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of text. There are, however, more than enough diagrams to make the text meaningful, while the instructor as part of the teaching process would supply any required additional material.

Each chapter has a summary of the salient points contained in that chapter and this is followed by both exercises and self-testing problems. There are a total of 621 such items for the student to exercise his or her mind as an adjunct to the learning process. These are unchanged from the previous edition, although the self-testing exercises in Chapter 11 have been rearranged to ensure the degree of difficulty increases as progress is made.

ACKNOWLEDGMENTS

The modifications incorporated in this fifth edition of *Electrical Principles for the [illegible] Trades* are the result of the work of many instructors from most Australian states. The work comprises the hard work undertaken by them in addition to their normal duties. Thanks to John [illegible] (Sydney Institute of TAFE), Bob Moore (Yeronga Institute of TAFE), Steve Brooks (West Coast College of TAFE), Pedro Batsiokis (Regency Institute of TAFE), Vince Franco (RMIT), Kerry Bellingham (North Point Institute of TAFE), John Ferguson (Southern Queensland Institute of TAFE), Doug Grant (Swinburne TAFE) and John Coleman (Barrier Reef Institute of TAFE), all of whose comments and suggestions were valued contributions to the revision process. Without them, the book wouldn't be what it is.

The organisation, production and success of such a book is due to the hard work of not only the editors but many other staff members of McGraw-Hill Australia, particularly Jennifer Speirs (Developmental Editor), Leanne Peters (Production Editor) and Karen Enkelaar (Freelance editor). Thank you all for your good work, advice and support; it is greatly appreciated.

Jim Jenneson
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Chapter 1

Units and physical quantities

1.1 INTRODUCTION

To understand many electrical principles, students should have a background in certain basic mechanical principles. This background in turn requires the adoption of a system of fundamental units and any derived units that may arise from the system. It is often necessary for students to have the ability to manipulate these units into other required forms by some mathematical process. In addition to knowing the usual basic mathematical processes, an understanding of graphs, graphical solution methods and trigonometry is also required for electrical studies. This chapter considers the units of measurement and mathematical processes that will aid the study of the technical subject matter in the remaining chapters.

1.2 BASE UNITS (SYSTÈME INTERNATIONALE)

There are six base units in the international metric system Système Internationale (SI), although there are many derived units of which the more relevant ones will be dealt with in section 1.3. An additional unit called a supplementary base unit is also relevant to the material in this book. It is the angle of rotation and is referred to as a plane angle.

For interest, brief definitions of these units are given here. More exact definitions are to be found in the Standards Australia publication AS.ISO 1000:1998 *The International system of units (SI) and its applications*.

Metre

A metre is equal in length to 1 650 763.73 wavelengths in a vacuum of the orange-red line spectrum for the isotope krypton-86.

Kilogram

The kilogram, first defined as the amount contained in 1000 millilitres of pure water at 0°C, is now the mass of a particular piece of platinum stored under special conditions in France.

Second

A second is an interval of time corresponding to 9 192 631 770 oscillations of a caesium-133 atom under specified conditions.

Table 1.1 • Base SI units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
length	l	metre	m
mass	m	kilogram	kg
time	t	second	s
electric current	I	ampere	A
temperature	T	kelvin	K
luminous intensity	I	candela	cd
angle of rotation (supplementary unit)	α	radian	rad

Ampere

An ampere is the current flowing in each of two parallel conductors of infinite length and negligible cross-section. When separated by a distance of 1 metre from each other in free space, 1 ampere produces between those conductors a force equal to 2×10^{-7} newton per metre length of conductor.

Kelvin

A kelvin is the unit of temperature equal to 1/273.16 of the triple-point temperature of water. The kelvin is used for absolute temperature measurements.

Candela

The candela is 1/60 of the lighting power emitted by 0.0001 square metre of a full radiator at the sea-level temperature of solidification of platinum.

Radian

A radian is the angle between two radii of a circle which mark off on the circumference an arc equal in length to the radius of the circle.

These base units, from which our other necessary units may be derived, are used to measure quantities that can vary considerably in magnitude. To avoid very large or small figures, prefixes representing multiples and submultiples are often used. The multiples and submultiples are listed in Table 1.5.

1.3 SI DERIVED UNITS

The six basic units are not sufficient to cater for all situations that arise in measurement. Derived units are used for all non-basic situations. Most derived units use the three basic units of length, mass and time in various combinations. The units used in this book can be subdivided into three groups: mechanical, electrical and magnetic, although it must be realised there are many more examples than those listed.

1.3.1 Mechanical

Newton

A newton is the force which, when applied to a mass of 1 kilogram, causes an acceleration of 1 metre per second per second.

Pascal

The pressure that occurs when a force of 1 newton is applied to an area of 1 square metre.

Energy and work

When a force of 1 newton is applied over a distance of 1 metre, the work done or energy expended is 1 joule.

Temperature

The temperature expressed in degrees Celsius (°C) is equal to the temperature expressed in kelvins (K) less 273.16. The intervals between °C and K are identical.

Angular velocity

In one revolution there are 2π radians. Angular velocity or speed of rotation is measured by revolutions per minute (r.p.m.) or by radians per second (rad/s).

Table 1.2 • Derived mechanical units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
force	F	newton	N
pressure	P	pascal	Pa
energy and work	W	joule	J
temperature	t	degree Celsius	°C
angular velocity[a]	ω	radians per second	rad/s
volume	V	cubic metres	m^3

[a] In accordance with AS/NZS 1046, angular velocity for practical cases can also be expressed as revolutions per minute and abbreviated as r/min.

Volume

The unit of volume is based on the unit of length and is the cubic metre. It is a large unit and for liquid measure the litre is used: 1000 litres = 1 cubic metre. The litre in turn has its submultiples such as the millilitre; that is, 1000 mL = 1 litre.

1.3.2 Electrical

Watt

A watt unit is the power used when energy is expended at the rate of 1 joule per second.

Coulomb

A coulomb is the quantity of electric charge transferred each second by a current of 1 ampere (nominally equal to 6.24×10^{18} electrons).

Hertz

A hertz is the number of periodic oscillations per second (frequency).

Volt

A volt is the potential difference existing between two points on a conductor carrying a current of 1 ampere when the power dissipated is 1 watt.

Farad

A farad is the capacity that exists between two plates of a capacitor if the transfer of 1 coulomb from one plate to the other creates a potential difference of 1 volt.

Table 1.3 • Derived electrical units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
power	P	watt	W
charge	Q	coulomb	C
frequency	f	hertz	Hz
potential	V	volt	V
capacity	C	farad	F

1.3.3 Magnetic

Weber

A weber was once a unit of 10^8 lines of force. Now it is the magnetic flux linking one turn that produces 1 volt if reduced to zero at a uniform rate in 1 second.

Tesla

A tesla is a magnetic flux of 1 weber per square metre.

Henry

An inductance has a value of 1 henry when an electromotive force (e.m.f.) of 1 volt is produced by a current changing uniformly at a rate of 1 ampere per second.

Table 1.4 • Derived magnetic units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
flux	Φ	weber	Wb
flux density	B	tesla	T
inductance	L	henry	H

1.3.4 Multiples and submultiples

In practical cases some SI values are inconveniently large or small. In order to choose values that are convenient to handle, multiples or submultiples are used. For example, if the resistance of an electrical installation is measured at 15 000 000 ohms, it is more convenient to refer to this value as 15 megohms; that is, 15 units, each consisting of one million ohms (see Table 1.5). Similarly, it is easier to refer to the output of a power station as 125 megawatts (125 MW) than 125 000 000 watts. The unit of capacity is the farad. This is a large unit for most applications so it is usual to refer to capacity in microfarads or picofarads.

Table 1.5 • SI multiples and submultiples

Grouping	Notation	Symbol	Example
tera	10^{12}	T	1 THz = 1 000 000 000 000 Hz
giga	10^9	G	1 GHz = 1 000 000 000 Hz
mega	10^6	M	1 MHz = 1 000 000 Hz
kilo	10^3	k	1 kHz = 1 000 Hz
Unit	**Notation**	**Symbol**	**Example**
milli	10^{-3}	m	1 mH = 0.001 H
micro	10^{-6}	μ	1 μH = 0.000 001 H
nano	10^{-9}	n	1 nH = 0.000 000 001 H
pico	10^{-12}	p	1 pH = 0.000 000 000 001 H

Example 1.1

How many millimetres are there in 1.47 metres (1.47 m)? The prefix 'milli' from Table 1.5 represents one-thousandth; therefore the number of millimetres = the number of metres × 1000. That is, 1.47 × 1000 = 1470 mm.

Example 1.2

How many farads are there in 125 picofarads (125 pF)? From Table 1.5 it can be seen that there are 1 000 000 000 000 picofarads in one farad. Therefore:

125/1 000 000 000 000 = 0.000 000 000 125 farads

The convenience of the former figure of 125 pF is self-evident.

1.3.5 Scientific notation

Another method of overcoming cumbersome rows of figures is to notate numbers to a value between 1 and 10 multiplied by 10 to some power. For example, 6 800 000 can be expressed as 6.8×10^6, and 1250 as 1.25×10^3.

For values less than unity a similar method is employed:

$0.0025 = 25/10\,000 = 2.5 \times 10^{-3}$

$0.0000047 = 4.7/1\,000\,000 = 4.7 \times 10^{-6}$

1.4 WORK, POWER AND ENERGY

The three terms work, power and energy are always closely associated but are separate and distinct entities.

Work is done when energy is converted from one form to another, for example, from fuel to heat.

Power is the rate of doing work.

Energy is the ability to do work.

Work

When a body is moved through a distance by a force acting on it, work is done. That is, if a force of F newtons acts through a distance of l m, then:

$$\boxed{\text{work} = Fl \text{ joules}}$$

Example 1.3

A force of 100 N is required to move a box 5 m along a horizontal surface. Find the value of work done.

work = $Fl = 100 \times 5 = 500$ J

The rate of doing work is called power. It can be found from the work value divided by the time in seconds, and is expressed in units as J/s = W (watts):

$$\boxed{\text{power} = \frac{\text{work}}{\text{time}}}$$

Example 1.4

If the box in example 1.3 was moved first in 10 s and later in 5 s, calculate the power used in both cases.

$$\text{power} = \frac{\text{work}}{\text{time}} = \frac{500}{10} = 50 \text{ W}$$

$$\text{power} = \frac{\text{work}}{\text{time}} = \frac{500}{5} = 100 \text{ W}$$

Energy

The potential to do work can be found from power multiplied by its time of application; that is:

$$\text{energy} = \text{power} \times \text{time}$$

$$= \frac{Flt}{t} \text{ joules}$$

that is, $\boxed{\text{energy} = Fl \text{ joules}}$

There is a direct equivalent in an electrical system, where the power rating of an appliance multiplied by the time it is switched on gives the electrical energy. However, it is then more common to express the value (which can be very large) in groups of 3 600 000 J to give kWh (kilowatt-hours). For more details, see section 2.9.

1.4.1 Torque

Torque is the term used to denote the effect of a force producing or tending to produce rotation of a body about a point. Common examples of torque are tightening a nut with a spanner or turning the steering wheel of a car. Torque can be present whether there is actual rotation, or only a tendency to rotation. The actual value of torque is due to a force acting at a perpendicular distance from an axis or pivot (see Fig. 1.1).

torque = Fr newton-metres

$$\boxed{T = Fr}$$

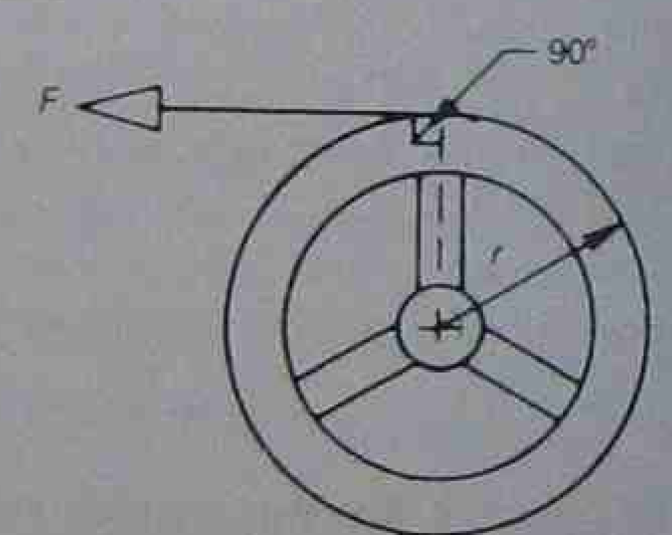


Figure 1.1 • Torque

When the restraining forces are not overcome, the turning moment Fr remains simply that. However, if the turning moment is great enough to overcome these forces, the torque can still be found from $T = Fr$, but at the same time it must be remembered that the force is now acting over a distance, and accordingly work is being done. For situations like this, where the movement is of a rotational type, it is usual to consider speed of rotation rather than distance moved, and since speed is a rate of rotation or a rate of doing work, power in watts becomes the more operative term; that is, power is proportional to rotational speed n and torque T:

$$\boxed{P \propto nT}$$

When dealing with rotation of a body or quantities derived from rotation, it is necessary to consider angular velocity; that is, the angle through which rotation occurs within a given time. In physics and related electrical work, angular velocity is expressed in radians per second (rad/s or rad.s^{-1}).

A radian is the angle subtended by an arc whose length is equal to the radius. See Figure 1.2, where:

angle θ = 1 radian

and r = radius = length of arc

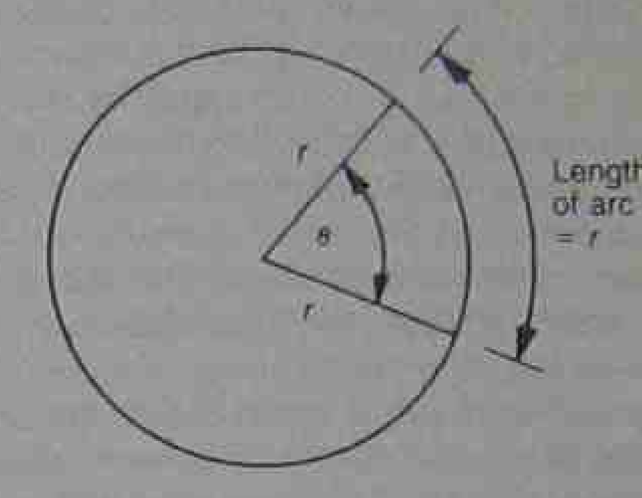


Figure 1.2 • The radian

Since the circumference C is found from $C = 2\pi r$, it follows that there are 2π radians in one complete revolution:

$$\therefore \boxed{360° = 2\pi \text{ radians}}$$

If the rotational speed is given as n rad/s, the angular velocity is proportional to $2\pi n$. In electrical terms, the rotational speed n is equivalent to the frequency f in hertz. The angular velocity in this case is $2\pi f$. Angular velocity ($2\pi n$ or $2\pi f$) is often denoted by the lower case omega, ω.

$$\therefore \boxed{\omega = 2\pi n} \text{ or } \boxed{\omega = 2\pi f}$$

From this, the rate of doing work (power) for a rotating body is found:

$$\boxed{P = 2\pi nT = \omega T}$$

where P = power in watts
n = revolutions per second (r/s)
T = torque in newton-metres (N m)

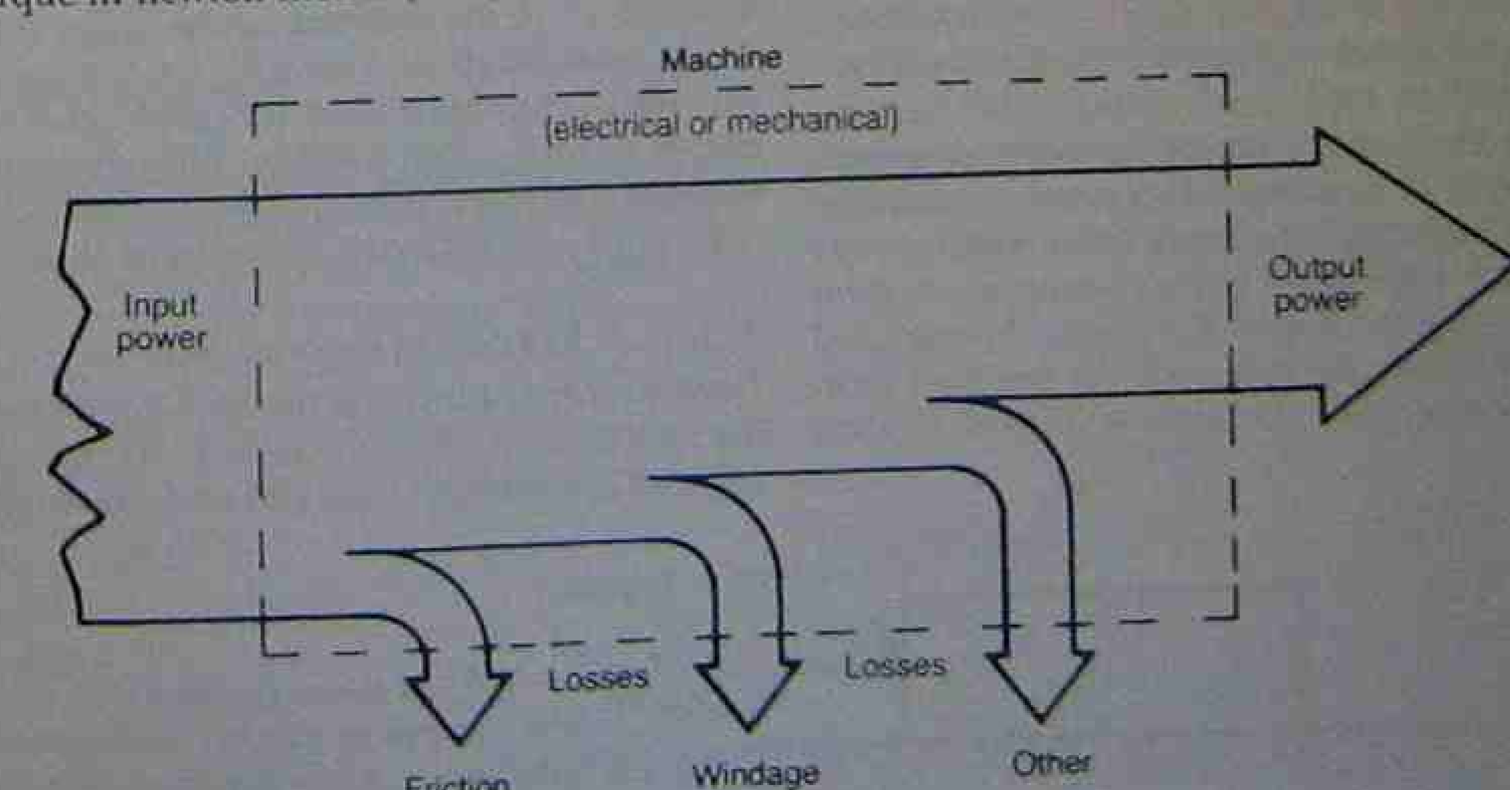


Figure 1.3 • Losses in a machine

Example 1.5

A force of 150 N is applied to the end of a spanner 0.4 m long in order to tighten a nut. Calculate the torque applied to the nut.

torque = $Fr = 150 \times 0.4 = 60$ N m

Example 1.6

Find the torque exerted by a 3 kW electric motor operating at 1440 r/min.

$$P = \omega T = 2\pi nT$$

$$\text{that is, } 3000 = \frac{2 \times \pi \times 1400 \times T}{60}$$

$$\therefore T = \frac{3000 \times 60}{2\pi \times 1400} = 19.9 \text{ N m}$$

1.4.2 Losses in a machine

The electric motor in example 1.6 is rated at 3 kW. The motor is said to have a nominal output of 3 kW, which is not a real indication of the electrical energy input to the motor. In both mechanical and electrical systems there are losses that can have an important bearing on the operation of the system. The losses consist of friction, windage and other forms, of which the major loss is usually friction.

Power in excess of the output has to be supplied to a system to compensate for these losses.

The action of friction in causing losses cannot always be considered bad, because some circumstances demand the use of friction for the satisfactory operation of a mechanical system. A simple example is that of a nut and bolt, where the friction between the male and female threads provides the basic need of the components. Without friction the nut would not remain tight but would come loose with little effort. However, for whatever reason, losses do exist and must be considered in the overall system.

Power put into the system equals power output plus losses. This statement can be expressed in other ways. The most common method is to express the ratio of the power output to the power input as a percentage (i.e. per hundred). This is called the efficiency of the system.

The usual symbol for efficiency is η (pronounced 'eta') and is expressed as a number followed by the per cent symbol; for example, efficiency can be expressed as 'efficiency = 89%' or 'η = 89%'. It is a ratio only and has no units:

$$\boxed{\eta = \left(\frac{P_{out}}{P_{in}} \times 100\right)\%}$$

Example 1.7

If a mechanical device has a power input of 160 W and a power output of 120 W, find the efficiency.

$$\text{efficiency} = \frac{\text{power output}}{\text{power input}} \times 100$$

$$\therefore \text{efficiency} = \frac{120}{160} \times 100 = 75\%$$

The loss is the difference between the power output and input, and in this example is 160 − 120 = 40 W loss.

Example 1.8

Find the efficiency of the electric motor in example 1.6 if the losses were found to be 357 W.

input power = 3000 + losses = 3357 W
output power = 3000 W (motor rating)

$$\eta = \frac{3000}{3357} \times 100 = 89.4\%$$

1.5 SCALAR AND VECTOR QUANTITIES

All quantities can be classified as being either scalar or vector quantities:

1. **Scalar** quantities are those with which no direction can possibly be associated (e.g. mass, volume, energy, time).
2. **Vector** quantities are those for which direction has importance; that is, they must be expressed in both magnitude and direction (e.g. velocity, acceleration, force as a 'push' or 'pull').

The examples given above are of mechanical quantities, but these types of quantities also occur in electrical theory. Electrical scalar quantities are dealt with in more detail in section 2.6.4. Electrical vector quantities are dealt with in detail starting in Chapter 8. Vectors when applied to electrical systems are called phasors, but the basic principles of their treatment are the same as for mechanical vector quantities.

1.5.1 Scalar quantities

A number and a unit are sufficient to specify many physical quantities. These quantities can be added by ordinary arithmetical means. For example, 5 seconds + 3 seconds = 8 seconds, or 1 km + 2.6 km = 3.6 km.

1.5.2 Vector quantities

Unlike scalar quantities, vector quantities cannot be satisfactorily specified without giving them a direction as well as a quantity and a unit. A vector quantity can be represented by a straight line, which when drawn out to a scale is able to represent both magnitude and direction. In many mechanical cases the vector quantity is a force, the unit of measurement being a derived unit based on mass, length and time. Weight, the gravitational attraction between two bodies, is a special case of force. The unit, kilogram, is used to measure mass, while the unit, newton, is used for force and weight. It is convenient for the purpose of this work to use force as a means of examining methods for solving vector problems. The characteristics of a force are shown in Figure 1.4.

1.5.3 Forces acting at point

Where more than one force acts on a body simultaneously, the forces can assist or oppose one another. Cyclists are familiar with the situation of the bicycle and the 'tail' or 'head' wind, but the problem can be more complicated with a wind blowing from neither of these directions. The cyclist then has to lean 'against' the wind to continue on a desired path, a situation usually associated with turning. A similar situation exists with a car in a cross-wind. The steering wheel has to be held at an angle to counteract the side forces. In effect the car is actually being steered across the road to counteract the side force and the resultant motion is along the road.

The resultant value of two forces acting on a body depends on the angle between the directions of the forces as well as their respective magnitudes. In Figure 1.5(a), F_1 is added directly to F_2 as a straight arithmetical addition and the resultant force is $F_1 + F_2$ (i.e. riding with a tail wind).

Similarly, in Figure 1.5(b) the resultant force is $F_1 - F_2$ and will act in the direction of the larger force (i.e. riding into a head wind). In Figure 1.5(c), the simple arithmetical process cannot be used. If F_1 and F_2 are acting at right angles to each other, a logical line of reasoning correctly suggests that the combination of F_1 and F_2 gives a resultant force F_R acting in the direction as shown.

1.5.4 Forces acting at 90°

The right-angled triangle method can be used to analyse the effects of two forces acting at 90° to each other. The value of the resultant force can be derived mathematically

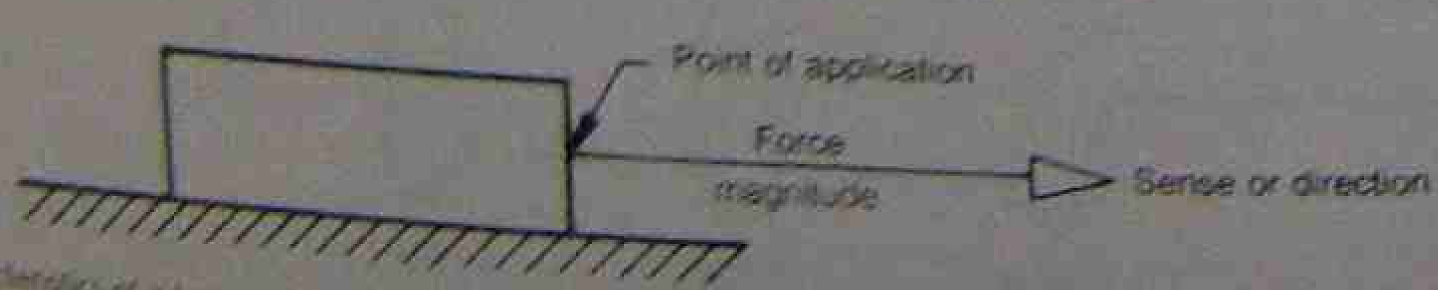


Figure 1.4 • Characteristics of a force

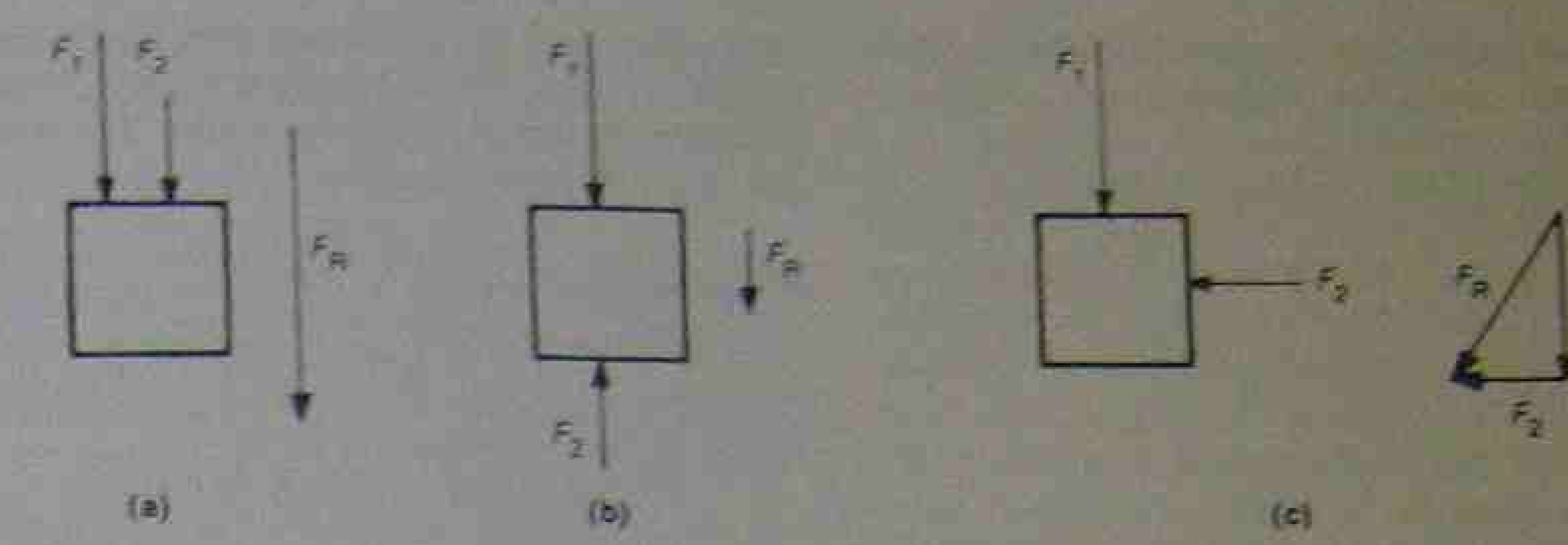


Figure 1.5 • Two forces acting on a body

by Pythagoras's theorem. The theorem was formulated around 540 BC by the Greek mathematician Pythagoras.

> *Pythagoras's theorem states that in a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.*

One of the best-known examples is the triangle with sides in the ratio of 3:4:5. It can be seen from Figure 1.6 that the square on the hypotenuse ($5^2 = 25$) is equal to the sum of the squares on the other two sides:

$$3^2 + 4^2 = 9 + 16 = 25$$

that is, $5^2 = 3^2 + 4^2$

or $\boxed{h = \sqrt{a^2 + b^2}}$

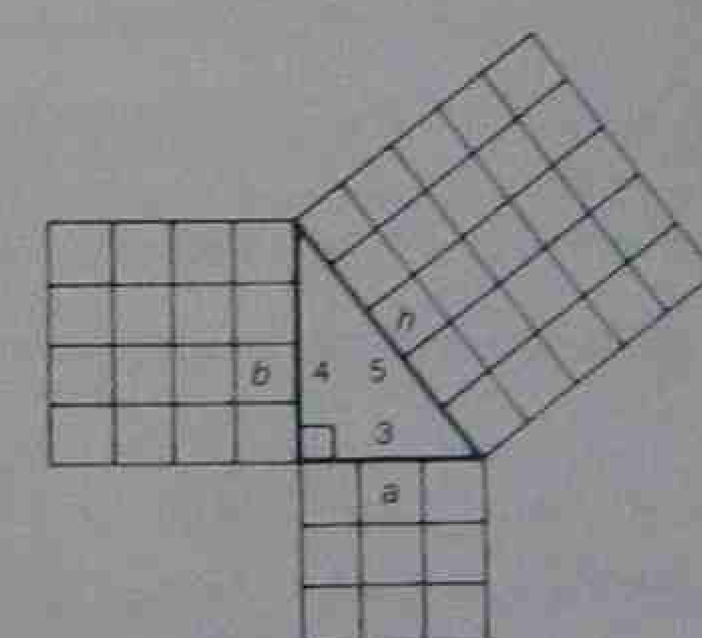


Figure 1.6 • Pythagoras's theorem

Example 1.9

Two forces (F_1 and F_2) each of 25 N act at right angles to each other on a body. Determine the value of the resultant force (F_R) acting on the body.

$$F_R = \sqrt{F_1^2 + F_2^2}$$
$$= \sqrt{25^2 + 25^2}$$
$$= \sqrt{625 + 625}$$
$$= \sqrt{1250}$$
$$= 35.35\text{ N}$$

Note that this method cannot be applied directly to the situation where the forces act at an angle other than 90° to each other.

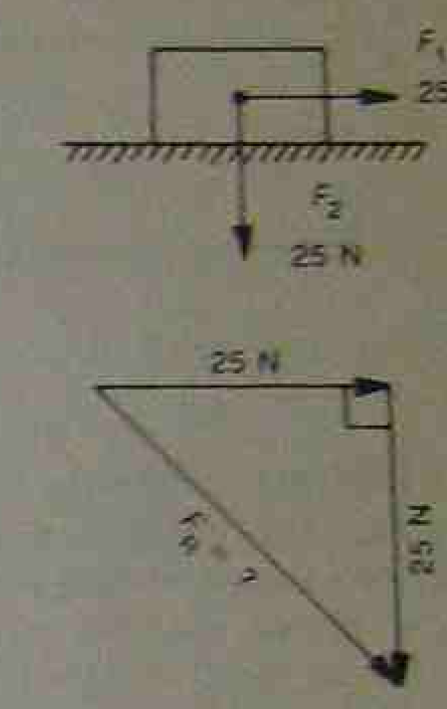


Figure 1.7 • Force diagrams for example 1.9

1.5.5 The parallelogram method

The parallelogram method can be used to solve both magnitude and direction of resultant forces, irrespective of the direction of the separate forces. The method consists of drawing two forces out to scale at the angle of application, completing the parallelogram and then drawing in the diagonal as the resultant force. This method is also known as a graphical or vector diagram method of solution.

In Figure 1.8, two forces OA and OB pull on point O as indicated by the arrowheads.

OA and OB are drawn out to scale at the appropriate angle to each other. AC is drawn equal in length and parallel to OB. BC is drawn equal in length and parallel to OA. The resultant of the two forces is OC acting in the direction indicated and the value of F_R is found from the scale to which the figure is drawn. Probably the most common method of construction is that of intersecting arcs with the aid of compasses, as used in Figure 1.8. The

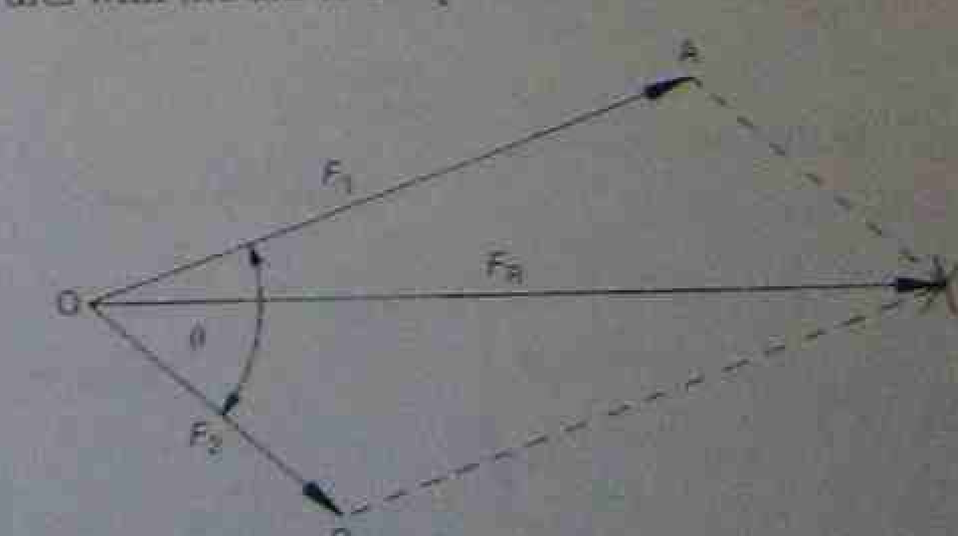


Figure 1.8 • Parallelogram method of adding vectors

parallelogram method is used extensively in Chapter 8 for solving power factor correction problems.

Example 1.10

Two forces of 8 N and 5 N act outwards from a point with an angle of 60° between them. Find the resultant force being exerted at the point and the angle at which it acts with respect to the 8 N force.

resultant = 11.4 N
angle to OB = 23°

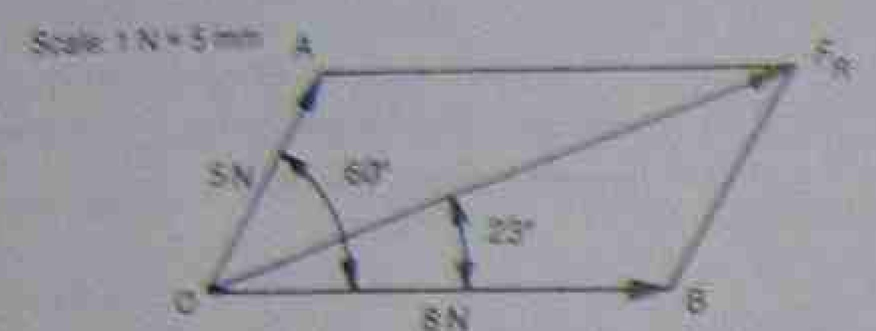


Figure 1.9 • Force diagram for example 1.10

1.5.6 Vector polygon method

The parallelogram method for finding the resultant of two forces becomes more cumbersome when it is used to solve problems involving a greater number of forces. The polygon method provides an easier approach in obtaining a resultant when three or more forces are involved. In Chapter 9 it is used for solving values of neutral current in three-phase systems.

This graphical method for finding the resultants of forces requires each vector to be drawn out to scale on the end of the previous one, due regard being given to its specified direction and magnitude. The vectors can be drawn in any order, provided the requirements of magnitude and direction are met. The resultant is the distance between the origin of the vector diagram and the end of the last vector. Note that the arrowheads of the vectors point progressively along the diagram until the resultant is met (pointing in the opposite direction). The method is best illustrated with an example.

Example 1.11

Three forces acting at a point are spaced 120° from each other. F_1 = 25 N; F_2 = 50 N; and F_3 = 20 N. Find the resultant force acting at the point.

The three forces are drawn out to scale in Figure 1.10(a). Commencing at the origin O in Figure 1.10(b), draw vector F_1 equal in length and parallel to F_1. On the arrowhead end of vector F_1, draw F_2 equal in length and parallel to F_2. On the arrowhead end of F_2, add F_3 in a similar fashion, equal in length and parallel to F_3. The resultant (F_R) of the three forces is the straight line between the origin O and the end of F_3. Note that the arrowhead of the resultant is opposite the flow of the arrowheads in the rest of the diagram.

The vector addition is repeated in Figure 1.10(c). The order of addition is altered, giving a differently shaped figure. It should be noted that the answers have the same magnitudes and relative directions in both cases (answer F_R = 27.8 N). The value and direction of F_R is the value and direction of a single force that could replace all three original forces and still produce the same effect. In all vector diagrams the resultant can be considered as an alternative means of arriving at the same end point as do the vectors; also, the arrowhead is opposite in direction to that of the final vector.

Example 1.12

Three forces, all 25 N acting at a point, are spaced 120° from each other. Find the resultant force acting at the point.

As in the previous example, the vectors are drawn out to scale with due regard to their direction from point O (Fig. 1.11(a)). F_1 is then drawn out parallel to F_1 from its origin O, as in Figure 1.11(b). On the end of F_1, draw F_2 equal in length and parallel to F_2. F_3 is then added to the end of F_2, also equal in length and parallel to F_3. The resultant (as in example 1.11) is the straight line between the origin and the end of F_3. In this example, however, F_3 joins back to the beginning of F_1 at point O. Because the distance between these two ends is zero, the resultant is 0 mm and the system is said to be balanced. Compare this result with that in example 1.11 where there was a resultant force of 27.8 N acting on point O with the unbalanced system of vectors.

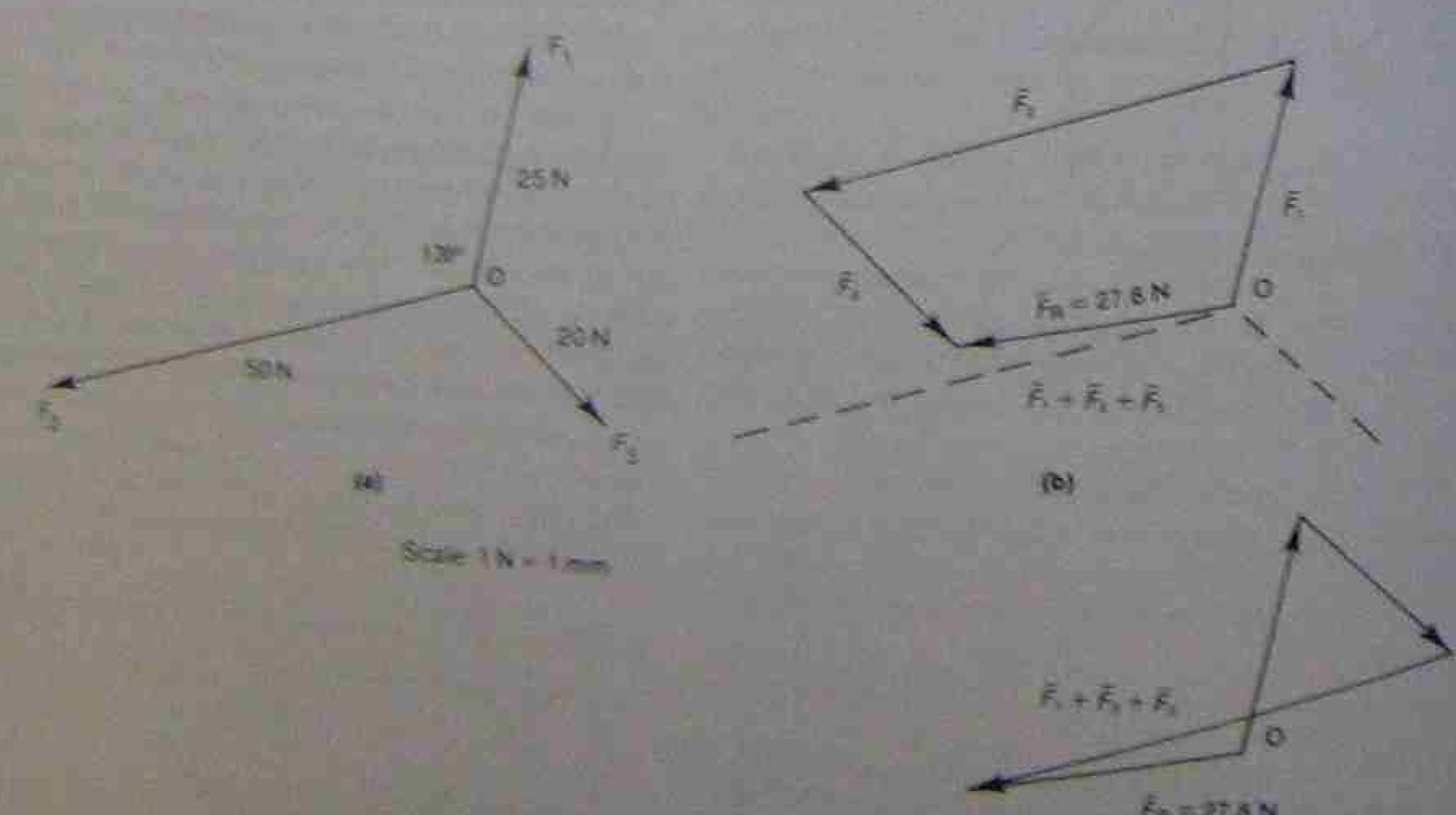


Figure 1.10 • Polygon method of adding vectors

1.5.7 Vector components

Vectors can also be added by a method of separating the vector into horizontal and vertical components. Figure 1.12(a) shows the same forces of Figure 1.10 redrawn with the addition of vertical and horizontal axes.

The dotted lines at right angles to the axes indicate the right-angled components of each force. Force F_1 can be described as being made up of two rectangular components: OA in the horizontal plane, acting to the right from the origin O and regarded as having a positive value as indicated in Figure 1.12(b); and OB in the vertical plane, acting upwards as also indicated in Figure 1.12(b) and regarded as having a positive sign.

On this basis, F_2 has horizontal and vertical components, both with negative values, while F_3 has a positive horizontal component and a negative vertical component. When all forces are separated into their components, simple algebraic addition gives the components for the resultant. For example, the components for the three forces in example 1.11 are given in Table 1.6.

Because these two totals are at right angles to each other, Pythagoras's theorem can be used to evaluate the resultant; that is:

$$F_R = \sqrt{(-27.32)^2 + (-5.33)^2}$$
$$= 27.8 \text{ N}$$

(as before)

Table 1.6 • Horizontal and vertical components

Force	Horizontal	Vertical
F_1	+4.34	+24.62
F_2	−46.98	−17.1
F_3	+15.32	−12.85
Totals	−27.32	−5.33

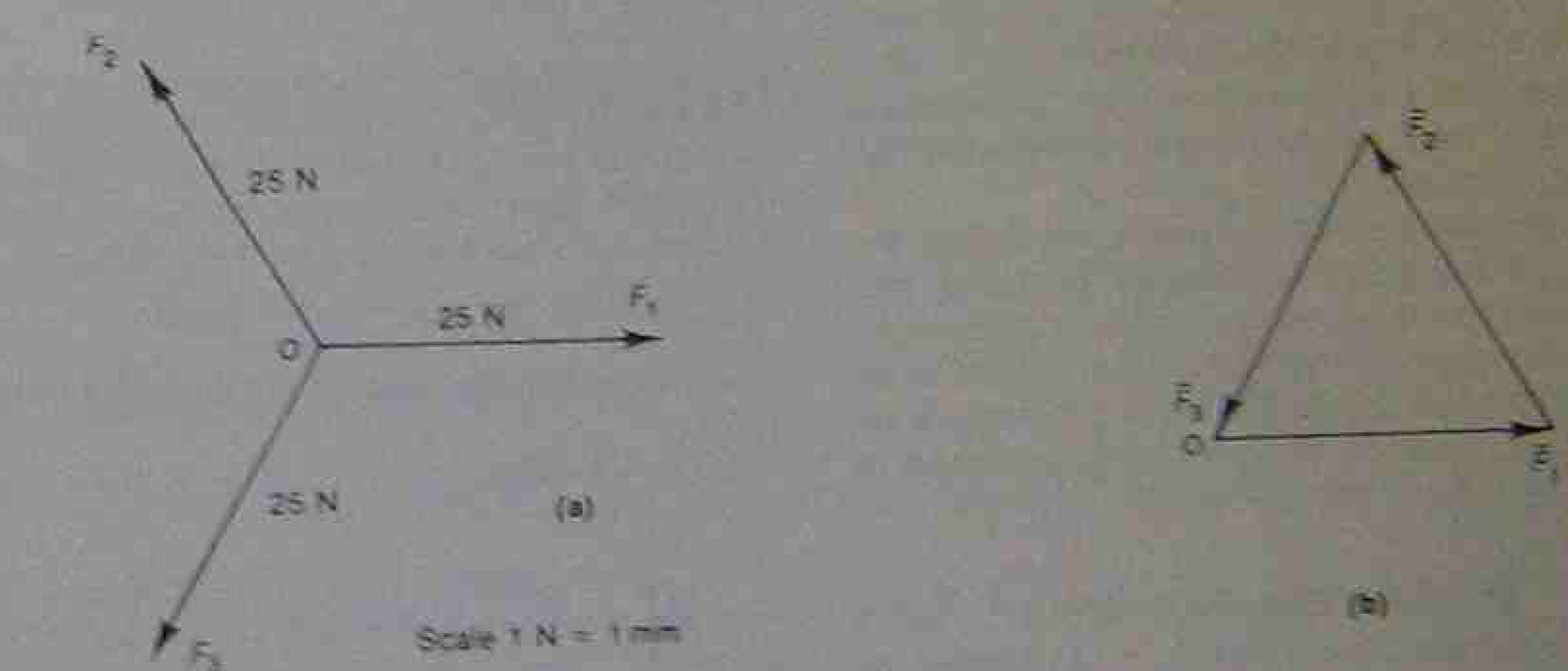


Figure 1.11 • Addition of a balanced system of vectors

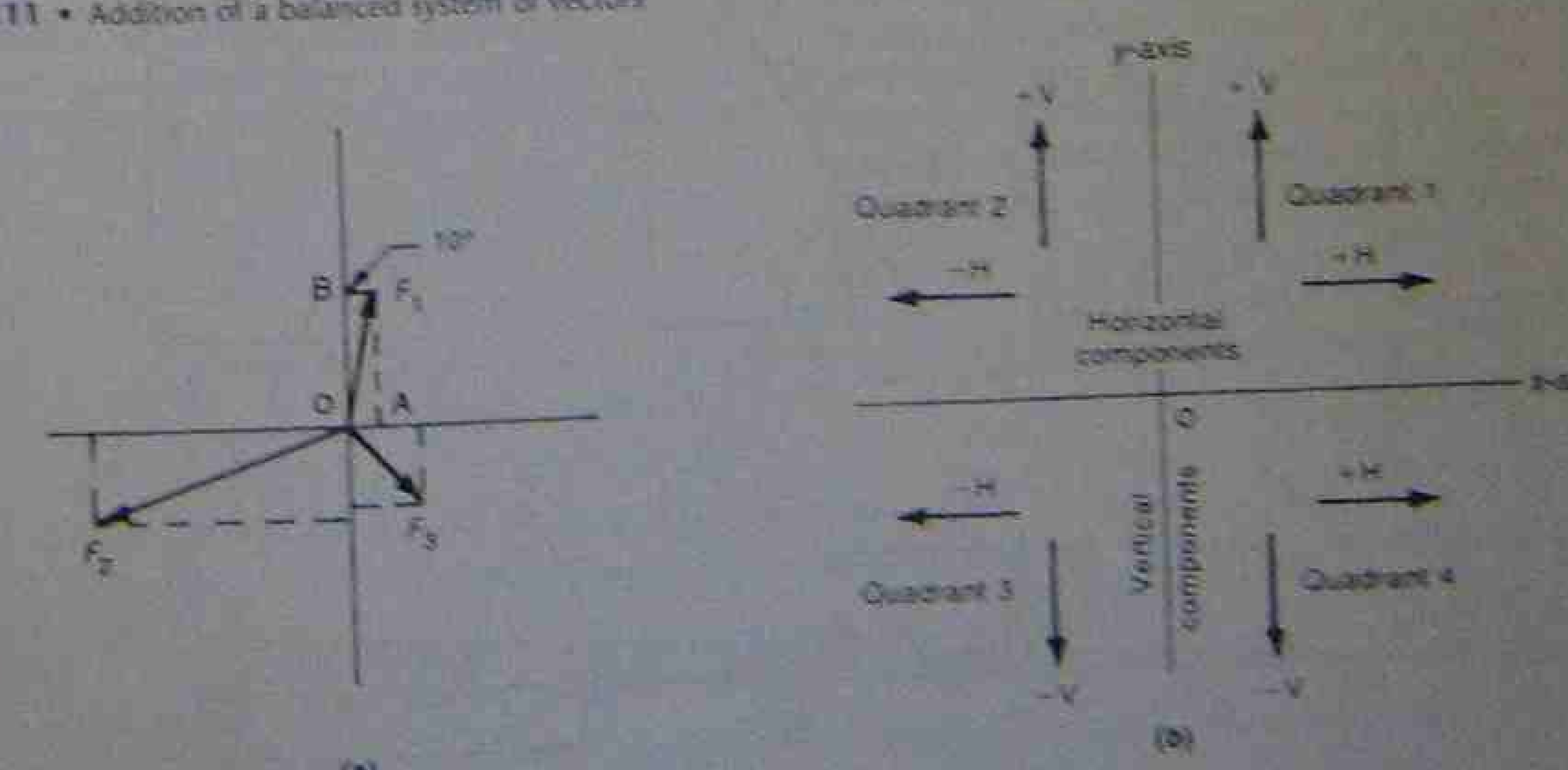


Figure 1.12 • Horizontal and vertical components of vectors

The direction of the resultant is downwards to the left, which is also the same as that found by the polygon method.

Mathematical values of vectors and vector components can be obtained by various means—in books of trigonometrical tables, or by use of a slide rule or a calculator. Appendix 1 gives an introduction to the use of calculators for solving electrical problems.

1.5.8 Rectangular component values

The component values for the three forces in the above problem could be obtained by graphical means (i.e. by drawing out to scale and measuring), but were in fact obtained by a branch of mathematics called trigonometry. Trigonometry for all practical purposes is a means of solving sides and angles in a right-angled triangle. In similar right-angled triangles, the ratios of the sides remain constant and these ratios are published as tables with the names of sine, cosine and tangent, depending on which two sides of the triangle are being considered.

1.6 TRIGONOMETRY

Trigonometry is a branch of mathematics based on the right-angled triangle, and can be used to solve the magnitude and direction of resultant forces with an accuracy greater than the graphical means.

1.6.1 Ratios of lengths of sides

The right-angled triangles shown in Figure 1.13 all have different lengths of sides, but the angles between any pair of sides remain constant.

Because all the corresponding angles are equal, the various figures are called similar triangles. Further to this, the ratios between the sides remain constant irrespective of the size of the triangle. A check of Figure 1.13 will show that the ratio of the horizontal side to the hypotenuse for each figure is 1:2 or ½ (0.5). Reference to trigonometrical tables shows that the cosine for 60° is 0.5. What this really means is that in the right-angled triangle the ratio of the two sides adjacent to a 60° angle will always be 1:2 or ½ or 0.5 (as it is usually expressed).

There are three commonly used ratios of sides for a right-angled triangle:

1. Sine of angle $= \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$$

2. Cosine of angle $= \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}}$$

3. Tangent of angle $= \dfrac{\text{opposite side}}{\text{adjacent side}}$

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}}$$

These sides are illustrated in Figure 1.14 and hold true even when the triangle is rotated into any position.

These ratios together with trigonometrical tables or scientific calculators can be used to solve problems without the necessity of drawing them out to scale.

Example 1.13

With the aid of a scientific calculator, find the ratio between the length of the adjacent side and the hypotenuse for the following enclosed angles:

2°, 20°, 35°, 50°, 34.6°, 85.2°, 89.9°

The ratio adj./hyp. is called the cosine of the enclosed angle:

cos 2° = 0.9994

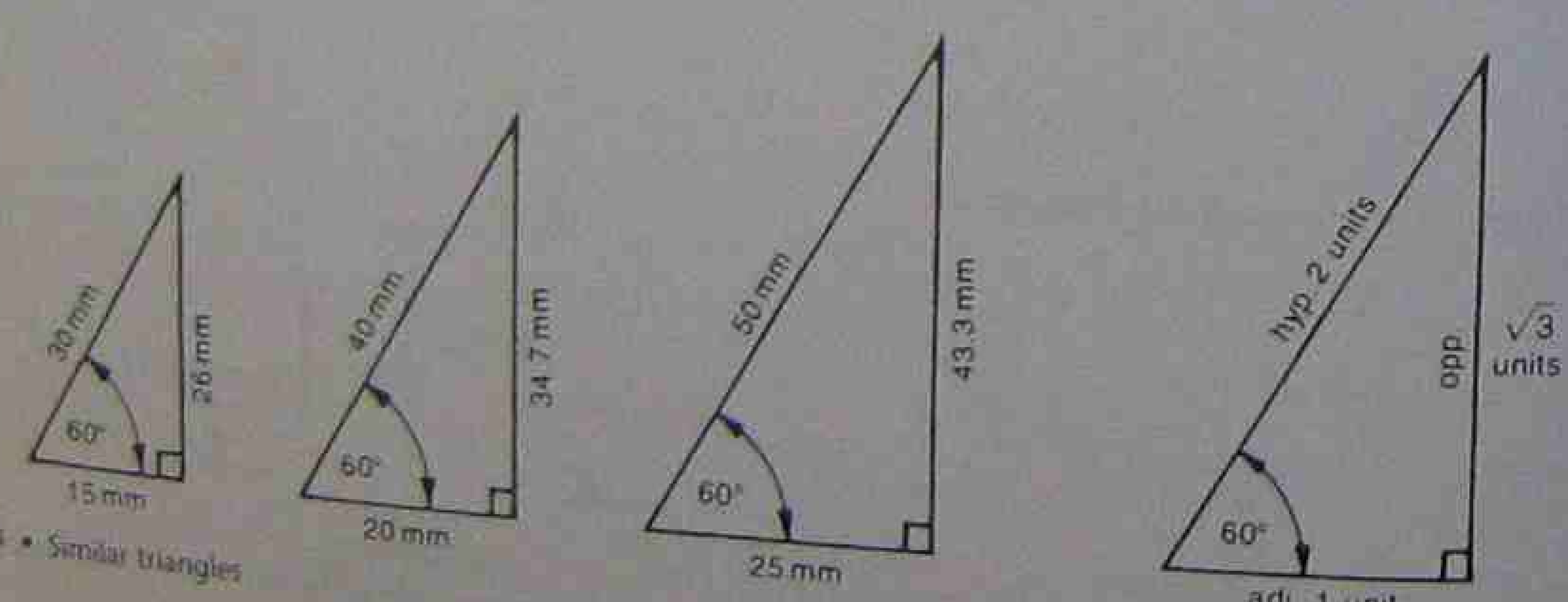


Figure 1.13 • Similar triangles

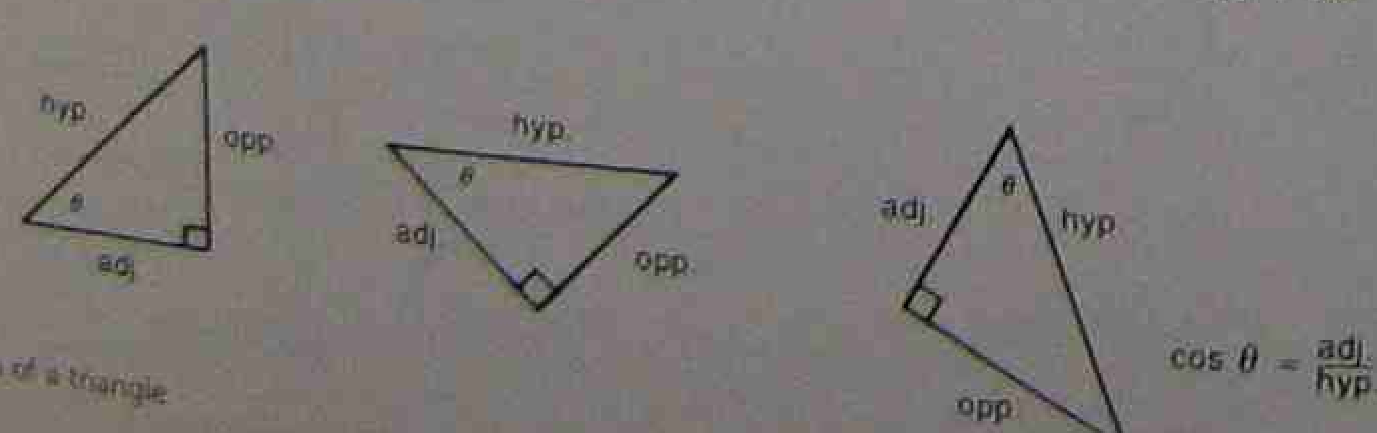


Figure 1.14 • Sides of a triangle

cos 20° = 0.9397, cos 35° = 0.8191
cos 50° = 0.6428, cos 34.6° = 0.8231
cos 85.2° = 0.8368, cos 89.9° = 0.00174

Note that the value approaches unity for small angles and nears zero for angles approaching 90°.

The reverse is true for sine values: sin 0° = 0 and sin 90° = 1. The tangent value varies between zero for 0° and very high numbers for angles approaching 90°.

Example 1.14

With the aid of trigonometry and a calculator, solve the unknown values of the sides in Figure 1.15.

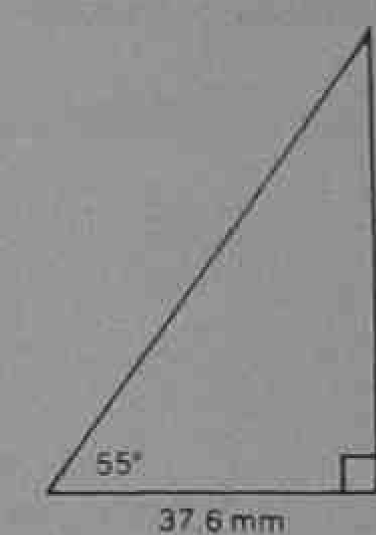


Figure 1.15 • Diagram for example 1.14

$$\frac{\text{adj.}}{\text{hyp.}} = \cos 55° \ (\cos 55° = 0.5736)$$

By transposition:

$$\text{hyp.} = \frac{\text{adj.}}{\cos 55°}$$

$$= \frac{37.6}{0.5763} = 65.55 \text{ mm}$$

$$\tan 55° = \frac{\text{opp.}}{\text{adj.}} \ (\tan 55° = 1.428)$$

$$\therefore \text{opp.} = \tan 55° \times \text{adj.} = 53.7 \text{ mm}$$

Example 1.15

Find the two remaining angles of a right-angled triangle, given the hypotenuse is 32.33 m long and the other two sides are 28 m and 16.16 m.

$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{28}{32.33} = 0.866$$

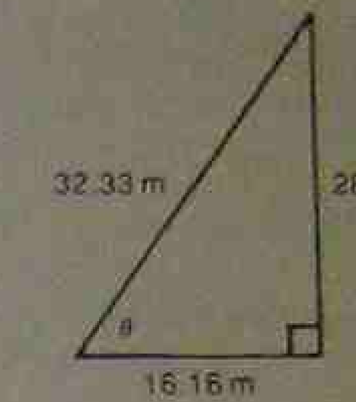


Figure 1.16 • Diagram for example 1.15

The problem is how to find the angle whose sine is 0.866.

Mathematically, this is expressed as either:

arc sin 0.866 or $\sin^{-1}$ 0.866

This value has to be found and translated to an angle in degrees. With a calculator that has trigonometrical facilities it is a matter of using the appropriate buttons. In the case of this example, arc sin (or $\sin^{-1}$) 0.866 = 60°. Since the second angle of the triangle is a right angle (90°), the third angle is 90° − 60° = 30°.

Example 1.16

A lathe operator is required to machine a tapered pin from high-tensile steel. It has to be 200 mm long with a diameter decreasing from 50 mm to 30 mm. At what angle must the lathe-slide be set?

From Figure 1.17, the taper is in the order of 10 mm in 200 mm (shaded area):

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{10}{200} = 0.05$$

$$\text{angle } \theta = \begin{bmatrix} \text{arc tan} \\ \text{or} \\ \tan^{-1} \end{bmatrix} 0.05 = 2.86°$$

1.6.2 Angles greater than 90°

An inspection of a book of trigonometrical tables will show that values are available only from 0° to 90°. This is also applicable to some hand-held calculators, although many are programmed to ratios of 360° and beyond, and include positive and negative signs where appropriate. Published tables do not do this directly, so a different approach becomes necessary for angles greater than 90°.

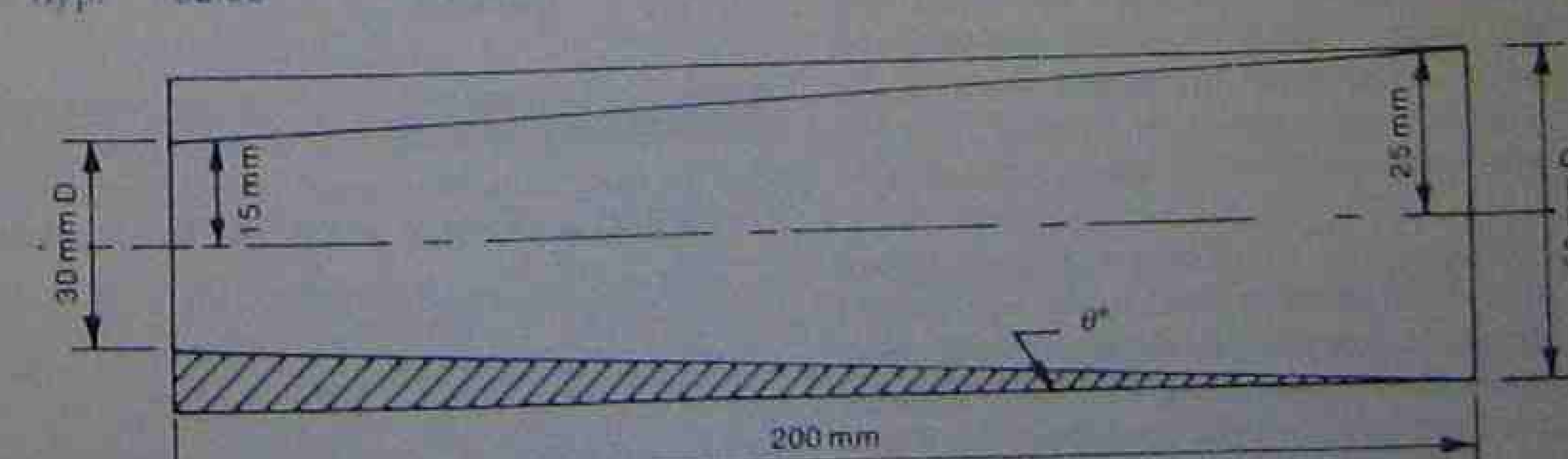


Figure 1.17 • Diagram for example 1.16

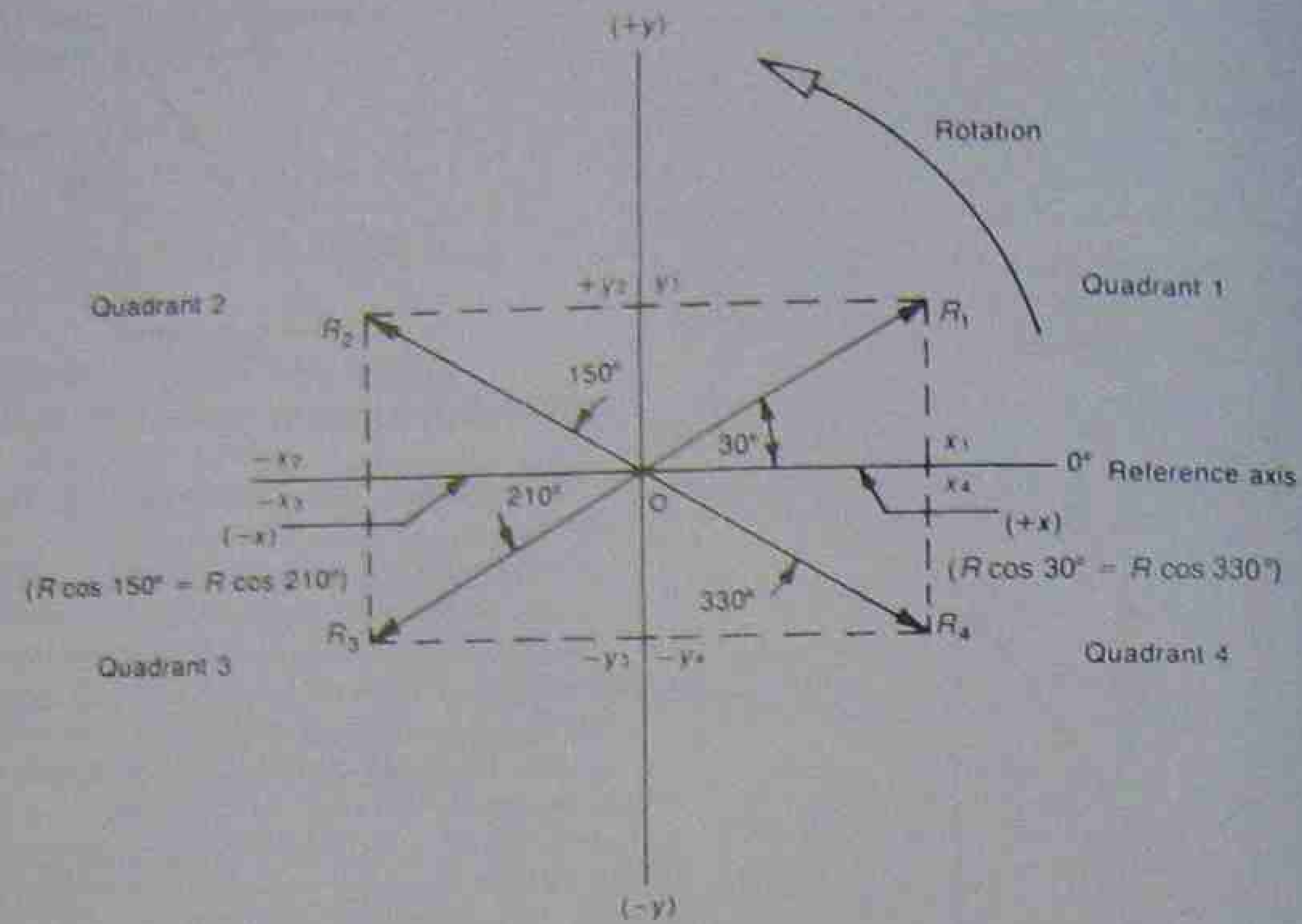


Figure 1.18 • Vector rotating in the four quadrants

In Figure 1.18, two rectangular axes are shown crossing each other at the origin O in a similar fashion to Figure 1.12. By convention, vectors are considered to rotate anticlockwise from a zero degree reference as shown. The same convention relates to electrical quantities, as in Chapters 8 and 9 for single- and three-phase circuits. The two axes are also conventionally known as x and y axes, with positive values to the right and upwards from the origin O.

