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Electric Power Systems

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Electric Power Systems

Generation and Transmission

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Distribution Systems

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...he Per-Unit System
Power Transfer and Reactive Power
Useful Network Theory
Problems

Components of a Power System

3.1 Introduction

Synchronous Machines

3.2 Characteristics

Electric Power Systems

Fourth Edition

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Preface to First Edition

In writing this book the author has been primarily concerned with the presentation of the basic essentials of power-system operation and analysis to students in the final year of first degree courses at Universities and Colleges of Technology. The emphasis is on the consideration of the system as a whole rather than on the engineering details of its constituents, and the treatment presented is aimed at practical conditions and situations rather than theoretical nicety.

In recent years the contents of many undergraduate courses in electrical engineering have become more fundamental in nature with greater emphasis on electromagnetism, network analysis, and control theory. Students with this background will be familiar with much of the work on network theory and the inductance, capacitance, and resistance of lines and cables, which has in the past occupied large parts of textbooks on power supply. In this book these matters have been largely omitted resulting in what is hoped is a concise account of the operation and analysis of electric power systems. It is the author's intention to present the power system as a system of interconnected elements which may be represented by models, either mathematically or by equivalent electrical circuits. The simplest models will be used consistent with acceptable accuracy and it is hoped that this will result in the wood being seen as well as the trees. In an introductory text such as this no apology is made for the absence of sophisticated models of plant (synchronous machines in particular) and involved mathematical treatments as these are well catered for in more advanced texts to which reference is made.

The book is divided into four main parts, as follows:

(a) Introduction, including the establishment of equivalent circuits of the components of the system, the performance of which, when interconnected, forms the main theme.

(b) Operation, the manner in which the system is operated and controlled to give secure and economic power supplies.

(c) Analysis, the calculation of voltage, power, and reactive power in the system under normal and abnormal conditions. The use of computers is emphasized when dealing with large networks.

(d) Limitations of transmittable power owing to the stability of the synchronous machine, voltage stability of loads, and the temperature rises of plant.

It is hoped that the final chapter will form a useful introduction to direct current transmission which promises to play a more and more important role in electricity supply.

The author would like to express his thanks to colleagues and friends for their helpful criticism and advice. To Mr. J. P. Perkins for reading the complete draft, to Mr. B. A. Carre on digital methods for load flow analysis, and to Mr. A. M. Parker on direct current transmission. Finally, thanks are due to past students who over several years have freely expressed their difficulties in this subject.

B. M. Weedy
Southampton, 1967

Preface to Third Edition

Since the appearance of the second edition the overall energy situation has changed considerably and this has created great interest in alternative sources of energy and energy conservation. Although this does not affect the basic theory and operation of power systems it does influence policies which have considerable impact on electric power supply. Chapter 1 has been enlarged and now includes a critical summary of new energy sources and conservation measures, and in particular their possible impact on the electricity supply industry. In addition, the influences of environmental constraints are included in the discussion of generation and transmission.

One object of the second edition was to provide a text, mainly at undergraduate level, which would cover a wide range of power-system engineering, not merely network analysis. In furthering this aim a new section on overhead line design is now combined with the previous material on underground cables.

A further major change is the bringing together of introductory network material into a new chapter called Basic Concepts. This includes a summary of three-phase theory which it is hoped will ease the transition of students into the practical world of power systems. All chapters have been revised to bring the material up to date and to improve clarity.

B. M. Weedy
Southampton, 1978

Preface to Third, Revised Edition

Overall the third edition has remained up to date since its publication. However, in certain areas significant changes have occurred and to include these a new edition has been prepared. These changes are small in number and comprise decoupled load-flows, a digital method for calculating system transients, and an introduction to state estimation security analysis.

With the new material it is hoped that the book now includes a comprehensive account of power-system engineering at the senior undergraduate level.

B. M. Weedy
Southampton, 1987

Preface to Fourth Edition

As a university teacher for 40 years, I have always admired the way that Dr. Birron Weedy's book has stood out from the numerous texts on the analysis and modelling of power systems, with its emphasis on practical systems rather than extensive theory or mathematics. Over the three previous editions and one revision, the text has been continually updated and honed to provide the essentials of electrical power systems sufficient not only for the final year of a first degree course, but also as a firm foundation for further study. As with all technology, progress produces new devices and understanding requiring revision and updating if a book is to be of continuing value to budding engineers. With power systems, there is another dimension in that changes in social climate and political thinking alter the way they are designed and operated, requiring consideration and understanding of new forms of infrastructure, pricing principles and service provision. Hence the need for an introduction to basic economics and market structures for electricity supply, which is given in a completely new Chapter 12.

In this edition, 10 years on from the last, a rewrite of Chapter 1 has brought in full consideration of CCGT plant, some new possibilities for energy storage, the latest thinking on electromagnetic fields and human health, and loss factor calculations. The major addition to system components and operation has been Flexible a.c. Transmission (FACT) devices using the latest semiconductor power switches and leading to better control of power and var flows. The use of optimization techniques has been brought into Chapter 6 with power-flow calculations but the increasing availability and use of commercial packages has meant that detailed code writing is no longer quite so important. For stability (Chapter 8), it has been necessary to consider voltage collapse as a separate phenomenon requiring further research into modelling of loads at voltages below 95 per cent or so of nominal. Increasingly, large systems require fast stability assessment through energy-like functions as explained in additions made to this chapter. Static-shunt variable compensators have been included in Chapter 9 with a revised look at h.v.d.c. transmission. Many d.c. schemes now exist around the world and are continually being added to so the description of an example scheme has been omitted. Chapter 11 now includes many new

sections with updates on switchgear, and comprehensive introductions to digital (numerical) protection principles, monitoring and control with SCADA, state estimation, and the concept of Energy Management Systems (EMS) for system operation.

Readers who have been brought up on previous editions of this work will realize that detailed design of overhead and underground systems and components has been omitted from this edition. Fortunately, adequate textbooks on these topics are available, including an excellent book by Dr. Weedy, and reference to these texts is recommended for detailed study if the principles given in Chapter 3 herein are insufficient. Many other texts (including some 'advanced' ones) are listed in a new organization of the bibliography, together with a chapter-referencing key which I hope will enable the reader to quickly determine the appropriate texts to look up. In addition, mainly for historical purposes, a list of significant or 'milestone' papers and articles is provided for the interested student.

Finally, it has been an honour to be asked to update such a well-known book and I hope that it still retains much of the practical flavour pioneered by Dr. Weedy. I am particularly indebted to my colleagues, Dr. Donald Macdonald (for much help with a rewrite of the material about electrical generators) and Dr. Alun Coonick for his prompting regarding the inclusion of new concepts. My thanks also go to the various reviewers of the previous editions for their helpful suggestions and comments which I have tried to include in this new edition. Any errors and omissions are entirely my responsibility and I look forward to receiving feedback from students and lecturers alike.

Brian Cory
Imperial College, London, 1998

Publisher's Note

Doctor B. M. Weedy sadly died in December 1997 during the production of this fourth edition.

Symbols

Bold symbols denote phasor or complex quantities

A,B,C,D = Generalized circuit constants
a–b–c = Phase rotation (alternatively R–Y–B)
a = Operator $1\angle 120^\circ$
C = Capacitance (farad)
c_p = Specific heat at constant pressure (J/gm per °C)
D = Diameter
E = e.m.f. generated
F = Cost function (units of money per hour)
f = Frequency (Hz)
G = Rating of machine
g = Thermal resistivity (°C m/w)
H = Inertia constant (seconds)
h = Heat transfer coefficient (W/m^2 per °C)
I = Current (A)
I^* = Conjugate of I
I_d = In-phase current
I_q = Quadrature current
j = $1\angle 90^\circ$ operator
K = Stiffness coefficient of a system (MW/Hz)
k = Thermal conductivity (W/(m°C))
L = Inductance (H)
ln = Natural logarithm
M = Angular momentum (J-s per rad or MJ-s per electrical degree)
N = Rotational speed (rev/min, rev/s, rad/s)
P = Propagation constant ($\alpha + j\beta$)
P = Power (W)
$\frac{dP}{d\delta}$ = Synchronizing power coefficient
p.f. = Power factor
p = Iteration number
Q = Reactive power (VAr)
q = Loss dissipated as heat (W)
R = Resistance (Ω); also thermal resistance (°C/W)
R–Y–B = Phase rotation (British practice)

S = Complex power = $P \pm jQ$
S = Siemen
s = Laplace operator
s = Slip
SCR = Short-circuit ratio
T = Absolute temperature (K)
t = Time
t = Off-nominal transformer tap ratio
Δt = Interval of time
Ω^{-1} = Siemen
U = Velocity
V = Voltage; ΔV scalar voltage difference
V = Voltage magnitude
W = Volumetric flow of coolant (m^3/s)
X' = Transient reactance of a synchronous machine
X'' = Subtransient reactance of a synchronous machine
X_d = Direct axis synchronous reactance of a synchronous machine
X_q = Quadrature axis reactance of a synchronous machine
X_s = Synchronous reactance of a synchronous machine
Y = Admittance (p.u. or Ω)
Z = Impedance (p.u. or Ω)
Z_0 = Characteristic or surge impedance (Ω)
α = Delay angle in rectifiers and inverters—d.c. transmission
α = Attenuation constant of line
α = Reflection coefficient
β = Phase-shift constant of line
β = $(180 - \alpha)$—in inverters
β = Refraction coefficient $(1 + \alpha)$
γ = Commutation angle in converters
δ = Load angle of synchronous machine or transmission angle across a system—(electrical degrees)
δ_0 = Recovery angle of semiconductor valve
ε = Permittivity
η = Viscosity (g/(cm-s))
θ = Temperature rise (°C) above reference or ambient
λ = Lagrange multiplier
ρ = Electrical resistivity (Ω-m)
ρ = Density (kg/m^3)
τ = Time constant
ϕ = Angle between voltage and current phasors (power factor angle)
ω = Angular frequency (rad/s)

Subscripts 1, 2, and 0 refer to positive, negative, and zero symmetrical components, respectively.

1

Introduction

1.1 Historical

In 1882 Edison inaugurated the first central generating station in the U.S.A. This had a load of 400 lamps, each consuming 83 W. At about the same time the Holborn Viaduct Generating Station in London was the first in Britain to cater for consumers generally, as opposed to specialized loads. This scheme comprised a 60 kW generator driven by a horizontal steam engine; the voltage of generation was 100 V direct current.

The first major alternating-current station in Great Britain was at Deptford, where power was generated by machines of 10 000h.p. and transmitted at 10 kV to consumers in London. During this period the battle between the advocates of alternating current and direct current was at its most intense and acrimonious level. During this same period, similar developments were taking place in the U.S.A. and elsewhere. Owing mainly to the invention of the transformer the advocates of alternating current prevailed and a steady development of local electricity generating stations commenced, each large town or load centre operating its own station.

The variation in rate of growth of electricity usage throughout the world and in the U.S.A. is shown in Figure 1.1. World growth continues at about 7 per cent per year, implying a doubling every 10 years. In the U.S.A. and in other industrialized countries there has been a tendency, since 1973, for the rate of increase to slow down, as shown by the trend in Figure 1.1. In the U.K., growth in consumption has been under 2 per cent per year for a number of years.

In 1926, in Britain, an Act of Parliament set up the Central Electricity Board with the object of interconnecting the best of the 500 stations then in operation with a high-voltage network known as the *grid*. In 1948 the British supply industry was nationalized and two organizations were set up: (1) the *area boards*, which are mainly concerned with distribution and consumer service; and (2) the *generating boards*, which are responsible for generation and the operation of the high-voltage transmission network or grid.

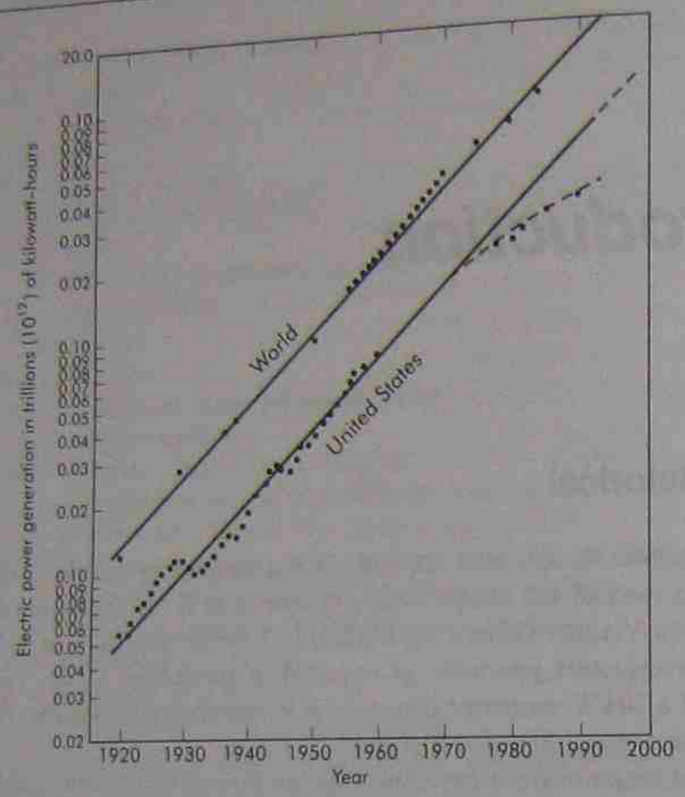


Figure 1.1 World growth rate for electricity (*Permission of Edison Electric Institute, Historic Studies of the Electric Industry*, 1975. Subsequent updating from *Power Engineering*, Oct. 1996, p. 54)

All of this changed radically in 1989/90 when the British Electricity Supply Industry (ESI) was privatized and *separate* companies were formed in England and Wales to provide competition in power generation (with the prospect of further independent generators entering the market) and to transmit electrical energy at high voltage (H.V.) (the National Grid Company, NGC, in England and Wales), and 12 Regional Electricity Companies (RECs) were formed to distribute and supply energy to consumers. In Scotland, Scottish Power and Hydroelectric companies were allowed to continue generating, transmitting, and distributing energy as 'vertically integrated' companies who could sell power to England and Wales competitively. Because transmission and distribution are recognized as *monopolies*, a Director General of Electricity Supply (the Regulator) was established to fix the profit that the NGC and RECs could earn on their monopoly businesses.

1.2 Characteristics Influencing Generation and Transmission

There are three main characteristics of electricity supply that, however obvious, have a profound effect on the manner in which it is engineered. They are as follows:

1. Electricity, unlike gas and water, cannot be stored and the supplier has small control over the load at any time. The control engineers endeavour to keep the output from the generators equal to the connected load at the specified voltage and frequency; the difficulty of this task will be apparent from a study of the daily load curves in Figure 1.2. It will be seen that the load consists basically of a steady component known as the 'base load', plus peaks that depend on the time of day, popular television programmes, and other factors. The effect of an unusual television programme is shown in Figure 1.3.
2. There is a continuous increase in the demand for power, as indicated in Figure 1.1. Although in many industrialized countries the rate of increase has declined in recent years, even the modest rate entails massive additions to the existing systems. A large and continuous process of adding to the system thus exists. Networks are evolved over the years rather than planned in a clear-cut manner and then left untouched. However, with increasing emphasis on the environment, overhead-line rights of way (wayleaves) in industrialized countries are ever more difficult to obtain, thereby forcing existing systems to be used more heavily and efficiently.
3. The distribution and nature of the *fuel* available. This aspect is of great interest as coal is mined in areas that are not necessarily the main load centres; hydroelectric power is usually remote from the large load centres. The problem of station siting and transmission distances is an involved exercise in economics. The development of gas grids enables modern power stations to be sited closer to load centres.

1.3 Energy Conversion

1.3.1 Steam

The combustion of coal, gas or oil in boilers produces steam, at high temperatures and pressures, which is passed to steam turbines. Oil has economic advantages when it can be pumped from the refinery through pipelines direct to the boilers of the generating station. Gas obtained directly from extraction platforms is becoming very useful. The energy from nuclear fission can also provide energy to produce steam for turbines. The axial-flow type of turbine is in common use with several cylinders on the same shaft.

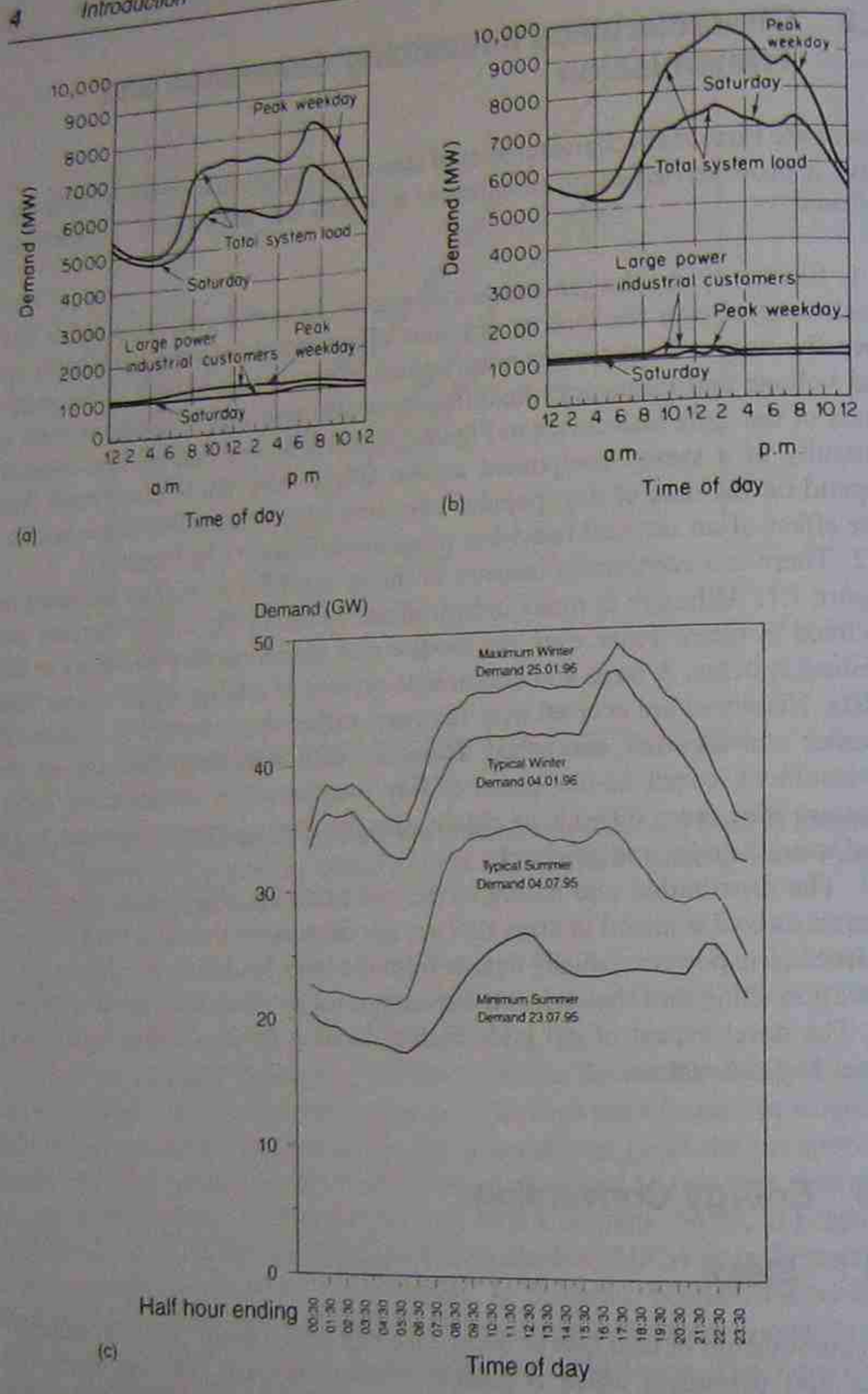


Figure 1.2 Daily load curves showing industrial component of a U.S. utility. (a) winter, (b) summer (*Copyright © 1976 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 13, No. 9 (Sept. 1976), pp. 50–53*), (c) Actual NGC summer and winter demands for 1995/96 (not weather corrected). Typical curves for England and Wales (*Courtesy of NGC*)

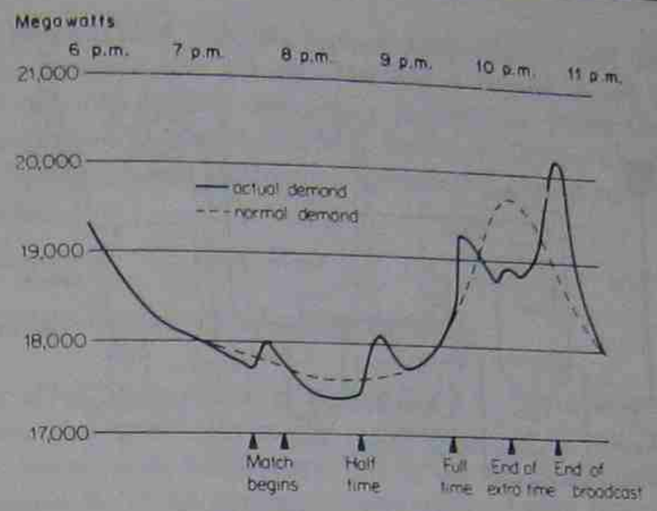


Figure 1.3 Effect of television programmes on demand—European Cup soccer match televised in Britain on 29 May 1968. Peaks caused by connection of kettles, etc., in intervals and at end (*Permission of Central Electricity Generating Board*)

The steam power-station operates on the Rankine cycle, modified to include superheating, feed-water heating, and steam reheating. Thermal efficiency results from the use of steam at the highest possible pressure and temperature. Also, for turbines to be economically constructed, the larger the size the less the capital cost. As a result, turbogenerator sets of 500 MW and more have been used. With steam turbines of 100 MW capacity and more, the efficiency is increased by reheating the steam, using an external heater, after it has been partially expanded. The reheated steam is then returned to the turbine where it is expanded through the final stages of blading. A schematic diagram of a coal-fired station is shown in Figure 1.4. In Figure 1.5 the flow of energy in a modern steam station is shown. Despite continual advances in the design of boilers and in the development of improved materials, the nature of the steam cycle is such that efficiencies are comparatively low and vast quantities of heat are lost in the condensate and atmosphere. However, the great advances in design and materials in the last few years have increased the thermal efficiencies of coal stations to about 40 per cent.

In coal-fired stations, coal is conveyed to a mill and crushed into fine powder, i.e. pulverized. The pulverized fuel is blown into the boiler where it mixes with a supply of air for combustion. The exhaust from the L.P. turbine is cooled to form condensate by the passage through the condenser of large quantities of sea- or river-water. Where this is not possible, cooling towers are used (see section on thermal pollution).

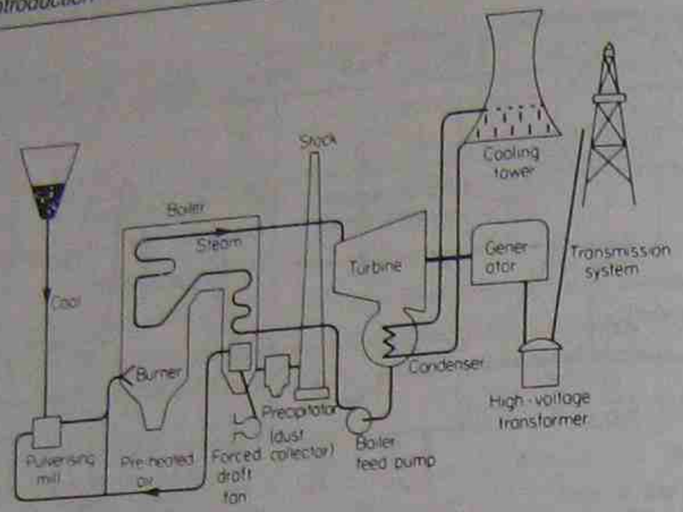


Figure 1.4 Schematic view of coal-fired generating station

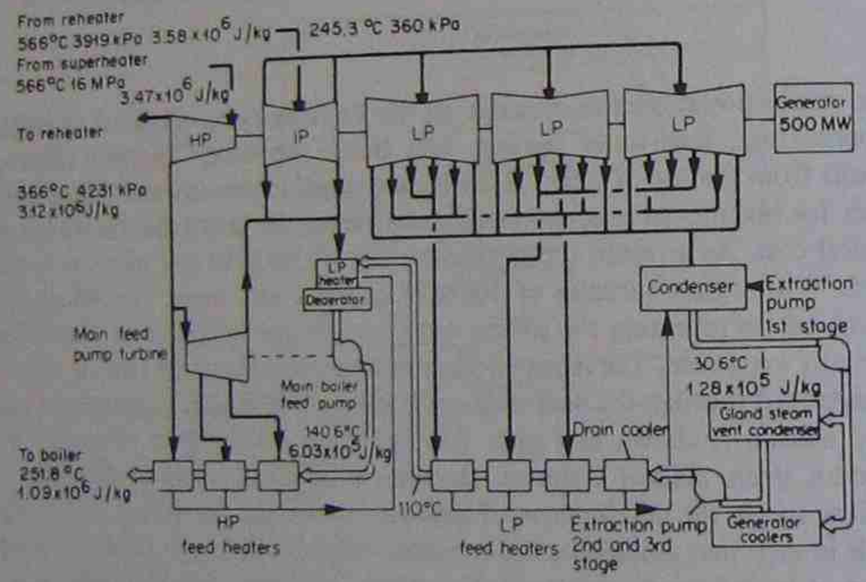


Figure 1.5 Energy flow diagram for a 500 MW turbogenerator (*Permission of Electrical Review*)

Fluidized-bed boilers

For typical coals, combustion gases contain 0.2–0.3 per cent sulphur dioxide by volume. If the gas flow rate through the granular bed of a grate-type boiler is increased, the gravity pull is balanced by the upward gas force and the fuel bed takes on the character of a fluid. In a travelling grate this increases the heat

output and temperature. The ash formed conglomerates and sinks into the grate and is carried to the ash pit. The bed is limited to the ash-sintering temperature of 1050–1200°C. Secondary combustion occurs above the bed where CO burns to CO_2 and H_2S to SO_2. This type of boiler is still undergoing extensive development and is attractive because of the lower pollutant level and better efficiency.

1.3.2 Energy conversion using water

Perhaps the oldest form of energy conversion is by the use of water power. In the hydroelectric station the energy is obtained free of cost. This attractive feature has always been somewhat offset by the very high capital cost of construction, especially of the civil engineering works. Today, however, the capital cost per kilowatt of small hydroelectric stations is becoming comparable with that of steam stations. Unfortunately, the geographical conditions necessary for hydro-generation are not commonly found, especially in Britain. In most highly developed countries, hydroelectric resources are used to the utmost. There still exists great hydroelectric potential in many underdeveloped countries and this will doubtless be utilized as their load grows, environmental considerations permitting.

An alternative to the conventional use of water energy, i.e. pumped storage, enables water to be used in situations which would not be amenable to conventional schemes. The utilization of the energy in tidal flows in channels has long been the subject of speculation. The technical and economic difficulties are very great and few locations exist where such a scheme would be feasible. An installation using tidal flow has been constructed on the La Rance Estuary in northern France, where the tidal height range is 9.2 m (30 ft) and the tidal flow is estimated at 18 000 m^3/s. Proposals for a 8000 MW tidal barrage in the Severn Estuary (U.K.) are still awaiting funding.

Before discussing the types of turbine used, a brief comment on the general modes of operation of hydroelectric stations will be given. The vertical difference between the upper reservoir and the level of the turbines is known as the head. The water falling through this head gains kinetic energy which it then imparts to the turbine blades. There are three main types of installation, as follows:

1. *High head or stored*—the storage area or reservoir normally fills in longer than 400 h;
2. *Medium head or pondage*—the storage fills in 200–400 h;
3. *Run of river*—the storage (if any) fills in less than 2 h and has 3–15 m head.

A schematic diagram for type 3 is shown in Figure 1.6.

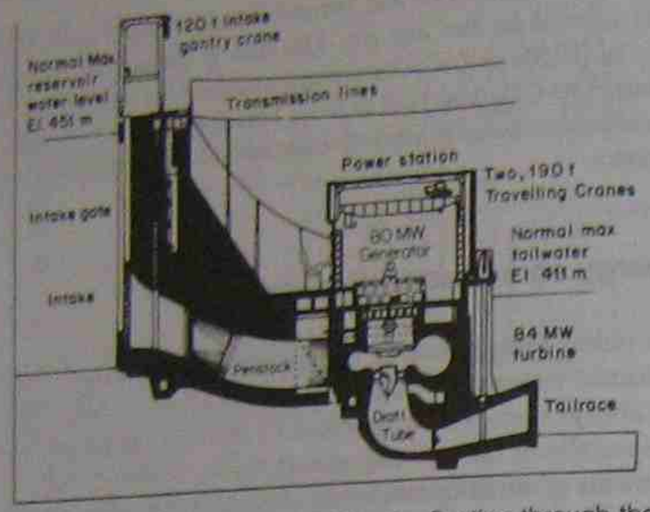


Figure 1.6 Hydroelectric scheme—Kainji, Nigeria. Section through the intake dam and power house. The scheme comprises an initial four 80 MW Kaplan turbine sets with the later installation of eight more sets. Running speed 115.4 r.p.m. This is a large-flow scheme with penstocks of 9 m diameter (*Permission of Engineering*)

Associated with these various heights or heads of water level above the turbines are particular types of turbine. These are:

1. *Pelton*—This is used for heads of 184–1840 m (600–6000 ft) and consists of a bucket wheel rotor with adjustable flow nozzles.
2. *Francis*—This is used for heads of 37–490 m (120–1600 ft) and is of the mixed flow type.
3. *Kaplan*—This is used for run-of-river and pondage stations with heads of up to 61 m (200 ft). This type has an axial-flow rotor with variable-pitch blades.

Typical efficiency curves for each type of turbine are shown in Figure 1.7. As the efficiency depends upon the head of water, which is continually fluctuating, often water consumption in cubic meters per kilowatt-hour is used and is related to the head of water. Hydroelectric plant has the ability to start up quickly and the advantage that no losses are incurred when at a standstill. It has great advantages, therefore, for power generation to meet peak loads at minimum cost, working in conjunction with thermal stations. By using remote control of the hydro sets, the time from the instruction to start up to the actual connection to the power network can be as short as 3 min.

At certain periods when water availability is low or when generation is not required from hydro sets, it may be advantageous to run the electric machines as motors supplied from the power system. These then act as synchronous

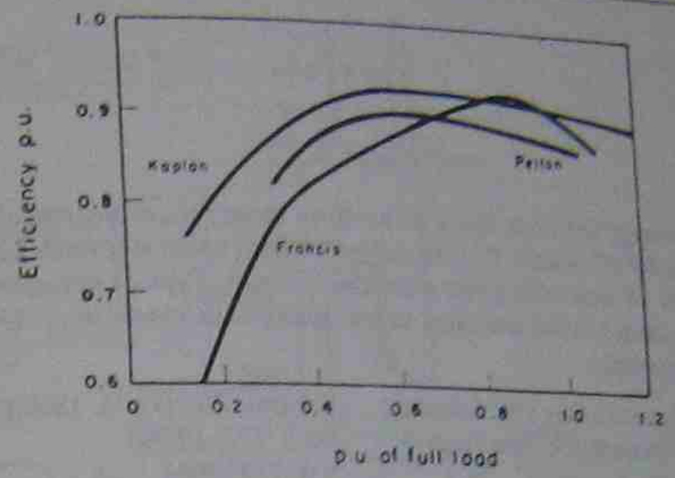


Figure 1.7 Typical efficiency curves of hydraulic turbines

compensators, to be discussed in Chapter 5. To reduce the amount of power required, the water is pushed below the turbine runner by compressed air. This is achieved by closing the water inlet valve and injecting compressed air which pushes the water towards the lower reservoir. The runner now rotates in air and thus requires much less motive power than in water.

The power available from a hydro scheme is given by

$$P = \rho g W H \text{ (W)}$$

where

W = flow rate (m^3/s) through the turbine;
ρ = density (1000 kg/m^3);
g = 9.81 m/s^2;
H = head, i.e. height of upper water level above the lower (m).

Substituting,

$$P = 9.81 W H \text{ (kW)}$$

Tides

An effective method of utilizing the tides is to allow the incoming tide to flow into a basin, thus operating the turbine, and then at low tide to release the stored water, again operating a set of turbines. This gives continuous, if varying, head operation. If the tidal range from high to low water is h (m) and the area of water enclosed in the basin is A (m^2), then the energy in the full basin

$$= \rho g A \int_0^h x \, dx$$

$$= \tfrac{1}{2}\rho g h^2 A$$

The total energy for both flows is therefore twice this value, and the average power is $\rho g A h^2 / T$, where T is the period of tidal cycle, normally 12 h 44 min. The number of sites with good potential is small. Typical examples of those which have been studied are listed below, along with values of h, A, and mean power, respectively.

Passamaquoddy Bay (N. America)	5.5 (m), 262 (km^2), 1800 (MW)
Minas-Cohequid (N. America)	10.7, 777, 19 900
San Jose (S. America)	5.9, 750, 5870
Severn (U.K.)	9.8, 70, 8000

1.3.3 Gas turbines

With the increasing availability of natural gas (methane) and its competitive price, prime movers based on the gas turbine as developed for aircraft are being increasingly used. Because of the high temperatures obtained by gas combustion, the efficiency of a gas turbine is comparable to that of a steam turbine, with the additional advantage that there is still sufficient heat in the gas-turbine exhaust to raise steam in a conventional boiler to drive a steam turbine coupled to another electricity generator. This is known as a combined-cycle gas-turbine (CCGT) plant, a schematic layout of which is shown in Figure 1.8. Combined efficiencies now being achieved are between 56 and 58 per cent.

The advantages of CCGT plant are the fast start up and shut down (2–3 min for the gas turbine, 20 min for the steam turbine), the flexibility possible for load following, the comparative speed of installation because of its modular nature and factory-supplied units, and its ability to run on oil (from local storage tanks) if the gas supply is interrupted. Modern installations are fully automated and require only a few operatives to maintain 24 h running or to supply peak load, if needed.

It is possible that with continuing further improvements demanded by aircraft in the high-temperature materials required for the gas turbine, even better efficiencies will be possible in the future. It is noteworthy that up to 10 per cent overload is available for short periods under emergency conditions to aid system recovery following stressed system operation. On the environmental side, CCGT plant with gas firing produces about 55 per cent of the carbon dioxide emission from a similarly rated coal/oil-fired plant. Many new, private operators are installing CCGT units either to produce electricity for sale

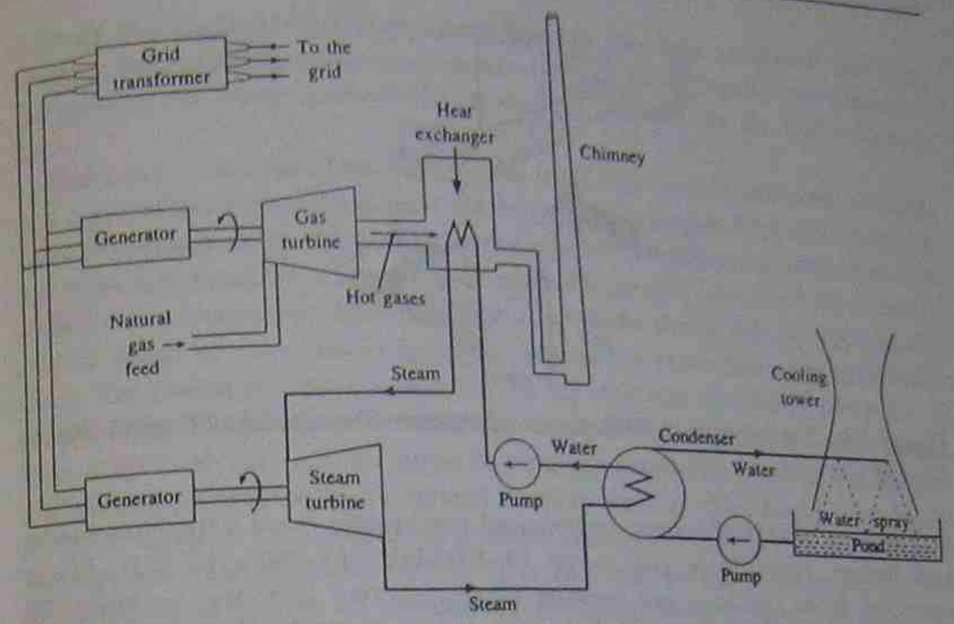


Figure 1.8 Schematic diagram of a combined-cycle gas-turbine power station (Reproduced by permission of Butterworth/Elsevier)

through the grid or as a combined heat and power (CHP) system for industrial or commercial use.

1.3.4 Magnetohydrodynamic (MHD) generation

Whether the fuel used is coal, oil, or nuclear, the result is the production of steam which then drives the turbine. Attempts are being made to generate electricity without the prime mover or rotating generator. In the magnetohydrodynamic method, gases at 2500°C are passed through a chamber in which a strong magnetic field has been created (Figure 1.9). If the gas is hot enough it is electrically slightly conducting (it is seeded with potassium to improve the conductivity) and constitutes a conductor moving in the magnetic field. An electromotive force (e.m.f.) is thus induced which can be collected at suitable electrodes. Nowadays, MHD is not seen as an economically viable option compared with CCGT alternatives.

1.3.5 Nuclear power

Fission

So far, power has been successfully obtained only from the fission reaction which involves the splitting of a nucleus. Compared with chemical reactions,

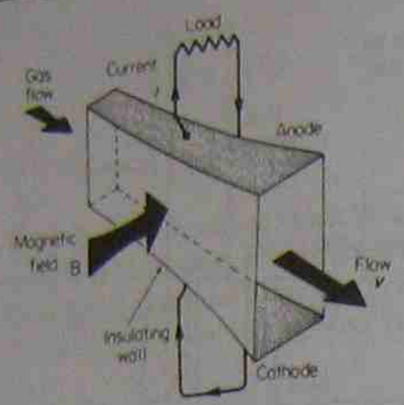


Figure 1.9 The principle of MHD power generation (*Permission of English Electric Co. Ltd*)

very large amounts of energy are released per atomic event with both fusion and fission, typically in the range 10–200 MeV ($15–300 \times 10^{-13}$ J). Metal extracted from the base ore consists mainly of two isotopes, uranium-238 (99.3 per cent by weight) and ^{233}U (0.7 per cent). Only ^{235}U is fissile, i.e. when struck by slow-moving neutrons its nucleus splits into two substantial fragments plus several neutrons and 3×10^{-11} J of kinetic energy. The fast-moving fragments hit surrounding atoms producing heat before coming to rest. The neutrons travel further, hitting atoms and producing further fissions. Hence the number of neutrons increases, causing, under the correct conditions, a chain reaction. In conventional reactors the core or moderator slows down the moving neutrons to achieve more effective splitting of the nuclei.

Fuels used in reactors have some component of ^{238}U. Natural uranium is sometimes used, and although the energy density is considerably less than for the pure isotopes it is still much better than fossil fuels. The uranium used at present comes from metal-rich ores, but these are a limited world resource (about 2×10^6 tons) and the requirement for a reactor which breeds fuel, the breeder reactor, is essential in the long term. The energy breakdown in the fission process is as follows (in MeV): kinetic energy of fission fragments 168, kinetic energy of neutrons 5, gamma radiation 5, beta and gammas emitted by fission products 7 and 6, respectively, and neutrinos 11.

When struck by the neutrons, certain non-fissile materials, if placed around the core of a reactor, are transformed into fissionable material. For example, uranium-238 and thorium-232 are converted into plutonium-239 and uranium-233. When more fissionable material is produced than is consumed the reactor is said to breed.

The basic reactor consists of the fuel in the form of rods or pellets situated in an environment (moderator) which will slow down the neutrons and fission products and in which the heat is evolved. The moderator can be light or heavy water or graphite. Also situated in the moderator are movable rods which absorb neutrons and hence exert control over the fission process. In some

reactors the cooling fluid is pumped through channels to absorb the heat, which is then transferred to a secondary loop in which steam is produced for the turbine. In water reactors the moderator itself forms the heat-exchange fluid.

There are a number of versions of the reactor in use with different coolants and types of fissile fuel. In Britain the Magnox reactor has been used, in which natural uranium in the form of rods is enclosed in magnesium-alloy cans. The fuel cans are placed in a structure or core of pure graphite made up of bricks (called the *moderator*). This graphite core slows down the neutrons to the correct range of velocities in order to provide the maximum number of collisions. The fission process is controlled by the insertion of control rods made of neutron-absorbing material; the number and position of these rods controls the heat output of the reactor. Heat is removed from the graphite via carbon dioxide gas pumped through vertical ducts in the core. This heat is then transferred to water to form steam via a heat exchanger. Once the steam has passed through the high-pressure turbine it is returned to the heat exchanger for reheating, as in a coal- or oil-fired boiler. A schematic diagram showing the basic elements of such a reactor is shown in Figure 1.10.

A reactor similar to the Magnox is the *advanced gas-cooled reactor* (AGR). A reinforced-concrete steel-lined pressure vessel contains the reactor and heat exchanger. Enriched uranium dioxide fuel in pellet form, encased in stainless steel cans, is used; a number of cans form a cylindrical fuel element which is placed in a vertical channel in the core. Carbon dioxide gas, at a higher pressure than in the Magnox type, removes the heat. The control rods are made of boron steel. Spent fuel elements when removed from the core are stored in a special chamber for about a week and then dismantled and lowered into a pond of water where they remain until the level of radioactivity has decreased sufficiently for them to be removed from the station.

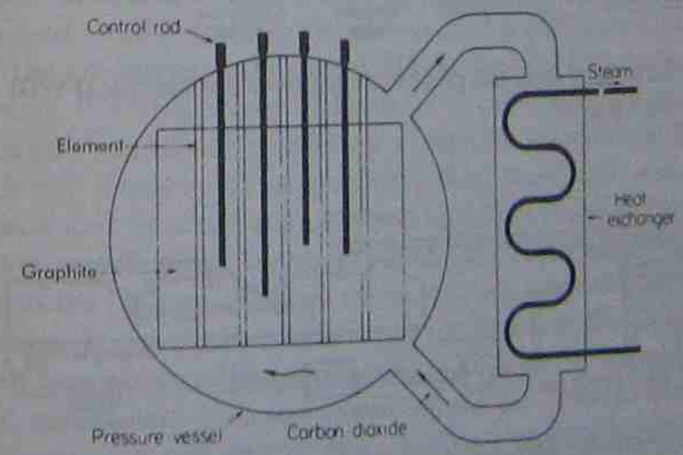


Figure 1.10 Schematic view, nuclear reactor—British Magnox type

In the U.S.A. and many other countries pressurized-water and boiling-water reactors are used. In the *pressurized-water type* the water is pumped through the reactor and acts as a coolant and moderator, the water being heated to 315°C. The steam pressure is greater than the vapour pressure at this temperature and the water leaves the reactor at below boiling point. The fuel is in the form of pellets of uranium dioxide in bundles of stainless steel tubing. The *boiling-water reactor* was developed later than the pressurized-water type and is now used extensively. Inside the reactor, heat is transferred to boiling water at a pressure of 690 N/cm^2. Schematic diagrams of these reactors are shown in Figures 1.11 and 1.12.

All reactors that use uranium produce plutonium in the reaction. Most of this is not utilized in the reactor. *Fast breeder reactors* breed new fuel in considerable quantities during the reaction, as well as producing heat. In the liquid-metal fast breeder reactor, shown in Figure 1.13, liquid sodium is the coolant, which leaves the reactor at 650°C at atmospheric pressure. The heat is then transferred to a secondary sodium circuit which transfers it via a heat exchanger to produce steam at 540°C. The cost and unreliability of these reactors has meant that, to date, they have only been trialled.

Both pressurized- and boiling-water reactors use *light water*. The practical pressure limit for the pressurized-water reactor is about 167 bar (2500 p.s.i.), which limits its efficiency to about 30 per cent. However, the design is relatively straightforward and experience has shown this type of reactor to be stable and

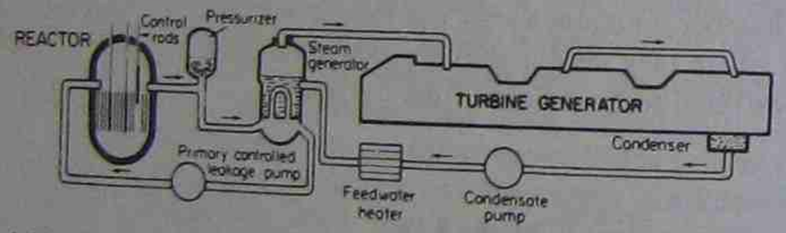


Figure 1.11 Schematic diagram of a pressurized-water reactor (PWR) (*Permission of Edison Electric Institute*)

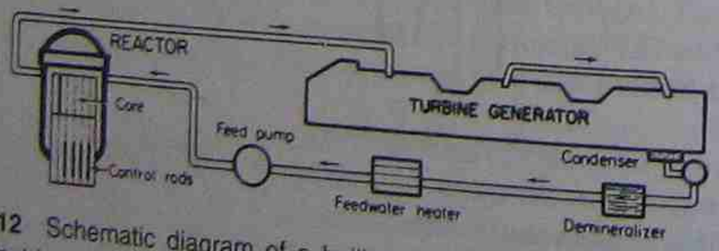


Figure 1.12 Schematic diagram of a boiling-water reactor (BWR) (*Permission of Edison Electric Institute*)

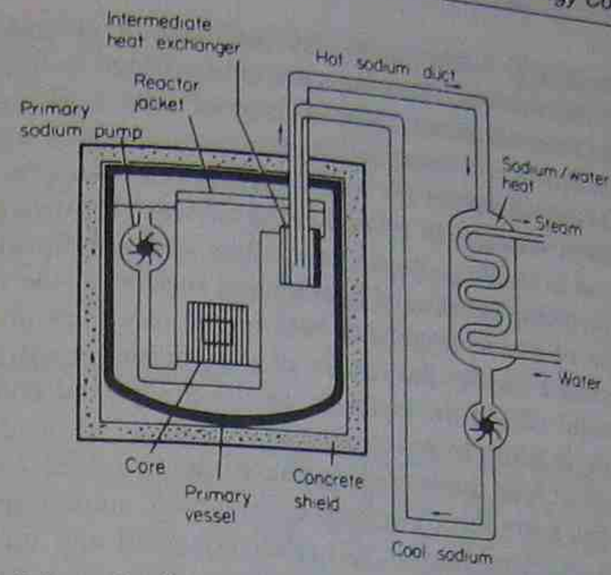


Figure 1.13 Schematic diagram of a liquid-metal fast breeder reactor (*Permission of Edison Electric Institute*)

dependable. In the boiling-water reactor the efficiency of heat removal is improved by use of the latent heat of evaporation. The steam produced flows directly to the turbine, causing possible problems of radioactivity in the turbine. The fuel for both water reactors is enriched to 1.44 per cent ^{235}U. These reactors are probably the cheapest to construct; however, the steam produced is saturated and requires wet-steam turbines. A further type of water reactor is the heavy-water CANDU type developed by Canada. Its operation and construction are similar to the light-water variety.

Safety and environmental considerations The translation of energy states in a nucleus creates the emission of γ- and β-rays, and α and fission fragments. The half-lives of the substances created are: $^{3}_{1}H_2$ (tritium) 12.26 years, ^{90}Sr (strontium) 28.8 years, ^{137}Cs (caesium) 30.2 years, ^{131}I (iodine) 8 days, ^{85}Kr (krypton) 10.76 years, and ^{133}Xe (xenon) 5.27 days. Generally, materials with a long half-life have a lower intensity of radioactivity than those with a short half-life. Tritium is produced in small amounts and mostly retained in the fuel. Xenon and krypton escape from fuel elements which have cladding defects and remain in free form in the coolant. Because of its long life krypton-85 constitutes the greater problem. In the water-cooled cores the fission and activation products are present in the coolant. The more active of such wastes are concentrated by evaporation, mixed with concrete, and shipped for storage. The lower level wastes are eventually released to the condenser cooling-water discharge at low concentration levels.

High-level wastes, e.g. strontium-90, are produced in processing the used fuel elements. At the moment, the wastes are concentrated in liquid form and stored in stainless steel containers. The storage of such wastes creates great controversy, the material still being active after centuries. Any mistakes made now will create serious problems for future generations. With future development such long-lived wastes will be converted to solid form (e.g. glass) and stored underground in stable geological situations such as salt domes.

Any accident involving substantial heating and rupture of the structure will involve the release of fission fragments held in the fuel rods into the atmosphere. With a breeder reactor the release of plutonium, an extremely radiotoxic material, would add to the problem. In the design and construction of reactors, great care is taken to cover every contingency. Many facilities, e.g. control systems, are at least duplicated and have alternative electrical supplies.

Over the past few years there has been considerable controversy regarding the safety of reactors. Experience is still relatively small and human error is always a possibility, such as happened at Three Mile Island in 1979 and Chernobyl in 1986. However, the health controls in the atomic power industry have, from the outset, been much more rigorous than in any other industry.

Fusion

Energy is produced by the combination of two light nuclei to form a single heavier one. Neutron emission is not required, the reaction being sustained by the very high temperature of the reactants which maintain continual collisions. The most promising fuels are isotopes of hydrogen known as deuterium (D) (mass 2) and tritium (T) (mass 3). The product of fusion is the helium isotope (mass 3), hydrogen, neutrons, and heat. As tritium is not a naturally occurring isotope it is produced in the reactor shield by the interaction of the fusion neutrons and the lithium isotope of mass 6. The deuterium–deuterium fusion requires higher temperatures than deuterium–tritium and the latter is more likely to be used initially.

Reserves of lithium have been estimated to be roughly equal to those of fossil fuels. Deuterium, on the other hand, is contained in sea-water of a concentration of about 34 parts per million. The potential energy-resource is therefore vast.

Based on radioactive considerations, the impact of fusion reactors would be much less than with fission reactors. In fission reactors the loss of coolant accident and the 'after-heat' generated after shut-down (fission which continues after full 'shut-down' control action) may lead to vaporization and dispersal of radioactive material. With fusion there is much less power density under these conditions, possibly 1/50th of the equivalent value for a fast breeder reactor. The main source of radioactive waste from fusion reactors would be the structural material which undergoes damage due to radiation and hence requires occasional renewal. This could be recycled after a 50-year period, compared with centuries for strontium-90 and casesium-137 from fission.

Intensive international research is still proceeding to develop materials and a suitable containment method, using either magnetic fields or powerful lasers, to produce the high temperatures ($\approx 8 \times 10^7$ K) and pressures (above 1000 bar) to initiate a fusion reaction. It is unlikely that a successful fusion reactor will be available before 2020.

1.3.6 Generation and fuel

With new and more efficient generating sets being brought into operation, there exists a wide range of plant available for use. As previously mentioned, the load consists of a base plus a variable element, depending on the time of day and other factors. Obviously the base load should be supplied by the more efficient plant which then runs 24 h per day, with the remaining load met by the less efficient stations. In addition to the machines supplying the load, a certain proportion of available plant is held in reserve to meet sudden contingencies. A proportion of this reserve must be capable of being brought into operation immediately and hence some machines must be run at, say, 75 per cent of full load to allow for this spare generating capacity, called 'spinning reserve'.

Reserve margins are allowed in the *total* generation plant available, to cope with unavailability of plant due to faults and maintenance. It is common practice to allow a planned margin of about 20 per cent over the annual peak demand. In a power system there is a mix of plants, i.e. hydro, coal, oil, and nuclear, and gas turbine. The optimum mix gives the most economic operation, but this is highly dependent on fuel prices which can fluctuate with time and from region to region. Hence in the U.S. in regions with plentiful coal resources, in the west–north–central and also in certain eastern areas, coal plants would be expected to be predominant. In the south and other areas not endowed with coal, nuclear or gas would tend to be dominant. Of course, hydro would be exploited wherever possible for cost and environmental reasons.

Typical plant and fuel costs for the U.K. and U.S.A. are given in Table 1.1. It should be noted that fuels bought on world markets at spot prices can vary in price, dependent upon political situations, especially in the Middle East and Russia.

1.4 Renewable Energy Sources

There is considerable international effort into the development of alternative energy sources to supplement fossil fuels. Many of the 'novel' sources (some of

Table 1.1 Typical plant and fuel costs

	Nuclear	Coal	Oil	Gas turbine	Wind
Plant cost £/kW	1600	800[a]	800[a]	300	1000
1995 US$/kW	2560	1300	1300	480	1600
Fuel cost p/kWh	0.2	1.6	3.0	2.0	0
1995 c/kWh	0.3	2.6	4.8	3.2	0

[a]Add 20 per cent if flue gas desulphurization is needed.
(Compiled from Power UK, FT Publications, London)

them, in fact, have been in use for centuries!) are manifestations of solar energy, e.g. wind, sea waves, ocean thermal gradients, and photosynthesis. The average incident solar energy received on the earth's surface is about 600 W/m^2, but the actual value, of course, varies considerably. In the following section the potentialities of various methods of utilizing this energy will be discussed.

1.4.1 Solar energy—thermal conversion

There are two distinct applications: (1) space and water heating on a domestic scale; and (2) central station, large-scale heat collection, used for steam raising to generate electricity; both of these influence power systems. The former affects the load demands and in particular the problem utilities will face in having to provide a sufficient back-up supply to customers who normally would use solar power, but in certain weather conditions would require large amounts of electricity. This involves the provision of the normal amount of utility plant but with much reduced sales of energy.

As the temperature of a solar collecting surface rises it radiates heat (infrared). The energy distributions with wavelength of solar energy and infrared radiation are shown in Figure 1.14. It is possible to design a selective cover plate over the collecting surface such that it would pass nearly all the solar radiation and reflect all the radiated infrared. Selective absorbers consist of a smooth metallic sheet covered with either a thin semiconducting surface or a finely divided metallic powder. The former reflects the infrared and provides a good thermal contact between the hot absorbing layer and cooling fluid. A diagram of a simple collecting system is shown in Figure 1.15.

The energy received by the collector per square meter (net):

$$q = I\alpha\tau - (\varepsilon_F + \varepsilon_B)\sigma(T^4 - T_0^4)$$

where

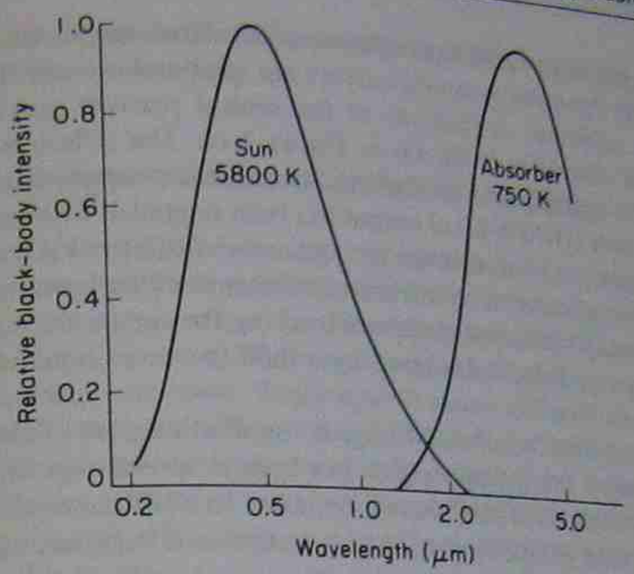


Figure 1.14 Relative black-body intensity of radiation with wavelength

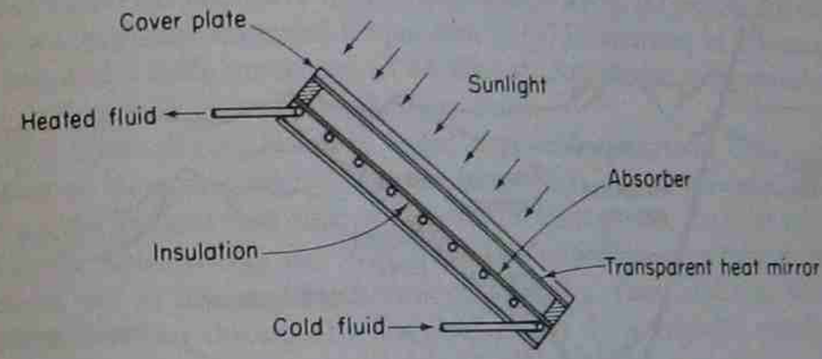


Figure 1.15 Simple solar energy panel for water heating

ε_F and ε_B = front and back emissivities of absorber;
σ = Stefan–Boltzmann constant = 5.67×10^{-8} W/m^{2}K^4;
τ = transmittance of cover plate (e.g. 0.93);
T_0 = temperature of cover plate (K);
I = incident radiation normal to surface;
$T(K)$ and α = temperature and absorptivity of absorbing panel.

In large-scale (central station) installations the sun's rays may be concentrated by lenses or mirrors. Both require accurately curved surfaces and steering mechanisms to follow the motion of the sun. Concentrators may be designed to follow the sun's seasonal movement, or additionally to track the sun throughout the day (double-axis system). The former is less expensive and

concentration factors of 30 have been obtained. However, in the French solar furnace in the Pyrenees, two-axis mirrors are used and a concentration factor of 16000 is achieved. A diagram of the central receiver system for major generation of electricity is shown in Figure 1.16. The reflectors concentrate the rays on to a single receiver (boiler), hence raising steam. A collector area of 1 km^2 for each 100 MW (e) of output has been suggested with capital costs of \$30/m^2 (mirrors, etc.) and thermal storage costs of \$15 per kWh of electricity. A less attractive alternative to this scheme (because of the lower temperatures) is the use of many individual absorbers tracking the sun in one direction only, the thermal energy being transferred by a fluid (water or liquid sodium) to a central boiler.

In all solar thermal schemes, storage is essential because of the fluctuating nature of the sun's energy, although it has been proposed that the schemes be used as pure fuel savers. This feature is common to all of the sources discussed, with the exception of geothermal, and constitutes a very serious drawback.

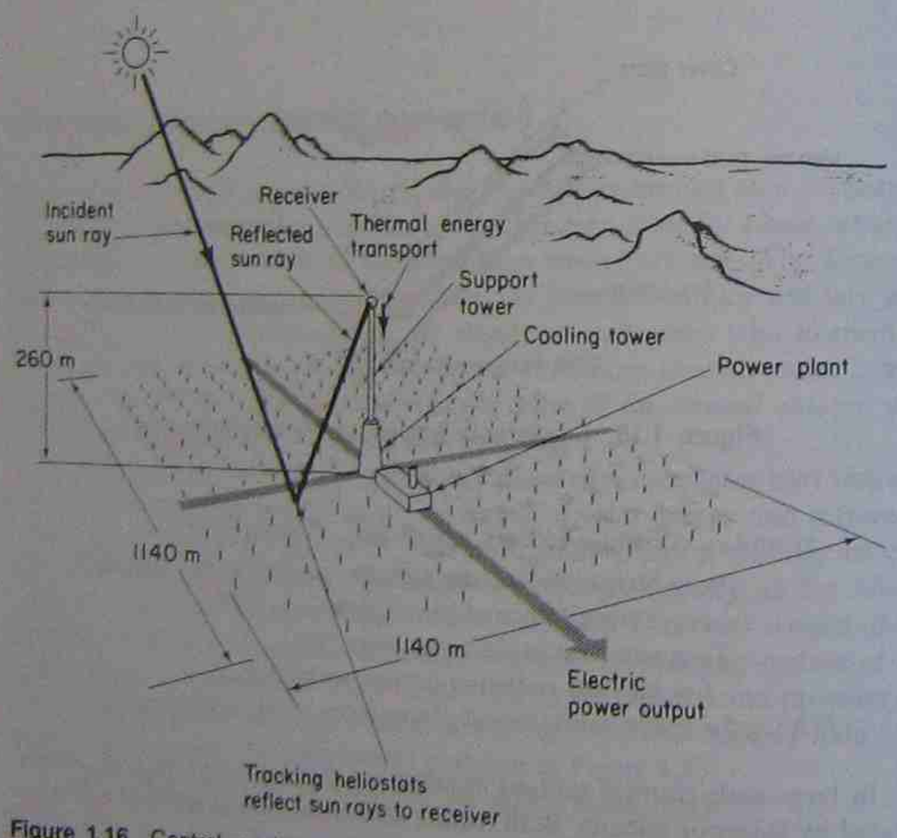


Figure 1.16 Central receiver scheme for electric power generated by solar energy (*Copyright © 1975 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 12, No. 12 (Dec. 1975), pp. 47–52*)

Fluctuating sources, as well as fluctuating loads, would complicate still further the process of electricity supply.

1.4.2 Solar energy—direct conversion to electricity

Photovoltaic conversion occurs in a thin layer of suitable material, e.g. silicon, when hole–electron pairs are created by incident solar photons and the separation of these holes and electrons at a discontinuity in electrochemical potential creates a potential difference. Whereas theoretical efficiencies are about 25 per cent, practical values are lower. Single-crystal silicon and gallium–arsenide cells have been constructed with efficiencies of 10 and 16 per cent, respectively. The cost of fabricating and interconnecting cells is high (used mainly, to date, in spacecraft). Polycrystalline silicon films having large-area grains (i.e. long continuous crystals) with efficiencies of over 16 per cent have been made by techniques amenable to mass production. Although these devices do not pollute, they will, in the large-power context, occupy large areas. It has been estimated that to produce 10^{12} kWh per year (about 65 per cent of the 1970 U.S. generation value) the necessary cells would occupy about 0.1 per cent of the U.S. land area (highways occupied 1.5 per cent in 1975), assuming an efficiency of 10 per cent and a daily insolation of 4 kWh/m^2. Automated cell production can now produce cells at around US \$5 per watt.

Other forms of conversion of lesser large-scale importance come under the heading of thermoelectricity. The Seebeck effect gives a potential difference between the hot and cold ends of joins of different metals, a typical value being 150 μV/K. Solar energy can heat a cathode of a diode-type tube from which electrons will be liberated by thermionic emission. These electrons drift to the anode and return through the external circuit. It is doubtful whether these devices will make any impact on the energy situation.

1.4.3 Wind generators

Wind power from horizontally mounted generators on 30–50 m high towers is now becoming economically viable. Sizes between 300 and 500 kW driven by two or, more effectively, three-bladed wind turbines are an optimum but larger turbines of 2–3 MW have been built for development purposes. However, the larger towers and blades for higher outputs must be traded against the extra capital costs.

The theoretical power in the wind is given by

$$P = \tfrac{1}{2}\rho A U^3 \text{ (W)}$$

where

ρ = density of air (1.201 kg/m^3 at NTP);
U = mean air velocity (m/s);
A = swept area of blades (m^2).

The range of operation of a wind turbine depends upon the wind speed and is depicted in Figure 1.17.

At low wind speeds, there is insufficient energy to operate the turbine coupled to the generator and no power is produced. At the 'cut-in' U_c speed, between 3 and 5 m/s on the diagram, power starts to be generated until rated power P_r is produced at rated wind speed U_r. After this point, the turbine is controlled, usually by altering the blade angle or 'pitch', to give rated output up to a maximum wind speed U_f, after which the blades are 'furled' and the unit is shut down to avoid excessive wind loading. Typically, wind turbines have rotors of 20 m diameter, rotate at 100–150 rpm, and are geared up to about 750 r.p.m. to drive an eight-pole induction generator excited by a 415 V three-phase (3 ph.), 50 Hz rural distribution system. If they are sited in 'windy' areas, normally found on exposed ridges, and can convert nearly half the theoretical power to electrical energy, a good site in the U.K. can produce 1800 kWh per kW of installed capacity per year.

Example 1.1

Calculate the number of wind generators required to produce the equivalent of a 600 MW CCGT operating at 80 per cent load factor. Assume average wind speed is 10 km/h (2.78 m/s), blade diameter is 20 m, and conversion efficiency is 45 per cent.

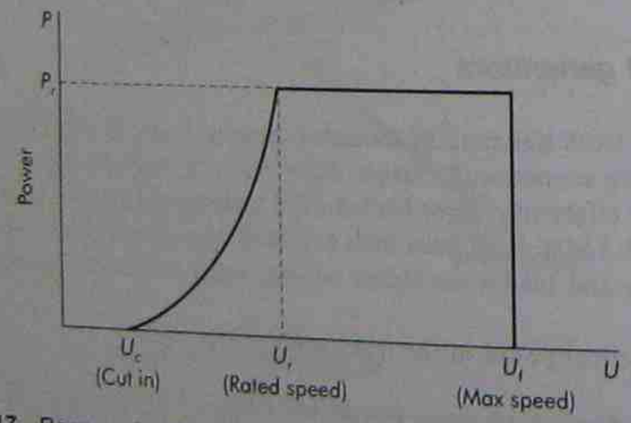


Figure 1.17 Range of operation of wind turbine (*Courtesy of I.E.E. Power Engineering Journal, Aug. 1995*)

Calculation

$$P_{\text{wind}} = \tfrac{1}{2} \cdot 1.201 \cdot \pi\left(\frac{20}{2}\right)^2 \cdot 2.78^3 \times 10^{-3} = 4053\,\text{kW}$$

$$P_{\text{generate}} = 4053 \times 0.45 = 1823\,\text{kW}$$

$$\therefore \text{No. of wind generators for 600 MW} = \frac{600}{1.83} = 330 \text{ generators}$$

From this calculation, it is apparent that many wind generators spread over a wide area would be required. Although the ground beneath them could be used for grazing, the proliferations and the acoustic noise can be detrimental to the environment. However, the saving in CO_2 emissions would be of the order of 12 000 t/day provided that the wind was always blowing!

1.4.4 Biofuels

Biofuels are derived from decaying vegetable matter produced by agriculture or forestry operations or from waste materials collected from industry, commerce, and residential households. As an energy resource, biomass used as a source of heat by burning wood, dung, etc., in developing countries is very important and contributes about 14 per cent of the world's energy requirements. Biofuel can be used to produce electricity in two ways:

1. by burning in a furnace to produce steam to drive turbines; or
2. by allowing fermentation in landfill sites or in special anaerobic tanks, both of which produce a methane-rich gas which can fuel a spark ignition engine or gas turbine.

It is interesting to note that if crops are cultivated for combustion, either as a primary source of heat or as a by-product of some other operation, they can be considered as CO_2 neutral, in that their growing cycle absorbs as much CO_2 as is produced by their combustion. In industrialized countries, biofuels have the potential to produce up to 5 per cent of electricity requirements if all possible forms are exploited, including household and industrial waste, sewerage sludge (for digestion), agricultural waste (chicken litter, straw, sugar cane, etc.).

1.4.5 Geothermal energy

In most parts of the world the vast amount of heat in the earth's interior is too deep to be tapped. In some areas, however, hot springs or geysers and molten lava streams are close enough to the surface to be used. Thermal energy from hot springs has been used for many years for producing electricity, starting in 1904 in Italy. In the U.S.A. the major geothermal power plants are located in northan California on a natural steam field called the Geysers. Steam from a number of wells is passed through turbines. The present utilization is about 500 MW and the total estimated capacity is about 2000 MW. Because of the lower pressure and temperatures the efficiency is less than with fossil-fuelled plants, but the capital costs are less and, of course, the fuel is free. New Zealand and Iceland also exploit their geothermal energy resources.

The Geysers in the U.S.A. represent a dry steam field which is preferable for power generation via steam turbines. Other basic types of geothermal energy reservoirs are: hot water, hot dry rock, geopressured water, and the normal thermal gradient in the earth's crust. It is more common for wells to produce a mixture of steam and hot water, this combination being much less useful than dry steam. Electricity may be generated from hot-water wells by passing the water, under pressure, through a heat exchanger, where it causes the vaporization of a volatile liquid such as freon. The latter expands through a turbine.

The largest potential is associated with the heat in deposits of dry rock. High-pressure water is forced down a deep shaft and creates cracks in the rock at the bottom. Pressurized water is then forced through the cracks to extract heat. Hot rocks contracts as it cools, thereby creating fresh cracks and extending the catchment volume. At Los Alamos in New Mexico a 780 m shaft was drilled and then hydraulic creation of cracks was achieved at pressures of about 100 atm. All parts of the earth's surface have heat in the rock beneath, but the temperature gradients are very modest. Although the potential amount of heat is vast, the technology involved to extract this heat successfully has still to be developed and shown to be economically viable.

1.4.6 Other renewable resources

Ocean temperature gradients

In 1881 D'Arsonval proposed the utilization of the temperature difference between the surface and lower layers of tropical seas. For practical purposes the layers need to be in reasonably close proximity to each other. The absorption of solar energy by the surface layers causes a thermal-syphon action, the warm surface water flowing towards the earth's poles from whence it moves

back towards the tropics as cold water at a greater depth. The Gulf Stream carries, on average, about 2 km^3/min of warm water, and the energy potential in this is vast. Warm and cold currents are close to each other off the Florida coast, the temperature difference range being 9–25°C. This gives a Carnot efficiency of about 3.4 per cent, and, with losses, an overall efficiency of, say, 2 per cent. Hence to produce large powers, vast quantities of water and large process plants are required. On the other hand an Arctic Ocean scheme could utilize the temperature differential between the water (at, say, +2°C) and the subzero air temperature.

A proposed plant would be situated 25 km east of Miami, where the temperature difference is 17.5°C, and this is shown in Figure 1.18. The best working fluids are ammonia (NH_3) or propane. A typical sea-water flow rate for a 400 MW installation is 115 000 m^3/min, and the diameter of the cold-water pipe would need to be 24 m and the length 500–600 m. The plant is submerged and anchored. The warm water boils a high-pressure working fluid and the vapour expands through vapour turbines and then condenses back to a liquid in the condenser.

In 1996 the cost of producing electricity from fossil plants was 1.5–2.0 p/kWh. Estimates for the sea thermal plant are now between 4 and 10 p/kWh. The surface temperature of the tropical oceans would be lowered by about 1°C if 60 000 GW of electricity is generated in this manner. Only if gas, oil, and coal increase in price to three times their present costs would an OTEC become worthwhile.

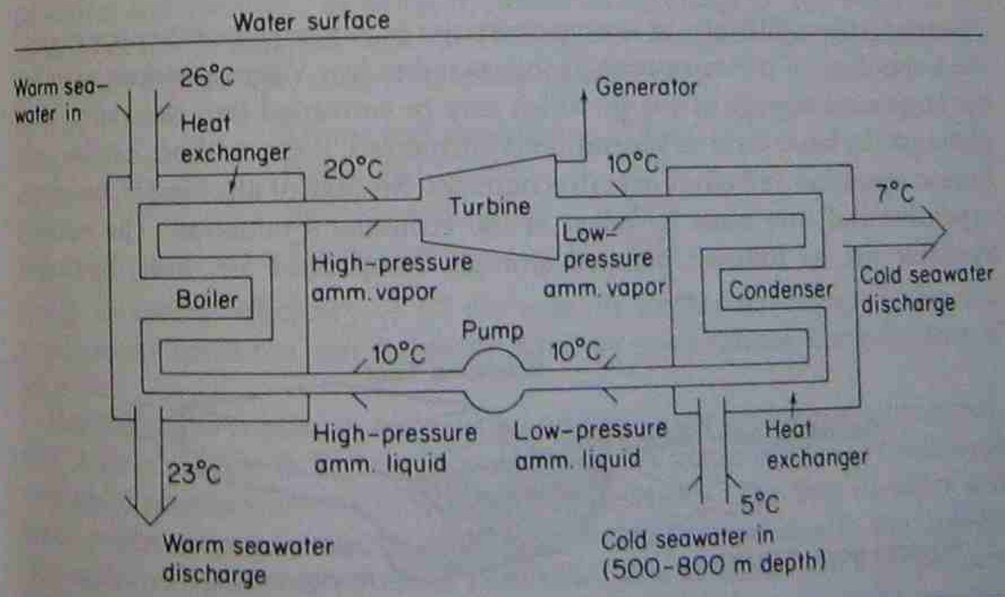


Figure 1.18 Concept of power plant using sea-temperature differences (ocean temperature energy converter, OTEC)

Wave power

The energy content of sea waves is very high. The Atlantic waves along the north-west coast of Britain have an average value of 80 kW/m of wave crest length. The energy is obviously very variable, ranging from greater than 1 MW/m for 1 per cent of the year to near zero for a further 1 per cent. Hence, over several hundreds of kilometres a vast source of energy is available.

The sea motion can be converted into mechanical energy in a number of ways. One method is by using the Salter cam (or duck!), a cross-section of which is shown in Figure 1.19. The cam rotates about its axis and is shaped to minimize back-water pressure. Lengths of the cam could be moored, say, 80 km off the coast. Conversion of the float movement to electricity is difficult because of the slow oscillations. If reciprocating generators of some form are used, a device producing a 1 MW while oscillating at 0.1 Hz with an amplitude of 15° would require a torque of 1200 t-m! Owing to the randomness of the waves the system must be rated much higher than the design average output level. This implies that the cost of the installation designed to withstand the strongest winter Atlantic waves will be very high. To avoid this, shore or anchored installations using an oscillating column of water propelling air through a turbine to drive a generator are possible alternatives. However, the cost and robustness of such devices has still to be assessed.

1.5 Energy Storage

The tremendous difficulty in storing electricity in any large quantity has shaped the technology of power systems as they stand today. Various options exist for the large-scale storage of energy which may be converted into electricity, and, although the basic form of generation is unchanged, these methods can be used to ease operation and effect overall economies. Storage of any kind is, however, expensive and care must be taken in the economic evaluation. The options available are as follows: pumped storage, compressed air, heat, hydrogen

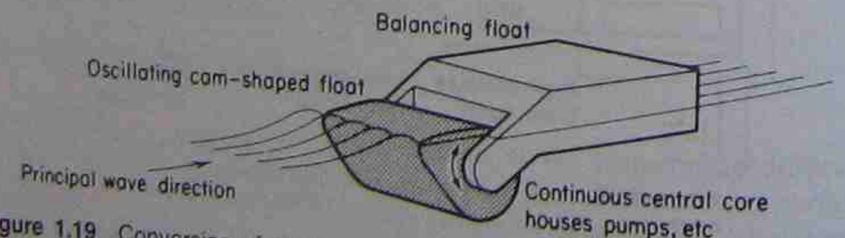


Figure 1.19 Conversion of wave energy by floating cam (*Permission of Central Electricity Generating Board*)

gas, secondary batteries, and, of doubtful promise, flywheels, and superconducting coils.

Although gas turbines are used to meet daily load peaks, very rapid changes in load (e.g. 1300 MW/min at the end of the 'Miss World' contest on British TV!) or the outage of lines or generators may occur. Gas turbines take 2 min to start up and a considerable amount of conventional steam plant must operate underloaded as a reserve. This is very expensive because there is a fixed heat loss for a steam turbogenerator regardless of output, and also the efficiency is reduced. A significant amount of storage capable of instantaneous use would be an effective method of meeting such loadings, and by far the most important method to date is that of pumped storage.

1.5.1 Pumped storage

A method of obtaining the advantages of hydro plant where suitable water supplies are not available is by the use of pumped storage. This consists of an upper and a lower reservoir and turbine-generators which can be used as motor-pumps. The upper reservoir has sufficient storage usually for 4–6 h of full-load generation, with a reserve of 1–2 h.

The sequence of operation is as follows. During times of peak load on the network the turbines are driven by water from the upper reservoir in the normal manner. The generators then change to synchronous-motor action and, being supplied from the general power network, drive the turbine which is now acting as a pump. During the night, when only base load stations are in operation and electricity is being produced at its cheapest, the water in the lower reservoir is pumped back into the higher one ready for the next day's peak load. Typical operating efficiencies attained are:

Motor and generator	96%
Pump and turbine	77%
Pipeline and tunnel	97%
Transmission	95%

giving an overall efficiency of 67 per cent. A further advantage is that the synchronous machines can be easily used as synchronous compensators if required.

Dams must be constructed if existing reservoirs are inadequate or non-existent. A large scheme in Britain uses six 330 MVA pump-turbine (Francis-type reversible) generator-motor units generating at 18 kV. The flow of water and hence power output is controlled by guide vanes associated with the turbine. The maximum pumping power is 1830 MW. The machines are 92.5 per cent efficient as turbines and 91.7 per cent efficient as pumps and the speed is 500 r.p.m. Such a plant can be used to provide fine frequency control for the

whole British system, when the machines will be expected to start and stop about 40 times a day.

1.5.2 Compressed-air storage

Air is pumped into large receptacles (e.g. underground caverns or old mines) at night and used to drive gas turbines for peak, day loads. A German utility has installed a 290 MW scheme at a capital cost in the order of £60 per kW (1975 prices). It generates 580 MWh of on-peak electricity and consumes 930 MWh of fuel plus 480 MWh of off-peak electricity. The energy stored is equal to the product of the air pressure and volume. The compressed air allows fuel to be burnt in the gas turbines at twice the normal efficiency. The general scheme is illustrated in Figure 1.20. Similar plant has been installed in the U.S.A.

One disadvantage of the above scheme is that much of the input energy to the compressed air manifests itself as heat and is wasted. Heat could be retained after compression, but there would be possible complications with the store walls due to the temperature of 450°C at 20 bar pressure. A solution would be to have a separate heat store that could comprise stacks of stones or pebbles which store heat cheaply and effectively. This would enable more air to be stored because it would now be cool. At 100 bar pressure, in the order of 30 m^3 of air is stored per megawatt-hour output.

1.5.3 Heat storage

No large-scale storage involving heat has yet evolved. Water has many advantages as a heat-storage medium, including good specific and latent heats. Liquid sodium has also been suggested and also various salts. In a steam generating plant, if the load is low, boilers may be kept at full output if the unwanted steam is bled to the feed-water heaters. During full-load output the

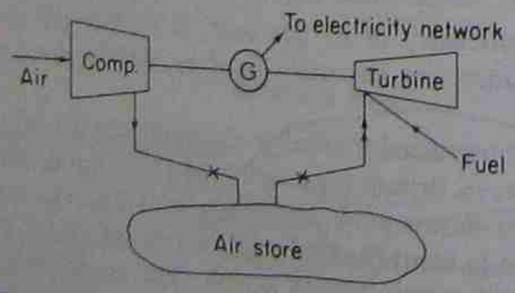


Figure 1.20 Storage using compressed air in conjunction with gas-turbine generator G

steam produced goes wholly to generation, whilst the hot water is drawn from the store to give heated feed water. A 500 MW(e) boiler requires 405 kg/s of feed water and would require a storage of 17 000 t of water.

1.5.4 Secondary batteries

Large-scale battery use is unlikely and the two areas where the use of secondary batteries will have impact are in the electric car and local fluctuating energy sources such as windmills or solar power. The popular lead–acid cell, although reasonable in price, has a low energy density (15 Wh/kg). Nickel–cadmium cells are better (40 Wh/kg) but much more expensive. Still under intensive development is the sodium–sulphur battery (200 Wh/kg), which has a solid electrolyte and liquid electrodes and operates at a temperature of 300°C. Other combinations of materials are under active development in attempts to increase output and storage per unit weight. A 3 MW battery storage plant has been installed in Berlin for frequency control in emergencies.

1.5.5 Fuel cells

A fuel cell converts chemical energy to electrical energy by electrochemical reactions. Fuel is continuously supplied to one electrode and an oxidant (usually oxygen) to the other electrode. Figure 1.21 shows a simple hydrogen–oxygen fuel cell, in which hydrogen gas diffuses through a porous metal electrode (nickel). A catalyst in the electrode allows the absorption of H_2 on the electrode surface as hydrogen ions which react with the hydroxyl ions in the electrolyte to form water ($2H_2 + O_2 \rightarrow 2H_2O$). A theoretical e.m.f. of 1.2 V at 25°C is obtained. Other fuels for use with oxygen are carbon monoxide (1.33 V at 25°C), methanol (1.21 V at 25°C), and methane (1.05 V at 25°C). In practical cells, conversion efficiencies of 80 per cent have been attained. A major use of the fuel cell could be in conjunction with a hydrogen energy system.

Intensive research and development is still proceeding on various types of fuel cell—the most successful to date for power generation being the phosphoric fuel cell (Figure 1.22). This uses methane as the input fuel and operates at about 200–300°C to produce 200 kW of electrical power plus 200 kW of heat energy, with overall efficiency of around 80 per cent. Compared with other forms of energy conversion (see Figure 1.23), fuel cells have the potential of being up to 20 per cent more efficient. Much attention is now being given to the high-temperature molten carbonate cell which has the highest efficiency yet conceived.

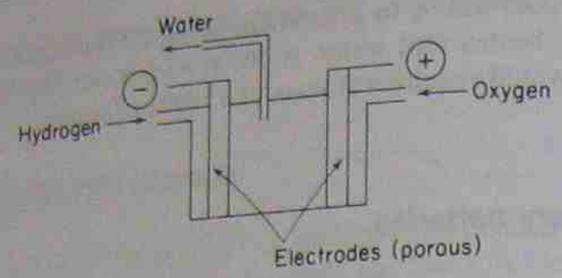


Figure 1.21 Hydrogen–oxygen fuel cell

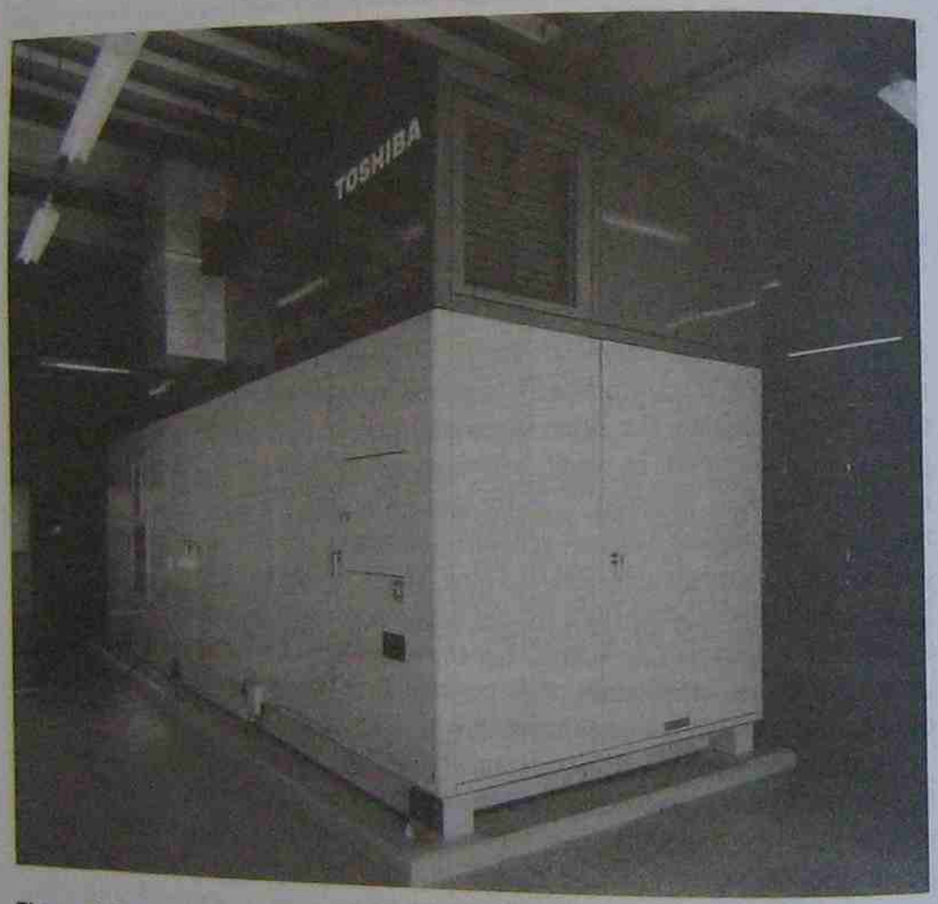


Figure 1.22 TEPCO's 200 kW fuel cell plant installed in the basement of the company's R&D Centre in Yokohama which went into operation in 1994. Manufactured by Toshiba, the plant is connected to TEPCO's grid and its waste heat is utilized for hot water supply and air conditioning for the R&D Centre building (*Courtesy of I.E.E. Power Engineering Journal*)

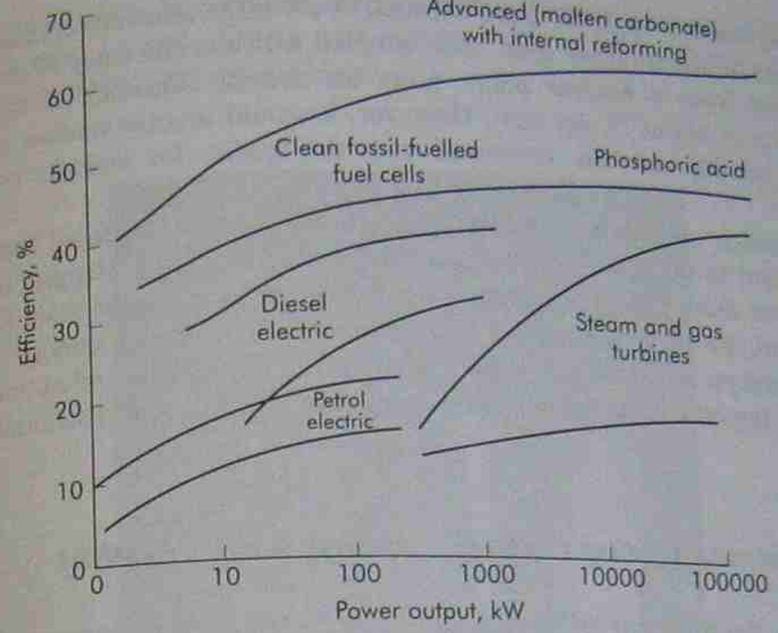


Figure 1.23 Simple-cycle efficiency of different technologies as a function of scale (*Courtesy of I.E.E. Power Engineering Journal, June 1995*)

1.5.6 Hydrogen energy systems

Transmission of natural gas via a network of pipes is well established. The transmission capacity of a pipe carrying natural gas is high compared with electrical links, the installed cost being about one tenth of an equivalent capacity H.V. overhead line.

For hydrogen gas the calorific value is 12×10^6 J m^3 (ATP).

Power transmitted = volumetric flow rate × volumetric calorific value (at working pressure)

For long transmission distances the pressure drop is compensated by booster compressor stations. A typical gas system uses a pipe of internal diameter 0.914 m, and with natural gas a power transfer of 12 GW is possible at a pressure of 68 atm and a velocity of 7 m/s. A 1 m diameter pipe carrying hydrogen gas can transmit 8 GW of power, equivalent to 4–400 kV, three phase transmission lines.

The major advantage of hydrogen is, of course, that it can be stored; the major disadvantage is that it must be produced from water via electrolysis. Alternative methods of production are under laboratory development, e.g. use of heat from nuclear stations to 'crack' water and so release hydrogen; how-

ever, temperatures of 3000°C are required. Very large electrolysers can attain efficiencies of about 60 per cent. This, coupled with the efficiency of electricity production from a nuclear plant, gives an overall efficiency of hydrogen production of about 21 per cent. However, it would involve nuclear plant at off-peak periods and this, combined with the facility for storage, could be attractive.

A schematic of such a system is shown in Figure 1.24. As fossil fuel prices increase, and allowing for industry's considerable needs for hydrogen, the idea will become more attractive, even if on a limited scale. Large-scale fuel cell installations can then be used for the conversion of hydrogen energy back to electrical energy. Also, the use of hydrogen as fuel for aircraft and automobiles could give impetus to its large-scale production, storage, and distribution.

1.5.7 Superconducting magnetic energy stores (SMES)

Continuing development of the so-called high-temperature superconductor, where the transition temperature can be around 60–80 K (K is degrees Kelvin where 0 K is absolute zero and 273 K is 0°C) has led to the possibility to store energy in the magnetic field produced by circulating a large current (over 100 kA) in an inductance. For a coil of inductance L in air, the stored energy is given by $\frac{1}{2}LI^2$ J, which can provide 100 MW for several seconds, if required to smooth out power demand peaks, with a coil diameter of the order of 20 m.

A big advantage of the high-temperature superconductor is that cooling by liquid nitrogen can be used, which is far cheaper than using helium to reach temperatures closer to absolute zero. Initially, it is expected that commercial units will be used to provide an uninterruptible supply for sensitive loads to guard against voltage sags or to provide continuity whilst emergency generators are started. Another use in transmission networks would be to provide fast

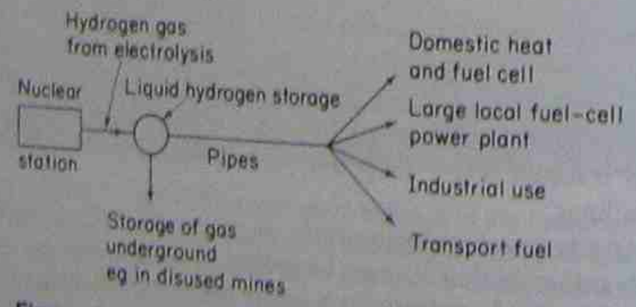


Figure 1.24 Hydrogen storage and transmission scheme

response for enhanced transient stability and improved power quality. Figure 1.25 illustrates an early SMES unit mounted in a large trailer for countering supply interruptions at remote sites.



Figure 1.25 Superconducting magnetic energy storage unit 750 kW (EPRI) *(Courtesy of EPRI, U.S.A.)*

Flywheels

The most compact energy store known is that of utilizing high-speed flywheels. Such devices coupled to an electrical generator/motor have been employed in buses on an experimental basis and also in special industrial applications. For power systems, very large flywheels constructed of composite high-tensile-resisting materials have been proposed, but their cost and maintenance problems have so far ruled them out of economic contention compared with alternative forms of energy supply.

Supercapacitors

The interface between an anode and cathode immersed in an electrolyte has a very high permittivity. This property can be exploited in a capacitor to produce a 25 V capsule with a capacitance of 0.1 F. Many units in series and parallel would have the capability of storing many MWh of energy, which can be quickly released for transient control purposes. To date, higher voltage forms of a commercially useful device have not got beyond their employment for pulse or actuator applications.

1.6 Environmental Aspects of Electrical Energy

Increasingly in industrialized nations, environmental pressure groups are having an impact on the development of energy resources, especially those involving electricity production, transmission, and distribution. Conversion of one form of energy to another produces unwanted side effects and, often, pollutants which need to be controlled and disposed of. In addition, safety and health are subject to increasing legislation by national and international bodies, thereby requiring all engineers to be aware of the laws and regulations governing the practice of their profession.

It must be appreciated that the extraction of fossil fuels from the earth is not only a hazardous business but also, nowadays, one controlled through licensing by governments, etc. Even hydro plants require careful study and investigation through modelling, widespread surveys, and impact statements to gain acceptance. Large installations of all kinds require often lengthy planning inquiries which are both time consuming and expensive, thereby delaying the start up of energy extraction and production. As a consequence, methods of producing electrical energy which avoid or reduce the inquiry process are to be favoured over those needing considerable consultation before receiving the go-ahead. This is likely to favour small-scale projects or the redevelopment of existing sites where industry or production facilities are already operating.

In recent years, considerable emphasis has been placed on 'sustainable development', by which is meant the use of technologies that do not harm the environment, particularly in the long term. It also implies that anything we do now to affect the environment should be recoverable by future generations. Irreversible damage, e.g. removal of the ozone layer or increase in CO_2 in the atmosphere, should be avoided.

1.6.1 Atmospheric pollution

The emissions associated with power plants are mainly sulphur oxides, particular matter, and nitrogen oxides. Of the former, sulphur dioxide accounts for about 95 per cent and is a by-product of the combustion of coal or oil. The sulphur content of coal varies from 0.3 to 5 per cent, and for generation purposes is specified by some U.S. state laws to be below a certain percentage. In the eastern U.S.A. this has led to the widespread use of coal from western states because of its lower sulphur content or the use of gas as an alternative fuel. Sulphur dioxide forms H_2SO_4 in the air which causes damage to buildings and vegetation. Sulphate concentrations of 9–10 $\mu g/m^3$ of air aggravate asthma and lung and heart disease. This level has been freqently exceeded in the past, a notorious episode being the London fog of 1952 (caused by domestic coal burning and not by the electricity industry). It should be noted that although sulphur does not accumulate in the air it does so in the soil.

Sulphur oxide emission can be controlled by:

- the use of fuel with, say, less than 1 per cent sulphur;
- the use of chemical reaction to remove the sulphur, in the form of sulphuric acid, from the combustion products, e.g. limestone scrubbers or fluidized-bed combustion;
- removing the sulphur from the coal by gasification or flotation processes.

In Europe (particularly Germany and the U.K.) the governments limit the amount of SO_2, NO_x, and particulate emission, as in the U.S.A. This has led to the retrofitting of 'flue gas desulphurization' (FGD) scrubbers to some coal-burning plants, thereby increasing their costs of production by up to 20 per cent. Emissions of NO_x can be controlled by fitting advanced technology burners which can ensure a more complete combustion process, thereby reducing the oxides going up the stack (chimney).

Particulate matter refers to particles in the air. These, in sufficient concentrations, are injurious to the respiratory system, and by weakening resistance to infection may well affect the whole body. Apart from settling on the ground or buildings to produce dirt, a further effect is the reduction of the solar radiation entering the polluted area. Reported densities (particulate mass in 1 m^3 of air)

are 10 $\mu g/m^3$ in rural areas to 2000 $\mu g/m^3$ in polluted areas. The average value in U.S. cities is about 100 $\mu g/m^3$.

About one-half of the oxides of nitrogen in the air in populated areas are due to power plants and originate in high-temperature combustion processes. At levels of 25–100 parts per million they can cause acute bronchitis and pneumonia. Increasingly, city pollutants are due to cars and lorries and not power plants.

A 1000 MW(e) coal plant burns approximately 9000 t of coal per day. If this has a sulphur content of 3 per cent the amount of SO_2 emitted per year is 2×10^5 t. Such a plant produces the following pollutants per hour (in kg): CO_2 8.5×10^5, CO 0.12×10^5, sulphur oxides 0.15×10^5, nitrogen oxides 3.4×10^3, and ash. Both SO_2 and NO_x are reduced considerably by the measures mentioned above. Gas-fired CCGT plants produce very little NO_x or SO_2 and their CO_2 output is about 55 per cent of an equivalent size coal-fired generator.

The concentration of pollutants can be reduced by dispersal over a wider area by the use of high stacks. If, in the stack, a vertical wire is held at a high negative potential relative to the wall, the expelled electrons from the wire are captured by the gas molecules moving up the stack. Negative ions are formed which accelerate to the wall, collecting particles on the way. When a particle hits the wall the charge is neutralized and the particle drops down the stack and is collected. Precipitators have particle-removing (by weight) efficiencies of up to 99 per cent, but this is misleading as performance is poor for small particles of, say, less than 0.1 μm in diameter. The efficiency based on number of particles removed is therefore less. Disposal of the resulting fly-ash is expensive, but the ash can be used for industrial purposes, e.g. building blocks. Unfortunately the efficiency of precipitators is enhanced by a reasonable sulphur content in the gases. For a given collecting area the efficiency decreases from 99 per cent with 3 per cent sulphur to 83 per cent with 0.5 per cent sulphur at 150 °C. This results in much larger and expensive precipitator units with the low-sulphur coal or the use of fabric filters in 'bag houses' situated before the flue gas enters the stack.

1.6.2 Thermal pollution

Steam from the low-pressure turbine is liquefied in the condenser at the lowest possible temperatures to maximize the steam-cycle efficiency. Where copious supplies of water exist the condenser is cooled by 'once-through' circulation of sea- or river-water. Where water is more restricted in availability, e.g. away from the coasts, the condensate is circulated in cooling towers in which it is sprayed in nozzles into a rising volume of air. Some of the water is evaporated, providing cooling. The latent heat of water is 2×10^6 J/kg compared with a sensible heat of 4200 J/kg per degree C in 'once-through' cooling.

A disadvantage of such towers is the increase in humidity produced in the local atmosphere.

Dry cooling towers in which the water flows through enclosed channels (similar to a car radiator), past which air is blown, avoid local humidity problems, but at a much higher cost than 'wet towers'. Cooling towers emit evaporated water to the atmosphere in the order of 75000 litres/min for a 1000 MW(e) plant. A crucial aspect of once-through cooling in which the water flows directly into the sea or river is the increased temperature of the latter due to the large volume per minute (typically 360 m^3/s for a coolant rise of 2.4 °C for a 2.4 GW nuclear station) of heated coolant. Because of their lower thermal efficiency, nuclear power stations require more cooling water than fossil-fuelled plants.

The chemical reaction rate doubles for each 10 °C rise in temperature, causing an increased demand for oxygen, but the ability of the water to dissolve oxygen is less at the higher temperatures. Hence extreme care must be taken to safeguard marine life, although the higher temperatures can be used effectively for marine farming if conditions can be controlled.

1.6.3 Electromagnetic radiation from overhead lines, cables, and equipment

The biological effects of electromagnetic radiation have produced considerable concern among the general public as to the possible hazards in the home and the workplace. Proximity of dwellings to overhead lines and even buried cables has also led to concerns of possible cancer-inducing effects, with the consequence that research effort has been needed to allay such fears to show that they are unfounded. Of course, it must be remembered that radiologists have employed ionizing and very-high-frequency radiation for medical treatment over many years, but the power frequencies used (50 or 60 Hz) are not considered harmful. The electric field and magnetic field strengths below typical H.V. transmission lines are given in Table 1.2.

Considerable international research and cooperative investigation has now been proceeding for over 20 years into low-frequency electric and magnetic field exposure produced by household appliances, video display terminals, and local power lines. To date there is no convincing evidence that they pose any demonstrable health hazards. Epidemiologic findings of an association between electric and magnetic fields and childhood leukaemia or other childhood or adult cancers are inconsistent and inconclusive; the same is likely to be true of birth defects or other reproductive problems.

It is important for the electrical and electronic engineer to put the human effects of radiation into perspective and to remember that many other environmental agents can produce far greater damage to health than does electromagnetic radiation from power equipment.

Table 1.2 Likely maximum electric and magnetic field strengths directly under overhead lines. Note that magnetic field is dependent upon current carried

Line voltage (V)	Electric field strength (V/m)	Magnetic flux density (μT)
400 000	11 000	40
275 000	6000	40
132 000	2000	11
33 000	350	7
11 000	120	7
415	< 1	1
National Radiation Protection Board Guidelines for Safety	10 000–15 000	1600
Earth's magnetic field	—	40–50

(*Source: Electricity Association Services Ltd, London 1996*)

Further aspects of fields produced by power lines are discussed in Chapter 3.

1.6.4 Visual and audible noise impacts

The presence of overhead lines constitutes an environmental problem (perhaps the most obvious one) on several counts.

1. Space is used which could be used for other purposes. The land allocated for the line is known as the right of way (or wayleave in Britain). The area used for this purpose is already very appreciable.
2. Lines are considered by many to mar the landscape. This is, of course, a subjective matter, but it cannot be denied that several lines converging on a substation or plant, especially from different directions, is offensive to the eye.
3. Radio interference (RI), audible noise (AI), and safety considerations must also be considered.

Although most of the above objections could be overcome by the use of underground cables, these are not free of drawbacks. The limitation to cable transmitting current because of temperature-rise considerations coupled with high manufacture and installation costs results in the ratio of the cost of transmitting energy underground to that for overhead transmission being between 10 and 20, which increases with operating voltage. With novel cables, such as superconducting, still under development it is hoped to reduce this

disadvantage. Estimates made of the increased costs resulting from the undergrounding of whole or large parts of a power system indicate that an intolerable financial burden would result on the utilities and hence the consumer. However, it may be expected that in future a larger proportion of circuits will be carried underground, especially in suburban areas. In large urban areas circuits are invariably underground, thereby posing increasing problems as load intensities increase.

Reduction of RI can be achieved in the same way that electromagnetic effects are reduced, but audible noise is a function of the line design. Careful attention to the tightness of joints, the avoidance of sharp or rough edges, and the use of earth screen shielding can reduce AI to acceptable levels at a distance dependent upon voltage.

Safety clearances dependent upon International Standards are, of course, extremely important and must be maintained in adverse weather conditions. The most important of these is the increased sag due to ice and snow build up in winter.

1.7 Transmission and Distribution Systems

1.7.1 Representation

Modern electricity supply systems are invariably three-phase. The design of distribution networks is such that normal operation is reasonably close to balanced three-phase working, and often a study of the electrical conditions in one phase is sufficient to give a complete analysis. Equal loading on all three phases of a network is ensured by allotting, as far as possible, equal domestic loads to each phase of the low-voltage distribution feeders; industrial loads usually take three-phase supplies.

A very useful and simple way of graphically representing a network is the schematic or line diagram in which three-phase circuits are represented by single lines. Certain conventions for representing items of plant are used and these are shown in Figure 1.26. A typical line or schematic diagram of a part of a power system is shown in Figure 1.27. In this, the generator is star connected, with the star point connected to earth through a resistance. The nature of the connection of the star point of rotating machines and transformers to earth is of vital importance when considering faults which produce electrical imbalance in the three phases. The generator feeds two three-phase lines (overhead or underground). The line voltage is increased from that at the generator terminals by transformers connected as shown. At the end of the lines the voltage is reduced for the secondary distribution of power. Two lines are provided to improve the *security* of the supply, i.e. if one line develops a fault and has to be switched out the remaining one still delivers power to the receiving end. It is

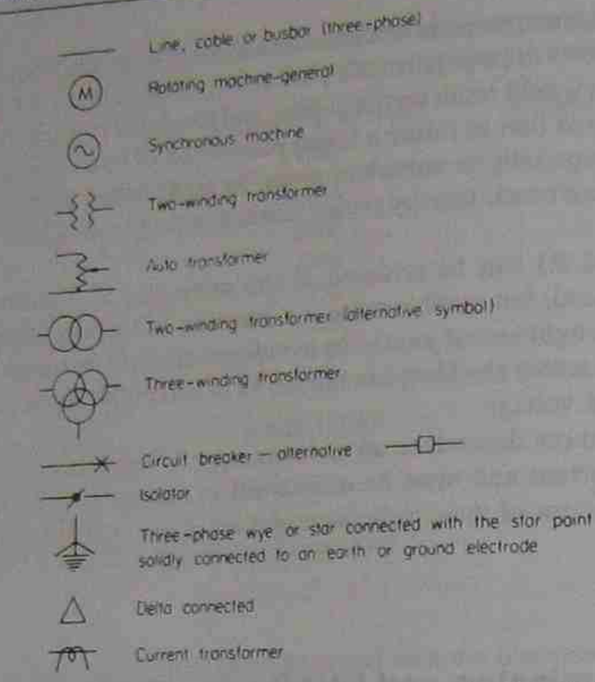


Figure 1.26 Symbols for representing the components of a three-phase power system

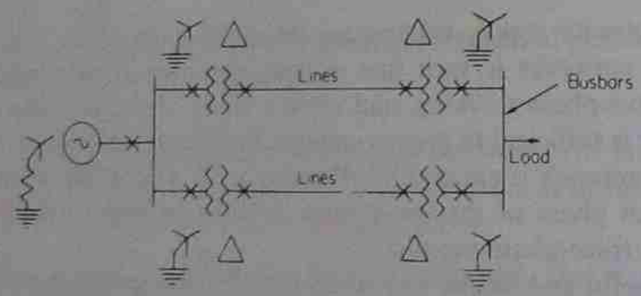


Figure 1.27 Line diagram of a simple system

not necessary in straightforward current and voltage calculations to indicate the presence of switches, etc., on the diagrams, but in some cases, such as stability calculations, the location of switches, current transformers, and protection is very useful.

Although the use of jargon is avoided as much as possible, certain contantly recurring terms may be rather vague to the newcomer to the subject. A short list of such terms follows, with explanations.

Systems This is used to describe the complete electrical network: generators, loads, and prime movers.

Load This may be used in a number of ways: to indicate a device or collection of devices which consume electricity; to indicate the power required from a given supply circuit; to indicate the power or current being passed through a line or machine.

Busbar This is an electrical connection of zero impedance joining several items, such as lines, loads, etc. Often this takes the form of actual busbars of copper or aluminium.

Earthing (Grounding) This is connection of a conductor or frame of a device to the main body of the earth. This must be done in such a manner that the resistance between the item and the earth is below prescribed limits. This often entails the burying of the large assemblies of conducting rods in the earth and the use of connectors of large cross-sectional area.

Fault This is a malfunctioning of the network, usually due to the short-circuiting of two conductors or live conductors connecting to earth.

Outage Removal of a circuit either deliberately or inadvertently.

Security of supply Provision must be made to ensure continuity of supply to consumers, even with certain items of plant out of action. Usually, two circuits in parallel are used and a system is said to be secure when continuity is assured. This is obviously the item of first priority in design and operation.

1.7.2 Transmission

Transmission normally implies the bulk transfer of power by high-voltage links between main load centres. Distribution, on the other hand, is mainly concerned with the conveyance of this power to consumers by means of lower voltage networks.

Generators usually produce voltages in the range 11–25 kV, which is increased by transformers to the main transmission voltage. At substations the connections between the various components of the system, such as lines and transformers, are made and the switching of these components is carried out. Large amounts of power are transmitted from the generating stations to the load-centre substations at 400 kV and 275 kV in Britain, and at 345 kV, 765 kV, and 500 kV in the U.S.A. The network formed by these very-high-

voltage lines is sometimes referred to as the *supergrid*. Most of the large and efficient stations feed through transformers directly into this network. This grid, in turn, feeds a subtransmission network operating at 132 kV in Britain and 115 kV in the U.S.A. Some of the older and sometimes less efficient stations feed into this system which, in turn, supplies networks which are concerned with distribution to consumers in a given area. In Britain these networks operate at 33 kV, 11 kV, or 6.6 kV and supply the final consumer feeders at 400–415 V three-phase, giving 230–240 V per phase. Other voltages exist in isolation in various places, e.g. the 66 and 22 kV London cable system. A typical part of a supply network is shown schematically in Figure 1.28. The

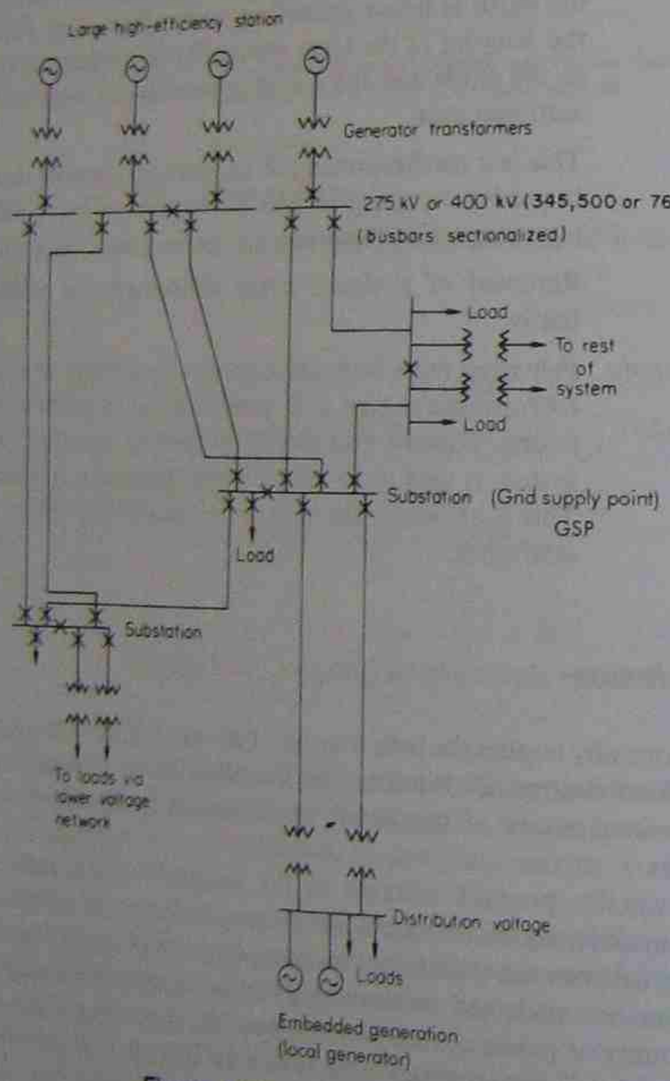


Figure 1.28 Part of a typical power system

power system is thus made up of networks at various voltages. There exist, in effect, voltage tiers as represented in Figure 1.29.

Summarizing transmission networks deliver to wholesale outlets at 132 kV and above; subtransmission networks deliver to retail outlets at voltages from 115 kV or 132 kV, and distribution networks deliver to final step-down transformers at voltages below 132 kV, usually operated as radial systems.

Reasons for interconnection

Many generating sets are large and 2000+ MW capacity stations are available to provide base or intermediate load. With CCGT units, their high efficiency and cheap long-term gas contracts, regardless of geographical position, mean that it is more economical to use these efficient stations to full capacity 24 h a day and transmit energy considerable distances than to use less efficient more local stations. The main base load therefore is met by these high-efficiency stations which must be interconnected so that they feed into the general system and not into a particular load.

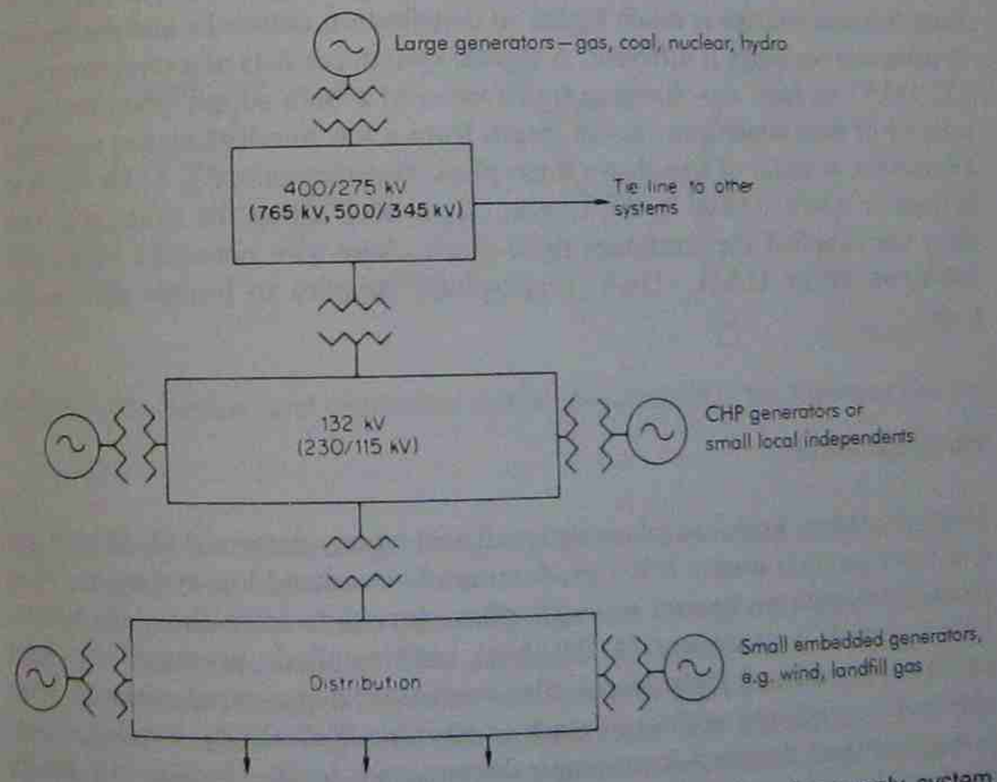


Figure 1.29 Schematic diagram of the constituent networks of a supply system. U.S.A. voltages in parentheses

To meet sudden increases in load a certain amount of generating capacity, known as the 'spinning reserve', is required. This consists of generators running at normal speed and ready to supply power instantaneously. If the machines are stationary a reasonable time is required (especially for steam turboalternators) to run up to speed; this can approach 1 h although gas turbines can be started and loaded in 3 min or less. Hydro generators can be even quicker. It is more economical to have certain stations serving only this function than to have each station carrying its own spinning reserve.

The electricity supplies over the entire country are synchronized and a common frequency exists: 50 Hz in Europe, 60 Hz in N. America.

Interconnection also allows for alternative paths to exist between generators and bulk supply points supplying the distribution systems. This provides security of supply should any one path fail.

1.7.3 Distribution systems

Although in this book there is a bias towards transmission networks and the associated problems, the general area of distribution should not be considered to be of secondary importance. Distribution networks differ from transmission networks in several ways, quite apart from voltage magnitude. The number of branches and sources is much higher in distribution networks and the general structure or topology is different. A typical system consists of a step-down (e.g. 132/11 kV) on-load tap-changing transformer at a bulk supply point feeding a number of lines which can vary in length from a few hundred metres to several kilometres. A series of step-down three-phase transformers, e.g. 11 kV/415 V in Britain or 4.16 kV/220 V in the U.S.A., are spaced along the route and from these are supplied the consumer three-phase, four-wire networks which give 240 V, or, in the U.S.A., 110 V, single-phase supplies to houses and similar loads.

Rural systems

In rural systems, loads are relatively small and widely dispersed (5–50 kVA per consumer group is usual). It is a predominantly overhead line system at 11 kV, three-phase, with no neutral or single phase for spurs from the main system. Pole-mounted transformers (5–200 kVA) are installed, protected by fuses which require manual replacement after operation; hence rapid access is desirable by being situated as close to roads as possible. Essentially, a radial system is supplied from one step-down point; distances up to 10–15 miles (16–24 km) are feasible with total loads of 500 kVA or so (see Figure 1.30), although in sparsely populated areas, distances of 50 miles may be fed by 11 kV. Single-

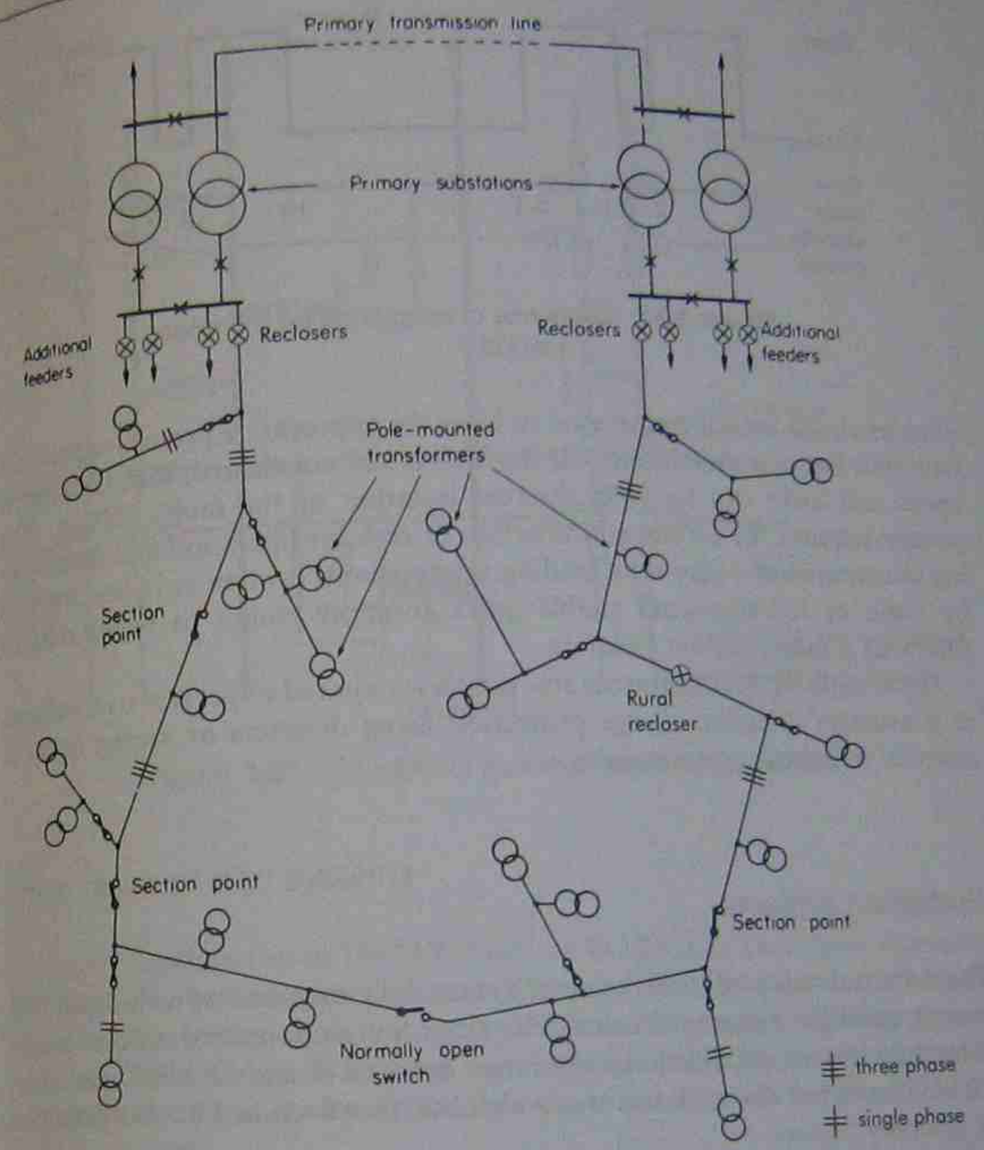


Figure 1.30 Typical rural distribution system (*Permission of the Electricity Council*)

phase earth-return systems operating at 20 kV are now being used increasingly in developing countries.

Over 80 per cent of faults are transitory due to flashover following some natural or man-made cause. This produces unnecessary fuse-blowing unless autoreclosers are employed on the main supply; these have been used with great success in either single- or three-phase form. The principle, as shown in Figure 1.31, is to open on fault before the fuse has time to operate and to reclose after 1–2 s. If the fault still persists, a second attempt is made to clear, followed by another reclose. Should the fault still not be cleared, the recloser

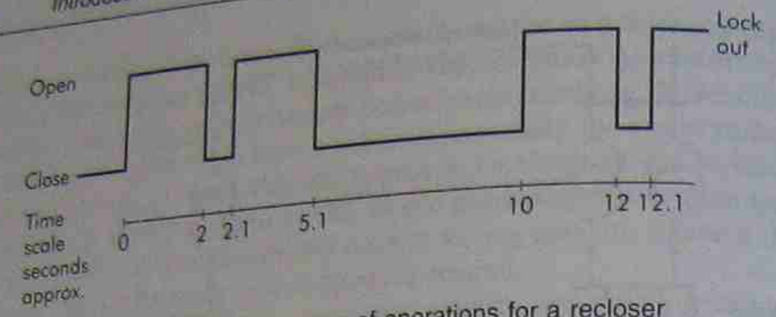


Figure 1.31 Sequence of operations for a recloser

remains closed for a longer period to blow the appropriate protective fuse (e.g. on a spur line or a transformer). If the fault is still not cleared, then the recloser opens and locks out to await manual isolation of the faulty section. This process requires the careful coordination of recloser operation and fuse-blowing characteristics where time grading is important. Section switches operated by radio or tele-command enable quick resupply routes to be established following a faulty section isolation.

Good earthing at transformer star points is required to prevent overvoltages at consumers' premises. Surge protection using diverters or arcing horns is essential in lightning-prone areas.

Surburban systems

These are a development of the rural system into ring mains, with much of it underground for amenity reasons. The rings are sectionalized so that simple protection can be provided. Loads range between 2 and 10 MW/mile^2 (0.8–4.0 MW/km^2) but diversity factor is wide (2–4) (see Section 1.8.1 for definition of diversity factor).

In high-density housing areas, the practice is to run the L.V. mains on either side of each road, interconnected at junctions by links for sectionalizing (see Figure 1.32). At appropriate points this network is fed by a step-down transformer of 200–500 kVA rating connected to an H.V. cable or overhead line network. Reinforcement is provided by installing further step-down transformers tapped from the H.V. network. It is very rare to up-rate L.V. cables as the load grows.

Short-circuit levels are fairly low due to long feeders from the bulk supply points. With the increasing cost of cables and undergrounding works, but with improved transformer efficiency and lower costs due to rationalization and standardization, an economic case can be made for reducing the L.V. network and extending the H.V. network so that fewer consumers are supplied from each transformer.

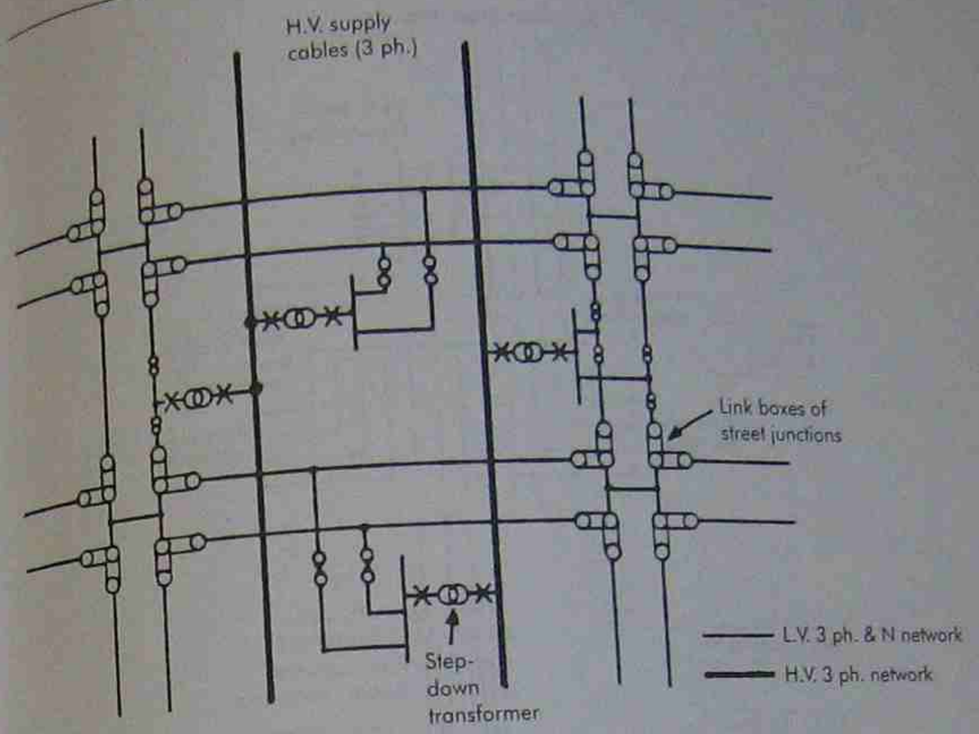


Figure 1.32 Principle of suburban distribution system

Urban (town or city) systems

Very heavy loadings (up to 100 MW/mile^2 or 40 MW/km^2) are usual, especially where high-rise buildings predominate. Extensive heating and air-conditioning loads as well as many small motors predominate. Fluorescent lighting reduces the power factor and leads to some waveform distortion but rectifier-fed loads and power electronic motor drives now cause considerable harmonics on all types of network.

Again, a basic L.V. grid, reinforced by extensions to the H.V. network, as required, produces minimum costs overall. The H.V. network is usually in the form of a ring main fed from two separate sections of a double busbar substation where 10–60 MVA transformers provide main supply from the transmission system (see Figure 1.33). The H.V. network is sectionalized to contain short-circuit levels and to ease protection problems. A high security of supply is possible by overlapping H.V. rings so that the same L.V. grid is fed from several transformers supplied over different routes. Failure of one portion of the H.V. system does not affect consumers, who are then supported by the L.V. network from adjacent H.V. supplies.

In the U.K. transformers of 500 or 750 kVA rating are now standard, with one H.V. circuit breaker or high rupturing capacity (HRC) fuse and two isolators either side to enable the associated H.V. cable to be isolated

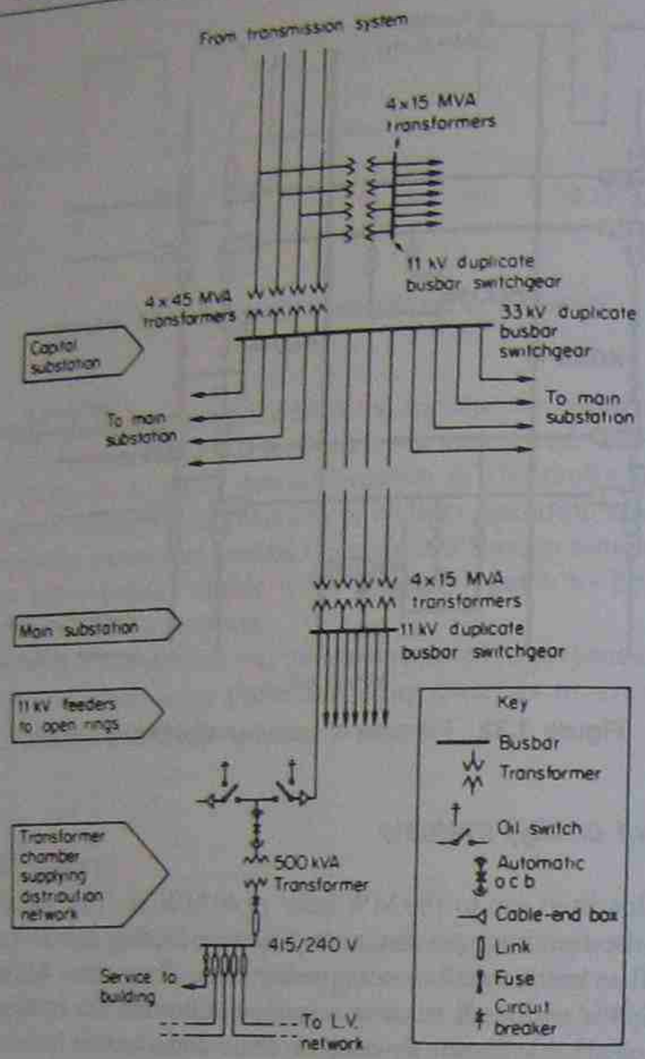


Figure 1.33 Typical arrangement of supply to an urban network—British practice (Reproduced by permission of the *Institution of Electrical Engineers*)

manually in the event of failure. Average H.V. feeder length is less than 1 mile and restoration of H.V. supplies is usually obtainable in under 1 h. Problems may arise due to back-feeding of faults on the H.V. system by the L.V. system, and in some instances reverse power relay protection is necessary.

In new developments it is essential to acquire space for transformer chambers and cable access before plans are finalized. High-rise buildings may require substations situated on convenient floors as well as in the basement.

Industrial systems

Apart from the supply of new industrial and housing estates or the electrification of towns and villages, a large part of the work of a planning engineer is involved with the up-rating of existing supplies. This requires good load forecasting over a period of 2–3 years to enable equipment to be ordered and access to sites to be established. One method of forecasting is to survey the demand on transformers by means of a maximum-demand indicator and to treat any transformer which has an average load factor of 70 per cent or above as requiring up-rating over the next planning period. Another method is to analyse consumers' bills, sectionalized into areas, and distributors and, by surveys and computer analysis, to relate energy consumed to maximum demand. This method could be particularly useful and quite economic where computerized billing is used.

In practice, good planning requires sufficient data on load demands, energy growth, equipment characteristics, and protection settings. All this information can be stored and updated from computer files at periodic intervals and provides the basis for the installation of adequate equipment to meet credible future demands without unnecessary load shedding or dangerous overloading.

1.7.4 Typical power systems

Throughout the world the general form of power systems follows the same pattern. Magnitudes of voltage vary from country to country, the differences originating mainly from geographical and historical reasons.

Several frequencies have existed, although now only two values—50 Hz and 60 Hz—remain: 60 Hz is used on the American continent, whilst most of the rest of the world uses 50 Hz, although Japan still has 50 Hz for the main island and 60 Hz for the northern islands. The value of frequency is a compromise between higher generator speeds (and hence higher output per unit machine volume) and the disadvantage of high system reactance at higher frequencies. Historically, the lower limit was set by the need to avoid visual discomfort from electric lamps.

The distance that a.c. trnasmission lines can transfer power is limited by the maximum permissible peak voltage between conductor and ground. As voltages increase, more and more clearance must be allowed in air to prevent the possibility of flashover or danger to people or animals on the ground. Unfortunately, the critical flashover voltage increases non-linearly with clearance, such that, with long clearances, proportionally less peak voltages can be safely used. Figure 1.34 shows this effect where critical flashover voltage is 4–6 times the rated voltage of the system.

An additional limitation is the power loading that can be applied to a transmission line dependent upon its surge impedance loading (SIL). The

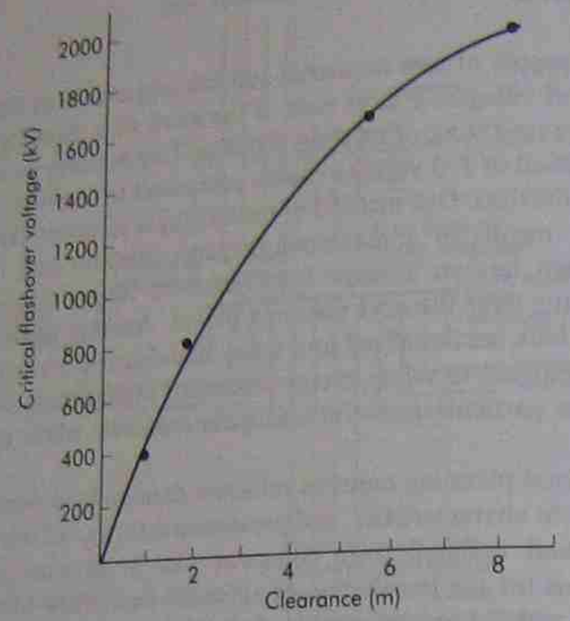


Figure 1.34 Critical flashover voltage for V-string insulators in a window tower

surge impedance is given by $Z_0 = \sqrt{L/C}$, where L is the series inductance and C is the shunt capacitance per unit length of the line (see Chapter 5). The curve and data given in Figure 1.35 show the capability for various transmission line-to-line voltages and conductor configurations.

As a consequence of these limitations, the highest a.c. voltage used for transmission is 750 kV, although experimental lines above 1000 kV have been built.

The topology of the power system and the voltage magnitudes used are greatly influenced by geography. Very long lines are to be found in North and South American nations and Russia. This has resulted in higher transmission voltages, e.g. 765 kV, with the possibility of voltages in the range 1000–1500 kV. In some South American countries, e.g. Brazil, vast amounts of hydroelectric resources are in the process of being developed, resulting in very long transmission links. In highly developed countries the available hydro resources have been utilized and a considerable proportion of new generation is nuclear. In geographically smaller countries, as exist in Europe, the degree of interconnection is much tighter, with shorter transmission distances, the upper voltage being about 420 kV.

Systems are universally a.c. with the use of high-voltage d.c. links used for specialist purposes, e.g. very long circuits, submarine cable connections, and back-to-back converters for variable-angle operation. The use of d.c. has been limited by the high cost of the conversion equipment. This requires overhead

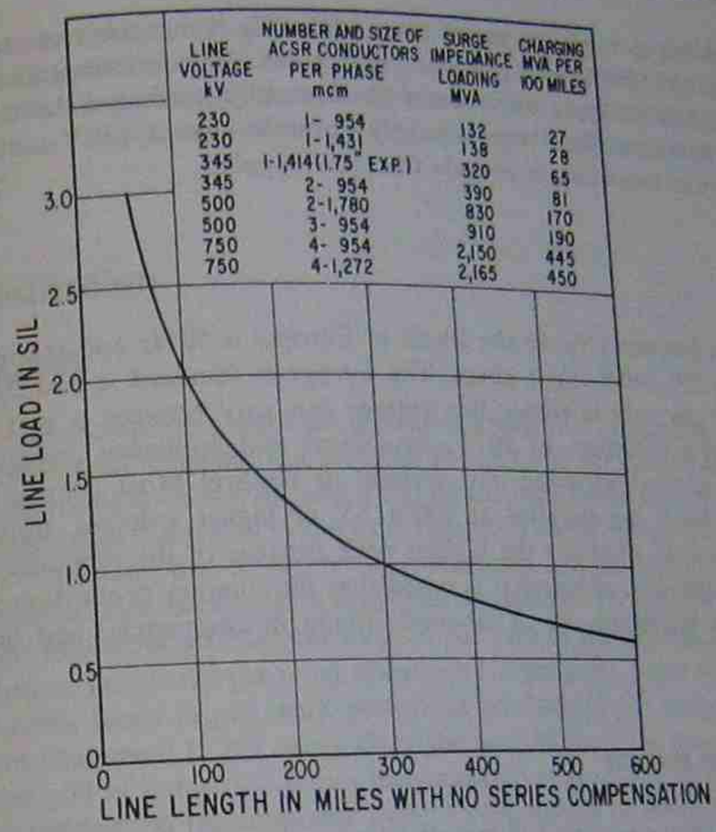


Figure 1.35 Transmission-line capability in terms of surge-impedance loading (*Permission of Edison Electric Institute, U.S.A.*)

line lengths of a few hundred kilometres or cables of 30–100 km to enable the reduced circuit costs to offset the conversion costs. Below are presented summaries of the systems of some countries.

U.S.A.

The systems in the U.S.A. are based on a comparatively few investor-owned generation/transmission utilities responsible for bulk transmission as well as operating and constructing new generation facilities when needed. Delivery is to smaller municipal or cooperative rural distribution companies or to the distribution arm of the transmission utility. The industry is heavily regulated by Federal (state) Agencies and the investor profits are carefully controlled.

The loads differ seasonally from one part of the country to another and load diversity occurs because of the time zones. Generally, the summer load is the

highest due to extensive use of air-conditioning. Consumers are supplied at either 110 V, 60 Hz, single phase for lighting and small-consumption appliances but at 220 V for loads above about 3 kW (usually cookers, water-heaters and air-conditioners). This type of supply normally uses a 220 V centre-tapped transformer secondary to provide the 110 V supply.

U.K.

Here, the frequency (as in the whole of Europe) is 50 Hz and the residential supply is 230–240 V single phase. The European standard is now 230 V, but with ±10 per cent tolerance this voltage can vary between a maximum of 253 V and a minimum of 207 V; equipment and appliance designers must take this allowed variation into account or beware! Most commercial and industrial loads are supplied at 400–415 V or higher voltages, three phase. The winter load produces the highest peak because of the preponderance of heating appliances, although it is noted that the summer proportion is growing due to increasing use of air-conditioning in commercial and industrial premises.

Continental Europe

Most continental countries still mainly have combined generator/transmission utilities, many of which cover the whole country and which are overseen by government control. Increasingly these systems are being 'unbundled', that is they are being separated into different functions (generation, transmission, distribution), each individually accountable such that private investors can enter into the electricity market (see Chapter 12). Interconnection across national boundaries enables electrical energy to be traded under agreed tariffs, and limited system support is available under disturbed or stressed operating conditions.

The daily load variation tends to be much flatter than in the U.K. because of the dominance of industrial loads with the ability to vary demand and because there is less reliance on electricity for heating in private households. Many German and Scandinavian cities have CHP plants with hot water distribution mains for heating purposes. Transmission voltages are 380–400 kV, 220 kV, and 110 kV with household supplies at 220–230 V, single phase.

Japan and the Pacific Rim

The fastest-growing systems are generally found in the Far East, particularly China, Indonesia, the Philippines and Malaysia. Voltages up to 500 kV are

used for transmission and 220–240 V are employed for households, etc. Hydroelectric potential is still quite large (particularly in China) but with gas and oil still being discovered and exploited, CCGT plant developments are underway. Increasing interconnection, perhaps by direct current, is a possibility.

1.8 Utilization

1.8.1 Loads

The major consumption groups are industrial, residential (domestic), and commercial. Industrial consumption accounts for up to 40 per cent of the total in many industrialized countries and a significant item is the induction motor. The percentage of electricity in the total industrial use of energy is expected to continue to increase due to greater mechanization and the growth of energy-intensive industries, such as chemicals and aluminium. In the U.S.A. the following six industries account for over 70 per cent of the industrial electricity consumption: metals (25 per cent), chemicals (20 per cent), paper and products (10 per cent), foods (6 per cent), petroleum products (5 per cent), and transportation equipment (5 per cent). Over the past 25 years the amount of electricity per unit of industrial output has increased annually by 1.5 per cent, but this is dependent on the economic cycle. Consumption of electricity in industrialized countries since about 1980 has been no more than 2 per cent per year due largely to the contraction of energy-intensive industries (e.g. steel manufacturing) combined with efforts to load manage and to make better use of electricity (see Chapter 12). Consequently, a tailing off of demand growth is seen, as shown in Figure 1.1. Residential loads are largely made up of refrigerators, fridge-freezers, freezers, cookers (including microwave ovens), space heating, water heating, lighting, and (increasingly in Europe) air-conditioning. Together these loads in the U.K. amount to around 40 per cent of total sales.

The commercial sector comprises offices, shops, schools, etc. The consumption here is related to personal consumption for services, traditionally a relatively high-growth quantity. In this area, however, conservation of energy measures are particularly effective and so modify the growth rate.

Quantities used in measurement of loads are defined as follows:

Maximum load The average load over the half hour of maximum output.

Load factor The units of electricity exported by the generators in a given period divided by the product of the maximum load in this period and the length of the period in hours. The load factor should be high; if it is unity, all

the plant is being used over all of the period. It varies with the type of load, being poor for lighting (about 12 per cent) and high for industrial loads (e.g. 100 per cent for pumping stations). Figure 1.36 illustrates the calculation of load factor.

Diversity factor This is defined as the sum of individual maximum demands of the consumers, divided by the maximum load on the system. This factor measures the diversification of the load and is concerned with the installation of sufficient generating and transmission plant. If all the demands occurred simultaneously, i.e. unity diversity factor, many more generators would have to be installed. Fortunately, the factor is much higher than unity, especially for domestic loads.

A high diversity factor could be obtained with four consumers by compelling them to take load as shown in Figure 1.36. Although compulsion obviously cannot be used, encouragement can be provided in the form of tariffs. An example is the two-part tariff in which the consumer has to pay an amount dependent on the maximum demand required, plus a charge for each unit of energy consumed. Sometimes the charge is based on kilovolt-amperes instead of power to penalize loads of low power factor.

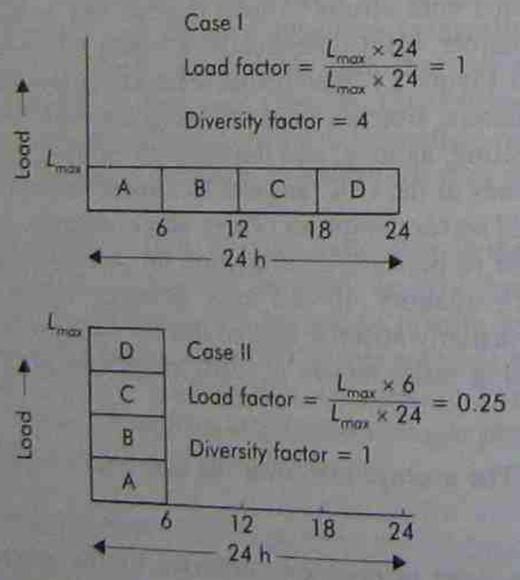


Figure 1.36 Two extremes of load factor and diversity factor in a system with four consumers

Load loss factor

The average losses in the system, e.g. transmission line losses, depend upon the load curves for any given period. The *load loss factor* is defined as the ratio of the actual losses over a period to the losses obtaining if the maximum load is maintained. This is an increasingly important factor because modern methods of charging for metered supplies depend upon accounting for losses between the generator output and the load input.

Referring to Figure 1.36, let us assume that in Case I, L_{max} is 25 kW and in Case II, L_{max} is 100 kW. Case I load is constant for 24 h but Case II is only 6 h duration and is zero for the rest of the day. Then the load factors are as given in the figure.

Now losses in the system, for most practical purposes, are given by the square of the current. But at nominal voltage and unity power factor the current is directly proportional to power in kilowatts. Hence the load loss factor (LLF) over 24 h of operation would be given by the time required for the peak load to produce the same losses as are produced by the actual load over the 24 h period. Thus

$$\text{LLF} = \frac{[\text{average demand (kW)}]^2}{[\text{peak demand (kW)}]^2}$$

In the example cases:

$$\text{Case I:LLF} = \frac{25^2}{25^2} = 1$$

$$\text{Case II:LLF} = \frac{25^2}{100^2} = 0 \cdot 0625$$

because the average demand = 100 ÷ diversity factor = 25 kW and the peak demand = 100 kW.

(Note that average demand can also be calculated as area under the demand–time curve divided by number of hours.)

In practice, the LLF varies with load factor in a quadratic manner dependent upon the daily load curve variation. It can be calculated by carrying out analytical studies on the load pattern over particular networks of known parameters. Typically, in the U.K. the LLF is given approximately by

$$\text{LLF} = 0.2\ (\text{load factor}) + 0.8\ (\text{load factor})^2$$

Tariffs

The setting of charges for the supply of electrical energy to consumers is one of the most important aspects of electricity economics. With the advent of competition in both generation and supply markets, the correct and prompt billing of customers is top priority in providing good customer services. Since such billing should now itemize the various charges imposed by the generators and the delivery networks (transmission and distribution systems), to achieve *fair* and *transparent* pricing from source to consumer requires a considerable communications and measurement infrastructure heavily dependent on efficient and compatible computer networking. Here is an excellent example of the application of Information Technology (IT) without which a modern competitive and economically efficient market in electricity supply could not be set up.

The setting of tariffs for generation, supply, and use-of-system (UoS) will be dealt with in more detail in Chapter 12. Briefly, it is worth noting here that two types of charge will be required, namely:

1. A *capacity charge*, to pay for the installed equipment required to produce electrical energy and to deliver it to the customers. This charge is often called the *standing charge* and is dependent, in principle, on the installed capacity assets of the various utilities handling the electrical power from generating sources, through transmission and distribution systems, and supplying the customer's premises. It is designated in monetary units per kilowatt of peak demand (£/kW or $/kW).
2. An *energy charge* to reflect the costs of the primary energy input (coal, oil, gas, hydro, etc.) and the cost of converting it to electrical energy, plus the losses incurred in delivering it to the customer. This charge is dependent upon the energy metered into the consumer's premises and is designated in monetary units per kilowatt-hour (p/kWh or c/kWh).

It should be remarked upon here that whereas the energy consumed is a cumulative quantity measured over a number of hours, days, or years, the peak demand is a 'spot' value which is often recorded only once a year. In practice, and so that the overload capability of equipment is taken into account, the peak kilowatt value is taken as the kilowatt-hour value consumed over, say, 30 min, divided by 0.5 (i.e. half-hour). Naturally, a high load factor (above 0.6) will lead to better utilization of system equipment than a low load factor as this often determines the fuel type which can attract investment.

Example 1.2

Compare the overall cost per unit generated by coal, oil, gas, and nuclear power stations given the following data:

Fuel type	Capital cost (£/kW)	Fuel cost (p/kWh)	Efficiency (%)
Coal	600	1.0	38
Oil	500	2.0	40
Gas	300	1.3	56
Nuclear	1500	0.2	30

Assume interest on capital and depreciation per annum is 8 per cent and that all have similar life times.

Solution

It is convenient to work on the basis of 1 kW output and over 1 year. The effect of load factor can be explored by repeating the calculation over a range of load factors. Table 1.3 has been compiled, using coal-fired generation data as an example.

Capital cost on an annual basis for each kW installed

$$= 600 \times \frac{8}{100} = £48$$

Energy generated per kW per annum

$$= \text{LF} \times 365 \times 24 = 0.4 \times 8760 = 3504\text{kWh}$$

Cost of 3504 kWh is given by

$$\frac{\text{Cost per unit}}{\text{Efficiency of conversion}} \times 3504 = \frac{1.0}{0.38} \times \frac{3504}{100} = £92.21$$

Total cost per annum to produce 3504 kWh is capital cost + generating cost

$$= 48 + 92.21 = £140.21$$

Hence, cost per unit generated is

$$\frac{140.21}{3504} = £0.0400 \quad \text{or 4p per unit}$$

Table 1.3 Cost in pence per unit at various load factors

	LF				
Fuel type	0.4	0.5	0.6	0.7	0.8
Coal	4.00	3.73	3.54	3.41	3.32
Oil	6.14	5.91	5.76	5.65	5.57
Gas	3.00	2.87	2.78	2.71	2.66
Nuclear	4.09	3.41	2.95	2.62	2.38

It is worth noting from Table 1.3 how nuclear generation is heavily dependent upon having a high LF to be able to compete with gas (using CCGT) and coal. With the prices quoted, gas is cheaper than coal at any load factor, hence emphasizing the importance of keeping coal costs down through long-term contracts.

Load management

Attempts to modify the shape of the load curve to produce economy of operation have already been mentioned. These have included tariffs, pumped storage, and the use of seasonal or daily diversity between interconnected systems. A more direct method would be the control of the load either through tariff structure or direct electrical control of appliances, the latter, say, in the form of remote on/off control of electric water-heaters where inconvenience to the consumer is least. For many years this has been achieved with domestic time switches, but some schemes use switches radio-controlled from the utility to give greater flexibility. This permits load reductions almost instantaneously and defers the hot-water and heating of air-conditioning load until after system peaks. This topic is considered further in Chapter 12.

Load forecasting

It is evident that load forecasting is a crucial activity in electricity supply. Forecasts are based on the previous year's loading for the period in question, updated by factors such as general load increases, major new loads, and weather trends. Both power demand and energy (kWh) forecasts are used, the latter often being the more readily obtained. Demand values may be determined from energy forecasts. Energy trends tend to be less erratic than peak power demands and are considered better growth indicators; however, load factors are also erratic in nature.

As weather has a much greater influence on residential than on industrial demands it may be preferable to assemble the load forecast in constituent parts to obtain the total. In many cases the seasonal variations in peak demand are caused by weather-sensitive domestic appliances, e.g. heaters and air-conditioning. A knowledge of the increasing use of such appliances is therefore essential. Several techniques are available for forecasting. These range from simple curve fitting and extrapolation to stochastic modelling and are given in detail in Sullivan, 1977. The many physical factors affecting loads, e.g. weather, national economic heath, popular TV programmes, public holidays, etc., make forecasting a complex process demanding experience and high analytical ability using probabilistic techniques including neural networks.

Problems

1.1 In the U.S.A. in 1971 the total area of right of ways for H.V. overhead lines was 16 000 km^2. Assuming a growth rate for the supply of electricity of 7 per cent per annum calculate what year the whole of the U.S.A. will be covered with transmission systems (assume area to approximate 4800 × 1600 km). Justify any assumptions made and discuss critically why the result is meaningless.
(Answer: 91.25 years)

1.2 The calorific value of natural gas at atmospheric pressure and temperature is 12×10^6 J/m^3. Calculate the energy transfer in a pipe of 1 m diameter with gas at 60 atm (gauge) flowing at 5 m/s. If *liquid* hydrogen was transferred at the same velocity calculate the energy transfer if the calorific value of liquid hydrogen is 650 times the value for gas at 1 atm.
(Answer: 2.8 GW, 30.6 GW)

1.3 (a) An electric car has a steady *output* of 10 kW over its range of 100 km when running at a steady 40 km/h. The efficiency of the car (including batteries) is 65 per cent. At the end of the car's range the batteries are recharged over a period of 10 h. Calculate the average charging power if the efficiency of the battery charger is 90 per cent.

(b) The calorific value of gasoline (petrol) is roughly 16 500 kJ/gallon. By assuming an average filling rate at a pump of 10 gallon/minute, estimate the rate of energy transfer on filling a gasoline-driven car. What range and what cost/km would the same car as (a) above produce if driven by gasoline with a 7 gallon tank? (Assume internal combustion engine efficiency is 60% and gasoline costs £3 per gallon)
(Answer: (a) 4.3 kW; (b) 2.75 MW, 76.8 km, 27p/km)

1.4 Discuss in a critical manner the possible impact of solar energy on the electrical supply industry (1 therm = 105.5×10^3 kJ)

(a) as a means of central generation;
(b) as a domestic energy source.

A solar panel (5 m × 2 m) is used for heating domestic water. The absorbing panel has an absorbtivity of 0.93, a top-plate emissivity of 0.9, and a back-plate emissivity of 0.05. The transmittance through the glass top cover is 0.85.

With a peak radiation of 700 W/m^2 the absorber temperature is 100°C and the top cover 60°C. If all the absorber heat is transferred to the water which flows through ducts in the absorber with negligible heat-transfer resistance, determine the water mass flow rate (kg/s) necessary to raise the water temperature from 15°C (inlet) to 60°C at peak radiation. State clearly any assumptions made. (Stefan–Boltzmann constant = 5.7×10^{-8} W/m^2 K^4.)
(Answer: 0.092 kg/s)

1.5 The variation of load (P) with time (t) in a power supply system is given by the expression,

$$P\,(\text{kW}) = 4000 + 8t - 0.0009t^2$$

where t is in hours over a total period of 1 year.

This load is supplied by three 10 MW generators and it is advantageous to fully load a machine before connecting the others. Determine:

(a) the load factor on the system as a whole;
(b) the total magnitude of installed load if the diversity factor is equal to 3;
(c) the minimum number of hours each machine is in operation;
(d) the approximate peak magnitude of installed load capacity to be cut off to enable only two generators to be used.
(Answer: (a) 0.74 (b) 65 MW (c) 8760, 7300, 2500 h (d) 5.4 MW)

1.6 (a) Explain why economic storage of electrical energy would be of great benefit to power systems.
(b) List the technologies for the storage of electrical energy which are available now and discuss, briefly, their disadvantages.
(c) Why is hydro power a very useful component in a power system?
(d) Explain the action of pumped storage and describe its limitations.
(e) A pumped storage unit has an efficiency of 78 per cent when pumping and 82 per cent when generating. If pumping can be scheduled using energy costing 2.0 p/kWh, plot the gross loss profit in p/kWh when it generates into the system with a marginal cost between 2p and 6p/kWh.
(f) Explain why out-of-merit generation is sometimes scheduled.
(Answer: (e) Overall efficiency 64 per cent; Profit max. 2.87 p/kWh; Loss max. 1.13 p/kWh)
(From Engineering Council (E.C.) Examination, 1996)

2
Basic Concepts

Before the modelling and analysis of power systems is pursued in detail, various basic ideas will be outlined. These include the nature of three-phase systems, the per-unit system, and circuit representation. Hopefully, some of this material will be revision for many readers.

2.1 Three-Phase Systems

The rotor flux of an alternating current machine induces sinusoidal e.m.f.s, the conductors forming the stator winding, which in a single-phase machine occupies slots over most of the circumference of the stator core. These e.m.f.s are not in phase and net winding voltage is less than the arithmetic sum of the individual conductor voltages. If this winding is replaced by three separate identical windings, as shown in Figure 2.1(a), each occupying one-third of the available slots, then the effective contribution of all the conductors is greatly increased, yielding a greatly enhanced power capability for a given machine size.

These three windings give voltages displaced in time or phase by 120°, as indicated in Figure 2.1(b). Also, because the voltage in the (a) phase reaches its peak 120° before the (b) phase and 240° before the (c) phase, the order of phase voltages reaching their maxima or *phase sequence* is a–b–c. Most countries use a,b, and c to denote the phases; however, R (Red), Y (Yellow), and B (Blue) is often also used. It is seen that the algebraic sum of the winding or phase voltages (and currents if the winding currents are equal) at every instant in time is zero. Hence, if the 'starts' of each winding are connected, then the electrical situation is unchanged and the three return lines can be dispensed with, yielding a three-phase, three-wire system, as shown in Figure 2.2(a). If the currents from the windings are not equal, then it is usual to connect a fourth wire (neutral) to the common connection or neutral point, as shown in Figure 2.2(b).

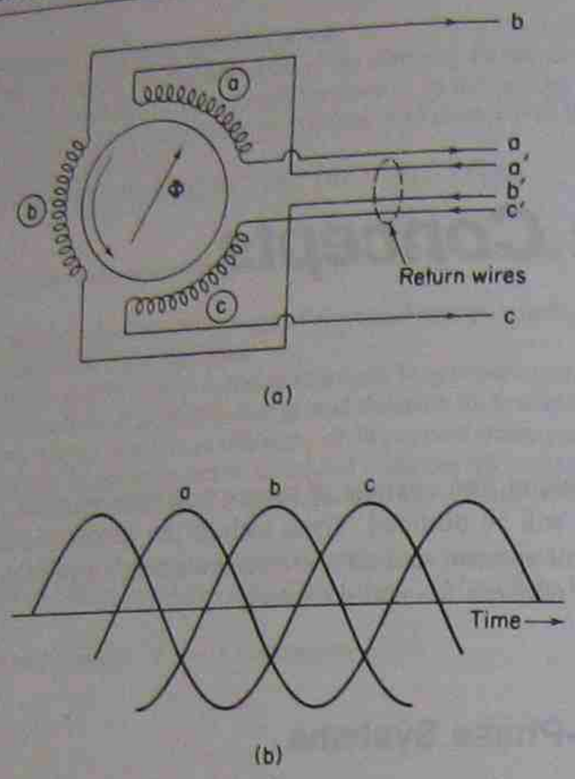


Figure 2.1 (a) Synchronous machine with three separate stator windings a, b, and c displaced physically by 120°. (b) Variation of e.m.f.s developed in the windings with time

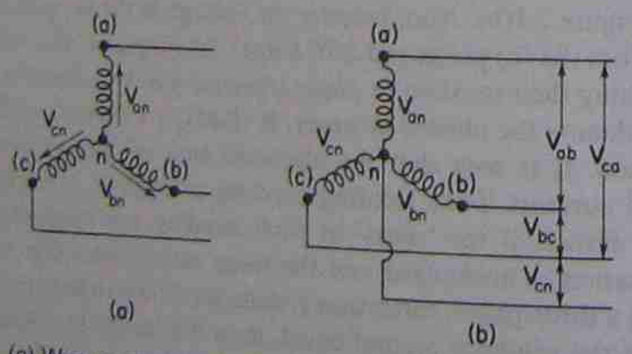


Figure 2.2 (a) Wye or star connection of windings. (b) Wye connection with neutral line

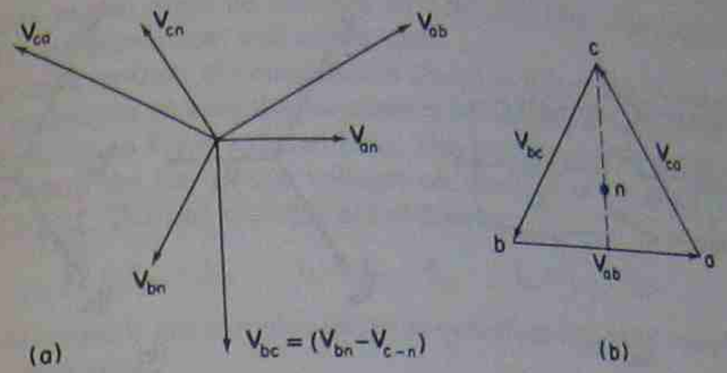


Figure 2.3 (a) Phasor diagram for wye connection. (b) Alternative arrangement of line-to-line voltages. Neutral voltage is at n, geometric centre of equilateral triangle

This type of winding connection is called 'wye' or 'star' and two sets of voltages exist:

1. the winding, phase, or line-to-neutral voltage, i.e. $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$; and
2. the line-to line voltages, $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{ca}$. The subscripts here are important, $\mathbf{V}_{ab}$, means the voltage of line or terminal (a) with respect to (b) ($\mathbf{V}_{ba} = \mathbf{V}_{ab}$). The corresponding phasor diagram is shown in Figure 2.3(a) and it can be shown that $V_{ab} = V_{bc} = V_{ca} = \sqrt{3}$ (phase voltage).

The phase rotation of a system is very important. Consider the connection through a switch of two voltage sources of equal magnitude and both of rotation a–b–c. When the switch is closed no current flows. If, however, one source is of reversed rotation (easily obtained by reversing two wires), as shown in Figure 2.4, i.e. a–c–b, a large voltage ($\sqrt{3}$ × phase voltage) exists across the switch contacts c′b and b′c, resulting in very large currents if the switch is closed. Also, on reversed phase rotation the rotating magnetic field set up by a three-phase winding is reversed in direction and an induction motor will rotate in the opposite direction, often with disastrous results to its mechanical load, e.g. a pump.

A three-phase load is connected in the same way as the machine windings. The load is *balanced* when each phase takes equal currents, i.e. has equal impedance. With the wye system the phase currents are equal to the current in the lines. The *four-wire* system is of particular use for low-voltage distribution networks in which consumers are supplied with a single-phase supply taken between a line and neutral. This supply is often 230 V or 240 V and the line-to-line voltage is 400 V or 415 V. In the U.S.A. the 220 V supply often comes into a house from a centre-tapped transformer, as shown in Figure 2.5, which in effect gives a choice of 220 V (large domestic appliances) or 110 V (lights, etc.).

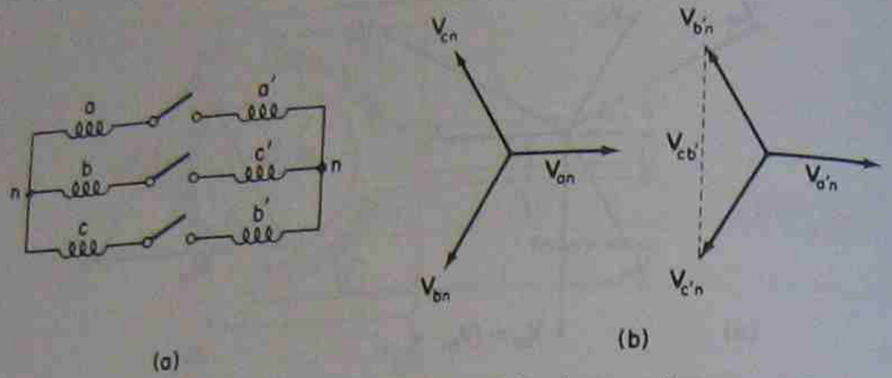


Figure 2.4 (a) Two generators connected by switch; phase voltages equal for both sets of windings. (b) Phasor diagrams of voltages. $\mathbf{V}_{cb'}$ = voltage across switch; $\mathbf{V}_{a'a} = 0$

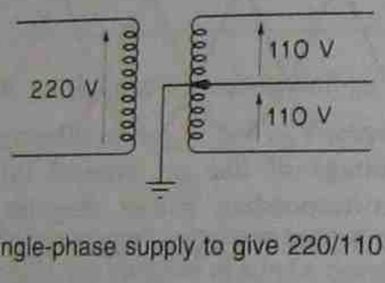


Figure 2.5 Tapped single-phase supply to give 220/110 V (centre-tap grounded)

The system planner will endeavour to connect the single-phase loads such as to provide balanced (or equal) currents in the supply lines. At any instant in time it is highly unlikely that consumers will take equal loads, and at this level of supply considerable unbalance occurs, resulting in currents in the neutral line. If the neutral line has zero impedance, this unbalance does not affect the load voltages. Lower currents flow in the neutral and it is usually of smaller cross-sectional area than the main line conductors. The combined or statistical effect of the large number of loads on the low-voltage network is such that when the next higher distribution voltage is considered (say 11 kV (line to line)), which supplies the lower voltage network, the degree of unbalance is small. This and the fact that at this higher voltage, large three-phase, balanced motor loads are supplied, allows the three-wire system to be used. It is used exclusively up to the highest transmission voltages, resulting in much reduced line costs and environmental impact.

In a balanced three-wire system a hypothetical neutral line may be considered and the conditions in only one phase determined. This is illustrated by the phasor diagram of line-to-line voltages shown in Figure 2.3(b). As the system is balanced the magnitudes so derived will apply to the other two phases but the

relative phase angles must be adjusted by 120° and 240°. This single-phase approach is very convenient and widely used.

An alternative method of connection is shown in Figure 2.6. The individual phases are connected (taking due cognizance of winding polarity in machines and transformers) to form a closed loop. This is known as the mesh or delta connection. Here the line-to-line voltages are identical to the phase voltages, i.e. $\mathbf{V}_{line} = \mathbf{V}_{phase}$. The line currents are as follows:

$$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca} \qquad \mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab} \qquad \mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc}$$

(Note: Bold symbols are complex (phasor) quantities requiring complex addition and subtraction.)

For balanced currents in each phase it is readily shown from a phasor diagram that $I_{line} = \sqrt{3} \times I_{phase}$. Obviously a fourth or neutral line is not possible with this connection. It is seldom used for rotating-machine stator windings, but is frequently used for the windings of one side of transformers for reasons to be discussed later. A line-to-line voltage transformation ration of $1:\sqrt{3}$ is obtained when going from a primary mesh to a secondary wye connection with the same number of turns per phase. Under balanced conditions the idea of the hypothetical neutral and single-phase solution may still be used (the mesh can be converted to a wye using the Δ–Y transformation).

It should be noted that three-phase systems are usually described by the *line-to-line voltage*.

2.1.1 Analysis of simple three-phase circuits

Four-wire systems

If the impedance voltage drops in the lines are negligible, then the voltage across each load is the source line or phase voltage.

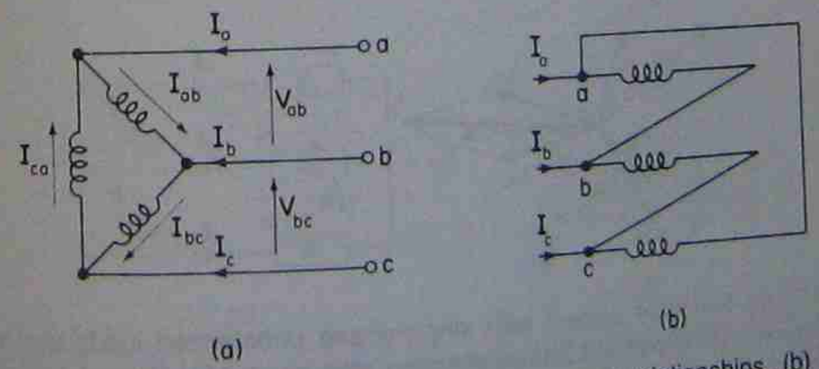


Figure 2.6 (a) Mesh or delta-connected load–current relationships. (b) Practical connections

Example 2.1

For the network of Figure 2.7(a), draw the phasor diagram showing voltages and currents and write down expressions for total line currents $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$, and the neutral current $\mathbf{I}_n$.

Solution

Note that the power factor of the three-phase load is expressed with respect to the phase current and voltage. The phasor diagram is shown in Figure 2.7(b). The line currents are as follows:

$$\mathbf{I}_a = \frac{V}{\sqrt{3}} \cdot \frac{1}{\mathbf{Z}_a} + I_m(\cos\phi - \mathrm{j}\sin\phi)$$

$$\mathbf{I}_b = \frac{V}{\sqrt{3}}(-0.5 - j0.866)\frac{1}{\mathbf{Z}_b} + I_m[-\cos(60-\phi) - \mathrm{j}\sin(60-\phi)]$$

$$\mathbf{I}_c = \frac{V}{\sqrt{3}}(-0.5 - j0.866)\frac{1}{\mathbf{Z}_c} + I_m[-\cos(60-\phi) - \mathrm{j}\sin(60-\phi)]$$

The neutral current

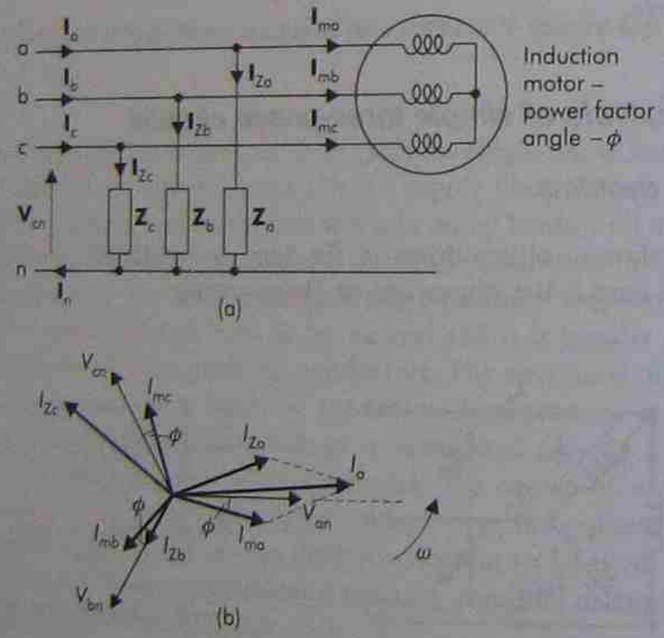


Figure 2.7 (a) Four-wire system with single-phase unbalanced loads and three-phase balanced motor load. (b) Phasor diagram: only $\mathbf{I}_{za}$ and $\mathbf{I}_{ma}$ have been phasorially added to show $\mathbf{I}_a$. $\mathbf{I}_b$, and $\mathbf{I}_c$ can be found in a similar way. Note: $\mathbf{V}_{an}$ is reference direction. Rotation is anticlockwise

$$\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = \frac{V}{\sqrt{3}}[\mathbf{Y}_a + \mathbf{Y}_b(0.5 - \mathrm{j}0.866) + \mathbf{Y}_c(-0.5 + \mathrm{j}0.866)]$$

since I_m does not contribute to I_n.

Three-wire balanced

The system may be treated as a single-phase system using phase voltages. It must be remembered, however, that the total three-phase power and reactive power are three times the single-phase values.

Three-wire unbalanced

In more complex networks the method of symmetrical components is used (see Chapter 7), but in simple situations conventional network theory can be applied. Consider the source load arrangement shown in Figure 2.8, in which $\mathbf{Z}_a \neq \mathbf{Z}_b \neq \mathbf{Z}_c$. From the mesh method of analysis, the following equation is obtained.

$$\begin{bmatrix} \mathbf{Z}_a + \mathbf{Z}_b & -\mathbf{Z}_b \\ -\mathbf{Z}_b & \mathbf{Z}_b + \mathbf{Z}_c \end{bmatrix}\begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} V \\ V(-0.5 - j0.866) \end{bmatrix}$$

Hence $\mathbf{i}_1$ and $\mathbf{i}_2$ are determined and from these are found $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ and the voltage of the neutral point (n) from ground (see Figure 2.8(b)). Depending on the severity of the imbalance, the voltage difference from (n) to ground can attain values exceeding the phase voltage, i.e. (n) can lie outside the triangle of line voltages. Such conditions produce damage to connected equipment and show the importance of earthing (grounding) the neutral.

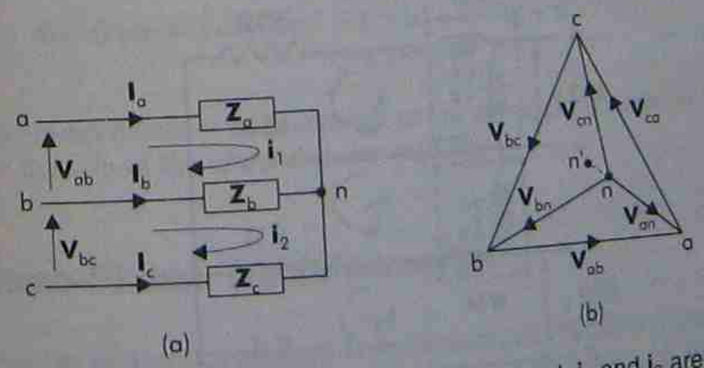


Figure 2.8 (a) Three-wire system with unbalanced load, $\mathbf{i}_1$ and $\mathbf{i}_2$ are loop currents. (b) Phasor diagram—voltage difference of neutral connection (n) from ground = n'n; n' = neutral voltage with balance

The power and reactive power consumed by a *balanced* three-phase load for both wye or mesh connections are given by

$$P = \sqrt{3} \cdot V \cdot I \cdot \cos\phi \qquad Q = \sqrt{3} \cdot V \cdot I \cdot \sin\phi$$

where

V = line-to-line voltage;
I = line current;
ϕ = angle between load phase current and load *phase voltage*.

Alternatively

$$P = 3V_{ph} \cdot I_{ph} \cdot \cos\phi \qquad \text{and} \qquad Q = 3V_{ph} \cdot I_{ph} \cdot \sin\phi$$

As the powers in each phase of a balanced load are equal, a single wattmeter may be used to measure total power. For unbalanced loads, two wattmeters are sufficient with three-wire circuits.

Example 2.2

A three-phase wye-connected load is shown in Figure 2.9. It is supplied from a 200 V, three-phase, four-wire supply of phase sequence a–b–c and the neutral line (n) has a resistance of 5 Ω. Two wattmeters are connected as shown. Calculate the power recorded on each wattmeter.

Solution

The mesh method will be used to determine the currents in the lines and hence in the wattmeter current coils.

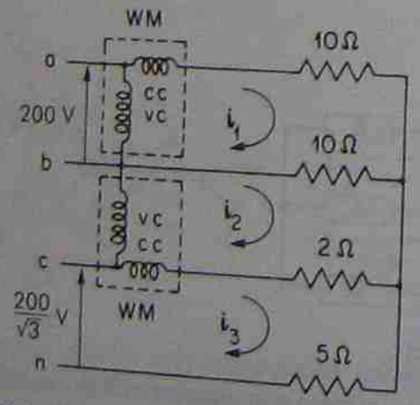


Figure 2.9 Use of two wattmeters to measure three-phase power. Arrangement for Example 2.2; cc = current coil, vc = voltage coil

The line voltage $\mathbf{V}_{ab}$ is used as a reference phasor. Hence,

$$\mathbf{V}_{bc} = (-0.5 - j0.866)200$$
$$\mathbf{V}_{ca} = (-0.5 + j0.866)200$$

and

$$\mathbf{V}_{cn} = +j\frac{200}{\sqrt{3}}$$

From Figure 2.9,

$$\begin{bmatrix} 20 & -10 & 0 \\ -10 & 12 & -2 \\ 0 & -2 & 7 \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \\ \mathbf{i}_3 \end{bmatrix} = \begin{bmatrix} 200 \\ -100 - j173.2 \\ j\frac{200}{\sqrt{3}} \end{bmatrix}$$

Eliminate $\mathbf{i}_1$.

$$\begin{bmatrix} 7 & -2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} \mathbf{i}_2 \\ \mathbf{i}_3 \end{bmatrix} = \begin{bmatrix} j173.2 \\ j\frac{200}{3} \end{bmatrix}$$

From which

$$\mathbf{i}_3 = j2.66\text{A}$$
$$\mathbf{i}_2 = -j24.0\text{A}$$
$$\mathbf{i}_1 = 10 - j12\text{A}$$

Power in wattmeter (1) = real part of $\mathbf{V}_{ab} \cdot \mathbf{I}_a^* = 200(10 + j12)$
= 2 kW

(see Section 2.5 for use of the conjugate $\mathbf{I}_a^*$)

Power in wattmeter (2) = real part of $\mathbf{V}_{cb} \cdot \mathbf{I}_c^* = (-\mathbf{V}_{bc})\mathbf{I}_c^*$
$= (100 + j173.2)(\mathbf{i}_3 - \mathbf{i}_2)^*$
$= (100 + j173.2)(-j26.66)$
= 4.612 kW

Actual power consumed $= \mathbf{I}_a^2 \cdot 10 + \mathbf{I}_b^2 \cdot 10 + \mathbf{I}_c^2 \cdot 2 + \mathbf{I}_n^2 \cdot 5$
= 6.337 kW

As the loads are not balanced and there is a return current in the neutral, the measured power is *not* the sum of the two wattmeter readings.

2.2 Three-Phase Transformers

The usual form of the three-phase transformer, i.e. the core type, is shown in Figure 2.10. If the magnetic reluctances of the three limbs are equal, then the sum of the fluxes set up by the three-phase magnetizing currents is zero. In fact, the core is the magnetic equivalent of the wye-connected winding. It is apparent

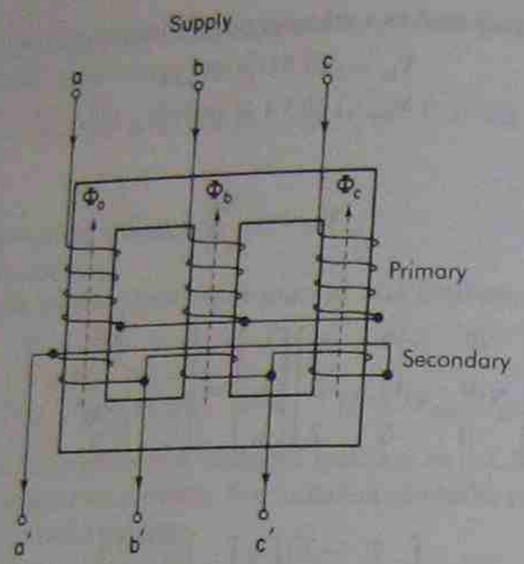


Figure 2.10 Three-phase core-type transformer. Primary connected in wye, secondary connected in mesh

from Figure 2.10 that the magnetic reluctances are not exactly equal, but in an introductory treatment may be so assumed. An alternative to the three-limbed core is the use of three separate single-phase transformers. Although more expensive (about 20 per cent extra), this has the advantage of smaller loads for transportation, and this aspect is crucial for large sizes. Also, with the installation of four single-phase units, a spare is available at reasonable cost.

The wound core, as shown in Figure 2.10, is placed in a steel tank filled with insulating oil or synthetic liquid. The oil acts both as electrical insulation and as a cooling agent to remove the losses from the windings and core. The low-voltage winding is situated over the core limb and the high-voltage winding is wound over the low-voltage one. The core comprises steel laminations insulated on one side (to reduce eddy losses) and clamped together.

Note that the voltages across the windings are phase voltages and it is these which are related by the turns ratio, but the secondary voltages are 30° out of phase with the primary voltages.

2.2.1 *Autotransformers* (Figure 2.11)

In the autotransformer only one winding is used per phase, the secondary voltage being tapped off the primary winding. There is obviously a saving in size, weight, and cost over a two-windings per phase transformer. It may be shown that the ratio of the weight of conductor on an autotransformer to that on a double-wound one is given by $(1 - N_2/N_1)$. Hence, maximum advantage

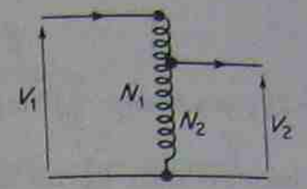


Figure 2.11 One phase of an autotransformer: $V_2/V_1 = N_2/N_1$

is obtained with a relatively small difference between the voltages on the two sides. There are, however, several requirements in a power system where connections from one voltage level to another do not entail large transformer ratios, e.g. 500 kV/345 kV, 500 kV/725 kV, and then the auto transformer is used. The effective reactance is reduced compared with the equivalent two-winding transformer and this can give rise to high short-circuit currents. The general constructional features of the core and tank are similar to those of double-wound transformers, but note that the primary and secondary voltages are now in-phase.

2.3 Harmonics in Three-Phase Systems

Harmonics may exist in three-phase systems and are created by non-linear loads, rectifiers, etc. Consider the instantaneous values of phase voltages in a balanced system containing harmonics up to the third:

$$v_a = V_1 \sin \omega t + V_2 \sin 2\omega t + V_3 \sin 3\omega t$$

$$v_b = V_1 \sin\left(\omega t - \frac{2\pi}{3}\right) + V_2 \sin 2\left(\omega t - \frac{2\pi}{3}\right) + V_3 \sin 3\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = V_1 \sin\left(\omega t - \frac{4\pi}{3}\right) + V_2 \sin 2\left(\omega t - \frac{4\pi}{3}\right) + V_3 \sin 3\left(\omega t - \frac{4\pi}{3}\right)$$

Here V_1, V_2 and V_3 are the magnitude of the harmonic voltages. The fundamental terms have the normal phase shifts (as do the fourth, seventh and tenth, etc., harmonics). However, the second harmonic terms possess a reversed phase-sequence (also fifth, eighth, eleventh, etc.), and the third harmonic terms are all in phase (also all multiples of three).

When substantial harmonics are present the $\sqrt{3}$ relation between line and phase quantities no longer holds. Harmonic voltages in successive phases are $2\pi n/3$ out of phase with each other and the resulting line voltage becomes

$$v_{ab} = V_n \sin n\omega t - V_n \sin n\left(\omega t - \frac{2\pi}{3}\right)$$

i.e.

$$v_{ab} = 2V_n \cos n\left(\omega t - \frac{\pi}{3}\right) \sin\left(\frac{n\pi}{3}\right)$$

When $n = 3, 6, 9$, etc., this becomes zero, i.e. no triple harmonics exist in the line voltages. The mesh connection forms a complete path for the triple harmonic currents which flow in phase around the loop.

When analysing the penetration of harmonics into the power network it is usual to assume that the effective reactance for the nth harmonic is n times the fundamental value.

2.4 Multiphase Systems

Although the three-phase system is universally used, attention has been given in recent years to the use of more than three phases for transmission purposes. In particular, six- and twelve-phase systems have been studied. Advantages relative to three-phase systems are as follows:

1. Increased thermal loading capacity of lines.
2. The higher the number of phases, the smaller the line-to-line voltage becomes relative to the phase voltage, resulting in increased utilization of rights of way because of less phase-to-phase insulation requirement.
3. For a given conductor size and tower configuration the stress on the conductor surface decreases with the number of phases, leading to reduced corona effects.
4. The transmission efficiency is higher. Existing double-circuit lines (two three-phase circuits on each tower) could be converted to single-circuit six-phase lines. It is advantageous to describe multiphase systems in terms of the phase voltage rather than line-to-line, as is the case for three-phase systems.

A six-phase supply is obtainable by suitable arrangement of the secondary windings of a three-phase transformer. Consider Figure 2.12. The windings on the three limbs of the transformer are centretapped with the taps mutually connected. This gives the phasor diagram shown in Figure 2.12(b). The line-to-line voltages are equal to the phase voltage in magnitude, i.e. $V_{af} = V_{an} - V_{fn} = V_{ph}$.

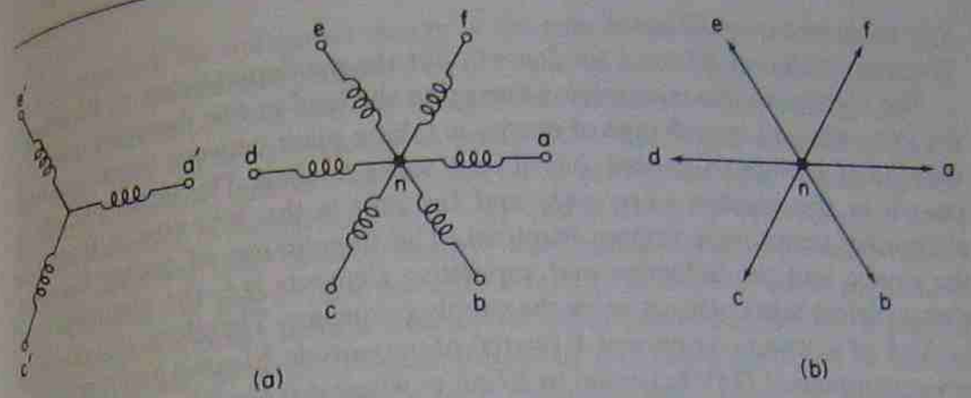


Figure 2.12 Six-phase system. (a) Transformer connections. (b) Phasor diagrams

2.5 Reactive Power

In the circuit shown in Figure 2.13, let the instantaneous values of voltage and current be

$$e = \sqrt{2}E \sin(\omega t + \phi)$$

and

$$i = \sqrt{2}I \sin(\omega t)$$

The instantaneous power

$$p = ei = EI \cos\phi - EI \cos(2\omega t + \phi)$$

also

$$-EI \cos(2\omega t + \phi) = -EI(\cos 2\omega t \cos\phi - \sin 2\omega t \cdot \sin\phi)$$

$$\therefore p = ei = (EI \cos\phi - EI \cos 2\omega t \cos\phi) + (EI \sin 2\omega t \sin\phi)$$

= (instantaneous real power) + (instantaneous reactive power)

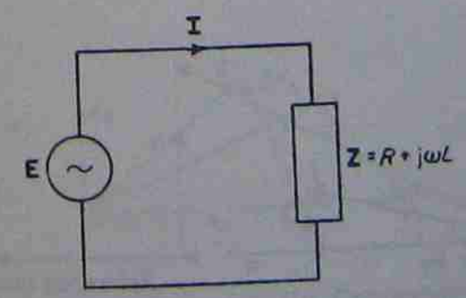


Figure 2.13 Voltage source and load

The mean power = $EI\cos\phi$.
The mean value of $EI\sin\phi\sin 2\omega t = 0$, but the maximum value = EI sin ϕ.

The voltage source is supplying energy to the load in one direction only. At the same time an interchange of energy is taking place between the source and the load of average value zero, but of peak value $EI\sin\phi$. This latter quantity is known as the *reactive power* (Q) and the unit is the VAr (taken from the alternative name, volt-ampere reactive). The interchange of energy between the source and the inductive and capacitive elements (i.e. the magnetic and electric fields) takes place at twice the supply frequency. Therefore, it is possible to think of a power component P (watts) of magnitude $EI\cos\phi$ and a reactive-power component Q (VAr) equal to $EI\sin\phi$, where ϕ is the power factor angle, i.e. the angle between **E** and **I**. It should be stressed, however, that the two quantities P and Q are physically quite different.

The quantity **S** (volt-amperes), known as the complex power, may be found by multiplying **E** by the conjugate of **I** or vice versa. Consider the case when **I** lags **E**, and assume $\mathbf{S} = \mathbf{E}^*\mathbf{I}$. Referring to Figure 2.14,

$$\begin{aligned}\mathbf{S} &= E\,e^{-j\phi_1} \times I\,e^{j\phi_2} = EI\,e^{-(\phi_1-\phi_2)}\\ &= EI\,e^{-\phi}\\ &= P - jQ\end{aligned}$$

Next assume

$$\begin{aligned}\mathbf{S} &= EI^*\\ &= EI\,e^{j(\phi_1-\phi_2)}\\ &= P + jQ.\end{aligned}$$

Obviously both the above methods give the correct magnitudes for P and Q but the sign of Q is different. The method used is arbitrarily decided and the convention to be adopted in this book is as follows:

> The volt-amperes reactive absorbed by an inductive load shall be considered positive, and by a capacitive load negative; hence $\mathbf{S} = \mathbf{EI}^*$. This convention is recommended by the International Electrotechnical Commission.

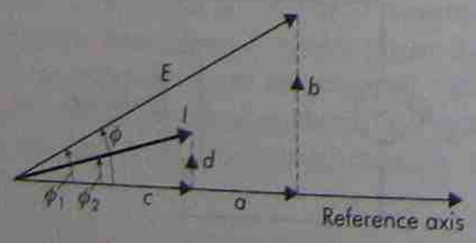


Figure 2.14 Phasor diagram

In a network the net stored energy is the sum of the various inductive and capacitive stored energies present. The net value of reactive power is the sum of the vars absorbed by the various components present, taking due account of the sign. Vars can be considered as being either produced or absorbed in a circuit; a capacitive load can be thought of as generating vars. Assuming that an inductive load is represented by $R + jX$ and that the $\mathbf{EI}^*$ convention is used, then an inductive load *absorbs* positive vars and a capacitive load *produces* vars. The reactive power flow towards a busbar is positive when the load is lagging; vars exported from a busbar are considered negative for a lagging power factor and positive for leading one. In this book, reference to the var will imply a lagging power factor, and inductive and capacitive vars will be distinguished by the sign.

The various elements in a network are characterized by their ability to generate or absorb reactive power. Consider a synchronous generator which can be represented by the simple equivalent circuit shown in Figure 2.15. When the generator is overexcited, i.e. its generated e.m.f. is high, it produces vars and a complex power, $P - jQ$. When the machine is underexcited the generated current leads V and the generator produces $P + jQ$. It can also be thought of as absorbing vars. The reactive power characteristics of various power-system components are summarized as follows: reactive power is generated by over-excited synchronous machines, capacitors, cables, and lightly loaded overhead lines; and absorbed by underexcited synchronous machines, induction motors, inductors, transformers, and heavily loaded overhead lines.

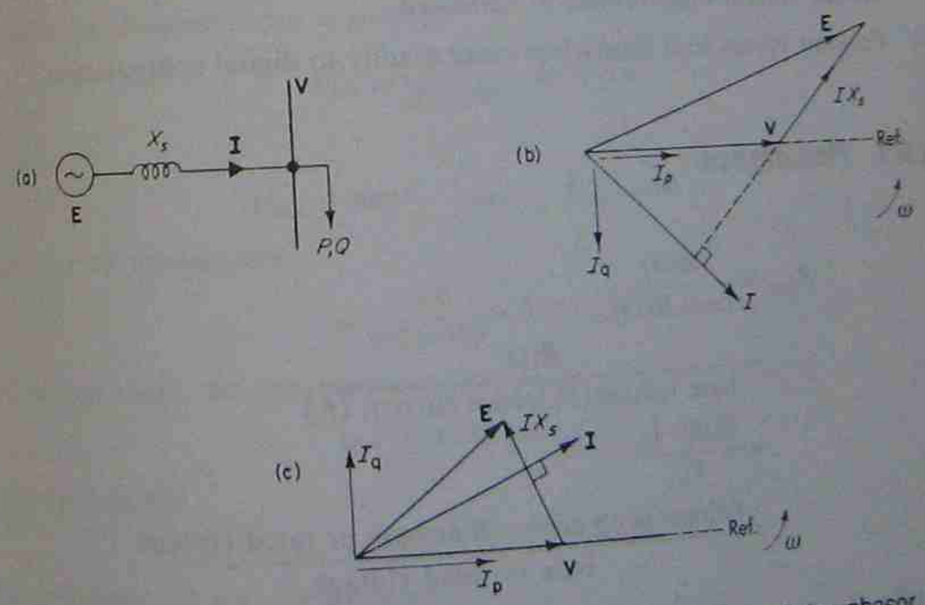


Figure 2.15 (a) Line diagram of system. (b) Overexcited generator–phasor diagram. (c) Underexcited generator–phasor diagram

The reactive power absorbed by a reactance of $X_L\Omega = I^2X_L$ where I is the current. Note that at a node, $\Sigma P = 0$ and $\Sigma Q = 0$ in accordance with Kirchhoff's law. A capacitance with a voltage V applied produces V^2B VAr, where $B = 1/X_c = \omega C$ if $X_c = 1/\omega C$.

2.6 The Per-Unit System

In the analysis of power networks, instead of using actual values of quantities it is usual to express them as fractions of reference quantities, such as rated or full-load values. These fractions are called per unit (denoted by p.u.) and the p.u. value of any quantity is defined as

$$\frac{\text{actual value (in any unit)}}{\text{base or reference value in the same unit}}$$

Some authorities express the p.u. value as a percentage. Although the use of p.u. values may at first sight seem a rather indirect method of expression there are, in fact, great advantages; they are as follows:

1. The apparatus considered may vary widely in size; losses and volt drops will also vary considerably. For apparatus of the same general type the p.u. volt drops and losses are in the same order, regardless of size.
2. As will be seen later, the use of $\sqrt{3}$'s in three-phase calculations is reduced.
3. By the choice of appropriate voltage bases the solution of networks containing several transformers is facilitated.
4. Per-unit values lend themselves more readily to digital computation.

2.6.1 Resistance

$$R_{\text{p.u.}} = \frac{R(\Omega)}{\text{base } R(\Omega)}$$

$$= \frac{R(\Omega)}{\text{base voltage}(V_b)/\text{base current } (I_b)}$$

$$= \frac{R(\Omega) \cdot I_b}{V_b}$$

$$= \frac{\text{voltage drop across } R \text{ at base or rated current}}{\text{base or rated voltage}}$$

Strictly, $R_{\text{p.u.}}$ is a non-dimensional ratio, but it is often given a unit (such as p.u. ohm) to provide a check on calculations.

Also

$$R_{\text{p.u.}} = \frac{R(\Omega)I_b^2}{V_bI_b}$$

$$= \frac{\text{power loss at base current}}{\text{base power or volt-amperes}}$$

$\therefore$ the power loss (p.u.) at base or rated current $= R_{\text{p.u.}}$

Power loss (p.u.) at $I_{\text{p.u.}}$ current $= R_{\text{p.u.}} \cdot I_{\text{p.u.}}^2$

Similarly

$$\text{p.u. impedance} = \frac{\text{impedance in ohms}}{\left(\frac{\text{base voltage}}{\text{base current}}\right)} = \frac{Z(\Omega).I_b}{V_b}$$

Example 2.3

A d.c. series machine rated at 200 V, 100 A has an armature resistance of 0.1 Ω and field resistance of 0.15 Ω. The friction and windage loss is 1500 W. Calculate the efficiency when operating as a generator.

Solution

Total series resistance in p.u. is given by

$$R_{\text{p.u.}} = \frac{0.25}{200/100} = 0.125 \text{ p.u.}$$

where

$$V_{\text{base}} = 200V \quad \text{and} \quad I_{\text{base}} = 100 \text{ A}$$

Friction and windage loss

$$= \frac{1500}{200 \times 100} = 0.075 \text{ p.u.}$$

At the rated load, the series-resistance loss

$$= 1^2 \times 0.125 \text{ p.u.}$$

and the total loss

$$= 0.125 + 0.075 = 0.2 \text{ p.u.}$$

As the output = 1 p.u., the efficiency

$$= \frac{1}{1 + 0.2} = 0.83 \text{ p.u.}$$

Three-phase circuits

A p.u. phase voltage has the same numerical value as the corresponding p.u. line voltage. With a line voltage of 100 kV and a rated line voltage of 132 kV, the p.u. value is 0.76. The equivalent phase voltages are $100/\sqrt{3}$ kV and $132/\sqrt{3}$ kV and hence the p.u. value is again 0.76. The actual values of R, X_L, and X_c for lines, cables, and other apparatus are phase values. When working with ohmic values it is less confusing to use the phase values of all quantities. In the p.u. system, three-phase values of voltage, current, and power can be used without undue anxiety about the result being a factor of $\sqrt{3}$ incorrect.

It is convenient in a.c. circuit calculations to work in terms of base volt-amperes, base VA. Thus,

$$\text{base } VA = V_{\text{base}} \times I_{\text{base}} \times \sqrt{3}$$

when V_{base} is the line voltage and I_{base} is the line current in a three-phase system.

Hence

$$I_{\text{base}} = \frac{\text{base } VA}{\sqrt{3} \cdot V_{\text{base}}} \tag{2.1}$$

Expression (2.1) shows that if base VA and V_{base} are specified, then I_{base} is also specified. Consequently, only *two* base quantities can be chosen from which all other quantities in a three-phase system can be calculated. Thus,

$$Z_{\text{base}} = \frac{V_{\text{base}}/\sqrt{3}}{I_{\text{base}}} = \frac{V_{\text{base}}/\sqrt{3}}{VA_{\text{base}}/(\sqrt{3} \cdot V_{\text{base}})} = \frac{V_{\text{base}}^2}{VA_{\text{base}}} \tag{2.2}$$

Hence

$$Z_{\text{p.u.}} = \frac{Z(\Omega)}{Z_{\text{base}}} = \frac{Z(\Omega) \times VA_{\text{base}}}{V_{\text{base}}^2} \tag{2.3}$$

Transformers

Consider a single-phase transformer in which the total series impedance of the two windings referred to the primary is Z_1 (Figure 2.16).

Then the p.u. impedance $= I_1 Z_1 / V_1$, where I_1 and V_1 are the rated or base values. The total impedance referred to the secondary

$$= Z_1 N^2$$

and this in p.u. notation is

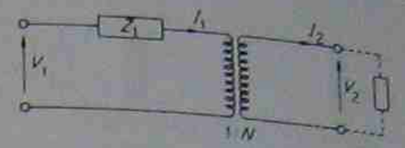


Figure 2.16 Equivalent circuit of single-phase transformer

$$= Z_1 N^2 \left(\frac{I_2}{V_2}\right)$$

$$= Z_1 N^2 \frac{I_1}{N} \cdot \frac{1}{V_1 N} = \frac{Z_1 I_1}{V_1}$$

Hence the p.u. impedance of a transformer is the same whether considered from the primary or the secondary side. But expression (2.3) shows that $Z_{\text{p.u.}}$ is directly proportional to the base VA and inversely proportional to the base voltage squared. Although the VA rating of a transformer is independent of the side to which it refers, the voltage rating is different. Consequently if we wish to vary the voltage base dependent upon which part of the network to which we are referring, so that all voltages are around 1 p.u., we must recalculate the corresponding value of $Z_{\text{p.u.}}$, as follows:

$$Z_{\text{p.u.}}(\text{new base}) = Z_{\text{p.u.}} \text{ (original base)} \times \left(\frac{\text{base}V_{\text{old}}}{\text{base}V_{\text{new}}}\right)^2 \tag{2.4}$$

Note that the VA base is unchanged. If we wish to calculate $Z_{\text{p.u.}}$ to a new VA base, then $Z_{\text{p.u.}}$ (new base) $= Z_{\text{p.u.}}$ (old base) $\times \left(\frac{\text{base}VA_{\text{new}}}{\text{base}VA_{\text{old}}}\right)$ (2.5)

Example 2.4

In the network of Figure 2.17, two single-phase transformers supply a 10 kVA resistance load at 200 V. Show that the p.u. load is the same for each part of the circuit and calculate the voltage at point D.

Solution

The load resistance is $(200^2/10 \times 10^3)$, i.e. 4 Ω.

In each of the circuits A, B, and C a different voltage exists, so that each circuit will have its own base voltage, i.e. 100 V in A, 400 V in B, and 200 V in C.

Although it is not essential for rated voltages to be used as bases, it is essential that the *voltage bases used be related by the turns ratios of the transformers*. If this is not so

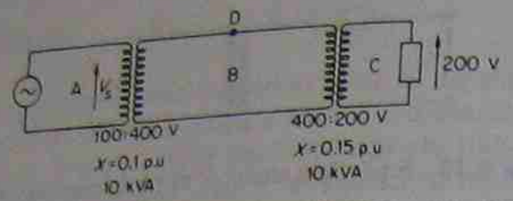


Figure 2.17 Network with two transformers—p.u. approach

the whole p.u. framework breaks down. The same volt-ampere base is used for all the circuits as $V_1 I_1 = V_2 I_2$ on each side of a transformer and is taken in this case as 10 kVA. The base impedance in C

$$= \frac{200^2}{10\,000} = 4\ \Omega$$

The load resistance (p.u.) in C

$$= \frac{4}{4} = 1 \text{ p.u.}$$

In B the base impedance

$$= \frac{400^2}{10\,000} = 16\ \Omega$$

and the load resistance referred to B

$$= 4 \times 2^2 = 16\ \Omega$$

Hence the p.u. load referred to B

$$= 1 \text{ p.u.}$$

Similarly, the p.u. load resistance referred to A is also 1 p.u. Hence, if the voltage bases are related by the turns ratios the load p.u. value is the same for all circuits. An equivalent circuit may be used as shown in Figure 2.18. Let the volt-ampere base be 10 kVA; the voltage across the load (V_R) is 1 p.u. (as the base voltage in C is 200 V, if the load voltage had been maintained at 100 V, then V_R would be 0.5 p.u.). The base current

$$= \frac{(VA)_b}{V_b} = \frac{10\,000}{200} = 50 \text{ A in C}$$

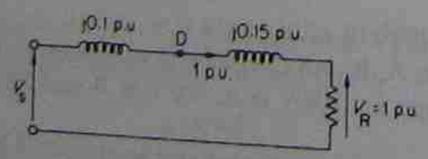


Figure 2.18 Equivalent circuit with p.u. values, of network in Figure 2.17

The corresponding currents in the other circuits are 25 A in B, and 100 A in A. The actual load current = 50/50 = 1 p.u. (in phase with V_R, the reference phasor). Hence the supply voltage V_s

$$= 1(j0.1 + j0.15) + 1 \text{ p.u.}$$
$$\therefore V_s = 1.03 \text{ p.u.}$$
$$= 1.03 \times 100 = 103 \text{ V}$$

The voltage at point D in Figure 2.17

$$= 1 + j0.15 \times 1 = 1 + j0.15$$
$$= 1.012 \text{ p.u.modulus}$$
$$= 1.012 \times 400 = 404.8 \text{ V}$$

It is a useful exercise to repeat this example using ohms, volts, and amperes.

Example 2.5

Figure 2.19 shows the schematic diagram of a radial transmission system. The ratings and reactances of the various components are shown, along with the nominal transformer line voltages. A load of 50 MW at 0.8 p.f. lagging is taken from the 33 kV substation which is to be maintained at 30 kV. It is required to calculate the terminal voltage of the synchronous machine. The line and transformers may be represented by series reactances. The system is three-phase.

Solution

It will be noted that the line reactance is given in ohms; this is usual practice. The voltage bases of the various circuits are decided by the nominal transformer voltages, i.e. 11, 132, and 33 kV. A base of 100 MVA will be used for all circuits. The reactances (resistance is neglected) are expressed on the appropriate voltage and MVA bases.

Base impedance for the line

$$= \frac{132^2 \times 10^6}{100 \times 10^6}$$
$$= 174\Omega$$

Hence the p.u. reactance

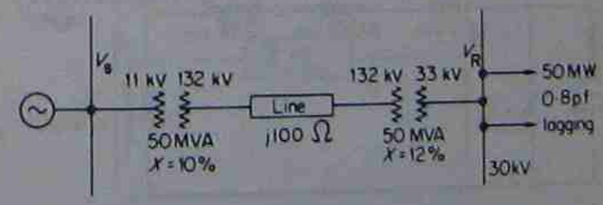


Figure 2.19 Line diagram of system for Example 2.5

$$= \frac{j100}{174} = j0.575$$

Per unit reactance of the sending-end transformer

$$= \frac{100}{50} \times j0.1 = j0.2$$

Per unit reactance of the receiving-end transformer

$$= j0.12 \times \frac{100}{50} = j0.24$$

Load current

$$= \frac{50 \times 10^6}{\sqrt{3} \times 30 \times 10^3 \times 0.8} = 1203 \text{ A}$$

Base current for 33 kV, 100 MVA

$$= \frac{100 \times 10^6}{\sqrt{3} \times 33 \times 10^3}$$
$$= 1750 \text{ A}$$

Hence the p.u. load current

$$= \frac{1203}{1750} = 0.687$$

Voltage of load busbar

$$= \frac{30}{33} = 0.91 \text{ p.u.}$$

The equivalent circuit is shown in Figure 2.20.
Also,

$$V_s = 0.687(0.8 - j0.6)(j0.2 + j0.575 + j0.24) + (0.91 + j0)$$
$$= 1.328 + j0.558 \text{ p.u.}$$
$$\therefore V_s = 1.44 \text{ p.u. or } 1.44 \times 11 \text{ kV} = 15.84 \text{ kV}$$

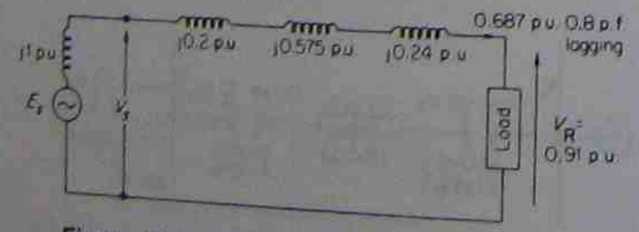


Figure 2.20 Equivalent circuit for Example 2.5

2.7 Power Transfer and Reactive Power

The circuit shown in Figure 2.21 is very important as it represents the simplest electrical model for a synchronous generator (with an e.m.f. (E) feeding into a power system represented by a voltage source (V)).

$$S_A = EI^* = E\left(\frac{E - V}{Z}\right)^*$$
$$= E\, e^{j\delta}\left(\frac{E\, e^{-j\delta} - V}{Z\, e^{-j\theta}}\right)$$
$$= \frac{E^2}{Z} e^{j\theta} - \frac{EV}{Z} e^{(j\theta + \delta)}$$

$$\left.\begin{aligned} P_A &= \frac{E^2}{Z}\cos\theta - \frac{EV}{Z}\cos(\theta + \delta) \\ Q_A &= \frac{E^2}{Z}\sin\theta - \frac{EV}{Z}\sin(\theta + \delta) \end{aligned}\right\} \quad (2.6)$$

Similarly,

$$S_B = V\left(\frac{V - Ee^{-j\delta}}{Ze^{-j\theta}}\right)$$

$$\left.\begin{aligned} P_B &= \frac{V^2}{Z}\cos\theta - \frac{EV}{Z}\cos(\theta - \delta) \\ Q_B &= \frac{V^2}{Z}\sin\theta - \frac{EV}{Z}\sin(\theta - \delta) \end{aligned}\right\} \quad (2.7)$$

The power output to V is a maximum when $\cos(\theta - \delta) = 1$, i.e. $\theta = \delta$.

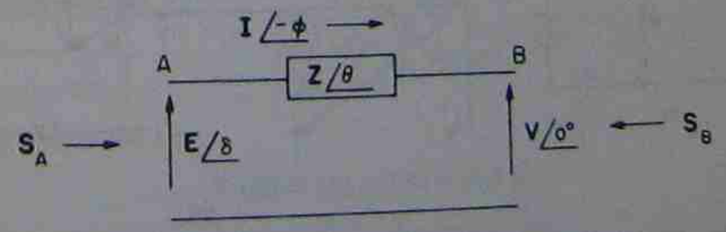


Figure 2.21 Power transfer between sources

2.7.1 Calculation of sending and received voltages in terms of power and reactive power

The determination of the voltages and currents in a network can obviously be achieved by means of complex notation, but in power systems usually power (P) and reactive power (Q) are specified and often the resistance of lines is negligible compared with reactance. For example, if $R = 0.1X$, the error in neglecting R is 0.49 per cent, and even if $R = 0.4X$ the error is 7.7 per cent.

A simple transmission link is shown in Figure 2.22(a). It is required to establish the equations for E, V, and δ. From Figure 2.22(b).

$$E^2 = (V + \Delta V_p)^2 + \Delta V_q^2$$
$$= (V + RI\cos\phi + XI\sin\delta)^2 + (XI\cos\phi - RI\sin\phi)^2$$
$$\therefore E^2 = \left(V + \frac{RP}{V} + \frac{XQ}{V}\right)^2 + \left(\frac{XP}{V} - \frac{RQ}{V}\right)^2 \tag{2.8}$$

Hence,

$$\Delta V_p = \frac{RP + XQ}{V} \tag{2.9}$$

and

$$\Delta V_q = \frac{XP - RQ}{V} \tag{2.10}$$

If

$$\Delta V_q \ll V + \Delta V_p$$

then

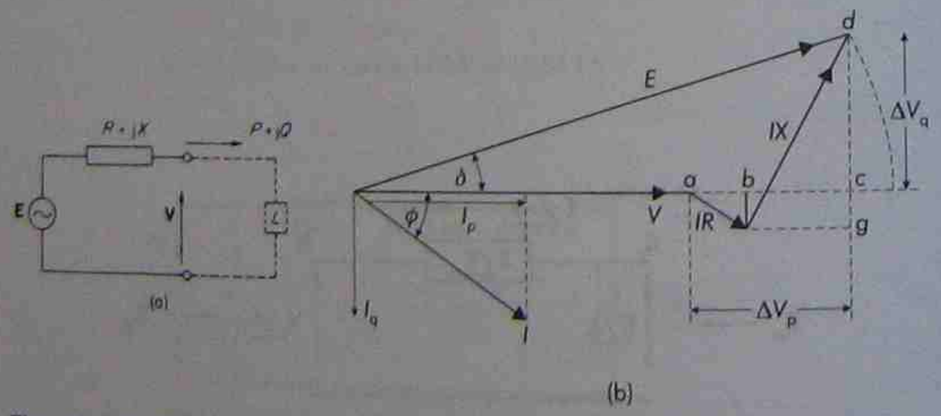


Figure 2.22 Phasor diagram for transmission of power through a series impedance

$$E^2 = \left(V + \frac{RP + XQ)}{V}\right)^2$$

and

$$E - V = \frac{RP + XQ}{V} = \Delta V_p$$

Hence the arithmetic difference between the voltages is given **approximately** by

$$\frac{RP + XQ}{V}$$

If

$$R = 0$$

then

$$E - V = \frac{XQ}{V} \tag{2.11}$$

i.e. *voltage depends only on* Q, and from equation (2.8) this is valid if $PX \ll V^2 + QX$. The angle of transmission δ is obtained from $\sin^{-1}(\Delta V_q/E)$, *and depends only on* P.

Equations (2.9) and (2.10) will be used wherever possible because of their great simplicity.

Example 2.6

Consider a 275 kV line of length 160 km (R and X per km $= 0.034$ and $0.32\ \Omega$, respectively). Obviously, $R \ll X$.

Compare the sending end voltage E in Figure 2.22(a) when calculated with the accurate and the approximate formulae. Assume a load of 600 MW, 300 MVAr and take a base of 100 MVA. (Note that 600 MVA is nearly the maximum rating of a $2 \times 258\ \text{mm}^2$ line at 275 kV.)

Solution

The base impedance is

$$\frac{275^2 \times 10^6}{100 \times 10^6} = 756\ \Omega$$

For line of length 160 km,

$$X\ (\Omega) = 160 \times 0.32 = 51.2\ \Omega$$
$$X = \frac{51.2}{756} = 0.0677\ \text{p.u.}\ (\Omega)$$

and let the received voltage be

$$V = 275 \text{ kV} = 1 \text{ p.u.}$$

then
$$PX = 6 \times 0.0677 = 0.406 \text{ p.u.}(V)$$

and
$$V^2 + QX = 1 + 3 \times 0.0677 = 1.203 \text{ p.u. } (V)$$

Hence the use of equation (2.9) will involve some inaccuracy

Using equation (2.8) with R neglected.

$$E^2 = \left(1 + \frac{0.0677 \times 3}{V}\right)^2 + \left(\frac{0.0677 \times 6}{V}\right)^2 = 1.203^2 + 0.406^2$$

$$E = 1.270 \text{ p.u. } (V)$$

An approximate value of E can be found from equation (2.11) as

$$E - V = \Delta V = \frac{0.0677 \times 3}{1} = 0.203$$

$$\therefore E = 1.203 \text{ p.u. } (V)$$

i.e. an error of 5.6 per cent.

With a 80 km length of this line at the same load (still neglecting R) gives

$$PX = 6 \times 0.0338 = 0.203 \quad \text{and} \quad QX = 3 \times 0.0388 = 0.101$$

The accurate formula gives

$$E = \sqrt{(1 + 0.101)^2 + 0.203^2} = 1.120 \text{ p.u. } (V)$$

and the approximate one gives

$$E = 1 + 0.101 = 1.101 \text{ p.u. } (V)$$

i.e. an error of 1.7 per cent.

In the solution above, it should be noted that p.u. values for power system calculations are conveniently expressed to three decimal places, implying a measured value to 0.1 per cent error. In practice, most measurements will have an error of at least 0.2 per cent and possibly 0.5 per cent.

With 132 kV lines on a 100 MVA base, the reactance of a 160 km 258 mm^2 line is $160 \times 0.4/174$ or 0.37 p.u. and the rated power is 1.5 p.u. ($Q = 0.75$ p.u. for same power factor). Hence

$$PX = 1.5 \times 0.37 = 0.555 \quad \text{and} \quad V^2 + XQ = 1 + 0.277 = 1.277$$

Hence, an error in the order of 5 per cent could be expected.

If E is specified and V is required, the phasor diagram in Figure 2.23 is used. From this,

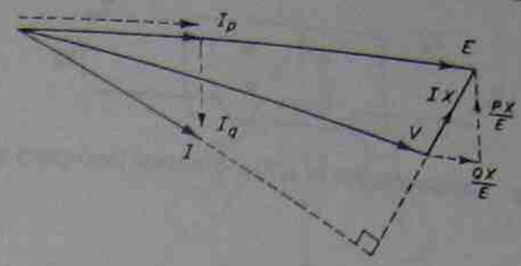


Figure 2.23 Phasor diagram when **E** is specified

$$V = \sqrt{\left[\left(E - \frac{QX}{E}\right)^2 + \left(\frac{PX}{E}\right)^2\right]} \tag{2.12}$$

when $R = 0$.

If

$$\frac{PX}{E} \ll \frac{E^2 - QX}{E}$$

then

$$V = E - \frac{QX}{E} \tag{2.13}$$

2.8 Useful Network Theory

2.8.1 Four-terminal networks

A lumped-constant circuit, provided that it is passive, linear, and bilateral, can be represented by the four-terminal network shown in the diagram in Figure 2.24. The complex parameters **A**, **B**, **C**, and **D** describe the network in terms of the sending- and receiving-end voltages and currents as follows:

$$\begin{aligned} \mathbf{V_S} &= \mathbf{AV_R} + \mathbf{BI_R} \\ \mathbf{I_S} &= \mathbf{CV_R} + \mathbf{DI_R} \end{aligned} \tag{2.14}$$

and it can be readily shown that $\mathbf{AD}-\mathbf{BC} = 1$.

A, **B**, **C**, and **D** may be obtained by measurement and certain physical interpretations can be made, as follows:

1. Receiving-end short-circuited:

$$\mathbf{V_R} = 0, \quad \mathbf{I_S} = \mathbf{DI_R} \quad \text{and} \quad \mathbf{D} = \frac{\mathbf{I_S}}{\mathbf{I_R}}$$

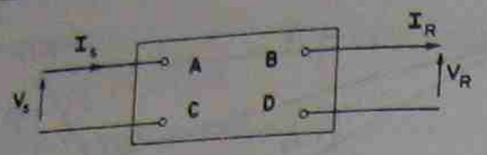


Figure 2.24 Representation of a four-terminal (two-port) network

Also,

$$\mathbf{V_S} = \mathbf{BI_R} \quad \text{and} \quad \mathbf{B} = \frac{\mathbf{V_S}}{\mathbf{I_R}} = \text{short-circuit transfer impedance}$$

2. Receiving-end open-circuited:

Here

$$\mathbf{I_R} = 0, \quad \mathbf{I_S} = \mathbf{CV_R} \quad \text{and} \quad \mathbf{C} = \left(\frac{\mathbf{I_S}}{\mathbf{V_R}}\right)$$

$$\mathbf{V_S} = \mathbf{AV_R}, \quad \mathbf{A} = \left(\frac{\mathbf{V_S}}{\mathbf{V_R}}\right)$$

Expressions for the constants can also be found by complex (i.e. magnitude *and* angle) measurements carried out solely at the sending end with the receiving end open and short-circuited.

Often it is useful to have a single four-terminal network for two or more items in series or parallel, e.g. a line and two transformers in series.

It is shown in most texts on circuit theory that the generalized constants for the combined network, $\mathbf{A_0}$,$\mathbf{B_0}$, $\mathbf{C_0}$, and $\mathbf{D_0}$ for the two networks (1) and (2) in cascade are as follows:

$$\mathbf{A_0} = \mathbf{A_1A_2} + \mathbf{B_1C_2} \qquad \mathbf{B_0} = \mathbf{A_1B_2} + \mathbf{B_1D_2}$$
$$\mathbf{C_0} = \mathbf{A_2C_1} + \mathbf{C_2D_1} \qquad \mathbf{D_0} = \mathbf{B_2C_1} + \mathbf{D_1D_2}$$

For two four-terminal networks in parallel it can be shown that the parameters of the equivalent single four-terminal network are:

$$\mathbf{A_0} = \frac{\mathbf{A_1B_2} + \mathbf{A_2B_1}}{\mathbf{B_1} + \mathbf{B_2}} \qquad \mathbf{B_0} = \frac{\mathbf{B_1B_2}}{\mathbf{B_1} + \mathbf{B_2}} \qquad \mathbf{D_0} = \frac{\mathbf{B_2D_1} + \mathbf{B_1D_2}}{\mathbf{B_1} + \mathbf{D_2}}$$

$\mathbf{C_0}$ can be found from $\mathbf{A_0D_0} - \mathbf{B_0C_0} = 1$.

Delta–star transformation

The delta network connected between the three terminals A, B, and C of figure 2.25 can be replaced by a star network such that the impedance measured

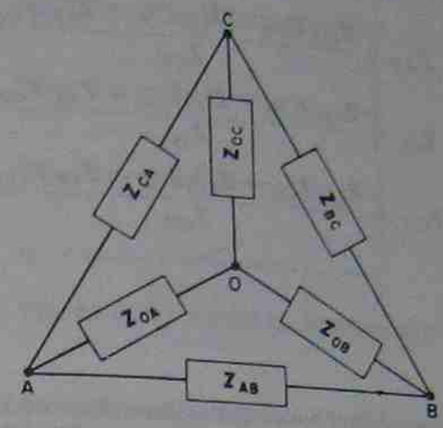


Figure 2.25 Star–delta, delta–star transformation

between the terminals is unchanged. The equivalent impedances can be found as follows:

Impedance between terminals AB with C open-circuited is given by

$$\mathbf{Z_{OA}} + \mathbf{Z_{OB}} = \mathbf{Z_{AB}}//(\mathbf{Z_{CA}} + \mathbf{Z_{BC}})$$

Similarly

$$\mathbf{Z_{OB}} + \mathbf{Z_{OC}} = \mathbf{Z_{BC}}//(\mathbf{Z_{AB}} + \mathbf{Z_{CA}})$$

and

$$\mathbf{Z_{OC}} + \mathbf{Z_{OA}} = \mathbf{Z_{CA}}//(\mathbf{Z_{BC}} + \mathbf{Z_{AB}})$$

From these three equations $\mathbf{Z_{OA}}$, $\mathbf{Z_{OB}}$, and $\mathbf{Z_{OC}}$, the three unknowns, can be determined as

$$\mathbf{Z_{OA}} = \frac{\mathbf{Z_{AB}Z_{CA}}}{\mathbf{Z_{AB}} + \mathbf{Z_{CA}} + \mathbf{Z_{BC}}}$$

$$\mathbf{Z_{OB}} = \frac{\mathbf{Z_{AB}Z_{BC}}}{\mathbf{Z_{AB}} + \mathbf{Z_{CA}} + \mathbf{Z_{BC}}} \qquad (2.15)$$

$$\mathbf{Z_{OC}} = \frac{\mathbf{Z_{BC}Z_{CA}}}{\mathbf{Z_{AB}} + \mathbf{Z_{CA}} + \mathbf{Z_{BC}}}$$

Star–delta transformation

A star-connected system can be replaced by an equivalent delta connection if the elements of the new network have the following values (Figure 2.25):

$$Z_{AB} = \frac{Z_{OA}Z_{OB} + Z_{OB}Z_{OC} + Z_{OC}Z_{OA}}{Z_{OC}}$$

$$Z_{BC} = \frac{Z_{OA}Z_{OB} + Z_{OB}Z_{OC} + Z_{OC}Z_{OA}}{Z_{OA}} \qquad (2.16)$$

$$Z_{CA} = \frac{Z_{OA}Z_{OB} + Z_{OB}Z_{OC} + Z_{OC}Z_{OA}}{Z_{OB}}$$

Problems

2.1 The star-connected secondary winding of a three-phase transformer supplies 415 V (line to line) at a load point through a four-conductor cable. The neutral conductor is connected to the winding star point which is earthed. The load consists of the following components:

Between a and b conductors	a 1 Ω resistor
Between a and neutral conductors	a 1 Ω resistor
Between b and neutral conductors	a 2 Ω resistor
Between c and neutral conductors	a 2 Ω resistor

Connected to the a, b, and c conductors is an induction motor taking a balanced current of 100 A at 0.866 power factor (p.f.) lagging. Calculate the currents in the four conductors and the total power supplied.
Take the 'a' to neutral voltage as the reference phasor. The phase sequence is a–b–c.
(Answer: 350 kW)

2.2 The wye-connected load shown in Figure 2.26 is supplied from a transformer whose secondary-winding star point is solidly earthed. The line voltage supplied to the load is 400 V. Determine (a) the line currents, and (b) the voltage of the load star point with respect to ground. The phase sequence is a–b–c.
(Answer: (a) $\mathbf{I}_a(200 - j69)$ A; (b) $V_{n-g} - 47$ V)

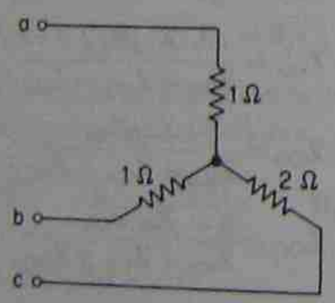


Figure 2.26 Circuit for Problem 2.2

2.3 Two capacitors, each of 10 μF, and a resistor R, are connected to a 50 Hz three-phase supply, as shown in Figure 2.27. The power drawn from the supply is the same whether the switch S is open or closed. Find the resistance of R.
(Answer: 143 Ω)

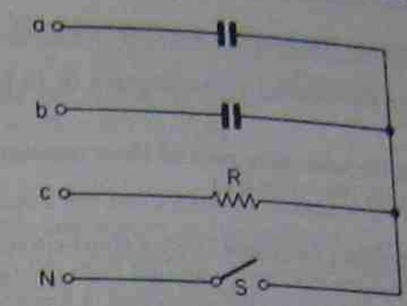


Figure 2.27 Circuit for Problem 2.3

2.4 The network of Figure 2.28 is connected to a 400 V three-phase supply, with phase sequence a–b–c. Calculate the reading of the wattmeter W.
(Answer: 2.24 kW)

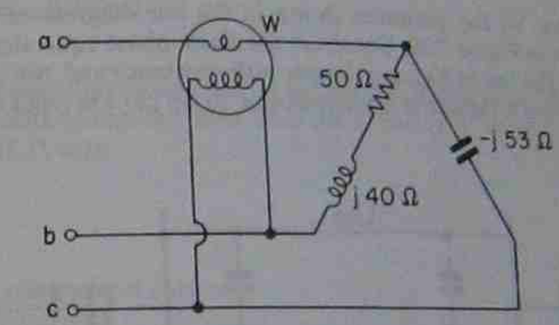


Figure 2.28 Circuit for Problem 2.4

2.5 A 400 V three-phase supply feeds a delta-connected load with the following branch impedances:

$$\mathbf{Z}_{RY} = 100\ \Omega \qquad \mathbf{Z}_{YB} = j100\ \Omega \qquad \mathbf{Z}_{BR} = -j100\ \Omega$$

Calculate the line currents for phase sequences (a) RYB; (b) RBY.
(Answer: (a) 7.73, 7.37, 4 A; (b) 2.07, 2.07, 4 A)

2.6 A synchronous generator, represented by a voltage source in series with an inductive reactance X_1, is connected to a load consisting of a fixed inductive reactance X_2 and a variable resistance R in parallel. Show that the generator power output is a maximum when

$$1/R = 1/X_1 + 1/X_2$$

2.7 A single-phase voltage source of 100 kV supplies a load through an impedance j100 Ω. The load may be represented in either of the following ways as far as voltage changes are concerned:

(a) by a constant impedance representing a consumption of 10 MW, 10 MVAr at 100 kV; or
(b) by a constant current representing a consumption of 10 MW, 10 MVAr at 100 kV.

Calculate the voltage across the load using each of these representations.
(Answer: (a) (90 – j8.2) kV; (b) (90 – j10) kV)

2.8 Show that the p.u. impedance (obtained from a short-circuit test) of a star–delta three-phase transformer is the same whether computed from the star-side parameters or from the delta side. Assume a rating of G (volt-amperes), a line-to-line input voltage to the star-winding terminals of V volts, a turns ratio of $1:N$ (star to delta), and a short-circuit impedance of Z (ohms) per phase referred to the star side.

2.9 A 11 kV/132 kV, 50 MVA, three-phase transformer has an inductive reactance of j0.005 Ω referred to the primary (11 kV). Calculate the p.u. value of reactance based on the rating. Neglect resistance.
(Answer: 0.002 p.u.)

2.10 Express in p.u. all the quantities shown in the line diagram of the three-phase transmission system in Figure 2.29. Construct the single-phase equivalent circuit. Use a base of 100 MVA. The line is 80 km in length with resistance and reactance of 0.1 and 0.5 Ω, respectively, and a capacitive susceptance of 10 μS per km (split equally between the two ends).

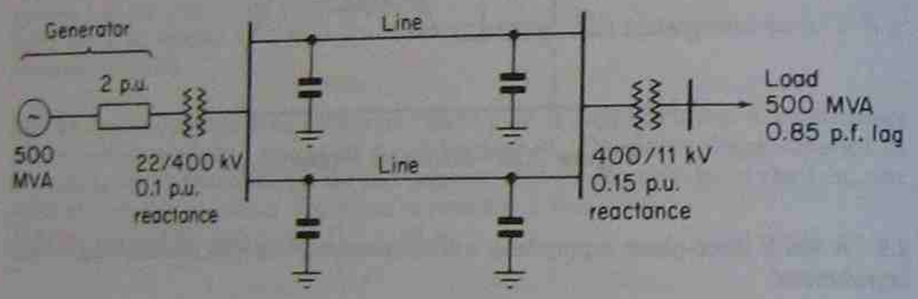


Figure 2.29 Circuit for Problem 2.10

2.11 A wye-connected load is supplied from three-phase 220 V mains. Each branch of the load is a resistor of 20 Ω. Using 220 V and 10 kVA bases calculate the p.u. values of the current and power taken by the load.
(Answer: 0.24 p.u.; 0.24 p.u.)

2.12 A 440 V three-phase supply is connected to three star-connected loads in parallel, through a feeder of impedance (0.1 + j0.5) Ω per phase. The loads are as follows: 5 kW, 4 kVAr; 3 kW, 0 kVAr; 10 kW, 2 kVAr. Determine:

(a) line current;
(b) power and reactive power losses in the feeder per phase;
(c) power and reactive power from the supply and the supply power factor.

(Answer: (a) 24.9 A; (b) 62 W, 310 VAR; (c) 18.06 kW, 6.31 kVAr, 0.94)

2.13 Two transmission circuits are defined by the following **ABCD** constants: 1, 50, 0, 1, and 0.9 ∠ 2°, 150∠ 79°, 9×10^{-4} ∠ 91°, 0.9∠ 2°. Determine the **ABCD** constants of the circuit comprising these two circuits in series.
(Answer: $\mathbf{A} = 45\angle 89.86$; $\mathbf{B} = 165.7\angle 63.6$)

2.14 A 132 kV overhead line has a series resistance and inductive reactance per phase per kilometre of 0.156 and 0.4125 Ω, respectively. Calculate the magnitude of the sending-end voltage when transmitting the full line capability of 125 MVA when the power factor is 0.9 lagging and the received voltage is 132 kV, for 16 km and 80 km lengths of line. Use both accurate and approximate methods.
(Answer: 136.92 kV (accurate), 136.85 kV; 157.95 kV (accurate), 156.27 kV)

2.15 A synchronous generator may be represented by a voltage source of magnitude 1.7 p.u. in series with an impedance of 2 p.u. It is connected to a zero-impedance voltage source of 1 p.u. The ratio of X/R of the impedance is 10. Calculate the power generated and the power delivered to the voltage source if the angle between the voltage sources is 30°.
(Answer: 0.49 p.u.; 0.44 p.u.)

2.16 A three-phase star-connected 50 Hz generator generates 240 V per phase and supplies three delta-connected load coils each having a resistance of 10 Ω and an inductance of 47.75 mH.

Determine:

(a) the line voltage and current;
(b) the load current;
(c) the total real power and reactive power dissipated by the load.

Determine also the values of the three capacitors required to correct the overall power factor to unity when the capacitors are (i) star- and (ii) delta-connected across the load.

State an advantage and a disadvantage of using the star connection for power factor correction.
(Answer: (a) 416 V, 39.96 A; (b) 23.07 A; (c) 15.77 kW, 24.09 kVAr; (i) 443.8 μF; (ii) 147.8 μF

(From E.C. Examination, 1996)

3

Components of a Power System

3.1 Introduction

In this chapter the essential characteristics of the components of a power system will be discussed. It is essential that these be fully understood before the study of large systems of interconnection of these components is undertaken. In all cases the simplest representation employing an equivalent circuit will be used not only to make the principles clearer but also because these simple models are used in practice. For more sophisticated treatments, especially of synchronous machines, the reader is referred to the more advanced texts given in the reference list at the end of the book.

It is assumed that the reader has completed introductory courses in circuit theory and machines (or electromechanical energy conversion). A knowledge of basic control theory will be needed for the section on the dynamic response of machines, although this may be omitted without essential loss of continuity in the text.

Loads are considered as components even though their exact composition and characteristics are not known with complete certainty. When designing a supply system or extending an existing one the prediction of the loads to be expected is required, statistical methods being used. This is the aspect of power supply known with least precision.

It is stressed that most of the equivalent circuits used are single phase and employ phase-to-neutral values. This assumes that the loads are balanced three-phase which is reasonable for normal steady-state operation. When unbalance exists between the phases, full treatment of all phases is required, and special techniques for dealing with this are given in Chapter 7.

Synchronous Machines

3.2 Characteristics

In this chapter those characteristics of the synchronous machine pertinent to power systems will be discussed. It is assumed that the reader has a basic knowledge of synchronous-machine theory. The two forms of rotor construction produce characteristics that influence the operation of the system to varying extents. In the round or cylindrical rotor the field winding is placed in slots cut axially along the rotor length as illustrated in Figure 3.1(a). The diameter is relatively small (1–1.5 m) and the machine is suitable for operation at high speeds (3000 or 3600 r.p.m.) driven by a steam or gas turbine. Hence it is known as a turbogenerator. In a salient pole rotor the poles project as shown in Figure 3.1(b) and low-speed operation driven by hydraulic turbines, wind, or diesel sets is usual. The frequency of the generated e.m.f. and speed are related by

$$f = \frac{np}{60}$$

where n is speed in r.p.m. and p is the number of *pairs* of poles. A hydraulic turbine rotating at 150–300 r.p.m., depending on type, thus needs many pole pairs to generate at normal frequencies.

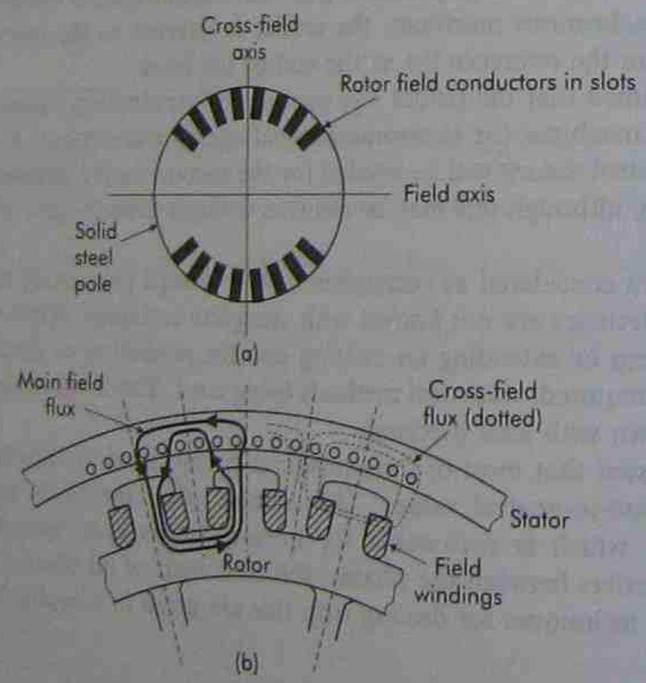


Figure 3.1 (a) Cylindrical rotor. (b) Salient pole rotor, and stator with flux paths

The three-phase currents in the stator winding or armature generate a rotating magnetic field which rotates with the synchronously rotating rotor and its field. The effect of the stator winding field on the rotor field is referred to as 'armature reaction'. Figure 3.1(b) indicates the paths of the main field fluxes in a salient pole machine from which it can be seen that these paths are mainly in iron, apart from crossing the air gap between stator and rotor. On the other hand, the cross-field flux has a longer path in air and hence the reluctance of its path is less than for the main field flux. Consequently, any e.m.f.s produced by the main field flux in the stator are larger than those produced by any cross-field fluxes.

Machine designers shape the poles in a salient machine and distribute the windings in a round-rotor machine to obtain maximum fundamental sinusoidal voltage on no-load in the carefully designed stator windings. Also, an acceptably low harmonic content is achieved in contrast to the cross-field flux which often has considerable harmonic content. However, the stator winding is designed and distributed such as to minimize the harmonic voltages induced in it and only the fundamental e.m.f. is significant for power system considerations.

Whether a synchronous machine has a cylindrical or salient-pole rotor, its action is the same when generating power and can be best understood in terms of the 'primitive' representation shown in Figure 3.2(a). The rotor current I_f

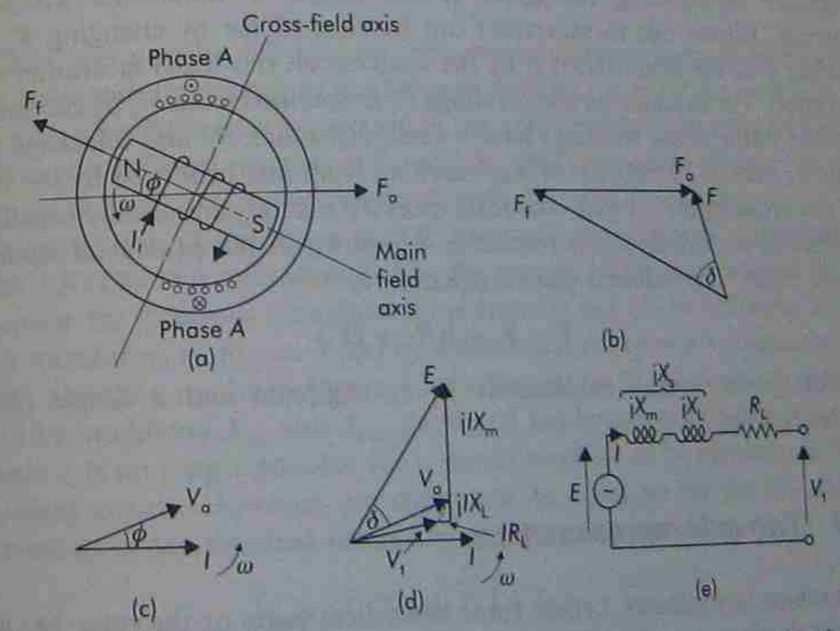


Figure 3.2 (a) Primitive machine at the instant of maximum current in phase A conductors and field m.m.f. F_f. (b) Corresponding space diagram of vector m.m.f.s with resultant F. (c) Terminal phasor diagram. (d) Phasor diagram with voltage representation of magnetic conditions. (e) Equivalent circuit for round-rotor machine

produces a magnetomotive force (m.m.f.) F_f ampere turns (AT) rotating with the rotor and producing sinusoidal-induced voltages in the stator windings on no-load. When the stator windings are closed through an external circuit, currents flow, thereby delivering power to the external load. These stator currents produce an m.m.f., shown in Figure 3.2(a) for phase A as F_a, which interacts with the rotor m.m.f. F_f, thereby producing a retarding force on the rotor that requires torque (and hence power) to be provided from the prime mover. The vector m.m.f.s in Figure 3.2(b) are drawn for a representative instant at which the current I in phase A is a maximum and the voltage is at an angle ϕ, the power factor angle, past its maximum as shown in Figure 3.2(c). Currents in phases B and C of the stator (equal and opposite at this instant) also contribute along the axis of F_a. The resultant air-gap m.m.f. is shown as $F = F_a + F_f$, using vectorial addition as in Figure 3.2(b), and this gives rise to a displaced air-gap flux such that the phase of the stator-induced e.m.f. E is delayed from that produced on no-load for the same rotor position by the torque angle shown as δ in Figure 3.2(d).

Detailed consideration of generator action taking full account of magnetic path saturation requires a simulation of the fluxes, including all winding currents and eddy currents. However, for power system calculations the m.m.f. summation of Figure 3.2(b) can be transferred to the voltage–current phasor diagram of Figure 3.2(d) by rotation through 90° clockwise and scaling F to V_a. Then, F_f becomes E and F_a corresponds to X_m where X_m is an equivalent reactance representing the effect of the magnetic conditions within the machine. Effects due to saturation can be allowed for by changing X_m and relating E to the field current I_f by the open circuit (no-load) saturation curve (see later). The machine terminal voltage V_t is obtained from V_a by recognizing that the stator phase windings have a small resistance R_L and a leakage reactance X_L (about 10 per cent of X_m) resulting from flux produced by the stator but not crossing the air gap. The reactances X_L and X_m are usually considered together as the synchronous reactance X_s, shown in the equivalent circuit of Figure 3.2(e). The machine phasor equation is:

$$\mathbf{E} = \mathbf{V}_t + \mathbf{I}(R_L + jX_s)$$

In most power system calculations R_L is neglected and a simple phasor diagram results.

3.2.1 Two-axis representation

If a machine has saliency, i.e. the rotor has salient poles or the rotor has non-uniform slotting in a solid rotor, then the main field axis and the cross-field axis have different reluctances. The armature reaction m.m.f., F_a in Figure 3.2(b), can be resolved into two components F_d and F_q, as shown in Figure 3.3(b) for the primitive machine shown in Figure 3.3(a). The m.m.f./flux/voltage transformation will now be different on these two component axes (known as the direct and quadrature axes) as less flux and induced e.m.f. will be produced on the q axis. This difference is reflected in the voltage phasor diagram of Figure 3.3(d) as two different reactances X_{ad} and X_{aq}, the component of stator current acting on the d axis I_d being associated with X_{ad}, and I_q with X_{aq}.

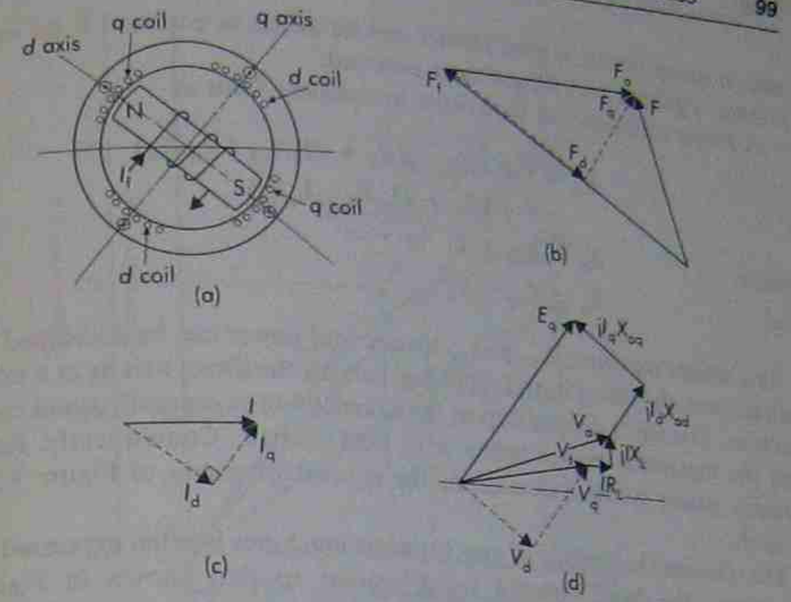


Figure 3.3 (a) Primitive machine- d and q axes. (b) The m.m.f. vectors resolved in d and q axes. F_d and F_q are components of F_a. (c) Current phasors resolved into I_d and I_q components. (d) Phasor diagram of voltages, E of Figure 3.2(d) now becomes E_q

The resolution of the stator m.m.f. into two components is represented in Figure 3.3(a) by two coils identified by the axes on which they produce m.m.f. Repeating the previous transformation from m.m.f. F to voltages, Figure 3.3(b) transforms to Figure 3.3(c) by considering the two-axis components of current and then results in Figure 3.3(d). Saturation of the two axes is represented by modifying X_{ad} and X_{aq}. Because I has been resolved into two components it is no longer possible for a salient machine to be represented by an equivalent circuit. However, equations can be obtained for the V_d and V_q components of the terminal voltage V_t as:

$$V_q = jI_d(X_{ad} + X_L) + I_qR_L - E_q$$
$$V_d = -jI_q(X_{aq} + X_L) - I_dR_L$$

It will be seen that the resistive terms (if included) contain the currents from the other axes. The term E_q is the e.m.f. representing the action of the field winding

and in many power system studies can be taken as constant if no automatic voltage regulator (AVR) action is assumed.

A single equation can be written in complex form as

$$\mathbf{E}_q = \mathbf{V} + \mathbf{I}R_L + \mathrm{j}\mathbf{I}X_L + \mathrm{j}(\mathbf{I}_d X_{ad} + \mathbf{I}_q X_{aq})$$
$$= \mathbf{V} + \mathbf{I}R_L + \mathrm{j}(\mathbf{I}_d X_d + \mathbf{I}_q X_q)$$

where $X_d = X_{ad} + X_L$

and $X_q = X_{aq} + X_L$

In a salient machine, $X_{ad} \neq X_{aq}$ torque and power can be developed with no field current, the stator m.m.f. 'locking' in with the direct axis as in a reluctance machine. The effect depends upon the saliency but is generally small compared with the maximum power rating with field current. Consequently, for multi-machine power system calculations the equivalent circuit of Figure 3.2(e) can be used.

The transient behaviour of synchronous machines is often expressed in two-axis terms, the basic concept being similar to that shown in Figure 3.3. However, a variety of conventions are employed and care must be taken to establish which direction of current is being taken as positive. In many texts, if a synchronous machine is running as a motor, the current entering the machine is taken as positive. Here we have assumed that current leaving a generator is positive.

To produce accurate voltage and current waveforms under transient conditions the two-axis method of representation with individually coupled coils on the d and q axes is necessary. Since these axes are at right-angles (orthogonal), then fluxes linking coils on the same axis do not influence the fluxes linking coils on the other axis, i.e. no cross-coupling is assumed. Differential equations can be set up to account for flux changes on both axes separately, from which corresponding voltages and currents can be derived through step-by-step integration routines. This form of calculation is left for advanced study. The use of finite elements can extend the accuracy of calculation still further.

3.2.2 Effect of saturation on X_s—the short-circuit ratio

The open-circuit characteristic is the graph of generated voltage against field current with the machine on open circuit and running at synchronous speed. The short-circuit chracteristic is the graph of stator current against field current with the terminals short-circuited; curves for a modern machine are shown in Figure 3.4. Here, X_s is equal to the open-circuit voltage produced by the same field current that produces rated current on short circuit, divided by this rated stator current. This value of X_s is constant only over the linear part of the open-circuit characteristic (the air-gap line) and ignores saturation. The actual

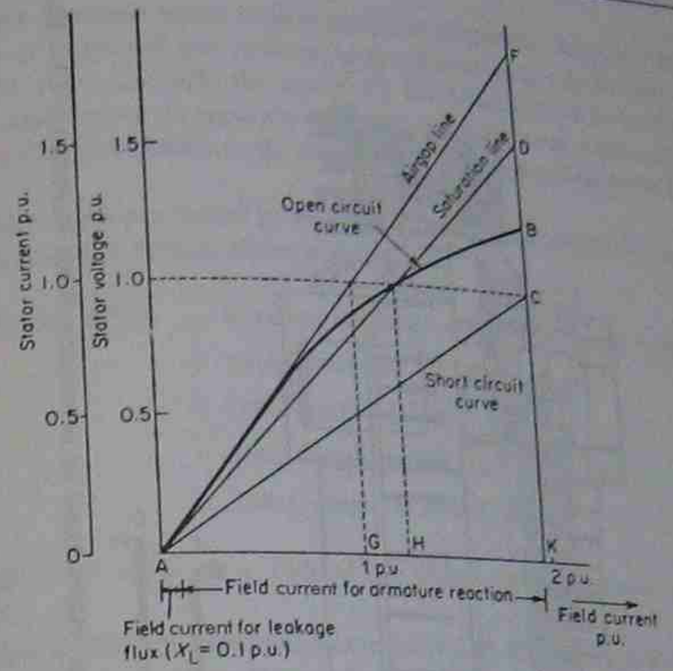


Figure 3.4 Open- and short-circuit characteristics of a synchronous machine. Unsaturated value of X_s = FK/CK. With operation near nominal voltage, the saturation line is used to give a linear characteristic with saturation

value of X_s at full-load current will obviously be less than this value and several methods exist to allow for the effects of saturation.

The short-circuit ratio (SCR) of a generator is defined as the ratio between the field current required to give nominal open-circuit voltage and that required to circulate full-load current in the armature when short-circuited. In Figure 3.4 the short-circuit ratio is AH/AK, i.e. 0.63. The SCR is also commonly calculated in terms of the air-gap line and the short-circuit curve, this giving an unsaturated value. To allow for saturation it is common practice to assume that the synchronous reactance is 1/SCR, which for this machine is 1.58 p.u. Economy demands the design of machines of low SCR and a value of 0.55 is common for modern machines. Unfortunately, transient stability margins are reduced as synchronous reactance increases (i.e. SCR reduces) so a conflict for design between economy and stability arises.

3.2.3 Turbogenerators

During the 1970s, the ratings of steam-driven turbine generators were reaching 1000 MW or over to obtain improvements in efficiency made possible by large

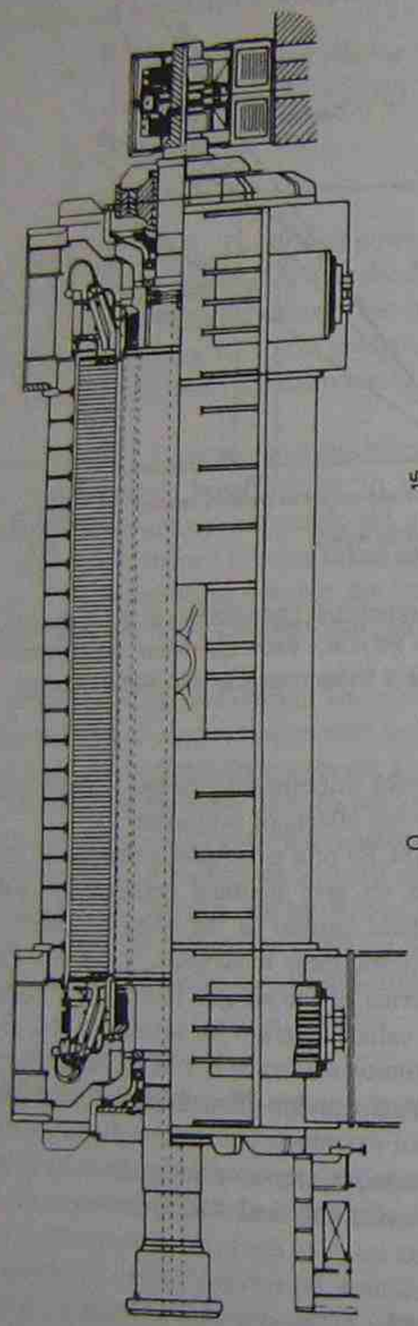


Figure 3.5 Sectional view of a 1000 MVA, 1800 r.p.m., 60 Hz generator (Reproduced by permission of the *Institution of Electrical Engineers*)

turbine plant and their lower capital costs per megawatt. Since the increasing availability of improved gas turbines with efficiencies of 55 per cent, even in 100–200 MW sizes, and with the use of gas for firing, air- or hydrogen-cooled generators with five or six separate machines in each power station are becoming common for new installations, particularly by independent plant producers (IPPs).

Large generators are cooled by hydrogen and water pumped through hollow stator conductors, a typical statistic being:

500 MW, 588 MVA : winding cooling—rotor, hydrogen; stator, water;
: rotor diameter 1.12 m;
: rotor length 6.2 m;
: total weight 0.63 kg/kVA;
: rotor weight 0.1045 kg/kVA.

A cross-sectional view of a 1000 MVA steam turbogenerator is shown in Figure 3.5.

The most important problems encountered in the development and use of large generators arise from (1) mechanical difficulties due to the rotation of large masses, especially stresses in shafts and rotors, critical speeds, and torsional oscillations; (2) the large acceleration forces produced on stator bars which must be withstood by their insulation (as much as 27 g in some designs); (3) the need for more effective cooling. Mica paper and glass bonded with epoxy resin are at present being used as stator insulation, the maximum permissible temperature being 135°C.

Semiconductor rectifiers are employed for excitation. Traditionally, sets had d.c. exciters mounted on the main shaft. This type was followed by an a.c. exciter from which the current is rectified so that a.c. and d.c. slip-rings are required. More recently, a.c. exciters with integral fused-diode or thyristor rectifiers are employed, thus avoiding any brush gear and consequent maintenance problems, the semiconductors rotating on the main shaft.

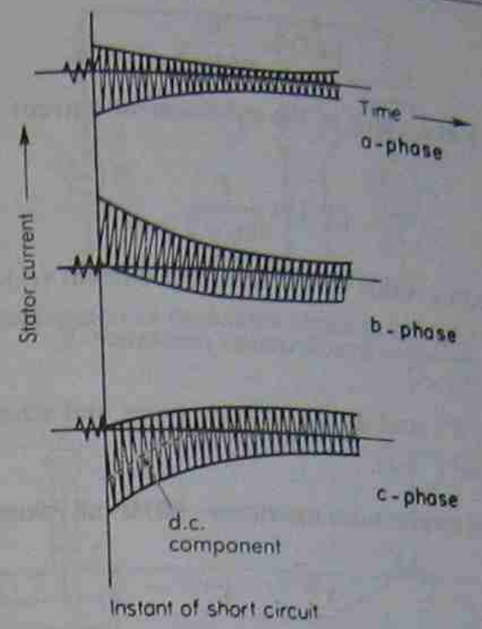
Superconducting field windings without ferromagnetic rotors would give magnetic flux densities of about three times the present 2–2.5 T and offer the possibility of reduced machine size and weight and an increase in operating efficiency of about 0.4 per cent; the machine reactance would be smaller (the synchronous reactance may be reduced by as much as four-fifths), but the short-circuit forces would be larger. Superconducting stator windings are impracticable because of the prohibitively large a.c. losses they would produce. Even if such machines are shown to be economic for large sizes their reliability must present major problems. Reliability is all-important, and the loss resulting from an outage of a large conventional generator for 1 day is roughly equivalent to 1 per cent of the initial cost of the machine.

3.3 Equivalent Circuit Under Balanced Short-Circuit Conditions

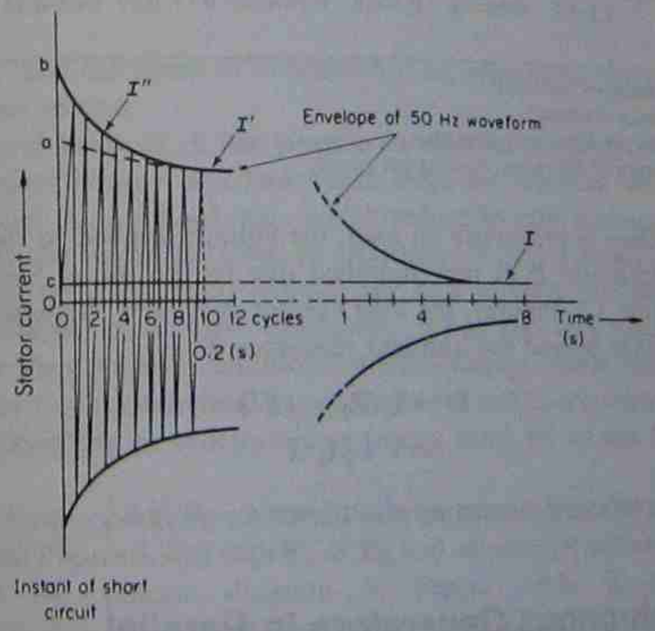
A typical set of oscillograms of the currents in three stator phases when a synchronous generator is suddenly short-circuited is shown in Figure 3.6(a). In all three traces a direct-current component is evident and this is to be expected from a knowledge of transients in R–L circuits. The magnitude of direct current in any phase present depends upon the instant at which the short circuit is applied and on the power factor of the circuit. As there are three voltages mutually at 120° it is possible for only one to have the maximum direct-current component. Often, to clarify the physical conditions, the direct component is ignored and a trace of short-circuit current, as shown in Figure 3.6(b), is considered. Immediately after the application of the short circuit the armature current endeavours to create an armature reaction m.m.f., as already mentioned, but the main flux cannot change to a new value immediately as it is linked with low-resistance circuits consisting of: (1) the rotor winding which is effectively a closed circuit; (2) the damper bars, i.e. a winding which consists of short-circuited turns of copper strip set in the poles to dampen oscillatory tendencies, and (3) the rotor body, often of forged steel. As the flux remains unchanged initially, the stator currents are large and can flow only because of the creation of opposing currents in the rotor and damper windings by what is essentially transformer action. Owing to the higher resistance, the current induced in the damper winding decays rapidly and the armature current commences to fall. After this, the currents in the rotor winding and body decay, the armature reaction m.m.f. is gradually established, and the generated e.m.f. and stator current fall until the steady-state condition on short circuit is reached. Here, the full armature reaction effect is operational and the machine is represented by the synchronous reactance X_s. These effects are shown in Figure 3.6, with the high initial current due to the damper winding and then the gradual reduction until the full armature reaction is established.

To represent the initial short-circuit conditions two new models must be introduced. If, in Figure 3.6(b), the envelopes of the 50 Hz waves are traced, a discontinuity appears. Whereas the natural envelope continues to point 'a' on the stator current axis, the actual trace finishes at point 'b'; the reasons for this have been mentioned above. To account for both of these conditions, two new reactances are needed to represent the machine, the very initial conditions requiring what is called the *subtransient reactance* (X'') and the subsequent period the *transient reactance* (X'). In the following definitions it is assumed that the generator is on no-load prior to the application of the short circuit and is of the round-rotor type. Let the no-load phase voltage of the generator be E volt (r.m.s.)

Then, from Figure 3.6(b), the subtransient reactance



(a)



(b)

Figure 3.6 (a) Oscillograms of the currents in the three phases of a generator when a sudden short circuit is applied. (b) Trace of a short-circuit current when direct-current component is removed

$$(X'') = \frac{E}{0b/\sqrt{2}}$$

where $0b/\sqrt{2}$ is the r.m.s. value of the subtransient current (I''); the transient reactance

$$(X') = \frac{E}{0a/\sqrt{2}}$$

where $0a/\sqrt{2}$ is the r.m.s. value of the transient current (I'), and finally

$$\frac{E}{0c/\sqrt{2}} = \text{synchronous reactance } X_s$$

Typical values of X'', X', and X_s for various types and sizes of machines are given in Table 3.1.

Table 3.1 Constants of synchronous machines—60 Hz (all values expressed as per unit on rating)

Type of machine	X_s (or X_d)	X_q	X'	X''	X_2	X_0	R_L
Turbo-alternator	1.2–2.0	1–1.5	0.2–0.35	0.17–0.25	0.17–0.25	0.04–0.14	0.003–0.008
Salient pole (hydroelectric)	0.16–1.45	0.4–1.0	0.2–0.5	0.13–0.35	0.13–0.35	0.02–0.2	0.003–0.0015
Synchronous compenstor	1.5–2.2	0.95–1.4	0.3–0.6	0.18–0.38	0.17–0.37	0.03–0.15	0.004–0.01

X_2 = negative sequence reactance.
X_0 = zero sequence reactance.
X' = and X'' are the direct axis quantities.
R_L = a.c. resistance of the stator winding per phase.

If the machine is previously on load, the voltage applied to the equivalent reactance, previously E, is now modified due to the initial load volt-drop. Consider Figure 3.7. Initially, the load current is I_L and the terminal voltage is V. The voltage behind the transient reactance X' is

$$\begin{aligned}\mathbf{E}' &= \mathbf{I}_L(\mathbf{Z}_L + \mathrm{j}X') \\ &= \mathbf{V} + \mathrm{j}\mathbf{I}_L X'\end{aligned}$$

and hence the transient current on short circuit $= \mathbf{E}'/\mathrm{j}X'$.

3.4 Synchronous Generators in Parallel

Consider two machines A and B (as shown in Figure 3.8(a)), the voltages of which have been adjusted to equal values by means of the field regulators, and the speeds of which are slightly different. In Figure 3.8(b) the phase voltages

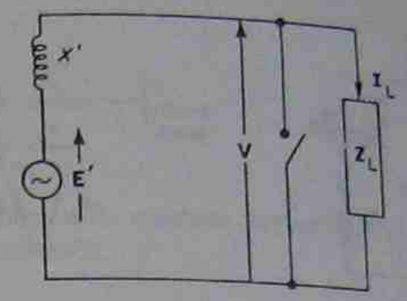


Figure 3.7 Modification of equivalent circuit to allow for initial load current

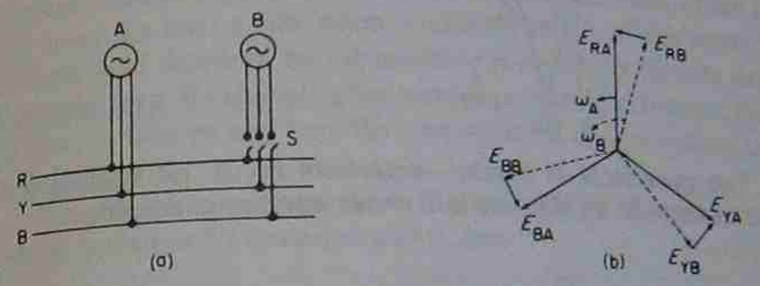


Figure 3.8 (a) Generators in parallel. (b) Corresponding phasor diagrams

are E_{RA}, etc., and the speed of machine A is ω_A radians per second and of B, ω_B radians per second.

If voltage phasors of A are considered stationary, those of B rotate at a relative velocity $(\omega_B - \omega_A)$ and hence there are resultant voltages across the switch S of $(\mathbf{E_{RA}} - \mathbf{E_{RB}})$, etc., which reduce to zero during each relative revolution. If the switch is closed at an instant of zero voltage, the machines are connected (synchronized) without the flow of large currents due to the resultant voltages across the armatures. When the two machines are in synchronism they have a common terminal-voltage, speed, and frequency. Any tendency for one machine to accelerate relative to the other immediately results in a retarding or synchronizing torque being set up due to the current circulated.

Two machines operating in parallel are represented by the equivalent circuits shown in Figure 3.9(a) with $\mathbf{E_A} = \mathbf{E_B}$ and on no external load. If A tries to gain speed the phasor diagram in Figure 3.9(b) is obtained and $I = E_R/(\mathbf{Z_A} + \mathbf{Z_B})$. The circulating current $\mathbf{I}$ lags $\mathbf{E_R}$ by an angle $\tan^{-1}(X/R)$ and, as in most machines $X \gg R$, this angle approaches 90°. This current is a generating current for A and a motoring current for B; hence A is generating power and tending to slow down and B is receiving power from A and speeding up. Therefore, A and B remain at the same speed, 'in step', or in synchronism.

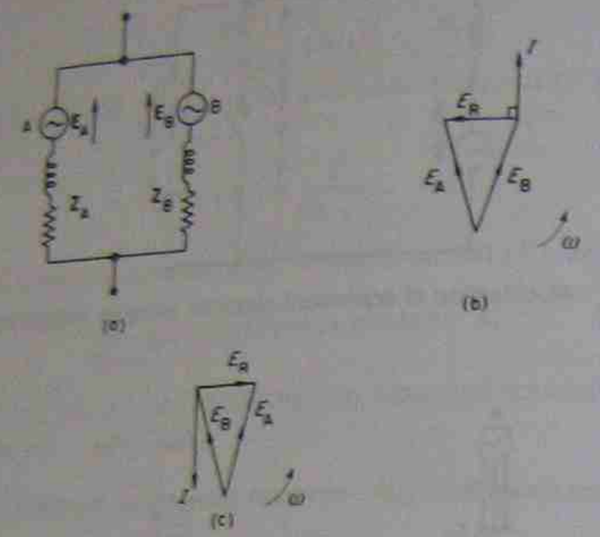


Figure 3.9 (a) Two generators in parallel—equivalent circuit. (b) Machine A in phase advance of machine B. (c) Machine B in phase advance of machine A

Figure 3.9(c) shows the state of affairs when B tries to gain speed on A. The quality of a machine to return to its original operating state after a momentary disturbance is measured by the *synchronizing power* and *torque*. It is interesting to note that as the impedance of the machines is largely inductive, the restoring powers and torques are large; if the system were largely resistive it would be difficult for synchronism to be maintained.

Normally, more than two generators operate in parallel and the operation of one machine connected in parallel with many others is of great interest. If the remaining machines in parallel are of such capacity that the presence of the generator under study causes no difference to the voltage and frequency of the other, they are said to comprise an *infinite busbar system*, i.e. an infinite system of generation. In practice, a perfect infinite busbar is never fully realized but if, for example, a 600 MW generator is removed from a 30 000 MW system, the difference in voltage and frequency caused will be very slight.

3.5 The Operation of a Generator on Infinite Busbars

In this section, in order to simplify the ideas as much as possible, the resistance of the generator will be neglected; in practice, this assumption is usually reasonable. Figure 3.10(a) shows the schematic diagram of a machine connected to an infinite busbar along with the corresponding phasor diagram. If losses are neglected the power output from the turbine is equal to the power output from

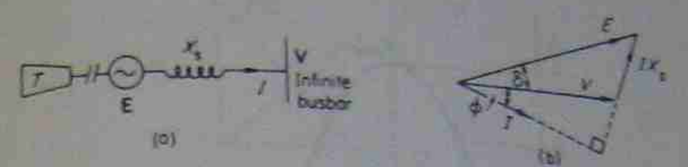


Figure 3.10 (a) Synchronous machine connected to an infinite busbar. (b) Corresponding phasor diagram

the generator. The angle δ between the E and V phasors is known as the load angle and is dependent on the power input from the turbine shaft. With an isolated machine supplying its own load the latter dictates the power required and hence the load angle; when connected to an infinite-busbar system, however, the load delivered by the machine is no longer directly dependent on the connected load. By changing the turbine output, and hence δ, the generator can be made to take on any load that the operator desires subject to economic and technical limits.

From the phasor diagram in Figure 3.10(b), the power delivered to the infinite busbar $= VI\cos\phi$ per phase, but

$$\frac{E}{\sin(90+\phi)} = \frac{IX_s}{\sin\delta}$$

hence

$$I\cos\phi = \frac{E}{X_s}\sin\delta$$

$$\therefore \text{ power delivered } = \frac{VE}{X_s}\sin\delta \quad \text{(W)} \tag{3.1}$$

This expression is of extreme importance as it governs, to a large extent, the operation of a power system. It could have been obtained directly from equation (2.6), putting $\alpha = 0$.

Equation (3.1) is shown plotted in Figure 3.11. The maximum power is obtained at $\delta = 90°$. If δ becomes larger than 90° due to an attempt to obtain more than P_{max}, increase in δ results in less power output and the machine becomes unstable and loses synchronism. Loss of synchronism results in the interchange of current surges between the generator and network as the poles of the machine pull into synchronism and then out again; i.e. the generator 'pole slips'.

If the power output of the generator is increased by small increments with the no-load voltage kept constant, the limit of stability occurs at $\delta = 90°$ and is known as the *steady-state stability limit*. There is another limit of stability due to a sudden large change in conditions, such as caused by a fault, and this is known as the *transient stability limit*; it is possible for the rotor to oscillate beyond 90° a number of times. If these oscillations diminish, the machine is

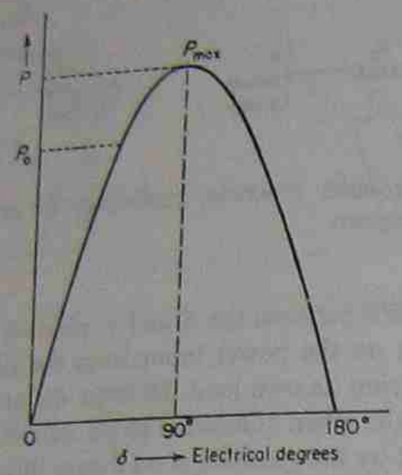


Figure 3.11 Power-angle curve of a synchronous machine. Resistance and saliency are neglected

stable. The load angle δ has a physical significance; it is the angle between like radial marks on the end of the rotor shaft of the machine and on an imaginary rotor representing the system. The marks are in identical physical positions when the machine is on no-load. The synchronizing power coefficient

$$= \mathrm{d}P/\mathrm{d}\delta \qquad \text{watts per radian}$$

and the synchronizing torque coefficient

$$= (1/\omega_s)/(\mathrm{d}P/\mathrm{d}\delta)$$

In Figure 3.12(a) the phasor diagram for the limiting steady-state condition is shown. It should be noted that in this condition, current is always leading. Figures 3.12(b), (c), and (d), show the phasor diagrams for various operational conditions. Another interesting operating condition is variable power and constant excitation. This is shown in Figure 3.13. In this case, as V and E are constant, when the power from the turbine is increased δ must increase and the power factor changes.

It is convenient to summarize the above types of operation in a single diagram or chart which will enable an operator to see immediately whether the machine is operating within the limits of stability and rating.

