# DIFFERENTIATION

$$y = x^n$$

$$\frac{dy}{dx} = \frac{d}{dx} x^n$$

$$\boxed{\frac{dy}{dx} = n x^{n-1}}$$

$$y = a x^n$$

$$\frac{dy}{dx} = \frac{d}{dx} (a x^n)$$

$$= a \frac{d}{dx} x^n$$

$$\frac{dy}{dx} = a n x^{n-1}$$

Ex $x^5 \to \frac{dy}{dx} = ?$

$$\frac{d}{dx} x^5 = 5 x^{5-1} = 5 x^4$$

Ex $0.6 x^7$

$$\frac{d}{dx} (0.6 x^7) = 0.6 \frac{d}{dx} x^7$$

$$= 0.6 \times 7 \times x^{7-1}$$

$$= 4.2 x^6$$

EXERCISE Find $\frac{dy}{dx}$ for

$2 x^{1.5}$

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## DIFFERENTIATION OF A SUM OF FUNCTIONS

THE DIFFERENTIATION OF A SUM OF FUNCTIONS IS EQUAL TO THE SUM OF THE INDIVIDUAL DIFFERENTIATIONS OF THE FUNCTIONS.

$$y = f_1(x) + f_2(x) + f_3(x)$$

$$\frac{dy}{dx} = \frac{d}{dx} f_1(x) + \frac{d}{dx} f_2(x) + \frac{d}{dx} f_3(x)$$

**Ex** $y = 5x^3 + 6x^2 + 7$

FIND $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx} 5x^3 + \frac{d}{dx} 6x^2 + \frac{d}{dx} 7$$

$$= 5\frac{d}{dx}x^3 + 6\frac{d}{dx}x^2 + 0$$

$$= 5 \times 3\, x^{3-1} + 6 \times 2 \times x^{2-1}$$

$$= 15x^2 + 12x$$

**Ex** $y = \sin x + \cos x$

$\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{d}{dx}\sin x + \frac{d}{dx}\cos x$$

$$= \cos x + (-\sin x)$$

$$= \cos x - \sin x$$

$$\boxed{\begin{aligned}\frac{d}{dx}\sin x &= \cos x \\ \frac{d}{dx}\cos x &= -\sin x\end{aligned}}$$

$5x^4$

$x^7$

$7 \times x^{7-1}$

$x^6$

$f(x)$

IN THIS BUILDING

Ex

$$y = \sin 3x + \cos 5x$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin 3x + \frac{d}{dx}\cos 5x$$

$$= \cos 3x \times \frac{d3x}{dx} + (-\sin 5x)\frac{d}{dx}5x$$

$$= \cos x \quad 3\frac{\cancel{dx}}{\cancel{dx}} - \sin 5x \quad 5\frac{\cancel{dx}}{\cancel{dx}}$$

$$= 3\cos x - 5\sin 5x$$

DIFFERENTIATION OF PRODUCT FUNCTIONS

$$y = u(x) \times v(x)$$

$$\frac{dy}{dx} = u(x)\frac{dv(x)}{dx} + v(x) \times \frac{du(x)}{dx}$$

Ex

$$y = \underbrace{(x+1)}_{u}\underbrace{(x+3)}_{v}$$

$$\frac{dy}{dx} = u\frac{dv(x)}{dx} + v\frac{du(x)}{dx}$$

$$= (x+1)\frac{d}{dx}(x+3) + (x+3)\frac{d}{dx}(x+1)$$

$$= (x+1)\left[\underbrace{\frac{dx}{dx}}_{1} + \underbrace{\frac{d3}{dx}}_{0}\right] + (x+3)\left[\underbrace{\frac{dx}{dx}}_{1} + \underbrace{\frac{d1}{dx}}_{0}\right]$$

$$= (x+1)(1) + (x+3)(1)$$

$$= x+1+x+3$$

$$= 2x+4$$

Ex

$$\frac{dy}{dx} =$$

$=$

$x)$

$+3)\frac{d}{dx}(x+1)$

$x+3)\left[\underbrace{\frac{dx}{dx}}_{1}+\underbrace{\frac{d1}{dx}}_{0}\right]$

$)(1)$

Ex $y=\underbrace{(x+1)^2}_{u}\underbrace{(x+3)^3}_{v}$

$$\frac{dy}{dx}=(x+1)^2\frac{d}{dx}(x+3)^3+(x+3)^3\frac{d}{dx}(x+1)^2$$

$$=(x+1)^2\times 3(x+3)^{3-1}\frac{d}{dx}(x+3)+(x+3)^3\times 2(x+1)^{2-1}\frac{d}{dx}(x+1)$$

$$=(x+1)^2\times 3(x+3)^2\left[\underbrace{\frac{d}{dx}x}_{1}+\underbrace{\frac{d3}{dx}}_{0}\right]+(x+3)^3\times 2(x+1)\left[\underbrace{\frac{d}{dx}x}_{1}+\underbrace{\frac{d}{dx}1}_{0}\right]$$

$$=3(x+1)^2(x+3)^2(1)+(x+3)^3\times 2(x+1)(1)$$

$$=3(x+1)^2(x+3)^2+2(x+3)^3(x+1)$$

$$=(x+1)(x+3)^2\left[3(x+1)+2(x+3)\right]$$

$$=(x+1)(x+3)^2\left[3x+3+2x+6\right]$$

$$=(x+1)(x+3)^2(5x+9)$$

DIFFERENTIATION OF QUOTIENT FUNCTIONS

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v(x)\frac{du(x)}{dx} - u(x)\frac{dv(x)}{dx}}{(v(x))^2}$$

Ex DIFFERENTIATE

$$y = \frac{x}{x+1}$$

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}x - x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x+1)\times 1 - x\left[\frac{d}{dx}x + \frac{d}{dx}1\right]}{(x+1)^2}$$

$$= \frac{x+1 - x(1)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

$$y = \frac{x^2+2x+1}{x^2-2x+1}$$

FIND $\frac{dy}{dx}$

ER $\longrightarrow$

ER

IE $\longrightarrow$

SUCCESSIVE DIFFERENTIATION

$$y = x^3 + 3x^2 + 4$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 + 3x^2 + 4\right]$$

$$= \left[\frac{d}{dx}x^3 + \frac{d}{dx}3x^2 + \frac{d}{dx}4\right]$$

$$= 3x^{3-1} + 3\times 2x^{2-1} + 0$$

$$= 3x^2 + 6x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[3x^2 + 6x\right]$$

$$= \frac{d}{dx}3\left[x^2 + 2x\right]$$

$$\frac{d^2y}{dx^2} = 3\left[\frac{dx^2}{dx} + \frac{d2x}{dx}\right]$$

$$= 3\left[2x^{2-1} + 2\right]$$

$$= 3\left[2x + 2\right]$$

$$= 6(x+1)$$

pb FIND THE FIRS

$$y = \frac{1}{2x+1}$$

$$\frac{dy}{dx} = \frac{d}{dx}\ (2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\ [-$$

$$= -2x$$

$$= 4\ ($$

$$= 8$$

pb FIND THE FIRST THREE DIFFERENTIAL COEFFICIENTS OF THE FUNCTION

$$y = \frac{1}{2x+1} = (2x+1)^{-1}$$

$$\frac{dy}{dx} = \frac{d}{dx}(2x+1)^{-1} = -1\,(2x+1)^{-1-1}\frac{d}{dx}(2x+1)$$

$$= -1\,(2x+1)^{-2} \times 2 = -2\,(2x+1)^{-2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-2\,(2x+1)^{-2}\right] = -2\,\frac{d}{dx}(2x+1)^{-2}$$

$$= -2 \times (-2)\,(2x+1)^{-2-1}\frac{d}{dx}(2x+1)$$

$$= 4\,(2x+1)^{-3} \times 2$$

$$= 8\,(2x+1)^{-3}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\,8\,(2x+1)^{-3}$$

$$= 8\,\frac{d}{dx}(2x+1)^{-3}$$

$$= 8\,(-3)\,(2x+1)^{-3-1}\frac{d}{dx}(2x+1)$$

$$= -24\,(2x+1)^{-4} \times 2$$

$$= -48\,(2x+1)^{-4}$$

$$= \frac{-48}{(2x+1)^4}$$ //

EXERCISES

① DIFFERENTIATE

(i) $y = x^{10}$

(ii) $y = x^{3}$

(iii) $y = m x^{k}$

(iv) $y = (x - 5)^{3}$

(v) $y = 15(x+4)^{7}$

(vi) $y = 4x^{-3.2}$

(vii) $y = 3.2(2x+3)^{1/5}$

② DIFFERENTIATE

(a) $y = 2x^{3/2} + x^{-3/2}$

(b) $y = 7x^{6} + 6x^{5} + 4x^{4} + 3x^{2} + 2x + 1$

(c) $y = \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^{2}}}$

(d) $y = -x^{-2} + \dfrac{1}{x} + x^{2}$

$+2x+1$

(3) THE LOSS IN ELECTRICAL MACHINE IS GIVEN BY

$$P = af + bf^2$$

WHERE P = POWER
f = FREQUENCY
a, b = CONSTANT

FIND $\frac{dP}{df}$

(4) THE INDUCED EMF OF A DC MACHINE IS GIVEN BY

$$E = 0.58 + 681.5 I_f - 461.3 I_f^2 + 46.3 I_f^3$$

$I_f$ = FIELD EXCITATION CURRENT

FIND $\frac{dE}{df}$

(5) DIFFERENTIATE

(i) $(x+1)^{-\frac{1}{2}}(x-5)$

(ii) $(2x+9)^3(4x^2-5)^2$

(iii) $(5x^2+6x+3)(5x-1)$

(iv) $\frac{4-x}{x-x^2}$

(v) $\frac{x^{1/4}}{x^{1/2}-1}$

SUCCESSIVE DIFFERENT

$y = x^3 + 3$

$\frac{dy}{dx} = \frac{d}{dx}[x^3$

$= \frac{d}{dx}x^3$

$= 3x^{3-1}$

$= 3x^2$

$\frac{d^2y}{dx^2} = \frac{d}{dx}$

$= \frac{d}{d}$

$$\frac{dy}{dx} = \frac{(x^2-2x+1)\frac{d}{dx}(x^2+2x+1) - (x^2+2x+1)\frac{d}{dx}(x^2-2x+1)}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)\left[\frac{d}{dx}x^2 + \frac{d}{dx}2x + \frac{d}{dx}1\right] - (x^2+2x+1)\left[\frac{d}{dx}x^2 - \frac{d}{dx}2x + \frac{d}{dx}1\right]}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)(2x^{2-1}+2+0) - (x^2+2x+1)[2x^{2-1}-2+0]}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)(2x+2) - (x^2+2x+1)(2x-2)}{(x^2-2x+1)^2}$$

$$= \frac{(x^2-2x+1)\times 2(x+1) - (x^2+2x+1)\,2(x-1)}{(x^2-2x+1)^2}$$

$$\frac{2(x-1)^2(x+1) - 2(x+1)^2(x-1)}{[(x-1)^2]^2}$$

$$\frac{2(x-1)(x+1)[(x-1)-(x+1)]}{(x-1)^4}$$

$$\frac{2(x-1)(x+1)[\cancel{x}-1-\cancel{x}-1]}{(x-1)^4}$$

$$\frac{2(x-1)(x+1)[-2]}{(x-1)^4}$$

$$\frac{-4(x+1)}{(x-1)^3} \quad //$$

IN THIS BUILDING

## DIFFERENTIATING TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}\sin(x) = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

## DIFFERENTIATING EXPONENTIAL FUNCTION

$$y = e^{x}$$

$$\frac{dy}{dx} = \frac{de^{x}}{dx} = e^{x}$$

If $y = e^{u}$

$$\frac{dy}{dx} = \frac{d}{dx}e^{u} = e^{u}\frac{du}{dx}$$

pb DIFFERENTIATE (i) $y = e^{ax}$

(ii) $y = e^{\frac{1}{2}bx^2 \ldots}$

(i)

$$y = e^{ax}$$

$$\frac{dy}{dx} = \frac{de^{ax}}{dx} = e^{ax}\frac{dax}{dx}$$

$$= e^{ax} \times a \ldots$$

$$= a e^{ax}$$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

$$\frac{dy}{dx} = \frac{d}{dx}e^{\frac{1}{2}bx^2 + \ldots}$$

$$= e^{(\frac{1}{2}bx^2 + x\ldots}$$

...ENTIAL FUNCTION

$e^{x}$

$e^{u} \frac{du}{dx}$

Qn DIFFERENTIATE (i) $y = e^{ax}$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

(i) $y = e^{ax}$

$\frac{dy}{dx} = \frac{de^{ax}}{dx} = e^{ax} \frac{dax}{dx}$

$= e^{ax} \times a \frac{\cancel{dx}}{\cancel{dx}}$

$= a e^{ax}$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

$\frac{dy}{dx} = \frac{d}{dx} e^{\frac{1}{2}bx^2 + x}$

$= e^{(\frac{1}{2}bx^2 + x)} \frac{d}{dx}(\frac{1}{2}bx^2 + x)$

$= e^{\frac{1}{2}bx^2 + x} \left[ \frac{d}{dx}\frac{1}{2}bx^2 + \frac{d}{dx} x \right]$

$= e^{\frac{1}{2}bx^2 + x} \left[ \frac{b}{2}\frac{d}{dx}x^2 + 1 \right]$

$= e^{\frac{1}{2}bx^2 + x} \left[ \frac{b}{2} \times 2x^{2-1} + 1 \right]$

$= e^{\frac{1}{2}bx^2 + x} \left[ bx + 1 \right]$ //

## DIFFERENTIATING LOGARITHMIC FUNCTION

$$y = \log_e u(x) = \ln u(x)$$

$$\boxed{\frac{dy}{dx} = \frac{1}{u(x)} \frac{d}{dx} u(x)}$$

DIFFERENTIATE

$\log_e x^2$

$\log_e (x^2 - 1)$

$\log_e \sin x$

(i) $$\frac{d}{dx} \log_e x^2 = \frac{1}{x^2} \frac{d}{dx} x^2$$

$$= \frac{1}{x^2} \times 2x^{2-1}$$

$$= \frac{2x}{x^2}$$

$$= \frac{2}{x}$$

(ii) $$\frac{d}{dx} \log_e (x^2 - 1) = \frac{1}{(x^2-1)} \frac{d}{dx} (x^2 - 1)$$

$$= \frac{1}{(x^2-1)} \left[ \frac{d}{dx} x^2 - \frac{d}{dx} 1 \right]$$

$$= \frac{1}{(x^2-1)} \left[ 2x \right] = \frac{2x}{x^2 - 1}$$

(iii) $\frac{d}{dx}\log_e \sin x = \frac{1}{\sin x}\frac{d}{dx}\sin x$

$= \frac{1}{\sin x}\cos x$

$= \frac{\cos x}{\sin x} = \cot x$

$\log_e u = \frac{\log_{10} u}{\log_{10} e} = \frac{\log_{10} u}{\log_{10} 2.718} = \frac{\log_{10} u}{0.434}$

$= 2.3 \log_{10} u$

$\log_{10} u = \frac{\log_e u}{2.3}$

$\frac{d}{dx}\log_{10} u = \frac{d}{dx}\left(\frac{\log_e u}{2.3}\right)$

$$\boxed{\frac{d}{dx}\log_{10} u = \frac{1}{2.3}\frac{d}{dx}\log_e u = \frac{1}{2.3}\times\frac{1}{u}\frac{du}{dx}}$$

