$e^{\sin x} dx$

$\cos x \, dx$

$\cos x \, dx$

$\sin x = e^{\sin x} + c$

$du = \ln u + c$

pb $\int \frac{4x\,dx}{2x^2+3}$

$d(2x^2+3) = d2x^2 + d3 = 2\,dx^2 + 0 = 2 \times 2\, x^{2-1} dx$

$= \underline{\underline{4x\,dx}}$

$\int \frac{d(2x^2+3)}{(2x^2+3)} = \ln(2x^2+3) + c$

ph INTEGRATE THE FOLLOWING LOGARITHMIC FUNCTIONS

(i) $\dfrac{x}{3x^2+2}$ (ii) $\dfrac{e^{ax}}{e^{ax}+4}$ (iii) $\dfrac{\cos x}{\sin x+4}$ (iv) $\dfrac{\sec^2 x+1}{\tan x+x}$

(i) $$\int \frac{x}{3x^2+2}\,dx$$

$$d(3x^2+2) = d\,3x^2 + d2 = 3dx^2 + 0 = 6x\,dx$$

$$\therefore x\,dx = \frac{d(3x^2+2)}{6}$$

$$\int \frac{\frac{d(3x^2+2)}{6}}{(3x^2+2)} = \frac{1}{6}\int \frac{d(3x^2+2)}{(3x^2+2)}$$

$$= \frac{1}{6}\ln(3x^2+2)+c$$

(ii) $$\int \frac{e^{ax}}{e^{ax}+4}\,dx$$

$$d(e^{ax}+4) = de^{ax} + d4 = e^{ax}\,dax$$
$$= ae^{ax}\,dx$$

$$\therefore e^{ax}\,dx = \frac{d(e^{ax}+4)}{a}$$

$$\int \frac{\frac{d(e^{ax}+4)}{a}}{e^{ax}+4} = \frac{1}{a}\int \frac{d(e^{ax}+4)}{e^{ax}+4}$$

$$= \frac{1}{a}\ln(e^{ax}+4)+c$$

(iii)

d(

OF VARIABLE

x

+1

+1

x+1)

+d1

dx

(i) $dx = u\,du$

(ii) $2x = u^2 - 1$

$$x = \frac{u^2-1}{2}$$

(iii) $\sqrt{2x+1} = u$

$$\int \frac{u^2-1}{2} \times u \times u\,du$$

$$\int \frac{(u^2-1)\,u^2\,du}{2}$$

$$\frac{1}{2}\int (u^4-u^2)\,du$$

$$\frac{1}{2}\left[\int u^4\,du - \int u^2\,du\right]$$

$$\frac{1}{2}\left[\frac{u^{4+1}}{4+1} - \frac{u^{2+1}}{2+1}\right] + c$$

$$\frac{1}{2}\left[\frac{u^5}{5} - \frac{u^3}{3}\right] + c$$

$$\frac{1}{2}\left[\frac{(\sqrt{2x+1})^5}{5} - \frac{(\sqrt{2x+1})^3}{3}\right] + c$$

$$\frac{1}{2}\left[\frac{(2x+1)^{5/2}}{5} - \frac{(2x+1)^{3/2}}{3}\right] + c \quad //$$

$$\int \frac{x}{\sqrt{5-x}}\,dx$$

$$u = \sqrt{5-x}$$

$$u^2 = 5-x$$

$$du^2 = d(5-x)$$

$$2u\,du = -dx$$

$$dx = -2u\,du \quad — 1$$

$$u^2 = 5-x$$

$$x = 5-u^2 \quad — 2$$

$$\int \frac{5-u^2}{u}\,(-2u\,du)$$

$$-\int \frac{(5-u^2)}{\cancel{u}}\,2\cancel{u}\,du$$

$$-\left[\int (10-2u^2)\,du\right]$$

$$-\left[\int 10\,du - \int 2u^2\,du\right]$$

$$-\left[10u - 2\frac{u^3}{3}\right] + c$$

$$-\left[10u - \frac{2}{3}u^3\right] + c$$

$-[\,10 \times \sqrt{5}$

$-[\,10\ (5$

Exerc

$$\int \frac{5-u^2}{u}\,(-2u\,du)$$

$$-\int \frac{(5-u^2)}{u}\,2u\,du$$

$$-\left[\int (10-2u^2)\,du\right]$$

$$-\left[\int 10\,du - \int 2u^2\,du\right]$$

$$-\left[10u - 2\,\frac{u^3}{3}\right]+c$$

$$-\left[10u - \frac{2}{3}u^3\right]+c$$

$$-\left[10 \times \sqrt{5-x} - \frac{2}{3}\left(\sqrt{5-x}\right)^3\right]+c$$

$$-\left[10\,(5-x)^{1/2} - \frac{2}{3}\,(5-x)^{3/2}\right]+c$$

EXERCISE INTEGRATE

$$\int x\sqrt{5x+4}\,dx$$

$$\int \frac{x}{\sqrt{3x+4}}\,dx$$

REVISION (1)

(1) DIFFERENTIATE THE FOLLOWINGS

(a) $y = \dfrac{x^2 + 3x + 2}{x^2 - 3x + 2}$

(b) $y = \log_e (x^3 - 1)$

(c) $y = x^{\sin x}$

(d) $y = \operatorname{cosec}^4 (x^3 + 1)$

(e) $y = \tan \sin^3 \theta$

(2) THE CURRENT GROWTH IN A RESISTIVE INDUCTIVE CIRCUIT IS $i = \dfrac{E}{R}\left(1 - e^{-Rt/L}\right)$

FIND RATE OF CHANGE OF CURRENT WITH RESPECT TO TIME

(3) $V_L = L \dfrac{di}{dt}$

IF $i = 10 \sin (314t + 70)$

FIND V_L

(4) DIFFERENTIATE

(a) $y = 2x^{5/2} + x^{-1/2}$

(b) $y = e^{-3ax}$

(c) $y = e^{x^3 + x^2}$

(a)

$\dfrac{dy}{dx}$

x

x

IN A RESISTIVE

$= \frac{E}{R}\left(1 - e^{-Rt/L}\right)$

OF CURRENT

$+ 7i)$

$- \frac{1}{2}$

$t\ x$

(a) $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(x^2-3x+2)\frac{d}{dx}(x^2+3x+2) - (x^2+3x+2)\frac{d}{dx}(x^2-3x+2)}{\left[(x^2-3x+2)\right]^2}$$

$$= \frac{(x^2-3x+2)(2x+3) - (x^2+3x+2)\left[2x-3\right]}{(x^2-3x+2)^2}$$

$x-2$

$x-1$

$$= \frac{(x-2)(x-1)(2x+3) - (x+2)(x+1)(2x-3)}{\left[(x-2)(x-1)\right]^2}$$

(b) $\frac{d}{dx}\log_e f(x) = \frac{f(x)'}{f(x)}$

$y = \operatorname{cosec}^4 u$

$= 4 \operatorname{cosec}$

$\frac{d}{dx}\log_e(x^3-1)$

(c) $y = x^{si}$

$\frac{dy}{dx} = \frac{d}{dx}$

(d) $y = c$

$\frac{dy}{dx} =$

$\frac{dv}{dx}$

$- (x^2+3x+2)\frac{d}{dx}(x^2-3x+2)$

$(x^2+3x+2)\,[2x-3]$

$+3) - (x+2)(x+1)(2x-3)$

$(x-1)]^2$

$f(x)'$

$f(x)$

$y = \operatorname{cosec}^4 u$

$= 4\operatorname{cosec}$

$$\frac{d}{dx}\log_e(x^3-1) = \frac{\frac{d}{dx}(x^3-1)}{x^3-1} = \frac{3x^2}{x^3-1}$$

(c) $y = x^{\sin x}$

$$\frac{dy}{dx} = \frac{d}{dx}x^{\sin x} = \sin x\; x^{\sin x - 1}\frac{dx}{dx}$$

$$= \sin x \times x^{\sin x - 1}$$

(d) $y = \operatorname{cosec}^4(x^3+1)$

$$\frac{dy}{dx} = \frac{d}{dx}\operatorname{cosec}^4(x^3+1)$$

$$= 4\operatorname{cosec}^3(x^3+1)\frac{d}{dx}\operatorname{cosec}(x^3+1)$$

$$= 4\operatorname{cosec}^3(x^3+1) \times [-\operatorname{cosec}(x^3+1)\cot(x^3+1)\frac{d}{dx}(x^3$$

$$= 4\operatorname{cosec}^3(x^3+1)\operatorname{cosec}(x^3+1)\cot((x^3+1)) \times 3x$$

(e)

$y = t$

$\frac{dy}{d\theta} = \frac{d}{d}$

$= \,$

$= \,$

$= \,$

$= \,$

(a) $y = \tan \sin^3\theta$

$$\frac{dy}{d\theta} = \frac{d}{d\theta} \tan \sin^3\theta$$

$$= \sec \sin^3\theta \; \frac{d}{d\theta} \sin^3\theta$$

$$= \sec \sin^3\theta \times 3 \sin^2\theta \times \frac{d}{d\theta} \sin\theta$$

$$= 3 \sin^2\theta \; \sec \sin^3\theta \times \cos\theta$$

$$= 3 \sin^2\theta \cos\theta \; \sec \sin^3\theta$$

(v) $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)\right]$$

$$= \frac{E}{R}\left[\frac{d}{dt}\left(1 - e^{-\frac{Rt}{L}}\right)\right]$$

$$= \frac{E}{R}\left[0 - \frac{d}{dt} e^{-\frac{Rt}{L}}\right]$$

$$= -\frac{E}{R} \times e^{-\frac{Rt}{L}} \frac{d}{dt}\left(-\frac{Rt}{L}\right)$$

$$= -\frac{E}{R} \times e^{-\frac{Rt}{L}} \times -\frac{R}{L}$$

$$= \frac{E}{L} e^{-\frac{Rt}{L}}$$

(3) V

4 (i)

