate maximum e.m.f. at the same instant.

Addition of further series-connected coils will result in a further increase in the value of e.m.f. generated.

8.4.2 Increase in flux with an iron core

The coils so far considered have only air cores, and the magnetic path between poles is one of high reluctance. Reluctance may be considerably reduced by winding the coils on iron cores. By using specially shaped cores and magnetic pole pieces, not only will the flux be increased but also it may be concentrated at the points where it is most required.

8.5 IRON LOSSES IN A.C. GENERATORS

The introduction of an iron core on which the coils are wound improves the voltage output, but causes two undesirable effects: eddy current losses and hysteresis losses.

8.5.1 Eddy current losses

Since iron is an electrical conductor, the rotation of an iron core within a magnetic field generates an e.m.f. in the core itself. The direction in which this voltage acts is shown in Figure 8.6.

Although the voltage is low in value, it is applied across

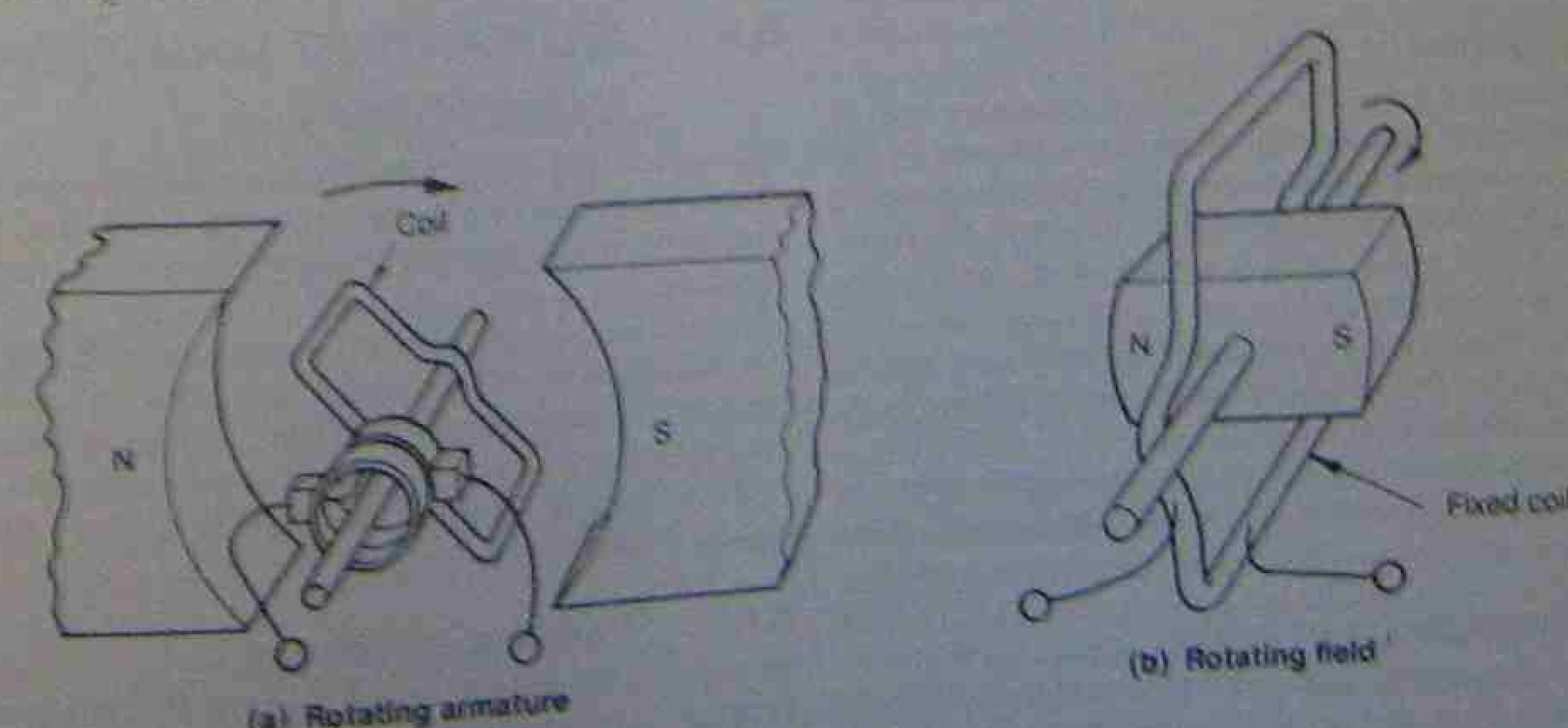


Figure 8.4 • Alternative alternator construction

8.3.2 Effect on e.m.f. as the loop rotates

The position of the loop shown in Figure 8.2(a) is such that the initial movement of the loop sides is parallel to the magnetic lines of force. No cutting of lines of force will occur and no e.m.f. will be generated.

When the loop begins rotating it tends to cut the lines of force at an acute angle, resulting in the generation of an e.m.f. in each loop side. Further rotation of the loop causes a corresponding increase in the value of generated e.m.f. because the angle at which the loop sides cut the flux becomes less acute as rotation proceeds. In the position shown in Figure 8.2(b), the loop sides cut the lines of force at 90°, resulting in the generation of maximum e.m.f. at this position. The direction of the generated e.m.f. is indicated by the arrows and may be confirmed by applying Fleming's right-hand rule (section 8.3.3).

Further rotation of the loop causes a gradual decrease in e.m.f. as the angle of cutting becomes increasingly acute until, at the position shown in Figure 8.2(c), the e.m.f. will again fall to zero.

As the loop rotates further from position (c) towards position (d), the relative direction of motion of the coil sides reverses. Since the field direction remains fixed, the direction of generated e.m.f. must reverse. This is indicated by the arrows in Figure 8.2(d).

In Figure 8.2(d) the e.m.f. will again reach a maximum, but in a direction opposite to that of the previous maximum. Further rotation from position (d) will cause a gradual decrease in e.m.f. until it again becomes zero, when the loop completes one revolution and returns to its original position shown in Figure 8.2(e).

As the voltage generated alternates in direction, the basic generator described above is known as an alternating current generator, or simply as an alternator.

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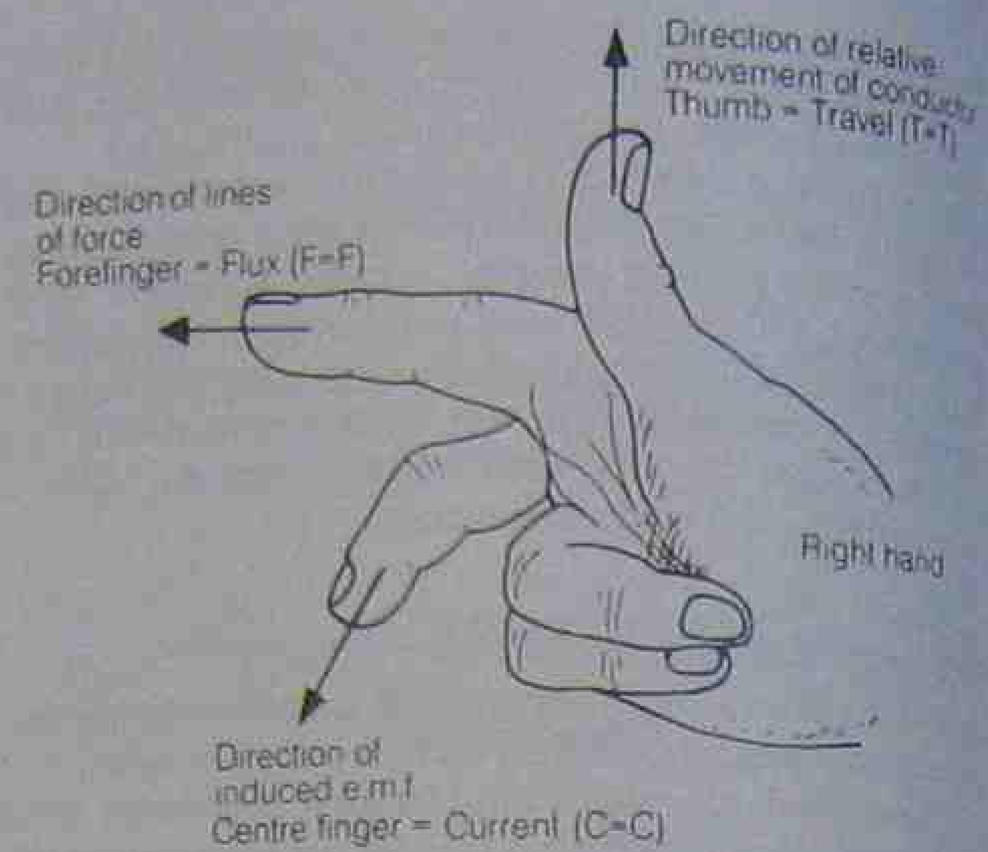

Figure 8.3 • Illustrating Fleming's right-hand rule

In practical machines it is easy to determine the rate of rotation of the machine and then take into account the comparative direction of the conductors in relation to the magnetic field. By combining these last two, the relative rate of motion can be found from the expression $v \sin \theta$. By combining all of these factors an equation can be developed; that is, the generated voltage can be found from:

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The introduction of an iron core on which the coils are wound improves the voltage output, but causes two undesirable effects: eddy current losses and hysteresis losses.

8.5.1 Eddy current losses

Since iron is an electrical conductor, the rotation of an iron core within a magnetic field generates an e.m.f. in the core itself. The direction in which this voltage acts is shown in Figure 8.6.

Although the voltage is low in value, it is applied across

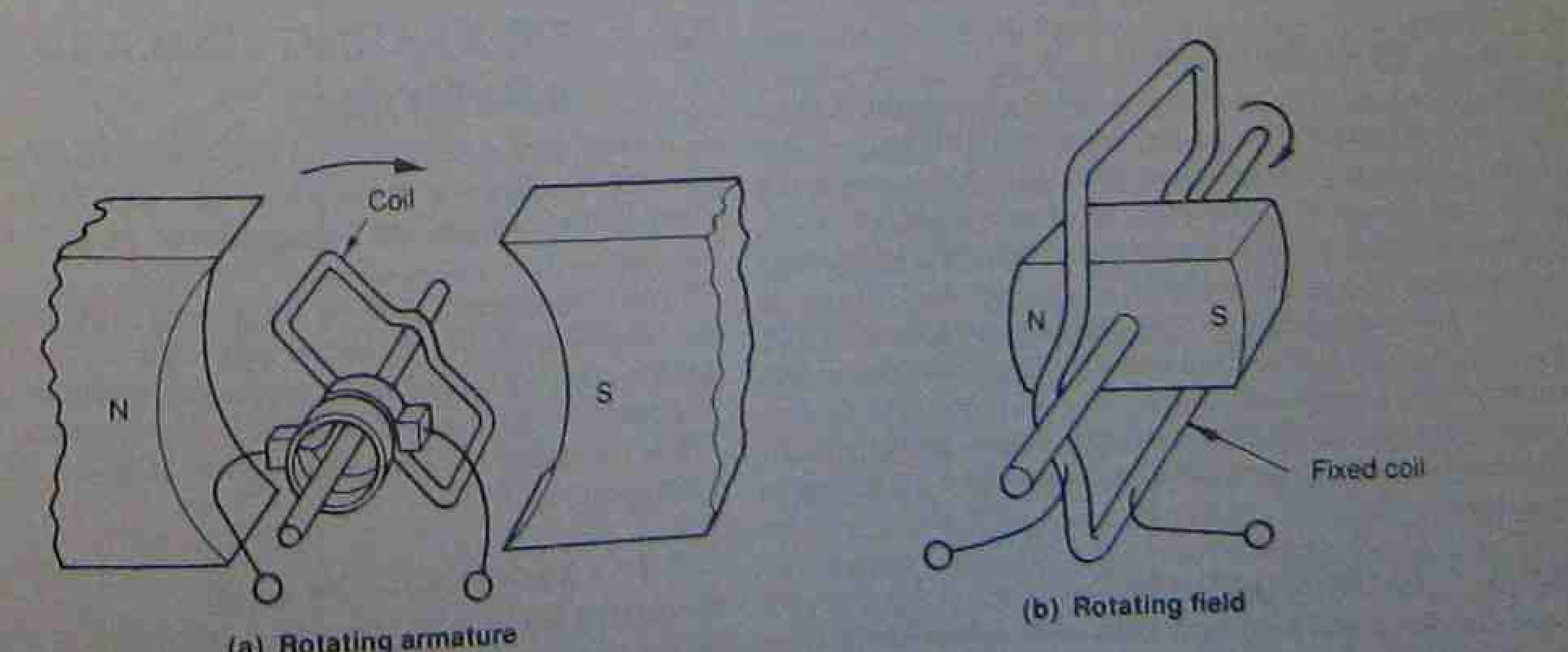
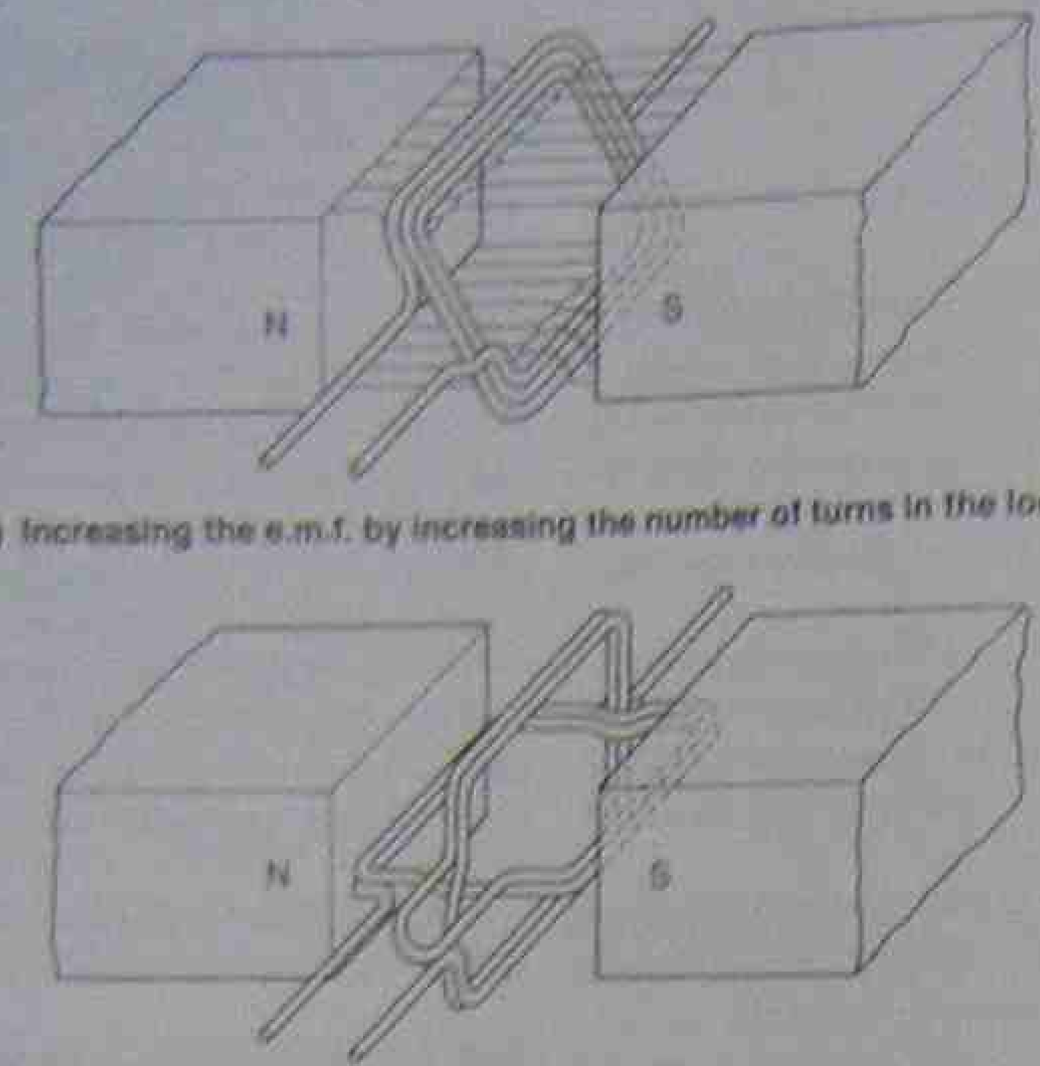


Figure 8.4 • Alternative alternator construction



(a) Increasing the e.m.f. by increasing the number of turns in the loop

(b) Increasing the e.m.f. by increasing the number of coils connected in series

Figure 8.5 • Two methods for increasing the generated e.m.f. in an alternator

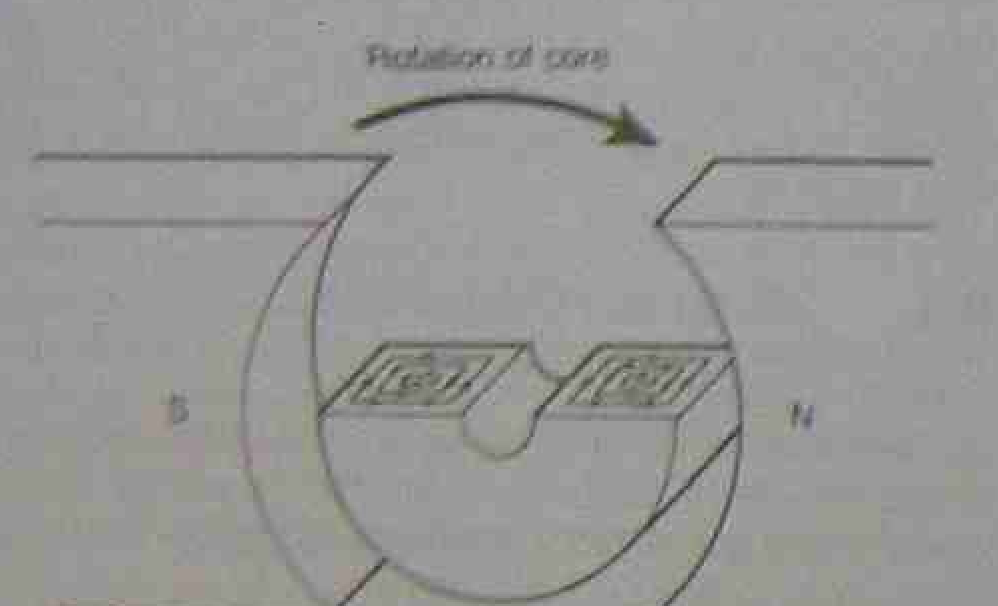


Figure 8.6 • Direction of eddy currents in a solid iron core

the very low resistance of the core, consequently heavy currents will flow. These currents, known as eddy currents, tend to heat the iron core. No useful work is done and the heat produced is a source of loss and is measured in watts.

8.5.2 Hysteresis losses

As the iron core of a generator rotates, a particular side of the core will come alternately under the influence of the north and south poles of the magnetic field. Consequently there will be rapid alternation of flux within the core, giving rise to the effect known as hysteresis, which has been discussed in Chapter 5.

Since hysteresis leads to the generation of heat within the core material, this effect results in another energy loss known as hysteresis loss.

Eddy current losses and hysteresis losses both occur in the iron core of a generator and are referred to jointly as iron losses.

8.5.3 Reducing iron losses

Eddy current losses may be greatly reduced by laminating the iron cores. That is, the cores are built up from many thin sheets of iron, which have an oxide scale on the surface. The scale is used deliberately to provide a relatively high resistance between the laminations and it breaks the core up into many separate conductors of thin section. This has a twofold effect:

1. The generated voltage within the core is broken up into much smaller voltages in each lamination.
2. These smaller voltages are applied across paths of very small section.

As the number of laminations increases, the value of e.m.f. across each path decreases, while the resistance of each path increases. The overall current flow will therefore reduce rapidly as the number of laminations used in a core of given length is increased.

Hysteresis losses can be kept within reasonable limits by using special steels. In particular, the addition of silicon to steel is found to be of advantage in this respect.

8.6 GENERATING SINUSOIDAL WAVEFORMS

If an e.m.f. that is sinusoidal in waveform is applied to the primary winding of a transformer, an e.m.f. that is also of sinusoidal waveform will be reproduced in the secondary winding.

Since transformers play an important part, not only in the distribution of electricity but also in industry in general, the sinusoidal waveform becomes the most practical waveform for use in general power supplies.

The following methods are used to achieve sinusoidal voltage waveforms.

8.6.1 Variation of air gaps

By varying the air gap across the pole face it is possible to produce a varying strength magnetic field, enabling a sinusoidal output to be generated (see Fig. 8.7).

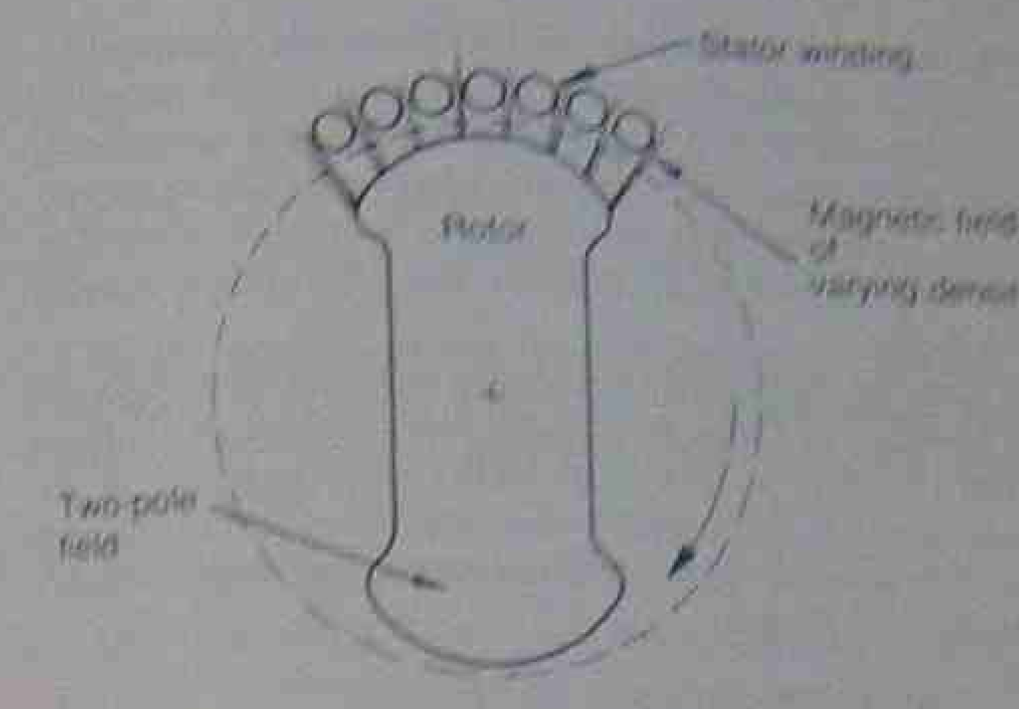


Figure 8.7 • Shaped pole (minimum air gap at centre of pole) for producing sinusoidal waveforms

8.6.2 Setting conductors at an angle

Figure 8.8 shows a conductor at an angle to the centre line of the pole face. This means that an initial flux linkage between the leading edge of the pole face and the conductor is low. Linkage reaches a maximum as the centre line of the pole passes the conductor and then tapers off again as the trailing edge of the pole reaches the conductor.

8.6.3 Distribution of windings

Figure 8.9 illustrates a distributed coil winding in the stator. Correct selection of the turns in each part of the winding produces a sinusoidal voltage output from the stator winding.

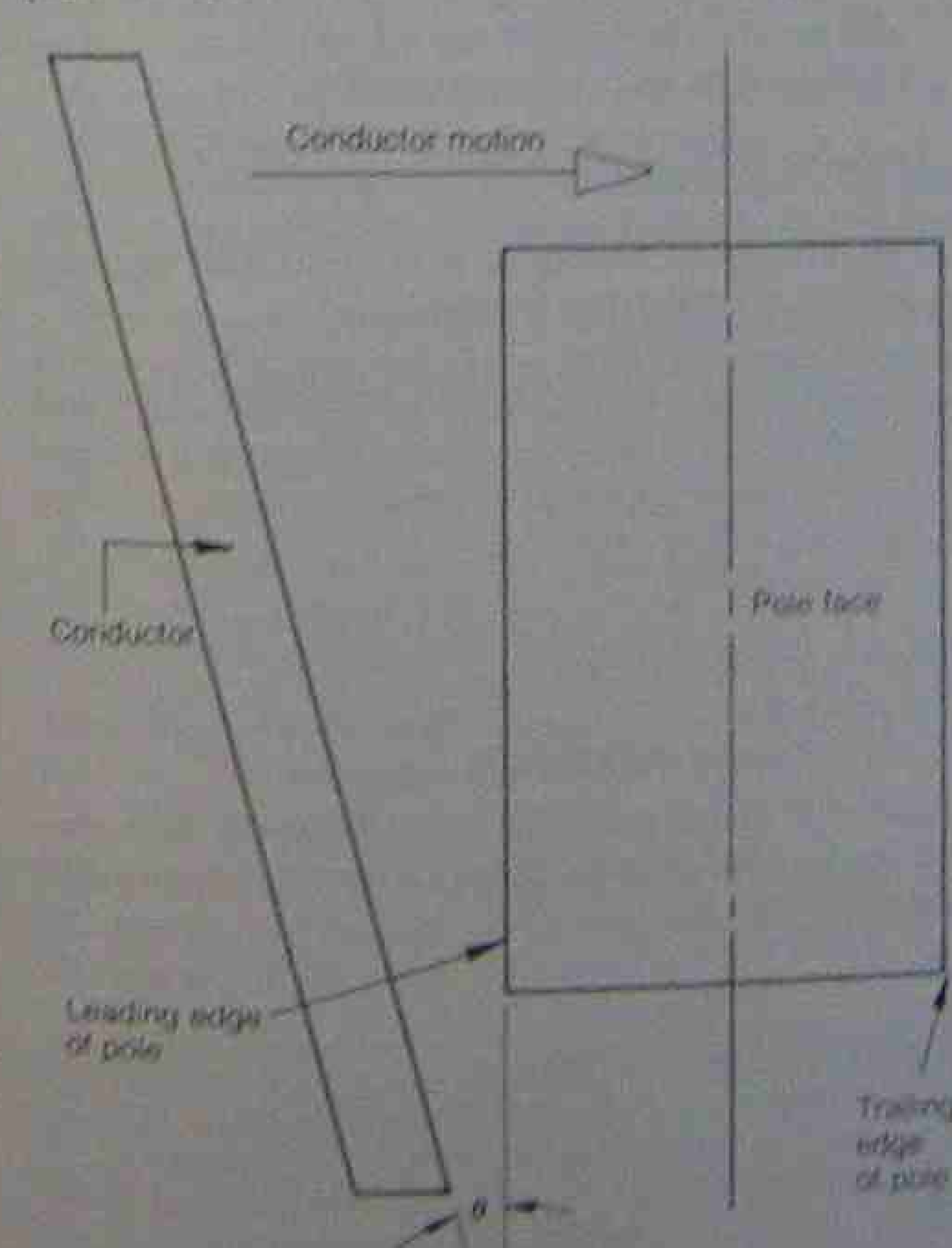


Figure 8.8 • Setting conductors at an angle to the pole face to produce sinusoidal waveforms

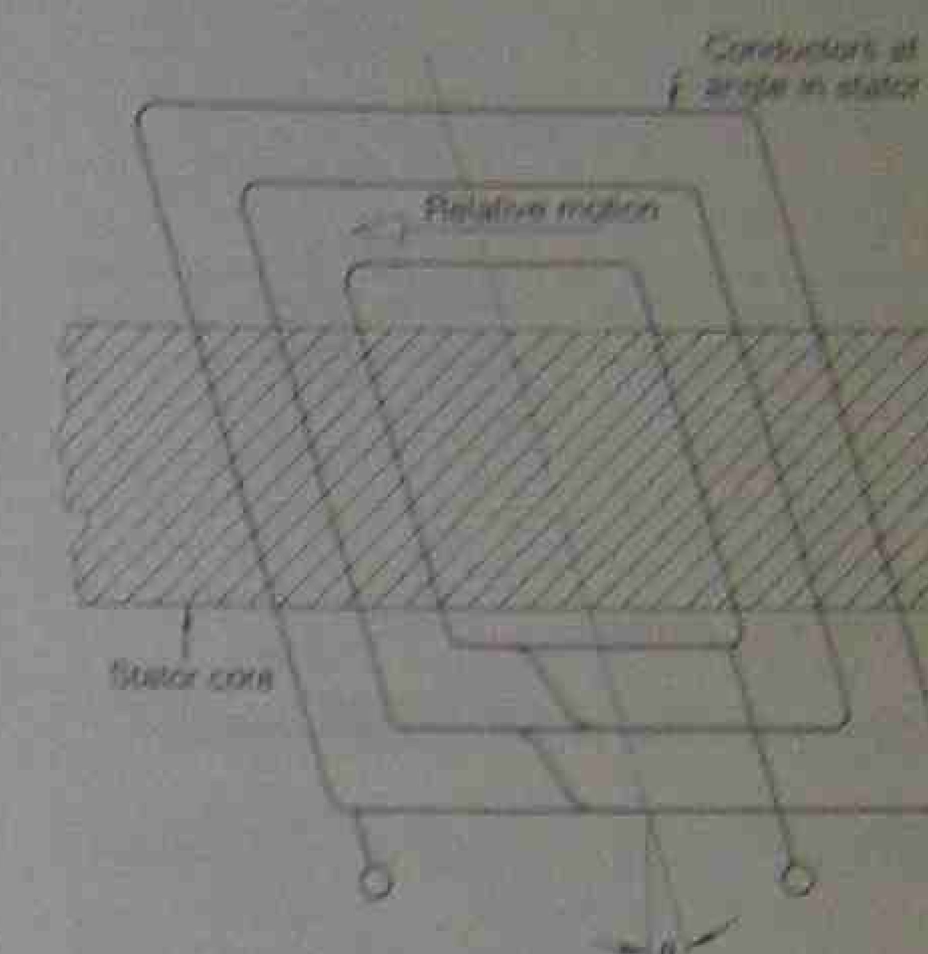


Figure 8.9 • Distributed windings at an angle to improve sinusoidal waveforms

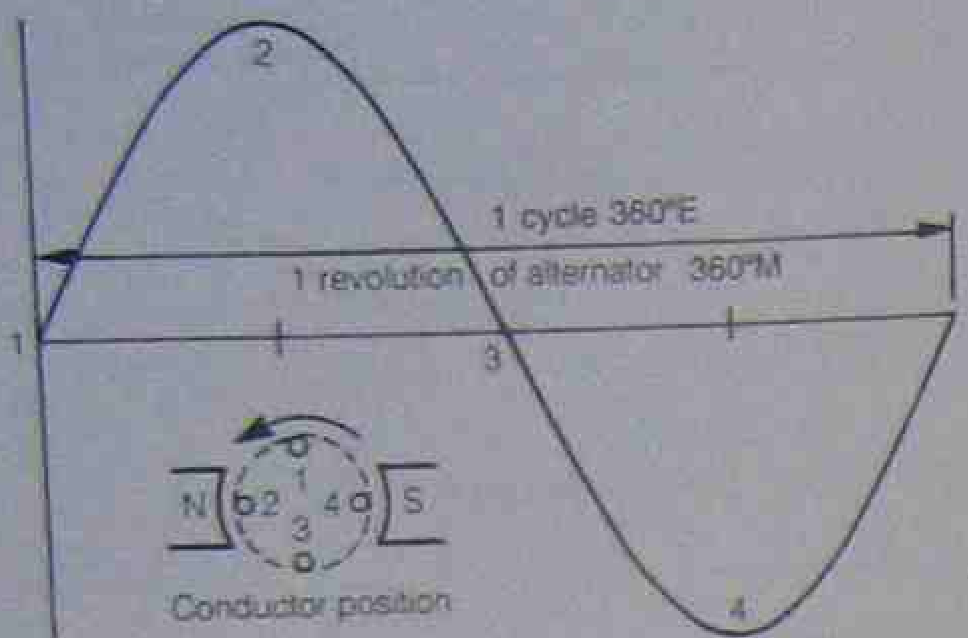
8.7 VOLTAGE AND CURRENT CYCLES

In general usage, the term cycle means a recurrent perio[d] of events. For example, the seasons of a year follow a de[f]inite cycle. In electrical terms, the output of an alternato[r] follows a definite cycle.

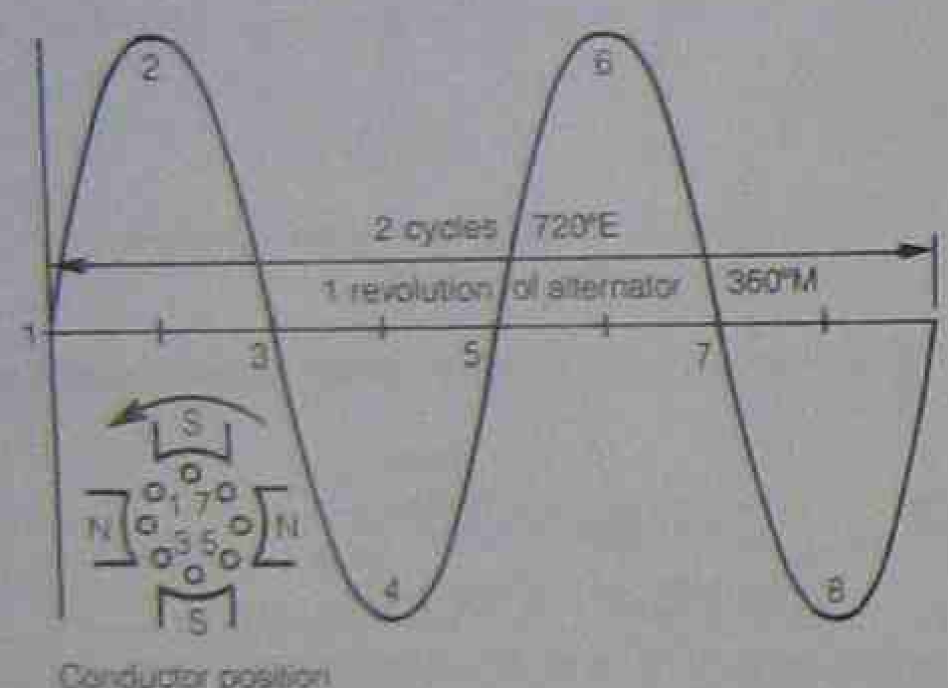
Figure 8.10(a) shows the output e.m.f. of a basic two pole alternator for one complete revolution. During th[e] period the e.m.f. is seen to start from zero, rise to [a] maximum in one direction, reverse to a maximum in th[e] opposite direction and finally return to zero. During th[e] period of one revolution the value of e.m.f. has made on[e] complete electrical cycle. Rotation of the coil throug[h] further revolutions causes a repetition of the above cycl[e] for each revolution.

It should be noted that there is not an abrupt change i[n] e.m.f. from zero to its maximum in either directio[n]. Because the conductors cut across the magnetic flux [at] various angles this cutting angle is sometimes referred [to] as the 'angle of attack'. This affects the relative speed [of] the cutting of those lines of force. In Figure 8.10(a), fo[r] example, conductors '1' and '3' are instantaneously trav[-]elling parallel to the lines of force and not actually cuttin[g] across them. Conductors '2' and '4' are cutting across th[e] magnetic lines of force at right angles at the same instan[t] and are able to generate their maximum e.m.f. Maxi[-]mum rate of cutting relates to the maximum induce[d] voltage. This is further discussed in section 8.7.2.

Consider now the effect when four poles are used in th[e] alternator. During one revolution, a particular side of th[e] coil will rotate from, say, a north pole to a south pole, the[n] to the next north pole and on to the next south pole, an[d] finally back to the original position. During this time th[e] output e.m.f. rises twice to a maximum in each directio[n] and thus completes two cycles, as shown in Figure 8.10(



(a) A 2-pole alternator gives 1 cycle for 1 revolution



(b) A 4-pole alternator gives 2 cycles for 1 revolution

Figure 8.10 • Relationship between number of poles and output frequency for one revolution of an alternator

Obviously, then, the number of completed cycles of e.m.f. in a given time depends on the number of poles that a rotor passes in that time and not on the number of mechanical degrees that the rotor rotates through in the same time.

8.7.1 Electrical degrees and frequency

Since the number of cycles in a given period is not directly related to the number of mechanical degrees of rotation, it is necessary to use another unit for this purpose. The unit used is the *electrical degree*.

Basically, 360 electrical degrees are equivalent to the amount of rotation necessary to produce one complete cycle; that is, one cycle is generated when a coil side rotates from one pole to the following pole of like polarity.

Table 8.1 shows the relationship between the terms electrical and mechanical degrees for alternating current machinery.

In general terms the word *frequency* is used to indicate the number of times a certain event recurs in a given period of time. In electrical terms the frequency of an e.m.f. indicates the number of complete cycles in a given time. It is usual to measure frequency in cycles per second or hertz (Hz); that is:

25 cycles per second = 25 Hz

These relationships may be shown by simple examples.

Consider a basic two-pole machine in which the rotor turns at 3000 r/min:

$$\text{number of cycles in 1 revolution} = \frac{\text{poles}}{2} = \frac{2}{2} = 1$$

number of cycles in 3000 rev = 3000
number of cycles per minute = 3000

$$\text{number of cycles per second} = \frac{3000}{60} = 50\text{ Hz}$$

Compare this with a second machine of four poles in which the rotor turns at 1500 r/min:

$$\text{number of cycles in 1 revolution} = \frac{\text{poles}}{2} = \frac{4}{2} = 2$$

number of cycles in 1500 rev = 1500 × 2
number of cycles per minute = 3000

$$\text{number of cycles per second} = \frac{3000}{60} = 50\text{ Hz}$$

From the above examples it is seen that frequency depends on the following relationship:

$$\text{frequency in Hz} = \frac{\text{speed in r/min} \times (\text{number of poles}/2)}{60}$$

that is, $\boxed{f = \dfrac{np}{120}}$

where f = frequency in Hz
n = r/min
p = number of field poles

Table 8.2 shows the relationship between rotor speed and the number of poles in an alternator for an output frequency of 50 Hz.

Table 8.1 • Relationships between electrical and mechanical degrees

Number of poles (p)	Mechanical degrees between like poles (°M)	Mechanical degrees in one revolution (°M)	Electrical degrees between like poles (°E)	Electrical degrees in one revolution (°E)	Cycles in one revolution
2	360	360	360	360	1
4	180	360	360	720	2
6	120	360	360	1080	3
8	90	360	360	1440	4
10	72	360	360	1800	[illegible]

Table 8.2 • Relationship between rotor speed and number of poles

Number of poles	2	4	6	8	10	12
Speed in r/min	3000	1500	1000	750	600	500

Where an a.c. waveform is repeated consistently and regularly, it is called a periodic wave and has a periodic function of time. It is the time required for one complete cycle of a.c.:

$$\boxed{\text{Periodic time for an a.c. waveform} = \frac{1}{f}\text{ seconds}}$$

where f = frequency, for example:

$$\begin{aligned}\text{periodic time for a 50 Hz waveform} &= \frac{1}{f}\\ &= \frac{1}{50}\\ &= 20\text{ ms}\end{aligned}$$

8.7.2 Instantaneous values of an alternating voltage

Figure 8.11 shows the sine wave of e.m.f. generated as an alternator coil is rotated through 360°E. In the example illustrated, the maximum voltage generated occurs when $\theta = 90°$ and is usually indicated by the symbol V_{max} or V_m. At this instant the coil cuts the flux at 90°.

An easy way to obtain the instantaneous voltage v is to use the sine ratio of θ at a particular value.

When a series of values is plotted on a curve, the shape obtained is called a sine wave, or is said to be sinusoidal in shape.

Since V_{max} occurs when $\theta = 90°$, V_{max} corresponds to a sine ratio of 1. The sine curve, however, varies between 0 and 1 as θ varies between 0° and 90°. The instantaneous voltage v must also vary between 0 and a maximum value as θ varies between 0° and 90°:

$$\boxed{v = V_{max} \sin\theta}$$

where V_{max} = maximum voltage
v = instantaneous voltage
θ = angle of rotation

In general, if an alternating voltage is applied to a load, the current that flows is also alternating, so the formula above is still applicable:

that is, $\boxed{i = I_{max} \sin\theta}$

where I_{max} = maximum current
i = instantaneous current

Trigonometrical sine values for angles from 0° to 360° were discussed in Chapter 1. It may be necessary to refer to the relevant section to revise the method for finding the value of the sine ratio for various angles, and to determine

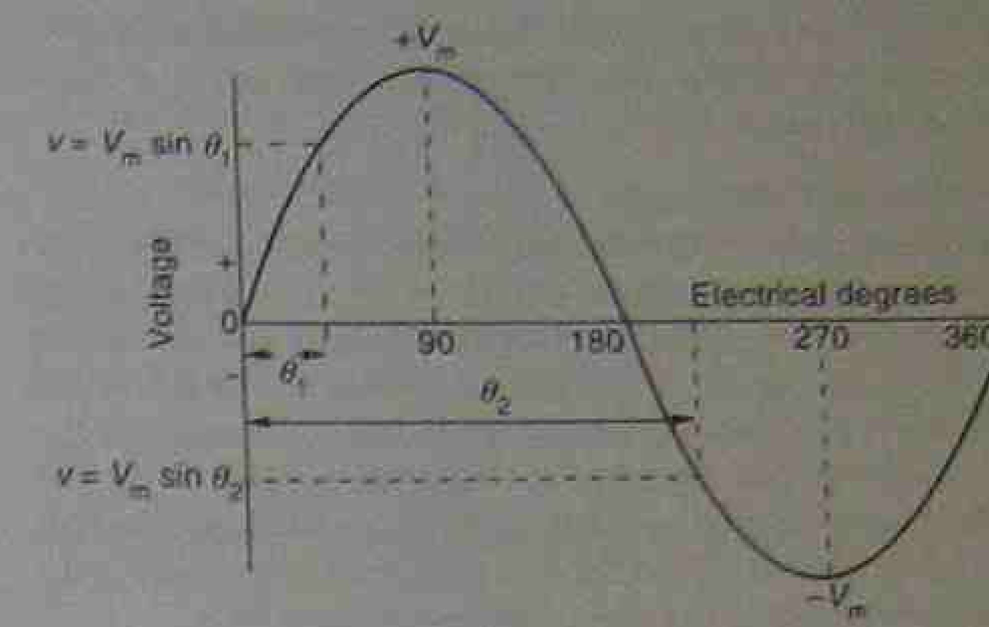


Figure 8.11 • Sinusoidal voltage waveform

Example 8.3

The maximum e.m.f. generated in an alternator coil is 200 V. Calculate the instantaneous voltage v for the following angles of rotation:

(a) $\theta = 30°$
(b) $\theta = 75°$
(c) $\theta = 150°$
(d) $\theta = 240°$
(e) $\theta = 310°$

$$v = V_{max} \sin\theta$$

(a) $v = 200 \sin 30°$
$= 200 \times 0.5$
$= 100\text{ V}$

(b) $v = 200 \sin 75°$
$= 200 \times 0.9659$
$= 193\text{ V}$

(c) $v = 200 \sin 150°$
$= 200 \sin (180 - 150)°$
$= 200 \sin 30°$
$= 200 \times 0.5$
$= 100\text{ V}$

(d) $v = 200 \sin 240°$
$= 200\,[-\sin (240 - 180)°]$
$= 200\,(-\sin 60°)$
$= 200 \times -0.87$
$= -173\text{ V}$

(e) $v = 200 \sin 310°$
$= 200\,[-\sin (360 - 310)°]$
$= 200\,(-\sin 50°)$
$= 200 \times -0.76$
$= -153\text{ V}$

8.8 CONSTRUCTION OF SINUSOIDAL CURVES

There are two methods for the construction of a sinusoidal waveform: the rotating line (vector) method, and the use of instantaneous values. The accuracy of the resultant curve naturally depends on the accuracy of construction and neither method has any distinct advantage over the other. It should be noted, however, that the curve produced is not simply two semicircles—nor has it straight lines in any section.

Rotating line method

Preliminary construction involves drawing a circle of some convenient diameter where the radius represents the maximum instantaneous voltage to some selected scale. The axes for the graph of the sine wave are drawn adjacent to the circle and the horizontal axis is extended to the left to become the diameter of the circle (see Fig. 8.12). The circle is then divided up with radii at equal-angle intervals (e.g. 12 angles of 30° each) and labelled accordingly, with 0° commencing at the zero degree reference axis.

On the graph, the horizontal axis is divided into the same number of equal spaces as the angle intervals in the circle.

