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Train-track-bridge dynamic interaction: a state-of-the-art review

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ABSTRACT

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Train-track-bridge dynamic interaction is a fundamental concern in the field of railway engineering, which plays an extremely important role in the optimal design of railway bridges, especially in high-speed railways and heavy-haul railways. This paper systematically presents a state-of-the-art review of train-track-bridge dynamic interaction. The evolution process of train-bridge dynamic interaction model is described briefly, from the simplest moving constant force model to the sophisticated train-track-bridge dynamic interaction model (TTBDIM). The modelling methodology of the key elements in the TTBDIM is systematically reviewed, including the train, the track, the bridge, the wheel-rail contact, the track-bridge interaction, the system excitation and the solution algorithm. The significance of detailed track modelling in the whole system is highlighted. The experimental research and filed test focusing on modelling validation, safety assessment and long-term performance investigation of the train-track-bridge system are briefly presented. The practical applications of train-track-bridge dynamic interaction theory are comprehensively discussed in terms of the system dynamic performance evaluation, the system safety assessment and train-induced environmental vibration and noise prediction. The guidance is provided on further improvement of the train-track-bridge dynamic interaction model and the challenging research topics in the future.

Q KEYWORDS: Train-track-bridge dynamic interaction train-bridge dynamic interaction dynamic model experimental investigation safety assessment engineering application

[< Previous article](#)[View issue table of contents](#)[Next article >](#)

1. Introduction

Research on train-bridge dynamic interaction (TBDI) could date back to the mid-nineteenth century, which has been a classic railway dynamics topic for quite a long time. The running trains induce severe vibrations of substructures, which may influence the normal service of track and bridge structures. In turn, the sharply dynamic behaviours of track and bridge can also affect the dynamics performance of trains. Thus, the train, the track and the bridge should be seriously considered and investigated simultaneously as a coupled system to evaluate the dynamic performance and the service life of the train-track-bridge system.

In this article

- Development of high-speed railways and heavy-haul railways, the train-track-bridge dynamic interaction (TTBDI)
- Attention of scholars worldwide in recent decades.

TBDI models were established to investigate the dynamic performance of the train-bridge system, which can be classified into the following categories by different simplifications of the dynamic system, especially the train subsystem.

The moving constant force model (MCFM) is the simplest and earliest model that can be conceived by regarding the train subsystem as constant forces, which is widely employed in researching train-induced bridge vibrations in early days. Adopting this model, Kolousek simulated the unbalanced forces of steam locomotives as single moving forces to capture the vertical vibration of continuous girder bridges in 1905 [3]. However, the computational accuracy is not good enough due to the excessive simplification of the train subsystem.

Then the moving harmonic force model (MHFM) was proposed in the early twentieth century, in which the eccentric forces of locomotives are considered as moving harmonic forces. With this model, Krylov [4] and Timoshenko [5] investigated the resonance problem of bridge structures. However, just as the above MCFM, the dynamic interaction between train and bridge is also not considered in this model, leading to the fact that the moving force model is only useful for the cases: (a) the weight of the train is much smaller than that of the bridge and (b) the dynamic behaviour of the train is not of interest.


Moreover, if the effect of vehicle cannot be ignored, the moving mass model (MMM) should be utilised instead by considering the mass and inertia of the running vehicle, which was first proposed and solved by Willis [1] and Stokes [2]. Adopting this model, Jeffcott [6], Inglis [7], etc. have done some valuable works and proposed several important suggestions.

On this basis, to better model the train loads, the moving spring-damping-mass model (MSDMM) was established by simply considering the vibration absorbing effect of the suspension system of the train, in which the suspension system is simplified to a moving mass supported by a spring-damping element, and a semi-analytical solution to this mechanical model was presented in 1964 by Biggs and Testa [8].

Further, simulating the vehicle as a 4 degrees of freedom (DOFs) multi-rigid-body system, the two-axle vehicle-bridge model (TAVBM) was proposed, which is more like the modern vehicle-bridge interaction model. In this model, the vertical and pitch motions of sprung part are considered. The dynamic responses of a two-axle vehicle or a bogie running through a bridge can be investigated with this model [9,10].

After the 1960s, as the rapid development of the Finite Element Method (FEM) and high-performance computers, the train-bridge dynamic interaction model (TBDIM) attracts scholars' attention worldwide. In this modern model, the theory of multi-body system dynamics (MBD) is adopted to simulate train subsystem, while the bridge subsystem is usually modelled based on FEM or continuous beam theories. The works conducted by Chu et al. [11], Bhatti [12], Olsson [13], Diana and Cheli [14], Green and Cebon [15], Yang and Lin [16], Xia et al. [17], etc. present plenty of meaningful conclusions, which have promoted the development and evolution of train-bridge interaction model.

With the deep research into train-bridge interaction, a limitation of the traditional TBDI model is gradually realised by scholars and engineers, that is the influence of track vibrations on the train-bridge system is not considered. As a matter of fact, trains operate on the track structures; and track structures are laid on bridge decks. Therefore, the train, the track and the bridge essentially form an integrated dynamic system, in which the train and the track are coupled by the wheel-rail interactive relationship, and the track and the bridge are linked through the track-bridge interaction. In other word, the track ties the train and the bridge together. Moreover, numerous types of track structures are adopted worldwide. When trains running on different track structures, the wheel-rail forces are different, resulting in different vibrations for different tracks, which will finally influence the vibration of bridges through the track-bridge interaction and vice versa. Therefore, incorporating a detailed track model into the TBDIM is of great necessities. Based on this concept, Zhai et al started to investigate the train-track-bridge system dynamics in 1995, which is an extension of the vehicle-track coupled dynamics [18]. In 1997, they established a train-track-bridge dynamic interaction model (TTBDIM) by including a detailed track model [19]. The evolution of the train-bridge interaction model is illustrated in Figure 1, in which, P is the moving load, M and m represent sprung mass and unsprung mass,

In this article  mass of the bogie and the carbody. It should be noted that the train model and vehicle model can be moving TTBDIM. nevertheless, the vehicle model is illustrated in this figure due to the limited space. In the

could be obtained.

Figure 1. Evolution of the train-bridge interaction model.



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With the rapid development of China's high-speed railway network in the twenty-first century, the dynamic interactions among the train-track-bridge system become severer. As a result, the former Railway Ministry of China in 2001 requested Zhai to lead a coordinated research group with participations from Southwest Jiaotong University, Beijing Jiaotong University, China Academy of Railway Sciences and Central South University to specifically study the high-speed train-track-bridge system dynamics, with the ultimate goal to provide an analytical methodology for simulating train passing bridge at high-speed operation. After a 10-year study, this goal was achieved and the TTBDIM became more and more perfect in this period, which was widely verified and applied in China's railway engineering [20–23].

Meanwhile, based on the idea of integrating the train, the track and the bridge as a coupled dynamic system, many other scholars have also conducted plenty of valuable works. Cheng et al. proposed a type of vehicle-track-bridge element to investigate the 2D interaction of the coupled system in 2001. In the established element, the vehicle is modelled as a mass-spring-damper system, while the rail and the bridge deck are all modelled as beam elements [24]. To analyse the dynamic performance of the train and the bridge, Wu and Yang built a vehicle-rail-bridge interaction model and three types of vehicle-rail interaction elements based on dynamic condensation method [25]. In 2005, a 2D TTBDIM was established by employing the substructure technology by Biondi et al., which was used to investigate the dynamic responses of multi-span simply supported girder bridges [26]. Then in 2007, a vertical TTBDIM was established by Lou, in which the train was modelled as a mass-spring-damper system while the rail and the bridge were simulated as Bernoulli-Euler beam elements [27]. To control the vibrations in the train-track-bridge system, Ziyaeifar proposed a mathematical model with the capability of representing supplementary vibrational control devices adopting tuned mass dampers, and a non-classical incremental eigen analysis was then proposed to explore the system behaviour [28]. In 2009, based on the theory of energy random analysis, Xiang and Zeng [29] established the vibration equations of the train-track-bridge system, and then investigated the mechanism of train derailment on bridge by applying the system dynamics stability concepts. In 2010, Martínez-Rodrigo developed a programme named DYNARET to study the transverse vibrations of railway bridges under resonant conditions [30]. By simulating the ballasted track structure with FEM, Rigueiro et al. presented an investigation of the dynamic response of medium span railway viaducts by considering the influence of the ballasted track [31]. To investigate the low-frequency noise, a TTBDIM was built based on the FEM by Li et al., adopting which various characteristics of the concrete bridge-borne low-frequency noise were obtained [32]. Meanwhile, by creating a 3D rail-ballast-beam finite element model, a 3D TTBDIM was established by Guo et al. [33]. With the established model, the dynamic responses of a bridge subjected to an Italian high-speed ETR500Y train were investigated. Then in 2015, assuming track irregularities to be fully coherent random excitations with the time lags between different wheels and seismic accelerations to be uniformly modulated, non-stationary random excitations with the time lags between different foundations, Zeng et al. [34] investigated the random vibrations of a high-speed train traversing on a continuous slab track bridge subjected to track irregularities and travelling seismic waves by using the pseudo-excitation method. In 2016, Yang and Hwang proposed a TTBDIM based on the direct stiffness method and the mode superposition method [35]. Zhu et al. [36] proposed a hybrid solution to solve the TTBDIM, which was compared with existing technologies. Moreover, the parameters in time-varying train-

In this article are inevitably subjected to uncertainty, leading to variability in its dynamic responses. Aiming at this issue, [37] provided an investigation on how uncertainty in the parameters influences the dynamic responses of time-

promoted the development of TTBDIM and also proposed many valuable suggestions to actual railway engineering.