3.5.1 The performance chart of a synchronous generator

Consider Figure 3.14(a), the phasor diagram for a round-rotor machine ignoring resistance. The locus of constant IX_s, I, and hence MVA, is a circle, and the locus of constant E is a circle. Hence, provided V is constant, then

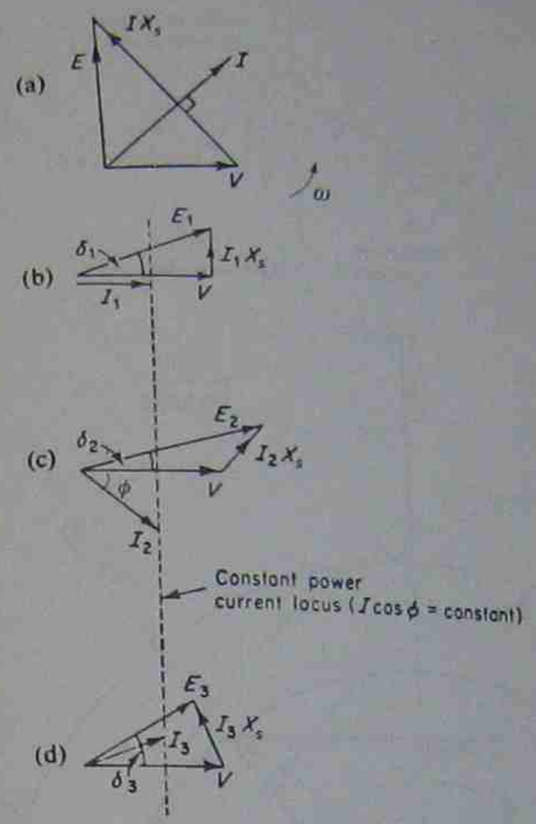


Figure 3.12 (a) Phasor diagram for generator at limit of steady-state stability. (b) to (d) Phasor diagrams for generator delivering constant power to the infinite busbar system but with different excitations. As V is constant, the in-phase component of **I** must be constant. As $(EV/X)\sin\delta)$ is constant as E changes, δ must change and

$$\delta_3 > \delta_1 > \delta_2$$

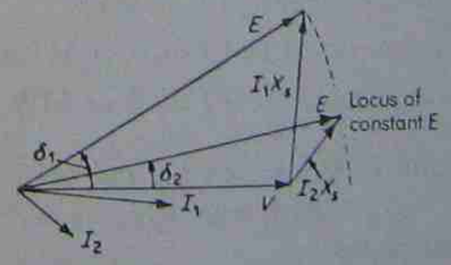
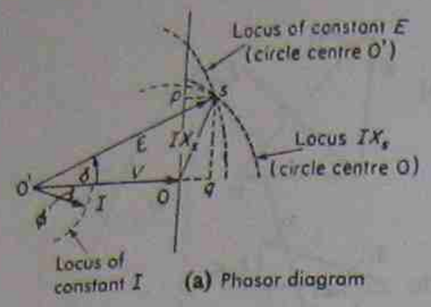


Figure 3.13 Operation at variable power and constant excitation



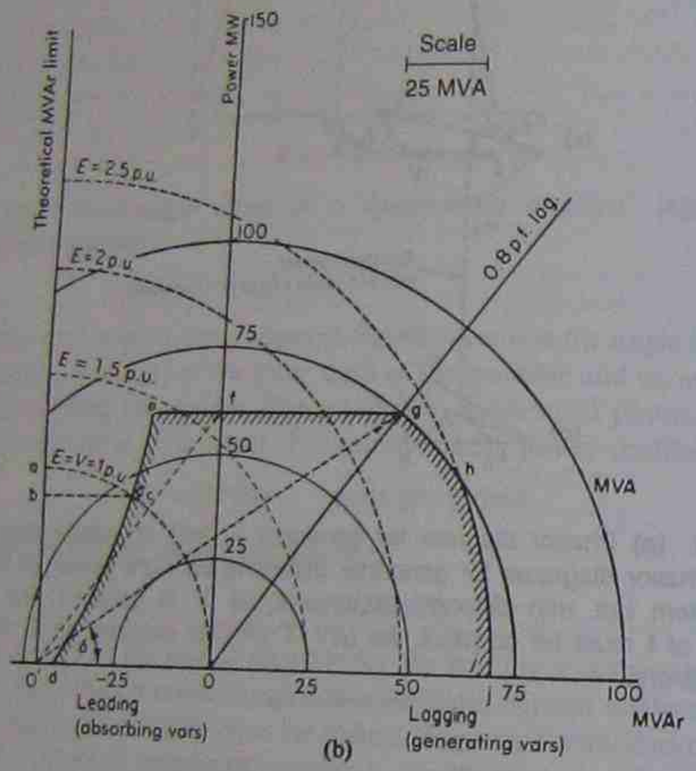


Figure 3.14 Performance chart of a synchronous generator

$0s$ is proportional to VI or MVA

ps is proportional to $VI \sin\phi$ or MVAr

sq is proportional to $VI \cos\phi$ or MW

To obtain the scaling factor for MVA, MVAr, and MW, the fact that at zero excitation, $E = 0$ and $IX_s = V$, is used, from which I is V/X_s at 90° leading to $00'$, corresponding to VAr/phase.

The construction of a chart for a 60 MW machine follows (Figure 3.14(b)).

Machine data

60 MW, 0.8 p.f., 75 MVA
11.8 kV, SCR 0.63, 3000 r.p.m.
Maximum exciter current 500 A

$$X_s = \frac{1}{0.63} \text{ p.u.} = 2.94\,\Omega/\text{phase}$$

The chart will refer to complete three-phase values of MW and MVAr. When the excitation and hence E are reduced to zero, the current leads V by 90° and is equal to (V/X_s), i.e. $11\,800/\sqrt{3} \times 2.94$. The leading vars correspond to this, given by $3V^2/X = 47.4$ MVAr.

With centre 0 a number of semicircles are drawn of radii equal to various MVA loadings, the most important being the 75 MVA circle. Arcs with 0′ as centre are drawn with various multiples of $00'$ (or V) as radii to give the loci for constant excitations. Lines may also be drawn from 0 corresponding to various power factors, but for clarity only 0.8 p.f. lagging is shown, i.e. the machine is generating vars. The operational limits are fixed as follows. The rated turbine output gives a 60 MW limit which is drawn as shown, i.e. line efg, which meets the 75 MVA line at g. The MVA arc governs the thermal loading of the machine, i.e. the stator temperature rise, so that over portion gh the output is decided by the MVA rating. At point h the rotor heating becomes more decisive and the arc hj is decided by the maximum excitation current allowable, in this case assumed to be 2.5 p.u. The remaining limit is that governed by loss of synchronism at leading power factors. The theoretical limit is the line perpendicular to $00'$ at 0′ (i.e. $\delta = 90°$), but, in practice, a safety margin is introduced to allow a further increase in load of either 10 or 20 per cent before instability. In Figure 3.14 a 10 per cent margin is used and is represented by ecd: it is constructed in the following manner. Considering point 'a' on the theoretical limit on the $E = 1$ p.u. arc, the power $0'a$ is reduced by 10 per cent of the rated power (i.e. by 6 MW) to $0'b$; the operating point must, however, still be on the same E arc and b is projected to c, which is a point on the new limiting curve. This is repeated for several excitations, finally giving the curve ecd.

The complete operating limit is shown shaded and the operator should normally work within the area bounded by this line, provided the generator is running at rated voltage. If the voltage is different from rated, e.g. at −5 per cent of rated, the whole operating chart will shrink by a pro-rata amount, except for the excitation and turbine maximum power line. Note, therefore, that for the generator to produce rated power at reduced voltage, the stator requires to operate on overcurrent. This may be possible if an overcurrent rating is given.

Example 3.1

Use the chart of Figure 3.14 to determine the excitation and the load angle at full load (60 MW, 0.8 p.f. lag) and at unity power factor, rated output. Check by calculation.

Solution

From the diagram, the following values are obtained:

	Excitation E	Angle δ
0.8 p.f. lag	2.3 p.u.	33°
u.p.f.	1.7 p.u.	50°

Check

$$\text{Power} = \frac{VE}{X_s}\sin\delta$$

$$\therefore 60 \times 10^6 = \frac{11\,800 \times (11\,800 \times 2.3)}{2.94}\sin\delta$$

$\delta = 33.4°$ at 0.8 p.f. lag

$\delta = 48.2°$ at u.p.f.

3.6 Salient-Pole Generators

The assumption that a generator has a single impedance X, as in Figure 3.2, is rarely true and the two-axis approach of Figure 3.3 with $X_d \neq X_q$ represents generator behaviour better, particularly as it gives a more accurate value for torque angle δ of a given load. Power as a function of load angle is derived below.

3.6.1 Output power

The output power is given by $VI\cos\phi$ where V is shown as V_t in Figure 3.3(d). If we redraw this figure neglecting R_L and splitting IX_L into two components I_qX_L and I_dX_L we obtain Figure 3.15.

From Figure 3.15(b) we see that

$$\mathbf{E} = \mathbf{V} + \mathrm{j}\mathbf{I}_d X_d + \mathrm{j}\mathbf{I}_q X_q$$
$$= \mathbf{V} + \mathrm{j}\mathbf{I}X_q + \mathrm{j}\mathbf{I}_d(X_d - X_q) \qquad (3.2)$$

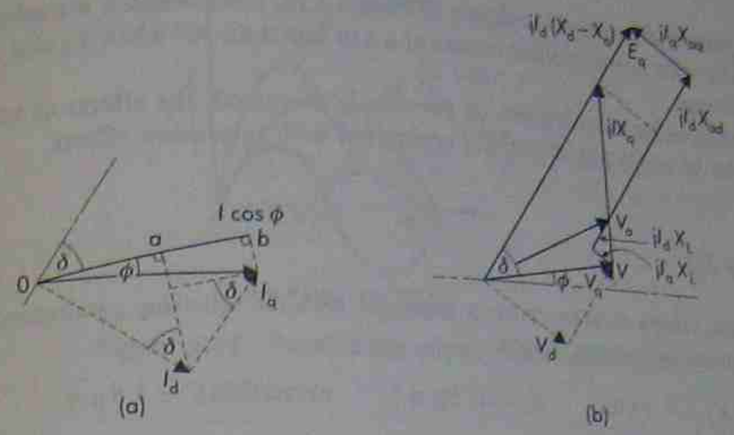


Figure 3.15 Current and voltage phasor diagrams for salient-pole generator. (a) Current phasors and components. (b) Voltage phasors and components

As $\mathrm{j}\mathbf{I}_d(X_d - X_q)$ is in phase with E, and we know V, I, and ϕ from the loading condition, IX_q can be drawn at right angles to the current I so that the direction of E can be determined. From this, the d and q axis directions can be drawn on both the current and voltage phasor diagrams and hence the remaining phasors can be established.

From Figure 3.15(a) we have

$$I\cos\phi = \text{oa} + \text{ab} = I_d\sin\delta + I_q\cos\delta$$

But

$$V\sin\delta = I_qX_q \qquad \text{and} \qquad V\cos\delta = E - I_dX_d$$

Hence

$$\text{Output power } P = V[I\cos\phi]$$
$$= V\left[\frac{V\sin\delta}{X_q}\cos\delta + \frac{E - V\cos\delta}{X_d}\sin\delta\right]$$
$$= \frac{VE}{X_d}\sin\delta + \frac{V^2(X_d - X_q)}{2X_dX_q}\sin 2\delta \qquad (3.3)$$

and

$$\frac{dP}{d\delta} = \frac{VE}{X_d}\cos\delta + \frac{V^2(X_d - X_q)}{X_dX_q}\cos 2\delta \qquad (3.4)$$

$$= \text{synchronizing power coefficient}$$

The plot of P against δ is shown in Figure 3.16, from which it is evident that the steady-state stability limit occurs at a δ of less than 90° when X_d and X_q are very different.

In steady-state calculations, as previously discussed, the effects of saliency may often be neglected, especially compared with saturation effects.

Example 3.2

Compare the effects of saliency for a generator with the following parameters when producing maximum power output.

$$X_d = 1.2\,\text{pu}; \qquad X_q = 0.9\,\text{p.u.}; \qquad \text{excitation} E = 1.5\,\text{p.u.}$$

Generator is connected to an infinite bus with voltage $V = 1\,\text{p.u.}$

Calculation

$$P_{max}\ (\text{saliency neglected}) = \frac{EV}{X_d}\sin\delta = \frac{1.5 \times 1.0}{1.2}$$
$$= 1.25\,\text{p.u.} \qquad (\text{W})$$

$$P_{max}\ (\text{with saliency}) = \frac{EV}{X_d}\sin\delta + V^2\frac{(X_d - X_q)}{2X_dX_q}\sin 2\delta$$
$$= 1.25\sin\delta + 0.14\sin 2\delta$$

The quickest way to solve this equation for P_{max} is to try several values of δ, and produce a table as follows:

δ (degrees)	P_{max}
90	1.25 (already calculated)
80	1.279
70	1.26 (less than before)
75	1.277
78	1.280 (Max value at 78°)

$$\text{Error is } \frac{1.28 - 1.25}{1.28} = 2.3 \text{ per cent}$$

Generally, provided that $E > V$, saliency can be neglected for most studies. The reader should try other values of E to check this out, but note that the P_{max} with saliency occurs at angles 10° or more less than without saliency.

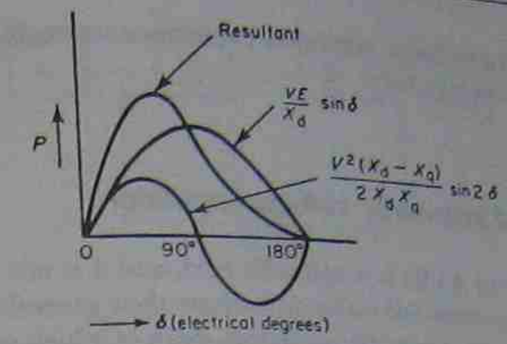


Figure 3.16 Power-angle curve—salient-pole machine

3.7 Automatic Voltage Regulators (AVRs)

Excitation of a synchronous generator is derived from a d.c. supply with variable voltage and requires considerable power to produce the required operating flux. Hence an exciter may require 1 MW or more to excite a large generator's field winding on the rotor. Nowadays, such a power amplifier is best arranged through an a.c. generator driven or overhung on the rotor shaft feeding the rotor through diodes, also mounted on the shaft and rotating with it, or by means of thyristors whose grids can be controlled by a feedback signal (see Figure 3.17).

Originally, the main exciter consisted of a d.c. shunt-controlled generator supplying the generator rotor winding through slip rings on the rotor shaft. The main exciter was itself excited from a pilot exciter, self-excited for reliability. The most important aspect of the excitation system is its speed of response between a change in the reference signal V_{ref} and the change in the excitation current I_f. Control system theory using Bode diagrams and phase

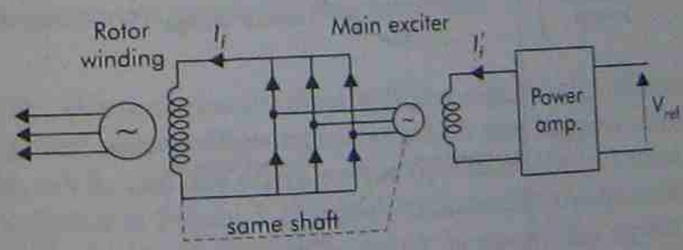


Figure 3.17 Excitation arrangements for a synchronous generator using diodes. Diodes can be replaced by thyristor controllers and the power amplifier can be eliminated

margins is required to design appropriate responses and stability boundaries of a generator excitation system.

3.7.1 Types of automatic voltage regulator

The detailed study of AVRs is a specialist field, and it is not intended here to discuss types in any detail but rather to indicate their general effect. There are two broad divisions of automatic regulators, both of which set out to control the output voltage of the synchronous generator by controlling the exciter. In general the deviation of the terminal voltage from a prescribed value is passed to control circuits and thus the field current is varied, and it is in the manner and speed in achieving this that the division occurs.

The first and older type can be broadly classed as *electromechanical*. A well-known variety of this is the carbon-pile regulator. In this, a voltage proportional to the deviation voltage operates a solenoid assembly to vary the pressure exerted on a carbon-pile resistor in the exciter field, thus varying its resistance. Another type depends upon the conversion of the deviation voltage into a torque by means of a 'torque motor'; according to the angular position of the shaft of the motor, certain resistors in a resistor chain are cut out of the circuit and hence the exciter-field current changes. There are various other types, including the popular vibrating-reed regulator. All these types suffer from the disadvantages of being relatively slow acting and possessing dead bands, i.e. a certain deviation must occur before the mechanism operates; this is illustrated in Figure 3.18.

The other main group of regulators is known as 'continuously acting' as in Figure 3.17 and these are faster than the above types and have no dead bands. A general block diagram of a typical control system is given in Figure 3.19.

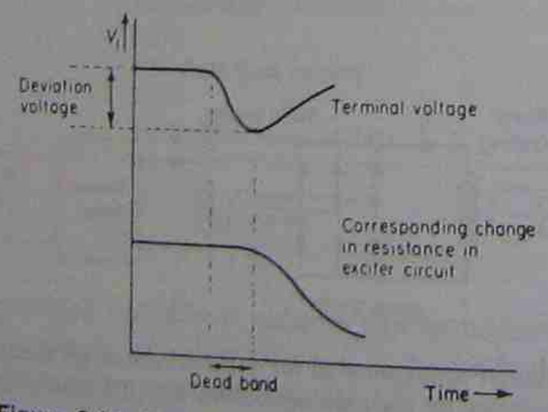


Figure 3.18 The effect of a dead band in a regulator

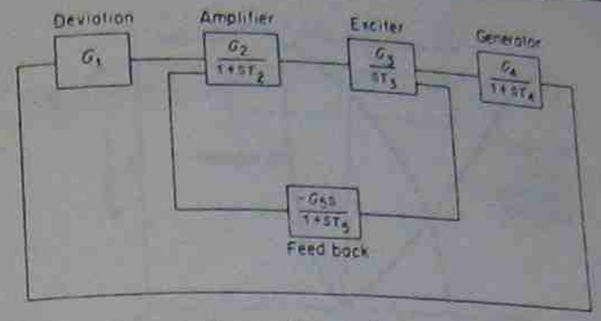


Figure 3.19 Block diagram of a continuously acting closed-loop automatic voltage regulator

3.7.2 Automatic voltage regulators and generator characteristics

The equivalent circuit used to represent the synchronous generator can be modified to account for the action of a regulator. Basically there are three conditions to consider:

1. Operation with fixed excitation and constant no-load voltage (E), i.e. no regulator action. This requires the usual equivalent circuit of E in series with Z_s.
2. Operation with a regulator which is not continuously acting, i.e. the terminal voltage varies with load changes. This can be simulated by E' and a reactance smaller than the synchronous value. It has been suggested by experience, in practice, that a reasonable value would be the transient reactance, although some authorities suggest taking half of the synchronous reactance. This mode will apply to most modern regulators.
3. Terminal voltage constant. This requires a very-fast-acting regulator and the nearest approach to it exists in the forced-excitation regulators (i.e. AVRs capable of reversing their driving voltage to suppress I_f quickly) used on generators supplying very long lines.

Each of the above representations will give significantly different values of maximum power output. The degree to which this happens depends on the speed of the AVR, and the effect on the operation chart of the synchronous generator is shown in Figure 3.20, which indicates clearly the increase in operating range obtainable. It should be noted, however, that operation in these improved leading power-factor regions may be limited by the heating of the stator winding. The actual power-angle curve may be obtained by a step-by-step process by using the gradually increasing values of E in $EV/X \sin\delta$.

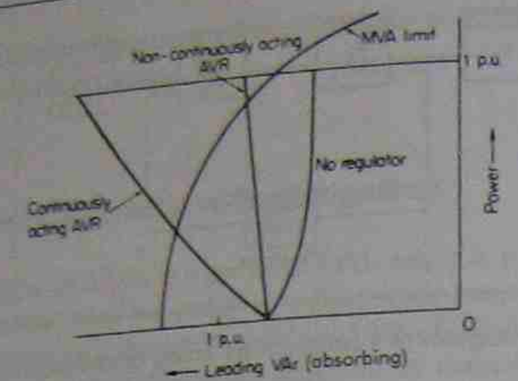


Figure 3.20 Performance chart as modified by the use of automatic voltage regulators

When a generator has passed through the steady-state limiting angle of $\delta = 90^\circ$ with a fast-acting AVR, it is possible for synchronism to be retained. The AVR, in forcing up the voltage, increases the power output of the machine so that instead of the power falling after $\delta = 90^\circ$, it is maintained and $dP/d\delta$ is still positive. This is illustrated in Figure 3.21(b), where the P–δ relation for the system in Figure 3.21(a) is shown. Without the AVR the terminal voltage of the machine (V_t) will fall with increased δ, the generated voltage E being constant and the power reaching a maximum at 90°. With a perfect AVR, V_t is maintained constant, E being increased with increase in δ. As $P = (EV_s/X)\sin\delta$, it is evident that power will increase beyond 90° until the excitation limit is reached, as shown in Figure 3.21(b).

Example 3.3

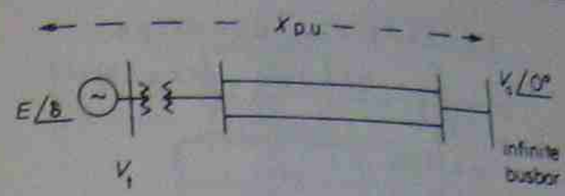
Determine the limiting powers for the system shown in Figure 3.22 for the three types of voltage regulation. All values are on 100 MVA, 132 kV bases. It may be assumed that the lines and transformers are each represented by a single-series reactance.

Solution

(a) No control, constant excitation voltage.

$$X = 1.5 + 0.15 + 0.1 + \frac{0.4}{2} = 1.95\,\text{p.u.}$$

From equation (2.8)



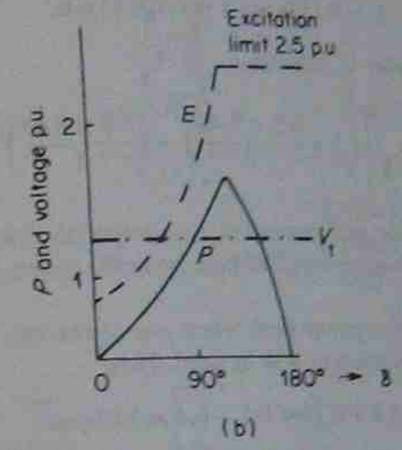


Figure 3.21 (a) Generator feeding into an infinite busbar. (b) Variations of output power P, generated voltage E, and terminal voltage V_t with load (transmission) angle δ. Perfect AVR

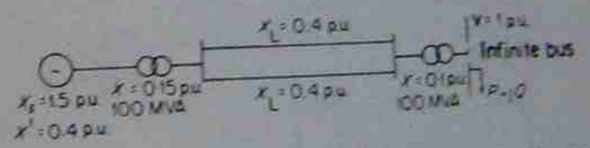


Figure 3.22 Line diagram of system for Example 3.3. Normal operating load $P = 0.8$ p.u., $Q = 0.5$ p.u.

$$E = \sqrt{\left[\left(1 + \frac{0.5 \times 1.95}{1}\right)^2 + \left(\frac{0.8 \times 1.95}{1}\right)^2\right]}$$

$$= 2.52\,\text{p.u.}$$

$$\therefore P_{max} = \frac{EV}{X} = \frac{2.52 \times 1}{1.95} = 1.29\,\text{p.u.}$$

(b) Non-continuously acting AVR.

$$X = 0.4 + 0.15 + 0.1 + 0.2 = 0.85\,\text{p.u.}$$

$$E' = \sqrt{\left[\left(1 + \frac{0.5 \times 0.85}{1}\right)^2 + \left(\frac{0.8 \times 0.85}{1}\right)^2\right]}$$

$$= 1.59\,\text{p.u.}$$

$$\therefore P_{max} = \frac{1.59 \times 1}{0.85} = 1.87\,\text{p.u.}$$

(c)

$$X = 0.15 + 0.1 + 0.2 = 0.45\,\text{p.u.}$$

Terminal voltage of generator V_t constant,

$$V_t = \sqrt{\left[\left(1 + \frac{0.5 \times 0.45}{1}\right)^2 + \left(\frac{0.8 \times 0.45}{1}\right)^2\right]}$$

$$= 1.28\,\text{p.u.}$$

$$P_{max} = \frac{1.28 \times 1}{0.45} = 2.85\,\text{p.u.}$$

It is interesting to compare the power limit when one of the two lines is disconnected; the transient-reactance representation will be used. Now

$$X = 0.4 + 0.15 + 0.1 + 0.4 = 1.05\,\text{p.u.}$$

$$E' = \sqrt{\left(1 + \frac{0.5 \times 1.05}{1}\right)^2 + \left(\frac{0.8 \times 1.05}{1}\right)^2}$$

$$= 1.74\,\text{p.u.}$$

$$P_{max} = \frac{1.74 \times 1}{1.05} = 1.65\,\text{p.u.}$$

Lines, Cables, and Transformers

3.8 Overhead Lines—Types and Parameters

Overhead lines are suspended from insulators which are themselves supported by towers or poles. The span between two towers is dependent upon the allowable sag in the line, and for steel towers with very-high-voltage lines the span is normally 370–460 m (1200–1500 ft). Typical supporting structures are shown in Figures 3.23 and 3.24. There are two main types of tower:

1. Those for straight runs in which the stress due to the weight of the line alone has to be withstood;

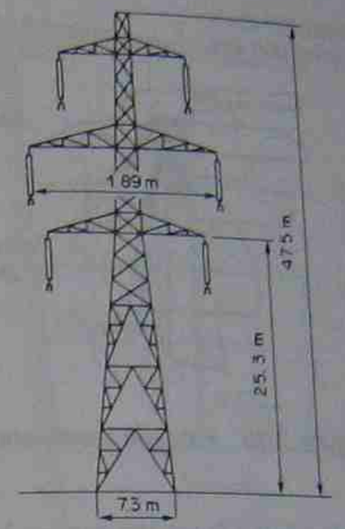


Figure 3.23 A 400 kV double-circuit overhead-line tower. Two conductors per phase (bundle conductors) (Reproduced by permission of *Institution of Electrical Engineers*)

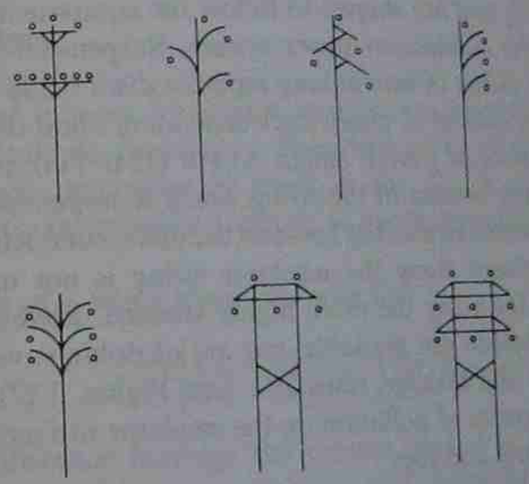


Figure 3.24 Typical pole-type structures

2. Those for changes in route, called deviation towers; these withstand the resultant forces set up when the line changes direction.

When specifying towers and lines, ice and wind loadings are taken into account, as well as extra forces due to a break in the lines on one side of a tower. For lower voltages and distribution circuits, wooden, reinforced concrete poles or, nowadays, glass fibre tubes are used with conductors supported in horizontal formations.

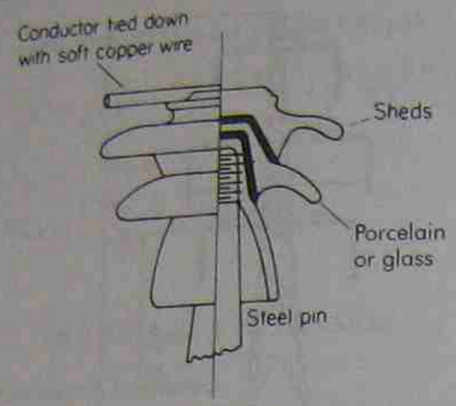


Figure 3.25 Pin-type insulators

The live conductors are insulated from the towers by insulators which take two basic forms: the *pin type* and the *suspension type*. The pin-type insulator is shown in Figure 3.25 and it is seen that the conductor is supported on the top of the insulator. This type is used for lines up to 33 kV. The two or three porcelain 'sheds' or 'petticoats' provide an adequate leakage path from the conductor to earth and are shaped to follow the equipotentials of the electric field set up by the conduction–tower system. Suspension insulators (Figure 3.26) consist of a string of interlinking separate discs made of glass or porcelain. A string may consist of many discs depending upon the line voltage; for 400 kV lines, 19 discs of overall length 3.84 m (12 ft 7 in) are used. The conductor is held at the bottom of the string which is suspended from the tower. Owing to the capacitances existing between the discs, conductor, and tower, the distribution of voltage along the insulator string is not uniform, the discs nearer the conductor being the more highly stressed. Methods of calculating this voltage distribution are available, but are of dubious value owing to the shunting effect of the leakage resistance (see Figure 3.27). This resistance depends on the presence of pollution on the insulator surfaces and is considerably modified by rain and fog.

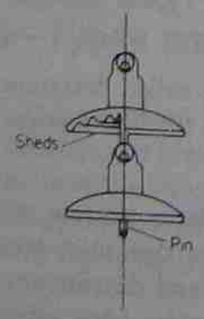


Figure 3.26 Suspension-type insulators

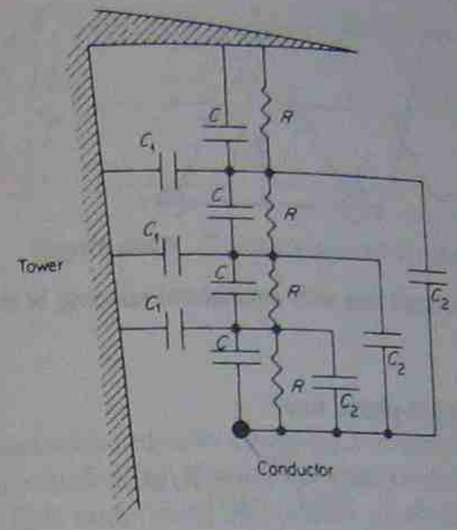


Figure 3.27 Equivalent circuit of a string of four suspension insulators. C = self-capacitance of disc; C_1 = capacitance disc to earth; C_2 = capacitance disc to line; R = leakage resistance

3.8.1 Parameters

The parameters of interest for circuit analysis are inductance, capacitance, resistance, and leakage resistance. The derivation of formulae for the calculation of these quantities is given in the *E.H.V. Transmission Line Reference Book*. It is intended here merely to quote these formulae and discuss their application.

The inductance of a single-phase two-wire line

$$= \frac{\mu_0}{4\pi}\left[1 + 4\ln\left(\frac{d-r}{r}\right)\right]\text{H/m} \tag{3.5}$$

where d is the distance between the centres, and r is the radius, of the conductors. When performing load flow and balanced-fault analysis on three-phase systems it is usual to consider one phase only, with the appropriate angular adjustments made for the other two phases. Therefore, phase voltages are used and the inductances and capacitances are the equivalent phase or line-to-neutral values. For a three-phase line with equilateral spacing (Figure 3.28) the inductance and capacitance with respect to the hypothetical neutral conductor are used, and this inductance can be shown to be half the loop inductance of the single-phase line, i.e. the inductance of one conductor.

The line–neutral inductance for equilateral spacing

$$= \frac{\mu_0}{8\pi}\left[1 + 4\ln\left(\frac{d-r}{r}\right)\right]\text{H/m} \tag{3.6}$$

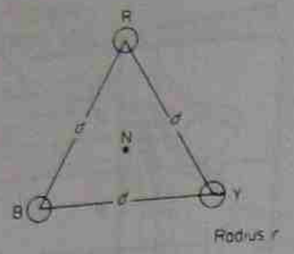


Figure 3.28 Overhead line with equiliateral spacing of the conductors

The capacitance of a single-phase line

$$C = \frac{\pi\varepsilon_0}{\ln[d-r)/r]}\text{F/m} \tag{3.7}$$

With three-phase conductors spaced equiliaterally, the capacitance of each line to be hypothetical neutral is double that for the two-wire circuit, i.e.

$$\frac{2\pi\varepsilon_0}{\ln[d-r)/r]}\text{F/m} \tag{3.8}$$

In practice, the conductors are rarely spaced in the equiliateral formation, and it can be shown that the average value of inductance or capacitance for any formation of conductors can be obtained by the representation of the system by one of equivalent equilateral spacing. The equivalent spacing d_{eq} between conductors is given by

$$d_{eq} = \sqrt[3]{(d_{12} \cdot d_{23} \cdot d_{31})}$$

Often, two three-phase circuits are electrically in parallel; if physically remote from each other, the reactances of the lines are identical. When the two circuits are situated on the same towers, however, the magnetic interaction between them should be taken into account. The use of bundle conductors, i.e. more than one conductor per insulator, reduces the reactance; it also reduces conductor–surface voltage gradients and hence corona loss and radio interference. Unsymmetrical conductor spacing results in different inductances for each phase which causes an unbalanced voltage drop, even when the load currents are balanced. The residual or resultant voltage or current induces unwanted voltages into neighbouring communication lines. This can be overcome by the interchange of conductor positions at regular intervals along the route, a practice known as *transposition* (see Figure 3.29). In practice, lines are rarely transposed at regular intervals and transposition is carried out where physically convenient, e.g. at substations. In short lines the degree of unbalance existing without transposition is small and may be neglected in calculations.

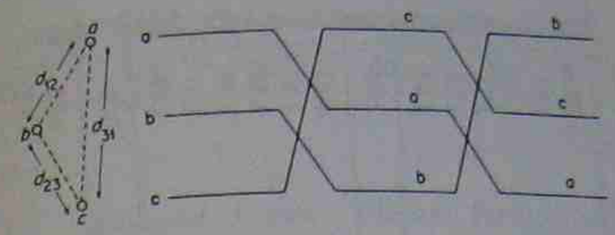


Figure 3.29 Transposition of conductors

Resistance

Overhead-line conductors usually comprise a stranded steel core (for mechanical strength) surrounded by aluminium wires which form the conductor. It should be noted that aluminium and ACSR (aluminium conductor steel reinforced) conductors are sometimes described by area of a copper conductor having the same d.c. resistance, i.e. their *copper equivalent*. In Table 3.2(a), 258 mm^2 (0.4 in^2) ACSR conductor implies the copper equivalent. The actual area of aluminium is approximately 430 mm^2, and including the steel core the overall cross-section of the line is about 620 mm^2, i.e. a diameter of 28 mm. Electromagnetic losses in the steel increase the effective a.c. resistance, which increases with current magnitude, giving an increase of up to about 10 per cent. With direct current the steel carries 2–3 per cent of the total current. The electrical resistivities of some conductor materials are as follows in ohm-metres at 20°C: copper 1.72×10^{-8}; aluminium 2.83×10^{-8} (3.52×10^{-8} at 80°C); aluminium alloy 3.22×10^{-8}.

Table 3.2(a) Overhead-line constants at 50 Hz (British) (per phase, per km)

Voltage:		132 kV		275 kV		400 kV	
No. and area of conductors (mm)^2:		1× 113	1× 258	2× 113	2× 258	2× 258	4× 258
Resistance (R)	Ω	0.155	0.068	0.022	0.034	0.034	0.017
Reactance (X_L)	Ω	0.41	0.404	0.335	0.323	0.323	0.27
Susceptance ($1/X_c$)	S × 10^{-6}	7.59	7.59	9.52	9.52	9.52	10.58
Charging current (I_c)	A	0.22	0.22	0.58	0.58	0.845	0.945
Surge impedance	Ω	373	371	302	296	296	258
Natural load	MW	47	47	250	255	540	620
X_L/R ratio	—	2.6	5.9	4.3	9.5	9.5	15.8
Thermal rating:							
Cold weather (below 5°C)	MVA	125	180	525	760	1100	2200
Normal (5–18°C)	MVA	100	150	430	620	900	1800
Hot weather (above 18°C)	MVA	80	115	330	480	790	1580

Table 3.2(b) Typical characteristics of bundled-conductor E.H.V. lines

Number of subconductors in bundle	Country	Line voltage and number of circuits (in parentheses)	Diameter of subconductors	Radius of circle on which subconductors are arranged	Resistance of bundle	Inductive reactance at 50 Hz	Susceptance at 50 Hz
		kV	mm	mm	Ω/km	Ω/km	μS/km
1	Japan	275 (2)	27.9	—	0.0744	0.511	3.01
	Canada	300 (2)	35.0	—	0.0451	0.492	2.33
	Australia	330 (1)	45.0	—	0.0367	0.422	2.78
	Russia	330 (1)	30.2	—	0.065	0.404	2.82
	Italy	380 (1)	50.0	—	0.0294	0.398	2.84
Average (one conductor):					0.048	0.442	2.75
2	Japan	275 (2)	25.3	200	0.0444	0.374	4.35
	Canada	360 (1)	28.1	299	0.04	0.314	3.57
	Australia	330 (1)	31.8	190.5	0.0451	0.341	3.34
	U.S.A.	345 (1)	30.4	228.6	0.0315	0.325	3.55
	Italy	380 (1)	31.5	200	0.0285	0.315	3.57
	Russia	330 (1)	28.0	200	0.04	0.321	3.43
Average (two conductors):					0.0040	0.323	3.58
3	Sweden	380 (1)	31.68	260	0.018	0.29	3.85
	Russia	525 (1)	30.2	230	0.0212	0.294	3.88
4	Germany	380 (2)	21.7	202	0.0316	0.260	4.3
	Germany	500 (1)	22.86	323	0.0285	0.272	4.0
	Canada	735 (1)	35.04	323	0.0121	0.279	3.9

Table 3.2(c) Overhead-line parameters—U.S.A.

Line voltage (kV)	345	345	500	500	735	735
Conductors per phase at 18 in spacing	1	2	2	4	3	4
Conductor code	Expanded	Curlew	Chuker	Parakeet	Expanded	Pheasant
Conductor diameter (inches)	1.750	1.246	1.602	0.914	1.750	1.382
Phase spacing (ft)	28	28	38	38	56	56
GMD (ft)	35.3	35.3	47.9	47.9	70.6	70.6
60 Hz inductive reactance Ω/mile X_A	0.3336	0.1677	0.1529	0.0584	0.0784	0.0456
60 Hz inductive reactance Ω/mile X_D	0.4325	0.4325	0.4694	0.4694	0.5166	0.5166
60 Hz inductive reactance Ω/mile $X_A + X_D$	0.7661	0.6002	0.6223	0.5278	0.5950	0.5622
60 Hz capacitive reactance MΩ/miles X'	0.0777	0.0379	0.0341	0.0126	0.0179	0.0096
60 Hz capacitive reactance MΩ/miles X'_D	0.1057	0.1057	0.1147	0.1147	0.1263	0.1263
60 Hz capacitive reactance MΩ/miles $X'_A + X'_D$	0.1834	0.1436	0.1488	0.1273	0.1442	0.1359
$Z_0(\Omega)$	374.8	293.6	304.3	259.2	276.4	
Natural loading MVA	318	405	822	965	1844	1955
Conductor d.c. resistance at 25°C (Ω/mile)	0.0644	0.0871	0.0510	0.162	0.0644	0.0709
Conductor a.c. resistance (60 Hz) at 50°C (Ω/mile)	0.0728	0.0979	0.0599	0.179	0.0728	0.0805

X_A = component of inductive reactance due to flux within a 1 ft radius.
X_D = component due to other phases.
Total reactance per phase = $X_A + X_D$.

$$X_A = 0.2794 \log_{10}\left(\frac{1}{[N(GMR)(A)^{N-1}]^{1/N}}\right) \Omega/\text{mile}.$$

$X_D = 0.2794 \log_{10}(GMD)\ \Omega$/mile.
X'_A and X'_D are similarly defined for capacitive reactance and

$$X'_A = 0.0683 \log_{10}\left(\frac{1}{[Nr(A)^{N-1}]^{1/N}}\right) \text{ MΩ-mile}; \quad X'_D = 0.0683 \log_{10}(GMD) \text{ MΩ-miles}.$$

GMR = geometric mean radius in feet;
GMD = geometric mean diameter in feet;
N = number of conductors per phase;
A = $S/2 \sin(\pi/N)$; $N > 1$;
A = 0; $N = 1$;
S = bundle spacing in feet;
r = conductor radius in feet.
(*Permission of Edison Electric Institute*)

Leakage resistance is usually negligible for most calculation purposes and is very difficult to assess because of its dependence on the weather. This resistance represents the combined effect of all the various paths to earth from the line. The main path is that presented by the surfaces of the line insulators, the resistance of which depends on the condition of these surfaces. This varies considerably according to location; in industrial areas there will be layers of dirt and soot whilst in coastal districts deposits of salt occur. The bushings of circuit breakers and transformers form other leakage paths. The leakage losses on 132 kV lines vary between 0.3 and 1 kW per mile.

Table 3.2(a) gives the parameters for various overhead-line circuits for the line voltages operative in Britain, Table 3.2(b) gives values for international lines, and Table 3.2(c) relates to the U.S.A.

3.9 Representation of Lines

The manner in which lines and cables are represented depends very much on their length and the accuracy required. There are three broad classifications of length: short, medium, and long. The actual line or cable is a distributed-constant circuit, i.e. it has resistance, inductance, capacitance, and leakage resistance distributed evenly along its length, as shown in Figure 3.30. Except for long lines the total resistance, inductance, capacitance, and leakage resistance of the line are concentrated to give a lumped-constant circuit. The distances quoted are only a rough guide.

3.9.1 The short line (up to 80 km, 50 miles)

The equivalent circuit is shown in Figure 3.31 and it will be noticed that both shunt capacitance and leakage resistance have been neglected. The four-terminal network constants are (see Section 2.8):

$$A = 1 \qquad B = Z \qquad C = 0 \qquad D = 1$$

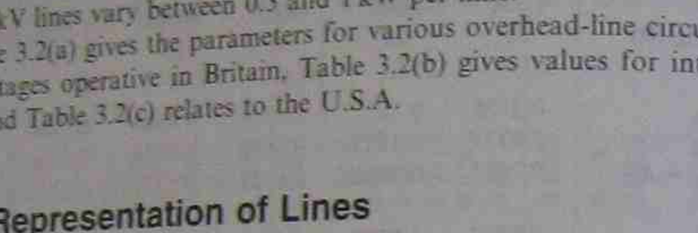


Figure 3.30 Distributed constant representation of a line: L = inductance line to neutral per unit length; r = a.c. resistance per unit length; C = capacitance line to neutral per unit length; R = leakage resistance per unit length

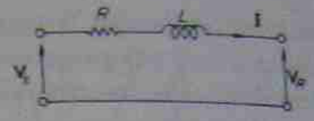


Figure 3.31 Equivalent circuit of a short line—representation under balanced three-phase conditions

The drop in voltage along a line is important and the *regulation* is defined as

$$\frac{\text{received voltage on no load}(V_0) - \text{received voltage on load } (V_R)}{\text{received voltage on load } (V_R)}$$

It should be noted that if $\mathbf{I}$ is leading $\mathbf{V}_R$ in phase, i.e. a capacitance load, then $V_R > V_S$, as shown in Figure 3.32.

3.9.2 Medium-length lines (up to 240 km, 150 miles)

Owing to the increased length the shunt capacitance is now included to form either a π or a T network. The circuits are shown in Figure 3.33. Of these two versions the π representation tends to be in more general use but there is little difference in accuracy between the two. For the π network:

$$\mathbf{V}_S = \mathbf{V}_R + \mathbf{IZ} \qquad \mathbf{I} = \mathbf{I}_R + \mathbf{V}_R\frac{\mathbf{Y}}{2} \qquad \mathbf{I}_S = \mathbf{I} + \mathbf{V}_S\frac{\mathbf{Y}}{2}$$

from which $\mathbf{V}_S$ and $\mathbf{I}_S$ are obtained in terms of $\mathbf{V}_R$ and $\mathbf{I}_R$ giving the following constants:

$$\mathbf{A} = \mathbf{D} = 1 + \frac{\mathbf{ZY}}{2} \qquad \mathbf{B} = \mathbf{Z} \qquad \text{and} \qquad \mathbf{C} = \left(1 + \frac{\mathbf{ZY}}{4}\right)\mathbf{Y}$$

Similarly, for the T network:

$$\mathbf{V}_S = \mathbf{V}_C + \frac{\mathbf{ZI}_S}{2} \qquad \mathbf{V}_C = \mathbf{V}_R + \frac{\mathbf{ZI}_R}{2} \qquad \mathbf{I}_S = \mathbf{I}_R + \mathbf{V}_C\mathbf{Y}$$

giving

$$\mathbf{A} = \mathbf{D} = 1 + \frac{\mathbf{ZY}}{2} \qquad \mathbf{B} = \left(1 + \frac{\mathbf{ZY}}{4}\right)\mathbf{Z} \qquad \text{and} \qquad \mathbf{C} = \mathbf{Y}$$

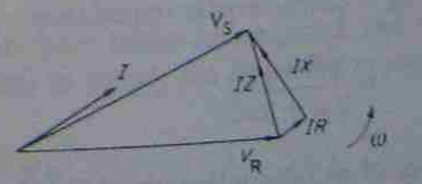


Figure 3.32 Phasor diagram for short line on leading load

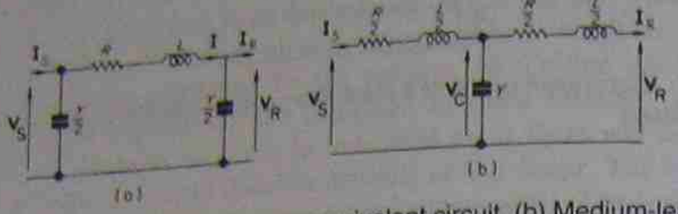


Figure 3.33 (a) Medium-length line—π equivalent circuit. (b) Medium-length line—T equivalent circuit

3.9.3 The long line (above 240 km, 150 miles)

Here the treatment assumes distributed parameters. The changes in voltage and current over an elemental length Δx of the line, x metres from the sending end, are determined, and conditions for the whole line are obtained by integration.

Let

$R =$ resistance/unit length — $\mathbf{z} =$ impedance/unit length

$L =$ inductance/unit length — $\mathbf{y} =$ shunt admittance/unit length

$G =$ leakage/unit length — $\mathbf{Z} =$ total series impedance of line

$C =$ capacitance/unit length — $\mathbf{Y} =$ total shunt admittance of line

The voltage and current x metres from the sending end are given by

$$\left.\begin{aligned} \mathbf{V}_x &= \mathbf{V}_S \cosh \mathbf{P}x - \mathbf{I}_S \mathbf{Z}_0 \sinh \mathbf{P}x \\ \mathbf{I}_x &= \mathbf{I}_S \cosh \mathbf{P}x - \frac{\mathbf{V}_S}{\mathbf{Z}_0} \sinh \mathbf{P}x \end{aligned}\right\} \tag{3.9}$$

where

$$\mathbf{P} = \text{propagation constant} = (\alpha + j\beta)$$
$$= \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{\mathbf{z} \cdot \mathbf{y}}$$

and

$$\mathbf{Z}_0 = \text{characteristic impedance} = \sqrt{\frac{R + j\omega \mathbf{L}}{G + j\omega C}} \tag{3.10}$$

$\mathbf{Z}_0$ is the input impedance of an infinite length of the line; hence if any line is terminated in $\mathbf{Z}_0$ its input impedance is also $\mathbf{Z}_0$.

The propagation constant $\mathbf{P}$ represents the changes occurring in the transmitted wave as it progresses along the line; α measures the attentuation, and β the angular phase-shift. With a loss-free line, where $R = G = 0$, $\mathbf{P} = j\omega\sqrt{LC}$ and $\beta = \sqrt{(LC)}$. With a velocity of propagation of 3×10^5 km/s the wavelength of the transmitted voltage and current at 50 Hz is 6000 km. Thus, lines are much shorter than the wavelength of the transmitted energy.

Usually conditions at the load are required when $x = l$ in equations (3.9).

$$\left.\begin{aligned} \therefore \mathbf{V}_R &= \mathbf{V}_S \cosh \mathbf{P}l - \mathbf{I}_S \mathbf{Z}_0 \sinh \mathbf{P}l \\ \mathbf{I}_R &= \mathbf{I}_S \cosh \mathbf{P}l - \frac{\mathbf{V}_S}{\mathbf{Z}_0} \sinh \mathbf{P}l \end{aligned}\right\} \tag{3.11}$$

Alternatively,

$$\left.\begin{aligned} \mathbf{V}_S &= \mathbf{V}_R \cosh \mathbf{P}l + \mathbf{I}_R \mathbf{Z}_0 \sinh \mathbf{P}l \\ \mathbf{I}_S &= \frac{\mathbf{V}_R}{\mathbf{Z}_0} \sinh \mathbf{P}l + \mathbf{I}_R \cosh \mathbf{P}l \end{aligned}\right\} \tag{3.12}$$

The parameters of the equivalent four-terminal network are thus,

$$\mathbf{A} = \mathbf{D} = \cosh \sqrt{\mathbf{ZY}}$$

$$\mathbf{B} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} \sinh \sqrt{\mathbf{ZY}}$$

and

$$\mathbf{C} = \sqrt{\frac{\mathbf{Y}}{\mathbf{Z}}} \sinh \sqrt{\mathbf{ZY}}$$

The easiest way to handle the hyperbolic functions is to use the appropriate series.

$$\mathbf{A} = \cosh \sqrt{\mathbf{ZY}} = 1 + \frac{\mathbf{YZ}}{2} + \frac{\mathbf{Y}^2\mathbf{Z}^2}{24} + \frac{\mathbf{Y}^3\mathbf{Z}^3}{720} + \cdots$$

$$\mathbf{B} = \mathbf{Z}\left(1 + \frac{\mathbf{YZ}}{6} + \frac{\mathbf{Y}^2\mathbf{Z}^2}{120} + \frac{\mathbf{Y}^3\mathbf{Z}^3}{5040}\right)$$

$$\mathbf{C} = \mathbf{Y}\left(1 + \frac{\mathbf{YZ}}{6} + \frac{\mathbf{Y}^2\mathbf{Z}^2}{120} + \frac{\mathbf{Y}^3\mathbf{Z}^3}{5040}\right)$$

Usually not more than three terms are required, and for (overhead) lines less than 500 km (312 miles) in length the following expressions for the constants hold approximately:

$$\mathbf{A} = \mathbf{D} = 1 + \frac{\mathbf{ZY}}{2} \qquad \mathbf{B} = \mathbf{Z}\left(1 + \frac{\mathbf{ZY}}{6}\right) \qquad \mathbf{C} = \mathbf{Y}\left(1 + \frac{\mathbf{ZY}}{6}\right)$$

An exact equivalent circuit for the long line can be expressed in the form of the π section shown in Figure 3.34. The application of simple circuit laws will show that this circuit yields the correct four-terminal network equations. If only the first term of the expansions is used, then

$$\mathbf{B} = \mathbf{Z} \qquad \text{and} \qquad \frac{\mathbf{A}-1}{\mathbf{B}} = \frac{\mathbf{Y}}{2}$$

i.e. the medium-length π representation. Figure 3.34 is only for conditions at the ends of the line; if intermediate points are to be investigated, then the full equations must be used.

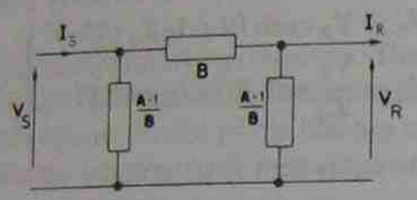


Figure 3.34 Equivalent circuit to accurately represent the terminal conditions of a long line

Example 3.4

The conductors of a 1.6 km (1 mile) long, 3.3 kV, overhead line are in horizontal formation with 762 mm (30 in) between centres. The effective diameter of the conductors is 3.5 mm. The resistance per kilometre of the conductors is 0.41 Ω. Calculate the line-to-neutral inductance of the line. If the sending-end voltage is 3.3 kV and the load is 1 MW at 0.8 p.f. lagging, calculate the voltage at the load busbar, the power loss in the line, and the efficiency of transmission.

Solution

The equivalent equilateral spacing is given by

$$d_e = \sqrt[3]{(d_{12}d_{23}d_{31})}$$

In this case

$$d_e = \sqrt[3]{(762 \times 762 \times 1524)} = 762\sqrt[3]{2} = 960\,\text{mm}$$

The inductance (line to neutral)

$$= \frac{\mu_0}{2 \times 4\pi}\left[1 + 4\ln\left(\frac{d-r}{r}\right)\right]\text{H/m}$$

$$= \frac{4\pi}{10^7 \times 2 \times 4\pi}\left[1 + 4\ln\left(\frac{960 - 1.75}{1.75}\right)\right]\text{H/m}$$

$$= \frac{1}{2 \times 10^7}\left(\frac{26.21}{1}\right)\text{H/m} = \frac{13.105}{10^7}\text{H/m}$$

Total inductance

$$= 1.0 \times 5280 \times 0.3048 \times \frac{13.105}{10^7} = 2.11 \times 10^{-3}\,\text{H}$$

Inductive reactance

$$= 2\pi fL = 2\pi \times 50 \times 2.11 \times 10^{-3} = 0.06\,\Omega$$

The line is obviously in the category of short and will be treated accordingly.

The load can be expressed as

$$P = 1\,\text{MW} \qquad Q = 0.75\,\text{MVAr}$$

The load can be represented by an equivalent shunt impedance, in which case P and $Q \propto V_R^2$. If P and Q are constant regardless of V_R, then an iterative procedure must be used. The latter will be used here. As the load voltage is unknown, to obtain the current the nominal voltage of 3.3 kV will be assumed. The current is then

$$\frac{10^6}{\sqrt{(3)} \times 3300 \times 0.8} = 218\angle -\cos^{-1}0.8\,\text{A}$$

$$\mathbf{V_S} = \mathbf{V_R} + \mathbf{IZ} \qquad \text{and} \qquad \mathbf{Z} = (0.41 \times 1.6 + \text{j}0.66)$$

$$\frac{3300}{\sqrt{3}} = \mathbf{V_R} + (218 \times 0.8 - \text{j}218 \times 0.6)(0.66 + \text{j}0.66)$$

$$= \mathbf{V_R} + 115.0 + 86.7 + \text{j}(115.0) - 86.7)$$

$$1900 = \mathbf{V_R} + 201.7 + \text{j}28.3$$

$$\mathbf{V_R} = 1698.3 - \text{j}28.3$$

$V_R = 1698$ V with negligible phase shift

A quicker method would be to use

$$\Delta V = \frac{RP + XQ}{V_R}$$

$$= \frac{\frac{1}{3} \times 0.66 \times 10^6 + \frac{1}{3} \times 0.66 \times 0.75 \times 10^6}{1900} = \frac{385 \times 10^3}{1.9 \times 10^3} = 202\,\text{V}$$

$$\therefore V_R = 1900 - 202 = 1698\,\text{V}$$

However, as the load is 1 MW at the receiving end, I should be recalculated and the above process repeated. As the approximate method is more convenient, it will be used.

$$\Delta V' = \frac{385}{1.698} = 227\,\text{V}$$

$$V_R' = 1900 - 227 = 1673\,\text{V}$$

Iterating again,

$$\Delta V'' = \frac{385}{1.673} = 230\,\text{V}$$
$$V_R'' = 1670\,\text{V}$$

Again,

$$\Delta V''' = \frac{385}{1670} = 230$$

The final value of V_R is therefore 1670 V.

Here, V_R can be obtained analytically without recourse to iterative procedures, but this is more lengthy in calculation. The type of load in this question in which P and Q remain constant regardless of the voltage is called *stiff*.

$$\text{Line loss} = 218^2 \times 0.66\ \text{W per phase}$$
$$= 94\ \text{kW for three phases}$$

$$\text{Efficiency of transmission} = \frac{\text{output}}{\text{output + losses}} = \frac{1000}{1000 + 94}$$
$$= 91.4\ \text{per cent}$$

This efficiency is typical of a rural distribution line or cable.

Example 3.5

A 150 km long overhead line with the parameters given in Table 3.2(a) for 400 kV, quad conductors is to be used to transmit 1800 MW (normal weather loading) at 0.9 p.f. lag. Calculate the required sending end voltage using three-line representation methods and compare the results.

Solution

From Table 3.2(a)

$$R = 0.017\,\Omega/\text{km}, \qquad X_L = 0.270\,\Omega/\text{km}, \qquad 1/X_c = 10.580 \times 10^{-6}\,\text{S/km}$$

1. Short-line representation:

$$R_{\text{total}} = 0.017 \times 150 = 2.550\,\Omega$$
$$X_{L\,\text{total}} = 0.270 \times 150 = 40.500\,\Omega$$
$$\mathbf{Z} = 2.550 + \text{j}40.500 = 40.580\,\underline{/86.4^\circ}\,\Omega$$

Receiving end power at nominal voltage

$$= 1800 \times 10^6\,\text{W}$$
$$= \sqrt{3} \times 400 \times 10^3 \times I \times \cos\theta$$

Hence

$$\mathbf{I} = 2887\ \text{A at angle}\cos^{-1}0.9\ \text{lag to}\ V_R$$
$$= 2887\underline{/-25.842^\circ}$$

$$\mathbf{V_S} = \mathbf{V_R} + \mathbf{IZ} = \frac{400 \times 10^3}{\sqrt{3}} + 2887\underline{/-25.842} \times 40.580\,\underline{/86.4}$$
$$= 230\,940 + 117\,154\,\underline{/60.56^\circ}$$
$$= 306\,031\underline{/19.47^\circ}\ V\ \text{(phase)}$$
$$= 530.061\ \text{kV (line)}$$

2. Medium-line representation:

$$\mathbf{A} = \mathbf{D} = 1 + \frac{\mathbf{ZY}}{2};\ \mathbf{B} = \mathbf{Z};\ \mathbf{Y}_{\text{total}} = \text{j}10.58 \times 10^{-6} \times 150 = \text{j}1587 \times 10^{-6}$$
$$\frac{\mathbf{ZY}}{2} = 40.580\,\underline{/86.4} \times 1587 \times 10^{-6}\underline{/90} = (-0.032 + \text{j}0.002)$$

Hence

$$\mathbf{A} = 1 - 0.032 + j0.002 = 0.968 + j0.002 = 0.968\underline{/0.12^\circ}$$
$$\mathbf{V_S} = \mathbf{AV_R} + \mathbf{BI_R} = 0.968\,\underline{/0.12^\circ} \times \frac{400 \times 10^3}{\sqrt{3}} + 40.58\underline{/86.4^\circ} \times 2887\underline{/-25.84^\circ}$$
$$= 223\,549.550 + \text{j}460.398 + 117\,154.476 + \text{j}0.030$$
$$= 340\,704.337\underline{/0.077}\ \text{V (phase)}$$
$$= 590.117\ \text{kV (line)}$$

(Note: It is important to work to three decimal places as some numbers can become quite small.)

3. Long-line representation:

$$\mathbf{A} = \mathbf{D} = \cosh\sqrt{\mathbf{ZY}} = 1 + \mathbf{YZ} + \frac{\mathbf{Y^2Z^2}}{24} + \cdots$$
$$\mathbf{B} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}}\sinh\sqrt{\mathbf{ZY}} = \mathbf{Z}\left(1 + \frac{\mathbf{YZ}}{6} + \frac{\mathbf{Y^2Z^2}}{120} + \cdots\right)$$
$$\mathbf{C} = \sqrt{\frac{\mathbf{Y}}{\mathbf{Z}}}\sinh\sqrt{\mathbf{ZY}} = \mathbf{Y}\left(1 + \frac{\mathbf{YZ}}{6} + \frac{\mathbf{Y^2Z^2}}{120} + \cdots\right)$$

Using three terms of the appropriate series approximation for cosh and sinh, we get

$$\mathbf{YZ} = 64400.460 \times 10^{-6}\underline{/176.4^\circ} = 0.0644\underline{/176.4^\circ}$$
$$\mathbf{Y^2Z^2} = -1587^2 \times 10^{-12} \times 40.580^2\underline{/2 \times 86.4^\circ} = -0.004\,\underline{/172.8^\circ}$$

Hence

$$\mathbf{A} = \mathbf{D} = 1 + \frac{0.0644}{2}\angle 176.4 + \frac{-0.004\angle 172.8^\circ}{24} = 0.968 + j0.002$$

$$\mathbf{B} = 40.58\angle 86.4^\circ\left(1 + \frac{0.0644}{6}\angle 176.4^\circ + \frac{-0.004\angle 172.8}{24}\right)$$

$$= 2.495 + j40.067 = 40.145\angle 86.4^\circ$$

$$\mathbf{V_S} = \mathbf{AV_R} + \mathbf{BI_R} = (0.968 + j0.002)\frac{400 \times 10^3}{\sqrt{3}}$$

$$+ 40.145\angle 86.4^\circ(2887\angle -25.84^\circ)$$

$$= 223\,550.024 + j461.880 + 56903.30 + j100966.540$$

$$= 298\,231.1\angle 19.88^\circ \text{ V (phase)}$$

$$= 516.551 \text{ kV (line)}$$

Summarizing these results:

Representation	V_R (kV line)	Error (%) on long-line representation	Overvoltage (%)
Short line	530	2.7	32.5
Medium line	590	14.3	47.5
Long line	516	—	29.0

Line–line voltage at sending end is

$$\sqrt{3} \times 255 = 442 \text{ kV}$$

(Note: This is an extremely high voltage and could not be tolerated as a 400 kV system would be designed for only about 10 per cent steady-state overvoltage, i.e. $400 + 40 = 440$ kV. In practice, an overhead line over 300 km would be equipped with series capacitors to reduce the total series reactance to a value such that the line voltage drop was about 10 per cent nominal phase voltage. In addition, the power factor at the receiving end would be raised to at least 0.95 lag by the use of shunt capacitors or synchronous compensators.)

3.9.4 The natural load

The characteristic impedance Z_0 (equation 3.10) is also known as the surge impedance. When a line is terminated in its characteristic impedance the power delivered is known as the *natural load*. For a loss-free line under natural-load conditions the reactive power absorbed by the line is equal to the reactive power generated, i.e.

$$\frac{V^2}{X_c} = I^2 X_L$$

and

$$\frac{V}{I} = Z_0 = \sqrt{X_L X_C} = \sqrt{\frac{L}{C}}$$

At this load, V and I are in phase all along the line and optimum transmission conditions are obtained; in practice, however, the load impedances are seldom in the order of Z_0. Values of Z_0 for various line voltages are as follows (corresponding natural loads are shown in parentheses): 132 kV, 150 Ω (50 MW); 275 kV, 315 Ω (240 MW); 380 kV, 295 Ω (490 MW). The angle of the impedance varies between 0 and −15°. For underground cables Z_0 is roughly one-tenth of the overhead-line value. Lines are operated above the natural loading, whereas cables operate below this loading.

3.10 Parameters of Underground Cables

In cables with three conductors contained within the lead or aluminium sheath, the electric field set up has components tangential to the layers of impregnated-paper insulation in which direction the dielectric strength is poor. Therefore, at voltages over 11 kV, each conductor is separately screened (Höchstadter type) to ensure only radial stress through the paper. Cables using cross-linked polyethylene (XPLE) are now coming into use. The capacitance of single-conductor and individually screened three-conductor cables is readily calculated. For three-conductor unscreened cables one must resort to empirical design data. A typical high-voltage paper insulated cable is shown in Figure 3.35. The capacitance (C) of single-core cables may be calculated from design data by the use of the formula

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(R/r)} \tag{3.13}$$

where r and R are the inner and outer radii of the dielectric and ε_r is the relative permittivity of the dielectric, often in the order of 3.5. This also holds for three-core cables with each conductor separately screened.

The various capacitances present in three-core unscreened cables may be represented as shown in Figure 3.36, in which the conductor-to-conductor capacitance is C_1 and the conductor-to-sheath capacitance is C_0. The equivalent circuit is shown in Figure 3.36(b) and circuits obtained by star–delta or delta–star transformations are shown in Figure 3.36(c) and (d). The value of

$$C_3 = \frac{C_0}{3}$$

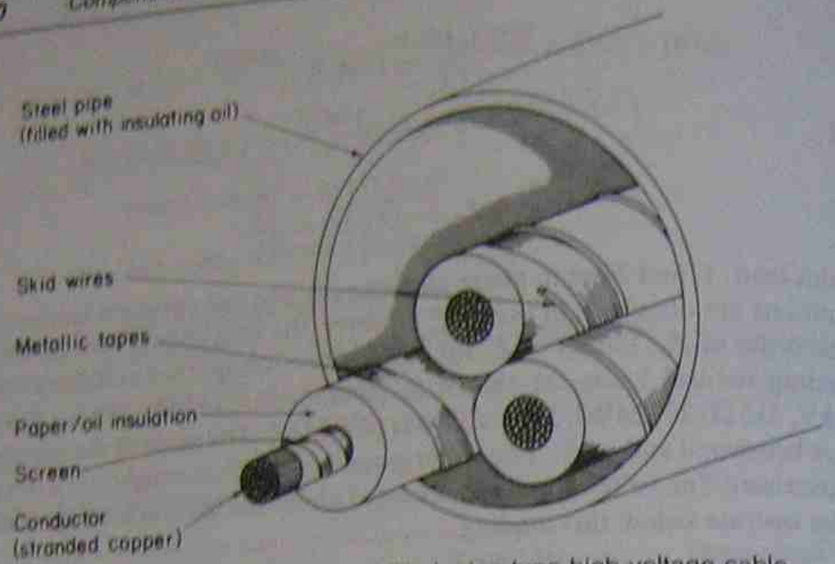


Figure 3.35 High-pressure, oil-filled, pipe-type high-voltage cable

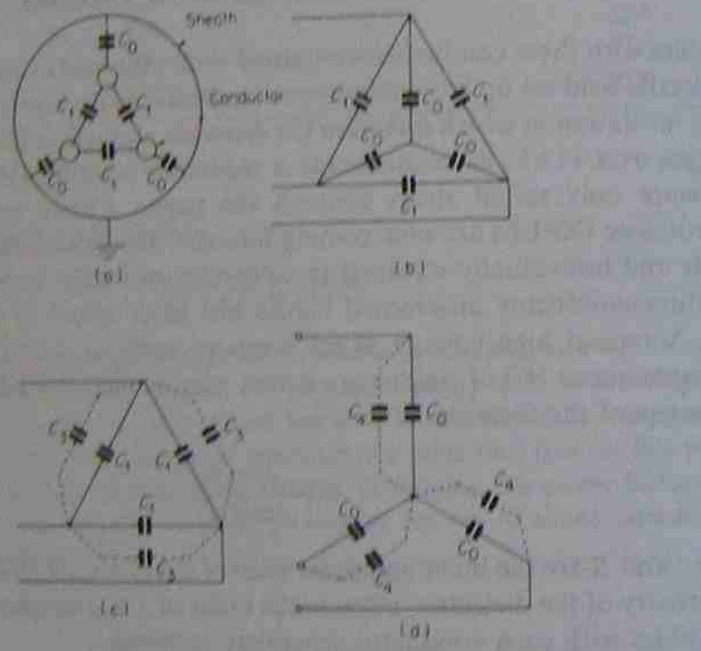


Figure 3.36 Capacitances in a three-core cable

and, similarly, $C_4 = 3C_1$. The parallel combination of C_4 and C_0 gives the equivalent line-to-neutral value of the cable capacitance. C_0 and C_1 may be measured as follows. The three cores are short-circuited and the capacitance between them and the sheath is measured; this measurement gives $3C_0$. Next, the capacitance between two cores is measured, the third being connected to

the sheath. It is easily shown that the measured value is now $C_1 + [(C_0 + C_1)/2]$/ hence C_0 and C_1 are obtained.

Owing to the symmetry of the cable the normal phase-sequence values of C and L are the same as the negative phase-sequence values (i.e. for reversed phase rotation). The series resistance and inductance are complicated by the magnetic interaction between the conductor and sheath.

The effective resistance of the conductor is the direct current resistance modified by the following factors: the skin effect in the conductor; the eddy currents induced by adjacent conductors (the proximity effect); and the equivalent resistance to account for the I^2R losses in the sheath. The determination of these effects is complicated and is left for advanced study.

The parameters of the cable having been determined, the same equivalent circuits are used as for overhead lines, paying due regard to the selection of the correct model for the appropriate length of cable. Owing to the high capacitance of cables the charging current, especially at high voltages, is an important factor in deciding the permissible length to be used. Table 3.3 gives the charging currents for self-contained low-pressure oil-filled (LPOF) cables.

Table 3.3 Undergound cable constants at 50 Hz (per km)

		132 kV		275 kV		400 kV
Size	mm^2	355	645	970	1130	1935
Rating (soil $g = 120°C$ cm/W)	A	550	870	1100	1100	1600
	MVA	125	200	525	525	1100
Resistance (R) at 85°C	Ω	0.065	0.035	0.025	0.02	0.013
Reactance (X_L)	Ω	0.128	0.138	0.22	0.134	0.22
Charging current (I_c)	A	7.90	10.69	15.70	17.77	23.86
X_L/R Ratio		2.0	4.0	8.8	6.6	16.8

3.11 Transformers

The equivalent circuit of one phase of a transformer referred to the primary winding is shown in Figure 3.37. The resistances and reactances can be found from the well-known open- and short-circuit tests. In the absence of complete information for each winding, the two arms of the T network can each be assumed to be half the total transformer impedance. Also, little accuracy is lost in transferring the shunt branch to the input terminals, forming a 'cantilever' circuit.

In power transformers the current taken by the shunt branch is usually a very small percentage of the load current and can be neglected.

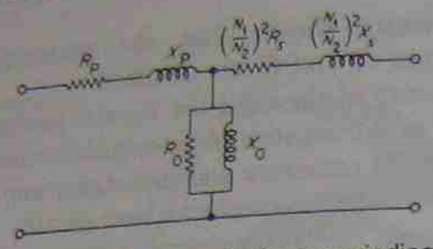


Figure 3.37 Equivalent circuit of a two-winding transformer

3.11.1 Phase shifts in three-phase transformers

Consider the transformer shown in Figure 3.38(a). The red phases on both sides are taken as reference and the transformation ratio is $1 : N$. The corresponding phasor diagrams are shown in Figure 3.38(b). Although no neutral point is available in the delta side, the effective voltages from line to earth are denoted by $\mathbf{E}_{R'n}$, $\mathbf{E}_{Y'n}$, and $\mathbf{E}_{B'n}$. Comparing the two phasor diagrams, the following relationships are seen:

$$\begin{aligned}\mathbf{E}_{R'n} &= \text{line–earth voltage on the delta side} \\ &= N\mathbf{E}_{Rn} \angle 30^\circ,\end{aligned}$$

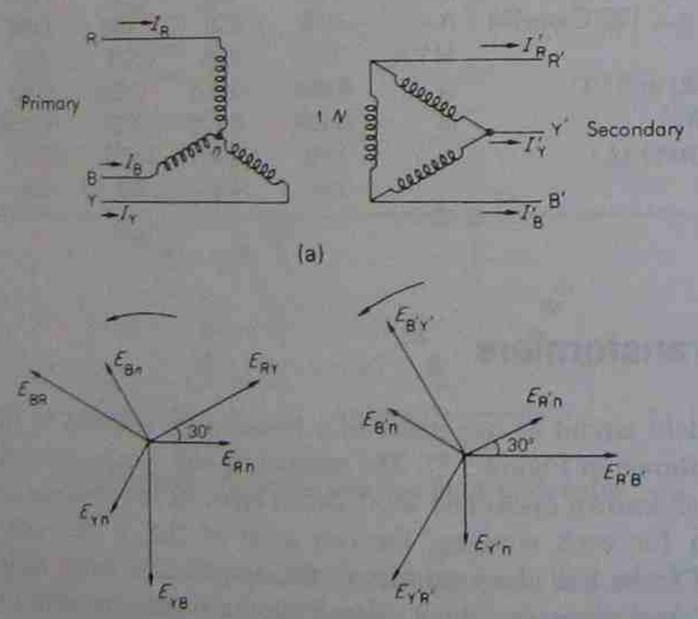


Figure 3.38 (a) Star–delta transformer with turns ratio $1 : N$. (b) Corresponding phasor diagrams (N taken as 1 in diagrams)

i.e. the positive sequence or normal balanced voltage of each phase is advanced through 30°. Similarly, it can be shown that the positive sequence currents are advanced through 30°.

By a consideration of the negative phase-sequence phasor diagrams (these are phasors with reversed rotation, i.e. R–B–Y) it will readily be seen that the phase currents and voltages are shifted through −30°. When using the per unit system the transformer ratio does not directly appear in calculations. In star–star and delta–delta connected transformers there are no phase shifts. Hence transformers having these connections and those with star–delta should not be connected in parallel. To do so introduces a resultant voltage acting in the local circuit formed by the usually low transformer impedances. Figure 3.39 shows the general practice on the British network with regard to transformers with phase shifts. It is seen that the reference phasor direction is different at different voltage levels. The larger than 30° phase shifts are obtained by suitable rearrangement of the winding connections.

Three-winding transformers

Many transformers used in power systems have three windings per phase, the third winding being known as the tertiary. This winding is provided to enable compensation equipment to be connected at an economic voltage (e.g. 13 kV) and to provide a circulating current path for third harmonics so that these currents do not appear outside the transformer.

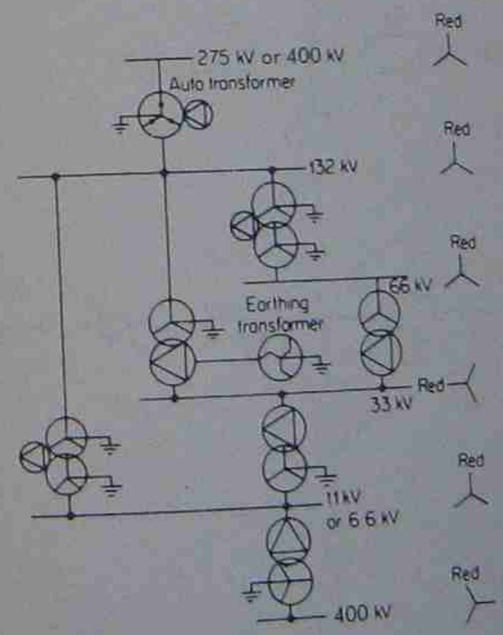


Figure 3.39 Typical phase shifts in a power system—British

The three-winding transformer can be represented under balanced three-phase conditions by a single-phase equivalent circuit of three impedances star-connected as shown in Figure 3.40. The values of the equivalent impedances Z_p, Z_s, and Z_t may be obtained by test. It is assumed that the no-load currents are negligible.

Let

Z_{ps} = impedance of the primary when the secondary is short-circuited and the tertiary open

Z_{pt} = impedance of the primary when the tertiary is short-circuited and the secondary open

Z_{st} = impedance of the secondary when the tertiary is short-circuited and the primary open

The above impedances are in ohms referred to the same voltage base. Hence,

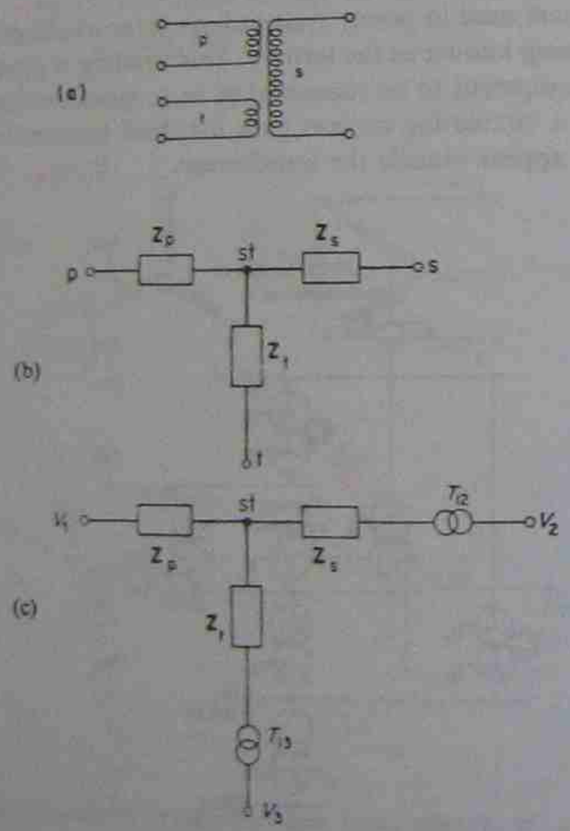


Figure 3.40 (a) Three-winding transformer (b) and (c) equivalent circuits

$$\left.\begin{aligned} Z_{ps} &= Z_p + Z_s \\ Z_{pt} &= Z_p + Z_t \\ Z_{st} &= Z_s + Z_t \\ Z_p &= \tfrac{1}{2}(Z_{ps} + Z_{pt} - Z_{st}) \\ Z_s &= \tfrac{1}{2}(Z_{ps} + Z_{st} - Z_{pt}) \\ Z_t &= \tfrac{1}{2}(Z_{pt} + Z_{st} - Z_{ps}) \end{aligned}\right\} \quad (3.14)$$

It should be noted that the star point st in Figure 3.40b is purely fictitious and that the diagram is a single-phase equivalent circuit. In most large transformers the value of Z_s is very small and can be negative. All impedances must be referred to common volt-ampere and voltage bases. The complete equivalent circuit is shown in Figure 3.40(c) where T_{12} and T_{13} provide any off nominal turns ratio.

Autotransformers

The symmetrical autotransformer may be treated in the same manner as two- and three-winding transformers. This type of transformer shows to best advantage when the transformation ratio is small and it is widely used for the interconnection of the supply networks working at different voltages, e.g 275 kV to 132 kV or 400 kV to 275 kV. The neutral point is solidly grounded, i.e. connected directly to earth without intervening resistance.

Earthing (grounding) transformers

A means of providing an earthed point or neutral in a supply derived from a delta-connected transformer may be obtained by the use of a zigzag transformer shown schematically in Figure 3.39. By the interconnection of two windings on each limb, a node of zero potential is obtained.

Harmonics

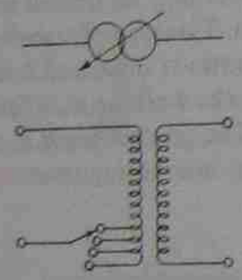
Due to the non-linearity of the magnetizing characteristic for transformers the current waveform is distorted and hence contains harmonics; these flow through the system impedances and set up harmonic voltages. In transformers with delta-connected windings the third and ninth harmonics circulate round the delta and are less evident in the line currents. An important source of harmonics is a rectifier load; this is becoming more prevalent as electronic devices proliferate.

On occasions, the harmonic content can prove important due mainly to the possibility of resonance occurring in the system, e.g. resonance has occurred with fifth harmonics. Also, the third-harmonic components are in phase in the three conductors of a three-phase line, and if a return path is present these currents add and cause interference in neighbouring communication circuits and increase the neutral return current.

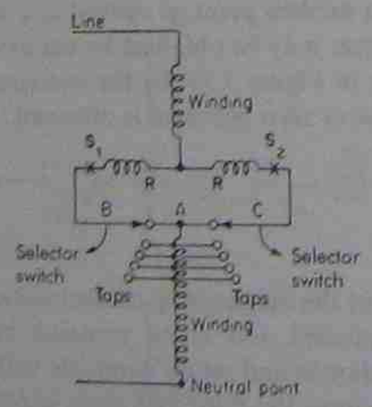
When analysing systems with harmonics it is sufficient to use the normal values for series inductance and shunt capacitance but one must remember to calculate reactance and susceptance at the frequency of the harmonic. The effect on resistance is more difficult to assess: however, it is usually only required to assess the presence of harmonics and the possibility of resonance.

Tap-changing transformers

A method of controlling the voltages in a network lies in the use of transformers, the turns ratio of which may be changed. In Figure 3.41(a) a schematic diagram of an off-load tap changer is shown; this, however, requires the



(a)



(b)

Figure 3.41 (a) Tap-changing transformer. (b) On-load tap-changing transformer. S_1, S_2 transfer switches, R centre-tapped reactor

disconnection of the transformer when the tap setting is to be changed. Many transformers now have on-load changers, the basic form of which is shown in Figure 3.41(b). In the position shown, the voltage is a maximum and the current divides equally in the two halves of reactor R, resulting in zero-resultant flux and minimum impedance. To reduce the voltage, S_1 opens and the total current passes through the other half of the reactor. Selector switch B then moves to the next contact and S_1 closes. A circulating current now flows in R superimposed on the load current. Now S_2 opens and C moves to the next tapping; S_2 then closes and the operation is complete. Six switch operations are required for one change in tap position. The voltage change between taps is often 1.25 per cent of the nominal voltage. This small change is necessary to avoid large voltage disturbances at consumer busbars. A schematic block diagram of the on-load tap systems is shown in Figure 3.42. The line drop compensator (LDC) is used to allow for the voltage drop along the feeder to the load point, so that the actual load voltage is seen and corrected by the transformer. The total range of tapping varies with the transformer usage, a typical figure for generator transformers is +2 to −16 per cent in 18 steps.

Typical parameters for transformers

The leakage reactances of two-winding transformers increase slightly with their rating for a given voltage, i.e. from 3.2 per cent at 20 kVA to 4.3 per cent at 500 kVA at 11 kV. For larger sizes, i.e., 20 MVA upwards, 10 per cent is a typical value at all voltages. For autotransformers the impedances are usually less than for double wound. Parameters for large transformers are as follows:

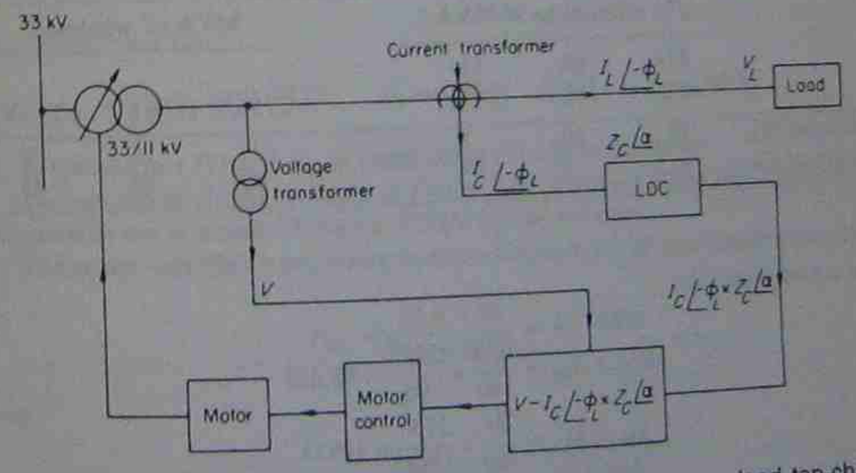


Figure 3.42 Schematic diagram of a control system for an on-load tap-changing transformer incorporating line drop compensation (LDC)

(1) 400/275 kV autotransformer, 500 MVA, 12 per cent impedance, tap range +10 to −20 per cent; (2) 400/132 kV double wound, 240 MVA, 20 per cent impedance, tap range +5 to −15 per cent. It should be noted that a 10 per cent reactance implies ten times rated current on short circuit. Winding forces depend upon current squared; the transformer must be designed to withstand short circuits.

Example 3.6

Two three-phase, 275/66/11 kV transformers, are connected in parallel and the windings are employed as follows:

- the 11 kV winding supplies a synchronous compensator (i.e. a synchronous motor running light);
- the 66 kV winding supplies a network with 80 MW of demand;
- the 275 kV winding is connected to the primary transmission system.

The winding data are as shown below. The line diagram is shown in Figure 3.43(a).

It is required to determine the division of power between the two 275/66 kV windings. Resistance may be neglected.

Solution

To obtain the equivalent star circuits the reactances will be expressed on common bases of 90 MVA, 275 kV.

	X% referred to 90 MVA			MVA of windings		
	275/66 kV	66/11 kV	11/275 kV	275 kV	66 kV	11 kV
Transformer (1)	10.5	20	8	95	90	60
Transformer (2)	10.5	5	15	90	85	45

For transformer (1),

$$X_{275} + X_{66} = \frac{10.5}{100} \times \frac{275^2}{90} = 88.4\,\Omega$$

$$X_{66} + X_{11} = \frac{20}{100} \times \frac{275^2}{90} = 168\,\Omega$$

$$X_{11} + X_{275} = \frac{8}{100} \times \frac{275^2}{90} = 67.2\,\Omega$$

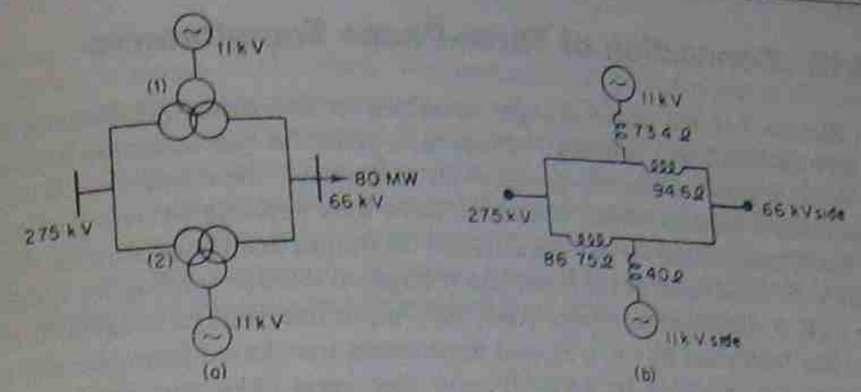


Figure 3.43 (a) Line diagram for system in Example 3.6. (b) Equivalent circuit: all reactance values refer to bases of 275 kV, 90 MVA

The corresponding quantities for transformer (2) are 88.4 Ω, 42.1 Ω, and 126.3 Ω, respectively.

For transformer (1),

$$X_p \text{ (275 kV side)} = \tfrac{1}{2}(X_{ps} + X_{pt} - X_{st})$$
$$= \tfrac{1}{2}(88.4 + 67.2 - 168)$$
$$= -6.2\,\Omega$$

Similarly,

$$X_S \text{ (66 kV side)} = 94.6\,\Omega$$

and

$$X_T \text{ (11 kV side)} = 73.4\,\Omega$$

For transformer (2),

$$X_p = 86.75\,\Omega \qquad X_S = 1.65\,\Omega \qquad \text{and} \qquad X_T = 40\,\Omega$$

X_p in transformer (1) and X_S in transformer (2) will be neglected.

The equivalent circuit is shown in Figure 3.43(b) from which it will be seen that two impedances are in parallel between 275 kV and 66 kV sides.

The power will divide according to the reactances of the two transformers. Hence,

$$P_{(1)} = \frac{94.6 \times 80}{94.6 + 86.75} = 41.7\,\text{MW}$$

and

$$P_{(2)} = \frac{86.75 \times 80}{86.75 + 94.6} = 38.3\,\text{MW}$$

3.12 Connection of Three-Phase Transformers

In Section 3.11 the phase changes occurring in star–delta transformers are briefly discussed and sufficient treatment is given for the following analysis to be understood. From the practical view, however, the connection of such transformers requires further discussion to be fully appreciated. It is seen that for the connection shown in Figure 3.38(a) the output line voltages on the delta side are 30° in advance of the input line voltages to the star winding for normal a–b–c (R–Y–B) phase notation. Also, the output line voltage is equal to the input line multiplied by $(N/\sqrt{3})$, and impedances transferred from one side to the other are modified by $(N/\sqrt{3})^2$ and vice versa. The true single-phase equivalent circuit is therefore as shown in Figure 3.44.

However, by suitable rearrangement of the winding connections, phase shifts of 30, 90, 150, 210, 270, or 330 electrical degrees may be obtained, and there is a need for standardization of nomenclature and procedure. Should a phase difference exist between the secondary output voltages of transformers connected to a common supply, dangerously large circulating currents will occur and the circuit protection should operate. For uniformity, the transformer windings of Figure 3.38(a) are shown in Figure 3.44 with capitals referring to the H.V. winding. It is seen that with the star–delta a shift of −30° can also be produced by a rearrangement of the terminal connections. British practice* involves the use of a 'vector group reference' number to describe transformer connections. The first number indicates the phase shift, e.g. 1 indicates 0°, 2 = 180°, 3 = (−30°), 4 = (+30°); the next number indicates the interphase connection of the secondary, e.g. 1 = star, 2 = delta, 3 = interconnected star. Letters y and D indicate star (y) or delta (D) connection, the H.V. winding coming first. Finally comes the phase displacement angle in the form of a clock-reference, the low-voltage winding-induced voltage being given by the hour hand and the H.V. voltage by the minute hand, e.g. 0° = 12 o'clock,

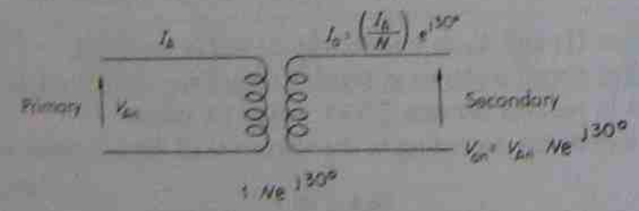


Figure 3.44 Equivalent single-phase circuit of a three-phase transformer with a phase shift from primary to secondary

*The American Standards Association adopt the standard that the high-voltage reference phase is always 30° ahead of the low-voltage reference phase, regardless of whether the three-phase connections are delta–star or star–delta.

180° = 6 o'clock, +30° = 11 o'clock and −30° = 1 o'clock; A_2 in the phasor diagrams is always at 12 o'clock. The application of this principle to the most widely used transformer connections is shown in Figure 3.45.

From an analytical viewpoint it may be desirable to have a phase shift of 90°, thus facilitating the use of operator 'j'. In this type of work the exact arrangement is not important provided the system adopted is used consistently and the winding arrangement is seldom the cause for concern. However, when dealing with the connection of actual equipment in the system, the correct arrangement of the transformer winding is of vital importance.

3.13 Voltage Characteristics of Loads

The variation of the power and reactive power taken by a load with various voltages is of importance when considering the manner in which such loads are to be represented in load flow and stability studies. Usually, in such studies, the load on a substation has to be represented and is a composite load consisting of industrial and domestic consumers. A typical composition of a substation load is as follows:

Induction motors	50–70%	(air-conditioners, freezers, washers, fans, pumps, etc.)
Lighting and heating	20–25%	(water heating, resistance heaters, etc.)
Synchronous motors	10%	
(Transmission and distribution losses	10–12%)	

3.13.1 Lighting

This is independent of frequency and consumes no reactive power. The power consumed does not vary as the (voltage)^2, but approximately as (voltage)$^{1.6}$. With fluorescent and sodium/mercury lamps, distorted currents are taken.

3.13.2 Heating

This maintains constant resistance with voltage change and hence the power varies with (voltage)^2.

The above loads may be described as static.

Phase displacement	Main group No	Vector group Ref No & symbol	Marking of line terminals and phasor diagram of induced voltages: H.V Winding	Marking of line terminals and phasor diagram of induced voltages: L.V Winding	Winding connections and relative position of terminals
Col 1	2	3	4	5	6
0°	1	11 Yy 0			
		12 Dd 0			
		13 Dz 0			
180°	2	21 Yy 6			
		22 Dd 6			
		23 Dz 6			
-30°	3	31 Dy 1			
		32 Yd 1			
		33 Yz 1			
+30°	4	41 Dy 11			
		42 Yd 11			
		43 Yz 11			

Figure 3.45 British Standard for connections of three-phase transformers and resulting phase shifts

3.13.3 Synchronous motors

The power consumed is approximately constant. For a given excitation the vars change in a leading direction with voltage reduction. The P–V, Q–V, generalized characteristics are shown in Figure 3.46.

3.13.4 Induction motors

The P–V, Q–V characteristics may be determined by the use of the simplified circuit shown in Figure 3.47. It is assumed that the mechanical load on the shaft is constant.

The electrical power,

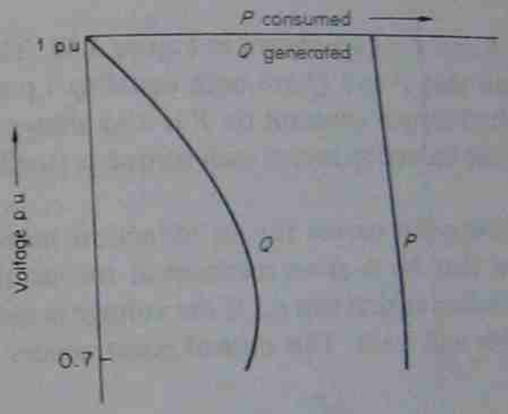


Figure 3.46 P–V and Q–V curves for a synchronous motor

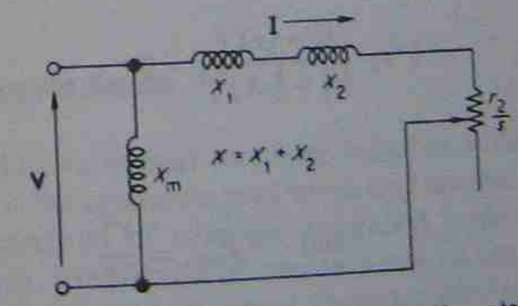


Figure 3.47 Equivalent circuit of an induction motor: X_1 = stator leakage reactance; X_2 = rotor leakage reactance referred to the stator; X_m = magnetizing reactance; r_2 = rotor resistance; s = slip p.u. Magnetizing losses have been ignored and the stator losses are lumped in with the line losses

$$P_{\text{electrical}} = P_{\text{mechanical}} = P$$
$$= 3I^2\frac{r_2}{s} = \text{constant}$$
$$\therefore\ s = \frac{3I^2 r_2}{P}$$

The reactive power consumed

$$= \frac{3V^2}{X_m} + 3I^2(X_1 + X_2) \tag{3.15}$$

Also, from Figure 3.47

$$P = 3I^2\frac{r_2}{s} = \frac{3V^2}{(r_2/s)^2 + X^2}\cdot\frac{r_2}{s}$$
$$= \frac{3V^2 r_2 s}{r_2^2 + (sX)^2} \tag{3.16}$$

The graphs of V–P and V–Q are shown in Figure 3.48. The P and Q scales have been so arranged that P and Q are both equal to 1 p.u. at a voltage of 1 p.u. The effect of shaft torque constant on P is also shown. Similar analysis can be performed for the induction motor with torque $\propto$ (speed)^2 and torque $\propto$ speed.

The well-known power-slip curves for an induction motor are shown in Figure 3.49. It is seen that for a given mechanical torque there is a critical voltage and a corresponding critical slip s_{cr}. If the voltage is reduced further the motor becomes unstable and stalls. This critical point occurs when

$$\frac{dP}{ds} = 0$$

i.e. when

$$V^2 r_2 \cdot \frac{r_2^2 - (sX)^2}{[r_2^2 + (sX)^2]^2} = 0$$

so that

$$s = \frac{r_2}{X} \quad \text{and} \quad P_{max} = \frac{V^2}{2X}$$

Alternatively, for a given output power P,

$$V_{\text{critical}} = \sqrt{2PX}$$

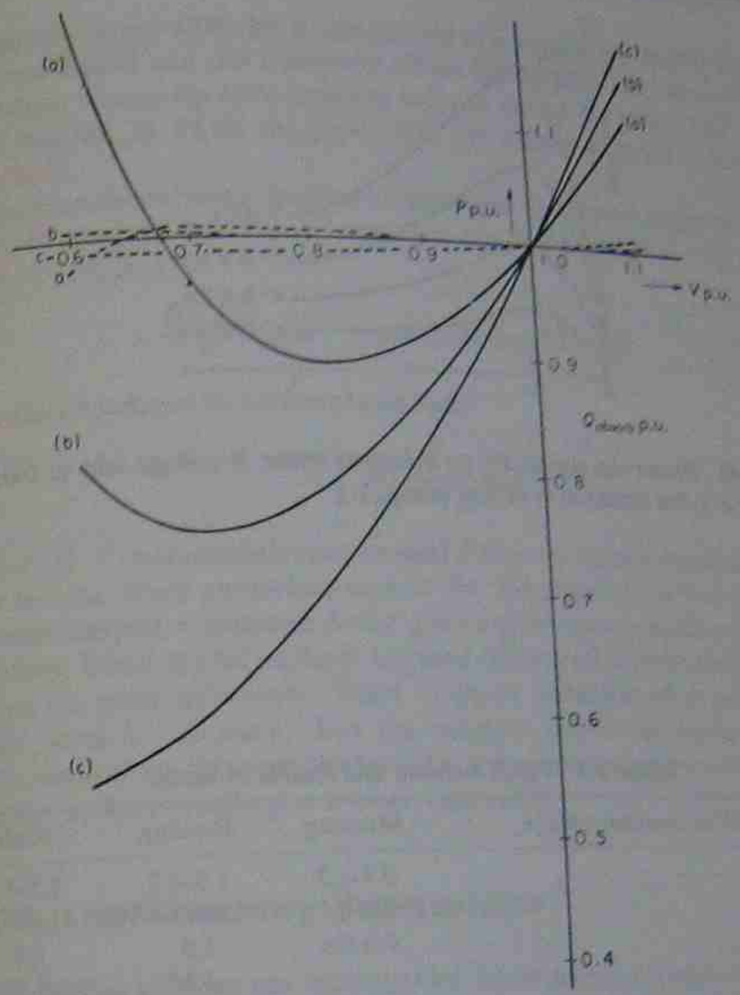


Figure 3.48 Graphs of P–V and Q–V for an induction motor—shaft torque constant. Curve (a) full load, (b) 0.75 full load, (c) 0.5 full load; – – – P, —— Q

3.13.5 Composite loads

Except in isolated cases, such as very large induction motor drives in refineries and steel mills, it is the composite load which is of interest. Tests to obtain P–V and Q–V characteristics of complete substation loads have been few (the consumers object!). The results of one such test on the Polish network are given in Table 3.4. Instead of actual characteristics, values of $\partial P/\partial V$ and $\partial Q/\partial V$ at 1 p.u. voltage are given. As will be seen later, these quantities are of considerable interest. Composite load characteristics can be obtained by summing the characteristics of the constituent loads.

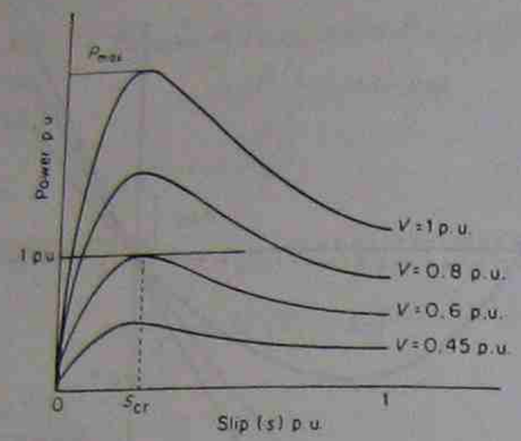


Figure 3.49 Power-slip curves for an induction motor. If voltage falls to 0.6 p.u. at full load $P = 1$, the condition is critical (slip s_{cr})

Table 3.4 Polish network test results of loads

dP/dV at nominal voltage	Morning	Evening	Night
Large towns	0.9–1.2	1.5–1.7	1.5–1.6
Small towns	0.6–0.7	1.4–1.6	1.4–1.6
Villages	0.5–0.6	1.5	1.5
Street lighting	—	1.55	1.55
Mines	0.6–0.76	0.75–0.89	0.65–0.78
Ironworks	0.6–0.7	0.6–0.75	0.6–0.7
Textile industries	0.5–0.6	0.6–0.7	0.6–0.65
Large chemical industries	0.6–0.7	0.6–0.75	0.6–0.7
Machine industry	0.5–0.55	0.6–0.65	0.6–0.65
Junction points in the H.V. network	0.6–0.7	0.8–1.1	0.8–1.0
dQ/dV at nominal voltage	Morning	Evening	Night
Large towns	2.5–3.0	2.2–2.5	2.4–3.0
Small towns	2.6–3.5	2.0–2.5	2.5–3.0
Villages	2.6–3.5	2.0–2.5	2.4–3.5
Industry (total)	1.8–2.7	1.8–2.3	1.8–2.7
Junction points in 6 kV network	2.2–2.9	1.8–2.5	2.0–2.6
Junction points in 110 kV network	1.8–2.6	1.5–2.2	1.7–2.3

(Source: Boyucki, A., and Wojcik, M., 'Static characteristics of loads', *Energetyka B.*, No. 3 (Polish), 1959.)

In practice, great difficulty is experienced in assessing the degree to which motors are loaded and this materially affects the shape of the characteristics. Loads which require the same power at reduced voltage are called 'stiff'; loads, such as heating, in which the power falls off rapidly at lower voltages are termed 'soft'.

In much analytical work, loads are represented by constant impedances, and hence it is assumed that $P \propto V^2$ and $Q \propto V^2$. If the load is $P + jQ$ p.u. the current at voltage V is

$$\frac{P - jQ}{V} \text{ p.u.} \qquad \text{(remembering } \mathbf{VI}^* = \mathbf{S}\text{)}$$

Hence the impedance to represent this load

$$= \frac{\mathbf{V}}{\mathbf{I}} = \frac{V^2}{P - jQ} \text{ p.u.}$$

The P–V, Q–V characteristics can be used if they are known, but this process is rather tedious. Some authorities suggest that the representation of the load by a constant-current consuming device gives a good approximation to practical conditions. When digital methods are used the correct representation of loads presents no great difficulties. When extensive reduction of a network to a simpler form is necessary, then the constant impedance representation is usually resorted to, although the load this represents seldom occurs in practice. However, it does usually give a worst-case result.

3.13.6 Load–frequency characteristics

Often, voltage changes are accompanied by frequency changes. Again, information regarding the characteristics of composite loads with frequency is scarce. With the small frequency changes allowed in practice, the effects on the power and reactive power consumed are usually neglected in calculations.

Problems

(*Note*: All machines, etc., are three-phase unless stated otherwise.)

3.1 When two four pole, 50 Hz synchronous generators are paralleled their phase displacement is 2° mechanical. The synchronous reactance of each machine is 10 Ω/phase and the common busbar voltage is 6.6 kV. Calculate the synchronizing torque. (Answer: 973.5 Nm)

3.2 A synchronous generator has a synchronous impedance of 2 p.u. and a resistance of 0.01 p.u. It is connected to an infinite busbar of voltage 1 p.u. through a transformer of reactance j0.1 p.u. If the generated (no-load) e.m.f. is 1.1 p.u. calculate the current

and power factor for maximum output.
(Answer: 0.708 p.u.; 0.74 leading)

3.3 A 6.6 kV synchronous generator has negligible resistance and synchronous reactance of 4 Ω/phase. It is connected to an infinite busbar and gives 2000 A at unity power factor. If the excitation is increased by 25 per cent find the maximum power output and the corresponding power factor. State any assumptions made.
(Answer: 31.6 MW; 0.96 leading)

3.4 A synchronous generator whose characteristic curves are given in Figure 3.4 delivers full load at the following power factors: 0.8 lagging, 1.0, and 0.9 leading. Determine the percentage regulation at these loads.
(Answer: 167, 119, 76 per cent)

3.5 A salient-pole, 75 MVA, 11 kV synchronous generator is connected to an infinite busbar through a link of reactance 0.3 p.u. and has $X_d = 1.5$ p.u. and $X_q = 1$ p.u., and negligible resistance. Determine the power output for a load angle 30° if the excitation e.m.f. is 1.4 times the rated terminal voltage. Calculate the synchronizing coefficient in this operating condition. All p.u. values are on a 75 MVA base.
(Answer: $P = 0.48$ p.u.; $\partial P/\partial\delta = 0.78$ p.u.)

3.6 A synchronous generator of open-circuit terminal voltage 1 p.u. is on no-load and then suddenly short-circuited; the trace of current against time is shown in Figure 3.6(b). In Figure 3.6(b) the current oc = 1.8 p.u., oa = 5.7 p.u., and ob = 8 p.u. Calculate the values of X_s, X', and X''. Resistance may be neglected. If the machine is delivering 1 p.u. current at 0.8 power factor lagging at the rated terminal voltage before the short circuit occurs, sketch the new envelope of the 50 Hz current waveform.
(Answer: $X_s = 0.8$ p.u.; $X' = 0.25$ p.u.; $X'' = 0.18$ p.u.)

3.7 Construct a performance chart for a 22 kV, 500 MVA, 0.9 p.f. generator having a short-circuit ratio of 0.55.

3.8 A 275 kV three-phase transmission line of length 96 km is rated at 800 A. The values of resistance, inductance, and capacitance per phase per kilometre are 0.078 Ω, 1.056 mH, and 0.029 μF, respectively. The receiving-end voltage is 275 kV when full load is transmitted at 0.9 power factor lagging. Calculate the sending-end voltage and current, and the transmission efficiency, and compare with the answer obtained by the short-line representation. Use the nominal π and T methods of solution. The frequency is 60 Hz.
(Answer: $V_s = 178$ kV per phase)

3.9 A 220 kV, 60 Hz three-phase transmission line is 320 km long and has the following constants per phase per km:

Inductance	1.3 mH
Capacitance	0.22 μF
Resistance	0.55 Ω

Ignore leakage conductance.
If the line delivers a load of 300 A, 0.8 power factor lagging, at a voltage of 220 kV, calculate the sending-end voltage. Determine the π circuit which will represent the line.

Calculate the error resulting from the use of the medium-length π section.
(Answer: Series $(11 + j116)\,\Omega$; shunt $(0.44 + j0.72) \times 10^{-3}\ \Omega^{-1}$)

3.10 In a three-core cable, the capacitance between the three cores short-circuited together and the sheath is 87 μF/km, and that between two cores connected together and the third core is 0.84 μF/km.
Determine the kVA required to keep 16 km of this cable charged when the supply is 33 kV, three phase, 50 Hz.
(Answer: 270 kVA)

3.11 Calculate the **A B C D** constants for a 275 kV overhead line of length 83 km. The parameters per kilometre are as follows:

Resistance	0.078 Ω
Reactance	0.33 Ω
Admittance (shunt capacitative)	9.53×10^{-6} S

The shunt conductance is zero.
(Answer: $[A = 0.8615 + j0.0317;\ B = 34.0 + j153;\ C = (-19.2 + j1693 \times 10^{-6})]$

3.12 A 132 kV, 60 Hz transmission line has the following generalized constants:

$$A = 0.9696\angle 0.49^\circ$$
$$B = 52.88\angle 74.79^\circ\ \Omega$$
$$C = 0.001177\angle 90.15^\circ\ \text{S}$$

If the receiving-end voltage is to be 132 kV when supplying a load of 125 MVA 0.9 p.f. lagging, calculate the sending-end voltage and current.
(Answer: 95.9 kV phase, 166.1 kV line 554 A)

3.13 Two identical transformers each have a nominal or no-load ratio of 33/11 kV and a leakage reactance of 2 Ω referred to the 11 kV side; resistance may be neglected. The transformers operate in parallel and supply a load of 9 MVA, 0.8 p.f. lagging. Calculate the current taken by each transformer when they operate five tap steps apart (each step is 1.25 per cent of the nominal voltage). Also calculate the kVAr absorbed by this tap setting.
(Answer: 307 A, 194 A for three-phase transformers, 118 kVAr)

3.14 An induction motor, the equivalent circuit of which is shown in Figure 3.50 is connected to supply busbars which may be considered as possessing a voltage and frequency which is independent of the load. Determine the reactive power consumed for various busbar voltages and construct the Q–V characteristic. Calculate the critical

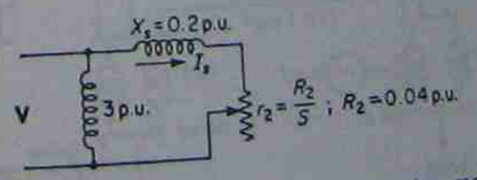


Figure 3.50 Equivalent circuit of 500 kW, 6.6 kV induction motor in Problem 3.14. All p.u. values refer to rated voltage and power ($P = 1$ p.u. and $V = 1$ p.u.)

voltage at which the motor stalls and the critical slip, assuming that the mechanical load is constant.
(Answer: $s_{cr} = 0.2$, $V_c = 0.63$ p.u.)

3.15 A 100 MVA round-rotor generator of synchronous reactance 1.5 p.u. supplies a substation (L) through a step-up transformer of reactance 0.1 p.u., two lines each of reactance 0.3 p.u. in parallel and a step-down transformer of reactance 0.1 p.u. The load taken at L is 100 MW at 0.85 lagging. L is connected to a local generating station which is represented by an equivalent generator of 75 MVA and synchronous reactance of 2 p.u. All reactances are expressed on a base of 100 MVA.

Draw the equivalent single-phase network. If the voltage at L is to be 1 p.u. and the 75 MVA machine is to deliver 50 MW, 20 MVAr, calculate the internal voltages of the machines.
(Answer: $E_1 = 2$ p.u.; $E_2 = 1.72$ p.u.; $\delta_{2L} = 35.45°$)

3.16 The following data applies to the power system shown in Figure 3.51.

Generating station A: Four identical turboalternators, each rated at 16 kV, 125 MVA, and of synchronous reactance 1.5 p.u. Each machine supplies a 125 MVA, 0.1 p.u. transformer connected to a busbar sectioned as shown.

Substation B: Two identical 150 MVW, three-winding transformers, each having the following reactances between windings: 132/66 kV windings 10 per cent; 66/11 kV windings 20 per cent; 132/11 kV windings 20 per cent; all on a 150 MVA base.

The secondaries supply a common load of 200 MW at 0.9 p.f. lagging. To each tertiary winding is connected a 30 MVA synchronous compensator of synchronous reactance 1.5 p.u.

Substation C: Two identical 150 MVA transformers, each of 0.15 p.u. reactance, supply a common load of 300 MW at 0.85 p.f. lagging.

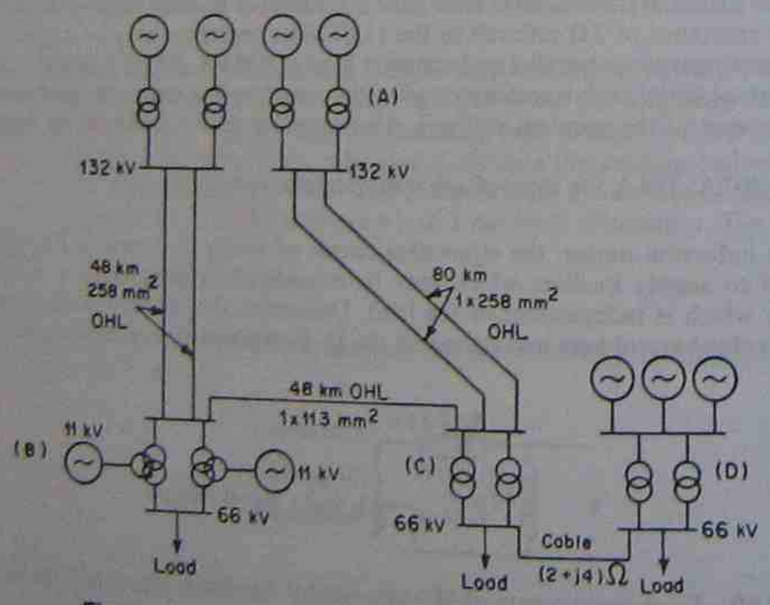


Figure 3.51 Line diagram of system in Problem 3.16

Generating station D: Three identical 11 kV, 75 MVA generators, each of 1 p.u. synchronous reactance, supply a common busbar which is connected to an outgoing 66 kV cable through two identical 100 MVA transformers. Load 50 MW, 0.8 p.f. lagging.

Determine the equivalent circuit for balanced operation giving component values on a base of 100 MVA. Treat the loads as impedances.

3.17 Distinguish between kW, kVA, and kVAr.
Explain why

(a) generators in large power systems usually run overexcited, 'generating' VAr.
(b) remote hydro-generators need an underexcited rating so that they can absorb VAr.
(c) loss of an overexcited generator in a power system will normally cause a drop in voltage at its busbar.

A load of 0.8 p.u. power and 0.4 p.u. VAr lagging is supplied from a busbar of 1.0 p.u. voltage through an inductive line of reactance 0.15 p.u. Determine the load terminal voltage assuming that p.u. current has the same value as p.u. VA.
(Answer: 0.934 p.u.)
(From E.C. Examination, 1996)

3.18 Sketch the performance chart of a synchronous generator indicating the main limits.

Consider a generator with the following nameplate data: 500 MVA, 20 kV, 0.8 p.f. (power factor), $X_s = 1.5$ p.u.

(a) Calculate the internal voltage and power angle of the generator operating at 400 MW with $\cos\phi = 0.8$ (lagging) with a 1 p.u. terminal voltage.
(b) What is the maximum reactive power this generator can absorb from the system?
(c) What is the maximum reactive power this generator can deliver to the system, assuming a maximum internal voltage of 2.25 p.u.
(d) Place the numerical values calculated on the performance chart.

A graphical solution is acceptable.
(Answer: (a) 2.23 p.u., 33.5°; (b) 333 MVAr; (c) 417 MVAr)
(From E.C. Examination, 1997)

4

Control of Power and Frequency

4.1 Introduction

Although to a certain extent the control of power and frequency is interrelated to the control of reactive power and voltage, it is hoped that by dealing with power and frequency separately from voltage control, a better appreciation of the operation of power systems will be obtained. In a large interconnected system, many generation stations, large and small, are synchronously connected and hence all have the same frequency. The following remarks refer mainly to networks in which the control of power is carried out by the decisions and actions of engineers, as opposed to systems in which the control and allocation of load to machines is effected completely automatically. The latter are sometimes based on a continuous load-flow calculation by analogue or digital computers. The allocation of the required power among the generators has to be decided before the advent of the load, which must therefore be predicted. An analysis is made of the loads experienced over the same period in previous years, account is also taken of the value of the load immediately previous to the period under study and of the weather forecast. The probable load to be expected, having been decided, is allocated to the various turbine-generators.

A daily load cycle is shown in Figure 1.2. It is seen that the rate of rise of the load is very high and varies between 2 MW/min per 1000 MW of peak demand in the early hours and about 8 MW/min per 1000 MW of peak demand between 07.00 and 08.00 hours. Hence the ability of machines to increase their output quickly from zero to full load is important. It is extremely unlikely that the output of the machines at any instant will exactly equal the load on the system. If the output is higher than the demand the machines will tend to increase in speed and the frequency will rise, and vice versa. Hence the

frequency is not a constant quantity but continuously varies; these variations are normally small and have no noticeable effect on most consumers. The frequency is continuously monitored against standard time-sources and when long-term tendencies to rise or fall are noticed, the control engineers take appropriate action by regulating the generator outputs.

Should the total generation available be insufficient to meet the demand, the frequency will fall. If the frequency falls by more than 1 Hz the reduced speed of power station pumps, fans, etc., may reduce the station output and a serious situation arises. In this type of situation, although the reduction of frequency will cause a reduction in power demand, voltage must be reduced, and if this is not sufficient then loads will have to be disconnected and continue to be disconnected until the frequency rises to a reasonable level. All utilities have a scheme of planned load shedding based on under-frequency relays set to reduce loads in blocks to prevent complete shut-down in extreme emergencies.

When a permanent increase in load occurs on the system, the speed and frequency of all the interconnected generators fall, since the increased energy requirement is met from the kinetic energy of the machines. This causes an increase in steam or water admitted to the turbines due to the operation of the governors and hence a new load balance is obtained. Initially, the boilers have a thermal reserve by means of which sudden changes can be supplied until the new firing rate has been established. Modern gas turbines have an overload capability for a few minutes which can be usefully exploited in emergency situations.

4.2 The Turbine Governor

In the following discussion of governor mechanisms, both steam and water turbines will be considered together and points of difference indicated where required. A simplified schematic diagram of a traditional governor system is shown in Figure 4.1. The sensing device, which is sensitive to change of speed,

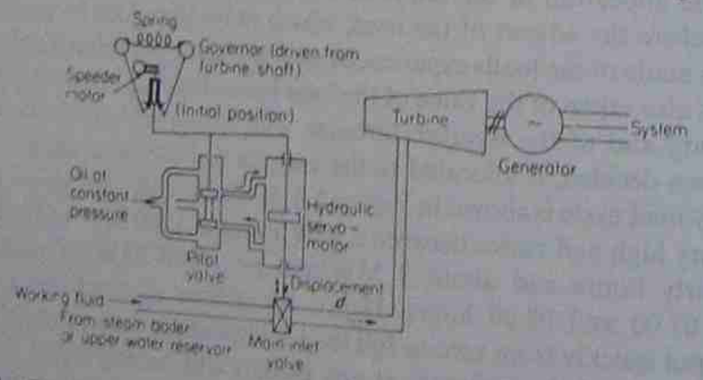


Figure 4.1 Governor control system employing the Watt governor as sensing device and a hydraulic servo-system to operate main supply valve. Speeder-motor gear determines the initial setting of the governor position

is the time-honoured Watt centrifugal governor. In this, two weights move radially outwards as their speed of rotation increases and thus move a sleeve on the central spindle. This sleeve movement is transmitted by a lever mechanism to the pilot-valve piston and hence the servo-motor is operated. A dead band is present in this mechanism, i.e. the speed must change by a certain amount before the valve commences to operate, because of friction and mechanical backlash. The time taken for the main steam valve to move due to delays in the hydraulic pilot-valve and servo-motor systems is appreciable, 0.2–0.3 s. The governor characteristic for a large steam turboalternator is shown in Figure 4.2 and it is seen that there is a 4 per cent drop in speed between no load and full load. Because of the requirement for high response speed, low dead band, and accuracy in speed and load control, the mechanical governor has been replaced in large modern turbogenerators by electrohydraulic governing. The method normally used to measure the speed is based on a toothed wheel (on shaft) and magnetic-probe pickup. The use of electronic controls requires an electrohydraulic conversion stage, using conventional servo-valves.

An important feature of the governor system is the mechanism by means of which the governor sleeve and hence the main-valve positions can be changed and adjusted quite apart from when actuated by the speed changes. This is accomplished by the speed changer, or 'speeder motor', as it is sometimes termed. The effect of this adjustment is the production of a family of parallel characteristics, as shown in Figure 4.3. Hence the power output of the generator at a given speed may be adjusted at will and this is of extreme importance when operating at optimum economy.

The torque of the turbine may be considered to be approximately proportional to the displacement d of the main inlet valve. Also, the expression for the change in torque with speed may be expressed approximately by the equation

$$T = T_0(1 - kN) \tag{4.1}$$

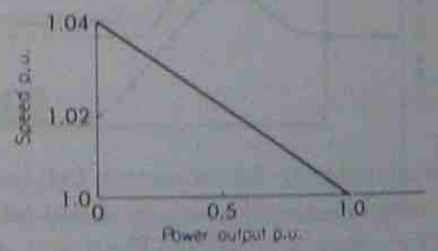


Figure 4.2 Idealized governor characteristic of a turboalternator with 4 per cent droop from zero to full load

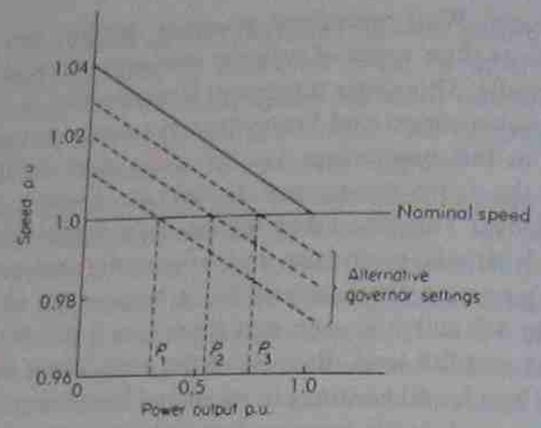


Figure 4.3 Effect of speeder gear on governor characteristics P_1, P_2, and P_3 = outputs at various settings but at same speed

where T_0 is the torque at speed N_0 and T the torque at speed N; k is a constant for the governor system. As the torque depends on both the main-valve position and the speed, $T = f(d, N)$.

There is a time delay between the occurrence of a load change and the new operating conditions. This is due not only to the governor mechanism but also to the fact that the new flow rate of steam or water must accelerate or decelerate the rotor in order to attain the new speed. In Figure 4.4 typical curves are shown for a turbogenerator which has a sudden decrease in the electrical power required, pehaps due to an external power network fault, and hence the retarding torque on the turbine shaft is suddenly much smaller. In the ungoverned

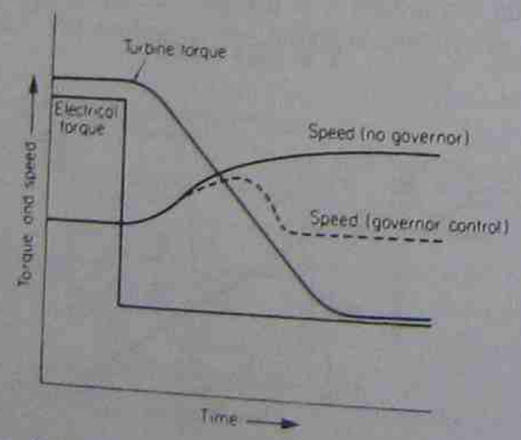


Figure 4.4 Graphs of turbine torque, electrical torque, and speed against time when the load on a generator suddenly falls

case the considerable time-lag between the load change and the attainment of the new steady speed is obvious. With the regulated or governed machine, due to the dead band in the governor mechanism, the speed–time curve starts to rise, the valve then operates, and the fluid supply is adjusted. It is possible for damped oscillations to be set up after the load change; this will be discussed in Chapter 8.

An important factor regarding turbines is the possibility of overspeeding, when the load on the shaft is lost completely, with possible drastic mechanical breakdown. To avoid this, special valves are incorporated to automatically cut off the energy supply to the turbine. In a turbogenerator normally running at 3000 r.p.m. this overspeed protection operates at about 3300 r.p.m.

Example 4.1

An isolated 75 MVA synchronous generator feeds its own load and operates initially at no-load at 3000 r.p.m., 50 Hz. A 20 MW load is suddenly applied and the steam valves to the turbine commence to open after 0.5 s due to the time-lag in the governor system. Calculate the frequency to which the generated voltage drops before the steam flow meets the new load. The stored energy for the machine is 4 kW-s per kVA of generator capacity.

Solution

For this machine the stored energy at 3000 r.p.m.

$$= 4 \times 75\,000 = 300\,000 \text{ kW–s}$$

Before the steam valves start to open the machine loses $20\,000 \times 0.5 = 10\,000$ kW-s of the stored energy in order to supply the load.

The stored energy $\propto$ (speed)^2. Therefore the new frequency

$$= \sqrt{\frac{300\,000 - 10\,000}{300\,000}} \times 50 \text{ Hz}$$

$$= 49.2 \text{ Hz}$$

4.3 Control Loops

The machine and its associated governor and voltage-regulator control systems may be represented by the block diagram shown in Figure 4.5. The nature of the voltage control loop has been discussed already in Chapter 3; accurate machine representation is required, involving two-axis expressions, as indicated in Chapter 2.

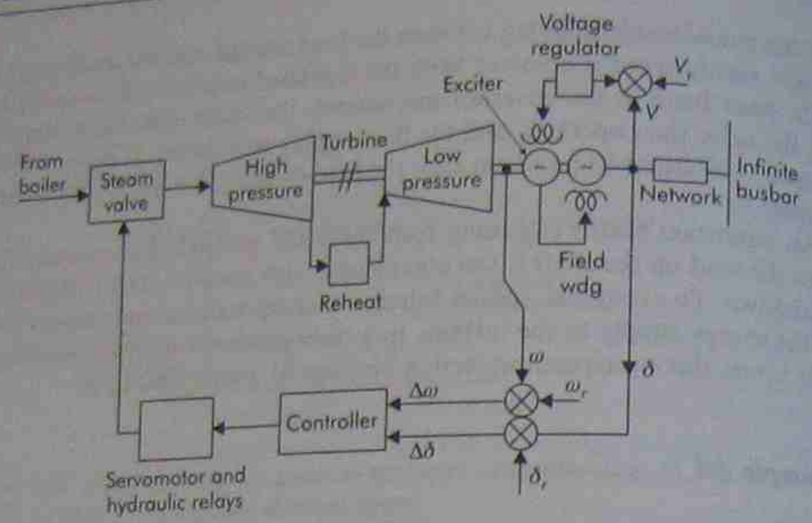


Figure 4.5 Block diagram of complete turboalternator control systems. The governor system is more complicated that that shown in Figure 4.1 owing to the inclusion of the load-angle δ in the control loop. Suffixes r refer to reference quantities and Δ to the error quantities. The controller modifies the error signal from the governor by taking into account the load angle

Two factors have a large influence on the dynamic response of the prime mover: (1) entrained steam between the inlet valves and the first stage of the turbine (in large machines this can be sufficient to cause loss of synchronism after the valves have closed); (2) the storage action in the reheater which causes the output of the low-pressure turbine to lag behind that of the high-pressure side. The transfer function

$$\frac{\text{prime mover torque}}{\text{valve opening}}$$

accounting for both these effects is

$$\frac{G_1 G_2}{(1+\tau_t s)(1+\tau_r s)} \tag{4.2}$$

where

G_1 = entrained steam constant;
G_2 = reheated gain constant;
τ_t = entrained steam time constant;
τ_r = reheater time constant.

The transfer function relating steam-valve opening d to changes in speed ω due to the governor feedback loop is

$$\frac{\Delta d}{\Delta \omega}(s) = \frac{G_3 G_4 G_5}{(1+\tau_g s)(1+\tau_1 s)(1+\tau_2 s)} \tag{4.3}$$

where

τ_g = governor-relay time constant
τ_1 = primary-relay time constant;
τ_2 = secondary-valve-relay time constant;
$(G_3 G_4 G_5)$ = constants relating system-valve lift to speed change.

By a consideration of the transfer function of the synchronous generator with the above expressions the dynamic response of the overall system may be obtained.

4.4 Division of Load Between Generators

The use of the speed changer enables the steam input and electrical power output at a given frequency to be changed as required. The effect of this on two machines can be seen in Figure 4.6. The output of each machine is not therefore determined by the governor characteristics but can be varied by the operating personnel to meet economical and other considerations. The governor characteristics only completely decide the outputs of the machines when a sudden change in load occurs or when machines are allowed to vary their outputs according to speed within a prescribed range in order to keep the frequency constant. This latter mode of operation is known as *free-governor action*.

It has been shown in Chapter 2 that the voltage difference between the two ends of an interconnector of total impedance $R + jX$ is given by

$$\Delta V = E - V = \frac{RP + XQ}{V}$$

Also the angle between the voltage phasors (i.e. the transmission angle) δ is given by

$$\sin^{-1}\frac{\Delta V_q}{E}$$

where

$$\Delta V_q = \frac{XP - RQ}{V}$$

When $X \gg R$, i.e. for most transmission networks,

$$\Delta V_q \propto P \quad \text{and} \quad \Delta V_p \propto Q$$

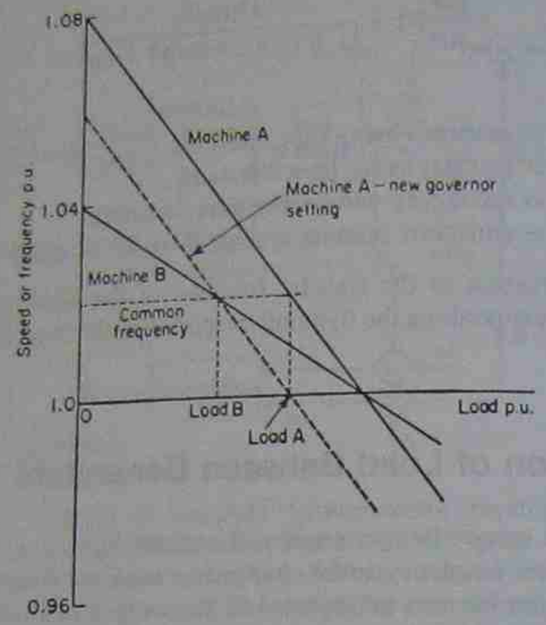


Figure 4.6 Two machines connected to an infinite busbar. The speeder gear of machine A is adjusted so that the machines load equally

Hence, (1) *the flow of power between two nodes is determined largely by the transmission angle;* (2) *the flow of reactive power is determined by the scalar voltage difference between the two nodes.*

These two facts are of fundamental importance to the understanding of the operation of power systems.

The angular advance of G_A (Figure 4.7) is due to a greater relative energy input to turbine A than to B. The provision of this extra steam (or water) to A is possible because of the action of the speeder gear without which the power outputs of A and B would be determined solely by the nominal governor characteristics. The following simple example illustrates these principles.

Example 4.2

Two synchronous generators operate in parallel and supply a total load of 200 MW. The capacities of the machines are 100 MW and 200 MW and both have governor droop characteristics of 4 per cent from no load to full load. Calculate the load taken by each machine, assuming free governor action.

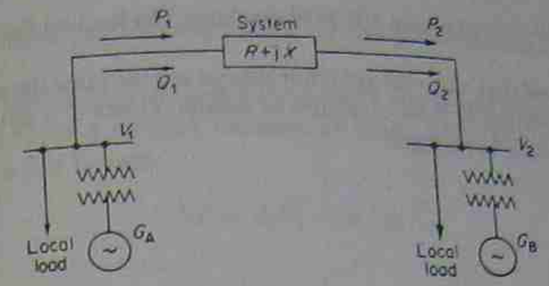


Figure 4.7 Two generating stations linked by an interconnector of impedance $(R+jX)\,\Omega$. The rotor of A is in phase advance of B and $V_1 > V_2$

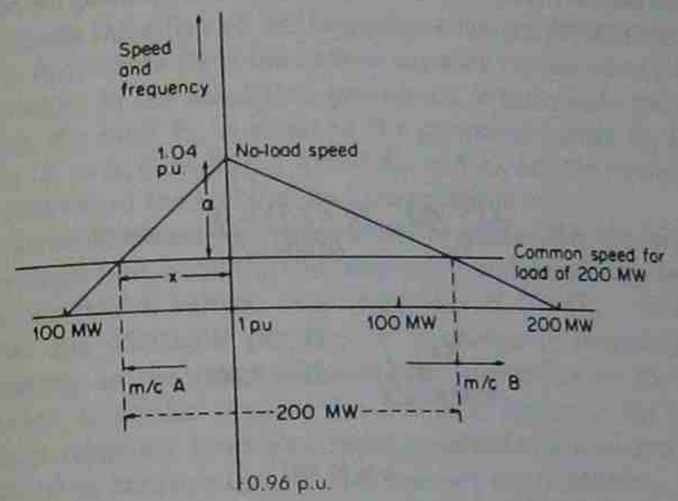


Figure 4.8 Speed–load diagram for Example 4.2

Solution

Let x megawatts be the power supplied from a 100 MW generator. Referring to Figure 4.8,

$$\frac{4}{100} = \frac{\alpha}{x}$$

For the 200 MW machine,

$$\frac{4}{200} = \frac{\alpha}{200 - x}$$

$$\therefore \frac{4x}{100} = \frac{800 - 4x}{200}$$

and $x = 66.6$ MW = load on the 100 MW machine. The load on the 200 MW machine = 133.3 MW.

It will be noticed that when the governor droops are the same the machines share the total load in proportion to their capacities or ratings. Hence it is advantageous for the droops of all turbines to be equal.

Example 4.3

Two units of generation maintain 66 kV and 60 kV (line) at the ends of an interconnector of inductive reactance per phase of 40 Ω and with negligible resistance and shunt capacitance. A load of 10 MW is to be transferred from the 66 kV unit to the other end. Calculate the necessary conditions between the two ends, including the power factor of the current transmitted.

Solution

$$\Delta V_q = \frac{XP - RQ}{V_R} = \frac{40 \times 3.33 \times 10^6}{60\,000/\sqrt{3}} = 3840\text{V}$$

Also,

$$\frac{\Delta V_q}{66\,000/\sqrt{3}} = \sin\delta = 0.101$$

$$\therefore \delta = 5^\circ\ 44'$$

Hence the 66 kV busbars are $5^\circ\ 44'$ in advance of the 60 kV busbars.

$$\Delta V_p = \frac{66\,000 - 60\,000}{\sqrt{3}} = \frac{RP + XQ}{V_R} = \frac{40Q}{60\,000/\sqrt{3}}$$

Hence

$$Q = 3 \text{ MVAr per phase (9 MVAr total)}$$

The p.f. angle $\phi = \tan^{-1} Q/P = 42^\circ$ and hence the p.f. $= 0.74$.

4.5 The Power–Frequency Characteristic of an Interconnected System

The change in power for a given change in the frequency in an interconnected system is known as the *stiffness* of the system. The smaller the change in frequency for a given load change the stiffer the system. The power–frequency characteristic may be approximated by a straight line and $\Delta P/\Delta f = K$, where

K is a constant (MW per Hz) depending on the governor and load characteristics.

Let ΔP_G be the change in generation with the governors operating 'free acting' resulting from a sudden increase in load ΔP_L. The resultant out-of-balance power in the system

$$\Delta P = \Delta P_L - \Delta P_G \tag{4.4}$$

and therefore

$$K = \frac{\Delta P_L}{\Delta f} - \frac{\Delta P_G}{\Delta f} \tag{4.5}$$

$\Delta P_L/\Delta f$ measures the effect of the frequency characteristics of the load and $\Delta P_G \propto (P_T - P_G)$, where P_T is the turbine capacity connected to the network and P_G the output of the associated generators. When steady conditions are again reached, the load P_L is equal to the generated power P_G (neglecting losses): hence, $K = K_1 P_T - K_2 P_L$, where K_1 and K_2 are the power-frequency coefficients relevant to the turbines and load respectively.

Here, K can be determined experimentally by connecting two large separate systems by a single link, breaking the connection and measuring the frequency change. For the British system, tests show that $K = 0.8P_T - 0.6P_L$ and lies between 2000 and 5500 MW per Hz, i.e. a change in frequency of 0.1 Hz requires a change in the range 200–550 MW, depending on the amount of plant connected. In smaller systems the change in frequency for a reasonable load change is relatively large and rapid-response electrical governors have been introduced to improve the power-frequency characteristic.

In 1977, owing to a series of events triggered off by lightning, New York City was cut off from external supplies and the internal generation available was much less than the city load. The resulting fall in frequency with time is shown in Figure 4.9, illustrating the time–frequency characteristics of an isolated power system.

4.6 System Connected by Lines of Relatively Small Capacity

Let K_A and K_B be the respective power–frequency constants of two separate power systems A and B, as shown in Figure 4.10. Let the change in the power transferred from A to B when a change resulting in an out-of-balance power ΔP occurs in system B, be ΔP_t, where ΔP_t is positive when power is transferred from A to B. The change in frequency in system B, due to an extra load ΔP and an extra input of ΔP_t from A, is $-(\Delta P - \Delta P_t)/K_B$ (the negative sign indicates a fall in frequency). The drop in frequency in A due to the extra load

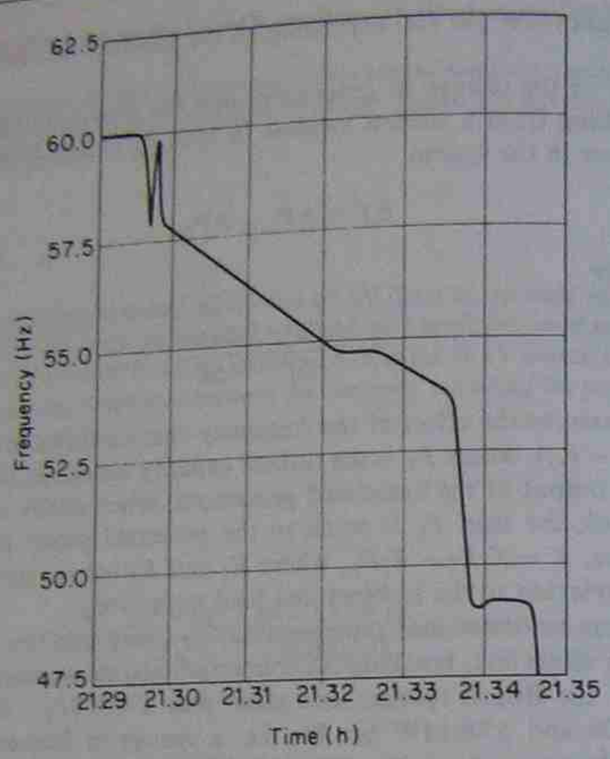


Figure 4.9 Decline of frequency with time of New York City system when isolated from external supplies (*Copyright © 1977 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 15, No. 2 (Feb. 1978) pp. 38–46*)

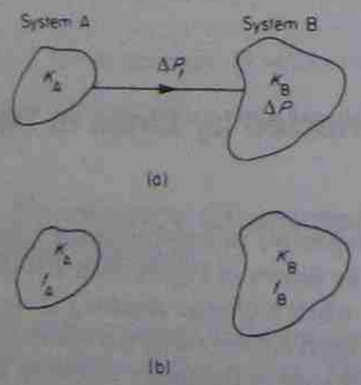


Figure 4.10 (a) Two interconnected power systems connected by a tie-line. (b) The two systems with the tie-line open

ΔP_t is $-\Delta P_t/K_A$, but the changes in frequency in each system must be equal, as they are electrically connected. Hence,

$$\frac{-(\Delta P - \Delta P_t)}{K_B} = \frac{-\Delta P_t}{K_A}$$

$$\therefore \Delta P_t = +\left(\frac{K_A}{K_A + K_B}\right)\Delta P \tag{4.6}$$

Next, consider the two systems operating at a common frequency f with A exporting ΔP_t to B. The connecting link is now opened and A is relieved of ΔP_t and assumes a freqency f_A, and B has ΔP_t more load and assumes f_B. Hence

$$f_A = f + \frac{\Delta P_t}{K_A} \quad \text{and} \quad f_B = f - \frac{\Delta P_t}{K_B}$$

from which

$$\frac{\Delta P_t}{f_A - f_B} = \frac{K_A K_B}{K_A + K_B} \tag{4.7}$$

Hence, by opening the link and measuring the resultant change in f_A and f_B the values of K_A and K_B can be obtained.

When large interconnected systems are linked electrically to others by means of tie-lines the power transfers between them are usually decided by mutual agreement and the power is controlled by regulators. As the capacity of the tie-lines is small compared with the systems, care must be taken to avoid excessive transfers of power and corresponding cascade tripping.

4.6.1 Effect of governor characteristics

A fuller treatment of the performance of two interconnected systems in the steady state requires further consideration of the control aspects of the generation process.

A more complete block diagram for the steam turbine-generator connected to a power system is shown in Figure 4.11. For this system the following equation holds:

$$Ms^2\delta + Ks\delta = \Delta P - \Delta P_L$$

where M is a constant depending on inertia (see Chapter 8); K is the stiffness or damping coefficient (i.e. change of power with speed) in MW/Hz or MW per rad/s

$$= \frac{\partial\,(\text{load power})}{\partial(s\delta)} - \frac{\partial\,(\text{turbine power})}{\partial(s\delta)}$$

where

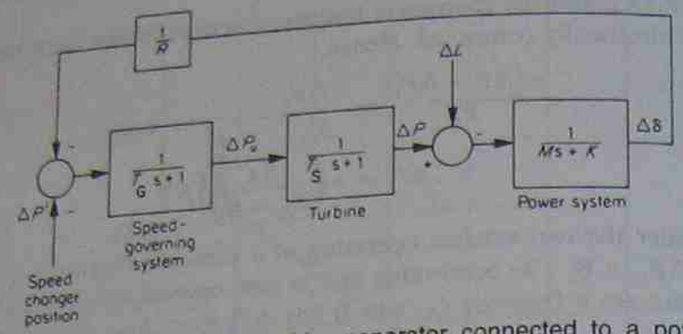


Figure 4.11 Block diagram for turbine-generator connected to a power system *(From Economic Control of Interconnected Systems, by L. K. Kirchmayer. Copyright © 1959 John Wiley & Sons Ltd. By permission of John Wiley & Sons Inc.)*

ΔP and ΔP_L = change in prime mover and load powers;

$\Delta\delta$ = change from initial angular position;

R = present speed regulation (or governor droop), i.e. drop in speed or frequency when combined machines of an area change from no load to full load, expressed as p.u. or Hz or rad/s per MW;

$\Delta P'$ = change in speed-changer setting.

Therefore, change from normal speed or frequency,

$$s\delta = \frac{1}{Ms + K}(\Delta P - \Delta P_L)$$

This analysis holds for steam-turbine generation; for hydro-turbines, the large inertia of the water must be accounted for and the analysis is more complicated.

The representation of two systems connected by a tie-line is shown in Figure 4.12. The general analysis is as before except for the additional power terms due to the tie-line. The machines in the inidividual power systems are considered to be closely coupled and to possess one equivalent rotor.

For system (1),

$$M_1 s^2\delta_1 + K_1 s\delta_1 + T_{12}(\delta_1 - \delta_2) = \Delta P_1 - \Delta P_{L1} \tag{4.8}$$

where T_{12} is the synchronizing torque coefficient of the tie-line.

For system (2),

$$M_2 s^2\delta_2 + K_2 s\delta_2 + T_{12}(\delta_2 - \delta_1) = \Delta P_2 - \Delta P_{L2} \tag{4.9}$$

The steady-state analysis of two interconnected systems may be obtained from the transfer functions given in the block diagram.

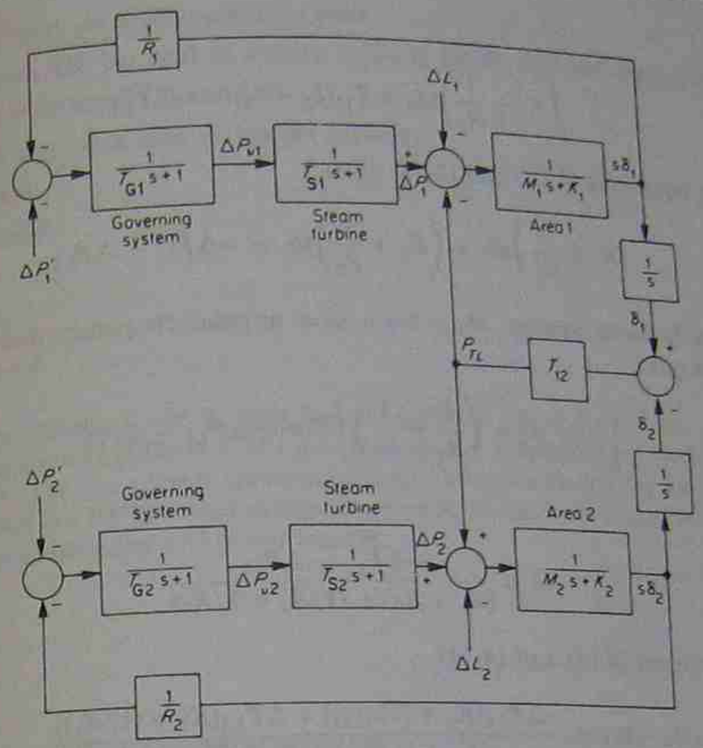


Figure 4.12 Block control diagram of two power systems connected by a tie-line *(From Economic Control of Interconnected Systems by L. K. Kirchmayer. Copyright © 1959 John Wiley & Sons Ltd. By permission of John Wiley & Sons Inc.)*

The speed governor response is given by

$$\Delta P_V = \frac{-1}{(T_G s + 1)}\left(\frac{1}{R}s\delta + \Delta P'\right) \tag{4.10}$$

In the steady state, from equation (4.10),

$$\Delta P_1 = \frac{1}{R_1}s\delta_1 \tag{4.11}$$

and

$$\Delta P_2 = -\frac{1}{R_2}s\delta_2 \tag{4.12}$$

Similarly, from equations (4.8) and (4.9), in the steady state,

$$\left(K_1 + \frac{1}{R_1}\right)s\delta_1 + T_{12}(\delta_1 - \delta_2) = -\Delta P_{L1} \tag{4.13}$$

and

$$\left(K_2+\frac{1}{R_2}\right)s\delta_2+T_{12}(\delta_2-\delta_1)=-\Delta P_{L2} \tag{4.14}$$

Adding equations (4.13) and (4.14) gives

$$\left(K_1+\frac{1}{R_1}\right)s\delta_1+\left(K_2+\frac{1}{R_2}\right)s\delta_2=-\Delta P_{L1}-\Delta P_{L2} \tag{4.15}$$

In a synchronous system, $s\delta_1=s\delta_2=s\delta=$ angular frequency and equation (4.15) becomes

$$\left[K_1+K_2+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]s\delta=-\Delta P_{L1}-\Delta P_{L2}$$

and

$$s\delta=\frac{-\Delta P_{L1}-\Delta P_{L2}}{(K_1+K_2)+(1/R_1+1/R_2)} \tag{4.16}$$

From equations (4.13) and (4.14),

$$T_{12}(\delta_1-\delta_2)=\frac{-\Delta P_{L1}[K_2+(1/R_2)]+\Delta P_{L2}[K_1+(1/R_1)]}{[K_2+(1/R_2)+K_1+(1/R_1)]} \tag{4.17}$$

It is normally required to keep the system frequency constant and to maintain the interchange through the tie-line at its scheduled value. To achieve this, additional controls are necessary to operate the speed-changer settings, as follows.

For area (1),

$$\begin{aligned}s\Delta P_1' &\propto \gamma_{1t}T_{12}(\delta_1-\delta_2)+\gamma_{1f}s\delta_1\\ &\propto \gamma_{1t}\left[T_{12}(\delta_1-\delta_2)+\frac{\gamma_{1f}}{\gamma_{1t}}s\delta_1\right]\end{aligned} \tag{4.18}$$

Similarly, for area (2),

$$s\Delta P_2' \propto \gamma_{2t}\left[T_{12}(\delta_2-\delta_1)+\frac{\gamma_{2f}}{\gamma_{2t}}s\delta_2\right] \tag{4.19}$$

where γ_t and γ_f refer to the control constants for power transfer and frequency, respectively. When a load change occurs in a given area the changes in tie-line power and frequency have opposite signs, i.e. the frequency falls for a load increase and the power transfer increases, and vice versa. In the interconnected area, however, the changes have the same sign. Typical system parameters have the following orders of magnitude:

K = 0.75 p.u. on system-capacity base

T_{12} = 0.1p.u. (10 per cent of system capacity results in 1 rad displacement between areas (1) and (2))

R = 0.04p.u. on a base of system capacity

$\gamma_{1f}=0.005$

$\gamma_{1t}=0.0009$

Example 4.4

Two power systems, A and B, each have a regulation (R) of 0.1 p.u. (on respective capacity bases) and a stiffness K of 1 p.u. The capacity of system A is 1500 MW and of B 1000 MW. The two systems are interconnected through a tie-line and are initially at 60 Hz. If there is a 100 MW load change in system A, calculate the change in the steady-state values of frequency and power transfer.

Solution

$$K_A=1\times 1500\text{ MW per Hz}$$

$$K_B=1\times 1000\text{ MW per HZ}$$

$$R_A=\frac{\Delta f\text{(no load to full load)}}{\text{full load capacity}}=\frac{0.1\times 60}{1500}$$

$$=6/1500\text{ Hz per MW}$$

$$R_B=(6/1000)\text{ Hz per MW}$$

From equation (4.16),

$$\Delta f=\frac{-\Delta P_1}{(K_1+1/R_1)+(K_2+1/R_2)}$$

$$=\frac{-100}{1500+\dfrac{1500}{6}+1000+\dfrac{1000}{6}}$$

$$=\frac{-600}{17500}=-0.034\text{ Hz}$$

$$P_{12}=T_{12}(\delta_1-\delta_2)=\frac{-\Delta P_1(K_2+1)/R_2)}{(K_1+1/R_1)+(K_2+1/R_2)}$$

$$=\frac{-100\left(\dfrac{7000}{6}\right)}{10500/6}=\frac{-7000}{105}=-6\text{ MW}$$

Note that without the participation of governor control.

$$P_{12} = \left(\frac{-K_B}{K_A + K_B}\right) \qquad \Delta P = \frac{1000}{2500} \times 100 = -40\,\text{MW}$$

4.6.2 Frequency-bias–tie-line control

Consider three interconnected power systems A, B and C, as shown in Figure 4.13, the systems being of similar size. Assume that initially A and B export to C, their previously agreed power transfers. If C has an increase in load the overall frequency tends to decrease and hence the generation in A, B, and C increases. This results in increased power transfers from A and B to C. These transfers, however, are limited by the tie-line power controller to the previously agreed values and therefore instructions are given for A and B to reduce generation and hence C is not helped. This is a severe drawback of what is known as straight tie-line control, which can be overcome if the systems are controlled by using consideration of both load transfer and frequency, such that the following equation holds:

$$\sum \Delta P + K\Delta f = 0 \tag{4.20}$$

where $\sum \Delta P$ is the net transfer error and depends on the size of the system and the governor characteristic, and Δf is the frequency error and is positive for high frequency. In the case above, after the load change in C, the frequency error is negative (i.e. low frequency) for A and B and the sum of ΔP for the lines AC and BC is positive. For correct control,

$$\sum \Delta P_A + K_A \Delta f = \sum \Delta P_B + K_B \Delta f = 0$$

Systems A and B take no regulating action despite their fall in frequency. In C, $\sum \Delta P_C$ is negative as it is importing from A and B and therefore the governor speeder motors in C operate to increase ouput and restore frequency. This system is known as frequency-bias–tie-line control and is often implemented automatically in interconnected systems.

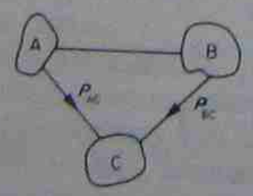


Figure 4.13 Three power systems connected by tie-lines

4.7 Economic Power-System Operation

4.7.1 Introduction

The extensive interconnection of power sources has made the operation of a system in the most economical manner a complex subject (see also Chapter 12). Economy must be balanced against considerations such as security of supply. The use of the *merit order* ensures that as far as possible the most economical generating sets are used. A knowledge of the flows of real and reactive power and other parameters in the network, and effective means of dealing with the analysis of large systems, is required for the operation to attain an economic optimum, although experienced operators certainly approach this aim.

Nowadays, the use of digital computers for load flows and fault calculations and the development of optimization techniques in control theory have resulted in much attention being given to this topic.

Apart from financial considerations, it is becoming difficult for operators to cope with the information produced by large complex systems in times of emergency, such as with major faults. Computers with on-line facilities can more readily digest this information and take correcting action by instructing control gear and settings, or by the suitable display of relevant information to enable human operators to take appropriate action. In the attempt to obtain economic optimization the limitations of the system, such as plant ratings and stability limits, must be observed.

Optimization may be considered in a number of ways according to the time scale involved, namely: daily, yearly (especially with hydro stations), and over much longer periods when planning for future developments, although this latter is not strictly operational optimization. In an existing system the various factors involved are the fixed and variable costs. The former includes labour, administration, interest and depreciation, etc., and the latter, mainly fuel. A major problem is the effective prediction of the future load whether it occurs in 10 minutes, a few hours, or in several year' time.

For operational planning, daily operation, and the setting of economic schedules, the following data is normally required.

- *For each generator:*
  1. maximum and economic output capacities;
  2. fixed and incremental heat rates;
  3. minimum shut-down time;
  4. minimum stable output, maximum run-up and run-down rates.

- *For each station:*
  1. cost and calorific value of fuel (thermal stations);
  2. factors reflecting recent operational performance of the station;
  3. minimum time between loading and unloading successive generators;
  4. any constraints on station output.

- *For the system:*
  1. load demand at given intervals for the specified period;
  2. specified constraints imposed by transmission capability;
  3. running-spare requirements;
  4. transmission circuit parameters, including maximum capacities and reliability factors.

The input–output characteristic of a turbine is of great importance when economical operation is considered. A typical characteristic is shown in Figure 4.14. The *incremental heat rate* is defined as the slope of the input–output curve at any given output. The graph of the incremental heat rate against output is known as the Willans line. For large turbines with a single valve, and for gas turbines, the incremental heat rate is approximately constant over the operating range (most steam turbines in Britain are of this type); with multivalve turbines (as used in the U.S.A.) the Willans line is not horizontal but curves upwards and is often represented by the closest linear law. The value taken for the incremental heat rate of a generating set is sometimes complicated because if only one or two shifts are being operated (there are normally three shifts per day) heat has to be expended in banking boilers when the generator is not required to produce output.

Instead of plotting incremental heat rate or fuel consumption against power output for the turbine-generator, the incremental fuel cost may be used. This is

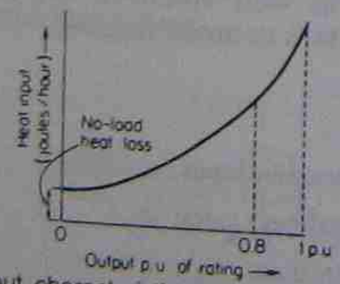


Figure 4.14 Input–output characteristic of a turbogenerator set. Often the curve above an output of 0.8 p.u. is steeper than the remainder and the machine has a maximum rating at 1 p.u. and a maximum economic rating at, say 0.8 p.u.

advantageous when allocating load to generators for optimum economy as it incorporates differences in the fuel costs of the various generating stations. Usually, the graph of incremental fuel cost against power output can be approximated by a straight line (Figure 4.15). Consider two turbine-generator sets having the following different incremental fuel costs, dC_1/dP_1 and dC_2/dP_2, where C_1 is the cost of the fuel input to unit number 1 for a power output of P_1, and, similarly, C_2 and P_2 relate to unit number 2. It is required to load the generators to meet a given requirement in the most economical manner. Obviously the load on the machine with the higher dC/dP will be reduced by increasing the load taken by the machine with the lower dC/dP. This transfer will be beneficial until the values of dC/dP for both sets are equal, after which the machine with the previously higher dC/dP now becomes the one with the lower value, and vice versa. There is no economic advantage in further transfer of load, and the condition when $dC_1/dP_1 = dC_2/dP_2$ therefore gives optimum economy; this can be seen by considering Figure 4.15. The above argument can be extended to several machines supplying a load. Generally, for optimum economy the *incremental fuel cost should be identical for all contributing turbine-generator sets* on free-governor action. In practice, most generators will be loaded to their maximum output.

The above reasoning must be modified when the distances of generating stations from the common loads are different; here the cost of transmission losses will affect the argument.

As important as the transmission losses is the optimum method of transporting fuel from the production centres to the generating stations. The transport of both electrical energy and fuel in the optimum manner forms *transportation problems*, which may be dealt with by special techniques or by the general method of linear programming. In a competitive situation the transportation costs will be included in the generator bid price.

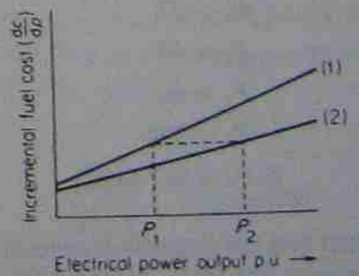


Figure 4.15 Idealized graphs of incremental fuel cost against output for two machines sharing a load equal to P_1 and P_2

Example 4.5

Four generators are available to supply a power system peak load of 472.5 MW. The cost of power $C(P_i)$ from each generator, and maximum output, is given in ($ U.S.) by:

$$C(P_1) = 200 + 15P_1 + 0.20P_1^2 \qquad \text{Max. output 100 MW}$$
$$C(P_2) = 300 + 17P_2 + 0.10P_2^2 \qquad \text{Max. output 120 MW}$$
$$C(P_3) = 150 + 12P_3 + 0.15P_3^2 \qquad \text{Max.output 160 MW}$$
$$C(P_4) = 500 + 2P_4 + 0.07P_4^2 \qquad \text{Max. output 200 MW}$$

The spinning reserve is to be 10 per cent of peak load and the transmission losses can be neglected.

Calculate the optimal loading of each generator and the cost of operating the system for 1 h at peak.

Solution

The generators' combined output is to be 472.5 MW.

Marginal costs are given (in U.S.$/MW) by:

$$\frac{dC(P_1)}{dP_1} = 15 + 0.40P_1 \qquad P_1 \leqslant 100\,\text{MW}$$
$$\frac{dC(P_2)}{dP_2} = 17 + 0.20P_2 \qquad P_2 \leqslant 120\,\text{MW}$$
$$\frac{dC(P_3)}{dP_3} = 12 + 0.30P_3 \qquad P_3 \leqslant 160\,\text{MW}$$
$$\frac{dC(P_4)}{dP_4} = 2 + 0.14P_4 \qquad P_4 \leqslant 200\,\text{MW}$$

The marginal cost curves are plotted in Figure 4.16 for eachof the generators up to their maximum output.

From the curves, at 30 $/MW the outputs are

$$P_1 = 23$$
$$P_2 = 64$$
$$P_3 = 60$$
$$P_4 = 200$$
$$\text{Total} = 347\,\text{MW}$$

Hence, P_4 runs at full output and the remaining generators must sum to 272.5 MW. At 40 $/MW the outputs sum to 446 MW. At 41 $/MW P_2 reaches its maximum of 120 MW, and $P_1 = 40$ MW and $P_3 = 98$ MW (total 458 MW).

This leaves 14.5 MW to be found from P_1 and P_3.

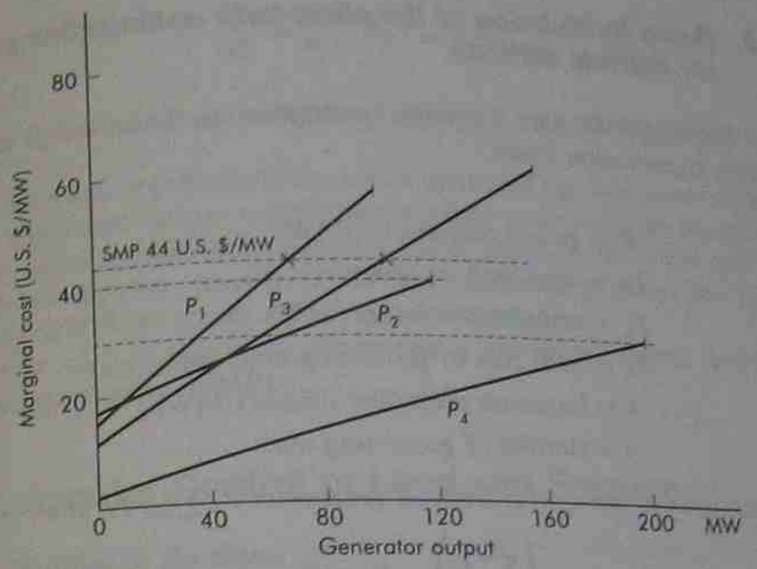


Figure 4.16 Marginal cost curves for four generators

Adjusting the marginal cost line now to 44 $/MW provides 44 MW from P_1 and 108 MW from P_3, giving a total of 472 MW, which is near enough when using this graphical method. The spinning reserve on these generators is 108 MW, more than enough to cover 10 per cent of 472.5 MW.

The cost of operating the system for 1 h at peak is

$$C(P_1) = 200 + 15 \times 44 + 0.2 \times 44^2 = \$1247$$

Similarly

$$C(P_2) = \$3780$$
$$C(P_3) = \$3195$$
$$C(P_4) = \$3700$$
$$\text{Total} = \$11922$$

i.e.

$$\frac{11922}{472.5} = 25.2\,\$/\text{MW}$$

(Note: This is the *average* cost per hour whereas the *marginal cost* was calculated as 44 $/MW. It can be seen that the use of large units on base load (e.g. the 200 MW generator) reduces system marginal price (SMP) considerably (see Chapter 12).)

4.7.2 Basic formulation of the short-term optimization problem for thermal stations

Kirchamayer (1958) uses Lagrange multipliers in formulating equations, including transmission losses.

Let

P_i = power output of i (MW)
P_R = total load on system (MW)
P_L = transmission losses (MW)
F_T = total cost of generating units (money/h)
λ = Lagrange multiplier ((money/MWh)
n = number of generating units

The total input to the system from all generators $P_T \sum_{i=1}^{n} P_i$, and

$$\left(\sum_{1}^{n} P_i\right) - P_L - P_R = 0$$

Using Lagrangian multipliers, the expression

$$\gamma = F_T - \lambda\left(\sum_{1}^{n} P_i - P_L - P_R\right)$$

is formulated, where for minimum cost (F_T), $\partial\gamma/\partial P_i = 0$ for all values of i. (Note the use of partial differentiation here.) This is given by

$$\frac{\partial\gamma}{\partial P_i} = \frac{dFi}{dP_i} - \lambda + \lambda\frac{\partial P_L}{\partial P_i} = 0$$

since P_R is assumed fixed. Hence

$$\frac{dF_i}{dP_i} + \lambda\frac{\partial P_L}{\partial P_i} = \lambda \tag{4.21}$$

In equation (4.21), $\partial P_L/\partial P_i$ is the incremental transmission loss. One way of solving the equations described by (4.21) is known as the penalty-factor method in which equation (4.21) is rewritten as

$$\frac{dF_i}{dP_i}L_i = \lambda \tag{4.22}$$

where

$$L_i = \left(\frac{1}{1 - \partial P_L/\partial P_i}\right) = \text{penalty factor of plant } i$$

where $i = 1 \ldots n$ (number of plants). In practice the determination of $\partial P_L/\partial P_i$ is difficult and the use of the so-called loss or 'B' coefficients is made, i.e.

$$\frac{\partial P_L}{\partial P_i} = \sum_{i} 2B_{mi}P_m + B_{i0}$$

where the B coefficients are determined from the network (see Kirchmayer, 1958).

There are many drawbacks to the above treatment e.g. limitations on power flows by equipment ratings transformer settings, and maximum phase angles allowable across transmission lines on stability grounds. Also, it is concerned only with active power, reactive power being neglected or taken into account by limiting MW flows across defined group boundaries.

The value of λ at any particular period is known as 'system lambda' or 'system marginal price' (SMP).

4.8 Computer Control of Load and Frequency

4.8.1 Control of tie lines

Automatic control of area power systems connected by tie-lines has already been discussed. The methods used will now be extended to include optimum economy as well as power transfer and frequency control. The basic systems described are typical of U.S. and European practice and have been comprehensively discussed by many authors. In the previous section, methods for economic analysis and optimization as developed by Kirchmeyer (1958) have been summarized. The choice of generating units to be operated is largely decided by spinning reserve, voltage control, stability, and protection requirments. The methods discussed decide the allocation of load to particular machines.

If transmission losses are to be neglected, it has been shown that optimum economy results when $dF_i/dP_i = \lambda$. Control equipment to adjust the governor speed-change settings such that all units comply with the appropriate value of dF_i/dP_i is incorporated in the control loops for frequency and power-transfer adjustment, as shown in Figure 4.17. The frequency and load-transfer control acts quickly, and once these quantities have been decided the slower acting economic controls then act. For example, if an increase in load occurs in the controlled area, a signal requiring increased generation transmits through the control system. These changes alter the value of λ and cause the economic control apparatus to call for generation to be operated at the same incremental cost. Eventually the system is again in the steady state, the load change having been absorbed, and all units operate at an identical value of incremental loss.

If transmission losses are included, the basic economic requirement calls for $dF_i/dP_i = \lambda/L_i$ where L_i is the penalty factor; $(1/L_i)$ signals are generated by a computer from a knowledge of system parameters in the form of the so-called B coefficients. However, in practice, most generators are being run at their

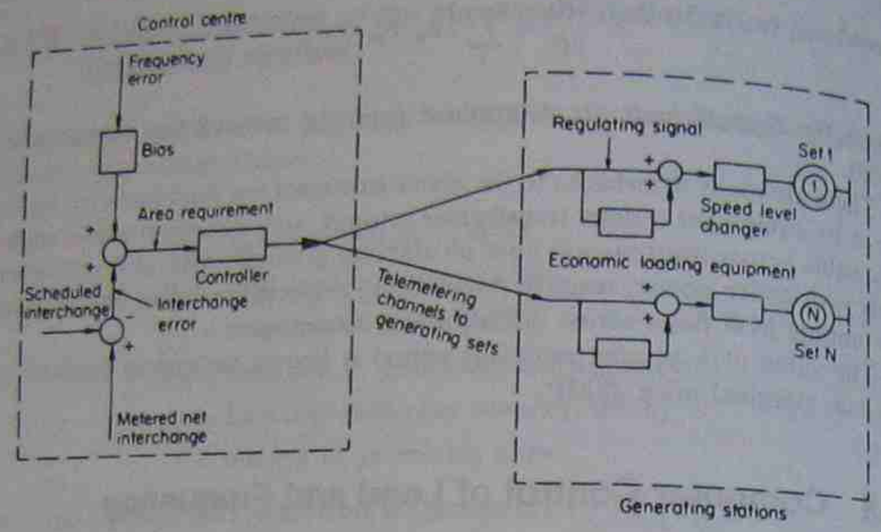


Figure 4.17 Schematic diagram of automatic control arrangements covering frequency, tie-line power flows, and economic loading of generating sets (*From Economic Control of Interconnected Systems by L. K. Kirchmayer. Copyright © 1959 John Wiley & Sons Ltd. By permission of John Wiley & Sons Inc.*)

maximum rated output because their cost is below the system λ. Only a few units on the cost margin are setting the value of λ.

In the U.S.A. the many separate power companies are connected into power pools. The aim is optimal control of the participating machines in each pool, within security constraints. Figure 4.18 shows a block diagram of the New York Power Pool. Among the advantages of such pools are; the more economic use of large generators, emergency assistance to neighbouring utilities, reduced spinning reserves, and lower overall generation costs. Each pool is connected to others by tie-lines, e.g. the New York Pool to Ontario Hydro, New England Power Exchange and the Pennsylvania–New Jersey–Maryland interconnection. Control of the generators in the pool is achieved either centrally or from each of the constituent areas (i.e. area control centres) as indicated in Figure 4.19

The central control mechanisms for pools are basically the same as for areas and the systems of Figure 4.17 may be used with 'pool' substituted for 'area'. Allocation between generators in an area, or between areas in a pool, may be accomplished by the use of base points or loadings and participation factors. The former gives the economic allocation for a specified total generation and they are normally established every few minutes or when loadings change. When the generation allocations are established they are compared with the actual values being generated and a control error formed. The unit participation factor (K_n) for any unit (n) in the pool is given by

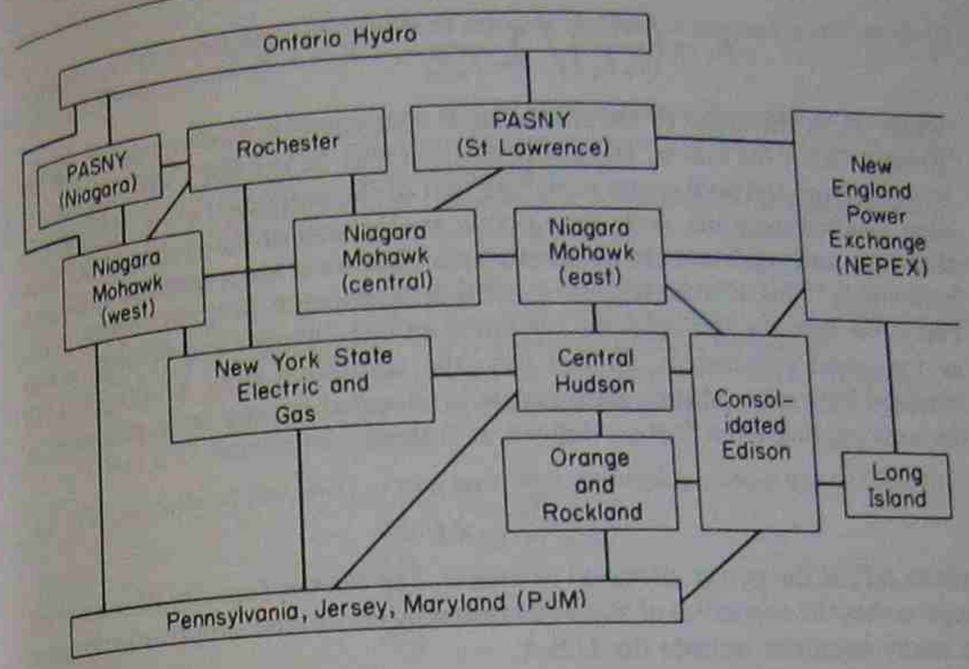


Figure 4.18 Block diagram showing the eight power companies within the New York Power Pool, and three other pools that are interconnected with it (*Copyright © 1973 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 10, No. 3, March 1973, pp. 54–61*)

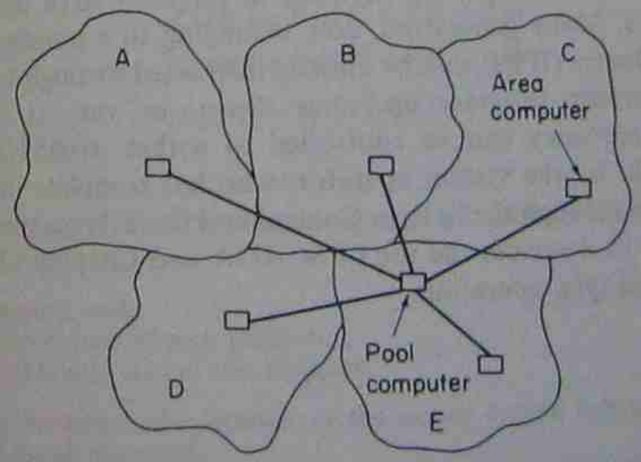


Figure 4.19 Multicomputer configuration formed by a five-area pool (*Copyright © 1973 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 10, No. 3, March 1973, pp. 54–61*)

$$K_n = \left(\frac{1}{F_n L_n}\right) \Big/ \sum \frac{1}{F_n L_n} \qquad n = 1, 2, 3, \ldots$$

where F_n is the slope of the incremental cost curve for unit n, and L_n is the penalty factor for unit n. The above method may be extended with the control of areas from the pool centre replacing that of the individual units; the areas, in turn, control the units. Area base points are the sum of the unit base points in that area and represent the economic area allocation for a specified total pool generation. This is achieved by a multi-area dispatch program which is run every few minutes and accounts for limits on interchanges and tie-lines, as well as the usual parameters. In this case the additional pool generation (ΔG) required between updating base points is allocated among areas according to the area participation factors defined as follows:

$$K_a = \frac{\Delta P_a}{\Delta G}$$

where ΔP_a is the power allocated to area a. The electric capacity of some pools approaches the capacities of state-run centrally controlled supply organizations in many countries outside the U.S.A.

In the U.K., following privatization in 1989/90, the England and Wales transmission network is run on a pool basis into which generators (including contributions from Scotland and France) bid a price for their electrical energy for the following day. The pool then computes an economic schedule of generation from a load forecast for the following day, predicting the system λ price for each half-hour if the schedule is adhered to as planned. Although some producers may be required to provide a variable output to cover reserves and contingencies, consumers can vary their requirements depending on the SMP as displayed over an information network to (mainly) large users and other interested parties. Many generators, now belonging to a number of independent power producers (IPPs), can be directly instructed through tele-control by the system operator to start up, shut down, or run at a part load. Consequently, frequency can be controlled to within ± 0.05 Hz throughout the day and night by the system operator, who has complete information at the control centre through the System Control and Data Acquisition (SCADA) system. Chapter 11 describes the use of SCADA and Chapter 12 outlines the economics of pool-type operation.

Problems

4.1 Two identical 60 MW synchronous generators operate in parallel. The governor settings on the machines are such that they have 4 per cent and 3 per cent droops (no-load to full-load percentage speed drop). Determine (a) the load taken by each machine for a total of 100 MW; (b) the percentage adjustment in the no-load speed to be made by the speeder motor if the machines are to share the load equally.

(Answer: (a) 42.8 and 57.2 MW; (b) 0.83 per cent increase in no-load speed on the 4 per cent droop machine)

4.2 (a) Explain how the output power of a turbine-generator operating in a constant frequency system is controlled by adjusting the setting of the governor. Show the effect on the generator power–frequency curve.

(b) Generator A of rating 200 MW and generator B of rating 350 MW have governor droops of 5 per cent and 8 per cent, respectively, from no-load to full-load. They are the only supply to an isolated system whose nominal frequency is 50 Hz. The corresponding generator speed is 3000 r.p.m. Initially, generator A is at 0.5 p.u. load and generator B is at 0.65 p.u. load, both running at 50 Hz. Find the no-load speed of each generator if it is disconnected from the system.

(c) Also determine the total output when the first generator reaches its rating.

(Answer: (b) Generator B 3156 r.p.m; generator A 3075 r.p.m; (c) 337 MW)

(From E C Examination, 1996)

4.3 The incremental fuel costs of two units in a generating station are as follows:

$$\frac{dF_1}{dP_1} = 0.003P_1 + 0.7$$

$$\frac{dF_2}{dP_2} = 0.004P_2 + 0.5$$

where F is in £/h and P is in MW.

Assuming continuous running with a total load of 150 MW calculate the saving per hour obtained by using the most economical division of load between the units as compared with loading each equally. The maximum and minimum operational loadings are the same for each unit and are 125 MW and 20 MW.

(Answer: $P_1 = 57$ MW, $P_2 = 93$ MW; saving £1.14 per hour)

4.4 What is the merit order used for when applied to generator scheduling?

A power system is supplied by three generators. The functions relating the cost (in £/h) to active power output (in MW) when operating each of these units are:

$$C_1(P_1) = 0.04P_1^2 + 2P_1 + 250$$

$$C_2(P_2) = 0.02P_2^2 + 3P_2 + 450$$

$$C_3(P_3) = 0.01P_3^2 + 5P_3 + 250$$

The system load is 525 MW. Assuming that all generators operate at the same marginal cost, calculate:

(a) the marginal cost;
(b) optimum output of each generator;
(c) the total hourly cost of this dispatch.

How is the economy of operation of the power system balanced against security requirements for all demands?

(Answer: (a) £10/MWh; (b) P_1 100 MW, P_2, 175 MW, P_3, 250 MW; (c) £4562.5/h)

(From E.C. Examination, 1997)

4.5 Two power systems A and B are interconnected by a tie-line and have power-frequency constants K_A and K_B MW/Hz. An increase in load of 500 MW on system A

causes a power transfer of 300 MW from B to A. When the tie-line is open the frequency of system A is 49 Hz and of system B 50 Hz. Determine the values of K_A and K_B, deriving any formulae used.
(Answer: K_A 500 MW/Hz; K_B 750 MW/Hz)

4.6 Two power systems, A and B, having capacities of 3000 and 2000 MW, respectively, are interconnected through a tie-line and both operate with frequency-bias–tie-line control. The frequency bias for each area is 1 per cent of the system capacity per 0.1 Hz frequency deviation. If the tie-line interchange for A is set at 100 MW and for B is set (incorrectly) at 200 MW, calculate the steady-state change in frequency.
(Answer: 0.6 Hz; use $\Delta P_A + \sigma_A \Delta f = \Delta P_B + \sigma_B \Delta f$)

4.7 (a) (i) Why do power systems operate in an interconnected arrangement?
(ii) How is the frequency controlled in a power system?
(iii) What is meant by the *stiffness* of a power system?

(b) Two 50 Hz power systems are interconnected by a tie-line, which carries 1000 MW from system A to system B, as shown in Figure 4.20. After the outage of the line shown in the figure, the frequency in system A increases to 50.5 Hz, while the frequency in system B decreases to 49 Hz.

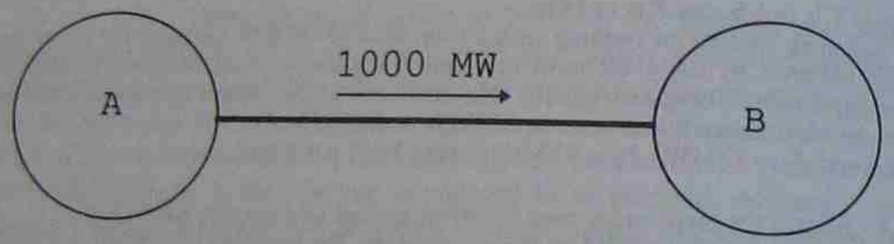


Figure 4.20 Interconnected systems of Problem 4.7 (b)

(i) Calculate the stiffness of each system.
(ii) If the systems operate interconnected with 1000 MW being transferred from A to B, calculate the flow in the line after outage of a 600 MW generator in system B.
(Answer: (b) (i) K_A 2000 MW/Hz, K_B 1000 MW/Hz; (ii) 1400 MW)
(From, E.C. Examination, 1997)

5
Control of Voltage and Reactive Power

5.1 Introduction

The approximate relationship between the scalar voltage difference between two nodes in a network and the flow of reactive power was shown in Chapter 2 to be

$$\Delta V = \frac{RP + XQ}{V} \tag{2.9}$$

Also it was shown that the transmission angle δ is proportional to

$$\frac{XP - RQ}{V} \tag{2.10}$$

For networks where $X \gg R$, i.e. most power circuits, ΔV, the voltage difference, determines Q. Consider the simple interconnector linking two generating stations A and B, as shown in Figure 5.1(a). The machine at A is in phase advance of that at B and V_1 is greater than V_2; hence there is a flow of power and reactive power from A to B. This can be seen from the phasor diagram shown in Figure 5.1(b). It is seen that I_d and hence P is determined by δ and the value of I_q, and hence Q mainly, by $V_1 - V_2$. In this case $V_1 > V_2$ and reactive power is transferred from A to B; by varying the generator excitations such that $V_2 > V_1$, the direction of the reactive power is reversed, as shown in Figure 5.1(c). Hence, power can be sent from A to B or B to A by suitably adjusting the amount of steam (or water) admitted to the turbine, and reactive power can be sent in either direction by adjusting the voltage magnitudes. These two operations are approximately independent of each other if $X \gg R$, and the flow of reactive power can be studied almost independently of the power flow. The phasor diagrams show that if a scalar voltage difference exists across a largely reactive link, the reactive power flows towards the node of lower voltage. From another point of view, if, in a network, there is

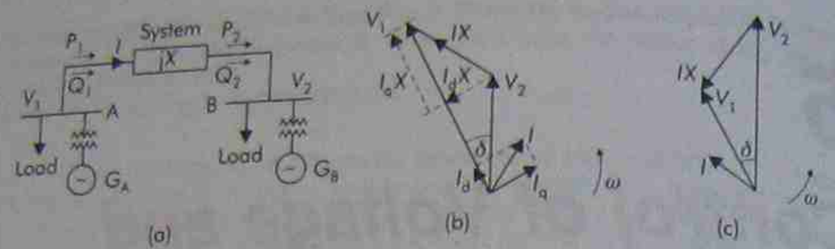


Figure 5.1 (a) System of two generators interconnected. (b) Phasor diagram when $V_1 > V_2$. I_d and I_q are components of I. (c) Phasor diagram when $V_2 > V_1$

a deficiency of reactive power at a point, this has to be supplied from the connecting lines and hence the voltage at that point falls. Conversely if there is a surplus of reactive power generated (e.g. lightly loaded cables generate positive vars), then the voltage will rise. This is a convenient way of expressing the effect of the power factor of the transferred current, and although it may seem unfamiliar initially, the ability to think in terms of var flows, instead of exclusively with power factors and phasor diagrams, will make the study of power networks much easier.

If it can be arranged that Q_2 in the system in Figure 5.1(a) is zero, then there will be no voltage drop between A and B, a very satisfactory state of affairs. Assuming that V_1 is constant, consider the effect of keeping V_2, and hence the voltage drop ΔV, constant. From equation (2.9)

$$Q_2 = \frac{V_2 \cdot \Delta V - R \cdot P_2}{X} = K - \frac{R}{X} P_2 \tag{5.1}$$

where K is a constant and R is the resistance of the system.

If this value of Q_2 does not exist naturally in the circuit then it will have to be obtained by aritificial means, such as the connection at B of capacitors or inductors. If the value of the power changes from P_2 to P_2' and if V_2 remains constant, then the reactive power at B must change to Q_2' such that

$$Q_2' - Q_2 = \frac{R}{X}(P_2' - P_2)$$

i.e. an increase in power causes an increase in reactive power. The change, however, is proportional to (R/X), which is normally small. It is seen that voltage can be controlled by the injection into the network of reactive power of the correct sign. Other methods of a more obvious kind for controlling voltage are the use of tap-changing transformers and voltage boosters.

5.2 The Generation and Absorption of Reactive Power

In view of the findings in the previous section, a review of the characteristics of a power system from the viewpoint of reactive power is now appropriate.

5.2.1 Synchronous generators

These can be used to generate or absorb reactive power. The limits on the capability for this can be seen in Figure 3.14. The ability to supply reactive power is determined by the short-circuit ratio (1/synchronous reactance) as the distance between the power axis and the theoretical stability-limit line in Figure 3.14 is proportional to the short-circuit ratio. In modern machines the value of this ratio is made low for economic reasons, and hence the inherent ability to operate at leading power factors is not large. For example, a 200 MW 0.85 p.f. machine with a 10 per cent stability allowance has a capability of 45 MVAr at full power output. The var capacity can, however, be increased by the use of continuously acting voltage regulators, as explained in Chapter 3. An overexcited machine, i.e. one with greater than normal excitation, generates reactive power whilst an underexcited machine absorbs it. The generator is the main source of supply to the system of both positive and negative vars.

5.2.2 Overhead lines and transformers

When fully loaded, lines absorb reactive power. With a current I amperes for a line of reactance per phase $X(\Omega)$ the vars absorbed are I^2X per phase. On light loads the shunt capacitances of longer lines may become predominant and the lines then become var generators.

Transformers always absorb reactive power. A useful expression for the quantity may be obtained for a transformer of reactance X_T p.u. and a full-load rating of $3V \cdot I_{rated}$.

The ohmic reactance

$$= \frac{V \cdot X_T}{I_{rated}}$$

Therefore the vars absorbed

$$= 3 \cdot I^2 \cdot \frac{V \cdot X_T}{I_{rated}}$$

$$= 3 \cdot \frac{I^2 V^2}{(IV)_{rated}} \cdot X_T = \frac{(VA \text{ of load})^2}{\text{Rated } VA} \cdot X_T$$

5.2.3 Cables

Cables are generators of reactive power owing to their high shunt capacitance. A 275 kV, 240 MVA cable produces 6.25–7.5 MVAr per km; a 132 kV cable roughly 1.9 MVAr per km; and a 33kV cable, 0.125 MVAr per km.

5.2.4 Loads

A load at 0.95 power factor implies a reactive power demand of 0.33 kVAr per kW of power, which is more appreciable than the mere quoting of the power factor would suggest. In planning a network it is desirable to assess the reactive power requirements to ascertain whether the generators are able to operate at the required power factors for the extremes of load to be expected. An example of this is shown in Figure 5.2, where the reactive losses are added for each item until the generator power factor is obtained.

Example 5.1

In the radial transmission system shown in Figure 5.2, all p.u. values are referred to the voltage bases shown and 100 MVA. Determine the power factor at which the generator must operate.

Solution

Voltage drops in the circuits will be neglected and the nominal voltages assumed.

Busbar A,

$$P = 0.5\,\text{p.u.} \qquad Q = 0$$

I^2X loss in 132 kV lines and transformers

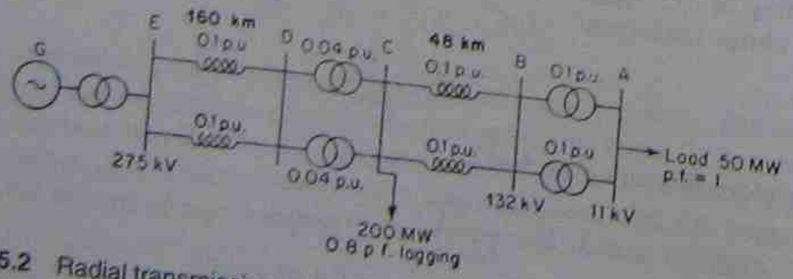


Figure 5.2 Radial transmission system with intermediate loads. Calculation of reactive-power requirement

$$= \frac{P^2 + Q^2}{V^2} X_{CA} = \frac{0.5^2}{1^2} \cdot 0.1$$
$$= 0.025\,\text{p.u.}$$

Busbar C,

$$P = 2 + 0.5 = 2.5\,\text{p.u.}$$
$$Q = 1.5 + 0.025\,\text{p.u.}$$
$$= 1.525\,\text{p.u.}$$

I^2X loss in 275 kV lines and transformers

$$= \frac{2.5^2 + 1.525^2}{1^2} 0.07$$
$$= 0.6\,\text{p.u.}$$

The I^2X loss in the large generator-transformer will be negligible, so that the generator must deliver $P = 2.5$ and $Q = 2.125$ p.u. and operate at a power factor of 0.76 lagging. It is seen in this example that, starting with the consumer load, the vars for each circuit, in turn, are added to obtain the total.

5.3 Relation Between Voltage, Power, and Reactive Power at a Node

The phase voltage V at a node is a function of P and Q at that node, i.e.

$$V = \phi(P, Q)$$

The voltage is also dependent on adjacent nodes and the present treatment assumes that these are infinite buses.

The total differential of V,

$$dV = \frac{\partial V}{\partial P} \cdot dP + \frac{\partial V}{\partial Q} \cdot dQ$$

and using

$$\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial P} = 1 \qquad \text{and} \qquad \frac{\partial Q}{\partial V} \cdot \frac{\partial V}{\partial Q} = 1$$

$$dV = \frac{dP}{(\partial P/\partial V)} + \frac{dQ}{(\partial Q/\partial V)} \tag{5.2}$$

It can be seen from equation (5.2) that the change in voltage at a node is defined by the two quantities

$$\left(\frac{\partial P}{\partial V}\right) \qquad \text{and} \qquad \left(\frac{\partial Q}{\partial V}\right).$$

If, at a particular system load, the line voltage of M falls below its nominal value by 5 kV, calculate the magnitude of the reactive volt-ampere injection required at M to re-establish the original voltage.

The p.u. values are expressed on a 500 MVA base and resistance may be neglected throughout.

Solution

The line diagram and equivalent single-phase circuit are shown in Figures 5.4 and 5.5.

It is necessary to determine the value of $\partial Q/\partial V$ at the node or busbar M; hence the current flowing into a three-phase short-circuit at M is required.

The base value of reactance in the 132 kV circuit is

$$\frac{132^2 \times 1000}{500\,000} = 35\,\Omega$$

Therefore the line reactances

$$= \frac{j50}{35} = j1.43 \text{ p.u.}$$

The equivalent reactance from M to N = $j0.5$ p.u.

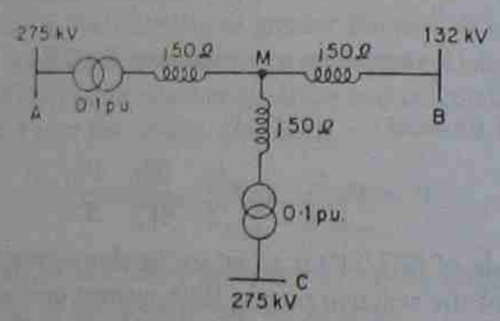


Figure 5.4 Schematic diagram of the system for Example 5.2

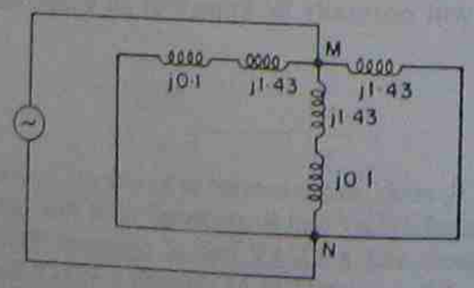


Figure 5.5 Equivalent single-phase network with the node M short-circuited to neutral (refer to Chapter 7 for full explanation of the derivation of this circuit)

Hence the fault MVA at M

$$= \frac{500}{0.5} = 1000 \text{ MVA}$$

and the fault current

$$= \frac{1000 \times 10^6}{\sqrt{(3)} \times 132\,000}$$
$$= 4380 \text{ A} \qquad \text{at zero power factor lagging}$$

It has been shown that $\partial Q_M/\sqrt{(3)}\partial V_M$ = three-phase short-circuit current when Q_M and V_M are three-phase and line values

$$\therefore \frac{\partial Q_M}{\partial V_M} = 7.6 \text{ MVAr/kV}$$

The natural voltage drop at M = 5 kV. Therefore the value of the injected vars required, ΔQ_M, to offset this drop

$$= 7.6 \times 5 = 38 \text{ MVAr}$$

5.4 Methods of Voltage Control: (i) Injection of Reactive Power

The background to this method has been given in the previous sections. This is the most fundamental method, but in transmission systems it lacks the flexibility and economy of transformer tap-changing. Hence it is only used in schemes when transformers alone will not suffice. The provision of static capacitors to improve the power factors of factory loads has been long established. The capacitance required for the power-factor improvement of loads for optimum economy is determined as follows.

Let the tariff of a consumer be

$$£A \times \text{kVA} + B \times \text{kWh}$$

A load of P_1 kilowatts at power factor ϕ_1 lagging has a kVA of $P_1/\cos\phi_1$. If this power factor is improved to $\cos\phi_2$, the new kVA is $P_1/\cos\phi_2$. The saving is therefore

$$£P_1 A\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi_2}\right)$$

The reactive power required from the correcting capacitors

$$= (P_1 \tan\phi_1 - P_1 \tan\phi_2)\text{kVAr}$$

Let the cost per annum in interest and depreciation on the capacitor installation be £C per kVAr or

$$£C(P_1 \tan\phi_1 - P_1 \tan\phi_2)$$

The net saving

$$= £\left[AP_1\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi_2}\right) - CP_1(\tan\phi_1 - \tan\phi_2)\right]$$

This saving is a maximum when

$$\frac{d(\text{saving})}{d\phi_2} = 0$$

i.e. when $\sin\phi_2 = C/A$.

It is interesting to note that the optimum power factor is independent of the original one. The improvement of load power factors in such a manner will help to alleviate the whole problem of var flow in the transmission system.

The main effect of transmitting power at non-unity power factors is as follows. It is evident from equation (2.9) that the voltage drop is largely determined by the reactive power (Q). The line currents are larger, giving increased I^2R losses and hence reduced thermal capability. One of the obvious places for the artificial injection of reactive power is at the loads themselves. In general, four methods of injection are available, involving the use of:

1. static shunt capacitors;
2. static series capacitors;
3. synchronous compensators;
4. current compensation by series injection.

5.4.1 Shunt capacitors and reactors

Shunt capacitors are used for lagging power-factor circuits, whereas reactors are used on those with leading power factors such as created by lightly loaded cables. In both cases the effect is to supply the requisite reactive power to maintain the values of the voltage. Capacitors are connected either directly to a busbar or to the tertiary winding of a main transformer and are disposed along the route to minimize the losses and voltage drops. Unfortunately, as the voltage falls, the vars produced by a shunt capacitor or reactor fall; thus, when needed most, their effectiveness falls. Also, on light loads when the voltage is high, the capacitor output is large and the voltage tends to rise to excessive levels, requiring some capacitors or cable circuits to be switched out by local overvoltage relays. Switched capacitors in parallel with semiconductor-controlled reactors provides modern variable var compensators for fast control of voltages in power systems.

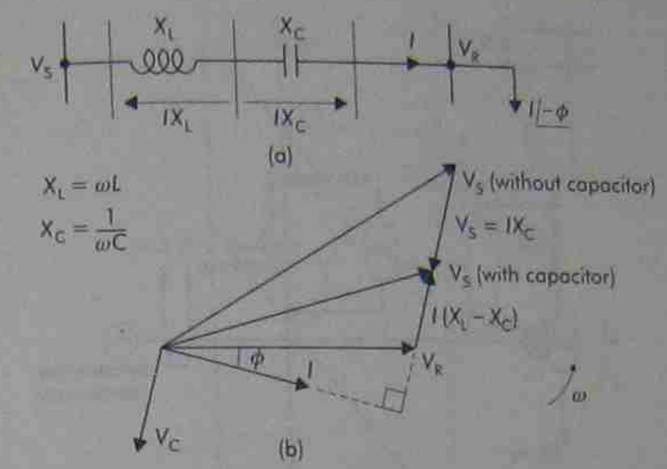


Figure 5.6 (a) Line with series capacitor, C load. (b) Phasor diagram for fixed V_R

5.4.2 Series capacitors

These are connected in series with the line conductors and are used to reduce the inductive reactance between the supply point and the load. One major drawback is the high overvoltage produced when a short-circuit current flows through the capacitor, and special protective devices need to be incorporated (e.g. spark gaps) and non-linear resistors. The phasor diagram for a line with a series capacitor is shown in Figure 5.6(b).

The relative merits between shunt and series capacitors may be summarized as follows:

1. If the load var requirement is small, series capacitors are of little use.
2. With series capacitors the reduction on line current is small; hence if thermal considerations limit the current, little advantage is obtained and shunt compensation should be used
3. If voltage drop is the limiting factor, series capacitors are effective; also, voltage fluctuations due to arc furnaces, etc., are evened out.
4. If the total line reactance is high, series capacitors are very effective and stability is improved.

5.4.3 Synchronous compensators

A synchronous compensator is a synchronous motor running without a mechanical load and, depending on the value of excitation, it can absorb or

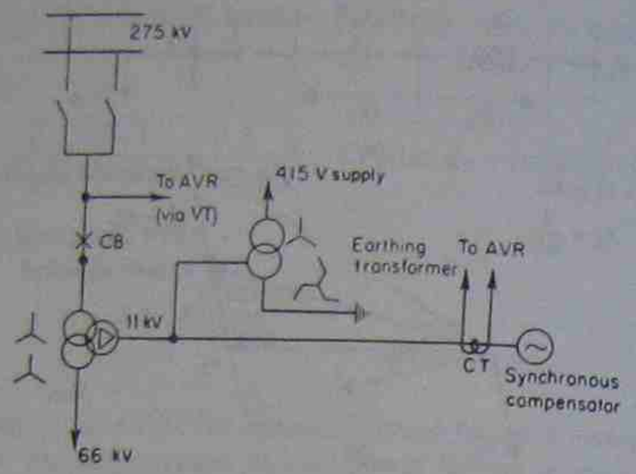


Figure 5.7 Typical installation with synchronous compensator connected to tertiary (delta) winding of main transformer. A neutral point is provided by the earthing transformer shown. The automatic voltage regulator on the compensator is controlled by a combination of the voltage on the 275 kV system and the current output; this gives a droop to the voltage-var output curve which may be varied as required

generate reactive power. As the losses are considerable compared with static capacitors, the power factor is not zero. When used with a voltage regulator the compensator can automatically run overexcited at times of high load and underexcited at light load. A typical connection of a synchronous compensator is shown in Figure 5.7 and the associated voltage–var output characteristic in Figure 5.8. The compensator is run up as an induction motor in 2.5 min and then sychronized.

A great advantage is the flexibility of operation for all load conditions. Although the cost of such installations is high, in some circumstances it is justified, e.g. at the receiving-end busbar of a long high-voltage line where transmission at power factors less than unity cannot be tolerated. Being a rotating machine, its stored energy is useful for riding through transient disturbances, including voltage sags.

5.4.4 Series injection

With the development of high-power, high-voltage semiconductor-controlled devices, including pulse turn-on and turn-off (e.g. IGBT, as discussed in Chapter 9), inverters are now being designed and constructed that can inject a voltage in series with a line whose angle can have any desired relation with the phase voltage. This is equivalent to the series capacitor case of Figure 5.6, except that V_C is not confined to being only 90° out of phase with the current

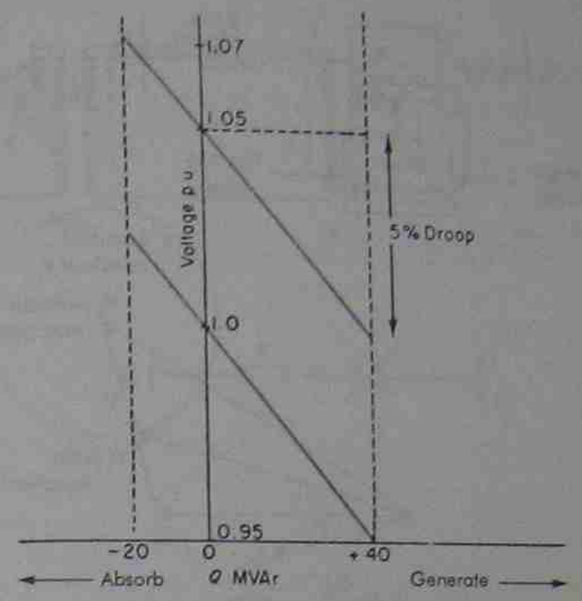


Figure 5.8 Voltage-reactive power output of a typical 40 MVAr synchronous compensator

and dependent upon the IX_C voltage rise. Such a regulator, known as a Universal Power Controller (UPC) is shown, in principle, in Figure 5.9.

It can be realised that if V_i is at 90° to the current I, then no energy is required from the source. At any other angle, energy is either drawn from the system or required from the source. Most conveniently, the source is a transformer connected to the system busbars feeding a rectifier, from which a sinusoidal injected voltage at the desired magnitude and angle is synthesized. Alternatively, the source could be a storage device (battery, capacitor, superconducting energy store, etc.), in which case some auxiliary charging may be necessary, but peak lopping of even uninterruptible power may also be provided.

5.5 Methods of Voltage Control: (ii) Tap-Changing Transformers

The basic operation of the tap-changing transformer has been discussed in Chapter 3; by changing the transformation ratio, the voltage in the secondary circuit is varied and voltage control is obtained. This constitutes the most popular and widespread form of voltage control at all voltage levels.

Consider the operation of a radial transmission system with two tap-changing transformers, as shown in the equivalent single-phase circuit of

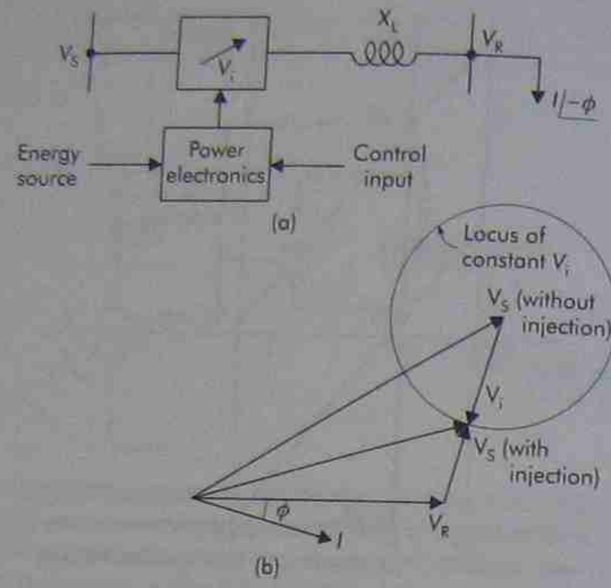


Figure 5.9 Principle of Universal Power Controller for series injection. (a) System diagram. (b) Phasor diagram

Figure 5.10. Here, t_s and t_r are fractions of the nominal transformation ratios, i.e. the tap ratio/nominal ratio. For example, a transformer of nominal ratio 6.6 to 33 kV when tapped to give 6.6 to 36 kV has a $t_s - 36/33 = 1.09$. V_1 and V_2 are the nominal voltages; at the ends of the line the actual voltages are t_sV_1 and t_rV_2. It is required to determine the tap-changing ratios required to completely compensate for the voltage drop in the line. The product t_st_r will be made unity; this ensures that the overall voltage level remains in the same order and that the minimum range of taps on both transformers is used.

(Note that all values are in per unit; t is the off-nominal tap ratio.)

Transfer all quantitites to the load circuit. The line impedance becomes $(R + jX)/t_r^2$; $V_s = V_1t_s$ and, as the impedance has been transferred, $V_r = V_1t_s$. The input voltage to the load circuit becomes V_1t_s/t_r and the equivalent circuit is as shown in Figure 5.10(c). The arithmetic voltage drop

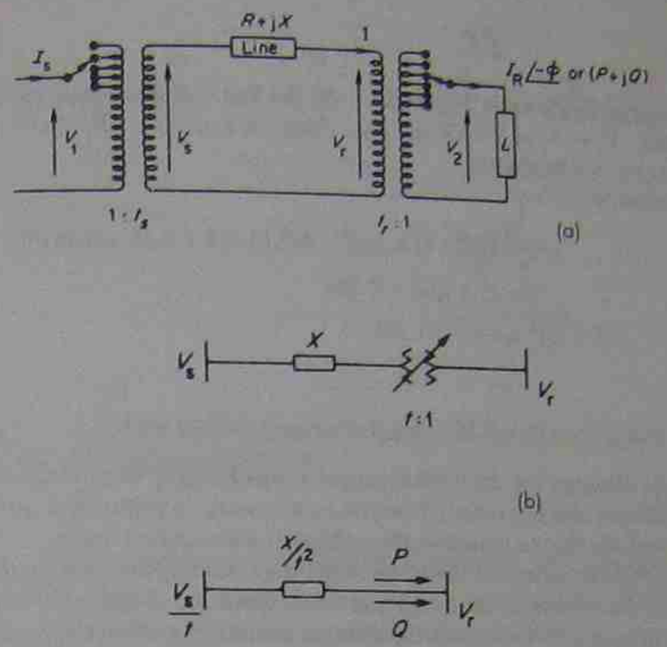
$$= (V_1t_s/t_r) - V_2 \approx \frac{RP + XQ}{t_r^2V_2}$$

When $t_r = 1/t_s$,

$$t_s^2V_1V_2 - V_2^2 = (RP + XQ)t_s^2$$

and

$$V_2 = \tfrac{1}{2}[t_s^2V_1 \pm t_s(t_s^2V_1^2 - 4(RP + XQ))^{\frac{1}{2}}] \qquad (5.7)$$



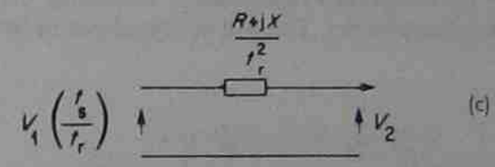


Figure 5.10 (a) Coordination of two tap-changing transformers in a radial transmission link (b) and (c) Equivalent circuits for dealing with off-nominal tap ratio. (b) Single transformer. (c) Two transformers

Hence if t_s is specified, there are two values of V_2 for a given V_1.

Example 5.3

A 13 kV line is fed through an 11/132 kV transformer from a constant 11 kV supply. At the load end of the line the voltage is reduced by another transformer of nominal ratio 132/11 kV. The total impedance of the line and transformers at 132 kV is (25 + j66) Ω. Both transformers are equipped with tap-changing facilities which are arranged so that the product of the two off-nominal settings is unity. If the load on the system is 100 MW at 0.9 p.f. lagging, calculate the settings of the tap-changers required to maintain the voltage of the load busbar at 11 kV. Use a base of 100 MVA.

Solution

The line diagram is shown in Figure 5.11. As the line voltage drop is to be completely compensated, $V_1 = V_2 = 132\,\text{kV} = 1\,\text{p.u.}$ Also, $t_s \times t_r = 1$. The load is 100 MW, 48.3 MVAr., i.e. $1 + j0.483$ p.u.

Using equation (5.7)

$$1 = \tfrac{1}{2}[(t_s^2 \cdot 1) \pm t_s(t_s^2 - 4(0.14 \times 1 + 0.38 \times 0.48))^{\frac{1}{2}}]$$
$$\therefore 2 = t_s^2 \pm t_s(t_s^2 - 1.28)^{\frac{1}{2}}$$
$$\therefore (2 - t_s^2)^2 = t_s^2(t_s^2 - 1.28)$$

Hence,

$$t_s = 1.21 \quad \text{and} \quad t_r = 1/1.21 = 0.83$$

These settings are large for the normal range of tap-changing transformers (usually not more than ±20 per cent tap range). It would be necessary, in this system, to inject vars at the load end of the line to maintain the voltage at the required value.

It is important to note that the transformer does not improve the var-flow position and also that the current in the supplying line is increased if the ratio is increased. In countries with long and inadequate distribution circuits, it is often the practice to boost the received voltage by a variable ratio transformer so as to maintain rated voltage as the power required increases. Unfortunately, this has the effect of increasing the primary supply circuit current by the transformer ratio, thereby decreasing the primary voltage still further until voltage collapse occurs. This will be considered in more detail later in this chapter.

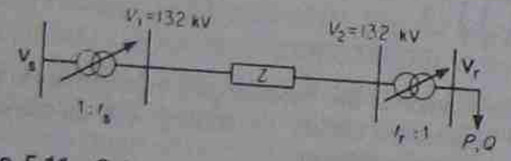


Figure 5.11 Schematic diagram of system for Example 5.3

5.6 Combined Use of Tap-Changing Transformers and Reactive-Power Injection

The usual practical arrangement is shown in Figure 5.12, where the tertiary winding of a three-winding transformer is connected to a compensator. For given load conditions it is proposed to determine the necessary transformation ratios with certain outputs of the compensator.

The transformer is represented by the equivalent star connection and any line impedance from V_1 or V_2 to the transformer can be lumped together with the transformer branch impedances. Here, V_n is the phase voltage at the star point of the equivalent circuit in which the secondary impedance (X_s) is usually

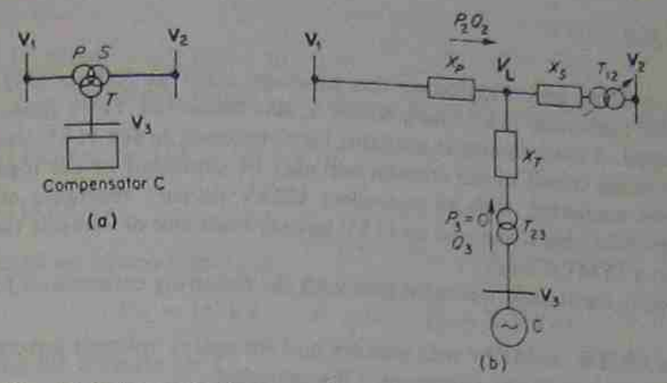


Figure 5.12 (a) Schematic diagram with combined tap-changing and synchronous compensation. (b) Equivalent network

approaching zero and hence is neglected. Resistance and losses are neglected. The allowable ranges of voltage for V_1 and V_2 are specified and the values of P_2, Q_2, P_3, and Q_3 are given; P_3 is usually taken as zero.

The volt drop V_1 to V_L

$$= \Delta V_p \approx X_p \frac{Q_2/3}{V_n}$$

or

$$\Delta V_p = X_p \frac{Q_2}{V_L\sqrt{3}},$$

where V_L is the line voltage $= \sqrt{(3)}V_n$, and Q_2 is the total vars. Also,

$$\Delta V_q = X_p \frac{P_2}{V_L\sqrt{3}}$$
$$\therefore (V_n + \Delta V_p)^2 + (\Delta V_q)^2 = V_1^2$$

(see phasor diagram of Figure 2.22; phase values used) and

$$\left(V_n + X_p \frac{Q_2}{V_L\sqrt{3}}\right)^2 + X_p^2\left(\frac{P_2^2}{3V_L^2}\right) = V_1^2$$
$$\therefore (V_L^2 + X_p Q_2)^2 + X_p^2 P_2^2 = V_{1L}^2 V_L^2$$

where V_{1L} is the line voltage $= \sqrt{(3)}V_1$

$$\therefore V_L^2 = \frac{V_{1L}^2 - 2X_pQ_2}{2} \pm \tfrac{1}{2}\sqrt{[V_{1L}^2(V_{1L}^2 - 4X_pQ_2) - 4 \cdot X_p^2P_2^2]}$$

Once V_L is obtained, the transformation ratio is easily found. The procedure is best illustrated by an example.

Example 5.4

A three-winding grid transformer has windings rated as follows: 132 kV (line), 75 MVA, star connected; 33 kV (line), 60 MVA, star connected; 11 kV (line), 45 MVA, delta connected. A compensator is available for connection to the 11 kV winding.

The equivalent circuit of the transformer may be expressed in the form of three windings, star connected, with an equivalent 132 kV primary reactance of 0.12 p.u., negligible secondary reactance, and an 11 kV tertiary reactance of 0.08 p.u. (both values expressed on a 75 MVA base).

In operation, the transformer must deal with the following extremes of loading:

1. Load of 60 MW, 30 MVAr with primary and secondary voltages governed by the limits 120 kV and 34 kV; compensator disconnected.
2. No load, Primary and secondary voltage limits 143 kV and 30 kV; compensator in operation and absorbing 20 MVAr.

Calculate the range of tap-changing required. Ignore all losses.

Solution

The value of X_p, the primary reactance (in ohms)

$$= 0.12 \times 132^2 \times 1000/75 \times 1000 = 27.8\ \Omega$$

Similarly, the effective reactance o the tertiary winding is 18.5 Ω. The equivalent star circuit is shown in Figure 5.13.

The first operating conditions are as follows:

$$P_1 = 60\,\text{MW} \qquad Q_1 = 30\,\text{MVAr} \qquad V_{1L} = 120\,\text{kV}$$

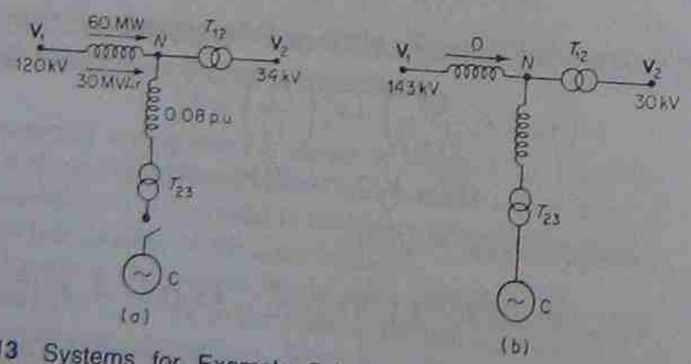


Figure 5.13 Systems for Example 5.4. (a) System with loading condition 1. (b) System with loading condition 2

Hence,

$$V_L^2 = \tfrac{1}{2}(120\,000^2 - 2 \times 27.8 \times 30 \times 10^6) \pm \tfrac{1}{2}\sqrt{[120\,000^2(120\,000^2 - 4 \times 27.8 \times 30 \times 10^6) - 4 \times 27.8^2 \times 60^2 \times 10^{12}]}$$

$$= \left(63.61 \pm \frac{122}{2}\right)10^8 = 124.4 \times 10^8$$

$$\therefore V_L = 111\,\text{kV}$$

The second set of conditions are:

$$V_{1L} = 143\,\text{kV} \qquad P_2 = 0 \qquad Q_2 = 20\,\text{MVAr}$$

Again, using the formula for V_L,

$$V_L = 138.5\,\text{kV}$$

The transformation ratio under the first condition

$$= 111/34 = 3.27$$

and, for the second condition

$$\frac{138.5}{30} = 4.61$$

The actual raio will be taken as the mean of these extremes, i.e. 3.94, varying by ±0.67 or 3.94 ± 17 per cent. Hence the range of tap-changing required is ±17 per cent.

A further method of var production is the use of adjustment of tap settings on transformers connecting large interconnected systems. Consider the situation in Figure 5.14(a), in which V_s and V_r are constant voltages representing the two connected systems. The circuit may be rearranged as shown in Figure 5.14(b), where t is the off-nominal (per unit) tap setting; resistance is zero. The voltage drop between busbars

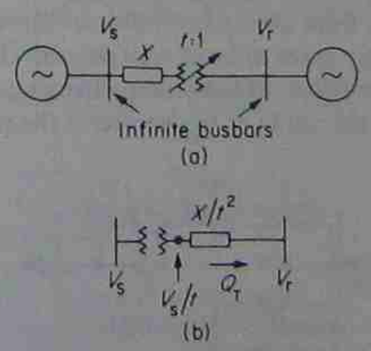


Figure 5.14 (a) Two power systems connected via a tap-change transformer. (b) Equivalent circuit with impedance transferred to receiver side

$$= \left(\frac{V_s}{t}\right) - V_r = \frac{X}{t^2} \cdot \frac{Q_T}{V_r}$$

Hence,

$$(V_s V_r t - V_r^2 t^2)1/X = Q_T$$

and

$$t(1-t)V^2/X = Q_T \qquad \text{when} \qquad V_s = V_r = V$$

Also, $Q_T = t(1-t)S$, where S = short-circuit level, i.e. V^2/X. When

$t < 1$, Q_T is positive, i.e. a flow of vars into V_r

$t > 1$, Q_T is negative, a flow of vars out of V_r

Thus, by suitable adjustment of the tap setting, an appropriate injection of reactive power is obtained.

The idea can be extended to two transformers in parallel between networks. If one transformer is set to an off-nominal ratio of, say, 1:1.1 and the other to 1:0.8 (i.e. in opposite directions), then a circulation of reactive power occurs round the loop, resulting in a net absorption of vars. This is known as 'tap-stagger' and is a comparatively inexpensive method of var absorption.

5.7 Booster Transformers

It may be desirable, on technical or economic grounds, to increase the voltage at an intermediate point in a line rather than at the ends as with tap-changing transformers, or the system may not warrant the expense of tap-changing. Here, booster transformers are used as shown in Figure 5.15. The booster can be brought into the circuit by the closure of relay B and the opening of A, and vice versa. The mechanism by which the relays are operated can be controlled from a change in either the voltage or the current. The latter method is the more sensitive, as from no-load to full-load represents a 100 per cent change in current, but only in the order of a 10 per cent change in voltage. This booster gives an in-phase boost, as does a tap-changing transformer. An economic advantage is that the rating of the booster is the product of the current

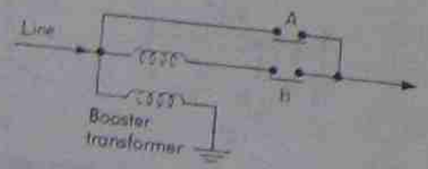


Figure 5.15 Connection of in-phase booster transformer. One phase only shown

and the injected voltage, and is hence only about 10 per cent of that of a main transformer. Boosters are often used in distribution feeders where the cost of tap-changing transformers is not warranted.

Example 5.5

In the system shown by the line diagram in Figure 5.16, each of transformers T_A and T_B have tap ranges of ± 10 per cent in 10 steps of 1.0 per cent. It is required to find the voltage boost needed on transformer T_A to share the power flow equally between the two lines.

The system data is as follows:

All transformers: $X_T = 0.1$ p.u.

Transmission lines: $R = R' = 0$, $X = 0.20$ p.u., $X' = 0.15$ p.u. } All to same base

$V_A = 1.1\underline{/5^\circ}$ $\quad V_B = 1.0\underline{/0^\circ}$

Solution

We must first calculate the current sharing in the two parallel lines:

$$I_1 = \frac{1.1\underline{/5^\circ} - 1.0\underline{/0}}{\text{j}0.40} = 0.2397 - \text{j}0.2397$$

$$I_2 = \frac{1.1\underline{/5^\circ} - 1.0\underline{/0}}{\text{j}0.35} = 0.2740 - \text{j}0.2740$$

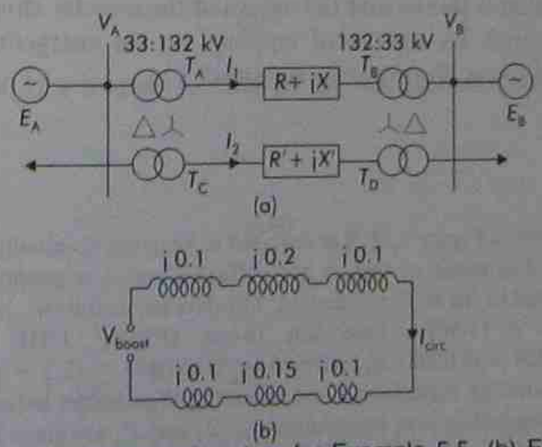


Figure 5.16 (a) Line diagram of system for Example 5.5. (b) Equivalent network with voltage boost V_{boost} acting

Any boost by transformer T_A will cause a current to circulate between the two busbars because the voltages V_A and V_B are assumed to be held constant by the voltage regulators on the generators.

To equalize the currents, a circulating current

$$I_{circ} = \frac{I_2 - I_1}{2}$$

is required, as in Figure 5.16(b), giving

$$I_{circ} = \frac{0.0343 - j0.0343}{2} = 0.0241 \angle -45^\circ$$

$$\therefore V_{Boost} = 0.0241\angle -45 \times j0.75$$

$$= 0.0180\angle 45^\circ \ V$$

To achieve this boost, ideally T_A should be equipped with a phase changer of 45° and taps to give 1.8 per cent boost. In practice, a tap of 2 per cent would be used in either an in-phase boost (such as obtainable from a normal tapped transformer) or a quadrature boost (obtainable from a phase-shift transformer—(see next section). In transmission networks it should be noted that because of the generally high X/R ratio, an in-phase boost gives rise to a quadrative current whereas a quadrature boost produces an in-phase circulating current, thereby adding to or subtracting from the real power flow. This technique has also been used to de-ice lines in winter by producing extra I^2R losses for heating. Alternatively, two transformers in parallel can be tap-staggered to produce I^2X absorption under light-load, high-voltage conditions.

5.7.1 Phase-shift transformer

A quadrature phase shift can be achieved by the connections shown in Figure 5.17(a). The booster arrangement shows the injection of voltage into one phase only; it is repeated for the other two phases. In Figure 5.17(b), the corresponding phasor diagram is shown and the nature of the angular shift of the voltage boost V'_{YB} indicated. By the use of tappings on the energizing transformer, several values of phase shift may be obtained.

Example 5.6

In the system shown in Figure 5.18, it is required to keep the nominally 11 kV busbar at constant voltage. The range of taps is not sufficient and it is proposed to use shunt capacitors connected to the tertiary winding. The data are as follows, per unit quantities being referred to a 15 MVA base: line 16 km, 115 mm^2, OHL (overhead line), 33 kV, $Z_L = (0.0304 = j0.0702)$; Z_L referred to 33 kV side $= (2.2 + j5.22)\,\Omega$.

For the three-winding transformer the measured impedances between the windings and the resulting equivalent star impedances Z_1, Z_2 and Z_3 are given in Table 5.1. The equivalent circuit referred to 33 kV is shown in Figure 5.18(b).

The voltage across the receiving-end load

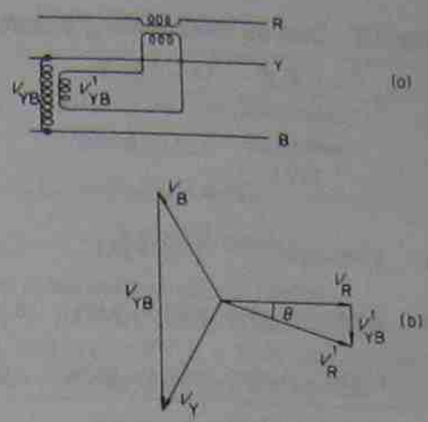
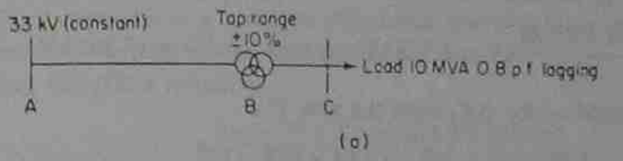


Figure 5.17 (a) Connections for one phase of a phase-shifting transformer. Similar connections for the other two phases. (b) Corresponding phasor diagram



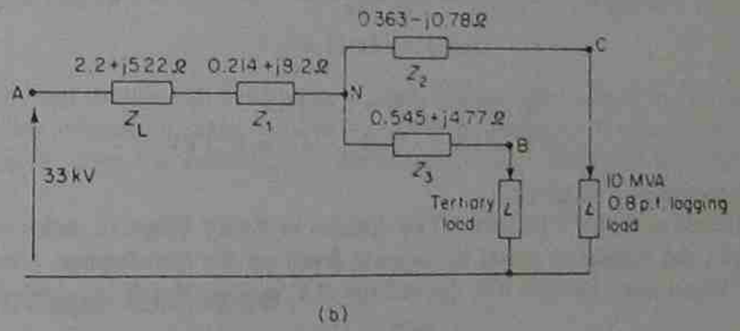


Figure 5.18 (a) Line diagram for Example 5.6. (b) Equivalent network—referred to 33kV

$$= \frac{33\,000}{\sqrt{3}} - \Delta V_p$$

where

$$\Delta V_p \approx \frac{RP + XQ}{V_C}$$

$$\therefore \Delta V_p \approx \frac{2.77 \times 8/3 \times 10^6 + 12.64 \times 6/3 \times 10^6}{33\,000/\sqrt{3}}$$

Table 5.1 Data for three-winding transformer

Winding	MVA	Voltage (kV)	p.u. Z referred to nameplate MVA	p.u. Z on 15 MVA base	(Z (Ω)) referred to 33 kV	Equivalent (Z (Ω)) referred to 33 kV
P–S	15	33/11	0.008 + j0.1	0.008 + j0.1	0.57 + j7.3	Z_1 = 0.214 + j8.2
P–T	5	33/1.5	0.0035 + j0.0595	0.0105 + j.179	0.76 + j4.32	Z_2 = 0.363 − j0.78
S–T	5	11/1.5	0.0042 + j0.0175	0.0126 + j0.0525	0.915 + j1.27	Z_3 = 0.545 + j4.77

As V_C referred to the 33 kV base is not known because of the system volt drop, 33 kV is assumed initially. The revised value is then used and the process is repeated.

$$\Delta V_p = \frac{7.4 + 25.28}{19}\text{kV} = 1.715\,\text{kV} \qquad \text{and} \qquad V_C = 17.285\,\text{kV}$$

Repeating the calculation for ΔV_p with the new V_C,

$$\Delta' V_p = \frac{2.77 \times 8/3 \times 10^6 + 12.64 \times 6/3 \times 10^6}{17.285} = 1.89\,\text{kV}$$

Hence

$$V_C' = 19 - 1.89 = 17.11\,\text{kV}$$
$$\Delta' V = 1.9 \qquad \text{and} \qquad V_C' = 17.1\,\text{kV}$$

This will be the final value of V_C.

V_C referred to 11 kV = 17.1/3 = 5.7 kV (phase) or 9.9 kV (line). In order to maintain 11 kV at C, the voltage is raised by tapping down on the transformer. Using the full range of 10 per cent, i.e. $t_r = 0.9$, the voltage at C is

$$\frac{29.7}{(33 \times 0.9)/11} = 11\,\text{kV}$$

The true voltage will be less than this as the primary current will have increased by (1/0.9) because of the change in transformer ratio. It is evident that the tap-changing transformer is not able to maintain 11 kV at C and the use of a static capacitor connected to the tertiary will be investigated.

Consider a shunt capacitor of capacity 5 MVAr (the capacity of the tertiary).

Assume the transformer to be at its nominal ratio 33/11 kV. The voltage drop

$$= \frac{2.414 \times 8/3 \times 10^6 + 13.42 \times 1/3 \times 10^6}{V_N (\approx 19\,\text{kV})}$$
$$= 0.587\,\text{kV}$$
$$V_N' = 19 - 0.587 = 18.413\,\text{kV (phase)}$$
$$\therefore \Delta' V_N = 0.606 \qquad \text{and} \qquad V_N' = 18.394\,\text{kV}$$

Therefore the volt drop N to C

$$\Delta V_C = \frac{0.363 \times 8/3 - 0.78 \times 6/3}{18.394}\text{kV}$$
$$= -0.032\,\text{kV}$$
$$\therefore V_C = 18.394 + 0.032$$
$$= 18.426\,\text{kV (phase)}$$

As ΔV_C is so small there is no need to iterate further.

Referred to 11 kV, $V_C = 10.55$ kV (line). Hence, to have 11 kV the transformer will tap such that $t_r = (1 - 0.35/11) = 0.97$, i.e. a 3 per cent tap change, which is well within the range and leaves room for load increases. On *no-load*

$$\Delta V_p = \frac{2.959 \times 0 + 18.19 \times (-5.3)}{19}\text{kV}$$
$$= \frac{30.3}{19} = -1.595\,\text{kV (phase)}$$

The shunt capacitor is a constant impedance load and hence as V_N rises the current taken increases, causing further volt increase.

Ignoring this effect initially,

$$V_C = 19 + 1.6 = 20.6\,\text{kV (phase)}$$

On the 11 kV side

$$V_C = 11.85\,\text{kV (line)}$$

therefore the tap change will have to be at least 7.15 per cent, which is well within the range. Further refinement in the value of V_C will be unnecessary. If an accurate value of V_C is required, then the reactance of the capacitor must be found and the current evaluated.

5.8 Voltage Collapse

Voltage collapse is essentially an aspect of system stability to be discussed in Chapter 8. As the voltages to be maintained in a system are influenced by system stability it is appropriate to discuss the subject here.

Consider the circuit shown in Figure 5.19(a). If V_s is fixed (i.e. an infinite busbar supply), the graph of V_R against P for given power factors is as shown in Figure 5.19(b). In Figure 5.19(b), Z represents the series impedance of a 160 km long, double-circuit, 400 kV, 260 mm^2 conductor overhead line. The fact that two values of voltage exist for each value of power is easily demonstrated by considering the analytical solution of this circuit. At the lower voltage a very high current is taken to produce the power. The seasonal thermal ratings of the line are also shown, and it is apparent that for loads of power factor less than unity the possibility exists that, before the thermal rating is reached, the operating power may be on that part of the characteristic where

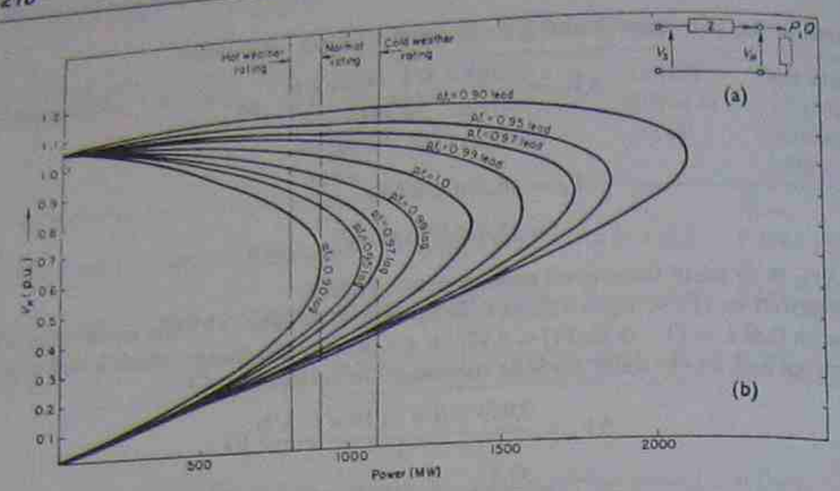


Figure 5.19 (a) Equivalent circuit of a line supplying a load $P + jQ$. (b) Relation between load voltages and received power at constant power factor for a 400 kV, $2 \times 260\,mm^2$ conductor line, 160 km in length. Thermal ratings of the line are indicated

small changes in load cause large voltage changes and *voltage instability* will have occurred. In this condition the action of tap-changing transformers is interesting. If the receiving-end transformers 'tap up' to maintain the load voltage, the line current increases, thereby causing further increase in the voltage drop. It would, in fact, be more profitable to 'tap down', thereby reducing the current and voltage drop. It is feasible therefore for a 'tapping-down' operation to result in increased secondary voltage, and vice versa.

The possibility of an actual voltage collapse depends upon the nature of the load. If this is stiff (constant power), e.g. induction motors, the collapse is aggravated. If the load is soft, e.g. heating, the power falls off rapidly with voltage and the situation is alleviated. Referring to Figure 5.19 it is evident that a critical quantity is the power factor; at full load a change in lagging power factor from 0.99 to 0.90 will precipitate voltage collapse. On long lines, therefore, for reasonable power transfers it is necessary to keep the power factor of transmission approaching unity, certainly above 0.97 lagging, and it is economically justifiable to employ var injection by static capacitors, synchronous compensators or UPC close by the load.

A problem arises with the operation of two or more lines in parallel, e.g. the system shown in Figure 5.20, in which the shunt capacitance has been represented as in a π section. If one of the three lines is removed from the circuit because of a fault, the total system reactance will increase from $X/3$ to $X/2$, and the capacitance, which normally improves the power factor, decreases to $2C$ from $3C$. Thus the overall voltage drop is greatly increased and, owing to the increased I^2X loss of the lines and the decreased generation of vars by the

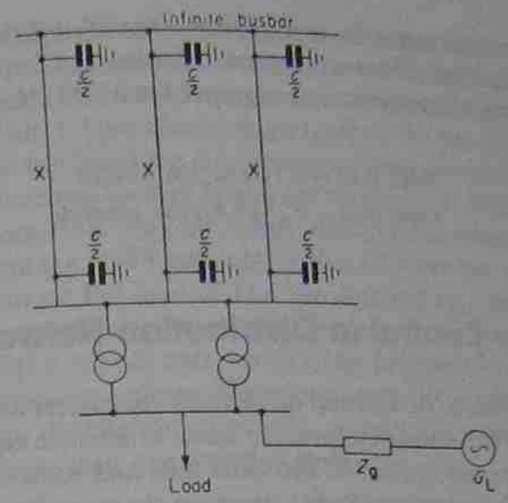


Figure 5.20 Line diagram of three long lines in parallel—effect of the loss of one line. G_L = local generators

shunt capacitances, the power factor decreases; hence the possibility of voltage instability. The same argument will, of course, apply to two lines in parallel.

Usually, there will be local generation or compensation feeding the receiving-end busbars at the end of long lines. If this generation is electrically close to the load busbars, i.e. low connecting impedance Z_g, a fall in voltage will automatically increase the local var generation, and this may be sufficient to keep the reactive power transmitted low enough to avoid large voltage drops in the long lines. Often, however, the local generators supply lower voltage networks and are electrically remote from the high-voltage busbar of Figure 5.20, and Z_g is high. The fall in voltage now causes little change in the local generator var output and the use of static-controlled capacitors at the load may be required. As Z_g is inversely proportional to the three-phase short-circuit level at the load busbar because of the local generation, the reactive-power contribution of the local machines is proportional to this fault level. When a static or synchronous compensator reaches its rated limit, voltage can no longer be controlled and rapid collapse of voltage can follow because any vars demanded by the load must now be supplied from sources further away electrically over the high-voltage system.

In the U.K. and some other countries where 'unbundled' systems are now extant, many generators are some distance from the load-centres. Consequently, the transmission system operator is required to install local flexible var controllers or compensators to maintain a satisfactory voltage at the delivery substations supplying the local distribution systems. Such flexible controllers, based on semiconductor devices which can vary the var absorption

in a reactor or generation in a capacitor, are called FACTS (Flexible a.c. Transmission System). These are discussed further in Chapter 9.

Typical values of compensation required for a 400 kV or 500 kV network are:

Peak load = 0.3 kVAr/kW absorb

Light load = 0.25 kVAr/kW generate

5.9 Voltage Control in Distribution Networks

Single-phase supplies to houses and other small consumers are tapped off from three-phase feeders. Although efforts are made to allocate equal loads to each phase the loads are not applied at the same time and some unbalance occurs.

In the distribution network (British practice) shown in Figure 5.21 an 11 kV distributor supplies a number of lateral feeders in which the voltage is 420 V, and then each phase, loaded separately.

The object of design is to keep the consumers' nominal 415 V supply within the statutory ±6 per cent of the declared voltage. The main 33/11 kV transformer gives a 5 per cent rise in voltage between no-load and full-load. The distribution transformers have taps that are only adjustable off-load, and a

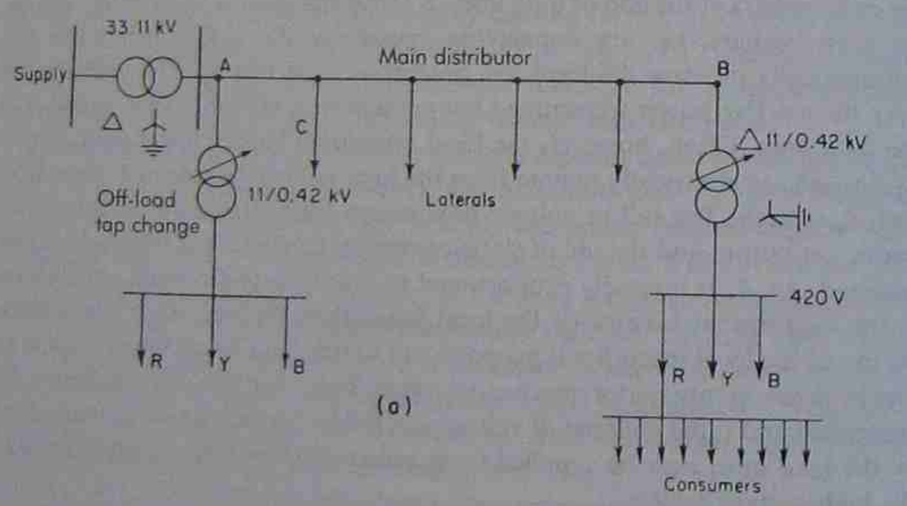


Figure 5.21 Line diagrams of typical radial distribution schemes

secondary phase voltage of 250 V which is 4 per cent high on the nominal value of 240 V. A typical distribution of voltage drops would be as follows: main distributor, 6 per cent; 11/0.42 kV transformer, 3 per cent; 420 V circuit, 7 per cent; consumer circuit, 1.5 per cent; giving a total of 17.5 per cent. On very light load (10 per cent of full load) the corresponding drop may be 1.5 per cent. To offset these drops, various voltage boosts are employed as follows: main transformer, +5 per cent (zero on light load); distribution transformer, inherent boost of +4 per cent (i.e. 250 V secondary) plus a 2.5 per cent boost. These add to give a total boost on full load of 11.5 per cent and on light load of 6.5 per cent. Hence the consumers' voltage varies between (−17.5 + 11.5), i.e. −6 per cent and (6.5 − 1.5) i.e. +5 per cent, which is just permissible. There will also be a difference in consumer voltage depending upon the position of the lateral feeder on the main distributor; obviously, a consumer supplied from C will have a higher voltage than one supplied from B.

5.9.1 Uniformly loaded feeder from one end

In areas with high load densities a large number of tappings are made from feeders and a uniform load along the length of a feeder may be considered to exist. Consider the voltage drop over a length dx of the feeder distant x metres from the supply end. Let iA be the current tapped per unit length and R and X be the resistance and reactance per phase per metre, respectively. The length of the feeder is L (m) (see Figure 5.21(b)).

The voltage across $dx = Rix\,dx\cos\phi + Xix\,dx\sin\phi$, where $\cos\phi$ is the power factor (assumed constant) of the uniformly distributed load.

The total voltage drop

$$= R\int_0^L ix\,dx\cos\phi + X\int_0^L ix\,dx\sin\phi$$

$$= R_i\frac{L^2}{2}\cos\phi + Xi\frac{L^2}{2}\sin\phi$$

$$= \frac{LR}{2}I\cos\phi + \frac{LX}{2}I\sin\phi$$

where $I = Li$, the total current load. Hence the uniformly distributed load may be represented by the total load tapped at the centre of the feeder length.

5.10 Long Lines

On light loads the charging volt-amperes of a line exceed the inductive vars consumed and the voltage rises, causing problems for generators. With very

long lines the voltage drop can be massive. A length of 1500 km at 50 Hz corresponds to a quarter-wavelength line. Series capacitors would normally be installed to improve the power capacity and these effectively shorten the line electrically. In addition, shunt reactors are switched in circuit at times of light load to absorb the generated vars.

A 500 km line can operate within ±10 per cent voltage variation without shunt reactors. However, with, say, a 800 km line, shunt reactors are essential and the effects of these are shown in Figure 5.22. For long lines in general, it is usual to divide the system into sections with compensation at the ends of each section. This controls the voltage profile, helps switching, and reduces short-circuit currents. Shunt compensation can be varied by switching discrete amounts of inductance. A typical 500 kV, 1000 km scheme uses compensation totalling 1200 MVAr.

Improvement in voltage profile may be obtained by compensation, using FACTS devices, at intermediate points, as well as at the ends of the line, as shown in Figure 5.22. If the natural load is transmitted there is, of course, constant voltage along the line with no compensation. If the various busbars of a sectioned line can be maintained at *constant voltage* regardless of load, each section has a theoretical maximum transmission angle of 90°. Thus, for a three-section line a total angle of much greater than 90° would be possible. This is illustrated in Figure 5.23 for a three-section, 1500 km line with a unity power-factor load.

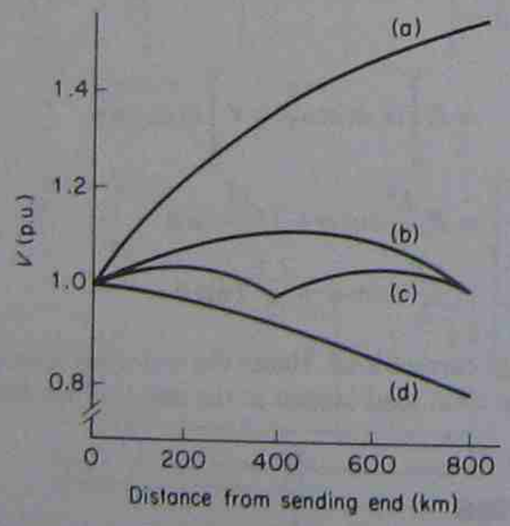


Figure 5.22 Voltage variation along a long line; (a) on no load with no compensation; (b) on no load with compensation at ends; (c) on no load with compensation at ends and at centre. (d) Transmitting natural load, compensation at ends and centre

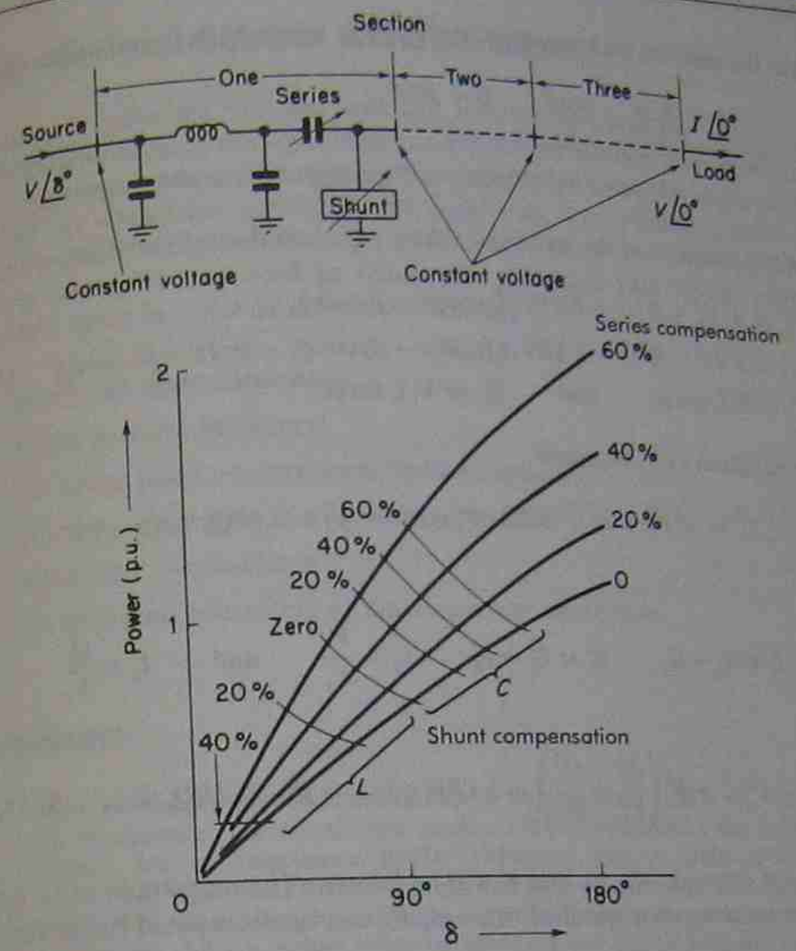


Figure 5.23 Power-angle curves for 1500 km line in three sections. Voltages at section-busbars maintained constant by variable compensation. Percentage of series and shunt compensation indicated (Reproduced by permission of the *Institution of Electrical Engineers*)

5.10.1 Reactive-power requirements for the voltage control of long lines

It is advantageous to use the generalized line equation,

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_r + \mathbf{B}\mathbf{I}_r$$

which takes into account the presence of transformers.

Let the received load current $\mathbf{I}_r$ lag $\mathbf{V}_r$ by ϕ_r where $\mathbf{V}_r$ is the reference phasor

$$\mathbf{A} = A\angle\alpha \quad \text{and} \quad \mathbf{B} = B\angle\beta$$
$$V_s\angle\delta_s = AV_r\angle\alpha + BI_r\angle(\beta - \phi_r)$$
$$\therefore V_s = AV_r\cos\alpha + jAV_r\sin\alpha + BI_r\cos(\beta - \phi_r) + jBI_r\sin(\beta - \phi_r)$$

As the modulus of the left-hand side is equal to that of the right

$$V_s^2 = A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r\cos(\alpha = \beta + \phi_r)$$
$$= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r[\cos(\alpha - \beta)\cos\phi_r - \sin(\alpha - \beta)\sin\phi_r] \quad (5.8)$$
$$P_r = V_rI_r\cos\phi_r \quad \text{and} \quad Q_r = V_rI_r\sin\phi_r$$

Hence equation (5.8) becomes

$$V_s^2 = A^2V_r^2 + B^2I_r^2 + 2ABP_r\cos(\alpha - \beta) - 2ABQ_r\sin(\alpha - \beta)$$

Also

$$\mathbf{I}_r = I_p - jI_q \qquad I_r^2 = I_p^2 + I_q^2 \qquad I_p = \frac{P_r}{V_r} \quad \text{and} \quad I_q = \frac{Q_r}{V_r}$$

$$\therefore V_s^2 = A^2V_r^2 + B^2\left(\frac{P_r^2}{V_r^2} + \frac{Q_r^2}{V_r^2}\right) + 2ABP_r\cos(\alpha - \beta) - 2ABQ_r\sin(\alpha - \beta) \quad (5.9)$$

For a given network, P_r, **A**, and **B** will be known. The magnitude of Q_r such that V_r is equal to, or a specified ratio of, V_S can be determined by the use of equation (5.9).

5.10.2 Subsynchronous oscillation

The combination of series capacitors and the natural inductance of the line (plus that of the connected systems) creates a resonant circuit of subsynchronous resonant frequency. This resonance can interact with the generator-shaft critical-torsional frequency, and a mechanical oscillation is superimposed on the rotating generator shaft that may have sufficient magnitude to cause mechanical failure.

Subsynchronous resonance has been reported, caused by line-switching in a situation where trouble-free switching was normally carried out with all capacitors in service, but trouble occurred when one capacitor bank was out of service. Although this phenomena may be a rare occurrence, the damage resulting is such that, at the design stage, an analysis of possible resonance effects is required.

5.11 General System Considerations

Because of increasing voltages and line lengths, and also the wider use of underground circuits, the light-load reactive-power problem for an interconnected system becomes substantial, particularly with modern generators of limited var absorption capability. At peak load, transmission systems need to increase their var generation, and as the load reduces to a minimum (usually during the night) they need to reduce the generated vars by the following methods, given in order of economic viability:

1. switch out shunt capacitors;
2. switch in shunt inductors;
3. run hydro plant on maximum var absorption;
4. switch out one cable in a double-circuit link;
5. tap-stagger transformers;
6. run base-load generators at maximum var absorption.

Problems

5.1. An 11 kV supply busbar is connected to an 11/132 kV, 100 MVA, 10 per cent reactance transformer. The transformer feeds a 132 kV transmission link consisting of an overhead line of impedance (0.014 + j0.04) p.u. and a cable of impedance (0.03 + j0.01) p.u. in parallel. If the receiving end is to be maintained at 132 kV when delivering 80 MW, 0.9 p.f. lagging, calculate the power and reactive power carried by the cable and the line. All p.u. values relate to 100 MVA and 132 kV bases.
(Answer: Line (23 + j38) MVA; cable (57 + j3.8) MVA)

5.2 A three-phase induction motor delivers 500 hp at an efficiency of 0.91, the operating power factor being 0.76 lagging. A loaded synchronous motor with a power consumption of 100 kW is connected in parallel with the induction motor. Calculate the necessary kVA and the operating power factor of the synchronous motor if the overall power factor is to be unity.
(Answer: 365 kVA, 0.274)

5.3 The load at the receiving end of a three-phase overhead line is 25 MW, power factor 0.8 lagging, at a line voltage of 33 kV. A synchronous compensator is situated at the receiving end and the voltage at both ends of the line is maintained at 33 kV. Calculate the MVAr of the compensator. The line has resistance of 5 Ω per phase and inductive reactance of 20 Ω per phase.
(Answer: 25 MVAr)

5.4 A transformer connects two infinite busbars of equal voltage. The transformer is rated at 500 MVA and has a reactance of 0.15 p.u. Calculate the var flow for a tap setting of (a) 0.85:1; (b) 1.1:1.

(Answer: (a) 427 MVAr; (b) −367 MVAr)

5.5 A three-phase transmission line has resistance and inductive reactance of 25 Ω and 90 Ω, respectively. With no load at the receiving end, a synchronous compensator there takes a current lagging by 90°; the voltage is 145 kV at the sending end and 132 kV at the receiving end. Calculate the value of the current taken by the compensator.

When the load at the receiving end is 50 MW, it is found that the line can operate with unchanged voltages at the sending and receiving ends, provided that the compensator takes the same current as before, but now leading by 90°.

Calculate the reactive power of the load.

(Answer: 83.5 A; Q_L 24.2 MVAr)

5.6 Repeat Problem 5.3 making use of $\partial Q/\partial V$ at the receiving end.

5.7 In Example 5.3, determine the tap ratios if the receiving-end voltage is to be maintained at 0.9 p.u. of the sending-end voltage.

(Answer: $1.19t_s$, $0.84t_r$)

5.8 In the system shown in Figure 5.24, determine the supply voltage necessary at D to maintain a phase voltage of 240 V at the consumer's terminals at C. The data in Table 5.2 apply.

(Answer: 33 kV)

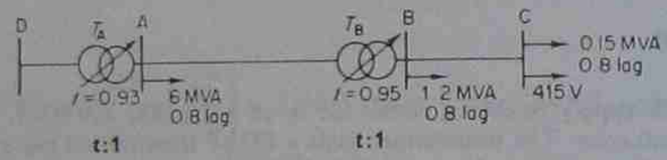


Figure 5.24 Line diagram for system in Problem 5.8

Table 5.2 Data for Problem 5.8

Line or/ transformers	Rated voltage (kV)	Transmission rating	Nominal tap ratio	Impedance (Ω)
BC	0.415			0.0127 + j0.00909
AB	11			1.475 + j2.75
DA	33			1.475 + j2.75
T_A	33/11	10 MVA	30.69/11	1.09 + j9.8 referred to 33 kV
T_B	11/0.415	2.5 MVA	10.450/0.415	0.24 + j1.95 referred to 33 kV

5.9 A load is supplied through a 275 kV link of total reactance 50 Ω from an infinite busbar at 275 kV. Plot the receiving-end voltage against power graph for a constant load power factor of 0.95 lagging. The system resistance may be neglected.

5.10 (a) Describe two methods of controlling voltage in a power system.

(b) Show how the scalar voltage difference between two nodes in a network is given approximately by:

$$\Delta V = \frac{RP + XQ}{V}$$

(c) Each phase of a 50 km, 132 km overhead line has a series resistance of 0.156 Ω/km and an inductive reactance of 0.4125 Ω/km. At the receiving end the voltage is 132 kV with a load of 100 MVA at a power factor of 0.9 lagging. Calculate the magnitude of the sending-end voltage.

(d) Calculate also the approximate angular difference between the sending-end and receiving-end voltages.

(Answer: (c) 144.5 kV; (d) 4.55°)

(*From E.C. Examination 1997*)

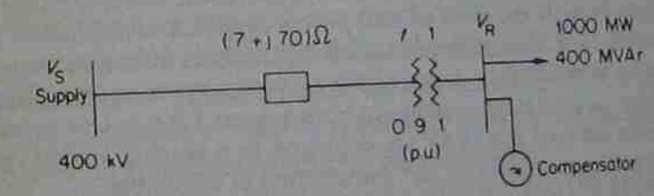
5.11 Explain the limitations of tap-changing transformers. A transmission link (Figure 5.25(a) connects an infinite busbar supply of 400 kV to a load busbar supplying 1000 MW, 400 MVAr. The link consists of lines of effective impedance (7 + j70) Ω feeding the load busbar via a transformer with a maximum tap ratio of 0.9:1. Connected to the load busbar is a compensator. If the maximum overall voltage drop is to be 10 per cent with the transformer taps fully utilized, calculate the reactive power requirement from the compensator.

(Answer: 148 MVAr)

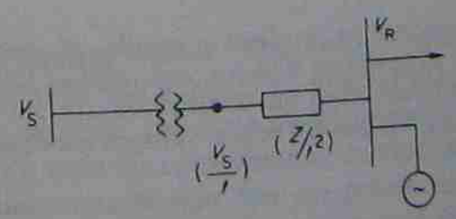
Note: Refer voltage and line Z to load side of transformer in Figure 5.25(b).

$$V_R = \frac{V_s}{t} - \left(\frac{\frac{RP}{t^2} + \frac{X}{t^2}Q}{V_R} \right)$$

5.12 A generating station consists of four 500 MW, 20 kV, 0.95 p.f. (generate) generators, each feeding through a 525 MVA, 0.1 p.u. reactance transformer onto a common busbar. It is required to transmit 2000 MW at 0.95 p.f. lagging to a substation



(a)



(b) Equivalent circuit

Figure 5.25 Circuits for Problem 5.11

maintained at 500 kV in a power system at a distance of 500 km from the generating station. Design a suitable transmission link of nominal voltage 500 kV to achieve this, allowing for a reasonable margin of stability and a maximum voltage drop of 10 per cent. Each generator has synchronous and transient reactances of 2 p.u. and 0.3 p.u. respectively, and incorporates a fast-acting automatic voltage regulator. The 500 kV transmission lines have an inductive reactance per phase of 0.4 Ω/km and a shunt capacitive reactance per phase of 0.3×10^6 Ω/km. Both series and shunt capacitors may be used if desired and the number of three-phase lines used should be not more than three—fewer if feasible. Use approximate methods of calculation, ignore resistance, and state clearly any assumption made. Assume shunt capacitance to be lumped at the receiving end only. (Use two 500 kV lines with series capacitors compensating to 70 per cent of series inductance.)

5.13 It is required to transmit power from a hydroelectric station to a load centre 480 km away using two lines in parallel for security reasons.

Assume sufficient bundle conductors are used such that there are no thermal limitations, and the effective reactance per phase per km is 0.44 Ω and that the resistance is negligible. The shunt capactive reactance of each line is 0.44 MΩ per phase per km, and each line may be represented by the nominal π-circuit with half the capacitance at each end. The load is 2000 MW at 0.95 lagging and is independent of voltage over the permissible range.

Investigate, from the point of view of stability and voltage drop, the feasibility and performance of the link if the sending-end voltage is 345, 500, and 765 kV assuming the transmission angle is not to exceed 30°.

The lines may be compensated up to 70 per cent by series capacitors and the load-end compensators of 120 MVAr capacity are available. The maximum permissible voltage drop is 10 per cent. As two lines are provided for security reasons, your studies should include the worst-operating case of only one line in use.

5.14 Explain the action of a variable-tap transformer, showing, with a phasor diagram, how reactive power may be despatched from a generator down a mainly reactive line by use of the taps. How is the level of real power despatch controlled?

Power flows down an H.V. line of impedance 0 + j0.15 p.u. from a generator whose output passes through a variable-ratio transformer to a large power system. The voltage of the generator and the distant large system are both kept at 1.0 p.u. Determine the tap setting if 0.8 p.u. power and 0.3 p.u. VAr are delivered to a lagging load at the power system busbar. Assume the reactance of the transformer is negligible.
(Answer: $t = 1.052$)
(From E.C. Examination, 1995)

5.15 Two substations are connected by two lines in parallel, of negligible impedance, each containing a transformer of reactance 0.18 p.u. and rated at 120 MVA. Calculate the net absorption of reactive power when the transformer taps are set to 1:1.15 and 1:0.85, respectively (i.e. tap-stagger is used). The p.u. voltages are equal at the two ends and are constant in magnitude.
(Answer: 30 MVAr)

6
Load Flows

6.1 Introduction

A *load flow* is power-system jargon for the steady-state solution of a network. This does not essentially differ from the solution of any other type of network except that certain constraints are peculiar to power supply. In previous chapters the manner in which the various components of a power system may be represented by equivalent circuits has been demonstrated. It should be stressed that the simplest representation should always be used, consistent with the accuracy of the information available. There is no merit in using very complicated machine and line models when the load and other data are known only to a limited accuracy, e.g. the long-line representation should only be used where absolutely necessary. Similarly, synchronous-machine models of more sophistication than given in this text are needed only for very specialized purposes, e.g. in some stability studies. Usually, the size and complexity of the network itself provides more than sufficient intellectual stimulus without undue refinement of the components. Often, resistance may be neglected with little loss of accuracy and an immense saving in computation.

The following combinations of quantities are usually specified at the system busbars for load-flow studies.

Slack, swing, or floating busbar One node is always specified by a voltage, constant in magnitude and phase. The effective generator at this node supplies the losses to the network; this is necessary because the magnitude of the losses will not be known until the calculation of currents is complete, and this cannot be achieved unless one busbar has no power constraint and can feed the required losses into the system. The location of the slack node can influence the complexity of the calculations; the node most closely approaching an infinite busbar should be used.

Load nodes The complex power $S = P \pm jQ$ is specified. Designated P, Q node.

Generation nodes The voltage magnitude and power are usually specified. Often, limits to the value of the reactive power are given depending upon the characteristics of individual machines. Designated P, V node. Load flow studies are performed to investigate the following features of a power system network:

1. Flow of MW and MVAr in the branches of the network;
2. Busbar (node) voltages;
3. Effect of rearranging circuits and incorporating new circuits on system loading;
4. Effect of temporary loss of generation and transmission circuits on system loading (mainly for security studies);
5. Effect of injecting in-phase and quadrature boost voltages on system loading;
6. Optimum system running conditions and load distribution;
7. Minimizing system losses;
8. Optimum rating and tap-range of transformers;
9. Improvements from change of conductor size and system voltage.

Studies will normally be performed for minimum-load conditions (possibility of instability due to high voltage levels and self-excitation of induction machines) and maximum-load conditions (possibility of instability). Having ascertained that a network behaves reasonably under these conditions, further load flows will be performed to attempt to optimize various quantities. The design and operation of a power network to obtain optimum economy is of paramount importance and the furtherance of this ideal is achieved by the use of centralized automatic control of generating stations through system control centres.

Although the same approach can be used to solve all problems, e.g. the nodal voltage method, the object should be to use the quickest and most efficient method for the particular type of problem. Radial networks will require less sophisticated methods than closed loops. In very large networks the problem of organizing the data is almost as important as the method of solution, and the calculation must be carried out on a systematic basis where the nodal-voltage method is the most advantageous. Such methods as network reduction combined with the Thevenin or superposition theorems are at their best with smaller networks. In the nodal method, great numerical accuracy is

required in the computation as the currents in the branches are derived from the voltage differences between the ends. These differences are small in well-designed networks so the method is ideally suited for computation using digital computers.

6.2 Radial and Simple Loop Networks

In radial networks the phase shifts due to transformer connections along the circuit are not usually important. The following examples illustrate the solution of this type of network.

Example 6.1

Distribution feeders with several tapped loads. A distribution feeder with several tapped inductive loads (or laterals), and fed at one end only, is shown in Figure 6.1(a). Determine the total voltage drop.

Solution

The current in AB

$$= (I_1 \cos\theta_1 + I_2 \cos\theta_2 + I_3 \cos\theta_3 + I_4 \cos\theta_4) - j(I_1 \sin\theta_1 + I_2 \sin\theta_2 + I_3 \sin\theta_3 + I_4 \sin\theta_4)$$

Similarly, the currents in the other sections of the feeder are obtained. The approximate voltage drop is obtained from $\Delta V = RI\cos\theta + XI\sin\theta$ for each section. That is

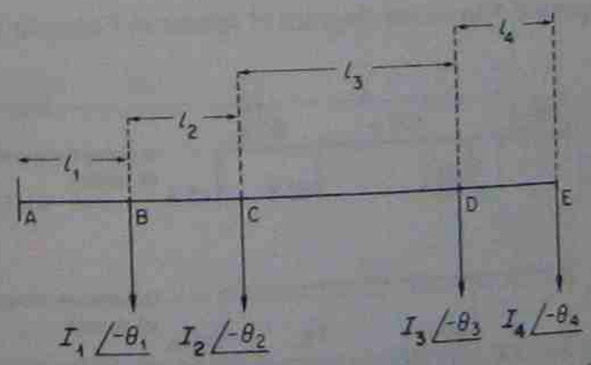


Figure 6.1(a) Feeder with several load tappings along its length

$$\Delta V_{AE} = R_1(I_1\cos\theta_1 + I_2\cos\theta_2 + I_3\cos\theta_3 + I_4\cos\theta_4) + R_2(I_2\cos\theta_2 + I_3\cos\theta_3 + I_4\cos\theta_4) + R_3(I_3\cos\theta_3 + I_4\cos\theta_4) + R_4(I_4\cos\theta_4) + X_1(I_1\sin\theta_1 + I_2\sin\theta_2 + I_3\sin\theta_3 + I_4\sin\theta_4), \text{ and so on}$$

Rearranging and letting the resistance per loop-metre be $r\,\Omega$ and the reactance per loop-metre be $x\,\Omega$ (the term loop-metre refers to single-phase circuits and includes the go and return conductors).

$$\Delta V_{AE} = r[I_1\cdot I_1\cos\theta_1 + I_2\cos\theta_2\cdot(I_1+I_2) + I_3\cos\theta_3(I_1+I_2+I_3) + I_4\cos\theta_4(I_1+I_2+I_3+I_4)] + x[I_1I_1\sin\theta_1 + I_2\sin\theta_2(I_1+I_2) + I_3\sin\theta_3(I_1+I_2+I_3) + I_4\sin\theta_4(I_1+I_2+I_3+I_4)]$$

In the system shown in Figure 6.1(b), calculate the size of cable required if the voltage drop at the end load on the feeder must not exceed 12 V (line value). Let the resistance and reactance per metre per phase be $r(\Omega)$ and $x(\Omega)$, respectively. Then, referring to Figure 6.1(c), voltage drop is given by

$$r[100\times 40 + 250\times 20 + 330\times 25] + x[100\times 30 + 250\times 0 + 330\times 25] = 12/\sqrt{3}$$

i.e.

$$r(4000 + 5000 + 8250) + x(3000 + 0 + 8250) = 12/\sqrt{3}$$

$$17250r + 11250x = 12/\sqrt{3}$$

The procedure now is to consult the appropriate overhead line or cable specifications to select a cross-section that gives values or r and x which best fit the above equation. Usually, if underground cables are used the size of cross-section is selected on thermal considerations and then the voltage drop is calculated. With overhead lines the volt drop is the prime consideration and the conductor size will be determined accordingly.

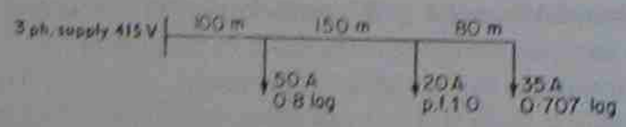


Figure 6.1(b) Line diagram of feeder in Example 6.1

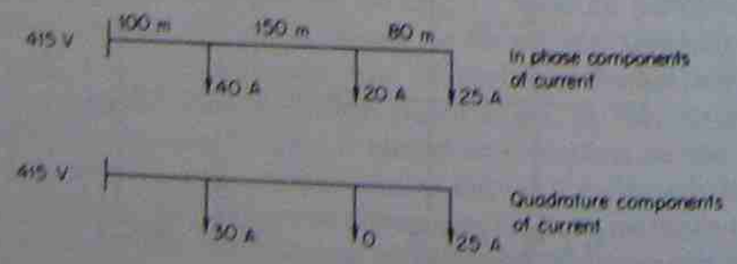


Figure 6.1(c) In-phase and quadrature quantities

Example 6.2

The system shown in figure 6.2(a) feeds two distinct loads, a group of domestic consumers, and a group of induction motors which, on starting, take 5× full-load current at zero power-factor lagging. This is a 'worst-case' calculation. The induction motor voltage is nominally 6 kV. Calculate the dip in voltage on the domestic-load busbar when the induction motor group is started.

Solution

The problem of the drop in the voltage to other consumers when an abnormally large current is taken for a brief period from an interconnected busbar is often serious. The present problem poses a typical situation. The 132 kV system is not an infinite busbar and is represented by a voltage source in series with a 0.04 p.u. reactance on a 50 MVA base. Using a 50 MVA base the equivalent single-phase circuit is as shown in Figure 6.2(b).