To locate the construction points for drawing the actual curve, the points on the circumference are projected to the right until they meet the corresponding point on the horizontal axis projected either upwards or downwards as required. Once the points are located, a curve joining the points is drawn in.

Calculated instantaneous values

Axes for a graph are constructed as in the previous method. The vertical axis is scaled for both positive and negative values of voltage (or current), while the horizontal axis is marked out in degrees of rotation.

The instantaneous values are then calculated for various angles from the formula $v = V_m \sin \theta$.

The values for v are then plotted against the corresponding angle, as shown in Figure 8.13. The curve is identical to that produced by the previous method.

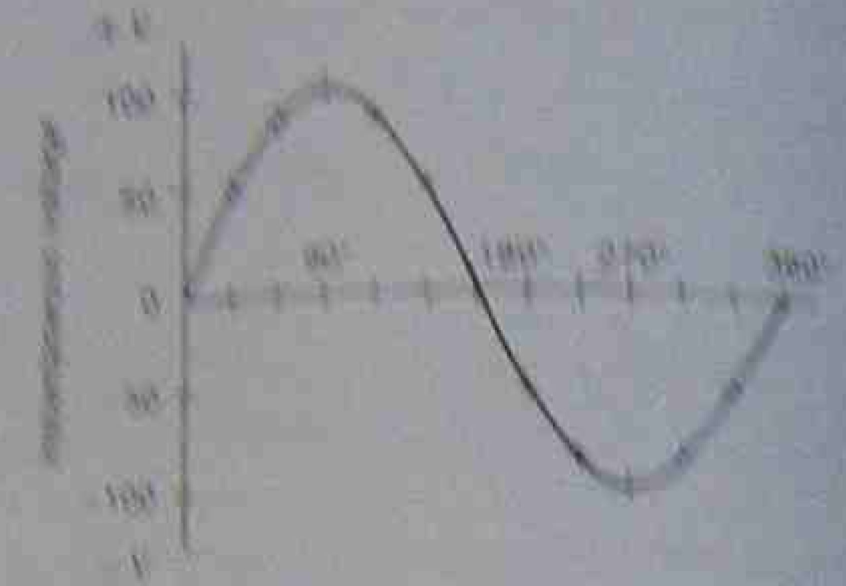


Figure 8.13 ▸ Instantaneous value method for producing a sine wave

8.9 SINUSOIDAL WAVE VALUES

Summarising some of the terms used so far to describe alternating waveforms:

1. The term *periodic function* has previously been used to describe an alternating wave that repeats itself.
2. The number of times a wave repeats itself in one second is called the *frequency* of that wave and is expressed in cycles per second or hertz.
3. The *periodic time* is the time taken for one cycle to complete itself and is equal to the reciprocal of the frequency (1/*f* seconds).
4. An *instantaneous value* of either a voltage or current waveform is a value expressed at only one instant of time.

Other values used to describe a sinusoidal waveform are now discussed.

8.9.1 Average values

An alternating sinusoidal waveform is continually varying in value, so a meaningful standard must apply when describing its value, whether as a voltage or a current. Mathematically the average value is found from one of the following expressions:

$$\boxed{V_{av} = 0.637\ V_{max}} \qquad \boxed{I_{av} = 0.637\ I_{max}}$$

Note: These expressions are accurate only for sinusoidal waveforms.

Each value applies for individual half-cycles and since the direction of flow alternates, the average flow for a complete cycle is zero.

Average values are more applicable in circuits involving the conversion of a.c. to d.c., than in the usual a.c. circuits. In some circumstances it might be necessary to know the average value of a supply.

Average values are not suitable as a general basis for comparing alternating waveforms with direct current. It is essential that there be a standard for comparing the real effects of alternating forces.

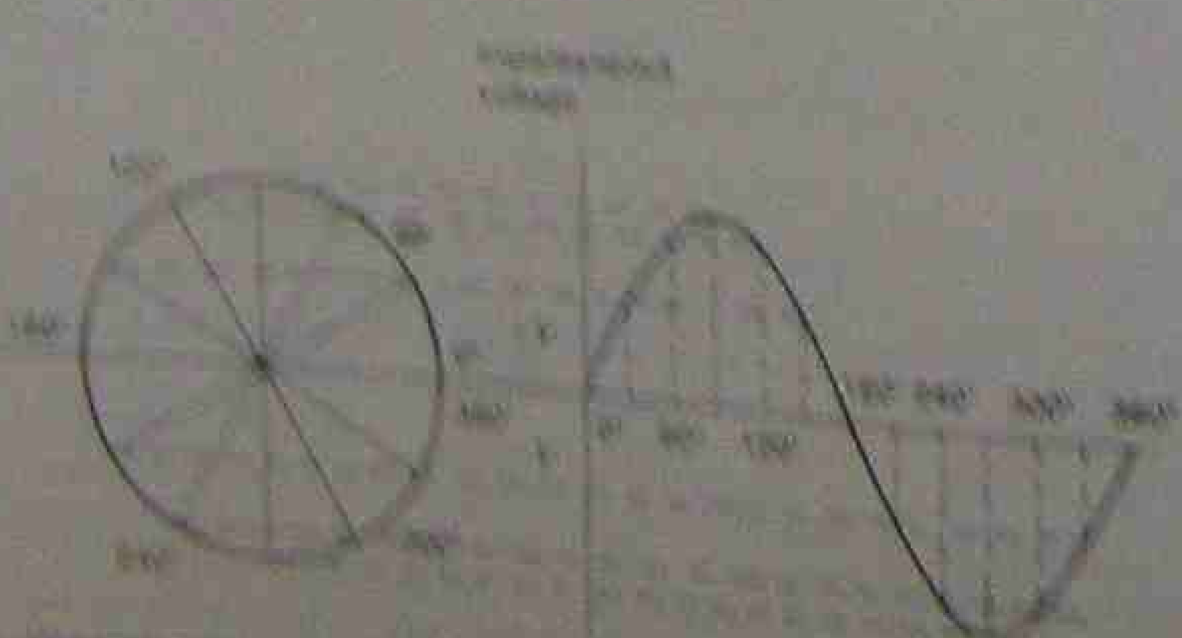


Figure 8.12 ▸ Rotating line—sine wave construction

8.9.2 Root-mean-square values

Since the average value of a.c. is not an entirely satisfactory solution to the problem, the heating effects of a.c. and d.c. have been used as a standard. The value of a sinusoidal waveform that produced the same heating effect as d.c. was obtained mathematically and called the root-mean-square value (r.m.s.) of that wave. (The term refers to the method by which it is calculated.) An a.c. current of 1 A (the r.m.s. value) will create the same heating effect as that of a d.c. current of 1 A. The effective voltage that will create a current flow to do this is called the r.m.s. voltage.

$$\boxed{V_{r.m.s.} = 0.707\ V_{max}} \qquad \boxed{I_{r.m.s.} = 0.707\ I_{max}}$$

Note: These expressions are accurate only for sinusoidal waveforms.

Unless specifically stated otherwise, any a.c. value should always be considered as the r.m.s. value. The normal domestic a.c. supply is referred to as 240 V. It is understood that without further qualification it is the r.m.s. value that is intended.

The maximum value of a 240 V r.m.s. supply can be found from the expression:

$$V_{r.m.s.} = 0.707\ V_{max}$$

$$V_{max} = \frac{V_{r.m.s.}}{0.707}$$

$$= \frac{240}{0.707}$$

$$= 340\ V$$

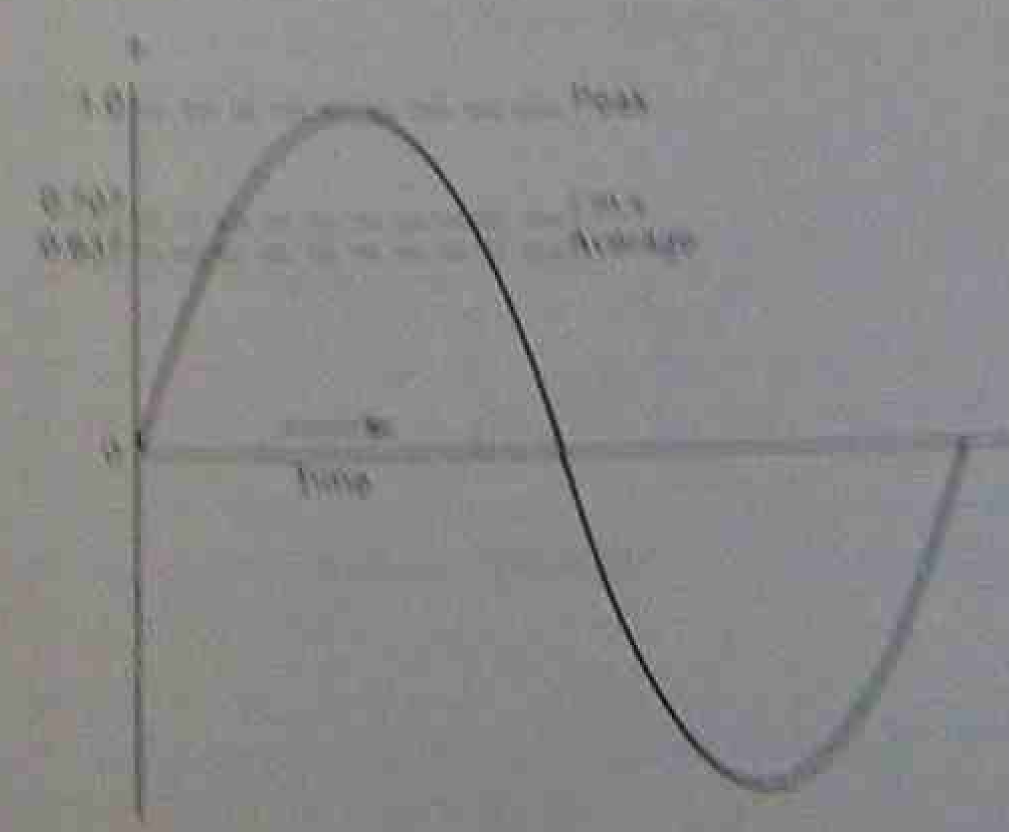


Figure 8.14 ▸ Values of a sine wave

8.9.3 Crest factor

The crest factor is used for waveforms in general and is simply a ratio that shows the relationship between the maximum value and the r.m.s. value for a particular waveform.

$$\boxed{\text{crest factor} = \frac{V_{max}}{V_{r.m.s.}}}$$

For example, for a sinusoidal waveform:

$$\text{crest factor} = \frac{V_{max}}{0.707\ V_{max}} = 1.414$$

Since the crest factor is only a ratio, there are no units.

The crest factor is useful for indicating the maximum e.m.f. that can be expected in a.c. waveforms of a known r.m.s. value, thus indicating the value of voltage the insulation must withstand.

8.9.4 Form factor

For any waveform, the form factor is defined as the ratio:

$$\boxed{\text{form factor} = \frac{\text{r.m.s. value}}{\text{average value}}}$$

and is useful for indicating the general shape of a.c. waveforms. For example, for a sine wave:

$$\text{form factor} = \frac{0.707\ \text{maximum value}}{0.637\ \text{maximum value}}$$

$$= 1.11$$

For waveforms in general, if the form factor is greater than 1.11, the waveform is more 'peaky' than a sine wave. On the other hand, if the form factor is less than 1.11, the waveform is more flat-topped than a sine wave. For example, the form factor of a square wave is 1.0.

8.9.5 Peak-to-peak values

There will be occasions where the value of a wave from its positive peak to its negative peak is important. With most alternating waveforms it is equal to twice the maximum value in either direction. For example, with a 240 V alternating current supply the maximum value of voltage is found by dividing 240 by 0.707.

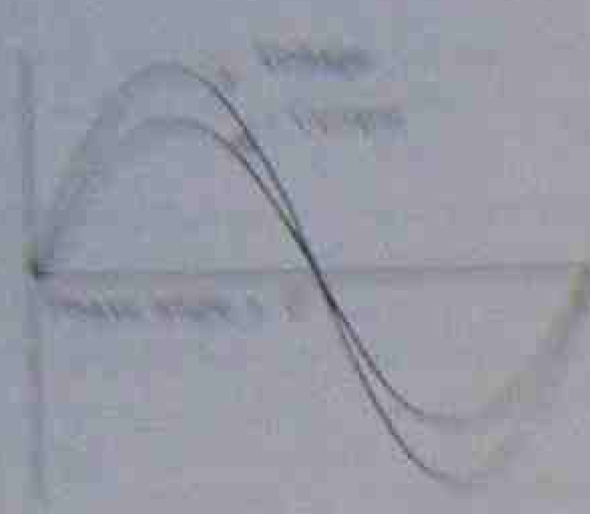
The same value can be obtained by multiplying 240 by 1.414 (the reciprocal of 0.707 is $\sqrt{2}$ = 1.4142). Either method gives 339.4 V as the peak value. Then the peak-to-peak value is twice this, or 678.8 V. A knowledge of this is useful when measuring values of alternating waveforms on a cathode ray oscilloscope.

8.10 PHASORS

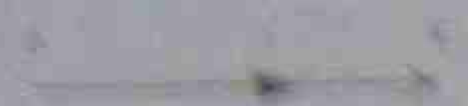
By definition, a phasor is a line that, by its length and direction, represents the magnitude and phase relationship of a.c. quantities. Formerly called a vector, for electrical analysis purposes the word phasor is used.

Alternating currents are caused by connecting an alternating voltage across a load. When the current and voltage curves pass through the zero position and increase to their maximum values in the same direction at the same time, the curves are in step or in phase with each other, as shown in Figure 8.15(a).

The phasor diagram for the in-phase condition is shown in Figure 8.15(b). A convention has been adopted in this book to distinguish between voltage and current phasors. The voltage phasor is drawn with an open arrowhead and the current phasor is drawn with a closed arrowhead. The two types of phasor conventions are illustrated in Figure 8.15(c), and should not be confused with arrows used for dimensioning purposes.



(a) Waveform diagram

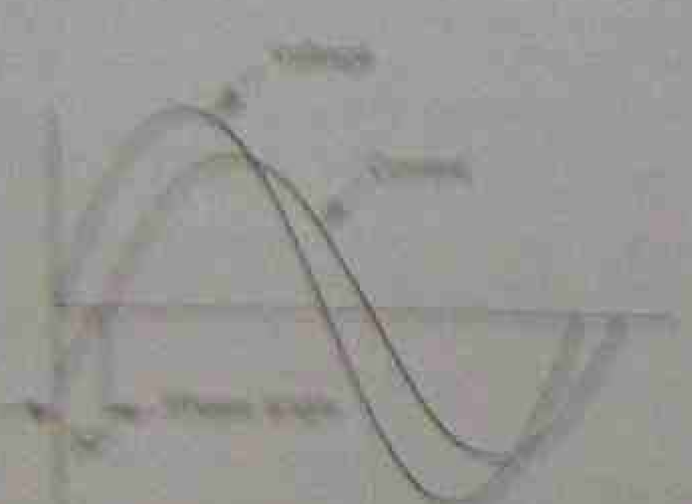


(b) Phasor diagram

(c) Phasors

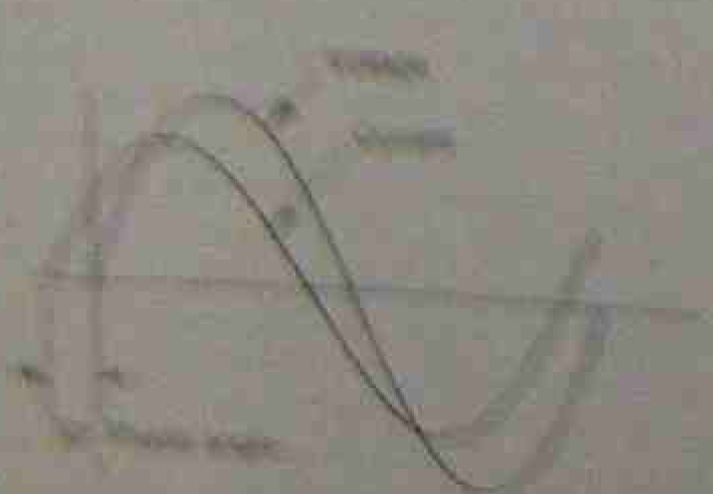
8.10.1 Phasor diagrams

8.10.2 Phasor addition



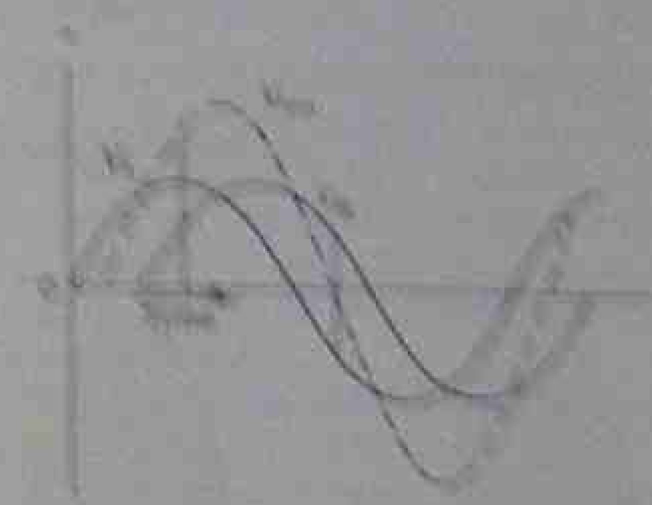
(a) Waveform diagram

(b) Phasor diagram



(a) Waveform diagram

(b) Phasor diagram

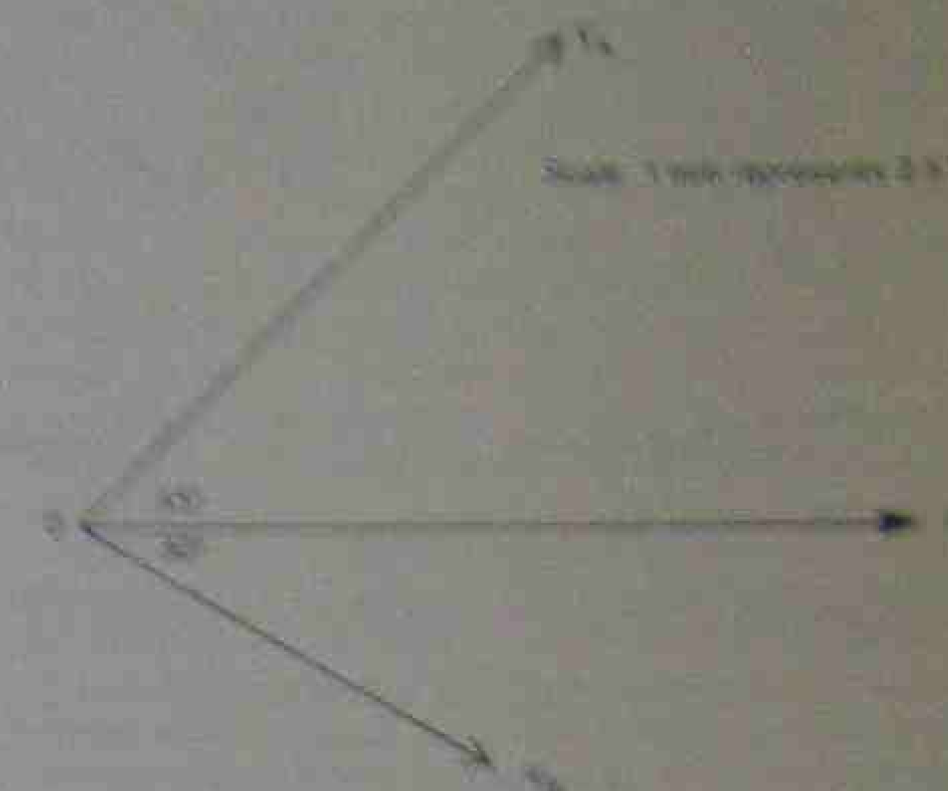


(a) Waveform diagram

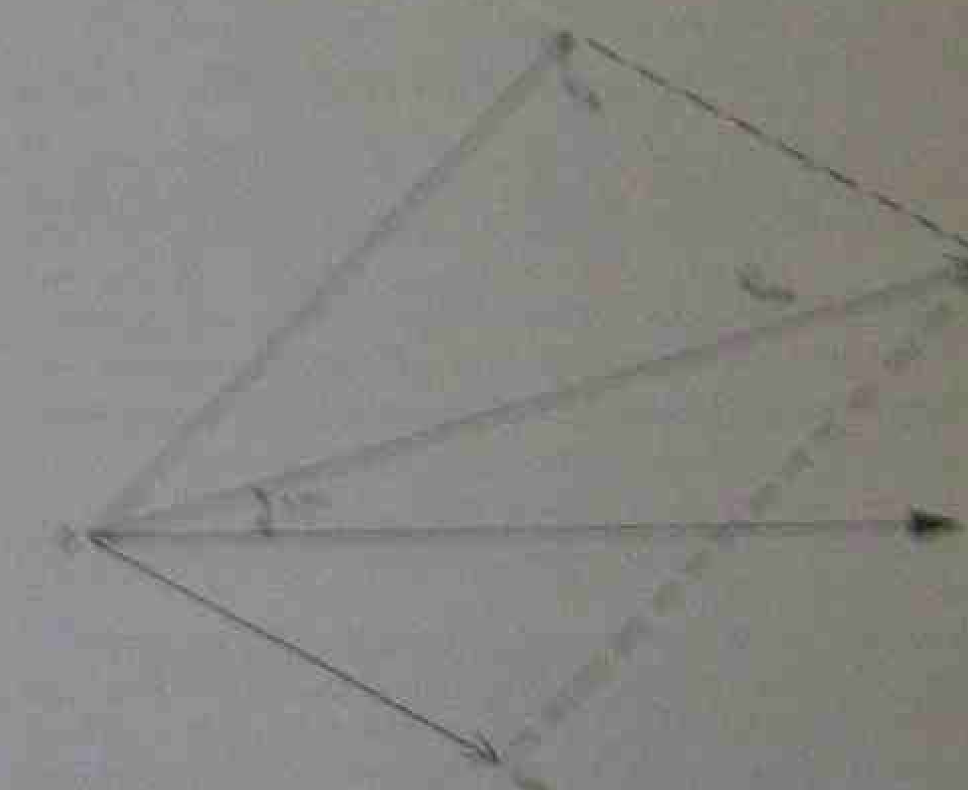
(b) Phasor diagram

Example 8.4

(a) Reference phasor for example 8.4



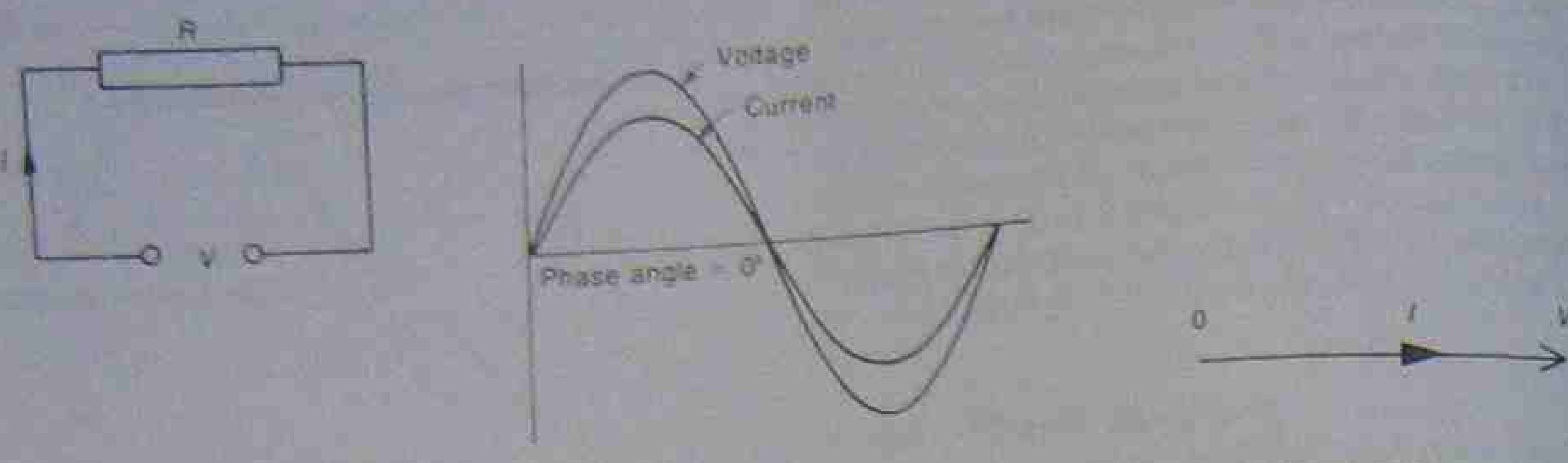
(b) Drawing in the voltage phasors



(c) Completing the phasor addition

8.11 RESISTANCE IN A.C. CIRCUITS

8.11.1 Power in a resistive circuit



(a) Circuit diagram (b) Waveform diagram (c) Phasor diagram

Figure 8.20 • Purely resistive circuit on a.c.

Example 8.5

A sinusoidal voltage of $V_{max} = 2$ V is applied to a resistance of 2 Ω. Draw a curve to show the value of power dissipated by the resistor at any given instant during one cycle.

The first step is the preparation of a table showing the values of v, i and P for angles of rotation between 0°E and 360°E. Since $V_{max} = 2$ V and $R = 2$ Ω, then $I_{max} = 1$ A (see Table 8.3).

The curves for voltage, current and power have been plotted in Figure 8.21 and the following details should be noted:

1. The power curve is sinusoidal in shape.
2. There are no negative values of power.
3. A horizontal line drawn 1 unit above the x axis forms the axis of power curve and therefore represents the average value of power during the cycle. The average power dissipated by the resistor is therefore 1 W.
4. The power curve completes two cycles for each complete cycle of current or voltage.

If values of $I_{r.m.s.}$ and $V_{r.m.s.}$ are taken:

$$I_{r.m.s.} = 0.707\, I_{max} = 0.707 \times 1 = 0.707 \text{ A}$$

$$V_{r.m.s.} = 0.707\, V_{max} = 0.707 \times 2 = 1.404 \text{ V}$$

$$\text{product of } I_{r.m.s.} \text{ and } V_{r.m.s.} = 1.414 \times 0.707 = 1 \text{ W}$$

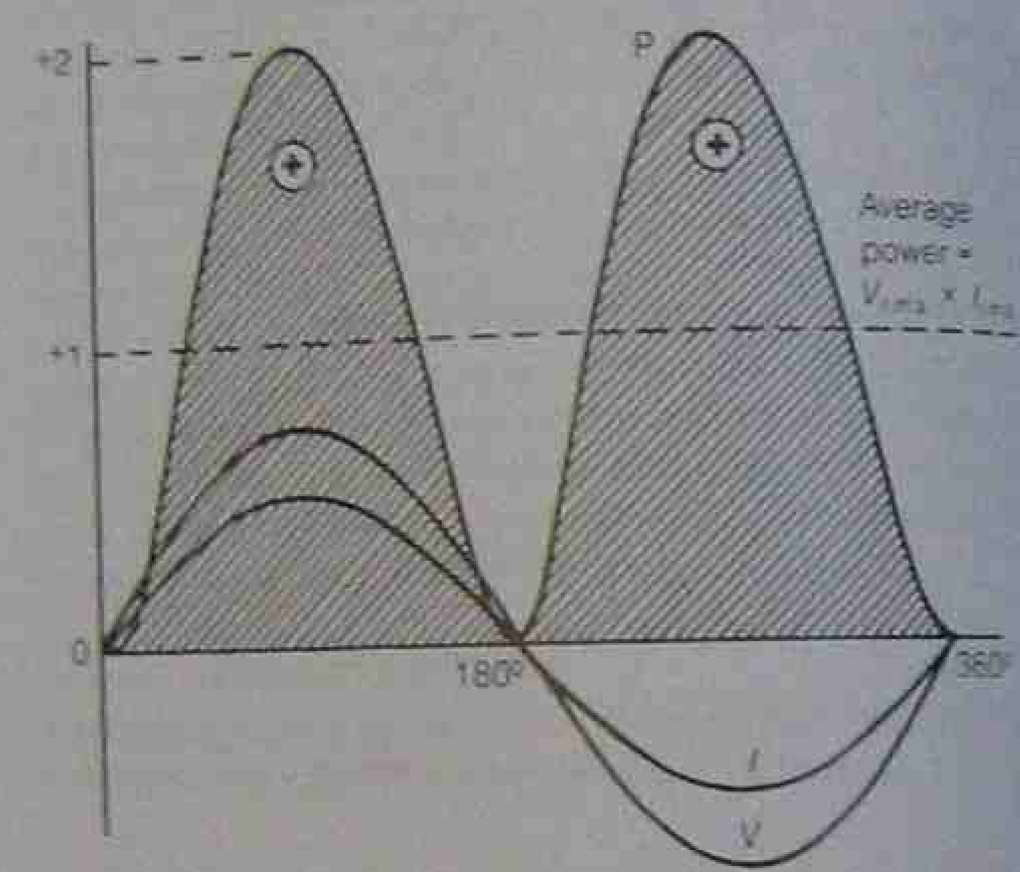


Figure 8.21 • Power consumed by a pure resistor on a.c.

This result corresponds with the value for average power shown in Figure 8.21, and indicates that average power can be found by taking the product of $V_{r.m.s.}$ and $I_{r.m.s.}$:

$$\text{average power} = V_{r.m.s.} \times I_{r.m.s.}$$

$$P = VI \quad \text{or} \quad P = \frac{V^2}{R} \quad \text{or} \quad P = I^2R$$

but with a.c., V and I must each be measured in r.m.s. values.

For sinusoidal waveforms only, it should be noted that if peak values are used to obtain a power rating, the average power value obtained is always half that of the peak power value, that is, average power equals half maximum instantaneous power. For a sinusoidal waveform:

Table 8.3 • Calculated instantaneous values for example 8.5

Angle of rotation θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$v = V_{max} \sin\theta$	0	1.0	1.73	2.0	1.73	1.0	0	−1.0	−1.73	−2.0	−1.73	−1.0	0
$i = I_{max} \sin\theta$	0	0.5	0.87	1.0	0.87	0.5	0	−0.5	−0.87	−1.0	−0.87	−0.5	0
$P = vi$	0	0.5	1.5	2.0	1.5	0.5	0	0.5	1.5	2.0	1.5	0.5	0

$$\text{Average power } P = V_{r.m.s.} \times I_{r.m.s.}$$

But the r.m.s. value of the wave = 0.707 of the peak value, that is:

$$V_{r.m.s.} = 0.707 \times V_{max} \quad \text{and} \quad I_{r.m.s.} = 0.707 \times I_{max}$$

Substituting maximum values in the power equation gives:

$$P = 0.707 \times V_{max} \times 0.707 \times I_{max} = 0.5\, P_{max}$$

8.11.2 Pure or non-inductive resistance

For practical purposes, all resistors on power-line frequencies, such as incandescent lamps, radiators and electric jug elements, can be considered as consisting of 'pure' or non-inductive resistance. Circuits containing inductors or capacitors discussed later in this chapter are specifically excluded from this category.

At higher frequencies, resistors have varying amounts of both inductive and capacitive effects.

The inductive effect is avoided comparatively easily. Half the resistor is wound on a non-magnetic former in a clockwise direction, and then the other half is wound on in an anticlockwise direction (see Fig. 8.22). The magnetic fields produced around the conductors tend to cancel each other and so prevent the production of self-induced voltages.

Capacitive effects cannot be neutralised by such direct means; consequently the aim is to minimise them. The main method used on long-distance transmission lines is to keep interchanging the positions of the conductors relative to each other and keeping them as far apart from each other as is economically possible.

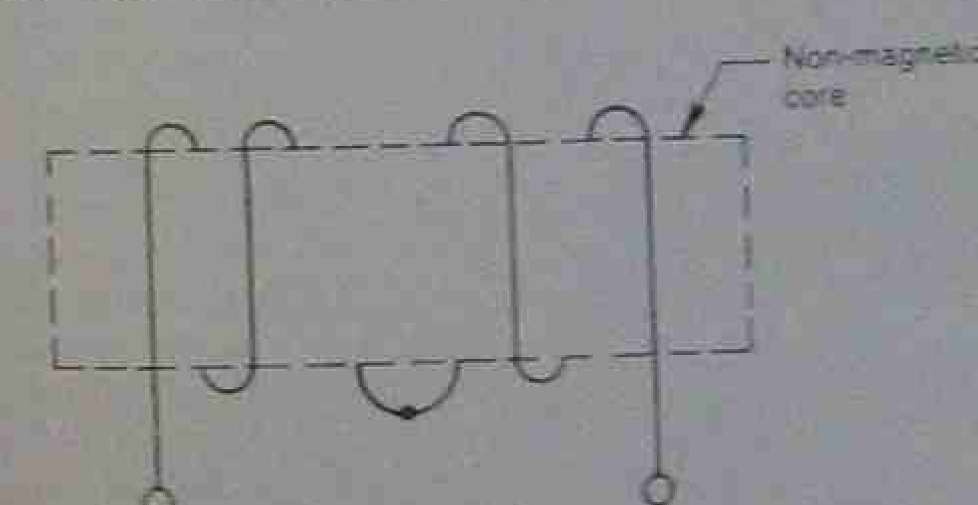


Figure 8.22 • Non-inductive resistor

8.12 INDUCTANCE IN A.C. CIRCUITS

It was shown in Chapter 5 that a change in current flow in an inductive circuit causes a change in flux linkages, thereby causing an induced e.m.f. which, according to Lenz's law, will oppose the change of current flow.

The value of an induced e.m.f. in a circuit depends upon the rate of change of flux linkages. In a d.c. inductive circuit the inductance affects the current flow only when the current is changing (e.g. when closing or opening the circuit). In an a.c. inductive circuit the current is continually changing in value and direction, so generating an induced e.m.f. that must continually oppose the change of current flow.

Figure 8.23(b) shows the relation between the current induced e.m.f. and supply voltage for a purely inductive circuit.

Figure 8.23(c) shows the phase relationships between voltage and current. Using the voltage phasor as the reference, the current phasor lags 90°E behind it. (If the current is used as the reference, the voltage leads the current by 90°E.)

8.12.1 Inductive reactance

On a.c., the change in current flow gives rise to an induced e.m.f. that opposes the current flow. The effect of this current opposition is called inductive reactance (symbol X_L), which is measured in ohms.

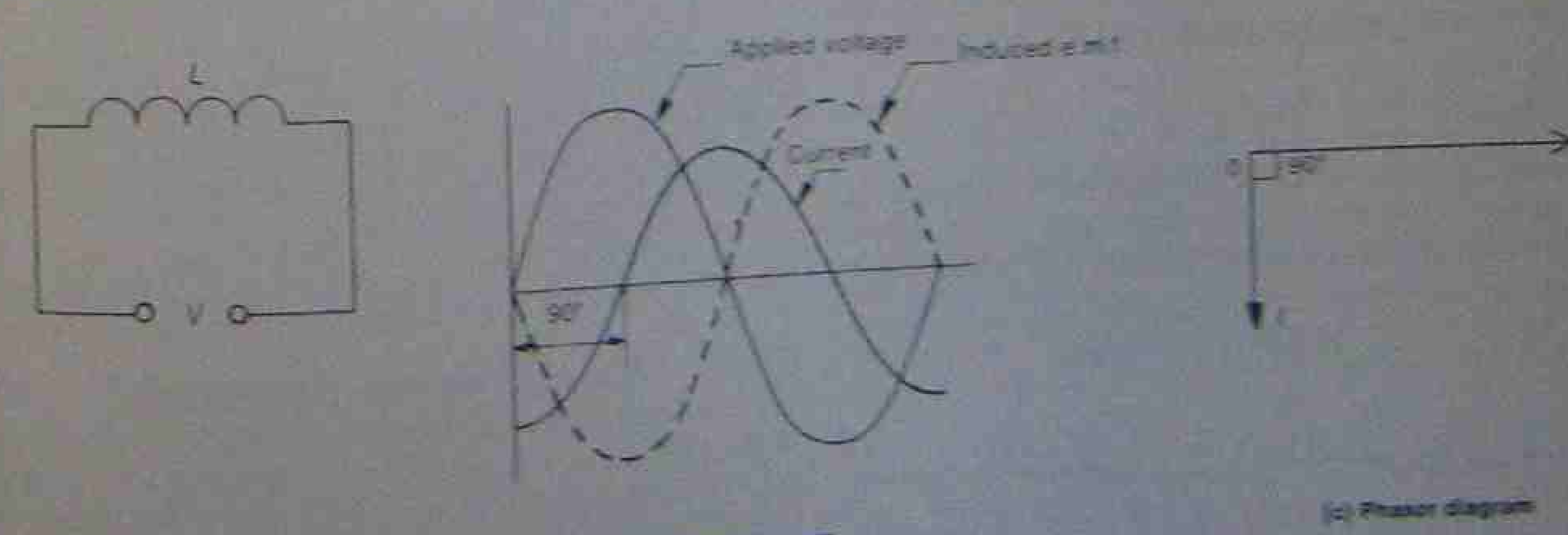
Ohm's law states, in effect, that the current is equal to the voltage divided by the opposition to current flow. Inductive reactance is a type of opposition to current flow:

that is, for pure inductance $\boxed{I = \dfrac{V}{X_L}}$

The value of induction reactance in a circuit depends on the inductance and the rate of change of current flow, which in turn depends on the supply frequency. Inductive reactance can be calculated from the formula:

$$\boxed{X_L = 2\pi f L}$$

where f = frequency in hertz
L = inductance in henrys



(a) Circuit diagram (b) Waveform diagram (c) Phasor diagram

Figure 8.23 • Current and voltage in a purely inductive circuit on a.c.

Example 8.6

Example 8.7

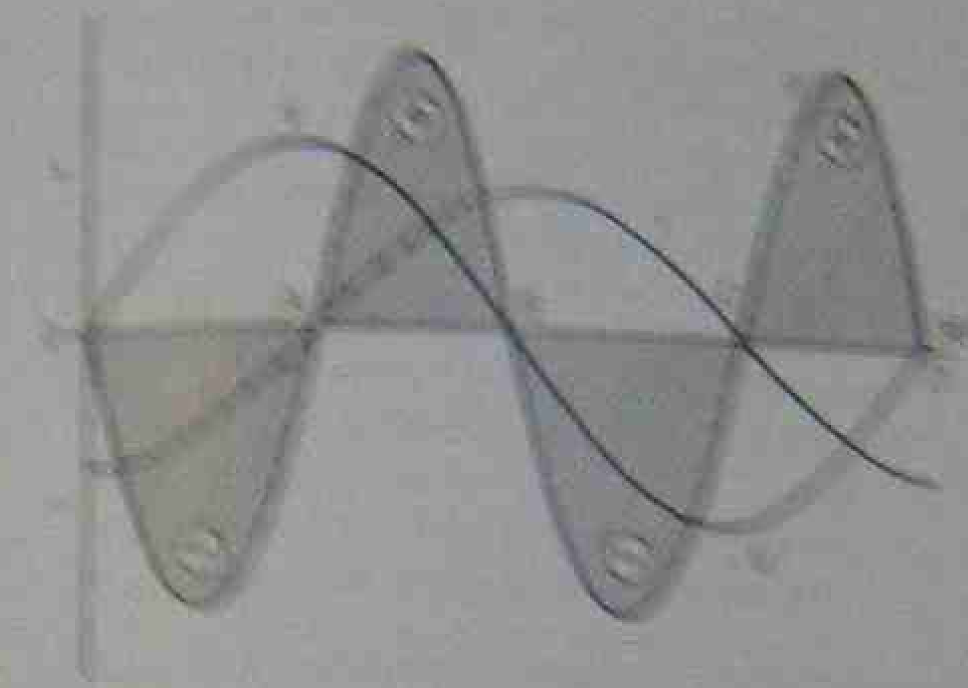
Example 8.8

8.12.2 Inductors in series

8.12.3 Inductors in parallel

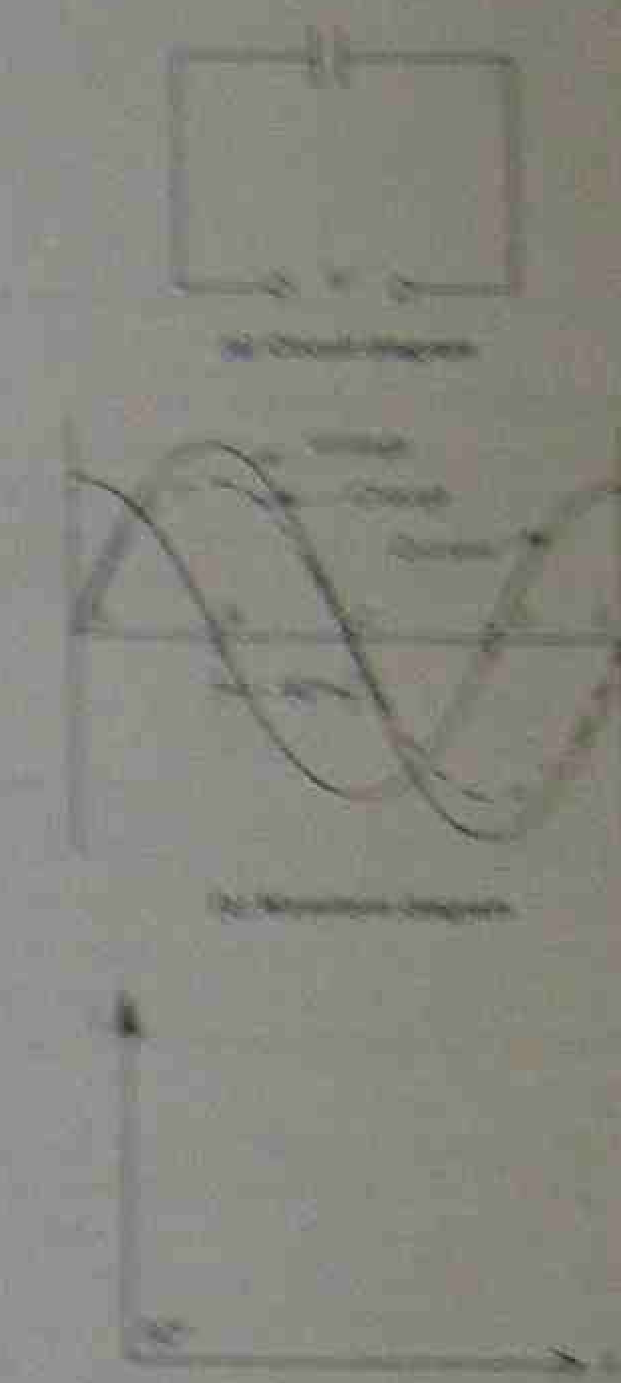
Example 8.9

8.12.4 Power in an inductive circuit



8.13 CAPACITORS ON ALTERNATING CURRENT

8.13.1 Capacitive reactance



Example 8.10

[illegible]

8.13.2 Capacitors in series

When two capacitors are placed in series, the resultant capacity is decreased. On an alternating current supply this effectively increases the opposition to a current flow in a similar fashion to that of resistors placed in series.

That is, $\boxed{X_{C\,total} = X_{C_1} + X_{C_2} + \ldots}$

Example 8.11

[illegible]

[illegible]

8.13.3 Capacitors in parallel

[illegible]

Example 8.12

[illegible]

[illegible]

8.13.4 Power in a capacitive circuit

The power consumed by a pure capacitor at any instant of time is equal to the product of the instantaneous values of voltage and current, as shown in Figure 8.28.

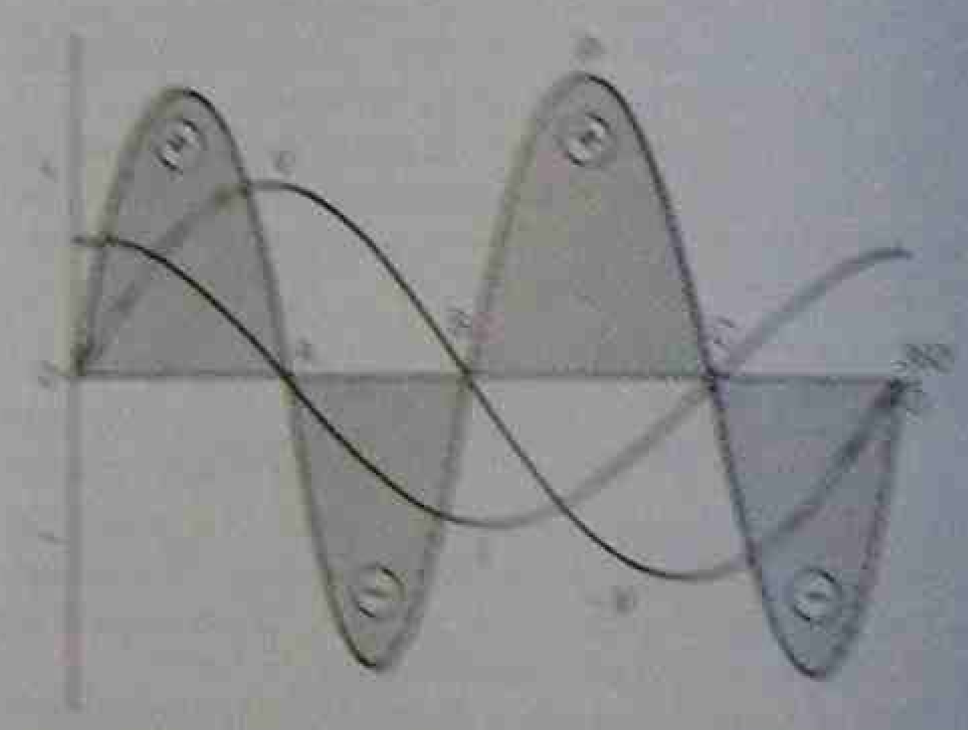


Figure 8.28 • Power consumed by a pure capacitor on a.c.