If a rotating vector R is shifted 30° in an anticlockwise direction from the reference axis to position R_1, the horizontal component Ox_1 is equal to $R \cos 30°$ and the vertical component $Oy_1 = R \sin 30°$. Effectively, for angles up to 90°, triangle Ox_1R_1 is a normal, right-angled triangle and is treated as that in section 1.6.1. It has normal sin, cos and tan values and is the triangle between the rotating vector and the nearest horizontal axis.

When R is rotated through 150° from the reference axis to position R_2, the horizontal and vertical components are represented by $-x_2$ and $+y_2$. The right-angled triangle is now Ox_2R_2, with the enclosed angle of 180° − 150° = 30°, as in the first quadrant. For angles between 90° and 180°, the effective triangle is that between the rotating vector and the nearest horizontal axis, as in the first quadrant. Numerically the values are equal for sin, cos and tan ratios, but it must be remembered that the horizontal component (adjacent side) has a negative sign. The vertical component (opposite side) is still positive, while the vector R is always considered positive.

In a similar fashion, the third and fourth quadrant ratios for sin, cos and tan also have numerical equivalents to the first quadrant. In Figure 1.18, the equivalent right-angled triangles are Ox_3R_3 and Ox_4R_4, and both relate to the angle between the rotating vector and the nearest horizontal axis, although the various ratios might have different signs of polarities; that is:

$$\cos 30° = -\cos 150° = -\cos 210° = \cos 330°$$
$$\sin 30° = \sin 150° = -\sin 210° = -\sin 330°$$
$$\tan 30° = -\tan 150° = \tan 210° = -\tan 330°$$

These are illustrated in Figure 1.19 and show the signs for the three ratios in each quadrant.

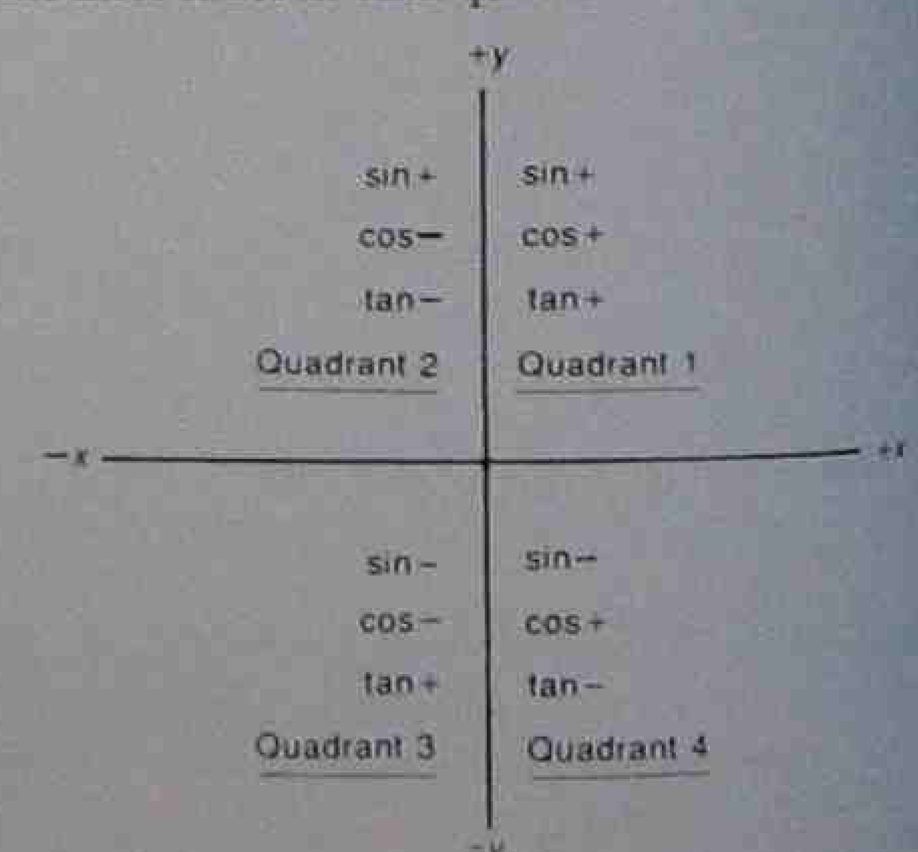


Figure 1.19 • Trigonometrical ratio signs or polarities in four quadrants

Example 1.17

Find the horizontal and vertical components for a vector of 29 N acting at an angle of 226.397° to the reference axis.

effective angle is 226.397 − 180 = 46.397°

$$\cos 46.397° = \frac{-x}{R}\left(= \frac{\text{adj.}}{\text{hyp.}}\right)$$

that is, $0.689 = \frac{-x}{29}$

$$\therefore -x = 20 \text{ N}$$

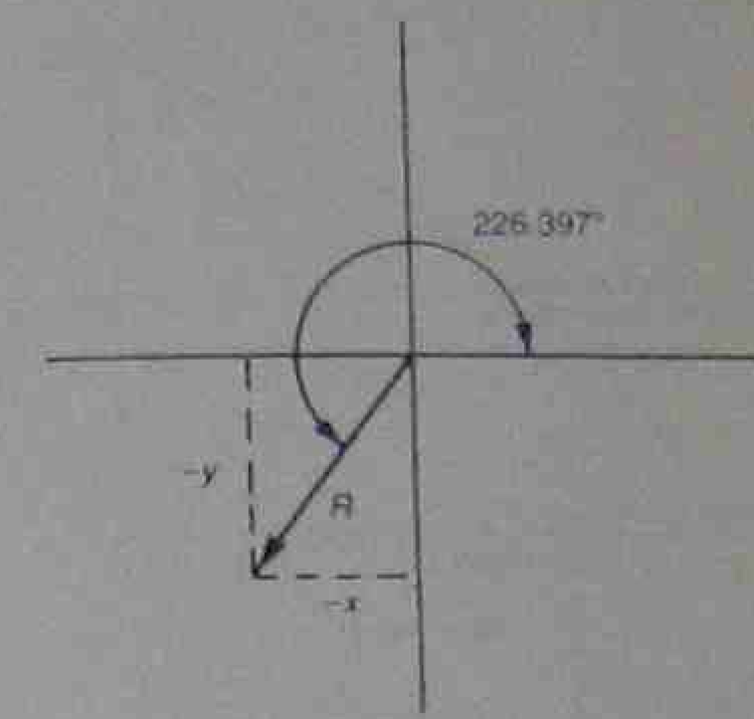


Figure 1.20 • Diagram for example 1.17

$$\sin 46.397° = \frac{-y}{R}\left(= \frac{\text{opp.}}{\text{adj.}}\right)$$

that is, $0.724 = \frac{-y}{29}$

$$\therefore -y = 21 \text{ N}$$

Example 1.18

Find the angle represented by a horizontal component of 45 units and a vertical component of −28 units. What is the length of the rotating vector?

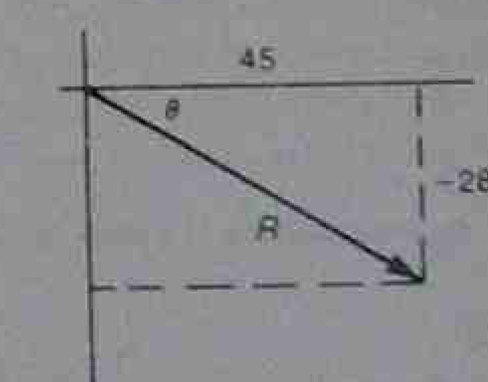


Figure 1.21 • Diagram for example 1.18

$$\tan \theta = \frac{x}{y}\left(= \frac{\text{opp.}}{\text{adj.}}\right) = \frac{-28}{45} = -0.6222$$

From the tables, this represents an angle of 31.89°.

The angle from the reference vector is:

360 − 31.89 = 328.11°

The length R can be calculated from trigonometrical values or by:

$$R = \sqrt{45^2 + (-28)^2}$$
$$= \sqrt{2025 + 784}$$
$$= \sqrt{2809}$$
$$= 53 \text{ units}$$

or $\cos \theta = \frac{45}{R}$

that is, $0.8490 = 45R$

$$\therefore R = 53 \text{ units}$$

Example 1.19

Evaluate the following: sin 97°, cos 184°, tan 215°, tan 290°, and cos 340°.

Sin 97°: 97° subtends an angle of 83° to the horizontal.
sin 83° = 0.9925
∴ sin 97° = +0.9925

Cos 184°: 4° to horizontal
cos 4° = 0.9975
∴ cos 184° = −0.9975

Tan 215°: 35° to horizontal
tan 35° = 0.77002
∴ tan 215° = +0.7002

Tan 290°: 70° to horizontal
tan 70° = 2.7475
∴ tan 290° = −2.7475

Cos 340°: 20° to horizontal
cos 20° = 0.9397
∴ cos 340° = +0.9397

Example 1.20

By means of rectangular components, find the result of the following forces. All angles given are according conventional rotation: F_1, 25 N at 80°; F_2, 50 N at 13 F_3, 15 N at 215°; and F_4, 35 N at 320°.

Table 1.7 • Rectangular components for example 1.20

Force	N	Angle to horizontal	Horizontal component F cos θ	Vertica compon F sin
F_1	25	80°	+4.34	+24.62
F_2	50	45°	−35.35	+35.35
F_3	15	35°	−12.29	−8.60
F_4	35	40°	+26.81	−22.80
	Totals	−16.49	+28.87	

The two components of the resultant can be plotted a vector diagram (Fig. 1.23).

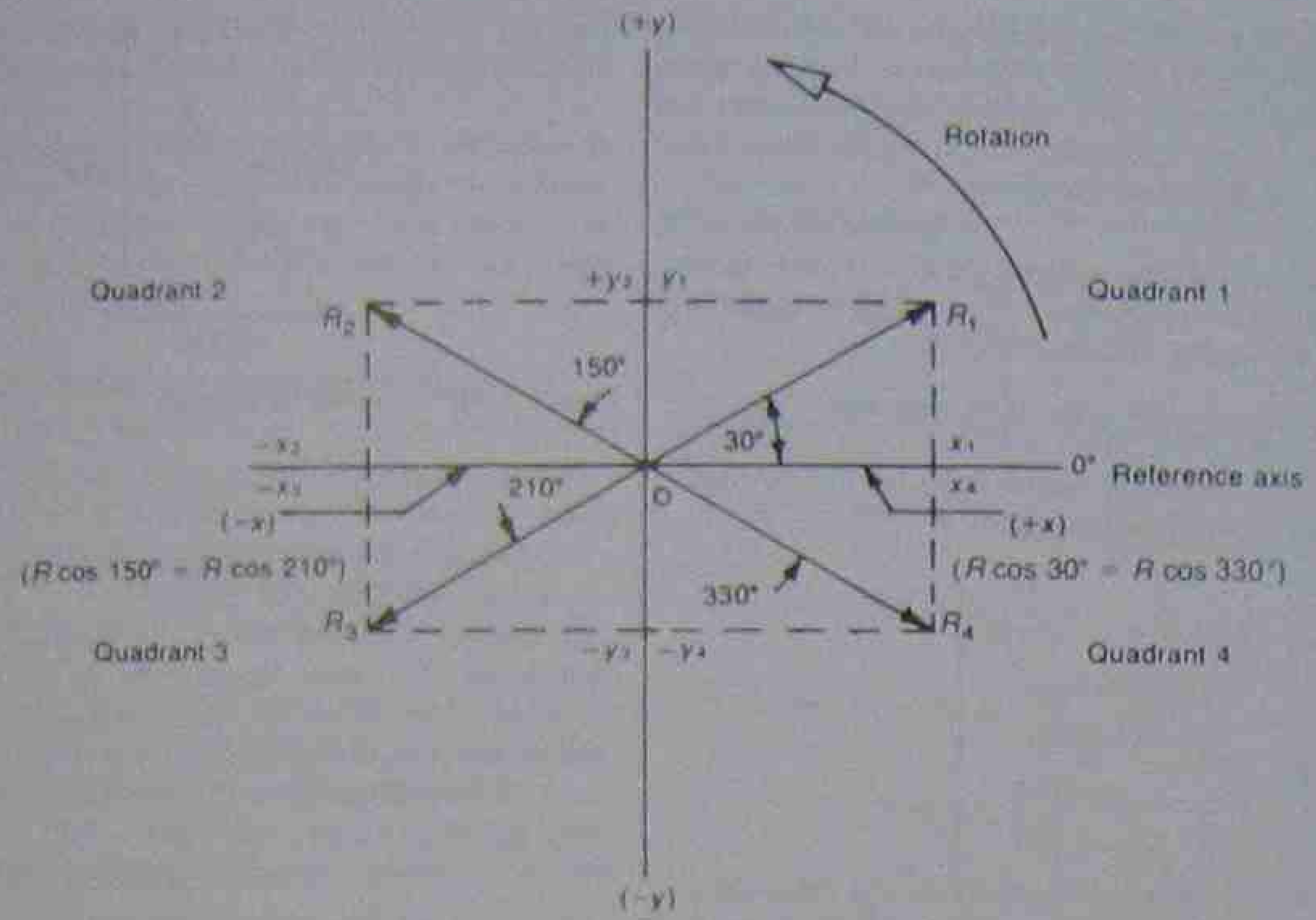
Value or magnitude of the resultant:

$$= OA$$
$$= \sqrt{(-16.49)^2 + (28.87)^2}$$
$$= 33.25 \text{ N}$$

Direction:

$$\text{tan of angle} = \frac{\text{opp.}}{\text{adj.}} = \frac{28.27}{-16.49} = -1.75$$

The angle with a tangent of 1.75 is 60.2°. This is angle of 60.2° to the horizontal in the second quad so the angle of force is 180 − 60.2 = 119.8° from the reference line.



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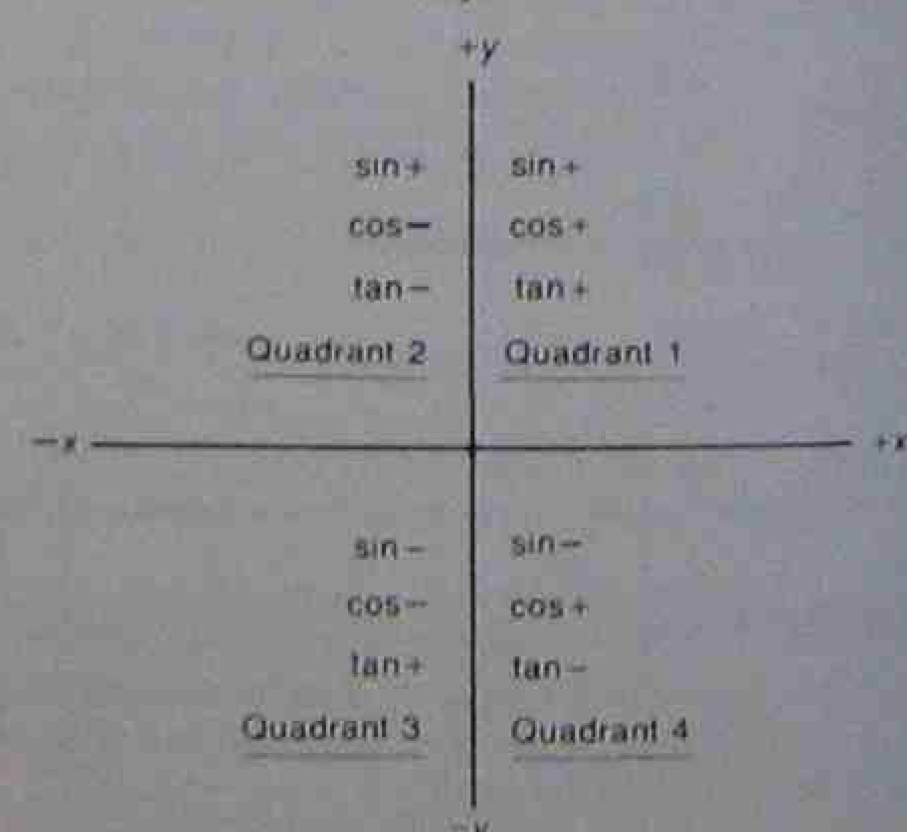

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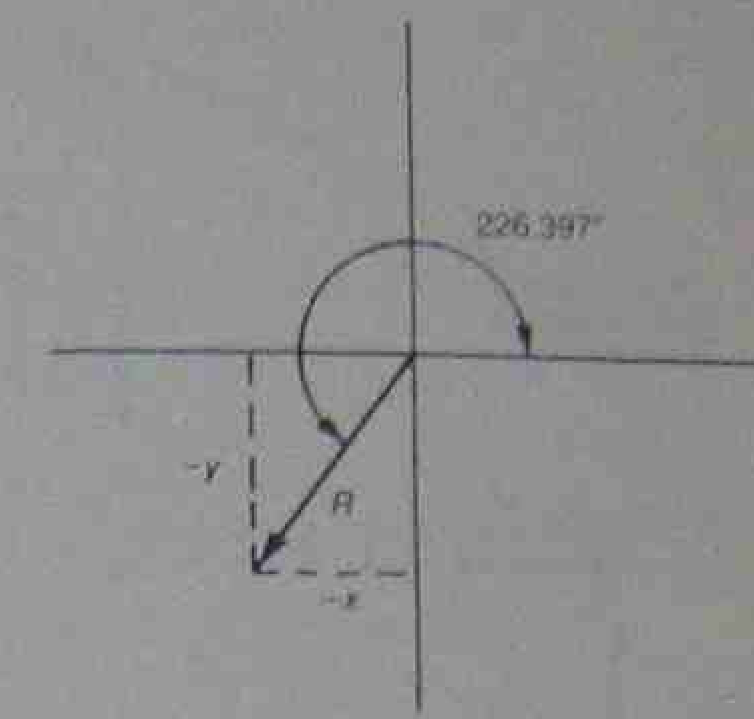

Figure 1.20 • Diagram for example 1.17

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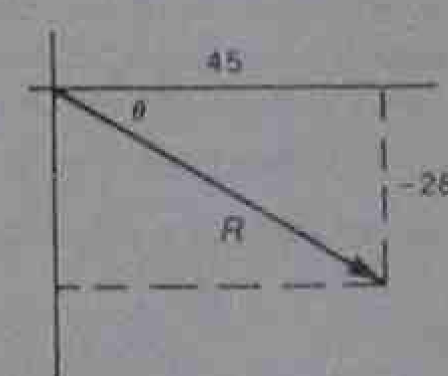

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From the tables, this represents an angle of 31.89°.

The angle from the reference vector is:

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The length R can be calculated from trigonometrical values or by:

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$$= \sqrt{2025 + 784}$$
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Evaluate the following: sin 97°, cos 184°, tan 215°, tan 290°, and cos 340°.

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By means of rectangular components, find the resultant of the following forces. All angles given are according to conventional rotation: F_1, 25 N at 80°; F_2, 50 N at 135°; F_3, 15 N at 215°; and F_4, 35 N at 320°.

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	Totals	−16.49	+28.87	

The two components of the resultant can be plotted on a vector diagram (Fig. 1.23).

Value or magnitude of the resultant:

$= OA$

$= \sqrt{(-16.49)^2 + (28.87)^2}$

$= 33.25$ N

Direction:

$$\tan \text{ of angle} = \frac{\text{opp.}}{\text{adj.}} = \frac{28.27}{-16.49} = -1.75$$

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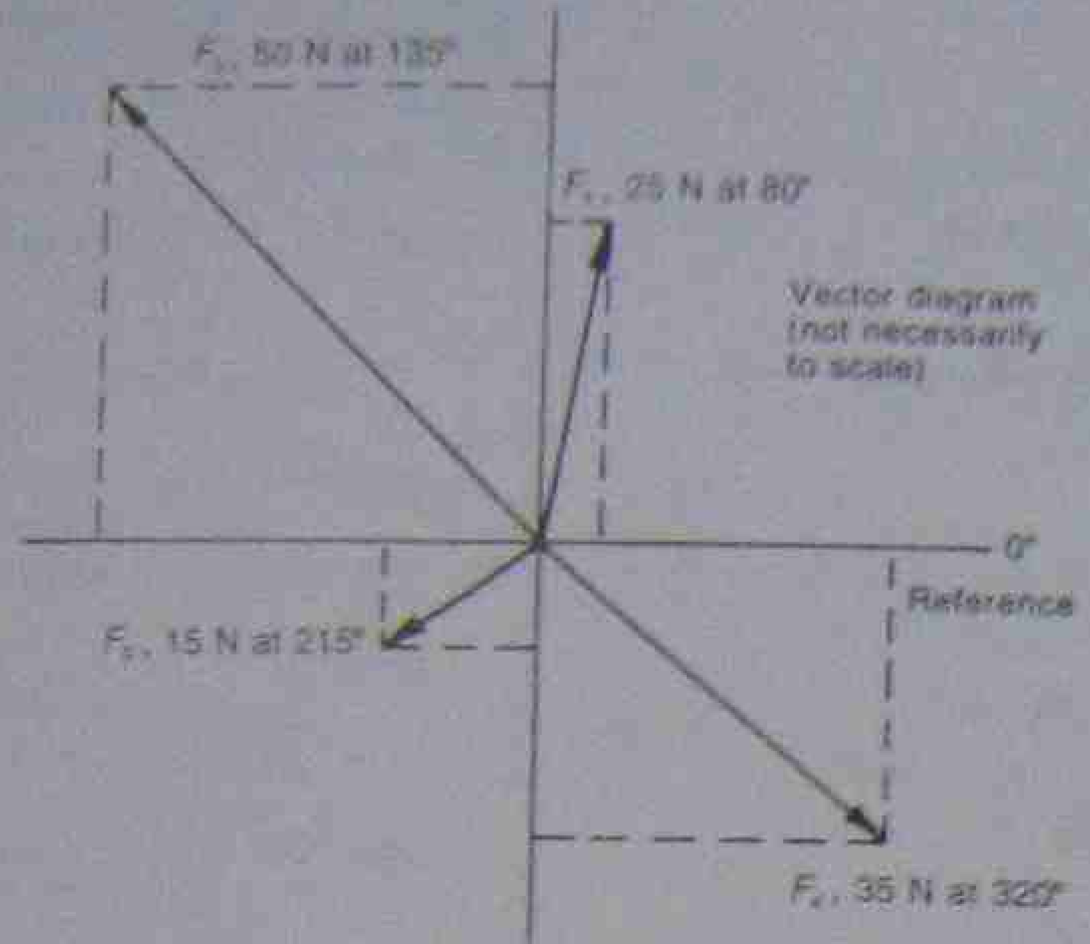
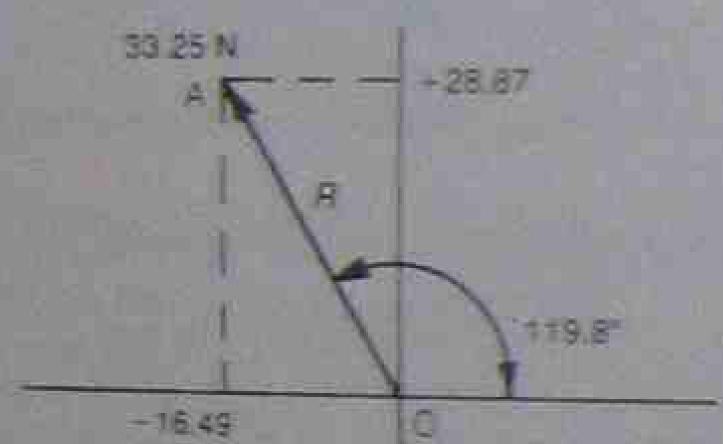


ure 1.22 • Force diagram for example 1.20



gure 1.23 • Diagram for example 1.20

.7 GRAPHS

he idea of a graph is widely understood as a means of onveying general impressions and characteristics. A aph usually illustrates the relationship between two sets f numbers and the method shows the connection as a he picture.

.7.1 Axes

hile a graph may be considered as a mathematical cture, it must follow certain conventions to be generally cepted and meaningful. One of the most important atters to be considered when preparing a graph is the lection of axes. The horizontal axis is chosen to represent e cause axis (independent variable), while the vertical xis represents the effect axis (dependent variable). When oney is placed in a bank account each week, the total ccumulates and can be illustrated with a graph. Note at the total increases over a period of time (cause), and he effect comes about because of time. The reverse cannot e true. Placing a sum of money in a bank account does ot make a week pass by. The actual situation allows time r money to be saved and placed in the account.

All other things being equal, a plant grows over a period of time. Time does not pass because the plant grows.

The graph in Figure 1.24 shows that the amount held ccumulates steadily for five weeks. The lack of a step at eek 6 also tells part of the story, suggesting that no money was banked that week.

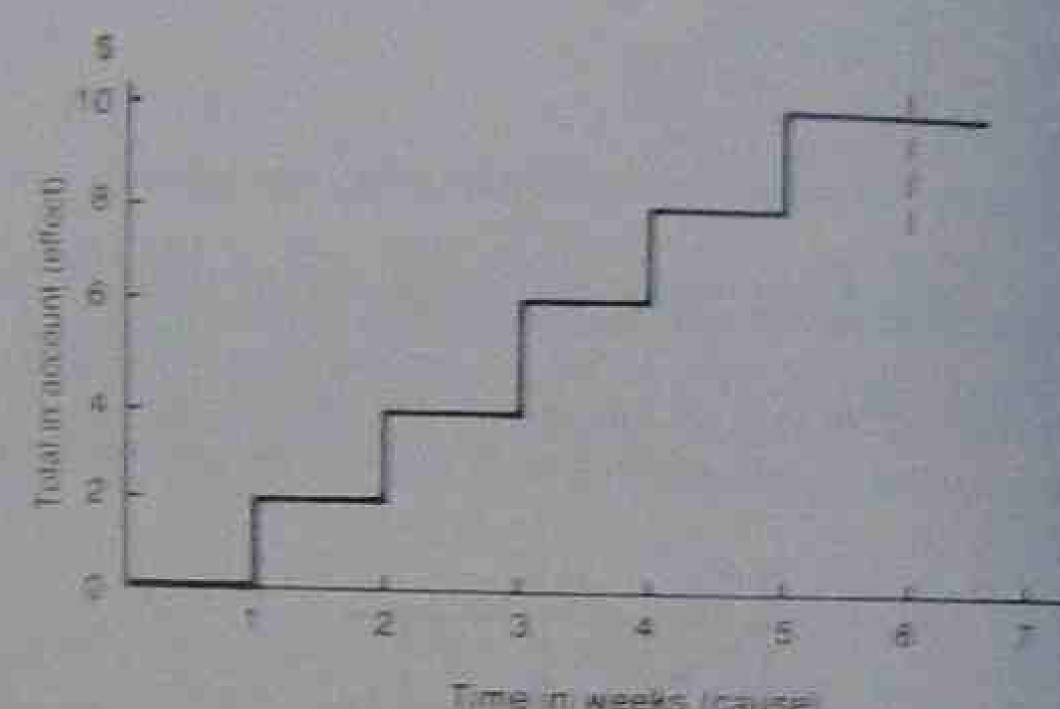


Figure 1.24 • Money accumulating in a bank account

1.7.2 Scales

Before a graph can be drawn, the axes need to be selected, followed by appropriate scales for the axes. A graph must be fitted into an available space in reasonable proportions, like any other type of picture. Other than in exceptional circumstances, an axis should have a zero position. A sense of proportion or a knowledge of characteristics can be destroyed without the zero reference point. Figure 1.25(a) appears to show a very rapid decrease in pressure as a load is applied; in fact, due to bad scale selection it appears at first glance to drop to zero at 10 units of load, but this is an incorrect impression. Reference to Figure 1.25(b) puts the graph in its correct perspective by showing that there is only a gradual decline, from no load to 10 units of load.

1.7.3 Steps and curves

Figure 1.24 shows a sum of money accumulating in steps of increments of $2 per week. The figures shown in Table 1.8 illustrate the growth of a plant and are derived from measurements taken over a period of time. The values are plotted as a series of marked points in Figure 1.26(a).

Figure 1.26(b) shows these points joined in a series of

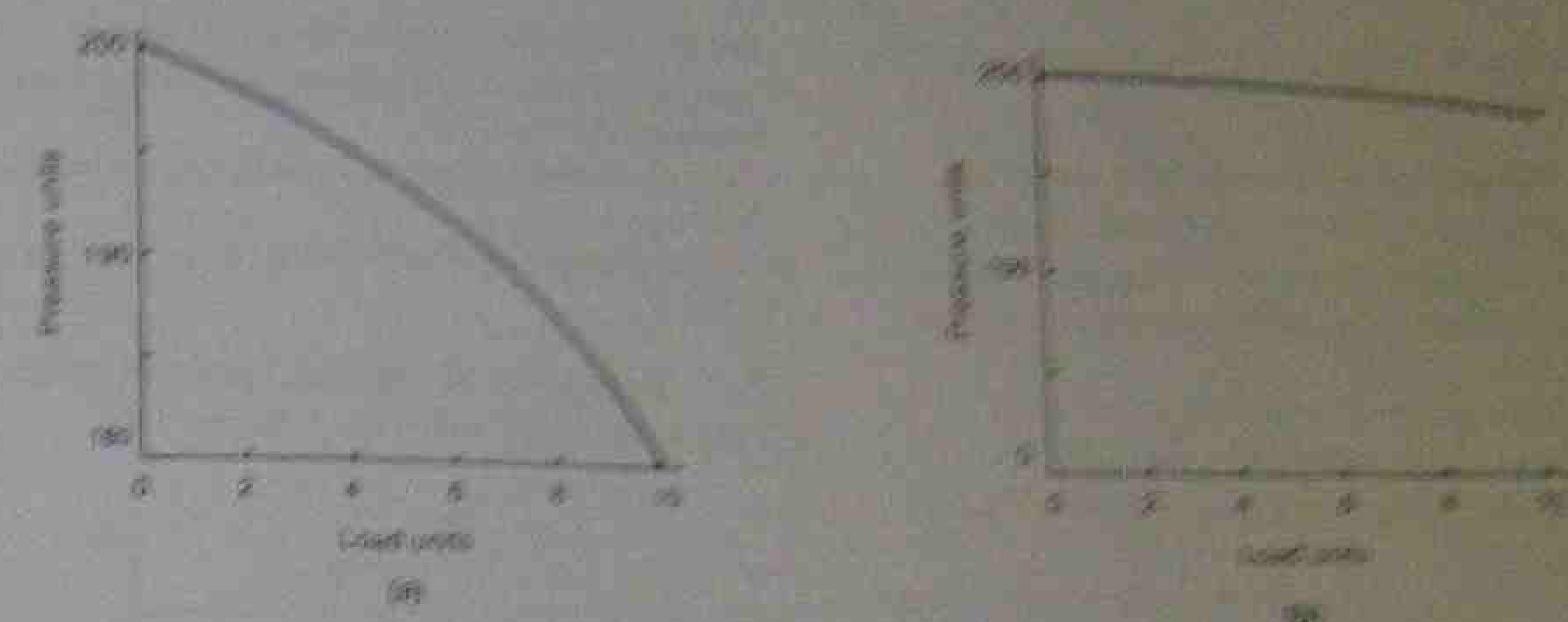


Figure 1.25 • Scale selection

Table 1.8 • Growth stages of a plant

Time (days)	0	5	10	20	30	40	50	60	70
Height (mm)	0	28	40	53	60	65	68	70	70

steps in a similar fashion to Figure 1.24. Obviously a plant does not grow in a series of steps and starts but has a gradual growth over a period of time. The points (coordinates) should therefore be joined by a smooth curve to indicate the gradual and steady growth of the plant, as in Figure 1.26(c).

The steps in Figure 1.24 show sudden increases of the total in line with the depositing of money. Care must be taken to ensure that points plotted on a graph are joined with the type of curve appropriate to the manner in which the values change. Examples of various types of curves are shown in section 1.7.4.

1.7.4 Lines and curves of best fit

When experimental data are taken, the results obtained are not always in line with theoretically calculated values. When these are plotted on a graph, it is not uncommon for discrepancies or inaccuracies to show up. It is usual to draw the graph as a smooth curve through the mean, or average, of the plotted points. Figure 1.27(a) shows a linear or straight-line graph drawn through the mean points.

The thin line of Figure 1.27(b) is the actual linear or non-straight-line graph and shows the effect of joining all the plotted points, which makes the graph meaningless for all practical purposes. The thick line is the correct version.

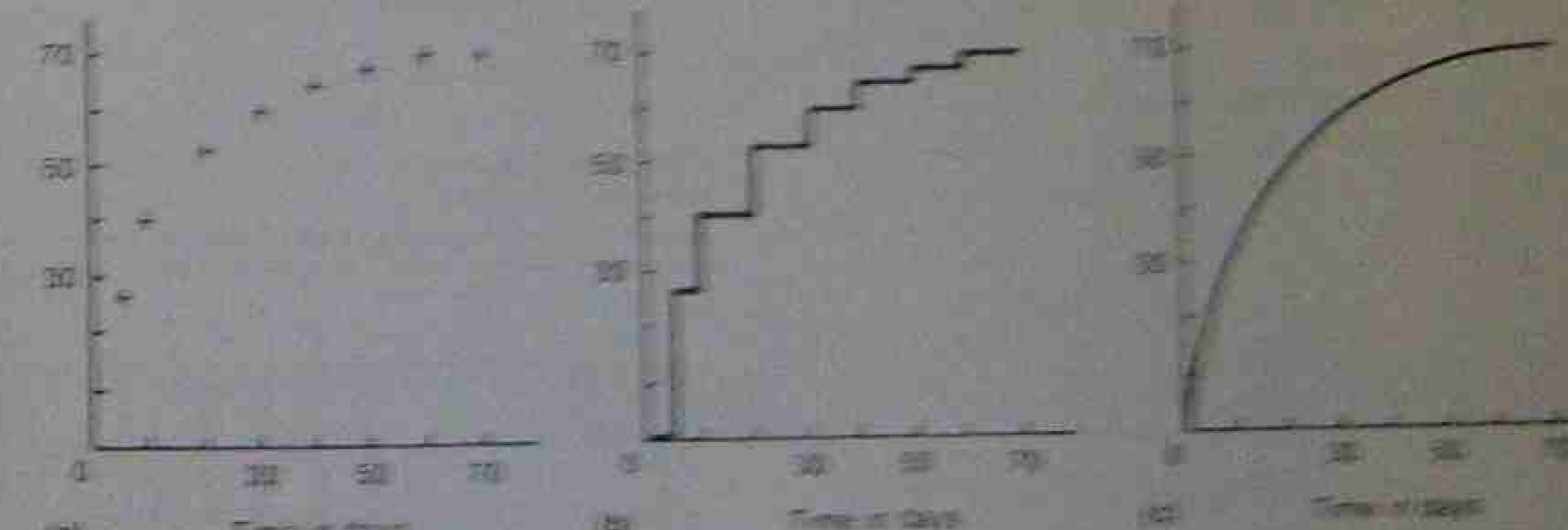


Figure 1.26 • Growth rate of a plant

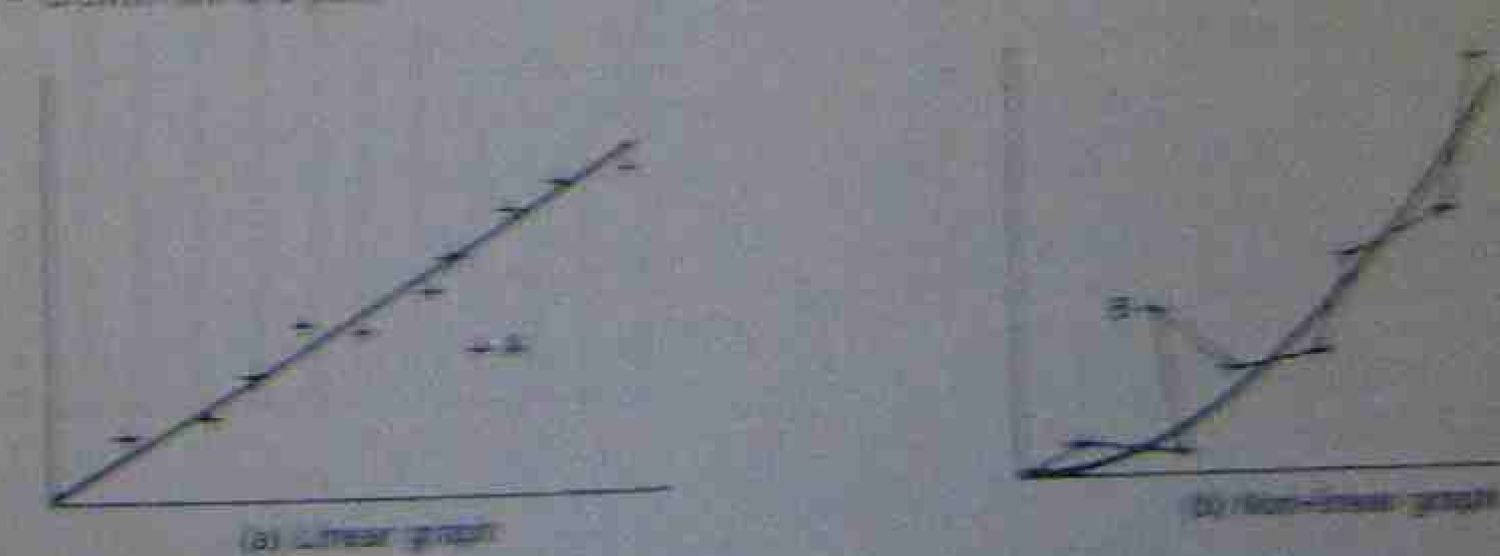


Figure 1.27 • Curves of best fit

Points A and B on the graphs are obvious errors and are ignored in drawing up the graphs.

1.7.5 Graph conventions and guides

1. Horizontal axis: cause or independent variable.
2. Vertical axis: effect or dependent variable.
3. Accuracy: length coordinates should be read to 0.5 mm or better.
4. Line thickness: equal to or less than 0.5 mm.
5. Curves on a graph should be continuous.
6. Lines to distinguish axes should be wider or thicker than grid lines.
7. The two main axes should intersect at zero.
8. Each of the axes should be labelled and the unit of measurement indicated.
9. Use curves of best fit where appropriate to the type of curve.

These conventions are shown in the examples of section 1.7.6.

1.7.6 Examples of various types of graphs

See Figures 1.28, 1.29, 1.30, 1.31, 1.32 and 1.33.

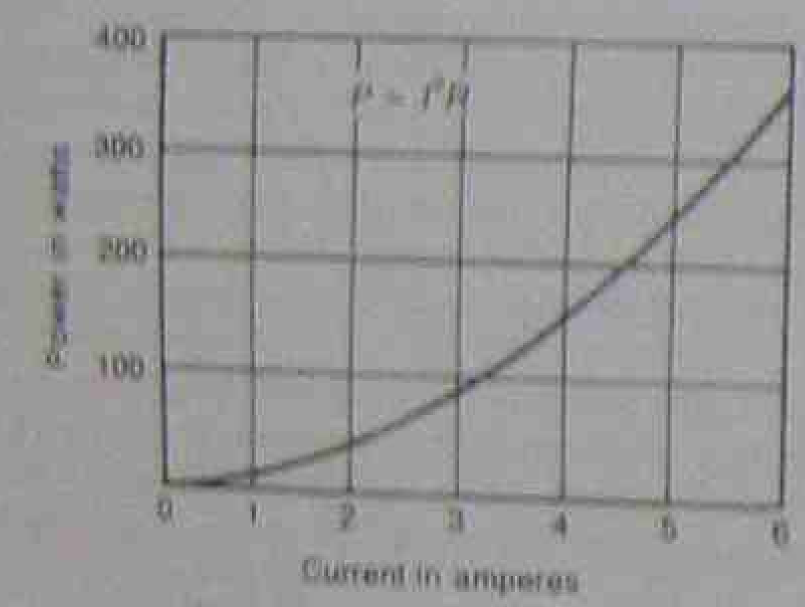


Figure 1.28 • Single curve graph

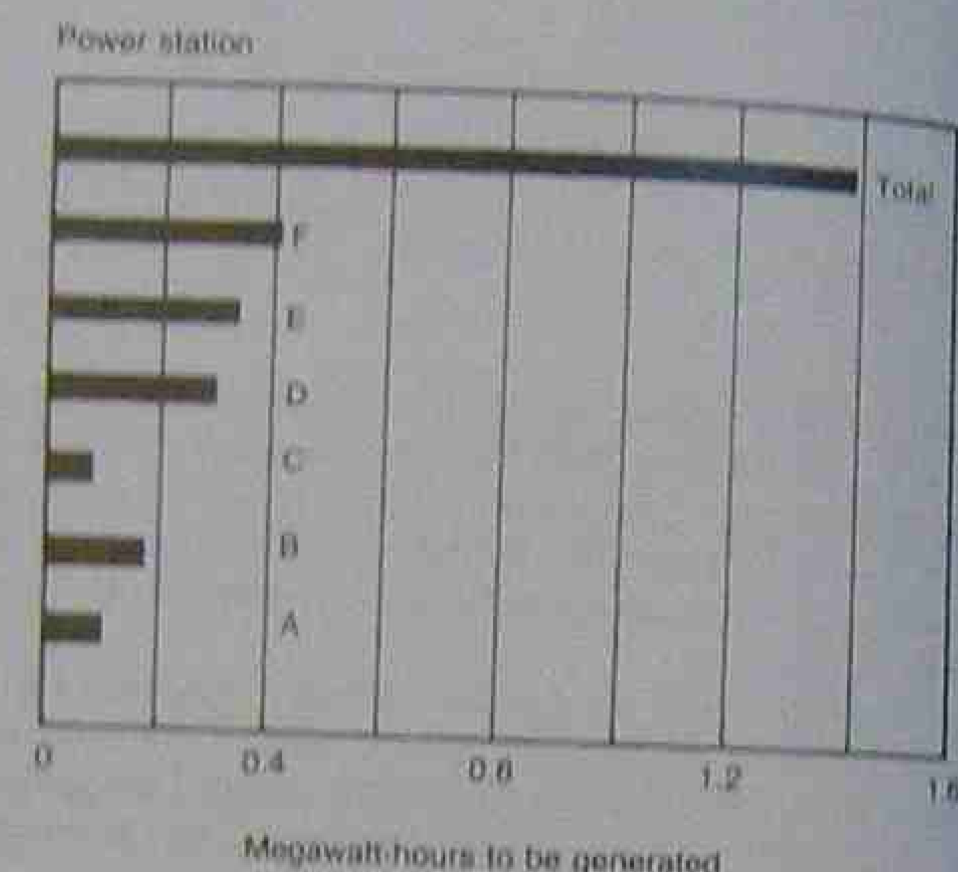


Figure 1.30 • Horizontal bar graph

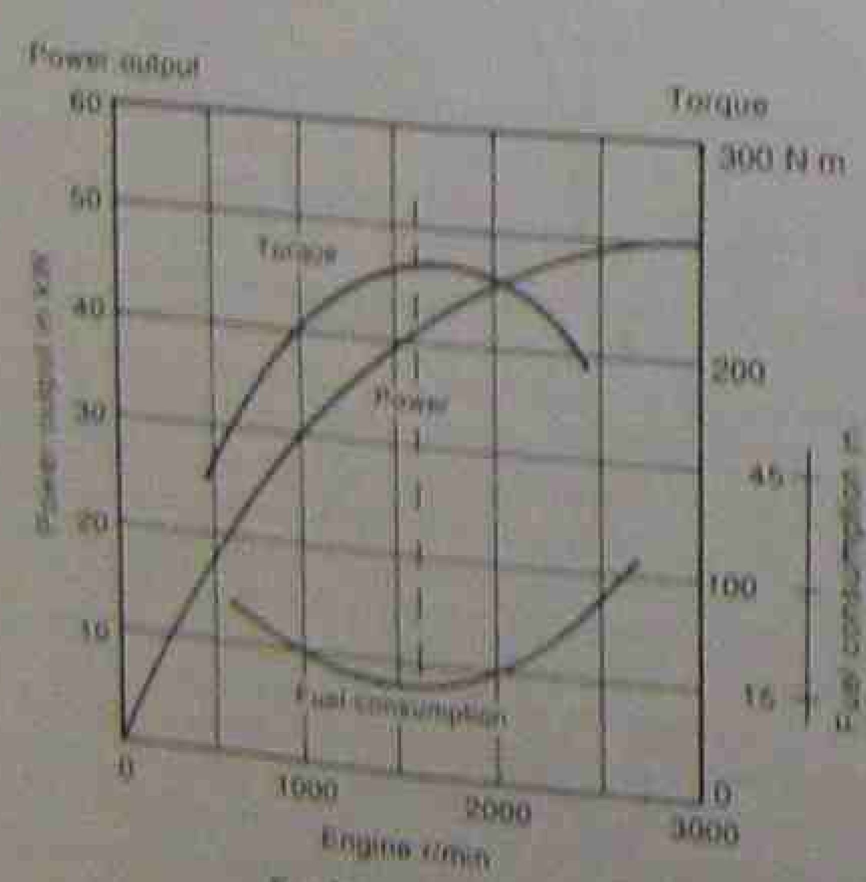


Figure 1.29 • Multiple curves graph on common axis

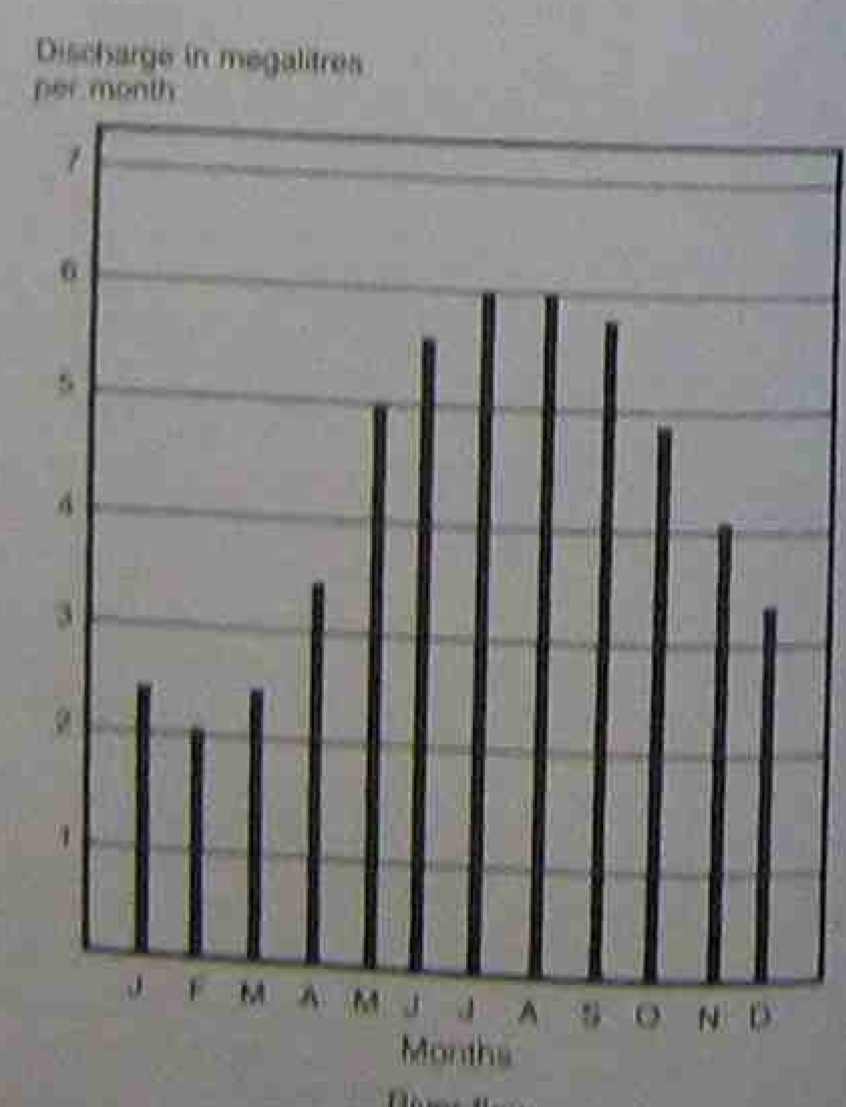


Figure 1.31 • Vertical bar graph

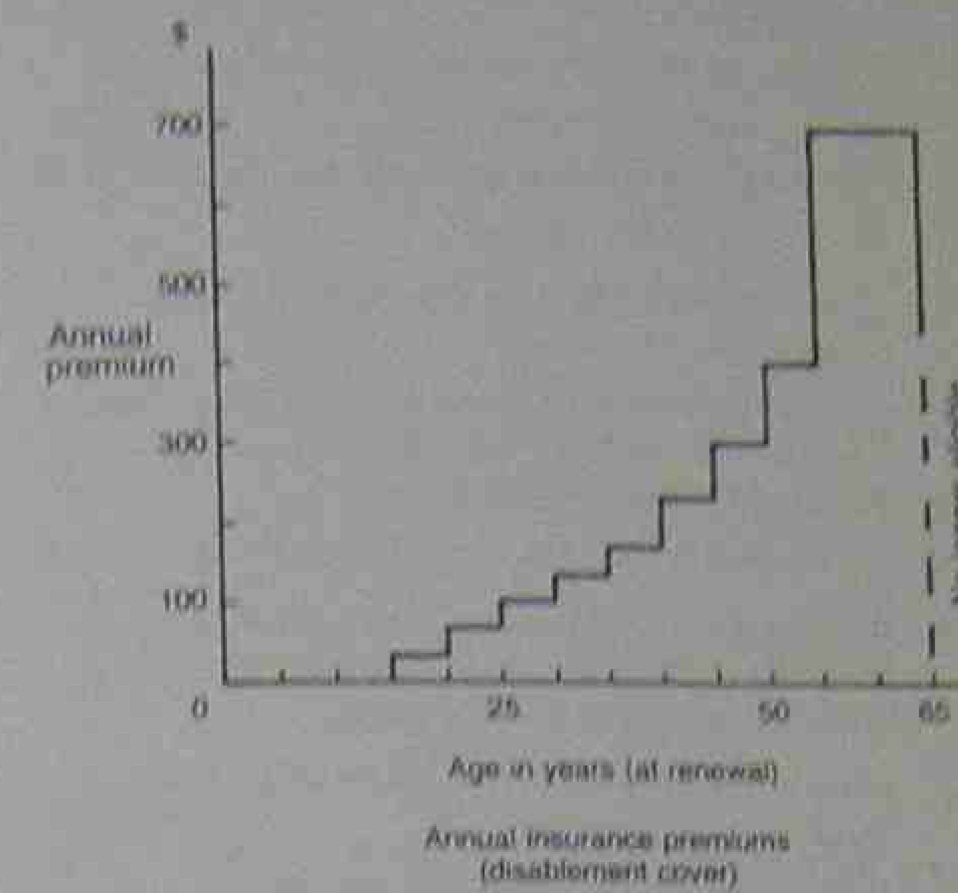


Figure 1.32 • Step graph

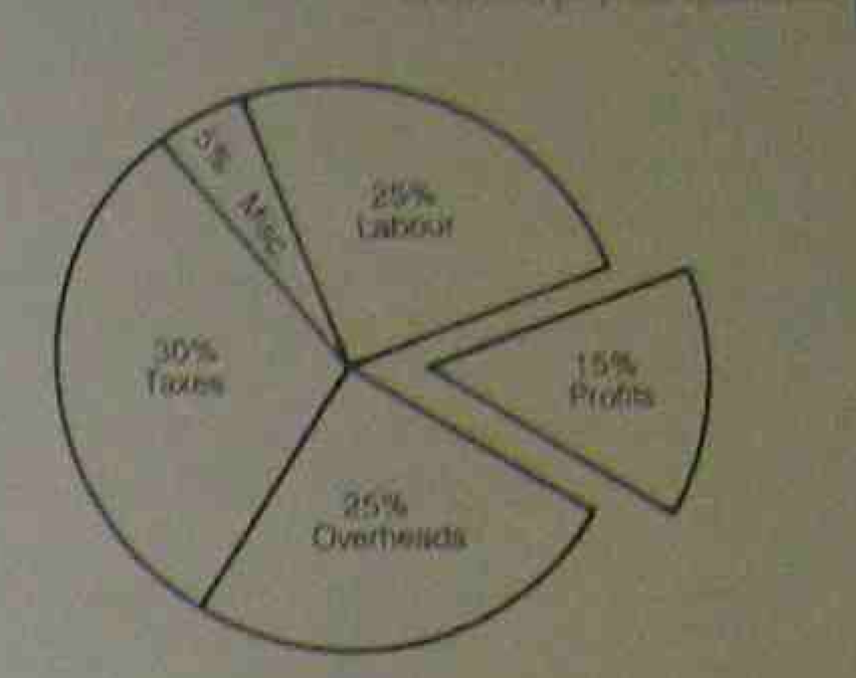


Figure 1.33 • Pie graph with 'exploded' section for emphasis

SUMMARY

- The SI method of units is a metric system and is a worldwide standard with very few exceptions.
- There are only six main units in the system, plus one supplementary.
- From these few units there are many derived units—mechanical, electrical, and magnetic.
- The system has standard multiples and submultiples with specific names.
- Scientific notation is an approved method for expressing quantities.
- Work, power, energy and torque are components of the system and apply to electrical and mechanical systems.
- All machines have losses. Efficiency is the ratio of the input and output values—expressed usually as a percentage.
- Any force can be expressed as a value and having direction.
- Mechanical forces are expressed as vectors.
- Electrical forces are expressed in terms of phasors.
- Combinations of forces give rise to a resultant force.
- A resultant force can be evaluated by drawing out to scale, with due regard to length and direction.
- Resultant forces can also be determined by calculation as rectangular quantities with the aid of trigonometry.
- Trigonometry is a mathematical system where angles are expressed in terms of ratios of the sides of a triangle.
- A graph is a pictorial representation of a series of quantities.
- Graphs have axes, and scales for those axes.
- Graphs can be drawn in several ways depending on what information is to be conveyed.

SELF-TESTING PROBLEMS

1.1 Convert the following to the units indicated:

1001 m to km	12 578 Pa to kPa
2582 km to m	125 J to MJ
25 043 MW to W	1400 kHz to MHz

1.2 Express the following numbers in scientific notation:

1 267 008	10 658
0.001	0.1
250	0.000 0026

1.3 Change the numbers listed below from scientific notation to regular numbers:

6.8×10^{-3}	7.47×10^{-3}
1.2×10^{6}	9.2×10^{-1}
1.2×10^{-6}	4.75×10^{3}

1.4 Calculate the amount of work done if a force of 520 N has to be exerted to move a body a distance of 4.3 m.

1.5 What power is being used if the operation in problem 1.4 above was done in:
(a) 5 s? (b) 30 s?

1.6 The work done in shifting a block of steel 2 m is 510 J. What force has to be exerted on the block?

1.7 A motor cycle is push-started against compression at a uniform speed of 4.5 m/s for 4 s. If the power expended is 600 W, what force was being exerted on the motor cycle's handle bars?

1.8 Two forces of 150 N and 200 N act at a point at 60° to each other. Using the parallelogram of forces

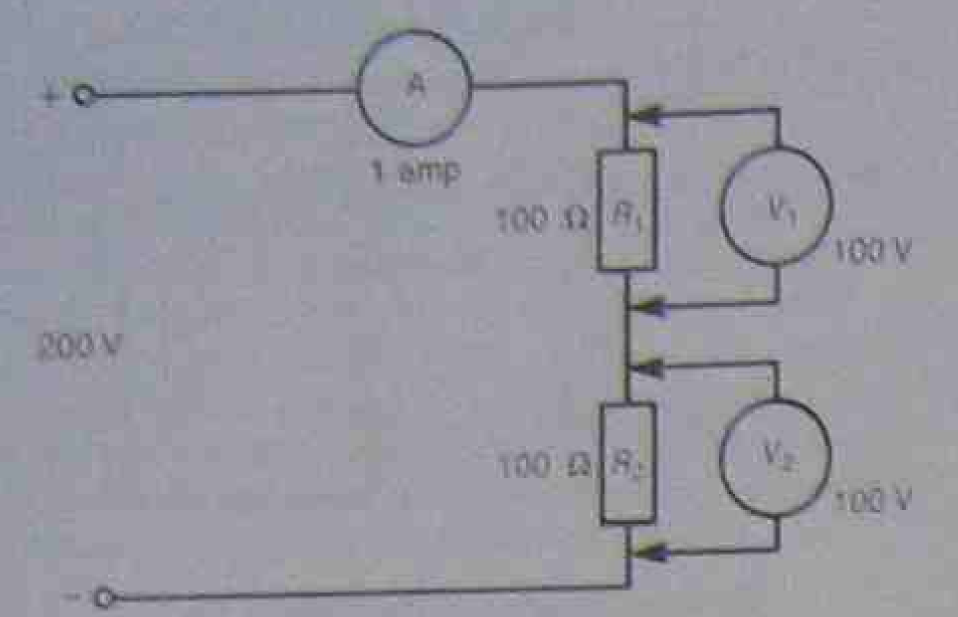


Figure 4.8 • Two loads connected in series to a supply source

the total drop across R_1, R_2 and R_3 is measured by V_5 and is equal to the applied battery voltage (V).

The ammeter measures the current flowing from the battery under a pressure of 6 V. With three equal-value resistors in series, the potential differences in each case will be equal ($V = IR = 2$ V in Fig. 4.9). If the resistors in series are not equal in value, the potential differences across each resistor will not be equal. Note, however, that the sum of their potential differences will always be equal to the supply voltage irrespective of whether the resistors are equal or not.

In this example the readings on each meter, V_1, V_2 and V_3 are the same and equal to one-third of the total applied voltage, or 2 V.

In section 4.3.1 it was shown that the resistance of a series circuit was equal to the sum of the individual resistances connected between these points:

$$R_{total} = R_1 + R_2 + \ldots$$

Between the terminals of V_4, R_1 and R_2 are connected in series, so the resistance between these two points is double the resistance between the terminals of either V_1, V_2 or V_3. Therefore V_4 should indicate $2 + 2 = 4$ V. Similarly, V_5 is equal to the sum of the voltage drops between its terminals: that is:

$$V_1 + V_2 + V_3 = 2 + 2 + 2 = 6 \text{ V}$$

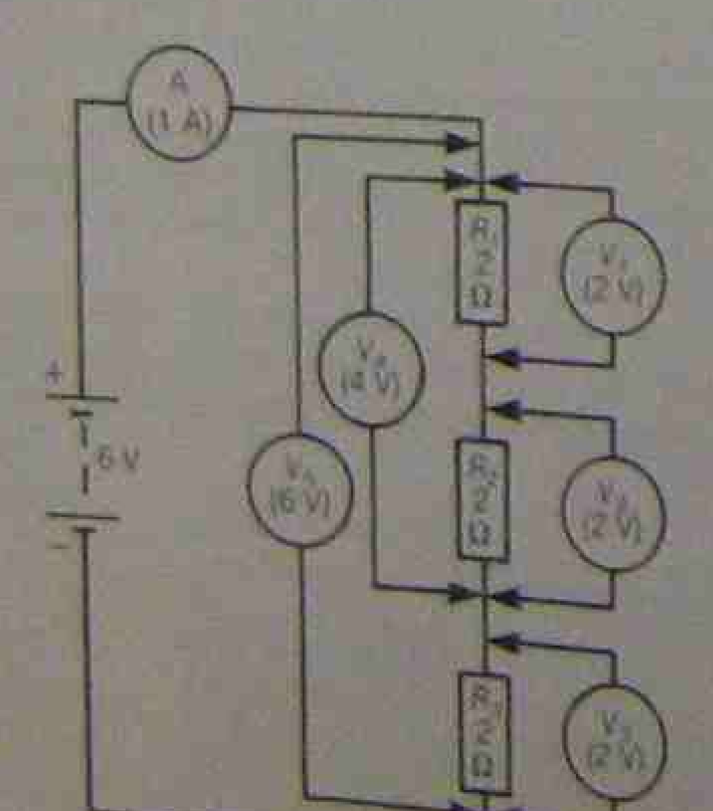


Figure 4.9 • Potential differences in a series circuit

These facts can be summarised by saying that when current is forced through a circuit against the resistance of that circuit, a fall of potential occurs across the resistance.

The voltage applied to a series circuit is equal to the sum of the individual voltage drops in the circuit. That is:

$$V_{total} = V_1 + V_2 + V_3 + \ldots$$

This is often expressed as Kirchhoff's voltage law:

The algebraic sum of the voltage drops around a circuit equals the applied voltage.

Example 4.3

What is the total voltage necessary to force a current of 5 A through the circuit of Figure 4.10?

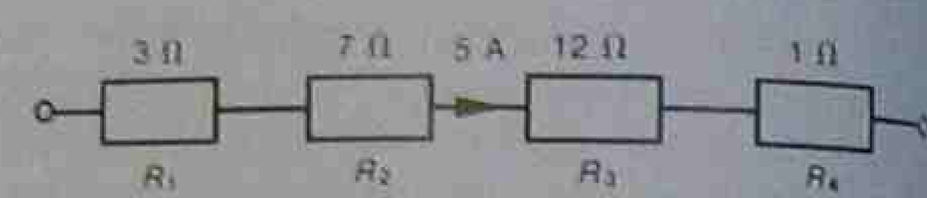


Figure 4.10 • Circuit diagram for example 4.3

The quickest method of calculating the value of V is to find the equivalent resistance of the circuit and multiply it by the total current:

$$\begin{aligned} R_{total} &= R_1 + R_2 + R_3 + R_4 \\ &= 3 + 7 + 12 + 1 \\ &= 23\ \Omega \\ V &= IR = 5 \times 23 = 115 \text{ V} \end{aligned}$$

This value can be checked by finding the sum of all the voltage drops:

$$\begin{aligned} V_1 &= IR_1 = 5 \times 3 &&= 15 \text{ V} \\ V_2 &= IR_2 = 5 \times 7 &&= 35 \text{ V} \\ V_3 &= IR_3 = 5 \times 12 &&= 60 \text{ V} \\ V_4 &= IR_4 = 5 \times 1 &&= 5 \text{ V} \\ V_{total} &= V &&= 115 \text{ V} \end{aligned}$$

4.3.4 Conditions for series circuits—summary

1. **Resistance**

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

2. **Current**
There is one path for current, so there is one constant value of current throughout the series circuit.

3. **Voltage**
The applied voltage equals the sum of the individual voltage drops around the series circuit:

$$V_{total} = V_1 + V_2 + V_3 + \ldots$$

4.3.5 Power in series circuits

In Chapter 2, reference ...

$$P = I^2R = VI = \frac{V^2}{R}$$

The same formulae are still applicable in this type of work.

However, it is necessary to be careful that the correct values are chosen.

Example 4.4

In Figure 4.11, three resistors are connected in series across a 240 V supply. Find the power consumption of each resistor and the total power consumed by the circuit.

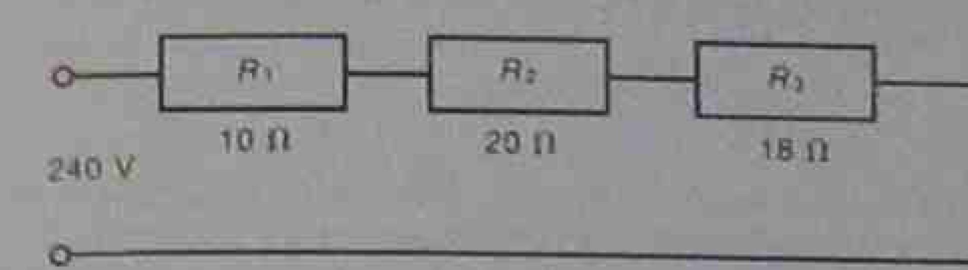


Figure 4.11 • Circuit diagram for example 4.4

There are two methods for finding the power consumption of a circuit:

1. Obtain the power consumption for each resistor and add those values to find the total.
2. Obtain the total resistance, total voltage and current, and then use these values to find the total power consumption.

Method 1

Each resistor is taken individually and the values used in the calculations apply only to that resistor. To use Ohm's law or the power formulae, two quantities must be known. The resistor value is known, so either the voltage across it or the current through it must be known.

$$\begin{aligned} R_{total} &= R_1 + R_2 + R_3 \\ &= 10 + 20 + 18 \\ &= 48\ \Omega \end{aligned}$$

$$I = \frac{V_{total}}{R_{total}} = \frac{240}{48} = 5 \text{ A}$$

That is, all resistors have 5 A flowing through them (a series circuit).

Power consumed in 10 Ω resistor:

$$P = I^2R = 5^2 \times 10 = 250 \text{ W}$$

Power consumed in 20 Ω resistor:

$$P = I^2R = 5^2 \times 20 = 500 \text{ W}$$

Power consumed in 18 Ω resistor:

$$P = I^2R = 5^2 \times 18 = 450 \text{ W}$$

Total power = 250 + 500 + 450 = 1200 W

Method 2

$$\begin{aligned} R_{total} &= R_1 + R_2 + R_3 \\ &= 10 + 20 + 18 \\ &= 48\ \Omega \end{aligned}$$

This second method gives the total power, but not the individual power ratings as in the first method.