Starting current of the induction motors

$$= -\mathrm{j}\frac{15000\times 10^3}{\sqrt{3}\times 6000}\times 5\,\mathrm{A}$$

$$= -\mathrm{j}7217\,\mathrm{A}$$

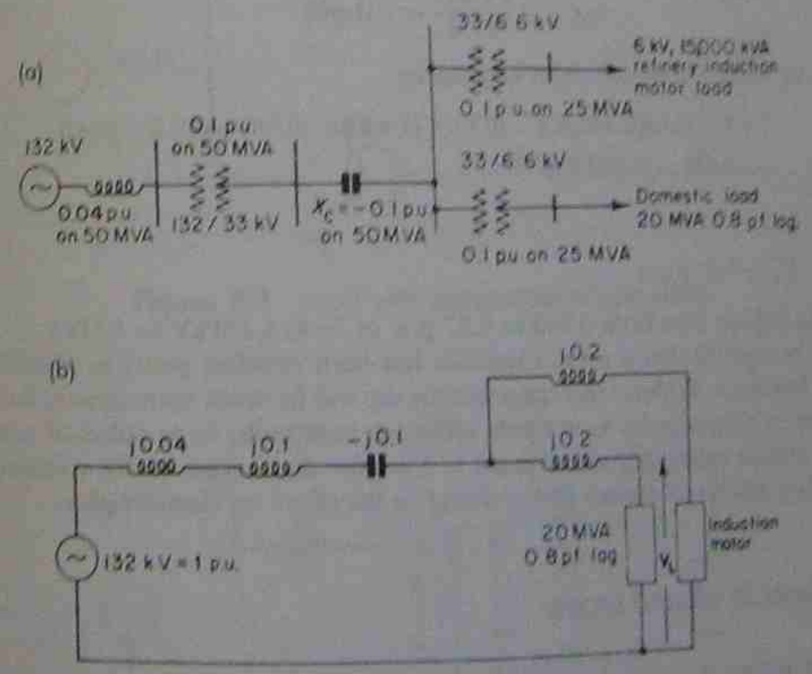


Figure 6.2 (a) Line diagram of the system in Example 6.2; (b) equivalent circuit of the system in Example 6.2

Domestic load voltage before the induction-motor load is switched on

$$\mathbf{V}_L = 1.0 - \mathbf{IZ}$$

where

$$\mathbf{Z} = j(0.04 + 0.1 - 0.1 + 0.2) = j0.24 \text{ p.u.}$$

and

$$\mathbf{I} = \frac{20 \times 10^6}{\sqrt{3} \times 6000} = 1925 \text{ A, } 0.8 \text{ p.f. lagging}$$

But base current at 6.6 kV

$$= \frac{50 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3}$$
$$= 4374 \text{ A}$$

$$\therefore \mathbf{I}_{\text{p.u.}} = \frac{1925}{4374} = 0.440 \text{ p.u. at 0.8 lag}$$

Hence

$$\mathbf{V}_L = 1.0 - 0.440(0.8 - j0.6)(j0.24) \text{ p.u.}$$
$$= 0.9366 - j0.0845 \text{ p.u.}$$
$$\therefore V_L = 0.9404 \text{ p.u.}$$

Starting current of the induction motors

$$= \frac{-j7217}{4374} = -j1.640$$

Voltage $\mathbf{V}_L'$ when the motors start is given by

$$= 1 - j0.04[0.440(0.8 - j0.6) - j1.640] - j0.2[0.440(0.8 - j0.6)]$$
$$= 0.871 - j0.084 \text{ p.u.}$$

and

$$V_L' = 0.875 \text{ p.u.}$$

Hence the voltage dips from 0.940 to 0.87 p.u. or from 6.204 kV to 5.74 kV.

It will be noticed that a series capacitor has been installed partly to neutralize the network reactance. Without this capacitor the dip will be much more serious; it is left to the reader to determine by how much. Often, in steel mills, large pulses of power are taken at regular intervals and the result is a regular fluctuation on the consumer busbars. This is known as *voltage flicker* owing to the effect on electric lights.

Load flows in closed loops

In radial networks any phase shifts due to transformer connections are not usually of importance as the currents and voltages are shifted by the same amount. In a closed loop, to avoid circulating currents, the product of the

transformer transformation ratios (magnitudes) round the loop should be unity and the sum of the phase shifts in a common direction round the loop should be zero. This will be illustrated by the system shown in Figure 6.3.

Neglecting phase shifts due to the impedance of components, for the closed loop formed by the two lines in parallel, the total shift and transformation ratio is

$$\left(\frac{33}{13.8}\angle 30^\circ\right)\left(\frac{132}{33}\angle -30^\circ\right)\left(\frac{13.8}{132}\angle 0^\circ\right) = 1\angle 0^\circ$$

In practice, the transformation ratios of transformers are frequently changed by the provision of tap-changing equipment. This results in the product of the ratios round a loop being no longer unity, although the phase shifts are still equal to zero. To represent this condition a fictitious autotransformer is connected as shown in Figure 6.4. The increase in voltage on the 132 kV line by tap-changing has the effect, in a largely reactive path, of changing the flow of reactive power and thus the power factor of the current. An undesirable effect is the circulating current set up around the loop, unless load sharing is required (see Section 5.7).

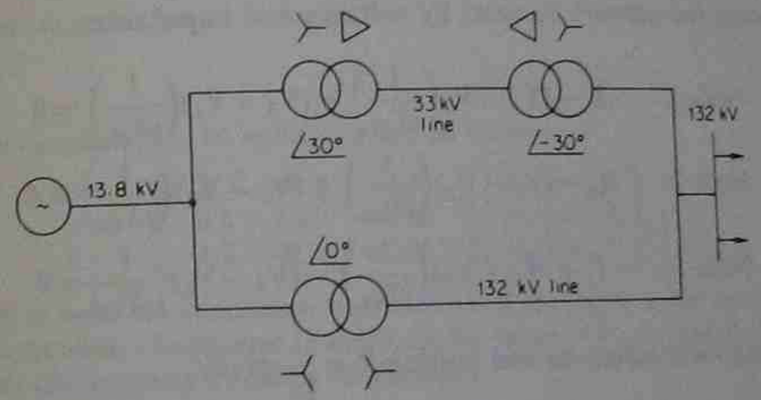


Figure 6.3 Loop with transformer phase shifts

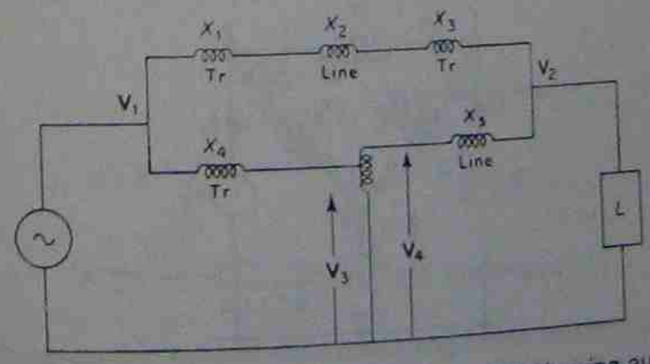


Figure 6.4 Equivalent circuit of network in Figure 6.3 showing autotransformer

Often, in lower voltage systems the out-of-balance voltage represented by the autotransformer can be neglected. If this is not the case, the best method of calculation is to determine the circulating current and consequent voltages due to the out-of-balance voltage acting alone, and then superpose these values on those obtained for operation with completely nominal voltage ratios.

6.3 Computation of Power Flows in a Network

Consider the three-node system of Figure 6.5.

With the voltages, currents, and line impedances as shown, we can formulate the following equations for each node using Kirchoff's laws:

$$\left.\begin{array}{ll} \text{Node 1:} & I_1 - I_{12} + I_{31} = 0 \\ \text{Node 2:} & I_2 - I_{23} + I_{12} = 0 \\ \text{Node 3:} & -I_3 + I_{23} - I_{31} = 0 \end{array}\right\} \quad (6.1)$$

(Note that currents flowing *into* each node are positive.)

Replacing the network currents by voltages and impedances, we obtain

$$\text{Node 1:} \quad \mathbf{I}_1 - (\mathbf{V}_1 - \mathbf{V}_2)\left(\frac{1}{\mathrm{j}X_{12}}\right) + (\mathbf{V}_3 - \mathbf{V}_1)\left(\frac{1}{\mathrm{j}X_{31}}\right) = 0$$

$$\text{Node 2:} \quad \mathbf{I}_2 - (\mathbf{V}_2 - \mathbf{V}_3)\left(\frac{1}{\mathrm{j}X_{23}}\right) + (\mathbf{V}_1 - \mathbf{V}_2)\left(\frac{1}{\mathrm{j}X_{12}}\right) = 0$$

$$\text{Node 3:} \quad -\mathbf{I}_3 + (\mathbf{V}_2 - \mathbf{V}_3)\left(\frac{1}{\mathrm{j}X_{23}}\right) - (\mathbf{V}_3 - \mathbf{V}_1)\frac{1}{(\mathrm{j}X_{32})} = 0$$

Rearranging these equations and putting $\mathbf{Y} = \dfrac{1}{\mathrm{j}X}$ gives:

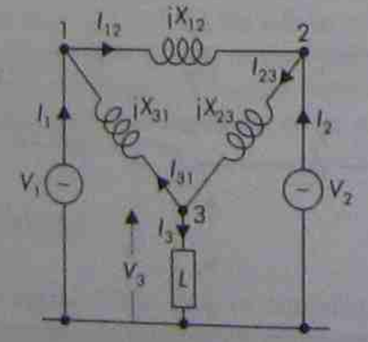


Figure 6.5 Three-node network for power-flow formulation

$$\mathbf{I}_1 = (\mathbf{Y}_{12} + \mathbf{Y}_{31})\mathbf{V}_1 + (-\mathbf{Y}_{12})\mathbf{V}_2 + (-\mathbf{Y}_{31})\mathbf{V}_3$$
$$\mathbf{I}_2 = (-\mathbf{Y}_{12})\mathbf{V}_1 + (\mathbf{Y}_{12} + \mathbf{Y}_{23})\mathbf{V}_2 + (-\mathbf{Y}_{23})\mathbf{V}_3$$
$$-\mathbf{I}_3 = (-\mathbf{Y}_{31})\mathbf{V}_1 + (-\mathbf{Y}_{23})\mathbf{V}_2 + (\mathbf{Y}_{31}\mathbf{Y}_{23})\mathbf{V}_3$$

These equations can be put into matrix form as:

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ -\mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} (\mathbf{Y}_{12} + \mathbf{Y}_{31}) & -\mathbf{Y}_{12} & -\mathbf{Y}_{31} \\ -\mathbf{Y}_{12} & (\mathbf{Y}_{12} + \mathbf{Y}_{23}) & -\mathbf{Y}_{23} \\ -\mathbf{Y}_{31} & -\mathbf{Y}_{23} & (\mathbf{Y}_{31} + \mathbf{Y}_{23}) \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} \quad (6.2)$$

It will be seen that the diagonal elements are the *sum* of the admittances connected to the node (known as self-admittances) whilst the off-diagonal elements are the admittances connected between the relevant nodes, and are negative. Consequently, if the currents on the left-hand side are known, the voltages can be calculated by inverting the Y matrix.

Example 6.3

Perform a load flow on the interconnected busbars shown in Figure 6.6.

Solution

The nodal equations may be written directly as follows:

$$\begin{bmatrix} 2 & -0.5 & -1.5 \\ -0.5 & 1.25 & -0.75 \\ -1.5 & -0.75 & +2.25 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

It is usual to write the admittance matrix direct. If the values in the current column vector are specified, it is required to determine the values of V_1, V_2, and V_3. [Y] may be inverted or the equations solved by elimination.

$$\text{As} \quad [\mathbf{Y}][\mathbf{V}] = [\mathbf{I}] \quad \text{then} \quad [\mathbf{Y}]^{-1}[\mathbf{I}] = [\mathbf{V}]$$

Hence it is required to invert the admittance matrix. A simple method will be used in this case:

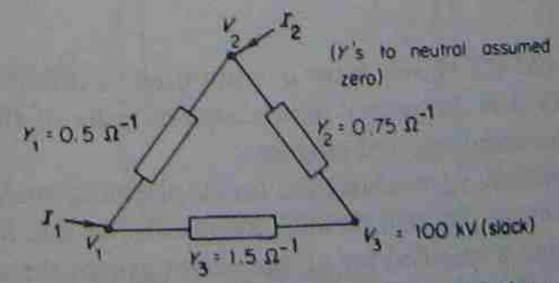


Figure 6.6 Three-busbar system–load flow using matrix inversion [note: Ω^{-1} is an alternative to *S* (siemen)]

$$V_1 = 0.5I_1 + 0.25V_2 + 0.75V_3$$

Substitute for V_1 in equations for I_2 and I_3:

$$-0.25I_1 + 1.125V_2 - 1.125V_3 = I_2$$
$$-0.75I_1 - 1.125V_2 + 1.125V_3 = I_3$$

Also, from the equation for I_2

$$V_2 = 0.222I_1 + 0.888I_2 + V_3$$

Substitute for V_2 in the other equations:

$$0.555I_1 + 0.222I_2 + V_3 = V_1$$
$$0.222I_1 + 0.888I_2 + V_3 = V_2$$
$$-I_1 - I_2 - 0V_3 = I_3$$

i.e.

$$\begin{bmatrix} 0.555 & 0.222 & 1.0 \\ 0.222 & 0.888 & 1.0 \\ -1 & -1 & 0.0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix}$$

The node at which the voltage is specified, the slack busbar, is node (3); hence the manipulation of the matrix into the form shown. Next, some loadings at the nodes will be specified:

$I_1 = 1000$ A (generator node)
$I_2 = 1500$ A (load node)
$V_3 = 100$ kV (phase value), slack busbar

Dealing in kiloamperes and kilovolts,

$$V_1 = 0.555(1) + 0.222(-1.5) + (+1)(100)$$
$$= 100.222 \text{ kV}$$
$$V_2 = 0.222(1) + 0.888(-1.5) + (1)(100)$$
$$= 98.89 \text{ kV}$$
$$I_3 = -(1)(1) + (1)(1.5) + 0(100)$$
$$= +0.5 \text{ kA i.e. a generator node}$$

(Note that $I_1 + I_2 + I_3 = 0$.)

Although Example 6.3 shows how a computed solution can be readily obtained, in practice it is the power inputs and outputs of the network that are specified, not the nodal injected currents.

A common calculation, particularly for quick planning studies and security calculations, is known as the 'real power', megawatt, or d.c. load flow. Here, the injected MW value is specified for all the nodes except the slack, where the voltage magnitude is given. It is required to find the MW flows in the network to satisfy the specified conditions.

Example 6.4

Perform a real-power load flow for the network shown in Figure 6.7.

Solution

An estimate may be obtained by assuming zero resistance in the network and using the fact that in a well-designed and well-operated system, all the voltages will be close to 1 p.u. such that P in p.u. is the same as I in p.u. Consequently, voltage drops given by PX around the network loops must sum to zero. Hence $\Sigma P_{ij}X_{ij} = 0$ where i and j refer to the branch between nodes i and j of the network. This follows from the fact that $\Delta V \propto /\delta \approx (XP/V)$ and the sum of the angular shifts round the loop is zero. Labelling the powers as in Figure 6.7, the following equations hold:

$$13.2P_1 + 2.06P_3 - 19.8P_4 - 6.6P_2 = 0$$
$$P_4 + P_3 = 100 \quad \text{or} \quad P_4 = 100 - P_3$$
$$P_2 - P_4 = 50 \quad P_1 = 60 + P_3$$
$$P_2 + P_1 = 210 \quad P_2 = 210 - 60 - P_3 = 150 - P_3$$
$$\therefore 13.2(60 + P_3) + 2.06(P_3) - 19.8(100 - P_3) - 6.6(150 - P_3) = 0$$

and

$$792 + 13.2P_3 + 2.06P_3 - 1980 + 19.8P_3 - 990 + 6.6P_3 = 0$$

From which $P_3 = 52.2$ MW, which is a reasonably accurate estimate.

Once P in BC is known, the flows in the other branches follow readily, i.e.

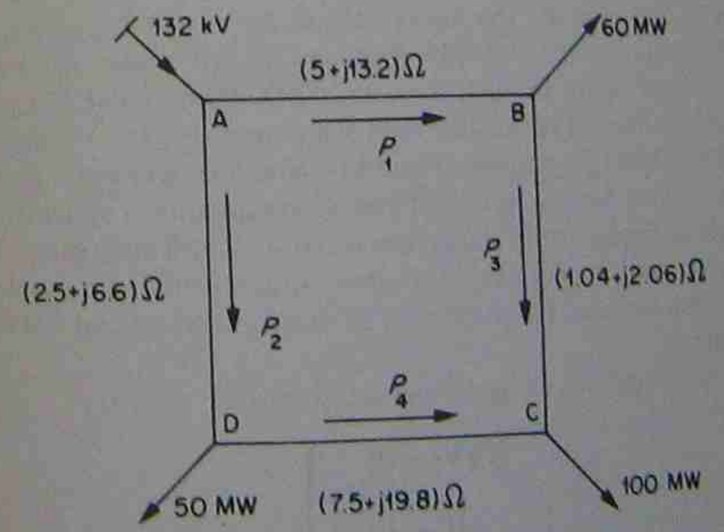


Figure 6.7 Line diagram of loop network for Example 6.4

$$P_4 = P_{DC} = 100 - 52.2 = 47.8\,\text{MW}$$
$$P_1 = P_{AB} = 60 + 52.2 = 112.2\,\text{MW}$$
$$P_2 = P_{AD} = 47.8 + 50 = 97.8\,\text{MW}$$

Quite often, for networks where $X/R > 10$, small changes to network flows, e.g. less than 10 per cent of existing flows, can be estimated by using the principle of superposition. For example, in the network of Figure 6.7, if node C supplied an extra 10 MW to the load, this would need to come from node A and flow over AD and DC in series, but in parallel with AB and BC in series. This 10 MW would divide in the ratio of (j13.2 + j2.06) to (j6.6 + j19.8) giving

$$10 \times \left(\frac{15.26}{15.26 + 26.4}\right)\text{MW} = 3.66\,\text{MW}$$

to be added to the path ADC. Hence P_2 would now become $97.8 + 3.66 = 101.46$ MW and $P_4 = 51.46$ MW. The remaining flow of $10 - 3.66 = 6.33$MW would be added to P_1 and P_3.

Using the same superposition principle it is possible to estimate the change in flows due to the removal of a circuit by injecting into the simplified network a flow equal and opposite to that previously in the removed circuit, and calculating the additional flows due to this negative injection. Such estimates are useful to assess the system security (see later in this chapter).

6.4 Complex Flows in Large Systems

In large practical power systems, for both planning and operational purposes it is necessary to carry out many load flows taking into account the complex impedances of the circuits, the limits caused by circuit capacities, and the voltages that can be satisfactorily provided at all nodes in the system. Systems consisting of up to 3000 busbars, 6000 circuits, and 500 generators may have to be solved in reasonable time scales (e.g. 1–2 min) with accuracies requiring 32 or 64 bits for numerical stability. Many system states possible in a day's operation may have to be considered. Consequently, a systematic way of computing the desired states of the system is required and an organized method for handling the data is necessary. Data base organization and rapid retrieval and modification are just as important as having an efficient and fast computation technique.

Basically, the equations to be solved are given by

$$\left.\begin{aligned}[\mathbf{Y}][\mathbf{V}] &= [\mathbf{I}] \\ [\mathbf{I}] &= \left[\frac{\mathbf{S}^*}{\mathbf{V}^*}\right]\end{aligned}\right\} \qquad (6.3)$$

where * denotes conjugate. This solution can only be obtained by iteration as the problem is non-linear, but iterative matrix inversion using Gaussian elimination can be employed as indicated in the next section.

6.4.1 Direct methods

Triangulation and partitioning

Direct methods solve only linear systems, i.e. $[\mathbf{Y}][\mathbf{V}] = [\mathbf{I}]$ where $[\mathbf{I}]$ is specified. The fact that powers are specified in practice makes the problem non-linear. A new value for $\mathbf{I}$ must be obtained from $\mathbf{S} = \mathbf{VI}^*$ after each direct solution and this value used to obtain a new one.

Partitioning is useful for handling large problems on small computers, for network reduction, and the elimination of unwanted nodes. A matrix

$$[\mathbf{Y}] = \left[\begin{array}{c|c|c} Y_{11} & Y_{12} & Y_{13} \\ \hline Y_{21} & Y_{22} & Y_{23} \\ \hline Y_{31} & Y_{32} & Y_{33} \end{array}\right] \qquad (6.4)$$

can be partitioned into four submatrices by the dividing lines shown in equation (6.4). Hence, in this typical admittance matrix for a network,

$$[\mathbf{Y}] = \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^t & \mathbf{D} \end{bmatrix}$$

where $\mathbf{C}^t$ is the transpose of $\mathbf{C}$,

$$\mathbf{B} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \quad \text{and so on}$$

A saving in computation time and storage is often obtained by manipulating the submatrices instead of the main one, and in the equation $[\mathbf{I}] = [\mathbf{Y}][\mathbf{V}]$ it is possible to eliminate nodes at which $\mathbf{I}$ is not injected. The matrix $[\mathbf{Y}]$ is partitioned so that the nodes to be eliminated are grouped together in a submatrix, e.g.

$$\begin{bmatrix} \mathbf{I}_w \\ \mathbf{I}_u \end{bmatrix} = \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^t & \mathbf{D} \end{bmatrix}\begin{bmatrix} \mathbf{V}_w \\ \mathbf{V}_u \end{bmatrix} \qquad (6.5)$$

where the subscript u indicates unwanted, and subscript w wanted, nodes. For example, if the currents at the unwanted nodes are zero,

$$\mathbf{I}_u = 0 \qquad -\mathbf{C}^t\mathbf{V}_w = \mathbf{D}\mathbf{V}_u \qquad \text{and} \qquad -\mathbf{D}^{-1}\mathbf{C}^t\mathbf{V}_w = \mathbf{V}_u$$

Substituting this value of $\mathbf{V}_u$ in the expression for $\mathbf{I}_w$ produces

$$\mathbf{I}_w = \mathbf{B}\mathbf{V}_w - \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^t\mathbf{V}_w$$

which gives a new admittance matrix containing only wanted nodes, i.e.

$$[\mathbf{Y}] = \mathbf{B} - \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^{t} \tag{6.6}$$

Methods are available for modifying the admittance matrix to allow for changes in the configuration of the network, say, due to line outages. These modifications can be performed such that a completely fresh inversion of the new admittance matrix is not necessary.

Most power-system matrices are symmetric, i.e. $\mathbf{Y}_{ij} = \mathbf{Y}_{ji}$, and it is only required to store elements on and above the diagonal. Frequently, most of the non-zero elements lie within a narrow band about the diagonal and, again, considerable savings in computer storage may be attained.

Accuracy When solving the set of equations $[\mathbf{A}][\mathbf{x}] = [\mathbf{b}]$, the residual vector $\mathbf{r} = [\mathbf{b}] - [\mathbf{A}][\mathbf{x}]$ is not zero because of rounding errors. Troubles arise when ill-conditioned equations are obtained in which, although the residual is small, the solution may be inaccurate. Such matrices are often large and sparse and for most rows the diagonal element is equal to the sum of the non-diagonal elements and of opposite sign.

6.4.2 Iterative methods

The two sets of equations of (6.3) are solved simultaneously as the iteration proceeds. A number of methods are available, and as three of these have gained wide adoption they will be described in detail. The first is the Gauss–Seidel method which has been widely used for many years and is simple in approach; the second is the Newton–Raphson method which, although more complex, has certain advantages. A third, which is almost an industry standard, is the fast decoupled load flow. The speed of convergence of these methods is of extreme importance as, apart from the cost of computer time, the use of these methods in schemes for the automatic control of power systems requires very fast load-flow solutions.

(i) The Gauss–Seidel method

In this method the unknown quantities are initially assumed and the value obtained from the first equation for, say V_1 is then used when obtaining V_2 from the second equation, and so on. Each equation is considered in turn and then the complete set is solved again until the values obtained for the unknowns converge to within required limits.

Application of the method to the simple network of Example 6.3 gives the nodal equations as

$$\begin{aligned} 2V_1 - 0.5V_2 - 1.5V_3 &= I_1 = 1 \\ -0.5V_1 + 1.25V_2 - 0.75V_3 &= I_2 = -1.5 \\ -1.5V_1 - 0.75V_2 + 2.25V_3 &= I_3 \end{aligned}$$

where the voltages are in kV and the currents in kA.

As V_3 is known, i.e. 100 kV (slack busbar voltage), it is necessary only to solve the first two equations. Initially, make $V_2 = 100$ kV.

$$\therefore {}^{1}V_1 = 0.5(1 + 150 + 50) = 100.5000$$

Using this value to evaluate V_2,

$${}^{1}V_2 = \frac{1}{1.25}(-1.5 + 75 + 50.25) = 99.0000$$

Using this value of V_2 to evaluate V_1,

$${}^{2}V_1 = 0.5(1 + 150 + 49.5) = 100.2500$$

$${}^{2}V_2 = \frac{1}{1.25}(-1.5 + 75 + 50.125) = 98.9000$$

$${}^{3}V_1 = 100.2250 \qquad {}^{2}V_2 = 98.8900$$

$${}^{4}V_1 = 100.2225 \qquad {}^{4}V_2 = 98.8890$$

$${}^{5}V_1 = 100.22225 \qquad {}^{5}V_2 = 98.8888$$

Iterations are now producing changes only in the fourth place of decimals and the process may be stopped; hence

$$I_3 = -1.5 \times 100.22225 - 0.75 \times 98.8888 + 2.25 \times 100 = 0.5\,\text{kA}$$

The solution has been obtained with much less computation than the more direct method used previously.

In the three-node system of Example 6.3, the iterative form of the three nodal equations with the nodal constraints that $\mathbf{S} = \mathbf{V}\mathbf{I}^{*}$ is obtained as follows (p indicates the iteration number and is *not* a power). For node 1:

$$\mathbf{I}_1 = \mathbf{V}_1\mathbf{Y}_{11} + \mathbf{V}_2\mathbf{Y}_{12} + \mathbf{V}_3\mathbf{Y}_{13}$$

$$\therefore \mathbf{V}_1 = -\mathbf{V}_2\frac{\mathbf{Y}_{12}}{\mathbf{Y}_{11}} - \mathbf{V}_3\frac{\mathbf{Y}_{13}}{\mathbf{Y}_{11}} + \frac{\mathbf{I}_1}{\mathbf{Y}_{11}}$$

$$\therefore \mathbf{V}_1^{*} = -\frac{\mathbf{Y}_{12}^{*}}{\mathbf{Y}_{11}^{*}}\mathbf{V}_2^{*} - \frac{\mathbf{Y}_{13}^{*}}{\mathbf{Y}_{11}^{*}}\mathbf{V}_3^{*} + \frac{\mathbf{I}_1^{*}}{\mathbf{Y}_{11}^{*}}$$

Substituting $\mathbf{I}_1^{*} = \mathbf{S}_1/\mathbf{V}_1$ and writing in the iterative form,

$$\mathbf{V}_1^{p+1} = -\frac{\mathbf{Y}_{12}}{\mathbf{Y}_{11}}\mathbf{V}_2^{p} - \frac{\mathbf{Y}_{13}}{\mathbf{Y}_{11}}\mathbf{V}_3^{p} + \frac{\mathbf{S}_1^{*}}{\mathbf{Y}_{11}}\frac{1}{\mathbf{V}_1^{*p}}$$

Similarly,

$$\mathbf{V}_2^{p+1} = -\frac{\mathbf{Y}_{21}}{\mathbf{Y}_{22}}\mathbf{V}_1^{p+1} - \frac{\mathbf{Y}_{23}}{\mathbf{Y}_{22}}\mathbf{V}_3^{p} + \frac{\mathbf{S}_2^*}{\mathbf{Y}_{22}}\frac{1}{\mathbf{V}_2^{*p}}$$

and

$$\mathbf{V}_3^{p+1} = -\frac{\mathbf{Y}_{31}}{\mathbf{Y}_{33}}\mathbf{V}_1^{p+1} - \frac{\mathbf{Y}_{32}}{\mathbf{Y}_{33}}\mathbf{V}_2^{p+1} + \frac{\mathbf{S}_3^*}{\mathbf{Y}_{33}}\frac{1}{\mathbf{V}_3^{*p}}$$

At any node i the already scanned nodes up to i will have new values appropriate to the $(p+1)$ iteration and the nodes yet to be scanned $(j > i)$ are appropriate to iteration p. Generally,

$$(\mathbf{V}_i^{p+1}) = +\frac{\mathbf{S}_i^*}{\mathbf{Y}_{ii}}\cdot\frac{1}{\mathbf{V}_i^{*p}} - \sum_{\substack{j>1\\ j\neq i}}\frac{\mathbf{Y}_{ij}}{\mathbf{Y}_{ii}}\mathbf{V}_j^{p} - \sum_{\substack{j>i\\ j\neq i}}\frac{\mathbf{Y}_{ij}}{\mathbf{Y}_{ii}}\mathbf{V}_i^{p+1} \qquad (6.7)$$

In this particular case, node 3 is the slack-bus, and as $\mathbf{V}_i$ is known, the equation for it is not required. It is seen in the above equations that the new value of $\mathbf{V}_i$ in the preceding equation is immediately used in the next equation, i.e. $\mathbf{V}_1^{p+1}$ is used in the $\mathbf{V}_2$ equation. Each node is scanned, in turn, over a complete iteration.

Equation (6.7) refers to a busbar with P and Q specified. At a generator node (i), usually V_i and P_i are specified with perhaps upper and lower limits to Q_i.

The magnitude of $\mathbf{V}_i$ is fixed, but its phase depends on Q_i. The values of $\mathbf{V}_i$ and Q_i from the previous iteration are not related by equation (6.7) because $\mathbf{V}_i$ has been modified to give a constant magnitude. It is necessary at the next iteration to calculate the value of Q_i corresponding to $\mathbf{V}_i$ from equation (6.7), i.e. Q_i = imaginary part of $\mathbf{S}_i$, i.e. of

$$\mathbf{Y}_{ii}^*\mathbf{V}_i^{p-1}\left[\mathbf{V}_i^{*p} + \sum_{\substack{j\neq i\\ j>i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p-1} + \sum_{\substack{j\neq i\\ j<i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p}\right]$$

This value of Q_i holds for existing value of $\mathbf{V}_i$ and is then substituted into

$$-\sum_{\substack{j\neq i\\ j>i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p} + \frac{P_i + \mathrm{j}Q_i}{\mathbf{Y}_{ii}^*\mathbf{V}_i^{p}}\sum_{\substack{j\neq i\\ j<i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p+1}$$

to obtain $(\mathbf{V}_i^{p+1})^*$.

The real and imaginary components of $\mathbf{V}_i^{p+1}$ are then multiplied by the ratio V_i/V_i^{p+1}, thus complying with the constant V_i constraint. This final step is a slight approximation, but the error involved is small and the saving in computation is large. The phase of $\mathbf{V}_i$ is thus found and the iteration can proceed to the next node. The process continues until the value of V^{p+1} at any node differs from V^p by a specified amount, a common figure being 0.0001 p.u. The study is commenced by assuming 1 p.u. voltage at all nodes except one; this exception is necessary in order that current flows may be obtained in the first iteration.

Acceleration factors The number of iterations required to reach the specified convergence can be greatly reduced by the use of acceleration factors. The correction in voltage from V^p to V^{p+1} is multiplied by such a factor so that the new voltage is brought closer to its final value, i.e.

$$\begin{aligned} {}^{1}\mathbf{V}^{p+1} &= \mathbf{V}^p + \alpha(\mathbf{V}^{p+1} - \mathbf{V}^p) \\ &= \mathbf{V}^p + \alpha\Delta\mathbf{V}^p \end{aligned}$$

where ${}^{1}\mathbf{V}^{p+1}$ is the accelerated new voltage. It can be shown that a complex value of α reduces the number of iterations more than a real value. However, up to the present, only real values seem to be used; the actual value depends on the nature of the system under study but a value of 1.6 is widely used. The Gauss–Seidel method with the use of acceleration factors is known as the method of successive overrelaxation.

Transformer tap-changing

Further changes which must be accommodated in the admittance matrix are those due to transformer tap-changing. When the ratio is at the nominal value the transformer is represented by a single series impedance, but when off-nominal, adjustments have to be made as follows.

Consider a transformer of ratio $t:1$ connected between two nodes i and j; the series admittance of the transformer is $\mathbf{Y}_t$. Referring to Figure 6.8(a), the following nodal-voltage equation holds:

$$\mathbf{I}_j = \mathbf{V}_j(\mathbf{Y}_{jr} + \mathbf{Y}_{j0} + \mathbf{Y}_{js} + \mathbf{Y}_t) - (\mathbf{V}_r\mathbf{Y}_{jr} + \mathbf{V}_x\mathbf{Y}_t + \mathbf{V}_s\mathbf{Y}_{js} + \mathbf{V}_0\mathbf{Y}_{j0})$$

As $\mathbf{V}_0 = 0$ and $\mathbf{V}_x = \mathbf{V}_i/t$,

$$\mathbf{I}_j + \mathbf{V}_s\mathbf{Y}_{js} = \mathbf{V}_j(\mathbf{Y}_{jr} + \mathbf{Y}_{j0} + \mathbf{Y}_{js} + \mathbf{Y}_t) - \left(\mathbf{Y}_{jr}\mathbf{V}_r + \frac{\mathbf{Y}_t\mathbf{V}_i}{t}\right)$$

where x is an artificial node between the voltage transforming element and the transformer admittance. From this last equation it is seen that for the node on

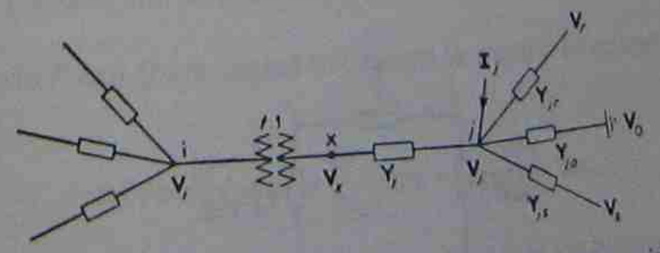


Figure 6.8(a) Equivalent circuit of transfofrmer with off-nominal tap ratio. Transformer series admittance on non-tap side

the off-tap side of the transformer (i.e. the more remote of the two nodes i and j), the following conditions apply:

when forming $\mathbf{Y}_{jj}$ use $\mathbf{Y}_t$ for the transformer, and when forming $\mathbf{Y}_{ij}$ use $\mathbf{Y}_t/t$ for the transformer.

It can similarly be shown for the tap-side node that the following conditions apply:

when forming $\mathbf{Y}_{ii}$ use $\mathbf{Y}_t/t^2$, and when forming $\mathbf{Y}_{ij}$ use $\mathbf{Y}_t/t$.

These conditions may be represented by the π section shown in Figure 6.8(b), although it is probably easier to modify the mutual and self-admittances directly.

(ii) The Newton–Raphson method

Although the Gauss–Seidel was the first popular method, the Newton–Raphson method was subsequently used increasingly. With some systems the latter gives a greater assurance of convergence and is at the same time economical in computer time.

The basic iterative procedure is as follows:

value at new iteration,

$$x^{p+1} = x^p - \frac{f(x^p)}{f'(x^p)}$$

extending this to a multi-equation system,

$$\mathbf{x}^{p+1} = \mathbf{x}^{(p)} - J^{-1}(\mathbf{x}^p) f(\mathbf{x}^p)$$

where the $\mathbf{x}$'s in f are column vectors and $J(\mathbf{x}^p)$ is a matrix known as the Jacobian matrix, of the form

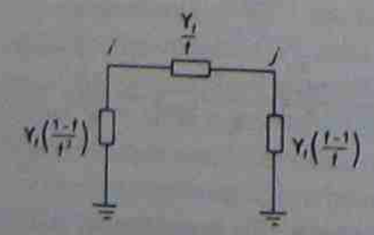


Figure 6.8(b) The π section to represent transformer with off-nominal tap ratio

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_k}{\partial x_k} & & \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} p\text{th iteration}$$

Consider now the application to an n-node power system, for a link connecting nodes k and j of admittance $\mathbf{Y}_{kj}$.

$$P_k + jQ_k = \mathbf{V}_k \mathbf{I}_k^* = \mathbf{V}_k \sum_{j=1}^{n-1} (\mathbf{Y}_{kj} \mathbf{V}_j)^*$$

Let

$$\mathbf{V}_k = a_k + jb_k \qquad \text{and} \qquad \mathbf{Y}_{kj} = G_{kj} - jB_{kj}$$

Then,

$$P_k + jQ_k = (a_k + jb_k) \sum_{1}^{n-1} [(G_{kj} - jB_{kj})(a_j + jb_j)]^* \tag{6.8}$$

from which,

$$P_k = \sum_{j=1}^{n-1} [a_k(a_j G_{kj} + b_j B_{kj}) + b_k(b_j G_{kj} - a_j B_{kj})] \tag{6.9}$$

$$Q_k = \sum_{j=1}^{n-1} [b_k(a_j G_{kj} + b_j B_{kj}) - a_k(b_j G_{kj} - a_j B_{kj})] \tag{6.10}$$

Hence, there are two non-linear simultaneous equations for each node. Note that $(n-1)$ nodes are considered because the slack node n is completely specified.

Changes in P and Q are related to changes in a and b by equations (6.9) and (6.10), e.g.

$$\Delta P_1 = \frac{\partial P_1}{\partial a_1} \Delta a_1 + \frac{\partial P_1}{\partial a_2} \Delta a_2 + \cdots \frac{\partial P_1}{\partial a_{n-1}} \Delta a_{n-1}$$

Similar equations hold in terms of ΔP and Δb, and ΔQ in terms of Δa and Δb.

The equations may be expressed generally in the following manner:

$$\begin{array}{|c|} \hline \Delta P_1 \\ \vdots \\ \Delta P_{n-1} \\ \hline \Delta Q_1 \\ \vdots \\ \Delta Q_{n-1} \\ \hline \end{array} = \begin{array}{|ccc|ccc|} \hline \frac{\partial P_1}{\partial a_1} & \cdots & \frac{\partial P_1}{\partial a_{n-1}} & \frac{\partial P_1}{\partial b_1} & \cdots & \frac{\partial P_1}{\partial b_{n-1}} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial P_{n-1}}{\partial a_1} & \cdots & \frac{\partial P_{n-1}}{\partial a_{n-1}} & \frac{\partial P_{n-1}}{\partial b_1} & \cdots & \frac{\partial P_{n-1}}{\partial b_{n-1}} \\ \hline \frac{\partial Q_1}{\partial a_1} & \cdots & \frac{\partial Q_1}{\partial a_{n-1}} & \frac{\partial Q_1}{\partial b_1} & \cdots & \frac{\partial Q_1}{\partial b_{n-1}} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial Q_{n-1}}{\partial a_1} & \cdots & \frac{\partial Q_{n-1}}{\partial a_{n-1}} & \frac{\partial Q_{n-1}}{\partial b_1} & \cdots & \frac{\partial Q_{n-1}}{\partial b_{n-1}} \\ \hline \end{array} \begin{array}{|c|} \hline \Delta a_1 \\ \vdots \\ \Delta a_{n-1} \\ \hline \Delta b_1 \\ \vdots \\ \Delta b_{n-1} \\ \hline \end{array} \tag{6.11}$$

For convenience, denote the Jacobian matrix by

$$\begin{array}{|c|c|} \hline J_A & J_B \\ \hline J_C & J_D \\ \hline \end{array}$$

The elements of the matrix are evaluated for the values of P, Q, and V at each iteration as follows.

For the submatrix J_A and from equation (6.9),

$$\frac{\partial P_k}{\partial a_j} = a_k G_{kj} - b_k B_{kj} \tag{6.12}$$

where $k \neq j$, i.e. off-diagonal elements.

Diagonal elements,

$$\frac{\partial P_k}{\partial a_k} = 2a_k G_{kk} + b_k B_{kk} - b_k B_{kk} + \sum_{\substack{j=1 \\ j \neq k}}^{n-1} (a_j G_{kj} + b_j B_{kj}) \tag{6.13}$$

This element may be more readily obtained by expressing some of the quantities in terms of the current at node k, $\mathbf{I}_k$, which can be determined separately at each iteration

Let

$$\mathbf{I}_k = c_k + jd_k = (G_{kk} - jB_{kk})(a_k + jb_k) + \sum_{\substack{j=1 \\ j \neq k}}^{n-1} (G_{kj} - jB_{jk})(a_j + jb_j)$$

from which,

$$c_k = a_k G_{kk} + b_k B_{kk} + \sum_{\substack{j=1 \\ j \neq k}}^{n-1} (a_j G_{kj} + b_j B_{kj})$$

and

$$d_k = b_k G_{kk} - a_k B_{kk} + \sum_{\substack{j=1 \\ j \neq k}}^{n-1} (b_j G_{kj} - a_j B_{kj})$$

So that,

$$\frac{\partial P_k}{\partial a_k} = a_k G_{kk} - b_k B_{kk} + c_k$$

For J_B,

$$\frac{\partial P_k}{\partial b_k} = a_k B_{kk} + b_k G_{kk} + d_k$$

and

$$\frac{\partial P_k}{\partial b_j} = a_k B_{kj} + b_k G_{kj} \qquad (k \neq j)$$

For J_C,

$$\frac{\partial Q_k}{\partial a_k} = a_k B_{kk} + b_k G_{kk} - d_k$$

and

$$\frac{\partial Q_k}{\partial a_j} = a_k B_{kj} + b_k G_{kj} \qquad (k \neq j)$$

For J_D,

$$\frac{\partial Q_k}{\partial b_j} = -a_k G_{kj} + b_k B_{kj} \qquad (k \neq j)$$

and

$$\frac{\partial Q_k}{\partial b_k} = -a_k G_{kk} + b_k B_{kk} + c_k$$

The process commences with the iteration counter 'p' set to zero and all the nodes except the slack-bus being assigned voltages, usually 1 p.u.

From these voltages, P and Q are calculated from equations (6.9) and (6.10). The changes are then calculated:

$$\Delta P_k^p = P_k \text{ (specified)} - P_k^p \text{ and } \Delta Q_k^p = Q_k \text{ (specified)} - Q_k^p$$

where p is the iteration number.

Next the node currents are computed as

$$I_k^p = \left(\frac{P_k^p + jQ_k^p}{V^p} \right)^* = c_k^p + jd_k^p$$

The elements of the Jacobian matrix are than formed, and from equation (6.11),

$$\begin{bmatrix}\Delta a\\ \Delta b\end{bmatrix} = \begin{bmatrix}J_A & J_B\\ J_C & J_D\end{bmatrix}^{-1}\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} \tag{6.14}$$

Hence, a and b are determined and the new values, $a_k^{p+1}k = a_k^p + \Delta a_k^p$ and $b_k^{p+1} = b_k^p + \Delta b_k^p$, are obtained. The process is repeated ($p = p + 1$) until ΔP and ΔQ are less than a prescribed tolerance.

The Newton–Raphson method has better convergence characteristics and for many systems is faster than the Gauss–Seidel method; the former has a much larger time per iteration but requires very few iterations (four is general), whereas the Gauss–Siedel requires at least 30 iterations, the number increasing with the size of system.

Acceleration factors may be used for the Newton–Raphson method. The quantities are frequently expressed in polar form.

The polar form of the equations has advantages and the equations are:

$$\begin{aligned} P_k &= P(V, \theta)\\ Q_k &= Q(V, \theta)\end{aligned} \tag{6.15}$$

The power at a bus is

$$\begin{aligned}\mathbf{S}_k &= P_k + jQ_k = \mathbf{V}_k\mathbf{I}_k^*\\ &= V_k\sum_{m\neq k} Y_{km}^* V_m^*\end{aligned} \tag{6.16}$$

$$P_k = \sum_{m\neq k} V_k V_m(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}) \tag{6.17}$$

$$Q_k = \sum_{m\neq k} V_k V_m(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km}) \tag{6.18}$$

where $\theta_{km} = \theta_k - \theta_m$.

For a load bus,

$$\begin{aligned}\Delta P_k &= \sum_{m\neq k}\frac{\partial P_k}{\partial\theta_m}\Delta\theta_m + \sum_{m\neq k}\frac{\partial P_k}{\partial V_m}\Delta V_m\\ \Delta Q_k &= \sum\frac{\partial Q_k}{\partial\theta_m}\Delta\theta_m + \sum\frac{\partial Q_k}{\partial V_m}\Delta V_m\end{aligned} \tag{6.19}$$

For a generator (P, V) busbar, only the ΔP_k equation is used as Q_k is not specified. The mismatch equation is

$$\begin{bmatrix}\Delta\mathbf{P}^{P-1}\\ \Delta\mathbf{Q}^{P-1}\end{bmatrix} = \begin{bmatrix}\mathbf{H}^{P-1} & \mathbf{N}^{P-1}\\ \mathbf{J}^{P-1} & \mathbf{L}^{P-1}\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{\theta}^P\\ \left(\dfrac{\Delta V^P}{\mathbf{V}^{P-1}}\right)\end{bmatrix} \tag{6.20}$$

$\Delta\theta^P$ is the correction to P, Q, and PV buses and $\Delta V^P/V^{P-1}$ is the correction to P, Q buses.

For buses k and m,

$$\begin{aligned}H_{km} &= \frac{\partial P_k}{\partial\theta_m} = V_k V_m(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km})\\ N_{km} &= V_m\frac{\partial P_k}{\partial V_m} = V_k V_m(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km})\\ J_{km} &= \frac{\partial Q_K}{\partial\theta_m} = -V_k V_m(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km})\\ L_{km} &= V_m\frac{\partial Q_m}{\partial V_m} = V_k V_m(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km})\end{aligned}$$

Also,

$$\begin{aligned}H_{kk} &= -Q_k - B_{kk}V_k^2\\ N_{kk} &= P_k + G_{kk}V_k^2\\ J_{kk} &= P_k^{\cdot} - G_{kk}V_k^2\\ L_{kk} &= Q_k - B_{kk}V_k^2\end{aligned}$$

In the above, admittance of the link km is $Y_{km} = G_{km} + jB_{km}$. The computational process can be enhanced by pre-ordering and dynamic ordering, defined as

- *Preordering*, in which nodes are numbered in sequence of increasing number of connections.
- *Dynamic ordering*, in which at each step in the elimination the next row to be operated on has the fewest non-zero terms.

(iii) Decoupled load flow

The coupling between P–θ an Q–V components is weak. Hence the equations can be reduced to

$$\begin{aligned}[\mathbf{P}] &= [\mathbf{T}][\theta]\\ [\mathbf{Q}] &= [\mathbf{U}][V - V_0]\end{aligned} \tag{6.21}$$

At reference node $\theta_0 = 0$ and $V_k = V_0$, so elements of **T** and **U** are given by

$$\begin{aligned}T_{km} &= -\frac{V_k V_m}{Z_{km}^2/X_{km}} \qquad U_{km} = -\frac{1}{Z_{km}^2/X_{km}}\\ T_{kk} &= -\sum_{m\neq k} T_{km} \qquad U_{kk} = -\sum_{m\neq k} U_{km}\end{aligned}$$

where Z_{km} and X_{km} are the branch impedance and reactance and $[U]$ is constant valued. Then

$$[\Delta \mathbf{P}] = [\mathbf{T}][\Delta\theta]$$
$$[\Delta \mathbf{Q}/\mathbf{V}] = [\mathbf{U}][\Delta \mathbf{V}]$$

These are solved alternatively. Advantage is obtained by using the following:

$$[\Delta \mathbf{P}/\mathbf{V}] = [\mathbf{A}][\Delta\theta]$$
$$[\Delta \mathbf{Q}/\mathbf{V}] = [\mathbf{C}][\Delta \mathbf{V}] \tag{6.22}$$

Fast decoupled load flow This makes the Jacobians of the decoupled method constant in value throughout the iteration. The following assumptions are made:

$$E_k = E_m = 1 \text{ p.u.}$$

$G_{km} \ll B_{km}$ and can be ignored; this is reasonable for lines and cables.

$$\cos(\theta_k - \theta_m) \approx 1$$
$$\sin(\theta_k - \theta_m) \approx 0$$

Hence,

$$[\Delta \mathbf{P}] = [\mathbf{B}][\Delta\theta]$$
$$[\Delta \mathbf{Q}] = [\mathbf{B}][\Delta \mathbf{V}]$$

where $B_{km} = -B_{km}$ for $m \neq k$, and

$$B_{kk} = \sum_{m \neq k} B_{km}$$

Further assumptions yield:

$$\left[\frac{\Delta P}{V}\right] = [B'][\Delta\theta]$$
$$\left[\frac{\Delta Q}{V}\right] = [B''][V] \tag{6.23}$$

where

$$B'_{km} = -\frac{1}{X_{km}} (m \neq k)$$
$$B'_{kk} = \sum_{m \neq k} \frac{1}{X_{km}}$$
$$B''_{km} = -B_{km} (m \neq k)$$
$$B''_{kk} = \sum_{m \neq k} B_{km}$$

Matrices B' and B'' are real and constant in value and need to be triangulated only once.

Example 6.5

Using the fast decoupled method calculate the angles and voltages after the first iteration for the three-node network described by the following admittance matrix (from Figure 6.9). V_1 is 230 kV, initially $V_2 = 220$ kV, $\theta_2 = 0$, and V_3 is 228 kV, $\theta_3 = 0$. Node 2 is a load consuming 200 MW, 120 MVAr; node 3 is a generator node set at 70 MW and 228 kV. V_1 is an infinite busbar.

$$\mathbf{Y} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.00819 - \text{j}0.049099 & -0.003196 + \text{j}0.019272 & -0.004994 + \text{j}0.030112 \\ 2 & -0.003196 + \text{j}0.019272 & 0.007191 - \text{j}0.043099 & -0.003995 + \text{j}0.02409 \\ 3 & -0.004994 + \text{j}0.030112 & -0.003995 + \text{j}0.02409 & 0.008989 - \text{j}0.053952 \end{array}$$

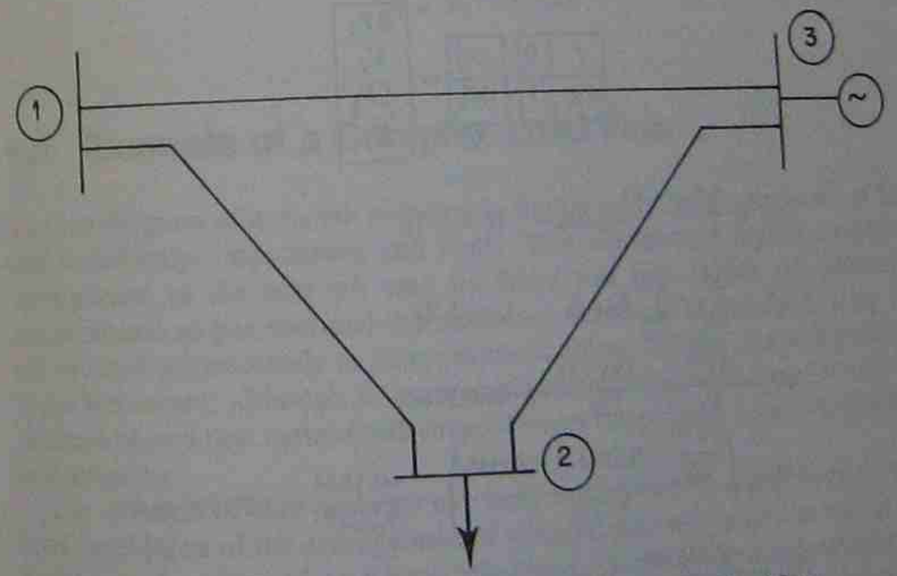


Figure 6.9 Three-node system for fast decoupled load flow

$$\begin{bmatrix} \frac{\Delta P_2}{V_2} \\ \frac{\Delta P_3}{V_3} \end{bmatrix} = \begin{array}{|cc|} \hline B'_{22} & B'_{23} \\ B'_{32} & B'_{33} \\ \hline \end{array} \begin{array}{|c|} \hline \Delta\theta_2 \\ \Delta\theta_3 \\ \hline \end{array}$$

$V_1 = 230\,\text{kV}, \quad V_2 = 220\,\text{kV}, \theta_2 = 0, \quad V_3 = 228\,\text{kV}, \theta_3 = 0$

$$B'_{22} = \frac{V_1}{X_{12}} + \frac{V_3}{X_{23}} = \frac{220}{50.5} + \frac{220}{40.4} = 9.80198$$

$$B'_{23} = -\frac{V_3}{X_{23}} = -\frac{220}{40.4} = -5.4455$$

$$B'_{32} = -\frac{V_2}{X_{23}} = -\frac{220}{40.4} = -5.4455$$

$$B'_{33} = \frac{V_1}{X_{13}} + \frac{V_2}{X_{23}} = \frac{220}{32.32} + \frac{220}{40.4} = 12.25247$$

$$C_{32} = L_{32} = \frac{B'_{32}}{B'_{22}} = -\frac{5.4455}{9.80198} = -0.5555$$

$$D_{32} = -B'_{22} = 9.80198$$

$$D_{33} = B'_{33} - \frac{B'_{32}B'_{23}}{B'_{22}} = 12.25247 - \frac{5.4455^2}{9.80198} = 9.2272$$

$$\Delta P_2^0 = -200 - 220(-0.003196 \times 230 + 0.00719 \times 220 - 0.003995 \times 228)$$
$$= -185.937\,\text{MW}$$

$$\Delta P_3^0 = 70 - 228(-0.004994 \times 230 - 0.003995 \times 220 + 0.008989 \times 228)$$
$$= 64.99\,\text{MW}$$

$$\begin{array}{|cc|} \hline D_{22} & \\ & D_{23} \\ \hline \end{array} \begin{array}{|cc|} \hline 1 & L_{23} \\ 0 & 1 \\ \hline \end{array} \begin{array}{|c|} \hline \Delta\theta_2 \\ \Delta\theta_3 \\ \hline \end{array} = \begin{array}{|c|} \hline \Delta\theta'_2 \\ \Delta\theta'_3 \\ \hline \end{array}$$

$$\begin{array}{|cc|} \hline 1 & 0 \\ C_{32} & 1 \\ \hline \end{array} \begin{array}{|c|} \hline \Delta\theta'_2 \\ \Delta\theta'_3 \\ \hline \end{array} = \begin{array}{|c|} \hline \frac{\Delta P_2}{V_2} \\ \frac{\Delta P_3}{V_3} \\ \hline \end{array}$$

$$\Delta\theta'_i = \frac{\Delta P_i}{V_i} - \sum_{k=2}^{i-1} C_{ik}\Delta\theta'_k \qquad (i = 2, 3)$$

$$\Delta\theta_i = \frac{\Delta\theta'_i}{D_{ii}} - \sum_{k=3}^{i+1} L_{ik}\Delta\theta_k \qquad (i = 3, 2)$$

$$\Delta\theta'_2 = \frac{\Delta P_2^{(0)}}{V_2} = \frac{-185.937}{220} = -0.845168$$

$$\Delta'\theta_3 = \left(\frac{\frac{64.99}{228} - 0.5555 \times 0.84516}{9.2272}\right) = \frac{-0.1844}{9.2272} = -0.1998\,\text{rad}$$

$$\Delta\theta'_2 = \frac{-0.845168}{9.80198} - 0.5555 \times 0.01988 = -0.09732\,\text{rad}$$

Hence,

$$\theta'_2 = \theta_2^{(0)} + \Delta\theta_2^{(0)} = -0.09732\,\text{rad} = -5.576^\circ$$
$$\theta'_3 = \theta_3^0 + \Delta\theta'_3 = -0.012\,\text{rad} = -1.14477^\circ$$

Considering reactive power at node 2,

$$\begin{aligned} \Delta Q_2^0 = Q_2 - V_2^0\{V_1[G_{21}\sin(\theta'_2 - \theta_1) - B_{21}\cos(\theta'_2 - \theta_1)] - V_2^0 B_{22} \\ + V_3^0[G_{23}\sin(\theta'_2 - \theta'_3) - B_{23}\cos(\theta'_2 - \theta'_3)]\} \\ = -120 - 220\{230[-0.003196\sin(-5.576 - 0) \\ - 0.019272\cos(-5.576 - 0)] - 220 \times 0.043099 \\ + 228[-0.003995\sin(-5.576 + 1.14477) \\ - 0.02409\cos(-5.576 + 1.14477)]\} = -59.476\,\text{MVAr} \end{aligned}$$

Hence,

$$\Delta V'_2 = \frac{\Delta Q_2^0}{B''_{22}V_2^0} = \frac{-59.476}{0.043049 \times 220} = -6.28\,\text{kV}$$

and

$$V_1 = 230\,\text{kV}$$
$$V'_2 = V_2^0 + \Delta V'_2 = 220 - 6.28 = 213.72\,\text{kV}$$
$$V'_3 = V_3^0 = 228\,\text{kV}$$

The process is repeated for the next iteration, and so on, until convergence is reached, when,

$$P_1 = 134.389\,\text{MW}$$
$$Q_1 = 56.77\,\text{MVAr}$$

6.5 Example of a Complex Load Flow

The line diagram of a system is shown in Figure 6.10, along with details of line and transformer impedances and loads. This represents a slightly simplified arrangement of the network used by Ward and Hale (1956). All quantities are expressed as per unit and the nodes are numbered as indicated. They are not ordered consecutively to show, in the solution, that the numbering system is not important, although in more sophisticated studies on larger systems it has been shown that certain orderings of nodes can produce faster convergence and solutions.

The solution of this problem has been carried out by digital computer; a brief description of the arrangement of essential data will be givn as well as the first iteration performed on a hand calculator. This program was developed for instructional purposes and is not as refined or sophisticated as commercial

Table 6.1

+5		n
+8		l
+1		Off-diagonal elements per row
+2		
+2		
+1		
+2		
+2		Column numbers
+3	+1	
+5	+2	
+5		
+4	+3	
+0.708238	−0.033932	Y_{12}/Y_{11}
+0.081853	+0.050025	Y_{23}/Y_{22}
+1.010063	−0/055033	Y_{21}/Y_{22}
+0.646931	−0.043097	Y_{35}/Y_{33} Off-diagonal elements $Y_{ij}/\mathbf{Y}_{ii}$
+0.353069	+0.043097	Y_{32}/Y_{33}
+0.640699	−0.052397	Y_{45}/Y_{44}
+0.726081	−0.090184	Y_{54}/Y_{55}
+0.292797	+0.087839	Y_{53}/Y_{55}
	Generation Table 1	
+1.000000	+0.000000	Voltage estimates, v_i
+1.000000	+0.000000	
+1.000000	+0.000000	
+1.000000	+0.000000	
−0.000369	+0.007706	$P+jQ/Y_{ii}^*$ values
−0.019534	+0.066298	
+0.179357	−0.162088	
−0.014702	+0.076068	
−0.046094	+0.058905	
+0.234539	+0.039069	$\mathbf{V}_S\mathbf{Y}_{is}/\mathbf{Y}_{ii}$ values slack-bus connections
+0	+0	
+0	+0	
+0.366206	+0.055921	
+0	+0	
+0.558269	−11.652234	$\mathbf{Y}_{ii}$ values
+0.444860	−8.164860	
+1.021401	−1.954524	
+0.433934	−5.306045	
+0.576541	−4.641795	
+0.223371	+0.037209	+101.000000 $\mathbf{Y}_{is}/Y_{ii}$ values + slacknumbers
+0.000000	+0.000000	+0.000000
+0.000000	+0.000000	+0.000000
+0.348767	+0.053259	+101.000000
+0.000000	+0.000000	+0.000000

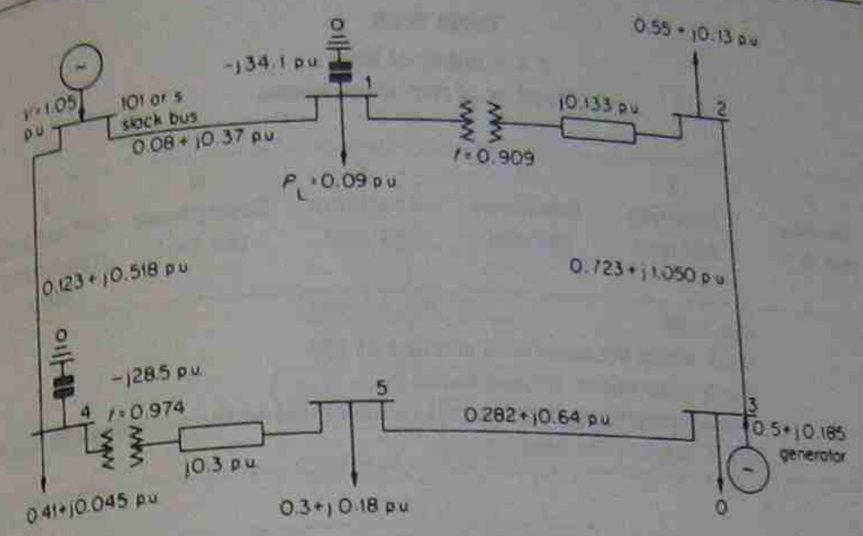


Figure 6.10 System for complex load flow using iterative method

programs; it is hoped, however, that the beginner will find it easier to understand than the latter. In view of the nature of equation (6.7), the basic system data will be modified to allow less computation during the actual solution of the equations; this will be obvious as the method is described. The system admittance-matrix [**Y**] is stored by specifying the following: order of matrix (n), number of off-diagonal elements, number of off-diagonal elements per row, column numbers of the off-diagonal elements in each row, a list of off-diagonal elements (Y_{ij}/Y_{ii}). Tables 6.1A and 6.1B show the tabular arrangement for input data and Table 6.1 shows the actual data.

In the tables, P and Q are considered positive for watts and lagging vars generated or supplied, and negative when received. It should be noted that the $\mathbf{Y}_{ij}$ values are, in fact, negative although they are shown as positive in the tables; this is useful as the $\mathbf{Y}_{ij}$ terms appear as negative in the nodal equations. The connections to the slack-busbar (V_s) are shown separately, no equation is necessary for the slack-nodes as the voltages are fully specified. The input data are shown in Table 6.1 and identification of the various quantities will be made easier by reference to the following system matrix:

$$\begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{Y}_{13} & \mathbf{Y}_{14} & \mathbf{Y}_{15} & \mathbf{Y}_{1s} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{Y}_{23} & \mathbf{Y}_{24} & \mathbf{Y}_{25} & \mathbf{Y}_{2s} \\ \mathbf{Y}_{31} & \mathbf{Y}_{32} & \mathbf{Y}_{33} & \mathbf{Y}_{34} & \mathbf{Y}_{35} & \mathbf{Y}_{3s} \\ \mathbf{Y}_{41} & \mathbf{Y}_{42} & \mathbf{Y}_{43} & \mathbf{Y}_{44} & \mathbf{Y}_{45} & \mathbf{Y}_{4s} \\ \mathbf{Y}_{51} & \mathbf{Y}_{52} & \mathbf{Y}_{53} & \mathbf{Y}_{54} & \mathbf{Y}_{55} & \mathbf{Y}_{5s} \end{bmatrix}$$

Table 6.1A

l = number of links
n = number of non-slack nodes

i Sending end note	j Receiving end note	R Resistance per unit	X Reactance per unit	B Susceptance per unit	t Off-nominal transformer ratio

n = 1–99
Slack nodes are numbered starting at 101
Earth connections are numbered 0
When representing transformers, node i must be the tap-side node and R and X refer to the non-tap side

Table 6.1B

l = number of links
n = number of non-slack nodes
s = number of slack nodes
(subscripts G = generation, subscript L = loads)

Node number	V_{real} per unit	V_{imag} per unit	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit

The initial part of the computer flow-diagram is shown in Figure 6.11.

Before proceeding with the first iteration, the calculation of the admittances associated with one of the transformers will be shown. Consider the transformer 1–2 of impedance j0.133 per unit and having $t = 0.909$. For the off-tap-side node 2, $\mathbf{Y}_{22}$ is formed using $\mathbf{Y}_t$ only, i.e.

$$\mathbf{Y}_{22} = \frac{1}{0.723 + \text{j}1.05} + \frac{1}{\text{j}0.133}$$
$$= 0.444860 - \text{j}8.164860$$
$$\mathbf{Y}_{21} = \frac{1}{\text{j}0.133} \cdot \frac{1}{0.909} = -\text{j}8.278000$$

The formation of the admittance matrix is necessary for both the Gauss–Seidel and Newton–Raphson methods. At this point, the methods differ, and the application of the Gauss–Seidel method will be given first with a complete computer solution, and then an outline will be given of the Newton–Raphson method applied to the problem. For node 1:

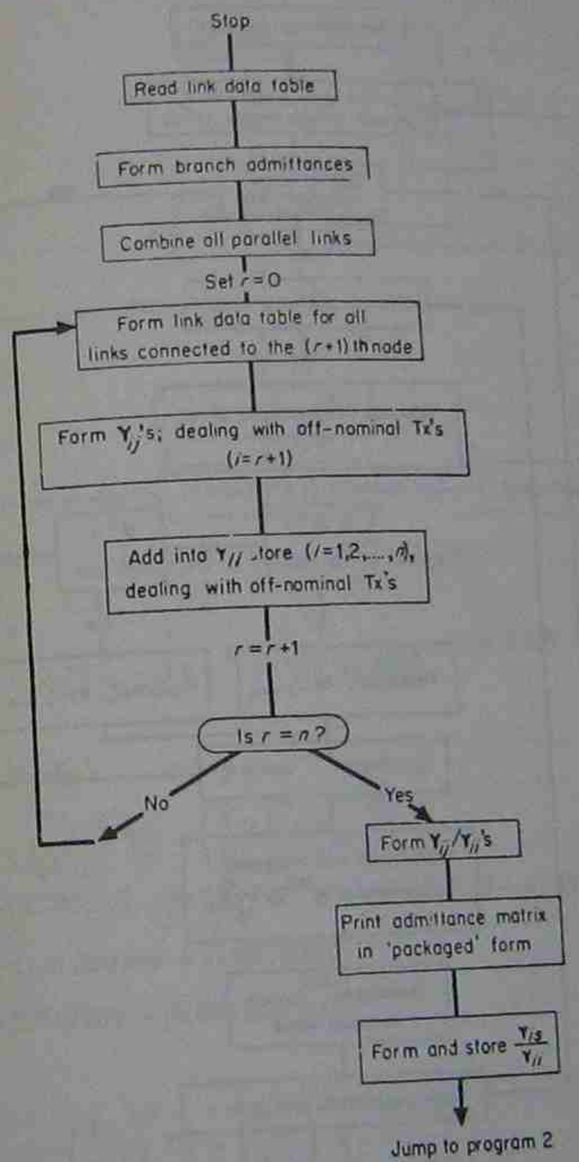


Figure 6.11(a) Initial part of the flow diagram for data processing of the load-flow program

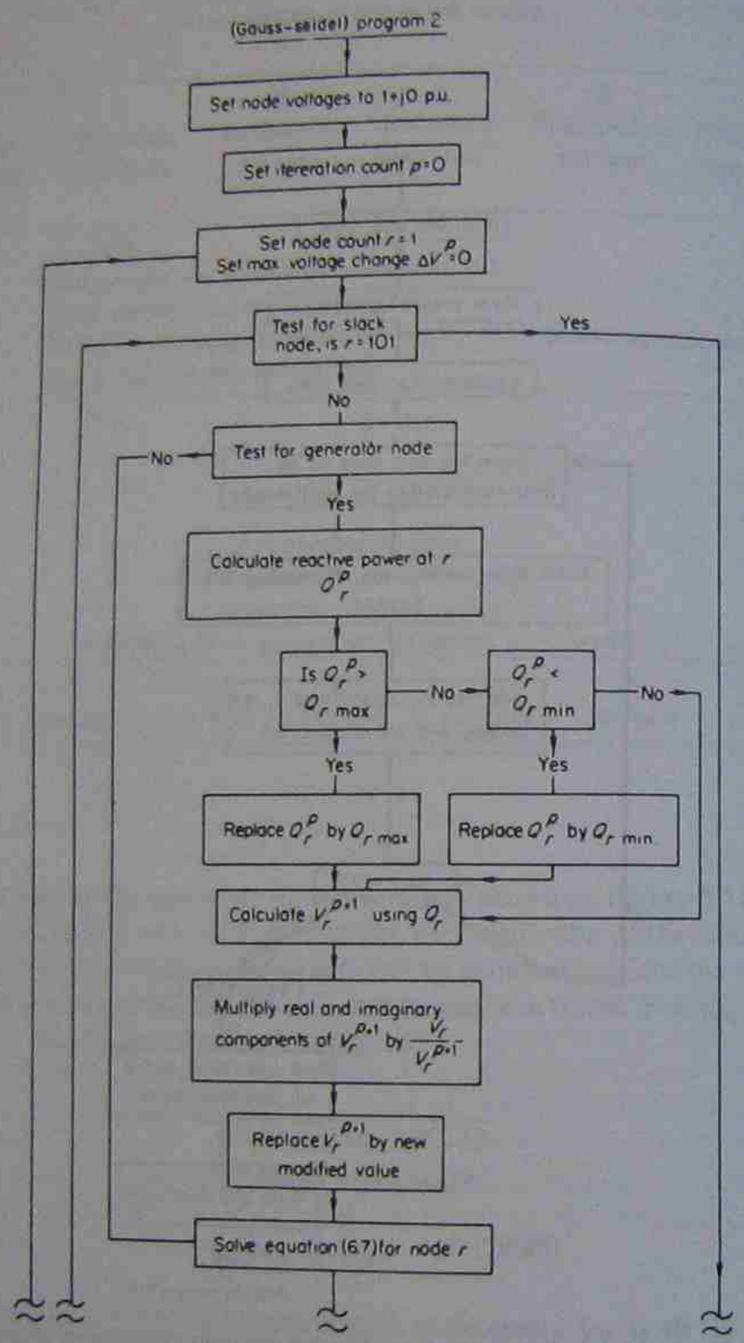


Figure 6.11(b) Flow diagram for Gauss–Seidel method *(continued opposite)*

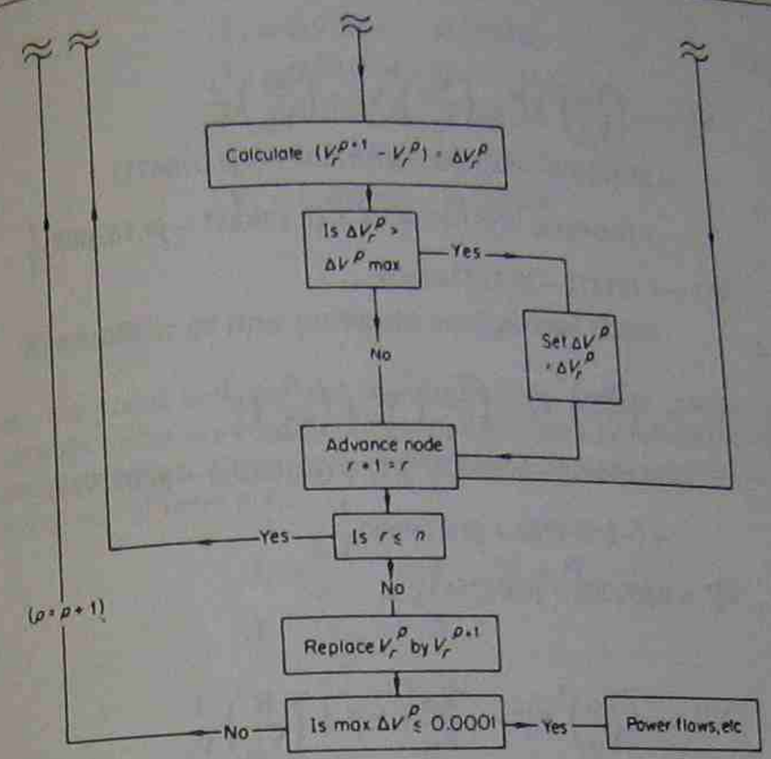


Figure 6.11(b) *(continued)*

$$Y_{11} = \frac{1}{0.08 + j0.37} + \frac{1}{-j34.1} + \frac{1}{j0.133} \cdot \frac{1}{0.909^2} = 0.558269 - j11.652234$$

$$Y_{12} = Y_{21}$$

First iteration. Node 1:

$$V_1^{1*} = -\left(\frac{Y_{12}}{Y_{11}}\right)^* V_2^* - \left(\frac{Y_{1s}}{Y_{11}}\right)^* V_s^* + \frac{S_1}{Y_{11}^*}\frac{1}{V_1}$$

$$= (0.708238 - j0.033932)^* 1^* + (0.234539 + j0.039069)^* 1.05^*$$

$$+ (-0.000369 + j0.007706)\frac{1}{1}$$

$$\therefore V_1^{1*} = 0.942409 + j0.002569 \text{ p.u.}$$

Node 2:

$$V_2^{1*} = -\left(\frac{Y_{21}}{Y_{22}}\right)^* V_1^1 - \left(\frac{Y_{23}}{Y_{22}}\right)^* V_3^* + \left(\frac{S_2}{Y_{22}^*}\right)\frac{1}{V_2}$$

$$= (1.010063 + j0.055033)(0.942409 + j0.002569)$$

$$+ (0.081853 - j0.050025)1 + (-0.019534 + j0.066298)\frac{1}{1}$$

$$\therefore V_2^{1*} = 1.014073 + j0.070671 \text{ p.u.}$$

Node 3:

$$\mathbf{V}_3^{1*} = -\left(\frac{\mathbf{Y}_{32}}{\mathbf{Y}_{33}}\right)^* \mathbf{V}_2^{1*} - \left(\frac{\mathbf{Y}_{35}}{\mathbf{Y}_{33}}\right)^* \mathbf{V}_5^* + \left(\frac{\mathbf{S}_3}{\mathbf{Y}_{33}^*}\right)\frac{1}{\mathbf{V}_3}$$

$$= (0.353069 - \text{j}0.043097)(1.014073 + \text{j}0.070671)$$

$$+ (0.646931 + \text{j}0.043097)1 + (0.179357 - \text{j}0.162088)\frac{1}{1}$$

$$\mathbf{V}_3^{1*} = 1.187371 - \text{j}0.137743 \text{ p.u.}$$

Node 4:

$$\mathbf{V}_4^{1*} = -\left(\frac{\mathbf{Y}_{45}}{\mathbf{Y}_{44}}\right)^* \mathbf{V}_5^* - \left(\frac{\mathbf{Y}_{4s}}{\mathbf{Y}_{44}}\right)^* \mathbf{V}_s^* + \left(\frac{\mathbf{S}}{\mathbf{Y}_{44}^*}\right)\frac{1}{\mathbf{V}_4}$$

$$= (0.640699 - \text{j}0.052397)(1) + (0.366206 - \text{j}0.055921)$$

$$+ (-0.014702 + \text{j}0.076068)\frac{1}{1}$$

$$\mathbf{V}_4^{1*} = 0.992202 + \text{j}0.072543 \text{ p.u.}$$

Node 5:

$$\mathbf{V}_5^{1*} = -\left(\frac{\mathbf{Y}_{54}}{\mathbf{Y}_{55}}\right)^* \mathbf{V}_4^{1*} - \left(\frac{\mathbf{Y}_{53}}{\mathbf{Y}_{55}}\right)^* \mathbf{V}_3^{1*} + \left(\frac{\mathbf{S}}{\mathbf{Y}_{55}^*}\right)\frac{1}{\mathbf{V}_5}$$

$$= (0.726081 + \text{j}0.090184)(1.010513 + \text{j}0.069748)$$

$$+ (0.292797 - \text{j}0.087839)(1.187371 - \text{j}0.137743)$$

$$+ (-0.046094 + \text{j}0.058905)\frac{1}{1}$$

$$\therefore \mathbf{V}_5^{1*} = 1.003342 - \text{j}0.056430 \text{ p.u.}$$

$$\mathbf{V}_s \text{ or } \mathbf{V}_{101} = 1.05000 + \text{j}0.000000 \text{ p.u.}$$

It will be noticed that the latest value of each nodal voltage is used. In this iteration no acceleration factor has been used (i.e. the factor = 1); if a factor of 1.6 is used, $\mathbf{V}_1^1$ becomes

$$\mathbf{V}_1 + 1.6(\mathbf{V}_1^1 - \mathbf{V}_1)$$

i.e.

$$1 + 1.6(-0.057591 - \text{j}0.002569)$$

or

$$0.907854 - \text{j}0.004110 \text{ p.u.}$$

This modified value of $\mathbf{V}_1^1$ should then be used to evaluate $\mathbf{V}_2^1$ when $\mathbf{V}_2^1$ is modified to $\mathbf{V}_2 + 1.6(\mathbf{V}_2^1 - \mathbf{V}_2)$, and so on. The busbar voltages after 30 iterations are

$$\mathbf{V}_1 = 0.918345 - \text{j}0.159312$$
$$\mathbf{V}_2 = 0.978674 - \text{j}0.221811$$
$$\mathbf{V}_3 = 1.101718 - \text{j}0.065242$$
$$\mathbf{V}_4 = 0.901468 - \text{j}0.194617$$
$$\mathbf{V}_5 = 0.903003 - \text{j}0.196604$$

6.5.1 Evaluation of line currents and power flows

Knowing the nodal voltages and admittances, the current, power, and var flows between nodes are readily obtained. Links with transformers, however, need special attention. Consider a transformer with its impedance referred to the non-tap side (Figure 6.8):

$$\mathbf{I}_i = \frac{\mathbf{I}_j}{t} \quad \text{and} \quad \mathbf{V}_i = t\mathbf{V}_x$$

$$\therefore \mathbf{I}_j = (\mathbf{V}_x - \mathbf{V}_j)\mathbf{Y}_t$$

$$= \left(\frac{\mathbf{V}_i}{t} - \mathbf{V}_j\right)\mathbf{Y}_t$$

The power at j is

$$P_j = \mathbf{V}_j\left[\left(\frac{\mathbf{V}_i}{t} - \mathbf{V}_j\right)\mathbf{Y}_t\right]^*$$

Also,

$$\mathbf{I}_i = \left(\frac{\mathbf{V}_i}{t^2} - \frac{\mathbf{V}_j}{t}\right)\mathbf{Y}_t$$

$$\therefore P_i = \mathbf{V}_i\left[\left(\frac{\mathbf{V}_i}{t^2} - \frac{\mathbf{V}_j}{t}\right)\mathbf{Y}_t\right]^*$$

Therefore power transferred i to j

$$= P_i - P_j$$

6.5.2 Application of the Newton–Raphson method to the system in Figure 6.10

The elements of the admittance matrix are determined as in the previous method, and it should be noted that for off-diagonal elements both G and B will be (-1) times the values derived from the network.

Let all the busbar voltages be assigned a voltage of $(1 + \text{j}0)$ p.u.

At node 3:

$$P_3 = 0.5 \qquad Q_3 = 0.185(\text{generated})$$

$$\mathbf{Y}_{32} = 0.444860 - \text{j}0.646063 \text{ p.u.}$$
$$\mathbf{Y}_{35} = 0.576541 - \text{j}1.308461 \text{ p.u.}$$
$$\mathbf{Y}_{33} = 1.021401 - \text{j}1.954524 \text{ p.u.}$$

$$a_3 = 1, b_3 = 0 \qquad a_2 = 1, b_2 = 0 \qquad a_5 = 1, b_5 = 0$$

First iteration ($p = 0$):

$$\begin{aligned} P_3^0 = {} & 1 \times 1 \times (-0.444860) + 1 \times 0 \times (-0.646063) \\ & + 0 \times 0 \times (-0.444860) - 0 \times 1 \times (-0.646063) \\ & + 1 \times 1 \times (+1.021401) + 1 \times 0 \times (1.954524) \\ & + 0 \times 0 \times (1.021401) - 0 \times 1 \times (1.954524) \\ & + 1 \times 1 \times (-0.576541) + 1 \times 0 \times (-1.308461) \\ & + 0 \times 0 \times (-0.576541) - 0 \times 1 \times (-1.308461) \\ = {} & 0.0 \text{ p.u.} \end{aligned}$$

Note that $\mathbf{Y}_{31}, \mathbf{Y}_{34}$ do not exist. (The above result would be expected in the initial case as all the involved voltages are equal to 1 p.u.) Similarly,

$$Q_3^0 = 0.0 \text{ p.u.}$$

Therefore,

$$\Delta P_3^0 = 0.5 - 0 = 0.5 \text{ p.u.}$$

and

$$\Delta Q_3^0 = 0.185 - 0 = 0.185 \text{ p.u.}$$

ΔP and ΔQ for the remaining non-slack nodes are similarly obtained.

$$\mathbf{I}_k^p = \frac{P_k^p - jQ_k^p}{(V_k^p)^*} \qquad \text{and} \qquad \mathbf{I}_3^0 = \frac{0.5 - j0.185}{1 - j0}$$

hence

$$c_3^0 = 0.5 \qquad \text{and} \qquad d_3^0 = -0.185$$

The elements of the Jacobian are determined next:

$$\begin{aligned} \frac{\partial P_3}{\partial a_3} &= a_3^0 G_{33} - b_3^0 B_{33} + C_3^0 \\ &= 1(1.021401) - 0.0 + 0.5 = 1.521401 \end{aligned}$$

$$\begin{aligned} \frac{\partial P_3}{\partial a_2} &= a_3^0 G_{32} - b_3^0 B_{32} \\ &= 1(-0.444860) + 0.0 \end{aligned}$$

$$\frac{\partial P_3}{\partial a_5} = -0.576541$$

$$\begin{aligned} \frac{\partial P_3}{\partial b_3} &= a_3^0 B_{33} + b_3^0 G_{33} + d_3^0 \\ &= 1(1.954520) + 0.0 + (-0.185) = 1.76952 \end{aligned}$$

$$\frac{\partial P_3}{\partial b_2} = a_3^0 B_{32} + b_3^0 G_{32} = 1(-0.646063) + 0.0$$

$$\frac{\partial P_3}{\partial b_5} = 1(-1.308461) + 0.0$$

Similarly, we obtain

$$\frac{\partial Q_3}{\partial a_3}, \quad \frac{\partial Q_3}{\partial a_2}, \quad \frac{\partial Q_3}{\partial a_5}, \quad \frac{\partial Q_3}{\partial b_3}, \quad \frac{\partial Q_3}{\partial b_2}, \quad \frac{\partial Q_3}{\partial b_5}$$

The Jacobian matrix for the first iteration is thus formed and inverted, and then Δa_k^0 and Δb_k^0 ($k = 1 \ldots 5$) are evaluated. hence, $V_k^{0+1} = V_k^0 + \Delta a^0 + j\Delta b^0$. The process is repeated until changes in real and reactive power at each bus are less than a prescribed amount, say 0.01 p.u.

6.5.3 Summary

The direct method involving matrix inverstion and a final iterative procedure to deal with the restraints at the nodes has advantages for smaller networks. For large networks the fast decoupled load flow (FDLF) method is preferable. System information (such as specified generation and loads, line and transformer series, and shunt admittances) is fed into the computer. The data are expressed in per unit on arbitrary MVA and voltage bases. The computer formulates the self- and mutual-admittances for each node.

If the system is very sensitive to reactive power flows, i.e. the voltages change considerably with change in load and network configuration, the computer program may diverge. It is preferable to allow the reactive-power outputs of generators to be initially without limits to ensure an initial convergence. Convergence having been attained, the computer evaluates the real and reactive power flows in each branch of the system, along with losses, absorption of vars, and any other information that may be required. Programs are available

which automatically adjust the tap settings of transformers to optimum values. Also, facilities exist for outputting only information regarding overloaded and underloaded lines; this is very useful when carrying out a series of load flows investigating the outages of plant and lines for security assessment purposes.

6.6 Optimal Power Flows

In Chapter 4, economic loading of generating plant, using 'B' coefficients to represent losses, was detailed using a differentiation and minimization method. In practice, non-linear minimization techniques with constraints are used to achieve a minimum of some function associated with the most economic operation of the system. Typical functions to be minimized are:

- total cost of system operation;
- total system losses;
- reactive requirements.

Nowadays, mathematical minimization techniques which guarantee a solution for many thousands of equations are available commercially. The emphasis is on speed, minimum data storage, accuracy, and easy data handling and interfacing. Normally, it is necessary to set up a scalar function, $f(x)$, to be minimized subject to constraints expressed as:

$$\min f(x)$$

subject to

$$g_i(x) \geqslant 0 \qquad \text{where } i = 1, 2 \ldots m$$

and

$$h_j(x) = 0 \qquad \text{where } j = 1, 2 \ldots p$$

In the case of a power system, $f(x)$ is often expressed as a cost function comprising the sum of generator costs plus the system losses; thus.

$$f(x) = \sum_{i=1}^{G} C_i P_i + \sum_{b=1}^{B} C_L P_L$$

where C_i is cost per unit of output of generation P_i and $C_L P_L$ is cost of losses in each circuit of B branches, subject to

$$\left.\begin{array}{l} \sum P_K = 0 \\ \text{and} \\ \sum Q_K = 0 \end{array}\right\} \text{at each node } k \text{ (Kirchoff's laws)}$$

and the following limits or constraints apply:

$$\left.\begin{array}{l} {}_{\min}P_i < P_i < P_{i\max} \\ {}_{\min}Q_i < Q_i < Q_{\max} \end{array}\right\} \quad \text{each generator } i$$

$${}_{\min}V_k < V_k < V_{k\max} \qquad \text{each node } k$$

$$S_{mn} < S_{mn\max} \qquad \text{for each circuit } m \text{ to } n$$

Additional constraints can be as follows:

$${}_{min}T_t < T_t < T_{t\max} \qquad \text{for each tap-change transformer t}$$
$${}_{\min}Q_c < Q_c < Q_{c\max} \qquad \text{for each compensator c}$$

Limits could also be added to ensure transient stability in the form:

$${}_{\min}\theta_{mn} < \theta < \theta_{mn\max} \qquad \text{where } \theta \text{ is an angle between node } m \text{ and node } n$$

More generally, it may be necessary to ensure that power P and var Q flows across defined boundaries in a network do not exceed pre-determined limits for security reasons. For example:

$$\sum_{mn\,\text{Set}S} P^s_{mn} \leqslant P_{\text{SpecC}} \qquad \text{where } P^S_{mn} \text{ is a set of flows whose sum must not exceed } P_{\text{SpecC}}$$

It can readily be seen that obtaining an optimum (minimum) solution is a complicated task, even with modern computers. The mathematics appropriate to this form of optimization is continually evolving and improving, including the use of novel techniques such as fuzzy logic, artificial neural networks, and genetic algorithms. Consequently, the employment of a computer library or commercial packages best suited to individual requirements is recommended.

Problems

6.1. A single-phase distributor has the following loads at the stated distances from the supply end: 10 kW at 10 m, 10 kW at 0.9 p.f. lagging at 16 m, 5 kW at 0.8 p.f. lagging at 91.5 m, and 20 kW at 0.95 p.f. lagging at 137 m. The loads may be assumed to be constant current at their nominal voltage values (240 V). If the supply voltage is 250 V and the maximum voltage drop is 5 per cent of the nominal value, determine the nearest commercially available conductor size.
(Answer: $r + 0.376x = 0.7 \times 10^{-3}$)

6.2 In the d.c. network shown in Figure 6.12, calculate the voltage at node B by inverting the admittance matrix. Check the answer by Thevenin's theorem.
(Answer: 247.7 V)

6.3 In the interconnected network shown in Figure 6.13, calculate the current in feeder BC.
(Answer: 26.75 A)

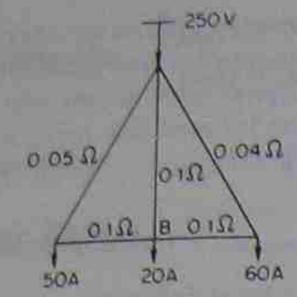


Figure 6.12 Network for Problem 6.2

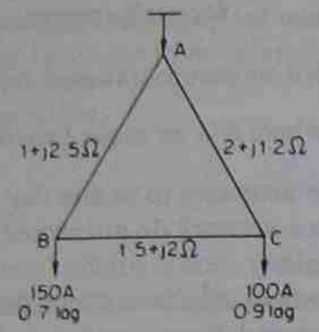


Figure 6.13 Network for Problem 6.3

6.4 In the network shown in Figure 6.14, the loads are represented by constant impedances $1 + j1$ p.u. Determine the current distributions in the network (a) when the transformer has its nominal ratio; (b) when the transformer is tapped up to 10 per cent.
(Note: determine the distribution with the off-nominal voltage alone and use superposition.)
(Answer: Transformer branch (a) 0; (b) $0.0735 - j0.075$ (p.u.))

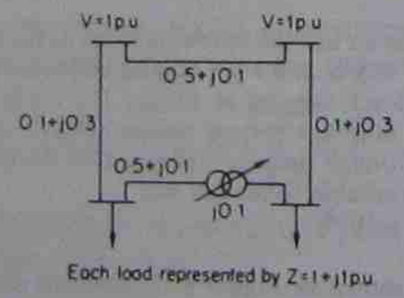


Figure 6.14 Network for Problem 6.4

6.5 Enumerate the information which may be obtained from a load-flow study. Part of a power system is shown in Figure 6.15. The line-to-neutral reactances and values of real

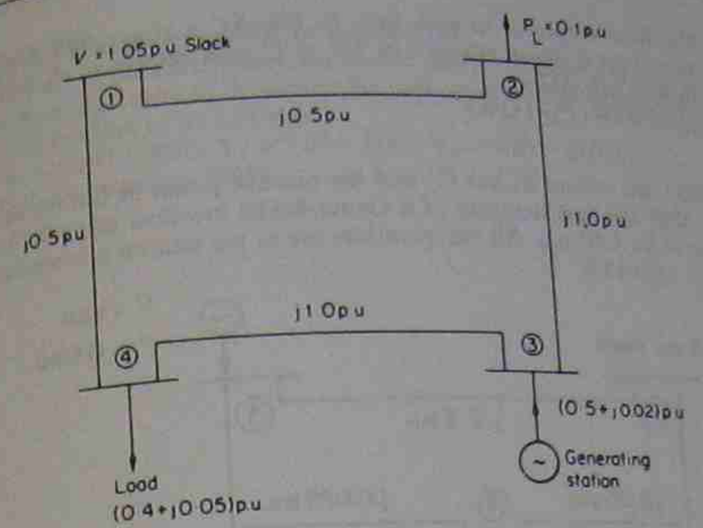


Figure 6.15 System for Problem 6.5

and reactive power (in the form $P \pm jQ$) at the various stations are expressed as per unit values on a common MVA base. Resistance may be neglected. By the use of an iterative method suitable for a digital computer, calculate the voltages at the stations after the first iteration without the use of an accelerating factor.
(Answer: $V_2 1.03333 - j0.03333$ p.u.; $V_3 1.11666 + j0.23333$ p.u.; $V_4 1.05556 + j0.00277$ p.u.)

6.6 A 400 kV interconnected system is supplied from bus A, which may be considered to be an infinite busbar. The loads and line reactances are as indicated in Figure 6.16.

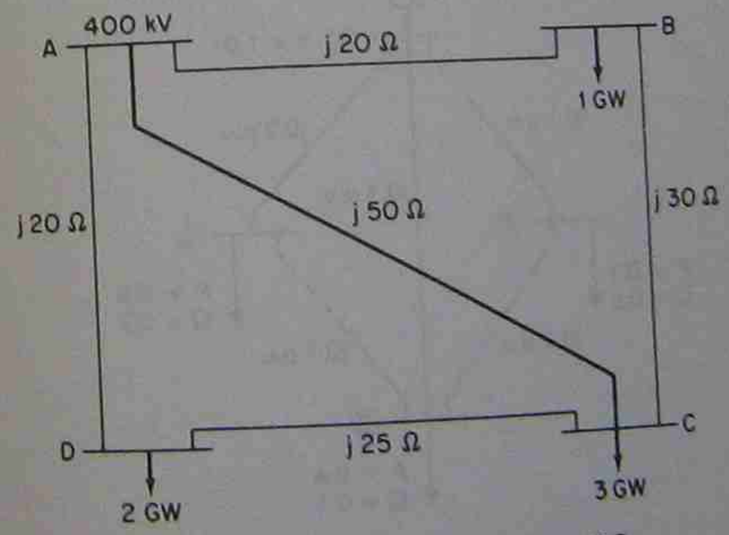


Figure 6.16 System for Problem 6.6

Determine the flow of power in each line. In line AC a phase shift transformer is installed which gives a phase advance of 10° at C with respect to A. Calculate the new power flow in line AC.
(Answer: P_{AC} 1.35 GW; P_{AC} 1 GW)

6.7 Determine the voltage at bus (2) and the reactive power at bus (3) as shown in Figure 6.17, after the first iteration of a Gauss–Seidel low-flow method. Assume the initial voltage to be 1∠0° p.u. All the quantities are in per unit on a common base.
(Answer: 0.99 − j0.0133)

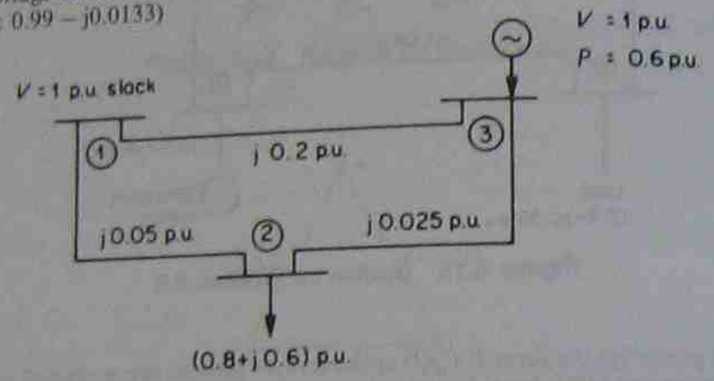


Figure 6.17 System for Problem 6.7

6.8 In Figure 6.18 the branch reactances and busbar loads are given in per unit on a common base. Branch resistance is neglected.

Explain briefly why an iterative method is required to determine the busbar voltages of this network.

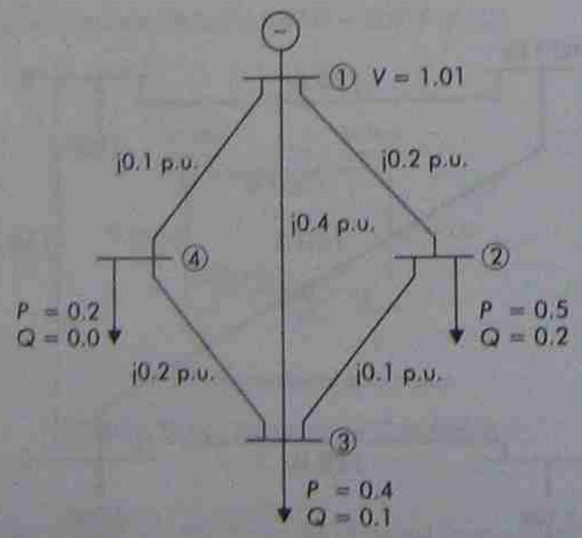


Figure 6.18 System for Problem 6.8

Form the bus admittance matrix for this network.
Using busbar 1 as the slack (reference) busbar, carry out the first iteration of a Gauss–Seidel load-flow algorithm to determine the voltage at all busbars. Assume the initial voltages of all busbars to be 1.01 p.u.
(Answer: $\mathbf{V}_2 = 0.997 - j0.03$, $\mathbf{V}_3 = 1.01 - j0.01$, $\mathbf{V}_4 = 1.01 - j0.026$)
(From E.C. Examination, 1995)

7
Fault Analysis

7.1 Introduction

An essential part of the design of a power supply network is the calculation of the currents which flow in the components when faults of various types occur. In a fault survey, faults are applied at various points in the network and the resulting currents are obtained by digital computation. The magnitude of the fault currents give the engineer the current settings for the protection to be used and the ratings of the circuit breakers. In some circumstances, the effect of open circuits may need investigation.

The types of fault commonly occurring in practice are illustrated in Figure 7.1, and the most common of these is the short circuit of a single conductor to ground or earth. Often, the path to earth contains resistance R_F in the form of an arc, as shown in Figure 7.1(f). Although the single-line-to-earth fault is the most common, calculations are frequently performed with the three-line, balanced short circuit (Figure 7.1(d) and (e)). This is the most severe fault and also the most amenable to calculation. The causes of faults are summarized in Table 7.1, which gives the distribution of faults, due to various causes, on the U.K. National Grid System in a typical year. Table 7.2 shows the components affected. In tropical countries the incidence of lightning is much greater than in the U.K., resulting in larger numbers of faults.

As well as fault current I_f, fault MVA is frequently used as a rating; this is obtained from the expression $\sqrt{(3)}V_L I_F \times 10^{-6}$, where V_L is the nominal line voltage of the faulted part. The fault MVA is often referred to as the *fault level*. The calculation of fault currents can be divided into the following two main types:

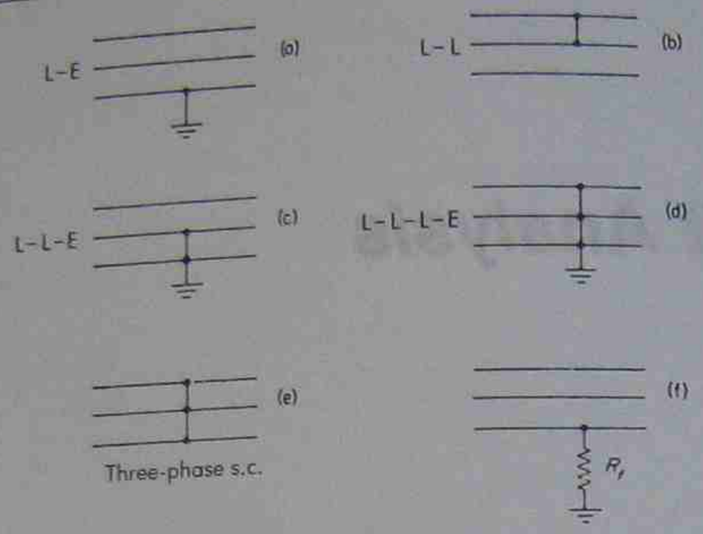


Figure 7.1 Common types of fault or short circuit (s.c.) L = line, E = earth

Table 7.1 Causes of overhead-line faults, British system 66 kV and above

	Faults/160 km of line/year
Lightning	1.59
Dew, fog, frost	0.15
Snow, ice	0.01
Gales	0.24
Salt spray	0.01
Total	2 faults per 160 km, giving a total of 232 faults on system/year

Table 7.2 Distribution of faults, NGC system

Type	Number of faults/year
Overhead lines	289
Cables	67
Switchgear	56
Transformers	59
Total	471

1. Faults short-circuiting all three phases when the network remains balanced electrically. For these calculations, normal single-phase equivalent circuits may be used as in ordinary load-flow calculations.
2. Faults other than three-phase short circuits when the network is electrically unbalanced. To facilitate these calculations a special method for dealing with unbalanced networks is used, known as the method of *symmetrical components*.

The main objects of fault analysis may be stated as follows:

1. to determine maximum and minimum three-phase short-circuit currents;
2. to determine the unsymmetrical fault current for single and double line-to-earth faults, line-to-line faults, and sometimes for open-circuit faults;
3. investigation of the operation of protective relays;
4. determination of rated rupturing capacity of breakers;
5. to determine fault-current distribution and busbar-voltage levels during fault conditions.

7.2 Calculation of Three-Phase Balanced Fault Currents

The action of synchronous generators on three-phase short circuits has been described in Chapter 3. There it was seen that, dependent on the time elapsing from the incidence of the fault, either the transient or the subtransient reactance should be used to represent the generator. For specifying switchgear, the value of the current flowing at the instant at which the circuit contacts open is required. It has been seen, however, that the initially high fault current, associated with the subtransient reactance, decays with the passage of time. Modern air-blast circuit breakers usually operate in 2.5 cycles and SF_6 breakers within 1.5 cycles of 60 Hz or 50 Hz alternating current, and are associated with extremely fast protection. Older circuit breakers and those on lower voltage networks usually associated with relatively cruder protection can take in the order of 8 cycles or more to operate. In calculations it is usual to use the subtransient reactance of generators and to ignore the effects of induction motors. The calculation of fault currents ignores the direct-current component, the magnitude of which depends on the instant in the cycle that the short circuit occurs. If the circuit breaker opens a reasonable time after the incidence of the fault, the direct-current component will have decayed considerably. With fast-acting circuit breakers the actual current to be interrupted is increased by the direct-current component and it must be taken into account. To allow for the

direct-current component of the fault current the symmetrical r.m.s. value is modified by the use of multiplying factors such as the following:

- 8-cycle circuit breaker opening time, multiply by 1;
- 3-cycle circuit breaker opening time, multiply by 1.2;
- 2-cycle circuit breaker opening time, multiply by 1.4.

Consider an initially unloaded generator with a short circuit across the three terminals, as shown in Figure 7.2. The generated voltage per phase is E and therefore the short-circuit current is $[E/Z(\Omega)]A$, where Z is either the transient or subtransient impedance. If Z is expressed in per unit notation,

$$Z\ (\text{p.u.}) = \frac{I_{FL} Z(\Omega)}{E}$$

$$\therefore\ Z\ (\Omega) = \frac{EZ\ (\text{p.u.})}{I_{FL}}$$

Taking I_{FL} (full-load or rated current) and E as base values of voltage and current, the short-circuit current

$$I_{s.c.} = \frac{E}{Z\,(\Omega)} = \frac{EI_{FL}}{EZ\,(\text{p.u.})} = \frac{I_{FL}}{Z\ (\text{p.u.})} \tag{7.1}$$

Also, the three-phase short-circuit volt-amperes

$$= \sqrt{(3)} V_L I_{s.c.} = \frac{\sqrt{(3)} V_L I_{FL}}{Z\,(\text{p.u.})}$$

$$= \frac{\text{Base volt-amperes}}{Z\,(\text{p.u.})} \tag{7.2}$$

Hence the short-circuit level is immediately obtained if the impedance Z (p.u.) from the source of the voltage to the point of the fault is known.

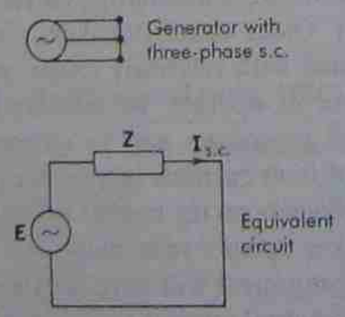


Figure 7.2 Voltage source with short circuit and equivalent circuit

Example 7.1

An 11.8 kV busbar is fed from three synchronous generators having the following ratings and reactances,

20 MVA, X' 0.08 p.u.; 60 MVA, X' 0.1 p.u.; 20 MVA, X' 0.09 p.u.

Calculate the fault current and MVA if a three-phase symmetrical fault occurs on the busbars. Resistance may be neglected. The voltage base will be taken as 11.8 kV and the VA base as 60 MVA.

Solution

The transient reactance of the 20 MVA machine on the above bases is $(60/20) \times 0.08$, i.e. 0.24 p.u., and of the 20 MVA machine $(60/20) \times 0.09$, i.e. 0.27 p.u. These values are shown in the equivalent circuit in Figure 7.3. As the generator e.m.f.s are assumed to be equal, one source may be used (Figure 7.3(c)). The equivalent reactance is

$$X_{eq} = \frac{1}{1/0.24 + 1/0.27 + 1/0.1} = 0.056\ \text{p.u.}$$

Therefore fault MVA

$$= \frac{60}{0.056} = 1071\ \text{MVA}$$

and fault current

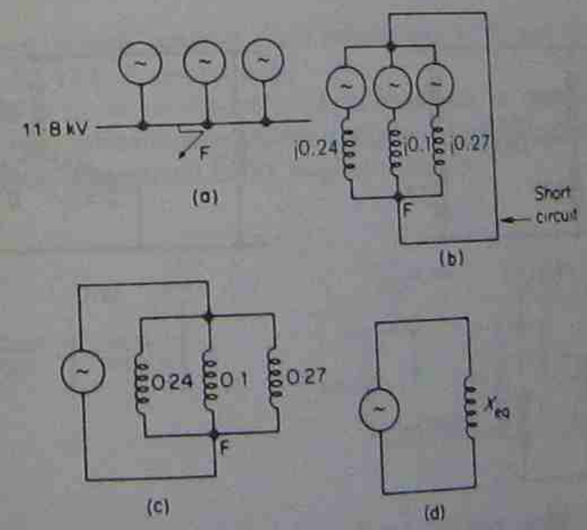


Figure 7.3 Line diagram and equivalent circuits for Example 7.1

$$= \frac{1071 \times 10^6}{\sqrt{(3)} \times 11\,800}$$
$$= 52\,402\,\text{A}$$

Example 7.2

In the network shown in Figure 7.4, a three-phase fault occurs at point F. Calculate the fault MVA at F. The per unit values of reactance all refer to a base of 100 MVA. Resistance may be neglected.

Solution

The equivalent single-phase network of generator and line reactances is shown in Figure 7.4(b). This is replaced by the network shown in Figure 7.4(c) by the use of the delta-

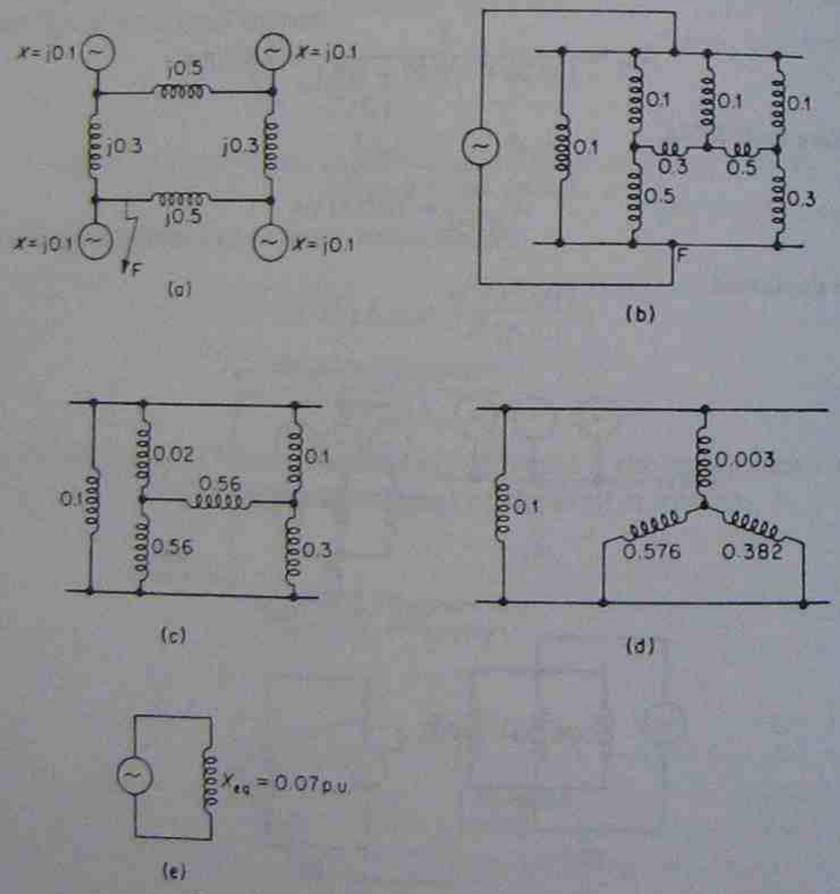


Figure 7.4 Line diagram and equivalent circuits for Example 7.2

star transformation. A further transformation is carried out on the network in Figure 7.4(d) to give the final single equivalent reactance,

$$X_{eq} = 0.07\,\text{p.u.}$$

The fault level at point F

$$= \frac{100}{0.07} = 1430\,\text{MVA}$$

This value is based on the assumption of symmetrical fault current. Allowing for a multiplying factor of 1.4, the fault level is 2000 MVA.

7.2.1 Current limiting reactors

The impedances presented to fault currents by transformers and machines when faults occur on substation or generating station busbars are low. To reduce the high fault current which would do considerable damage mechanically and thermally, artificial reactances are sometimes connected between bus sections. These current-limiting reactors usually consist of insulated copper strip embedded in concrete formers; this is necessary to withstand the high mechanical forces produced by the current in neighbouring conductors. The position in the circuit occupied by the reactor is a matter peculiar to individual designs and installations (see Figure 7.5(a) and (b)).

Example 7.3

A 400 kV power system contains three substations A, B, and C having fault levels (GVA) of 20, 20, and 30, respectively.

The system is to be reinforced by three lines each of reactance j5 Ω connecting together the three substations as shown in Figure 7.6(a). Calculate the new fault level at C (three-phase symmetrical fault). Neglect resistance.

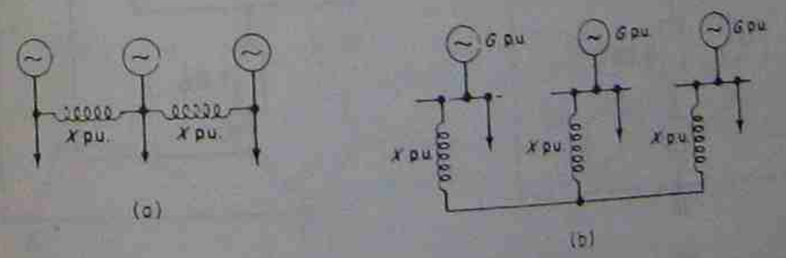


Figure 7.5 Connection of artificial reactors: (a) ring system; (b) tie-bar system. Transient reactance of machines, *G* p.u.; reactance of artificial reactors *X* p.u.

Solution

The equivalent circuits for the new system are shown in Figure 7.6(a) and (b). The substations are represented by a voltage source (400 kV) in series with the value of reactance to give the specified initial fault level, e.g. for A, the fault current (before reinforcement)

$$= \frac{20 \times 10^9}{\sqrt{3} \times 400\,000}\ \text{A}$$

Hence the effective reactance 'behind' A when subject to a three-phase short circuit

$$= \frac{400\,000}{\sqrt{3}} \times \frac{\sqrt{(3)} \times 400\,000}{20 \times 10^9} = j8\,\Omega$$

Similarly, the effective reactance at B = j8 Ω and at C = j5.33 Ω.

The 5 Ω mesh is transformed into a star with arms of value 1.66 Ω and the equivalent circuit with the fault on C after reinforcement is shown in Figure 7.6(b). Fault current at C with equivalent reactance j2.9 Ω gives:

$$= \frac{400\,000}{\sqrt{3}} \times \frac{1}{j2.9} = -j79.635\,\text{kA}$$

Fault level at C

$$= \sqrt{3} \times 400 \times 79.6\,\text{MVA} = 55\,148\,\text{MVA}$$

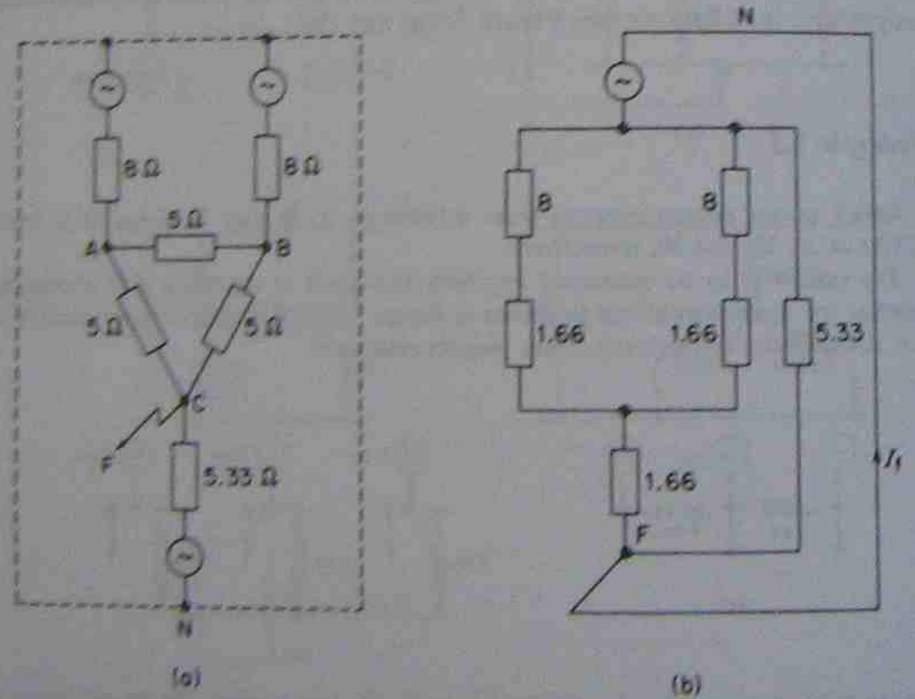


Figure 7.6 (a) Equivalent networks for Example 7.3. (b) Reduced network

Note that this value is very high and that the result of reinforcement is always to increase the fault levels. Maximum rating of a 400 kV circuit breaker is about 50 GVA, hence either a busbar must be run split between two incoming feeders or current limiters must be installed.

7.3 Method of Symmetrical Components

This method formulates a system of three separate phasor systems which, when superposed, give the unbalanced conditions in the circuit. It should be stressed that the systems to be discussed are essentially artificial and used merely as an aid to calculation. The various sequence-component voltages and currents do not exist as physical entities in the network, although they could be monitored by special filters.

The method postulates that a three-phase unbalanced system of voltages and currents may be presented by the following three separate systems of phasors:

1. a balanced three-phase system in the normal a-b-c (red-yellow-blue) sequence, called the *positive phase-sequence* system;
2. a balanced three-phase system of reversed sequence, i.e. a-c-b (red-blue-yellow), called the *negative phase-sequence* system;
3. three phasors equal in magnitude and phase revolving in the positive phase rotation, called the *zero phase-sequence* system.

In Figure 7.7 an unbalanced system of currents is shown with the corresponding system of symmetrical components. If each of the red-phase phasors are added, i.e. $\mathbf{I}_{1R} + \mathbf{I}_{2R} + \mathbf{I}_{0R}$, the resultant phasor will be I_R in magnitude and direction; similar reasoning holds for the other two phases.

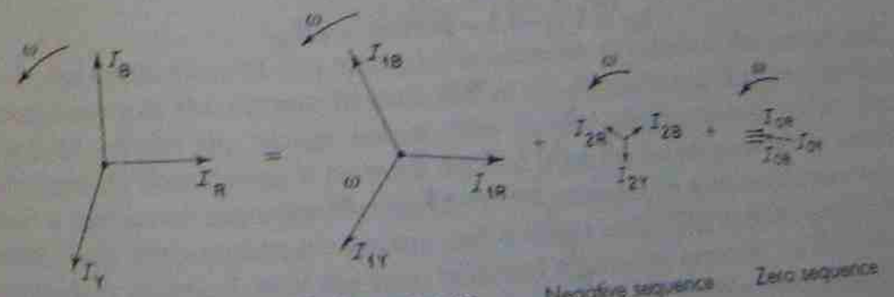


Figure 7.7 Real system and corresponding symmetrical components

To express the phasors algebraically, use is made of the complex operator 'a' (sometimes denoted by λ or $\mathbf{h}$) which denotes a phase-shift operation of $+120°$ and a multiplication of unit magnitude; i.e.

$$V\angle\phi \times \mathbf{a} = V\angle\phi \times 1\angle 120° = V\angle\phi + 120°, \mathbf{a} = e^{j2\pi/3} \text{ and } \mathbf{a}^3 = e^{j3\times 2\pi/3} = 1$$

Also,

$$\mathbf{a}^2 + \mathbf{a} = (-0.5 - j0.866) + (-0.5 + j0.866) = -1$$

$$\therefore \mathbf{a}^3 + \mathbf{a}^2 + \mathbf{a} = 0$$

and

$$1 + \mathbf{a} + \mathbf{a}^2 = 0$$

For positive-sequence phasors, taking the red phasor as reference,

$$\mathbf{I}_{1R} = \mathbf{I}_{1R}e^{j0} = \text{ reference phasor} \qquad \text{(Figure 7.8)}$$
$$\mathbf{I}_{1Y} = \mathbf{I}_{1R}(-0.5 - j0.866) = \mathbf{a}^2\mathbf{I}_{1R}$$

and

$$\mathbf{I}_{1B} = \mathbf{I}_{1R}(-0.5 + j0.866) = \mathbf{a}\mathbf{I}_{1R}$$

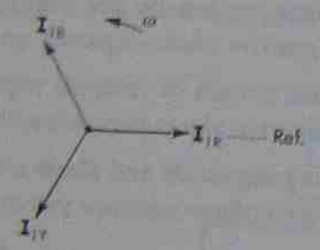


Figure 7.8 Positive-sequence phasors

For negative-sequence quantities,

$$\mathbf{I}_{2R} = \mathbf{I}_{2R}(1 + j0) \qquad \text{(Figure 7.9)}$$
$$\mathbf{I}_{2Y} = \mathbf{I}_{2R}(-0.5 + j0.866) = \mathbf{a}\mathbf{I}_{2R}$$
$$\mathbf{I}_{2B} = \mathbf{I}_{2R}(-0.5 - j0.866) = \mathbf{a}^2\mathbf{I}_{2R}$$

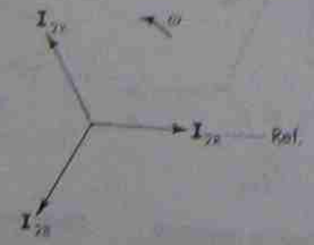


Figure 7.9 Negative-sequence phasors

Returning to the original unbalanced system of currents, $\mathbf{I}_R$, $\mathbf{I}_Y$, and $\mathbf{I}_B$,

$$\mathbf{I}_R = \mathbf{I}_{1R} + \mathbf{I}_{2R} + \mathbf{I}_{0R}$$
$$\mathbf{I}_Y = \mathbf{I}_{1Y} + \mathbf{I}_{2Y} + \mathbf{I}_{0Y} = \mathbf{a}^2\mathbf{I}_{1R} + \mathbf{a}\mathbf{I}_{2R} + \mathbf{I}_{0R}$$
$$\mathbf{I}_B = \mathbf{I}_{1B} + \mathbf{I}_{2B} + \mathbf{I}_{0B} = \mathbf{a}\mathbf{I}_{1R} + \mathbf{a}^2\mathbf{I}_{2R} + \mathbf{I}_{0R}$$

Hence in matrix form,

$$\begin{bmatrix} \mathbf{I}_R \\ \mathbf{I}_Y \\ \mathbf{I}_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{0R} \\ \mathbf{I}_{1R} \\ \mathbf{I}_{2R} \end{bmatrix} \tag{7.3}$$

Inverting the matrix,

$$\begin{bmatrix} \mathbf{I}_{0R} \\ \mathbf{I}_{1R} \\ \mathbf{I}_{2R} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{I}_R \\ \mathbf{I}_Y \\ \mathbf{I}_B \end{bmatrix} \tag{7.4}$$

The above also holds for voltages, i.e.

$$[\mathbf{E}_{\text{actual}}] = [\mathbf{T}_s][\mathbf{E}_{1,2,0}]$$

where $[\mathbf{T}_s]$ is the symmetrical component transformation matrix,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}$$

In a three-wire system the instantaneous voltages and currents add to zero and therefore no single-phase component is required. A fourth wire or connection to earth must be provided for single-phase currents to flow. In a three-wire system the zero phase-sequence components are replaced by zero in equations (7.3) and (7.4). Also,

$$\mathbf{I}_{0R} = \frac{\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B}{3} = \frac{\mathbf{I}_N}{3}$$

where $\mathbf{I}_N$ is the neutral current.

$$\therefore \mathbf{I}_N = 3\mathbf{I}_{0R} = 3\mathbf{I}_{0Y} = 3\mathbf{I}_{0B}$$

In the application of this method it is necessary to calculate the symmetrical components of the current in each line of the network and then to combine them to obtain the actual values. The various phase-sequence values are obtained by considering a network derived from the actual network in which only a particular sequence current flows; for example, in a zero-sequence network, only zero-sequence currents and voltages exist. The positive-sequence network is identical with the real, balanced equivalent network, i.e. it is the same as used for three-phase symmetrical short-circuit studies. The negative-sequence network is almost the same as the real one except that the values of

impedance used for rotating machines are different and there are no generated voltages from an ideal machine. The zero-sequence network is considerably different from the real one.

The above treatment assumes that the respective sequence impedances in each of the phases are equal, i.e. $\mathbf{Z}_{1R} = \mathbf{Z}_{1Y} = \mathbf{Z}_{1B}$, etc. Although this covers most cases met in practice, unequal values may occur in certain circumstances, e.g. an open circuit on one phase. The following equation applies for the voltage drops across the phase impedances:

$$\begin{bmatrix} \mathbf{V}_R \\ \mathbf{V}_Y \\ \mathbf{V}_B \end{bmatrix} = [\mathbf{T}_S] \begin{bmatrix} \mathbf{V}_{0R} \\ \mathbf{V}_{1R} \\ \mathbf{V}_{2R} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_R & & \\ & \mathbf{Z}_Y & \\ & & \mathbf{Z}_B \end{bmatrix} [\mathbf{T}_S] \begin{bmatrix} \mathbf{I}_{0R} \\ \mathbf{I}_{1R} \\ \mathbf{I}_{2R} \end{bmatrix}$$

From which,

$$\mathbf{V}_{0R} = \tfrac{1}{3}\mathbf{I}_{1R}(\mathbf{Z}_R + \mathbf{a}^2\mathbf{Z}_Y + \mathbf{a}\mathbf{Z}_B) + \tfrac{1}{3}\mathbf{I}_{2R}(\mathbf{Z}_R + \mathbf{a}\mathbf{Z}_Y + \mathbf{a}^2\mathbf{Z}_B) + \tfrac{1}{3}\mathbf{I}_{0R}(\mathbf{Z}_R + \mathbf{Z}_Y + \mathbf{Z}_B)$$

and, similarly, expressions $\mathbf{V}_{1R}$ and $\mathbf{V}_{2R}$ may be obtained.

It is seen that the voltage drop in each sequence is influenced by the impedances in all three phases. If, as previously assumed, $\mathbf{Z}_R = \mathbf{Z}_Y = \mathbf{Z}_B = \mathbf{Z}$, then the voltage drops become $\mathbf{V}_{1R} = \mathbf{I}_{1R}\mathbf{Z}$, etc., as before.

7.4 Representation of Plant in the Phase-Sequence Networks

7.4.1 The synchronous machine (see Table 3.1)

The positive-sequence impedance Z_1 is the normal transient or subtransient value. Negative-sequence currents set up a rotating magnetic field in the opposite direction to that of the positive-sequence currents and which rotates round the rotor surface at twice the synchronous speed; hence the effective impedance (Z_2) is different from Z_1. The zero-sequence impedance Z_0 depends upon the nature of the connection between the star point of the windings and the earth and the single-phase impedance of the stator windings in parallel. Resistors or reactors are frequently connected between the star point and earth for reasons usually connected with protective gear and the limitation of overvoltages. Normally, the only voltage sources appearing in the networks are in the positive-sequence one, as the generators only generate positive-sequence e.m.f.s.

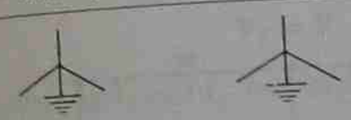
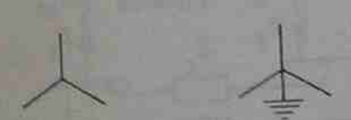
7.4.2 Lines and cables

The positive- and negative-sequence impedances are the normal balanced values. The zero-sequence impedance depends upon the nature of the return path through the earth if no fourth wire is provided. It is also modified by the presence of an earth wire on the towers that protects overhead lines against lightning strikes. In the absence of detailed information the following rough guide to the value of Z_0 may be used. For a single-circuit line $(Z_0/Z_1) = 3.5$ with no earth wire and $Z_0/Z_1 = 2$ with one earth wire. For a double-circuit line, $(Z_0/Z_1) = 5.5$. For underground cables, $(Z_0/Z_1) = 1 - 1.25$ for single-core and 3–5 for three-core cables.

7.4.3 Transformers

The positive- and negative-sequence impedances are the normal balanced ones. The zero-sequence connection of transformers is, however, complicated, and depends on the nature of the connection of the windings. Table 7.3 lists the zero-sequence representation of transformers for various winding arrangements. Zero-sequence currents in the windings on one side of a transformer must produce the corresponding ampere-turns in the other. But three in-phase

Table 7.3

Connections of windings		Representation per phase	Comments
Primary	Secondary		
			Zero-sequence currents free to flow in both primary and secondary circuits
			No path for zero-sequence currents in primary circuits
			Single-phase currents can circulate in the delta but not outside it
			No flow of zero-sequence currents possible
			No flow of zero-sequence currents possible
			Tertiary winding provides path for zero-sequence currents

currents cannot flow in a star connection without a connection to earth. They can circulate round a delta winding, but not in the lines outside it. Owing to the mutual impedance between the phases, $Z_0 \neq Z_1$. For a three-limb transformer, $Z_0 < Z_1$; for a five-limb transformer, $Z_0 > Z_1$.

An example showing the nature of the three sequence networks for a small transmission link is shown in Figure 7.10.

7.5 Types of Fault

In the following, a single voltage source in series with an impedance is used to represent the power network as seen from the point of the fault. This is an extension of Thevenin's theorem to three-phase systems. It represents the general method used for manual calculation, i.e. the successive reduction of

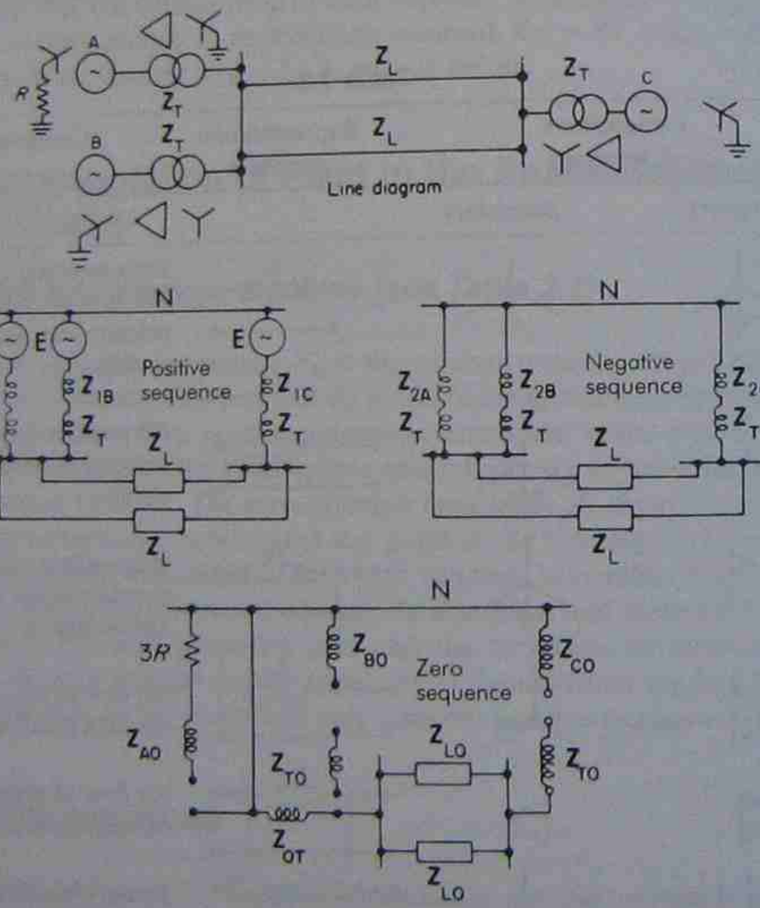


Figure 7.10 Typical transmission link and form of associated sequence networks

the network to a single impedance and voltage or current source. The network is assumed to be initially on no-load before the occurrence of the fault, and linear, so that superposition applies.

7.5.1 Single-line-to-earth fault

The three-phase circuit diagram is shown in Figure 7.11, where the three phases are on open-circuit at their ends.

Let $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_0$ be the symmetrical components of $\mathbf{I}_R$ and let $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_0$ be the components of $\mathbf{V}_R^f$. For this condition, $\mathbf{V}_R^f = 0$, $\mathbf{I}_B = 0$, and $\mathbf{I}_Y = 0$. Also, $\mathbf{Z}_R$ includes components $\mathbf{Z}_1$, $\mathbf{Z}_2$, and $\mathbf{Z}_0$.

From equation (7.4)

$$\mathbf{I}_0 = \tfrac{1}{3}(\mathbf{I}_R + \mathbf{I}_B + \mathbf{I}_Y)$$
$$\mathbf{I}_1 = \tfrac{1}{3}(\mathbf{I}_R + \mathbf{a}\mathbf{I}_Y + \mathbf{a}^2\mathbf{I}_B)$$
$$\mathbf{I}_2 = \tfrac{1}{3}(\mathbf{I}_R + \mathbf{a}^2\mathbf{I}_Y + \mathbf{a}\mathbf{I}_B)$$

Hence,

$$\mathbf{I}_0 = \frac{\mathbf{I}_R}{3} = \mathbf{I}_1 = \mathbf{I}_2 \qquad (\text{as } \mathbf{I}_B = \mathbf{I}_Y = 0)$$

Also,

$$\mathbf{V}_R = \mathbf{E} - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_0\mathbf{Z}_0 = 0$$

Eliminating $\mathbf{I}_0$ and $\mathbf{I}_2$, we obtain

$$\mathbf{E} - \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_0) = 0$$

hence

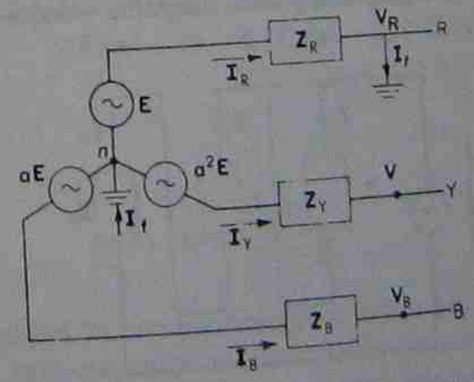


Figure 7.11 Single line-to-earth fault—Thevenin equivalent of system at point of fault

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_0} \tag{7.5}$$

The fault current,

$$\mathbf{I}_f = \mathbf{I}_R = 3\mathbf{I}_1$$

so

$$\mathbf{I}_f = \frac{3\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_0} \tag{7.6}$$

The e.m.f. of the Y phase $= \mathbf{a}^2\mathbf{E}$, and (from equation (7.4))

$$\mathbf{I}_Y = \mathbf{I}_0 + \mathbf{a}^2\mathbf{I}_1 + \mathbf{a}\mathbf{I}_2$$

$$\therefore \mathbf{V}_Y = \mathbf{a}^2\mathbf{E} - \mathbf{I}_0\mathbf{Z}_0 - \mathbf{a}^2\mathbf{I}_1\mathbf{Z}_1 - \mathbf{a}\mathbf{I}_2\mathbf{Z}_2$$

The pre-fault and post-fault phasor diagrams are shown in Figure 7.12, where it should be noted that only $\mathbf{V}_{YB}$ remains at its pre-fault value.

It is usual to form an equivalent circuit to represent equation (7.5) and this can be obtained from an inspection of the equations. The circuit is shown in Figure 7.13 and it will be seen that $\mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I}_0$, and

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_0}$$

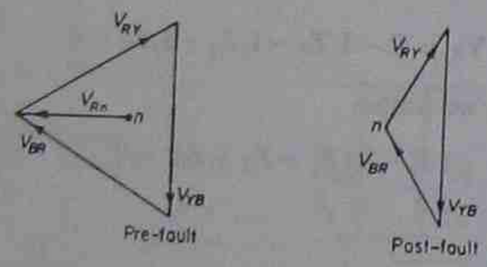


Figure 7.12 Pre- and post-fault phasor diagrams—single-line-to-earth fault

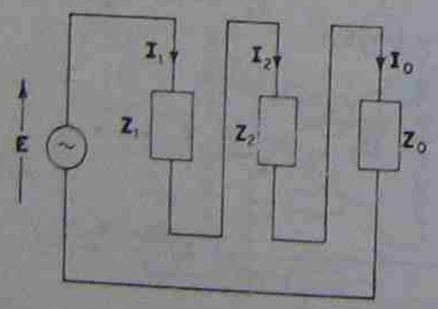


Figure 7.13 Interconnection of positive-, negative-, and zero-sequence networks for single-line-to-earth faults

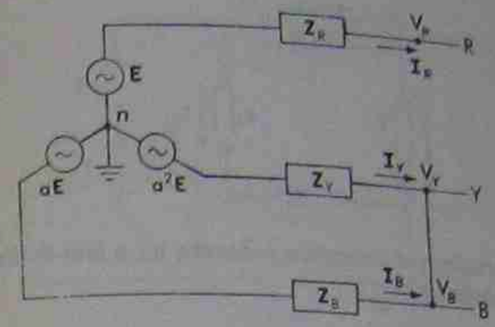


Figure 7.14 Line-to-line fault on phases Y and B

7.5.2 Line-to-line fault

In Figure 7.14, $\mathbf{E}$ = e.m.f. per phase and the R phase is again taken as the reference phasor. In this case, $\mathbf{I}_R = 0$, $\mathbf{I}_Y = -\mathbf{I}_B$, and $\mathbf{V}_Y = \mathbf{V}_B$.

From equation (7.4),

$$\mathbf{I}_0 = 0$$

$$\mathbf{I}_1 = \tfrac{1}{3}\mathbf{I}_Y(\mathbf{a} - \mathbf{a}^2)$$

and

$$\mathbf{I}_2 = \tfrac{1}{3}\mathbf{I}_Y(\mathbf{a}^2 - \mathbf{a})$$

$$\therefore \mathbf{I}_1 = -\mathbf{I}_2$$

As $\mathbf{V}_Y = \mathbf{V}_B$ (but not equal to zero),

$$\mathbf{a}^2\mathbf{E} - \mathbf{a}^2\mathbf{I}_1\mathbf{Z}_1 - \mathbf{a}\mathbf{I}_2\mathbf{Z}_2 = \mathbf{a}\mathbf{E} - \mathbf{a}\mathbf{I}_1\mathbf{Z}_1 - \mathbf{a}^2\mathbf{I}_2\mathbf{Z}_2$$

$$\therefore \mathbf{E}(\mathbf{a}^2 - \mathbf{a}) = \mathbf{I}_1[\mathbf{Z}_1(\mathbf{a}^2 - \mathbf{a}) + \mathbf{Z}_2(\mathbf{a}^2 - \mathbf{a})]$$

because $\mathbf{I}_1 = -\mathbf{I}_2$

hence

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \tag{7.7}$$

This can be represented by the equivalent circuit in Figure 7.15, in which, of course, there is no zero-sequence network. If the connection between the two lines has an impedance $\mathbf{Z}_f$ (the fault impedance), this is connected in series in the equivalent circuit.

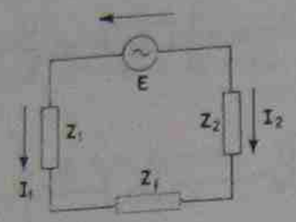


Figure 7.15 Interconnection of sequence networks for a line-to-line fault (including fault impedance Z_f, if present)

7.5.3 Line-to-line-to-earth fault (Figure 7.16)

$$\mathbf{I}_R = 0 \qquad \mathbf{V}_Y = \mathbf{V}_B = 0$$

and

$$\mathbf{I}_R = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_0 = 0$$

$$\therefore \mathbf{a}^2\mathbf{E} - \mathbf{a}^2\mathbf{I}_1\mathbf{Z}_1 - \mathbf{a}\mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_0\mathbf{Z}_0 = \mathbf{V}_Y = 0$$

and

$$\mathbf{a}\mathbf{E} - \mathbf{a}\mathbf{I}_1\mathbf{Z}_1 - \mathbf{a}^2\mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_0\mathbf{Z}_0 = \mathbf{V}_B = 0$$

Hence,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1 + [\mathbf{Z}_2\mathbf{Z}_0/(\mathbf{Z}_2 + \mathbf{Z}_0)]} \tag{7.8}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 \cdot \frac{\mathbf{Z}_0}{\mathbf{Z}_2 + \mathbf{Z}_0} \tag{7.9}$$

and

$$\mathbf{I}_0 = -\mathbf{I}_1 \cdot \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + \mathbf{Z}_0} \tag{7.10}$$

These can be represented by the equivalent circuit as shown in Figure 7.17.

The inclusion of impedances in the earth path, such as the star-point-to-earth connection in a generator or transformer, modifies the sequence diagrams. For a line-to-earth fault an impedance $\mathbf{Z}_g$ in the earth path is represented by an impedance of $3\mathbf{Z}_g$ in the zero-sequence network. $\mathbf{Z}_g$ can include the impedance of the fault itself, usually the resistance of the arc. As $\mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I}_0$ and $3\mathbf{I}_1$ flows through $\mathbf{Z}_g$ in the physical system, it is necessary to use $3\mathbf{Z}_g$ to obtain the required effect. Hence,

$$\mathbf{I}_f = \frac{3\mathbf{E}}{\mathbf{Z}_1 + 3\mathbf{Z}_g + \mathbf{Z}_2 + \mathbf{Z}_0}$$

Again, for a double-line-to-earth fault an impedance $3\mathbf{Z}_g$ is connected as shown in Figure 7.18. $\mathbf{Z}_g$ includes both machine neutral impedances and fault impedances.

The phase shift introduced by star–delta transformers has no effect on the magnitude of the fault currents, although it will affect the voltages at various points. It is shown in Chapter 3 that the positive-sequence voltages and currents are advanced by a certain angle and the negative-sequence quantities are retarded by the same angle for a given connection.

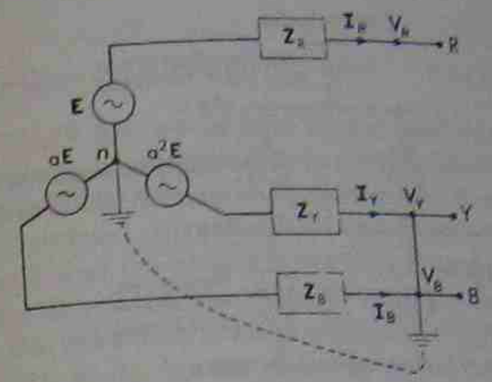


Figure 7.16 Line-to-line-to-earth fault

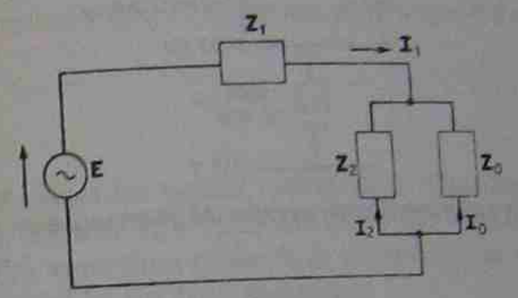


Figure 7.17 Interconnection of sequence networks—double line-to-earth fault

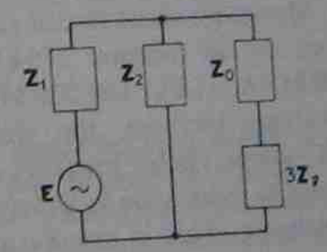


Figure 7.18 Modification of network in Figure 7.17 to account for neutral impedance Z_g

7.6 Fault Levels in a Typical System

In Figure 7.19, a section of a typical system is shown. At each voltage level the fault level can be ascertained from the reactances given. It should be noted that the short-circuit level will change with network conditions, and there will normally be two extreme values: that with all plant connected and that with

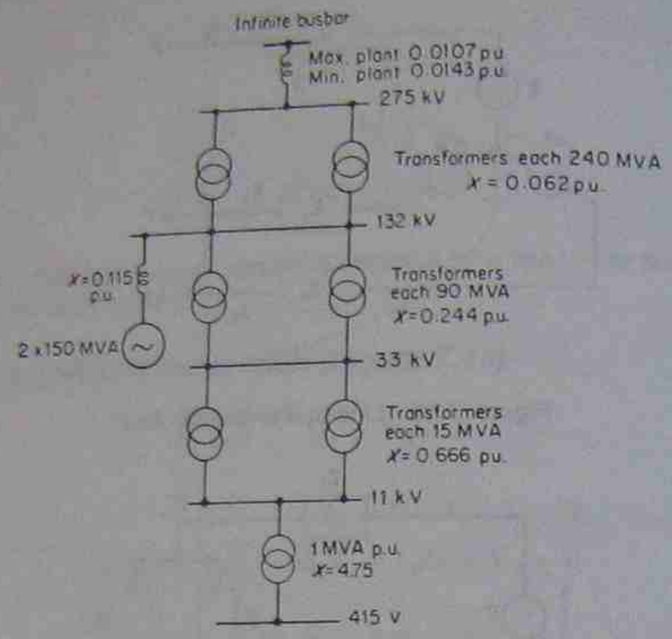


Figure 7.19 Typical transmission system. All reactances on a 100 MVA base

the minimum plant normally connected. The short-circuit MVA at 275 kV busbars in Britain is normally 10 000 MVA, but drops to 7000 MVA with minimum plant connected. Maximum short-circuit (three-phase) levels experienced in the British system are as follows: 275 kV, 15 000 MVA; 132 kV, 3500 MVA; 33 kV, 750/1000 MVA; 11 kV, 150/250 MVA; 415 V, 30 MVA.

As the transmission voltages increase, the short-circuit currents also increase, and for the 400 kV system, circuit breakers of 35 000 MVA breaking capacity are required. In order to reduce the fault level the number of parallel paths is reduced by sectionalizing. This is usually achieved by opening the circuit breaker connecting two sections of a substation or generating station busbar. One great advantage of direct-current transmission links in parallel with the alternating-current system is that no increase in the short-circuit currents results.

7.6.1 Circuit parameters with faults

Fault resistance

The resistance of the fault is normally that of the arc and may be approximated by

$$R_a(\Omega) = 44\,(V \text{ in kV})/I_f \qquad \text{for} \qquad V < 110\,\text{kV}$$

and

$$R_a(\Omega) = 22\,(V \text{ in kV})/I_f \qquad \text{for} \qquad V > 110\,\text{kV}$$

For example, a 735 kV line would have an arc resistance of 4 Ω with a fault current of 4 kA, assuming no resistance in ground return path.

The overall grounding resistance depends on the footing resistance (resistance of tower metalwork to ground) of the towers (R_T) and also on the resistance per section of the ground wires (R_s), where present. The situation is summarized in Figure 7.20, where usually the effective grounding resistance is smaller than the individual tower footing resistance. There is normally a spread in the values of R_T, but normally R_T should not exceed 10 Ω (ground wires present).

Typical values of fault resistance with fault location are as follows: at source, 0 Ω; on line with ground wires, 15 Ω; on line without ground wires, 50 Ω.

X/R ratio

The range of X/R values for typical voltage class (Canadian) are as follows: 735 kV, 18.9–20.4; 500 kV, 13.6–16.5; 220 kV, 2–25; 110 kV, 3–26. The X/R value decreases with separation of the fault point from the source and can be substantially decreased by the fault resistance.

For distribution circuits, X/R is lower, and, although data is limited, typical values are 10 (at source point) and 2–4 on an overhead line.

Example 7.4

A synchronous machine A generating 1 p.u. voltage is connected through a star–star transformer, reactance 0.12 p.u., to two lines in parallel. The other ends of the lines are connected through a star–star transformer of reactance 0.1 p.u. to a second machine B, also generating 1 p.u. voltage. For both transformers, $X_1 = X_2 = X_0$.

Calculate the current fed into a double-line-to-earth fault on the line-side terminals of the transformer fed from A.

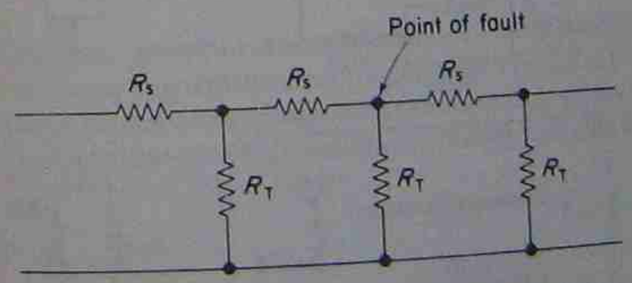


Figure 7.20 Equivalent network of towers with footing resistance (R_T) and ground wire sections (resistance R_s)

The relevant per unit reactances of the plant, all referred to the same base, are as follows:

For each line $X_1 = X_2 = 0.30$, $X_0 = 0.70$.

	X_1	X_2	X_0
Machine A	0.30	0.20	0.05
Machine B	0.25	0.15	0.03

The star points of machine A and of the two transformers are solidly earthed.

Solution

The positive-, negative-, and zero-sequence networks are shown in Figure 7.21. All per unit reactances are on the same base. From these diagrams the following equivalent reactances up to the point of the fault are obtained: $\mathbf{Z}_1 = \text{j}0.23$ p.u., $\mathbf{Z}_2 = \text{j}0.18$ p.u., and $\mathbf{Z}_0 = \text{j}0.17$ p.u.

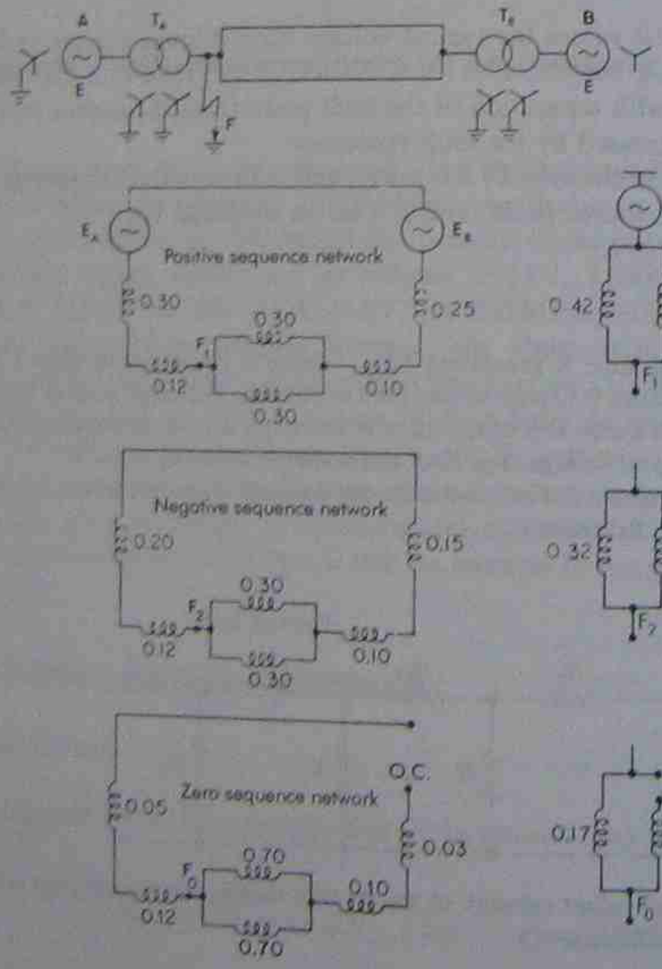


Figure 7.21 Line diagram and sequence networks for Example 7.4

The red phase is taken as reference phasor and the blue and yellow phases are assumed to be shorted at the fault point. From the equivalent circuit for a line-to-line fault,

$$\mathbf{I}_1 = \frac{1}{\mathbf{Z}_1 + [\mathbf{Z}_0\mathbf{Z}_2/(\mathbf{Z}_0 + \mathbf{Z}_2)]} \text{ p.u.}$$

$$= \frac{1}{\text{j}\{0.23 + [0.17 \times 0.18/(0.17 + 0.18)]\}} \text{ p.u.}$$

$$= -\text{j}3.15 \text{ p.u.}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 \frac{\mathbf{Z}_0}{\mathbf{Z}_0 + \mathbf{Z}_2}$$

$$= \text{j}3.15 \times \frac{0.17}{0.17 + 0.18}$$

$$= \text{j}1.53 \text{ p.u.}$$

$$\mathbf{I}_0 = -\mathbf{I}_1 \frac{\mathbf{Z}_2}{\mathbf{Z}_0 + \mathbf{Z}_2}$$

$$= \text{j}3.15 \times \frac{0.18}{0.18 + 0.17}$$

$$= \text{j}1.62 \text{ p.u.}$$

$$\mathbf{I}_Y = \mathbf{I}_0 + \mathbf{a}^2\mathbf{I}_{R1} + \mathbf{a}\mathbf{I}_{R2}$$

$$= \text{j}1.62 + (-0.5 - 0.866\text{j})(-\text{j}3.15) + (-0.5 + \text{j}0.866)(\text{j}1.53)$$

$$= -4.05 + \text{j}2.43 \text{ p.u.}$$

$$\mathbf{I}_B = j1.62 + (-0.5 + \text{j}0.866)(-\text{j}3.15) + (-0.5 - \text{j}0.866)(\text{j}1.53)$$

$$= 4.05 + \text{j}2.43.$$

$$\mathbf{I}_Y = \mathbf{I}_B = 4.72 \text{ p.u.}$$

The correctness of the first part of the solution can be checked as

$$\mathbf{I}_R = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_0 = -\text{j}3.15 + \text{j}1.53 + \text{j}1.62 = 0$$

Example 7.5

An 11 kV synchronous generator is connected to a 11/66 kV transformer which feeds a 66/11/3.3 kV three-winding transformer through a short feeder of negligible impedance. Calculate the fault current when a single-phase-to-earth fault occurs on a terminal of the 11 kV winding of the three-winding transformer. The relevant data for the system are as follows:

Generator $X_1 = \text{j}0.15$ p.u., $X_2 = \text{j}0.1$ p.u., $X_0 = \text{j}0.03$ p.u., all on a 10 MVA base; star point of winding earthed through a 3 Ω resistor.

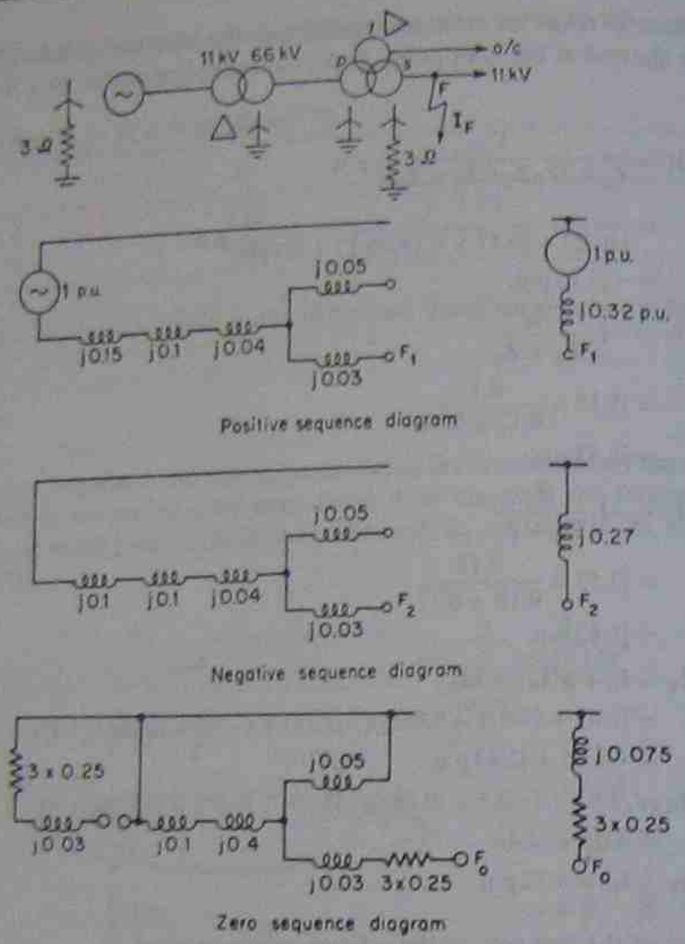


Figure 7.22 Line diagram and sequence networks for Example 7.5

11/66 kV Transformer $X_1 = X_2 = X_0 = j0.1$ p.u. on a 10 MVA base; 11 kV winding delta connected and the 66 kV winding star connected with the star point solidly earthed.

Three-winding transformer A 66 kV winding, star connected, star point solidly earthed; 11 kV winding, star connected, star-point earthed through a 3Ω resistor; 3.3 kV winding, delta connected; the three windings of an equivalent star connection to represent the transformer have sequence impedances, 66 kV winding $X_1 = X_2 = X_0 = j0.04$ p.u., 11 kV winding $X_1 = X_2 = X_0 = j0.03$ p.u., 3.3 kV winding $X_1 = X_2 = X_0 = j0.05$ p.u., all on a 10 MVA base.

Resistance may be neglected throughout.

Solution:

The line diagram and the corresponding positive-, negative-, and zero-sequence networks are shown in Figure 7.23. A 10 MVA base will be used. The 3Ω earthing resistor has the following p.u. value:

$$\frac{3 \times 10 \times 10^6}{(11)^2 \times 10^6} \quad \text{or} \quad 0.25 \text{ p.u.}$$

Much care is needed with the zero-sequence network owing to the transformer connections. For a line-to-earth fault, the equivalent circuit shown in Figure 7.13 is used, from which

$$\mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I}_0 \quad \text{and} \quad \mathbf{I}_f = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_0$$

Hence

$$\mathbf{I}_f = \frac{3 \times 1}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_0 + 3\mathbf{Z}_n}$$
$$= \frac{3}{j0.32 + j0.27 + j0.075 + 0.75}$$
$$= \frac{3}{0.75 + j0.66} = \frac{3}{1\angle 41^\circ} \text{ p.u.}$$
$$\therefore \mathbf{I}_f = \frac{3 \times 10 \times 10^6}{\sqrt{(3)} \times 11\,000}$$
$$= 1575 \text{ A}$$

7.7 Power in Symmetrical Components

The total power in a three-phase network

$$= \mathbf{V}_a\mathbf{I}_a^* + \mathbf{V}_b\mathbf{I}_b^* + \mathbf{V}_c\mathbf{I}_c^*$$

where $\mathbf{V}_a$, $\mathbf{V}_b$ and $\mathbf{V}_c$ are phase voltages and $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ are line currents. In phase (a),

$$P_a + jQ_a = (\mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2})(\mathbf{I}_{a0}^* + \mathbf{I}_{a1}^* + \mathbf{I}_{a2}^*)$$
$$= (\mathbf{V}_{a0}\mathbf{I}_{a0}^* + \mathbf{V}_{a1}\mathbf{I}_{a1}^* + \mathbf{V}_{a2}\mathbf{I}_{a2}^*)$$
$$+ (\mathbf{V}_{a0}\mathbf{I}_{a1}^* + \mathbf{V}_{a1}\mathbf{I}_{a2}^* + \mathbf{V}_{a2}\mathbf{I}_{a0}^*)$$
$$+ (\mathbf{V}_{a0}\mathbf{I}_{a2}^* + \mathbf{V}_{a1}\mathbf{I}_{a0}^* + \mathbf{V}_{a2}\mathbf{I}_{a0}^*)$$

with similar expressions for phases (b) and (c).

In extending this to cover the total three-phase power it should be noted that

$$\mathbf{I}_{b1}^* = (\mathbf{a}^2\mathbf{I}_{a1})^* = (\mathbf{a}^2)^*\mathbf{I}_{a1}^* = \mathbf{a}\mathbf{I}_{a1}^*$$

Similarly, $\mathbf{I}_{b2}^*$, $\mathbf{I}_{c1}^*$, and $\mathbf{I}_{c2}^*$ may be replaced.

$$\text{The total power} = 3(\mathbf{V}_{a0}\mathbf{I}^*_{a0} + \mathbf{V}_{a1}\mathbf{I}^*_{a1} + \mathbf{V}_{a2}\mathbf{I}^*_{a2}) \tag{7.11}$$

i.e. 3× (the sum of the individual sequence powers in any phase).

7.8 Systematic Methods for Fault Analysis in Large Networks

The methods described so far become unwieldy when applied to large networks and a systematic approach utilizing digital computers is used. The digital methods used for load flows are also suitable for three-phase symmetrical fault calculations, which after all are merely load flows under certain network conditions. The input information must be modified so that machines are represented by the appropriate reactances. The generators are represented by their no-load voltages in series with the subtransient reactances. If loads are to be taken into account they are represented by the equivalent shunt admittance to neutral. This will involve some modification to the original load-flow admittance matrix, but the general form will be as in the normal load flow.

A current of 1 p.u. is injected at the fault point and removed at the neutral. If the voltage at the fault becomes $\mathbf{V}_f$ then $1/\mathbf{V}_f$ is the fault level (i.e. short-circuit admittance) at node f. Also, the flow along branch kf into the fault is given by

$$\frac{(\mathbf{V}_f - \mathbf{V}_k)}{\mathbf{V}_f}\mathbf{Y}_{fk}$$

The voltages due to the injected currents can be obtained by the same numerical techniques as used for load flows. As loads are represented by admittance the problem is linear and iterative methods are not required.

Three-phase fault studies are performed in conjunction with load flows. For example, if the fault level on a solid busbar is too high the busbar will be sectioned, i.e. split into two or more sections by opening switches, and new load are flows required.

The nodal or admittance method may be applied to large networks. It is preferable, on grounds of storage and time, not to invert the matrix but to use Gauss elimination methods. The computation efficiency may also be improved by utilizing the sparsity of the Y matrix. The mesh or loop (impedance matrix) method may be used, although the matrix is not so easily formed.

The following example illustrates a method suitable for determination of balanced three-phase fault currents in a large system by means of a digital computer.

Example 7.6

Determine the fault current in the system shown in Figure 7.4 for the balanced fault shown.

Solution

As already stated, generators are represented by their voltages behind the transient reactance, and normally the system is assumed to be on no-load before the occurrence of the three-phase balanced fault. The voltage sources and transient reactances are converted into current sources and the admittance matrix is formed (including the transient reactance admittances). The basic equation $[Y][V] = [I]$ is formed and solved with the constraint that the voltage at the fault node is zero. A current may be injected into the faulted node, as explained in the previous section, or the voltage made zero and the remaining nodal voltages calculated.

The system in Figure 7.4 is replaced by the equivalent circuit shown in Figure 7.23, in which $\mathbf{V}_D = 0.0$ p.u.

Nodal admittance matrix

$$-\mathrm{j}\begin{bmatrix} 15.33 & -2.00 & & -3.33 \\ -2.00 & 15.33 & -3.33 & \\ & -3.33 & 15.33 & -2.00 \\ -3.33 & & -2.00 & 15.33 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_B \\ \mathbf{V}_C \\ 0 \end{bmatrix} = -\mathrm{j}\begin{bmatrix} 10 \\ 10 \\ 10 \\ \mathbf{I}_F \end{bmatrix}$$

As all the Y values are reactive, the j values will be omitted for simplicity. If the bottom row is eliminated as explained in Chapter 6, we get:

$$\begin{bmatrix} 23 & -3 & 0 \\ -3 & 23 & -5 \\ 0 & -5 & 23 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_B \\ \mathbf{V}_C \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix}$$

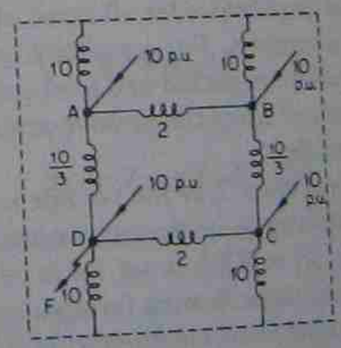


Figure 7.23 Equivalent circuit of the system of Figure 7.4. All values are admittances (i.e. $-\mathrm{j}Y$). The generators, i.e. 1 p.u. voltage behind $-$j10 p.u. admittance, transform to $-$j10 p.u. current sources in parallel with $-$j10 p.u. admittance

from which

$$\begin{bmatrix} 520 & -115 \\ -5 & 23 \end{bmatrix} \cdot \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 385 \\ 15 \end{bmatrix}$$

Thus,

$$V_B = 0.92950 \qquad V_C = 0.85510 \qquad \text{and} \qquad V_A = 0.77495\,\text{p.u.}$$

and

$$I_F = -j\left(10 + \frac{0.77495}{0.3} + \frac{0.88510}{0.5}\right)\text{p.u.} = -j14.2985\,\text{p.u.}$$

Fault MVA $= 14.2985 \times 100 = 1429.85\,\text{MVA}$

7.9 Bus Impedance (Short-Circuit Matrix) Method

This method is used for computer fault analysis and has the following advantages:

1. Matrix inversion is avoided, resulting in savings in computer storage and time.
2. The matrices for the sequences quantities are determined only once and retained for later use; they are readily modified for system changes.
3. Mutual impedances between lines are readily handled.
4. Subdivisions of the main system may be incorporated.

The system is represented by the usual symmetrical component sequence networks and, frequently, the positive and negative impedances are assumed to be identical. Balanced phase impedances for all items of plant are assumed as are equal voltages for all generators. For simplicity, in this treatment, mutual coupling between lines will be neglected and load currents assumed negligible. The method will be explained using the positive-sequence network as this will be most familiar to the reader.

In the system shown in Figure 7.24 the voltage source supplies a common bus, and four busbars of interest in the passive network are identified. The network loop matrix, i.e. $[e] = [Z][i]$, is set up in terms of the various loop currents which must include those flowing through busbars (1)–(4) to the neutral bus, i.e. these busbars are short-circuited to N. In the matrix equation so formed only nodes 1–4 are of interest, and by the process of partitioning, described in the section on load flows, a new equation is obtained involving only currents from these busbars. $E = 1$ p.u.

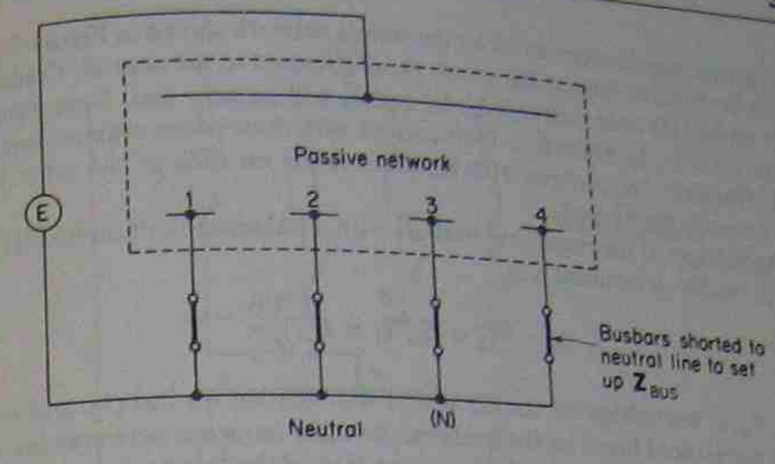


Figure 7.24 Identification of busbars of interest in a large network—single-phase equivalent network

$$\begin{matrix} (e) \\ \end{matrix} \qquad \begin{matrix} \quad (Z_A) \quad\quad\quad\quad\quad\quad (Z_B) \end{matrix}$$

$$\left[\begin{array}{c} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ \hline 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{array}\right] = \left[\begin{array}{cccc:c} Z_{11} & Z_{12} & Z_{13} & Z_{14} & \cdots Z_{1n} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & \cdots Z_{2n} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & \cdots Z_{3n} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & \cdots Z_{4n} \\ \hline \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \\ Z_{n1} & Z_{n2} & Z_{n3} & Z_{n4} & \cdots Z_n \end{array}\right] \cdot \left[\begin{array}{c} i_1 \\ i_2 \\ i_3 \\ i_4 \\ \hline \cdot \\ \cdot \\ \cdot \\ i_n \end{array}\right]$$

$$(Z_C) \qquad (Z_D)$$

i.e.

$$[e] = [Z_{sc}]\,[i]$$

where

$$Z_{sc} = Z_A - Z_B Z_D^{-1} Z_C$$

The new matrix is called the *bus impedance* or short-circuit matrix and the equation is given by

$$\begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{21} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \qquad (7.12)$$

This matrix may be represented by the simple network shown in Figure 7.25, in which the buses of interest are shown short-circuited to the neutral. Consider a fault on bus (1) only, currents $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_4$ will be zero and, from equation $\mathbf{e} = \mathbf{zi}$, $1.0 = \mathbf{z}_{11}\,\mathbf{I}_1$, where $\mathbf{I}_1$ = fault current with three-phase symmetrical fault on 1. Similarly, the currents with balanced faults on each of the other buses may be easily determined.

The voltages at the remaining busbars with a balanced fault on bus (1) may also be readily determined, e.g.

$$1.0 - \mathbf{V}_{n2} = \mathbf{Z}_{21}\,\mathbf{I}_1 = \mathbf{Z}_{21}\frac{1.0}{\mathbf{Z}_{11}}$$

where $\mathbf{V}_{n2}$ is the voltage on bus (2) with a short circuit on bus (1), and so on.

For unbalanced faults on the busbars, the three sequence networks are used in the normal way. In Figure 7.26 the connection of the three reduced-sequence networks for a line-to-ground fault on bus (1) is illustrated, where superscripts +, − and 0 denote the sequence impedances. As usual, the sequence networks refer to phase (a) of the real network.

For a L–E fault on bus (1),

$$\mathbf{I}_1 = 3/(2\mathbf{z}_{11}^{+} + \mathbf{z}_{11}^{0})$$

and

$$\mathbf{I}_1^{+} = \mathbf{I}_1^{-} = \mathbf{I}_1^{0} = 1/(2\mathbf{z}_{11}^{+} + \mathbf{z}_{11}^{0})$$

The assumption is made that the positive- and negative-sequence impedances are equal. This aids computation and does not produce major errors.

Also, the voltage on bus (3),

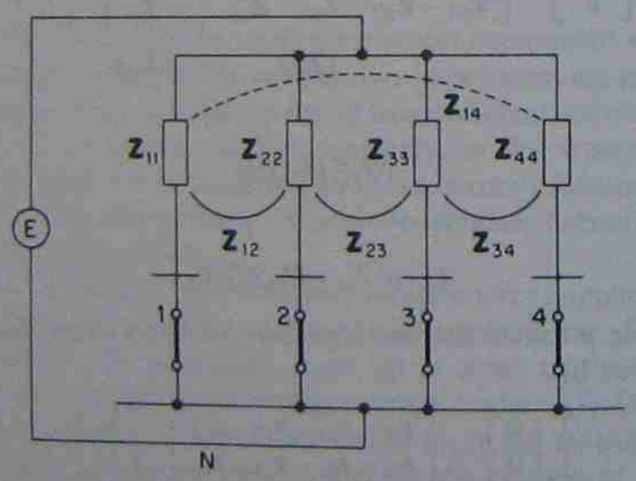


Figure 7.25 Equivalent circuit of the Z_{bus} matrix equation—single-phase equivalent, i.e. balanced-fault conditions

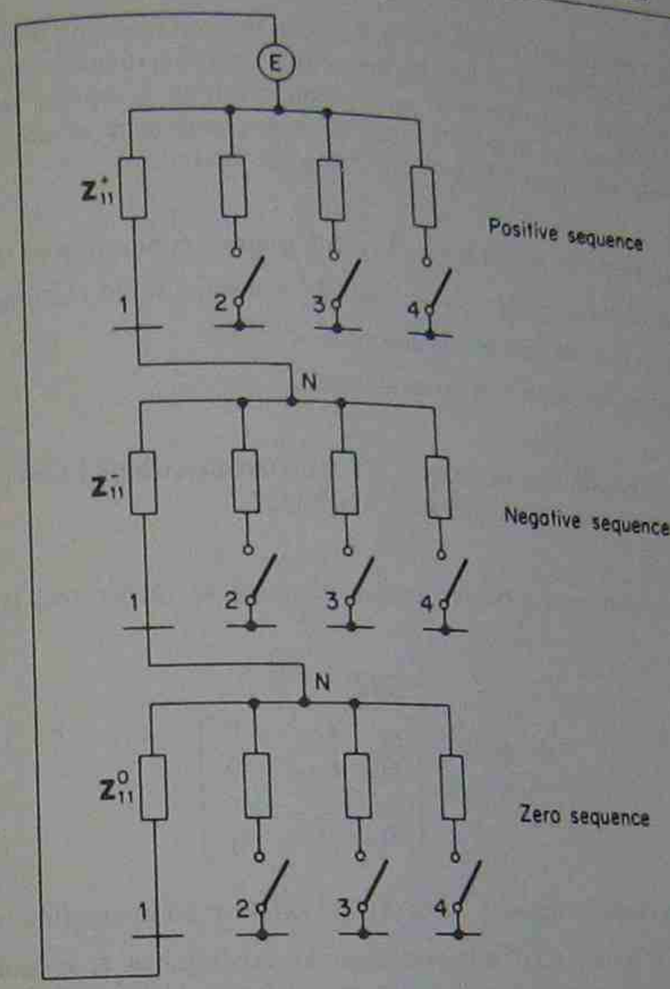


Figure 7.26 Equivalent network using Z_{bus} method for single-line-to ground fault on busbar (1) of the network in Figure 7.24

$$\mathbf{V}_3^{+} = 1 - \frac{\mathbf{z}_{31}^{+}}{(2\mathbf{z}_{11}^{+} + \mathbf{z}_{11}^{0})}$$

$$\mathbf{V}_3^{-} = -\frac{\mathbf{z}_{31}^{-}}{2\mathbf{z}_{11}^{+} + \mathbf{z}_{11}^{0}}$$

$$\mathbf{V}_3^{0} = -\frac{\mathbf{z}_{31}^{0}}{(2\mathbf{z}_{11}^{+} + \mathbf{z}_{11}^{0})}$$

and

$$\mathbf{V}_3 = \mathbf{V}_3^{+} + \mathbf{V}_3^{-} + \mathbf{V}_3^{0} = 1 - \left(\frac{2\mathbf{z}_{12}^{+} + \mathbf{z}_{13}^{0}}{2\mathbf{z}_{11}^{+} + \mathbf{z}_{11}^{0}}\right)$$

The bus impedance matrix has so far been derived from the full-loop impedance matrix obtained either by inspection or by systematic methods. An alternative approach, which is more economical in computer usage, is to form the matrix one step at a time as each component of the network is added. These additions fall into four categories as follows:

1. A new generator bus or a new bus with a direct connection to the neutral.
2. A generator (or connection to neutral) connected to an existing bus.
3. Connection from an existing bus to a new bus.
4. Connection between two existing buses.

To illustrate the addition of buses, a 2×2 matrix describing buses (1) and (2) will be assumed to have already been assembled.

1. New bus with connection to neutral through $\mathbf{z}_L$ (generator) (see Figure 7.27).

$$\text{New } \mathbf{z}_B = \begin{array}{c} \text{Old} \\ \left[\begin{array}{cc:c} \mathbf{z}_{11} & \mathbf{z}_{12} & 0 \\ \mathbf{z}_{21} & \mathbf{z}_{22} & 0 \\ \hdashline 0 & 0 & \mathbf{z}_L \end{array}\right] \end{array}$$

Note that currents through buses (1) and (2) have no interaction in bus (3).

2. Connection to neutral of generator from an existing bus. $\mathbf{z}_L$ is connected to bus (2) and is associated with loop current $\mathbf{i}_3$, as in Figure 7.28. The new matrix is

$$\begin{array}{cc} \text{(A)} & \text{(B)} \\ \begin{array}{ccc} ① & ② & ③ \end{array} & \\ \left[\begin{array}{cc:c} \mathbf{z}_{11} & \mathbf{z}_{12} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} & \mathbf{z}_{22} \\ \hdashline \mathbf{z}_{21} & \mathbf{z}_{22} & \mathbf{z}_{22}+\mathbf{z}_L \end{array}\right] & \begin{array}{c} (\mathbf{i}_1) \\ (\mathbf{i}_2) \\ (\mathbf{i}_3) \end{array} \\ \text{(C)} & \text{(D)} \end{array}$$

Note that the $\mathbf{i}_1$ influences loop $\mathbf{i}_3$ via $\mathbf{z}_{12}$. The internal loop is eliminated by applying $\mathbf{z} = \mathbf{z}_A - \mathbf{z}_B\mathbf{z}_D^{-1}\mathbf{z}_C$ to the partitioned matrix, giving a 2×2 matrix.

3. Connection of impedance between an existing bus and a new bus. New line $\mathbf{z}_L$ connected to bus (2). Consider loop current $\mathbf{i}_3$ as shown in Figure 7.29.

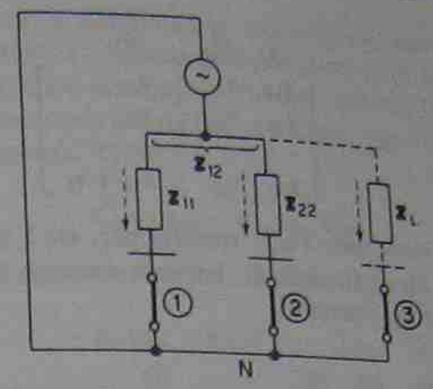


Figure 7.27 New busbar with connection to neutral through $\mathbf{z}_L$, e.g. generator

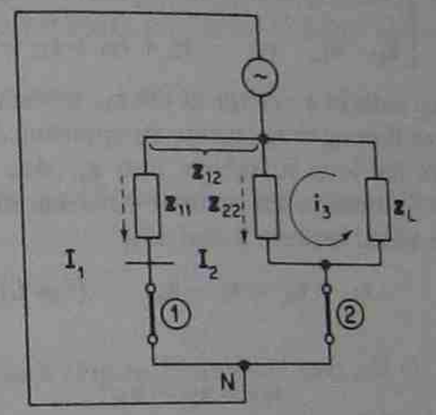


Figure 7.28 Connection to neutral of generator from existing bus (2)

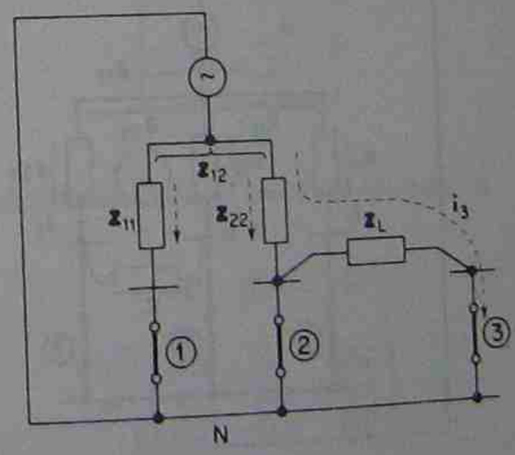


Figure 7.29 Connection of impedance ($\mathbf{z}_L$) between existing bus (2) and new bus (3)

$$\mathbf{Z}_{bus} = \begin{array}{c} \\ \textcircled{1} \\ \textcircled{2} \\ \\ \textcircled{3} \end{array} \begin{array}{c} \begin{array}{cc} \textcircled{1} & \textcircled{2} \end{array} \\ \left[\begin{array}{cc|c} \mathbf{z}_{11} & \mathbf{z}_{12} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} & \mathbf{z}_{22} \\ \hline \mathbf{z}_{21} & \mathbf{z}_{22} & \mathbf{z}_{22}+\mathbf{z}_L \end{array}\right] \end{array}$$

4. Connection of impedance (line, transformer, etc.) between two existing buses. Consider the new link $\mathbf{z}_L$ between existing buses (2) and (3) in Figure 7.30 and loop current $\mathbf{i}_L$.

$$\begin{array}{c} \\ \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \\ \textcircled{L} \end{array} \begin{array}{c} \begin{array}{ccc} \textcircled{1} & \textcircled{2} & \textcircled{3} \end{array} \\ \left[\begin{array}{ccc|c} \mathbf{z}_{11} & \mathbf{z}_{12} & \mathbf{z}_{13} & (\mathbf{z}_{1L}=\mathbf{z}_{12}-\mathbf{z}_{13}) \\ \mathbf{z}_{21} & \mathbf{z}_{22} & \mathbf{z}_{23} & (\mathbf{z}_{2L}=\mathbf{z}_{22}-\mathbf{z}_{23}) \\ \mathbf{z}_{31} & \mathbf{z}_{32} & \mathbf{z}_{33} & (\mathbf{z}_{3L}=\mathbf{z}_{23}-\mathbf{z}_{33}) \\ \hline \mathbf{z}_{L1} & \mathbf{z}_{L2} & \mathbf{z}_{L3} & \mathbf{z}_L+\mathbf{z}_{33}+\mathbf{z}_{22}-2\mathbf{z}_{23} \end{array}\right] \end{array}$$

Note that $\mathbf{i}_L$ in $\mathbf{z}_{22}$ induces a voltage in the $\mathbf{z}_{33}$ branch due to $\mathbf{z}_{23}$. Similarly, it induces in $\mathbf{z}_{22}$ when flowing in $\mathbf{z}_{33}$, giving an opposing double voltage $2\mathbf{i}_L \times \mathbf{z}_{23}$.

As $\mathbf{i}_L$ traverses the loop it induces into $\mathbf{z}_{11}$ due to $\mathbf{z}_{12}$ and $\mathbf{z}_{13}$. Hence $\mathbf{z}_{L1} = \mathbf{z}_{12} - \mathbf{z}_{13}$ or, generally, the mutual between the loop current $\mathbf{i}_L$ and node i with a line added between k and m is

$$\mathbf{z}_{Li} = \mathbf{z}_{iL} = \mathbf{z}_{ki} - \mathbf{z}_{mi} \qquad (i \neq L)$$

e.g. for node 3,

$$\mathbf{z}_{L3} = \mathbf{z}_{23} - \mathbf{z}_{33}$$

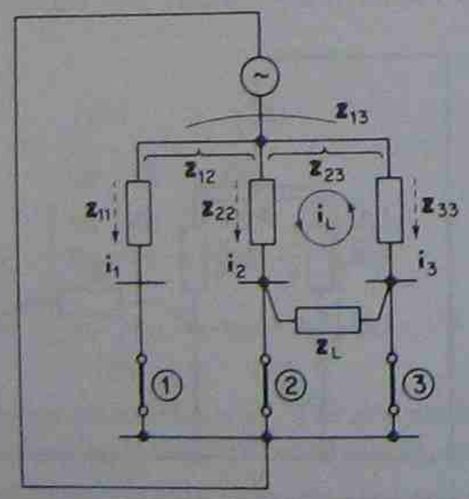


Figure 7.30 Connection of impedance ($\mathbf{z}_L$) between two existing buses, (2) and (3)

Again, the internal loop is eliminated by partitioning, leaving a 3×3 matrix.

It can be shown that a *single-loop* elimination may be accomplished by operation on the individual elements as follows. Let $\mathbf{z}'_{mn}$ be an element in the new bus impedance (reduced) matrix (i.e. column n, row m) and $\mathbf{z}_{mn}$ an element in the original loop matrix. Then,

$$\mathbf{z}'_{mn} = \mathbf{z}_{mn} - \mathbf{z}_{mL}\,\mathbf{z}_{LL}^{-1}\mathbf{z}_{Ln} \tag{7.13}$$

For example, in the above 4×4 matrix the new term, $\mathbf{z}'_{23} = \mathbf{z}_{23} - \mathbf{z}_{2L}\,\mathbf{z}_{LL}^{-1}\mathbf{z}_{L3}$

$$= \mathbf{z}_{23} - (\mathbf{z}_{22} - \mathbf{z}_{23})(\mathbf{z}_L + \mathbf{z}_{33} + \mathbf{z}_{22} - 2\mathbf{z}_{23})^{-1}(\mathbf{z}_{23} - \mathbf{z}_{33})$$

Similarly, the other eight terms are obtained.

If, in existing networks, lines or generators are to be removed, this can be achieved as outlined above, but with the use of negative impedances. The same general approach is made to systems with coupling between parallel lines and also the splitting of large networks into several smaller networks.

Example 7.7

For the system shown in Figure 7.31, assemble the $\mathbf{Z}_{bus}$ matrix, branch by branch.

Solution

First consider the j0.2 branch (Figure 7.31(b)) and then add the j0.1 branch (Figure 7.31(c)), giving the matrix

$$\begin{array}{c} \\ \textcircled{1} \\ \textcircled{2} \end{array} \begin{array}{c} \begin{array}{cc} \textcircled{1} & \textcircled{2} \end{array} \\ \left[\begin{array}{cc} \text{j0.2} & 0 \\ 0 & \text{j0.1} \end{array}\right] \end{array}$$

Finally, the j0.4 branch connectes nodes 1 and 2 and the matrix is

$$\begin{array}{c} \\ m \\ \\ n \end{array} \begin{array}{c} \begin{array}{ccc} \textcircled{1} & \textcircled{2} & \textcircled{L} \end{array} \\ \left[\begin{array}{ccc} \text{j0.2} & 0 & \text{j0.2} \\ 0 & \text{j0.1} & -\text{j0.1} \\ \text{j0.2} & -\text{j0.1} & \text{j0.7} \end{array}\right] \end{array}$$

Eliminate the internal loop, using equation (7.13).

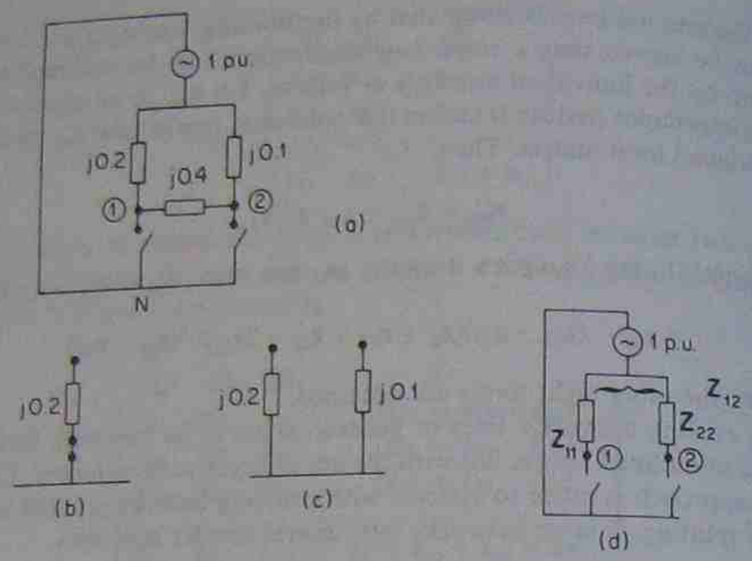


Figure 7.31 Network for Example 7.7

$$\mathbf{Z}_{11}^{1} = \text{j}0.2 - (\text{j}0.2)(\text{j}0.7)^{-1}(\text{j}0.2) = \text{j}0.143$$
$$\mathbf{Z}_{12}^{1} = \text{j}0.1 - (-\text{j}0.1)(\text{j}0.7)^{-1}(-\text{j}0.1) = \text{j}0.086$$
$$\mathbf{Z}_{12}^{1} = 0 - (\text{j}0.2)(\text{j}0.7)^{-1}(-\text{j}0.1) = \text{j}0.029$$
$$\mathbf{Z}_{21}^{1} = \text{j}0.029$$

Therefore

$$\begin{matrix} & ① & ② \end{matrix}$$
$$\mathbf{Z}_{\text{bus}} = \text{j}\begin{bmatrix} 0.143 & 0.029 \\ 0.029 & 0.086 \end{bmatrix}$$

and

$$|\mathbf{E}| = |\mathbf{Z}_{\text{bus}}||\mathbf{I}|$$

(see Figure 7.31(d)).

With a three-phase short circuit on bus (1)

$$\mathbf{I}_1 = \frac{1.0}{\mathbf{z}_{11}} = \frac{1.0}{\text{j}0.143} = 6.99 \text{ p.u.}$$
$$(\mathbf{I}_2 = 0)$$

Similarly, for a fault on bus (2),

$$\mathbf{I}_2 = \frac{1.0}{\mathbf{z}_{22}}$$
$$\mathbf{I}_2 = \frac{1}{\text{j}0.086} = 11.63 \text{ p.u.}$$

7.10 Neutral Grounding

7.10.1 Introduction

From the analysis of unbalanced fault conditions it has been seen that the connection of the transformer and generator neutrals greatly influences the fault currents and voltages. In most high-voltage systems the neutrals are solidly grounded, i.e. connected directly to the ground, with the exception of generators which are grounded through a resistance to limit stator fault currents. The advantages of solid grounding are as follows:

1. Voltages to ground are limited to the phase voltage.
2. Intermittent ground faults and high voltages due to arcing faults are eliminated.
3. Sensitive protective relays operated by earth fault currents clear these faults at an early stage.

The main advantage in operating with neutrals isolated is the possibility of maintaining a supply with a ground fault on one line which places the remaining conductors at line voltage above ground. Also, interference with telephone circuits is reduced because of the absence of zero-sequence currents. With normal balanced operation the neutrals of an unground or isolated system are held at ground potential because of the presence of the system capacitance to earth. For the general case shown in Figure 7.32, the following analysis applies:

$$\frac{\mathbf{V}_{ag}}{\mathbf{Z}_{ag}} + \frac{\mathbf{V}_{bg}}{\mathbf{Z}_{bg}} + \frac{\mathbf{V}_{cg}}{\mathbf{Z}_{cg}} = 0 \tag{7.14}$$

Also,

$$\mathbf{V}_{ag} = \mathbf{V}_{an} + \mathbf{V}_{ng}$$

where

$\mathbf{V}_{an}$ = voltage of line (a) to neutral
$\mathbf{V}_{ng}$ = voltage of neutral to ground

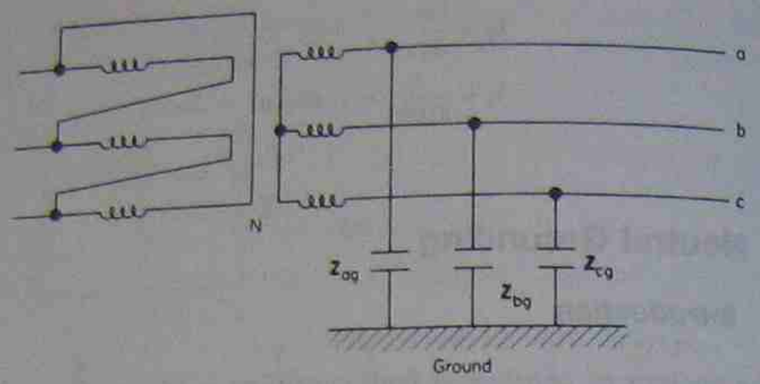


Figure 7.32 Line-to-ground capacitances in an unground system

Similarly,

$$\mathbf{V}_{bg} = \mathbf{V}_{bn} + \mathbf{V}_{ng}$$

and

$$\mathbf{V}_{cg} = \mathbf{V}_{cn} + \mathbf{V}_{ng}$$

Substituting in equation (7.14) and separating terms,

$$\frac{\mathbf{V}_{an}}{\mathbf{Z}_{ag}} + \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{bg}} + \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{cg}} + \mathbf{V}_{ng}\left(\frac{1}{\mathbf{Z}_{ag}} + \frac{1}{\mathbf{Z}_{bg}} + \frac{1}{\mathbf{Z}_{cg}}\right) = 0 \qquad (7.15)$$

The equation

$$\left(\frac{1}{\mathbf{Z}_{ag}} + \frac{1}{\mathbf{Z}_{bg}} + \frac{1}{\mathbf{Z}_{cg}}\right) = \mathbf{Y}_g$$

gives the ground capacitance admittance of the system.

7.10.2 Arcing faults

Consider the single-phase system in Figure 7.33 at the instant when the instantaneous voltages are v on line (a) and $-v$ on line (b), where v is the maximum instantaneous voltage. The sudden occurrence of a fault to ground causes line (b) to assume a potential of $-2v$ and line (a) to become zero. Because of the presence of both L and C in the circuit, the sudden change in voltages by v produces a high-frequency oscillation of peak magnitude $2v$ superimposed on the power frequency voltages (see Chapter 10) and line (a) reaches $-v$ and line (b) $-3v$, as shown in Figure 7.34. These oscillatory voltages attenuate quickly due to the resistance present. The current in the arc to earth on line (a) is approximately 90° ahead of the fundamental voltage, and when it is zero the

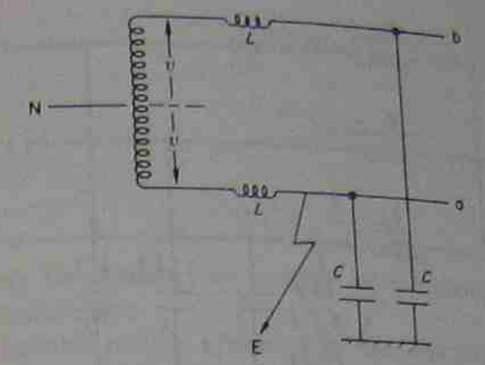


Figure 7.33 Single-phase system with arcing fault to ground

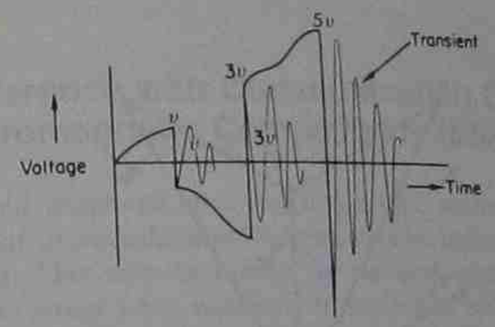
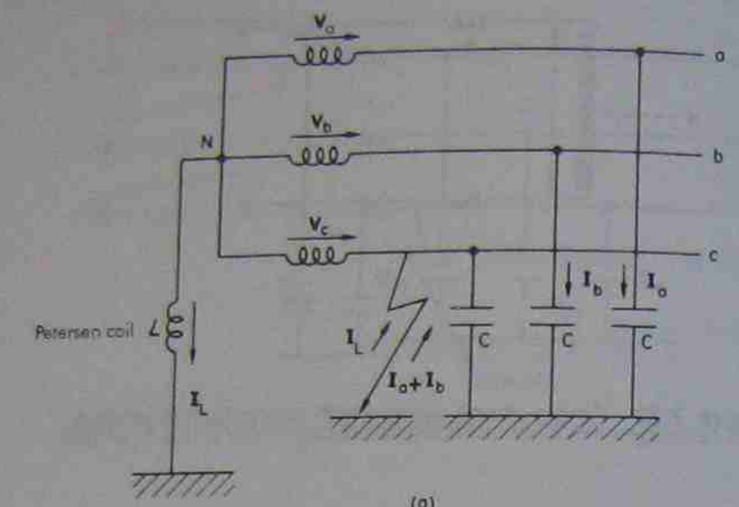


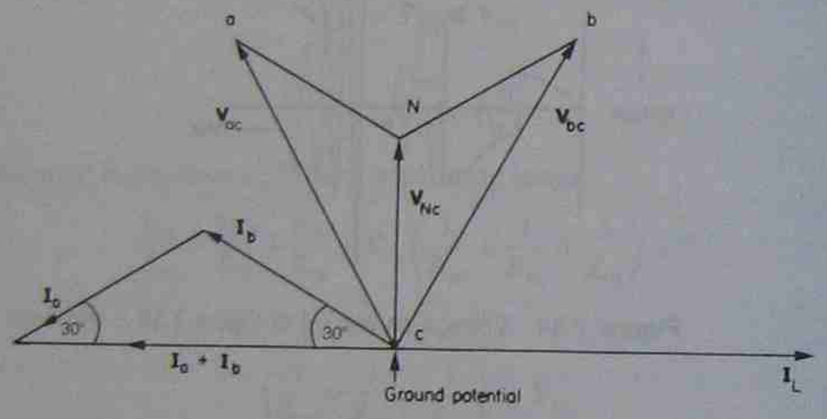
Figure 7.34 Voltage on line (a) of Figure 7.33

voltage will be at a maximum. Hence, if the arc extinguishes at the first current zero, the lines remain charged at $-v$ for (a) and $-3v$ for (b). The line potentials now change at power frequency until (a) reaches $-3v$, when the arc could restrike causing a voltage change of $-3v$ to 0, resulting in a transient overvoltage of $+3v$ in line (a) and $+5v$ in line (b). This process could continue and the voltage build up further, but the resistance present usually limits the peak voltage to under $4v$. A similar analysis may be made for a three-phase circuit, again showing that serious overvoltages may occur with arcing faults because of the inductance and shunt capacitance of the system.

This condition may be overcome in an isolated neutral system by means of an *arc suppression* or Petersen coil. The reactance of this coil, which is connected between the neutral and ground, is made in the range 90–110 per cent of the value required to neutralize the capacitance current. The phasor diagram for the network of Figure 7.35(a) is shown in Figure 7.35(b) if the voltage drop across the arc is neglected.



(a)



(b)

Figure 7.35 (a) System with arc suppression coil. (b) Phasor diagram of voltages and currents in part (a)

$$\mathbf{I}_a = \mathbf{I}_b = \sqrt{(3)}\mathbf{V}\omega C$$

and

$$\mathbf{I}_a + \mathbf{I}_b = \sqrt{(3)} \times \sqrt{(3)}\mathbf{V}\omega C = 3\mathbf{V}\omega C$$

also,

$$\mathbf{I}_L = \frac{\mathbf{V}}{\omega L}$$

For compensation of the arc current,

$$\frac{\mathbf{V}}{\omega L} = 3\mathbf{V}\omega C$$

$$\therefore L = \frac{1}{3\omega^2 C}$$

and

$$X_L = \frac{1}{3\omega C} \tag{7.16}$$

This result may be obtained by analysis of the ground fault by means of symmetrical components.

Generally, isolated neutral systems give rise to serious arcing-fault voltages if the arc current exceeds the region of 5–10 A, which covers most systems operating above 33 kV. If such systems are to be operated with isolated neutrals, arc-suppression coils should be used. Most systems at normal transmission voltages have grounded neutrals.

7.11 Interference with Communication Circuits—Electromagnetic Compatibility (EMC)

When power and telephone lines run in parallel, under certain conditions voltages sufficient to cause high noise levels may be induced into the communication circuits. This may be caused by electromagnetic and electrostatic unbalance in the power lines, especially if harmonics are present. The major problem, however, is due to faults to ground producing large zero-sequence currents in the power line, which inductively induce voltage into the neighbouring circuits. The value of induced voltage depends on the spacing, resistivity of the earth immediately below, and the frequency. If the telephone wires are a twisted pair or are situated close together and transposed, no voltage is induced between the communication conductors. However, a voltage can exist between the pair of wires and ground. These induced longitudinal voltages can be controlled by connecting the communication circuits to ground through an inductance which produces little attenuation at communication frequencies of 400–3500 Hz.

Capacitive coupling can occur if open communication circuits are run along the same route as power lines. Interference from underground cable circuits is much less (10 per cent) than that from overhead lines.

Because of right-of-way constraints, telephone and power distribution lines run parallel along the same street in many urban areas. However, the interference in rural areas is often greater because communication lines and plant may be unshielded or have higher shield resistances, and unlike urban areas there is no extensive network of water and gas pipes to share the ground return currents.

Resistive coupling between power and communication circuits can exist:

- because of physical contact between them;
- via paths through the soil between telephone and power grounds.

Various formulae exist to calculate the value of mutual inductance—in H/m between circuits with earth return. These assume that ε_r (soil) is unity, displacement currents are much less than conduction currents, and the length of conductor is infinite. A typical formula is that due to Carson (1926).

During line-to-ground faults, induced voltages into communication circuits may be sufficient to be a shock hazard to personnel. Although in transmission circuits, equal current loading may be assumed in the phases, this is not the case in the lower voltage distribution circuits where significant residual currents may flow. Most telephone circuit standards now require up to 15 kV isolation if communication circuits are to be connected into substations for monitoring, control, and communication purposes. Particular care must be taken with bonding the sheath of communication circuits brought into buildings where the power distribution system is also bonded to earth and to the building structure.

Problems

7.1 Four 11 kV generators designated A, B, C, and D each have a subtransient reactance of 0.1 p.u. and a rating of 50 MVA. They are connected in parallel by means of three 100 MVA reactors which join A to B, B to C, and C to D; these reactors have per unit reactances of 0.2, 0.4, and 0.2, respectively. Calculate the volt-amperes and the current flowing into a three-phase symmetrical fault on the terminals of machine B. Use a 50 MVA base.
(Answer: 940 MVA; 49 300 A)

7.2 Two 100 MVA, 20 kV turbogenerators (each of transient reactance 0.2 p.u.) are connected, each through its own 100 MVA, 0.1 p.u. reactance transformer, to a common 132 kV busbar. From this busbar, a 132 kV feeder, 40 km in length, supplies an 11 kV load through a 132/11 kV transformer of 200 MVA rating and reactance 0.1 p.u. If a balanced three-phase short circuit occurs on the low-voltage terminals of the load transformer, determine, using a 100 MVA base, the fault current in the feeder and the rating of a suitable circuit breaker at the load end of the feeder. The feeder impedance per phase is $(0.035 + j0.14)\,\Omega$/km.
(Answer: 482 MVA)

7.3 Two 60 MVA generators of transient reactance 0.15 p.u. are connected to a busbar designated A. Two identical machines are connected to another busbar B. A feeder is supplied from A through a step-up transformer rated at 30 MVA with 10 per cent reactance.

Calculate the reactance of a reactor to connect A and B if the fault level due to a three-phase fault on the feeder side of the transformer is to be limited to 240 MVA. Calculate also the voltage on A under this condition if the generator voltage is 13 kV (line).
(Answer: $X = 0.075$ p.u.; $V_A = 10.4$ kV)

7.4 A single line-to-earth fault occurs on the red phase at the load end of a 66 kV transmission line. The line is fed via a transformer by 11 kV generators connected to a common busbar. The line side of the transformer is connected in star with the star point earthed and the generator side is in delta. The positive-sequence reactances of the transformer and line are $j10.9\,\Omega$ and $j44\,\Omega$, respectively, and the equivalent positive- and negative-sequence reactances of the generators, referred to the line voltage, are $j18\,\Omega$ and $j14.5\,\Omega$, respectively. Measured up to the fault the total effective zero-sequence reactance is $j150\,\Omega$. Calculate the fault current in the lines if resistance may be neglected. If a two-line-to-earth fault occurs between the blue and yellow lines, calculate the current in the yellow phase.
(Answer: 392 A; 488 A)

7.5 A single-line-to-earth fault occurs in a radial transmission system. The following sequence impedances exist between the source of supply (an infinite busbar) of voltage 1 p.u. to the point of the fault: $Z_1 = 0.3 + j0.6$ p.u., $Z_2 = 0.3 + j0.55$ p.u., $Z_0 = 1 + j0.78$ p.u. The fault path to earth has a resistance of 0.66 p.u. Determine the fault current and the voltage at the point of the fault.
(Answer: $I_f = 0.732$ p.u.; $V_f = (0.43 - j0.23)$ p.u.)

7.6 Develop an expression, in terms of the generated e.m.f. and the sequence impedances, for the fault current when an earth fault occurs on phase (A) of a three-phase generator, with an earthed star point. Show also that the voltage to earth of the sound phase (B) at the point of fault is given by

$$V_B = \frac{-j\sqrt{3}E_A[Z_2 - aZ_0]}{Z_1 + Z_2 + Z_0}$$

Two 30 MVA, 6.6 kV synchronous generators are connected in parallel and supply a 6.6 kV feeder. One generator has its star point earthed through a resistor of $0.4\,\Omega$ and the other has its star point isolated. Determine: (a) the fault current and the power dissipated in the earthing resistor when an earth fault occurs at the far end of the feeder on phase (A); and (b) the voltage to earth of phase (B). The generator phase sequence is ABC and the impedances are as follows:

	Generator p.u./p.h.	Feeder Ω/p.h.
To positive-sequence currents	j0.2	j0.6
To negative-sequence currents	j0.16	j0.6
To zero-sequence currents	j0.06	j0.4

Use a base of 30 MVA.
(Answer: (a) $5459\angle -52.6°$ A; 11.92 MW; (b) $(-1402 + j2770)$ V)

7.7 An industrial distribution system is shown schematically in Figure 7.36. Each line has a reactance of j0.4 p.u. calculated on a 100 MVA base; other system parameters are given in the diagram. Choose suitable short-circuit ratings for the oil circuit breakers, situated at substation A, from those commercially available, which are given in the table below.

Short circuit (MVA)	75	150	250	350
Rated current (A)	100	300	400	600

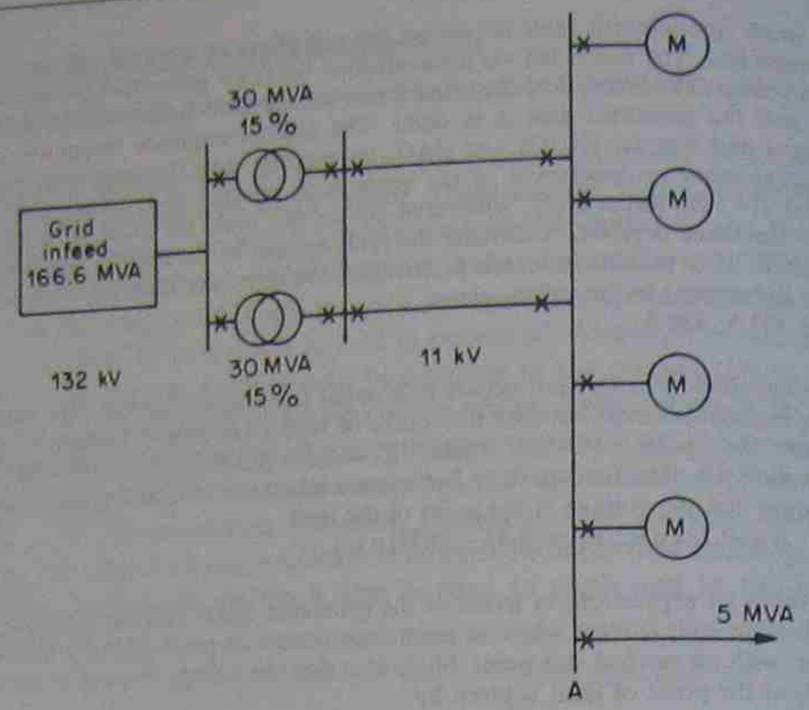


Figure 7.36 System for Problem 7.7

The industrial load consists of a static component of 5 MVA and four large induction motors each rated at 6 MVA. Show that only three motors can be started simultaneously given that, at starting, each motor takes five times full-load current at rated voltage, but at 0.3 p.f.

7.8 Explain how the Method of Symmetrical Components may be used to represent any 3 p.h. current phasors by an equivalent set of balanced phasors.

A chemical plant is fed from a 132 kV system which has a 3 p.h. symmetrical fault level of 4000 MVA. Three 15 MVA transformers, connected in parallel, are used to step down to an 11 kV busbar from which six 5 MVA, 11 kV motors are supplied. The transformers are delta–star connected with the star point of each 11 kV winding, solidly earthed. The transformers each have a reactance of 10 per cent on rating and it may be assumed that $X_1 = X_2 = X_0$. The initial fault contribution of the motors is equal to five times rated current with 1.0 p.u. terminal voltage.

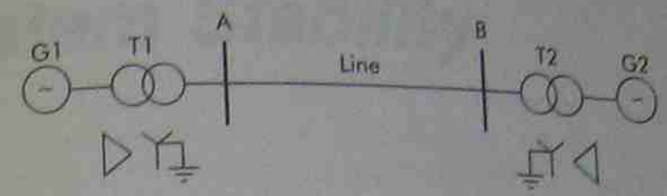
Using a base of 100 MVA,

(a) calculate the fault current (in A) for a line-to-earth short circuit on the 11 kV busbar with no motors connected;
(b) calculate the 3 p.h. symmetrical fault level (in MVA) at the 11 kV busbar if all the motors are operating and the 11 kV busbar voltage is 1.0 p.u.
(Answer: (a) 21.97 kA; (b) 544.45 MVA)
(*From E.C. Examination, 1996*)

7.9 Describe the effect on the output current of a synchronous generator following a solid three-phase fault on its terminals.

For the system shown in Figure 7.37 calculate (using symmetrical components):

(a) the current flowing in the fault for a three-phase fault at busbar A;
(b) the current flowing in the fault for a one-phase-to-earth fault at busbar B;
(c) the current flowing in the faulted phase of the overhead line for a one-phase-to-earth fault at busbar B.



Generators G1 and G1: $X_1'' = X_2'' = j0.1$ p.u.; 11 kV
Transformers T1 and T2: $X_1 = X_2 = X_0 = j0.1$ p.u.; 11/275 kV
(Earthed star–delta)
Line: $Z_1 = Z_2 = j0.05$ p.u., $Z_0 = j0.1$ p.u.; 275 kV
(All p.u. values are quoted on a base of 100 MVA)

Figure 7.37 Circuit for Problem 7.9

Assume the pre-fault voltage of each generator is 1 p.u. and calculate the symmetrical fault currents (in amps) immediately after each fault occurs.
(Answer: (a) 1.89 kA; (b) 2.18 kA; (c) 0.89 kA)
(*From E.C. Examination, 1997*)

7.10 Why is it necessary to calculate short-circuit currents in large electrical systems?

A generator rated at 400 MW, 0.8 power factor, 20 kV has a star-connected stator winding which is earthed at its star point through a resistor of 1 Ω. The generator reactances, in per unit on rating, are:

$$X_1 = 0.2 \qquad X_2 = 0.16 \qquad X_0 = 0.14$$

The generator feeds a delta–star-connected generator transformer rated at 550 MVA which steps the voltage up to a 275 kV busbar. The transformer star-point is solidly earthed and the transformer reactance is 0.15 p.u. on its rating. The 275 kV busbar is connected only to the transformer. Assume that for the transformer $X_1 = X_2 = X_0$.

Using a base of 500 MVA calculate the base current and impedance of each voltage level.

Calculate the fault current in amperes for:

(a) a 275kV busbar three-line fault;
(b) a 275 kV single-line-to earth fault on the busbar;
(c) a 20 kV three-line fault on the generator terminals;
(d) a 20 kV single-line-to-earth fault on the generator terminals.

(Answer: (a) 3.125 kA; (b) 4.1 kA; (c) 72.15 kA; (d) 11.44 kA)
(*From E.C. Examination, 1995*)

8
System Stability

8.1 Introduction

The stability of a system of interconnected dynamic components is its ability to return to normal or stable operation after having been subjected to some form of disturbance. The study of stability is one of the main concerns of the control engineer whose methods are applied to electric power systems.

When the rotor of a synchronous generator advances beyond a certain critical angle, the magnetic coupling between the rotor and the stator fails. The rotor, no longer held in synchronism with the rotating field of the stator currents, rotates relative to the field and pole slipping occurs. Each time the poles traverse the angular region where stability obtains, synchronizing forces attempt to pull the rotor into synchronism. It is general practice to disconnect the machine from the system if it commences to slip poles. However, a generator having lost synchronism may operate successfully as an induction generator for some time and then be resynchronized; the possibility of allowing controlled pole-slipping for limited periods is thus possible. It should be remembered, however, that an induction generator takes its excitation requirements from the network, which must be capable of supplying the requisite reactive power, and the rotor must not overheat and fail.

There are two forms of instability in power systems: the loss of synchronism between synchronous machines, and the stalling of asynchronous loads. Synchronous stability may be divided into two regimes: steady-state and transient. Reference to the former has already been made when discussing sychronous-machine characteristics. It is basically the ability of the power system, when operating under given load conditions, to retain synchronism when subject to *small* disturbances, such as the continual changes in load or generation and the switching out of lines. It is most likely to result from the changes in source-to-load impedance resulting from changes in the network configuration or system state. Often, this is referred to as *dynamic stability*.

Transient stability is concerned with sudden and large changes in the network condition, such as those brought about by faults. The maximum power transmittable, the stability limit under a fault condition, is less than that for the corresponding steady-state condition.

The stability of an asynchronous load is controlled by the voltage across it; if this becomes lower than a critical value, induction motors may become unstable and stall. This is, in effect, the voltage instability problem already mentioned. In a power system it is possible for either synchronous or voltage instability to occur. The former is more onerous and hence has been given much more attention in the past. Recently, with the increasing use of static compensators, the study of voltage collapse has become important (see Section 8.8)

8.2 Equation of Motion of a Rotating Machine

Before the equation is considered, a revision of the definitions of certain quantities is given. The kinetic energy absorbed by a rotating mass $= \frac{1}{2}I\omega^2$ joules, and the angular momentum is $M = I\omega$ joule-seconds per radian where ω is the synchronous speed of the rotor (radians/second) and I is the moment of inertia (kilogram-metre^2). The inertia constant (H) is defined as the stored energy at synchronous speed per volt-ampere of the rating of the machine. In power systems the unit of energy is taken as the kilojoule or megajoule. If G megavolt-amperes is the rating of the machine, then

GH =stored energy (megajoules) or kinetic energy

$= \frac{1}{2}I\omega^2 = \frac{1}{2}M\omega$ and as $\omega = 180/\pi$ (pole pairs) or $360f$ electrical degrees per second, where f is the system frequency (Hz)

then

$$GH = \tfrac{1}{2}M\,(360f)$$

and

$$M = GH/180f \text{ megajoule-seconds/electrical degree}$$

The inertia constant H for steam or gas turbogenerators decreases from 10 kW-s/kVA for machines up to 30 MVA to values in the order of 4 kW-s/kVA for large machines, the value decreasing as the capacity increases. For salient-pole water-wheel machines H depends on the number of poles; for machines in the range 200–400 r.p.m. the value increases from about 2 kW-s/kVA at 10 MVA rating to 3.5 kW-s/kVA at 60 MVA. A mean value for synchronous motors is 2 kW-s/kVA. The net accelerating torque on the rotor

of a machine, ΔT = mechanical torque input − electrical torque output $= I\,d^2\delta/dt^2$.

$$\therefore \frac{d^2\delta}{dt^2} = \frac{\Delta T}{I} = \frac{(\Delta T\omega)\omega}{2 \times I\omega^2/2} = \frac{\Delta P \cdot \omega}{2 \times \text{kinetic energy}}$$

where ΔP = net power corresponding to ΔT, i.e. $P_{mech} - P_{elect}$.

$$\therefore \frac{d^2\delta}{dt^2} = \frac{\Delta P}{M} \tag{8.1}$$

In equation (8.1) a negative change in power output results in an increase in δ. Sometimes, ΔP is considered as the change in *electrical power output* and increase in ΔP_{elect} results in increase in angle δ. As the power input is assumed to be constant, the equation of motion now becomes

$$\frac{d^2\delta}{dt^2} = -\frac{\Delta P}{M} \quad \text{or} \quad M\frac{d^2\delta}{dt^2} + \Delta P = 0$$

8.3 Steady-State Stability—Theoretical Considerations

The power system forms a group of interconnected electromechanical elements, the motion of which may be represented by the appropriate differential equations. With large disturbances in the system the equations are non-linear, but with small changes the equations may be linearized with little loss of accuracy. The differential equations having been determined, the characteristic equation of the system is then formed, from which information regarding stability is obtained. The solution of the differential equation of the motion is of the form

$$\delta = k_1 e^{a_1 t} + k_2 e^{a_2 t} + \cdots + k_n e^{a_n t}$$

where $k_1, k_2 \ldots k_n$ are constants of integration and $a_1, a_2 \ldots a_n$ are the roots of the characteristic equation as obtained through the well-known eigenvalue methods. If any of the roots have positive real terms then the quantity δ increases continuously with time and the original steady condition is not re-established. The criterion for stability is therefore that all the real parts of the roots of the characteristic equation, i.e. eigenvalues, be negative; imaginary parts indicate the presence of oscillation. Figure 8.1 shows the various types of motion. The determination of the roots is readily obtained through an eigenvalue analysis package, but indirect methods for predicting stability have been established, e.g. the Hurwitz–Routh criterion in which stability is predicted without the actual solution of the characteristic equation. No

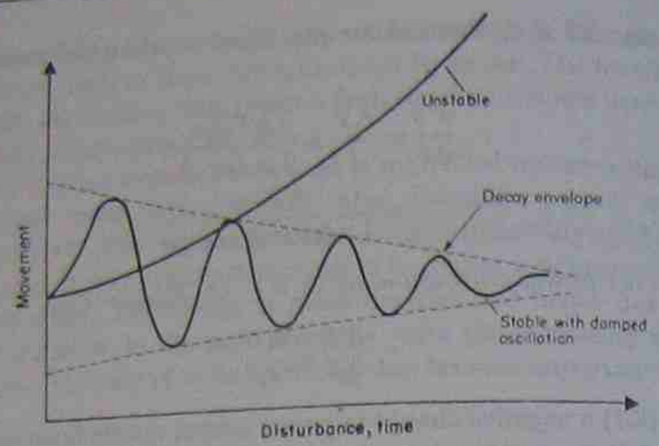


Figure 8.1 Types of response to a disturbance on a system

information regarding the degree of stability or instability is obtained, only that the system is, or is not, stable. One advantage of using eigenvalues is that the characteristics of the control loops associated with governors and automatic voltage regulators may be incorporated in the general treatment.

For a generator connected to an infinite busbar through a network of zero resistance it has been shown in equation (3.1) that $P = (VE/X)\sin\delta$. With operation at P_0 and δ_0 (Figure 8.2), we can write

$$M\frac{d^2\Delta\delta}{dt^2} = -\Delta P = -\Delta\delta\left(\frac{\partial P}{\partial\delta}\right)_0$$

where change in P causing increase in δ is positive and refers to small changes in the load angle δ such that linearity may be assumed.

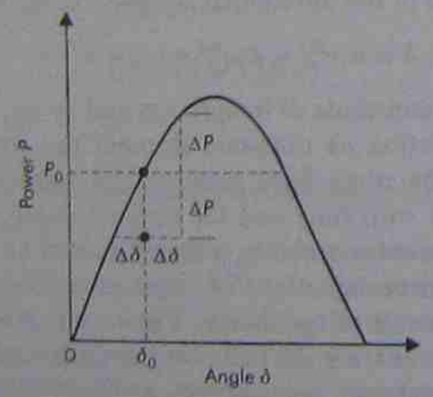


Figure 8.2 Small disturbance—initial operation on power-angle curve at P_0, δ_0. Linear movement assumed about P_0, δ_0

$$\therefore\ Ms^2\Delta\delta + \left(\frac{\partial P}{\partial\delta}\right)_0\Delta\delta = 0 \tag{8.2}$$

where

$$s \equiv d/dt$$

Here, $Ms^2 + (\partial P/\partial\delta)_0 = 0$ is the characteristic equation which has two roots

$$\pm\sqrt{\frac{-(\partial P/\partial\delta)_0}{M}}$$

When $(\partial P/\partial\delta)_0$ is positive, both roots are imaginary and the motion is oscillatory and undamped; when $(\partial P/\partial\delta)_0$ is negative, both roots are real, and positive and negative, respectively, and stability is lost. At $\delta = 90^\circ$, $(\partial P/\partial\delta)_{90} = 0$, and the system is at the limit. If damping is accounted for, the equation becomes

$$Ms^2\Delta\delta + K_d s\Delta\delta + \left(\frac{\partial P}{\partial\delta}\right)\Delta\delta = 0 \tag{8.3}$$

and the characteristic equation is

$$Ms^2 + K_d s + \left(\frac{\partial P}{\partial\delta}\right) = 0 \tag{8.4}$$

where K_d is the damping coefficient, assumed to be constant, independent of δ.

Again, if $(\partial P/\partial\delta)$ is negative, stability is lost. The frequency of the oscillation is given by the roots of the characteristic equation.

If the excitation of the generator is controlled by a fast-acting automatic voltage regulator without appreciable dead zone, the excitation voltage E is increased as increments of load are added. Hence the actual power-angle curve pertaining is no longer that for constant E (refer to Chapter 3) and the change of power may be obtained by linearizing the P–V characteristic at the new operating point (1), when

$$\Delta P = \left(\frac{\partial P}{\delta E}\right)_1\Delta E$$

The complete equation of motion is now

$$Ms^2\Delta\delta + K_d s\Delta\delta + \left(\frac{\delta P}{\partial\delta}\right)_1\Delta\delta + \left(\frac{\partial P}{\partial E}\right)_1\Delta E = 0 \tag{8.5}$$

Without automatic voltage control the stability limit is reached when $\delta = 90^\circ$; with control the criterion is obtained from the characteristic equation of (8.5).

Example 8.1

A synchronous generator of reactance 1.5 p.u. is connected to an infinite busbar system ($V = 1$ p.u.) through a line and transformers of total reactance 0.5 p.u. The no-load voltage of the generator is 1.1 p.u. and the inertia constant $H = 5$ MW-s per MVA All per unit values are expressed on the same base; resistance and damping may be neglected. Calculate the frequency of the oscillations set up when the generator operates at a load angle of 60° and is subjected to a small disturbance. The system frequency is 50 Hz.

Solution

The nature of the movement is governed by the sign of the quantity under the root sign in the equation for s_1 and s_2 (equation (8.4)). This changes when $K_d^2 = 4M(\partial P/\partial\delta)_1$, in this case $K_d = 0$. The roots of the characteristic equations give the frequency of oscillation; when $\delta_0 = 60°$,

$$\left(\frac{\partial P}{\partial \delta}\right)_{60°} = \frac{1.1 \times 1}{2}\cos 60$$

$$= 0.275 \text{ p.u.}$$

$$s_1 \text{ and } s_2 = \pm j\sqrt{\left(\frac{\partial P}{\partial \delta}\right)\cdot\frac{1}{M}}$$

$$= \pm j\sqrt{\frac{0.275}{5 \times \left(\frac{1}{\pi \times 50}\right)}}$$

$$= \pm j\sqrt{8.64}$$

Therefore frequency of oscillation

$$= 2.94 \text{rad/s} = \frac{2.94}{2\pi} \text{ Hz}$$

$$= 0.468 \text{ Hz}$$

and the periodic time

$$= \frac{1}{0.468} = 2.14 \text{ s}$$

The stability of a two-machine system

The terms M_1 and M_2 refer to machines 1 and 2 which are connected in parallel through an impedance.

The equations of motion are (for small changes, damping neglected)

$$M_1 s^2 \Delta\delta_1 + \left(\frac{\partial P_1}{\partial \delta_{12}}\right)\Delta\delta_{12} = 0$$

and

$$M_2 s^2 \Delta\delta_2 + \left(\frac{\partial P_2}{\partial \delta_{12}}\right)\Delta\delta_{12} = 0$$

As

$$\Delta\delta_1 - \Delta\delta_2 = \Delta\delta_{12}$$

then

$$s^2\Delta\delta_{12} + \left[\frac{(\partial P_1/\partial\delta_{12})}{M_1} - \frac{(\partial P_2/\partial\delta_{12})}{M_2}\right]\Delta\delta_{12} = 0$$

The characteristic equation has two roots,

$$s_1, s_2 = \pm j\sqrt{\frac{(\partial P_1/\partial\delta_{12})}{M_1} - \frac{(\partial P_2/\partial\delta_{12})}{M_2}}$$

Stability is assured if the quantity under the square root is positive. Hence the stability limit for small disturbances is not the same as the maximum power limit discussed in Chapter 3 and is, in fact, always larger. The difference, however, is never large, and when one machine is effectively an infinite busbar, the two limits coincide.

Effects of governor action

In the above analysis the oscillations set up with small changes in load on a system have been considered and the effects of governor operation ignored. After a certain time has elapsed the governor control characteristics commence to influence the powers and oscillations, as explained in Section 4.3. It is now the practice to represent both the excitation system and the governor system with the dynamic equations of the generator in the control-state space form, from which the eigenvalues of the complete system with feedback can be determined. Using well-established control design techniques, appropriate feedback paths and time constants can be established for a range of generating conditions and disturbances, thereby assuring adequate dynamic stability margins. Further information on these design processes can be found in advanced-control textbooks. Here we will consider the simpler but important aspects of so-called steady-state stability.

8.4 Steady-State Stability—Practical Considerations

The steady-state stability limit is the maximum power that can be transmitted in a network between sources and loads when the system is subject to small disturbances. The power system is, of course, constantly subjected to small changes as load variations occur. To obtain the limiting value of power, small increments of load are added to the system; after each increment the generator excitations are adjusted to maintain constant terminal voltages and a load flow is carried out. Eventually, a condition of instability is reached.

The stability limits of synchronous machines have been discussed in Chapter 3. It was seen that provided the generator operates within the 'safe area' of the performance chart (Figure 3.15), stability is assured; usually, a 20 per cent margin of safety on the var absorb side is allowed and the limit is extended by the use of automatic voltage regulators. Often, the performance charts are not used directly and the generator-equivalent circuit employing the synchronous impedance is used. The normal operating load angle for modern machines is in the order of 60 electrical degrees, and for the limiting value of 90° this leaves 30° to cover the transmission network. In a complex system a reference point must be taken from which the load angles are measured; this is usually a point where the direction of power flow reverses.

The simplest criterion for steady-state synchronous stability is $(\partial P/\partial\delta) > 0$, i.e. the synchronizing coefficient must be positive. The use of this criterion involves the following assumptions:

1. generators are represented by constant impedances in series with the no-load voltages;
2. the input torques from the turbines are constant;
3. changes in speed are ignored;
4. electromagnetic damping in the generators is ignored;
5. the changes in load angle δ are small.

The degree of complexity to which the analysis is taken has to be decided, e.g. the effects of machine inertia, governor action, and automatic voltage regulators can be included; these items, however, greatly increase the complexity of the calculations. The use of the criterion $(\partial P/\partial\delta) = 0$ alone gives a pessimistic or low result and hence an inbuilt factor of safety.

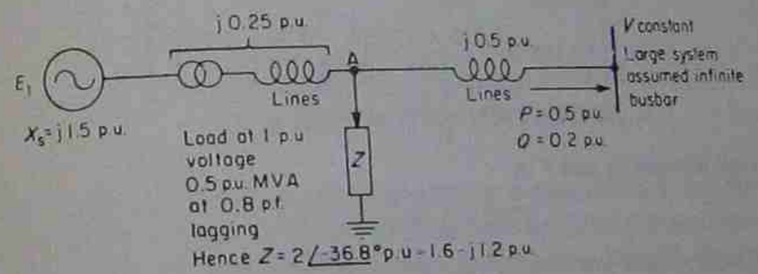
In a system with several generators and loads, the question as to where the increment of load is to be applied is important. A conservative method is to assume that the increment applies to one machine only, determine the stability, and then repeat for each of the other machines in turn. Alternatively, the power outputs from all but the two generators having the largest load angles are kept constant.

For calculations made without the aid of computers it is usual to reduce the network to the simplest form that will keep intact the generator nodes. The values of load angle, power, and voltage are then calculated for the given conditions, $\partial P/\partial\delta$ determined for each machine, and, if positive, the loading is increased and the process repeated.

In a system consisting of a generator supplying a load through a network of lines and transformers of effective reactance, X_T the value of $(dP/d\delta) = (EV/X_T)\cos\delta$, where E and V are the supply- and receiving-end voltages and δ the total angle between the generator rotor and the phasor of V. The power transmitted is obviously increased with higher system voltages and lower reactances, and it may be readily shown that line series capacitance increases the stability limit. The determination of $dP/d\delta$ is not very difficult if the voltages at the loads can be assumed to be constant or if the loads can be represented by impedances. Use can be made of the P–V, Q–V characteristics of the load if the voltages change appreciably with the redistribution of the power in the network; this process, however, is extremely tedious.

Example 8.2

For the system shown in Figure 8.3, investigate the steady-state stability.



(a) Line diagram of system for Example 8.2

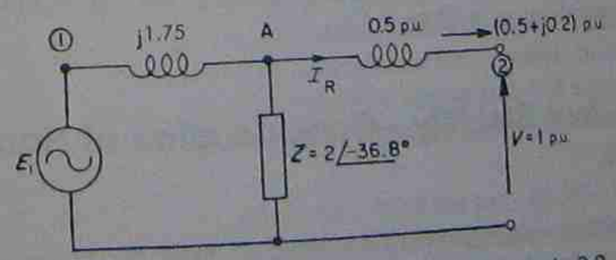


Figure 8.3 (b) Equivalent circuit for Example 8.2

Solution

Voltage at the load is given by

$$V_A = \sqrt{\left[\left(V + \frac{QX}{V}\right)^2 + \left(\frac{PX}{V}\right)^2\right]}$$

$$= \sqrt{\left[\left(1 + \frac{0.2 \times 0.5}{1.0}\right)^2 + \frac{(0.5 \times 0.2)^2}{1.0}\right]}$$

$$= 1.105 \text{ p.u. at } 5.19^\circ \text{ to } V$$

Reactive power absorbed by line A to (2)

$$= I_R^2 X = \left(\frac{P^2 + Q^2}{V^2}\right) X = \left(\frac{0.5^2 + 0.2^2}{1.0^2}\right) 0.5$$

$$= 0.145 \text{ p.u.}$$

The actual load taken by A (if represented by an impedance) is given by

$$\frac{V_A^2}{Z} = \frac{1.105^2}{2\angle -36.8^\circ} = 0.61\angle 36.8^\circ = (0.49 + j0.37) \text{ p.u.}$$

Total load supplied by link from generator to A

$$= (0.5 + 0.49) + j(0.2 + 0.145 + 0.37)$$

$$= (0.99 + j0.715) \text{ p.u.}$$

Internal voltage of generator E_1

$$= \sqrt{\left[\left(1.105 + \frac{0.715 \times 1.75}{1.105}\right)^2 + \left(\frac{0.99 \times 1.75}{1.105}\right)^2\right]}$$

$$= \sqrt{[5.006 + 2.458]} = 2.73 \text{ at } 35.02^\circ \text{ to } V_A$$

Hence, angle between E_1 and V is

$$35.02 + 5.19 = 40.21^\circ$$

Since this angle is much less than 90°, the system is stable.
(Note that the formula $P = (E_1 V/X) \sin\delta$ is not valid here as the load at A provides a resistance in the equivalent network. If the angle between E_1 and V by the approximate calculation shown above is nearly equal to 90°, then the power will need to be calculated from the exact equations as in Chapter 3.)

8.5 Transient Stability—Consideration of Rotor Angle

Transient stability is concerned with the effect of large disturbances. These are usually due to faults, the most severe of which is the three-phase short circuit

which governs the transient stability limits in the U.K. Elsewhere, limits are based on other types of fault, notably the single-line-to-earth fault which is by far the most frequent in practice.

When a fault occurs at the terminals of a synchronous generator the power output of the machine is greatly reduced as it is supplying a mainly inductive circuit. However, the input power to the generator from the turbine has not time to change during the short period of the fault and the rotor endeavours to gain speed to store the excess energy. If the fault persists long enough the rotor angle will increase continuously and synchronism will be lost. Hence the time of operation of the protection and circuit breakers is all-important.

An aspect of importance is the use of *autoreclosing* circuit breakers. These open when the fault is detected and automatically reclose after a prescribed period (usually less than 1 s). If the fault persists the circuit breaker reopens and then recloses as before. This is repeated once more, when, if the fault still persists, the breaker remains open. Owing to the transitory nature of most faults, often the circuit breaker successfully recloses and the rather lengthy process of investigating the fault and switching in the line is avoided. The length of the autoreclose operation must be considered when assessing transient stability limits; in particular, analysis must include the movement of the rotor over this period and not just the first swing, as is often the case. If, in equation (8.1), both sides are multiplied by $2(d\delta/dt)$, then

$$\left(2\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right) = \frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2\Delta P}{M}\left(\frac{d\delta}{dt}\right)$$

$$\therefore \left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} \Delta P \, d\delta \qquad (8.6)$$

The rotor will swing until its angular velocity is zero, in which case the machine remains stable; if $d\delta/dt$ does not become zero the rotor will continue to move and synchronism is lost. The integral of $\Delta P \, d\delta$ in equation (8.6) represents an area on the P–δ diagram. Hence the criterion for stability is that the area between the P–δ curve and the line representing the power input P_0 in Figure 8.2 must be zero. This is known as the *equal-area criterion*. It should be noted that this is based on the assumption that synchronism is retained or lost on the first swing or oscillation of the rotor, which in certain cases is subject to doubt. Physically, the criterion means that the rotor must be able to return to the system all the energy gained from the turbine during the acceleration period.

A simple example of the equal-area criterion may be seen by an examination of the switching out of one of two parallel lines which connect a generator to an infinite busbar (Figure 8.4). For stability to be retained the two

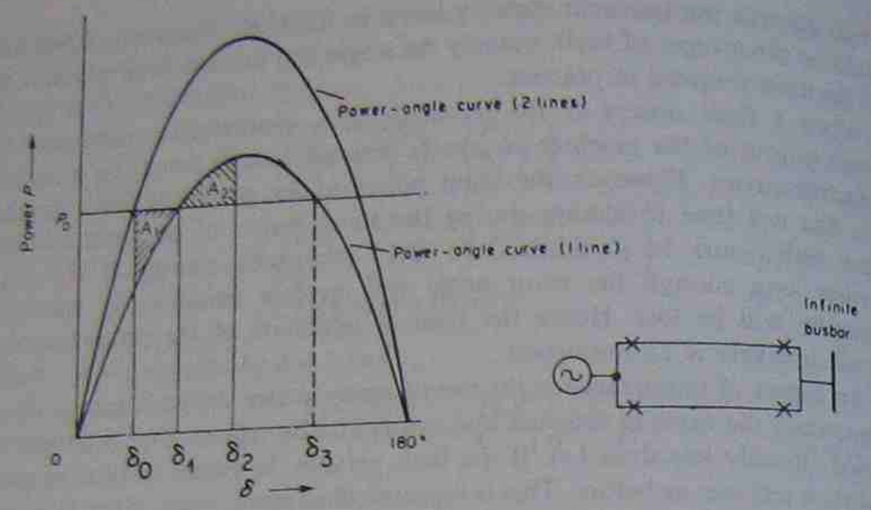


Figure 8.4 Power-angle curves for one line and two lines in parallel. Equal-area criterion. Resistance neglected

shaded areas (A_1 and A_2) are equal and the rotor comes initially to rest at angle δ_2, after which it oscillates until completely damped. In this particular case the initial operating power and angle could be increased to such values that the shaded area between δ_0 and δ_1 (A_1) could be equal to the area between δ_1 and δ_3, where $\delta_3 = 180 - \delta_1$; this would be the condition for maximum input power. Beyond δ_3, energy would again be absorbed by the rotor from the turbine.

The power-angle curves pertaining to a fault on one of two parallel lines are shown in Figure 8.5. The fault is cleared in a time corresponding to δ_1 and the shaded area δ_0 to δ_1 (A_1) indicates the energy stored. The rotor swings until it reaches δ_2 when the two areas A_1 and A_2 are equal. In this particular case P_0 is the maximum operating power for a fault clearance time corresponding to δ_1, and, conversely, δ_1 is the *critical clearing angle* for P_0. If the angle δ_1 is decreased it is possible to increase the value of P_0 without loss of synchronism. The general case where the clearing angle δ_1 is not critical is shown in Figure 8.6. Here, the rotor swings to δ_2, where the shaded area from δ_0 to δ_1 (A_1) is equal to the area δ_1 to δ_2 (A_2). Critical conditions are reached when $\delta_2 = 180 - \sin^{-1}(P_0/P_2)$. The time corresponding to the critical clearing angle is called the *critical clearing time* for the particular (normally full-load) value of power input. The time is of great importance to protection and switchgear engineers as it is the maximum time allowable for their equipment to operate without instability occurring. The critical clearing angle for a fault on one of two parallel lines may be determined as follows:

Applying the equal-area criterion to Figure 8.5:

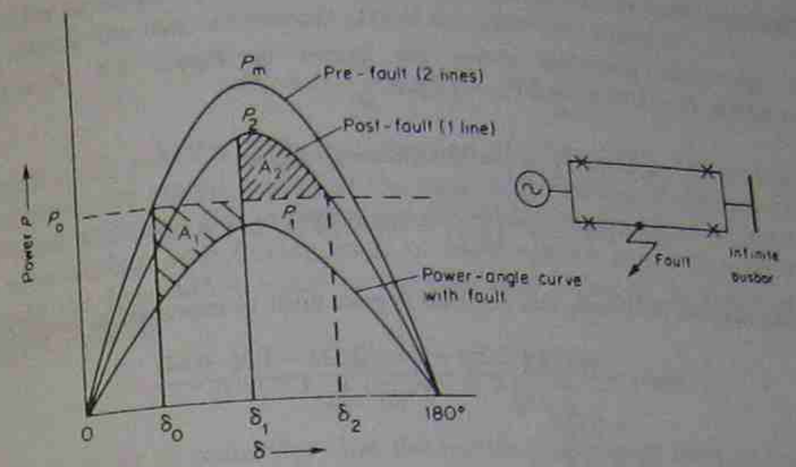


Figure 8.5 Fault on one line of two lines in parallel. Equal-area criterion. Resistance neglected. δ_1 is critical clearing angle for input power P_0

$$\int_{\delta_0}^{\delta_1} (P_0 - P_1 \sin\delta)\,d\delta + \int_{\delta_1}^{\delta_2} (P_0 - P_2 \sin\delta)\,d\delta = 0$$

$$\therefore [P_0\delta + P_1 \cos\delta]_{\delta_0}^{\delta_1} + [P_0\delta + P_2 \cos\delta]_{\delta_1}^{\delta_2} = 0$$

from which,

$$\delta_2 = 180 - \sin^{-1}\left(\frac{P_0}{P_2}\right)$$

and

$$\cos\delta_1 = \frac{P_0(\delta_0 - \delta_2) + P_1 \cos\delta_0 - P_2 \cos\delta_2}{P_1 - P_2} \qquad (8.7)$$

Hence the critical clearing angle δ_1 is determined.

Example 8.3

A generator operating at 50 Hz delivers 1 p.u. power to an infinite busbar through a network in which resistance may be neglected. A fault occurs which reduces the maximum power transferable to 0.4 p.u., whereas before the fault this power was 1.8 p.u. and after the clearance of the fault it is 1.3 p.u. By the use of the equal-area criterion, determine the critical clearing angle.

Solution

The appropriate load-angle curves are shown in Figure 8.6. $P_0 = 1$ p.u., $P_1 = 0.4$ p.u., $P_2 = 1.3$ p.u., and $P_m = 1.8$ p.u.

$$\delta_0 = \sin^{-1}\left(\frac{1}{1.8}\right) = 33.8 \text{ electrical degrees}$$

$$\delta_2 = 180 - \sin^{-1}\left(\frac{1}{1.3}\right) = 180 - 50.28^\circ = 129.72^\circ$$

Applying equation (8.7) (note that electrical degrees must be expressed in radians),

$$\cos\delta_1 = \frac{1(0.59 - 2.26) + 0.4 \times 0.831 - 1.3(-0.64)}{0.4 - 1.3}$$

$$= 0.562$$

$$\therefore \delta_1 = 55.8^\circ$$

In a large system it is usual to divide the generators and loads into a single equivalent generator connected to an equivalent motor. The main criterion is that the machines should be electrically close when forming an equivalent generator or motor. If stability with faults in various places is investigated, the position of the fault will decide the division of machines between the equivalent generator and motor. A power system (including generation) at the receiving end of a long line would constitute an equivalent motor if not large enough to be an infinite busbar. It should be noted that a three-phase short circuit on the generator terminals or on a closely connected busbar absorbs zero power and prevents the generator outputting any power to the system. Consequently, $P_1 = 0$ in Figure 8.6.

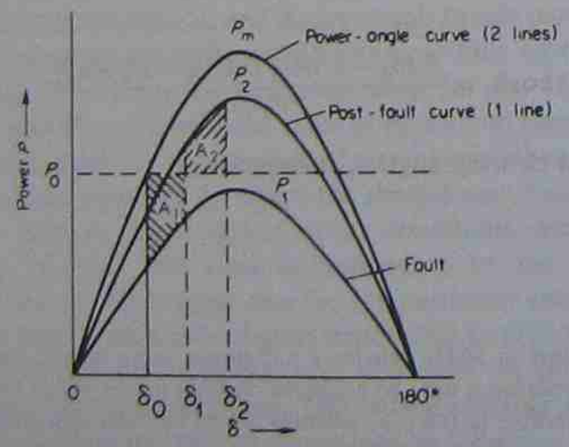


Figure 8.6 Situation as in Figure 8.5, but δ_1 not critical

Reduction to simple system With a number of generators connected to the same busbar the inertia constant (H) of the equivalent machine,

$$H_e = H_1\frac{S_1}{S_b} + H_2\frac{S_2}{S_b} \cdots H_n\frac{S_n}{S_b}$$

where $S_1 \ldots S_n$ = MVA of the machines and S_b = base MVA. This is obtained by equating the stored energy of the equivalent machine to the total of the individual machines. For example, consider six identical machines connected to the same busbar, each having an H of 5 MW-s/MVA and rated at 60 MVA. Making the base MVA equal to the combined rating of the machines, the effective value of

$$H = 5 \times \frac{60}{6 \times 60} \times 6 = 5\frac{\text{MW-s}}{\text{MVA}}$$

It is important to remember that the inertia of the loads must be included; normally, this will be the sum of the inertias of the induction motors and their mechanical loads connected.

Two synchronous machines connected by a reactance may be reduced to one equivalent machine feeding through the reactance to an infinite busbar system. The properties of the equivalent machine are found as follows.

The equation of motion for the two-machine system is

$$\frac{d^2\delta}{dt^2} = \frac{\Delta P_1}{M_1} - \frac{\Delta P_2}{M_2} = \left(\frac{1}{M_1} + \frac{1}{M_2}\right)(P_0 - P_m \sin\delta)$$

where δ is the relative angle between the machines. Note that

$$\Delta P_1 = -\Delta P_2 = (P_0 - P_m \sin\delta)$$

where P_0 is the input power and P_m the maximum transmittable power.

For a single generator of M_e and the same input power connected to the infinite busbar system,

$$M_e\frac{d^2\delta}{dt^2} = P_0 - P_m \sin\delta$$

therefore,

$$M_e = \frac{M_1 M_2}{M_1 + M_2} \tag{8.8}$$

This equivalent generator has the same mechanical input as the actual machines and the load angle δ it has with respect to the busbar is the angle between the rotors of the two machines.

Often, the maximum powers transferable before, during, and after a fault need to be calculated from the system configuration reduced to a network between the relevant generators. The use of network reduction by nodal elimination is most valuable in this context; it only remains then for the transfer

reactances to be calculated, as any shunt impedance at the reduced nodes does not influence the power transferred. With unbalanced faults more power is transmitted during the fault period than with three-phase short circuits and the stability limits are higher.

Effect of automatic voltage regulators and governors

These may be represented in the equation of motion as follows,

$$M\frac{d^2\delta}{dt^2} + K_d\frac{d\delta}{dt} = (P_0 - \Delta P_0) - P_e \qquad (8.9)$$

where K_d = damping coefficient;

P_0 = power input;

ΔP_0 = change in input power due to governor action;

P_e = electrical power output modified by the voltage regulator.

Equation (8.9) is best solved by digital computer.

8.6 Transient Stability—Consideration of Time

The swing curve

In the previous section, attention has been mainly directed towards the determination of rotor angular position; in practice, the corresponding times are more important. The protection engineer requires allowable times rather than angles when specifying relay settings. The solution of equation (8.1) with respect to time is performed by means of numerical methods and the resulting time–angle curve is known as the swing curve. A simple step-by-step method will be given in detail and references will be made to more sophisticated methods suitable for digital computation.

In this method the change in the angular position of the rotor over a short time interval is determined. In performing the calculations the following assumptions are made:

1. The accelerating power ΔP at the commencement of a time interval is considered to be constant from the middle of the previous interval to the middle of the interval considered.
2. The angular velocity is constant over a complete interval and is computed for the middle of this interval.

These assumptions are probably better understood by reference to Figure 8.7. From Figure 8.7,

$$\omega_{n-\frac{1}{2}} - \omega_{n-\frac{3}{2}} = \frac{d^2\delta}{dt^2}\Delta t = \frac{\Delta P_{n-1}}{M}\Delta t$$

The change in δ over the $(n-1)$th interval, i.e. from times $(n-1)$ to $(n-2)$

$$= \delta_{n-1} - \delta_{n-2} = \Delta\delta_{n-1} = \omega_{n-\frac{1}{2}}\Delta t$$

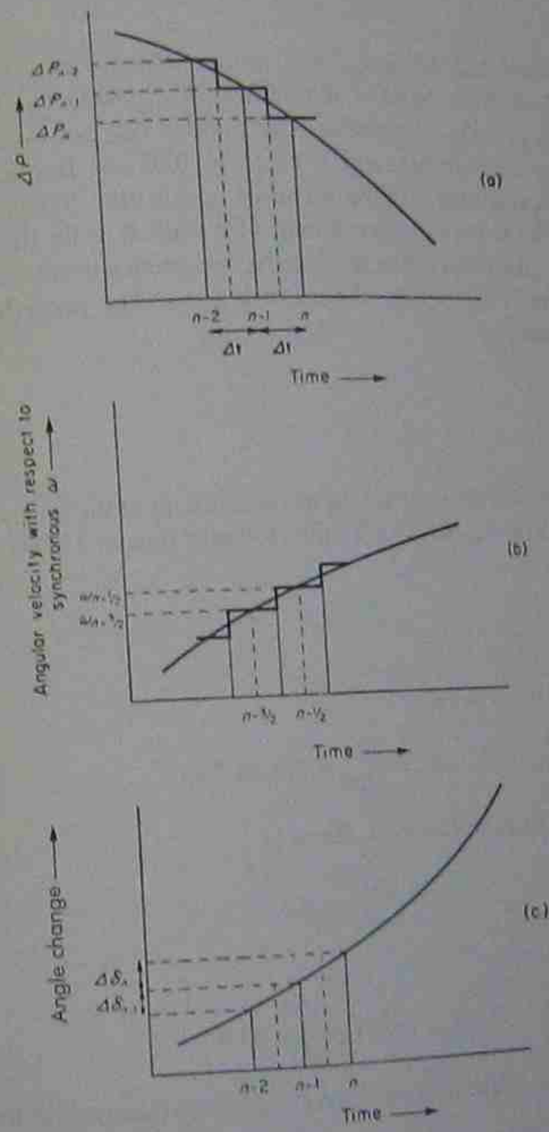


Figure 8.7 (a), (b), and (c) Variation of ΔP, ω, and $\Delta\delta$ with time. Illustration of step-by-step method to obtain δ time curve

as $\omega_{n-\frac{1}{2}}$ is assumed to be constant. Over the nth interval,

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \omega_{n-\frac{1}{2}}\Delta t$$

From the above,

$$\Delta\delta_n - \Delta\delta_{n-1} = \Delta t\left(\omega_{n-\frac{1}{2}} - \omega_{n-\frac{3}{2}}\right)$$
$$= \Delta t \cdot \Delta t \cdot \frac{\Delta P_{n-1}}{M}$$
$$\therefore\ \Delta\delta_n = \Delta\delta_{n-1} + \frac{\Delta P_{n-1}}{M}(\Delta t)^2 \qquad (8.10)$$

It should be noted that $\Delta\delta_n$ and $\Delta\delta_{n-1}$ are the *changes* in angle.

Equation (8.10) is the basis of the numerical method. The time interval Δt used should be as small as possible (the smaller Δt, however, the larger the amount of labour involved), and a value of 0.05 s is frequently used. Any change in the operational condition causes an abrupt change in the value of ΔP. For example, at the commencement of a fault ($t = 0$), the value of ΔP is initially zero and then immediately after the occurrence it takes a definite value. When two values of ΔP apply, the mean is used. The procedure is best illustrated by an example.

Example 8.4

In the system described in Example 8.3 the inertia constant of the generator plus turbine is 2.7 p.u. Obtain the swing curve for a fault clearance time of 125 ms.

Solution

$H = 2.7$ p.u., $f = 50$ Hz, $G = 1$ p.u.

$$\therefore\ M = \frac{HG}{180f} = 3 \times 10^{-4}\ \text{p.u.}$$

A time interval $\Delta t = 0.05$ s will be used. Hence

$$\frac{(\Delta t)^2}{M} = 8.33$$

The initial operating angle

$$\delta_0 = \sin^{-1}\left(\frac{1}{1.8}\right) = 33.8^\circ$$

Just before the fault the accelerating power $\Delta P = 0$. Immediately after the fault,

$$\Delta P = 1 - 0.4\sin\delta_0$$
$$= 0.78\ \text{p.u.}$$

The first value is that for the middle of the preceding period and the second is for the middle of the period under consideration. The value to be taken for ΔP at the commencement of this period is (0.780/2), i.e. 0.39 p.u. At $t = 0$, $\delta = 33.8^\circ$.

Δt_1: $\Delta t_1 = 0.05$ s $\quad \Delta P = 0.39$ p.u.

$$\therefore\ \Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M}\Delta P_{n-1}$$
$$\therefore\ \Delta\delta_n = 8.33 \times 0.39 = 3.25^\circ$$
$$\therefore\ \delta_{0.05} = 33.8 + 3.25 = 37.05^\circ$$

Δt_2:
$$\Delta P = 1 - 0.4\sin 37.05^\circ = 0.76\ \text{p.u.}$$
$$\therefore\ \Delta\delta_n = 3.25 + (8.33 \times 0.76) = 9.58^\circ$$
$$\therefore\ \delta_{0.1} = 37.05 + 9.58 = 46.63^\circ$$

Δt_3:
$$\Delta P = 1 - 0.4\sin 46.63^\circ = 0.71\ \text{p.u.}$$
$$\Delta\delta_n = 9.58 + (8.33 \times 0.71) = 15.49^\circ$$
$$\delta_{0.15} = 46.63 + 15.49 = 62.12^\circ$$

The fault is cleared after a period of 0.125 s. As this discontinuity occurs in the middle of a period (0.1–0.15 s), no special averaging is required (Figure 8.8).

If, on the other hand, the fault is cleared in 0.15 s, an averaging of two values would be required.

From $t = 0.15$ s onwards,

$$P = 1 - 1.3\sin\delta \qquad \text{(note change to } P_2 \text{ curve of Figure 8.6)}$$

Δt_4:
$$\Delta P = 1 - 1.3\sin 62.12^\circ = -0.149\ \text{p.u.}$$
$$\Delta\delta_n = 15.49 + 8.33(-0.149) = 14.25^\circ$$
$$\therefore\ \delta_{0.2} = 14.25 + 62.12 = 76.37^\circ$$

Δt_5:
$$\Delta P = 1 - 1.3\sin 76.4^\circ = -0.26\ \text{p.u.}$$
$$\Delta\delta_n = 14.25 - (8.33 \times 0.26) = 12.08^\circ$$

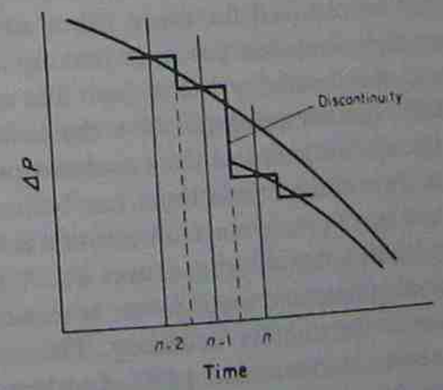


Figure 8.8 Discontinuity of ΔP in middle of a period of time

and $\delta_{0.25} = 88.39°$

Δt_6 $\Delta P = 1 - 1.3 \sin 88.39 = -0.3$ p.u.

$\Delta\delta_n = 12.09 - 2.5 = 9.59°$

and $\delta_{0.3} = 97.98°$

If this process is continued, δ commences to decrease and the generator remains stable.

If computed by hand, a tabular calculation is recommended, as shown in Figure 8.9.

t(s)	$P = P_1 \sin\delta$	ΔP	$\frac{(\Delta t)^2}{M} \cdot \Delta P$	$\Delta\delta_n°$	$\delta_n°$
0−	1.0	0.00	—	—	33.80
0+		0.78	—	—	33.80
0.05	0.23	0.39	3.25	3.25	37.05
0.10	0.24	0.76	6.33	9.58	46.63
0.15	0.29	0.71	5.91	15.49	62.12
0.20	1.149	−0.149	−1.24	14.25	76.37
0.25	1.26	−0.26	−2.17	12.08	88.39
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮

Figure 8.9 Tabular calculation of δ_n

This calculation should be continued for at least the peak of the first swing, but if switching or autoreclosing is likely to occur somewhere in the system, the calculation of $\delta°_n$ needs to be continued until oscillations are seen to be dying away. A typical swing curve shown in Figure 8.10 illustrates this situation.

Different curves will be obtained for other values of clearing time. It is evident from the way the calculation proceeds that for a sustained fault, δ will continuously increase and stability will be lost. The critical clearing time should be calculated for conditions which allow the least transfer of power from the generator. Circuit breakers and the associated protection operate in times dependent upon their design; these times can be in the order of a few cycles of alternating voltage. For a given fault position a faster clearing time implies a greater permissible value of input power P_0. A typical relationship between the critical clearing time and input power is shown in Figure 8.11 — this is often referred to as the stability boundary. The critical clearing time increases with increase in the inertia constant (H) of turbine generators. Often, the first swing of the machine is sufficient to indicate stability.

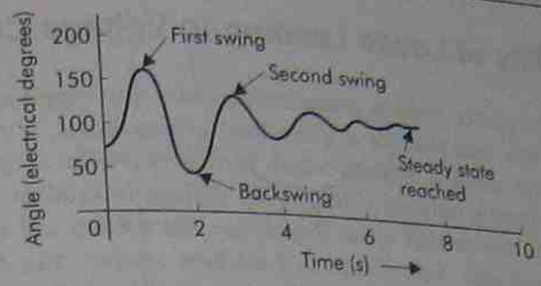


Figure 8.10 Typical swing curve for generator

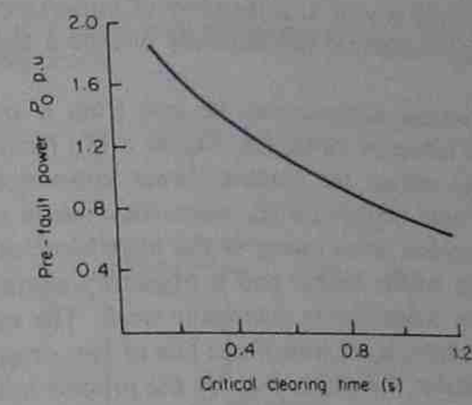


Figure 8.11 Typical stability boundary

8.7 Transient Stability Calculations by Computer

It is obvious that a digital computer program can be readily written to carry out the simple studies of Section 8.6. If a load-flow program is readily available, then improved accuracy will be obtained if, for each value of δ_n, the actual power output of the generators is calculated. At the same time, the effect of the excitation system and the governor movement can be included. Such calculations make use of numerical integration packages based on mathematical concepts. Techniques such as trapezoidal integration provide fast and sufficiently accurate results for many stability studies; more accurate techniques, such as Runge–Kutta (fourth order), predictor–corrector routines, etc., can be employed if the improved accuracy and longer run times can be economically justified. Most commercial stability programs offer various options for inclusion of generator controls, system switching and reclosing, compensator modelling, and transformer tap-change operation, according to some input criteria. Packages dealing with 1000 generators, 2000 lines, and 1500 nodes are available.

8.8 Stability of Loads Leading to Voltage Collapse

In Chapter 5 the power–voltage characteristics of a line supplying a load were considered. It was seen that for a given load power-factor, a value of transmitted power was reached, beyond which further decreases of the load impedance produce greatly reduced voltages, i.e. voltage instability. If the load is purely static, e.g. represented by an impedance, the system will operate stably at these lower voltages. Sometimes, in load-flow studies, this lower voltage condition is unknowingly obtained and unexpected load flows result. If the load contains non-static elements, such as induction motors, the nature of the load characteristics is such that beyond the critical point the motors will run down to a standstill or stall. It is therefore of importance to consider the stability of composite loads that will normally include a large proportion of induction motors.

The process of voltage collapse may be seen from a study of the V–Q characteristics of an induction motor (see Figure 3.48), from which it is seen that below a certain voltage the reactive power consumed increases with decrease in voltage until $(dQ/dV) = \infty$, when the voltage collapses. In the power system the problem arises owing to the impedance of the connection between the load and infinite busbar and is obviously aggravated when this impedance is high, i.e. connection is electrically weak. The usual cause of an abnormally high impedance is the loss of one line of two or more forming the connection. It is profitable, therefore, to study the process in its basic form—that of a load supplied through a reactance from a constant voltage source (Figure 8.12)

Already, two criteria for load instability have been given, i.e. $(dP/d\delta) = 0$ and $(dQ/dV) = \infty$; from the system viewpoint, voltage collapse takes place when $(dE/dV) = 0$ or $(dV/dE) = \infty$. Each value of E yields a corresponding value for V, and the plot of E–V is shown in Figure 8.13; also the plot of V–X for various values of E is shown in Figure 8.14. In these graphs the critical operating condition is clearly shown and the improvement produced by higher values of E is apparent, indicating the importance of the system-operating voltage from the load viewpoint.

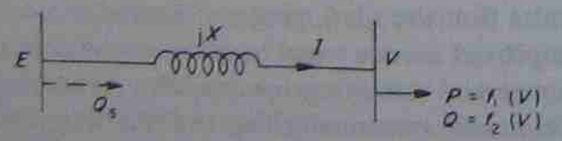


Figure 8.12 System with a load dependent on voltage as follows: $P = f_1(V)$ and $Q = f_2(V)$; Q_s = supply vars = $Q + I^2X$; E = supply voltage

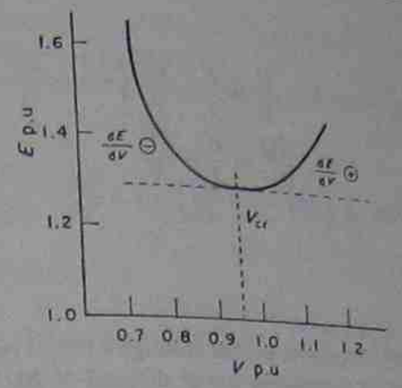


Figure 8.13 The E–V relationship per system in Figure 8.12. V_{cr} = critical voltage after which instability occurs

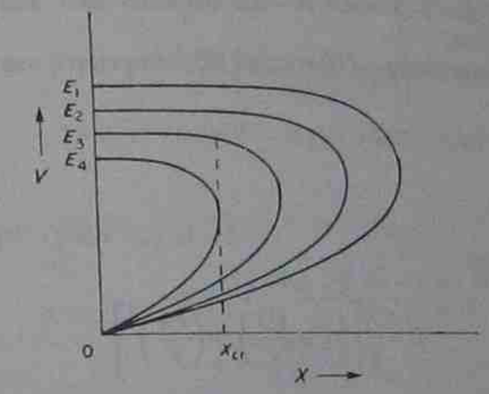


Figure 8.14 The V–X relationship. Effect of change in supply voltage E. X_{cr} = critical reactance of transmission link

In the circuit shown in Figure 8.12, and from equation 2.11,

$$E = V + \frac{QX}{V} \quad \text{if} \quad \frac{PX}{V} \ll \frac{V^2 + QX}{V}$$

$$\therefore \frac{dE}{dV} = 1 + \left(\frac{dQ}{dV} \cdot XV - QX\right)\frac{1}{V^2}$$

which is zero at the stability limit and negative in the unstable region.

At the limit,

$$\frac{dE}{dV} = 0 \quad \text{and} \quad \frac{dQ}{dV} = \left(\frac{QX}{V^2} - 1\right)\frac{V}{X} = \frac{Q}{V} - \frac{V}{X}$$

From equation (2.11),

$$\frac{Q}{V}=\frac{E}{X}-\frac{V}{X}$$

$$\therefore \frac{dQ}{dV}=\frac{E}{X}-\frac{2V}{X} \qquad (8.11)$$

Example 8.5

A load is supplied from a 275 kV infinite busbar through a line of reactance 70 Ω phase-to-neutral. The load consists of a constant power demand of 200 MW and a reactive power demand Q which is related to the load voltage V by the equation:

$$(V-0.8)^2=0.2(Q-0.8)$$

This is shown in Figure 8.15, where the base quantities for V and Q are 275 kV and 200 MVAr, respectively.

Examine the voltage stability of this system, indicating clearly any assumptions made in the analysis.

Solution

It has been shown that

$$E=\sqrt{\left[\left(V+\frac{QX}{V}\right)^2+\left(\frac{PX}{V}\right)^2\right]}$$

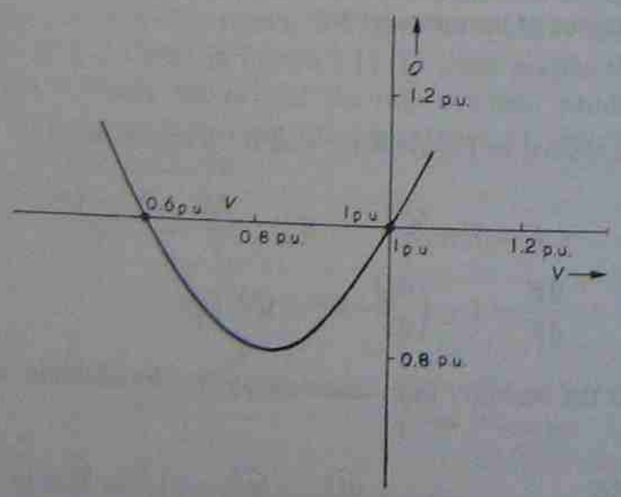


Figure 8.15 Example 8.5. Reactive power–voltage characteristic

If

$$\frac{PX}{V}<\frac{V^2+QX}{V} \quad \text{then} \quad E=\frac{V^2+QX}{V}$$

and

$$\frac{dE}{dV}=1+\left(\frac{dQ}{dV}XV-QX\right)\frac{1}{V_2}=0$$

In this problem

$$\left.\begin{matrix} P=200\,\text{MW} \\ Q=200\,\text{MVAr} \end{matrix}\right\} \text{1 p.u. three-phase values}$$

and

$$X=\frac{70\times 200}{275^2}\,\text{p.u.}$$

$$=0.185\,\text{p.u.}$$

$$E=V+\frac{QX}{V}$$

$$\therefore 1=V+\frac{0.185}{V}\left(\frac{(V-0.8)^2}{0.2}+0.8\right)$$

$$\therefore V^2=V-0.925V^2-0.59+1.48V-0.148$$

giving

$$1.925V^2-2.48V+0.74=0$$

$$\therefore V=\frac{+2.48\pm\sqrt{(2.48^2-4\times 1.925\times 0.74)}}{2\times 1.925}$$

$$=\frac{2.48\pm 0.67}{3.85}$$

Taking the upper value, $V=0.818$ p.u.,

$$Q \text{ at this } V=\frac{(0.818-0.8)^2}{0.2}+0.8$$

$$=0.8016\,\text{p.u.}$$

$$\frac{dE}{dV}=1+[10(0.818-0.8)\times 0.185\times 0.818-0.8016\times 0.185/V^2]$$

$$=1+\left(\frac{0.027-0.148}{0.818^2}\right)$$

which is positive, i.e. the system is stable.

(Note: $PX/V\approx 0.16$ and $(V^2+QX)/V\approx 1$; therefore approximation is reasonable.)

When the reactance between the source and load is very high, the use of tap-changing transformers is of no assistance. Large voltage drops exist in the supply lines and the 'tapping-up' of transformers increases these because of the increased supply currents. Hence the peculiar effects from tap-changing

noticed when conditions close to a voltage collapse have occurred in practice, i.e. tapping-up reduces the secondary voltage, and vice versa. One symptom of the approach of critical conditions is sluggishness in the response of tap-changing transformers.

The above form of instability is essentially a form of steady-state instability, and a full assessment of the latter should not only take into account the synchronous criterion $(dP/d\delta) \geq 0$, although this is the more likely to occur, but also the criterion for load instability $(dV/dE) = \infty$.

Recent studies into the voltage collapse phenomenon in interconnected power systems have shown how difficult it is to predict its occurrence or, more usefully, estimate the margin available to collapse at any given operating condition. One approach is to increase all loads at constant power factor until the load flow diverges. Unfortunately, this does not provide a realistic case since (1) any increase in real-power load will result in reserve generation being brought on line at various nodes of the system not previously used for generation input; and (2) as voltage falls, loads behave non-linearly (see Figure 8.15) but our knowledge of load behaviour below about 0.9 p.u. voltage is extremely sparse. Consequently, only worst-case situations can be assumed if voltage margins are to be estimated. Further data from actual situations and improved study methods, e.g. using eigenvalues, are being sought.

8.9 Further Aspects

8.9.1 Faults on the feeders to induction motors

A more common cause of the stalling of induction motors (or the low-voltage releases operating and removing them from the supply) occurs when the supply voltage is either zero or very low for a brief period because of a fault on the supply system, commonly known as 'voltage sag'. When the supply voltage is restored the induction motors accelerate and endeavour to attain their previous operating condition. In accelerating, however, a large current is taken, and this, plus the fact that the system impedance has increased due to the loss of a line, results in a depressed voltage at the motor terminals. If this voltage is too low the machines will stall or cut out of circuit. An uninterruptible power supply (UPS) or voltage injection from a compensator could help to alleviate this problem.

8.9.2 Steady-state instability due to voltage regulators

Consider a generator supplying an infinite busbar through two lines, one of which is suddenly removed. The load angle of the generator is instantaneously

unchanged and therefore the power output decreases due to the increased system reactance, thus causing the generator voltage to rise. The automatic voltage regulator of the generator then weakens its field to maintain constant voltage, i.e. decreases the internal e.m.f., and synchronous instability may result.

8.9.3 Dynamic stability

The control circuits associated with generator AVRs, although improving steady-state stability, can introduce problems of poorly damped response and even instability. For this reason, dynamic stability studies are performed, i.e. steady-state stability analysis, including the automatic control features of the machines (see Section 8.5). The stability is assessed by determining the response to small step changes of rotor angle, and hence the machine and control-system equations are often linearized around the operating point, i.e. constant machine parameters and linear AVR characteristic. The study usually extends over several seconds of real-system time.

8.10 Multimachine Systems and Energy-Type Functions

In networks interconnecting many generators and dynamic loads, it is a complicated task, even with a fast digital computer, to solve a set of dynamic equations to establish if a given state of the system is stable or unstable following a credible disturbance. Obviously, many hundreds of studies may need to be run with different faults and the output data assessed in some way. It must be remembered that if each dynamic machine (generator or load) is to be represented by its swing trajectory, starting from an initial steady-state angle (as in Figure 8.10), then a multitude of trajectories up to 5 s will be presented by the computer output. Some machines will probably not be affected by the disturbance and their angles remain within a few degress of their initial angle. Others will show oscillations but their mean angle could gradually diverge from the more or less unaffected machines—these diverging machines would indicate that the system is unstable and, if continued, would split up, by action of the interconnecting circuit-protection systems, into two or more 'islands'. A common feature of such studies often shows that machine angles oscillate at different frequencies but that they *all* gradually change their angles in unison from the initial angles, implying that the system is stable but that, subsequent to the initiating disturbance, the frequency of the whole system is either increasing or decreasing.