[illegible]

8.14 RESISTANCE AND REACTANCE IN SERIES ON ALTERNATING CURRENT

[illegible]

small amount of inductive reactance will behave like a resistive circuit rather than an inductive one, but the influence of the inductive reactance, however small, will still be there. Where an a.c. circuit has both inductive and capacitive reactance, their effects tend to cancel each other and cause the circuit to have the characteristics of a resistive circuit. The resulting reactance in the circuit is the difference between the two.

The three basic components and their effects are:

1. Resistance: no phase shift between voltage and current—$P_{av} = V_{r.m.s.} \times I_{r.m.s.}$
2. Inductance: 90° phase shift: current lagging—$P_{av} = 0$.
3. Capacitance: 90° phase shift: current leading—$P_{av} = 0$.

In an a.c. circuit the combined opposition to a current flow of both resistance and reactance is called impedance (symbol Z) and is measured in ohms. Ohm's law is still applicable:

$$\boxed{Z = \frac{V}{I}}$$

where V and I are r.m.s. values.

8.14.1 Series R-L circuits

In a series circuit the current is common to all parts and the total voltage is the phasor sum of the individual voltage drops (Fig. 8.29).

Figure 8.29 • R-L series circuit

[illegible]

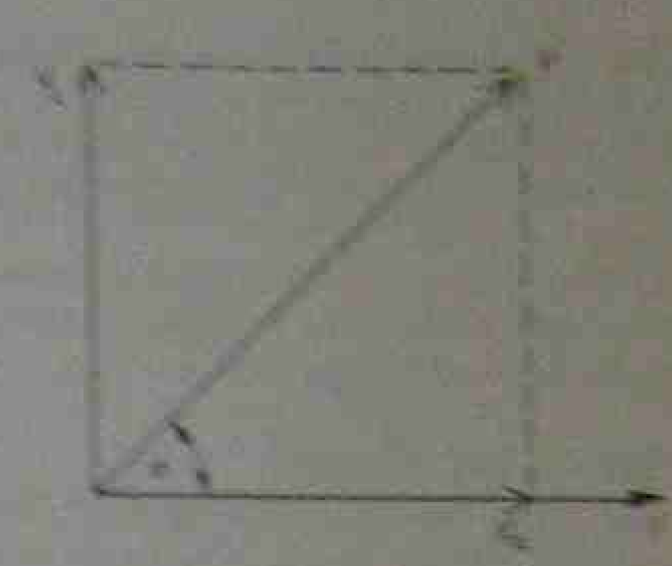
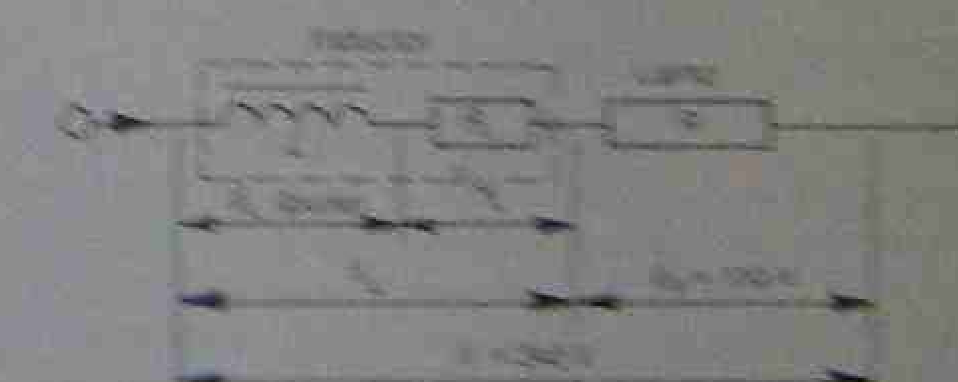


Figure 8.30 • Phasor diagram for a circuit containing resistance and inductance in series

Figure 8.31 • Inductor in series with an incandescent lamp

[illegible]



(a) Circuit diagram

(b) Phasor diagram

Figure 8.32 • Practical inductor in series with a lamp both iron and copper losses. Its value will not necessarily equal the resistance of the windings. The power loss associated with it is the sum of both iron and copper losses. For example, 6 W of iron loss and 4 W of copper loss will show on a wattmeter as 10 W.

Figure 8.32(b) shows the relationship between the respective voltages in this series R-L circuit. The current I lags behind the applied voltage V by ϕ.

8.14.2 Series R-C circuits

When current flows through a series R-C circuit (Fig. 8.33(a)) it causes a voltage drop V_R (due to the resistance), which is in phase with the current, and a voltage drop V_C (due to the capacitive reactance), which lags the current by 90°. The total voltage V is the phasor sum of the two voltage drops V_R and V_C as shown in Figure 8.33(b). The current leads the applied voltage by ϕ.

Commercially made capacitors are considered as pure capacitance for all practical purposes and the problem of losses as shown in the series R-L circuits does not normally occur.

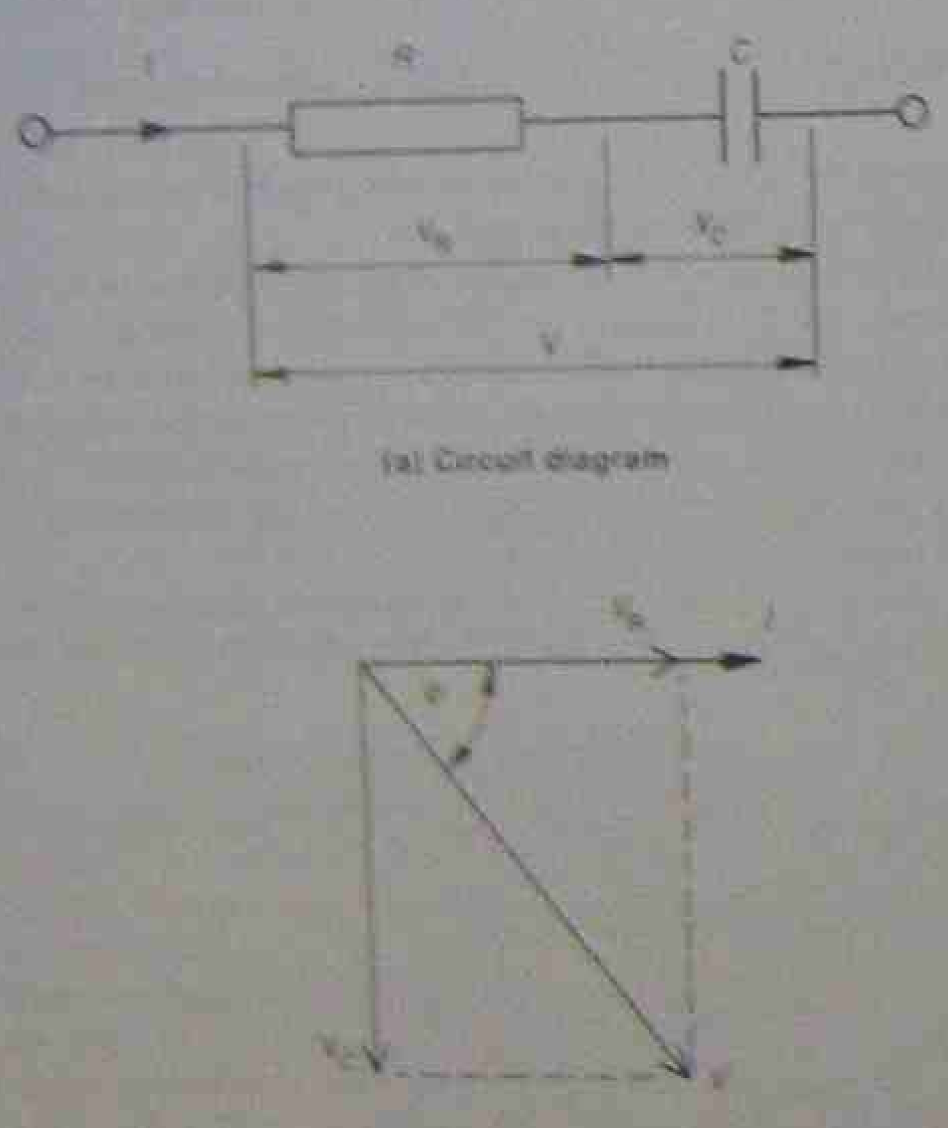


Figure 8.33 • R-C series circuit

8.14.3 Series R-L-C circuits

In Figure 8.34(a) the voltage V_R is in phase with the current, V_L leads the current by 90° and V_C lags the current by 90°.

The two voltages V_L and V_C are 180° out of phase with each other, and so the voltage drop across the total reactance is $V_L - V_C$, as shown in Figure 8.34(b). The total voltage V is the phasor addition of V_R and $(V_L - V_C)$ and ϕ is the phase angle, with the current lagging the applied voltage by ϕ°. If V_C had been greater in value than V_L, the current would have led the voltage by some angle.

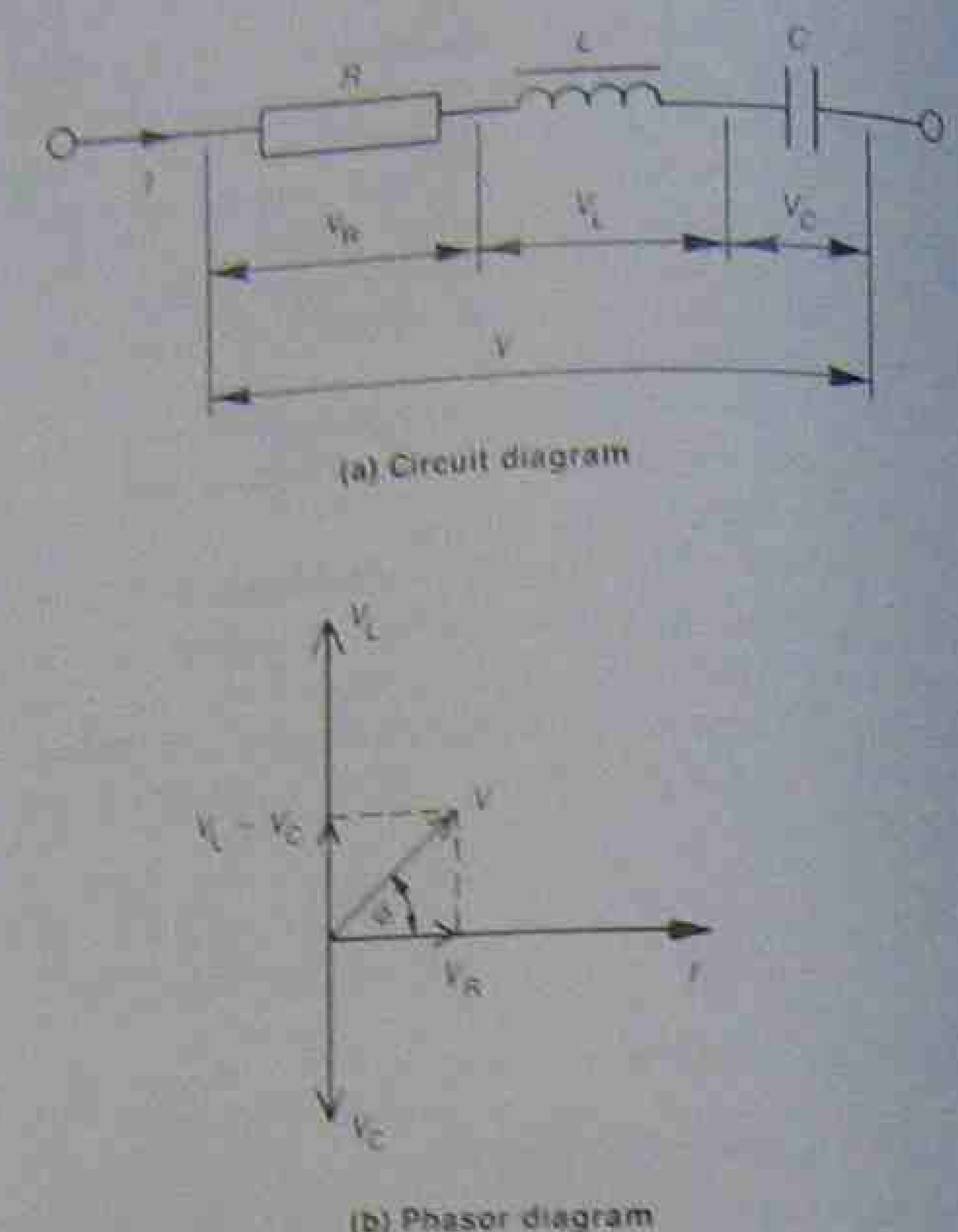


Figure 8.34 • R-L-C series circuit

8.14.4 Series circuit impedance (impedance triangle)

The voltage phasors in Figure 8.35(a) can also be represented as in Figure 8.35(b).

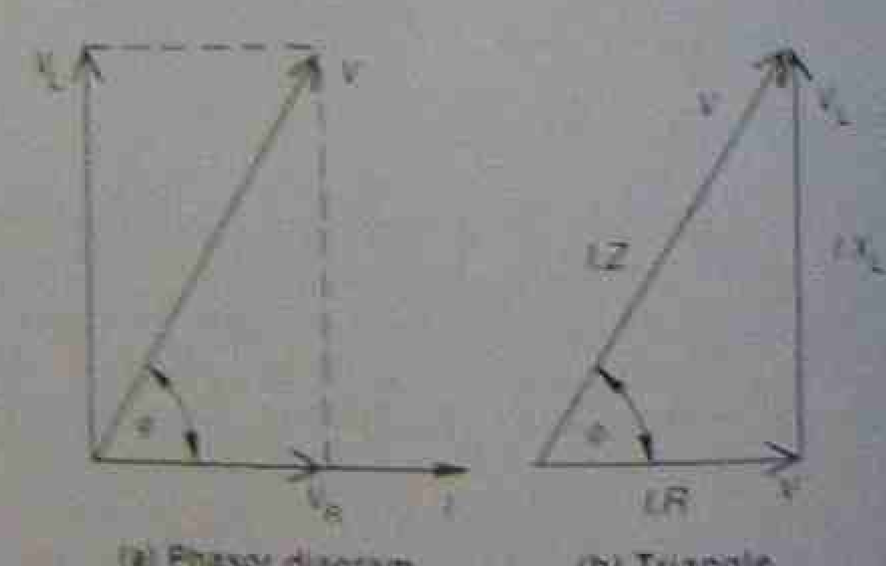


Figure 8.35 • Using a voltage triangle in place of a phasor diagram

Each voltage drop can be expressed as a factor of current; that is, $V = IZ$, $V_R = IR$ and $V_L = IX_L$.

Because the same value of current is common to each component, the triangle in Figure 8.35(b) can also represent R, X_L and Z, as in Figure 8.36. Since it is a right-angled triangle, Pythagoras's theorem is applicable; that is, $Z^2 = R^2 + X_L^2$

That is, $\boxed{Z = \sqrt{R^2 + X_L^2}}$

The angle ϕ between the adjacent sides R and Z has the same value as the angle ϕ between the voltage phasors V and V_R because the two triangles ...

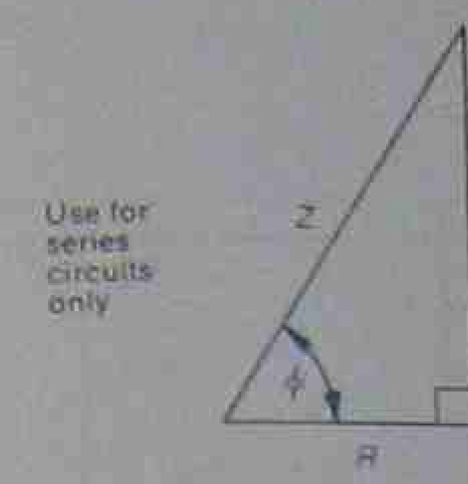


Figure 8.36 • Impedance triangle

similar triangles. In the first case the ratio V_R/V gives the cosine of the angle of phase shift, while in the second it is found from the ratio R/Z.

If it is a series R-L-C circuit, the effective reactance is $(X_L - X_C)$. In this case, then:

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

If X_C has a greater value than X_L then the value of $X_L - X_C$ is a minus quantity. However, this has no effect on the value of Z because a minus number when squared becomes a positive number.

Capacitive reactance tends to produce a current leading the voltage by 90°. Inductive reactance tends to produce a current lagging the voltage by 90°. When the values of capacitive and inductive reactance are equal, they cancel each other out, leaving only resistance as an effective circuit component. Under this condition the circuit is said to be *resonant*. The circuit then behaves as a purely resistive circuit. See section 8.17 for further details on resonance.

Example 8.13

A resistance of 30 Ω is connected in series with an inductive reactance of 60 Ω and a capacitive reactance of 20 Ω. What is the impedance of the circuit?

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{30^2 + (60 - 20)^2} \\ &= \sqrt{900 + 1600} \\ &= \sqrt{2500} \\ &= 50\ \Omega \end{aligned}$$

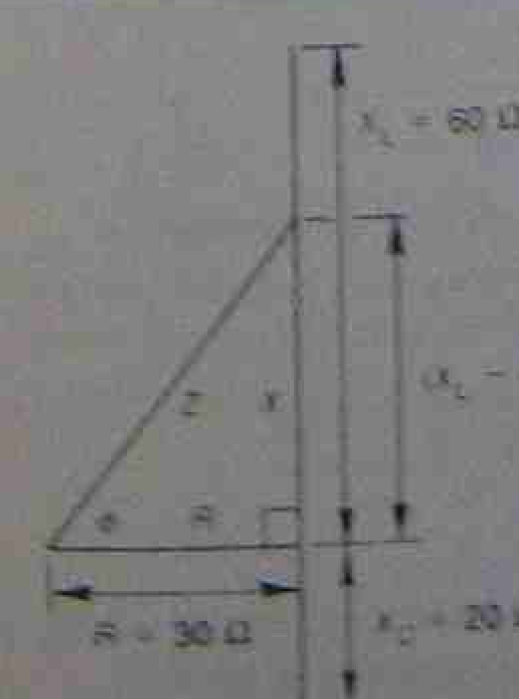


Figure 8.37 • Diagram for example 8.13

Example 8.14

The circuit in Figure 8.38 is connected to a 240 V 50 Hz supply. Determine the impedance of the circuit, the current flowing and its phase angle.

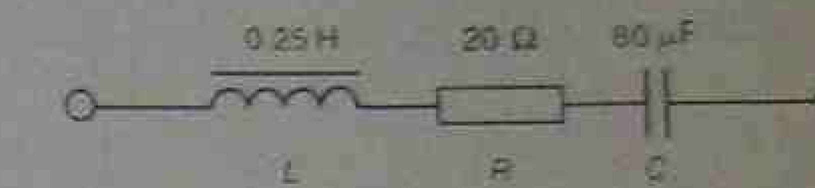


Figure 8.38 • Circuit diagram for example 8.14

$$R = 20\ \Omega$$

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.25 = 78.5\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2 \times \pi \times 50 \times 80} = 39.8\ \Omega$$

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{20^2 + (78.5 - 39.8)^2} \\ &= \sqrt{400 + 1500} \\ &= \sqrt{1900} \\ &= 43.6\ \Omega \end{aligned}$$

$$I = \frac{V}{Z} = \frac{240}{43.6} = 5.5\ \text{A}$$

$$\cos\phi = \frac{R}{Z} = \frac{20}{43.6} = 0.459$$

$$\therefore \phi = 62.7° \text{ (current lags voltage by } 62.7°)$$

8.15 RESISTANCE AND REACTANCE IN PARALLEL ON ALTERNATING CURRENT

8.15.1 Parallel R-L circuits

In a parallel circuit the voltage is common to all the components and the total current comprises the individual components, as shown in Figure 8.39(a).

In drawing the phasor diagram the voltage V is used as the reference phasor. Assuming R and L are both pure values, then: I_R is in phase with V; I_L lags V by 90°; I_{total} is equal to the phasor sum of I_R and I_L (as shown in Fig. 8.39(b), where I_{total} lags V by ϕ).

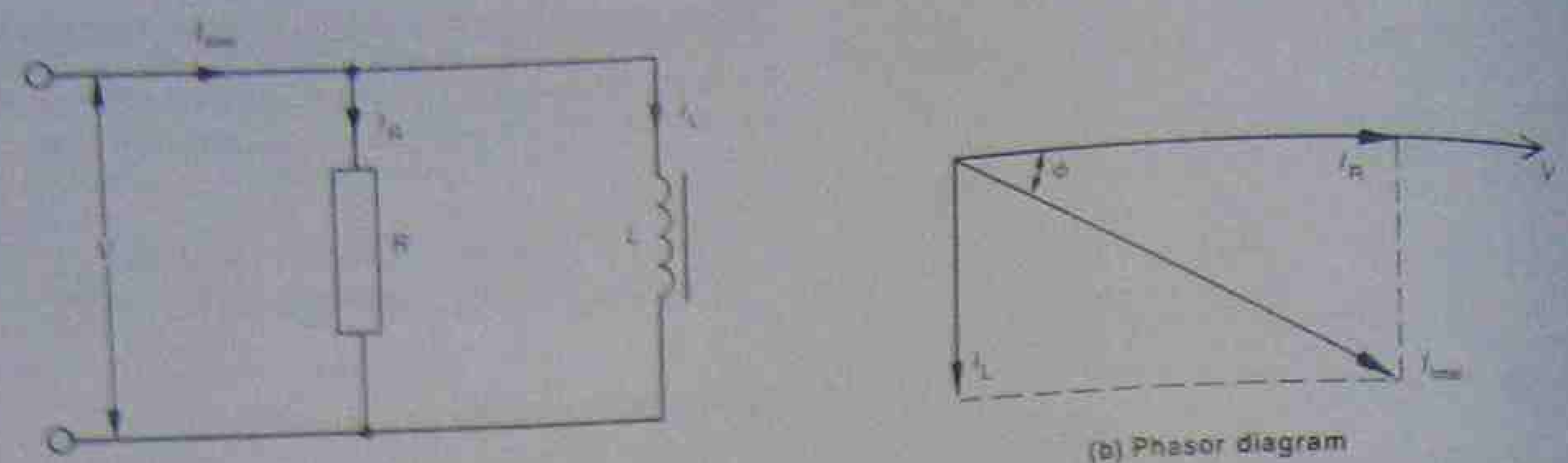
In practice, inductors have resistance associated with them; therefore a more detailed circuit would be as shown in Figure 8.40(a) and the phasor representation would be as in Figure 8.40(b).

The current I_R is in phase with V but the current I_L through the inductor lags the voltage by ϕ_1. Because the inductor is basically a series R-L circuit, the angle $\phi_1 = R_L/Z_L$.

The phasor addition of I_R and I_L represents the total current I_{total} which lags the voltage V by ϕ_2.

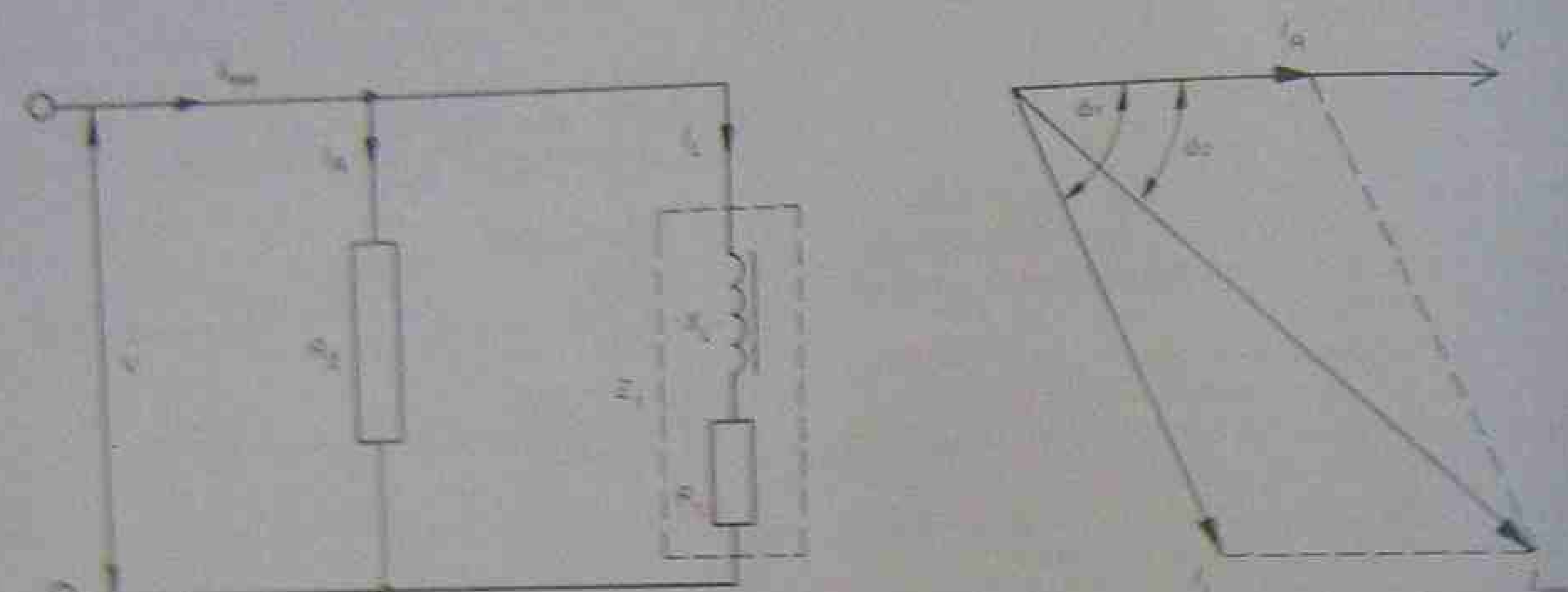
8.15.2 Parallel R-C circuits

An R-C parallel circuit is shown in Figure 8.42. For all practical purposes the resistor current I_R is in phase with



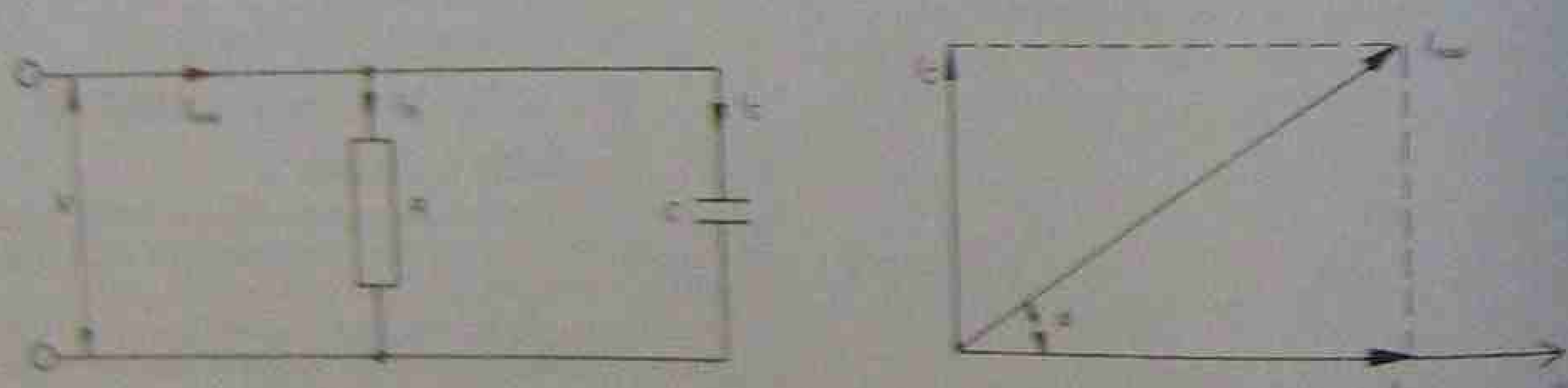
(a) Circuit diagram (b) Phasor diagram

Figure 8.39 • R-L parallel circuit



(a) Circuit diagram (b) Phasor diagram

Figure 8.40 • R-L circuit in parallel with R



(a) Circuit diagram (b) Phasor diagram

Figure 8.41 • R-C parallel circuit

the applied voltage V and the capacitor current leads both by 90°. The total current is the phasor sum of the two individual currents and leads V by φ.

8.15.3 Parallel R-L-C circuits

The total current in this type of circuit is equal to the phasor addition of I_R, I_L and I_C (see Fig. 8.42). When L and C are pure quantities, the two currents I_L and I_C are 180° out of phase and can therefore be subtracted (i.e. $I_L - I_C$). In this case I_{total} lags the voltage by φ. A greater value of I_C would cause the total current to lead the applied voltage.

8.15.4 Parallel circuit impedance

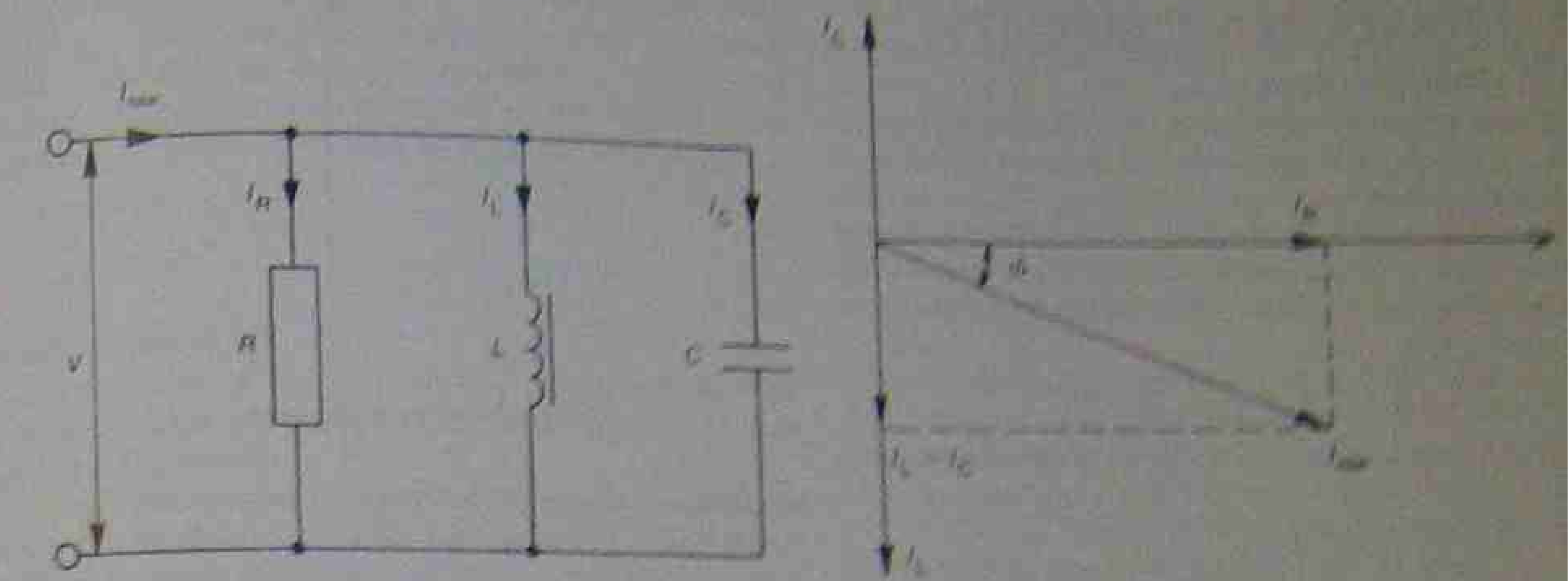
In determining impedances in parallel circuits, first find the value of total current and then apply Ohm's law:

$$\text{impedance} = \frac{\text{applied voltage}}{\text{total current}}$$

that is, $Z = V/I$

The impedance triangle method for series circuits is meaningless here and must not be used.

The method for finding the impedance of a parallel circuit is illustrated in example 8.15.



(a) Circuit diagram (b) Phasor diagram

Figure 8.42 • R-L-C parallel circuit

Example 8.15

An inductor (with an inductance of 0.12 H and resistance of 10 Ω), a resistor (of 60 Ω) and a capacitor (of 25 μF) are connected in parallel across a 240 V 50 Hz supply. Calculate:

(a) the total current
(b) the total impedance
(c) the phase angle between the total current flowing and the applied voltage

The total current I_{total} is the phasor sum of I_1, I_2 and I_3 (see Fig. 8.43):

$$I_1 = \frac{V}{R} = \frac{240}{60} = 4\text{ A}$$

I_1 and V are in phase, $\therefore \phi_1 = 0°$ (I_1 and V are in phase with each other)

Using the formulae $I_2 = \frac{V}{Z}$ and $Z = \sqrt{R^2 + X_L^2}$, find I_2:

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.12$$
$$= 37.7\ \Omega$$
$$Z = \sqrt{10^2 + 37.7^2}$$
$$= \sqrt{100 + 1420}$$
$$= 39\ \Omega$$
$$\therefore I_2 = \frac{240}{39} = 6.15\text{ A}$$

$$\cos \phi_2 = \frac{R}{Z} = \frac{10}{39} = 0.256$$

$\phi_2 = 75°$ lagging (that is, I_2 lags V by 75°)

$$I_3 = \frac{V}{X_C}$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2 \times \pi \times 50 \times 25}$$
$$= 127\ \Omega$$
$$\therefore I_3 = \frac{240}{127} = 1.88\text{ A}$$

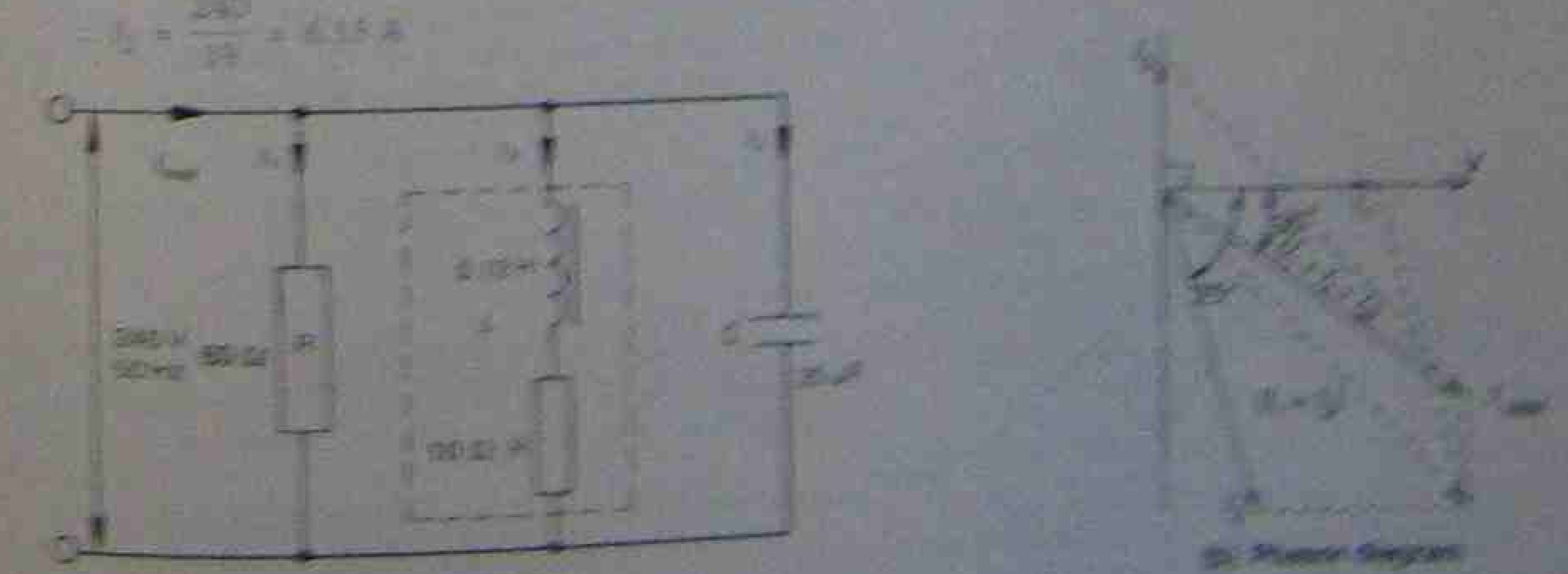
I_3 leads V by 90°, $\therefore \phi_3 = 90°$ leading

These values of current must now be added by means of phasor representation to obtain the total current flowing. First, I_1 and I_2 are added to give the [illegible] of $(\bar{I}_1 + \bar{I}_2)$ (the bar above the symbols [illegible] each is a phasor sum and not a simple arithmetic sum). To this is now added I_3 and therefore

(a) From the phasor diagram drawn to scale I_{total} [illegible]

(b) $Z = \frac{V}{I_{total}} = \frac{240}{[illegible]} = 54.8\ \Omega$

(c) From the phasor diagram in Figure 8.43(b), [illegible] lagging (i.e. I lags V by 40°).



(a) Circuit diagram (b) Phasor diagram

Figure 8.43 • Diagrams for example 8.15

POWER IN A.C. CIRCUITS

Effects of low power factor

Example

Figure 8.47 • Phasor diagram for example 8.16

determines the amount and type of insulation required and the current determines the size of the conductors.

For example, a transformer is designed for 250 V and 40 A. The rating of this transformer is 250 × 40 = 10 000 VA = 10 kVA.

If it were used to supply an electric furnace (unity power factor), it could supply a maximum of 10 × 1 = 10 kW of power. If the same transformer were used to supply a load at 0.8 power factor, the maximum power it could supply would be 10 × 0.8 = 8 kW. By rating the transformer or any electrical equipment in VA, a value is given that is independent of the power factor of the load.

8.16.5 Causes of low power factor

In practice most circuits are inductive and as a result cause currents to lag. A circuit with a leading current (i.e. a capacitive circuit) is seldom found, although the effect tends to occur in long-distance transmission lines.

The major causes of low power factor are lightly loaded electric motors and transformers, and fluorescent lighting circuits.

Motors and transformers should be designed to run at or near full load.

For fluorescent lighting circuits, as well as other special cases, steps have to be taken to improve the power factor situation.

Electricity generating authorities, under their conditions of supply, usually stipulate conditions for the use of equipment at poor power factor values.

8.16.6 Determining power factor

Measurement

The power factor of a circuit can be obtained by using a voltmeter, ammeter and wattmeter. Figure 8.48 shows the circuit for finding the power factor of a single-phase a.c. motor.

The power factor can then be found from:

$$\text{power factor } (\lambda) = \frac{P}{S} = \frac{\text{watts}}{\text{volt-amperes}}$$

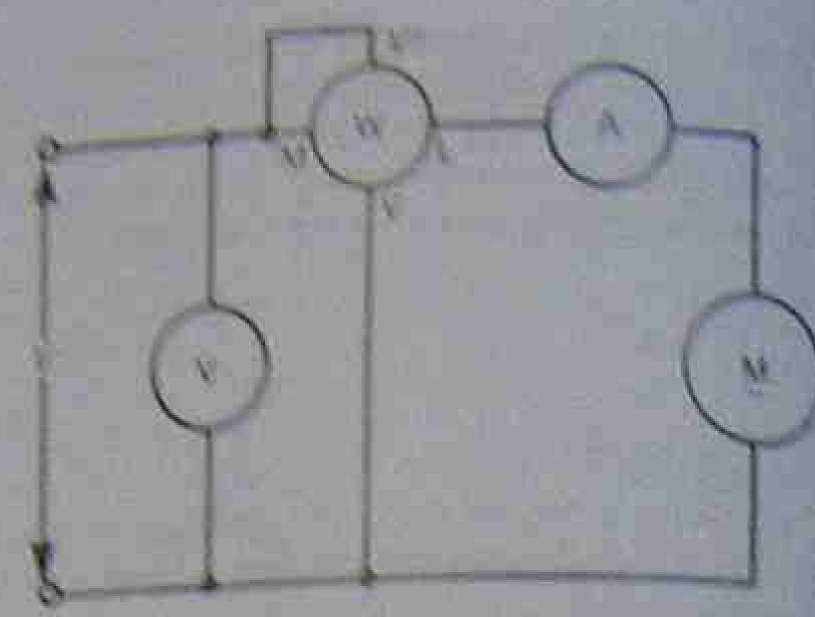


Figure 8.48 • Circuit to obtain the power factor of a single-phase motor

Example 8.17

A single-phase motor draws 2.7 A on 240 V and a wattmeter in the circuit reads 486 W. Find the power factor.

$$\text{power factor } (\lambda) = \frac{P}{S} = \frac{486}{240 \times 2.7}$$

$$= \frac{486}{648}$$

$$= 0.75$$

Power factor can also be measured by using a power factor meter (cos φ), as shown in Figure 8.49. The meter is connected in the same manner as a wattmeter, and the scale is calibrated in values of power factor.

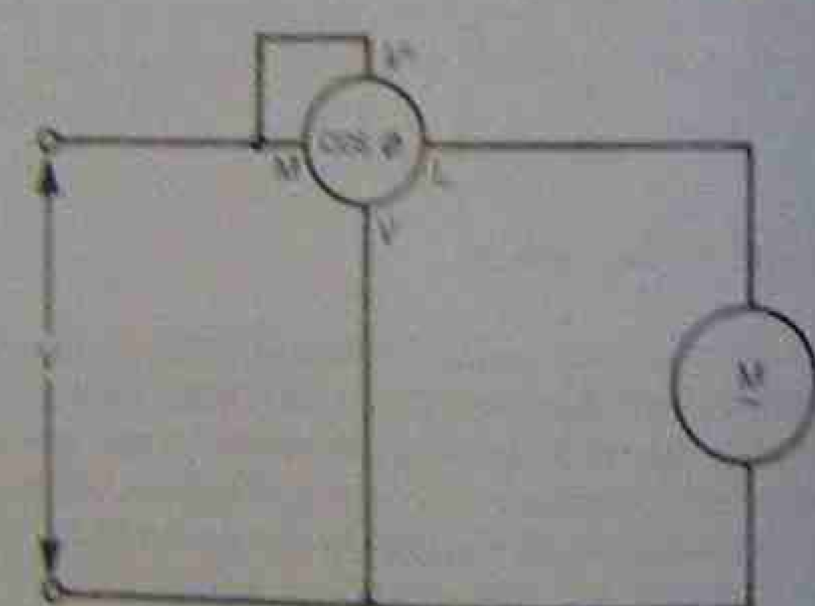


Figure 8.49 • Alternative circuit to find the power factor of a single-phase motor

Calculation

In a series circuit, the power factor can be obtained by using the values of resistance and impedance.

Example 8.18

An inductor draws 20 A on 240 V d.c. and 10 A on 240 V a.c. Calculate the angle of lag when on a.c.

In a d.c. circuit:

$$R = \frac{V}{I} = \frac{240}{20}$$

$$= 12\ \Omega$$

In an a.c. circuit:

$$Z = \frac{V}{I} = \frac{240}{10}$$

$$= 24\ \Omega$$

For a series circuit:

$$\lambda = \cos \phi$$

$$= \frac{R}{Z} = \frac{12}{24} = 0.5$$

$$\therefore \phi = 60°$$

Phasors

Another method of obtaining the power factor is by drawing a phasor diagram to scale, measuring the phase angle, and then determining the cosine of the angle.