Considering the vibrations of track structures, the traditional train–bridge interaction model has been extended to the train–track–bridge dynamic interaction model. As seen in Table 1, the TTBDIM can be utilised in an extremely wide range, from the basic issues to the application in the research field of train–track–bridge dynamic interaction. On this basis, this paper presents a state-of-the-art review of train–track–bridge dynamic interaction in detail. Primarily, the theoretical modelling method of train–track–bridge dynamic interaction is clearly described in Section 2. Then, the experimental investigation and actual application of train–track–bridge dynamic interaction are presented in Section 3 and Section 4 respectively. Finally, concluding remarks are summarised and some feasible future research works are proposed in Section 5.

Table 1. Research topic of TTBDIM.

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2. Theoretical modelling of train–track–bridge dynamic interaction

When a train passes a bridge, the train will induce vibration of the bridge and bring dynamic impact to the bridge structure, which can in reverse influence the running safety and the ride comfort of the train travelling on the bridge. Obviously, the train and the bridge are essentially coupled with each other.

The train and bridge structure dynamically interact by virtue of the wheel–rail interaction as well as the track–bridge interaction. On the one hand, dynamic loads of vehicles are exerted on the rail, and transmitted downward to sleepers (or track slabs) via fasteners, and eventually delivered to the bridge deck. As a result, vibrations of track and bridge are induced. On the other hand, the vibration and deformation of the bridge affect the vibration of the track structure via the track–bridge interface, which causes the changes of the wheel–rail contact geometry and wheel–rail interactive forces. Once the wheel–rail forces vary, the dynamic behaviour of the train system will be affected. From the mechanism in the above two dynamic processes, it could be concluded that the train, the track and the bridge interact alternately and deeply, as shown in Figure 2. With the increase of train speed, the dynamic interactions of the train, the track and the bridge aggravate. Severe dynamic interactions will not only affect the running safety and the ride comfort of the train passing through the bridge but also have a direct influence on the bridge vibration. Therefore, it is necessary to extend the study of the train–bridge dynamic interaction into the investigation of the train–track–bridge interaction to be able to get a better understanding of the dynamic performance of the entire system.

Figure 2. Train–track–bridge dynamic interaction model [22].



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Figure 3 illustrates the major elements and their relationships involving in the train–track–bridge dynamic interactions [22]. The entire system consists of three subsystems namely the train subsystem, the track subsystem and the bridge subsystem, which are coupled through the wheel–rail interaction and the track–bridge interaction, respectively. Under possible excitations such as track irregularities, non-round wheels and environmental wind loads, the train–track–bridge coupled system generate dynamic

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In this chapter, an overview is given of the rich literature on the theoretical modelling methods of the train–track–bridge dynamic interaction system and its major components. The literature about the theoretical methods for the key subsystems including the train, the track, the bridge, the dynamic interaction between these components, the system excitations and the numerical solution is presented.

2.1. Train modelling method in TTBDIM

In the development history of TTBDIM, several types of train model were proposed with different complexity in the railway vehicle dynamics. As shown in Figure 1, these models include the moving force model (moving constant force model and moving harmonic force model), the moving mass model and the moving vehicle system model [10,42,43].

The moving force model was formulated based on the assumption of the inertia of the moving train load to be small compared to the inertia of the bridge [3–5]. The moving force model is the simplest ‘train–bridge interaction’ model, so it has the advantages of fast calculation and easy to use. However, the moving force model is only suitable to investigate the dynamic response and strength of railway bridge because the dynamic behaviors of moving trains are not considered. The moving mass model was extended from the moving force model by considering the inertia of the bridge and the dynamic interaction between the bridge and the train [7]. Because the suspension systems of the trains are not included, the moving mass model still cannot simulate the dynamic behaviour of trains running on bridges and the train dynamic effects on bridge response. The moving vehicle system model introduces the vehicle mass and suspension system [8–11,13–17,20,21] and thus represents a TBDI formulation. There exist a number of moving vehicle system models for the study of TBDI according to the complexity in the vehicle model. The simplest model within the family of moving vehicle system models is the sprung mass model. The moving vehicle system models considering two axles of a vehicle, a whole vehicle and a series of vehicles have been reported for different concerns [8–17,20,21,42,44,45].

In general, the moving vehicle system models are established based on the theory of MBD, which implies that most of the vehicle degrees of freedom, or equations of motion, are assigned to the motions of the vehicle bodies [22,46]. A comprehensive literature review of railway vehicle MBD modelling is provided in [46–48]. Here the main focus is placed on MBD approaches for vehicle dynamics simulations in TTBDIM. With respect to the complexity of the railway vehicle model in TTBDIM, the DOFs of the vehicle model can range from several to dozens according to the issue of interest [10,22,42,49–51]. The vertical TTBDIM usually include several DOFs that related to the vertical and pitch motions of the railway vehicle system [20,52]. In the vertical TTBDIM presented by Vu-Quoc and Olsson [53], the longitudinal DOFs of the vehicle components were also included to simulate the speed variation of the train crossing the bridge. The vertical-lateral or 3D TTBDIM are widely applied with the fast development of computer technology [20,22,42,49,50]. These models fully consider the 3D coupling behaviour between railway vehicle components via bogie suspensions.



In this article  moving vehicle system model [22,51] is shown in Figure 4. The 3D vehicle model consists of seven rigid  carbody, bogies and wheelsets. The model completely considers the linear or nonlinear stiffness and

Figure 4. A three-dimensional vehicle model in TTBDIM: (a) end view of the model [22]; (b) nonlinear characteristic of the yaw damper; (c) nonlinear characteristic of the secondary lateral bump-stop [54].



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It's worth noting that such a model is capable of simulating the 3D dynamic response of railway vehicles travelling on bridges, as well as evaluating the ride comfort and running safety of the train. This model can, of course, be further extended to the 3D TTBDIM by considering the longitudinal DOFs of the vehicle components. An example has been shown by Ling et al. [41], in which the rigid bodies of the locomotive and wagon were assigned with six DOFs.

One issue should be noted for the train model in TTBDIM is the solution of the nonlinear components. There are a few important nonlinear elements in the railway vehicle dynamics system including the wheel–rail contact and suspension components. To some extent, the accuracy of rail vehicle multi-body models is mainly affected by the model of wheel–rail contact and by the models of vehicle suspension components [55,56]. This is especially true regarding the high-speed trains equipped with a lot of hydraulic dampers and freight wagons with friction elements [22,50,57,58]. However, these nonlinearities are rarely considered in the reported TBDIMs and TTBDIMs. In Ref. [22,50], several important nonlinear elements existing in the suspension systems of a high-speed railway vehicle was adopted in the 3D TTBDIM by Zhai et al. The saturation nonlinearity of the hydraulic yaw dampers was modelled as shown in Figure 4(b), while the nonlinear model of the secondary lateral bump-stop is presented in Figure 4(c). Ref. [59] presented an example of considering the nonlinear characteristics of friction elements in freight wagon dynamics. For a three-piece bogie comprising one bolster and two side frames, there are no spring and no viscous damping component in the primary suspensions. The side frames directly contact with the wheel axles in the vertical direction. There are longitudinal clearances δ_x and lateral clearance δ_y between the axle box and the side frame, see Figure 5(a). In consideration of the combination effect of the Coulomb friction and the clearances between the axle box and the side frame, the nonlinear force-displacement characteristics in these places can be described in Figure 5(b,c).

Figure 5. Modelling of the axle box connection between side frame and wheelset [60]: (a) clearances in axle box, (b) longitudinal force characteristic, and (c) Lateral force characteristic.



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while a great influence on the vehicle response and the ride comfort of trains travelling on bridges.

2.2. Track modelling method in TTBDIM

The railway track is a key structure guiding the train and distributing the load to the bridge structure or embankment. Although the ballasted track is widely used around the world, a large variety of non-ballasted track structures with different components has been developed to meet the requirements of different railway transportation systems. Therefore, rail bridges can contain either the ballasted track or the non-ballasted tracks. Figure 6 shows the schematics of a conventional ballasted track and three typical non-ballasted tracks built in China. The ballasted track system (Figure 6(a)) consists of rail, railpad and fastening, sleeper and ballast. The type 'Double-block' track (Figure 6(b)) consists of rail, fastening and railpad, concrete sleeper, concrete slab and concrete base. The rails are fastened onto sleepers in two blocks, which are directly concreted into a cast-in-place slab. There is no elasticity between the slab and the concrete base or bridge deck. Thus, only the vibration of rails is important for train-track interaction for this type of non-ballasted track. The type 'Elastic-supporting-block' track (Figure 6(c)) consists of rail, fastening and railpad, concrete block, block pad and rubber boot and concrete base. The track system elasticity existing in the fastening system and the rubber boot, which leads to that the vibrations of rails and concrete blocks are both momentous for the train-track interaction. The slab tracks (Figure 6(d)) are widely used in high-speed railways, which consists of rail, fastening and railpad, concrete slab, CAM layer and concrete base. The slab elastic bearings separate the track from the bridge deck, thus constituting a floating slab track. Both the vibrations of the rails and slabs have significant effects on the train-track interaction.