To enable a digital study to assess its own results, criteria need to be built into the software. One of these is whether or not the system angles are within a norm. Another is, in the case of instability, which parts of the system are splitting away from other parts. To achieve this assessment, a concept known as 'Centre of Inertia' (COI) is employed. This is similar to determining the centre of gravity of a mechanical system by writing

$$M_{\text{tot}} \cdot \delta_{\text{COI}} = \sum_{i=1}^{i=n} M_i \delta_i \tag{8.12}$$

where M_{tot} is the total area angular momentum $= \sum_{i=1}^{i=n} M_i$ and δ_{COI} is the average angle of the machines in the system.

Consequently, we now have, for an area of the system, that

$$M_{\text{tot}} \cdot \frac{\mathrm{d}^2\delta_{\text{COI}}}{\mathrm{d}t^2} + \Delta P_{\text{area}} = 0 \qquad \text{(see equation (8.1))}$$

where ΔP_{area} is the combined power being input (negative if output) into that area. By use of conditional statements in the software used for post-analysis of a stability study, areas of the system which swing together, i.e. whose angles are within specified limits around the δ_{COI} (known as *coherency*), can be identified. Note also that the power transfer across the area boundary ΔP_{area} before the disturbance can also be established for each area, once the vulnerable areas are known. To ensure stability, further studies need to be undertaken with revised generator scheduling such that critical ΔP_{area} flows are reduced to a sustainable level. System operators should then observe these area flow limits to ensure stability under credible system contingencies. In most systems, by proper design of transient machine controllers or reinforcement of the transfer capability of the vulnerable circuits, only a few critical areas remain where power transfer is limited by stability considerations. For example, in the U.K., the Scotland–England transfer over two double-circuit lines is often limited in this way; similarly under some conditions, circuits from North Wales to the rest of the National Grid system are flow-restricted.

8.10.1 Transient Energy Functions (TEF)

A fast stability study in large systems is essential to establish viable operating conditions. The use of Lyapunov-type functions for this purpose is left for advanced study. However, the equal-area criterion (EAC) is a form of energy function which can be used as a screening tool to enable a fast assessment of multimachine stability to be made.

Figure 8.16 shows three curves similar to those of Figure 8.5. From equation (8.8) we have that

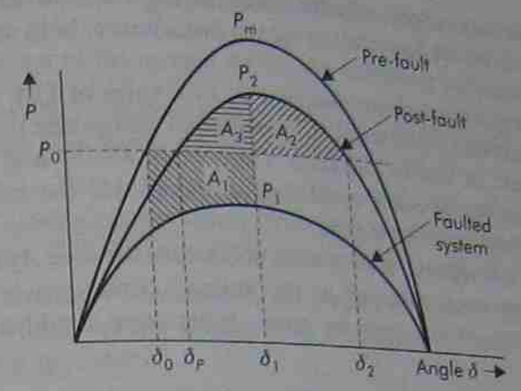


Figure 8.16 Equal-area criterion applied to an equivalent machine connected to infinite busbar through a system

$$M_e \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_0 - P_2 \sin\delta$$

We have also that

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \omega \qquad \text{and} \qquad \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{(P_0 - P_2 \sin\delta)}{M_e}$$

noting that ω is the incremental speed from the steady-state speed ω_s. At any moment, the *energy* in this system consists of two components, namely:

$M_e\omega$ — the incremental kinetic energy stored in the rotating masses of the machines at any instant;

$(P_0 - P_2 \sin\delta)$ — the potential energy due to the excess electrical power at any instant.

The energy balance in the system during the disturbance period can be obtained by integrating the two energy components over the period. However, we must remember that prior to the disturbance under steady-state conditions the energy put into the system equalled the energy being taken out. Our development of the EAC only took into account the energy increments or decrements from steady-state conditions; therefore we need only concern ourselves with the *incremental* kinetic and potential energy functions. We can compute a value for the total transient energy in the system, TEF, by using

$$\text{TEF} = \int_0^{\omega_1} M_e\omega \,\mathrm{d}\omega - \int_{\delta_s}^{\delta_1} (P_0 - P^2 \sin\delta)\,\mathrm{d}\delta \qquad \text{(Joules)} \tag{8.13}$$

where ω_1 is the incremental angular speed at angle δ_1 and δ_P is the angle at which the system settles down following the disturbance; both angles are shown in Figure 8.16.

In fact, for any intermediate value of ω or δ the value of TEF is an indication that there is a surplus or deficiency of incremental energy and that the system is still in a dynamic or oscillation state. Only when $\delta = \delta_P$ and $\omega = 0$, i.e. no increment on the steady-state synchronous speed, will the oscillations have died away and the system be stable.

If the system is to regain a steady-state condition, the area A_2 must equal A_1, and the maximum angle attained at the limiting conditions is δ_2, where the incremental speed ω would again be zero. Under these conditions we see that

$$\text{TEF}_{\text{limit}} = -\int_{\delta_P}^{\delta_1} (P_0 - P_2 \sin\delta)\,\mathrm{d}\delta \qquad \text{since } \omega = 0$$

TEF_{limit} represents the maximum energy that the system can gain due to a disturbance and subsequently dissipate by transfer over the system, if it is to remain stable. Knowing the value of $\text{TEF}_{\text{limit}}$ from the above equation enables a quick determination of stability by calculating TEF as the disturbance proceeds. This can be done readily as the step-by-step integration proceeds because values of ω and δ will be available. Provided that the value of TEF remains less than $\text{TEF}_{\text{limit}}$ the system is stable and will settle down to a new angle δ_P. It is usual to compute TEF at fault clearance and assume afterwards that no more energy is added (rather, it is dissipated by system damping and losses), thereby checking that $\text{TEF} < \text{TEF}_{\text{limit}}$.

It is worth noting that in Figure 8.16:

1. the kinetic energy-like term $\int_0^{\omega_1} M_e \omega\,\mathrm{d}\omega$ is the area A_1;
2. the potential energy-like term $\int_{\delta_P}^{\delta_1} (P_0 - P_2 \sin\delta)\,\mathrm{d}\delta$ is the area A_3;
3. TEF is equal to areas $A_1 + A_3$;
4. $\text{TEF}_{\text{limit}}$ is the area $A_2 + A_3$;
5. Equating TEF and $\text{TEF}_{\text{limit}}$ produces

$$A_1 + A_3 = A_2 + A_3 \qquad \text{i.e. } A_1 = A_2$$

which is the EAC for the system.

This shows that for stability the total energy gained during a disturbance must equal the capability of the system to transfer that energy away from the system afterwards. Considerable effort has been put into the determination of dynamic system boundaries through improved calculations of the TEF.

8.10.2 Improvement of system stability

Apart from the use of fast-acting AVRs the following techniques are in use:

1. Reduction of fault clearance times, 80 ms is now the norm with SF_6 circuit breakers and high-speed protection (see Chapter 11).
2. Turbine fast-valving by bypass valving—this controls the accelerating power by closing steam valves. Valves which can close or open in 0.2 s are available. CCGTs can reduce power by fuel control within 0.2 to 0.5 s.
3. Dynamic braking by the use of shunt resistors across the generator terminals; this limits rotor swings. The switching of such resistors can be achieved by thyristors.
4. High-speed reclosure (independent pole tripping) in long (point-to-point) lines. In highly interconnected systems the increase in overall clearance times on unsuccessful reclosure makes this technique of dubious value. Delayed autoreclose (DAR) schemes with delays of 12–15 s are preferable if the voltage sag can be tolerated.
5. Increased use of H.V. direct-current links using thyristors and GTOs also alleviates stability problems.
6. Semiconductor-controlled static compensators enabling oscillations following a disturbance to be damped out.
7. Energy-storage devices, e.g. batteries, superconducting magnetic energy stores (SMES) with fast control, providing the equivalent of a UPS.

Problems

8.1 A round-rotor generator of synchronous reactance 1 p.u. is connected to a transformer of 0.1 p.u. reactance. The transformer feeds a line of reactance 0.2 p.u. which terminates in a transformer (0.1 p.u. reactance) to the *LV* side of which a synchronous motor is connected. The motor is of the round-rotor type and of 1 p.u. reactance. On the line side of the generator transformer a three-phase static reactor of 1 p.u. reactance per phase is connected via a switch. Calculate the steady-state power limit with and without the reactor connected. All per unit reactances are expressed on a 10 MVA base and resistance may be neglected. The internal voltage of the generator is 1.2 p.u. and of the motor 1 p.u.
(Answer: 5 MW and 3.14 MW for shunt reactor)

8.2 In the system shown in Figure 8.17, two equivalent round-rotor generators feed into a load which may be represented by the constant impedance shown. Determine Z_{11}, Z_{12}, and Z_{22} for the system and calculate δ_{12} if P_1 is 1.2 p.u. All per unit values are expressed on the same base and the resistance of the system (apart from the load) may be neglected.
(Answer: $Z_{11} = 0.78\angle 87.8°$; $Z_{12} = 1.47\angle 112°$; $Z_{22} = 0.22\angle 62.5°$)

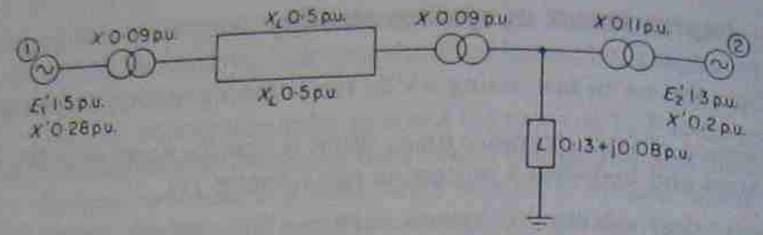


Figure 8.17 Line diagram of system in Problem 8.2

8.3 A hydroelectric generator feeds a load through two 132 kV lines in parallel, each having a total reactance of 70 Ω phase-to-neutral. The load consists of induction motors operating at three-quarters of full load and taking 30 MVA. A local generating station feeds directly into the load busbars. Determine the parameters of an equivalent circuit, consisting of a single machine connected to an infinite busbar through a reactance, which represents the above system when a three-phase symmetrical fault occurs halfway along one line.

The machine data are as follows:

Hydroelectric generator:	rating 60 MW at power factor 0.9 lagging; transient reactance 0.3 p.u.; inertia constant 3 kW-s/kVA.
Induction motors:	composite transient reactance 3 p.u.; inertia constant 1 kW-s/kVA.
Local generator:	rating 30 MVA; transient reactance 0.15 p.u. inertia constant 10 kW-s/kVA.

(Answer: $X = 9.9$ p.u., $M = 0.000121$ p.u., on a 100 MVA base)

8.4 For the system described in Example 8.3 determine the swing curves for fault clearance times of 250 and 500 ms.

8.5 The P–V, Q–V characteristics of a substation load are as follows:

V	1.05	1.025	1	0.95	0.9	0.85	0.8	075
P	1.03	1.017	1	0.97	0.94	0.92	0.98	0.87
Q	1.09	1.045	1	0.93	0.885	0.86	0.84	0.85

The substation is supplied through a link of total reactance 0.8 p.u. and negligible resistance. With nominal load voltage, $P = 1$ and $Q = 1$ p.u. By determining the supply voltage–received voltage characteristic, examine the stability of the system by the use of dE/dV. All quantities are per unit.

8.6 An induction motor load is supplied through a transformer of 0.1 p.u. reactance from a receiving-end substation which is supplied from a generator through a transmission link of total reactance 0.3 p.u. Plant data are as follows: generator $X_s = 1.1$, $X' = 0.3$. Induction motor load $X_s = 0.2$, $R_2 = 0.03$, mechanical power output is independent of speed. All per unit values are on a 60 MVA base. Examine the stability of the load when the motor takes 50 MW when the generator has (a) no AVR, (b) a continuously acting AVR.

8.7 A large synchronous generator, of synchronous reactance 1.2 p.u., supplies a load through a link comprising a transformer of 0.1 p.u. reactance and an overhead line of initially 0.5 p.u. reactance; resistance is negligible. Initially, the voltage at the load busbar is 1 p.u. and the load $P + jQ$ is $(0.8 + j0.6)$ p.u. regardless of the voltage. Assuming the internal voltage of the generators is to remain unchanged, determine the value of line reactance at which voltage instability occurs.
(Answer: Unstable when $X = 2.0$ p.u.

8.8 A load is supplied from an infinite busbar of voltage 1 p.u. through a link of series reactance X p.u. and of negligible resistance and shunt admittance. The load consists of a constant power component of 1 p.u. at 1 p.u. voltage and a per unit reactive power component (Q) which varies with the received voltage (V) according to the law

$$(V - 0.8)^2 = 0.2(Q - 0.8)$$

All per unit values are to common voltage and MVA bases.

Determine the value of X at which the received voltage has a unique value and the corresponding magnitude of the received voltage.

Explain the significance of this result in the system described. Use approximate voltage-drop equations.
(Answer: $X = 0.25$ p.u.; $V = 0.67$ p.u.)

8.9 Explain the criterion of stability based on the equal-area diagram.

A synchronous generator is connected to an infinite busbar via a generator transformer and a double-circuit overhead line. The transformer has a reactance of 0.15 p.u. and the line an impedance of $0 + j0.4$ p.u. per circuit. The generator is supplying 0.8 p.u. power at a terminal voltage of 1 p.u. The generator has a transient reactance of 0.2 p.u. All impedance values are based on the generator rating and the voltage of the infinite busbar is 1 p.u.

(a) Calculate the internal transient voltage of the generator.
(b) Determine the critical clearing angle if a three-phase solid fault occurs on the sending (generator) end of one of the transmission line circuits and is cleared by disconnecting the faulted line.

(Answer: (a) $0.98 + j0.28$ p.u.; (b) 65.2°
(From E.C. Examination, 1995)

8.10 A 500 MVA generator with 0.2 p.u. reactance is connected to a large power system via a transformer and overhead line which have a combined reactance of 0.3 p.u. All p.u. values are on a base of 500 MVA. The amplitude of the voltage at both the generator terminals and at the large power system is 1.0 p.u. The generator delivers 450 MW to the power system.

Calculate

(a) the reactive power in MVAr supplied by the generator at the transformer input terminals;
(b) the generator internal voltage;
(c) the critical clearing angle for a 3 p.h. short circuit at the generator terminals.

(Answer: (a) 62 MVAr; (b) 1.04 p.u.; (c) 86°)
(From E.C. Examination, 1996)

9

Direct-Current Transmission

9.1 Introduction

Alternating-current transmission using three phases is the well-established method of transmitting large blocks of electrical energy over both short and long distances. However, there is a limit to the distance that a.c. can be transmitted by overhead lines because of the surge impedance limitation producing high voltages at the receiving end unless some form of compensation is employed. For long overhead lines an alternative form of transmission is use of direct current. If undersea crossings greater than 45 km are required, then, because of the charging current limitations in a.c. cables, d.c. is the only alternative.

The choice of d.c. is, however, based on economics. With the use of high-voltage, high-current semiconductor devices, converter stations and their controls are becoming cheaper and more reliable, and comparable to many a.c. components and systems. Consequently, the main technical reasons for high-voltage direct-current (h.v.d.c.) transmission are:

1. interconnection of two large a.c. systems without having to ensure synchronism and stability margins between them (the U.K.–France cross-channel link of 2000 MW is of this nature);
2. interconnection between systems of different frequency (e.g. the connection between north and south islands in Japan, which have 50 Hz and 60 Hz systems, respectively);
3. long overland transmission of high powers where a.c. transmission towers, insulators, and conductors are more expensive than using h.v.d.c. (e.g. Nelson River scheme in Manitoba—total of 4000 MW over 600+ km).

The main advantages of h.v.d.c. compared with h.v.a.c. are:

1. two conductors, positive and negative to ground, are required instead of three, thereby reducing tower or cable costs;
2. the direct voltage can be designed equivalent to the peak of the alternating voltage for the same amount of insulation to ground (i.e. $V_{d.c.} = \sqrt{2}V_{a.c.}$)
3. the voltage stress at the conductor surface can be reduced on d.c., thereby reducing corona loss, audible emissions, and radio interference;
4. h.v.d.c. infeeds do not raise the short-circuit capacity required of switchgear on the a.c. sides;
5. fast control of converters can be used to damp out connected a.c. system oscillations.

Disadvantages of h.v.d.c. are:

1. the higher cost of converter stations compared with an a.c. transformer substation;
2. the need to provide filters and associated equipment to ensure acceptable waveform and power factor on a.c. side;
3. limited ability to form multiterminal d.c. networks because of the need for coordinated controls and no acceptable d.c. circuit breaker.

9.2 Semiconductor Valves for High-Voltage Direct-Current Converters

The rapid growth in the use of direct current since about 1980 has been due to the development of high-voltage, high-current semiconductor devices. These have taken over from the previously used complex and expensive mercury arc valve, employing a mercury pool as cathode and a high-voltage graded column of anodes, with the whole enclosed in steel and ceramic to provide a vacuum-tight enclosure. Nowadays, the semiconductor units are stacked to form a group which is able to withstand the designed voltages and to pass the desired maximum currents—this group still being termed a 'valve' in h.v.d.c. parlance.

Since h.v.d.c. converters operate connected to active a.c. systems, line commutation is the cheapest and most reliable method to employ, requiring devices which can be triggered on and turned off by reducing the current through them to zero. The main device used for this purpose is the thyristor.

9.2.1 Thyristors

Thyristors are manufactured from pure silicon wafers and are four-layer versions of the simple rectifier P–N junction, as shown in Figure 9.1(a). The P layer in the middle is connected to a gate terminal biassed such that the whole unit can be prevented from passing current, even when a positive voltage exists on the anode. By applying a positive pulse to the gate, conduction can be started, after which the gate voltage control has no effect until the main forward current falls below its latching value (see Figure 9.1(c)). This current must be kept below the latching value for typically 100 μs before the thyristor is able to regain its voltage hold-off properties. (Note that in forward conduction there is still a small voltage across the P–N junctions, implying that power is being dissipated—hence the semiconductor devices must be cooled and their losses accounted for.)

In practice, many devices, each of rating 5000 V and up to 2000 A, are stacked in a valve to provide a rating of, say, 200 kV, 2000 A. Valves are connected in series to withstand direct voltages up to 500 or 600 kV to earth on each 'pole'. Each semiconductor thyristor can be 8–10 cm in diameter and 2 cm depth with its anode and cathode terminals. A typical device is shown in Figure 9.2(a) and a valve in Figure 9.2(b).

A recent development has been the insulated gate bipolar thyristor (IGBT) which is a development of the mosfet, in which removal of current from the gate switches off the through current, thereby allowing power to be switched on or off throughout an a.c. cycle. This provides a means of controlling currents in relation to the voltage in an a.c. system so that vars may be generated or absorbed. Current injection into a.c. systems for power-flow control is achieved using these devices. Ratings up to 4 kV and 1000 A are possible with IGBTs, although the on-state voltage drop is generally larger than with a thyristor.

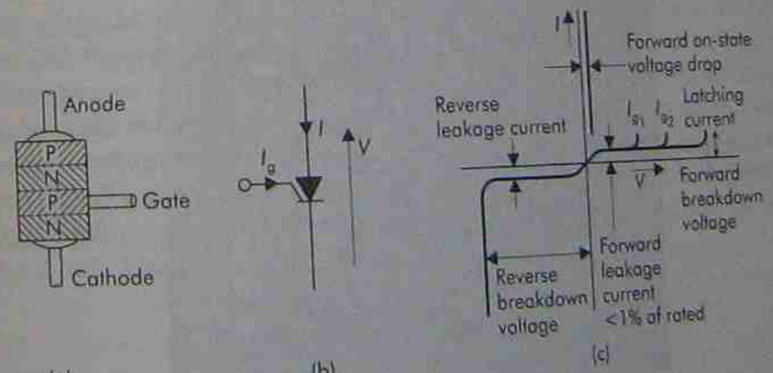
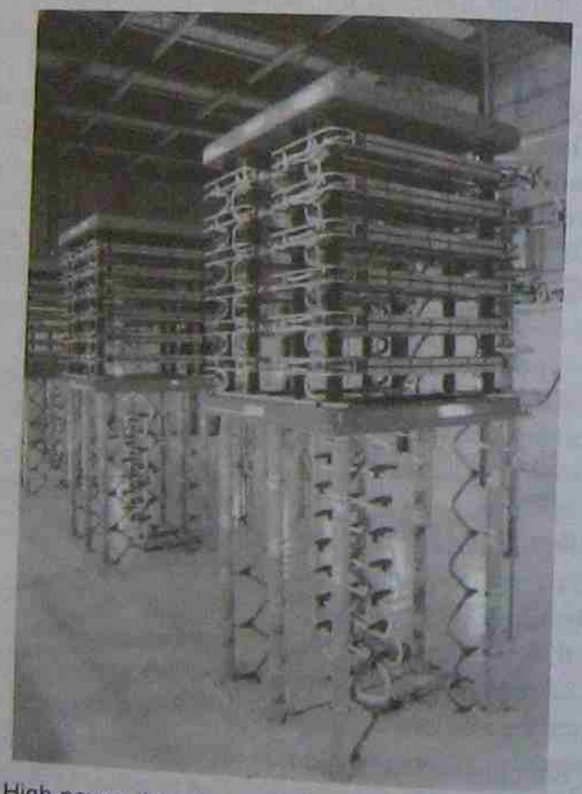


Figure 9.1 (a) Structure of a four-layer thyristor. (b) Symbol. (c) Thyristor characteristic: I_g gate current to switch thyristor on at forward voltage



(a)



(b)

Figure 9.2 (a) High-power thyristor silicon device (*Courtesy E.P.R.I.—U.S.A.*). (b) Thyristor valves in converter station (*Couresy of I.E.E. Power Engineering Journal*)

9.3 Basic Converter and Direct-Current System Operation

A converter is required at each end of a d.c. line and operates as a rectifier (a.c. to d.c.) or an inverter (power transfer from d.c. to a.c.). The valves at the sending end of the link rectify the alternating current, providing direct current which is transmitted to the inverter. Here, it is converted back into alternating current which is fed into the connected a.c. system (Figure 9.3(a)). If a reversal of power flow is required, the inverter and rectifier exchange roles and the direct voltages at each end are reversed (Figure 9.3(b)). This is necessary because the direct current can flow in one direction only (anode to cathode in the valves), so to reverse the direction (or sign) of power the voltage direction must be reversed.

The alternating-current waveform injected by the inverter into the receiving-end a.c. system, and taken by the rectifier, is roughly trapezoidal in shape, and thus produces not only a fundamental sinusoidal wave but also harmonics of an order dependent on the number of valves. For a six-valve bridge the order is $6n \pm 1$, i.e. 5, 7, 11, 13, etc. Filters are incorporated to tune out harmonics up to the 25th.

In the following sections the processes of rectification and inversion will be analysed and the control of the complete link discussed.

9.4 Rectification

Transformer secondary connections may be arranged to give several phases for supplying the valves. Common arrangements are three, six, and twelve phases and higher numbers are possible. Six-phase is popular because of the better output-voltage characteristics. To begin with, a three-phase arrangement will be described, but most of the analysis will be for n phases so that results are readily adaptable for any system. In Figure 9.4(a), a three-phase rectifier is shown and Figure 9.4(b) shows the current and voltage variation with time in the three phases of the supply transformer. With no gate control, conduction will take place between the cathode and the anode of highest potential. Hence the output-voltage wave is the thick line and the current output is continuous. In an n-phase system the anode changeover occurs at $(\pi/2 - \pi/n)$ degrees at a voltage $\hat{V} \sin \pi/n$ and the mean value of the direct-output voltage ($\hat{V}$ is the peak a.c. side voltage) is

$$V_O = \frac{1}{2\pi/n} \int_{\pi/2-\pi/n}^{\pi/2+\pi/n} \hat{V} \sin \omega t \, \mathrm{d}(\omega t)$$

$$= \frac{\hat{V} \sin(\pi/n)}{\pi/n} \tag{9.1}$$

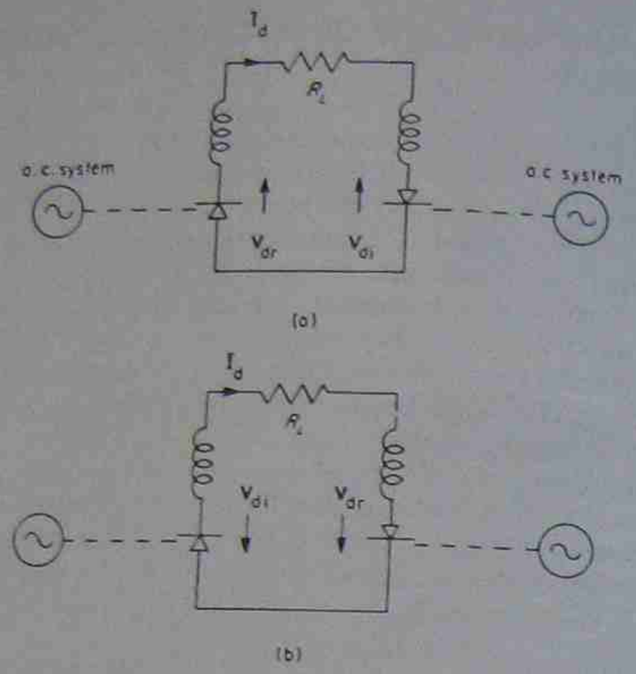


Figure 9.3 (a) Symbolic representation of two alternating current systems connected by a direct-current link; V_{dr} = direct voltage across rectifier, V_{di} = direct voltage across inverter. (b) System as in part (a) but power flow reversed

For three phases,

$$V_O = \frac{\hat{V}\sin 60^\circ}{\pi/3}$$
$$= \hat{V}\frac{3\sqrt{3}}{2\pi} = 0.83\hat{V}$$

and for six phases,

$$V_O = \frac{3\sqrt{2}}{\pi}V = \frac{3}{\pi}\hat{V} = 0.955\hat{V}$$

where V = r.m.s. voltage.

Owing to the inductance present in the circuit, the current cannot change instantaneously from $+I_d$ to 0 in one anode and from 0 to I_d in the next. Hence, two anodes conduct simultaneously over a period known as the commutation time or overlap angle (γ). When valve b commences to conduct, it short-circuits the (a) and (b) phases, the short-circuit current eventually becoming zero in valve a and I_d in valve b. This is shown in Figure 9.5.

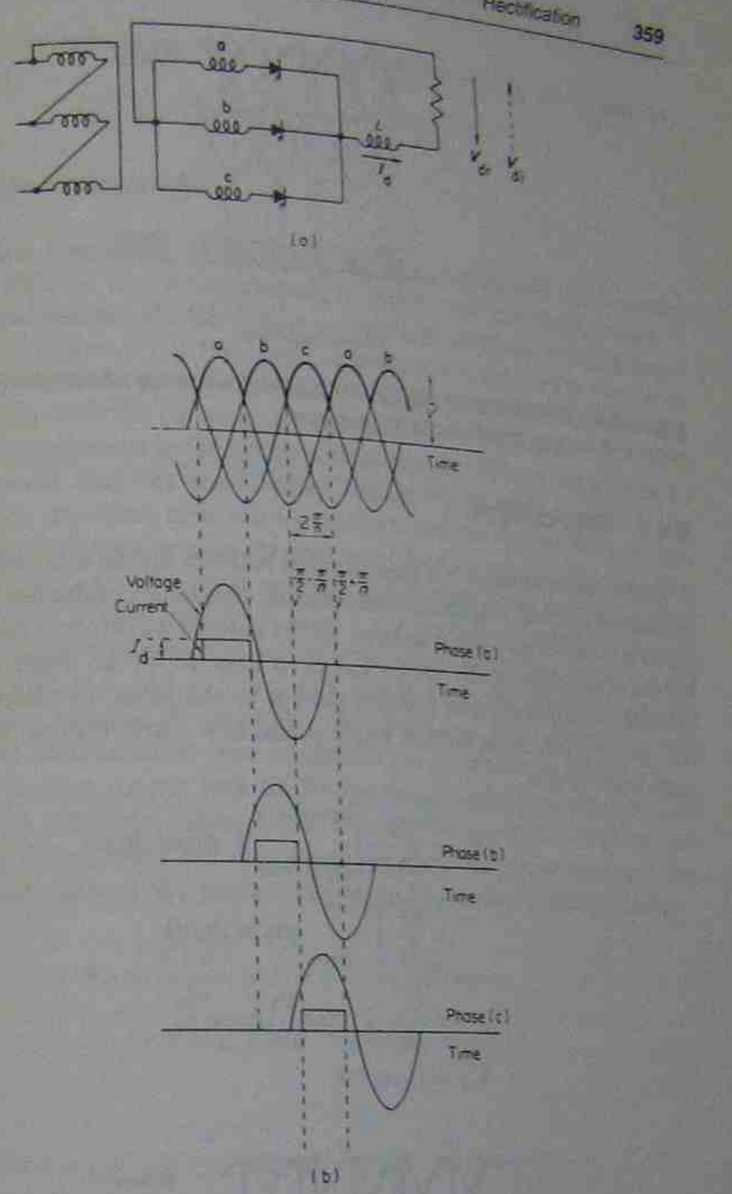


Figure 9.4 (a) Three-phase rectifier; V_{di} = voltage of inverter. (b) Waveforms of anode voltage and rectified current in each phase

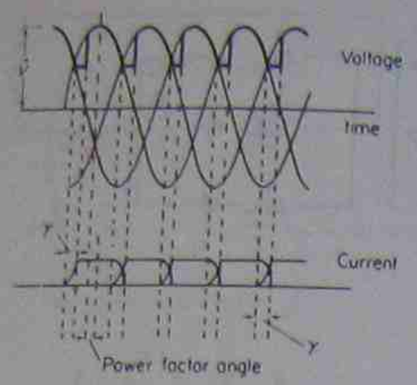


Figure 9.5 Waveforms of voltage and current showing effect of the commutation angle γ. A lagging power factor is produced

9.4.1 Gate control

A positive pulse applied to a gate situated between anode and cathode controls the instant at which conduction commences, and once conduction has occurred the gate exercises no further control. In the voltage waveforms shown in Figure 9.6 the conduction in the valves has been delayed by an angle α by suitably delaying the application of positive voltage to the gates. Considering n phases and ignoring the commutation angle γ, the new direct-output voltage with a delay angle of α is,

$$V'_O = \frac{1}{2\pi/n}\int_{\pi/2-\pi/n+\alpha}^{\pi/2+\pi/n+\alpha} \hat{V}\sin\omega t\,\mathrm{d}(\omega t)$$

$$= \frac{n\hat{V}}{2\pi}\int_{-\pi/n+\alpha}^{\pi/n+\alpha} \cos\omega t\,\mathrm{d}(\omega t)$$

$$= \frac{n\hat{V}}{2\pi}\cdot 2\cdot\sin\left(\frac{\pi}{n}\right)\cos\alpha$$

$$\therefore\ V'_O = V_O\cos\alpha \qquad (9.2)$$

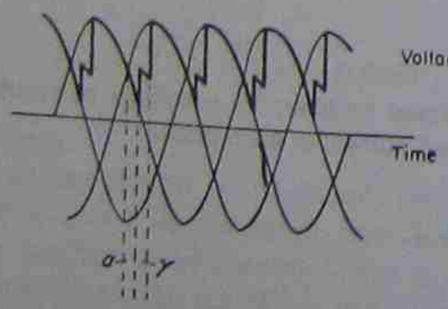


Figure 9.6 Waveforms of rectifier with instant of firing delayed by an angle α by means of gate control

where V_O is the maximum value of direct-output voltage as defined by equation (9.1).

9.4.2 Bridge connection

To avoid undue complexity in describing the basic operations, the converter arrangement used so far in this chapter is simple and has disadvantages in practice. Mainly because the d.c. output voltage is doubled, the bridge arrangement shown in Figure 9.7 is favoured, in which there are always two valves conducting in series. The corresponding voltage waveforms are shown in Figure 9.8 along with the currents (assuming ideal rectifier operation).

The sequence of events in the bridge connection is as follows (see Figures 9.7 and 9.8). Assume that the transformer voltage V_A is most positive at the beginning of the sequence, then valve 1 conducts and the current flows through valve 1 and the load then returns through valve 6 as V_B is most negative. After this period, V_C becomes the most negative and current flows through valves 1 and 2. Next, valve 3 takes over from valve 1, the current still returning through valve 2. The complete sequence of valves conducting is therefore: 1 and 6; 1 and 2; 3 and 2; 3 and 4; 5 and 4; 5 and 6; 1 and 6. Control may be obtained in exactly the same manner as previously described, and the voltage waveforms with delay and commutation time accounted for are shown in Figure 9.9.

The direct-voltage output with the bridge may be calculated either by using the line voltage (phase-to-phase) in the formula for six-valve, six-phase rectification or by determining the magnitude for three-valve operation and doubling it, as both sides of the bridge contribute to the direct voltage. Hence,

$$V_0 = \frac{\hat{V}3\sqrt{3}}{2\pi}\times 2 = \sqrt{2}V\times 3\sqrt{3}/\pi$$

$$= \sqrt{2}V_L\times 3/\pi \qquad (\text{as } V_L = \sqrt{3}\,V)$$

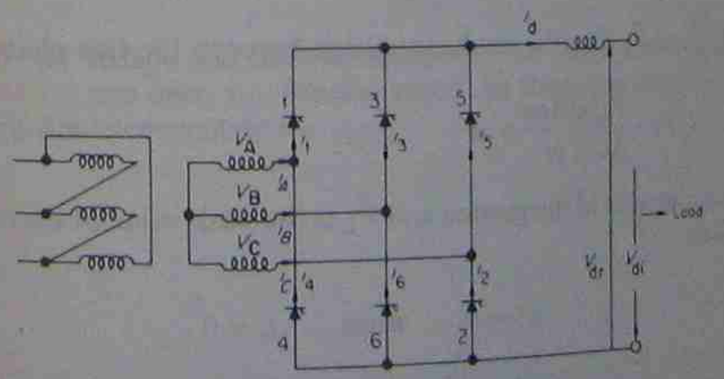


Figure 9.7 Bridge arrangement of valves

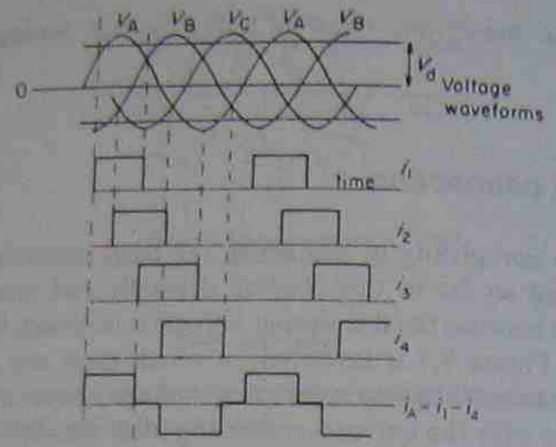


Figure 9.8 Idealized voltage and current waveforms for bridge arrangement

If the analysis used to obtain equation (9.2) is repeated for the bridge, it will be shown that $V_d = V_0 \cos\alpha$, where V_0 is the maximum direct-voltage output for the bridge connection.

9.4.3 Current relationships in the bridge circuit

During the commutation process when two valves are conducting simultaneously, the two corresponding secondary phases of the supply transformer are short-circuited and if the voltage drop across the valves is neglected the following analysis applies.

When two phases of the transformer each of leakage inductance L henries are effectively short-circuited, the short-circuit current (i_s) is governed by the equation,

$$2L\frac{di_s}{dt} = \hat{V}_L \sin\omega t = \text{resultant voltage between the two phases}$$

$$\therefore i_s = -\frac{\hat{V}_L}{2L}\frac{\cos\omega t}{\omega} + A$$

where A is a constant of integration and $\hat{V}_L$ is the peak value of the line-to-line voltage. Now

$$\omega t = \alpha \quad \text{when} \quad i_s = 0$$

$$\therefore A = \frac{\hat{V}_L}{2\omega L}\cos\alpha$$

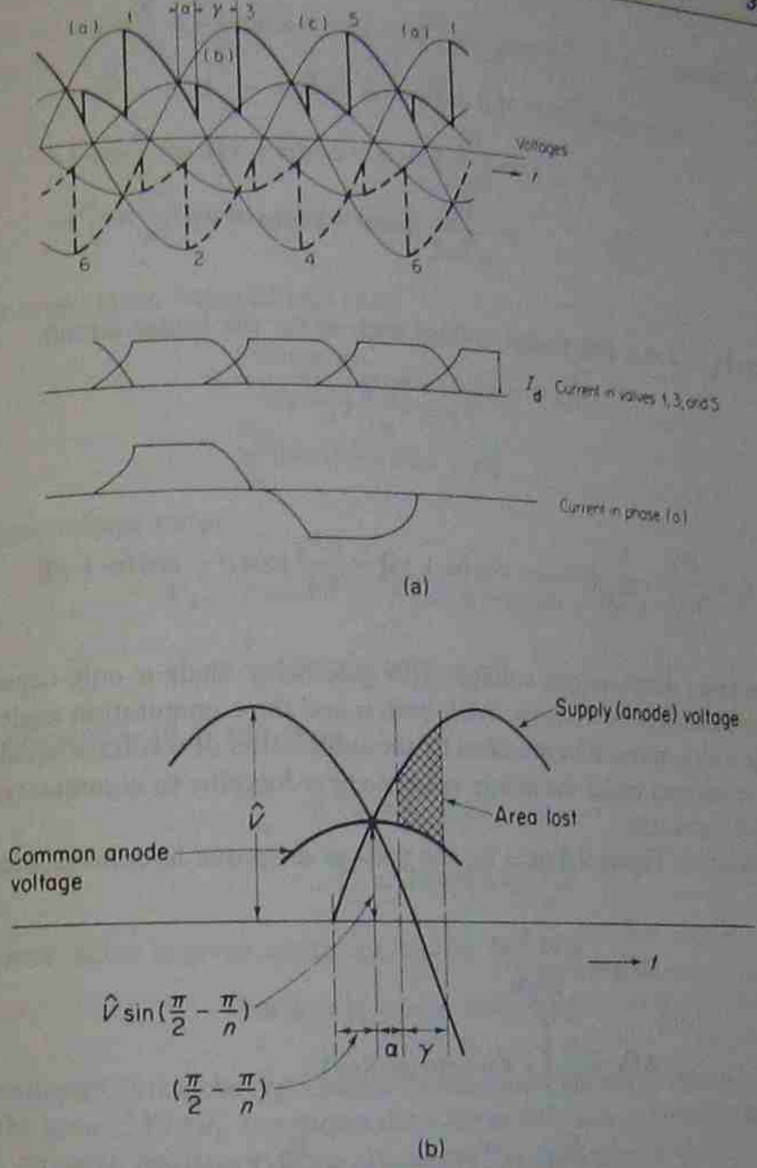


Figure 9.9 (a) Voltage and current waveforms in the bridge connection, including commutation (γ) and delay (α). Rectifier action. (b) Expanded waveforms showing voltage drop due to commutation

Also, when

$$\omega t = \alpha + \gamma \qquad i_s = I_d$$

$$\therefore I_d = \frac{\hat{V}_L}{2\omega L}[\cos\alpha - \cos(\alpha+\gamma)]$$

$$= \frac{V_L}{\sqrt{2}\omega L}[\cos\alpha - \cos(\alpha+\gamma)]$$

where V_L = r.m.s. line-to-line voltage and, as for the bridge circuit,

$$V_0 = \frac{3\sqrt{2}}{\pi} V_L$$

and

$$I_d = \frac{\pi V_0}{3\sqrt{2}} \cdot \frac{1}{\sqrt{2}X}[\cos\alpha - \cos(\alpha+\gamma)] = \frac{\pi V_0}{6X}[\cos\alpha - \cos(\alpha+\gamma)] \qquad (9.3)$$

The mean direct-output voltage, with gate delay angle α only considered, has been shown to be $V_0 \cos\alpha$. With both α and the commutation angle γ, the voltage with α only will be modified by the subtraction of a voltage equal to the mean of the area under the anode voltage curve lost due to commutation (see Figure 9.5 and 9.9).

Referring to Figure 9.5 ($\alpha = 0$), the voltage drop due to commutation,

$$\Delta V_0 = \frac{\text{area lost}}{2\pi/n}$$

$$\Delta V_0 = \frac{n}{2\pi}\int_0^{\gamma} \hat{V}\sin\frac{\pi}{n}\sin\omega t\,\mathrm{d}(\omega t)$$

$$= \frac{n}{2\pi}\hat{V}\sin\frac{\pi}{n}(1-\cos\gamma) = \frac{V_0}{2}(1-\cos\gamma)$$

When $\alpha > 0$ (see Figure 9.9(b)) the voltage drop is obtained as follows: area between input voltage wave and common anode voltage is

$$\int_{\pi/2-\pi/2+\alpha}^{\pi/2-\pi/n+\gamma+\alpha} \hat{V}\sin\omega t\,\mathrm{d}(\omega t) - \int_{\alpha}^{\alpha+\gamma} \hat{V}\sin\left(\frac{\pi}{2}-\frac{\pi}{n}\right)\cos\omega t\,\mathrm{d}(\omega t)$$

$$= \hat{V}\left[-\cos\left(\frac{\pi}{2}-\frac{\pi}{n}+\alpha+\gamma\right)+\cos\left(\frac{\pi}{2}-\frac{\pi}{n}+\alpha\right)\right.$$

$$\left.-\sin\left(\frac{\pi}{2}-\frac{\pi}{n}\right)\sin(\alpha+\gamma)+\sin\left(\frac{\pi}{2}-\frac{\pi}{n}\right)\sin\alpha\right]$$

$$= \hat{V}\left[-\sin\left(\frac{\pi}{n}-\alpha-\gamma\right)+\sin\left(\frac{\pi}{n}-\alpha\right)-\cos\frac{\pi}{n}\sin(\alpha+\gamma)\right.$$

$$\left.+\cos\frac{\pi}{n}\sin\alpha\right]$$

$$= \hat{V}\left[\sin\frac{\pi}{n}\right][\cos\alpha - \cos(\alpha+\gamma)]$$

Voltage drop (mean value of lost area)

$$= \frac{\hat{V}\sin(\pi/n)}{2(\pi/n)}[\cos\alpha - \cos(\alpha+\gamma)].$$

$$= \frac{V_0}{2}[\cos\alpha - \cos(\alpha+\gamma)]$$

The direct-voltage output,

$$V_d = V_0\cos\alpha - \frac{V_0}{2}[\cos\alpha - \cos(\alpha+\gamma)]$$

$$= \frac{V_0}{2}[\cos\alpha + \cos(\alpha+\gamma)] \qquad (9.4)$$

Adding equations (9.3) and (9.4),

$$\frac{I_d 3X}{\pi} + V_d = V_0\cos\alpha$$

$$\therefore V_d = V_0\cos\alpha - \frac{3XI_d}{\pi} \qquad (9.5)$$

The power factor is given approximately by

$$\cos\phi = \tfrac{1}{2}[\cos\alpha + \cos(\alpha+\gamma)] \qquad (9.6)$$

Equation (9.5) may be represented by the equivalent circuit shown in Figure 9.10, the term $(3X/\pi)I_d$ represents the voltage drop due to commutation and not a physical resistance drop. It should be remembered that V_0 is the

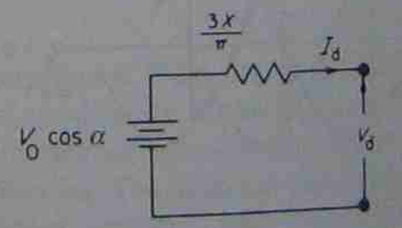


Figure 9.10 Equivalent circuit representing operation of a bridge rectifier. Reactance per phase X (Ω)

theoretical maximum value of direct-output voltage and it is evident that V_d can be varied by changing V_0 (control of transformer secondary voltage by tap changing) and by changing α.

9.5 Inversion

With rectifier operation the output current I_d and output voltage V_d are such that power is absorbed by a load. For inverter operation it is required to transfer power from the direct-current to the alternating-current systems, and as current can flow only from anode to cathode (i.e., in the same direction as with rectification), the direction of the associated voltage must be reversed. An alternating-voltage system must exist on the primary side of the transformer, and gate control of the converters is essential.

As the bridge connection is in common use it will be used to explain the inversion process. If the bridge rectifier is given progressively greater delay the output voltage decreases, becoming zero when α is 90°, as shown in Figure 9.11. With further delay the average direct voltage becomes negative and the applied direct voltage (from the rectifier) forces current through the valves against this negative or back voltage. The converter thus receives power and inverts. The inverter bridge is shown in Figure 9.12(a) and the voltage and current waveforms in Figure 9.12(b). Commutation from valve 5 to valve 1 is possible only when phase (a) is positive with respect to phase (b), and the current changeover must be complete before (F) by a time (δ_0) equal to the recovery time of the valves. From the current waveforms it is seen that the current supplied by the inverter to the a.c. system *leads* the voltage and hence the inverter may be considered as an absorber of vars.

The power factor $\cos\phi \simeq (\cos\delta + \cos\beta)/2$, where δ and β are defined in Figure 9.12(a). Valve 5 is triggered at time A and as the cathode is held negative to the anode by the applied direct voltage (V_d), current flows, limited

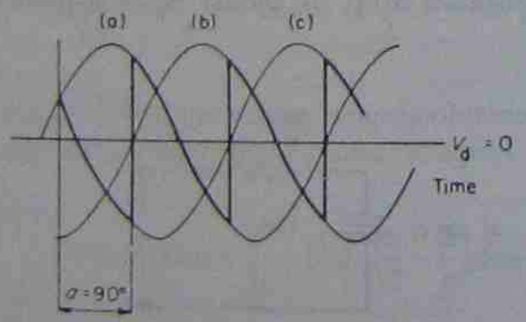


Figure 9.11 Waveforms with operation with $\alpha = 90°$, direct voltage zero. Transition from rectifier to inverter action

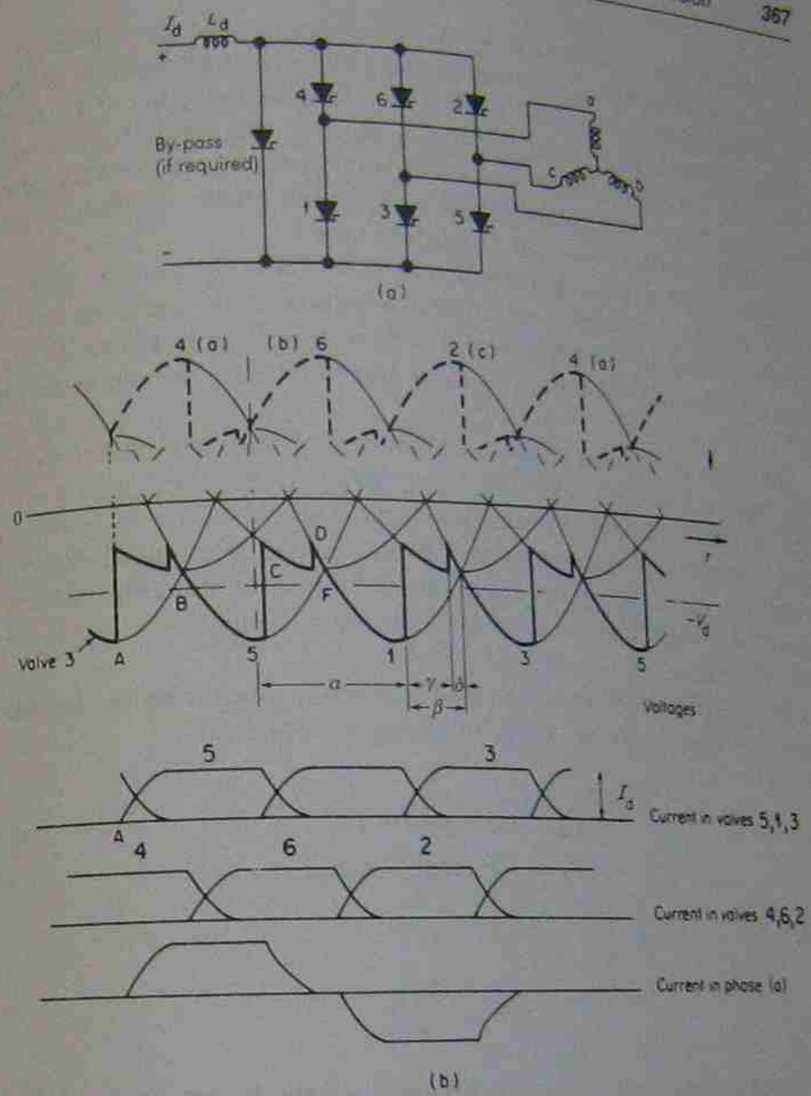


Figure 9.12 (a) Bridge connection—inverter operation. (b) Bridge connection—inverter voltage and current waveforms

only by the circuit impedance. If the voltage drop across the valve is neglected, then the cathode and anode are at the same potential in valve 5. When time B is reached, the anode-to-cathode open-circuit voltage is zero and the valve endeavours to cease conduction. The large d.c.-side inductance L_d, however, which has previously stored energy, now maintains the current constant ($e = -L(di/dt)$ and if $L_d \to \infty$, $di/dt \to 0$). Conduction in valve 5 continues until time C, when valve 1 is triggered. As the anode-to-cathode voltage for

valve 1 is greater than for valve 5, valve 1 will conduct, but for a time valves 5 and 1 conduct together (commutation time), the current gradually being transferred from valve 5 to valve 1 until valve 5 is non-conducting at point D. If triggering is delayed to point F, valve 5, which is still conducting, would be subject to a positively rising voltage and would continue to conduct into the positive half-cycle with breakdown of the inversion process. Hence, triggering must allow cessation of current flow before time F.

The angle (δ) between the extinction of valve 1 and the point F, where the anode voltages are equal, is called the extinction angle, i.e. sufficient time must be allowed for the gate to regain control. It is usual to replace the delay angle α by $\beta = 180 - \alpha$, hence β is equal also to $(\gamma + \delta)$. The minimum value of δ is δ_0.

The action of the inverter is essentially that of the rectifier, but with the delay angle α greater than 90° the direct voltage output (V_d) is in a certain direction; as α increases, V_d decreases; and when $\alpha = 90°$, $V_d = 0$ V; with further increase in α, V_d reverses and inverter action is obtained. Hence the change from rectifier to inverter action, and vice versa, is smoothly obtained by control of α. This may be seen by consulting Figures 9.5 ($\alpha = 0$), 9.11 ($\alpha = 90°$), and 9.12(a) ($\alpha > 90°$).

Equations (9.3) and (9.4) may be used to describe inverter action. Replacing α by $(180 - \beta)$ and γ by $(\beta - \delta)$ the following are obtained:

$$V_d = -[V_0 \cos\beta + I_d R_c] \tag{9.7}$$

$$V_d = -[V_0 \cos\delta - I_d R_c] \tag{9.8}$$

where

$$R_c = \frac{3X}{\pi}$$

Therefore two equivalent circuits are obtained for the bridge circuit as shown in Figure 9.13(a) for constant β and Figure 9.13(b) for constant δ.

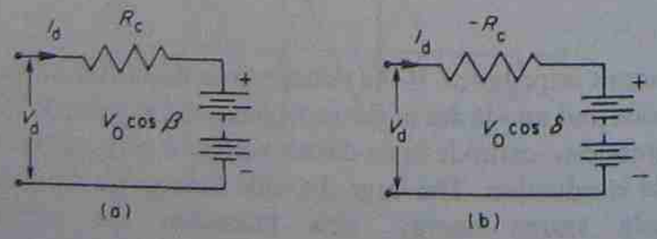


Figure 9.13 (a) Equivalent circuit of inverter in terms of angle β. (b) Equivalent circuit of inverter in terms of angle δ

9.6 Complete Direct-Current Link

The complete equivalent circuit for a d.c. transmission link under steady-state operation is shown in Figure 9.14. If both inverter and rectifier operate at constant delay angles the current transmitted

$$= I_d = \frac{V_{dr} - V_{di}}{R_L}$$

or

$$I_d = \frac{V_{0r}\cos\alpha - V_{0i}\cos\beta}{R_L + R_{cr} + R_{ci}}$$

where R_L is the loop resistance of the line or cable, and R_{cr} and R_{ci} are the effective commutation resistances of the rectifier and inverter, respectively.

The magnitude of direct current can be controlled by variation of α, β, V_{0r} and V_{0i} (the last two by tap-changing of the supply transformers). Inverter control using constant delay angle has the disadvantage that if δ and hence β are too large, excessively high reactive-power demand results (it will be seen from Figure 9.12(b) that the inverter currents are considerably out of phase with the anode voltages and hence a large requirement for reactive power is established). Also, a reduction in the direct voltage to the inverter results in an increase in the commutation angle γ, and if β is made large to cover this the reactive-power demand will again be excessive. Therefore it is more usual to operate the inverter with a constant δ, which is achieved by the use of suitable control systems (called *compounding*).

The equations governing the operation of the inverter may be summarized as follows:

$$V_d = \frac{3\sqrt{2}V_L}{\pi}\cos\beta + \frac{3\omega L}{\pi} I_d \qquad \text{(constant } \beta\text{)}$$

$$= \frac{3\sqrt{2}V_L}{\pi}\cos\delta - \frac{3\omega L}{\pi} I_d \qquad \text{(constant } \delta\text{)}$$

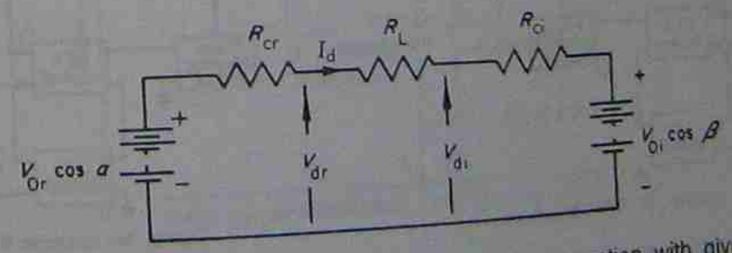


Figure 9.14 Equivalent circuit of complete link with operation with given delay angles for rectifier and for inverter

(Note that L is the leakage inductance per phase of the inverter transformer, and the power factor is $\cos\phi = (\cos\beta + \cos\delta)/2$ leading.)

The advantages of operating with δ fixed and as small as possible have been discussed; it is also advisable to incorporate a facility for constant-current operation. The rectifier and inverter change roles as the required direction of power flow dictates, and it is necessary for each device to have dual-control systems. A schematic diagram of the control systems is shown in Figure 9.15. In Figure 9.16 the full characteristics of the two converters of a link are shown, with each converter operating as rectifier and inverter in turn. In the top half of the diagram, converter (A) acts as a rectifier and the optimum characteristic with an α value of zero is shown. With constant-current control, α is increased and the output voltage–current characteristic crosses the $V_d = 0$ axis at point D, below which converter (A) acts as an inverter and can be operated on constant-δ control. A similar characteristic is shown for converter (B), which commences as an inverter and with constant-current control (β increasing) eventually changes to rectifier operation. It is seen that the current setting for converter (A) is larger than that for converter (B).

In Figure 9.17 the inverter characteristic for a power flow from A to B is shown drawn in the upper half of the graph and this will facilitate the discussion of the operational procedure.

9.6.1 Methods of control

To operate an inverter at a constant δ, the instant of valve firing is controlled by a computer which takes into account variations in the instantaneous values of voltage and current. The computer then controls the firing times such that the extinction angle δ is slightly larger than the recovery angle of the valve. As

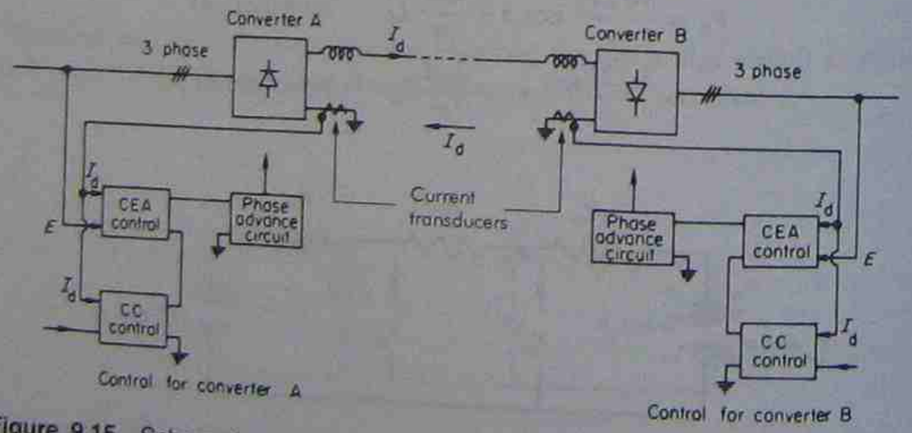


Figure 9.15 Schematic diagram of control of an h.v.d.c. system with ground return. CEA = constant extinction angle; CC = constant current

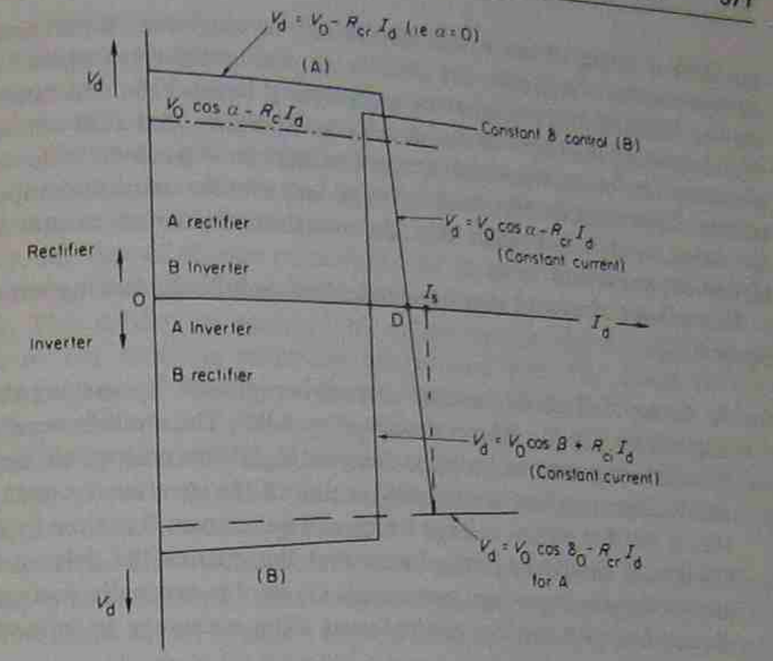


Figure 9.16 Voltage–current characteristics of converters with compounding

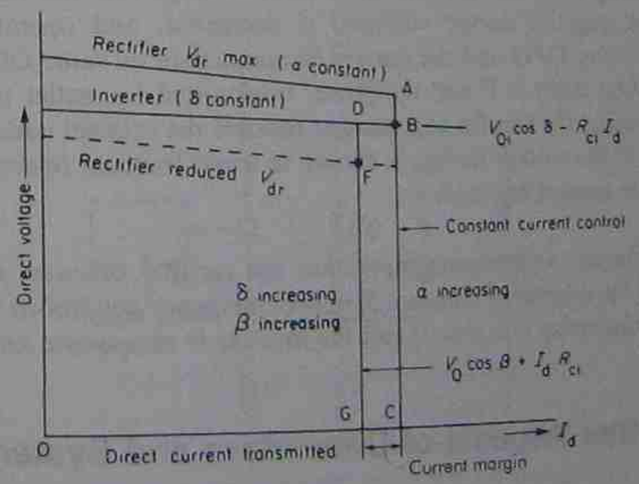


Figure 9.17 Inverter and rectifier operation characteristics with constant-current compounding. Point of operation is where the two characteristics intersect

the current rating of the valves should not be exceeded, some measure of current control is desirable, the ideal being constant-current operation. This may be achieved in the inverter by increasing β beyond the constant-δ value, thus decreasing the back-voltage developed. Similarly, constant-current compounding can be incorporated in the rectifier. In Figure 9.17 the voltage-current characteristics with both inverter and rectifier current-compounding are shown, and it is evident that the transmitted current cannot increase beyond the prescribed value.

The methods of control may be summarized as follows, making reference to Figure 9.17.

1. In the rectifier the transmitted current is regulated by varying the delay angle and hence V_{dr}. This is described by ABC. The rectifier tap-changing transformer is used to keep the delay angle within reasonable limits and also to give an excess in voltage over that of the inverter to cover sudden falls in rectifier output voltage which can be recovered in time by the tap-changer. It should be remembered that the greater the delay angle the greater the reactive power consumed. Control is normally performed by the rectifier, with inverter control used when necessary as follows.
2. The inverter transformer gives the required back direct-voltage by tap-changing. Again, this is slow acting and constant-current control is required, giving a characteristic indicated by DG. The reference value of current is lower than that for the rectifier (a margin of 10–20 per cent of the rated current).
3. Point B shows the operating condition with normal rectifier control. Should the rectifier voltage fall below the allowed margin to avoid I_d becoming zero, the inverter back direct-voltage (i.e. the mean voltage of the negative anode voltages) is decreased, and operation makes place along DFG and the current is maintained at value OG. The new operating point is F and the power transmitted is smaller than before. Eventually, the rectifier tap-changer restores the original conditions. The value of the voltage margin is chosen to avoid frequent operations in the inverter control region.

To summarize, with normal operation the rectifier operates at constant current and the inverter at constant δ; under emergency conditions the rectifier is at zero firing-delay (i.e. $\alpha = 0$) and the inverter is at constant current.

9.7 Further Aspects of Converters and Systems

The problem of obtaining a uniform voltage distribution with many thyristors in series is well known in low-voltage techniques. Because of the widely

differing capacitances, the low-voltage technique of connecting R–C circuits in parallel with each thyristor does not suffice, and additional capacitance must be incorporated. Also, it is necessary to retain uniform transient voltage distributions with time. To achieve this, inductors with a non-linear characteristic may be used. On a sudden rise in voltage the inductor first absorbs the voltage, the capacitors are charged in the opposite direction with the inductor magnetizing current only, and the voltage rise across the thyristor is delayed. Similarly, the rate of change of current may be controlled on switching off.

Obviously, it is advantageous to have as high a voltage per thyristor as possible. This cannot be achieved by increasing the silicon wafer thickness because of the resulting increased temperature rise. The device can only block the voltage if the temperature is below 125°C. The voltage collapse across the wafer when a positive impulse is applied to the gate must not be accompanied by relatively high currents until conduction exists over the entire wafer. Only about a third of the possible reverse voltage is used for normal working because of overvoltages, etc.

The electrical circuitry associated with each thyristor is shown in Figure 9.18. The inductance limits the current rise upon firing and upon the discharge of C_2. The chain R_1, C_1 bypasses current from the thyristor, which is not yet or is no longer conducting, thus avoiding unacceptable voltages. At the end of the working interval the current is allowed to flow in the reverse direction until a barrier layer is established in the wafer that enables voltage to be built up and current to cease (see Figure 9.19). After this, the element can withstand negative voltage, but the current must fall to a very low value (T_2 in Figure 9.19) if the thyristor is to block positive voltage whilst waiting for a new gate firing impulse. Extreme care must be taken to avoid thermal overload of the thyristors because of the resulting damage. Thyristors with integrated gate

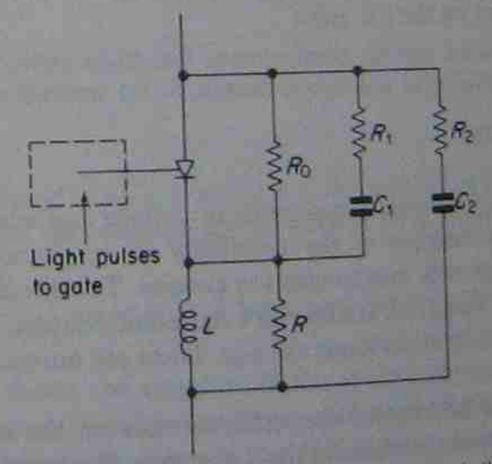


Figure 9.18 Circuitry associated with each thyristor

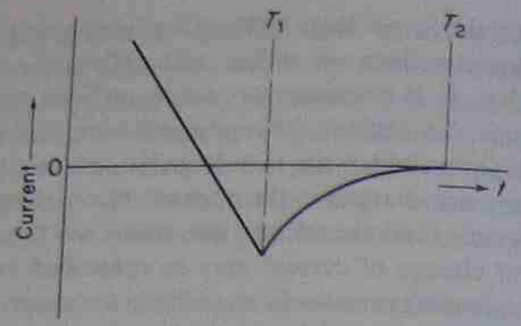


Figure 9.19 End of conduction period in thyristor—recovery of blocking property

circuitry pulsed through fibre-optic channels enable ready voltage isolation to be obtained.

9.7.1 Transmission systems

The cheapest arrangement is a single conductor with a ground return. This, however, has various disadvantages. The ground-return current results in the corrosion of buried pipes, cable sheaths, etc., due to electrolysis. With submarine cables the magnetic field set up may cause significant errors in ships' compass readings, especially when the cable runs north–south. This system is shown in Figure 9.20(a). Two variations on two-conductor schemes are shown in Figure 9.20(b) and (c). The latter has the advantage that if the ground is used in emergencies, a double-circuit system is formed to provide some security of operation. Note that in the circuits of Figure 9.20(c), transformer connections provide six-phase operation, thereby reducing harmonics generated on the a.c. sides.

9.7.2 Harmonics

A knowledge of the harmonic components of voltage and current in a power system is necessary because of the possibility of resonance and also the enhanced interference with communication circuits. The direct-voltage output of a converter has a waveform containing a harmonic content, which results in current and voltage harmonics along the line. These are normally reduced by a smoothing choke.

The currents produced by the converter currents on the a.c. side contain harmonics. The current waveform in the a.c. system produced by a delta–star transformer bridge converter is shown in Figures 9.9 and 9.12. The order of the

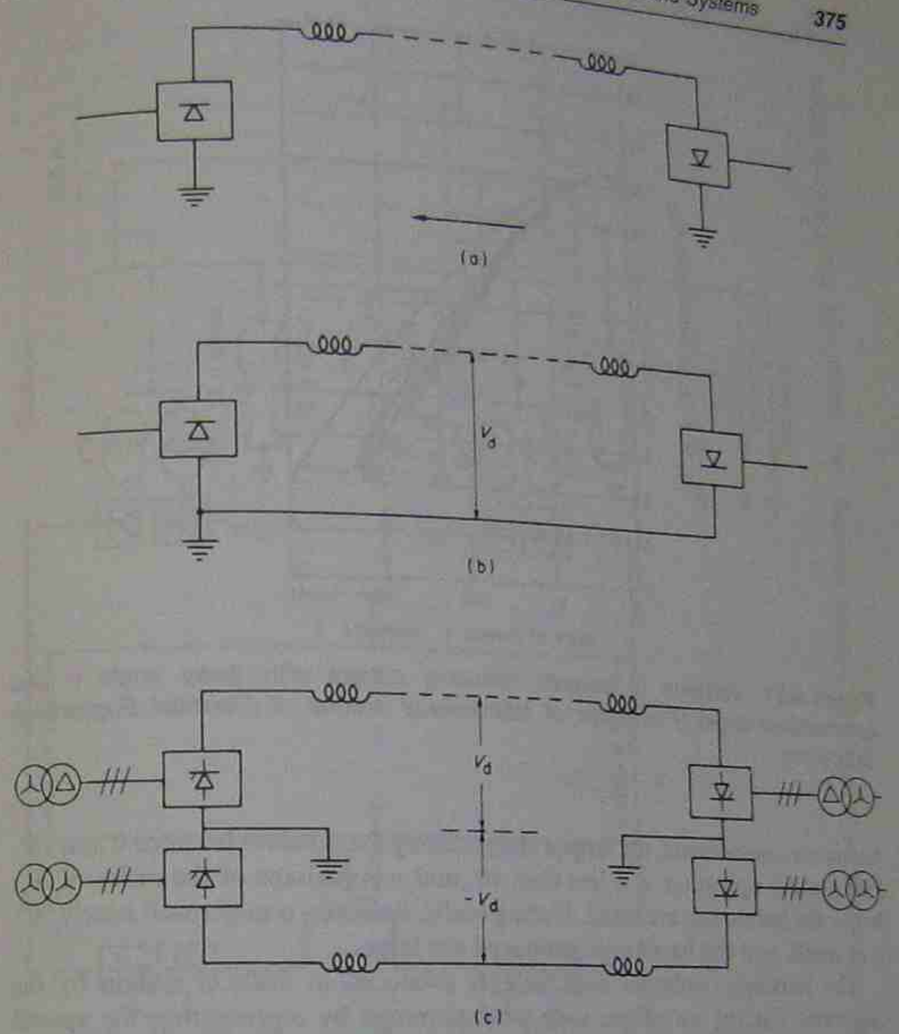


Figure 9.20 Possible conductor arrangements for d.c. transmissions: (a) ground return; (b) two conductors, return earthed at one end; (c) double-bridge arrangement

harmonics produced is $6n \pm 1$, where n is the number of valves. By the use of the Fourier series the equation for the current (i) is given by

$$i = \frac{2\sqrt{3}}{\pi} I_d \left(\cos \omega t + \frac{1}{5} \cos 5\omega t - \frac{1}{7} \cos 7\omega t - \frac{1}{11} \cos 11\omega t + \cdots \right)$$

Figure 9.21 shows the variation of the seventh harmonic component with both commutation (overlap) angle (γ) and delay angle (α). Generally, the harmonics decrease with decrease in γ this being more pronounced at higher harmonics. Changes in α for a given γ value do not cause large decreases in the

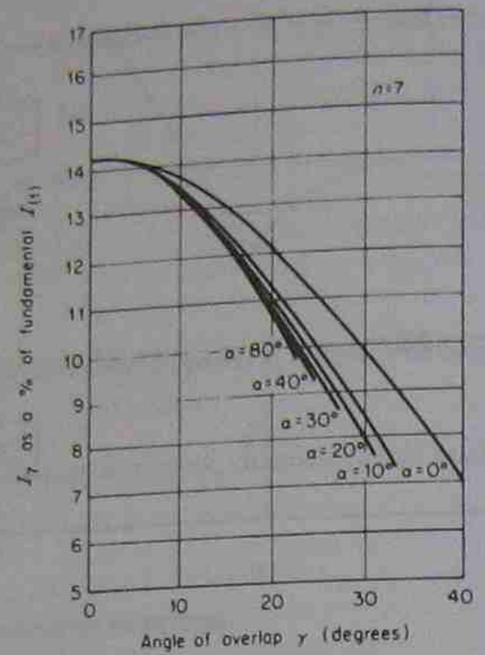


Figure 9.21 Variation of seventh harmonic current with delay angle α and commutation angle (*Permission of International Journal of Electrical Engineering Education*)

harmonic components, the largest change being for α values between 0 and 10°. For normal operation, α is less than 10° and γ is perhaps of the order of 20°; hence the harmonics are small. During faults, however, α may reach nearly 90°, γ is small, and the harmonics produced are large.

The harmonic voltages and currents produced in the a.c. system by the converter current waveform may be determined by representing the system components by their reactances at the particular harmonic frequency. Most of the system components have resonance frequencies between the fifth and eleventh harmonics.

It is usual to provide filters (L–C shunt resonance circuits) tuned to the harmonic frequencies. A typical installation is shown in detail in Figure 9.22. At the fundamental frequency the filters are capacitive and help to meet the reactive-power requirements of the converters.

9.7.3 Variable compensators

Manual control of shunt units can achieve a reasonably constant voltage profile under steady-state conditions, but cannot cope with transient conditions. Similarly, synchronous compensators are too slow to help materially under

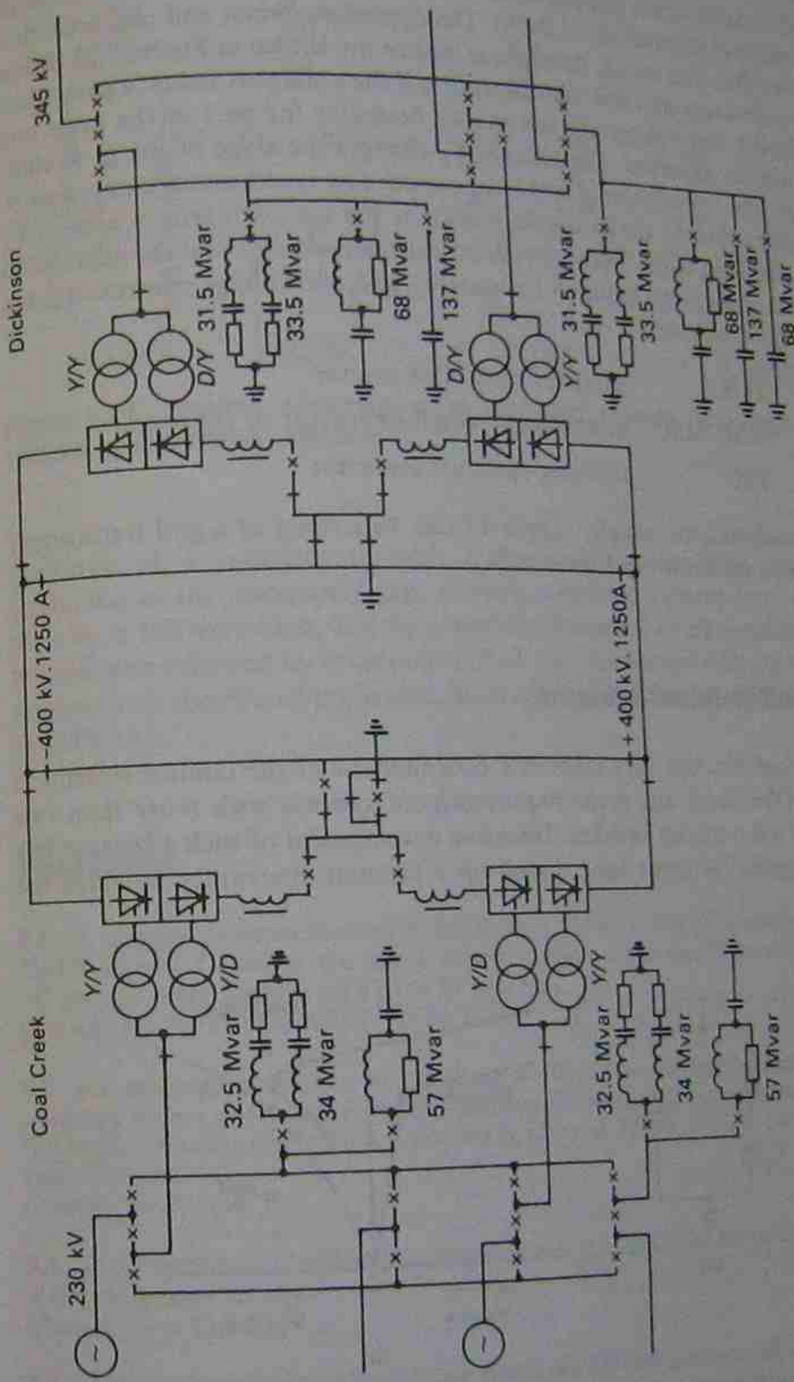


Figure 9.22 Single-line diagram of the main circuit of an h.v.d.c. scheme showing filter banks and shunt compensation. (By kind permission of I.E.E.E.)

transient conditions and also contribute to fault currents. A method of providing fast variable-shunt compensation lies in the use of saturated reactors, which have response times of about 0.03 s. The equivalent circuit and characteristics of a saturable, iron-cored, three-phase reactor are shown in Figure 9.23. It has a high inductance and low var absorption if the voltage is below a prescribed level. Above this voltage the inductance decreases for part of the cycle and more vars are absorbed. Capacitance C_0 changes the slope of the Q–V characteristic. The overall cost is in the same order as a synchronous compensator.

An alternative to the saturable reactor is the use of thyristor control by varying the firing angle. Many variations exist depending upon the application and speed of response required for stability and overvoltage control. Among the possible types are:

TCR:	thyristor-controlled reactor
MSR/MSC:	mechanically switched reactor or capacitor
TSC:	thyristor-switched capacitor

Typical combinations, usually supplied from the tertiary of a grid transformer at 13–15 kV, are shown in Figure 9.24.

9.7.4 Multiterminal systems

These are feasible, but very elaborate coordination of the control systems is necessary. Obviously, the great requirement of systems with more than two terminals is a d.c. circuit breaker. Intensive development of such a breaker has been in progress for some time, based on a vacuum interrupter in which the

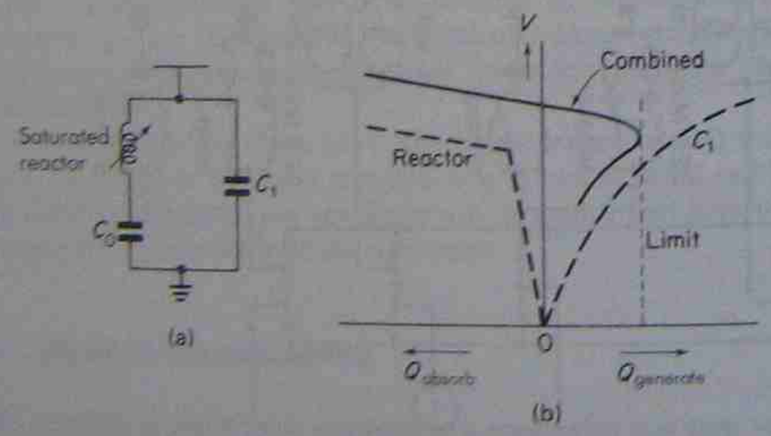


Figure 9.23 Saturated reactor for voltage control: (a) equivalent circuit; (b) V–Q characteristics

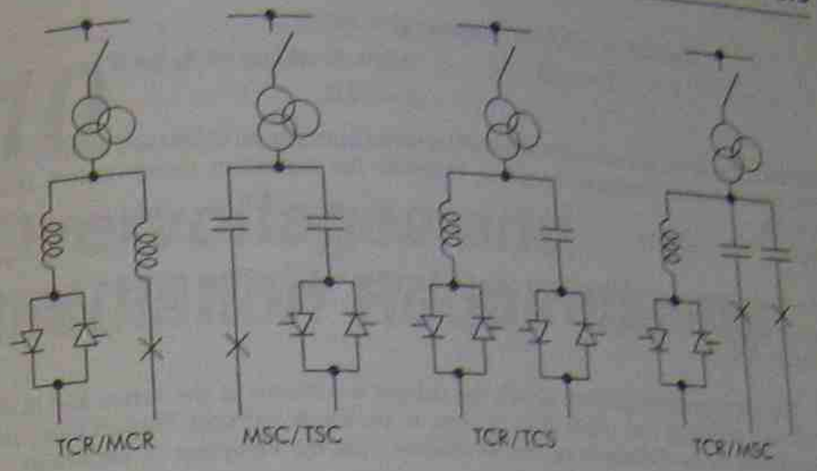


Figure 9.24 Possible combinations of controlled reactors and capacitors for variable compensation purposes

discharge of a capacitor through a triggered vacuum gap produces an oscillation in the interrupter gap, thereby creating a current zero. However, the cost is still very high, and by coordinated control of all terminals, a brief voltage zero followed by isolator-switched and -controlled start-up of remaining circuits is considered preferable. Such systems operate in Italy Sardinia, and in California.

Problems

9.1 A bridge-connected rectifier is fed from a 230 kV/120 kV transformer from the 230 kV supply. Calculate the direct voltage output when the commutation angle is 15° and the delay angle is (a) 0°; (b) 30°; (c) 60°.
(Answer: (a) 160 kV; (b) 127 kV; (c) 61.5 kV)

9.2 It is required to obtain a direct voltage of 100 kV from a bridge-connected rectifier operating with $\alpha = 30°$ and $\gamma = 15°$. Calculate the necessary line secondary voltage of the rectifier transformer, which is nominally rated at 345 kV/150 kV; calculate the tap ratio required.
(Answer: 94 kV line; 1.6)

9.3 If the rectifier in Problem 9.2 delivers 800 A d.c., calculate the effective reactance X (Ω) per phase.
(Answer: $X = 13.0\,\Omega$)

9.4 A d.c. link comprises a line of loop resistance 5 Ω and is connected to transformers giving secondary voltage of 120 kV at each end. The bridge-connected converters operate as follows:

Rectifier: $\alpha = 10°$ $X = 15\,\Omega$ Inverter: $\delta_0 = 10°$ Allow 5° margin on δ_0 for δ $X = 15\,\Omega$

Calculate the direct current delivered if the inverter operates on constant δ control. If all parameters remain constant, except α, calculate the maximum direct current transmittable.
(Answer: 610; 1104 A)

9.5 The system in Problem 9.4 is operated with $\alpha = 15°$ and on constant β control. Calculate the direct current for $\gamma = 15°$.
(Answer: 550 A)

9.6 For a bridge arrangement, sketch the current waveforms in the valves and in the transformer windings and relate them, in time, to the anode voltages. Neglect delay and commutation times. Comment on the waveforms from the viewpoint of harmonics.

9.7 Draw a schematic diagram of an existing or proposed direct-current transmission scheme. Give vital parameters, e.g. voltage, rating of valves, etc.

9.8 A direct current transmission link connects two a.c. systems via converters, the line voltages at the transformer–converter junctions being 100 kV and 90 kV. At the 100 kV end the converter operates with a delay angle of 10°, and at the 90 kV end the converter operates with a δ of 15°. The effective reactance per phase of each converter is 15 Ω and the loop resistance of the link is 10 Ω. Determine the magnitude and direction of the power delivered if the inverter operates on constant-δ control. Both converters consist of six valves in bridge connection. Calculate the percentage change required in the voltage of the transformer, which was originally at 90 kV, to produce a transmitted current of 800 A, other controls being unchanged. Comment on the reactive power requirements of the converters.
(Answer: 1.55 kA; 207 MW; 6.5 per cent)

9.9 A thyristor rectifier supplies direct current to a load. On no-load the voltage across the load is 1.5 kV and the firing or delay angles 35°. The delay is controlled by a feedback system which holds the load voltage constant with changes in current. With this control the transformer reactance causes the ratio (voltage drop/no-load voltage) to be 0.08 p.u.

By considering only the fundamental components of the three-phase supply currents, plot a curve of volt–amperes reactive as a function of load current. Neglect the commutation angle.

9.10 Draw a schematic diagram showing the main components of an h.v.d.c. link connecting two a.c. systems. Explain briefly the role of each component and how inversion into the receiving end a.c. system is achieved.

Discuss two of the main technical reasons for using h.v.d.c. in preference to a.c. transmission and list any disadvantages.
(From E.C. Examination, 1996)

10

Overvoltages and Insulation Requirements

10.1 Introduction

An area of critical importance in the design of power systems is the consideration of the insulation requirements for lines, cables, and stations. At first glance this may appear to be a simple matter once the operating voltage of the system is decided, but unfortunately this is far from so. As well as the normal operating voltages, transients occur, causing overvoltages in the system as a result of switching, lightning strokes, and other causes; the peak values of these can be much in excess of the operating voltage. Because of this, devices must be provided to protect items of plant. The term extra-high voltage (E.H.V.) has generally been accepted as describing systems of 230 kV up to 765 kV, and for voltages above 765 kV the term ultra-high voltage (U.H.V.) is applied; below 230 kV, high voltage (H.V.) is in use.

Until recent years, lightning has largely determined the insulation requirements, i.e. size of bushings, number of insulators per string, and tower clearances of the system, and the insulation of equipment has been tested with voltages of a waveform approximately that of a lightning surge. With the much higher operating voltages now in use, and projected, the voltage transients or surges due to switching, i.e. the opening and closing of circuit breakers, have become the major consideration. In this context it is of interest to note that voltages of 500 kV and 765 kV are now in use, and active discussion and research are taking place to decide the next voltage level, likely to be in the range 1000–1500 kV if rights of way can be found.

A factor of major importance is the *contamination* of insulator surfaces caused by atmospheric pollution. This considerably modifies the performance of insulation, which becomes difficult to assess precisely. The presence of dirt,

salt, etc., on the insulator discs or bushing surfaces results in these surfaces becoming slightly conducting, and hence flashover occurs.

A few terms frequently used in high-voltage technology need definition. They are as follows.

1. *Basic impulse insulation level or basic insulation level* (BIL)—reference levels expressed in impulse crest (peak) voltage with a standard wave not longer than a $1.2 \times 50\,\mu s$ wave (see Figure 10.1). Apparatus insulation as demonstrated by suitable tests shall be equal or greater than the BIL. The two standard tests are the power frequency and 1.2/50 impulse-wave withstand tests. The *withstand voltage* is the level that the equipment will withstand for a given length of time or number of applications without disruptive discharge occurring, i.e. a failure of insulation resulting in a collapse of voltage and passage of current (sometimes termed 'sparkover' or 'flashover' when the discharge is on the external surface). Normally, several tests are performed and the number of flashovers noted. The BIL is usually expressed as a per unit of the peak (crest) value of the normal operating voltage to earth; e.g. for a maximum operating voltage of 362 kV,

$$1\,\text{p.u.} = \sqrt{2} \times \frac{362}{\sqrt{3}} = 300\,\text{kV}$$

so that a BIL of 2.7 p.u. = 810 kV.

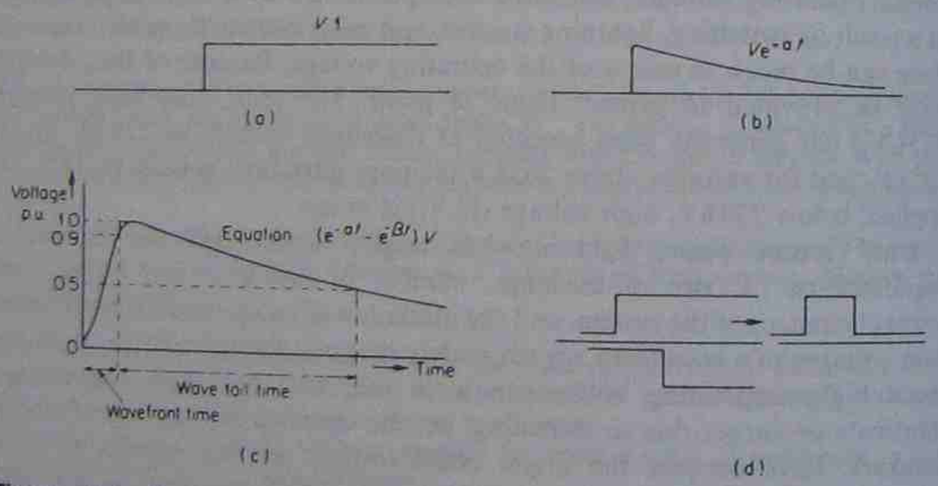


Figure 10.1 Basic impuse waveforms; (a) and (b) frequently used in calculations. (c) Shape of lightning and switching surges; the former have a rise time of, say, $1.2\,\mu s$ and a fall time of half maximum value of $50\,\mu s$ (hence 1.2/50 wave). Switching surges are much longer, the duration times varying with situation; a typical wave is $175/3000\,\mu s$. Equation for 1/50 wave is $v = 1.036\,(e^{-0.0146t} - e^{-2.56t})$. (d) Use of two unit functions to form a wave of finite duration

2. *Critical flashover voltage* (CFO)—the peak voltage for a 50 per cent probability of flashover or disruptive discharge (sometimes denoted by V_{50}).
3. *Impulse ratio* (for flashover or puncture of insulation)—impulse peak voltage divided by the crest value of power-frequency voltage to cause flashover or puncture.

Impulse tests are normally performed with a voltage wave which rises in 1.2 μs and falls to half the peak value in $50\,\mu s$ (see Figure 10.1); this is known as a 1.2/50 μs wave and typifies the lightning surge. In most impulse generators the shape and duration of the wave may be modified. Basic impulse waves are shown in Figure 10.1. Switching surges consist of damped oscillatory waves, the frequency of which is determined by the system configuration and parameters; they are normally of amplitude 2–2.8 p.u., although they can exceed 4 p.u. (per-unit values based on peak line-to-earth operating voltage as before).

10.2 Generation of Overvoltages

10.2.1 Lightning surges

A thundercloud is bipolar, often with positive charges at the top and negative at the bottom, usually separated by several kilometres. When the electric field strength exceeds the breakdown value a lightning discharge is initiated. The first discharge proceeds to the earth in steps (stepped leader stroke). When close to the earth a faster and luminous return stroke travels along the initial channel, and several such leader and return strokes constitute a flash. The ratio of negative to positive strokes is about 5 : 1 in temperate regions. The magnitude of the return stroke can be as high as 200 kA, although an average value is of the order of 20 kA.

Following the initial stroke, after a very short interval, a second stroke to earth occurs, usually in the ionized path formed by the original. Again, a return stroke follows. Usually, several such subsequent strokes (known as dart leaders) occur, the average being between three and four. The complete sequence is known as a multiple-stroke lightning flash and a representation of the strokes at different time intervals is shown in Figure 10.2. Normally, only the heavy current flowing over the first $50\,\mu s$ is of importance and the current–time relationship has been shown to be of the form $i = i_{peak}(e^{-\alpha t} - e^{-\beta t})$.

When a stroke arrives on an overhead conductor, equal current surges of the above waveform are propagated in both directions away from the point of impact. The magnitude of each voltage surge set up is therefore $\frac{1}{2} \cdot Z_0 \cdot i_{peak}$ $(e^{-\alpha t} - e^{-\beta t})$, where Z_0 is the conductor surge impedance. For a current peak of 20 kA and a Z_0 of $350\,\Omega$ the voltage surges will have a peak value of $(350/2) \times 20 \times 10^3$, i.e. 3500 kV.

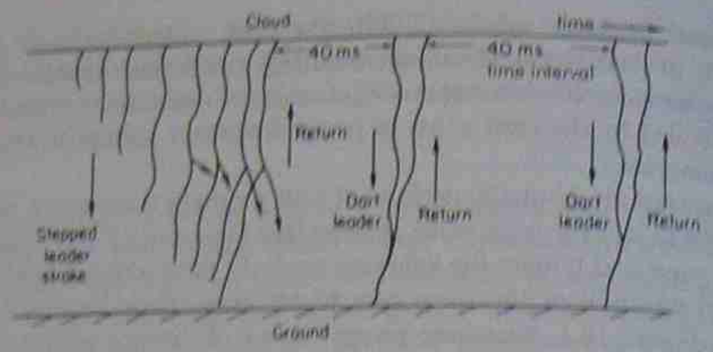


Figure 10.2 Sequence of strokes in a multiple lightning stroke

When a ground or earth wire exists over the overhead line, a stroke arriving on a tower or on the wire itself sets up surges flowing in both directions along the wire. On reaching neighbouring towers they are partially reflected and transmitted further. This process continues over the length of the line as towers are encountered. If the towers are 300 m apart the travel time between towers and back to the original tower is $(2 \times 300)/(3 \times 10^8)$, i.e. 2 μs, where the speed of propagation is 3×10^8 m/s. The voltage distribution may be obtained by means of the Bewley lattice diagram, to be described in section 10.6.

If an indirect stroke strikes the earth near a line, the induced current, which is normally of positive polarity, creates a voltage surge of the same waveshape which has an amplitude dependent on the distance from the ground. With a direct stroke the full lightning current flows into the line producing a surge that travels away from the point of impact in all directions. A direct stroke to a tower can cause a *back flashover* due to the voltage set up across the tower inductance and footing resistance by the rapidly changing lightning current (typically 10 kA/μs); this appears as an overvoltage between the top of the tower and the conductors (which are at a lower voltage).

10.2.2 Switching surges—interruption of short circuits and switching operations

When the arc between the circuit-breaker contacts breaks, the full system voltage (recovery voltage) suddenly appears across the open gap and hence across the *R–L–C* circuit comprising the system. The simplest form of single-phase equivalent circuit is shown in Figure 10.3(a) and (b). The resultant voltage appearing across the circuit is shown in Figure 10.3(c). It consists of a high-frequency component superimposed on the normal system voltage, the total being known as the restriking voltage and constituting a switching surge. The equivalent circuit of Figure 10.3 may be analysed by means of the Laplace transform. When the circuit breaker opens, the cessation of current may be

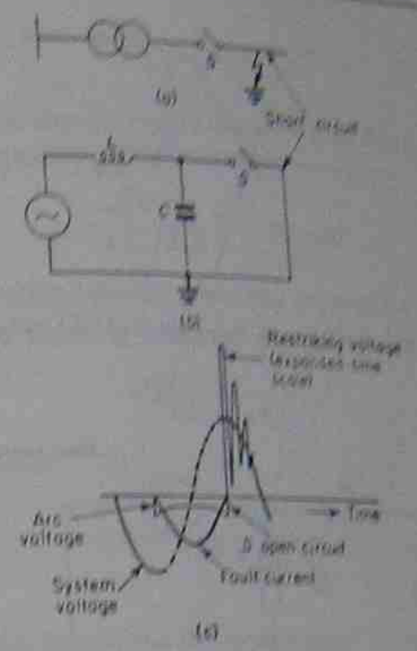


Figure 10.3 Restriking voltage set up on fault interruption. (a) System diagram. (b) Equivalent circuit. L = transformer leakage inductance; C = transformer and connections capacitance to ground. (c) Current and voltage waveforms

simulated by the injection of an equal and opposite current into the system at time zero. Although this current is sinusoidal in form it may be approximated by a ramp function as shown in Figure 10.4. Let the fault current, $i = \sqrt{2}I \sin \omega t$.

The arc is extinguished at a current pause or zero (see Chapter 11), at which

$$\frac{\mathrm{d}i}{\mathrm{d}t} = (\sqrt{2}I\omega \cos \omega t)_{t=0}$$
$$= \sqrt{2}I\omega$$

The short-circuit current $= I = V/\omega L$ (r.m.s. value). Hence the equation for the ramp function of injected current is

$$\left(\frac{\mathrm{d}i}{\mathrm{d}t}\right)_{t=0} \cdot t = \sqrt{2}\left(\frac{V}{L}\right)t \qquad (10.1)$$

The transform of the circuit after breaker opening is shown in Figure 10.5. Transform of the ramp function

$$= i(s) = \frac{\sqrt{2}V}{Ls^2} \qquad (10.2)$$

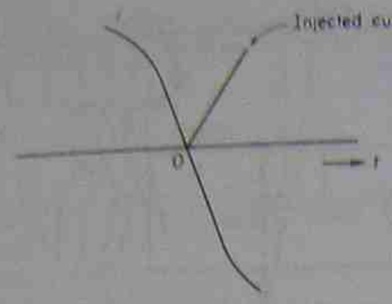


Figure 10.4 Application of Laplace transform to circuit of Figure 10.3. Ramp function of current

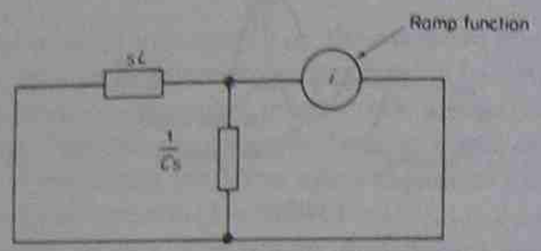


Figure 10.5 Equivalent circuit with ramp applied

Voltage across the breaker contacts, i.e. across C in Figure 10.3(b) is

$$v(s) = \frac{\sqrt{2}V}{Ls^2} \cdot \frac{L/C}{Ls + 1/Cs}$$

$$= \sqrt{2}V \frac{\omega_0^2}{s(s^2 + \omega_0^2)}$$

where $\omega_0^2 = 1/LC$.

$$\therefore v(s) = \sqrt{2}V\left(\frac{A}{s} + \frac{Bs + D}{s^2 + \omega_0^2}\right)$$

$$= \sqrt{2}V\left(\frac{1}{s} - \frac{s}{s^2 + \omega_0^2}\right)$$

since $A = 1$, $B = -1$, $D = 0$ from initial conditions.

From a table of transforms,

$$v(t) = \sqrt{2}V[1 - \cos(\omega_0 t)] \tag{10.3}$$

Equation (10.3) assumes that the full-system voltage V exists across the open switch; strictly the voltage across C is

$$V\left(\frac{X_c}{X_c - X_L}\right)$$

where X_c and X_L are the capacitive and inductive reactances at system angular frequency ω so that equation (10.3) becomes

$$\frac{\sqrt{2}V}{[1 - (X_L/X_c)]}(1 - \cos\omega_0 t) \tag{10.4}$$

The difference, in practice, between these two expressions is small.

When series resistance ($R\,\Omega$) is significant, equation (10.1) becomes,

$$\sqrt{2}\left(\frac{V\omega}{R + j\omega L}\right)t$$

and the ramp expression

$$i(s) = \frac{\sqrt{2}\omega V}{(R + j\omega L)s^2} \tag{10.5}$$

and $v(s) = i_s Z(s)$.

The analysis of this case yields the expression,

$$v(t) = \omega IL(1 - e^{-\alpha t}\cos\omega_0 t) \tag{10.6}$$

where ω is the system angular frequency, ω_0 is the natural angular frequency of the circuit, and I is the short-circuit current prior to the breaker opening,

$$\alpha = \frac{R}{2L}, \qquad (\alpha^2 + \omega_0^2) = \frac{1}{LC}$$

and

$$V \approx I\omega L$$

From equation (10.6)

$$(dv/dt)_{\text{maximum}} \approx \omega I\sqrt{\left(\frac{L}{C}\right)}e^{-\alpha t} \tag{10.7}$$

The restriking voltage (v) can rise to a maximum value of $2V$, where V is the peak value of the system recovery voltage. A similar expression is obtained when a line is suddenly energized by the system voltage.

If a resistance R_s be connected across the contacts of the circuit breaker, the surge will be critically damped when $R_s = \frac{1}{2}\sqrt{(L/C)}$, and this offers an important method of reducing the severity of the transient. The initial rate of rise of the surge is very important as this determines whether the contact gap, which is highly polluted with arc products, breaks down again after the initial open circuit occurs. In the system shown in Figure 10.6(a), a double-frequency transient (Figure 10.6(b)) is set up, the two frequencies being determined by the circuit on each side of the switch, i.e.

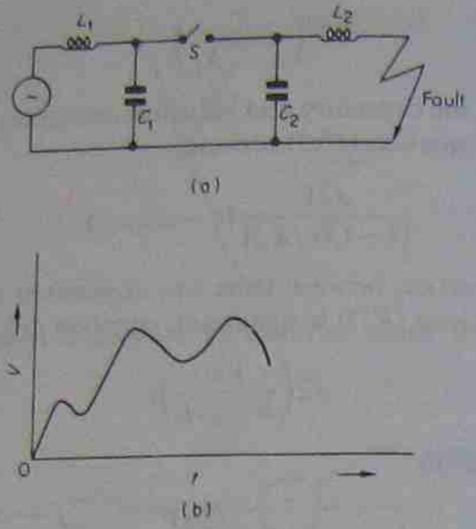


Figure 10.6 (a) System with double-frequency restriking transient. (b) Typical waveform of double frequency, although initial amplitude is low, rate of rise is high

$$\omega_1 = \frac{1}{\sqrt{(L_1 C_1)}} \quad \text{and} \quad \omega_2 = \frac{1}{\sqrt{(L_2 C_2)}}$$

Example 10.1

Determine the relative attenuation occurring in 5 cycles in the overvoltage surge set up on a 66 kV cable fed through an air-blast circuit breaker when the breaker opens on a system short circuit. The breaker incorporates resistance switching, i.e. an optimum resistance switched in across the contact gap on opening. The network parameters are as follows:

$$R = 7.8\,\Omega \qquad L = 6.4\,\text{mH} \qquad C = 0.0495\,\mu\text{F}$$

Solution

$$\text{Switching resistance} = \tfrac{1}{2}\sqrt{(L/C)} = 180\,\Omega$$

$$\alpha = \frac{1}{2} \cdot \frac{R}{L} = 610$$

$$\omega = \frac{1}{\sqrt{LC}} = 5.61 \times 10^4\ \text{rad/s}$$

i.e. transient frequency = 8.93 kHz.

Hence

$$\text{time for 5 cycles} = \frac{1}{8.93 \times 10^3} \times 5\,\text{s}$$

and

$$e^{-\alpha t} = \exp\left(-610 \times \frac{5}{8.93 \times 10^3}\right) = 0.712$$

The maximum theoretical voltage set up with the circuit breaker opening on a short-circuit fault would be

$$2 \times \frac{66\,000}{\sqrt{3}} \times \sqrt{2}$$

i.e. 107.5 kV, and in 5 cycles (i.e. 560 μs) this becomes $107.5 \times (1 - 0.712)$, i.e. 31 kV.

(Note that resistance switching lowers the current to be broken and raises the power factor so that the voltage is not at peak value at opening.)

10.2.3 Switching surges—interruption of capacitive circuits

The interruption of capacitive circuits is shown in Figure 10.7. At the instant of arc interruption, the capacitance is left charged to value v_m but half a cycle later the system voltage is $-v_m$, giving a gap voltage of $2v_m$. If the gap breaks down, an oscillatory transient is set up (as previously discussed), which can increase the gap voltage still further, as shown in Figure 10.7.

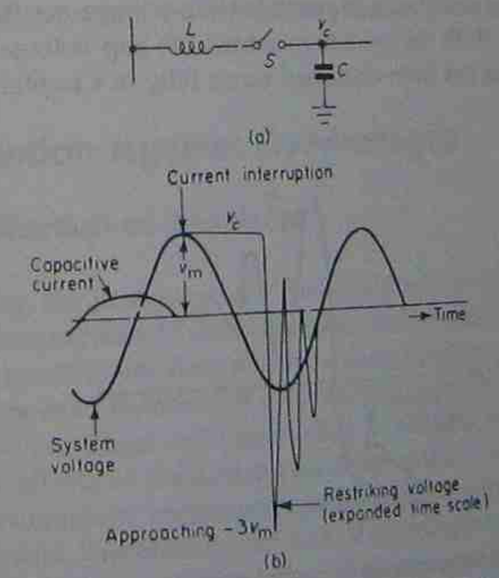


Figure 10.7 Voltage waveform when opening a capacitive circuit

Current chopping

Current chopping arises with air-blast or puffer-assisted circuit breakers which operate on the same gas pressure and velocity for all values of interrupted current. Hence, on low-current interruption the breaker tends to open the circuit before the current natural-zero, and the electromagnetic energy present is rapidly converted to electrostatic energy, i.e.

$$\tfrac{1}{2}Li_0^2 = \tfrac{1}{2}Cv^2 \qquad \text{and} \qquad v = i_0\sqrt{\frac{L}{C}} \tag{10.8}$$

The voltage waveform is shown in Figure 10.8.

An extension of equation (10.8) to include resistance and time yields

$$v_r = i_0\sqrt{\frac{L}{C}}\mathrm{e}^{-\alpha t}\sin\omega_0 t \tag{10.9}$$

where $\omega_0 = 1\sqrt{LC}$ and i_0 is the value of the current at the instant of chopping. High transient voltages may be set up on opening a highly inductive circuit such as a transformer on no-load.

Faults

Overvoltages may be produced by certain types of asymmetrical fault, mainly on systems with unground neutrals. The voltages set up are of normal operating frequency. Consider the circuit with a three-phase earth fault as shown in Figure 10.9. If the circuit is not grounded the voltage across the first gap to open is $1.5\,V_{\text{phase}}$. With the system grounded the gap voltage is limited to the phase voltage. This has been discussed more fully in Chapter 7.

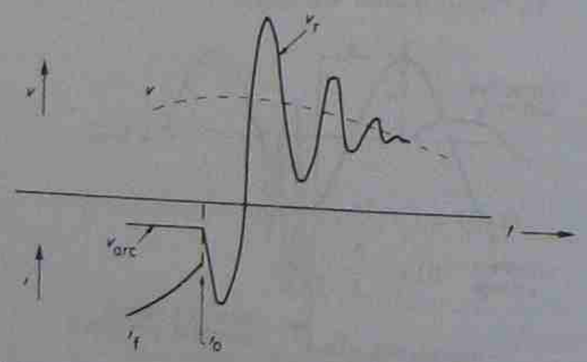


Figure 10.8 Voltage transient due to current chopping: i_f = fault current; v = system voltage, i_0 = current magnitude at chop, v_r = restriking voltage

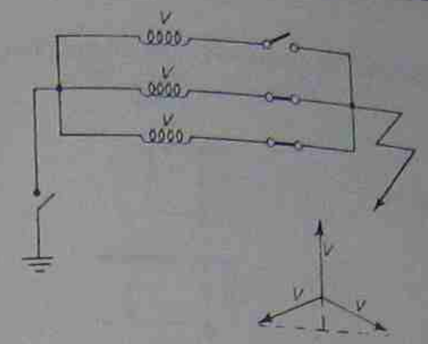


Figure 10.9 Three-phase system with neutral earthing (grounding)

Resonance

It is well known that in resonant circuits severe overvoltages occur, dependent on the resistance present; the voltage at resonance across the capacitance can be high. Although it is unlikely that resonance in a supply network can be obtained at normal supply frequencies, it is possible to have this condition at harmonic frequencies. Resonance is normally associated with the capacitance to earth of items of plant and is often brought about by an opened phase caused by a broken conductor or a fuse operating.

In circuits containing windings with iron cores, e.g. transformers, a condition due to the shape of the magnetization curve, known as *ferroresonance*, is possible. This can produce resonance with overvoltages and also sudden changes from one condition to another.

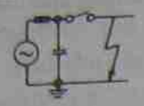
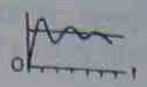
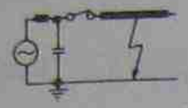
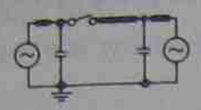
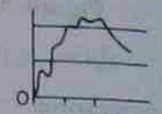
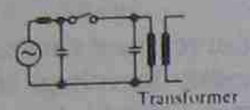
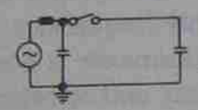
A summary of important switching operations is given in Table 10.1.

10.3 Protection Against Overvoltages

10.3.1 Modification of transients

When considering the protection of a power system against overvoltages, the transients may be modified or even eliminated before reaching the substations, or, if this is not possible, the lines and substation equipment may be protected, by various means, from flashover or insulation damage. By the use of overhead earth (ground) wires, phase conductors may be shielded from direct lightning strokes and the effects of induced surges from indirect strokes lessened. The shielding is not complete, except perhaps for a phase conductor immediately below the earth wire. The effective amount of shielding is often described by an angle ϕ, as shown in Figure 10.10; a value of 35° appears to agree with practical experience. Obviously, two earth wires horizontally separated provide much

Table 10.1 Summary of the more important switching operations

Switching operation	System	Voltage across contacts
1. Terminal short circuit		
2. Short line fault		
3. Two out-of-phase systems—voltage depends on grounding conditions in systems		
4. Small inductive currents, current chopped (unloaded transformer)	 Transformer	
5. Interrupting capacitive currents—capacitor banks, lines, and cables on no-load		See Figure 10.7

(*Permission of* Brown Boveri Review, *December 1970.*)

better shielding. Often, for reasons of economy, earth wires are installed over the last kilometre or so of line immediately before it enters a substation. It has already been seen that the switching-in of resistance across circuit-breaker contacts reduces the high overvoltages produced on opening, especially on capacitive or low-current inductive circuits.

An aspect of vital importance, quite apart from the prevention of damage, is the maintenance of supply, especially as most flashovers cause no permanent damage and therefore a complete and lasting removal of the circuit from operation is not required. This may be achieved by the use of *autoreclosing* circuit breakers.

A *surge modifier* may be produced by the connection of a shunt capacitor between the line and earth or an inductor in series with the line; oscillatory effects may be reduced by the use of fast-acting voltage or current injections. It is uneconomical to attempt to modify or eliminate most overvoltages, and means are required to protect the various items of power systems. Surge divertors are connected in shunt across the equipment and divert the transient to earth; surge modifiers are connected in series with the line entering the substation and attempt to reduce the steepness of the wavefront—hence reduce its severity.

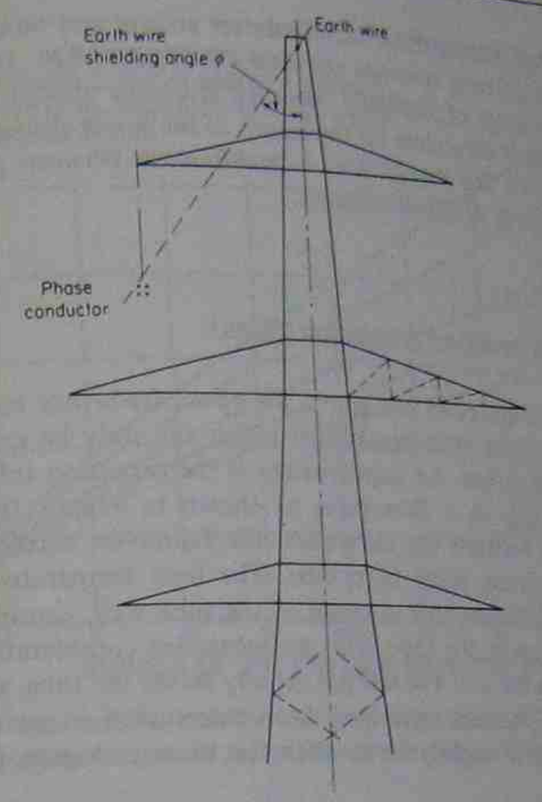


Figure 10.10 Single earth wire protection; shielding angle ϕ normally 35

10.3.2 Surge diverters

The basic requirements for diverters are that they should pass no current at normal voltage, interrupt the power frequency follow-on current after a flash-over, and break down as quickly as possible after the abnormal voltage arrives. The simplest form is the *rod gap* connected across a bushing or insulator (as shown in Figure 10.48). This may also take the form of rings (arcing rings) around the top and bottom of an insulator string. The breakdown voltage for a given gap is polarity dependent to some extent. It does not interrupt the post-flashover follow-on current (i.e. the power-frequency current which flows in the path created by the flashover), and hence the circuit protection must operate, but it is by far the cheapest device for plant protection against surges. It is usually recommended that a rod gap be set to a breakdown voltage not less than 30 per cent below the voltage-withstand level of the protected equipment. For a given gap, the time for breakdown varies roughly inversely with the applied voltage; there exists, however, some dispersion of values. The times for positive voltages are lower than those for negative ones.

Typical curves relating the critical flashover voltage and time to breakdown for rod gaps of different spacings are shown in Figure 10.11. In Figure 10.12, the flashover voltage of vertically mounted insulator strings is shown. The flashover voltage is dependent on the length of the lower clearance to ground. For low values of this length there is a difference between positive (lower values) and negative flashover voltages.

Expulsion gaps or tubes (protector tubes)

A disadvantage of the plain rod gap is the power-frequency current (follow-on current) that flows after breakdown which can only be extinguished by circuit-breaker operation. An improvement is the expulsion tube which consists of a spark gap in a fibre tube, as shown in Figure 10.13. When a sparkover occurs between the electrodes, the follow-on current arc is contained in the relatively small fibre tube. The high temperature of the arc vaporizes some of the organic material of the tube wall, causing a high gas pressure to build up in the tube. This gas possesses considerable turbulence and it extinguishes the arc. The hot gas rapidly leaves the tube, which is open at the ends. Very-high currents have been interrupted in such tubes. The breakdown voltage is slightly lower than for plain rod gaps for the same spacing.

An improved but more expensive surge diverter is the *lightning arrester*. A porcelain bushing contains a number of spark gaps in series with silicon carbide discs, the latter possessing low resistance to high currents and high resistance to low currents, i.e. it obeys a law of the form $V = aI^{(0.2)}$, where a depends on the material and its size. The overvoltage breaks down the gaps and then the power-frequency current is determined by the discs and limited to such a value that the gaps can quickly interrupt it at the first current zero. The voltage–time characteristic of a lightning arrester is shown in Figure 10.14; the gaps break down at S and the characteristic after this is determined by the current and the discs; the maximum voltage at R should be in the same order as at S. For high voltages a stack of several such units is used. The basic arrangement of discs and gaps, which are housed inside a porcelain bushing, is shown in Figure 10.15(a).

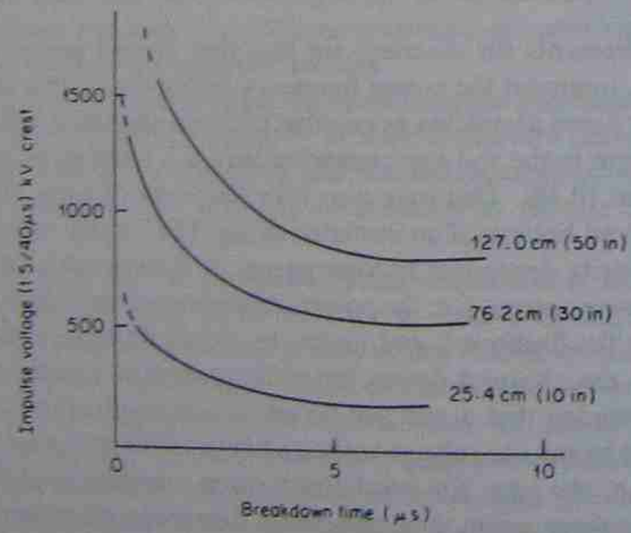


Figure 10.11 Breakdown characteristics of rod gaps

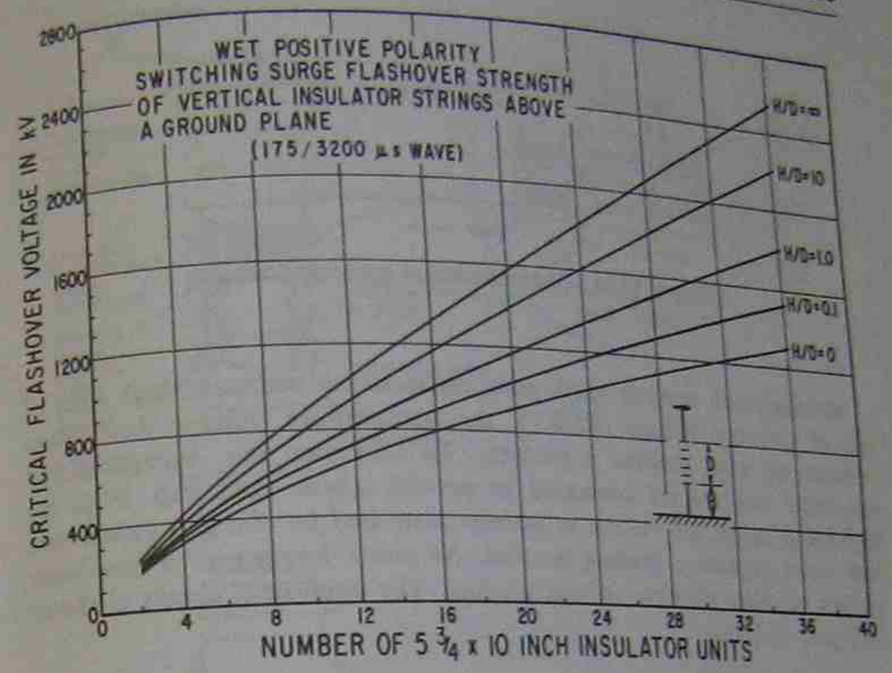


Figure 10.12 Wet positive-polarity switching-surge flashover strength of vertical insulator strings above a ground plane *(Permission of the Edison Electric Institute)*

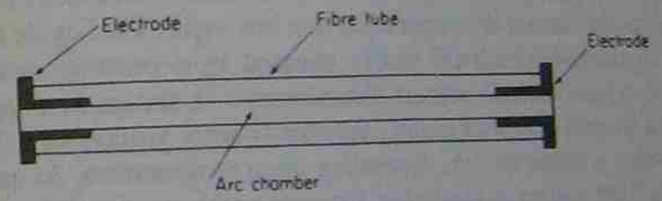


Figure 10.13 Expulsion tube

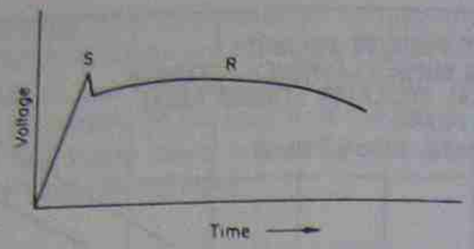


Figure 10.14 Characteristic of lightning arrester

Although with multiple spark gaps, diverters can withstand high rates of rise of recovery voltage (RRRV), the non-uniform voltage distribution between the gaps presents a problem. To overcome this, capacitors and non-linear resistors are connected in parallel across each gap. With the high-speed surge, the voltage is mainly controlled by the gap capacitance and hence capacitive grading is used. At power-frequencies a non-linear resistor provides effective voltage grading. The equivalent circuit is shown in Figure 10.15(b).

Metal oxide (MO) arresters

Developments in materials have produced an arrester which does not require gaps to reduce the current after operation to a low value. Zinc oxide material is employed in stacked cylindrical blocks encased in a ceramic weatherproof insulator. A typical current–voltage characteristic is shown in Figure 10.16, where it can be seen that with a designed steady-state voltage of 4.25 kV, the residual current is below 10^{-2} A, depending upon temperature. At an effective rating of 3 kV the current is negligible. Consequently, no action is required to extinguish the current after operation, as may be required with a gapped arrester.

Metal oxide arresters up to the highest transmission voltages are now employed almost to the exclusion of gapped devices, although many of these latter still remain in operation.

10.4 Insulation Coordination

The equipment used in a power system comprises items having different breakdown or withstand voltages and different voltage–time characteristics. In order that all items of the system be adequately protected there is a need to consider the situation as a whole and not items of plant in isolation, i.e. the insulation

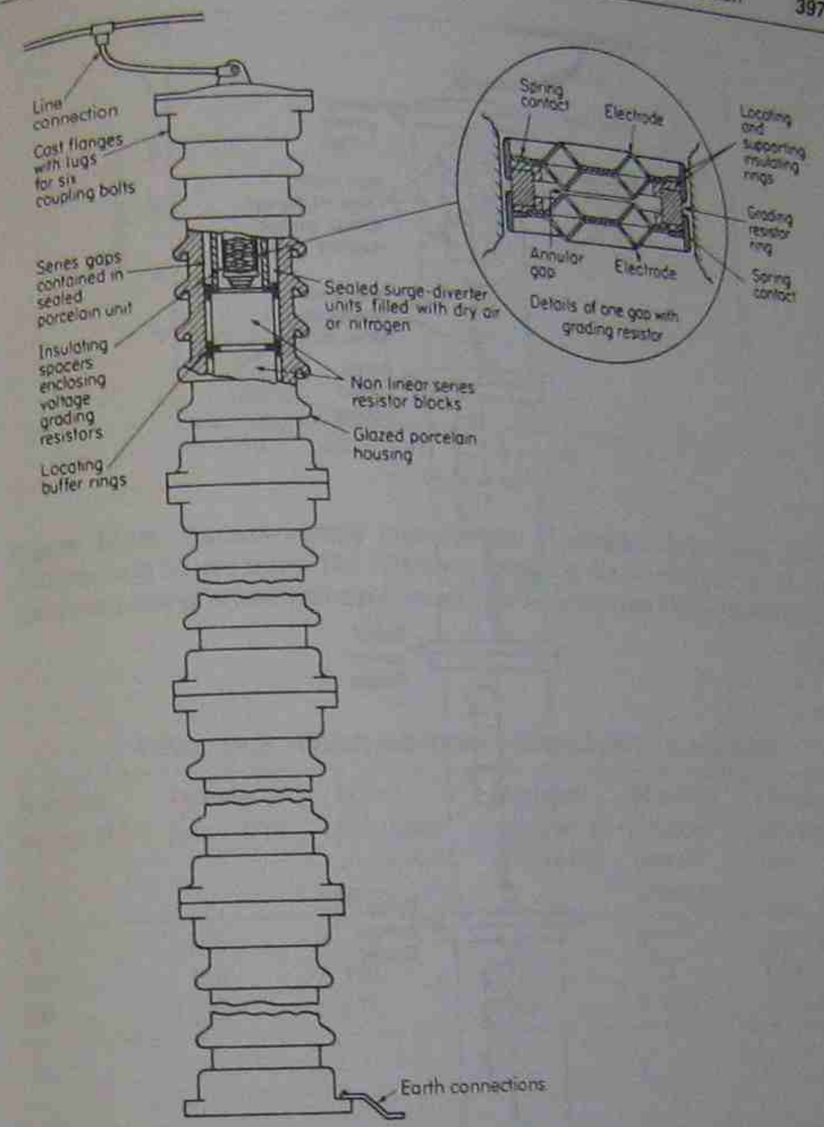


Figure 10.15 (a) Diagram of a lightning arrester—four-unit stack showing part section of one unit (*Permission of the Electricity Council*)

protection must be coordinated. To assist this process, standard insulation levels are recommended and these are summarized in Tables 10.2 and 10.3. Reduced basic insulation impulse levels are used when considering switching surges and these are also summarized in Table 10.3.

Coordination is rendered difficult by the different voltage–time characteristics of plant and protective devices, e.g. a gap may have an impulse ratio of 2 for a 20 μs front wave and 3 for a 5 μs wave. At the higher frequencies (shorter wavefronts) corona cannot form in time to relieve the stress concentration

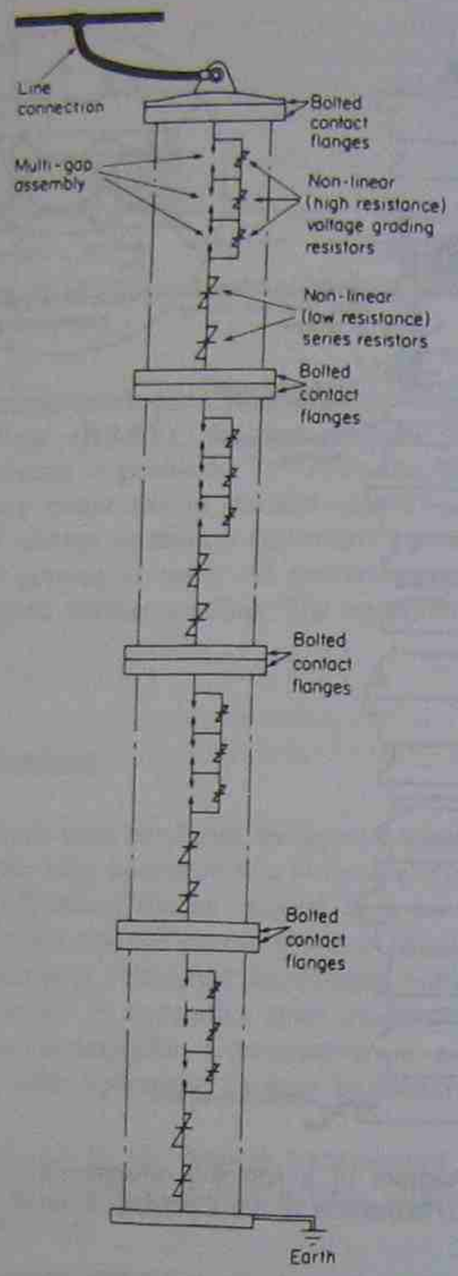


Figure 10.15 (b) Equivalent circuit of components comprising a single-phase four-unit surge-diverter stack (*Permission of the Electricity Council*)

on the gap electrodes. With a lightning surge a higher voltage can be withstood because a discharge requires a certain discrete amount of energy as well as a minimum voltage, and the applied voltage increases until the energy reaches this value. Figure 10.17 shows the voltage–time characteristics for the system

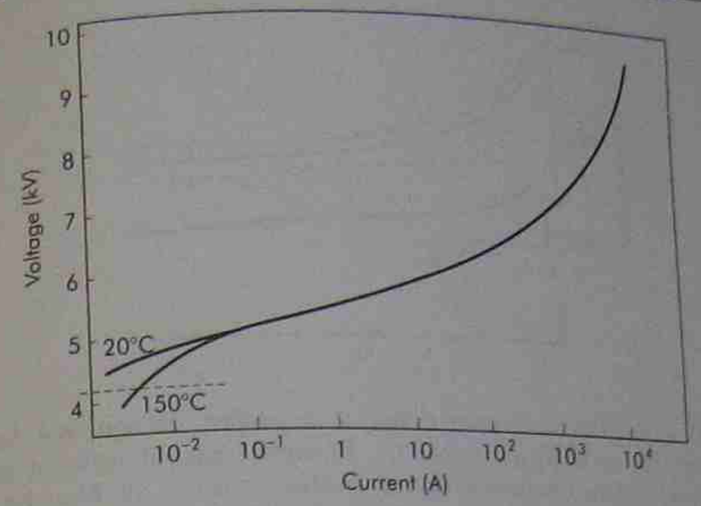


Figure 10.16 Current–voltage characteristic of standard ZnO blocks, 80 mm in diameter and 35 mm thick. The reference voltage of the block is 4.25 kV d.c. and its continuous rating is 3kV (effective value) (*Permission of the Electricity Association*)

Table 10.2 British substation practice (now I.E.C. standard)

Nominal voltage (kV)	Impulse (peak kV)	Power frequency withstand (peak kV)	Minimum clearance to ground (m)	Minimum clearance between phases (m)	Coordinated gap setting (m)
11	100	29	0.2	0.25	2 × 0.03
132	550	300	1.1	1.25	0.66
400	1425	675	3.05	3.55	1.5–1.8

Table 10.3 Recommended BILs at various operating voltages (U.S. practice)

Voltage class (kV)	15	23	34.5	46	69	92	115
BIL (kV)	110	150	200	250	350	450	550 / 450
Reduced BIL (kV)			125				350
Voltage class (kV)	138	161	196	230	287	345	500
BIL (kV)	650	750	900	1050	1300	1550	1800
Reduced BIL (kV)	550 / 450	650 / 550		900 / 825 / 750	1175 / 1050 / 900	1425 / 1300 / 1050	1675 / 1550 / 1300

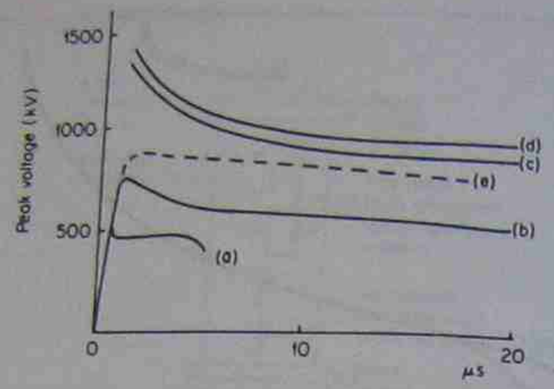


Figure 10.17 Insulation coordination in an H.V. substation. Voltage–time characteristics of plant for a 1.5/40 μs wave. (a) Characteristics of lightning arrester. (b) Transformer. (c) Line insulator string. (d) Busbar insulation. (e) Maximum surge applied waveform

elements comprising a substation, and the protection of the weakest items by the arrester is illustrated.

Up to an operating voltage of 345 kV the insulation level is determined by lightning, and the standard impulse tests suffice, along with normal frequency tests. Above this value, however, the overvoltages resulting from switching are higher in magnitude and therefore they decide the insulation. The characteristics of air gaps and some solid insulations are different for switching surges than for the standard impulse waves, and closer coordination of insulation is required because of the lower attenuation of switching surges, although their amplification by reflection is less than with lightning. Recent work indicates that, for transformers, the switching impulse strength is of the order of 0.95 of the standard value while for oil-filled cables it is 0.7 to 0.8.

The design withstand level is selected by specifying the risk of flashover, e.g. for 550 kV towers a 0.13 per cent probability has been used. At 345 kV, design is carried out by accepting a switching impulse level of 2.7 p.u., which corresponds to the lightning level. At 500 kV, however, a 2.7 p.u. switching impulse would require 40 per cent more tower insulation than that governed by lightning. The tendency is therefore for the design switching impulse level to be forced lower with increasing system operating voltage and for control of the surges to be made by the more widespread use of resistance switching in the circuit breakers or use of surge diverters. For example, for the 500 kV network the level is 2 p.u. and at 765 kV it is reduced to 1.7 p.u.; if further increases in system voltage occur, it is hoped to decrease the level to 1.5 p.u.

The problem of switching surges is illustrated in Figure 10.18, in which flashover probability is plotted against peak (crest) voltage; *critical flashover voltage* (CFO) is the peak voltage for a particular tower design for which there is a 50 per cent probability of flashover. For lightning, the probability of

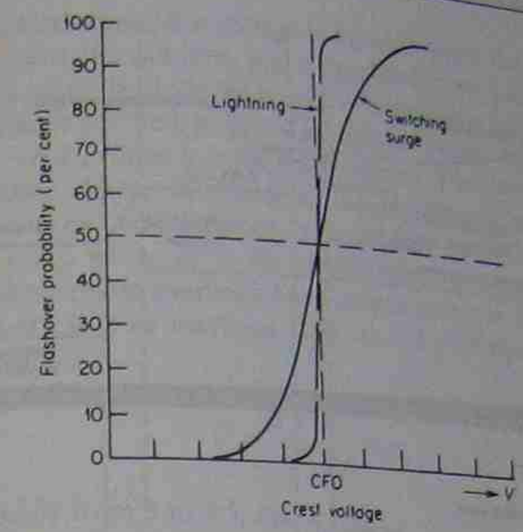


Figure 10.18 Flashover probability—peak surge voltage. Lightning flashover is close-grouped near the critical flashover voltage (CFO), whereas switching-surge probability is more widely dispersed and follows normal Gaussian distribution (*Permission of the Westinghouse Electrical Corporation, East Pittsburgh, Pennsylvania, U.S.A.*)

flashover below the CFO is slight, but with switching surges having much longer fronts the probability is higher, with the curve following the normal Gaussian distribution; the tower clearances must be designed for a CFO much higher than the maximum transient expected.

An example of the application of lightning-arrester characteristics to system requirements is illustrated in Figure 10.19. In this figure, variation of crest voltage for lightning and switching surges with time is shown for an assumed BIL of the equipment to be protected. Over the impulse range (up to 10 μs) the fronts of wave and chopped-wave peaks are indicated. Over the switching surge region the insulation withstand strength is assumed to be 0.83 of the BIL (based on transformer requirements). The arrester maximum sparkover voltage is shown along with the maximum voltage set up in the arrester by the follow-on current (10 or 20 per cent). A protective margin of 15 per cent between the equipment withstand strength and the maximum arrester sparkover is assumed.

10.5 Propagation of Surges

The basic differential equations for voltage and current in a distributed-constant line are as follows:

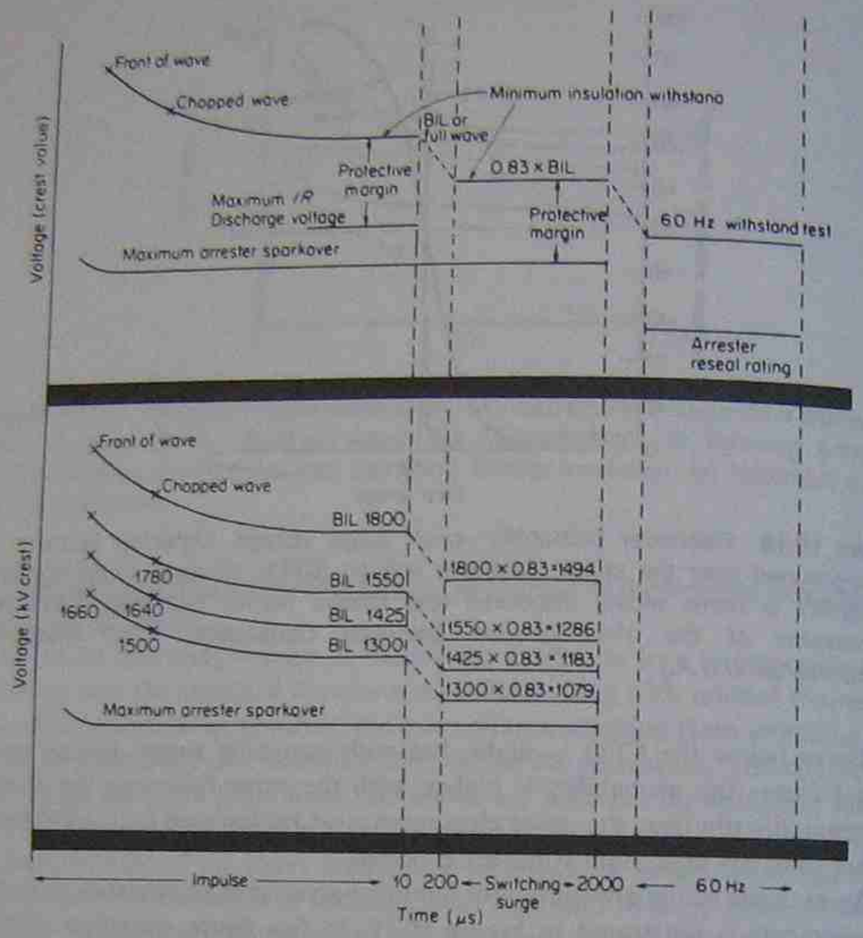


Figure 10.19 Coordination diagrams relating lightning-arrester characteristics to system requirements Top: Explanation of characteristics. Bottom: Typical values for 500k V system (300 kV to ground) (*Permission of the Ohio Brass Co.*)

$$\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2} \qquad \text{and} \qquad \frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\mathrm{d}t^2}$$

and these equations represent travelling waves. The solution for the voltage may be expressed in the form,

$$v = F_1(t - x\sqrt{LC}) + F_2(t + x\sqrt{LC})$$

i.e. one wave travels in the positive direction of x and the other in the negative. Also, it may be shown that because $\partial v/\partial x = -L(\partial i/\partial t)$, the solution for current is

$$i = \sqrt{\frac{C}{L}}[F_1(t - x\sqrt{LC}) - F_2(t + x\sqrt{LC}) \qquad \text{noting that } \sqrt{\frac{C}{L}} = \frac{1}{Z_0}$$

In more physical terms, if a voltage is injected into a line (Figure 10.20) a corresponding current i will flow, and if conditions over a length $\mathrm{d}x$ are considered, the flux set up between the go and return wires is equal to $iL\,\mathrm{d}x$, where L is the inductance per unit length. The induced back e.m.f. is $-\mathrm{d}\Phi/\mathrm{d}t$, i.e. $-Li(\mathrm{d}x/\mathrm{d}t)$ or $-iLU$, where U is the wave velocity. The applied voltage v must equal iLU. Also, charge is stored in the capacitance over $\mathrm{d}x$, i.e. $Q = i\mathrm{d}t = vC\,\mathrm{d}x$ and $i = vCU$. Hence, $vi = viLCU^2$ and $U = 1/\sqrt{LC}$. Also, $i = v\sqrt{C/L} = v/Z_0$, where Z_0 is the characteristic or surge impedance. For single-circuit three-phase overhead lines (conductors not bundled) Z_0 lies in the range 400–600 Ω. For overhead lines, $U = 3 \times 10^8$ m/s, i.e. the speed of light, and for cables

$$U = \frac{3 \times 10^8}{\sqrt{\varepsilon_r \mu_r}}\ \mathrm{m/s}$$

where ε_r is usually from 3 to 3.5, and $\mu_r = 1$.

From the above relations,

$$\tfrac{1}{2}Li^2 = \tfrac{1}{2}(iLU)\left(\frac{i}{U}\right)$$

$$= \tfrac{1}{2}\left(\frac{vi}{U}\right) = \tfrac{1}{2}Cv^2$$

The incident travelling waves of v_i and i_i, when they arrive at a junction or discontinuity, produce a reflected current i_r and a reflected voltage v_r which travel back along the line. The incident and reflected components of voltage and current are governed by the surge impedance Z_0, so that

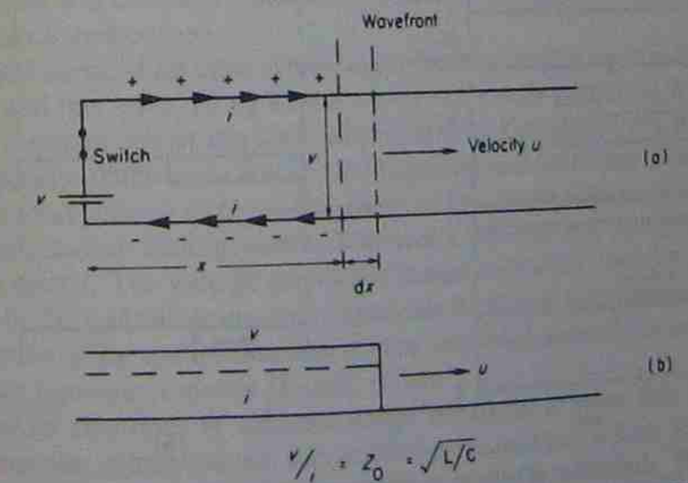


Figure 10.20 Distribution of charge and current as wave progresses along a previously unenergized line. (a) Physical arrangement. (b) Symbolic representation

$$v_i = Z_0 i_i \qquad \text{and} \qquad v_r = -Z_0 i_r$$

In the general case of a line of surge impedance Z_0 terminated in Z (Figure 10.21), the total voltage at Z is $v = v_i + v_r$ and the total current is $i = i_i + i_r$.

Also,

$$(v_i + v_r) = Z(i_r + i_i)$$
$$Z_0(i_i - i_r) = Z(i_r + i_i)$$

and hence

$$i_r = \left(\frac{Z_0 - Z}{Z_0 + Z}\right) i_i \tag{10.10}$$

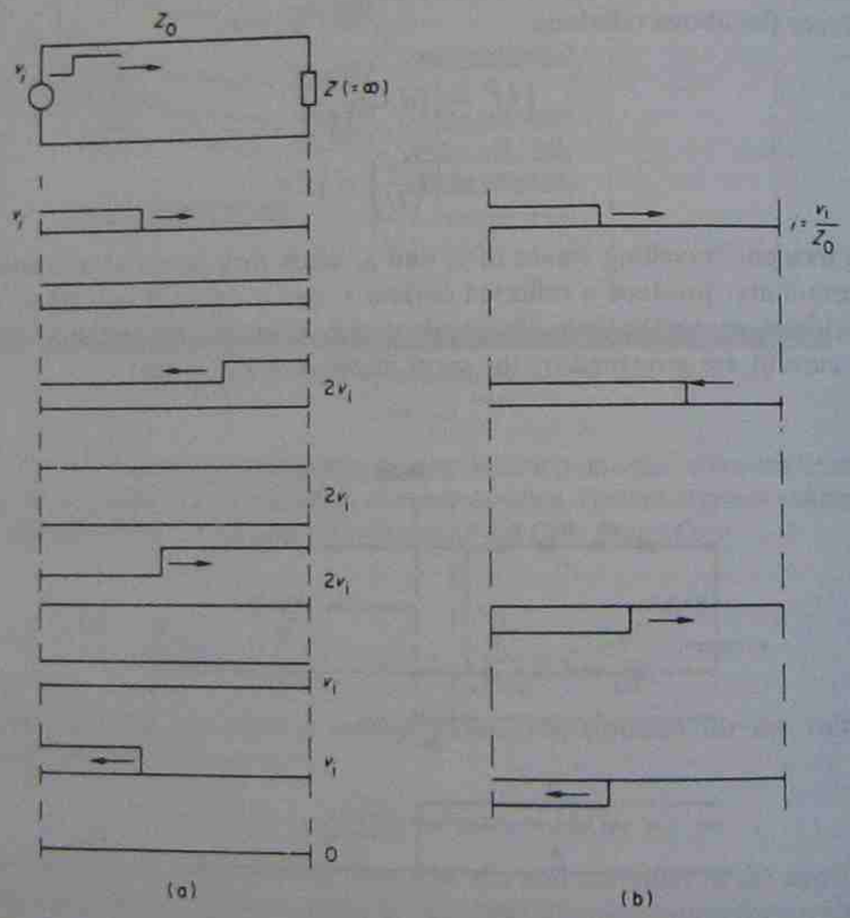


Figure 10.21 Application of voltage to unenergized loss-free line on open circuit at far end. (a) Distribution of voltage. (b) Distribution of current. Voltage source is an effective short circuit

Again,

$$v_i + v_r = Z(i_i + i_r)$$

but, as $v_r = -Z_0 i_r$, we have

$$v_i + v_r = Z\left(\frac{v_i - v_r}{Z_0}\right)$$

or,

$$v_r = \left(\frac{Z - Z_0}{Z + Z_0}\right) v_i = \alpha v_i \tag{10.11}$$

where α is the *reflection coefficient* equal to

$$\left(\frac{Z - Z_0}{Z + Z_0}\right)$$

Hence,

$$v = \left(\frac{2Z}{Z + Z_0}\right) v_i \tag{10.12}$$

and

$$i = \left(\frac{2Z_0}{Z + Z_0}\right) i_i \tag{10.13}$$

From the above, if $Z \to \infty$, $v = 2v_i$ and i = 0. Also, if $Z = Z_0$ (matched line), $\alpha = 0$, i.e. no reflection. If $Z > Z_0$, then v_r is positive and i_r is negative, but if $Z < Z_0$, v_r is negative and i_r is positive. The reflected waves will travel back and forth along the line, setting up, in turn, further reflected waves at the ends, and this process will continue indefinitely unless the waves are attenuated because of resistance and corona.

Summarizing, at an open circuit the reflected voltage is equal to the incident voltage and this wave, along with a wave $(-i_i)$, travels back along the line; note that at the open circuit the total current is zero. Conversely, at a short circuit the reflected voltage wave is $(-v_i)$ in magnitude and the current reflected is (i_i), giving a total voltage at the short circuit of zero and a total current of $2i_i$. For other termination arrangements, Thevenin's theorem may be applied to analyse the circuit. The voltage across the termination when it is open-circuited is seen to be $2v_i$ and the equivalent impedance looking in from the open-circuited termination is Z_0; the termination is then connected across the terminals of the Thevenin equivalent circuit (Figure 10.22).

Consider two lines of different surge impedance in series. It is required to determine the voltage across the junction between them (Figure 10.23):

$$v_{AB} = \left(\frac{2v_i}{Z_0 + Z_1}\right) Z_1 = \beta v_i \tag{10.14}$$



Figure 10.22 Analysis of travelling waves—use of Thevenin equivalent circuit. (a) System. (b) Equivalent circuit



Figure 10.23 Analysis of conditions at the junction of two lines or cables of different surge impedance

The wave entering the line Z_1 is the refracted wave and β is the *refraction coefficient*, i.e. the proportion of the incident voltage proceeding along the second line (Z_1).

$$v_r = v_i\alpha = v_i\left(\frac{Z_1 - Z_0}{Z_1 + Z_0}\right) \quad \text{and} \quad i = \frac{v_{AB}}{Z_1} = \frac{2v_i}{Z_1 + Z_0} = \text{refracted current}$$

When several lines are joined to the line on which the surge originates (Figure 10.24), the treatment is similar, e.g. if there are three lines having equal surge impedances (Z_1), then

$$i_A = \frac{2v_i}{(Z_0 + Z_1/3)} \quad \text{and} \quad v_{AB} = \left(\frac{2v_i}{Z_0 + Z_1/3}\right)\left(\frac{Z_1}{3}\right) \qquad (10.15)$$

An important practical case is that of the clearance of a fault at the junction of two lines and the surges produced. The equivalent circuits are shown in Figure 10.25; the fault clearance is simulated by the insertion of an equal and opposite current (I) at the point of the fault. From the equivalent circuit, the magnitude of the resulting voltage surges (v)

$$= I\left(\frac{Z_1 Z_2}{Z_1 + Z_2}\right)$$

and the currents entering the lines are

$$I\left(\frac{Z_1}{Z_1 + Z_2}\right) \quad \text{and} \quad I\left(\frac{Z_2}{Z_1 + Z_2}\right)$$

The directions are as shown in Figure 10.25(c).

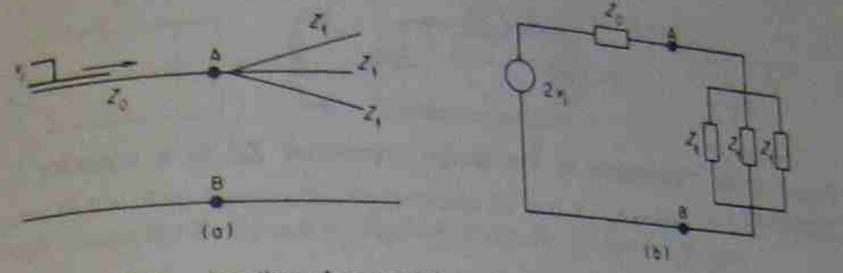


Figure 10.24 Junction of several lines. (a) System. (b) Equivalent circuit

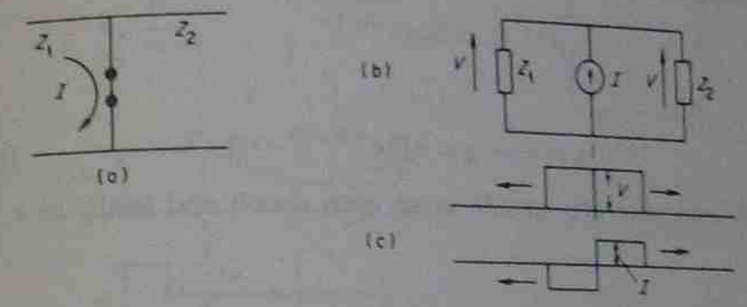


Figure 10.25 Surge set up by fault clearance. (a) Equal and opposite current (I) injected in fault path. (b) Equivalent circuit. (c) Voltage and current waves set up at a point of fault with direction of travel

10.5.1 Termination in inductance and capacitance

Shunt capacitance

Using the Thevenin equivalent circuit as shown in Figure 10.26, the voltage rise across the capacitor C is $v_c = 2v_i(1 - e^{-t/Z_0C})$, where t is the time commencing with the arrival of the wave at C. The current through C is given by,

$$\left(\frac{2v_i}{Z_0}\right)e^{-t/Z_0C}$$

The reflected wave,

$$v_r = v_c - v_i = 2v_i(1 - e^{-t/Z_0C}) - v_i = v_i(1 - 2e^{-t/Z_0C}) \qquad (10.16)$$

As to be expected, the capacitor acts initially as a short circuit and finally as an open circuit.

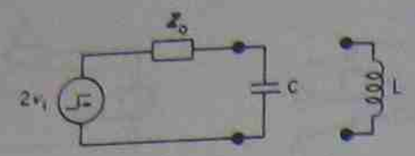


Figure 10.26 Termination of line (surge impedance Z_0) in a capacitor C or inductance L

Shunt inductance

Again, from the equivalent circuit the voltage across the inductance is

$$v_L = 2v_i e^{-(Z_0/L)t}$$

and

$$v_r = v_L - v_i = v_i[2e^{-(Z_0/L)t} - 1] \qquad (10.17)$$

Here, the inductance acts initially as an open circuit and finally as a short circuit.

Capacitance and resistance in parallel (Figure 10.27(a))

Open-circuit voltage across AB (Figure 10.27(b))

$$= \left(\frac{2v_i}{Z_0 + R}\right)R$$

Equivalent Thevenin resistance

$$= \frac{RZ_0}{R + Z_0}$$

Voltage across R and C is

$$v = \frac{2v_i}{Z_0 + R}(1 - e^{-[t(R+Z_0)]/RZ_0C}) \qquad (10.18)$$

This is the solution to the practical system shown in Figure 10.27(c), where C is used to modify the surge. The reflected wave is given by $(v - v_i)$.

Example 10.2

An overhead line of surge impedance 500 Ω is connected to a cable of surge impedance 50 Ω through a series resistor (Figure 10.28(a)). Determine the magnitude of the resistor

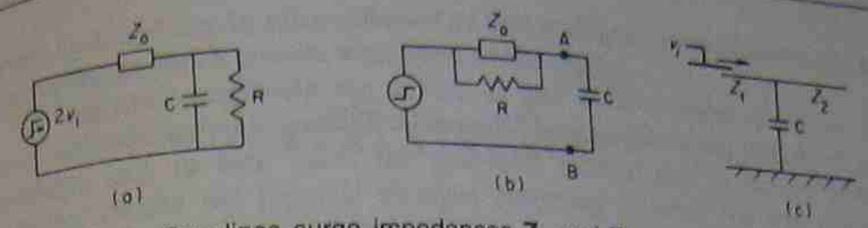


Figure 10.27 Two lines surge impedances Z_1 and Z_2 grounded at their junction through a capacitor C. (a) and (b) Equivalent circuits. (c) System diagram

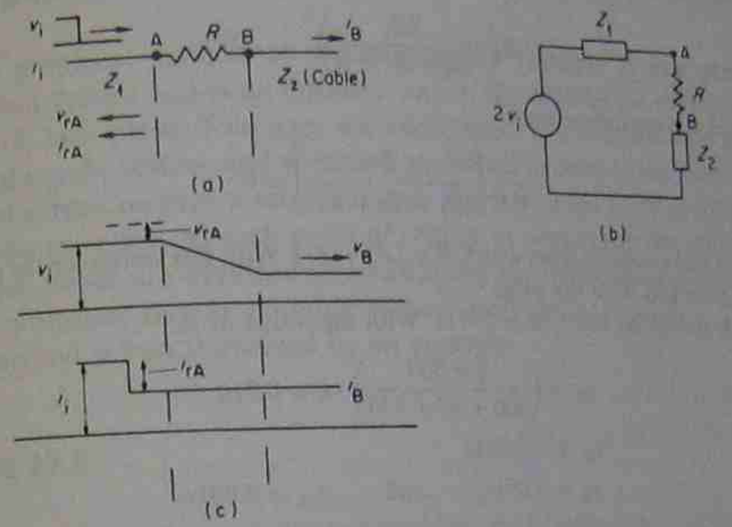


Figure 10.28 (a) System for Example 10.2. (b) Equivalent circuit. (c) Voltage and current surges

such that it absorbs maximum energy from a surge originating on the overhead line and travelling into the cable. Calculate:

- the voltage and current transients reflected back into the line, and
- those transmitted into the cable, in terms of the incident surge voltage;
- the energies reflected back into the line and absorbed by the resistor.

Let the incident voltage and current be v_i and i_i, respectively. From the equivalent circuit (Figure 10.28(b)),

$$v_B = \frac{2v_i \cdot Z_2}{Z_1 + Z_2 + R}$$

and the reflected voltage at A

$$v_{rA} = \frac{2v_i(Z_2 + R)}{Z_1 + Z_2 + R} - v_i = \frac{Z_2 + R - Z_1}{Z_1 + Z_2 + R} \cdot v_i$$

As

$$v_i = Z_1 i_i$$
$$i_B = \frac{2v_i}{Z_1 + Z_2 + R} = \frac{2Z_1 i_i}{Z_1 + Z_2 + R}$$

and

$$i_{rA} = i_i \frac{(Z_1 - Z_2 - R)}{Z_1 + Z_2 + R}$$

Power absorbed by the resistance

$$= R\left(i_i \frac{2Z_1}{Z_1 + Z_2 + R}\right)^2 \quad \text{(W)}$$

This power is a maximum when

$$\frac{\mathrm{d}}{\mathrm{d}R}\left[\frac{R}{(Z_1 + Z_2 + R)^2}\right] = 0$$

if i_i is given and constant; from which $R = (Z_1 + Z_2)$. With this resistance the maximum energy is absorbed from the surge.

Hence R should be $500 + 50 = 550\,\Omega$. With this value of R,

$$i_B = \left(\frac{2 \times 500}{500 + 50 + 550}\right) \cdot i_i = 0.91 i_i$$
$$i_{rA} = -0.091 i_i$$
$$v_B = 0.091 v_i \quad \text{and} \quad v_{rA} = 0.091 v_i$$

Also, the surge energy entering Z_2

$$= v_B i_B = 0.082 v_i i_i$$

The energy absorbed by $R = (0.91 i_i)^2 \times 550$

$$= 455\left(\frac{v_i}{Z_1}\right) i_i = 0.91 v_i i_i$$

and the energy reflected $= v_i i_i (1 - 0.082 - 0.91)$

$$= 0.008 v_i i_i$$

The waveforms are shown in Figure 10.28(c).

10.6 Determination of System Voltages Produced by Travelling Surges

In the previous section the basic laws of surge behaviour were discussed. The calculation of the voltages set up at any node or busbar in a system at a given instant in time is, however, much more complex than the previous section would suggest. When any surge reaches a discontinuity its reflected waves travel back and are, in turn, reflected so that each generation of waves sets up further waves which coexist with them in the system.

To describe completely the events at any node involves, therefore, an involved book-keeping exercise. Although many mathematical techniques are available and, in fact, used, the graphical method due to Bewley (1961) indicates clearly the physical changes occurring in time, and this method will be explained in some detail.

10.6.1 Bewley lattice diagram

This is a graphical method of determining the voltages at any point in a transmission system and is an effective way of illustrating the multiple reflections which take place. Two axes are established: a horizontal one scaled in distance along the system, and a vertical one scaled in time. Lines indicating the passage of surges are drawn such that their slopes give the times corresponding to distances travelled. At each point of change in impedance the reflected and transmitted waves are obtained by multiplying the incidence-wave magnitude by the appropriate reflection and refraction coefficients α and β.

The method is best illustrated by an example.

Example 10.3

A loss-free system comprising a long overhead line (Z_1) in series with a cable (Z_2) will be considered. Typically, Z_1 is $500\,\Omega$ and Z_2 is $50\,\Omega$.

Solution

Referring to Figure 10.29, the following coefficients apply:

Line-to-cable reflection coefficient,

$$\alpha_1 = \frac{50 - 500}{50 + 500} = -0.818 \quad \text{(note order of values in numerator)}$$

Line-to-cable refraction coefficient,

$$\beta_1 = \frac{2 \times 50}{50 + 500} = 0.182$$

Cable-to-line reflection coefficient,

$$\alpha_2 = \frac{500 - 50}{500 + 50} = 0.818$$

Cable-to-line refraction coefficient,

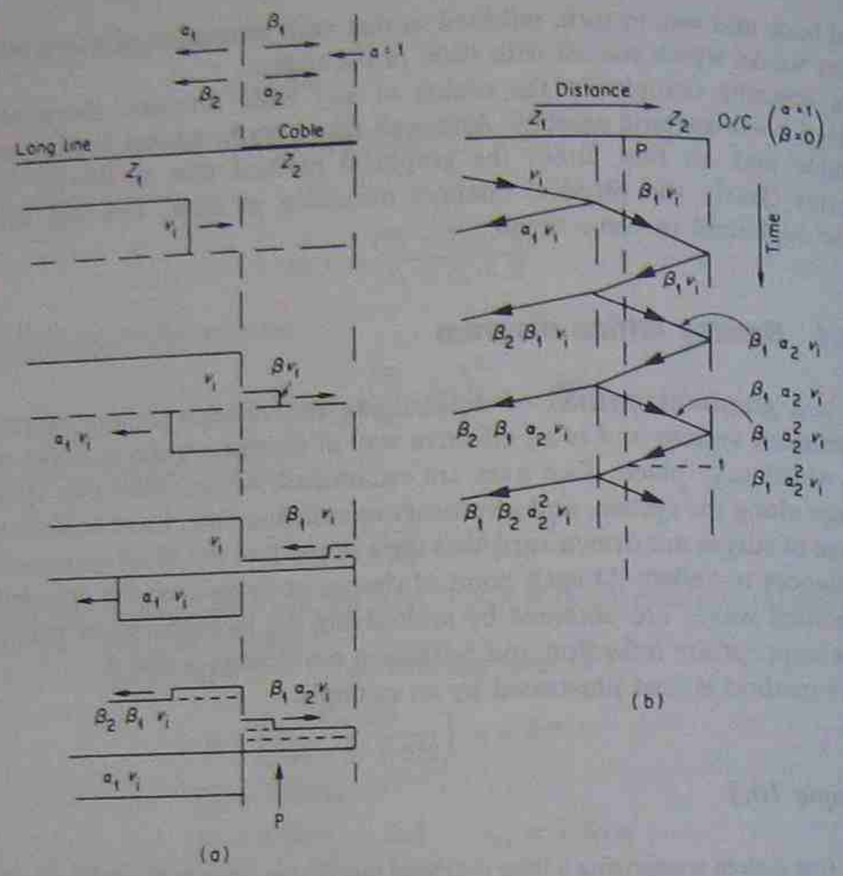


Figure 10.29 Bewley lattice diagram—analysis of long overhead line and cable in series. (a) Position of voltage surges at various instants over first complete cycle of events, i.e. up to second reflected wave travelling back along line. (b) Lattice diagram

$$\beta_2 = \frac{2 \times 500}{500 + 50} = 1.818$$

As the line is long, reflections at its remote (sending) end will be neglected. The remote end of the cable is considered to be open-circuited, giving an α of 1 and a β of zero at that point.

When the incident wave v_i (see Figure 10.29) originating in the line reaches the junction, a reflected component travels back along the line ($\alpha_1 v_i$), and the refracted or transmitted wave ($\beta_1 v_i$) traverses the cable and is reflected from the open-circuited end back to the junction ($1 \times \beta_1 v_i$). This wave then produces a reflected wave back through the cable ($1 \times \beta_1 \alpha_2 v_i$) and a transmitted wave ($1 \times \beta_2 \beta_1 v_i$) through the line. The process continues and the waves multiply as indicated in Figure 10.29(b). The total voltage at a point P in the cable at a given time (t) will be the sum of the voltages at P up to time t, i.e. $v_i\beta_1(2 + 2\alpha_2 + 2\alpha_2^2)$, and the voltage at infinite time will be $2v_i\beta_1$ $(1 + \alpha_2 + \alpha_2^2 + \alpha_2^3 + \alpha_2^4 + \cdots)$.

The voltages at other points are similarly obtained. The time scale may be determined from a knowledge of length and surge velocity; for the line the latter is of the order of 300 m/μs and for the cable 150 m/μs. For a surge 50 μs in duration and a cable 300 m in length there will be 25 cable lengths traversed and the terminal voltage will approach $2v_i$. If the graph of voltage at the cable open-circuited end is plotted against time, an exponential rise curve will be obtained similar to that obtained for a capacitor.

The above treatment applies to a rectangular surge waveform, but may be modified readily to account for a waveform of the type illustrated in Figure 10.1(b) or (c). In this case the voltage change with time must also be allowed for and the process is more complicated.

10.6.2 Short-line faults

A particular problem for circuit breakers on h.v. systems is interrupting current if a solid short circuit occurs a few kilometres from the circuit breaker. This situation is illustrated in Figure 10.30.

It will be recalled that in Figure 10.3, fault interruption gives rise to a restriking transient on the supply side of the breaker. The sudden current interruption sends a step surge down the short line to the fault, which is reflected at the short-circuit point back to the breaker. In a manner similar to the Bewley lattice calculation of Figure 10.29, a surge is propagated back and forth in Z_0 as shown in Figure 10.30(b). The resulting combination of restriking voltage on the supply side and the continuously reflected surge on the line side of the breaker produces a steeply rising initial restriking voltage, greater than the restriking transient of Figure 10.3. Since this occurs just following current zero, it provides the most onerous condition of all for the circuit breaker to deal with, and must be included in the series of proving tests on any new design.

The first peak of the line-side component of the transient recovery voltage may be obtained from a knowledge of the fault current I_f and the value of Z_0. Assuming ideal interruption at current zero, the rate of rise of transient recovery voltage is

$$\frac{dv}{dt} = Z_0 \frac{di}{dt} = \sqrt{2}\omega Z_0 I_f \times 10^{-6}\ \text{V}/\mu\text{s}$$

The amplitude of the first peak of reflected wave is

$$V = t \frac{dv}{dt}\ (\text{V})$$

where t is twice the transit time of the surge through Z_0 and ω is the system angular frequency. To this value should be added any change of v_s on the supply side, calculated as given by equation (10.6).

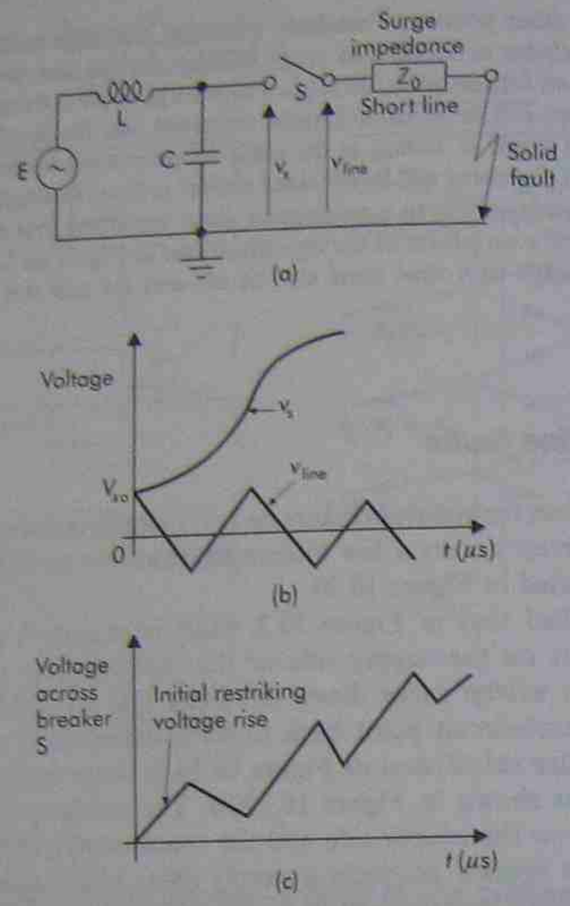


Figure 10.30 Short-line fault. (a) System diagram. (b) Supply-side and line-side transients. (c) Transient voltage across circuit breaker contacts

10.6.3 Effects of line loss

Attenuation of travelling waves is caused mainly by corona which reduces the steepness of the wavefronts considerably as the waves travel along the line. Attenuation is also caused by series resistance and leakage resistance and these quantities are considerably larger than the power-frequency values. The determination of attenuation is usually empirical and use is made of the expression $v_x = v_i e^{-\gamma x}$, where v_x is the magnitude of the surge at a distance x from the point of origination. If a value for γ is assumed, then the wave magnitude of the voltage may be modified to include attenuation for various positions in the lattice diagram. For example, in Figure 10.29(b), if $e^{-\gamma x}$ is equal to a_L for the length of line traversed and a_c for the cable, then the magnitude of the first reflection from the open circuit is $a_L v_i a_c \beta_1$ and the voltages at subsequent times will be similarly modified.

Considering the power and losses over a length dx of a line of resistance and shunt conductance per unit length $R(\Omega)$ and $G(\Omega^{-1})$, the power loss

$$dp = i^2 R\,dx + v^2 G\,dx \quad \text{(W)}$$

also,

$$p = vi = i^2 Z_0 \qquad \text{and} \qquad dp = 2iZ_0\,di.$$

As dp is a loss it is considered negative and

$$-2iZ_0\,di = (i^2 R + v^2 G)\,dx$$

hence,

$$\frac{di}{i} = -\tfrac{1}{2}\left(\frac{R + Z_0^2 G}{Z_0}\right)dx$$

giving

$$i = i_i \exp\left[-\frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)x\right] \tag{10.19}$$

where i_i = surge amplitude (A).

Also, it may be shown that

$$v = v_i \exp\left[-\frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)x\right] \tag{10.20}$$

and the power at x is

$$v_x i_x = vi = v_i i_i e^{-[(R/Z_0)+GZ_0]x} \tag{10.21}$$

If R and G are realistically assessed (including corona effect), attenuation may be included in the travelling-wave analysis.

10.6.4 Digital methods

The lattice diagram becomes very cumbersome for large systems and digital methods are usually applied. Digital methods may use strictly mathematical methods, i.e. the solution of the differential equations or the use of Fourier or Laplace transforms. These methods are capable of high accuracy, but require large amounts of data and long computation times. The general principles of the graphical approach described above may also be used to develop a computer program which is very applicable to large systems. Such a method will now be discussed in more detail and an example considered.

A rectangular wave of infinite duration is used. The theory developed in the previous sections is applicable. The major role of the program is to scan the nodes of the system at each time interval and compute the voltages. In Figure 10.31 a

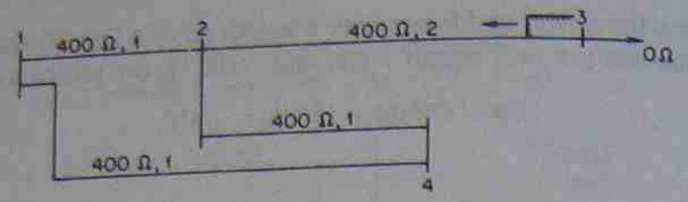


Figure 10.31 Application of digital method. Each line is labelled with surge impedance and surge travel time (multiples of basic unit), e.g. 400 Ω 1

particular system (single-phase representation) is shown and will be used to illustrate the method.

The relevant data describing the system are given in tabular form (see Table 10.4). Branches are listed both ways in ascending order of the first nodal number and are referred to under the name BRANCH. The time taken for a wave to travel along a branch is recorded in terms of a positive integer (referred to as PERIOD) which converts the basic time unit into actual travel time. Reflection coefficients are stored and referred to as REFLECT and the corresponding refraction coefficients are obtained, i.e. $(1 + \alpha_{ij})$. Elements of time as multiples of the basic integer are also shown in Table 10.4.

The method is illustrated by examining the system after the arrival of the rectangular wave at node 3 at TIME (0). This voltage (magnitude 1 p.u.) is entered in the BRANCH (3, 2), TIME (0) element of the BRANCH–TIME matrix. On arrival at node 2 at time equal to zero plus PERIOD (3, 2), two waves are generated, on BRANCH (2, 1) and BRANCH (2, 4), both of magnitude $1(1 + \alpha_{32})$,

Table 10.4 Data for lattice calculations

BRANCH i / j	1 / 2	1 / 4	2 / 1	2 / 3	2 / 4	3 / 2	4 / 1	4 / 2
PERIOD	1	1	1	2	1	2	1	1
REFLECT αij	$-\frac{1}{3}$	0	0	−1	0	$-\frac{1}{3}$	0	$-\frac{1}{3}$
$1 + \alpha ij$	$\frac{2}{3}$	1	1	0	1	$\frac{2}{3}$	1	$\frac{2}{3}$
TIME 0						1		
1								
2			$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$			
3	0	$\frac{2}{3}$						
4								

i.e. 2/3. A reflected wave is also generated on BRANCH (2, 3), of magnitude $1 \times \alpha_{32}$, i.e. −1/3. These voltages are entered in the appropriate BRANCH in the TIME (2) row of Table 10.4. On reaching node 1, TIME (3), a refracted wave of magnitude $\frac{2}{3}(1 + \alpha_{21})$, i.e. 2/3, is generated on BRANCH (1, 4) and a reflected wave $\frac{2}{3} \times \alpha_{21}$, i.e. 0, is generated on BRANCH (1, 2). This process is continued until a specified time is reached. All transmitted waves for a given node are placed in a separate node–time array; a transmitted wave is considered only once even though it could be entered into several BRANCH–TIME elements. Current waves are obtained by dividing the voltage by the surge impedance of the particular branch. The flow diagram for the digital solution is shown in Figure 10.32.

It is necessary for the programs to cater for semi-infinite lines (i.e. lines so long that waves reflected from the remote end may be neglected) and also inductive/capacitive terminations. *Semi-infinite* lines require the use of artificial nodes labelled (say) 0. For example, in Figure 10.33, node 2 is open-circuited and this is accounted for by introducing a line of infinite surge impedance between nodes 2 and 0, and hence, if

$$Z \to \infty \qquad \alpha_{12} = \frac{Z - Z_0}{Z + Z_0} \to 1$$

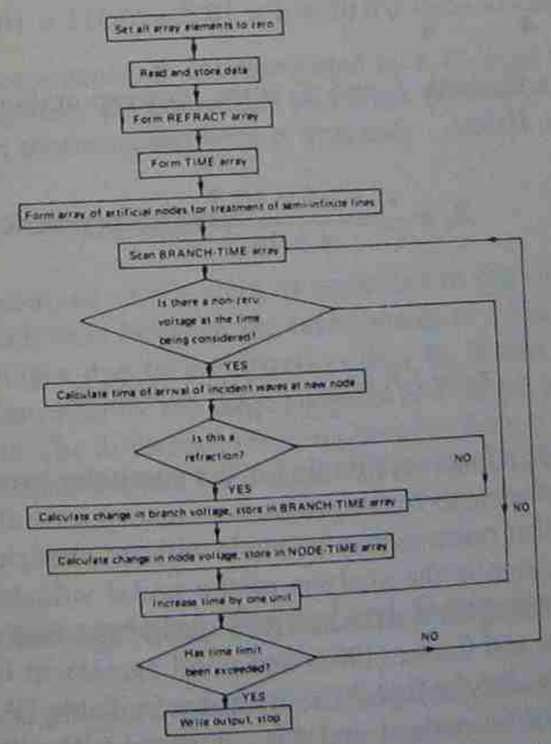


Figure 10.32 Flow diagram of digital method for travelling-wave analysis



Figure 10.33 Treatment of terminations—use of artificial node 0 and line of infinite impedance to represent open circuit at node 2

Although the scanning will not find a refraction through node 2, it is necessary for the computer to consider there being one to calculate the voltage at node 2. Lines with short-circuited nodes may be treated in a similar fashion with an artificial line of zero impedance.

Inductive/capacitive terminations may be simulated by *stub lines*. An inductance L (henrys) is represented by a stub transmission line short-circuited at the far end and of surge impedance $Z_L = L/t$, where t is the travel time of the stub. Similarly, a capacitance C (farads) is represented by a line open-circuited and of surge impedance $Z_c = t/C$. For the representation to be exact, t must be small and it is found necessary for Z_L to be of the order of 10 times and Z_c to be one-tenth of the combined surge impedance of the other lines connected to the node.

For example, consider the termination shown in Figure 10.34(a). The equivalent stub-line circuit is shown in Figure 10.34(b). Stub travel times are chosen to be short compared with a quarter cycle of the natural frequency, i.e.

$$\frac{2\pi\sqrt{LC}}{4} = \frac{2\pi}{4}\sqrt{0.01 \times 4 \times 10^{-8}} = 0.315 \times 10^{-4}\,\text{s}$$

Let $t = 5 \times 10^{-6}$ s for both L and C stubs (corresponding to the total stub length of 1524 m). Hence,

$$Z_c = \frac{t}{C} = \frac{2.5 \times 10^{-6}}{4 \times 10^{-8}} = 67.5\,\Omega$$

and

$$Z_L = L/t = \frac{0.01}{2.5 \times 10^{-6}} = 4000\,\Omega$$

The configuration in a form acceptable for the computer program is shown in Figure 10.34(c). Refinements to the program to incorporate attenuation, waveshapes, and non-linear resistors may be made without changing its basic form.

A typical application is the analysis of the nodal voltages for the system shown in Figure 10.31. This system has previously been analysed by a similar method by Barthold and Carter (1961) and good agreement found. The printout of nodal voltages for the first 20 μs is shown in Table 10.5, and in Figure 10.35 the voltage plot for nodes 1 and 4 is compared with a transient analyser solution obtained by Barthold and Carter.

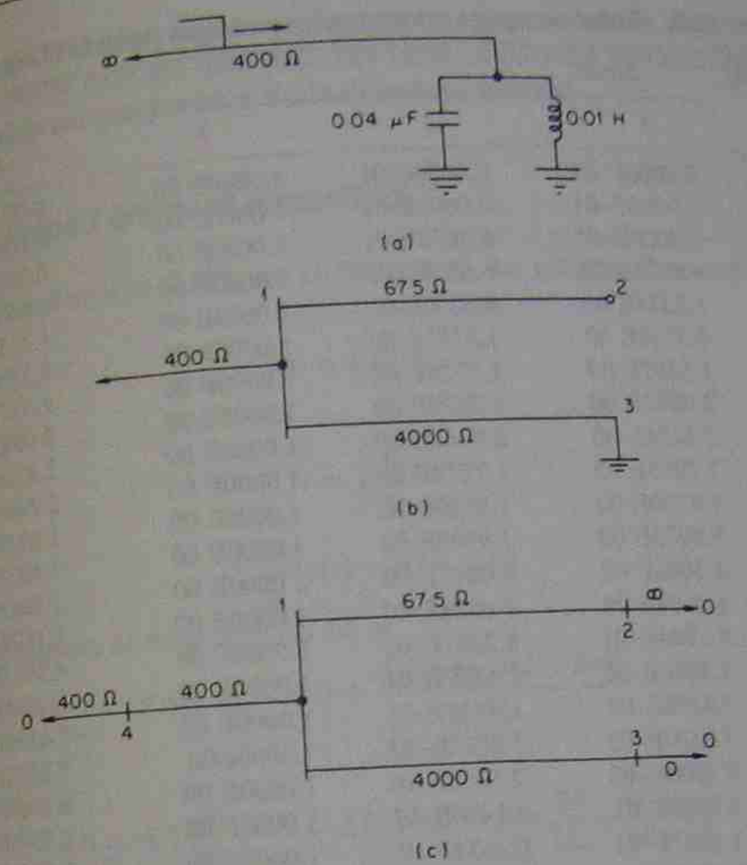


Figure 10.34 Representation of line terminated by L–C circuit by means of stub lines. (a) Original system. (b) Equivalent stub lines. (c) Use of artificial nodes to represent open- and short-circuited ends of stub lines

10.6.5 Three-phase analysis

The single-phase analysis of a system as presented in this chapter neglects the mutual effects which exist between the three phases of a line, transformer, etc. The transient voltages due to energization may be further increased by this mutual coupling and also by the three contacts of a circuit breaker not closing at the same instant. The difference resulting between the use of three-phase and single-phase representation has been discussed by Bickford and Doepel (1967).

10.7 Electromagnetic Transient Program (EMTP)

The digital method previously described is very limited in scope. A much more powerful method has been developed by the Bonneville Power Administration and is known as EMTP. This is widely used, especially in the U.S.A.

Table 10.5 Digital computer printout: node voltages (system of Figure 10.31)

Time (μs)	Node			
	1	2	3	4
0	0.0000E–01	0.0000E-01	1.0000E 00	0.0000E–01
1	0.0000E–01	0.0000E-01	1.0000E 00	0.0000E–01
2	0.0000E–01	6.6670E-01	1.0000E 00	0.0000E–01
3	6.6670E–01	6.6670E-01	1.0000E 00	6.6670E–01
4	1.3334E 00	6.6670E-01	1.0000E 00	1.3334E 00
5	1.3334E 00	1.5557E 00	1.0000E 00	1.3334E 00
6	1.5557E 00	1.7779E 00	1.0000E 00	1.5557E 00
7	2.0002E 00	1.7779E 00	1.0000E 00	2.0002E 00
8	2.2224E 00	2.0743E 00	1.0000E 00	2.2224E 00
9	2.2965E 00	1.7779E 00	1.0000E 00	2.2965E 00
10	1.8520E 00	1.8520E 00	1.0000E 00	1.8520E 00
11	1.4075E 00	1.9508E 00	1.0000E 00	1.4075E 00
12	1.5062E 00	1.0617E 00	1.0000E 00	1.5062E 00
13	1.1604E 00	7.6534E–01	1.0000E 00	1.1604E 00
14	4.1944E–01	8.2297E–01	1.0000E 00	4.1955E–01
15	8.2086E–02	2.6307E–01	1.0000E 00	8.2086E–02
16	–7.4396E–02	1.6435E–01	1.0000E 00	–7.4396E–02
17	7.8720E–03	1.0732E–02	1.0000e 00	7.8720E–03
18	9.3000E–02	–2.5556E–01	1.0000E 00	9.3000E–02
19	–1.7043E–01	4.1410E–01	1.0000E 00	–1.7043E–01
20	1.5067E–01	6.2634E–01	1.0000E 00	1.5067E–01

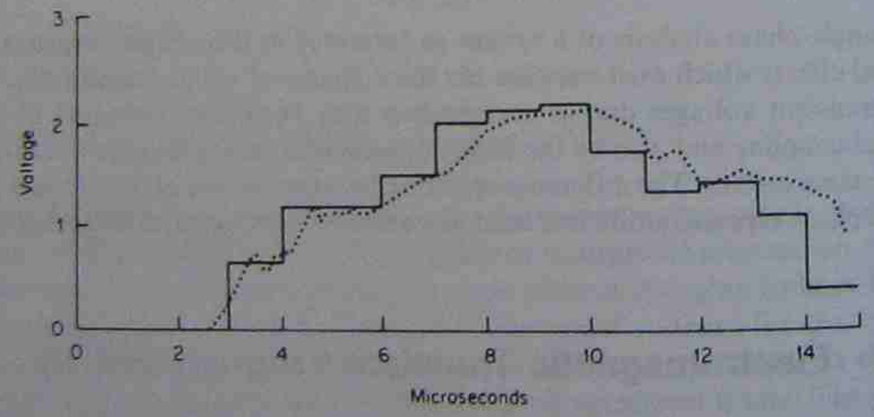


Figure 10.35 Voltage–time relationship at nodes 1 and 4 of system in Figure 10.31. ——, computer solution; . . . ., transient analyser
(*Permission of the Institute of Electrical and Electronic Engineers*)

It is assumed that the variables of interest are known at the previous time step $t - \Delta t$, where Δt is the time step. The value of Δt must be small enough to give reasonable accuracy with a finite difference method.

10.7.1 Lumped element modelling

Consider an inductance L (Figure 10.36(a)) and the voltage–time curve shown in 10.36(b).

$$v = L\,\mathrm{d}i/\mathrm{d}t$$

and

$$i = \int \frac{v}{L}\,\mathrm{d}t$$

$$\tfrac{1}{2}\frac{\Delta t}{L}[v(t) - v(t - \Delta t)]$$

Hence the current at time interval (t) is

$$i(t) = i(t - \Delta t) + \frac{\Delta t[v(t) - v(t - \Delta t)]}{2L}$$

or

$$i(t) = v(t)\frac{\Delta t}{2L} + i(t - \Delta t) + v(t - \Delta t)\frac{\Delta t}{2L} = \frac{v(t)}{R} + I \qquad (10.22)$$

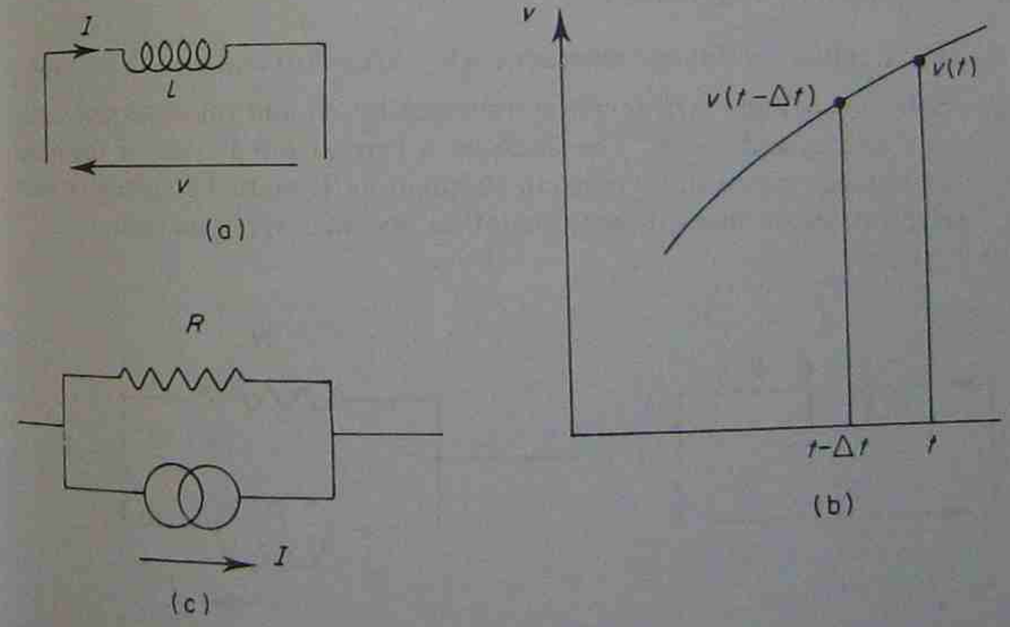


Figure 10.36 (a) Lumped inductance. (b) Voltage–time curve for inductance. (c) Equivalent circuit

where

$$R = \frac{2L}{\Delta t} \quad \text{and} \quad I = i(t - \Delta t) + v(t - \Delta t)/R$$

Here, R is constant and I varies with time. The equivalent circuit is shown in Figure 10.36(c).

A similar treatment applies to capacitance (C) (see Figure 10.37(a)). Here,

$$v = \int \frac{i}{C}\,\mathrm{d}t$$

and, over the interval Δt,

$$i(t) = i(t - \Delta t) + \frac{2C}{\Delta t}[v(t) - v(t - \Delta t)]$$

Again, if

$$R = \Delta t/2C$$

then

$$I = -i(t - \Delta t) - \frac{v(t - \Delta t)}{R} \tag{10.23}$$

giving the equivalent circuit of Figure 10.37(b).

Resistance is represented directly by a piecewise linear curve, as in Figure 10.39(a).

The procedure is as follows:

1. From initial conditions, determine $i(t - \Delta t) = i(0)$ and $v(t - \Delta t) = v(0)$.
2. Solve for $i(t)$ and $v(t)$. Increase time step by Δt and calculate new values of i and v, and so on. The analysis is carried out by use of the nodal admittance matrix and Gaussian elimination. If mutual coupling between elements exists then the representation becomes very complex.

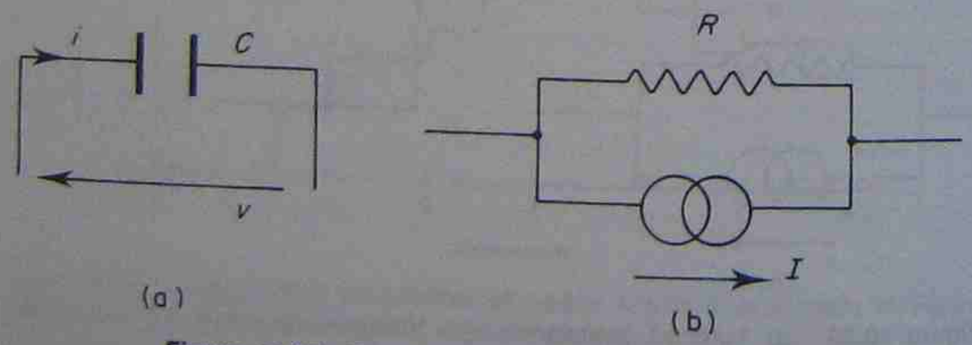


Figure 10.37 Equivalent circuits for transient analysis

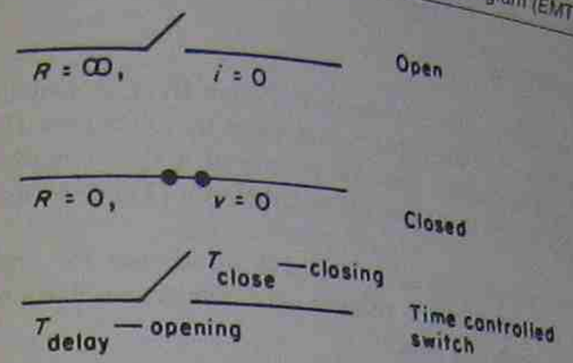


Figure 10.38 Representation of switch

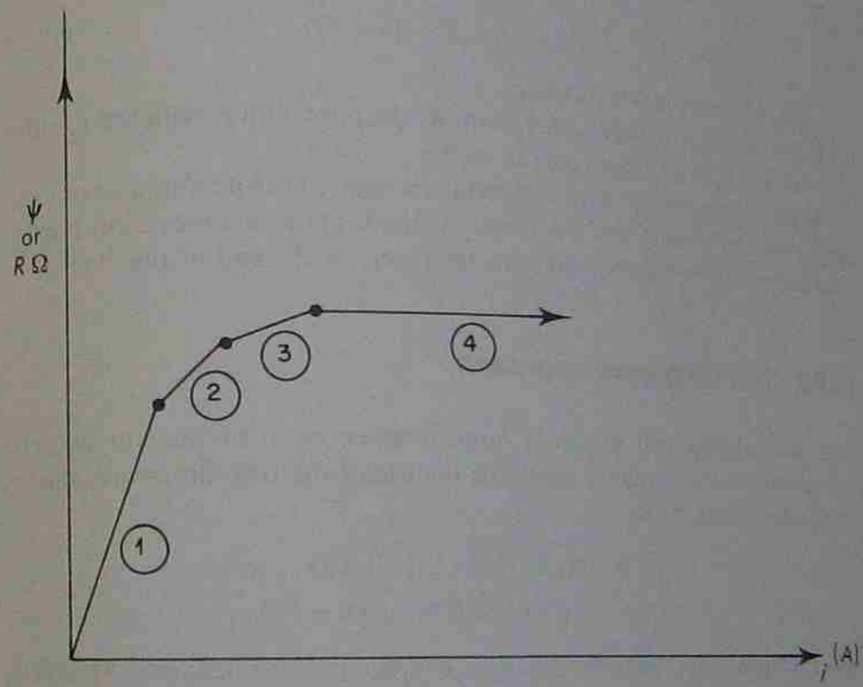


Figure 10.39 (a) Non-linear characteristic; (b) circuit representation

10.7.2 Switching

The various representations are shown in Figure 10.38. A switching operation changes the topology of the network and hence the $[Y]$ matrix. If $[Y]$ is formed with all the switches open, then the closure of a switch is obtained by the adding together of the two rows and columns of $[Y]$ and the associated rows of $[i]$.

Another area where switching is used is to account for non-linear $\psi - i$ characteristics of transformers, reactors, etc. The representation is shown in Figure 10.39(a) and (b), in which

$$R = 2b_k/\Delta t$$

and

$$I = v(t - \Delta t)/R + i(t - \Delta t) \tag{10.24}$$

where b_k = incremental inductance.

If ψ is outside of the limits of segment K, the operation is switched to either $k-1$ or $k+1$. This changes the $[Y]$ matrix.

Because of the random nature of certain events, e.g. switching time or lightning incidence, Monte Carlo (statistical) methods are sometimes used. Further information can be obtained from the references at the end of this book.

10.7.3 Travelling-wave approach

Lines and cables would require a large number of π circuits for accurate representation. An alternative would be the use of the travelling-wave theory.

Consider Figure 10.40,

$$i(x, t) = f_1(x - Ut) + f_2(x + Ut)$$
$$v(x, t) = z_0 f_1(x - Ut) + z_0 f_2(x + Ut)$$

where s = speed of propagation and z_0 = characteristic impedance. At node k,

$$i_k(t) = \frac{v_k(t)}{z_0} + I_k \tag{10.25}$$

where

$$I_k = -i_m(t - \tau) - v_m(t - \tau)/z_0$$

and

$$\tau = \frac{d}{U} = \text{travel time}$$

The equivalent circuit is shown in Figure 10.41. An advantage of this method is that the two ends of the line are decoupled. The value of I_k depends

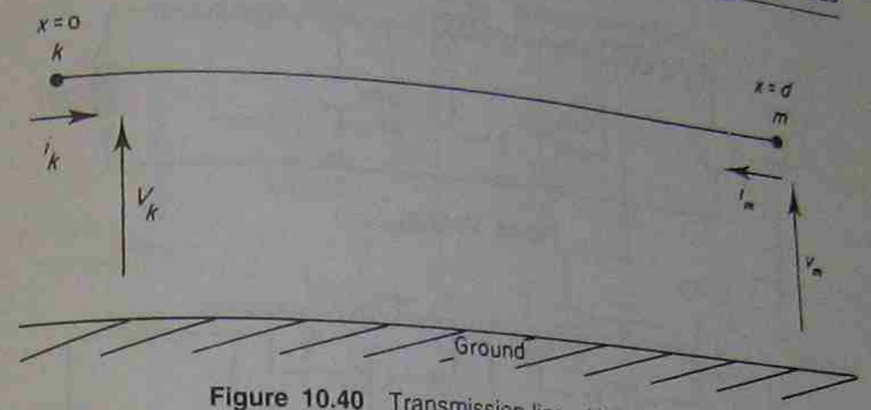


Figure 10.40 Transmission line with earth return

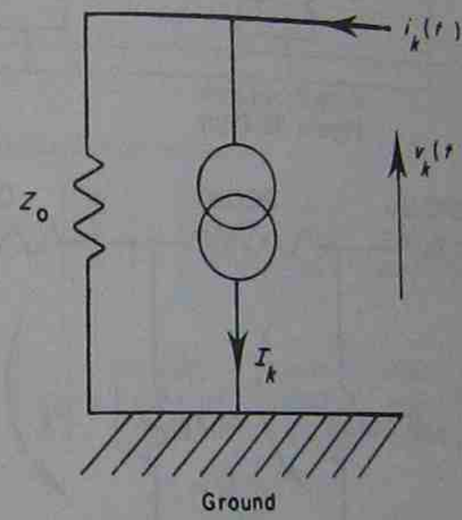


Figure 10.41 Equivalent circuit for single-phase line

on the current and voltage at the other end of the line τ seconds previously, e.g. if $\tau = 0.36$ ms and $\Delta t = 100\,\mu$s, storage of four previous times is required.

Detailed models for synchronous machines and h.v.d.c. converter systems are given in the references.

Example 10.4

The equivalent circuit of a network is shown in Figure 10.42. Determine the network which simulates this network for transients using the EMTP method after the first time step of the transient of 5 μs.

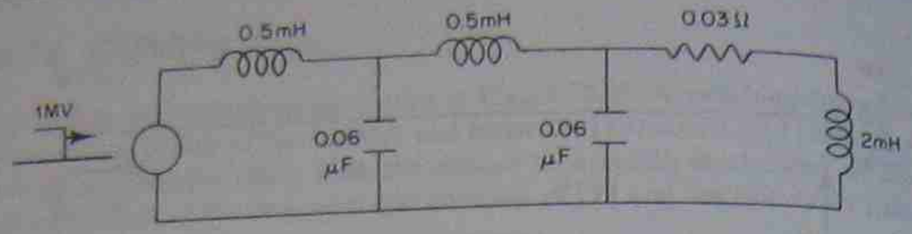


Figure 10.42(a)

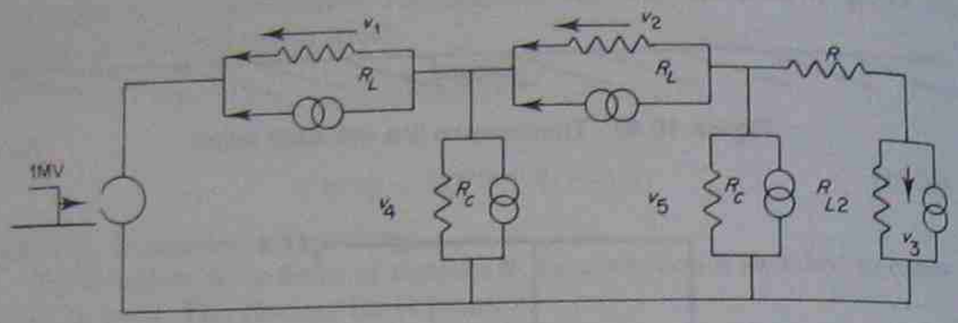


Figure 10.42(b)

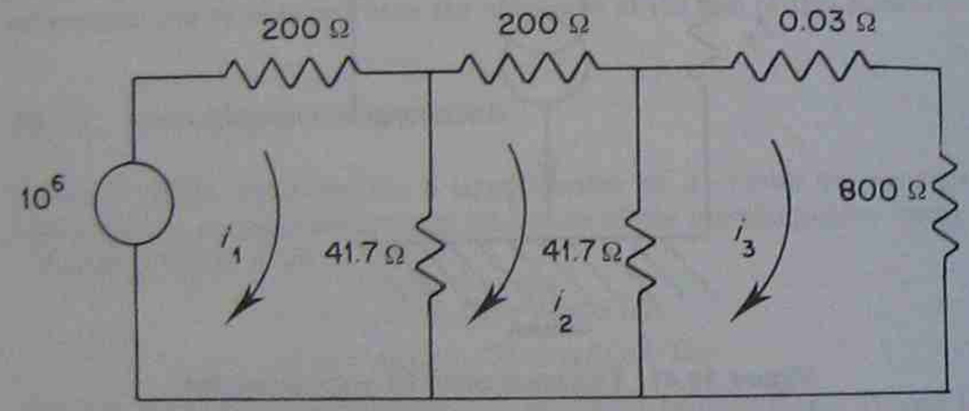


Figure 10.42(c)

Solution

$$R_L = \frac{2 \times 0.0005}{5 \times 10^{-6}} = 200\,\Omega$$

$$R_c = \frac{5 \times 10^{-6}}{2 \times 0.06 \times 10^{-6}} = 41.7\,\Omega$$

$$R_{L2} = \frac{2 \times 0.002}{5 \times 10^{-6}} = 800\,\Omega$$

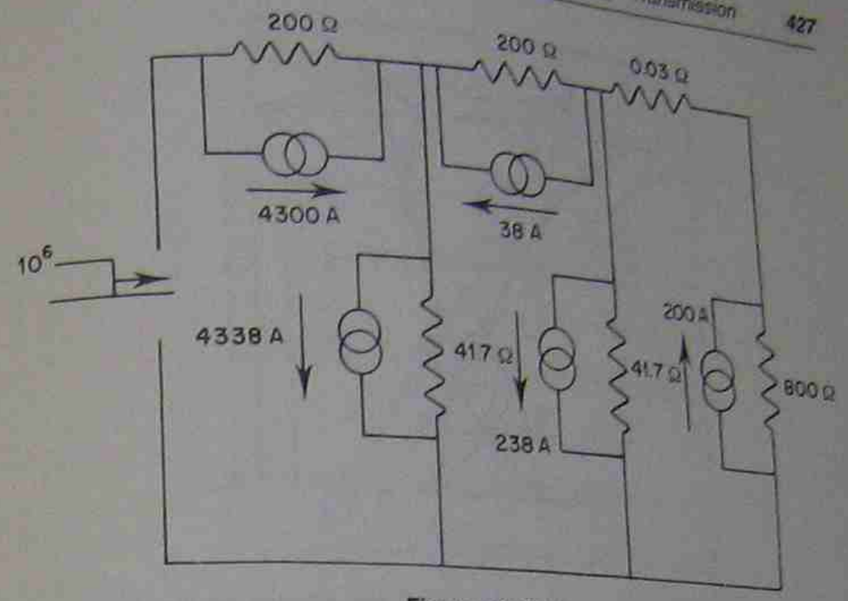


Figure 10.42(d)

At $t = 0$, note current sources are zero.

$$\begin{bmatrix} 10^6 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 241.7 & -41.7 & 0 \\ -41.7 & 241.7 & -41.7 \\ 0 & -41.7 & 800 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Invert

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.0043 & 0.0043 & 0.0007 \\ 3.8 \times 10^{-5} & 0.0007 & 0.0043 \\ 0.0002 & 3.8 \times 10^{-5} & 0.0002 \end{bmatrix} \begin{bmatrix} 10^6 \\ 0 \\ 0 \end{bmatrix}$$

$$i_1 = 4300\,\text{A}$$

$$i_2 = -38, \qquad i_3 = -200$$

The equivalent circuit after 5 μs is shown in Figure 10.42(d).

The process is then repeated for the next 5 μs step using Figure 10.42(d) as the starting condition.

10.8 Ultra-High-Voltage Transmission

In this section, arrangements for extra-high voltage (E.H.V.) lines will be reviewed and the factors influencing designs for U.H.V. discussed. At the present time, for E.H.V. lines, 'V'-type insulator strings are used to support bundle conductors in horizontal formation on steel lattice towers with two earth (ground) wires; a typical structure is shown in Figure 10.43. In this figure, dimensions of interest are labelled and some critical distances are shown. Span

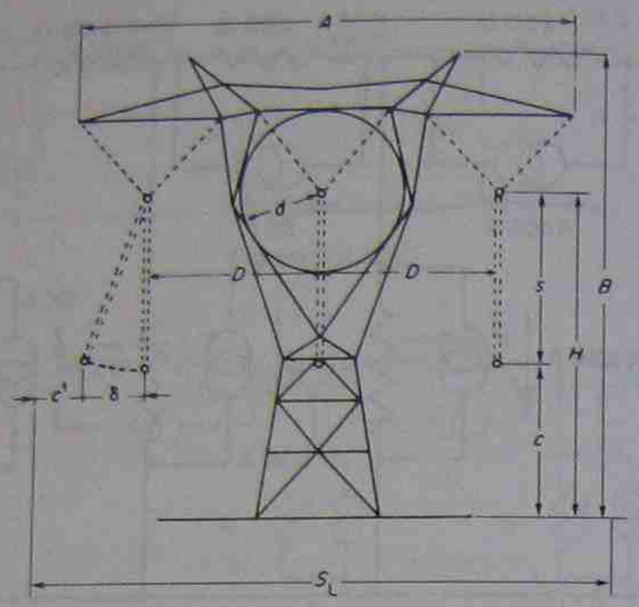


Figure 10.43 U.H.V. tower—critical dimensions (*Permission of the Institute of Electrical and Electronics Engineers*)

lengths (i.e. distances between towers) are of the order of 400–500 m at present and are expected to continue in this range. The number of subconductors in each bundle is determined by radio-interference and audible noise levels stipulated along the route.

Tower dimensions are determined by insulation requirements and the importance of the switching surge in this connection has already been discussed. The tower surge withstand voltage is set below the critical flashover (i.e. 50 per cent probability) voltage to give a reasonable value of flashover probability. When designing a tower, the insulation string length, insulator surface creep distance, tower grounding, and strike distances must be decided upon. Tower strike distance is the distance from the conductor to the tower structure. These dimensions are chosen to make the insulation strength such that the applied surge results in an acceptable surge flashover rate.

An example of laboratory assessment of the strength of tower insulation (V-string insulators) is shown in Figure 10.44. It has been shown (Figure 10.18) that for a given insulation the flashover voltage follows a Gaussian cumulative distribution curve to at least four standard deviations (σ_f) below the CFO; σ_f is about 4.6–5 per cent of the CRO. The procedure is to equate the switching-surge voltage to the withstand strength of the insulation where withstand is defined as that voltage which results in a 0.13 per cent flashover probability, i.e. $3\sigma_f$ below the CFO. The CFO is determined from a knowledge of σ_f and the withstand voltage, and from it the strike distances are obtained. This process is illustrated in Figure 10.45 for CFOs under wet and dry conditions.

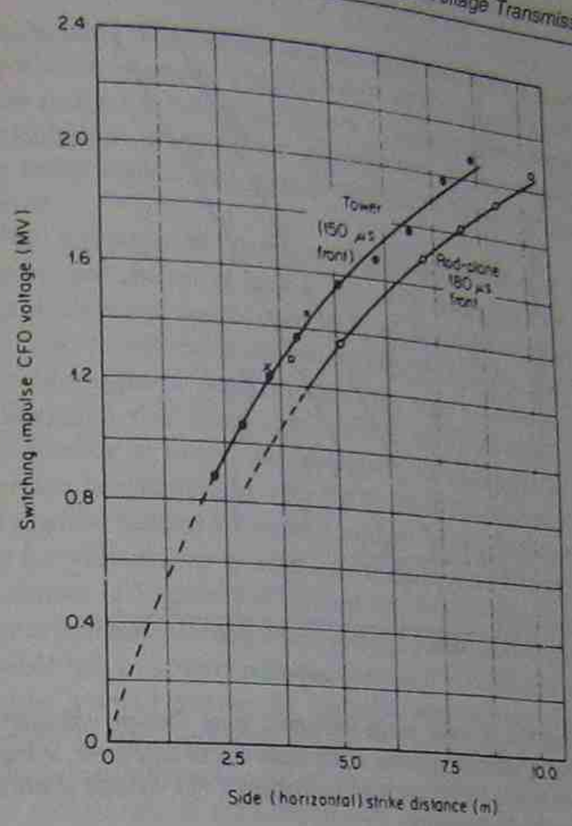


Figure 10.44 Maximum obtainable switching impulse CFO voltage for a specific side strike distance (*Permission of the Westinghouse Electric Corporation*)

Contamination requirements are expressed in terms of the shortest distance along the insulator disc from cap to pin (creep distance); the creep distance required per kilovolt for flashover-free operation with normal practice is, at present, between 0.83 in (2.1 cm) and 1 in (2.54 cm) per kilovolt. The various factors involving flashover and insulation requirements are summarized in Figure 10.46, in which system operating voltage is related to the minimum distance for flashover between the phase conductor and tower sides (strike distance) for a 'V' formation of string insulators. The switching-surge curves refer to various per-unit peak values of switching surge, control over which may be exercised by circuit-breaker resistance switching. It is seen that increases in system voltages require progressively larger increases in the strike distance (conductor-to-tower minimum distance), and the tower dimensions would become intolerable from both economic and appearance standpoints unless the per-unit value of the surge is reduced.

The method illustrated in this section has been questioned for the U.H.V. region. It does not produce an estimate of the switching-surge flashover rate and only matches the withstand voltage to the maximum switching surge. In

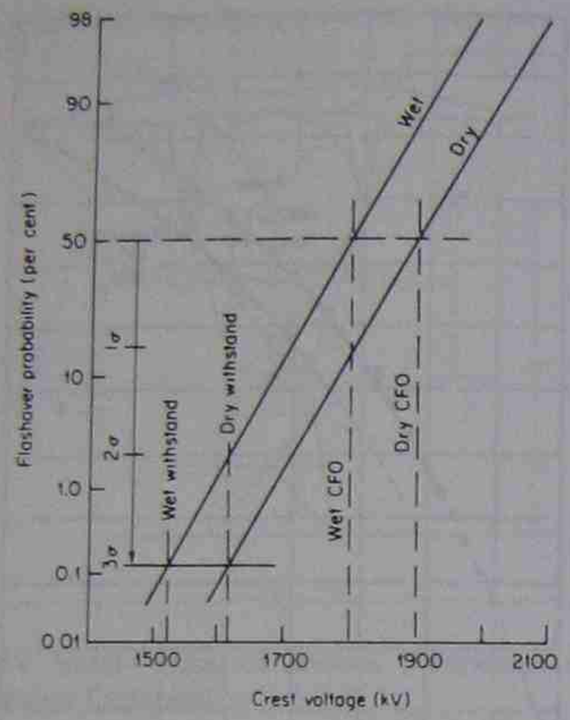


Figure 10.45 Selection of tower surge withstand level. Design withstand level for a tower is established by specifying some acceptable risk of flashover, in this case three standard deviations below the 50 per cent probability CFO voltage (*Permission of the Westinghouse Electric Corporation*)

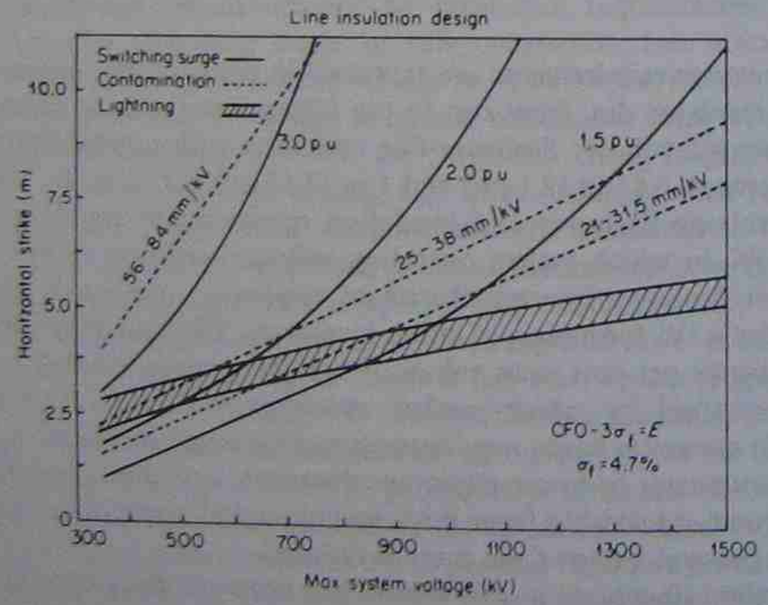


Figure 10.46 Estimates of E.H.V.–U.H.V. tower insulation requirements (*Permission of the Westinghouse Electric Corporation*)

fact, two probability distributions are involved; one for surge magnitude, the other for insulation strength. The exact form of the surge magnitude probability curves for systems is not as yet known to an acceptable degree of accuracy for the calculation of flashover probability. This complete probability approach is being studied and revised versions of the curves shown in Figure 10.46 obtained.

Paris (1969) has suggested various criteria for the design of U.H.V. towers, e.g. dimensions are chosen in such a way as to make the cost of the inactive components a constant ratio (about 0.8) of the active components (i.e. the conductors). The tower size is defined by two parameters: the 'line size' giving the width of the 'right of way' (width of the strip of ground required underneath the line) and the 'tower size' defined as the product of tower width, height, and number of towers per kilometre. As the height of the conductors will not increase proportionally with the voltage, the magnitude of the electric field at the ground surface becomes a critical factor. Paris's analysis suggests that towers for voltages up to 1500 kV are feasible and some of his suggested towers are shown in Figure 10.47, based on line characteristics given in Table 10.6 and the criteria mentioned above. The guyed structures already in use lead to the possible use of towers able to withstand vertical and transverse forces only, with just a small number of special towers able to withstand the longitudinal forces as well, say one special tower to five light two-dimensional towers. Other possibilities for reducing size at high voltages include the use of insulated cross-arms of the form shown in Figure 10.48.

10.9 Design of Insulation by Digital Computer

The design of the complete system involves many parameters and variables and the design-analysis procedure is complex. Some factors, e.g. loss, radio interference (RI), and the swinging of insulator strings and conductors, depend on the weather conditions. Hence, some form of statistical analysis dependent on the geographical position of the line is important to obtain an economical design. Although the presentation here is of an introductory nature, flow diagrams of typical programs will be presented mainly to indicate the general manner in which design studies are developing at the present time. In particular, such programs endeavour to give a realistic account of weather conditions and perform computations to cover a long time-period, i.e. 10–20 years, to obtain reasonable statistical distributions. As well as weather conditions, it is also necessary statistically to account for switching-surge magnitudes and flashover occurrences. A general critical-path diagram covering the steps in the design of an E.H.V. line is shown in Figure 10.49.

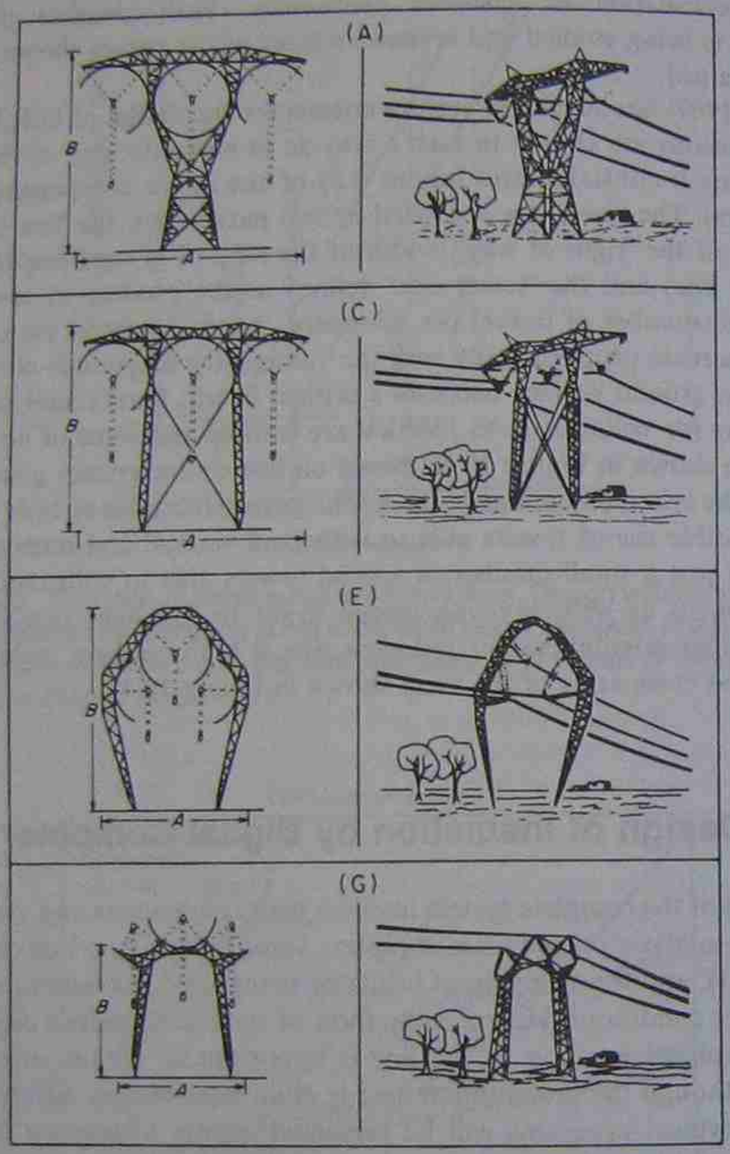


Figure 10.47 Proposed types of tower for 1500 kV lines. Proposed new tower types, C–H. Traditional tower types, A and B (*Permission of the Institute of Electrical and Electronics Engineers*)

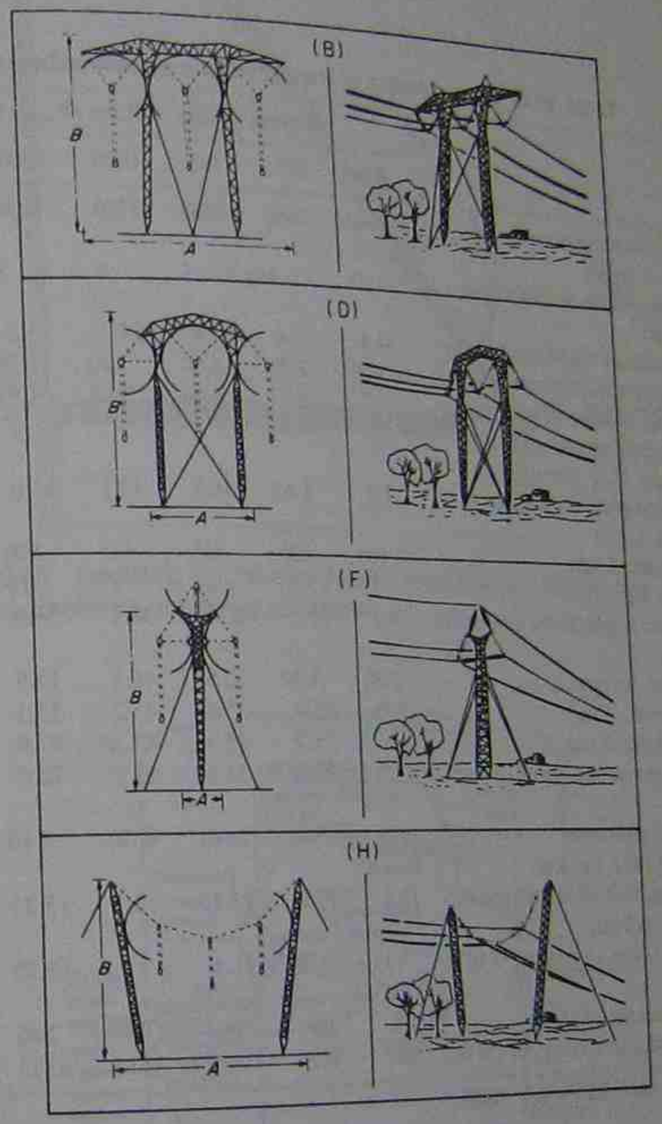


Figure 10.47 (*continued*)

Table 10.6 Characteristics of lines at various system voltages

	Highest system voltage (V_m)(kV)					
	420	525	765	1000	1300	1500
Overall aluminium section per phase, S (mm^2)	1240	1660	2680	3780	5250	6300
Number of subconductors per phase, n	2	3	4	6	8	8
Subconductor diameter, ϕ (mm)	34.5	32.4	35.8	34.7	35.5	38.8
Conductor-tower clearance, d (m)	3.00	3.90	5.60	7.20	8.50	9.40
Switching impulse 50 per cent discharge voltage of tower insulation, $V_{50\%}$ (per unit)	3.2	2.95	2.60	2.25	1.95	1.80
Conductor-ground clearance at midspan, C (m)	7.2	8.45	10.8	13.1	15.0	16.2
Span length, L (m)	400	420	445	475	500	515
Midspan sag, s (m)	12	13.5	15	17	19	20
Conductor height at the tower, H (m)	19.2	21.7	25.8	30.1	34.0	36.2
Interphase distance, D (m)	7.30	9.20	12.8	16.1	19.0	20.8
Tower width, A (m)	20.0	25.4	35.6	45.2	53.3	58.4
Tower height, B (m)	24.6	28.2	35.5	42.25	47.9	51.5
Line-size parameter (right of way), S_L (m)	35.5	42.3	52.0	62.5	72.0	76.5
Tower-size parameter, $S_T = 1000AB/L$ (m^2/km)	1230	1700	2840	4020	5110	5840
RI limit gradient of lateral phase conductor, (kV/m)	15.8	15.7	15.35	15.5	15.25	14.85
Voltage gradient at ground, G (kV/m)	7.35	9.50	11.4	13.1	16.55	17.55
Surge impedance, Z_s(Ω)	284	268	264	249	240	245
Surge impedance loading, P_s (MW)	560	925	1970	3615	6335	8265

(Permission of I.E.E.E.—Paris, 1969)

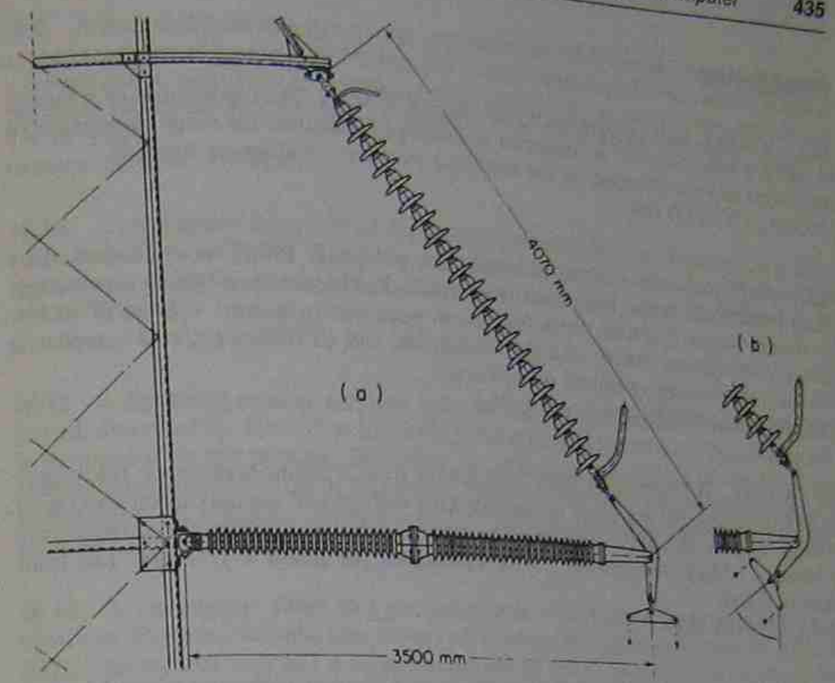


Figure 10.48 Insulating cross-arm for a double-circuit 420 kV line (Italian). (a) Normal conditions. (b) Windy conditions

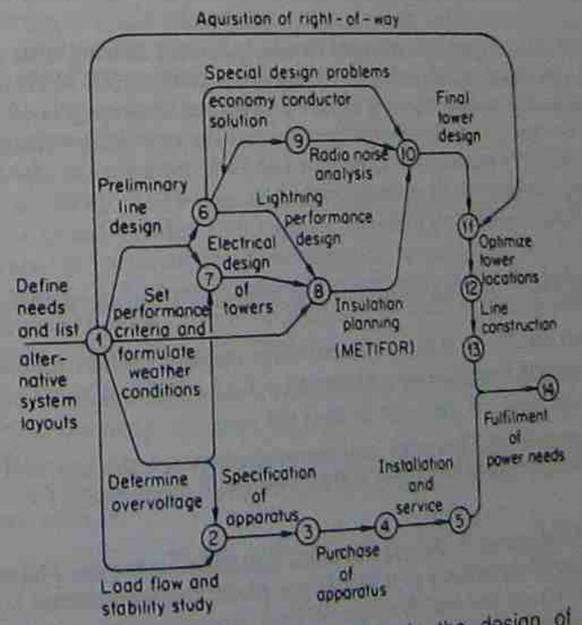


Figure 10.49 Critical-path diagram for steps in the design of an E.H.V. line (*Permission of the Edison Electric Institute*)

Problems

10.1 A 345 kV, 60 Hz system has a fault current of 40 kA. The capacitance of a busbar to which a circuit breaker is connected is 25 000 pF. Calculate the surge impedance of the busbar and the frequency of the restriking (recovery) voltage on opening.
(Answer: 674 Ω, 875 Hz)

10.2 A highly capacitive circuit of capacitance per phase 100 μF is disconnected by circuit breaker, the source inductance being 1 mH. The breaker gap breaks down when the voltage across it reaches twice the system peak line-to-neutral voltage of 38 kV. Calculate the current flowing with the breakdown, and its frequency, and compare it with the normal charging current of the circuit.
(Answer: 34 kA, 503 Hz; note $\hat{I} = 2V_p/Z_0$)

10.3 A 10 kV, 64.5 mm^2 cable has a fault 9.6 km from a circuit breaker on the supply side of it. Calculate the frequency of the restriking voltage and the maximum voltage of the surge after 2 cycles of the transient. The cable parameters are (per km), capacitance per phase = 1.14 μF, resistance = 5.37 Ω, inductance per phase = 1.72 mH. The fault resistance is 6 Ω.
(Answer: 374 Hz; 16 kV)

10.4 The effective inductance and capacitance of a faulted system as viewed by the contacts of a circuit breaker are 2 mH and 500 μF, respectively. The circuit breaker chops the fault current when it has an instantaneous value of 100 A. Calculate the restriking voltage set up across the circuit breaker. Neglect resistance.
(Answer: 200 kV)

10.5 A 132 kV circuit breaker interrupts the fault current flowing into a symmetrical three-phase-to-earth fault at current zero. The fault infeed is 2500 MVA and the shunt capacitance, C, on the source side is 0.03 μF. The system frequency is 50 Hz. Calculate the maximum voltage across the circuit breaker and the restriking-voltage frequency.
If the fault current is prematurately chopped at 50 A, estimate the maximum voltage across the circuit breaker on the first current chop.
(Answer: 215.5 kV; 6.17 kHz; 45 kV)

10.6 Repeat Example 10.2 but with the surge travelling from the cable into the overhead line.
(Answer: Current into line = 0.091× incident surge current; current reflected back into cable = 0.91× incident current, reflected energy = 0.83 × incident surge energy)

10.7 Repeat Example 10.2 but with zero resistance between the line and cable.
(Answer: Energy reflected back to line = 0.67× incident surge energy)

10.8 A cable of inductance 0.188 mH per phase and capacitance per phase of 0.4 μF is connected to a line of inductance of 0.94 mH per phase and capacitance 0.0075 μF per phase. All quantities are per km. A surge of 1 p.u. magnitude travels along the cable towards the line. Determine the voltage set up at the junction of the line and cable.
(Answer: 1.85 p.u.)

10.9 A long overhead line has a surge impedance of 500 Ω and an effective resistance at the frequency of the surge of 7 Ω/km. If a surge of magnitude 500 kV enters the line at a certain point, calculate the magnitude of this surge after it has traversed 100 km and calculate the resistive power loss of the wave over this distance. The wave velocity is 3×10^5 km/s.
(Answer: 250 kV; 375 MW)

10.10 A rectangular surge of 2 μs duration and magnitude 2 p.u. travels along a line of surge impedance of 350 Ω. The latter is connected to another line of equal impedance through an inductor of 800 μH. Calculate the value of the surge transmitted to the second line.
(Answer: $v = v_i[1 - e^{-(2Z_0/L)t}]$ (i.e. 1.67 p.u.))

10.11 A lightning arrester employs a thyrite material possessing a resistance characteristic described by $R = (72 \times 10^3)/(I^{0.75})$. An overhead line of surge impedance 500 Ω is terminated by the arrester. Determine the voltage across the end of the line when a rectangular travelling surge of magnitude 500 kV travels along the line and arrives at the termination. (A graphical method using the voltage–current characteristics is useful.)
(Answer: 375 kV)

10.12 A rectangular surge of 1 p.u. magnitude strikes an earth (ground) wire at the centre of its span between two towers of effective resistance to ground of 200 Ω and 50 Ω. The ground wire has a surge impedance of 500 Ω. Determine the voltages transmitted beyond the towers to the earth wires outside the span.
(Answer: 0.44v_i from 200 Ω tower and 0.18v_i from 50 Ω tower)

10.13 A system consists of the following elements in series: a long line of surge impedance 500 Ω, a cable (Z_0 of 50 Ω), a short line (Z_0 of 500 Ω), a cable (Z_0 of 50 Ω), a long line (Z_0 of 500 Ω). A surge takes 1 μs to traverse each cable (they are of equal length) and 0.5 μs to traverse the short line connecting the cables. The short line is half the length of each cable. Determine, by means of a lattice diagram, the p.u. voltage of the junction of the cable and the long line if the surge orginates in the remote long line.
(Answer: see Figure 10.50)

10.14 A 3 p.h., 50 Hz, 11 kV star-connected generator, with its star point earthed, is connected via a circuit breaker to a busbar. There is no load connected to the busbar. The capacitance to earth on the generator side terminals of the circuit breaker is 0.007 μF per phase. A 3 phase-to-earth short circuit occurs at the busbar with a symmetrical subtransient fault current of 5000 A. The fault is then cleared by the circuit breaker. Assume interruption at current zero.

(a) Sketch the voltage across the circuit breaker terminals of the first phase to clear.
(b) Neglecting damping, calculate the peak value of the transient recovery voltage of this phase.
(c) Determine the time to this peak voltage and hence the average rate of rise of recovery voltage.

(Answer: (b) 17.96 kV; (c) 76.7 μs, 1.075 kV/μs)
(*From E.C. Examination, 1996*)

10.15 A very long transmission line AB is joined to an underground cable BC of length 5 km. At end C, the cable is connected to a transmission line CD of 15 km length. The

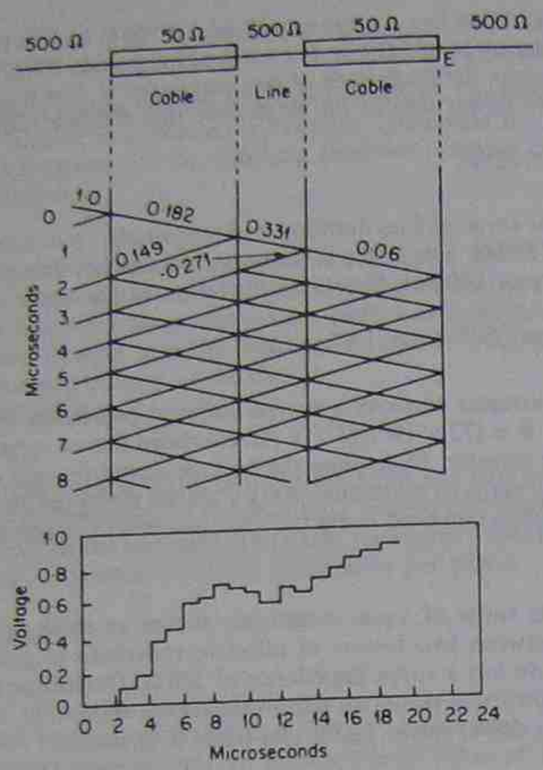


Figure 10.50 Solution of Problem 10.13

transmission line is open-circuit at D.

The cable has a surge impedance of 50 Ω and the velocity of wave propagation in the cable is 150×10^6 m/s. The transmission lines each have a surge impedance of 500 Ω. A voltage step of magnitude 500 kV is applied at A and travels along AB to the junction B with the cable.

Use a lattice diagram to determine the voltage at:

(a) D shortly after the surge has reached D;
(b) D at a time 210 μs after the surge first reaches B;
(c) B at a time 210 μs after the surge first reaches B.

Sketch the voltage at B over these 210 μs.
(Answer: (a) 331.2 kV)
(*From E.C. Examination, 1995*)

11

Substations and Protection

11.1 Introduction

In Chapter 7 attention was confined to the analysis of various types of faults which may occur in a power system. Although the design of electrical plant is influenced by a knowledge of fault conditions, the major use of fault analysis is in the specification of switchgear and protective gear, both being housed in 'substations'. Circuit-breaker ratings are determined by the fault MVA at their particular locations. The maximum circuit-breaker rating is of the order of 50 000–60 000 MVA, and this is achieved by the use of several interrupter heads in series per phase. Not only has the circuit breaker to extinguish the fault-current arc, with the substation connections it has also to withstand the considerable forces set up by short-circuit currents, which can be very high.

A knowledge of the currents resulting from various types of fault at a location is essential for the effective operation of what is known as 'system protection'. If faults occur on the system, the control engineers, noting the presence of the fault, can operate the appropriate circuit breakers to remove the faulty line or plant from the network. This, however, takes considerable time and experience. Faults on a power system resulting in high currents and also possible loss of sychronism must be removed in the minimum of time. Automatic means, therefore, are required to detect abnormal currents and voltages and, when detected, to open the appropriate circuit breakers. It is the object of protection to accomplish this. In a large interconnected network, considerable design knowledge and skill is required to remove the faulty part from the network and leave the healthy remainder working intact.

There are many varieties of automatic protective systems, ranging from simple overcurrent electromechanical relays to sophisticated electronic systems

transmitting high-frequency signals along the power lines. The simplest but extremely effective form of protection is the electromechanical relay, which closes contacts and hence energizes the circuit-breaker opening mechanisms when currents larger than specified pass through the equipment.

The protection used in a network can be looked upon as a form of insurance in which a percentage of the total capital cost (about 5 per cent) is used to safeguard apparatus and ensure continued operation when faults occur. In a highly industrialized community the maintenance of an uninterrupted supply to consumers is of paramount importance and the adequate provision of protection systems is essential.

Summarizing, protection and the automatic tripping (opening) of associated circuit breakers has two main functions: (1) to isolate faulty equipment so that the remainder of the system can continue to operate successfully; and (2) to limit damage to equipment caused by overheating, mechanical forces, etc.

11.2 Switchgear

Some of the functions of the switches or circuit breakers are obvious and apply to any type of circuit, others are peculiar to high-voltage equipment. For maintenance to be carried out on plant, it must be isolated from the rest of the network and hence switches must be provided on each side. If these switches are not required to open under working conditions, i.e. with fault or load current and normal voltage, a cheaper form of switch known as an *isolator* can be used; this can close a live circuit but not open one. Owing to the high cost of circuit breakers, much thought is given, in practice, to obtaining the largest degree of flexibility in connecting circuits with the minimum number of switches. Popular arrangements of switches are shown in Figure 11.1 and a typical feeder bay layout in Figure 11.2.

High-voltage circuit breakers, 11 kV and above, take five basic forms: oil immersed, air blast, small oil volume, sulphur hexafluoride (SF_6), and vacuum. In addition, air circuit breakers with long contact gaps and large arc-splitter plates have been used for up to 6.6 kV, but they are bulky and expensive compared with the five types listed above. These five types will now be described in more detail.

11.2.1 The bulk-oil circuit breaker

A cross-section of an oil circuit breaker with all three phases in one tank is shown in Figure 11.3(a). There are two sets of contacts per phase. The lower and moving contacts are usually cylindrical copper rods and make contact with the upper fixed contacts. The fixed contacts consist of spring-loaded copper

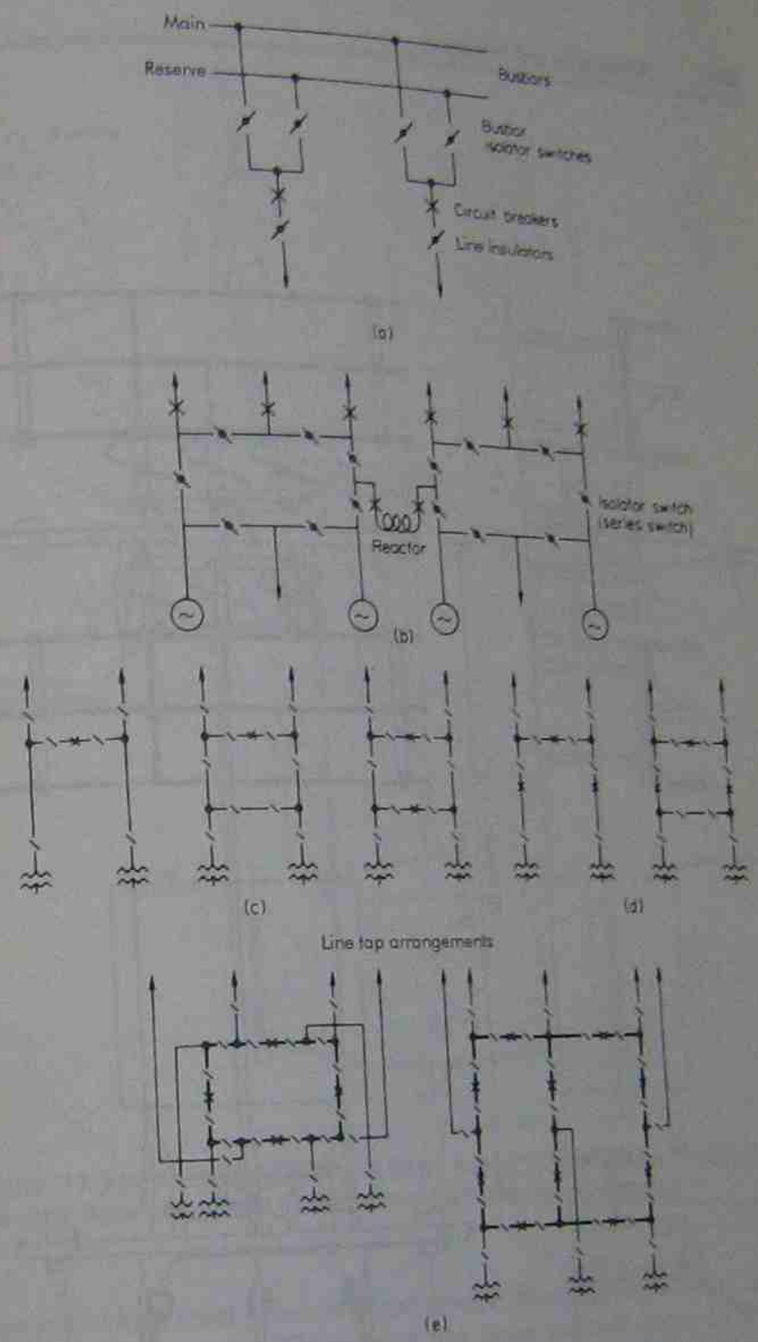


Figure 11.1 Possible switchgear arrangements. (a) Double-busbar selection arrangements. (b) Double-ring busbars with connecting reactor busbars can be isolated for maintenance, but circuits cannot be transferred from one side of the reactor to the other. (c) Open-mesh switching stations, transformers not switched. (d) Open-mesh switching stations, transformers switched. (e) Closed-mesh switching stations. (Isolators are sometimes called series switches.) Arrangements can be indoors or outdoors

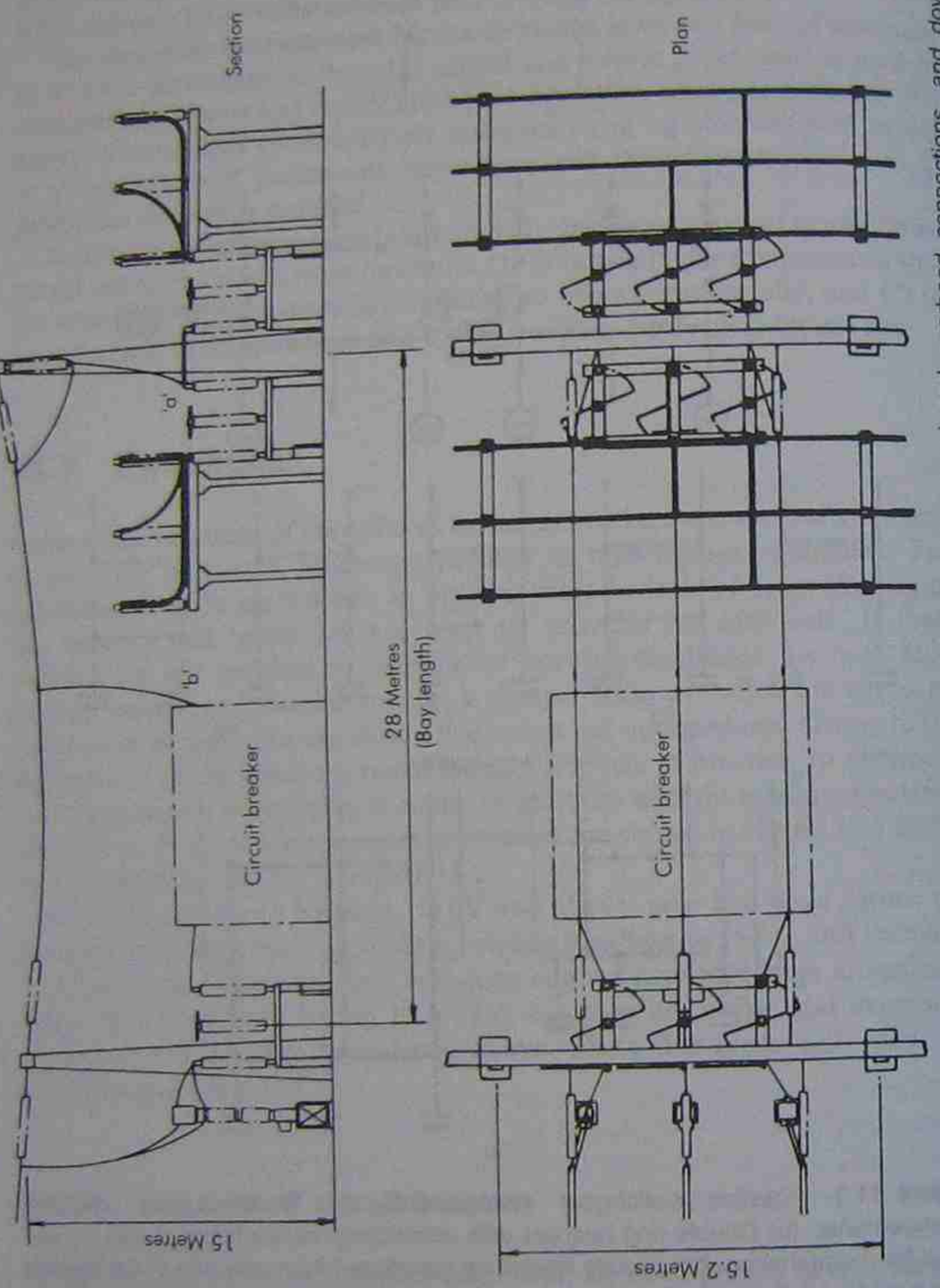


Figure 11.2 Typical arrangement of 275 kV feeder bay using gantries, tensioned overhead connections, and downdroppers (*Permission of I.E.E.*)

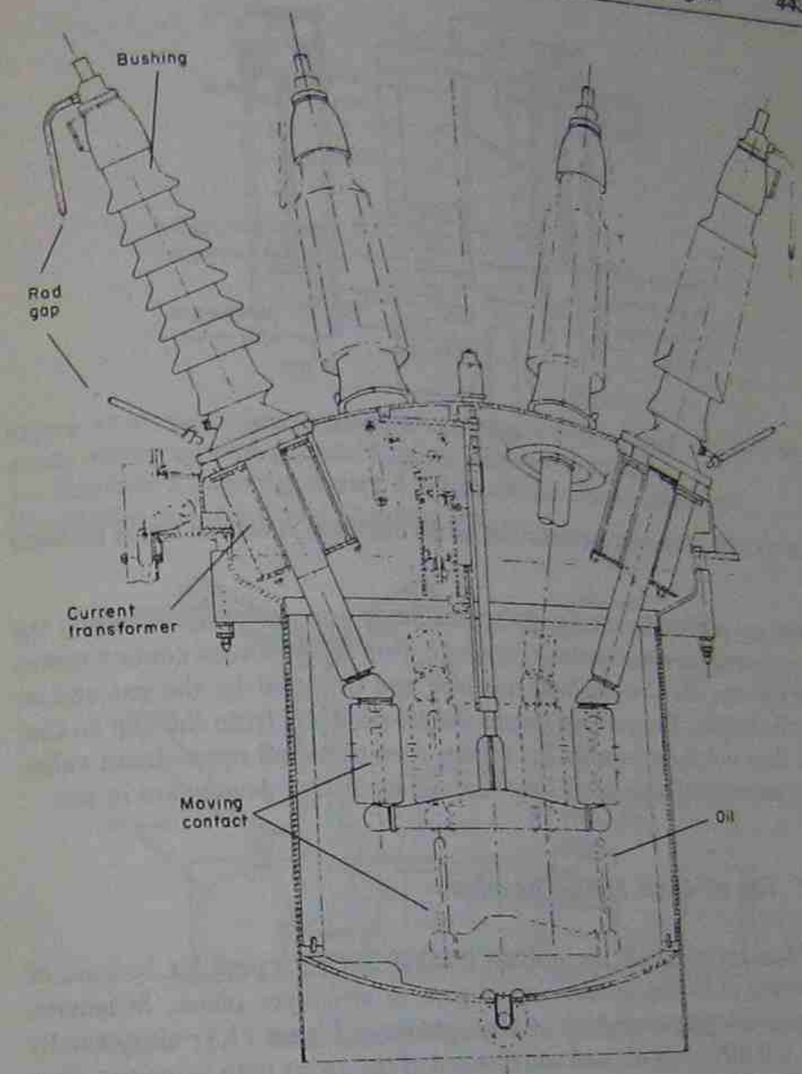


Figure 11.3 (a) Cross-section of a 66 kV bulk-oil circuit breaker. Three phases in one tank (Now obsolete) (*Permission of English Electric Co. Ltd*)

segments which exert pressure on the lower contact rod, when closed, to form a good electrical contact. On opening, the lower contacts move rapidly downwards and draw an arc. When the circuit breaker opens under fault conditions many thousands of amperes pass through the contacts and the extinction of the arc (and hence the effective open-circuiting of the switch) are major engineering problems. Effective opening is only possible because the instantaneous voltage and current per phase reduces to zero during each alternating current cycle. The arc heat causes the evolution of a hydrogen bubble in the oil and this high-

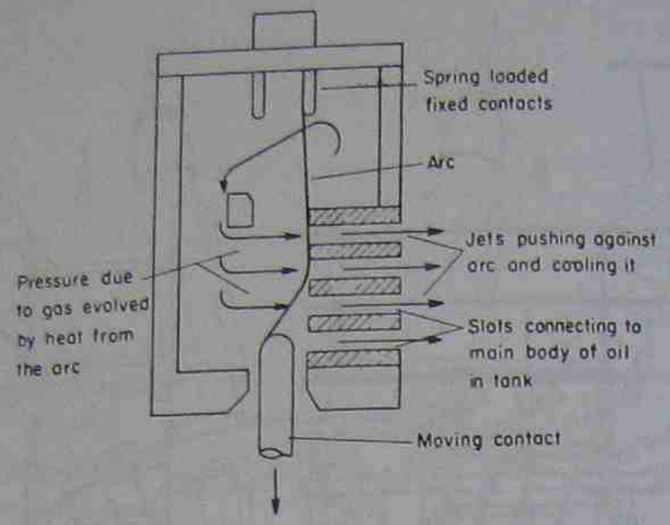


Figure 11.3(b) Cross-jet explosion pot for arc extinction in bulk-oil circuit breakers

pressure gas pushes the arc against special vents in a device surrounding the contacts and called a 'turbulator' (Figure 11.3(b)). As the lowest contact moves downwards the arc stretches and is cooled and distorted by the gas and so eventually breaks. The gas also sweeps the arc products from the gap so that the arc does not re-ignite when the voltage rises to its full open-circuit value. Various patentable forms of oil-immersed arc-extinction devices are in use.

11.2.2 The air-blast circuit breaker

For voltages above 120 kV the air-blast breaker has been popular because of the feasibility of having several contact gaps in series per phase. Schematic diagrams of two types of air-blast head are shown in Figure 11.4. Air normally stored at 1.38 MN/m^2 is released and directed at the arc at high velocities, thus extinguishing it. The air also actuates the mechanism of the movable contact. Figure 11.5 shows a 132 kV air-blast breaker with two breaks (interrupters) per phase and its associated isolator or series switch. Although air-blast circuit breakers have been developed and installed up to the highest voltages they have now been largely superseded by the SF_6 gas circuit breaker (see later).

11.2.3 Small- or low-oil-volume circuit breakers

Here, the quenching mechanisms are enclosed in vertical porcelain insulation compartments and the arc is extinguished by a jet of oil issuing from the moving (lower contact) as it opens. The volume of oil is much smaller than

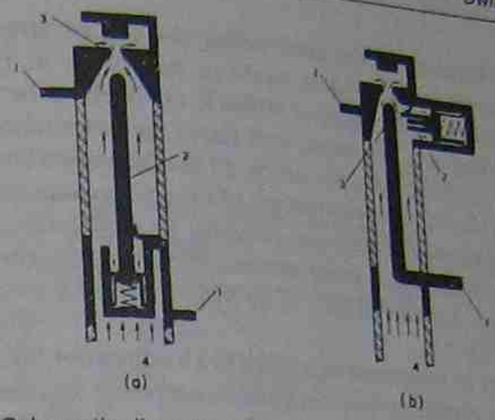


Figure 11.4 Schematic diagrams of two types of air-blast head. (a) Axial flow with axially moving contact. (b) Axial flow with side-moving contact.
1 = Terminal, 2 = moving contact, 3 = fixed contact, and 4 = blast pipe
(*Permission of A. Reyrolle & Company Ltd*)

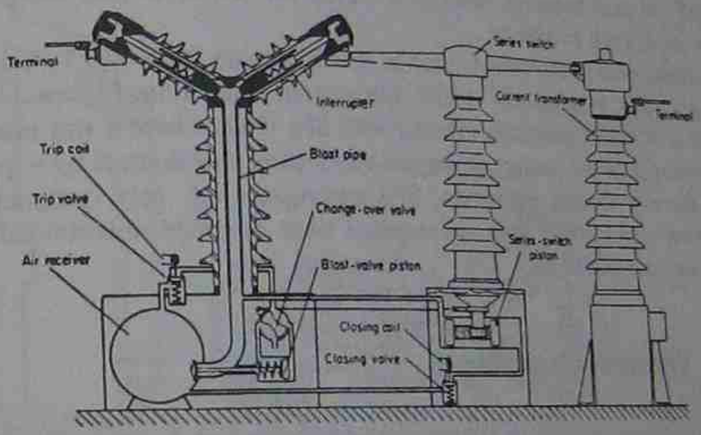


Figure 11.5 Schematic arrangement for typical air-blast circuit breaker

in the bulk-oil type, thereby reducing hazards from explosion and fire. Although many oil circuit breakers are in use, particularly at distribution voltages, modern practice is to use SF_6 or vacuum breakers to avoid the presence of flammable liquids for circuit-interruption purposes.

11.2.4 Sulphur hexafluoride (SF_6) gas

The advantages of using sulphur hexafluoride (SF_6) as an insulating and interrupting medium in circuit breakers arise from its high electric strength and outstanding arc-quenching characteristics. SF_6 circuit breakers are much

smaller than air breakers of the same rating, the electric strength of SF_6 at atmospheric pressure being roughly equal to that of air at the pressure of 10 atm. Temperatures of the order of 30 000 K are likely to be experienced in arcs in SF_6 and these are, of course, well above the dissociation temperature of the gas (about 2000 K); however, nearly all the decomposition products are electronegative so that the electric strength of the gas recovers quickly after the arc has been extinguished. Filters are provided to render the decomposition products harmless and only a small amount of fluorine reacts with metallic parts of the breaker. A sectional view of an SF_6, 420/525 kV switchgear unit is shown in Figure 11.6.

The arrangement of equipment in a 380/110 kV substation SF_6 is illustrated in Figure 11.7. The switchgear also contains all necessary measuring and other facilities as follows: SF_6-insulated, toroidal current transformers and voltage transformers, cable terminations, gas storage cylinders, cable isolator, grounding switches, bus isolator, and bus system. Such substations are of immense value in urban areas because of their greatly reduced size compared with air blast. SF_6 circuit breakers rated at 45 GVA are available and designs for 1300 kV have been produced.

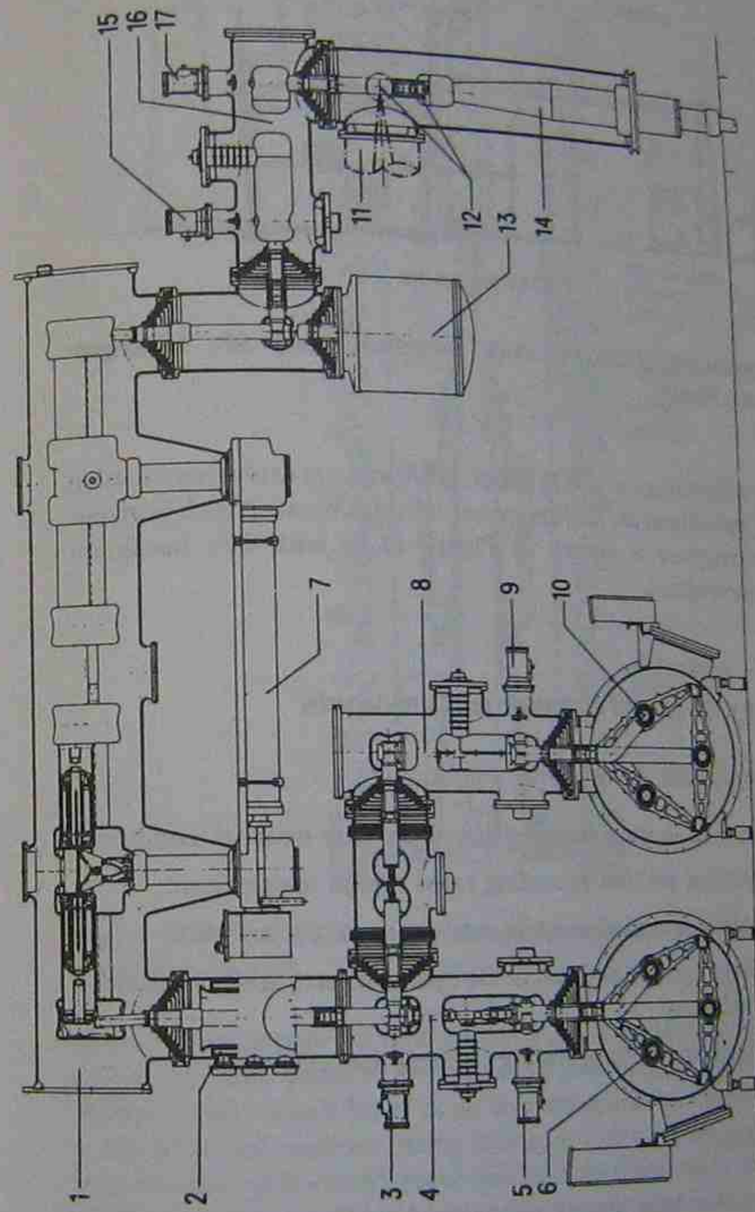
SF_6 switchgear is now widely used at lower voltages from 6.6 kV to 132 kV for distribution systems. A typical interrupter, shown in Figure 11.8, is enclosed in a sealed porcelain cylinder with SF_6 under about 5 atm pressure. The movement of the contact forces gas into the opening contact by a 'puffer' action, thereby forcing extinction. SF_6 interrupters are very compact and robust; they require very little maintenance when mounted in metal cabinets to form switchboards.

11.2.5 Vacuum interrupter

A pair of contacts opening in vacuum draws an arc which burns in the vaporized contact material. Consequently, the contact material and its arcing root shape are crucial to the design of a commercial interrupter. A typical design is illustrated in Figure 11.9.

The main advantages of a vacuum interrupter are: (1) the very small damage normally caused to the contacts on operation, so that a life of 30 years can be expected without maintenance; (2) the small mechanical energy required for tripping; and (3) the low noise caused on operation. The nature of the vacuum arc depends on the current; at low currents the arc is diffuse and can readily be interrupted, but at high currents the arc tends to be constricted. Electrode contour geometries have been produced to give diffuse arcs with current densities of 10^6–10^8 A/cm^2; electron velocities of 10^8 cm/s are experienced in the arc and ion velocities of 10^6 cm/s.

Considerable progress has been made in increasing the current-breaking capacity of vacuum interrupters, but not their operating voltage. Today,



1 Circuit breaker
2 Current transformer
3 Earthing switch
4 Bus insulator
5 Bus earthing switch
6 Bus system
7 Hydraulic storage cylinder
8 Bus isolator
9 Bus earthing switch
10 Bus system
11 Arrangement for voltage testing
12 Splitting points for testing purposes
13 Voltage transformer
14 Cable sealing end
15 Earthing switch
16 Cable isolator
17 Make-proof earthing switch

Figure 11.6 Sectional view of an SF_6-insulated 420/525 kV switchgear unit (*By kind permission of I.E.E.*)

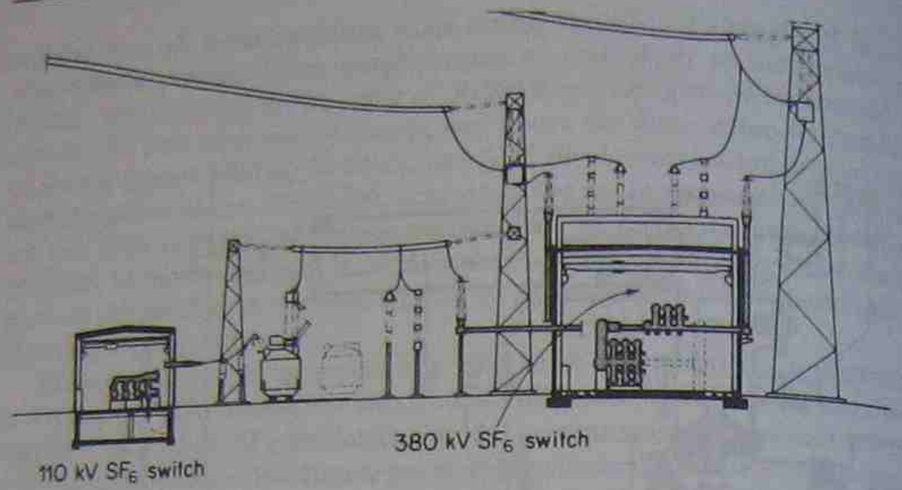


Figure 11.7 Arrangement of 380 kV/110 kV substation using SF_6 switchgear (*Permission of Brown Boveri*)

they are available with ratings of 500 MVA at 30 kV and are extensively used in 11, 20, and 25 kV switchboards. A three-phase vacuum breaker with horizontally mounted interrupters is shown in Figure 11.10 with SF_6 insulation surrounding the interrupters.

11.2.6 Summary of circuit-breaker requirements

A circuit breaker must fulfil the following conditions:

1. open and close in the shortest possible time under any network condition;
2. conduct rated current without exceeding rated design temperature;
3. withstand, thermally and mechanically, any short-circuit currents;
4. maintain its voltage to earth and across the open contacts under both clean and polluted conditions;
5. not create any large overvoltage during opening and closing;
6. be easily maintained;
7. be not too expensive.

Although air has now been largely overtaken by SF_6 as an interrupting medium at high voltages, it has given good performance over the years as it was easily able to achieve 2-cycle ($\approx$ 40 ms) interruption after receipt of a tripping signal to the operating coil. Increasing need, at voltages up to 725 kV, to reduce interruption times to $1\frac{1}{2}$ cycles to maintain stability and to

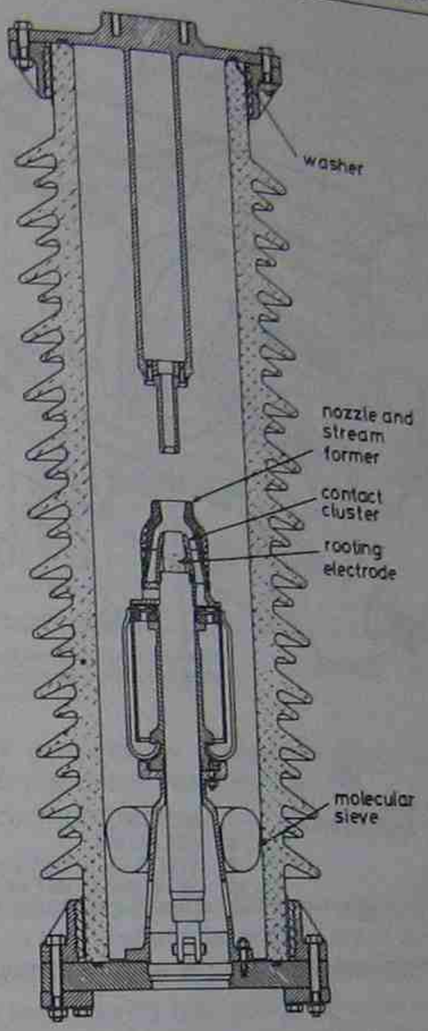
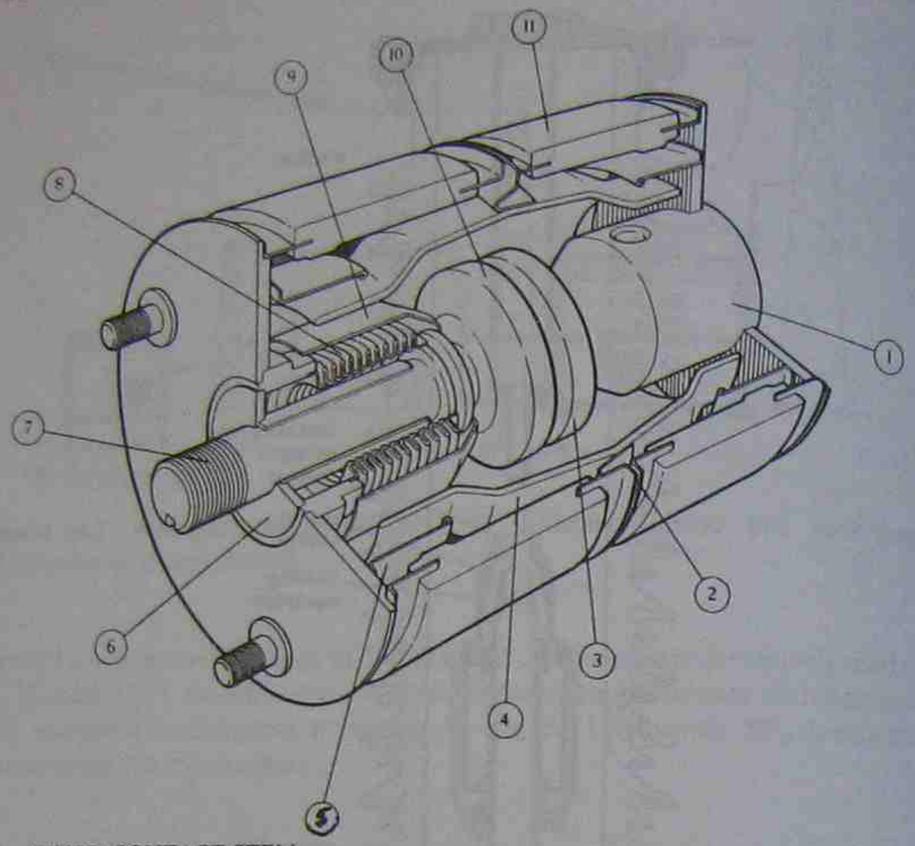


Figure 11.8 An SF_6 single-pressure puffer-type interrupter (*By kind permission of A. Reyrolle & Co. Ltd*)

reduce fault damage has meant that SF_6 switchgear has been required. A 60 kA rating has been possible with resistive switching to avoid unnecessary overvoltages, and a much lower noise level on operation has further confirmed SF_6 as the preferred medium. Being factory-sealed and fully enclosed has considerably reduced radio-interference and audible noise due to corona discharge.

The increasing performance and low maintenance has led to the development of on-line monitoring of the health of switchgear and the associated connections and components (known as 'condition monitoring'). This is particularly required for SF_6 enclosed units as even small dust particles can cause breakdown. Fast protection gear is essential to match these developments.



1. FIXED-CONTACT STEM
2. SPUTTER-SHIELD MOUNTING
3. FIXED CONTACT
4. SPUTTER SHIELD
5. GRADING SHIELD
6. MOVING-CONTACT GUIDE
7. MOVING-CONTACT STEM
8. BELLOWS
9. SPUTTER SHIELD FOR BELLOWS
10. MOVING CONTACT
11. GLASS-CERAMIC BODY

Figure 11.9 Constructional features of an 11 kV vacuum interrupter (*Courtesy of A. Reyrolle & Co. Ltd*)

11.3 Qualities Required of Protection

A few terms often used to describe the effectiveness of protective gear will now be described.

1. *Selectivity or discrimination*—its effectiveness in isolating only the faulty part of the system.
2. *Stability*—the property of remaining inoperative with faults occurring outside the protected zone (called external faults).
3. *Speed of operation*—this property is more obvious. The longer the fault current continues to flow, the greater the damage to equipment. Of great importance is the necessity to open faulty sections before the connected synchronous generators lose synchronism with the rest of the system. This aspect is dealt with in detail in Chapter 8. A typical fault clearance time in H.V. systems is 80 ms, and this requires very high-speed relaying as well as breaking.
4. *Sensitivity*—this is the level of magnitude of fault current at which operation occurs, which may be expressed in current in the actual network (primary current) or as a percentage of the current-transformer secondary current.
5. *Economic consideration*—in distribution systems the economic aspect almost overrides the technical one, owing to the large number of feeders, transformers, etc., provided that basic safety requirements are met. In transmission systems the technical aspects are more important. The protection is relatively expensive, but so is the system or equipment protected, and security of supply is vital. In transmission two separate protective systems are used, one main (or primary) and one back-up. In some instances, two out of three systems must operate before circuit-breaker tripping is initiated.
6. *Reliability*—this property is self-evident. A major cause of circuit 'outages' is mal-operation of the protection itself. On average, in the British

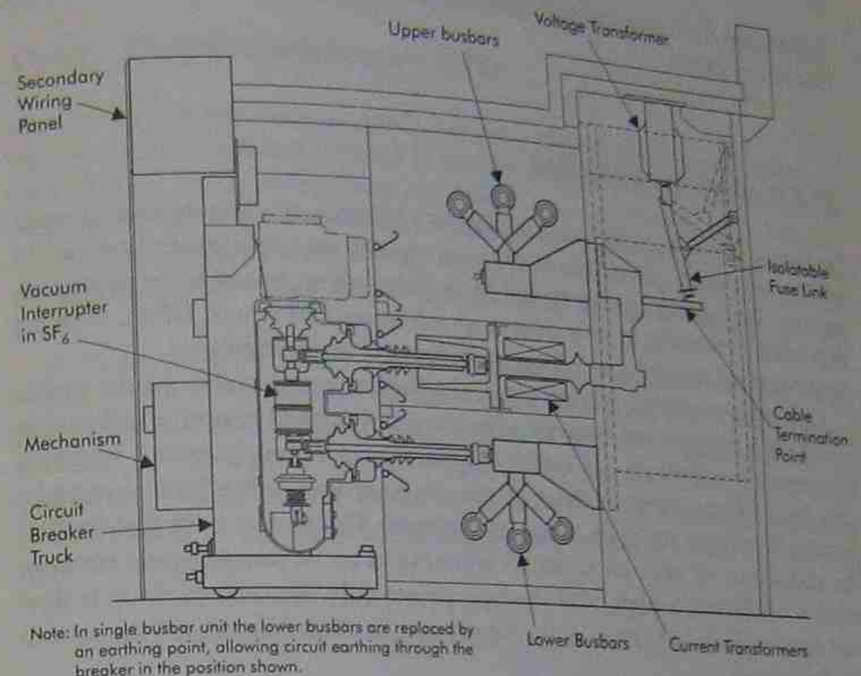


Figure 11.10 HMX double-busbar configuration (*Reproduced by permission of G.E.C. Alsthom T&D Distribution Switchgear Ltd.*)

system (not including faults on generators), nearly 10 per cent of outages are due to this cause.

11.3.1 Back-up protection

Back-up protection, as the name implies, is a completely separate arrangement which operates to remove the faulty part should the main protection fail to operate. The back-up system should be as independent of the main protection as possible, possessing its own current transformers and relays. Often, only the circuit-breaker tripping and voltage transformers are common.