Example 8.19

Two single-phase motors are connected in parallel across a 240 V 50 Hz supply. One motor draws 12 A at a power factor of 0.4, the other draws 16 A at a power factor of 0.8. Determine the total line current and power factor. (See Fig. 8.50.)

$$\lambda_1 = \cos \phi_1 = 0.4$$

$$\therefore \phi_1 = 66°$$

$$\lambda_2 = \cos \phi_2 = 0.8$$

$$\therefore \phi_2 = 37°$$

From the phasor diagram, I_{total} is 27 A and ϕ = 49°.

$$\lambda = \cos \phi$$

$$\therefore \text{power factor} = \cos 49° = 0.66$$

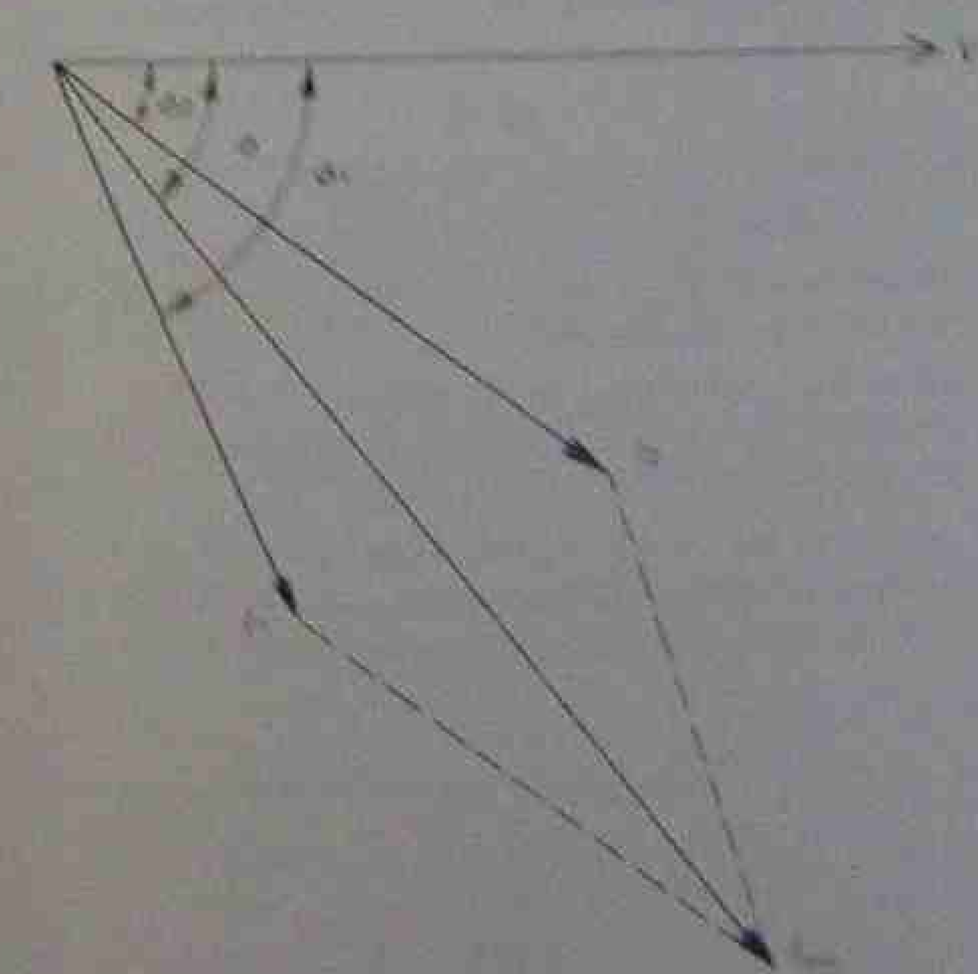


Figure 8.50 • Phasor diagram for example 8.19

8.16.7 Power factor correction

A system running at a low power factor increases the current, which in turn leads to other disadvantages. For example, a 480 V single-phase installation supplies a number of motors requiring 90 kW of power. Reduced power factor, perhaps due to oversize motors operating at less than full load, causes a sharp rise in current, as shown in Table 8.4.

Table 8.4 • A constant output load of 90 kW requires an increasing current as the power factor decreases

Power factor	1	0.9	0.8	0.7	0.6	0.5
Load (kW)	90	90	90	90	90	90
Current (A)	187	208	234	268	312	375

If the power factor of the load were increased from 0.6 to 0.8, the current would decrease from 312 A to 234 A, a decrease of 25 per cent for the same input power. This illustrates the need for improving the power factor. For economic reasons, the recommended value of power factor is 0.8. Below this, the current increases rapidly, whereas above this, the cost of the necessary equipment is too great when compared with the overall benefit gained.

To correct the power factor it is necessary to reduce the phase angle between the line current and voltage, without affecting the values of voltage or load current.

Most low-power factor problems are caused by inductive loads, such as induction motors and transformers. One method used to improve the power factor in this type of circuit is to connect a capacitor in parallel with the load, as shown in Figure 8.51.

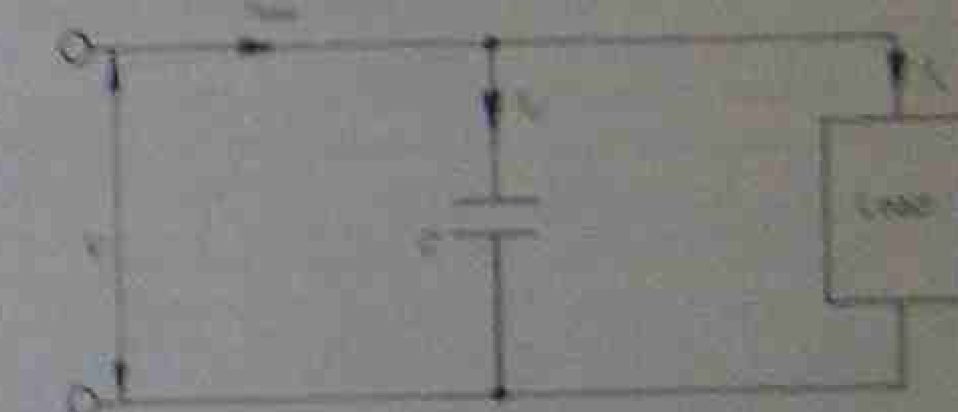


Figure 8.51 • Connecting a capacitor to improve power factor

In this way, I_L remains the same and each load operates at its own power factor, but the overall power factor of the combined circuit is improved. A pure capacitor is a load that operates at a leading zero power factor and, when connected across an inductive load, tends to oppose the lagging effect of the inductance—but without consuming any power.

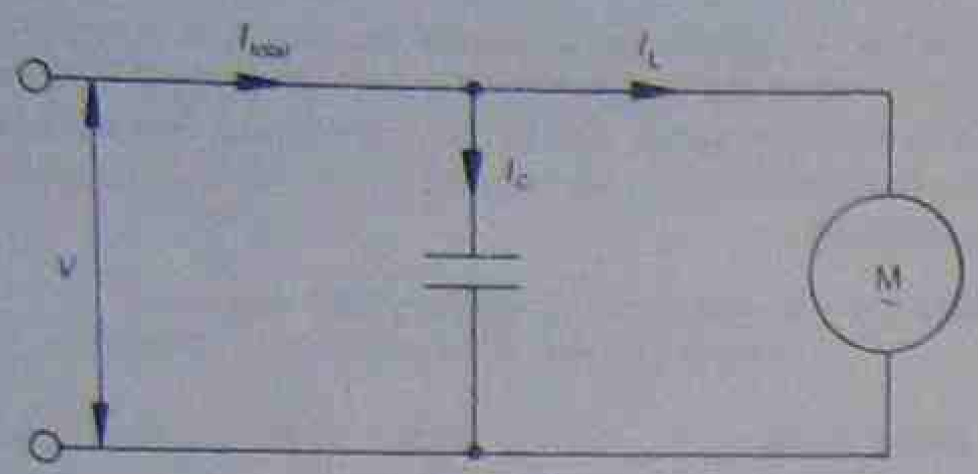
Example 8.20

A single-phase induction motor is connected across a 250 V 50 Hz supply and draws 15 A at 0.6 power factor. Determine the line current and power factor when an 80 μF capacitor is connected across the line. (See Fig. 8.52.)

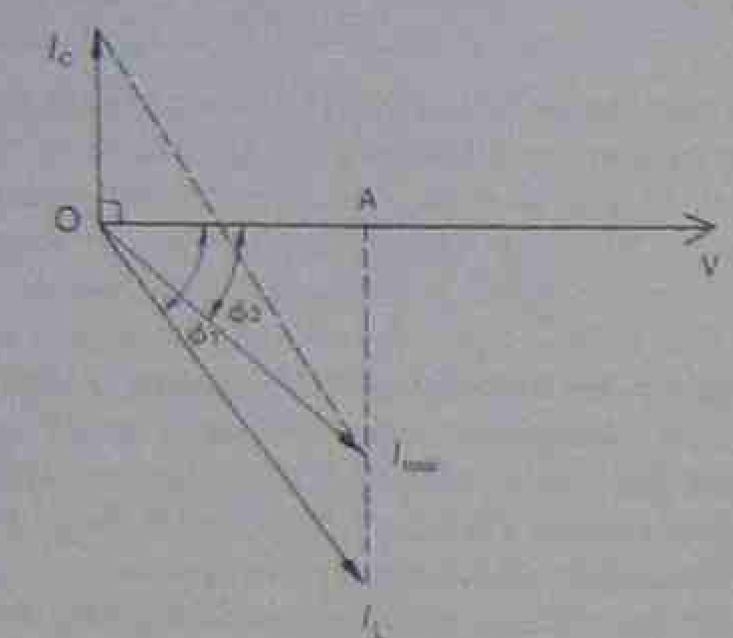
power factor (λ) = 0.6, therefore ϕ_1 = 53°

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 80} = 39.8\ \Omega$$

$$I_C = \frac{V}{X_C} = \frac{250}{39.8} = 6.28\ \text{A}$$



(a) Circuit diagram



(b) Phasor diagram

Figure 8.52 • Diagrams for example 8.20

From the phasor diagram (Fig. 8.52(b)):

$$I_{total} = 10.6\text{ A}$$
$$\phi_2 = 32°$$
$$\therefore \text{power factor } (\lambda) = \cos\phi_2 = 0.85$$

Points to note in this example are:

1. Although an extra load (the capacitor) has been added, the line current has been reduced, while the in-phase or power component of current (OA) has retained its original value.
2. The motor current and power factor remain unaltered.
3. The power factor of the resultant combined load has been improved.
4. The power consumption remains at its original value.
5. The volt-ampere rating of the combined circuit has been reduced.
6. The reactive power rating of the combined circuit has been reduced.

The purpose of using capacitors to improve power factor is to provide a leading current to counteract the lagging current drawn by the load, and at the same time not increase the value of power consumed. Supply authorities try to keep as high a value of power factor as is economically possible in their supply systems. They do this by regulating the minimum allowable value of power factor for any load connected to the supply, and accordingly there is regular demand for power factor correction capacitors.

Power factor correction problems can be handled as shown in example 8.20 by obtaining the value of capacity required to improve the overall power factor. The problem is sometimes dealt with by using the reactive power approach. The value of current flowing in the capacitor, multiplied by the voltage across it, gives the reactive power value.

For example, in example 8.20 the capacitor was rated at 250 V 80 μF. On 250 V, the current drawn is 6.28 A:

$$250 \times 6.28 = 1500 \text{ volt-amperes reactive}$$
$$= 1.5 \text{ kvar}$$

(*Note:* 'Kilo-volt-amperes reactive' is usually abbreviated to 'kvar'.)

In the following example on power factor correction, the first method has been used and the kvar rating of the capacitor noted. The reactive power method is more applicable to another means of power factor correction where the values of reactive power required are economically beyond the range of capacitors.

A type of motor called a synchronous motor has the characteristic, under certain conditions, of drawing a leading current. Synchronous motors are usually installed in large industrial premises and used to drive an air compressor or other similar machines where a constant service is required throughout the plant. While providing the service, the motor is also correcting the power factor of the complete factory.

For installations requiring 20 Mvar (20 million var) or more of power factor correction, the synchronous motor is installed without a load connected to it and it is then usually called a synchronous capacitor or condenser.

Example 8.21

A motor takes a current of 10 A at 0.65 power factor, lagging, from a 240 V 50 Hz supply. What size of capacitor is required to improve the power factor to 0.9 lagging?

$$\cos\phi_1 = 0.65, \quad \therefore \phi_1 = 49°$$
$$\cos\phi_2 = 0.9, \quad \therefore \phi_2 = 26°$$

The following steps produce Figure 8.53:

1. Draw the phasor $I_L = 10$ A at 49° lagging V (OB).
2. From the end of I_L construct a line perpendicular to the reference phasor AB.
3. Draw the phasor I_{total} at 26° lagging V until it intersects the perpendicular. By measurement, then $I_{total} = 7.22$ A.
4. Construct the parallelogram as shown and the vertical phasor I_C represents the value of current flowing through the capacitor ($I_C = 4.45$ A).
5. $X_C = \dfrac{V}{I_C} = \dfrac{240}{4.45} = 53.9\ \Omega$
6. $X_C = \dfrac{1}{2\pi fC}$

$$\therefore C = \frac{1}{2\pi fX_C}$$
$$= \frac{10^6}{2 \times \pi \times 50 \times 53.9}\ \mu\text{F}$$
$$= 59\ \mu\text{F}$$

The kvar rating of the capacitor is $240 \times 4.45 = 1.07$ kvar.

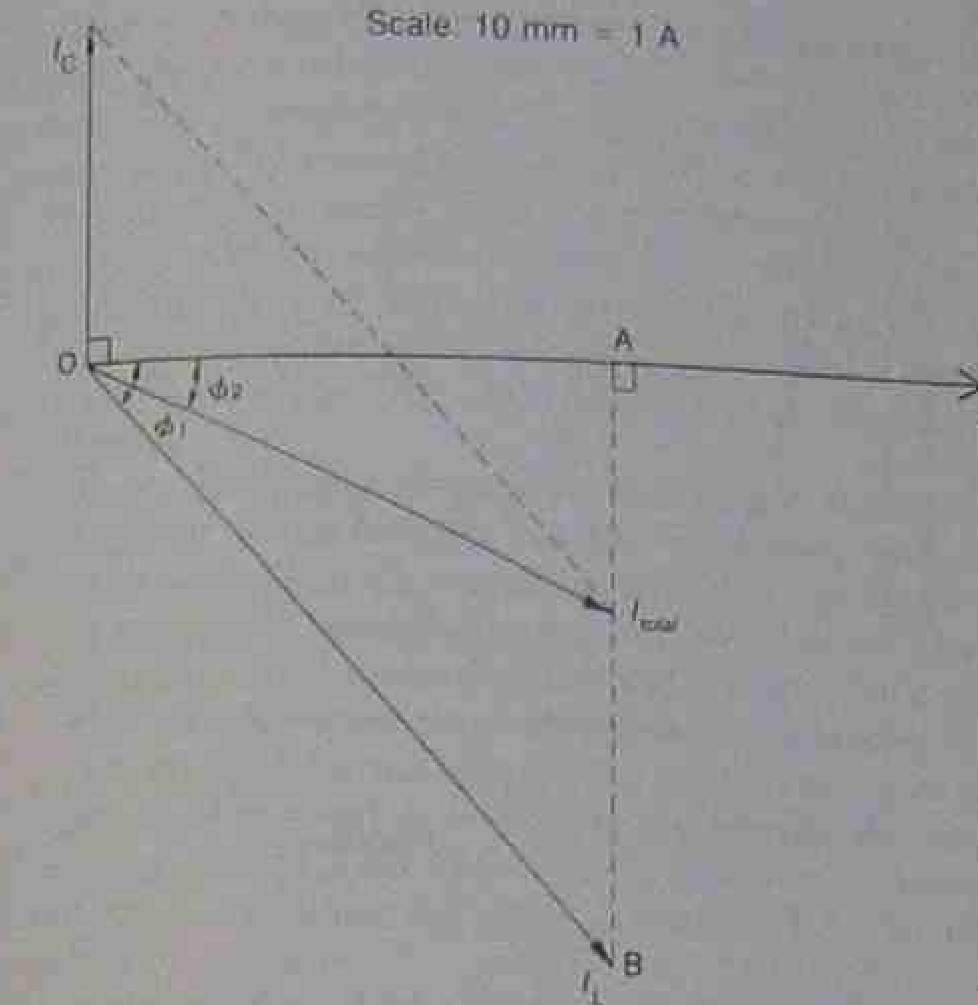


Figure 8.53 • Phasor diagram for example 8.21

Example 8.22

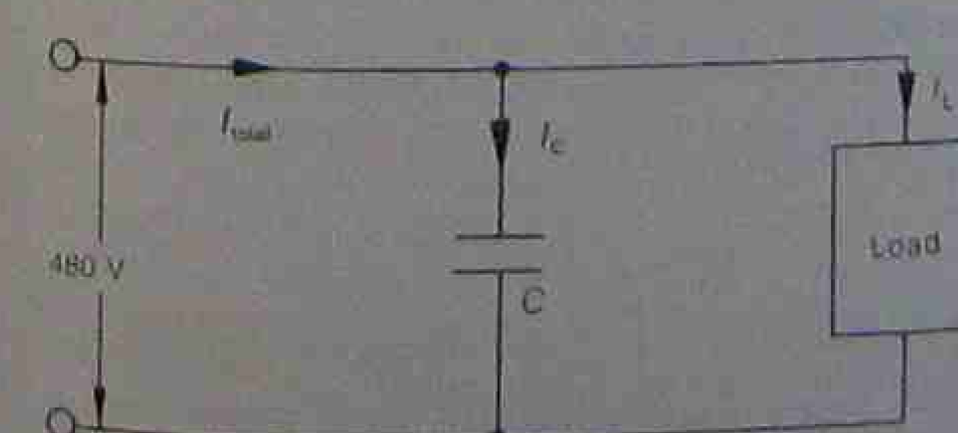
A 480 V single-phase supply feeds an installation that draws 54 A at a power factor of 0.55. Determine the kvar rating of the capacitor required to improve the power factor to 0.8. (See Fig. 8.54.)

1. Determine the phase angles of the load before and after correction:

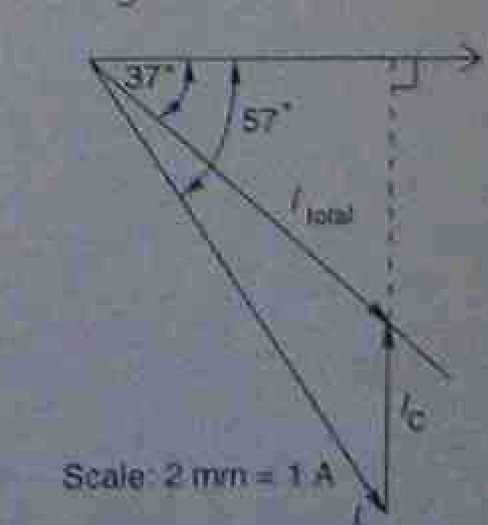
   $$\cos\phi_1 = 0.55, \quad \therefore \phi_1 = 57°$$
   $$\cos\phi_2 = 0.8, \quad \therefore \phi_2 = 27°$$

2. Draw the phasor diagram with V as the reference phasor, scaling I_L to 54 A lagging V by 57°, and I_{total} (value unknown) lagging V by 37°.
3. From the end of I_L draw a line perpendicular to the reference phasor, and where this line intersects the I_{total} phasor, it represents the value of I_C.
4. From the phasor diagram $I_C = 23$ A:

   $$\therefore \text{kvar} = V_C \times I_C$$
   $$= \frac{480 \times 23}{1000}$$
   $$= 11 \text{ kvar}$$



(a) Circuit diagram



(b) Phasor diagram

Figure 8.54 • Diagrams for example 8.22

8.17 RESONANCE

When an electrical circuit has its power factor corrected to unity, the current is brought into phase with the voltage and, although the circuit could still contain both capacitive and inductive reactances, the circuit behaves as a purely resistive circuit.

In both series and parallel resonant circuits, capacitive reactance is equal to inductive reactance. The energy stored in the magnetic field of the inductor for one part of a cycle appears in the electrostatic field of the capacitor in the next part of the cycle. This energy transfer from one part of the circuit to another produces different effects, depending on whether it is a series or parallel circuit.

8.17.1 Series-circuit resonance

The major characteristics of the series resonant circuit are a power factor of unity and a minimum impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Since at resonance $X_L = X_C$, then $X_L - X_C = 0$:

that is, $Z = \sqrt{R^2}$ or $Z = R$

It is important to note that because impedance is a minimum, the current flow will be a maximum. In many alternating current circuits the actual resistance may be quite low. Consequently the currents encountered may be quite high. Due care must be taken with this type of circuit and the current flow limited to safe values.

The majority of a.c. circuits are connected to the power supply mains with their massive power backup capability. The low resistance of the series circuit means that the current flow will be correspondingly high. These current values are capable of creating a large amount of damage.

For general electrical work the series resonant circuit is one that should be avoided unless special precautions are taken. One way is to ensure that adequate series resistance is in the circuit, so ensuring that the current is limited. Where circuits are operated such that voltages higher than the supply voltage are generated, the circuit must be insulated for the higher voltages.

These voltage values can be many times the applied voltage and in some cases create a hazardous situation. Some specialised equipment is manufactured to make use of these high voltages.

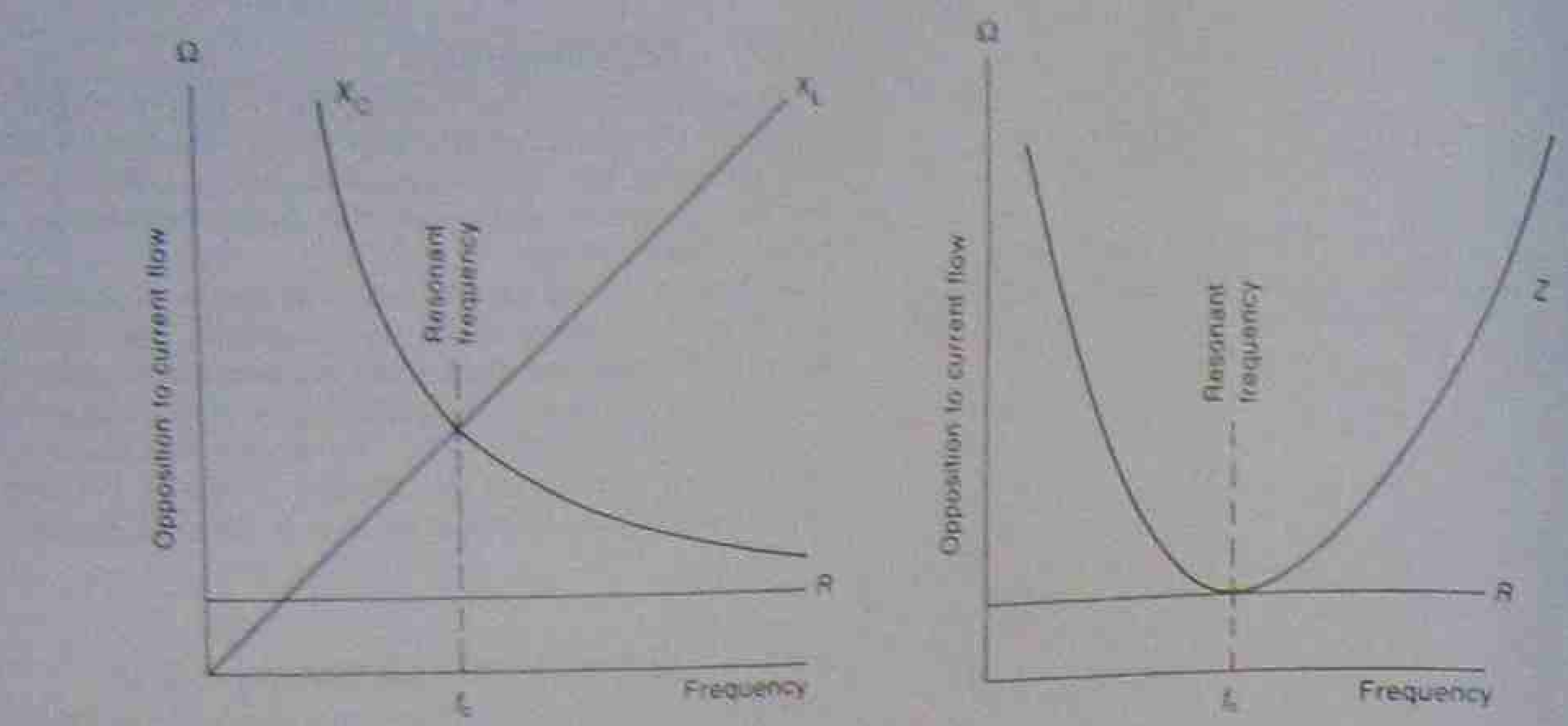


Figure 8.55 • Series resonant circuit characteristics

Example 8.23

A 10 Ω resistor, 0.25 H inductor and a 40.52 μF capacitor are connected in series across a 240 V 50 Hz supply. Calculate the current flowing and voltage drop across each component. (See Fig. 8.56.)

Figure 8.56 • Circuit diagram for example 8.23

$$R = 10\ \Omega$$
$$X_L = 2\pi fL = 2\pi \times 50 \times 0.25 = 78.54\ \Omega$$
$$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50 \times 40.52} = 78.54\ \Omega$$
$$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{10^2 + (78.54 - 78.54)^2}$$
$$= 10\ \Omega$$

that is, $Z = R = 10\ \Omega$

$$\therefore I = \frac{V}{Z} = \frac{240}{10} = 24\ \text{A}$$
$$V_R = IZ = 24 \times 10 = 240\ \text{V}$$
$$V_L = IX_L = 24 \times 78.54 = 1885\ \text{V}$$
$$V_C = IX_C = 24 \times 78.54 = 1885\ \text{V}$$

A useful characteristic of the series resonant circuit is its ability to be frequency sensitive. At only one frequency will the capacitive reactance be equal to the inductive reactance. Other frequencies encounter much higher impedances and become filtered out of the circuit. (See Fig. 8.55.)

When the frequency of the supply is varied, the resistance is unchanged, but both X_L and X_C will vary, as shown in Figure 8.55:

$$X_L \propto f \text{ and } X_C \propto \frac{1}{f}$$

At resonant frequency f_0, X_C and X_L are equal and the impedance Z is equal to the resistance R. The value of resonant frequency can be calculated as follows:

$$X_L = X_C$$
$$2\pi fL = \frac{1}{2\pi fC}$$
$$\therefore (2\pi f)^2 = \frac{1}{LC}$$
$$2\pi f = \frac{1}{\sqrt{LC}}$$
$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

8.17.2 Parallel-circuit resonance

When an inductor and capacitor are connected in parallel and their respective reactances are equal, the reactive currents are equal but 180° out of phase with each other. Theoretically, then, the current drawn from the supply would be zero, while a comparatively high circulating current would oscillate backwards and forwards between the inductor and capacitor at the supply frequency.

However, in practice the inductor has some finite value of resistance associated with it. The parallel resonant circuit would then appear as a purely resistive one, and some current in phase with the voltage would flow from the supply.

The resonant frequency in a parallel circuit with zero resistance is found by using the same formula as for series resonant frequency:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

For a practical circuit, the resonant frequency of a parallel resonant circuit is more exactly:

$$\boxed{f_0 = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}}$$

This is because the conductors of any inductance operating on power line frequencies have an inherent resistance. The conductors making up that inductance are in series with the inductive reactance while the capacitor is connected in parallel with the combined effect of the inductor and its resistance. For parallel resonant circuits operating on very high frequencies the inductive reactance can be achieved with very short lengths of conductors. The resistance can then be ignored for all practical purposes.

When the supply frequency is varied, the resistance in the circuit is unchanged but the impedance will be a maximum only at the resonant frequency (see Fig. 8.57).

At resonance, energy is being transferred from the electromagnetic field to the electrostatic field and back again via the circulating current, which cannot be related to the input current from the supply. The supply current at resonance is at a minimum because the impedance is at its maximum, and is sufficient only to make up the losses in the circuit. The lower the losses, the lower will be the value of input current.

In electronic circuits, a capacitance in parallel with an inductance can be used to obtain the highest possible impedance for a particular frequency across that parallel section. Making either the capacitor or the inductor adjustable enables selection of a particular resonant frequency.

With a variety of frequencies presented to a parallel circuit, only the resonant frequency is passed to the next part of the circuit. It is the method used in tuning a radio to a set frequency. In other words, the non-resonant frequencies are presented with a low impedance path to earth and so removed from the circuit.

Test equipment sometimes uses the principle of parallel resonance for measuring capacitance. Transducers known to be frequency dependent also use the phenomenon as a means of monitoring and regulating frequency.

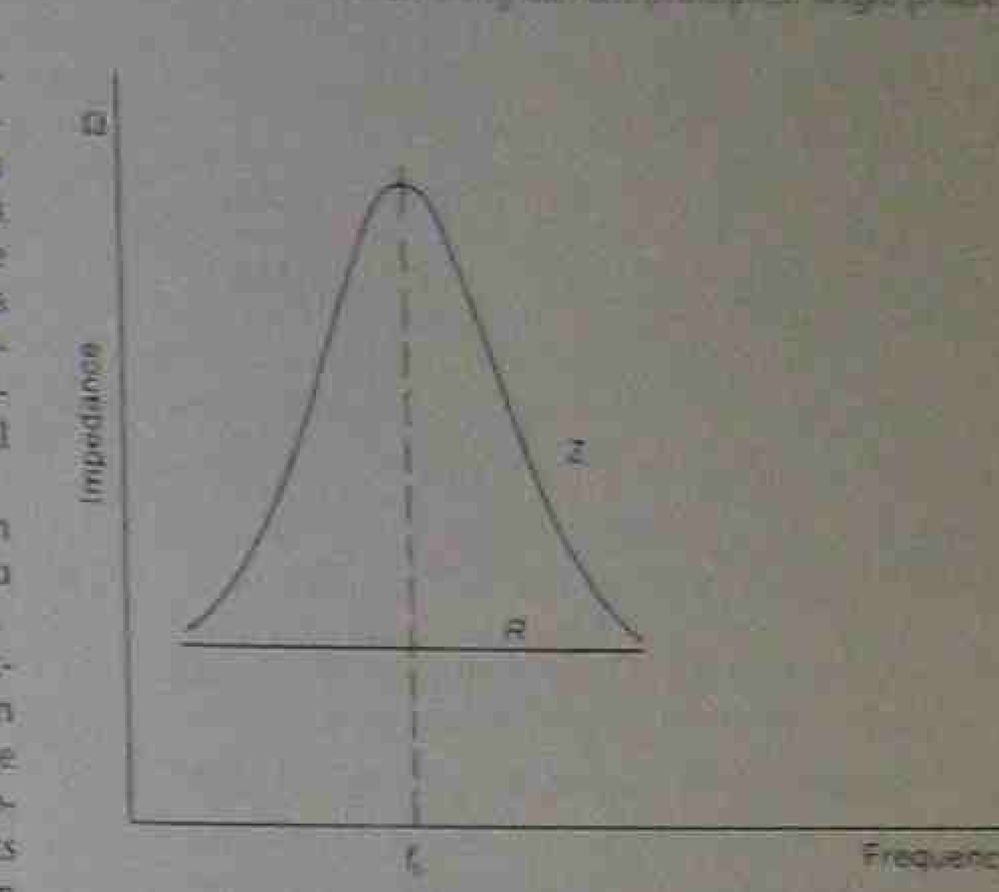


Figure 8.57 • Parallel resonant circuit impedance characteristic

SUMMARY

- A loop rotating in a magnetic field produces an alternating voltage waveform. The shape used depends on the use to which it is put.
- The most common waveform is sinusoidal. It is the only one used in a.c. power work because the current that flows has the same waveform as the voltage.
- An alternator produces an alternating voltage.
- There are two types of alternators—conductors rotated inside a stationary magnetic field and a magnetic field rotated inside stationary conductors.
- The direction of the induced voltage can be determined from Fleming's right-hand rule. The magnitude of the voltage can be found from $V = Blv \sin\theta$.
- The output voltage of an alternator can be increased by either increasing the number of conductors or increasing the magnetic flux. Adding an iron core to an alternator is the easiest way to increase the magnetic flux, but it gives rise to iron losses—eddy currents and hysteresis. Because frequency is usually fixed, the speed of the machine is also fixed ($f = np/120$).
- One cycle of alternating current occupies 360 electrical degrees. It is not always equal to mechanical degrees. Terms used are:
  —*Periodic function*—any wave that repeats itself.
  —*Periodic time*—the time taken in seconds to complete one cycle ($t = 1/f$) seconds.
  —*Frequency*—the number of times a wave repeats itself in one second (unit = hertz).
  —*Crest factor*—a ratio ($V_{max}/V_{r.m.s.}$ = 1.414 for a sine wave).
  —*Form factor*—a ratio (V_{max}/V_{av} = 1.11 for a sine wave).
  —V_{p-p} and I_{p-p}—twice the maximum value.
  —*Average value*—0.637 of maximum for a sine wave.
  —*r.m.s. value*—0.707 of maximum for a sine wave.
  —*Instantaneous values*: $v = V_{max} \sin\theta$ for voltage, and $i = I_{max} \sin\theta$ for current.
- With alternating current, the current might not be in step with the voltage. For resistive circuits, both are in phase. For purely inductive circuits the current lags the voltage by 90°. For purely capacitive circuits the current leads the voltage by 90°.
- For circuits containing more than one of these components, the phase displacement of the current depends on the relative proportions of each component (ohms) in the circuit.
- If inductive reactance is equal to capacitive reactance, the circuit behaves as purely resistive and is said to be resonant at that frequency.
- Resonance can be dangerous in series circuits.
- The total opposition of a circuit to a current flow is called *impedance* and may comprise resistance, capacitive reactance and inductive reactance.
- An impedance triangle can be used with series circuits but not with parallel circuits.
- The cosine of the angle between the voltage and current is called the power factor of a circuit.
- For single phase the power consumed in a circuit is dependent on the power factor: $P = VI \cos\phi$.
- When V and I are in phase, $\phi = 0°$ and $\cos\phi = 1$ and $P = VI$.

- When V and I are 90° out of phase, $\cos\phi = 0$ and $P = 0$.
- A power triangle can be used for an a.c. circuit by the components *true power (P)*, *apparent power (S)* and *reactive power (Q)*. True power divided by apparent power gives the power factor.
- Poor power factor leads to 'wattless' components of current flowing in distribution lines—it causes poor voltage regulation, excess fuel consumption and useless loading of alternators.
- For individual loads, capacitors are connected in parallel to improve the power factor.
- The power factor can be obtained by measuring with a voltmeter, ammeter, and wattmeter and then calculating the value. It can be measured directly with a power factor meter.

EXERCISES

8.1 Briefly outline the generation of a voltage by a rotating loop in a fixed field.

8.2 Explain why field poles are specially shaped.

8.3 The rotating-magnet system is much less complicated in construction than the rotating-coil system. Explain why.

8.4 The use of an iron core in a rotating-coil generator improves the output, but at the same time problems are introduced.
(a) Show how output is improved.
(b) Briefly describe the problems.
(c) Describe how these problems can be overcome.

8.5 Briefly compare rotating-field and rotating-armature alternators. Point out any advantage that one type might have in comparison with the other. Indicate an application of each type.

8.6 With reference to a sine wave, what is meant by the terms *cycle, frequency, mechanical degrees, electrical degrees* and *periodic time*?

8.7 List the characteristics of a voltage sine wave. Refer to r.m.s. and average values in your answer.

8.8 What is meant by a periodic function?

8.9 Compare the relationships between voltage and current if a.c. is applied to pure resistance, inductance and capacitance.

8.10 What is a non-inductive resistor? Explain how this is achieved.

8.11 Explain why a pure inductor does not consume power.

8.12 With the aid of sketches, describe the construction of an inductor and a capacitor.

8.13 What is meant by the time constant of an R-C circuit?

8.14 If the voltage to an R-C circuit is doubled, how will it affect the time constant of the circuit? Explain your answer.

8.15 List the factors affecting the capacity of a capacitor and explain how each factor affects the capacity.

8.16 Which value of an a.c. sinusoidal waveform should be used when selecting insulation for an electrical appliance? Explain your answer.

8.17 An iron-cored inductor and a capacitor connected in series are supplied from a variable-frequency power source. Draw a characteristic curve of current flow and explain the reason for its shape.

8.18 Explain why an R-L series circuit will always have a lagging current flow.

8.19 Why does an inductor never cause the current to lag by 90°?

8.20 Discuss how an inductive circuit on a.c. always consumes some power.

8.21 Explain the differences between true power, apparent power and reactive power.

8.22 What is power factor? Give its maximum and minimum values.

8.23 What are the effects of poor power factor on a distribution system?

8.24 Explain why a capacitor can correct poor power factor.

8.25 List the advantages of a high power factor.

8.26 Describe the dangers of a series resonant circuit when it is connected to a 240 V 50 Hz supply.

8.27 Why does a parallel resonant circuit draw minimum current from the supply? What is its power factor?

SELF-TESTING PROBLEMS

8.28 The output characteristics of alternators depend on the number of poles. Fill in all missing details of Table 8.5.

Table 8.5 • The relationship between the number of poles and cycles per revolution in an alternator

Number of poles	Mechanical degrees between poles	Electrical degrees in 1 revolution	Cycles in 1 revolution
2			
		720°E	
			4
	30°		

8.29 Calculate the output frequency of a twenty-pole alternator turning at 300 r/min.

8.30 The maximum voltage generated in an alternator coil is 400 V. Calculate the instantaneous voltage at the following angles of rotation:
(a) $\theta = 45°$
(b) $\theta = 130°$
(c) $\theta = 210°$
(d) $\theta = 250°$
(e) $\theta = 300°$

8.31 The value of V_{max} for a given four-pole alternator is 600 V. Prepare a table showing values of instantaneous voltage against suitable values of the angle of rotation θ (θ being in electrical degrees) during one complete revolution of the alternator. Use these values to plot the output waveform of the alternator during one revolution. Assume that the waveform is sinusoidal.
From the graph, determine:
(a) the values of θ for which v has greatest positive value
(b) the values of θ for which v is zero
(c) the value of v when $\theta = 600°E$
(d) the values of θ for which $v = 450$ V.

8.32 If the alternator of problem 8.31 is connected to a resistive load of 30 Ω, calculate:
(a) the instantaneous current when $\theta = 120°$
(b) the instantaneous value of current when $\theta = 30°E$.

8.33 Prepare a table showing the speeds at which alternators of 2, 4, 6, 8, 10, 12, 16 and 20 poles must be driven to generate an output frequency of 60 Hz.

8.34 In industry it is common practice to use motor-alternator sets to generate a local supply at 180 Hz for use with portable drills and grinders.
Calculate the number of poles in such an alternator if the driving motor rotates at 2700 r/min.

8.35 A sinusoidal alternating current with a peak value of 25 A increases from zero to 15 A in 2.5 ms. Find:
(a) the angle through which the wave has rotated
(b) the periodic time of the wave
(c) the frequency
(d) the instantaneous value of the current after a further 5 ms.

8.36 An item of electrical equipment is given an insulation rating value of 1000 V d.c. Find the maximum r.m.s. value of a sinusoidal waveform to which the equipment can be safely subjected.

8.37 The average value of an alternating current waveform is 9.4 A. The maximum value is 14 A while its r.m.s. value is 11 A. Find the form and crest factors.

8.38 (a) The average value of an alternating current is 16.6 A. Its maximum value is 25 A and its r.m.s. value is 19 A. Determine the form factor and the crest factor of the wave.
(b) Is this alternating current sinusoidal? Give a reason for your answer.

8.39 In a system of distribution used in country areas, a voltage of 19 kV r.m.s. is used in the transmission line. What is the least value of voltage for which the associated installation must be insulated?

8.40 The maximum value of an a.c. voltage is 1700 V. Assume that the waveform is sinusoidal and determine the r.m.s. and average values of this voltage.

8.41 If an a.c. supply with a peak voltage of 340 V is applied to resistors of 20, 25 and 35 Ω connected in series, calculate:
(a) the r.m.s. value of the current
(b) the maximum value of the current
(c) the voltage drop across each resistor.

8.42 Resistors of 12, 15 and 20 Ω are connected in parallel. If a sinusoidal voltage of 170 V_{max} is connected to them, calculate:
(a) the total current flow
(b) the current in each resistor.

8.43 A resistive circuit is connected to an alternating supply. If the resistance is 6.8 Ω and the voltage 240 V, calculate the current flow and the power absorbed.

8.44 An electric heater, with a resistance of 35 Ω, is supplied from 230 V 50 Hz alternating current mains. Assuming the mains voltage to be sinusoidal, determine:
(a) the average power
(b) the maximum instantaneous power.

8.45 What would be the inductance of a choke coil that would take the same current from a 25 Hz supply as a non-inductive resistance of 20 Ω?

8.46 It is necessary to change the power supply of a purely inductive coil from 50 Hz a.c. to d.c. and maintain the coil current at the same value. If the coil has an inductance of 0.2 H and the a.c. supply voltage is 240 V, calculate the amount of resistance to be used in series with the coil when energised by a 200 V d.c. supply.

8.47 Capacitors rated at 3, 5 and 8 μF are connected in:

(a) series

(b) parallel.

Find the combined capacity in each case.

8.48 What current will be drawn by a 3.5 μF capacitor when connected to a 240 V 50 Hz supply?

8.49 A capacitor has a reactance of 3.183 MΩ at 50 Hz. Find its capacity in:

(a) microfarads

(b) picofarads.

8.50 Two capacitors are rated at 16 μF and 9 μF. They are connected in series to a 240 V 50 Hz supply. Find:

(a) the effective capacity

(b) the current drawn in mA.

8.51 Capacitive reactances of 2000 Ω and 500 Ω are connected in:

(a) series

(b) parallel to a 250 V 50 Hz supply.

In each case find the total current.

8.52 A 100 μF capacitor is connected in series with a [illegible] resistor. Find the time constant of the circuit. If the combination is connected to a 100 V d.c. supply, find:

(a) the initial charging current

(b) the final charge in the capacitor

(c) the energy stored in the capacitor when it is charged to 100 V.

8.53 A 20 μF capacitor has an insulation resistance of 50 MΩ. If charged to 250 V d.c., find the minimum time required for the voltage across its terminals to fall to zero.

8.54 Calculate the inductive reactance of a 0.1 H inductor on a 50 Hz supply.

8.55 A 0.32 H inductor has a reactance of 50 Ω. Find the frequency of the supply.

8.56 Find the inductance of a coil on 50 Hz when its reactance is 314 Ω.

8.57 An inductor of 0.1 H, when connected to a 110 V supply, draws a current of [illegible] A. Find the frequency of the supply.

8.58 An 8 μF capacitor is connected to a 250 V 100 Hz supply. Find the current drawn.

8.59 What is the capacitive reactance of a 100 μF capacitor when it is connected to a 50 Hz supply?

8.60 Find the capacity of a capacitor if its reactance at 50 Hz is 314 Ω.

8.61 A series circuit has a resistance of 20 Ω and an inductive reactance of 16 Ω. Determine the impedance of the circuit and the angle of phase difference.

8.62 What resistance must be connected in series with an inductance of 0.2 H in order that the phase difference for the circuit may be 45° when the frequency is 60 Hz?

8.63 What resistance must be connected in series with a choke of 1 H inductance so that a current of 1 A may flow when connected to 480 V 50 Hz mains?

8.64 A circuit has a resistance of 8 Ω and an inductance of 0.025 H. At what frequency will a supply of 100 V send a current of 12 A through this circuit?

8.65 A choke takes a current of 3 A from a 240 V 50 Hz supply at a power factor of 0.4 lagging. Sketch the phasor diagram illustrating this. Determine the inductance of the coil in henrys. What current would flow if the coil were connected to a 240 V d.c. supply?

8.66 A coil has a resistance of 50 Ω and a reactance of 220 Ω on a 60 Hz supply. Determine the current if the coil is connected to a 240 V 50 Hz supply and also calculate the power factor of the circuit.

8.67 A single-phase motor draws 3150 W from a supply of 240 V 50 Hz. A power factor meter reading in the circuit is 0.54. Determine the current taken from the supply.

8.68 A 400 V single-phase alternator is rated at 32 kVA. What would be the maximum safe power output of the machine:

(a) at unity power factor?

(b) at a power factor of 0.8?

8.69 An iron-cored choke is connected to a 240 V 50 Hz supply. It has an impedance of 360 Ω at 0.2 power factor.

(a) Using an impedance triangle, find its resistance and reactance.

(b) What current would it draw on a 220 V 40 Hz supply?

8.70 A circuit consisting of a resistor and an inductor in parallel draws a current of 4 A at 240 V, the circuit phase angle being 60°. Find the values of resistance and reactance.

8.71 The following two circuits are connected in parallel across a 240 V supply:

Circuit A: 2.75 A at a power factor of 0.98 lagging

Circuit B: 4.30 A at a power factor of 0.68 lagging

Find the supply current and power factor.

8.72 A welder draws 40 A at a power factor of 0.22 from a 240 V supply. If it operates continuously for half an hour, how many kilowatt-hours of energy will it consume?