$\frac{d}{d}$

$-12 \operatorname{cosec}^4(x^3+1) \cot(x^3+1)\, x^2$

$^3+1)$

$) \cot(x^3+1) \frac{d}{dx}(x^3+1)$

$+1) \cot(x^3+1) \times 3x^2$

(3) $V_L = L \frac{di}{dt}$

$= L \frac{d}{dt} \left(10 \sin(314t + 70) \right]$

$= L \frac{d}{dt} 10 \sin(314t + 70)$

$= L \times 10 \cos(314t + 70) \frac{d}{dt}(314t + 70)$

$= 10L \cos(314t + 70) \times 314$

$= 3140L \cos(314t + 70)$ //

4 (i) $y = 2x^{5/2} + x^{-1/2}$

$\frac{dy}{dx} = 2 \times \frac{5}{2} x^{5/2 - 1} + (-1/2) x^{-1/2 - 1}$

$= 5 x^{1/2} - \frac{1}{2} x^{-3/2}$

(ii) $y = e^{-3ax}$

$\frac{dy}{dx} = \frac{d}{dx} e^{-3ax} = -3$

$= -3$

$= 9$

(iii) $y = e^{x^3 + x^2}$

$\frac{dy}{dx} = \frac{d}{dx} e^{x^3 + x^2} =$

$=$

$=$

$\cdot 70)\,]$

$0)$

$\frac{d}{dt}(314t+70)$

314

$70)$

$(-\frac{1}{2})\,x^{-\frac{1}{2}-1}$

$\frac{3}{2}$

(ii) $y = e^{-3ax}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{-3ax} = -3a\, e^{-3ax-1} \frac{d}{dx}(-3ax)$$

$$= -3a\, e^{-3ax-1} \times -3a$$

$$= 9a^2 e^{-3ax-1}$$

(iii) $y = e^{x^3+x^2}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{x^3+x^2} = e^{x^3+x^2}\left[\frac{d}{dx}(x^3+x^2)\right]$$

$$= e^{x^3+x^2}\left[3x^2+2x\right]$$

$$= x\left[3x+2\right] e^{x^3+x^2}$$

$$\int x^n \, dx = x^{n+1}$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \tan x \, dx = \ln \sec x + c$$

$$\int \cot x \, dx = \ln \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$\int e^u \, du = e^u + c$$

$$\int \sin^2 x \, dx = ?$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx$$

$$= \int \frac{dx}{2} - \int \frac{\cos 2x}{2} \, dx$$

$$= \frac{x}{2} - \frac{1}{2} \int \frac{\cos(2x) \, d(2x)}{2}$$

$$= \frac{x}{2} - \frac{1}{4} \sin 2x + c$$

$\sin A \cos B =$

$\int \sin 3x \cos$

$\sin 3x \cos 4x$

$\int \sin 3x$

$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$

$\int \sin 3x \cos 4x \, dx = ?$

$\sin 3x \cos 4x = \frac{1}{2}\left[\sin(3x+4x) + \sin(3x-4x)\right]$

$= \frac{1}{2}\left[\sin 7x + \sin(-x)\right]$

$= \frac{1}{2}\left[\sin 7x - \sin x\right]$

$\int \sin 3x \cos 4x \, dx = \int \frac{1}{2}\left[\sin 7x - \sin x\right] dx$

$= \frac{1}{2}\left[\int \sin 7x \, dx - \int \sin x \, dx\right]$

$= \frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} + \cos x\right]$

$\int \frac{\cos 2x \, d2x}{2}$

$\sin 2x + C$

$= \frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + C$

pb FIND THE AREA UNDER THE CURVE $y = 3\sin x + 10$

BETWEEN π AND 2π

$\int_{\pi}^{2\pi} y \, dx = \int_{\pi}^{2\pi} (3\sin x + 10) \, dx$

$= \int_{\pi}^{2\pi} 3\sin x \, dx + \int_{\pi}^{2\pi} 10 \, dx$

$= 3\left(-\cos x\right)_{\pi}^{2\pi} + 10$

$2\pi - \pi)$

$\times \pi$

3.1416

6

EXERCISE

FIND THE AREA UNDER THE CURVE

$y = 4\sin x + 3$

BETWEEN π AND 2π

$\int e^u du = e^u + c$

INTEGRATE

$5e^{-\frac{1}{4}x}$

$\int 5e^{-\frac{1}{4}x} dx$

$\int \frac{5e^{-\frac{1}{4}x} d(-\frac{1}{4}x)}{-\frac{1}{4}}$

$-\frac{5}{\frac{1}{4}} \int e^{-\frac{1}{4}x} d(-\frac{1}{4}x)$

$-20 e^{-\frac{1}{4}x} + c$

INTEGRATE

① $-3x^2 e^{-x^3}$

② $\cos x \, e^{\sin x}$

① $\int -3x^2 e^{-x^3} dx$

$d(-x^3) = -3x^{3-1} dx$

$= -3x^2 dx$

$\int e^{-x^3} \times -3x^2 dx$

$\int e^{-x^3} d(-x^3)$

$e^{-x^3} + c$

② $\int \cos$

$d\sin x =$

$\int e^{\sin x}$

$\int e^{\sin x}$

$\int \frac{1}{4}$

① $\int -3x^2 e^{-x^3} dx$

$d(-x^3) = -8\, x^{3-1}\, dx$

$= -3x^2 dx$

$\int e^{-x^3} \times -3x^2 dx$

$\int e^{-x^3} d(-x^3)$

$e^{-x^3} + c$

② $\int \cos x \, e^{\sin x} dx$

$d\sin x = \cos x \, dx$

$\int e^{\sin x} \cos x \, dx$

$\int e^{\sin x} d\sin x = e^{\sin x} + c$

$$\boxed{\int \frac{1}{u} du = \ln u + c}$$

ph $\int \frac{4x\,dx}{2x^2+3}$

$d(2x^2+3) = d2x^2 + d3 = 2\,dx^2 + 0 = 2\times 2\, x^{2-1} dx$

$= 4x\,dx$

$\int \frac{d(2x^2+3)}{(2x^2+3)} = \ln(2x^2+3) + c$

$+ d4 = e^{ax} dax$

$= ae^{ax} dx$

$+4)$

$\frac{1}{a} \int \frac{d(e^{ax}+4)}{e^{ax}+4}$

$= \frac{1}{a} \ln(e^{ax}+4) + c$

(iii) $\int \frac{\cos x}{\sin x + 4} dx$

$d(\sin x + 4) = d \sin x + d4$

$= \cos x \, dx$

$\int \frac{d(\sin x + 4)}{(\sin x + 4}$

$= \ln(\sin x + 4) + c$

(iv) $\int \frac{\sec^2 x + 1}{\tan x + x} dx$

$d(\tan x + x) = d\tan x + dx$

$= \sec^2 x \, dx + dx$

$= (\sec^2 x + 1) dx$

$\int \frac{d(\tan x + x)}{\tan x + x} = \ln(\tan x + x) + c$

INTEGRATION BY CHANGE OF VARIABLE

$$\int x\sqrt{2x+1}\,dx$$

SUBSTITUTE $u = \sqrt{2x+1}$

$$u^2 = 2x+1$$

$$du^2 = d(2x+1)$$

$$2u^{2-1}\,du = d2x + d1$$

$$2u\,du = 2dx$$

(i) $dx = u\,du$

(ii) $2x = u^2 - 1$

$$x = \frac{u^2-1}{2}$$

(iii) $\sqrt{2x+1} = u$

$$\int \frac{u^2-1}{2} \times u \times u\,du$$

$$\int \frac{(u^2-1)\,u^2\,du}{2}$$

$$\frac{1}{2}\int (u^4-u^2)\,du$$

$$\frac{1}{2}\left[\int u^4\,du - \int\right.$$

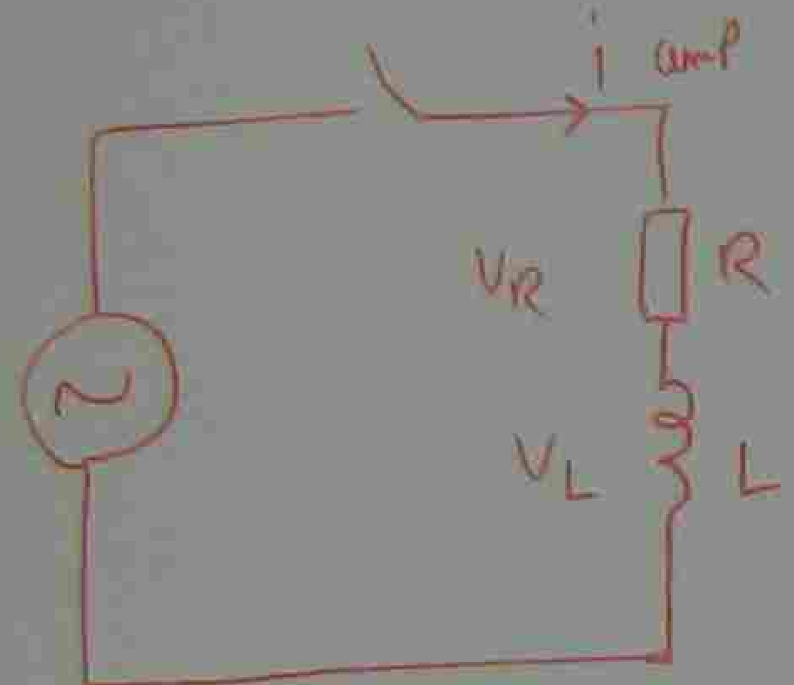
$$\frac{1}{2}\left[\frac{u^{4+1}}{4+1} - \right.$$