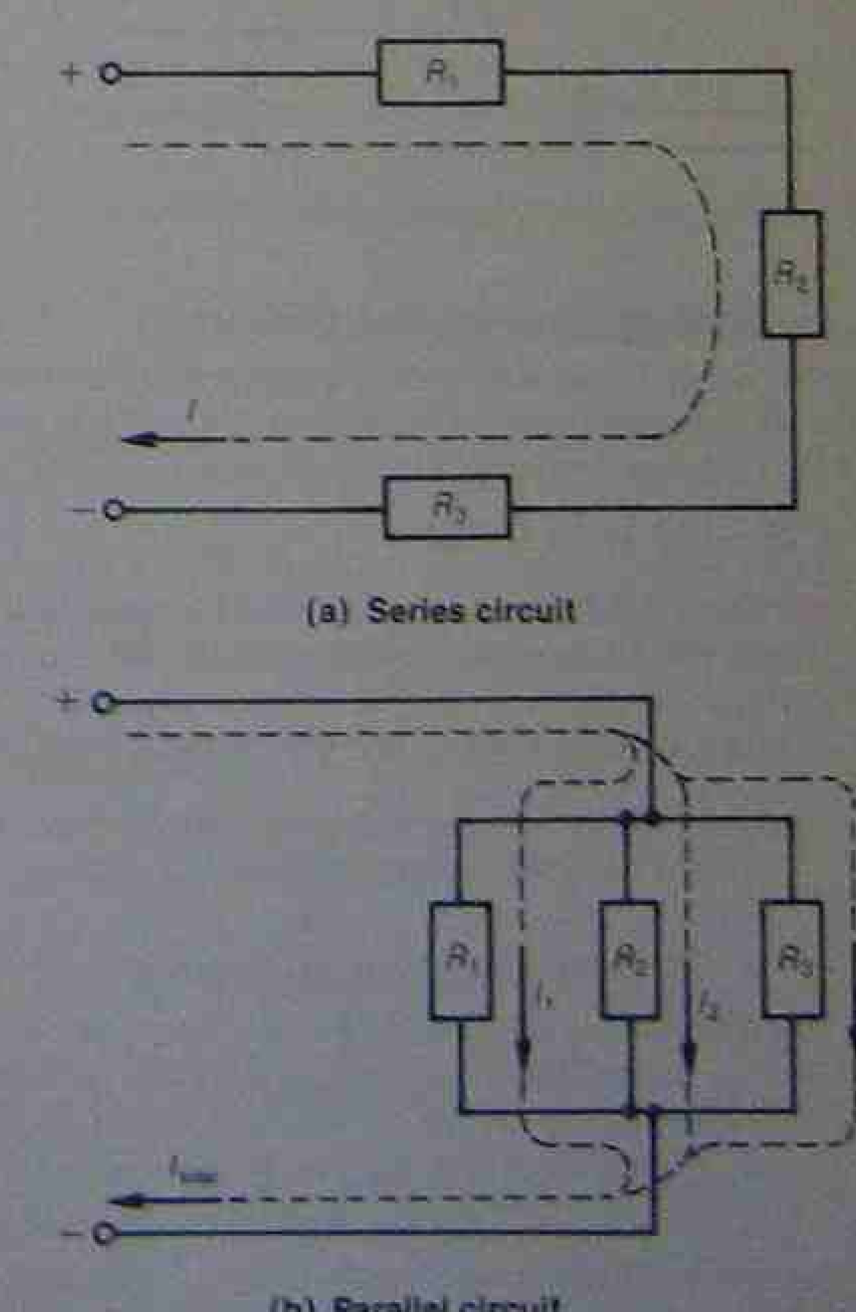
In a circuit with only one current path, any break in the circuit will cause all current to stop flowing. Circuit power consumption is reduced to zero and the full supply voltage appears across the ends of the open circuit. Where more than one series resistor is involved, the voltage across the break may exceed the original voltage drop across individual resistors. The circuit will remain inoperative until the cause of the open circuit is found and rectified.

A partial short-circuiting of one of the series resistors is equivalent to presenting a lower resistance to the supply source, and current flow will increase. If the short-circuit occurs nearer the supply source, the current flow could then increase to a value that might cause damage to the circuit or its conductors. If the circuit is fitted with circuit protection, then under normal conditions the protection will operate and the complete circuit will be isolated from the supply source.

4.4 PARALLEL CIRCUIT ANALYSIS

Parallel circuits are multiple circuits, in that they have more than one current path between the two terminals of the power source.

The major difference between series and parallel circuits is illustrated in Figure 4.12. In Figure 4.12(a) only one current path is shown, while in Figure 4.12(b) it can be seen that the current from the source divides into several components when the parallel section is reached and recombines again to flow to the source. For each path $I = V/R$, where R is the resistance of that path.



4.4.1 Current in parallel circuits

Figure 4.12(b) shows the current flowing from the source of supply as I_{total} and the currents flowing through the individual resistors as I_1, I_2 and I_3. Since I_{total} splits into three components and then recombines:

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

This expression can be expressed as Kirchhoff's current law:

> *The algebraic sum of all the currents entering a junction equals the algebraic sum of all the currents leaving that junction.*

This is illustrated in Figure 4.13 where a total current of 30 A is flowing. When the current reaches the parallel section, it divides into three 10 A components and afterwards combines to give the total of 30 A again:

current entering junction A = 30 A

current leaving junction A = 10 + 10 + 10 = 30 A

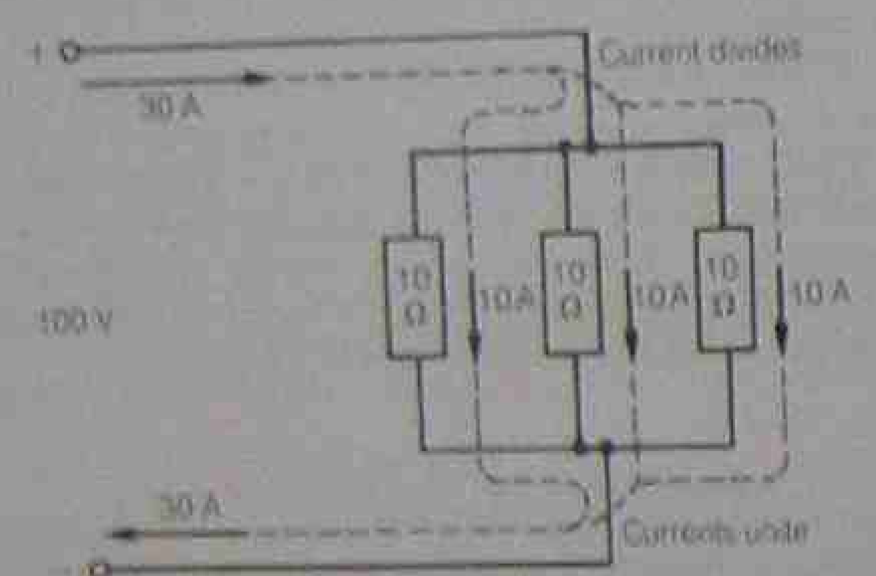


Figure 4.13 • Currents in a parallel circuit

4.4.2 Voltage in parallel circuits

If each resistor in Figure 4.13 were considered in isolation as a single 10 Ω resistor, the application of Ohm's law would show that each has the same voltage across it. That is:

$$V = IR = 10 \times 10 = 100 \text{ V}$$

In a parallel circuit, each resistor has the same voltage across it and the voltage is a constant value for that parallel section.

In Figure 4.13, each resistor has 100 V potential difference across it and, because each has a resistance value of 10 Ω, each also has the same value of current flowing through it. In practice this is seldom found; the resistors often have varying values and the current values are therefore different.

Example 4.5

Three resistors of 2 Ω, 4 Ω and 8 Ω are connected in parallel across a 12 V supply as in Figure 4.14. Calculate the total current drawn from the supply.

Take each resistor singly and apply Ohm's law to it as shown in Figure 4.15. (Each resistor has a potential difference of 12 V across it.)

$$I_{total} = I_1 + I_2 + I_3$$
$$= 6 + 3 + 1.5$$
$$= 10.5 \text{ A}$$

That is, the total current is the sum of the individual currents. This example also shows one other point: the three resistors in parallel each have 12 V across them. Stated in general terms, this means that loads (resistors) connected in parallel always have the same voltage across them.

4.4.3 Resistance in parallel circuits

When the total current flowing in the circuit of example 4.5 is considered and Ohm's law is applied to it, a value of resistance is found that has no apparent connection with any of the three resistors. For example:

$$R = \frac{V}{I_{total}} = \frac{12}{10.5}$$
$$= 1.14 \; \Omega$$

When considered more carefully it becomes apparent that a resistor of 1.14 Ω placed across a 12 V supply draws the same amount of current as the three resistors in example 4.5. If the three resistors (2 Ω, 4 Ω, 8 Ω) were to be replaced by their equivalent resistance, the current drawn from the supply would still be 10.5 A (see Fig. 4.16).

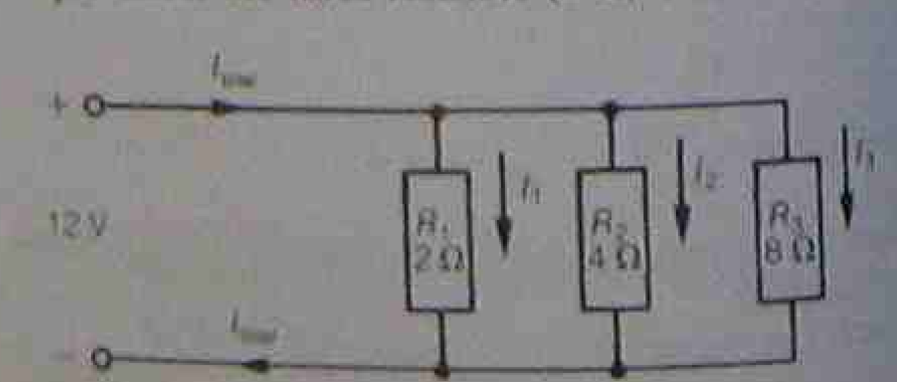
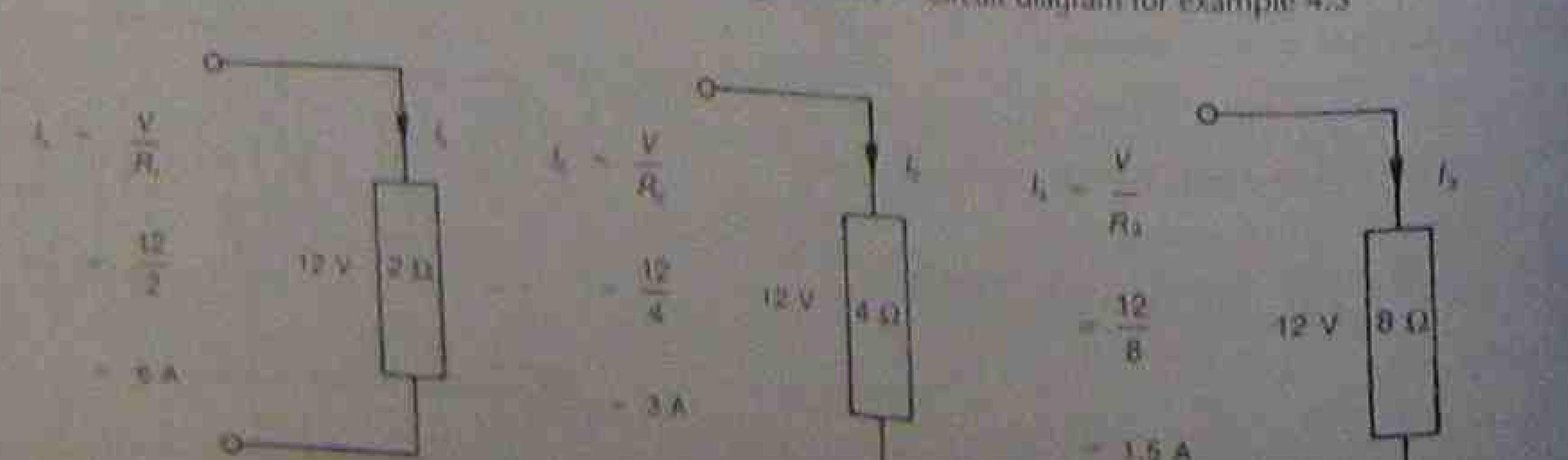


Figure 4.14 • Circuit diagram for example 4.5



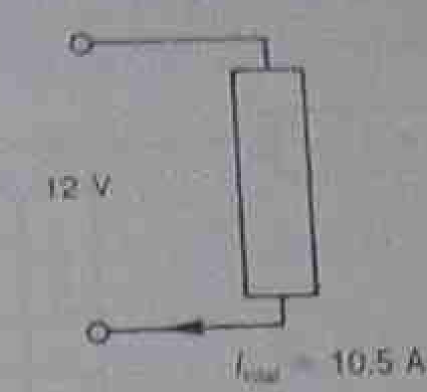


Figure 4.16 • Equivalent circuit for example 4.5

The equivalent or total resistance of a parallel circuit can be found from:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Example 4.6

Using the above formula for resistances in parallel, calculate the total resistance of the three resistors in example 4.5. The circuit is shown in Figure 4.17.

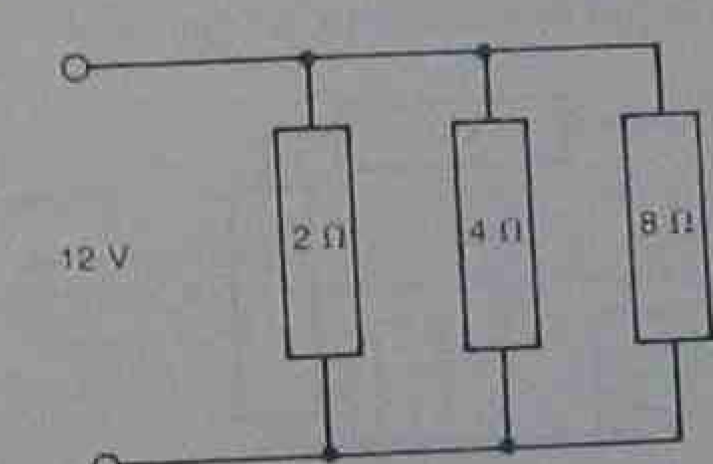


Figure 4.17 • Circuit diagram for example 4.6

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$
$$= \frac{4 + 2 + 1}{8}$$

that is,
$$\frac{1}{R_{total}} = \frac{7}{8}$$
$$\therefore \frac{R_{total}}{1} = \frac{8}{7}$$
$$\therefore R_{total} = 1.14 \; \Omega$$

Note: It must be remembered that $1/R_{total}$ must be processed mathematically to obtain the value of R_{total} (the reciprocal).

Example 4.7

Find the total current flowing when a 25 Ω resistor and a 5 Ω resistor are connected in parallel to a 50 V supply.

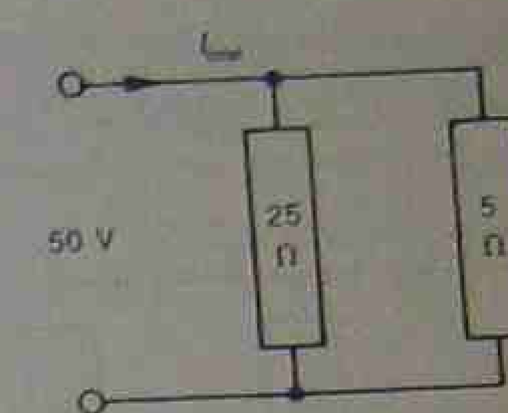


Figure 4.18 • Circuit diagram for example 4.7

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{25} + \frac{1}{5}$$
$$= \frac{1 + 5}{25}$$
$$= \frac{6}{25}$$
$$\therefore R_{total} = \frac{25}{6} = 4.167 \; \Omega$$
$$\therefore I_{total} = \frac{V_{total}}{R_{total}} = \frac{50}{4.167}$$
$$= 12 \text{ A}$$

The answer can be checked by finding the individual currents:

that is, 25 Ω resistor on 50 V: $I = \frac{V}{R} = \frac{50}{25} = 2$ A

5 Ω resistor on 50 V: $I = \frac{V}{R} = \frac{50}{5} = 10$ A

$\therefore I_{total} = I_1 + I_2 = 2 + 10 = 12$ A

4.4.4 Conditions for parallel circuits—summary

1. **Resistance**

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

2. **Current**

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

3. **Voltage**

The voltage across the parallel section is constant across each resistor in that section.

4.4.5 Power in parallel circuits

The total power consumption in a parallel circuit can be found by addition of the individual power consumptions of each resistor in the circuit. The total can be found in either of two ways, as with series circuits:

1. Add the individual values.
2. Find the total resistance, then the total current, and calculate the total power directly. Both methods are shown in example 4.8.

Example 4.8

Two resistors of 8 Ω and 24 Ω respectively are connected to a 20 V supply. Find the total power consumption. (See Fig. 4.19.)

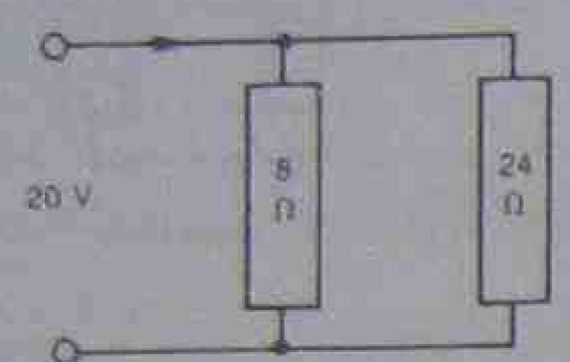


Figure 4.19 • Circuit diagram for example 4.8

Method 1

8 Ω resistor: $P = \frac{V^2}{R} = \frac{20^2}{8} = 50$ W

24 Ω resistor: $P = \frac{V^2}{R} = \frac{20^2}{24} = 16.67$ W

Total power consumption = 66.67 W.

Method 2

$$\frac{1}{R_{total}} = \frac{1}{8} + \frac{1}{24} = \frac{3+1}{24} = \frac{4}{24}$$

$$\therefore R_{total} = \frac{24}{4} = 6\ \Omega$$

$$\therefore P_{total} = \frac{V^2}{R} = \frac{20^2}{6} = 66.67\ \text{W}$$

In a circuit with several current paths, a circuit fault in one of the loads, or the conductors leading to that load, will cause current to stop flowing only in that section. Circuit power consumption is reduced to zero only in that part of the circuit, and the full supply voltage will appear across the ends of the open circuit. Where more than one paralleled load is involved, all loads connected to the supply after the fault are affected. The remainder of the connected loads operate normally.

In the event of a load becoming faulty and its resistance value being decreased, current flow to that load increases. If the current flow increases beyond a certain point, circuit protection would normally operate and isolate that circuit from the supply source. For partial short-circuits, other paralleled loads would continue to operate until subcircuit protection occurs.

4.5 COMBINED SERIES—PARALLEL CIRCUITS

The combined series–parallel-type of circuit is commonly found in practice. It may consist of parallel loads, with the series resistance being that of the supply lines, or it may consist of far more complicated circuits. In analysing combined circuits it is important to consider the components as separate parts of the whole and apply only those values to them that apply according to Ohm's law. The circuit 'rules' are those of the normal series or parallel circuits, as summarised in sections 4.3.4 and 4.4.4.

Example 4.9

Find the total current flowing and the voltages across each resistor in the circuit of Figure 4.20.

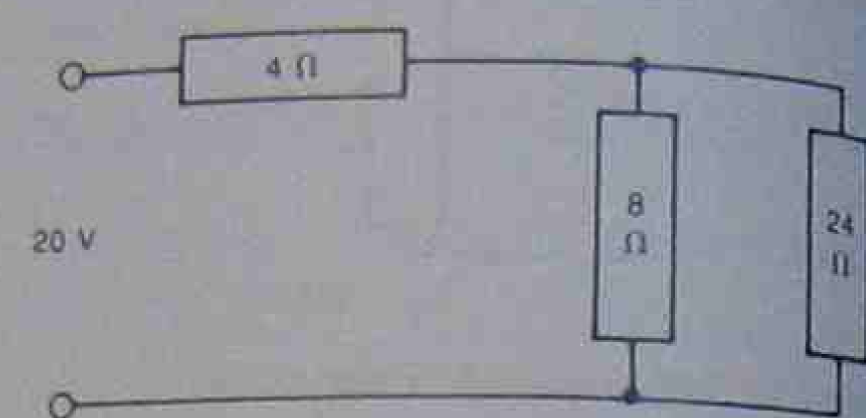


Figure 4.20 • Circuit diagram for example 4.9

Step 1

Reduce the parallel section to one equivalent resistance.

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{8} + \frac{1}{24} = \frac{3+1}{24} = \frac{4}{24}$$

$$\therefore R_{total} = \frac{24}{4} = 6\ \Omega$$

The circuit can be redrawn, as in Figure 4.21.

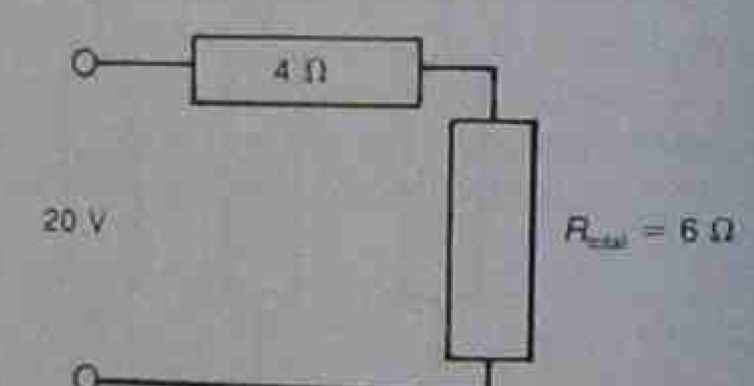


Figure 4.21 • Equivalent circuit—step 1

Step 2

Find the total resistance of the series circuit.

$$R_{total} = R_1 + R_2$$

$$= 4 + 6 = 10\ \Omega$$

Step 3

The circuit can then be redrawn as in Figure 4.22.

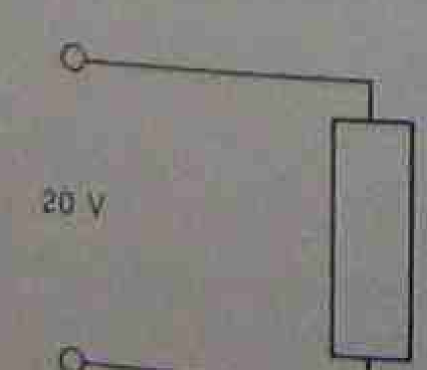


Figure 4.22 • Equivalent circuit—step 3

Find the total current flowing.

$$I_{total} = \frac{V}{R} = \frac{20}{\ldots}$$

Step 4

The total current flowing is 2 A and should then be applied to the series circuit in Figure 4.21. For the 6 Ω resistor ringed with a dotted line in Figure 4.23:

$$V = IR$$

$$= 6 \times 2 = 12\ \text{V}$$

Similarly for the 4 Ω resistor

$$V = IR = 4 \times 2 = 8\ \text{V}$$

(Check: $V_{total} = V_1 + V_2 = 12 + 8 = 20$ V)

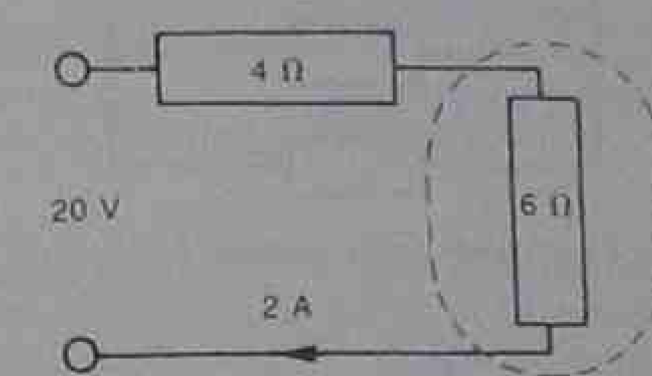


Figure 4.23 • Equivalent circuit—step 4

Step 5

The above values should be considered in terms of the complete circuit, which is redrawn in Figure 4.24 with the above values on it.

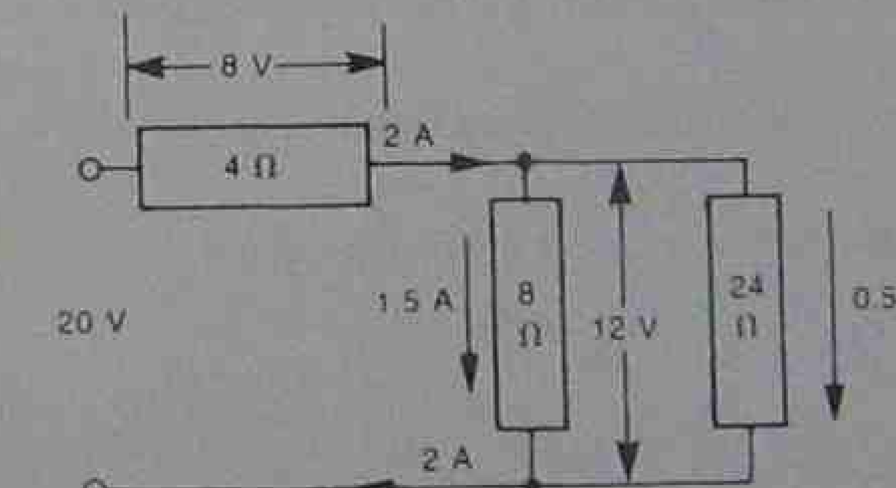


Figure 4.24 • Complete circuit for example 4.9 showing all values

Step 6 (check)

Current through 8 Ω resistor:

$$I = \frac{V}{R} = \frac{12}{8} = 1.5\ \text{A}$$

For 24 Ω:

$$I = \frac{V}{R} = \frac{12}{24} = 0.5\ \text{A}$$

that is, $I_{total} = 1.5 + 0.5 = 2$ A

4.6 WORKED EXAMPLES

Example 4.10

Three resistors of 20 Ω, 50 Ω and 30 Ω are connected in series to a generator. If the current flowing is 2.5 A, calculate:

(a) the generator voltage
(b) the voltage across each resistor
(c) the power consumption of each resistor
(d) the total power consumed by the circuit

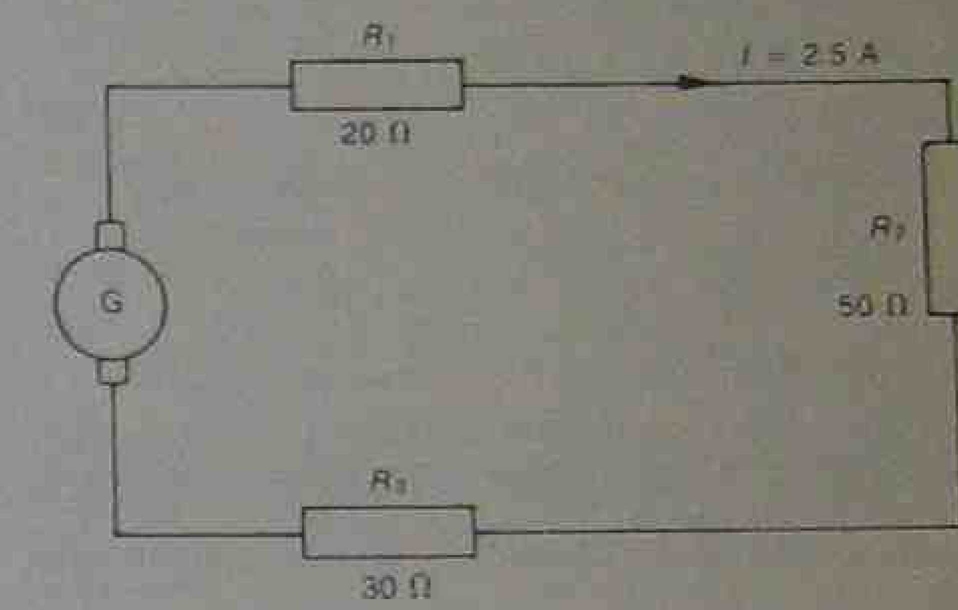


Figure 4.25 • Circuit diagram for example 4.10

(a) Generator voltage = applied voltage

$$= V_{total}$$

$$V_{total} = I_{total} \times R_{total}$$

$$R_{total} = R_1 + R_2 + R_3 \text{ (series circuit)}$$

$$= 20 + 50 + 30$$

$$= 100\ \Omega$$

$$V_{total} = I_{total} \times R_{total}$$

$$= 2.5 \times 100$$

$$= 250\ \text{V}$$

(b) p.d. across $R_1 = IR_1$

$$= 2.5 \times 20$$

$$= 50\ \text{V}$$

p.d. across $R_2 = IR_2$

$$= 2.5 \times 50$$

$$= 125\ \text{V}$$

p.d. across $R_3 = IR_3$

$$= 2.5 \times 30$$

$$= 75\ \text{V}$$

These values can be checked by finding their sum which should be equal to the applied voltage:

($50 + 125 + 75 = 250$ V $= V_{total}$)

(c) Power in $R_1 = P_1 = V_1 I$

$$= 50 \times 2.5$$

$$= 125\ \text{W}$$

Power in $R_2 = P_2 = V_2 I$

$$= 125 \times 2.5$$

$$= 312.5\ \text{W}$$

Power in $R_3 = P_3 = V_3 I$

$$= 75 \times 2.5$$

$$= 187.5\ \text{W}$$

These values can be checked by using the formula $P = I^2R$.

(Checking P_1 only)

$$\begin{aligned} P_1 &= I^2R_1 \\ &= (2.5)^2 \times 20 \\ &= 2.5 \times 2.5 \times 20 \\ &= 125\ \text{W} \end{aligned}$$

(d) Total power delivered by the generator = P_{total}

$$\begin{aligned} P_{total} &= V_{total} \times I_{total} \\ &= 250 \times 2.5 \\ &= 625\ \text{W} \end{aligned}$$

Note: By adding together the individual values of power obtained in (c) above, the total power also equals this value of 625 W, that is, 125 + 312.5 + 187.5 = 625 W.

Example 4.11

What is the total resistance of a 2 Ω and a 1 kΩ resistor connected in parallel? (See Fig. 4.26.)

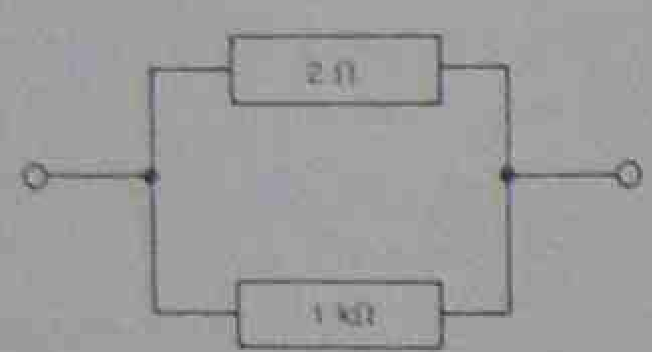


Figure 4.26 • Circuit diagram for example 4.11

$$\begin{aligned} \frac{1}{R_{total}} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{2} + \frac{1}{1000} \\ &= \frac{500 + 1}{1000} = \frac{501}{1000} \\ \therefore R_{total} &= \frac{1000}{501} = 1.99\ \Omega \end{aligned}$$

Note: The total value is always less than the value of any of the resistors in the parallel circuit. When each resistor has the same value, the total resistance can be found by dividing the resistance of one by the number of resistors in parallel.

Example 4.12

What is the total resistance of five 10 Ω resistors connected in parallel? (See Fig. 4.27.)

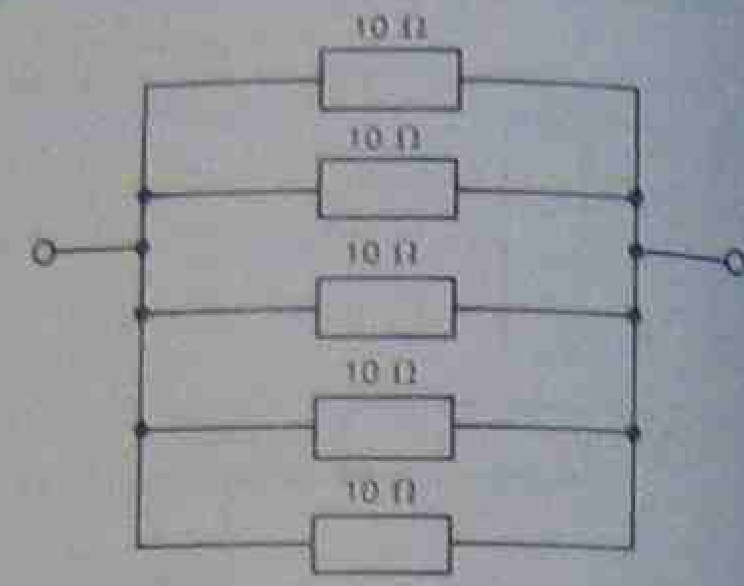


Figure 4.27 • Circuit diagram for example 4.12

$$\therefore R_{total} = \frac{10}{5} = 2\ \Omega$$

Example 4.13

Find the total resistance between the terminals A and B in the circuit of Figure 4.28.

Step 1
Draw the circuit diagram and mark in all known values.

Step 2
Letter the connections between the groups.

Step 3
Calculate the resistance of parallel group a–b.

$$\frac{1}{R_{a\text{-}b}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$$\therefore R_{a\text{-}b} = \frac{10}{2} = 5\ \Omega$$

Step 4
Calculate the resistance of parallel group c–d.

$$\frac{1}{R_{c\text{-}d}} = \frac{1}{20} + \frac{1}{30} = \frac{3 + 2}{60} = \frac{5}{60}$$

$$\therefore R_{c\text{-}d} = \frac{60}{5} = 12\ \Omega$$

Step 5
Redraw the circuit using the equivalent total values calculated, as shown in Figure 4.29.

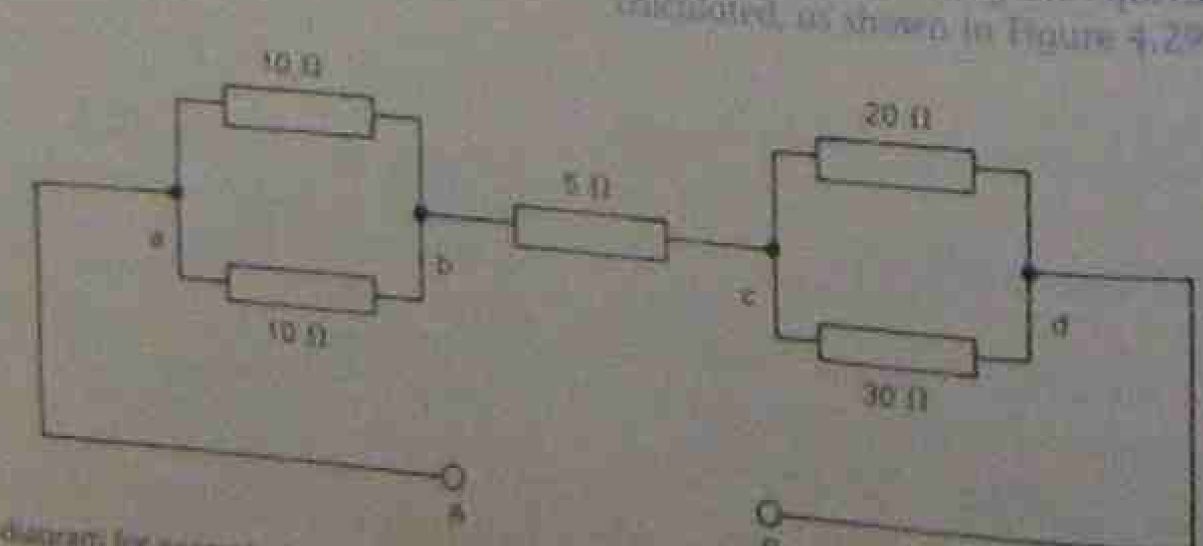


Figure 4.28 • Circuit diagram for example 4.13

Figure 4.29 • Equivalent circuit diagram for example 4.13

Step 6
Calculate the total resistance of the equivalent series circuit.

$$R_{A\text{-}B} = 5 + 5 + 12 = 22\ \Omega$$

Example 4.14

Calculate the voltage required to cause a total current of 5 A to flow in the circuit in Figure 4.30.

Step 1
Replace each parallel group with its equivalent value.

Note. One parallel branch contains two resistors in series, and by series circuit 'rules', is the equivalent of 10 Ω.

Step 2
Calculate the resistance of parallel group B–C.

$$\frac{1}{R_{B\text{-}C}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$$\therefore R_{B\text{-}C} = \frac{10}{2} = 5\ \Omega$$

Step 3
Calculate the resistance of parallel group C–D.

$$\begin{aligned} \frac{1}{R_{C\text{-}D}} &= \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + \frac{1}{10} \\ &= \frac{1 + 1 + 1 + 2}{20} = \frac{5}{20} \end{aligned}$$

$$\therefore R_{C\text{-}D} = \frac{20}{5} = 4\ \Omega$$

Step 4
Redraw the circuit using these values, as in Figure 4.31.

Figure 4.31 • Equivalent series circuit for example 4.14

$$R_{total} = 11 + 5 + 4 = 20\ \Omega$$

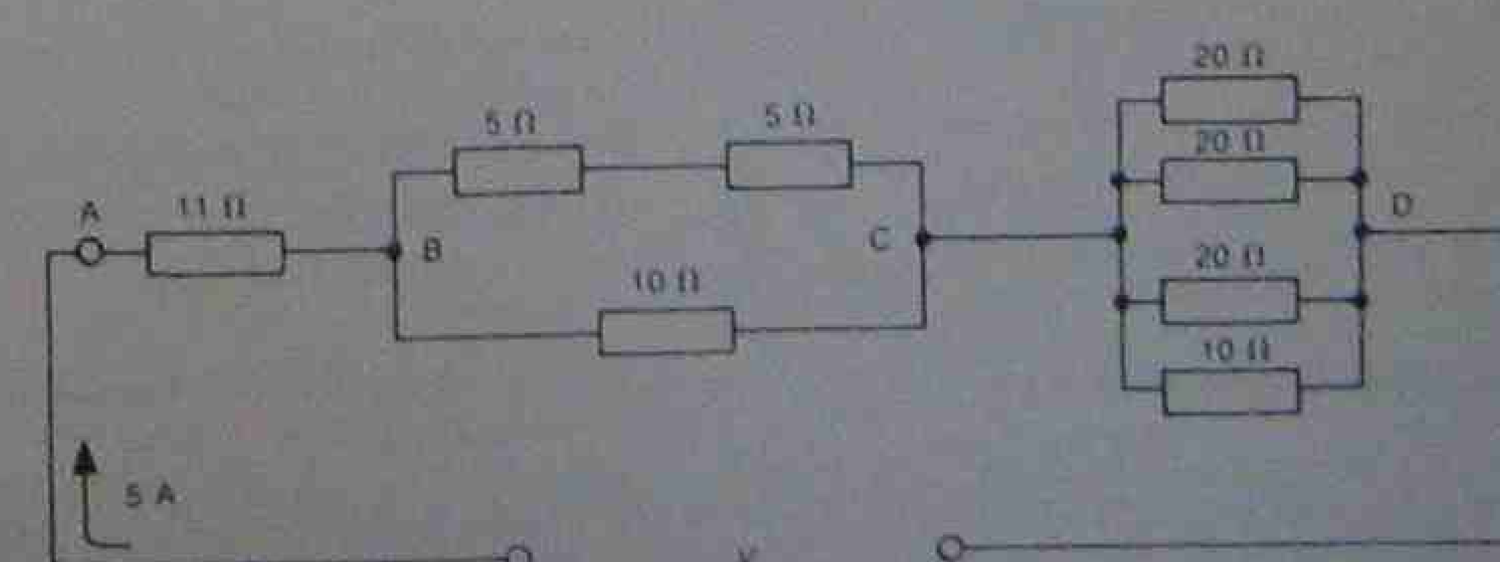


Figure 4.30 • Circuit diagram for example 4.14

Step 5
Total voltage:

$$\begin{aligned} V_{total} &= IR_{total} \\ &= 5 \times 20 \\ &= 100\ \text{V} \end{aligned}$$

Example 4.15

Find the total resistance of the circuit shown in Figure 4.32.

Note that one branch of the parallel circuit B–C contains a further parallel circuit (X–Y) in series with a 3 Ω resistance.

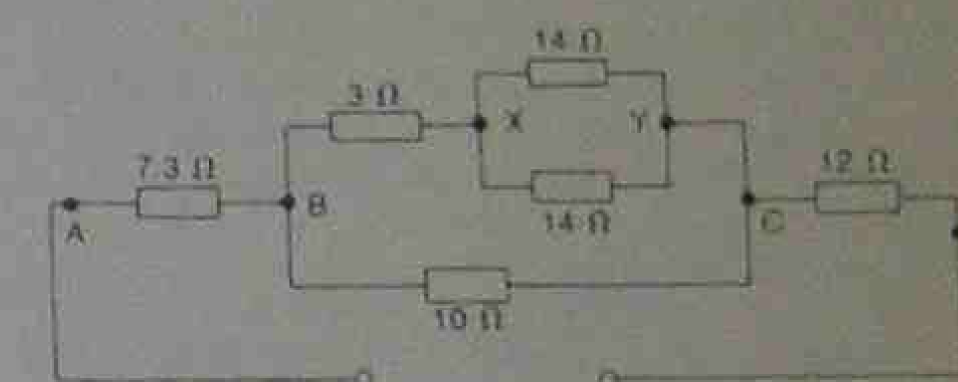


Figure 4.32 • Circuit diagram for example 4.15

Whenever one parallel circuit is contained within another parallel circuit the inner parallel group must be replaced with its equivalent series resistance before attempting to calculate the equivalent resistance of the outer parallel group.

Step 1

$$R_{X\text{-}Y} = \frac{14}{2} = 7\ \Omega$$

Step 2
Redraw the parallel group B–C, as in Figure 4.33.

Step 3
The total resistance of the 3 Ω and 7 Ω leg is 10 Ω, so the circuit becomes as shown in Figure 4.34.

$$R_{B\text{-}C} = \frac{10}{2} = 5\ \Omega$$

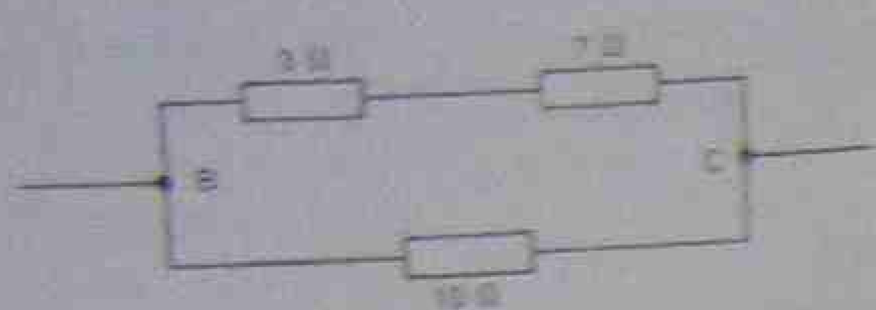


Figure 4.33 • First step in simplifying circuit—section B–C

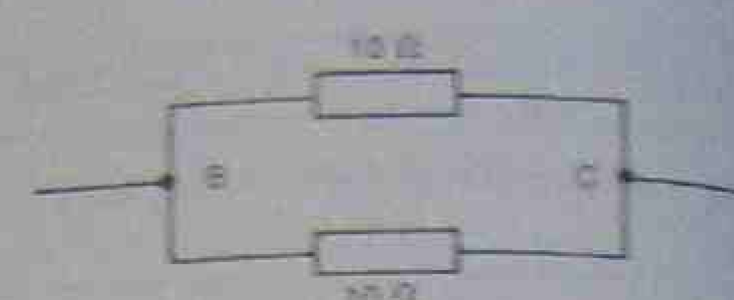


Figure 4.34 • Equivalent circuit—section B–C

Step 4

Draw the equivalent series circuit as shown in Figure 4.35.

$$R_{total} = 7.3 + 5 + 12 = 24.3\ \Omega$$

Figure 4.35 • Equivalent series circuit for example 4.15

SUMMARY

- There are two main types of circuits—series and parallel.
- When a circuit has parts of both types of circuit, it is treated as a combined, or a combination circuit.
- Combined series and parallel loads have to be treated in sections to find equivalent values.
- Series circuit characteristics are:
  - —One current path only. Current is a constant value.
  - —The sum of the individual potential differences is equal to the applied voltage.
  - —The total series circuit resistance is equal to the sum of the individual resistances in the circuit.
  - —Power consumed in a resistor $= VI = I^2R = \dfrac{V^2}{R}$.
  - —The total power is the sum of the power consumed by the individual loads.
- Parallel circuit characteristics are:
  - —More than one current path.
  - —The total current flowing is the sum of the currents in the individual loads.
  - —The supply voltage is constant and is applied to all loads.
  - —Total parallel circuit resistance is found from:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

  - —R_{total} can replace all loads and still consume the same current and power as all the individual loads combined.
  - —The power consumed is the sum of all the power consumed by the individual loads.

SELF-TESTING PROBLEMS

4.1 (a) State the distinctive features of current and voltage in series circuits.

(b) Determine the power in each resistor connected in the circuit shown in Figure 4.36.

(c) What is the total power in the circuit?

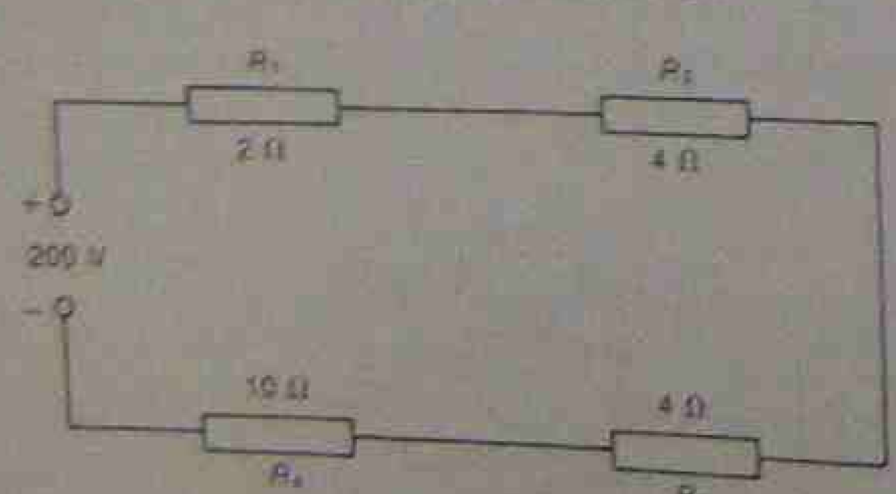


Figure 4.36 • Circuit diagram for problem 4.1

4.2 Resistors of 27 Ω, 15 Ω, 17 Ω and 3 Ω are connected in series. Find the total equivalent resistance presented to the supply source.

4.3 What value resistor must be added in series with a 12 Ω resistor to give the circuit a total resistance of 28 Ω?

4.4 Three resistors of 2 Ω, 4 Ω and 8 Ω are connected in series. If the potential difference across the 4 Ω resistor is 10 V, find:

(a) the p.d. across the other resistors

(b) the power consumed by each resistor.

4.5 Find the total circuit resistance of each of the following groups of resistors:

(a) 12 Ω, 15 Ω, 21 Ω, all connected in series

(b) 6 Ω, 7.5 Ω, 3.85 Ω, all connected in series

(c) 0.25 Ω, 0.225 Ω, 0.425 Ω, 0.55 Ω, all connected in series.

4.6 (a) State all the distinctive features of current and voltage in parallel circuits.

(b) Calculate, in the circuit shown in Figure 4.37: (i) the total circuit resistance; (ii) the total circuit current; and (iii) the current in R_2.

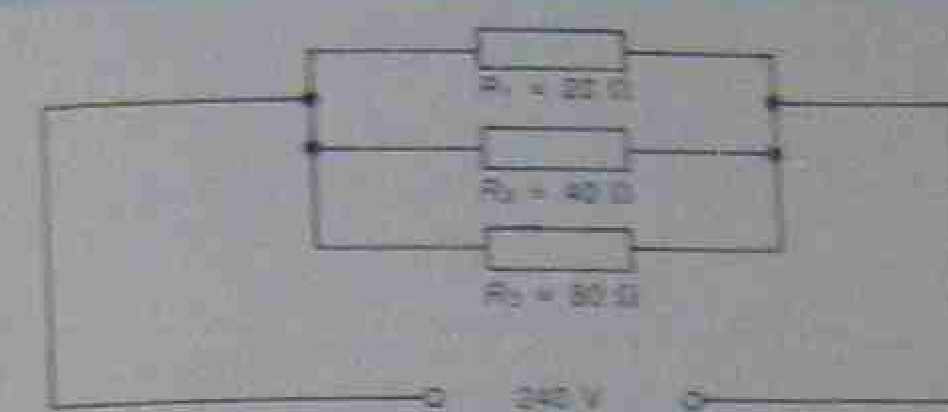


Figure 4.37 • Circuit diagram for problem 4.6

4.7 Find the total resistance and the total current in the circuit shown in Figure 4.38.

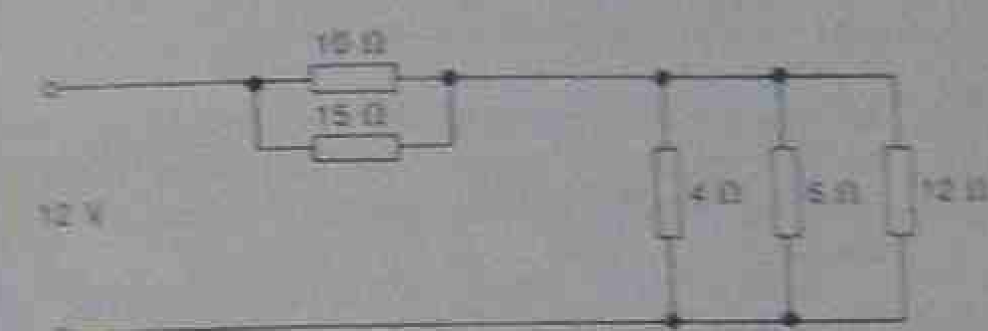


Figure 4.38 • Circuit diagram for problem 4.7

4.8 Three resistors are connected in parallel, two of them being 9 Ω and 18 Ω. If the total resistance of the three in parallel equals 4 Ω, what is the value of the unknown third resistor?

4.9 Calculate the p.d. across each resistor in the circuit shown in Figure 4.39.

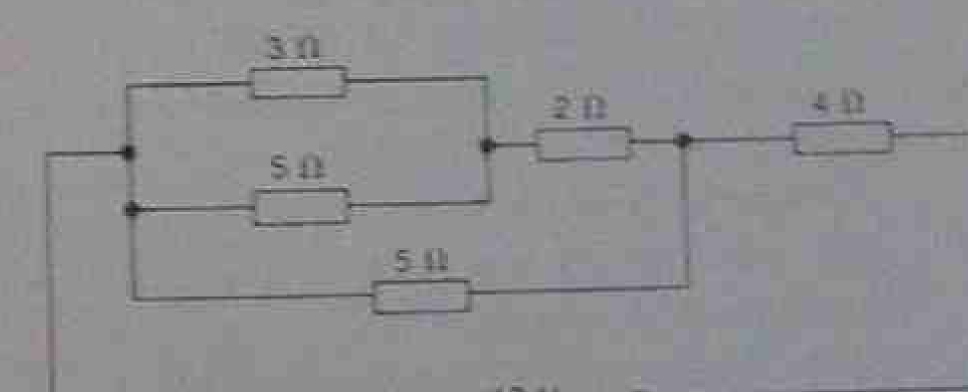


Figure 4.39 • Circuit diagram for problem 4.9

4.10 (a) What is the total current flowing in the circuit shown in Figure 4.40?

(b) Calculate the voltage across the 3 Ω resistor.

(c) Find the power consumption in the 2 Ω resistor.

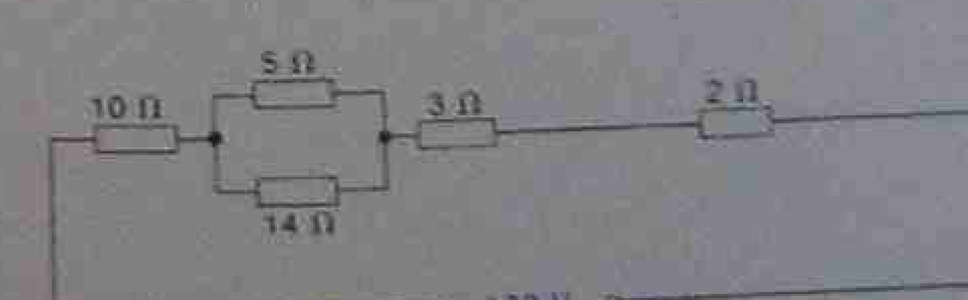


Figure 4.40 • Circuit diagram for problem 4.10

4.11 Three resistors of 15 Ω, 25 Ω and 60 Ω are connected first in series and then in parallel across a 240 V supply. Calculate for each case the current drawn and the power consumed.

4.12 Two resistors of 10 Ω and 40 Ω are connected in parallel and a third resistor of 16 Ω is connected in series with this parallel combination. A supply of 240 V is connected across the complete circuit.

Calculate:

(a) the total circuit resistance

(b) the current drawn from the supply

(c) the total power consumed

(d) the current flowing through the 10 Ω resistor.

4.13 Three resistors of 3 Ω, 6 Ω and 10 Ω, all connected in parallel, are supplied from a 30 V battery. Calculate the current flowing through each resistor and the total power consumed by the complete circuit.

4.14 Two resistors of 8 Ω and 4 Ω are connected in series across a 24 V supply. Calculate:

(a) the current flowing

(b) the power dissipated in the 4 Ω resistor.

4.15 An electric motor is supplied by means of a two-core cable, each core having a resistance of 0.4 Ω. If the motor is consuming 4.8 kW and the p.d. across the motor terminals is 240 V, calculate the motor current and the power loss in the cable.

4.16 A factory is lit by 200 lamps, each having an operating resistance of 500 Ω. Calculate the power consumed by each lamp and the total current drawn by the lighting load if each lamp is supplied at 250 V.

4.17 A 240 V two-bar electric radiator is rated at 1 kW per bar. Calculate the resistance in the radiator circuit when:

(a) one bar is switched on

(b) both bars are switched on.

4.18 A 240 V installation consists of nine 100 W lighting points, a 7.7 kW cooker, two 2 kW radiators, and a 1.5 kW electric kettle. Find:

(a) the total power consumed

(b) the current taken by the cooker

(c) the cost of running the cooker for one hour if the relevant tariff is 9.8¢/kWh.

4.19 A circuit consists of 40 parallel connected lamps, each of which takes a current of 0.25 A when supplied by 240 V. If the lamps are in use for a period of half an hour, calculate:

(a) the energy consumed in kWh

(b) the total cost if the tariff is 14.8¢/kWh.

4.20 An electric radiator consisting of three elements in parallel is connected to a 240 V supply. If each element has a resistance of 60 Ω, calculate the total current drawn by the radiator and the power rating of the device.

4.21 A two-core cable in an installation has a resistance of 0.25 Ω. Calculate the power loss in the cable when a current of 12 A is flowing.

4.22 (a) What would be the voltage across resistor R_5 in the circuit shown in Figure 4.41 if the supply voltage is 200 V?

(b) What would be the effect on the total circuit current if R_4 and R_5 were removed?

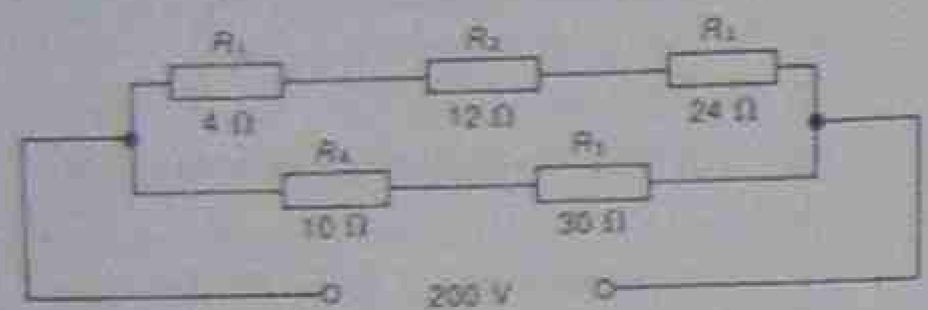


Figure 4.41 • Circuit diagram for problem 4.22

4.23 Calculate the following quantities for the circuit shown in Figure 4.42:

(a) the total circuit resistance

(b) the total power consumed by the circuit.

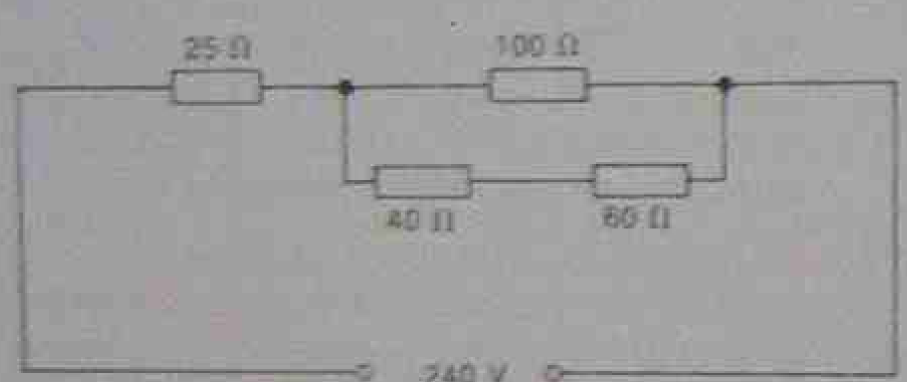


Figure 4.42 • Circuit diagram for problem 4.23

4.24 Calculate the following quantities for the circuit shown in Figure 4.43:

(a) the total resistance of the circuit

(b) the total power consumed

(c) the current in R_4 and R_5

(d) the values of resistors R_1, R_3, and R_4,

given:

voltage across R_1 = 100 V

voltage across R_3 = 50 V

current in R_1 = 20 mA

current in R_2 = 15 mA

current in R_3 = 10 mA

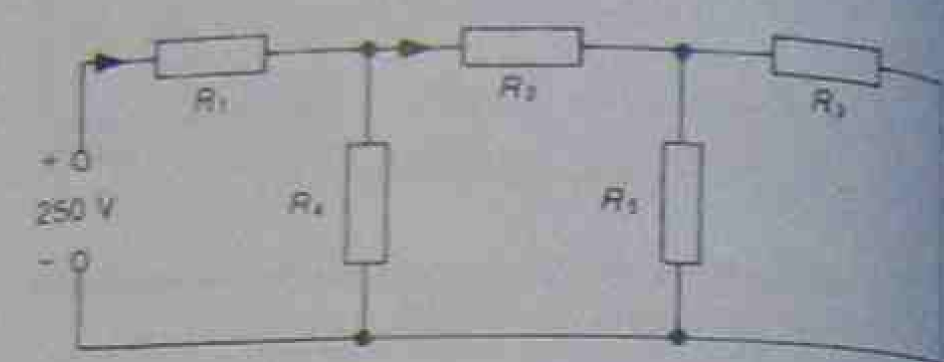


Figure 4.43 • Circuit diagram for problem 4.24

4.25 An incandescent lamp rated at 110 V and 40 W is to be run from a 240 V supply. Calculate the value of resistance required to be connected in series with the lamp.

4.26 Two resistors of 40 Ω and 60 Ω are connected in series with an unknown resistor to a 240 V supply. The voltage drop across the 60 Ω is 30 V. Find the value of the unknown resistor and the quantity of power being dissipated in it.

4.27 A d.c. motor of 0.5 kW output operates with an efficiency of 78 per cent on a terminal voltage of 225.6 V. Find the maximum value of the resistance of each supply conductor that would permit the motor to be supplied with power at a total voltage drop of 6 per cent of the supply voltage.

4.28 Two resistors of 30 Ω and 40 Ω are connected in parallel to a 12 V supply. Calculate:

(a) the current flowing in each resistor

(b) the total current flowing

(c) the power dissipation in each resistor

(d) the total equivalent resistance of the circuit.

4.29 A 55 Ω resistor is connected in parallel with an 85 Ω resistor. Find the total equivalent resistance.

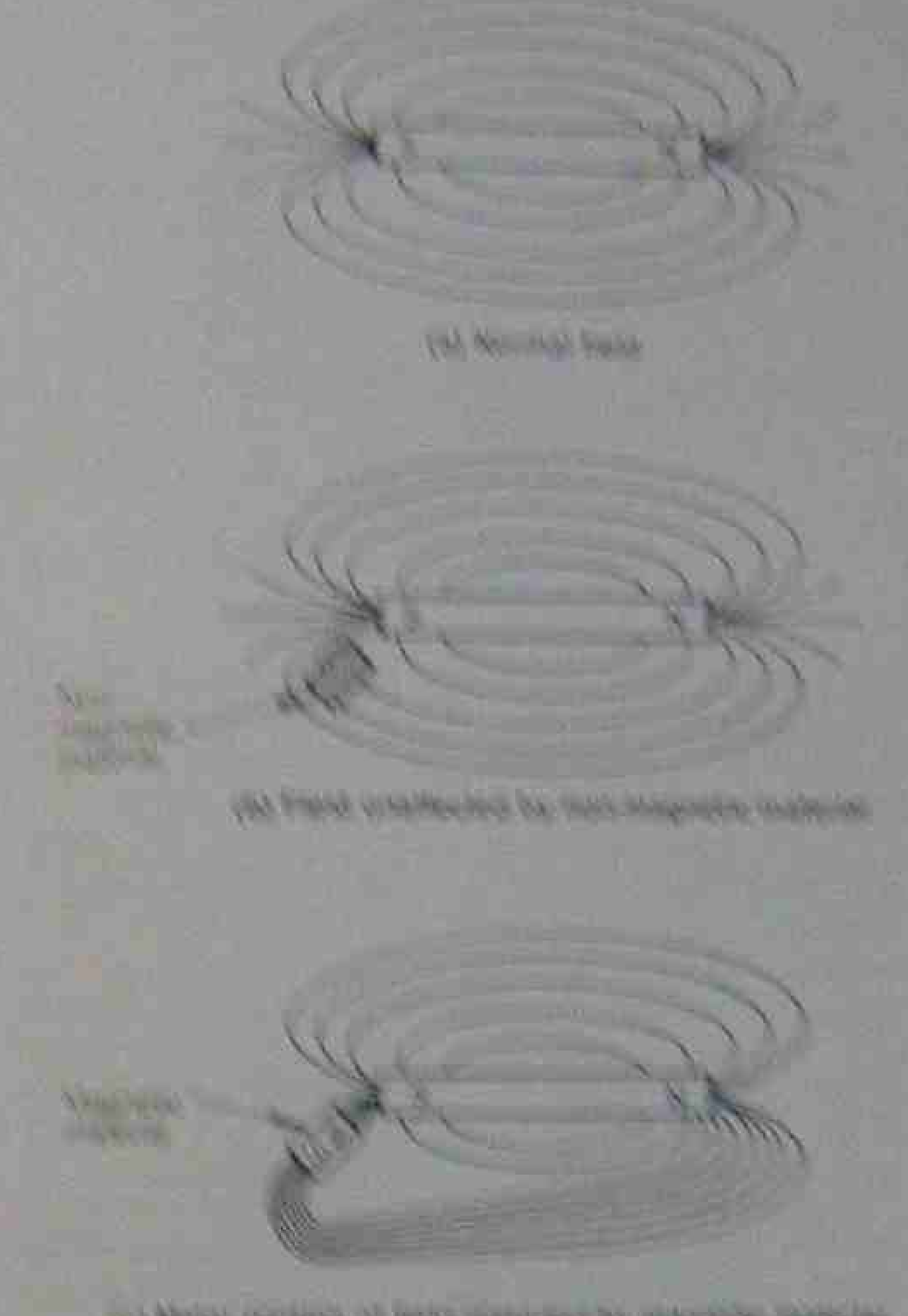
Chapter 5

Inductors and magnetism

5.1 INTRODUCTION

5.2 NATURAL MAGNETS

5.3 PERMANENT MAGNETS



(a) Normal field

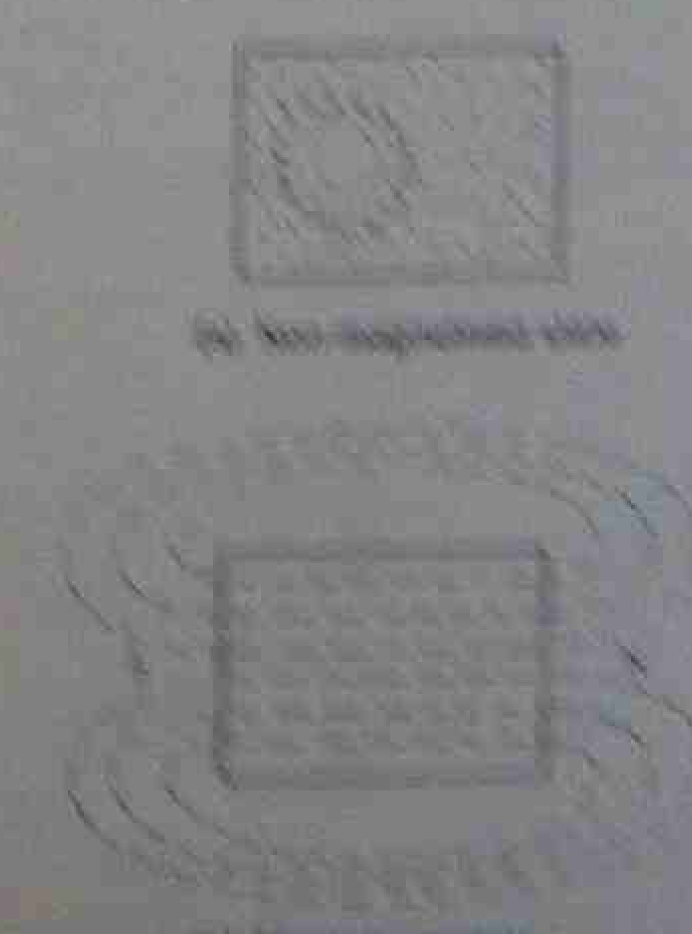
(b) Field unaffected by non-magnetic material

(c) Major portion of field attracted by magnetic material which creates a low reluctance path for the flux

5.3.1 Molecular theory of magnetism

5.3.2 Magnetic induction

5.3.3 Types of magnetic materials



(a) Non-magnetised state

(b) Magnetised state

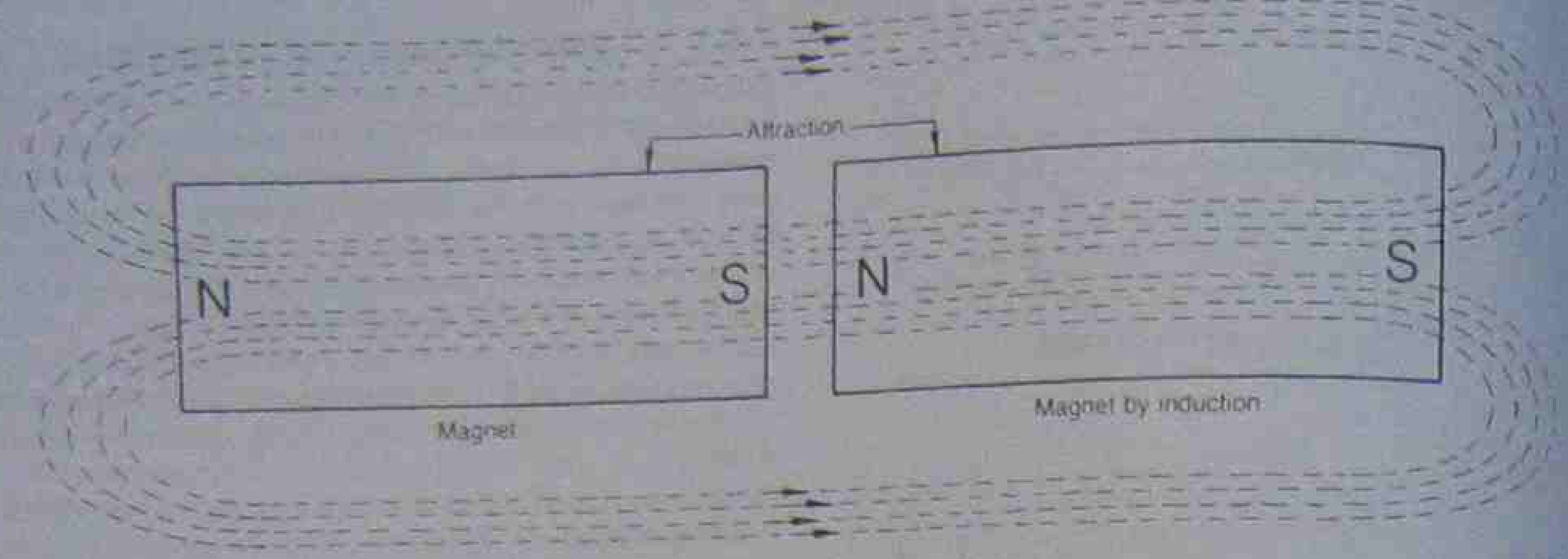


Figure 5.5 • Magnetic induction

Many of these are called rare earth magnets—not because the material of which they are made is rare but because of the difficulty of isolating the element from the surrounding material. There are several of these elements, all of which come from the *Lanthanide* group of elements (see the Periodic Table's list of elements at the back of the book). Various of these elements have been tried but the most promising appears to be element number 60 called *Neodymium*. To the present time it appears to make the strongest magnets currently available. It has a very high melting point, which adds to the difficulty of working it. It also requires an extremely high magnetic force to magnetise it, but it also has the advantage of requiring a higher magnetic force to de-magnetise it. The magnet tends to be rather brittle and if roughly handled can shatter without warning. Rare earth magnets can be obtained in a variety of shapes. Applications for these magnets include electric motor fields for quite large motors up to several kilowatts. It is used in disk drive heads and arms for computer drives and optical disks.