FIRST ORDER DERIVATIVE $\rightarrow \frac{dy}{dx}$

SECOND ORDER DERIVATIVE $\rightarrow \frac{d^2}{d}$

SUCCESS

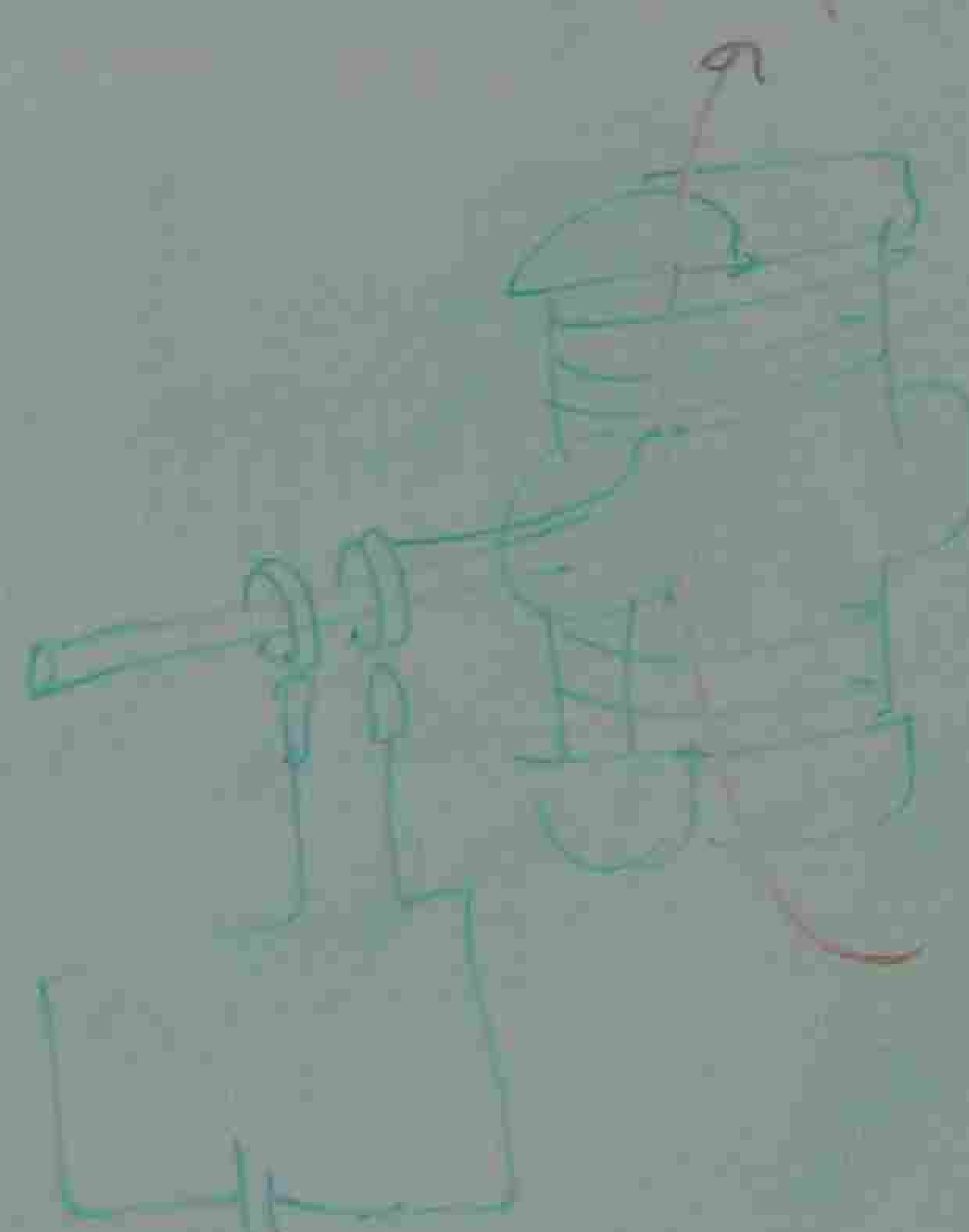
y

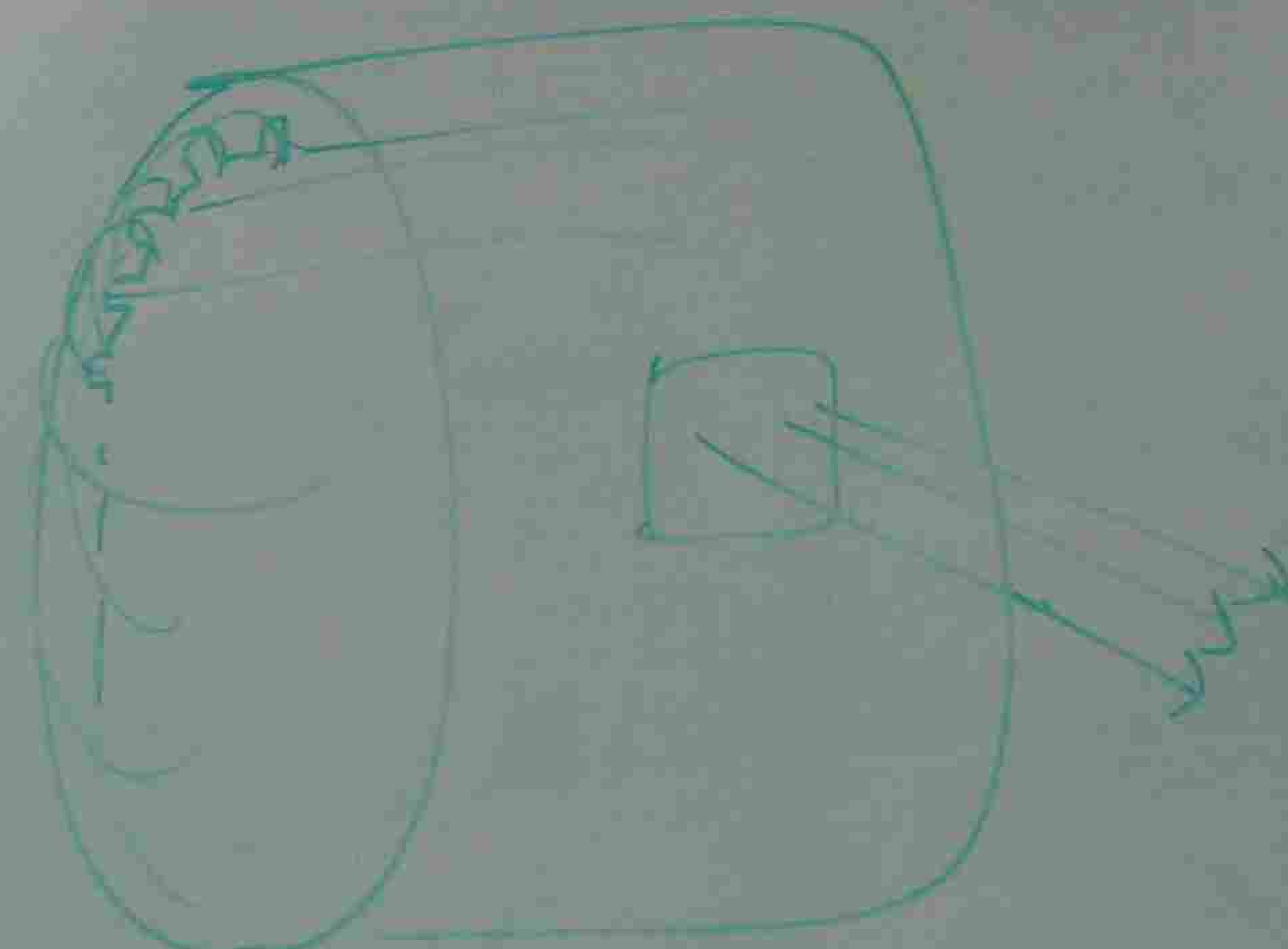
of A SALIENT POLE

IS THE ANGULAR

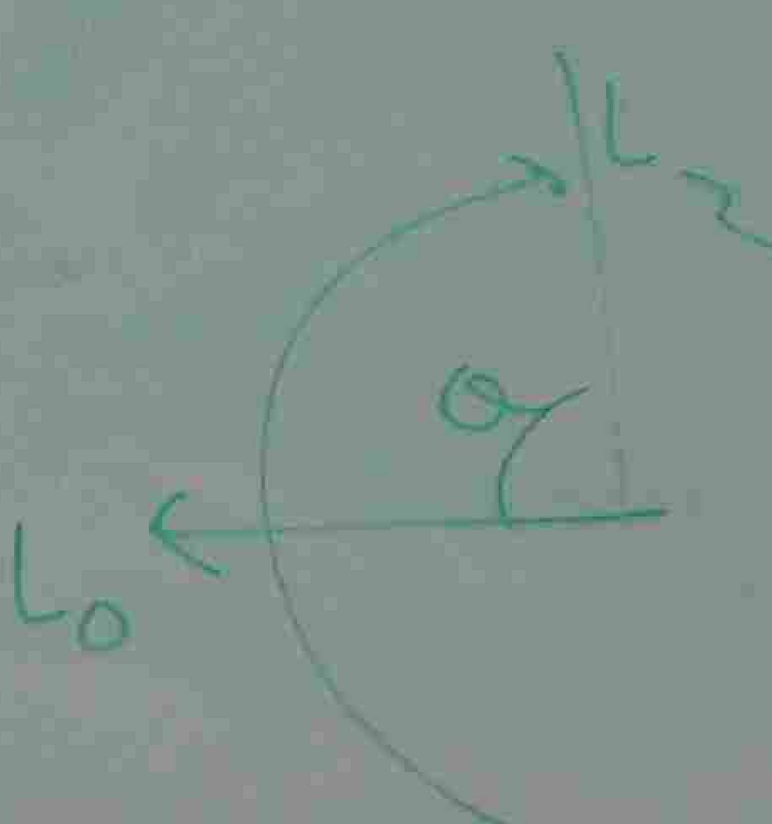
OF CHANGE OF

$-2L_2 \sin 2\theta$




$$N = \frac{120f}{P}$$




DIFFERENTI

LOGARITHMIC

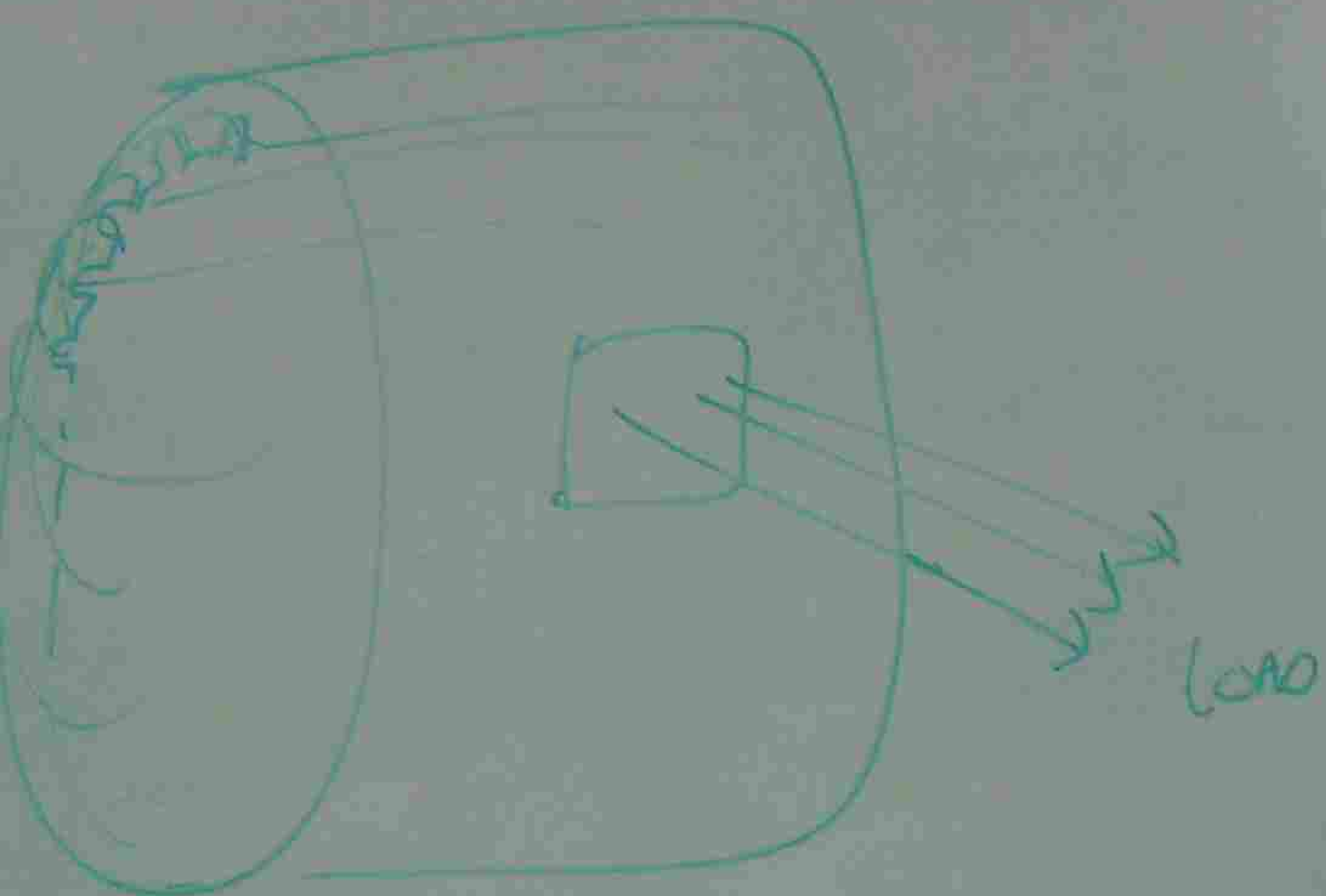
(i) $e^{-ax}$

(ii) $e^{(x^2}$

(iii) $\log_e ($

(iv) $a^{x^2+}$

(v) $x^{\sin}$




$= \frac{120 f}{p}$

pb DIFFERENTIATE THE FOLLOWING EXPONENTIAL, LOGARITHMIC AND POWER FUNCTIONS.