$$\frac{1}{2}\left[\frac{u^5}{5} - \right.$$

$$\frac{1}{2}\left[\frac{(\sqrt{2x+1})}{5}\right.$$

$$\frac{1}{2}\left[\frac{(2x+}{}\right.$$

RESPONSE OF RL AND RC CIRCUITS TO AC VOLTAGE



$$e = E_m \,\text{sim}\, (\omega t + \theta)$$

$$V_R = iR$$

$$V_L = L\frac{di}{dt}$$

$$V_R + V_L = e$$

$$iR + L\frac{di}{dt} = E_m \,\text{sim}\,(\omega t + \theta)$$

$$i = i_c + i_p$$

i_c: COMPLEMENTARY COMPONENT

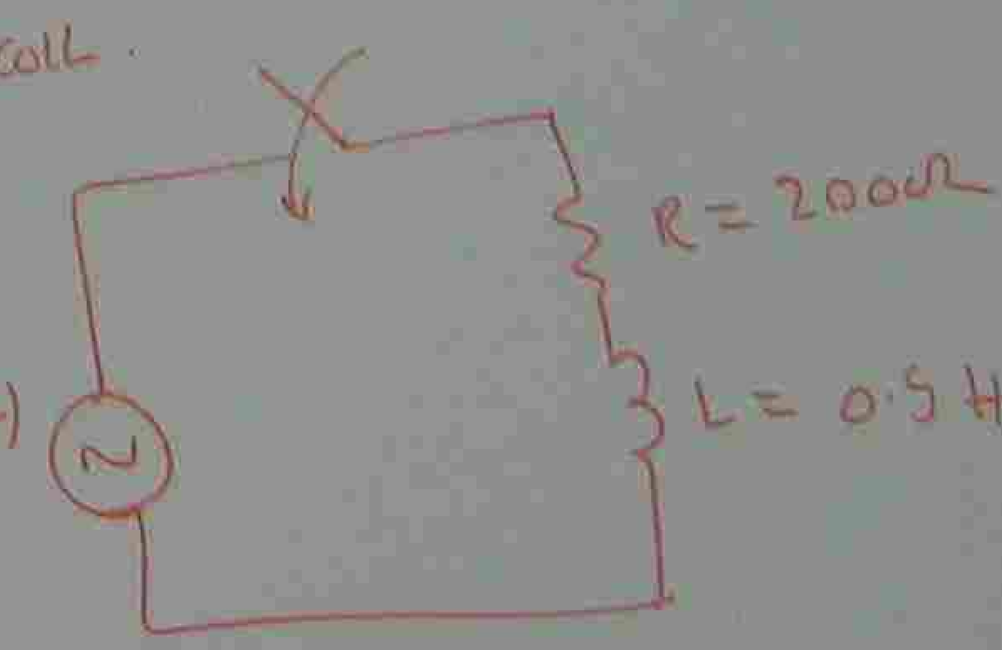
i_p: PARTICULAR COMPONENT

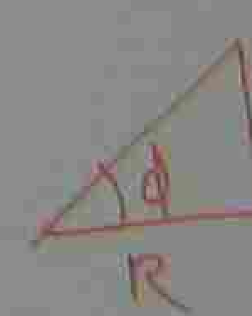
GENERAL SOLUTION

$$i = A e^{-\frac{t}{\tau}} + I_p \,\text{sim}\,(\omega t + \theta - \phi)$$

pb

AN E.M.F $e = 100 \,\text{sim}\,(314t + \theta)$ volT IS APPLIED TO A COIL OF RESISTANCE $200\,\Omega$ AND INDUCTANCE 0.5 HENRY WHEN θ is 30. DETERMINE THE EQUATION OF THE RESULTING CURRENT IN THE COIL.





$$X_L = \omega L$$
$$= 314 \times 0.$$
$$= 157$$

$$Z = R + jX_L = 200 + j157$$

$$\phi = \tan^{-1}\frac{157}{200} = 38.$$

$i =$

$\tau =$

$\theta -$

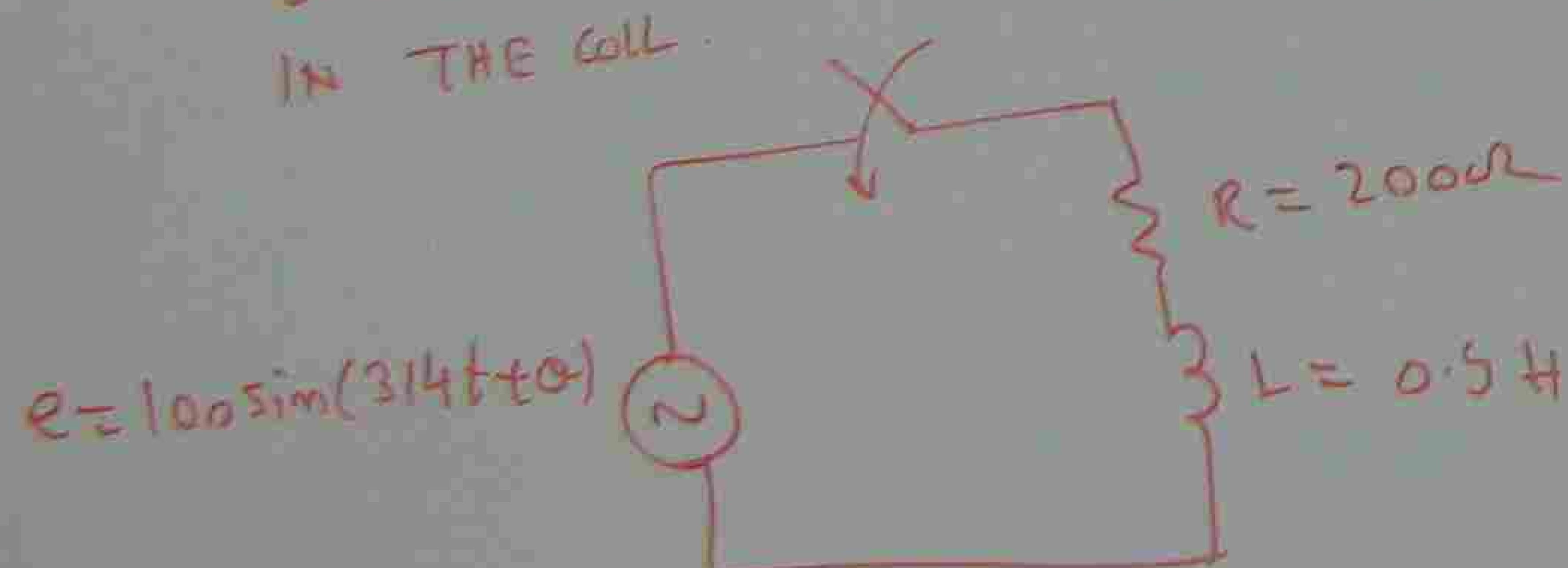
C VOLTAGE

GENERAL SOLUTION

$$i = A e^{-\frac{t}{\tau}} + I_p \sin(\omega t + \theta - \phi)$$

pb

AN E.M.F $e = 100 \sin(314t + \theta)$ VOLT IS APPLIED TO A COIL OF RESISTANCE 200 Ω AND INDUCTANCE 0.5 HENRY WHEN θ IS 30. DETERMINE THE EQUATION OF THE RESULTING CURRENT IN THE COIL



$X_L = \omega L$

$= 314 \times 0.5$

$= 157$

$$Z = R + jX_L = 200 + j157$$

$$\phi = \tan^{-1}\frac{157}{200} = 38.13$$

$$i = Ae^{-t/\tau} + I_p \sin(\omega t + \alpha - \phi)$$

$$\tau = \frac{L}{R} = \frac{0.5}{200} = 0.0025 = 2.5 \text{ ms}$$

$$\theta - \phi = 30 - 38.13 = -8.13$$

$$i = A e^{-\frac{t}{2.5\times10^{-3}}} + I_p \sin(314t - 8.13)$$

$$I_p = \frac{E}{Z} = \frac{100}{\sqrt{R^2 + X_L^2}} = \frac{100}{\sqrt{200^2 + 157^2}} = 0.\ldots$$

$$i = A e^{-400t} + 0.392 \sin(314t - 8.13)$$

$$t = 0 \rightarrow i = 0$$

$+\theta)$

R

NT

IN THIS BUILDING

$)$

ms

$(314t - 8.13)$

$= \frac{100}{\sqrt{200^2+157^2}} = 0.3932$

$2 \sin(314t - 8.13)$

$$0 = A e^{-400 \times 0} + 0.392 \sin(314 \times 0 - 8.13)$$

$$0 = A e^{0} + 0.392 \sin(-8.13)$$

$$0 = A - 0.392 \sin 8.13$$

$$A = 0.0556$$

$$i = 0.0556 e^{-400t} + 0.392 \sin(314t - 8.13)$$

RC circuit

$$e = E_m \sin(\omega t + \alpha)$$

$$v_R = iR$$

$$v_c = \frac{1}{C}\int i\,dt$$

$$v_R + v_c = e$$

$$iR + \frac{1}{C}\int i\,dt = e = E_m \sin(\omega t + \alpha)$$

$$\frac{d}{dt}(iR) + \frac{d}{dt}\left[\frac{1}{C}\int i\,dt\right] = \frac{d}{dt}\left[E_m \sin(\omega t + \alpha)\right]$$

$$R\frac{di}{dt} + \frac{1}{C}\frac{d}{dt}\int i\,dt = E_m \frac{d}{dt}\sin(\omega t + \alpha)$$