Figure 6. Typical track structures on bridges: (a) ballasted track, (b) double-block track, (c) elastic-supporting-block track, and (d) slab track.



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It is obvious that these different tracks laid on railway bridges have different structural and material properties and therefore have different dynamic behaviours. However, the elastic and dynamic effects of the track structure are not always taken into account in traditional bridge dynamic assessments [10,22,42,49–51]. It is well known that the track dynamic behaviour has a significant effect on train running safety and ride comfort [46–48,51]. It is also reported that the overall stiffness and the dynamic response of bridges under moving trains have both been shown to be sensitive to the changes in track stiffness, especially for short-span bridges [43]. Therefore, a detailed track model considering the elastic and dynamic effects of track structure is a necessary condition for the accurate analysis of the TTBDIs.

A survey of track dynamic modelling methods for train-track interaction can be found in [46–48,51,64]. The reported track system models for analysis of TTBDIs can be grouped into three categories, namely: (i) lumped parameter models; (ii) continuous beam models; and (iii) discrete beam models. The lumped parameter model is the simplest model that includes rigid mass or massless elements for rails supported on sleepers and/or ballasts connected by spring and damping elements [46–48,64]. In continuous models, the rails are usually represented by infinite Euler or Timoshenko beams resting on Winkler elastic foundations [46–48] or discrete sleepers along the track [22,42,46–48,64]. Discrete models usually include finite rails

In this article  finite element (FE) method [26,43,46–48,65–67]. In FE modelling, two classes of track models can be  spring-dashpot models (with the rails as beam elements) [31,33,43,68] and solid models (which may also

fastenings, the sleepers, the slabs and even the ballast [43,46,64,70].



The number of layers of the track models in TTBDIM can be one up to four. The single-layer track model (Figure 7) usually includes a finite/infinite rail beam laid on the bridge [24,26,65]. Such a model can be used to represent the track system with a high elastic fastening that separates the rails from the track base that being precast into the bridge deck. For the double-block non-ballasted track shown in Figure 7(b), the single-layer model is considered to represent the dynamic behaviour of the track system fairly. If the track components like sleepers and track slabs sustaining the rails that also have important effects on the track system dynamic behaviour, a two-layer model including rail and sleeper/slab masses is further needed. To simulate the dynamic effects of the elastic-supporting-block track system (Figure 7(c)) and the non-ballasted slab track system (Figure 7(d)), the two-layer track model should be justified and efficient. In the two-layer track models [31–33,68], the concrete sleepers or blocks are usually modelled as rigid bodies, while the elastic bearing layers connecting the bridge deck and these rigid bodies can be simplified as linear or nonlinear viscoelastic elements. The track slabs can be assumed as finite length beams in 2D models, while described as elastic rectangle plates supported on the viscoelastic foundation in 3D models [22,51,54]. The three-layer or four-layer track models are mainly developed for the ballasted track system, which consists of rails, sleepers, ballast and/or subgrade. In the three-layer or multi-layer track models, the ballast and subgrade layers beneath the sleepers are usually treated as uniformly distributed equivalent mass elements that connected with each other [22,43,46,64]. For example, a five-parameter model of the ballast block under each rail supporting point is adopted in [22,51,54], which is formulated based on the hypothesis that the load-transmission from a sleeper to the ballast approximately coincides with the cone distribution [71].

Figure 7. Typical track models with increasing complexity (a)–(d): (a) single-layer model, (b) two-layer model for elastic-supporting-block track, (c) two-layer model for slab track, and (d) multi-layer model for ballasted track.



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Some comparative studies on the ballasted track models with different layers have been reported in the literature [24,26,31,43,64]. The general conclusion from these studies is that the track model has little influence on the bridge response, while a larger influence on the vehicle response and the wheel–rail contact forces. Rigueiro et al. [31] compared three ballasted track models with one to three-layer on three slab bridges with span lengths in the range 11–24 m. Only small differences in

In this article  ion were found in the frequency range 0–15 Hz between these track models. But above 15 Hz, the track  act as filters, reducing the high-frequency vibrations. It should be noted that a variety of defects (such as

At last, it is believed that the bridge frequency will decrease while its vibration amplitude will increase if the nonlinear characteristics of the track components like pads and ballast are included [43,72,73]. However, almost all the current track models in the reported TTBDIMs are linear as the consideration of the track nonlinearities is still an open problem for most types of track models. Such a consideration seems to be impossible for the models in the frequency domain [46–48]. By introducing some newer time-domain models raised in train-track interaction, the TTBDIMs taking into account those nonlinearities become available, which needs further study.

2.3. Bridge modelling method in TTBDIM




For the modelling of bridge in TTBDIM, two main approaches including the analytical method and the finite element method (FEM) have been widely used. Research on the vibration of bridges by using analytical or semi-analytical approaches is abundant. In these studies, the analytical or semi-analytical models for bridges are established based on the beam theories. A beam that is simply supported at both ends is the most popular structure that has ever been adopted in the study of vehicle-bridge interaction [3–5,42,43]. Timoshenko [5] derived an enormous number of approximate solutions to the problem of simple beams under moving loads. Similar models were adopted by Vellozzi [74] in studying the vibration of suspension bridges. Zhai et al. [20] formulated a semi-analytical bridge model for studying the vertical train-track-bridge interaction. In their model, the bridges were described as simply supported Bernoulli-Euler beams, while the differential equations of the beams were solved by using the modal superposition method [51,64]. The analytical methods have the advantage of high computation efficacy and easy to use, but it is impossible to represent the complicated bridge structures accuracy using these simplified models.

With the advent of computers with high performances, various numerical methods become a highly effective tool for simulating train-bridge coupling vibration, playing an important role in this field, which are used by more and more researchers. The commonly used numerical simulation methods include the FEM, the BEM (boundary element method) and the F-B hybrid method [42,43,50,51]. Owing to its high accuracy and versatility, the FEM is extensively used to formulate the bridge models in TTBDIM. For different types of bridge structures, the spatial beam element, the spatial pole element, the plate/shell element and other special elements can be used for modelling of specific components. The simply supported bridge can be easily modelled by 3D beam elements [22,42,43,50,51], while the FE models for the huge complicated bridges usually consist of a mixture of the spatial beam element, the spatial pole element, the plate element and other special elements [22,42,43,50,51,75]. The direct integration approaches are widely adopted to solve the equations of the bridge FE models with arbitrary bridge geometry, in which the choice of time step and numerical damping parameters affect the solution [43]. The limitation of the direct integration for the complicated bridge FE models is very time-consuming due to the large number of DOFs. Therefore, some approximation assumptions such as the modal superposition method have been adopted in the FE modelling of the complicated huge bridges [50,51]. A lot of recent researches used these simplified approaches rather than the full FEM to deal with the dynamic analysis of train-track-bridge systems [50,51]. But it should be recognised that the chosen modes in these approximation assumptions provide an automatic cut-off of higher frequencies, which may have a significant effect on the acceleration response [43].

2.4. Modelling method of dynamic interaction between subsystems in TTBDIM

2.4.1. Wheel-rail dynamic interaction model

The wheel-rail dynamic interaction model is an essential element that couples the train subsystem with the track subsystem at the wheel-rail interfaces. The wheel-rail contact model addresses two basic issues: the geometric relationship and the contact forces between the wheel and the rail. When dealing with the geometric problem of wheel-rail contact system, the 'no loss of contact' or 'rigid contact' assumption that the displacement of a wheel is always equal to that of the bridge/rail beam under the wheel has been often adopted in the conventional train-bridge dynamic interaction analyses [22,25,50,51,76,77]. Such a

In this article   lects the nonlinear geometric relationships of wheel-rail contact due to the curved wheel/rail profiles, the  tics of wheel/rail materials and the no loss of contact. In fact, the track structure laid on the bridge is an

under normal conditions (Figure 8(a)), the instantaneous loss of contact between one side wheel of a wheelset and the rail would appear (Figure 8(b)), and loss of contact at both sides of the wheelset (Figure 8(c)) could also happen in some particular conditions [22,51].

Figure 8. Wheel–rail contact states: (a) constant contact, (b) instantaneous loss of contact at one side, and (c) loss of contact at two sides [24].