(i) $e^{-ax}$

(ii) $e^{(x^2+2x)}$

(iii) $\log_e (x^2+2x+3)$

(iv) $a^{x^2+1}$

(v) $x^{\sin x}$

(i) $\frac{d}{dx} e^{-ax}$

(ii) $\frac{d}{dx} e^{x^2}$

(iii) $\frac{d}{dx}$

CANVAS SHOES
ARE NOT
APPROPRIATE
FOOTWEAR
IN THIS BUILDING

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x^2-1)\frac{d}{dx}2x - 2x\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{2(x^2-1) - 2x \times 2x}{(x^2-1)^2}$$

$$= \frac{2x^2-2-4x^2}{(x^2-1)^2}$$

$$= \frac{-2(x^2+1)}{(x^2-1)^2}$$

(ii) $y = \sin^2 x$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^2 x = 2\sin^{2-1} x\,\frac{d\sin x}{dx} = 2\sin x\cos x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \times 2\sin x\cos x$$

$$= 2\left[\sin x\frac{d}{dx}\cos x + \cos x\frac{d}{dx}\sin x\right]$$

$$= 2\left[\sin x(-\sin x) + \cos x \times \cos x\right]$$

$$= -2\left[\sin^2 x - \cos^2 x\right]$$

$$= 2\left[-\sin^2 x + \cos^2 x\right]$$

$$= 2\left[-\sin^2 x + 1 - \sin^2 x\right]$$

$$2\left[1 - 2\sin^2 x\right]$$

(ii

FOOTWEAR
N THIS BUILDING

(iii) $y = e^{\frac{x^2}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\frac{x^2}{2}} = e^{\frac{x^2}{2}} \frac{d}{dx} x^2/2$$

$$= e^{\frac{x^2}{2}} \times \frac{1}{2} \times 2x$$

$$= x e^{x^2/2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} x e^{\frac{x^2}{2}}$$

$$= x \frac{d}{dx} e^{\frac{x^2}{2}} + e^{\frac{x^2}{2}} \frac{d}{dx} x$$

$$= x \times e^{\frac{x^2}{2}} \frac{d}{dx} x^2/2 + e^{\frac{x^2}{2}}$$

$$= x e^{x^2/2} \times \frac{\cancel{2}x}{\cancel{2}} + e^{x^2/2}$$

$$= e^{x^2/2}\left[x^2+1\right] \quad //$$

(iv) $y = a^x$

$$\frac{dy}{dx} = \frac{d}{dx} a^x = x a^{x-1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[x \, a^{x-1}\right]$$

$$= x \frac{d}{dx} a^{x-1} + a^{x-1} \frac{dx}{dx}$$

$$= x \times (x-1)\, a^{x-1-1} + a^{x-1}$$

$$= x(x-1)\, a^{x-2} + a^{x-1}$$

ph) THE CURRENT GROWTH IN A RESISTIVE-INDUCTIVE CIRCUIT FROM A SUDDENLY APPLIED BATTERY EMF IS

$$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

WHERE $E$, $R$, $L$ ARE CONSTANTS, $t$ IS TIME

FIND THE RATE OF CHANGE OF CURRENT WITH RESPECT TO TIME.

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)\right]$$

$$= \frac{E}{R}\left[-\frac{d}{dt}e^{-\frac{Rt}{L}}\right]$$

$$= -\frac{E}{R} \times e^{-Rt/L} \times \frac{d}{dt}\left(-\frac{Rt}{L}\right)$$

$$= -\frac{E}{R} \times e^{-\frac{Rt}{L}} \times \left(-\frac{R}{L}\right)$$

$$\left\{ \frac{di}{dt} = \frac{E}{L} \times e^{-\frac{Rt}{L}} \right.$$

\#

## DIFFERENTIATION OF A FUNCTION OF A FUNCTION

$y = \sin^2 (x^2+1)$

Ex $y = \sin^3 (2x^2-1)$ FIND $y'$ $\left(\frac{dy}{dx} = ??\right)$

LET $u = 2x^2 - 1$

$\frac{du}{dx} = \frac{d}{dx}(2x^2-1)$

$= 2 \times 2 x^{2-1}$

$= 4x$

$y = \sin^3 u$

$\frac{dy}{du} = 3 \sin^{3-1} u \frac{d \sin u}{du}$

$= 3 \sin^2 u \times \cos u$

$\frac{dy}{dx}$

BUT $u$

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}}$$

$$= 4x \times 3\sin^2 u \cos u$$

But $u = 2x^2 - 1$

$$= 4x \; 3\sin^2(2x^2-1) \cos 2x^2 - 1$$

$$= 12x \sin^2 2x^2 - 1 \; \cos 2x^2 - 1$$

\#

DIFFERENTIATION OF IMPLICIT FUNCT

pb

$x^2 + y^2 = 9$ Find $\frac{dy}{dx}$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 9$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = 0$$

$$2x^{2-1} \cancel{\frac{dx}{dx}} + 2y^{2-1} \frac{dy}{dx} = 0$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \times \frac{dy}{dx} = -2x$$

E NOT
OPRIATE
TWEAR
BUILDING

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

6b DIFFERENTIATE FOLLOWING FUNCTIONS WITH RESPECT TO $x$.

a) $x^2 + y^2 = 4$

b) $y \log_e x = 2$

(a) $\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 4$

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = 0$$

$$2x^{2-1} + 2y \frac{dy}{dx} = 0$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

(b) $y \log_e x = 2$

$$\frac{d}{dx}\left[y \log_e x\right] = \frac{d}{dx} 2$$

$$\boxed{\frac{d}{dx} u.v = u\frac{dv}{dx} + v\frac{du}{dx}}$$

$$\frac{d}{dx} \log_e f(x) = \frac{f'(x)}{f(x)}$$

$$y \frac{d}{dx} \log_e x + \log_e x \frac{dy}{dx} = 0$$

$$y \times \frac{\frac{dx}{dx}}{x} + \log_e x \times \frac{dy}{dx} = 0$$

$$\frac{y}{x} + \log_e x \times \frac{dy}{dx} = 0$$

$\log_e x \times \frac{dy}{dx}$

$\frac{dy}{dx} =$

$) = \frac{d}{dx} 4$

$0$

$= 0$

$0$

(b) $y \log_e^x = 2$

$\frac{d}{dx}\left[ y \log_e^x \right] = \frac{d}{dx} 2$

$$\boxed{\frac{d}{dx} u.v = u \frac{dv}{dx} + v \frac{du}{dx}}$$

$$\boxed{\frac{d}{dx} \log_e f(x) = \frac{f(x)'}{f(x)}}$$

$y \frac{d}{dx} \log_e^x + \log_e^x \frac{dy}{dx} = 0$

$y \times \frac{\frac{dx}{dx}}{x} + \log_e^x \times \frac{dy}{dx} = 0$

$\frac{y}{x} + \log_e^x \times \frac{dy}{dx} = 0$

$\log_e^x \times \frac{dy}{dx} = -\frac{y}{x}$

$\frac{dy}{dx} = \frac{-y/x}{\log_e^x}$

DIFFERENTIATE THE FOLLOWING TRIGONOMETRIC FUNCTIONS

(i) $\sin(10x+4) + \cos(7x+1)$

(ii) $\tan^2 3\theta$

(iii) $\sec x \tan x$

(iv) $\csc^4(x^2+1)$

(v) $\cot 5x \sin 6x$

(vi) $1 - 10\sin 10t + 5\sin 20t + 2.5\sin 30t$

(i) $\frac{d}{dx}\left[\sin(10x+4) + \cos(7x+1)\right]$

$\frac{d}{dx}\sin(10x+4) + \frac{d}{dx}\cos(7x+1)$

$\cos(10x+4)\frac{d}{dx}(10x+4) + (-\sin(7x+1))\frac{d}{dx}(7x+1)$

$\cos(10x+4) \times 10 - \sin(7x+1) \times 7$

$10\cos(10x+4) - 7\sin(7x+1)$

(ii) $u = \tan 3\theta$

$u^2 = \tan^2 3\theta$

$\frac{d}{d\theta}u^2 = 2u\frac{du}{d\theta}$

$= 2u\frac{d}{d\theta}\tan 3\theta$

$2u \sec^2 3\theta \times$

$2\tan 3\theta \times \sec^2 3\theta$

$6\tan 3\theta \sec^2 3\theta$

NG TRIGONOMETRIC FUNCTIONS

+1)

$t + 2.5\ \sin 30t$

(i) $\frac{d}{dx}\left[\sin(10x+4) + \cos(7x+1)\right]$

$\frac{d}{dx}\sin(10x+4) + \frac{d}{dx}\cos(7x+1)$

$\cos(10x+4)\frac{d}{dx}(10x+4) + (-\sin(7x+1))\frac{d}{dx}(7x+1)$

$\cos(10x+4)\times 10 - \sin(7x+1)\times 7$

$10\cos(10x+4) - 7\sin(7x+1)$

(ii) $u = \tan 3\theta$

$u^2 = \tan^2 3\theta$

$\frac{d}{d\theta}u^2 = 2u\frac{du}{d\theta}$

$= 2u\frac{d}{d\theta}\tan 3\theta$

$2u \quad \sec^2 3\theta \times \frac{d\,3\theta}{d\theta}$

$2\tan 3\theta \times \sec^2 3\theta \times 3$

$6\tan 3\theta\ \sec^2 3\theta$ //

(iii) $\frac{d}{dx}\sec x.\tan x = \sec x$

$= \sec$

$= S$

$= S$

(iv) $\frac{4}{\operatorname{cosec}(x^2+1)}$

$\operatorname{cosec}$

$u = x^2+1 \longrightarrow$

$4$

$\frac{d}{dx}\operatorname{cosec}^4 u = 4\operatorname{cosec}$

$= 4\operatorname{cosec}^3 u$

$= 4\operatorname{cosec}^3$

$= 8x$

$+ \cos(7x+1)\}$

$\cos(7x+1)$

$+ \left(-\sin(7x+1)\right) \frac{d}{dx}(7x+1)$

$\sin(7x+1) \times 7$

$\sin(7x+1)$ //

$2u \quad \sec^2 3\theta \times \frac{d}{d\theta} 3\theta$

$2\tan 3\theta \times \sec^2 3\theta \times 3$

$6 \tan 3\theta \sec^2 3\theta$ //

$\frac{d}{d\theta} \tan 3\theta$

(iii) $\frac{d}{dx} \sec x \tan x = \sec x \frac{d}{dx}\tan x + \tan x \frac{d}{dx} \sec x$

$= \sec x \times \sec^2 x + \tan x \times \sec x \tan x$

$= \sec^3 x + \sec x \tan^2 x$

$= \sec x \left(\sec^2 x + \tan^2 x\right)$ //

(iv) $\operatorname{cosec}^4 (x^2+1)$

$u = x^2 + 1 \longrightarrow \operatorname{cosec}^4 u$

$\frac{d}{dx} \operatorname{cosec}^4 u = 4 \operatorname{cosec}^{4-1} u \frac{d}{dx} u$

$= 4 \operatorname{cosec}^3 u \times \frac{d}{dx}(x^2+1)$

$= 4 \operatorname{cosec}^3 (x^2+1) \times 2x$

$= 8x \operatorname{cosec}^3 (x^2+1)$

IN THIS BUILDING

(v) $\cot 5x \sin 6x$

$$\frac{d}{dx}\cot 5x.\sin 6x = \cot 5x \frac{d}{dx}\sin 6x + \sin 6x \frac{d}{dx}\cot 5x$$

$$= \cot 5x \cos 6x \frac{d}{dx}6x + \sin 6x \times (-\operatorname{cosec}^2 5x)\frac{d}{dx}5x$$

$$= 6\cot 5x \cos 6x - 5\sin 6x \operatorname{cosec}^2 5x$$

(vi) $\frac{d}{dt}\left[1 - 10\sin 10t + 5\sin 20t + 2.5\sin 30t\right]$

$$\frac{d}{dt}1 - \frac{d}{dt}10\sin 10t + \frac{d}{dt}5\sin 20t + \frac{d}{dt}2.5\sin 30t$$

$$0 - 10\cos 10t\frac{d}{dt}10t + 5\cos 20t\frac{d}{dt}20t + 2.5\cos 30t\frac{d}{dt}30t$$

$$-100\cos 10t + 100\cos 20t + 75\cos 30t$$

pb THE POTENTIAL DIFFE[...] OF SELF INDUCTANCE L [...]

$$V_L = L\frac{di}{dt}$$

IF $i = 10\sin(314t + [...]$

$V_L = L\frac{d}{dt}[10\sin( [...]$

$= L \times 10 \cos(31[...]$

$= 10L\cos(31[...]$

$= 3140L\cos( [...]$

$6x \frac{d}{dx} \cot 5x$

$\sin 6x \times (-\mathrm{cosec}^2 5x) \frac{d}{dx} 5x$

$-5 \sin 6x \,\mathrm{cosec}^2 5x$

$\sin 30t \,]$

$5 \sin 30t$

$2.5 \cos 30t \frac{d}{dt} 30t$

pb THE POTENTIAL DIFFERENCE ACROSS AN INDU[...] OF SELF INDUCTANCE $L$ IS

$$V_L = L \frac{di}{dt}$$

IF $i = 10 \sin(314t + 60)$, FIND THE POTENTIAL DIFFERENCE $V_L$.

$$V_L = L \frac{d}{dt}\left[10 \sin(314t+60)\right]$$

$$= L \times 10 \cos(314t+60) \times \frac{d}{dt}(314t+60)$$

$$= 10L \cos(314t+60) \times 314$$

$$= 3140L \cos(314t+60)$$

Pb THE SELF INDUCTANCE OF A ROTOR WINDING OF A SALIENT POLE SYNCHRONOUS MACHINE IS

$$L = L_0 + L_2 \cos 2\theta$$

WHERE $L_0$ AND $L_2$ ARE CONSTANT AND $\theta$ IS THE ANGULAR POSITION OF THE ROTOR. FIND THE RATE OF CHANGE OF INDUCTANCE WITH ANGULAR POSITION.

$$\frac{dL}{d\theta} = \frac{d}{d\theta}\left(L_0 + L_2 \cos 2\theta\right)$$

$$= 0 + L_2 \frac{d}{d\theta} \cos 2\theta$$

$$= L_2(-\sin 2\theta) \times \frac{d}{d\theta} 2\theta$$

$$-2L_2 \sin 2\theta$$

ONENTIAL)

(i) $\frac{d}{dx} e^{-ax} = e^{-ax} \frac{d}{dx}(-ax)$

$= -a e^{-ax}$

(ii) $\frac{d}{dx} e^{x^2+2x} = e^{x^2+2x} \frac{d}{dx}(x^2+2x)$

$= e^{x^2+2x} \left[\frac{d}{dx} x^2 + \frac{d}{dx} 2x\right]$

$= e^{x^2+2x} \left[2x + 2\right]$

$= 2 e^{x^2+2x} \left[x+1\right]$

(iii) $\frac{d}{dx} \log_e (x^2+2x+3) = \frac{\frac{d}{dx}(x^2+2x+3)}{x^2+2x+3}$

$= \frac{2x+2}{x^2+2x+3} = \frac{2(x+1)}{x^2+2x+3}$

(iv) $\frac{d}{dx} a^{x^2+1} = x^2+1$

$= (x^2+1$

$= 2x$

(v) $\frac{d}{dx} x^{\sin x} =$

$=$

$=$

$$\text{(iv)}\quad \frac{d}{dx} a^{x^2+1} = x^2+1 \; a^{x^2+1-1} \; \frac{d}{dx} x^2+1$$

$$= (x^2+1) \; a^{x^2} \; 2x$$

$$= 2x(x^2+1) \; a^{x^2}$$

$$\text{(v)}\quad \frac{d}{dx} x^{\sin x} = \sin x \; x^{\sin x-1} \; \frac{d}{dx} \sin x$$

$$= \sin x \; x^{\sin x-1} \; \cos x$$

$$= \sin x \cos x \; x^{\sin x-1}$$

$2x \}$

$$\frac{\ldots(x^2+2x+3)}{x^2+2x+3}$$

$$\frac{\ldots x+2}{\ldots+2x+3} = \frac{2(x+1)}{x^2+2x+3}$$

Pb FIND FIRST TWO DIFFERENTIAL COEFFICIENTS OF THE FOLLOWING FUNCTIONS

(i) $\log_e(x^2-1)$

(ii) $\sin^2 x$

(iii) $e^{\frac{x^2}{2}}$

(iv) $a^x$

(i) $y = \log_e(x^2-1)$

$$\frac{dy}{dx} = \frac{d}{dx}\log_e(x^2-1)$$

$$= \frac{\frac{d}{dx}(x^2-1)}{(x^2-1)}$$

$$= \frac{2x}{x^2-1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{2x}{x^2-1}$$