$$R\frac{di}{dt} + \frac{1}{C} \times i = E_m \omega \cos(\omega t + \alpha)$$

$E_m \sin(\omega t + \alpha)$

$\frac{d}{dt}\left[E_m \sin(\omega t+\alpha)\right]$

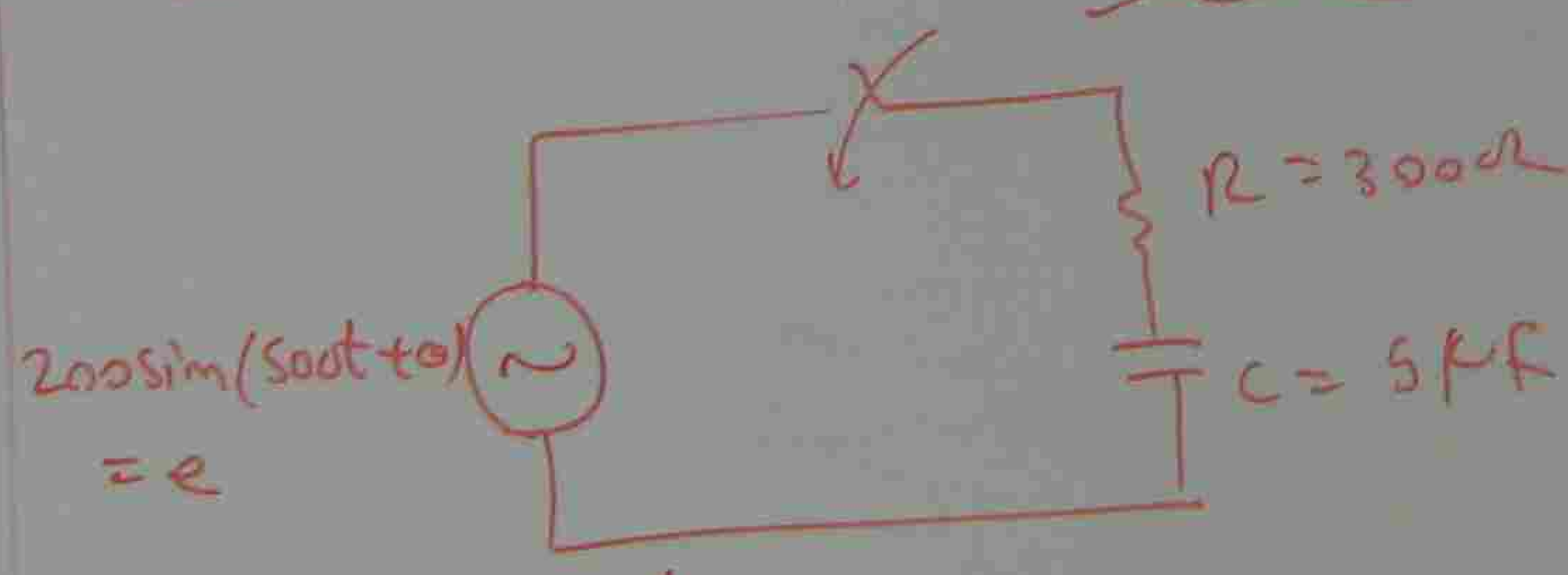
$= E_m \frac{d}{dt} \sin(\omega t + \alpha)$

$\ldots \omega \cos(\omega t + \alpha)$

THE GENERAL SOLUTION

$$i = A e^{-t/\tau} + I_p \sin(\omega t + \theta - \phi)$$

pb AN E.M.F OF $e = 200 \sin(500t + \theta)$ VOLT IS APPLIED TO AN RC CIRCUIT WHERE $R = 300\ \Omega$ AND $C = 5\ \mu F$ WHEN θ IS 60°. DETERMINE THE EQUATION OF THE CIRCUIT IF THE INITIAL CHARGE ON THE CAPACITOR IS 250 MICRO COULOMBS.



$$i = A e^{-t/\tau} + I_p \sin(\omega t + \theta - \phi)$$

$$\tau = R.C = 300 \times 5 \times 10^{-6} = 1.5 \times 10^{-3}$$

$$\theta = 60°$$

$$Z = R - jX_c = 300 - j\frac{1}{2\pi f c}$$

$$= 300 - j\frac{1}{500 \times 5 \times \ldots}$$

$$= 300 - j400\ \Omega$$

$$= \sqrt{300^2 + 400^2} \angle \ldots$$

$$Z = 500 \angle -53.1$$

$$\phi = -53.1$$

$$I_p = \frac{E}{Z} = \frac{200}{500} = \ldots$$

$$i = A e^{-\frac{t}{1.5 \times 10^{-3}}} + 0.4 \sin\ldots$$

$$i = A e^{-666.7t} + 0.4 \sin\ldots$$

$$t = 0 \rightarrow i(0) = A e^{-666\ldots}$$

$$i(0) = A + 0.4\ldots$$

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$$Z = R - jX_C = 300 - j\frac{1}{2\pi f c}$$

$$= 300 - j\frac{1}{500 \times 5\times10^{-6}}$$

$$= 300 - j400 \ \Omega$$

$$= \sqrt{300^2 + 400^2} \angle -\tan^{-1}\frac{400}{300}$$

$$Z = 500 \angle -53.1 \ \Omega$$

$$\phi = -53.1$$

$$I_p = \frac{E}{Z} = \frac{200}{500} = 0.4$$

$$i = A e^{-\frac{t}{1.5\times10^{-3}}} + 0.4 \sin(500t + 60 - (-53.1))$$

$$i = A e^{-666.7t} + 0.4 \sin(500t + 113.1) \ \underline{\underline{\text{Amp}}}$$

$$t=0 \rightarrow i(0) = A e^{-666.7\times0} + 0.4 \sin(500\times0 + 113.1)$$

$$i(0) = A + 0.4 \sin 113.1 \text{ —— } (1)$$

$$q = \int i\,dt = 250\ \mu C$$

INITIAL

$$iR + \frac{1}{C}\int i\,dt = e$$

$$iR + \frac{1}{C}\times q = e$$

$$i\times300 + \frac{250\times10^{-6}}{5\times10^{-6}} = 200\sin 60$$

$$i\times300 + 50 = 200\sin 60$$

$$i(0) = \frac{200\sin 60 - 50}{300} = 0.4107$$

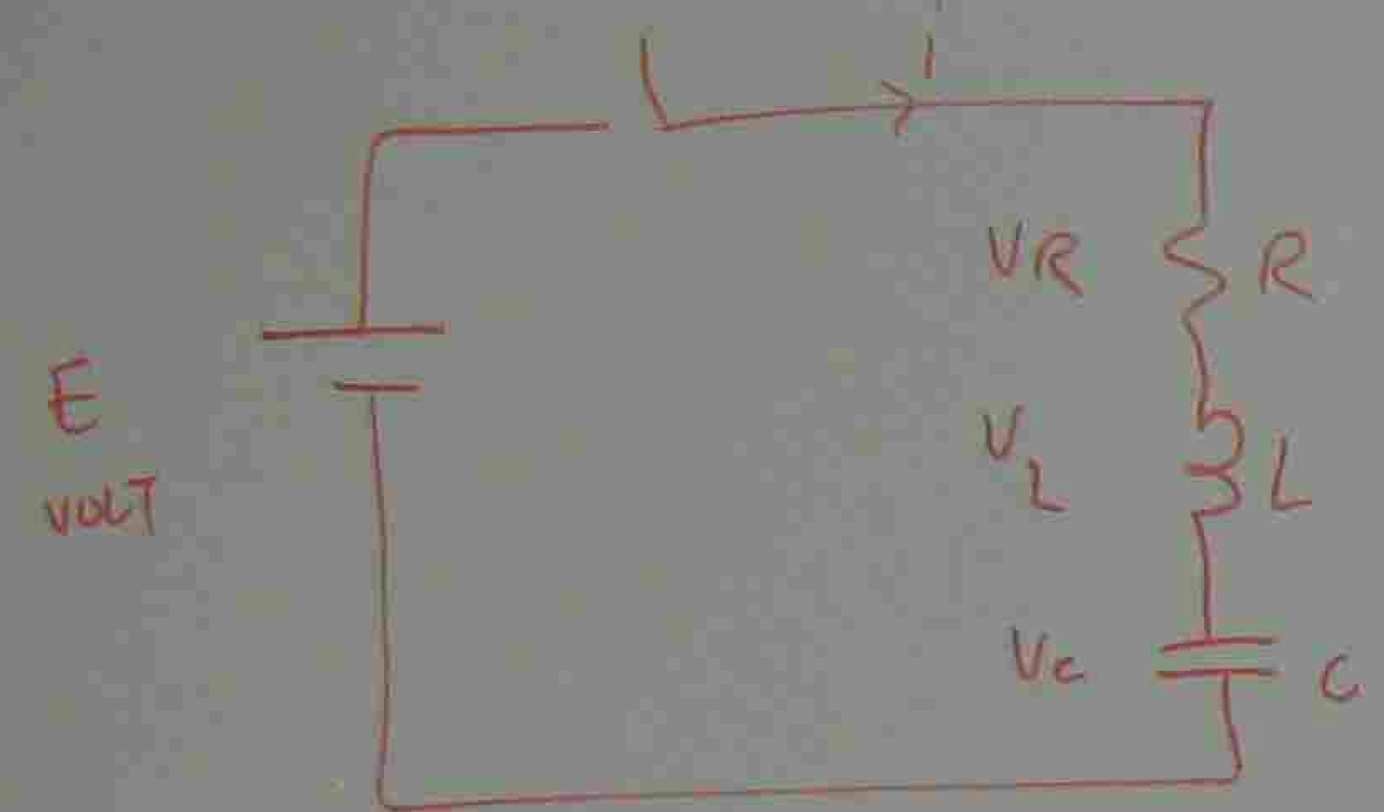
(1)

$$0.4107 = A + 0.4\sin 113.1$$

$$A = 0.0433$$

$$i(t) = 0.0433 e^{-666.7t} + 0.4(500t + 113.1)$$

SOLVING THE SECOND ORDER DIFFERENTIAL EQUATION



$$V_R + V_L + V_c = E$$

$$iR + L\frac{di}{dt} + \frac{1}{c}\int i\,dt = E$$

$$\frac{d}{dt}iR + \frac{d}{dt}L\frac{di}{dt} + \frac{d}{dt}\frac{1}{c}\int i\,dt = \frac{d}{dt}E$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{c}\times i = 0$$

$$\frac{R}{L}\frac{di}{dt} + \frac{d^2i}{dt^2} + \frac{1}{Lc}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{Lc}i = 0$$