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To consider these important nonlinear characteristics of wheel/rail geometric contact, a nonlinear wheel–rail spatially dynamic coupling model was firstly introduced into the TTBDIM by Zhai et al. [22,51]. Recently, some similar nonlinear models allowing for the loss and subsequent re-establishment of wheel–rail contact were reported in Ref. [32,79,79]. The improved model is capable of considering three kinds of rail motions in vertical, lateral and torsional directions and dealing with the situation that the wheel loses its contact with the rail. The wheel/rail nonlinear contact relationship due to the curved and worn wheel/rail profiles is included as well.

The wheel–rail contact forces include the normal contact force and the tangent creep forces. The nonlinear Hertzian elastic contact theory is usually used to calculate the wheel–rail normal contact forces [22,51]. For lateral dynamics analyses of the train–track–bridge system, such as train running stability and safety under crosswinds or earthquakes, advanced contact theories like the Kalker linear or nonlinear creep theories should be adopted. For the calculation of wheel–rail nonlinear creep forces, the Kalker's 3D exact theory (CONTACT) is currently considered the 'golden standard' for wheel–rail contact evaluation. Some recent works show that the 3D finite element method (FEM) can also solve the 3D wheel/rail contact characteristics close to the CONTACT [80–82]. But these two exact models are only suitable for one wheel/rail rolling contact simulation due to the very expensive computation cost at the current stage. Thus, some approximate models, such as the FASTSIM, USETAB, Vermeulen–Johnson, Shen–Hedrick–Elkins, Polach methods, are commonly used for fast calculation applications of the train–track–bridge dynamic simulations [23,32,78,79,83–85].

It is noted that the linear creep theory by Kalker that widely used in TTBDIMs is only available for small creepage, where no slip occurs at the wheel–rail interface, and the creepages are in the region of adhesion [64]. But when the creepages are in the regions of creep and slip, large creepages appear in the wheel–rail contact patches and the relationship between the creep forces and creepages are nonlinear. In such a situation, the linear creep force model is not suitable. The nonlinear models such as the Shen–Hedrick–Elkins model [86] or the Kalker FASTSIM algorithm [87] should be applied to the calculation of the creep forces of the wheel and the rail under the large and small creepages.

Antolín et al. [79] compared different wheel–rail contact algorithms and demonstrated that differences arise mainly in the lateral response of the vehicle and the lateral wheel–rail forces. Due to the unsprung wheel mass, slightly higher vertical bridge deck accelerations are obtained with a rigid contact assumption than with Hertzian contact in the presence of track irregularities. This kind of comparisons should be also performed for different wheel–rail creep force models, the train–track–bridge system subjected to earthquakes and crosswinds and the high pier and curved bridges, where the lateral interaction are the main concern.

2.4.2. Track–bridge dynamic interaction model

In this article  interaction models are established to determine the track–bridge interaction forces. For the bridges  tracks, the train loads are transmitted to the bridge deck through the ballast layers. While for the non-

[22,51].

An example of a slab track–bridge dynamic interaction model is shown in Figure 9, in which the connection between the track slab and the bridge deck is treated as linear spring and damping element. As shown in Figure 9, the lateral track–bridge interaction forces F_{sHi} at the i th support section and the vertical track–bridge interaction forces F_{svij} at the j th support point can be calculated by:

$$F_{sHi}=K_{sh}(Y_{si}-y_{bh}-\phi_{bH}b)+C_{sh}(\dot{y}_{si}-\dot{y}_{bh}-\dot{\phi}_{bH}b)$$
$$F_{svij}=K_{sv}(Z_{sij}-Z_{bv}+\phi_{bX}x_{sij})+C_{sv}(\dot{z}_{sij}-\dot{z}_{bv}+\dot{\phi}_{bX}x_{sij})$$

(1)

where K_{sh} and C_{sh} represent the lateral stiffness and damping of track slab, respectively; K_{sv} and C_{sv} denote the stiffness and damping of elastic layer under the track slab including the CAM and/or the rubber pad; y_{si} is the lateral displacement of the slab; z_{sij} is the vertical displacement of the slab at the j th support point; and x_{sij} represents the lateral distance between the bridge section centroid and the j th support point of the i th support section under the slab.

Figure 9. Slab track–bridge dynamic interaction model [22].



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2.5. Description of system excitations in TTBDIM

There is a variety of excitation sources that generating the vibration of the TTBDI system. The important excitation sources are summarised in Table 2, which includes the geometry defects at the wheel–rail interface, the bridge deformations due to the pier settlement and temperature variation, and the external impact loads such as crosswind, earthquake, vessel/car collision, etc.

Table 2. Main excitation sources in TTBDI system.

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Track irregularities are the most important excitation source for the TTBDI system. The track spectrums described by power spectral density (PSD) functions of the track geometry quality are commonly used to generate the random track irregularities. Various PSD functions generally derived from field measurements are used by different railway authorities. The commonly used PSD functions in the TTBDI research field include the Federal Railroad Administration (FRA) spectrum [42,51,64], the German track spectrum [10,42,51,64], the SNCF track spectrum [10,43,88], the Chinese track spectrum [22,51,64,89] and PSD functions proposed by ISO 3095 [90,91]. For time-domain analyses, realisations of the PSD functions are obtained by performing inverse Fourier transforms with random phases. Detailed explanations on how to generate profile realizations from a PSD can be found in Zhai and Xia [51], Zhai [64], Claus and Schiehlen [92], Dinh et al. [78]. Measured track irregularities, as used in Ref. [77,102], for example, are another relevant way of including random track irregularities. Unfortunately, these data are obviously not available in the design phase.

In this article

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or defect at wheel–rail interfaces is another important excitation source for the vibration of the TTBDI

ail defects usually generate sharp peak responses to the TTBDI system. Some defects cause periodic

model the periodic defects by harmonic waves such as cosine functions represented in Ref. [51,64]. These periodic geometry defects are treated as an addition to the elastic compression at the wheel-rail contact zones. On the other hand, the localised geometry defects can be represented by cosine displacements [41,51,64] or the impact velocities [51,64] to the wheels passing through the defective rails.

The external impact sources such as crosswind, earthquake, vessel/car collision threaten the running safety of trains travelling on bridges, which is the main concern of the analysis of the TTBDI system. It is a common practice to introduce the time series of turbulent wind velocity into the TTBDIM when the dynamic responses of the train-track-bridge system subjected to crosswinds are investigated [42,50,51,94–97]. As the measured velocity profiles of natural crosswinds are always scarce, the artificial wind velocity time series is often adopted as the input [50,51]. To ensure the rationality of simulation results, the property of the artificial wind velocity should be close to that of the natural crosswinds as much as possible. There are four common approaches available to simulate a stochastic wind velocity field, including the spectral representation method, linear filtering method, the wavelet method and the artificial wind field based on observed records [50]. It should be noted that it is important not only to reproduce the spectral components of wind velocity, but also the correlation of the time histories describing wind velocity at different locations along the span and at different heights.

When the dynamic response of a train-track-bridge system subjected to an earthquake is studied, the time histories of the seismic motions around the bridge piers are essential. The seismic ground motion for the analysis of TTBDI can be obtained from strong seismic motion records [17,40,42,98,101], which are usually rare and insufficient. As a result, the application of artificial ground motions generated by numerical simulations is a good option. The seismic ground motion is a typical non-stationary process, and the motion samples can be obtained by using the random field theory based on spectral method [50,100,101]. Two numerical methods namely the unconditional and conditional simulations have been widely used to obtain the random field of seismic ground motion [50]. Besides, if a long bridge is considered, the effects of the spatial variation of the seismic ground motion should be considered.

The collision loads on bridge structure concerned in the dynamic analysis of TTBDI mainly include the impacts due to vessel collision, vehicle collision and ice-floe collision [50,102,103]. These collision loads are very complicated, with their respective characteristics in the time history, the maximum impact force and the acting position. The impact forces on bridge formed during the collision process are also influenced by the mass and velocity of the moving object, the contact area, location and angle of collision, the deformation stiffness of the colliding object itself and the collided structure, and the boundary conditions. Some of the collision loads adopted in TTBDIM are achieved by field tests [102,104], while most of the vessel, vehicle and ice-floe collision loads are obtained by FE simulation due to the high cost-effective [50,105–108].

The bridge structural deformations due to the pier settlement and temperature variation are also important excitations to the TTBDI system, which can be modelled as additional displacements to the bridge pier or deck in the simulations [38,39,50,108].

2.6. Numerical solution method

The train-track-bridge dynamic interaction system has a large number of DOFs and strong nonlinearities arising from the wheel-rail contact and suspension elements that formulating very complicated mass, damping and stiffness matrices. Therefore, it is crucial to seek a high-efficiency numerical algorithm to solve such a large-scale system. A majority of the methods used in solving the TTBDIMs are time-domain-based due to its strong capacity to solve the nonlinear problems [42,43,50,109–112]. Frequency domain methods with high computational efficiency have also been presented by Green and Cebon [15] and Lombaert and Conte [113].