$$\frac{d}{dx}\frac{u}{v} =$$

$=$

$=$

$\cos x \quad \frac{d}{dx} \sin x$

$\cos x * \cos x$

$$\boxed{\begin{aligned}
\sin A \cos B &= \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right] \\
\cos A \sin B &= \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right] \\
\cos A \cos B &= \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] \\
\sin A \sin B &= \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]
\end{aligned}}$$

pb $\int \sin 3x \cos 4x \, dx = ?$

$$\sin 3x \cos 4x = \frac{1}{2}\left[\sin(3x+4x) + \sin(3x-4x)\right]$$

$$= \frac{1}{2}\left[\sin 7x + \sin(-x)\right]$$

$$= \frac{1}{2}\left[\sin 7x - \sin x\right]$$

$$\int \frac{1}{2}\left[\sin 7x - \sin x\right] dx$$

$$\frac{1}{2}\left[\int \sin 7x \, dx - \int \sin x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} - (-\cos x)\right]$$

$$\frac{1}{2}\left[\frac{(-\cos 7x)}{7} + \cos x\right] + c$$

$$\frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + c$$

$$-\frac{\cos 7x}{14} + \frac{\cos x}{2} + c$$

EXERCISE

INTEGRATE THE FOLLOWING TRIGONOMETRIC FUNCTIONS

(i) $\sec^2 x$

(ii) $\tan x$

(iii) $\cos 3x$

(iv) $\sin 6x$

(v) $\cot x$

(vi) $\csc x \cot x$

(vii) $\sec x \tan x$

pb $\int \frac{1}{5} \cos^2 5x \, dx = ?$

$$\cos 2x = 2\cos^2 x - 1$$

$$\therefore \cos 10x = 2\cos^2 5x - 1$$

$$\cos^2 5x = \frac{\cos 10x + 1}{2}$$

$$\int \frac{1}{5}\left(\frac{\cos 10x + 1}{2}\right) dx$$

$$\frac{1}{10}\int (\cos 10x + 1)\, dx$$

$$\frac{1}{10}\left[\int \cos 10x \, dx + \int dx\right]$$

$$\int \sin x \sin 3x \, dx$$

$$\sin x \sin 3x = \frac{1}{2}\left[\cos(x-3x) - \cos(x+3x)\right]$$

$$= \frac{1}{2}\left[\cos(-2x) - \cos 4x\right]$$

$$= \frac{1}{2}\left[\cos 2x - \cos 4x\right]$$

$$\int \frac{1}{2}\left[\cos 2x - \cos 4x\right] dx$$

$$\frac{1}{2}\left[\int \cos 2x \, dx - \int \cos 4x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\cos 2x \, d2x}{2} - \int \frac{\cos 4x \, d4x}{4}\right]$$

$$\frac{\sin 2x}{4} - \frac{\sin 4x}{8} + C$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

INTEGRATION OF EXPONENTIAL FUNCTION

$\int e^u du = e^u + c$

ph

$\int e^{3x} dx$

$\int \frac{e^{3x} d3x}{3}$

$\frac{e^{3x}}{3} + c$

ph

$\int x e^{x^2} dx \longrightarrow \int e^{x^2} x \, dx$

$dx^2 = 2x^{2-1} dx$

$= 2x\,dx$

$\therefore x\,dx = \frac{dx^2}{2}$

$\int e^{x^2} \frac{dx^2}{2}$

$\frac{1}{2} \int e^{x^2} dx^2$

$\frac{1}{2} e^{x^2} + c$

EXER

INT

(i

(ii

(

(

$e^{x^2}\, dx \longrightarrow \int e^{x^2}\, x\, dx$

$= 2x^{2-1}\, dx$

$= 2x\, dx$

$dx = \frac{dx^2}{2}$

$$\int e^{x^2}\, \frac{dx^2}{2}$$

$$\frac{1}{2}\int e^{x^2}\, dx^2$$

$$\frac{1}{2} e^{x^2} + c$$

EXERCISE

INTEGRATE THE FOLLOWING EXPONENTIAL FUNCTION

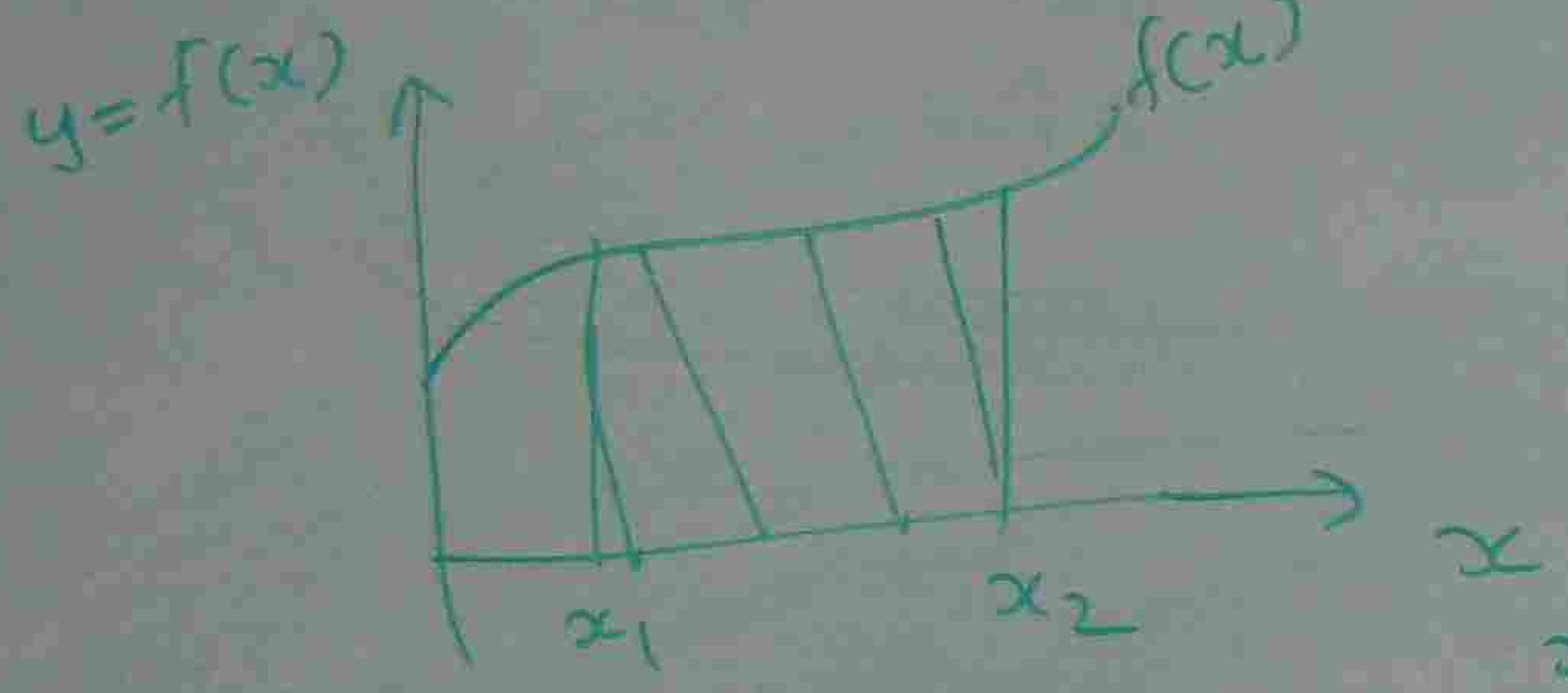
(i) $e^{-3x}$

(ii) $2e^{4x}$

(iii) $5e^{-\frac{1}{3}x}$

(iv) $3xe^{x^2}$

# INTEGRATION




AREA UNDER CURVE $= \int_{x_1}^{x_2} f(x)\,dx$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

Ex

$$\int x^7\,dx = \frac{x^{7+1}}{7+1} + c$$

$$= \frac{x^8}{8} + c$$

Ex

$$\int x^{\frac{1}{5}}\,dx = \frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+1} + c$$

$$\frac{x^{6/5}}{6/5} + c$$

$$\frac{5}{6}x^{6/5} + c$$

Ex $\int x^{-3}\,dx = \dfrac{x^{-3+1}}{-3+1} + c$

$= \dfrac{x^{-2}}{-2} + c$

$= -\dfrac{1}{2}x^{-2} + c$

Ex $\int x^{-1/2}\,dx = \dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$= \dfrac{x^{1/2}}{1/2} + c$

$= 2x^{1/2} + c$

$\int (x^n + x^m)\,dx = \int x^n\,dx + \int x^m\,dx$

$= \dfrac{x^{n+1}}{n+1} + \dfrac{x^{m+1}}{m+1} + c$

Ex $\int (x^4 + 2x^3)\,dx$

$\int x^4\,dx + \int 2x^3\,dx$

$\dfrac{x^{4+1}}{4+1} + 2\int x^3\,dx$

$\dfrac{x^5}{5} + 2 \times \dfrac{x^{3+1}}{3+1} + c$

$\dfrac{x^5}{5} + 2\,\dfrac{x^4}{4} + c$

$\frac{x^5}{5} + \frac{x^4}{2} + c$ //

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

$\int u^n \, du = \frac{u^{n+1}}{n+1} + c$

$\int (2x+3)^n \, d(2x+3) = \frac{(2x+3)^{n+1}}{n+1} + c$

$\int (2x+3)^n \, dx \neq \frac{(2x+3)^{n+1}}{n+1} + c$

Ex $\int (2x+3)^3 \, dx = ?$

$d(2x+3) = d2x + d3$

$= 2\,dx + 0$

$= 2\,dx$

$dx = \frac{d(2x+3)}{2}$

$\int (2x+3)^3 \, \frac{d(2x+3)}{2}$

$\frac{1}{2} \int (2x+3)^3 \, d(2x+3)$

$$\frac{1}{2}\,\frac{(2x+3)^{3+1}}{3+1}+c$$

$$\frac{1}{2}\times\frac{(2x+3)^{4}}{4}+c$$

$$\frac{1}{8}(2x+3)^{4}+c$$

Ex $\int(-3x+2)^{-1/3}\,dx$

$$d(-3x+2)=d(-3x)+d2$$

$$d(-3x+2)=-3dx$$

$$dx=-\frac{1}{3}\,d(-3x+2)$$

$$\int(-3x+2)^{-\frac{1}{3}}\left(-\frac{1}{3}\,d(-3x+2)\right)$$

$$-\frac{1}{3}\int(-3x+2)^{-\frac{1}{3}}\,d(-3x+2)$$

$$-\frac{1}{3}\times\frac{(-3x+2)^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}+c$$

$$-\frac{1}{3}\times\frac{(-3x+2)^{2/3}}{2/3}+c$$

$$-\frac{1}{3}\times\frac{3}{2}(-3x+2)^{2/3}+c$$

$$-\frac{1}{2}(-3x+2)^{2/3}+c$$

## EXERCISE

$$\int (5x+8)^{-2}\, dx$$

## INTEGRATE THE FOLLOWINGS

(i) $x^5$

(ii) $\frac{1}{3} x^{-1/2}$

(iii) $6x^{-2}$

(iv) $3x^{1/5}$

(v) $\frac{1}{2} x^{-1/3} + x^2$

(vi) $(2x+3)^3$

(vii) $(1+x)^{-4}$

(viii) $(3-x)^{1/2}$

(ix) $(5x+6)^{-1/3}$

(x) $(3x-2)^{-3/2}$

## INTEGRATION OF TR

$\frac{d}{dx} \sin x = \cos$

$\frac{d}{dx} \cos x = -\text{si}$

$\frac{d}{dx} \tan x = \text{se}$

$\frac{d}{dx} \cot x = -$

$\frac{d}{dx} \sec x = \text{se}$

$\frac{d}{dx} \text{cosec}\, x = -$

## INTEGRATION OF TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}\sin x = \cos x \longleftrightarrow \int \sin x\,dx = -\cos x + c$$

$$\frac{d}{dx}\cos x = -\sin x \longleftrightarrow \int \cos x\,dx = \sin x + c$$

$$\frac{d}{dx}\tan x = \sec^2 x \longleftrightarrow \int \sec^2 x\,dx = \tan x + c$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x \longleftrightarrow \int \operatorname{cosec}^2 x\,dx = -\cot x + c$$

$$\frac{d}{dx}\sec x = \sec x \tan x \longleftrightarrow \int \sec x \tan x = \sec x + c$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x \longleftrightarrow \int \operatorname{cosec} x \cot x = -\operatorname{cosec} x + c$$

$x + c$

$x + c$

$\tan x + c$

$= -\cot x + c$

$x = \sec x + c$

$\cot x = -\operatorname{cosec} x + c$

$$\frac{d}{dx}\sin^2 x = 2\sin^{2-1} x \frac{d}{dx}\sin x = 2\sin x\cos x$$

$$\int \sin^2 x\,dx$$

$$\boxed{\begin{aligned}\cos 2x &= 2\cos^2 x - 1\\ \cos 2x &= 1 - 2\sin^2 x\end{aligned}}$$

$$\int \sin^2 x\,dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x \longrightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\int \frac{1-\cos 2x}{2}\,dx = \int \frac{1}{2}\,dx - \int \frac{\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos 2x\,\frac{d2x}{2}$$

$$= \frac{x}{2} - \frac{1}{4}\int \cos 2x\,d2x$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + C$$

pb $\int \cos^2 x\,dx = ?$

$$\cos 2x = 2\cos^2 x - 1$$

$$2\cos^2 x = 1 + \cos 2x$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \frac{1+\cos 2x}{2}\,dx$$

$$\int \frac{1}{2}\,dx + \int \frac{\cos 2x\,dx}{2}$$

$$\frac{x}{2} + \int \frac{\cos 2x\,d2x}{2\times 2}$$

$\frac{x}{2}$ -

$\frac{x}{2}$

pb

S

c

$$\frac{x}{2} + \frac{1}{4}\int \cos 2x \, d2x$$

$$\frac{x}{2} + \frac{1}{4} \quad \sin 2x + c$$

pb

$$\boxed{\int \tan^2 x \, dx = ?}$$

$$\boxed{\begin{aligned} \sec^2 x &= \tan^2 x + 1 \\ \operatorname{cosec}^2 x &= \cot^2 x + 1 \end{aligned}}$$

$$\sec^2 x = \tan^2 x + 1$$

$$\therefore \tan^2 x = \sec^2 x - 1$$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$$

$$= \int \sec^2 x \, dx - \int dx$$

$$= \tan x - x + c$$

$dx$

$2$

$d2x$

$2 \times 2$

$\sin 2x = 2\sin x\cos x$

ph $\int \sin x \cos x \, dx = ?$

But $\sin 2x = 2\sin x\cos x$

$\therefore \sin x\cos x = \frac{\sin 2x}{2}$

$$\int \frac{\sin 2x}{2}\,dx = \int \frac{\sin 2x \; d2x}{2 \times 2}$$

$$= \frac{1}{4}\int \sin 2x \, d2x$$

$$= -\frac{\cos 2x}{4} + C$$

$$\frac{d}{dx}\sin x\cos x = \sin x\frac{d}{dx}\cos x + \cos x\frac{d}{dx}\sin x$$