COMPARE WITH

$$a\frac{d^2i}{dt^2} + b\frac{di}{dt} + ci = 0$$

$$am^2 + bm + c = 0$$

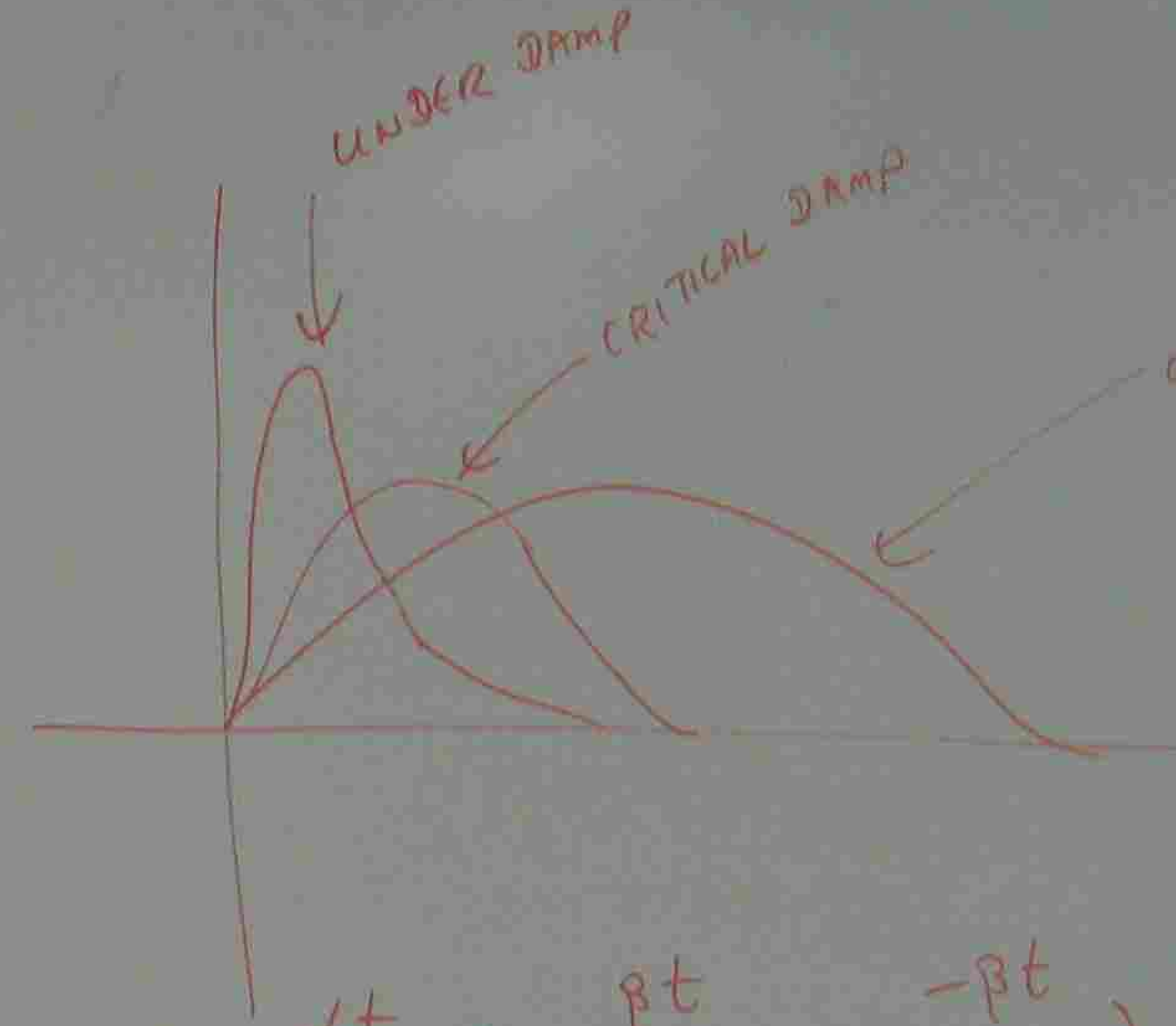
$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

LET

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$



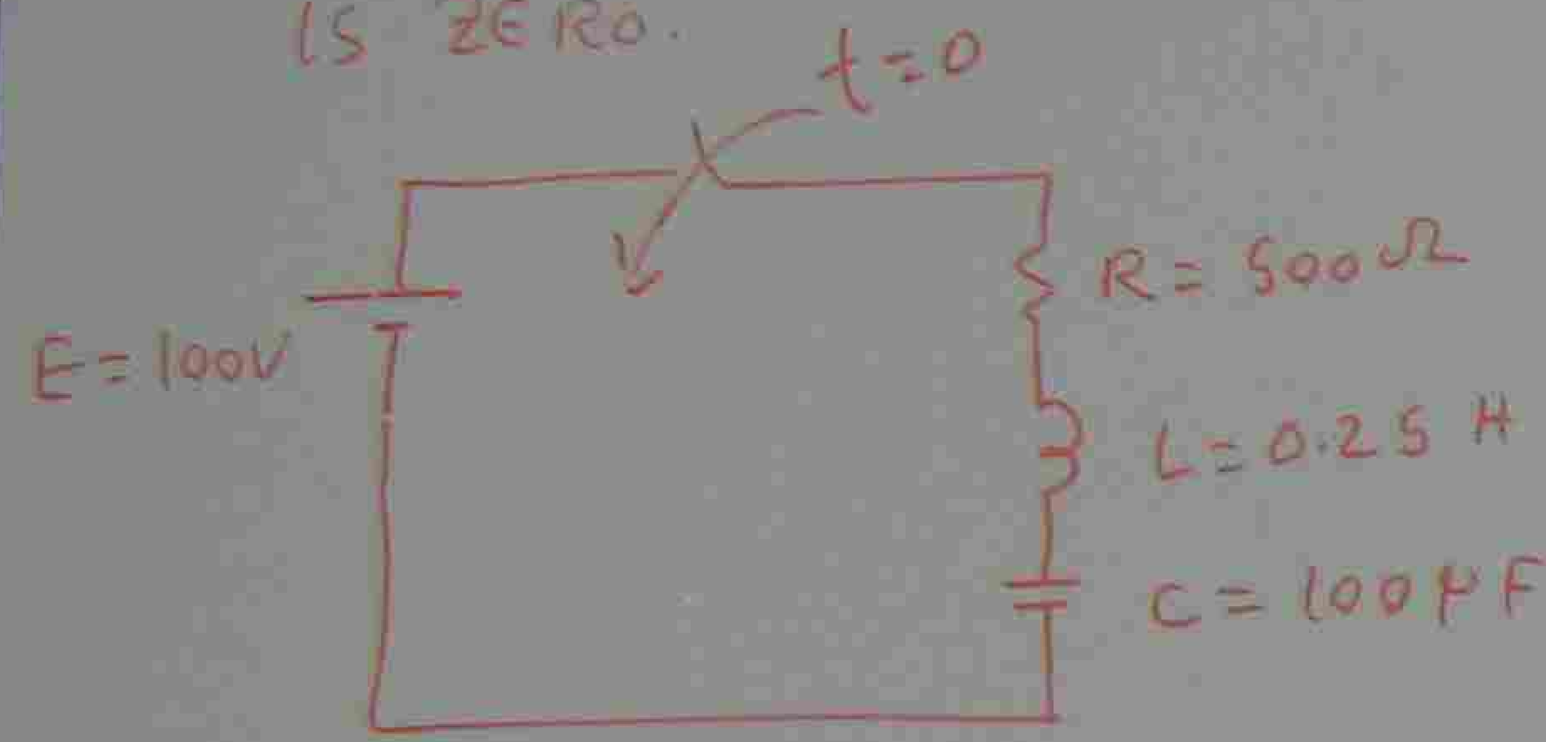
IF $\alpha > \omega_0 \Longrightarrow$ OVER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$

IF $\alpha = \omega_0 \Longrightarrow$ CRITICAL DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 + A_2 t\right)$

IF $\alpha < \omega_0 \Longrightarrow$ UNDER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 \sin \beta t + A_2 \cos \beta t\right)$

pb IN A GIVEN CIRCUIT, E = 100V R = 500Ω, L = 0.25 H, C = 100 μF.