8.73 Two separate factories are supplied from the same a.c. mains. One factory takes a load of 120 A at a power factor of 0.79. The combined load is 200 A at a power factor of 0.62. Determine the current drawn and power factor of the other factory.

8.74 Three motors run in parallel from an a.c. supply. One motor takes 12 A at a power factor of 0.4, and another takes 16 A at a power factor of 0.8. If the total current is 40 A at a power factor of 0.6, determine the current and power factor of the third motor.

8.75 A resistor of 100 Ω is connected in parallel with a 50 μF capacitor to a 250 V 50 Hz supply. Determine the total current, impedance and power factor for the complete circuit.

8.76 What capacitance must be connected in series with a resistance of 100 Ω so that the phase difference for the circuit may be 60° when the frequency is 40 Hz?

8.77 A resistor of 240 Ω is connected in series with a 20 μF capacitor.

(a) Determine the impedance at 50 Hz.

(b) At what frequency would the impedance be 400 Ω?

8.78 A choke takes a current of 5 A at a power factor of 0.2 when connected to a 200 V 50 Hz supply. Determine the current and power factor when a 120 μF capacitor is connected in series with it to the same supply.

8.79 Find the current in each branch of and the total current for Figure 8.58.

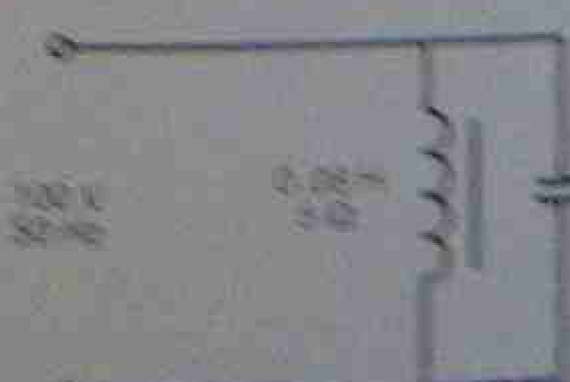


Figure 8.58 ▪ Circuit for problem 8.79

8.80 A resistor of 50 Ω, an inductor of 0.08 H and a capacitor of 40 μF are all connected in parallel to a 250 V 100 Hz supply. Determine the total current and the power factor.

8.81 At what frequency would resonance occur in a series [illegible]

8.82 A mercury-vapour lamp draws a current of 3.7 A at a power factor of 0.5 from a 240 V 50 Hz supply. What capacitor value is required to correct the power factor to 0.8?

8.83 A dairy farmer draws a current of 18 A at a power factor of 0.7 from a 480 V 50 Hz supply. What would be the kvar rating of a capacitor that would improve the power factor to 0.9?

8.84 A lighting circuit draws a current of 9.1 A at 0.5 power factor from a 240 V 50 Hz supply. It is desired to use an 8 A circuit breaker for these lamps. What is the minimum size of capacitor that will reduce the current below 8 A when connected in parallel with the load?

8.85 Three inductive coils as follows are connected in series:

Coil 1: Resistance 12 Ω, reactance 40 Ω

Coil 2: Resistance 20 Ω, reactance 25 Ω

Coil 3: Resistance 30 Ω, reactance 16 Ω

(a) Find the impedance and angle of lag for each inductor.

(b) Calculate the impedance and power factor for the total circuit.

8.86 A series R-L circuit draws 4 A on a 240 V 50 Hz supply at a power factor of 0.5. Find the values of resistance and inductance in the circuit.

8.87 An impedance connected to a 240 V 50 Hz supply consumes 1728 W and draws 12 A. Find the resistive and inductive values of the circuit.

8.88 Two impedances are connected in parallel to a 240 V 50 Hz supply. The resistive and inductive values are:

Impedance 1: 24 Ω resistance and 0.12 H inductance

Impedance 2: 60 Ω resistance and 0.08 H inductance

Find:

(a) The impedance of each unit.

(b) The current and power factor of each unit.

(c) The total current flowing in the supply lines.

(d) The power factor of the complete circuit.

8.89 A resistor of 20 Ω is connected in parallel with an inductive reactance of 12 Ω across a 50 Hz supply. Find the resistance and inductance of a single inductor which when placed across the same supply would take the same amount of current at the same power factor.

8.90 A resistor of 20 Ω is connected in parallel with an inductor of 0.3 H to a 240 V 25 Hz supply. Calculate the total current flowing and the power factor of the total circuit.

8.91 A resistor and inductor when connected in series across a 240 V supply draw a current of 2.4 A at a [illegible] power factor of 0.7. Find the total current drawn and the total circuit power factor when reconnected in parallel across the same supply.

8.92 Calculate the resonant frequency of an inductor of 0.1 H inductance connected in series with a 5 μF capacitor.

8.93 A series circuit consisting of a 1.29 H inductor and an 8 μF capacitor are connected in series across a 240 V 50 Hz supply. If the winding resistance is 3 Ω calculate:

(a) the current flowing.

(b) the power factor of the complete circuit.

8.94 A 160 Ω resistor is connected in parallel with a capacitor across a 240 V 50 Hz supply. If the current drawn by the circuit is 2 A, find the value of the capacitor and the circuit power factor.

8.95 A 0.015 H inductor with a coil resistance of 18 Ω is connected to a 240 V 50 Hz supply. Determine the value of capacitor required to bring the circuit to resonance.

Chapter 9

Alternating current principles: three phase

9.1 INTRODUCTION

The study of a.c. circuits to this point has been confined to those containing a single a.c. voltage source. Such circuits are called single-phase a.c. circuits. Circuits containing more than one a.c. voltage source at fixed phase displacements to each other are called multiphase or polyphase systems.

These are important because virtually all higher power systems operate as multiphase systems. They can be considered as a combination of similar a.c. alternators driven from the one common source, each producing equal sinusoidal voltage waveforms.

When more than one a.c. voltage source is available, the possibility exists that they can be interconnected in various ways, so leading to different types of multiphase systems.

9.2 ALTERNATORS IN PARALLEL

If the load on an alternator exceeds the alternator's rating, a second machine (preferably one of the same type) may be connected to the same load; the two machines then share the load.

The alternators, like batteries, may be connected in either series or parallel, but more precautions are needed with alternators than with batteries.

If two batteries are connected in parallel, care must be taken to ensure that similar polarities are joined together and both batteries are of the same voltage. Similarly, alternators may be operated in parallel, and again voltage and polarity are important.

Equal voltages

Should one machine have a higher voltage than the other, it will provide current, which is driven into the machine having the lower voltage, making it run as a motor. The higher voltage machine has an additional load—the other machine. Conditions are worse than with only one machine.

The polarity of an alternator is not as simple as for batteries—the alternator polarity is changing repeatedly.

Assuming identical waveforms, the two factors that decide whether the polarities of the alternators agree are frequency and phase.

Voltages in phase

The two alternators must be in phase; if they are not, there will be times when the polarities are not the same. Also the voltages of the two machines will often differ; first one and then the other machine will have higher voltage, resulting in heavy circulating currents. The extreme case is when the machines are 180° out of phase. Connecting machines in parallel under these conditions can cause great damage. The voltage driving circulating current between the alternators is the phasor difference of the alternator voltages, and the circulating currents can be very high. The voltage differences at varying phase angles are illustrated in Figure 9.1. The waveforms indicate the outputs of the two alternators. The thick line in (a), (b) and (c) shows the voltage difference driving a circulating current between the two alternators.

- When cells are connected in parallel, ends of the same polarity must be connected together (Fig. 9.2(a)).
- If one of the parallel cells is reversed, a large current circulates through the cells (as shown in Fig. 9.3(a)).

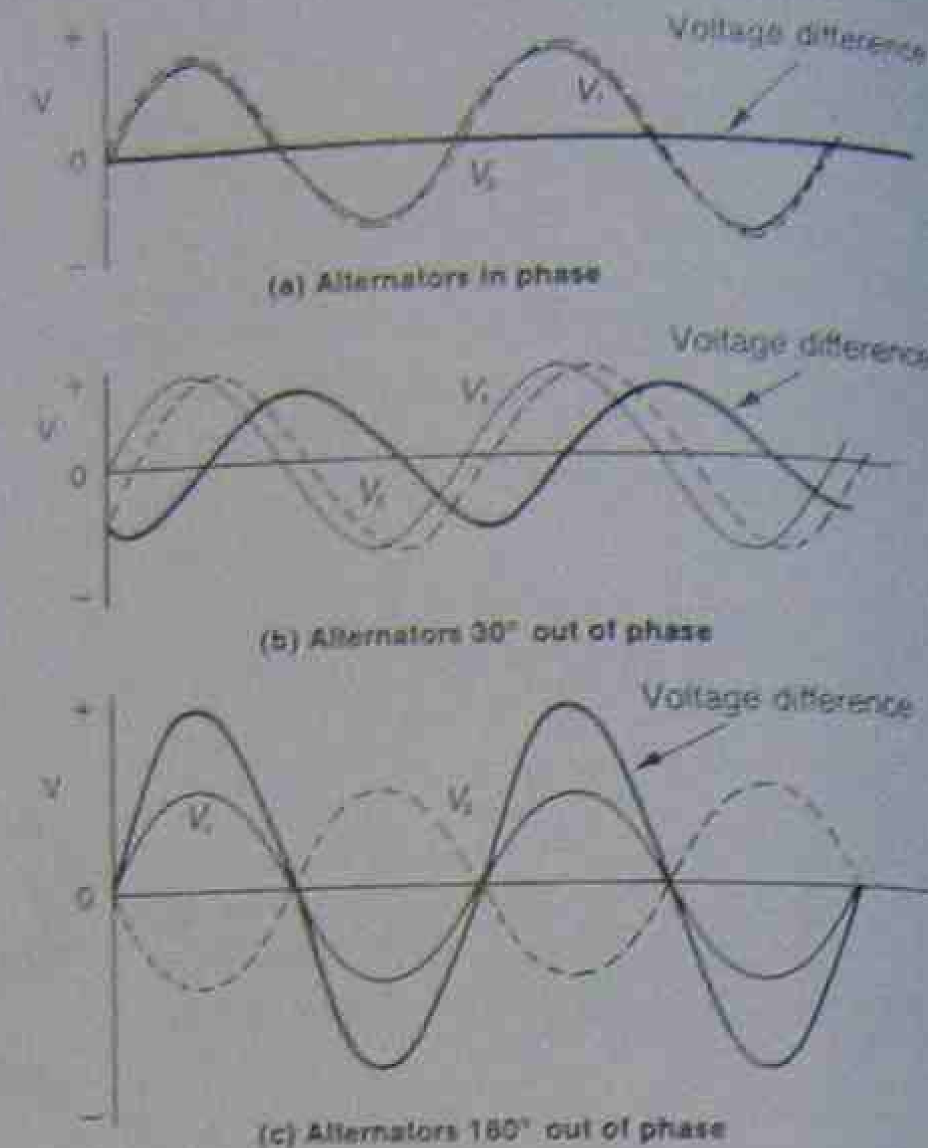


Figure 9.1 • Single-phase alternators in parallel

- When two alternators are connected in parallel, the winding ends that are connected together must be the same polarity at all times (Fig. 9.2(b)).
- Similarly, if the alternator connections are reversed large circulating currents result (Fig. 9.3(b)).

Same frequency

When the frequency of the two machines is different, the phase difference between them is constantly changing—and parallel operation is impossible. The voltage causing a circulating current varies constantly from zero to twice normal alternator voltage as the phase relationships change. Figure 9.4 illustrates the effect when the alternators have frequencies of 40 Hz and 60 Hz.

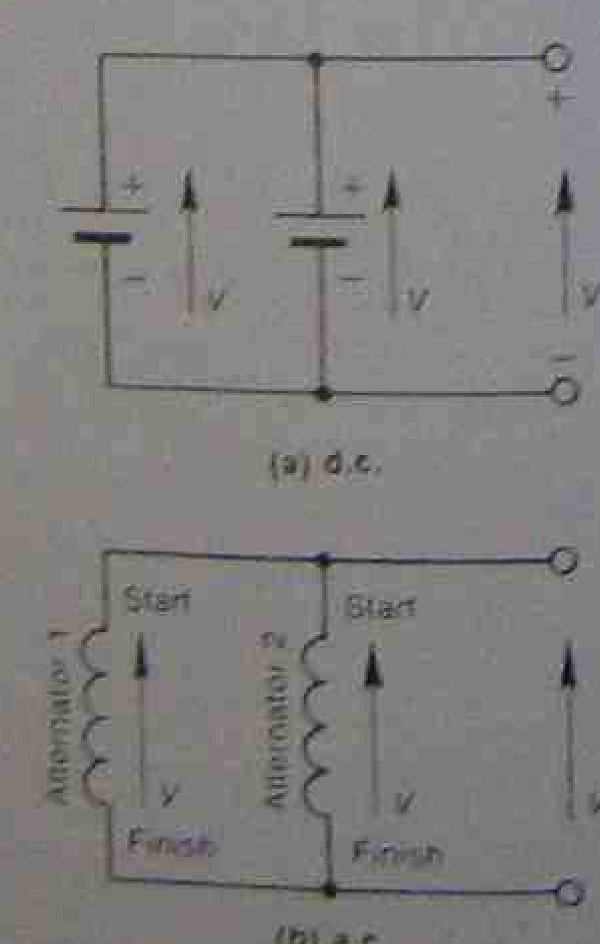


Figure 9.2 • Correct connections for voltage sources in parallel

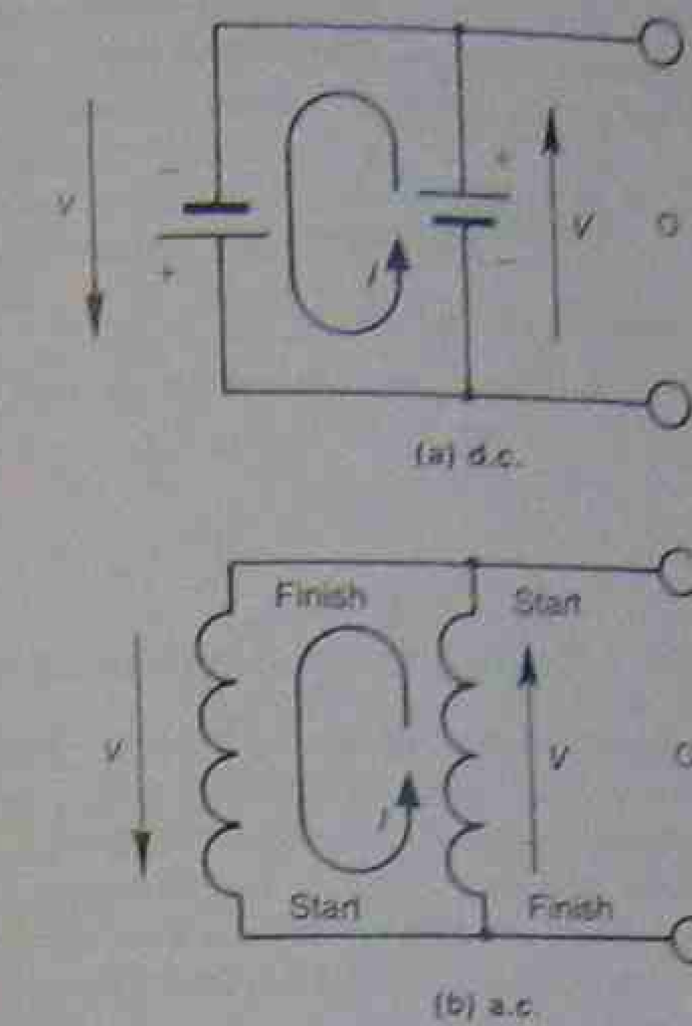


Figure 9.3 • Circulating current paths for reversed connections

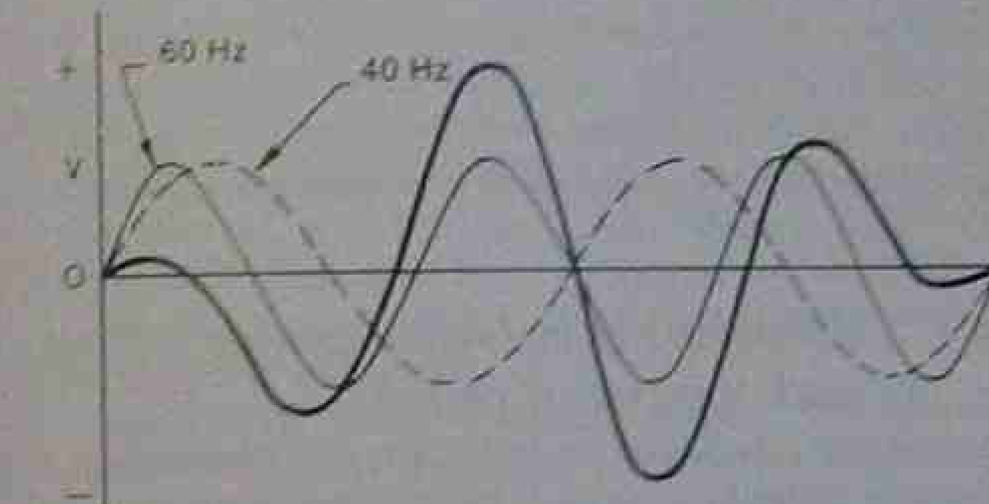


Figure 9.4 • Frequency difference in parallel alternators

Waveform

A similar effect to that of differing frequencies is produced when the shape of the two waveforms is not identical. The irregular resultant waveform produces an irregular current waveform that might not even be similar to the shape of the voltage wave. As well as causing an unsatisfactory load performance, a circulating current between the two power sources is created. This is a further reason for using the sinusoidal waveform as the standard on all power generation equipment.

9.3 ALTERNATORS IN SERIES

Provided they have the same voltage, frequency, waveform and phase relationships, the alternators may be connected as shown in Figure 9.5. The system gives a choice of voltages: 240 V across either alternator for general light and power, and 480 V across the two actives for larger loads. The centre connection is earthed, to ensure that no wire is more than 240 V above earth potential, making the system safer to use. The line at earth potential is known as the neutral.

When two windings are connected in series and dissimilar ends are connected at the midpoint (see Fig. 9.6(a)), the total voltage is the phasor sum of the winding voltages as shown in Figure 9.6(b). This is also illustrated in the waveform diagram of Figure 9.6(c).

If the series-connected windings have similar ends connected (see Fig. 9.7(a)), the resultant voltage is the phasor difference, as shown in Figure 9.7(b).

Figure 9.7(c) is a waveform diagram illustrating the phasor difference voltage when similar winding ends are joined. Note that reversing the winding connections reverses the phase of the voltage—a phase shift of 180°. To indicate this reversal, the second voltage is shown as $-V_2$.

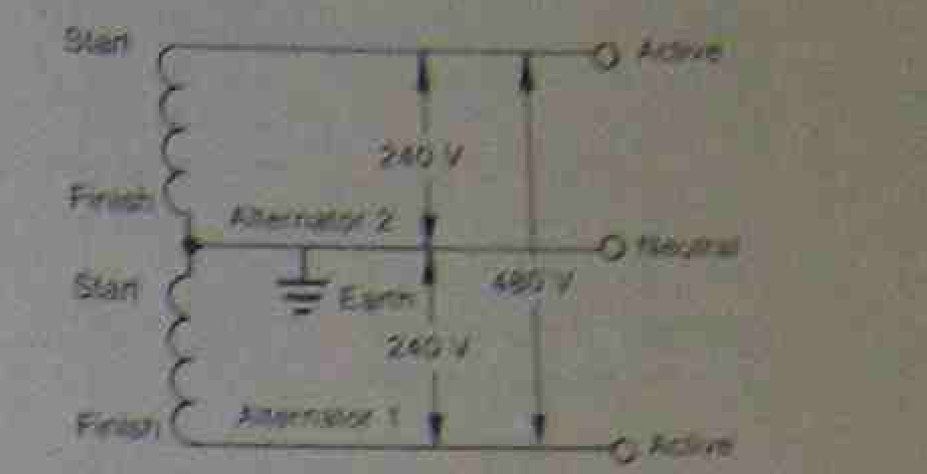


Figure 9.5 • Series alternators

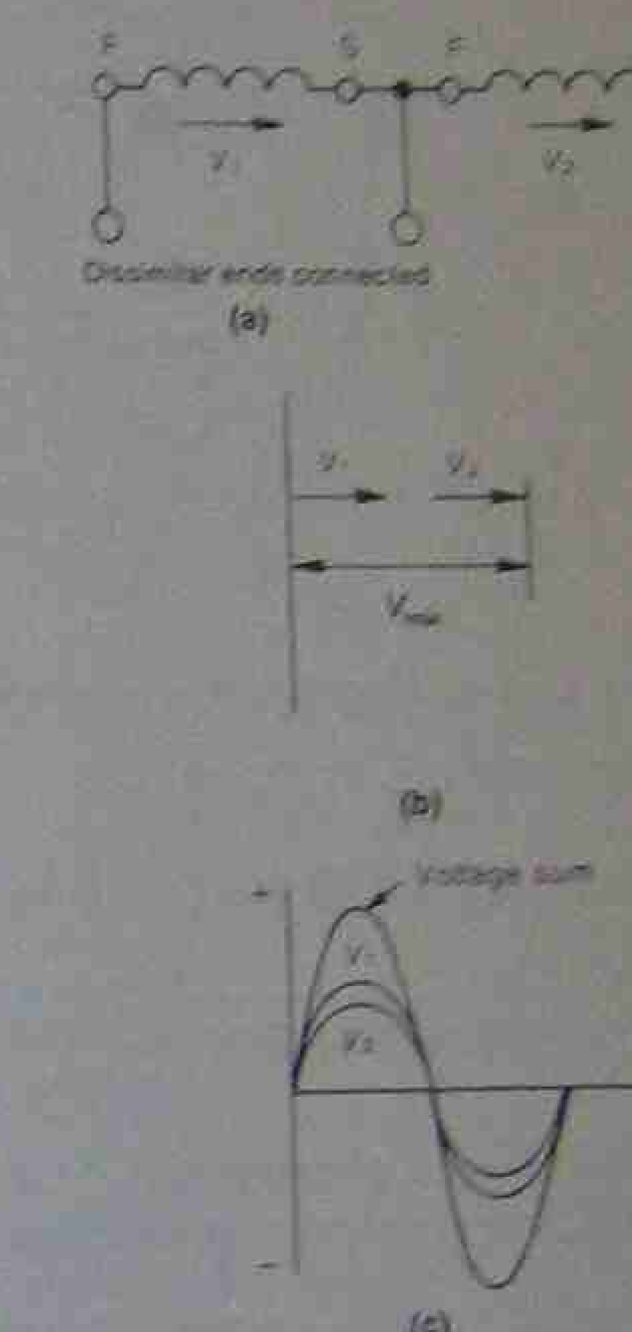


Figure 9.6 • Phasor sum of two a.c. supplies

9.4 TWO-PHASE SYSTEMS

If the two armatures of alternators 1 and 2 are physically displaced by 90°E, as shown in Figure 9.8, the result will be a 90°E phase displacement between the two output voltages.

When the windings are in series (Fig. 9.9(b)) the voltages add, but in this case they are not in phase and must be added by means of phasors.

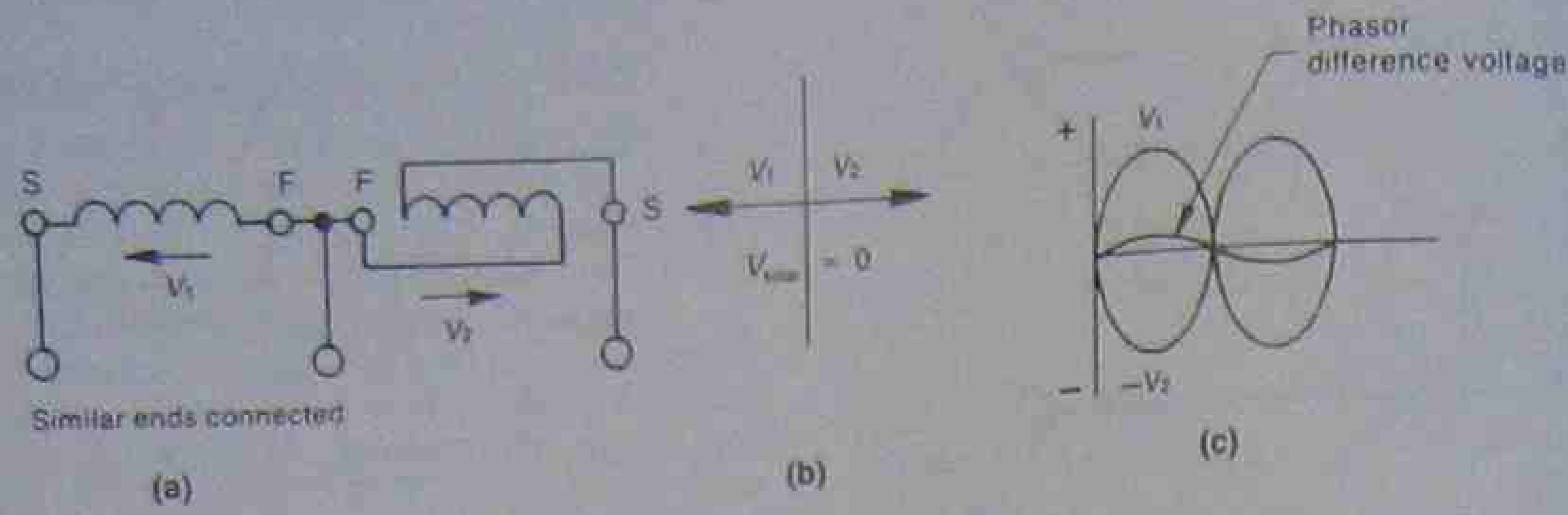


Figure 9.7 • Phasor difference of two a.c. supplies

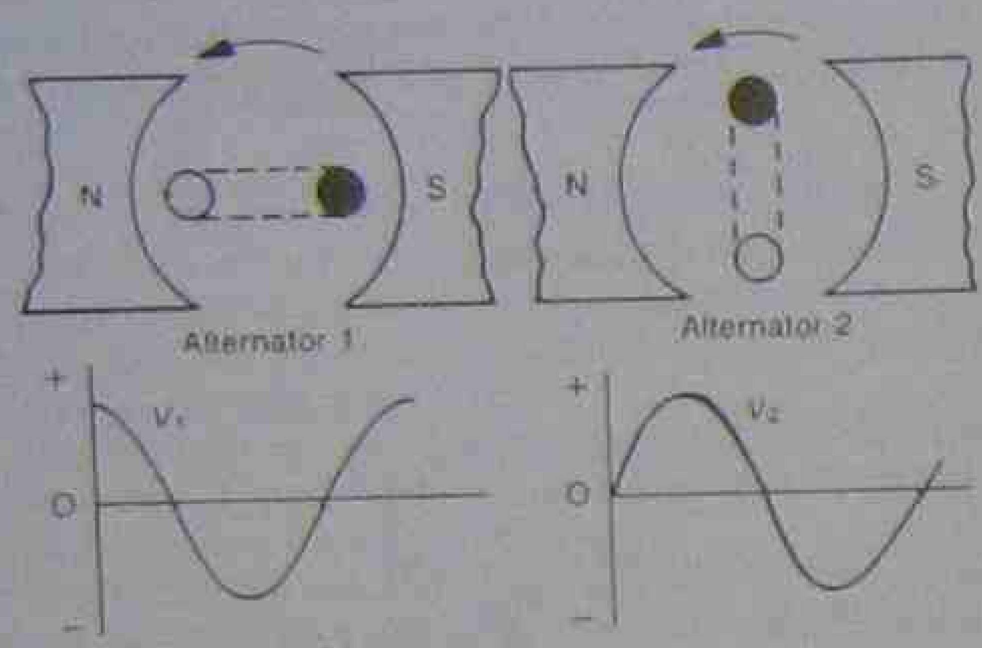


Figure 9.8 • Two single-phase alternators at 90° to each other

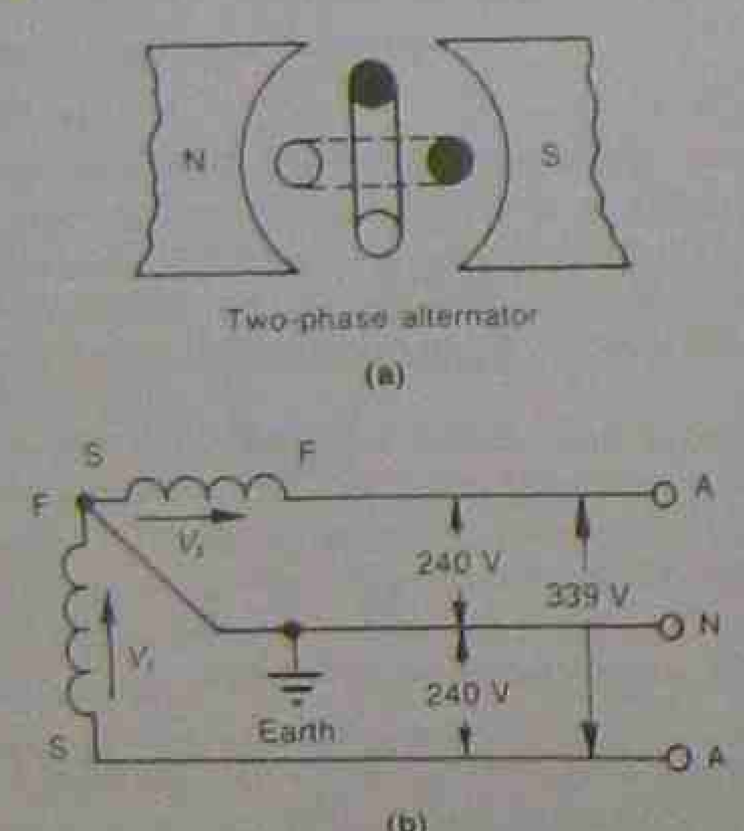


Figure 9.9 • Connections for a two-phase alternator

This is shown in Figure 9.10 where the two voltages (V_2) are shown at 90°E to each other:

$$V_{line} = \sqrt{2}\,V_{phase}$$
$$= 1.414\,V_{phase}$$

It is not necessary to have two alternators to produce a two-phase supply. The most common method is to use one alternator and have two windings on the armature, the windings being displaced by half a pole-pitch, or 90°E. This is illustrated in Figure 9.9(a). The windings are drawn at right angles to each other in Figure 9.9(b) to indicate that they are placed 90° apart in the machine, and that the voltages are 90° out of phase.

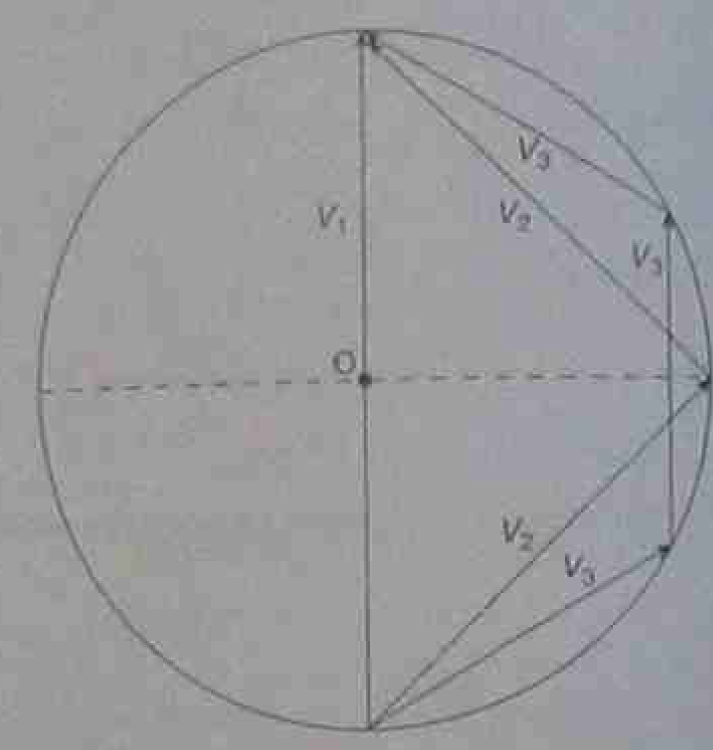


Figure 9.10 • Alternator voltages for one-, two- and three-phase systems

9.4.1 Relationship between power output and number of phases

For any given size alternator there are two limitations—a maximum current determined by the size of the conductors in the windings, and a limit to the amount of power that can be obtained from the alternator with that size frame. For simplicity, assume unity power factor.

If all the windings are fully utilised, then for a:

1. *Single-phase alternator*
   Generated voltage = V_1 and current = I.
   Total power $P_{total} = V_1 I$.
2. *Two-phase alternator*
   Half the total windings are used for each phase.
   Generated voltage = V_2 in each half winding; current = I. The two voltages are at 90° E to each other so the total output voltage = $\sqrt{2}V_2$.
   But $\sqrt{2}V_2 = V_1$ (see phasors in Fig. 9.10).
   With a vector polygon, if all the sides are equal, the polygon can be enclosed in a circle. See section 1.5.6.
   Rearranging this equation:
   $$V_2 = \frac{V_1}{\sqrt{2}}$$
   $$P_{total} = 2V_2 I$$
   So substituting for V_2:
   $$P_{total} = \frac{2V_1 I}{\sqrt{2}} = 1.414 V_1 I$$
   That is, total available power output for a two-phase machine is 41.4 per cent greater than the equivalent-size single-phase machine.
3. *Three-phase alternator*
   One third of the winding is used for each phase.
   Generated voltage is V_3 in each part of the windings; current = I.
   The voltages are at 120°E to each other so that the total voltage output is now:
   $$V_1 = \bar{V}_3 + \bar{V}_3 + \bar{V}_3 = 2V_3 \text{ (see phasors in Fig. 9.10)}$$
   That is, rearranging the equation:
   $$V_3 = \frac{V_1}{2}$$
   $$P_{total} = 3V_3 I$$
   so substituting for V_3, $P_{total} = \dfrac{3V_1 I}{2} = 1.5 V_1 I$

That is, total power available is 50 per cent greater than that for a single-phase machine. From a vector polygon it can be seen that it eventually approximates a semi-circle for a large number of phases so that maximum power output for any one size machine cannot exceed $\pi/2$ or 1.57 times that of a single-phase machine.

9.5 THREE-PHASE SYSTEMS

A three-phase system has three voltages, each out of phase with the other two. The ideal spread of these voltages occurs when the three phases are spread uniformly through the cycle. To achieve this, the phases must start one-third of a cycle apart, or 360/3 = 120°E apart. Figure 9.11 illustrates three phases A, B and C starting at 120°E intervals. At 360°E, phase A is starting again and the sequence continues.

Figure 9.11 is not complete, because portions of the waves of B and C have been left out to emphasise the starts of the cycles. The completed diagram is shown in Figure 9.12(a). The degrees scale is based on the cycles of phase A. To show such a system as this by phasors, the three phasors must be shown 120° apart, as in Figure 9.12(b).

The phasor of voltage V_A has been shown as the reference phasor, drawn horizontally to the right.

The voltages on the phasor diagram at 0° correspond to those in the waveform diagrams at 0°, so in both diagrams:

$$V_A = 0\text{ V}$$

If the phasors are rotated through 120° in an anticlockwise direction, another phasor is in line with the reference axis. This second phasor must represent the voltage, which starts its cycle next after V_A (i.e. voltage V_B). After another 120° rotation, phasor V_C is now in line with the reference axis.

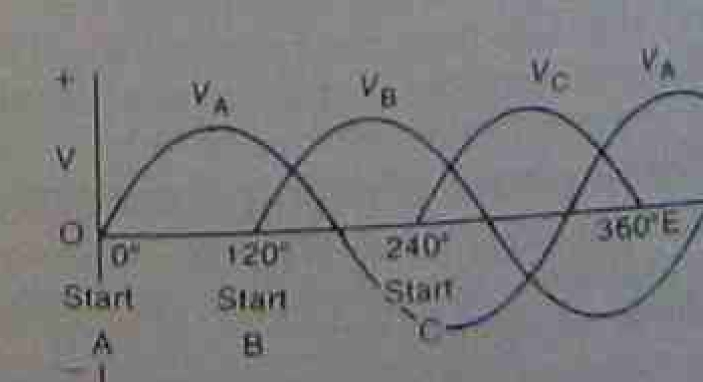


Figure 9.11 • Three voltages at 120° phase displacement to one another

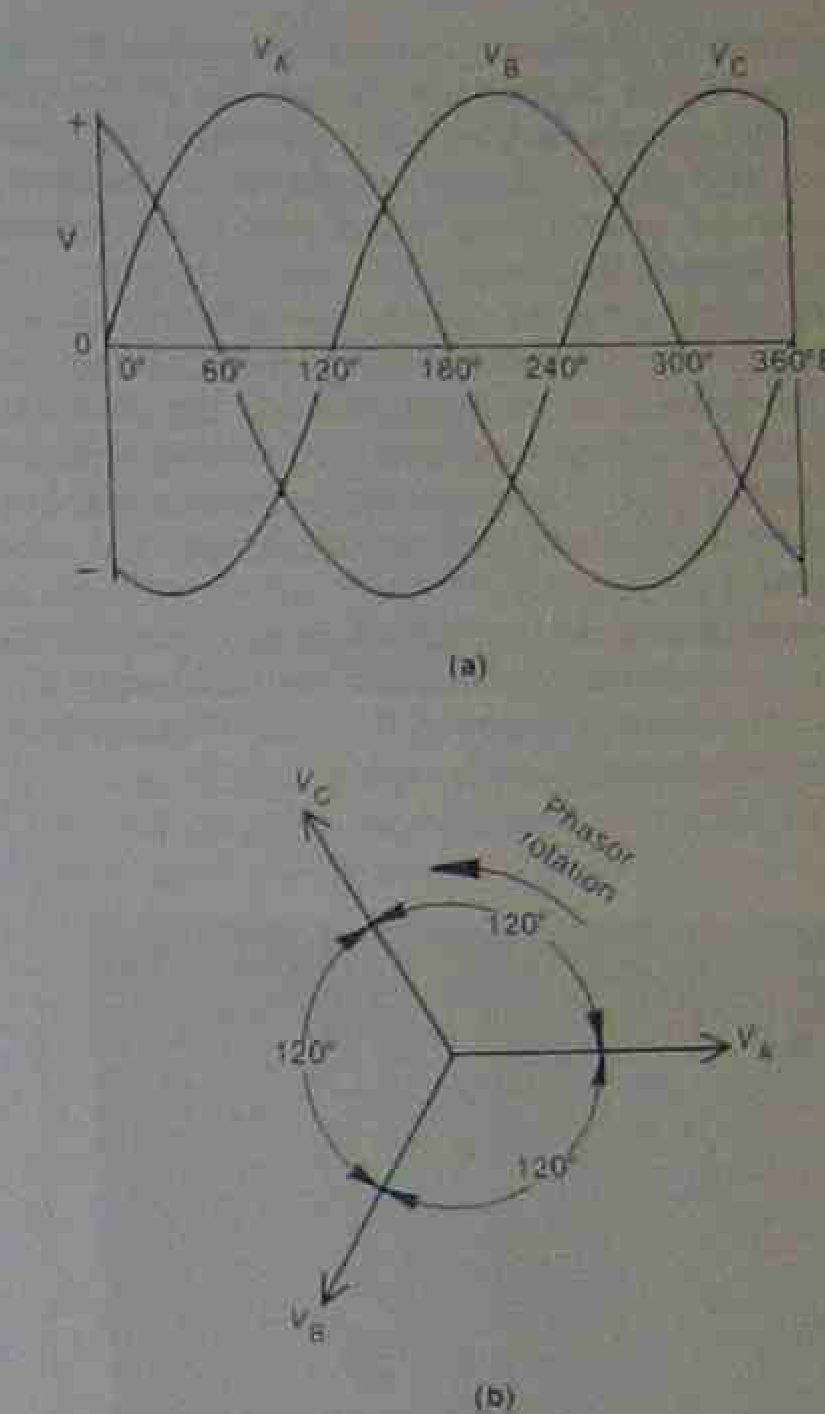


Figure 9.12 • Waveform and phasor diagrams for a three-phase system

9.5.1 Generating a three-phase supply

A three-phase supply may be produced with three alternators locked together; more commonly, however, one machine contains three windings, as shown in Figure 9.13. The positions of the three coils correspond to the voltages at the left-hand end of the graph of Figure 9.12(a) and the phasor diagram of Figure 9.12(b). The corresponding coil sides of the windings in Figure 9.13 have been blocked in, and it can be seen that these are 120° apart.

Coil A is in the magnetic neutral plane and its voltage is zero. This corresponds with 0°E in Figure 9.12. Coil B is 120° behind coil A, and is approaching a maximum negative value of voltage under the south pole. Phasor V_B is also approaching negative maximum—the vertically downward position. Coil C is lagging a further 120° and is

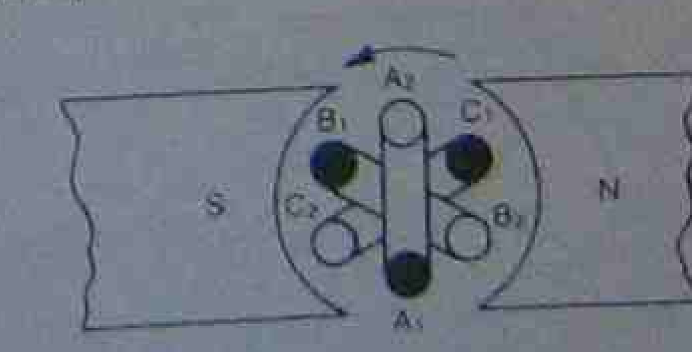


Figure 9.13 • Elementary three-phase alternator

just leaving the position of maximum positive voltage—the centre of the north pole. Phasor V_C is shown just past the maximum positive point, the vertically upward position. The three voltage waveforms in Figure 9.14 are shown in positions to match the waveforms and phasors shown in Figure 9.12.

The three phases must often be identified, to allow the sequence to be stated and to enable loads to be balanced across the phases. In some applications the letters A, B and C are used, while for general-purpose supply and distribution, the phases are given the colour coding red, white and blue. (Prior to 1981 the colours were red, yellow and blue.)

As the phasors of Figure 9.15 rotate, they pass through the reference position in the order red, white, and blue. This sequence must be followed in connecting equipment on three-phase circuits.

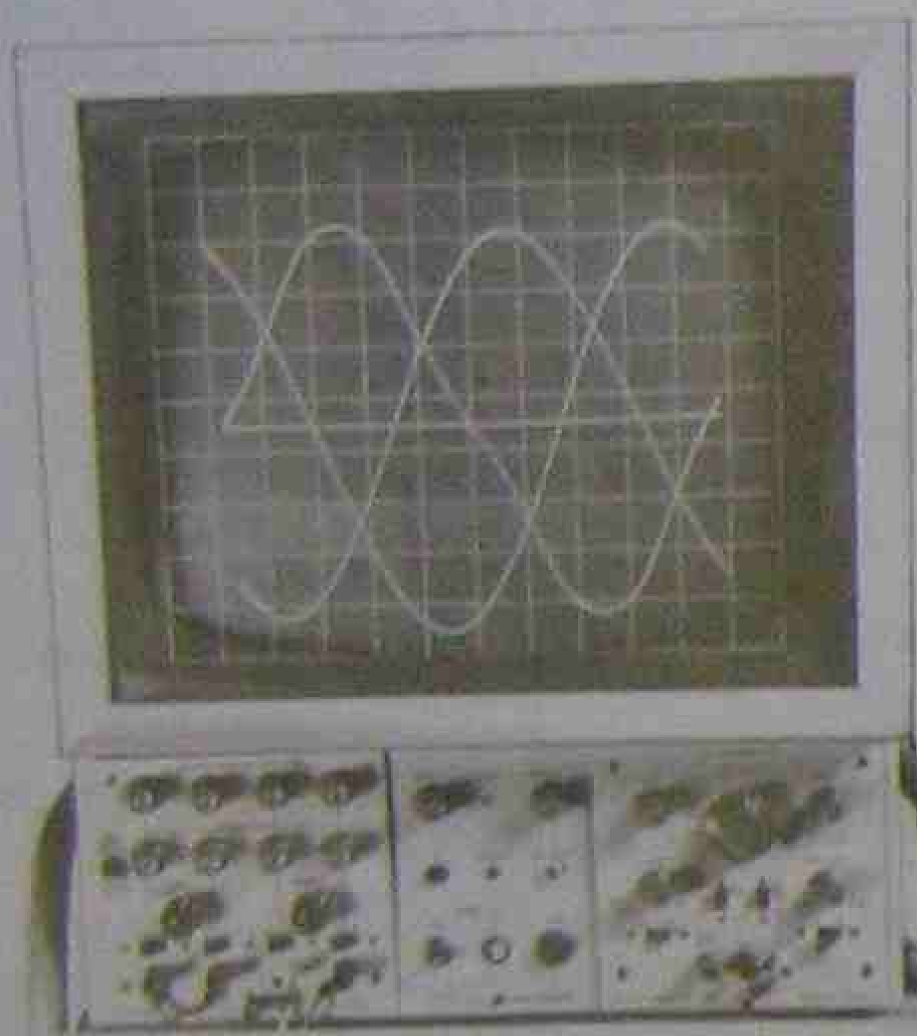


Figure 9.14 • Three voltages of a three-phase system as shown on a four-beam cathode-ray oscilloscope. The fourth input, shown as a horizontal line (zero voltage), has been included to show a base or reference line. (Compare with Fig. 9.12(a).)