$$= \sin(-\sin x) + \cos x * \cos x$$

$$= -\sin^2 x + \cos^2 x$$

$$\boxed{\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}}$$

$1$

$dx$

$dx$

$\int dx \Big]$

$$\frac{1}{10}\left[\int \frac{\cos 10x \, d10x}{10} + x\right]$$

$$\frac{1}{10}\left[\frac{\sin 10x}{10} + x\right] + c$$

$$\frac{\sin 10x}{100} + \frac{x}{10} + c$$

pb $\int \tan^2 3x \, dx = ?$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\sec^2 x = \tan^2 x + 1$$

$$\therefore \sec^2 3x = \tan^2 3x + 1$$

$$\tan^2 3x = \sec^2 3x - 1$$

$$\int (\sec^2 3x - 1) \, dx$$

$$\int \sec^2 3x \, dx - \int dx$$

$$\int \frac{\sec^2 3x \, d3x}{3} - x$$

$$\frac{\tan 3x}{3} - x + c$$

pb $\int \sin 6x \cos x \, dx$

$$\sin 6x \cos x = \frac{1}{2}\left[\sin(6x+x) + \sin(6x-x)\right]$$

$$= \frac{1}{2}\left[\sin 7x + \sin 5x\right]$$

$$\int \frac{1}{2}\left[\sin 7x + \sin 5x\right] dx$$

$$\frac{1}{2}\left[\int \sin 7x \, dx + \int \sin 5x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} + \int \frac{\sin 5x \, d5x}{5}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 7x}{7} - \frac{\cos 5x}{5}\right] + C$$

$$-\frac{\cos 7x}{14} - \frac{\cos 5x}{10} + C$$

pb $\int \cos 3x \cos 5x \, dx = ?$

$$\cos 3x \cos 5x = \frac{1}{2}\left[\cos(3x+5x) + \cos(3x-5x)\right]$$

$$= \frac{1}{2}\left[\cos 8x + \cos(-2x)\right]$$

$$= \frac{1}{2}\left[\cos 8x + \cos 2x\right]$$

$$\int \frac{1}{2}\left[\cos 8x + \cos 2x\right] dx$$

$$\frac{1}{2}\left[\int \cos 8x \, dx + \int \cos 2x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\cos 8x \, d8x}{8} + \int \frac{\cos 2x \, d2x}{2}\right]$$

$$\frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right] + C$$

$$\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$$

pb $\int$ s

$\sin x$

$\frac{1}{2}$

$\frac{1}{2}$

pb $\int \sin x \sin 3x \, dx$

$$\sin x \sin 3x = \frac{1}{2}\left[\cos(x-3x) - \cos(x+3x)\right]$$

$$= \frac{1}{2}\left[\cos(-2x) - \cos 4x\right]$$

$$= \frac{1}{2}\left[\cos 2x - \cos 4x\right]$$

$$\int \frac{1}{2}\left[\cos 2x - \cos 4x\right] dx$$

$$\frac{1}{2}\left[\int \cos 2x \, dx - \int \cos 4x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\cos 2x \, d2x}{2} - \int \frac{\cos 4x \, d4x}{4}\right]$$

$$\frac{\sin 2x}{4} - \frac{\sin 4x}{8} + C \quad \#$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$-9x)]$

$d2x$

$C$

pb/ $\int \sin 6x \cos x \, dx$

$$\sin 6x \cos x = \frac{1}{2}\left[\sin(6x+x) + \sin(6x-x)\right]$$

$$= \frac{1}{2}\left[\sin 7x + \sin 5x\right]$$

$$\int \frac{1}{2}\left[\sin 7x + \sin 5x\right] dx$$

$$\frac{1}{2}\left[\int \sin 7x \, dx + \int \sin 5x \, dx\right.$$

$$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} + \int \frac{\sin 5x \, d5x}{5}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 7x}{7} - \frac{\cos 5x}{5}\right] + C$$

$$-\frac{\cos 7x}{14} - \frac{\cos 5x}{10} + C$$

pb/ $\int \cos 3x \cos 5x \, dx = ?$

$$\cos 3x \cos 5x = \frac{1}{2}\left[\cos(3x+5x) + \cos(3x-5x)\right]$$

$$= \frac{1}{2}\left[\cos 8x + \cos(-2x)\right]$$

$$= \frac{1}{2}\left[\cos 8x + \cos 2x\right]$$

$$\int \frac{1}{2}\left[\cos 8x + \cos 2x\right] dx$$

$$\frac{1}{2}\left[\int \cos 8x \, dx + \int \cos 2x \, dx\right]$$

$$\frac{1}{2}\left\{\int \frac{\cos 8x \, d8x}{8} + \int \frac{\cos 2x \, d2x}{2}\right]$$

$$\frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right] + C$$

$$\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$$

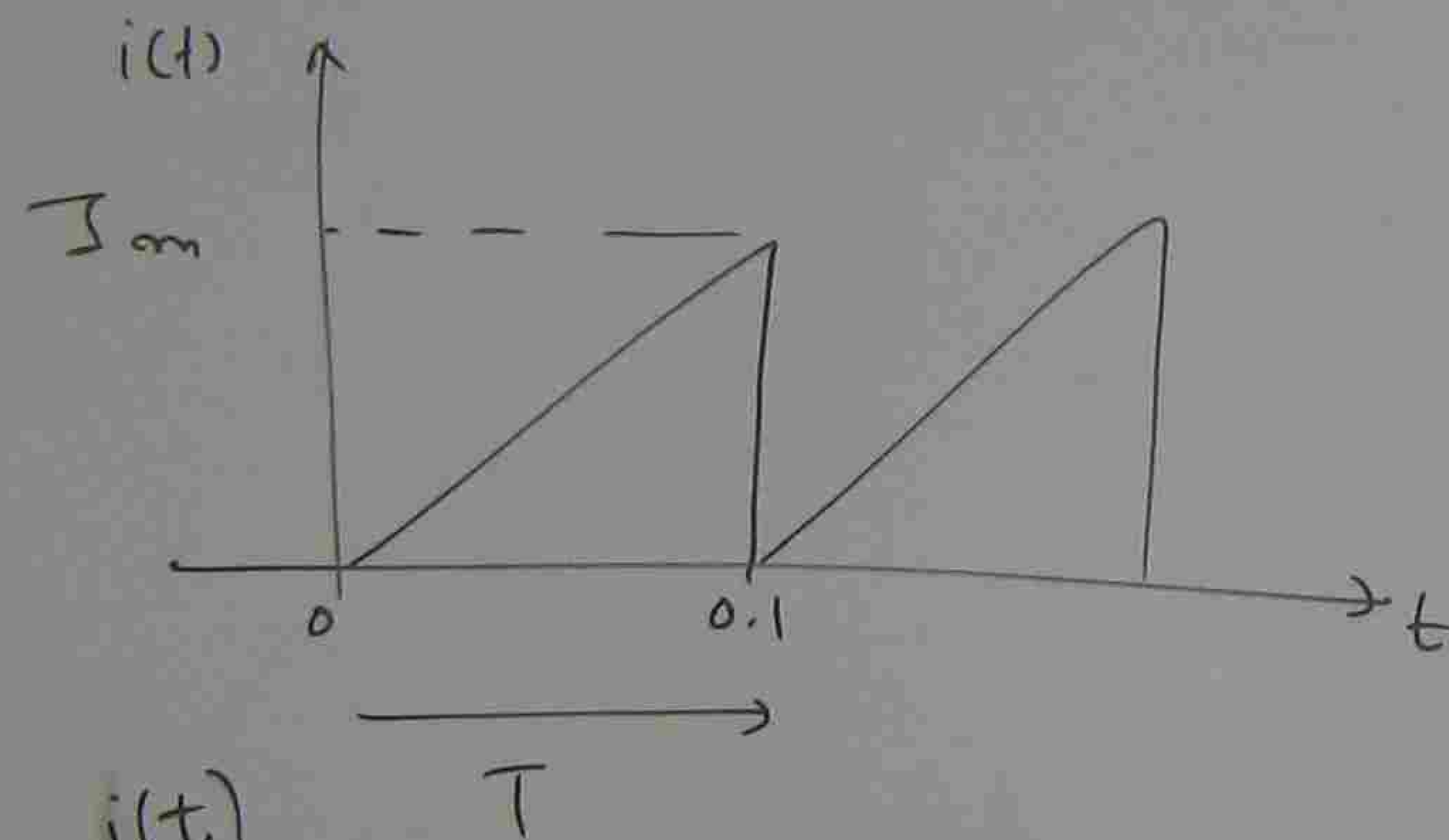
$\sin x$ ...

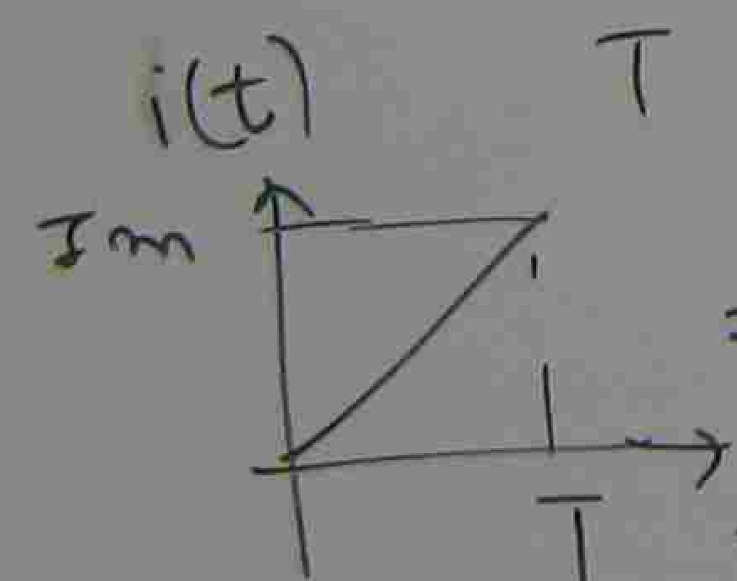
$\frac{1}{2}$

$\frac{1}{2}$

## INTEGRATION OF ELECTRICAL WAVE FORMS

### AVERAGE VALUE




$$= i(t) = \frac{I_m}{T} t$$

$$I_{AVE} = \frac{1}{T} \int_0^T i(t)\, dt$$

$$I_{AVE} = \frac{1}{T} \int_0^T \frac{I_m t}{T}\, dt$$

$$= \frac{I_m}{T^2} \int_0^T t\, dt$$

$$= \frac{I_m}{T^2} \left[\frac{t^2}{2}\right]_0^T$$

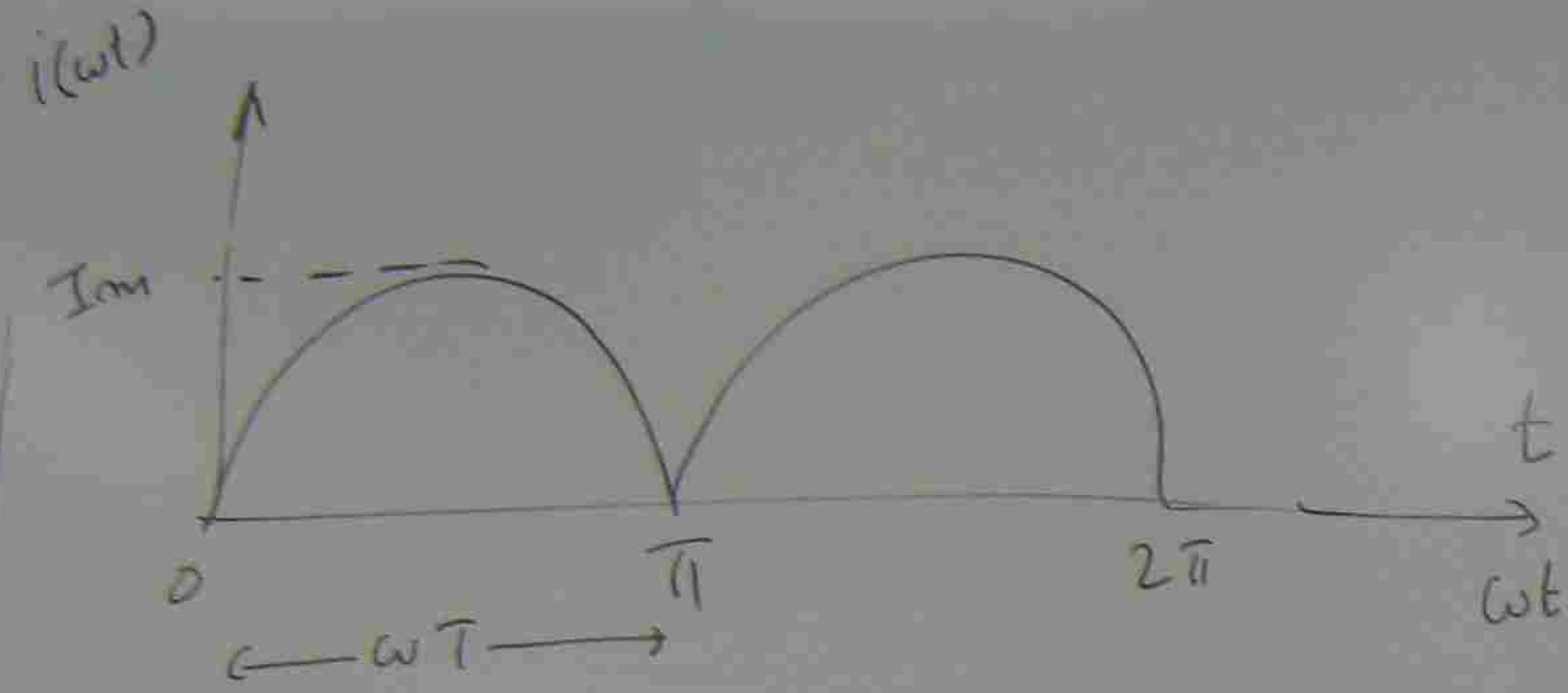
$$= \frac{I_m}{2T^2} \left[T^2 - 0\right]$$

$$= \frac{I_m}{2T^2} \times T^2$$

$$= \frac{I_m}{2}$$

i(ωt)