To solve the equations of the TTBDIM in the time domain, a time stepping integration solution of the train, the track and the bridge subsystems is widely employed. Commonly adopted time integration schemes include the Newmark method [110], the

In this article the Wilson method [112] and the Zhai method [109]. To achieve maximum computational efficiency, many used for a specific problem, often including analytically derived equations of motion for the vehicle and

implicit integration scheme was proposed by Zhai et al [22,51] for the TTBDIM. In Ref. [22], a simple fast explicit two-step integration method namely 'Zhai method' was adopted to solve the train-track system dynamics, while the Newmark- β implicit integration method was employed for analysis of the bridge structure dynamics. It has been proved that the Zhai method is very efficient for dynamic analyses of the vehicle-track coupled system with strong nonlinearities, and the Newmark- β method is suitable for complex structure dynamics analysis. Numerical integration processes for the train-track system and for the bridge system can be harmonised through the track-bridge interaction. The practical calculation has shown the effectiveness of the hybrid explicit-implicit scheme.

The basic principle of Zhai's explicit two-step method is as follows: the displacements and velocities of a system at next time step ($t=(n+1)\Delta t$) can be estimated based on the dynamic responses at previous two steps ($t=n\Delta t, t=(n-1)\Delta t$), and the accelerations at time step ($t=(n+1)\Delta t$) are calculated from the equations of motion. The integration scheme to calculate the displacement and the velocity of a system is expressed as [109]:

$$\begin{aligned} u_{n+1} &= u_n + u'_n \Delta t + (1/2 + \psi) u''_n \Delta t^2 - \psi u''_{n-1} \Delta t^2 \\ u'_{n+1} &= u'_n + (1 + \phi) u''_n \Delta t - \phi u''_{n-1} \Delta t \end{aligned} \quad (2)$$

where u , u' and u'' are the generalised displacement, velocity and acceleration of the system, respectively; Δt is the time step; ϕ and ψ are free parameters that control the stability and numerical dissipation of the algorithm.

3. Experimental investigation of train-track-bridge dynamic interaction


Before the 1960s, the investigation on bridge vibration caused by running train was mainly conducted with experimental technology due to the poor computational devices. From 1907 to 1921, large-scale experiments were performed in America, through which the bridge resonance was found to be mainly caused by the unbalanced hammer force of a steam locomotive, and the concepts of 'impact coefficient' and 'critical speed' were first proposed [51]. Then in the following 100 years, experimental technology has become an extremely important method to investigate the dynamic responses of the train-track-bridge system. Especially in the twenty-first century, valuable suggestions were proposed through experimental investigation ascribed to the rapid development of sensor technology.

According to the target of test research, three aspects are introduced in the present review work, namely the field tests for validating simulation models, the situ tests for assessing the system operation safety and the experimental investigations for the long-term monitoring of the system performances.

3.1. Field test for validating train-track-bridge dynamic model

Field tests for validating the TTBDIM can be mainly adopted in two aspects: (a) tests for determining parameters required in the models and (b) tests for validating simulation models.

In some cases, the key dynamic parameters used in the models are unclear, and field tests should be further performed to determine these values. In addition, some parameters are different with those in the design stage due to the long-term operation, hence relevant experiments should be conducted to obtain the evolved parameters. For instance, to accurately investigate the responses of the 7-span Sesia viaduct subjected to both ambient vibration and the excitation of Italian ETR500Y high-speed trains, a situ test was carried out to obtain the modal properties of the bridge by Liu et al. [70]. To estimate the mechanical properties to be used for numerical analyses of railway bridges, Rauert et al. [114] performed tests on railway ballast within the framework of the European research project DETAILS (Design for optimal life cycle costs of high-speed railway

In this article  monitoring systems). Besides, Kim et al. [115] and Feng et al. [116] have also obtained key parameters

show the capability of accurately simulating the train–track–bridge system. In 2003, Xia and Zhang proposed a computational model for a train–bridge system, which was validated by the test results of the China-Star high-speed train running through a bridge with 24 m-span PC box girders on the Qin-Shen Special Passenger Railway in China [77]. Over the following years, Su et al. [76], Guo et al. [33], Galvín et al. [117], Chen et al. [38] and Ticona Melo et al. [118] have performed many high-quality field experiments to validate their TTBDIMs.

From 29 May to 30 June in 2008, an all-around field experiment was performed on the Beijing–Tianjin intercity railway, the first high-speed railway in China [23]. The tested bridge, Yangcun Bridge, is composed of the 32 m simply supported beams, with CRTS-II slab track. The CRH2 and the CHR3 high-speed EMUs were adopted as the tested trains in the test. The tested train and bridge are shown in Figure 10(a), and the measurement point arrangement is displayed in Figure 10(b,c). Further, the acquired test results were adopted to validate the established TTBDIM, as illustrated in Figure 10(d), which indicates that the established TTBDIM is reliable in exploring the dynamic behaviours of the train–track–bridge system.

Figure 10. Field test on the Yangcun Bridge of the Beijing–Tianjin high-speed railway: (a) test site, (b) arrangement of sensors on the track, (c) arrangement of sensors on the bridge, and (d) an example of the comparison between calculated and measured results.



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3.2. Test for assessing safety operation of the train–track–bridge system

With the rapid development of high-speed railway and heavy-haul railway, the influences of faster speed and heavier weight on the running safety and ride comfort of trains travelling on bridges deserve serious concern. Although the theoretical study can be adopted to assess the safety operation in some cases, the accuracy is doubtful due to the facts that (a) the dynamic model is simplified and (b) the parameters used in the calculation are usually different with those in the actual situation. As a consequence, some scholars prefer to evaluate such problems with experimental technology.

In 2003, to investigate the dynamic responses of the Antoin Bridge located on the high-speed railway line between Paris and Brussels subjected to Thalys trains, Xia et al. performed a test by measuring the modal parameters, deflection, acceleration, strain of girder concrete and so on, which can be a reference for the study and the design of high-speed railway bridge [119]. During the next 15 years, plenty of research conducted by Shin et al. [120], Guo et al. [121], Wang et al. [122] and so on, played an important role in assessing the running safety and ride comfort of the train–track–bridge dynamic system. The authors also have done many remarkable works in this field, one of which is illustrated for instance.

In this article, it was performed to assess the operation performance of the Beijing–Shanghai high-speed railway led by the Research Institute (TTRI), in which the dynamic behaviours of track and bridge structures were measured. The

range, indicating that the running safety of this railway can be ensured at high speed.

Figure 11. Field test on the Beijing-Shanghai high-speed railway: (a) test site and (b) an example of test results.



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It should be emphasised that a comprehensive experiment should be conducted before the operation of a new-constructed high-speed railway, which is the most important experiment to ensure of the safe operation of the railway. The experiment usually lasts for more than three months. In this test, all the subsystems attached to the new high-speed railway should be tested synchronously, including the high-speed trains, traction power system, communication system, railway subgrade/bridge/tunnel/track structures, dynamic behaviour and vibration-induced noise, passenger service system and so on. Only when all the test results meet the relevant standards, the high-speed railway is then allowed to operate for commercial use.

3.3. Experiment for investigating long-term performance of the train-track-bridge system

Another aspect of the experimental investigation is to explore the long-term performance of the train-track-bridge system, i.e. the evolution of the characteristics and parameters in the system. The longer the service time is, the more degenerate the characteristic of the railway system is, which may influence the safe operation if the performance of the railway line substantially changes. For instance, the time-dependent pier settlement and bearing damage deteriorate the mechanical properties of the train-track-bridge system, which directly influences the track-bridge interaction, and further aggravate the vibrations in the system.

Moreover, the long-term monitoring of the train-track-bridge system guides the state-based maintenance for the track and bridge structures. With the monitored data, the degradation of the structures is easy to be obtained. Then compared with the standards, the reasonable maintenance plan is convenient to be made. Hence, it is very important to monitor the evolution of the property of the train-track-bridge coupled system.

Research on this topic is very hot. Many scholars have reported a lot of method and technology for the long-term monitoring of the train-track-bridge system. From January 2004 to May 2007, Kwark et al. [123] conducted a long-term field test on a high-speed railway bridge under the actual service condition in South Korea and found that some bridge dynamic indicators exceed the standards. On this basis, a vibration control measure was carried out on the bridge to ensure the safety operation of the railway. With fibre Bragg grating sensors, Chan et al. [124] presented an experimental investigation on the Hong Kong's landmark Tsing Ma Bridge, which is a 1377 m-long suspension bridge carrying both railway and regular road traffic. In addition, Ni et al. [125], Cruz and Salgado [126], Leander et al. [127] and Karoumi et al. [128] have also made a contribution to the development of the structural health monitoring technology for the train-track-bridge coupled system. It is believed that, with the rapid development of high-speed railways, the long-term monitoring of the train-track-bridge system will continue being a very valuable research topic in the following decades, which deserves more and more attention worldwide.

train-track-bridge system under special excitations, and the investigation of environmental vibration and bridge-borne noise problem.