A material that is magnetically hard requires a very strong magnetising force to induce the molecular magnets to align themselves with this force, but when the force is removed the molecular magnets tend to remain in their new alignment. This leaves the material in a magnetised condition that is reasonably permanent. It follows that all permanent magnets are made from materials that are magnetically hard. A magnetically soft metal can be magnetised by induction with a relatively low magnetising force. However, when the magnetising force is removed, the molecular magnets do not maintain their alignment, but tend to form closed magnetic circuits within the metal; that is, the material tends to demagnetise itself. Any magnetism that remains is called *residual magnetism*.

5.3.4 Applications for permanent magnets

Probably the most common application of permanent magnets is the magnetic compass, followed closely by the use of permanent magnets to provide a constant magnetic flux for certain classes of meters. Today the permanent magnet is used for a variety of purposes ranging from credit cards, fridge door seals and magnets, proximity relays, alarms, bus and train tickets, small generators, sump plugs and printed circuit motors. In the United States (which will probably extend to Australia), permanent magnets are imbedded in false teeth, helping to hold the teeth in place by being attracted to small magnetic blocks permanently installed in the patient's mouth.

Another application is the magnetic chuck. This device uses the holding power of its magnets to retain magnetic materials firmly in position on the worktable of a machine during machining processes. One of the advantages of permanent magnet chucks over similar appliances using electromagnets is the fact that no electrical connections are needed and, in the event of an electrical failure, the material being machined is not accidentally released and allowed to move and cause damage.

Permanent magnet chucks for large jobs have been possible mainly because of the discovery of special alloys which can be magnetised to a very high flux density and can retain this state of magnetisation.

Where greater holding power is necessary, electromagnets are used to provide the required force. When the chuck has to move or rotate, the connection between the magnetising coils and the electric supply complicates the design.

Because it is not possible to 'switch off' the magnetism of a permanent magnet, some other means must be used to release articles held by the chuck. This is achieved by shunting (bypassing) the magnetic flux through a low-reluctance bridging piece.

Figure 5.6 illustrates a typical permanent magnet chuck.

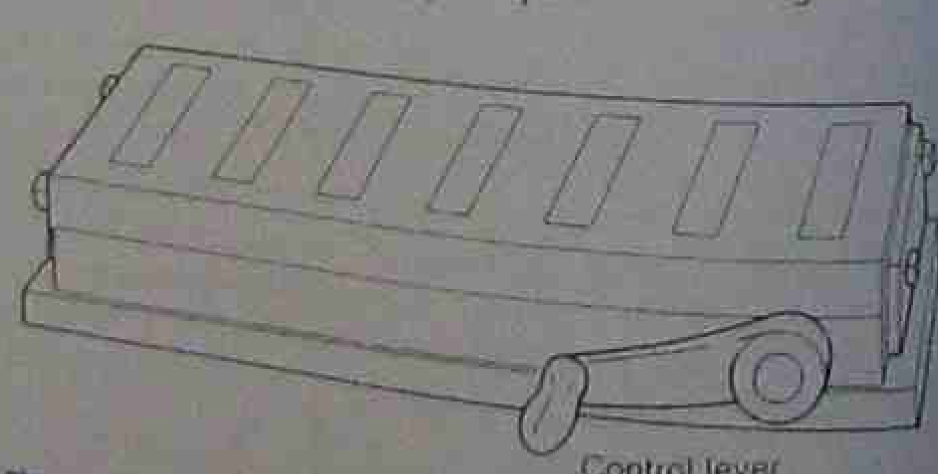


Figure 5.6 • Rectangular form of magnetic chuck

5.4 ELECTROMAGNETISM

It was stated in section 2.16.3 that one of the effects of an electric current was magnetism. From the initial discovery linking the two in 1819 the uses for the effect have grown steadily. The electromagnetic effect is the basis of operation of various types of motors and generators, transformers, microphones, loud speakers, indicators such as doorbells, and car ignition coils. The effect is also used in operating coils of devices such as contractors, relays, inductors, solenoids, chokes, and ballasts. The effect can be further manipulated by varying the number of turns on a coil, or by using different types of magnetic cores.

Special types of resistors and semiconductors have been developed to detect either an electric current or the magnetic fields produced by it. Instruments have been developed to detect the magnetic fields produced by conductors buried in a wall or in the ground.

5.4.1 Magnetic fields around a straight conductor

By sending a current through a piece of straight wire passing vertically through a horizontal piece of cardboard covered with iron filings, a map of the magnetic field around the conductor can be seen (Fig. 5.7). A compass can be used to indicate the direction of the lines of force, which form concentric circles around the conductor. If the direction of current in the conductor is reversed, the pattern of the filings remains unchanged, so indicating no change in the position or strength of the magnetic flux.

However, the compass will now point in the opposite direction, indicating that the lines of force are now acting in the reverse direction around the conductor.

A simple rule can be learnt to help determine in which direction the lines of force act for a given direction of current flow:

> **Right-hand thumb rule—straight conductor:**
> *Grasp the conductor in the right hand, as shown in Figure 5.8(a), with the thumb pointing in the direction of current flow. The fingers point in the direction in which the magnetic force is acting.*

The strength of the magnetic field around a straight conductor depends on the value of the current in the conductor. Doubling the current results in double the field strength. That is, the field strength is proportional to the current strength. However, the field strength is not uniform throughout the magnetic field; the further away from the conductor, the weaker the field intensity.

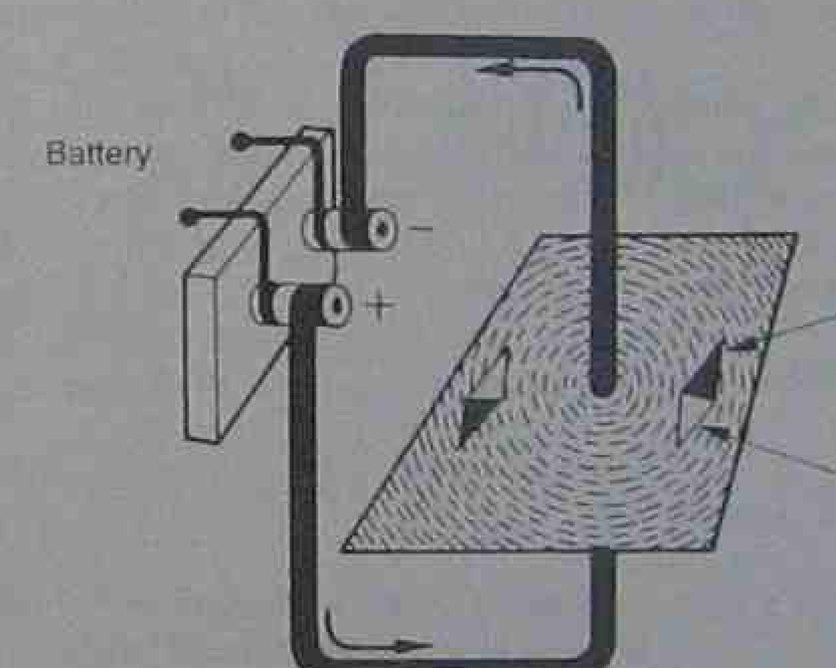


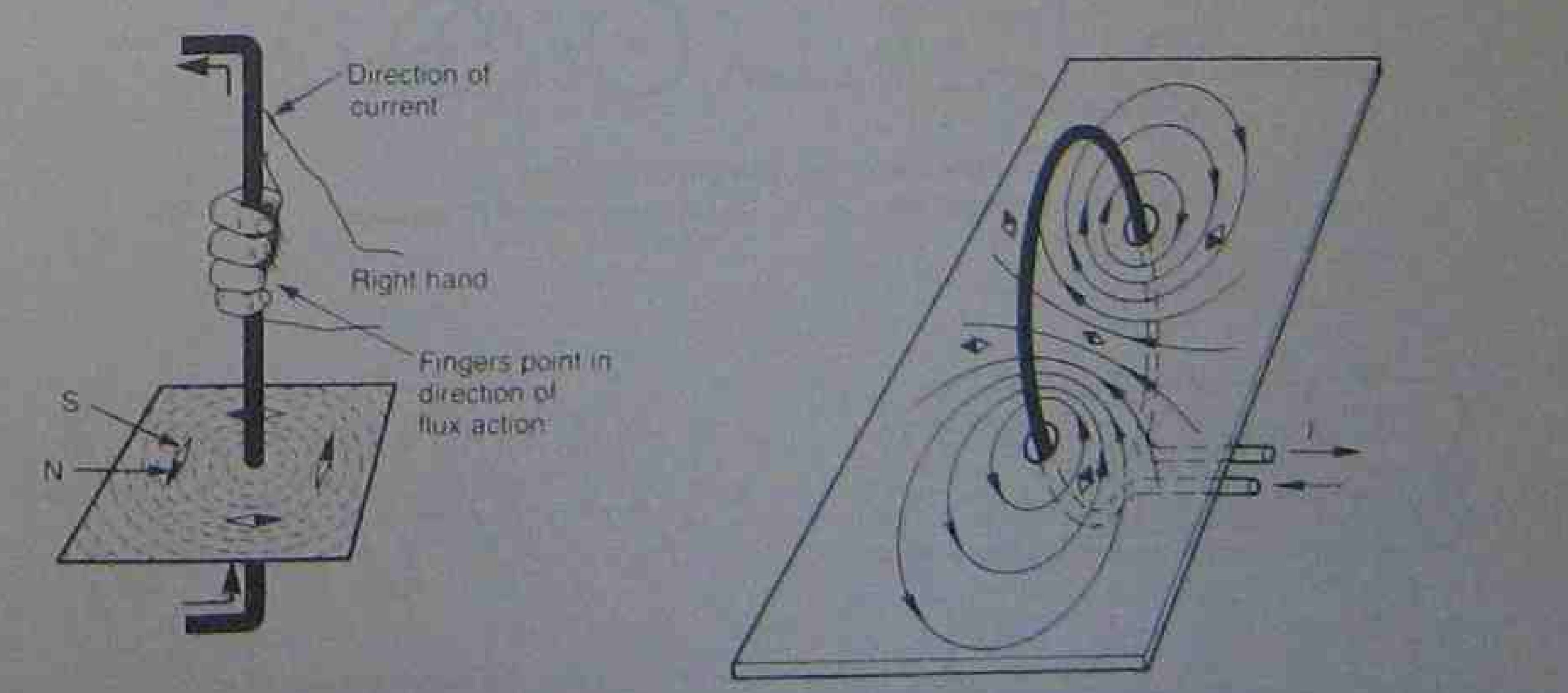
Figure 5.7 • Magnetic field around a straight conductor

5.4.2 Magnetic field within a loop and a solenoid

If a straight conductor is bent to form a loop, as shown in Figure 5.8(b), the strength of the flux inside the loop is doubled.

By winding the conductor into a coil of many turns, the field strength is increased in proportion to the number of turns in the coil.

Figure 5.9 illustrates the convention by which the direction of current flow is indicated. If an arrow is imagined in the conductor as pointing in the direction of current flow, and cross-sections are taken at the point and the flight of the arrow, they would resemble symbols (a) and (b). The cross sign represents the flight of the arrow (and also the current) moving away, while the dot represents



(a) The right-hand thumb rule

(b) Magnetic field around a loop

Figure 5.8 • Direction of magnetic fields created around conductors by a current flowing through them

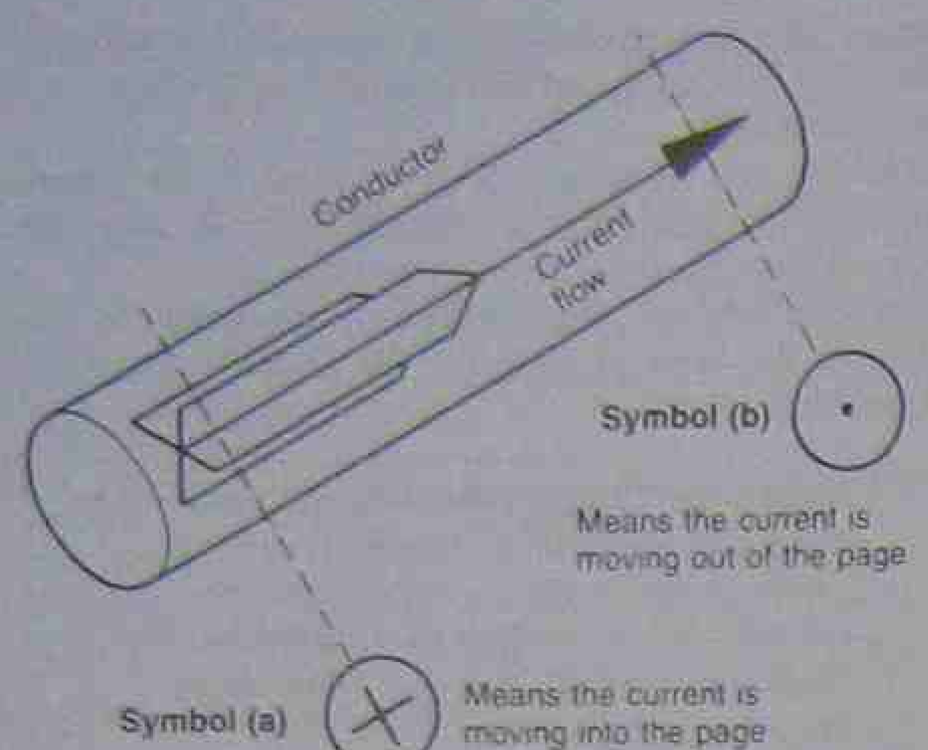


Figure 5.9 • Derivation of current direction symbols

the point of the arrow approaching, and indicates current flow towards an observer.

Figure 5.10(a) shows how the flux around two straight conductors carrying current in the same direction unites to form a single flux around both conductors. This action will not occur if the current in each conductor is in opposite directions, as shown in Figure 5.10(b).

Figures 5.10(c) and (d) show how the current in each adjacent turn of a coil flows in the same direction. The magnetic fields around each loop will combine to form a single magnetic flux embracing all the turns of the coil. The strength of the resultant flux will be equal to the total of all the separate fields set up by each coil turn. As a result the flux inside the coil will be proportional to the number of turns in the coil.

With reference to Figure 5.10(d), note that the flux flowing through the centre of the coil establishes a north pole at the end where it leaves the inside of the coil and a south pole at the end where it enters the coil. The field around the outside of the coil is similar to the field around a bar magnet. Such a flux-producing coil is commonly referred to as an electromagnet because the magnetic field is produced by the current flowing through the coil.

If the turns of the coil are wound in the opposite direction, or if the current flows through the coil in the opposite direction, the magnetic polarity of the coil is reversed.

A simple rule to determine the polarity of an electromagnet when the directions of current and coil windings are known is given below.

A coil, or electromagnet, whose length is many times greater than its diameter is called a *solenoid*:

> **Right-hand thumb rule—solenoid:**
> *Grasp the solenoid in the right hand, as shown in Figure 5.11, so that the fingers point in the direction of the current flowing through the coil. Extend the thumb at right angles to these fingers and the thumb will point in the direction of the north pole.*

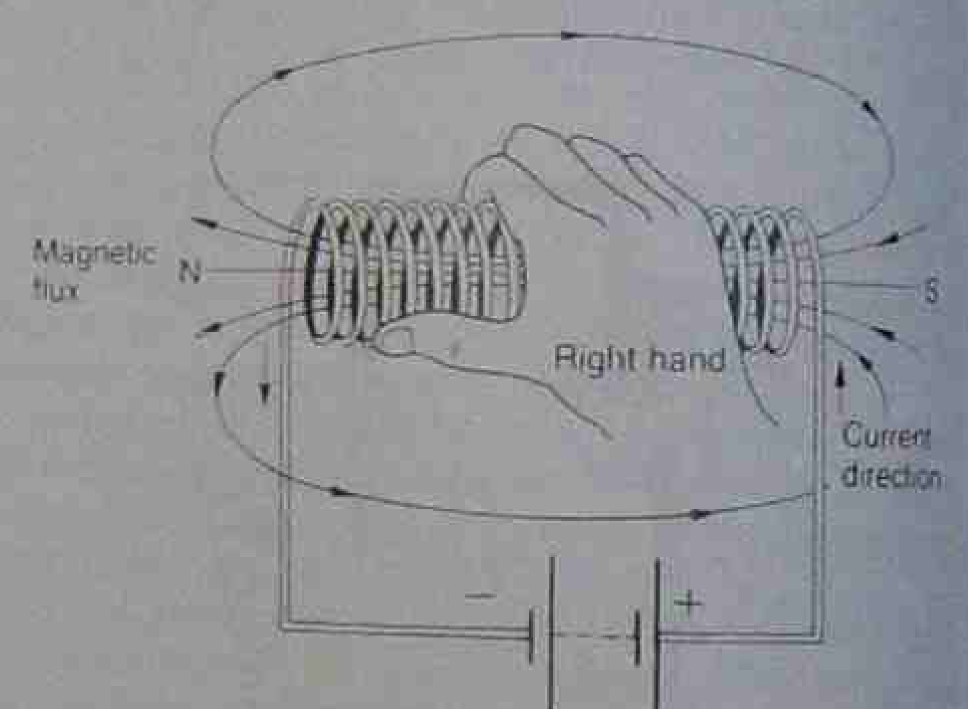


Figure 5.11 • Right-hand thumb rule for a solenoid

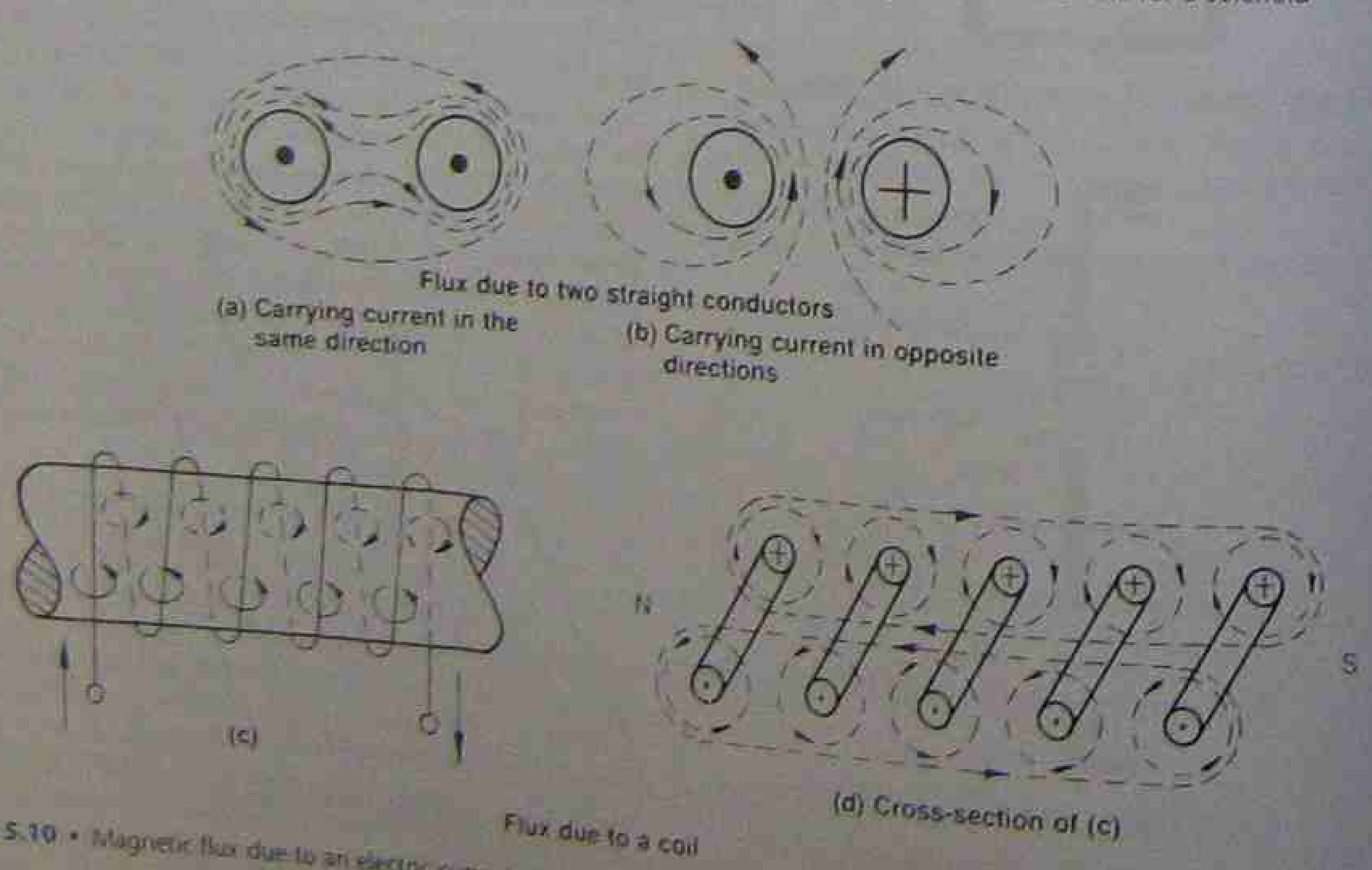


Figure 5.10 • Magnetic flux due to an electric current

To magnetise air or other material within a solenoid, a magnetising influence is needed. This force, called the magnetomotive force, acts like the electromotive force in an electric circuit and is the total magnetic force necessary to set up a flux in a magnetic circuit.

5.4.3 Forces between straight conductors carrying current

Figure 5.12(a) shows a cross-sectional view of two conductors carrying currents in opposite directions. The lines of force between the two conductors act in the same direction and hence tend to repel each other.

As the current strengths are increased, the force of repulsion will tend to move the conductors apart and reduce the compression of the two fields.

Conductor movement will occur when the magnetic force of repulsion exceeds the physical forces holding the conductors in position. This situation can occur in practice with heavy-current switchgear and machinery, and precautionary measures may be needed. The effects are not always undesirable, and electric motors, measuring instruments and tractive-type electromagnets depend upon these forces for their operation.

The force between conductors carrying current in the same direction causes attraction between the conductors (Fig. 5.12(b)).

It follows that if the currents are flowing in opposite directions a force of repulsion will be created. The magnitude of this force has been mentioned briefly in section 1.2 where the ampere was defined as the current that would cause a force of 2×10^{-7} newtons per metre between conductors placed one metre apart.

Combining these factors produces the equation:

$$\boxed{F = 2 \times 10^{-7} \times I_1 I_2 / s}$$

where F = force between conductors
$I_1 I_2$ = product of currents flowing
s = distance separating the conductors in metres

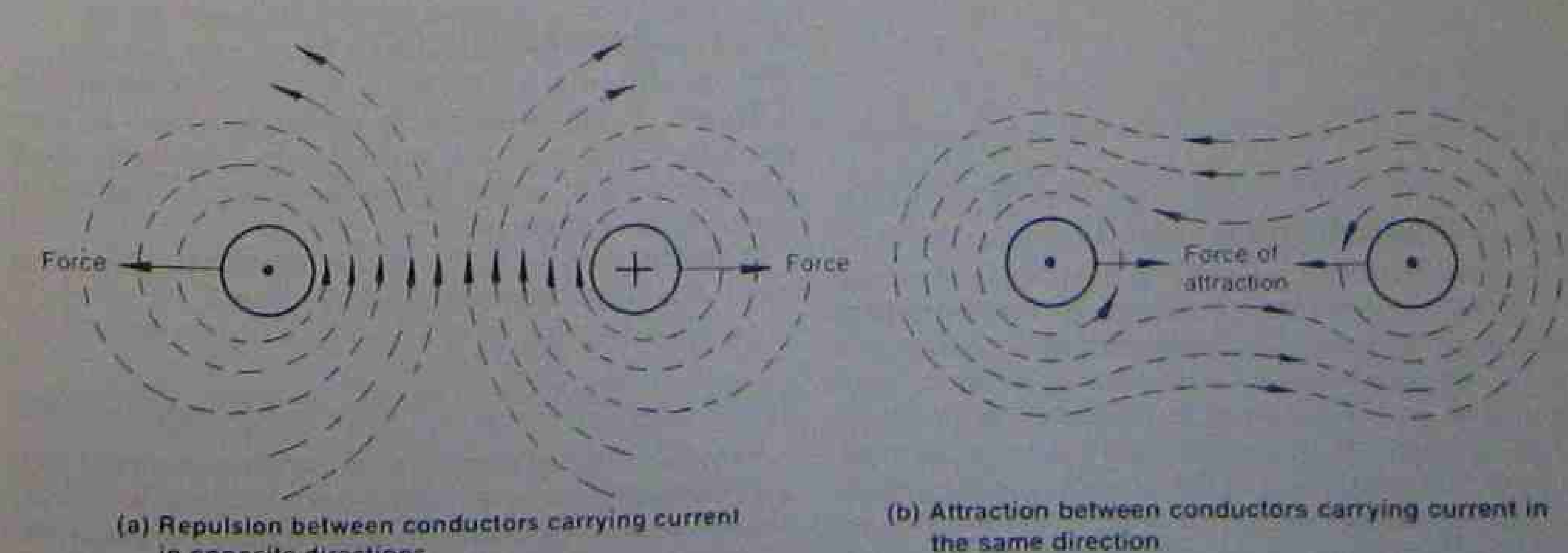


Figure 5.12 • Forces exerted by conductors carrying a current

Example 5.1

Two long parallel conductors 0.1 m apart each carry a current of 35 A in opposite directions. Calculate the force of repulsion between them.

$$F = 2 \times 10^{-7} \times I_1 I_2 / s$$

$$= \frac{2 \times 35 \times 35}{10^7 \times 0.1} = 2.45 \times 10^{-5}\ \text{N}$$

Because the magnetic field is created at right angles to the conductor, placing a conductor in a magnetic field at right angles to that conductor will also enable the creation of a force when a current is passed through it. (See Fig. 5.13.) The value of the force can be found from:

$$\boxed{F = BIl}$$

where F = force in newtons
B = flux density
l = conductor length in metres
I = current in amperes

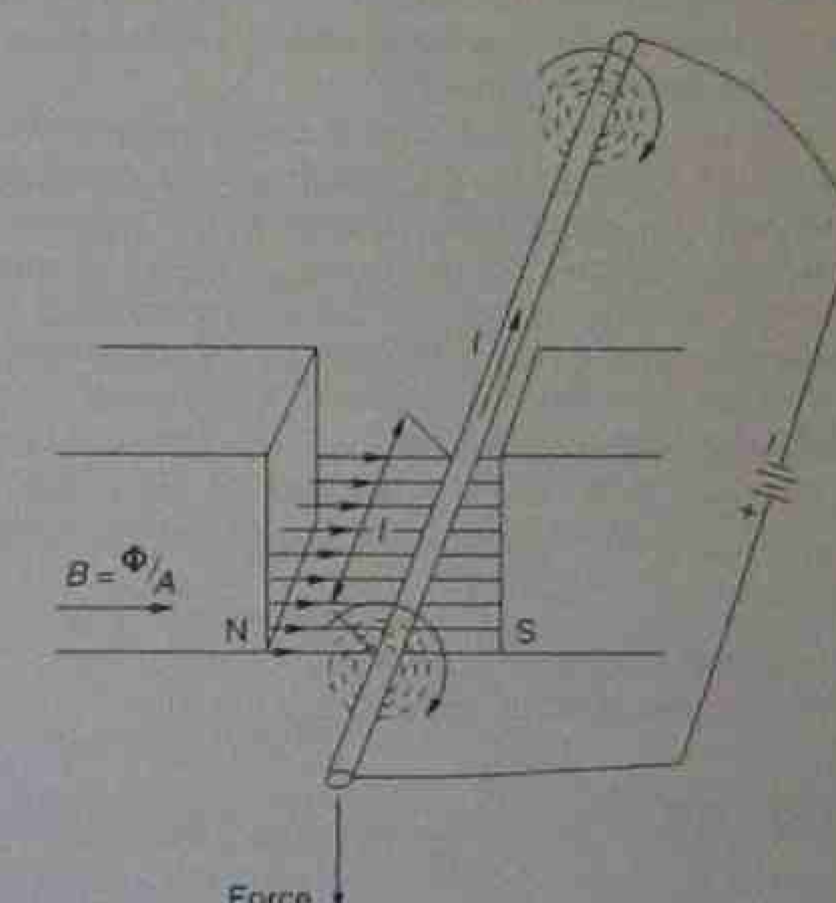


Figure 5.13 • Creating a force with the interaction of two magnetic fields

Example 5.2

A conductor 0.1 m long is placed at right angles to a magnetic field that has a flux density of 0.65 T. If a current of 25 A is passed through the conductor calculate the force exerted on the conductor.

$$F = BIl$$
$$= 0.65 \times 25 \times 0.1 = 1.63\ \text{N}$$

5.5 MAGNETIC UNITS

5.5.1 Magnetomotive force (*Fm*)

Earlier in this chapter it was stated that doubling the current through a conductor doubles the flux created by the conductor. Doubling the current in a solenoid will double the flux created by the solenoid in a similar fashion. If both the current and the number of turns are doubled, the flux created is four times as great. The field strength of a coil is proportional to the product of the current and the number of turns in the coil. This product is called the magnetomotive force, which is abbreviated as m.m.f. with a general symbol *Fm*.

In pure SI units the m.m.f. unit is the ampere because the number of turns in a coil or solenoid is considered dimensionless. In calculations, however, the number of turns has to be included. For this book, an m.m.f. quantity will be given directly as ampere-turns (abbreviated *IN*), and where a number of units is specified the abbreviation used will be At.

Magnetomotive force (m.m.f.):

$$\boxed{Fm = IN}$$

where I = current flowing in amperes
N = number of turns in coil

Example 5.3

If a current of 5 A is flowing in a coil of 120 turns, find the value of m.m.f. creating a magnetic flux.

$$Fm = IN$$
$$= 5 \times 120$$
$$= 600\ \text{At}$$

5.5.2 Magnetising force (*H*)

The m.m.f. required to magnetise a unit length of a magnetic path is termed the magnetising force for that portion of the magnetic circuit. It is applicable only to that section made of the one material and with a constant cross-section. The unit is expressed in ampere-turns per metre and the symbol is *H*. In a similar fashion to magnetomotive force, the 'turns' part of the expression is dimensionless and should be omitted, leaving it as amperes per metre. For all practical purposes, however, the more descriptive term is ampere-turns per metre, that is:

$$\boxed{H = \frac{IN}{l}}$$

where l = length of magnetic circuit in metres

The magnetising force must not be confused with magnetomotive force. One is simply magnetomotive force (= *IN*), while the other expresses the m.m.f. per metre of the magnetic circuit.

5.5.3 Flux density (*B*)

So far in this chapter, magnetic fields have been considered in terms of the total flux. In many instances it is helpful to know the density of the flux rather than its total strength. Density refers to the numbers of lines of force per unit area.

The general symbol for flux density is *B* and the unit is the weber (Wb) per square metre. One weber is 10^8 lines of force and one weber per square metre is called a tesla (T).

If both the total flux and the area of the magnetic path are known, the flux density is found from:

$$\boxed{B = \frac{\Phi}{A}}$$

where Φ = webers (group of 10^8 lines of force)
B = flux density in teslas (Wb/m²)
A = area in m²

Example 5.4

A magnetic circuit has a cross-sectional area of 100 mm² and a flux density of 0.001 T. Calculate the total flux in the circuit.

By transposition:

$$\Phi = BA$$
$$= 0.001 \times \frac{100}{10^6}$$
$$= 1 \times 10^{-7}\ \text{Wb}$$

Note: The answer is expressed in webers and not in lines of force.

5.5.4 Permeability (μ)

The ease with which a material allows flux to be created is known as the permeability of the material. To give meaning to the term it is necessary to have some fixed standard against which the permeability of individual materials may be compared.

In SI units the standard is the permeability of free space and it has been assigned the value:

$$\boxed{\mu_0 = 4\pi \times 10^{-7}}$$

To compare the permeability of any given material with the permeability of space, it is necessary to use a ratio which is known as the relative permeability of the material concerned. For air and other non-magnetic materials μ_r has the value of unity ($\mu_r = 1$).

If the non-magnetic core of a solenoid is replaced with a magnetic material, the flux produced by the same number of ampere-turns is greatly increased. The ratio of the flux produced by the magnetic core to that produced by the non-magnetic core is called the relative permeability of the magnetic material. For some magnetic materials μ_r can have a value in thousands.

For any one magnetic material the value of relative permeability can vary considerably, being dependent on the flux density in the material. Relative permeability is higher at low values of flux density.

To find the actual permeability of a material it is necessary to use the following equation:

$$\boxed{\mu = \mu_r \mu_0}$$

where μ = actual permeability
μ_r = relative permeability
μ_0 = permeability of free space

5.5.5 Reluctance (R_m)

Some materials require high magnetising forces to align their atomic magnets in the same direction, while others are readily magnetised by smaller forces. All materials offer some opposition to being magnetised and the term used to describe this opposition is magnetic reluctance. Reluctance is comparable with resistance in an electric circuit and, like resistance, depends on a number of different factors:

1. ***Length of a magnetic circuit.*** Reluctance varies directly as the mean length of a magnetic circuit and is similar in this respect to electrical resistance.

$$\boxed{R_m \propto l}$$

2. ***Cross-sectional area of a magnetic circuit.*** Reluctance varies inversely as the cross-sectional area of a magnetic circuit.

$$\boxed{R_m \propto \frac{1}{A}}$$

3. ***Permeability of the circuit material.*** The term permeability is used as a measure of the ease with which materials may be remagnetised. Reluctance, on the other hand, is a measure of the opposition to flux:

$$\boxed{R_m = \frac{l}{\mu_r \mu_0 A}}$$

where R_m = reluctance in an ampere-turns per weber
l = length of circuit in metres
A = cross-sectional area in square metres

Example 5.5

The total mean length of path of an iron core is 200 mm. The core is rectangular in cross-section with dimensions 15 mm × 10 mm. If the core has a relative permeability of 830 at a certain flux density, calculate the reluctance of the core.

$$R_m = \frac{l}{\mu_r \mu_0 A}$$
$$= \frac{0.2 \times 10^7}{830 \times 4\pi \times 0.015 \times 0.010}$$
$$= 1.278 \times 10^6\ \text{At/Wb}$$

5.5.6 Ohm's law applied to magnetic circuits

Ohm's law, when applied to electrical circuits, gave the following formula:

$$\boxed{I = \frac{V}{R}}$$

where I = current flow in amperes
V = electromotive force
R = circuit opposition to flow or resistance

A similar version can be applied to magnetic circuits, that is:

$$\boxed{\Phi = \frac{IN}{R_m}}$$

where Φ = magnetic flow of lines of force (webers)
IN = magnetomotive force (ampere-turns)
R_m = magnetic opposition or reluctance (*IN*/Wb)

From the above equation it can be seen that increasing either the current or turns of a solenoid will increase the flux. A decrease in R_m would also increase the flux.

Ohm's law for magnetic circuits may be given in three variations to suit particular problems:

$$\Phi = \frac{IN}{R_m}\ \text{webers}$$

$$R_m = \frac{IN}{\Phi}\ (IN/\text{Wb})$$

$$IN = \Phi R_m\ \text{ampere-turns}$$

Electrical equivalent:

$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$

$$V = IR$$

Example 5.6

An electromagnet has 600 turns and the total reluctance of the magnetic core is 800 units. Calculate the flux produced when 10 A flows through the coil.

Using the magnetic version of Ohm's law in the form:

$$\Phi = \frac{IN}{R_m}\ \text{webers}$$
$$= \frac{600 \times 10}{800}$$
$$= 7.5\ \text{Wb}$$

Example 5.7

A contractor coil has 7200 turns, which are wound on an iron core, rectangular in section, and having dimensions of 20 mm × 30 mm. If the flux density in the magnetic circuit is 1.2 T, find the reluctance of the magnetic core. The current drawn is 0.1 A.

$$\Phi = BA$$
$$= 1.2 \times 0.02 \times 0.03$$
$$= 0.000\,72\ \text{Wb}$$

Using the magnetic version of Ohm's law in the form:

$$R_m = \frac{IN}{\Phi}$$
$$= \frac{7200 \times 0.1}{0.000\,72}$$
$$= 1\,000\,000\ \text{At/Wb}$$

5.6 MAGNETISATION CURVES

5.6.1 Magnetisation curve for a non-magnetic material

Reluctance of non-magnetic materials is not affected by the density of flux in those materials. Flux Φ therefore will vary directly as the m.m.f. (IN), and flux density B will consequently vary directly as the magnetising force H.

For non-magnetic materials, B varies directly as H and therefore the graph B against H will be a straight line. The magnetisation curve for air and non-magnetic materials is shown in Figure 5.14.

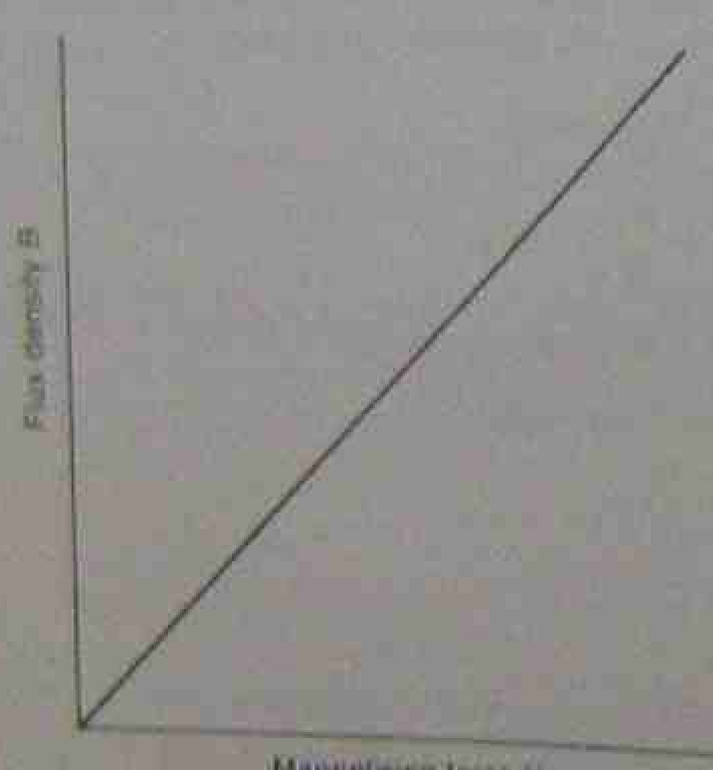


Figure 5.14 • Magnetisation (B/H) curve for non-magnetic materials

5.6.2 Magnetisation curve for a magnetic material

When values of B are plotted against values of H for a magnetic material it is found that the resulting graph is in the form of a curve. Table 5.1 shows figures for an iron sample.

A graph plotted from these figures is shown in Figure 5.15. Since values of B are plotted against values of H, the graph is known as a B/H curve. These curves are commonly used as a means of comparing the magnetic characteristics of different types of ferromagnetic materials.

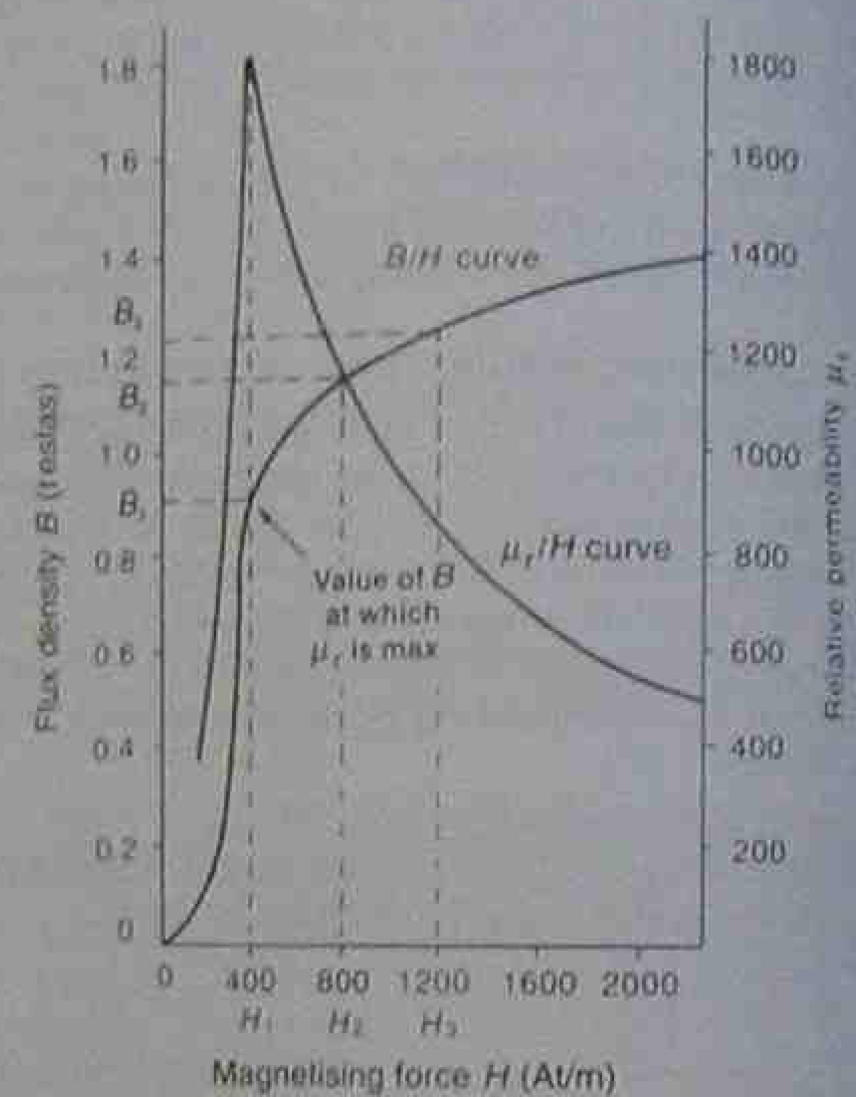


Figure 5.15 • Magnetisation (B/H) and permeability (μ_r/H) curves for a sample of iron

5.6.3 Magnetic saturation

Reference to the B/H curve in Figure 5.15 shows that when the value of H is low, small increases in the value of H will produce large increases in the value of B. This is shown by the section of the curve that slopes steeply.

For higher values of H and B it can be seen that increases in H will produce progressively smaller increases in B. When H increases from zero to H_1, it is seen that flux density B increases from zero to B_1. If H is now doubled to the value H_2, then B increases by a smaller amount from B_1 to B_2. Further increases of H to the value H_3 will result in an even smaller increase of B from B_2 to B_3.

Table 5.1 • Magnetisation curve for a magnetic material

H (IN/l)	100	200	300	400	500	600	700	800	900	1000	1200	1400	1600	2000
B (Wb/m²)	0.04	0.12	0.40	0.90	1.00	1.06	1.11	1.15	1.18	1.21	1.25	1.29	1.32	1.36

From the above facts it is apparent that a stage of magnetisation will be reached where an increase in H will have negligible effect on B. Magnetic saturation is the term used to describe this effect.

Saturation occurs at a flux density near the centre of the 'knee' of the B/H curve. In practice, it is not economical to magnetise steel to a degree of flux density that is very far beyond the point of magnetic saturation. If this is attempted, a large increase in ampere-turns produces only a small increase in flux density. To increase coil current beyond a certain value results in a waste of electrical power, without achieving any useful increase in flux.

Permeability of ferromagnetic materials changes with differing values of flux density. It can be shown that for a given flux density, μ is equal to the ratio B/H, where the values of B and H are those that apply for that particular flux density.

The above statement may be proved by applying the basic magnetic equation in the following manner:

$$\Phi = \frac{IN}{R_m}$$

but $\Phi = BA$ (section 5.5.3) and

$R_m = l/\mu_r\mu_0 A$ (section 5.5.5)

By substitution:

$$BA = \frac{IN\mu_r\mu_0 A}{l}$$

$$\therefore B = \frac{IN\mu_r\mu_0 A}{lA} = \frac{IN\mu_r\mu_0}{l}$$

Since $H = IN/l$ (section 5.5.2):

$$B = H\mu_r\mu_0$$

or $\mu_r\mu_0 = \dfrac{B}{H}$

From section 5.5.4:

$\mu_r\mu_0$ = permeability (μ)

that is, $\boxed{\mu = \dfrac{B}{H}}$

Table 5.1 gives values for B and H for iron. It is possible to use these values to calculate permeability for each particular flux density and magnetising force. In Table 5.2, values for μ have been calculated from the given values of B and H.

The values of μ_r have been plotted against values of H in Figure 5.15 to give the μ_r/H curve. It can be seen that the permeability curve rises steeply to a peak. Beyond this point of maximum permeability the curve slopes away quite rapidly. This indicates that permeability becomes progressively less as H is increased beyond the value that causes magnetic saturation.

5.6.4 Comparison of B/H magnetisation curves

Figure 5.16 illustrates the magnetisation curves for silicon steel, cast steel and cast iron.

The following points should be noted:

1. The materials tend to become magnetically saturated in the region that corresponds with the centres of the 'knees' of the respective curves.
2. When the value of H is in the lower ranges, much greater flux density will be produced in silicon steel compared with cast steel or cast iron.
3. Silicon steel saturates at a slightly lower value of flux density than cast steel.
4. Cast iron saturates at much lower values of flux density than either silicon steel or cast steel. It is also much harder to magnetise than either of the above materials.

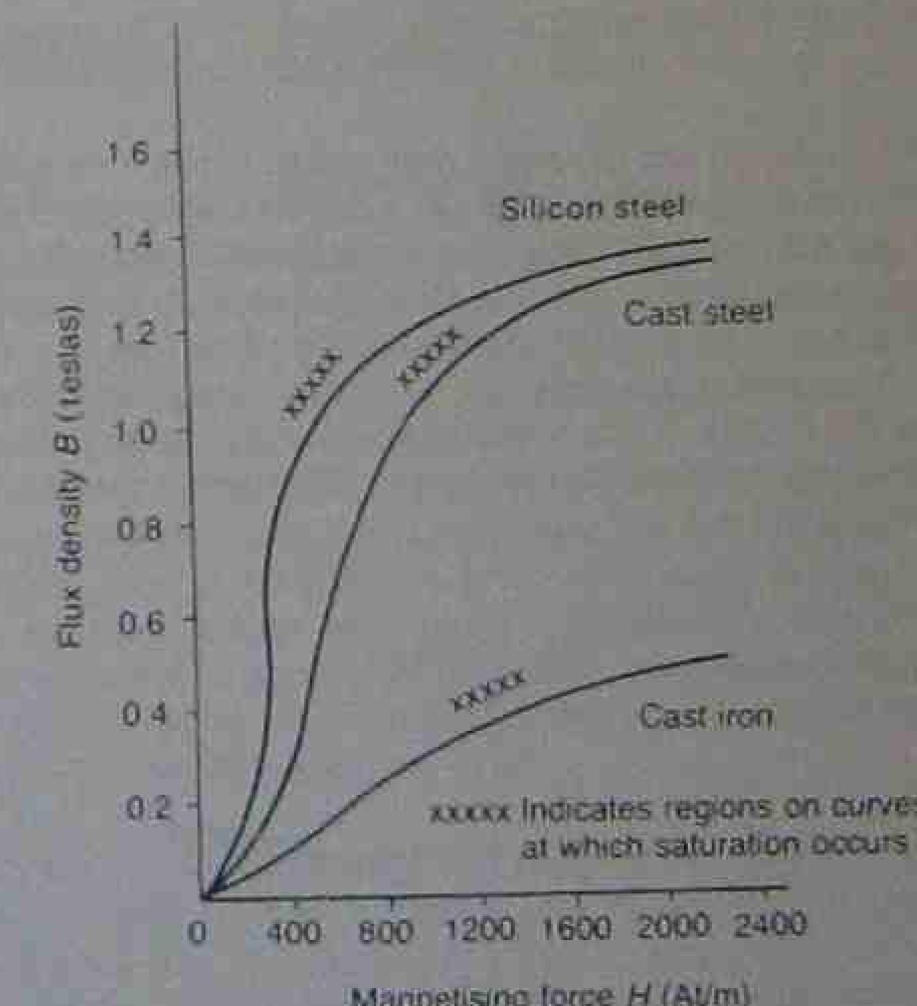


Figure 5.16 • Comparison of B/H magnetisation curves for ferromagnetic materials

Table 5.2 • Permeability values derived from magnetisation figures

H	100	200	300	400	500	600	700	800	900	1000	1200	1400	1600	2000
B	0.04	0.12	0.40	0.90	1.00	1.06	1.11	1.15	1.18	1.21	1.25	1.29	1.32	1.36
$\frac{B}{H} = \mu$	0.00040	0.00060	0.00133	0.00225	0.00200	0.00177	0.00159	0.00144	0.00131	0.00121	0.00104	0.00092	0.00083	0.00068
$\mu_r = \frac{\mu}{\mu_0}$	318	477	1058	1790	1591	1408	1265	1146	1042	963	828	732	660	541

5.7 MAGNETIC HYSTERESIS

The term hysteresis means to lag. In electrical terminology it is used to describe the lag between a change in value or direction of the magnetising force and the resulting change in value or direction of flux.

Residual magnetism is that portion of the flux that remains in a ferromagnetic material when the magnetising force is removed. In order to remove residual magnetism it is necessary to use a force that acts in the opposite direction to the original magnetising force. This force used to overcome residual magnetism is known as the coercive force.

Each time a ferromagnetic material is magnetised first in one direction and then in the other it is necessary to use coercive force to overcome the effect of residual magnetism. The amount of coercive force required depends on the type of magnetic material in use.

5.7.1 Hysteresis loops

If magnetisation curves are plotted to show the variation of flux density in a ferromagnetic material for both increasing and decreasing values of magnetising force, it is found that the curves do not coincide. This deviation between the curves is caused by hysteresis.

Figure 5.17 illustrates the curves that result when a sample of ferromagnetic material is subjected to a magnetising force which varies in both magnitude and direction.

Section AC shows the curve that results when a steadily increasing magnetising force (+H) is applied to the material that was initially in a completely demagnetised state. Point C represents the value of flux density that occurs when it reaches a maximum value in the positive direction.

If H is now gradually decreased, the magnetisation curve CE results. AE indicates the value of residual flux density. This residual flux remains in the material when H is reduced to zero.

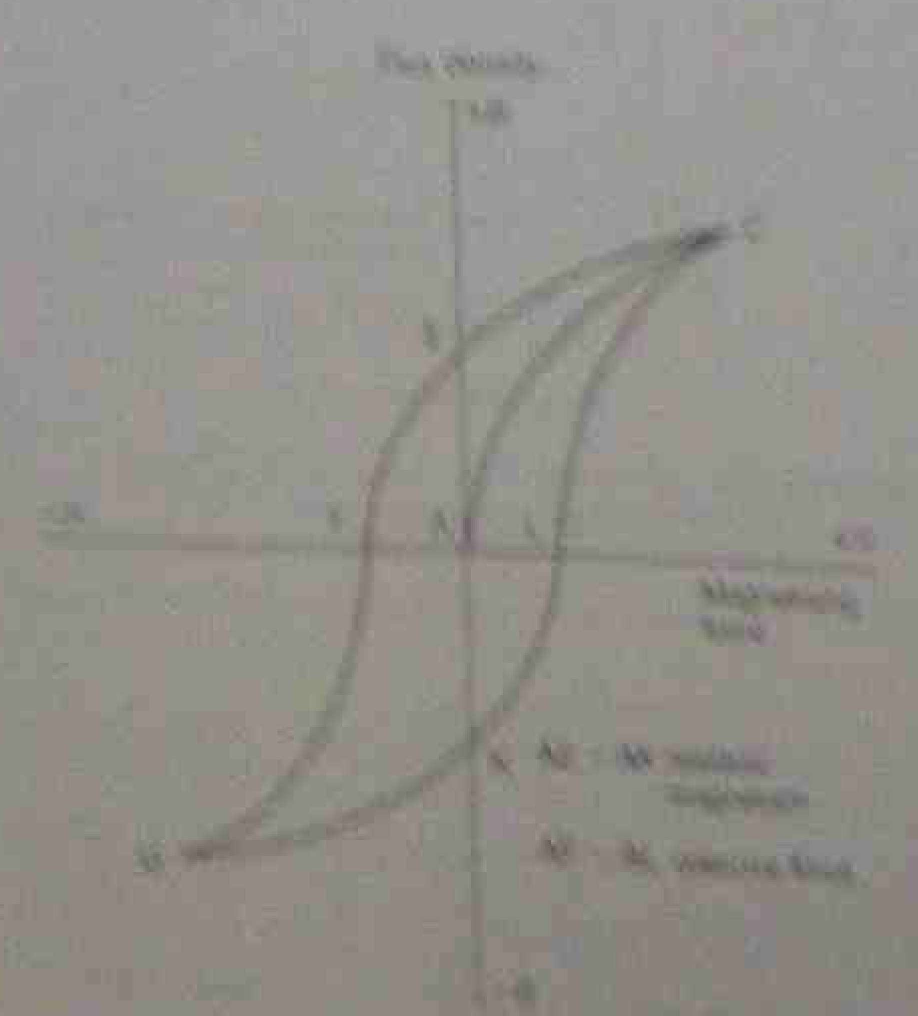


Figure 5.17 • Hysteresis loop for a magnetic material

Application of a magnetising force in the opposite direction to the original magnetising force will reduce the residual flux to zero. The value of this coercive force is indicated by AF.

By increasing H beyond point F in a negative direction, flux may be set up in the opposite direction to that of the original flux. The curve FG represents the flux density resulting from application of −H. Point G represents the maximum value of flux density in the negative direction.

If the magnetising force is again reversed, and changed from maximum in the negative direction to maximum in the positive direction, it is found that associated changes in flux density will be shown by the portion of the curve GKLC.

Thus the combined magnetisation curves form a closed loop, CEFGKLC. This loop is commonly known as a hysteresis loop for a ferromagnetic material.

5.7.2 Hysteresis losses

Changing magnetic flux causes changes in the alignment of molecules in a magnetic material. This results in the generation of heat within the material. Such heat generation in a magnetic material represents a loss of energy which is known as hysteresis loss.

The amount of hysteresis loss for a particular material varies directly with the area within the hysteresis loop for that material. It follows that the hysteresis loop gives an important indication of the suitability of a magnetic material for a particular application. In equipment that is subjected to a rapidly changing flux, it is important that the material used in the magnetic core has a hysteresis loop of small area.

Because hysteresis loss appears in the form of heat in the magnetic cores of equipment it is known as an iron loss. Electrical power has to be consumed to make up for iron loss and it is therefore usual to give values of iron loss, for a particular material, in watts per kilogram.

The areas of hysteresis loops, obtained from tests on samples of magnetic materials, give important information on the suitability of a material for a particular application. Figure 5.18 compares the curves of transformer steel and carbon steel. It shows that the hysteresis loop for transformer steel is comparatively small in area, which indicates that transformer steel will give a relatively small iron loss. This is an important factor in the choice of core material for transformers, because rapidly reversing fluxes occur in this application. Carbon steel would not be suitable because of the large iron loss that would occur with this material.

5.7.3 Other magnetic losses

Magnetic leakage

In many practical magnetic circuits it is desirable to have the maximum value of flux that may be economically obtained, across a particular section of the circuit. Unfortunately there is a tendency for some lines of force to leave the core material and to return to their source through the surrounding air. This results in the loss of a portion of the flux between the source of m.m.f. and the point at which the flux is needed.

Figure 5.19 illustrates part of a magnetic circuit in which the source of m.m.f. is a permanent magnet. In this

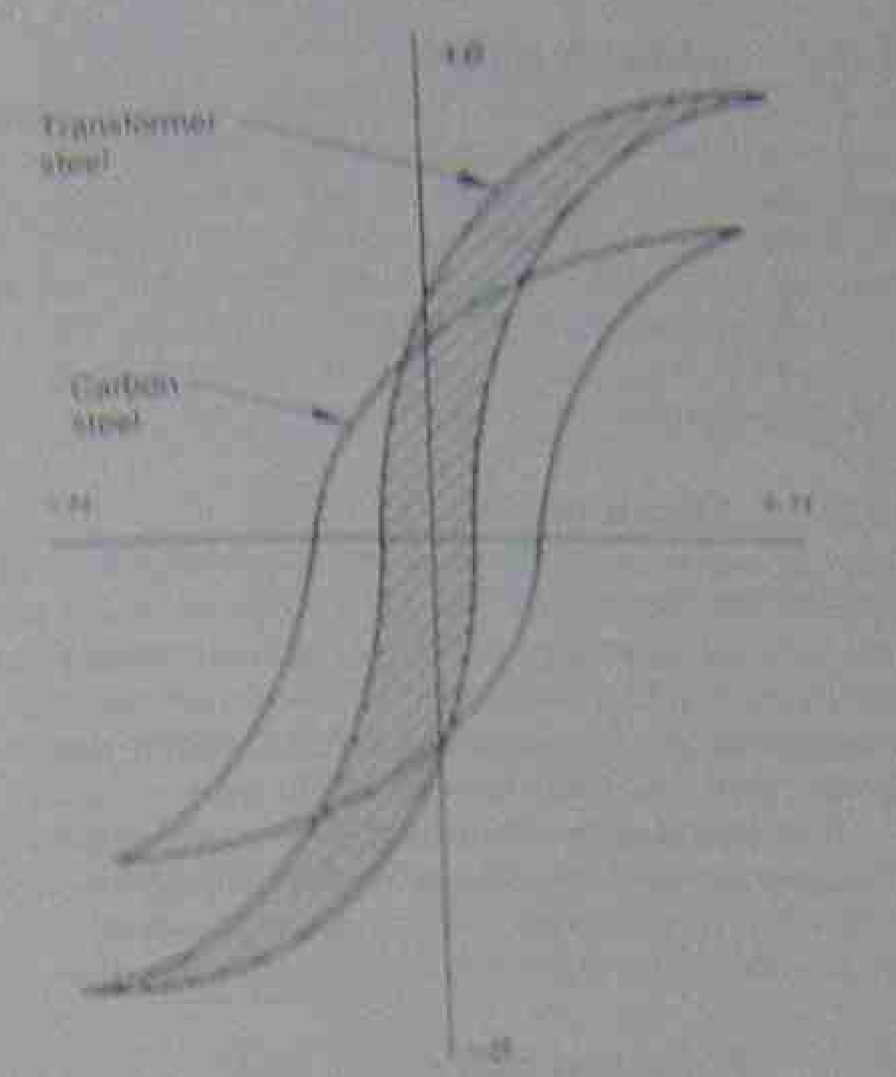


Figure 5.18 • Comparison of hysteresis loops for two magnetic materials

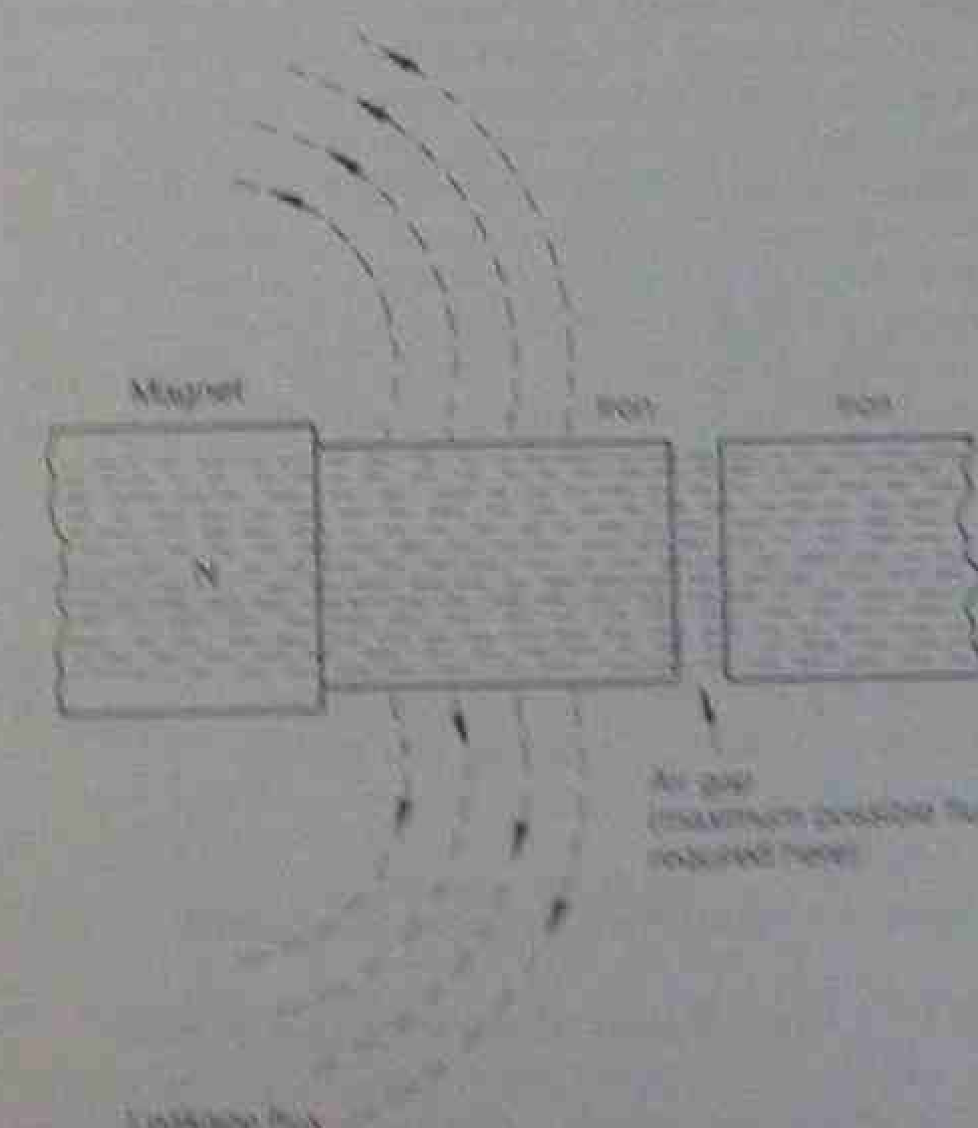


Figure 5.19 • Magnetic leakage

particular circuit, maximum possible flux is desirable at the air gap. Thus the flux must pass through a section of iron core before reaching the air gap.

The total flux produced at the source of m.m.f. does not reach the air gap because a portion of it leaves the iron core and returns through the surrounding air. This portion of flux which leaves the main path is known as the leakage flux.

The tendency for part of the flux to stray from the desired path is known as magnetic leakage. When designing magnetic circuits, it is necessary to allow for magnetic leakage when calculating the values of flux required.