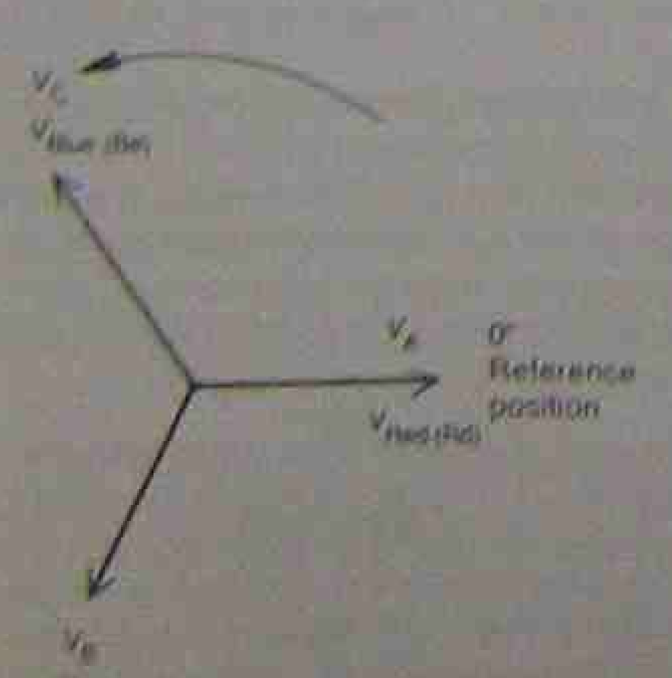


Figure 9.15 • Sequence of voltages in a three-phase system

When three-phase phasors are drawn, the usual practice is to draw the red phasor in the reference position, although, if there is a particular advantage, either the white or the blue phasor may be used as the reference as long as the sequence is still correct.

9.5.2 Advantages of a three-phase system

The advantages of a three-phase system are as follows:

1. As mentioned in section 9.4 on two-phase systems, the one size of machine can produce higher outputs as the number of phases increases. For example, for any one machine frame the available output is:

Phases	*Output*
1	64 per cent
2	90 per cent
3	96 per cent
n	100 per cent

where n = infinite number of phases

These figures can be obtained by manipulating the figures calculated in section 9.4.1 for the various number of phases:

- For n number of phases, the maximum power output is an absolute 100 per cent.
- For 1 phase, maximum available power is 100 per cent divided by $\pi/2$ (= 1.57); that is, the power available = 100/1.57 = 63.7 per cent = 64 per cent.
- For two phases, the maximum power available is $63.7 \times \sqrt{2} = 90$ per cent.
- For three phases, the maximum power available is $63.7 \times 1.5 = 95.5$ per cent = 96 per cent.

In economic terms, the three-phase system gives the biggest increase for the addition of only one extra conductor, when compared to the single-phase system. When compared to the two-phase system there is no extra conductor needed.

2. Another advantage is that the power delivered to or taken from a three-phase system is a more constant value. In a single-phase system the power curve 'pulses' at twice the line frequency. With three phases, the pulses are six times the line frequency and do not cross the zero axis like the single-phase system. Since the power is more constant, the torque of a rotating machine is more constant and this results in less vibration from the machine.
3. With one type of three-phase connection there are two voltages available—a selection can be made depending on the type of load.
4. A three-phase machine can be smaller than a single-phase one for the same power output.
5. In a distribution system the total quantity of material needed for three conductors is less than that required for the equivalent single-phase system.

9.5.3 Three-phase winding arrangements

A method for providing three identical windings in a three-phase machine, each displaced 120°E from one another, is shown in Figure 9.16.

Phase A is drawn in with the coils on the outside, and the connections on the inside of the circle, which represents the core and slots. It can be seen that the phase has four poles and occupies one-third of the total number of slots.

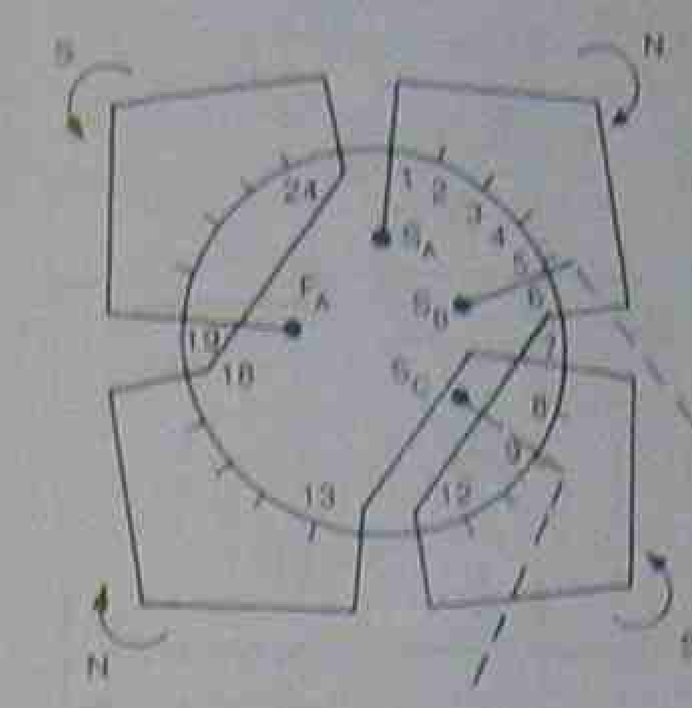


Figure 9.16 • Three-phase winding layout

A four-pole machine has 720°E in one complete rotation. In this particular case the core has 24 slots, so the degrees per slot are 720/24 = 30°E.

The start of B phase (S_B) is displaced four slots from S_A ($4 \times 30 = 120$°E) and commences at the fifth slot. Phase B is identical to phase A in arrangement and connections, the only exception being the phase displacement. The winding also occupies one-third of the total number of slots.

Similarly, phase C is identical to phases A and B, except that it is displaced a further 120°E and commences at the ninth slot.

9.5.4 Alternator construction

The principles of alternator construction are discussed in Chapter 11, but basically an alternator consists of coils rotating in a magnetic field. In an alternative form, which has many advantages, the a.c. windings are stationary and the magnetic field system rotates. The same basic principles apply equally to both single- and three-phase alternators, the only real difference being whether there is one winding or three identical windings. The single-phase alternator was discussed in Chapter 8, together with alternative means for constructing sine waves.

9.5.5 Three-phase sine wave construction

The method of construction of a single sine wave was discussed in section 8.8. The construction of the three sine wave curves for a three-phase system uses the same method, provided due regard is paid to the phase displacement of 120°E between the phases.

The method is illustrated in Figure 9.17 using the rotating line method described in section 8.8.

Three voltage phasors are drawn as shown, with the red phase on the reference axis. Because the phase sequence is normally RWB and the rotation is anticlockwise, the white and blue phasors are drawn in the positions indicated. While the three phasors are fixed in their relationship to one another, they can rotate as a unit. First the red phase is drawn in, using the rotating line method as described previously. Note that the horizontal axis is marked off in degrees for the red phase (the reference).

There are two ways of obtaining the curves for the other two phases:

1. To draw in the voltage curve for the white phase, the red voltage curve can be copied by adding 120° to each rotational reference.

   At 0° for the red phase:

   $V_R = 0$

   Therefore, at 0° for the white phase:

   $V_W = 0$ but is located at 0 + 120 = 120° on the red phase scale

   At 90° for the red phase:

   V_R = maximum positive value

   Therefore, at 90° for the white phase:

   V_W = the maximum positive value but is located at 90 + 120 = 210° on the red phase scale

   When sufficient points are located, the curve can be drawn in and the method repeated for the blue phase by adding 240°E each time. The result is three sine-wave curves similar to Figure 9.11. The parts of the curve between 360°E and 600°E have to be relocated to get a full set of curves within one cycle of the red phase.

2. An alternative method is to transfer locating points directly across to the red phase cycle. Phasor V_W is approaching its most negative value when the red phase is at the 0° reference axis. This value can be located on the 0° line at the appropriate level (point M). When the three phasors rotate 30°, V_W is then at its most negative value and this value is transferred across to meet the 30° line (point N). The method is repeated at intervals until sufficient

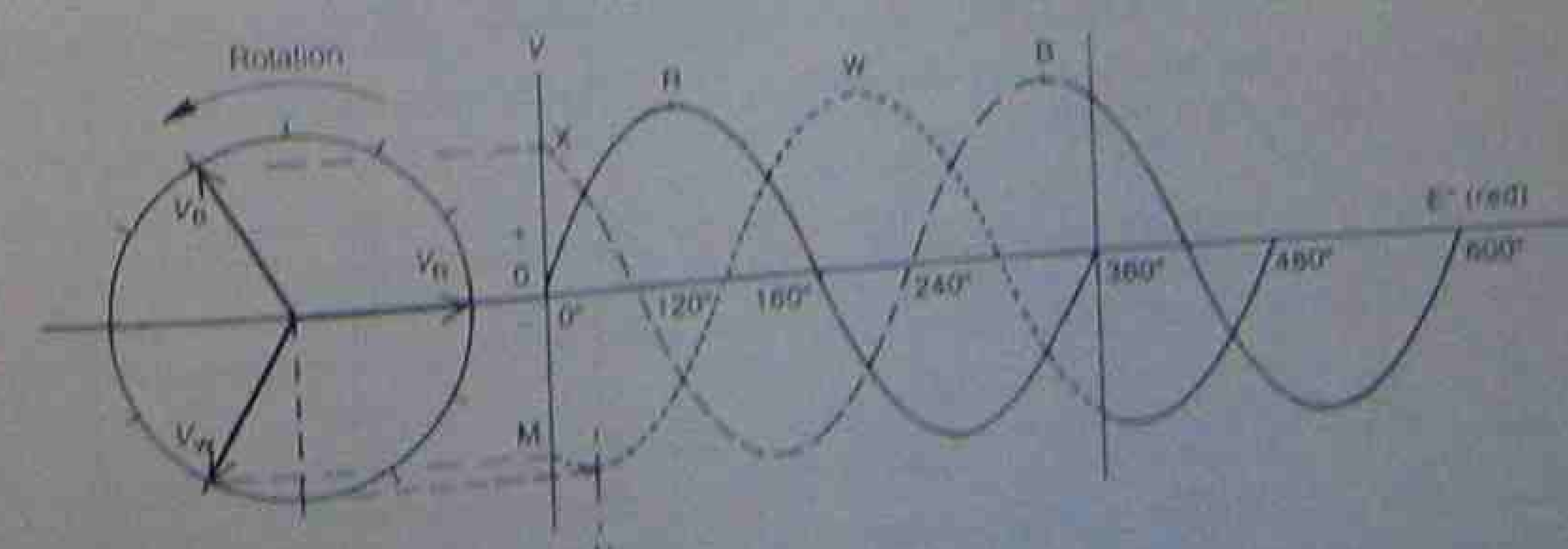


Figure 9.17 • Construction of three-phase sine waves

9.6 THREE-PHASE CONNECTIONS

9.6.1 Phase sequence

9.6.2 Three-phase star connections

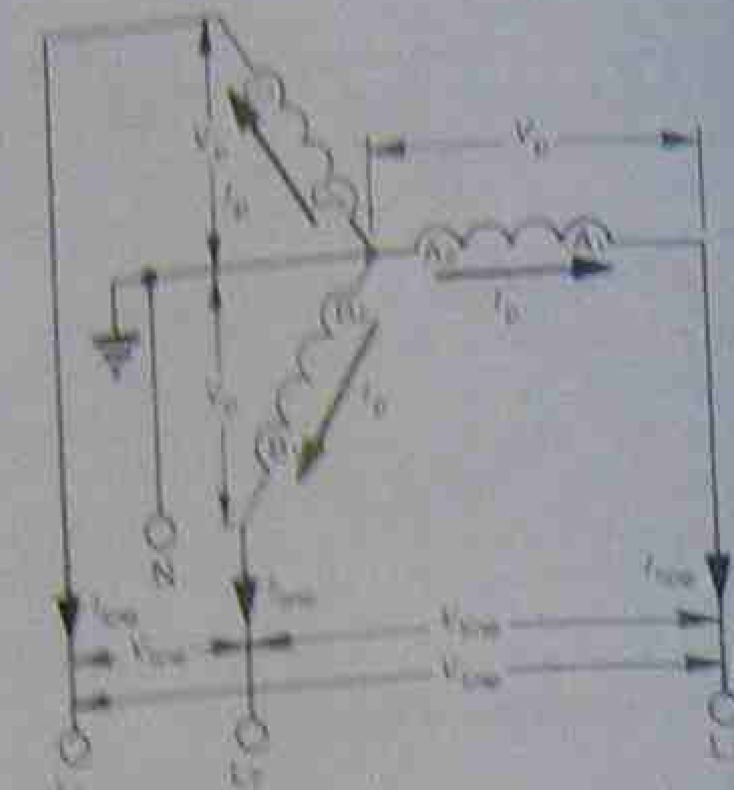


Figure 9.19 • Line and phase values for a star system

The neutral, which is connected to the star point and earthed, is not considered to be a line.

The line voltage is not equal to the phase voltage because two phase windings are connected across each pair of lines. The voltage across a single-phase winding is therefore called the phase voltage (V_p) to distinguish it from the line voltage. Similarly, the current flowing through the phase winding is called the phase current (I_p).

At point K in Figure 9.20, the voltage in A phase is at its maximum positive value and the current flows in the direction A_2 to A_1 as shown in Figure 9.19. The corresponding point for B phase occurs 120° later at L, when the white phase current flows from B_2 to B_1. Similarly a further 120° later at point M, the C phase current flows from C_2 to C_1.

The three-phase currents therefore do not reach their maximum values at the same time, but at regular intervals 120°E apart (i.e. the currents are 120°E out of phase with one another).

Because the A phase winding and the red line are in series, there is only one path for the current. This is applicable for all three phases. That is, in a star-connected system, the line current equals the phase current:

$$\boxed{I_{line} = I_p \text{ (star)}}$$

The line voltage V_{line} is a combination of two phase voltages, as shown in Figures 9.20 and 9.21.

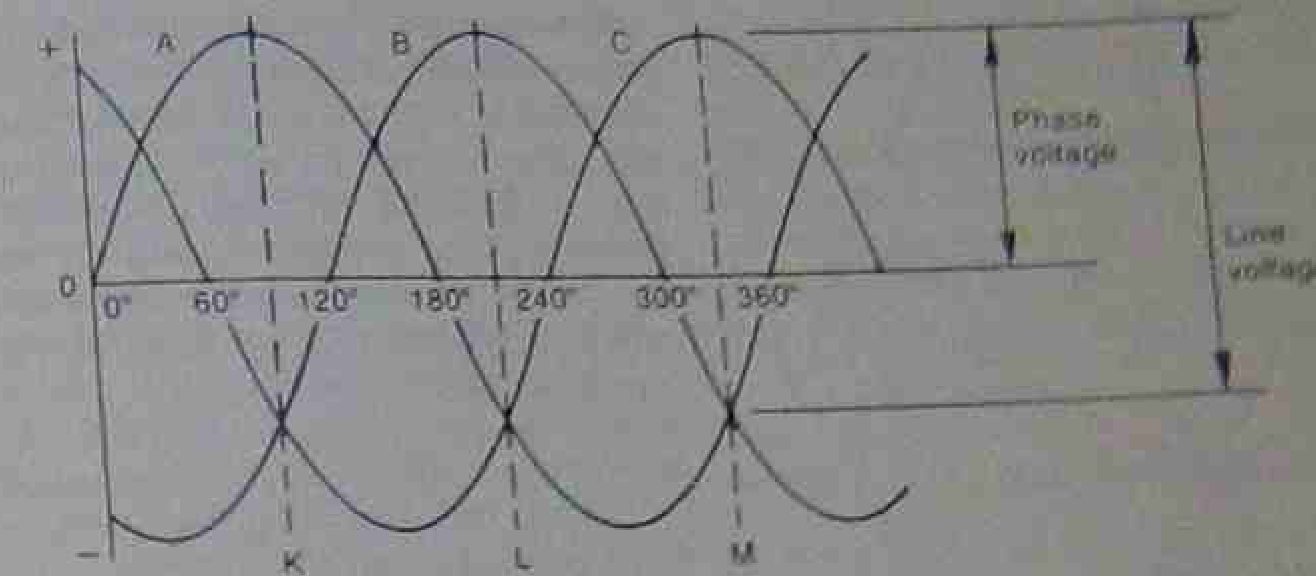


Figure 9.20 • Line and phase voltages for a three-phase star connection

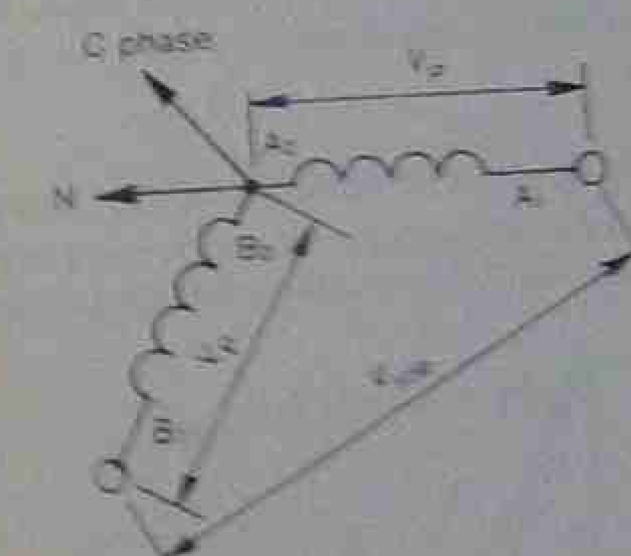


Figure 9.21 • Line voltage in a star system

In section 9.3 it was shown that when similar ends of two windings are connected, the voltage across the actives is equal to the phasor difference between the two voltages in the windings. In Figure 9.21, the phase windings are connected with similar ends, therefore V_{line} is equal to the phasor difference between the two phase voltages.

The phasor difference is found by subtracting one phase voltage from another by means of phasors.

Example 9.1

Figure 9.22 • Phasor diagram for example 9.1

Step 3

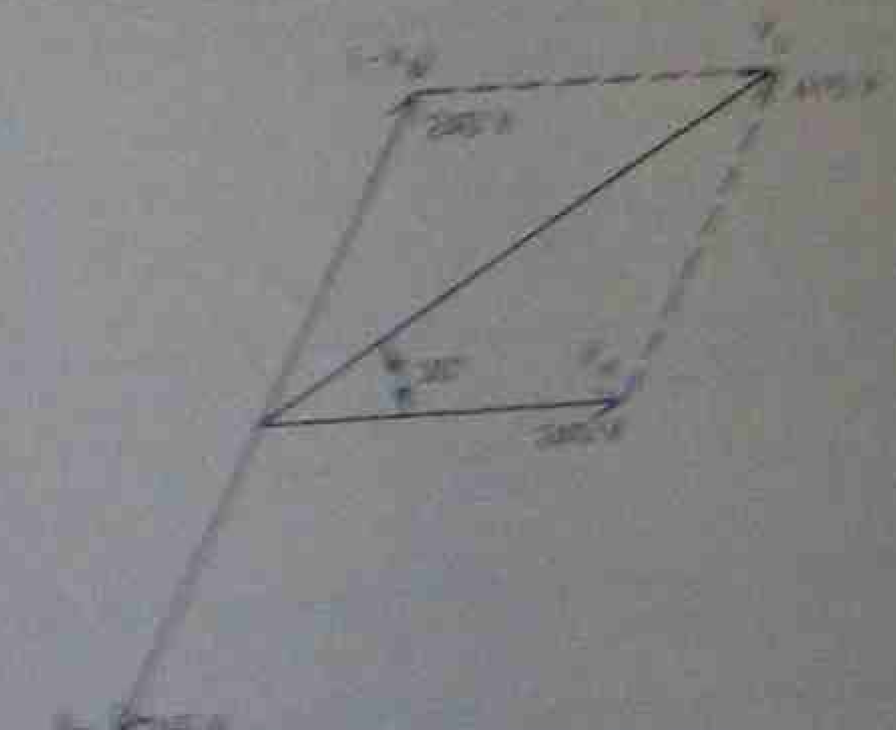


Figure 9.24 shows all three line voltages obtained by finding the phasor difference between each pair of phase voltages.

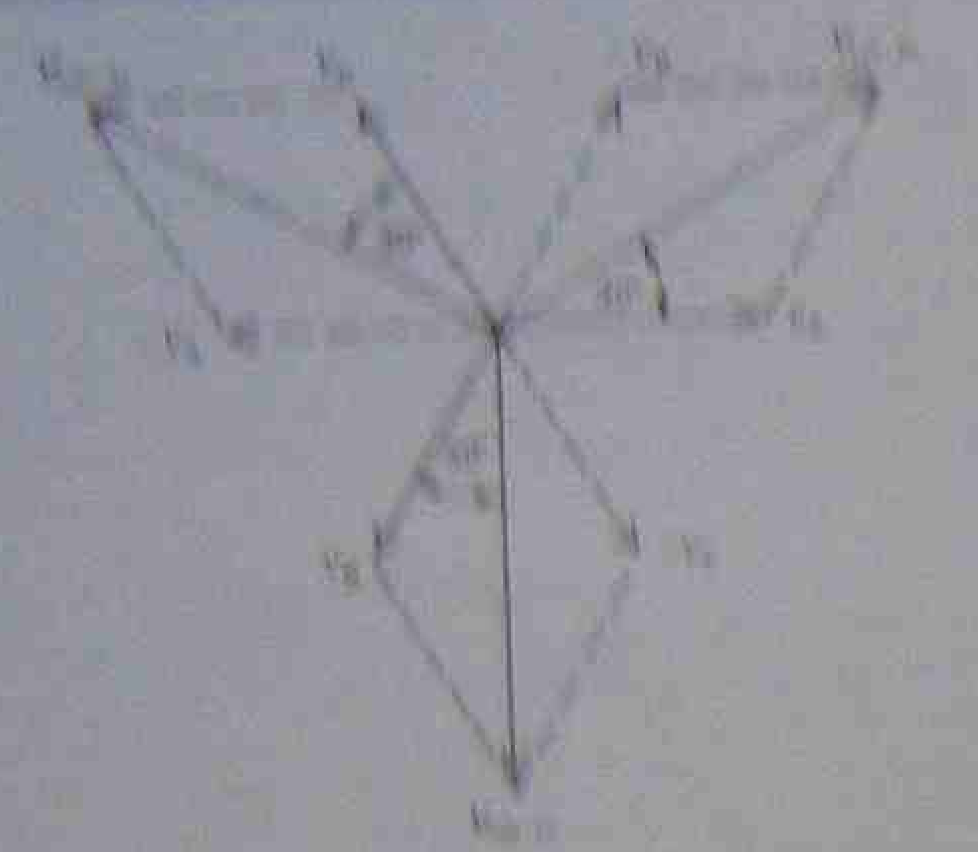


Figure 9.24 ◂ Relationships between line and phase voltages in a star-connected system

9.6.3 Effect of phase reversal on a three-phase star system

If the ends of one phase winding are reversed, the start ends of the three windings are now no longer 120° apart. If, for example, phase winding C is reversed, the phase voltage of C has a phase shift of 180°. The waveforms of the three phase voltages then have a displacement of 120° between A and B, and 60° between A and C, and between C and B, as shown in Figure 9.25.

Figure 9.25 ◂ Waveform diagram for a three-phase system with phase C reversed

The above can be represented by the phasors V_A, V_B and V_C as in Figure 9.26.

Figure 9.26 ◂ Phasor diagram for reversed phase C in a star-connected system

Although the phase voltages are equal, they are not in the correct phase sequence, and the line voltages are also affected. In Figure 9.27 the line voltage V_{A-B} is the phasor difference between phase voltages V_A and V_B and equals $\sqrt{3}\,V_P$. The line voltage V_{B-C} is the phasor difference between phase voltages V_B and V_C, and the line voltage between phase voltages [illegible] V_{C-A} is the phasor difference between phase voltages V_C and V_A. Both line voltages V_{B-C} and V_{C-A} are equal to the phase voltage.

Figure 9.27 ◂ Line voltages for a three-phase, star-connected system with phase winding C reversed

The three line voltages are also shown in Figure 9.26 in dotted lines. It can be seen that they are not all equal, nor are they all 120° apart. Therefore, if one phase winding is reversed, the system is unsatisfactory for a balanced three-phase supply.

9.6.4 Three-phase delta connections

The windings of a three-phase alternator can also be connected, as shown in Figure 9.28(a), forming a closed loop

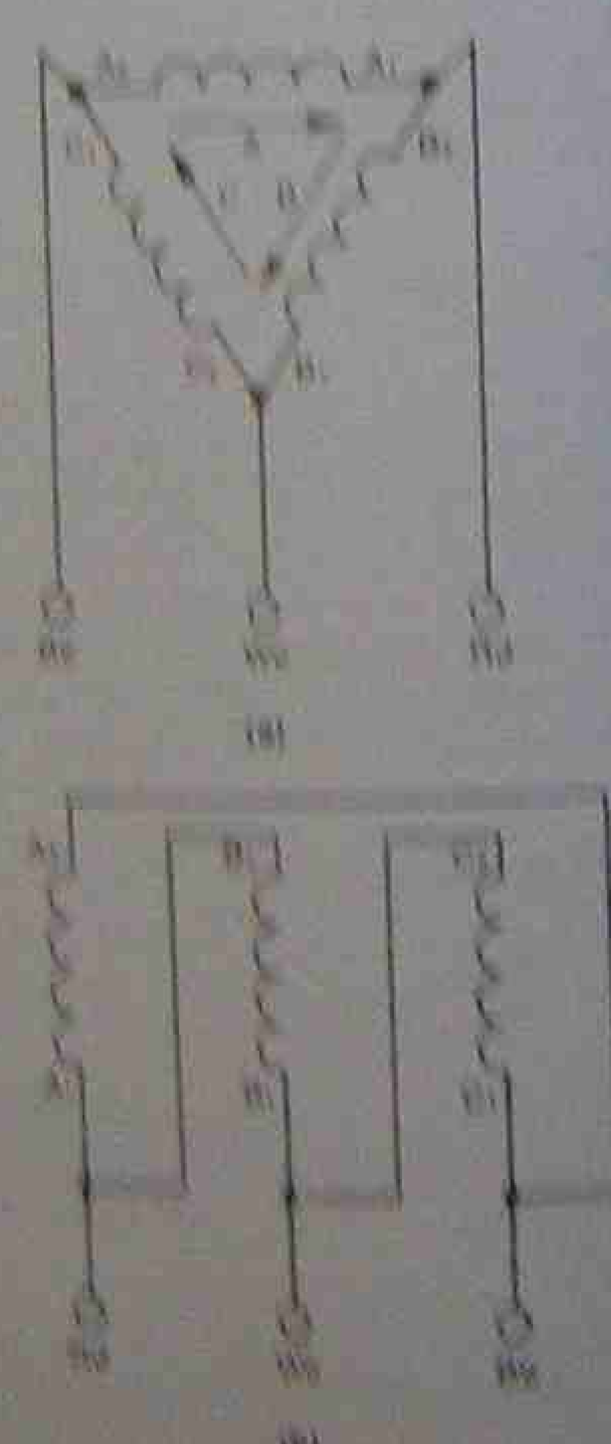


Figure 9.28 ◂ Three-phase delta connection

with dissimilar ends joined together, and the lines are connected to these junctions. This system is called a 'delta' connection because the shape of the diagram represents the Greek letter Δ (delta).

An alternative method of drawing the windings and connections is shown in Figure 9.28(b).

Because dissimilar ends of the phase windings are joined, the resultant voltage is equal to the phasor sum of the individual voltages. It can be seen from the phasor diagram in Figure 9.29(a) that the phasor sum of the voltages V_B and V_C is equal and opposite to the phase voltage V_A. The phasor sum of these three is therefore zero.

This can also be seen from the phasor diagram in Figure 9.29(b), where the phase voltage V_B is added to the end of V_A, and V_C is added to the end of V_B. The resultant voltage is represented by the distance between the end of V_C and the start of V_A (i.e. zero voltage).

In a delta connection each phase winding is connected across two lines, therefore the line voltage equals the phase voltage:

that is, $\boxed{V_L = V_P \text{ (delta)}}$

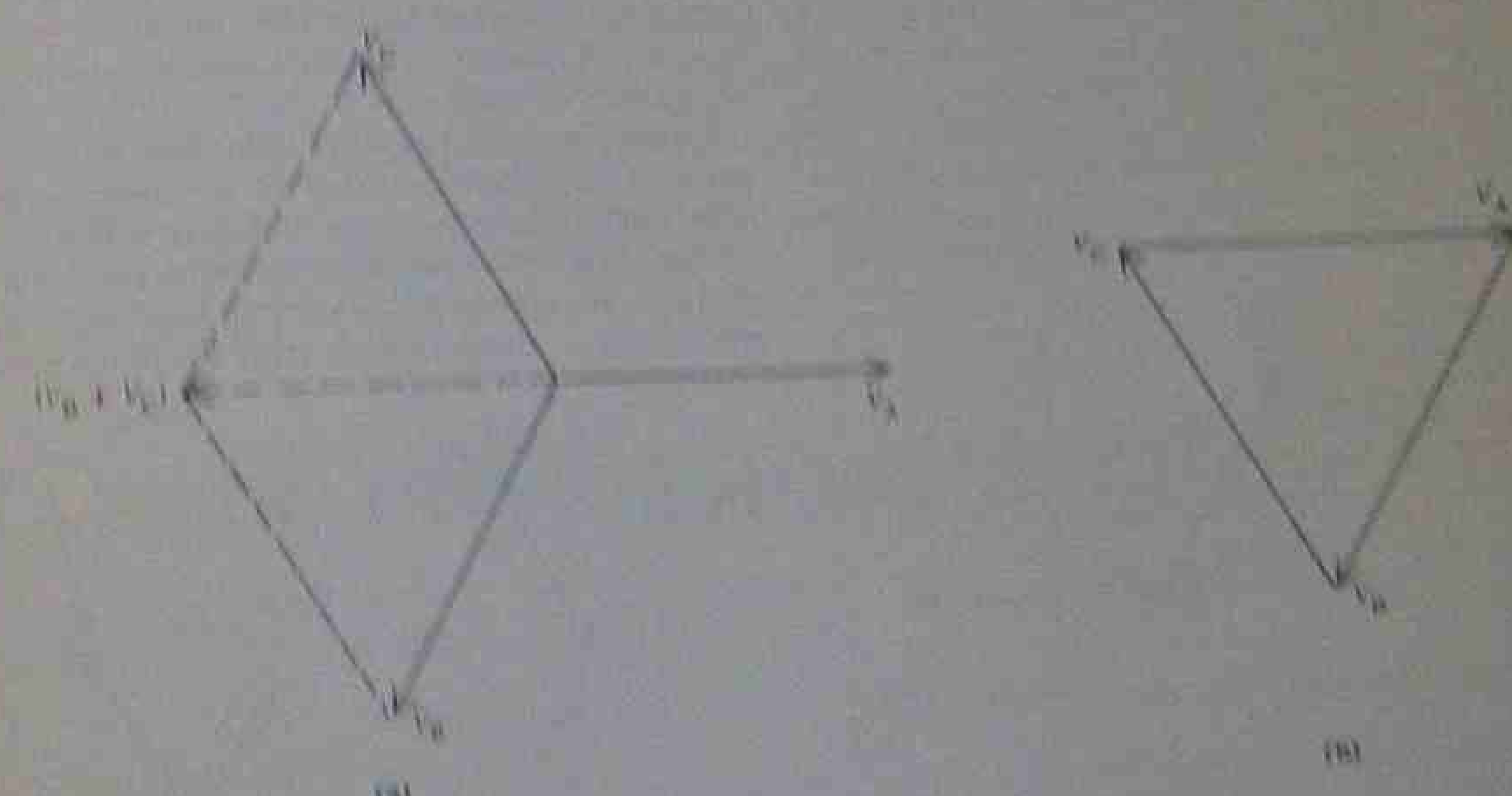


Figure 9.29 ◂ Voltages in a delta system

The phase currents in Figure 9.30(a) are I_A, I_B and I_C, and the line currents are I_1, I_2 and I_3.

To find the line current I_1, it is necessary to add the phasors of the two phase currents I_A and I_B. Since I_B is negative with respect to I_A, then:

$$I_1 = I_A + (-I_B) = I_A - I_B$$

where $I_A - I_B$ = phasor difference

This can also be seen in Figure 9.30(a), with the arrows at the junctions of the delta; the line current is a combination of these currents. If the phase currents are of equal value, then the line current is equal to $\sqrt{3}$ times the phase current:

that is, $\boxed{I_L = \sqrt{3}\,I_P \text{ (delta)}}$

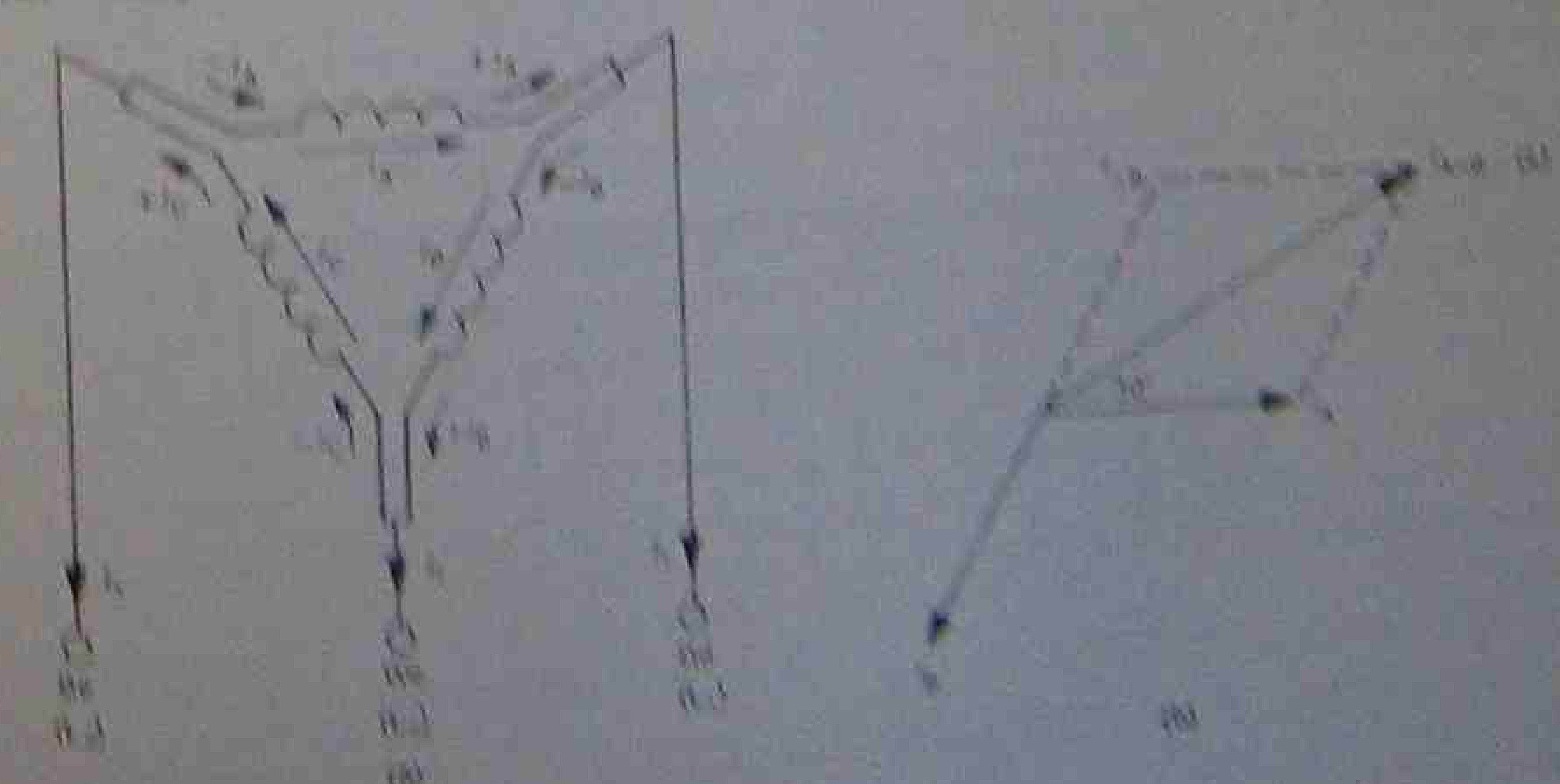


Figure 9.30 ◂ Currents in a delta system

Also there is a 30° phase displacement, as shown in Figure 9.30(b), where I_{A-B} leads I_A by 30°.

The phasor diagrams for line currents I_2 and I_3 are shown in Figure 9.31.

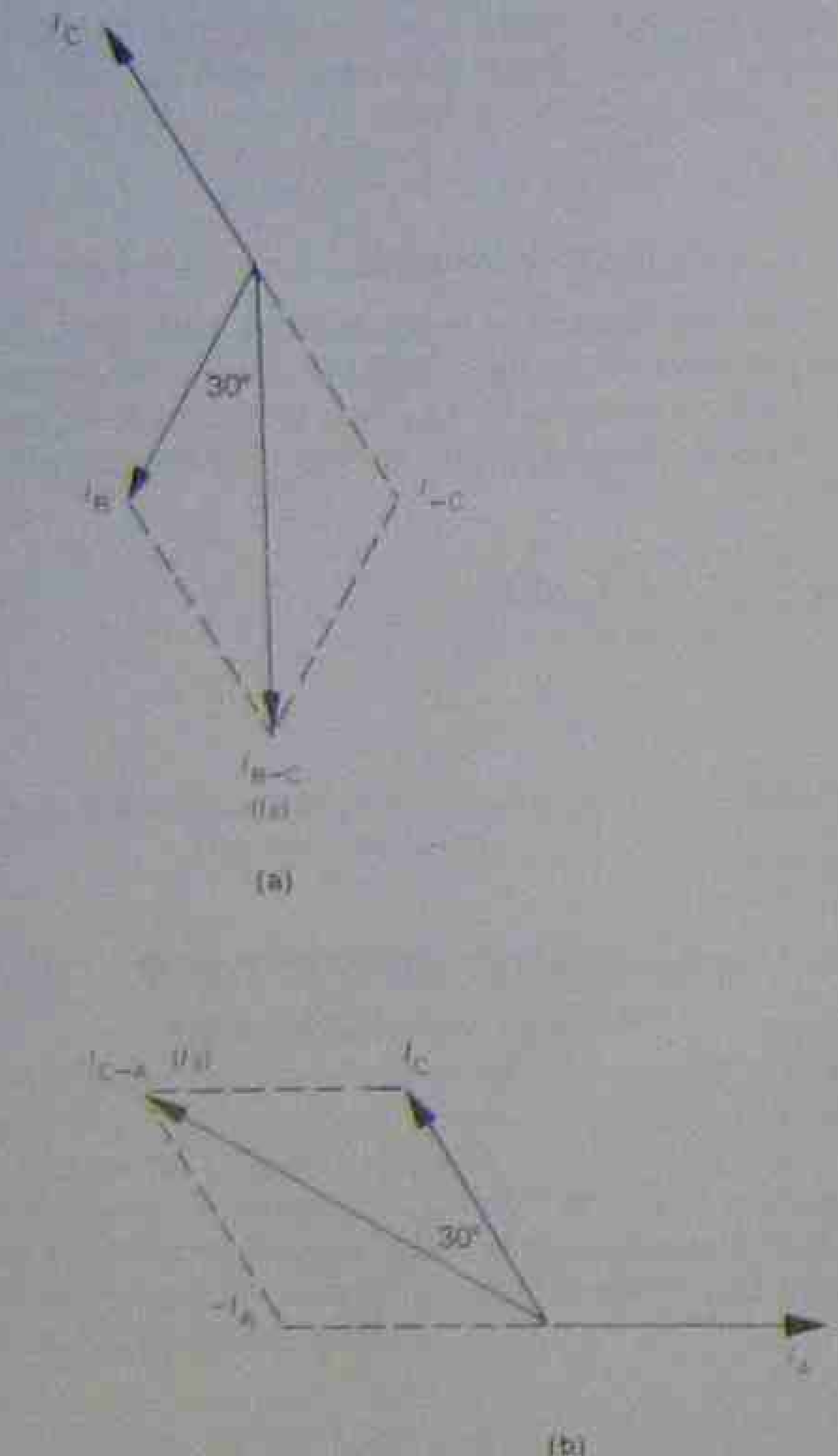


Figure 9.31 • Line currents in a delta system

It should be noted that the star connection gives the line voltage a phase shift, causing it to lead one of the phase voltages by 30°, whereas the delta connection causes the line current to lead one of the phase currents by 30°.

This effect is put into use with interchanging star and delta connections when larger numbers of phases are required (as with large industrial rectifiers, where filtering costs become prohibitive). The major differences between star and delta configurations are compared in Table 9.1.

9.6.5 Effect of phase reversal on a three-phase delta system

If the ends of one phase winding are reversed, the phase is given a phase shift of 180°E. The phases are no longer in their correct sequence or phase relationship. When the phasors are added, the resulting voltage is no longer equal to zero. From the phasor diagram of Figure 9.32(a) it can be seen that the resultant voltage V_R is equal to twice the phase voltage.

Because this higher voltage is generated within the closed circuit of the windings, heavy circulating currents will flow in the windings and cause them to burn out quickly. Great care must be taken when the phase windings are connected in delta to ensure that dissimilar ends are joined.

A simple method of testing the connections is to leave one junction open and connect a voltmeter across the open winding ends, as shown in Figure 9.32(b).

If the connections are made correctly, the voltmeter will read zero. Should one phase winding be reversed, the meter will register a voltage equal to twice the phase voltage.

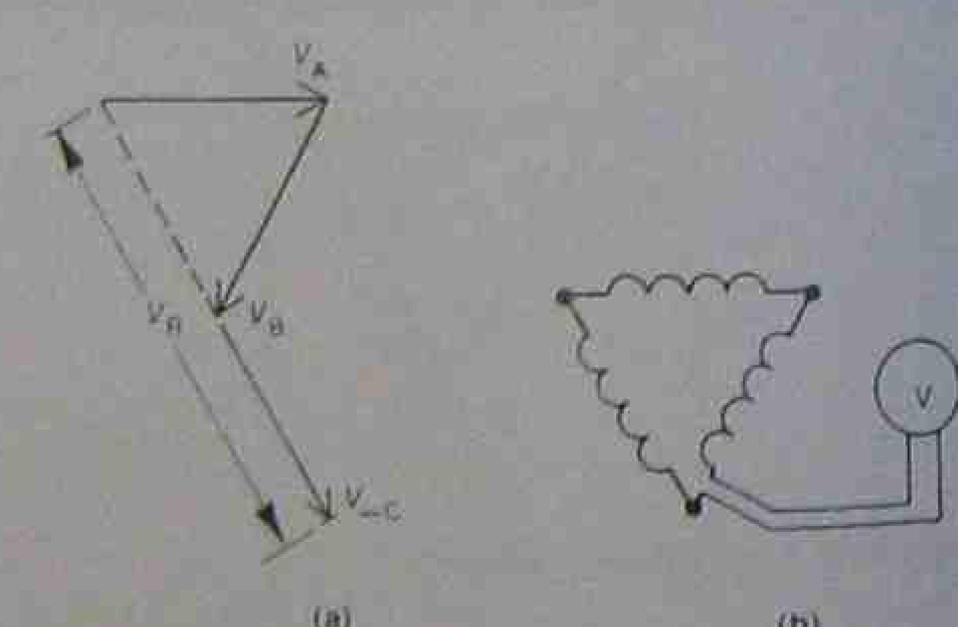


Figure 9.32 • Phase reversal in a delta system

Table 9.1 • Comparisons between star and delta systems*

Star	Delta
Similar ends connected	Dissimilar ends connected
For balanced or unbalanced loads	More suitable for balanced loads
$V_L = \sqrt{3}V_P$	$V_L = V_P$
$I_L = I_P$	$I_L = \sqrt{3}I_P$
Two values of voltage available	One common voltage only available
Has common connection available for earthing	No common earthing point
V_L leads V_P by 30°E	I_L leads I_P by 30°E
Suited to long-distance power transmission	Suited to locally operated machinery

* Both connections have their advantages in the distribution of electrical power and both are used in situations dependent on local conditions and requirements.