Im




$$i(\omega t) = I_m \sin \omega t \qquad 0 \leq \omega t \leq \pi$$

$$I_{AVE} = ?$$

$$I_{AVE} = \frac{1}{T} \int_0^T i(\omega t)\, dt$$

$$\boxed{T = \frac{\pi}{\omega}}$$

$$T = \pi$$

$$i(\omega t) = I_m \sin \omega t$$

$$I_{AVE} = \frac{1}{\pi/\omega} \int_0^{\pi} I_m \sin \omega t \, dt$$

$$I_{AVE} = \frac{\omega}{\pi} \cdot \int_0^{\pi} I_m \sin \omega t \, dt$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin \omega t \, d\omega t$$

$$= \frac{I_m}{\pi} \int_0^{\pi} \sin \omega t \, d\omega t$$

$$= \frac{I_m}{\pi} \Big[ -\cos \omega t \Big]_0^{\pi}$$

$$= \frac{I_m}{\pi} (-) \Big[ \cos \pi - \cos 0 \Big]$$

$$= \frac{I_m}{\pi} \times (-) \Big[ -1 - 1 \Big]$$

$$= \frac{2 \cdot I_m}{\pi}$$

$$= 0.636\, I_m$$

RM
I
RM
T
I

$m \sin \omega t \, dt$

$\pi$

$I_m \sin \omega t \, d\omega t$

$\int^{\pi} \sin \omega t \, d\omega t$

$0 \qquad \pi$

$\left[ -\cos \omega t \right]_0$

$\left[ \cos \pi - \cos 0 \right]$

$\times (-) \left[ -1 - 1 \right]$

$\frac{I_m}{\pi}$

$36 I_m$

RMS VALUE

$$I_{Rms} = \sqrt{\frac{1}{T} \int_0^T [i(t)]^2 dt}$$

TRIANGULAR WAVE

$$i(t) = \frac{I_m}{T} t$$

$$I_{Rms} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{I_m t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T \frac{I_m^2 t^2}{T^2} dt}$$

$$= \sqrt{\frac{1}{T} \times \frac{I_m^2}{T^2} \int_0^T t^2 dt}$$

$$= \sqrt{\frac{I_m^2}{T^3} \left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3 - 0}{3}}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3}{3}}$$

$$= \sqrt{\frac{I_m^2}{3}}$$

$$I_{rms} = \frac{I_m}{\sqrt{3}}$$

SINUSOIDAL WAVE

$$i(t) = I_m \sin \omega t$$

$$T = \frac{\pi}{\omega}$$

$$I_{rms} = \sqrt{\frac{1}{T}}$$

$$= \sqrt{\frac{1}{\frac{\pi}{\omega}}}$$

$$= \sqrt{\frac{\omega}{\pi}}$$

## SINUSOIDAL WAVE

$$i(t) = I_m \sin \omega t$$

$$T = \frac{\pi}{\omega}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T (I_m \sin \omega t)^2 \, dt}$$

$$= \sqrt{\frac{1}{\frac{\pi}{\omega}} \int_0^{\pi} I_m^2 \sin^2 \omega t \, dt}$$

$$= \sqrt{\frac{\omega}{\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \, dt}$$

$$= \sqrt{\frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \sin^2 \omega t \, d\omega t}$$

$$\cos 2\omega t = 1 - 2 \sin^2 \omega t$$

$$\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \frac{1 - \cos 2\omega t}{2} \, d\omega t}$$

$m^2 \omega t \, d\omega t$

$^2 \omega t \, d\omega t$

$in^2 \omega t$

$\omega t$

$- \cos 2\omega t \over 2$ $d\omega t$

$$= \sqrt{\frac{I_m^2}{\pi \times 2} \int_0^{\pi} (1 - \cos 2\omega t) \, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[ \int_0^{\pi} d\omega t - \int_0^{\pi} \cos 2\omega t \, d\omega t \right.}$$

$\pi = 180$

$\sin 180 = 0$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[ \omega t - \frac{1}{2} \sin \omega t \right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[ (\pi - \tfrac{1}{2} \underbrace{\sin \pi}_{0}) - (\underbrace{0 - \tfrac{1}{2} \sin 0}_{0}) \right.}$$

$$= \sqrt{\frac{I_m^2}{2\cancel{\pi}} \times \cancel{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707 \, I_m$$

## Rms value of complex wave forms

period $T = \frac{2\pi}{\omega}$

$$i(t) = I_0 + A_1 \cos\omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + \cdots + B_1 \sin\omega t + B_2 \sin 2\omega t + B_3 \sin 3\omega t +$$

$$I_{Rms} = \sqrt{I_0^2 + \frac{1}{2}(A_1^2 + A_2^2 + A_3^2 + \cdots) + \frac{1}{2}(B_1^2 + B_2^2 + B_3^2 + \cdots)}$$

$$= \sqrt{I_{dc}^2 + A_{1\,Rms}^2 + A_{2\,Rms}^2 + \cdots + B_{1\,Rms}^2 + B_{2\,Rms}^2 + \cdots}$$

$$= \sqrt{Dc^2 + (\text{sum of Rms})^2}$$

$\sin 3\omega t + - - -$

ph

$i(t) = 10 + 15 \sin 100t + 5 \sin 200t$ Amp, Find $I_{Rms}$ = ?

$$I_{Rms} = \sqrt{10^2 + \frac{1}{2}(15^2 + 5^2)} = 15 \text{ Amp}$$

## Form Factor

$$\text{Form Factor} = \frac{\text{Rms value}}{\text{Average value}} = \frac{I_{Rms}}{I_{Ave}}$$

| Wave Shape | Form Factor |
|---|---|
| Rectified Square Wave | 1 |
| Rectified Sine Wave | 1.11 |
| Triangular Wave | 1.15 |

ph

A voltage wave form is represented by the equation

$v(t) = 10 - 10 \sin \omega t$ volt

DC component + AC component

Determine the followings

(a) Average value

(b) Rms value

(c) Form factor

$$V_{Ave} = \frac{1}{T}\int_0^T v(t)\,dt$$

(OR)

$$V_{Ave} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\,d\theta$$

$\omega = 2\pi f$

$\theta = \omega t$

ENTED

$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\int_0^T (10 - 10\sin\omega t)\,dt$$

$$= \frac{1}{T}\left[\int_0^T 10\,dt\right] - \int_0^T 10\sin\omega t\,dt\Big]$$

$$= \frac{1}{T}\left[10(t)_0^T - \int_0^T \frac{10\sin\omega t\,d\omega t}{\omega}\right]$$

$$= \frac{1}{T}\left[10T - \frac{10}{\omega}\int_0^T \sin\omega t\,d\omega t\right]$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}\left[\cos\omega t\right]_0^T\right.$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}(\cos\omega T - \cos 0)\right.$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}(\cos 2\pi - 1)\right)$$

$$\boxed{\omega T = 2\pi}$$

$$\frac{1}{T}\Big(10T + \underbrace{\frac{10}{\omega}(1-1)}_{0}$$

$$= 10\ V$$

$$V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\,d\theta$$

$$\frac{1}{2\pi}\int_0^{2\pi}(10 - 10\sin\theta)\,d\theta$$

$$\frac{1}{2\pi}\left[\int_0^{2\pi} 10\,d\theta - \int_0^{2\pi} 10\sin\theta\,d\theta\right]$$

$$\frac{1}{2\pi}\left[10(\theta)_0^{2\pi} + 10(\cos\theta)_0^{2\pi}\right]$$

$$\frac{1}{2\pi}\left[20\pi + 10\underbrace{(\cos 2\pi - \cos 0)}\right]$$

$$= 10V$$

$V_{Rms} =$

$V_{Rms} =$

$= \int$

$= \int$

$=$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T (v(t))^2 \, dt}$$

(or)

$$V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [v(\theta)]^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [10 - 10\sin\theta]^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 10^2 (1-\sin\theta)^2 \, d\theta}$$

$$= \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1-\sin\theta)^2 \, d\theta}$$

$$(1-x)^2 = 1 - 2x + x^2$$

$$= \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta) \, d\theta}$$

$$= \sqrt{\frac{50}{\pi}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} 2\sin\theta \, d\theta + \int_0^{2\pi} \sin^2\theta \, d\theta\right.}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2(\cos\theta)_0^{2\pi} + \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2\,\cancel{[\cos 2\pi - \cos 0]} \to 0 + \int_0^{2\pi} \frac{1}{2} d\theta - \int_0^{2\pi} \frac{\cos 2\theta}{2} d\theta\right.}$$

$$\frac{\cos 2\theta \, d2}{4}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[3\pi - \cancel{\frac{\sin 4\pi}{4}} \to 0\right.} = \sqrt{150} = 12.25 \text{ V}$$

$$= \sqrt{\frac{50}{\pi} \int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{50}{\pi} \left[ \int_0^{2\pi} d\theta - \int_0^{2\pi} 2\sin\theta\, d\theta + \int_0^{2\pi} \sin^2\theta\, d\theta \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[ 2\pi + 2\,(\cos\theta)_0^{2\pi} + \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[ 2\pi + 2\left[\cancel{\cos 2\pi - \cos 0}\right]_{\to 0} + \int_0^{2\pi} \frac{1}{2}\, d\theta - \int_0^{2\pi} \frac{\cos 2\theta}{2}\, d\theta \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[ 2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi} \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[ 3\pi - \cancel{\frac{\sin 4\pi}{4}}_{\to 0} \right.} = \sqrt{150} = 12.25\ \text{V}$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\frac{\cos 2\theta\, d2\theta}{4}$

$(t))]^2\,dt$

$v(\theta)]^2\,d\theta$

$10 - 10\sin\theta]^2\,d\theta$

$(1-\sin\theta)^2\,d\theta$

$1-\sin\theta)^2\,d\theta$

$-2x+x^2$

$$= \sqrt{\frac{50}{\pi}\int_0^{2\pi}(1-2\sin\theta+\sin^2\theta)\,d\theta}$$

$$= \sqrt{\frac{50}{\pi}\left[\int_0^{2\pi}d\theta - \int_0^{2\pi}2\sin\theta\,d\theta + \int_0^{2\pi}\sin^2\theta\,d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2(\cos\theta)_0^{2\pi} + \int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2\underbrace{\cancel{[\cos 2\pi - \cos 0]}}_{0} + \int_0^{2\pi}\frac{1}{2}d\theta - \int_0^{2\pi}\frac{\cos 2\theta}{2}\,d\theta\right.}$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\frac{\cos 2\theta\, d2\theta}{4}$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[3\pi - \underbrace{\cancel{\frac{\sin 4\pi}{4}}}_{0}\right.} = \sqrt{150} = 12.25\ V$$

(c) $\text{FORM FACTOR} = \frac{V_{rms}}{V_{AVE}} = \frac{12.25}{10}$

$= 1.225$

pb

DETERMINE THE AVERAGE VALUE, RMS VALUE AND FORM FACTOR FOR EACH WAVE FORM SHOWN IN FIGURE

USE INTEGRAL CALCULUS

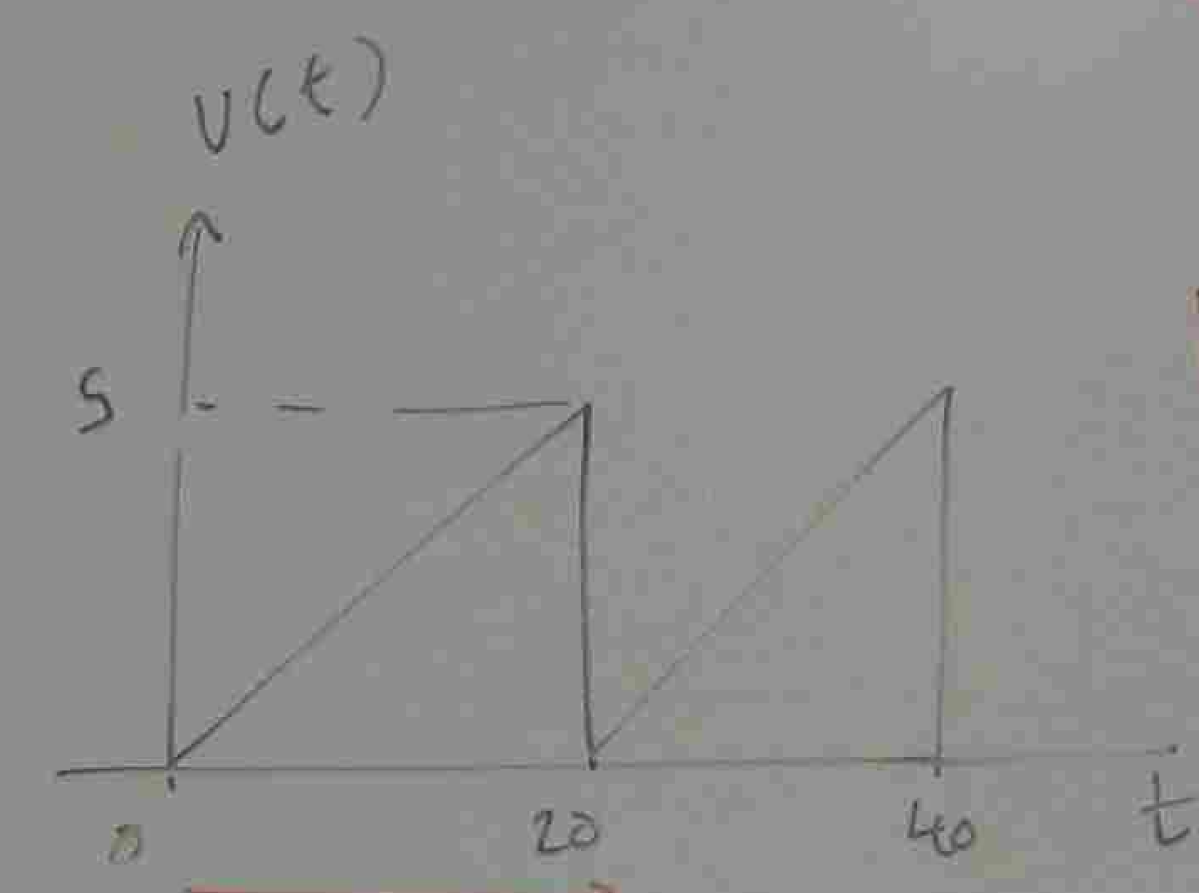
$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$V_{rm} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

(a)

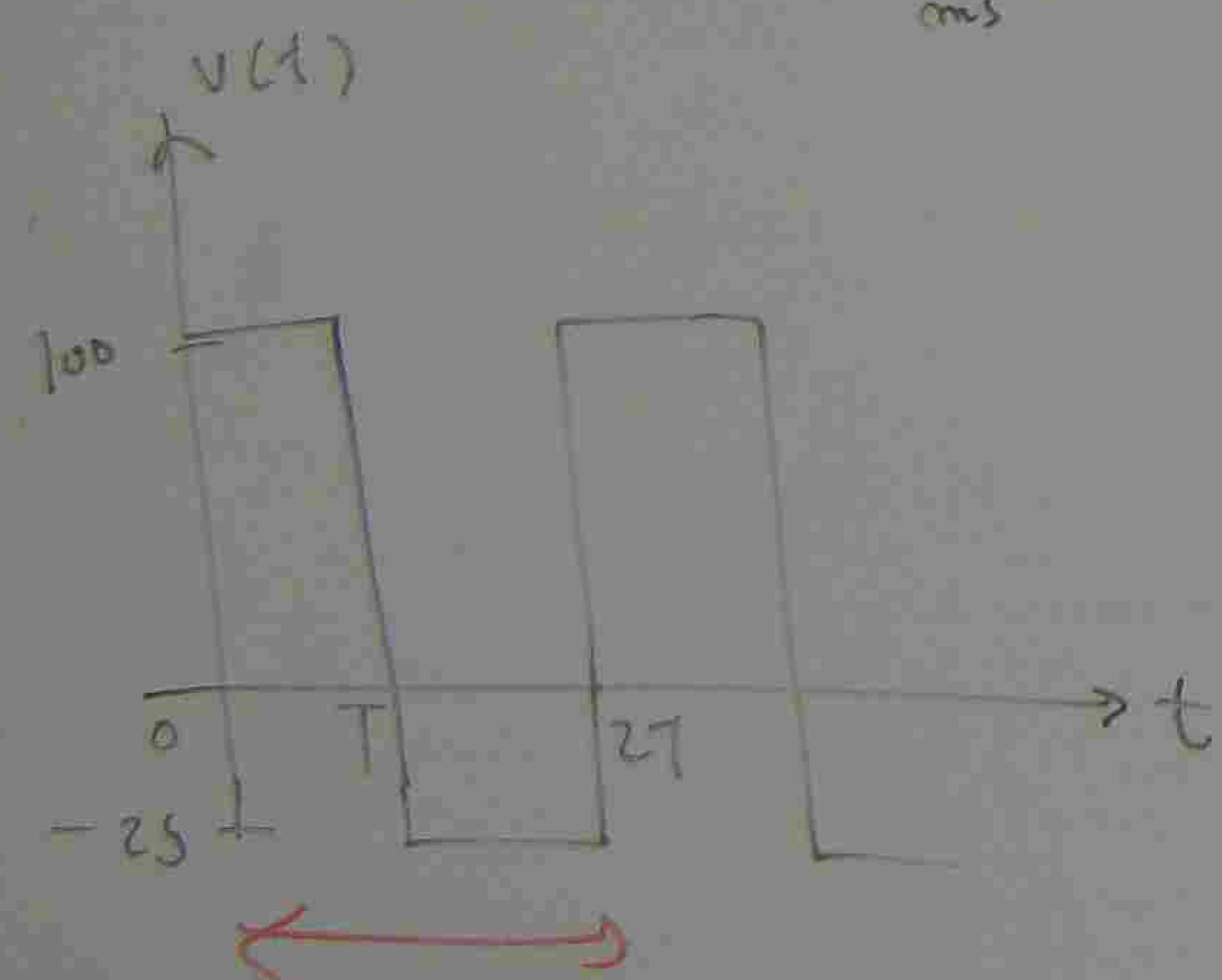

T = 20

(b)