Each main protective scheme protects a defined area or *zone* of the power system. It is possible that between adjacent zones a small region, e.g. between the current transformers and circuit breakers, may be unprotected, in which case the back-up scheme (known as remote back-up), will afford protection because it overlaps the main zones, as shown in Figure 11.11. In distribution the application of back-up is not as widespread as in transmission systems; often it is sufficient to apply it at strategic points only. Remote back-up is slow and usually disconnects more of the supply system than is necessary to remove the faulty part.

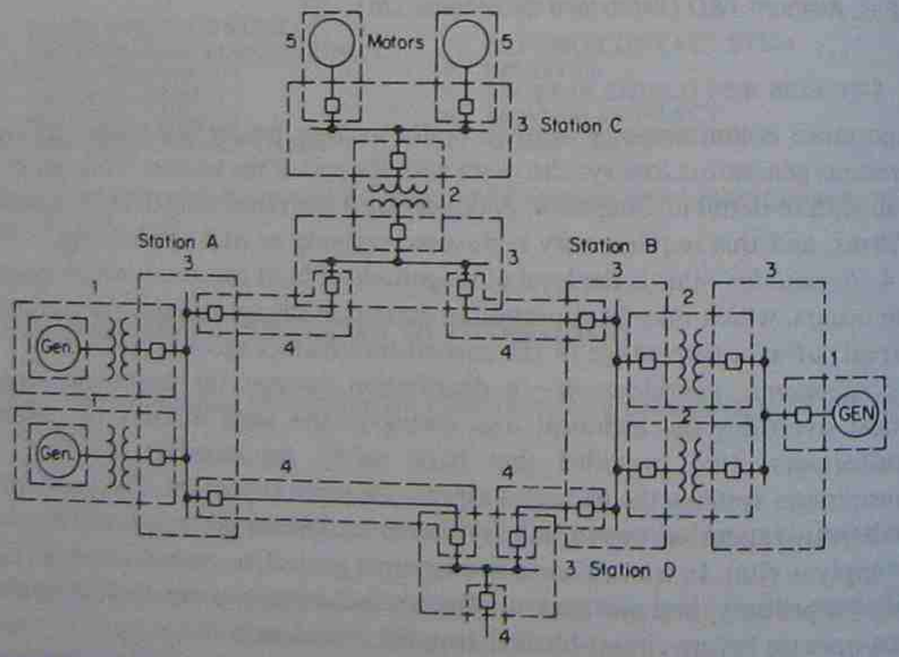


Figure 11.11 Line diagram of a typical system and the overlapping zones of protection (*Permission of Westinghouse Electrical Corporation*)

11.4 Components of Protective Schemes

11.4.1 Current transformers (CTs)

In order to obtain currents which are proportional to the system (primary) currents, and which can be used in control circuits, current transformers are used. Often the primary conductor itself, e.g. an overhead line, forms a single primary turn (bar primary). Whereas instrument current transformers have to remain accurate only up to slight overcurrents, protection current transformers must retain proportionality up to at least 20 times normal full load.

A major problem exists when two current transformers are used which should retain identical characteristics up to the highest fault current, e.g. in pilot wire schemes. Because of saturation in the silicon steel used and the possible existence of a direct component in the fault current, the exact matching of such current transformers is difficult. The nominal secondary current rating of current transformers is now usually 1 A, but 5 A has been used in the past.

Linear couplers

The problems associated with current transformers have resulted in the development of devices called linear couplers, which serve the same purpose but, having air cores, remain linear at the highest currents. These are also known as Rogowski coils and are particularly suited to digital schemes.

11.4.2 Voltage (or potential) transformers (VTs or PTs)

These provide a voltage which is much lower than the system voltage, the nominal secondary voltage being 110 V. There are two basic types: the wound (electromagnetic), virtually a small power transformer, and the capacitor type. In the latter a tapping is made on a capacitor bushing (usually of the order of 12 kV) and the voltage from this tapping is stepped down by a small voltage transformer. The arrangement is shown in Figure 11.12; the reactor (X) and the capacitor (C) constitute a tuned circuit which corrects the phase-angle error of the secondary voltage. In H.V. systems, the capacitor divider is a separate unit mounted within an insulator. It can also be used as a line coupler for high frequency signalling (power line carrier, see Section 11.10).

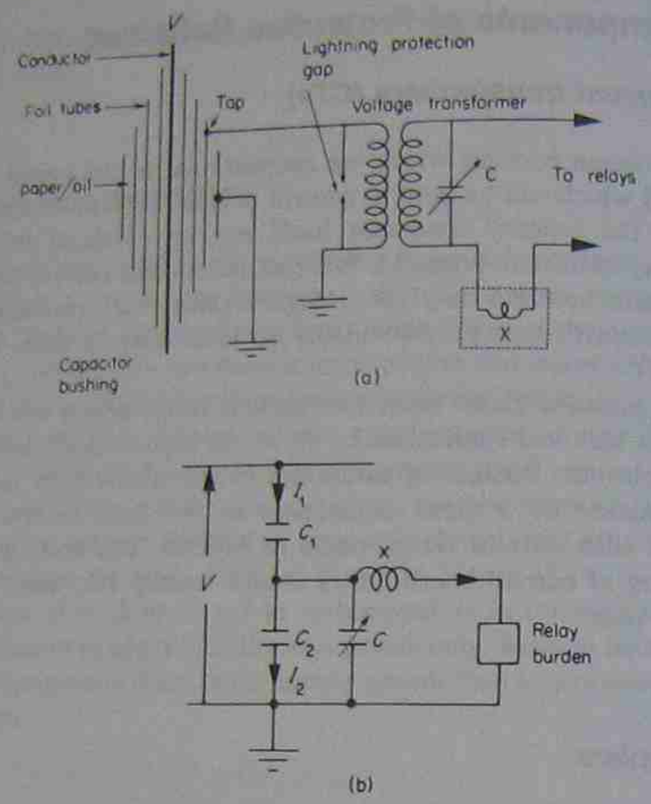


Figure 11.12 Capacitor voltage transformer. (a) Circuit arrangement. (b) Equivalent circuit—burden = impedance of transformer and load referred to primary winding

11.4.3 Relays

A relay is a device which, when energized by appropriate system quantities, indicates an abnormal condition. When the relay contacts close, the associated circuit-breaker trip-circuits are energized and the breaker contacts open, isolating the faulty part from the system. There are two main forms of relay: electromagnetic and semiconductor, including digital (numerical). For some purposes (e.g. overload protection) a bimetallic-strip thermal action is used.

The basic forms of the electromagnetic type comprise induction disc, induction cup, hinged armature, and plunger action. The hinged-armature and plunger-type devices (Figure 11.13) are the simplest and rely on the attraction of an armature or plunger due to an electromagnet which may be energized by a.c. or d.c.

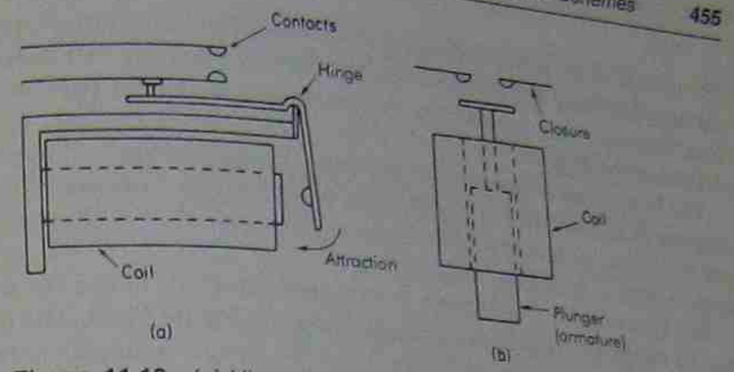


Figure 11.13 (a) Hinged-armature relay. (b) Plunger-type relay

Induction-disc relay

Here, a copper or aluminium disc is free to rotate between the poles of an electromagnet which produces two alternating magnetic fields displaced in phase and space. The eddy currents due to one flux and the remaining flux interact to produce a torque on the disc. In early relays the flux displacement was produced by a copper band around part of the magnet pole (shading ring) which displaced the flux contained by it. Modern relays employ a wattmetric principle in which two electromagnets are employed, as shown in Figure 11.14.

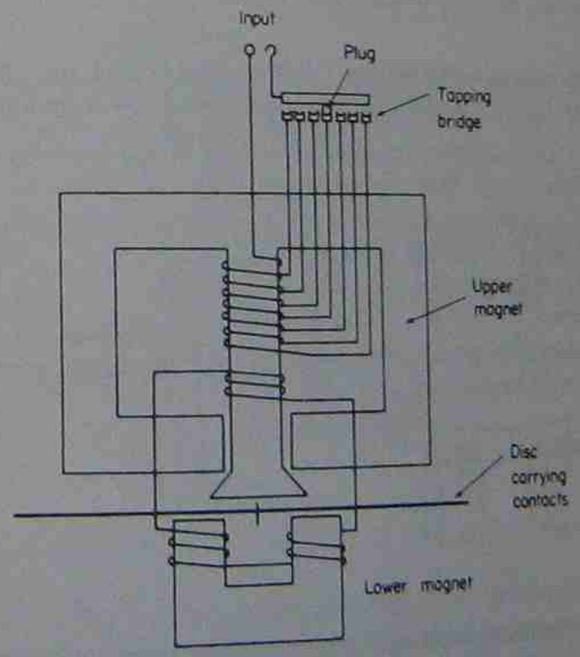


Figure 11.14 Induction-disc relay

The current in the lower electromagnet is induced by transformer action from the upper winding and sufficient displacement between the two fluxes results. This, however, may be adjusted by means of a reactor in parallel with the secondary winding.

The basic mode of operation of the induction disc is indicated in the phasor diagram of Figure 11.15. The torques produced are proportional to $\Phi_2 i_1 \sin\alpha$ and $\Phi_1 i_2 \sin\alpha$, so that the total torque is proportional to $\Phi_1 \Phi_2 \sin\alpha$ as Φ_1 is proportional to i_1 and Φ_2 to i_2.

This type of relay is fed from a current transformer (CT) and the sensitivity may be varied by the plug arrangement shown in Figure 11.14. The operating characteristics are shown in Figure 11.16. To enable a single characteristic curve to be used for all the relay sensitivities (plug settings) a quantity known as the current (or plug) setting multiplier is used as the abscissa instead of current magnitude, as shown in Figure 11.16. To illustrate the use of this curve (usually shown on the relay casing) the following example is given.

Example 11.1

Determine the time of operation of a 1 A, 3 s overcurrent relay having a plug setting of 125 per cent and a time multiplier of 0.6. The supplying CT is rated 400 : 1 A and the fault current is 4000 A.

Solution

The relay coil current = (4000/400) × 1 = 10 A. The normal relay coil current is 1 × (125/100) = 1.25 A. Therefore the relay fault current as a multiple of the plug

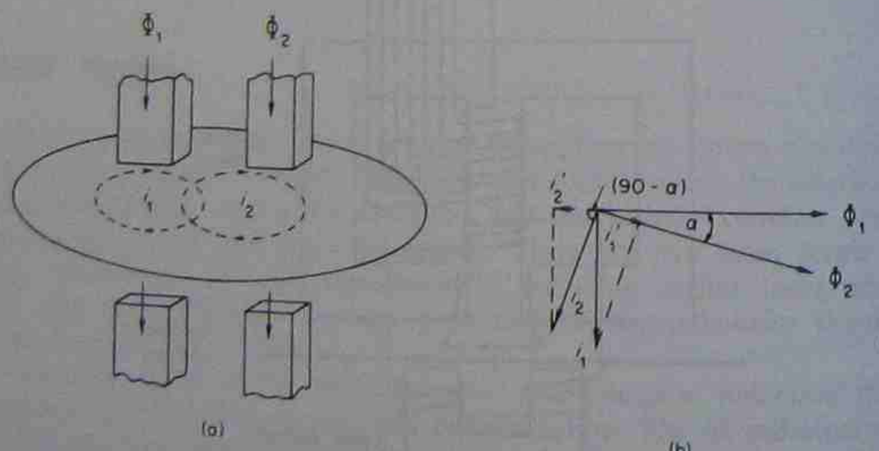


Figure 11.15 Operation of disc-type electromagnetic relay. (a) Fluxes. (b) Phasor diagram. i_1 and i_2 are induced currents in disc

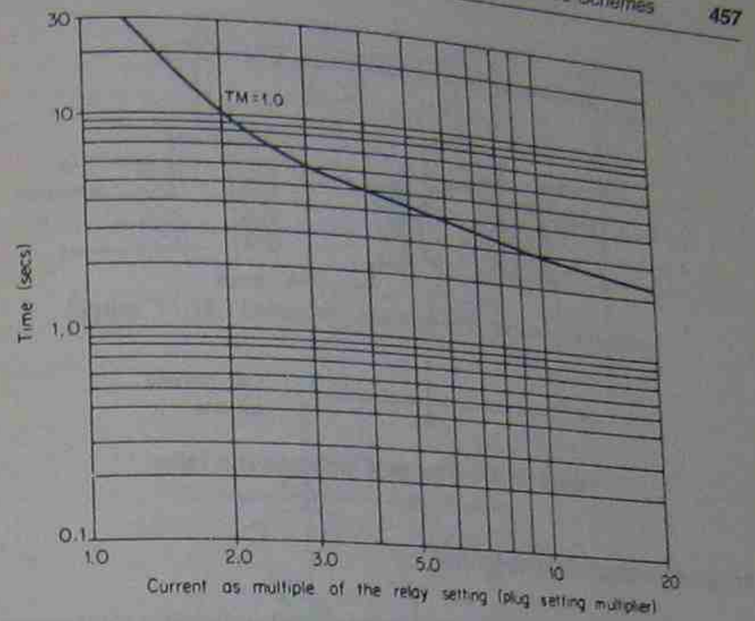


Figure 11.16 Time–current characteristics of a typical induction disc in terms of the plug-setting multiplier. TM = time multiplier. (Note: Time is along the vertical axis)

setting = (10/1.25) = 8. From the relay curve (Figure 11.16), the time of operation is 3.3 s for a time setting of 1. The time multiplier (TM) controls the time of operation by changing the angle through which the disc moves to close the contacts. The actual operating time = 3.3 × 0.6 = 2.0 s.

Induction-disc relays may be made responsive to power flow by feeding the upper magnet winding in Figure 11.14 from a voltage via a potential transformer and the lower winding from the corresponding current. As the upper coil will consist of a large number of turns, the current in it lags the applied voltage by 90°, whereas in the lower (small number of turns) coil they are almost in phase. Hence, Φ_1 is proportional to **V**, and Φ_2 is proportional to **I**, and torque is proportional to $\Phi_1 \Phi_2 \sin\alpha$, i.e. to $\Phi_1 \Phi_2 \sin(90 - \phi)$, or $VI\cos\phi$ (where ϕ is the angle between **V** and **I**).

The direction of the torque depends on the power direction and hence the relay is directional. A power relay may be used in conjunction with a current-operated relay to provide a directional property.

Induction-cup relay

The operation is similar to the induction disc; here, two fluxes at right angles induce eddy currents in a bell-shaped cup which rotates and carries the moving contacts. A four-pole relay is shown in Figure 11.17.

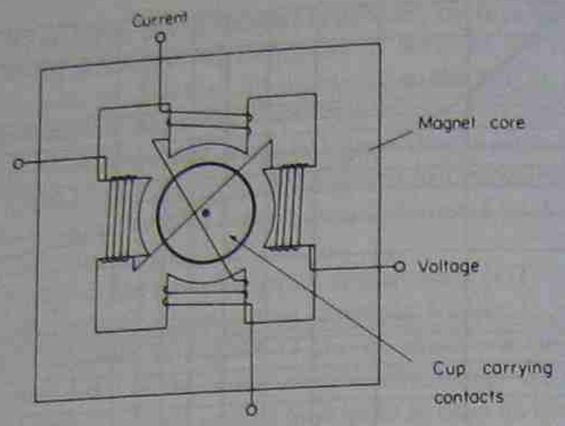


Figure 11.17 Four-pole induction-cup relay

Permanent-magnet moving coil

The action in one type is similar to a moving-coil indicating instrument with the moving-coil assembly carrying the contacts. In a second type the action is basically that of the loudspeaker in which the coil moves axially in the gap of a permanent magnet. The time–current characteristic is inverse with a definite minimum time.

Balanced beam

The basic form of this relay is shown in Figure 11.18. The armatures at the ends of the beam are attracted by electromagnets which are operated by the appropriate parameters, usually voltage and current. A slight mechanical bias is incorporated to keep the contacts open, except when operation is required.

The pulls on the armatures by the electromagnets are equal to K_1V^2 and K_2I^2, where K_1 and K_2 are constants, and for operation (i.e. contacts to close), i.e.

$$K_1V^2 > K_2I^2$$

then

$$\frac{V}{I} < \sqrt{\frac{K_2}{K_1}} \quad \text{or} \quad Z < \sqrt{\frac{K_2}{K_1}}$$

This shows that the relay operates when the impedance it 'sees' is less than a predetermined value. The characteristic of this relay, when drawn on R and jX axes, is a circle, as shown in Figure 11.19.

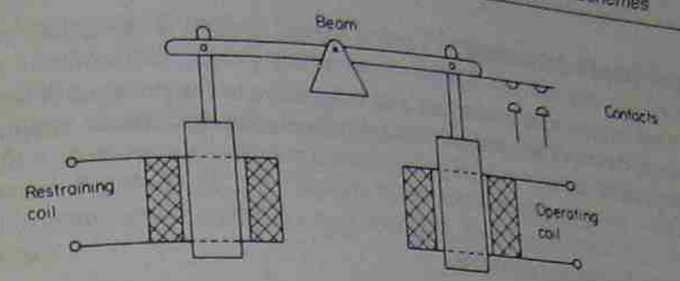


Figure 11.18 Schematic diagram of balanced-beam relay

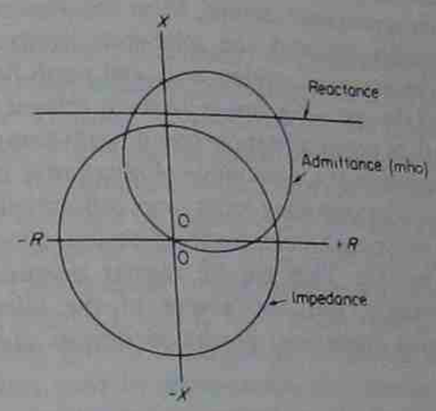


Figure 11.19 Characteristics of impedance, reactance, and admittance (mho) relays shown on the R–X diagram

Distance relays

The balanced-beam relay, because it measures the impedance of the protected line, effectively measures distance. Two other relay forms also may be used for this purpose:

1. the reactance relay which operates when $(V/I)\sin\phi \leqslant$ constant, having the characteristic shown in Figure 11.19;
2. the mho or admittance relay, the characteristic of which is also shown in Figure 11.19.

The above relays operate with any impedance phasor lying inside the characteristic circle or below the reactance line. The mho characteristic may be obtained by devices balancing two torque-producing elements and an induction cup could be used. A rectifier bridge supplying a moving-coil relay is also often used.

Negative-phase sequence

This is used in generator protection and is sensitive to the presence of negative-sequence currents. The protection comprises a bridge circuit supplying a current-operated relay.

Solid-state (passive) devices

These relays are extremely fast in operation, having no moving parts and are very reliable. Detection involving phase angles and current and voltage magnitudes are made with appropriate circuits. Most required current–time characteristics may be readily obtained and solid-state devices are now firmly established. Inverse-characteristic, overcurrent, and earth-fault relays have a minimum time lag and the operating time is inversely related to some power of the input (e.g. current). In practical static relays it is advantageous to choose a circuit which can accommodate a wide range of alternative inverse time characteristics, precise minimum operating levels, and definite minimum times.

Electromechanical relays are vulnerable to corrosion, shock vibration, and contact bounce and welding. They require regular maintenance by skilled personnel. Not surprisingly, with the advent of the microprocessor and integrated circuits, digital (numerical) protection devices are now the norm.

Digital (numerical) relaying

Having monitored currents and voltages through primary transducers (CTs and VTs), these analogue quantities can be sampled and converted to digital form for numerical manipulation, analysis, display, and recording. This process provides a flexible and very reliable relaying function, thereby enabling the same basic hardware units to be used for almost any kind of relaying scheme. With the continuous reduction in digital-circuit costs and increases in their functionality, considerable cost–benefit improvement ensues. It is now often the case that the cost of the relay housing, connections, and EMC protection dominates the hardware but, as is usual in digital systems, the software development and proving procedure are the most expensive items in the overall scheme.

Since digital relays can store data, they are particularly suited to post-fault analysis and can therefore be used in a self-adaptive mode, which is impossible with conventional devices. Additionally, they are capable of self-monitoring and communication with hierarchical controllers. By these means, not only can fast and selective fault clearance be obtained, but also fault location can be flagged to mobile repair crews. Minor (non-vital) protection-system faults can also be indicated for maintenance attention. With the incorporation of a satellite-timing signal receiver using Global Positioning Satellite (GPS) to give a 1 μs synchronized signal, faults on overhead circuits can be located to within 300 m.

The basic hardware elements of a digital relay are indicated in Figure 11.20 and a flow chart for the software is outlined in Figure 11.21. A digital relay unit is shown in Figure 11.22. For the advanced student, many good references to digital or numerical relaying are now available (see the references at the end of this book).

Summation transformer

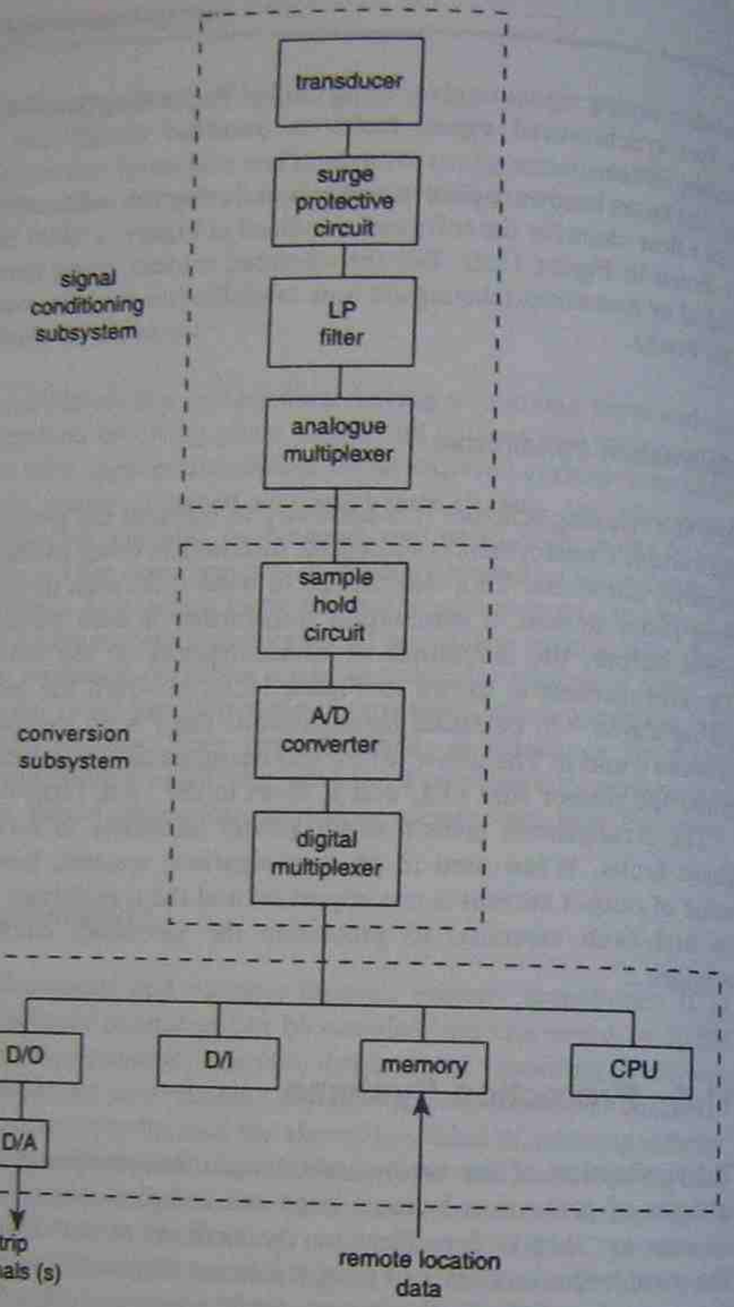
In some relaying schemes it is necessary to transmit the secondary currents of the current transformers considerable distances in order to compare them with currents elsewhere. To avoid the use of wires from each of the three CTs in a three-phase system, a summation transformer is used which gives a single-phase output, the magnitude of which depends on the nature of the fault. The arrangement is shown in Figure 11.23, in which the ratios of the turns are indicated. On balanced through-faults there is no current in the winding between c and n. The phase (a) current energizes the 1 p.u. turns between a and b and the phasor sum of $\mathbf{I}_a$ and $\mathbf{I}_b$ flows in the 1 p.u. turns between b and c.

The arrangement gives a much greater sensitivity to earth faults than to phase faults. When used in phase-comparison systems, however, the actual value of output current is not important and the transformer usually saturates on high-fault currents, so protecting the secondary circuits against high voltages.

11.5 Protection Systems

The application of the various relays and other equipment to form adequate schemes of protection forms a large and complex subject. Also, the various schemes are largely dependent on the methods of individual manufacturers. The main intention here is to present a survey of general practice and to outline the principles of the methods used. Some schemes are discriminative to fault location and involve several parameters, e.g. time, direction, current, distance, current balance, and phase comparison. Others discriminate according to the type of fault, e.g. negative-sequence relays, and some use a combination of location and type of fault.

A convenient classification is the division of the systems into *unit* and *non-unit* types. Unit protection signifies that an item of equipment or zone is being uniquely protected independently of the adjoining parts of the system. Non-unit schemes are those in which several relays and associated equipment are



LP = low pass
A/D = analogue-to-digital
D/A = digital-to-analogue
CPU = central processor unit
D/I = data input
D/O = data output

Figure 11.20 Basic components of a digital relay (*Permission of Peter Peregrinus, I.E.E.*)

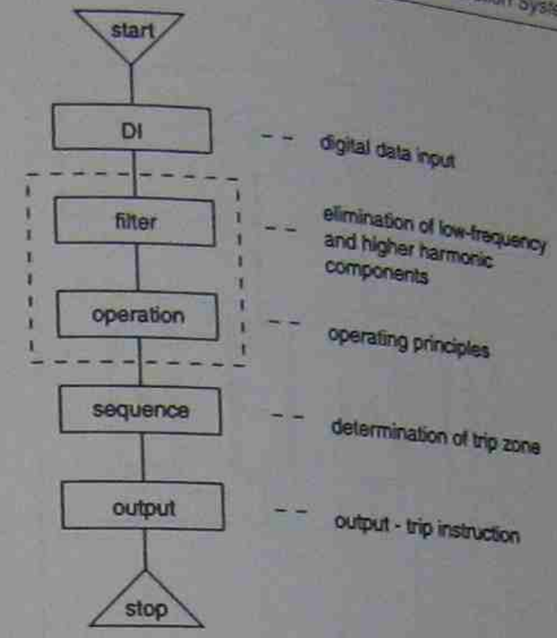


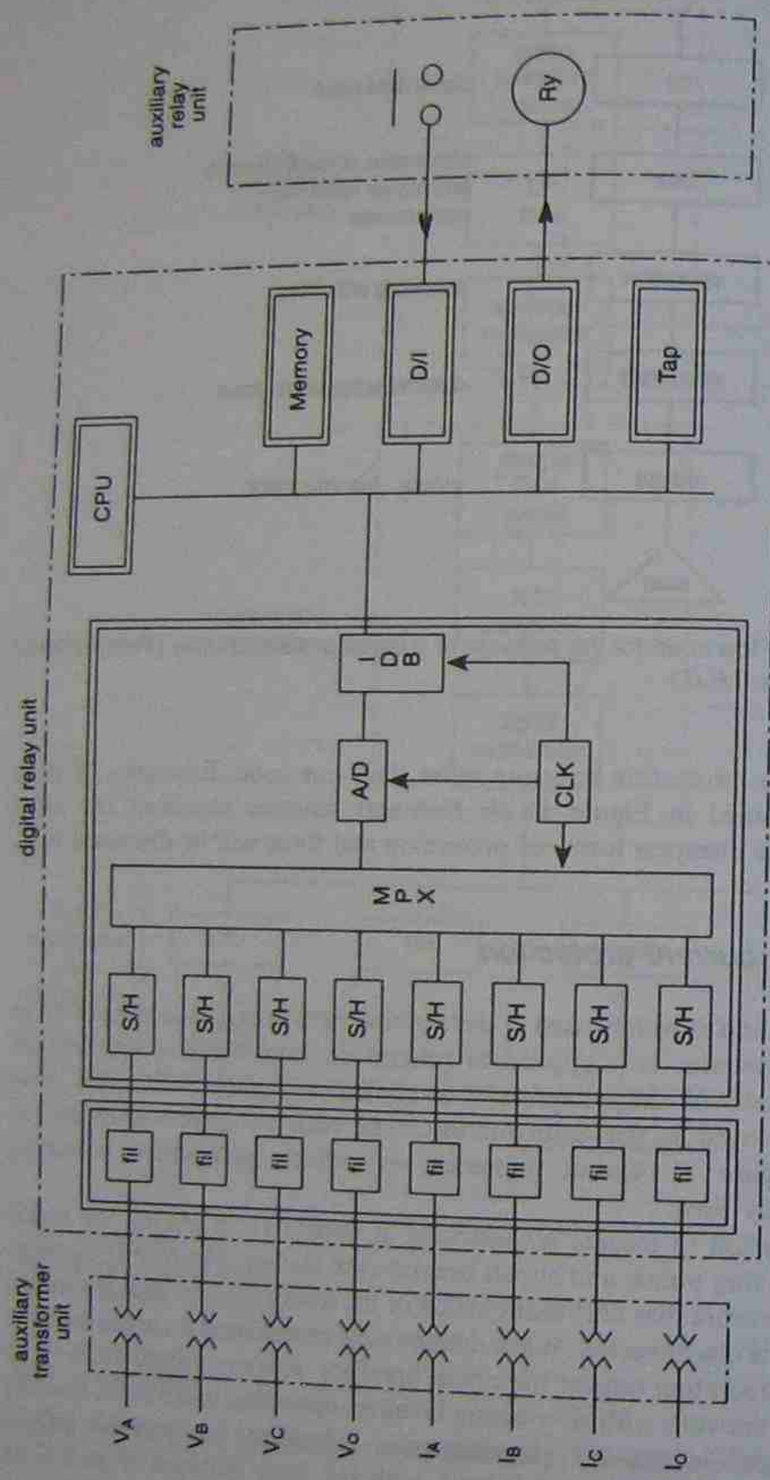
Figure 11.21 Flow chart for the software of a digital protective relay (*Permission of Peter Peregrinus, I.E.E.*)

used to provide protection covering more than one zone. Examples of both types are classified in Figure 11.24. Non-unit schemes represent the most widely used and cheapest forms of protection and these will be discussed first.

11.5.1 Overcurrent protection

This basic method is widely used in distribution networks and as a back-up in transmission systems. It is applied to generators, transformers, and feeders. The arrangement of the components is shown in Figure 11.25. The relay normally employed is the induction-disc type with two electromagnets, as shown in Figure 11.14, but numerical overcurrent relays are becoming more frequently used.

The application to feeders is illustrated in Figure 11.26. Along the radial feeder the relaying points and circuit breakers are shown. The operating times are graded to ensure that only that portion of the feeder remote from the infeed side of a fault is disconnected. When determining selectivity, allowance must be made for the operating time of the circuit breakers. Assume Figure 11.26 to be a distribution network with slow-acting breakers operating in 0.3 s and that the relays have true inverse-law characteristics. Selectivity is obtained with a through-fault of 200 per cent full load, with the fault between D and E as



filt = filter
MPX = multiplexer 720 600 samples/s
CLK = clock, 10 MHz
CPU = central processor unit
S/H = sample-and-hold-circuit, 12 bit
A/D = analogue-to-digital converter
IDB = input data buffer
Ry = trip relay
D/I = digital input status
D/O = output to trip relay
Tap = transformer tap setting

Figure 11.22 Digital relay unit, 16 bit (*Permission of Peter Peregrinus, on behalf of I.E.E., Johns and Salman, 1995*)

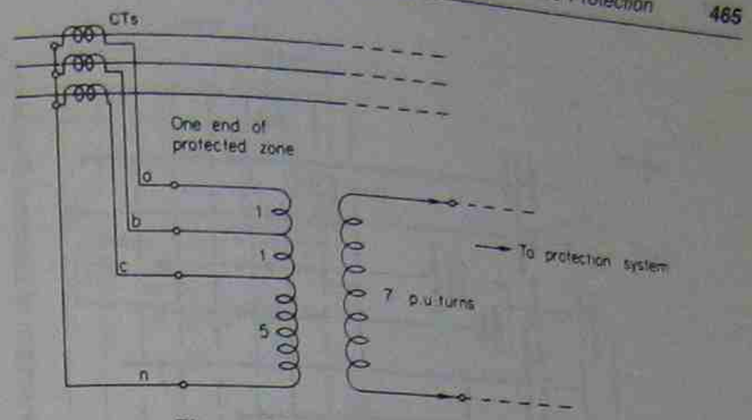


Figure 11.23 Summation transformer

illustrated because the time difference between relay operations is greater than 0.3 s. Relay D operates in 0.5 s and its circuit breaker trips in 0.8 s. The fault current ceases to flow (normal-load current is ignored for simplicity) and the remaining relays do not close their contacts. Consider, however, the situation when the fault current is 800 per cent of full load. The relay operating times are now: A, 0.5 s [i.e. $2 \times (200/800)$]; B, 0.375 s; C, 0.25 s; D, 0.125 s; and the time for the breaker at D to open is $0.125 + 0.3 = 0.425$ s. By this time, relays B and C will have operated and selectivity is not obtained. This illustrates the fundamental drawback of this system, i.e. that for correct discrimination to be obtained the times of operation close to the supply point become large.

11.5.2 Overcurrent and directional

To obtain discrimination in a loop or networked system, relays with an added directional property are required. For the system shown in Figure 11.27. directional and non-directional overcurrent relays have time lags for a given fault current as shown. Current feeds into fault at the location indicated from both directions, and the first relay to operate is at B (0.6 s). The fault is now fed along route ACB only, and next the relay at C (1 s) operates and completely isolates the fault from the system. Assuming a circuit-breaker clearance time of 0.3 s, complete selectivity is obtained at any fault position. Note, however, that directional relays require a voltage input, non-directional ones do not.

11.6 Distance Protection

The shortcomings of graded overcurrent relays have led to the widespread use of distance protection. The distance between any point in the feeder and the

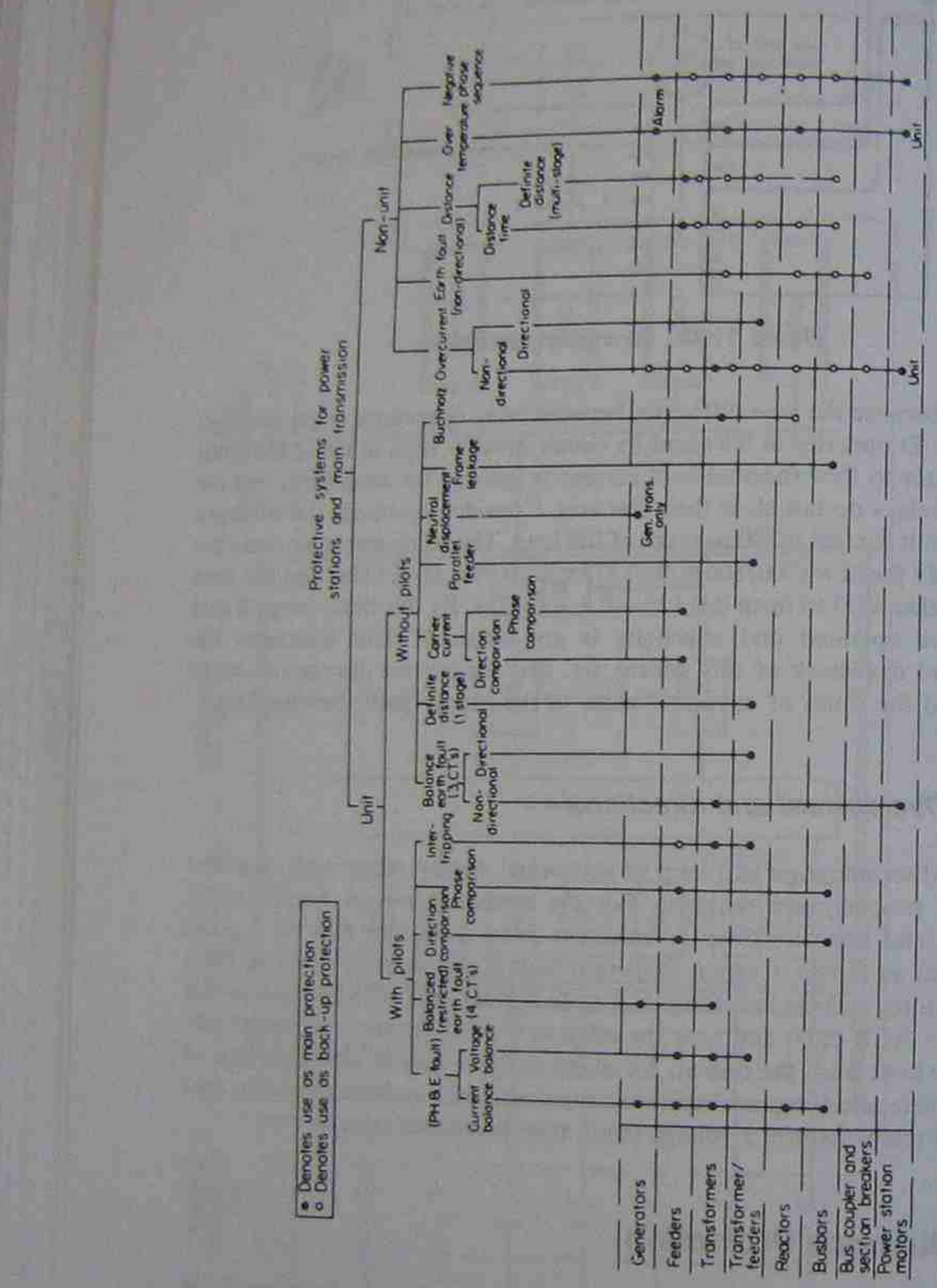


Figure 11.24 Classification of protection schemes (Reproduced by permission of the *Institution of Electrical Engineers*)

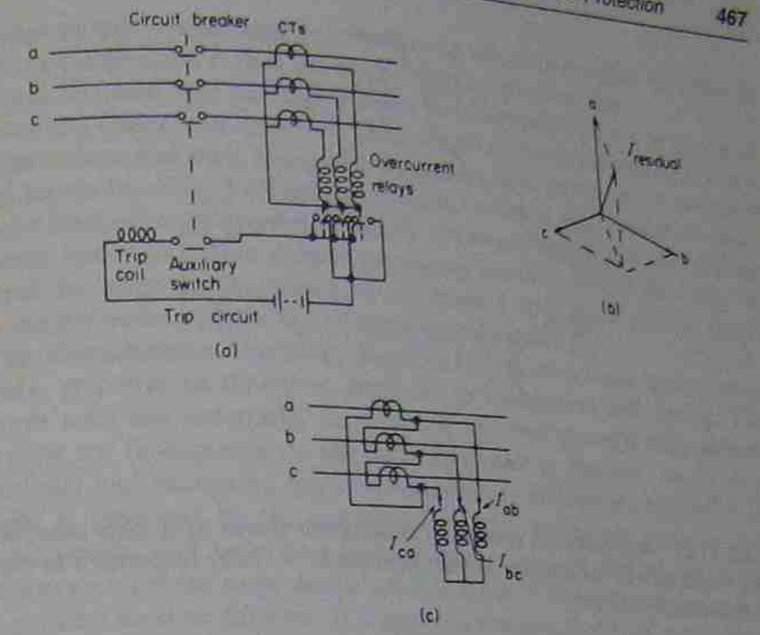


Figure 11.25 Circuit diagram of simple overcurrent protection scheme. (a) CTs in star connection. (b) Phasor diagram of relay currents, star connection. (c) CTs in delta connection

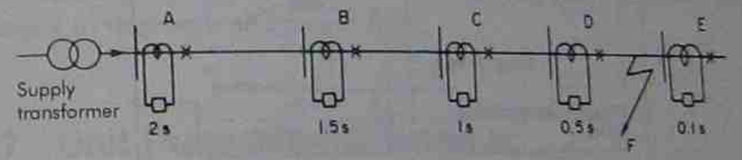


Figure 11.26 Application of overcurrent relays to feeder protection

fault is proportional to the ratio (voltage/current) at that point and relays responsive to impedance, admittance (mho), or reactance may be used. Although a variety of time–distance characteristics are available for providing correct selectivity, the most popular one is the stepped characteristic shown in Figure 11.28. Here, A, B, C, and D are distance relays with directional properties and A and C only measure distance when the fault current flows in the indicated direction. Relay A trips its associated breaker if a fault occurs within the first 80 per cent of the length of feeder 1. For faults in the remaining 20 per cent of feeder 1 and the initial 30 per cent of feeder 2 (called the stage 2 zone), relay A initiates tripping after a short time delay. A further delay in relay A is introduced for faults further along feeder 2 (stage 3 zone). Relays B and D have similar characteristics when the fault current flows in the opposite direction.

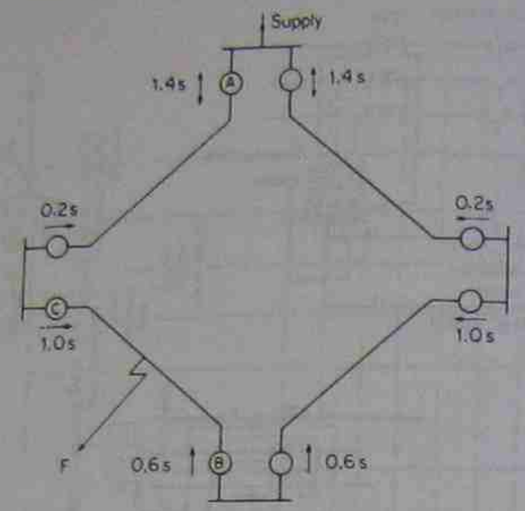


Figure 11.27 Application of directional overcurrent relays to a loop network. ↔ Relay responsive to current flow in both directions; →, relay responsive to current flow in direction of arrow

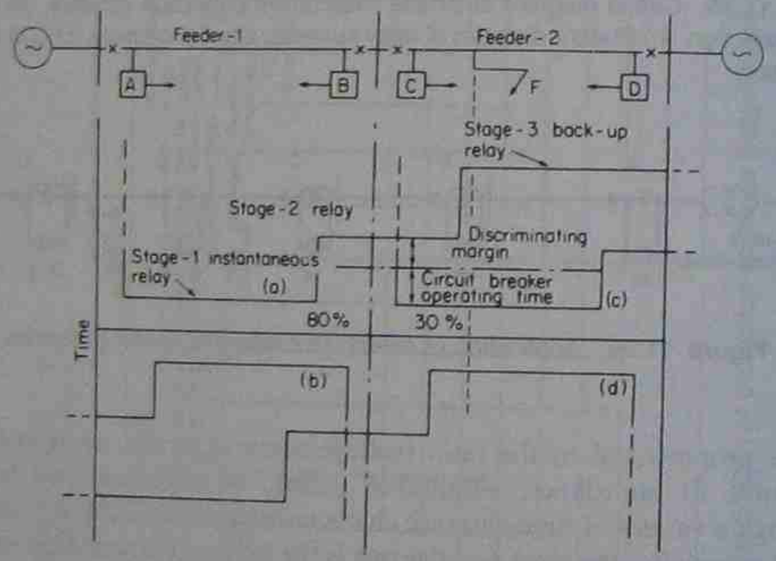


Figure 11.28 Characteristic of three-stage distance protection

The selective properties of this scheme can be understood by considering a fault such as at F in feeder 2, when fault current flows from A to the fault. For this fault, relay A starts to operate, but before the tripping circuit can be completed, relay C trips its circuit breaker and the fault is cleared. Relay A then resets and feeder 1 remains in service. The margin of selectivity provided is

indicated by the vertical intercept between the two characteristics for relays A and C at the position F, less the circuit-breaker operating time.

It will be noted that the stage 1 zones are arranged to extend over only 80 per cent of a feeder from each end. The main reason for this is because practical distance relays and their associated equipment have errors, and a margin of safety has to be allowed if incorrect tripping for faults which occur just inside the next feeder is to be avoided. Similarly, the stage 2 zone is extended well into the next feeder to ensure definite protection for that part of the feeder not covered by stage 1. The object of the stage 3 zone is to provide general back-up protection for the rest of the adjacent feeders.

The characteristics shown in Figure 11.28 require three basic features: namely, response to direction, response to impedance, and timing. These features need not necessarily be provided by three separate relay elements, but they are fundamental to all distance protective systems. As far as the directional and measuring relays are concerned, the number required in any scheme is governed by the consideration that three-phase, phase-to-phase, phase-to-earth, and two-phase-to-earth faults must be catered for. For the relays to measure the same distance for all types of faults, the applied voltages and currents must be different. It is common practice, therefore, to provide two separate sets of relays, one set for phase faults and the other for earth faults, and either of these caters for three-phase faults and double-earth faults. Each set of relays is, in practice, usually further divided into three, since phase faults may concern any pair of phases, and, similarly, any phase can be faulted to earth. With digital relays, a pre-selection of relaying quantities allows just one processor to deal with all types of fault.

11.7 Unit Protection Schemes

With the ever-increasing complexity of modern power systems the methods of protection so far described may not be adequate to afford proper discrimination, especially when the fault current flows in parallel paths. In unit schemes, protection is limited to one distinct part or element of the system that is disconnected if any internal fault occurs. On the other hand, the protected part should remain connected with the passage of current flowing into an external fault.

11.7.1 Differential relaying

At the extremities of the zone to be protected the currents are continuously compared and balanced by suitable relays. Provided that the currents are equal in magnitude and phase, no relay operation will occur. If, however, an internal

fault (inside the protected zone) occurs, this balance will be disturbed (see Figure 11.29) and the relay will operate. The current transformers at the ends of each phase should have identical characteristics to ensure perfect balance on through-faults. Unfortunately, this is difficult to achieve and a restraint, or bias, is applied (see Figure 11.30) that carries a current proportional to the full system current and restrains the relay operation on large through-fault currents. The corresponding characteristic is shown in Figure 11.30. This principle (circulating current) may be applied to generators, feeders, transformers, and busbars, and provides excellent selectivity. By suitable connections and current summation, teed or multi-ended circuits can be protected using the same principles.

11.8 Generator Protection

Large generators are invariably connected to their own step-up transformer and the protective scheme usually covers both items. A typical scheme is shown

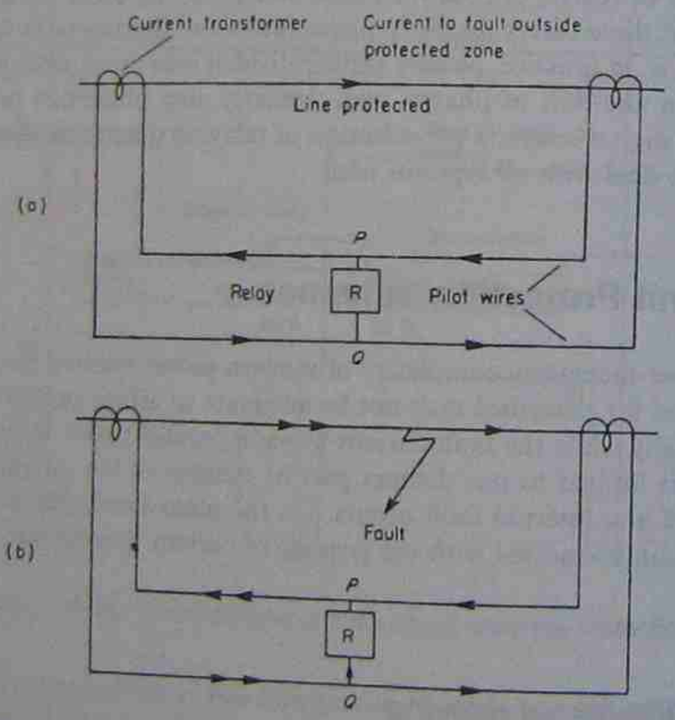


Figure 11.29 Circulating current, differential protection (one phase only shown). (a) Current distribution with through-fault—no current in relay. (b) Fault on line, unequal currents from current transformers and current flows in relay coil. Relay contacts close and trip circuit breakers at each end of the line

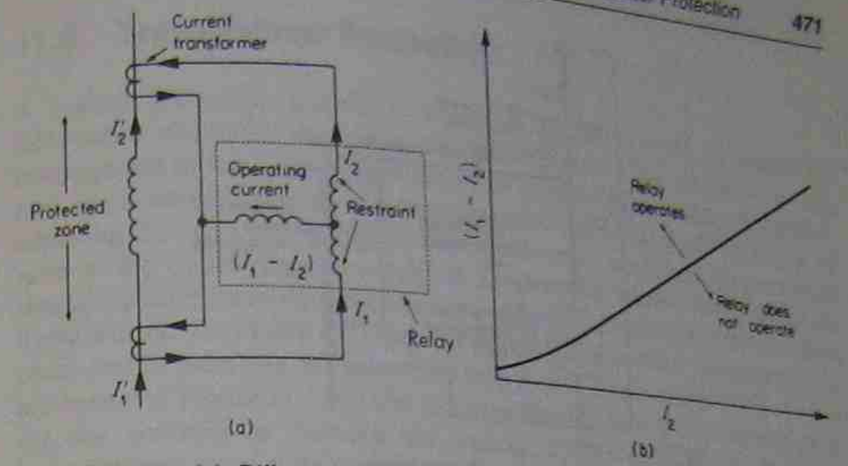


Figure 11.30 (a) Differential protection—circuit connections (one phase only)—relay with bias. (b) Characteristic of bias relay in differential protection. Operating current plotted against circulating or restraint current

in Figure 11.31, in which separate differential circulating-current protections are used to cover the generator alone and the generator plus transformer. When differential protection is applied to a transformer the current transformer on each side of a winding must have ratios which give identical secondary currents. In many countries the generator neutral is often grounded through a distribution transformer. This energizes a relay which operates the generator main and field breakers when a ground fault occurs in the generator or transformer. The ground fault is usually limited to about 10 A by the distribution transformer or a resistor, although an inductor has some advantages. The field circuit of the generator must be opened when the differential protection operates in order to avoid the machine feeding the fault.

The relays of the differential protection on the stator windings (see Figure 11.32) are set to operate at about 10–15 per cent of the circulating current produced by full-load current in order to avoid current-transformer errors. If the phase e.m.f. generated by the winding is E, the minimum current for a ground fault at the star-point end, and hence with the whole winding in circuit, is E/R, where R is the neutral effective resistance. For a fault at a fraction x along the winding from the neutral (Figure 11.32(c)), the fault current is xE/R and 10–15 per cent of the winding is unprotected. With the neutral grounded via the transformer, R is high and earth faults are detected by a sensitive relay across the transformer secondary. With an interturn fault (turn-to-turn short circuit) on a phase of the stator winding (Figure 11.32(b)), current balance at the ends is retained and no operation of the differential relay takes place. the relays operate only with phase-to-phase and ground faults.

On unbalanced loads or faults the negative-sequence currents in the generator produce excessive heating on the rotor surface and generally ($I_2^2 t$)

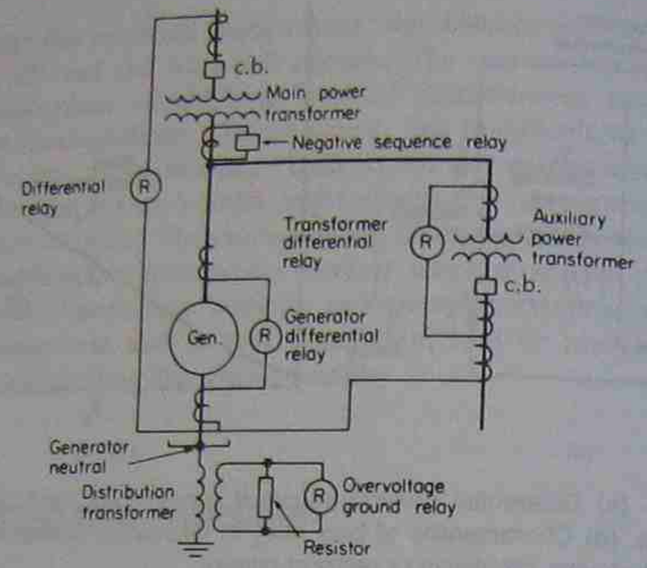


Figure 11.31 Protection scheme for a generator and unit transformer

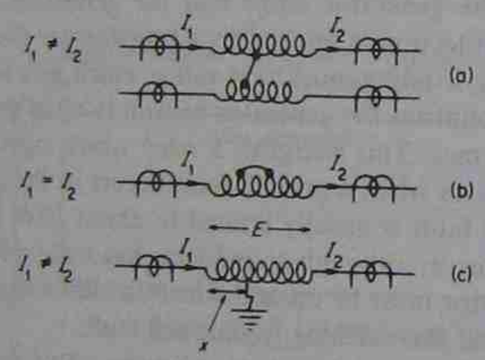


Figure 11.32 Generator winding faults and differential protection. (a) Phase-to-phase fault. (b) Interturn fault. (c) Phase-to-earth fault

must be limited to a certain value for a given machine (between 3 and 4 $(\text{p.u.})^2$ s for 500 MW machines), where t is the duration of the fault in seconds. To ensure this happens, a relay is installed which detects negative-sequence current and trips the generator main breakers when a set threshold is exceeded. When loss of excitation occurs, reactive power (Q) flows into the machine, and if the system is able to supply this the machine will operate as an induction generator, still supplying power to the network. The generator output will oscillate slightly as it attempts to lock into synchronism. Relays are connected to isolate the machine when a loss of field occurs, which can be readily detected by a reactance relay.

11.9 Transformer Protection

A typical protection scheme is shown in Figure 11.33(a), in which the differential circulating-current arrangement is used. The specification and arrangement of the current transformers is complicated by the main transformer connections and ratio. Current-magnitude differences are corrected by adjusting the turns ratio of the current transformers to account for the voltage ratio at the transformer terminals. In a differential scheme the phase of the secondary currents in the pilot wires must also be accounted for with star–delta transformers. In Figure 11.33(a) the primary-side current transformers are connected in delta and the secondary ones in star. The corresponding currents are shown in Figure 11.33(b) and it is seen that the final (pilot) currents entering the connections between the current transformers are in phase for balanced-load conditions and hence there is no relay operation. The delta current–transformer connection on the main transformer star-winding also ensures stability with through earth-fault conditions which would not be obtained with both sets of current transformers in the star connection. The distribution of currents in a Y–Δ transformer is shown in Figure 11.34.

Troubles may arise because of the magnetizing current inrush when energization operates the relays, and often restraints sensitive to third-harmonic components of the current are incorporated in the relays. As the inrush current has a relatively high third-harmonic content the relay is restrained from operating.

Faults occurring inside the transformer tank due to various causes give rise to the generation of gas from the insulating oil or liquid. This may be used as a means of fault detection by the installation of a gas/oil-operated relay in the pipe between the tank and conservator. The relay normally comprises hinged floats and is known as the *Buchholz* relay (see Figure 11.35). With a small fault, bubbles rising to flow into the conservator are trapped in the relay chamber, disturbing the float which closes contacts and operates an alarm. On the other hand, a serious fault causes a violent movement of oil which moves the floats, making other contacts which trip the main circuit breakers.

11.10 Feeder Protection

11.10.1 Differential pilot wire

The differential system already described can be applied to feeder protection. The current transformers situated at the ends of the feeder are connected by insulated wires known as pilot wires. In Figure 11.29, P and Q must be at the electrical midpoints of the pilot wires, and often resistors are added to obtain a geographically convenient midpoint. By reversing the current-transformer

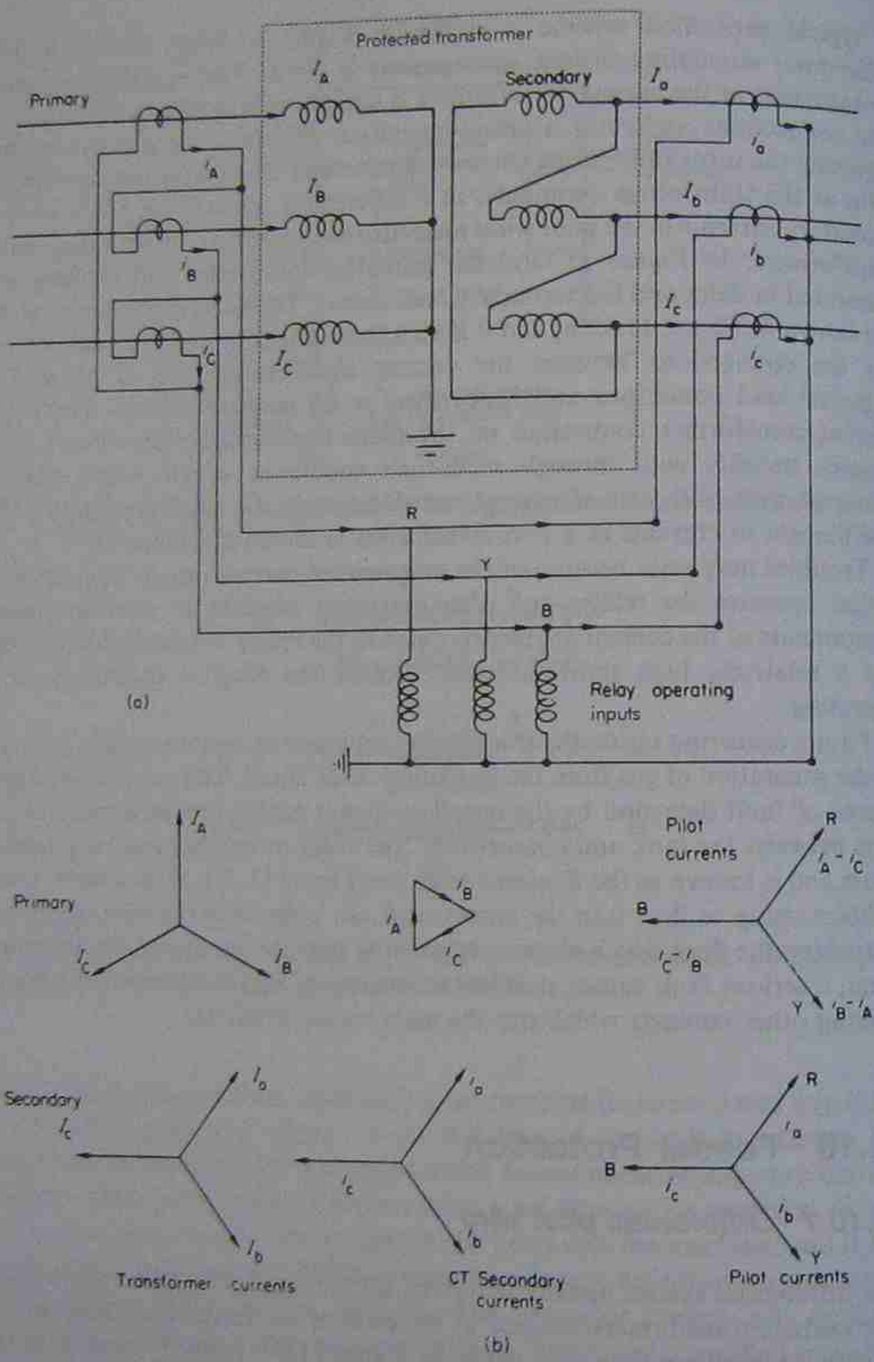


Figure 11.33 Differential protection applied to Y–Δ transformers. (a) Current-transformer connections. (b) Phasor diagrams of currents in current transformers

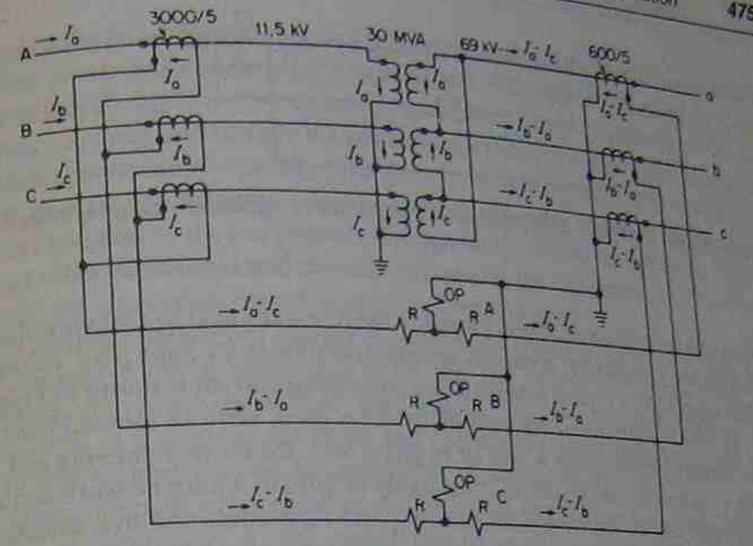


Figure 11.34 Currents in Y–Δ transformer differential protection. OP = operate input; R = restraint input

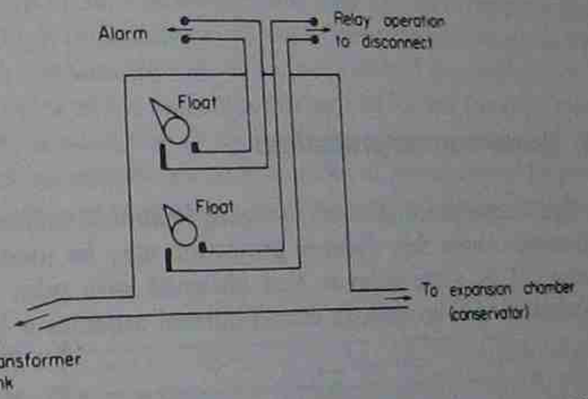


Figure 11.35 Schematic diagram of Buchholz relay arrangement

connections (Figure 11.36) the current-transformer e.m.f.s oppose and no current flows in the wires on normal or through-fault conditions. This is known as the *opposed voltage* method. As, under these conditions, there are no back ampere-turns in the current-transformer secondaries, on heavy through-faults the flux is high and saturation occurs. Also, the voltages across the pilot wires may be high under this condition and unbalance may occur due to capacitance currents between the pilot wires. To avoid this, sheathed pilots are used.

Pilot wires may be installed underground or strung on towers. In the latter method, care must be taken to cater for the induced voltages from

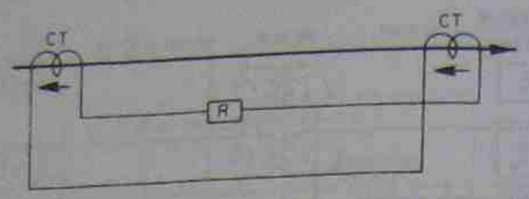


Figure 11.36 Pilot-wire differential feeder protection—opposed voltage connections

the power-line conductors. Sometimes it is more economical to rent wires from the telephone companies, although special precautions to limit pilot voltages are then required. A typical scheme using circulating current is shown in Figure 11.37, in which a mixing or summation device is used. With an internal fault at F_2 the current entering end A will be in phase with the current entering end B, as in H.V. networks the feeder will inevitably be part of a loop network and an internal fault will be fed from both ends. V_A and V_B become additive, causing a circulating current to flow, which causes relay operation. Thus, this scheme could be looked on as a phase-comparison method. If the pilot wires become short-circuited, current will flow and the relays can give a false trip. In view of this, the state of the wires is constantly monitored by the passage of a small d.c. current.

11.10.2 Carrier-current protection

Because of pilot capacitance, pilot wire relaying is limited to line lengths below 40 km (30 miles). Above this, distance protection may be used, although for discrimination of the same order as that obtained with pilot wires, carrier-current equipment may be used. In carrier-current schemes a high-frequency

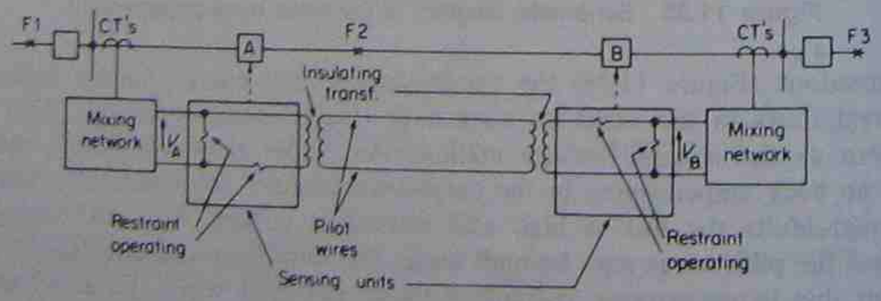


Figure 11.37 Differential pilot-wire practical scheme using mixing network (or summation transformer) and biased relays. $\mathbf{V_A} = \mathbf{V_B}$ for external faults, e.g. at F1 and F3; $V_A \neq V_B$ for internal faults, e.g. at F2

signal in the band 80–500 kHz and of low power level (1 or 2 W) is transmitted via the power-line conductors from each end of the line to the other. It is not convenient to superimpose signals proportioned to the magnitude of the line primary current, and usually the phases of the currents entering and leaving the protected zone are compared. Alternatively, directional and distance relays are used to start the transmission of a carrier signal to prevent the tripping of circuit breakers at the line ends on through faults or external faults. On internal faults other directional and distance relays stop the transmission of the carrier signal, the protection operates, and the breakers trip.

A further application, known as transferred tripping, uses the carrier signal to transmit tripping commands from one end of the line to the other. The tripping command signal may take account of, say, the operation or non-operation of a relay at the other end (permissive intertripping) or the signal may give a direct positive instruction to trip alone (intertripping).

Carrier-current equipment is complex and expensive. The high-frequency signal is injected on to the power line by coupling capacitors and may be coupled either to one phase conductor (phase-to-earth) or between two conductors (phase-to-phase), the latter being technically better but more expensive. A schematic diagram of a phase-comparison carrier-current system is shown in Figure 11.38. The wave or line trap is tuned to the carrier frequency and presents a high impedance to it, but a low impedance to power-frequency currents; it thus confines the carrier to the protected line. Information regarding the phase angles of the currents entering and leaving the line is transmitted from the ends by modulation of the carrier by the power current, i.e. by blocks of carrier signal corresponding to half-cycles of power current (Figure 11.39). With through faults or external faults the currents at the line ends are equal in

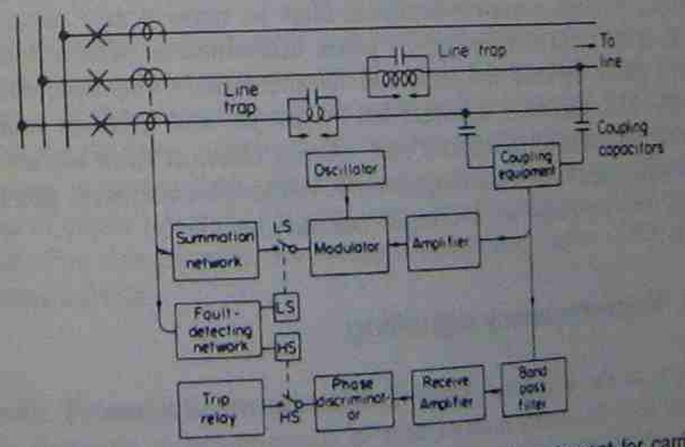


Figure 11.38 Block diagram measuring and control equipment for carrier-current phase-comparison scheme. LS = low-set relay; HS = high-set relay

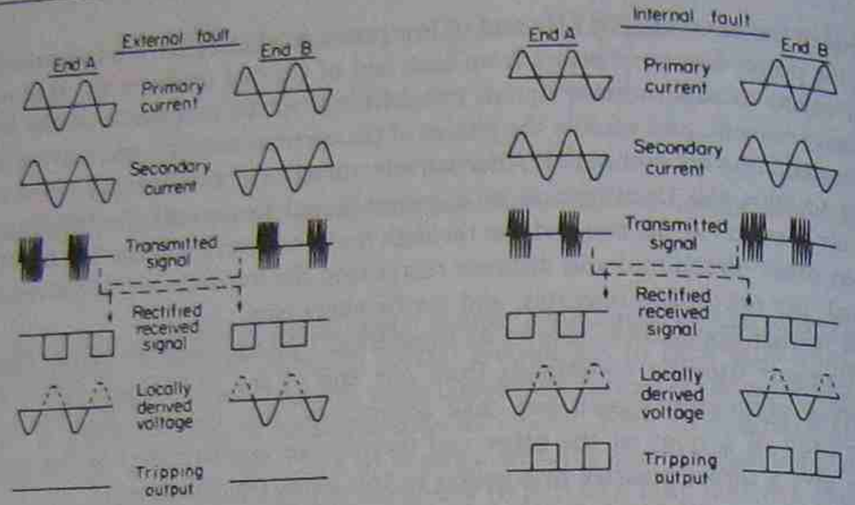


Figure 11.39 Waveforms of transmitted signals in carrier-type line protection

magnitude but 180° phase-displaced (i.e. relative to the busbars, it leaves one bus and enters the other). The blocks of carrier occur on alternate half-cycles of power current and hence add together to form a continuous signal, which is the condition for no relay operation. With internal faults the blocks occur in the same half-cycles and the signal comprises non-continuous blocks; this is processed to cause relay operation (Figure 11.39).

The currents from the current transformers are fed into a summation device which produces a single-phase output that is fed into a modulator (Figure 11.38). This combines the power frequency with the carrier to form a chopped 100 per cent modulated carrier signal, which is then amplified and passed to the line-coupling capacitors. The carrier signal is received via the coupling equipment, passed through a narrow-bandpass filter to remove any other carrier signals, amplified, and then fed to the phase discriminator which determines the relative phase between the local and remote signals and operates relays accordingly. The equipment is controlled by low-set and high-set relays that start the transmission of the carrier only when a relevant fault occurs. These relays are controlled from a starting network. Although expensive, this form of protection is very popular on overhead transmission lines.

11.10.3 Voice-frequency signalling

Increasingly, as the telephone network expands with the use of fibre-optic cabling and high-frequency multiplexing of communication channels, it is no longer possible to obtain a continuous metallic connection between the ends of unit-protection schemes. Consequently, differential protection, utilizing voice

frequencies (600–4000 Hz), has been developed. In Figure 11.37 the pilot wires (after the insulating transformers) feed into a voltage-to-frequency (v.f.) converter with send and receive channels. The summated or derived 50 (or 60) Hz relaying signal is frequency-modulated onto the channel carrier and demodulated at the far end. The signal is then compared with the local signal and if a discrepancy is detected then relay operation occurs. It is important to ensure that both signal magnitude and phase are faithfully reproduced at each end after demodulation, independently of the channel characteristics which can change or distort during adverse transmission conditions. The usual methods built into the v.f. channel relays to ensure reliability utilize an automatic control of signal level and a regular measurement (say, every 100 ms) of channel delay so that phase correction can be applied to the demodulated signal.

11.11 System Monitoring and Control

11.11.1 Introduction

For a power system to be able to supply all its customers within normal voltage and frequency limits, it must be able to ride through unavoidable disturbances, some of which could be quite abnormal. Examples of abnormal, but nevertheless credible, disturbances are:

- shunt faults and consequent line outages;
- equipment failure with subsequent isolation, e.g. generators, transformers, busbars;
- switching surges and lightning strikes;
- mechanical damage, e.g. double-circuit line tower failure.

Some of these disturbances can be dealt with by protective devices, as discussed in previous sections, and the system restored to normal within a few cycles. In these cases no further control action is needed. Others may cause transient oscillations which could last for several seconds, producing large oscillations in power flow, abnormal voltages and frequency, and subsequent tripping of plant items. If tripping occurs, then corrective control actions are required. For this purpose, an Energy Management System (EMS) is vital for any power system.

11.11.2 Energy Management System

An EMS enables engineers to operate and control the network in real time and includes facilities to capture the current state of the system and to instruct

generating plant and other controllable system components such that all consumers are supplied, at least-cost, with security. Considerable back-up facilities are necessary, including special software programs, displays, and support staff. The hierarchy of a power system with EMS is shown in Figure 11.40. At level 0 is the power system with its isolators, switchgear, interconnections, transmission lines, cables, transformers, etc.; substation (local) controls are situated at level 1. These may include protection relays, tap-change controllers, and compensator controls, with operating channels to the level 0 units. Level 1 controls often comprise digital/electronic devices for voltage and current measurement, interlocking and facilities for receiving and sending data to the next level up (area concentrators). In many cases, level 1 consists of racks of electronics within cabinets called remote terminal units (RTUs).

At level 2 (area), man–machine interfacing and data concentrators enable control and maintenance to be exercised so that the whole system can be kept in reliable and efficient condition. At the top level (3), the Supervisory Control and Data Acquisition (SCADA) system resides, usually in a single control centre (variously called Pool, National, or System control). The SCADA system accepts data from the various level 1 collectors and displays it in a meaningful way to the control engineers or operators, usually by a one-line mimic diagram on a colour video screen. With powerful computer processing, the SCADA feeds into an Alarm Management subsystem to supplement automatic relay operation and to give warning about any system abnormality that is not able to be detected locally at levels 1 or 2.

The EMS processes SCADA data in various ways, including topology identification by using the dynamic data from switchgear, isolators, and

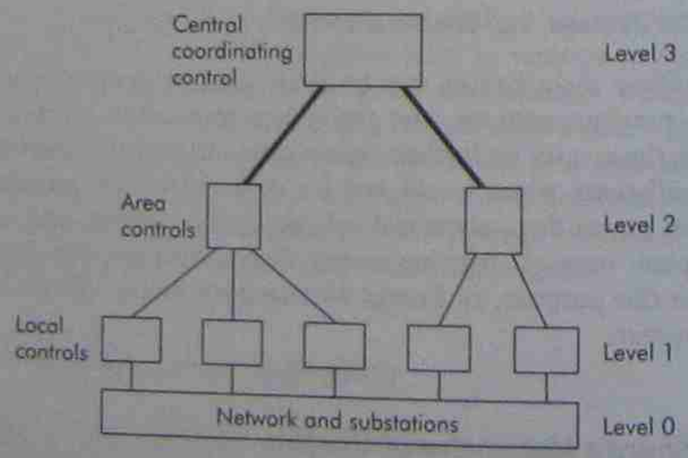


Figure 11.40 Hierarchy of controls required for an Energy Management System (EMS)

other system connectors. This 'switch' data can also be combined, in both logical and estimated fashion, with current and voltage measurements to determine the system 'state'. Such a procedure, using mathematical methods, is called 'state estimation'. In a typical system, flows and connection status (i.e. circuit breaker and isolator position) data may involve many thousands of items on which surveillance is required. Bearing in mind that this data may change minute by minute, it is obvious that the most efficient way of receiving and recording it in the control centre is by on-line digital processors.

If this data is displayed to the operator, e.g. on a video display, then, because the operator has knowledge via other sources, such as the telephone, and he has experience of similar situations arising before, he can correlate items together. If a few data items are wrong or missing, no great harm results. On the other hand, if the incoming data is to be used automatically for further computer analysis, such as contingency checking, economic scheduling, and automatic frequency control (as required in modern computer-aided control systems), then the telemetered system data must be checked for consistency. Missing or erroneous items must be found *before* they will be worth using for further processing. If the data is checked by simple and, perhaps, ad hoc methods, such as limit checking and redundant measurement comparison (e.g. two busbar voltages should be of similar value if the busbars are connected together), then the process is called *data validation*. It can be performed within a single set of measurements from a substation at level 1 before being telemetered to the control centre, or it can be carried out at the control centre before being stored in the data base.

11.11.3 The basis of Power-System State Estimation (PSSE)

State estimation is used in modern control theory to enable processes to be controlled, mainly by an on-line digital computer. In a power system, the control engineer is primarily concerned with operation of the network in the steady state. Hence the process of 'static-state estimation' is normally applied. As computer methods and hardware continue to evolve (particularly powerful microprocessors), it may be possible to control the system in the dynamic or transient state, for which 'dynamic state estimation' could be employed.

Power-System State Estimation is defined as follows:

> PSSE is a process whereby data, telemetered from network measuring points to a central computer, can be formed into a set of reliable data (the 'data base') for control and recording purposes.

A *static-state estimator* is obtained from measurements taken within a short time interval (e.g. 5–10 s) and these values are the only ones used for

the estimate (this is sometimes called the 'snapshot' estimator, for obvious reasons).

More measurements are actually taken than are required to identify the system at any given moment, and redundant measurements are available to 'observe' the system. In Chapter 6, load flows were calculated from given data which was just enough for a unique solution to be obtained. Common sense tells us that if we were to take a snapshot of our power system, a unique set of flows and voltages would be measured. And yet common practice is to measure, for example, power and reactive-power flows at both ends of an overhead line for which we know the impedance. Consequently, we strictly only require flow measurements at the *near* end of each line—we can then calculate the expected flows at the far end and compare them with the measured values.

Averaging of the two values provides an 'estimate' of the flows which, if our method is any good, can give us a better result than either of the original measurements. This, then, is our estimation process; namely, an attempt to minimize the errors in our measurements by statistical calculation. Note that we must assume that measurement errors are randomly distributed among our system measurements and that, if we take enough measurements, we can minimize the error through the estimator.

Once we have estimated all our measurements for one system 'snapshot', then the measurements can be stored in a data base and used for further calculation or even for updating the display to the operator. Control action required as a result of this latest information can then be made on the power system, thus completing the loop, as depicted in Figure 11.41.

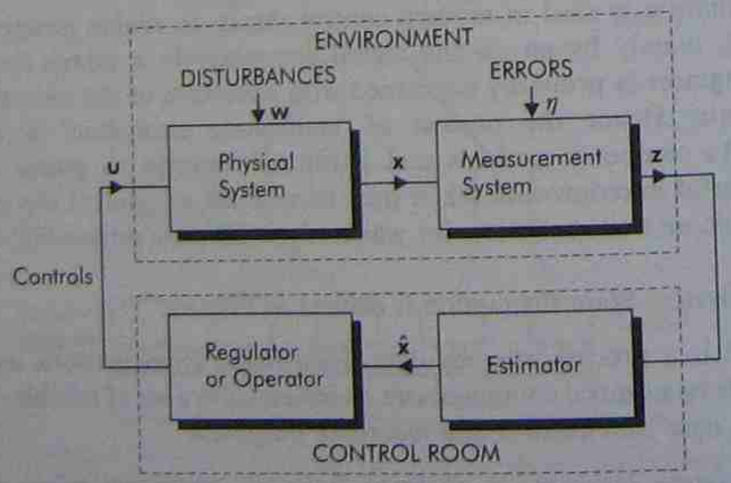


Figure 11.41 Outline of estimation system

11.11.4 Mathematical description of the PSSE process

The practical application of PSSE requires four main subroutines, as indicated by the flow diagram of Figure 11.42.

In PSSE the system states are usually denoted by the vector **x**. It is important to define these states and to make sure that there are enough, but only *just* enough, to uniquely describe the system. Now, in the load-flow problem, it is usual to calculate the nodal voltages from the given data and to use them subsequently to calculate line flows, and generator and load injections. We know that, provided we have knowledge of the network configuration and its nodal admittances, the complex nodal voltages define the state of the system. We therefore let

$$\mathbf{x}^t \equiv [a_1, a_2, a_3 \ldots a_n, b_2, b_3 \ldots b_n]$$

or

$$\mathbf{x}^t \equiv [x_1, x_2, x_3 \ldots x_n]$$

where voltage at node k is $V_k = a_k + jb_k$ and there are n nodes in the network. Note that we have omitted b_1 from $\mathbf{x}^t$ because node 1 is taken as the reference node, where $b_1 = 0$. The vector **x** contains $(2n-1) = N$ elements for which, in a load-flow solution, we would require $(2n-1)$ equations to determine. Of course, **x** can be expressed in polar form, if desired, in which case it will contain the same number of elements but there would be n voltage magnitudes and $(n-1)$ angles.

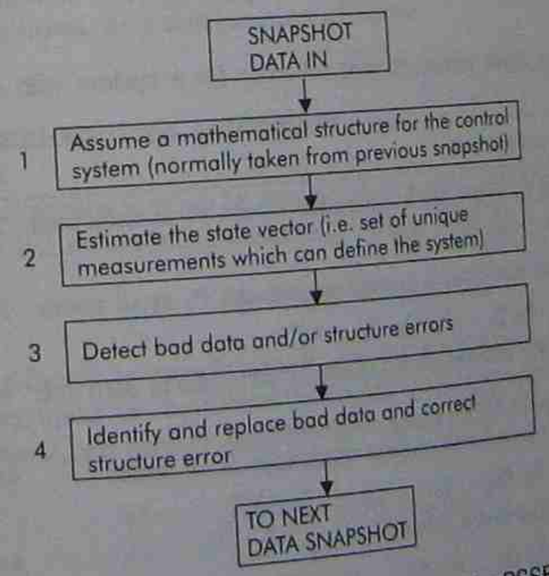


Figure 11.42 Subroutines required for PSSE

In PSSE the input-data is not necessarily in the form of busbar voltages because these are not always easy to measure with accuracy. In any case, measurement of phase angle between substations many miles apart is quite expensive. It is usually much cheaper and more direct to measure P and Q flows on many three-phase circuits, generator outputs, transformer feeds, etc., and to telemeter these to the control centre. Consequently, measurements could consists of any combination of P and Q flows or injections, busbar voltages, or perhaps even current flows where they are measured in preference to P or Q, e.g. on cables or in distribution systems.

As well as a mixture of measurements, say M in number, we know that, in practice, each measurement contains an error η, which we hope will be randomly distributed around a mean value. Consequently, we can state that our M measurements (represented by $\mathbf{z}$) must be a non-linear function $h(\mathbf{x})$ of the state variable $\mathbf{x}$ with a 'noise' vector $\boldsymbol{\eta}$ so that

$$\mathbf{z} = h(\mathbf{x}) + \eta \tag{11.1}$$

This is our fundamental PSSE equation. It contains the load-flow equations developed in Chapter 6, using equations (6.9) and (6.10), from which flows P_{kj} and Q_{kj} from node k to node j can be derived.

Table 11.1 gives five ways of making up the vector $\mathbf{z}$ from sets of measurements. Case 1 corresponds to the load-flow calculation whilst case 5 assumes that all possible measurements are taken so that the redundancy of the measurements is a maximum. In practice, the redundancy will be somewhere between these two extremes, defined as

$$\text{Redundancy} = \frac{\text{number of measurements}}{(2n-1)}$$

Table 11.1 Five basic measurement schemes for a system with n buses and m lines

Case	Components of $\mathbf{z}$	Dimension $\mathbf{z}$
1	P_k and Q_k injections at all nodes (except P_1 at slack bus) $\mathbf{z} = [P_2 \ldots P_k, Q_1 \ldots Q_k]$	$2n-1$
2	Same as case 1, with voltage magnitudes V_k at all nodes $\mathbf{z} = [\ldots P_k, Q_k \ldots P_n, Q_n, V_1 \ldots V_k]$	$3n-1$
3	Active P_{kj} and reactive Q_{kj} flows at both ends of each line $\mathbf{z} = [\ldots P_{kj}, Q_{kj}]$	$4m$
4	Same as case 3 plus voltage magnitude at all nodes $\mathbf{z} = [\ldots P_{kj}, Q_{kj} \ldots V_1 \ldots V_n]$	$4m+n$
5	Full-measurement system $\mathbf{z} = [\ldots P_k, Q_k \ldots P_{kj}, Q_{kj} \ldots V_1 \ldots V_n, \theta_2 \ldots \theta_n]$	$4n-1+4m$

For any system, the economic case for PSSE depends upon the cost of redundant measurements weighed against the savings obtained due to better system control. Since, in most cases, the economic justification for control is difficult to establish, considerable off-line study needs to be done before a PSSE scheme is installed.

11.11.5 Minimization technique for PSSE

The fundamental PSSE equation has already been derived:

$$\mathbf{z} = h(\mathbf{x}) + \eta \tag{11.1}$$

and since this contains an error vector η, it cannot be 'solved' to give a unique solution as in the csse of the load-flow equations of Chapter 6. However, if it is possible to reduce the errors to a minimum, a good value of the state vector $\mathbf{x}$ can be determined. However, these will only be 'best' or 'optimal' estimates of $\mathbf{x}$ (denoted by $\hat{\mathbf{x}}$) and will never be the 'true' values, although we would hope that they could approach the true values if we could take enough measurements. In practice, the control engineer is happy if he/she feels that he/she knows their flows and voltages to between 1–2 per cent accuracy, but it must be remembered that if we are determining economic operating conditions, an error of 1 per cent in voltage could make a large difference in the calculation of losses on the system. Consequently, it is worthwhile spending some time and effort on obtaining the best error minimization for on-line application in a particular system. This method could well depend upon the system characteristics (closely meshed or long lines), as well as the type of measurements taken (injections, line flows, or a combination of both).

11.11.6 Least-squares estimation

The optimal estimate $\hat{\mathbf{x}}$ is given by that value of $\mathbf{x}$ for which the scalar sum J of the squares of the errors has a minimum. This is given by

$$\text{min. } J = [\mathbf{z} - h(\mathbf{x})]^T R^{-1} [\mathbf{z} - h(\mathbf{x})] \tag{11.2}$$

where R^{-1} is a factor or 'weight' which is inserted to obtain a better minimization depending upon the nature of the 'noise' or 'error' vector η.

To minimize J, we need to take its derivative and set it equal to zero, thus

$$\frac{\partial J}{\partial x} = -2\left[\frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}\right]^T R^{-1}[\mathbf{z} - h(\mathbf{x})] = 0 \tag{11.3}$$

where

$$\left[\frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}\right]$$

is the Jacobian matrix, as used in the Newton–Raphson load flow, consisting of elements as follows:

$$\left[\frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}\right] = H(\mathbf{x})\Delta \begin{bmatrix} \frac{\partial h_1(x)}{\partial x_1} & \cdots & \frac{\partial h_1(x)}{\partial x_N} \\ \vdots & & \vdots \\ \frac{\partial h_M(x)}{\partial x_M} & & \frac{\partial h_m(x)}{\partial x_N} \end{bmatrix} \tag{11.4}$$

and is of dimensions M (no. of measurements) $\times$ N $(= 2n - 1)$.

Now we have to solve equation (11.3) by any means at our disposal in order to obtain $\mathbf{x}$ (our state variable defining the system), which will actually be $\hat{\mathbf{x}}$, i.e. our best estimate of the system states.

Many methods are available to solve equation (11.3) and these are reported in the literature, but it is worthwhile to note the following points.

1. The dimensions of $\mathbf{z}$ and $\mathbf{x}$ can be quite large (e.g. 200 nodes, 500 measurements).
2. The Jacobian $H(\mathbf{x})$ is sparse.
3. Elements of $H(\mathbf{x})$ and $h(\mathbf{x})$ have similar properties so that algorithms written generally to obtain $h(\mathbf{x})$ can be used to obtain $H(\mathbf{x})$.
4. Sparsity programming and optimal ordering can be used effectively to minimize storage and computation time. Yet again, it pays to study the system before implementing PSSE.

An efficient solution is as follows:

Use the iteration process of

$$G^p[\mathbf{x}^{p+1} - \mathbf{x}^p = H(\mathbf{x})^p R^{-1}[z - h(\mathbf{x})^p] \tag{11.5}$$

where superscript p denotes the iteration number and $\mathbf{G}$ is a 'gain' matrix which must be chosen, usually by trial and error, as in the case of an acceleration factor for a Gauss–Seidel load-flow solution. The unit matrix of full rank can be used if no experience of the measurement system is available beforehand.

An initial guess x^0 is required to start the process and, at convergence, $\mathbf{x}^{p+1} = \mathbf{x}^p = \hat{\mathbf{x}}$, although as usual, convergence is assumed if $|x^{p+1} - x^p| <$ tolerance. As noted in Chapter 6, since equation (11.5) is very sparse, sparsity techniques can be used to speed up solution time and reduce storage. Figure 11.43 gives a flow diagram for this computation.

It should be noted that simplifications of the algorithm for $\hat{\mathbf{x}}$ calculation can often be obtained if the 'decoupling' between P–θ and Q–V is utilized. In some closely meshed networks, as in the NGC system for example, it may only be

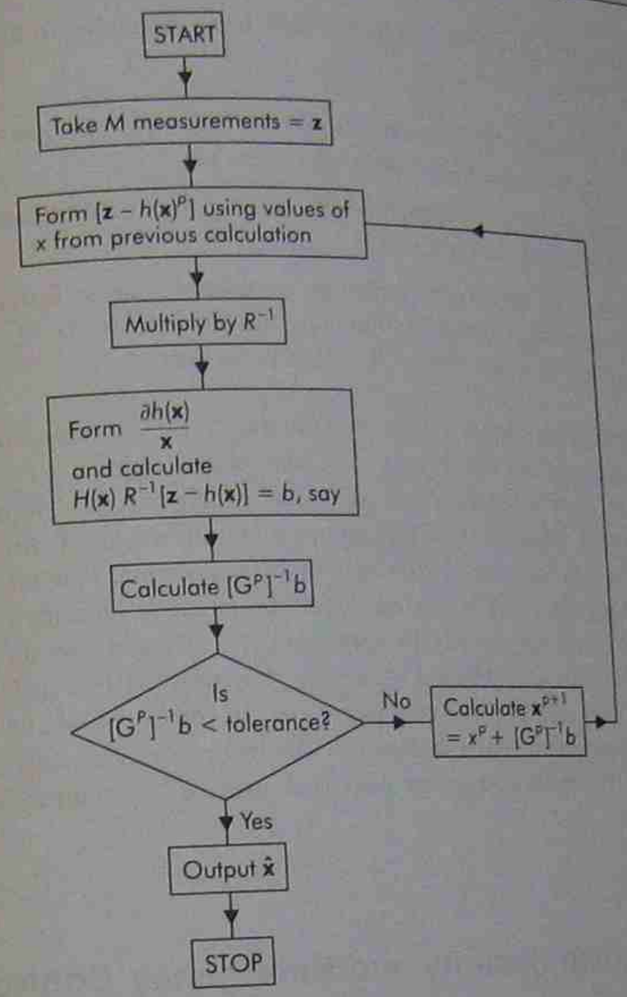


Figure 11.43 Flow diagram for calculation of $\hat{\mathbf{x}}$. (Note: For Gauss–Seidel calculations, an inner loop, changing one element of $\mathbf{x}$ each time, is required)

necessary to estimate P flows from a P–θ calculation using only P measurements plus one or two voltages. This considerably eases the on-line computation problem, as well as reducing the amount of telemetered data required.

11.11.7 Errors and detection in PSSE

The criterion for a good estimator is not only that it is suitable for on-line calculation, but also that it can detect data errors, identify the cause, and apply a correction to the estimation process automatically. A full appreciation of the possibilities requires a knowledge of probability theory, but an indication of what can be done with practical estimators will be given in this section.

An estimate $\hat{\mathbf{x}}$ of the state variables may be in error due to any or all of the following causes:

1. metering errors, e.g. non-zero time constant of some instruments, non-linearities, noise on telemetering channels, etc.;
2. bias error in the measurement—this should be constant and it could be accounted for in the data if it is known;
3. bad data, e.g. observations during transients, major failures of meter/communication, errors in assumed system parameters G, B, etc., network structure errors unknown to the data processor.

Consequently, to *detect* errors, a statistical test is required during the process of determining the estimate of the system states. In practice, it is convenient to have a single 'residual' value which can be compared with a pre-set threshold value after each estimate $\hat{\mathbf{x}}$ is completed. If this threshold is exceeded then a process of *identifying* which measurement or set of measurements is causing the error is started. Once the cause is identified, the estimator has to be *corrected*, usually by removing the affected measurement, after which the estimator can proceed as before, but now with a little less redundancy. Obviously, the control engineer must be informed of the removal of some measurements so that the cause of the discrepancy can be investigated and put right before that measurement is reinserted, by manual action, into the estimator.

11.12 System Security and Emergency Control

It has already been stated that the reason for designing and operating a system in a meshed form is to provide a path from every generator to every load, despite the possibility that one or two circuits could be outaged. A network configuration and loading state which enables any one circuit to be outaged without loss of supply to any load is called $n-1$ secure. To determine the secure network configuration and state (i.e. loading) required for each hour of the day is a daunting task, even with large and powerful computers. Normally, secure network states are calculated for a few representative loading conditions (usually up to 6 per day), including the daily peak, night minimum, and intermediate subpeak conditions.

Nowadays, optimal power flows with many constraints are employed to determine the secure network configuration and economic loading, including any constrained-on or constrained-off plant. Security calculations are made at least 24 h ahead by many a.c. or d.c. load flows, taking out critical circuits one-by-one. Adjustments to the generator commitment and loading are made by

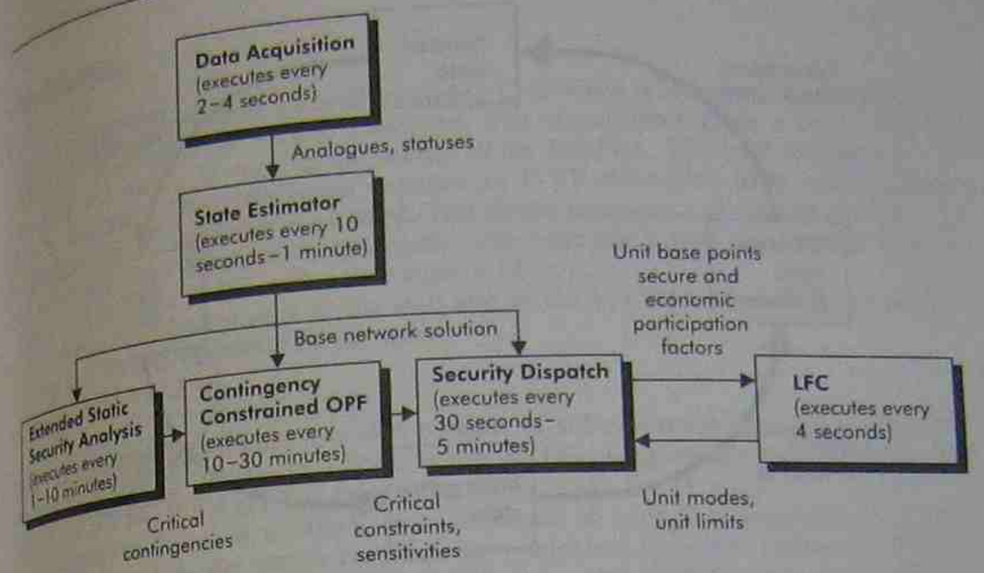


Figure 11.44 Optimal power flow (OPF) in secure optimal automatic generation control. LFC, load frequency control. (*Permission of Siemens Power System Controls, U.S.A.*)

feedback rules or by a 'contingency-constrained optimal power flow (OPF), which outputs a generator schedule, as illustrated in Figure 11.44.

Other constraints within which the operator must work are the requirements (1) to hold enough VAr reserves at strategic nodes in the system such that voltages can be maintained despite loss of a circuit, compensator, or generator; and (2) that across pre-defined boundaries, flows should not exceed certain values, otherwise a credible fault could plunge the system into instability. Both voltage-secure and stability-secure situations require considerable pre-event study and the use of sophisticated planning and operating software utilizing data from SCADA. Many programs to automatically derive secure dispatch and network configuration schedules are still under development and this provides a fruitful area for research.

11.12.1 *Emergency control* (Figure 11.45)

The following system-operating states and transition between states may be identified.

1. *Normal* to *alert*—reductions of security level. This could be caused by unexpected load increases, loss of generating units, derating of plant due to environmental constraints, and rescheduled maintenance.

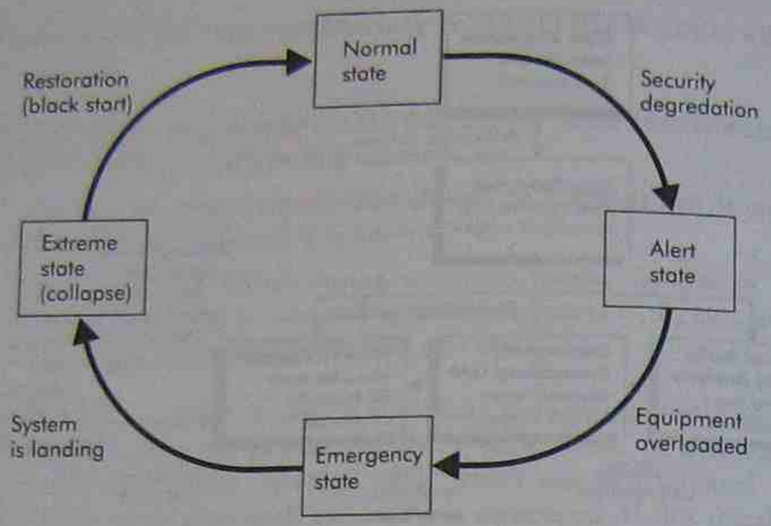


Figure 11.45 System collapse and restoration process

2. *Alert* to *emergency*—inability of parts of system to meet requirements, e.g. lines (emergency ratings), voltage levels, frequency, machine and bus voltage angles. Caused by malfunction of protection, lightning, etc.
3. *Emergency* to *extreme condition* (collapse)—loss of integrity. Caused by loss of ties resulting in isolated generation islands which are unable to carry their internal loads. This is triggered by prolonged overloading of critical ties, malfunction of protection, and successive disturbances.

Corresponding control measures to meet the above situations are as follows.

1. *Alert*—This involves restoration of reserve margins, increased generation reserves, rescheduling of tie-lines, and voltage reduction.
2. *Emergency*—This involves fast valving on steam-turbines, dynamic braking of generators, load control, capacitor switching, and immediate action to clear equipment overloads.
3. *Extreme*—This involves all of the above plus load shedding and controlled operation of isolated power groups.

Following these emergencies action is taken to re-establish a viable system. This involves restarting and synchronization of generation units, load restoration, and resynchronization of all areas. Although action to prevent transient instability of individual generators has been a major factor in emergency control, such instabilities are not necessarily a major factor in system-extreme emergencies. Local immediate (reflexive) action may prevent damage to the equipment involved, but the resulting system security may be reduced to a dangerous level.

Problems

11.1 A 132 kV supply feeds a line of reactance 13 Ω which is connected to a 100 MVA, 132/33 kV transformer of 0.1 p.u. reactance. The transformer feeds a 33 kV line of reactance 6 Ω which, in turn, is connected to an 80 MVA, 33/11 kV transformer of 0.1 p.u. reactance. This transformer supplies an 11 kV substation from which a local 11 kV feeder of 3 Ω reactance is supplied. This feeder energizes a protective overcurrent relay through 100/1 A current transformers. The relay has a true inverse-time characteristic and operates in 10 s with a coil current of 10 A.

If a three-phase fault oocurs at the load end of the 11 kV feeder, calculate the fault current and time of operation of the relay.
(Answer: 1575 A; 6.35 s)

11.2 A ring-main system consists of a number of substations designated A, B, C, D, and E, connected by transmission lines having the following impedances per phase (Ω): AB (1.5 + j2); BC (1.5 + j2); CD (1 + j1.5); DE (3 + j4); EA (1 + j1).

The system is fed at A at 33 kV from a source of negligible impedance. At each substation, except A, the circuit breakers are controlled by relays fed from 1500/5 A current transformers. At A, the current transformer ratio is 4000/5. The characteristics of the relays are as follows:

Current (A):		7	9	11	15	20
Operating time(s)	relays at A, D, and C:	3.1	1.95	1.37	0.97	0.78
	relays at B and E:	4	2.55	1.8	1.27	1.02

Examine the sequence of operation of the protective gear for a three-phase symmetrical fault at the midpoint of line CD.

Assume that the primary current of the current transformer at A is the total fault current to the ring and that each circuit breaker opens 0.3 s after the closing of the trip-coil circuit. Comment on the disadvantages of this system.

11.3 The following currents were recorded under fault conditions in a three-phase system:

$$I_R = 1500\angle 45^\circ \text{ A}, \qquad I_Y = 2500\angle 150^\circ \text{ A}, \qquad I_B = 1000\angle 300^\circ \text{ A}.$$

If the phase sequence is R–Y–B, calculate the values of the positive, negative, and zero phase-sequence components for each line.
(Answer: $I_0 = (-200 + j480)$ A)

12
Basic Power-System Economics and Management

12.1 Introduction

It is important that all power-system engineers understand the economic context within which the generation, transmission, distribution, and supply of electrical energy to customers operates. Figure 12.1 illustrates the hierarchy that exists between the whole economy of a country (the *macroeconomy*) and the *electricity subsector*. Note that at the *intermediate level*, competition cannot exist between the different forms of energy unless some form of centralized energy-sector planning is in place. Economists and national planners (i.e. governments) attempt to model the interactions between sectors and subsectors so that development plans and financial investments can be undertaken. Each sector has its 'participants' who are usually experts in their own particular field.

Economic theory tries to coordinate the decisions of participants to produce the 'best' outcome. Unfortunately, 'best' in this context is almost impossible to define because participants in the energy game will have different and often conflicting objectives. This section therefore gives, first, a brief outline of what is known as welfare economics and how they can be applied to the electricity industry in order to provide maximum *social welfare* for the various consumers. It is assumed throughout that the well-being of the consumer is paramount and that pricing policies are directed towards producing maximum benefits in consuming electrical energy in efficient and effective ways. However, a good pricing policy should not only provide maximum benefit to consumers, it should also signal any desirable reallocation of resources in society. This

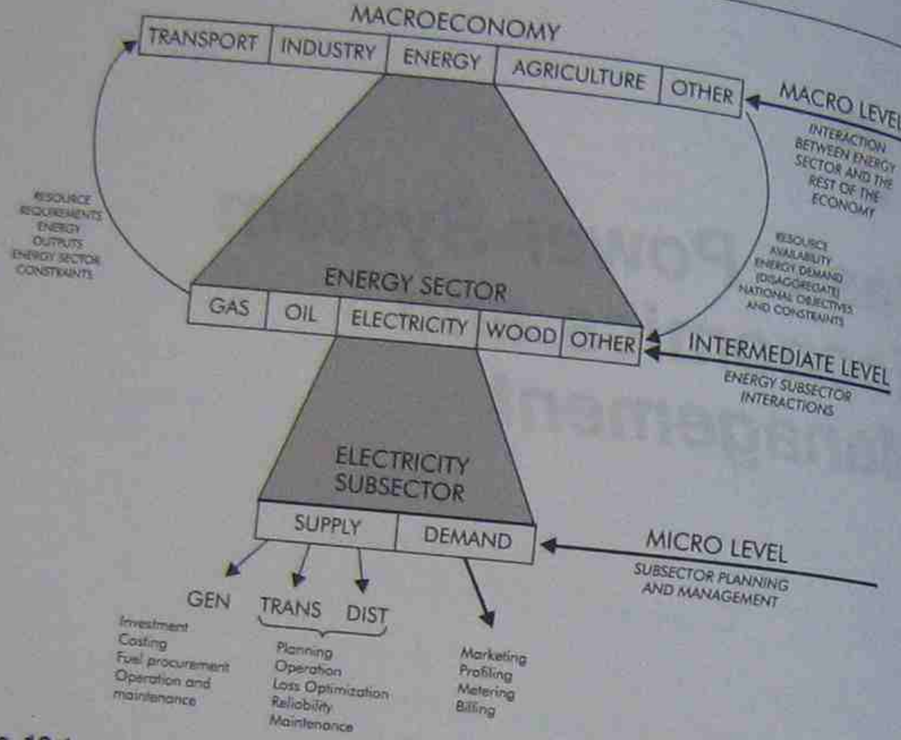


Figure 12.1 Hierarchy of energy within the national economy showing electricity subsector aspects (*Modified from Monasinghe, Electric Power Economics, Butterworths, 1990. Acknowledgement to Dr Mohan Monasinghe, World Bank*)

latter objective of pricing is extremely difficult to achieve with simple and easy-to-understand tariffs.

As electrical energy cannot be conveniently stored, its production and use are simultaneous. Hence the well-known supply–demand equation must always apply. On the *supply side* are the methods of generation (thermal—coal, oil, gas; nuclear, hydro, wind, solar, etc.), transmission, and distribution of electrical energy; whilst on the *demand side* are the consumers with their large varieties of end-use equipment. Whereas the supply side consists generally of large companies or utilities, the demand side ranges from individual residential consumers to large industries whose existence in a competitive market place depends crucially on stable and economic prices for the energy consumed.

12.2 Basic Pricing Principles

Pricing methods (such as a high-day but cheap-night tariffs) can cause consumers, over a period of time, to change their consumption pattern so that

their costs are minimized. However, presumably the supplier will still charge sufficient to cover the cost of production and overheads, so it would seem that one group of participants (the consumers) could well benefit without any change to the other group of participants (the suppliers). Such a change, in which one group benefits without affecting any others, is called a *Pareto improvement*, named after the nineteenth-century economist and philosopher, Vilfredo Pareto.

There is obviously a limit to the gains which can be made by the consumer without affecting the supplier, so economic efficiency is said to be maximized where no Pareto improvements are possible, i.e. no alternative pricing policy can make one group of participants better off and no other group worse off. Thus, it is tempting to argue that economic efficiency should enable everyone to benefit in an ideal world. Unfortunately, having everyone on an equal footing does not necessarily mean that everyone is getting good value for money, so a measure of total benefit (supplier plus consumer) is needed.

This produces the idea of measuring *social welfare* as the sum total of the monetary benefits accruing to every individual and group from a given tariff, including many intangible, but nevertheless useful, benefits, reckoned in monetary terms which both suppliers and consumers can experience. A good measure of useful benefit is the willingness to pay for it, leading to the idea of optional alternatives or *choice* for the supplier and/or the consumer.

The willingness to pay is often reflected, in the consumer, by the *sensitivity* to price changes known as *demand elasticity*. For electricity, where the energy consumed is related to income (above a certain minimum level), *income demand elasticity* is given by

$$\eta = \frac{(\Delta Q/Q)}{(\Delta m/m)} \tag{12.1}$$

where $(\Delta Q/Q)$ is the percentage change in consumption as a result of a given percentage change in income $(\Delta m/m)$.

It is useful to have a measure of demand elasticity in deciding upon pricing policies, but, unfortunately, it is extremely difficult to measure because of the many conflicting effects of any income change, e.g. purchase of substitute fuels, changing social climate. These *cross elasticities*, as they are called, also need to be accounted for in any tariff study if the outcome is to be valid. In Figure 12.1, the energy sector would need to be considered for its interactions via cross elasticities.

On the supply side, the generator of electrical energy presumably has a full knowledge of costs of production dependent upon percentage of available capacity that is actually producing electricity and the time schedule. Now, total revenue (R) is the sum of *variable costs* (VC), dependent upon output; *fixed costs* (FC), which are independent of output; and *profit* (P).

Thus

$$P = R - VC - FC \tag{12.2}$$

and if P is to be maximized, then *marginal revenue* (MR), defined as increase in revenue per unit increase in quantity sold, should be equal to *marginal cost* (MC), the corresponding change in variable costs.

Figure 12.2 shows a *demand curve* D in terms of quantity (kWh) bought by consumers in a given period plotted against *price* that consumers are willing to pay. It is assumed that as the quantity of electrical energy sold increases, its price decreases. The marginal cost of production is shown by line MC, which crosses the demand line D at a quantity of electricity (over a period) Q_1 at a price P_1. The *consumer surplus* is then taken as being proportional to area CS, and producer or supplier surplus as proportional to area PS. The total surplus is $(CS + PS)$ but this is, of course, divided between a number of consumers and suppliers, any of whom can try to increase their own 'surplus' by varying their demand or their 'price'. It should be noted that by reducing MC, the proportion of surplus to suppliers can be increased, thereby producing more income for producers.

For a supplier to maximize profit, then optimal theory suggests that (from equation (12.2))

$$\frac{dP}{dQ} = 0 \text{ for max.} = \frac{dR}{dQ} - \frac{dVC}{dQ} - \frac{dFC}{dQ}$$

where dQ is the change in output.

Now (dVC/dQ) is known as the *short-run marginal cost* (SRMC) and is an important factor in any pricing policy. If (dFC/dQ) is not zero, then

$$\left(\frac{dVC}{dQ} + \frac{dFC}{dQ}\right)$$

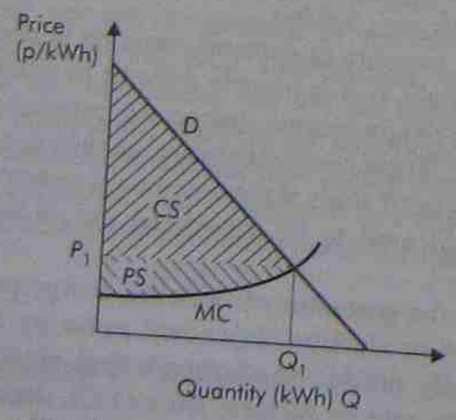


Figure 12.2 Price/quantity relationship. D = Demand by consumers; MC = marginal cost of generation; CS = consumer surplus; PS = producer surplus

must be equal to (dR/dQ) for profit maximization, and this is the *long-run marginal cost* (LRMC) which takes into account the changes in the fixed costs (e.g. the cost of installing more generators) to make a profit in the future. The nearer utilities can structure themselves to providing output to SRMC, the more likely they are to make an adequate profit as the actual consumer demand changes. *Economic efficiency* is achieved when a supplier's income is equal to the marginal cost of production.

12.2.1 Ramsey pricing

In 1928, Ramsey published a famous paper (Ramsey, 1928) which gave a rule for the pricing of a commodity when SRMC pricing was not possible. Sometimes, SRMC is called *first-best* pricing and any other form of pricing is *second-best*, particularly where utilities are unable to make a profit from SRMC and need to price above it to provide for new or replacement plant.

In the same way that we defined income elasticity η, we can define *demand elasticity* ε as

$$\varepsilon = \frac{(\Delta Q/Q)}{(\Delta P/P)} \tag{12.3}$$

where $(\Delta Q/Q)$ is change in consumption and $(\Delta P/P)$ is the corresponding change in price (rather than income). To increase revenue, a supplier could increase price, but preferably only to those consumers (or markets) which are least sensitive to price changes, i.e. their demand elasticity is small. The 'mark-up' in price for the least-sensitive consumers should then alter as little as possible from the SRMC pricing policy. So a reasonable formula for a second-best pricing policy would be to set the price change according to some proportion of the demand elasticity ε. This can be expressed as

$$\text{Price increase (called mark-up)} = \frac{P_i - MC_i}{P_i} = \frac{\lambda}{\varepsilon_i}$$

where P_i = price to consumer group i;
MC_i = the marginal cost for consumer group i;
ε_i = the demand elasticity;
λ = a weighting factor.

If several groups of consumers are involved then we require that

$$\left(\frac{P_i - MC_i}{P_i}\right)\varepsilon_i = \left(\frac{P_j - MC_j}{P_j}\right)\varepsilon_j = \lambda$$

for the second-best pricing policy in the ith and jth groups. (This can be proved by using Lagrangian multipliers in a mathematical formulation.) This rule is

called the *Inverse Elasticity Rule* (IER) and the pricing policy which maximizes total surplus is known as *Ramsey pricing*.

In a *benefit maximization* calculation, taking into account the various types of consumer, the cost–quantity relationships on the supply side are combined with price elasticity on the demand side to maximize total benefit subject to constraints such as limits on supply capacity, cost constraints imposed by government, and maximum price for 'lifeline' supplies, i.e. a cap to the maximum price any consumer can be asked to pay. Obviously, if electricity supply is a *monopoly* (or any part of supplying consumers is a monopoly business, e.g. transmission), then some imposed constraints are necessary to prevent customers being 'ripped off'. Either direct government controls are required or a 'Regulator' is appointed to carry out the price-control function. If market competition can be set up, then no price regulation is necessary, in theory, except to ensure a 'level playing field' for all participants.

12.3 Supply-Side and Demand-Side Options

Because electricity demand and consumption have been notoriously difficult to forecast over the past 15–20 years, it is now recognized that *uncertainty* in forecasting must be accepted. This uncertainty arises for the following reasons:

1. The price of fuel now fluctuates on worldwide markets almost daily.
2. Costs of construction vary due to property and land price variations.
3. Environmental constraints are forever changing, dependent upon pressure groups and 'green' policies.
4. Uses of electricity for comfort and convenience can change with local climatic conditions.
5. The effects of economic and political changes are increasingly difficult to estimate.

This uncertainty requires a change in approach, particularly as in the U.K., privatization is producing a rethink of supply-side policies. In fact, one possible approach is to consider supply-side and demand-side options on an equal basis. Economic theory would then tend to support this approach as a way of ensuring an *optimal social outcome*.

12.3.1 Supply-side options

The most important option available to the supplier is that of price. It can be viewed as the main coordinating signal, not only to the consumer to influence investment decisions, but also to other suppliers in a competitive economy. By

setting prices using the principles given in Section 12.2, a number of varying objectives can be achieved. For example:

- Economic efficiency of the suppliers can be guaranteed.
- Government policies, such as safeguarding future supplies from indigenous resources, subsidies to rural areas, and target rates of return or profit on assets, can be achieved.
- Revenue can be raised to cover costs of investment, management, operations, and maintenance.
- The market share of electricity can be influenced compared with other fuels.

Depending upon the supplier's policy, the options open include:

- scheduling generation to maximize income;
- investing in new plant for future returns;
- purchasing energy from neighbouring utilities or local generators;
- diversifying the business by, for example, supplying heat, aggregates from ash, by-products of flue cleaning, etc.

12.3.2 Demand-side options

On the demand or consumers' side of the electricity meter, the options available should endeavour to contribute to the economic efficiency of the system as a whole. These endeavours should be guided mainly by the pricing policies set by the suppliers. Opportunities to exploit demand-side options include:

- use of alternative fuels (i.e. fuel substitution) to achieve objectives, e.g. minimum annual cost, better quality product;
- self-generation or private-contract purchase of electricity;
- improvement in end-use efficiency, e.g. more insulation in buildings and appliances;
- lifestyle changes because of conservation issues;
- end-use storage (hot and cold, compressed gas, etc.).

With proper choice of options, the user theoretically can obtain cheaper operating costs and, because of the shorter lead times compared with increasing generator capacity, the user can end up with less risk-taking on capital investments than on the supply side. Thus, one answer to coping with uncertainty is to encourage consumers to exercise their available options to maximize both supply-side and demand-side benefits.

12.4 Load Management and Spot Pricing

One way of maximizing benefits is to have some central coordinated control of demand so that it always matches the power being generated by the suppliers, who, in turn, have a means of measuring the benefit of that demand to the economy. It is well known that such a benefit cannot be measured for every consumer because of the large diversity of needs and objectives. As we have seen, the only appropriate and acceptable measure is that of price and willingness to pay. Consequently, according to competition economic theory, a market in electricity is required corresponding to the markets in oil, coal, gas, etc. The difficulty, of course, is that the infrastructure necessary to inform all consumers of the price fluctuations in the purchase of the next kWh of electrical energy is very costly, since the price will vary with each generator on the system, implying a price change every half-hour at least. There are basically two ways of achieving the objective:

1. The consumer can enter into an agreement with the supplier to exercise direct physical control of electrical appliances when it is advantageous to do so, e.g. because no more generation or transmission capacity is available (peak lopping) or because starting up another generator would push up the price of electricity beyond that which the consumer is willing to pay. This is called *load management* and usually implies direct control of consumer load by the supplier. The methods which can be used are:
   - telecommunication networks—two-way is possible, using telephone, video cabling, etc.
   - radio communications—two-way is expensive, but teleswitching over a radio broadcast band by modulating the carrier can give extensive one-way coverage very cheaply.
   - mains-borne signalling—restricted to areas around injection point on network.
2. Alternatively, the price can be transmitted to the consumer and he/she then determines how to react, either by manual or by automatic control of selected loads.

In both cases, a variation of price broadcast by the supplier is an advantage, known as *spot pricing*. In general, the spot price should follow the SRMC since it should depend upon the availability of generators and the state of the end user. However, it can be shown that SRMC pricing is essential if economic efficiency is to be obtained. It thus tracks the supply–demand balance. Schweppe et al. (1988) have written at length on the various methods and technologies now available for spot-pricing policies. Spot pricing cannot be

pre-determined because of changing system conditions, thus leading to *probabilistic forecasting* of possible price variations.

Flexible consumers or suppliers who can respond to changing prices will be able to improve their profits by reducing consumption or increasing cogeneration when prices are high and by increasing consumption or reducing generation at times of low prices. Inflexible suppliers who cannot adjust their operations activity to suit evolving prices will experience untoward fluctuations in cash flow. For some participants, the advantages of paying low prices most of the time would not be justified by the risk of occasional periods of higher prices. For these participants, financial instruments can be used for risk sharing. For example, under a system of *forward contracts*, a price is offered for electrical energy transactions at each time period in the future. Thus, consumers can avoid the risk of higher prices by purchasing their future requirements at the fixed price. However, that price will be higher than the usual spot price since it contains a *risk premium* to cover the possible system conditions with higher values of SRMC. Consumers thus pay extra for the increased certainty in the price. However, financial instruments, such as *options*, can be used to reduce the investment risk, if desired.

12.5 Electricity Pricing and Markets

Until the early 1980s, almost every electricity utility in the world believed that generation, transmission, and, in many cases, distribution of energy should be an integrated business with one price to the consumer. The actual price must depend upon the average costs to the utility of producing and delivering this energy, so differential prices for large, medium, and small customers was the norm. With the breaking up of the large integrated utilities (unbundling) into separate businesses, as in the U.K. and elsewhere, pricing of electricity services, including the cost of energy, has become much more commercial. The following are practical guidelines for pricing objectives, which are generally acceptable to all types of customer.

1. Prices should be based on economic efficiency, costing resources in terms of fuel, conversion costs, and effects on the environment, not just in purely monetary terms.
2. Prices should be firmly set in accordance with costs, i.e. no hidden or cross subsidies.
3. Prices should ensure commercial viability.
4. Equity between different classes of consumers should be maintained.
5. Tariffs should be as simple as possible and transparent to all customers.

As we have seen, fixed capacity (kW) and variable (kWh) costs are incurred by a utility, but, in addition, there are fixed costs, such as billing, meter installation and reading, and customer services (answering queries and dealing with complaints), which depend on the number of customers. Therefore, a three-part tariff for each customer, made up of a fixed service charge, a kW-capacity charge, and an energy kWh charge, would appear to be the most appropriate. Many tariffs will lump the two fixed charges together for simplicity, leading to the familiar domestic two-part tariff based on household size plus the monthly or quarterly metered kWh (unit) consumption. Larger customers (shops, offices, small industries, workshops, etc.) will have a maximum-demand meter as well as an energy meter, so their tariff will include a price for kW of maximum demand over the year. Additionally, large consumers could have their var consumption (kVArh) metered and charged-for separately, leading to the concept of a separate reactive payment.

12.5.1 Electricity market models

The electricity industry is, in principle, just like any other producer or manufacturer of goods in that the electrical product (kWh) is made at a particular location and needs to be transported to the customer who has contracted to purchase that product at a certain price. Normally, some form of a market is set up in a democratic society so that purchasers can 'shop around' for the product that best suits them, at the price they consider to give the best value. For efficiency and expendiency, trading markets have a recognized infrastructure whereby intermediaries purchase and resell commodities in varying quantities and arrange for delivery to varying locations. In the case of electrical energy, there is only one product, and at every time instant the amount produced must equal the amount consumed, otherwise the frequency will vary. Consequently, looked at from a market-structure viewpoint, there must be a direct and continuous connection between generators and the consumers, but the various market intermediaries, such as coordinators or controllers and energy brokers or traders, can be considered as facilitators in this process, as illustrated in Figure 12.3.

Hunt and Shuttleworth (1996) have postulated four market models as illustrating the possible ways that an electricity market can be modelled. These models correspond to varying degrees of monopoly and competition in the electricity industry and represent possible structures for a country to organize its industry. The models are shown in Figures 12.4–12.7 and their characteristics are summarized in Table 12.1.

The four models have these characteristics:

- *Model 1*—Monopolistic or vertically integrated. Generation, transmission, distribution, and supply to the consumer (retailing) are all in one company or utility and there is no choice for anyone. This requires government control or a 'Regulator' to ensure fairness in pricing.
- *Model 2*—Purchasing Agency. This allows a single buyer to purchase energy from competing electricity generators or independent power producers (IPPs). The buyer arranges for delivery over monopoly

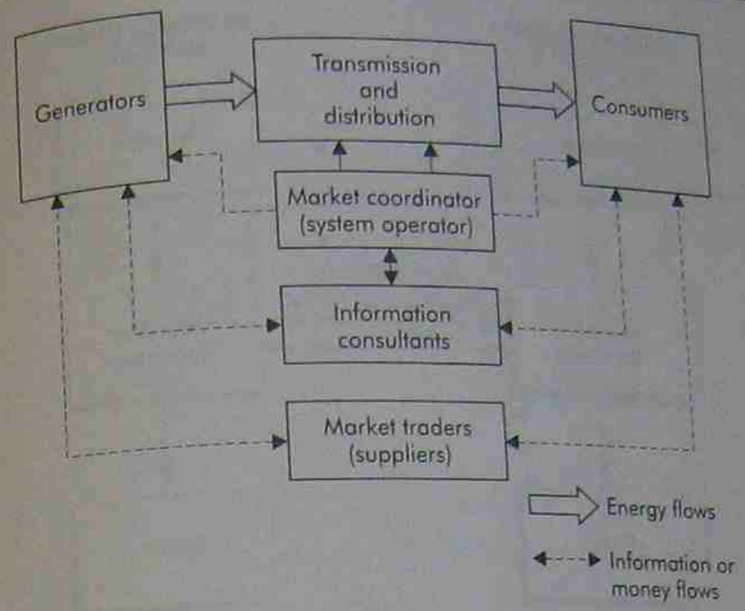


Figure 12.3 The electrical energy market

Table 12.1 Structural alternatives

Characteristic	Model 1—Monopoly	Model 2—Purchasing agency	Model 3—Wholesale competition	Model 4—Retail competition
Definition	Monopoly at all levels	Competition in generation—single buyer	Competition in generation and choice for Discos[a]	Competition in generation and choice for final consumers
Competing generators	No	Yes	Yes	Yes
Choice for retailers?	No	No	Yes	Yes
Choice for final customers?	No	No	No	Yes

[a]Discos, distribution companies.
(Source: Hunt and Shuttleworth (1996). Reproduced by kind permission of the authors of National Economics Research Association (New York & London))

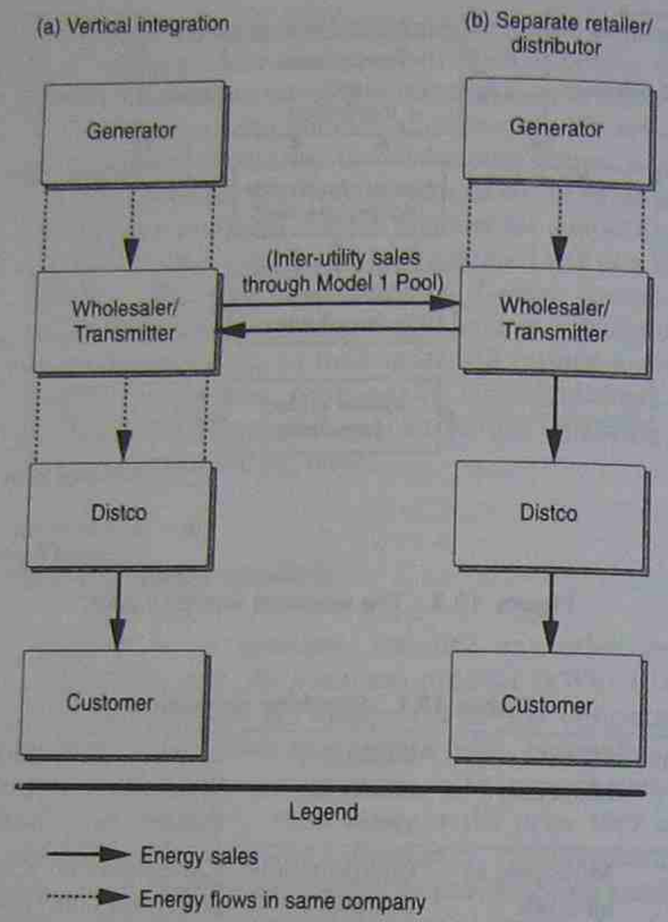


Figure 12.4 Model 1—monopoly (*Source: Hunt and Shuttleworth, Competition and Choice in Electricity, Wiley, 1996*)

transmission and distribution systems and sells at advertised tariffs to all consumers (large and small). The Purchasing Agency would purchase by tender or bidding system from the IPPs to ensure the best price is obtained throughout the day and year.

- *Model 3*—Wholesale competition. In this model the distribution companies (Distcos) can buy direct from the producers and deliver to their own supply points over the transmission networks. These networks will allow any Distco to contract for their use, thereby allowing 'open access'. The Distco, however, has a monopoly over supplying the consumers.
- *Model 4*—Retail competition. Here, all consumers can choose their supplier and pay for delivery over the transmission and distribution networks, which are on 'open access'. This is the most that can be obtained by competition,

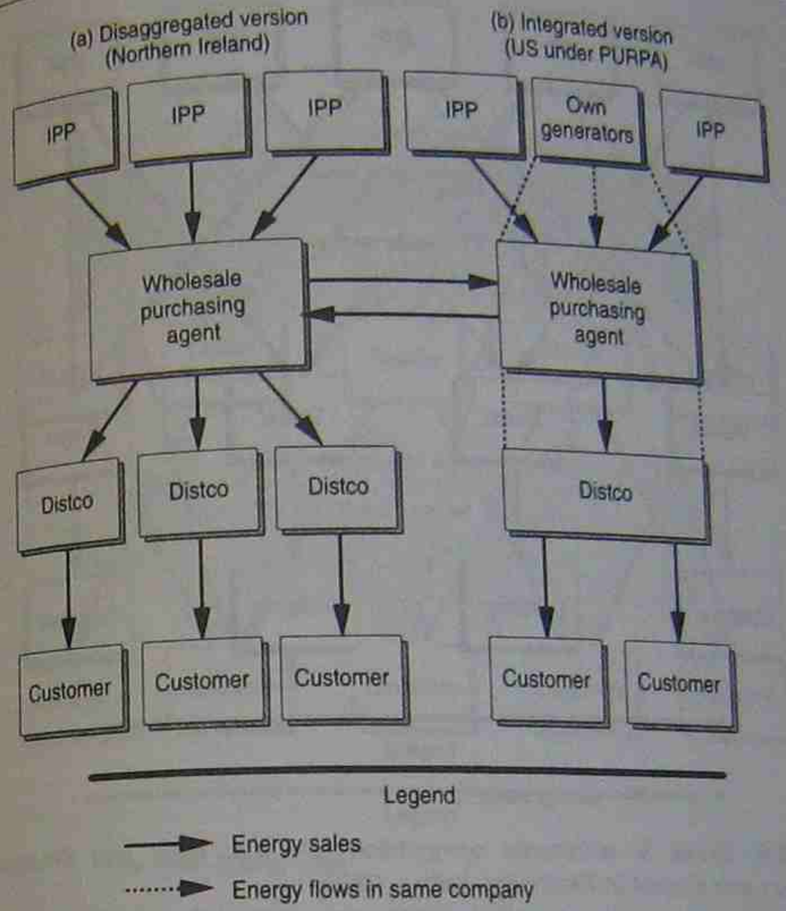


Figure 12.5 Model 2—purchasing agent (*Source: Hunt and Shuttleworth, Competition and Choice in Electricity, Wiley, 1996*)

although a Regulator is still required to agree the monopoly transmission and distribution charges, unless alternative delivery networks are built. The Regulator is also required to maintain a fair and equitable market structure for consumers to exercise their choice of supplier.

12.5.2 Comments on market models

Most countries are somewhere between Model 1 (monopoly, vertically integrated structure) and Model 4 (completely open, competitive energy trading). There are many problems and difficulties in transferring from one model to another, not least of which is the need to set up an appropriate infrastructure, including the necessary control, metering, and information technology to achieve a free market at reasonable cost. In addition, a considerable re-

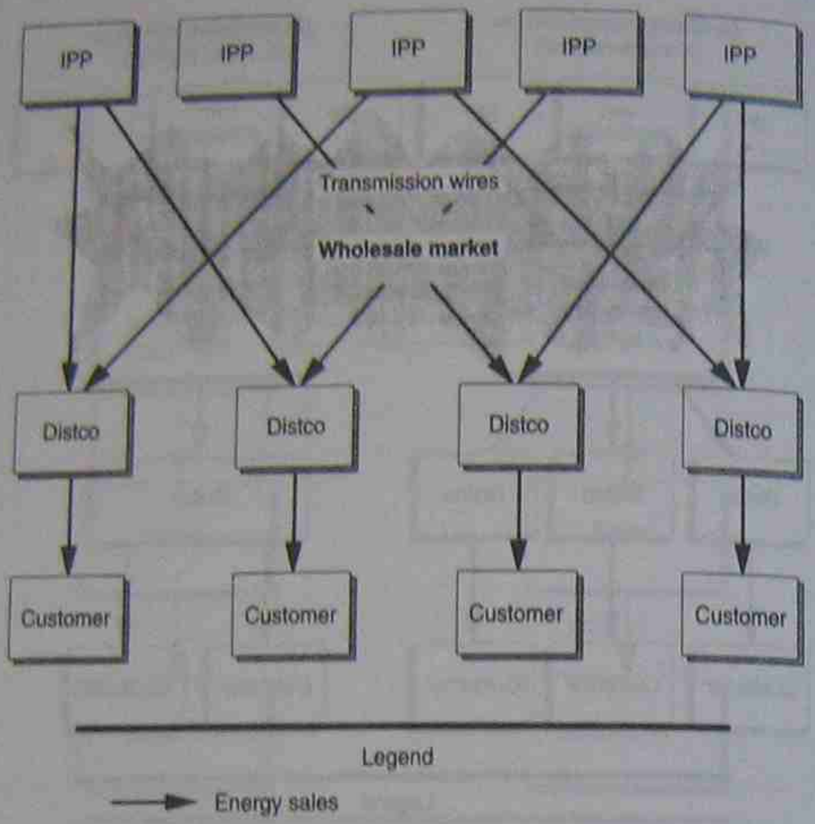


Figure 12.6 Model 3—wholesale competition (*Source: Hunt and Shuttleworth, Competition and Choice in Electricity, Wiley, 1996*)

education and re-training of utility employees is required to enable the 'new' philosophies of marketing and improved customer services to be provided. From the U.K. experience, it is apparent that the technical aspects of providing a reliable electrical energy supply of the specified quality becomes a smaller proportion of the overall activity within the overall supply business, with many of the technical matters being overtaken by energy buying and selling, customer relations, services to the individual consumer (appliance maintenance, billing, data transfer, improved end-use efficiency, etc.), and the need to maintain a healthy share price and public relations image. These aspects require business and management skills combined with a good technical knowledge and understanding.

12.5.3 Reactive market

Large consumers are encouraged by their tariff to take supplies as near to unity power factor as possible, thereby helping to reduce the I^2R loss in supply

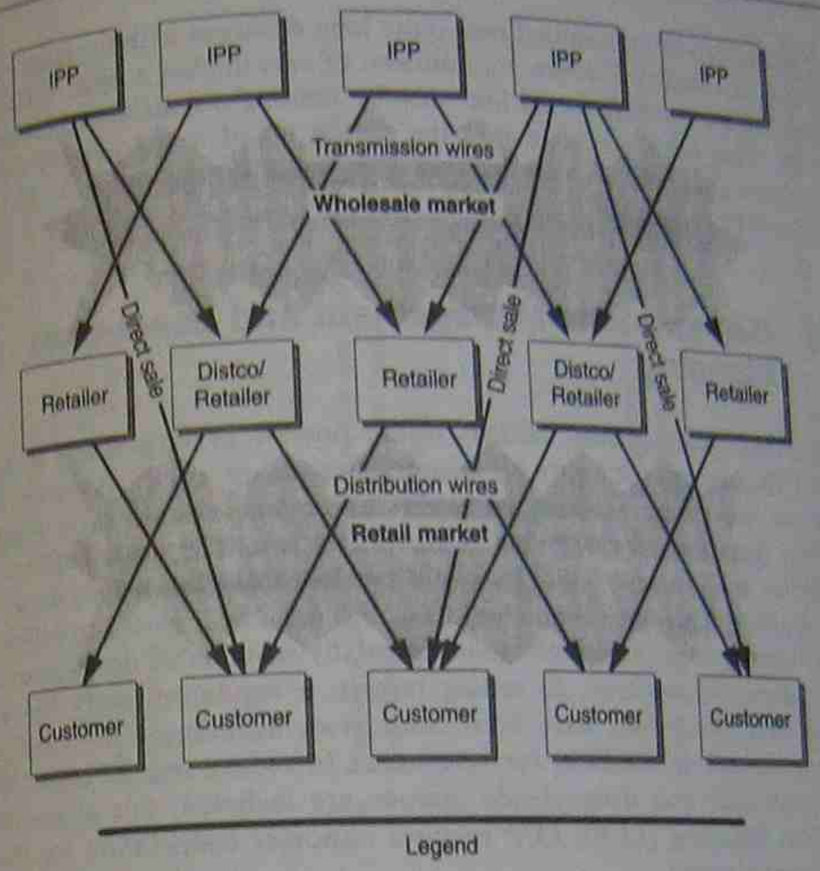


Figure 12.7 Model 4—retail competition (*Source: Hunt and Shuttleworth, Competition and Choice in Electricity, Wiley, 1996*)

circuits. It is therefore reasonable to expect kVArh to be charged for as a contribution to the increased power loss incurred, particularly in distribution circuits. On the transmission network, reactive power is required to maintain voltage (see Chapter 5) and this has to be provided either from generators by excitation adjustment or by the installation of var compensators (series or shunt) at strategic busbars in the system. If vars are provided by generators through overexcitation (i.e. generated vars), then not only is additional heating of the rotor occurring but also increased losses because of additional stray flux and end-winding eddy currents produced in the stator. To compensate the generator owner for the extra deterioration of the machine, a payment proportional to the MVArh produced is made by the power-system operator to the generator or IPP. Obviously, the transmission network requires to cover investment in reactive compensation and the extra I^2R losses incurred by its operation. Therefore, it can be argued that a reactive market could also be established along similar lines as a real-power pool. Unfortunately, whereas

real power can be transmitted over quite long distances without large voltage changes, as Chapter 5 shows, transmission of vars implies a large voltage difference on an uncompensated line, thereby limiting the distance between var sources. This complication makes the setting up of a var market difficult. Negotiated contracts for vars between the power system operator and IPPs could well be preferable.

12.6 Demand-Side Management and Least-Cost Planning

Demand-Side Management (DSM) is the process of influencing customer demand throughout 24 hours by encouraging use of electrical energy when prices are low or by assisting customers to employ conservation measures such that their overall energy bill is acceptably low. The main problem in a competitive environment is that both of these desirable goals reduce the income to the supplier. Therefore, regulatory measures must be imposed to achieve set targets, usually associated with environmentally sustainable developments or socially desirable practices. In setting targets, a regulatory body has to be satisfied that all options have been considered, including both supply-side and demand-side possibilities for investment to reduce overall costs. When both supply-side and demand-side options are included, this is known as Least-Cost Planning (LCP). LCP requires customer cooperation for overall benefit maximization.

Figure 12.8 shows the various possibilities for influencing the load-demand shape over the normal day. Most large consumers (especially electrical-energy-intensive industries, such as chemical, steel, cement) have individually negotiated tariffs dependent upon their production needs and demand profile. Metering and billing is a very small part of the overall costs, but cheap energy is often essential for success in their markets. Overall energy/production cost optimization is normally part of their business plan.

12.6.1 Benefits of DSM

Considerable benefits can flow from DSM measures, but these need to be realistically costed by surveys or modelling. The following aspects should be considered:

- reduction of peak demand, thereby producing savings in new investment on transmission and distribution capacity;
- avoidance of load shedding where available plant is limited;
- improved load factor on existing plant by making better use of capacity;

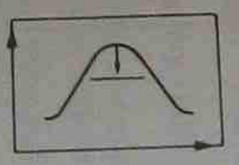
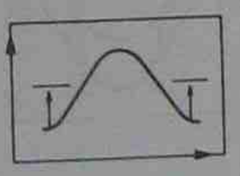
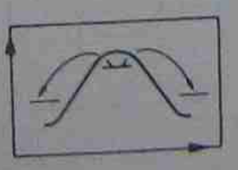
Utility load shape objectives		Examples of customer options		
		Residential	Commerical	Industrial
Peaking clipping, or reduction of load during peak periods, is generally achieved by directly controlling customers' appliances. This direct control can be used to reduce capacity requirements, operating costs, and dependence on critical fuels.		• Accept direct control of air-conditioners	• Accept direct control of water-heaters	• Subscribe to interruptible rate
Valley filling, or building load during off-peak periods, is particularly desirable when the long-run incremental cost is less than the average price of electricity. Adding properly priced off-peak load under those circumstances can decrease the average price.		• Use off-peak water-heating	• Store hot water to augment space-heating	• Add night-time operations
Load shifting, which accomplishes many of the goals of both peak clipping and valley filling, involves shifting load from on-peak to off-peak periods, allowing the most efficient use of capacity.		• Subscribe to time-of-use rates	• Install cool-storage equipment	• Shift operations from daytime to night-time

Figure 12.8 Load shape and demand-side alternatives (*Source: Cory, B. J. 'Load management by direct control', internal paper, Department of Electrical Engineering, Imperial College, 1985, Figure 1. Reproduced by permission of EPRI Journal, December 1994, pp 6–15*)

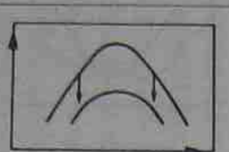
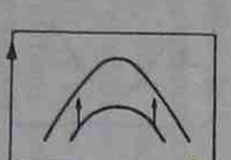
Utility load shape objectives		Residential	Examples of customer options	
			Commerical	Industrial
Strategic conservation involves a reduction in sales, often including a change in the pattern of use. The utility planner must consider what conservation actions would occur naturally and then evaluate the cost-effectiveness of utility programs intended to accelerate or stimulate conservation actions.		• Supplement home-insulation	• Reduce lighting use	• Install more efficient processes
Strategic load growth, a targeted increase in sales, may involve increased market share of loads that are or can be served by competing fuels, as well as developments of new markets. In the future, load growth will include greater electrification — electric vehicles, automation, and industrial process heating.		• Switch from gas to electric water-heating	• Install heat pumps	• Convert from gas to electric process heating

Figure 12.8 (*continued*)

- better load forecasting, if elasticity of demand is monitored;
- lower supply costs to customers, including improved cash flow with improved billing and metering facilities.

The largest savings are often due to reduced generation costs as the most commercial and cost-effective generators can be scheduled. The avoidance of investment in new plant has considerable impact, particularly when interest rates are high. If demand elasticities are being continuously monitored for billing purposes, load-forecasting techniques can be further improved with this data. Improved load factors avoid extra start-up, shut-down, and part-load costs on generators. Also, better use of the network reduces use-of-system costs (see later).

Typical DSM methods for residential and small commercial consumers include the following:

- Improved thermal insulation of building structures (loft and wall insulation, double glazing, draught proofing) in cold/temperate climates. In tropical zones, better solar protection and natural cooling is required.
- Use of electrically efficient equipment (heat pumps, microwave ovens, improved refrigerators and freezers with energy labelling to guide purchase, washing machines and dishwashers, low-energy and long-life lamps, etc).
- Better control of electrical appliances, either by local optimal controllers, proximity switches, temperature controllers, etc., or by load management agreements for direct control by the electricity supplier (see Section 12.4).

In evaluating DSM measures, the key factors are as follows:

- load profile and coincidence of peaks, hence the need to disaggregate customers into different groups geographically;
- need to differentiate between groups by consumer type (commercial, domestic, etc.);
- ability to reconfigure supply system to avoid overloads;
- measures are often required to maintain customer participation, e.g. by loyalty cards, bonuses, etc.;
- strategies are required to assist customers to change habits of energy use, e.g. discounts for energy-saving devices.

To become really effective, DSM requires good and continuous contact between suppliers and customers. Ideally, a two-way communication link between the customer's meter, with a wall display, and the supplier enables consumption to be monitored and, if desired, an update on electricity prices and accounting details can be downloaded to the customer. This link can be used for load management control of water-heaters, space-heating,

air-conditioning, etc., or the customers can install their own controllers responding to price messages from the supplier. Various communication methods have already been listed but, unfortunately, none are particularly inexpensive relative to an average domestic annual bill of £400 (US$650) or so.

12.6.2 Least-cost planning (LCP)

By definition, LCP is a maximization of supplier plus consumer welfare at least cost (see Stoll *et al.*, 1989). Hence there is a need for *integration* of investments by both supplier and consumer to include:

- load management;
- energy conservation;
- generation, including CHP and renewables;
- transmission and distribution capacity;
- interutility energy transfers.

The planning processes should consider *all* the options available to meet the desired objectives, namely:

- minimum cost investment over a given period;
- robustness against changes in market rates, social climate, environmental constraints, etc.;
- maximum cooperation between utility and customer;
- financial viability.

A typical procedure for LCP is illustrated in Figure 12.9, in which various DSM measures are placed in order of ascending cost. Figure 12.9 is sometimes called a *scoping* or *screening* diagram and should include any marginal costs of new generation plus any reinforcements required to the transmission and distribution systems to supply the DSM equipment.

In theory, investment in new plant should not occur until the cost of alternative measures is equivalent to new plant costs. In practice, new plant may be required because of ageing, security of supply, new development, economic boom, etc., as determined by a risk-and-contingency analysis, including uncertainties in forecasts.

12.6.3 Comments on DSM

DSM, which may, or may not, include direct load-management control, requires considerable supplier–customer interaction. New metering technolo-

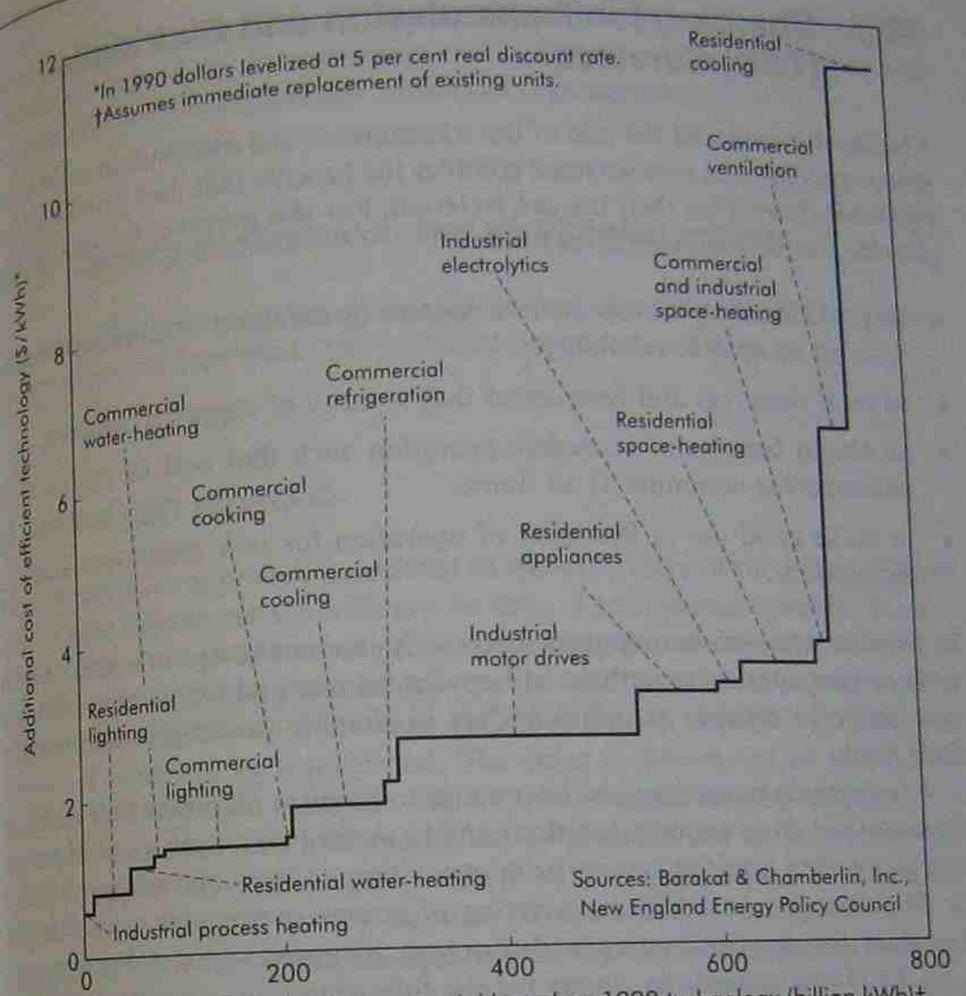


Figure 12.9 Scoping diagram for LCP. Vertical scale is the capital and installation costs amortized over the life of the replacement unit (*Permission of E.P.R.I., U.S.A*)

gies and communications will facilitate this process, but is it important to keep good relations with the customer through efficient billing and the easy availability of discounts, rebates, etc. Utilities need to set up information technology (IT) infrastructures and to maintain an up-to-date customer data base, including comprehensive profiling of customer demand throughout the day, season, and year. With continuously remotely read meters, the profiling data is readily collected, perhaps by a separate data collection organization, but if regular (monthly or quarterly) meter reading is employed, comprehensive sampling techniques and surveys must be conducted to enable appropriate charges to be levied. These charges should include not only fuel costs related to generation but also delivery charges over the transmission and distribution system.

12.7 Charging for Transmission and Distribution (T&D) Services

Chapter 1 introduced the role of the transmission and distribution networks in power systems, but now we must consider the benefits that they provide so that equitable charges for their use can be levied. For this purpose, the services they provide can be summarized as follows:

- to provide energy transport from sources (generators) to consumers (loads) with an acceptable reliability;
- to pool resources and reserves so that security of supply is achieved;
- to obtain benefits of economic operation such that cost of energy to all consumers is minimum at all times;
- to make good use of flexibility of operation for new generators and load connections.

In practice, transmission systems are mostly interconnected in a grid or mesh form so that alternative paths exist between sources and loads. Many distribution networks operate as radial feeders to simplify protection and minimize fault levels.

For charging purposes, it is the practice to monitor all power flows (kW) on these services from major substations and to meter (kWh over specified period) energy injected into the system from generating sources and energy delivered to all customers. In addition, metering of energy over major circuits is also necessary for detailed costing and charging purposes.

A T&D utility needs to charge for the following:

- losses incurred in transporting energy (in kWh);
- operation and maintenance of the system (probably a fixed cost dependent upon capacity or rating);
- reinforcement and extension to maintain customer service and quality (planned by extensive studies and future plausible scenarios);
- new connections and services (related mainly to individual customers when connection and service is negotiated and contract agreed).

The organization required to provide these functions and to allocate the charges is very complex and varies from utility to utility. The 'technical' requirements of monitoring and metering are considered as the job of the engineering staff and associated planners; the collection and collation of data from the system is part of the operation of the system, while the billing and collection of monies falls on the accounting and business division of the utility. Obviously, of vital importance is the data communication between monitoring/

metering points in the network and the operation/control centre, and the information technology infrastructure available between the data centre and the settlement/accounting groups within the organization.

12.7.1 Costing transmission and distribution access

Several approaches to costing transmission and distribution (T&D) services and access can be identified. The principal ones are given here.

(i) Contract path approach

The contract path approach originated in the early days of the industry when systems were interconnected with few tie lines. The approach specifies an electrically continuous path from a generator (or a point of receipt if more than one utility involved) to a point of delivery (i.e. the buyer or an adjacent utility system). The path chosen must have sufficient unused capability to carry the amount of power to be transported. The price is determined to recover the necessary rate of return from the T&D assets assigned for the transport and any other additional costs incurred by the T&D utility due to the transaction. For transport between closely meshed utilities with strong tie lines, this method is not suitable mainly because of the inability to ensure that the power flows follow the contracted paths. Another serious deficiency with this approach is that it entirely ignores outages of elements in the path, imposing power flows through neighbouring parallel systems which are not parties to the contract. This method is still widely used, particularly in the U.S.A., for pricing wheeling transactions between utilities based on payment per MW or MWkm.

(ii) Postage stamp method

The postage stamp method differs from the contract path method in that it considers system-wide average T&D costs rather than the costs of specially selected facilities. This method is often referred to as the postage stamp method because it results in the same transport charge irrespective of distance or location. The main argument for this method is that an integrated T&D system is designed to achieve maximum efficiency and reliability at minimum cost on a system-wide basis, and that all users should receive the benefits in such an integrated system.

(iii) Method based on usage

This method basically concentrates on two measurements: the amount of T&D capacity used, and the per unit cost of that capacity. Measurement of capacity used can be as simple as total megawatts of the transaction (such as the postage stamp method) or involve calculation of estimated or actual power flows and related transmission data. Cost of T&D capacity can be calculated on an embedded, incremental, or marginal basis. It may also be either short-run cost (when there is insufficient time to change capacity) or long-run cost. This method is suitable for transaction pricing between utilities and for the user-specific cost streams in a fully deregulated electricity market.

Cost of the facility is defined as the revenue requirement for the facility on an embedded, incremental, or marginal basis, as defined below.

Embedded cost This is the revenue required to pay for all existing T&D facilities or any added to the system during the life of the transmission contract. It can be calculated from

$$\mathrm{ECC}_i = \sum_{f \in F} \frac{|\Delta \mathrm{MW}_{f,i}| \times \mathrm{EC}_f}{\sum_{s \in S} |\Delta \mathrm{MW}_{f,s}|}$$

where

ECC_i = annual embedded capacity cost of transaction i;
$\Delta \mathrm{MW}_{f,i}$ = change in megawatt flow due to contracted transaction i on facility or circuit f;
$\Delta \mathrm{MW}_{f,s}$ = change in megawatt flow due to all contracted transactions on facility or circuit f;
EC_f = annual embedded cost of facility f consisting of depreciation, annuitized cost of capital, taxes and operating/maintenance costs;
S, F = sets of all the transactions S and facilities or circuits F considered in any year.

Incremental cost This is the revenue requirement that is specifically attributed to the T&D service customer. These facilities must be identified for all years across the life of the contract for service. The T&D services customer pays the full cost for any new facilities that the transaction requires; if a new facility would have been built for other reasons at a later time, then the T&D services customer pays the cost to advance the facility's in-service date. If a facility is needed by more than one T&D service customer, then the cost of the facility can be allocated to the incremental customers by usage methods.

Marginal cost This can be defined as the revenue requirement needed to pay for any new capacity on the T&D system. The T&D service customer pays an allocated share of the cost for any new facility that the system requires.

The difference between the three types of cost focuses on the usage allocation function of the facility. Embedded capacity cost includes existing and new customers and charges the embedded cost of all existing and new facilities, thereby providing an average rate for all customers. Incremental capacity cost applies only to new customers who forced the construction of new facilities, and charges the full cost of these facilities over the life time of the facilities. 'New facilities' includes only facilities that are specifically identified as being needed to serve the new customers. Marginal capacity cost applies to all customers requiring new T&D facilities and charges the marginal cost of new facilities anywhere in the system, thereby providing the same marginal rate in £/MW for all marginal customers.

12.7.2 Transmission facilities including security

While most distribution facilities can be identified for existing or new customers, since mainly radial circuits and associated components are involved, on the multimesh transmission network this distinction is no longer feasible because of the many paths available between the generators and the loads.

The cost of service provided by a transmission system is spatially and temporally differentiated to take into account the configuration of the network and its inherent capability to operate at times of peak flow. Marginal-cost-based service pricing provides the incentives needed for the network utility to operate and to invest for increased system efficiency, and for generating units and end users to locate efficiently within the network. Unfortunately, it may not necessarily guarantee a sufficiently large return to provide for a financially survivable company.

The transmission network can be seen as one of two businesses:

1. It is a fixed-asset-based business put in place to guarantee sufficient transmission capability to maintain a reliable and secure supply. Defined in this manner, the business is defined in kW capacity, and is billed to the user (generator or distributor/end user) as a function of their location and their peak demand.
2. It is a transportation business whose objective is to transport energy at profit. Defined in this manner, the business is denominated in kWh, and is billed to the users as a function of the amount of energy, the location, and the timing of energy input to, or extracted from, the network.

Regardless of the manner in which the transmission business is defined, operating uncertainties exist. These are increased under privatization or unbundling because the network utility no longer has direct control over the decisions of either the supply side or the demand side. Nowadays, in England and Wales, for example, network expansion planning has to be carried out in an environment in which the City financial market information about future generator locations plays an important role. The incentive is for the network utility to provide for the most efficiently constructed and flexibly controlled system.

12.7.3 System losses

With a given system and patterns of generation and loads, the estimated losses in kWh per km of system can be calculated. These losses can be costed at the marginal price/kWh for each node or zone of the system and the costs apportioned to users on a kWh basis.

In estimating losses in use-of-system pricing there are a number of possible ways this can be done. These are:

- *Empty system*—the losses are estimated by calculating the flows in all the circuits from a single generator input to a single consumer load as if no other flows are present. Any contract between the generator and load includes a charge for the expected 'copper' losses because of the transport between the two.
- *Vector difference*—contract transport is added to the existing system flows and the difference in losses is included in the cost of that contract. Note that flows opposite to existing flows actually produces a negative incremental loss.
- *Critical path*—the path that the contract transport is expected to take through the system is costed for losses by assuming that the flow is added on top of existing normal flow in the system.

Most of these methods relate to specified flows when a contract in terms of kWh for a specified period up to a given kW maximum is being fulfilled. When many different contracts over many different paths are being accommodated, the complexity of system operation is apparent. For an integrated system, such as NGC, a posted tariff for use-of-system based on capacity is considered to be preferable.

For planning and pricing purposes, network studies based on economic pricing principles are required. Transportation algorithms can provide an insight into the major power flows but to account for security, voltage control, and the necessary stability margins, network studies over a range of loading and generation scenarios are required. These studies can be done through fast d.c. load flows with added group constraints, or through OPFs using full a.c.

equations. From the studies, revenue recovery can be calculated with various pricing policies applied, such as a fixed charge per MW km of system or using a maximization of net benefit formula including security. Whatever basis of charging is employed, for a monopoly utility the total cost recovery per year must be agreed with a regulating authority. It is becoming established that this cost recovery is set at a percentage return on assets to provide a profit margin dependent upon a measure of retail prices (such as the Retail Price Index (RPI) in the U.K.) minus a given percentage factor X per year to provide an incentive towards achieving greater efficiency. This is known as the RPI-X formula and, if set for a fixed number of years, it provides an incentive for the utility to invest such that costs are reduced and profits increase, despite the X penalty.

12.7.4 Ancillary services

To maintain reliability and quality of supply on an interconnected transmission system, reserves of real power and reactive power are required, strategically located around the network. In addition, in the event of a dire emergency, such as a system shut-down, a self-start (called 'black-start') capability is required. These reserves, including reactive power provision for voltage control, if not bought by a 'reactive market' process, and plant held in readiness for a 'black-start', need to be purchased by the system operator and charged to system users on some equitable basis.

The ancillary services required are defined in more detail as:

- *Spinning reserves*—fast-response capability held on part-loaded synchronized generators to check any sudden frequency change due to loss of an infeed. Generator capability requires monitoring and periodic assessment to ensure contracted response is available. Response is classified into primary (available within 10 s) and secondary (available within 30 s). Note that if a frequency fall cannot be contained by scheduled part-loaded generators, then automatic load shedding by under-frequency relays is a further safeguard.
- *Standing reserve*—unsynchronized, fast start-up plant (hydro, gas turbine) available within 5 min or less. Demand reduction by operator instruction can also be used in this category.
- *Replacement reserve*—plant that can be brought into service with longer notice in the event of a shortfall in standing reserve. This may be hot-standby plant or deferred-start plant, designated by the system operator.
- *Reactive power (voltage control)*—the requirement of generators to operate at an instructed power factor within registered capabilities and to have a prescribed excitation response on change of voltage. The cost of MVArh is negotiated with individual suppliers, dependent upon type and design of

plant. Note that the power-system operators can provide their own reactive power by installing compensators at strategic nodes in the system.

- *'Black-start' capability*—a requirement that a generator can start one or more units without external electrical supply within a given period. Requires provision of self-contained power supply from batteries, diesel generators, gas turbines, etc., to start up power-station auxiliaries.

Most of the ancillary services that are contracted for can be separately and continuously metered or monitored by inspection, thereby the payments for the services can be made through the pool or settlement system. The recovery of these costs from customers is dependent upon the methods adopted for sharing the charges for use-of-system. Hence they can be allocated according to the monitored kW demand or according to the kWh consumed in each billing period.

Finally, it is worth noting that in a competitive environment, contracted services, such as detailed here, can be bid for, using an auction process to ensure that the total ancillary service provision is obtained at minimum cost. Such auctions can include consumers as well as generators, particularly where demand reductions or power factor correction can be beneficial to the overall operation of the power network. In principle, therefore, bidding could be on a daily basis, together with the submission of marginal prices for energy under the pool system. In the U.S.A., electronic auction systems are being established for daily bidding, including MW capacity rights on transmission corridors (known as 'congestion contracts') so that energy trading can flourish, thus leading to minimum prices for all customers.

Appendix I
Synchronous Machine Reactances

Table AI.1 Typical percentage reactances of synchronous machines at 50 Hz—British

Type and rating of machine	Positive sequence			Negative sequence	Zero sequence	Short-circuit ratio
	X''	X'	X_s	X_2	X_0	
11 kV Salient-pole alternator without dampers	22.0	33.0	110	2.0	6.0	—
11.8 kV, 60 MV, 75 MVA Turboalternator	12.5	17.5	201	13.5	6.7	0.55
11.8 kV, 56 MW, 70 MVA Gas-turbine turboalternator	10.0	14.0	175	13.0	5.0	0.68
11.8 kV, 70 MW, 87.5 MVA Gas-turbine turboalternator	14.0	19.0	195	16.0	7.5	0.55
13.8 kV, 100 MW, 125 MVA Turboalternator	20.0	28.0	206	22.4	9.4	0.58
16.0 kV, 275 MW, 324 MVA Turboalternator	16.0	21.5	260	18.0	6.0	0.40
18.5 kV, 300 MW, 353 MVa Turboalternator	19.0	25.5	265	19.0	11.0	0.40
22 kV, 500 MW, 588 MVA Turboalternator	20.5	28.0	255	20.0	6.0–12.0	0.40
23 kV, 600 MW, 776 MVA Turboalternator	23.0	28.0	207	26.0	15.0	0.50

Table AI.2 Approximate reactance values of three-phase 60-HZ generating equipment—United States (Values in per unit on rated kVA base)

Apparatus	Positive sequence						Negative sequence X_2		Zero sequence X_0	
	Synchronous X_s		Transient X'		Subtransient X''					
	Average	Range	Average	Range	Average	Range	Average	Range	Average	Range
2-pole turbine generator (45 psig inner-cooled H_2)	1.65	1.22–1.91	0.27	0.20–0.35	0.21	0.17–0.25	0.21	0.17–0.25	0.093	0.04–0.14
2-pole turbine generator (30 psig H_2 cooled)	1.72	1.61–1.86	0.23	0.188–0.303	0.14	0.116–0.17	0.14	0.116–0.17	0.042	0.03–0.073
4-pole turbine generator (30 psig H_2 cooled)	1.49	1.36–1.67	0.281	0.265–0.30	0.19	0.169–0.208	0.19	0.169–0.208	0.106	0.041–0.1825
Salient-pole generator and motors (with dampers)	1.25	0.6–1.5	0.3	0.2–0.5	0.2	0.13–0.32	0.2	0.13–0.32	0.18	0.03–0.23
Salient-pole generator (without dampers)	1.25	0.6–1.5	0.3	0.2–0.5	0.3	0.2–0.5	0.48	0.35–0.65	0.19	0.03–0.24
Synchronous condensers (air-cooled)	1.85	1.25–2.20	0.4	0.3–0.5	0.27	0.19–0.3	0.26	0.18–0.4	0.12	0.025–0.15
Synchronous condensers (H_2 cooled at $\frac{1}{2}$ psig rating)	2.2	1.5–2.65	0.48	0.36–0.6	0.32	0.23–0.36	0.31	0.22–0.48	0.14	0.03–0.18

(Permission of Westinghouse Corp.)

Table AI.3 Principal data of 200–500 MW turbogenerators—Russian

	Units	Values of parameters of turbogenerators of various types					
		1	2	3	4	5	6
Power	MW/MVA	200/235	200/235	300/353	300/353	500/588	500/588
Cooling of winding							
(stator)		Water	Hydrogen	Water	Hydrogen	Water	Water
(rotor)		Hydrogen[c]	Hydrogen	Hydrogen	Hydrogen	Hydrogen	Water
Rotor diameter	m	1.075	1.075	1.075	1.120	1.125	1.120
Rotor length	m	4.35	5.10	6.1	5.80	6.35	6.20
Total weight	kg/VA (N/kVA)	0.93 (9.1)	1.3 (12.7)	0.98 (9.6)	1.05 (10.3)	0.64 (6.26)	0.63 (6.17)
Rotor weight	kg/kVA (N/kVA)	0.18 (1.76)	0.205 (2.01)	0.16 (1.57)	0.158 (1.55)	0.11 (1.08)	0.1045 (1.025)
X_s[a]	%	188.0	184.0	169.8	219.5	248.8	241.3
X'	%	27.5	29.5	25.8	30.0	36.8	37.3
X''	%	19.1	19.0	17.3	19.5	24.3	24.3
X_q	%	188.0	184.0	169.8	219.5	248.8	241.3
X_0	%	8.5	8.37	8.8	9.63	15.0	14.6
τ_1^0	s	2.3	3.09	2.1	2.55	1.7	1.63

[a]For all reactances, unsaturated values are given.
[b]Without the turbine.
[c]Hydrogen pressure for columns 1–4 is equal to 3 atm (304,000 N/m^2) and for column 5 4 atm (405,000 N/m^2).
(Source: Glebov, I. A, C.I.G.R.E., 1968, Paper 11-07.)

Appendix II

Typical Transformer Impedances

Table A.2.1 Standard impedance limits for power transformers above 10 000 kVA (60 Hz)

Highest voltage winding (BIL kV)	Low-voltage winding (BIL kV)	At kVA base equal to 55°C, rating of largest capacity winding							
		Self-cooled (OA), self-cooled/forced-air cooled (OA/FA) self-cooled/forced-air, forced-oil cooled (OA, FOA)				Forced-oil cooled (FOA and FOW)			
	For intermediate BIL use value for next higher BIL listed	Standard impedance (%)				Standard impedance (%)			
		Ungrounded neutral operation		Grounded neutral operation		Ungrounded neutral operation		Grounded neutral operation	
		Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
110 and below	110 and below	5.0	6.25			8.25	10.5		
150	110	5.0	6.25						
200	110	5.5	7.0			8.25	10.5		
	150	5.75	7.5			9.0	12.0		
250	150	5.75	7.5			9.75	12.75		
	200	6.25	8.5			9.5	12.75		
350	200	6.25	8.5			10.5	14.25		
	250	6.75	9.5			10.25	14.25		
450	200	6.75	9.5	6.0	8.75	11.25	15.75		
	250	7.25	10.75	6.75	9.5	11.25	15.75	10.5	14.5
	350	7.75	11.75	7.0	10.25	12.0	17.25	11.25	16.0
550	200	7.25	10.75	6.5	9.75	12.75	18.0	12.0	17.25
	350	8.25	13.0	7.25	10.75	12.0	18.0	10.75	16.5
	450	8.5	13.5	7.75	11.75	13.25	21.0	12.0	18.0
650	200	7.75	11.75	7.0	10.75	14.0	22.5	12.75	19.5
	350	8.5	13.5	7.75	12.0	12.75	19.5	11.75	18.0
	450	9.25	14.0	8.5	13.5	14.0	22.5	12.75	19.5
						15.25	24.5	14.0	22.5
750	250	8.0	12.75	7.5	11.5	13.5	21.25	12.5	19.25
	450	9.0	13.75	8.25	13.0	15.0	24.0	13.75	21.5
	650	10.25	15.0	9.25	14.0	16.5	25.0	15.0	24.0
825	250	8.5	13.5	7.75	12.0	14.25	22.5	13.0	20.0
	450	9.5	14.25	8.75	13.5	15.75	24.0	14.5	22.25
	650	10.75	15.75	9.75	15.0	17.25	26.25	15.75	24.0
900	250			8.25	12.5			13.75	21.0
	450			9.25	14.0			15.25	23.5
	750			10.25	15.0			16.5	25.5
1050	250			8.75	13.5			14.75	22.0
	550			10.0	15.0			16.75	25.0
	825			11.0	16.5			18.25	27.5
1175	250			9.25	14.0			15.5	23.0
	550			10.5	15.75			17.5	25.5
	900			12.0	17.5			19.5	29.0
1300	250			9.75	14.5			16.25	24.0
	550			11.25	17.0			18.75	27.0
	1050			12	18.25			20.75	30.5

(Permission of Westinghouse Corp.)

Appendix III

Typical Overhead Line Parameters

Table A.3.1 Overhead-line parameters—50 Hz (British)

Parameter	275 kV		400 kV	
	$2 \times 113\,mm^2$	$2 \times 258\,mm^2$	$2 \times 258\,mm^2$	$4 \times 258\,mm^2$
$Z_1\,\Omega/km$	$0.09 + j0.317$	$0.04 + j0.319$	$0.04 + j0.33$	$0.02 + j0.28$
$Z_0\,\Omega/km$	$0.2 + j0.87$	$0.14 + j0.862$	$0.146 + j0.862$	$0.104 + j0.793$
$Z_{mo}\,\Omega/km$	$0.114 + j0.487$	$0.108 + j0.462$	$0.108 + j0.45$	$0.085 + j0.425$
$Z_p\,\Omega/km$	$0.127 + j0.5$	$0.072 + j0.5$	$0.075 + j0.507$	$0.048 + j0.45$
$Z_{pp}\,\Omega/km$	$0.038 + j0.183$	$0.033 + j0.182$	$0.035 + j0.177$	$0.028 + j0.172$
$B_1\,\mu mho/km$	3.60	3.65	3.53	4.10
$B_0\,\mu mho/km$	2.00	2.00	2.00	2.32
$B_{m0}\,\mu mho/km$	5.94	7.00	7.75	8.50

Z_1 Positive-sequence impedance.
Z_0 Zero-sequence impedance of a d.c. 1 line.
Z_{m0} Zero-sequence mutual impedance between circuits.
Z_p Self-impedance of one phase with earth return.
Z_{pp} Mutual impedance between phases with earth return.
B_1 Positive-sequence susceptance.
B_0 Zero-sequence susceptance of a d.c. 1 line.
B_{m0} Zero-sequence mutual susceptance between circuits.
Note: A d.c. 1 line refers to one circuit of a double-circuit line in which the other circuit is open at both ends.
The areas quoted are copper equivalent values based on $0.4\,in^2$ and $0.175\,in^2$.

Table A.3.2 Overhead line data—a.c. (60 Hz) and d.c. lines

Region:	500–550 kV—a.c.		700–750 kV	h.v.d.c.	
	Pacific	Canada	Canada	Mountain	Pacific
Utility:	So. California Edison Co.	Ontario Hydro	Quebec Hydroelectric Co.	U.S. Bureau of Reclamation	Los Angeles Dept. of W. & P.
Line name or number	Lugo–Eldorado	Pinard–Hanmer	Manics.–Levis	Oregon–Mead	Dalles–Sylmar
Voltage (nominal), kV; a.c. or d.c.	500; a.c.	500; a.c.	735; a.c.	750 (±375); d.c.	750 (±375); d.c.
Length of line, miles	176	228	235.92	560	560
Originates at	Lugo sub	Pinard TS	Manicouagan	Oregon border	Oregon border
Terminates at	Eldorado sub	Hanmer TS	Levis sub	Mead sub	near, Los Angeles
Year of construction	1967–69	1961–63	1964–65	1966–70	1969
Normal rating/cct, MVA	1000	—	1700	1350 mW	1350 mW
STRUCTURES					
(S)teel, (W)ood, (A)lum	S	S, A	S	S	S or A
Average number/mile	4.21	3.7	3.8	4.5	4.5
Type: (S)q, (H), guy (V) or (Y)	S (S-56)	V (A-51, S-51)	–; (S-71)	S, T (S-72, 73)	S, T (S-72, 73)
Min 60-Hz flashover, kV	850	1110	—	—	915
Number of circuits: Initial; Ultimate	1; 1	1; 1	1; 1	1; 1	1; 1
Crossarms (S)teel, (W)ood, (A)lum	—	S, A	S	S	S or A
Bracing (S)teel, (W)ood, (A)lum	—	—	S	S	S or A
Average weight/structure, lb	19 515	10 800; 4730	67 3000	11 000	—
Insulation in guys (W)ood, (P)orc, kV	No guys	Nil	—	None	—
CONDUCTORS					
AI, ACSR, ACAR, AAAC, 5005	ACSR 84/19	ACSR 18/7	ACSR 42/7	ACSR 96/19	ACSR 84/19
Diameter, in.; MCM	1.762; 2156	0.9; 583	–; 1361	–; 1857	1.82, 2300
Weight 1 cond./ft, lb	2.511	0.615	1.468	2.957	2.68
Number/phase; Bundle spacing, in	2; 18	4; 18	4; 18	2; 16	2, 18

Table A.3.2 (*Continued*)

Spacer (R)igid, (S)ring, (P)reform	S	R	R	S or P	—
Designed for—A/phase	2400	2500	—	1800	3380
Ph. config. (V)ertical, (H)orizontal, (T)riangular	H	H	H	H	H
Cond. offset from vertical, ft	—	—	—	—	—
Phase separation, ft	32	40	50	38–41	39–40
Span: Normal, ft; Maximum, ft	1500; 2764	1400; 2550	1400; 2800	1150; 1800	1175; –
Final sag, ft; at °F; Tension, 10^3 lb	46 at 130F; 19.1	45.0 at 60F; 3.4	226 at 120F; 6.2	40 at 120F; 12.4	35 at 60F; 13.0
Minimum clearance: Ground, ft; Structure, ft	40; 12.5	33; 10.5	45; 18.3	35; 7.75	35; 7.8
Type armour at clamps	Performed	Nil	None	—	None
Type vibration dampers	Stockbridge	Spacer damper	Tuned spacer	—	Stockbridge
Line altitude range, ft	1105–4896	800–1300	50–2500	2000–7000	400–8000
Max corona loss, kW/three-ph. mile	5	—	74 ($\frac{1}{2}$ in. snow/h)	—	—
INSULATION					
Basic impulse level, kV	2080	1800	—	—	—
Tangent: (D)isk or (L)ine-post	D	D	D	D	D
Number strings in (V) or (P)arallel	2V	1 or 2P	4V	Single	Single
Number units; Size; Strength, 10^3 lb	27; $5\frac{3}{4} \times 10$; 30	23; $5\frac{3}{4} \times 10$; 25	35; 5×10; 15	–; –; –	–; –; –
Angle: (D)isk or (L)ine-post	D	D	D	D	D
Number strings in (V) or (P)arallel	2V	3P	4P	Single	—
Number units; Size; Strength; 10^3 lb	25; $6\frac{1}{4} \times 10\frac{3}{8}$; 40	26; $5\frac{3}{4} \times 10$; 25	35; $6\frac{1}{4} \times 10$; 36	–; –; –	–; –; –
Insulators for struts	None	None	—	—	None
Terminations: (D)isk or (L)ine-post	D	D	—	D	D
Number strings in (P)arallel	2P	3P	—	—	4P
Number units; Size; Strength, 10^3 lb	25; $6\frac{1}{4} \times 10\frac{3}{8}$; 40	26; $5\frac{3}{4} \times 10$; 25	–; –; –	–; –; –	–; –; –
BIL reduction, kV or steps	3.5 steps	—	—	—	—
BIL of terminal apparatus, kV	1150–1425	1675–1900	2050–2200	—	1300
PROTECTION					
Number shield wires; Metal and Size	2; 7 No. 6 AW	2; $\frac{3}{16}$ in. galv.	2; $\frac{7}{16}$ in. galv.	1;–	2;–

Table A.3.2 (*Continued*)

Region:	500–550 kV—a.c.		700–750 kV	h.v.d.c.	
	Pacific	Canada	Canada	Mountain	Pacific
Utility:	So. California Edison Co.	Ontario Hydro	Quebec Hydroelectric Co.	U.S. Bureau of Reclamation	Los Angeles Dept. of W. & P.
Shield angle, deg; Span clear, ft	24.5; 28	20; 48	20; 50	30;–	20;–
Ground resistance range, Ω	—	15	—	—	30 max.
Counterpose: (L)inear, (C)rowfoot	C	L	L	—	—
Neutral gdg: (S)ol, (T)sf;	S; 0	S;–	S;–	S;–	S;–
Arrester rating, kV; Horn gap, in	420; None	480/432; None	636; –	–; –	–; –
Arc ring diameter, in.: Top; Bottom	None	None	–; –	–; –	–; –
Expected outages/100 miles/year	0.5	1	—	—	0.2
Type relaying	Ph. compar., carr.	Dir'l compar.	—	—	—
Breaker time, cyc; Reclos, cycles	2; None	3; None	6; 24	–; –	–; –
VOLTAGE REGULATION					
Nominal, %	±10	—	—	Variable	Grid control
Synch. cond. MVA; Spacing, miles	None	None	–; –	–; –	None
Shunt capac. MVA; Spacing, miles	None	None	330; 236	–; –	On a.c. terminals
Series capac. MVA; Compens., %	None	None	None	–; –	–; –
Shunt reactor, MVA; Spacing, miles	–; –	—	—	—	—
COMMUNICATION					
(T)elephone circuits on structures	None	None	None	None	None
(C)arrier; Frequency, kHz	C; –	C; 50–98	C; 42.0 and 50.0	None	None
(P)ilot wire	None	—	P	None	None
(M)icrowave; Frequency, MHz	M; 6700	–; –	M; 777.35	M; –	—

(*Permission of Edison Electric Insitute*)

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FOREWORD

When I was first approached to write this foreword there was no need to be threatened with ancient Arab curses for I had known and worked with the author for nearly 20 years. I had the pleasure of seeing his first book on this subject developed and become very successful both here in Australia and in the United States of America.

Arthur C Clark once wrote, 'Any sufficiently advanced technology is indistinguishable from magic.' Technology is now so well developed that it is no longer necessary to treat it as magic—in this modern world we can live with it, or we can study to understand it.

Whether you study, buy, build or service DC power supplies and oscillators, this book helps you to understand their operation and to analyse them. This easy-to-read book explains the basic theory and functions in clear, understandable terms. It gives you the latest information you need to design, construct, install and properly adjust DC power supplies and oscillators.

Barry Davis's book is recommended as a text or reference book for those studying electronics in TAFE colleges from vocational to advanced certificate technician levels, and for those already in the electronics industry.

Only those who read and then advance to application of the knowledge will appreciate this book's beneficial effects.

Robert G Comrie
Technician/Teacher/Writer

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ABOUT THE AUTHOR

Barry Davis began his career in electronics as a radio technician in the Royal Air Force in 1960. Since then he has been involved in many areas of electronic servicing and design.

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Advancing technological change and the acceptance of his previous two books has resulted in them being revised and combined into this edition. During the 1980s he served as a specialist consultant in electronics involved in a skills upgrading program for overseas teachers.

His achievements in writing and technical training over the years have resulted in a listing with the American Biographical Institute and an honorary position as a member of their research board of advisors.

Curriculum development and regular liaison with industry are still high on his list of priorities.

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PREFACE

This textbook is a second edition of *Understanding DC Power Supplies* combined with the first section of *Understanding Oscillators*. It is intended for students studying electronics at the basic level through to the Advanced Certificate, who already have a basic knowledge of AC theory and semi-conductor devices. It will also be of interest to the hobbyist and the radio amateur. The objectives of the National Engineering Curriculum modules NE04, 06, 37, 178 and 181 are also satisfied by this text and a modular grid is available to Instructors on application to Prentice Hall.

Each chapter has the objectives listed and concludes with review questions (multiple choice and written response types), with the solutions given in an appendix. These questions are designed to test the level of understanding achieved from each chapter.

Section 1 is a comprehensive study of power supplies covering rectification, filtering, regulation and voltage multipliers. It then explores the 'state of the art' technology used in linear integrated voltage regulators, with tips on how to use them for various tasks and the precautions to be observed. This is followed by an in-depth look at switched mode power supplies, both discrete and fully integrated. The final chapter in this section dicusses fault-finding techniques and procedures, with many practical examples, plus a look at test and current television switched mode power supply circuits.

Section 2 initially discusses the principles of negative and positive feedback, with examples to illustrate the advantages and disadvantages of each. This is followed by a comprehensive study of many types of oscillator circuits including phase shift, LC, crystal controlled and digital types.

Many practical circuit diagrams (designed around readily available components) are included throughout the text and these enable the reader to build and test the circuits under discussion. This is a very important part of 'reinforcing' the theory, which is presented in a logical manner that is easy to follow. The use of mathematics has been kept to a minimum and any essential or interesting mathematical relationships have been documented in an appendix.

I have 'been fishing' and my wife didn't believe me when I said 'this will be the last book'. I would like to thank her for hours of letter crunching on the computer which has been an invaluable help.

Electron or conventional current?

A famous scientist named Michael Faraday performed a series of experiments in the 1830s involving the 'flow of current' through a solution. The experiments involved the use of a silver bar and a nickel bar immersed in a silver nitrate solution. The silver bar was the anode and was connected to the positive side of a battery; while the nickel bar was the cathode and was connected to the negative side of the battery. A reaction took place which caused silver to be lost from the anode and desposited on the cathode. This was the principle behind electroplating as shown in Figure 1.

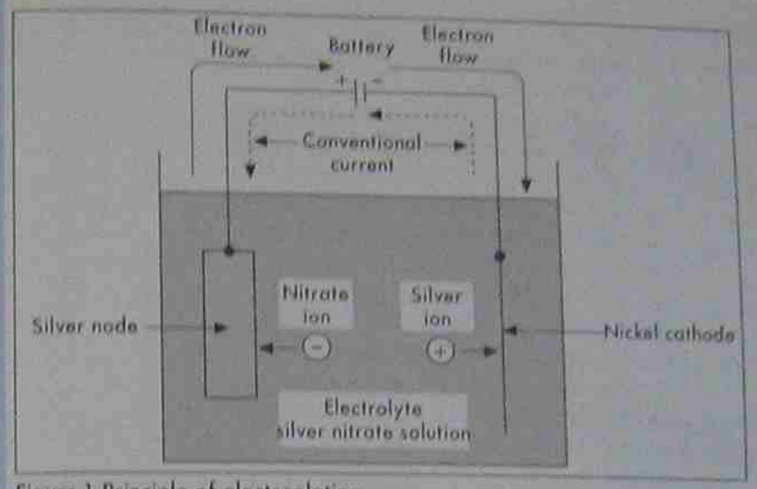


Figure 1: Principle of electroplating

In the 1830s the existence of the electron was unknown and the logical conclusion from the experiments was that the 'electricity passing through the solution' carried the silver to the cathode. Unfortunately, the conclusion is not that simple. Positive silver ions move from the anode to the cathode, while negative nitrate ions move in the opposite direction through the electrolyte solution.

In the external connecting circuit, negative electrons leave the negative terminal of the battery, travel through the solution, and enter the positive terminal. The actual 'charge past a point', that is, current or electron flow, is therefore in the opposite direction to Faraday's assumption.

Futhermore, it is commonly accepted that 'current' in a copper wire is due to the motion of negatively charged electrons. This 'fact' can be demonstrated by placing a rectangular sheet of copper in a strong magnetic field. The field should be perpendicular to the direction of current. The magnetic field exerts a force on the electrons flowing in the copper, with a direction that is perpendicular to both the magnetic field and the right to left motion of the electrons in the wire—similar to the beam deflection in the cathode ray tube of a television receiver. Figure 2 shows the action of the magnetic force (B) on the electrons (⊖) in the copper sheet.

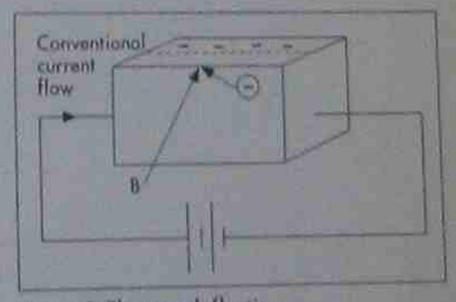


Figure 2: Electron deflection

Electrons are deflected upwards, and the upper edge of the sheet becomes negatively charged. Charge accumulates until the electrostatic force of repulsion acting downwards, balances or equalises the magnetic force acting upwards. The electrons now move through the sheet from right to left without being deflected. A condition now exists where a voltage can be measured between the top and bottom edges of the copper sheet. It is only microvolts but the polarity is negative, indicating that the charge carriers are negative, hence electrons!

This effect is known as the 'Hall Effect' and the voltage generated between the top and bottom edges is called the Hall voltage, as first identified in 1879 by Professor Edwin Hall.

Is the polarity of the voltage correct?

Figure 3 is the *same* as Figure 2 in terms of conventional current flow and magnetic field.

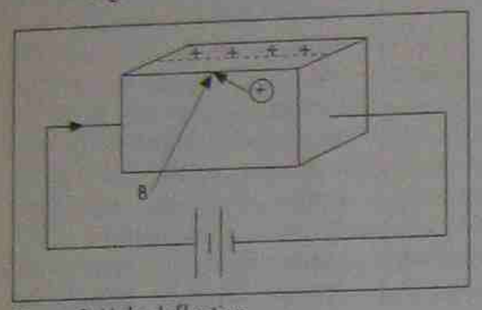


Figure 3: Hole deflection

The current carriers are assumed to be of the opposite polarity (positive) and move from left to right. They would be diverted upwards and charge the edge positive. This fact was *not observed*, thereby Figure 2 is correct and the polarity of the voltage is *negative*.

However, after all that, conventional or mathematical current flow (positive to negative) is still used, and we have had over a century to accept that electron flow is a scientific fact. Albeit the end result is the same. This text, may Mr Faraday forgive me, uses electron flow.

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CHAPTER 1

RECTIFIERS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Explain the purpose of, and the function of, each subsystem in the DC power supply of a piece of electronic equipment.
- Sketch and describe the operation of a half-wave rectifier circuit.
- Calculate the peak and average output voltage and current for a given half-wave rectifier circuit.
- Determine the output polarity from the schematic diagram of a half-wave rectifier.
- Determine the peak inverse voltage (PIV) for a diode used in a half-wave rectifier.
- Sketch and describe the operation of a centre-tapped full-wave rectifer.
- Calculate the peak and average output voltage and current for a given centre-tapped full-wave rectifier circuit.
- Determine the PIV for a diode used in a centre-tapped full-wave rectifier.
- Sketch and describe the operation of a full-wave bridge rectifier.
- Calculate the peak and average voltage and current for a given full-wave bridge rectifier circuit.
- Determine the PIV for a diode used in a bridge rectifier.
- Compare the three types of rectifier circuit and list the advantages and disadvantages of each.
- Select a diode from a data sheet to perform a particular task.
- List the faults experienced with diodes and transformers.
- Demonstrate how to test a diode with an ohmmeter.
- Briefly explain a fault-finding procedure for rectifier circuits.
- Diagnose faults in rectifier circuits from given circuit conditions.

1.1 Introduction

Electrical power is generally available in Australia, New Zealand, the United Kingdom and South Africa in the form of an alternating current (AC) supply at a fixed frequency of 50 Hz (hertz). The nominal voltage is generally 240 V (volts), and this represents its 'effective' or RMS value. However, a direct current (DC) voltage is usually required to operate most pieces of electronic equipment: for example, a stereo amplifier may require 24 V DC for satisfactory operation. Other types of equipment may require a higher or lower voltage than this, but the need for a steady voltage is common in all fields of electronics. DC power supplies are the circuits that change the 240 V 50 Hz (AC) mains supply into the DC voltage we require. In the United States the nominal voltage is 110 V and the frequency is 60 Hz; however, the principles of operation discussed in this text still apply.

There are four basic subsystems to be considered when discussing **power supply** circuits:

1. *Voltage transformation*: the use of a transformer to step the 240 V 50 Hz, or 110 V 60 Hz input voltage up or down to the level required. It *does not* change the 240 V AC to DC.
2. *Rectification*: the conversion of the AC voltage, with alternating polarity, into a voltage of constant polarity. The result is a **pulsating DC** voltage.

3. *Filtering*: the process of 'smoothing out' the fluctuations in the rectified voltage to give a 'smooth' DC voltage.
4. *Regulation*: the ability of the power supply to maintain a fixed level of DC voltage with a varying load, that is, the current demand is not constant.

This chapter discusses basic rectifier circuit arrangements, their operation and expected output waveforms. The requirements, advantages and disadvantages of the circuits are compared, and examples of relevant calculations are given. In Chapter 2 we consider how the pulsating DC voltage is applied to a filter circuit to smooth out the fluctuations. The types of circuits available to perform this function are shown, and their operation, merits and effect on circuit performance are also discussed. Regulation is a very important parameter in a power supply, and voltage regulators, both discrete and integrated, are given thorough treatment in Chapters 3 and 4. Basic regulator circuit operation is discussed in Chapter 3, and this leads to a consideration of three-terminal integrated circuit (IC) regulator circuits in Chapter 4. Handling precautions and circuit design ideas are also discussed there.

A list of notation used in this chapter is shown in Table 1.1.

1.2 Half-wave rectification

A PN junction, or semi-conductor diode, presents a low resistance to current in one direction and a high resistance to current in the opposite direction. Thus, it can perform **rectification**. Many types of semi-conductor diodes are available, and they vary in size from the very tiny, as used in sub-miniature circuitry, to very large 500 A (ampere) rectifiers, as used in large power supplies.

A simple **half-wave rectifier** circuit using a semi-conductor diode (D_1)is shown in

Table 1.1: Abbreviations

V_p	Voltage across primary winding (RMS value)
V_s	Voltage across secondary winding (RMS value)
V_{pm}	Maximum or peak primary voltage
V_{sm}	Maximum or peak secondary voltage
V_{Lm}	Maximum or peak voltage across the load
V_L or V_{out}	Average load voltage
I_{Lm}	Maximum or peak load current
I_L	Average load current
RMS	Root mean square
PIV	Peak inverse voltage (the reverse voltage rating of a diode also termed V_{RM})
R_L	Resistance
R_L	Average load resistance

Note: The term 'current' in this text refers to electron flow, that is, from negative to positive, unless otherwise stated. This is in accordance with standards set down by the Electronic Trades Standing Committee.

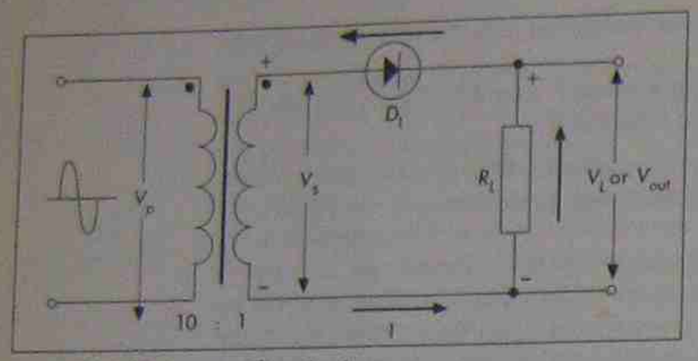


Figure 1.1: Half-wave rectifier circuit

Figure 1.1. V_p is the generator voltage representing 240 V AC, and is applied to the transformer primary winding. The transformer is connected so that no phase reversal takes place between primary and secondary windings (indicated by the dots at the top of the windings).

When the top of the primary is made positive by the input signal, the induced voltage will make the top of the secondary positive. This makes the anode of D_1 positive, forward biasing it, and D_1 conducts. Electrons flow from the negative end of the transformer secondary winding up through R_L, through the diode and back to the top of the secondary winding. This current causes a voltage drop across the load resistor, with a negative polarity at the bottom and positive at the top. If we assume that D_1 is an 'ideal diode' (that is, it behaves as a short circuit when forward biased), V_{out} will be equal to V_s. In practice, an ideal diode is not available, and therefore V_{out} or V_L will be less than V_s by the amount of voltage dropped over the diode (approximately 700 mV (millivolts)).

On the negative half cycle of input voltage, the top of the secondary winding is made negative. This reverse biases D_1, and the diode does not conduct. An output is available only when the input voltage forward biases the diode.

The transformer in Figure 1.1 is shown as a 10:1 step down, thereby reducing the input voltage to the rectifier. The waveforms for primary and secondary voltages for this are shown in Figure 1.2.

V_p is 240 V AC. This is the RMS value; therefore, the peak input waveform, or peak primary voltage (V_{pm}), can be calculated as follows:

RMS = 0.707 × peak value
peak value or V_{pm} = RMS/0.707

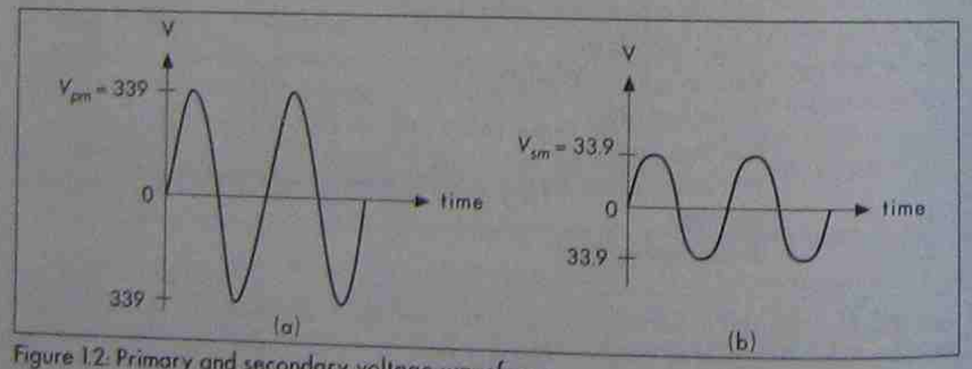


Figure 1.2: Primary and secondary voltage waveforms

= RMS × 1.414
= 240 × 1.414
= 339 V (1.414 is also referred to as $\sqrt{2}$)

The transformer will step the peak load voltage down to 1/10 of V_{pm}, giving the peak secondary voltage, or V_{sm}:

V_{sm} = 339/10
= 33.9 V

The transformer has thus changed the magnitude of the AC voltage to the desired level, and this is applied to the diode. As explained above, the diode conducts only on the positive half of the cycle, and the output waveform shown in Figure 1.3 results. Since only one-half of the input waveform is represented in the output, this circuit is known as a half-wave rectifier, that is, one half of the input signal is removed.

Care must be taken in power supply circuits not to exceed the maximum current and voltage ratings of the components used. We need to consider the capabilities of the load, the diode and the transformer. First, the current passing through the load will be considered. The peak load voltage (V_{Lm}) in the example is 33.9 V, and if we assume the value of R_L to be 200 Ω (ohms), the peak load current can be found thus:

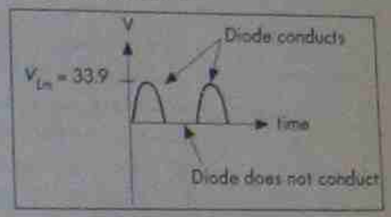


Figure 1.3: Output waveform of a half-wave rectifier circuit

$I_{Lm} = V_{Lm}/R$
= 33.9/200
= 0.1695 A
= 170 mA

Since the current through the load must follow the variations in V_L, it will also be half wave. Figure 1.4 shows the output voltage and current waveforms with amplitudes.

For a full sine wave (current or voltage) the relationships between peak, RMS and average values are given by the following formulae. (For discussion and derivation of these and other formulae, see Appendix C.)

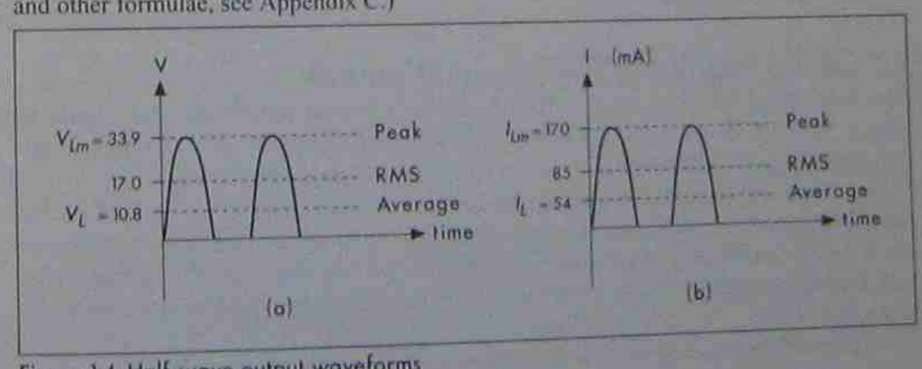


Figure 1.4: Half-wave output waveforms

RMS value = 0.707 × peak value
Average value = 0.637 × peak value

However, different relationships apply for a half-wave rectified sine wave, because one-half of the input waveform is clipped off:

RMS value = 0.5 × peak value
Average value = 0.318 × peak value

The power dissipated in the load can be calculated from the RMS values of load voltage and current:

$$\text{Power} = V \times I$$
$$= 17 \times 85 \times 10^{-3}$$
$$= 1.45 \text{ W}$$

The load resistor, or R_L, should therefore have a power rating greater than 1.5 W.

The diode must also be considered from the standpoint of its breakdown voltage. It conducts on one half cycle only and is non-conducting (off) on the other half cycle. During this time the input voltage (V_s) reverse biases the diode, and there is no voltage across R_L. The full transformer secondary peak voltage is across the diode; therefore, a diode with a breakdown voltage rating greater than 33.9 V must be used. This voltage rating is often termed the **peak inverse voltage** (PIV) or V_{RM}. The maximum and average current rating of the diode must also be taken into consideration, and this data is available from the manufacturer's specification sheets. Thus, the diode used in Figure 1.1 must meet the following minimum specifications:

Peak current = 170 mA
Average current = 54 mA
PIV = 33.9 V

Finally, the transformer has a maximum performance rating also. This is usually specified in terms of a VA (volt-ampere) rating, which indicates the product of the RMS voltage and current that can be safely handled.

1.3 Centre-tapped full-wave rectification

A **centre-tapped full-wave rectifier** circuit permits a current path in the same direction through the load for the full cycle of input. Two silicon diodes (D_1 and D_2) used in the circuit configuration shown in Figure 1.5 will perform full-wave rectification.

An alternating voltage is fed to the primary of the transformer. When point A on the transformer secondary is positive, point C will be negative. Diode D_1 will be forward biased, and D_2 will be reverse biased. Electrons will flow from the centre tap to earth, up through the load, through D_1, and back to the top of the transformer. This current path is indicated by the heavy arrows.

On the next half cycle, point A will be driven negative and point C positive, therefore D_1 will be reverse biased, and D_2 will conduct. Electrons will flow from the centre tap into earth, up through R_L, through D_2, and back to the bottom of the transformer. This current path is indicated by the dotted arrows.

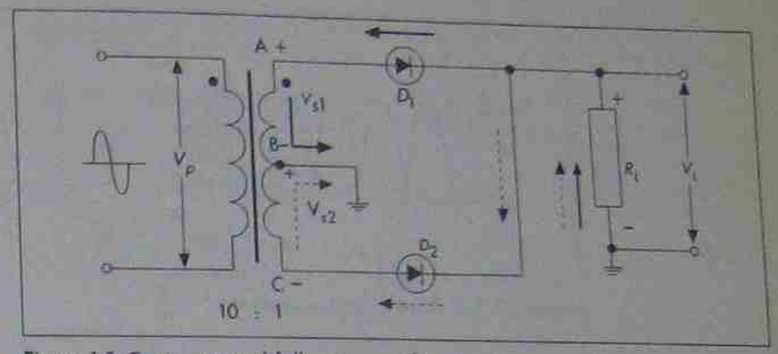


Figure 1.5: Centre-tapped full-wave rectifier circuit

It is important to note that the input voltage to each diode is one-half of the overall secondary voltage, and that the output waveform has the same polarity regardless of which diode is conducting. The input and output waveforms are shown in Figure 1.6. The primary voltage is 240 V RMS and if a 10:1 step-down transformer is used again, the full V_s will be 33.9 V. This is divided into two to provide each diode with its own secondary voltage. Therefore V_{s1} and V_{s2} will each be 16.95 V.

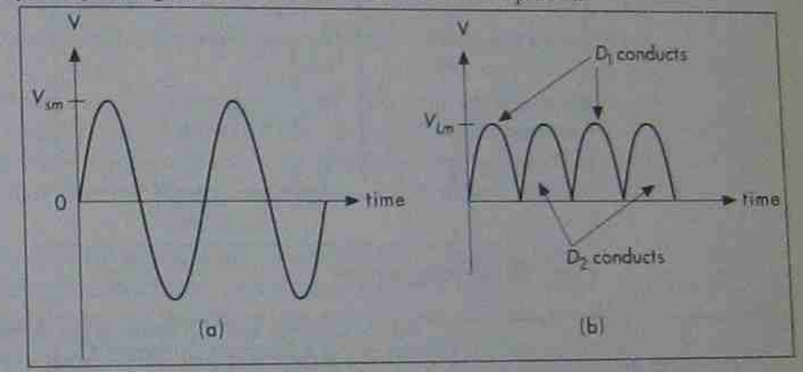


Figure 1.6: Secondary and output voltage waveforms

A full-wave rectifier circuit conducts for the full input cycle. The values of the voltages can be calculated thus:

V_{RMS} = 0.707 × peak value
$V_{average}$ = 0.637 × peak value

The peak load current will be:

$$I_{Lm} = V_{Lm}/R_L$$

If R_L is assumed to be 200 Ω, then:

$$I_{Lm} = 17/200$$
$$= 0.085 \text{ A}$$
$$= 85 \text{ mA}$$

Figure 1.7 shows the output voltage and current waveforms with amplitudes.

To calculate the peak inverse voltage (PIV) for a full-wave rectifier circuit, we must take into account the fact that one diode conducts while the other is off. For example, in Figure 1.8, when point A is positive, D_1 is conducting, D_2 is off, and $V_L = V_{s1}$. Diode D_2 then

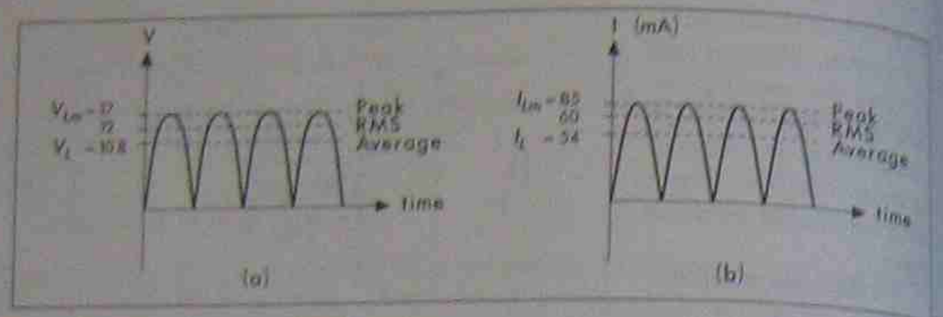


Figure 1.7. Full-wave output waveforms

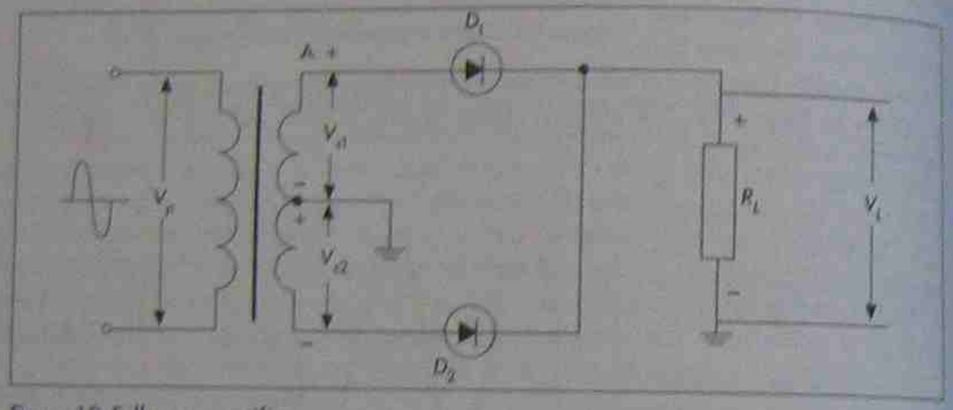


Figure 1.8. Full-wave rectifier

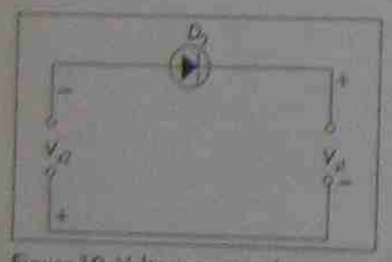


Figure 1.9. Voltage across the non-conducting diode

has V_{s1} in series with V_{s2} across it (as shown in Figure 1.9) because D_1 is forward biased, and virtually no voltage is dropped across it. Therefore, each diode must have a PIV equal to at least twice the peak load voltage (V_{Lm}).

1.4 Full-wave bridge rectifiers

A full-wave **bridge rectifier** uses a configuration of four diodes (D_1, D_2, D_3 and D_4), in which two diodes conduct on one-half of the cycle, and the other two conduct on the next half of the cycle. An example is shown in Figure 1.10. The transformer secondary is connected across the bridge at points A and B, and the output is taken across points C and D.

When the top of the transformer is positive, point A will be positive and point B will be negative. This means that the D_3 anode is positive, and the D_4 cathode is negative, or forward biased. The D_1 cathode is positive, and the D_2 anode is negative, or reverse biased. Therefore, on the positive half cycle D_3 and D_4 conduct, and on the negative half cycle D_1 and D_2 conduct. Electrons flow from the bottom of the transformer through D_4 to the common line,

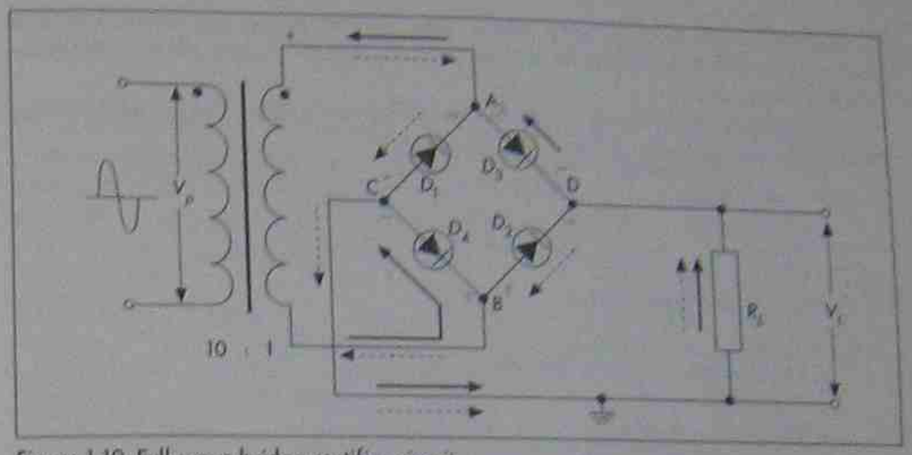


Figure 1.10. Full-wave bridge rectifier circuit

up through R_L, and through D_2 to the top of the transformer. This path is indicated by the heavy arrows.

On the next half cycle, D_3 and D_4 are off, and electrons flow from the top of the transformer through D_1 to the common line, up through R_L, and through D_2 to the bottom of the transformer. This path is indicated by the dotted arrows. Current passes through the load in the same direction for both half cycles of input voltage. The input and output waveforms are shown in Figure 1.11.

One advantage of the bridge rectifier is that, with a given transformer, the bridge circuit produces a voltage output nearly twice that of the centre-tapped full-wave rectifier circuit. This increase in voltage is due to the fact that the bridge does not use a centre-tapped secondary. The bridge rectifier circuit therefore applies the full secondary voltage to the rectifiers, while the centre-tapped full-wave rectifier circuit only applies the voltage between the centre tap and one end of the secondary to the rectifiers.

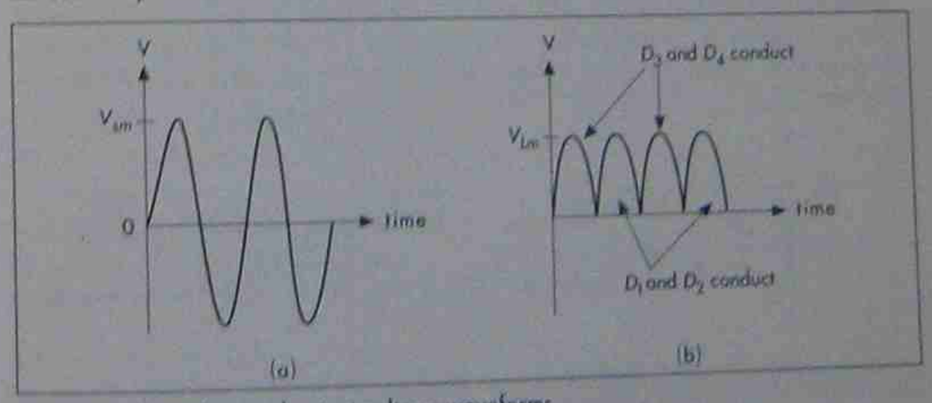


Figure 1.11. Secondary and output voltage waveforms

A further advantage of the bridge circuit is that the peak inverse voltage (PIV) across a diode is only one-half the PIV across a diode in a centre-tapped full-wave circuit designed for the same output voltage.

If the same circuit conditions are assumed for the bridge rectifier circuit as were used for the centre-tapped full-wave rectifier circuit earlier, then:

V_{rms} = 240 V RMS
= 339 V peak
V_m = 33.9 V

The peak load current is therefore:

$I_{Lm} = V_{Lm}/R$
= 33.9/200
= 0.1695 A
= 170 mA

Figure 1.12 shows the output voltage and current waveforms of a bridge rectifier circuit with amplitudes.

1.5 Advantages and disadvantages of rectifier circuits

If we compare the three rectifier circuits given in this chapter under the same transformer voltage and circuit load conditions, then it can be seen that the full-wave bridge rectifier circuit gives the highest average output voltage and current levels.

While the half-wave rectifier circuit has the advantage of simplicity and low cost, it also has several disadvantages. It is not very efficient since only half the input wave is used, and the average output voltage is low.

The centre-tapped full-wave rectifier circuit is more efficient than a half-wave rectifier circuit because it operates on both half cycles of the input wave. Its disadvantage is that it requires a centre-tapped transformer, and the PIV of the diode must therefore be higher.

Finally, the full-wave bridge rectifier circuit, even though it requires four diodes, is relatively inexpensive. The transformer does not require a centre tap and, given the same secondary input voltage, the output voltage will be twice that of a centre-tapped full-wave

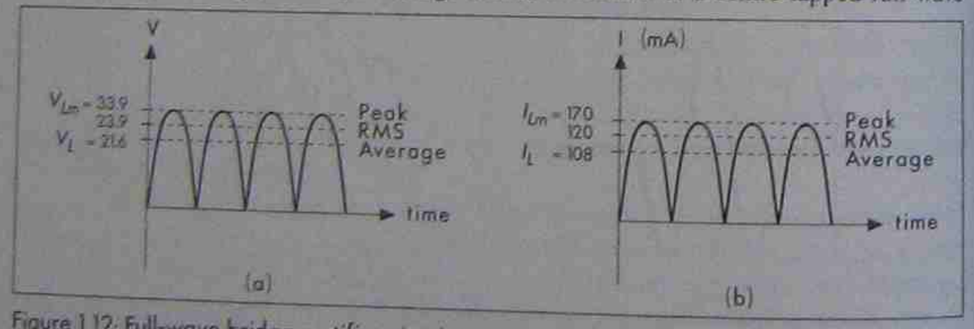


Figure 1.12: Full-wave bridge rectifier circuit output waveforms

rectifier circuit. However, the bridge rectifier circuit is less efficient than a centre-tapped rectifier circuit, particularly at low voltages with high load currents.

1.6 Diode characteristics

Diode characteristics have their own language of abbreviation. Unfortunately, there are no standardised abbreviations used by all manufacturers. Table 1.2 shows diode terminology and the most commonly used abbreviations. Figure 1.13 shows the structure of a diode and the symbol which represents it in a circuit diagram.

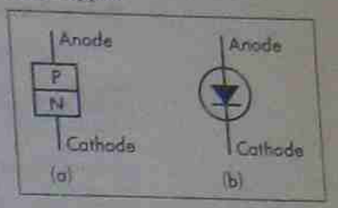


Figure 1.13: Diode structure and symbol

Table 1.2: Diode terminology

Average forward current	$I_{F(av)}$	The current to be handled by the diode under normal operating conditions.
Peak forward current	I_{FM}	The absolute maximum current at any instant (often called I_{peak}).
Surge current	I_{FSM}	The maximum current the diode can handle for one second.
Forward voltage	V_{FM}	The operating voltage at a particular forward current ($I_{F(av)}$) (often called V_{peak}).
Reverse current	I_{RM}	The maximum reverse current at maximum reverse voltage.
Power dissipation		Permissible power at 25°C; the power rating is less above 25°C.
Peak inverse voltage	PIV	The maximum voltage the diode can stand when in its non-conducting state, also referred to as the maximum peak reverse voltage, or V_{RM}.

1.6.1 Choosing the correct device

All the information concerning particular families or styles of diodes is contained in the specification (or 'spec') sheet. Tables 1.3 and 1.4 are examples of such sheets for International Rectifiers. Note the peak inverse voltage (PIV) is shown on these not as V_{RM} but V_{RRM} (maximum repetitive reverse voltage). Hint: Read the acronym in reverse.

To use the specification sheet you have to first know the parameters of the device you want to use; namely, $I_{F(av)}$ and V_{RM}. Check the $I_{F(av)}$ column for the value you need, say 1.5 A at 75°C (note that temperature may be important in a hostile environment). Determine the V_{RM} required, say 400 V, and look across from this value to the $I_{F(av)}$ column. The diode required is a 10D4 or its direct equivalent if it is not available.

1.7 Faults in rectifier circuits

We will now consider possible faults in the components which are common to the power supply rectifier circuits discussed in this chapter: diodes and transformers.

1.7.1 Diodes

Diodes are semi-conductor devices and are therefore susceptible to current surges. The faults encountered with diodes are of three types:

Table 1.3: Diode rectifier specification sheet
Source: Reproduced with permission of International Rectifier

Silicon Rectifiers

SILICON RECTIFIERS — GENERAL PURPOSE

Silicon rectifiers are available from less than 1 Amp to 3000 Amps, and voltages to 3000 volts. In addition to the standard industry packages, there are a number of other packages available as required. Two of these are pictured on page 11. These are ruggedly built devices with excellent reputations for reliability and performance.

AXIAL LEAD SILICON RECTIFIERS – 750 mA TO 6 AMPS

IF(AV) (A) @ Max. T_C (°C)	750 mA @ 25	1 @ 75	1 @ 100	1.5 @ 75	1.5 @ 40	3 @ 150	3 @ 125	4 @ 120	6 @ 85
I_{FSM} (A)	22	30	30	50	50	50	150	300	400
Notes	(1) (2)	(1) (2)	(1) (2) (3)	(1) (2)	(1) (2)	(2) (4)	(2) (4)	(2) (4)	(2) (4)
Case Style	DO-41	DO-41	DO-41	DO-39	DO-39	DO-39	C-12	C-16	C-15
				Part Number					
V_{RRM} 50 Volts	–	1N4001	1N7001PT	1N5000	1N4816	2N5905	–	–	60D05
100 Volts	–	1N4002	1N7002PT	1N51	1N4817	2N51	3051	4001	60S1
150 Volts	–	–	–	–	–	–	–	–	–
200 Volts	1N2069, A	1N4003	1N7003PT	1N52	1N4818	2N52	3052	4002	60S2
250 Volts	–	–	–	1N53	1N4819	–	3053	–	–
300 Volts	1N2070, A	1N4004	1N7004PT	1N54	1N4820	2N54	3054	4004	60S4
350 Volts	–	–	–	1N55	1N4821	–	3055	–	60S5
400 Volts	1N2071, A	1N4005	1N7005PT	1N56	1N4822	2N56	3056	4006	60S6
450 Volts	–	–	–	–	1N5052	–	–	–	–
500 Volts	–	1N4006	1N7006PT	1N58	1N5053	2N58	3058	4008	60S8
1000 Volts	–	1N4007	1N7007PT	1N5818	1N5054	2N5818	3058 10	–	60S10
Data Sheet (PD-)	2.002	2.005	#	2.001	2.004	2.004	2.008	2.029	2.029

SEE PAGE 12 FOR NOTES

AXIAL LEAD HIGH VOLTAGE 0.2 AMPS

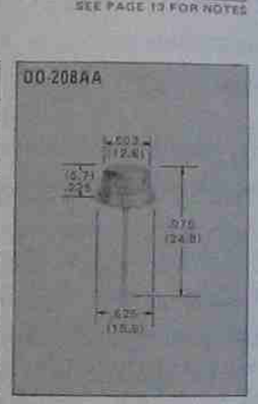
IF(AV) (A) @ Max. T_C (°C)	0.2 @ 50
Max. t_{rr} (µs)	1
Max. $I_{RM(REC)}$ (mA)	10
I_{FSM} (A)	3.5
Notes	(4)
Case Style	C-17
	Part No.
800 Volts	2D18
1000 Volts	2DL10
1500 Volts	2DL15
–	–
2000 Volts	2DL20
Data Sheet (PD-)	#

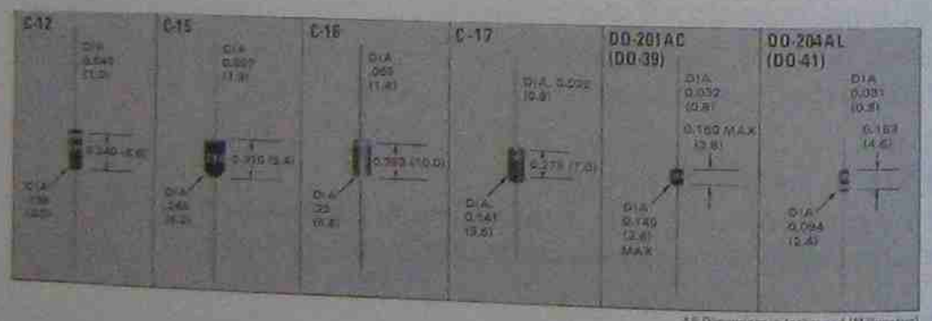
SEE PAGE 12 FOR NOTES

STUD MOUNTED[5] AND PRESS FIT SILICON RECTIFIERS 12 TO 35 AMPS

IF(AV) (A) @ Max. T_C (°C)	12 @ 150	12 @ 150	12 @ 150	15 @ 150	16 @ 150	25 @ 100	35 @ 140
I_{FSM} (A)	200	240	250	250	300	300	500
Notes	(6)	(6)	(6)	(6)	(6)	(6)	(6)
Case Style	DO-4	DO-4	DO-4	DO-5	DO-4	DO-208AA	DO-5
				Part Number			
V_{RRM} 50 Volts	12F5	1N1183A	12F5R	1N3208	16F5	1N3491	1N1183
100 Volts	12F10	1N1184A	12F10R	1N3209	16F10	1N3492	1N1184
150 Volts	–	1N1185A	–	–	–	–	1N1185
200 Volts	12F20	1N1202A1	12F20R	1N3210	16F20	1N3493	1N1186
300 Volts	–	1N1203A	–	1N2211	–	1N3494	1N1187
400 Volts	12F40	1N1204A1	12F40R	1N2212	16F40	1N3495	1N1188
500 Volts	–	1N1205A	–	1N3213	–	–	1N1189
600 Volts	12F60	1N1206A1	12F60R	1N3214	16F60	–	1N1190
700 Volts	–	1N3670A	–	–	–	–	1N3765
800 Volts	12F80	1N3671A	12F80R	–	16F80	–	1N3766
900 Volts	–	1N3672A	–	–	–	–	1N3767
1000 Volts	12F100	1N3673A	12F100R	–	16F100	–	1N3768
Data Sheet (PD-)	2.009	2.009	2.009	2.010	2.011	#	2.014

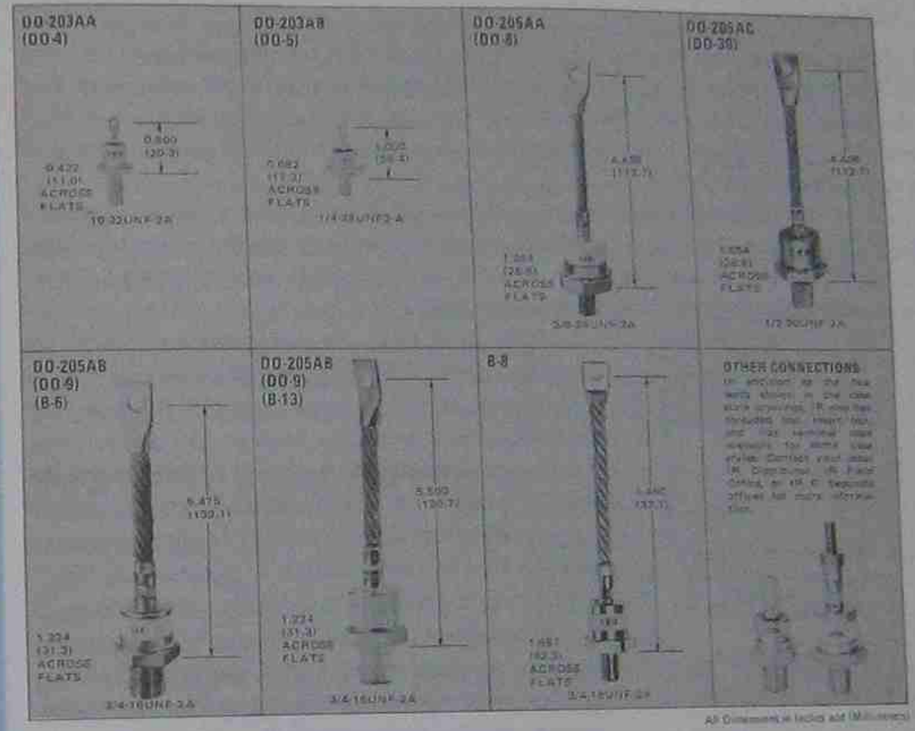
SEE PAGE 12 FOR NOTES





All Dimensions in Inches and (Millimeters)

Table 1.4: Diode rectifier specification sheet



All Dimensions in Inches and (Millimeters)

STUD MOUNTED[5] SILICON RECTIFIERS – 40 TO 150 AMPS

IF(AV) (A) @ Max. T_C (°C)	40 @ 140	40 @ 150	60 @ 150	60 @ 115	60 @ 140	70 @ 140	70 @ 115	100 @ 140	100 @ 140	150 @ 150
I_{FSM} (A)	500	600	700	700	900	1,050	1,200	1,600	2,000	3,000
Notes	(6)	(6)	(6)	(6)	(6)	(6) (8)	(4) (8)	(4) (8)	(4) (8)	(4) (8)
Case Style	DO-5	DO-5	DO-5	DO-5	DO-5	DO-5	DO-5	DO-8	DO-8	DO-30
					Part Number					
V_{RRM} 50 Volts	40HF5	1N1183A	1N2128	–	1N2128A	–	70H5A	–	–	1N3311
100 Volts	40HF10	1N1184A	1N2129	60HF10	1N2129A	–	70H10A	1N3288	1N3288A	1N3305
150 Volts	–	1N1185A	1N2130	–	1N3130A	–	–	–	–	–
200 Volts	40HF20	1N1186A	1N2131	60HF20	1N2131A	–	70H20A	1N3289	1N3289A	1N3306
250 Volts	–	–	1N2132	–	1N2132A	–	–	–	–	–
300 Volts	40HF30	1N1187A	1N2133	60HF30	1N2133A	–	70H30A	1N3290	1N3290A	1N3307
350 Volts	–	–	1N2134	–	1N2134A	–	–	–	–	–
400 Volts	40HF40	1N1188A	1N2135	60HF40	1N2135A	–	70H40A	1N3291	1N3291A	1N3308
450 Volts	–	–	1N2136	–	1N2136A	–	–	–	–	–
500 Volts	40HF50	1N1189A	1N2137	60HF50	1N2137A	–	70H50A	1N3292	1N3292A	1N3309
600 Volts	40HF60	1N1190A	1N2138	60HF60	1N2138A	–	70H60A	1N3293	1N3293A	1N3310
700 Volts	–	–	–	–	–	–	–	–	–	–
800 Volts	40HF80	–	–	60HF80	–	70HF80	70H80A	1N3294	1N3294A	1N3081
900 Volts	–	–	–	–	–	–	–	–	–	–
1000 Volts	40HF100	–	–	60HF100	–	70HF100	70H100A	1N3295	1N3295A	1N3082
1200 Volts	–	–	–	–	–	70HF120	–	1N3296	1N3296A	1N5102
Data Sheet (PD-)	2.014	2.028	2.028	#	2.028	#	2.015	2.016	2.016	2.017

SEE PAGE 13 FOR NOTES

1. open circuit;
2. short circuit; and
3. high resistance.

All of the conditions can be caused by excessive current. An open-circuit condition exists when the PN junction of the diode has been blown apart. A **back-to-front ratio** check will show an ohmmeter reads infinity in both directions. When the PN junction of the diode is fused together, it creates a short-circuit condition. A back-to-front ratio check gives a reading of zero ohms in both directions.

A condition of high resistance can exist within the diode if the junction has been partially damaged. The diode will then have a higher than normal voltage drop in the forward direction due to the higher level of resistance ($V = I \times R$). The back-to-front ratio will reveal this by giving a reading of a lower than expected output voltage.

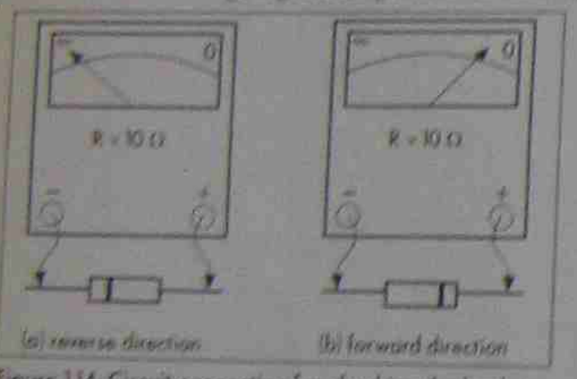


Figure 1.14 Circuit connection for checking the back-to-front ratio of a diode

Figure 1.14 shows the multimeter connections for checking a diode. Note that most analog multimeters have a negative polarity at the positive socket! This being the case, in order to forward bias the diode, the cathode end of the diode (indicated by the bar) must be connected to the negative polarity. In the forward direction (cathode negative and anode positive) and with the ohmmeter range set to $R \times 10$, a forward resistance of approximately 100 Ω to 200 Ω will be shown. With the diode reversed, the reading will be infinity. The readings in the forward direction vary between diode types, that is, germanium, silicon, signal or power diodes. However, the ratio of high reverse resistance and low forward resistance is true for all.

1.7.2 Transformers

Transformers are basically two coils of wire with magnetic linkage between them. Faults experienced are:

1. open circuit primary winding, where an ohmmeter check reads infinity ohms;
2. open circuit secondary winding, where again the ohmmeter reads infinity; or
3. short circuited turns in either the primary or secondary winding.

The DC resistance of the transformer primary and secondary is quite low, therefore ohmmeter checks must be carried out on the $R \times 1$ range. If a short circuited turn exists, the DC resistance of the winding will be changed from its normal value. However, more importantly, the inductance of the coil will be reduced. The inductance is reduced because the field in the primary winding of the transformer is partially cancelled by the current induced into the short circuited turn. The transformer operates with an AC voltage. With a reduced inductance the AC current in the coil will be higher, and this will result in the transformer getting physically hot.

1.8 Summary

The following items are a summary of the major points covered in this chapter:

- The transformer changes the level of AC voltage only (steps it up or down).
- The rectifier changes the AC voltage into a pulsating DC.
- The polarity of the pulsating DC depends on the orientation of the diodes.
- The input voltage to the rectifier is AC, and therefore is an RMS value.
- When a diode is in its non-conducting state, it has a peak voltage across it.
- The peak inverse voltage (PIV) of a diode is equal to 1.414 times the RMS voltage across it, and is dependent on the rectifier configuration.
- The choice of diode depends on the reverse voltage and average forward current it has to withstand under operating conditions.
- Rectifier faults include open circuit, short circuit and high resistance.
- A good diode has a high back-to-front ratio.
- Transformer faults include open circuit and short circuited turn windings.
- A full-wave bridge rectifier circuit gives the highest DC output voltage level.

Key terms

average load current (I_L)
average load voltage (V_L)
back-to-front ratio
bridge rectifier
centre-tapped full-wave rectifier
half-wave rectifier
maximum reverse voltage (V_{RM})
peak inverse voltage (PIV)
peak load current (I_{LM})
peak load voltage (V_{LM})
peak secondary voltage (V_{SM})
pulsating DC
power supply
rectifier
rectification

1.9 Troubleshooting problems

1. The DC output of the circuit in Figure 1.1 is strange. Tests show that the output signal is the same as V_s. What is the problem? (Answer: D_1 is short circuited)
2. The DC output of the circuit in Figure 1.5 is half what it should be. Tests show the output waveform to be the same as Figure 1.3. What is the probable cause? (Answer: D_1 or D_2 is open circuit.)
3. The circuit shown in Figure 1.10 has an output voltage of 18 V. Tests with an oscilloscope show that one half cycle of the output waveform is lower in amplitude than the other. What is the most probable fault? (Answer: One diode in the bridge is damaged and has a high resistance.)
4. The DC voltage across R_L of Figure 1.16 is 0 V and the primary voltage measures 240 V. What

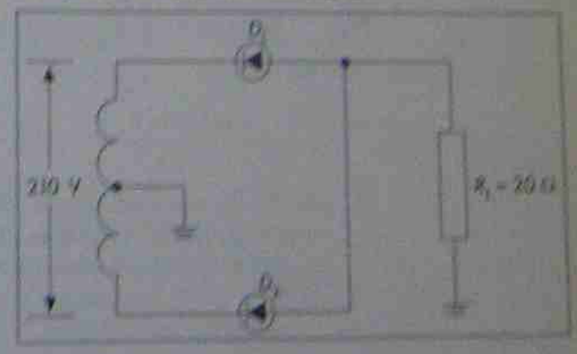


Figure 1.15

are the possible causes? (Answer: V_p is correct, therefore the fuse is intact. The possibilities are that the transformer secondary winding is open circuit or two or more diodes are open circuit.)

1.10 Review questions (answers in Appendix A)

1. Figure 1.1 shows the polarity of the output voltage across the load resistor R_L. How can this polarity be reversed?
2. A rectifier circuit is named according to the way it performs.
   (a) A half-wave rectifier passes _____ of the AC input.
   (b) A full-wave rectifier passes _____ of the AC input.
3. What is the anticipated forward voltage drop across a diode?
4. For the circuit in Figure 1.1, assume $V_{pri} = 163$ V, the transformer turns ratio, primary to secondary, is 1.15:1, and the load is 10 Ω. Determine:
   (a) the RMS secondary voltage of the transformer;
   (b) the minimum reverse voltage rating of the diode;
   (c) the peak and RMS load current; and
   (d) the power dissipated by the load.
5. What is the average value of the output from a half-wave rectifier if the peak-to-peak secondary voltage is 100 V?
6. (a) What is the polarity of the output voltage in the circuit shown in Figure 1.15?
   (b) Calculate the following:
   (i) peak secondary voltage for D_1 and D_2;
   (ii) the average output voltage;
   (iii) the average load current; and
   (iv) the PIV.

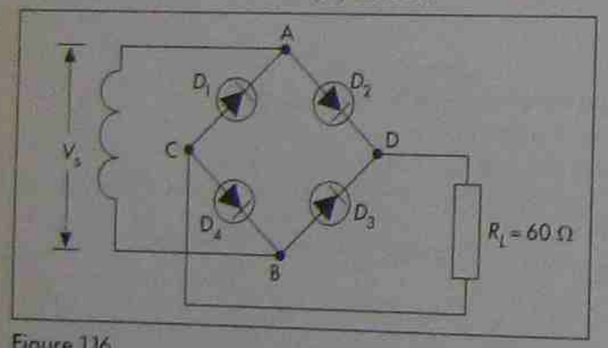


Figure 1.16

7. (a) What type of circuit is shown in Figure 1.16?
   (b) Which diode or diodes conduct when point A is positive with respect to point B, and what is the polarity of the voltage at point D with respect to point C?
   (c) Assume V_s is 40 V RMS. Calculate the value of:
   (i) the average output voltage;
   (ii) the current through the load; and
   (iii) the PIV of the diodes.
8. Assume the secondary voltage rating of a transformer is 250 V-0-250 V (that is, 500 V centre-tapped) and it is used in a full-wave circuit configuration.
   (a) Calculate the peak voltage across the load and the PIV of the diodes.
   (b) If the circuit was changed to a bridge configuration, would the values found for (a) remain the same? Explain.

9. Briefly explain the fault conditions you could experience with:
   (a) diodes; and
   (b) transformers.
10. A diode is required for a full-wave centre-tapped supply from a 110 V-0-110 V (that is, 220 V centre-tapped) secondary. It must be able to handle an average forward current ($I_{F(av)}$) of 2 A. Refer to Tables 1.3 and 1.4 and choose a diode to do the job.

Multiple-choice questions

1. Half-wave rectifiers are more likely to be used in:
   (a) portable equipment
   (b) digital power supplies
   (c) battery chargers for calculators
   (d) DC power supplies
2. The PIV of the diode in a half-wave rectifier circuit will be:
   (a) 2 times V_{peak}
   (b) V_{peak}
   (c) 1.414 times V_{peak}
   (d) 2.828 times V_{peak}
3. If the secondary voltage supplying the diode in question 2 is 100 V and the diode became short circuited, the output voltage will be approximately:
   (a) 0 V AC
   (b) 50 V DC
   (c) 100 V DC
   (d) 100 V AC
4. The total secondary voltage of a transformer in a full-wave centre-tapped rectifier circuit is 30 V peak-to-peak. The output voltage (unfiltered) measured with a digital voltmeter (DVM) will be:
   (a) 4.778 V
   (b) 5.303 V
   (c) 9.555 V
   (d) 10.607 V
5. The PIV of the diodes in a full-wave centre-tapped rectifier circuit will be:
   (a) 2 times V_{peak}
   (b) V_{peak}
   (c) 1.414 times V_{peak}
   (d) 2.828 times V_{peak}
6. Refer to Figure 1.17. The circuit configuration is:
   (a) full-wave rectifier
   (b) half-wave rectifier
   (c) full-wave bridge rectifier
   (d) half-wave voltage doubler

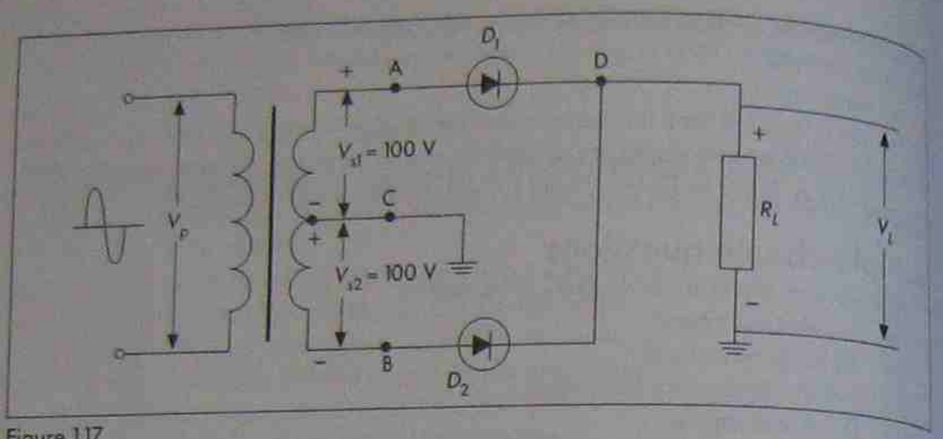


Figure 1.17

7. Refer to Figure 1.17. Diode D_1 will conduct when:
   (a) point B is positive with respect to point C
   (b) point A is negative with respect to point D
   (c) point C is positive with respect to point D
   (d) point A is positive with respect to point B
8. If D_1 in Figure 1.17 became open circuit, the DC voltage across R_L, measured with a DVM, will be approximately:
   (a) 32 V
   (b) 45 V
   (c) 64 V
   (d) 90 V
9. The minimum PIV for the diodes in Figure 1.17 is:
   (a) 70.7 V
   (b) 200 V
   (c) 141.4 V
   (d) 282.8 V
10. If R_L in Figure 1.17 is 20 Ω, the peak current rating for the diode must be a minimum of:
   (a) 3.53 A
   (b) 7.07 A
   (c) 10 A
   (d) 14.14 A

CHAPTER 2

FILTERS

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2.4 Full-wave rectifier with capacitor filter 26
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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Explain the need for filtering in a power supply.
- Briefly describe the three main filter configurations.
- Calculate the percentage ripple in a power supply circuit from given data.
- Explain how to reduce the ripple level of a power supply even further.
- Describe the effect of a capacitor input filter on output voltage level and diode current.
- List the precautions to be taken for diode protection when using a capacitor input filter.
- Calculate the percentage voltage regulation of a power supply circuit from given data.
- List practical ways to improve the regulation of a basic rectifier/filter circuit.
- State the ripple frequency of filtered half-wave and full-wave circuit.
- Define line regulation.

2.1 Introduction

As shown in Chapter 1, alternating current (AC) can be converted into a pulsating direct current (DC) by the use of rectifier circuits. However, most electronic equipment requires a 'smooth' DC supply, and this can be accomplished by the use of properly designed **filters**.

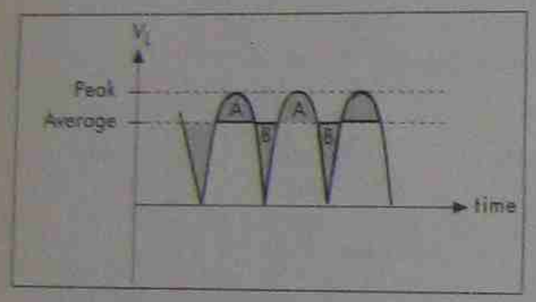


Figure 2.1 Unfiltered output of a full-wave rectifier

The unfiltered output of a full-wave rectifier is shown in Figure 2.1.

The polarity of the output voltage does not reverse but its magnitude fluctuates about an average level as successive pulses of energy are delivered to the load. The average voltage for the waveform shown in Figure 2.1 can be defined as the level at which area A is equal to area B. In this case the average level is 0.637 of the peak value. The voltage fluctuations above and below this average value are called '**ripple**'. The **frequency ripple** of a full-wave circuit is twice the frequency of the voltage that is being rectified. In the case of 50 Hz mains, the ripple is 100 Hz. For a half-wave circuit the ripple frequency is the same frequency as the input voltage, that is, for 50 Hz mains the ripple is 50 Hz. The higher the ripple frequency, the easier it is to **filter** the power supply, because the fluctuations occur at a faster rate, so the time period between pulses is therefore reduced.

2.2 Percentage ripple

The output of a rectifier is composed of direct voltage and an alternating (or ripple) voltage. The amount of ripple that can be tolerated varies with different applications and is designated by a percentage. (For the mathematics involved see Appendix B.)

The percentage of ripple for **percentage ripple** is 100 times the ratio of the Root Mean Square (RMS) value of the **ripple voltage** at the rectifier output to the average value of the total output voltage (V_{out}), that is:

$$\% \text{ ripple} = (V_{RMS}/V_{out}) \times 100$$

where $V_{RMS} = V_r \sqrt{3}$

and V_r = peak value of the ripple voltage

Figure 2.2 indicates graphically how the percentage of ripple may be determined. The ripple voltage is assumed to be sinusoidal for purpose of explanation, in fact it resembles a sawtooth as you will see in Figure 2.5, and hence the RMS value is therefore:

$V_{r\,peak}/\sqrt{3}$

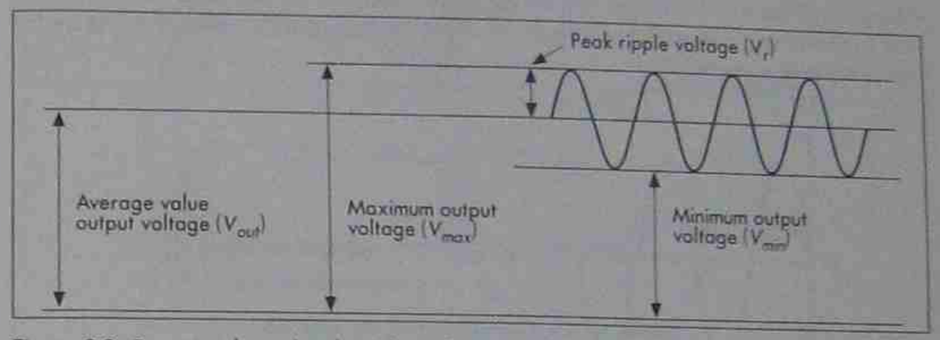


Figure 2.2: Output voltage levels with ripple

2.3 Half-wave rectifier with capacitor filter

To obtain a reasonably smooth voltage output we require an electronic component that will oppose changes in voltage. A **capacitor** is such a component, and it becomes our basic filter. Figure 2.3 shows a half-wave rectifier circuit with a capacitor (C) across the output.

Adding the capacitor to the circuit modifies the circuit operation and output waveform. When the diode conducts, current passes through the load and through C. The capacitor, C, charges almost instantly because there is negligible resistance in its charge path. There is only the resistance of the transformer winding. After one-quarter of an input cycle, C is charged to the peak voltage value. During the next quarter cycle, the input is going negative and C tries to discharge. The discharge path is through the load and the time taken for C to discharge is

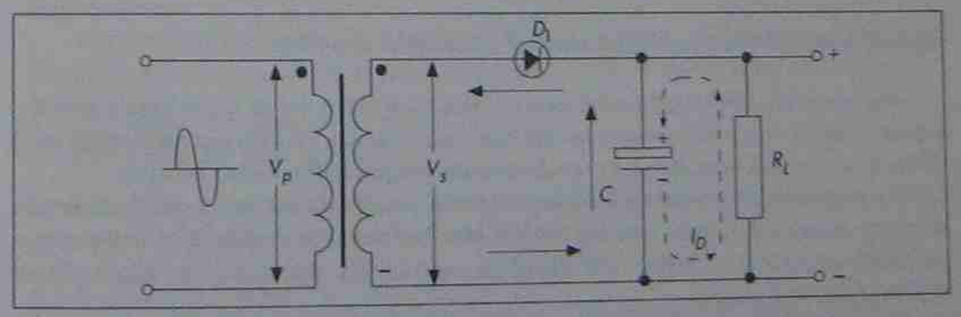


Figure 2.3: Half-wave rectifier with a capacitive filter

governed by the value of the load. The **time constant** ($C \times R_L$ where R_L is the load resistor) is the time taken for the voltage across the capacitor to change by 63% of its former steady value; therefore, the larger the value of R_L, the longer the time constant.

At first glance it appears that C could discharge through the D_1 diode and the transformer. However, C is charged, and therefore the D_1 cathode is at the potential of V_m (the peak secondary voltage), and shortly after one-quarter of a cycle of input (90°), D_1 is switched off, that is, the cathode is more positive than the anode. The capacitor, C, begins to discharge through the load, and the voltage across it decreases. Before it can discharge completely, the next cycle of input voltage forward biases D_1, and C recharges to the peak value. Figure 2.4 shows this action graphically.

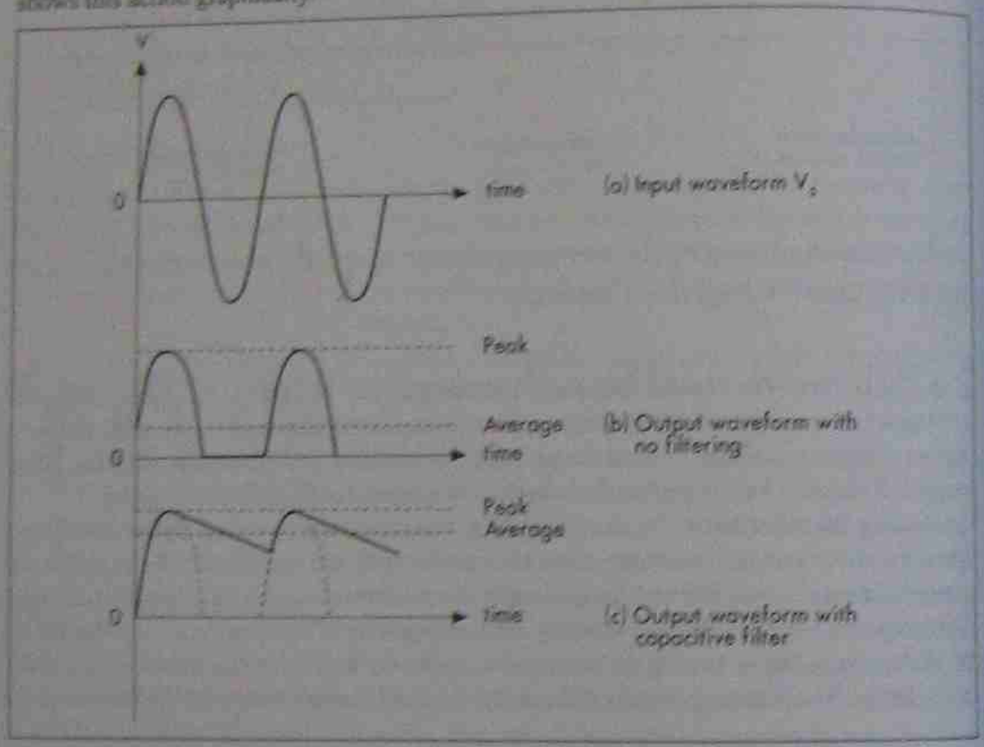


Figure 2.4: Waveforms showing the action of a capacitor as a filter

The heavy line showing the waveform in Figure 2.4(c) represents the voltage across the capacitor, and it shows that not only has the level been reduced, but also that the average level of the waveform has been raised, hence the **average output voltage** level is higher.

If a larger capacitor were used, the time constant would be even larger, and C would not discharge as much during the time the diode is off. Therefore, the average level of the output waveform would be even higher. The size of the capacitor also determines the length of time that the diode will conduct.

Figure 2.5 shows a comparison of conducting time for a large and a small value capacitor.

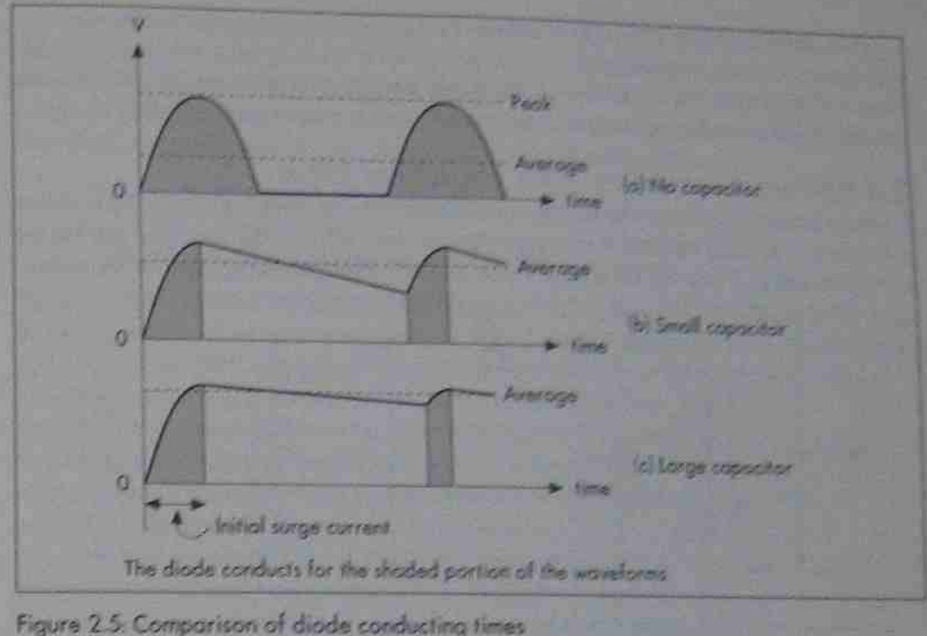


Figure 2.5: Comparison of diode conducting times

When the diode is not conducting, the capacitor is supplying the current through the load. Therefore, if a high load current is required the capacitor needs to be very large. This indicates the limitations of the capacitive filter—it is most useful at low current.

If the load demand on the circuit is excessive, the capacitor will discharge to a lower voltage level, and the diode will therefore have to conduct longer and harder. A fully discharged capacitor appears as a short circuit to the rectifier, and the initial **surge current** can be extremely large which could damage the diode.

With a large capacitor the **diode conduction time** is short, but the peak diode current is much higher. It is good practice to have a few ohms in series with the diode to protect it from the initial surge current. The transformer secondary resistance is usually sufficient with modern diodes.

The time between 'peaks' for a half-wave filtered waveform is 20 ms (milliseconds). The amplitude of the ripple voltage (V_r) can be determined providing we know two things:

1. the load current demand (I_L); and
2. the value of the filter capacitor.

So, if the half-wave rectified output is filtered by a 470 µF (microfarad) capacitor (C) and the load current is 30 mA, then V_r can be found from:

$$V_r = (I_L \times t)/C$$

where t = the time between charging peaks or 1/ripple frequency, that is, 20 ms

$V_r = 30 \times 10^{-3} \times 20 \times 10^{-3} / 470 \times 10^{-6}$
$= 1.27$ V peak-to-peak (p-p)

This is fairly excessive, so to reduce the ripple amplitude further we can increase the capacitor value providing it does not cause excessive surge current for the diode.

2.4 Full-wave rectifier with capacitor filter

A graphical representation of the waveforms obtained from the circuit in Figure 2.6 is shown in Figure 2.7. It can be seen that the capacitor has the same effect in this circuit as it did in the previous example. It reduces the level of ripple and increases the average level of the output.

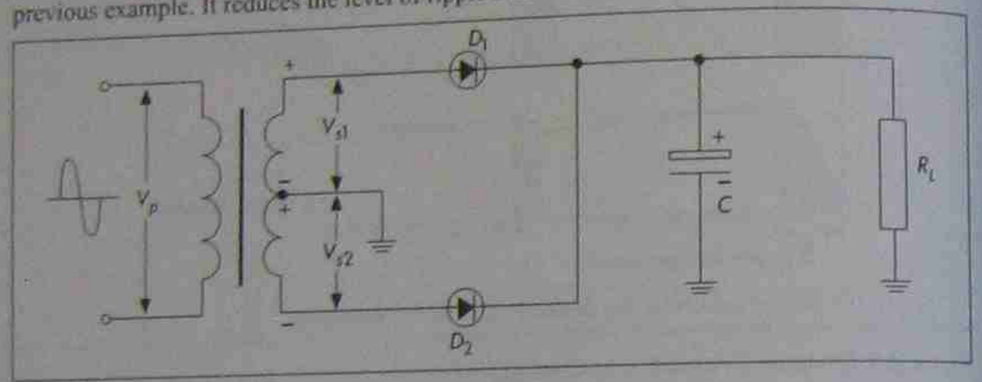


Figure 2.6: Full-wave rectifier with capacitive filter

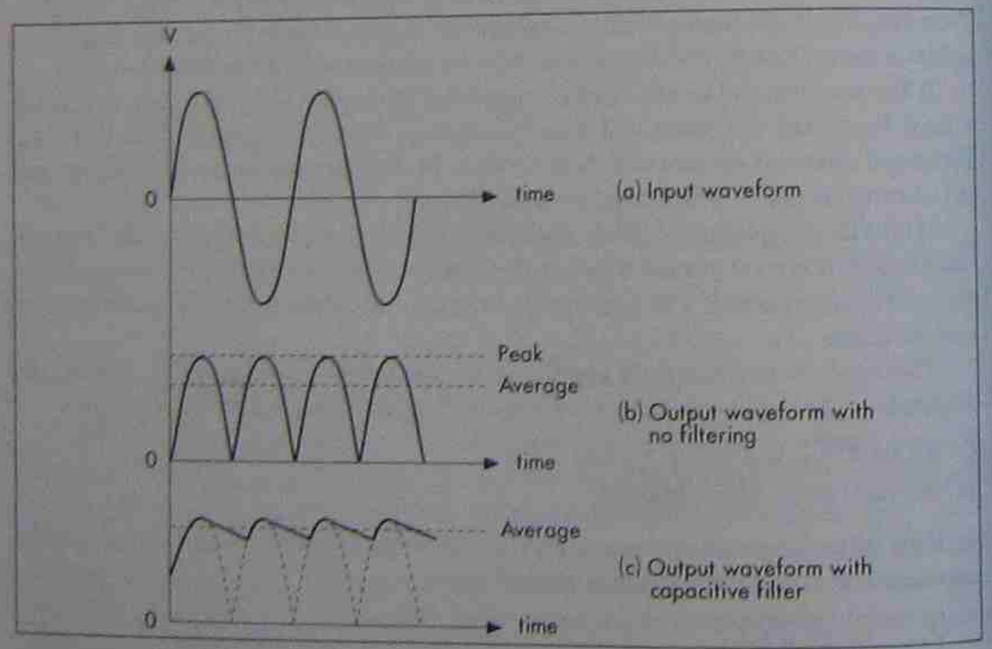


Figure 2.7: Waveforms showing the action of a capacitor as a filter in a full-wave circuit

Once again, the size of the capacitor determines the length of time the diode conducts. Because the voltage peaks occur twice as often as with the half-wave circuit the capacitor does not discharge very much before the next pulse occurs. This means that a given value of capacitor will filter a full-wave circuit better than it will a half-wave circuit. The ripple amplitude for the full-wave filtered circuit can also be determined from:

$V_r = (I_L \times t)/C$ but t is now 10 ms (100 Hz)
$= 30 \times 10^{-3} \times 10 \times 10^{-3} / 470 \times 10^{-6}$
$= 638$ mV p-p which is exactly half that of the half-wave circuit

Another consideration is the effect of the capacitor on the **peak inverse voltage** (PIV) of the diode. From Figure 2.8 it can be seen that with the capacitor charged to the peak of the secondary voltage (V_s) and with V_s changing polarity (in the next half cycle of input), the diode will have V_s + V_C in series across it in the reverse direction. This is true for both the full-wave and half-wave rectifier and, therefore, the PIV must be sufficient to handle the sum of V_s and V_C (that is, twice the peak voltage) when a **capacitor filter** is used. With a full-wave bridge rectifier circuit, however, this situation does not apply.

As can be seen in Figure 2.9, the diodes are never exposed to more than the voltage across secondary (V_s). The diode D_3 appears to have twice the peak voltage across it, but D_1 is conducting and point A is effectively at ground potential. The D_3 diode, therefore, has only the peak voltage across it. Thus, the PIV rating required for a bridge circuit is less than that for a full-wave circuit.

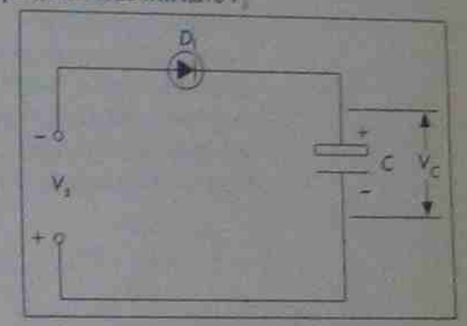


Figure 2.8: D_1 reverse biased with V_s & V_C across it

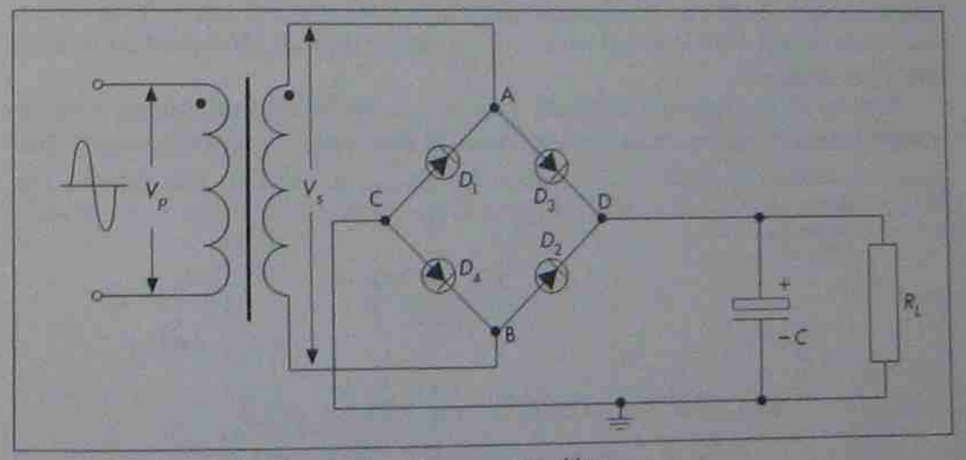


Figure 2.9: Full-wave bridge circuit with a capacitor filter

2.5 Choice of input capacitor

For high voltage supplies, a general 'rule of thumb' for choosing **capacitance value** is to use 1 μF of capacitance for every milliamp of current. However, the capacitive reactance at the ripple frequency must be less than the value of the load resistor, R_L.

With an output voltage of 200 V and 1 mA of current through the load, the load resistance will be:

$$R = V/I$$
$$= 200/1 \times 10^{-3}$$
$$= 200\ \text{k}\Omega$$

If a 1 μF capacitor is used as a filter, then the 'discharge' time constant will be:

$$t = C \times R_L$$
$$= 1 \times 10^{-6} \times 200 \times 10^{3}$$
$$= 200\ \text{ms}$$

To determine the filter input frequency, we must consider the type of rectifier employed. If a full-wave circuit is used on 50 Hz mains voltage, then the frequency is 100 Hz, and the 'period' of input is equal to 1/frequency, or 10 ms. The filter capacitor, which delivers energy to the load, has its charge 'topped up' every 10 ms. The time constant of the output is very long compared to the input charge period and, therefore, the capacitor does not discharge very much between input periods due to the high value of load resistance (200 kΩ). However, if the output voltage is reduced to say 5 V and 1 mA of current through the load, then:

$$R_L = V/I$$
$$= 5/1 \times 10^{-3}$$
$$= 5\ \text{k}\Omega$$

With a 1 μF capacitor, the discharge time constant will be $C \times R_L$. The capacitor is still 'topped up' every 10 ms, but the shorter time constant of the output circuit discharges it to a much lower voltage before the next input period. Figure 2.10 shows the current paths of the circuit just described.

From the two examples just discussed, it can be seen that for a low voltage power supply a larger value of capacitor is required to make the discharge time constant longer. This

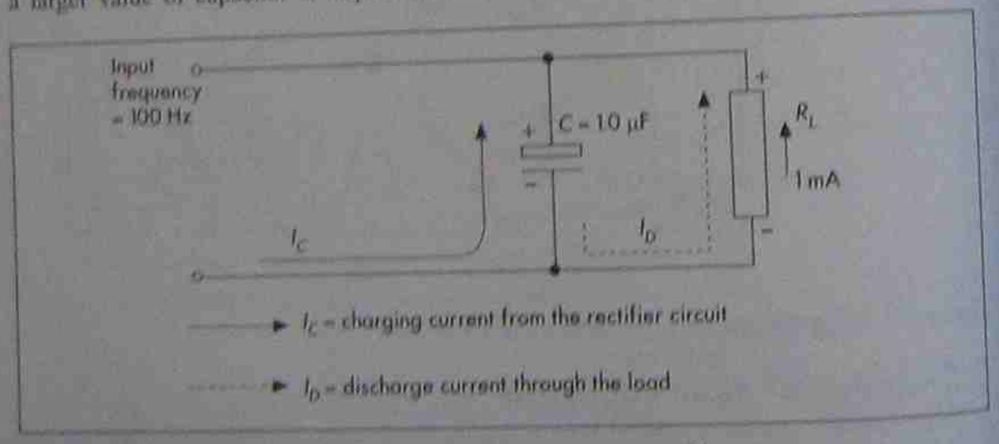


Figure 2.10: Charge and discharge currents through a capacitor filter

enables the input circuit to keep the capacitor charged more fully, and hence reduces the amplitude of the ripple voltage.

The discharge time constant must be long compared to the period of the input voltage frequency. The larger the value of the load resistance (R_L), the smaller the value of the filter capacitor required for a given ripple, because ripple amplitude is related to the discharge time constant.

A distinct disadvantage of **capacitor input filters,** including pi section filters (which will be discussed shortly), is that high load current (low resistance loads) results in short discharge time constants. This means that the capacitor discharges to a lower voltage level before the rectifier circuit conducts; the ripple amplitude is therefore increased, and the diode current is greater.

The 'technical' way to determine the appropriate value of filter capacitor to use is to diagnose the rectified output waveform. First, determine the peak level (this is V_s peak less the number of diode voltage drops). We also need to know the load current and the level of the DC output voltage required.

For example, a half-wave rectified output has a peak voltage level (V_p) of 10 V. A capacitor is required to filter this voltage to give a DC output voltage (V_{DC}) of 7 V and provide a load current (I_L) of 2 A.

From $V_{DC} = V_p - (I_L/4fC)$ we can transpose the formula for C
$C = (I_L/4f) \times (1/V_p - V_{DC})$ where f = the ripple frequency
So, $C = 2 / (4 \times 50) \times (1/10 - 7)$
$= 3333\ \mu\text{F}$

A figure of 2000 μF per ampere of load current is used as a practical rule for determining the value of C and works quite satisfactorily.

2.6 L-C section filters

A further improvement to a filtering circuit can be made by adding an inductor between the diode output and the capacitor. This type of circuit is called an 'L section' because it resembles an inverted 'L'. A schematic diagram of this type of filter is shown in Figure 2.11.

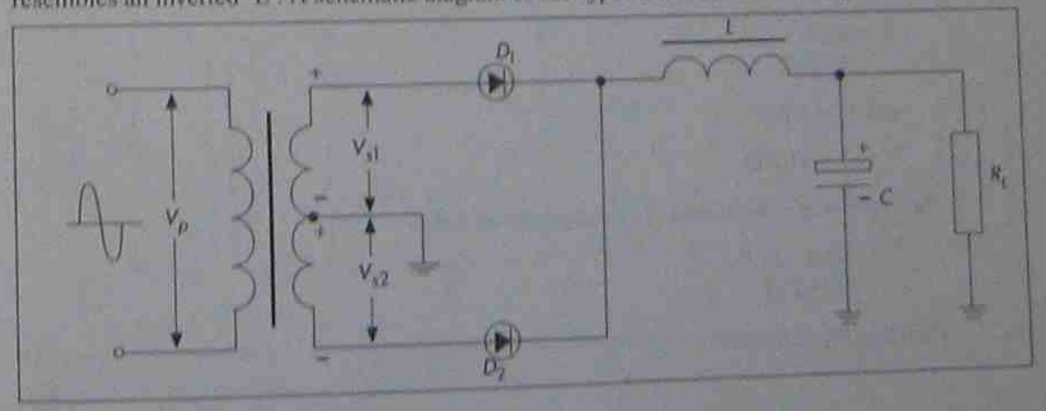


Figure 2.11: L-C section filter

The inductor or 'choke' opposes change in current, and energy is stored in the inductor (L) in the form of a magnetic field. During the interval between pulses, energy comes out of L as the field collapses. If a half-wave circuit were used with an **L-C section filter**, energy would be released from L during the second half cycle of input. The counter electromotive force set up by the changing current will tend to oppose the change and try to keep the current constant. The result is a smoother DC voltage, which is then used to charge the filter capacitor (C). An inductor offers a high impedance to AC and a low impedance to DC. The action of an L-C section filter is shown in Figure 2.12.