DETERMINE THE EQUATION OF THE CURRENT IF THE INITIAL CHARGE ON CAPACITOR IS ZERO.



$$\alpha = \frac{R}{2L} = \frac{500}{2\times 0.25} = 1000$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25\times 100\times 10^{-6}}} = 200$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$
$$= \sqrt{1000^2 - 200^2}$$
$$= 979.8$$

$\alpha > \omega_0 \Rightarrow$ OVER DAMP

$$i = e^{-\alpha t}\left[A_1 e^{\beta t} + A_2 e^{-\beta t}\right]$$

$$= A_1 e^{(-\alpha+\beta)t} + A_2 e^{(-\alpha-\beta)t}$$

$$= A_1 e^{(-1000+979.8)t} + A_2 e^{(-1000-979.8)t}$$

$$i = A_1 e^{-20.2t} + A_2 e^{-1979.8t}$$

I $t=0 \rightarrow i=0$

$$0 = A_1 e^{-20.2\times 0} + A_2 e^{-1979.8\times 0}$$

$$A_1 + A_2 = 0 \quad \text{—— } (1) \Rightarrow A_1 = -A_2$$

$\ldots - 979.8)t$

$\times 0$

$\Rightarrow A_1 = -A_2$

$$i = A_1 e^{-20.2t} + A_2 e^{-1979.8t}$$

$$\frac{di}{dt} = A_1 \frac{d}{dt} e^{-20.2t} + A_2 \frac{d}{dt} e^{-1979.8t}$$

$$\frac{di}{dt} = -20.2 A_1 e^{-20.2t} + (-1979.8) A_2 e^{-1979.8t}$$

$$\frac{di}{dt} \quad (1)$$

II

$L \frac{di}{dt}$ = INDUCTOR VOLTAGE

$$t=0 \rightarrow L\frac{di}{dt} = E$$

$$\frac{di}{dt} = \frac{E}{L} = \frac{100}{0.25} = 400$$

$$400 = -20.2 A_1 e^{-20.2\times 0} - 1979.8 A_2 e^{-1979.8\times 0}$$

$$-20.2 A_1 - 1979.8 A_2 = 400$$

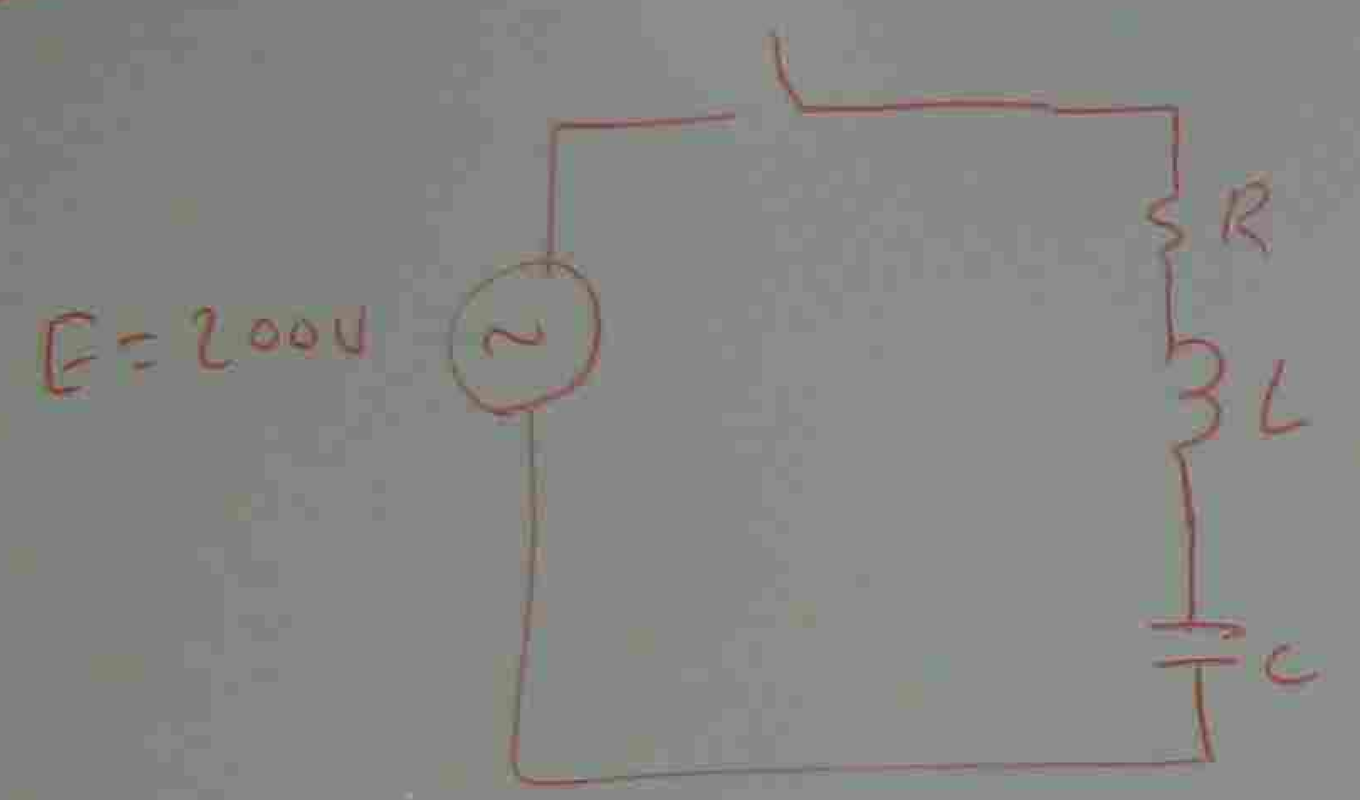
$$-20.2(-A_2) - 1979.8 A_2 = 400$$

$$A_2 = -0.2041$$

$$\therefore A_1 = 0.02041$$

$$i = 0.02041 e^{-20.2t} - 0.204 e^{-1979.8t}$$

EXERCISE



E = 200V, R = 100Ω L = 0.5 HENRY

C = 200μF

FIND THE EQUATION OF THE CURRENT

IF THE INITIAL CHARGE ON CAPACITOR IS

20 MILLICOULOMBS

$\alpha = \frac{R}{2L}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

COMPARE α & ω_0 SELECT THE EQUATION

CONDITION (1) $t = 0 \longrightarrow i = 0$

CONDITION 2

$q = 20\ mC$

$V_c = \frac{q}{C} = \frac{20 \times 10^{-3}}{200 \times 10^{-6}}$

$= 100V$

$V_c = \frac{1}{C}\int i\,dt$

$t = 0 \left. \frac{di}{dt} \right. = \frac{E}{L} = \frac{100V}{0.5}$

$= 200$

IF

IF

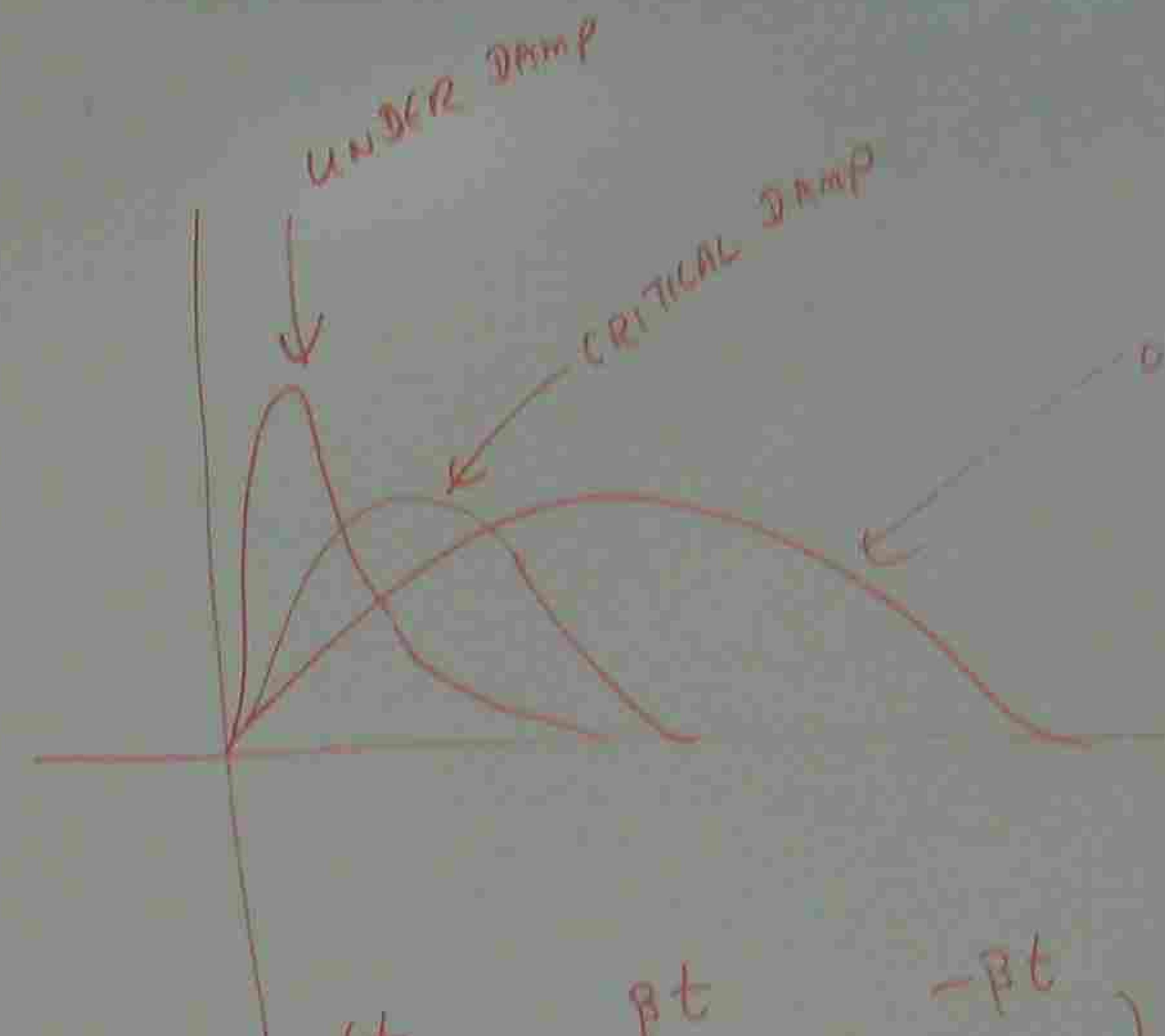
IF

$$m = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

LET

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$



IF $\alpha > \omega_0 \Rightarrow$ OVER DAMP $\longrightarrow i = e^{-\alpha t}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$

IF $\alpha = \omega_0 \Rightarrow$ CRITICAL DAMP $\longrightarrow i = e^{-\alpha t}\left(A_1 + A_2 t\right)$

IF $\alpha < \omega_0 \Rightarrow$ UNDER DAMP $\longrightarrow i = e^{-\alpha t}\left(A_1 \sin\beta t + A_2 \cos\beta t\right)$