Magnetic fringing

Figure 5.20 shows the magnetic field that exists across an air gap in a magnetic circuit. The lines of force near the centre line of the flux path are straight. Lines of force at the edges of the field tend to curve outwards in the air gap. As a result, the area of the flux path is greater in the air gap than in the material of the magnetic circuit. The flux density of the air gap will in consequence be less than that in the magnetic material on either side of the gap. This effect is known as magnetic fringing and must be allowed for when designing magnetic circuits in which there are air gaps.

Figure 5.20 • Magnetic fringing

5.8 MAGNETIC RELAYS

A magnetic relay consists of a solenoid that has an iron core, part of which is movable. Some force, such as gravity or a spring, is provided to hold the movable portion (the armature) away from the fixed portion (the core) when the solenoid is de-energised.

Relays can be used to operate electrical contacts for closing or opening a circuit, or they can operate a plunger whose movement is used to effect a mechanical operation.

Magnetic relays are usually controlled by electromagnets alone or in conjunction with permanent magnets. The latter type are mainly polarised relays. Typical examples of both are described below.

5.8.1 Simple electromagnetic relays

Figure 5.21 shows a simple attracted-armature type of relay used to open or close an electrical circuit.

When current flows in the operating coil it sets up a magnetic flux in the soft iron core. If the air gap between the core and the armature is reasonably small, most of the core flux will pass through the armature and induce polarities in the pole faces of the armature as shown, and a force of attraction will exist between the armature and the core. If this tractive force is greater than the force holding the armature in the open position, it will pull the armature close up to the core and hold it there as long as the flux in the core is great enough to support the armature against the forces tending to open it. The tractive force exerted on the armature in the closed position (minimum air gap) will be many times greater than when the armature is in the fully open position (maximum air gap).

In the case of a coil connected to a direct current supply

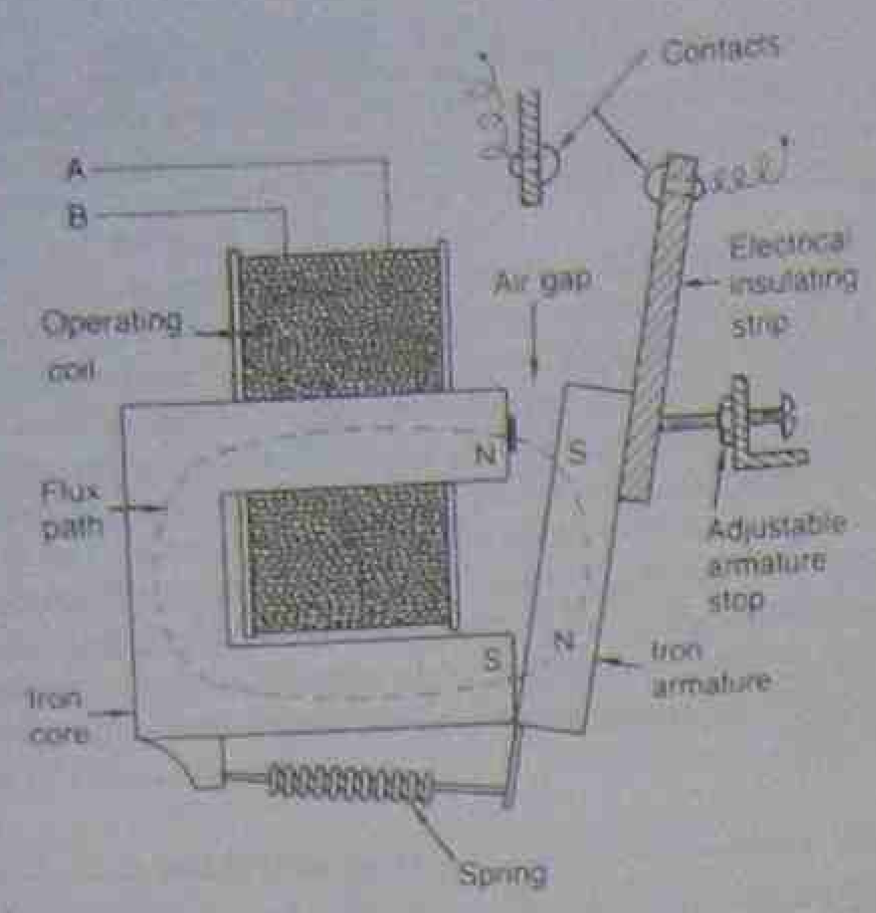


Figure 5.21 • Typical relay construction

the current will be constant for all positions of the armature and only the circuit reluctance will change with each change in the length of the air gap. With the coil connected to an alternating supply, conditions are somewhat different due to the coil current being dependent on the flux and the reluctance of the magnetic circuit. The reasons for this are considered later.

If the ampere-turns are great enough to create the tractive force necessary to close the armature through a large air gap, then this same force will often leave a residual flux in the magnetic circuit which is strong enough to keep the armature closed even when the coil current is switched off. This difficulty can be overcome by using a non-magnetic stop, or spacing piece, on one pole face to ensure that a certain minimum air gap is left in the magnetic circuit when the armature is in the fully closed position. The length of this gap must be such that the residual magnetism is not sufficient to maintain the armature in the closed position.

The photograph in Figure 5.22 shows typical small relays.

Figure 5.22 • Typical relays. (a) This relay operates several sets of contacts at current ratings of 100 mA. (b) This relay operates only one set of changeover contacts at a current rating of 2 A

5.8.2 No-volt relays

No-volt relays are circuit-closing relays with an operating coil connected to the supply voltage and the armature and contacts are held closed as long as the voltage across the coil is above a certain percentage of the normal circuit voltage. When the circuit voltage falls below this minimum value the armature is released and the relay contacts open the control circuit of the apparatus being protected against low voltage.

5.8.3 Overload relays

Another common electromagnetic protective relay is the magnetic overload relay. The operating coil of this relay is connected in series with the circuit to be protected against over-current. When the current exceeds a preset value, the armature is attracted and the relay contacts disconnect the overloaded circuit from the supply.

In this type of relay, the important feature is the length of air gap between the core and armature poles, because it controls the minimum value of the ampere-turns necessary to attract the armature and operate the relay.

5.8.4 Polarised relays

One form of polarised relay is the reverse-current relay with its normal magnetic polarity controlled by a voltage-operated coil; however, it is also subject to the influence of a current-operated coil. Should the current flow in the correct direction, both voltage and current coils produce fluxes that act in the same direction around the common magnetic circuit. If the current should flow in the reverse direction, it tends to demagnetise the iron core and allows the control spring to release the armature, so interrupting the flow of reverse current through the circuit. A common example of this form of relay is the reverse current cut-out in automobile battery-charging circuits. It is shown in Figure 5.23.

5.9 GENERATION OF A VOLTAGE

An e.m.f. can be induced in a conductor if it cuts or is cut by lines of force. The following combination of factors is required to produce an e.m.f. by this means:

- conductors
- magnetic field
- relative motion between conductor and field.

5.9.1 Magnitude of an induced e.m.f.

The value of an induced e.m.f. depends on the rate of change of the flux linkages. This in turn depends on the number of conductors, the quantity of flux and the rate of the relative motion. A flux linkage occurs when one unit of flux cuts across one conductor.

1. The value of induced e.m.f. varies directly with the number of conductors connected in series. An increase in the number of conductors will result in an increase in the value of voltage.
2. The value of induced e.m.f. varies directly with the quantity of flux. An increase in flux will cause an increase in voltage.
3. The value of induced e.m.f. varies directly with the

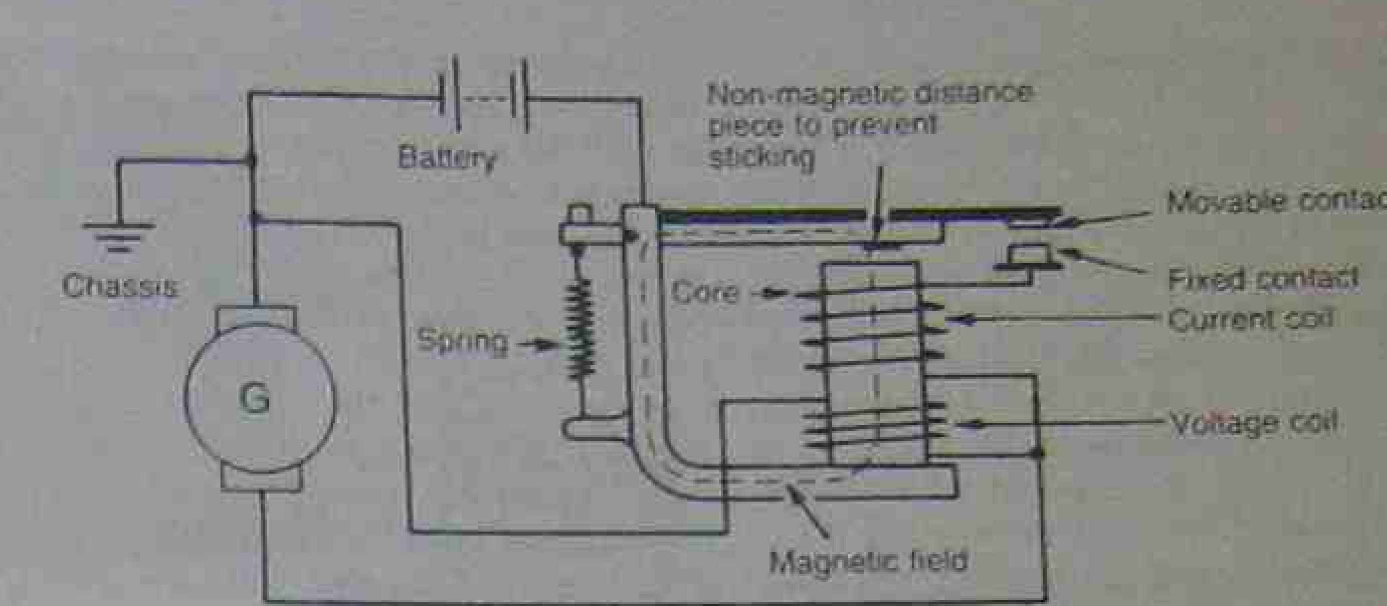


Figure 5.23 • Typical reverse-current relay circuit

rate at which flux linkages change. The rate of change in turn depends on the rate of relative motion between the conductors and the magnetic field.

The preceding three factors can be related as follows:

$$V = N.\frac{\text{change in flux}}{\text{time change}}$$

that is, $$\boxed{V = N.\frac{\Delta\Phi}{\Delta t}}$$

where V = induced voltage
$\Delta\Phi$ = flux change in webers
Δt = time change in seconds

This is the mathematical expression of Faraday's law of induced e.m.f.:

> *The value of the e.m.f. induced in a circuit depends on the number of conductors in the circuit and the rate of change of the magnetic flux linking the conductors.*

Example 5.8

A coil of 600 turns has a flux of 0.000 08 Wb passing through it. If the flux is reduced to 0.000 03 Wb in 15 ms, find the average induced voltage.

$$V = N.\frac{\Delta\Phi}{\Delta t}$$
$$= 600 \times \left(\frac{0.000\,08 - 0.000\,03}{0.015}\right)$$
$$= \frac{600 \times 0.000\,05}{0.015}$$
$$= 2\text{ V}$$

Example 5.9

A coil of 500 turns has a permanent magnet moved into it such that 0.2 Wb cuts across the coil in 4 s.

$$V = N.\frac{\Delta\Phi}{\Delta t}$$
$$= \frac{500 \times 0.2}{4}$$
$$= 25\text{ V}$$

It has been seen that an e.m.f. can be produced by thermo-couples (Ch. 2), by chemical action (Ch. 3) and the relative movement between magnetic lines of force and a conductor. This third method of dynamically induced e.m.f. is undoubtedly the most important method and has quite a long and innovative history. The e.m.f. is produced by the continuous movement of a conductor in a magnetic field. The method and magnitude of the induced voltage is further discussed in Chapter 8 section 8.3.4.

5.9.2 Lenz's law

Current flow that results from an induced e.m.f. will produce a field about a conductor in which the e.m.f. is induced. The direction of action of such an induced field is defined in Lenz's law:

> *The direction of an induced e.m.f. is such that the resulting current flow will produce a magnetic field which tends to oppose the original motion causing the induced e.m.f.*

This is the electrical counterpart of Newton's third law, which states that:

> *Action and reaction are equal and opposite.*

The application of Lenz's law is illustrated in Figures 5.24 and 5.25. Figure 5.24 represents downward movement of a conductor through a magnetic field as a result of mechanical force acting in this direction. This downward movement through the field will induce an e.m.f. in the conductor and the resulting current will create a circular magnetic field around the conductor.

For Lenz's law to apply, the induced field must oppose the motion. To do this, the direction of the induced field must be such that the following reaction will occur between the main and induced fields:

1. The main field will be strengthened below the conductor and weakened above it.
2. The resulting magnetic force should then act upwards in opposition to the motion of the conductor.

If the above conditions are to apply, it is necessary for the induced field to be in the anticlockwise direction

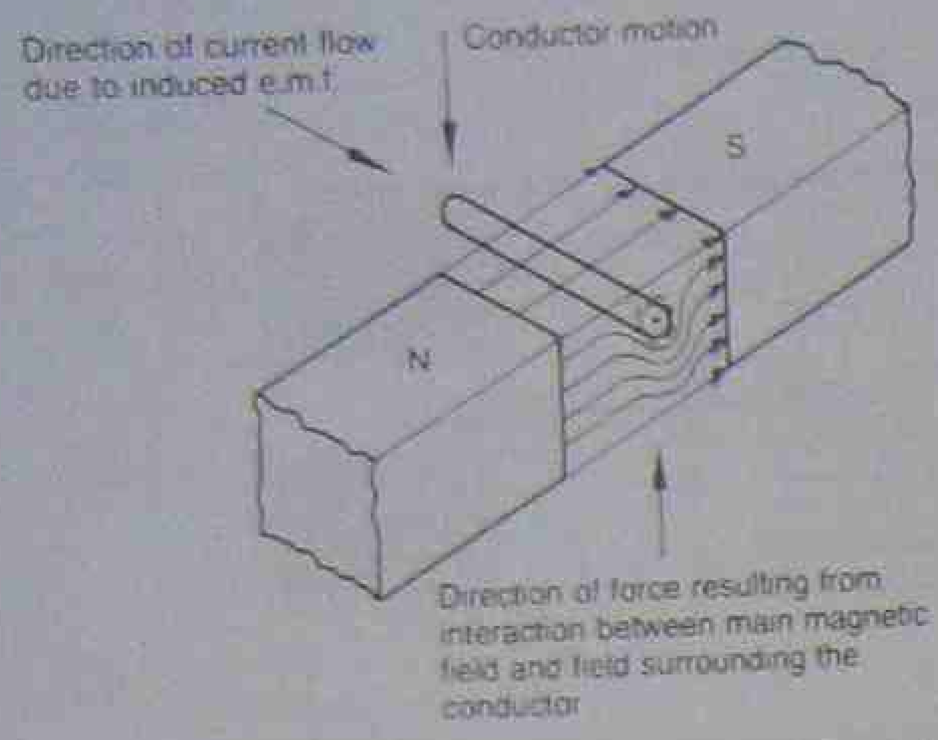


Figure 5.24 • Illustrating Lenz's law—conductor moving

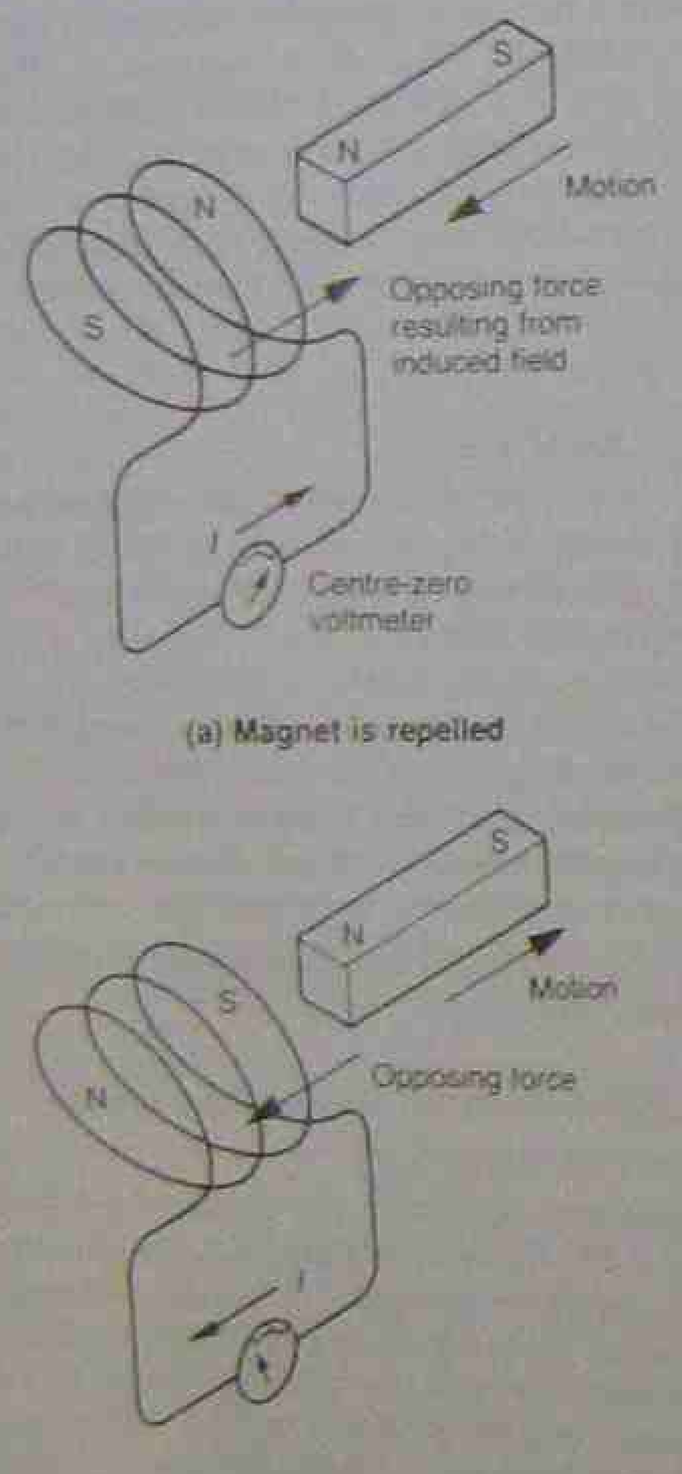


Figure 5.25 • Illustrating Lenz's law—field moving

illustrated. It will follow that the induced current, which produces the field, must flow in the direction shown.

As a further illustration of Lenz's law, Figure 5.25(a) shows a permanent magnet which may be moved along the axis of a coil. The action of moving the magnet into the coil will induce an e.m.f. in the coil.

By applying Lenz's law, it can be seen that the induced current flow must be in such a direction that the resulting field will oppose the original motion. The polarity of the field that is induced about the coil will be as shown, that is, with the north pole of the induced field adjacent to the north pole of the permanent magnet. The force created by this induced field will then oppose the original motion, as shown by the arrow. Note that this is consistent with the fundamental law of magnetism which states that like poles repel.

In Figure 5.25(b) the direction of movement has been reversed. As the magnet is withdrawn from the coil, the induced field will resist the motion. This opposing action will continue until the magnet moves beyond the area of influence of the induced field surrounding the coil.

5.10 INDUCTANCE

Inductance is the property of a circuit that enables an e.m.f. to be induced in it.

An inductance coil has the property of inductance, so that it is designed to generate an e.m.f.

In practice, various names are given to coils that are classified as inductance coils. For example, the inductance coil used in automotive ignition systems is known as an ignition coil. In other circuits they can be known as a choke. For the purpose of this section it is proposed that they be known by the general term 'inductors', having the property of inductance.

5.10.1 Inductors

The value or quantity of inductance in an inductor is a function of the number of turns in the coil, the magnetic effects of the core, and the flux density at which it is operating.

At very high frequencies (50 MHz and above, for example) core losses are a cause of concern. Losses can become prohibitive at high frequencies, so a choice of core material is an important one.

At power-line frequencies, metallic cores have advantages, while at radio frequencies, iron-dust cores have reduced losses. At still higher frequencies, air-cored coils are essential.

Air-cored inductors

An air-cored inductor symbol is shown in Figure 5.26(a). This is the general symbol for all inductors. The basic symbol is four half-loops and, only if more information is required is an indication of the core type given.

Iron-powder cores

Sometimes called ferrite cores, iron-powder cores are manufactured from very fine iron powder mixed with an insulating bonding medium and allowed to set in preformed shapes. Owing to the small size of the particles, iron losses are greatly reduced. Their use, however, is still restricted to much lower frequencies than air-cored inductors and transformers.

In radios they can be used as cores up to about 2 MHz or 3 MHz. The central iron-powder core position is often adjustable so that the unit can be tuned to resonance by altering the inductance of the core. In rod form, these can often be seen in domestic radios as a component of the antenna system.

Iron-powder cores are also used in small transformers at

Figure 5.26 • Inductors

power-line frequencies and in special-purpose inductors operating at higher frequencies.

To indicate an iron-powder core is being used, the inductor symbol shown in Figure 5.26 would have a dashed or broken line added above the four loops.

Iron cores

Iron cores are available in two types—laminated and solid. Solid cores are used for direct current pole-pieces, or in the rotors of synchronous machines where the magnetic polarity does not change. The inductance of the coils associated with this use is usually so high that the poles are restricted to frequencies below approximately 5 Hz. The iron losses are prohibitively high if the magnetic polarity of the pole changes at a higher frequency.

Laminated cores and transformers are discussed in greater detail in Chapter 14. The cores are laminated to reduce iron losses mainly at power-line frequencies. Special types of ferrous materials have been developed to reduce these losses further. Some laminations are made into packs and preformed, and then stress relieved to form what are commonly called 'C-cores'. It is a descriptive term relating to their shape.

Everyday mild-steel sheeting can be used to make laminations for smaller transformers, but for larger distribution transformers the more expensive electrical sheet-steel alloy is used. The special steel allows higher flux densities and a smaller iron core for the same power output. Owing to the lower iron losses there is a lesser problem with heat generated in the cores.

If it is necessary to indicate that an iron core has been used, the inductor symbol is modified by adding a solid line above the four loops of the inductor symbol. This is shown in Figure 5.26.

5.10.2 Unit of inductance

The unit of inductance is the henry. It is defined by the AS/NZS 1000 Standard as:

> *A henry is the inductance of a closed circuit in which an e.m.f. of one volt is produced when the electric current flowing in the circuit varies uniformly at the rate of one ampere per second.*

The basic unit is the henry and it is quite common to use submultiples of the unit such as milli-henrys (mH) and micro-henrys (μH). Multiples of the henry are virtually unknown because of the large physical sizes involved.

In Table 1.4 the general symbol for inductance was given as L, for which the units were henrys (H).

If an inductor has an inductance of L henrys and the current changes from i_2 to i_1 in t seconds:

average rate of change of current

$$= \frac{(i_2 - i_1)}{t} \text{ amperes/second}$$

average induced e.m.f.

$$= L \times \text{rate of change of current}$$

that is, $V = L\dfrac{(i_2 - i_1)}{t}$

or $\boxed{V = L\dfrac{\Delta I}{\Delta t}}$

where L = inductance in henrys
V = induced e.m.f. in volts
ΔI = change in current in amperes
Δt = time interval in seconds

Example 5.10

If the current through an inductor of 1.5 H is reduced from 5 A to 1 A in 0.5 s, find the average value of the induced e.m.f.

$$V = L\frac{\Delta I}{\Delta t}$$

$$= \frac{1.5 \times (5 - 1)}{0.5} = \frac{1.5 \times 4}{0.5}$$

$$= 12 \text{ V}$$

Example 5.11

An inductor of 0.05 H has a current flowing through it of 2 A. If the current is reduced to zero in 1 ms, find the induced voltage across the terminals.

$$V = L\frac{\Delta I}{\Delta t} = \frac{0.5 \times 2}{0.001}$$

$$= 100\ \mathrm{V}$$

Inductance can also be expressed in terms of the change in flux linkages brought about by a change of current:

$$\boxed{L = N\frac{\Delta\Phi}{\Delta I}}$$

where L = inductance in henrys
$\Delta\Phi$ = change in flux linkage
ΔI = change in current
N = number of conductors

For a simple cylindrical solenoid of length greater than its diameter, a relatively close estimation of its inductance can be calculated from the equation:

$$\boxed{L = \frac{N^2\mu_r\mu_0 A}{l}}$$

where L = inductance in henrys
N = number of turns on solenoid
A = cross-sectional area of the solenoid
l = length of solenoid
μ_r = relative permeability
μ_0 = absolute permeability ($= 4\pi \times 10^{-7}$)

5.10.3 Self-inductance

The term self-inductance is used when a conductor or coil has a voltage induced in it by its own magnetic field.

Figure 5.27(a) shows the field that surrounds a conductor in which current flows at a steady value. Because the current flow is steady, the field also is steady and there is no relative movement between the field and the conductor.

As current flow in a conductor increases, the flux density of the surrounding field will also increase. This leads to crowding of the lines of force and results in a tendency for the field to expand outwards from the conductor, thus causing relative movement between field and conductor. This condition is illustrated in Figure 5.27(b).

Under the conditions illustrated in Figure 5.27(b), the conductor is fixed but the field is moving. It is convenient, however, to consider that the conductor moves relative to the field when determining the direction of induced e.m.f. Relative conductor movement is in the direction indicated by the broken arrow.

When current flow in a conductor decreases, flux density also decreases, and the magnetic field tends to contract. The relative movement in this case is opposite in direction and is shown in Figure 5.28.

The relative movement between conductor and field that results from current changes in the conductor will induce an e.m.f. in the conductor. This induced e.m.f. results directly from current change within the same conductor and it is therefore known as a self-induced e.m.f. A circuit in which self-induced voltages occur is said to have the property of self-inductance.

In inductors, the property of self-induction is inherent in their method of construction. When power is applied, the magnetic field builds up and in doing so produces a generated voltage that opposes the applied voltage.

When the current from the power source reaches a steady value, the relative movement of the field ceases and no induced voltage is generated.

When the power is switched off, the current has to reduce to zero and so the magnetic field has to collapse. While it is decreasing, an induced e.m.f. is produced in the opposite direction and this voltage opposes the decrease.

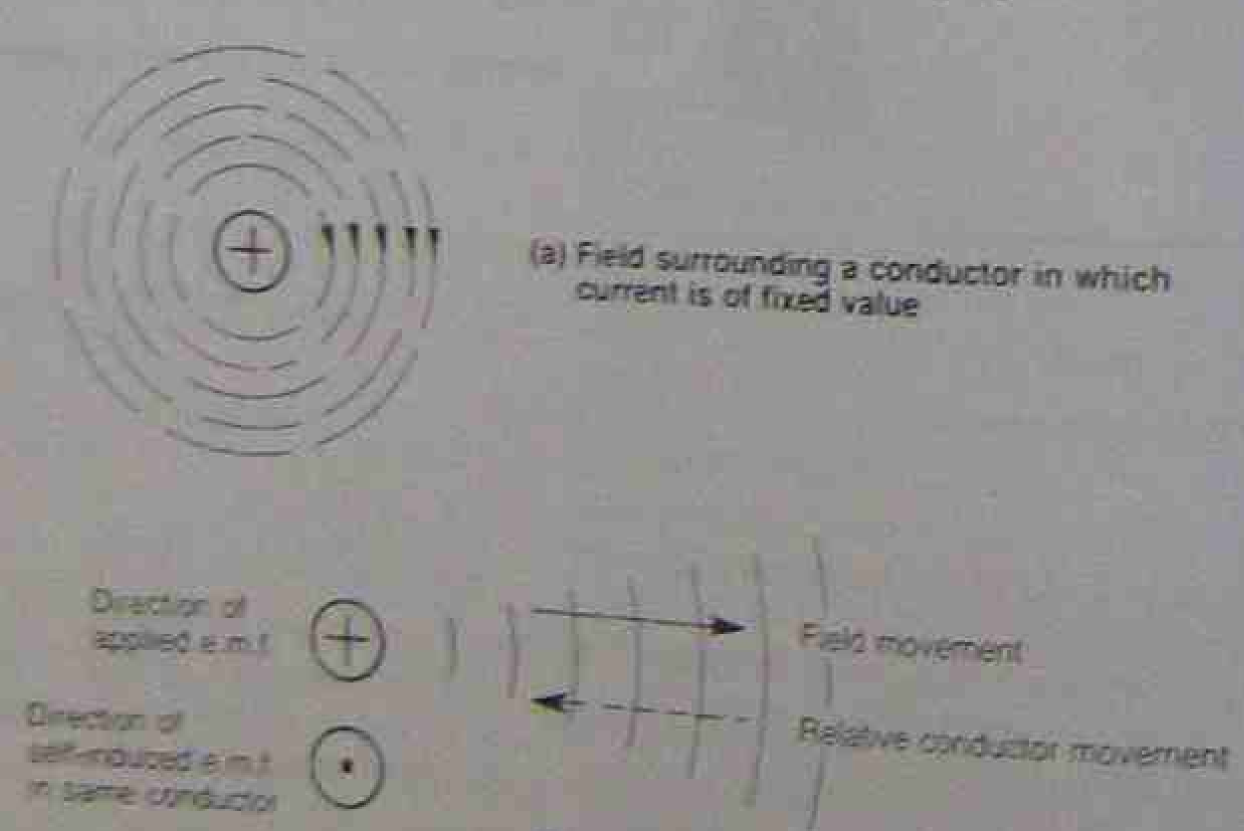


Figure 5.27 • Relative movement of a conductor in a magnetic field as the current increases

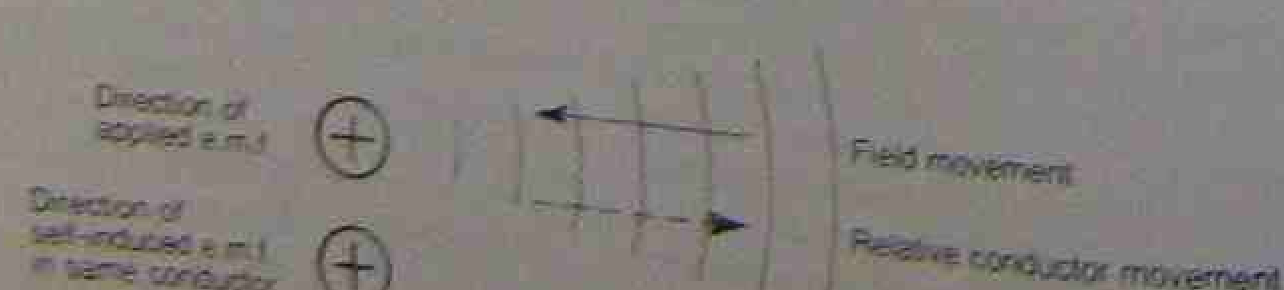


Figure 5.28 • Relative movement of a conductor in a magnetic field as the current decreases

5.10.4 Factors affecting the value of a self-induced voltage

In section 5.9.1 it was stated that the value of induced voltage in general depends on flux strength, the number of conductors and the relative rate of motion between them. These same factors also affect the value of a self-induced voltage.

The inclusion of an iron core within a coil greatly increases the field strength produced when a given current flows through the coil. Coils with iron cores will have much greater self-induced voltages than coils without iron cores.

Because the value of an induced voltage is dependent on the number of turns connected in series, the self-induced voltage produced by a coil of many turns will be greater than that produced by a coil of few turns. An increase in the number of turns in a coil will give an increase in self-induced voltage for a given rate of change of current flow. The reason for this is that the flux around any one conductor cuts not only that conductor, but others also.

The value of any induced voltage depends on the rate of motion, or the rate of change of flux linkages. The rate of change of flux linkages that cause self-induced voltages depends in turn on the rate of change of current. The more rapidly current changes in value or direction, the greater will be the self-induced voltage.

A good example is provided by comparing the conditions that apply during the making and breaking of an inductive circuit. The collapse of a magnetic field surrounding an inductor occurs much more rapidly at switching off than does the building up during switching on.

Figure 5.29 illustrates the relative directions and values of induced voltage during 'circuit break' and 'circuit make'. The following important points should be noted:

1. The self-induced voltage opposes current build-up during switching on.
2. The maximum value of self-induced voltage during switching on is less than the applied voltage.
3. The self-induced voltage also opposes current collapse at switching off.
4. The value of self-induced voltage is greater at circuit break than circuit make, and can be many times the value of applied voltage.
5. The greatest values of self-induced voltage occur at points where the current curves have the steepest slope. At these points the rate of change of current is greatest.
6. Where there is no current change, there is no self-induced voltage.

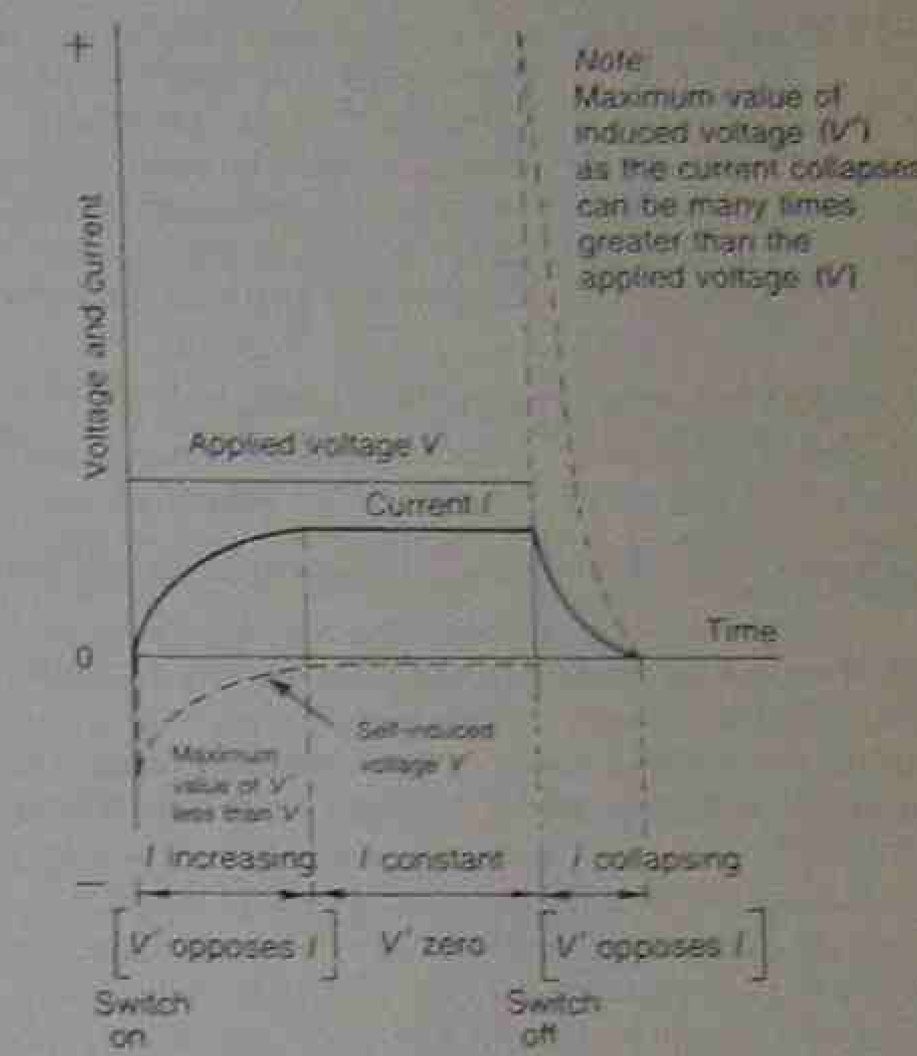


Figure 5.29 • Values and directions of self-induced voltages for different current conditions

The rate of change of current or voltage has no connection with the actual value of current or voltage. In Figure 5.29, for instance, when the current I is at its maximum value, the rate of change is zero.

The high self-induced voltage, which occurs during breaking of the circuit, will tend to maintain current flow because its direction of action opposes the collapse of the current. There will be a tendency for an arc to form as the contacts open, and in practice it is necessary to use special devices to decrease this arcing effect.

When opening highly inductive circuits it is necessary to use a bypass circuit through which the high self-induced voltage may be discharged. Reference to Figure 5.29 will show that this induced voltage can reach values many times the normal operating voltage of equipment. This could easily lead to failure of the circuit insulation if precautions are not taken. This 'bypass' circuit is usually called a snubber circuit. The effects of switching off inductive circuits are referred to as 'commutation'. Some of the circuit components are shown in Figure 5.30.

Figure 5.30(a) shows the relay; Figure 5.30(b) shows a diode connected across the coil. It is suitable only for d.c. circuits and is connected such that no current flows through it in normal operation. On switching off the relay, the self-induced voltage that attempts to keep the current flowing is shorted out by the diode. Figures 5.30(c), (d), (e) show variations of snubbing circuits with other components such as inductors, resistors, and capacitors.

5.10.5 Time constant

An inspection of Figure 5.29 shows that the applied voltage is at a maximum when switched on, but the current flow takes a period of time to reach its maximum value. Similarly, when the circuit is switched off, the

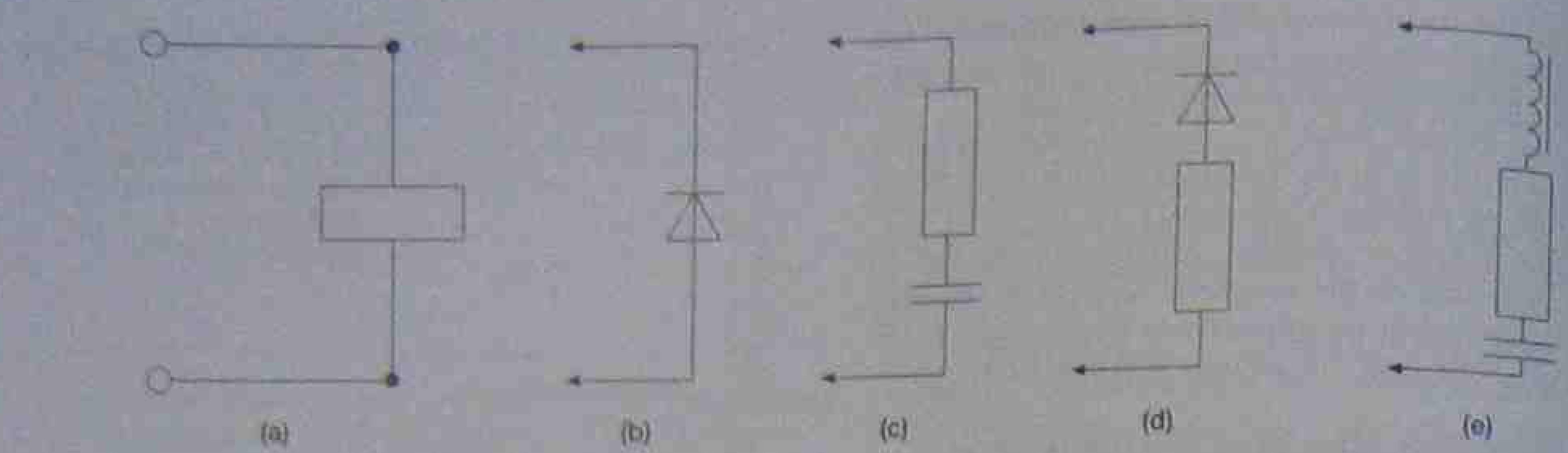


Figure 5.30 • Various snubber circuits designed to avoid high transient voltages when switching off inductive circuits

current does not immediately drop to zero but decreases rapidly at first and then more slowly.

An inductor is said to have a time constant when it is connected to a direct current supply. The time constant is found from:

$$\boxed{\tau = \frac{L}{R}}$$

where τ = time constant in seconds
L = inductance in henrys
R = resistance of inductor

From this expression it can be seen that the greater the inductance and the lower the resistance, the longer will be the time constant.

In general terms the time constant represents the time taken for the current to reach 63 per cent of its final value. It is the same period of time that the current would take if it continued to increase at its initial rate. Because it is a curve, the rate of increase slows down and finally stops.

As a guide the full period of time is approximately five times as long as the time constant (see Fig. 5.31).

When the current is decreasing, it decreases at the same rate as the increase. In L/R seconds it reduces to 37 per cent of the maximum value (the decrease is 63%). It is this initially rapid decrease in current (and flux) which causes induced voltages to be generated with values many times greater than the applied voltage. Because of the usually open-circuit conditions that exist with the current decrease, the sparking referred to earlier occurs at the contacts of the switch, unless steps are taken to prevent it.

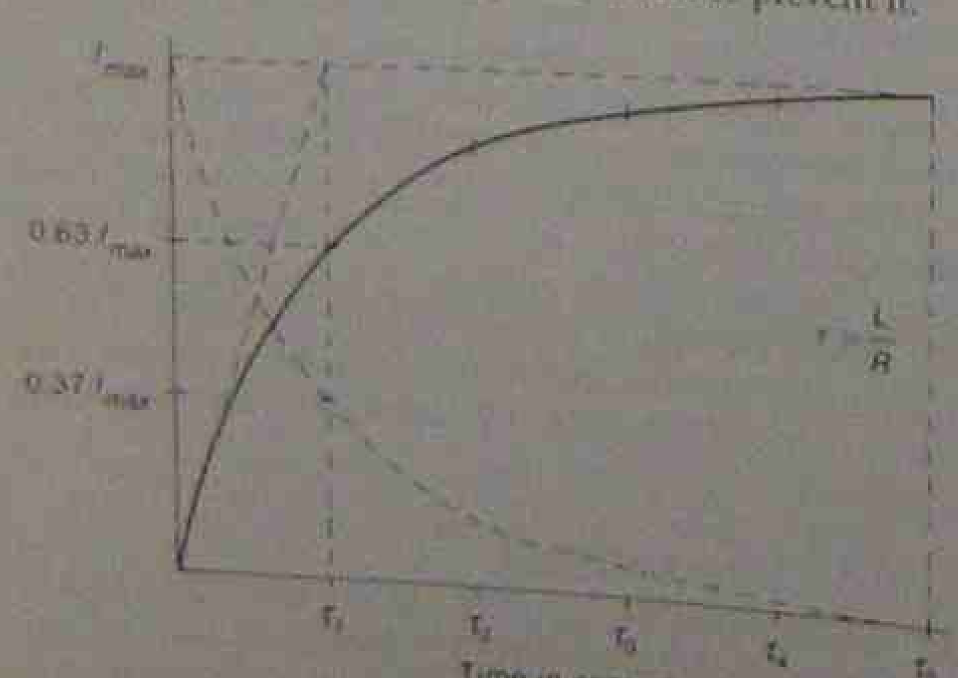


Figure 5.31 • Time constant for an inductive circuit

5.10.6 Energy stored in a magnetic field

If the arcing referred to above is allowed to occur, a current will flow. Because a current flows, energy is expended and this energy can only come from the magnetic field.

Energy stored in a magnetic field is:

$$\boxed{W = \frac{1}{2}LI^2}$$

where W = energy in joules
L = inductance in henrys
I = current flow in amperes

Example 5.12

A 10 H choke with a resistance of 15 Ω has a current flowing through it of 5 A. Find:

(a) the time constant of the choke

(b) the energy stored in the magnetic field.

$$\tau = \frac{L}{R}$$

$$= \frac{10}{15} = 0.67 \text{ seconds}$$

$$W = \frac{1}{2}LI^2$$

$$= \frac{1}{2} \times 10 \times 5^2$$

$$= 125 \text{ J}$$

5.10.7 Mutual induction between conductors

The term mutual induction is used to describe the effect when variation of current flow in a conductor causes an induced e.m.f. in a neighbouring conductor. It is not necessary to have an electrical connection between the conductors.

Figure 5.32 shows two parallel conductors. AB is connected in series with a switch and battery. The conductor CD and a centre-zero milli-voltmeter form a separate circuit.

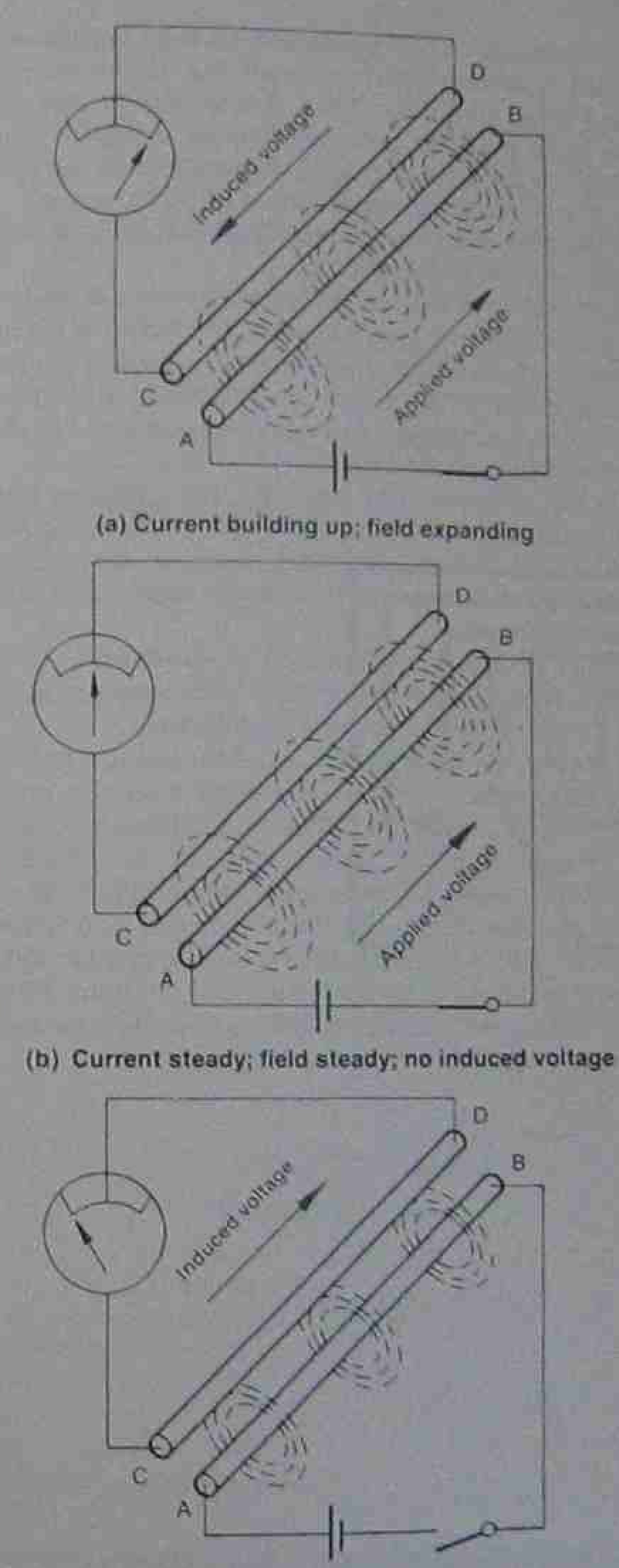
Immediately the switch is closed, current will build up in accordance with the curve shown in Figure 5.31. While current is building up in AB, the field about AB is increasing in strength and this will produce an induced voltage in conductor CD. This induced voltage will act in the direction shown in Figure 5.32(a).

Figure 5.32(b) shows the condition that applies when the current flow in AB is at a steady value. The field surrounding AB will be at a fixed strength and no induced voltage will occur in CD.

Figure 5.32(c) shows what happens immediately the switch is opened. The current flow in AB will collapse and, in doing so, will cause the field about AB also to collapse. This will induce a voltage in CD in the direction shown.

Conductor AB is termed the primary winding because it is connected to a source of power. Conductor CD is termed the secondary because it has an e.m.f. induced in it by the magnetic flux from the primary.

The terms are still applicable if multi-turn coils were used instead of single conductors. Note that if conductor CD had the power connected to it, it would be the primary winding, and conductor AB would be the secondary winding because it generated the induced e.m.f.



(a) Current building up; field expanding

(b) Current steady; field steady; no induced voltage

(c) Current switched off; field collapsing

Figure 5.32 • Mutual induction between parallel conductors

5.10.8 Mutual induction between coils

This effect will be more easily understood if reference is made to Figure 5.25 where it was shown that a voltage might be induced in a fixed coil by moving a permanent magnet into the coil. When the magnet was withdrawn, a voltage of opposite polarity was induced.

Consider what happens if the permanent magnet is replaced by an inductive coil, as illustrated in Figure 5.33. The secondary coil is in the same relative position as the coils illustrated in Figure 5.25. The primary coil, however, replaces the permanent magnet.

In Figure 5.25, relative movement between coil and magnetic flux was achieved by moving the magnet. The positions of the primary and secondary coils shown in Figure 5.33 are fixed, but relative movement between the flux of the primary coil and the inductors of the secondary coil results when the current in the primary coil changes in magnitude or direction.

When the current is first switched on, the field produced by the primary coil will have the polarities shown in Figure 5.33(a). Note the polarities in the secondary coil and it will be seen that this corresponds with those shown in Figure 5.25(a).

For steady values of current in the primary coil there will be no relative field movement and therefore no induced voltage in the secondary coil (Fig. 5.33(b)).

When the current is switched off, the primary field collapses and the lines of force again move in relation to the inductors of the secondary coil. The relative direction of movement will, however, be opposite to that which applied during switching on, and the polarity of the induced voltage in the secondary coil will also be reversed as illustrated.

5.10.9 Applications of mutual inductance

The principle of mutual inductance is used in generating high voltages in induction coils. An early version of an induction coil was called the Ford coil because it was used to provide ignition in an early model car (the T model Ford).

In an induction coil, an armature is magnetically vibrated at a rapid rate by the current flowing through the primary winding. Contacts attached to the vibrating armature then make and break the current to the primary winding, causing a rapid change in the flux linkage coupling both windings. By virtue of the large number of turns wound on the secondary winding, a high voltage is induced in that winding. It is normal practice to connect a capacitor across the opening and closing contacts to reduce sparking. An induction coil is shown in Figure 5.

Later car ignition systems used the Kettering system which is based on the same principles. In ignition systems the current is interrupted rapidly at precise times by a cam-operated switch. This breaking of the current in the primary coil causes a rapid change in the flux linkage between the primary and secondary windings. To ens

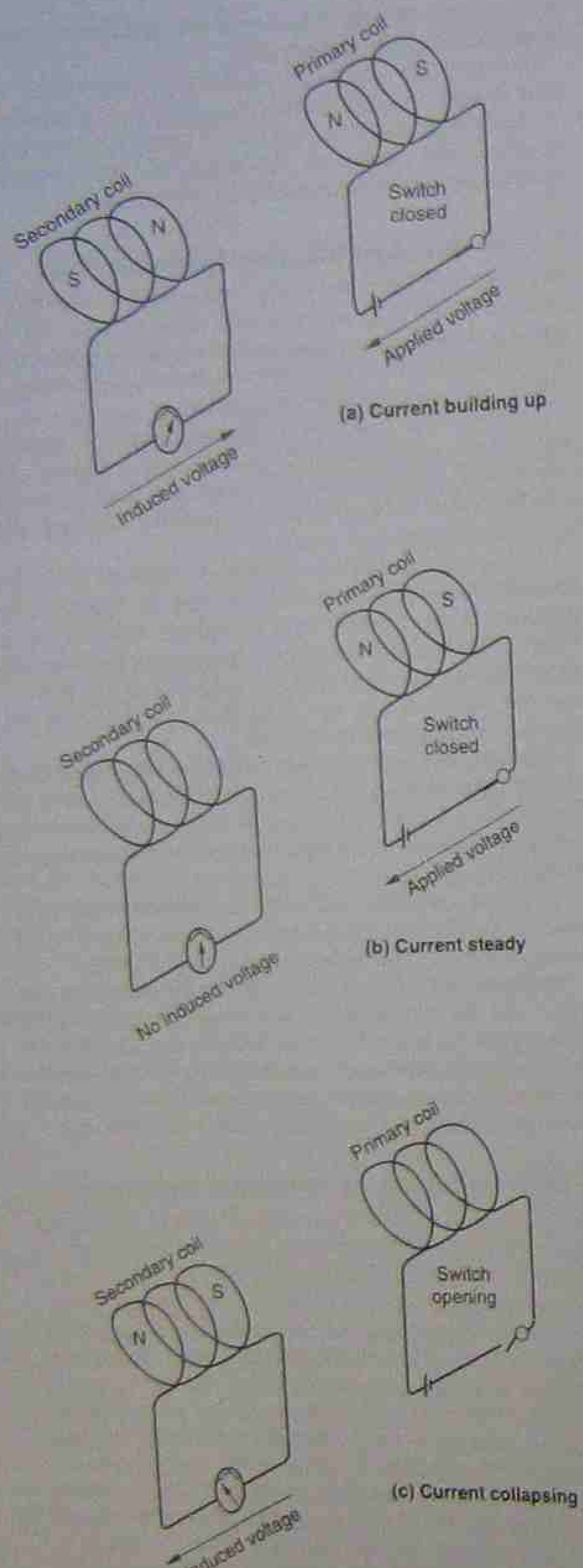


Figure 5.33 • Mutual induction between adjacent coils

the maximum rate of change in current, a capacitor is connected across the contacts of the switch to reduce arcing to a minimum. Since the secondary has many more turns than the primary winding, a high voltage is induced in the secondary and transmitted via the distributor to each cylinder in the correct firing sequence.

Figure 5.35 is a pictorial representation of the Kettering system of automobile ignition.

The trend is to replace the points method of switching the primary winding current with an electronic circuit to interrupt the current flow without moving contacts. Electronic control gives increased reliability with reduced maintenance costs and extends the operating life of the ignition system many times.

One car manufacturer currently uses impulses from a unit mounted on the front of the crankshaft to trigger the ignition coil. Many stationary engines use a similar system with the components mounted adjacent to and on the flywheel.

Further development is aimed at eliminating the distributor entirely, and replacing it with separate ignition coils for each cylinder or pairs of cylinders.

At present the most common use for mutual induction is in transformers. Instead of interrupting the primary current with contacts, an alternating voltage is applied to the primary winding. This results in an alternating current flow which is continually changing in both magnitude and direction. It eliminates the need for a system of points which are subject to arcing and have a limited life. The two windings of the induction coil in Figure 5.34 are the primary and secondary windings of the transformer. These are shown linked by the magnetic flux.

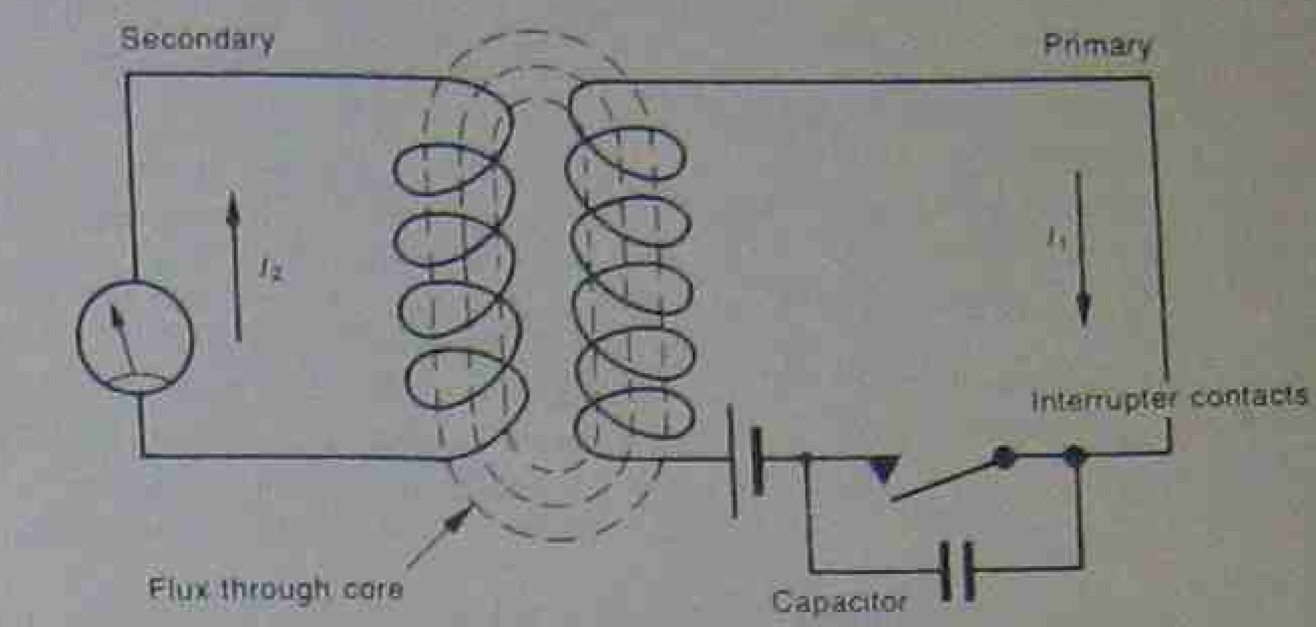


Figure 5.34 • Basic induction coil

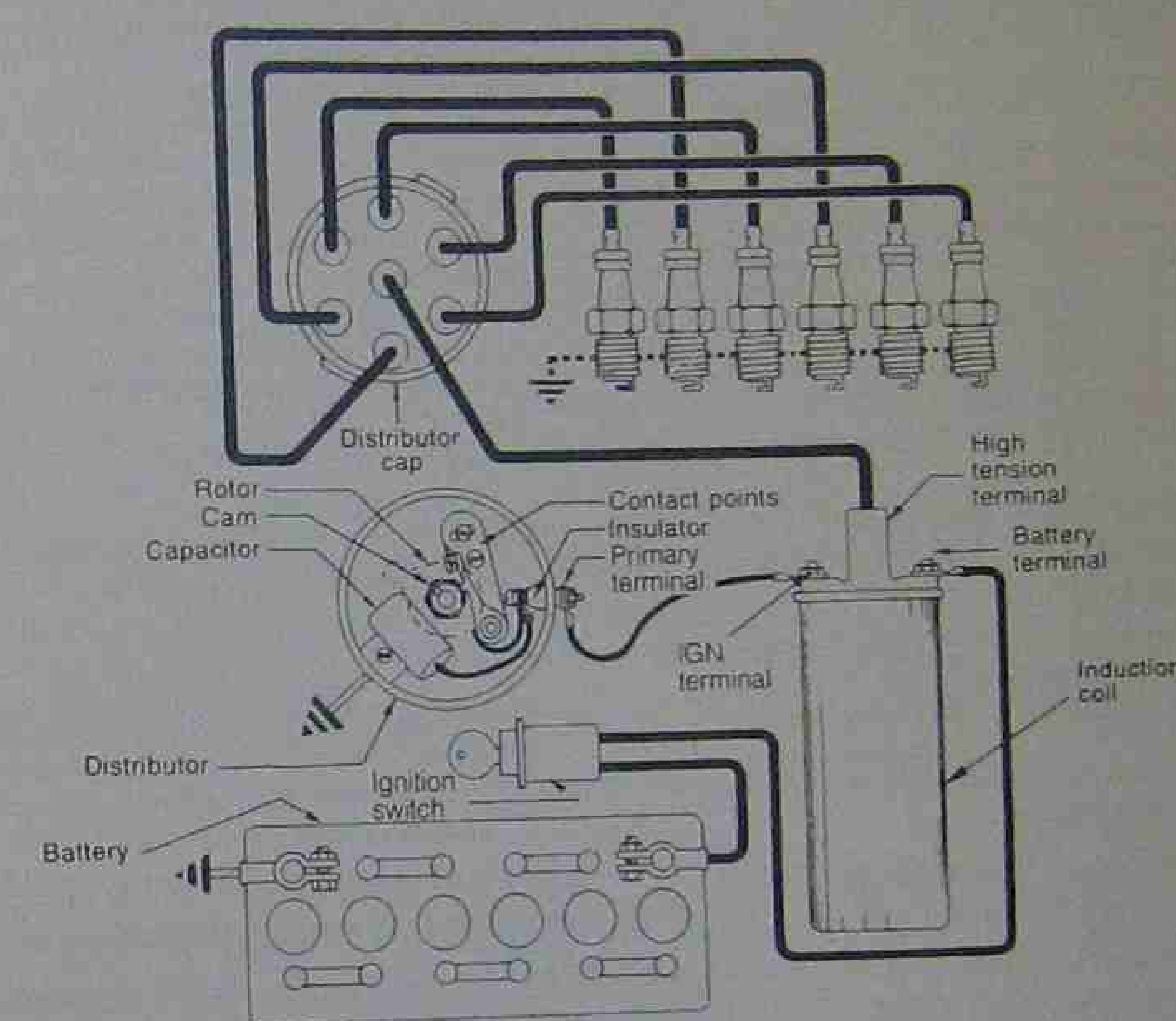


Figure 5.35 • Car ignition system

SUMMARY

- A magnet has two poles (north and south) and lines of force exist in the space surrounding it.
- Lines of force are assumed to act outwards at a north pole and inwards at the south pole.
- Magnetic lines of force form closed loops.
- Magnetic lines of force take the line of least resistance and never cross each other.
- Magnetic lines of force existing outside the desired magnetic path are called leakage flux.
- Like poles repel each other—unlike poles attract.
- Magnets can induce magnetism in other magnetic materials.
- The permeability of a magnetic material indicates the ease with which magnetic induction can occur in a material.
- Ferromagnetic materials have high values of permeability and are termed *magnetic materials*.
- Materials which are used for permanent magnets are called *hard*, while those that can easily be induced to exhibit magnetic properties but lose it when the magnet is removed are called *soft* materials.
- The degree of magnetism in magnetically soft materials is easily changed and is used in electromagnetism.
- Magnetically hard materials are used to form permanent magnets.
- Magnetism apparent in a material after the magnetising force has been removed is called *residual magnetism*.
- No material can be a magnetic insulator.
- There are two right-hand thumb rules for magnetism:

—the right-hand thumb rule for a straight conductor
—the right-hand thumb rule for a solenoid.

- Parallel conductors carrying currents in the same direction exert a force of attraction on each other.
- Parallel conductors carrying currents in opposite directions exert a force of repulsion on each other.
- The force created by conductors in a magnetic field can be found from $F = BIl$.
- Magnetomotive force: $Fm = IN$ (ampere-turns)
- Magnetising force: $H = IN/l$ (ampere-turns/metre)
- Flux density: $B = \dfrac{\Phi}{A}$ (webers/m^2)
- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$
- Permeability (actual): $\mu = \mu_r \times \mu_0$ (for air, $\mu = 1$) $= \dfrac{B}{H}$
- Reluctance of a magnetic circuit: $R_m = \dfrac{l}{\mu_r \mu_0 A}$
- The magnetisation curve for a non-magnetic material is a straight line. For magnetic materials, it is a curve with a pronounced 'knee' at saturation.
- Magnetic hysteresis is the difference between the magnetisation curves for increasing and decreasing degrees of magnetisation.
- The volume of a hysteresis curve is indicative of the losses in a magnetic material.
- When lines of force cut a conductor, an induced voltage is created.
- By manipulating the number of turns and the number of lines of force, the induced voltage can be increased. This leads to the use of inductors.
- Induced voltage:

$$V = N.\frac{\Delta\Phi}{\Delta t}$$

$$= \frac{\text{change in flux}}{\text{change in time}} \times \text{conductors}$$

- Inductors can be air cored, iron cored, or iron-powder cored, depending on the use to which they are put.
- The unit of inductance is the henry (H).
- The inductance of a magnetic circuit is:

$$L = \frac{N^2 \mu_r \mu_0 A}{l}$$

- Induced voltage: $V = L\dfrac{\Delta I}{\Delta t}$
- Time constant of an inductive circuit: $\tau = \dfrac{L}{R}$
- Energy stored in a magnetic field: $W = \frac{1}{2}LI^2$ joules
- A circuit in which a change of current causes an e.m.f. to be induced within the circuit itself is said to have the property of *self-inductance*. The direction of the induced voltage is such that it opposes the change in current (Lenz's Law).
- Mutual inductance occurs when the lines of force generated by conductors cut conductors in an adjacent circuit.
- Three factors are required in order to produce an induced voltage:
  1. a conductor
  2. a magnetic field
  3. relative movement between a conductor and the magnetic field.

EXERCISES

5.1 Describe fully how you could determine whether a piece of iron was magnetised.

5.2 State the rule for determining the polarity of a solenoid.

5.3 State whether the current flows from terminal A to B or vice versa, in the solenoid below.

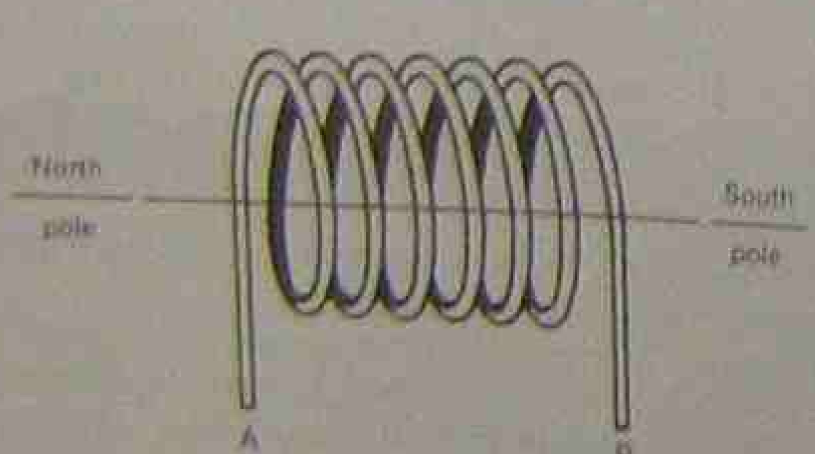


Figure 5.36 • Solenoid diagram for exercise 5.3

5.4 What is meant by the magnetic reluctance of a circuit?

5.5 In what manner do the strength of the magnetic poles and their distance apart affect the force exerted by magnets?

5.6 Describe the forces between two conductors carrying current and located parallel to each other.

5.7 State clearly the difference in the meaning of the terms *flux density* and *total flux*.

5.8 What is meant by the term *magnetic permeability* of a material?

5.9 What is the important difference between the permeability of a ferromagnetic material and that of air?

5.10 Explain why the length of air gaps in a magnetic circuit has a much greater effect on the total reluctance than does the length of the magnetic core.

5.11 Sketch a typical *B/H* curve for a magnetic core made from silicon steel. Label the axes with the correct units. Mark the estimated saturation region on the curve. What is the significance of this saturation region?

5.12 Draw a typical hysteresis loop for transformer stampings that may be expected to have low iron loss. Indicate the coercive force. Indicate the residual magnetism. On the same axes, draw the hysteresis loop for a material that has a greater iron loss.