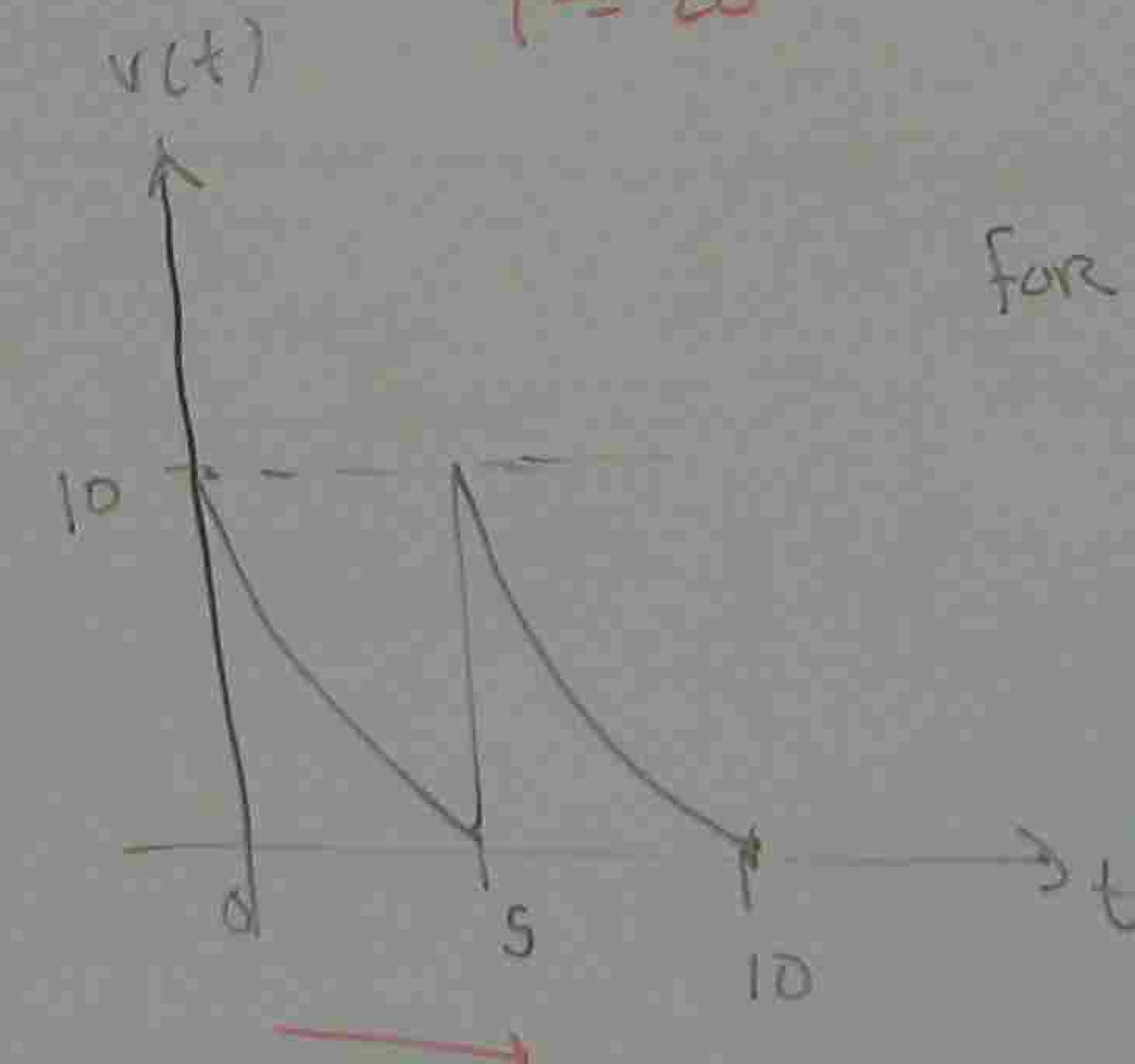

T = 20

(c)




T = 2T

(d)




T = 5

for $0 \le t \le 5\,ms$

$v(t) = 10e^{-1000t}$

(a)

V Rm

(a) $V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{20}\left[\int_0^{10} 100\,dt + \underbrace{\int_{10}^{20} 0\,dt}_{0}\right]$

$= \frac{1}{20}\left[100(t)_0^{10}\right]$

$= \frac{1}{20}\times 100 \times 10$

$= \frac{1000}{20} = 50 \text{ V}$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$= \sqrt{\frac{1}{20}\left[\int_0^{10} 100^2\,dt + \underbrace{\cancel{\int_{10}^{20} 0^2\,dt}}_{0}\right]}$

$= \sqrt{\frac{1}{20}\times 10000\,(t)_0^{10}}$

$= \sqrt{\frac{1}{20}\times 100{,}00 \times 10}$

$= \sqrt{5000}$

$= 70.7 \text{ V}$

Form factor $= \frac{V_{Rms}}{V_{AVG}}$

$= \frac{70.7}{50} = 1.4142$

(b) $v(t) = \frac{V_m}{T}\times t$

$= \frac{5}{20}t = 0.25t$

$V_{AVG} = \frac{1}{T}\int_0^T v(t)\,dt$

$V_{AVE} = \frac{1}{20}\int_0^{20} 0.25t\,dt$

$= \frac{1}{20}\times 0.25\left[\frac{t^2}{2}\right]_0^{20}$

$= \frac{1}{20}\times 0.25\times\frac{20^2}{2}$

$= \frac{0.25\times 20}{2} = 2.5 \text{ V}$

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$= \sqrt{\frac{1}{20}\int_0^{20}(0.25t)^2\,dt}$

$= \sqrt{\frac{1}{20}\times 0.0625\times\left[\frac{t^3}{3}\right]_0^{20}}$

$dt$

$\Big]_0^{20}$

$V$

$(t)^2\,dt$

$dt$

$\Big]_0^{20}$

$$= \sqrt{\frac{1}{20} \times 0.0625 \times \frac{20^3}{3}}$$

$$= \sqrt{\frac{20^2 \times 0.0625}{3}}$$

$$= 0.25 \times 20 \times \frac{1}{\sqrt{3}}$$

$$= \frac{5}{1.7321} = 2.887\ V$$

$$\text{Form Factor} = \frac{V_{Rms}}{V_{Avg}} = \frac{2.887}{2.5} = 1.155$$

(c) $$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{2T}\left[\int_0^T 100\,dt + \int_T^{2T}(-25)\,dt\right]$$

$$\frac{1}{2T}\left[\left[100\,(t)\right]_0^T - 25\,(t)_T^{2T}\right]$$

$$\frac{1}{2T}\left[100T - 25\left[2T - T\right]\right]$$

$$\frac{1}{2T}\left[100T - 25T\right]$$

$$\frac{75T}{2T} = 37.5\ V$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

$$= \sqrt{\frac{1}{2T}\left[\int_0^T 100^2\,dt + \int_T^{2T}(-25)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{2T}\left[100{,}00\,T + 625\,(2T - T)\right]}$$

$$= \sqrt{\frac{1}{2T}\left[10000\,T + 6\ldots\right.}$$

$$= \sqrt{5312}$$

$$= 72.89\ V$$

$$\text{Form Factor} = \frac{72.89}{37.5}$$

(d) $$V_{AVE} = \frac{1}{T}\int_0$$

$$= \frac{1}{5/1000}\int_0$$

$$= 200\int_0$$

$[10000\,T + 625\,T]$

$12$

$.89\,V$

$_2 = \frac{72.89}{37.5} = 1.94$

$_E = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{5/1000}\int_0^{5\times10^{-3}} 10\times e^{-1000t}\,dt$

$= 200\int_0^{5\times10^{-3}} 10\times e^{-1000t}\,\frac{d(-1000)}{-1000}$

$= \frac{2000}{-1000}\left[e^{-1000t}\right]_0^{5\times10^{-3}}$

$= -2\times\left[e^{-1000\times5\times10^{-3}} - e^{0}\right]$

$= -2\times\left[e^{-5} - 1\right]$

$= -2\times\left[\frac{1}{148} - 1\right]$

$= -2\times -0.993$

$= 1.98\,V.$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$V_{Rms} = \sqrt{\frac{1}{\frac{5}{1000}}\int_0^{5\times10^{-3}}\left[10e^{-1000t}\right]^2 dt}$

$= \sqrt{\frac{1000}{5}\times10\int_0^{5\times10^{-3}} e^{-2000t}\,dt}$

$= \sqrt{2000\times\int_0^{5\times10^{-3}} \frac{e^{-2000t}\,d(-2000t)}{-2000}}$

$= \sqrt{-\int_0^{5\times10^{-3}} e^{-2000t}\,d(-2000t)}$

$\sqrt{-\left[e^{-2000t}\right]_0^{5\times10^{-3}}}$

$\sqrt{-[e^{-5} - 1]}$

$\sqrt{1 - e^{-5}}$

$0.996\,V$

$= \sqrt{\frac{1}{2T}\left[10000T + 625T\right]}$

$= \sqrt{5312}$

$= 72.89\,V$

Form Factor $= \frac{72.89}{37.5} = 1.94$

(d) $V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{5/1000}\int_0^{5\times10^{-3}} 10\times e^{-1000t}\,dt$

$= 200\int_0^{5\times10^{-3}} 10\,e^{-1000t}\,\frac{d(-1000t)}{-1000}$

$= \frac{2000}{-1000}\left[e^{-1000t}\right]_0^{5\times10^{-3}}$

$= -2\times\left[e^{-1000\times5\times10^{-3}} - e^{0}\right]$

$= -2\times\left[e^{-5} - 1\right]$

$= -2\times\left[\frac{1}{148} - 1\right]$

$= -2\times -0.993$

$= 1.98\,V$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$FF = \frac{3.1}{1.98} = 1.59$

$V_{Rms} = \sqrt{\frac{1}{5/1000}\int_0^{5\times10^{-3}}\left[10e^{-1000t}\right]^2 dt}$

$= \sqrt{\frac{1000}{5}\times100\int_0^{5\times10^{-3}} e^{-2000t}\,dt}$

$= \sqrt{20000\times\int_0^{5\times10^{-3}} e^{-2000t}\,\frac{d(-2000t)}{-2000}}$

$= \sqrt{-10\int_0^{5\times10^{-3}} e^{-2000t}\,d(-2000t)}$

$= \sqrt{-10\left[e^{-2000t}\right]_0^{5\times10^{-3}}}$

$\sqrt{\left[e^{-5} - 1\right]}\times3.16$

$\sqrt{1 - e^{-5}}\times3.16$

$3.1$

Ph

IF A 100 Ω RESISTOR DISSIPATES AN AVERAGE POWER OF 1000 WATT, DETERMINE

(a) RMS VALUE OF CURRENT

(b) MAXIMUM VALUE OF CURRENT IF THE WAVE FORM IS SINUSOIDAL

(c) MAXIMUM VALUE OF CURRENT IF THE WAVE FORM IS TRIANGULAR.

(a) $POWER = I^2 R$

$I_{rms}^2 R = POWER$

$I_{rms}^2 \times 100 = 1000$

$I_{rms}^2 = 10$

$I_{rms} = \sqrt{10} = 3.16$ Amp.

$I_{rms}$

SI

I

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I(t)^2\,dt}$$

SINUSOIDAL => $I_{rms} = \sqrt{\frac{1}{T}\int_0^T I(t)^2\,dt}$

(OR)

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I(\theta)^2\,d\theta}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (I_{max}\sin\theta)^2\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi} I_m^2 \int_0^{2\pi} \sin^2\theta\,d\theta}$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\sqrt{\frac{1}{2\pi} I_m^2 \int_0^{2\pi}\left[\frac{1-\cos 2\theta}{2}\right]d\theta}$$

$$I_m\sqrt{\frac{1}{2\pi}\left\{\left[\frac{1}{2}\theta\right]_0^{2\pi} - \left[\int_0^{2\pi}\frac{\cos 2\theta\, d2\theta}{4}\right]\right.}$$

$$\sqrt{\frac{1}{2\pi}\left[\frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}\right.}$$

$$I_m\sqrt{\frac{1}{2\pi}\left[\pi - \frac{\sin 4\pi - \sin 0}{4}\right]}$$

$$I_m \times \frac{1}{\sqrt{2}} = 3.16$$

$I_m = \sqrt{2}\times 3.16 = 4.46$ Amp.