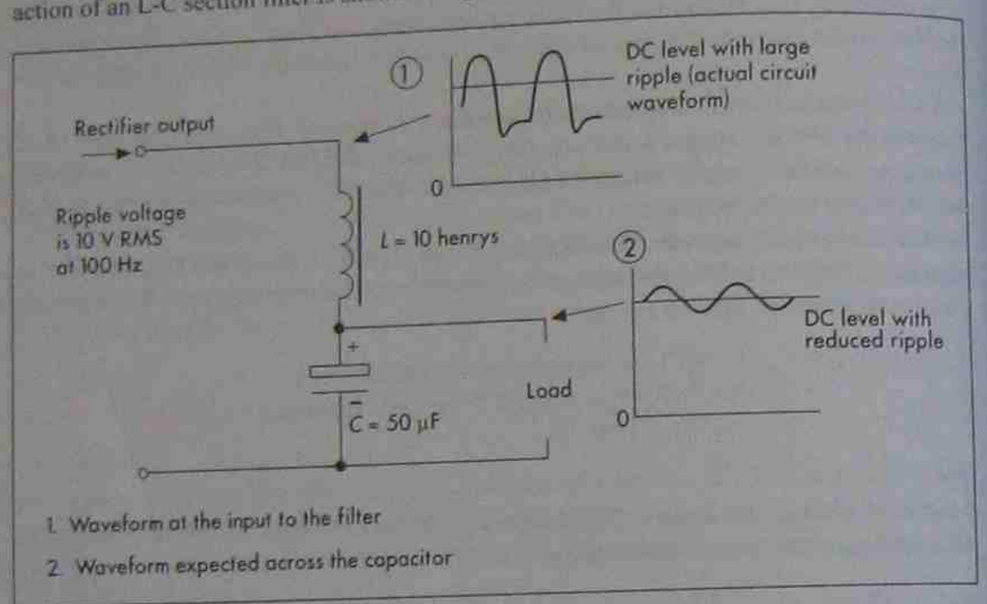


Figure 2.12: L-C section filter operation

Using the values from Figure 2.12, with 10 V RMS of ripple at 100 Hz, the L-C section filter behaves as a voltage divider:

$$X_L = 2\pi fL$$
$$= 6280\ \Omega$$

and $X_C = 1/2\pi fC$

$$= 31.8\ \Omega$$

The impedance of the circuit (Z) is calculated as follows:

$$Z = 6280 - 31.8$$
$$= 6248.2\ \Omega$$

Therefore the current is:

$$I = V/Z$$
$$= 10/6248.2$$
$$= 0.0016\ \text{A}$$
$$= 1.6\ \text{mA}$$

With 1.6 mA of current in the circuit, we end up with a ripple voltage across C approximately equal to $I \times X_C$:

$$V_C = I \times X_C$$
$$= 1.6 \times 10^{-3} \times 31.8$$
$$= 0.05\ \text{V}$$
$$= 50\ \text{mV (RMS)}$$

This has reduced the ripple level in the output by 200 times. The ratio of X_L to X_C must be high to achieve the best filtering, and also X_C should be much less than the load resistance (R_L) so that the capacitor provides the easier path for current.

The choke input filter allows a more continuous current from the rectifier diodes, rather than the pulsating current demanded by the capacitor filter and, because of this, it has better voltage regulation. Maximum current is available to the load with this type of operation, so this filter is more suited to higher load current demand.

On the debit side, the output voltage of the filter is equal to the average of the AC voltage delivered from the rectifier, rather than the peak value. It can also be costly, because a physically large, low resistance inductance is required for good regulation.

If a high degree of filtering is required with a high current, then two L-C section filters may be used in series. The level of the ripple on the DC output can be further reduced by including a capacitor prior to the inductor. This type of filter circuit is shown in Figure 2.13 and is called a **pi section filter** (π).

Capacitor C_1 performs the same function as the capacitor input filter, selected to have a

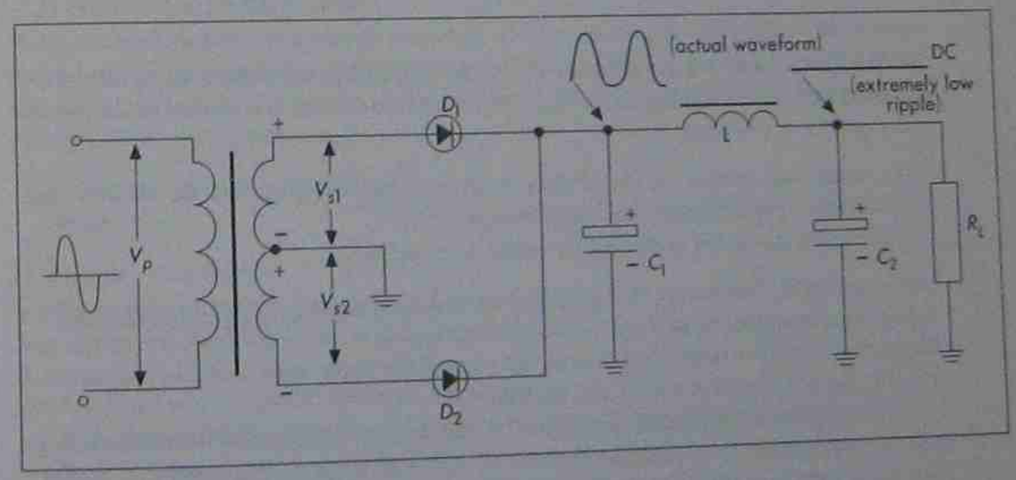


Figure 2.13: Pi section filter

low reactance (X_C) at the ripple frequency. The charge stored in C_1 is then released into an L-C section filter, L and C_2. The output voltage of this type of filter is very nearly pure DC.

Although a pi section filter is a more efficient filter, C_1 is still connected across the rectifier output, and high pulses of current are required to keep it charged. If the load demand is too great, the high current pulses may damage the diodes. The pi section filter is used on medium current equipment—it does produce a higher output voltage than the L-C section filter, but regulation is not as good.

2.7 C-R section filters

The inductor in either of the previous filters may be replaced by a resistor to form a **C-R section filter**. This type of filter is less expensive. A full-wave bridge circuit with a C-R section filter is shown in Figure 2.14.

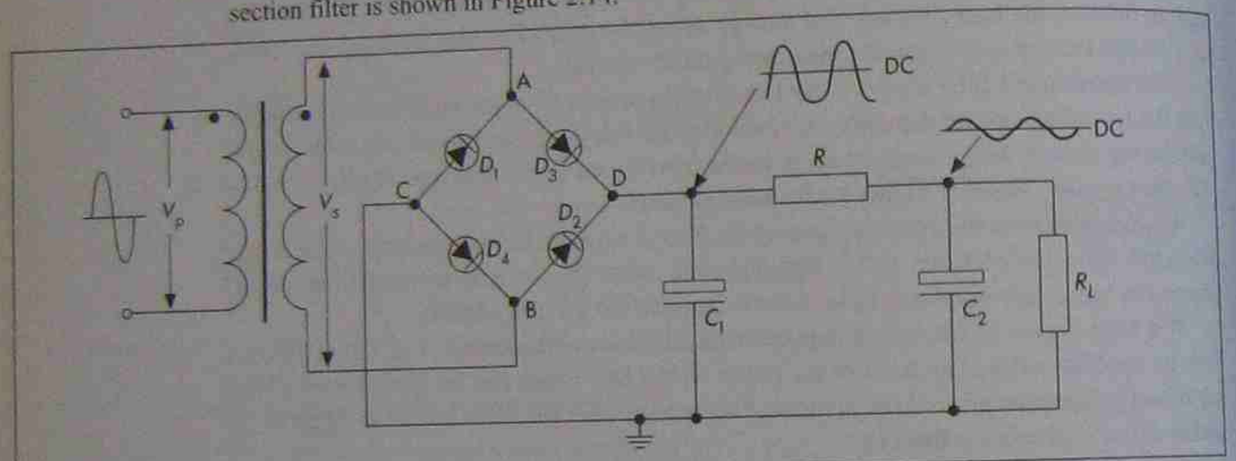


Figure 2.14: C-R section filter

The filtering action is not as good with a C-R section filter, and there is some loss of DC voltage at the output because of the voltage drop across the resistor. It is limited to low current equipment for two reasons:

1. A large load current would produce a large voltage drop across the resistor and, therefore, severely reduce the output voltage.
2. The power dissipated in the resistor would be excessive.

The capacitor C_1 behaves as in the other circuits discussed, and the ripple voltage sees a voltage divider consisting of R and C_2. The reactance of C_2 is low at the ripple frequency, and the ratio of R to X_C is large. This ensures that most of the ripple voltage is dropped across R and very little is present across C_2 and the load.

If the value of the components in Figure 2.14 are known, the expected DC output voltage and ripple amplitude can be calculated where $C_1 = C_2 = 20\ \mu F$, $R = 200\ \Omega$, and $R_L = 5000\ \Omega$. The input voltage is 100 V DC with 10 V (RMS) of AC ripple. The DC output voltage is

determined by R and R_L acting as a voltage divider, therefore:

$$\begin{aligned}\text{DC } V_{out} &= (R_L/R + R_L) \times V_{in}\ (\text{DC})\\ &= (5000/5200) \times 100\\ &= 96.15\ \text{V}\end{aligned}$$

The ripple amplitude calculation requires the value of X_C. Capacitor C_1 is equal to C_2, therefore:

$$\begin{aligned}X_{C1} = X_{C2} &= 1/2\pi fC\\ &= 76.9\ \Omega\end{aligned}$$

The value of X_C is about one-third the value of R and therefore must be taken into account.

$$\begin{aligned}\text{AC ripple voltage} &= (X_C/\sqrt{R^2 + X_C^2}) \times V_{in}\ (\text{RMS})\\ &= (79.6/\sqrt{200^2 + 79.6^2}) \times 10\\ &= 3.69\ \text{V}\end{aligned}$$

If X_C were much smaller than R (that is, a larger value capacitor), the amplitude of the ripple at the output would be considerably reduced. If $C_2 = 50\ \mu F$, then:

$$\begin{aligned}X_{C2} &= 1/2\pi fC\\ &= 31.8\ \Omega\\ \text{AC ripple voltage} &= (31.8/\sqrt{200^2 + 31.8^2}) \times 10\\ &= 1.57\ \text{V}\end{aligned}$$

Generally, R must be much less than R_L (to maintain the level of DC output voltage), X_{C2} must be much less than R (to reduce the level of AC ripple in the output), and X_{C2} must be about one-fifth the value of R_L, so that the capacitor provides the low resistance path for the AC current.

2.8 Voltage regulation

Voltage regulation is a measure of a circuit's ability to maintain a constant output voltage under varying load conditions. If a power supply delivers 100 V with no load connected, and this output drops to 80 V when the load is connected, then the regulation of the power supply can be expressed as a percentage:

$$\text{Percentage voltage regulation} = (V_{NL} - V_{FL})/V_{FL} \times 100$$

where V_{NL} = no load voltage and V_{FL} = full load output voltage

In the case given above:

$$\begin{aligned}\text{Percentage voltage regulation} &= [(100 - 80)/80] \times 100\\ &= 25\%\end{aligned}$$

The lower the percentage, the better the regulation.

If the regulation of a simple rectifier and filter circuit is poor, it may be improved by:

- using full-wave rectification;

- ensuring minimum resistance in both transformer and filter;
- using an L-C section filter if the current is high; or
- ensuring that solid-state diodes are used and that they have a low voltage drop across them.

The other parameter of importance is called **line regulation**, and this is the ability of the circuit to maintain a constant output voltage under a varying AC supply voltage condition.

2.9 Faults in filter circuits

Two main faults can be experienced with the inductor in a filter circuit:

1. open circuit coil; and
2. short circuited turn in the coil.

The most common fault is an open circuit, and the filter choke is tested in the same manner as a transformer primary or secondary.

The capacitor on the other hand can present three unserviceable conditions:

1. open circuit;
2. short circuit; and
3. leaks.

If it is a case of short circuit or open circuit, the capacitor is unserviceable because it cannot store charge. A leaky capacitor has a weakened dielectric and, therefore, it has a lower resistance than normal.

A further point to bear in mind is a change in capacitance value over time. This is especially noticeable with electrolytic capacitors. As they age, they dry out and their capacitance tends to decrease. This leads to an increase in capacitive reactance. They then offer more resistance to AC, and the circuit operation is affected as a result.

If an electrolytic capacitor changes value in a filtering circuit, the reactance ratio is changed and the ripple amplitude increases. This can cause faults such as hum in audio equipment, bending of straight edges in a television scene and general instability in operation. Electrolytic capacitors can be tested with an ohmmeter set to the R × 1000 range, as shown in Figure 2.15.

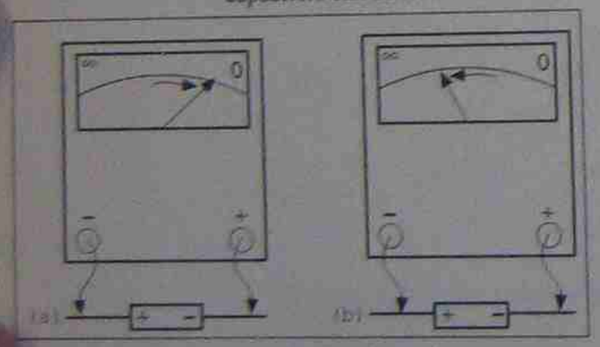


Figure 2.15 Testing a capacitor with an ohmmeter

With a good capacitor the needle moves quickly towards the low resistance side of the scale as the capacitor charges (Figure 2.15(a)) and then it slowly drops back towards infinity. The final resting point of the needle indicates the insulation resistance of the capacitor (Figure 2.15(b)). The insulation resistance of an electrolytic capacitor varies with the size, but approximately 1 megohm and upwards is common.

If the capacitor has a short-circuit condition, the ohmmeter will go to zero and stay there. If the capacitor is open circuit, then there will not be any charging action and the ohmmeter needle will read a very high resistance. However, this fault should be double-checked by reversing the meter leads and checking in the opposite direction.

2.10 Summary

The following items are a summary of the major points covered in this chapter:

- The pulsating DC has to be 'smoothed' or filtered to remove the fluctuations.
- The ratio of ripple amplitude to DC output level is called the ripple factor and the lower this figure is the better the filtering.
- The filter capacitor charges to the peak level of the pulsating DC.
- The average level of the DC output voltage is determined by the value of the load resistance, and hence the output time constant.
- With capacitor input filters the value of the peak inverse voltage (PIV) for the diodes has to be considered.
- Voltage regulation is the circuit's ability to maintain a constant output voltage under an increasing load condition.
- Line regulation is the circuit's ability to maintain a constant output voltage under a varying AC supply voltage condition.
- Ripple can be reduced and regulation improved by increasing the value of the filter capacitor.
- Main fault conditions include open circuit inductors, open circuit capacitors, short circuit capacitors and leaky capacitors.
- A practical rule of thumb for choosing a filter capacitor in low voltage supplies is one with a value of approximately 2000 μF per ampere of load current.

Key terms

average output voltage	peak inverse voltage (PIV)
capacitor filter	percentage ripple
capacitor input filter	pi section filter
capacitor value	ripple
C-R section filter	ripple frequency
diode conduction time	ripple voltage
filter	surge current
line regulation	time constant
L-C section filter	voltage regulation

2.11 Troubleshooting problems

1. The DC output of the circuit in Figure 2.9 is zero volts. Tests show the primary fuse is blown and an ohmmeter check from point D to ground (power off) reads zero ohms. What is the problem? (Answer: Capacitor *C* is a short circuit.)

2. The DC output of the circuit in Figure 2.11 is zero volts. Tests show the voltage at D_1/D_2 cathodes is acceptable. $R_L = 200\ \Omega$ and an ohmmeter check across R_L is acceptable. What is the fault? (Answer: Inductor L is an open circuit.)
3. The DC output of the circuit in Figure 2.13 is very unstable due to the high ripple level measured with an oscilloscope across C_2. V_{C1} is acceptable, so what is the most likely problem? (Answer: C_2 is open circuit or aged and gone low in value.)

2.12 Review questions (answers in Appendix A)

1. What is the purpose of filtering?
2. A 240 V DC voltage has a 2 V peak-to-peak ripple voltage present on it. What is the percentage ripple?
3. The regular capacitor in a capacitor-input filter is replaced with a capacitor of a larger value. What effect does this have on:
   (a) the average output level?
   (b) the diode current?
4. Which diode specification should be considered when using a capacitor input filter arrangement?
5. A high voltage power supply has an unloaded output of 350 V. If the loaded output is 330 V, what is the percentage voltage regulation?
6. If high current was the prime consideration of a power supply rather than voltage, what type of filter would you use?
7. From the circuit shown in Figure 2.16, calculate:
   (a) inductive reactance X_L;
   (b) filter current; and
   (c) peak output ripple voltage.
8. List three practical ways to improve the regulation of a basic rectifier/filter circuit.
9. What is the ripple frequency of:
   (a) a full-wave circuit operating at 100 Hz?
   (b) a half-wave circuit operating at 400 Hz?

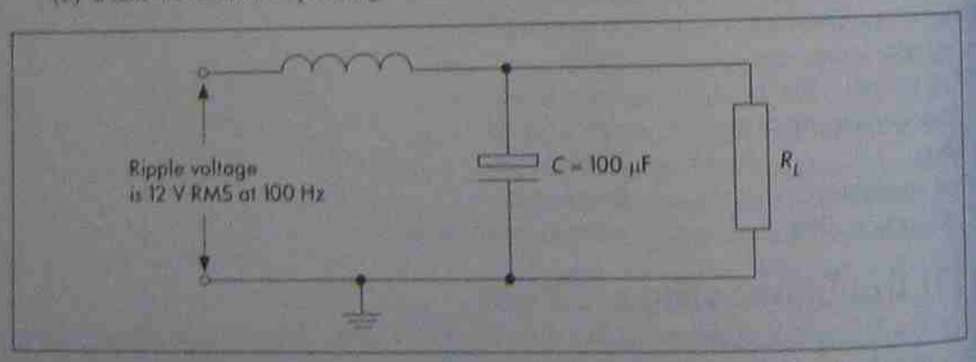


Figure 2.16

Multiple-choice questions

1. Filters are used in power supplies to:
   (a) reduce the rectifier diode PIV rating
   (b) increase the peak output voltage level
   (c) shunt high frequencies to ground
   (d) reduce variations in the DC output voltage
2. The ripple frequency of a full-wave rectified output voltage will be:
   (a) the same as the input frequency
   (b) twice the input frequency
   (c) unrelated to the input frequency
   (d) much higher than the input frequency
3. The inital surge current that charges the filter capacitor can be reduced by:
   (a) increasing the capacitor's value
   (b) reducing the capacitor's value
   (c) using higher $I_{F(av)}$ rated diodes
   (d) ensuring that the capacitor value is greater than 1500 µF
4. If the DC output voltage of 15 V has an AC ripple level of 500 mV on it, what is the percentage ripple?
   (a) 0.33%
   (b) 2.35%
   (c) 3.33%
   (d) 33%
5. If the value of the load resistance decreases, the average level of the DC output voltage will:
   (a) increase
   (b) stay the same
   (c) be zero
   (d) decrease
6. The ripple amplitude can be reduced further by:
   (a) placing another capacitor in series with the main filter
   (b) placing another capacitor in parallel with the main filter
   (c) changing the rectifier from a centre tap to a bridge type
   (d) placing a resistor in series with the filter capacitor
7. The output of a bridge rectifier is 10 V peak, and a DC level of 8 V is required. If the load current is to be 3 A, what value of filter capacitor is required?
   (a) 2000 µF
   (b) 5500 µF
   (c) 7500 µF
   (d) 10 000 µF
8. A power supply delivers 50 V with no load. When the load is connected, the level drops to 48 V. What is the percentage regulation?

(a) 4.39%
(b) 5%
(c) 6.34%
(d) 8.2%

9. The voltage regulation of a power supply may be improved by:
(a) decreasing the value of the load resistance
(b) using half-wave rectification
(c) ensuring minimum series resistance in the output current path
(d) inserting a filter choke in series with the output

10. An audio amplifier develops a hum in the speaker. When tested, the voltages were acceptable but the ripple level is 500 mV higher than normal. The most likely fault is:
(a) an open circuit filter capacitor
(b) a short circuited filter capacitor
(c) an open circuit filter choke
(d) a dried out electrolytic filter capacitor

CHAPTER 3

VOLTAGE REGULATORS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Explain the need for a voltage regulator in a power supply circuit.
- Sketch the circuit and describe the operation of a zener diode shunt regulator.
- List the disadvantages of a shunt regulator.
- Sketch the circuit and describe the operation of a simple series regulator.
- Determine the efficiency of a regulator.
- Define the term 'closed loop' regulator, and explain the circuit's operation.
- Show two techniques for providing overload current protection.
- Explain the operation of a current regulator.
- State the minimum input-output voltage difference required to hold regulation.

3.1 Introduction

The need for voltage regulation can be seen by examining the simulated regulator in Figure 3.1. With 20 V from the rectifier and filter circuit, 15 V is required across the load and 5 V is dropped across the variable resistance, R_A.

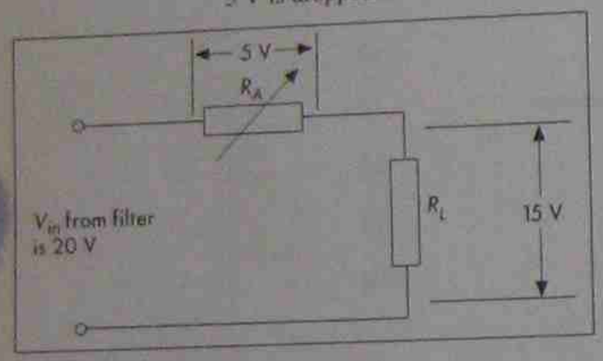


Figure 3.1: Simulated voltage regulator

If the input voltage suddenly increases, the voltage drop across R_A would have to increase to maintain a constant voltage across R_L; that is, the resistance of R_A would have to be increased. On the other hand, a decrease in the value of R_L (more current required by the load) would increase the load current and therefore increase the voltage across R_A. To maintain the output voltage constant, R_A would have to decrease in value. An increase in the value of R_L would cause less current through the load. The voltage drop across R_A would decrease, and the value of R_A would have to be increased to hold the voltage across R_L at 15 V. It can be seen from these examples that the output voltage can be held constant by varying R_A to suit the circuit conditions. To keep adjusting R_A manually would be impossible, so a device called a **voltage regulator** is used to perform the task electronically. Figure 3.2 shows the format of the final power supply circuit.

3.2 Zener diode shunt regulators

The simplest form of voltage regulator circuit is the **zener diode shunt regulator** (Z_D) as shown in Figure 3.3.

A zener diode works in a 'reverse breakdown' or 'avalanche mode', with the cathode connected to the positive terminal of the supply. When the diode is in breakdown, its voltage remains fairly close to the **breakdown voltage value**, even if the current varies. The reverse characteristic curve for a zener diode is shown in Figure 3.4.

The zener diode will operate between its knee voltage (I_{zk}) and its maximum current

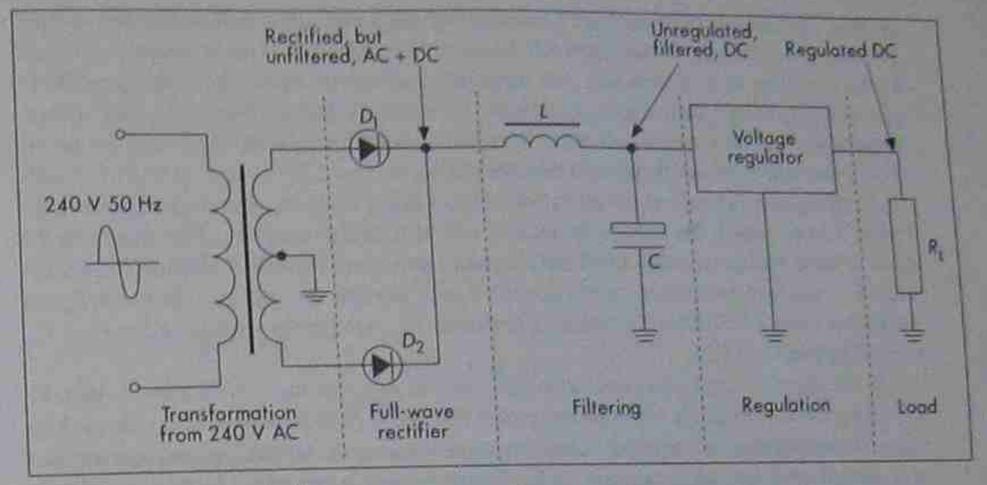


Figure 3.2: Power supply diagram

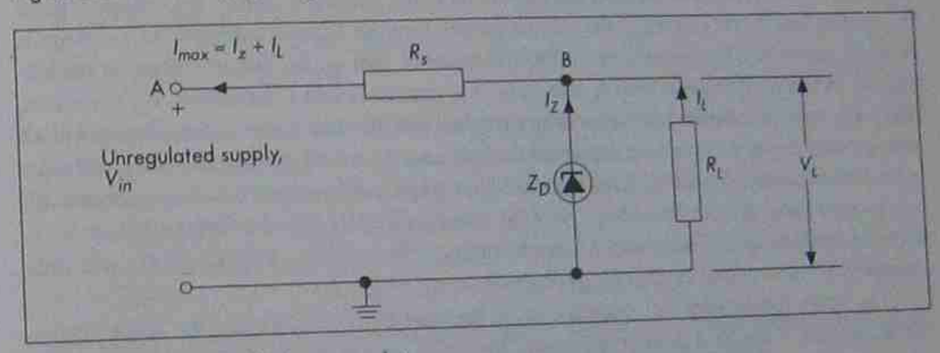


Figure 3.3: Zener diode shunt regulator

rating (I_{zt}). When the load is minimum or removed, the zener diode, being a shunt regulator, must be able to handle the load current plus its **knee current**. Therefore, $I_{zt} = I_L + I_{zk}$. The load is in parallel with the zener diode and therefore the load voltage will remain fairly constant providing the zener current stays between the two level limits of I_{zk} and I_{zt}. If the unregulated DC voltage input increases, the current through the series resistor (R_s) also increases. This extra current is diverted through the diode

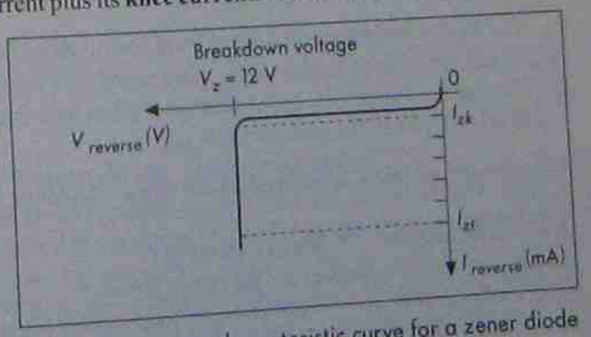


Figure 3.4: Reverse characteristic curve for a zener diode

instead of passing through the load. The diode voltage is virtually unaffected by the increase in current, and the load voltage therefore remains fairly constant. If more current is required by the load (that is, R_L is reduced), then the zener diode circuit allows the extra current to be supplied, providing I_z does not drop below the level of I_{zk}, without affecting the load voltage. In a practical circuit a capacitor is often found in parallel with the zener diode to filter out the noise generated by the diode when it 'avalanches'.

Looking more closely at the operation of this circuit, if the input voltage at point A (in Figure 3.3) increases, the voltage at point B will also tend to increase. This increases the diode reverse voltage which immediately causes more zener current. Providing I_z does not exceed I_{zm}, this increased zener current through R_s will increase the voltage drop across R_s and the output voltage will remain constant. Remember V_{out} = the zener voltage (V_z) = $V_{in} - V_{R_s}$ (from Kirchhoff's law).

If the input voltage increases, then the reverse zener voltage is increased, thereby increasing the diode current. More current passes through R_s, which increases the voltage drop across it, and this returns the output voltage to its previous value. All this happens in a fraction of a second, and thus little variation in the output voltage is recorded. In the second case mentioned, if R_L is reduced the load current will increase, tending to reduce the voltage at point B, causing less diode current. Less current passes through R_s, and therefore less voltage is dropped across it. This restores the voltage at point B, that is, the output voltage is back to normal. When R_L decreases then I_L increases, V_{RL} decreases and I_z decreases, V_{R_s} decreases, hence V_{out} remains constant. It is important to realise that the description of the operation of all voltage regulator circuits is time expanded to show exactly what happens. In reality, a change in the output condition is barely noticeable, and in complex regulators it is not noticeable at all.

In summary, the output voltage will be constant as long as the voltage across R_s is constant (assuming a steady level of input voltage). The voltage drop across R_s will only change if the total circuit current (I_{max}) changes, so $I_{max} = I_z + I_L$

The zener has to keep I_{max} constant so, if the load current increases, the zener current decreases and, if the load current decreases, the zener current increases—this causes a see-saw effect between the two parallel branches.

When considering zener diode shunt regulators, two important factors must not be overlooked:

1. The diode must never come out of breakdown, where I_z drops below I_{zk}, or regulation will be lost.
2. The current through the zener must never be so high that maximum **power dissipation** is exceeded, power dissipation = $V_z \times I_{max}$.

These factors are taken into consideration when calculating the value of the series resistor R_s.

3.2.1 Calculation of series resistor

Assume the following values for the circuit shown in Figure 3.3: V_{in} = 20 V, zener voltage = 12 V, load current (I_L) = 55 mA (average), load current variation = ± 5 mA (I_L varies between 50 mA and 60 mA) and zener current, I_{zk} (from the specification sheet) = 10 mA.

The maximum current (I_{max}) through R_s is calculated as follows:

$$I_{max} = I_z + I_L$$
$$= 60 + 10$$
$$= 70 \text{ mA}$$

Then R_s will drop 70 mA × R_s, and this must be equal to $V_{in} - V_z$. Therefore:

$$R_s = V_{in} - V_z / I_{max}$$
$$= 20 - 12 / 70 \times 10^{-3}$$
$$= 114\ \Omega$$

Power dissipated by $R_s = I_{max}^2 \times R_s$
$= (70 \times 10^{-3})^2 \times 114$
$= 559$ mW

The power dissipated by the zener diode when no current is being drawn by the load can be calculated. With no load current being drawn, I_z equals the current through R_s, that is I_{max}:

Power dissipated = $V_Z \times I_{max}$
$= 12 \times 70 \times 10^{-3}$
$= 840$ mW

If the zener diode characteristics are not known, a general rule of thumb for calculating the value of R_s is to regard I_{max} as 1.3 times the average load current. Using this rule for the previous example, we have I = 55 mA, V_{in} = 20 V and V_z = 12 V, so:

$$R_s = (20 - 12) / (1.3 \times 55 \times 10^{-3})$$
$$= 8/71.5 \times 10^{-3}$$
$$= 112\ \Omega$$

This is very close to the value calculated with all the circuit information known, but it gives a slight increase in the maximum power to be dissipated by the zener diode.

Power dissipated = $V_Z \times I_{max}$
$= 12 \times 71.5 \times 10^{-3}$
$= 858$ mW

This is still within the power rating of the chosen zener diode, which would have been rated at one watt.

3.2.2 Disadvantages of a zener diode shunt regulator

One disadvantage of a zener diode shunt regulator circuit is that the output voltage varies if the load current is subject to considerable variations. Zener diodes are also very sensitive to temperature changes, and in this case the output will vary. The environment that the zener diode will work in also needs to be considered when selecting a device. The power losses in R_s and in the zener diode itself are quite high when high current is supplied, and this reduces the circuit's **efficiency**.

3.2.3 Selection of a zener diode

Tables 3.1 and 3.2 are extracts from an *International Rectifier Diode Data* book. When selecting a zener diode for a job you need to consider the following parameters: V_z, I_{zm}, power rating and the operating temperature.

For example, if a zener diode is required to provide a 10 V output and handle a maximum load current of 25 mA and it has to work in an enviroment where the operating temperature will be 200°C, which type will you use?

First, delve into the data book (Tables 3.1 and 3.2 for this exercise) and locate the zener diode section, look at the nominal voltage column (left-hand side vertical one) and find the 10 V line. Now look across the top line and find the I_{zt} column (at the same time take note of the maximum operating temperature). Table 3.2 has the diode we want—the second column across is the 200°C and the diode number is 1N4740A. To be absolutely certain you should check the power rating:

$$\begin{aligned}\text{Power dissipation} &= V_z \times I_{max} \\ &= 10 \times 25 \times 10^{-3} \\ &= 250\ \text{mW}\end{aligned}$$

The 1N4740A is rated at 1 W which appears to be too high for our use, and could be over-engineered! The 10 V, 400 mW diodes in Table 3.1 which may suit our needs are the 1N758A and the 1N961B, but they cannot handle the current or temperature. Likewise, the 500 mW 1N5240B is fine in the temperature area but it cannot handle the current.

3.3 Simple transistor shunt regulators

A simple shunt voltage regulator with a transistor (Q_1) in parallel with the load is shown in Figure 3.5. If the load resistance increases, the output voltage would tend to rise (that is, become more positive) raising the base voltage and hence the forward bias on Q_1. With more forward bias (V_{BE}) the collector current (I_C) increases, increasing the voltage drop across R_1 and the output voltage is reduced to its former value.

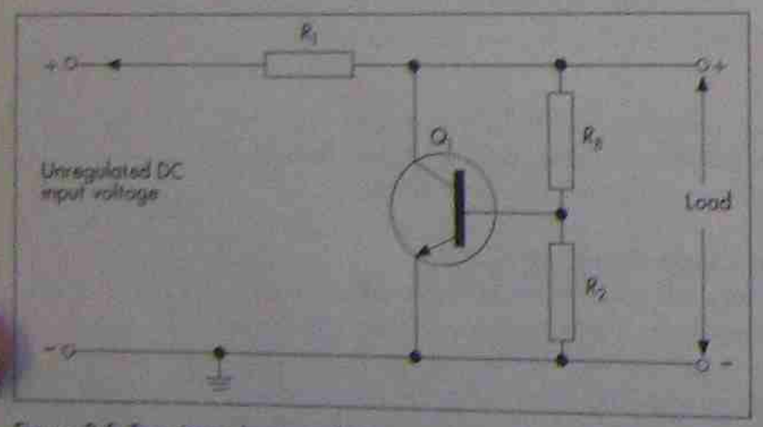


Figure 3.5 Transistor shunt regulator

Figure 3.6 shows an improvement on the simple circuit. The zener diode maintains a constant voltage between base voltage and collector current of Q_1 which is equal to its breakdown voltage.

If the load resistance increases, the output voltage tends to become more positive. There will be more voltage across the zener diode (Z_D), R_3 leg, and the zener current will increase, increasing the voltage drop across R_1. This will increase the forward bias on Q_1, and hence on Q_2

Table 3.1 Zener diode specification sheet (low power)

Zener Voltage Regulators



International Rectifier has long been recognized as a leader in the development and production of standard, selected, and special value Zener Voltage Regulators.

Temperature Compensated Voltage Reference are specially constructed Zeners which offer predictable variations in voltage with respect to temperature. These conditions are stable for any given set of parameters specified.

These regulators and references are complementary to IR's power-oriented line of quality semiconductors.

ZENER VOLTAGE REGULATORS

Power Rating	400mW	400mW	1Watt	1W
Tolerance (%)	5	5	5	5,10
Notes	(4)	(4)	(4)	(6)
Case Style	DO-7/DO-35	DO-7/DO-35	DO-7/DO-35	DO-204AL

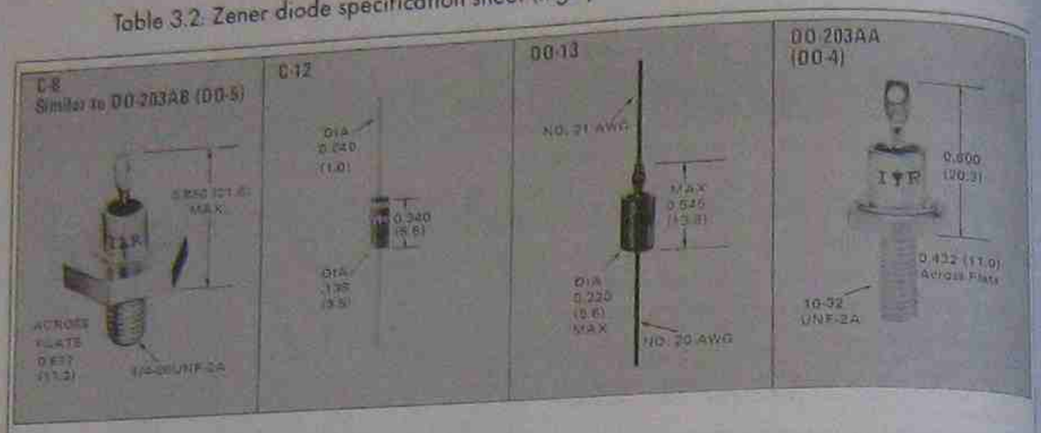
Nominal Vz	Part No.	Izt (mA)	Part No.	Izt (mA)	Part No.	Izt (mA)	Part No.	Izt (mA)
3.3	1N746A	20	-	-	1N5226B	20	1N4728A	76
3.6	1N747A	20	-	-	1N5227B	20	1N4729A	69
3.9	1N748A	20	-	-	1N5228B	20	1N4730A	64
4.3	1N749A	20	-	-	1N5229B	20	1N4731A	58
4.7	1N750A	20	-	-	1N5230B	20	1N4732A	53
5.1	1N751A	20	-	-	1N5231B	20	1N4733A	49
5.6	1N752A	20	-	-	1N5232B	20	1N4734A	45
6.0	-	-	-	-	1N5233B	20	-	-
6.2	1N753A	20	-	-	1N5234B	20	1N4735A	41
6.8	1N754A	20	1N965A	18.5	1N5235B	20	1N4736A	37
7.5	1N755A	20	1N966A	16.5	1N5236B	20	1N4737A	34
8.2	1N756A	20	1N967A	15.5	1N5237B	20	-	-
8.4	-	-	-	-	1N5238B	20	-	-
8.7	1N757A	20	1N968A	14.5	1N5239B	20	-	-
9.1	1N758A	20	1N969A	12.5	1N5240B	20	-	-
10	-	-	1N970A	11.5	1N5241B	20	-	-
11	1N759A	20	1N971A	10.5	1N5242B	20	-	-
12	-	-	1N972A	9.5	1N5243B	20	-	-
13	-	-	1N973A	8.5	1N5244B	9.5	-	-
14	-	-	-	-	1N5245B	8.5	-	-
15	-	-	1N974A	8.5	1N5246B	8.5	-	-
16	-	-	1N975A	7.8	1N5247B	7.8	-	-
17	-	-	-	-	1N5248B	7.4	-	-
18	-	-	1N976A	7.0	1N5249B	7.0	-	-
19	-	-	-	-	1N5250B	6.6	-	-
20	-	-	1N977A	6.2	1N5251B	6.2	-	-
22	-	-	1N978A	5.6	1N5252B	5.6	-	-
24	-	-	1N979A	5.2	1N5253B	5.2	-	-
25	-	-	-	-	1N5254B	5.0	-	-
27	-	-	1N980A	4.6	1N5255B	4.6	-	-
28	-	-	1N981A	4.5	1N5256B	4.5	-	-
30	-	-	1N982A	4.2	1N5257B	4.2	-	-
33	-	-	1N983A	3.8	1N5258B	3.8	-	-
36	-	-	1N984A	3.4	1N5259B	3.4	-	-
39	-	-	1N985A	3.2	1N5260B	3.2	-	-
43	-	-	1N986A	2.9	1N5261B	2.9	-	-
47	-	-	1N987A	2.7	1N5262B	2.7	-	-
51	-	-	1N988A	2.5	1N5263B	2.5	-	-
56	-	-	1N989A	2.3	1N5264B	2.3	-	-
60	-	-	-	-	1N5265B	2.1	-	-
62	-	-	1N990A	2.0	1N5266B	2.0	-	-
68	-	-	1N991A	1.8	1N5267B	1.8	-	-
Data Sheet (PD -)	1.002		1.003		1.003		1.004	

TEMPERATURE COMPENSATED VOLTAGE REFERENCES
(See Data Sheet No. PD-1.311)

Power Rating	Tolerance (%)	Vz (Volts)	Izt (mA)	Izm (mA)	Part No.	Notes
400	0.5-2.5	7.5	10	53	1.01	(1)
					1.02	
					1.03	
					1.04	
					1.05	
					1.06	
400	0.4-2.5	7.5	10	53	1.07	(2)
					1.08	
					1.09	
					1.10	
					1.11	
					1.12	
400	0.4-2.5	6.2	20	64	1.13	(3)
					1.14	
					1.15	
					1.16	
					1.17	
500	0.4-2.5	7.5	10	66	1.18	(4)
					1.19	
					1.20	
					1.21	
					1.22	
					1.23	
500	0.4-2.5	6.2	20	80	1.24	(5)
					1.25	
					1.26	
					1.27	
					1.28	
					1.29	
500	0.25-0.5	7.5	10	66	1.30	(6)
					1.31	
					1.32	
					1.33	
					1.34	
					1.35	
500	0.25-0.5	6.2	20	80	1.36	(7)
					1.37	
					1.38	
					1.39	
					1.40	
					1.41	

Notes:
(1) V_{zt} @ I_{zt} = 10mA, T_A = 25°C
(2) V_{zt} @ I_{zt} = 10mA, T_A = 25°C
(3) V_{zt} @ I_{zt} = 20mA, T_A = 25°C
(4) V_{zt} @ I_{zt} = 10mA, T_A = 25°C
(5) V_{zt} @ I_{zt} = 20mA, T_A = 25°C
(6) V_{zt} @ I_{zt} = 10mA, T_A = 25°C
(7) V_{zt} @ I_{zt} = 20mA, T_A = 25°C

Table 3.2: Zener diode specification sheet (high power)



ZENER VOLTAGE REGULATORS (Continued)

Power Rating	1W		1W		5W		10W		50W	
Max. Op. Temp. (°C)	175		200		175		175		175	
Tolerance (%)			5, 10							
Notes	(4)(1)		(5)		(4)		(6)(1)		(6)	
Case Style	DO-13		DO-204AL		C-12		DO-4		C-8	
Nominal Vz	Part No.	Izt (mA)	Part No.	Izt (mA)	Part No.	Izt (mA)	Part No.	I_{zt} (mA)	Part No.	I_{zt} (mA)
	Part Number									
6.2	1N3016B†	31	1N4736A	31	1N5344B	150	1N2972B†	305	1N3307B	1500
6.7	–	–	–	–	1N5345B	150	–	–	–	–
6.8	1N3019B†	28	1N4739A	28	1N5346B	150	1N2973B†	275	1N3308B	1370
7.1	1N3020B†	25	1N4740A	25	1N5347B	125	1N2974B†	250	1N3309B	1200
10	1N3021B†	23	1N4741A	23	1N5348B	125	1N2975B†	230	1N3310B	1100
11	1N3022B†	21	1N4742A	21	1N5349B	100	1N2976B†	210	1N3311B	1000
12	1N3023B†	19	1N4743A	19	1N5350B	100	1N2977B†	190	1N3312B	960
13	–	–	–	–	1N5351B	100	1N2978B†	180	1N3313B	890
14	1N3024B†	17	1N4744A	17	1N5352B	75	1N2979B†	170	1N3314B	830
15	1N3025B†	16.5	1N4745A	15.5	1N5353B	75	1N2980B†	155	1N3315B	780
16	–	–	–	–	1N5354B	70	1N2981B†	145	1N3316B	740
18	1N3026B†	14.0	1N4746A	14.0	1N5355B	65	1N2982B†	140	1N3317B	700
19	–	–	–	–	1N5356B	65	1N2983B†	130	1N3318B	660
20	1N3027B†	12.5	1N4747A	12.5	1N5357B	65	1N2984B†	125	1N3319B	630
22	1N3028B†	11.5	1N4748A	11.5	1N5358B	50	1N2985B†	115	1N3320B	570
24	1N3029B†	10.5	1N4749A	10.5	1N5359B	50	1N2986B†	105	1N3321B	520
25	–	–	–	–	1N5360B	50	1N2987B†	100	1N3322B	500
27	1N3030B†	9.5	1N4750A	9.5	1N5361B	50	1N2988B†	95	1N3323B	460
28	–	–	–	–	1N5362B	50	–	–	–	–
30	1N3031B†	8.5	1N4751A	8.5	1N5363B	40	1N2989B†	85	1N3324B	420
33	1N3032B†	7.5	1N4752A	7.5	1N5364B	40	1N2990B†	85	1N3325B	380
36	1N3033B†	7.0	1N4753A	7.0	1N5365B	30	1N2991B†	70	1N3326B	350
39	1N3034B†	6.5	1N4754A	6.5	1N5366B	30	1N2992B†	65	1N3327B	320
43	1N3035B†	6.0	1N4755A	6.0	1N5367B	30	1N2993B†	60	1N3328B	290
45	–	–	–	–	–	–	1N2994B†	55	1N3329B	280
47	1N3036B†	5.5	1N4756A	5.5	1N5368B	25	1N2995B†	55	1N3330B	270
50	–	–	–	–	–	–	1N2996B	50	1N3331B	250
51	1N3037B†	5.0	1N4757A	5.0	1N5369B	25	1N2997B†	50	1N3332B	245
52	–	–	–	–	–	–	1N2998B	50	1N3333B	240
56	1N3038B†	4.5	1N4758A	4.5	1N5370B	20	1N2999B†	45	1N3334B	220
60	–	–	–	–	1N5371B	20	1N2999B†	–	–	–
62	1N3039B†	4.0	1N4759A	4.0	1N5372B	20	1N3000B†	40	1N3335B	200
68	1N3040B†	3.7	1N4760A	3.7	1N5373B	20	1N3001B†	37	1N3336B	180
75	1N3041B†	3.3	1N4761A	3.3	1N5374B	15	1N3002B†	33	1N3337B	170
82	1N3042B†	3.0	1N4762A	3.0	1N5375B	15	1N3003B†	30	1N3338B	150
82	–	–	–	–	1N5376B	15	–	–	–	–
91	1N3043B†	2.8	1N4763A	2.8	1N5377B	15	1N3004B†	28	1N3339B	140
100	1N3044B†	2.5	1N4764A	2.5	1N5378B	15	1N3005B	25	1N3340B	120
Data Sheet (PD-)	1.004		1.005		1.008		1.009		1.013	

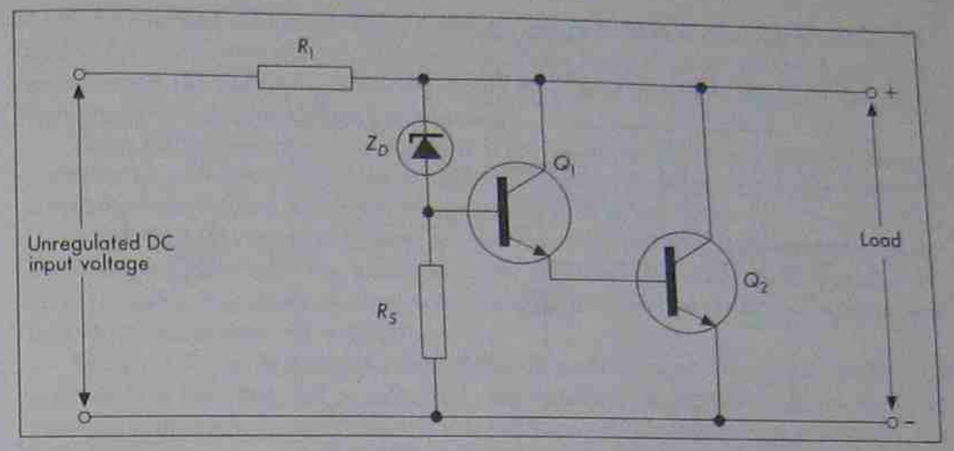


Figure 3.6: Improved transistor shunt regulator

The collector current of Q_2 will increase, and therefore more voltage will be dropped across R_1, returning the output voltage back to its former value.

When the load resistance decreases, the circuit operates in reverse, but the voltage across the collector base junction of Q_1 always equals the zener diode voltage. The output voltage will fall, I_z will be less, this will make the voltage across the series resistor (R_S) less and, therefore, Q_1 forward bias will be reduced. Less collector current flows through R_1 reducing the voltage drop across it and restoring the output voltage.

The problem with shunt regulators is the waste of power in the regulator itself. When the load is disconnected, the transistor must dissipate all the power that the load would otherwise have dissipated. When the full load is connected, the regulator must draw just enough current to keep it operating. All the heating of R_1, which is present regardless of the load condition, is also a waste of power. Shunt regulators are usually only used when a small current is to be controlled. One advantage of the shunt regulator is its ability to withstand a short circuit without being damaged; the only consequence is that the series resistor gets hot.

3.4 Series transistor regulators

As shown in Figure 3.1, an automatically varying impedance in series with the power supply can handle all of the circuit demands for current. A **series transistor regulator** does just that, and it can vary from a simple to a highly complex circuit. Figure 3.7 shows the circuit diagram of a simple series regulator. Q_1 is referred to as a **series pass transistor**.

The series resistor (R_s) is chosen to allow the necessary reverse current for the zener diode to operate just in the breakdown region. This ensures that the base of transistor Q_1 is fixed at the zener diode voltage (in this case 12 V). The regulated output voltage will be slightly less than the zener voltage ($V_{ZD} - V_{BE}$). The average $V_{BE} = 600$ mV, and therefore the output voltage will be 11.4 V. The difference between the input voltage and the output voltage is dropped across the transistor (V_{CE}), therefore:

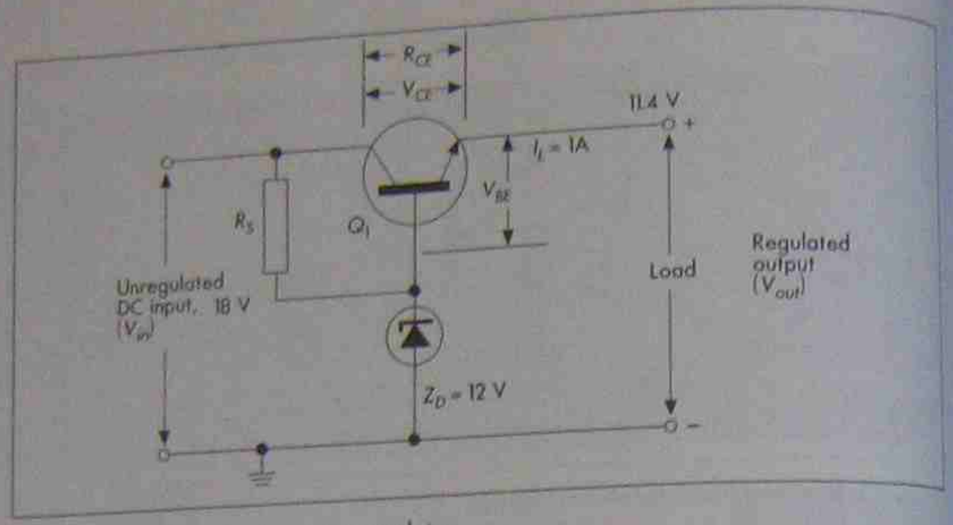


Figure 3.7: Simple series transistor regulator

$$V_{CE} = V_{in} - V_{out}$$
$$= 18 - 11.4$$
$$= 6.6 \text{ V}$$

For a 1 A load current, transistor Q_1 must dissipate power, the value of which is found in the following way:

$$\text{Power dissipation} = V_{CE} \times I_c$$
$$= 6.6 \times 1$$
$$= 6.6 \text{ W}$$

This is wasted power which reduces the **efficiency** of the power supply:

$$\text{Power output} = V_{out} \times I_c$$
$$= 11.4 \times 1$$
$$= 11.4 \text{ W}$$
$$\text{Power input} = V_{in} \times (I_c + I_Z)$$
$$= 18 \times (I_c + I_Z) \text{ and } I_z = 50 \text{ mA}$$
$$= 18 \times 1.05$$
$$= 18.9 \text{ W}$$
$$\text{Efficiency (power output/power input)} \times 100$$
$$= 11.4/18.9 \times 100$$
$$= 60.3\%$$

This low level of efficiency is one of the drawbacks with this type of circuit. Another problem is the heat dissipation in the series pass transistor Q_1. A high operating temperature can severely degrade the transistor's reliability, so a heatsink should be used.

If the load resistance in the circuit shown in Figure 3.7 decreases, the output voltage will tend to fall (that is, become less positive). This means that the emitter voltage (V_E) of Q_1 is reduced, and the emitter potential (V_E)moves away from the fixed base potential (V_B); therefore, the base-mitter voltage (V_{BE}) increases.

When the forward bias of a transistor is increased, the transistor's collector emitter resistance (R_{CE}) decreases. The load current is through R_{CE}; therefore, less voltage is dropped across the transistor, and the output voltage returns to its former value.

When the load current decreases, the output voltage will tend to increase towards the base voltage. This reduces V_{BE}, which causes the transistor's resistance R_{CE} to increase. With more resistance, V_{CE} increases, more voltage is dropped across the series pass transistor, and the output voltage returns to normal.

Similarly, if there is a sudden change in the unregulated input voltage, the regulator will respond in a way that will maintain the output voltage at its predetermined level. For example, if the input voltage increased, the output voltage would also tend to increase. This would bring the emitter voltage of Q_1 closer to the base voltage, thereby reducing V_{BE}. The transistor's resistance would increase, dropping more voltage across Q_1, and the output would return to normal.

3.4.1 Feedback regulators

The circuit in Figure 3.7 is called an **open-loop** regulator, because there is no way to ensure that the output voltage will stay absolutely constant under varying operational conditions, such as temperature variation and component tolerances. Also, it always takes some error signal to initiate the action of the regulator. If the error signal is amplified, the circuit becomes more sensitive to voltage variations, and this results in better voltage regulation. It also makes the output voltage adjustable. A block diagram of such a system is shown in Figure 3.8.

The regulator in Figure 3.8 is a **closed loop** regulator, in which the base current of a series pass transistor is altered to compensate for any changes in the output voltage. A portion of the regulator output voltage is fed to a **comparator** along with a fixed DC reference. These two voltages are compared to each other. Any difference between the feedback and the reference voltages (the error signal) is used to bias an amplifier (known as an **error amplifier**) which increases the signal's amplitude. The amplified error signal is then fed to the base of the series transistor to adjust its forward bias and hence its conductivity. A practical circuit which applies this principle is shown in Figure 3.9.

The zener diode (Z_D) and the series resistor (R_s) are again used to supply a reference voltage which maintains the emitter voltage of the error amplifier Q_2 constant. The voltage divider R_1, V_{R1}, and R_2 across the output senses the value of the regulated voltage and feeds a suitable proportion of it to the base of Q_2. If the zener voltage is 12 V, a suitable potential for Q_2 base to initiate conduction would be approximately 12.6 V. Resistor R_B puts transistor Q_1 into forward bias, and the second resistor (for the voltage divider bias) is made up of Q_2 and the zener diode (Z_D). The equivalent circuit is shown in Figure 3.10. The voltage across the resistor V_R is an electronically controlled variable resistance.

Any change in the resistance of Q_2 will change the bias on Q_1 in Figure 3.9. If V_{R1} is set

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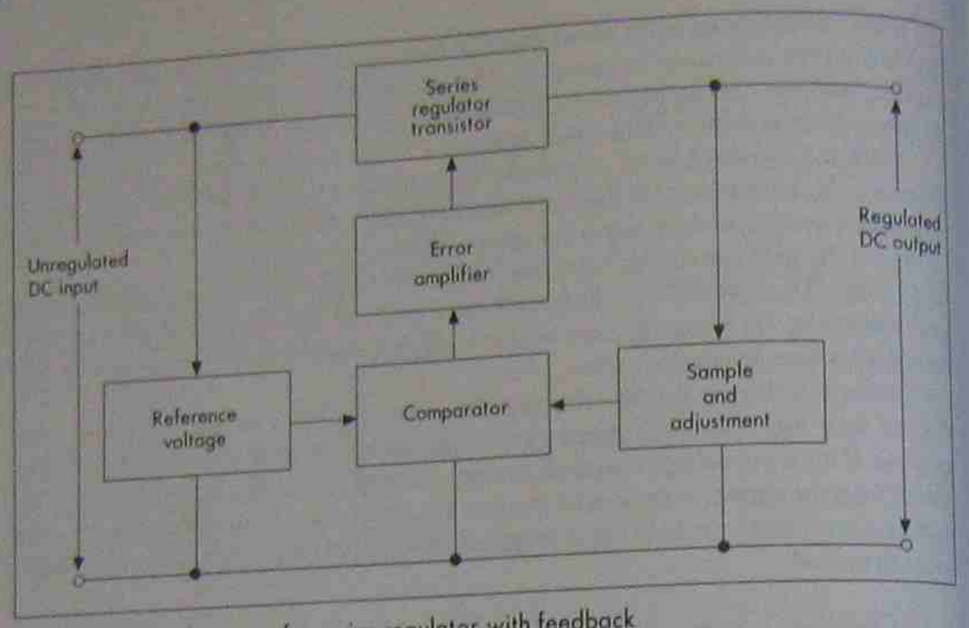


Figure 3.8: Block diagram of a series regulator with feedback

to give a small value of collector current in Q_2, then the forward bias of Q_1 will be reduced to a state of conduction that may produce 15 V across the load.

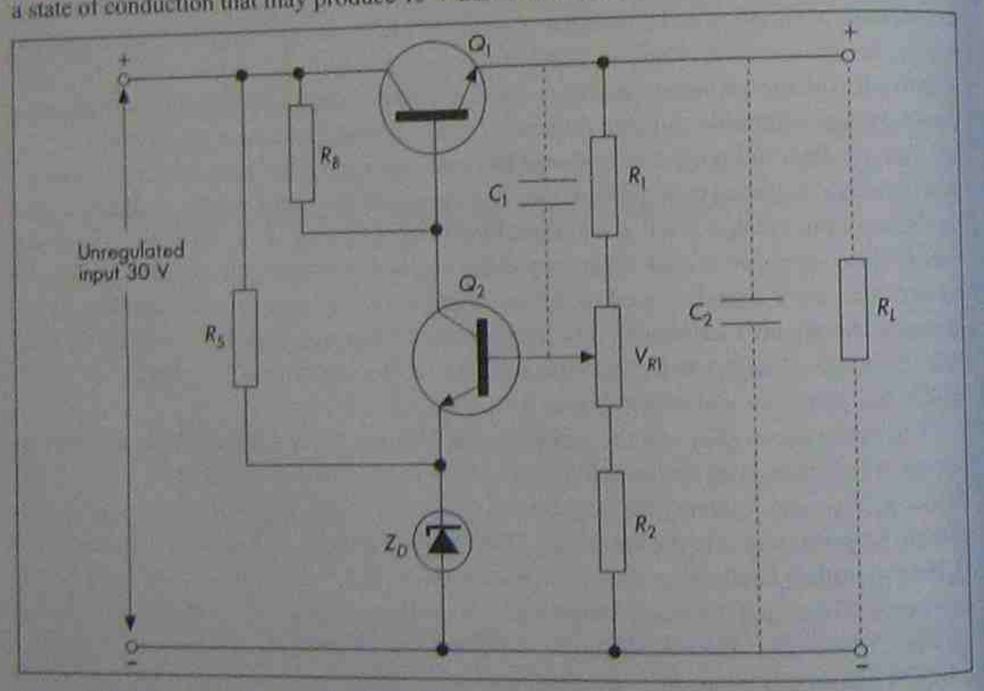
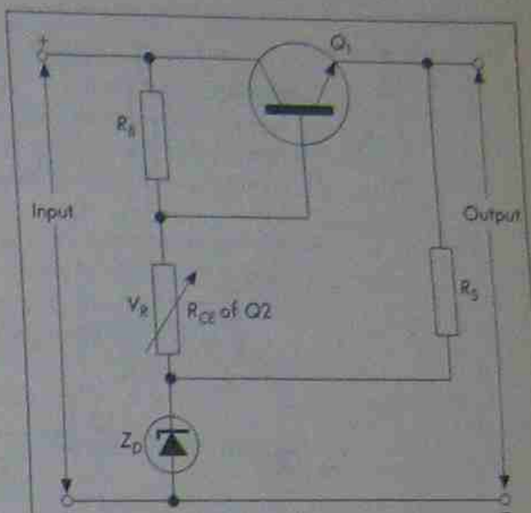


Figure 3.9: Series regulator with adjustable output voltage

If the load resistance decreases, the emitter voltage of Q_1 tends to decrease. The reduction in load voltage also reduces the forward bias on Q_2, increasing the transistor's resistance. The collector voltage of Q_2 will rise, increasing the forward bias on Q_1, and therefore decreasing the collector-emitter resistance (R_{CE}) of Q_1. With the resistance decreased, less voltage is dropped across Q_1, and the output voltage is restored to its previous level.

On the other hand, if the load resistance is increased, and there is less requirement for current, the emitter voltage of Q_1 will tend to increase. This will increase the forward bias on Q_2, reduce its resistance, and the collector voltage will decrease. The base voltage on Q_1 is also reduced, and hence its forward bias is reduced. With less forward bias on Q_1, R_{CE} is increased, more voltage is dropped across the transistor, and the output voltage is returned to its predetermined value.

The output voltage is adjustable over a range of voltages from slightly above the zener diode value to about three-quarters of the unregulated input voltage. This is done by adjusting V_{R1}. In Figure 3.10 the bias resistor (R_s) for the zener diode is taken from the lower voltage on the output side of the series pass transistor Q_1. This is an ideal situation, because the voltage drop across the resistor is less. Also, there is less chance of the zener diode coming out of its operating region, because the output voltage has little, if any, fluctuation. To allow the variation to start from zero volts, the zener diode and V_{R1} must be connected to a negative supply.



V_R represents the R_{CE} (collector-remitter resistance) of transistor Q_2, and R_S is taken to the output

Figure 3.10: Equivalent circuit of a variable output regulator

Transistor Q_1 is a low frequency power transistor and can follow variations in the load up to a frequency of approximately 8 kHz. Above this, it has trouble following the input signals, so a 'speed-up' capacitor (C_1) can be added between Q_2 base and Q_1 emitter (shown in dashed lines in Figure 3.9). This will improve regulation.

A well regulated supply has a low impedance, and with low frequency transistors, the impedance will be low at low frequencies. However, at higher frequencies the impedance increases. To combat this, a capacitor C_2, whose reactance decreases as frequency increases, is connected across the output. The result is a reasonably well regulated power supply from DC to high frequencies, with a low output impedance and a ripple of perhaps 5 mV. The ability of the regulator to oppose any changes in its output voltage can be further utilised to perform electronic smoothing. Any residual mains ripple on the output of the regulator is fed back to Q_2 base via the R_1/C_1 phase shift network, shown in heavy lines in Figure 3.11. Transistor Q_2 sees the ripple as a fluctuation in the output, and the circuit takes corrective action resulting in a much smoother output voltage.

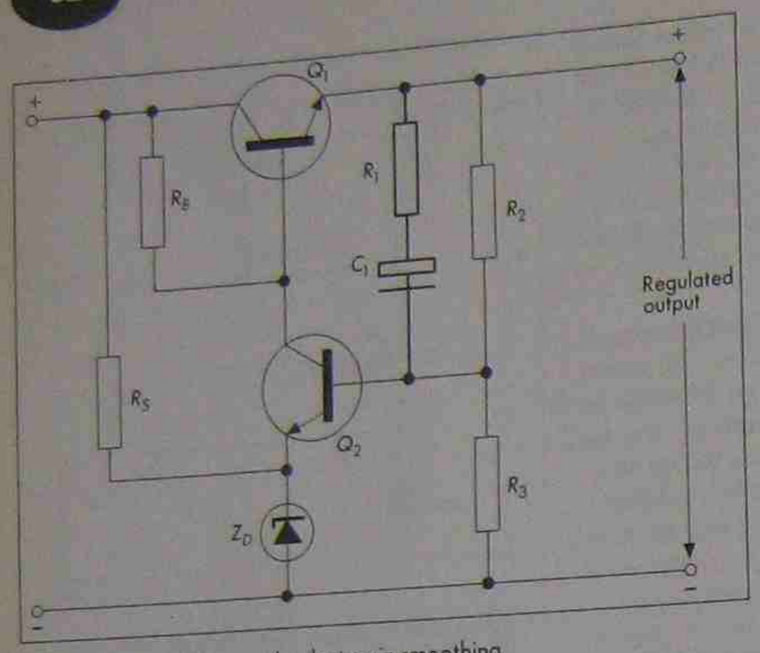


Figure 3.11: R_1 and C_1 provide electronic smoothing

3.5 Overload protection

Several methods are available to protect a power supply from burnout due to excessively heavy current. A fuse in series with the transformer primary, secondary, or the DC output circuit can be used, but these require replacement when blown. Overload relays and thermal relays that trip when there is excessive current can be inserted in the DC line, but these require manual resetting. An automatic electronic overload **protection circuit** is shown in Figure 3.12.

In this circuit Q_1 is a silicon transistor and V_{BE} = 600 mV. Two silicon diodes and a 1 Ω resistor are connected between the base and emitter of the current passing transistor. The load is connected to the junction of the diodes and the resistor, ensuring that the load current passes through the 1 Ω resistor.

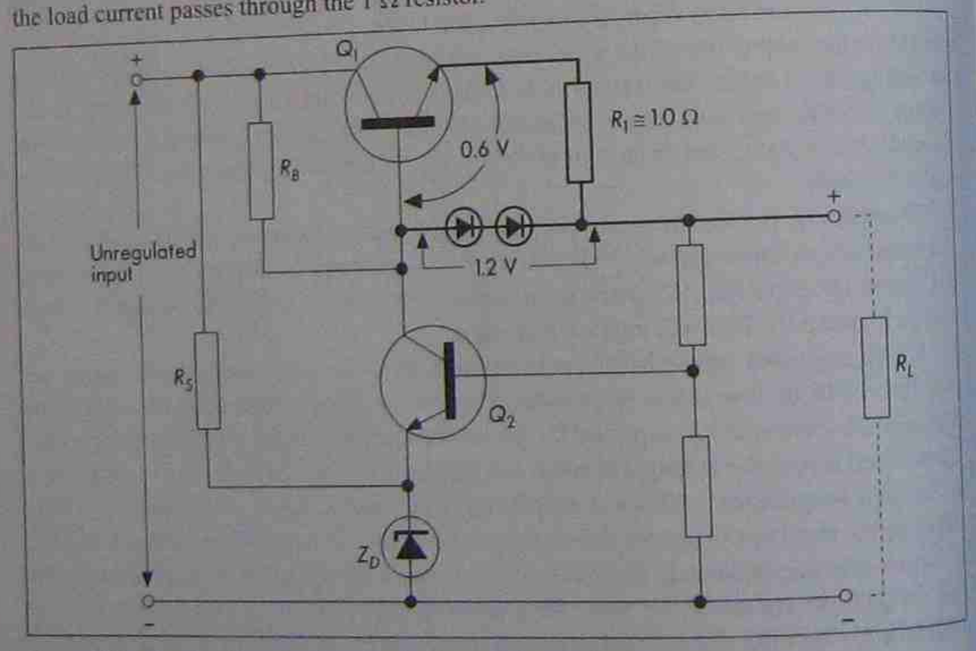


Figure 3.12: Overload protection circuit shown in heavy lines

The diodes will not conduct until the voltage drop across them exceeds 1.2 V. With V_{BE} of Q_1 equal to 600 mV, the load current must produce a 600 mV drop across R_1. If the load is drawing 500 mA, the voltage drop across R_1 is $1 \times 500 \times 10^{-3} = 500$ mV, so the diodes will not conduct. As the load increases the current also increases, and the voltage drop across R_1 exceeds 600 mV causing the diodes to conduct. Transistor Q_1 will now only pass the selected value of current, and even if the output is short circuited, there is insufficient current through Q_1 to damage it.

A similar method of limiting current to prevent damage to the current passing transistor involves replacing the diodes with a transistor. This is shown in Figure 3.13. When an excessive current passes through the current-sensing resistor R_1, the transistor Q_3 turns on. This decreases the forward bias voltage on Q_1 and limits the current flow through it.

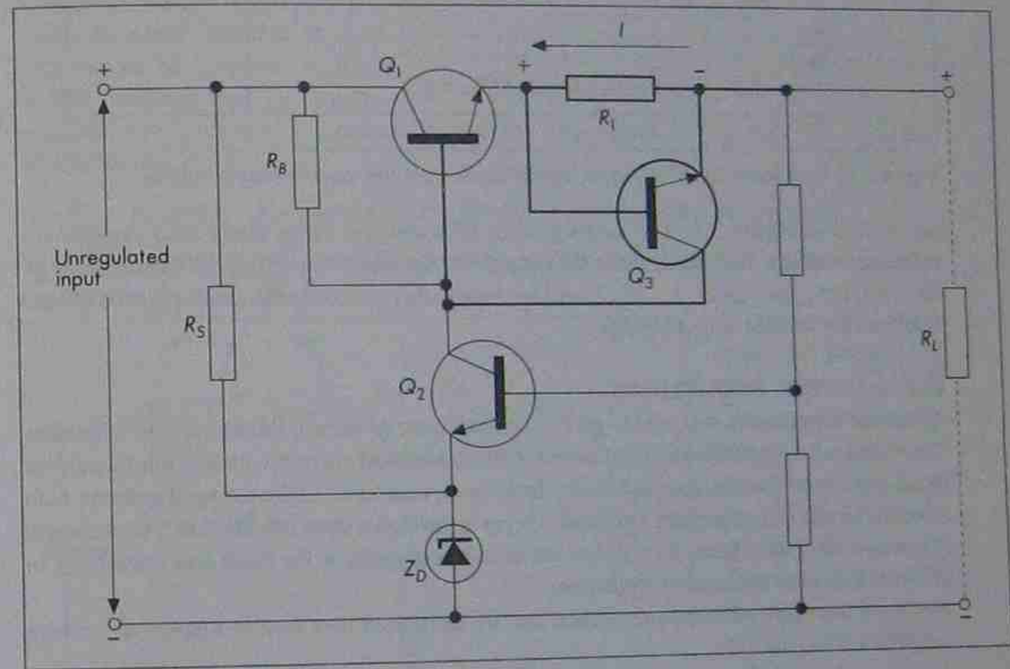


Figure 3.13: Transistor regulator with current-limiting control

3.6 Operational amplifiers in regulators

Monolithic integrated circuit operational amplifiers, which are specifically designed for voltage regulation, can be found in better quality voltage regulator circuits. They retain the series current passing transistor, but the error amplifier takes the form of an operational amplifier. Such a circuit configuration is shown in Figure 3.14.

The sample of the output voltage is obtained from voltage divider R_2/R_3. The ratio can be adjusted to provide the required output voltage level; or, alternatively, an adjustable leg can

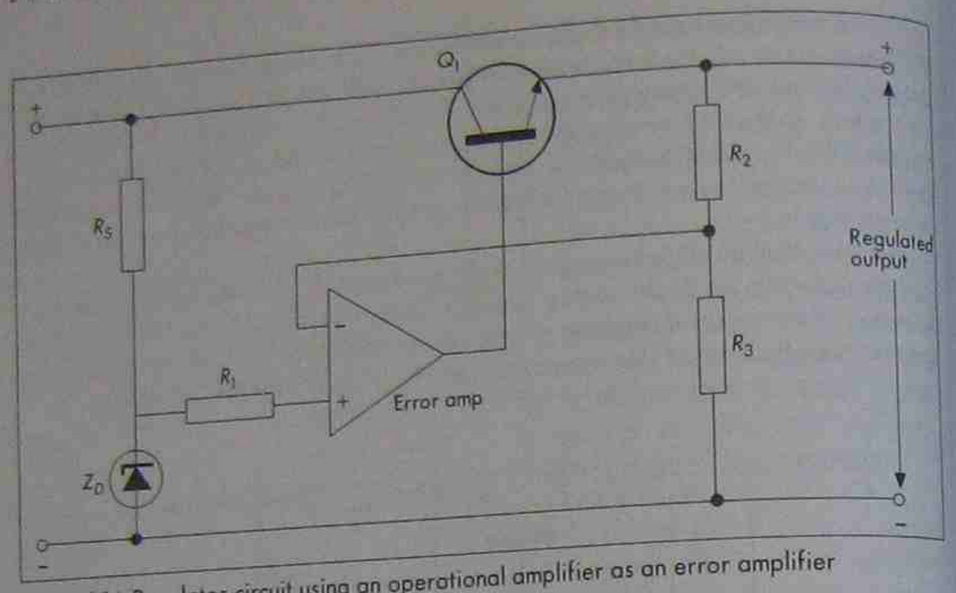


Figure 3.14: Regulator circuit using an operational amplifier as an error amplifier

be used as in Figure 3.9. The series resistor (R_s) and the zener diode (Z_D) provide the **reference voltage**. Any variation in the output voltage results in an opposite reaction at Q_1 base. So, if V_{out} decreases, V_{BE} of Q_1 will increase—this increases the conduction of Q_1 and stabilises the level of output voltage.

3.7 Current regulators

A **current regulator** is designed to give a constant value of current through a load which has a varying load voltage. Ideally, the current with no load and current with full load would be equal, but in practice this does not occur. Remember, current will take the least resistive path around a circuit and, therefore, a no load current is through a short circuit, and a load current is through the load. Figure 3.15 shows the circuit arrangement for these two conditions in order to determine the level of regulation.

The percentage of current regulation can be calculated in a similar manner as voltage regulation. Thus:

$$\% \text{ current regulation} = (I_{NL} - I_{FL})/I_{FL} \times 100$$

where I_{NL} = no load current and I_{FL} = full load current

Figure 3.16 shows the circuit of a constant current regulator.

A zener diode (Z_D) is connected across the series combination of R_1 and the base-emitter junction. The emitter current (I_E) produces a voltage drop across R_1. The emitter current is limited to a value which causes a potential difference across R_1 which, when added to V_{BE} of Q_1, equals the breakdown voltage of the zener diode. Neglecting the 600 mV V_{BE}:

$$I_E = V_Z/R_1$$

The resistor R_1 can be made variable to allow adjustment of the current level. The load current (I_L) equals the collector current (I_C), which in turn equals the emitter current (I_E) minus the base current (I_B). Hence:

$$I_L = I_C = I_E - I_B$$

and $I_E = I_C + I_B$

Therefore, a decrease in I_L (due to a drop in load voltage) would cause an equal decrease in I_C. This would be reflected in the emitter current, and I_E would decrease. Any decrease in I_E reduces the voltage drop across R_1 ($V_{R1} = I_E \times R_1$), and this affects the bias on Q_1:

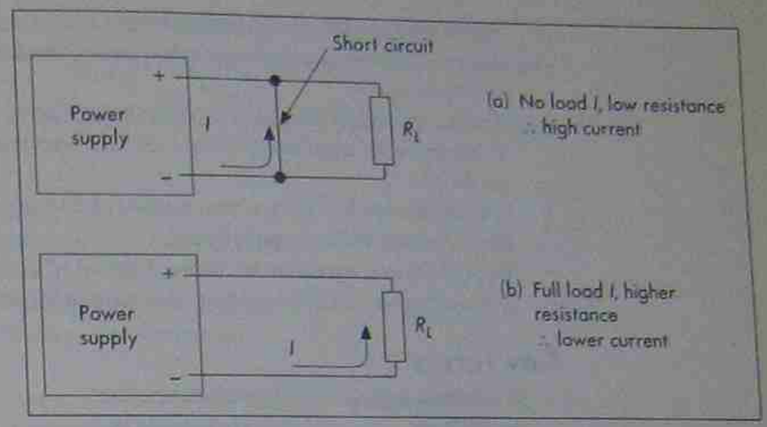


Figure 3.15: Current regulation circuit arrangement

$$V_{BE} = V_z \text{ (fixed)} - V_{R1} \text{ (Note: Observe the circuit polarities they are opposing).}$$

Therefore, a decrease in V_{R1} would increase V_{BE} of Q_1, and the transistor will conduct more, maintaining the level of I_L fixed.

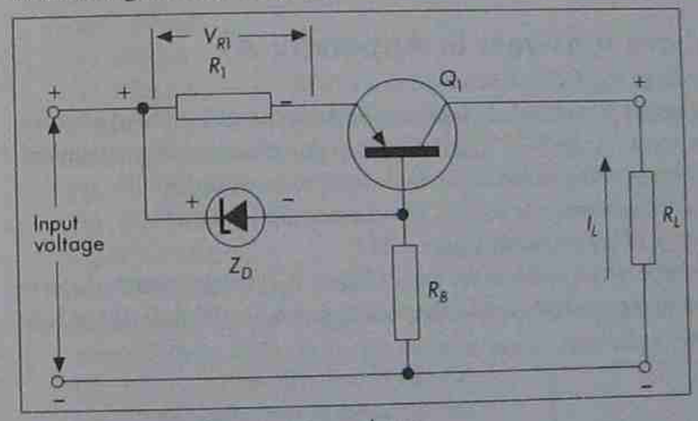


Figure 3.16: A series current regulator

If the load voltage were to increase, I_L (and hence I_E) would also increase. The voltage drop across R_1 would increase, and V_{BE} of Q_1 would decrease. This reduces the conductivity of the transistor, and I_L would be reduced to its former value.

3.8 Summary

The following items are a summary of the major points covered in this chapter:

- Regulation is required to hold a power supply output voltage constant when the load condition varies or the input condition varies.
- Zener diode regulators are shunt type.

- Shunt type regulators dissipate maximum power under no load conditions.
- Series regulators dissipate maximum power when the output voltage is much less than the input voltage.
- Series regulators have poor efficiency, but are better than shunt regulators.
- A 'closed loop' regulator provides the best operation by monitoring the output voltage condition.
- Current overload protection uses a small resistor in series with the load current to sense the magnitude of the output current.
- Two diodes or a transistor are forward biased when load current is excessive.
- Current regulators ensure a constant current to the load.

Key terms

breakdown voltage
closed loop
comparator
current regulator
error amplifier
efficiency
knee current
knee voltage
open loop
power dissipation
protection circuit
reference voltage
series pass transistor
series regulator
shunt regulator
voltage regulator
zener diode shunt regulator (Z_D)

3.9 Review questions (answers in Appendix A)

1. Briefly explain why voltage regulation is required in a circuit.
2. An 18 V zener diode requires 15 mA to hold breakdown, and a circuit current of 85 mA is 5 mA is required. If the input voltage is 25 V, calculate the value of series resistor required.
3. What is the minimum power rating of the zener diode used in question 2?
4. Why are transistor shunt regulators only used in low current applications?
5. What is the output voltage of the circuit in Figure 3.17?
6. A series transistor regulator circuit (such as the one in Figure 3.7) has an input voltage of 15 V. The zener diode voltage is 12.5 V, and the zener current is 100 mA. If the load current is 1 A, calculate:
   (a) the circuit efficiency;
   (b) the zener power rating; and
   (c) the power dissipated by the transistor.
7. Briefly explain the circuit operation of a series transistor regulator when:
   (a) the load resistance decreases; or
   (b) the input voltage increases.
8. How can a series regulator be protected against overload currents?
9. What is the difference between an 'open-loop' and a 'closed-loop' regulator circuit?

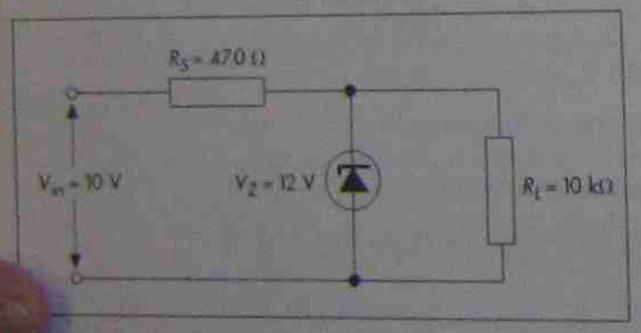


Figure 3.17

10. What is the purpose of a constant current regulator? If the current changed from 1.5 A under no load to 1.2 A under full load, what would the percentage regulation be? Is this percentage satisfactory?
11. Briefly explain the operation of a constant current regulator when the load voltage suddenly increases.

Multiple-choice questions

1. The purpose of a voltage regulator in a power supply is to:
   (a) ensure V_{out} does not vary with load variation
   (b) ensure V_{out} does not vary with input voltage variation
   (c) keep V_{out} steady with minimum power dissipation
   (d) all of the above
2. A zener regulator is required to give 15 V and handle the power generated by a 120 mA current. The maximum power dissipation rating for the diode must be:
   (a) 180 mW
   (b) 1.5 W
   (c) 2 W
   (d) 18 W
3. A zener diode has a power dissipation rating of 500 mW and a voltage rating of 8.2 V. What is the maximum current rating (I_{ZJ}) for the diode?
   (a) 60.9 mA
   (b) 6.09 mA
   (c) 609 μA
   (d) 60.9 μA
4. A zener diode is required to provide 12 V and the I_{ZJ} rating is 80 mA. From Tables 3.1 and 3.2, select the diode you would use:
   (a) 1N759A
   (b) 1N5349B
   (c) 1N4742A
   (d) 1N5242B
5. A simple series pass regulator has an input voltage of 16 V DC. If the reference base zener diode is 12 V, the output voltage level is approximately:
   (a) 11.4 V
   (b) 12.0 V
   (c) 12.6 V
   (d) 15.4 V
6. If the load current is 1 A for the circuit in question 5, the transistor power dissipation will be approximately:
   (a) 600 mW
   (b) 4.0 W
   (c) 4.6 W
   (d) 11.4 W

7. The percentage efficiency of the circuit discussed in questions 5 and 6 will be approximately:
   (a) 40%
   (b) 50%
   (c) 60%
   (d) 70%
8. Refer to Figure 3.12 or 3.13. If the output current is to be limited to 750 mA, the value of current sensing resistor (R_1) required is:
   (a) R75
   (b) R8
   (c) 1R
   (d) 1R5
9. Refer to Figure 3.11. If the value of the load resistor decreases, the bias voltages of Q_1 and Q_2 respectively will:

	Q_1	Q_2
(a)	decrease	decrease
(b)	increase	increase
(c)	increase	decrease
(d)	decrease	increase

10. Refer to Figure 3.11. If R_2 = 4k7 Ω, R_3 = 1 kΩ and the zener diode is 3.9 V, the output voltage will be approximately:
    (a) 3.3 V
    (b) 12.0 V
    (c) 15.5 V
    (d) 18.8 V

CHAPTER 4

INTEGRATED VOLTAGE REGULATORS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- List the advantages of a three-terminal regulator.
- Recognise a three-terminal regulator in a circuit.
- Identify different component packages.
- Select a three-terminal regulator to perform a specific task.
- Calculate the values of resistors required to set a particular output voltage.
- Explain the difference between fixed and variable regulators.
- Explain the heatsinking requirements of three-terminal regulators.
- Define the term 'drop-out' voltage.
- State why the input-output voltage differential must not be excessive.
- Explain why the earth return or adjust connection should be kept intact while the power is on.
- Show how to protect the integrated circuit from a short circuit condition.
- Explain how to increase the current capability of integrated circuit regulators.
- Define the terms 'line regulation' and 'load regulation'.

4.1 Introduction

The trend in voltage regulation is towards localised regulation with integrated circuit (IC) regulators. They have the following advantages:

- small size;
- low cost;
- low current; and
- fixed voltage.

The IC regulators require minimal or no **heatsinking**, and few, if any, external components.

In the past, high power regulators supplied all areas of an electronic system. These were large, costly and inefficient. Line impedance caused voltage drops which varied throughout the system. Unwanted coupling existed between critical parts of the system, and the excessive decoupling which this necessitated caused degraded local regulation. Now, recently developed three-terminal devices supplying 1A to 5A can be placed in various areas of a system to satisfy the local area power requirements. These regulators are available in a variety of positive and negative voltages at various current ratings. Packages can be as small as the TO-92 plastic small-signal transistor or as large as the TO-3 power transistor. With this variety to choose from, it is now possible to select a regulator that is appropriate for each application. This will significantly reduce costs over other more established approaches. The IC regulators are excellent at maintaining a constant output voltage, even when there is a change in mains input voltage (**line regulation**), or output current demand (**load regulation**).

Note that data operation and application notes in this chapter are reproduced with the kind permission of National Semiconductor from their *Voltage Regulator Handbook*.

4.2 Packages

Available regulator packages are listed in Table 4.1. The current capability of a regulator can usually be determined by the type of package it is presented in, but the final selection of voltage and current capabilities should be made by referring to the manufacturer's selection chart or table. A typical chart presentation, which lists data about three-terminal regulators, is shown in Table 4.2.

Table 4.1: Regulator current ratings and packages

Current	Package
100 mA	TO– 92, TO– 39
250 mA	TO– 39, TO–202
500 mA	TO– 39, TO–202
1.5 A	TO–220, TO– 3
3.0 A	TO– 3
5.0 A	TO– 3

Once the voltage and current capability of the IC regulator is established, attention should be given to the terminal connections. It is important to remember that similar packages *do not* have similar circuit connections. Unless you are familiar with a particular regulator, always check the terminal connections by referring to the manufacturer's data.

4.3 Connection diagrams and coding

The type of package, and hence the terminal connection, is determined by the order number of the regulator. For example, the LM117/LM217/LM317 family is available in a 500 mA rating (LM317M), and a 1.5 A rating (LM317). This gives a choice of four packages as shown in Figure 4.1.

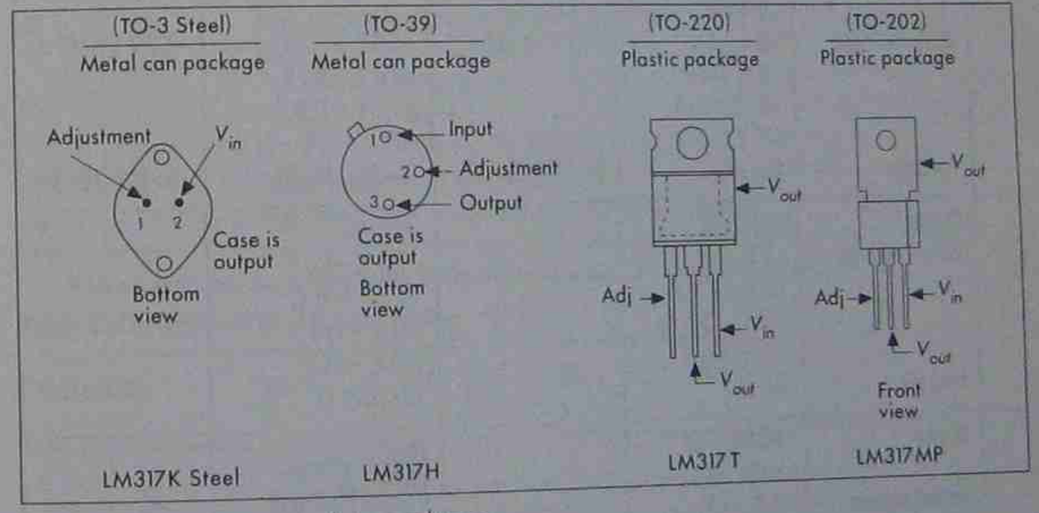


Figure 4.1: Four common regulator packages

A similar regulator, LM340T, is also available in a TO-220 package. Its specifications are a fixed output three-terminal regulator with an output voltage range of +5 V to +24 V at 1.5 A. It looks identical to an LM317T, but the terminal connections are different. The terminal connections of the LM317T from left to right (see Figure 4.1) are adjustment, V_{out} and V_{in}. However, the LM340T connections from left to right are V_{in}, common, V_{out}.

Table 4.2: Three-terminal regulator selection guide

Output current		Positive output voltage		Negative output voltage	
		Fixed output voltage	Adjustable output voltage	Fixed output voltage	Adjustable output voltage
5 A	Device Output voltage Package		LM338 +1.2 V to +33 V TO–3		
3 A	Device Output voltage Package	LM323 +5.0 V TO–3	LM350 +1.2 V to +33 V TO–3	LM345 –5.0 V, –5.2 V TO–3	
1.5 A	Device Output voltage Package	LM340–XX, LM78XX +5 V, +6 V +8 V, +10 V +12 V, +15 V, +18 V, +24 V TO–3, TO–220	LM317 +1.2 V to +37 V High Voltage (HV) +1.2 V to +57 V TO–3, TO–220	LM320–XX, LM79XX –5.0 V, –5.2 V, –6.0 V, –8.0 V, –9.0 V, –12 V, –15 V, –18 V, –24 V TO–3, TO–220	LM337 –1.2 V to –37 V High Voltage (HV) –1.2 V to –47 V TO–3, TO–220
500 mA	Device Output voltage Package	LM341-XX, LM78MXX +5 V, +6 V, +8 V, +10 V, +12 V, + 15 V, +18 V, +24 V TO–202	LM317M +1.2 V to +37 V TO–202, TO–39	LM320M,LM79MXX –5.0 V, –5.2 V, –6.0 V, –8.0 V, –9.0 V, –12.0 V, –15 V, –18 V, –24 V TO–202, TO–39	LM337M –1.2 V to –37 V TO–202, TO–39
250 mA	Device Output voltage Package	LM342–XX +5 V, +6 V, +8 V, +10 V, +12 V, +15 V, +18 V, +24 V TO–202		LM320ML –5.0 V, –6.0 V, –8.0 V, –10 V, –12 V, –15 V, –18 V, –24 V TO–202	
100 mA	Device Output voltage Package	LM340LA–XX LM78L–XX +5 V, +6 V, +8 V, +10 V, +12 V, +15 V, +18 V, +24 V TO–39, TO–92		LM320L–XX, LM79L–XX, –5 V, –6 V, –8 V, –9 V, –12 V, –15 V –18 V, –24 V TO–92, TO–39	

4.3.1 Type designation or coding

The following information applies to National Semiconductor products only, and may be subject to variation.

The part number of the integrated circuit (IC) regulator is designed to give the following information:

- type of device (linear);
- temperature range of operation;
- device code (fixed, variable, etc);
- case style;
- nominal output voltage; and
- tolerance (operating specification).

An example of this type of coding is given in Table 4.3.

Table 4.3: Example of IC regulator coding

Part number		LM 1	3 2	40 3	K 4	2 5	A 6
1	LM	Linear monolithic					
2	3	Temperature range:	0°C to +70°C (1) –55°C to + 125°C (2) –25°C to + 85°C				
3	40	Basic part number (positive regulator)					
4	K	Case style:	K TO-3 Steel H Metal can N D.I.P. P Plastic (TO-202)				
5	12	Output voltage:	+5 V to +24 V				
6	A	High accuracy:	R = relaxed accuracy, no letter = standard				

Some regulators use a somewhat different designation, as shown in Table 4.4.

Table 4.4: Example of IC regulator coding

Part number		LM 1	78L 3	05 5	AC 6	Z 4
1	LM	Linear monolithic				
3	78L	Basic part number				
5	05	Output voltage				
6	AC	Improved accuracy: C = standard				
4	Z	Case style Z: TO-92 Plastic There is no selection of operating temperatures, the only range being 0°C to + 70°C.				

4.4 Heatsinking

Heatsinking is the technique used to transfer the heat created by power dissipation away from the IC regulator. If the regulator has a large difference between input and output voltage, then a fair amount of power will be dissipated by the IC regulator itself. Assume that a 12 V regulator handling a constant load current of 1.5 A has an input voltage of 30 V. The power dissipated would be:

$$\begin{aligned} P &= V \times I \\ &= (V_{in} - V_{out}) \times I \\ &= (30 - 12) \times 1.5 \\ &= 27 \text{ W} \end{aligned}$$

This may well exceed the power dissipation rating of the regulator; if not, it will certainly cause it to run much hotter than is desirable and will cause automatic shut down. The excess heat can be reduced by attaching the regulator to a heatsink.

Heat can be transferred from the regulator package by three methods:

1. conduction;
2. convection; and
3. radiation.

Conduction is the most effective method of moving heat from the IC junction to the case, and from the case to the heatsink. It can be aided by the use of a heat conductive paste placed between the regulator package and the heatsink.

Convection is an effective method of cooling both the regulator case and the heatsink. It is dependent on the type of convecting agent (which in the case of a regulator is air), and it can be improved, if necessary, by increasing the air flow with the aid of a fan.

Radiation is the transfer of heat from the cooling fins of the heatsink to the air. The choice and finish of material has a large bearing on the ability of the heatsink to get rid of the heat. A polished surface is not as efficient as an anodised or black-painted surface.

Care must be taken when connecting the regulator to the heatsink not to short circuit the regulator output to ground. The TO-3 and TO-220 packages have the case connected to the output terminal (see Figure 4.1). They both require a mica insulating washer between the package and the heatsink when mounted.

There are many heatsinks available commercially, but sometimes it is more convenient and economical to mount the regulator to the chassis of the power supply or to an aluminium extrusion. Whatever the method chosen, there are three basic rules for efficient heatsinking:

1. Mount the cooling fin vertically, where practical, for best convective heat flow.
2. Anodise or paint the fin surface area for better radiation.
3. Use 2 mm or thicker fins to provide low thermal resistance and hence better heat transfer from the regulator case.

4.5 Basic three-terminal regulators

A standard three-terminal regulator connection is shown in Figure 4.2. All devices have short-circuit protection, automatic thermal shut down, on-chip series pass transistors and internal references. No external components are normally required.

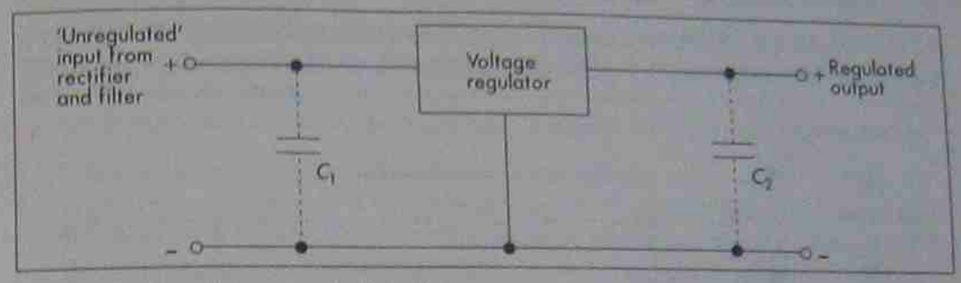


Figure 4.2: Normal three-terminal regulator connection

If the regulator is located more than 5 cm from the supply filter capacitor, a supply bypass capacitor (C_1) is required to maintain stability.

Capacitor C_1 (shown within dotted lines in Figure 4.2) should be a 100 nF to 200 nF (nanofarad) ceramic disc, or a 2 µF (microfarad) solid tantalum disc capacitor. It prevents any pick-up on the long input lead from influencing the regulator's operation.

Capacitor C_2 is not absolutely necessary for stable operation, but a 1 µF tantalum capacitor gives the regulator both improved **transient** response and noise rejection. In some circumstances a 100 nF capacitor may be required across C_2 to minimise high frequency noise. (A 1 µF solid tantalum capacitor is equivalent to a 25 µF aluminium electrolytic capacitor at high frequencies.) With the basic regulator circuit connected as shown in Figure 4.2, a reliable, low noise, well regulated output voltage is obtained, provided that three other important conditions are met.

First, the maximum power dissipation for the regulator must not be exceeded. Heat dissipated is wasted power, and a sensible input-output voltage differential is desirable. If the regulator is operated at too high a temperature, it will automatically shut down.

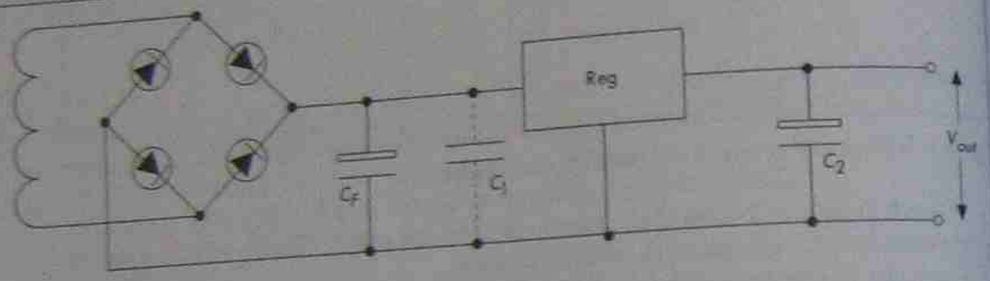
Second, the regulator must be operated above its **'drop-out' voltage**. The drop-out voltage is the input-output **voltage differential** at which the circuit stops regulating. It is dependent on load current, and the regulator's temperature. A typical value for drop-out voltage is 1.5 V to 2 V. From these two conditions it can be seen that there is an optimum range for the input and output voltage difference. The difference must not be too great, otherwise excessive temperature will shut the regulator down, and it must not be too small, otherwise it will fall below the drop-out voltage and the circuit will cease to regulate.

Third, the amplitude of the ripple on the rectified, but unregulated input voltage must be considered. It must not fall into the 'drop-out' region on the negative-going half cycle or the regulator will falter in operation. Figure 4.3(a) shows a complete power supply circuit using a regulator.

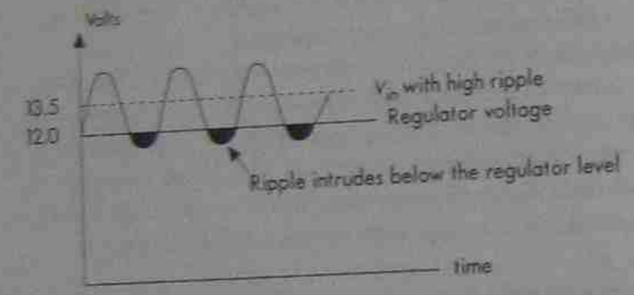
The choice of capacitor for C_1 and C_2 has already been discussed; we now turn to the filter capacitor (C_F) which must be large for two reasons:

1. to make the discharge time constant long (refer to Chapter 2);
2. to keep the amplitude of the ripple low.

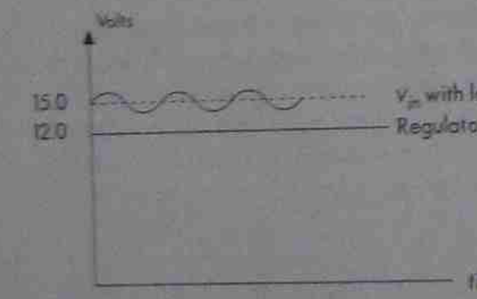
If a 12 V regulator is operated at a minimum input-output voltage differential of 1.5 V and the ripple amplitude is high (say 3 V peak), then we have the situation shown in Figure 4.3(b).



(a) Complete regulator power supply



(b) Graph showing a 12 V regulator with minimum input-output voltage differential, and a high ripple voltage



(b) Graph showing ideal operating conditions. The input–output voltage differential is small, and ripple amplitude is low

Figure 4.3: Regulator operating conditions

The ripple intrudes below the regulator voltage and upsets the operation. To reduce the ripple amplitude, the value of the filter capacitor needs to be increased.

In Figure 4.3(c), two things have happened: the input-output voltage differential has been increased slightly (to 3 V) and the ripple amplitude has been reduced. However, the regulator still operates quite satisfactorily.

The input-output voltage differential is sensible, and only a slight amount of power is dissipated. A typical value for the filter capacitor is 1000 µF at a voltage rating in excess of the rectifier output voltage.

Choke-input filters or pi section filters (see Chapter 2) are rarely used for two reasons:

1. It is desirable to eliminate the weight and cost of a choke.
2. Integrated circuit regulator's have a high rejection to ripple, called the **common mode rejection ratio** (CMRR) with a typical value of 60 dB, and this compensates for the shortcomings of capacitor input systems.

The line and load regulation figures are also very good for these regulators, with line regulation often being 0.005% per volt, and load regulation 5 mV for a 5 V output. To be absolutely certain of these figures check the specification sheet. A typical electrical characteristics sheet is shown in Table 4.5.

4.5.1 Regulator operation

The circuit functions shown in Figure 4.4 are common to all three-terminal regulators. The reference voltage is a temperature-stabilised voltage, developed from a zener diode or a

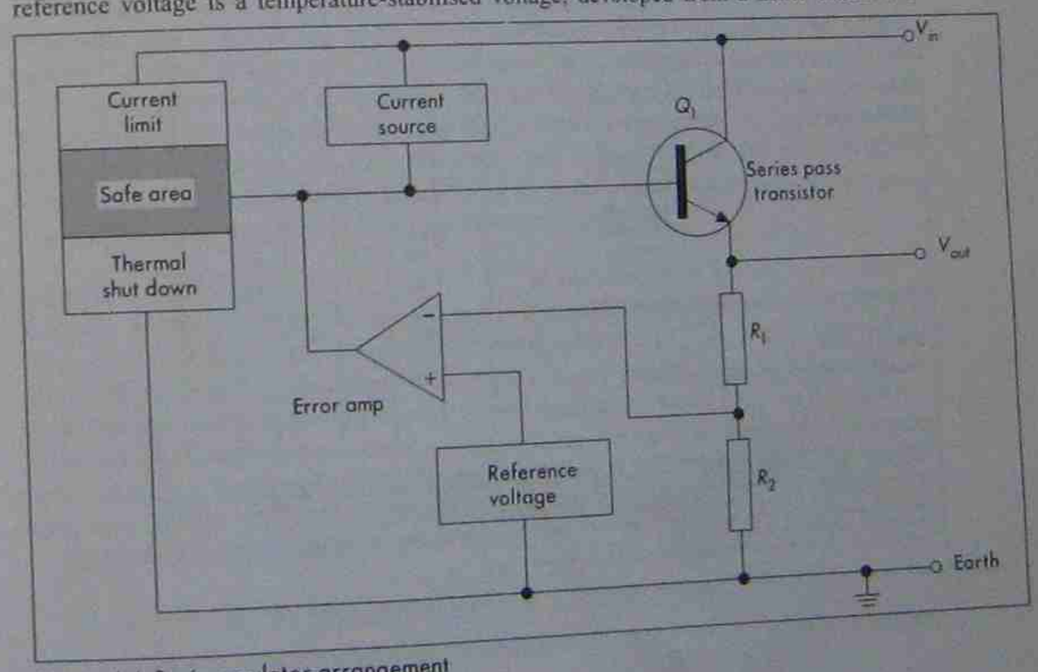


Figure 4.4: Basic regulator arrangement

Table 4.5. Electrical characteristics sheet for an LM317

absolute maximum ratings

Power Dissipation	Internally limited
Input–Output Voltage Differential	40V
Operating Junction Temperature Range	
LM117	−55°C to +150°C
LM217	−25°C to +150°C
LM317	0°C to +125°C
Storage Temperature	−65°C to +150°C
Lead Temperature (Soldering, 10 seconds)	300°C

electrical characteristics (Note 1)

PARAMETER	CONDITIONS	LM117/217			LM317			UNITS
		MIN	TYP	MAX	MIN	TYP	MAX	
Line Regulation	T_A = 25°C, (Note 2)		0.01	0.02		0.01	0.04	%/V
Load Regulation	T_A = 25°C, 10 mA ≤ I_{OUT} ≤ I_{MAX}							
	V_{OUT} ≤ 5V, (Note 2)		5	15		5	25	mV
	V_{OUT} ≥ 5V, (Note 2)		0.1	0.3		0.1	0.5	%
Adjustment Pin Current			50	100		50	100	µA
Adjustment Pin Current Change	10 mA ≤ I_L ≤ I_{MAX} 2.5V ≤ $\|V_{IN}-V_{OUT}\|$ ≤ 40V		0.2	5		0.2	5	µA
Reference Voltage	3 ≤ $\|V_{IN}-V_{OUT}\|$ ≤ 40V, (Note 3) 10 mA ≤ I_{OUT} ≤ I_{MAX}, P ≤ P_{MAX}	1.20	1.25	1.30	1.20	1.25	1.30	V
Line Regulation	10 mA ≤ I_{OUT} ≤ I_{MAX}, (Note 2)		0.02	0.05		0.02	0.07	%/V
Load Regulation	10 mA ≤ I_{OUT} ≤ I_{MAX}, (Note 2)							
	V_{OUT} ≤ 5V		20	50		20	70	mV
	V_{OUT} ≥ 5V		0.3	1		0.3	1.5	%
Temperature Stability	T_{MIN} ≤ T_j ≤ T_{MAX}		1			1		%
Minimum Load Current	$V_{IN}-V_{OUT}$ = 40V		3.5	5		3.5	10	mA
Current Limit	$V_{IN}-V_{OUT}$ ≤ 15V							
	K and T Package	1.5	2.2		1.5	2.2		A
	H Package	0.5	0.8		0.5	0.8		A
	$V_{IN}-V_{OUT}$ = 40V							
	K and T Package		0.4			0.4		A
	H Package		0.07			0.07		A
RMS Output Noise, % of V_{OUT}	T_A = 25°C, 10 Hz ≤ f ≤ 10 kHz		0.003			0.003		%
Ripple Rejection Ratio	V_{OUT} = 10V, f = 120 Hz		65			65		dB
	C_{ADJ} = 10µF	66	80		66	80		dB
Long Term Stability	T_A = 125°C		0.3	1		0.3	1	%
Thermal Resistance, Junction to Case	H Package		12	15		12	15	°C/W
	K Package		2.3	3		2.3	3	°C/W
	T Package					5		°C/W

Note 1: Unless other wise specified, these specifications apply −55°C ≤ T_j ≤ +150°C for the LM117, −25°C ≤ T_j ≤ +150°C for the LM217 and 0°C ≤ T_j ≤ +125°C for the LM317; $V_{IN}-V_{OUT}$ = 5V and I_{OUT} = 0.1A for the TO-5 package and I_{OUT} = 0.5A for the TO-3 package and TO-220 package. Although power dissipation is internally limited, these specifications are applicable for power dissipations of 2W for the TO-5 and 20W for the TO-3 and TO-220. I_{MAX} is 1.5A for the TO-3 and TO-220 package and 0.5A for the TO-5 package.

Note 2: Regulation is measured at constant junction temperature. Changes in output voltage due to heating effects must be taken into account separately. Pulse testing with low duty cycle is used.

Note 3: Selected devices with tighterst tolerance reference voltage available.

special **'band gap' circuit** (a very stable internal reference voltage source, typically 1.23 V). The error amplifier compares the reference voltage with a portion of the output voltage. Resistors R_1 and R_2 act as a voltage divider, and the fraction of the output voltage that is fed to the error amplifier is determined by the feedback ratio of $R_2 / (R_1 + R_2)$. The error amplifier output controls the base drive of the series pass transistor (Q_1) to provide regulation.

All the regulator **protection circuits** (current limiting, thermal shut down and safe area) limit or turn off the base drive of the series pass transistor when activated. This either reduces the available output current or turns the series pass transistor off completely.

4.5.2 Operation of internal protection circuits

Current limiting

When the difference between V_{in} and V_{out} is less than the 6 V breakdown of the zener diode, there is no current in R_3 and only a small Q_2 base current in R_4. The base-emitter voltage of the current limit transistor Q_2 equals the voltage developed across the current-sensing resistor (R_{CL}) (see Figure 4.5). As the regulator's output current increases, the voltage across the current-sensing resister (R_{CL}) and hence the V_{BE} of transistor Q_2, also increase. Eventually, Q_2 turns on. This prevents additional base drive from reaching the series pass transistor (Q_1), and therefore limits the output current.

Thermal shut down

Transistor Q_3 in Figure 4.5 is the 'thermal shut down' transistor. It is physically located next to the series pass transistor Q_1, which is the major heat source in the regulator package.The base of Q_3 is held just below its turn-on voltage at room temperature (approximately 400 mV). As the temperature of the package increases, the voltage required to turn Q_3 on will decrease to 400 mV. At this point, the transistor Q_3 turns on; this removes the base drive from the series pass transistor and turns the output off. Regulators have thermal shut down temperatures ranging from 150°C to 190°C. If the package body exceeds the temperature rating, the integrated circuit regulator automatically turns off.

Safe area protection.

When the difference between V_{in} and V_{out} is greater than the breakdown voltage of the zener diode, a current which is proportional to this voltage difference ($V_{in} - V_{out}$) passes through the zener diode, R_3 and R_4 to the output. This causes the base-emitter voltage of transistor Q_2 to be greater than the voltage drop across the current-sensing resistor (R_{CL}). Refer to Figure 4.5. Transistor Q_2, therefore, turns on at a lower output current through the current-sensing resistor, and the current limit point of the regulator is reduced. The rate of reduction of current limit, with increase in input-output voltage differential, is equal to:

$$\text{Rate of reduction of current limit} = \Delta I_{CL} / \Delta (V_{in} - V_{out})$$
$$= R_4 / (R_3 \times R_{CL})$$

where Δ is 'delta' and means 'a change in', and the units are amperes per volt

The curves shown in Figure 4.6 indicate the safe area characteristics. They show a reduction

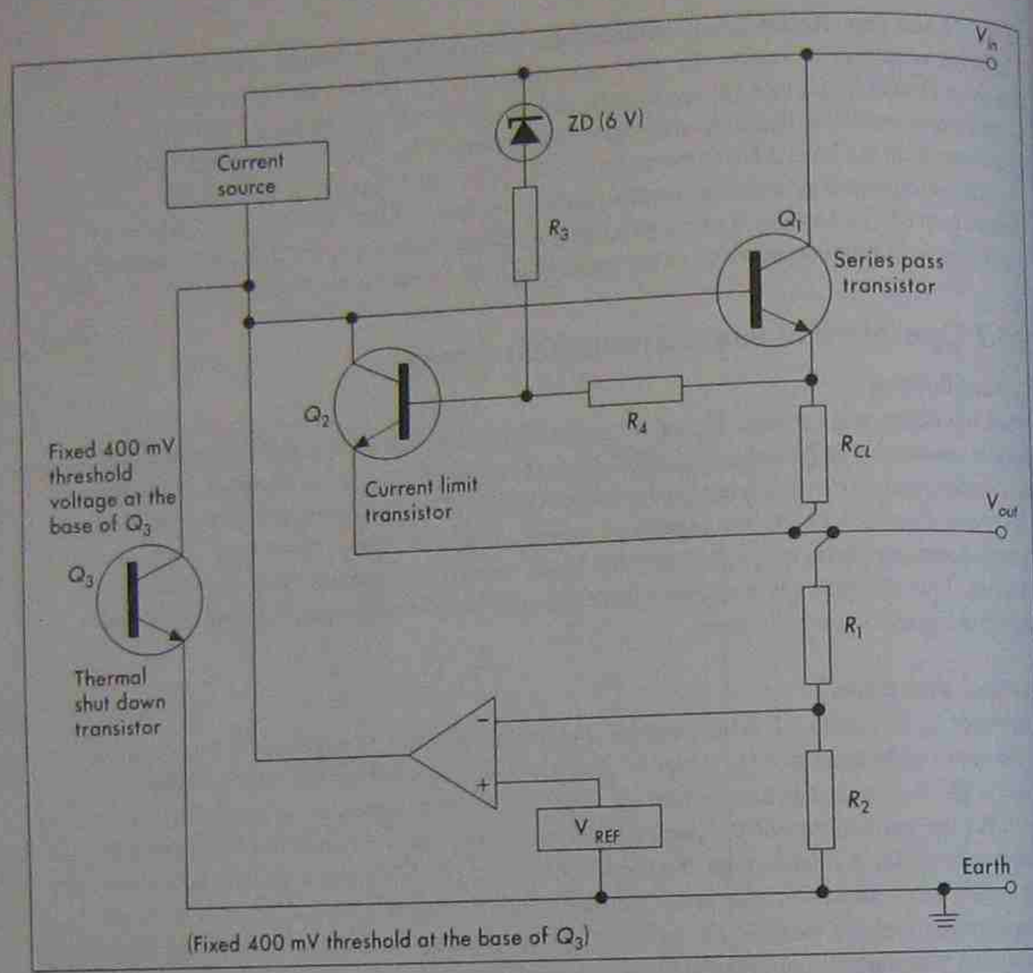


Figure 4.5: Basic regulator with protection circuits

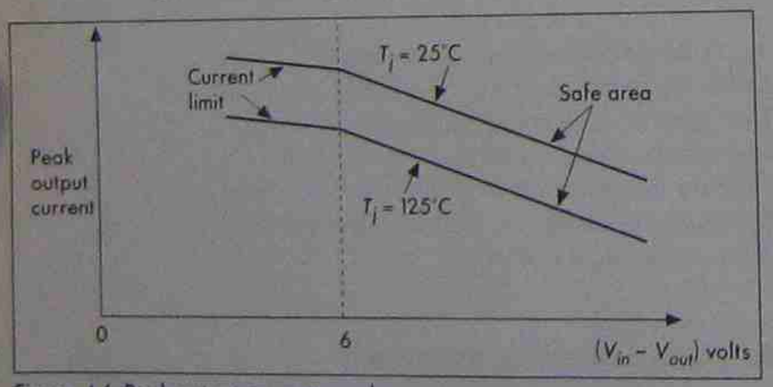


Figure 4.6: Peak output current graph

in output current with increased junction temperature (T_j). A lower base-emitter voltage is required to turn on the current limit transistor as the temperature increases. A large voltage difference ($V_{in} - V_{out}$) can cause excessive temperature and therefore reduce the output current.

This reinforces the earlier statement that a sensible input-output voltage differential is desirable. It is important to note that the 'safe area' circuitry causes the maximum output current to drop significantly with large input-output voltage differentials. Many of the important specification limits of high power regulators are determined by thermal characteristics rather than the electrical characteristics.

4.6 Applications

Voltage regulator use can be extended beyond that of the simple three-terminal fixed-voltage device discussed in section 4.5. Some useful practical circuits will now be described. Equations relative to the circuit operation are included which will enable the concepts to be applied to the entire regulator family. But first, three handling precautions are given.

4.6.1 Precautions when using regulators

A range of voltage regulators are designed with current limiting, thermal shut down, and safe area protection. However, with any integrated circuit regulator it is necessary to take precautions to ensure that the regulator is not inadvertently damaged.

Short circuiting the regulator input

When using large capacitors at the output of regulators that have an output voltage greater than 6 V, a precaution is required. A protection diode *D* (shown in Figure 4.7) connected between input and output may be required if the input is short circuited to ground.

An **input short circuit** will pull the input to ground potential, while the output remains close to V_{out} because of the charge stored in the output capacitor. The capacitor will try to discharge through the low impedance junction inside the regulator. If the energy is large enough, both the junction and the regulator will be destroyed. However, a fast diode (*D* in Figure 4.7) would shunt the capacitor's discharge current around the regulator and protect it.

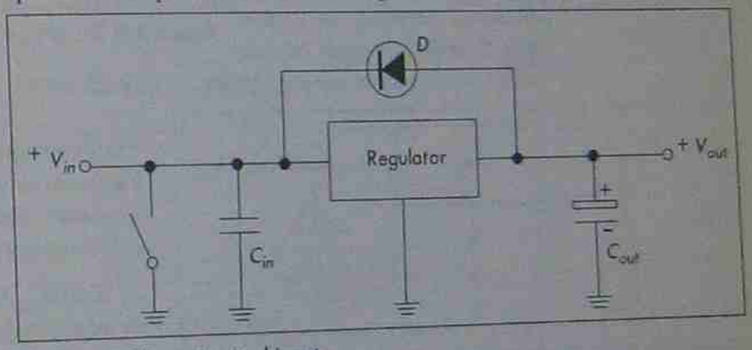


Figure 4.7: Short circuited input

If the output voltage of the regulator is raised above the input voltage, then the internal current paths of the regulator will be damaged in a manner similar to that just described. The diode shown would also protect the integrated circuit by becoming forward biased and shunting the regulator's internal circuitry.

Regulator with floating ground

If the ground pin alone becomes disconnected, during servicing or construction (as shown in Figure 4.8), the output approaches the unregulated input voltage level. This could cause damage to any circuit connected to V_{out}. If the open ground connection is remade with the

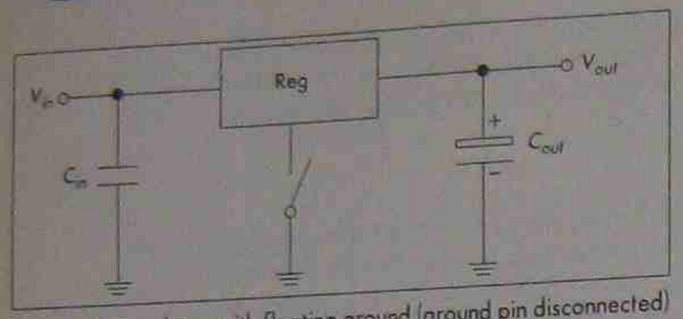


Figure 4.8: Regulator with floating ground (ground pin disconnected)

power on, the regulator itself may be damaged. This fault is most likely to occur when plugging regulators (or modules with on-card regulators) into powered up sockets. Power should be turned off prior to removal or insertion of integrated circuit regulators.

Transient voltages

If input **transients** are of excessive amplitude and exceed the maximum input voltage of the regulator, they will damage the regulator. The regulator will also be damaged if the transients reach more than 800 mV below ground, that is, a negative input. See Figure 4.9.

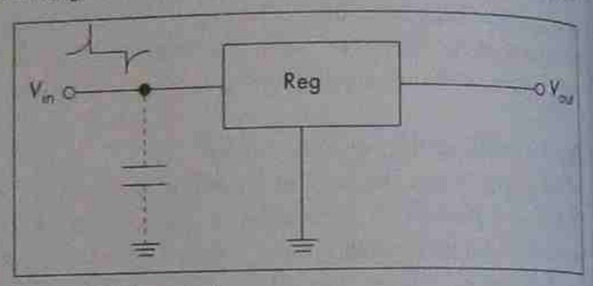


Figure 4.9: Transients

4.7 Positive regulator circuits

Figure 4.10 is the basic three-terminal regulator as described in section 4.5. As previously discussed, the choice of capacitors C_1 and C_2 depends on the circuit's environment and the constructor's circuit layout.

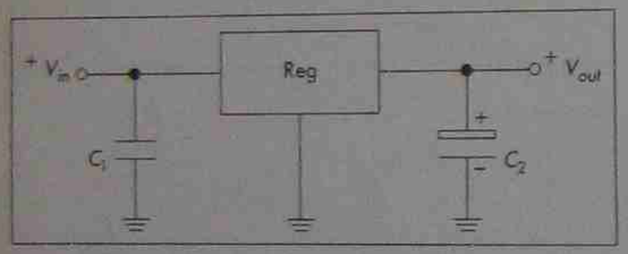


Figure 4.10: Basic regulator as described in section 4.5

Positive regulator as a current source

Figure 4.11 shows an integrated circuit (IC) regulator used as a current source. A constant output current (I_L) is delivered to a variable load:

$I_L = (V_{reg} / R) + I_Q$

where I_Q is the **quiescent current**, and V_{reg} is the regulator voltage rating.

The impedance (Z) to be used as a load can be calculated from:

$$Z = [V_{in} - (V_{reg} + V_{drop\text{-}out})] / I_L$$

High current regulator

The current boost circuit shown in Figure 4.12 takes advantage of the internal current-limiting characteristics of the regulator. It provides short circuit current protection for the series pass transistor as well.

The IC regulator handles one-fifth of the output current and provides voltage regulation, while Q_1 handles the remaining higher output current. The ratio of current sharing between the regulator and the transistor is set by the ratio of R_1/R_2, provided that:

$V_D = V_{BE}$ of Q_1

where V_D is the voltage drop across the diode

$I_1 = (R_2 / R_1) \times I_{reg}$

If the output is short circuited (s/c), then:

$I_1(s/c) = (R_2 / R_1) \times I_{reg}(s/c)$

The thermal protection of the regulator will also be extended to the transistor if the following two conditions are met:

1. the regulator and Q_1 have the same thermal resistance, that is, junction-to-ambient temperatures the same; and
2. the pass transistor heatsink has R_2/R_1 times the thermal capacity of the regulator heatsink.

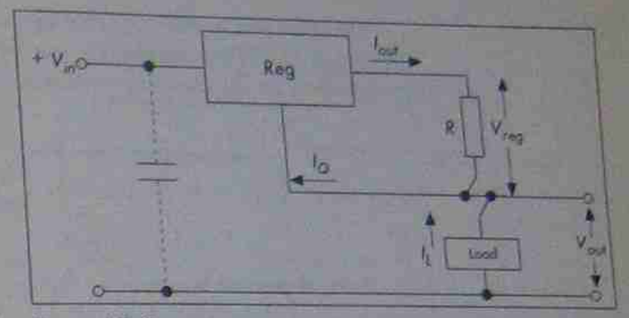


Figure 4.11: Current source

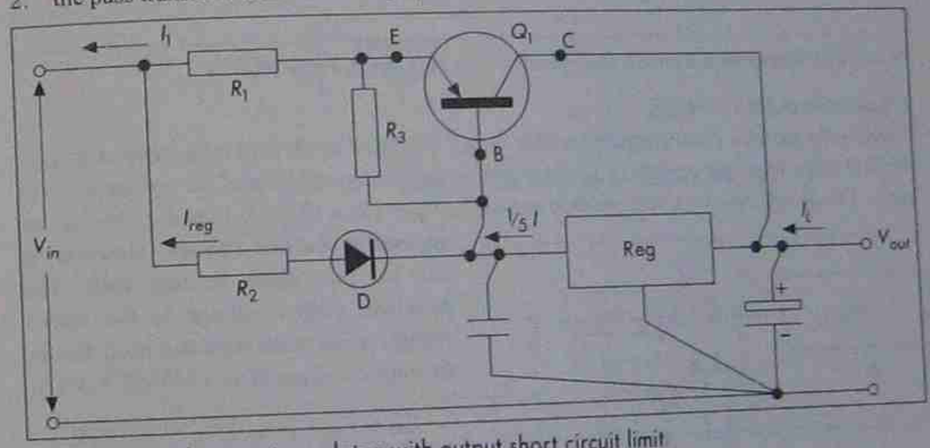


Figure 4.12: High current regulator with output short circuit limit

Component suggestions are listed in Table 4.6.

Table 4.6: Component suggestions

Q_1	D	I_1	I_{reg}	R_2/R_1	R_3
2N4398	1N4719	≥3 A	1 A	≥3	5-10 Ω
NSD32	1N4719	2 A	1 A	2	5-10 Ω
NSDU51A	1N4003	1 A	500 mA	2	5-10 Ω

Using the circuit configuration in Figure 4.12, there is an increase in the minimum input-output voltage differential of the regulator. The voltage drop across the diode and R_1 must be taken into consideration. The minimum input-output voltage differential is increased by one diode voltage drop (that is, V_{BE} of Q_1), plus the voltage developed across resistor R_1 ($I_1 \times R_1$).

The series pass transistor (Q_1) for high current application is an expensive PNP type. A low-priced PNP/NPN combination can be used in its place. Figure 4.13 shows the configuration, and the circuit connection points are E, B and C.

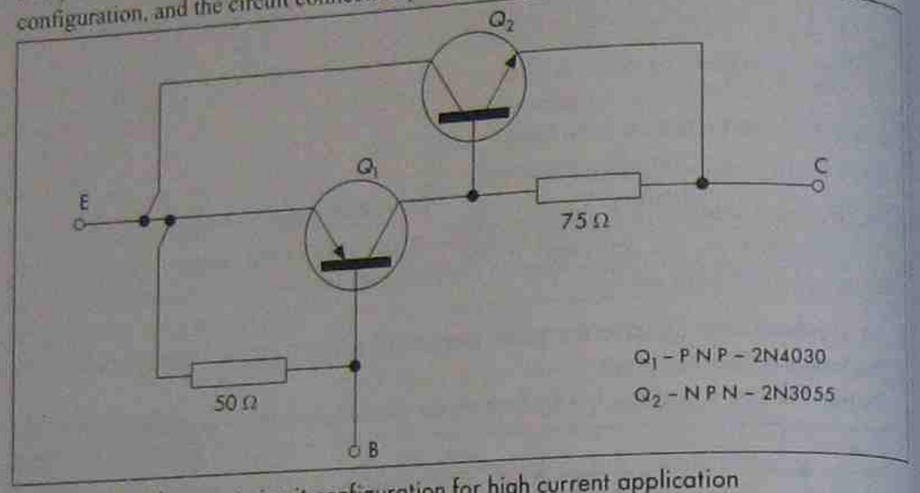


Figure 4.13: Replacement circuit configuration for high current application

Adjustable output voltage

Figure 4.14 shows a fixed output regulator circuit modified to give an adjustable output. A fraction of the regulator current (V_{reg} / R_1) is used to raise the ground pin of the regulator above earth. This is achieved by a voltage drop across R_2, $V_{R2} = [R_2 \times (V_{reg} / R_1)]$, and it provides an **adjustable output voltage**. However, it will not turn down to zero volts. The minimum output voltage is the output voltage rating of the regulator used, that is, the output voltage of an LM340T-5 is 5 V.

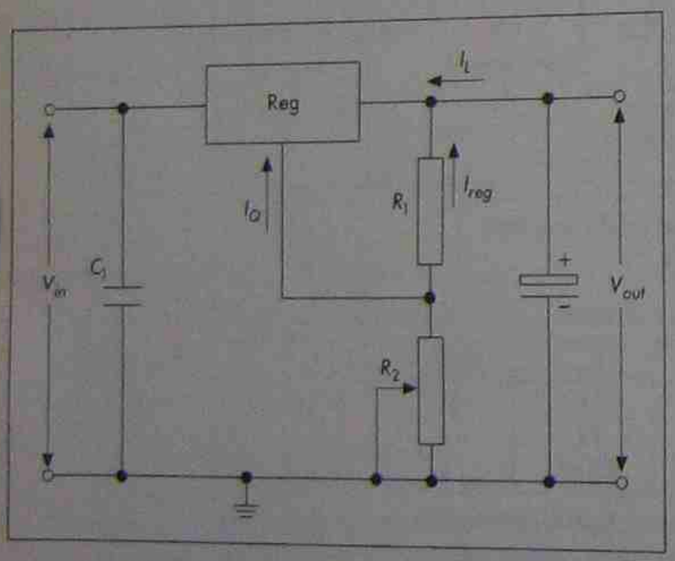


Figure 4.14: Adjustable output voltage

$$V_{out} = V_{reg} + [(V_{reg} / R_1) + I_Q)] \times R_2$$

where V_{reg} = rated regulator voltage and
I_Q = quiescent operating current

These are but a few applications of the fixed output voltage regulator, and further ideas can be obtained from data published by National Semiconductor, which are available from their distributors. One circuit that is becoming very popular in modern equipment is the switching regulator, and this will be discussed in Chapter 6.

4.8 Negative regulator circuits

With the advent of operational amplifiers and microprocessors requiring dual-polarity power supplies, a range of fixed negative voltage regulators is also available. Figure 4.15 shows a three-terminal negative regulator with basic decoupling components and input short-circuit protection.

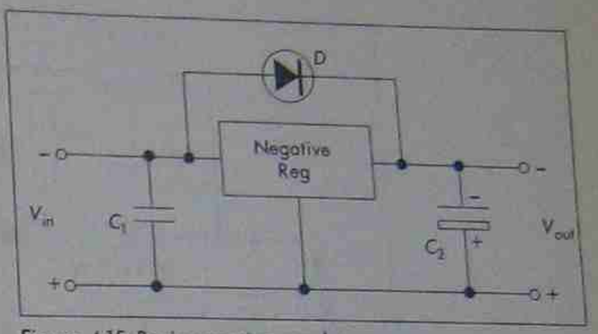


Figure 4.15: Basic negative regulator

Capacitors C_1 and C_2 and diode D perform the same function as they do in a positive regulator circuit. The only variation is the reverse connection of the diode and the polarity of the bypass capacitors. The basic positive regulator circuits shown in Figures 4.11, 4.12 and 4.14 can be duplicated with a negative regulator as shown in Figures 4.16, 4.17 and 4.18. However, the output voltage is negative, and the current path is in the opposite direction.

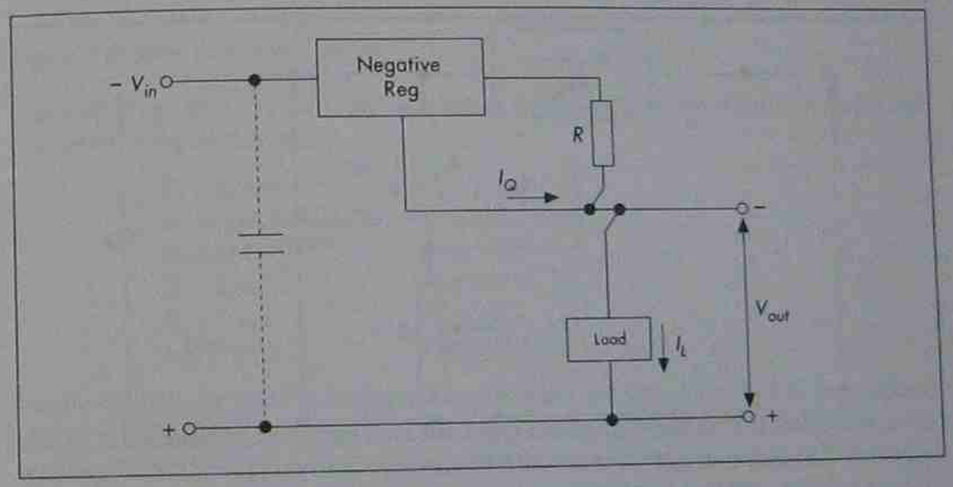


Figure 4.16: Current source

4.9 Dual power supply

A positive regulator, such as LM340, can be connected with a negative regulator, LM320, to provide a non-tracking **dual power supply**. Each regulator exhibits the specifications and limitations of the individual device. The circuit connection is shown in Figure 4.19. Diodes D_1 and D_2 are protection diodes to allow the regulator to work with a common load, and they should be rated at the short-circuit current rating of the regulator.

Typical combinations of positive and negative regulators are:

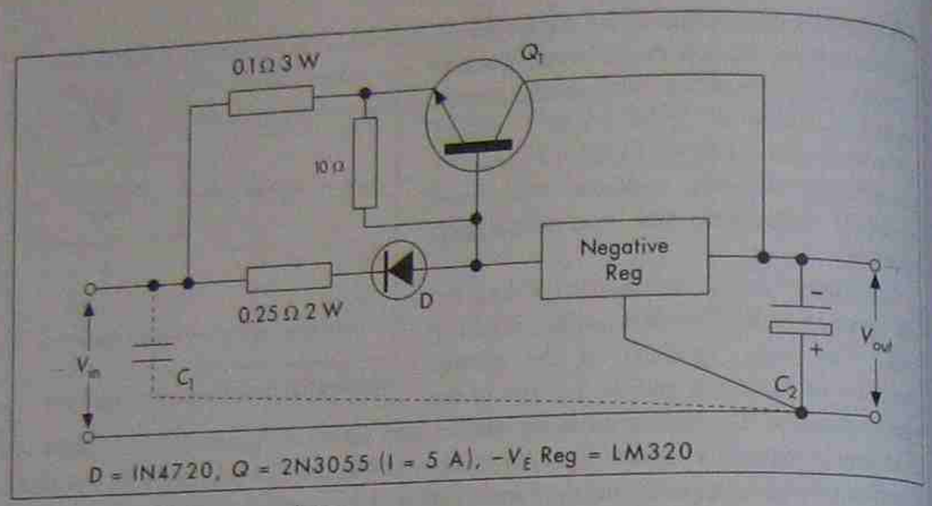


Figure 4.17: High current regulator

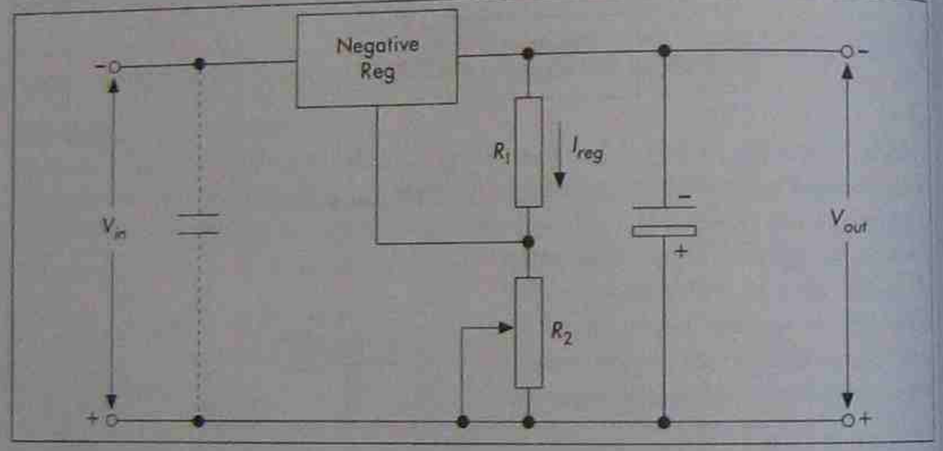


Figure 4.18: Adjustable output

- ± 15 V supply, 1 A common load: LM340T-15, LM320T-15 (D_1 and D_2 IN4720)
- ±12 V supply, 1 A common load: LM340T-12, LM320T-12 (D_1 and D_2 IN4720)
- ± 15 V supply, 200 mA common load: LM342H-15, LM320H-15 (D_1 and D_2 IN4001)

It is possible when using the circuit in Figure 4.19 to get a wider than desirable tolerance between the output voltages: one may be +15.5 V and the other −14.5 V. This can be overcome by modifying the basic circuit by providing a means of trimming the output voltages to a closer tolerance.

In the circuit in Figure 4.20, potentiometers R_4 and R_6 allow a trimming voltage to be developed across resistors R_1 and R_2. Resistors R_3 and R_5 help to linearise the adjustment and prevent short circuiting between the regulator ground pin and the opposite polarity output voltage. D_3 is a 'germanium' diode that may be required to enable the positive regulator to

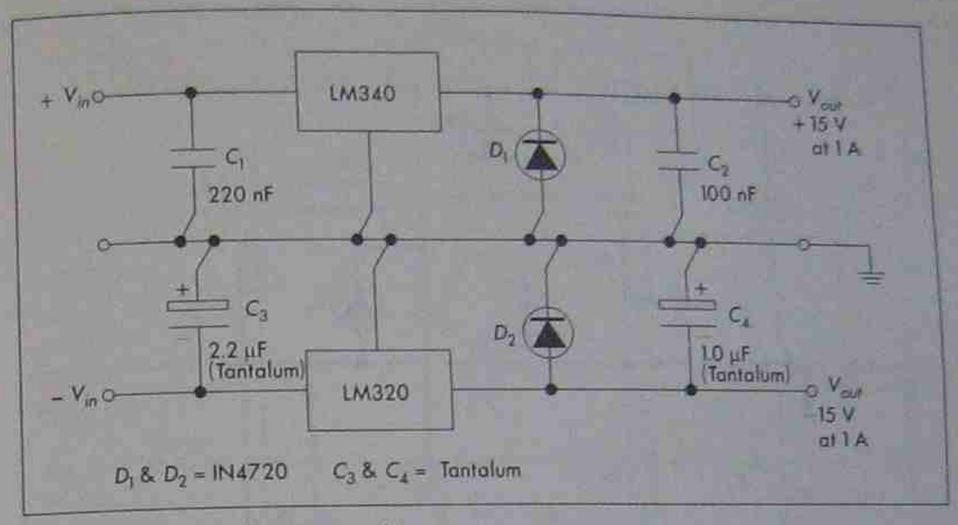


Figure 4.19: Basic dual power supply

start with a high load. Typical component values for those who would like to experiment with the circuit in Figure 4.20 are:

C_1 = 220 nF	C_2 = 2.2 µF
D_1 and $D2$ = 1N4720	D_3 = 1N91
R_1 = 33 Ω	R_2 = 33 Ω
R_3 = 220 Ω	R_5 = 470 Ω
R_4 = 5 kΩ	R_6 = 2 kΩ
LM340T-5	LM320T-5.2

Ideally, as with our basic transistor voltage regulator, we want a circuit that automatically adjusts to compensate for variations in the output condition. When using a dual power supply we want the two voltages to lock together, and if there is any variation in one, a corresponding variation takes place in the other. The circuit that operates in this fashion is called a 'tracking dual power supply'.

4.9.1 Tracking dual power supply

The circuit of a tracking dual power supply is shown in Figure 4.21. The positive regulator 'tracks' the negative regulator. Diodes D_1, D_2 and D_3 are the same as those used in Figure 4.20. Resistors R_1 and R_2 are identical and of close tolerance (1%) for close tracking. Point V_A is at virtual ground potential under steady-state operating conditions, and transistor Q_2 passes the quiescent current of the positive regulator. If the output voltage of the negative regulator falls, point V_A follows accordingly, forward biasing the collector-base junction of transistor Q_1, and the emitter voltage (point V_B) of Q_1 decreases. The forward bias on transistor Q_2 is reduced, and the collector voltage therefore increases. This increase in collector voltage is

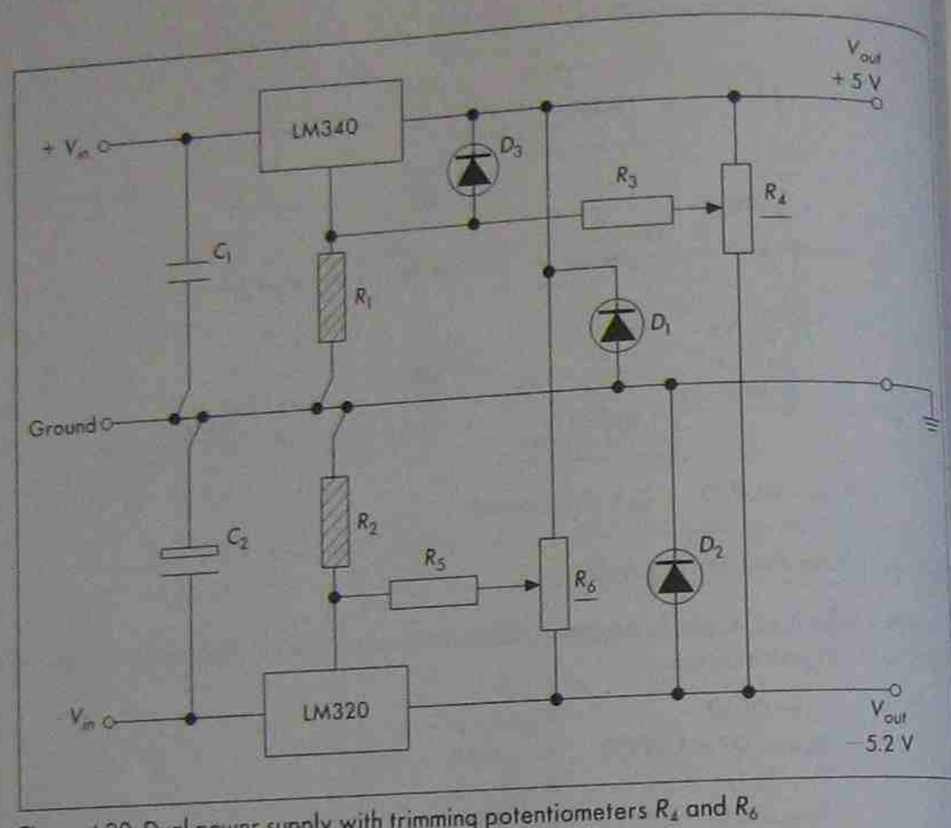


Figure 4.20: Dual power supply with trimming potentiometers R_4 and R_6

also an increase in the positive regulator output voltage. The voltage divider of R_1 and R_2 is across the positive-negative output, and therefore an increase in V_{out} restores the voltage at point V_A to its former level.

A typical combination for a ±15 V tracking dual power supply is LM340T-5, LM320T-15. The LM340T will track the LM320T within 100 mV if R_1 is matched to R_2 to within 1%.

Tracking regulators are also available fully encapsulated in a dual-in-line package complete with heatsink if required. The only connections required to the package, other than earth and the plus and minus voltage inputs and outputs, are the completion of the output current paths for plus and minus current limiting.

4.10 Adjustable power regulators

A later concept in three-terminal adjustable regulators is the power regulator. The LM317 is capable of supplying in excess of 1.5 A over a 1.2 V to 37 V output range, or up to 57 V in the high voltage mode. The LM350 is capable of supplying in excess of 3 A over a 1.2 V to 33 V output range. The LM338 is capable of supplying in excess of 5 A over a 1.2 V to 33 V output range.

These and other regulators, both fixed and adjustable, both positive and negative, are

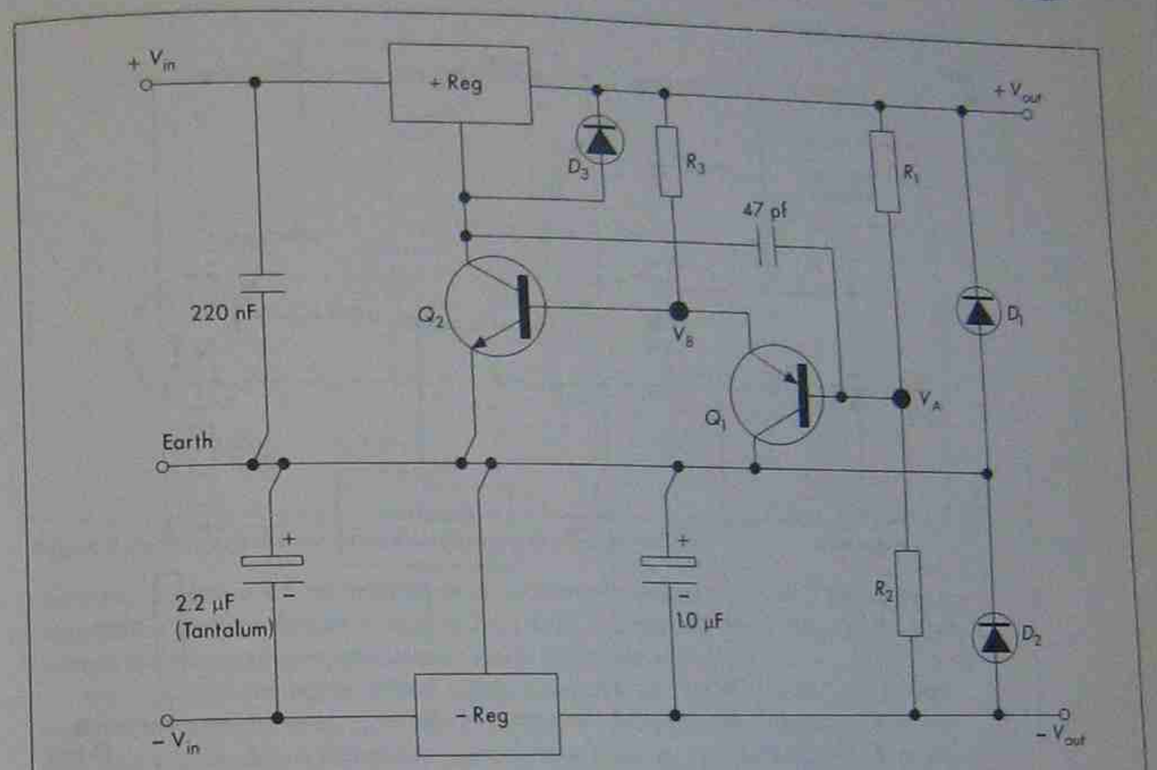


Figure 4.21: Tracking dual regulator

listed in the selection guide in Table 4.2. They are easy to use and require only two external resistors to set the output voltage. Regulation is extremely good, and all include the standard current limiting, thermal shut down and safe area protection features of the basic fixed output regulator. However, high current circuits need special design consideration and this will be discussed in section 4.10.2 using the 5 A LM338 as the example.

4.10.1 Operation

The adjustment of a three-terminal regulator is shown in Figure 4.22. An operational amplifier connected as a buffer drives a power Darlington. The circuitry is arranged so that all the quiescent current is delivered to the regulator output (rather than earth), eliminating the need for a separate ground terminal. A reference diode D_1 inserts approximately 1.2 V between the non-inverting input of the operational amplifier and the adjustment terminal. Approximately 50 µA is required to bias the reference diode. When operating, the output of the regulator is the voltage at the adjustment terminal plus 1.2 V. If the adjustment terminal is grounded, the circuit behaves as a 1.2 V regulator.

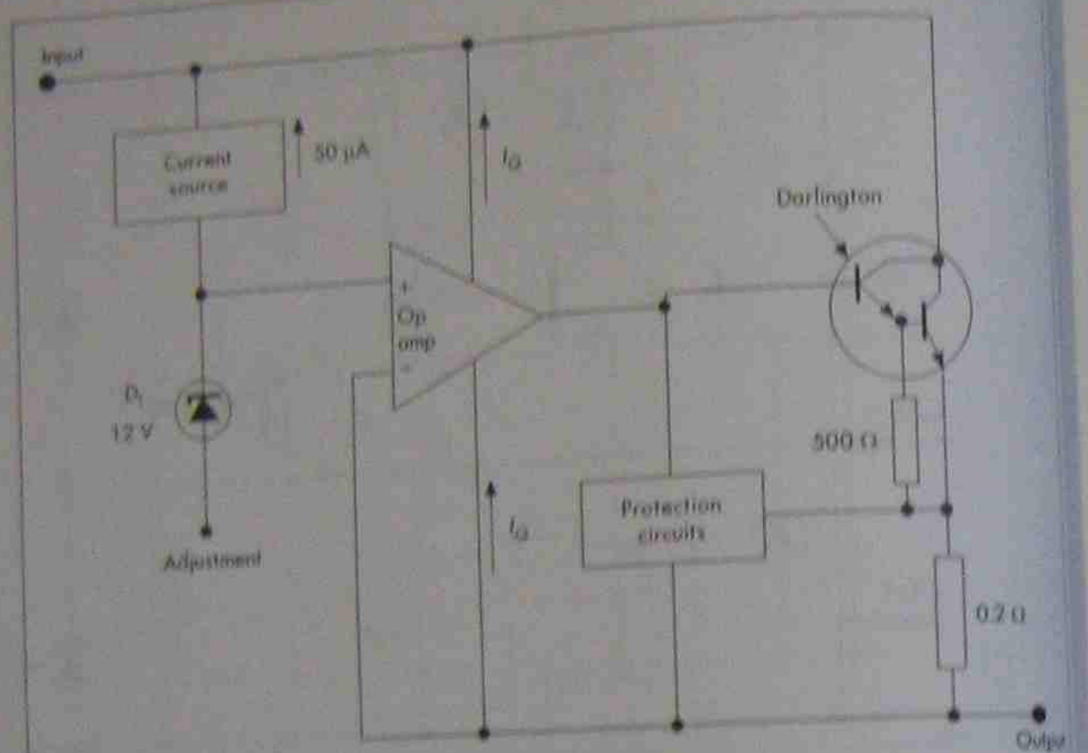


Figure 4.22: Functional diagram of an adjustable regulator (LM317)

To increase the output voltage, a voltage divider R_1 and R_2 is connected from the output to ground as shown in Figure 4.23. The 1.2 V reference across R_1 forces about 5 mA of current flow. This 5 mA flows through R_2, increasing the voltage at the adjustment terminal and hence the output voltage can be determined from:

$$V_{out} = V_{ref} + [1 + (R_2/R_1) + (I_b \times R_2)]$$

where V_{ref} is the internal reference voltage (that is, band gap circuit) for the integrated circuit regulator, and is generally between 1.2 V and 1.25 V, I_b is the internal bleed current and is 50 μA

Note that (50 μA × R_2) is a fairly insignificant value and is often omitted from the calculation of V_{out}. The formula then becomes:

$$V_{out} = V_{ref} + (1 + R_2/R_1)$$

Capacitor C_1 can be included to improve ripple rejection, and diode D will protect the regulator against short circuits at the input or output. The circuit shown in Figure 4.23 makes quite a suitable basic 1.5 A workshop power supply. The values of C_{in} and C_{out} are

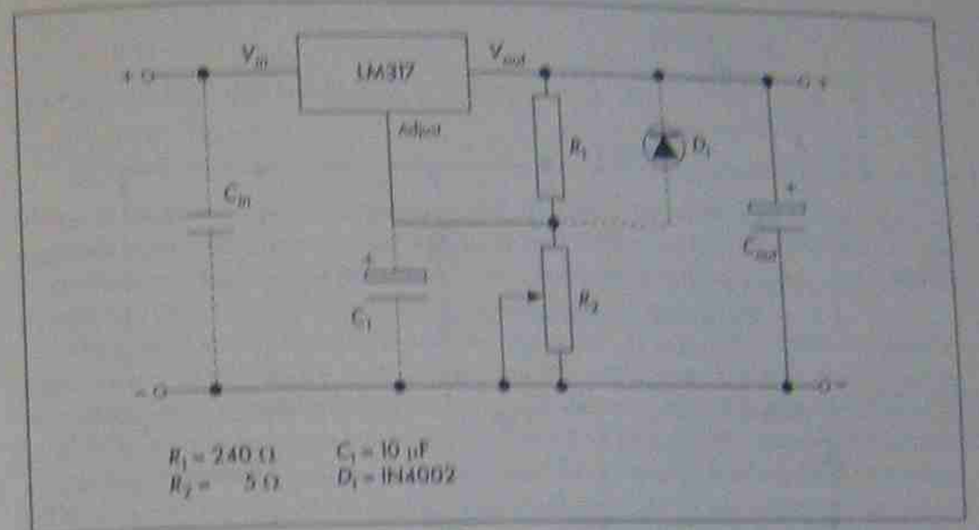


Figure 4.23: Voltage divider added to increase the output voltage

discussed in section 4.5 but referred to as component numbers C_1 and C_2. The supply is adjustable from the reference voltage (1.2 V) to 25 V, depending on the value of the input voltage, and the maximum input-output voltage differential is 40 V.

If a higher current, higher voltage supply is required, an LM350 or LM338 can be used. However, to ensure complete protection of the regulator, the precautions mentioned in section 4.6.1 should be heeded. A further feature would be to make the regulator adjustable from zero volts. This can be achieved by returning the adjustment pin to a negative reference voltage. Figure 4.24 shows a typical circuit with full protection.

4.10.2 Using the LM388 as an example of high current regulators

The LM338 regulator has all the protection features that have been taken for granted in the lower power LM117/317 family. A unique feature of this regulator is that the current limit circuitry will allow 12 A of peak current for a short period of time. The output voltage is adjustable over the range of 1.25 V to 32 V. The maximum input-output voltage differential is 35 V, and higher output voltages are possible providing that this parameter is not exceeded. A full load current of 5 A is available at all output voltages, however, the maximum power dissipation is internally limited. The features of the regulator are:

- 5 A guaranteed output current;
- adjustable output from 1.25 V to 32 V;
- 100% burn-in thermal limit;
- internal current power limiting;
- input-output voltage differential is 35 V maximum;
- drop-out voltage is approximately 2.1 V; and
- TO-3 package.

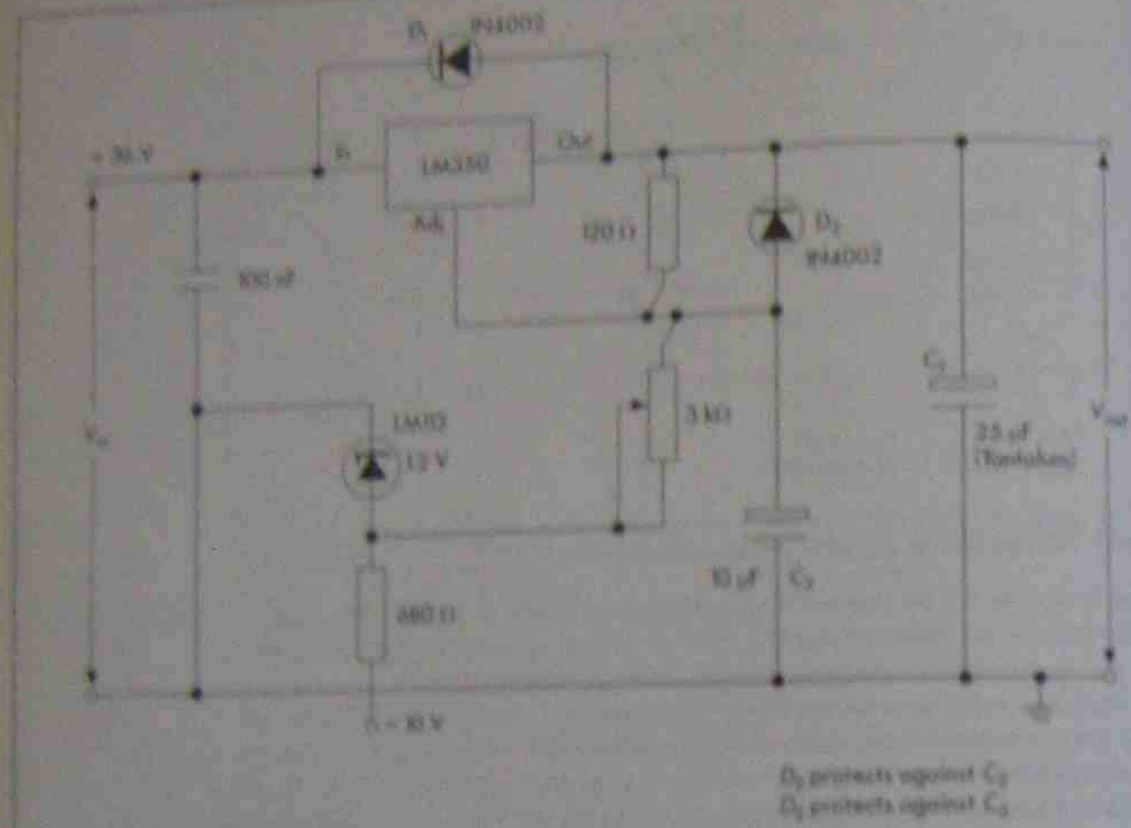


Figure 4.24 0 V to 30 V regulator at 3 A with full protection

4.10.3 Application precautions

Heatsinking

The major limitation in the output current capability of the power regulator is heatsinking. The power regulator has high power dissipation providing that the maximum junction temperature limited is not exceeded. Careful attention must be paid to all junction thermal resistances. A good heat-conductive paste should be used when mounting the regulator on the heatsink. The regulator must also be bolted down tightly. To ensure the selection of the correct heatsink, the procedure is to first calculate the worst case continuous average power dissipation in the regulator from the formula:

$$P = (V_{in} - V_{out}) \times I_L$$

The voltage and current characteristics of the unregulated input must be accurate. A small change in input voltage can result in a large increase in the power dissipated by the regulator. For example, normal operating conditions are $V_{out} = 10$ V, $V_{in} = 14$ V, $I_{out} = 5$ A, therefore:

$$P = (14 - 10) \times 5$$
$$= 20 \text{ W}$$

If the input voltage increases by 10% to 15.4 V:

$$P = (15.4 - 10) \times 5$$
$$= 27 \text{ W}$$

This is an increase in power dissipation of 35%. Therefore, the power supply circuit up to the regulator input (that is, transformer, rectifier diodes, filter capacitor) plays an important role in the successful operation of the regulator itself. It should be built and tested to determine its average DC output voltage under full load with maximum input voltage.

The choice of filter capacitor is also very important. At high current levels the capacitor ripple current (RMS) is two to three times the DC output current. If the capacitor has an **equivalent series resistance** (ESR) of 0.05 Ω, this can cause internal power dissipation (I^2R) of 10 W to 20 W at an output current of 5 A. The life of the capacitor 'derates' with increase in operating temperature, and the choice of a small-value capacitor is asking for trouble (about 2000 μF is used for the LM317 circuit). A value of some 2000 μF per ampere of load current is the minimum recommended value. Large values of capacitor will have longer life and will also reduce the ripple level. This allows a lower DC input voltage to the regulator, which will result in savings in transformer and heatsink costs. A further idea is to place several capacitors in parallel. This increases the capacitance, reduces the net series resistance and increases the heat dissipating area (that is, shares it among the capacitors). The heatsink resistance generally falls into the range of 0.2°C per watt to 1.5°C per watt at a temperature of 60°C. There are large heatsinks such as the Philips 45D6CB, 55D6CB, and the large Minifin. These must be mounted for best convection cooling and could also be cooled by a fan.

Transformers

Correct transformer ratings are extremely important in high current supplies. If the secondary voltage is too high, power will be wasted and cause unnecessary power dissipation in the regulator. However, if the secondary voltage is too low it may cause loss of regulation if the input voltage fluctuates excessively. The following formula can be used to calculate the secondary voltage required using a centre-tapped full-wave rectifier:

$$V(_{RMS}) = [(V_{out} + V_{reg} + V_{rect} + V_{ripple}) \sqrt{2}] \times (V_{prim}/V_{sec}) \times 1.1$$

where 1.1 is the factor accounting for load regulation of the transformer

V_{out} = DC regulated output voltage

V_{reg} = minimum $V_{in} - V_{out}$

V_{rect} = voltage drop (forward) across the diode at $3 \times I_L$

V_{ripple} = peak capacitor ripple voltage (1/2 p-p)

The current rating required is calculated from:

$$I(_{RMS}) = I_L \times 1.2$$

The value of filter capacitor required can be determined from:

$$C = (5.3 \times 10^{-3}) \times I_L / (2 \times V_{ripple}) \text{ microfarads}$$

The best way to appreciate these formulas in use is to calculate the values required for a power supply circuit. If we design a good mobile radio power supply, say 13.8 V at 5 A, where $V_{out} = 13.8$ V, $I_{out} = 5$ A, $V_{reg} = 2.2$ V, $V_{rect} = 1.2$ V, $V_{ripple} = 2\left(\frac{Vp-p}{2}\right)$, $V_{norm} = 240$ V and $V_{low} = 220$ V:

$$V_{(RMS)} = \left(\frac{V_{out} + V_{reg} + V_{rect} + V_{ripple}}{\sqrt{2}}\right) \times \left(\frac{V_{norm}}{V_{low}}\right) \times 1.1$$

$$= [(13.8 + 2.2 + 1.2 + 1)/\sqrt{2}] \times (240/220) \times 1.1$$

$$= (18.2/\sqrt{2}) \times 1.09 \times 1.1$$

$$= 12.869 \times 1.09 \times 1.1$$

$$= 15.4 \text{ V (RMS)}$$

$$I_{(RMS)} = I_L \times 1.2$$

$$= 5 \times 1.2$$

$$= 6 \text{ A}$$

The transformer must therefore be 240:30 V centre-tapped at 6 A. The centre tap will provide 15 V secondary voltage for each diode. The size of the filter capacitor required can be calculated from:

$$C = \frac{(5.3 \times 10^{-3}) \times I_L}{(2 \times V_{ripple})}$$

$$= \frac{(5.3 \times 10^{-3}) \times 5}{(2 \times 1)}$$

$$= 13250\ \mu F$$

The transformer, rectifier and filter circuit is shown in Figure 4.25.

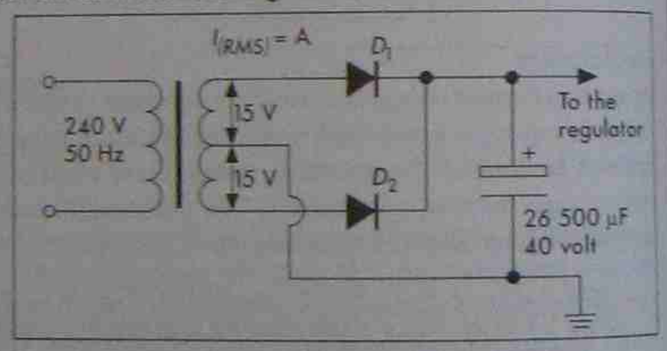


Figure 4.25: Rectifier and filter circuit

Diodes

The diodes used in the circuit must have a high DC current rating. The capacitor input filter draws high peak current pulses that are considerably higher than the average DC current. With a 5 A supply the average current is 2.5 A. The current pulse's duration and amplitude result in a long-term diode heating of approximately 5 A DC. Therefore, the diodes should have a rating of at least 5 A to 7.5 A. Also, the power supply may have to survive a short circuit and average current could rise considerably.

Another important factor in the choice of diode is the surge current at switch on. The peak surge current is about 10 to 20 times the DC output current (that is, 50 A to 100 A for a 5 A supply). (Note that smaller transformers and filter capacitors may be used in lower current supplies. This will reduce the surge current. Unless you are sure of the worst case surges, do not economise on diodes.) Axial lead diodes can be used, such as International Rectifier 60S05 or 60S1 silicon rectifers. Remember to choose the correct peak inverse voltage for the type of transformer in use.

Wiring

High load currents produce higher than normal voltage drops across the resistance of the wiring. It is suggested that 16-18 gauge wire is used for input and output connections, and the length is kept to a minimum.

The two resistors used to set the output voltage level are connected directly to a common point earth and directly to the output of the regulator, as shown in Figure 4.26.

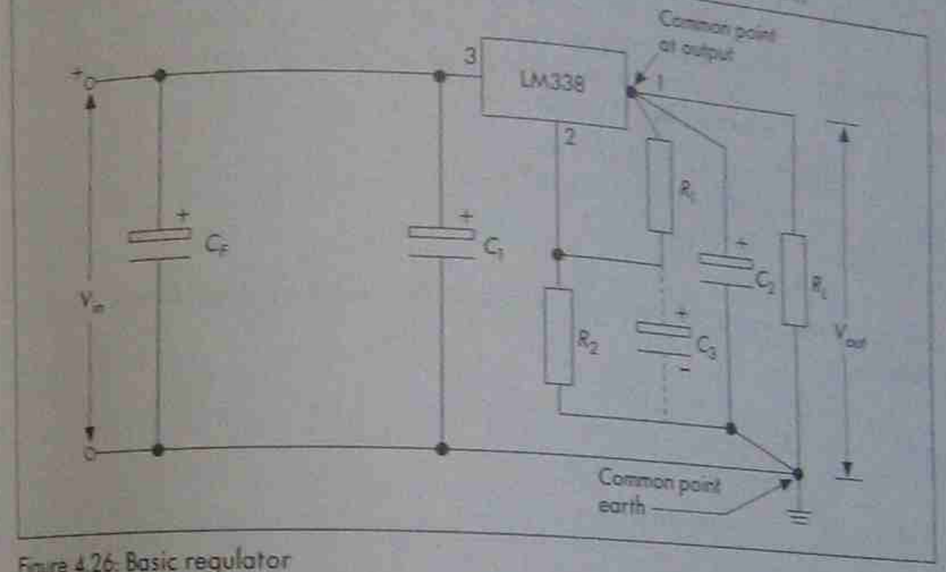


Figure 4.26: Basic regulator

The components in Figure 4.26 are:

- C_F = main filter capacitor of 13 250 µF.
- C_1 = 4.7 µF tantalum. It is only necessary if the main filter capacitor is more than 150 mm away from the regulator. Connecting wire is 18 gauge or larger.
- C_2 = 4.7 µF tantalum. It is not absolutely necessary, but is recommended to maintain low output impedance at high frequencies.
- C_3 = 25 µF. Improves ripple rejection, output impedance and noise. (Capacitor C_2 should be close to the regulator if C_3 is used.)
- R_1 = 120 Ω. It should be a wire-wound or metal-film resistor, tolerance 1% or better.
- R_2 = calculated to set V_{out}; the same type of resistor as R_1. The value of R_2 can be calculated from the formula:

$$R = [(V_{out}/1.25) \times R_1] - R_1$$

where $V_{out} = 13.8$ V, $R_1 = 120\ \Omega$ and $R_2 = (13.8/1.25) \times 120 - 120$, so

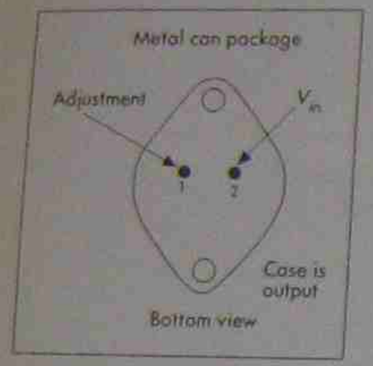


Figure 4.27: Connection diagram for the LM338

so, $R = (11.04 \times 120) - 120$
$= 1324.8 - 120$
$= 1204.8\ \Omega$

As stated earlier, the package is a TO-3 and the connections are shown in Figure 4.27.

4.10.4 Typical applications

Applications for a high current adjustable voltage regulator include temperature controller, high stability regulator, high current regulator, adjustable current regulator and many others. Circuits for these applications are shown in the *National Semiconductor Voltage Regulator Handbook;* however, a couple of useful and interesting ones are included here for experiments.

The output voltage for the circuit in Figure 4.28 can be determined from the formula used in section 4.10.1:

$V_{out} = [1.2 + (1 + R_2/R_1) + (50\ \mu A \times R_2)$
$= 1.2 + (1 + 1100/240) + (50 \times 10^{-6} \times 1.1 \times 10^{-3})$
$= 1.2 + 5.58 + 55 \times 10^{-3}$
$= 6.8\ V$

The 0.3 Ω resistor sets the maximum current of the circuit by controlling the bias of transistor Q_1.

$V_{BE} = I \times R$
$= 2 \times 0.3$
$= 600\ m\ V$

Therefore the voltage across the battery is 6.2 V.

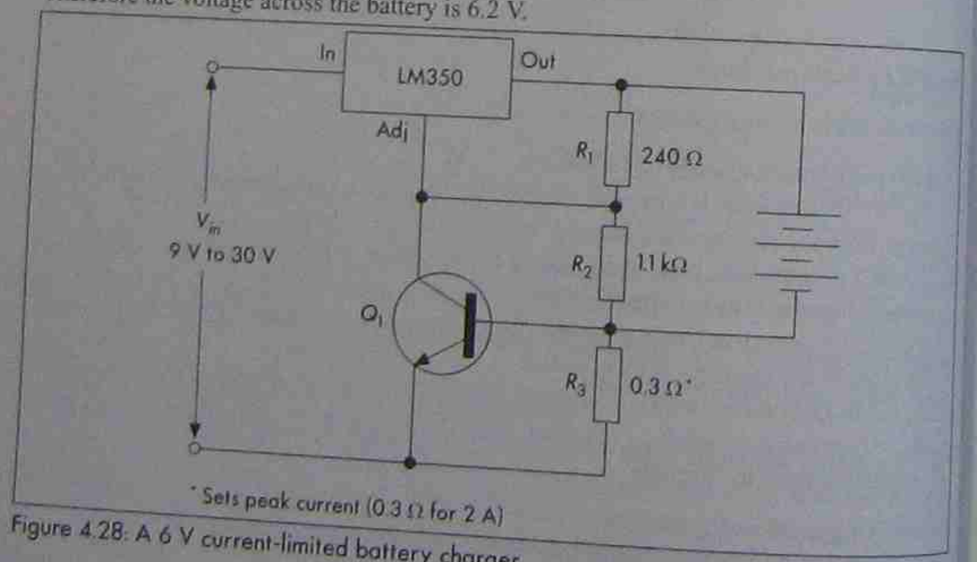


Figure 4.28: A 6 V current-limited battery charger

If the current was to suddenly try to increase, the voltage developed across the 0.3 Ω resistor would increase. This would increase the forward bias (V_{BE}) on Q_1. Transistor Q_1 is in parallel with $R_2 + R_3$ and, therefore, as its conduction increases, the total resistance of the parallel network decreases. The voltage developed across this network is fed to the adjustment pin of the regulator to set the output voltage, and therefore this also decreases. The output of the regulator is thereby reduced, protecting the battery. For example, if the resistance of the parallel network drops to 500 Ω, the output of the regulator decreases to approximately 4 V.

4.11 Basic power supply circuits

The purpose of this section is to provide some practical circuit configurations, showing the complete circuit from input to output. The final choice of transformer, rectifier configuration

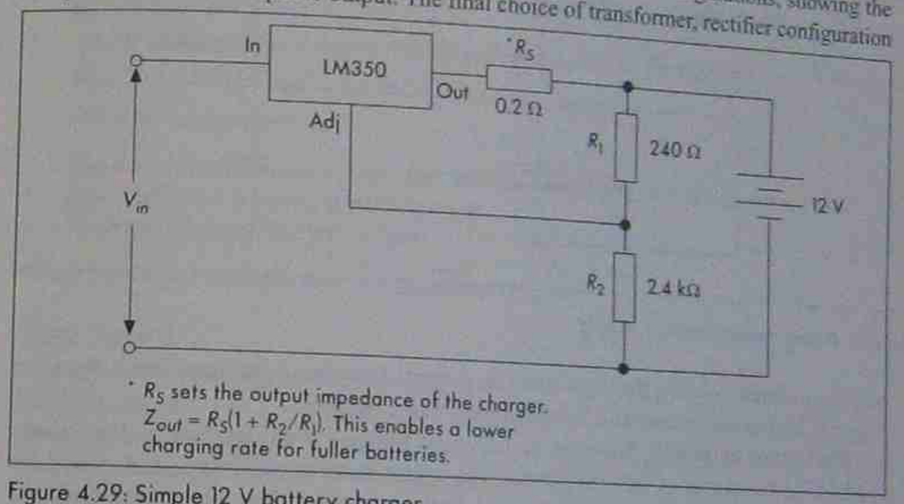


Figure 4.29: Simple 12 V battery charger

and filter is up to the circuit builder, but the following circuits should provide some interesting possibilities. A 12 V battery charger is shown in Figure 4.29.

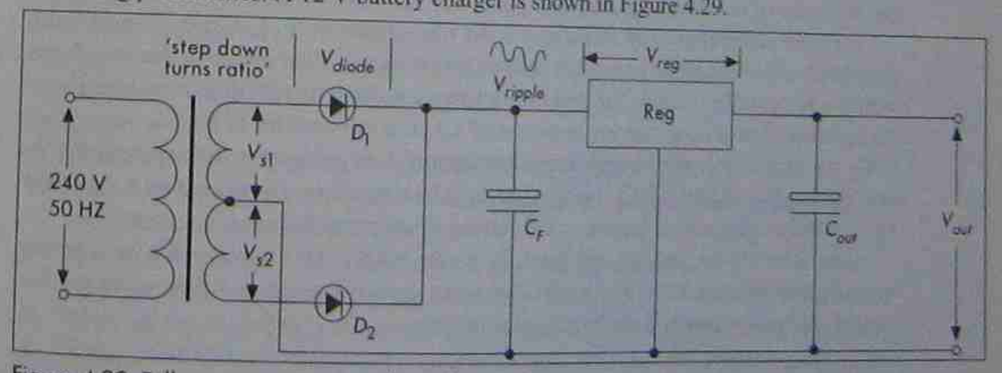


Figure 4.30: Full-wave centre-tapped rectifier circuit

The circuit in Figure 4.30 represents a full-wave centre-tapped rectifier using a capacitive filter. It is the most common selection for medium power, regulated DC supplies.

The full-wave bridge circuit, as shown earlier in Figure 4.3(a), can also be used. This configuration is popular, especially given the availability of bridge rectifier packages.

A circuit not previously mentioned in this book is a dual full-wave circuit. It is called a 'dual complementary rectifier circuit', and it is a very efficient way of obtaining two identical

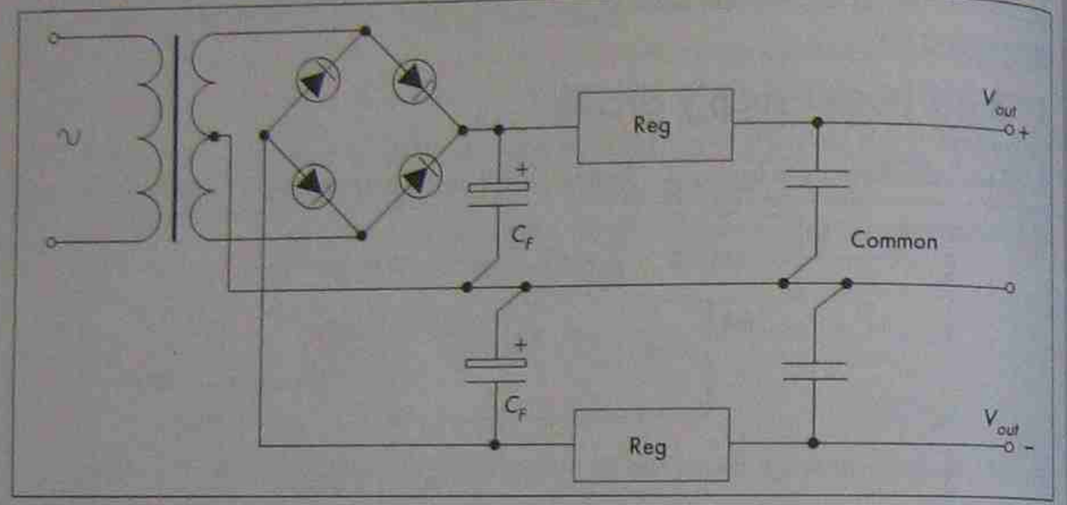


Figure 4.31 A dual complementary supply

outputs of reverse polarity, sharing a common ground. This circuit configuration is shown in Figure 4.31. It is also known as a 'centre-tapped bridge rectifier'.

The choice of straight full-wave or full-wave bridge circuit depends on the voltage available at the output of the transformer. The bridge rectifier makes the best use of the transformer, but it requires four diodes and hence a voltage drop across the diodes of approximately 2 V. A centre-tapped full-wave circuit loses only 1 V across the diode, and so may be preferred in low voltage supplies.

The other component to be considered is the filter capacitor (C_F). A specification rarely considered with electrolytic capacitors used in power supply filtering is the heating factor. With low values of DC current, the peak-to-peak ripple voltage usually determines the size of the capacitor. However, at currents in excess of 1.5 A, a fair amount of heating takes place inside the capacitor itself. Manufacturers put temperature ratings on their capacitors; for example, a rating of 1000 µF/63 V at 65°C is capable of handling approximately 3 A (RMS). As the ambient temperature increases, the current capability of the capacitor derates.

Capacitors are the number one cause of power supply failure, often due to a hostile operating environment. Careful consideration when choosing a value of filter capacitor can prevent your power supply from becoming a statistic.

4.12 Summary

The following items are a summary of the major points covered in this chapter. Integrated voltage regulators have the following characteristics:

- Integrated voltage regulators are small, low cost and reliable.
- They are available with a fixed or variable output voltage, positive or negative polarities and a 100 mA to 5 A current range.
- $V_{in} - V_{out}$ differential must not be less than 2 V to maintain regulation.
- Internal current protection is a feature of all the ICs.
- Never open circuit the ground/adjust leg with the power on the circuit.
- The output voltage for a fixed regulator is predetermined.
- A fixed regulator output voltage can be increased by raising the ground connection above ground potential.
- An adjustable regulator output voltage is determined from the formula: $V_{out} = 1.2 + (1 + R_2/R_1) + (50\ \mu A \times R_2)$.

Other features include:

- Good rejection of ripple voltage, common mode rejection ratio is typically 60 dB.
- Line regulation typically 0.005% per volt.
- Load regulation typically 5 mV.

These figures are available from the specification sheet.

Key terms

adjustable output voltage	heatsinking
band gap circuit	ratio
common mode rejection	line regulation
common point earth	load regulation
drop-out voltage	protection circuits
dual power supply	quiescent current
equivalent series resistance	short-circuited input
fixed output voltage	transients
floating ground	voltage differential
high current	voltage regulator

4.13 Review questions (answers in Appendix A)

1. Why is an input capacitor required on the regulator when the regulator is more than 5 cm away from power supply filter capacitor?
2. (a) Is an output capacitor essential for stable regulator operation?
   (b) What is the purpose of it when included in the circuit?
3. What are the basic things to consider when heatsinking an integrated circuit regulator?
4. Name three factors that finally determine how efficient and reliable the regulator operation is going to be. Briefly explain what is meant by each.

5. As the operating temperature of the regulator increases, what happens to the output current capability? Why does this happen?
6. How can a regulator be protected against a short-circuited input? Can this protection serve any other purpose?
7. Draw a basic three-terminal positive regulator with input and output decoupling and full protection.
8. How can the output of a fixed three-terminal positive regulator be made adjustable?
9. Is it true to say that the external circuitry for a negative regulator is the same as that for a positive regulator? Explain your answer.
10. Draw a basic dual-polarity power supply including the transformer, rectifier and filter components.
11. What is meant by the term 'tracking regulator'? What are its advantages over a standard dual circuit?
12. What is a typical value for the 'drop-out' voltage of an integrated circuit regulator?
13. A 5 V regulator is to draw a steady load current of 1.0 A. If the input voltage is 45 V:
    (a) What is the power dissipated by the regulator, and how will this affect its operation?
    (b) How can this disadvantage be remedied?

Multiple-choice questions

1. The advantage of three-terminal regulators over discrete voltage regulators is:
   (a) small size and cost
   (b) wide current range (100 mA to 10A)
   (c) automatic current protection
   (d) all of the above
2. A 'heatsink' is required for the integrated circuit regulator if:
   (a) used at currents greater than 3 A
   (b) a large input-output voltage differential exists
   (c) $V_{in} - V_{out}$ equals 2 V
   (d) a large package (TO-3) is used
3. A fixed output LM 340T-12 has a DC output voltage of:
   (a) 1.2 V
   (b) 5 V
   (c) 10 V
   (d) 12 V
4. To raise the output voltage of a fixed output three-terminal regulator, you would connect the ground pin to:
   (a) the centre of a voltage divider connected from the input to ground
   (b) a resistor connected to ground
   (c) the centre of a voltage divider connected from the output to ground
   (d) none of the above
5. An LM317T regulator is required to give an output voltage of 13.8 V to power a CB radio. The input voltage is 18 V DC and the resistor between the V_{out} and 'adjust' pins is 220 Ω. What approximate value of resistor is required to be connected from the 'adjust' pin to ground?
   (a) 2k3 Ω
   (b) 1k2 Ω
   (c) 1k Ω
   (d) 120 Ω
6. 'Load regulation' means the output voltage will be relatively unaffected (up to design limitations) by:
   (a) variations in input voltage
   (b) variations in output resistance
   (c) variations in filter capacitor
   (d) variations in drop-out voltage
7. Referring to the data sheet for a LM138 regulator (see Table 4.7). The maximum input-output voltage differential is:
   (a) 1.2 V
   (b) 2 V
   (c) 35 V
   (d) 45 V
8. Refer to Figure 4.23. What will be the approximate output voltages when R_2 is set to first 0 Ω and then 3k9 Ω where (R_1 = 240 Ω) ?

	$R_2 = 0\ \Omega$	$R_2 = 3k9\ \Omega$
(a)	0 V	20.7 V
(b)	1.2 V	20.7 V
(c)	0 V	19.5 V
(d)	1.2 V	19.5 V

9. A three-terminal regulator is required to provide –8 V at 1.5 A. From Table 4.2, which unit would you select?
   (a) LM320-8
   (b) LM340-8
   (c) LM323
   (d) LM345
10. A three-terminal regulator is set to give a 5 V output. If the input-output voltage differential is 35 V, the maximum allowable input voltage will be:
    (a) 10 V
    (b) 20 V
    (c) 30 V
    (d) 40 V

Table 4.7: Electrical characteristics sheet for an LM138/338

Absolute Maximum Ratings

	Internally limited
Power Dissipation	35V
Input–Output Voltage Differential	
LM138	−55°C to +150°C
LM238	25°C to +150°C
LM338	0°C to +125°C
	−55°C to +150°C
Storage Temperature	
Lead Temperature (Soldering, 10 seconds)	300°C

Preconditioning

Burn-In in Thermal Limit — All Devices 100%

Electrical Characteristics (Note 1)

PARAMETER	CONDITIONS	LM138/LM238			LM338			UNITS
		MIN	TYP	MAX	MIN	TYP	MAX	
Line Regulation	$T_A = 25°C$, $3V \le V_{IN} - V_{OUT} \le 35V$, (Note 2)		0.005	0.01		0.005	0.03	%/V
Load Regulation	$T_A = 25°C$, $10mA \le I_{OUT} \le 5A$		5	15		5	25	mV
	$V_{OUT} \le 5V$, (Note 2)		0.1	0.3		0.1	0.5	%
	$V_{OUT} \ge 5V$, (Note 2)		0.002	0.01		0.002	0.02	%/W
Thermal Regulation	Pulse = 20 ms		45	100		45	100	µA
Adjustment Pin Current			0.2	5		0.2	5	µA
Adjustment Pin Current Change	$10mA \le I_L \le 5A$, $3V \le V_{IN} - V_{OUT} \le 35V$							
Reference Voltage	$3 \le \lvert V_{IN} - V_{OUT} \rvert \le 35V$, (Note 3) $10mA \le I_{OUT} \le 5A$, $P \le 50W$	1.19	1.24	1.29	1.19	1.24	1.29	V
Line Regulation	$3V \le V_{IN} - V_{OUT} \le 35V$, (Note 2)		0.02	0.04		0.02	0.06	%/V
Load Regulation	$10mA \le I_{OUT} \le 5A$, (Note 2)		20	30		20	50	mV
	$V_{OUT} \le 5V$		0.3	0.5		0.3	1.0	%
	$V_{OUT} \ge 5V$		1			1		%
Temperature Stability	$T_{MIN} \le T_J \le T_{MAX}$		3.5	5		3.5	10	mA
Minimum Load Current	$V_{IN} - V_{OUT} = 25V$							
Current Limit	$V_{IN} - V_{OUT} \le 10V$							
	DC	5.0	8		5.0	8		A
	0.5 ms Pulse	7	12		7	12		A
	$V_{IN} - V_{OUT} = 30V$		1			1		A
RMS Output Noise, % of V_{OUT}	$T_A = 25°C$, $10 Hz \le f \le 10 kHz$		0.003			0.003		%
Ripple Rejection Ratio	$V_{OUT} = 10V$, $f = 120 Hz$		60			60		dB
	$C_{ADJ} = 10 \mu F$	60	75		60	75		dB
Long Term Stability	$T_A = 125°C$		0.3	1		0.3	1	%
Thermal Resistance, Junction to Case	K Package		1.8	2.0		1.8	2.0	°C/W

CHAPTER 5

VOLTAGE MULTIPLIERS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Define the term 'voltage multiplier circuit' and state an application for such a circuit.
- Sketch the circuit and explain the operation of a half-wave voltage doubler, a full-wave voltage doubler and a voltage tripler.
- State the ripple frequency of the various voltage multiplier circuits.
- Determine the working voltage required for capacitors used in voltage multiplier circuits.
- Calculate the peak inverse voltage required for diodes used in voltage multiplier circuits.
- State the main disadvantage associated with voltage multiplier circuits.

5.1 Introduction

Voltage multiplier circuits are used to develop DC voltages with a higher value than the peak AC input voltage without the use of a step-up transformer. These circuits can double, triple or quadruple the input voltage if desired, but they are only able to supply a small amount of current. A television picture tube is an example of a device requiring a high value of DC voltage. The current requirement is small, and therefore the high DC voltage can be supplied more economically by means of a voltage multiplier circuit than by a transformer.

The basic half-wave rectifier circuit (as discussed in Chapter 1) can be used as a simple voltage multiplier circuit, with careful selection of a capacitor-resistor (CR) circuit. Figure 5.1 shows a typical circuit with a capacitor across the output, that is, a capacitor input filter.

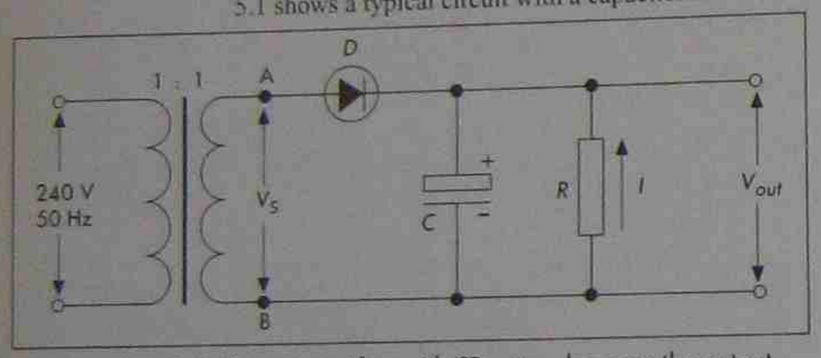


Figure 5.1: Simple half-wave rectifier with CR network across the output

The transformer in Figure 5.1 has a 1:1 turns ratio and is used to isolate the circuit from the AC mains. In some countries the mains input is fed directly to points A and B. This is called a *'transformerless supply', and it is potentially dangerous for the technician.* The secondary voltage, therefore, is in this case 240 V RMS. When point A is positive with respect to point B, the diode *D* will conduct, and current passes through the resistor. The capacitor will charge up to the **peak value** of 339.4 V (that is, 1.414 × 240 V), where 1.414 = $\sqrt{2}$ = conversion factor for RMS to peak level.

On the next half cycle of input, point A will be negative with respect to point B, and the diode will not conduct. The capacitor will discharge through the resistor *R*. This action repeats itself every half cycle when the diode is forward biased, and it results in a varying DC voltage across *R*. Each time the diode conducts, the capacitor charges to the peak value of V_S; however, the extent to which the capacitor discharges on the negative half cycle of input is determined by the output time constant ($C \times R$).

Figure 5.2 shows the output waveforms for three different time constants. With 240 V

applied across the secondary voltage (V_S), it is possible to obtain a DC output voltage greater than the input by using a rectifier and a suitable CR circuit.

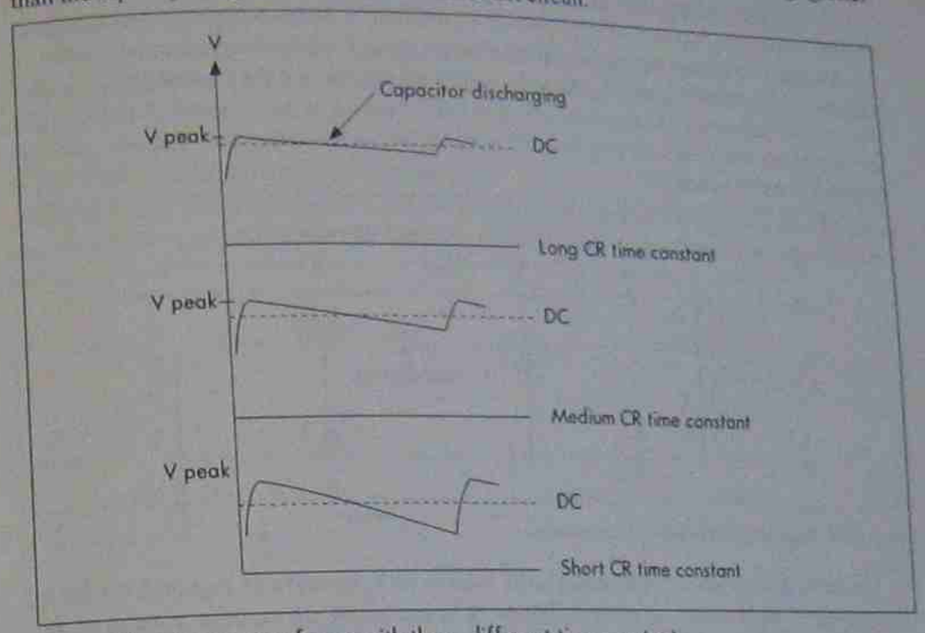


Figure 5.2: Output waveforms with three different time constants

5.2 Voltage doublers

5.2.1 Half-wave voltage doublers

A modification to the capacitor input filter allows us to build up an output voltage that may be two, three or four times the secondary voltage. Figure 5.3 shows the circuit of a **half-wave voltage doubler**, often referred to as a 'cascade voltage doubler'.

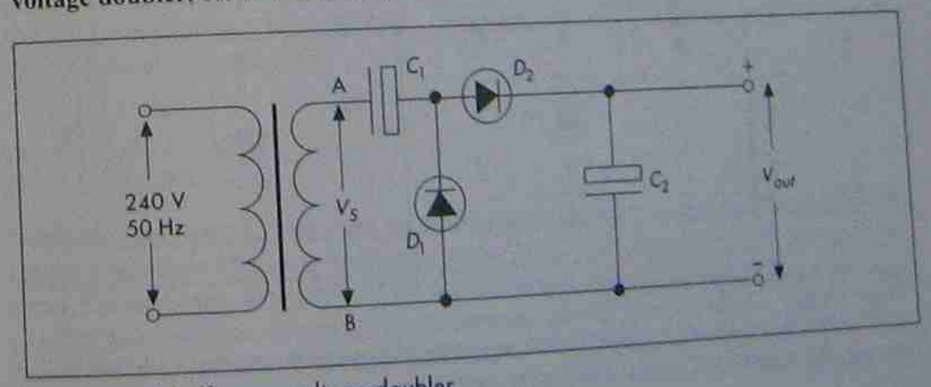


Figure 5.3: A half-wave voltage doubler

The value of the secondary voltage (V_s) is determined by the turns ratio of the transformer, and the operation of the circuit is best understood by looking at each half cycle of input separately. Figure 5.4 shows the circuit with 240 V 50 Hz input and a turns ratio of 1:1. During the negative half cycle of input voltage, point A of the transformer secondary is negative with respect to point B. Diode D_1 is forward biased, and the current path is in the direction of the arrow. The capacitor C_1 charges to the peak value of V_s (that is, 339.4 V) with the polarity shown. (There is very little forward voltage drop across a good diode, and for simplicity we will ignore it.)

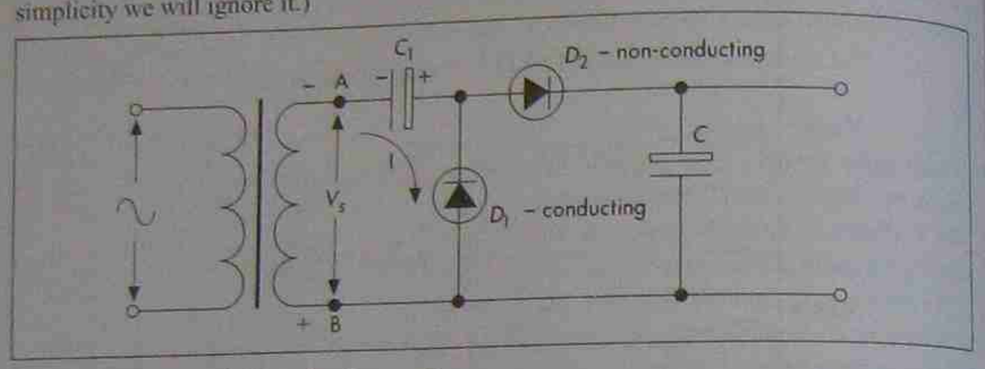


Figure 5.4: Negative half cycle of operation

On the next half cycle of input voltage, the polarity of V_s reverses. In Figure 5.5 point A is now positive with respect to point B. Diode D_1 is cut off by the positive voltage on its cathode. Diode D_2 has the secondary voltage (V_s) in series with the voltage stored in C_1 applied to its anode. Diode D_2 conducts and the current path is in the direction indicated, charging C_2 to twice V_s:

$$V_{C2} = V_s + V_s$$
$$= 339.4 + 339.4$$
$$= 678.8 \text{ V}$$

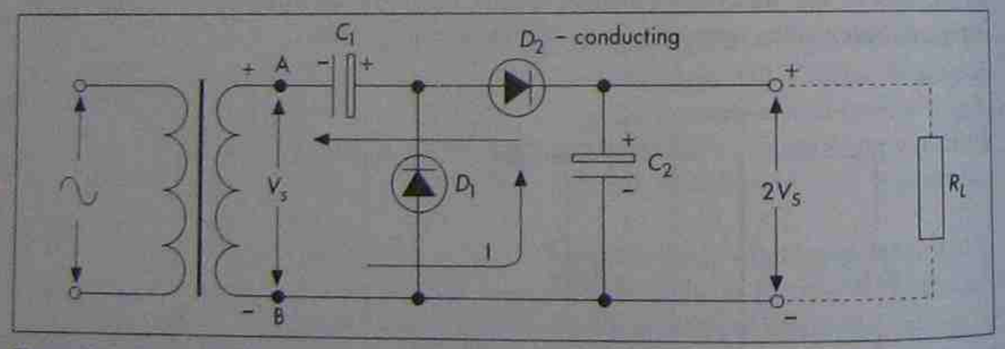


Figure 5.5: Positive half cycle of operation

If no load is connected, capacitor C_1 will stay charged to V_s and capacitor C_2 to twice V_s. However, the idea of the circuit is to supply voltage to a load. Capacitor C_2 discharges

through the load on the next negative half cycle and is recharged to twice V_s during the positive half cycle. An output voltage of reverse polarity can be obtained by simply reversing the diodes and the capacitors.

The output waveform across capacitor C_2 is the same as that of a half-wave rectifier with a capacitor filter. The peak inverse voltage across each diode is twice V_s peak, and the **ripple frequency** is the same as the input.

5.2.2 Full-wave voltage doublers

Another doubler circuit uses the full-wave principle. Two capacitors are individually charged, and the output is the sum of the two voltages. Figure 5.6 shows a **full-wave voltage doubler** circuit.

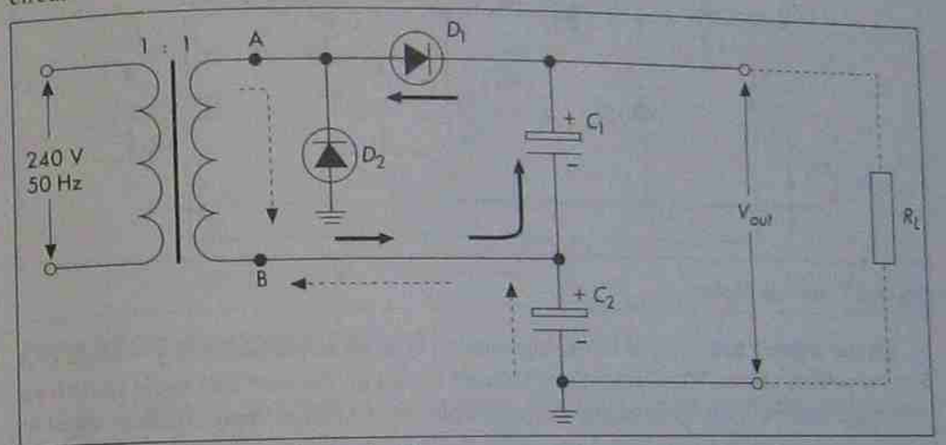


Figure 5.6: A full-wave voltage doubler

When point A is positive with respect to point B, diode D_1 conducts. Current is in the direction shown by the heavy arrows, and capacitor C_1 charges up to 339.4 V (peak) (1.414 × 240); this is V_{C1}. When point A is negative with respect to point B, diode D_2 conducts. Current follows the path of the dotted arrows, and capacitor C_2 charges up to 339.4 V (peak) (1.414 × 240); this is V_{C2}. The output voltage (V_{out}) is therefore the sum of the voltages stored in C_1 and C_2:

$$V_{out} = V_{C1} + V_{C2}$$
$$= 339.4 + 339.4$$
$$= 678.8 \text{ V (peak)}$$

When current is drawn by the load, the voltage across capacitors C_1 and C_2 is the same as that across a capacitor filter in a full-wave rectifier circuit. However, because C_1 and C_2 are in series, the total capacitance is less, and this results in a poorer filtering action.

The peak inverse voltage (PIV) across each diode is twice V_s (peak), and the ripple frequency is twice the input. Both the half and full-wave voltage doubler circuits provide an output that is twice the peak voltage of the transformer secondary. No centre tap is required

for the transformer and, as expected, the minimum diode peak inverse voltage rating is two
the peak secondary voltage. The voltage regulation is poor in these circuits, however, so
they are only suitable for low current applications.

5.3 Voltage triplers

The **voltage tripler** concept is an extension of the half-wave voltage doubler. It makes use
the voltage stored in a capacitor charged by the previous alternation and transfers it to a
capacitor. Figure 5.7 shows a voltage triple-circuit that is an extension of the doubler sho
in Figure 5.3.

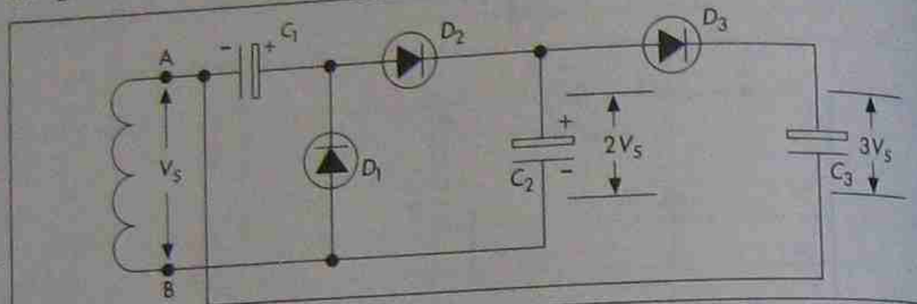


Figure 5.7: Voltage tripler

On the negative half cycle of input, capacitor C_1 charges to the secondary voltage (
with the polarity shown. When diode D_2 is forward biased on the next half cycle (positive
the voltage stored in C_1 is series-aiding the input voltage. A voltage approximately equal
twice V_s is then transferred and stored in C_2 with the polarity as shown in Figure 5.7. So f
this is the same as the action of the voltage doubler circuit in Figure 5.3.

On the next negative half cycle of input, point B is positive with respect to point A
Therefore, V_s is now series-aiding the voltage stored in C_2, as shown in Figure 5.8. Th
positive potential on D_2 cathode biases the diode off. The anode of D_3 is at a potential of two
V_s and is therefore forward biased. Current follows the direction of the arrow, and C_3 charg

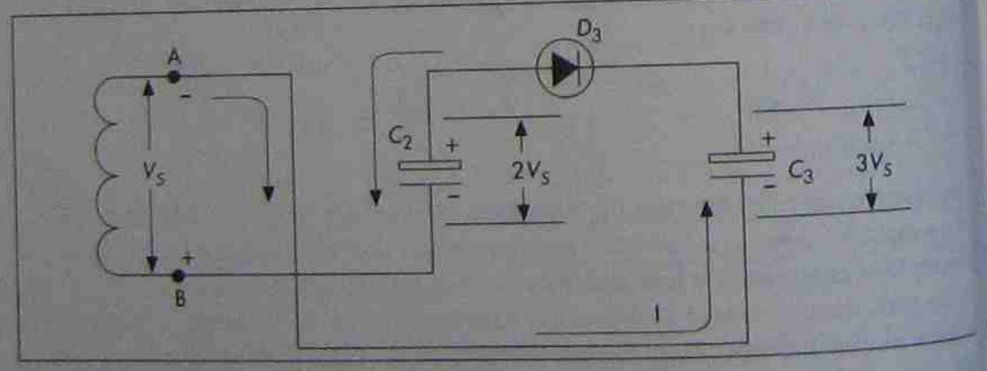


Figure 5.8: V_s in series with V_{C2}

to three times V_s. At the same time, diode D_1 (as shown in Figure 5.7) is forward biased due to the negative polarity at point A, and capacitor C_1 recharges to V_s. This prepares it for the next positive half cycle.

The practical approach is to charge the third capacitor to twice V_s and tap the triple voltage off across two capacitors in series ($V_s + 2V_s = 3V_s$). Figure 5.9 shows this type of circuit arrangement.

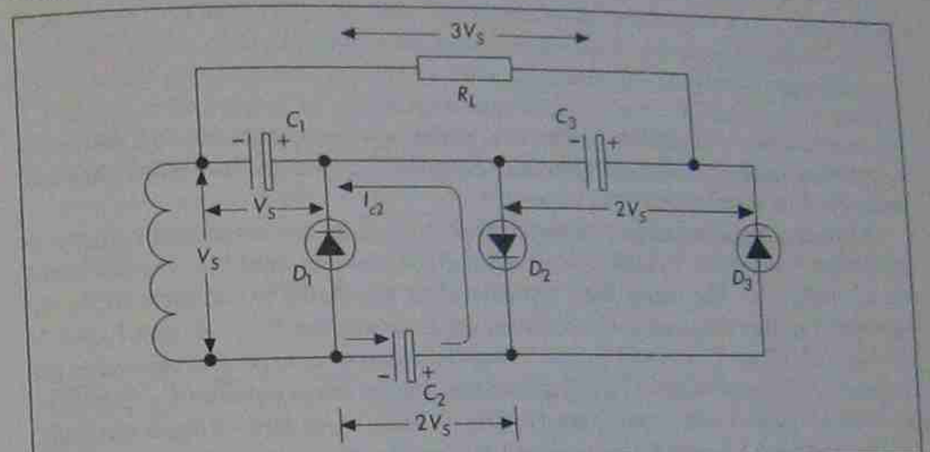


Figure 5.9: Practical voltage tripler circuit

On the negative half cycle of input voltage, capacitor C_1 charges to V_s peak through diode D_1. Capacitor C_2 charges to twice the peak voltage ($V_s + V_{C1}$) during the positive half cycle of input voltage. During the next negative half cycle D_3 conducts, and the voltage across C_2 charges C_3 to the same value of twice V_s, as shown in Figure 5.10. At the same time D_1 is again forward biased, and capacitor C_1 is recharged.

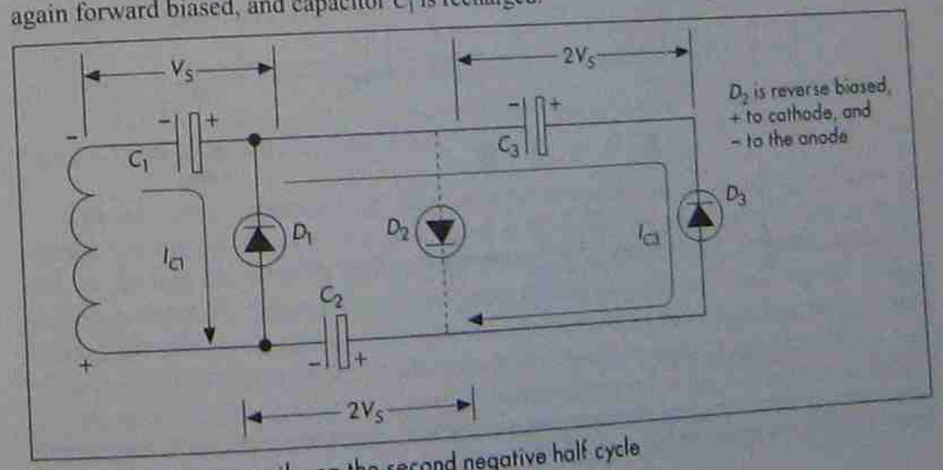


Figure 5.10: Current paths on the second negative half cycle

In summary then, after switch-on, D_1 conducts on the first negative half cycle charging C_1 to V_s. On the first positive half cycle, D_2 conducts. The charge in C_1 is in series with V_s and C_2 charges to twice V_s. During the second negative half cycle, D_1 conducts 'topping up' capacitor C_1, diode D_3 conducts and C_3 charges C_3 to twice V_s. The output voltage is taken across capacitors C_1 and C_3 in series, and it equals three times the input voltage.

If V_s = 240 V AC, then the peak value is 339.4 V.

$$V_{out} = V_{C1} + V_{C3}$$
$$= 339.4 + 678.8$$
$$= 1018.2 \text{ V} = 3V_s$$

Let us now follow a bit further the operation of the voltage tripler circuit just described. Assume the first complete cycle of operation has passed. Capacitor C_1 charges to V_s, and this series-aids V_s to charge capacitor C_2 to twice V_s.

At the commencement of the next negative half cycle, point A is negative with respect to point B (see Figure 5.11). Two things now happen simultaneously: diode D_1 is forward biased and C_1 'recharges'. The charge from capacitor C_2 is transferred to capacitor C_3 in the following way. (For the purpose of explanation, let us assume that V_s = 1 V; then V_{C1} = 1 V and V_{C2} = 2 V.) Point C (anode of D_2) is 1 V positive with respect to point A because of the charge in C_1. The input voltage V_s is series-aiding the voltage across capacitor C_2. Therefore, point D is 3 V positive with respect to point A. The 3 V on D_3 anode forward biases the diode, and therefore point E is also at a potential of 3 V. Capacitor C_3 now has 1 V at point C and 3 V at point E. It will charge to the difference (3 – 1 = 2 V) or the equivalent to twice V_s. As current is drawn by the load, the voltage across C_1 drops slightly, forward biasing D_1 and allowing it to conduct. This recharges C_1.

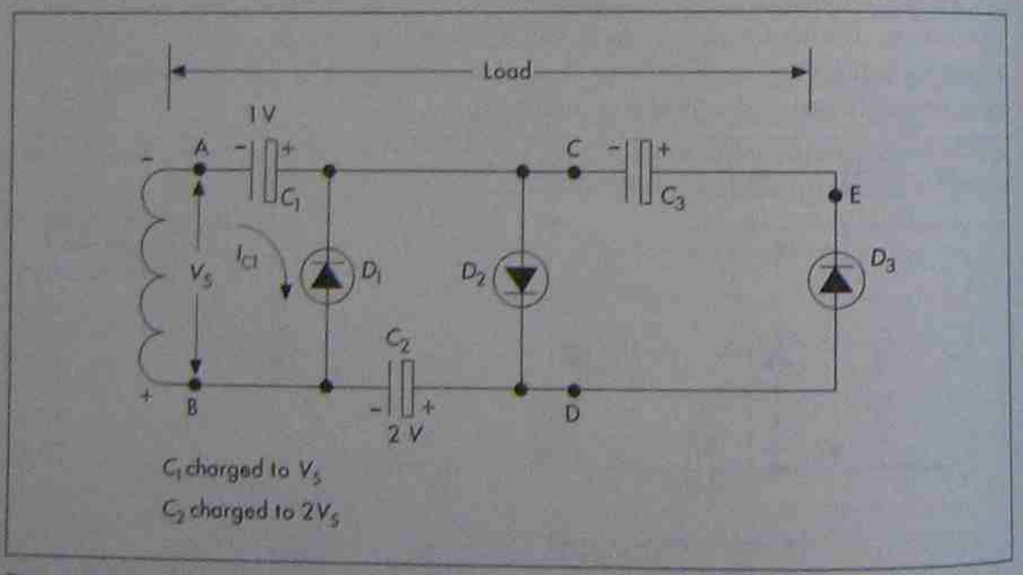


Figure 5.11: Second negative half cycle of input

It takes one cycle of operation (approximately 20 ms) to set the circuit up, that is, C_1 charges to V_s and C_2 charges to twice V_s. After that, diodes D_1 and D_3 conduct on the negative half cycle of input, charging C_1 and C_3. Diode D_2 conducts on the positive half cycle of input, charging C_2.

5.4 Voltage quadruplers

If we follow even further the operation of the voltage tripler circuit just described and assume the first complete cycle of operation has passed. Capacitor C_1 charges to V_s, and this series-aids V_s to charge capacitor C_2 to twice V_s. It is now apparent that by means of the addition of a diode/capacitor network we can charge a capacitor to twice V_s. An output of any multiple of V_s can therefore be obtained by tapping the voltage off selected capacitors in series. Figure 5.12 shows the circuit extended to give an output of four times V_s, that is, a **quadrupler**.

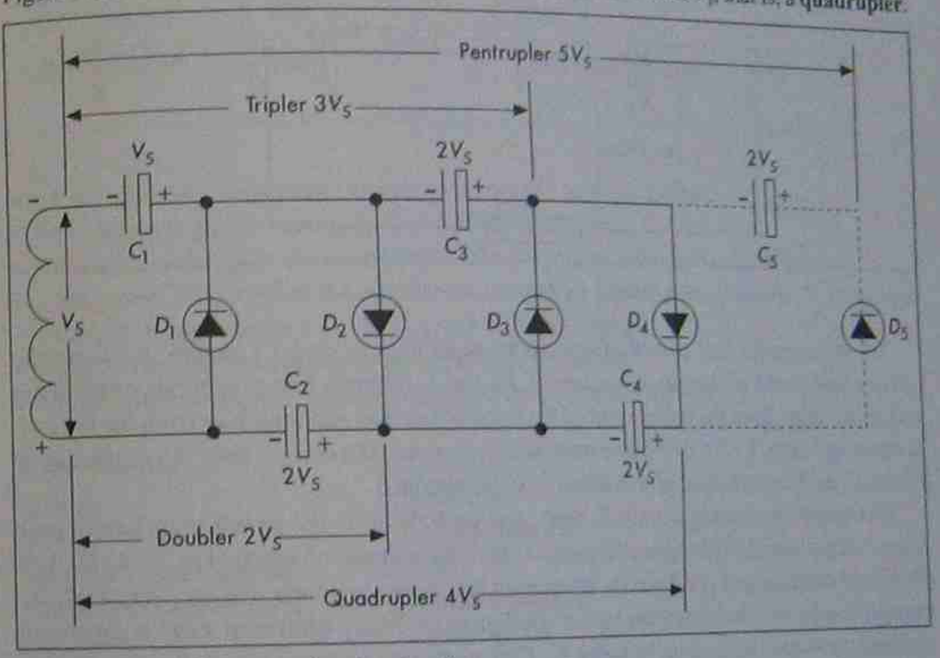


Figure 5.12: A voltage quadrupler circuit

The full circuit operation for the quadrupler is as follows. In operation, capacitor C_1 charges through diode D_1 to the peak value of V_s during the negative half cycle of input voltage. During the positive half cycle of input, V_{C1} is series-aiding V_s, and capacitor C_2 charges to twice V_s. Diode D_3 conducts during the next negative half cycle, and the voltage across capacitor C_2 charges capacitor C_3 to the peak value of twice V_s. Capacitor C_1 is also 'topped up'. On the next positive half cycle, diodes D_2 and D_4 conduct with capacitor C_3, and they charge capacitor C_4 to twice V_s peak. This area of the circuit operation is shown in Figure 5.13. Its behaviour is exactly the same as the transfer of voltage from C_2 to C_3 in the tripler circuit. However, the input voltage is now series-aiding the voltage across C_1 and C_3. Point B

is at a potential of $V_s + V_{C1} + V_{C3} = 4V_s$. This forward biases D_4, and therefore point C is also at four times V_s. Point D is at a potential of twice V_s because of the charge in C_2 and C_4 charges to the difference of $4V_s - 2V_s = 2V_s$.

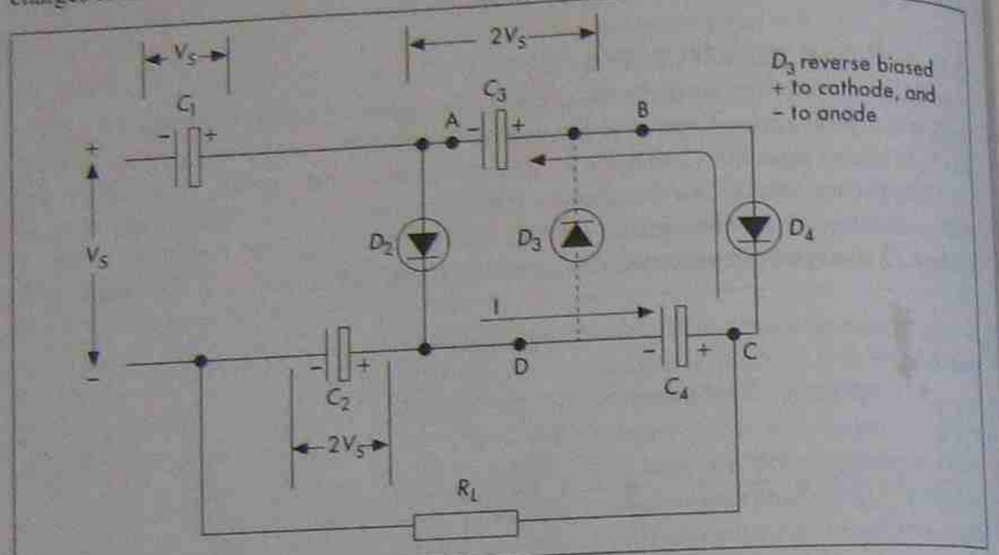


Figure 5.13: C_3 charging capacitor C_4 to $2V_S$

On the negative half cycles of operation, capacitors C_1 and C_3 charge, while on the positive half cycles of operation, capacitors C_2 and C_4 charge. Even multiples of the input voltage are taken from the bottom end of the transformer and odd multiples from the top end, as shown in Figure 5.12. One further stage is shown within the dashed lines. The inclusion of D_5 and C_5 enables an output of five times V_s to be obtained.

The transformer rating is only V_s peak, and each diode in the circuit must have a peak inverse voltage equal to two times the peak value of the source voltage, that is, $2 \times V_s\sqrt{2}$. With small load currents and good quality capacitors that have very little leakage, very high DC voltages can be obtained from this circuit configuration. Every additional stage is capable of stepping the output voltage up by twice V_s. Care must also be taken when choosing the DC **working voltage (DCWV)** of the capacitors used in voltage multiplier circuits. They are often charged to two times the peak input voltage.

5.5 Features of voltage multipliers

A table is a good way to obtain an overall view of voltage multipliers, their features and component requirements. Table 5.1 shows a brief comparison between four basic circuits.

5.6 Summary

The following items are a summary of the major points covered in this chapter:

- Step up the DC output voltage without using a step-up transformer.

Table 5.1: Comparison of voltage multiplier features

Circuit type	Half-wave doubler	Full-wave doubler	Half-wave tripler and quadrupler
Ripple frequency	Same as V_s	Twice V_s	Same as V_s
PIV of diodes (minimum)	$2 \times V_S\sqrt{2}$	$2 \times V_S\sqrt{2}$	$2 \times V_S\sqrt{2}$
Capacitor working voltage (minimum)	1@ $2\ V_S$ 1@ $2 \times V_S\sqrt{2}$	Both $V_S\sqrt{2}$	1@ $\sqrt{2} \times V_S$ others $2 \times V_S\sqrt{2}$
Output voltage (approx)	$2 \times V_S\sqrt{2}$	$2 \times V_S\sqrt{2}$	$\sqrt{2} \times 3\ V_S$ $\sqrt{2} \times 4\ V_S$
Current capability	Low	Low	Low
Voltage regulation	Poor	Better than half-wave doubler	Poor

- Each capacitor's operation working voltage has to be considered.
- Minimum capacitor charge is V_s and maximum is twice V_s.
- Capacitors charge to the peak value of the secondary voltage, that is, $\sqrt{2}\ V_s$.
- Maximum output when the output time constant is long.
- Output voltage can be ×2, ×3, ×4 or ×5 the input voltage.
- Full-wave operation provides better voltage regulation.
- Main disadvantage of voltage multipliers is low current capability.

Key terms

doubler
DCWV
full-wave voltage doubler
half-wave voltage doubler
peak valve
quadrupler
ripple frequency
tripler
voltage multiplier
voltage tripler

5.7 Review questions (answers in Appendix A)

1. Why are voltage doublers and triplers required?
2. With the aid of diagrams, explain the operation of a full-wave voltage doubler circuit.
3. What would be the specifications of the following for a half-wave voltage quadrupler circuit?
   (a) PIV (minimum)
   (b) DCWV of the capacitors
   (c) ripple frequency
4. Draw the circuit of a multiplier that will step up the input voltage by six times. Explain how each capacitor obtains its charge.

e. What is the main disadvantage of a multiplier circuit?
f. What is a practical application for a voltage multiplier?

Multiple-choice questions

1. Voltage multiplier circuits produce a high DC voltage from an AC source
   (a) with the use of a step-up transformer
   (b) without the use of a step-up transformer
   (c) by special circuit arrangement of inductors and diodes
   (d) by special circuit arrangement of capacitors and inductors
2. An AC voltage of 120 V fed to a half-wave doubler will produce a DC output voltage of approximately
   (a) 120 V
   (b) 214 V
   (c) 169 V
   (d) 32 V
3. The ripple frequency seen on an oscilloscope from a half-wave doubler operating at 2 kHz is
   (a) 30 Hz
   (b) 1 kHz
   (c) 2 kHz
   (d) 4 kHz
4. The minimum peak inverse voltage rating for diodes used in any voltage doubler is
   (a) 2 × V_p (peak)
   (b) 2 × V_p (RMS)
   (c) V_p (peak)
   (d) V_p (RMS)
5. The output of a full-wave voltage doubler operating from a 240 V AC mains is approximately
   (a) 480 V
   (b) 520 V
   (c) 340 V
   (d) 470 V
6. The DCWV of capacitors used in a full-wave doubler circuit must have a minimal rating of
   (a) V_p (RMS)
   (b) 2 × V_p (peak)
   (c) 2 × V_p (RMS)
   (d) V_p (peak)
7. If a voltage tripler was supplied from a 240 V AC source, the DCWV of the capacitor in the final diode/capacitor section should be a minimum of
   (a) 340 V
   (b) 480 V

   (c) 680 V
   (d) 1020 V
8. The output ripple frequency from a full-wave doubler operated from the 50 Hz mains will be
   (a) 50 Hz
   (b) 100 Hz
   (c) 150 Hz
   (d) 200 Hz
9. The output ripple frequency from a half-wave tripler circuit operated at 10 kHz will be
   (a) 10 kHz
   (b) 20 kHz
   (c) 45 kHz
   (d) undetermined
10. The current capability and voltage regulation of a full-wave doubler compared to a half-wave doubler is:

Current capability	Voltage regulation
(a) high	poor
(b) low	better than half-wave
(c) better than half-wave	good
(d) low	good

CHAPTER 6

SWITCHED MODE POWER SUPPLIES

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- List the advantages of a switched mode power supply (SMPS) over a linear regulated supply.
- State the main disadvantage of an SMPS.
- Explain the 'special components' required for an SMPS in terms of transformers, diodes and capacitors.
- Sketch the circuit and explain the operation of a step-down or buck SMPS, a step-up or boost SMPS, and an isolated output SMPS.
- Calculate the output voltage from each type of SMPS from given data.
- Explain the effect on the output voltage of varying the driver mark-to-space ratio.
- Discuss why soft-start circuits are found in SMPS circuits.
- Explain how over-voltage protection is provided.
- Explain how current limiting is achieved.
- Show how primary/secondary isolation is maintained in the SMPS.
- Explain why a radio frequency interference filter is found in SMPS circuits.

6.1 Introduction

The series pass transistor in a conventional series regulator operates as a variable resistance which keeps the output voltage constant despite changes in input voltage and load current. This transistor must be able to dissipate the power due to the voltage difference between the input and output voltages at the load current. If the input-output voltage differential is large, or if the input voltage has a high ripple content, the power dissipated can be excessive. This power is radiated as heat, and extensive heatsinking is therefore required (refer to Chapter 4).

During the past couple of years or so, another type of regulator circuit has arrived on the scene which overcomes these disadvantages. This new type of circuit is called either a **switched mode power supply** (SMPS) or a **chopper** power supply and can be encountered in either discrete or integrated format.

The switching mode type of regulator is basically a DC-to-DC converter that operates with a minimum number of components, and at high efficiency (it is often said that an SMPS is a power supply designed by an accountant). The transistor switches from saturation to cut-off, and therefore very little voltage is dropped across it. Saturation voltage across the transistor is very low, and the power dissipated by the transistor is greatly reduced. This is the main advantage over linear voltage regulators.

Heatsinking is kept to a minimum because the device runs much cooler than high current regulators using series pass transistors. High efficiency operation is maintained even with quite a large input-output voltage differential. In fact the voltage differential has very little effect on the efficiency. Additional advantages of this type of regulator include the fact that the output voltage can be stepped-up, stepped-down or inverted.

One of the disadvantages of switching regulators is that they are more complex than linear regulators. However, this increase in electronic complexity must be weighed against

the thermal and design complexity of high current linear regulators. A further disadvantage is higher output ripple amplitude, but this can be reduced with adequate filtering. Lastly, switching regulators throw current transients back into the unregulated supply which are somewhat larger than the maximum load current. These can be troublesome unless reduced by filtering.

(Note that basic circuit concepts used in this chapter are reproduced with the kind permission of National Semiconductor from their LM3524 *Pulse Width Modulator* publication.)

6.2 Switching versus linear regulators

Most power supplies operate from AC mains and supply a low voltage regulated DC. These power supply units perform three essential functions:

1. they reduce the level of the input voltage and provide isolation from the AC mains;
2. they provide a means of storing energy; and
3. they regulate or stabilise the output against variations in either input or load conditions.

Ideally, these functions are performed by a power supply that is small, light-weight and efficient. We will briefly consider each of these main functions in more detail.

Voltage reduction and isolation from the mains are performed by a transformer. The size, and hence the weight, of the transformer is inversely proportional to the operating frequency (within such design parameters as turns ratio, core material and losses). To keep the size down to a minimum, the transformer should operate at the highest possible frequency.

The reservoir or filter capacitor performs two functions: it stores energy and it filters the rectifier output voltage. The energy stored is proportional to $1/2\ CV^2$. Therefore, the higher the voltage level at which the energy can be stored, the smaller need be the size of the capacitor.

Regulation is the third essential function to be considered. Regulation was discussed at some length in Chapters 3 and 4. Both discrete and integrated approaches were presented. They both come into the category of 'linear regulators'. Regulation is obtained by having more power than the load requires and dissipating the excess energy in the form of heat. The physical size is large, due to the heatsinking requirements, and the efficiency is poor. Figure 6.1 shows a block diagram of the linear system.

In contrast, the power dissipated by the regulating device in a switching system is very small. Minimum, if any, heatsinking is required and, because there is negligible power loss, efficiency is high. However, due to the method of controlling the switching device, the average value of the output waveform has to be recovered by rectification and filtering. This results in double rectification, but the power dissipation is nevertheless reasonable and, overall, the switching system is smaller and more efficient. Figure 6.2 shows a block diagram of the switching system.

Having discussed the basic requirements for a power supply circuit, we can see that the 'ideal' system would have:

- a high frequency transformer;

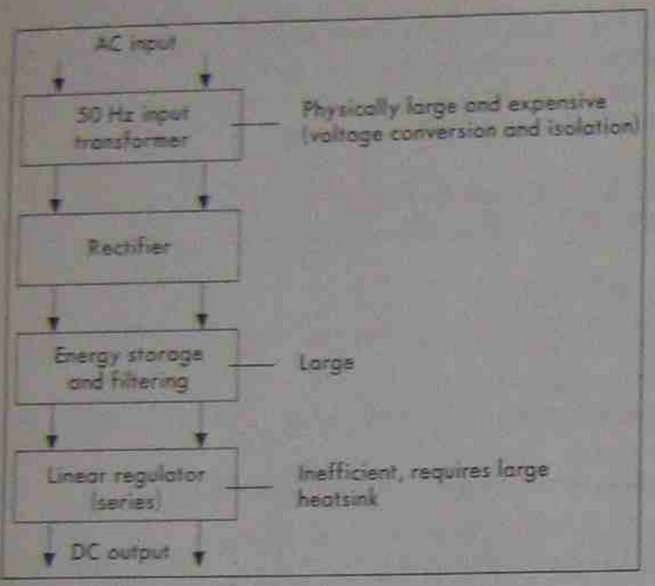


Figure 6.1: Linear regulator system

- a high voltage reservoir capacitor; and
- a switching regulator

If we compare Figures 6.1 and 6.2, it is interesting to note that the linear system is the exact opposite to the ideal system. The transformer operates at a low frequency (50 Hz) and is large and expensive. The reservoir capacitor is charged at a low voltage and is therefore quite large. Regulation is by energy dissipation and is very 'lossy'. Large heat-sinks are required and efficiency is low.

6.2.1 Further practical considerations

Low frequency transformers are readily available off the shelf and, therefore, another approach would be to use such a transformer followed by a switching regulator. Switching the current-controlling device limits the dissipation stresses and widens the choice of devices available. The **efficiency** of the system would be improved, but it would still require a large transformer and filter capacitor. Either thyristors or triacs may be considered as switching devices, but these are not capable of operating at the same high frequencies as transistors. Size advantage is lost, and a further disadvantage is the slower response to transients. In low frequency design where good transient response is required, it is common practice to use a switching preregulator followed by a linear regulator. In such circuits the switching preregulator keeps the voltage across the linear regulator at a low constant value, irrespective of other circuit variations. This results in the reasonable efficiency and good transient response of a linear regulator.

The choice of the 'type' of output capacitor is also important. No capacitor is entirely free of resistance, but the type of capacitors used in switch mode power supplies (SMPS) must exhibit a low **equivalent series resistance** (ESR). Normal electrolytic capacitors may not be suitable. The equivalent series resistance is determined by the construction techniques and such capacitors are more expensive than the conventional type that have a much higher series resistance.

Generally, ordinary power diodes are also not suitable; the type used in an SMPS must have fast switching characteristics. They are called **Schottky diodes**, and they also have a low forward voltage drop as a bonus.

6.2.2 Basic switching regulator concept

The basic operation of a switching regulator can best be understood with the aid of Figure 6.3. Transistor Q_1 is the switching element which is turned on and off by a pulse waveform with a fixed duty cycle (on/off period). The diode D provides a continuous path for the inductor current when the transistor turns off. The voltage waveform on the emitter of the

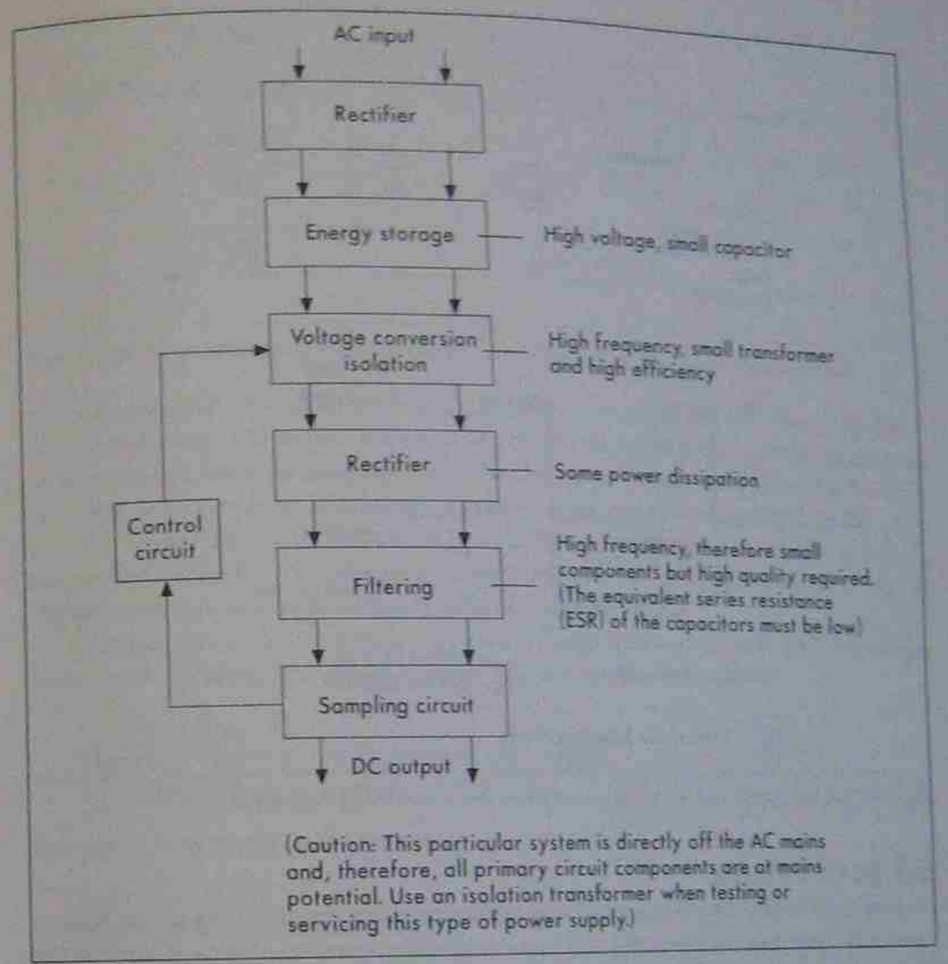


Figure 6.2: Switching regulator system

transistor (at point X) will be as shown in Figure 6.3(b). Neglecting any voltage drop across the transistor and the diode, the output of the LC (inductor-capacitor) network will be the average value of the switched waveform.

$V_{out} = V_{in} \times (t_{on} / T)$

where T = total period

t_{on} = time 'on'

V_{out} is independent of the load current

It can be seen from the equation above that any changes in the input voltage (V_{in}) can be compensated for by varying the duty cycle of the switched waveform. This is what happens in a switching regulator circuit and forms the basis of its operation. The control pulses are referred to as **pulse-width modulated** (PWM).

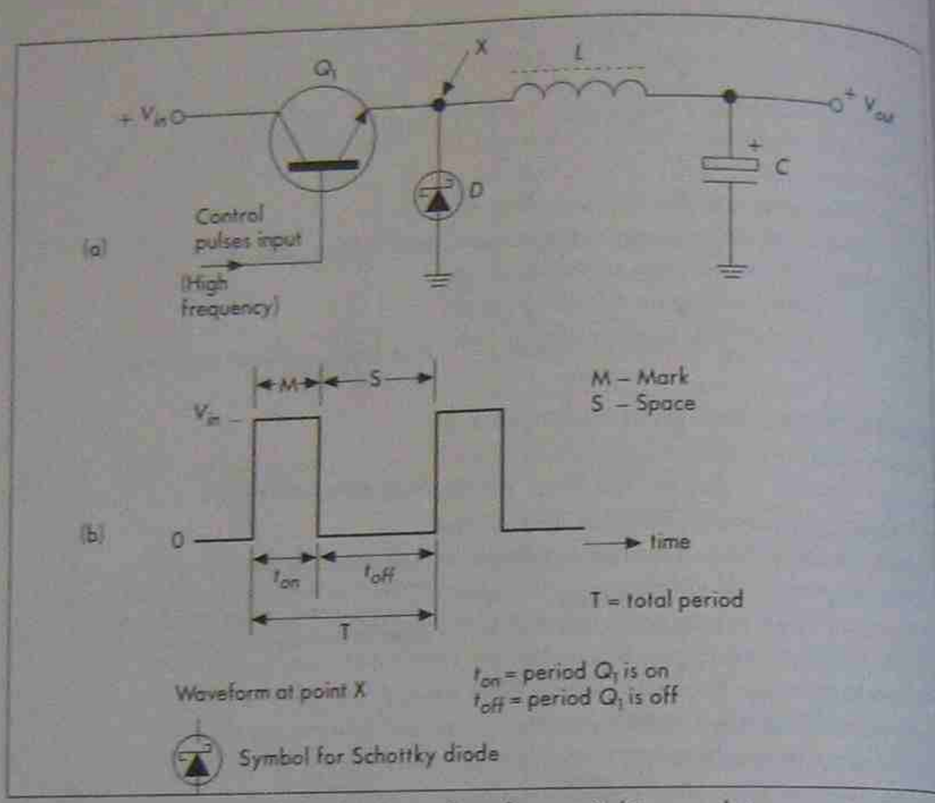


Figure 6.3: Basic circuit and idealised waveform for a switching regulator

6.3 Basic circuits

As mentioned earlier, the advantages of a switching regulator circuit are its ability to step-down, step-up or invert the input. We will now examine the operation of each of these circuit configurations in detail.

6.3.1 Basic step-down switching regulators

The basic circuit of a **'step-down' switching regulator** (also referred to as a 'buck' regulator) is shown in Figure 6.4. Transistor Q_1 is the switching element which has the 'on' and 'off' times controlled by a pulse-width modulator. When Q_1 is on, power is drawn from V_{in} and supplied to the load. Current passes through the inductor L, and point V_A is at approximately the potential of V_{in}, that is, Q_1 is saturated. This positive potential on D_1 cathode reverse biases the diode, and the output capacitor (C_o) is charging. When Q_1 turns off, the magnetic field stored in L collapses. The polarity of the voltage across L reverses, forcing point V_A negative, and forward biasing the diode D_1. Current continues through the load and the inductor via the diode.

The control circuit 'samples' the output voltage and automatically increases the 'on' time

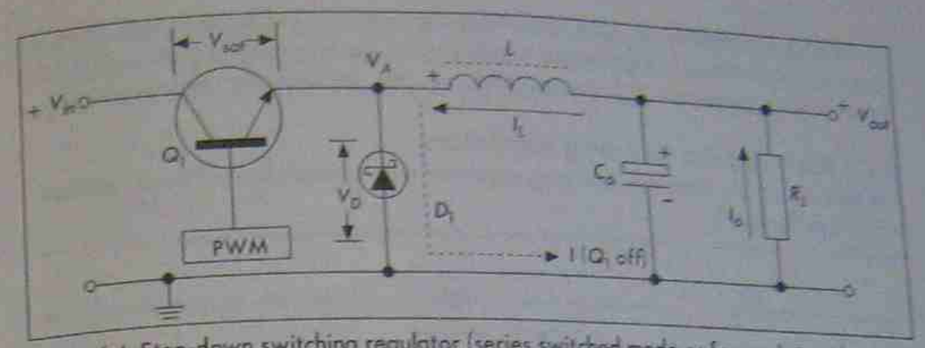


Figure 6.4: Step-down switching regulator (series switched mode or forward circuit)

of the transistor if the output decreases. Likewise, it increases the 'off' time if the output voltage increases. The output voltage is filtered by the LC network and maintained at an almost constant level. As the energy stored in the inductor is recovered during the transistor 'off' time when no current is being taken from the input, the circuit operates at high efficiency.

The current through the inductor is equal to the nominal DC load current plus a changing current that results from the changing voltage across the inductor. The peak-to-peak value of the changing current is approximately 40% of the output current.

This type of circuit can generate a relatively large amount of noise in the input line due to the rapid switching. The output voltage (V_{out}) is approximately equal to:

$$V_{out} = V_{in} - (V_{sat} + V_L)$$

where V_{sat} = saturation voltage across the transistor

V_L = voltage across the inductor

and can be determined by the formula:

$$V_{out} = V_{in} \times (t_{on}/T)$$

where T = total period for one cycle

t_{on} = period Q_1 is on

The voltage and current waveforms are shown in Figure 6.5.

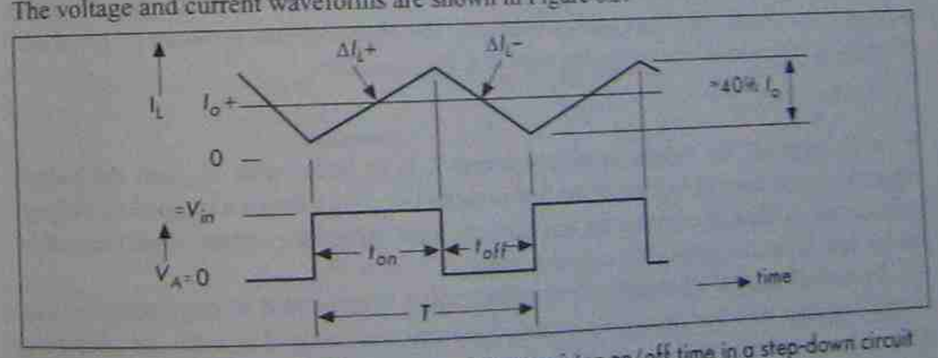


Figure 6.5: Relationship of inductor current to transistor on/off time in a step-down circuit

Efficiency approaches the ideal 100% when the transistor and diode voltage drops are small. The output ripple is dependent on the duty cycle, the frequency of switching, and the size of the output capacitor (C_o). The larger the capacitor, the lower the ripple amplitude.

6.3.2 Basic step-up switching regulators

The basic circuit of a **'step-up' switching regulator** (also referred to as a 'boost' regulator) is shown in Figure 6.6.

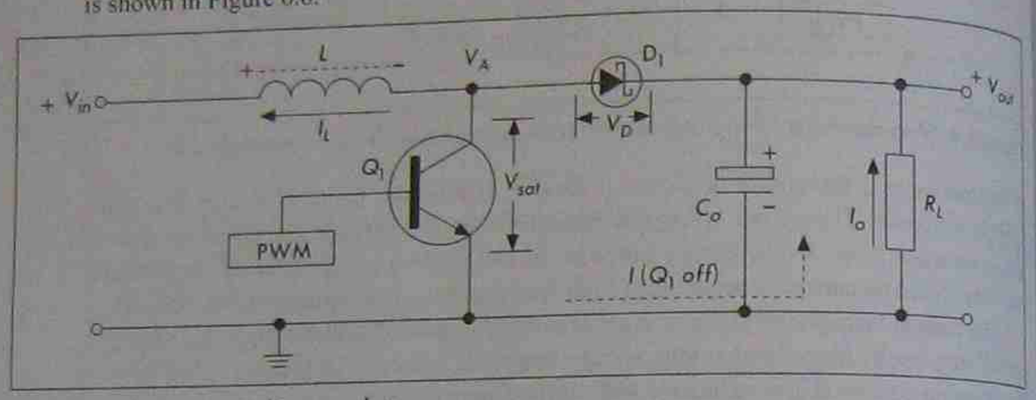


Figure 6.6: Step-up switching regulator

Assume the output capacitor (C_o) is charged. Transistor Q_1 is the switching element and is used to apply the input voltage across the inductor during the 'on' time. When Q_1 is on, energy is drawn from V_{in} and stored in the inductor. The diode is reverse biased because point V_A is forced less positive than V_{out}, and the output current is supplied to the load from the charge previously stored in the output capacitor (C_o). When Q_1 turns off, the voltage across the inductor reverses polarity due to the collapsing field. Point V_A is forced positive, which forward biases the diode, and the energy stored in the inductor will produce an output current. The current passes through the inductor L and the diode D_1 to the load, and replenishes any charge lost by the capacitor during the transistor's 'on' time. The control circuit automatically times the switching so that the 'on' time of D_1 is approximately equal to the Q_1 'on' time for normal operation. If the load resistor R_L decreases then the Q_1 'on' time increases to maintain the output voltage constant. The charging time constant (Υ) is short. When Q_1 is switched on, the energy is stored in L very quickly:

$$\Upsilon = L / (R \text{ of } Q_1)$$

When Q_1 turns off, the voltage developed across L is in series with V_{in}, and the output capacitor charges towards the sum of the two voltages ($V_{in} + V_L$); hence a stepped-up voltage at the output. In this type of circuit the input current must exceed the output current to enable energy transfer, and so step-up the output voltage.

This circuit has the advantage that very little noise is generated in the input line. The inductor isolates the input from the switching transistor and helps to smooth out switching noise. Also, current is present in the load circuit during both the 'on' and 'off' states of the transistor, again reducing the switching noise. Unfortunately, this makes output filtering more difficult. The voltage and current waveforms are shown in Figure 6.7. The relationship between input voltage, output voltage and duty cycle can be seen from the equation:

$$V_{out} = V_{in} [1 + (t_{on}/t_{off})]$$

where t_{on} = period Q_1 is on

t_{off} = period Q_1 is off

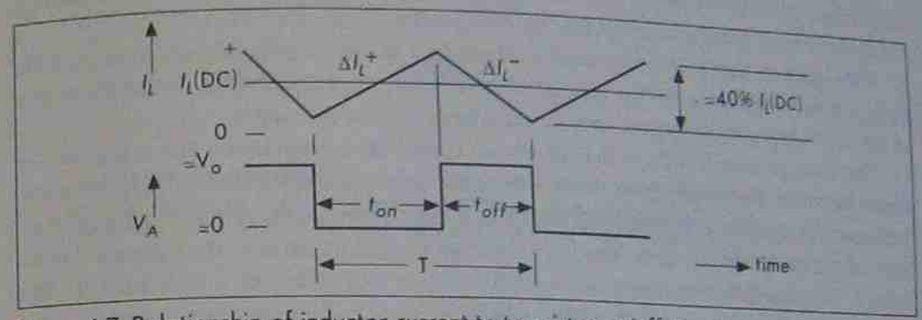


Figure 6.7: Relationship of inductor current to transistor on/off time in a step-up circuit

When the voltage drops across the transistor and the diode are small compared to V_{in} and V_{out}, the efficiency approaches 100%. Ripple can be reduced by using a large value of output capacitor.

6.3.3 Basic inverting switching regulators

The basic circuit of an **'inverting' switching regulator** (also referred to as a 'buck-boost' regulator) is shown in Figure 6.8. This circuit is also known as a shunt switched mode circuit or **'flyback'** type circuit. It provides a negative output voltage from a positive input voltage. The magnitude of the output voltage may be either larger or smaller than the input, depending on circuit design. Transistor Q_1 is again the switching element, and applies the input voltage across the inductor during the 'on' time. When Q_1 is on, energy is drawn from the supply and stored in the inductor (L). When Q_1 turns off, the voltage across the inductor reverses due to

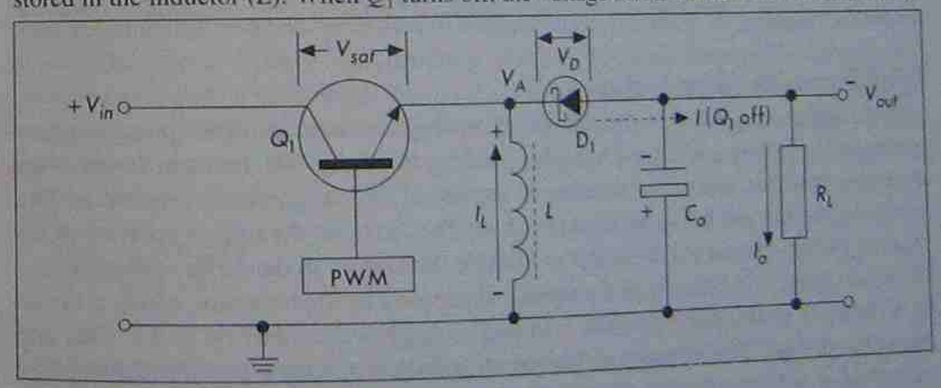


Figure 6.8: Inverting switching regulator

the collapsing field, and the diode is forward biased. Point V_4 is thus forced negative. The energy stored in the inductor produces an output current in the load in the opposite direction and charges the output capacitor (C_o).

The control circuit automatically adjust the transistor's 'on' and 'off' time to produce an almost constant output voltage of the opposite polarity. In this type of circuit both the input and output circuits are quite noisy at the switching frequency. Normal electrolytics are insufficient for filtering due to their inductance at high frequencies, and therefore a small value capacitor is required to shunt the high frequencies.

Efficiency is high if the saturation voltage (V_{sat}) is low during the 'on' time and the voltage drop across the diode (V_D) is small when the energy is transferred to the load. The size of the output capacitor will again determine the amplitude of the ripple.

The shunt switched mode, or flyback, circuit is the only configuration that is suitable for **mains isolation**. Figure 6.9 shows the circuit modifications and waveforms to achieve mains isolation. The inductor is replaced by a transformer which isolates the primary and secondary circuit from each other. A turns ratio is selected which will increase or decrease the output voltage; also, multiple secondaries can be used to derive more than one output voltage. This configuration is very popular in colour television switched mode power supply units. However, to ensure isolation is maintained, feedback from the output to the control circuit is via an 'opto coupler'. When the transistor is on, the DC input voltage is fed to the transformer primary, and a linear rising current results. The time constant for the 'on' time is short and is determined basically by the resistance of the transformer primary winding. When the transistor switches off, the primary current falls to zero, and the voltages across the transformer windings change polarity (signs shown inside the circles in Figure 6.9(a)).

Diode D_1 in the secondary circuit is now forward biased, and current passes to the output capacitor and load. The secondary current falls linearly with time, dissipating the stored magnetic energy. The secondary time constant is long compared to the primary time constant. This means that the secondary current will not fall to zero before the transistor conducts again. The circuit operation then repeats itself.

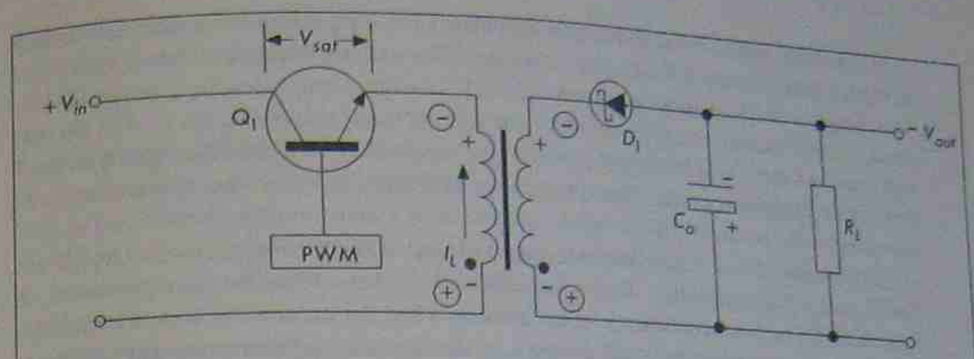
By choosing the phase relationship of the transformer windings and diode polarity, the output can be inverted or non-inverted. The switching transistor can also be reversed to operate on a negative DC input voltage; this applies to both series and parallel configurations.

6.3.4 Summary of operation

Both the series and parallel configurations of switched mode or 'chopper' power supplies (shown in Figures 6.4 and 6.8) use a single switching transistor. The transistor is one of the main sources of loss, and should therefore be capable of switching at high frequencies. This means that the rise and fall times should be short. The diode for the reverse recovery of the inductor's energy should also be as fast as possible (Schottky). In the series configuration a DC input voltage is obtained from the mains and chopped by the transistor, which is driven by a square or rectangular wave. The 'chopped' voltage is smoothed by an LC filter and passed to the load. A **commutating (or 'catch') diode** provides a path for the inductor current when the transistor is switched off. If the on/off time (often called the **mark-to-space ratio**) or duty cycle of the switching transistor is varied, the output voltage can be controlled over quite a wide range. The peak voltage handled by the transistor is equal to the input voltage, and if the mark-to-space ratio is 1:1, then the output voltage will be approximately one-half the input voltage. If V_{in} = 24 V, t_{on} = 1.0 µs and T = 2.0 µs (that is, mark-to-space = 1:1), then:

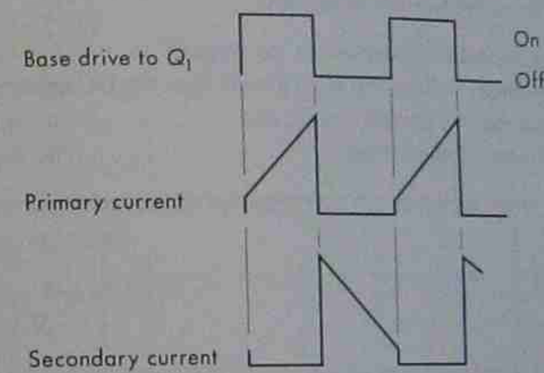
$$V_{out} = V_{in} \times (t_{on}/T)$$
$$= 24 \times 1/2$$
$$= 12 \text{ V}$$

In the shunt configuration, energy from the rectified AC supply is stored in the inductor when the transistor conducts. When the transistor is off, energy is transferred via the diode to the



(a) Circuit

(Note: With the isolated type of switching transformer, care must be taken with the design so that all the primary turns are 'mutually coupled' to the secondary turns, that is, no 'leakage inductance', as this puts extra stress on the chopper transistor at switch off.)



(b) Waveforms

Figure 6.9: Isolated parallel switching regulator and 'basic' waveforms

load. The output voltage will be the opposite polarity to the input unless transformer coupling is used. Again, the output voltage can be controlled by varying the mark-to-space ratio of the switching circuit. If the mark-to-space ratio is 1:1, then the output voltage will be equal to the input voltage, because energy is stored for one half cycle and released to the load on the next half cycle. As mentioned in Chapter 3, shunt type regulator circuits are more heavily stressed than series regulator circuits. This also applies to SMPS, but they have the advantage of providing mains isolation if the inductor is replaced by a transformer.

Finally, the choice exists between **forward** and **flyback** configurations. A forward SMPS circuit transfers energy to the inductor and the load when the transistor is on. A flyback SMPS circuit stores energy in the inductor when the transistor is on and transfers it to the load when the transistor is off. Figure 6.10 shows a graph of the general voltage-power limits within which each type of circuit is more efficient.

In the series step-down switched mode circuit, the inductor or 'choke' is connected in series with the load, and energy is transferred to the load while the transistor is 'on'. In the shunt switched mode circuit, the inductor is in parallel with the load. Energy is stored in the choke while the transistor is on, and delivered to the load when the transistor is off.

Advantages of the series type circuit include:

- switching that is limited to the transistors 'on' period;
- a peak collector-emitter voltage that is no greater than the DC input voltage;
- a smaller inductor than the parallel circuit; and
- a lower level of output ripple current.

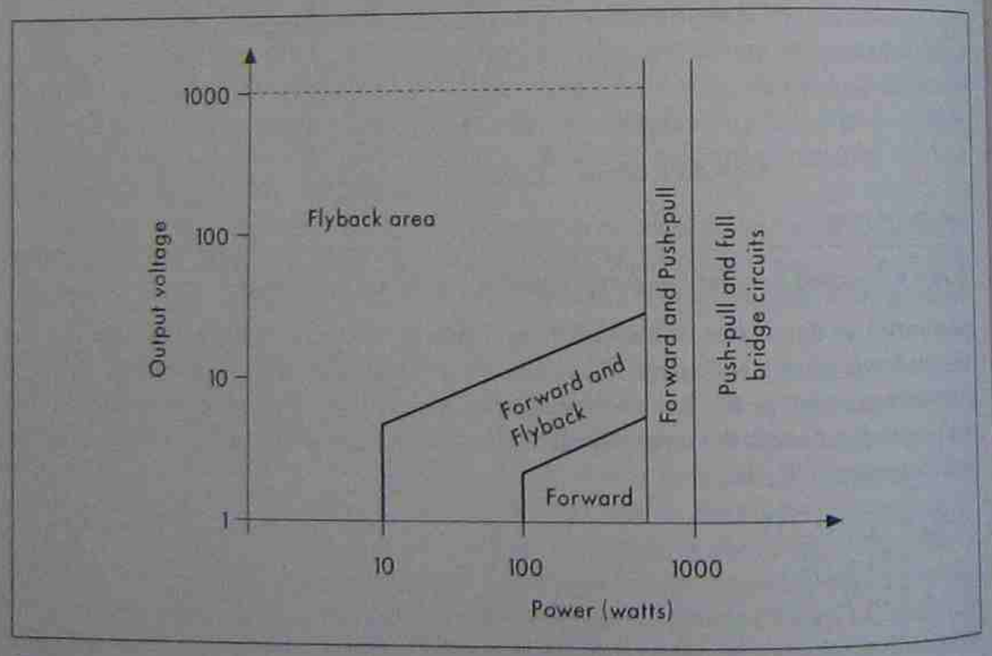


Figure 6.10: Voltage-power recommendations for different circuit configurations

The disadvantages include impossibility of obtaining mains isolation and a distinct possibility that the full input voltage will appear across the load if the switching transistor short circuits.

6.4 Design information

(Note that basic design information in this section is reproduced with the kind permission of National Semiconductor.)

When building a switched mode power supply, careful consideration must be given to the values of various circuit components. Different circuit configurations require slightly different components. This section will enable the reader to calculate the values of inductance and output capacitance required, as well as the approximate efficiency of the circuit.

6.4.1 Step-down regulators

The basic circuit and switching waveforms of a step-down regulator are shown in Figure 6.11. Neglecting V_{sat}, V_D and the settling current ($\Delta I_L \pm$), we can determine the output voltage from the formula:

$V_{out} = V_{in} \times t_{on}/(t_{on} + t_{off})$
which simplifies to $V_{out} = V_{in} \times (t_{on}/T)$
where T = total period
t_{on} = period Q_1 is on
t_{off} = period Q_1 is off

The formula shows the relationship between V_{in}, V_{out} and duty cycle. Transistor Q_1 only conducts during the 'on' time:

Efficiency = $V_{out} / (V_{out} + 1)$
for $V_{sat} = V_D = 1$ V

The maximum efficiency will decrease further due to switching losses for Q_1. The switching transistor should therefore have very fast rise and fall times for all circuit configurations.

Calculating value of inductor (L)

The mathematics is fairly complex, and for this reason the step-by-step derivation has been omitted.

$L = 2.5 \times V_{out} (V_{in} - V_{out}) (I_{out} \times V_{in} \times f)$
where L is in henrys
f is the switching frequency in hertz

Calculating value of capacitor (C_o)

Figure 6.11(b) shows the inductor's current with respect to the transistor's on/off time. This current path must be through the load and output capacitor (C_o). The current in the capacitor will then be the difference between the inductor current (I_L) and the output current (I_o):

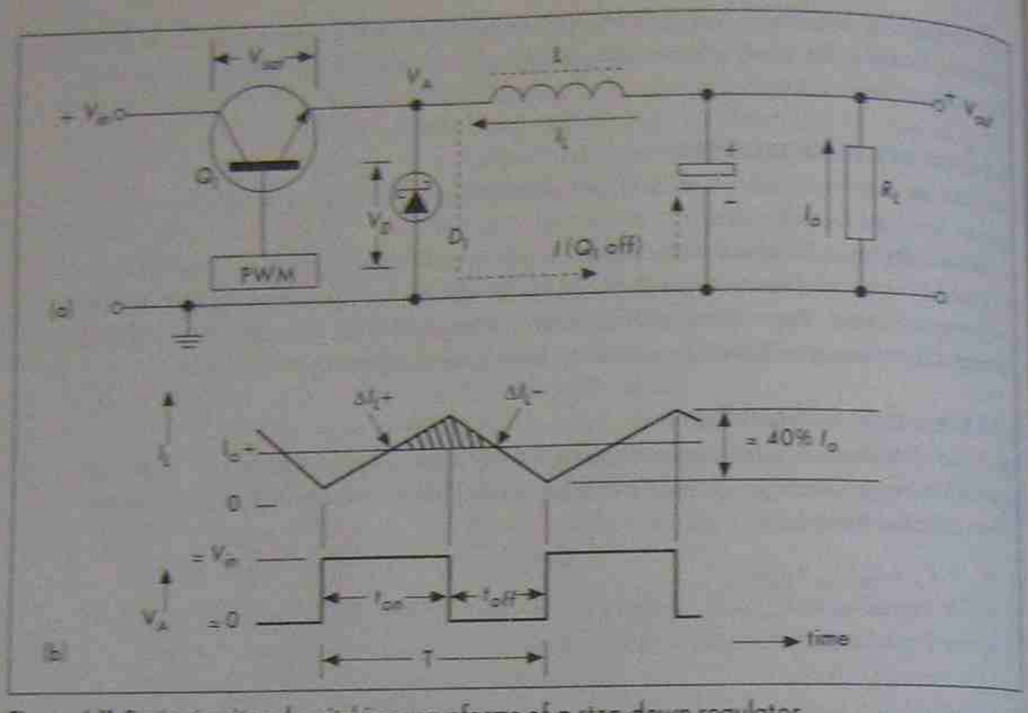


Figure 6.11: Basic circuit and switching waveforms of a step-down regulator

$$I_{Co} = I_L - I_o$$

From Figure 6.11(b) it can be seen that current will flow into C_o for the second half of t_{on} and for the first half of t_{off}; that is, $t_{on}/2 + t_{off}/2$ (the shaded portion of the waveform). The resulting current flow for this period of time is $\Delta I_L/4$.

Once again the step-by-step derivation has been omitted, and the formula for calculating the value of C_o is:

$$C_o = [(V_{in} - V_{out})V_{out}] \times T^2 / (8\Delta V_o \times V_{in} \times L)$$

where C is in farads
T is 1/switching frequency
ΔV_o is the peak-to-peak output ripple

6.4.2 Step-up regulators

The basic circuit and switching waveforms for a step-up regulator are shown in Figure 6.12. Neglecting V_{on} and V_D, and noting that $\Delta I_L+ = \Delta I_L-$, then:

$$V_{out} = V_{in}(1 + t_{on}/t_{off})$$

To calculate the input current I_{in} (DC), which equals I_L (DC), 100% efficiency must first be assumed:

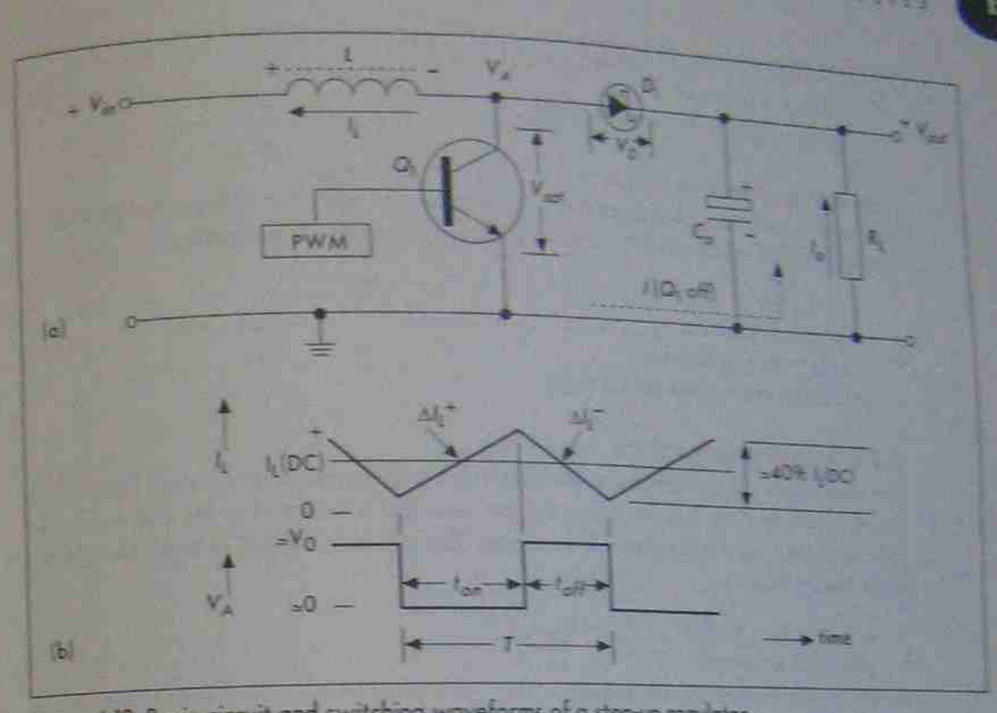


Figure 6.12: Basic circuit and switching waveforms of a step-up regulator

$$P_{in} = I_{in}(DC) \times V_{in}$$
$$P_{out} = I_{out} \times V_{out}$$
$$= I_{out} \times [V_{in}(1 + t_{on}/t_{off})]$$

where P_{in} is input power
P_{out} is output power

If efficiency is 100%:

$P_{out} = P_{in}$, so
$I_{out} \times [V_{in}(1 + t_{on}/t_{off})] = I_{in}(DC) \times V_{in}$
$I_{in}(DC) = I_{out}(1 + t_{on}/t_{off})$

This shows that the input, or inductor current, is larger than the output, as stated earlier, by the factor $(1 + t_{on}/t_{off})$. This is the same relationship as V_{in} and V_{out}:

$V_{out} = V_{in}(1 + t_{on}/t_{off})$
Efficiency = P_{out}/P_{in}
Maximum efficiency = $V_{in}/(V_{in} + 1)$
for $V_{out} = V_D = 1$ V

Calculating value of inductor (L)

Again, the final formula, without its derivation, is given:

$L = 2.5 \times V_{in}^2 \times (V_{out} - V_{in}) / I_{out} \times V_{out}^2 \times f$
where L is in henrys
f is the switching frequency in hertz

Calculating value of capacitor (C_o)

The output capacitor (C_o) supplies current during the 'on' time. The voltage change on C_o during this time will be equal to the value of the output ripple of the regulator:

$C_o = I_{out} \times (V_{out} - V_{in}) / (\Delta V_o \times V_{out} \times f)$
where C_o is in farads
f is the switching frequency
ΔV_o is the peak-to-peak output ripple

6.4.3 Inverting regulators

The basic circuit and switching waveforms of an inverting regulator are shown in Figure 6.13. Energy builds up in the inductor during the 'on' time and is delivered to the load during the 'off' time. The circuit is therefore a flyback type. The values of L and C_o are calculated from the following formulae:

$L = 2.5 \times V_{in} \times V_{out} / [(V_{out} + V_{in}) \times I_{out} \times f]$
where L is in henrys
f is the switching frequency in hertz
$C_o = I_{out} \times V_{out} / [\Delta V_o \times f \times (V_{out} + V_{in})]$
where C_o is in farads
f is the switching frequency in hertz
ΔV_o is the peak-to-peak ripple frequency

The output voltage is dependent on V_{in}, the duty cycle and the inductor. Neglecting V_{sat} and V_D:

$V_{out} = V_{in} (t_{on} / t_{off})$

6.5 Control circuitry

The basic operation of switched mode power supply (SMPS) configurations has been discussed in some detail. All of the circuits use a form of automatic control to sample the output voltage and adjust the mark-to-space ratio of the control circuit accordingly. An overall block diagram of the switched mode system is shown in Figure 6.14.

The switching element is operated at a fixed high frequency (approximately 20 kHz). A variable mark-to-space ratio generator accepts this 20 kHz, and drives the switching element. The mark-to-space ratio is controlled by a DC level fed to the generator. This level is the amplified error signal obtained by comparing the output from the power supply with a DC reference voltage.