9.7 THREE-PHASE POWER TRANSMISSION

Because power is proportional to the product of voltage and current, any quantity of electrical power can be transmitted using either low voltage and high current, or high voltage and low current. For large amounts of power the line current can be reduced by increasing the transmission voltage. This means a reduction in conductor size for transmission lines and a reduced power loss in the line.

In general economic terms, the higher the voltage used for transmission, the lower the cost of installing and maintaining the transmission lines becomes. For long distances between the source of supply and the consumer, voltages between 7 kV and 576 kV are used for power transmission and distribution. One arrangement for such distribution is shown in Figure 9.33(a). Typical voltages are shown on the diagram, although these values vary from state to state and might vary within the one state.

It can be seen that the voltages of a transmission system are far higher than that normally used by the consumer, and the final or local distribution voltage is obtained through transformers at substations. Figure 9.33(b) shows a simplified arrangement for a local distribution network.

Most main transmission and sub-transmission lines are duplicated, and alternative routes provided, so that different localities can be fed by other lines and substations in the event of essential maintenance or breakdowns.

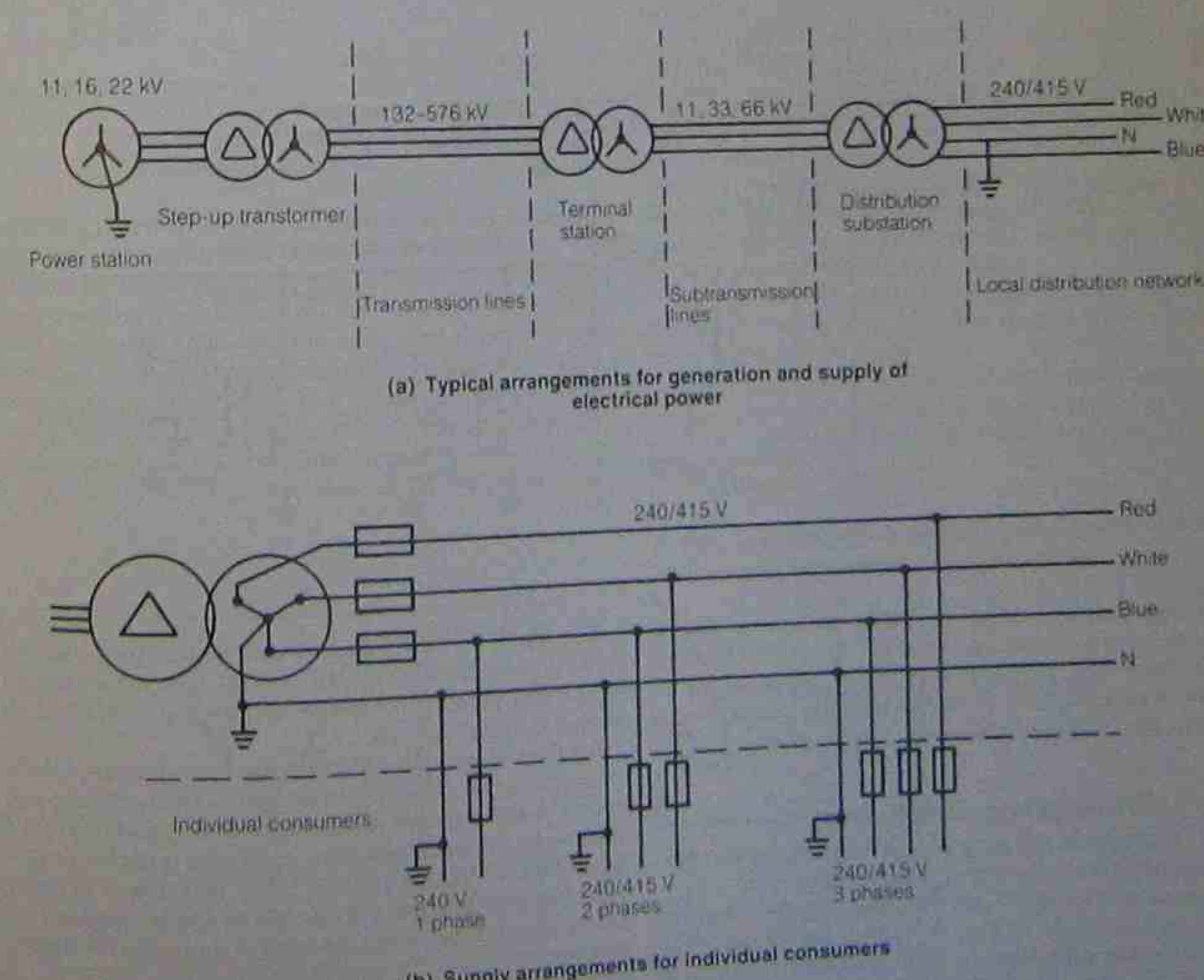
Figure 9.34 shows a typical suburban substation for local voltage transformation and power distribution.

9.7.1 SWER distribution

One method of providing electrical power in rural areas is the single-wire earth-return (SWER) system, as shown in Figure 9.35. Rural areas are generally a special case, where load units are comparatively small and their points of application widely dispersed. The diagram shows an isolating transformer connected to the sub-transmission lines. The secondary voltage of the transformer is connected to a single conductor that traverses the countryside. The voltage of this line can vary between states or between localities. If the isolating transformer is omitted, and the SWER line fed directly from one conductor of a three-phase 33 kV line, the phase voltage to earth is 19 kV. In some localities the other phases are led out in different directions, so providing a balancing effect on the line.

Because of the high voltage, both the line currents and the earth return currents are small. In areas where large amounts of metal are buried in the ground, special bonding arrangements might have to be undertaken to limit the amount of electrolytic corrosion taking place. These ground currents effectively prevent the system being used in larger population centres.

Transformers are provided at the point of distribution to



(a) Typical arrangements for generation and supply of electrical power

(b) Supply arrangements for individual consumers

Figure 9.33 • Distribution details, generator to consumer

Figure 9.34 • Small suburban transformer yard

Courtesy of EnergyAustralia

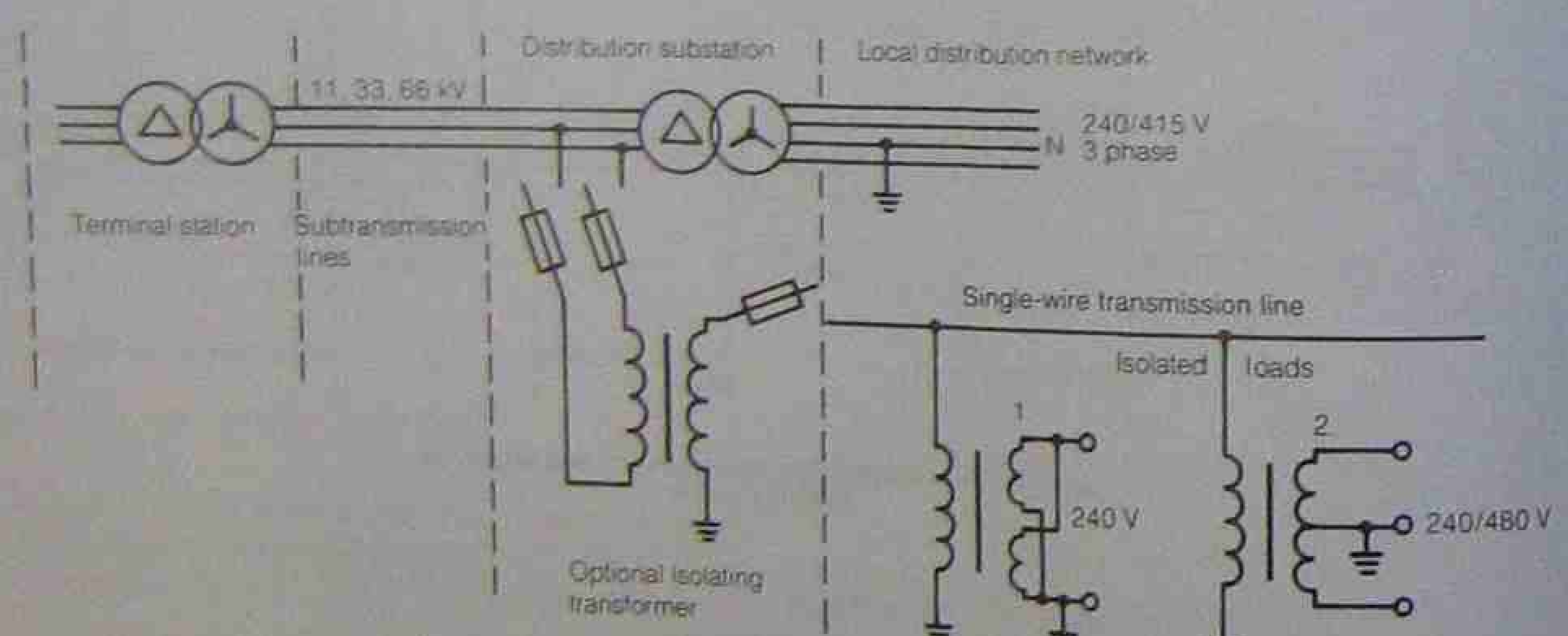


Figure 9.35 • Single-wire earth-return (SWER) system

reduce the supply voltage to either 240 V or 480 V. Effectively, there are two secondaries on an SWER transformer; they can be connected either in series or in parallel, depending on the use to which the transformer is to be put (see also Fig. 9.36).

9.7.2 Three-phase distribution

For general purposes, three-phase power may be supplied by using either a three-wire or a four-wire system.

The three-wire system is one that uses only the three line conductors, as shown in Figure 9.37(a). The phase windings are shown connected in delta, but they can also be connected in star, with the star point usually earthed.

The four-wire distribution system is one that used the three line conductors plus the neutral, which is connected to the star point of the phase windings and earthed, as shown in Figure 9.37(b).

For the supply authority itself there is no great problem in supplying power with star or delta systems. For the consumer, the star system is safer and more versatile because it provides an earth reference point and a choice of voltages. Balancing (see section 9.7.3) becomes a problem with the introduction of single-phase loads. Single-phase loads, however, are more convenient and safer for smaller consumers.

A major problem in power distribution is voltage drop resulting in those consumers furthest from the supply

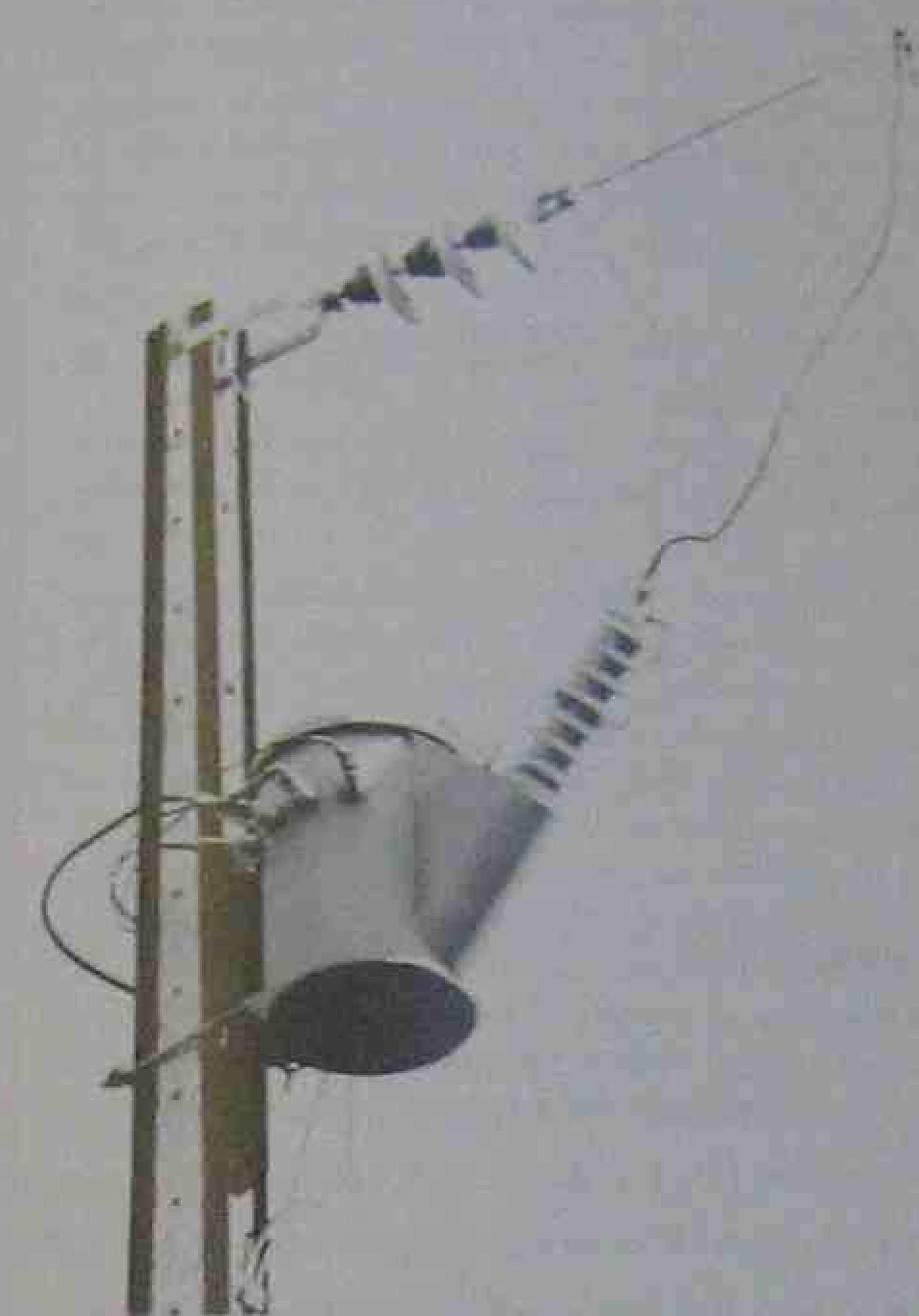


Figure 9.36 • 'End' transformer on a SWER transmission line. The link joining the two secondaries can be seen, enabling a 240/480 V system to be used, as illustrated in Figure 9.35

receiving power at a reduced voltage. Various techniques are used in attempts to overcome this difficulty. One method for small groups of consumers is to supply them from a centrally located transformer fed by a high voltage primary feeder. Radiating from the central point are the secondary or supply mains consisting of three lines and a neutral. Characteristically, the mains become smaller in size as the distance from the transformer increases. This method is typical for very small country towns.

For higher density distribution, as in the suburbs of a larger town, it is more common to have each block of consumers supplied from a transformer and have the mains from various blocks interconnected so that it is possible for any one consumer to be supplied from a number of sources. This method has the disadvantages of needing more protection devices for the transformers and it becomes more difficult to isolate sections of mains for maintenance purposes.

The ring main method is useful where there are large isolated groups of consumers. They are encircled by a main, which can be fed from a number of sources. A highly reliable source of supply is provided but many protective devices are required. This method must not be confused with the European ring main method where the same supply source feeds both ends of the one main. It is then necessary to fuse each individual appliance at its appropriate rating because the protection for the ring main has a rating suited to the main and not the appliance.

9.7.3 Balanced loads

The loading on a three-phase system is said to be balanced when the three line currents have the same magnitude and power factor.

Under these conditions the line currents are 120° out of phase with one another. Figure 9.38 shows the waveform diagram for a balanced load with three phase currents I_A, I_B and I_C.

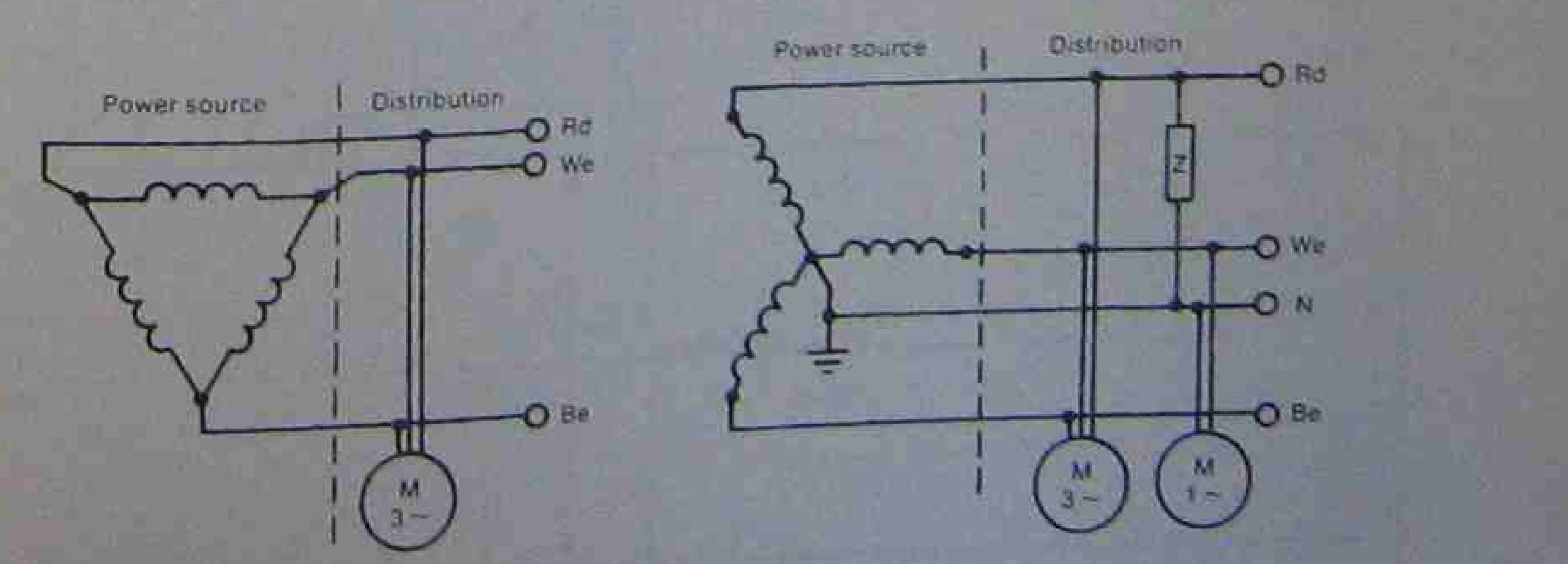
At the point K, the current I_A is a maximum at +10 A, and the currents I_B and I_C are both −5 A. At point L, the current I_A is zero, and I_B is +8.66 A and I_C is −8.66 A.

Although the three currents are all changing in value and direction, the phasor sum of the instantaneous currents is zero ($\bar{i}_A + \bar{i}_B + \bar{i}_C = 0$). Therefore the current flowing through the neutral in a balanced circuit is also zero.

The same result can be obtained from the phasor sum of the line currents using r.m.s. values.

For a balanced load (e.g. a three-phase motor) the neutral current is zero, thus the neutral wire is unnecessary and is usually omitted.

Where a distribution system is subject to load changes on one phase, so putting the system out of balance, a neutral conductor becomes necessary and must be installed.



(a) Three wire

(b) Four wire

Figure 9.37 • Three-phase distribution systems

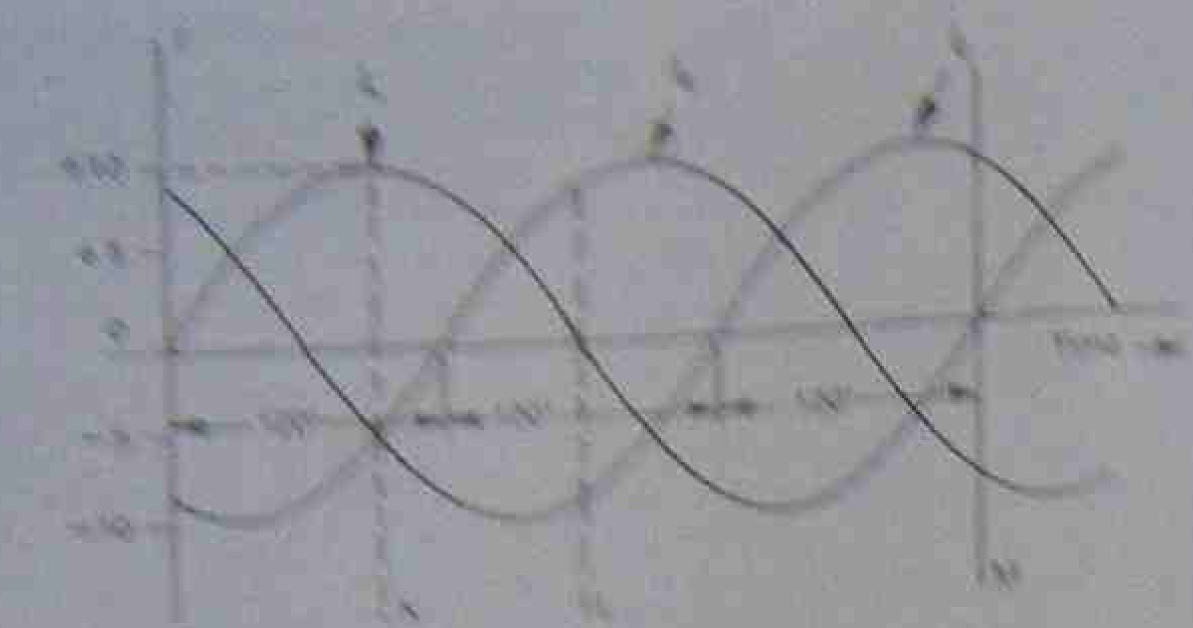


Figure 9.38 • Phase currents for a balanced three-phase load

The two requirements for a balanced load are:

1. same current loading on each phase
2. each current must have the same power factor.

When both requirements are met, the neutral current is zero.

9.7.4 Unbalanced loads

When the two conditions for balance are not met, a system is said to be unbalanced and a current may flow in the neutral (Fig. 9.39).

Because the generated voltage is assumed to be positive when acting from the star point to the line terminal, the phase current is also regarded as positive when flowing in the same direction.

In Figure 9.39, i_R, i_W and i_B are the instantaneous values of the currents in the three lines and i_N is the instantaneous value of current in the neutral.

The value of i_N is equal to the phasor sum of i_R, i_W and i_B, but the direction of current flow in the neutral wire is reversed.

Current in a neutral conductor
The value of the neutral current of any three-phase, four-wire system is equal to minus the phasor sum of the line currents.

$$\boxed{I_N = -(\bar{I}_R + \bar{I}_W + \bar{I}_B)}$$

Example 9.2

Find the value of current flowing in the neutral conductor for the following load on a three-phase, star-connected distribution system.

Red phase 125 A at power factor (λ) = 0.79 lagging

White phase 147 A at power factor (λ) = 0.85 lagging

Blue phase 215 A at power factor (λ) = 0.80 lagging

The procedure for finding the neutral current value is probably best illustrated by a series of numbered steps (the method used is the addition of phasors by parallelogram):

1. Draw the three voltage phasors with the red phase on the reference axis (Fig. 9.40).
2. Convert the individual power factors to angles; that is, 0.79 lagging corresponds to 38° of lag
   0.85 lagging corresponds to 32° of lag
   0.80 lagging corresponds to 37° of lag.
3. Draw current phasors at angles to each phase voltage and of lengths to represent the current value to some appropriate scale (I_R, I_W, I_B in Fig. 9.40).
4. The next step is to add these three phasors. They can be added in any order and, although the shapes and sizes of the constructions may be different, the answer will still be the same. Here the order of addition is I_R, I_B, I_W.
5. To add I_R and I_B complete the parallelogram; phasor $I_{(R+B)}$ is the sum.
6. With the phasors $I_{(R+B)}$ and I_W, complete another parallelogram. The phasor marked $I_{(R+W+B)}$ is the phasor sum.
7. The value when measured against the scale is 68 A and is equal to the value of current flowing in the neutral. However, to satisfy the requirement of 'minus' the phasor sum, the neutral current I_N must be drawn 180° away, or directly opposite the phasor sum. When required, the angle between it and one of the active lines can be used as a reference. In this particular case the neutral current is 68 A and it lags the red line voltage by 80°.
8. The polygon method of addition will be seen from an inspection of this phasor diagram. A phasor equal in length and parallel to I_B has been added on to the end of I_R. At this junction, a phasor equal in length and parallel to I_W has also been added. The distance back to the origin of the diagram when measured against the scale gives the same value of neutral current as before with the parallelogram method.

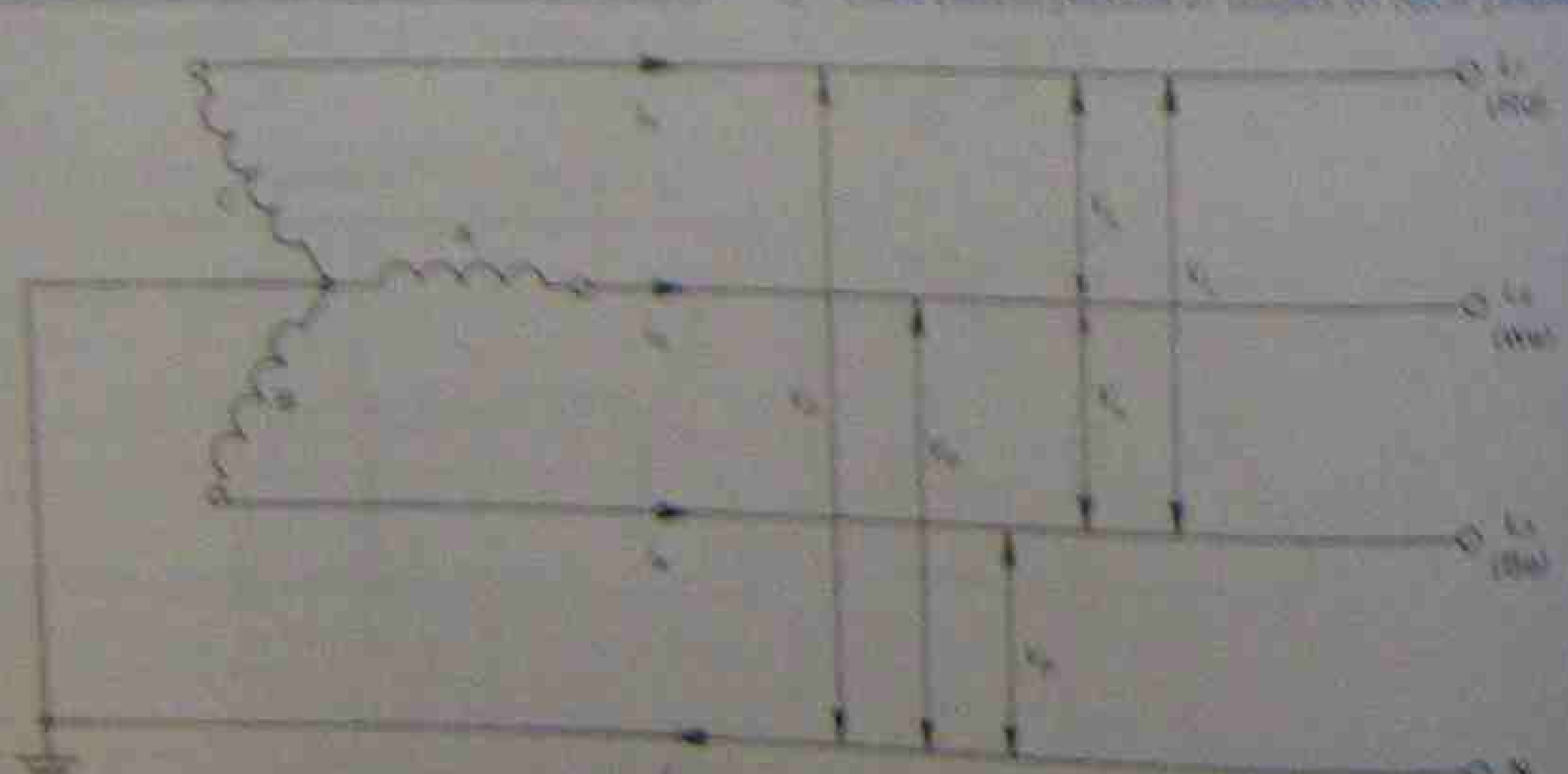


Figure 9.39 • Voltages and currents in a four-wire star connection

Example 9.3

What is the value of the neutral current in a three-phase circuit in which I_R = 32 A at unity power factor, I_W = 20 A at a power factor of 0.8 lagging, and I_B = 20 A at unity power factor?

1. The three voltage reference phasors are drawn and from them the line current phasors are constructed to a convenient scale (e.g. 5 mm represents 1 A). (The 0.8 lagging represents an angle of 37° lag.)
2. Any two current phasors (e.g. I_R and I_B) are added and the resultant of these is added to the third current phasor. (See Fig. 9.41.)
3. The neutral current is drawn equal to, but directly opposite, the resultant of the three current phasors. To ensure approximate balance of a system, single-phase loads are connected to phases in such a way that the currents in each phase are approximately

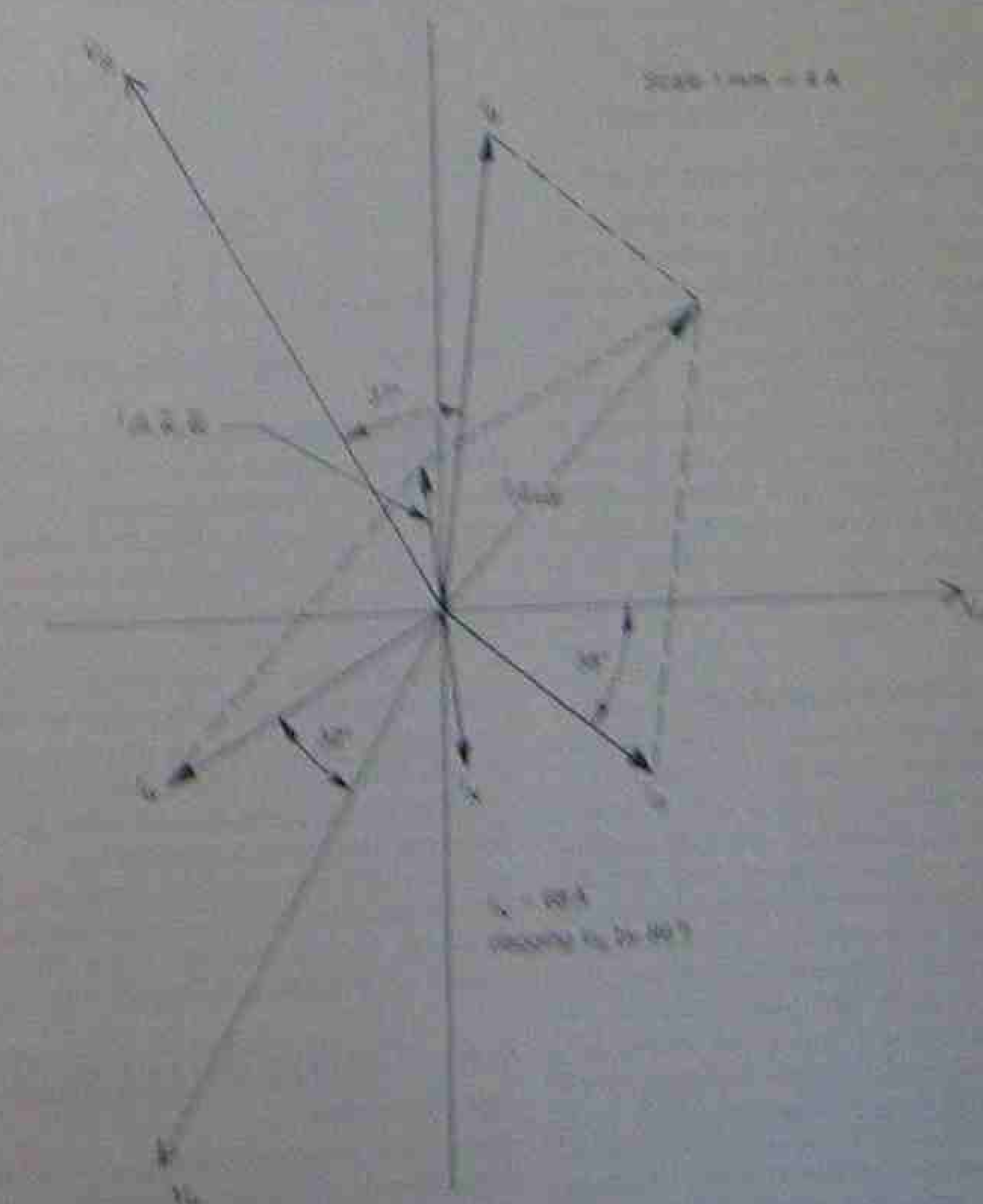


Figure 9.40 • Phasor diagram for example 9.2

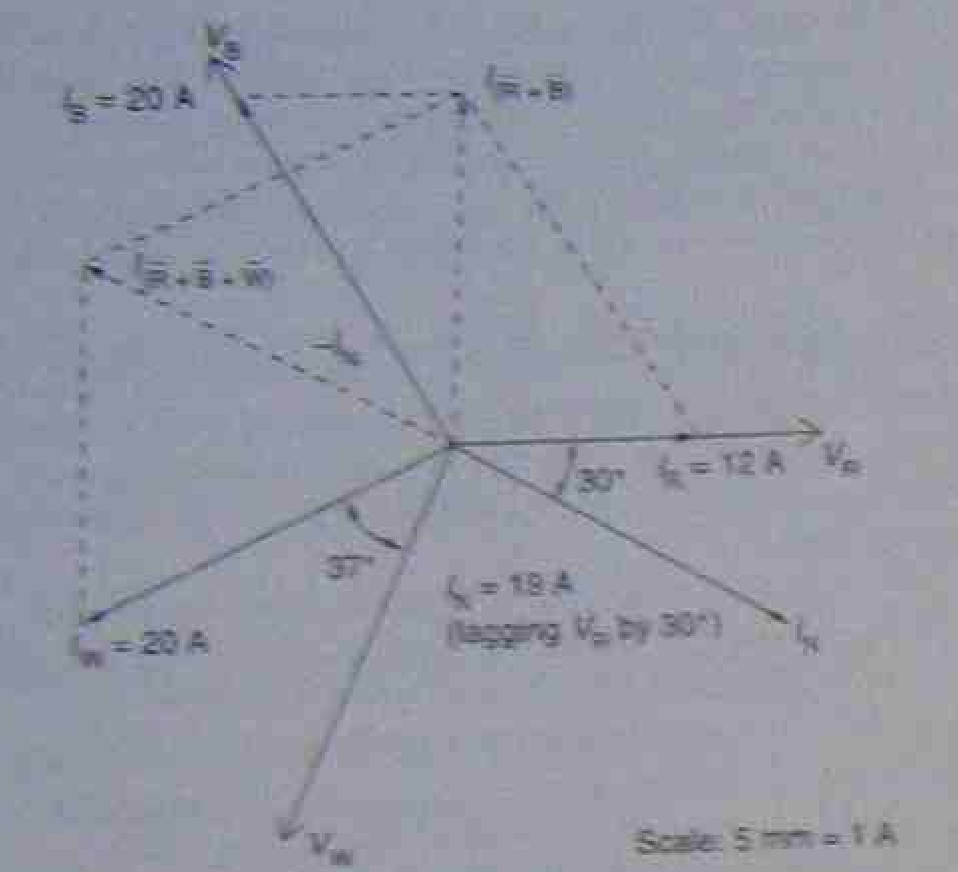


Figure 9.41 • Phasor diagram for example 9.3

equal. In general, with a domestic-type load, the power factor problem is of no great consequence because most households consume power at approximately the same power factor. Somewhere in the region of one-third of the total number of houses are connected to each phase. A check on the value of neutral current flowing in the main neutral conductor indicates whether a degree of balance has been achieved in the loading.

Standards Australia, in its publication on wiring rules, recognises that the neutral current can be reduced by balancing single-phase loads on a three-phase distribution system. The rules permit a reduction in the size of the neutral conductor in certain circumstances.

In the section of AS/NZS 3000 on size and type of conductors, it is specified that the neutral conductor shall be of the same size and material as the active conductors up to a current rating of 100 A. Below this figure the neutral must be the same size and material as the active conductors. Above this figure the neutral current rating can be reduced to one-half that of the active conductors, provided it is never less than a 100 A rating.

9.7.5 Effects of a broken neutral

With single-phase loads on three-phase, four-wire systems it is essential that the neutral not be open-circuited.

One major reason for this precaution is personal safety. With equipment operating on 240 V, an open-circuited neutral can effectively prevent equipment from working. The appliance, while it appears to be without power, still has the active conductor connected. It is possible for a person to contact the active conductor and to complete the circuit via the body to earth and be electrocuted.

Another reason for the neutral not to be broken is to protect the loads applied to the supply. Consider a 240/480 V single-phase, three-wire rural supply with loads connected as shown in Figure 9.42. If a break occurs in the neutral at the point indicated, then 480 V is applied across the loads of R_3 and R_4 in series. If R_1 and R_3 are both 100 Ω

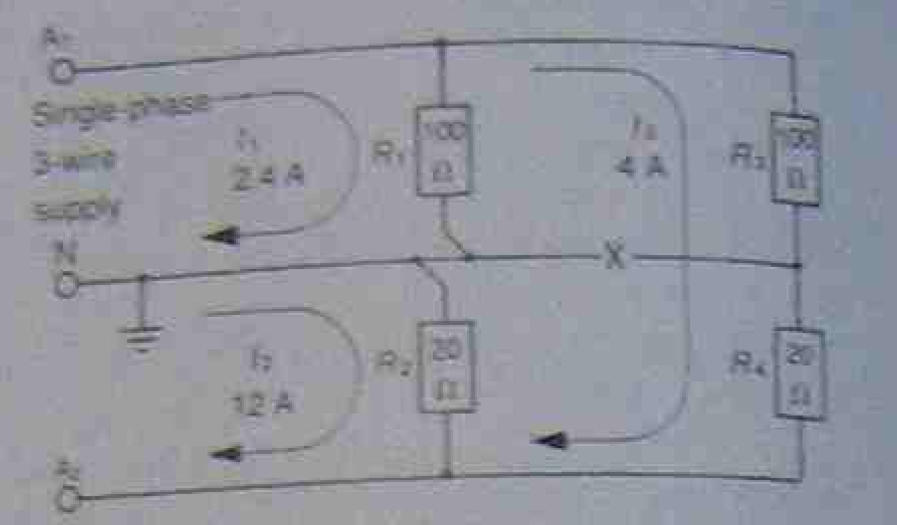


Figure 9.42 • Effects of a broken neutral

and R_2 and R_4 are both 20 Ω, then I_1 = 2.4 A, I_2 = 12 A and I_3 = 4 A.

Applying Ohm's law, the voltage drop across R_3 is 400 V and across R_4 is 80 V. R_3 has a higher current and voltage than normal, whereas R_4 has lower values. Thus R_3 is overloaded and would burn out, while R_4 is not able to function correctly. Figure 9.43 shows the variation in voltages that can occur if there is a break in the neutral conductor.

A dangerous voltage (160 V in this case) can develop across the break caused by the change in potential at the junction of R_3 and R_4.

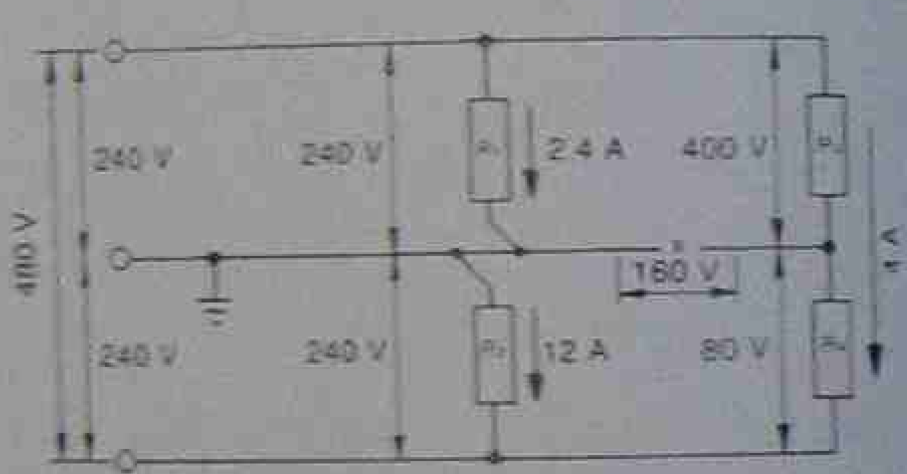


Figure 9.43 • Change in voltages with a broken neutral

Where the multiple-earthed neutral (MEN) system is used, a break in the neutral can also create a hazardous situation. The potential of earthed appliances in some circumstances can rise to levels approaching that of the mains voltage.

With three-phase unbalanced loads, a broken neutral causes several effects. The voltage across the lightest load increases, that across the heaviest load decreases, while the voltage across the third phase might shift either up or down. At the same time the power factor of the individual line currents might also be affected. Any load before the neutral break is not affected but single-phase loads and unbalanced three-phase loads after the break are affected. These are illustrated in Figure 9.44.

To prevent the neutral from being open-circuited, neutral switches and links in substations are often locked or bolted, and neutrals are never fitted with fuses under normal installation conditions.