5.13 Briefly explain what is meant by the term *magnetic leakage*.

5.14 The flux density in the air gap of a magnetic circuit is usually slightly less than the flux density in the adjacent magnetic materials. Explain why this is so.

5.15 Three factors determine the value of self-induced e.m.f. List these factors. Discuss how each affects the value of self-induced e.m.f.

5.16 Briefly state the meaning of the term *mutual inductance*. What is meant by the term *primary*? What is meant by the term *secondary*? Explain the meaning of *coupling*. How does coupling affect mutual inductance?

5.17 What happens when a highly inductive circuit is quickly opened? What adverse effect could result from this?

5.18 What is meant by the time constant of a circuit and by what two quantities is it measured?

SELF-TESTING PROBLEMS

5.19 Determine the m.m.f. necessary to create a flux of 0.2 Wb in a magnetic core which has a reluctance of 2000 At/Wb.

5.20 A magnetic circuit has a reluctance of 750 At/Wb. The coil, having 800 turns, carries 0.5 A. Find the total flux produced.

5.21 A magnetic circuit has a reluctance of 75 000 At/Wb. The total flux produced is 0.01 Wb. If there are 1500 turns on the coil, find the value of current being drawn.

5.22 A magnetic circuit has a reluctance of 1200 At/Wb and a total flux of 57 mWb. The circuit current is restricted to 1.8 A. Calculate the necessary number of turns on the coil.

5.23 The mean length of a magnetic path is 600 mm. The cross-sectional area is 800 mm^2. The relative permeability is 600. Determine the reluctance of the magnetic path.

5.24 In a test to determine the characteristics of a ferromagnetic material, the following procedure was used:

A sample of the material was placed in a magnetometer and readings of compass deflection were recorded as the current was first increased and then decreased in steps for both positive and negative directions of current flow. The readings of deflection were then converted into terms of flux density in the sample and Table 5.3 was compiled.

Using the current as the magnetising force, draw the hysteresis curve of these results and determine the values of residual magnetism and coercive force for these conditions.

Table 5.3 • Magnetometer test results for problem 5.24

Core initially demagnetised, I increasing in positive direction									
Current *I* in amperes	0	0.1	0.2	0.3	0.4	0.6	0.8	1.0
Flux density *B* in teslas	0	0.175	0.55	0.85	1.0	1.2	1.3	1.35
I in positive direction but decreasing in value								
Current *I* in amperes	1.0	0.8	0.6	0.4	0.2	0		
Flux density *B* in teslas	1.35	1.32	1.27	1.15	0.95	0.55		
I in negative direction but increasing in value								
Current *I* in amperes	−0.1	−0.2	−0.4	−0.6	−0.8	−1.0		
Flux density *B* in teslas	+0.2	−0.1	−0.7	−1.0	−1.25	−1.35		
I in negative direction but decreasing in value								
Current *I* in amperes	−1.0	−0.8	−0.6	−0.4	−0.2	0		
Flux density *B* in teslas	−1.35	−1.32	−1.27	−1.15	−0.95	−0.55		
I in positive direction and again increasing in value								
Current *I* in amperes	0.1	0.2	0.4	0.6	0.8	1.0		
Flux density *B* in teslas	−0.2	0.1	0.7	1.0	1.25	1.35		

5.25 An inductor has a current flowing through it of 4 A. When the current is reduced to zero in 1.8 s, the voltage appearing across the coil terminals is 22.5 V. What is the inductance of the coil?

5.26 A 12 H inductor has a resistance of 300 Ω. When working on a 50 V supply, a voltage of 500 V appears across its terminals each time it is switched off. Calculate the time taken for the current to decrease to zero.

5.27 An inductor of 1 H has a resistance of 1.1 Ω.

(a) What is its time constant?

(b) When switched on, approximately how long will the current take to reach its maximum value?

5.28 An ignition coil of 0.35 H inductance draws 3 A when the engine is stationary. Calculate the energy stored in the coil's magnetic field.

5.29 Calculate the flux density in the air gap of a permanent magnet if the pole piece is 5 cm × 4 cm and the total flux in the air gap is 1.5 mWb.

5.30 A magnetic flux of 9.5 mWb is present in an iron-cored solenoid with a circular cross-section of 80 cm^2. Find the flux density.

5.31 Find the inside diameter of a circular solenoid in which there is a magnetic flux of 11 mWb with a flux density of 0.85 T.

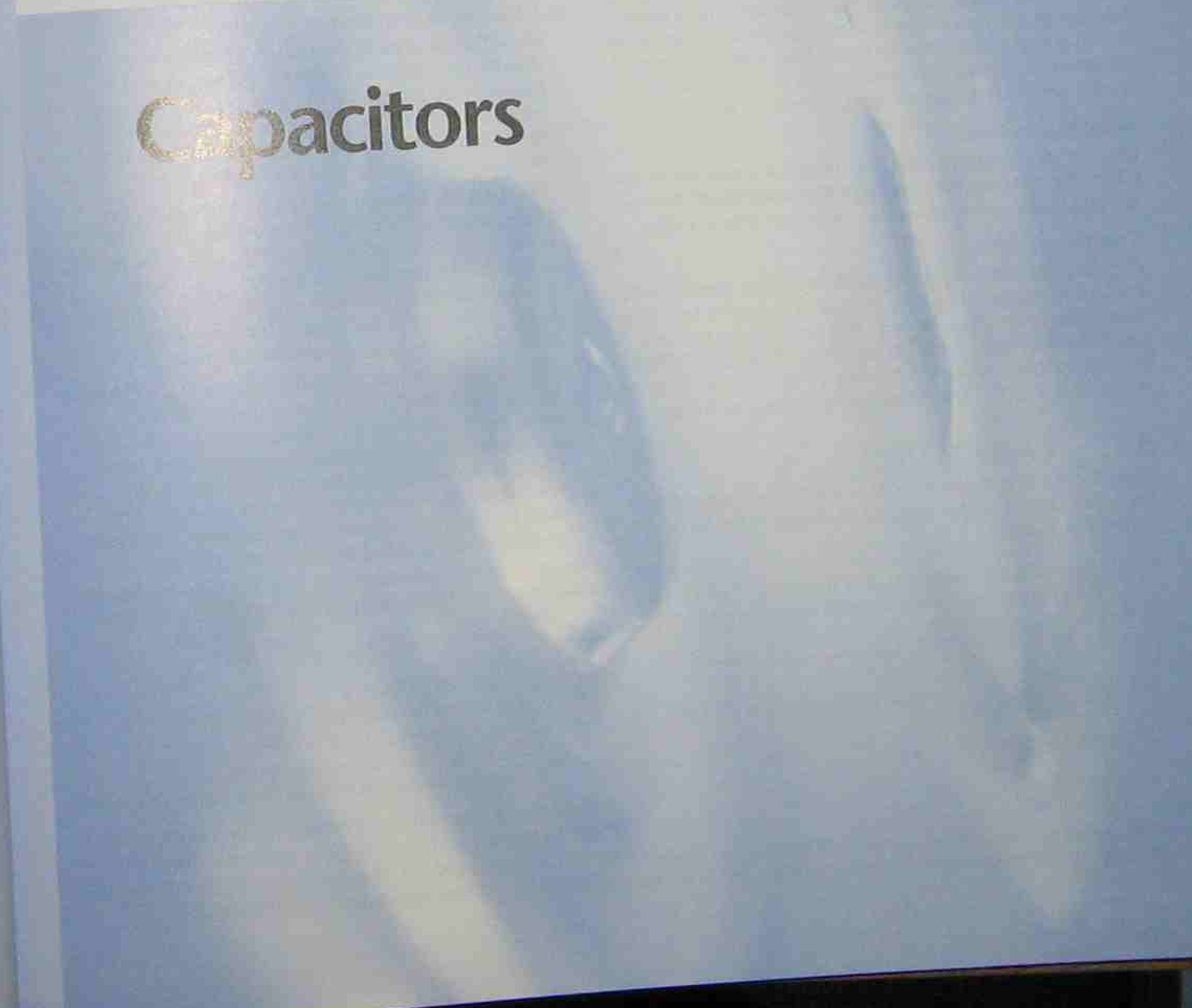
5.32 The magnetising force in an iron ring is 1500 At/m. It creates a flux density of 0.95 T. Find the relative permeability for these conditions.

5.33 A long solenoid of 0.8 m has a current of 2 A flowing through a coil of 2000 turns. What is the magnetising force?

5.34 An ignition coil has an inductance of 3 H and has a current of 2 A flowing when the points are closed. If the current has to reduce to zero in 0.01 s, calculate the induced voltage created in the coil.

Chapter 6

Capacitors



6.1 INTRODUCTION

A capacitor is a device that stores energy in the form of an electric charge. It consists of two conducting surfaces called plates, separated by an insulating material called the dielectric. There are five main types.

6.2 CAPACITOR TYPES

Figures 6.1 and 6.2 illustrate the five main types of capacitors.

Stacked-plate capacitors

Alternate plates and dielectric are stacked in a pile and alternate plates are joined to form two large surface areas (Fig. 6.1(a)). These capacitors are sometimes referred to as mica capacitors because of the dielectric material used.

Rolled capacitors

Aluminium-foil sheets are separated by a layer of thin, impregnated paper, which serves as the dielectric. Because both the foil and paper are so flexible, they can be rolled up to a convenient size and then enclosed in a plastic, cardboard or metallic case (Fig. 6.1(b)).

Electrolytic capacitors

These are similar in construction to rolled capacitors, except that the dielectric is an absorbent material impregnated with a borax solution. They provide a large capacitance in proportion to physical size but, if a voltage of the wrong polarity is applied to them, they can be destroyed. Therefore the main application is for d.c. work, although special types are available for a.c. use (Fig. 6.1(c)).

Variable capacitors

One set of capacitor plates can be moved relative to the other. By altering the spacing or surface area between the plates, the capacitance can be varied (Fig. 6.1(d)).

Ceramic capacitors

Ceramic disc capacitors are made by silver-plating both sides of a ceramic form, as shown in Figure 6.1(e). It is then encapsulated as shown. It is possible to achieve a much larger capacitance in a smaller volume. Ceramic capacitors have values from 0.5 pF (0.5×10^{-12} F) to 0.1 μF (0.1×10^{-6} F) and voltage ratings up to 2.0 kV.

6.3 CAPACITANCE

Capacitance is a measure of the ability of a capacitor to hold an electric charge. The unit of charge is the farad (F), defined as follows:

> *A farad is the capacitance of a capacitor which stores a charge of one coulomb at a potential difference of one volt.*

That is, $\dfrac{\text{charge in coulombs}}{\text{applied voltage}} = \text{capacitance in farads}$

Expressed in symbols:

$$\frac{Q}{V} = C$$

Transposing: $\boxed{Q = VC}$

where Q = charge in coulombs
V = voltage
C = capacity in farads

The farad is a very large unit and submultiples are generally used. The two most common submultiples are:

- microfarads, μF (= 10^{-6} F)
- picofarads, pF (= 10^{-12} F).

The quantity of charge held in a capacitor is dependent on both capacitance as defined above, and the voltage across the capacitor. The same quantity of charge can be held in a large capacitor at a low voltage as in a small capacitor at a high voltage.

Example 6.1

A charged 200 μF capacitor has a potential difference of 30 V. Calculate the charge on the plates.

$$Q = VC$$
$$= 30 \times \frac{200}{10^6}$$
$$= 0.006 \text{ coulomb}$$

6.3.1 Factors affecting capacitance

The following factors affect capacitance:

1. ***The effective area of the plates.*** The capacitance is directly proportional to the effective area and is improved by increasing the number of plates (e.g. stacked plates) or the size of the plates (e.g. rolled capacitors). Effective area means the surface area adjacent to a plate of the opposite polarity.
2. ***The distance between the plates.*** As the spacing decreases, the effect that one plate has upon the other is increased, so the closer the plates are to each other, the greater the capacitance. This is why electrolytic capacitors have a large capacity for a small physical size due to the thinness of the dielectric for the same breakdown voltage.
3. ***The type of dielectric.*** The capacitance is affected by the type of material used as the dielectric. For example, if glass is used as a dielectric instead of air, the capacitance increases approximately six times. Glass, as do some other materials, increases the breakdown voltage of the capacitor considerably. The ratio by which the dielectric can increase the charge relative to air, is called the dielectric constant.

For a capacitor consisting of two parallel plates, the capacitance can be found from the following equation:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

where: C = capacity in farads
ε_0 = absolute permittivity (= 8.85×10^{-12})
ε_r = relative permittivity (see Table 6.1)
A = area of plates in square metres
d = distance between two opposite plates in metres

The farad is a very large unit and to find a capacitor's value expressed in farads is quite unusual. The value is normally expressed in microfarads, so in using this equation the answer has to be multiplied by 10^6.

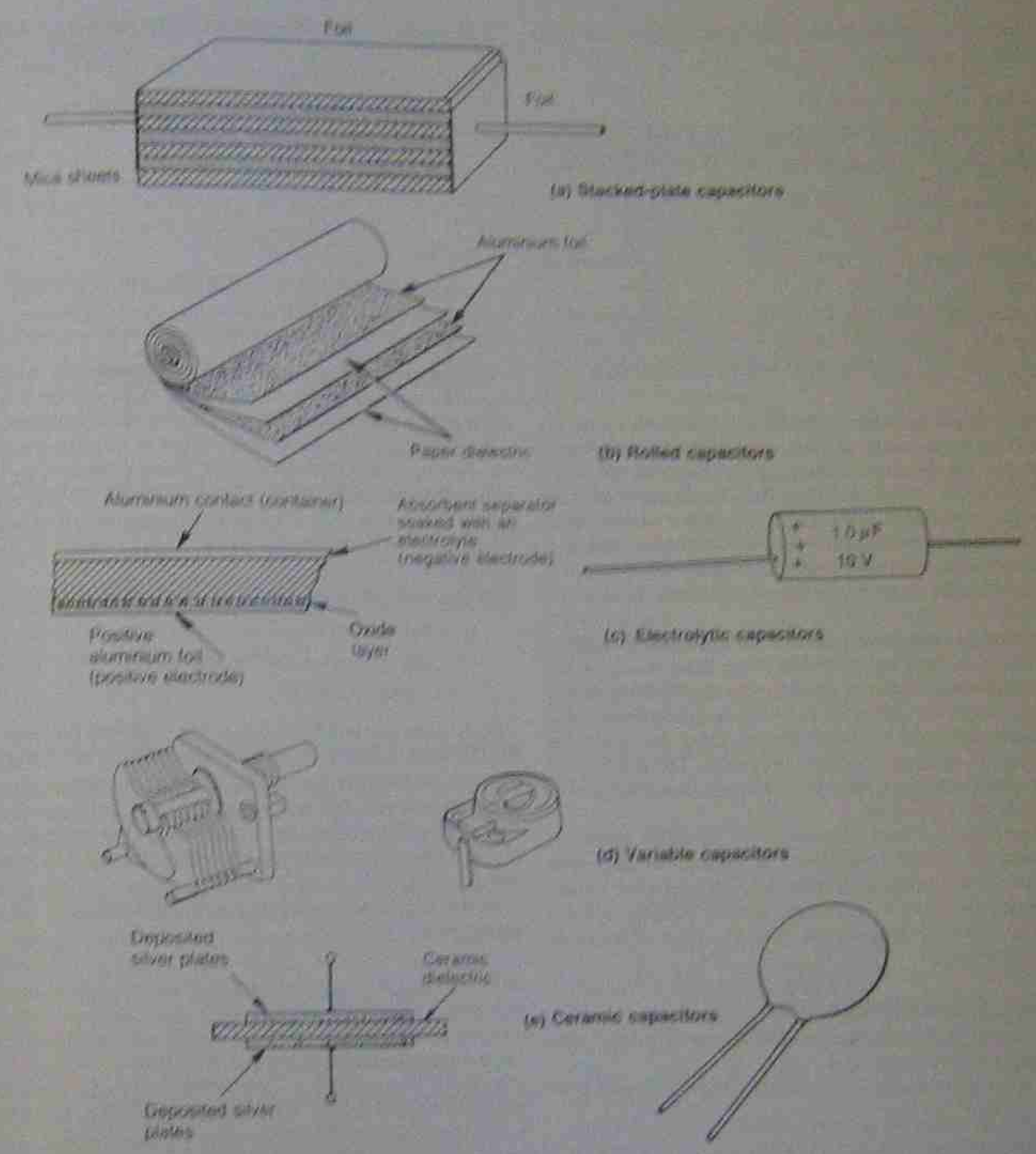


Figure 6.1 • Types of capacitor construction

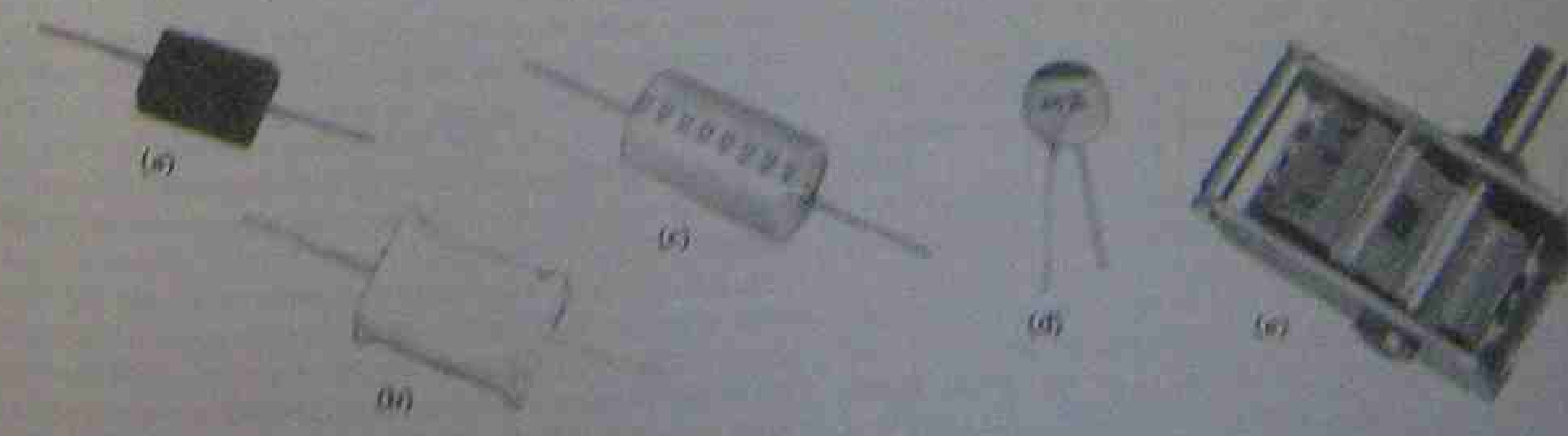
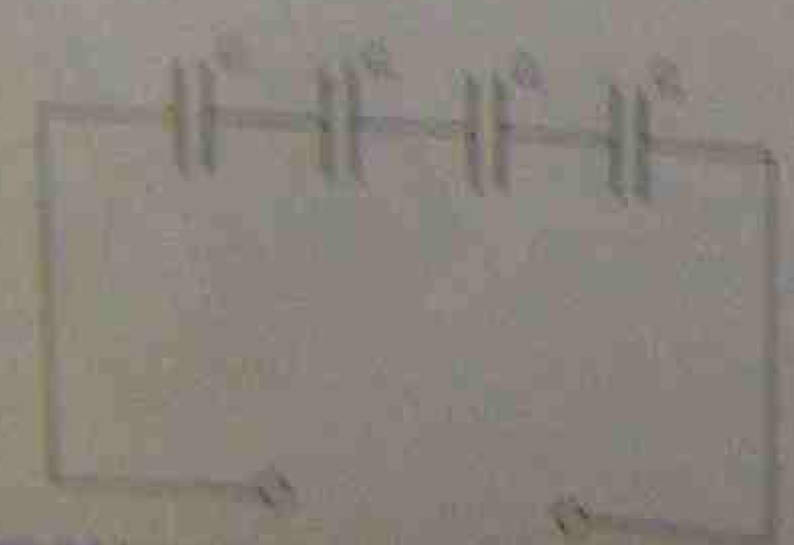


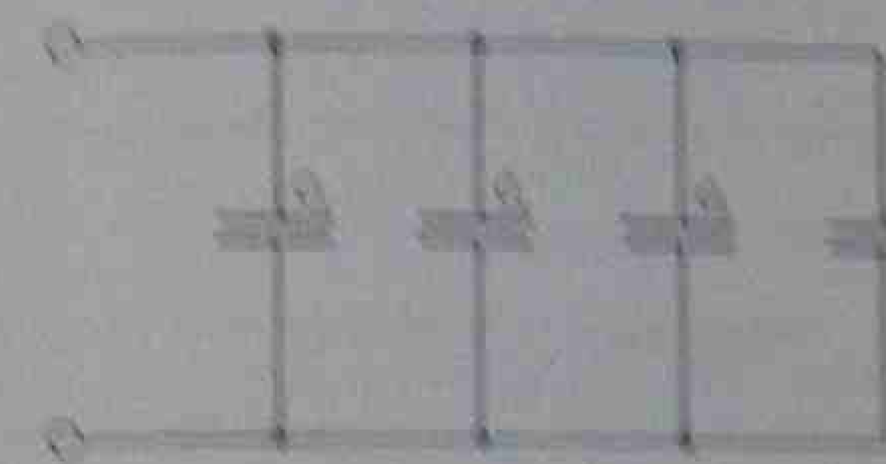
Figure 6.2 • Actual physical appearance of capacitors in approximately the same layout as that shown in Figure 6.1: (a) stacked-plate capacitor (mica), (b) rolled capacitor, (c) electrolytic capacitor, (d) ceramic capacitor (marked 20 per cent), (e) variable capacitor

Example 6.2

Capacitors in series

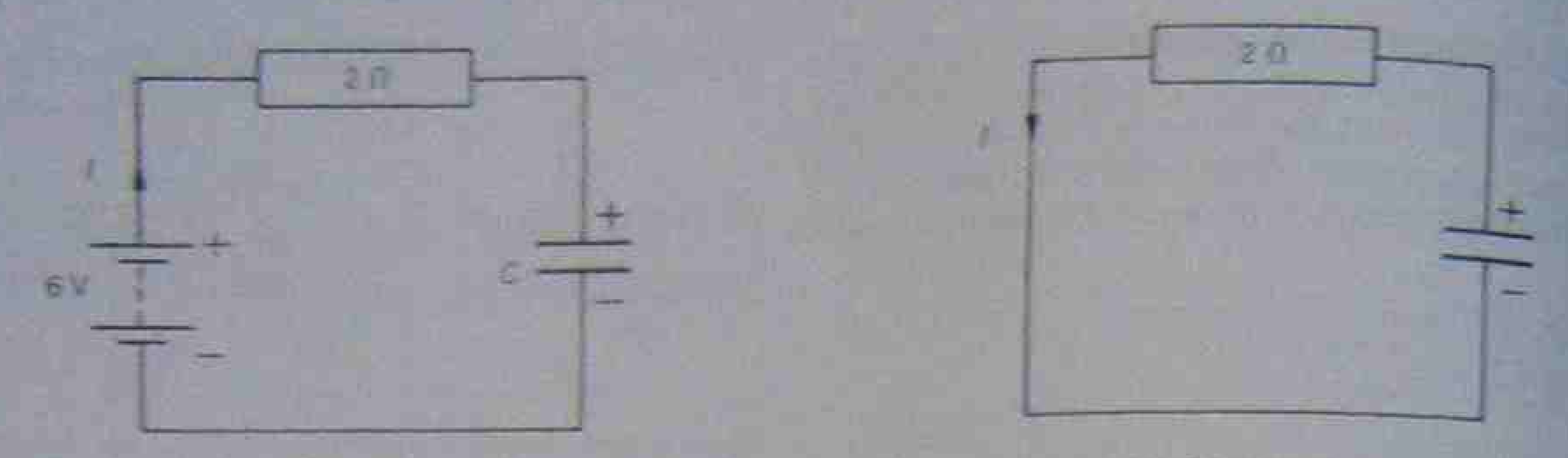


Capacitors in parallel



Example 6.3

6.4 CAPACITORS ON DIRECT CURRENT



(a) Charging (b) Discharging

Figure 6.6 • Resistance connected in series with a capacitor to limit the current flow. The direction of current flow is different for both conditions

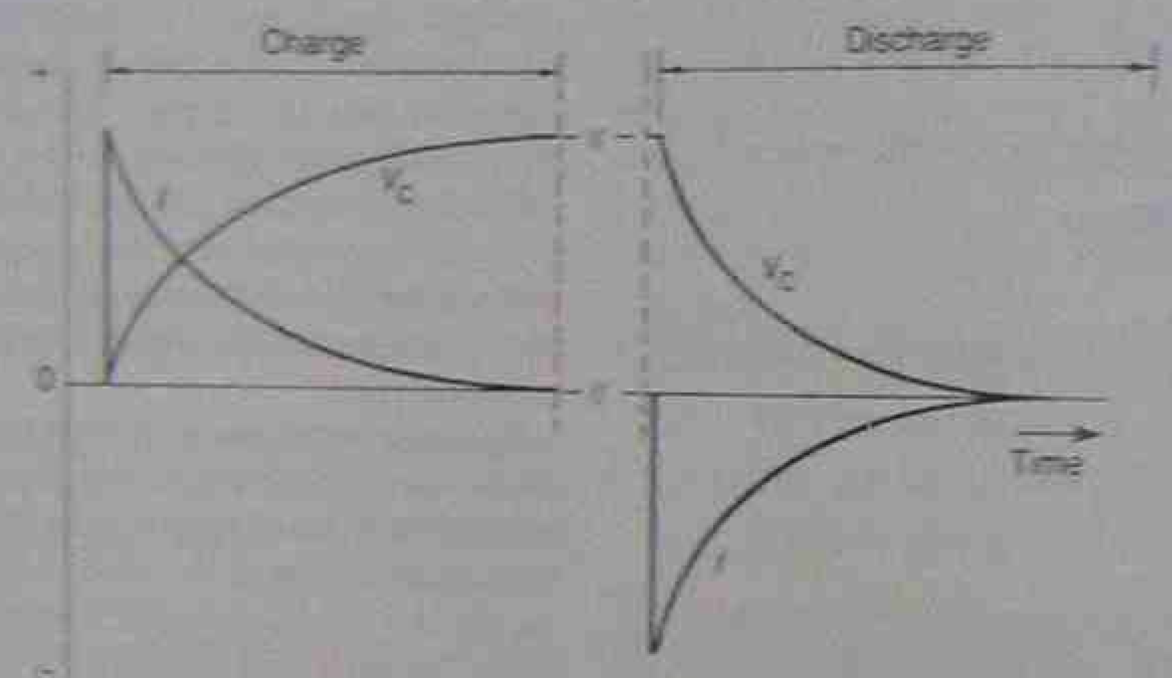


Figure 6.7 • Charge and discharge curves

The curves show that the current is at a maximum when the voltage is changing most rapidly (i.e. at the start of charging and discharging). The current is zero when the voltage is steady, even though the voltage may be at a maximum value, as at the end of the charging period.

Because no dielectric is a perfect insulator, a charged capacitor will slowly lose its charge as current leaks from one plate to another. However, a good capacitor may hold its charge for some time. Many circuits have a high-value resistor connected across the capacitor to reduce the charge to a safe limit within one minute (see section 6.4.3 for further details).

6.4.1 Time constant

In the previous section it was shown that the curves had a characteristic shape. The L–R curves shown in Chapter 5 (Fig. 5.29) have the same characteristic shape.

A resistor–capacitor (R–C) has a time constant that can be found from:

$$\boxed{\tau = RC}$$

where τ = time in seconds
R = resistance in ohms
C = capacitance in farads

The higher the value of resistance and the greater the capacitance, the longer are the time constants. The curve is shown in Figure 6.8 for the voltage across a charging capacitor. The voltage across the capacitor increases to 63 per cent of its final value in the period of time corresponding to the time constant.

Because of the regularity of the time constant, and the proportions of final values reached, the R–C charge and discharge circuit is a valuable basic timing circuit that is independent of the applied voltage.

The L–R and R–C curves show further similarity: the final steady-state values are achieved in both circuits at approximately 5τ seconds.

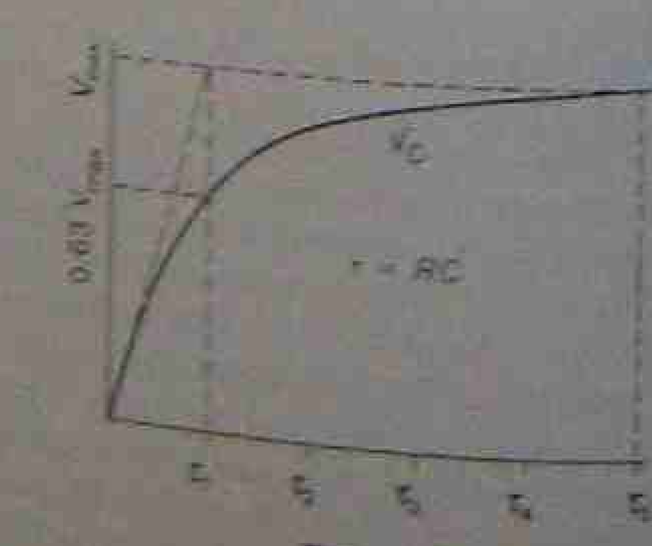


Figure 6.8 • Time constant for an R–C circuit

Example 6.4

Find the time constant for a circuit consisting of a 100 kΩ resistor in series with a 500 μF capacitor. How long would it take for the voltage across the capacitor to reach the value of the supply?

$$\tau = RC$$
$$= 100\,000 \times \frac{500}{10^6}$$
$$= 50\ \text{s}$$

$$\text{Steady-state time} = 5\tau$$
$$= 5 \times 250$$
$$= 250\ \text{s}$$

6.4.2 Energy stored in a capacitor

When a capacitor is charged, there exists between the plates a static electrical field. This electrostatic field contains energy resulting from the applied voltage and the initial current flow into the capacitor. The actual amount of energy stored in the field depends on the applied voltage and the capacitance of the capacitor. It can be found from the formula:

$$\boxed{W = \tfrac{1}{2}CV^2}$$

where W = energy in joules
C = capacitance in farads
V = voltage across capacitor

Example 6.5

A 1 μF capacitor is charged from a 300 V d.c. supply. Find the energy stored in the capacitor.

$$W = \tfrac{1}{2}CV^2$$
$$= \frac{\frac{1}{2} \times 1 \times 300^2}{10^6}$$
$$= 0.045\ \text{J}$$

6.4.3 Dangers of a charged capacitor

The value of energy stored in the capacitor of example 6.5 is certainly low, but because the potential difference across the terminals is 300 V, an unpleasant if not dangerous electric shock can be given to an operator.

Depending on actual circumstances, a high discharge current can flow. To illustrate this point, consider a resistor of 4 Ω connected across the above capacitor while it is in a charged state.

From Ohm's law:

$$I = \frac{V}{R}$$
$$= \frac{300}{4} = 75\ \text{A}$$

This current lasts for only a very short period of time before deteriorating to a lower value, but it can cause damage to a circuit that is not built to withstand such current surges.

A circuit of this type is the basis of capacitor-discharge ignition circuits used in internal combustion engines. A capacitor is charged up to 200–300 V and discharged into an ignition coil in a very short period of time.

Before handling capacitors or working on circuits where capacitors are used, it is a sensible precaution to ensure that the capacitors have been discharged. Small values of capacitors can be discharged directly with a short-circuit, but larger ones might need a discharge resistor to control the value of current during discharge. As mentioned in an earlier section, some circuits have high-value discharge resistors permanently connected to ensure a controlled discharge. This applies particularly in higher-voltage circuits.

6.4.4 Applications of capacitors

Stacked-plate capacitors (a.c. or d.c.)

These capacitors contain mica, a comparatively thick dielectric, so their use is generally restricted to high-voltage low-capacity applications. In the electrical field their main use is for suppression of voltage surges and contact arcing.

Rolled capacitors (a.c. or d.c.)

Some rolled capacitors are d.c. only and are used mainly in electronics. They have various type names and construction methods. For electrical use the dielectric is often porous paper. The capacitor is usually placed in a metal can, often topped up with a special insulating oil called transformer oil. The main uses of rolled capacitors are in motor starting and running capacitors, and for general power-factor correction purposes.

Electrolytic capacitors (d.c.)

The true electrolytic capacitor has a special liquid dielectric which tends to be self-sealing in the event of breakdowns. A so-called 'dry' electrolytic capacitor has been developed that is smaller and more suitable for lower voltages. It is important that electrolytic capacitors be connected correctly. Connecting with the wrong polarities usually causes a rapid breakdown of the dielectric.

Variable capacitors (a.c. or d.c.)

Variable capacitors have their main application in the electronic communications industry.

Ceramic capacitors (a.c. or d.c.)

These capacitors are often constructed for high-capacity low-voltage use. There are types of construction that enable them to be used on higher voltages at lower capacities. For electrical work their main use is voltage-surge suppression and arc suppression, as in fluorescent tube starters. Some types are d.c. only.

SUMMARY

- Capacitors are two conducting electrodes separated by an insulating medium called a *dielectric*.
- Capacitors can store an electric charge whose amount depends on the cross-sectional area of the plates and their separation. Capacitance is also affected by the insulating material used to separate the two plates (Table 6.1). The quantity of charge $Q = VC$.
- A parallel-plate capacitor's capacity can be found from the equation:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

- There are various methods of assembly, depending on the intended use of the capacitor, the voltage rating, the capacitance required and whether it is to be used on a.c. or d.c. or both.
- Capacitors in series have potential differences across each and the sum of these is equal to the applied voltage. Total capacity is obtained from:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

- Each capacitor in series has the same quantity of charge as the total charge for the circuit:

$$Q_{total} = V_{total}C_{total} = V_1C_1 = V_2C_2 = \ldots$$

- Capacitors in parallel have the same voltage across each one in a similar fashion to parallel resistor circuits. The quantity of charge on each capacitor will depend on its capacity:

$$C_{total} = C_1 + C_2 + \ldots$$

- Capacitors charge at a rate depending on the applied voltage and the circuit resistance.
- The time constant of a resistance/capacitor combination in series is the time taken to charge up the capacitor to 63 per cent of the final voltage (the applied voltage).
- When charging, a capacitor starts from zero voltage while the charging current starts at a maximum value depending on circuit resistance. The voltage gradually increases but the charging current gradually decreases.
- The time constant of a circuit is found from:

$$\tau = RC$$

- The energy stored in a charged capacitor is found from:

$$W = \frac{1}{2}CV^2$$

- Charged capacitors can deliver lethal shocks and should be discharged before handling.

SELF-TESTING PROBLEMS

6.1 Find the equivalent capacitance of an 8 μF capacitor and a 33 μF capacitor connected in series.

6.2 Find the equivalent capacitance of an 8 μF and a 16 μF capacitor connected in parallel.

6.3 Two capacitors, each of 27 pF, are connected in series with a 100 pF capacitor. Find the circuit capacitance.

6.4 A 27 pF capacitor is connected in series with a 0.1 μF capacitor. Find the circuit capacitance.

6.5 Determine the amount of charge held in a 100 μF capacitor at a voltage of 100 V d.c.

6.6 If the quantity of charge held on a 1000 μF capacitor is one coulomb, what would be the voltage across its terminals?

6.7 Three capacitors of 8 μF, 16 μF and 100 μF are connected in series across a 100 V d.c. supply. Calculate the effective capacitance of the circuit and the voltages across each capacitor.

6.8 Capacitors of 2.2 μF, 0.27 μF and 0.01 μF are connected in series. Find the equivalent circuit capacitance. If the circuit is then to be connected across a 250 V d.c. supply and the 0.01 μF capacitor fails by becoming short-circuited, calculate the potential differences across the other two.

6.9 Three 8 μF capacitors, each rated at 600 V, are connected in series. Find the circuit capacitance and the maximum voltage the combination would withstand.

6.10 Find the time constant of a circuit consisting of a 3.3 μF capacitor in series with a 1 [illegible] Ω resistor.

6.11 Calculate the time constant of a circuit consisting of a 27 μF capacitor connected in series with an 82 kΩ resistor.

6.12 A 500 μF capacitor is connected in series with a resistor across 24 V d.c. What would be the value of the resistor if the voltage across the capacitor rose to 15.1 V in 34 s?

6.13 A 33 μF capacitor reaches 63 per cent of its final voltage in 15 s. Calculate the value of the series resistance in the circuit.

6.14 A time constant of 10 s is required. Given a capacitor of 33 μF, find the value of series resistance required.

6.15 Calculate the energy stored in a 33 μF capacitor at a voltage of 100 V.

6.16 Find the energy stored in a capacitor if it has a capacitance of 1000 μF and a terminal voltage of 1000 V.

6.17 Find the amount of energy stored in a 100 μF capacitor if it is charged up to a potential of 24 V. Estimate the maximum instantaneous current discharge possible if it is discharged into a resistance of 0.25 Ω.

Chapter 7

Test equipment

7.1 INTRODUCTION

Previous chapters have introduced certain basic concepts concerning electrical circuits and components. Circuits were shown to consist of three basic elements: load, source of electricity and a complete circuit between the source and the load.

The load itself had three possible characteristics. It could be resistive, inductive or capacitive in effect. It could also be a mixture of any or all of these three.

For a load to operate correctly, the circuit between the source and the load had to be without fault, the correct voltage had to be applied, and the load had to be in good working order. Only then would the correct amount of current flow, as dictated by the individual load.

When a circuit or load is not working properly, steps have to be taken to rectify the problem. This inevitably involves the use of some type of test equipment to find the cause so that the equipment and circuit can be restored to correct working order.

In many cases the test material needed can be simple and cheap. For complex faults, more involved testing equipment might be needed.

7.2 CIRCUIT INDICATORS

Much of the available equipment is fitted with indicator lights to draw the operator's attention to the fact that certain conditions exist. Usually in the form of lights, and sometimes meters, their function and indications are many and varied.

Fortunately most circuit indicators can provide self-evident indications. For example, a steady glowing red indicator usually indicates that power is available at the associated equipment. An indicator with a flashing function is designed to attract attention and might indicate that a problem exists.

A modern motor vehicle is fitted with many of these lights. Indicator lights provide a visual confirmation of power, others indicate such functions as lack of oil pressure, high temperature, loss of brake fluid, and so forth.

In an aircraft, indicator lights show green when its landing gear is up. Below a critical height, or slower than a critical speed, they flash red and are often accompanied by an audible warning noise. The indicators are meant to attract a pilot's attention and warn that all is not well and a landing should not be attempted. However, indicators alone cannot prevent an aeroplane landing. They can only warn that a problem exists.

It should be borne in mind, however, that on occasion, an indicator light can signify other conditions, for example:

1. Power is available.
2. Power is not available.
3. A working temperature has been reached.
4. A working temperature has not been reached.
5. A fault has occurred.

7.3 SIMPLE VOLTAGE TESTING EQUIPMENT

Any testing equipment used by a technician needs to be relevant to the task. It needs to be relatively simple in many cases and as well it needs to be reliable and consistent with its indications.

7.3.1 Series test lamps

A test lamp is one of the most elementary units for an electrical technician. It is used to test for the presence of voltages. It can indicate that the circuit voltage is present and the probability exists that the circuit is intact up to the test point.

Since two different voltages (240 V and 415 V) are often encountered by technicians, the test lamps must be capable of working satisfactorily on the higher voltage. The arrangement is shown in Figure 7.1. Two 240 V, 15 W lamps are connected in series. On the odd occasion when 480 V supplies are encountered, the two-lamp unit will still be satisfactory.

It is important that both lamps be identical in both wattage and voltage. When non-identical lamps are used, the smaller wattage lamp is in danger of failing, owing to excessive voltage. The smaller wattage lamp always has the highest voltage across it.

Two leads are brought out to act as test leads. Since potentially lethal voltages are being tested, it is necessary that the leads be insulated accordingly. Bared ends stripped for actual contact with live terminals should be kept as short as possible to prevent accidental contact with other live terminals.

The two lamps are often mounted on a base plate with a surround of insulating material for convenience and lamp protection. Another form of protection is to mount two batten holders back to back inside a short length of plastic conduit. The ends of the tube can be covered or otherwise, as dictated by the user. However, the lamps need to be seen by the user, because no audible indication is given.

It should be noted that the essential components are lamps made of thin glass with enclosed filaments. To ensure reliability they must be protected against mechanical damage.

It is also good practice to test the lamps on a known source of power before accepting that a circuit has no power applied. On 240 V, the lamps will not shine with full brilliance, although for all practical purposes they will be at almost full brilliance on 415 V. With experience a technician can use lamp brilliance as a rough guide to the voltage at the test point.

While test lamps are simple and practical pieces of test equipment, their fragility makes them vulnerable and their reliability rather questionable.

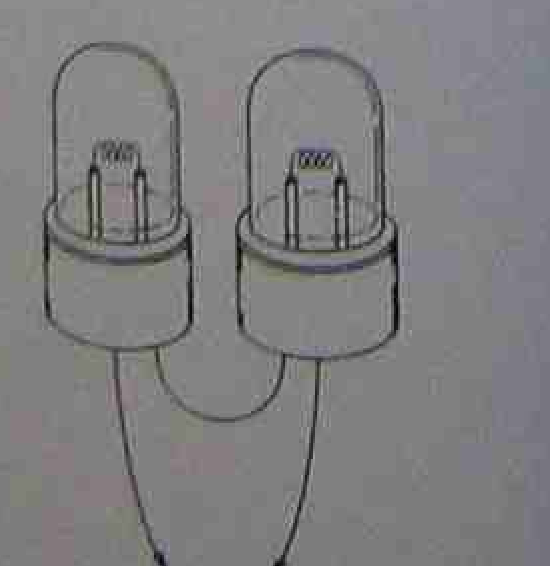


Figure 7.1 • Test lamps consisting of two 240 V, 15 W lamps connected in series

7.3.2 Single-filament test lamps

Another version of the test lamp has been the introduction of a single-filament test lamp mounted in a shockproof rubber housing. The replaceable lamp is a specially produced rough-service lamp of the 240 V, 15 W, pygmy style. Incorporated in the housing is a 4000 Ω series resistor, which gives it a working range of 110 V to 500 V a.c. The complete unit and lead are double-insulated.

It differs from the test lamps discussed previously in that one probe is an integral part of the lamp housing. The unit is held in one hand and only one wandering lead is required. Provision is made for storing the wandering lead in the main body of the unit. It is worth observing that the makers also recommend that the unit be tested on a known power source before relying on its readings.

7.3.3 Vibrating testers

Commonly called by the name Wigger or Wiggy testers, these types of testers have been adopted by several manufacturers and made on a commercial basis.

Rather more substantial and slightly more complex than a pair of lamps in series, they can withstand far rougher handling. Handled correctly, their life expectancy is many years. The construction of one version is shown in Figure 7.2.

The coil wound on the spool has an impedance of approximately 2500 Ω and limits the current to less than 100 mA on 240 V.

As with low wattage test lamps, this can be a limiting factor in some circuits. A high series-resistance fault can easily be missed when testing circuits and equipment with such low currents.

With the version shown in Figure 7.2, the test leads and probes are loose and can be easily damaged. In a later model the body of the tester tends to be rectangular in cross-section. Provision is made for tucking the leads into the body of the unit when not in actual use. A third version has one probe built into the body of the unit so that only one probe is free to be shifted from contact point to contact point.

The vibrating tester is relatively small and portable. Its vibration can be felt and heard. It also gives a visual indication against a scale. In terms of voltage, the reading is very approximate and by no means accurate. It is rugged and reliable, particularly for those engaged in maintenance work where the technician must travel from job to job. The tester reads both a.c. and d.c. up to voltages of 500 V—usually on different scales moulded in the body of the tester.

7.3.4 Electronic-based voltage detectors

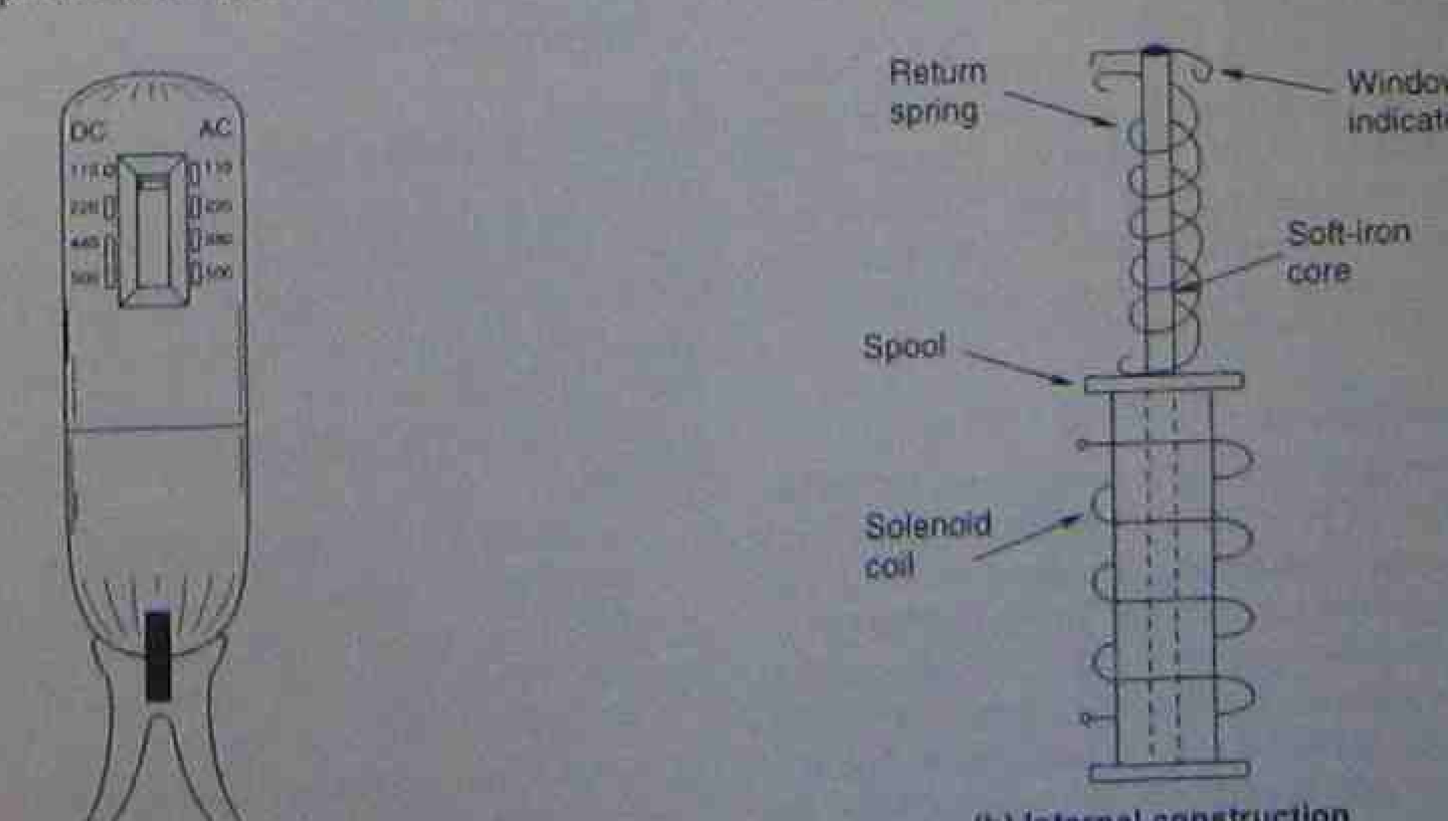
With the advent and development of modern electronics the vibrating tester has been almost completely superseded by a more compact battery powered design that is lighter and small enough to fit comfortably into a shirt pocket. Many of the later designs do not even have an on/off switch. By touching the ends of the probes together or putting the probe on to a voltage source they automatically turn themselves on. After a set period of inactivity they turn off.

Made by several manufacturers, the testers mostly cover both a.c. and d.c. voltages to about 600 V. There are as many variations in design as there are manufacturers. Some register a voltage as a digital readout and might indicate voltage polarity, while others simply have a bar of indicating lights. Some models automatically select a voltage range or even switch to a continuity reading mode. Well insulated, they are designed to protect the user up to about 1000 V.

7.3.5 Plug-style testers

The plug-style tester is designed for Australian-type standard three-pin sockets. Three neon lamps are mounted in a protective package of hardened plastic. In use it simply plugs into a three-pin socket, which is then switched on. Various combinations of neon lamps light up, indicating the condition of the circuit and the connections to the socket.

The lamp combinations are shown in Figure 7.3. Since one neon lamp indicates the condition of the earthing



(a) Complete assembled unit

(b) Internal construction

Figure 7.2 • Vibrating-type voltage tester

Figure 7.3 • Lamp conditions for a three-pin socket tester

system, the unit cannot be used in those rare situations where power is supplied through an isolating transformer. At least one model had a pushbutton installed, which when pressed activates a leakage current to test any residual current devices installed.

7.3.6 Neon testers

1. Screwdrivers

The screwdriver neon testers vary in style and size according to a manufacturer's preferences. Probably the best-known version is the small neon tube with a series resistor moulded into the handle of a screwdriver. Some models have a metal cap on the end of the handle, others do not. In use the metal end of the screwdriver is placed in contact with the circuit to be tested and the capacitive effect of the user's hand on the handle provides the return path to earth. The current through the lamp is so small that no electric shock effect is felt.

The neon tube will either glow or not. There are no degrees of brilliance and no way of estimating a voltage at the point of contact. In bright sunlight it is difficult to determine if the neon is glowing at all.

As with other neon tube voltage testers, they are subject to capacitive and inductive effects. They will occasionally glow in the presence of a radio frequency field radiating from a transmitter without actually being in contact with a circuit.

2. Other neon light versions

Several variations of the neon tester are available. Apart from the screwdriver type, most use two leads. One model has a neon lamp built into each probe. Under certain conditions the lamps will light, indicating a voltage when there is merely an inductive or capacitive effect present. Some versions also have fuses built into the probes.

3. Neon voltage indicators

A neon tester comprising four neon lamps was manufactured in an attempt to provide a more accurate voltage indication. The neon lamps had different value series resistors and the tube that emitted light indicated an approximation of the line voltage present. The range was 50 V to 500 V. For example, if two lamps glowed, the indicated voltage would be between 50 V and 250 V. Again it is worth observing that the makers recommend that the unit be tested on a known power source before relying on its readings.

7.3.7 Logic probes

Another form of voltage tester is the logic probe. With the advent of more and more logic circuits being used to control electrical power systems, logic module repair work is becoming more common. The probes are made for specific purposes and cannot be used in general electrical maintenance work as a voltage tester.

A logic probe can be self-powered with batteries or can be powered by the circuit under test with flexible leads and alligator clips. In general the probe consists of circuitry driving two light-emitting diodes. One is coloured green, the other red.

Logic circuits derive their output voltage from either having a voltage or not having a voltage at specific points in their circuit. These points are classified as being high (logic 1) or low (logic 0) without a specific voltage being meant.

For example, the green diode in the probe might glow indicating that the circuit is low at the point the probe is touching. All this means is that the voltage at that point will be in the range 0.8 V to 1.0 V. The value may be positive or negative. If the probe is touched at a point on the circuit and the red diode glows, then that point in the circuit is classed as high and indicates a voltage of between 2 V and 5 V.

The logic probe will operate satisfactorily at 20 MHz which is a far higher frequency than power line frequencies. The maximum voltage that can be applied to a probe is usually about 100 V.

7.4 METERS

To measure a voltage with greater accuracy than can be achieved with the voltage testers described above, a voltmeter is used. The face of the meter is calibrated in volts and the value read off against a scale. Meters are made with varying degrees of accuracy. The greater the accuracy, the higher the cost, and the more care that must be taken to protect the meter. For most purposes, meters can be classified by one of two operating principles. The two major movements used are:

1. moving-coil meters
2. moving-iron meters.

7.4.1 Moving-coil meters

Figure 7.4 is an exploded view of a moving-coil movement. It can be seen that a coil free to rotate is suspended in the field of a permanent magnet. The coil ends are connected to a suspension system so that current can be passed through the coil.

The suspension system may consist of one of two methods:

1. A coiled spring as shown in Figure 7.4. Sometimes called a hair spring, the outer end is attached to an adjustable arm so that the pointer of the movement can be adjusted to align itself up with the zero on the meter scale.
2. The second method is called *taut band suspension* and is considered a more robust method for suspending the moving coil. With this method the rigid coil pivot is replaced with two separate thin metal strands under tension. The hair springs are no longer necessary so are usually removed. Zero adjustment of the meter is achieved by a similar movable arm attached to one of the bands.

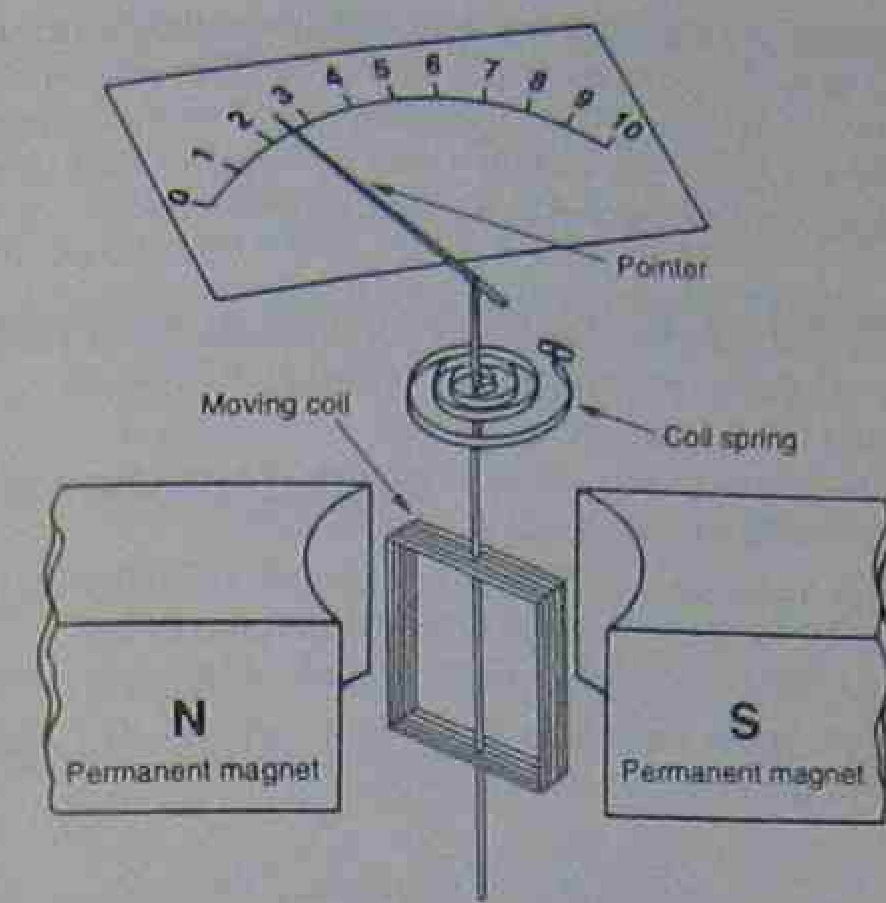


Figure 7.4 • Exploded view of a moving-coil movement

It must be noted that a current is passed through the coil—not a voltage. The current that flows through the coil is governed by the value of the applied voltage. The coil sets up its own field which reacts with that of the permanent magnet and causes the coil to rotate. A pointer attached to the coil gives a voltage reading against a scale.

The meter movement can only work satisfactorily on direct current. If alternating current is applied to the movement, it tries to turn the coil rapidly in the opposite directions with the result that the coil effectively remains stationary.

The meter can only operate on alternating current if the a.c. is rectified to direct current before it flows through the meter. Owing to these factors, the moving-coil meter always reads average values of current.

7.4.2 Moving-iron meters

Figure 7.5(a) is an exploded view of a moving-iron meter to illustrate its operating principle. In practice the construction is slightly different and is shown in Figure 7.5(b).

There are two magnetically soft iron vanes in the movement. One vane is fixed and the other pivoted and free to rotate. A pointer attached to the moving vane moves across a scale as an indicator.

When an electric current is passed through the coil, both the fixed and moving vanes are magnetised and have like poles at adjacent ends. Like poles repel each other and the movable vane moves away from the fixed vane. The attached pointer then indicates a value against a calibrated scale. A restraining spring provides opposing torque so that the vane movement can be stabilised.

Like the moving-coil instrument, the moving-iron meter is current operated. The current that flows through the coil is governed by the applied voltage. As a voltmeter, the coil impedance is very low when compared with the required

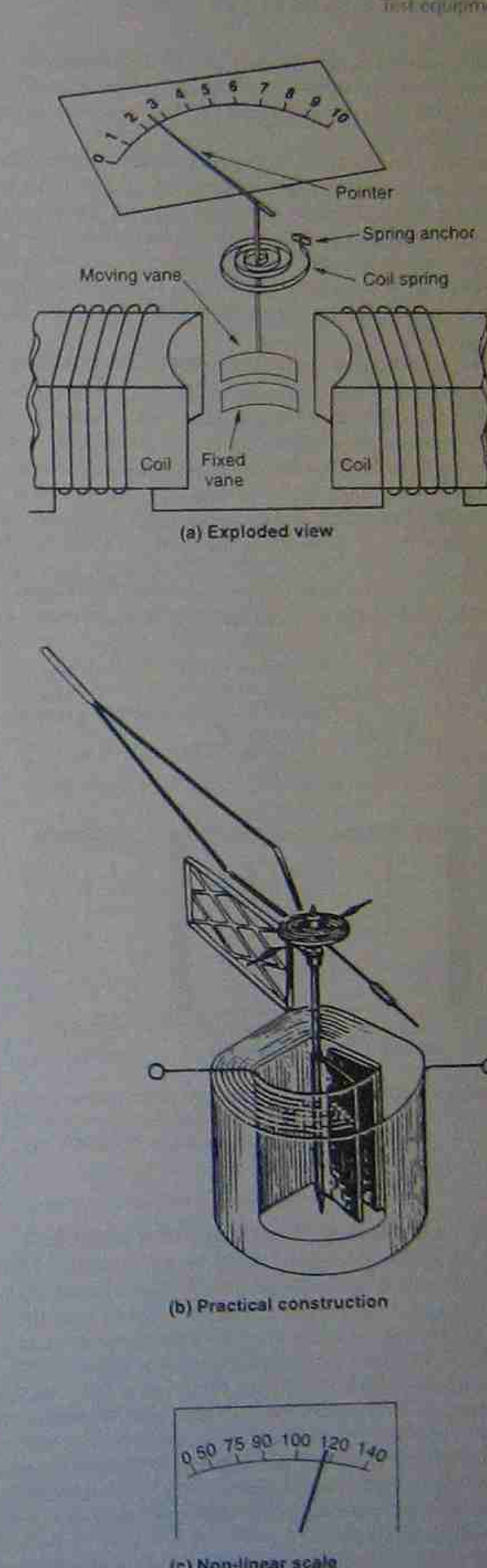


Figure 7.5 • Moving-iron meter movement

series resistance. Consequently the meter movement can be considered as resistive only and the current through the meter is directly proportional to the applied voltage (Ohm's law).

The meter will operate on both d.c. and a.c., although it might need to be calibrated differently. Because the two vanes are magnetised by the same current, the moving-iron meter operates on root-mean-square (r.m.s.) values of current. This term is discussed in Chapter 8. The major difference in scale calibration is that the moving-iron meter has a non-linear scale. This is illustrated in Figure 7.5(c).

7.4.3 Reading a meter—error of parallax

For accurate readings of a meter scale, precautions should be taken to ensure that the reading is taken with the eye vertically above the pointer. Figure 7.6 shows how errors can be made in reading meters by reading the scale from one side or the other. Error introduced in this fashion is called error of parallax. For many readings this factor can be accepted but if accuracy is required it should not be ignored.

In addition, scales have to be examined for the reader to be able to distinguish between major and minor divisions.

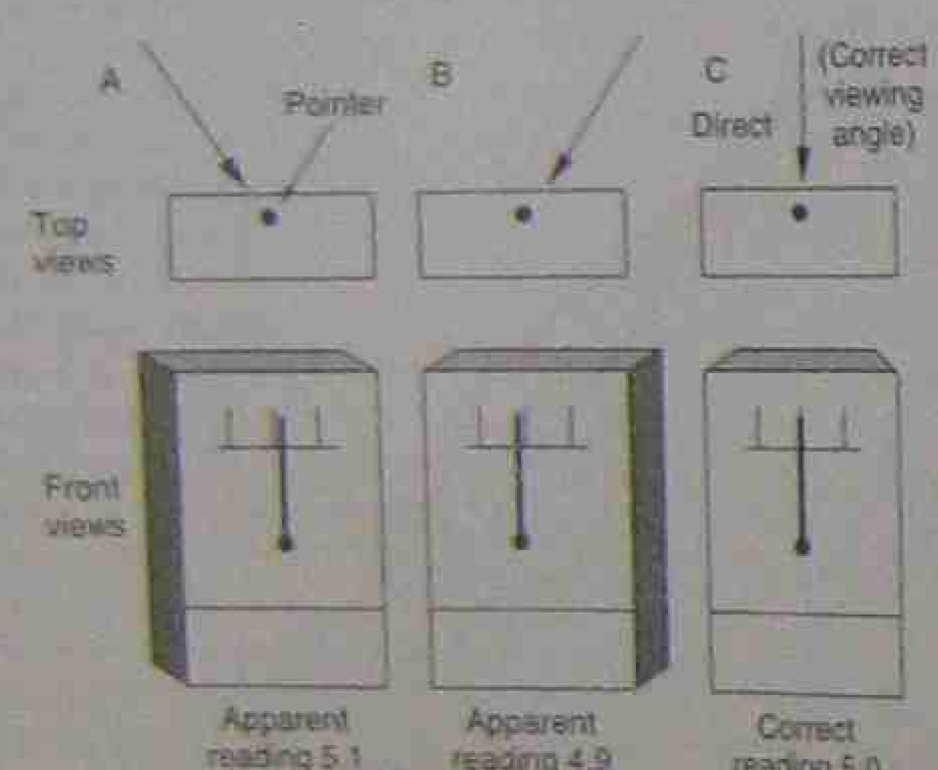


Figure 7.6 • Parallax. The eye at A sees the meter pointer over the 5.1 mark. The eye at B sees the pointer over the 4.9 mark. If the meter were observed directly, the pointer would show 5.0.

7.4.4 Extending the range of voltmeters

It was stressed in sections 7.4.1 and 7.4.2 that both moving-iron and moving-coil meters were current operated and relied on magnetic effects for their operation. However, the meter scales may be calibrated as voltage because of the direct ratio between voltage and current.

The basic movements of both types often have only a small voltage drop across the operating coils. Typically for a modern moving-coil meter, this voltage is in the order of a few millivolts. While the moving-iron type has a slightly higher voltage drop it is still very low and this factor limits the uses of these meters unless steps are taken to extend their operating range.

For any given voltage, adding series resistance decreases the current flow through the meter. If a moving-coil movement with 100 Ω coil resistance has an extra 10 kΩ added in series, the total meter resistance circuit would be 10.1 kΩ. From Ohm's law the current is now reduced approximately ten times. To restore the operating current to its original value, ten times the voltage must be applied; that is, the voltage range of the meter is extended ten times.

Example 7.1

A moving-coil meter as shown in Figure 7.7 has an internal resistance of 100 Ω. Find:

(a) The voltage drop across the meter if 1 mA gives a full-scale deflection (f.s.d.) of the pointer.

(b) If a resistance of 10 kΩ is connected in series with the meter, find the voltage that would have to be applied to give full-scale deflection of the movement.

(c) If the series resistance is replaced with one of 1 MΩ find the new full-scale voltage.

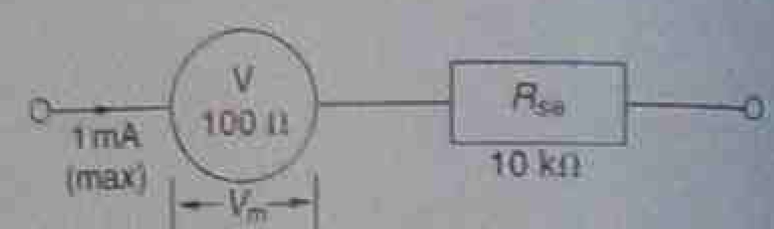


Figure 7.7 • Circuit for example 7.1

(a) Voltage across meter:

$$V_M = IR$$
$$= 1 \times 10^{-3} \times 100$$
$$= 0.1\ \text{V}$$

(b) $R_{se} = 10$ kΩ.

Maximum current of meter is 1 mA.
Total resistance = 10 000 + 100 = 10 100 Ω

$$V = IR$$
$$= 1 \times 10^{-3} \times 10\,100$$
$$= 10.1\ \text{V}$$

(c) $R_{se} = 1$ MΩ

Maximum current is still 1 mA.
Total resistance = 1 000 000 + 100 = 1 000 100 Ω

$$V = IR$$
$$= 1 \times 10^{-3} \times 1\,000\,100$$
$$= 1\,000.1\ \text{V}$$

The full answers for voltage have been given in (b) and (c) but in normal practice they would be rounded off to 10 V and 1000 V.

In summary, for a moving-coil meter, a series resistor of 10 kΩ enables the meter to be used as a 10 V meter. Similarly, a series resistance of 1 MΩ enables the meter to have a full-scale rating of 1000 V.

For moving-iron meters used as voltmeters, a similar situation applies. Once the full-scale current value and the internal resistance of the movement are known, series resistors can be added to extend the maximum voltage range of the meter. These two facts are often inscribed on the face of the meter for identification purposes. These added series resistors are known as multiplying resistors or simply multipliers.

Example 7.2

A moving-iron meter has the following inscribed on its face: 'fsd 5 mA, Resis 25 Ω'. Find the full-scale voltage ranges for series resistors of 10 kΩ and 1 MΩ.

Voltage across meter at full-scale deflection:

$$V = IR$$
$$= 5 \times 10^{-3} \times 25$$
$$= 0.25\ \text{V}$$

The 10 kΩ series resistor:

$$V = IR$$
$$= 5 \times 10^{-3} \times (10\,000 + 25)$$
$$= 50\ \text{V}$$

The 1 MΩ series resistor:

$$V = IR$$
$$= 5 \times 10^{-3} \times (1\,000\,000 + 25)$$
$$= 5\,000\ \text{V}$$

7.4.5 Extending the range of ammeters

The operating current of an ammeter is generally very low to ensure adequate sensitivity for measuring small values of current. The internal resistance of the meter is also kept as low as possible to reduce the voltage drop across the meter itself.