The oscillator, variable mark-to-space generator, comparator and DC amplifier all come under the broad heading of '**control circuitry**'. The control circuitry is what makes a switched mode supply more complex than a linear supply, and it can be found in both discrete

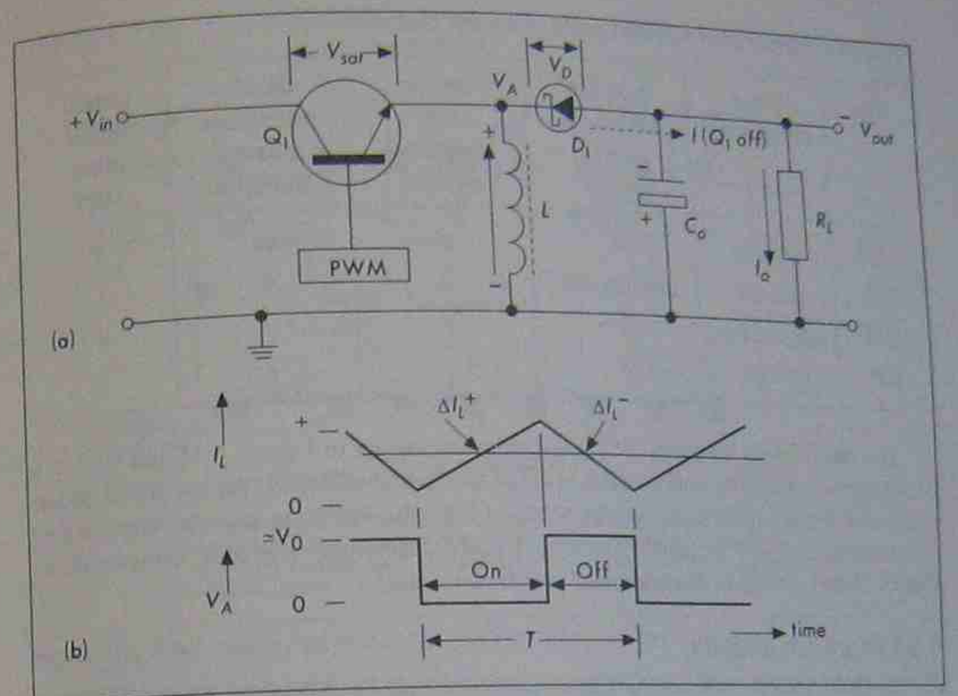


Figure 6.13: Basic circuit and switching waveforms of an inverting regulator

or integrated forms. With discrete control circuits, the oscillator is usually a transistor multivibrator or unijunction transistor. The comparator/DC amplifier is a transistor amplifier or differential pair, and the output circuit has series current limiting with **over-voltage protection**. There are many components in this type of circuit, but new technology has meant the whole lot can be incorporated in one integrated circuit. This has made switched mode power supplies a viable proposition by reducing their complexity. Many companies are producing switched mode control chips, and Table 6.1 lists the characteristics of a few of them.

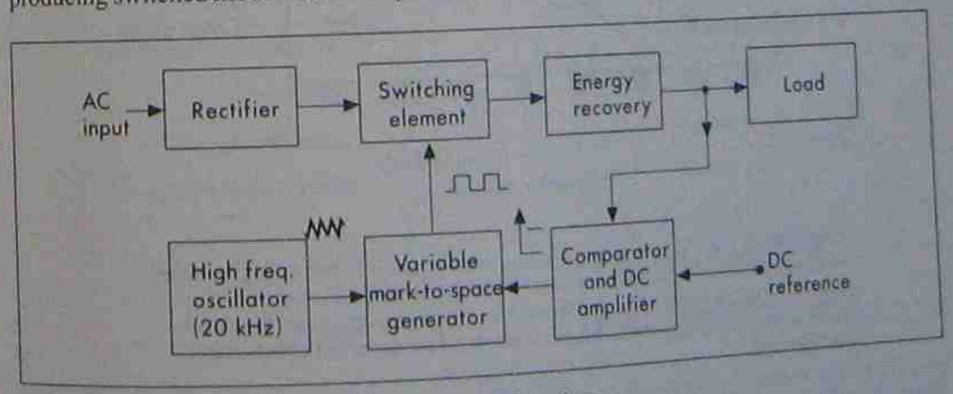


Figure 6.14: Block diagram of a complete SMPS system

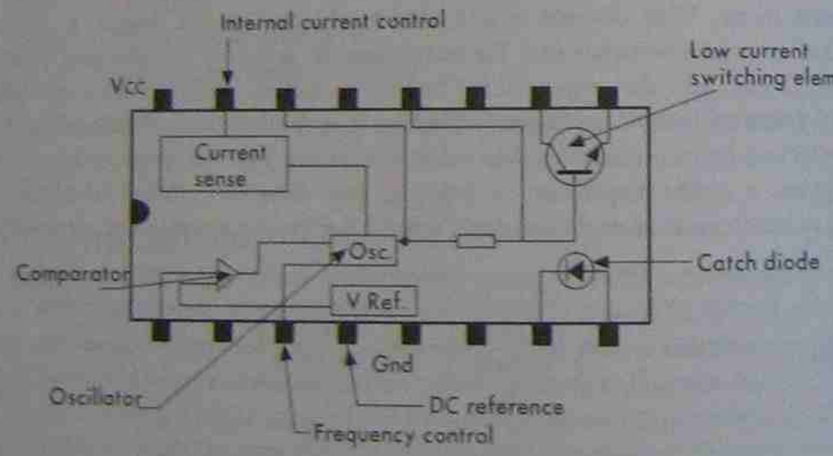
Table 6.1 Comparison of switched mode controller characteristics

Type	Output configuration	Accessible emitter and collector outputs	On-chip error operational amplifier	On-chip over-voltage or current protection	Current-limited output stage
494	push-pull	✓	✓	✓	X
3520	push-pull	collectors only	X	X	X
5560	single-ended	✓	✓	✓	✓
1524	push-pull	✓	✓	✓	X

Two basic 'block diagrams' of control chips are shown in Figures 6.15 and 6.16. It is impossible to cover every variation and make of switched mode controller integrated circuits in the text. Further information can be obtained from the following manufacturers or their representatives: Exar, Fairchild, Ferranti, Hitachi, Motorola, National Semiconductor, Plessey Ducon, Philips Electronics and Texas Instruments.

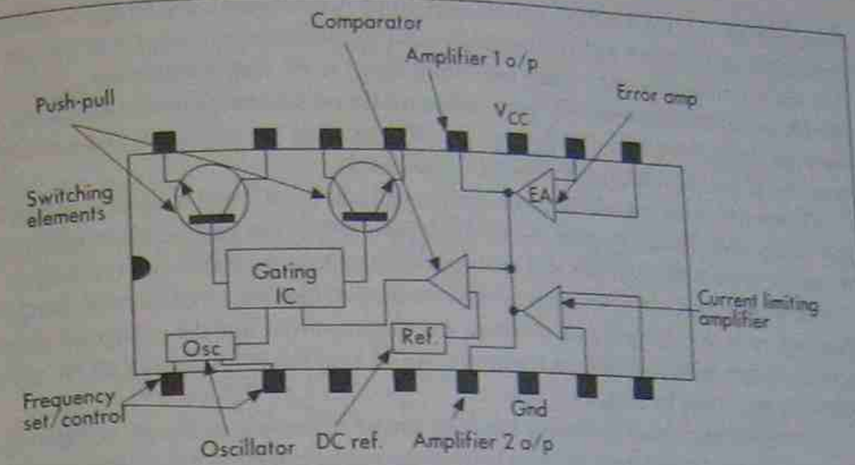
6.5.1 Practical circuits

(Note that the circuits in this section are reproduced with the kind permission of National Semiconductor. They all use the LM3524 controller integrated circuit.) We will now examine



Integrated circuit (IC) containing basic control circuitry and a single-ended switching element. Approximate output current is 200 mA and, if higher output current is required, an external transistor is used. Even the catch diode is incorporated inside this IC, and very few external components are required.

Figure 6.15: Basic integrated circuit switched mode controller



Integrated circuit containing the control circuitry. The switching output is push-pull and, if higher output current is required, an external transistor can be used. The oscillator frequency is set by an external control circuitry (CR) network, and may be as high as 100 kHz.

Figure 6.16: Basic integrated circuit switched mode controller

one example of the integrated circuit (IC) mentioned above. The National LM3524 is a typical integrated switched mode power supply controller. It is available in a dual-in-line package, and the pin connections are shown in Figure 6.17. The LM3524 integrated regulating **pulse-width modulator** contains all of the circuitry that is required to control a switched mode power supply. The device has a control amplifier, an oscillator, a pulse-width modulator, a phase-splitting flip-flop, dual-output transistors and a current-limiting shut down circuit. It also contains a built-in 5 V voltage regulator that has a current capability of

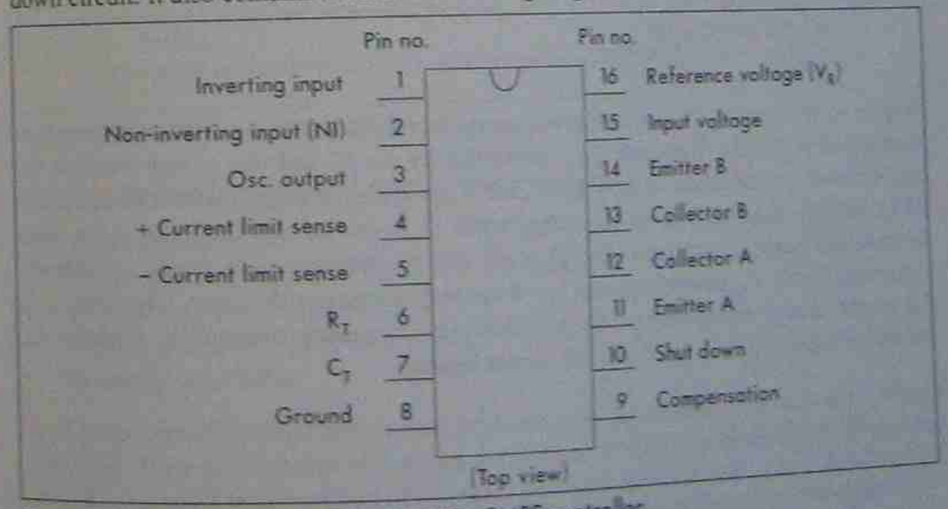


Figure 6.17: Pin connections of the LM3524 SMPS controller

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50 mA (at pin 16). The oscillator frequency is set by two external components: a resistor R_T and a capacitor C_T (pins 6 and 7). The output of the oscillator (pin 3) triggers an internal flip-flop, which provides the pulse-width modulated signal to the output transistors.

The output transistors are NPN types with a maximum current capability of 100 mA. The bases are driven internally, 180 degrees out of phase, but the collectors and emitters are accessible at pins 12 and 13, and 11 and 14 respectively.

The error amplifier has a differential input through pins 1 and 2, and the gain is set by a feedback voltage divider taken from the output. The error amplifier can be overridden by applying a DC voltage to pin 9 of the IC; this will reduce the duty cycle of the pulse-width modulator.

Likewise, the current-limiting circuit takes control of the error amplifier and changes the pulse duty cycle. A resistor is used to 'sense' the output current, and the voltage developed is applied between the ± current limit sense pins (4 and 5) of the IC. A voltage of 200 mV will reduce the duty cycle by 25%.

A complete step-down switching regulator circuit schematic is shown in Figure 6.18. Transistors Q_1 and Q_2 have been included to boost the output current to 1 A — normally 80 mA is available from the IC internal transistors. The output voltage is determined by the formula:

$$V_{out} = V_{NI}(1 + R_1/R_2)$$

where V_{NI} is the voltage at the error amplifier's non-inverting input

As can be seen in Figure 6.18, V_{NI} is set by voltage dividing the internal 5 V reference at pin 16 of the IC:

$$V_{NI} = 5 \times R_4/(R_4 + R_5)$$
$$= 5 \times 5 \times 10^3/10 \times 10^3$$
$$= 2.5\ V$$

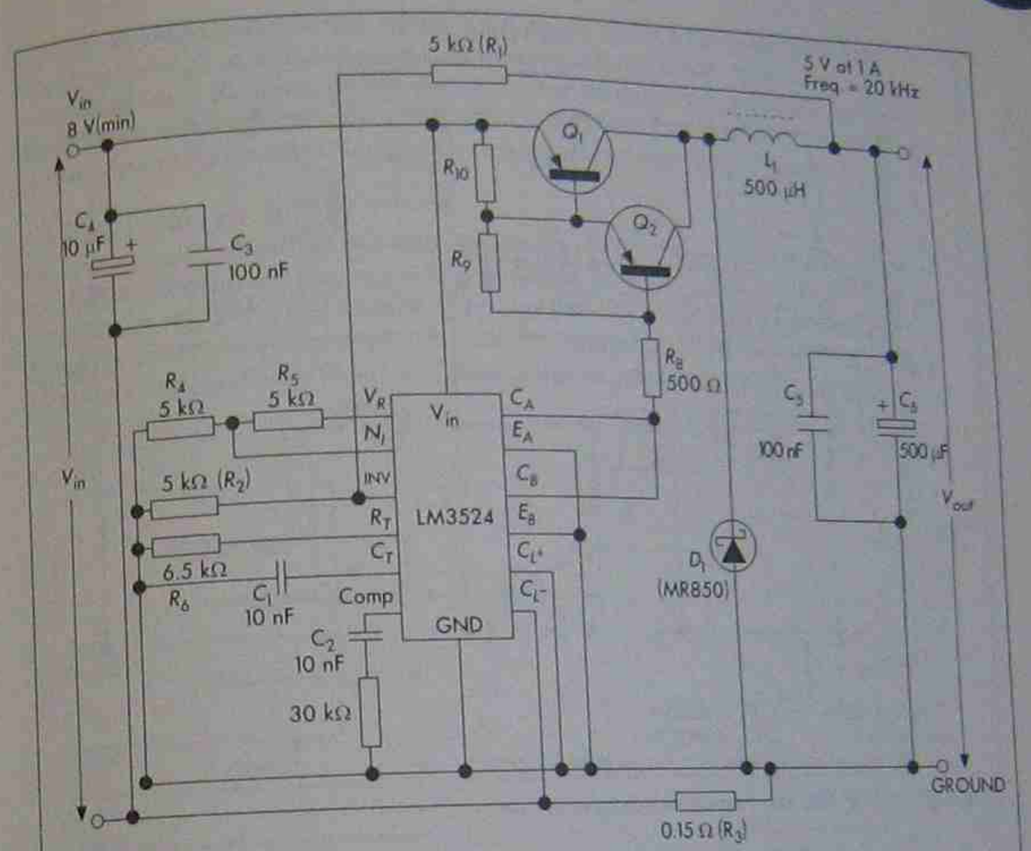
The inverting input (that is, the feedback voltage) is set by voltage divider R_1 and R_2:

$$V_{INV} = V_{out} \times R_2/(R_1 + R_2)$$
$$= 5 \times 5 \times 10^3/10 \times 10^3$$
$$= 2.5\ V$$

Resistor R_6 and capacitor C_1 are connected to pins 6 and 7 to set the oscillator frequency at approximately 20 kHz. Transistor Q_2 is controlled by the IC and this in turn controls the conduction of the series pass element Q_1 (R_9 and R_{10} provide the initial forward bias required for Q_1 and Q_2). Inductor L stores the energy while Q_1 is conducting. Diode D_1 is the catch diode for the output current path when Q_1 is off, and the magnetic field around L collapses. Capacitors C_3 and C_4 provide high and low frequency filtering at the input. Capacitor C_6 is the output storage capacitor and C_5 provides high frequency decoupling. Resistor R_3, connected between pins 4 and 5, sets the current limiting.

$$I = V_s/R_3$$
$$= 200 \times 10^{-3}/0.15$$
$$= 1.3\ A$$

Output duty cycle = 25%, when V_s = 200 mV



Q_1 = BD 344 (mounted on a small heatsink)
Q_2 = 2N5023
L_1 = Approx. 40 turns No. 22 wire on a Ferrox cube K300502 core

Figure 6.18: A 5 V, 1 A step-down switching regulator

This circuit should display the following characteristics:

- output voltage of 5 V;
- operating frequency of 20 kHz;
- short circuit current limit of 1.3 A;
- output ripple 10 mV peak-to-peak; and
- an efficiency of 80%.

A step-up switching regulator is our next example, again using the National LM3524 controller. The circuit schematic is shown in Figure 6.19, and this circuit steps an unregulated 5 V input up to 15 V at a current of 500 mA. This time the input voltage is divided into two to bias the error amplifier's non-inverting input, and the output voltage is:

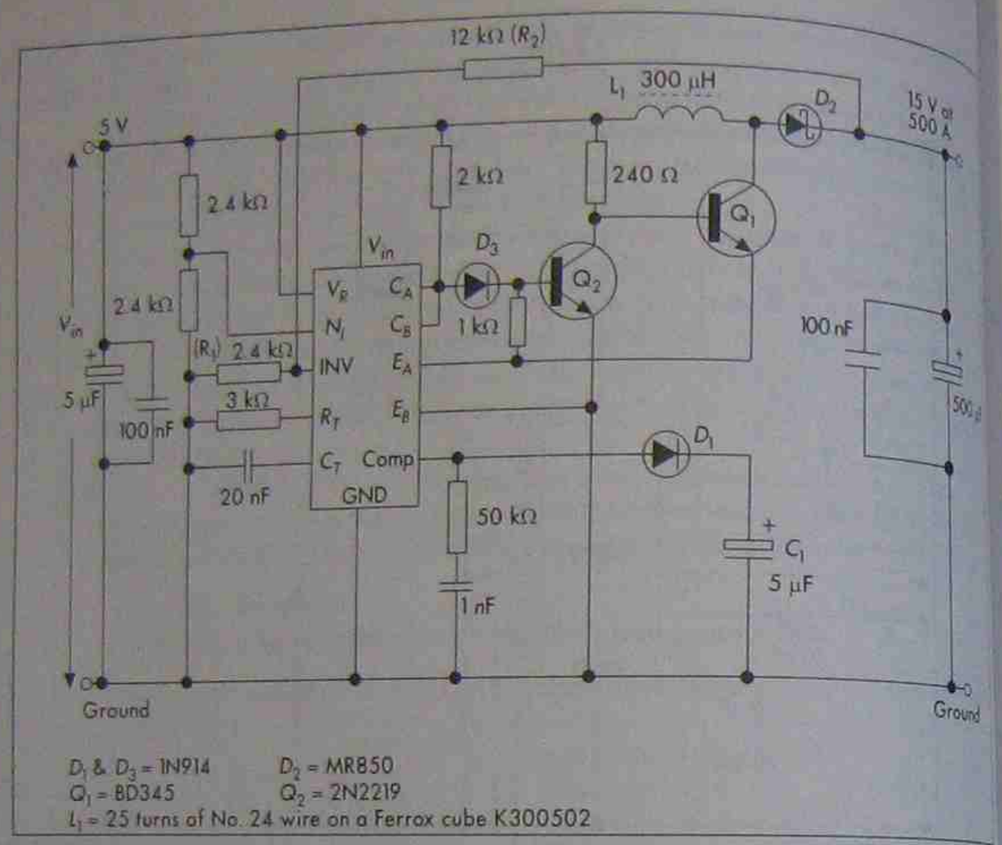


Figure 6.19: A 15 V, 500 mA step-up switching regulator

$$V_{out} = (1 + R_2/R_1) \times V_{im}$$
$$= (1 + 12 \times 10^3/2.4 \times 10^3) \times 2.5$$
$$= 15\ V$$

A new circuit concept is introduced here: a **slow- or soft-start circuit**. The D_1/C_1 network forms the slow-start circuit. At switch-on the circuit holds the output of the error amplifier low, reducing the duty cycle to a minimum. This prevents the inductor from saturating at switch-on. The output capacitor is initially discharged, and the high peak current required to charge it may cause the inductor to saturate; hence the precaution of a slow-start circuit. Soft-start circuits will be discussed in the next section.

6.6 Protection circuits

The switch-on surge in switched mode power supplies is usually contained to the first couple of charging current pulses. When the circuit is first switched on, the large value output capacitor has to be charged, and this causes the output voltage to be low. The comparator in the control circuit sees this as a drop in output voltage and therefore increases the duty cycle

to restore the output level. The high current that is present for the initial turn-on period may saturate the inductor or damage the switching element itself. The sudden increase in voltage could also overload the device, blow the fuse or damage the capacitor. It is desirable that the output voltage should rise gradually to the normal working level over a period of a couple of seconds. The circuit that controls this gradual rise is called a 'soft-start' or 'slow turn-on' circuit.

6.6.1 Soft-start circuits

The basic concept of all **soft- or slow-start circuits** is to hold the output of the error amplifier low, thereby keeping the duty cycle to a minimum. The duty cycle of the switching transistors, or element, is then allowed to increase gradually to the normal operating point during the circuit's power-up time. This function is particularly easy to implement with integrated circuit controllers. Figure 6.20 shows a typical circuit arrangement.

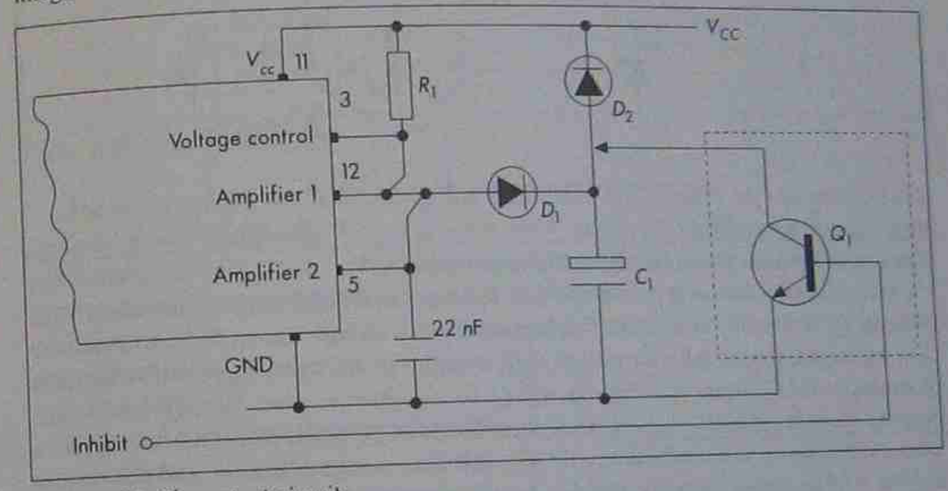


Figure 6.20: Slow-start circuit

During power-up, the voltage at pin 3 rises exponentially from zero volts towards the supply voltage V_{cc}, with a time constant of $R_1 \times C_1$. This allows a gradual increase in the duty cycle. The transistor Q_1 (inside the dotted area) can further extend the soft-start capability to provide an inhibit control, or a soft-start reset. When an inhibit command is received, Q_1 will shunt capacitor C_1, hence discharging it and reducing the voltages at pins 3, 5 and 12. Diode D_2 allows capacitor C_1 to reset (discharge) when the power is turned off.

Some integrated circuits have the soft-start feature incorporated in them. They only require an external capacitor for the time constant and a discharge resistor to reset the soft start when the power is turned off.

Another approach is to control the conduction of a transistor in a similar manner. The circuit in Figure 6.21 shows this type of operation. The slow-start circuit is connected to the emitter of transistor Q_1 making it conduct heavily to reduce the trigger pulse level and to allow the switched mode power supply output voltage to rise gradually.

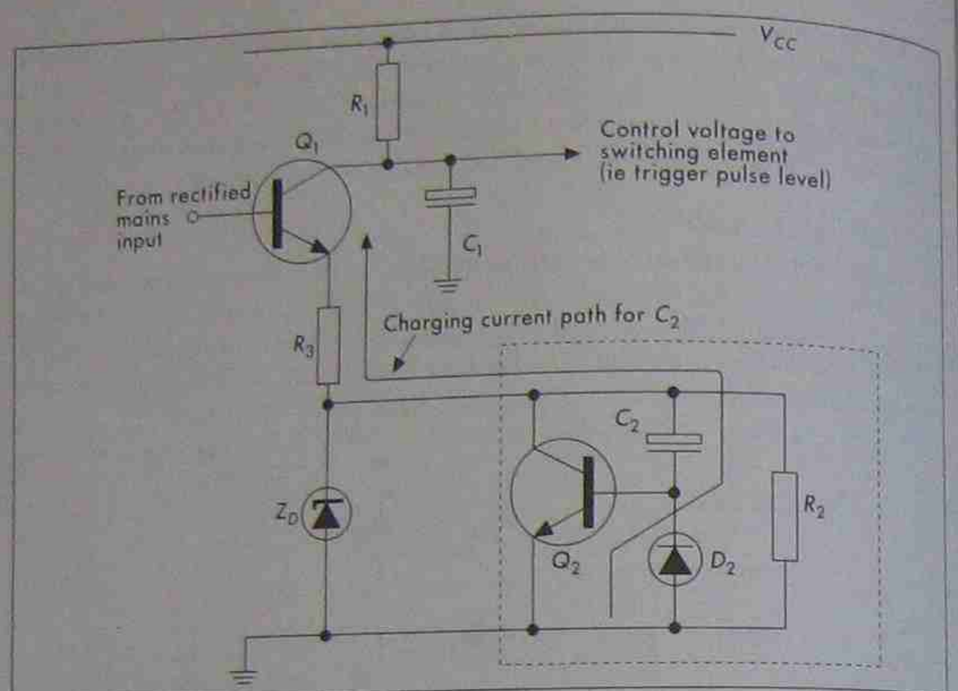


Figure 6.21: Transistor slow-start circuit (inside dotted lines)

Transistor Q_2 is connected in parallel with the zener diode (Z_D) in Q_1 emitter circuit. The transistor Q_2 is normally non-conducting because there is no base bias voltage. At switch-on C_2 is discharged, and the collector voltage of Q_2 is applied to the base of Q_2 via C_2. Transistor Q_2 conducts, and C_2 begins to charge through Q_2 base emitter junction. With Q_2 conducting, the zener diode is bypassed, causing the emitter voltage of Q_1 to decrease. Transistor Q_1 has a sharp increase in V_{BE} and conducts heavily. The collector voltage decreases, and this ensures that the voltage across capacitor C_1 takes longer to reach the required voltage level. As capacitor C2 charges, the base-emitter voltage of Q_2 decreases, turning Q_2 slowly off and allowing the zener diode to work again. When C_2 is charged, current ceases to flow in Q_2 base-emitter; Q_2 is 'off', reducing the emitter current in Q_1. This allows the collector voltage of Q_1 to rise and charge capacitor C_1 to the voltage required to trigger the switching element. When the equipment is switched off, C_2 discharges through resistor R_2 and the diode D_2. It is thus ready for the next soft start.

6.6.2 Current limiting

There are several simple methods to ensure that electronic equipment is protected against excessive current. Some of these include anti-surge fuses, thermal cut-outs, fusible resistors and circuit breakers. All of these devices can be inserted in the primary or secondary side of the power supply, whether it be a switched mode supply or linear supply. However, it is much better to limit the current to a predetermined maximum value, as we did with linear regulators in Chapter 3, and similar circuits can be used with switched mode power supplies (SMPS).

The output current is controlled indirectly by sensing the voltage drop across a low value series resistor. The resultant voltage is then used to bias on a protection transistor, or circuit, which switches off or reduces the conduction of the regulator device. Figure 6.22 shows the application of this current limiting technique as used in an SMPS.

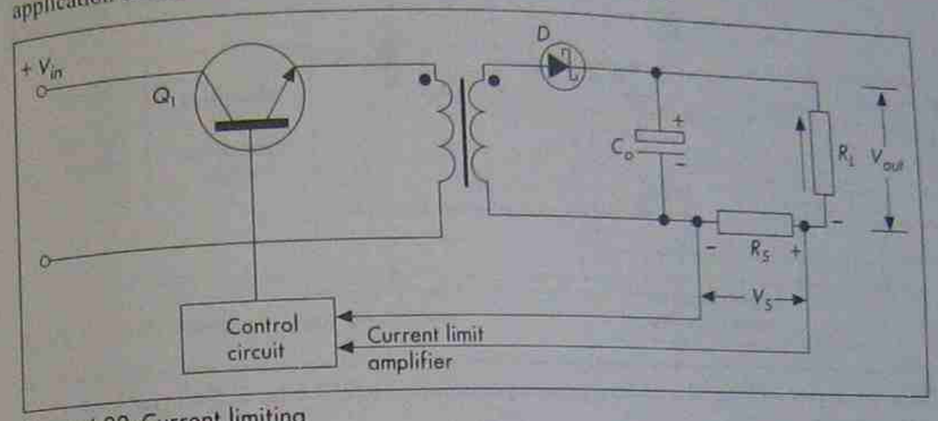


Figure 6.22: Current limiting

The value of the series resistor (R_s) and the voltage drop across it are kept small so as to minimise power dissipation. A typical value for V_s is 50 mV, thus keeping the maximum power dissipated in R_s down to about 1% of the output power. However, this value varies with circuit design. The current limiting amplifier overrides the output of the error amplifier and takes control of the pulse width. The duty cycle decreases as V_s increases, and V_{out} is reduced accordingly.

A typical **current-limiting** characteristic curve is shown in Figure 6.23, and with careful design the short-circuit current can be reduced to a safe level.

Another way to protect against over-current is to use the sensed voltage across R_s to latch up the oscillator, and so remove the drive from the switching device. The output of the power supply is then removed very rapidly.

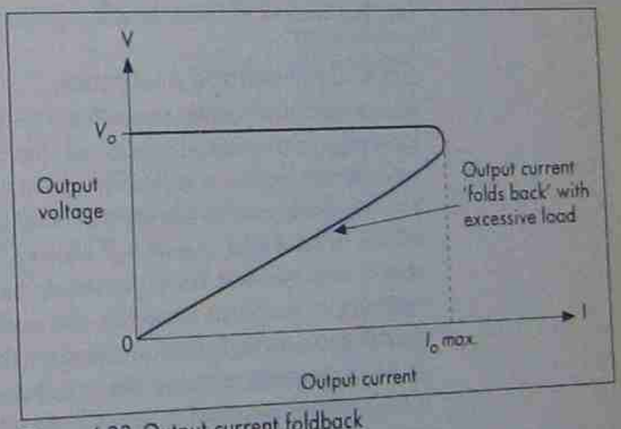


Figure 6.23: Output current foldback

6.6.3 Input surge current protection

Many switched mode power supplies are operated off the 240 V AC line with capacitor input filters. Therefore, some method of preventing the rectifiers from being destroyed by a high 'surge current' is usually required. Figure 6.24 shows a circuit that is capable of limiting the surge current, and then allowing full voltage to the rectifiers.

The series resistor (R_s) is used as the element for surge current limiting, and hence

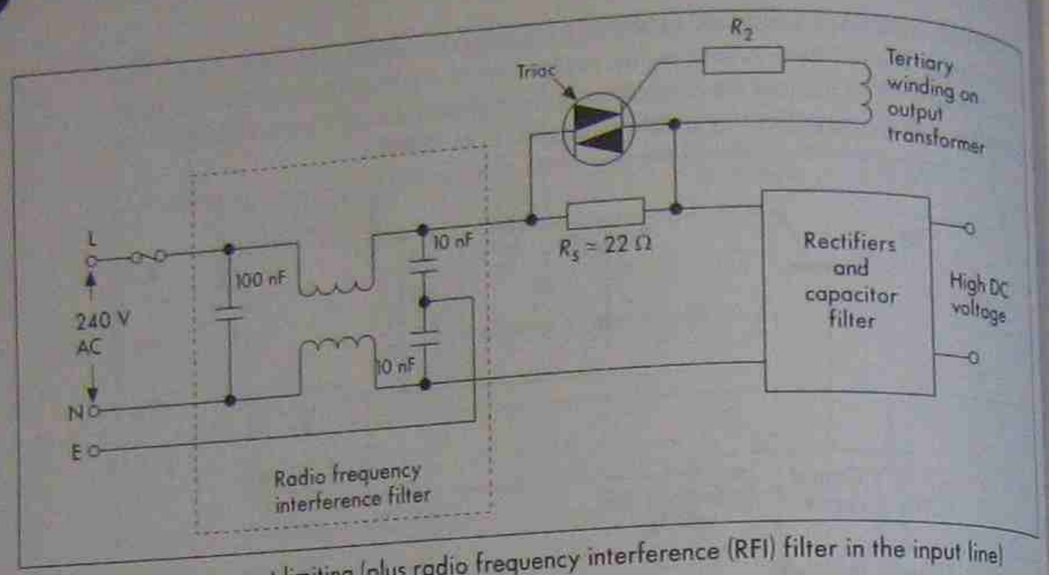


Figure 6.24: Surge current limiting (plus radio frequency interference (RFI) filter in the input line)

protects the rectifier diodes at switch-on. Once the input filter capacitor has charged and the switched mode circuit is operating, a voltage is induced in a tertiary winding on the output transformer. This voltage switches on a triac, which shunts R_s and virtually eliminates any power dissipation in R_s. The triac is not brought into conduction until the input capacitor is charged, and the surge current associated with initially charging the capacitor is therefore very low. Resistor R_2 limits the gate current of the triac.

6.6.4 Over-voltage protection

Some form of **over-voltage protection** is necessary because an excessive output voltage may cause damage to integrated circuits and other semi-conductor devices in the load circuit. One very effective method of protecting against over-voltage is 'electronic crowbar protection'. This is basically a circuit that applies a short circuit to the power supply if the output voltage exceeds its rated level. (Figure 6.25 shows a **'crowbar' circuit**.) The excessive current can blow a fuse, operate a cut-out or initiate current foldback as discussed in section 6.6.2 depending on the circuit connection. The avalanche voltage of the zener diode is chosen so that it does not normally conduct; therefore, the silicon controlled rectifier (SCR) is also off.

If V_{out} becomes excessive due to a fault condition, the zener diode (Z_D) conducts. A positive voltage is applied to the SCR gate, and the SCR switches on, causing the power supply to shut down. Capacitor C_1 absorbs any sudden fluctuations (transients) in the load circuit that may inadvertently trigger the SCR. With integrated controllers, a simple comparator can be used to keep a check on the output voltage condition. The output is monitored, and a portion of it is fed back to a comparator to be compared with a reference voltage. At a predetermined voltage 'difference', the comparator triggers the protection circuit, and the switching drive waveform is either reduced in duty cycle or taken to zero which switches the circuit off. Figure 6.26 shows this circuit arrangement.

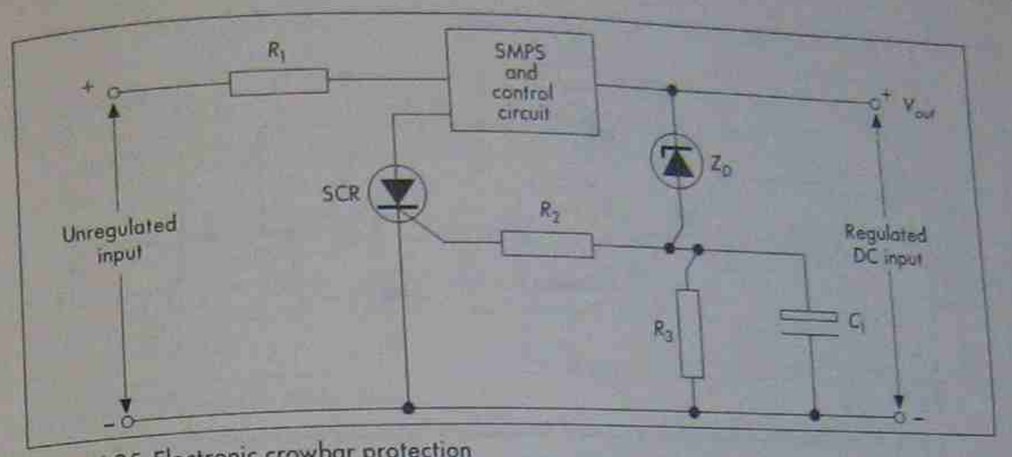


Figure 6.25: Electronic crowbar protection

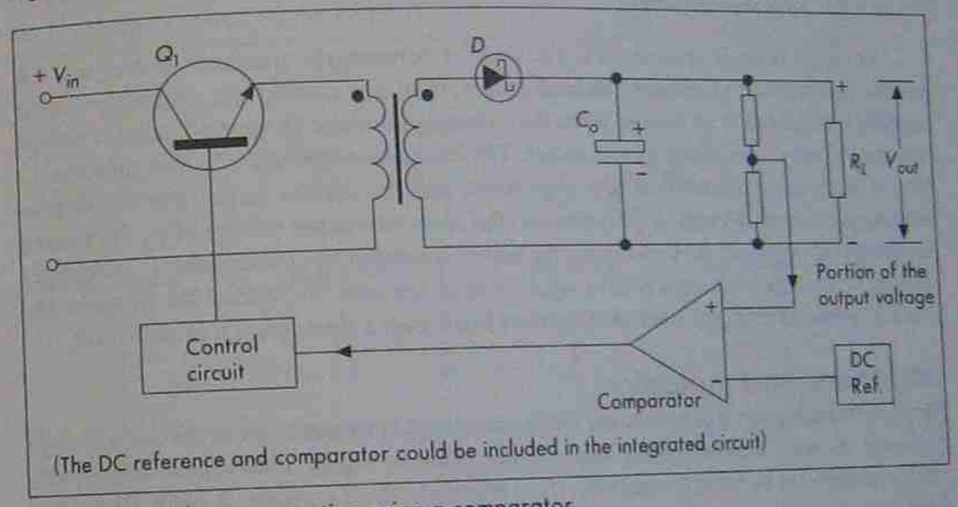


Figure 6.26: Voltage protection using a comparator

6.6.5 Switching transistor protection

In practice, the switching transistor cannot turn off the instant the drive is removed. As soon as the collector current falls, the collector voltage will start its swing to the opposite value. It depends on the circuit configuration as to whether it swings positive or negative. If there is a negative input voltage to the emitter, the collector voltage swings positive on switch-off. If there is a positive input voltage to the collector (as shown in the figures in this text), the emitter voltage swings negative on switch-off.

Whichever the case, precautions are required to prevent the transistor from dissipating too much power during the transition. This is termed keeping the transistor in its '**safe operating area**' (SOAR). The components C_1, D_1, and R_1 in Figure 6.27 are included in circuits for this purpose (some variations will be encountered in different designs).

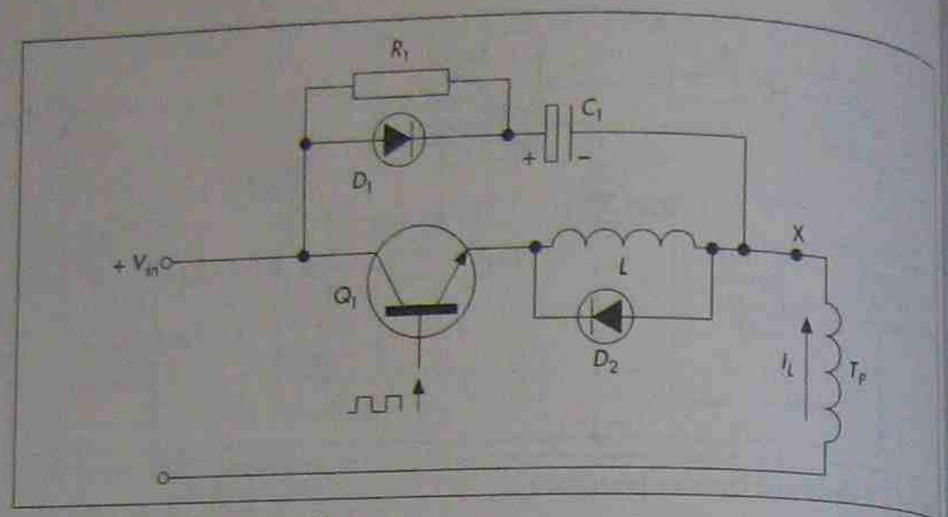


Figure 6.27: Transistor protection

The small value of inductance (L) is inserted between the transistor emitter and the transformer primary to prevent reflected spikes from the transformer secondary circuit reaching transistor Q_1 at switch-on. When the switching transistor Q_1 turns off, point X swings negative, which causes diode D_1 to conduct. The transformer primary (T_p) and capacitor C_1 form a series-tuned circuit with low impedance, and the current surges into C_1, thereby reducing the current through Q_1. This prevents the collector-emitter voltage of Q_1 from rising too quickly at switch-off, hence reducing the power dissipated by Q_1. Capacitor C_1 discharges through R_1 when point X goes positive again, that is, the next 'on' period for transistor Q_1. Diode D_2 prevents excessive back electromotive force from L damaging Q_1 at switch-off.

6.6.6 Mains input protection

The high switching speed of a switched mode power supply causes a few problems with high frequency currents flowing in the mains input. This is highly undesirable and several precautions need to be taken to minimise, if not eliminate, these currents. A **radio frequency filter** consisting of two coupled inductors and three capacitors is inserted in the mains input lead. This circuit is shown in Figure 6.28.

Capacitors C_1, C_2 and C_3 form the filter circuit with inductors L_1 and L_2. The inductors are mounted on ferrite cores; they are connected so that the two leakage fluxes are in opposition in order to prevent magnetic radiation. A further precaution is to bypass the rectifier diodes with small value capacitors (C_4 to C_7 in Figure 6.28). These capacitors suppress high frequency (HF) and very high frequency (VHF) radiation caused by the diodes switching on and off at the mains rate.

6.7 Three-terminal regulators in switched mode circuits

Low cost adjustable switching regulators can be made using three-terminal integrated circuit regulators as the control device. The simplest circuit configuration is shown in Figure 6.29.

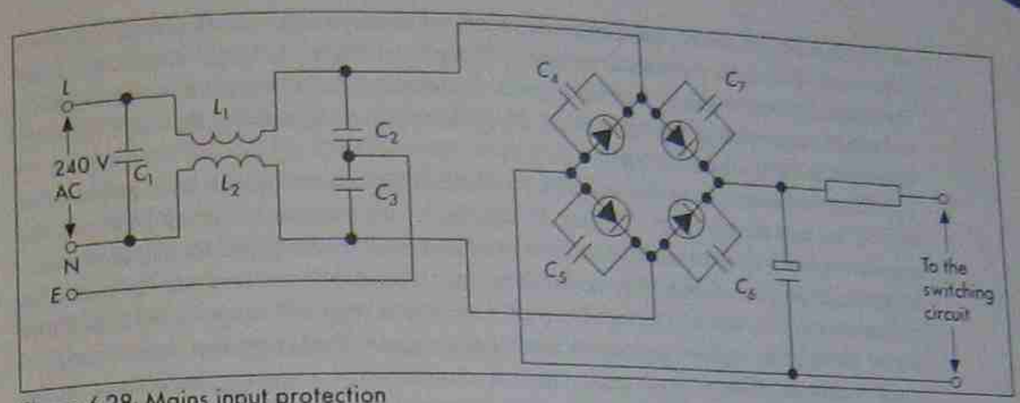
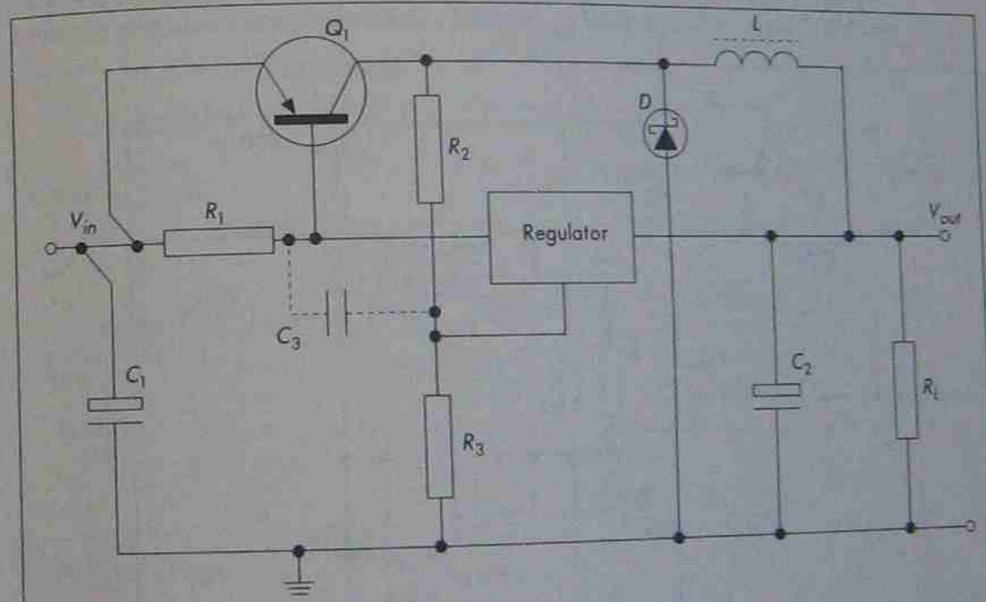


Figure 6.28: Mains input protection

V_{in} is a rectified low voltage input. Voltage divider R_2 and R_3 is the feedback loop from the output. It sets the voltage on the adjustment pin of the integrated circuit, which determines the output voltage (refer to section 4.7).



V_{in} = 10 V	R_1 = 500 Ω
V_{out} = 5 V	R_2 = 1 Ω
I_{out} = 300 mA	R_3 = 10 Ω
L = 500 µH	C_1 = 2.2 µF
f = 37 kHz	C_2 = 100 µF
M:S ratio = 1:1	C_3 = 100 µF
Q_1 = 2N2905	D = 1N5807

Figure 6.29: Switching regulators using a three-terminal regulator

Transistor Q_1 is the switching element, inductor L is the energy storage device, and diode D provides the current path for the energy returning from L when Q_1 is off. The normal input capacitor is C_1, the output storage capacitor is C_2, which is much larger than normal, and C_3 is included to minimise the ripple level at the junction of R_2 and R_3, that is, filters any variations in the feedback voltage.

When the power supply is switched on, the load draws current through the regulator. This turns Q_1 on and allows current to pass through L. As the current through L increases, the regulator supplies less and less current to the load and finally turns Q_1 off. The field around L collapses, the polarity of the voltage across L reverses, forward biasing the diode D, and current continues in the output, even though Q_1 is off. Once the energy in L is spent, the load draws current through the regulator and turns transistor Q_1 on again. The action then repeats itself.

A further development is to make the output adjustable by varying the voltage on the adjustment pin. A circuit using this technique is shown in Figure 6.30. It operates in a similar manner to the circuit in Figure 6.29, but uses an adjustable voltage power regulator (LM317) which enables a much higher output current to be obtained.

A power PNP is used as the switch, driving the L_1/C_4 filter. Positive feedback is applied to the LM317 via R_5. When transistor Q_1 switches, a small square-wave voltage is generated

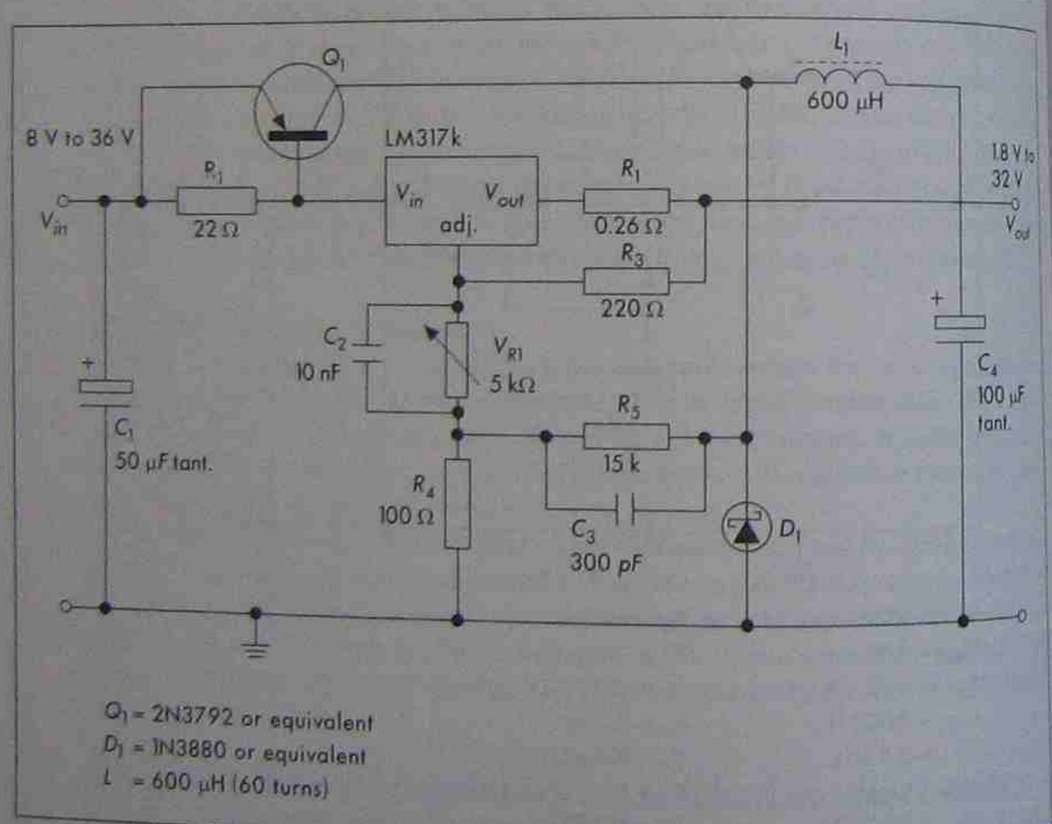


Figure 6.30: Low cost 3 A switching regulator for experimenters

across resistor R_4. This is level-shifted and applied to the adjustment terminal of the regulator by V_{R1} and C_2, causing it to switch 'on' or 'off'. Negative feedback is taken from the output through resistor R_3. Capacitor C_3 acts as a speed-up capacitor, increasing switching speed, while R_2 limits the peak drive current to Q_1. A switching regulator which cannot be blown-out is shown in Figure 6.31. The PNP switching transistor has been replaced by a PNP-NPN combination using LM395s as the NPN transistors. The LM395 is an integrated circuit which acts as a NPN transistor with overload protection.

Included in the integrated circuit is current limiting, thermal shut down and safe area protection (as discussed in Chapter 4), making the device virtually immune to any type of overload. Efficiency for the regulator ranges from 65% to 85%, depending on the output voltage. At low voltages, fixed power losses are a greater percentage of the total output power, so efficiency is lowest. Operating frequency is approximately 30 kHz, and the ripple is about 150 mV, depending on the input voltage.

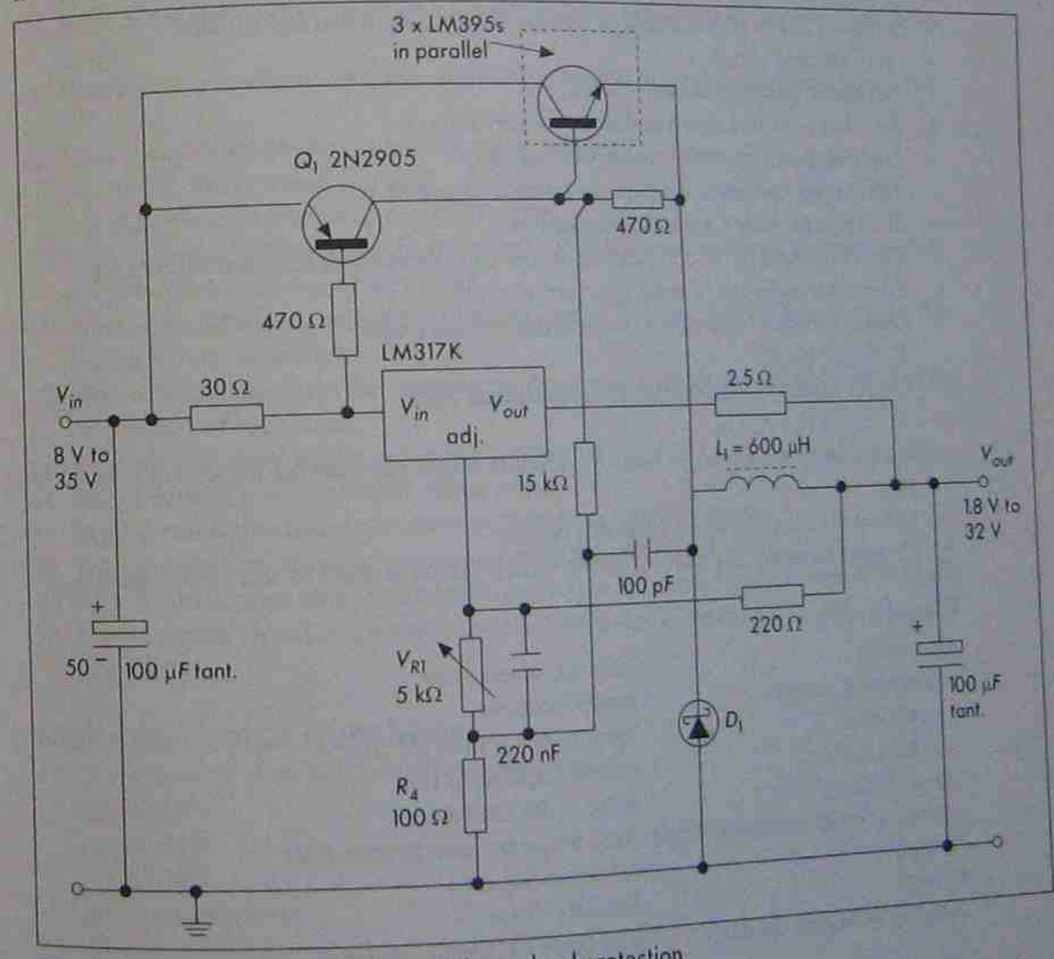


Figure 6.31: 4 A switching regulator with overload protection

One of the more unique applications for these switching regulators is as a tracking preregulator. The only DC connection to ground on either circuit is through the 100 Ω resistor R_4. Instead of tying this resistor to ground, it can be connected to the ouput of a linear regulator, so that the switching regulator maintains a constant input-output voltage differential on the linear regulator. This technique was discussed in section 6.2.1. The switching regulator would be set to hold the input voltage to the linear regulator about 3 V higher than the output. The operating conditions of the linear regulator are now held constant, because any fluctuations in input or output voltage are taken care of by the switching preregulator.

6.8 Summary

The following items are a summary of the major points covered in this chapter. The characteristics of a switched mode power supply are as follows:

- It is more efficient than a linear regulator.
- Styles of SMPS circuit available include step up, step down and isolated.
- Very efficient circuit.
- No mains 'power transformer' used.
- Requires a 1:1 isolation transformer for servicing.
- High frequency operation makes filtering easy.
- Fairly noisy operation and requires suitable suppression.
- Diodes must be fast, such as Schottky type.
- Capacitors used for filtering should be low equivalent series resistance (ESR) types.
- Chopper transformer is small and 'ferrite' cored.
- Output voltage of step-down circuit is determined from:
  $V_o = V_{in} \times (t_{on}/T)$
- Output voltage of step-up circuit is determined from:
  $V_o = V_{in} \times (1 + t_{on}/t_{off})$
- Uses a soft-start circuit to limit the mark-to-space ratio during the circuit's power-up time.
- Circuit is controlled by an integrated circuit.
- Chopper transistor is protected during switch-off to maintain its safe operating area.

Key terms

catch diode
commutating chopper
control circuit
crowbar circuit
current limiting
equivalent series resistance (ESR)
forward
flyback
inverting switching regulator
isolation transformer
mains isolation
mark-to-space ratio (M:S)
over-voltage protection
pulse-width modulator
radio frequency interference filter
safe operating area
Schottky diode
step-down switching regulator
step-up switching regulator
soft-start circuit
switched mode power supply (SMPS)
switching regulator

6.9 Review questions (answers in Appendix A)

1. When making a comparison between linear and switching regulators, what are the three main requirements for an 'ideal' system?
2. What are the main advantages of a switched mode power supply (SMPS)?
3. In your opinion what are the disadvantages of an SMPS? How can they be overcome?
4. What danger is encountered with some SMPS systems that is not normally encountered with linear power supplies?
5. Is there any special requirement in output capacitors with an SMPS? If so, what is it?
6. Draw the circuit arrangement for a basic series-switching regulator and explain its operation. Show drive voltage waveforms.
7. What is the main difference in circuit configuration between a step-up and a step-down SMPS?
8. Which of the two circuits in question 7 exhibit the least noise characteristics? Why is this so?
9. (a) Can an SMPS be used to provide mains isolation?
   (b) Which circuit would you use?
   (c) Draw the circuit and explain its operation.
10. In an 'inverting' SMPS where is the energy stored when the switching element is turned on? What is the purpose of the diode?
11. What is the difference between a 'forward' and a 'flyback' SMPS? Which one has the highest power capability?
12. Draw a block diagram of the control circuitry required for an SMPS and briefly explain the function of each block.
13. Why is a 'soft-start' circuit required in an SMPS?
14. Briefly explain how a soft-start circuit works.
15. Explain two methods that may be used to 'current limit' an SMPS.
16. What is meant by 'crowbar' protection? How does the circuit work?
17. Why do precautions have to be taken to protect the switching transistor when it turns off?
18. Draw a typical circuit you would expect to find in the AC input of an SMPS. What is its function?

Multiple-choice questions

1. The efficiency of an SMPS is often in the range of:
   (a) 40-50%
   (b) 50-60%
   (c) 60-70%
   (d) greater than 80%

2. If a basic step-down SMPS had an input voltage of 16 V DC and was switched by a signal with a mark-to-space ratio of 3:1, the output voltage will be approximately:
   (a) 4 V
   (b) 12 V
   (c) 14 V
   (d) 16 V
3. A step-up SMPS has a DC input of 20 V. It is driven from a chopper signal with a mark-to-space ratio of 5:2. The output voltage will be approximately:
   (a) 40 V
   (b) 50 V
   (c) 70 V
   (d) 160 V
4. Refer to Figure 6.12(a). The primary purpose of the inductor (L) is to:
   (a) store energy when Q_1 is off
   (b) stop noise radiation when Q_1 is on
   (c) store energy when Q_2 is on
   (d) stop noise radiation when Q_2 is off
5. If the load current of on SMPS circuit increased in order to maintain the level of the output voltage, the chopper drive signal would:
   (a) increase the 'mark' time
   (b) trigger the 'crowbar' circuit
   (c) increase the 'space' time
   (d) change the oscillator frequency
6. Refer to Figure 6.13(a). The catch diode D_1 is forward biased when:
   (a) Q_1 is conducting
   (b) Q_1 is non-conducting
   (c) the flux in L is rising
   (d) C_o is fully discharged
7. The 'style' of SMPS that is more efficient for high power levels is:
   (a) push-pull and forward
   (b) forward only
   (c) forward and flyback
   (d) flyback only
8. The main disadvantage of a series type SMPS if Q fails (that is, a short circuited collector-emitter junction) is the:
   (a) catch diode is destroyed
   (b) load could be damaged
   (c) filter capacitor DC working voltage (DCWV) will be exceeded
   (d) power dissipation of L will be exceeded
9. The advantage of a parallel flyback SMPS is that:
   (a) secondary and primary circuits are isolated
   (b) secondary voltage may be stepped up
   (c) load will not be damaged if Q has a collector-emitter short circuit
   (d) all of the above
10. Refer to Figure 6.13. If the input voltage is 18 V and the mark-to-space ratio is 5:2, the output voltage will be:
    (a) 12.8 V
    (b) 18 V
    (c) 45 V
    (d) 63 V

CHAPTER 7

INTEGRATED SWITCHED MODE POWER SUPPLIES

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Explain the terms 'equivalent series inductance' and 'equivalent series resistance' and their importance in switched mode power supply (SMPS) circuits.
- Discuss the advantage of an integrated SMPS over its discrete component counterpart.
- List the external components required to make the integrated circuit operational.
- Calculate the external resistor values required to set a specific output voltage.
- Modify a fixed voltage output circuit to make the output adjustable over a specified range.
- Use data sheets to select an inductor value for a given output voltage and load current.

7.1 Introduction

As anticipated, the **switched mode power supplies (SMPS)** have advanced in technology and become integrated. They require a minimum number of external components, mainly an inductor and a catch diode, and are available in variety of packages (for example, TO-220, TO-3, 16 pin DIP and 24 pin Surface Mount). Information and specification sheets on these integrated circuits can be obtained from National Semiconductor and their distributors. (Note that data and application notes in this chapter are reproduced with the kind permission of National Semiconductor.)

These regulators offer a high efficiency replacement for the three-terminal regulators discussed in Chapter 4 and the discrete SMPS circuits of Chapter 6, and are simple to use. They feature internal frequency compensation with a fixed frequency oscillator at a typical frequency of 52 kHz. Heatsinking requirements are substantially reduced due to the switching technique, and often no heatsinking is required at all.

The step-down version is available as a fixed output of 5 V, 12 V and 15 V or an adjustable output of 1.23 V to 37 V, with current capability of 1 A and 3 A. The step-up version is available as a fixed voltage of 12 V at 3 A or adjustable up to 60 V at 3 A. A chart listing data about the regulators is shown in Table 7.1.

The LM1575/LM2575 series has a maximum input of 45 V, the high voltage version LM2575HV has a maximum input of 63 V.

Table 7.1: Integrated SMPS selection guide

Output		Step-down regulator		Step-up regulator	
Current		Fixed	Adjustable	Fixed	Adjustable
1 A	Device Output voltage	LM2575 5 V, 12 V, 15 V	LM2575 Adj 1.2 V to 31 V		
3 A	Device Output voltage	LM2576 5 V, 12 V, 15 V	LM2576 Adj 1.23 V to 37 V	LM1577 12 V	LM1577 Adj 12 V to 60 V

7.2 Packages

Available switched mode power supply (SMPS) regulator packages are shown in Figure 7.1. Note the different connections between step-down and step-up regulators.

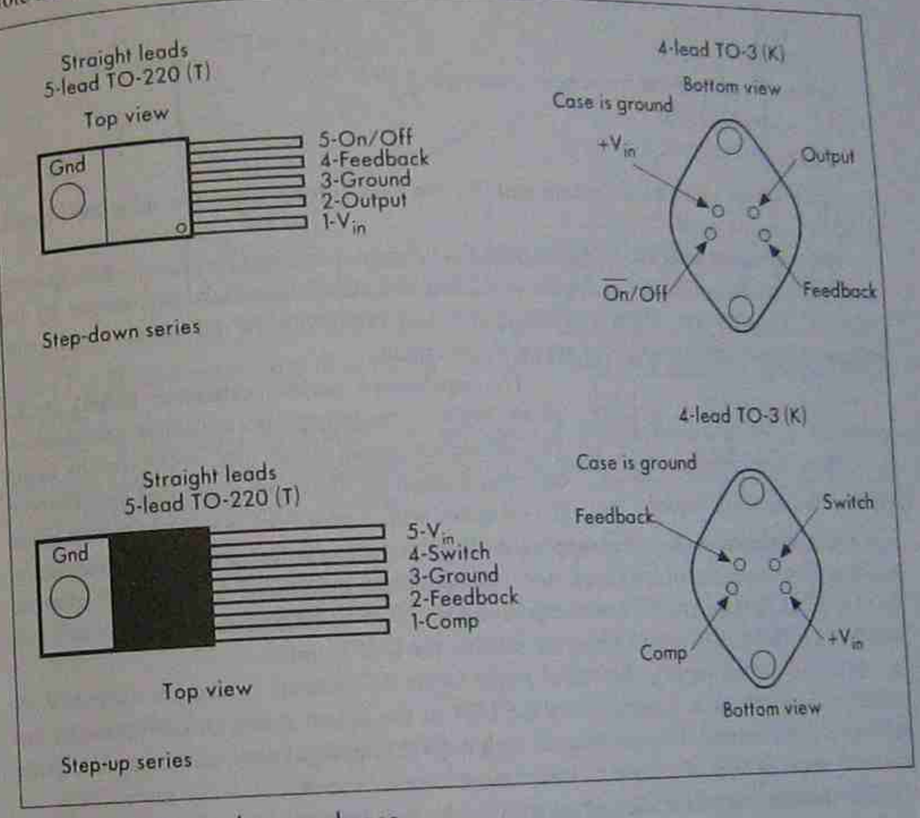


Figure 7.1: SMPS regulator packages

7.3 Definition of terms

As with any new technology there is a multitude of new terms. This section revises the glossary of terms used when discussing switched mode power supplies.

A **buck regulator** is a switching regulator in which a high voltage is converted to a lower voltage, that is, a **step-down regulator**.

A **boost regulator** is a switching regulator in which a low voltage is converted to a higher voltage, that is a **step-up regulator**.

A **buck-boost regulator** is a switching regulator where a positive input voltage is converted to a negative output voltage without the use of a transformer.

A **duty cycle (*D*)** is the ratio of 'on time' to the period of one cycle. For a buck regulator:

$$\beta = t_{on} T$$
$$= V_{out} / V_{in}$$

For a buck-boost regulator:

$$\beta = t_{on} T$$
$$= V_{out} (V_{out} + V_d)$$

where T = 1/oscillator frequency
$= 1/52 \times 10^3$
$= 19\,\mu s$

The **catch diode** provides the return path for the load current when the integrated switch (LM2575) switch is off.

The **equivalent series inductance (ESL)** is the pure inductive component of a capacitor. The equivalent circuit is shown in Figure 7.2). The inductance is mainly determined by the capacitor's construction. When considering switching regulators, the inductance is important because it causes voltage spikes to appear at the output.



Figure 7.2 Capacitor model

The **equivalent series resistance (ESR)** is the purely resistive component of a capacitor's impedance. This resistance causes power loss, which results in the capacitor heating and therefore degrading its operation. If a capacitor with a high ESR is used as a filter at the output of a switched mode power supply (SMPS), the ripple current is increased. General-purpose standard aluminium electrolytic capacitors around 420 μF to 2000 μF have an ESR of the order 0.1 Ω to 0.3 Ω. Special low ESR rated capacitors of 220 μF to 1000 μF usually come in at less than 0.15 Ω. Philips ESR to ESS series are suitable for SMPS units.

As in any power supply, the output ripple is the AC content on the DC output. In switching regulators it is determined by the ESR of the output capacitor multiplied by the inductor's ripple current. This can be quite high in switching regulators, often a peak-to-peak sawtooth ripple of 30% of maximum load current is experienced.

The **standby current** is the current required by the integrated circuit when in stand-by mode. The on/off pin is driven to TTL high voltage, thus switching the output switch off.

Inductor saturation exists when the inductor cannot hold any more magnetic flux. When an inductor saturates, the inductor appears less inductive and the resistive component is dominant. Therefore, the inductor current is limited by the DC resistance of the coil and the source.

7.4 Fixed step-down regulators

The operation of a step-down switching regulator has been well established in Chapter 6. The block diagram of a fixed output switched mode power supply (SMPS) regulator is shown in Figure 7.3.

The output of this regulator is +15 V. This fixed output voltage is established by the internal resistor ratio at the input of the error amplifier (11kΩ to 1kΩ for 15 V, 8kΩ to 1kΩ for 12 V, and 3kΩ to 1kΩ for 5 V).

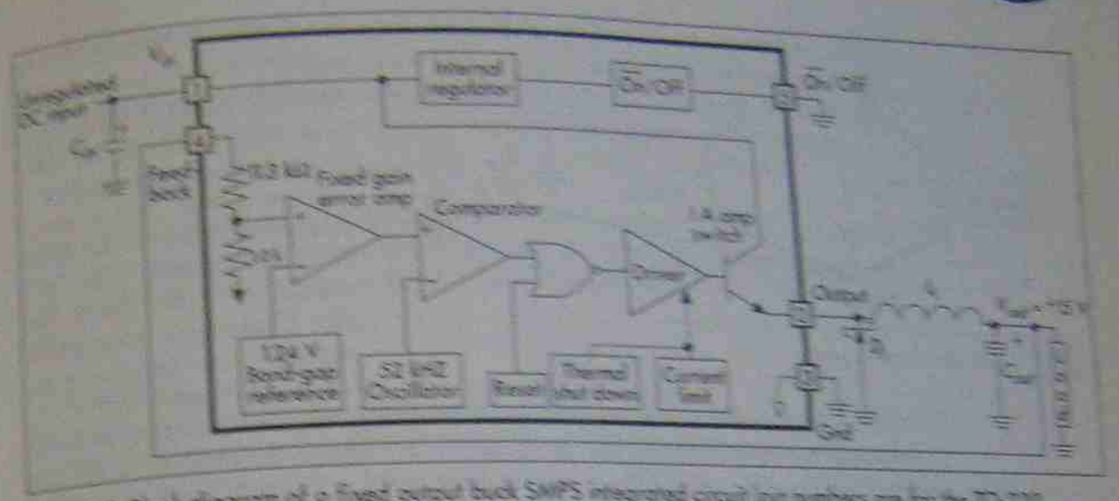


Figure 7.3 Block diagram of a fixed output buck SMPS integrated circuit (pin numbers are for the TO-220 package)

The 1 A version is the LM2575-15 and the 3 A version the LM2576-15. The only difference between the two integrated circuits is the final stage current switch feeding the output. All have the following features:

- a guaranteed output within ±2% maximum over varying line and load conditions;
- guaranteed 1 A or 3 A output current;
- 52 kHz internal oscillator;
- 80% to 88% efficiency;
- thermal shut down;
- current limit protection;
- low standby current (typically < 200 μA);
- only four external components required; and
- wide input voltage range (note that this varies with the different output voltages, so check the data sheet).

As with any switching regulator, wiring and layout is important. Rapidly switching currents associated with inductive properties can create problem voltage transients. **Single point wiring** technique, or ground plane construction should be used as shown in Figure 7.4.

Stray inductance and earth loop problems will be minimised if the heavy leads indicated in Figure 7.4 are kept short in length.

7.4.1 Selection of the external components

Inductor

To enable the correct choice of inductor, two circuit parameters must be known:

1. maximum input voltage (V_{in} (max)); and

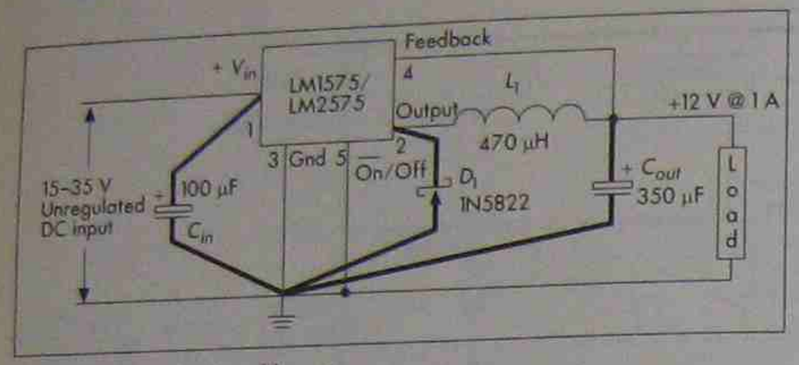


Figure 7.4: Test circuit and layout

The H prefix represents high power, the L prefix low power, and the number the unit of inductance in microhenrys (µH) that can operate at the switching frequency of 52 kHz, and may be used if it has a current rating of 1.15 × I_L (max). For example, where V_{in} (max) = 25 V, I_L (max) = 1 A and L = H470 = 470 µH:

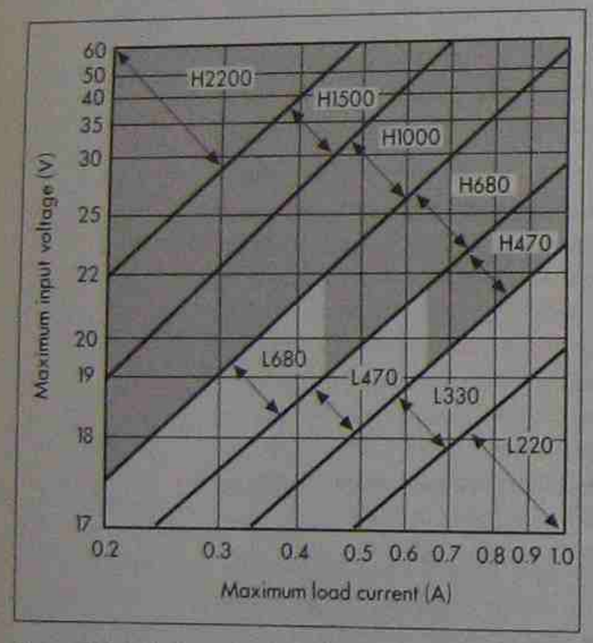


Figure 7.5: Inductor selection chart

current rating = 1.15 × I_L (max) 1.15 × 1
= 1.15 A

This is only an approximate guide and exact details for specific output voltage are in the data sheets.

Some hints on choosing inductor cores follow.

- Ferrite pot core inductors have low electromagnetic interference (EMI), small physical size and low power dissipation. Care must be taken not to exceed their maximum peak current rating or the core will saturate.
- Powered iron toroid core inductors have low EMI and can stand peak currents beyond their rated value better than ferrite cores.
- Ferrite bobbin core inductors are low cost and excellent for handling excess current. However, they generate more EMI and may cause interference in equipment with low signal levels.

Output capacitor

The combination of inductor and capacitor is important for stable operation and minimum output ripple. Typical SMPS ripple figures are V_r (peak-to-peak) = 1% of the output voltage. Capacitor values between 330 µF and 1000 µF are typical. As we already know, output ripple is a function of the equivalent series resistance (ESR) of the capacitor; therefore the lower the ESR the lower the ripple level. However, we also know from our previous studies that a larger capacitor value reduces the ripple level. This is good because generally the lower value capacitors have the higher ESR figures. Capacitors in the 200 µF to 680 µF range allow an output ripple of about 50 mV to 150 mV, and larger value capacitors will reduce this even further.

$$V_r \text{ (p-p) (approx)} = 0.3 \times I_L \text{ (max)} \times \text{ESR}$$

If too low an ESR is used then instability can occur. It is for this reason that tantalum capacitors are not used as the sole output capacitor. The working voltage of the capacitor must be above the rated output voltage. A typical minimum value is 1.25 times the output voltage.

Catch diode

The recovery or catch diode is under the most stress in an overload or short circuited output situation. The forward current rating must be at least 1.2 times the maximum load current figure. Also the peak inverse voltage rating should be adequate, about 1.25 times the maximum input voltage.

Speed is also important because of the fast switching rate, therefore **Schottky diodes** are preferred. They are fast and have a low forward voltage drop. However, any fast recovery diode is suitable as long as it has a fast but soft recovery characteristic. Fast turn-off characteristics may cause some instability, and standard power supply diodes such as 1N4004 etc, are *not* suitable.

7.5 Ajustable step-down regulators

From the selection chart (Table 7.1) we can see that there are two current ratings available 1 A and 3 A. We will discuss the LM2576 which provides an adjustable output voltage from 1.23 V to 37 V and can deliver 3 A of load current. Adjustment is made possible by sampling the output via a voltage divider and feeding this voltage level back to the feedback input of the integrated circuit. The block diagram for this arrangement is show in Figure 7.6.

As with linear regulators, the divider across the output determines the output voltage level:

$$V_{out} = V_{ref}(1 + R_2/R_1)$$

where V_{ref} = 1.23 V

R_1 is typically any value between 1k Ω and 10k Ω, so the value of R_2 can be determined from:

$$R_2 = R_1 (V_{out}/V_{ref}) - 1$$

One per cent metal-film resistors give the best stability.

7.5.1 Component selection

The first step is to determine the values of the voltage divider resistors. If the supply has the following specifications: V_{out} = 10 V, V_{in} (max) = 25 V, I_L (max) = 3 A, R_1 = 1 k Ω, then we can calculate R_2:

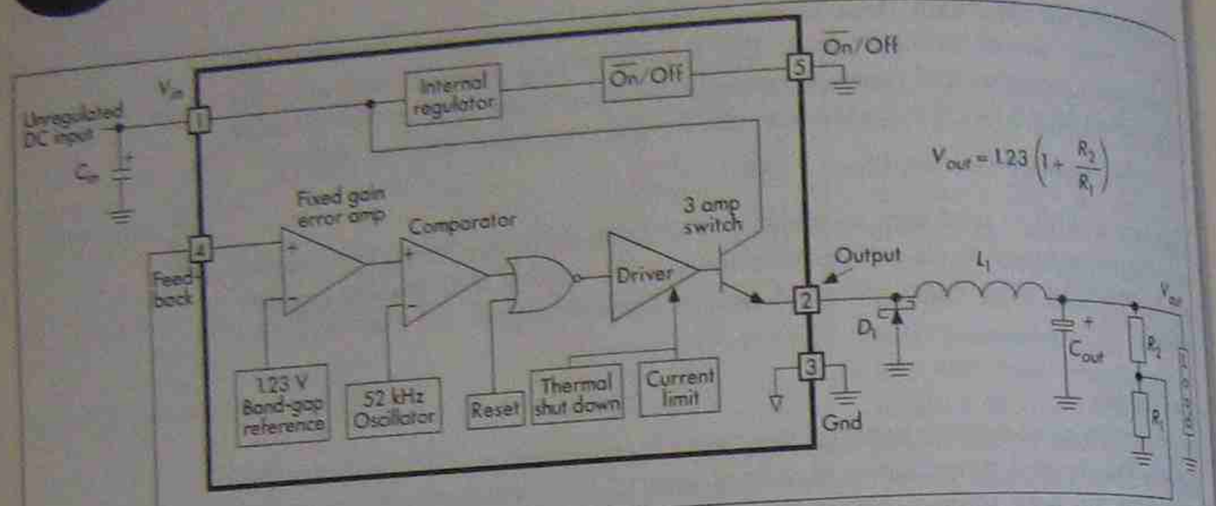


Figure 7.6: Block diagram of an adjustable output SMPS (note pin numbers are for the to-220 package)

$$R_2 = R_1 (V_{out}/V_{ref}) - 1$$
$$= 1 \times 10^3 (10/1.23) - 1$$
$$= 7.13\ \text{k}\Omega \text{ or } 7\text{k}13\Omega$$

The nearest preferred value at 1% tolerance is 7.15 kΩ or 7k15Ω. The inductor selection is similar to the LM2575-15, but uses the rate of change of voltage (V/μs) not the V_{in} (max). Rate of change of voltage can be determined from:

$$\Delta V = (V_{in} - V_{out}) \times (V_{out}/V_{in}) \times (1000/f \text{ where } f \text{ is in kHz})$$
$$= 25 - 10 \times (10/25) \times (1000/52)$$
$$= 115\ \text{V}/\mu\text{s}$$

From the chart in Figure 7.7, the inductor value can be found for a 3 A load current: an H150 inductor with an inductance of 150 μH.

The output capacitor which will determine the ripple level must be a low equivalent series resistance type, around 680 μF to 1000 μF, and have a voltage rating of at least 1.25 times greater than the output voltage V_{out}. The catch diode must be a fast Schottky type and be able to handle the rated load current, that is, 3 A.

7.5.2 Adjustable output voltage

A 3 A adjustable power supply using the LM2576-Adj is shown in Figure 7.8. A similar technique for choosing R_2 is used, but R_2 is now a potentiometer which allows output variation from 1.23 V to 35 V:

$$R_2\,(\text{min}) = R_1 (V_{out}/V_{ref}) - 1$$
$$= 1.8 \times 10^3 (1.23/1.23) - 1$$
$$= 1.8\ \text{k}\Omega \text{ or } 1\text{k}8\ \Omega.$$

$$R_2\,(\text{max}) = 1.8 \times 10^3 (35/1.23) - 1$$
$$= 49.4\ \text{k}\Omega \text{ or } 49\text{k}4\ \Omega$$

An optional LC filter of 20 μH and 100 μF can be put across the output to reduce the ripple level by about a factor of 10.

7.5.3 Soft start

A soft-start function for the switched mode power supply regulator is shown in Figure 7.9.

The regulator is held off until the input voltage reaches a certain level. This operation is sometimes called 'lock-out'. The threshold voltage (V_{th}) is determined by the zener voltage (V_Z):

V_{th} (approx) = V_z + base-emitter voltage of Q1

7.6. Fixed step-up regulators

Obviously, the step-up regulator produces an output which is greater than the input. If you remember the basic theory from Chapter 6, you will know that the increase in output is established by the counter electromotive force (cemf) of the inductor aiding the input voltage. The basic circuit using an LM1577-12 is shown in Figure 7.10.

So, energy stored in the inductor during switch-on time is transferred to the output during switch-off. The level of output voltage is determined by how much energy is recovered, and this in turn is set by the mark-to-space ratio of the pulse width modulator.

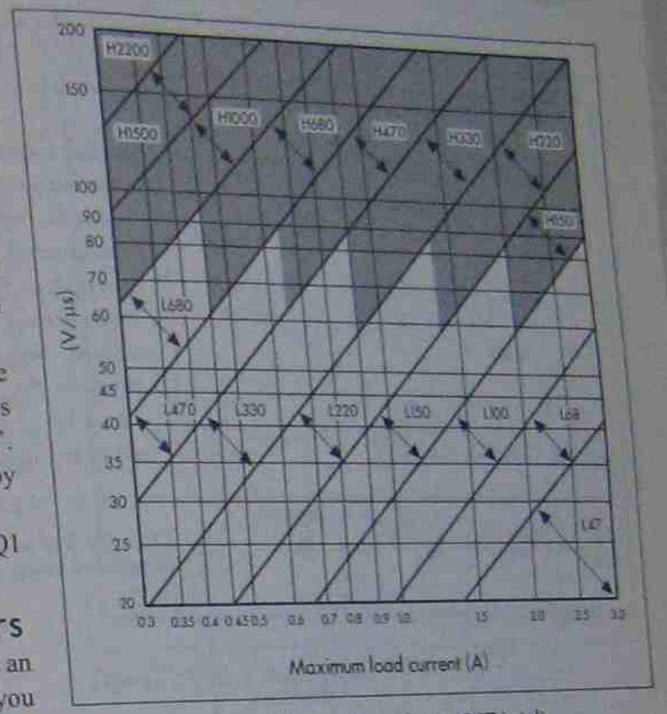


Figure 7.7: Inductor selection chart for LM2576-Adj

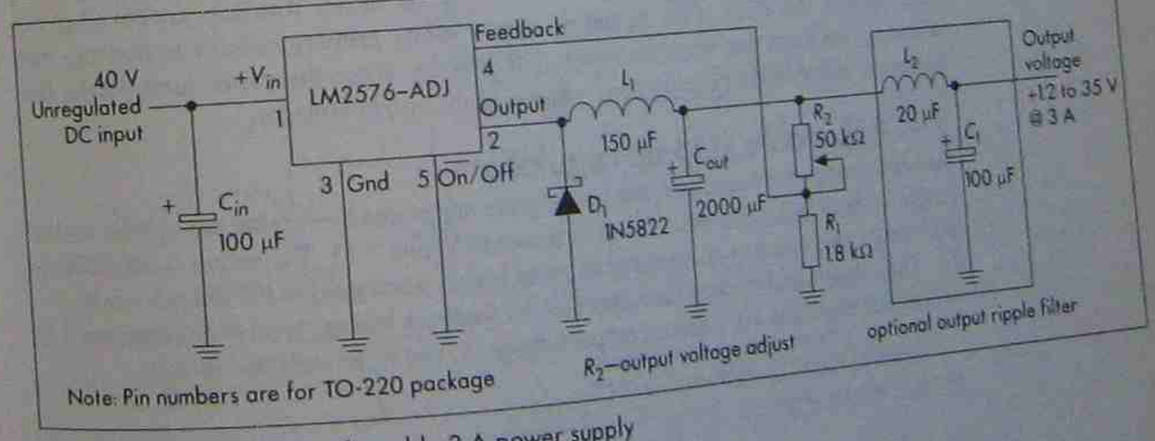


Figure 7.8: 1.23 V to 35 V adjustable 3 A power supply

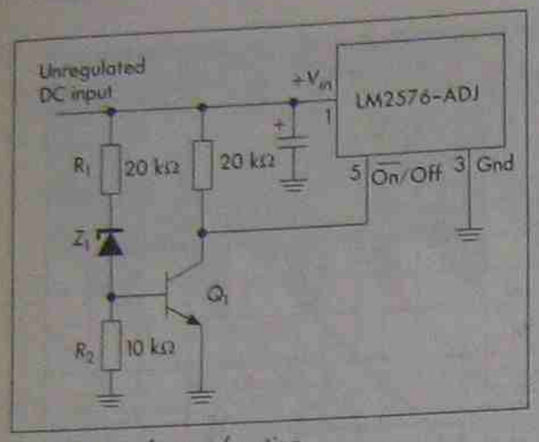


Figure 7.9: Soft-start function

The block diagram of the internal structure of the LM1577-12 is shown in Figure 7.11. Note the current switch is 3 A.

7.6.1 Component selection

The selection of external components is a little more scientific with the step-up regulator. **Compensation components** are required for stability and these also have an effect on the internal soft-start circuit. Reference to the manufacturer's design procedure is essential for step-up circuits because of the other external components. However, the compensation components connected to pin 1 have $R < 3$ kΩ and $C > 220$ nF (this can be seen in the circuit in Figure 7.10).

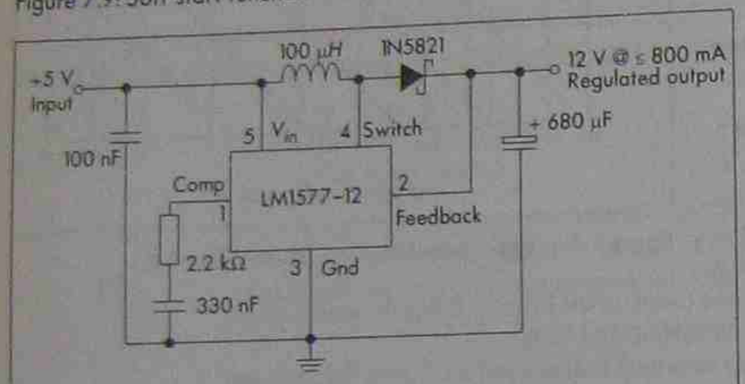


Figure 7.10: Step-up regulator

7.6.2 Flyback mode

Used in the flyback mode configuration the regulator can produce single or multiple output voltages that are either greater or lower than the input voltage. The magnitude depends on the turns ratio of the transformer and the polarity depends on the orientation of the diode. A dual polarity is also possible by using a centre-tapped transformer as shown in Figure 7.12.

Note that the primary and secondary windings are out of phase. This ensures that there is no secondary current while primary current is flowing, thus allowing maximum flux when the switch (pin 4) is on. When the switch turns off the flux collapses, and secondary current flows, producing the output voltages.

7.7 Adjustable step-up regulators

As with the step-down regulator, this unit is made adjustable by varying the voltage applied to the feedback input. The basic circuit is shown in Figure 7.13. The output is adjustable to 60 V when used as a step-up regulator and even higher when used in the flyback mode.

The voltage divider across the output sets the feedback voltage level to be compared with the internal reference. For a desired output voltage, R_1 and R_2 are selected so that:

$$R_2/R_1 = (V_{out}/V_{ref}) - 1, \text{ so}$$
$$R_2 = R_1 (V_{out}/V_{ref}) - 1$$

So for a 15 V output where R_1 = 2k2 Ω, the value of R_2 required is:

$$R_2 = 2.2 \times 10^3 \times (15/1.23) - 1$$
$$= 24.6 \text{ k}\Omega$$

When the input voltage is greater than 10 V, or when using a transformer with primary inductance (L_p) > 200 µH, a **snubber circuit** is required. This circuit is fairly traditional and is used in many circuits with inductive loads. Its job is to clamp the voltage spike from the transformer primary that is created when the output switch turns off. If the snubber is not fitted the counter electromotive force may exceed the maximum switch stand-off voltage of 65 V. Figure 7.14 shows a snubber circuit. It is made up from a fast recovery diode and a parallel CR circuit.

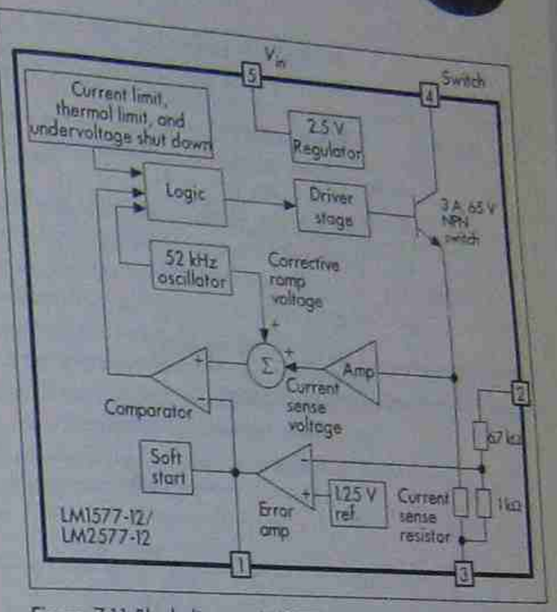


Figure 7.11: Block diagram LM1577

7.8 Integrated television power supplies

A range of integrated power supply circuits are appearing in television receivers. These are called STRs and the internal circuitry, with pin connections, for an STR 30125 is shown in Figure 7.15.

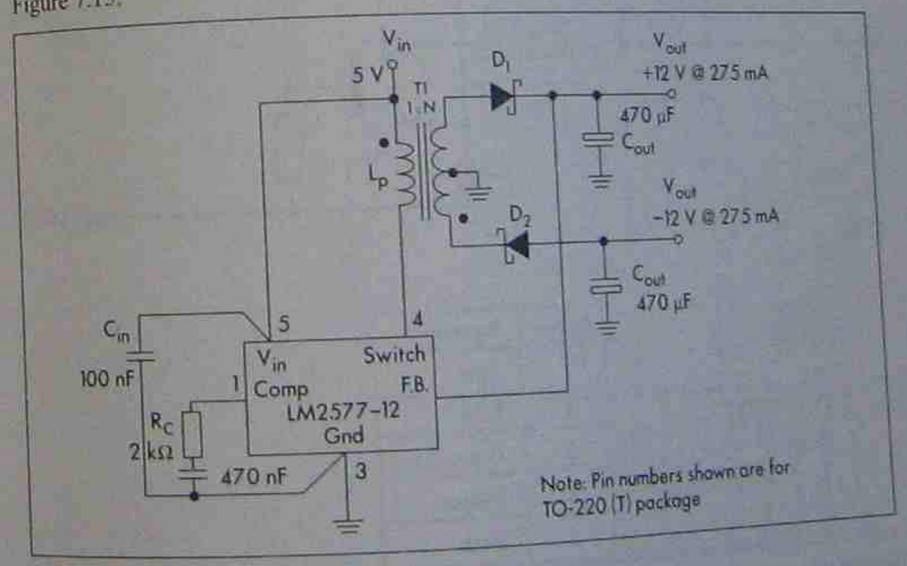


Figure 7.12: Dual output flyback regulator

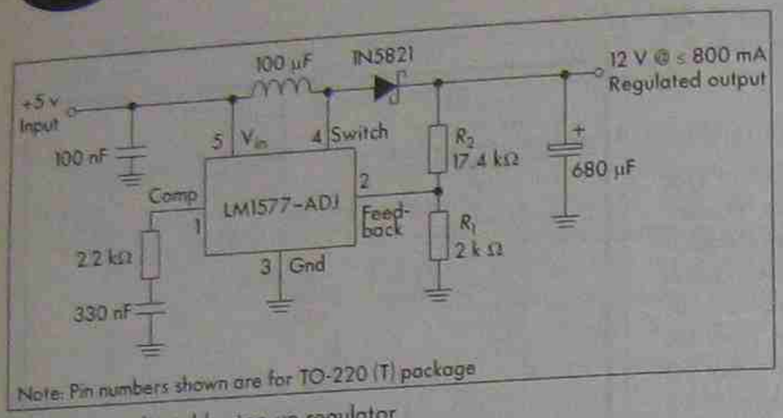


Figure 7.13: Adjustable step-up regulator

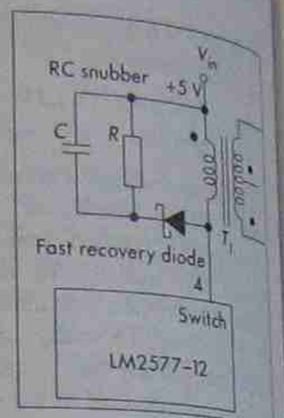
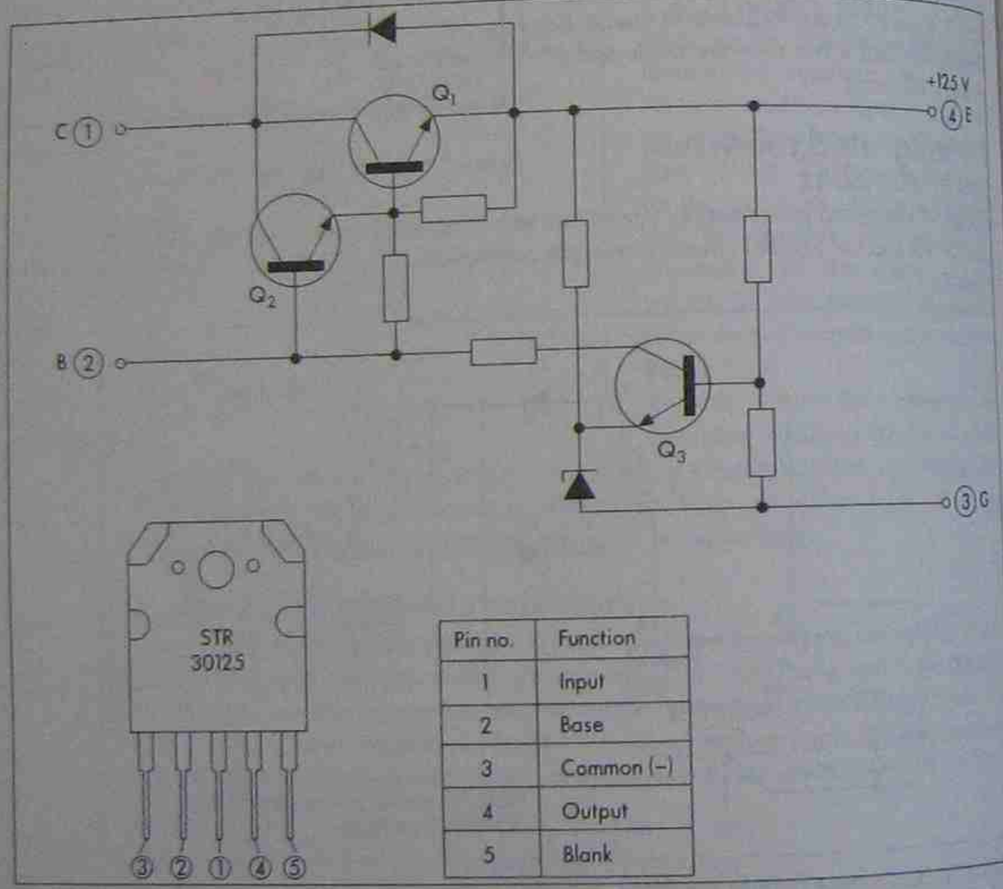


Figure 7.14: Snubber circuit



Pin no.	Function
1	Input
2	Base
3	Common (–)
4	Output
5	Blank

Figure 7.15: Internal circuit of an STR

Generally, they contain the error amplifier (Q_3), the driver (Q_2) and the power output stage (Q_1), with current-voltage overload protection circuits external. The STR is connected in a self-oscillating arrangement by the correct 'phasing' of the switching transformer main winding and the feedback windings (shown by dots). They are referred to as a **self-oscillating power supply** (SOPS). A number of circuit arrangements can be found, but the principle of operation is shown in Figure 7.16.

At switch-on, C_1 charges to establish the main input voltage for the STR. A high value resistor R_1 provides the initial 'start-up' base current to the chopper transistor inside the integrated circuit (IC). Electron flow is from ground, through R_2 into pin 4 of the IC, out of pin 3 into the lower winding (pins 4-5) of the transformer T_1 and back to the positive supply line, setting up a magnetic flux. The oscillator feedback winding is the top winding (1-3). It provides a positive drive into pin 2 of the IC, via the drive circuit aiding the original conduction (positive feedback). This drive swamps the start-up current and takes over control of the circuit. The voltage induced into winding 1-2 is rectified and filtered and provides the supply voltage for the internal IC circuitry at pin 1. It also develops a sample voltage, proportional to V_{out} which is fed back to the error amplifier via pin 5.

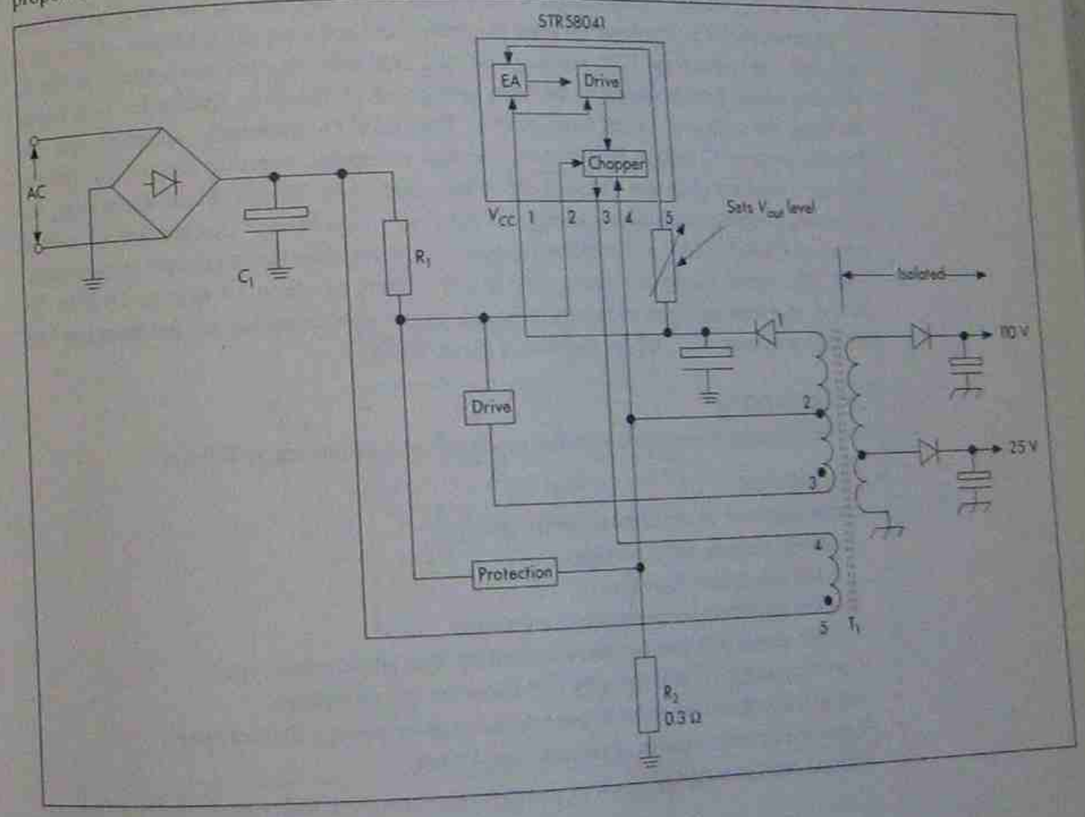


Figure 7.16: Isolated SOPS

When this sample voltage is large enough, the drive transistor inside the IC switches the chopper off. The magnetic field around T_1 collapses, inducing a voltage in the secondary circuits which is rectified, filtered and used to power other areas of the receiver. The circuit then continues to operate between the chopper-on, chopper-off conditions. If the sample voltage level to pin 5 varies, then the mark-to-space ratio automatically adjusts to maintain a constant level of output voltage.

An important feature of this supply is the isolation of the secondary voltages. Any excessive secondary load causes a higher primary current through the transformer. This is sensed by resistor R_2. The voltage across R_2 increases and the protection circuit trips, grounding the input to pin 2 of the IC. The STR is now switched off.

A variation on the previous circuit is shown in Figure 7.17, which is basically the power supply used in the Panasonic M15L chassis television receiver. Note that this chassis is live!

This circuit generates a 113 V supply for the horizontal output stage and a 16 V supply for the audio section of the receiver. All other voltages are obtained from the operation of the horizontal output stage, that is, 180 V, 24 V, 12 V, 5 V and the cathode ray tube (CRT) filament voltage.

When the receiver is switched on, a start-up current is supplied via the R803 to the base of Q_3 inside the STR. Transistor Q_3 starts to conduct and collector current flows, setting up a magnetic field around the P_1-P_2 winding of T801. This induces a positive voltage in the F_2 winding which further increases the base voltage of Q_3 (positive feedback), and hence increases the collector current even further. Transistor Q_3 eventually saturates and the induced positive F_2 voltage falls to zero, the flux collapses, reversing the polarity of the voltage. Transistor Q_3 is now switched off by the negative F_2 voltage. At the same time, the collapsing magnetic field releases energy through the F_1-F_3 winding. This charges the output capacitor, C808, to 113 V and provides the operating voltage for the horizontal output stage, Q501. The circuit operation repeats itself at a frequency of about 14 kHz to 16 kHz. To prevent, or at least minimise switching noise, horizontal flyback pulses are fed from pin 1 of the flyback transformer (FBT) to the base of Q_3 via D805.

7.9 Summary

The characteristics of integrated switched mode power supplies are as follows:

- High efficiency.
- Fully integrated switched mode power supply.
- Fixed and variable output voltages.
- 1 A and 3 A current capability.
- Use low equivalent series resistance capacitors.
- Use fast diodes with low forward voltage drop, that is, Schottky type.
- Output capacitor DCWV typically 1.25 times the output voltage.
- Adjustable output voltage set by the external resistor voltage divider ratio.
- Requires minimum number of external components.
- Internal oscillator at 52 kHz.

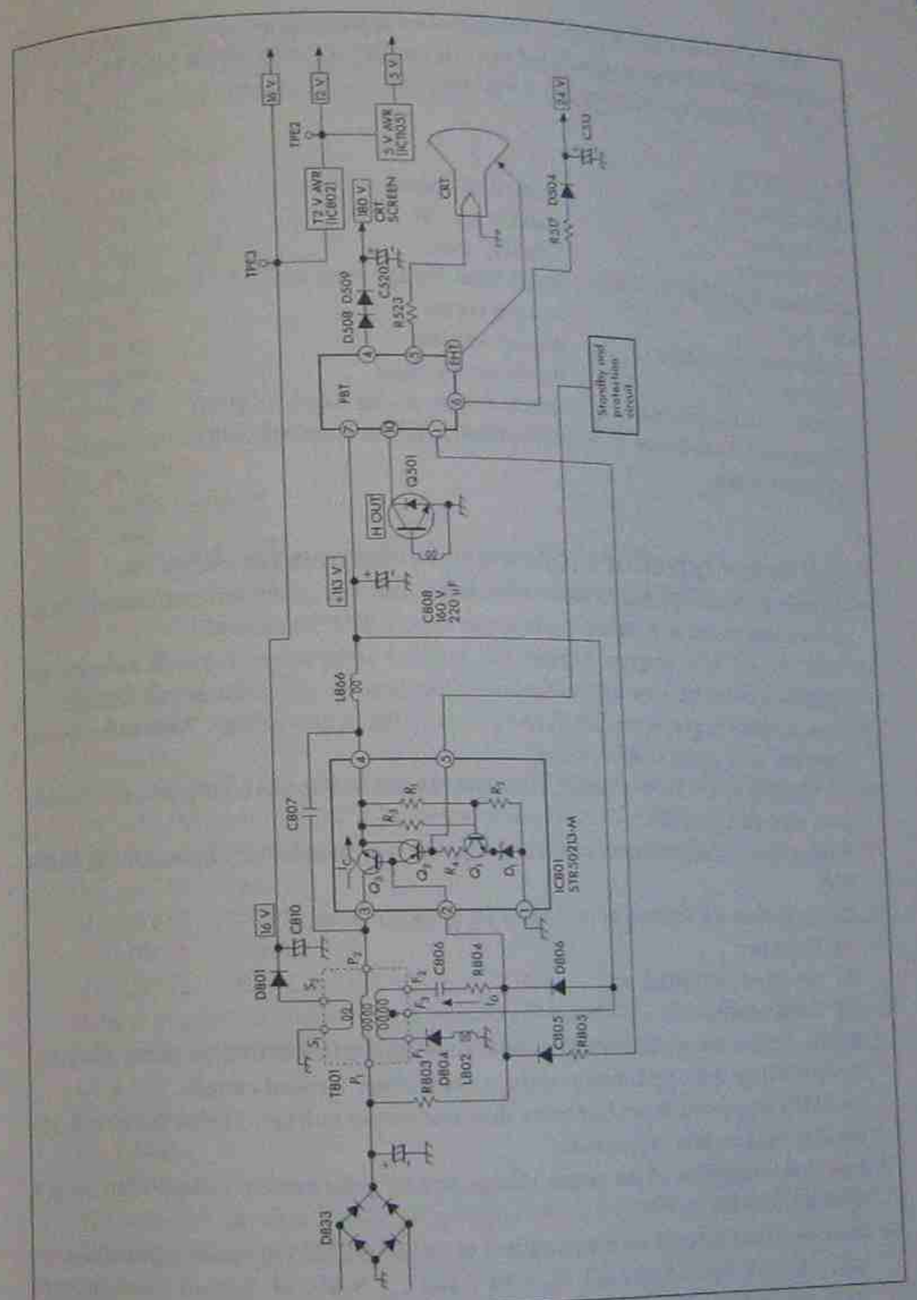


Figure 7.17: Live chassis SOPS (reproduced courtesy of Panasonic Australia)

- Step-up regulator circuit requires compensation to pin 1.
- An integrated circuit may be found in a self-oscillating power supply (SOPS).
- A SOPS may have an isolated or a live chassis.

Key terms

adjustable output voltage
buck regulator
boost regulator
buck-boost regulator
catch diode
compensation components
duty cycle
equivalent series resistance
equivalent series inductance
fixed output voltage
integrated circuit
standby current
Schottky diode
single point wiring
snubber circuit
step-up regulator
step-down regulator
switched mode power supply (SMPS)
self-oscillating power supply (SOPS)

7.10 Review questions (answers in Appendix A)

1. Explain the terms 'equivalent series inductance' and 'equivalent series resistance'. What is their importance in switched mode power supply (SMPS) circuits?
2. With the aid of a diagram explain the external components required to make the integrated circuit (IC) operational for both a fixed and an adjustable output voltage.
3. Typical SMPS ripple figures are V_r (p-p) = 1% of the output voltage. What is the typical capacitor value range to achieve this?
4. An LM2575 IC is used in a circuit. How would you tell if it was configured as a fixed or as an adjustable output?
5. What is the oscillator frequency used and what are typical efficiency figures for all SMPS ICs?
6. Briefly explain the features of the following inductor cores:
   (a) ferrite pot;
   (b) powdered iron toroid; and
   (c) ferrite bobbin.
7. Briefly explain the requirements of the diodes used in an SMPS in terms of speed, forward voltage drop, peak inverse voltage and average forward current.
8. An SMPS is required to produce more than one output voltage. Sketch the circuit you would use and explain its operation.
9. How is the magnitude of the output voltage and its polarity determined when using a flyback SMPS IC?
10. Draw the IC and external circuitry required to make an LM2576 circuit adjustable from 1.23 V to 30 V where R_1 = 1 kΩ, V_{in} = 30 V and C_{out} = 680 µF. List all the component values required for a load current of 3 A.

Multiple-choice questions

1. An integrated SMPS is required to give an adjustable output of 1.2 V to 20 V from a 30 V DC source. The load current required is a maximum of 3 A, which device would you use?
   (a) LM2575 Adj
   (b) LM2576 Adj
   (c) LM1577
   (d) LM1577 Adj
2. Refer to Figure 7.4. If the input is 25 V, the power dissipated by the IC will be:
   (a) 10 W
   (b) 12 W
   (c) 13 W
   (d) 25 W
3. Schottky diodes are used in an SMPS because they are:
   (a) cheaper
   (b) readily available
   (c) faster
   (d) faster and have a low forward voltage drop
4. Refer to Figure 7.6. If V_{ref} = 1.23 V and R_1 = 1k5 Ω, the value for R_2 to give a 15 V output at 1.5 A will be:
   (a) 6k8 Ω
   (b) 14k Ω
   (c) 16k8 Ω
   (d) 21k Ω
5. Refer to Figure 7.8. An output voltage range of 5 V to 30 V is required from a 35 V input. If R_1 = 2k2 Ω, the values of R (min) and R (max) for R_2 will be:

	Minimum	Maximum
(a)	6k7 Ω	51k5 Ω
(b)	1k9 Ω	22k2 Ω
(c)	8k9 Ω	54k Ω
(d)	2k3 Ω	22k6 Ω

6. Refer to Figure 7.6. For the same circuit as in question 4, where V_{in} = 35 V, what is the value of inductor required (using the chart in Figure 7.7)?
   (a) L150 µH
   (b) H680 µH
   (c) L470 µH
   (d) H470 µH
7. The soft-start function is used as in Figure 7.9. If the zener voltage is 6V8, what is the threshold voltage?
   (a) 6.8 V
   (b) 6.1 V

(c) 7.5 V
(d) 8.2 V

8. When using an LM1577 IC the power supply will be a _____ type and require _____. Fill in the blanks.
   (a) step-down, compensation
   (b) step-up, compensation
   (c) step-up, soft start
   (d) step-down, soft start
9. In an SMPS, if a filter capacitor with a high equivalent series resistance is used, the ripple amplitude of the output will:
   (a) increase
   (b) double in frequency
   (c) decrease
   (d) be unaffected
10. The purpose of a 'snubber' circuit is to:
   (a) clamp voltage spikes at switch-off
   (b) clamp voltage spikes at switch-on
   (c) tune the Q of the inductor-capacitor circuit
   (d) produce a lower V_{out} from a high level V_{in}

CHAPTER 8



FAULT FINDING, TEST AND PRACTICAL TELEVISION CIRCUITS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- List the faults you expect to experience with diodes, resistors, capacitors, transistors and transformers.
- State the safety precautions to observe for yourself and the safety of the equipment and the customer.
- Explain the reason for using an isolation transformer when servicing switched mode power supplies (SMPS).
- Make minor repairs to printed circuit boards.
- Explain a basic procedure to adopt when fault finding.
- Describe the 'final checks' to make after a power supply has been repaired.
- Diagnose faults from given data in the following circuits:
  - a basic series regulator;
  - a closed loop regulator;
  - a three-terminal regulator; and
  - an SMPS.
- Discuss the operation of television switching power supply circuits.

8.1 Introduction

A high percentage of faults in electronic equipment are usually tracked down to the power supply. After all, the power supply is probably the hardest working area of any piece of equipment. It has to supply sufficient voltage and current to keep the device it is connected to working satisfactorily. It has to suffer the stress of abuse, because it must occasionally supply excess current due to a fault condition in its load, or dissipate excess power due to a fault condition in its input circuit. Under fault conditions, the life of a power supply is not a happy one, and if insufficient precautions are taken to protect it, it will fail through no fault of its own.

The basic assumption to make when servicing a power supply unit is that a fault has occurred in one of its components, and when the faulty component is located and replaced, the power supply will function correctly again. However, one thing that must be considered is did the power supply fail by itself, or was the failure induced by a fault in the circuits connected to it? This chapter outlines some fault-finding techniques and procedures for locating and rectifying failures in the circuits discussed in the preceding chapters.

8.2 Circuit measurements and component faults

8.2.1 Voltage measurements

Voltage measurements are made to check the AC input and/or DC output voltages of the power supply. They are the simplest to perform and do not necessitate any component removal. The instrument used is a voltmeter with a high sensitivity (greater than 20 000 Ω/V), so that the circuit under test is not loaded excessively by the meter. Remember to choose the range on the meter to suit the voltage you expect to find in the circuit. If in any doubt, start on the highest range to avoid damaging the meter movement.

8.2.2 Resistance measurements

Resistance measurements are not usually made with the components in circuit, since parallel paths can often give a misleading reading. However, when servicing power supplies, or circuits connected to power supply outputs, checking the output line for any obvious short circuit can be useful for diagnosis. *Remember* that all resistance measurements in a circuit *must* be carried out with the equipment unplugged and with any large capacitors discharged to avoid damaging the meter. A further point to be aware of is the polarities on the test probes when measuring areas containing diodes and transistors: the potential at the test probes may easily forward bias the rectifier diodes in a power supply and thus indicate a false result.

8.2.3 Current measurements

Current measurements are usually only made when unusual faults occur because it is often difficult to break the circuit and insert the meter—especially on printed circuit boards. However, they are an essential measurement in the final testing of the power supply in order to determine the DC output current available.

8.2.4 Oscilloscope measurements

An oscilloscope can be an invaluable piece of test equipment and can quickly give a visual indication of a circuit failure. For example, if the waveform of a half-wave rectifier circuit appears at the output of a full-wave rectifier circuit, then the area of the fault is located instantly. It is essential to use an oscilloscope in the **final testing** of the power supply to check the amplitude of the AC ripple on the DC output voltage.

8.2.5 Component faults

Capacitors

Capacitors usually have one of three faults, as discussed in Chapter 2:

1. *Open circuit:* it has little if any capacitance, and no meter deflection is given when checked with an ohmmeter.
2. *Short circuit:* in this case it will read zero ohms when checked with an ohmmeter.
3. *Leaky:* in this case it will have a resistance value that is considerably less than infinity.

Resistors

Resistors can change in value, due to excessive current being passed through them, or they may be completely open circuit.

Transistors

Transistors suffer from the same problem as diodes, and these were discussed in Chapter 1. The usual problems encountered with semi-conductor devices are:

1. *Short-circuited junction,* caused by a high-voltage surge.
2. *Open-circuited junction,* usually caused by a heavy overload, and hence excessive current.
3. *High leakage current,* usually accompanied by low gain or a high noise level.

Transformers and inductors

Transformers and inductors generally develop an open circuit, but sometimes a short circuited turn may be found. The testing procedure was discussed in Chapters 1 and 2. A device that may prove useful when checking coils is a shorted turns tester. This is an oscillator circuit in which the coil under suspicion is put across the winding. If there is no fault, the frequency and current drawn will only vary slightly. However, if it has a shorted turn, the oscillator will stop, and a large current will be drawn through the circuit.

8.3 Safety precautions

There are three aspects to consider when it comes to **safety** in the workshop or on the job:

1. *Safety of individuals:* this means your safety and, in most cases, you are responsible for it.
2. *Safety of the equipment:* your employers do not want their precious, and probably expensive, test equipment damaged.
3. *The personal safety of the customer:* the customer must not be able to touch any bare metal surfaces.

Electricity is a silent medium and should be treated with caution and respect. If you are careless when handling electronic equipment, quite often the first thing you know about it is a terrible sensation up your arm or across your chest. Very small electric currents of 100 mA, often less, through the body are potentially lethal. It is important to remember that it is not a matter of how much voltage is required to produce this current, rather, the crucial factor is your body resistance. From the famous Ohm's law, $I = V/R$, we can determine this important information. If your body resistance is high, say 10 000 Ω, then you would require 1000 V to produce 100 mA of current. As your body resistance decreases, so does the amount of voltage required to kill you.

Technicians and servicers who work on 'low voltage' equipment are often lulled into a false sense of security. In a situation where body resistance might be as low as 100 Ω, then 10 V is potentially lethal. Such a situation might be unlikely in practice, but consider this: if you suddenly switched from servicing a low voltage power supply to a high voltage supply, would you also remember to switch your safety precautions? The probable answer is 'No'; therefore, you are in danger of becoming a statistic.

In electronics you are likely to encounter power supplies of all shapes and sizes. You will certainly experience a 'transformerless' circuit or a switched mode power supply, working directly from the AC (as in Figure 6.2 of Chapter 6). With these circuits there is always the possibility of the 'live' and 'neutral' wires of the AC input becoming swapped over. If this happens in a 'transformerless' circuit, then the chassis of the equipment could be 'live'.

Therefore, adopt a sensible attitude to safety, do not be careless, and do not fall into bad habits, such as unsoldering components from energised circuits.

Always use an isolation transformer between the AC mains and the equipment you are working on. This will prevent a direct short circuit being placed across the mains and will protect both the technician and the test equipment. If you are building a workbench, an **isolation transformer** is an essential piece of equipment for the workshop. It must be wired between the AC mains input and a designated **general power outlet (GPO)** on your workbench (make sure you label it). The volt per amp rating of the isolation transformer must be sufficient to handle the power requirements of the equipment you intend to service on the workbench, and a typical value is 240 V/A (or watts).

Only one AC outlet should be used on the transformer secondary because, if two pieces of equipment (such as two television receivers) are plugged into the isolated side, the transformer no longer provides effective isolation. The inputs are floating and, depending on the configuration of the mains input lead and the rectifier circuit, contact between the two chassis could be lethal. The isolation transformer will only protect against the mains earth neutral (MEN), it will *not prevent an electric shock* from within the equipment itself. Therefore, remember the old 'keep one hand in the pocket' rule to avoid the risk of the most dangerous through the heart shock! A further safety suggestion is to use a residual current device (RCD) or earth leakage circuit breaker (ELCB) in the primary side of the transformer.

Equipment damage usually occurs as a result of carelessness or sloppy practices. Before using a meter, check the position of the range selector. Be sure you do not check the continuity of a coil and then measure a DC voltage. Keep the workplace tidy. There is no excuse for a meter being knocked off the bench because it got caught up in a power lead when a piece of equipment was being removed after servicing.

8.4 Component replacement and circuit board repairs

8.4.1 Component replacement

When replacing faulty components in power supply circuits it is important to obey two rules:

1. Replace the faulty component with an identical or equivalent component.
2. Position the replacement component in the exact physical location from which the faulty component was removed.

It is important to make sure that the replacement component is the same or equivalent because some power supplies use special components. A three-terminal regulator circuit uses tantalum capacitors. They have a smaller physical size, but they also have a much higher frequency capability than standard electrolytics. The capacitors used in a switched mode power supply must have a low equivalent series resistance (ESR). Likewise, transistors and diodes used in switching circuits must have fast rise and fall times (such as Schottky diodes). Some power supplies will use wire-wound resistors in place of filter chokes because they exhibit a small amount of inductance and aid filtering. The DC working voltage of

replacement components should also be watched. The working voltage of the replacement may be higher than the original, but never lower.

Concern with the exact positioning of the component may seem a little unnecessary. Some might say that if it is wired in, then it must work. This may be true, but certain components such as filter chokes have quite a large magnetic field associated with their operation. When a power supply is designed, magnetic radiation is taken into account, and components are placed in the optimum working positions. If this position is altered, interference may result, giving rise to hum or higher ripple amplitude. Fortunately, printed circuit boards have almost superseded the choice of physical locality: in these circuits the new part must go back where the faulty part came from. However, there are still a few custom-built power supplies around which do not use printed boards, and positioning of components and leads can be an important consideration.

8.4.2 Printed board repair

When printed circuit boards are used in power supplies, excessive current can be a problem. The high current heats the printed copper track, and this can melt the adhesive with the result that the copper track lifts off the board. Small-scale repairs, such as resticking loose copper track or bypassing hairline cracks, can be economically carried out, but more complicated repairs often justify replacing the whole board. If the board has been subjected to a heavy current due to excessive load conditions, the latter is the more practical solution. New components should also be fitted to the board rather than removing them from the old board and resoldering them to the new board.

Loose copper tracks can usually be restuck to the board by carefully applying a quick-setting epoxy and holding the track to the board until the adhesive has set. Hairline cracks can be bypassed with a short length of wire soldered to the copper track. The track must be clean and the heat should be applied only as long as necessary. If a long 'jumper' wire is required, it should be insulated to prevent accidental short circuits to other parts of the circuit. Care should be taken when soldering not to short out printed tracks that are in close proximity to the track being repaired.

8.4.3 Component replacement on printed circuit boards

As mentioned in the previous section, heat can melt the adhesive on the board and cause the copper track to lift off. Sometimes a component can be very stubborn during removal, and the heat applied to free it could become excessive and damage the board. In this case, note the value and working voltage (if applicable) of the component, ensure that you have a replacement, and then cut the component from its mounting leads. The mounting leads are then left protruding through the top of the board. These can be cleaned, the new component leads trimmed to size and soldered to the protruding leads, using a minimum amount of heat.

Ideally, the solder should be removed from the printed circuit and the new component fitted directly to the board. Special desoldering irons are available which suck the solder away from the board, but these are relatively expensive. A mechanical spring-loaded 'solder sucker' and 'solder wick' (flux-coated copper braid) are also very good for removing solder

from the printed circuit. However, whichever method is used, heat must be kept to a minimum to ensure that there is no lifting of the copper track.

8.5 Fault-finding procedure and testing techniques

When a power supply unit requires service, the fault must be isolated to a particular area (as with all electronic equipment), and a **logical procedure** is required for efficient and economical servicing. General troubleshooting techniques are as follows, and these may be applied to power supplies.

1. Basic assumption

As with all radio, television and electronic servicing, the basic assumption is that only one fault is causing the malfunction of the power supply. When the fault is located, and the defect rectified, normal operation will be restored. If normal operation is not restored, a second source of trouble should be investigated.

2. Isolate the defective section

Refer to any service data available, and familiarise yourself with the circuit layout and characteristics. A visual inspection of the suspect section will reveal any obvious faults, such as broken printed track, broken components, burnt out resistors and, if it is a valve unit, unlit valves. If the fault is not found visually, checks should be made at certain test points to determine the circuit DC voltage conditions.

3. Isolate the defective stage

When the defective section has been located, test equipment is used to isolate the exact fault locality.

4. Isolate the defective component

After the faulty stage has been located, voltage and resistance checks are used to locate the defective component. Substitution of known good components for suspect components may be required to help in the diagnosis, especially on more complex circuits.

5. Correct the defect

The defective part is replaced with the exact same or equivalent component.

6. Preventive maintenance

When operation has been restored to normal, other defects that may cause trouble later—such as cracked resistors, overheated and leaky capacitors—should be replaced. If it is a valve unit, valves that flash or arc when lightly tapped are also a potential source of trouble.

When defective components are replaced, bear the following points in mind:

- Keep the same positions for earth return and connecting leads.
- Lead-dress (that is, the physical layout of the wiring) is critical with high frequencies to prevent feedback, also at high voltages to prevent arcing.

- Resistors may be replaced with ones of a higher wattage, but not lower.
- Capacitors may be replaced with the same type of higher working voltage, but not lower.
- Mica, ceramic or polyester capacitors should not be replaced with paper capacitors, as their inductance will alter the coupling or bypassing at high frequencies.

When the circuit operation appears to be satisfactory, the power supply parameters (operational statistics) should be tested.

8.5.1 Power supply testing

The main points which should be tested after a power supply has been repaired are:

- DC output voltage;
- DC output current available;
- ripple voltage amplitude at the output;
- voltage regulation, or current regulation if a constant current supply; and
- circuit stability with variations in the mains input voltage.

A power supply can be tested for satisfactory operation by the connection of an AC voltmeter, a DC voltmeter, a DC ammeter and an oscilloscope as shown in Figure 8.1.

The AC input voltage should be checked and noted. Unfortunately, the only means available for adjusting the AC input is to connect the unit to the mains supply via a 'variac' (adjustable auto-transformer). The input voltage should be adjusted to the nominal voltage setting on the power supply: 220 V, 230 V or 240 V. The DC voltage should be measured under full load conditions. With the load set to maximum resistance, note the output voltage, then slowly decrease the resistance until the current in the load reaches the maximum value—there should be little change in the output voltage. Take care not to short circuit the output voltage by adjusting the variable resistance to zero ohms. The power supply should now be delivering maximum current at the rated output voltage.

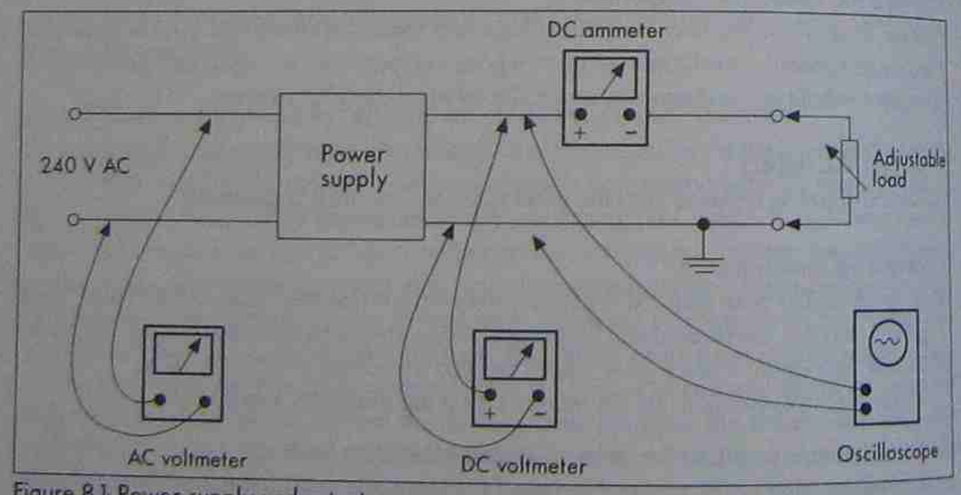


Figure 8.1: Power supply under test

The peak-to-peak ripple amplitude can be checked with the oscilloscope. Set the oscilloscope to AC and adjust the vertical amplitude to a sensitive range. The peak-to-peak amplitude depends on the type of supply and filtering arrangement, but should be quite low—typically in the range of 5 mV to 20 mV.

Voltage regulation is determined by measuring the output voltage no-load and full-load conditions. Likewise, with current regulation, the output current is measured under no-load and full-load conditions as explained in Chapters 2 and 3. The formula used expresses the regulation as a percentage:

$$\% \text{ regulation} = [(V \text{ or } I_{NL}) - (V \text{ or } I_{FL})] \times 100/(V \text{ or } I_{FL})$$

where NL = no load and FL = full load

If the regulation is good (that is, the percentage is low), the difference between the no-load and full-load conditions will be very small. This being the case, a digital meter may be more accurate in recording the values. Ensure that the AC input voltage is kept constant when making regulation measurements.

Finally, the stability of the circuit should be checked. This gives an indication of how much the DC output voltage changes with a change in AC input voltage. Most electronic equipment must be able to handle a swing in AC input voltage of approximately ±8%. This means the AC input can vary from 220 V to 260 V, about an average of 240 V AC. The input voltage is adjusted by varying the variac and monitoring the AC voltmeter across the input. As the input voltage is varied from 220 V to 260 V, the DC output voltage is noted, with the load set at maximum. If a swing of ±8% at the AC input causes the DC output to vary by 50 mV from a rated value of 12 V, then the output changes by:

$$\frac{\text{voltage change}}{\text{original voltage}} \times 100 = (50 \times 10^{-3})/12 \times 100 = \text{approx } 0.4$$

The stabilisation is expressed as a ratio: ±8% is a total possible variation of 16%. Therefore, a variation of 16% at the input causes a variation of 0.4% at the output. The line stabilisation is thus 16:0.4 or 40:1.

8.5.2 Fault condition and symptom tables

In Figure 8.2 a fault may be in the transformer, the rectifier diodes, the filter section or the

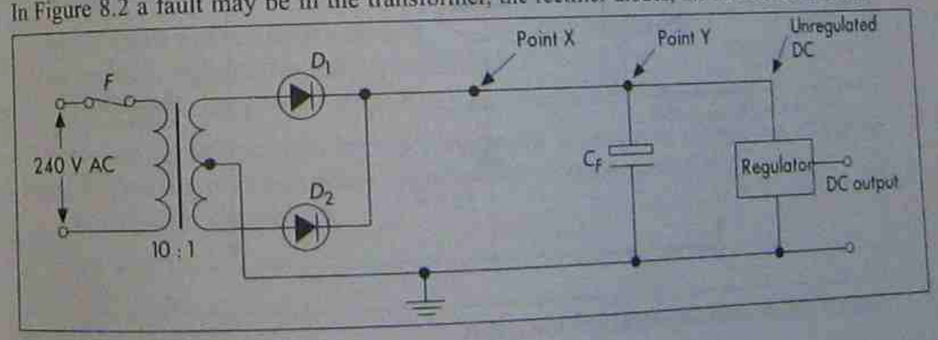


Figure 8.2: Power supply circuit

regulator. Diagnosis should start with a few obvious checks. First, ensure that the AC mains is plugged in and switched on. Check the DC output voltage. If no output, check that the AC is reaching the transformer secondary. If no AC at the secondary, three faults are possible:

1. open-circuit primary or secondary;
2. open-circuit mains wire inside the plug; or
3. blown fuse.

To check these, unplug the mains and perform continuity checks with an ohmmeter. Do not rely on a visual check for the fuse—use the ohmmeter. Alternatively, the fuse may be checked by measuring the voltage across it (using the correct range). If a voltage appears across the fuse it is open circuit.

If the fuse has blown, a fault condition in the power supply circuit has caused it. This fault must be located and repaired prior to fitting a new fuse.

Ensure that the main filter capacitor (C_F) is discharged, and perform a continuity check from the junction of diodes D_1 and D_2 to earth (watch the polarity of the test probes). If a short circuit or very low resistance results, disconnect the output line just before the filter capacitor at point X and test again—this isolates the fault area. If the same reading results, the fault is either in the rectifier diodes or in the transformer; if there is a different reading, the fault is either the capacitor C_F or the regulator. Disconnect the circuit at point Y and perform continuity checks across C_F and the regulator. Should the capacitor check out satisfactorily, the problem will be in the regulator circuitry. Voltage readings are then taken around the various test points in the regulator to determine the faulty component.

Typical faults and their symptoms in the area from the AC mains input up to the regulator input are listed in Table 8.1.

The next area of concern in the power supply unit is the regulator itself. A series regulator circuit is shown in Figure 8.3. Fault finding in the regulator is usually straightforward.

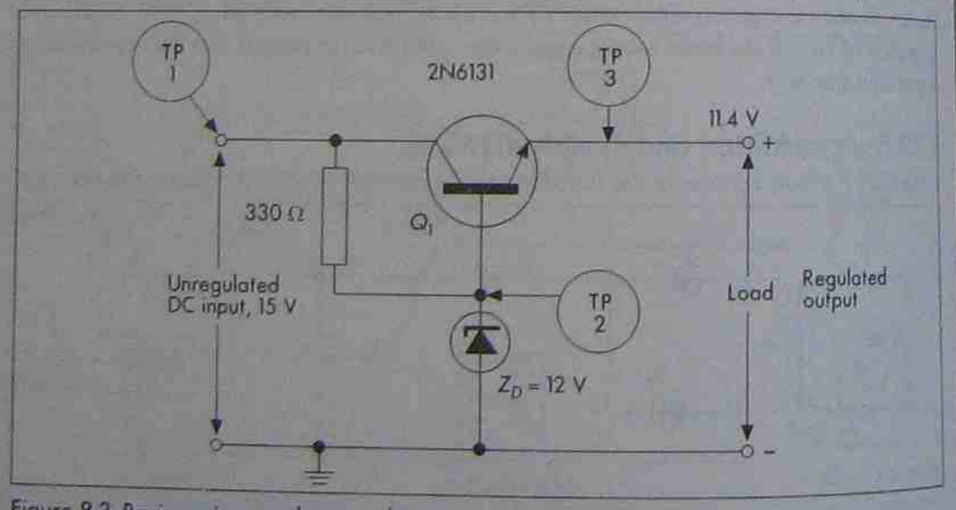


Figure 8.3: Basic series regulator with test points

Table 8.1: Table of faults and symptoms in the transformer, rectifier and filter stages of a power supply

Symptoms	Faults
Transformer faults	
Zero DC output, and no secondary voltage. Low resistance readings on both primary and secondary.	Open-circuit AC input lead or blown fuse.
Zero DC output, and no secondary voltage. Infinity resistance reading across primary, and low resistance reading across secondary.	Open-circuit primary on mains transformer.
Zero DC output, and no secondary voltage. Infinity resistance reading across secondary, and low resistance reading across primary.	Open-circuit secondary on mains transformer.
Low DC output, and transformer excessively hot. (Fuse may also blow.)	Short-circuited turn on mains transformer primary or secondary.
Rectifier faults (full-wave and bridge)	
Low DC output with 50 Hz ripple. Poor regulation.	Open-circuit rectifier diode.
Fuse blown due to excessive current. Resistance check on transformer OK.	Short-circuit rectifier diode.
Circuit operates OK, but DC output is lower than expected.	High resistance diode. (High forward voltage drop.)
Filter capacitor faults	
Low DC output, with high level of ripple. Very poor regulation.	Filter capacitor open circuit.
Fuse blown. Resistance of the unregulated DC line is low in both directions.	Filter capacitor short circuit.
Lower DC output, with increase in ripple level. Poor regulation. (Equipment function may be unstable.)	Filter capacitor leaky.

Typical working DC voltages are compared to the actual circuit voltage, and a diagnosis made. The voltage reference points are referred to as **'test points'** (TP).

The test-point voltages are:

- TP 1 = 15 V
- TP 2 = 12 V
- TP 3 = 11.4 V

A table of faults in regulator circuits and their symptoms is shown in Table 8.2.

Table 8.2: Table of faults and symptoms in regulator circuits

Symptoms	Faults
Zero DC output, and the unregulated DC input slightly higher than normal (no current drawn).	Series transistor base-emitter open circuit.
Input and output voltages equal, may blow a fuse, or destroy the transistor due to excessive current.	Series transistor collector-emitter short circuit.
Low DC output, series transistor hot because $V_{CE} = V_{in} - V_{out}$	Zener diode short circuit.
No reference voltage. Output voltage may be high or low, depending on circuit configuration (see Figure 8.3), V_{BE} increased, DC output high. No regulation.	Zener diode open circuit.
Higher than normal DC output, no regulation and no circuit control.	Open circuit error amplifier (transistor or IC).
Reference diode hot due to excessive current. No circuit control, DC output low: 600 mV lower than zener voltage.	Short-circuit error amplifier.

8.6 Fault diagnosis

In this section we will discuss the diagnosis of faults from information that is known about the DC circuit conditions at the test points.

8.6.1 Fault diagnosis on a series regulator

The first circuit is a basic series regulator with an error amplifier. Its operation was described in section 3.4, and the circuit under investigation is shown in Figure 8.4. The normal DC voltages are measured with a 30 000 Ω/V multimeter.

Normal working conditions

The test-point voltages for **normal working conditions** are TP 1 = 30 V, TP 2 = 20 V, TP 3 = 10 V, TP 4 = 10.8 V and TP 5 = adjustable voltage. (Ripple voltage level = 80 mV peak-to-peak. TP 5 set to 20 V for this exercise.)

Test-point voltages under fault conditions

1. All test point voltages are zero volts. Therefore, the fault must exist in a stage prior to the regulator, that is, the transformer, the rectifier, the filter or the fuse.

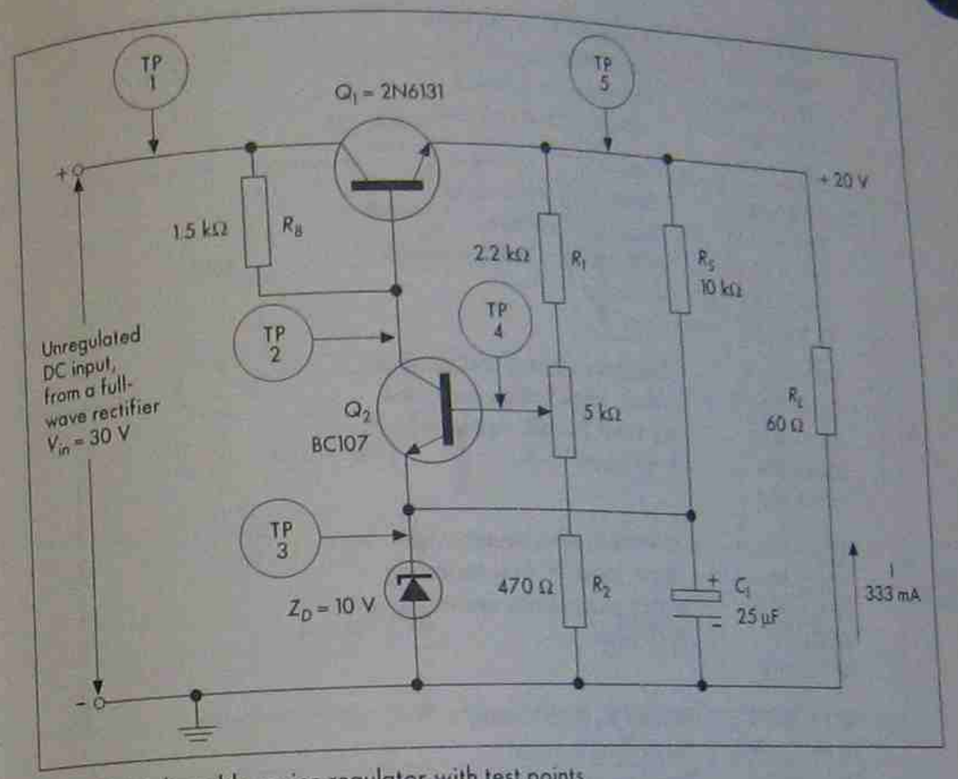


Figure 8.4: Adjustable series regulator with test points

2. TP 1 = 33 V — R_B open circuit.
   TP 2 = 0 V — No initial forward bias
   TP 3 = 0 V — for transistor Q_1. Input
   TP 4 = 0 V — DC high, indicating no current
   TP 5 = 0 V — being drawn.

3. TP 1 = 30 V — Transistor Q_2 open
   TP 2 = 24 V — circuit. Ripple level
   TP 3 = 10 V — very high. No adjustment.
   TP 4 = 13 V
   TP 5 = 23 V

4. TP 1 = 28.5 V — Transistor Q_1 short circuit.
   TP 2 = 21 V — Ripple level high.
   TP 3 = 10 V — Input voltage equals output
   TP 4 = 11.2 V — voltage.
   TP 5 = 28.5 V

5. TP 1 = 32 V
   TP 2 = 2.5 V
   TP 3 = 0 V
   TP 4 = 0 V
   TP 5 = 2.0 V
   Capacitor C_1 or the zener diode short circuit. No reference, no feedback voltage, output very low.

6. TP 1 = 30 V
   TP 2 = 24 V
   TP 3 = 22 V
   TP 4 = 13 V
   TP 5 = 23 V
   Zener diode open circuit. High ripple voltage. Reference too high, Q_1 turned on harder.

7. TP 1 = 33 V
   TP 2 = 33 V
   TP 3 = 0 V
   TP 4 = 0 V
   TP 5 = 0 V
   Transistor Q_1 base-emitter open circuit. Input voltage increased. TP 1 and 2 equal, no *I* through R_8 to Q_1 base.

8. TP 1 = 29.5 V
   TP 2 = 24 V
   TP 3 = 10 V
   TP 4 = 1 V
   TP 5 = 23 V
   Resistor R_1 open circuit. High ripple. Input DC low, indicating higher current being drawn. No drive to Q_2 base.

9. TP 1 = 30.5 V
   TP 2 = 11.5 V
   TP 3 = 10 V
   TP 4 = 11 V
   TP 5 = 11 V
   Resistor R_2 open circuit. Ripple level reduced. Transistor Q_2 on harder, and Q_1 turned off.

8.6.2 Fault diagnosis on a current-limited series regulator

The next circuit to consider is a current-limited regulator. Figure 8.5 shows the circuit configuration that was discussed in Chapter 3. The output voltage is 20 V at a maximum useable current of 500 mA.

Normal working conditions

The test-point voltages for normal working conditions are TP 1 = 30 V, TP 2 = 20 V, TP 3 = 10 V, TP 4 = 10.8 V, TP 5 = 20 V and TP 6 = 20 V. (Ripple voltage level = 20 mV peak-to-peak and maximum current = 500 mA.)

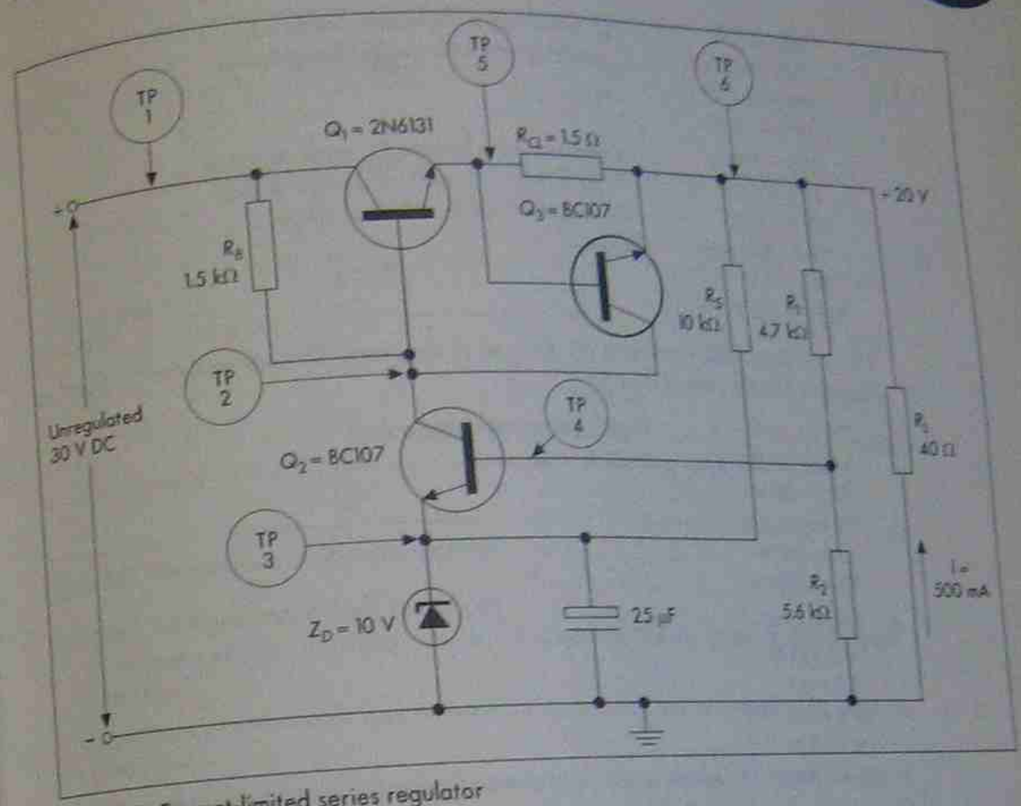


Figure 8.5: Current-limited series regulator

Test-point voltages under fault conditions

1. The value of R_L is decreased, and therefore excess current is demanded (I = 570 mA).
   TP 1 = 30 V
   TP 2 = 23 V
   TP 3 = 10 V
   TP 4 = 6.5 V
   TP 5 = 12.5 V
   TP 6 = 11.8 V
   The excessive current flow through current-sensing resistor R_{CL} increases the voltage across Q_3 base-emitter junction. Transistor Q_3 turns on, and bypasses a portion of Q_1 base current. Less current is available to Q_1 base, therefore TP 2 voltage increases. Transistor Q_1 has its conduction reduced, and the output voltage is decreased.

2. TP 1 = 29 V
   TP 2 = 20 V
   TP 3 = 10 V
   TP 4 = 10 V
   TP 5 = 19 V
   TP 6 = 19 V
   Excessive current is demanded (750 mA) and Q_3 base-emitter is open circuit. Ripple level is higher, DC output is only reduced slightly. Load could be damaged because circuit does not reduce the output voltage availability.

3. TP 1 = 32 V
   TP 2 = 1.2 V
   Transistor Q_3 has a short circuited base-emitter. Ripple level high. TP 2 pulled down and

TP 3 = 10 V
TP 4 = 750 mV
TP 5 = 1.2 V
TP 6 = 1.2 V

Q_1 turned off. Input voltage increased, indicating very little current being drawn.

4. TP 1 = 32V
TP 2 = 2.4 V
TP 3 = 2 V
TP 4 = 600 mV
TP 5 = 2 V
TP 6 = 1.2 V

Current-sensing resistor R_{C1} open circuit. Input voltage increased, no current drawn. No output circuit, therefore no output voltage.

The next three fault conditions are for you to diagnose. The answers can be found in Appendix A at the back of the book.

5. TP 1 = 30 V
TP 2 = 21.5 V
TP 3 = 20 V
TP 4 = 11.4 V
TP 5 = 21 V
TP 6 = 20.5 V

Higher than normal ripple level. Fault: ?

6. TP 1 = 33 V
TP 2 = 33 V
TP 3 = 0 V
TP 4 = 0 V
TP 5 = 0 V
TP 6 = 0 V

Fault: ?

7. TP 1 = 30 V
TP 2 = 21.5 V
TP 3 = 10 V
TP 4 = 11.4 V
TP 5 = 21.5 V
TP 6 = 21 V

Higher than normal ripple level. Fault: ?

8.6.3 Fault diagnosis on a three-terminal regulator

A slightly different technique is adopted when diagnosing faults in three-terminal regulator circuits. The actual components of the device are not accessible for testing, and therefore test points are limited to the input, output and third terminals. Figure 8.6 shows an adjustable three-terminal regulator circuit with test points. The output is adjusted to 25 V and the load current is 500 mA.

Normal working conditions

The test-point voltages for normal working conditions are TP 1 = 30 V, TP 2 = 13 V and TP 3 = 25 V. (Ripple voltage level = 10 mV peak-to-peak and output current = 500 mA.

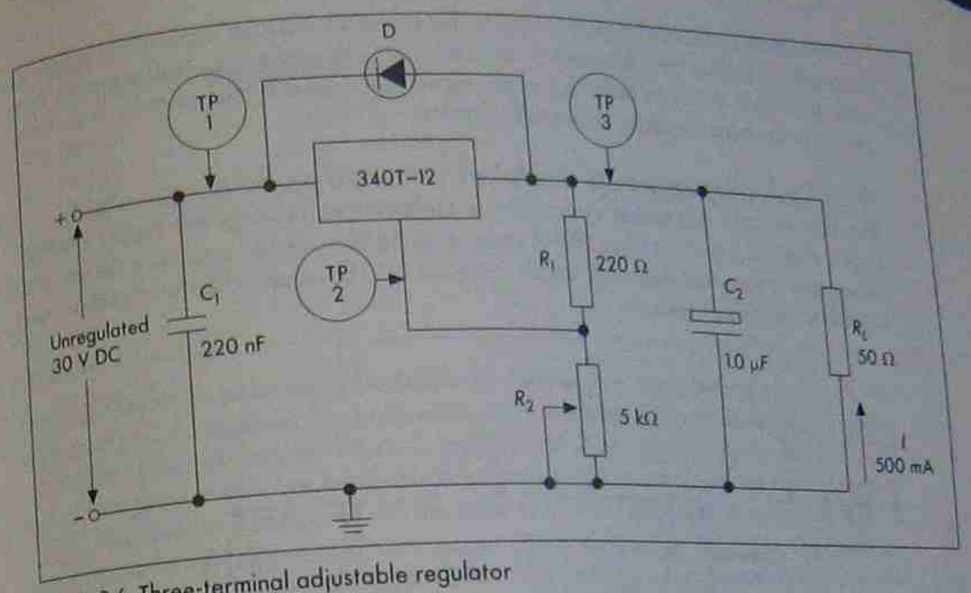


Figure 8.6: Three-terminal adjustable regulator

Test-point voltages under fault conditions

1. Diode open circuit: no change in circuit operation, but no input or output circuit protection.
2. Capacitor C_1 open circuit: may increase the ripple level, but no change in DC working conditions.
3. Capacitor C_2 open circuit: transient response poor, but no change in working conditions.
4. TP 1 = 0 V
TP 2 = 0 V
TP 3 = 0 V

Fuse blown upon inspection, capacitor C_1 short circuit.

5. TP 1 = 28 V
TP 2 = 0 V
TP 3 = 0 V

No DC output or adjustment voltage. Capacitor C2 short circuit, or integrated circuit (IC) open circuit.

6. TP 1 = 31 V
TP 2 = 1.2 V
TP 3 = 13 V

Resistor R_1 open circuit. Input DC increased, less current being drawn. No adjustment voltage feedback.

7. TP 1 = 30 V
TP 2 = 26.5 V
TP 3 = 26.5 V

Resistor R_2 open circuit. Full adjustment voltage fed back. Output voltage and current increased. Ripple high.

8. TP 1 = 29 V
TP 2 = 17 V
TP 3 = 29 V

Input and output voltages equal. Ripple high. Diode or IC short circuit.

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With regulator circuits that employ feedback, that is, the 'closed loop' type, it is important to realise that a fault in the comparator or error amplifier can give a false fault symptom. In a complex circuit it may be necessary to disconnect the feedback loop in order to perform checks on the basic regulating circuit test points.

8.6.4 Fault diagnosis on a simulated switched mode power supply

The final circuit configuration to consider is the switched mode power supply (SMPS) circuit. Figure 8.7 shows a basic SMPS concept, using the variable resistance feature of a transistor to 'pulse-width modulate' an NE 555 timer IC. The output of the NE 555 drives the base of the switching transistor.

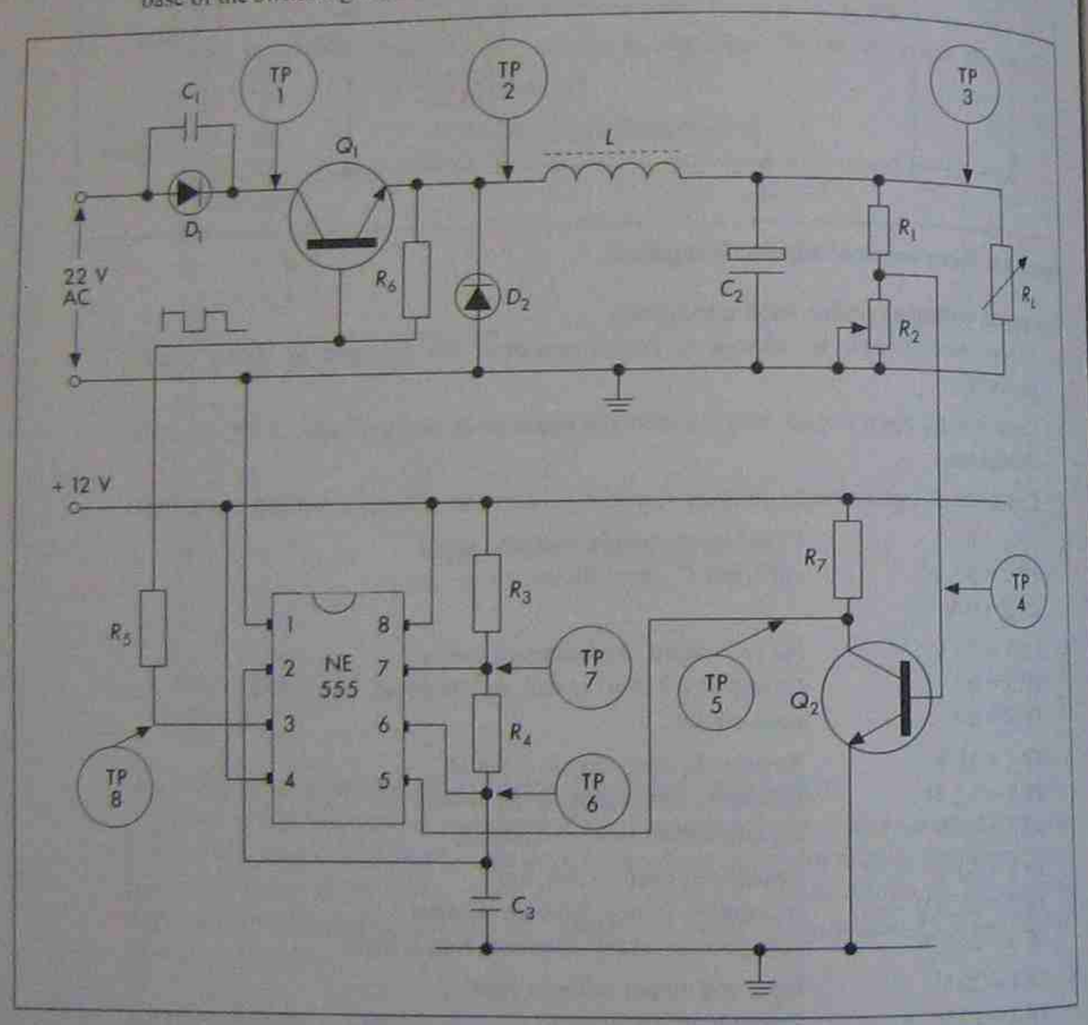


Figure 8.7: Switched mode power supply

Parts list

The parts list for the SMPS circuit follows:

Q_1 = 2N6131
Q_2 = BC107
R_1 = 2.2 kΩ
R_2 = 10 kΩ potentiometer
R_3 = 10 kΩ
R_4 = 68 kΩ
R_5 = 100 Ω
R_6 and R_7 = 1 kΩ
D_1 and D_2 = power diodes
L = 41 mH (approx)
C_1 = 2.2 nF
C_2 = 1000 μF
C_3 = 10 nF

The NE 555 can be pulse-width modulated by varying the voltage applied to pin 5. As the voltage at pin 5 is increased, the width of the mark increases. Resistors R_1 and R_2 provide an adjustable 'feedback voltage' which is proportional to the output voltage. Resistor R_2 is adjusted to provide sufficient forward bias for transistor Q_2 to set the voltage level at pin 5 of the IC, and so drive transistor Q_1. As the value of the load resistor R_L is decreased, there is more demand on the power supply for current. The output voltage level falls, and hence the feedback voltage at Q_2 base is reduced. The forward bias (V_{BE}) of Q_2 is reduced, less collector current is drawn, and the potential at Q_2 collector rises. This increase in voltage is applied to pin 5 of the IC. The mark width increases, thereby increasing the 'on' time of transistor Q_1. Transistor Q_1 conducts for a slightly longer period of time, and this restores the output voltage to its former value.

Normal working conditions

The test-point voltages for normal working conditions are:

TP 1 = 12 V
TP 2 = 5.2 V
TP 3 = 5.0 V
TP 4 = 660 mV
Output voltage = 5 V
TP 5 = 8.2 V
TP 6 = triangular wave
TP 7 = trapezoid wave
TP 8 = spiky square wave

The spikes indicate that the oscillator is working.

Test-point voltages under fault conditions

1.	TP 1 = 10 V TP 2 = 0 V TP 3 = 0 V TP 4 = 0 V	TP 5 = 9.5 V TP 6 = no oscillation (osc) TP 7 = no osc TP 8 = no osc	Multivibrator (NE 555) failed. No drive to switching transistor base.
2.	TP 1 = 10 V TP 2 = 0 V TP 3 = 0 V TP 4 = 0 V	TP 5 = 9.5 V TP 6 = osc TP 7 = osc TP 8 = osc	Diode D_2 short circuit. Transformer buzzes due to excess I. Mark-to-space ratio increases.

3.	TP 1 = 12.5 V TP 2 = 5.2 V TP 3 = 5 V TP 4 = 660 mV	TP 5 = 8.2 V TP 6 = osc TP 7 = osc TP 8 = osc	Diode D_2 open circuit, unstable oscillator, inductive spikes top and bottom, pulsing waveform.
4.	TP 1 = 20 V TP 2 = 20 V TP 3 = 20 V TP 4 = 800 mV	TP 5 = 0 V TP 6 = no osc TP 7 = no osc TP 8 = no osc	Transistor Q_1 short circuit collector-emitter Input equals the output voltage. High feedback voltage shuts the oscillator off.
5.	TP 1 = 15 V TP 2 = 7 V TP 3 = 0 V TP 4 = 0 V	TP 5 = 9.5 V TP 6 = osc TP 7 = osc TP 8 = osc	Inductor L open circuit. No inductive spike at TP 8.
6.	TP 1 = 10 V TP 2 = 3 V TP 3 = 0 V TP 4 = 0 V	TP 5 = 9.5 V TP 6 = osc TP 7 = osc TP 8 = osc	Capacitor C_2 short circuit. Mark-to-space ratio increases to try to increase output voltage.
7.	TP 1 = 11.8V TP 2 = 3.8 V TP 3 = 3.6 V TP 4 = 500 mV	TP 5 = 9.5V TP 6 = osc TP 7 = osc TP 8 = osc	Capacitor C_2 open circuit. Mark-to-space ratio decreased.
8.	TP 1 = 15 V TP 2 = 6V TP 3 = 5.8 V TP 4 = 1.5 V	TP 5 = 9.5 V TP 6 = osc TP 7 = osc TP 8 = osc	Transistor Q_2 open circuit, no control on the IC. Mark-to-space ratio increases.

The test-point voltages in faults 9 and 10 have been left blank, so that you can diagnose the expected voltages from the faults indicated. The answers can be found in Appendix A at the back of the book.

9.	TP 1 = TP 2 = TP 3 = TP 4 =	TP 5 = TP 6 = TP 7 = TP 8 =	Transistor Q_2 short circuit.
10.	TP 1 = TP 2 = TP 3 = TP 4 =	TP 5 = TP 6 = TP 7 = TP 8 =	Resistor R_5 open circuit. No inductive spike on the oscillator waveform.

8.6.5 Experimental switched mode circuit

An SMPS supplied from a full-wave rectifier circuit and an IC comparator is shown in Figure 8.8. The output voltage is 'sampled' and compared to a reference voltage. The comparator output then controls the multivibrator circuit (NE 555). This circuit is easy to build and is ideal for the experimenter. If you want to observe the circuit action, the IC can be replaced with a potentiometer connected as shown in Figure 8.9.

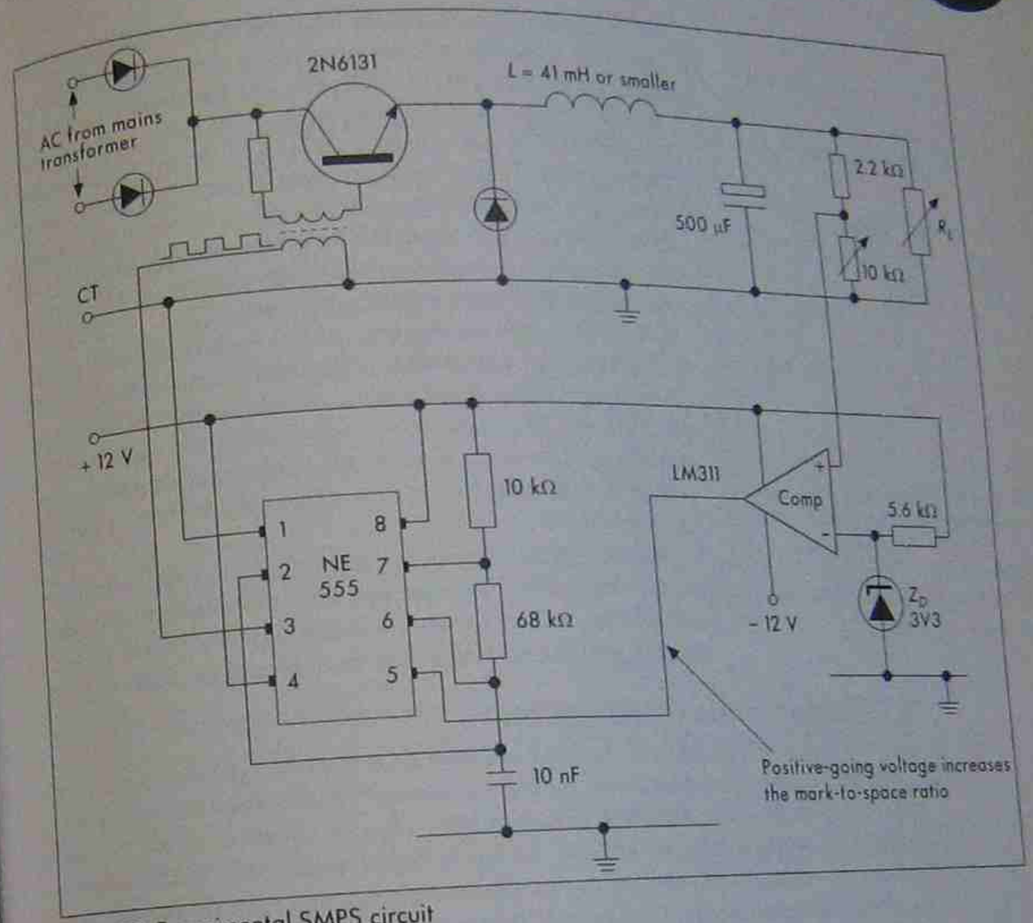


Figure 8.8: Experimental SMPS circuit

Testing procedure

Adjust potentiometer V_{R1} to give the desired output voltage. Decrease the load resistance, and you will notice a drop in the output voltage. Monitor the output waveform of the NE 555 and slowly increase the positive potential at pin 5 by varying V_{R1}. The mark-to-space ratio of the output waveform will increase, and this in turn will restore the output voltage level. Likewise, if the potential at pin 5 of the NE 555 is reduced, the mark-to-space ratio will decrease, and the output voltage will also decrease. This proves the theory that the longer the switching transistor conducts, the higher will be the output voltage, within the limitations of the input voltage.

8.6.6 Protection circuits

An experimental circuit for testing the effect of over-voltage and excess current protection is shown in Figure 8.10. Transistor Q_1 is the switching transistor, and Q_2 is the driver, which in turn is switched on and off by a multivibrator circuit (MV).

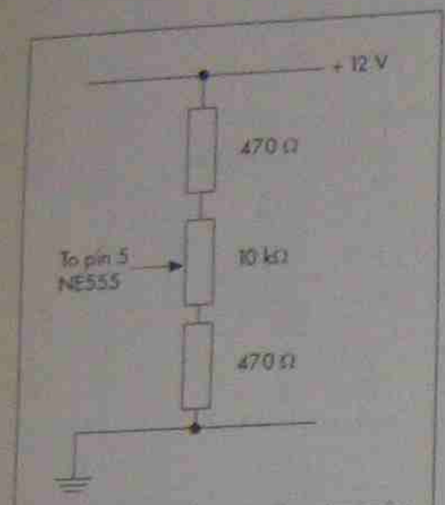


Figure 8.9: Replacement circuit for the comparator in Figure 8.8.

A silicon-controlled rectifier (SCR_1) is the crowbar protection to switch the input 'off' in the event of excess output voltage (this was discussed in Chapter 6). The SCR_2 provides protection in the case of excessive current. Pulses from Q_1 collector are fed to SCR_2 cathode via diode D_2. The gate voltage is set by V_{R1} to a potential just under the firing level. If a fault condition exists, excessive current will flow through Q_1. The negative-going current pulses will drive the cathode of SCR_2 negative with respect to the preset gate voltage. The SCR_2 will fire, and short out the supply to the collector of Q_2, thereby removing the drive to the switching transistor.

The over-voltage crowbar protection is connected directly across the supply voltage to the switching transistor. Normally the SCR will not conduct, but if the output voltage exceeds the zener diode voltage the zener will fire, capacitor C_3 will charge, and SCR_1 will be switched on. This puts a short circuit on the input supply from the cathode on D_1 to earth. Excess current flows and the fuse, or thermal trip, will blow.

With the advent of switching transistors and silicon-controlled rectifiers, the rectifier diodes are now more prone to destruction than they were before. High transient voltages exist in these circuits and, therefore, precautions need to be taken to protect the rectifier diodes.

8.7 Diode transient protection

The peak inverse voltage rating of the rectifier diodes must often be greater than the value calculated. This is necessary because of the likely presence of high voltage transients on the power line. Voltage spikes (transients) as high as 1000 V can be experienced, and these are the result of switching inductive loads on the power line. Typical loads which cause this problem are motors, transformers, switching regulator circuits and even silicon-controlled rectifier (SCR) light dimmers. The transients appear on the power transformer primary and are coupled to the secondary, therefore the rectifier diodes may experience a very high peak voltage.

The simplest way to protect the diodes against these transients is to use high current diodes, but these can be expensive. However, alternative methods are available, and these involve placing some line impedance in the transformer primary or secondary circuit. This is slightly detrimental to the circuit's basic regulation, but it can be overcome.

Transient protection relies on shunting the transient around the rectifier diodes to dissipate the transient energy in the protection circuit. Figure 8.11 shows a basic half-wave rectifier circuit complete with capacitive filter C_F. Resistor R, inductor L, and capacitors C_1 and C_2 all afford some protection to the diode. The combinations of components to provide **transient protection** are:

(a) Series resistor R and shunt capacitor C_1 in the primary circuit.
(b) Series inductor L and shunt capacitor C_1 in the primary circuit.

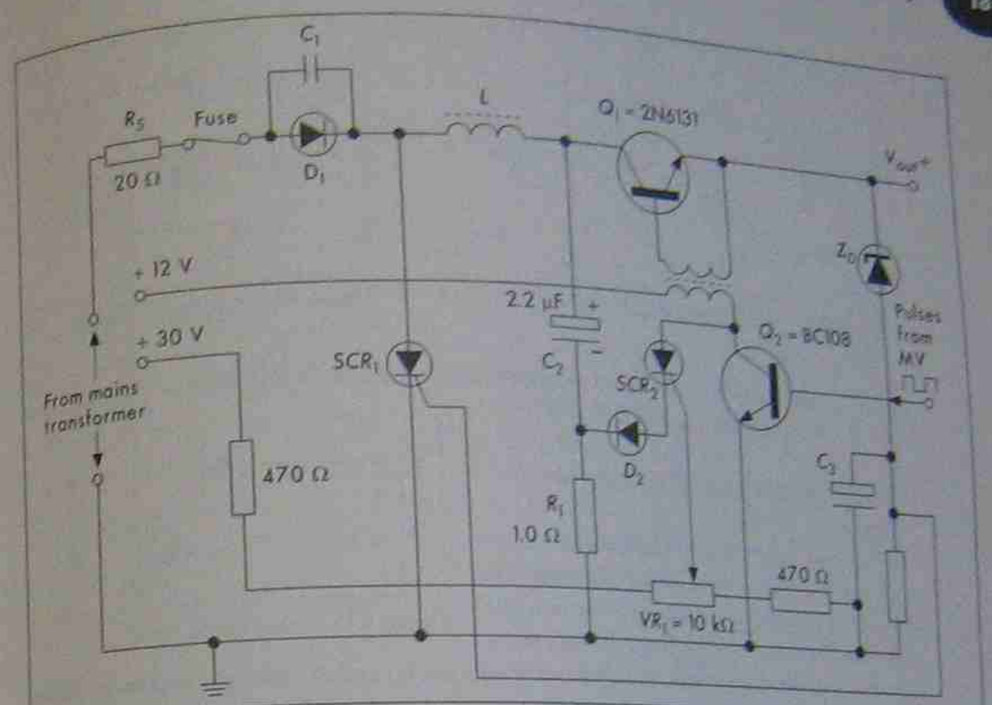


Figure 8.10: Experimental voltage and current-protection circuit

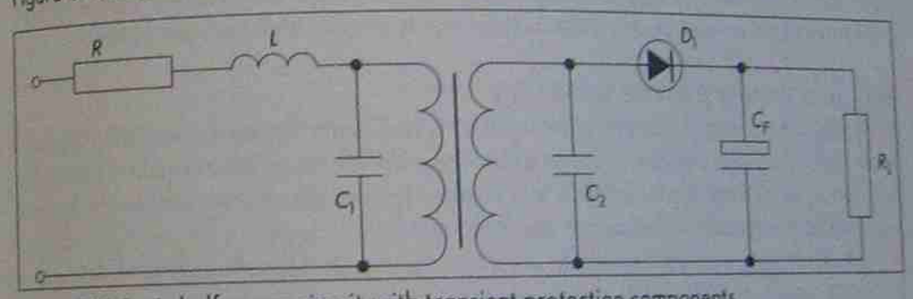


Figure 8.11: Basic half-wave circuit with transient protection components

(c) Shunt capacitor C_2 across the secondary winding.
(d) Further protection can be obtained by shunting the diode with a capacitor (see Figure 8.12).

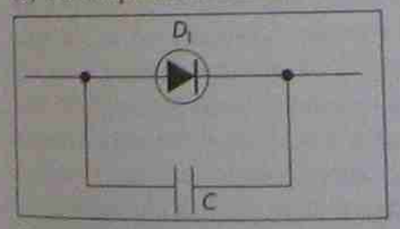


Figure 8.12: Rectifier diode shunted by C

(e) A surge suppression varactor can be placed across the diode (see Figure 8.13). This is very effective, but can be costly.
(f) A shunt-connected dynamic clipper circuit, consisting of a resistor, capacitor and another diode (see Figure 8.14).

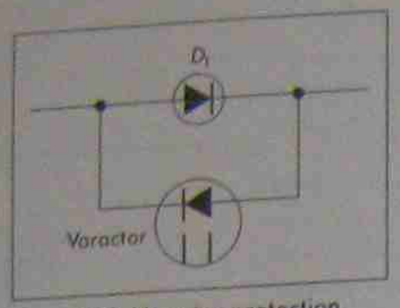


Figure 8.13: Varactor protection

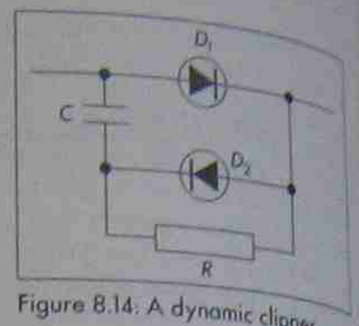


Figure 8.14: A dynamic clipper

(g) A zener diode shunt across the rectifier diode. A series resistance may be included in the zener branch (see Figure 8.15).

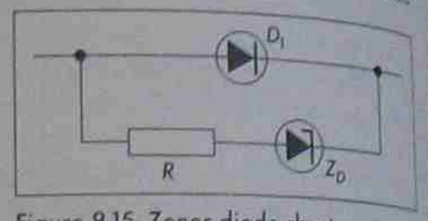


Figure 8.15: Zener diode shunt

8.7.1 Comparison of protection circuits

Circuits (a), (b), (c) and (d) above are the cheapest, but they are limited in their ability to provide complete protection. Circuit (d) is the one that provides the best protection for the money, and it is quite suitable for low current supplies. Circuits (e), (f) and (g) are more expensive circuits because they use a further active device. However, they do provide the greatest protection, and their use is worthwhile on high current supplies. They eliminate the need for expensive high peak inverse voltage rated diodes, and they successfully combat high transient voltages.

8.8 Television power supplies

In this section we will have a look at practical switched mode power supplies (SMPS) used in television receivers. Figure 8.16 shows a shunt type SMPS, where the secondary side of the transformer is isolated, but the primary side is not. An isolation transformer should be used when servicing receivers with this type of power supply.

The switching or chopper transistor is Q_1 and the circuit is controlled by an integrated circuit (IC) (such as a TDA 4600 series). The main supply from the bridge rectifier is fed to Q_1 via the transformer T_1 primary. This supply is filtered by capacitor C_{10} and also provides the start-up voltage for the control IC via resistor R_1. At switch-on a 'soft start' circuit operates. The capacitor C_8 charges up via R_1. When the voltage reaches 6 V the IC is enabled and feeds drive pulses to Q_1 base. Current now flows through T_1 primary which sets up a magnetic flux to couple to the secondary windings and the feedback winding (3-4). The voltage developed in the feedback winding is rectified by diode D_5 and provides the 'run power' for the IC. It also provides the feedback voltage which controls the mark-to-space ratio of the pulse-width modulator (PWM) to keep the output voltage constant. Excessive secondary current is reflected into the primary circuit via the magnetic flux and changes the feedback voltage to the IC.

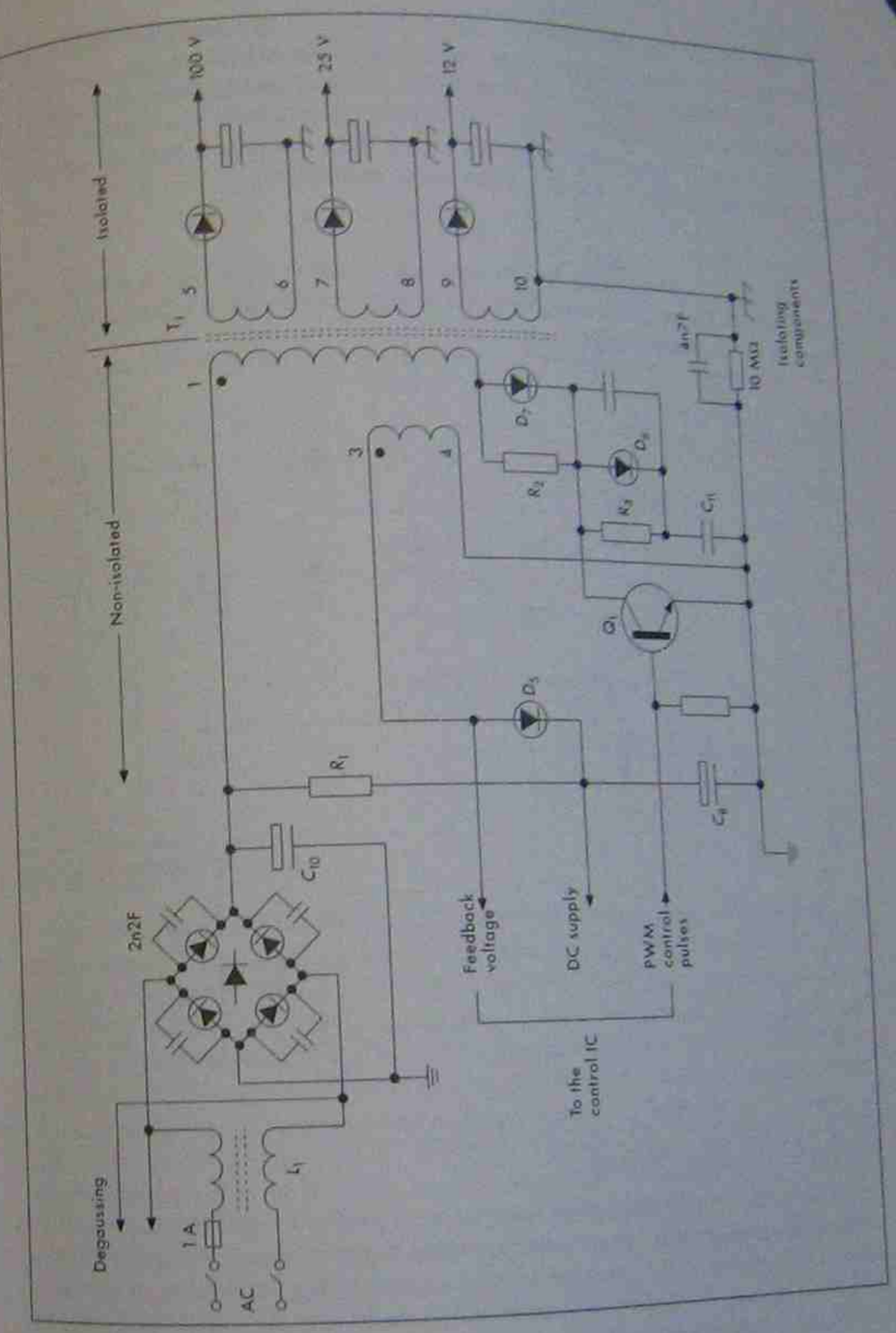


Figure 8.16: Shunt SMPS

The chopper transistor is protected against excessive counter electromotive force by the network C_{11}, R_3 and D_6, and any transformer ringing is damped by D_7 and R_2.

Three output voltages 12 V, 25 V and 100 V are developed by the switching action to supply the voltages for other areas of the receiver (for example, horizontal and vertical outputs, luminance/chrominance integrated circuits and audio stages). Note the small value capacitors across the rectifier diodes and the mains input filter L_1.

8.8.1 Isolation

A problem with a power supply with isolated outputs is how to provide 'feedback' for the control circuit. Figure 8.16 used magnetic flux linkage. An alternative technique is shown in Figure 8.17—an **'opto-coupler'**.

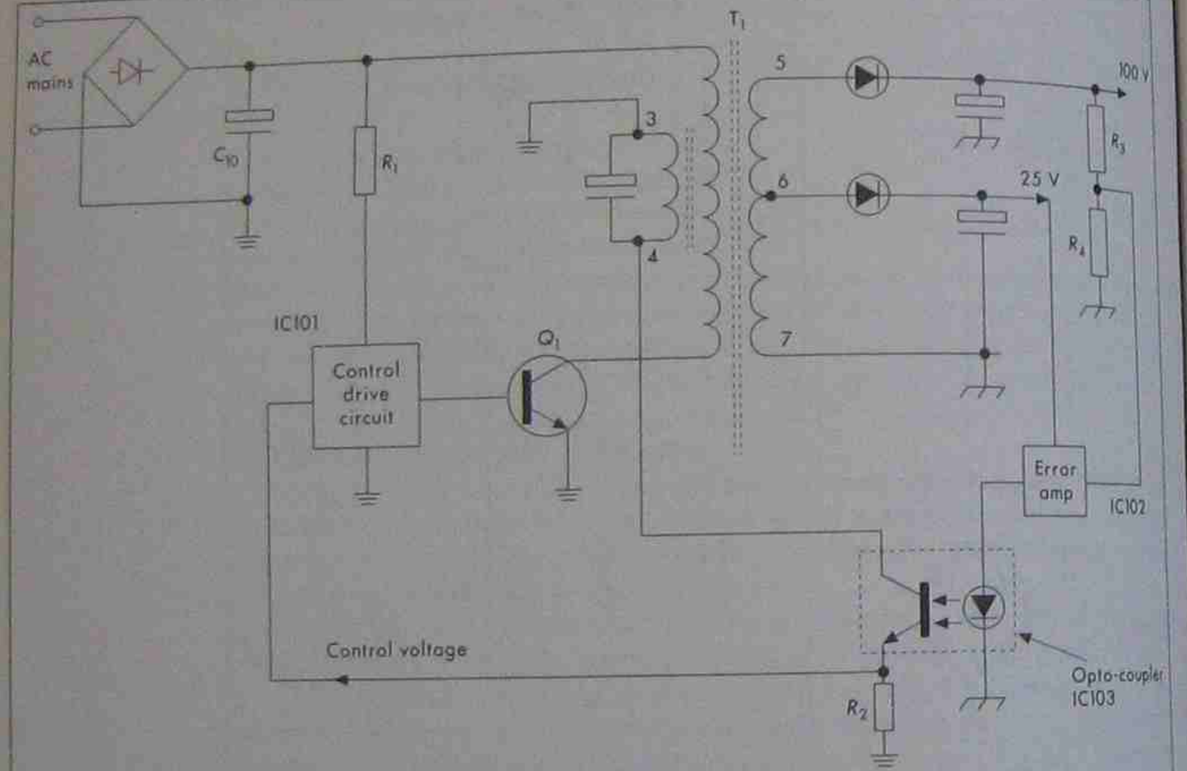


Figure 8.17: Opto-coupler feedback

In this circuit a portion of the output is compared to a reference voltage (as in a standard error amplifier arrangement). The error detector output is then used to drive the light emitting diode (LED) section of the opto-coupler, which controls the conduction of the photo transistor. Therefore, the conduction is determined by the level of the output voltage, and the control circuit receives a feedback voltage proportional to this level. Opto-couplers are very sensitive and react to small changes, thus providing very accurate control.

8.8.2 MOSFET chopper

A fairly new innovation in SMPS design is to use a MOSFET as a chopper. This design can be found in European television receivers and a basic circuit is shown in Figure 8.18.

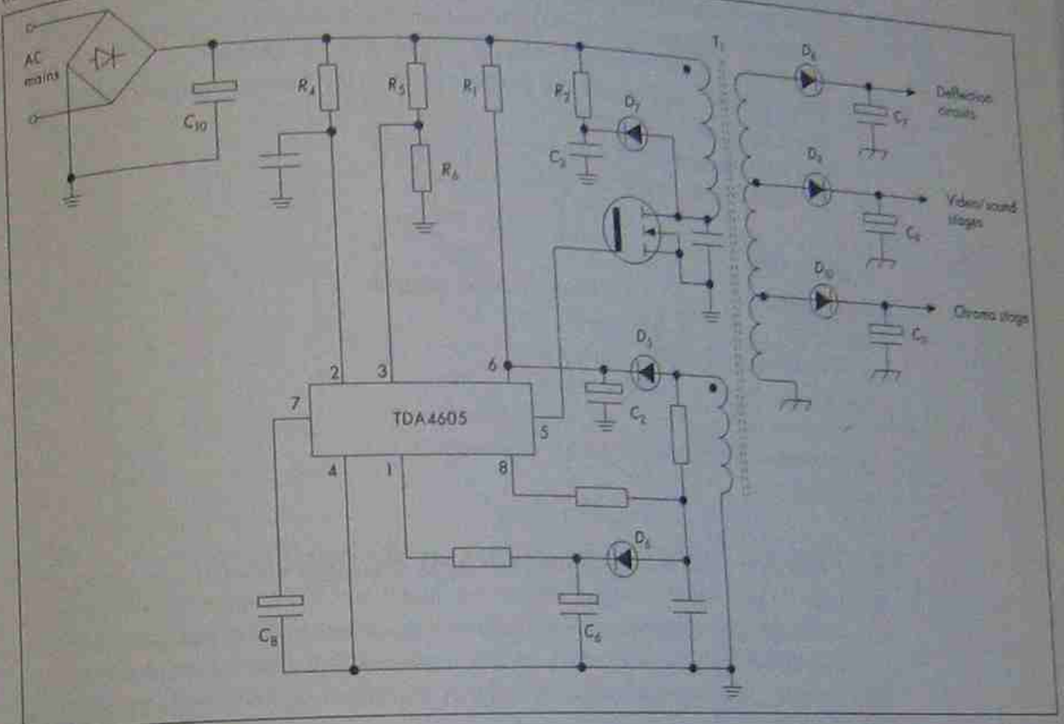


Figure 8.18: MOSFET chopper

Again this supply is controlled by a TDA 4605 IC. It operates in a similar manner to the circuit in Figure 8.16. Resistor R_1 provides the start-up voltage, while D_5 and C_2 provide the 'run power' for the IC after C_8 has charged up slowly (soft start). Feedback voltage for regulation is produced by D_6/C_6 and applied to pin 1 for comparison with an internal reference voltage—typically 3 V. The switching operation produces an AC waveform at pin 8, where a zero crossing detector picks up when the transformer core is de-energised. At this point in time the MOSFET must switch on again, so the relationship between the signals on pins 1 and 8 determines the circuit's duty cycle. This then controls the PWM drive to the MOSFET from pin 5 to the IC. Some circuit variations may be found where the feedback to pin 1 is via an opto-coupler. Any increase in mains voltage results in a rise in voltage at pin 6, and this shuts down the IC. If the mains input is too low, the voltage at pin 3 drops below about 1.4 V and again the IC shuts down.

8.9 Summary

The following items are a summary of the major points covered in this chapter:

- Initial checks should be visual.
- Use an isolation transformer if no mains transformer is fitted to the unit.
- Adopt safe working practices for both high and low voltage equipment.
- Replace faulty component with identical or equivalent component.
- Make voltage measurements and compare them to the 'normal working conditions' to help make your fault diagnosis.
- Check the mains isolation before returning a repair.
- Excessive output voltage may damage the load.

Key terms

fault condition
final testing
general power outlet (GPO)
isolation transformer
logical procedure
normal working conditions
opto-coupler
resistance measurement
safety
symptom
test point
transient protection
voltage measurement

8.10 Review questions (answers in Appendix A)

In the text we have looked at several circuit configurations, their operation and protection circuits. For the review exercise, a new circuit configuration that has not been previously discussed is shown in Figure 8.19. It is a current-limited, adjustable output voltage regulator using a slightly different technique for error-signal amplification. Following is a list of parts for Figure 8.19:

D_1-D_4 = power diodes
Q_1-Q_4 = BC 107 or BC 108
Q_5 = 2N6131 or 2N3055
D_5 = 6.2 V zener diode
C_1 = 6400 µF 50VW
C_2 = 25 µF 10VW
C_3 = 470 nF
R_1 = 1k5 Ω
R_2 = 2k2 Ω
R_3 = 1 kΩ
R_4 = 470 Ω
R_5 = 68 Ω
R_6 = 1 Ω
R_7 = 1 kΩ
R_8 = 1 kΩ
V_{R1} = 1 kΩ
V_{R2} = 100Ω

Answer the following questions with reference to Figure 8.19.

1. Briefly explain the purpose of *all the components* listed in the parts list.
2. Explain the operation of the circuit if the load resistance decreases.

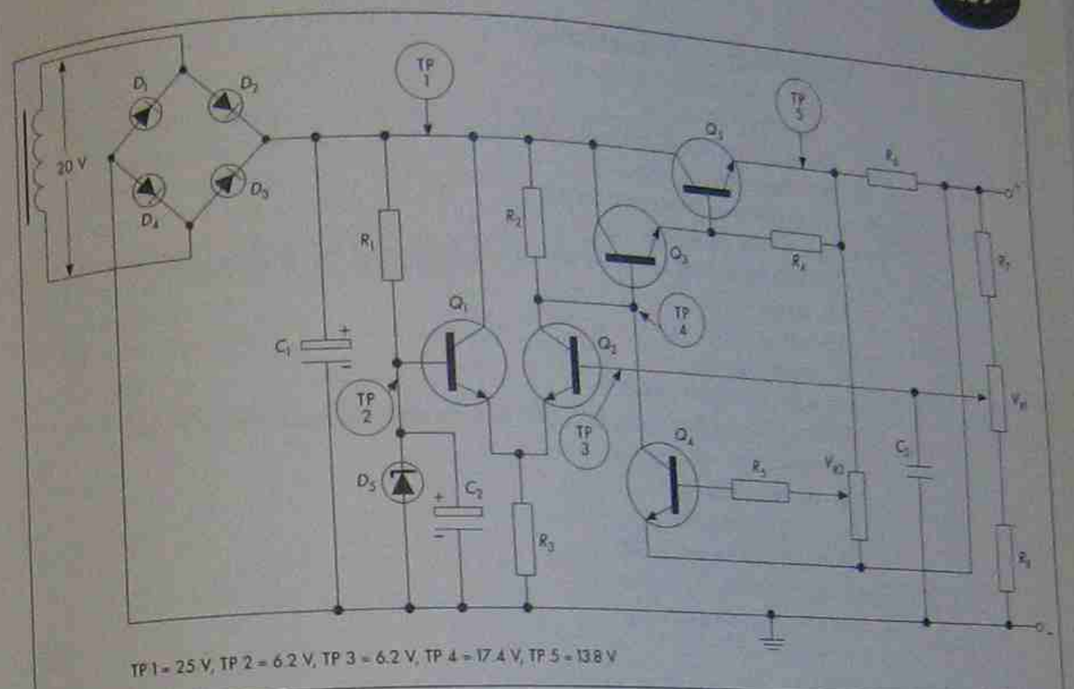


Figure 8.19: Current-limited, adjustable output voltage regulator

3. A fault develops in the power supply and the following test-point voltages result:
   TP 1 = 23.8 V TP 4 = 23.6 V
   TP 2 = 23.6 V TP 5 = 20.5 V
   TP 3 = 11.9 V
   What is the fault? Give reasons for your answer.
4. Diagnose the fault given the following test-point voltages:
   TP 1 = 25.6 V TP 4 = 0 V
   TP 2 = 6.2 V TP 5 = 0 V
   TP 3 = 0 V
   Give reasons for your diagnosis.
5. The power supply suddenly fails and the following test-point voltage readings are taken:
   TP 1 = 25.6 V TP 4 = 25.6 V
   TP 2 = 6.2 V TP 5 = 0 V
   TP 3 = 0 V
   What is the fault? Give your reasons.
6. Diagnose the fault from the following information and support your diagnosis.
   TP 1 = 24.5 V TP 4 = 16 V
   TP 2 = 0 V TP 5 = 10.4 V
   TP 3 = 4 V

Multiple-choice questions

Questions 1 to 3 refer to Figure 8.16.

1. The purpose of L_1 is to:
   (a) transform the AC to a higher voltage
   (b) change the AC to DC
   (c) minimise radiation into the AC
   (d) limit the degaussing current
2. The 2n2F capacitors across the rectifier diodes protect the diodes against:
   (a) surge current
   (b) voltage spikes
   (c) reverse biasing
   (d) a short-circuited output
3. Feedback information about the 'state' of the output voltage is sensed by the:
   (a) transformer winding 1-2
   (b) voltage across D_5
   (c) charge in capacitor C_8
   (d) transformer winding 3–4

Questions 4 to 6 refer to Figure 8.17.

4. The intial start-up resistor is:
   (a) R_1
   (b) R_2
   (c) R_3
   (d) R_4
5. Feedback for this circuit is provided via:
   (a) winding 3–4
   (b) winding 1-2
   (c) IC 102
   (d) IC 103
6. If R_3 went open circuit, the missing voltage would be the:
   (a) 100 V output voltage
   (b) reference voltage
   (c) anode voltage for IC 102
   (d) control voltage

Questions 7 to 10 refer to Figure 8.18.

7. The 'soft start' component is:
   (a) C_6
   (b) C_2
   (c) C_8
   (d) C_{10}
8. The 'run power' is provided by:
   (a) D_6/C_6
   (b) D_7/C_3
   (c) D_9/C_9
   (d) D_5/C_2
9. If R_5 went high in value, the IC would shut down because of:
   (a) too low a voltage at pin 3
   (b) excessive voltage at pin 3
   (c) excessive voltage at pin 6
   (d) no feedback voltage to pin 1
10. An open circuit in D_5 results in the loss of:
    (a) run power
    (b) start-up power
    (c) sound
    (d) colour (chroma)

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CHAPTER 9



FEEDBACK FUNDAMENTALS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Define positive feedback.
- Define negative feedback.
- State the desensitising factor for negative feedback.
- List the advantages derived from using negative feedback.
- State the disadvantages experienced from using negative feedback.
- Define voltage and current feedback.
- State the effect on input impedance from series and shunt-fed feedback.

9.1 Introduction

Feedback is used in audio amplifiers, industrial control systems and oscillators, to name but a few applications. This chapter discusses general aspects of feedback: the types, the various configurations and their effect on circuit parameters. This will form an introduction to our study of oscillators.

Feedback means 'to return a proportion of the output signal of a circuit to the input of the same or preceding stage'. If the signal fed back is the opposite phase, and hence opposes the original input signal, the feedback is said to be negative or degenerative. However, if the signal fed back is in phase, and thereby aids the original signal, the feedback is said to be positive or regenerative.

A general feedback amplifier arrangement is shown in Figure 9.1. The output signal is fed back to the input via a feedback network. The Greek letter beta (ß) is used to signify the

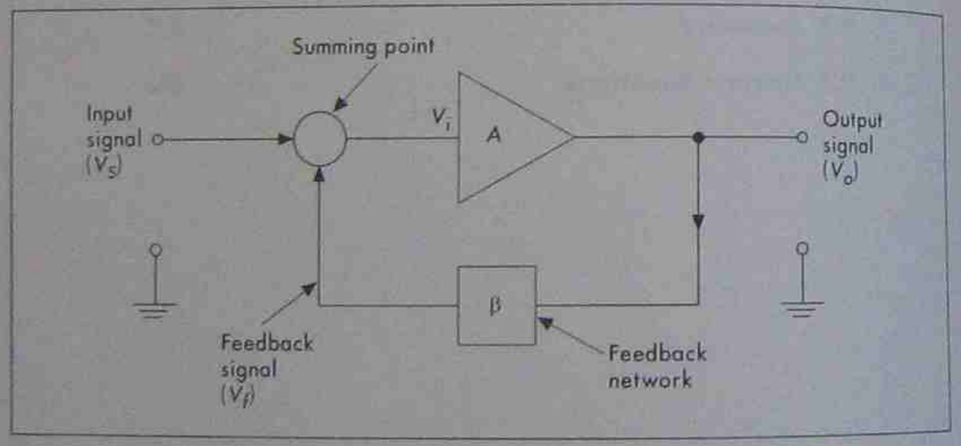


Figure 9.1: Feedback amplifier arrangement

percentage of output signal that is fed back. This is called the 'feedback factor' and is often expressed as a decimal, for example, 10% feedback = 0.1. The feedback may be in the form of voltage or current, and the difference between these will be discussed below.

At the summing point, the original input signal (V_s) and the feedback signal (V_f) are

summed together to provide a new input signal to the amplifier (V_i) (where V_f is a proportion of the output signal V_o):

$V_i = V_s \pm V_f$ (± determined by the type of feedback)
but $V_f = \beta V_o$
therefore $V_i = V_s \pm \beta V_o$ (equation 9.1)

9.2 Negative feedback

When the feedback voltage is 180° out of phase with the input voltage, the conditions are ideal for **negative or degenerative feedback**. The signal fed back to the amplifier input opposes the original input signal, and therefore the feedback factor ß is taken as negative. A block diagram showing the negative feedback connection is shown in Figure 9.2. (Note that V_f opposes V_s.)

The feedback network takes a portion of the output signal and inverts it, if necessary, prior to feeding it back to the input. The signal, fed back, must have the correct phase to reduce the level of the amplifier's input signal (V_i). The polarities indicated on Figure 9.2 show this arrangement.

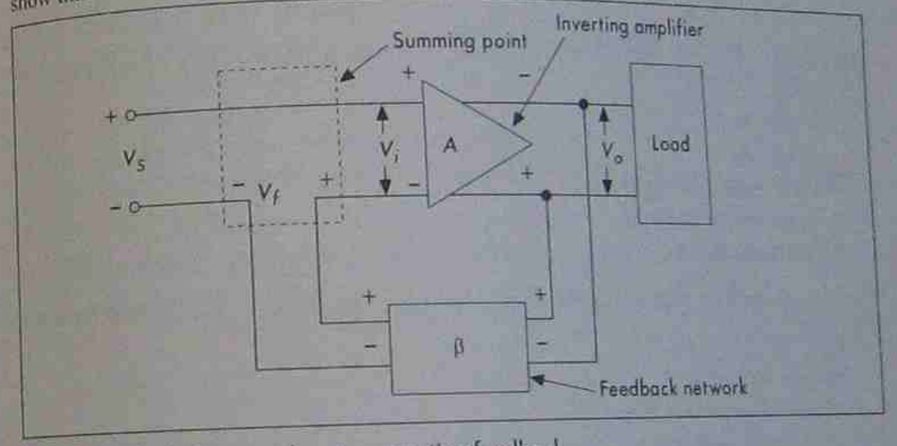


Figure 9.2: Block diagram showing negative feedback

The voltage fed back is $-\beta V_o$ and the net input voltage (V_i) becomes:

$V_i = V_s - \beta V_o$ (equation 9.2)
because $V_f = \beta V_o$

In order to produce an output voltage (V_o), the net input signal multiplied by the amplifier gain (A) must be equal to V_o. Therefore:

$V_o = A(V_s - \beta V_o)$ (equation 9.3)
where $-\beta V_o = V_f$ (the feedback signal)
V_s = the input signal

The voltage gain of the overall system including feedback is A' (where the **prime** indicates voltage gain feedback and is sometimes designated A_f):

$A' = V_o/V_s$ (equation 9.4)

The output voltage is given by:

$V_o = A' (V_s - \beta V_o)$ (equation 9.5)

The expression for voltage gain with feedback (A') is derived using equation 9.2 and 9.4.

Normal amplifier gain $(A) = V_{output}/V_{input}$

In the case of Figure 9.2, this becomes:

$A = V_o/(V_s - \beta V_o)$, so
$= V_o A (V_s - \beta V_o)$
$= AV_s - \beta V_o A$
$AV_s = V_o + \beta V_o A$
$= V_o (1 + \beta A)$
$A = [V_o (1 + \beta A)/V_s$

However, the circuit has feedback and, as we say in equation 9.4, $A' = V_o/V_s$. So, substituting A' for V_o/V_s:

$A = A' (1 + \beta A)$
$A' = A/1 + \beta A$
where A' = gain with feedback
A = gain without feedback
ß = the feedback factor

This is the formula for negative feedback only because ß is negative. The general feedback formula is:

$A' = A/[1 - (-\beta A)]$
where + ß = positive feedback
– ß = negative feedback

Therefore, for negative feedback:

$A' = A/1 - (-\beta A)]$
$= A/(1 + \beta A)$

9.2.1 Effect on gain

The gain with feedback therefore depends on two factors:

1. The amplifier gain (A).
2. The value of the feedback factor (ß).

If an amplifier has a gain of 80 without feedback, and a feedback factor of 0.1 is applied, the gain will drop considerably. The gain with feedback (A') is:

$A' = A/(1 + \beta A)$
$= 80/[1 + (0.1 \times 80)]$
$= 80/9$
$= 8.88$ or 8.9

The gain of the amplifier is reduced by a factor of 9, that is, negative feedback. The greater the feedback factor, the lower the overall gain. If the feedback factor is 1.0, the gain (A') would be less than one. We can therefore make the general assumption that if A' is less than A, then the feedback is negative.

Even though negative feedback reduces the gain of the amplifier, there are improvements that can be obtained from this type of feedback, such as:

- better gain stability;
- improved frequency response (wider bandwidth);
- less phase distortion; and
- tailoring of input and/or output impedances.

9.2.2 Effect on gain stability

It can be shown that the gain of an amplifier with feedback remains relatively constant when there is a change in working conditions. This could be important in an industrial control situation, for example, where a change in the gain of an amplifier due to a variation in working conditions might mean that an entire production lot could be lost.

Assume an increase in temperature causes the gain of the amplifier in the example above to increase from 80 to 90:

$$\text{Change in gain} = \frac{\text{difference}}{\text{original}} \times 100$$
$= (90 - 80)/80 \times 100$
$= 10/80 \times 100$
$= 12.5\%$

The gain with feedback is:

$A' = A/1 + \beta A$
$= 90/1 + (0.1 \times 90)$
$= 90/10$
$= 9$

Using the figures from the previous example, this represents a change in gain from 8.9 to 9—that is, 1.11%—an improvement of better than a factor of 10. In addition, since amplitude and frequency distortion are caused by changes in gain, both of these effects are in general reduced with negative feedback. Table 9.1 shows a comparison of gain stability with negative feedback. The 'net input' is the effective input actually reaching the amplifier and, since negative feedback reduces the input signal, the 'total input' voltage must be increased to maintain the 'net input' constant. Nominal gain is the gain of the amplifier without feedback.

Table 9.1: Gain stability by negative feedback

Example	Nominal gain (A)	Output voltage (V_o)	Percentage feedback (β)	Feedback voltage (V_f)	Total input (V_s)	Overall gain (A')	Net input (V_i)
(a)	40 10	40 10	10% 10%	4 1	5 2	8 5	1 1
(b)	400 100	400 100	10% 10%	40 10	41 11	9.7 9.1	1 1
(c)	800 200	800 200	10% 10%	80 20	81 21	9.87 9.55	1 1
(d)	200	200	5%	10	11	18.3	1
(e)	200	200	20%	40	41	4.88	1
(f)	200	200	30%	60	61	3.28	1

Several points emerge from Table 9.1. In examples (a), (b) and (c), the amplifier gain is varied over a 4:1 range. This can be caused by amplitude/frequency distortion or any change in circuit parameters. Using a small amount of feedback (say, for example, 10%), in the worst case this variation in gain is reduced to 2:1, as in example (a). Example (b) shows an improvement and, in example (c), there is practically no change at all. Note also that in examples (b) and (c) with 10% feedback, the gain is approximately 10, and it is stabilised to this value. From this, it appears that the maximum gain possible with feedback is 1/ß. With ß equal to 10%, the theoretical maximum gain is approximately 10. Examples (d), (e) and (f) further verify this conclusion. All three cases have a 'nominal gain' of 200, but the feedback factors vary from 5% to 30%. However, the overall gain with feedback (A') is still approximately 1/ß.

When ßA is large compared to unity, due to high gain or high ß, the denominator (1 + ßA) is approximately equal to ßA, and the equation for gain with feedback simplifies to:

$A' = A/1 + ßA$
$= A/ßA$
$= 1/ß$ (for $ßA >> 1$)

9.2.3 Effect on distortion

Amplitude and frequency **distortion** are often caused by variations in gain. Any technique that stabilises gain must therefore also correct, or at least minimise, distortion.

With negative feedback, the gain is reduced:

$A' = A/1 + ßA$

It can be shown that distortion will be reduced by the same factor:

$D' = 1 + ßA$
where D = the stage distortion without feedback
D' = the distortion with feedback

With amplitude and frequency distortion being minimised by the use of negative feedback, it follows that any variations in phase will also be affected. This is best shown with the use of a phasor diagram, as shown in Figure 9.3.

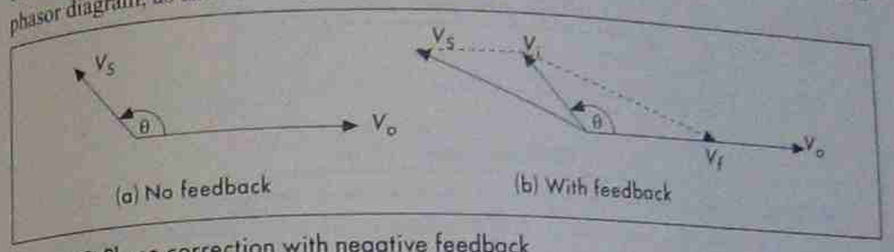


Figure 9.3: Phase correction with negative feedback

At a particular frequency, an amplifier without feedback has a phase shift between the actual input (V_s) and the output (V_o) of less than the expected 180°. This represents a certain amount of phase distortion and is represented by the angle theta (θ), as shown in Figure 9.3(a).

The reference is the output voltage (V_o). When feedback (V_f) is applied, it is in phase with V_o (shown in Figure 9.3(b)) and the value is determined by the feedback factor (ß). Feedback (V_f) combines with the actual input voltage (V_s) to produce a new net input. The net input voltage (V_i) is the algebraic sum of V_s and V_f. To recover the actual input voltage (V_s) the feedback voltage must be subtracted from the net input voltage.

Completing the parallelogram (Figure 9.3(b)) builds up the complete phasor diagram, and it can be seen that V_i is the algebraic sum of V_s and V_f. When the feedback voltage is taken into account, the phase angle of the actual input voltage (V_s) and the output voltage (V_o) is much closer to 180°. Phase distortion has therefore been reduced.

9.2.4 Effect on bandwidth

The amplifier gain with feedback has been shown to be $A' = 1 + ßA$. It is also approximately 1/ß, as long as ßA is much larger than one, that is, $ßA >> 1$. In a practical amplifier circuit, with single low (f_l) and high frequency (f_h) **breakpoints**, the gain drops off at high frequencies. This is due to active device and circuit capacitances. At low frequencies, gain may also drop off if the amplifier stages are capacitively coupled. As soon as the **open-loop gain** (A) drops low enough, the factor ßA is no longer greater than one. This means that A' being approximately equal to 1/ß is no longer true.

Figure 9.4 shows frequency responses and gains of an amplifier with and without feedback. The gain of the amplifier without feedback is 0.707A (where A is the amplifier gain), and the **bandwidth** is between the two half-power points f_1 and f_2. The amplifier with feedback has a gain of 0.707A', and its bandwidth is between the two points f_1' and f_2'.

$f_1' = \dfrac{f_1}{(1 + \beta A)}$ and is lower,

while $f_2' = f_2 \times (1 + \beta A)$ and is therefore higher

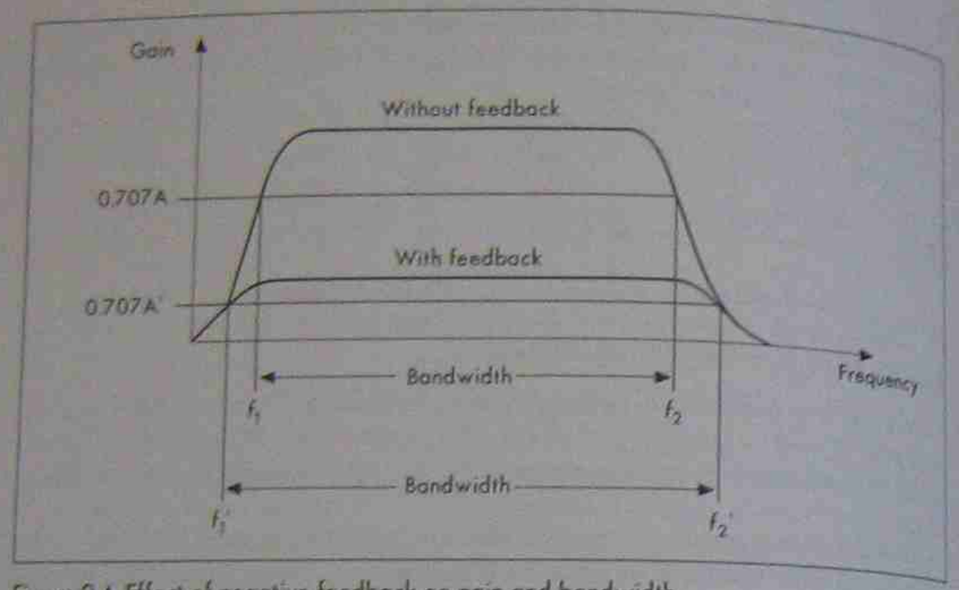


Figure 9.4. Effect of negative feedback on gain and bandwidth

9.2.5 Effect on input and output impedance

There are two general types of circuit used for negative feedback:

1. voltage-derived feedback; and
2. current-derived feedback.

With voltage feedback, a portion of the output voltage is returned to the input via a voltage divider. A typical circuit arrangement is shown in Figure 9.5. The feedback voltage is obtained by a voltage divider that is across the output circuit of the amplifier. This parallel connection is called **shunt-derived**, and its effect is to reduce the output impedance of the amplifier.

Figure 9.6 shows a block diagram of the amplifier, complete with **internal input and output resistances** (R_i and R_o). Resistor R_s is the internal resistance of the signal source V_s. The feedback voltage V_f is in series with the input signal V_s, and therefore the net input voltage V_i is the algebraic sum of V_s and V_f. It can be seen that without feedback, the output circuit is voltage divider R_o/R_L, with V_o being developed across R_L:

$$V_o = R_L/(R_o + R_L) \times AV_i$$

With the feedback network connected, the equivalent output circuit is shown in Figure 9.7.

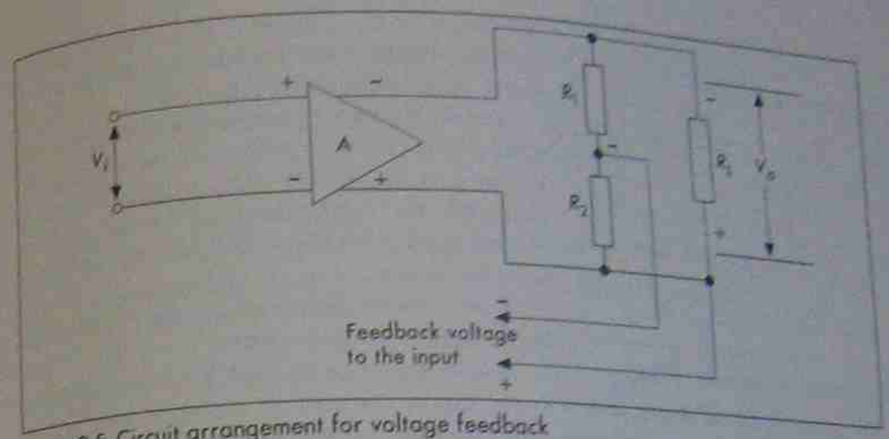


Figure 9.5. Circuit arrangement for voltage feedback

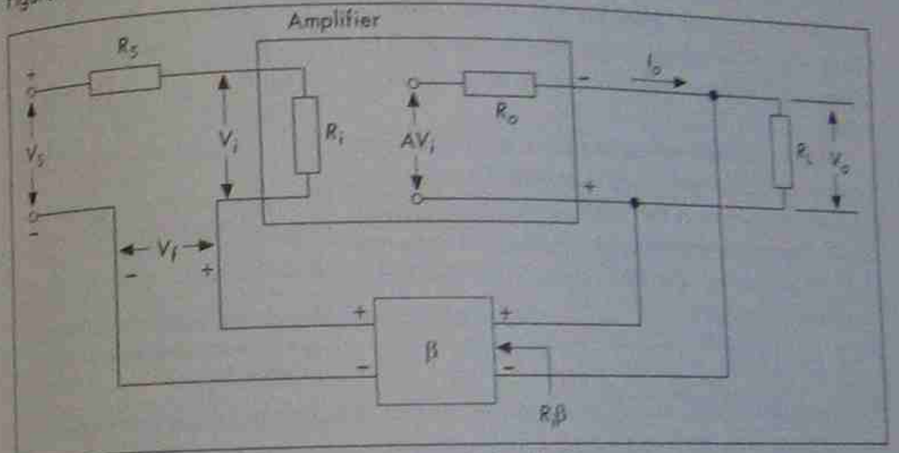


Figure 9.6. Block diagram of an amplifier with feedback

The input resistance of the feedback network ($R_f\beta$) is in parallel with R_i. However, AV_i is a low impedance (as shown by the dotted line) and therefore the three resistors are in parallel, so $Z_o = R_{of} = R_o//R_L//R_f\beta$ which is less than the original output impedance. It can be shown mathematically that R_o with feedback (R_{of}) = $R_o/(1 + \beta A)$.

Current feedback is a voltage fed back to the input that is directly proportional to the current flowing in the load circuit. This circuit arrangement is shown in Figure 9.8.

The feedback voltage is developed across the resistor R_f. This is called **series-derived feedback**, and its effect is to increase the output impedance. The equivalent output circuit is shown in Figure 9.9. AV_i is a low impedance and

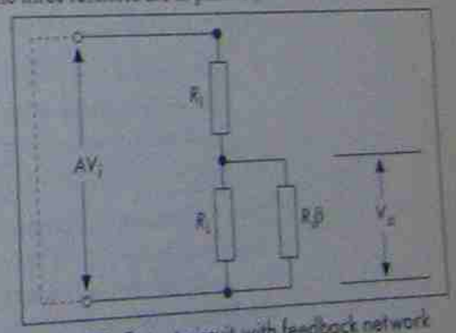


Figure 9.7. Output circuit with feedback network connected

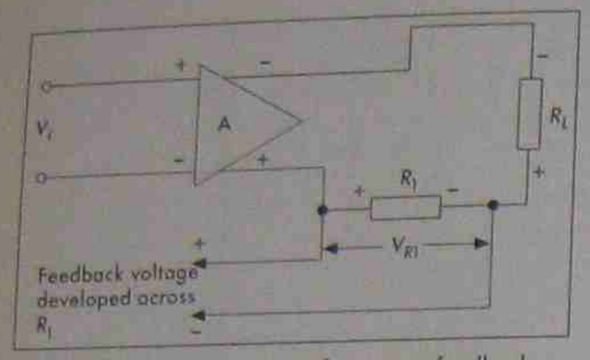


Figure 9.8: Circuit arrangement for current feedback

therefore $Z_o = R_{of} = R_o//(R_L + R_1\beta)$, which is greater than the original output impedance. It can be shown that R_o with feedback $(R_{of}) = R_o(1 + \beta A)$

The same reasoning can be used to determine the effect on the input impedance of the amplifier R_i. Referring to Figure 9.6, if the input impedance is **series-fed**, the output resistance of the feedback network ($R_1\beta$) is in series with R_i, and this network is in parallel with R_s.

It can be shown mathematically that R_i with series-fed feedback $(R_{if}) = R_i(1 + \beta A)$.

In order for series feedback to be most effective, the circuit should be driven from a source with a low internal resistance. The internal resistance (R_s) of the source must be small compared to the input resistance (R_i) of the amplifier. If R_s is large compared with R_i, the feedback voltage (V_f) will be voltage divided between the two. This means V_f will not have full effect on the amplifier input voltage.

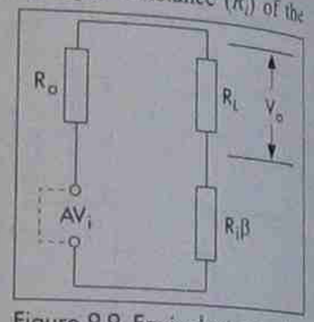


Figure 9.9: Equivalent output circuit with series-derived feedback

With the feedback voltage connected in parallel with the input signal, the feedback is **shunt-fed**. The effect on the input impedance is the same as the shunt-derived circuit. The output resistance of the feedback network ($R_1\beta$) is across the amplifier input, and the input impedance is decreased:

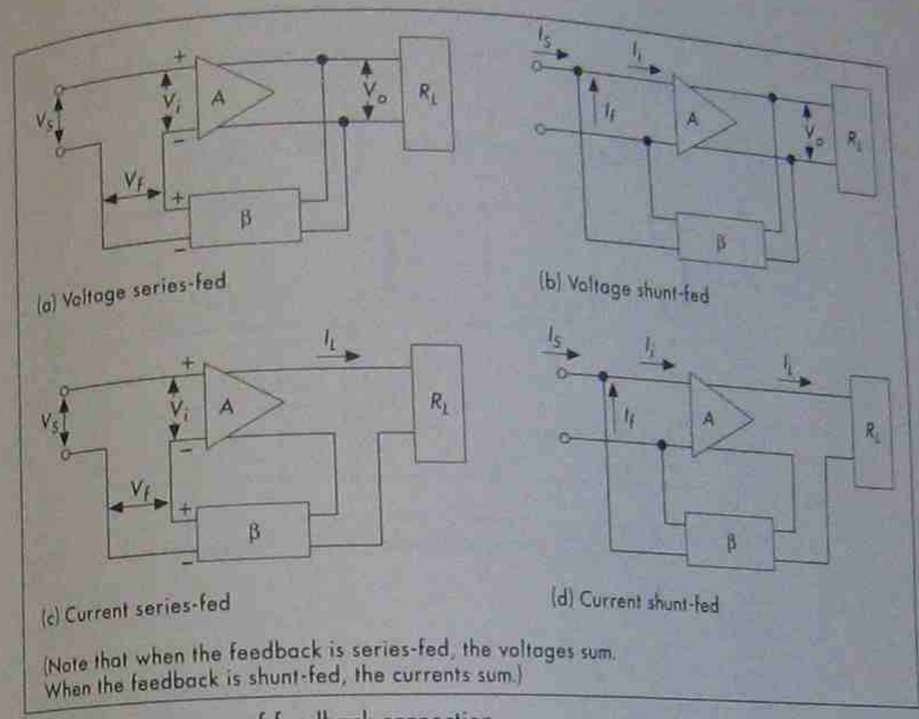
$$R_{if} = R_i/(1 + \beta A)$$

For shunt feedback to be most effective, the circuit should be driven from a constant current source or from a source with a high internal resistance. The internal resistance (R_s) of the source must be high compared to the input resistance (R_i) of the amplifier. If R_s is small compared with R_i, the feedback current would flow through R_s instead of R_i, and any feedback control would be severely minimised.

The four basic ways of deriving and feeding back both voltage and current feedback are:

1. shunt-derived, series-fed (voltage sample);
2. shunt-derived, shunt-fed (voltage sample);
3. series-derived, series-fed (current sample); and
4. series-derived, shunt-fed (current sample).

These are shown in Figure 9.10(a) to (d) and the effects listed in Table 9.2. The main amplifying device used in this chapter is the transistor. However, the theory of operation is essentially the same no matter which active device is used, for example, FET or operational amplifier.



(a) Voltage series-fed

(b) Voltage shunt-fed

(c) Current series-fed

(d) Current shunt-fed

(Note that when the feedback is series-fed, the voltages sum. When the feedback is shunt-fed, the currents sum.)

Figure 9.10: Four types of feedback connection

Table 9.2: The effects on input and output impedance

Figure 9.10	Type of feedback	R_{if}	R_{of}
(a)	shunt-derived, series-fed	increase	decrease
(b)	shunt-derived, shunt-fed	decrease	decrease
(c)	series-derived, series-fed	increase	increase
(d)	series-derived, shunt-fed	decrease	increase

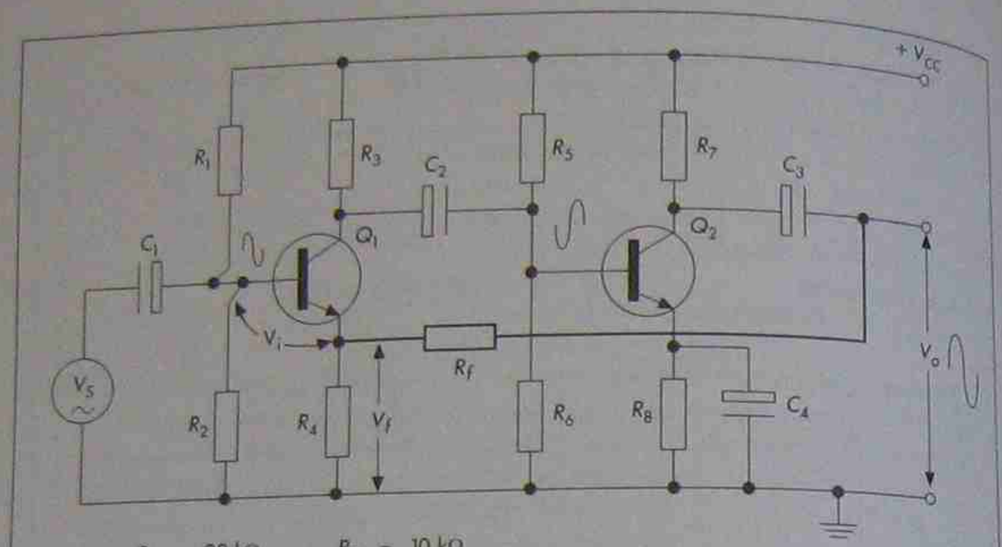
From the previous examples you can see that the expression $1 + \beta A$ has an effect on every circuit parameter. Because of this it is often referred to as the **desensitising factor**.

9.3 Practical circuits

This section shows practical amplifier circuits and illustrates the four types of feedback configurations just discussed. The value of the supply voltage V_{cc} in all the practical circuits is adjusted for a non-distorted output signal, and is generally between 10 V and 18 V.

9.3.1 Shunt-derived, series-fed voltage feedback

A two-stage amplifier is shown in Figure 9.11. Resistors R_1 to R_8 and capacitors C_1 to C_4 are typical audio amplifier components. Transistor Q_1 amplifies and inverts the input signal V_s.



R_1	= 22 kΩ	R_7	= 10 kΩ
R_2	= 4k7 Ω	R_8	= 2k2 Ω
R_3	= 10 kΩ	R_f	= 47 kΩ
R_4	= 2k2 Ω	C_1	= 50 μF
R_5	= 22 kΩ	C_2	= 50 μF
R_6	= 4k7 Ω	C_3	= 50 μF
		C_4	= 100 μF

Q_1 and Q_2 are BC 107 or equivalent. BC 107 is a general purpose amplifier with h_{fe} = 100.

Figure 9.11: Two-stage transistor amplifier with voltage series negative feedback

applying it to the base of Q_2. The signal is again amplified and inverted to give the output signal shown. A portion of this output signal voltage (V_o) is fed back to the emitter of Q_1 via the feedback resistor R_f. The amount of voltage applied to the emitter of Q_1 is determined by the voltage divider R_f and R_4:

$$V_f = V_o \times (R_4/R_f + R_4)$$

As the input signal V_s drives the base of Q_1 positive, the feedback voltage V_f also drives the emitter of Q_1 positive. The amplification of the transistor is determined by the input signal's influence on the forward bias V_{BE}. Therefore, as the base goes positive (influenced by V_i), the emitter also goes positive (influenced by V_f). The net result is a reduction in forward bias (V_{BE}) and therefore a reduction in gain (V_o is less), that is, the negative feedback. The 'addition' of these two signals is shown in Figure 9.12.

Note that only the difference signal is able to be viewed on an oscilloscope. However, the net input voltage (V_i) can be calculated if the values of V_f and V_{out} are known.

To calculate net input voltage where V_s = 1 V peak, V_{out} = 10 V peak, using the resistor values in Figure 9.11:

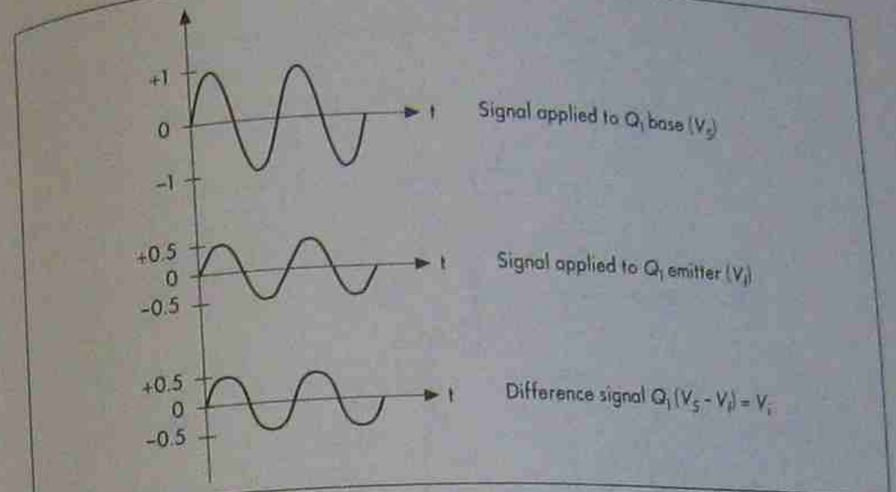


Figure 9.12: The summing of signals for voltage series feedback

$V_f = [R_4/(R_f + R_4)] \times V_o$
$= 2k2/(47k + 2k2) \times 10$
$= 0.447$ V peak or 500 mV peak
Net input voltage $= V_s - V_f$
$= 1 - 0.5$
$= 0.5$ V peak or 500 mV peak

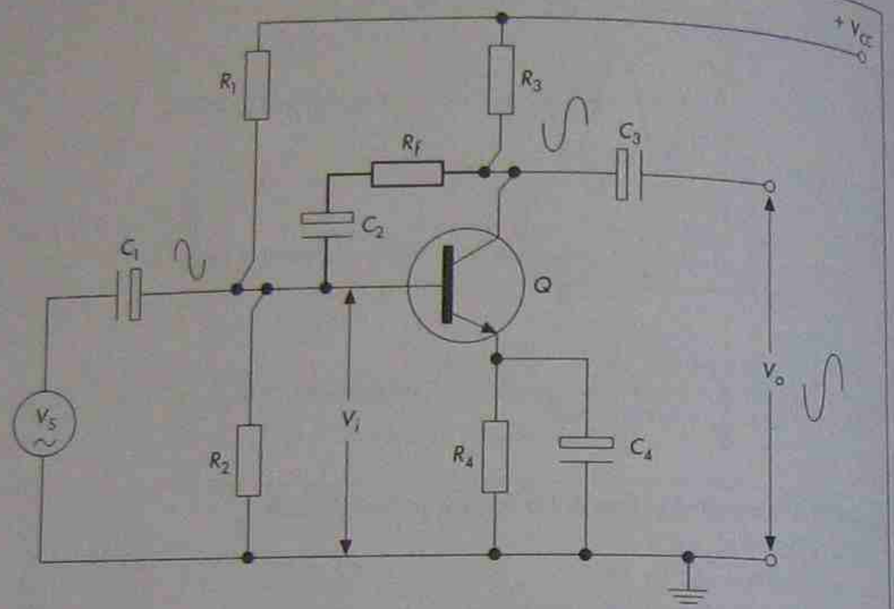
With the use of negative voltage series feedback, the input voltage (V_i) has been decreased to half its former value. Output resistance with feedback (R_{of}) is decreased (because of the parallel connection) and R_{if} is increased (because of the series connection).

9.3.2 Shunt-derived, shunt-fed voltage feedback

A single-stage amplifier with voltage feedback is shown in Figure 9.13. The transistor amplifies and inverts the input signal (V_s). A portion of the output signal, which is opposite in polarity to the input, is developed across R_f relative to V_i. The value of R_f determines the amount of current fed back to the input. Capacitor C_2 prevents the DC bias of the transistor from being upset by blocking the collector voltage and allowing only the AC signal to be fed to the base.

The input signal V_s drives the base positive, the feedback (which is 180° out of phase) reduces the net input level. The addition of these two signals is shown in Figure 9.14. This is a shunt-fed circuit and therefore the currents sum.

The net input current is reduced and less gain results. Output resistance with feedback (R_{of}) is decreased, R_{if} is decreased, and V_o is also decreased. The amount of current fed back is determined by the voltage (V_o), R_f and the net input resistance of the amplifier.



R_1	= 22 kΩ	C_1	= 50 µF
R_2	= 4k7 Ω	C_2	= 10 µF
R_3	= 10 kΩ	C_3	= 50 µF
R_4	= 2k2 Ω	C_4	= 100 µF
R_f	= 1.0 kΩ	Q	= BC107

(V_S has a high value of R_S)

Figure 9.13: Amplifier with voltage shunt negative feedback

9.3.3 Series-derived, series-fed current feedback

The circuits in Figures 9.11 and 9.13 have sampled the output voltage and fed a portion of this voltage back to the input. With current feedback, the current is sampled and a voltage proportional to the current is fed back to the input.

In Figure 9.15 the transistor amplifies the input signal (V_s). The resulting output current (I_o), which is dependent on V_s, passes through the emitter resistor R_4. This is because C_3, the low impedance path for the AC signal, is removed. This results in a voltage being developed across R_4 which is proportional to I_o. The emitter end is positive, and this feedback voltage reduces the level of the net input voltage.

The summing of V_s and V_f is shown in Figure 9.16. The circuit action is the same as for series-fed voltage feedback. As the base voltage goes positive, the output current develops a voltage across R_4 (the emitter voltage), which is also going positive. The forward bias (V_{BE}) of the transistor is decreased, the gain is less, and V_o is reduced. Output and input resistance with feedback (R_{of} and R_{if}) are both increased because of the series connections.

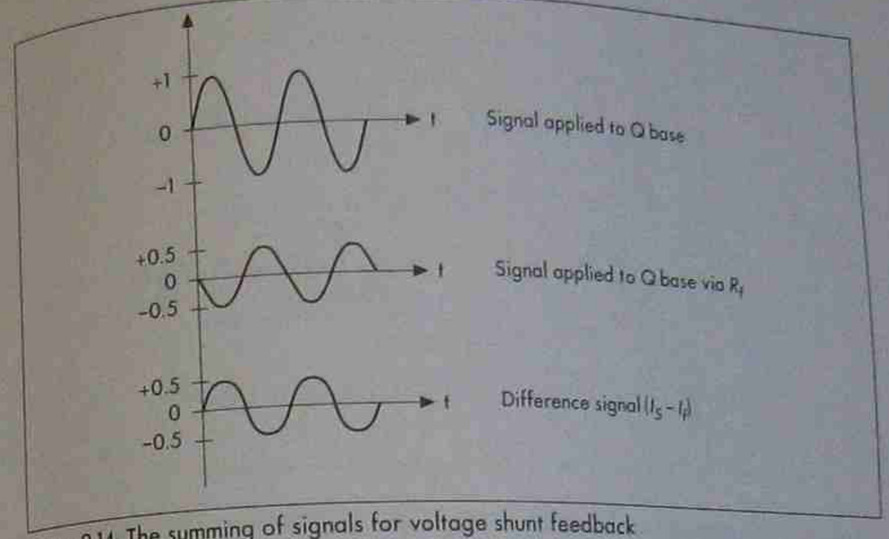
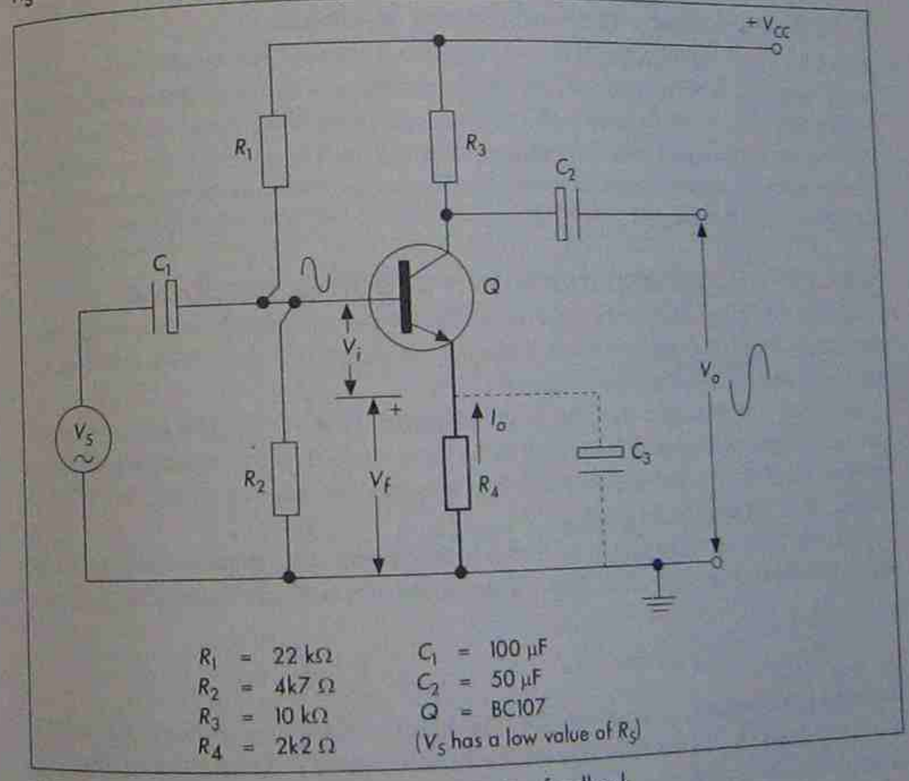


Figure 9.14: The summing of signals for voltage shunt feedback



R_1	= 22 kΩ	C_1	= 100 µF
R_2	= 4k7 Ω	C_2	= 50 µF
R_3	= 10 kΩ	Q	= BC107
R_4	= 2k2 Ω	(V_S has a low value of R_S)	

Figure 9.15: Amplifier with current series negative feedback

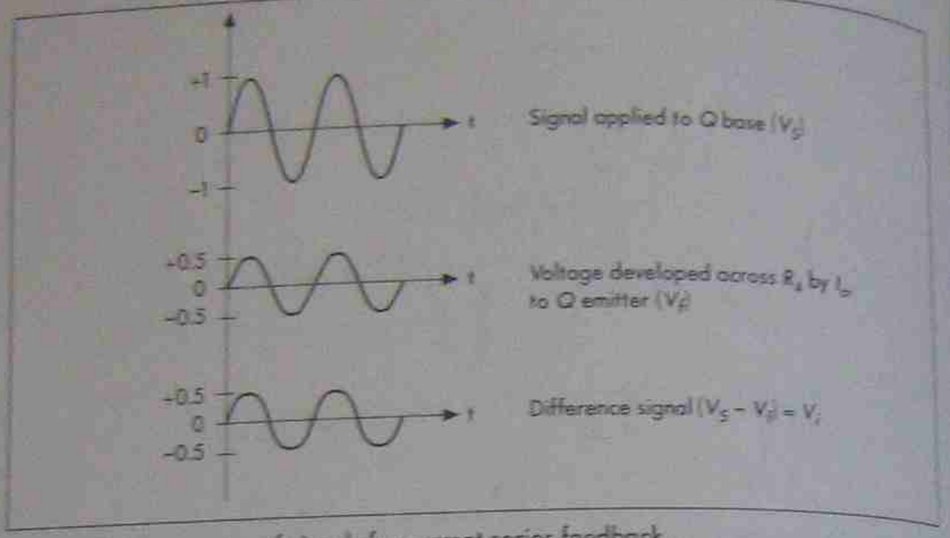


Figure 9.16: The summing of signals for current series feedback

9.3.4 Series-derived, shunt-fed current feedback

Figure 9.17 is a two-stage amplifier with negative feedback. The emitter resistor of the second stage (R_4) is unbypassed and provides the current path for the output current (I_b). The voltage developed across R_4 is fed back and shunt connected to the input, via R_f and C_3. The summing of the two signal currents is shown in Figure 9.18. The net input current is reduced, thus reducing the amplifier gain and hence V_o. The output resistance with feedback (R_{of}) is increased and R_{if} is decreased.

9.4 Operational amplifiers with negative feedback

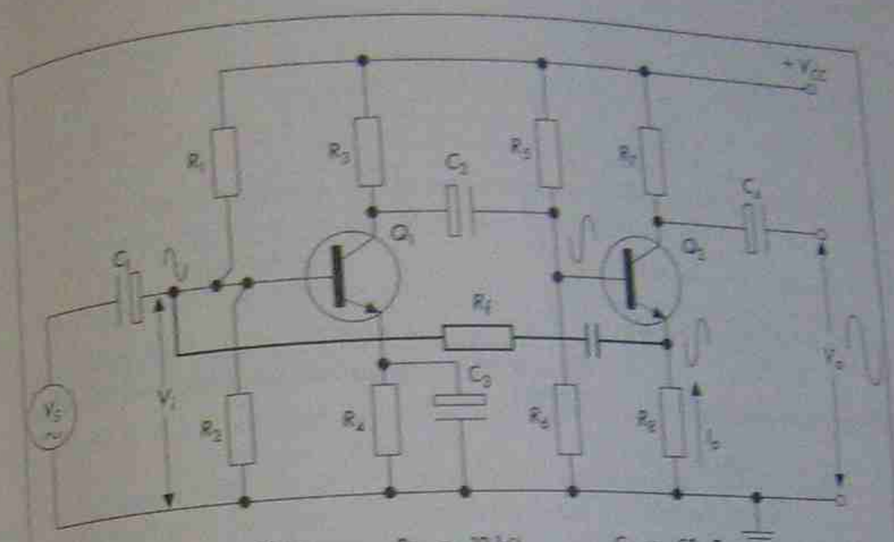
As stated earlier in this chapter, the negative feedback theory discussed can also be applied to operational amplifiers (op-amps). Three examples are shown in Figures 9.19, 9.20 and 9.21 (the power supply connections are omitted for clarity).

In Figure 9.19 the input V_i is fed to the inverting input via R_1. The output is phase-inverted, and part of this output is fed back to the input via R_f. The two signal currents mix at the input resulting in a net input (V_i) which is less than the original input. This is shunt-derived, shunt-fed negative feedback.

Likewise, the circuit in Figure 9.20 shows an op-amp with shunt-derived, series-fed negative feedback. A portion of the output is developed across R_f and fed back to the input:

$$V_f = \beta V_o$$
$$V_i = V_s - V_o$$

Current-derived feedback can be obtained by the circuit configuration shown in Figure 9.21. The output current (I_o) is passed through the feedback resistor (R_f). The feedback voltage (V_f) is proportional to this current:



R_1 = 22 kΩ
R_2 = 4k7 Ω
R_3 = 10 kΩ
R_4 = 2k2 Ω
R_5 = 22 kΩ
R_6 = 4k7 kΩ
R_7 = 10 kΩ
R_8 = 2k2 Ω
R_f = 47 kΩ
C_1 = 100 µF
C_2 = 50 µF
C_3 = 100 µF
C_4 = 50 µF
C_f = 10 µF
Q_1 = BC107
Q_2 = BC107

(V_S has a high value of R_5)

Figure 9.17: Amplifier with current shunt negative feedback

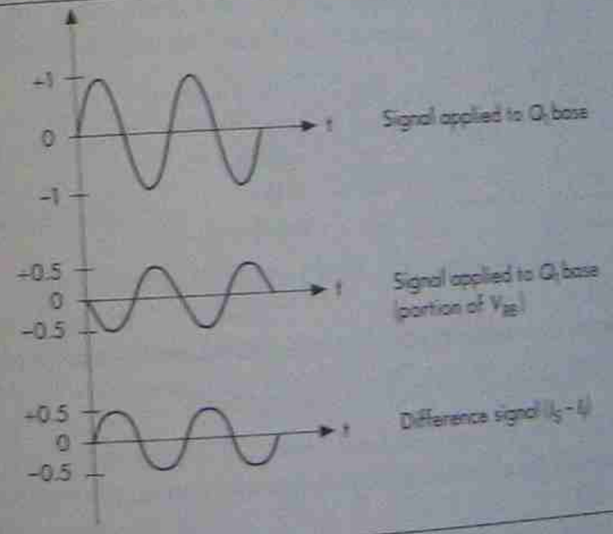


Figure 9.18: The summing of signals for current shunt feedback

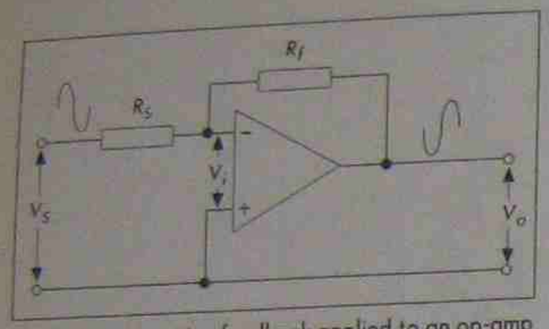


Figure 9.19. Negative feedback applied to an op-amp

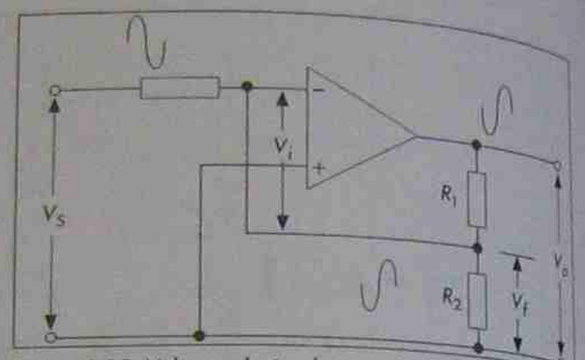


Figure 9.20. Voltage-derived series feedback around an op-amp

$$V_f = I_o \times R_f$$

The net input decreases:

$$V_i = V_s - (I_o \times R_f)$$
$$= V_s - V_f$$

From these examples it can be seen that exactly the same technique is used for feedback with any type of amplifying device. The input is phase-inverted at the output, so that part of the output is returned to the input *opposing* the original input signal. The resultant input is the algebraic sum of the original input signal and the feedback signal. The feedback may be either voltage or current derived, resulting in a decrease and an increase in R_{of} respectively.

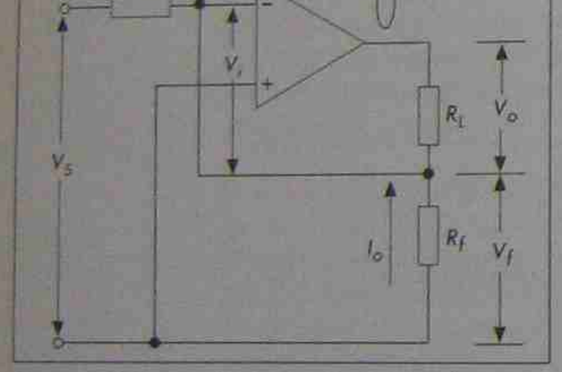


Figure 9.21. Current-derived series feedback

For those wishing to build op-amp circuits, a general purpose op-amp is the LM741. It requires a voltage supply with a maximum value of ±18 V. The pin connections for an 8 pin mini DIP package are:

pin 2—inverting input
pin 3—non-inverting input
pin 4—negative V_{cc}
pin 6—output
pin 7—positive V_{cc}

9.5 Summary

The following items are a summary of the main points covered in this chapter:

- Desensitising factor can be calculated by the equation:
  $1 + \beta A$
- Voltage gain is stabilised but reduced:
  $A' = A/(1 + \beta A)$

- Input voltage:
  $V_i = V_s - \beta V_o$
- Output voltage:
  $V_o = A(V_s - \beta V_o)$
- Input and output impedances may be increased or decreased, depending on the type of connection. For example, in a voltage feedback, series-fed circuit:
  $R_{if} = R_i(1 + \beta A)$ (increased)
  $R_{of} = R_o/(1 + \beta A)$ (decreased)
- Distortion is reduced by the factor:
  $D' = D/(1 + \beta A)$
- Bandwidth is increased with negative feedback applied.
- Rules for recognising type of feedback.
- If the feedback is taken directly from the output, the feedback is voltage and R_{of} is decreased.
- If the feedback is not taken directly from the output, the feedback is current and R_{of} is increased.
- If the feedback is connected directly to the input, the feedback is shunt-fed and R_{if} is decreased.
- If the feedback is not connected directly to the input, the feedback is series-fed and R_{if} is increased.

Key terms

bandwidth
current feedback
degenerative feedback
desensitising factor
distortion
feedback
gain without feedback (A)
gain with feedback (A')
high frequency breakpoint (f_h)
input resistance
internal resistance
low frequency breakpoint (f_l)
negative feedback
prime (')
series-derived/fed
shunt-derived/fed feedback
voltage feedback

9.6 Review questions (answers in Appendix A)

1. What is meant by the term 'feedback'?
2. If the feedback decreases the net input voltage to the amplifier, what type of feedback is used?
3. What is the main disadvantage of negative feedback, and how is this disadvantage outweighed?
4. What is the general formula for feedback? How does the feedback formula for negative feedback differ from the general formula and why is this so?
5. Explain the term 'feedback factor'.
6. If an amplifier has a gain of 500 without feedback, and 12% negative feedback is applied, what is the new gain value?

7. Explain why R_{if} and R_{of} are increased when current series feedback is used.
8. What type of feedback is used in the circuit in Figure 9.22?
9. Explain why a constant current source is required when using shunt feedback.

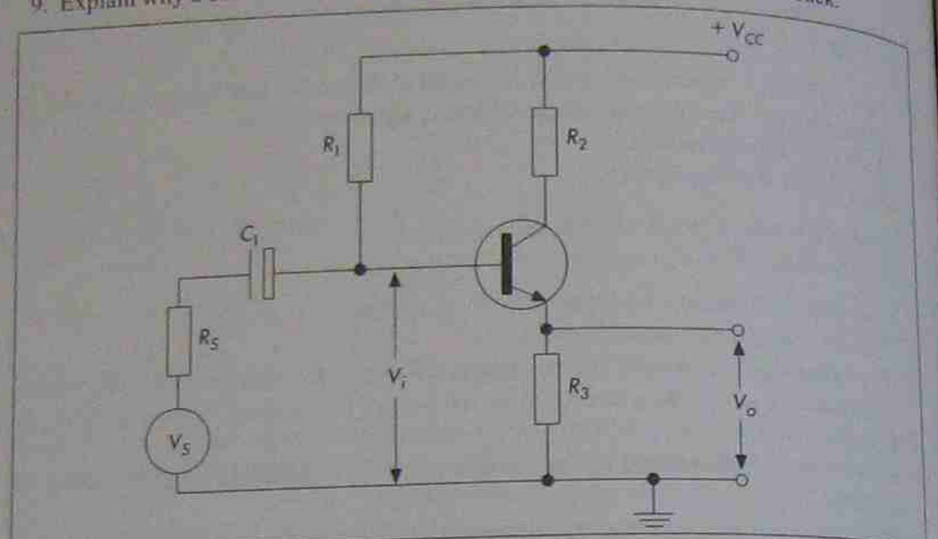


Figure 9.22

10. Draw the four basic connection configurations used for feedback.
11. The following questions are based on Figure 9.23.
    (a) What type of feedback is used?
    (b) Is it series or shunt-fed?
    (c) Is it series or shunt-derived?
    (d) What effect has the feedback had on R_{if} and R_{of}?
    (e) Briefly explain the signal summing procedure.
    (f) We are given the following values for the circuit in Figure 9.23:
    V_s = 500 mV peak
    V_o = 12 V peak-to-peak
    R_3 = 3k3 Ω
    R_5 = 58 kΩ
    R_6 = 10 kΩ

    What is the value of V_i?

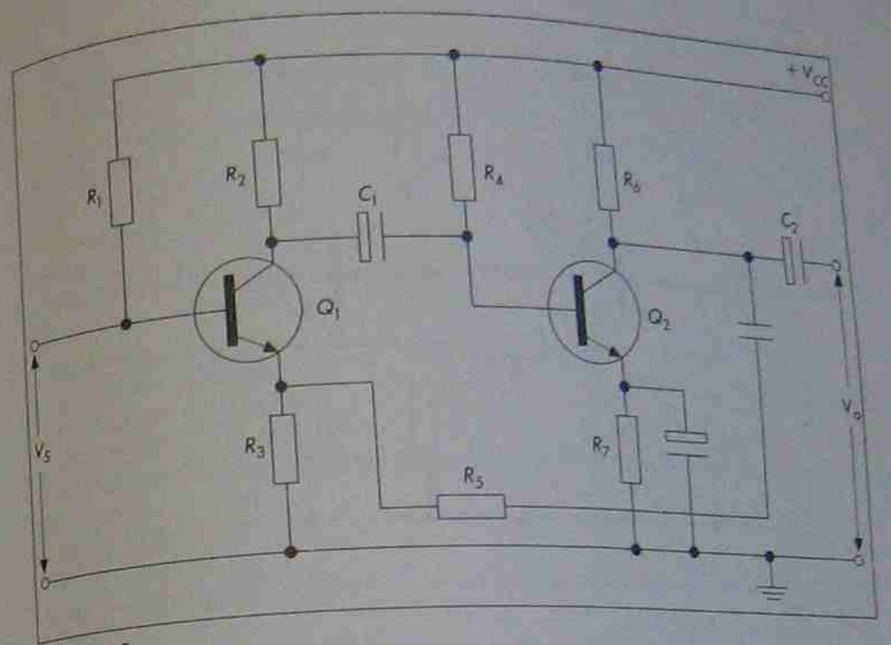


Figure 9.23

Multiple-choice questions

1. The application of negative feedback to an amplifier will cause the gain and bandwidth to:

	Gain	Bandwidth
(a)	increase	increase
(b)	decrease	increase
(c)	increase	decrease
(d)	decrease	decrease

2. Negative feedback occurs when:
   (a) part of the output is fed back to the input so that it aids the input signal
   (b) part of the input is fed back to the output so that it aids the output signal
   (c) part of the output is fed back to the input so that it opposes the input signal
   (d) part of the input is fed back to the output so that it opposes the output signal

Questions 3 to 5 refer to Figure 9.24.

3. The type of sampling and injection shown are:

	Sampling	Injection
(a)	current	shunt
(b)	current	series
(c)	voltage	shunt
(d)	voltage	series

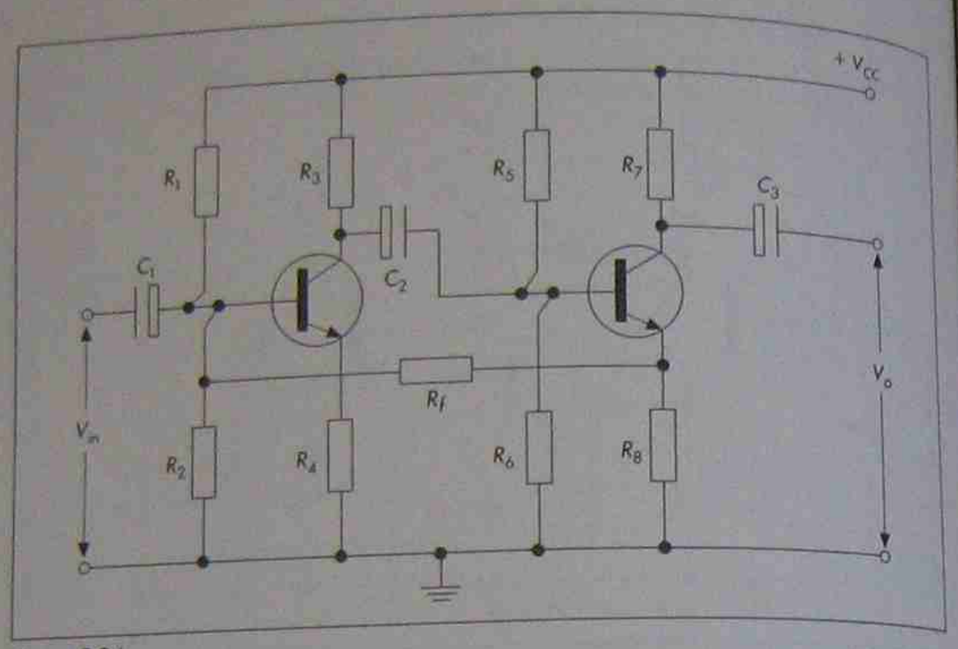


Figure 9.24

4. The effect of the feedback on input and output impedance will be:

	Z_{out}	Z_{in}
(a)	decrease	decrease
(b)	decrease	increase
(c)	increase	increase
(d)	increase	decrease

5. Refer to Figure 9.24. If the value of R_f is decreased, the amount of feedback applied will:
   (a) decrease
   (b) increase
   (c) cause V_o to increase
   (d) remain unchanged

Questions 6 to 8 apply to this amplifier. A common emitter amplifier has the following circuit parameters: voltage gain = 100, distortion = 2%, high frequency limit (f_h) = 25 kHz. Negative feedback is applied to the circuit that has a ß of 0.12.

6. The new gain (A_v) figure is:
   (a) 76.9
   (b) 13
   (c) 8.33
   (d) 7.69
7. The new high frequency limit (f_h') will be:
   (a) 2.5 MHz
   (b) 500 kHz
   (c) 325 kHz
   (d) 300 kHz
8. The percentage distortion figure will now be:
   (a) 0.154%
   (b) 0.167%
   (c) 1.85%
   (d) 26%
9. When series-fed voltage feedback is used, the effect on input and output impedance is:

	Z_{in}	Z_{out}
(a)	increase	increase
(b)	increase	decrease
(c)	decrease	decrease
(d)	decrease	increase

10. The formula to determine the low frequency limit of an amplifier with feedback is:
    (a) $f_l' = f_l \times (1 + \beta A)$
    (b) $f_l' = f_l / (1 + \beta A)$
    (c) $f_l = f_l' \times (1 + \beta A)$
    (d) $f_l = f_l' / (1 + \beta A)$

CHAPTER 10



POSITIVE FEEDBACK AND OSCILLATOR FUNDAMENTALS

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10.2.1 Tuned circuit action 222

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Draw a block diagram to illustrate positive feedback.
- Explain the operation of 'tuned circuit action' in an inductor-capacitor (LC) circuit.
- Calculate the oscillating frequency of an LC circuit.
- Define the term 'damped oscillation'.
- List the three main circuit requirements for oscillation to be sustained.
- State the term used when $\beta A = 1$ in a feedback circuit.
- Explain the difference between an amplifier and an oscillator.
- State the formula for calculating an oscillator frequency.
- Recognise a positive feedback connection on a circuit diagram.

10.1 Introduction

When the signal fed back from the output is in phase, and aids the original signal, we have **positive feedback** or **regeneration**. The circuit arrangement in Figure 10.1 shows the feedback voltage (V_f) and the signal voltage (V_s) arriving at the summing point with the same polarities. The net input voltage (V_i) is the algebraic sum of the two. Therefore:

$V_i = V_s + V_f$ but $V_f = \beta V_o$, so
$V_i = V_s + \beta V_o$

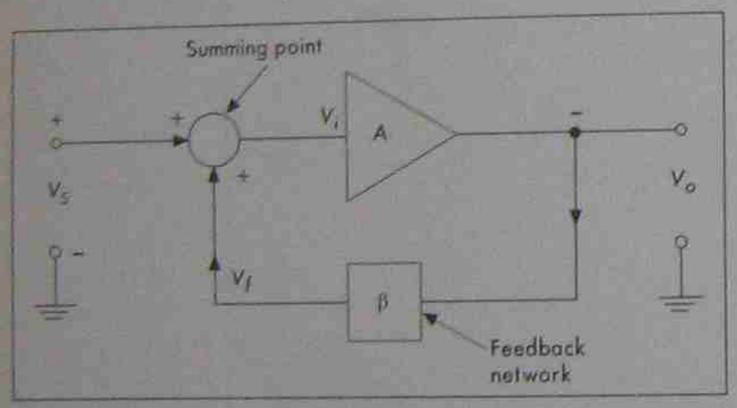


Figure 10.1 Feedback arrangement for positive feedback

The feedback factor (ß) is positive and is such that it makes the **closed-loop gain** (A') greater than the open-loop gain (A) (that is, $A' > A$).

A **'closed-loop' circuit** is a circuit where some sort of monitoring of the output is carried out. A signal which bears a relationship to the output, either in voltage, phase or frequency, is then fed back into the circuit, as with negative and positive feedback. When there is no feedback the circuit is said to be an 'open-loop' circuit.

If βA is equal to 1, the feedback formula:

$A' = A/(1 - (+\beta A))$
gives $A' = A/1 - 1$
which equals $A/0 = \infty$ (infinity)

With infinite amplification, there is an output with no input.

Pick-up, noise, and so on, at the input or output results in the amplifier supplying its own V_i via the feedback network from V_o. The amplifier is now an **oscillator**. In other words, it

produces a continuous alternating output signal when connected to a DC power source. AC power is generated at the expense of the DC power absorbed. A comparison between an amplifier and an oscillator is shown in Figure 10.2.

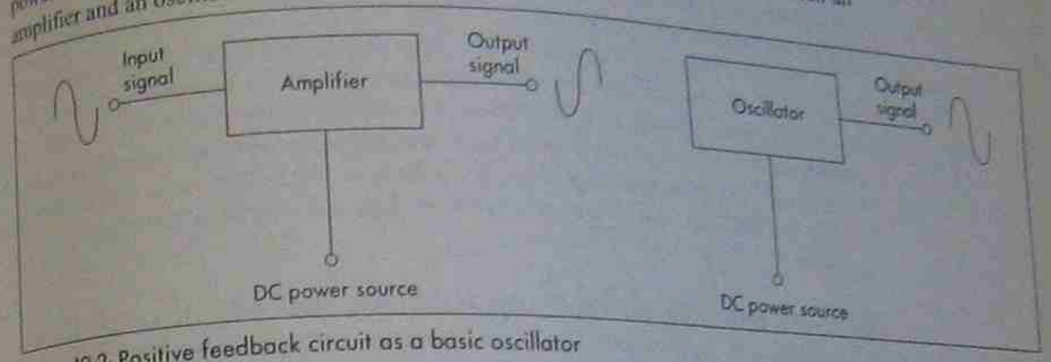


Figure 10.2: Positive feedback circuit as a basic oscillator

The operation of an amplifier and its feedback network as an oscillator can best be seen by referring to Figure 10.3. When switch S_1 is closed, electron-hole combination (noise) inside the amplifying device creates a small current, generating the initial input voltage (V_i). This voltage is fed into a high gain amplifier, and the result is:

$V_o = AV_i$

The output voltage is fed into the feedback network, and a feedback voltage ($V_f = \beta A V_i$) is passed back to the input of the amplifier. If the feedback network gives a βA value with the correct amplitude and phase, the feedback voltage can be made equal to V_i.

As stated earlier, V_i is now derived from V_o via the feedback circuit with the correct phase ($\beta A = 1$), and this is known as the **Barkhausen criterion for oscillation**. When $\beta A = 1$, the system will be self-sustaining. In practice, however, βA is made slightly greater than 1 to ensure self-starting from the noise voltage. The output waveform is sinusoidal in nature, but has slight distortion due to amplifier saturation.

Oscillators are classified in terms of the output waveform, frequency range or circuit configuration. If the output waveform is sinusoidal, the oscillator is referred to as a harmonic oscillator. An oscillator giving a non-sinusoidal output is called a relaxation oscillator, and its output waveform may be square, triangular or sawtooth.

The components used in oscillator circuits are both active and passive. Active components, such as transistors, field effect transistors, Schottky and tunnel diodes, provide the medium

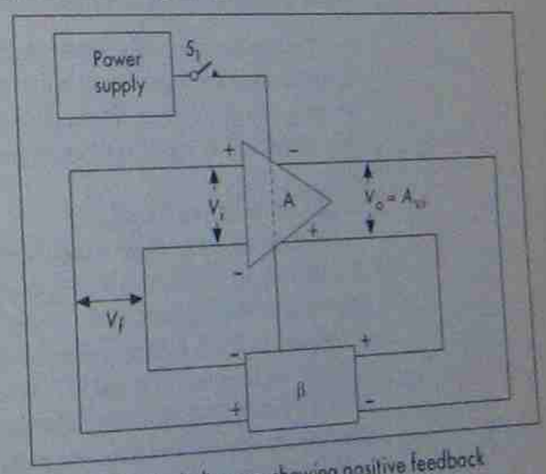


Figure 10.3: Block diagram showing positive feedback

for energy conversion and gain. The active device may be biased in class A, B, or C. The output of an oscillator biased in class A is relatively free from harmonics. Class C is the most efficient operating area, but the harmonic content can be high, and good filtering is required to attenuate the undesirable frequencies.

A resonant circuit may be included at the output to select a desired frequency and reject all others. Passive components (resistors, capacitors, inductors) normally determine the frequency of operation and, to a large extent, the stability of the circuit. An oscillator is generally called upon to provide a controlled output frequency and constant amplitude, but the limits on these parameters depend on the application. Where frequency and amplitude stability are important, the choices of component type and operating tolerances become important.

10.2 The inductor and the capacitor as sources of oscillation

A basic sinusoidal oscillator can be visualised using two components that have completely opposite AC characteristics. The inductor and capacitor shown in Figures 10.4(a) and (b) are reactive components, and as such are capable of storing energy (a tuned circuit).

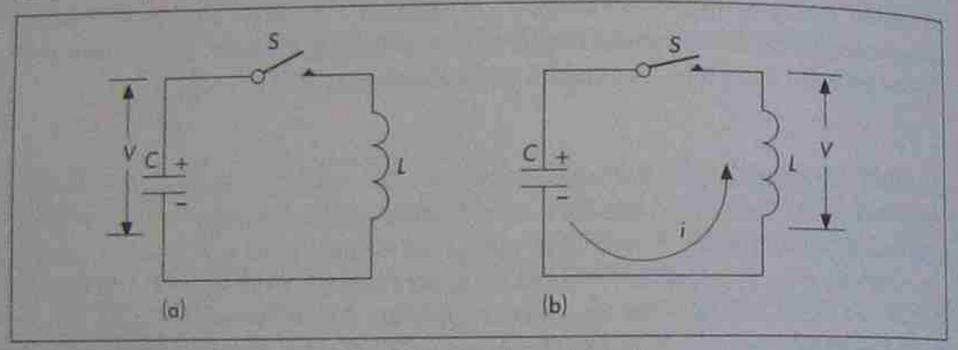


Figure 10.4: Basic inductor-capacitor (LC) circuit (tuned circuit)

The inductor stores energy in the form of a magnetic field whenever current passes through it, and the capacitor stores energy in the form of an electrostatic field whenever there is voltage across its plates. However, for the purpose of explanation, it must be assumed that both the inductor (L) and the capacitor (C) have an infinite Q value, and therefore have no losses.

10.2.1 Tuned circuit action

A **tuned circuit** can be made to oscillate by initially charging the capacitor with a voltage V (Figure 10.4(a)). With switch S open, capacitor C remains charged and there is no current in the circuit. When S closes (Figure 10.4(b)), the capacitor discharges through the inductor L, setting up a magnetic field around its windings. The electrostatic energy previously stored in C is now magnetic energy stored in L. When the capacitor's discharge current is zero, that is, no more changing current, the magnetic field around L collapses. The collapsing field generates an electromotive force, the polarity of which is determined by Lenz's law, and the capacitor charges again, but in the opposite direction.

This sequence of events is shown pictorially in Figures 10.5(a) to (e). Once the energy stored in L is spent, the capacitor discharges again through L and the cycle repeats itself. A continuous alternating output signal is produced, and the circuit behaves in an oscillating fashion.

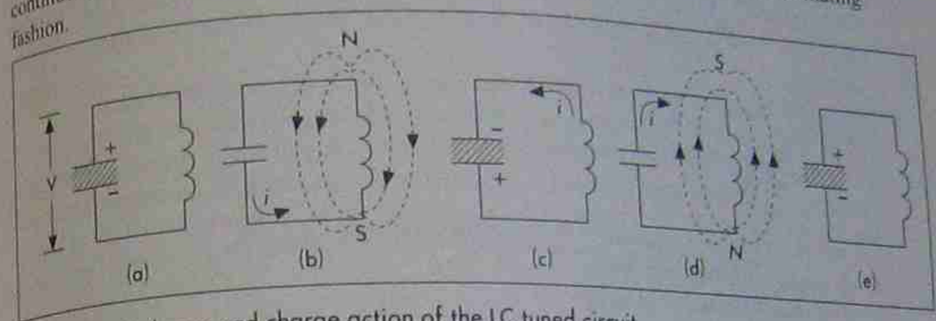


Figure 10.5: Discharge and charge action of the LC tuned circuit

The diagram in Figure 10.6 shows the relationship between capacitor voltage and circuit current for the action previously described. Both the voltage and current are sinusoidal, and the waveform is developed from a DC voltage stored in a capacitor:

(a) Charged capacitor is placed across L.
(b) C discharges through L, creating a magnetic field around L.
(c) Discharge current from C ceases, the magnetic field collapses and charges C with the opposite polarity.
(d) C discharges through L again, creating a magnetic field around L.
(e) Discharge current ceases, the field collapses, and C charges again. Note the polarity — a complete cycle of events has taken place.