4.1. Dynamic performance evaluation of the track-bridge system

Performance evaluation of bridge and track by simulations in the design stage has been proved to be a significant way to find out structure design defects from the dynamic perspective, to prevent potential risks in operation and to save time or budget for the design and construction. The key advantage of train-track-bridge dynamics simulation lies in the ability to perform a comprehensive and intensive dynamic analysis of bridge and track structures as well as the running trains.

Early in the design period, the performance evaluations of basic characteristics of structures under operation speed range, such as bridge and track responses as well as bridge resonance characteristics, are essential issues. Such works will be introduced in Section 4.1.1. During long-term service of bridges, structural health problem would inevitably appear due to structure fatigue under repetitive actions of moving vehicles, and it might threaten regular operation of the railway system, therefore evaluations of fatigue life and improvement measures of bridges are concerned issues with the help of TTBDIM, as presented in Section 4.1.2 and Section 4.1.3, respectively. In addition, the uncertain factors objectively existing in the system parameters or the system excitations are usually ignored in conventional works. But in recent years, stochastic approaches for the train-track-bridge dynamic analysis with special concerns on the uncertainties have been proposed in plenty of papers, as reviewed in Section 4.1.4.

4.1.1. Evaluation of basic characteristics of the system

To ensure the operation safety of a railway line, it is an extremely important issue to investigate basic characteristics of bridge and track structures including the dynamics indices under actual operation speed as well as the resonance characteristics.

(i) Dynamic performance of bridge and track under operation condition.

Evaluation of the system dynamic performance under operation condition is an essential issue to reasonably design or maintain a bridge. For example, in China, such evaluations are required for every new-constructed bridge, especially for the important or extra-large bridges, as recorded in ‘Code for Design of High-Speed Railway (TB 10621-2014)’.

Considering actual operation cases, various types of bridges were evaluated with dynamics models in past years, including the most commonly used simply supported girder concrete bridge [20,21,23], the continuous girder bridge [129], the steel-concrete composite bridge [33,130], the curved railway bridge [131] as well as complicated bridges such as the cable-stayed bridge [51] and the steel arch bridge [23,35,132]. The general information of typical researches by means of TTBDIM is listed in Table 3.

Table 3. Dynamic performance evaluation of typical bridges.

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Among the existing studies about the dynamic responses of bridges subjected to running trains, one typical example introduced by Zhai et al. [23] is briefly introduced below, which is an application of TTBDIM simulating a CRH3 EMU passing through the Jinan Yellow River Bridge in the Beijing-Shanghai high-speed line. The bridge is a large steel truss arch bridge supported by piers made of the reinforced concrete structures (Figure 12(a)). It consists of five spans with a total length of 728 m (112 m + 3 × 168 m + 112 m), crossing the wide watercourse of Yellow River and providing stable structure foundation for the high-speed railway with an operation speed of 350 km/h. With the help of FEM, the bridge model was established mainly by beam elements and spatial plate elements (Figure 12(b)). The track was in the form of ballasted track, for which the modal superposition method was adopted to simulate the rails regarded as Bernoulli-Euler beams. The measured track irregularity



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Moreover, by employing a TTBDIM with track vibrations considered in detail, Zhang et al. [133] performed an evaluation of the continuous CRTS-II non-ballast track slab, one of the most commonly used track slabs in Chinese high-speed railways. Xia et al. [134] investigated the ladder track system, a new type of track structure adopted in urban railway transit for vibration reduction, by means of numerical train–track–viaduct dynamic model. Li et al. [135] studied the vertical dynamic performance of CRTS I slab track on long-span plate-truss cable-stayed bridges subjected to the train at different speeds and proposed a reasonable value of the stiffness of elastic vibration-damping pad, which provides a meaningful reference for the design of the track.

(ii) Resonance characteristic of the bridge


As one important natural characteristic, resonance property of the bridge is required to better guide the bridge design. Approaches to carry out resonance investigations can be categorised into three types: the analytical approach, the semi-analytical approach and the numerical approach:

The analytical approach is an efficient approach, through which the closed-form solution to the response of a bridge can be derived for interpreting the mechanism behind the key phenomena [136]. In the analytical model, a train can be simplified as moving loads with inertial forces neglected. The bridge also should be modelled as a linearly elastic beam without the consideration of damping [136–138].

The semi-analytical approach is another proficient method through which the vertical and pitching resonances of the vehicles can also be investigated [138,139].

The numerical approaches are usually required for further investigation of the resonance phenomena. It could be realised by employing sophisticated TTBDIM with excitations input such as track irregularities. Thus, comprehensive solutions at a certain operation speed could be obtained step-by-step in the time domain. Based on the numerical solutions in the cases of different running speeds, one could analyse the mechanism and the effects of the resonance not only on the bridge system but also on the train system [138,140] as well as on the track system [25,139,141].

With the developed research techniques, the mechanism of resonance occurred in the train–track–bridge system has been discussed in the past decades. As presented in Refs. [25,142], bridge resonance would not only occur in the vertical modes but

In this article  the torsional modes. In such cases, the health of the bridge structure may be threatened, while the riding comfort of the train may also be affected. Xia et al. [143] classified that there are three basic

phenomenon can also occur in the vehicle system due to the repetitive excitations induced by the stiffness variation of the bridge, which may result in the discomfort of running trains [138,139]. Moreover, as revealed by Refs. [25,139,144], the track structure also has an influence on the resonance of the system.

The resonance investigation is of great practical usages. One is to obtain the key parameters associated with the resonance effects of the system, which is helpful to develop optimal design criteria for the bridge under a certain operation speed [137]. For this purpose, the parameters such as the stiffness of the beam [142], the property of the bearings [136,142] and the characteristics of the damping ratio of the bridge were investigated [145]. Another one is to identify the resonant speed that must be avoided. For example, Guo et al. [33] introduced a resonance analysis of a high-speed railway bridge which constitutes of seven simply supported composite spans which showed that the peak value of the vertical responses of the bridge appears at the speed of 340–350 km/h and the bridge has a perfect dynamic performance under the passage of a high-speed train. Similar works were also introduced in Ref. [141,146]. Moreover, resonance investigation is also practical in other scenarios, for instance, the obtained critical speed could be adopted in fatigue analysis and stochastic analysis [65,88,141].

4.1.2. Evaluation of fatigue life of bridge

During normal operation, fatigue damage of bridge structures will occur inevitably with the increases of service time due to the repeated actions of train loads. However, conventional analysis with isolated factors considered is not adequate to assess the fatigue life of the bridge. For one thing, in the design codes, sometimes the conventional consideration of the factors for fatigue assessment is confusing due to the regardless of real operation conditions [147]. For another, the stress obtained from the conventional moving load approach is not accurate enough compared with the measured data [148]. By contrast, the systematic approach with TTBDI considered is more reasonable and practical.

The fatigue assessment of one structure requires its accurate dynamics responses, especially the local stress in a significant location. The accuracy of local dynamics response and the global computation cost are conflicts with each other which require a balance. For a regular bridge, FEM could be adopted with the significant parts using shell element and other parts using beam element to achieve high accuracy for damage analysis efficiently. Nevertheless, to model a complicated bridge like a suspension bridge with a long span for fatigue life assessment, the mode superposition approach is more reasonable than FEM [149]. As presented in Ref. [150], based on the stress range and its cycle count of each train passage were obtained by dynamic analysis, the fatigue reliability was achieved by solving a fatigue limit-state function through the S-N approach. Such work enabled the discussion of the residual service life of a bridge [151] and the investigation of fatigue behaviour in the weak part of a bridge such as the welded connection of a composite steel-concrete railway girder bridge [152].

4.1.3. Evaluation of measures for improving bridge dynamic property

The development of railway transportation requires an improvement in the serviceability of bridges. For example, the speed-up project of railway transportation in China was conducted around the year of 2000, aiming to upgrade existing railway bridges to meet the operation need at higher speed levels. In Europe, as reported [153], a large proportion of existing railway bridges have served for more than 50 years or even 100 years, which need to be improved to ensure the reliability of the current operation condition with higher speed and larger loads.

Strengthening and vibration reduction are the two main approaches to improve the dynamic property of a bridge. In both two approaches, dynamics analysis of the train-track-bridge system played an important role. In Ref. [51], applications of TTBDIM for bridge strengthening were introduced. The bridge was initially designed for the operation speed of 120 km/h. Through the strengthening, the stiffness of the bridge increased a lot and enabled the operation of the CRH train with the speed of 250 km/h (Figure 13). Xia et al. [154] also conducted a simulation to assist the design of reinforcement schemes for steel girders. Wallin et al. [153] performed a dynamic assessment of two schemes for strengthening an aged bridge.

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The vibration reduction approaches such as tuned mass damper (TMD), fluid viscous damper (FVD) and active control methods are practical to improve the performance of a bridge. But the adoption of such approaches requires an optimal design. Dynamics simulation of the train–track–bridge system considering vibration reduction components enables the optimal design for vibration control by providing the variations of dynamic responses of the system with different parameters of the components [28,155–158] or by assessing the extension of fatigue life of the system [159].