-3

0

-6

0

$= \frac{100V}{0.5}$

00

$$\pi \quad - \quad 20 \times -\left[\cos\theta\right]_0^{2\pi} \Big]$$

$$\pi + 20\left[\cos 2\pi - \cos 0\right]$$

$$20\left[1 - 1\right]$$

$$\to 0$$

20 unit

$$dt \quad (\text{OR}) \quad V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v(\theta)^2\, d\theta}$$

$$\int_0^{2\pi} (20 - 20\sin\theta)^2\, d\theta$$

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left(400 - 800\sin\theta + 400\sin^2\theta\right)d\theta}$$

$$\cos 2\theta = 1 - 2\sin^2\theta \longrightarrow \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} 400\, d\theta - \int_0^{2\pi} 800\sin\theta\, d\theta + 400\int_0^{2\pi}\left(\frac{1 - \cos 2\theta}{2}\right)d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[400 \times 2\pi + 800\left(\cos\theta\right)_0^{2\pi} + \frac{400}{2}\left(\theta\right)_0^{2\pi} - \frac{400}{4}\left(\sin 2\theta\right)_0^{2\pi}\right.}$$

$$= \sqrt{\frac{1}{2\pi}\left[\left(800\pi + 800\cancel{(\cos 2\pi - \cos 0)}_{\to 0}\right) + 200 \times 2\pi - \frac{400}{4}\cancel{\left(\sin 4\pi - \sin 0\right)}_{\to 0}\right]}$$

$$= \sqrt{\frac{1}{2\pi} \times \left(800\pi + 400\pi\right)}$$

$$V_{RMS} = \sqrt{600} = 24.49$$

(c)

$$\text{FORM FACTOR} = \frac{V_{RMS}}{V_{AVG}} = \frac{24.49}{20} = 1.225$$

REVISION

1. $$\int_{\pi}^{2\pi} (3\sin x + 10)\,dx$$

$$\int_{\pi}^{2\pi} 3\sin x\,dx + \int_{\pi}^{2\pi} 10\,dx$$

$$3\int_{\pi}^{2\pi} \sin x\,dx + 10\int_{\pi}^{2\pi} dx$$

$$3\times\left[-\cos x\right]_{\pi}^{2\pi} + 10\left[x\right]_{\pi}^{2\pi}$$

$$-3\left[\cos 2\pi - \cos\pi\right] + 10\left[2\pi - \pi\right]$$

$$-3\left[\cos 360 - \cos 180\right] + 10\times\pi$$

$$-3\left[1-(-1)\right] + 10\times 3.1416$$

$$-3\times 2 + 31.416$$

$$25.416$$

② $\int (5x+7)^3\,dx$

$d(5x+7) = d5x + d7$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \swarrow 0$

$= 5\,dx$

$\therefore dx = \frac{d(5x+7)}{5}$

$\int (5x+7)^3 \frac{d(5x+7)}{5}$

$\frac{1}{5}\int (5x+7)^3\,d(5x+7)$

$\frac{1}{5} \times \frac{(5x+7)^{3+1}}{3+1} + c$

$\frac{1}{5}\frac{(5x+7)^4}{4} + c = \frac{1}{20}(5x+7)^4 + c$

③ $\int \sin^2 2x\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 4x = 1 - 2\sin^2 2x$

$2\sin^2 2x = 1 - \cos 4x$

$\sin^2 2x = \frac{1-\cos 4x}{2}$

$\int \frac{1-\cos 4x}{2}\,dx$

$\frac{1}{2}\int [1-\cos 4x]\,dx$

$\frac{1}{2}\left[\int dx - \int \cos 4x\,dx\right]$

$d4x = 4\,dx$

$\therefore dx = \frac{d4x}{4}$

$\frac{1}{2}\left[x - \int \cos 4x \times \frac{d4x}{4}\right]$

$\frac{1}{2}\left[x - \frac{\sin 4x}{4}\right] + c$

$\frac{x}{2} - \frac{\sin 4x}{8} + c$

(4) $\int \sin 3x \cos 4x \, dx$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\sin 3x \cos 4x = \frac{1}{2}\left[\sin(3x+4x) + \sin(3x-4x)\right]$

$= \frac{1}{2}\left[\sin 7x + \sin(-x)\right]$

$= \frac{1}{2}\left[\sin 7x - \sin x\right]$

$\int \frac{1}{2}\left[\sin 7x - \sin x\right] dx$

$\frac{1}{2}\left[\int \sin 7x \, dx - \int \sin x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} - (-\cos x)\right]$

$\frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + C$

$\frac{\cos x}{2} - \frac{\cos 7x}{14} + C$

(5) $\int x \sqrt{2x+1} \, dx$

LET

$\sqrt{2x+1} = u$

$2x+1 = u^2$

$2x = u^2 - 1$

$x = \frac{u^2-1}{2}$

$dx = d\left(\frac{u^2-1}{2}\right)$

$dx = \frac{du^2}{2} - d\frac{1}{2} \to 0$

$= \frac{2u\,du}{2}$

$dx = u\,du$

$+ c$

$\int \frac{u^2-1}{2} \times u \times u \, du$

$\frac{1}{2}\left[\int (u^2-1)\, u^2 \, du\right]$

$\frac{1}{2}\left[\int (u^4-u^2)\, du\right]$

$= \frac{1}{2}\left[\int u^4 du - \int u^2 du\right]$

$= \frac{1}{2}\left[\frac{u^5}{5} - \frac{u^3}{3}\right] + c$

$= \frac{u^5}{10} - \frac{u^3}{6} + c$

$= \frac{(\sqrt{2x+1})^5}{10} - \frac{(\sqrt{2x+1})^3}{6} + c$

$\frac{(2x+1)^{5/2}}{10} - \frac{(2x+1)^{3/2}}{6} + c$

6 $\int \frac{x}{\sqrt{7-x}} dx$

$u = \frac{1}{\sqrt{7-x}}$

$u = (7-x)^{-1/2}$

$u^2 = (7-x)^{-1}$

$u^2 = \frac{1}{7-x}$

$7 - x = \frac{1}{u^2}$

$x = 7 - \frac{1}{u^2}$

$dx = d\left(7 - \frac{1}{u^2}\right)$

$dx = -d\, u^{-2}$

$= -(-2)\, u^{-2-1} du$

$dx = 2 u^{-3} du$

$= \frac{2\,du}{u^3}$

$\int \left(14 - \frac{2}{u^2}\right) \times u \times \frac{2\,du}{u^3}$

$\int \left(14 - \frac{2}{u^2}\right) \times \frac{1}{u^2} du$

$\int \frac{14}{u^2} du - \frac{2}{u^4} du$

$14 \int u^{-2} du - 2\int u^{-4} du$

$14 \times \frac{u^{-2}}{-2}$

$14\ u$

$-\frac{14}{u}$

$-\frac{1}{\sqrt{7}}$

$-$

$-$

$$\frac{(2x+1)^{5/2}}{10} - \frac{(2x+1)^{3/2}}{6} + c$$

⑥ $\displaystyle\int \frac{x}{\sqrt{7-x}}\,dx$

$$u = \frac{1}{\sqrt{7-x}}$$

$$u = (7-x)^{-1/2}$$

$$u^2 = (7-x)^{-1}$$

$$u^2 = \frac{1}{7-x}$$

$$7-x = \frac{1}{u^2}$$

$$x = 7 - \frac{1}{u^2}$$

$$dx = d\left(7 - \frac{1}{u^2}\right)$$

$$dx = -d\,u^{-2}$$

$$= -(-2)\,u^{-2-1}\,du$$

$$dx = 2\,u^{-3}\,du$$

$$= \frac{2\,du}{u^3}$$

$$\int \left(14 - \frac{2}{u^2}\right) \times u \times \frac{2\,du}{u^3}$$

$$\int \left(14 - \frac{2}{u^2}\right) \times \frac{1}{u^2}\,du$$

$$\int \frac{14}{u^2}\,du - \frac{2}{u^4}\,du$$

$$14\int u^{-2}\,du - 2\int u^{-4}\,du$$

$$14 \times \frac{u^{-2+1}}{-2+1} - 2\frac{u^{-4+1}}{-4+1} + c$$

$$14\,\frac{u^{-1}}{-1} - \frac{2u^{-3}}{-3} + c$$

$$-\frac{14}{u} + \frac{2}{3u^3} + c$$

$$-\frac{14}{\frac{1}{\sqrt{7-x}}} + \frac{2}{3\times\left(\frac{1}{\sqrt{7-x}}\right)^3} + c$$