9.8 THREE-PHASE POWER

Star connection

The amount of power consumed by a load connected to a single-phase a.c. supply is found by using the formula:

$$P = VI\lambda$$

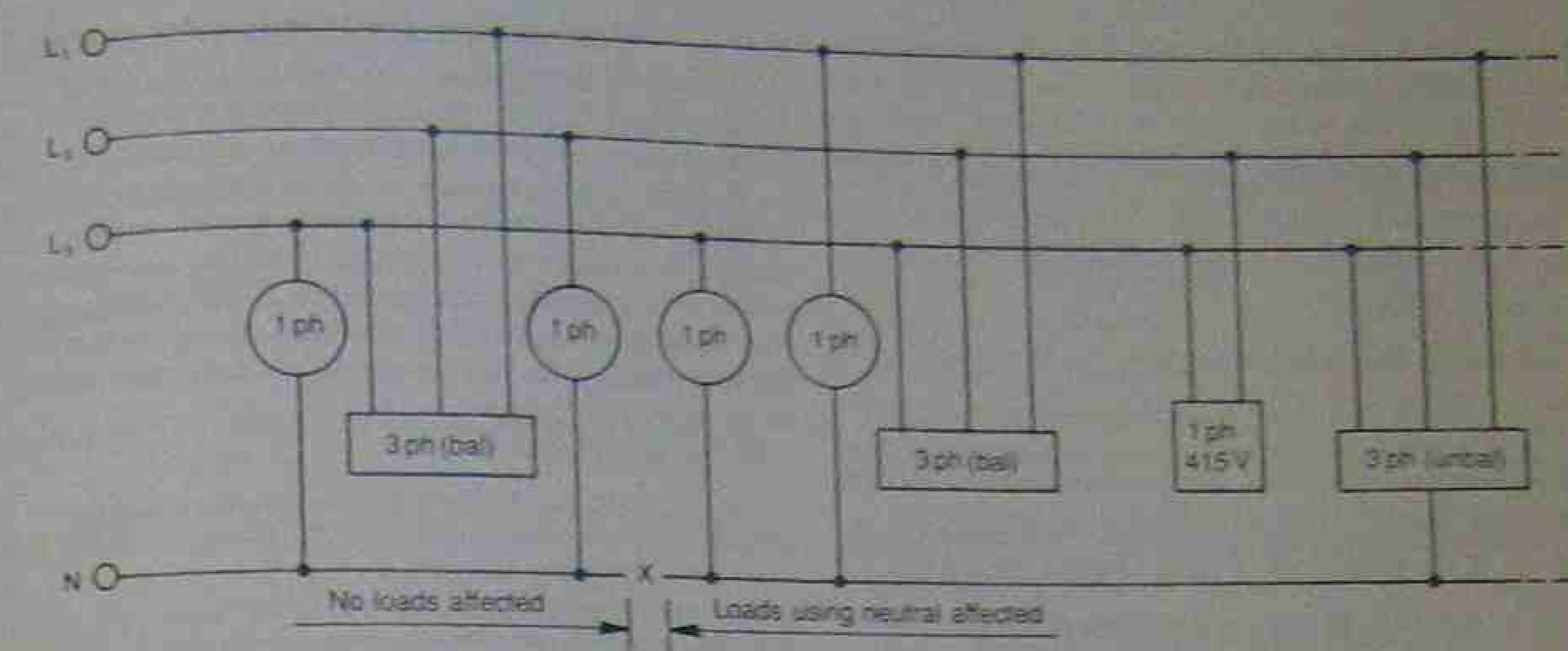


Figure 9.44 • Loads affected by a broken neutral

If this same load is one of three loads connected in star to a three-phase supply, as shown in Figure 9.45, the power consumed by load K is:

$$P_K = V_p I_p \cos \phi_K$$

If the three loads are balanced then $P_K = P_L = P_M$ and the total power drawn from the three-phase supply is:

$$\begin{aligned} P_K + P_L + P_M &= 3P_K \\ &= 3V_p I_p \cos \phi \\ &= 3V_p I_p \lambda \end{aligned}$$

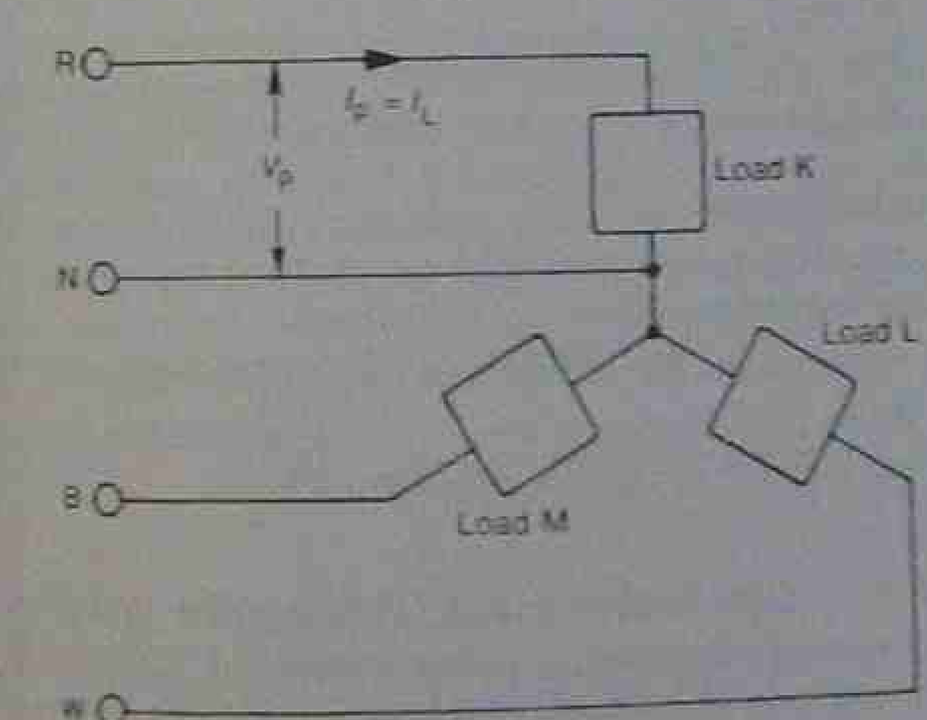


Figure 9.45 • Star-connected loads on a three-phase supply

In general, it is simpler to obtain line values of V and I rather than phase values, and use these instead. In a star-connected system, $I_L = I_p$ and $VL = \sqrt{3}.V_p$. Substituting line values for phase values:

$$P = 3\frac{V_L}{\sqrt{3}} I_L \lambda = \sqrt{3} V_L I_L \lambda$$

where P = total three-phase power $(P_K + P_L + P_M)$

This is normally expressed as:

$$\boxed{P = \sqrt{3} V I \lambda}$$

where V and I are assumed to be line values

Delta connection

If the three individual loads are connected in delta, as shown in Figure 9.46, the power consumed by load K is still:

$$P = V_p I_p \cos \phi_K$$

For a balanced condition, the total power is still:

$$3P = 3V_p I_p \lambda$$

In a delta system, $V_L = V_p$ and $I_L = \sqrt{3} I_p$.

Substituting these:

$$\begin{aligned} \text{total power } P &= 3V_L \frac{I_L}{\sqrt{3}} \cos \phi \\ &= \sqrt{3} V_L I_L \lambda \end{aligned}$$

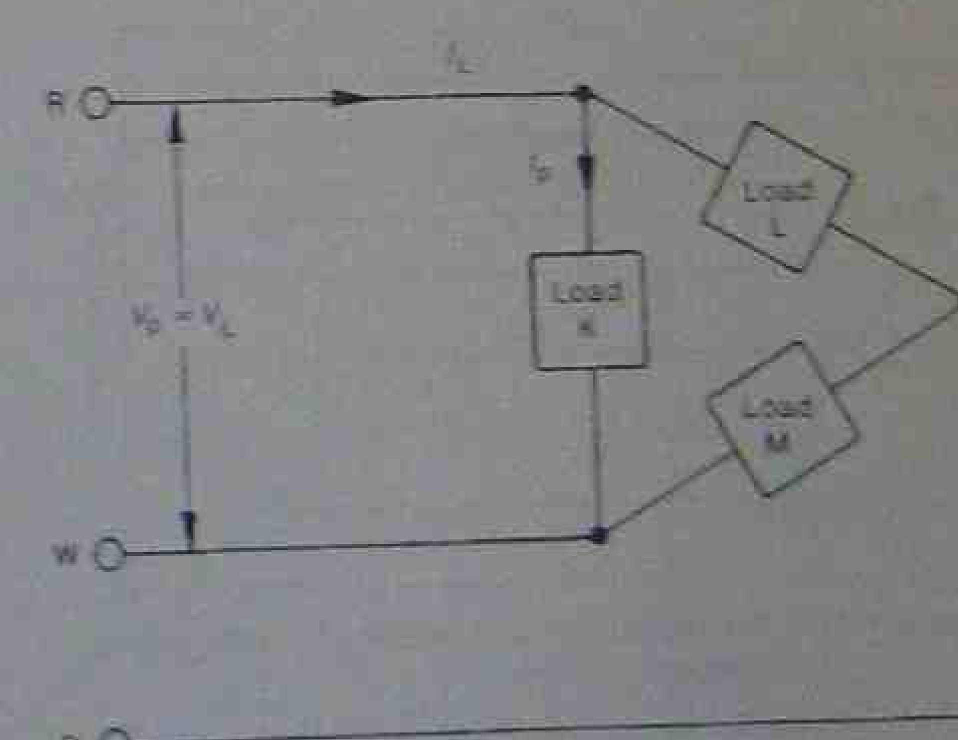


Figure 9.46 • Delta-connected loads on a three-phase supply

In general terms:

$$\boxed{P = \sqrt{3} V I \lambda}$$

where V and I are assumed to be line values

Note that this formula is as it was for a balanced star-connected load, so it must be applicable for both star- and delta-connected balanced three-phase loads. It is not to be used for unbalanced loads.

Example 9.4

A three-phase 415 V motor draws 10 A at 0.8 power factor. How much power does it consume?

$$P = \sqrt{3}VI\lambda$$
$$V = 415\text{ V}, I = 10\text{ A}, \lambda = 0.8$$
$$\therefore P = \sqrt{3} \times 415 \times 10 \times 0.8$$
$$= 5.75\text{ kW}$$

9.9 METHODS OF THREE-PHASE POWER MEASUREMENT

9.9.1 One wattmeter (four-wire system)

The wattmeter is connected between one line and neutral. (See Fig. 9.47.) The total power drawn from a three-phase supply is found by adding the separate values of power consumed by each phase. In the case of a balanced load (an equal load and power factor on each phase):

$$P_{total} = 3P_A = 3P_B = 3P_C = 3 \text{ times the wattmeter reading}$$

For an unbalanced load, the wattmeter has to be connected or switched into each phase in turn and the individual power readings added.

$$\boxed{P_{total} = P_A + P_B + P_C}$$

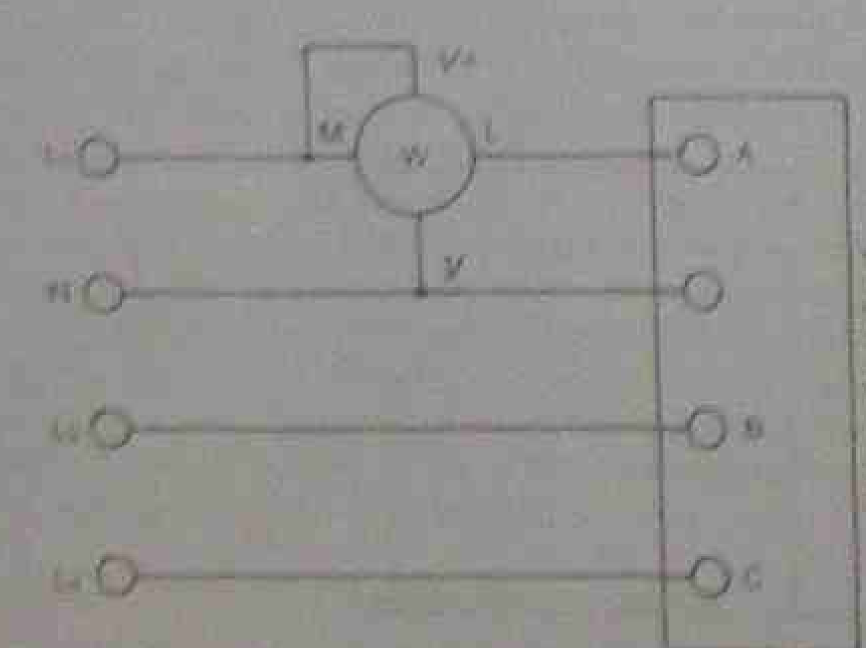


Figure 9.47 • Connections for one wattmeter in a three-phase, four-wire system

Advantages

1. One wattmeter only is required.
2. Suitable for balanced and unbalanced loads.

Disadvantages

1. Neutral connection required for the wattmeter.
2. Not accurate for unbalanced fluctuating loads.
3. The wattmeter has to be connected or switched into each phase in turn for unbalanced loads.

9.9.2 One wattmeter (three-wire system)

With a three-wire system, no neutral is available and because there is a 30° phase shift between line and phase voltages, it is necessary to provide an artificial star point so that the correct voltage at the correct phase angle is applied to the wattmeter. Two impedances, each matching the impedance of the voltage circuit in the wattmeter, must be connected in star with the meter voltage circuit (Fig. 9.48) and to the other two lines. Resistors matching the resistance of the voltage circuit are very close approximations for this purpose.

For balanced loads only:

$$P_{total} = 3P_A = 3 \text{ times the wattmeter reading}$$

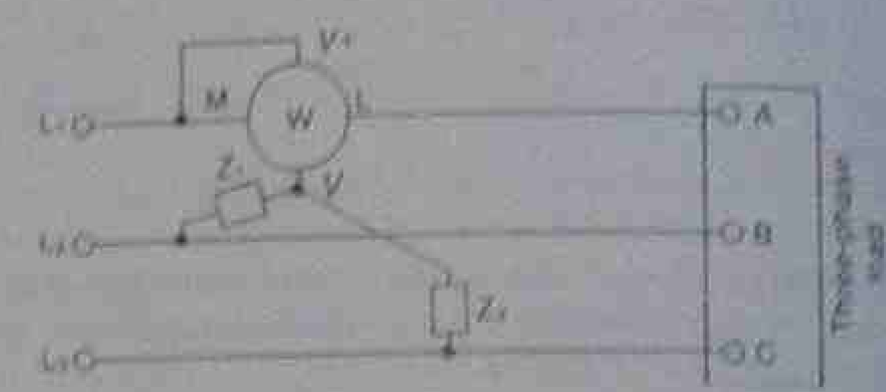


Figure 9.48 • Connections for one wattmeter in a three-phase, three-wire system

For balanced and unbalanced loads:

$$\boxed{P_{total} = P_A + P_B + P_C}$$

Advantages

1. One wattmeter only is required.
2. Suitable for balanced and unbalanced loads.

Disadvantages

1. Two matching impedances are required to provide an artificial neutral.
2. Not accurate for unbalanced fluctuating loads.
3. The wattmeter has to be connected or switched into each phase in turn for unbalanced loads.

9.9.3 Two wattmeters (three-wire system)

A method for measuring power consumed in a three-phase, three-wire circuit is shown in Figure 9.49. The two meters have their current windings in any two lines and both voltage windings are connected to the third line. Neither meter alone indicates the total power in the circuit, but the two meters together, by their algebraic sum, indicate the power consumed:

that is, $\boxed{P_{total} = W_1 + W_2}$

For a balanced load with unity power factor, both meter readings will be equal. For all other conditions the meters will show different readings.

If the lower-value meter indication is W_1 and the higher one W_2, then, as the power factor decreases, W_1 registers less and less of the total power. When the power factor is 0.5 with a balanced load, W_1 will read zero and W_2 will

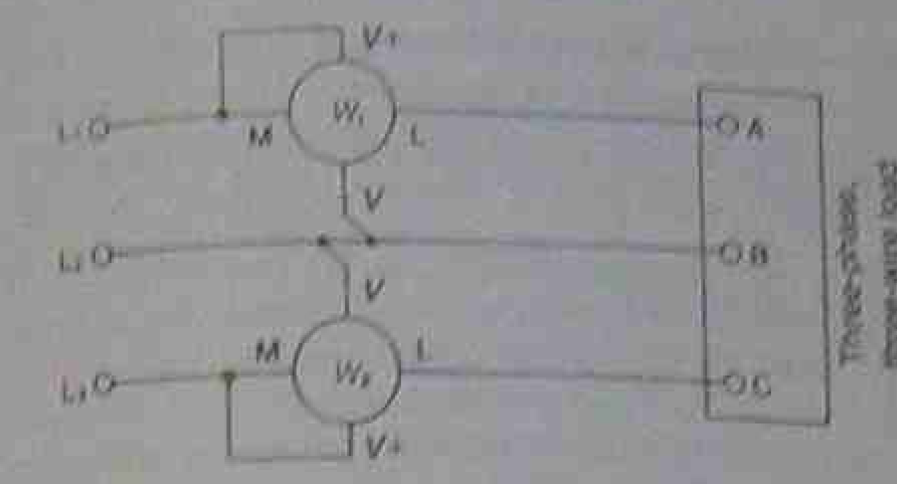


Figure 9.49 • Connections for two-wattmeter measurement of power

read the total power. Should the power factor fall further (to, say, 0.3), W_1 will read even less (i.e. W_1 will attempt to read a negative value of power consumption). If the current or voltage connections to the meter are reversed, the numerical value of W_1 can be obtained, but it must be noted that it is a negative value. The total power in this case is still the algebraic sum of W_1 and W_2. That is:

$$P_{total} = (-W_1 + W_2)$$
$$= W_2 - W_1$$

Should the power be reduced to zero with a balanced load (pure capacitance or inductance), W_1 would read a negative value equal numerically to W_2 and the algebraic sum would be zero:

$$P_{total} = -W_1 + W_2 = 0$$

This also agrees with the concept of a purely reactive circuit.

The two-wattmeter method may be used on three-phase, three-wire systems to obtain load power values whether the load is balanced or unbalanced, or star or delta connected, but it cannot be used on a four-wire, star-connected system because a single-phase component of current might be flowing in the line (and neutral) having no wattmeter current-coil connection, and the power being consumed would not be recorded.

Only when the three-phase load is balanced is it possible to find the power factor of the load from the wattmeter readings. The tangent of the angle of lag or lead is found from:

$$\boxed{\tan \phi = \sqrt{3}\left[\frac{W_2 - W_1}{W_2 + W_1}\right]}$$

(balanced loads and sinusoidal waveforms only)

The angle ϕ is obtained from $\tan^{-1} \phi$, and the cosine of this angle gives the power factor of the load.

Example 9.5

When connected to a three-phase motor, two wattmeters gave readings of 5 kW and −1 kW. Find:

(a) the total power being consumed

(b) the power factor of the motor.

(a) $P_{total} = W_1 + W_2 = -1 + 5$
$$= 4\text{ kW}$$

(b) $\tan \phi = \sqrt{3}\left[\frac{W_2 - W_1}{W_2 + W_1}\right]$
$$= \sqrt{3}\left[\frac{5 - (-1)}{5 + (-1)}\right]$$
$$= \sqrt{3}\left[\frac{5 + 1}{5 - 1}\right]$$
$$= \sqrt{3} \times \frac{6}{4} = \sqrt{3} \times 1.5$$
$$= 2.598$$
$$\therefore \phi = 68.9°$$
$$\lambda = \cos \phi = 0.3592 = 0.36$$

A second method for obtaining the power factor is by the use of the curve shown in Figure 9.50.

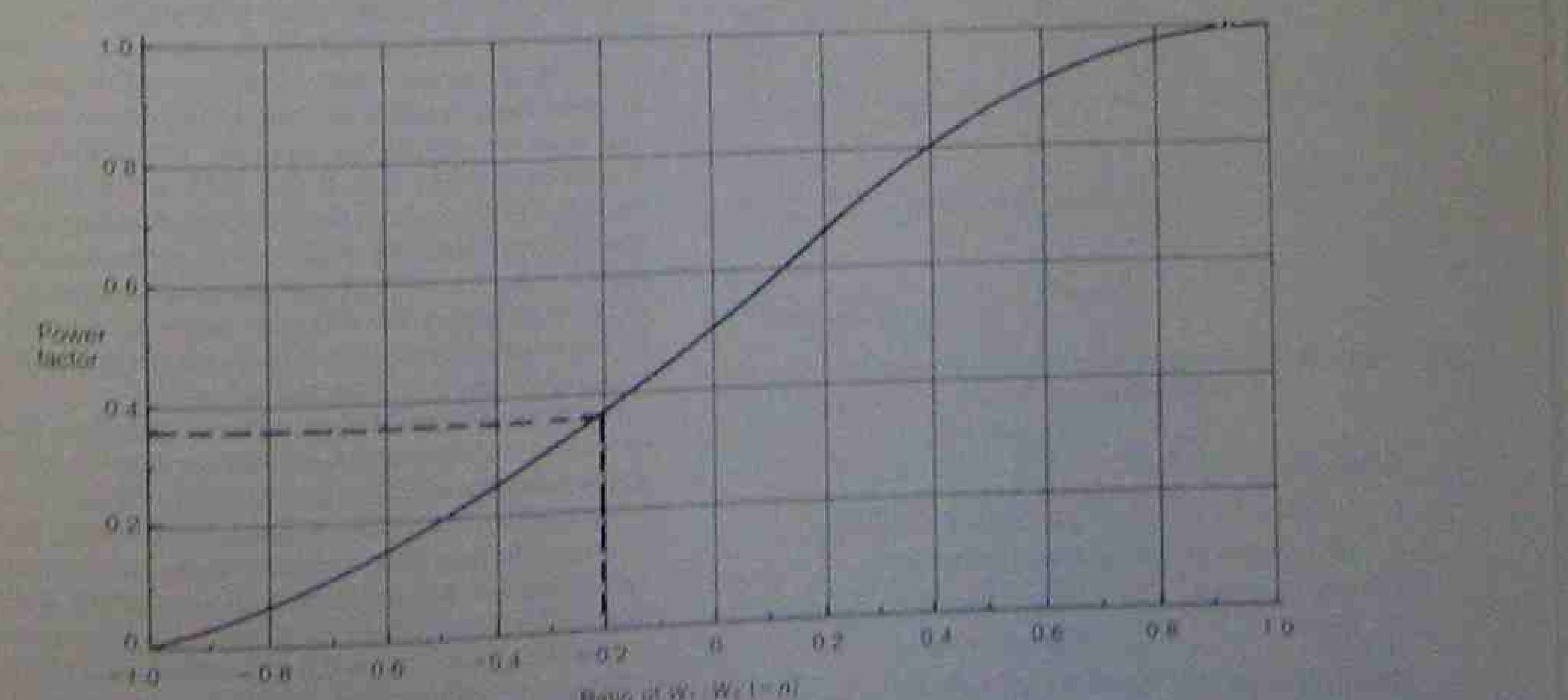


Figure 9.50 • Power factor curve for the two-wattmeter method of measuring power

A figure for n is obtained from:

$$n = \frac{W_1}{W_2}$$

In the example above, $W_1 = -1$ and $W_2 = 5$:

$$n = \frac{-1}{5} = -0.2$$

The dotted lines on Figure 9.50 show how n is transferred via the curve to a value of power factor. The power factor is 0.36, as in the example above.

Advantages

1. Only two wattmeters are required.
2. Useful for balanced and unbalanced three-phase, three-wire loads.
3. The power factor can be obtained for balanced loads.
4. No neutral connection is required.

Disadvantages

1. Suitable only for three-phase, three-wire loads.
2. Care must be used in determining the polarity of W_1.
3. The power factor cannot be obtained for unbalanced loads.
4. Not suitable for power or power factor readings with three-phase, four-wire systems.

9.9.4 Three wattmeters (three-wire system)

With a three-wire system, no neutral is available, so an artificial neutral must be provided. However, if identical wattmeters are used, the three voltage circuits can be connected to provide a star point, as shown in Figure 9.51:

$$\boxed{P_{total} = W_1 + W_2 + W_3}$$

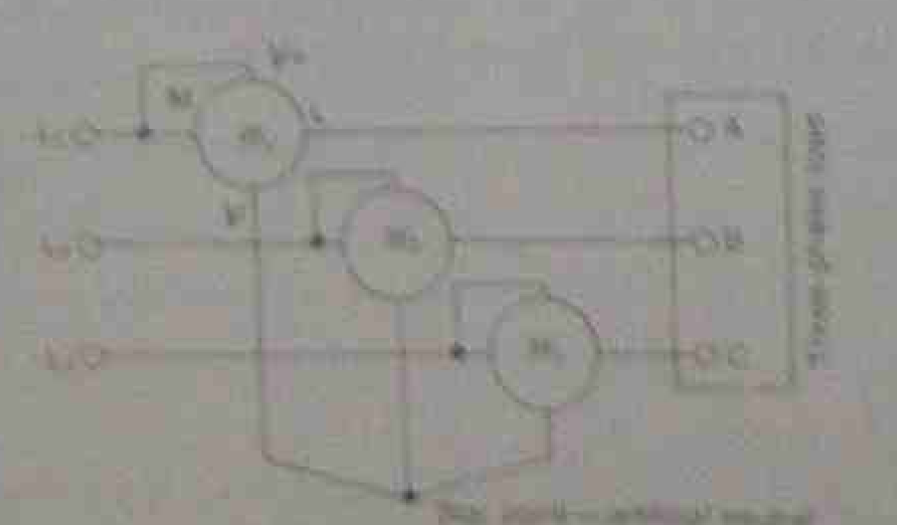


Figure 9.51 • Connections for three wattmeters to a three-phase, three-wire system

Advantages

1. Suitable for balanced and unbalanced loads.
2. Convenient for obtaining total power.
3. More accurate than one wattmeter for fluctuating loads.

Disadvantage

Three wattmeters are needed.

9.9.5 Three wattmeters (four-wire system)

The three-phase, four-wire system is basically three separate supplies with only a common neutral. The total power is obtained by connecting three wattmeters, as shown in Figure 9.52.

$$\boxed{P_{total} = W_1 + W_2 + W_3}$$

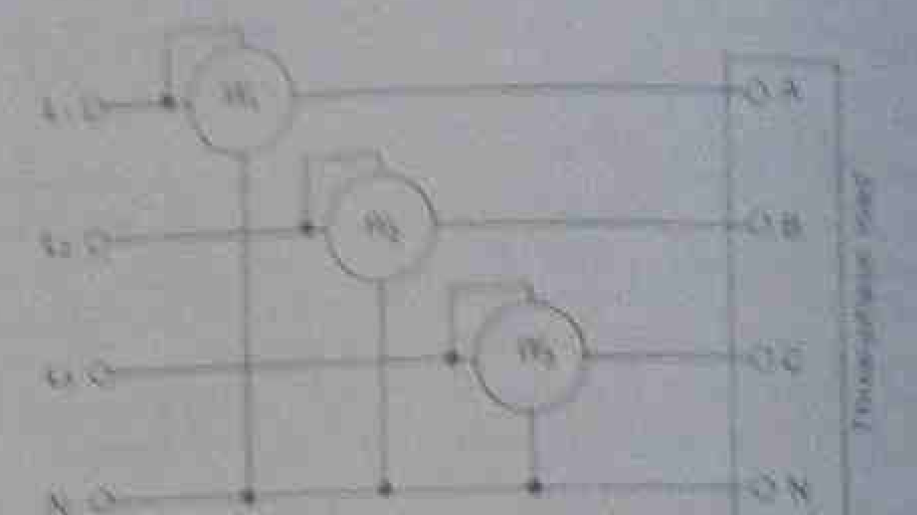


Figure 9.52 • Connections for three wattmeters to a three-phase, four-wire system

Advantages

1. Suitable for balanced and unbalanced loads.
2. Convenient for obtaining total power.
3. More accurate than one wattmeter for fluctuating loads.

Disadvantage

Three wattmeters are needed.

9.10 REACTIVE VOLT-AMPERE MEASUREMENT

In power stations operating under normal conditions the values of voltage and current are generally too high and unwieldy to be used directly with portable instruments. Because the operating system is usually a three-phase four-wire star-connected system, the following refers mainly to such a system. It should be noted, however, that for long-distance transmission of large quantities of electrical power, star/delta or delta/star connected transformers may also be used. This may make a third or tertiary winding necessary to eliminate harmonics in the transformer when a neutral is not used. For further information on these connections refer to Chapter 14.

The measurement of output power and var under these conditions means that the registering instruments have to be permanently installed in a suitable location such as a power station control room. Thus, of necessity, there may be a considerable distance between the point of measurement and the actual instrument registering the values. Under some circumstances, potential and current transformers can be used, but the modern trend is to the use of transducers—many with outputs of low-value direct current. Circuits are given in section 9.10.1 for both transformer and transducer measurement.

The circuit for power and var measurement using instrument transformers is an adaptation of those circuits already given in section 9.9 on three-phase power measurement and in the circuits for instrument transformers discussed in Chapter 14.

With dedicated measuring equipment, the registering meter is calibrated to take into account any multiplying factors. This enables a direct reading to be taken, avoids confusion and reduces possible errors.

Reactive power measurement is often required when a power station supplies a grid system. There might also be more than one generating station supplying power to the same grid. It can be desirable to have direct knowledge of the amount of reactive power that might be circulating between power stations and the grid. Of direct consequence is the amount of reactive power entering or leaving an individual power station. To keep track of this a read-out is provided in a power station's control room.

Under 'Conditions of Supply to Consumers', supply authorities try to ensure that the overall power factor shall be no worse than 0.8 lagging, to reduce the amount of var circulating in their supply mains. Somewhat ironically, at the same time, due to the capacitive effect between long-distance transmission lines, the power station also has trouble with leading power factors. The var load up the transmission lines with a circulating current and can cause unwanted rises in voltage at the end remote to the power station. The trouble can be exacerbated with underground transmission lines.

To measure reactive power, connections similar to that of three-phase power measurement are used. Usually a three-phase meter or transducer is used, rather than three individual meters. The connections and method are similar to power measurement, with the exception of the need to use a voltage at 90°E to the usual voltage used for power measurement. In this context the displaced voltage is sometimes referred to as a cross-phase voltage.

9.10.1 Cross-phase voltages

There are three general methods for obtaining the cross-phase voltage. The one chosen depends on local circumstances and the preferences of the station operators.

Method 1

The diagram in Figure 9.53 illustrates some of the phasors for a three-phase four-wire load. All irrelevant phasors have been omitted for purposes of clarity. By using the difference in potential between phase voltages B and C, a line voltage V_{B-C} is created. It is at 90°E to the reference phase voltage V_A. Phase current I_A is shown lagging $\phi°$ behind V_A and so it is at an angle of $(90 - \phi)°$ to V_{B-C}.

Note that $\cos(90 - \phi) = \sin\phi$.

In section 8.5.2 it was stated that the value of single phase var could be obtained by using the formula var = $VI \sin\phi$. For three-phase working, the formula is adapted in a similar fashion to that for obtaining the power consumption, and applies to both balanced and unbalanced systems.

That is, var $= \sqrt{3}\, VI \sin\phi$

where ϕ is the angle between voltage and current.

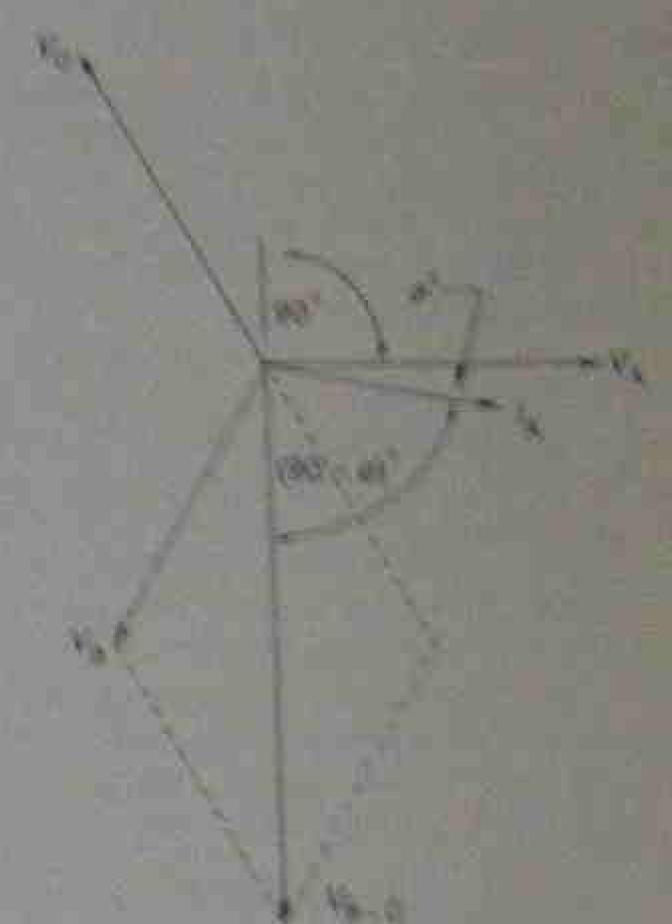


Figure 9.53 • Phasors showing production of a cross-phase voltage

Method 2

A cross-phase voltage can be produced with a specially built phasing transformer. It provides three voltages at 90°E to the individual phase voltages. It is a separate unit and still requires either a three-phase meter or transducer, together with potential and current transformers. See Figure 9.54.

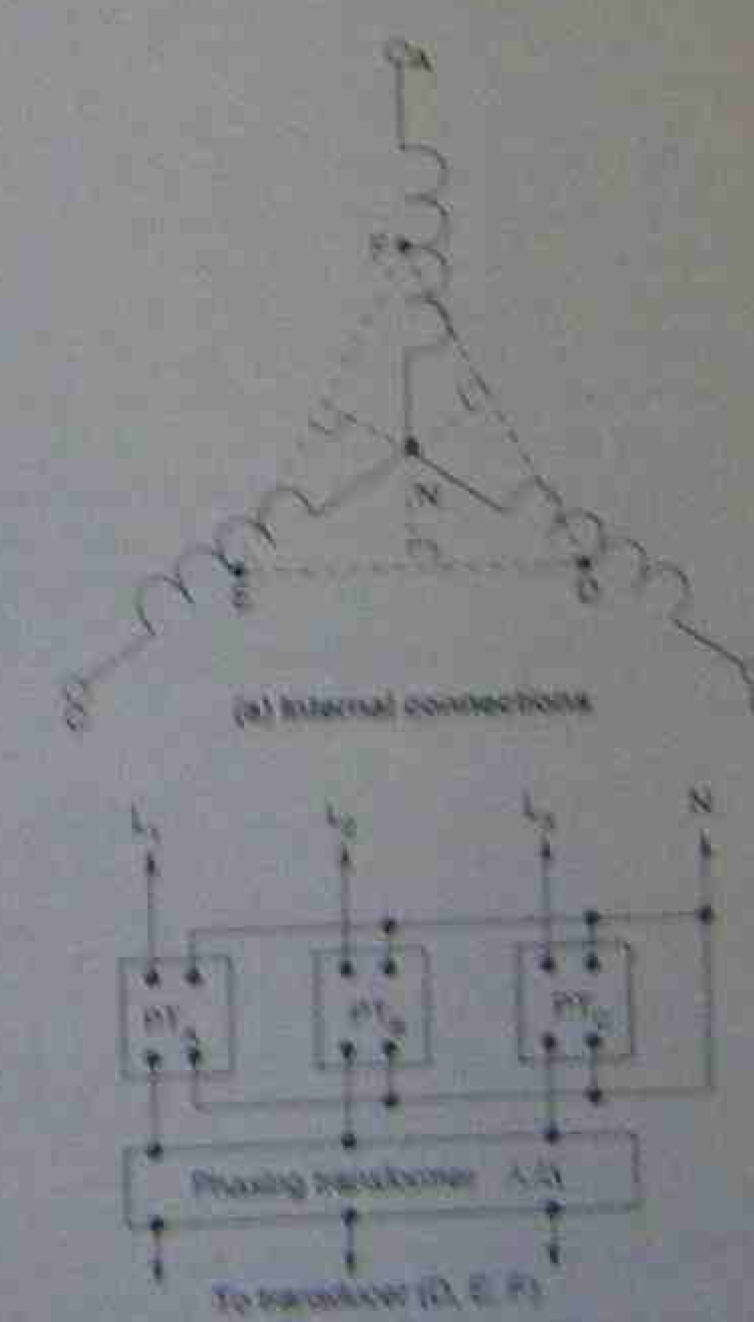
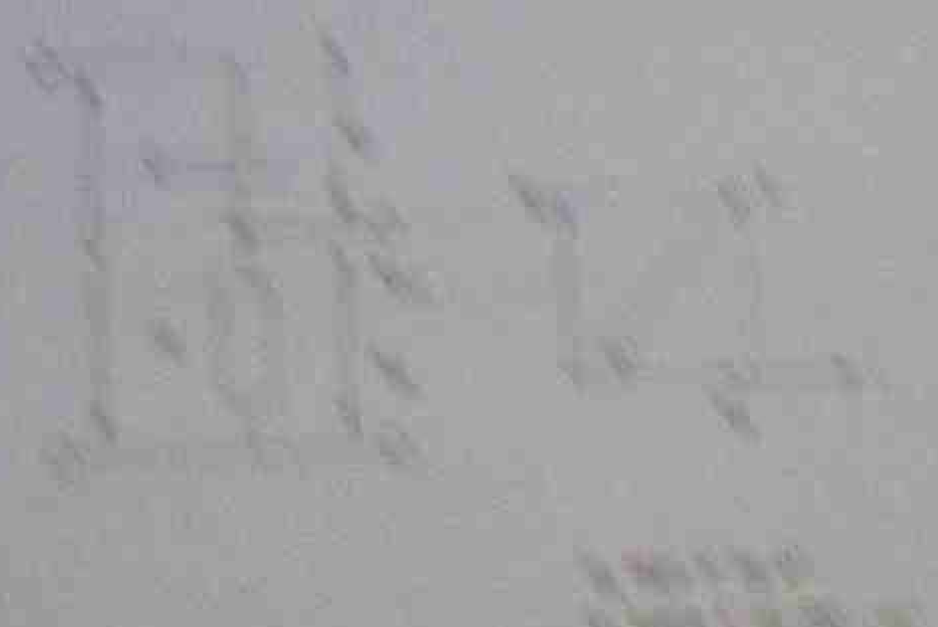
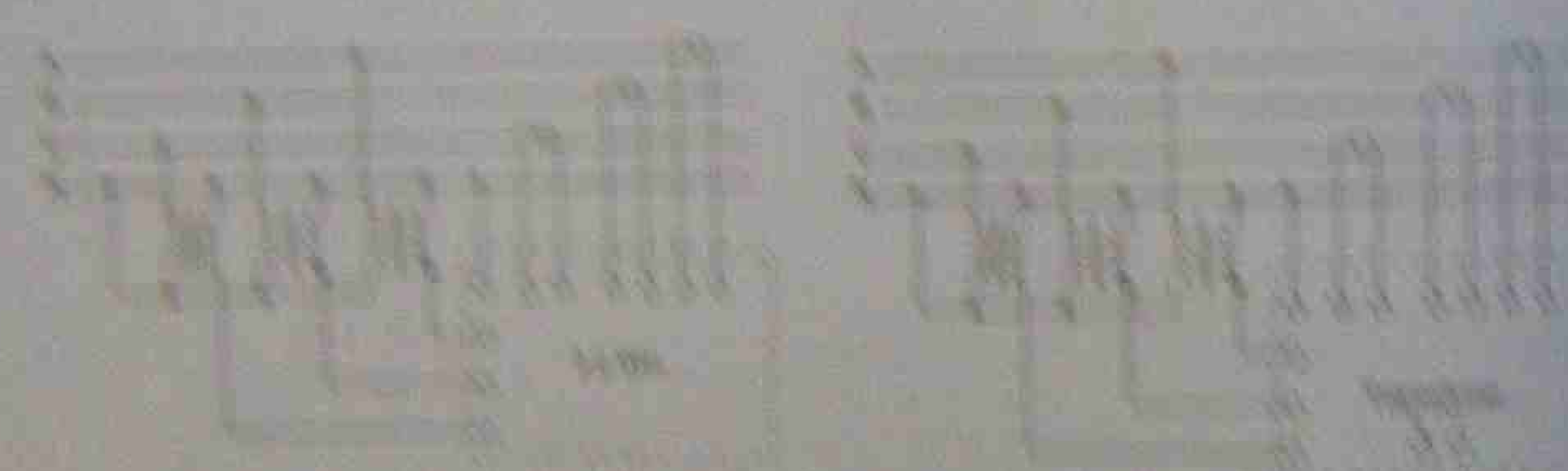
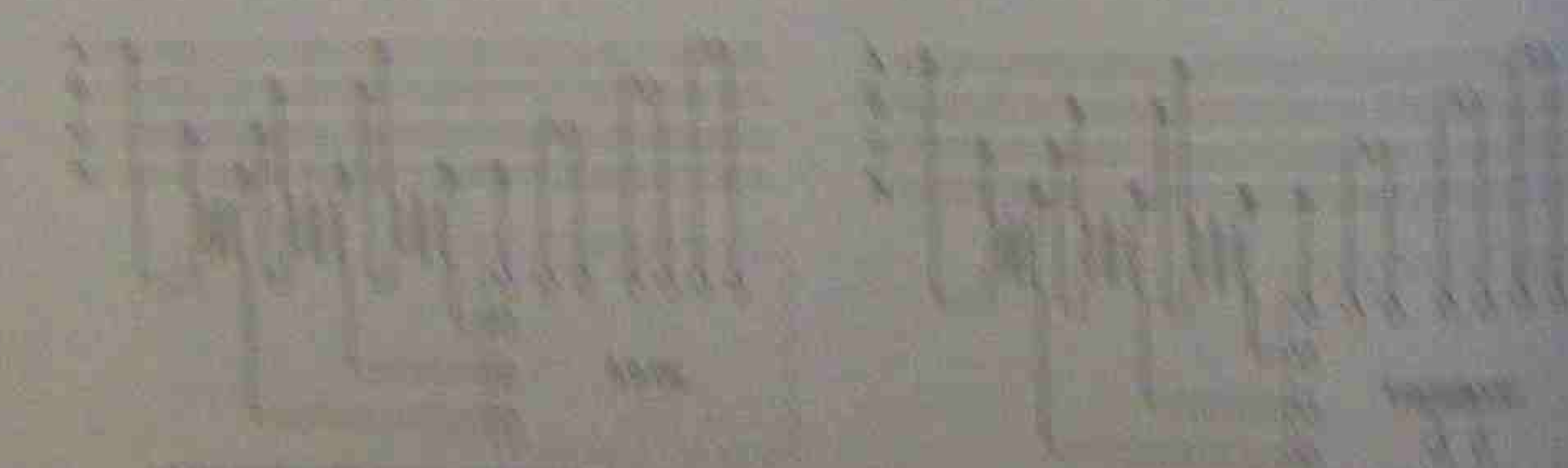


Figure 9.54 • Phasing transformer







SELF-TESTING PROBLEMS

9.10 A two-phase, three-wire supply has a voltage of 340 V between actives. Find the voltage from active to neutral.

9.11 A star-connected supply has a phase voltage of 200 V. Find the line voltage by phasors, and then check the answer using $V_L = \sqrt{3}.V_P$

9.12 A three-phase star system has a line voltage of 415 V. Find the phase voltage by calculation.

9.13 A three-phase, star-connected alternator supplies a delta-connected induction motor at a line voltage of 600 V. The current in each line is 40 A. Find:

(a) the phase voltage of the alternator

(b) the current in each phase of the motor.

9.14 A three-phase machine is connected in delta. If the line currents are 5 A each, find the phase current in each winding.

9.15 Three resistors, each having a resistance of 25 Ω, are connected to a 415 V, three-phase supply. Determine the voltage across each resistor and the current through each resistor when they are connected:

(a) in star configuration

(b) in delta configuration.

9.16 Three heating elements are connected in star across a three-phase, 415 V line. If 10 A flows in each line wire, determine:

(a) the resistance of each heating element

(b) the power drawn from the supply by the three elements.

9.17 An 11 kV three-phase alternator supplies a balanced load with 400 A in each line, the current lagging 38° behind the line voltage. Determine the power output.

9.18 Three resistors of 10, 20 and 30 Ω are connected to a 415 V, three-phase supply in:

(a) star configuration

(b) delta configuration.

Find the total power drawn from the supply in each case.

9.19 A delta-connected heater, consisting of three elements of resistance 15 Ω each, is connected to a three-phase, 415 V supply. Find the line current, and from this find the total power consumed.

9.20 A 415/240 V supply delivers power to 30 filament lamps, each rated at 60 W, 240 V. If the system is balanced, determine the current in each line.

9.21 The currents in the three lines of a three-phase, four-wire system are each 100 A, the phase differences being 30°, 45° and 60° lagging respectively. Determine the value of the current flowing in the neutral conductor.

9.22 A three-phase, four-wire system has line currents of 40 A, 50 A and 60 A, lagging 45°, 30° and 20° respectively. Find the value of current in the neutral conductor.

9.23 The power input to a three-phase induction motor is measured by the two-wattmeter method. The wattmeters show readings of 13.5 kW and 7.5 kW, both positive. Calculate the power input to the motor.

9.24 The power input to a three-phase induction motor is determined by the two-wattmeter method. On no load, the readings are 0.26 kW and 1.04 kW; on full load the meters read 1.68 kW and 3.5 kW. Find the power input and power factor at:

(a) no load

(b) full load.

9.25 A three-phase, four-wire distribution system carried the following unbalanced loads:

- red phase 45.0 A at 10° lagging
- white phase 87.5 A at 42° lagging
- blue phase 62.5 A at 27° lagging.

(a) Draw a scaled phasor diagram of this loading

(b) Determine the current in the neutral wire.

9.26 The power input to an unloaded three-phase induction motor is measured by the two-wattmeter method. The meter readings are 450 W and 220 W (reversed). Determine the power input and, by using the tan ϕ formula, find the power factor of the motor.

9.27 A balanced three-phase circuit has its power measured by two wattmeters. The first wattmeter reads 400 W and the second wattmeter reads 2000 W. Calculate:

(a) the total power consumed

(b) the power factor of the load, using the curve in Figure 9.50.

9.28 A three-phase, four-wire load has its power input measured by three wattmeters. The first reading is 2.7 kW, the second 8.7 kW and the third 9.3 kW. Find the total power consumed.

9.29 A 33 kV transmission line carries a current of 100 A at a power factor of 0.85 leading. Determine the amount of power being transmitted down the line.

9.30 What is the voltage to earth of a three-phase star-connected transmission line if its line voltage is 11 kV?

9.31 What is the power output of a 240/415 V star-connected alternator if it has a line current of 9 A and a power factor of 0.8 lagging? Find also the kV A rating if the above values are the full load figures.

9.32 A 240/415 V alternator has a load of three 240 V, 2 kW radiators connected in star configuration for test-run purposes. Find:

(a) the total power consumption

(b) the resistance of each radiator element

(c) each line current

(d) the new power consumption and line currents if the radiators were reconnected in delta configuration.

Would the radiators be overloaded when connected in delta configuration?

9.33 The input to a three-phase motor is 9 A on a 415 V supply. Given a power factor of 0.8 and a load of 4240 W calculate the motor's efficiency.

9.34 Readings taken by the two-wattmeter method for measuring power are: W_1, 11.7 kW; W_2, 15.3 kW. Find the total power consumption and the power factor of the load.

9.35 A three-phase balanced load, when measured by the two-wattmeter method for measuring power, gave a total of 12 kW of power at a power factor of 0.78 lagging. Calculate the readings of the two wattmeters.