4.1.4. Evaluation of the system with uncertainties

Conventional studies usually neglected the uncertain factors existed in the system parameters and the external excitations such as the randomness of the track irregularities, the uncertainties in the vehicle suspension parameters, and the inhomogeneity of structural material and geometrical status. In recent years, scholars have attempted to take uncertain factors into consideration when performing the train–track–bridge dynamics simulation.

Monte-Carlo method (MCM) is the most fundamental approach to perform dynamics analysis with random parameters. Arvidsson et al. [49] conducted parametric investigations of the dynamic performance of an existing railway line with more than 200 bridges and trains running over these bridges under random operating conditions. Rocha et al. [88,160] carried out a safety assessment for a short-span bridge in a high-speed railway line and obtained the failure probability of the bridge through the MCM-based dynamic simulations with the uncertainties of the train–track–bridge system taken into account.

Though considered as the most stable stochastic dynamics method, the MCM-based dynamic simulation is not acceptable in computation cost sometimes, especially for the train–track–bridge system. To perform the stochastic analysis efficiently, some improvement approaches were proposed. Yu and Mao [161] adopted the probability density evolution method in the stochastic dynamic analysis, based on which the evolutions over time of the probability density functions in terms of different responses or dynamic indices could be acquired. Wan et al. [37] developed the Gaussian-process-model-based approach to study the uncertainty influence of individual parameters. The pseudo-excitation method (PEM) is also a widely used method to quantify the influence of uncertainties on the dynamic system and has been adopted in the train–track–bridge dynamics analysis in some studies [100,162,163], which has been proved to be an efficient method, just as Zeng et al. presented [162].

Another mentionable proficient stochastic model for train–track–bridge dynamic interaction system was developed by Xu and Zhai [141]. They applied the Karhunen–Loève Expansion to save the total computational quantities and achieved fast uncertainty quantification of system parameters and adopted the track irregularity probabilistic model (TIPM) [164] to generate random samples of track irregularities with the full amplitude–frequency characteristics considered. By integrating the efficient stochastic method, the system dynamics simulation, as well as the spectra evolution method, the long-term behaviours of train–track–bridge systems with the evolution of the track irregularities, could be predicted. As shown in Figure 14, the upper limit, the lower limit and the general value of the amplitudes of the accelerations were obtained.



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4.2. Safety assessment of train-track-bridge dynamic system under special excitations

The above studies just considered the regular operation situations only involving track irregularities. However, actual operation situations are more complicated, thus the regular analyses mentioned above are not adequate. Further studies have also been conducted by researchers with accounting abnormal working conditions induced by special excitations. The causes for the abnormal working conditions of the train-track-bridge dynamic system could be classified into two categories: the deformations inside the system and the transient dynamic excitations such as earthquake, wind and collision. Detailed safety assessments of the train-track-bridge system under such excitations are introduced as below.

4.2.1. Deformation-induced excitation inside the system



Deformations inside the system are various which can be resulted from different resources. They may be induced by dynamic interactions with the service age increases, as in the forms of the polygonal wear of wheel [165] and the rail corrugation [41]; may be generated by inevitable environmental actions, e.g. the deformation of bridge and track subjected to sunlight or temperature change periodically [166]; also may be evolved with time due to nature characteristic of materials or geological conditions and so on, such as the deformation of bridge structures due to the creepage or shrinkage of concrete [167] and the pier settlement appears after construction [38,39,168]. Figure 15 shows the deformations of track and bridge induced by pier settlement and concrete shrinkage and creep effect.

Figure 15. Typical deformations of track and bridge due to (a) pier settlement [168] and (b) concrete shrinkage and creep [167].



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For the typical deformations stated above, mentionable works through the TTBDIM-based simulations had been conducted, and valuable suggestions were proposed from different aspects. Ling et al. [41] investigated the effects of derail impacts resulting from track defects through a train-track-bridge dynamics simulation with detailed wheel-rail dynamic contact geometric considered and presented abnormal wheel-rail interaction process. Han et al. [165] analysed the effects of the polygonal wheel on the train-track-bridge system by adopting a 2D TTBDIM considering the wheel polygonal wear and found that the high order components of polygonal wear would results in a serious threat to the running safety of the vehicle and even activate the abnormal vertical vibration of the bridge in middle or high-frequency ranges. Chen et al. [38,39,168] used TTBDIM to study the influences induced by single pier settlements and multi-pier settlements on the dynamic behaviours of the train-track-bridge system. Settlement of bridge pier is inevitable with the evolution of infrastructures. According to the analysis, it would cause the deformation of bridge structures and further distorts track structures placed on the bridge decks, and greatly affect the service life of the tracks and the train running safety and should be controlled strictly. Chen and Han [167] analysed the influences of the shrinkage and creep effect on dynamic responses of the coupled system which showed that the dynamic responses of the train system are mainly affected, while responses of other structures are almost not influenced. Besides, as

In this article ed, the continuous rail structure can smooth the creep deformation of the bridge, indicating that the effect ould not be neglected in the system dynamic analysis.

paralyze the high-speed railway transportation. Great concern was raised about the threat of the earthquake on the operation safety of the rail transportation system.

Many scholars have attempted to take the seismic effects into account to analyse the vibration of bridges as well as the train running over it. For Example, Wang and Zhai [98] performed the dynamic responses analysis of the high-speed train–track–bridge system excited by seismic waves with the help of a three-dimensional TTBDIM. The seismic waves adopted were the normalised El Centro seismic waves for lateral and vertical dynamic inputs and act on the base nodes of the piers. The results indicated that the seismic effects on the high-speed train–track–bridge system are much more significant, especially those on lateral vibrations. Related works were performed by Yang and Wu [169], Tanabe et al. [170] and so on, and meaningful results were obtained. Stochastic methods can also be adopted to the safety analysis of the system under earthquake excitation. By combining the TTBDIM with the PEM, considering track irregularity and earthquake as random excitations, Zeng et al. [34,171] performed a stochastic analysis of the train–track–bridge system under earthquakes. Corresponding with the conclusion of Ref. [98], the study in Ref. [171] also draw the point of view that earthquake contributes more than the track irregularity to the vibration of the bridge. The adoption of PEM enabled a comparison between the influence of track irregularity and earthquake excitation in the frequency–time domain. The sensitivity comparison of the track irregularity-induced and earthquake-induced responses to the train speed could also be performed.

Moreover, the design of anti-seismic devices is another hot research topic which can also be investigated furtherly with the help of train–track–bridge dynamic interaction theory. As presented in Ref. [40,172,173], several anti-seismic measures were designed and compared, and the optimal measure for the T-beam bridges in Sichuan-Tibet railway was suggested. According to the simulations presented in [40], tuned mass dampers (TMDs) which were properly designed to be installed on the pier-top of the bridge would significantly suppress the bridge vibration and increase the safety of the train in operation. Figure 16 displays the comparison of the effectiveness of different anti-seismic measures, in which VRR is short for vibration reduction rate, and d , a , m and s denote successively the displacement and the acceleration of the pier-top, the bending moment and the stress of the pier-bottom, respectively [40]. The paper [172] introduced another anti-seismic design, named as bridge-based designed TMD and then studied the effect of the proposed design on the dynamic responses of the train–track–bridge systems. The conclusions emphasised the necessity of the train–track–bridge dynamic simulation for structures designed to suppress the earthquake-induced vibration because the influence on the running train is not always positive.

Figure 16. Comparison of the effectiveness of different anti-seismic measures [40].



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The scaled model experiments conducted in wind tunnels were presented in Refs. [121,122,174–176]. Through wind tunnel tests, some important parameters for numerical simulation, such as the aerodynamic coefficient of a vehicle and the flutter derivatives of a bridge, could be measured. However, these papers also revealed that in wind tunnel tests, designing and making a detailed physical model of the train were rather difficult. Details of the suspension system and the wheel–rail contact were usually neglected or simplified. Moreover, measuring the responses of the train was also a complicated job [174]. Hence, numerical simulation is still vital for the safety assessment of the running train and the bridge.

FEM and the modal superposition method are the most universal methods to model one bridge. In addition to the regular concern of the model, researchers usually paid special attention to the nonlinear effects in the bridge model and the wheel–rail interaction. For example, Olmos and Astiz [97] took the nonlinear wheel–rail interaction as well as the geometric nonlinearity of the high pier into account. Wang et al. [177] considered nonlinearities of the internal force effect and the sagging effect of the suspension cable when modelling a suspension bridge. With the train–track–bridge system model prepared, the numerical simulation of the system under the wind excitations could be performed by three steps in general:

1. Acquiring the time history value of the wind loads, including the static wind loads, the buffeting loads and the self-excited loads that act on the bridge and the train [96].
2. Considering the wind force in the dynamic equations of the train and bridge systems.
3. Performing the simulation through numerical integration method.

The numerical simulation of the train–track–bridge system under the wind excitation have been applied to assess high-speed railway bridges crossing large river or valley [96,97], to investigate the influence of turbulent winds on the running safety of the train on a bridge and obtained the threshold curve of wind velocity and train speed [178,179], and to evaluate the improvement of the dynamic performance of the system with the wind barrier [121,175,180,181] or TMDs [180] installed on the bridge for achieving an optimal design of vibration control measure under the wind excitation.