To measure values of current higher than that required to give full-scale deflection, a resistor called a shunt is placed in parallel with the meter. It allows some of the current to bypass the meter. Any fixed value of current flowing into a parallel circuit will divide according to the resistance of the paths. The less the resistance the greater will be the current flow in that path. It is better explained with an example, using the same meter movement as that in example 7.1.

Example 7.3

A moving-coil meter movement has a full-scale current rating of 1 mA and an internal resistance of 100 Ω. Calculate the resistance of a shunt to be placed in parallel with it so that currents of up to 1 A can be measured. The circuit is shown in Figure 7.8.

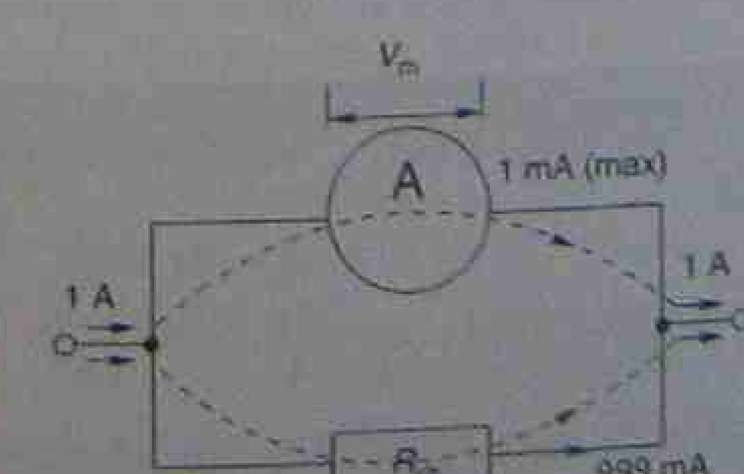


Figure 7.8 • Circuit for example 7.3

The intention is to bypass 999 mA around the meter with R_{sh}, so leaving only a maximum of 1 mA to flow through the meter.

Meter voltage at f.s.d.:

$$V = IR$$
$$= 1 \times 10^{-3} \times 100$$
$$= 0.1\ \text{V}$$

Because the shunt resistor and the meter are in parallel, the voltage across the parallel section will be the same for both resistors: that is, the voltage across the shunt = 0.1 V. This means that two of the three values required to apply Ohm's law are known. (It has already been established that 999 mA must bypass the meter.) Thus:

$$R_{sh} = \frac{V}{I}$$
$$= \frac{0.1}{0.999}$$
$$= 0.1\ \Omega$$

Example 7.4

A meter movement has a full-scale deflection current of 500 μA and an internal resistance of 40 Ω. Calculate the value of a shunt resistance to extend the meter range to 300 mA. The circuit will be similar to that of Figure 7.8, although the values will be different.

Current to bypass meter:

$$= 300\ \text{mA} - 50\ \mu\text{A}$$
$$= (300 \times 10^{-3}) - (50 \times 10^{-6})$$
$$= 0.299\,95\ \text{A}$$

Voltage across the meter at f.s.d.:

$$V = IR$$
$$= 50 \times 10^{-6} \times 40$$
$$= 0.002\ \text{V}$$

Resistance of shunt:

$$R = \frac{V}{I}$$
$$= \frac{0.002}{0.299\,95}$$
$$= 0.0066\ \Omega$$

In practical terms, to extend the range of an ammeter 1000 times, a shunt resistor must be placed in parallel with the meter. Its resistance will be one-thousandth that of the meter movement.

It should be appreciated that the values indicated on the meter scale have to be corrected for the new range of the meter; that is, at an indicated half-scale reading of 0.5 mA, the actual current would be 500 mA.

For some meters, and particularly d.c. meters, shunts of very low values of resistance are required. Small errors in resistance can lead to much bigger errors in current readings. Moving-iron meters on a.c. often use a special type of transformer called a current transformer to eliminate the possibility of shunt calibration error. In conjunction with voltage transformers used to extend the range of voltmeters, these are discussed in greater detail in section 14.10.

7.5 NON-CONTACT TESTING

Meters have been made for testing for voltages and currents that make no electrical contact with the circuit under test. Several versions are on the market.

7.5.1 Voltage testers

A commercially made unit that senses voltage and does not rely on lamps or vibrating solenoids is a device sensitive to the electrostatic fields produced by the circuit voltage. It is the basis of the finder used to locate live conductors buried in walls up to a depth of 2.5 cm. Most units give both audible and visual signals.

The units are battery operated and incorporate a self-test check, which should be used regularly as a safety measure. One model can also detect magnetic fields.

7.5.2 Current testers

Current testers are marketed under various names, most of which are trade related. They are called tong testers, clamp meters, clip-on testers, link-test meters and so on.

Clamp-action meters are generally used to measure currents without having to interrupt the circuit being tested. Most meters have accuracies within 1 per cent of full-scale deflection and on frequencies ranging from direct current ($f = 0$) to about 1 kHz.

Originally there were only two types. One worked on the repulsion principle of the moving-iron meter while the other used a transformer combined with a switch to select the desired current ranges. These can still be obtained, but many other versions are now available.

1. Repulsion type movement (a.c. and d.c.)

The operating principle was that of the moving-iron meter. A variety of current ranges were catered for with plug-in modules. On being placed around the conductor to be measured, the magnetic field created by the current set up repulsion between the meter elements and caused the moving section with pointer attached to rotate. They were capable of use on both a.c. and d.c.

2. Transformer operated (a.c. only)

Different current ranges were catered for by using a transformer with tappings connected to a range switch. The transformer prevented it being used on d.c. The basic principle is shown in Figure 7.9.

The indicating meter could be a direct-current-operated meter by connecting a rectifying unit between the transformer output and the meter movement. With a d.c. meter the scale then becomes linear. With a moving-iron meter the scale was non-linear.

In use, the jaws of the instrument were opened with a lever and then placed around the chosen conductor. The jaws were then allowed to close. The magnetic field around the conductor entered the low permeability path of the iron, and the meter movement responded according to the strength of that magnetic field.

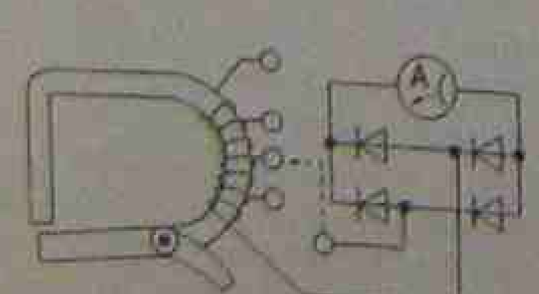


Figure 7.9 • Internal circuit arrangement of a current transformer type, link-test ammeter suitable only for a.c. work

3. Modern versions

Current models can be switched to indicate peak, r.m.s. and average values for many different waveforms, as well as indicating d.c. values. Some models will also indicate circuit voltage by measuring the strength of the electrostatic field around the conductor.

The number of available functions on any one meter has a considerable bearing on the price of the instrument. The range of options is impressive. The trend is for instruments to have digital readouts and auto-ranging facilities.

Two testers are illustrated in Figure 7.10. The finger-operated trigger can be seen. In Figure 7.10(a) a digital readout model is illustrated. This model is auto-ranging and will read currents in the range 2 A to 2000 A a.c. It can also be used to measure voltages, both a.c. and d.c. and resistance values of 40 kΩ with a pair of plug-in test leads.

The unit in Figure 7.10(b) has an analogue readout and a pointer which has to be read against a scale. Varying ranges are provided by a series of scales, which are rolled around in a window. It is an a.c.-only instrument and reads up to a maximum of 500 A. To read voltages or resistance values, test leads are plugged into the base of the unit. A replaceable battery is enclosed and the meter is protected by a fast-acting fuse.

(a) Digital clamp meter

(b) Analogue clamp meter

Figure 7.10 • Non-intrusive ammeters

Other clamp-on meter movements use a Hall-effect device to detect the presence and quantity of current flowing. The phenomenon was first recorded in 1879 but its use has only become practical with the development of integrated-circuit modules.

When a current is passed through the opposite edges of a thin piece of foil, a magnetic field applied at right angles to the current will produce a voltage across the remaining two edges. This voltage is proportional to the current flow through the piece of foil. By maintaining a constant current flow, the voltage produced is directly proportional to the strength of the magnetic field.

When the magnetic field is the result of a current flowing through a conductor the device becomes in effect an ammeter. Electronic circuitry has to be employed to maintain a constant current flow and to measure and amplify the voltage produced. A separate power supply (usually batteries) also has to be used.

The method is an excellent and accurate one but the additional components tend to make a meter of this type expensive. Available models can be obtained in current ranges from milliamps to 2000 A. Their sensitivity to low values of current and their accuracy make them ideal for measuring leakage currents.

7.6 POWER METERS FOR ALTERNATING CURRENT

Power being consumed in a circuit is measured with a wattmeter. Wattmeters are often constructed with a dynamometer movement. This type of movement usually has two internal electrical circuits.

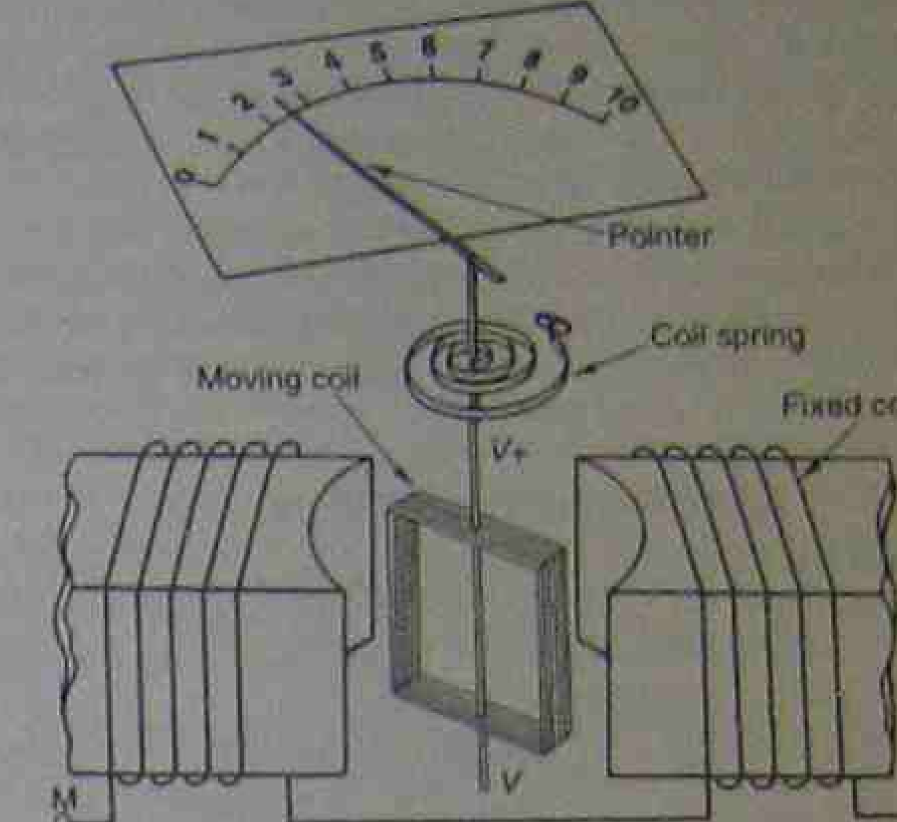
7.6.1 Dynamometer-movement meters

An exploded view of a dynamometer movement is shown in Figure 7.11(a). The meter has two circuits. One is for voltage, the other for current. The model illustrated has a soft-iron core around which is wound a low resistance coil to carry the circuit current. This coil produces a magnetic flux proportional to the current flowing in a circuit. Not all dynamometer-movement meters have an iron core, some models are air cored.

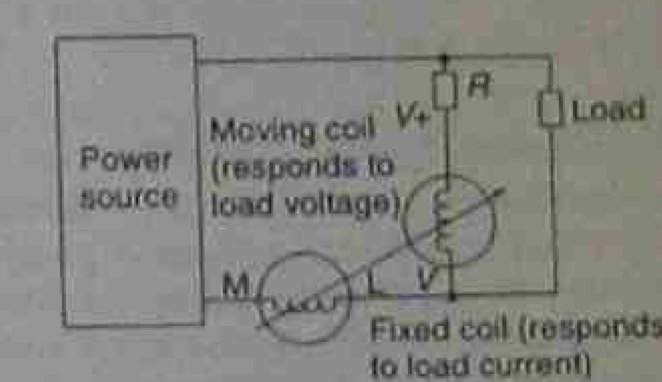
The meter's second circuit consists of a coil with a series resistance of high value. This is the voltage circuit and produces a magnetic field proportional to the applied voltage.

The direct multiplication of voltage and current values in an alternating current circuit to obtain a power value can at best be only an approximation. With some electrical components, the voltage and current can be out of step with each other and this effect will be discussed more fully in Chapter 8. This type of meter construction with its two magnetic fields takes into account any displacement between voltage and current and gives a true power reading.

With alternating current supplies, the values of voltage and current are continually changing so a true power reading indicates the average power being consumed over a finite period of time, rather than at one instant. (See sections 8.11 and 8.16 for more detailed information.)



(a) Exploded view



(b) Connections. The moving coil with the series resistance R carries a current proportional to load voltage. Meter deflection is proportional to the product of voltage and current, which is power. This meter may be used for a.c. or d.c. measurements

Figure 7.11 • A dynamometer movement

Dynamometer movements find their greatest number of applications in alternating current work because they integrate both current and voltage values and give a true power reading with a high degree of accuracy. The meter-movement principle is also applied to other types of meters.

While wattmeters can be used on direct current, it is not the usual practice. Good voltmeters and ammeters can give quite high accuracy for direct current work by multiplying the two meter values together. Figure 7.11(b) shows how the meters are connected into an alternating current circuit. Dynamometers and their applications are discussed further in Appendix 1.

7.6.2 Hand-held wattmeters

Hand-held wattmeters are similar in size and shape to a multimeter. Battery powered, they operate electronically and provide a digital readout. Range selection is by a rotary switch. The meter uses r.m.s. values of current and voltage irrespective of the actual waveform and has an accuracy of about 5 per cent of the readout, a figure which is sufficiently accurate for a portable instrument.

The rotary switch has three sections—voltage, current, and power. Maximum ranges are up to about 750 V and 20 A, giving a power range from 400 W to 15 kW. The instrument has the added advantage of being comparatively accurate from 15 Hz to 1 kHz.

Because there are three groups of readings, individual readings of voltage, current and power can be obtained. Some models also indicate the displacement, if any, between voltage and current. The meter may also be referred to as a power analyser.

7.6.3 Bench-type wattmeters

Equipment of the bench type has a far higher degree of accuracy than the portable version and is normally never taken into the field. Expensive to purchase, they are kept in a workshop for better protection.

They are usually 240 V mains powered, but later models are electronically operated. With an analogue readout, the operating frequency is usually from d.c. ($f = 0$) to about 15 kHz. Current ranges are up to 10 A with a maximum voltage of 1000 V. This gives a maximum power range from 250 mW to 10 kW.

7.6.4 High-frequency wattmeters

Wattmeters intended for use on frequencies well above power-line frequencies use different principles of operation. Most rely on the heating effect of the current flowing in the circuit. The heat produced generates a voltage proportional to the temperature of a thermocouple. The voltage is then processed and indicates on a meter, whether analogue or digital. One wattmeter of this type is discussed in more detail in Appendix 1.

7.6.5 Ultra high-frequency wattmeters

For frequencies in excess of 400 MHz, parallel-line meters are used. One of the parallel lines has the load current flowing through it and the second line has a voltage induced in it. This voltage is rectified and read against the scale of a meter calibrated for that frequency.

7.7 CONTINUITY AND RESISTANCE TESTING

The testing of electrical circuits requires that measuring instruments be able to cope with very high and very low values of resistance. What may be suitable for measuring high resistances is not necessarily suitable for measuring low resistances. High-value resistors and insulation testing need one type of device, while low values of resistance and continuity testing need another type.

To check a circuit, two factors have to be taken into account—continuity and resistance. The test required determines the type of testing device to be used.

7.7.1 Low-value resistance and continuity testers

Low-value resistance testers can range from elementary to quite complex. The choice often depends on the degree of accuracy required.

For simple continuity testers, all that is required is a low-voltage source of power and an indicator, for example, a 6 V battery and a 6 V lamp or buzzer. Two test leads, the lamp, and the battery are connected in series and, when testing a circuit, the continuity of a conductor is indicated when the lamp lights.

This type of tester had the advantage of being simple, easily made up when required, and inexpensive. The disadvantages are that regular battery replacement is required and the low voltage will not provide sufficient power to give a positive indication when the circuit is complete but contains an appreciable amount of resistance. For example, if a circuit is complete but has a relay coil resistance of 100 Ω the lamp will not light, thus falsely indicating an open circuit.

Whenever circuits and test equipment indicate a possible fault is present, further testing is necessary. More sophisticated equipment might have to be used to indicate the extent and type of the problem. It may well be that resistance is present in the circuit, so giving the doubtful indication. The fault might be more extensive than that, of course, and that is the purpose of the testing function.

One attempt to get around this problem was the introduction of small hand-cranked generating sets used in conjunction with a suitable bell. When the crank was turned and the test leads touched together the bell would ring.

The voltage was often around 150 V but was also a.c. More costly than a battery and a buzzer, the system was rugged, still relatively simple to use and operated on low resistances. Excellent for checking out multiple conductors in a new installation, it also had one big disadvantage. Since it worked on alternating current, its indication was subject to high inductances in the circuit path. No indication would be given under some circumstances. On the other hand, with very long runs it would give a false indication of continuity because of the capacity between adjacent conductors in the circuit.

7.7.2 Ohmmeters

There are two basic types of ohmmeter circuits—series and parallel. The name relates to the position of the unknown resistance in the circuit. A series circuit has the unknown resistor connected in series with the meter. A parallel ohmmeter circuit has the unknown resistor connected in parallel with the meter. Each connection has its own advantages. Both circuits can give continuity checks as well as indicate relative values of resistance. Unlike the continuity testers mentioned above, no audible or visual indication is given. A meter scale has to be read.

1. Series ohmmeters

The series circuit is the more common and is shown in Figure 7.12(a). It consists of a battery, a fixed resistor, and an adjustable resistor. The fixed resistor is a current-limiting resistor to provide some form of protection for the meter. It is often called a ballast resistor.

Figure 7.12(b) shows that the meter scale is the reverse of that of a normal scale. Zero ohms is indicated at full-scale deflection and infinity at the zero end. It should be noted also that the scale is non-linear.

In use, the meter must be adjusted to indicate zero before switching on, and then the probes joined together and the meter adjusted to read full scale. When the probes are open-circuited, the meter reading will fall to zero again. The unknown resistor is then placed in contact with the probes and the value read against the meter scale. For the values shown in Figure 7.12, the meter would indicate a resistance of 60 kΩ at half-scale or 25 μA meter current. At 10 μA the indicated resistance would be 300 kΩ. Both values are shown on the meter scale.

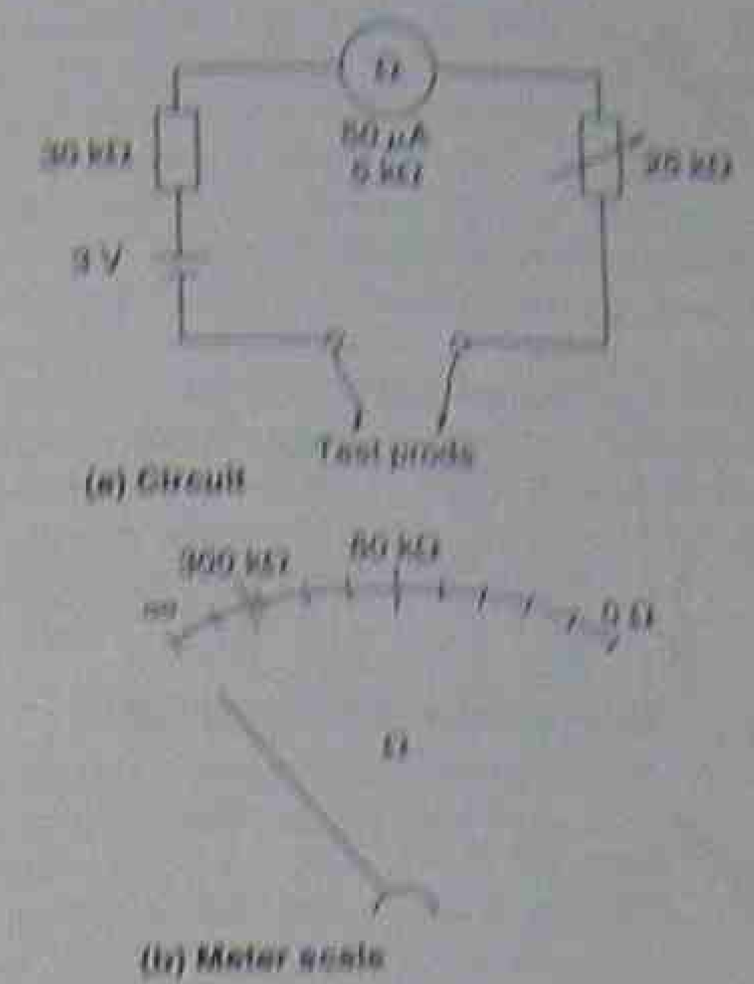


Figure 7.12 • Simplified circuit for a series ohmmeter

2. Parallel ohmmeters

A typical circuit is shown in Figure 7.13(a). It includes a switch to ensure that the battery is not left on when the meter is not in use. This is usually done with some type of trigger or finger-operated switch. The idea is that, when the leads are put down, the battery is automatically switched off.

The same operating adjustments have to be made as for the series meter to ensure that zero and full-scale deflections occur at the correct scale positions. When the probes short out the meter, it should indicate zero. When the probes are open-circuited, the meter should read full-scale deflection.

The meter scale is shown in Figure 7.13(b). It is also non-linear but the values progress across the scale in the normal manner. For comparison purposes the indicated position for the 60 kΩ resistor is marked on the scale at a position corresponding to 46 μA. Half scale or 25 μA then corresponds to 5 kΩ.

The 300 kΩ position is not shown, but corresponds to a current of 49.2 μA. This would be indistinguishable from a full-scale reading. If the two half-scale values are compared it can be seen that the parallel-type circuit is better suited to lower values of resistance than the series-type circuit.

The parallel-type circuit is less popular and is used for measuring lower values of resistance than the series-type circuit. It does not readily lend itself to being part of a multimeter circuit like the series-type resistance circuit.

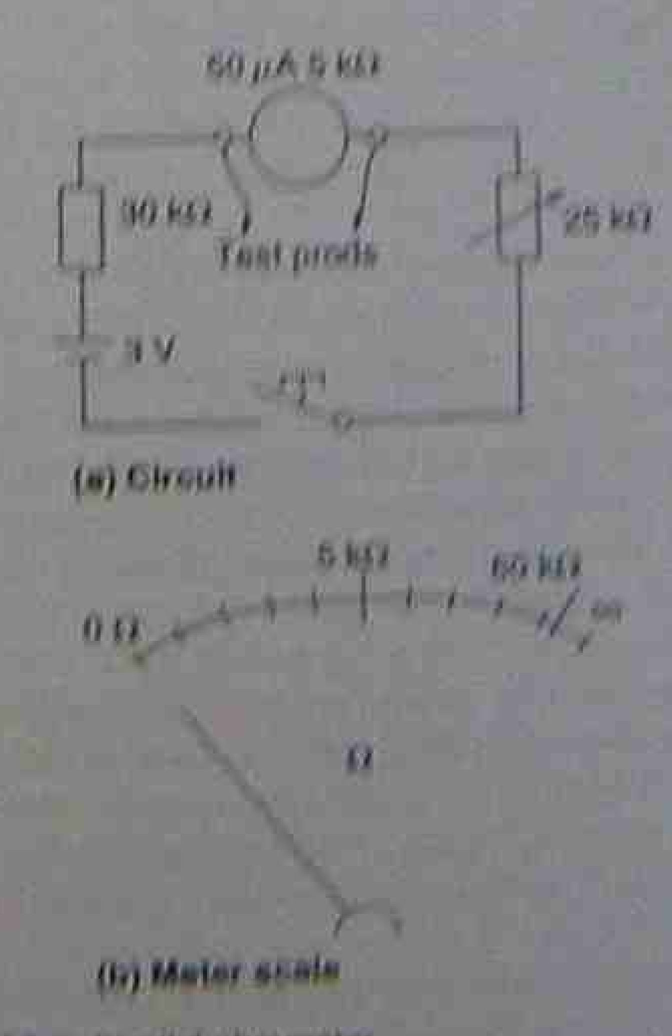


Figure 7.13 • Parallel ohmmeter

7.8 ANALOGUE MULTIMETERS

Sometimes called a volt-ohm meter (a VOM), or an ammeter, voltmeter, ohmmeter (an AVO, which is also a registered trade name) it is actually a single meter with switching arrangements to connect it as either a voltmeter, ammeter, or ohmmeter. The normal practice is to provide several ranges for each function.

7.8.1 The voltmeter section

Figure 7.14 shows a moving coil 50 μA meter with an internal resistance of 5 kΩ connected to a five-position switch. Each switch position connects a different value resistor in series (a multiplier) with the meter to provide a range of voltages.

The moving-coil meter movement can read only direct

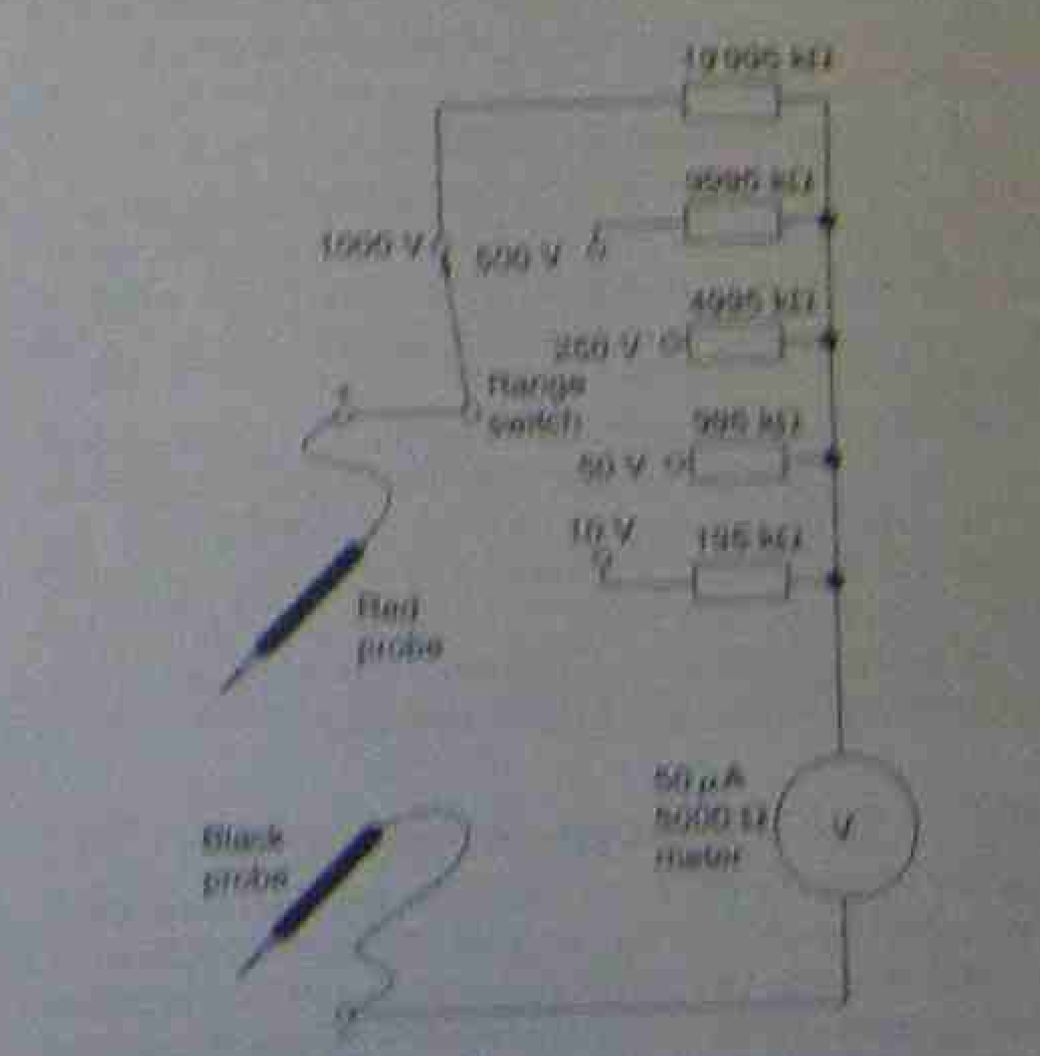
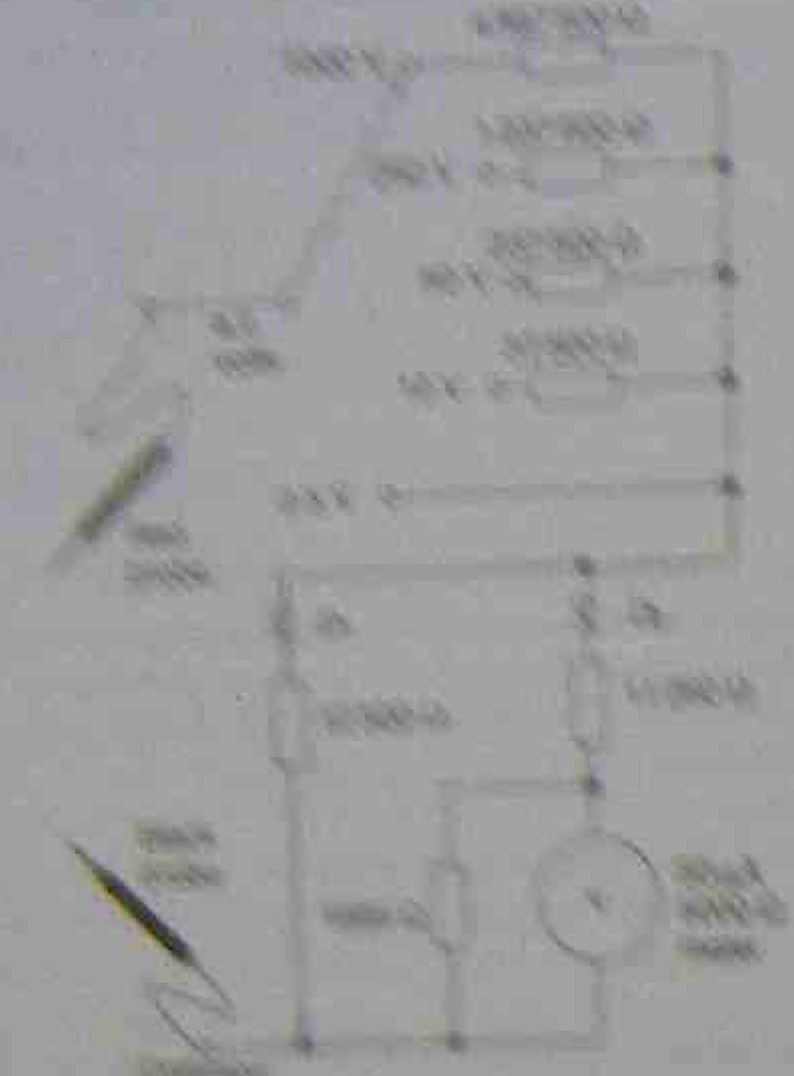
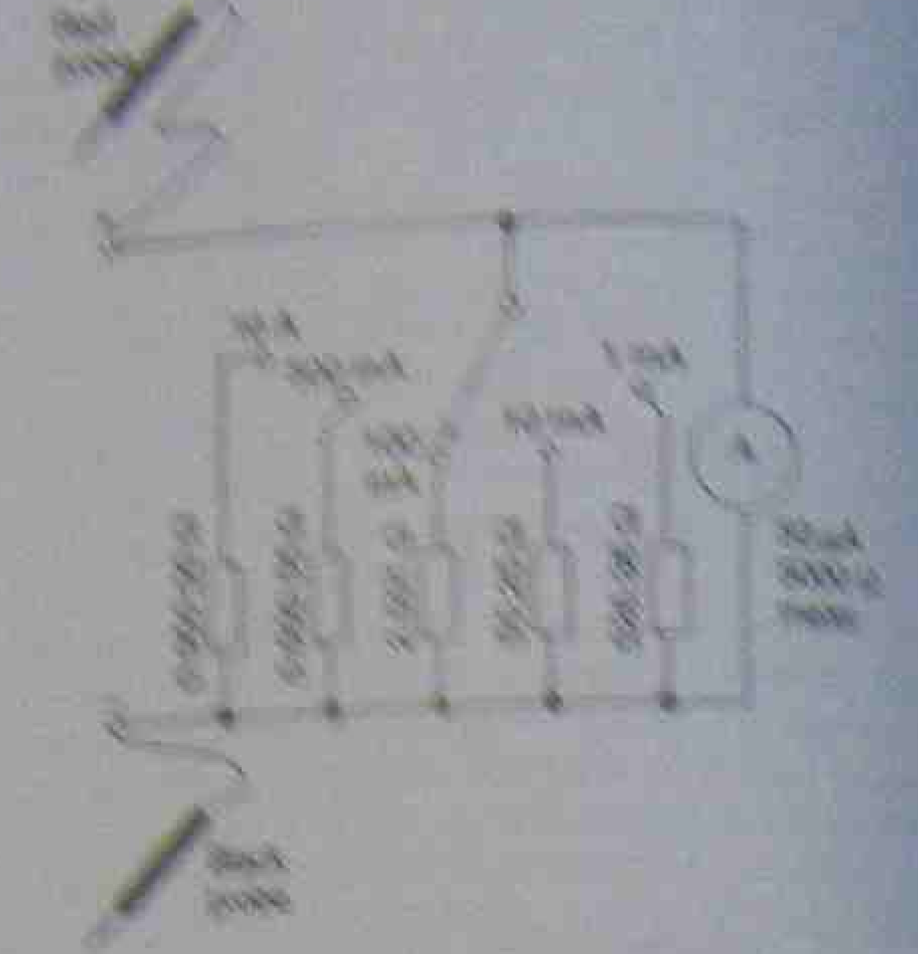


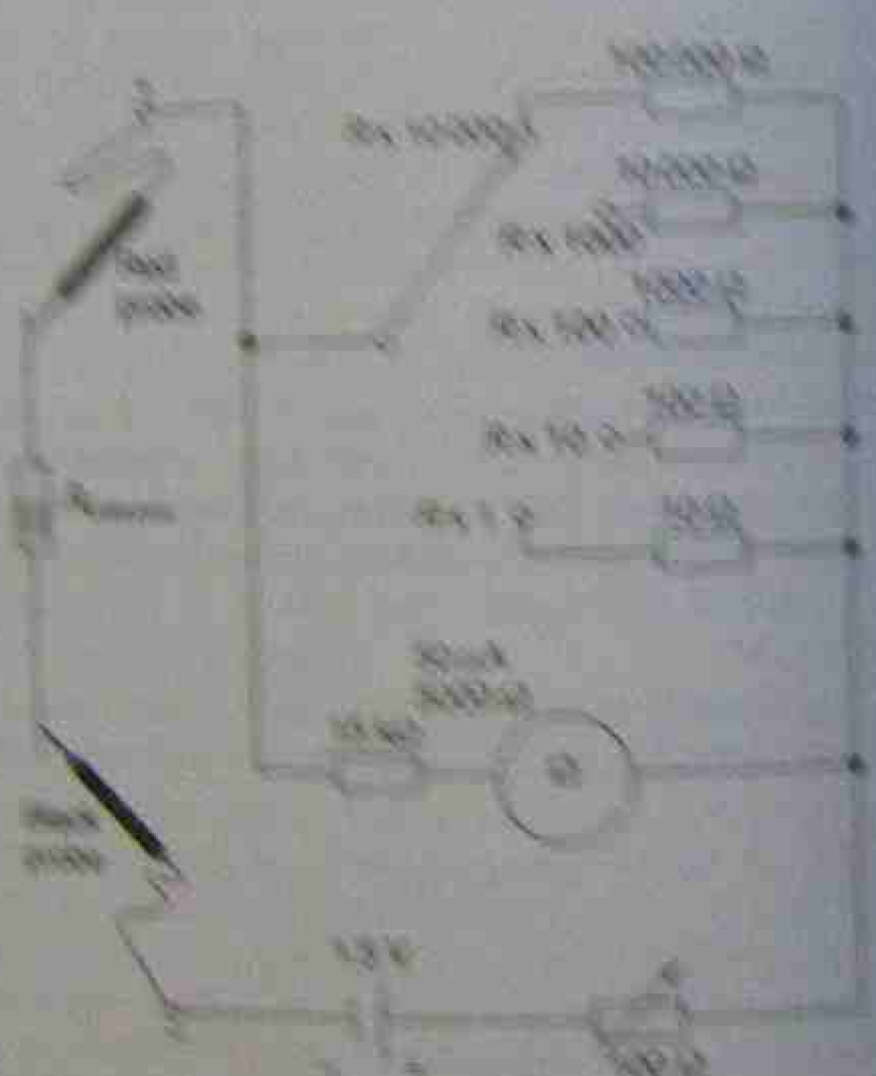
Figure 7.14 • Simplified VOM voltmeter schematic diagram. This voltmeter uses one 50 μA, 5000 Ω meter movement, multiplier resistors, and one range switch





7.8.2 The ammeter section

7.8.3 The ohmmeter section



7.8.4 The complete multimeter

7.9 DIGITAL READING METERS

7.9.1 Digital voltmeters

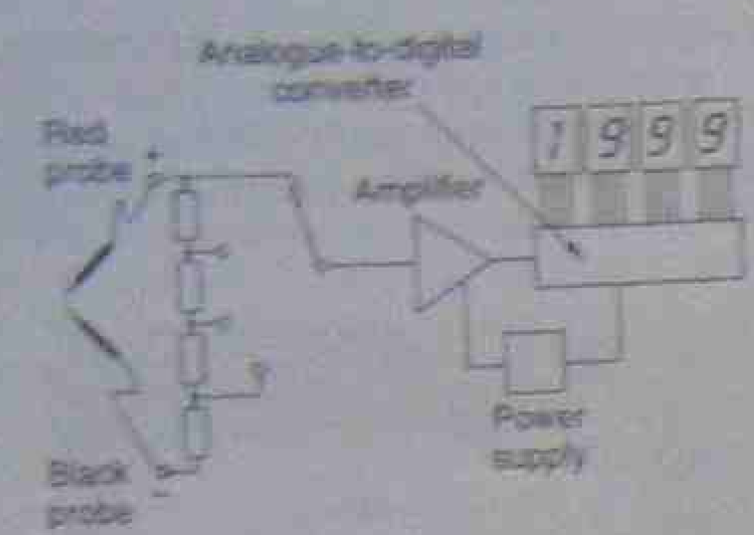


Figure 7.19 • Digital voltmeter circuit principles

voltage according to the values selected by the range switch.

The output is connected to a converter, which processes the voltage value and sends output signals to the appropriate bars of the display.

7.9.2 Readout display

Each numeral in a display group is made up of a maximum of seven bars. Full scale is usually specified as 199 or 1999 and increases in multiples of 10 (decades) as the number of digits is increased. A 199 display can also be described as a 2½ digit display. Similarly, the 1999 display is sometimes called a 3½ digit display.

Originally the range required had to be selected, and the decimal point shifted itself along the display to match. Figure 7.20 is a meter of this type. Modern digital displays are auto-ranging and the decimal point shifts along to suit the input. The electronic circuits are accordingly more complex.

Now the only adjustment is to select the resistance, current, or voltage function, and the rest is done automatically. Even the polarity of d.c. voltages and currents is catered for with suitable indicators.

Digit sizes can range from 5 mm to 12 mm or larger, depending on the manufacturer and the targeted use of the meter. Display size can be important, for example if a reading has to be made from a distance.

Display types can range from light-emitting diodes to liquid crystal displays. Vacuum and gaseous displays are also popular. Gaseous displays are excellent for reading under very weak illumination, while a liquid crystal display (LCD) is almost useless without a supplementary light source.

In bright sunlight a gaseous discharge display is hard to read, while the liquid crystal display can be easily read. The LCD display is, however, more temperature dependent and if the temperature of the display gets too high the whole screen goes either blank or black and cannot be read. Permanent damage can occur under these conditions.

7.9.3 Digital multimeters

Digital multimeter circuits of the modern type are far more complicated than analogue meter circuits, so only a basic circuit has been shown in Figure 7.20. With this circuit, the range and type of measurement has to be selected by the user. However, it does serve to illustrate the principles behind the operation of such a meter.

Digital multimeter inputs are made up by an input attenuator and a function selection switch. The attenuator is sometimes automated and combines with the auto-ranging function. The converter and digital readout are not always sensitive enough for multimeter use so an amplifier is often provided. It also serves the additional function of providing isolation between the converter and the attenuator. This prevents possible loading of the circuit being tested.

7.9.4 Comparison of digital and analogue meters

1. For normal operation the digital instrument is more accurate. The more expensive the meter, the greater is the accuracy.

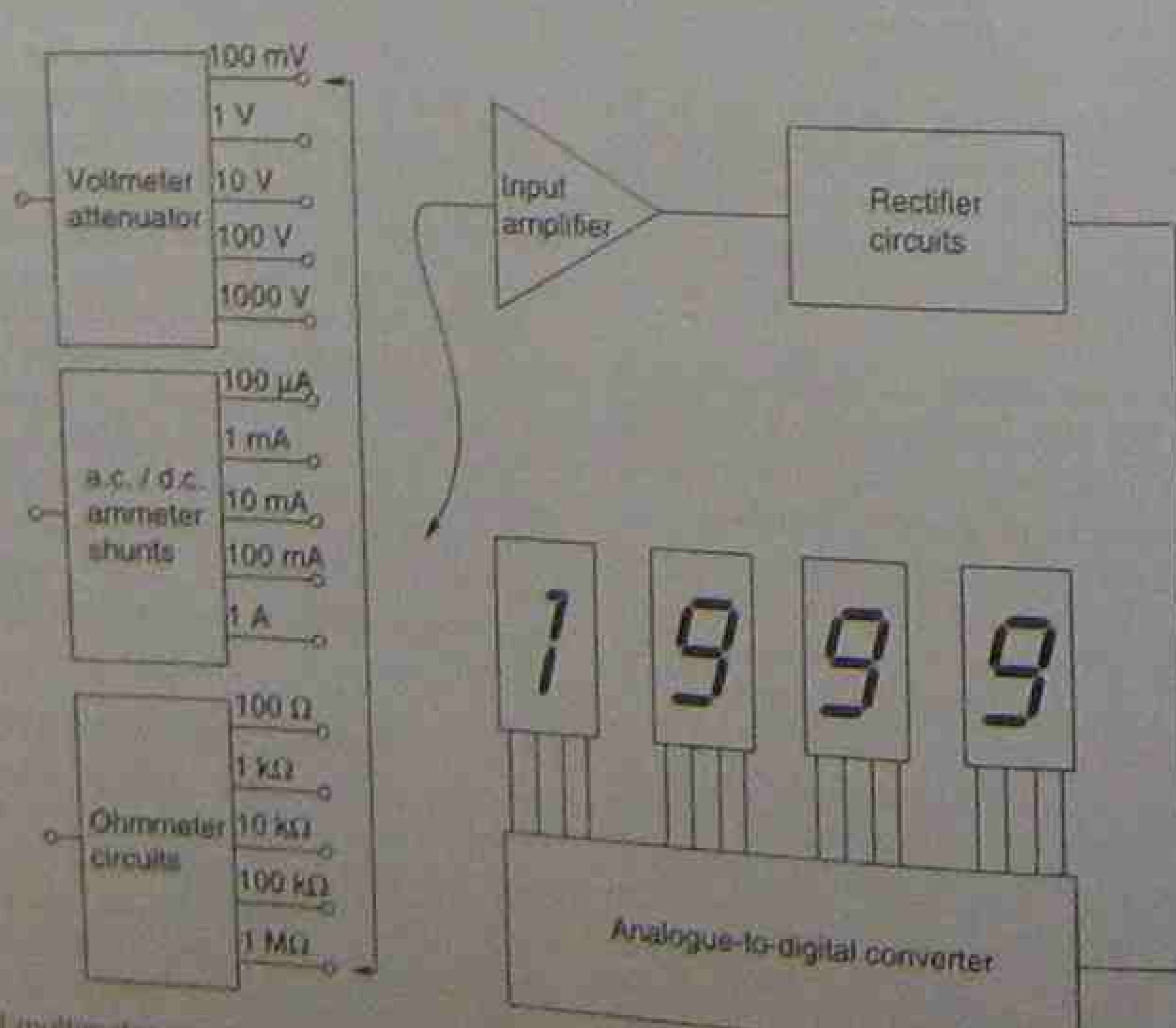


Figure 7.20 • Digital multimeter circuit principles

2. Both types need an internal battery source of power.
3. Both types use a rectifier to convert a.c. to d.c. but the analogue meter has to use a separate scale for a.c. voltages. The digital meter has to have a correcting circuit to compensate for this.
4. Ohmmeter functions in an analogue meter use a non-linear scale. The digital meter has no scale and its non-linear tendency has to be corrected with a special constant-current circuit.
5. The input impedance of a digital meter is far higher than that of an analogue-type meter circuit; that is, for an analogue meter it may be 20 kΩ/V while for the digital meter it may be 20 MΩ/V. This means less interference or effect on the circuit being tested.
6. A digital multimeter is subject to large errors in the presence of a radio-frequency field. Normally this has minimal effect on the analogue meter.
7. The digital meter is far more sensitive to circuit conditions than the analogue meter. In some circumstances this can lead to misleading readings.
8. Analogue meters used on resistance readings can have negative polarities (from the internal battery) on the positive probe. This has to be checked before use because of possible directional errors in current-sensitive devices such as diodes.
9. Digital meters, on the other hand, have constant polarities on the resistance ranges and cause no confusion.
10. An analogue meter is more responsive to changing values than a digital instrument. The digital device, because of its circuit configuration, takes an appreciably longer time to respond to the new value and settle again.

7.10 RESISTANCE MEASURING CIRCUITS

In previous sections, both parallel and series ohmmeter circuits were introduced as a means of determining values of resistance. Both circuits, however, have limits beyond which their accuracy is questionable.

Parallel ohmmeter circuits perform better than series circuits for low values of resistance, but as resistance values get still lower, errors can creep in. The resistance of the terminal connections and that of the leads connecting the resistor under test begin to have an appreciable effect.

Similarly the series ohmmeter circuit outperforms the parallel circuit for higher values of resistance, but as resistance values increase, the current flowing through the resistor and the meter decreases. A point is eventually reached where the meter is no longer able to indicate a value accurately.

7.10.1 Volt-ammeter testing

For most purposes a resistance measurement can be obtained with sufficient accuracy by passing current through the resistor under test and at the same time measuring both the current through and the voltage across it. The resistance is then calculated by applying Ohm's law. For greater accuracy, care must be taken in the

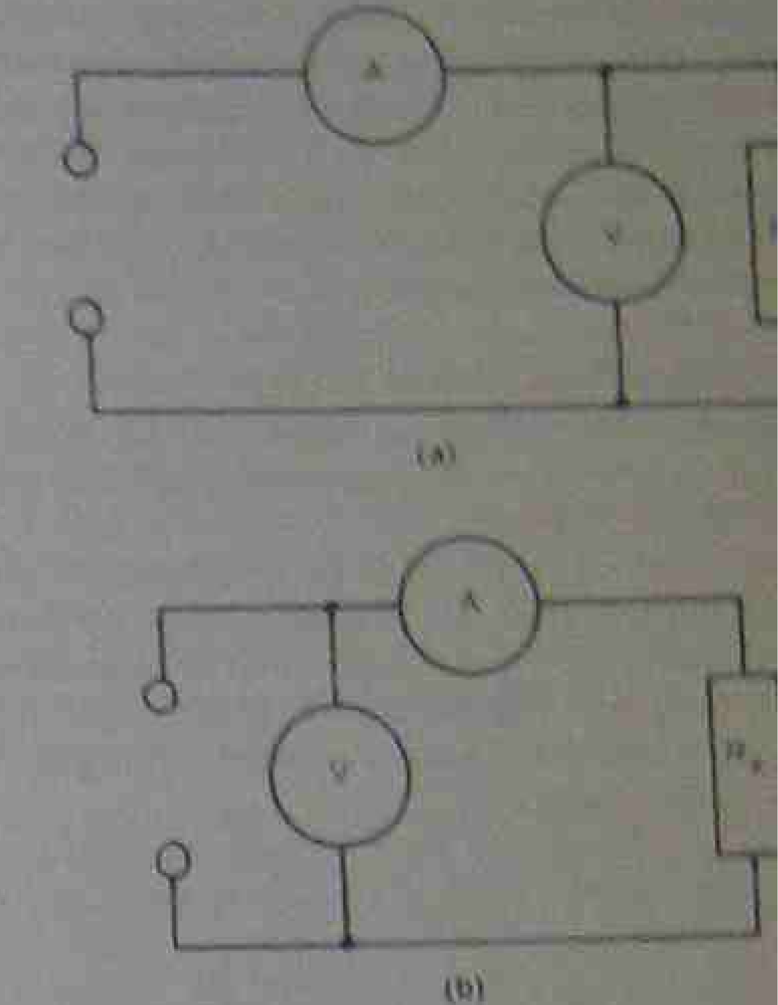


Figure 7.21 • Volt-ammeter circuits for determining resistance

connection of the circuit. Figure 7.21 shows t[...]
circuits.

In Figure 7.21(a) the ammeter will measure [...] flowing through the voltmeter as well as that [...] resistor. If the resistor has a comparatively l[...] resistance when compared to the voltmeter re[...] discrepancy in ammeter reading can be i[...] example, if the current flowing through the re[...] 1 A and the current flowing through the [...] 50 µA, the ammeter is unable to discrimin[...] readings of 1 A and 1.000 05 A.

On the other hand, if the current flowing [...] resistor is 100 µA, a suitable ammeter w[...] current flow of 150 µA ($I_R + I_m$) and an er[...] ciable percentage would result. Figure 7.21(a[...] a more suitable circuit for measuring lo[...] values.

Figure 7.21(b) shows the voltmeter connect[...] supply source. The ammeter now reads onl[...] flowing through the resistor. With this circ[...] meter reads the supply source or the sum o[...] drops across the resistor and the ammeter in [...] $V = V_m + V_R$.

For accuracy, with this circuit the volta[...] resistor must be far greater than the voltag[...] the ammeter. Assuming that the ammeter [...] movement and the internal resistance is 5 k[...] drop across the meter is $V = IR = 50\ \mu A \times$ 5[...] For best results the p.d. across the resistor h[...] times this value, that is, a value around 60 [...] lower voltage values the error increases con[...]

These methods for obtaining resistance [...] serious limitations as regards both accuracy [...] ity. Two meters are employed and this m[...] possibilities for introducing errors are gre[...] Voltages and currents also have to be a[...] resistor and meter ratings.

The diagram in Figure 7.21(a) is the circuit in general use, provided a good quality analogue multimeter with a high internal impedance is used. Some analogue multimeters have input impedances of 10 MΩ or higher and as a consequence the loading on the circuit is minimal. A good quality digital multimeter can be substituted provided it is of sufficient accuracy.

7.10.2 The Wheatstone bridge

The Wheatstone bridge circuit was first described by S. H. Christie in 1833. It was virtually ignored until taken up by Wheatstone in 1843.

It was first intended for accurate measurement of values of resistance but was adapted and modified to suit many different circuits able to measure other values. Figure 7.22(a) illustrates the original circuit. It comprises three known resistors and the unknown resistor making up the fourth arm of the bridge.

Figure 7.22(b) shows one modification for measuring inductance and Figure 7.22(c) is another modification for measuring capacitance. There are many variations of these three circuits, usually named after the people who developed them. All are based on the original bridge circuit.

With good quality equipment the bridge circuit will measure accurately down to 0.01 Ω. For the standard bridge circuit this is about its lower limit. Further modifications to the original circuit enable the bridge to be used to measure accurately down to 0.001 Ω.

Altering the ratios of resistors R_1 and R_2 in the arms of the bridge enables resistances of much higher or lower values to be measured. The value of voltage used to supply the bridge is immaterial because the circuit is adjusted correctly when the meter indicates a null, that is, when the meter pointer remains stationary on zero. The bridge is then said to be balanced.

Higher or lower values of voltage mean only that the higher or lower values of current flow through the bridge and make the meter susceptible to possible overload and damage in the unbalanced state. The meter used in the circuit is usually a sensitive moving-coil meter with a centre-zero scale. It is sometimes referred to as a galvanometer.

When the bridge is balanced, the voltage drop across the unknown resistor (R_X in Fig. 7.22(a)) is equal to the voltage drop across resistor R_1. This also means that the potentials across resistors R_2 and R_3 are also equal to each other, that is:

At null: $I_1R_1 = I_2R_X$ (1)

Similarly: $I_1R_2 = I_2R_3$ (2)

Dividing (1) by (2): $\dfrac{I_1R_1}{I_1R_2} = \dfrac{I_2R_X}{I_2R_3}$

Then, cancelling the I's the equation becomes:

$$\frac{R_1}{R_2} = \frac{R_X}{R_3}$$

$$\boxed{R_X = \frac{R_1R_3}{R_2}}$$

In practice, resistance arms R_1 and R_2 are usually made adjustable in ratios of 0.1, 1, 10, 100 and 1000. This approach enables the bridge circuit's range to be expanded to cover a wider range of values.

It also means that when using the bridge to measure resistances, multiplying factors might have to be taken into account for calculation purposes. In practical Wheatstone bridges the actual values of R_1 and R_2 might not be known. It is usual to have them labelled as a switch with its position indicating the ratio between them. R_X is read directly off the values set by the adjustable resistor R_3. The value obtained by R_3 at the null position is then multiplied or divided by the indicated ratio.

The following two examples illustrate the calculations involved in using a bridge circuit.

Example 7.5

The resistances on a bridge read R_1 = 10 kΩ, R_2 = 1 kΩ and R_3 = 3.92 kΩ. Find the value of the resistance being measured.

$$R_X = \frac{R_1R_3}{R_2}$$

$$= \frac{10\,000 \times 3920}{1000}$$

$$= 39.20 \text{ k}\Omega$$

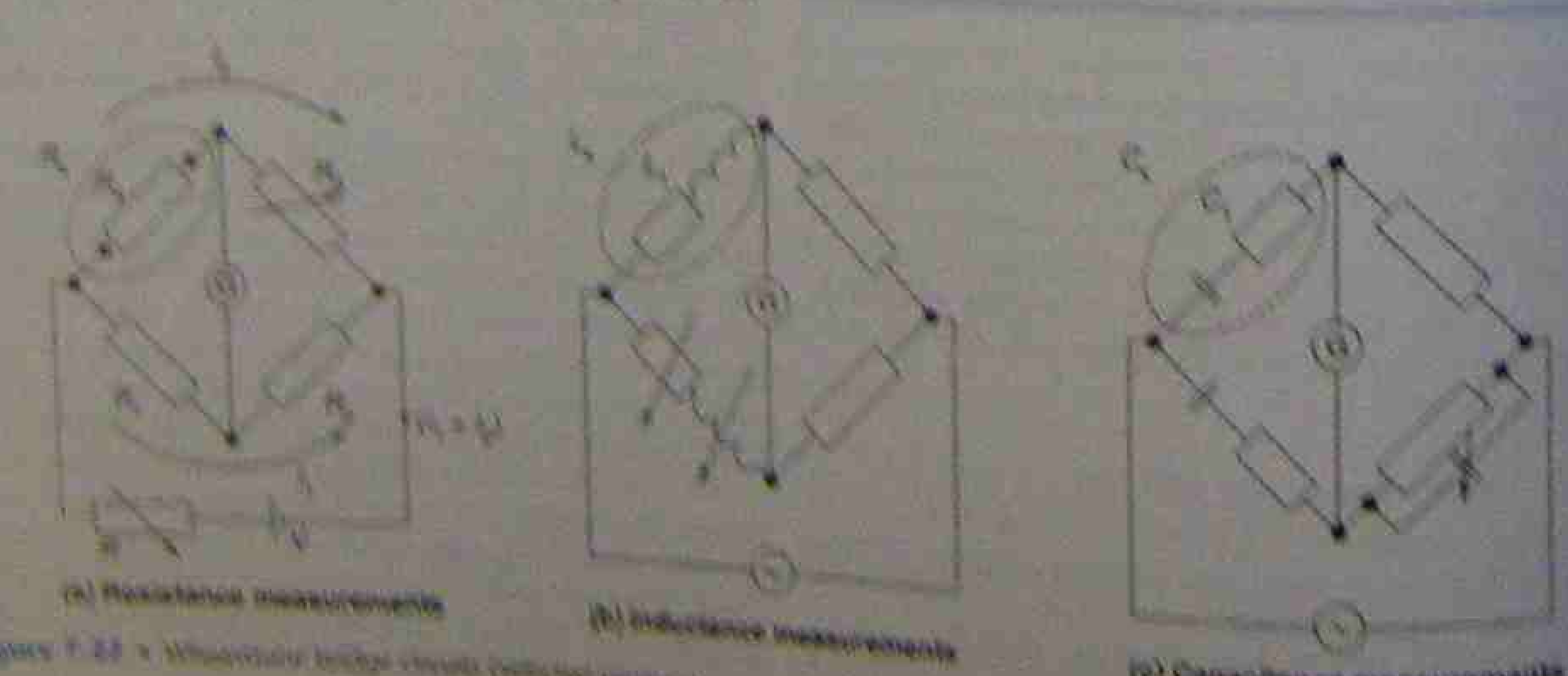


Figure 7.22 • Wheatstone bridge circuits (selected variations)

Example 7.6

The resistances on a bridge read R_1 = 100 Ω, R_2 = 1000 Ω, and R_3 = 68.4 Ω. Find the value of the resistance being measured.

$$R_X = \frac{R_1R_3}{R_2}$$

$$= \frac{100 \times 68.4}{1000}$$

$$= 6.84 \text{ k}\Omega$$

7.10.3 Insulation resistance

In practical terms a battery-operated circuit as described above is neither convenient, accurate nor practical for measuring very high values of resistance.

A high voltage is required to ensure that a reasonable amount of current is able to flow in the circuit being tested. This is necessary to enable a meter to be able to give a more positive indication.

Australian Standards specify minimum voltages for testing circuits. As an approximation it is about double the operating voltage of the circuit. That is, on a 240 V circuit, the specified test voltage is 500 V.

Probably the original maker of an instrument for tests of this nature was an English firm known today as Megger Instruments Ltd. The original name was Evershed and Vignoles and the trade name for the unit was 'Megger'. The trade name has become a generic term and is almost universally used to describe instruments of this type. It should be noted that other firms manufacture similar test instruments and all should be known as insulation testers.

There are two general arrangements for obtaining the necessary voltages: one a hand-cranked generator, the other dry cells and an electronic circuit.

1. Generator-powered insulation testers

To make the instrument truly portable there has to be an inbuilt power source. This was achieved in the original megger with a generator cranked by hand. Earlier models used a d.c. generator, while later models took advantage of the strength of modern magnets rotating inside a coil to produce an alternating voltage. The resulting a.c. was converted to d.c. internally and used to operate the instrument. The circuit of a direct current generator and insulation tester is shown in Figure 7.23.

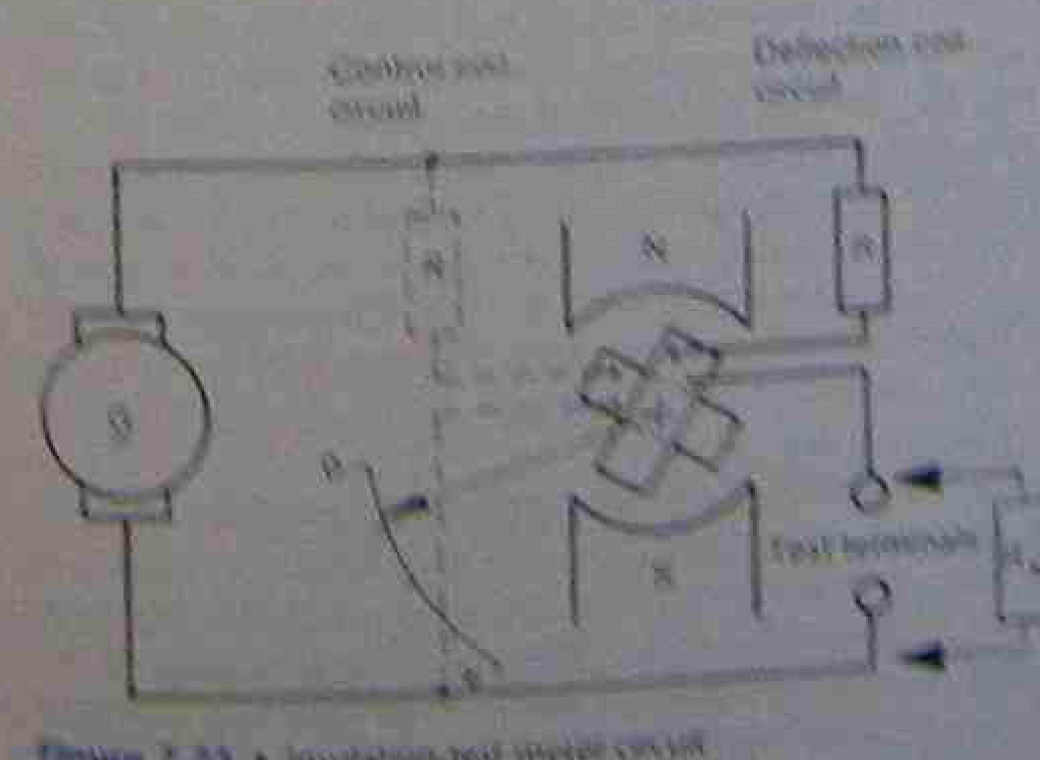


Figure 7.23 • Insulation-test meter circuit

Analysis of the circuit will show that it is a variation of the series ohmmeter circuit, since the generator, deflecting coil, deflecting coil resistor and the resistance being tested are all in series.

The moving coil with attached pointer is actually two coils fixed at right angles to each other. The second coil is connected in series with another resistor directly across the voltage source powering the instrument. This second coil acts in place of a return spring to return the pointer to zero. There is no other restraining spring against which torque must be exerted.

The final position of the coil depends on the relative amounts of current flowing in each coil. One current is governed by the voltage supply and the other by the current flowing through the circuit being tested. Normal measuring ranges are 0–200 MΩ, with a fair degree of accuracy.

2. Battery-powered insulation testers

Many modern instruments made by other manufacturers use a bank of dry cells to energise an electronic circuit. The output from this circuit is high-voltage, high-frequency a.c., which is then rectified and used to operate the instrument.

The complete unit is usually much smaller and lighter than a generator-powered instrument but care must be taken to ensure that the batteries are in good condition for satisfactory operation of the tester.

Once a suitable value of d.c. voltage is obtained, the operation of the unit is much the same as the generator-powered unit. The normal operating resistance range is 0–200 MΩ. Standard models are available in voltages of 100 V, 250 V, 500 V, or 1000 V and accuracy is equivalent to that of the megger.

Because the instruments are battery powered it is normal for one of the test probes to include a switch that must be held down while the instrument is actually in operation. On release of the probe the battery is disconnected.

Later models of the battery-powered insulation testers have digital readouts. The readout is supplied directly with power from the batteries. See Figure 7.26.

7.10.4 Bridge meggers

Several variations of the basic megger circuit have been produced. One version includes a built-in bridge circuit for measurement of lower values of resistance. The instrument is much larger, heavier and more expensive than the megger described above. A circuit of a bridge megger is shown in Figure 7.24.

It can be seen that adjustable resistors are connected into the circuit. This is achieved with the aid of the switches on the side of the instrument housing. (See also Fig. 7.28.) On the top surface are the four rotary switches, which operate the resistance box component of the bridge-measuring circuit.

When used as an insulation tester the bridge megger is a direct-reading series ohmmeter for high resistance values. When switched to the bridge configuration the circuit has to be balanced by adjusting the rotary switches. The infinity end of the scale is taken as the null or balanced position. In the bridge-balanced position the pointer rests directly above the infinity symbol.