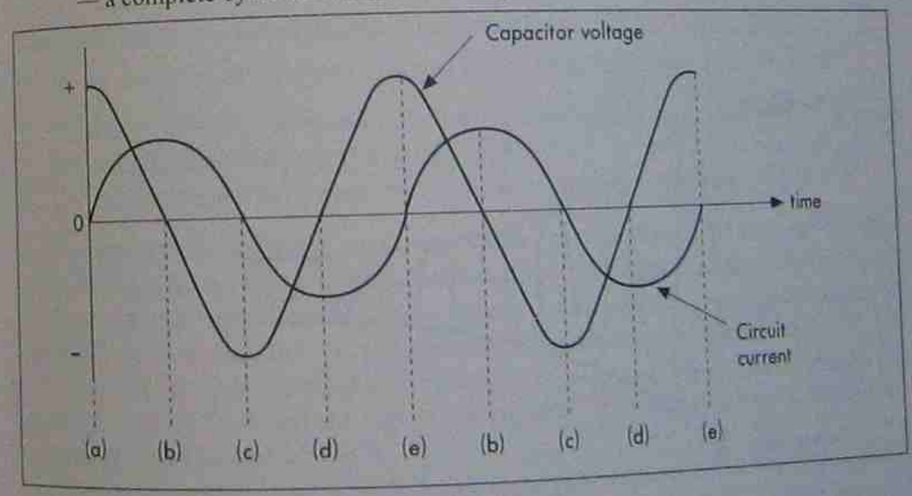


Figure 10.6: Two cycles of circuit action

At the resonant frequency (f_r) of the tuned circuit, the inductive reactance (X_L) and capacitive reactance (X_C) are equal. Thus:

$X_L = X_C$, therefore
$2\pi fL = 1/2\pi fC$

Solving for f:

$2\pi fL \times 2\pi fC = 1$, so
$f^2 = 1/4\pi^2 LC$
$f = \sqrt{1/4\pi^2 LC}$
$f_r = 1/2\pi\sqrt{LC}$
where f_r = resonant frequency (frequency of oscillation) in hertz
L = value of inductor in henrys
C = value of capacitor in farads
2π = constant (approximate value of 6.28)

If the value of C is 100 nF and L is 500 mH, the resonant frequency is:

$f_r = 1/2\pi\sqrt{LC}$
$= 1/6.28\sqrt{(500 \times 10^{-3} \times 100 \times 10^{-9})}$
$= 1/1404.25 \times 10^{-6}$
$= 712.12$ Hz

The circuit just described is not a practical oscillator. We had to assume a Q of infinity and, even if ideal components with no losses were available, energy could not be extracted without putting a load on the circuit. When energy is drawn from the circuit, the oscillations slowly decay. This effect is called **damped oscillations**; an example of the effect is shown in Figure 10.7.

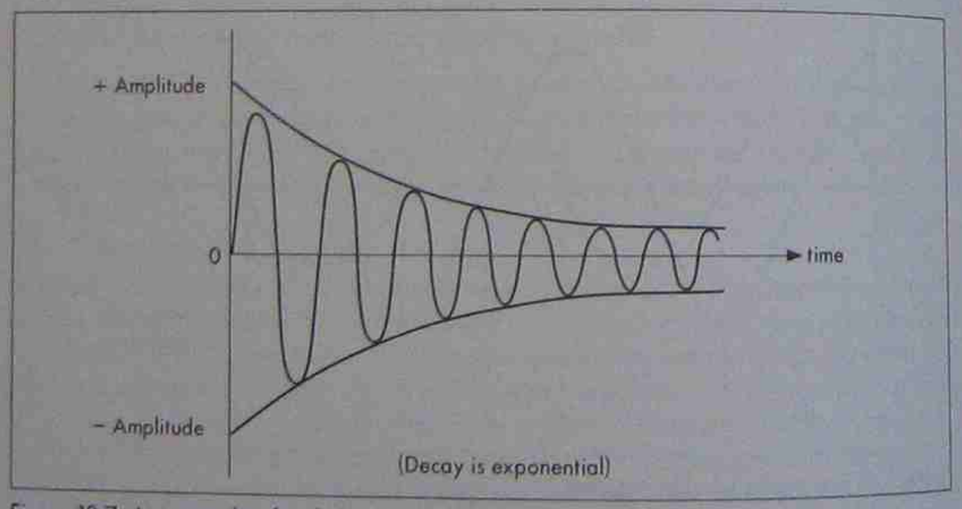


Figure 10.7: An example of a damped oscillation

Eventually, all the energy is consumed and the circuit stops oscillating. If too heavy a load is placed on the circuit, overdamping will occur, and all the energy will be used before even one cycle of oscillation can take place. The losses in a practical circuit are mainly due to the resistive component present in the physical construction of the inductor. This resistance reduces the figure of merit Q of the coil ($Q = X_L/R$) but does not greatly affect the **resonant frequency**.

If the energy losses could be replaced from an external source, the circuit would continue to oscillate and the amplitude of the sinusoidal waveform would remain constant. The theoretical operation of such a circuit can best be seen by referring to Figure 10.8. Normally, the output would be damped. However, if switch S is closed each time the tuned circuit develops the polarity indicated in the circles across the capacitor C, energy will be returned to C. This will replace the energy taken by circuit losses and the output.

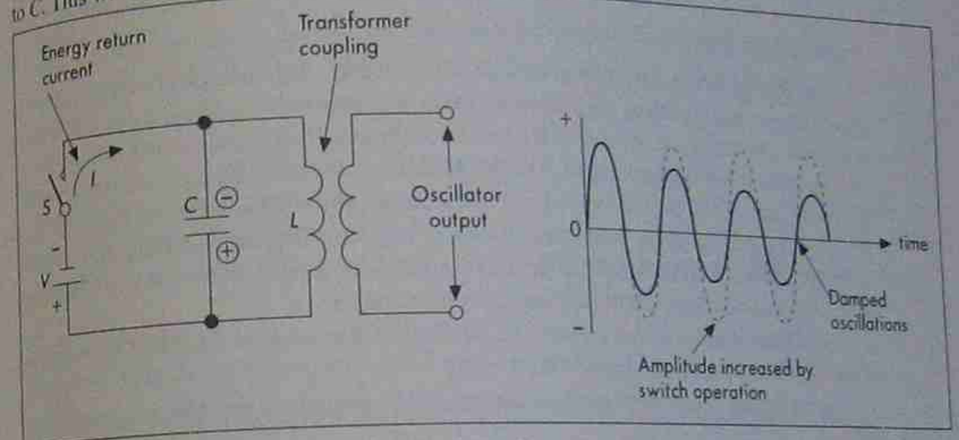


Figure 10.8: Energy replacement to the tuned circuit

The two main requirements of this type of circuit operation are:

1. The energy must be returned at the correct time, so that the voltage or current being returned *aids* the oscillating voltage or current.
2. The energy returned must equal the *total* energy lost from the circuit.

In a functional circuit, the active device is controlled to return the correct amount of energy at the correct time. Such a circuit can be represented by the block diagram shown in Figure 10.9.

The amplitude of the oscillation is limited by the circuit's own non-linearities, but conditions could be such that the active device swings between saturation and cut-off. This will square-off the top and bottom of the sinusoidal (sine) wave. The gain of the active device should be just enough to start and maintain oscillations. If a sine wave output is required, the amplifier must not be overdriven.

The frequency-determining circuit, which may be a tuned circuit or a CR network, determines the frequency of oscillation. It also ensures that harmonics and other unwanted signals which may occur in the output signal are rejected.

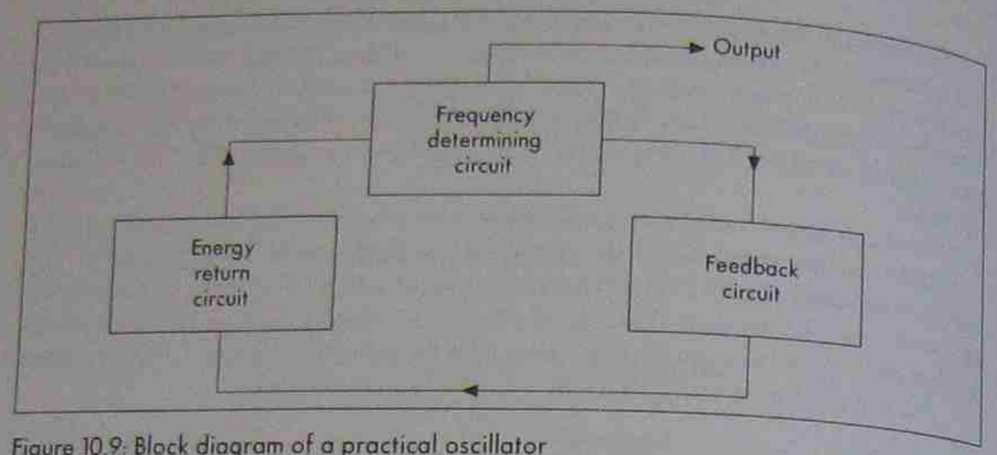


Figure 10.9: Block diagram of a practical oscillator

The **energy return** circuit must replace circuit losses. This energy is supplied from a power source and applied to an amplifier which maintains the oscillations at a constant amplitude. The amplitude of the output signal is determined by the gain of the amplifier and the amplitude of the feedback signal. The feedback circuit returns the feedback energy with the correct timing and polarity to maintain oscillations. From Chapter 9 we learn that this feedback is a portion of the output signal, and it may require phase shifting 180°. Phase inversion of the feedback signal can be accomplished by a transformer or an LC voltage divider. These techniques will be discussed in a later chapter.

These basic building blocks can now be clarified further to show the phase relationships of the signals as illustrated in Figure 10.10.

The conditions necessary to produce and maintain oscillations can now be summarised:

- The active device must be an amplifier with sufficient gain to return energy to the circuit (greater than 1).

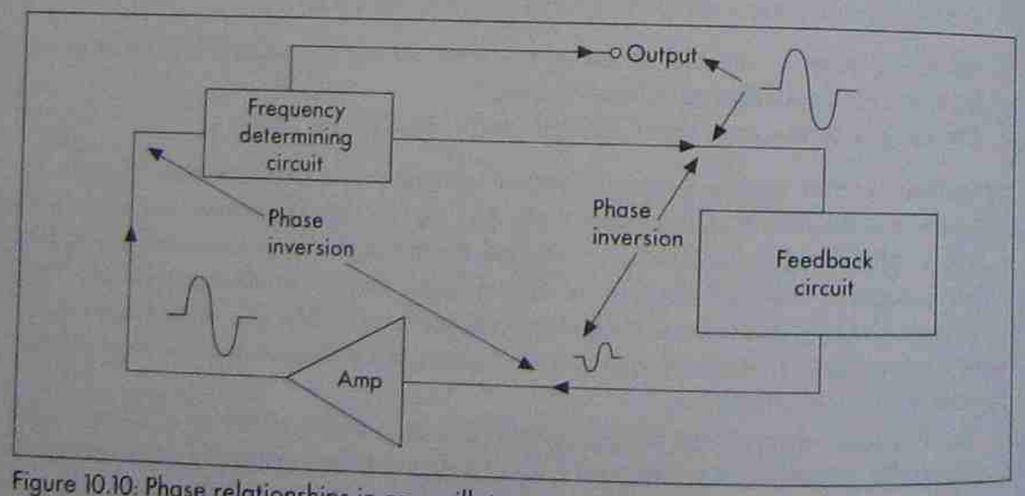


Figure 10.10: Phase relationships in an oscillator

- A frequency-determining circuit must be used to maintain the correct frequency.
- A feedback circuit is required to return energy and produce a further 180° **phase shift** if necessary. This phase shift, when added to the amplifier's phase shift of 180°, gives the required 360° shift (that is, feedback in phase with V_i) for positive feedback.

10.3 Operational amplifiers with positive feedback

An example of positive feedback on an operational amplifier (op-amp) is shown in Figure 10.11. The input (V_s) is applied to the non-inverting input. Part of the output is taken across a voltage divider and fed back to the input. The two signals (V_f and V_s) are in phase and the net input voltage increases:

$$V_i = V_s + \beta V_o$$
$$= V_s + V_f$$

The closed-loop gain (A') is greater than the open-loop gain (A). When $\beta A = 1$, the Barkhausen criterion for oscillation exists and the op-amp becomes an oscillator. The input signal (V_s) is no longer required. However, it is still necessary to control the frequency of oscillation and return lost energy to the circuit. The ways in which this can be achieved are discussed in Chapters 11 and 12.

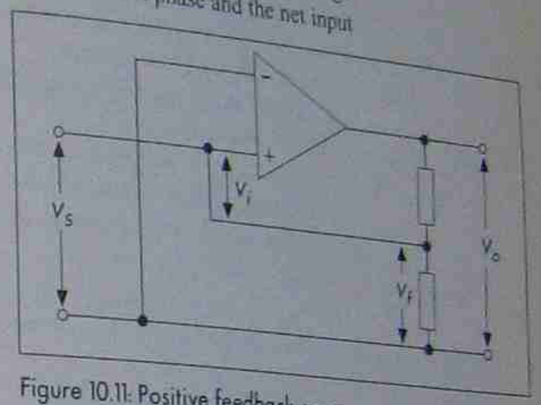


Figure 10.11: Positive feedback on an op-amp

10.4 Summary

The following items are a summary of the major points covered in this chapter:

- Feedback signal must be in phase with the original input voltage (V_i) to produce positive feedback.
- Positive feedback causes oscillation.
- $V_i = V_s + \beta V_o$
- An oscillator does not require an input signal unlike an amplifier.
- An oscillation developed by an inductor-capacitor (LC) circuit is a 'damped oscillation'.
- Energy return to the circuit must have the correct timing to aid the circuit and must equal the total energy lost from the circuit.
- The active device must not be overdriven or clipping will occur.
- When $\beta A = 1$, the Barkhausen criterion for oscillation exists.
- $f_r = 1 / 2\pi\sqrt{LC}$

Key terms

Barkhausen criterion
closed-loop circuit
damped oscillation
energy return
oscillator
phase shift
positive feedback
regeneration

open-loop gain
oscillation
resonant frequency
tuned circuit

10.5 Review questions (answers in Appendix A)

1. What is the main purpose of an oscillator?
2. With the aid of a diagram, explain how an amplifier can oscillate.
3. (a) Explain how an LC circuit may be used to produce a sinusoidal waveform.
   (b) Is this a practical situation?
   (c) Explain your answer to (b).
4. A parallel circuit contains an inductor L and a capacitor C. If $L = 20$ mH and $C = 10$ nF, what is the oscillator frequency?
5. With the aid of a diagram, explain three necessary functions required to ensure stable continuous oscillations.
6. What is the term used to describe the effect of reduced amplitude with loading?
7. State the two main requirements of the feedback voltage.
8. Will the circuit shown in Figure 10.12 oscillate? Explain the reason for your answer.

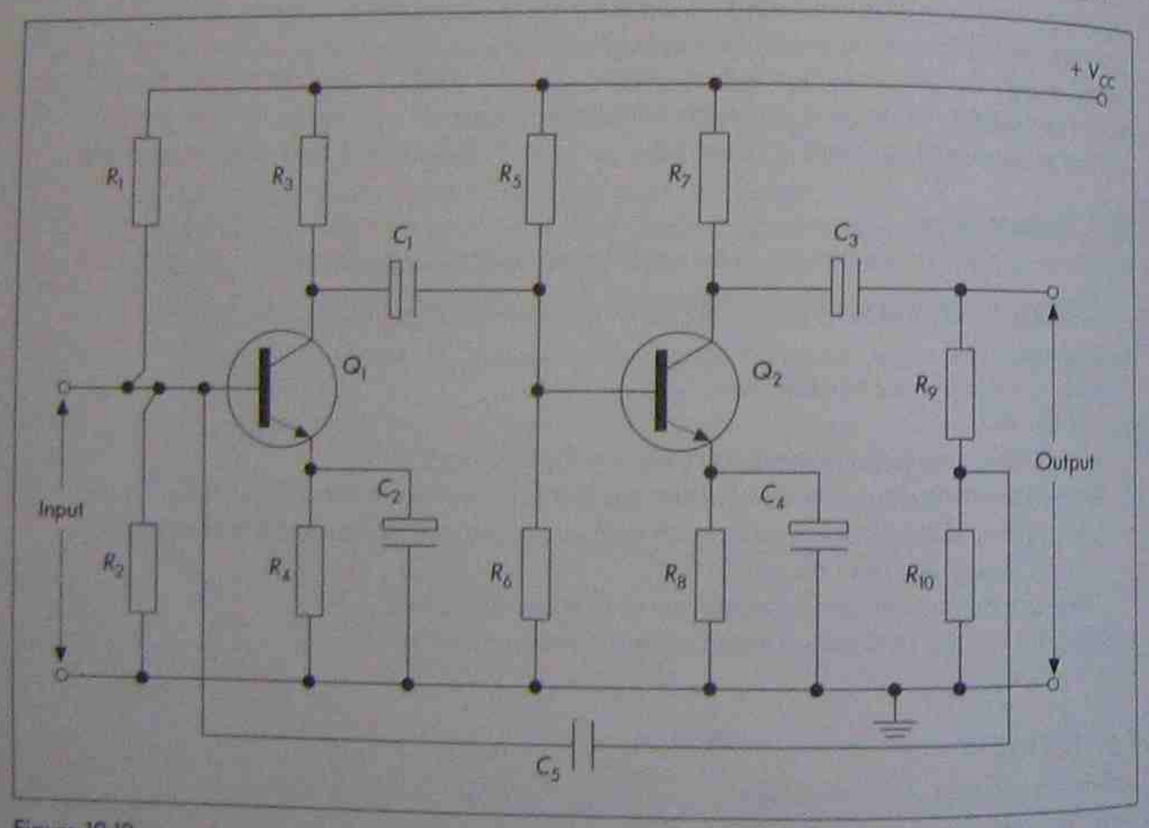


Figure 10.12

Multiple-choice questions

1. Positive feedback occurs when:
   (a) part of the output is fed back to the input so that it aids the input signal
   (b) part of the input is fed back to the output so that it aids the output signal
   (c) part of the output is fed back to the input so that it opposes the input signal
   (d) part of the input is fed back to the output so that it opposes the output signal
2. When $BA = 1$ the circuit will:
   (a) have maximum negative feedback
   (b) oscillate freely
   (c) behave as an amplifier
   (d) produce a square wave output
3. In a parallel LC circuit that has 'no losses' the exchange of energy between L and C is called:
   (a) negative feedback
   (b) amplifying
   (c) degeneration
   (d) tuned circuit action
4. In an LC circuit where $X_L = X_C$, the voltage across the capacitor:
   (a) lags the circuit current by 90°
   (b) is in phase with V_L
   (c) leads the circuit current by 90°
   (d) leads V_L by 90°
5. An LC circuit has $L = 500$ mH and $C = 10$ nF. The resonant frequency will be approximately:
   (a) 712 Hz
   (b) 924 Hz
   (c) 1125 Hz
   (d) 2250 Hz
6. The conditions necessary to produce and sustain oscillation are:
   (a) the active device must have sufficient gain to replace lost energy
   (b) a frequency-determining circuit is required
   (c) a feedback circuit and 180° phase shift network is required
   (d) all of the above
7. With reference to Figure 10.11, the type of feedback shown is:
   (a) voltage, shunt-derived
   (b) current, shunt-derived
   (c) voltage, series-derived
   (d) current, series-derived

Questions 8 and 9 refer to Figure 10.12.

8. If C_c went open circuit, the circuit would:
   (a) cease to function
   (b) be overdriven by excessive feedback
   (c) behave as an amplifier
   (d) produce a very low level output due to degeneration
9. If the value of R_9 was increased by 10 times, the amount of feedback will:
   (a) decrease
   (b) increase
   (c) remain unchanged
   (d) cause V_o to be distorted
10. An oscillator circuit:
   (a) converts AC to DC
   (b) converts DC to AC
   (c) requires an input signal
   (d) generates its own power supply voltage

CHAPTER 11

PHASE-SHIFT OSCILLATORS

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11.3 Four-section feedback networks 236

11.4 Variable frequency phase-shift oscillators 236

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Draw the block diagram of a basic phase-shift oscillator.
- State the degree of phase shift required for three and four-section oscillators.
- Calculate the frequency of three and four-section phase-shift oscillators.
- Explain how the required phase shift is obtained.
- Define the terms 'graded' and 'ungraded' sections.
- List the gain figures required for three and four-section oscillators.
- Describe the operation of the following:
  - a three-section phase-shift oscillator;
  - a four-section phase-shift oscillator; and
  - a Wien bridge oscillator.
- Calculate the operating frequency of a Wien bridge oscillator.
- Show how an operational amplifer can be connected to function as a Wien bridge oscillator.

11.1 Introduction

To achieve oscillation, the signal fed back from the output (V_o) must be in phase with the input. The amplifier provides a 180° phase difference between input and output; therefore, the feedback network must phase-shift the output signal a further 180°. This ensures that the ideal condition for oscillation is met, that is, the feedback voltage (V_f) is in phase with the input voltage (V_i). The amplifier must also have sufficient gain to replace any circuit losses.

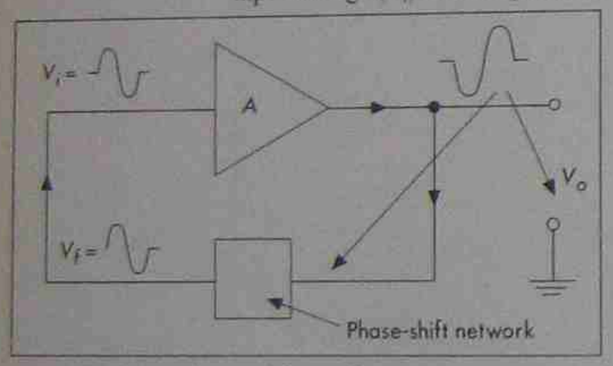


Figure 11.1: Basic phase-shift oscillator

The **phase-shift oscillator** is made up of two major sections as shown in Figure 11.1 The phase shift can be produced by a network of capacitors and resistors. At one frequency only, the network will produce an overall phase shift of 180° and, therefore, it will also determine the frequency of oscillation. The capacitor-resistor (CR) phase-shift circuit is usually used for frequencies up to about 100 kHz.

11.2 Three-section phase-shift networks

In a series CR network, the current through the capacitor and resistor leads the applied voltage. The phase angle is determined by the ratio of capacitive reactance (X_C) to the resistance (R). The circuit and the phasor diagram are shown in Figure 11.2.

The output of the circuit is taken across the resistor (V_R), and this voltage is in phase with the circuit current (I). However, V_R leads the input signal voltage (V_s) by an **angle theta (θ)**:

$$\tan\theta = V_C/V_R$$
$$= (I \times X_C)/(I \times R)$$

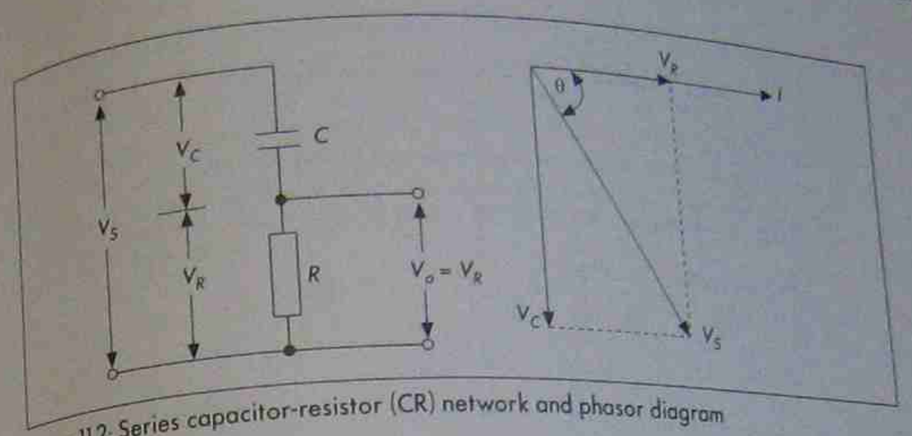


Figure 11.2: Series capacitor-resistor (CR) network and phasor diagram

But the circuit current is common (series circuit), therefore:

$$\tan\theta = X_C/R$$

If $X_C = 8140\ \Omega$ at the operating frequency and $R = 4700\ \Omega$, then the phase angle would be:

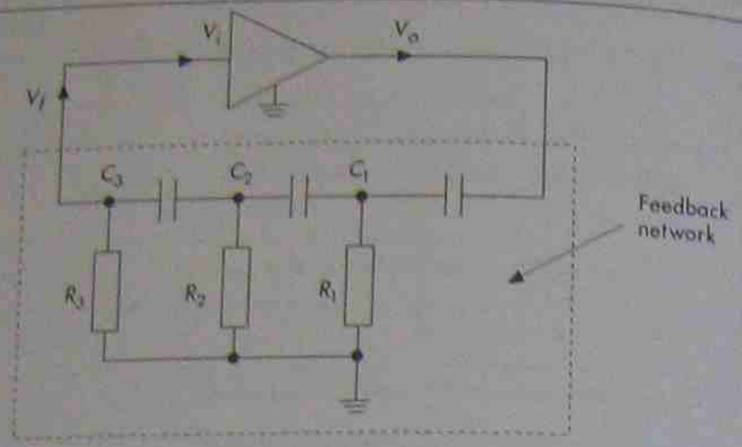
$$\tan\theta = X_C/R$$
$$= 8140/4700$$
$$= 1.732$$

The artan (that is, $\tan^{-1}$ of $1.732 = 60°$. Therefore, three sections should in theory give an **overall phase shift** of $3 \times 60° = 180°$. This is the idealised situation. The circuit and the phasor diagram for this are shown in Figures 11.3(a) and (b). Note the greatly reduced output of the loaded phasor.

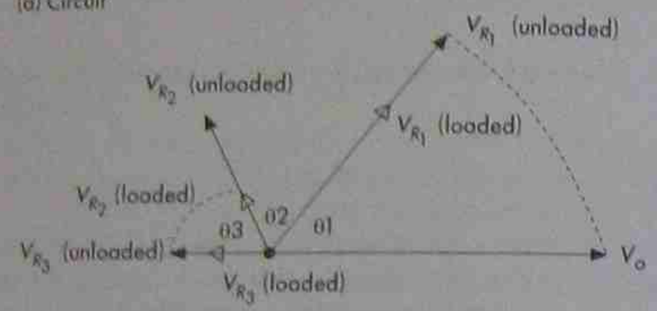
The output of the amplifier (V_o) is fed to the CR network (C_1R_1). The voltage developed across R_1 leads the original output voltage by the phase angle θ (see Figure 11.2). This voltage across R_1 is the input voltage to the next CR stage (C_2R_2) and the voltage developed across R_2 leads the original V_o by 2 θ. The voltage across R_2 is then fed into a third stage (C_3R_3) which phase-shifts it a further θ. The original V_o has now been phase-shifted $3\theta = 3 \times 60° = 180°$. The feedback voltage V_f is now 180° out of phase with V_o, and the ideal situation for oscillation is met, providing that the amplifier has sufficient gain to replace the circuit losses.

The **CR sections** can all be equal ($R_1 = R_2 = R_3$ and $C_1 = C_2 = C_3$) or graded ($R_3 = 10R_2 = 100R_1$ and $C_3 = 0.1C_2 = 0.01C_1$). If **graded sections** are used, the loading effect on each stage upon the previous one is negligible. The circuit will oscillate at one frequency only: the frequency where the phase shift from V_o to V_f is 180°. With negligible loading, this is the ideal situation where each CR section provides 60° of phase shift. See the unloaded vector of Figure 11.3(b). The **frequency of oscillation** (f_r) can be calculated by network analysis (the mathematics of this is not covered in this text). It can be shown that:

$$f_r = 1 / 2\pi CR\sqrt{3}$$ (for graded sections only)



(a) Circuit



(b) Phasor diagram
(open arrow-heads indicate loaded voltages)

Figure 11.3: Three-section capacitor-resistor (CR) phase-shift network

The output voltage is attenuated by a factor of 8 through the feedback network and, therefore, the amplifier must have an open loop gain (A) greater than 8 to ensure sufficient gain for oscillation (that is, all losses in the circuit are replaced).

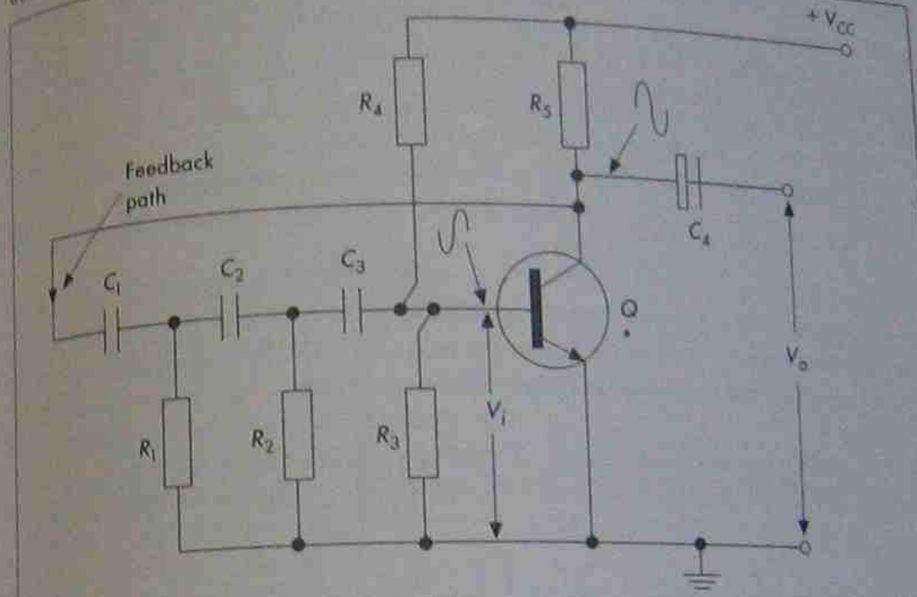
Graded sections are rarely used in practice because the loading effects of input and output impedance cannot be neglected. The idealised phase shift of 60° per section at one particular frequency is not present. If the phase shifts per section were measured, it would be seen that each section is not identical. However, the overall phase shift betwen V_o and V_f is 180° and, therefore, oscillation does occur.

The frequency of oscillation can again be calculated from network analysis. It shows that the frequency of oscillation can be found from the formula:

$f_r = 1/2\,CR\sqrt{6}$
where $R = R_1 = R_2 = R_3$
$C = C_1 = C_2 = C_3$

The open loop gain (A) must be greater than 29 (see Appendix D) because the circuit losses are greater in a loaded circuit. (See the loaded phasor of Figure 11.3(b).)

If exactly 60° phase shift is required per section with equal values of R and C, then an emitter-follower circuit (buffer) would be required after each CR section. This is not practical because four active devices would be required in the circuit, and unnecessary complexity would result. A practical three-section CR phase-shift oscillator is shown in Figure 11.4.



$C_1 = C_2 = C_3 = 1.0$ nF
$R_1 = R_2 = R_3 = 47$ kΩ
$R_4 = 330$ kΩ
$R_5 = 4k7$ Ω
$C_4 = 50$ µF

Q = BC109 or a Darlington pair

* Q may be replaced by two transistors connected as a 'Darlington pair'. This arrangement has a very high input resistance, and the loading effect is kept to a minimum.

If temperature stability is a problem, an emitter resistor of 1 kΩ and a bypass capacitor may be included.

Figure 11.4: Practical three-section CR phase-shift oscillator (with input and output waveforms)

The frequency of oscillation is:
$f_r = 1/2\pi CR\sqrt{6}$
$= 1/6.28 \times 1 \times 10^{-6} \times 47 \times 10^{3} \times 2.45$
$= 1382.8$ Hz
$= 1.4$ kHz

11.3 Four-section feedback networks

The greater the phase shift in the CR section, the greater the losses, and the smaller the output voltage level. The graded three-section oscillator requires an amplifier with a gain of 8. If the resistor values and capacitor value are made equal, that is, **ungraded sections** the gain requirement increases to 29. Thus, the number of CR sections can be increased to four in order to reduce the amount of gain required by the amplifier. The overall phase shift must be 180°, therefore each section ideally provides a 45° shift. The operation of the circuit is the same as discussed previously, but the amplifier gain is reduced to 18, because there is less loss in a four-section circuit. A four-section phase-shift oscillator using a field effect transmitter (FET) is shown in Figure 11.5.

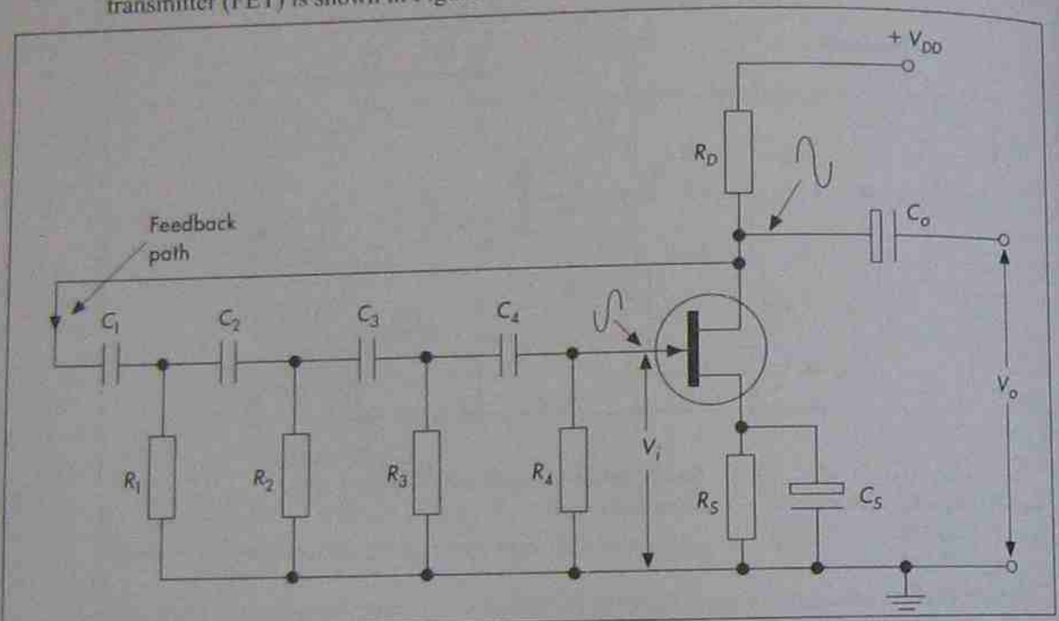


Figure 11.5: Four-section CR phase-shift oscillator using a high gain FET (with input and output waveforms)

The CR section (C_1R_1) phase-shifts the output (V_o) by 45°, developing a voltage across R_1 that leads V_o by 45°. This voltage is then phase-shifted three more times by approximately 45°, providing a voltage across R_4 that is 180° out of phase with V_o. If the gain of the FET is greater than 18, the circuit will oscillate at the frequency where V_o and the input voltage (V_i) are 180° out of phase. The high input impedance of an FET reduces the loading on the CR circuit, and the frequency of oscillation can be calculated from:

$$f_r = 1/2\pi CR\sqrt{1.4}$$

11.4 Variable frequency phase-shift oscillators

To adjust the frequency of a phase-shift oscillator, the values of the feedback capacitors or the feedback resistors must be varied simultaneously. The versatility of such an oscillator can be

expanded by switching R_1, R_2 and R_3. This provides a coarse frequency range switch. Capacitors C_1, C_2 and C_3 can then be ganged together so that the values are kept the same. Adjusting the ganged capacitors provides **a fine frequency control**.

However, a disadvantage of varying frequency is that changing the value of a feedback component also changes the value of the feedback voltage. This means that the output signal amplitude changes as the frequency changes. Also, the resistor across which V_i is developed is the lower leg of the voltage divider bias network, and care must be taken not to bias the active device on a non-linear portion of its characteristic.

A phase-shift oscillator is operated in class A bias to keep distortion as low as possible. The gain is such that a sinusoidal waveform is provided at the output of the amplifier. However, the output must not be connected directly to a low impedance circuit, otherwise the oscillator output will be loaded down, and oscillations could cease. An emitter-follower or FET stage (high impedance) should be included in the oscillator's output circuit to provide negligible loading. This stage is often referred to as a buffer.

11.5 Wien bridge oscillators

In the CR phase-shift oscillator, the amplifier provided 180° of phase shift, and the CR network provided a further 180°. This puts the output signal back in phase with the input signal. An alternative way of achieving a 360° phase shift is to pass the signal through two stages of amplification as shown in Figure 11.6.

The input is phase-shifted 180° by amplifier A_1. The output signal of A_1 is fed to amplifier A_2 and this amplifier provides a further 180° of phase shift. The output is now back in phase

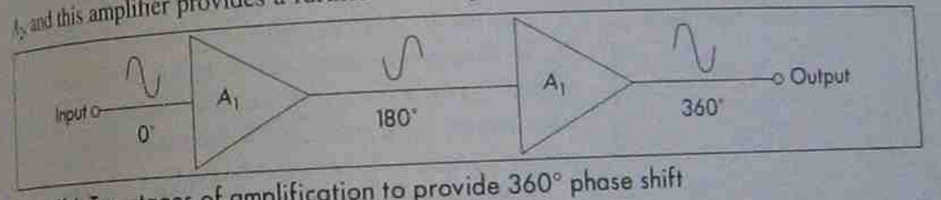


Figure 11.6: Two stages of amplification to provide 360° phase shift

with the input as shown. If the output is connected directly to the input, the amplifiers are overdriven, and a distorted square wave results. A sinusoidal output can be obtained by allowing maximum positive feedback at *one frequency only* and by having some control of the system's gain. A bridge network is inserted between the output of A_2 and the input of A_1 to satisfy these requirements. Figure 11.7 shows the block diagram of such an arrangement. It is referred to as a **Wien bridge**.

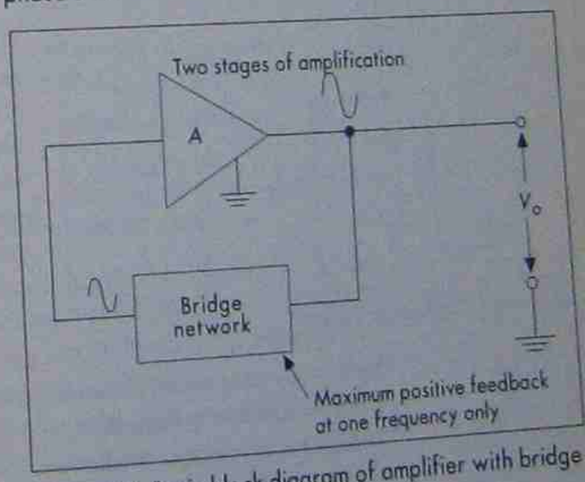


Figure 11.7: Basic block diagram of amplifier with bridge

11.5.1 Bridge circuits

The bridge circuit is connected to the amplifier to provide **positive feedback** for oscillation and

negative feedback for gain control. The basic block diagram can be expanded further to Figure 11.8, which shows the complete system.

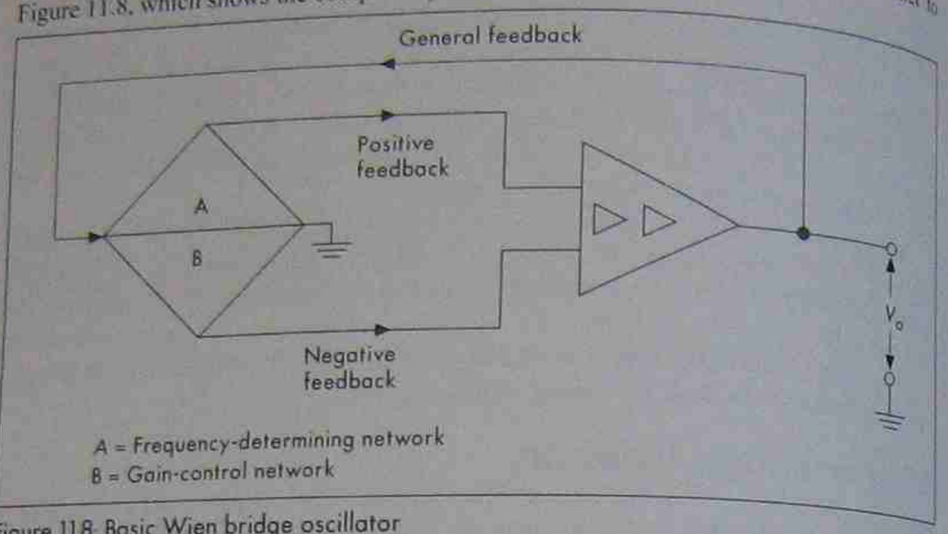


Figure 11.8. Basic Wien bridge oscillator

Part A of the bridge is the whole secret of obtaining maximum positive feedback at one frequency only (as in the CR phase-shift oscillator), and a CR network is used to perform a similar function. The operation of the frequency-determining circuit can best be understood by referring to Figure 11.9.

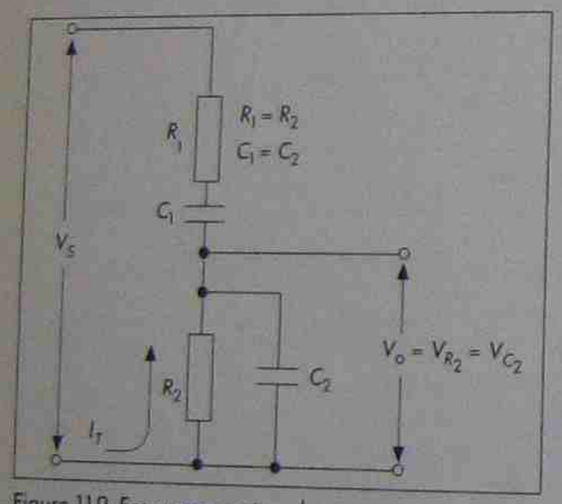


Figure 11.9. Frequency network

The usual circuit condition is to have $R_1 = R_2$ and $C_1 = C_2$. To sustain oscillations, the output voltage (taken across R_2 and C_2) must be in phase with the input voltage (V_s). In other words, the phase angle is the same between the total current (I_T) and V_s as between I_T and V_o.

With low frequencies, the capacitive reactance is large compared to the values of R_1 and R_2. Current in the circuit is determined by the value of X_{C1}, and it leads V_s by a phase angle slightly less than 90°. The ratio of X_{C2} to R_2 is large, therefore the current takes the easier path through R_2. The voltage developed across R_2 leads by the same amount, and cannot therefore sustain oscillations. The equivalent circuit and the phasor diagrams showing the phase relationships are shown in Figures 11.10(a), (b) and (c).

At high frequencies, the capacitive reactance is small compared to R_1 and R_2. Current in the circuit is determined by the resistance, and is almost in phase with V_s. However, the ratio of X_{C2} to R_2 is small, the circuit is effectively capacitive, and the voltage developed across C_2 lags the current. This voltage cannot sustain oscillations either. The equivalent circuit and the phasor diagrams showing the phase relationships are shown in Figures 11.11(a), (b) and (c).

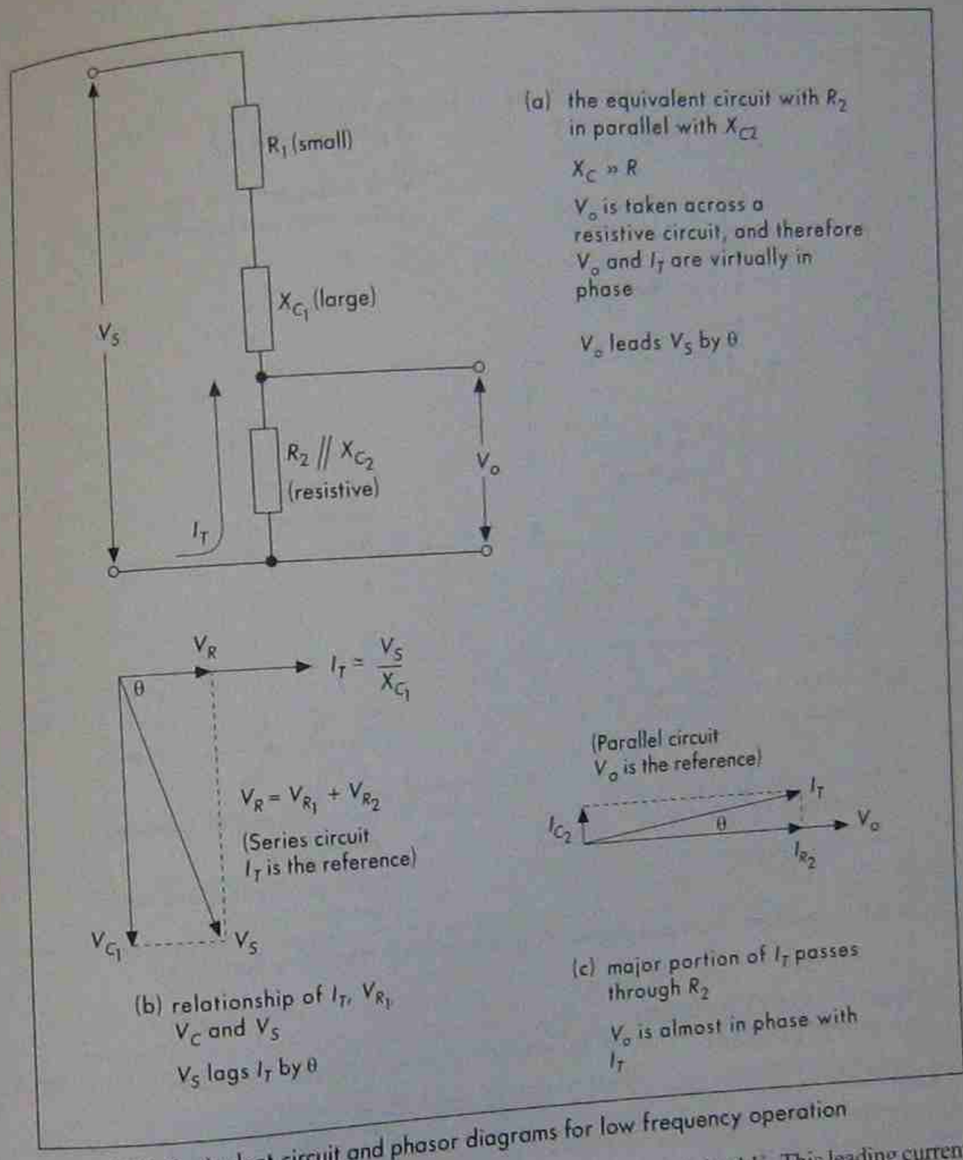
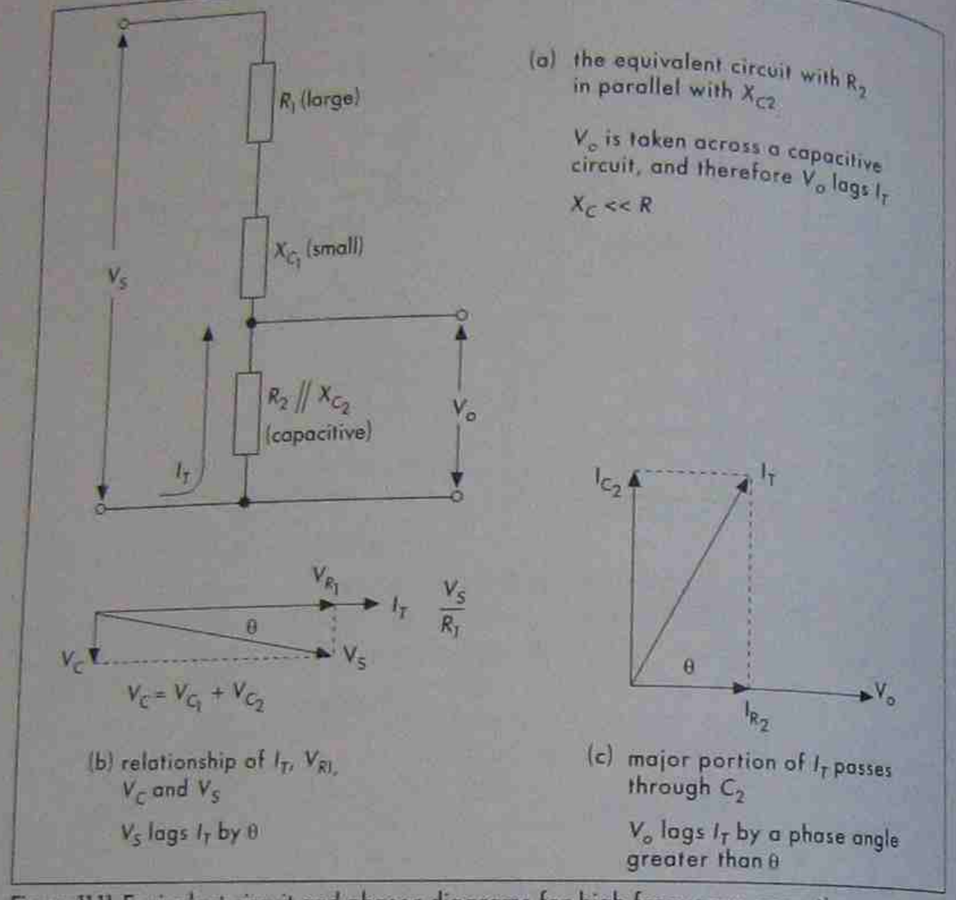


Figure 11.10: Equivalent circuit and phasor diagrams for low frequency operation

At *one frequency only* the reactance of C_1 causes the current to lead V_s. This leading current produces a lagging voltage across the R_2C_2 network, because it is still slightly capacitive. The output voltage (V_o) is in phase with the input voltage (V_s), and oscillations are sustained. The equivalent circuit and phasor diagrams are shown in Figures 11.12(a), (b) and (c).

As stated earlier, if $R_1 = R_2$ and $C_1 = C_2$, the frequency of oscillation can be calculated from:



(a) the equivalent circuit with R_2 in parallel with X_{C2}

V_o is taken across a capacitive circuit, and therefore V_o lags I_T

$X_C << R$

(b) relationship of I_T, V_{R1}, V_C and V_S

V_S lags I_T by θ

(c) major portion of I_T passes through C_2

V_o lags I_T by a phase angle greater than θ

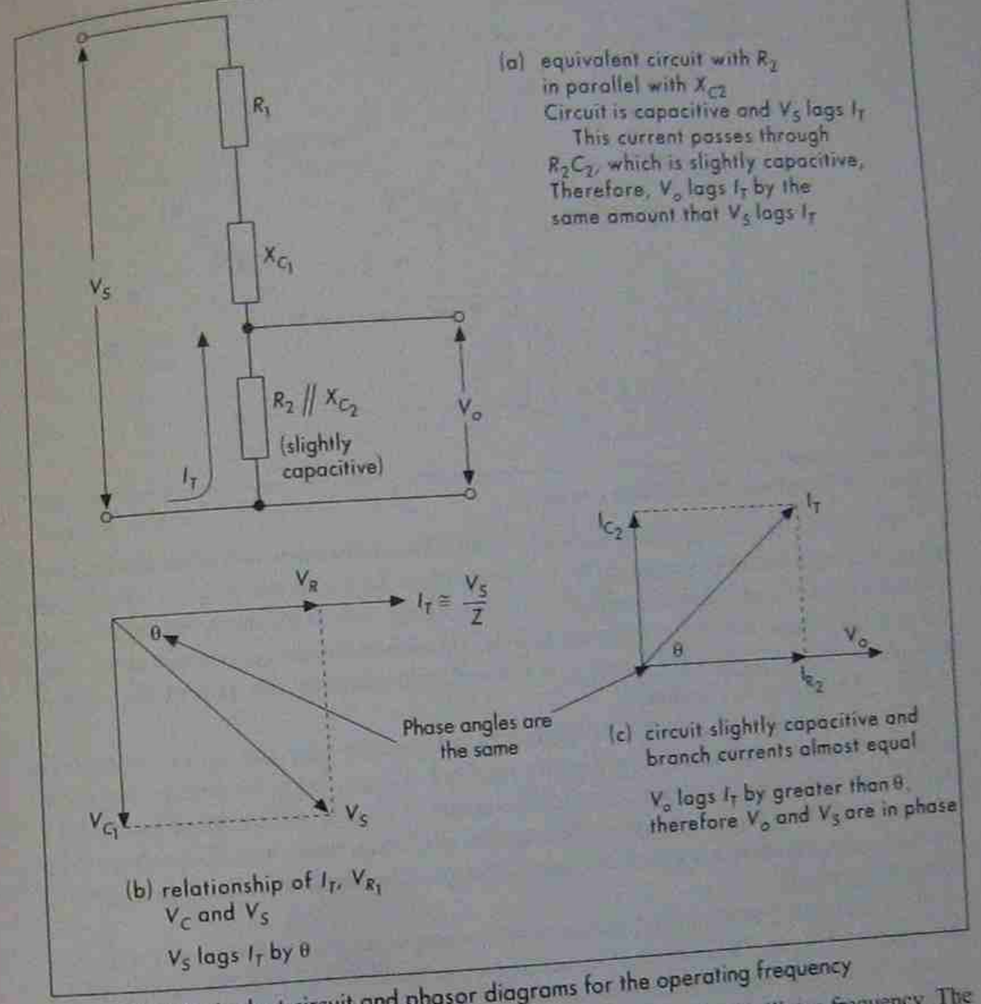
Figure 11.11: Equivalent circuit and phasor diagrams for high frequency operation

$f_r = 1/2\pi CR$
where $C = C_1 = C_2$ (in farads)
$R = R_1 = R_2$ (in ohms)

In a practical circuit, the lower leg of the voltage divider (R_2C_2 network) could be in parallel with another resistive network. This would have to be taken into account when calculating the resonant frequency (f_r)

The base-emitter junction resistance of a transistor, in series with the emitter resistor, may be in parallel with R_2. If R_2 is greatly influenced by this network, then the formula for calculating the frequency of oscillation is:

$f_r = 1/2\ \pi\sqrt{R_1C_1R_2C_2}$
where R_2 is the total resistance of the above network



(a) equivalent circuit with R_2 in parallel with X_{C2}
Circuit is capacitive and V_S lags I_T
This current passes through R_2C_2, which is slightly capacitive, Therefore, V_o lags I_T by the same amount that V_S lags I_T

(b) relationship of I_T, V_{R_1} V_C and V_S

V_S lags I_T by θ

(c) circuit slightly capacitive and branch currents almost equal

V_o lags I_T by greater than θ, therefore V_o and V_S are in phase

Figure 11.12: Equivalent circuit and phasor diagrams for the operating frequency

In fact, any circuit connected to the R_2C_2 junction will affect the oscillator frequency. The output from the CR network (V_o) has been shown to be in phase with V_s.

The negative feedback that is derived from the general feedback is used to perform the functions discussed in Chapter 9;

- control gain;
- maintain stability; and
- reduce distortion.

The complete circuit is shown in Figure 11.13.

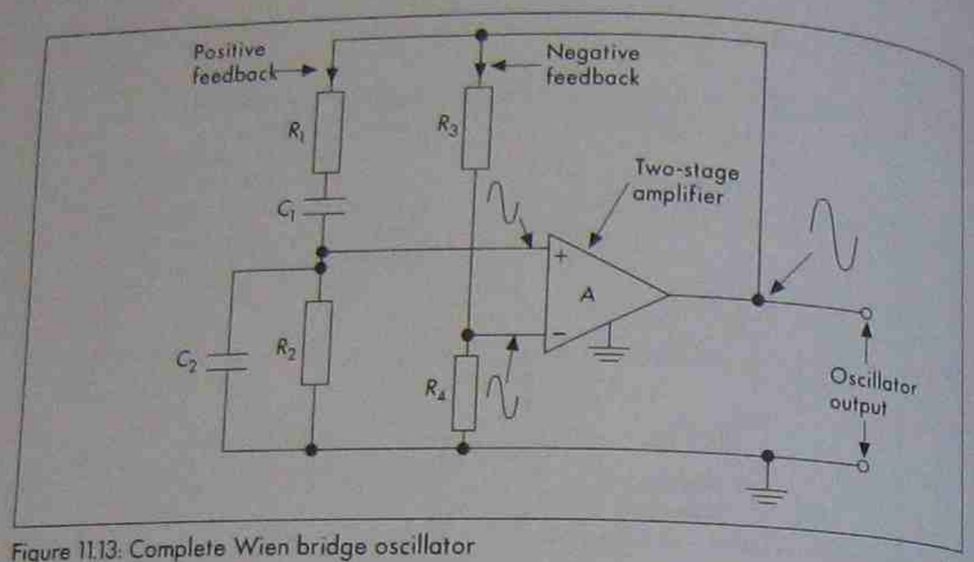


Figure 11.13: Complete Wien bridge oscillator

If the amplifier A is a two-stage common emitter circuit, the positive feedback is fed from the second stage output (collector) to the first stage input (base). At one frequency the circuit oscillates and the base is driven positive. At the same time, part of the output signal is series-fed to the first stage, that is, to the emitter circuit. This drives the emitter positive at the same time as the base. The forward bias V_{BE} is reduced, and therefore the gain of the stage is reduced (that is, negative feedback).

The circuit is designed so that the difference between the base and emitter voltages is sufficient to sustain oscillations. The negative feedback voltage, fed to the emitter, is derived from the oscillator output voltage. Any increase in output voltage will therefore increase the negative feedback. This in turn reduces the amplifier gain and returns the output voltage to its previous value. Likewise, a decrease in output will reduce the negative feedback to restore the output voltage level.

This monitoring of the output voltage level ensures a constant output, even when the frequency is changed. The frequency of oscillation can be altered in a similar manner to the phase-shift circuit. The two resistors R_1 and R_2 can be switched to provide a **coarse frequency control**, and capacitors C_1 and C_2 can be ganged to give a fine frequency control. The negative feedback components have no effect on the frequency.

The Wien bridge, therefore, consists of two parts. A frequency-determining network (R_1C_1, R_2C_2) whose output is fed to the input of the active device, and a stabilising network (R_3, R_4) connected as series-fed negative feedback. The bridge connections are shown in Figure 11.14, and it can be seen that the circuit can be used with any amplifying device.

11.6 Oscillators using operational amplifiers

The phase-shift circuits discussed may also be constructed using operational amplifers (op-amps). Figure 11.15 shows a three-section CR phase-shift oscillator using an LM741 op-amp.

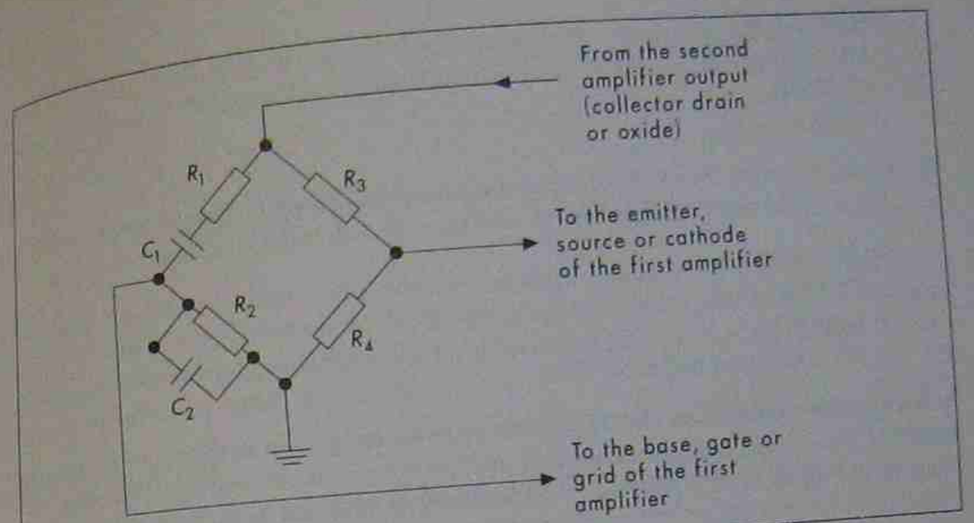


Figure 11.14: Wien bridge connections

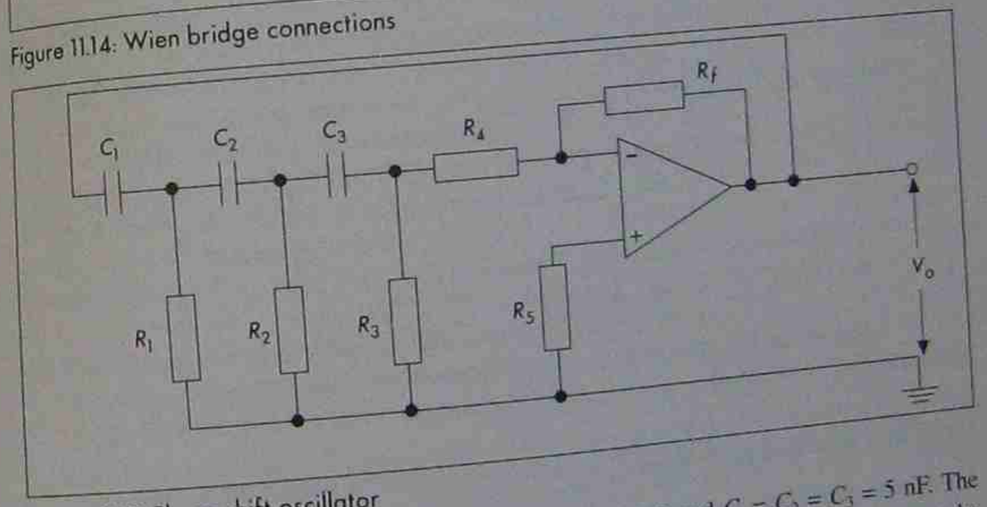


Figure 11.15: Phase-shift oscillator

The CR sections are not graded: $R_1 = R_2 = R_3 = 10\ k\Omega$ and $C_1 = C_2 = C_3 = 5$ nF. The attenuation through the CR network is 1/29, therefore the gain of the op-amp must be sufficient to replace this loss, that is, slightly greater than 29 (see Appendix D). The gain of the op-amp = R_f/R_4. If R_4 is chosen to provide minimum load on the CR network, say 100 kΩ and a gain of 30 is required to ensure operation, then the value of R_f can be calculated as follows:

$$R_f = R_4 \times \text{gain}$$
$$= 100\ k\Omega \times 30$$
$$= 3\ M\Omega$$

The frequency of oscillation uses the same formula discussed earlier:

$f_r = 1/2\pi CR\sqrt{6}$
$= 1/(6.28 \times 5 \times 10^{-9} \times 10 \times 10^{3} \times 2.45)$
$= 1300$ Hz
$= 1.3$ kHz

Resistor (R_3) corrects for any error in input bias current. Its value depends on the network used, but it should equal R_f in parallel (//) with $(R_1 + R_2 + R_3 + R_4)$, therefore:

$R_3 = R_f$ //130 kΩ
$= 3.0$ MΩ//130 kΩ
$= 124.6$ kΩ
$= 120$ kΩ (nearest preferred value)

A Wien bridge oscillator can also be constructed using an op-amp as the active device (see Figure 11.16). Positive feedback is required, and two transistors are used to provide the 360° phase shift. With an op-amp, however, the non-inverting input can be used to achieve this.

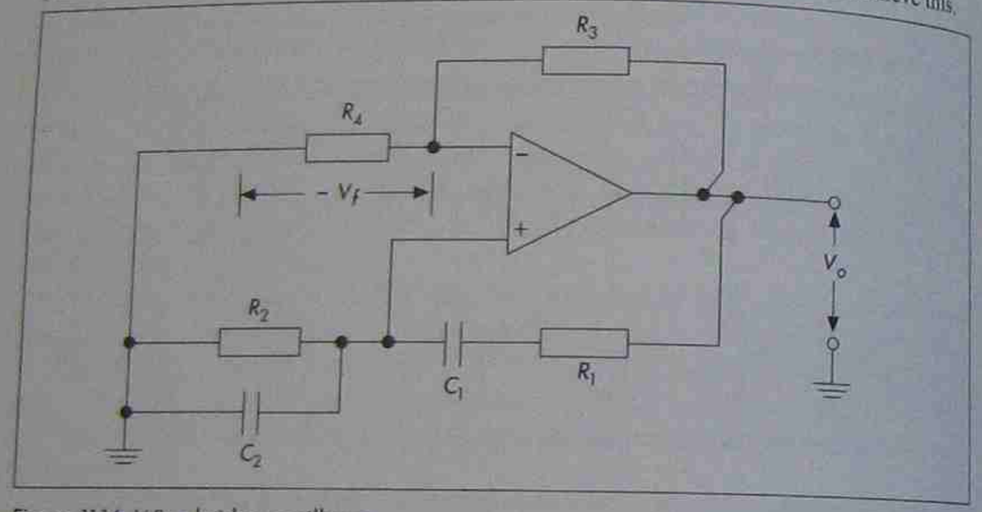


Figure 11.16: Wien bridge oscillator

A conventional balanced bridge is used to control the frequency of oscillation, and it is connected between the output and the non-inverting input. With $C = C_1 = C_2$ and $R = R_1 = R_2$, the frequency of oscillation is:

$f_r = 1/(2\pi CR)$

The negative feedback to provide gain **stability** is obtained by feeding part of the output back to the inverting input. The amount of negative feedback voltage can be calculated from:

$-V_f = [R_4/(R_3 + R_4) \times V_o$

Ideally, some sort of automatic adjusting negative feedback is required. Resistor R_3 can be replaced with a negative temperature coefficient (NTC) thermistor (Type R53) to achieve

this. The value of $-V_f$ will increase as the negative feedback current increases (that is, V_o increases), thereby stabilising the circuit. Practical values for the circuit in Figure 11.16 are:

$R_1 = R_2 = 1$ kΩ
$C_1 = C_2 = 47$ nF
$R_3 =$ Thermistor Type R53
$R_4 = 1$k5 Ω

Without the thermistor in the circuit, the signal amplitude is limited by the saturation of the amplifier. This results in distortion. The bridge resistor R_3 must be about the same value as the equivalent resistance connected to the inverting input of the op-amp. This ensures that any error in input bias current is minimised.

11.7 Summary

The following items are a summary of the major points covered in this chapter.

The characteristics of a phase-shift oscillator are as follows:

- The CR phase-shift oscillator shifts the phase of V_o by 180° and applies it to the input.
- Three stages shift approximately 60° per section, and a gain of at least 29 is required.
- Four stages shift approximately 45° per section, and a gain of at least 18 is required.
- Switching resistors provides a coarse frequency control.
- Varying the ganged capacitors provides a fine frequency control.
- Output signal level varies as frequency is varied.
- The resonant frequency $f_r = 1/2\pi CR\sqrt{6}$
  where $R_1 = R_2 = R_3$ and $C_1 = C_2 = C_3$ in a three-section phase-shift network

The characteristics of a Wien bridge oscillator are as follows:

- Two stages of amplification are needed for 360° input-to-output phase shift.
- Bridge network ensures that maximum positive feedback is available at one frequency only.
- The output is monitored and fed back to the input in the form of negative feedback, thereby providing gain stability.
- Switching resistors provides a coarse frequency control.
- Varying the ganged capacitors provides a fine frequency control.
- Output signal is constant as frequency is varied.
- The resonant frequency $f_r = 1/2\pi CR$
  where $C = C_1 = C_2$ and $R = R_1 = R_2$

Key terms

coarse frequency control
CR section
fine frequency control
frequency of oscillation
graded sections
phase angle θ
positive feedback
stability

ungraded sections — *variable frequency*
overall phase shift — *Wien bridge*
phase-shift oscillator

11.8 Review questions (answers in Appendix A)

1. What two conditions must be satisfied to obtain sustained oscillations?
2. (a) What type of circuit is shown in Figure 11.17?
   (b) What is the phase shift across the C_1R_1 network?
   (c) What is the approximate gain required by Q_1?
   (d) If $C_1 = C_2 = C_3 = 1$ nF and $R_1 = R_2 = R_3 = 4k7\ \Omega$, what is the approximate frequency of oscillation?
   (e) Explain why transistor Q_2 is required in the circuit.

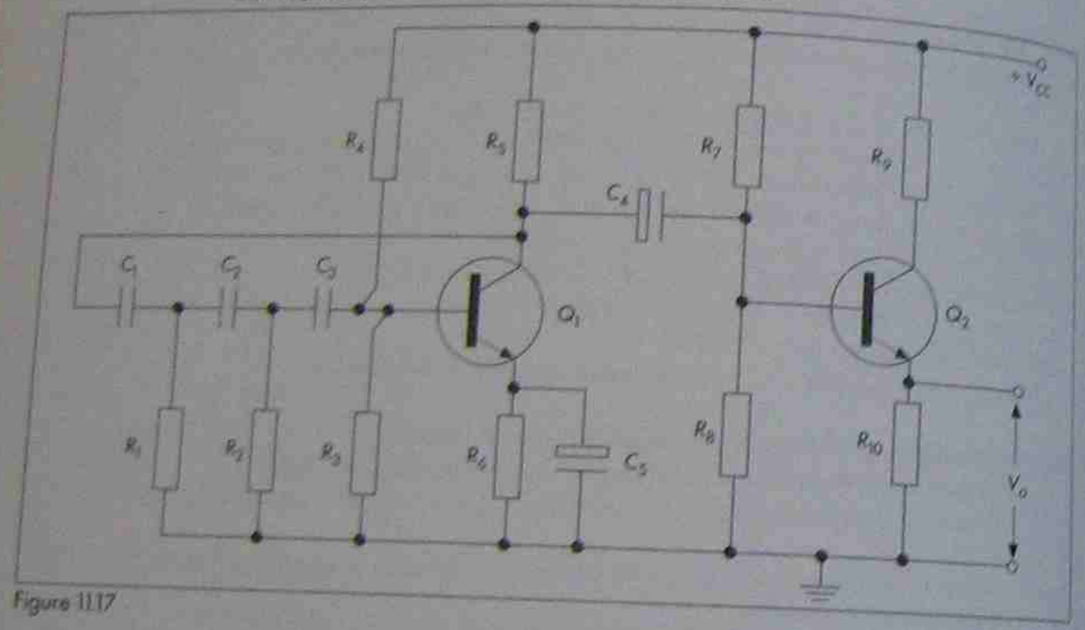


Figure 11.17

3. How can the frequency of oscillation of the circuit in Figure 11.17 be controlled? Are there any disadvantages with the frequency being variable?
4. What is the effect on phase shift and on amplifier gain when more CR sections are used?
5. How is the 360° input-to-output phase shift achieved in a Wien bridge oscillator?
6. (a) What type of circuit is shown in Figure 11.18?
   (b) Which components determine the frequency of oscillation?
   (c) Which components control output amplitude stability, and how is it achieved?
   (d) What is the phase of the signal at point A with respect to point B?
   (e) $C_1 = 10$ nF $\quad C_2 = 10$ nF $\quad C_3 = 50\ \mu$F
   $R_1 = 47\ k\Omega \quad R_2 = 4k7\ \Omega \quad R_3 = 2k7\ \Omega$

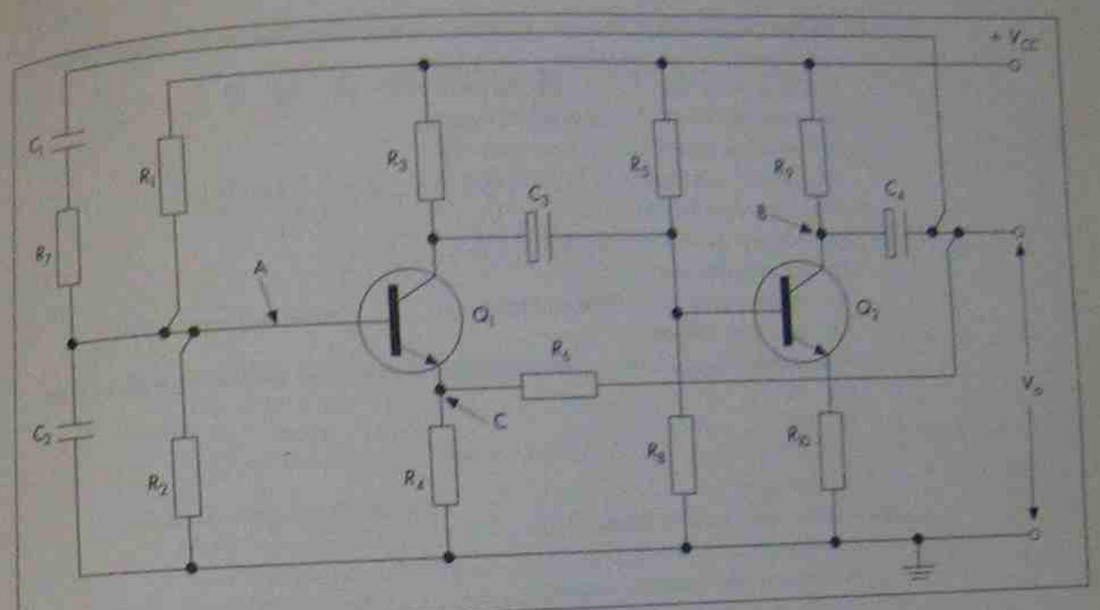


Figure 11.18

   $R_4 = 1k5\ \Omega \quad R_6 = 27\ k\Omega \quad R_7 = 4k7\ \Omega$
   Given the above values, what is the approximate frequency of operation?
   (f) If the oscillator does not oscillate at the frequency calculated in (e), which other circuit parameters need to be considered?
7. What is the phase of the signal at point C with respect to point A in Figure 11.18? What type of feedback does this represent and why?

Multiple-choice questions

Questions 1 to 4 refer to Figure 11.17.

1. If one more identical CR section were fitted, the phase shift per section and gain required would be:

	Phase shift	Gain
(a)	60°	18
(b)	60°	29
(c)	45°	18
(d)	45°	29

2. The new oscillator frequency would be approximately:
   (a) 13.9 kHz
   (b) 14.3 kHz
   (c) 22.6 kHz
   (d) 28.6 kHz

3. To make the oscillator frequency variable, coarse and fine controls are required. These are:

Coarse	Fine
(a) ganged resistors	switched capacitors
(b) switched resistors	ganged capacitors
(c) switched capacitors	ganged resistors
(d) ganged capacitors	switched resistors

4. If C_3 went open circuit:
   (a) negative feedback is applied
   (b) the gain increases
   (c) the frequency of oscillation is increased
   (d) the output will distort
5. The active device in a Wien bridge oscillator must provide the following phase shift:
   (a) 90°
   (b) 180°
   (c) 270°
   (d) 360°

Questions 6 and 7 refer to Figure 11.18.

6. The purpose of R_4 and R_6 is to:
   (a) increase the operating frequency
   (b) decrease the operating frequency
   (c) provide negative feedback
   (d) provide positive feedback
7. If R_1 went open circuit, the result would be the:
   (a) oscillator stops working
   (b) frequency increases
   (c) frequency decreases
   (d) gain increases
8. Refer to Figure 11.15. If the value of R_f is decreased the result will be:
   (a) an increase in frequency
   (b) a distorted output (clipped)
   (c) a decrease in frequency
   (d) a sinusoidal wave output with low amplitude
9. Refer to Figure 11.16. The frequency of oscillation is determined by:
   (a) $C_1R_3C_2R_4$
   (b) $C_1R_3C_2R_2$
   (c) $C_1R_1C_2R_2$
   (d) $C_1R_1C_2R_4$
10. In a Wien bridge circuit, maximum positive feedback is only available:
    (a) below the resonant frequency
    (b) above the resonant frequency
    (c) between (a) and (b) above
    (d) at the resonant frequency

CHAPTER 12

INDUCTOR-CAPACITOR (LC) OSCILLATORS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Explain the principle of reactive voltage dividing.
- Recognise the circuit connections for an inductive and a capacitive reactive divider.
- Sketch a basic Hartley oscillator.
- Determine the feedback and energy return components for a Hartley oscillator.
- Calculate the operating frequency of a Hartley oscillator.
- List the advantages and disadvantages of a Hartley oscillator.
- Sketch a basic Colpitts oscillator.
- Determine the feedback and return components for a Colpitts oscillator.
- Calculate the operating frequency for a Colpitts oscillator.
- List the advantages and disadvantages of a Colpitts oscillator.
- Show how to convert a Colpitts oscillator to a Clapp oscillator.
- State the advantage of a Clapp over a Colpitts oscillator.
- Explain the circuit connections required for transformer feedback.
- Describe the operation of a tuned base and a tuned collector oscillator.

12.1 Introduction

The oscillating action of a tuned circuit was discussed in Chapter 10. If this exchange of energy can be controlled and divided into useable proportions, it can be used to provide positive feedback and thereby replace circuit losses. It can therefore control an oscillator.

The tuned circuit consists of two components which display opposite characteristics to AC, namely an inductor (L) and a capacitor (C). They make up the LC tuned or **'tank' circuit**, and are connected in a manner that provides **positive feedback** and voltage division.

12.2 Reactive voltage dividers

When a capacitor and an inductor are connected in series, as shown in Figure 12.1, and an AC voltage is applied to the circuit, two things happen:

1. current flows in the circuit; and
2. voltages are developed across both the capacitor and the inductor.

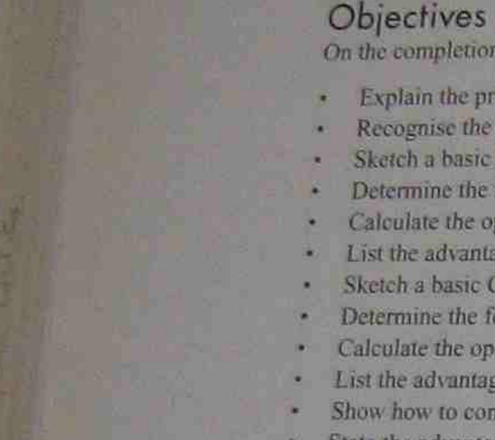


Figure 12.1 An AC voltage applied to L and C in series

The current is common to both components, but the voltage developed across the component is the product of the current (I) and the reactance of the component (X). Thus:

$$V_L = I \times X_L$$
$$V_C = I \times X_C$$

The voltage developed is proportional to the value of the reactance, and therefore the circuit behaves as a **reactive voltage divider**. The ratio of X_L to X_C determines the output voltage (see

Figures 12.2(a) and (b)). The output voltage (V_o) is taken across the component with the least reactance, and therefore V_o is a portion of the input voltage (V_{in}).

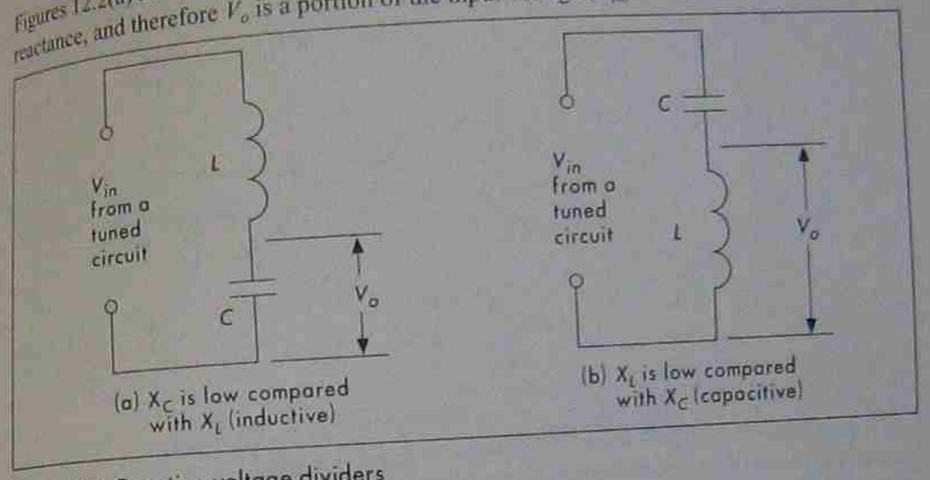


Figure 12.2: Reactive voltage dividers

12.2.1 Inductive dividers

If we assume for Figure 12.2(a) that $V_{in} = 10$ V AC, $X_L = 2500\ \Omega$ at 5000 Hz and $X_C = 150\ \Omega$ at 5000 Hz, then the output across capacitor (C) can be calculated as follows:

Total reactance $X_T = X_L - X_C$
$= 2500 - 150$
$= 2350\ \Omega$
Circuit current $I = V_{in}/X_T$
$= 10/2350$
$= 0.00425$ A
$= 4.3$ mA

Therefore V_o, which is taken across the capacitor and which equals V_C, is equal to:

$V_o = I \times X_C$
$= 4.3 \times 10^{-3} \times 150$
$= 0.645$ V
$= 645$ mV

This circuit is an **inductive voltage divider** because X_L is much greater than X_C. In an inductive circuit the current lags the voltage.

In a capacitive circuit the current leads the voltage. Therefore, the voltages developed across the capacitor and the inductor are 180° out of phase. These can be represented by the phasor diagram and graph shown in Figures 12.3(a) and (b).

The net circuit voltage is $V_L - V_C$, and this leads I by 90°. The output voltage is taken across C. The voltage across the capacitor (V_C) lags I and is phase-shifted a further 90°. The output voltage is now 180° out of phase with the input. This relationship can be seen by referring to the $V_L - V_C$ and V_C graphs in Figure 12.3(b).

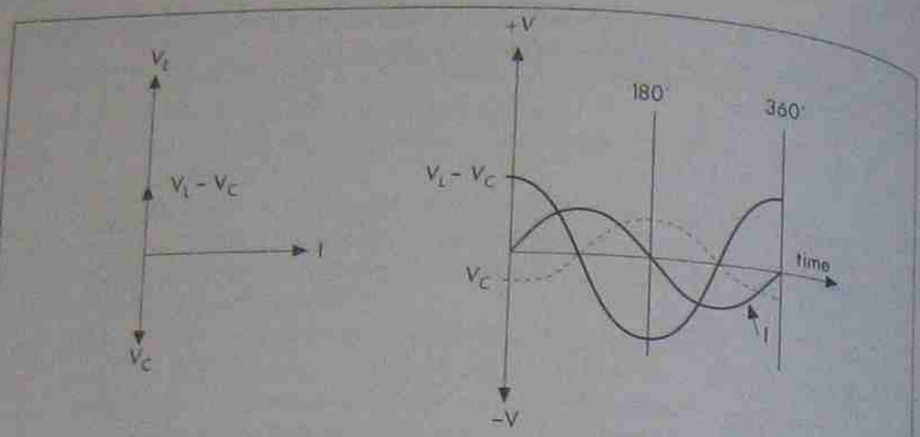


Figure 12.3: Phasor diagram and graph for an inductive divider (see also Figure 12.2(a))

12.2.2 Capacitive dividers

If we assume for Figure 12.2(b) that $V_{in} = 10$ V AC, $X_L = 150\ \Omega$ at 300 Hz and $X_C = 2500\ \Omega$ at 300 Hz, then the output across L can be calculated in a manner similar to that shown in section 12.2.1:

Total reactance $X_T = X_C - X_L$
$= 2500 - 150$
$= 2350\ \Omega$
$= V_{in}/X_T$

Circuit current $I = V_{in}/X_T$
$= 10/2350$
$= 4.3$ mA

$V_o = V_L$
$= I \times X_L$
$= 4.3 \times 10^{-3} \times 150$
$= 645$ mV

This circuit is a **capacitive voltage divider** because X_C is much greater than X_L. The phasor diagram and graph of the phase relationships of V_L and $V_C - V_L$ are shown in Figures 12.4(a) and (b).

The net circuit voltage is $V_C - V_L$ and it lags I by 90°. The output voltage is taken across L. The voltage across the inductor (V_L) leads I and is phase-shifted a further 90°. The output voltage is now 180° out of phase with the input. This relationship can be seen by referring to the $V_C - V_L$ and V_L graphs in Figure 12.4(b).

12.2.3 Reactive dividers coupled to an amplifier

We have established that by using a reactive divider, the input voltage can be reduced in

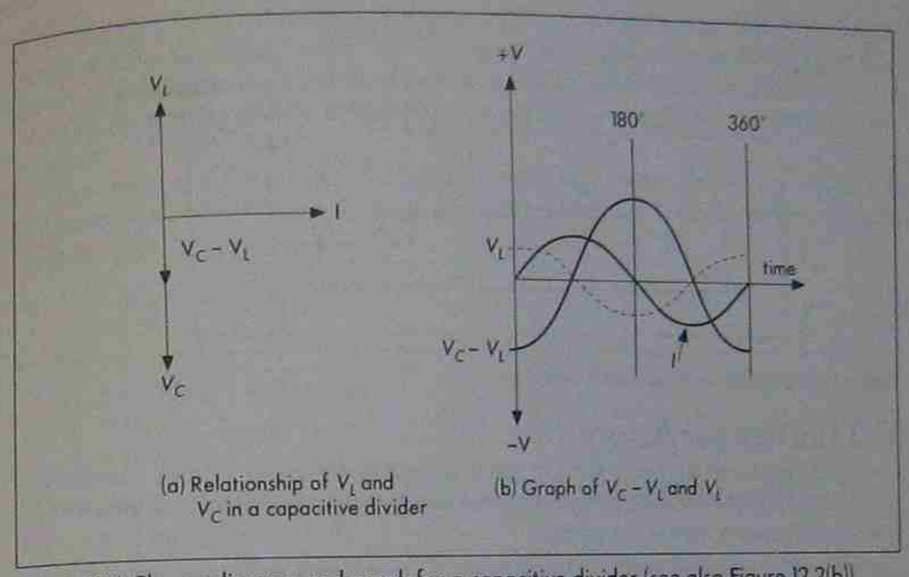


Figure 12.4: Phasor diagram and graph for a capacitive divider (see also Figure 12.2(b))

amplitude and, more importantly, phase-shifted 180°. This enables either a capacitive or an inductive reactive divider to provide a feedback voltage with the correct phase for an oscillator. The block diagram of such a circuit is shown in Figure 12.5.

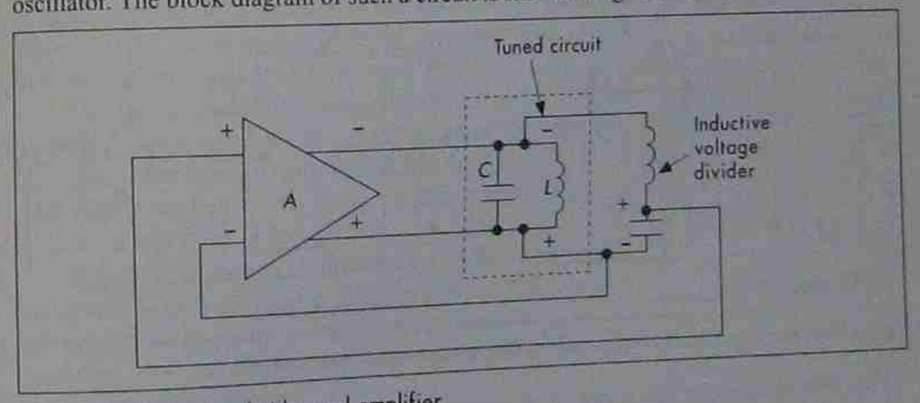


Figure 12.5: Inductive divider and amplifier

The output of the amplifier is connected to a tuned circuit. The oscillations are tapped off via a reactive voltage divider (inductive in Figure 12.5). A signal with the correct amplitude and phase relationship for positive feedback is fed back to the amplifier input. The circuit losses are replaced and the circuit oscillates. The inductive divider can be replaced with a capacitive divider to provide the necessary feedback to sustain oscillations. This circuit arrangement is shown in Figure 12.6.

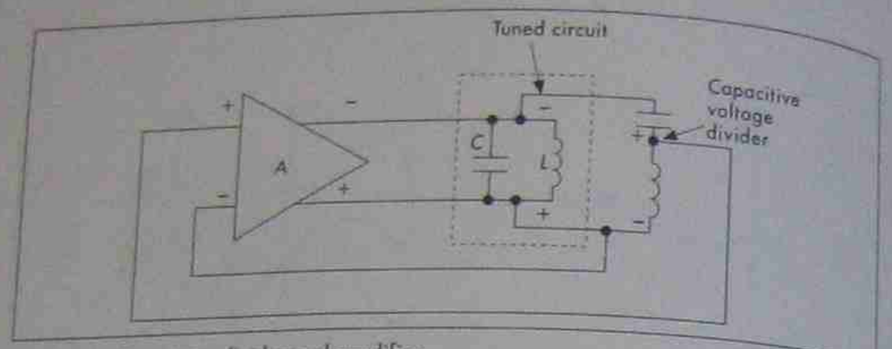


Figure 12.6: Capacitive divider and amplifier

12.3 Hartley oscillators

Oscillator circuits which use capacitive reactive dividers are called Hartley circuits. The tuned circuit/reactive divider combination shown in Figure 12.6 is simplified, and part of the tuned circuit itself is used as the divider.

The positive feedback required for operation of the oscillator is obtained from the tuned circuit and fed to the base of the transistor. Feedback is achieved by tapping the inductor, and **Hartley oscillators** are identified by this characteristic. The original circuit configuration is shown in Figure 12.7(a) and the simplified version in 12.7(b).

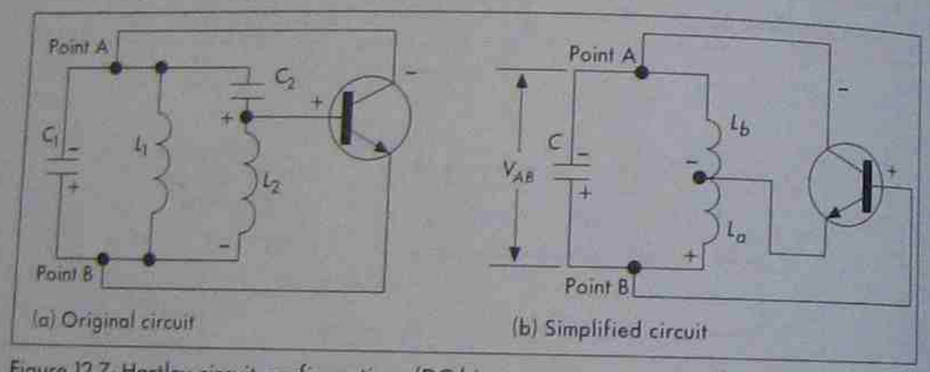


Figure 12.7: Hartley circuit configurations (DC bias components omitted)

As discussed earlier, the frequency of oscillation (f_r) is determined by the formula:

$$f_r = 1/2\pi\sqrt{LC}$$

The addition of inductor L and capacitor C_2 to the tuned circuit changes the frequency of oscillation (see Figure 12.7(a)). The circuit in Figure 12.7(b) splits L into two parts: L_a and L_b.

Capacitor C and inductor L_a become the capacitive reactive divider, and the ratio of X_{La} to X_{Lb} determines the actual amount of feedback (V_f is divided between L_a and L_b). At

resonance $X_L = X_C$, the current through both parallel branches is the same, and $V_L = V_C$, which is also equal to the voltage across the tuned circuit (the voltage measured from point A to point B is V_{AB}). The reactance of X_{Lb} is much greater than that of X_{La} (approximately 5:1), and therefore about one-sixth of the phase-reversed feedback voltage is used to forward bias the transistor.

For example, if V_{AB} in Figure 12.7(b) is equal to 12 V, then V_C and V_L also equal 12 V. The **reactance ratio** of X_{La} to X_{Lb} is 1:5. The value of the circuit voltages can be calculated by proportion:

$$V_{La} = (V_{AB}/(X_{La} + X_{Lb})) \times X_{La}$$
$$= 12/6 \times 1$$
$$= 2\text{ V}$$
$$V_{Lb} = (V_{AB}/(X_{La} + X_{Lb})) \times X_{Lb}$$
$$= 12/6 \times 5$$
$$= 10\text{ V}$$

The voltages around the tuned circuit are, therefore, $V_C = V_{AB} = 12$ V, $V_{La} = 2$ V, $V_{Lb} = 10$ V and the sum of V_{La} and $V_{Lb} = V_{AB}$.

The transistor is across the tuned circuit and it must be switched 'on' with the correct timing to return energy to the tuned circuit. This action can best be seen by referring to Figures 10.8 in Chapter 10 and Figure 12.7.

Assume that the circuit in Figure 12.7(b) is oscillating, and point A is negative with respect to point B. A positive-going signal at the transistor base will be phase-inverted by the transistor and give a negative-going signal at the collector.

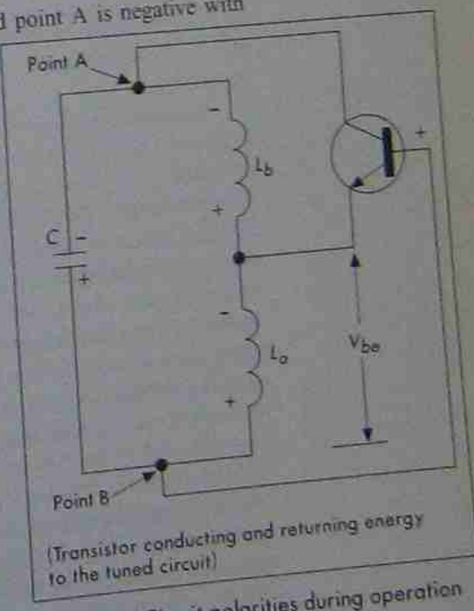
The collector-emitter voltage of the transistor has the correct polarity across C and L_a, and the capacitor recharges. This prevents the damped oscillation from taking place and returns energy to the circuit in the manner shown in Figure 10.8 in Chapter 10.

Figure 12.8 shows the circuit action with instantaneous polarities around the circuit. The tuned circuit phase-inverts the signal between points A and B, while the ratio of X_{La} to X_{Lb} develops the correct forward bias voltage to drive the transistor into conduction.

When the transistor conducts harder, there is an increase in transistor collector current. This increase in current performs two functions:

1. it passes through the tuned circuit, returning energy to the circuit; and
2. it passes through the reactive divider-inverter C and L_a.

The voltage developed across L_a is phase-reversed, and



(Transistor conducting and returning energy to the tuned circuit)

Figure 12.8: Circuit polarities during operation when point A is negative

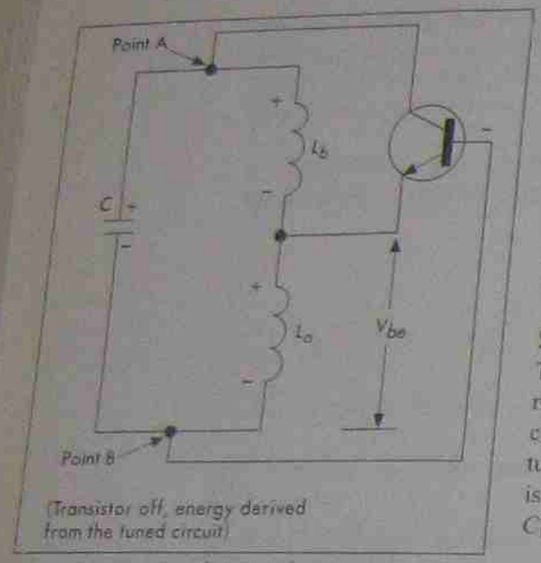


Figure 12.9: Circuit polarities when point A is positive

continues to drive the base of the transistor with the correct polarity to sustain oscillation, that is, positive feedback. The polarities for the other half cycle of operation, where point A is positive with respect to point B, are shown in Figure 12.9.

12.3.1 Practical circuits

There are two basic circuit configurations for the Hartley oscillator (series-fed and shunt-fed), and these will now be discussed.

Series-fed oscillators

The Hartley oscillator circuit shown in Figure 12.10 is referred to as a **series-fed oscillator** because there is a DC current path through the tuned circuit, in this case L_b. The tuned circuit is across the collector-base of AC circuit, that is, it is connected to the base of transistor Q via capacitor C_1 and to the collector of Q via C_2.

The tapped inductor is connected directly to the emitter. Energy return to the tuned circuit is via the emitter current passing through coil L_b, and positive feedback is developed across the component connected between base and emitter (L_a). Resistors R_1 and R_2 provide the bias for transistor Q, resistor R_4 provides temperature stability and capacitor C_3 prevents degeneration.

The operation of the oscillatory and **energy-return** circuit is as follows. Assume point B in Figure 12.10 swings positive. This changing positive potential is coupled to the base of transistor Q by capacitor C_1. The forward bias of Q is increased, causing an increase in collector current. This results in an increase in emitter current through the feedback winding L_b. More energy is supplied to the tuned circuit by the increase in current. Point B becomes more positive and therefore the forward bias on Q increases further. This regenerative action continues until circuit non-linearities cause the rate of change of current through L_b to decrease. When this happens, less energy is supplied to the tuned circuit, and **tuned circuit action** (see section 10.2 in Chapter 10) causes the potential at point B to decrease. The transistor forward bias is reduced and the collector and emitter currents decrease. The potential across the tuned circuit decreases to zero. The tuned circuit energy is stored in the magnetic field of the coil, and the oscillator has now completed one half cycle of operation.

The magnetic field begins to collapse, charging the capacitor in the opposite direction. Point B swings negative (as in Figure 12.9). This negative potential is coupled to the base of the transistor Q via C_1. The forward bias is decreased, causing both the collector and emitter currents to decrease even further. When the signal reaches its maximum negative value, the collector and emitter currents are minimum. The magnetic field of the coil has completely

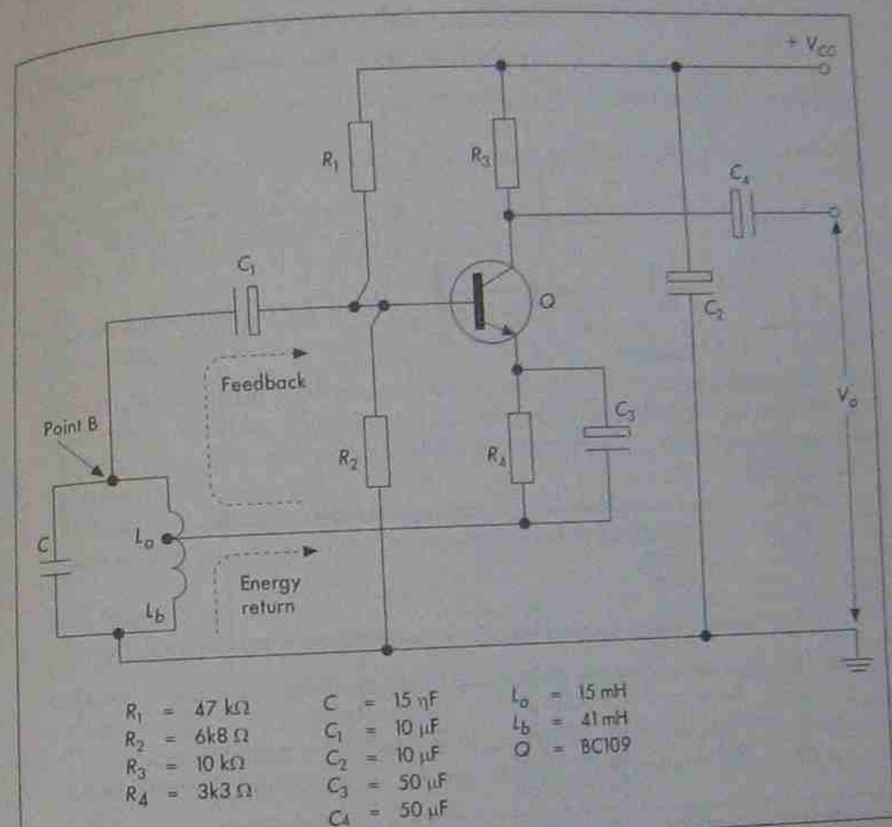


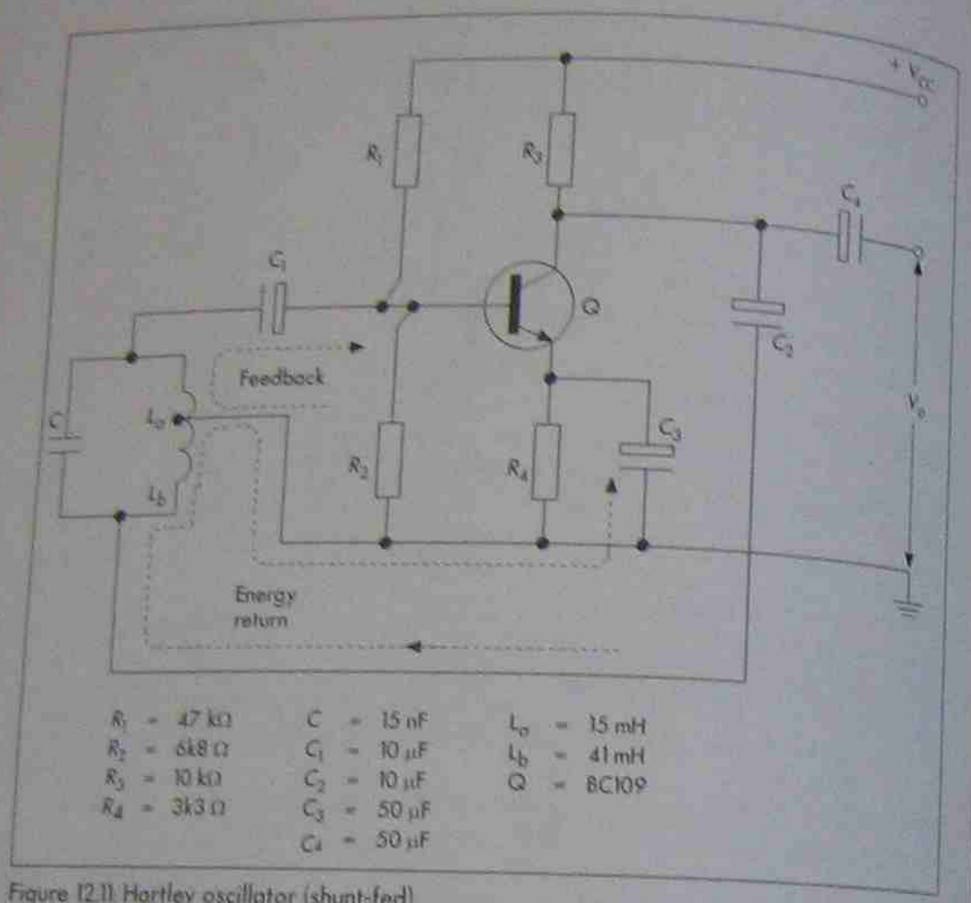
Figure 12.10: Hartley oscillator (series-fed)

collapsed and the oscillator has completed three-quarters of a cycle of operation. The circuit energy is now stored in the capacitor.

The capacitor begins to discharge. The negative potential at point B decreases, and the forward bias starts to increase again. Emitter current through L_b increases, thereby returning energy to the tuned circuit to replace the energy dissipated. If the energy replaced is equal to, or slightly greater than, the energy dissipated, oscillations will be sustained.

Shunt-fed oscillators

The Hartley oscillator shown in Figure 12.11 is referred to as a **shunt-fed oscillator**. The DC current path from earth to the power supply V_{cc} is via R_4, Q and R_3. The tuned circuit is isolated from the DC current path by C_1 and C_2. Apart from this slight difference, the action of the tuned circuit is essentially the same as the series-fed Hartley oscillator.



R_1	= 47 kΩ	C	= 15 nF	L_a	= 15 mH
R_2	= 6k8 Ω	C_1	= 10 µF	L_b	= 41 mH
R_3	= 10 kΩ	C_2	= 10 µF	Q	= BC109
R_4	= 3k3 Ω	C_3	= 50 µF		
		C_4	= 50 µF		

Figure 12.11 Hartley oscillator (shunt-fed)

When calculating the frequency of oscillation, the mutual coupling between the two coils in the tuned circuit must be taken into consideration. The total inductance (L_T) can be calculated from:

$$L_T = L_a + L_b + 2L_M$$

where L_M = mutual inductance calculated from $L_M = k\sqrt{L_a L_b}$

k = coefficient of coupling between the two series-aiding fluxes

The actual frequency of oscillation (f_r) is then determined by the formu

$$f_r = 1/2\pi\sqrt{L_T C}$$

12.3.2 Advantages of the Hartley oscillator

The advantages of the Hartley oscillator are as follows:

1. The frequency of oscillation may be easily varied by making the capacitor C in the tuned circuit variable.
2. The amplitude of the output signal remains constant when the oscillator is tuned over the designed frequency range.
3. The feedback ratio of X_{La} to X_{Lb} is not varied during tuning provided C is varied. If L is varied, the ratio of X_{La} to X_{Lb} must be maintained.

12.3.3 Disadvantages of the Hartley oscillator

The disadvantages of the Hartley oscillator are as follows:

1. The **feedback voltage** is developed across an inductor, the reactance of which increases as frequency increases. Harmonics are multiples of the oscillator resonant frequency, and therefore the amplitude of the harmonics (developed across X_{La}) will also increase as the frequency increases.
2. The output is rich in **harmonic content** and the Hartley oscillator is therefore not suitable for applications where a pure sinusoid/wave is required.

12.3.4 Summary

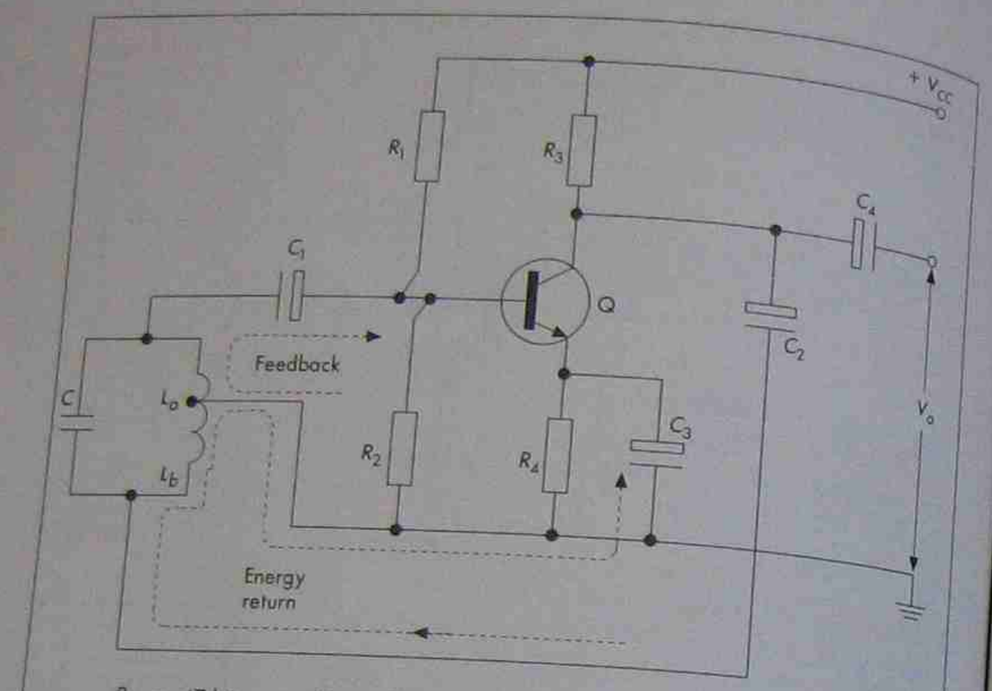
In summary, the main features of the Hartley oscillator circuit are as follows:

- The tuned circuit is connected between the collector and base of the transistor (output to input).
- The inductor is tapped and the tap is connected to the emitter. Energy is returned to the tuned circuit by emitter current passing through L_b (inductor between collector and emitter).
- The positive feedback is developed across the inductor between the base and emitter of the transistor (L_a).
- The amount of feedback is determined by the reactance ratio X_{La} to X_{Lb}.
- The output amplitude is constant over the designed frequency range.
- The output signal is rich in harmonic content.
- The frequency of oscillation is determined by the formula:

$$f_r = 1/2\pi\sqrt{L_T C}$$

m of a Hartley oscillator can now be represented by an amplifier and three
hown in Figure 12.12:
quency of the Hartley oscillator circuit can be calculated from the

s for Figures 12.10 and 12.11: L_a = 15 mH, L_b = 41 mH, C =



R_1	= 47 kΩ	C	= 15 nF	L_a	= 15 mH
R_2	= 6k8 Ω	C_1	= 10 µF	L_b	= 41 mH
R_3	= 10 kΩ	C_2	= 10 µF	Q	= BC109
R_4	= 3k3 Ω	C_3	= 50 µF		
		C_4	= 50 µF		

Figure 12.11: Hartley oscillator (shunt-fed)

When calculating the frequency of oscillation, the mutual coupling between the two coils in the tuned circuit must be taken into consideration. The total inductance (L_T) can be calculated from:

$L_T = L_a + L_b + 2L_M$

where L_M = mutual inductance calculated from $L_M = k\sqrt{L_a L_b}$

k = coefficient of coupling between the two series-aiding fluxes

The actual frequency of oscillation (f_r) is then determined by the formula:

$f_r = 1/2\pi\sqrt{L_T C}$

12.3.2 Advantages of the Hartley oscillator

The advantages of the Hartley oscillator are as follows:

1. The frequency of oscillation may be easily varied by making the capacitor C in the tuned circuit variable.
2. The amplitude of the output signal remains constant when the oscillator is tuned over the designed frequency range.
3. The feedback ratio of X_{La} to X_{Lb} is not varied during tuning provided C is varied. If L is varied, the ratio of X_{La} to X_{Lb} must be maintained.

12.3.3 Disadvantages of the Hartley oscillator

The disadvantages of the Hartley oscillator are as follows:

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2. The output is rich in **harmonic content** and the Hartley oscillator is therefore not suitable for applications where a pure sinusoid/wave is required.

12.3.4 Summary

In summary, the main features of the Hartley oscillator circuit are as follows:

- The tuned circuit is connected between the collector and base of the transistor (output to input).
- The inductor is tapped and the tap is connected to the emitter. Energy is returned to the tuned circuit by emitter current passing through L_b (inductor between collector and emitter).
- The positive feedback is developed across the inductor between the base and emitter of the transistor (L_a).
- The amount of feedback is determined by the reactance ratio X_{La} to X_{Lb}.
- The output amplitude is constant over the designed frequency range.
- The output signal is rich in harmonic content.
- The frequency of oscillation is determined by the formula:
  $f_r = 1/2\pi\sqrt{L_T C}$

The block diagram of a Hartley oscillator can now be represented by an amplifier and three reactance blocks as shown in Figure 12.12.

The resonant frequency of the Hartley oscillator circuit can be calculated from the formula:

$f_r = 1/2\pi\sqrt{L_T C}$

Assume the following values for Figures 12.10 and 12.11: L_a = 15 mH, L_b = 41 mH, C = 15 nF, then:

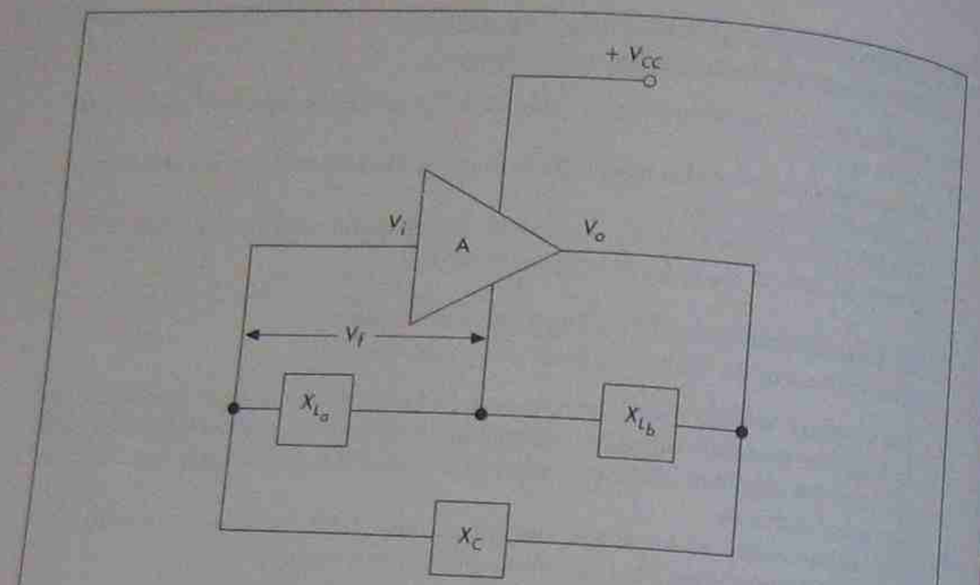


Figure 12.12: Block diagram of a Hartley oscillator (no biasing components)

$L_T = L_a + L_b + 2L_M$
where $L_M = k\sqrt{L_a L_b}$

L_T can only be approximated because the coupling factor (k) is not known, Therefore:

$L_T = L_a + L_b$ (assuming $k = 0$)
$= 15 \times 10^{-3} + 41 \times 10^{-3}$
$= 56$ mH
$f_r = 1/2\,\pi\sqrt{L_T C}$
$= 1/6.28\,\sqrt{56 \times 10^{-3} \times 15 \times 10^{-9}}$
$= 1/6.28 \times 28.98 \times 10^{-6}$
$= 5494.8$ Hz
$= 5.5$ kHz

12.4 Colpitts oscillators

Oscillator circuits which use inductive reactive dividers are called **Colpitts oscillators**. Again, the basic tuned circuit/reactive divider combination is simplified and the divider is part of the tuned circuit. The Colpitts oscillator is identified by the two tapped capacitors in the tuned circuit. The original circuit configuration is shown in Figure 12.13(a) and the simplified version in 12.13(b).

The circuit in Figure 12.13 splits the tuned circuit capacitor (C) into two parts, C_1 and C_2. Inductor L and capacitor C_2 become the inductive reactive divider, and the ratio of the reactance X_{C1} to X_{C2} determines the actual amount of feedback (V_{AB} is divided between C_1 and C_2).

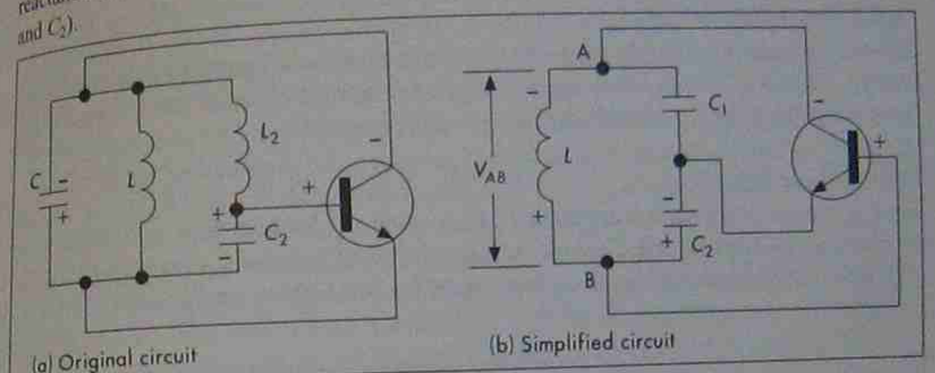


Figure 12.13: Colpitts circuit configuration (DC bias components omitted)

At resonance $X_L = X_{CT}$, the current through both branches is the same, and V_L = the voltage across the total capacitance (V_{CT}) which is equal to the voltage across the tuned circuit (V_{AB}). The reactance X_{C1} is much greater than X_{C2} (approximately 5:1), and therefore about one-sixth of the phase-reversed feedback voltage is used to forward bias the transistor. For example, if V_{AB} in Figure 12.13(b) is equal to 12 V, then V_L and V_{CT} also equal 12 V. The ratio of X_{C2} to X_{C1} is 1:5, so:

$V_{C2} = (V_{AB}/(X_{C1} + X_{C2})) \times X_{c2}$
$= 12/6 \times 1$
$= 2$ V
$V_{C1} = (V_{AB}/(X_{C2} + X_{C1})) \times X_{c1}$
$= 12/6 \times 5$
$= 10$ V

The voltages around the tuned circuit are, therefore, $V_L = V_{AB}$ = 12 V, V_{C1} = 10 V and V_{C2} = 2 V, and the sum of V_{C1} and V_{C2} = V_{AB}. The conduction of the transistor is controlled with the correct timing to return energy to the tuned circuit.

The tuned circuit action is similar to that of the Hartley oscillator. Referring to Figure 12.14, the tuned circuit is between the collector and base of the transistor. Capacitors C_1 and C_2 are tapped and the tap is connected to the emitter of the transistor. The feedback voltage is developed across capacitor C_2. When the transistor is driven further into conduction, the collector current increases. This increase in current performs two functions:

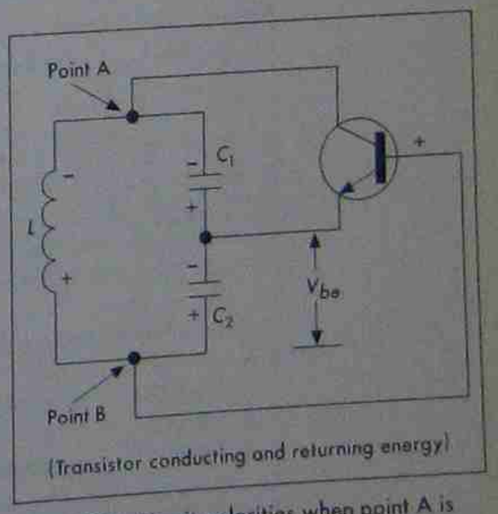


Figure 12.14: Circuit polarities when point A is negative

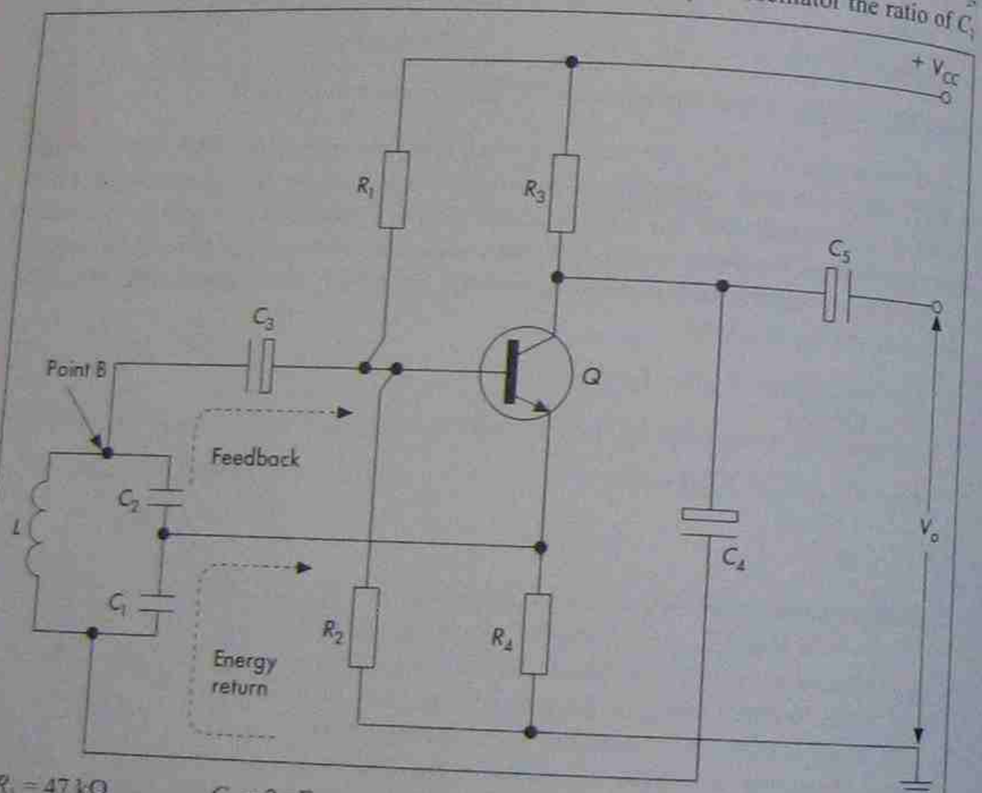
1. it passes through the tuned circuit returning energy to it; and
2. it passes through the reactive divider L and C_2. The voltage across C_2 is phase-reversed and provides positive feedback for the transistor.

12.4.1 Practical circuits

As with the Hartley oscillator, there are two basic circuit configurations available for the Colpitts oscillator. The oscillator may be shunt-fed or series-fed (determined by the DC current path), and these two configurations are now discussed.

Shunt-fed oscillators

The Colpitts oscillator shown in Figure 12.15 is shunt-fed because there is no DC current path through the tuned circuit. The tuned circuit consists of inductor L, capacitors C_1 and C_2, with the feedback voltage being developed across C_2. In a Colpitts oscillator the ratio of C_1



$R_1 = 47\ k\Omega$ $C_1 = 2\ nF$ $L = 15\ mH$
$R_2 = 5k6\ \Omega$ $C_2 = 10\ nF$ $Q = BC109$
$R_3 = 10\ k\Omega$ $C_3 = 10\ \mu F$
$R_4 = 1\ k\Omega$ C_4 and $C_5 = 100\ \mu$

Figure 12.15: Colpitts oscillator (shunt-fed)

to C_2 is important. To maintain a specific feedback voltage the ratio of C_1 to C_2 must remain the same (typically 1:5). Therefore, if the frequency is adjusted by varying the capacitance, the two capacitors must be varied together to ensure that the 1:5 ratio is maintained. If this ratio is not maintained, the feedback voltage will vary and oscillation may cease.

Resistors R_1, R_2, R_3 and R_4 are typical amplifier components. Capacitors C_3 and C_4 provide the AC signal path from the active device to the tuned circuit. Capacitors C_3 and C_4 also provide DC isolation between collector and base circuits.

The circuit action is very similar to that of the Hartley oscillator. Assuming that the circuit is oscillating and the energy is being released from inductor L, then the circuit operation is as follows. As the magnetic field around L collapses, point B is driven positive (as in Figure 12.14). The collapsing field increases the charge in capacitor C_2. This increase in positive potential at point B is coupled to the transistor base by capacitor C_3 (see Figure 12.15). The forward bias of transistor Q increases, resulting in an increase in collector and emitter currents and a decrease in collector voltage. Capacitor C_4 couples the increase in current through C_1 and Q, and this current aids the tuned circuit circulating current by recharging C_1. With C_1 recharging (negative at the C_1-C_4 junction) energy is replaced in the tuned circuit.

This circuit action continues until the magnetic field of the coil has completely collapsed (C_1 and C_2 are now charged). At this point the circulating current is zero, collector current and voltage are no longer changing, and the tuned circuit voltage is at its peak positive value.

The capacitors C_1 and C_2 begin to discharge. The circulating current reverses direction and the potential at point B decreases. The forward bias on Q and the collector current decrease, resulting in an increase in voltage across capacitor C_4. Capacitor C_4 charges, aiding the tuned circuit action. Capacitors C_1 and C_2 are discharged and the energy is stored in the coil. When there is no rate of change of current in the coil, the field collapses again, charging C_2 and C_1 in the opposite direction, that is, negative potential at point B. The tuned circuit action continues, positive feedback is applied to the transistor base, energy is returned to the circuit via C_1 and oscillations are sustained.

Series-fed oscillators

The Colpitts oscillator circuit shown in Figure 12.16 is series-fed because the DC current path is through the inductor in the tuned circuit. One side of the tuned circuit is connected directly to the collector and the other side to the base via C_3. The centre tap of C_1 and C_2 is connected to the emitter. Inductor L and capacitor C_2 make up the reactive divider, and energy is returned to the tuned circuit by the charge in C_1. The operation is the same as the shunt-fed circuit in Figure 12.15.

When calculating the frequency of oscillation, the total capacitance (C_T) of C_1 and C_2 in series must be taken into consideration:

$$C_T = (C_1 \times C_2)/(C_1 + C_2)$$

The actual frequency of oscillation (f_r) is then determined by the formula:

$$f_r = 1/2\pi\sqrt{LC_T}$$

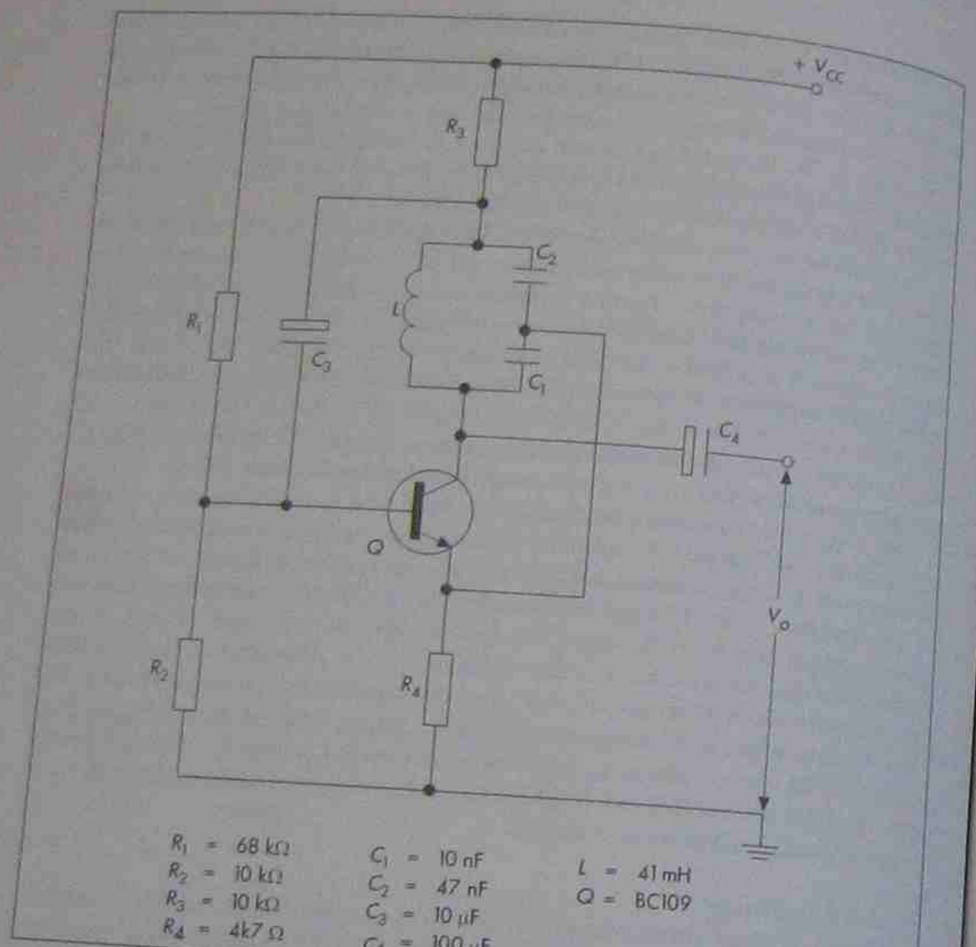


Figure 12.16: Colpitts oscillator (series-fed)

The frequency of oscillation can be affected by the interelement capacitances of the transistor. In Figure 12.15 capacitor C_2 is across the base-emitter capacitance (C_{be}) of the transistor. Capacitor C_1 is in series with C_4 across the collector-emitter capacitance (C_{ce}) of the transistor. The effect of these interelement capacitances is frequency instability, and it can be minimised by making C_1 and C_2 large compared to C_{ce} and C_{be}.

12.4.2 Advantage of the Colpitts oscillator

The advantage of the Colpitts oscillator is that the feedback voltage is developed across a capacitor, the reactance of which decreases as frequency increases. This reduces the harmonic content of the output, and therefore the Colpitts is widely used in test equipment.

12.4.3 Disadvantages of the Colpitts oscillator

The disadvantages of the Colpitts oscillator are as follows:

1. The amplitude of the output signal varies as the frequency is varied. To keep the output relatively constant, the ratio of C_1 to C_2 must be maintained over the designed frequency range. Tuning may be accomplished by varying L (permeability tuning).
2. There is frequency instability due to the shunting effect of transistor capacitances and the temperature effects on the active devices.

12.4.4 Summary

In summary, the main features of the Colpitts oscillator are as follows:

- The tuned circuit is connected between output and input of the active device.
- The positive feedback is developed across the capacitor between base and emitter of the transistor.
- The amount of positive feedback is determined by the reactance ratio of X_{C1} to X_{C2}.
- The capacitor in the tuned circuit is split, and the centre tap is connected to the emitter of the transistor. Energy is returned to the tuned circuit by the charge developed in C across the collector-emitter junction.
- Output amplitude usually varies over the designed frequency range, unless the tracking ratio of C_1 and C_2 can be maintained.
- There is low harmonic content in the output signal.
- The frequency of oscillation is determined by the formula:

$$f_r = 1/2\pi\sqrt{LC_T}$$

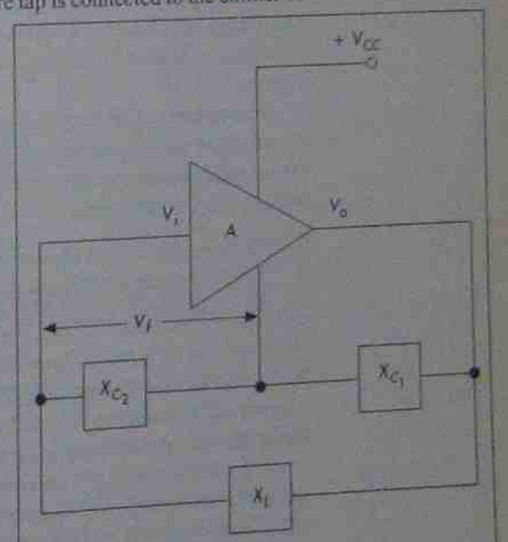
The block diagram of a Colpitts oscillator can be represented by an amplifier and three reactance blocks as shown in Figure 12.17.

The resonant frequency of the Colpitts oscillator shown in Figure 12.16 can be calculated from the formula:

$$f_r = 1/2\pi\sqrt{LC_T}$$

Assume the following values for Figure 12.16:

$C_1 = 10$ nF, $C_2 = 47$ nF, $L = 41$ mH. Then:



X_L made adjustable to vary the resonant frequency, or X_{C_2} and X_{C_1} ganged to maintain the feedback ratio

Figure 12.17: Block diagram of a Colpitts oscillator (no biasing components)

$$C_T = (C_2 \times C_3)/(C_1 + C_2)$$
$$= (10 \times 47)/(10 + 47)$$
$$= 8.25 \text{ nF}$$
$$f_r = 1/2\pi\sqrt{LC}$$
$$= 1/6.28 \times \sqrt{41 \times 10^{-3} \times 8.25 \times 10^{-9}}$$
$$= 1/6.28 \times 18.39 \times 10^{-6}$$
$$= 8658 \text{ Hz}$$
$$= 8.7 \text{ kHz}$$

The product of the inductance and capacitance is referred to as the LC product, and it determines the frequency of oscillation. For a given value of L, the value of C can be calculated to provide oscillations at one particular frequency. The formula is derived from:

$$f_r = 1/2\pi\sqrt{LC}$$
$$C = 1/(2\pi f_r)^2 \times L$$

Likewise, if the value of L is required for a given C, then:

$$L = 1/(2\pi f_r)^2 \times C$$

12.5 Clapp oscillators

The **Clapp oscillator** is a series-tuned modified shunt-fed Colpitts circuit and it has the advantage of greater frequency stability. The modification is in the tuned circuit as shown in Figure 12.18.

The variable capacitor C_6 is added to the tuned circuit but, apart from that, the circuit in Figure 12.18 is the same as that in Figure 12.15. When C_6 is added in series with the tuned circuit inductor, the oscillator is known as a Clapp oscillator. It has the following advantages over a basic Colpitts circuit:

1. frequency stability is improved;
2. the effect of the transistor capacitances on the tuned circuit is practically eliminated; and
3. the circuit can be capacitively tuned by varying C_6 (the ratio of C_1 to C_2 is maintained).

If the value of C_6 is low compared to C_1 and C_2, the resonant frequency is mainly determined by C_6. (Actual tuned circuit C_T is now C_1, C_2 and C_6 in series.) With C_6 in series with the tuned circuit inductor, the DC current path is blocked. Thus the Clapp oscillator configuration cannot be used in a series-fed Colpitts circuit. Capacitors C_1 and C_2 could be omitted and the tuned circuit coupled directly to the collector and base of the transistor, because capacitors C_2 and C_6 provide DC isolation.

Clapp oscillators are used at radio frequencies. The circuits discussed so far oscillate at audio frequency. However, if an oscillator is to be used at radio frequencies, a radio frequency choke (RFC) must be included in the collector circuit to prevent the oscillations from entering the power supply.

The block diagram of a Clapp oscillator can be represented by an amplifier and reactance blocks, and this is shown in Figure 12.19.

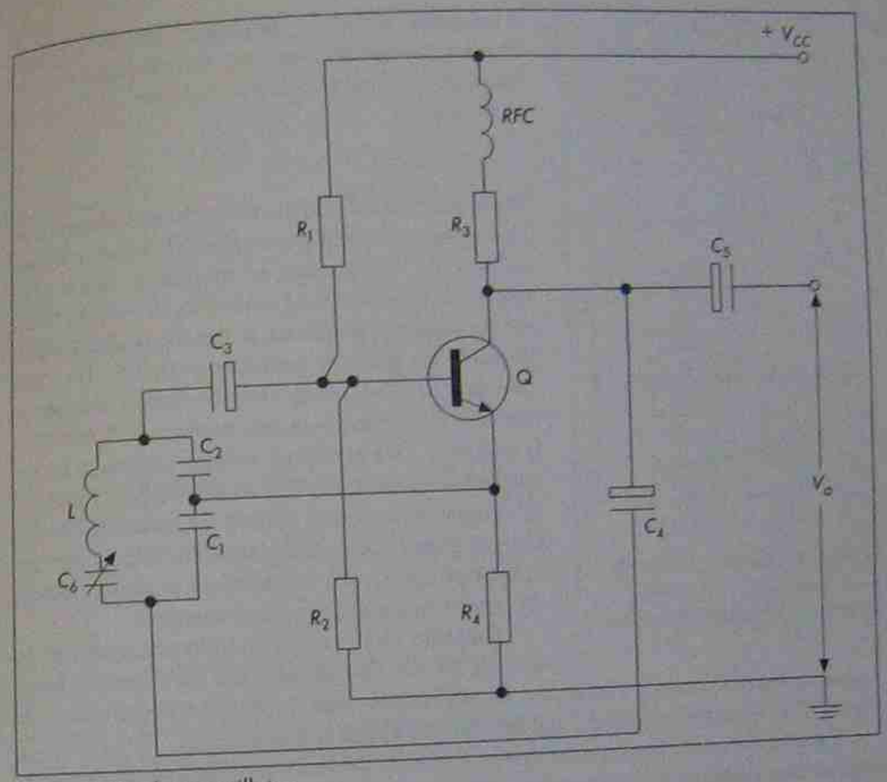


Figure 12.18: Clapp oscillator

12.6 Transformer feedback

A transformer with the correct phase relationship between the primary and secondary winding can also be used to provide feedback. A circuit arrangement showing **transformer feedback** is shown in Figure 12.20.

The amplifier provides the initial 180° of phase shift. The output voltage (V_o) is fed to transformer (T) which is wound to provide a further phase inversion (indicated by the dots). Through transformer action, a secondary voltage (V_f) is developed which is in the same phase as the original input (V_i).

A tuned circuit can be made using the inductance of either the primary or secondary transformer winding, and the amount of feedback is determined by the turns ratio of the transformer. If the tuned circuit is incorporated with the primary winding, the result is a **tuned collector oscillator**. A further secondary winding can be used in the transformer to develop the output signal.

A tuned collector circuit with a transformer coupled output is shown in Figure 12.21. Resistors R_1 and R_2 provide the initial forward bias for the transistor. Resistor R_4 is included

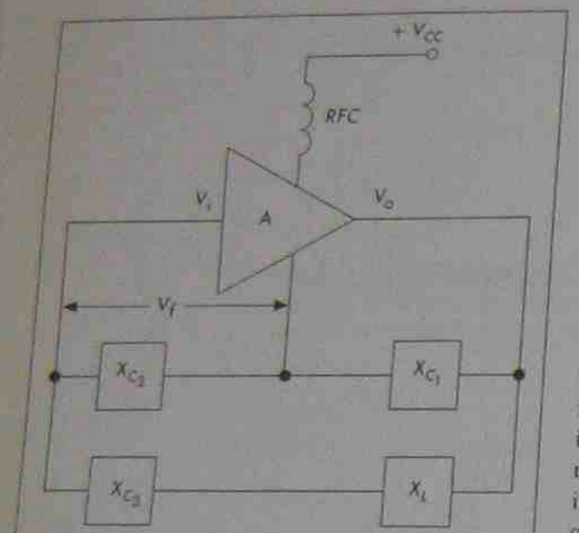


Figure 12.19: Block diagram of a Clapp oscillator (no biasing components)

in the emitter circuit for temperature stability, capacitor C_3 prevents degeneration, and R_3 is the load resistor. The tuned circuit C_1L_1 determines the frequency of oscillation (f_r):

$$f_r = 1/2\pi\sqrt{L_1C_1}$$

The initial forward bias increases the collector current. This collector current passes through inductor L_1 and replaces the energy lost in the tuned circuit. The associated magnetic field around L_1 is coupled to L_2, and the windings are phased so that the feedback signal increases the collector current even further. The turns ratio of L_1 to L_2 ensures that the correct amount of feedback is returned to sustain oscillation. When there is no further 'rate of change' of current through L_1, the magnetic field collapses. The reversed polarity induced in L_2 turns the transistor off and the tank capacitor C_1 charges. Tuned circuit action maintains the oscillations in the tank circuit, and the transistor is turned on with the correct timing to replace lost energy.

Capacitor C_2 provides a low impedance path along with C_3 for the base current, and the output is taken across the other secondary winding L_3. The DC current path through the circuit shown in Figure 12.21 is via the inductor L_1, and therefore the circuit is series-fed.

The alternative is to associate the tuned circuit with the secondary winding. The result is a **tuned base oscillator** and the circuit is shown in Figure 12.22.

The operation of the tuned base oscillator is similar to that of the tuned collector oscillator. The tuned circuit is L_2C_2 and the oscillations are coupled to the transistor base by

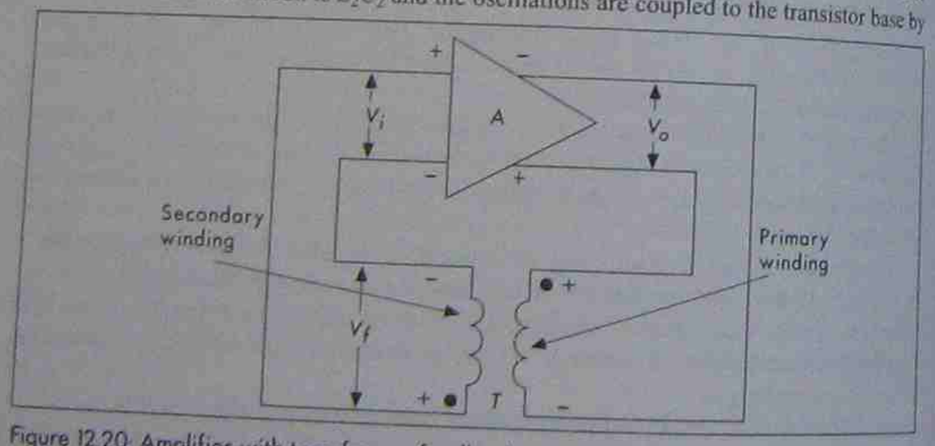


Figure 12.20: Amplifier with transformer feedback

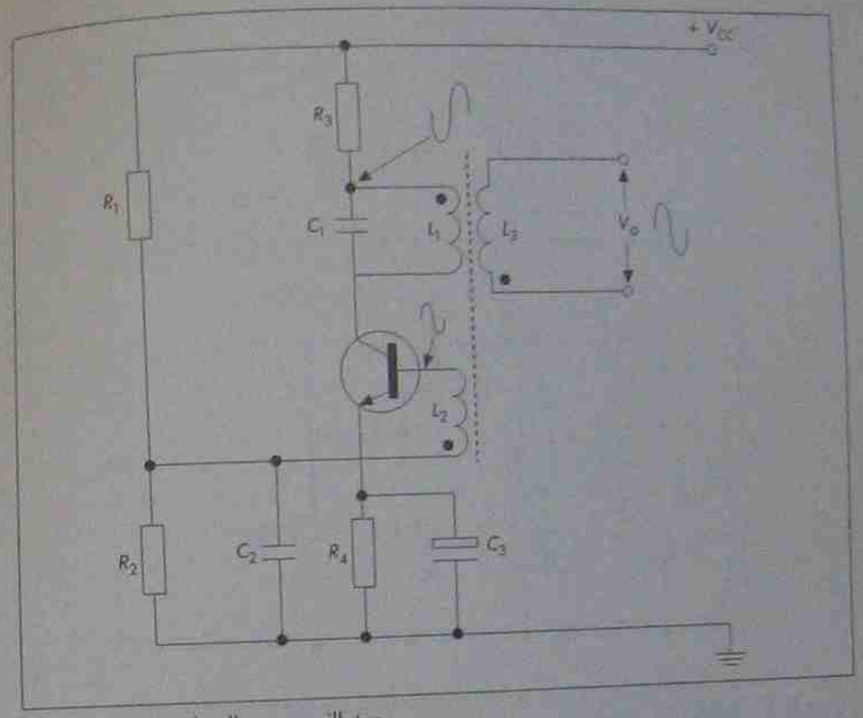


Figure 12.21: Tuned collector oscillator

C_4. The output is fed back via C_1 to the transformer primary L_1. The signal induced into the transformer secondary is phase-inverted through the transformer, and the amount of feedback is determined by the turns ratio of L_1 to L_2. Normal amplifier operating conditions are obtained from resistors R_1, R_2, R_3, R_4 and capacitor C_3.

The coil that the feedback voltage is developed across is called a "tickler" coil. Oscillator circuits that use tickler coils to provide regeneration to the input of the amplifying device are called Armstrong oscillators.

Key terms

capacitive divider
Clapp oscillator
Colpitts oscillator
energy return
feedback voltage
harmonic content
Hartley oscillator
inductive reactive dividers
inductive voltage divider
permeability tuning
positive feedback
reactance ratio
reactive voltage divider
series-fed oscilator
shunt-fed oscilator
tank circuit

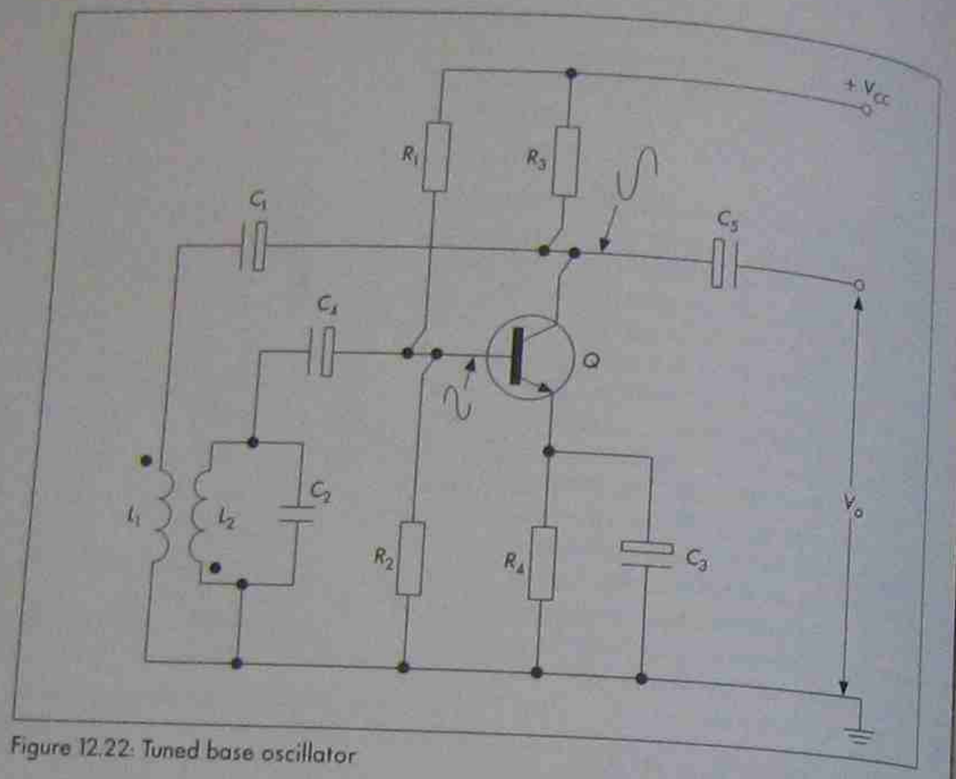


Figure 12.22: Tuned base oscillator

transformer feedback
tuned base
tuned circuit action
tuned collector

12.7 Review questions (answers in Appendix A)

1. A reactive voltage divider is connected to a tuned circuit whose resonant frequency is 100 kHz. The voltage input to the divider is 4 V AC, the inductance is 10 mH and the capacitance is 20 nF.
   (a) Determine the value of V_L.
   (b) Determine the value of V_C.
   (c) Is the divider inductive or capacitive?
2. What is the difference between a series-fed and a shunt-fed oscillator?
3. What is the characteristic that enables a Hartley oscillator to be recognised?
4. Draw a basic Hartley oscillator circuit (without biasing components).
   Describe: (a) the tuned-circuit action; and (b) how the feedback voltage is developed.
5. (a) What type of circuit is shown in Figure 12.23?
   (b) Is it shunt-fed or series-fed?
   (c) Which components determine the resonant frequency?

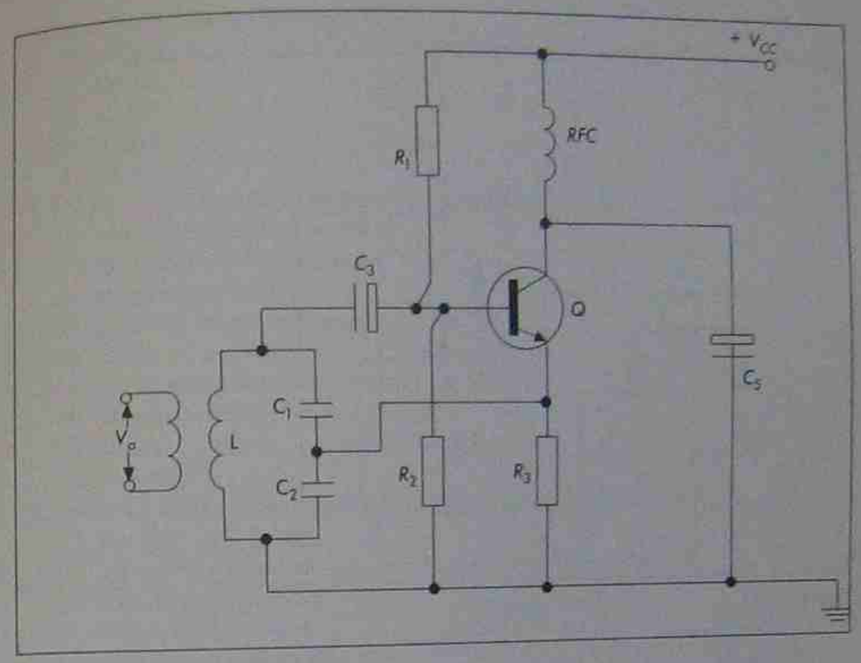


Figure 12.23

   (d) Which component has the feedback voltage developed across it?
   (e) What is the purpose of: (i) C_3 and (ii) C_5?
   (f) Which component is used to return energy to the tank circuit?
   (g) What is the ratio that must remain constant in order to keep the feedback factor constant?
   (h) How can this circuit be modified to improve the frequency stability?
   (i) Is the output voltage (V_o) rich in harmonic content? Explain.
6. If a pure sine wave output was of little importance, which type of oscillator would you use? Why?
7. What is meant by the term 'tickler' coil?
8. An oscillator tuned circuit is required to oscillate at 12 kHz. If the capacitor is 5 nF, what value of inductor is required?
9. Draw a circuit to represent a tuned collector/tuned base oscillator.
   Note that this circuit is *not* included in this text, you must develop it yourself.

Multiple-choice questions

Questions 1 to 5 refer to Figure 12.24.

1. If the value of C_1 is increased, the oscillating frequency will:
   (a) increase
   (b) decrease

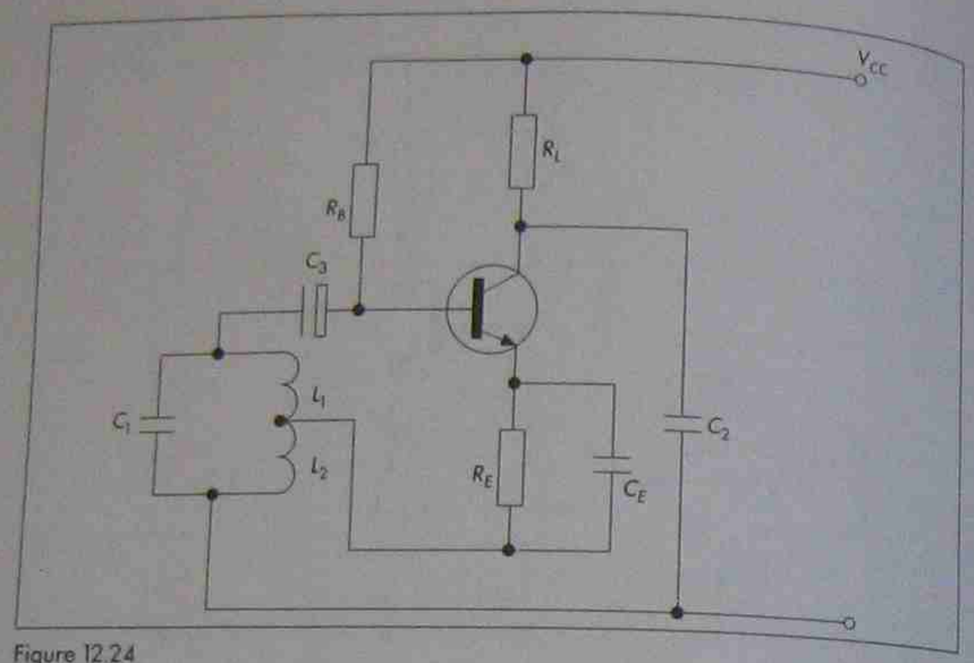


Figure 12.24

(c) stay the same
(d) cease

2. The feedback voltage level is determined by the reactance ratio of:
   (a) L_1 and L_2 to C_1
   (b) L_2 to C_2
   (c) L_1 to L_2
   (d) C_1 to L_1
3. Positive feedback for Q_1 is developed across:
   (a) L_1
   (b) L_2
   (c) C_1
   (d) L_2 and C_1
4. The type of oscillator is a:
   (a) series-fed Colpitts
   (b) shunt-fed Hartley
   (c) shunt-fed Colpitts
   (d) series-fed Hartley
5. Energy is returned to the tank circuit via:
   (a) L_1
   (b) L_2
   (c) C_1
   (d) L_1 and C_1

Questions 6 to 8 refer to Figure 12.23.

6. If $C_1 = 10$ nF, $C_2 = 47$ nF and $L = 41$ mH, the frequency of operation will be approximately:
   (a) 33 kHz
   (b) 8.7 kHz
   (c) 3.3 kHz
   (d) 870 Hz
7. If a small change in load condition caused the output current to increase, then the:
   (a) transistor conduction increases
   (b) base-emitter voltage increases
   (c) output voltage increases
   (d) oscillator frequency decreases
8. The circuit can be converted to a Clapp oscillator by:
   (a) adding a capacitor in series with C_1 and C_2
   (b) adding a variable C to the L branch
   (c) making L variable
   (d) removing C_2
9. A Hartley and Colpitts oscillator use the following reactive dividers:

Hartley	Colpitts
(a) capacitive	inductive
(b) inductive	capacitive
(c) inductive	inductive
(d) capacitive	capacitive

10. If a Colpitts oscillator (as in Figure 12.23) had 10 V AC across the tank, what would be the approximate values of V_{C1} and V_{C2}?

V_{C1}	V_{C2}
(a) 7 V	3 V
(b) 8.33 V	1.66 V
(c) 3 V	7 V
(d) 1.66 V	8.33 V

CHAPTER 13

CRYSTAL-CONTROLLED OSCILLATORS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- *Explain the advantage of a crystal-controlled oscillator over a conventional oscillator.*
- *Describe the piezoelectric effect.*
- *Interpret the operating points on a crystal characteristic diagram.*
- *Define the term 'overtone operation'.*
- *Explain how to slightly change the resonant frequency of a crystal.*
- *Sketch oscillator circuits where the crystal has replaced the tank circuit.*
- *List the predominant circuit features of a Pierce oscillator and a Miller oscillator.*

13.1 Introduction

In the oscillator circuits described in previous chapters, the frequency is controlled by the electrical parameters of the circuit. Frequency stability is affected by the DC resistance of the components, and the DC resistance of the inductor determines the Q factor, a figure of merit, of the tuned circuit. The sharpness of the tuned circuit response curve is limited by the DC resistance and, therefore, circuits which require a high degree of frequency stability use a quartz crystal to control oscillations.

13.2 Crystal characteristics

Before proceeding with the discussion of **crystal-controlled** oscillators, it is necessary to study the **characteristics** and behaviour of the crystal itself. Certain crystalline materials, such as rochelle salts, tourmaline and quartz, develop a potential difference between the opposite plates of the crystal when subjected to mechanical stress. This action is called a **piezoelectric effect**. Crystals used in electronics are cut into thin slices, and quartz is most commonly used.

The piezoelectric effect may be observed in two ways:

1. If a mechanical stress is applied to the flat surfaces of a crystal slice, a potential difference will be generated across the surfaces. The polarity of the potential will be reversed if the mechanical stress is reversed.
2. If a voltage is applied across the flat surfaces of a crystal slice, the electric field causes the crystal to change physical shape and distort. If the polarities are reversed, then the physical stress also reverses.

When an alternating voltage is applied across the two surfaces, the crystal will vibrate at a frequency equal to the resonant frequency of the crystal as determined by its structural characteristics. The physical size of the crystal can be a rough guide to the operating frequency—the smaller the crystal the higher its frequency.

The two basic crystal cuts are X and Y: an **X-cut crystal** is when the slice is cut so that the flat surfaces are perpendicular to its electrical axis; and a **Y-cut** is when the slice is cut so that the surfaces are perpendicular to its mechanical axis. The crystal can be cut in different ways to display different characteristics, for example:

- various or zero temperature coefficients;
- characteristics suitable for oscillators; and
- characteristics suitable for filters.

Figure 13.1 shows a **quartz** crystal with its two basic cuts.

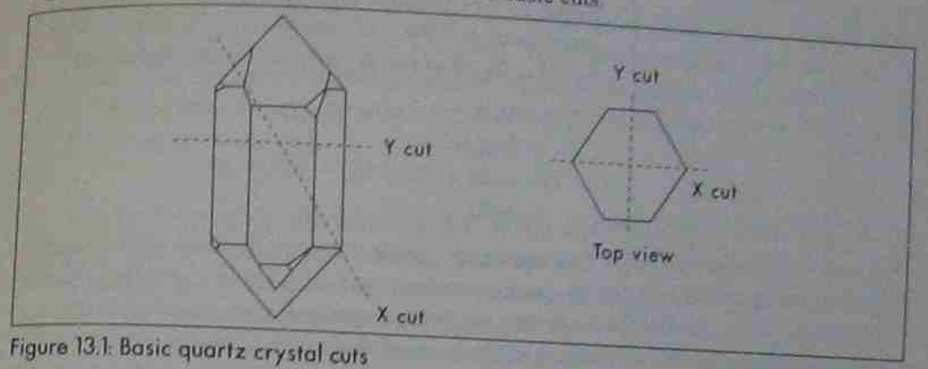


Figure 13.1: Basic quartz crystal cuts

A high quality crystal can maintain the frequency of oscillation so that any variation will be less than one part per million, that is, a drift of less than 1 Hz in 1 MHz. Crystals can be made to resonate at fundamental frequencies ranging from a few kilohertz to several megahertz. It is also possible to make a crystal which will oscillate strongly at its odd harmonic frequencies, and this can extend the crystal frequency range up to 100 MHz or more. This is called '**overtone operation**', and a crystal will resonate at f_r, $3f_r$, $5f_r$ and so on (where f_r is the frequency of oscillation).

A crystal designed to resonate at low frequencies is relatively thick physically compared to a high frequency crystal. The resonant frequency will vary with temperature change. The number of hertz change per megahertz per degree Celsius change in temperature is termed the temperature coefficent of the crystal. Common frequency tolerance specifications are 0.005% to 0.0025% from −55°C to +105°C. An X-cut crystal has a negative temperature coefficient, so the resonant frequency will decrease with a temperature increase. A Y-cut crystal has a positive temperature coefficient. This results in an increase in resonant frequency with a temperature increase.

Two methods may be used to stabilise the resonant frequency against changes with temperature:

1. Enclose the crystal in a temperature controlled box, this is referred to as a crystal **oven**.
2. Vary the angle of the cut of the crystal to obtain a zero temperature coefficient.

The quartz crystal symbol and its equivalent circuit is shown in Figures 13.2(a) and (b). In Figure 13.2(b) the series combination inductor inductor L, capacitor C and resistor R represent the quartz crystal. Capacitor C_M represents the shunt capacitance of the mounting electrodes and the crystal holder. Inductor L is a function of the **mass** of the quartz, while capacitor C is associated with its **compliance**, that is, its stiffness. The resistance R represents friction losses in the quartz itself and any losses due to mounting.

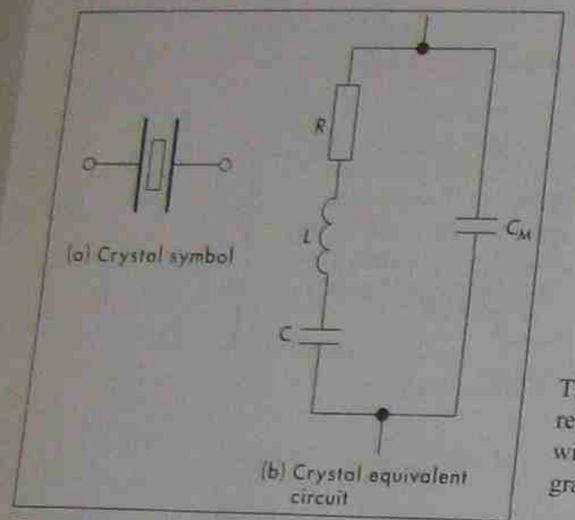


Figure 13.2: Symbol and equivalent circuit of a crystal

The value of R is small, and because of this the Q of the crystal is high, hence approaching the situation of high Q discussed in Chapter 10.

A crystal is required to resonate at 500 kHz. At this frequency its equivalent inductance is 2.5 H, its equivalent capacitance is 0.0405 pF, and its equivalent resistance is 7250 Ω. The **Q of the crystal** can then be calculated:

$$Q = X_L/R$$
$$= 2\pi fL/R, \text{ so}$$
$$Q = (6.28 \times 500 \times 10^{+3} \times 2.5)/7250$$
$$= 1083$$

The equivalent circuit shown in Figure 13.2(b) therefore represents a series-tuned circuit (the crystal) in parallel with the mounting capacitance. A reactance/frequency graph of the net parallel circuit is shown in Figure 13.3.

It can be seen from the graph that, as the frequency increases, the combined $X_L - X_C$ branch of the parallel

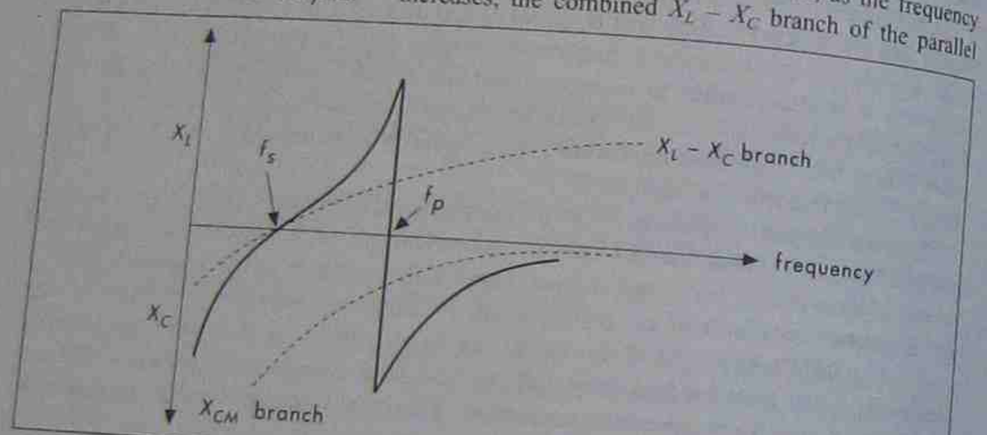


Figure 13.3: Reactance characteristic of the circuit in Figure 13.2(b)

circuit becomes inductive while the reactance of the other branch decreases. When X_L equals X_C, the condition of **series resonance** is met and this point is shown as f_s. When $X_L - X_C$ equals X_{CM}, the condition of **parallel resonance** is met and this point is shown as f_p. The heavy line shows the total crystal reactance. Thus, when a crystal is used in an oscillator circuit, five different conditions may exist which are dependent on the applied signal frequency:

1. Frequencies below f_s cause the crystal to exhibit capacitive reactance.
2. Frequencies at f_s cause the crystal to exhibit low resistance, that is, a series resonant circuit.

3. Frequencies between f_s and f_p cause the crystal to exhibit inductive reactance.
4. Frequencies at f_p cause the crystal to exhibit a very high resistance, that is, a parallel resonant circuit.
5. Frequencies greater than f_p cause the crystal to exhibit capacitive reactance.

Oscillator circuits generally rely on the crystal behaving as either a low resistance series resonant circuit or as an inductive reactance. These two conditions exist only over an extremely narrow range of frequencies and, therefore, the frequency stability of the oscillator is high. The crystal may also be used to replace all or part of the tuned circuit. These circuits are discussed in section 13.3.

13.3 Controlling an oscillator with a crystal

A crystal may be made to operate in its series resonant mode by connecting it as a series element in the feedback path of the oscillator circuit. At the series resonant frequency of the crystal, the crystal's resistance is minimal and therefore the feedback voltage is maximised. At any other frequency the crystal presents a high reactance, and the high losses prevent the circuit from oscillating. The circuit in Figure 13.4 shows a crystal employed in this manner. The tuned circuit L_1C_1 is adjusted close to the operating frequency of the crystal. The exact oscillating frequency is controlled and stabilised by the crystal itself. Capacitor C_3 allows the resonant frequency of the crystal to be altered slightly if necessary. Figures 13.5(a) and (b) show a Hartley and a Colpitts oscillator controlled in a similar manner.

13.3.1 Replacing the tuned circuit

There are a number of configurations in which a crystal replaces a tuned circuit and one of these is called the **Pierce oscillator** (essentially a modified Colpitts). The inductor is replaced by a crystal used in its inductive mode (f_r is between f_s and f_p) as shown in Figure 13.6(a) and the AC equivalent circuit is shown in Figure 13.6(b).

The frequency of oscillation is determined by the series resonance of the circuit made up of C_1C_2 and the equivalent LC circuit of the crystal. Capacitors C_1 and C_2 are much greater than the equivalent crystal capacitance (C_s). Therefore, the resonant frequency is almost entirely determined by the value of C_s. The input capacitance of the amplifier and the capacitance of the tuned circuit are overridden by C_s. The resonant frequency is approximately equal to the series value of the crystal. However, the feedback ratio is still X_{C2}:X_{C1} (as for a Colpitts oscillator).

When energy is drawn from the circuit in the

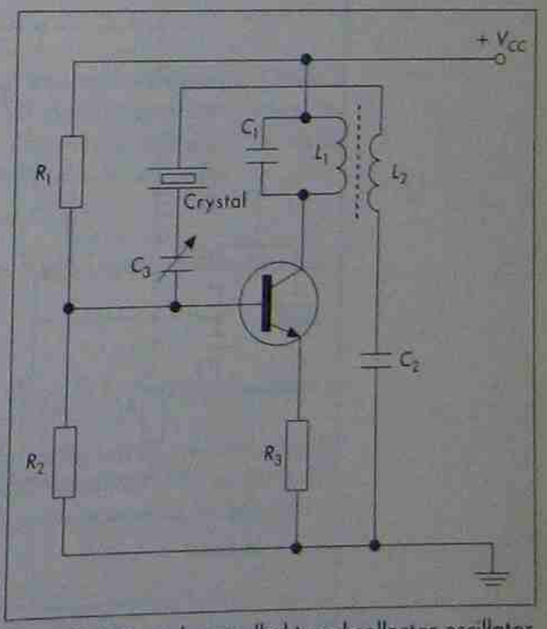


Figure 13.4: Crystal-controlled tuned collector oscillator

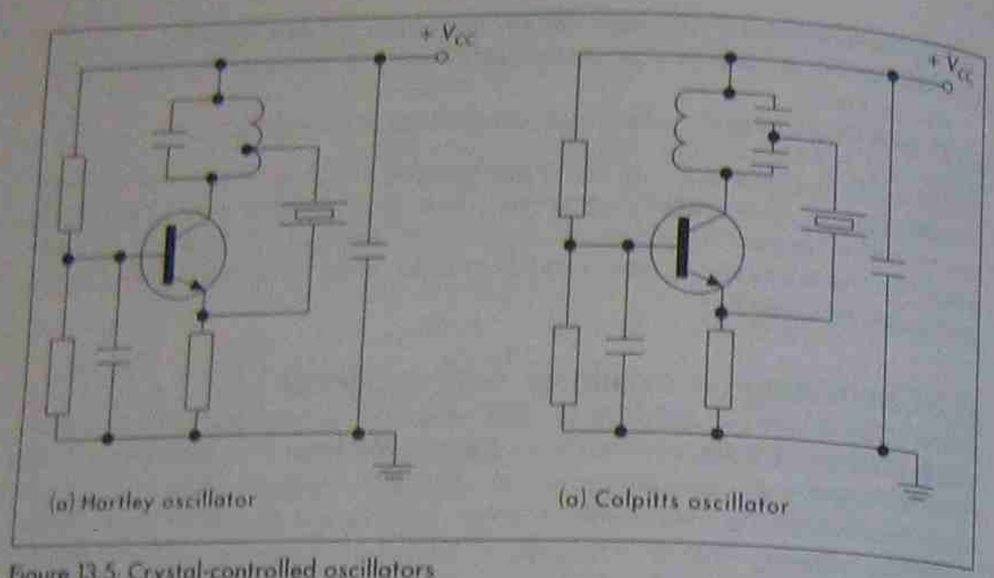


Figure 13.5: Crystal-controlled oscillators

usual manner, the Q of the crystal is reduced. Special coupling can be used to minimise the loading effect as shown in Figure 13.7.

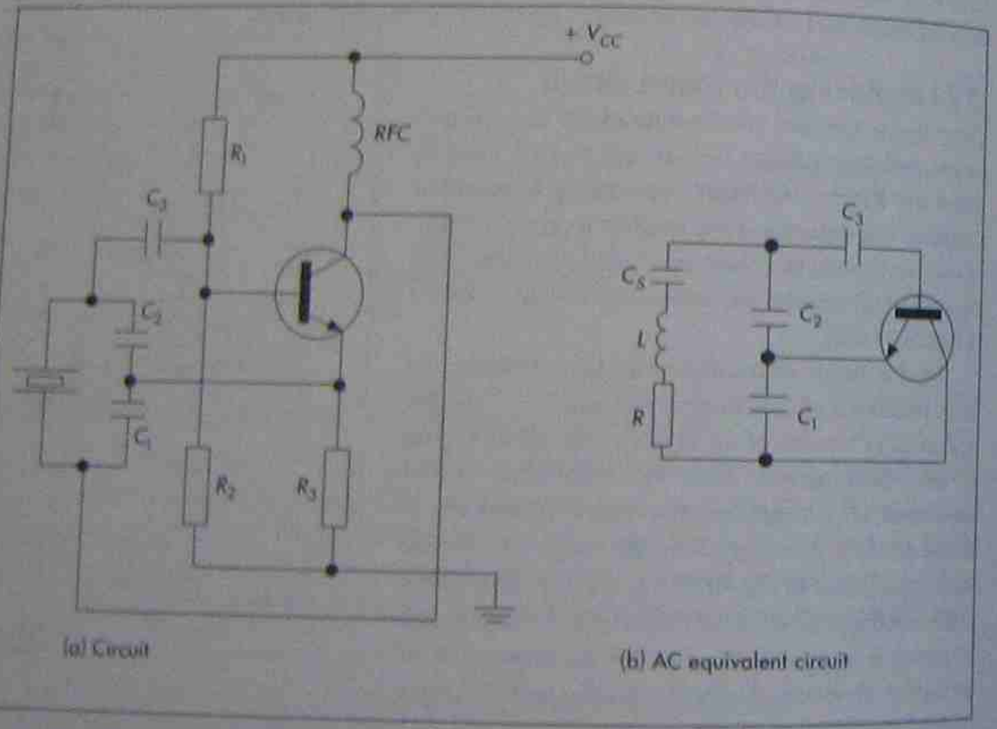


Figure 13.6: Pierce oscillator (L is replaced by a crystal)

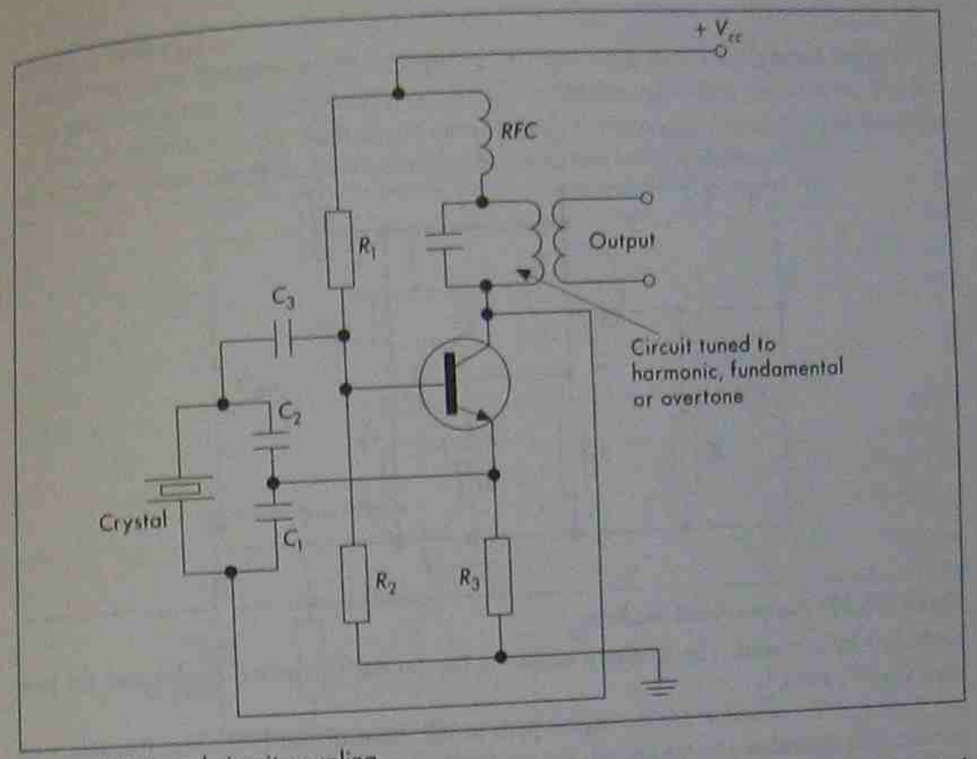


Figure 13.7: Tuned circuit coupling

A parallel-tuned circuit, tuned to the fundamental or the required frequency, is inserted in the collector circuit. The following stage is coupled through transformer action and therefore does not greatly affect the Q of the crystal. The tuned circuit also provides the capability of using the oscillator in its overtone mode, that is, $3f_r$, $5f_r$, and so on.

13.3.2 Other crystal-controlled circuits

A crystal can be used to control the frequency of any oscillator and this section discusses a few of the more commonly found types.

Pierce oscillators

A crystal-controlled FET **Pierce oscillator** is shown in Figure 13.8. The Pierce oscillator is commonly found in communications circuits.

The Colpitts feedback circuit is the crystal (inductive mode) and the two capacitors C_1 and C_2. Capacitor C_1 can be external, or the input to ground capacitance of the amplifier (C_{ie} for a transistor, C_g, for FET, C_{gk} for a valve). Capacitor C_2 may be external, or the output to ground capacitance of a further stage.

Crystal-controlled Clapp oscillators

Another communications oscillator, shown in Figure 13.9, is an FET Clapp oscillator

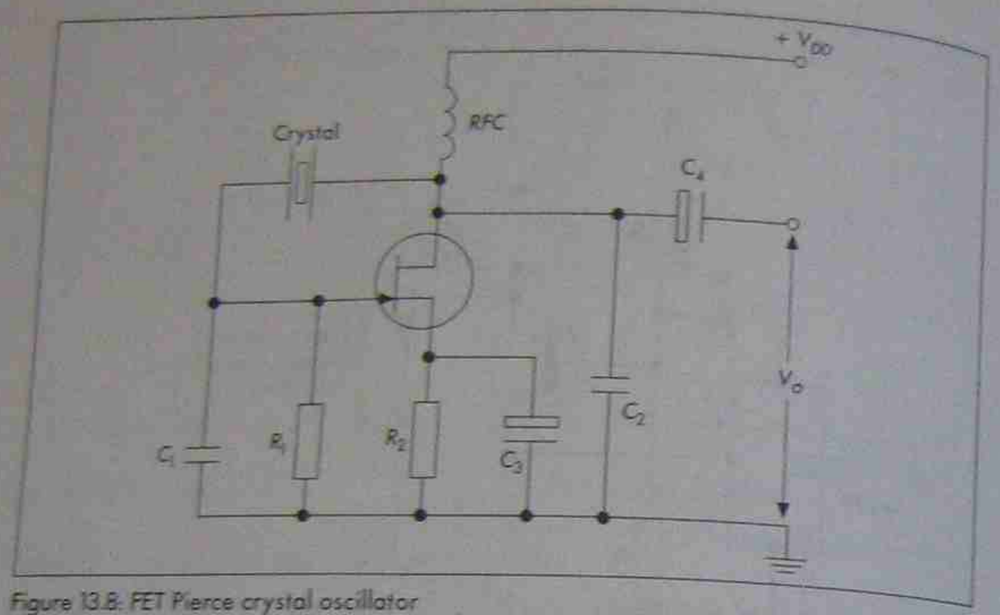


Figure 13.8: FET Pierce crystal oscillator

controlled by a crystal. The feedback circuit is the crystal (inductive mode) and the two capacitors C_2 and C_3.

Capacitor C_1 provides the AC signal path to the input of the FET. The frequency of oscillation is determined by the crystal, and the feedback voltage is developed across C_3. The ratio of X_{C2} to X_{C3} still determines the feedback ratio, as in a Colpitts oscillator.

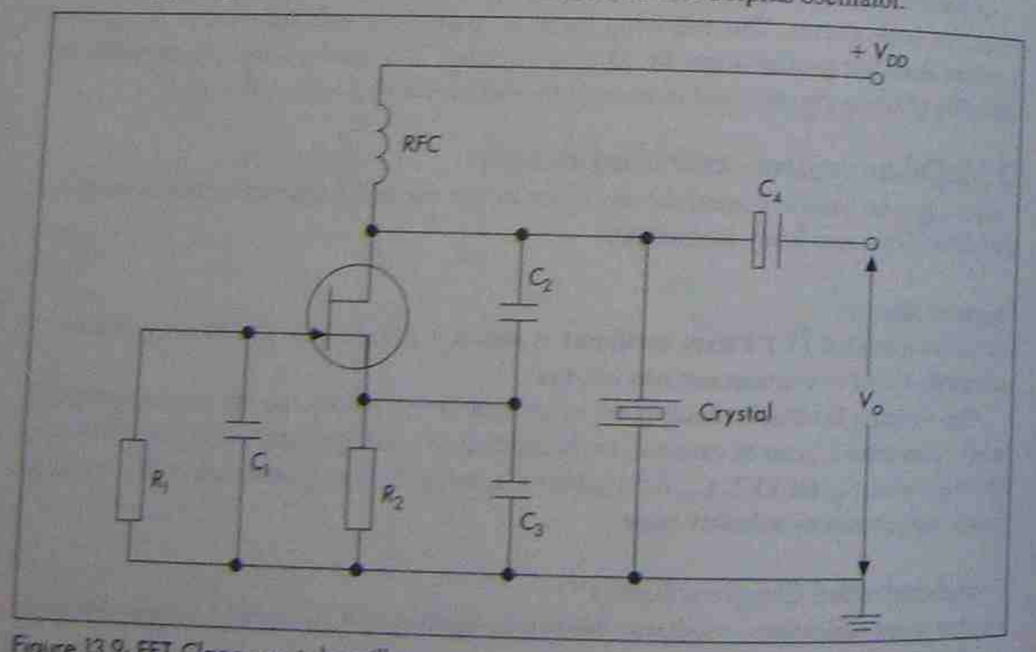


Figure 13.9: FET Clapp crystal oscillator

Miller oscillators

A **Miller oscillator** circuit is shown in Figure 13.10. The basic circuit has a tuned drain, and a tuned gate circuit which is replaced by a crystal. The Miller equivalent circuit is the Hartley oscillator. However, with the drain circuit tuned exactly to the crystal frequency, both the drain and gate circuits are resistive. All the reactances cancel and no oscillations occur.

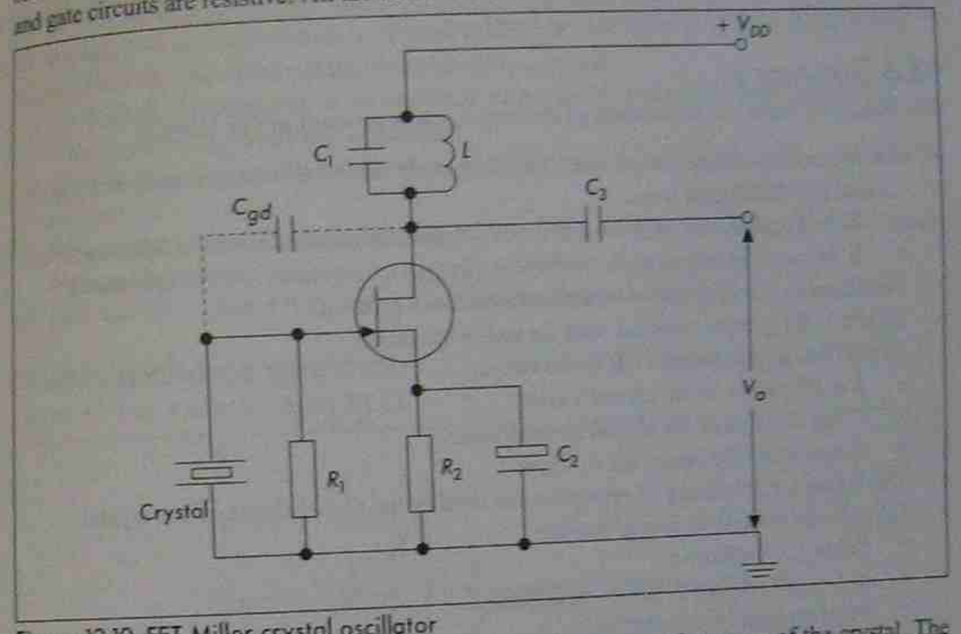


Figure 13.10: FET Miller crystal oscillator

To ensure oscillation, the drain is tuned above the resonant frequency of the crystal. The crystal frequency is now lower than the drain tank circuit. Both the drain circuit and the crystal exhibit inductive characteristics and, along with the gate/drain capacitance (C_{gd}), form a Hartley oscillator. The AC and equivalent circuits are shown in Figures 13.11(a) and (b).

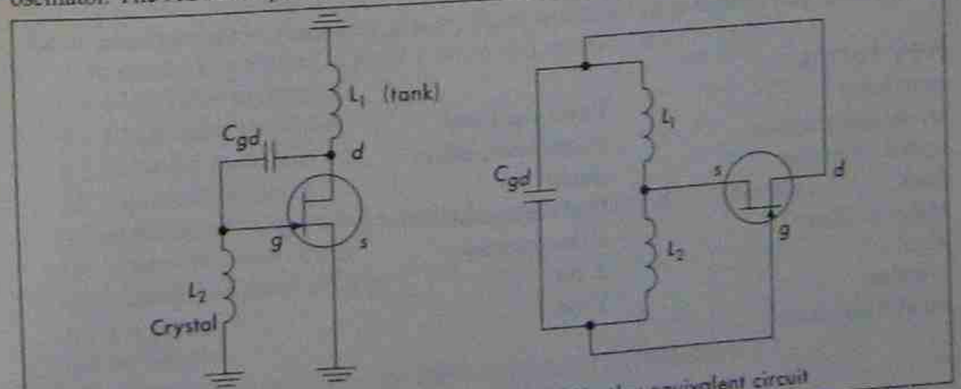


Figure 13.11: AC and Hartley equivalent circuits

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Inductors L_1 and L_2 and capacitor C_{tune} form the tank circuit. Energy is returned to the tuned circuit by drain/source current passing through L_1. The feedback voltage to sustain oscillation is developed across inductor L_2 (the gate/source of the FET) and the feedback ratio is X_{L2}:X_{C2}.

The capacitor in the Miller oscillator can be external, or the input to output capacitance of the particular amplifying device (C_{iss}, C_{gd}, C_{gs}).

13.4 Summary

The following items are a summary of the major points covered in this chapter.

- An alternating voltage causes the crystal to vibrate and vice versa (piezoelectric effect).
- Crystal is cut different ways:
  - X for negative temperature coefficient (frequency decreases with temperature); and
  - Y for positive temperature coefficient (frequency increases with temperature).
- Stabilised by keeping at a constant temperature in an oven.
- Quartz is the general material used for stable operation.
- Crystal has an equivalent LCR series circuit.
  - L is a function of the crystal's mass.
  - C is a function of the crystal's compliance.
  - R represents the losses due to friction.
- The higher the frequency of operation, the smaller the physical size of the crystal.
- Crystal may operate in several modes:
  - below f_s = capacitive;
  - at f_s = low resistance (series resonance);
  - between f_s and f_p = inductive;
  - at f_p = high resistance (parallel resonance); and
  - above f_p = capacitive.
- The crystal can replace the inductor in the tank, or provide a low impedance feedback path.
- A crystal may operate in its 'overtone' mode (that is, frequency of the odd harmonics).

Key terms

compliance	Pierce oscillator
crystal characteristic	piezoelectric effect
crystal controlled	quartz
mass	Q of the crystal
Miller oscillator	series resonance
oven	X cut
overtone	Y cut
parallel resonance	

13.5 Review questions (answers in Appendix A)

1. Explain the 'piezoelectric' effect.
2. How can temperature stability be maintained when using a crystal?
3. What is the main advantage of replacing a tuned circuit with a crystal?
4. In what mode is the crystal operating when f_s is greater than f, but less than f_p?
5. Explain how a crystal can be used in its series-resonant mode to control an oscillator.
6. (a) How can the tuned circuit be replaced by a crystal?
   (b) Explain the operation of an oscillator where the crystal replaces the tank circuit.
7. What is meant by 'overtone' operation?
8. What is the effect on crystal Q when an oscillator circuit is loaded? How can this be minimised?
9. What is the main difference between a Pierce and a Miller oscillator? List the circuit features of each.
10. How can the resonant frequency of a crystal be changed slightly?

Multiple-choice questions

Questions 1 to 3 refer to Figure 13.12.

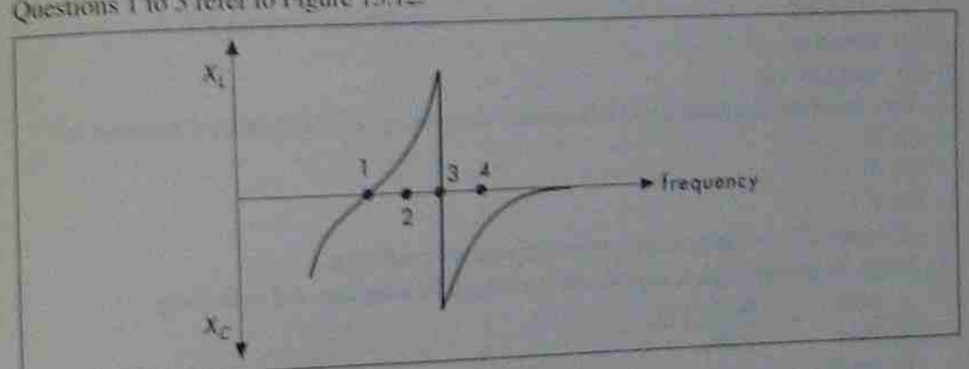


Figure 13.12

1. If the crystal is operating at point 1 on its characteristic, it will exhibit:
   (a) a low value resistance
   (b) a high value resistance
   (c) inductive reactance
   (d) capacitive reactance
2. When the crystal is operating at point 3 on the characteristic, it behaves as:
   (a) a series resonant circuit
   (b) a parallel resonant circuit
   (c) a capacitor
   (d) an inductor
3. When operating at point 2 on the characteristic, the crystal behaves as:
   (a) a capacitor

(b) a resistor
(c) an inductor
(d) an open circuit

4. If a crystal has an equivalent inductance of 100 mH at 1 MHz and an equivalent resistance of 2 kΩ, the *Q* will be approximately:
(a) 31.4
(b) 62.8
(c) 188.4
(d) 314

5. When a crystal is used to replace the inductor in the tank circuit, the oscillator is known as:
(a) a Miller oscillator
(b) a Pierce oscillator
(c) a Clapp oscillator
(d) an Armstrong oscillator

6. When an oscillator is operating in its 'overtone' mode, the frequency will be:
(a) the fundamental (f_r)
(b) twice f_r
(c) three times f_r
d) four times f_r

7. The crystal cut that results in the frequency decreasing with temperature increase is:
(a) X
(b) Y
(c) Z
(d) none, all crystals have a positive temperature coefficient

8. A high frequency, high *Q* crystal will have the following size and selectivity:

Size	Selectivity
(a) large	wide
(b) small	wide
(c) large	narrow
(d) small	narrow

9. With reference to Figure 13.7, the reactance ratio of X_{C1} to X_{C3} should be approximately:
(a) 1:5
(b) 2:1
(c) 1:2
(d) 5:1

10. The crystal material used for oscillators that require a high degree of frequency stability is:
(a) rochelle salt
(b) quartz
(c) tourmaline
(d) silicon

CHAPTER 14

DIGITAL OSCILLATORS

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Objectives

On the completion of this chapter you should be able to achieve the following objectives:

- Determine the 'natural' operating frequency of a digital oscillator.
- Explain how to control the output frequency of an oscillator.
- Draw a basic crystal-controlled TTL/CMOS gate oscillator.
- Explain the operation of the components in a TTL/CMOS gate digital oscillator.
- Explain how to 'fine-tune' a digital oscillator.
- Show the circuit modifications required to produce a sine wave output from a digital oscillator.
- Explain why 'buffer' arrangements in the oscillator output.
- Design the circuit of a digital oscillator with multiple output frequencies derived from one crystal.

14.1 Introduction

With digital prototypes coming to the forefront of technology in the microprocessor and computer industry, it is not surprising to see digital oscillators arriving on the scene. Many modern communication, electronic, audio and industrial control systems use digital oscillators to derive their output frequencies. Other forms are at present higher frequency, but require an internal down to the lower frequency. A digital oscillator produces a sine output at the required frequency. The output can then be applied to power amplifiers, or other circuitry as a single well-defined frequency source.

14.2 Logic gate oscillators

In basic terms, every gate oscillator can be produced by connecting the output device to a Schmitt oscillator with a logic gate. The gate can be an **inverter**, a TTL/CMOS gate or a logic gate.

The gate provides the required gain and 180° of phase shift. Any odd number of inverting logic gates will oscillate if they are connected in a ring, as shown in Figure 14.1.

These logic gates are to represent a three-input inverter, each with a certain time lag between input and output, **propagation delay**. If the input to gate A is a logic '1' condition, the output of the gate will be logic 0 in zero. This output will be processed by gate B and C to produce a logic '0' (zero) at the output of gate C (see Figure 14.1). If this output is fed back to the input of gate A, the original '1' circuit itself around the ring and the network oscillates. The frequency of oscillation is determined by the total delay time through the whole circuit, and can be calculated from:

$$f = \frac{1}{2nT}$$

where f = frequency of oscillation in hertz
n = number of gates
T = delay time per gate in seconds

This is the natural or free running frequency of the network and it could be as high as 20 MHz. It is highly dependent on such factors as:

- supply voltage;
- temperature; and
- external loading capacitance.

The greater the number of gates, the longer the propagation delay time and the lower the **natural frequency**. In order to build a useful stable oscillator, it is necessary to add other circuit elements to control the frequency of oscillation.

14.3 Single capacitor–resistor (CR) oscillators

Any inverting TTL or CMOS gate or combination of gates, can be used to make a stable capacitor–resistor (CR) oscillator. Figure 14.2 shows a three-gate CR oscillator with a duty cycle of approximately 50%. The frequency of oscillation can be calculated from:

$$f = \frac{1.2}{R_2 \times (0.405 \times R_1/R_2 + 0.693) \times C}$$

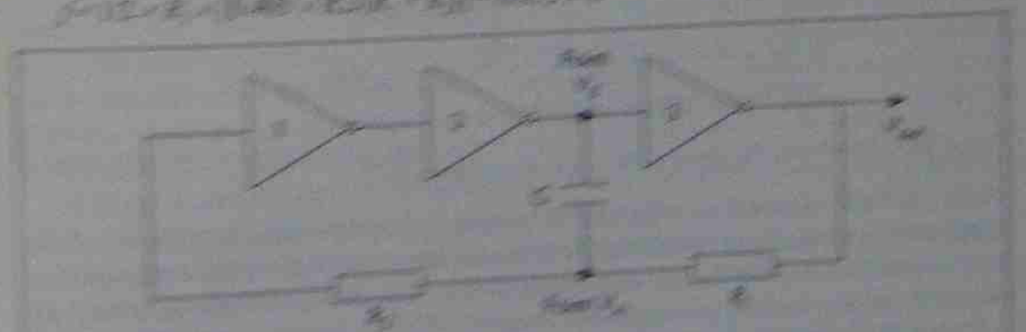


Figure 14.2 Three-gate CR oscillator

The R_1/R_2 network can be simplified down to R_3, where:

$$R_3 = \frac{R_1 \times R_2}{(R_1 + R_2)}$$

and $f = \frac{1.2}{R_2 \times (0.405 \times R_3 + 0.693) \times C}$

The relationship of R_1 to R_2 has the greatest bearing on the frequency, and the following three cases apply:

1. If $R_1 = R_2 = R_3$, then
   $f = 0.350/C \times R$.
2. When R_1 is much greater than R_2, then
   $f = 0.455/C \times R$.
3. When R_1 is much smaller than R_2, then
   $f = 0.722/C \times R_1$.

These relationships may be useful to experimenters. The ideal output waveform and the voltage waveform at the charging node for C (point V_A) are shown in Figures 14.3(a) and (b).

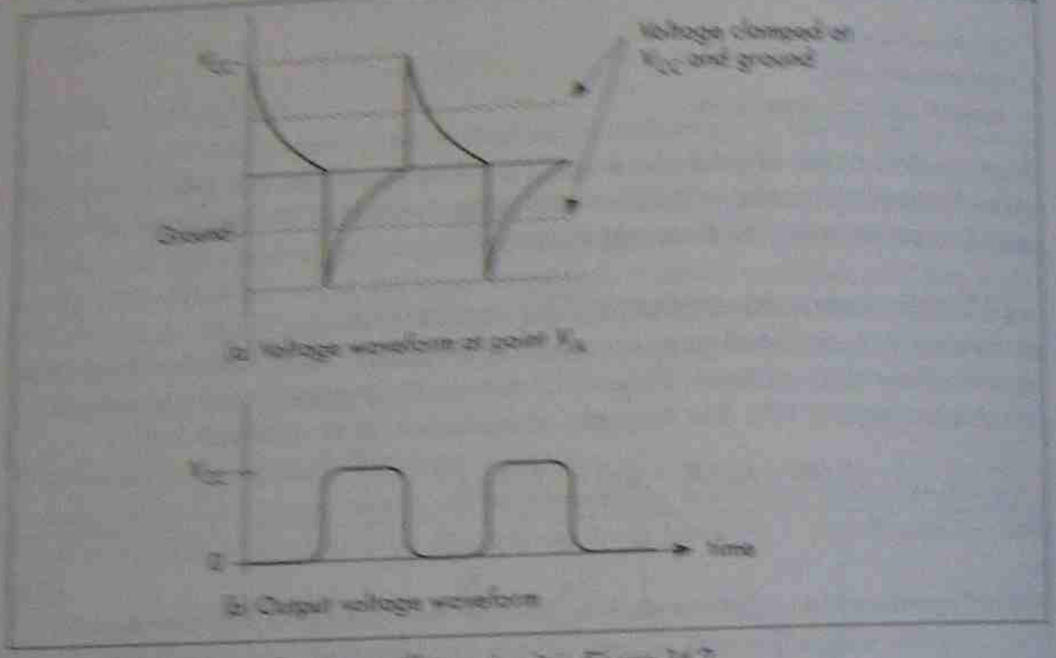


Figure 14.3: Waveforms for the oscillator circuit in Figure 14.2

The internal construction of the MM74C04 is such that the voltage at point V_A will be clamped by the input diodes when point V_A is greater than V_{CC} or more negative than ground. Current passes through R_2 during this portion of the cycle, but at all other times the only current through R_2 is a very small leakage current.

When the voltage at point V_A exceeds about 50% of the supply voltage, and the input to inverter 3 begins to change, point V_A swings in a direction that aids the switching action. This action is positive feedback which makes the oscillating frequency more predictable and stable.

Note that data on the basic logic gate oscillator is reproduced with the kind permission of National Semiconductor.

14.3 Crystal-controlled logic gate oscillators

To ensure stable and accurate oscillation, a crystal may be used as in the manner discussed in Chapter 11. The crystal can be used to ensure feedback at one frequency only across one or two gates. Figure 14.4 shows a single gate **crystal-controlled** oscillator.

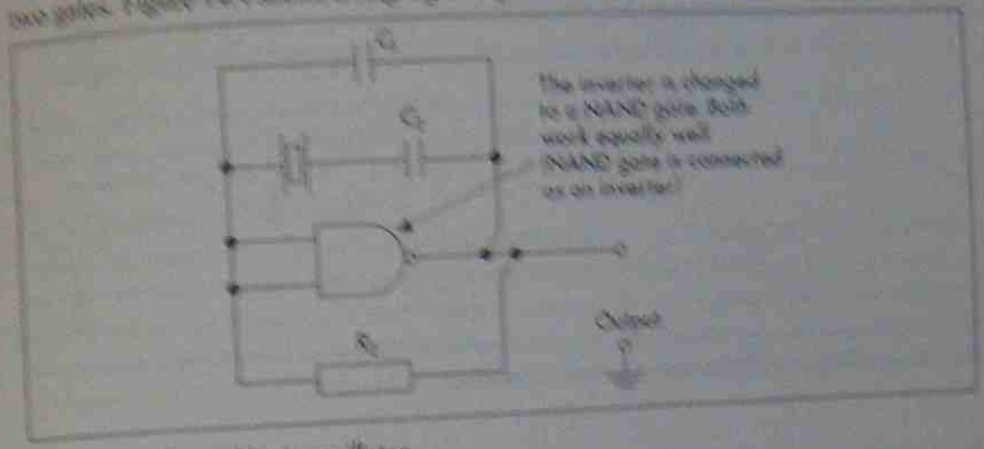


Figure 14.4: Crystal logic oscillator

Resistor R_1 ensures a 'DC' current path around the gate to provide the input current required by the internal transistor. It should be high in value (2 MΩ to 10 MΩ) because it is in parallel with the crystal.

A low value of R_1 will reduce the Q (figure of merit) of the crystal and this is undesirable. Capacitor C_1 tends to lower the oscillator frequency and C_2 tends to increase the frequency; that is, they pull the crystal frequency up and down.

The propagation delay must not be too long, otherwise the oscillator cannot keep in pace with the crystal. However, if the propagation delay is too short the oscillator may operate in its overtone mode (harmonics). At low frequencies, this problem is overcome by increasing the propagation delay through the ring to a point where the circuit will not oscillate at the harmonic frequency, but continues to operate at the fundamental frequency. A circuit to achieve this delay is shown in Figure 14.5.

Resistor R_2 and capacitor C_3 along with inverters 2, 3, 4 and 5 increase the propagation delay. The inverters also **buffer** the output. The output of a crystal-controlled logic gate oscillator should be buffered to prevent the external circuit from loading down the oscillator. This is easily achieved because there is usually a spare gate or two in the integrated circuit package. Figure 14.6 shows a NAND gate oscillator used to generate a **colour television subcarrier** frequency. Resistor R_1 provides a high impedance path from output to input to initiate oscillation.

The crystal specifications require its parallel resonant load to be 30 pF (C_3). The frequency of oscillation is mainly determined by the crystal. The C_3R_2 network serves three purposes:

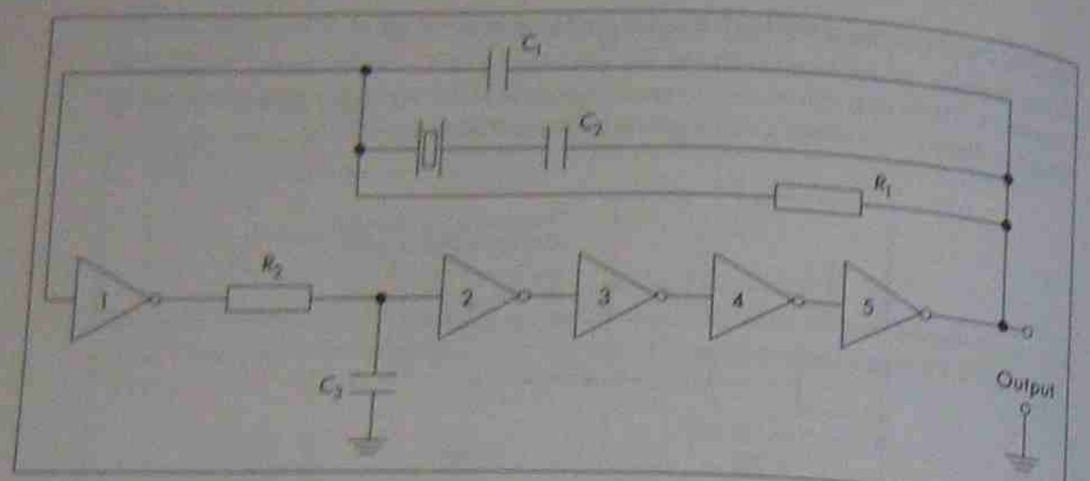
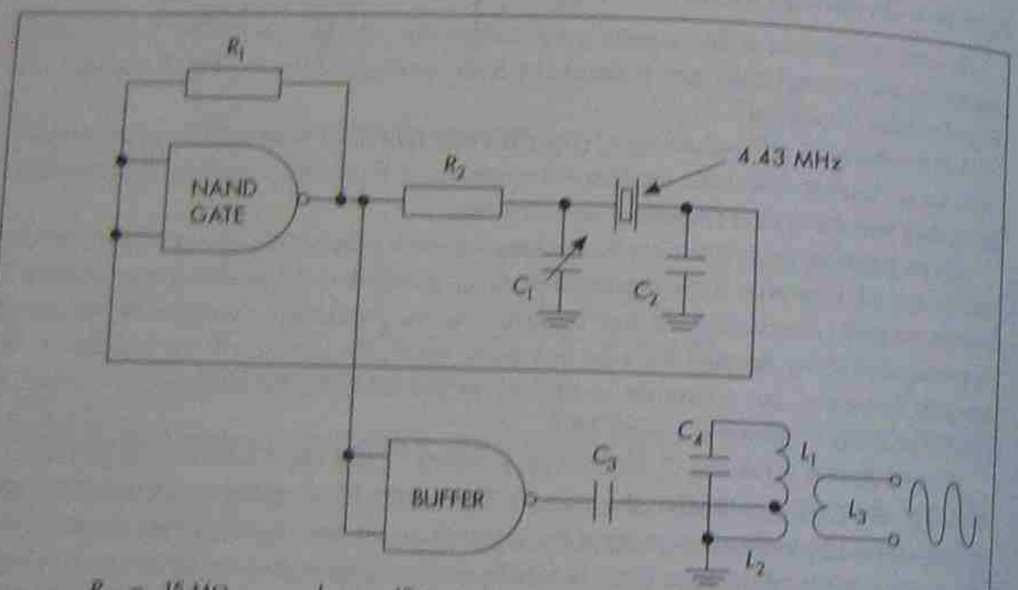


Figure 14.5. Crystal logic oscillator with overtone protection



R_1 = 15 MΩ
R_2 = 470 Ω
C_1 = 30 pF
C_2 = 30 pF
C_3 = 10 nF
C_4 = 220 pF

L_1 = 12 turns
L_2 = 4 turns
L_3 = 2 turns
IC = DM7400 (TTL) or MM74C00 (CMOS)
V_{CC} pin 14, earth pin 7

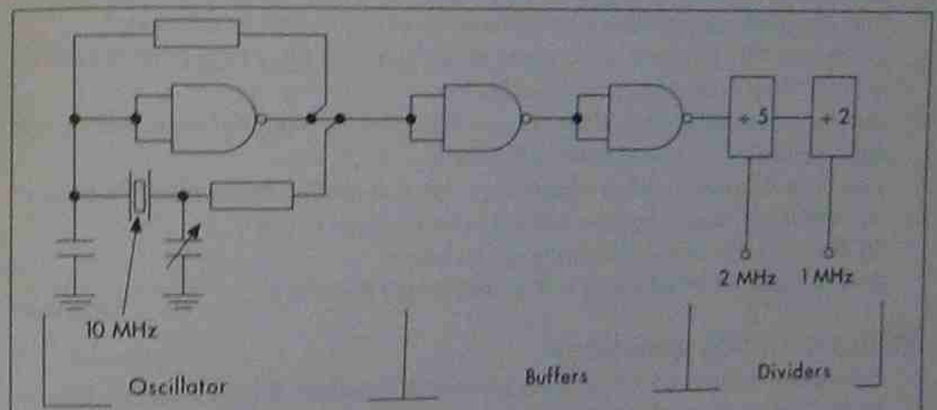
Figure 14.6. A 4.43 MHz NAND gate oscillator

1. It increases the propagation delay.
2. It provides phase shift.
3. It behaves as a low pass filter to reduce the chance of the crystal operating in its overtone mode. Capacitor C_1 is adjustable to enable 'fine' control of the output frequency, hence offsetting any crystal manufacturing variations. This enables the output frequency to be adjusted to exactly 4.43361875 MHz (the critical value for colour television operation).

The output is buffered to prevent loading. Inductors $L_1 + L_2$ form a tuned circuit with C_4, and a sine wave output at 4.43361875 MHz is taken from L_3.

14.3.1 Frequency division

Further frequency stability can be gained by selecting the crystal frequency a multiple above the desired operating frequency and by using **frequency dividers** to reduce the oscillator frequency to the final output frequency. If there is any frequency drift (due to temperature variations, and so on), only a proportion of the error will be detected at the outputs of the dividers. The circuit in Figure 14.7 shows a 1 MHz and a 2 MHz output being obtained from a 10 MHz crystal.



(The gate is a MM74C00, ÷ 5 and 2 ÷ 2 is a decade counter IC DM2490)

Figure 14.7: Frequency stability using dividers

Other digital oscillators such as voltage-controlled oscillators can be found incorporated in other integrated circuits. For example, the 4046 phase-locked loop contains an internal voltage-controlled oscillator circuit which requires only an external resistor and capacitor to set the oscillator frequency.

14.4 Summary

The following items are a summary of the major points covered in this chapter:

- Any odd number of digital gates in a ring will oscillate.
- The natural oscillating frequency can be found from:
  $f = 1/2nT$

- The circuit can be stabilised with a CR network or crystal.
- A digital gate requires a large value resistor across the input/output to provide DC feedback.
- The oscillator is usually run at a frequency above the desired value and then divided down to the value required. This provides good stability.
- The output of the oscillator should be buffered to prevent loading.

Key terms

buffer
crystal controlled
frequency divider
inverter gate
NAND gate
natural frequency
NOR gate
propagation delay
colour television subcarrier

14.5 Review questions (answers in Appendix A)

1. Can a sine wave output be obtained from a digital oscillator? Explain your answer.
2. What determines the natural oscillating frequency of a three-gate inverter loop?
3. A three-gate TTL inverter with an input-output delay time of 12 ns is to be used as an oscillator. What is the frequency of oscillation?
4. How can the free running frequency of an oscillator loop be controlled to provide a lower output frequency? Give an example of the circuit.
5. What precaution must be taken when using a crystal to stabilise the oscillator frequency?
6. (a) Why should buffer stages be included in the oscillator circuit?
   (b) Do they perform any other functions? List them.
7. Draw a circuit that would ensure a very stable output frequency.

Multiple-choice questions

1. If three inverter gates are connected in series, with the output of gate 3 connected to the input of gate 1 and each gate has a delay time of 10 μS, the natural frequency will be approximately:
   (a) 8 kHz
   (b) 12 kHz
   (c) 17 kHz
   (d) 33 kHz
2. The type of digital gate required to make an oscillator is:
   (a) an inverter
   (b) a NAND gate
   (c) a NOR gate
   (d) any of the above
3. The output waveform of a purely digital oscillator will be:
   (a) sine
   (b) square
   (c) triangular
   (d) sawtooth
4. A large value resistor is required across the input/output of a digital gate to provide:
   (a) an AC current path
   (b) a negative feedback path
   (c) a DC current path
   (d) a low Q circuit

Questions 5 to 8 refer to Figure 14.8.

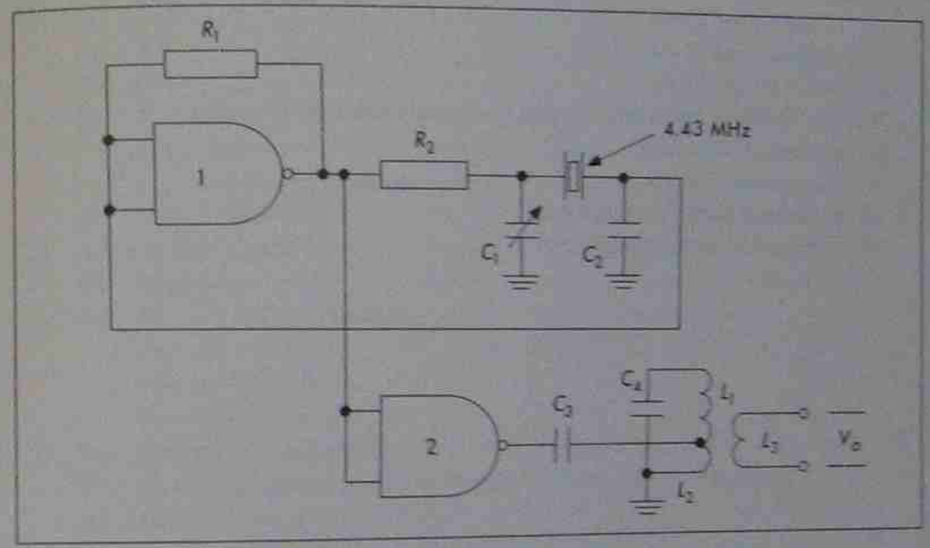


Figure 14.8

5. Gate number 2:
   (a) is required for the oscillator to operate
   (b) prevents the output of gate 1 from being loaded down
   (c) acts as a buffer which enables a crystal to be used to control the frequency
   (d) converts the output from a sine wave to a square wave
6. The purpose of C_1 is to:
   (a) provide the correct capacitive load for the crystal
   (b) set the crystal to 4.43 MHz
   (c) prevent gate 1 from being loaded
   (d) allow trimming of the output frequency
7. The purpose of the tuned circuit L_1 L_2 C_4 is to:
   (a) convert the square wave into a sine wave
   (b) isolate the oscillator output from the load
   (c) increase the level of the output voltage
   (d) convert the sine wave into a square wave

8. Capacitor C_2 has a value of 30 pF to:
   (a) meet the crystal's output specifications
   (b) trim the oscillator frequency
   (c) match the value of C_1
   (d) to make a high pass filter with R_2
9. The output of a 500 kHz digital oscillator is fed into a ÷ 32 integrated circuit, and the output of the circuit is fed into a ÷ 625 circuit. The resulting frequencies will be:
   (a) 7812.5 Hz and 50 Hz
   (b) 15 625 Hz and 25 Hz
   (c) 15 625 Hz and 50 Hz
   (d) 7812.5 Hz and 25 Hz
10. If the DC feedback path across the gate goes open circuit, the result is:
   (a) an increase in frequency
   (b) a decrease in frequency
   (c) an increase in output voltage level
   (d) a decrease in output voltage level

APPENDIX A

ANSWERS TO REVIEW QUESTIONS

Chapter 1

1. By reversing the diode
2. (a) Half
   (b) All
3. 700 mV
4. (a) $V_p = 100$ V
   (b) PIV = 142 V
   (c) $I_{Lm} = 14.14$ A and I_L(RMS) = 7.07 A
   (d) Power = 500 W
5. $0.318 \times 50 = 15.9$ V
6. (a) Negative
   (b) (i) $V_{s1} + V_{s2} = 148.49$ V
   (ii) $V_L = 94.5$ V
   (iii) $I_L = 4.72$ A
   (iv) PIV = 296.98 V
7. (a) Full-wave bridge circuit
   (b) D_2 and D_4 conduct and point D is positive with respect to point C.
   (c) (i) $V_L = 36$ V
   (ii) $I_L = 600$ mA
   (iii) PIV = 56.6 V
8. (a) Load voltage = 353.5 V
   PIV of diodes = 707 V
   (b) Load voltage = 2×353.5 because the full secondary voltage is twice that of (a)
9. (a) Open circuit, short circuit and high forward resistance
   (b) Open circuit winding in either primary or secondary or short circuited turn in either winding.
10. 110 V-0-110 V = 220 V (RMS) × 1.414 = 311 V peak
    Diode is a 20D4

Multiple-choice answers. 1c 2b 3d 4c 5b 6a 7d 8b 9d 10b

Chapter 2

1. To reduce the amount of AC ripple present in the load.
2. 0.29%
3. (a) The average level increases.
   (b) The diode current is reduced.

4. Peak inverse voltage
5. 6%
6. L-C section filter
7. (a) $X_L = 3140\ \Omega$
   (b) $I = 3.8$ mA
   (c) Ripple voltage = 86.26 mV (peak)
8. (a) Use L-C section filter.
   (b) Use full-wave rectification.
   (c) Use solid-state diodes.
9. (a) 200 Hz
   (b) 400 Hz

Multiple-choice answers. 1d 2b 3b 4c 5d 6b 7c 8a 9c 10d

Chapter 3

1. To maintain a constant output voltage with varying current demand on the circuit.
2. 66.6 Ω
3. 1.89 W
4. Power wastage is high.
5. 9.5 V
6. (a) 72%
   (b) 1.25 W
   (c) 3.1 W
7. (a) $R_L \downarrow I_L \uparrow V_E \downarrow V_{BE} \uparrow R_{CE} \downarrow V_{CE} \downarrow$ and V_E goes back to normal.
   (b) $V_{in} \uparrow V_E \uparrow V_{BE} \downarrow R_{CE} \uparrow V_{CE} \uparrow$ and V_E goes back to normal.
8. By shunting the base-emitter junction with a circuit that senses the load current and reduces V_{BE} when required.
9. Closed-loop circuit has an error sensing amplifier and responds to very small changes in load variation.
10. To provide constant current with a varying load.
    25%
    No
11. $I_L \uparrow I_E \uparrow V_{RS} \uparrow V_{BE} \downarrow$ and I_L is reduced to normal value.

Multiple-choice answers. 1d 2c 3a 4b 5a 6c 7d 8b 9c 10d

Chapter 4

1. To decouple any noise on the input line.
2. (a) No
   (b) Improves transient response and noise rejection.
3. (a) Mount for best convective cooling.
   (b) Anodise or paint the heatsink surface.

   (c) Use a thick heatsink material.
   (d) Use a heat conductive paste on the intergrated circuit pack.
   (e) Insulate the integrated circuit from the heatsink if necessary.
4. (a) Input-output voltage differential must not be too high or there will be excessive power dissipation by the regulator itself.
   (b) 'Drop-out' voltage must not be too low or the regulator will lose its ability to regulate.
   (c) Amplitude of the ripple must not be excessive or it will intrude into the regulator drop-out voltage region.
5. The output current capability decreases, because the current-limiting transistor requires a lower V_{BE} to turn it on at higher temperature levels.
6. A back-biased diode shunted across the regulator input/output.
   Yes, if V_{out} happened to exceed V_{in}, then the internal circuit of the regulator is protected.
   Current path is via the diode.
7. Refer to Figure 4.7.
8. By raising the ground pin above earth potential using a voltage divider from the output pin (see Figure 4.14).
9. No, input and output capacitors require reverse polarity, and the short-circuit input protection diode must be reversed.
10. See Figure 4.31.
11. A tracking regulator makes automatic adjustment for circuit variations. The positive regulator will follow or track the negative regulator or vice versa. The advantage is that the tracking regulator automatically keeps the positive/negative potentials within 100 mV of each other.
12. 1.5 V to 2.0 V
13. (a) Power = $(V_{in} - V_{out}) \times I_L$
    = 40 W
    This will cause excessive heat to be dissipated by the regulator, thereby limiting its output current.
    (b) (i) Good heatsinking and extra convection cooling, such as a fan
    (ii) Reduce V_{in} to lower the $V_{in} - V_{out}$ differential

Multiple-choice answers. 1d 2b 3d 4c 5a 6b 7c 8b 9a 10d

Chapter 5

1. To step up DC voltages cheaply without the use of a step-up transformer.
2. Refer to section 5.2.
3. (a) $\sqrt{2} \times 2V_s$
   (b) $\sqrt{2} \times 2V_s$ for 3, $\sqrt{2} \times V_s$ for 1
   (c) Same as the input voltage frequency
4. Refer to Figure 5.12, and add one more section with diode polarity the same as D_2 and D_4 and with the capacitor C_6 on the bottom.

5. It is only suitable for small load currents.
6. Television picture tube (high V, low I)

Multiple-choice answers. 1b 2d 3c 4a 5d 6d 7c 8b 9a 10b

Chapter 6

1. (a) High frequency operation
   (b) High voltage output capacitor
   (c) Switching regulator
2. (a) Efficient
   (b) Output voltage may be easily stepped up or down.
3. Current transients thrown back into the supply which means that heavy suppression is required.
4. The primary components could be at AC mains potential, that is, not isolated.
5. Low equivalent series resistance (ESR)
6. Refer to Figure 6.3 and section 6.2.2.
7. Step-down: transistor in series
   Step-up: transistor in parallel
8. Step-up circuit, because I flows in the load during both the 'on' and 'off' states of the transistor.
9. (a) Yes
   (b) Inverting circuit with L replaced by a transformer.
   (c) Refer to Figures 6.8 and 6.9.
10. Energy is stored in the inductor. The diode provides a current path through the load when the magnetic field around L collapses.
11. A 'forward' SMPS transfers energy to the inductor and load when the transistor is on. A 'flyback' SMPS stores energy when the transistor is on and transfers it to the load when transistor is off.
    The 'forward' circuit has the highest power capability.
12. Refer to Figure 6.14 and section 6.5.
13. To prevent L from saturating due to the intial high current required to charge C_o.
14. It overrides the error amplifier output to keep the duty cycle of the drive waveform to a minimum. This allows C_o to charge before the control circuit takes control of the circuit.
15. (a) Sensing resistor to provide a voltage proportional to output current, which then controls the error amplifier.
    (b) Use the sensed voltage to inhibit the oscillator, and so remove the drive.
16. Refer to section 6.6.4 and Figure 6.25.
17. The transistor is slow turning off. This dissipates power and may destroy the transistor.
18. Refer to Figure 6.28 and section 6.6.6.

Multiple-choice answers. 1d 2b 3c 4c 5a 6b 7a 8b 9d 10c

Chapter 7

1. The equivalent series inductance (ESL) and the equivalent series resistance (ESR) are related to the capacitor's construction and need to be kept at a minimum. The ESL will cause spikes to appear at the output, and the ESR results in power loss due to internal heating.
2. For a fixed output the components required are an inductor, a catch diode and input/output filter capacitors (refer to Figure 7.3). The adjustable output requires the above, plus the selection of a voltage divider at the output. One resistor in the external divider is adjustable to allow V_{out} to vary (refer to Figure 7.8).
3. Capacitor range is 330 µF to 1000 µF.
4. The feedback line from the output is directly from V_{out} if fixed, and from a potentiometer if adjustable.
5. Oscillator frequency is 52 kHz and efficiency is 80% to 88%.
6. (a) Ferrite pot cores give low electromagnetic interference (EMI), are physically small and have low power dissipation. Core is easily saturated by high current.
   (b) Powdered iron cores also have low EMI but can stand peak currents better than ferrite pot.
   (c) Ferrite bobbin cores generate more EMI and may cause interference. They are low cost and can handle high current.
7. Diodes used in SMPS circuits must be fast and have a low forward voltage drop (V_f). They must have a peak inverse voltage and $I_{F(av)}$ about 1.2 to 1.25 times the maximum input voltage and load current. Schottky types are used.
8. Refer to Figure 7.12.
9. The phasing and turns ratio of the transformer winding, plus the orientation of the catch diode, determines the V_{out} level and polarity.
10. Refer to Figures 7.6 and 7.8 for the circuit diagram.
    $C_{out} \cong 680$ µF (chosen because no V_r known)
    DCWV = 37.5 V
    R_1 given to be 1 kΩ, then:
    R_2 (max) $= R_1((V_{out}/V_{ref}) - 1)$
    $= 1 \times 10^3 \times ((30/1.23) - 1)$
    $= 23\text{k}4\ \Omega$
    Inductor $\Delta V = (V_{in} - V_{out}) \times V_{out}/V_{in} \times 1000/f$
    $= (35 - 30) \times 30/35 \times 1000/52$
    $= 82.4$ V/µs
    From Figure 7.7:
    L = H150
    = 150 µH

Multiple-choice answers. 1b 2b 3d 4c 5a 6d 7c 8b 9a 10a

Chapter 8

8.6.2 Faults in the current-limited series regulator

Fault 5: Open-circuit zener diode
Fault 6: Base-emitter of Q_1 open circuit
Fault 7: Error-amplifier transistor Q_2 open circuit

8.6.4 Faults in the SMPS circuit

Fault 9: TP 1 = 10 V TP 5 = 0 V
TP 2 = 0 V TP 6 = no osc
TP 3 = 0 V TP 7 = no osc
TP 4 = 0 V TP 8 = no osc

Fault 10: TP 1 = 10 V TP 5 = 9.5 V
TP 2 = 0 V TP 6 = osc
TP 3 = 0 V TP 7 = osc
TP 4 = 0 V TP 8 = osc (no inductive spike)

8.10 Review questions

1. Purpose of the components in Figure 8.19, in order through the circuit:
   D_1–D_4 = full-wave bridge rectifier
   C_1 = filter capacitor
   R_1 = series-biasing resistor for the zener diode D_5
   D_5 = Reference diode to hold the base of Q_1 steady at 6.2 V
   Q_1 and Q_2 = differential pair error amplifier
   R_3 = common emitter-resistor
   R_2 = load resistor for error amplifier
   Q_3 = control transistor which drives the series transistor Q_5. It in turn has its conduction determined by the output of the Q_1/Q_2 pair
   Q_4 = current limiting device which when, forward biased, draws current away from Q_1 base
   Q_5 = series pass transistor
   R_4 = fixed leg of the voltage divider, R_4/Q_3, which controls the forward bias on Q_5
   R_5 = base current limiting resistor for Q_4
   R_6 = current sensing resistor, across which the forward bias for Q_4 is developed
   R_7 and R_8 = fixed top and bottom legs of the voltage divider across the output. They prevent the output from being shorted to earth when the output voltage is adjusted
   V_{R1} = adjustment for the output voltage. Varies the voltage fed to the base of Q_2 which is compared at the base of Q_1
   V_{R2} = adjustment for setting the current limit. It is in parallel with the sensing resistor R_6
   C_2 = suppresses zener fixing noise.
   C_3 = high frequency bypass capacitor

Circuit operation

The reference diode is connected to transistor Q_1 base and holds it at a steady 6.2 V DC. A portion of the output voltage, determined by the setting of V_{R1}, is applied to the base of Q_2. Any difference between these two voltage levels is amplified and then applied to the base of Q_3.

The conduction of transistor Q_3 is varied, and this adjusts the forward bias on Q_5, thereby controlling the output level. Transistors Q_3 and Q_5 are connected in Darlington configuration, giving a high gain to the circuit and making it very sensitive. Current limiting is provided by R_6 in parallel with V_{R2}. The voltage developed across this resistor network adjusts the forward bias on Q_4. As Q_4 conducts harder, it draws current away from Q_3 base. This reduces the conduction of Q_3, and hence its resistance (R_{CE}) increases. The forward bias or Q_5 is developed across R_4 (the lower leg of the Q_3/R_4 voltage divider) and is also reduced, therefore the conduction of Q_5 is less, resulting in a reduced output voltage.

2. If the load is decreased, the output current demand is less, and therefore Q_4 does not conduct, resulting in no current limiting.

   If the load current increases sufficiently, the voltage developed across R_6/V_{R2} will increase above 600 mV, and forward bias Q_4. Base current will be drawn away from Q_3, and it will conduct less, increasing in resistance. The forward bias on Q_5 is therefore reduced; the resistance increases, resulting in more voltage being dropped across Q_5, and the output voltage is reduced.
3. Fault: zener diode open circuit. The voltage at TP 1 has dropped, indicating that more current is being drawn. All the other voltages have increased. Voltage at TP 2 has increased, indicating that the zener diode is inoperative. Transistor Q_1 conducts heavily, Q_2 conduction ceases, and therefore the base voltage at Q_3 increases. Transistor Q_5 is turned on harder and the output voltage increases.
4. Fault: Q_5 open circuit. The voltage at TP 1 has increased, indicating that less current is being drawn. The reference voltage, derived from the input, is normal, and all other voltages are zero. Transistor Q_5 is non-conducting because it is open circuit, therefore there is no output voltage.
5. Fault: transistor Q_3 open circuit. The voltage at TP 1 has increased indicating that less current is being drawn. TP 2 is normal, but TP 4 is high, indicating no current through R_2. The output and feedback voltages are zero because Q_5 is not being driven by Q_3.
6. Fault: zener diode or C_2 short circuit. The voltage at TP 1 has dropped indicating that more current is being drawn. TP 2 is zero. The output of the differential pair (TP 4) is low, therefore Q_3 and hence Q_5 have their conduction reduced. The output voltage is low. Ohmmeter check from TP 2 to earth shows a short circuit.

Multiple-choice answers. 1c 2b 3d 4a 5d 6b 7c 8d 9a 10c

Chapter 9

1. Feedback means to return a portion of the output signal of a circuit to the input of the circuit.
2. Negative feedback
3. Disadvantage is low gain. Advantages are that stability is gained, there is less distortion, wider bandwidth, and higher or lower input and output impedances.
4. $A' = A/1 - \beta A$ (general formula)
   $A' = A/1 + \beta A$ (negative feedback formula)
   $1 + \beta A$ because β is negative, that is, $1 - (-\beta A)$
5. The term feedback factor means the proportion of the output that is fed back to the input. Usually expressed as a decimal.
6. $A' = A/1 + \beta A$
   $= 500/1 + (0.12 \times 500)$
   $= 500/61$
   $= 8.2$
7. Series-fed
   $R_i = R_i \| R_s$
   $R_i f = R_s (R_i + R\beta)$
   Therefore $R_i f > R_i$ and input impedance has increased.
   $R_i f = R_i(1 + \beta A)$
   Series-derived
   $R_o = R_o \| R_L$
   $R_o f = R_o(R_L + R_f\beta)$
   Therefore, $R_o f > R_o$
   $R_o f = R_o(1 + \beta A)$
8. Output is across R_3 (V_f) and this opposes the input voltage V_s. Feedback is voltage shunt-derived and it is series-fed.
9. If R_s of the source is low, the feedback (shunt-fed) would go through R_s rather than R_i. Therefore, a high R_s is required, that is, a constant current source.
10. See Figure 9.11.
11. (a) Voltage feedback
    (b) Series-fed
    (c) Shunt-derived
    (d) R_{if} increases, R_{of} decreases
    (e) V_s going positive results in V_o going positive. Part of this in-phase voltage is fed to Q_1 emitter. Therefore V_b goes positive and at the same time V_e goes positive. This results in a decrease in V_{be}, hence less gain and less V_o.
    (f) $V_f = V_o \times [R_3/(R_3 + R_5)$
    $= 12 \times 3.3/61.3$
    $= 0.65$ V peak-to-peak
    $= 650$ mV peak-to-peak
    $V_s = 500$ mV peak

Therefore, $V_i = Vs - V_f$
$= 1 - 0.65$
$= 0.35$ V
$= 350$ mV peak-to-peak

Multiple-choice answers. 1b 2c 3a 4d 5b 6d 7c 8a 9b 10a

Chapter 10

1. To provide a continuous alternating output signal, with constant amplitude and frequency, when connected to a DC power source.
2. Refer to section 10.1 and Figure 10.2.
3. (a) A charged C placed across L (both ideal, infinite Q components). C discharges through L building up a magnetic field. Field collapses and charges C with opposite field; when C discharged, field collapses and recharges C as it was initially.
   (b) No
   (c) Resistance present in L and also load, therefore oscillations die away.
4. $f_r = 1/2\pi\sqrt{LC}$
   $= 1/6.28\sqrt{20 \times 10^{-3} \times 10 \times 10^{-9}}$
   $= 11259.66$ Hz
   $= 11.3$ kHz
5. Refer to Figure 10.9 or 10.10.
6. Damped oscillations
7. Feedback voltage must:
   (a) return sufficient energy to the circuit to replace all losses; and
   (b) have the correct phase or timing to aid the oscillating voltage.
8. Yes. With positive-going input, the output is an amplified positive-going signal. Part of this is fed back to the input base circuit aiding the original signal. Therefore, positive feedback, and the circuit will oscillate if gains of Q_1 and Q_2 are sufficient.

Multiple-choice answers. 1a 2b 3d 4c 5d 6d 7a 8c 9a 10b

Chapter 11

1. (a) The feedback must be in phase with the original input signal (360° phase shift).
   (b) The amplifier must have enough gain to replace the circuit losses.
2. (a) CR phase-shift oscillator
   (b) 60°
   (c) gain = 29
   (d) $C = 1$ nF
   $R = 4$k7 Ω
   $f_r = 1/2\pi CR\sqrt{6}$
   $= 13.8$ kHz
   (e) Transistor Q_2 prevents the oscillator from being loaded down by any external circuit connected to it. Transistor Q_2 is connected as an emitter follower.

3. Resistors R_1, R_2, R_3 may be switched; capacitors C_1, C_2, C_3 may be ganged or vice versa. The disadvantage is a change in the output level as the frequency is changed.
4. Phase shift is reduced to 45°, and the gain required is less (gain = 18).
5. Two stages of amplification provide 360° phase shift.
6. (a) Wien bridge oscillator
   (b) $C_1 R_1$, $C_2 R_2$
   (c) R_4 and R_6. Negative feedback proportional to V_o is fed back to Q_1. As V_o increases, V_f increases and the output is reduced.
   (d) Positive at points A and B, therefore in phase
   (e) $f_r = 1/2\pi CR$
   $1/(6.28 \times 10 \times 10^{-9} \times 4.7 \times 10^{-3})$
   $= 3.39$ kHz
   (f) The resistance of Q_1 base-emitter junction and R_4 are in parallel with R_2. This affects the resonant frequency.
7. Points A and C are in phase. This is negative feedback because both the base and emitter are going positive at the same time. The net result is a reduction in V_{be}.

Multiple-choice answers. 1c 2d 3b 4a 5d 6c 7a 8b 9c 10d

Chapter 12

1. (a) $X_L = 6280\ \Omega$, therefore, $V_L = 4.02$ V
   (b) $X_C = 79.6\ \Omega$, therefore, $V_C = 51$ mV
   (c) Inductive
2. In a series-fed oscillator the circuit DC current path is through the tuned circuit. In a shunt-fed circuit the DC is blocked by capacitors.
3. Tapped inductor
4. (a) Refer to section 12.3, Figure 12.7(b) and text.
   (b) The feedback voltage is developed by the circuit current passing through the inductive component connected between the base and emitter of the transistor:
   $V_L = I \times X_L$
5. (a) Colpitts oscillator
   (b) Shunt-fed
   (c) L, $C_1$and C_2
   (d) Capacitor C_1
   (e) (i) C_3 couples the AC signal from the tank to the transistor base. It also blocks the DC bias from the tank.
   (ii) C_4 provides the AC signal path from the collector to the tuned circuit via the earth line.
   (f) C_2
   (g) X_{C1}:X_{C2}
   (h) A capacitor placed in the tuned circuit in series with L will improve the stability. This is a Clapp circuit.

   (i) No, the feedback is developed across capacitor C. As the frequency increases, the capacitive reactance decreases and therefore the harmonic content is also reduced.
6. Hartley oscillator because it is easy to tune by varying C, and the ratio of L_1 to L_2 is not altered in any way.
7. A 'tickler' coil is the coil in a transformer feedback oscillator across which the feedback voltage is developed.
8. $f_r = 1/2\pi\sqrt{LC}$
   $L = 1/(2\pi f_r)^2 \times C$
   $1/((6.28 \times 12 \times 10^{-3})^2 \times 5 \times 10^{-9})$
   $= 35.2$ mH
9. See Figure A.1

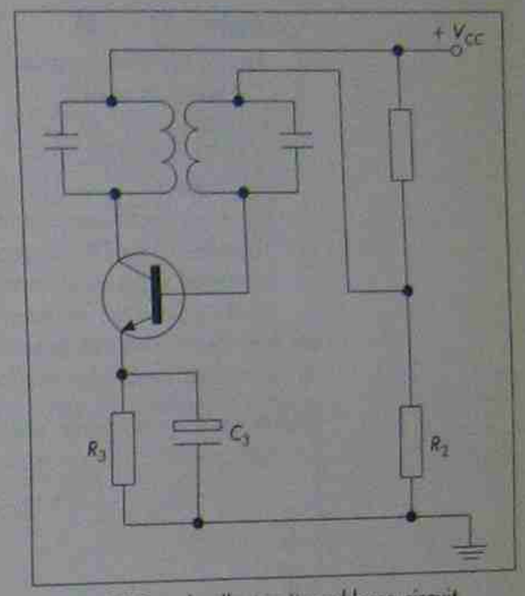


Figure A.1: Tuned collector/tuned base circuit

Multiple-choice answers. 1b 2c 3a 4d 5b 6b 7a 8b 9a 10d

Chapter 13

1. Piezoelectric effect means:
   (a) A potential difference is developed between the opposite plates of the crystal when the crystal is subjected to mechanical pressure.
   (b) A vibration is set up in the crystal when a voltage is applied between the opposite plates.
2. (a) House the crystal in a temperature-controlled oven.
   (b) Slope the angle of cut to give a zero temperature coefficient.
3. The crystal has a low value of resistance and therefore has a high Q.
4. Inductive mode
5. Connected in the series feedback path. At f_r the resistance is very low and V_f is maximum. See Figure 13.4 or 13.5.
6. (a) Crystal used in its inductive mode.
   (b) Refer to section 13.3.
7. Using the crystal at one of its odd harmonic frequencies, such as $3f_r$, $5f_r$ and so on.
8. Q is reduced. This can be minimised by transformer coupling as per Figure 13.7.
9. The Pierce circuit uses the crystal in its inductive mode, coupled with two circuit capacitances. The equivalent circuit is a Colpitts. The crystal is connected between drain and gate of a FET or collector and base of a transistor. The Miller circuit has the crystal between base and earth. It is a tuned base (crystal), tuned collector circuit. The two 'tuned' circuits are inductive, and the circuit interelement capacity, C_{gd} or C_{cb}, makes up a Hartley equivalent circuit. The tuned circuit is tuned above the resonant frequency of the crystal.
10. By placing a capacitor in series or parallel with the crystal.

Multiple-choice answers. 1a 2b 3c 4d 5b 6c 7a 8d 9d 10b

Chapter 14

1. Yes, by applying the pulse output to a tuned circuit.
2. The propagation delay
3. $f = 1/2nT$
   $= 2 \times 3 \times 12 \times 10^{-9}$
   $= 13.9$ MHz
4. By increasing the propagation time (see Figure A.2).
5. The value of the DC current path resistor in parallel with the crystal is kept high so that the crystal's Q is not reduced.
6. (a) To prevent the oscillator from being loaded by the external circuit.
   (b) Yes, they increase propagation delay time and also increase the loop gain.
7. Frequency divider circuit as in Figure 14.6

Multiple-choice answers 1c 2d 3b 4c 5b 6d 7a 8a 9b 10d

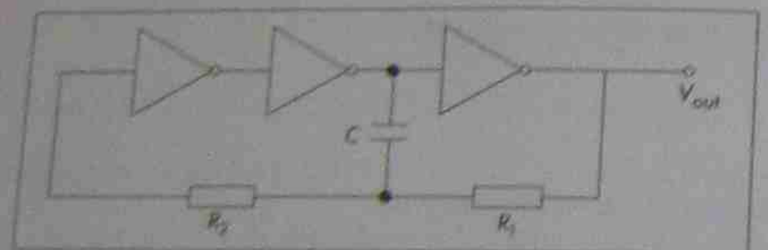


Figure A.2

APPENDIX B

CALCULATION OF RIPPLE AND FORM FACTORS FOR UNFILTERED HALF-WAVE AND FULL-WAVE RECTIFIERS

The output of a power supply contains both AC and DC voltage components. During the design stage of a power supply, we must be aware of the relationship between the AC voltage component and the DC output voltage. Figure B.1 shows the output of a half-wave rectifier circuit.

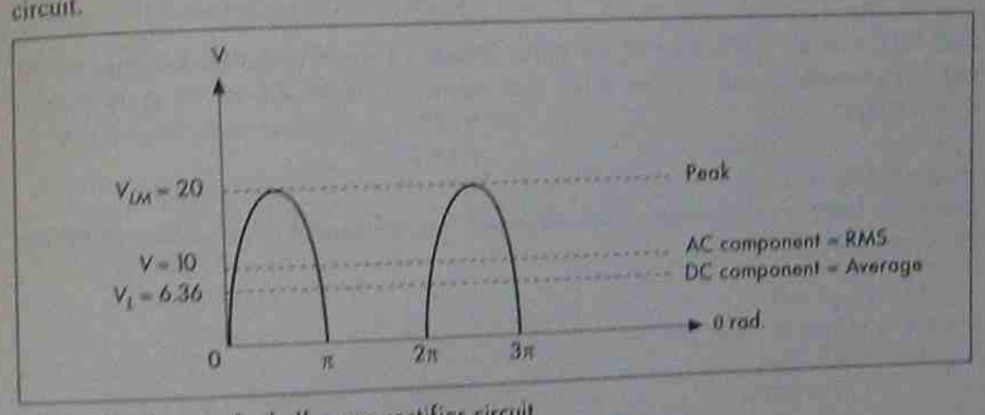


Figure B.1 Output of a half-wave rectifier circuit

The peak voltage of the waveform in Figure B.1 is 20 V, therefore:

DC component = 0.318×20
= 6.36 V

The RMS value of the waveform (complete load voltage) is, therefore:

$V_{load} = 0.5 \times 20$
= 10 V

See Appendix C for details on how these values are derived.

This value is the RMS value of the complete waveform, which contains both the AC and DC voltages. Complex mathematics can show that the RMS value of the whole waveform is equal to the square root of the sum of the squares of each RMS component in the waveform:

$$V = \sqrt{V_1^2 + V_2^2}$$

Therefore:

$V_{load} = \sqrt{V_{AC}^2 + V_{DC}^2}$

where V_{AC} = RMS value of AC component

V_{DC} = RMS value of DC component

V_{DC} = 6.36 V and, since the RMS value of a DC component is equal to the DC component itself, then:

$V_{load} = \sqrt{V_{AC}^2 + 6.36^2}$

so, $10 = \sqrt{V_{AC}^2 + 6.36^2}$

Solving the equation:

$V_{AC}^2 = 10^2 - 6.36^2$

$= 100 - 40.45$

$= 59.55$

$V_{AC} = 59.55$

$= 7.71$ V

This 7.71 V is the RMS value of the ripple voltage. Therefore, the load voltage of the unfiltered rectifier output contains a DC component = 6.36 V, and an AC component = 7.71 V (RMS)

Ripple factor (RF)

The ripple factor is an important circuit parameter. It is the ratio of ripple voltage (RMS) to DC component:

$RF = V_{AC}/V_L(DC)$

For the waveform in Figure B.I:

$RF = 7.71/6.36$

$= 1.21$ or 121%

Form factor (F)

A further measurement of power supply performance is the ratio of RMS load voltage to the DC component This is called the form factor. For the waveform in Figure B.1, the RMS load voltage (complete AC and DC components) is equal to 10 V, and the DC component is equal to 6.36 V.

Therefore:

$F = V/V_L$ or $V_L(AC + DC)/V_L(DC)$

$= 10/6.36$

$= 1.57$

The same theory applies in determining RF and F for a full-wave rectifier circuit (see Figure B.2).

In an ideal power supply the AC component would be zero, resulting in a form factor (F) of 1. However, in practice this does not happen, and an efficient filtering circuit simply reduces the amplitude of the ripple in the DC output.

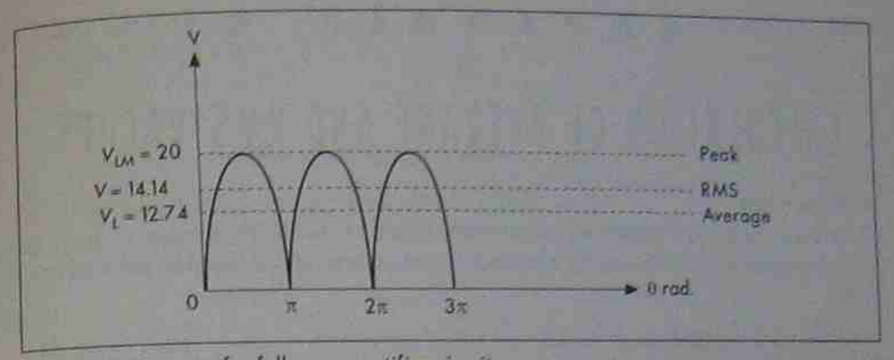


Figure B.2: Output of a full-wave rectifier circuit

In order to reinforce the difference between a half and a full-wave rectifier, the same peak voltage will be used to calculate RF and F:

DC component = 0.637 × 20

= 12.74 V

RMS value of the waveform = 0.707 × 20

= 14.14 V

$V_{load} = \sqrt{V_{AC}^2 - V_{DC}^2}$

$14.14 = \sqrt{V_{AC}^2 - 12.74^2}$

$(V_{AC})^2 = 14.14^2 - 12.74^2$

$= 199.94 - 162.3$

$= 37.63$

$V_{AC} = \sqrt{37.63}$

$= 6.13$ V

The ripple factor is:

$RF = V_{AC}/V_L(DC)$

$= 6.13/12.74$

$= 0.48$ or 48%

The form factor is:

$F = V/V_L$

$= 14.14/12.74$

$= 1.1$

The ripple factor has been reduced considerably and this leads to better filtering, while the form factor now approaches the ideal.

APPENDIX C

CALCULATION OF AVERAGE AND RMS VALUES

Average value

The average value of a repetitive waveform is also referred to as the DC component. It can be found by algebraically summing the area under the waveform, and dividing by the total time period:

$$F(av) = 1/T \int_0^T f(t)\,dt$$

$F(av)$ = average value of $f(t)$

Example 1: Find the average value of the waveforms in Figures C.1(a) and (b).

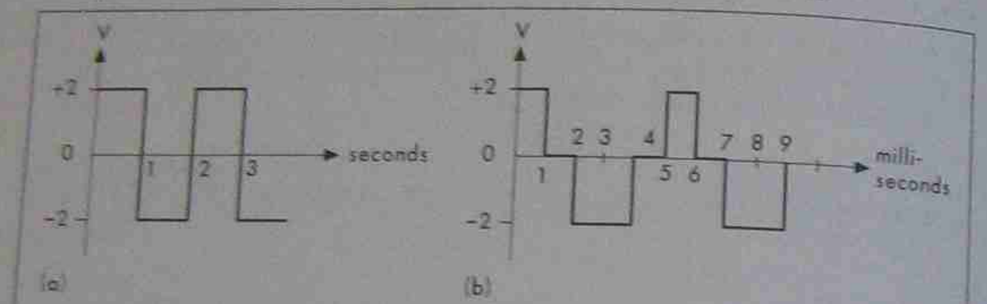


Figure C.1: Example waveforms

(a) Total period for one repetition (T) is 2 seconds. Finding the values for individual time segments, we get:

from 0 to 1 = 1 × 2 V
= 2 V
from 1 to 2 = 1 × –2 V
= –2 V

Therefore, the DC component (average value) is found to be:

total period/period = 2 – 2/2
= 0

(b) Total period for one repetition is 5 milliseconds. The values for individual time segments are:

from 0 to 1 = $1 \times 10^{-3} \times 2$ V
= 2×10^{-3} V

from 1 to 2 = 0
from 2 to 4 = $2 \times 10^{-3} \times -2$ V
= -4×10^{-3} V
from 4 to 5 = 0

Therefore, the DC component (average value) is:

DC component (average value) = total period / period
= $[(2 \times 10^{-3}) - (4 \times 10^{-3})$ V$]/5 \times 10^{-3}$
= – 0.4 V
= – 400 mV

With power supplies, we are chiefly concerned with sinusoidal waveforms. A sine wave is symmetrical above and below the zero axis; therefore, the positive-going region cancels the negative-going region (as in Figure C.1(a)) and the DC component equals zero.

Our concern with the DC component is to determine the average voltage output level from a full or a half-wave rectifier. Figure C.2 shows the two output voltage waveforms.

Example 2: Find the average value of the waveforms in Figure C.2(a) and (b).

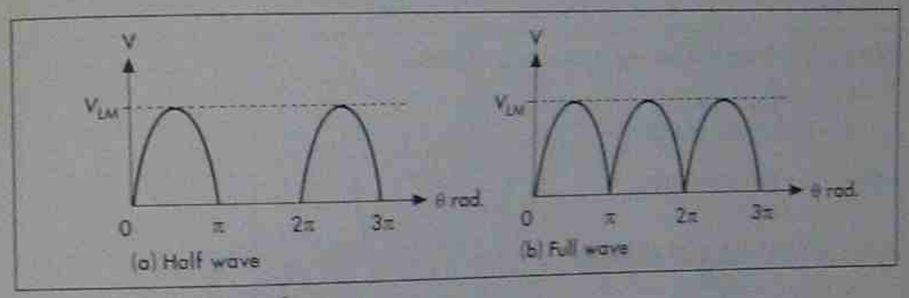


Figure C.2: Example waveforms

(a) Total period for one repetition is 2π radians. From 0 to π = 2 times the maximum load voltage ($2V_{LM}$) (from integration). From π to 2π = 0. Therefore, the DC component (average value) is:

$$2L_m/2\pi = 0.318\,V_{Lm}$$

Hence, the average value of a half-wave rectified sine wave is equal to 0.318 of the maximum value.

(b) Total period for one repetition is 2π radians. From 0 to $\pi = 2V_{Lm}$ (from integration). From π to $2\pi = 2V_{Lm}$. Therefore, the DC component (average value) is:

$$4V_{Lm}/2\pi = 0.6366$$
= 0.637 (to 3 decimal places)

Hence, the average value of a full-wave rectified sine-wave is equal to 0.637 of the maximum value.

RMS value

The RMS value of a repetitive waveform is also referred to as its effective or working value. It can be found in the following manner:

1. square the waveform;
2. find the average value of the squared waveform; and
3. find the square root of the average value of the squared waveform. Parts (a) and (b) of Figure C.3 show the output waveforms of a full and half-wave rectifier.

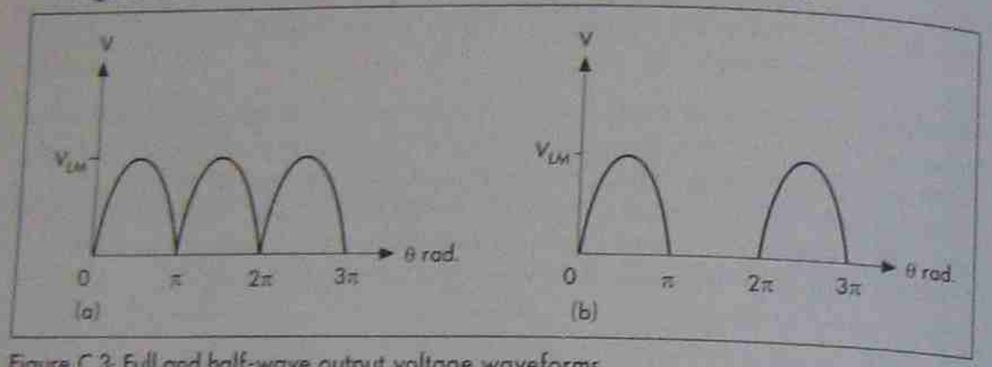


Figure C.3: Full and half-wave output voltage waveforms

$F = \sqrt{[1/T]}\int_0^T f^2(t)\,dt$
(F = RMS value of $f(t)$)

Equation for the waveform:
$v = V_{max} \sin\theta$

Equation for the squared waveform:
$v^2 = V_{max}^2 \times \sin\theta^2$

Solution: (a) The waveform in Figure C.3(a) is the output of a full-wave rectifier. The first step is to square this waveform, as shown in Figure C.4.

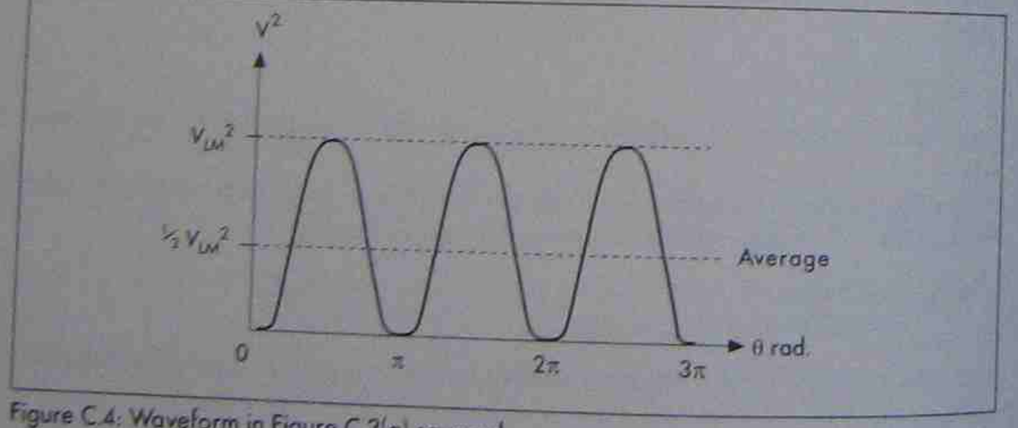


Figure C.4: Waveform in Figure C.3(a) squared

It has the same RMS as a full sine wave. The waveform is symmetrical, therefore the average is half way between 0 and V_{Lm}^2, or $V_{Lm}^2/2$. Then we take the square root of the average value:

$\sqrt{V_{Lm}^2/2} = V_{Lm}/\sqrt{2}$
$= 0.707 \times V_{max}$

This is true for both a full sinusoidal waveform, and a full-wave rectified sinusoidal waveform.

(b) The waveform in Figure C.3(b) is the output of a half-wave rectifier. Again, the first step is to square this waveform. Due to the absence of each second half cycle from the waveform, the average must be half the average of a squared full wave (see Figure C.5). Therefore, the average value = $1/4\ V_{Lm}^2$.
The square root of the average value:

$\sqrt{1/4\ V_{Lm}^2} = 1/2\ V_{Lm}$

Therefore, the RMS value of a half-wave rectified sinusoidal waveform is $0.5 \times V_{max}$.

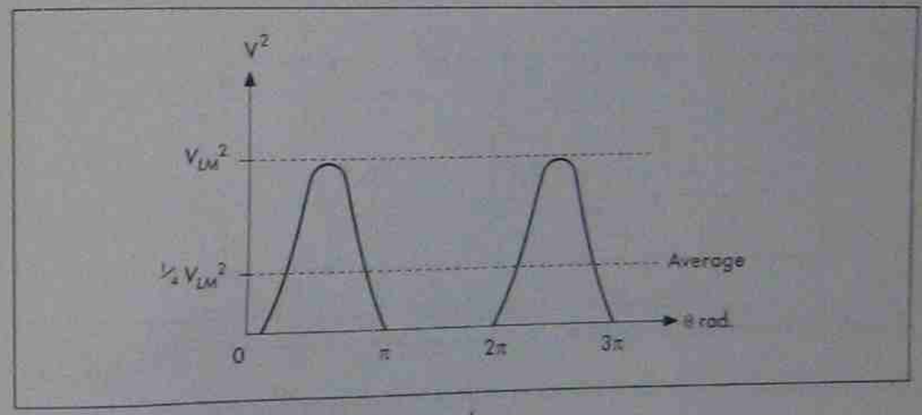


Figure C.5: Waveform in Figure C.3(b) squared

APPENDIX D

LOSSES IN A CR NETWORK

Chapter 11 states the various levels of gain required with different numbers of CR networks. The most common phase-shift oscillator is the three-section with $R_1 = R_2 = R_3$ and $C_1 = C_2 = C_3$. This appendix will prove two factors:

1. $f_r = 1/2\pi CR\sqrt{6}$
2. The gain must be 29 or greater.

Figure D.1 shows a three-section CR network to be used with an amplifier to make a phase-shift oscillator. If $C_1 = C_2 = C_3 = 1$ nF and $R_1 = R_2 = R_3 = 47$ kΩ, then if:

$f_r = 1/2\pi CR\sqrt{6}$
$= 1.4$ kHz

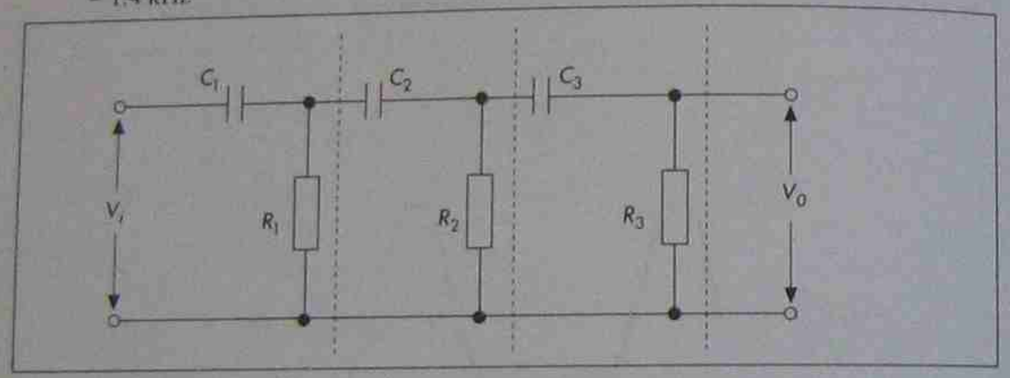


Figure D.1: Three-section CR network

At a frequency of 1.4 kHz the reactance of each capacitor (X_C) will be equal to:

$X_c = 1/2\pi fC$
$= 113\,682\ \Omega$
$= 113.6\ \text{k}\Omega$

Treating the network as an AC circuit the following analysis can be made:

1. X_{C3} is in series with $R_3 = Z_4$—Figure D2.
2. Z_4 is in parallel with $R_2 = Z_3$—Figure D3.
3. Z_3 is in series with $X_{C2} = Z_2$—Figure D4.
4. Z_2 is in parallel with $R_1 = Z_1$—Figure D5.
5. Z_1 and X_{C1} make up a reactive voltage divider—Figure D5.

Using our AC theory we can solve the network as follows:

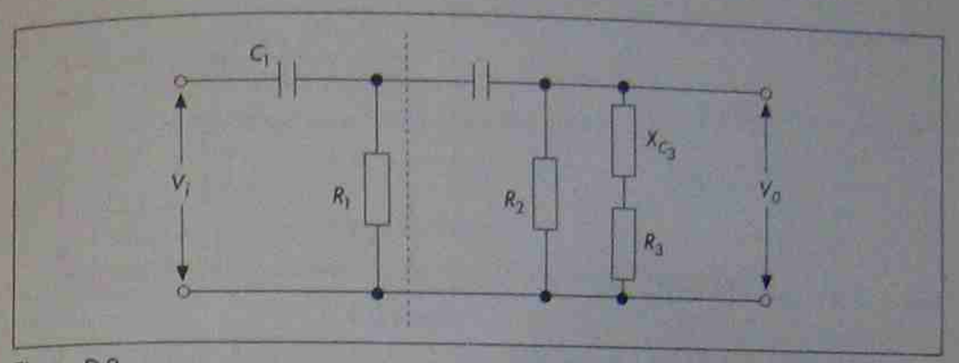


Figure D.2

Step 1: R_3 is in series with $X_{C3} = Z_4$.

$Z_4 = \sqrt{R_3^2 + C_3^2}$
$= \sqrt{47\,000^2 + 113\,682^2}$
$= 123\,015\ \Omega$
$= 123\ \text{k}\Omega$

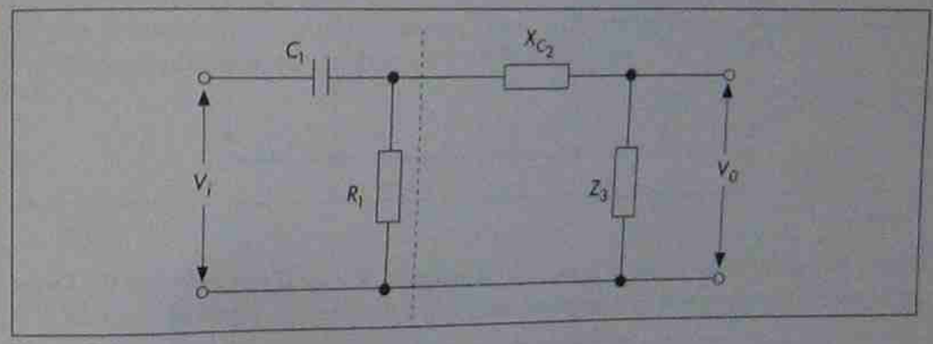


Figure D.3

Step 2: Z_4 is in parallel with $R_2 = Z_3$.

$Z_3 = (Z_4 \times R_2)/Z_4 + R_2$
$= (47\,000 \times 123\,015)/(47\,000 + 123\,015)$

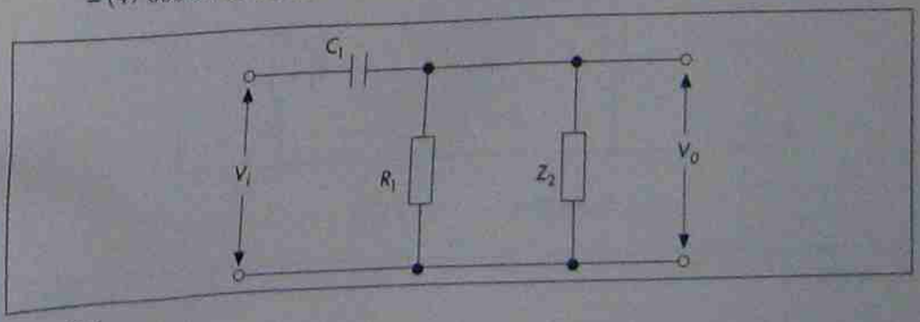


Figure D.4

$= 34007\ \Omega$
$= 34\ k\Omega$

Step 3: Z_3 in series with $X_{C2} = Z_2$.

$Z_2 = \sqrt{Z_3^2 + XC_2^2}$
$= \sqrt{340\,07^2 + 113\,682^2}$
$= 118\,659\ \Omega$
$= 118.7\ k\Omega$

Step 4: Z_2 is in parallel with $R_1 = Z_1$.

$Z_1 = (Z_2 \times R_1)/(Z_2 + R_1)$
$= 118\,659 \times 47\,000/118\,659 + 47\,000$
$= 33\,665\ \Omega$
$= 33.7\ k\Omega$

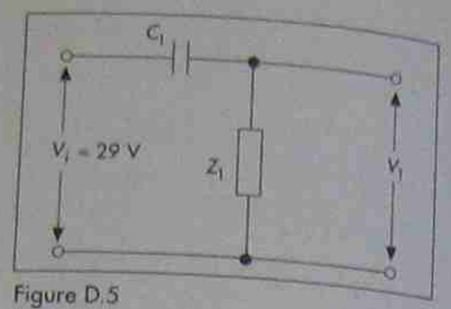


Figure D.5

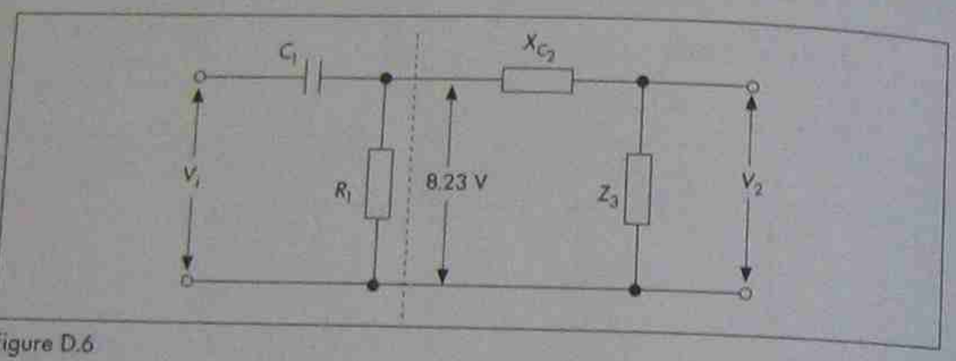


Figure D.6

Step 5: This gives us the resultant reactive voltage divider X_{C1}: Z_1. If $V_i = 29$ V, then the losses in the circuit should reduce V_o to approximately 1 V if the figure for a gain of 29 is true.

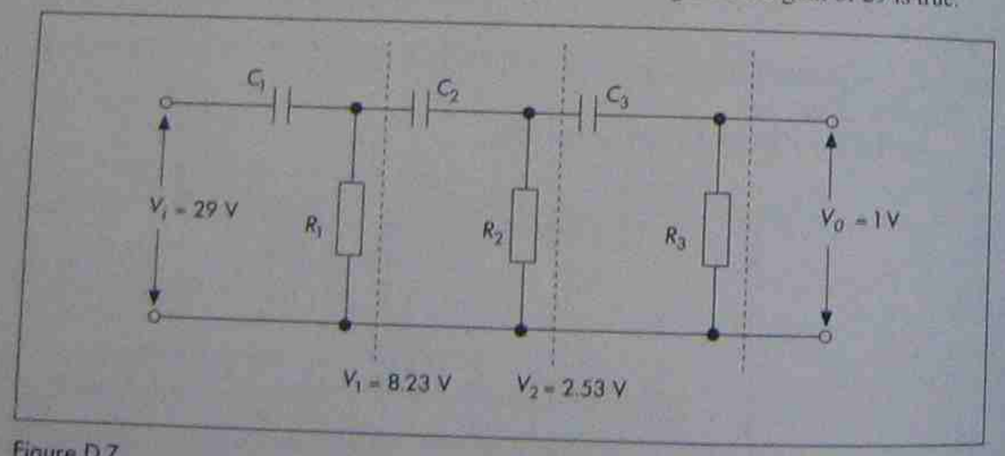


Figure D.7

$V_1 = [Z_1/\sqrt{Z_1^2 + X_{C1}^2}] \times V_i$
$= 33\,665/(\sqrt{33\,665^2 + 113\,682^2}) \times 29$
$= 8.23$ V (this is the input to the next CR section)

Step 6:

$V_2 = [Z_3/\sqrt{Z_3^2 + X_{C2}^2}] \times V_1$
$= 34\,007/(\sqrt{34\,007^2 + 113\,682^2}) \times 8.23$
$= 2.53$ V (this in the input to the next CR section). (Refer to Figure D.6.)

Step 7:

$V_o = R_3/(\sqrt{R_3^2 + X_{C3}^2}) \times V_2$
$= 47\,000/(\sqrt{47\,000^2 + 113\,682^2}) \times 2.53$
$= 0.97$ V
$= 1.0$ V (to 2 decimal places). (Refer to Figure D.7.)

Conclusion

1. The 29 V input has been reduced to a level of 1 V by the losses in the CR networks. Therefore, the voltage gain required AV is 29.
2. The formula derived by mathematical analysis ($f_r = 1/2\pi CR\sqrt{6}$) was used in the proof of the stage gain requirement, therefore it is also true.

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- a chapter summary and list of key terms at the end of each chapter
- essay-review and multiple-choice questions at the end of each chapter
- a minimal use of mathematics throughout the text
- many practical and tested circuits
- an extensive use of circuit diagrams.

This book reflects the learning outcomes of the TAFE National Engineering Curriculum modules NF04, [illegible] and 184.

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