$$-14\sqrt{7-x} + \frac{2(7-x)^{3/2}}{3} + c$$

$$-14(7-x)^{1/2} + \frac{2}{3}(7-x)^{3/2} + c$$

7 $\displaystyle\int \frac{e^{ax}}{e^{ax}+4}\,dx$

$u = e^{ax}+4$

$du = d\,e^{ax} + d4$

$du = e^{ax}\,d\,ax$

$du = e^{ax} \times a\,dx$

$\therefore e^{ax} = u-4$

$du = (u-4)\times a \times dx$

$dx = \dfrac{du}{(u-4)\times a}$

$\displaystyle\int \frac{u-4}{u} \times \frac{du}{(u-4)\times a}$

$\displaystyle\int \frac{du}{u \times a}$

$\displaystyle\frac{1}{a}\int \frac{du}{u}$

$\displaystyle\frac{1}{a}\int \frac{1}{u}\,du$

$\dfrac{1}{a}\ \mathrm{Ln}\,u + c$

$\dfrac{1}{a}\ \ln(e^{ax}+4)+c$ //

8 $\int$

$d\sin x$

$\int e^{\sin}$

$\int e^{s}$

s

e

⑨ $$\int_2^5 \frac{1}{x}\,dx = \left[\ln x\right]_2^5$$

$$= \ln 5 - \ln 2 = 1.609 - 0.693 = 0.915$$

⑩ $$\int_1^2 e^{-3x}\,dx$$

$$d(-3x) = -3\,dx$$

$$dx = -\frac{1}{3}\,d(-3x)$$

$$\int_1^2 e^{-3x}\left(-\frac{1}{3}\,d(-3x)\right)$$

$$-\frac{1}{3}\int_1^2 e^{-3x}\,d(-3x)$$

$$-\frac{1}{3}\left[e^{-3x}\right]_1^2$$

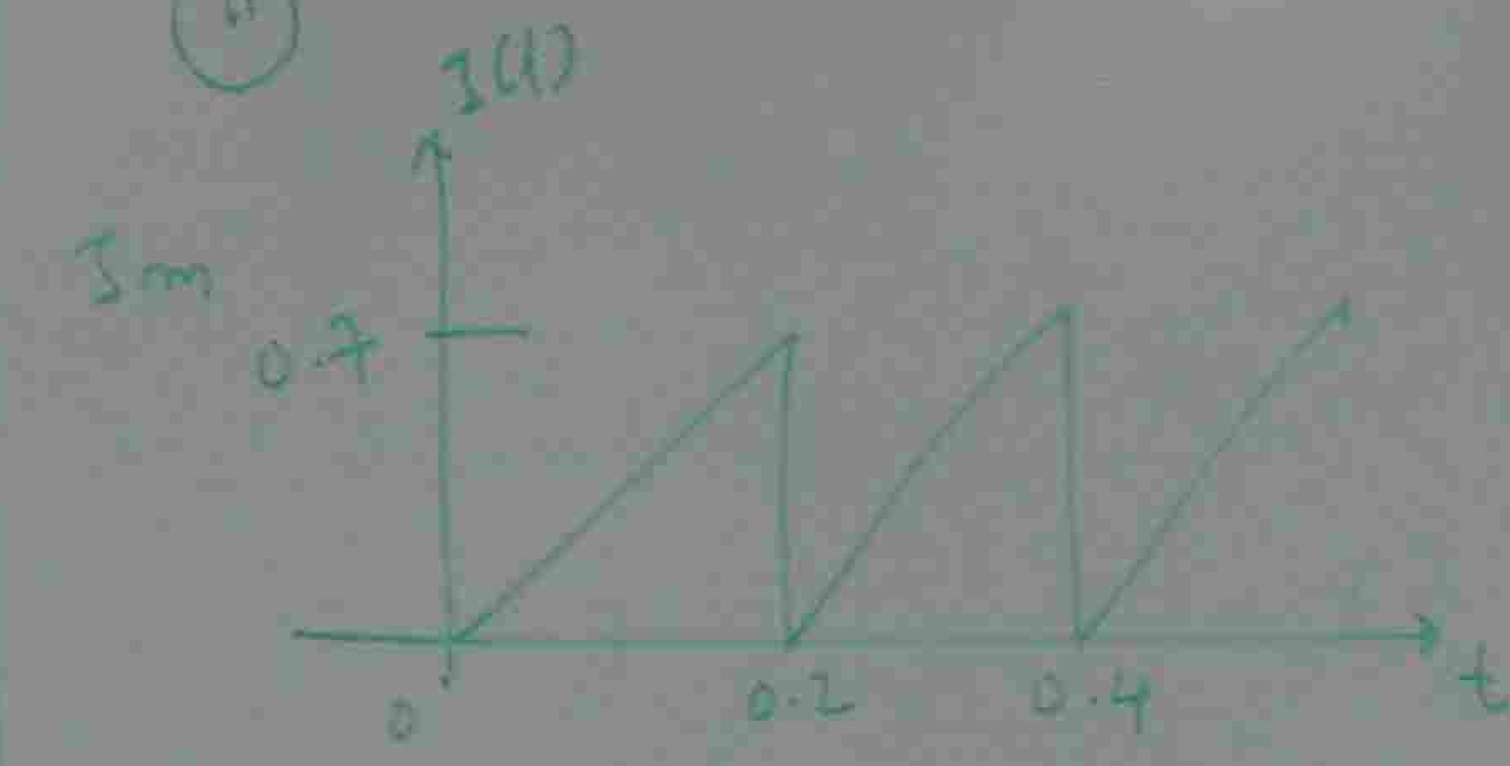
$$-\frac{1}{3}\left[e^{-6} - e^{-3}\right]$$

$$-\frac{1}{3}\left[2.478\times10^{-3} - 0.049\right]$$

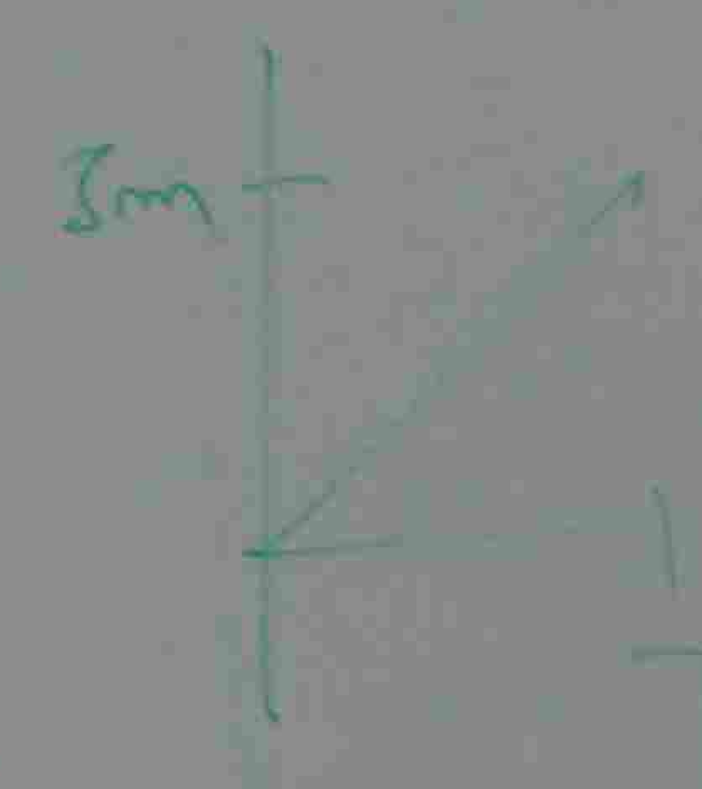
$$-\frac{1}{3}\left[-0.046\right]$$

$$= 0.0157$$

⑪



FIND I_{AVG} FOR GIVEN WAVE.



$$f(t) = \frac{I_m}{T}\times t$$

$I_{AV} = \frac{1}{T}\int_0^T f(t)\,dt$

$I_{AVE} = \frac{1}{T}\int_0^T \frac{I_m}{T}\, t \; dt$

$= \frac{I_m}{T^2}\int_0^T t\,dt$

$= \frac{I_m}{T^2}\left[\frac{t^2}{2}\right]_0^T$

$= \frac{I_m}{T^2} \times \frac{T^2}{2}$

$= \frac{I_m}{2}$

$= \frac{0.7}{2}$

$= 0.35$

(12) A VOLTAGE WAVE FORM IS REPRESENTED BY THE EQUATION

$V(t) = 20 - 20\sin\omega t$

DETERMINE THE FOLLOWINGS

(a) AVERAGE VALUE (b) RMS VALUE (c) FORM FACTOR

$\omega t = \theta$

$V_{AVE} = \frac{1}{T}\int_0^T f(t)\,dt$ (OR) $V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\,d\theta$

$v(t) = 20 - 20\sin\omega t$ (OR) $V(\theta) = 20 - 20\sin\theta$

$V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi}(20 - 20\sin\theta)\,d\theta$

$= \frac{1}{2\pi}\left[\int_0^{2\pi} 20\,d\theta - \int_0^{2\pi} 20\sin\theta\,d\theta\right]$

$= \frac{1}{2\pi}\left[20(\theta\right.$

$= \frac{1}{2\pi}\left[20\right.$

$= \frac{1}{2\pi}\left(40\right.$

$V_{AVG} = \frac{1}{2\pi} \times 40\pi$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0}$

$V_{Rms} = \sqrt{\frac{1}{2}}$

$$= \frac{1}{2\pi}\left[20\,(\theta)_0^{2\pi} - 20\times-\left[\cos\theta\right]_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[20\times 2\pi + 20\left[\cos 2\pi - \cos 0\right]\right]$$

$$= \frac{1}{2\pi}\left[40\pi + \underbrace{20\left[\cancel{1 - 1}\right]}_{0}\right]$$

$$V_{AVG} = \frac{1}{2\pi}\times 40\pi = 20 \text{ unit}$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt} \quad (\text{OR}) \quad V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v(\theta)^2\,d\theta}$$

$$V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(20 - 20\sin\theta)^2\,d\theta}$$

$$V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(400 - 800\sin\theta + 400\sin^2\theta)\,d\theta}$$

$$\cos 2\theta = 1 - 2\sin^2\theta \longrightarrow \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} 400\,d\theta - \int_0^{2\pi} 800\sin\theta\,d\theta + 400\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left(400\times 2\pi + 800(\cos\theta)_0^{2\pi} + \frac{400}{2}(\theta)_0^{2\pi} - \frac{1}{4}(\sin 2\theta)_0^{2\pi}\right.}$$

$$= \sqrt{\frac{1}{2\pi}\left[\left(800\pi + \underbrace{800(\cancel{\cos 2\pi - \cos 0})}_{0} + 200\times 2\pi - \underbrace{\frac{1}{4}(\cancel{\sin 4\pi - s}}_{0}\right.\right.}$$

$$= \sqrt{\frac{1}{2\pi}\times(800\pi + 400\pi)}$$

$$V_{Rms} = \sqrt{600} = 24.49$$

(c)

$$\text{FORM FACTOR} = \frac{V_{RMS}}{V_{AVG}} = \frac{24.49}{20} = 1.225$$