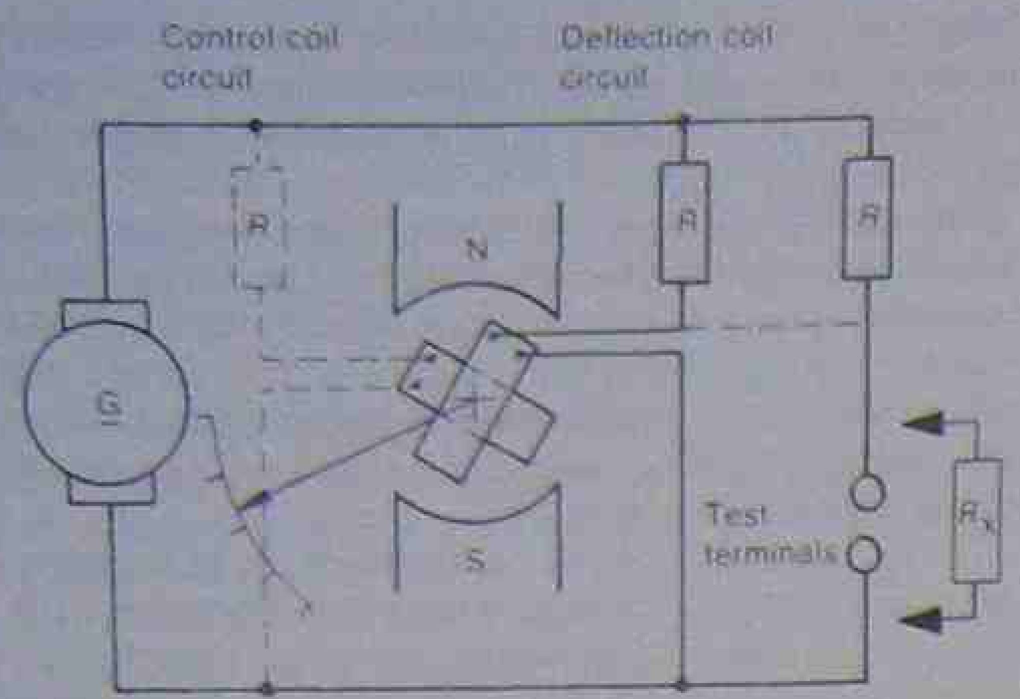


Figure 7.24 • Basic bridge megger circuit

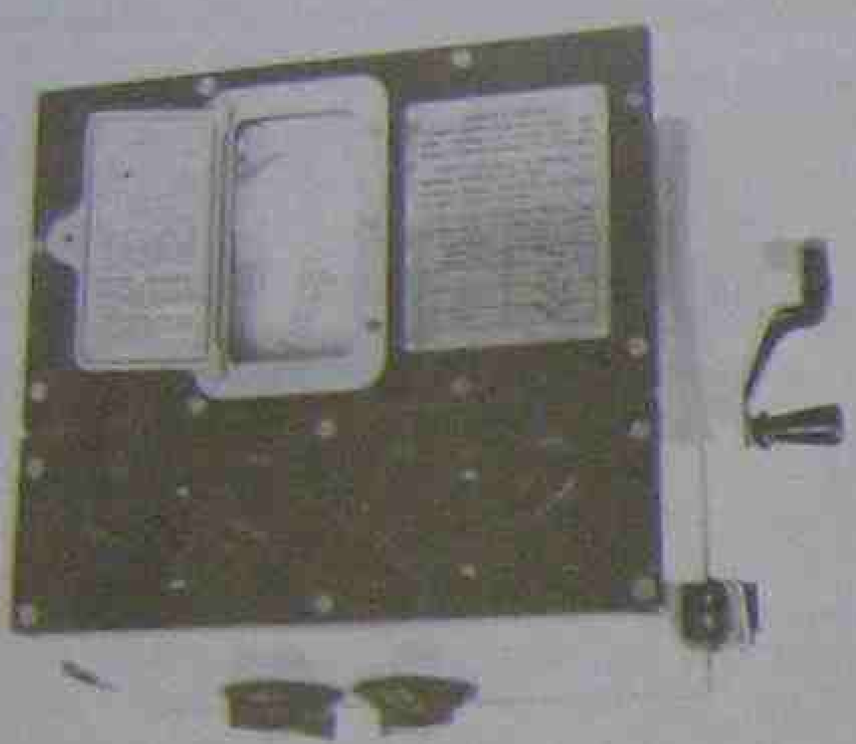


Figure 7.25 • A 500 V bridge megger

H Rowe & Co.

One of the major applications of a bridge megger instrument is the location of cable faults. The resistance between the ends of cables and the fault can be accurately measured and, by calculation, the distance to the location of the fault can be obtained. The circuit connection most often used has been given the name 'Varley Loop' after the man who proposed the method.

7.10.5 Continuity testing

Another variation of the megger circuit is intended for testing the continuity and resistance of conductors in an installation. While the standard megger operates as a series ohmmeter for resistance ranges of 0–100 MΩ, or 0–200 MΩ in some instances, the continuity tester operates like a parallel-connected ohmmeter. Its normal resistance range is 0–20 Ω, although this may vary from manufacturer to manufacturer.

7.10.6 Care in the use of meggers

A 500 V megger can cause an electric shock unless care is taken when using it. Inadvertent contact with the test leads can give rise to an unpleasant electric shock. Similarly, other technicians handling the same conductors being tested can be affected.

Underground cables are often tested with higher voltages. Meggers generating 3000 V are commonly used for

Figure 7.26 • Modern battery-powered insulation tester

Courtesy of AVO International

the purpose. Apart from direct electric shocks from the instrument, there is an additional danger created by the capacitance effects of the cable. This applies particularly to armoured or metal-sheathed underground cables.

The capacitance created by the method of construction of the cable enables an electric charge to be stored on the cable. The charge is created from the d.c. of the megger and generally exists between the conductor and the armoured metal sheath protecting the cable. Because the actual capacitance can vary widely from cable to cable, it is usually expressed in microfarads per unit length. One typical cable has a capacitance of approximately 0.2 μF/300 m. Others may have higher or lower capacitances.

At 3000 V, this capacitance relates to an energy storage of around 9 J. This quantity of charge at this voltage can cause enough of a shock to immobilise a technician for a time. Sometimes medical attention is needed.

On an aerodrome, for example, there can be many kilometres of underground cable, so the scope for an electric shock is considerable. Even with a comparatively short length of underground cable this equates to an energy content at a voltage that can kill.

Before relying on readings taken by a megger on an installation that contains capacitance, the operator should ensure the installation is charged up to the voltage of the megger. This is generally done by extended testing on any one conductor for a period of time. The meter reading usually indicates that this has been achieved when the reading stabilises at one value.

For example, when reading the resistance of one conductor in an underground cable to its sheath, the megger may show a reading which indicates a low resistance path to earth. On persisting with the test the megger reading will generally climb to a much higher—and more satisfactory—reading.

7.10.7 Fault loop impedance testers

Under current self-regulatory conditions for electrical workers, the worker can be expected to check the fault loop conditions for an installation. The fault loop is the path from the installation's main earth back to the supply point where the main neutral is also grounded.

Fault loop impedance testers are designed to give an indication of the impedance of that path.

The tester itself may be a combination of megger, continuity tester, and other functions such as testing the operation and effectiveness of any residual current device in that installation. The loop test places a known resistor between one phase of the installation and measures both the no-fault voltage and the voltage under simulated fault conditions. This is then processed in the instrument and provides a reading in ohms impedance for the loop back to the supply via the earth return.

For safety and residual current devices a good earth return is generally considered a must. Particularly with the later electronic devices, the testing of the loop impedance can cause tripping at least and a possibility of damage to the device itself. In either case an interruption to the supply service can vary from a minor nuisance in domestic cases to a major problem in existing industrial cases involving safety.

7.11 POTENTIOMETERS

The primary purpose of a potentiometer is calibration of the voltage of cells. However, the device can be adapted to the calibration of voltmeters, ammeters, resistors, wattmeters and pyrometers. Strictly speaking it does not measure anything; instead it compares known values with unknown values, which can be referred to a linear measurement.

A sensitive voltmeter such as a good moving-coil instrument can measure the voltage of a cell. However, no matter how small the current taken by the meter, there is always a voltage drop within the cell due to its internal resistance. Thus the meter will always register a voltage slightly less than the cell's true potential.

For greater accuracy, a method has to be used where no current is taken from the cell when the voltage is measured. This method is known as the potentiometer method and employs the bridge circuit described earlier. When the bridge is balanced, no current is being taken from the cell under test. The basic circuit is shown in Figure 7.27(a).

A length of resistance wire is laid out against a scale and a sliding contact is provided so that it can make contact with the wire anywhere along its length. This resistance wire is shown between points A and B in Figure 7.27(a). A variable resistor is connected in series with the resistance wire and a d.c. supply source. The resistor controls the current flowing through the wire and limits it to a value that will enable the wire to remain relatively cool and hence its resistance constant.

If point B is taken as a reference point, the voltage gradient along the wire will vary from zero (at point B) to the voltage of the supply source.

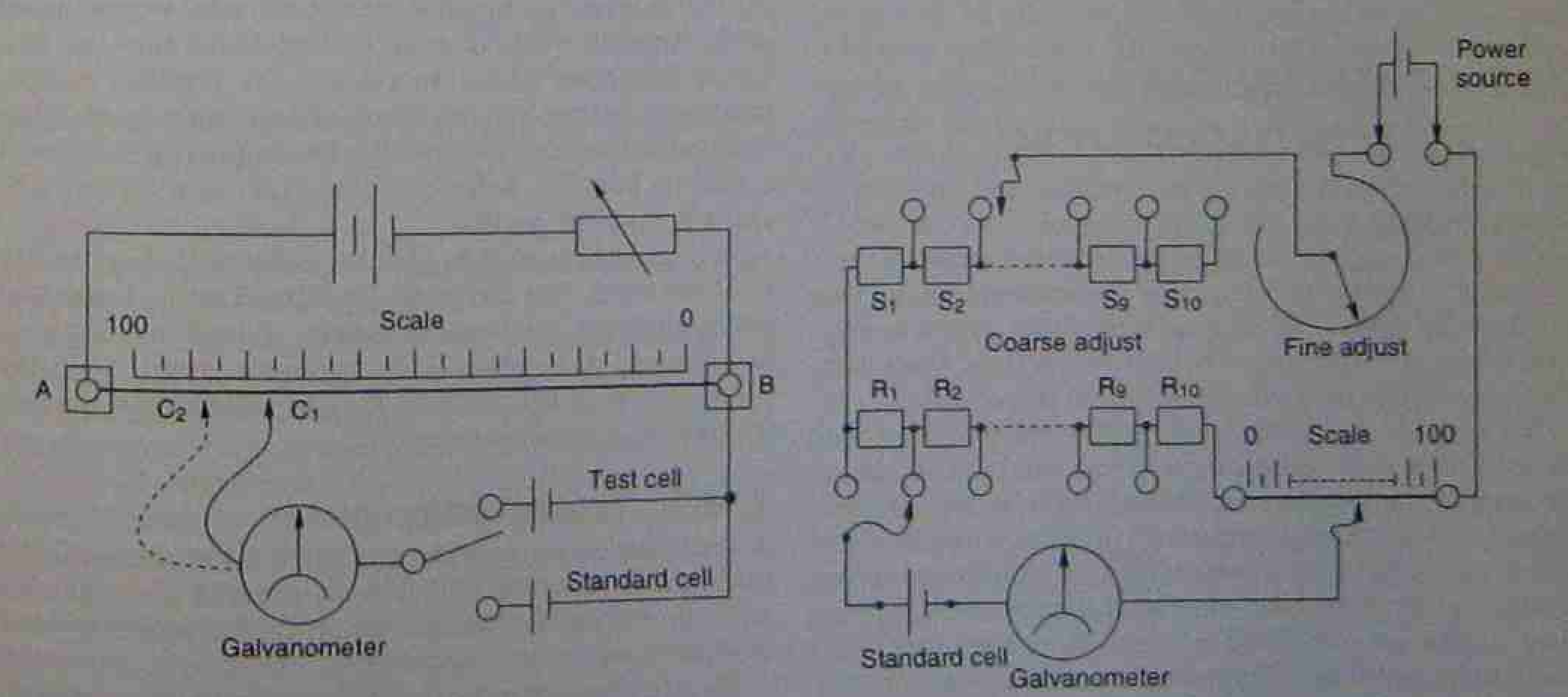
A cell of known voltage is connected in series with a galvanometer between the sliding contact and point B. This cell is usually one called a *Weston standard cell* (see Fig. 7.28). Its no-load voltage is 1.0183 V at 20°C. It is a reliable cell with a constant voltage and is relatively temperature tolerant. The voltage error is usually about 1 in 40 000.

A point C_1 can be found on the length of resistance wire where the galvanometer will register zero voltage. This is noted against the attached scale as a distance. Effectively it is another way of stating that the length of resistance wire corresponding to BC_1 bears a direct relationship between the voltage gradient along the wire and the voltage of the Weston cell.

The cell to be tested then replaces the Weston cell. (In Figure 7.27(a) a changeover switch has been provided.) A balance is again found by adjusting the sliding contact (at point C_2). By calculation, these lengths can now be transformed into the required voltages by simple ratio.

Example 7.7

A potentiometer was used to test the voltage of a cell. The distance for balance with a Weston cell was found to be 77.8 cm and the length for the test cell was 58.3 cm. Find the voltage of the test cell.



(a) Principle

(b) Practical form

Figure 7.27 • Potentiometer circuits

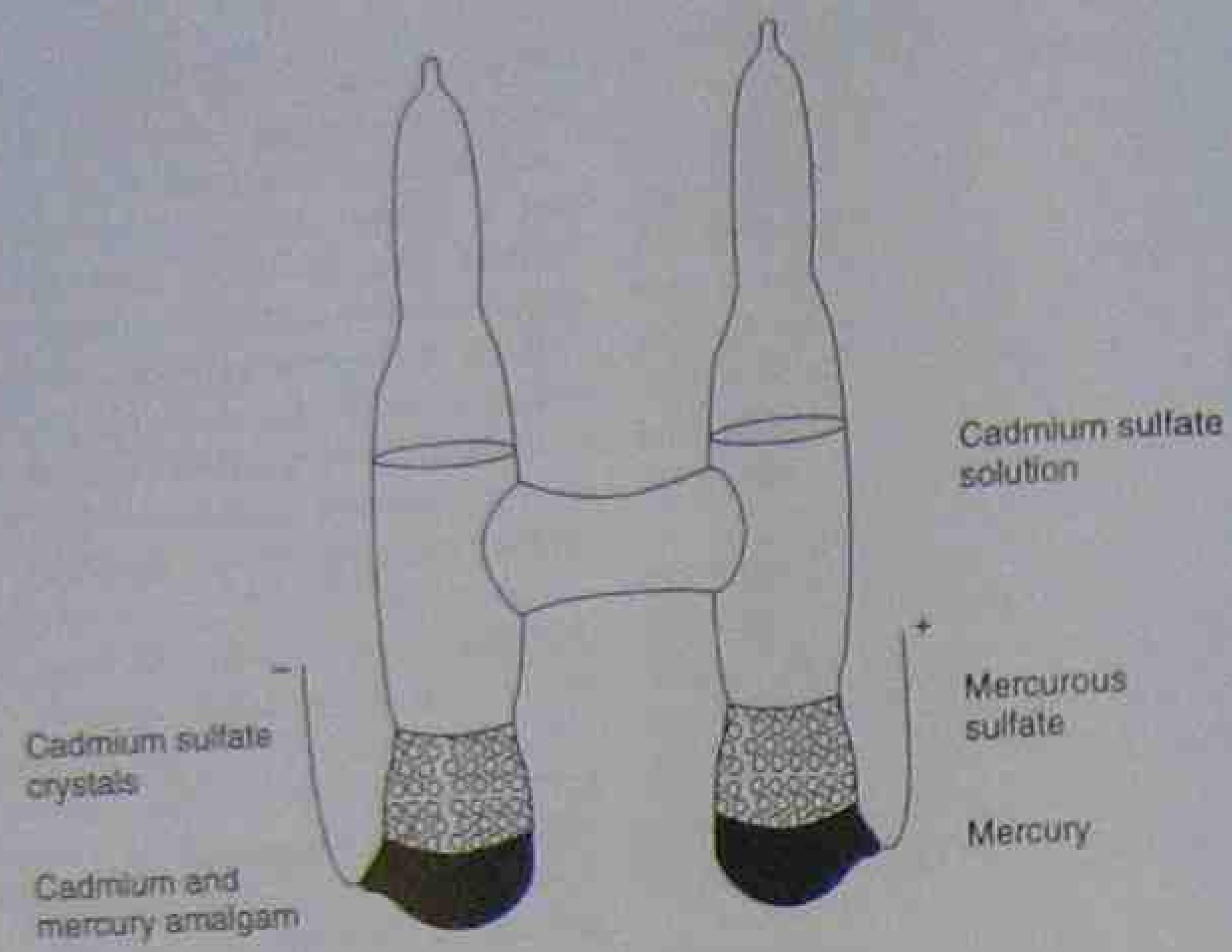


Figure 7.28 • Weston standard cell

$$\text{unknown cell voltage} = \frac{58.3}{77.8} \times 1.0183 = 0.76\ \text{V}$$

The above description illustrates the principle of the potentiometer. Modifications have been introduced to simplify its use and to improve its accuracy. First the length of the slide wire section was increased to something like 7 m. This made the instrument rather cumbersome to use and calibration tedious. Next the wire was split into sections, requiring only the last section to be calibrated in terms of length.

The modern version uses standard resistors, which take up far less room. The sliding scale is retained. This can be seen in Figure 7.27(b). Note also from the diagram that instead of a variable resistor for the power source, two adjustable resistors are used. One is for coarse, the other for fine adjustment. Each controls the amount of current taken from the power source.

7.12 CATHODE RAY OSCILLOSCOPE (CRO)

In general terms a cathode ray oscilloscope presents an electronic signal as a picture or graph on a screen. It shows how a voltage changes over a short period of time. If there is a need to examine any electrical signal, all that needs to be done is to change that signal into a voltage and it can be examined at length on a screen. The instrument is complicated only because it is so versatile.

The cathode ray oscilloscope (CRO) was first developed in the 1930s and rapid progress was made in its design because of the development of radar. CROs are used universally in electronic laboratories and are virtually essential in automotive repair workshops for diagnosing engine faults and for the correct tuning of modern fuel injection and engine management systems on car engines. They have extensive medical applications and special versions are made for the interpretation of outgoing signals from television stations.

In industrial servicing situations where any appreciable amount of electronic equipment is installed, it is essential to have a cathode ray oscilloscope available for maintenance purposes.

7.12.1 Cathode-ray tube (CRT)

The cathode-ray tube is central to the operation of the oscilloscope. It consists of an elongated glass tube with a glass faceplate at the viewing end and that coated internally with a phosphor compound. This coating glows when bombarded by an electron beam. When the electron beam is removed, the glowing spot will remain for a short period of time and gradually fade. This time is an indication of the retentivity or persistence of the screen, just as the human eye retains its last image for a short period.

At the other end of the tube is an electron gun designed to produce the electron stream and direct it towards the coated screen or faceplate. The *gun* section is made of a heater inside a metal tube called a cathode. Surrounding the cathode and on top of it are circular wire-wound metal grids. Between the grids and the face of the tube are four metal deflection plates to control the electron stream. External controls on the front of the instrument allow final adjustment or focus of the beam to ensure as small a spot as possible. Refer to Figure 7.29 for a typical tube construction and layout.

The filament is used to heat the cathode and cause it to emit electrons. The electrons are formed into a beam and accelerated by electrostatic forces created by applying voltages to the grids. This beam is directed at the phosphor-coated end of the tube and where it hits the face it shows as a small glowing dot.

7.12.2 The deflection plates

A single dot glowing on the end of a CRO contains no real information. Two sets of deflection plates in pairs are used to move the dot to different places on the screen by electrostatic force.

An electron stream has a negative charge, so by applying voltages to the plates the stream can be bent in any desired direction. This is done by attraction and repulsion as illustrated in Figure 7.30.

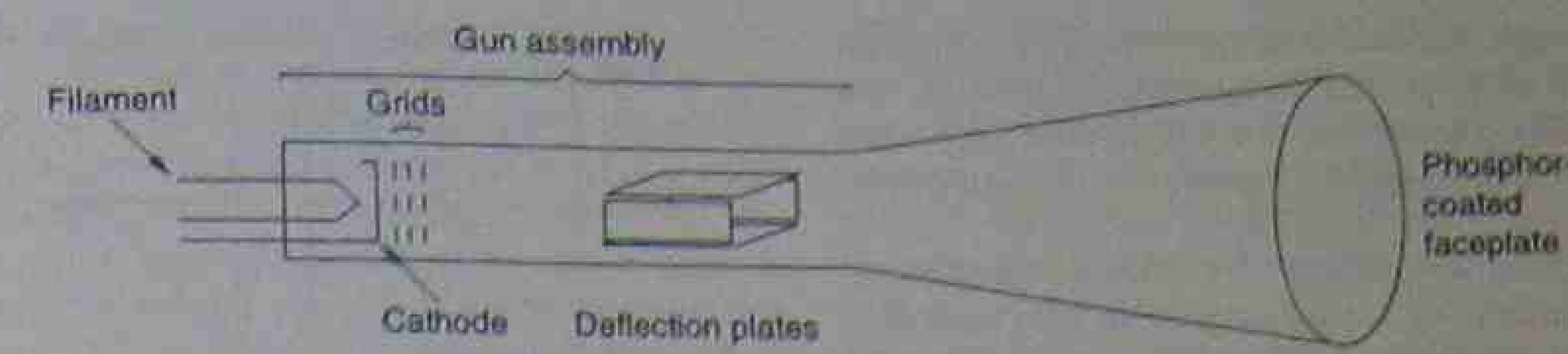


Figure 7.29 • Cathode-ray tube

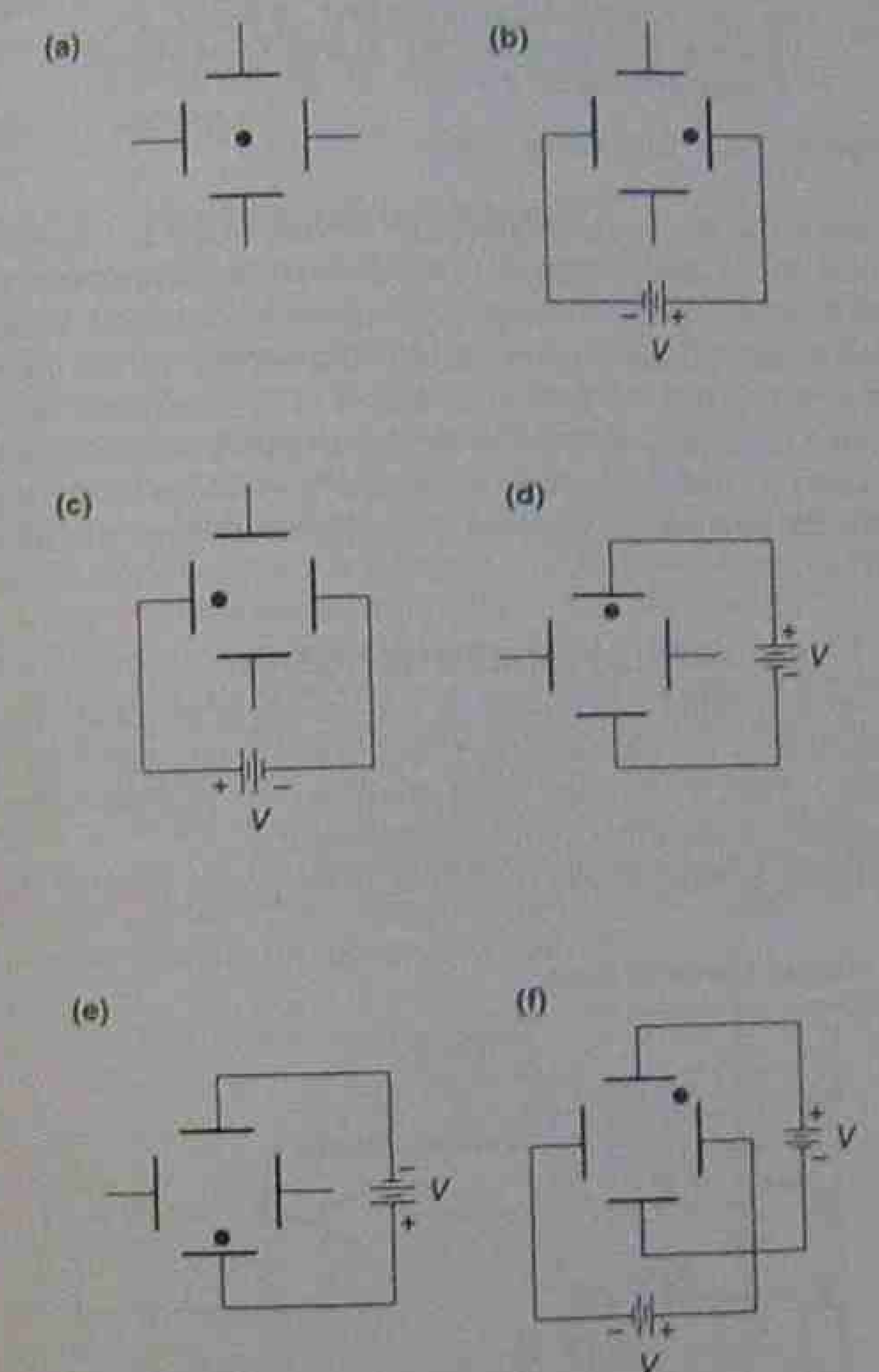


Figure 7.30 • Horizontal and vertical plates deflect the electron stream

In Figure 7.30(a) no voltage has been applied to either set of plates and the dot indicating the electron stream remains centred on the screen.

In Figure 7.30(b) a voltage has been applied to one pair of plates and the beam has been attracted horizontally to the right by the positive applied voltage. It is assisted by the electrostatic repulsion created by the left-hand plate. It is said that the beam has been swept horizontally by these plates. Accordingly they are called the horizontal deflection plates because they create horizontal movement.

In Figure 7.30(c) the polarity has been reversed and the beam has been swept in the opposite direction. The distance the spot moves depends on the value of voltage applied to the deflection plates. For example, if 100 V sweeps the spot 2 cm to the right, then 200 V will sweep the spot 4 cm to the right; that is, the deflection capability is 50 V/cm.

A similar situation exists in Figures 7.30(d) and 7.30(e). When a voltage is applied to the other set of plates the beam can be swept vertically up or down according to the applied polarity. These are called the vertical deflection plates.

By applying voltages to both sets of plates at the same time, simultaneous vertical and horizontal deflection can be achieved. This is shown in Figure 7.30(f) where a positive polarity has been applied to the top vertical plate and the right-hand horizontal plate. The beam is swept to the upper right-hand corner of the screen.

If the voltages applied to the plates are allowed to vary, the glowing spot will move around to suit the new voltages. If the voltages change fast enough, the spots on the CRT will continue to glow even though the spot has moved.

This leads to the screen appearing to show continuous lines. If the applied voltage is changed slowly, the image will be of a dot moving around on the screen. If the change is rapid, the persistence of both the screen and the human eye makes it appear as though a line is showing on the screen.

7.12.3 Block diagram of a CRO

A block diagram of a CRO is shown in Figure 7.31. It shows the essential components of all CROs.

1. Vertical amplifiers

In practice the CRO input is often attenuated to protect the instrument and to isolate the CRO from the circuit under test. In addition, the vertical scale is set in V/cm as a form of calibration. The size of the signal on the screen can be adjusted to take up as much or as little of the screen as required.

By isolating the oscilloscope from the circuit, it cannot

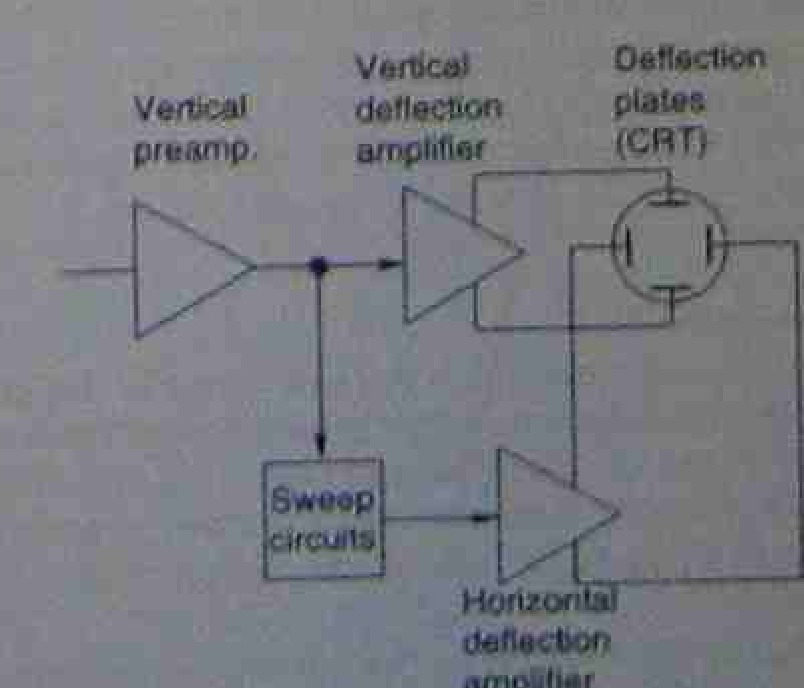


Figure 7.31 • Simplified block diagram showing the basic electronic blocks of all oscilloscopes

load the circuit and give possible false readings. It might also mean that after circuit isolation or buffering, the incoming signal might have to be amplified again. That is the reason for the preamplifier before the vertical deflection amplifier.

Vertical deflection amplifiers are voltage amplifiers. They boost the incoming voltage signal so that sufficient deflection of the beam can occur. The amplification of the signal must be equal at all frequencies over which the instrument is expected to operate.

A positive incoming signal will cause the beam to deflect upwards from the centre line of the screen and a negative signal will cause it to deflect downwards. The vertical display is always calibrated as a voltage.

If the centre line of the screen is not needed then a vertical position control is usually added to set the centre line of the display up or down as required (see Fig. 7.32, where the display centre is shown centre, up and then down).

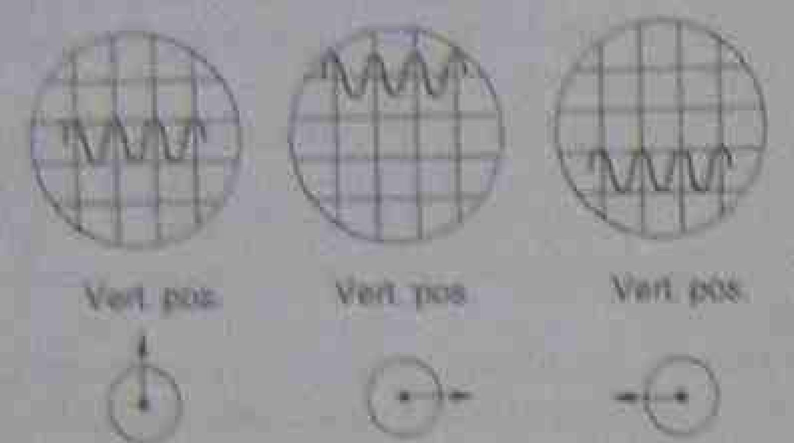


Figure 7.32 • Vertical position control

2. Horizontal amplifiers

The horizontal deflection plates are driven by the horizontal deflection amplifier. It is similar in action to the vertical amplifier in that it is used to produce sufficient voltage to ensure adequate horizontal deflection of the electron beam.

Like the vertical amplifier, it deflects the beam in accordance with the voltages imposed on the plates. The horizontal display can also be adjusted so that it can be shifted to the left or right as required.

For most of the work with an oscilloscope the horizontal deflection circuit is driven by a sweep generator and not by some applied external signal. The sweep circuit unit generates a sawtooth wave, which moves the electron beam linearly from left to right across the screen. Only in special cases is an external signal applied to the horizontal deflection plates. That method is discussed in section 7.12.9. The horizontal display is always calibrated in seconds or parts of a second, depending on the speed selected.

3. The sweep circuit

The sweep circuit generates sawtooth waves at designated frequencies. The electron beam is swept from left to right by the voltage of the sawtooth wave at the selected frequency. Since frequency is time related, the sweep is expressed in seconds per centimetre (s/cm), that is, it is a time base.

Once the beam reaches the right-hand side of the screen it is blanked off and must be returned to the left-hand side as quickly as possible. Figure 7.33 illustrates the basic sweep signal.

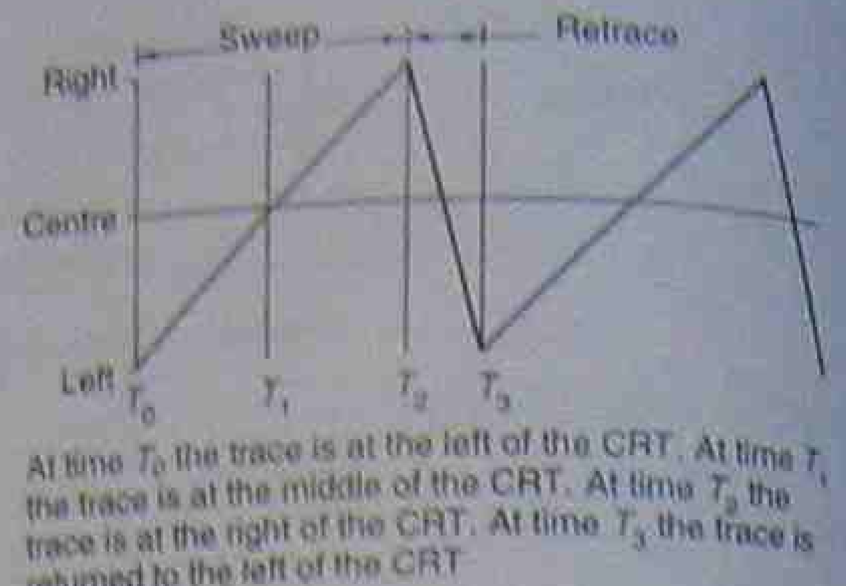


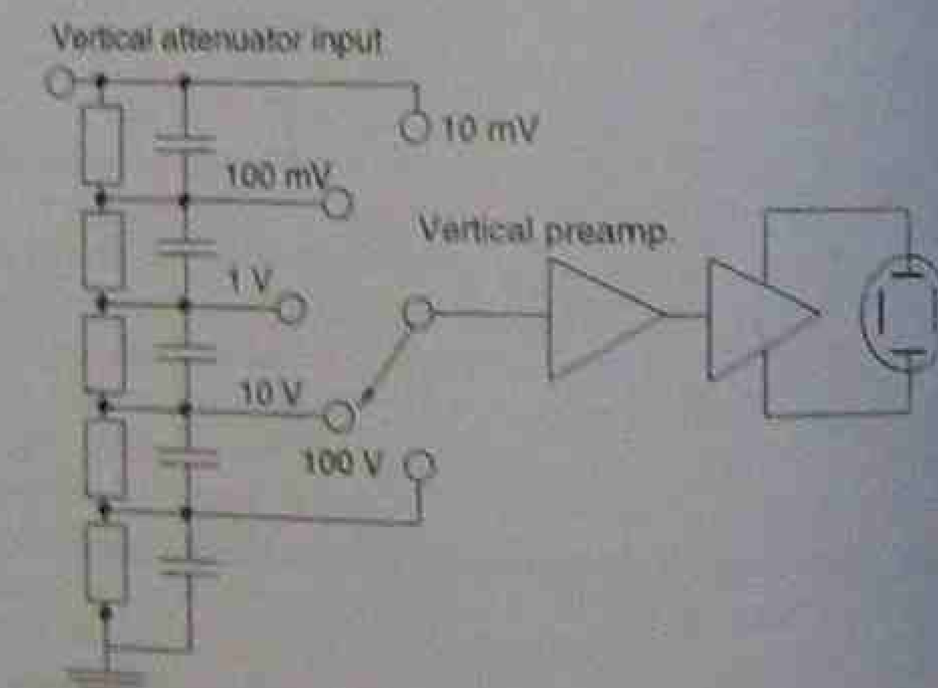
Figure 7.33 • Basic sweep signal

Initially, the voltage polarity of the wave is negative (T_0 in Fig. 7.33) to start the beam on the left-hand side of the screen. As the voltage goes more positive the beam moves at constant speed across the screen to the right until it reaches the end of its travel (T_2). The beam must then be returned quickly to its starting point. The wave is shaped so that it drops to its negative value in as short a time as possible to position T_3 and commences the next travel.

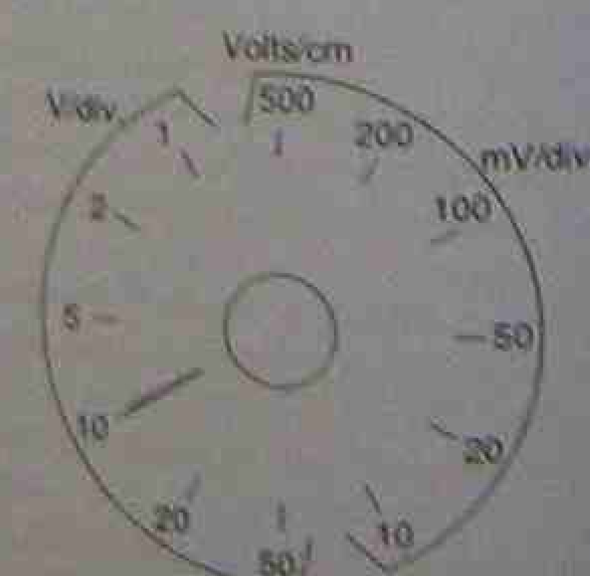
7.12.4 Vertical input attenuator

A vertical input attenuator is a network of resistors connected in a circuit such that they can be selected with a rotary switch. Figure 7.34(a) shows a representative diagram of the layout and connections.

Figure 7.34(b) shows a typical front panel view of the



(a) Schematic diagram



(b) Front-panel control

Figure 7.34 • Vertical attenuator

control switch. At the extreme left-hand position of the panel switch, each vertical centimetre on the face of the CRO represents 50 V. At the right-hand end of the switch, each centimetre represents 10 mV/cm. In both diagrams the 10 V/cm switch position has been selected.

The usual input impedance of most oscilloscopes is around 1 MΩ shunted by an input capacitance in the range 20–50 pF. These values are generally accepted as an industry standard and oscilloscope accessories are made to match these values. A value of 1 MΩ does not cause any appreciable loading effect on many circuits and the capacitors connected across the network of resistors stabilise the input.

7.12.5 Horizontal time base

To improve the performance of an oscilloscope, the horizontal time base has to be calibrated. That is, the time taken for the electron beam to travel across the screen has to be known exactly. The unit is controlled by an oscillator in the sweep circuit module (shown in Fig. 7.31) with a sawtooth waveform as shown in Figure 7.33.

The time-base control on the front panel enables the period of time for each complete cycle to be selected. The time can be shortened or lengthened as required by rotating the panel switch for the time base.

In Figure 7.35, a front-panel layout is shown for a time-base switch. In this particular instance the slowest sweep time is 500 ms/cm; that is, the beam travels from left to right at a rate of one half-second for each centimetre of travel (2 cm/s). The fastest sweep time is 0.1 μs/cm. There is also provision for an external sweep signal to be inserted into the oscilloscope circuit.

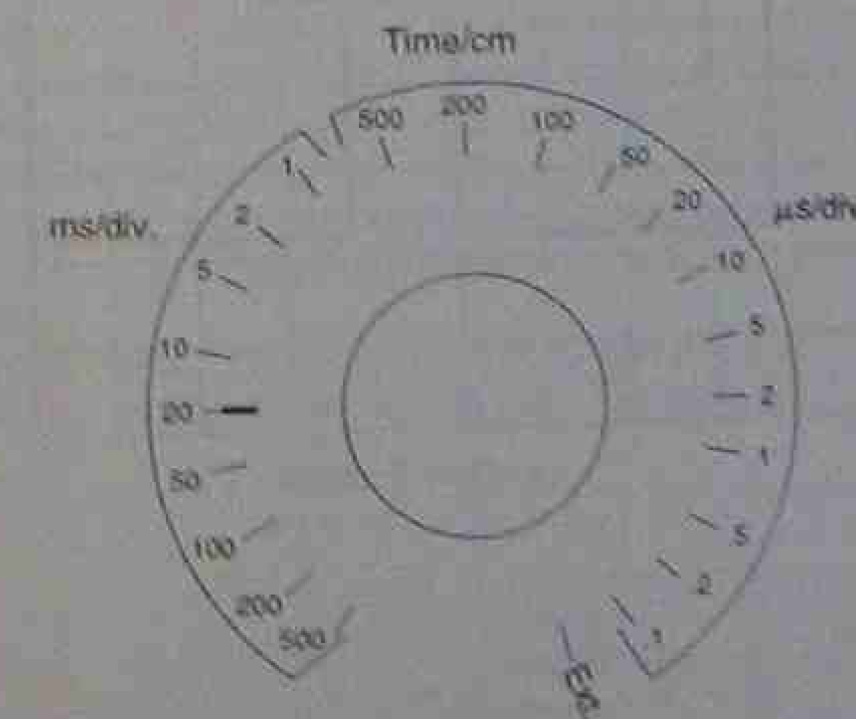


Figure 7.35 • Horizontal time-base control. There are 21 time-base settings and an external input position

7.12.6 Triggering the time base

The vertical movement of the electron beam is controlled by a voltage input to the vertical attenuator. The horizontal movement of the beam is controlled by a purpose-built oscillator.

One causes the beam to sweep horizontally across the screen; the other causes the beam to rise or fall according to the applied input voltage. The combined result of these two entirely separate circuits is a graph drawn on the face of the oscilloscope.

When a CRO is to be used to check the waveform of an alternating voltage supply, the cycles of a.c. keep repeating themselves as long as there is an input. Each time a cycle occurs it will draw a trace on the screen. Unless the sweep is controlled, it might draw the input voltage later or earlier in the cycle. The result is a meaningless jumble of lines on the screen.

To enable an oscilloscope to draw a graph that makes sense, a trigger circuit is provided. The trigger controls the instant when the horizontal sweep commences its travel across the screen.

Trigger circuits provide the means for the oscilloscope to synchronise the beam movements. They ensure that the horizontal sweep always starts at the instant the vertical input reaches a specific voltage. For example, if inspecting a sinusoidal waveform, there is only one part of a cycle when the voltage is not only going positive but is also positive in value.

A trigger circuit can be adjusted to sense this condition and send a pulse to the sweep circuit so that it commences its horizontal travel. The level of positive voltage required can also be selected, for example, going positive and +1 V. The result is that only one line is swept or repeated continuously on the screen. It is not the same cycle but is the same waveform traced over and over. The trace appears to remain stationary and can then be examined.

7.12.7 Interpreting the CRO screen

Most oscilloscope screens have a transparent cover fixed in front of the screen with a graticule inscribed on it in centimetre squares. This is the normal measurement and it is usual for the screen to be 8 cm high by 10 cm wide. If the instrument is calibrated correctly, measurements taken by means of the graticule can be translated into values.

Figure 7.36 shows two cycles of a sinusoidal waveform as they would appear on an oscilloscope screen. Using horizontal and vertical reference values, the sinusoidal waveform can be evaluated.

As an example, use the values set on the vertical attenuator from Figure 7.34(b) and the horizontal time base from Figure 7.35. This gives the oscilloscope screen values of 10 V/cm vertically and 20 ms/cm horizontally.

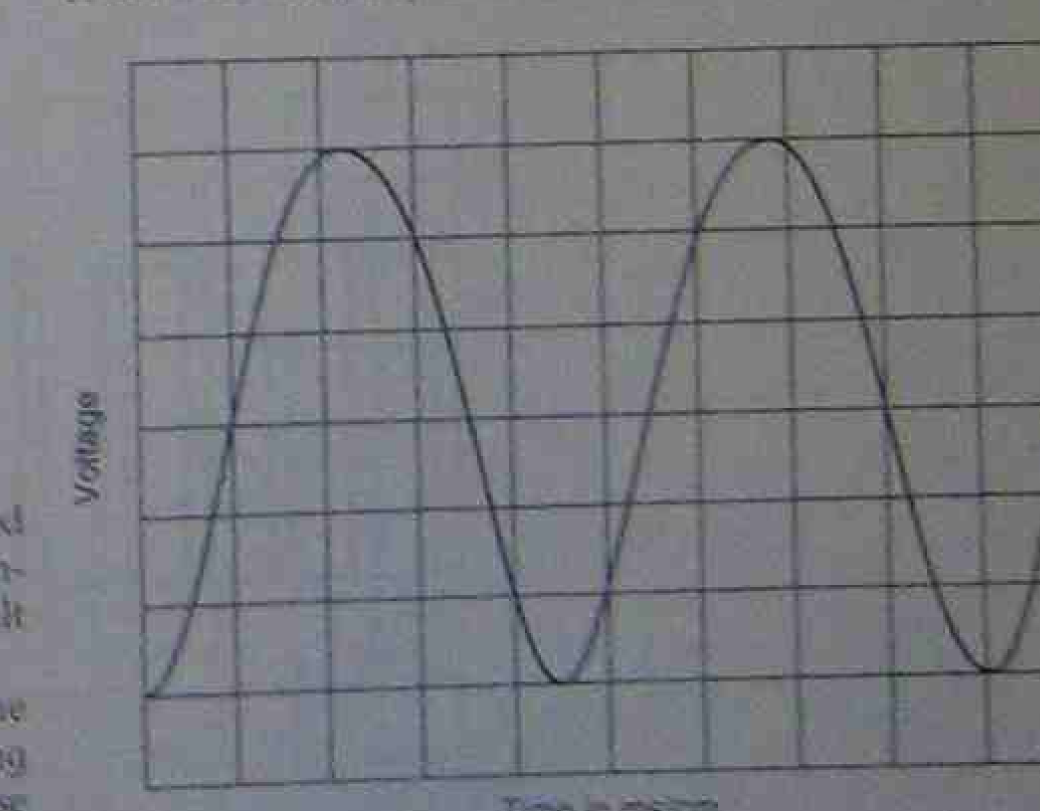


Figure 7.36 • Sinusoidal waveform on an oscilloscope screen

The peak-to-peak value of the voltage wave is 6 cm. Since each centimetre vertically represents 10 V, this can be translated as 6 × 10 = 60 V_{p-p} or 30 V_{max} from the centre line to either peak. As will be discussed in section 6.9.2, this can then be converted to an effective value of 30 × 0.707 = 21.2 V, that is, the waveform shown on the oscilloscope has a value of 21 V.

Two cycles of alternating current take up a horizontal distance of 9 cm, which is equivalent to 4.5 cm per cycle. Since the time base is set at 20 ms/cm, this represents a time of 4.5 × 20 = 90 ms; that is, one cycle takes up a period of 90 ms or 0.09 s. Taking the reciprocal of this figure gives the frequency in cycles per second. In this instance the frequency is 11.1 Hz.

Frequency is the inverse of the time in seconds. Expressed as a formula this becomes:

$$\boxed{f = \frac{1}{t}}$$

Example 7.8

Find the peak-to-peak voltage and frequency of the sinusoidal waveform in Figure 7.36 if the vertical attenuator is set on 50 V/cm and the time base is set on 5 ms/cm.

Two cycles take 9 cm horizontally. This is equivalent to 4.5 cm/cycle.

The horizontal time for 1 cycle = 4.5 × 5 = 22.5 ms

$$\text{Frequency } f = \frac{1}{0.0225} = 44.4 \text{ Hz}$$

The peak-to-peak value of the wave is 6 cm.

The value in voltage = 6 × 50 = 300 V

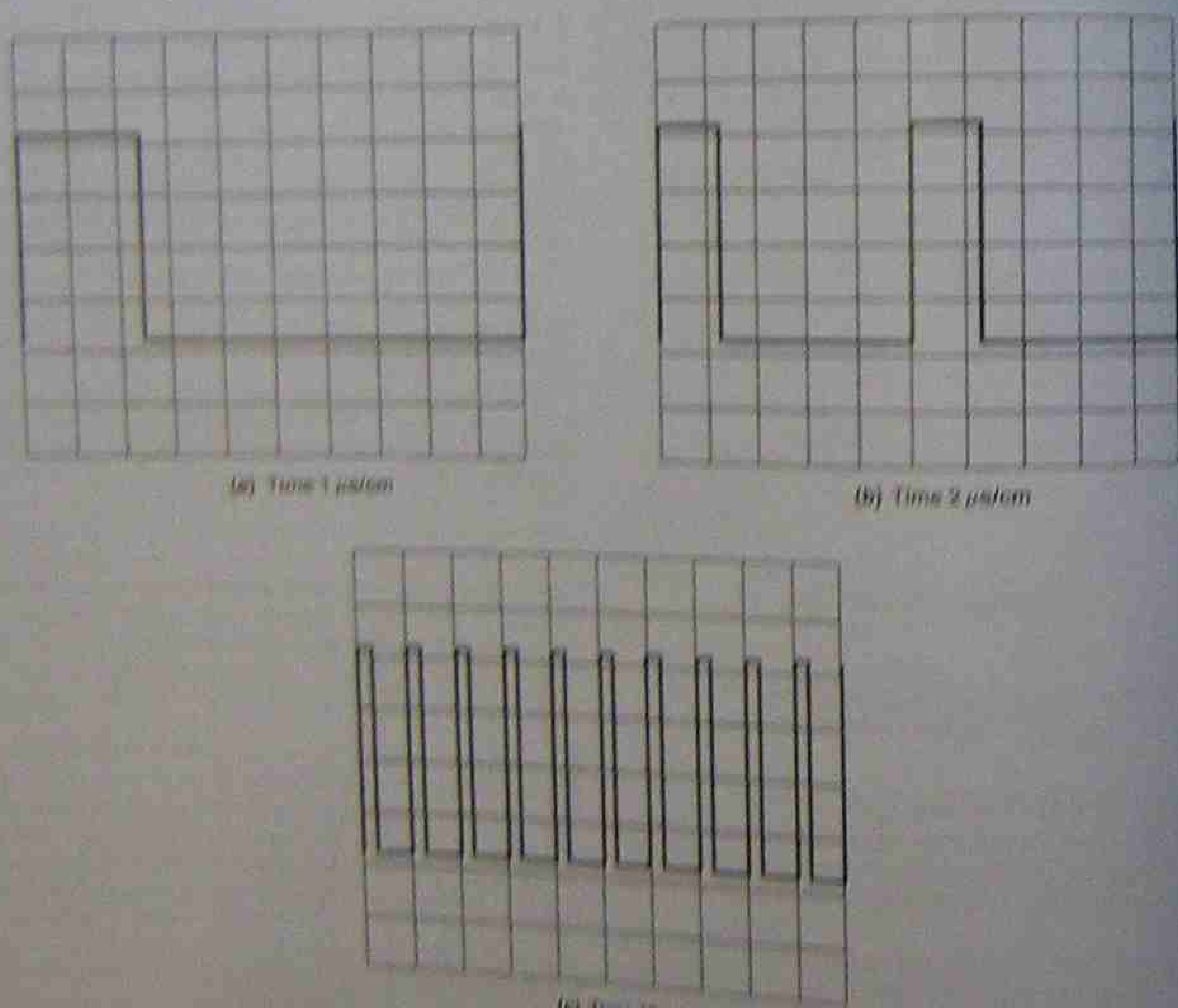
Of course all waves are not sinusoidal. Figure 7.37 shows one version of a square wave. The same wave appears in each of the diagrams but with different time bases. It can be seen that the apparent frequency differs considerably. However, when the horizontal calibrations are taken into account, all are still the same. The voltage is constant in each.

In Figure 7.37(a) the time base is set at 1 μs/cm. One complete cycle takes 10 cm or 10 μs. The voltage is high for 2.5 μs and low for 7.5 μs. This translates to a frequency of 100 kHz.

In Figure 7.37(b), given a time base of 2 μs/cm, a complete cycle takes 5 cm or 5 × 2 = 10 μs as before. Although the wave looks different it is still the same wave at the same frequency. It is still high for 2.5 μs and low for 7.5 μs.

In Figure 7.37(c) the time base has been altered to 10 μs/cm. Each cycle is completed in 1 cm and still gives a cycle time of 10 μs and a frequency of 100 kHz. However, this time it is more difficult to determine the high and low periods of each cycle.

Altering the timing of the horizontal time base from one value to another can give either an overall picture of events or enlarge a cycle for analysis purposes.


(a) Time 1 μs/cm

(b) Time 2 μs/cm

(c) Time 10 μs/cm

Figure 7.37 • Effect of altering the sweep frequency

7.12.8 Dual trace oscilloscopes

Operators sometimes need to compare waveforms. For example, the input waveform to an electronic circuit might need to be compared with the output waveform. When manipulating a.c. waveforms, they may become displaced in time from each other. If both waves can be shown on the screen at the same time, their relationship to each other can be compared.

With a normal oscilloscope, each waveform can be shown in turn but it does not allow a direct comparison between the two, nor does it show any relationship between them.

There are two methods by which this comparison can be achieved. One is by making a cathode-ray tube with two electron guns and connecting each wave shape to its own input. In television cathode-ray tubes there are three guns for the three primary colours. The other and more common way for oscilloscopes is with the aid of a high-speed electronic switch enabling portions of each wave to be displayed in turn. The circuit is shown in block form in Figure 7.38.

There are two vertical inputs but still only a single time base circuit. Samples of each input signal are shown on the oscilloscope screen in turn against the same time interval.

Owing to the persistence of vision of the human eye and the persistence of the oscilloscope screen, the trace on the screen stays there until the next piece of information is placed on the end of the previous piece. The result is that two traces appear on the screen against a common time base. A proper comparison between the traces can then be made.

Theoretically there is no limit to the number of traces that can be shown at the one time, but in practical terms four appears to be a reasonable limit for most types of oscilloscopes. Even then, the upper operational frequency limit is reduced because of the conflict between the numbers of traces, the number of samples, the speed of the electronic switch and the screen persistence.

7.12.9 Other CRO measurements

In sections 7.12.3 and 7.12.5 mention was made of applying an external signal to the horizontal time base. If a sinusoidal waveform is applied to both the horizontal and vertical deflection plates simultaneously, specific patterns can be displayed on the oscilloscope screen.

The resulting pattern depends on the relative amplitudes, frequencies, and phases of the two voltages. Stationary patterns can be seen if the ratios of these values are kept constant.

1. Phase displacements

If the same voltage and frequency are applied to both sets of plates, the resulting pattern will be a straight line inclined upwards to the right at 45°. If the pattern obtained is a circle, then the two voltages are at 90° to each other. Some of these patterns are shown in Figure 7.39 for 30° intervals.

Actual phase differences between two voltages can be found quite accurately by measuring parts of the curve against the graticule. The ratio between two of the measurements is related to the angle of phase displacement as follows:

$$\sin \theta = y/Y$$

Example 7.9

Find the phase displacement between two voltages if the display gives measurements of 11.8 mm and 13.6 mm (see Fig. 7.40).

$$\sin \theta = \frac{11.8}{13.6}$$

$$= 0.867$$

that is, $\theta = 60°$

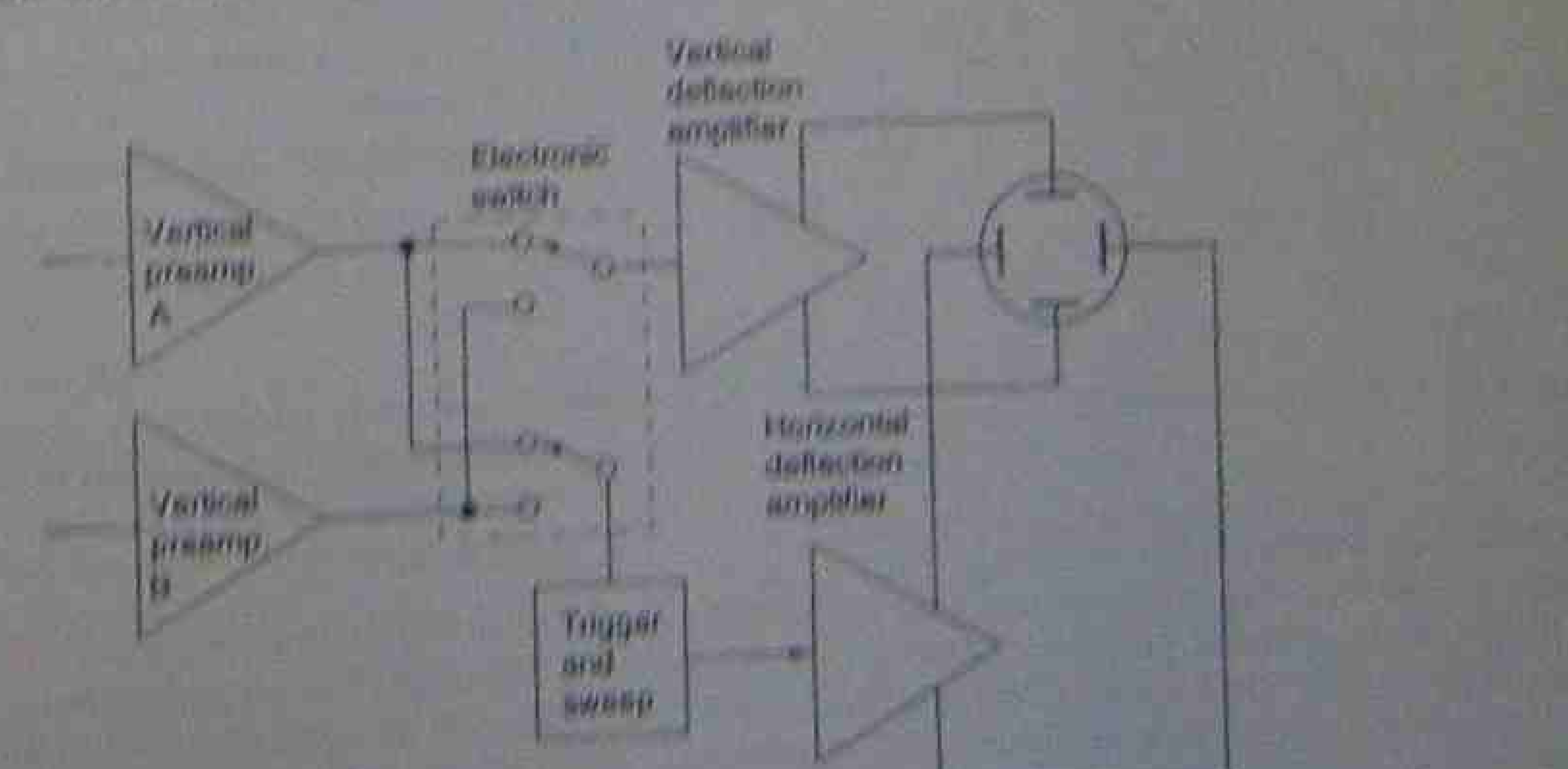
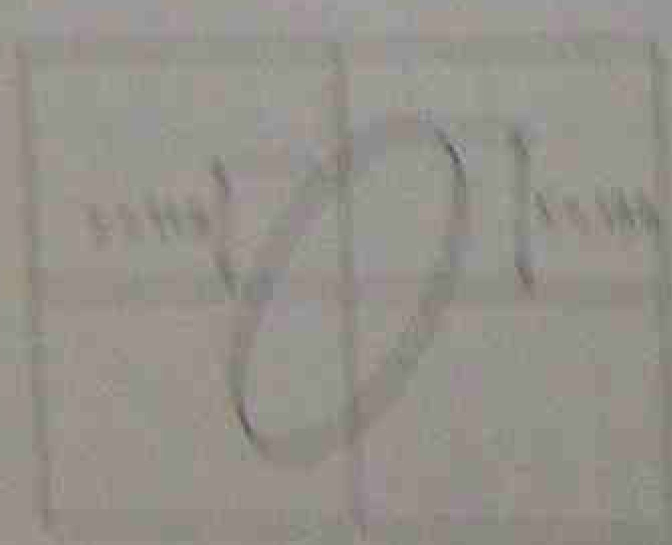


Figure 7.38 • Simplified block diagram of a dual-trace oscilloscope. Note that the outputs of the two vertical preamplifiers are connected to the vertical deflection amplifier with an electronic switch



Oscilloscope applications

CARE, SELECTION AND PROTECTION OF INSTRUMENTS

Use of instruments

Selection of instruments

Reading position

If a meter is calibrated to be read in a horizontal position, it should be read in that position for the best results. When read in any other position the readings must be treated with caution. Similarly, the same conditions apply where meters are calibrated to be mounted on magnetic or non-magnetic panels as designated.

Meter ranges

Where possible, meters should be chosen so that indicated values are read well up the scale. A 2 per cent tolerance in accuracy is proportionately a far greater value at a lower scale reading than at full scale. An additional consideration is the linearity of the scale. Lower scale values can be read reasonably well on the linear scale of a moving-coil meter, but on the moving iron or dynamometer movements, the lower part of the scale is suspect in its accuracy and because of this is often left blank.

7.13.3 Care and protection of instruments

Physically, care and protection of instruments implies careful handling, cleanliness and protection from knocks; however, in the electrical sense the problem is more involved. In general any instrument correctly and permanently installed can be expected to operate correctly and have a lengthy service life. Portable meters are subject to possible damage during transportation, and each time they are connected into a circuit the instruments face the possibility of electrical damage. The permanently installed meter is usually installed only once, while the portable meter is effectively installed each time it is connected into a circuit. Possible causes of damage are:

1. overload—either current or voltage ranges being exceeded
2. wrong connections—for example, an ammeter connected across a voltage source
3. d.c. meters connected to an a.c. power source
4. some meters need a separate power source for their operation and often one of the instrument's test terminals is permanently earthed. The application of this terminal into a circuit at a point that is not earthed can cause damage to the instrument's internal connections. The correct approach is to earth that terminal to the test circuit's earth and use only one test lead.

Multimeters are light, compact and have a variety of ranges. These advantages on their own give rise to the popularity of the multimeter as a portable test meter. Because of its portability and its number of different ranges it is susceptible to misuse and damage. Some precautions to take with multimeters are as follows:

1. Always leave a multimeter on the highest a.c. voltage range when not in use. The most common reason for damage to multimeters is the connecting of a meter into a circuit without prior inspection of the multimeter range setting. Leaving a meter on the high a.c. voltage range reduces the possibility of damage.
2. When checking an unknown voltage (or current) always start with the highest range. If the reading is too low, a quick check will soon show if a lower range is more suitable. Many operators disconnect a multimeter from a power source before changing ranges because of the possibility of arcing occurring between contacts during the changeover.
3. Never attempt to take a resistance reading in a circuit while there is power applied to the circuit. Similarly, capacitors in a circuit often hold a charge that can damage a meter. The capacitors should be short-circuited temporarily after the power source is removed, to discharge them, before using the ohmmeter.
4. A similar problem to that in point 3 exists for insulation testers, continuity testers, bridge meggers and the later model battery-operated insulation testers. Each has its own inbuilt power supply and any connection to an external power source can lead to its destruction through excessive current flow.

SUMMARY

- Test equipment ranges from simple lamp indicators to voltage testers and more complicated instruments.
- Lamp indicators warn of conditions. They can indicate that all is well, or warn that something is wrong.
- The simplest voltage testing equipment is usually a lamp or a pair of lamps in series with test leads to test for the presence of a voltage. They are fragile and need constant care.
- Buzzers and bells can also be used but are mostly used for continuity testing. They have serious limitations.
- Simple voltage testing equipment ranges from neon testers to vibrating testers. These are more rugged in construction but care should still be taken.
- Plug testers can test for a voltage, but also check for correct connections in a socket-outlet.
- Neon testers such as screwdrivers are convenient but under some conditions can indicate the presence of a voltage when none is present. The illumination is not easily seen in bright or sunny conditions.
- A logic probe is a voltage tester and indicates relative high or low conditions. It is designed for low voltage conditions only and is intended for electronic logic circuits.
- Analogue reading meters are in two general categories—moving coil and moving iron.
- Moving-coil meters will work only on d.c. and need a rectifier component to read a.c. values, often on a different scale to the d.c. scale. The meters will work on low values of current.
- Moving-coil meters have a linear scale.
- Moving-iron meters will work on either d.c. or a.c.
- In general a moving-iron meter is not as sensitive as a moving-coil meter but is also current operated.
- Moving-iron meters generally have a non-linear scale.
- Both types of meters can be adapted to read voltage or current.
- Multiplying resistors are used to extend the meter range when used as a voltmeter. They limit current flow through the meter.
- Shunts are used to bypass current around the meter so that it can indicate higher values of current.
- Non-contact testers do not make electrical contact with the circuit being tested. They are available to measure current or voltage. One type operates on the magnetic field produced by current flow, the other on the electrostatic field produced by voltage.
- The most common non-contact tester is the tong or clamp tester. Some models work on a.c. and d.c. Some work only on a.c. Some work only on d.c.
- Power meters generally use a dynamometer movement. It has connections to access the current flowing and a moving coil operated by the supply voltage.
- Small hand-held wattmeters are available and most have a digital readout.
- Bench-type wattmeters are usually operated at 240 V and are not taken into the field.
- Special wattmeters are available for high-frequency work—often thermocouple operated.
- For ultra-high-frequency work special parallel-line wattmeters are used.
- For simple continuity testing, a lamp and a battery is sufficient, although a buzzer or bell can be used. They are suitable only for low resistance circuits such as the continuity of conductors.
- Ohmmeters are generally series-type circuits. The zero end of the scale is at the right-hand end and infinity at the left-hand end. They are not accurate for very low resistances and are also limited at the upper end of the scale.
- Parallel-type ohmmeter circuits are better at lower resistance values but their accuracy is less at higher values.
- Multimeters have a series of ranges for voltage, current and resistance built into the one instrument.
- Digital multimeters are more accurate than analogue multimeters but are also more subject to outside influences such as heat and strong radio-frequency fields.
- Accurate resistance measuring circuits use variations of the Wheatstone bridge measuring circuit where a galvanometer is used to obtain a balanced reading.
- A higher voltage is used to measure higher resistance values such as insulation resistance.
- The voltage for most instruments is either 500 V or 1000 V.
- The voltage can be obtained from a built-in generator (a.c. or d.c.) or it can be obtained by an electronic oscillating circuit. The latter obtains its power from a battery supply carried within the instrument.
- A continuity tester is available as a bridge circuit with a hand-cranked generator for measuring low resistances.
- A potentiometer is a development of the bridge circuit and balances voltages against each other so that a no-load voltage value can be obtained.
- The CRO produces an electron beam that can be manipulated to show graphs or pictures on the face or screen of the instrument.
- Many CROs have two traces so that different waves can be viewed and compared at the same time.
- The CRO can be used to measure voltage, current, power, frequency, and phase displacement.
- Test instruments have to be well looked after and protected, to give reliable and accurate indications.
- The correct instruments should be used at all times and set on the correct scales and adjustments before being used on live circuits.

EXERCISES

7.1 Explain why a moving-iron meter can operate on a.c., while a moving-coil meter cannot.

7.2 A resistor of value 0.1 Ω is to be measured using the volt-ammeter method. Draw a circuit diagram showing how the instruments are to be connected. Give reasons for the positions selected for the meters in the circuit.

7.3 With the aid of sketches, describe the construction and principles of operation of a permanent-magnet moving-coil meter.

7.4 With the aid of sketches, describe the construction and principles of operation of a moving-iron repulsion-type meter movement.

7.5 Discuss the major differences between a series-type ohmmeter and a shunt-type ohmmeter. Use circuits and sketches to illustrate your answer.

7.6 State why a current transformer-type linkage-type meter can be used only on an alternating current circuit.

7.7 Draw a circuit diagram of an insulation-tester meter and explain its operation when:

(a) the leads are open-circuited
(b) the leads are short-circuited.

7.8 When used to test the continuity of a circuit or conductor, the insulation tester is connected as a shunt ohmmeter. Draw a circuit diagram of this connection.

7.9 Explain why the accuracy of a Wheatstone bridge circuit is not affected by the cell voltage.

7.10 What is the purpose of the variable resistor in a series ohmmeter circuit?

7.11 Which type of ohmmeter circuit would be best suited for measuring a resistance of 0.9 Ω? Draw the circuit and give a short explanation to justify your answer.

7.12 What are the advantages and disadvantages of using test lamps when checking for a voltage?