4.2.4. Collision excitation

According to Ref. [104], 18% of bridge collapses resulted from collision excitations. When it occurs to bridges, the great impact could lead to dislocation of bearings and girders. If there were trains passing through at that time, high-risk in running safety would be generated. Therefore, the safety assessment of the train–track–bridge system subjected to collision loads is of great importance.

Collision loads can be categorised by the sources introduced in Section 2.5. For the bridge crossing the river, the vessel, the ice-floe and other floating objects are the most common sources of collisions. For the bridges or viaducts over the ground, the uncontrolled truck is a potential source. The above cases have all been studied in existing references [102,104,106]. However, to accurately obtain the collision load is still quite difficult. For one thing, the collisions (except the ice load, as presented in Xia [102]) usually occurred unexpectedly, which are impractical and dangerous to measure. For another, a full-scale test is very costly. Thus, the most realistic approach is to simulate the collision process through numerical method. The related research reveals the degree of the influence on the system subjected to collision loads, as well as the threshold curves for the train speed and the collision density [102,104].

4.3. Investigation of environmental vibration and bridge-borne noise problem

With the rapid development of urban rail transits and elevated bridges. The environmental vibration resulting from the TTBDI and the bridge-borne noise in the low-frequency range is drawing increasing attention by worldwide scholars.

According to related papers, simulation schemes usually consist of two parts [182–184], one is to obtain bridge vibrations through TTBDI, and the other is to illustrate the vibration propagation from the bridge to the ground through the pier-foundation-ground system which is usually established by FE method. In the work of Li et al. [182], a scheme shown in Figure 17 was verified by field tests. On that basis, the elastic bearing was studied by comparing with a rigid bearing upon the influence on environmental vibrations.

Figure 17. Calculation scheme of environmental vibration caused by train-track-bridge system [182].



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4.3.2. Bridge-borne noise

The sources of bridge-borne noise are the vibrations of bridge beams, piers and other components generated from the interaction between wheel and rail, thus TTBDIM has been widely applied to investigate the bridge-borne noise problem. As presented in plenty of works [32,185–189], vibrations of the bridge deck in the time domain should be obtained first as the source of bridge-borne noise by means of the TTBDIM. Then it comes to the study of noise radiation, for which, methods such as boundary element method (BEM) [32,185,188], two-dimensional infinite element model, power flow method [189], statistical energy analysis (SEA) [186] are adopted by different scholars.

5. Conclusions and discussions about future research

5.1. Conclusions

With the track structure and the detailed wheel-rail interaction model included in the dynamics analysis of the train-track-bridge system since 1997 [19], the train-track-bridge dynamic interaction model (TTBDIM) became increasingly powerful and widely used due to its expansibility and comprehensiveness. Especially after the framework of the train-track-bridge dynamic interaction theory being established [22,23,51], the investigation in this area has aroused increasing attention in the past years. The current development of the TTBDIM can be concluded as the following aspects:

1. The improvement of the TTBDIM is supported by the related theory of each part of the train-track-bridge system. The consideration of track structure makes the system dynamics analysis more practical and more convincing because the vibration propagation of the system can be captured in a more detailed way. Additionally, plenty of practical problems of the track on the bridge can be investigated with the track modelled elaborately in the TTBDIM. For the vehicle dynamics model, it has made much development in the past years and become versatile with fully considering all possible DOFs of
- In this article
- ting the nonlinearity or even the flexibility of key components in the model. To model a bridge, the finite
 - and the modal superposition method are widely adopted with the consideration of precision and

2. The introduction of the detailed wheel-rail contact model has made it possible to precisely illustrate the interaction between the train and the track in every simulation step and present a more reliable assessment of the running safety and ride comfort of the train. System excitations can be described with a high precision through measured data or increasingly accurate model with the development of experimental methods and related subjects like the statistics, earthquake science, wind engineering, etc. The numerical algorithm has also shown great capability for large-scale computation. The hybrid integration method combining the explicit and implicit integration methods is specially recommended for the dynamic interaction analysis of the train-track-bridge system.

3. The TTBDIM has exhibited its advantages in various engineering applications. For the bridge and track structures under design or in service, the dynamics performance evaluation through the TTBDIM has become a common practice. Fatigue life assessment of bridge is also getting more reliable with the help of better description of its actual service condition through the train-track-bridge dynamic analysis. By integrating with the stochastic theory, the TTBDIM has been able to consider system uncertainties and can give a probability analysis of the system dynamic behaviour. The safety of the train-track-bridge system under various special excitations like the structure deformation, the earthquake, the wind excitation and the collision loads, have been assessed due to the versatile of the systematic dynamics model. Moreover, TTBDIM is also a practical tool to evaluate the environmental vibration problems induced by the operation of urban rail transit.

5.2. Future works

The TTBDIM is getting more and more perfect and competent to practical applications through further improvement of modelling method and integrating with other theories. Nevertheless, the train-track-bridge dynamic interaction theory is still a promising and energetic research subject with a lot of interesting and challenging works to pursue. Future research topics are suggested as listed below from three aspects.

(1) *Elaboration of the model.* In the most existing studies using the TTBDIM, the vehicle model is considered as a multi-rigid-body system without accounting for its component flexibility, and the nonlinearities involved in the vehicle-track-bridge system are not fully considered and their effects on the system dynamic performance are not yet thoroughly revealed. However, the vehicle component flexibility, such as the wheelset elasticity that could have certain effects on the wheel-rail interaction when analysing the wheel-rail wear evolution. And the system considering component nonlinearities would exhibit significantly different dynamic performance compared with that of neglecting these nonlinear elements. Additionally, the dynamic behaviour of trains running on bridges has been shown to be influenced by changes in track stiffness. Although long-span railway bridges have been studied extensively by means of the TBIM, only a few works can be found which took detailed track structure into consideration. Meanwhile, the long-span bridges, such as cable-stayed bridge and the steel truss arch bridge that has already been adopted into high-speed railway lines in China, may have a significant influence on the track geometry. The use of non-ballasted track structures for such long-span railway bridges and how about the dynamic performance of this train-track-bridge system are still challenging tasks for railway engineers. Therefore, the model involved in the train-track-bridge system should be further elaborated with the following challenge issues:

- Consideration of vehicle system flexibility in specific cases.
- The inclusion of component nonlinearities of the train-track-bridge system.
- Modelling detailed track structures on long-span railway bridges.

(2) *Comprehensive analysis of the system.* Most of the existing researches are not adequate to be considered as comprehensive analyses of the train-track-bridge system. For the compatibility problems in the train-track-bridge system, it is a significant topic to realise the system optimum performance that is regarded as an essential need of railway engineering. Although the compatibility between train and track has been investigated extensively in the past, the bridge was not taken into account which

contact relationship considered in the existing models would be a restriction of a thorough and comprehensive study. For the environment vibration problem, the vibrations resulting from the wheel-rail interaction can also propagate to the ground via the pier-pile-soil interaction. There are some experimental studies about the environment vibration of the train-track-bridge system, but convincing theoretical studies are quite hard to be found in the literature, due to the complexity in the modelling related to the piers, piles and the soil foundations and the increasing computation efforts. Therefore, the dynamic analysis of the train-track-bridge system should be more elaborate and comprehensive by addressing the following points:

- Compatibility analysis of the train-track-bridge system to avoid the system resonance and abnormal vibrations.
- Dynamic performance analysis of the train-track-bridge system under external extreme excitations.
- Influence of introducing additional components (countermeasures for vibration reduction and control) to bridge structures on the system dynamic performance.
- Consideration of pier-pile-soil interaction in the system dynamics to investigate environmental vibrations and bridge-borne noise.

(3) *Diversification of application.* With the rapid development of modern railway transportation, the TTBDIM needs to address more engineering requirements and many new emerging open problems. For instance, a large number of railway bridges have been constructed in high-speed railways which are in an increasing need of reliability assessment. To this end, worldwide scholars have conducted the reliability assessment through a relatively simplified method. However, more detailed analysis based on the TTBDIM would be required for predicting structure reliability, especially when it concerns the track structures. Additionally, when an unexpected extreme excitation occurs, for example, a collision from a vessel to the pier, the running safety of trains on the bridge would be greatly threatened. How to respond to such an extreme condition should be seriously concerned. The simulation by TTBDIM may provide some practical guidance on the emergency measures. Moreover, to prevent potential threats due to damage or failure of system components in advance, damage identification of the system could be possible with the help of the TTBDIM and artificial intelligence. Some researchers have performed preliminary works on this interesting topic, but many technical difficulties in the accuracy and reliability of the simulation and signal processing still need to be overcome in the future. Therefore, intensive attention should be paid to the following aspects to diversify the application of the TTBDIM and to meet more demanding engineering requirements:

- Evolution of the system dynamic performance and reliability assessment.
- Identification of degradation, defects and failure of system components.
- Analysis of emergency measures or countermeasures for extreme excitations.

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