$-2\sin^2\theta$

$\theta = \frac{1-\cos 2\theta}{2}$

## TRIANGULAR

$$i(t) = \frac{I_m t}{T}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{I_m t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \times \frac{I_m^2}{T^2}\int_0^T t^2\, dt}$$

$$= \sqrt{\frac{I_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3}{3}}$$

$$I_{rms} = \frac{I_m}{\sqrt{3}}$$

$$3.16 = \frac{I_m}{1.7321}$$

$$I_m = 3.16 \times 1.7321$$

$$= 5.47 \text{ Amp.}$$

$= \sqrt{\frac{1}{2T}\left[10000\ T\right.}$

$= \sqrt{5312}$

$= 72.89\ V$

FORM FACTOR $= \frac{72.}{37}$

(d) $V_{AVG} =$

$= \frac{1}{5/1000}$

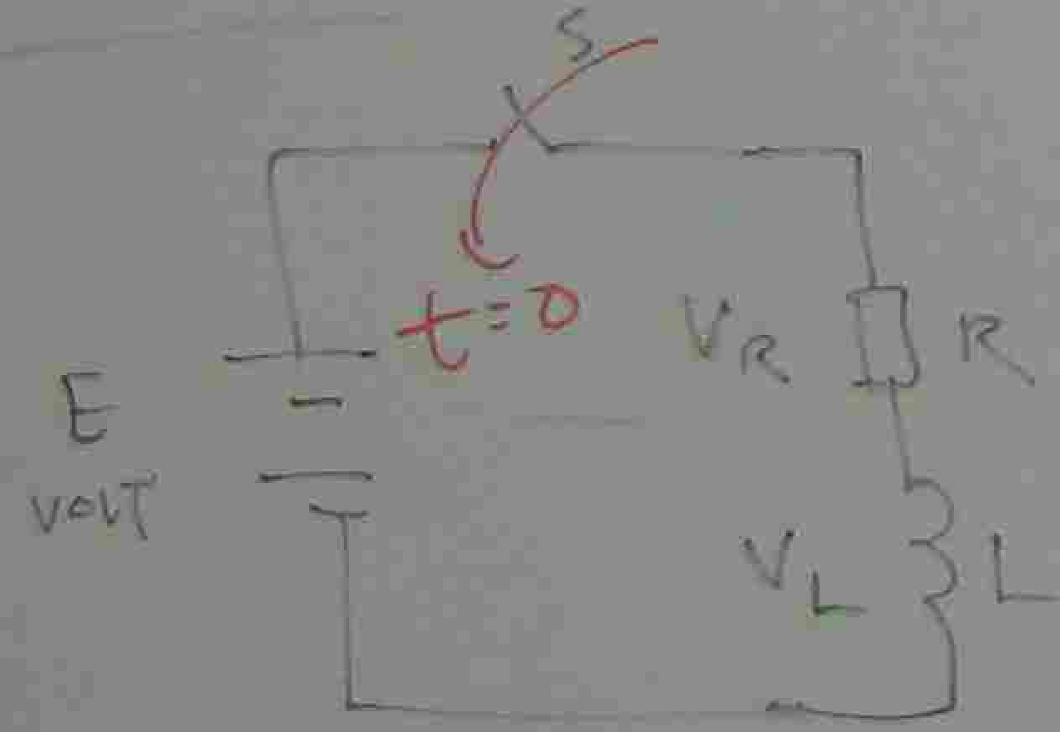
$= 200$

## DIFFERENTIAL EQUATION & APPLICATION OF DIFFERENTIAL EQUATION IN ELECTRICAL CIRCUIT CALCULATION

$$\frac{dy}{dt} + 3y = 4 \quad \text{(FIRST ORDER)}$$

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 4 = 0 \quad \text{(SECOND ORDER)}$$

RL CIRCUIT




$V_R = IR$

$V_L = L\frac{di}{dt}$

$$E = V_R + V_L$$

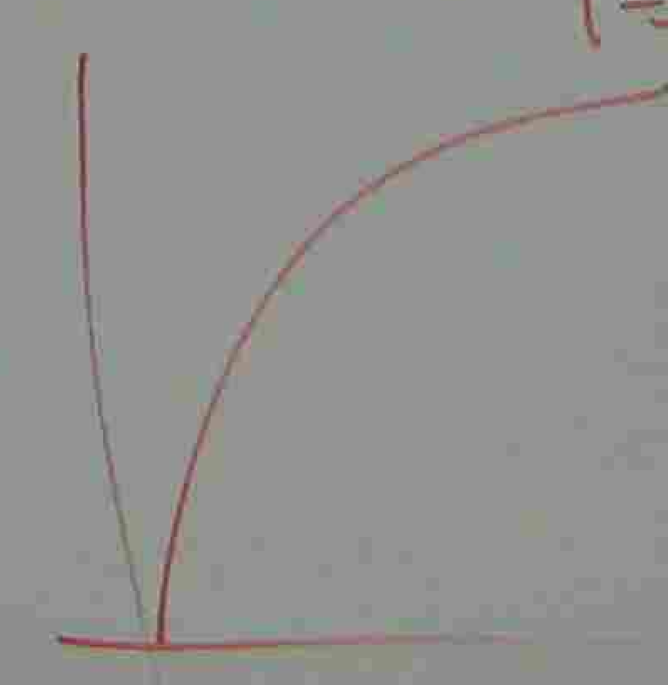
$$E = IR + L\frac{di}{dt}$$

$$i = I\left(1 - e^{-\frac{R}{L}t}\right)$$

VALUE OF CURRENT AT $t=0$

SWITCHING ON $t=0$

$i = I(1 - e^{-\frac{Rt}{L}})$



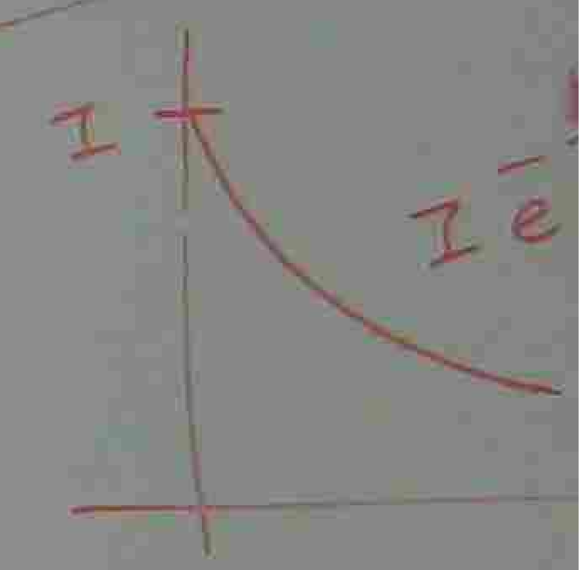
$$i = I(1 - e^{-\frac{Rt}{L}})$$

$$= I - Ie^{-\frac{Rt}{L}}$$

CONSTANT

VARIABLE WITH e FUNCTION

SWITCHING OFF




$Ie^{-}$

GENERALIZ

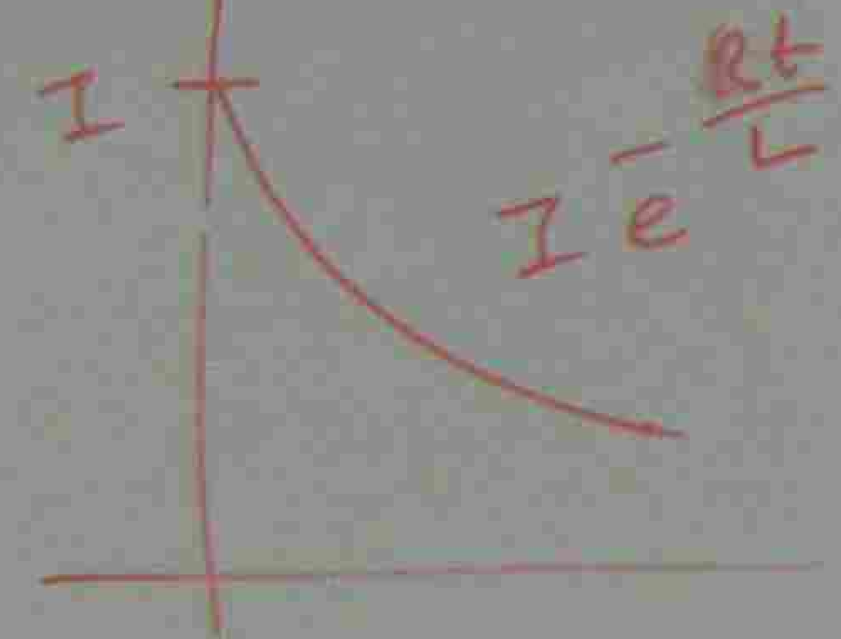
FOR ANS

THE EQU

L

i(t)

SWITCHING OFF




GENERALIZED FORMAT FOR ANSWER OF THE EQUATION

$$L\frac{di}{dt} + IR = E$$

$$\boxed{i(t) = I_p + A e^{-\frac{t}{\tau}} \qquad \tau = \frac{L}{R}}$$

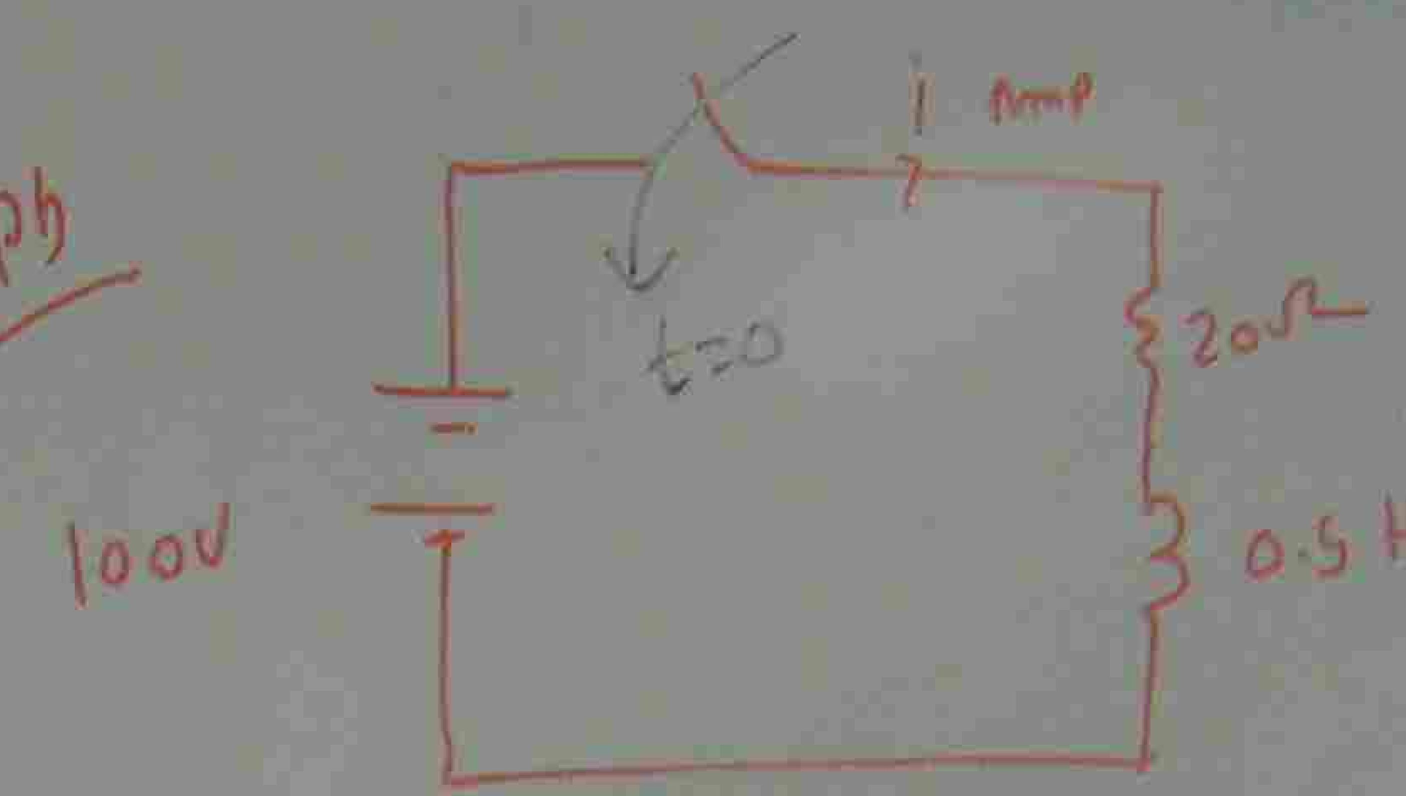
$(1 - e^{-\frac{Rt}{L}})$

$I - I e^{-\frac{Rt}{L}}$

...NSTANT

VARIABLE WITH e FUNCTION

pb




FOR THE CIRCUIT SHOWN ABOVE, DETERMINE THE FOLLOWING VALUES AFTER THE SWITCH HAS BEEN CLOSED.

(a) THE FINAL VALUE OF CURRENT
(b) THE INITIAL VALUE OF CURRENT
(c) TIME CONSTANT OF THE CIRCUIT
(d) THE EQUATION OF THE CURRENT
(e) THE INITIAL RATE OF CHANGE OF CURRENT.

(a) SWITCH CLOSED, INDUCTOR IS SHORT CIRCUITED

$$I_{FINAL} = \frac{V}{R} = \frac{100}{20} = 5A$$

(b) $I_{INITIAL} = 0$

(c) TIME CONSTANT $(\tau) = \frac{L}{R} = \frac{0.5}{20} = 0.025$ sec $= 25$ ms

(d) $E = IR + L\frac{dI}{dt}$

$100 = 20I + 0.5\frac{dI}{dt}$ CIRCUIT EQUATION

$$i(t) = I\left(1 - e^{-t/\tau}\right)$$

FINAL CURRENT

$$= 5\left(1 - e^{-\frac{t}{0.025}}\right)$$

$$i(t) = 5\left(1 - e^{-40t}\right)$$

(e) $\frac{di(t)}{dt} = \frac{d}{dt}\,5\left(1 - e^{-40t}\right)$

$= \underbrace{\cancel{\frac{d}{dt}5}}_{0} - \frac{d}{dt}5e^{-40t}$

$= -5\times(-40)\,e^{-40t}$

RATE OF CHANGE OF CURRENT $= 200\,e^{-40t}$ A/s

pb

+ 200V

DE T

CURR

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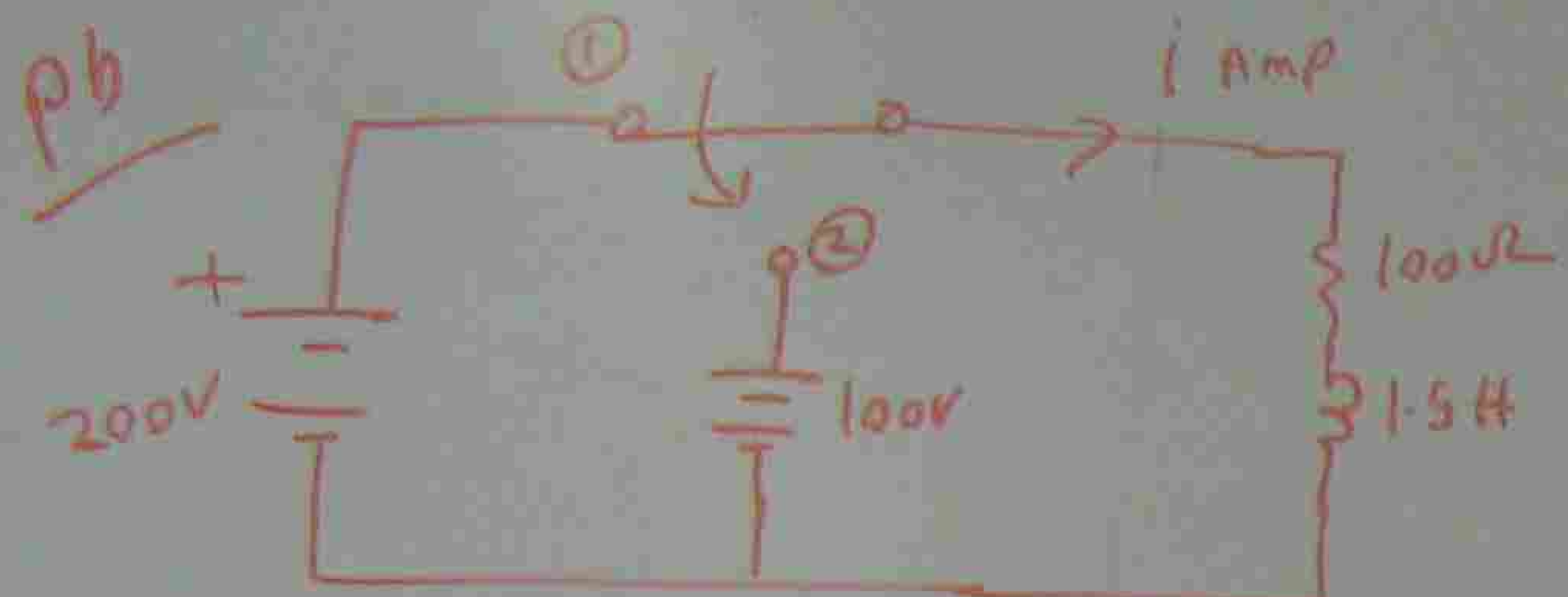
INITI

(OR

WHEN SWITCH

2

pb

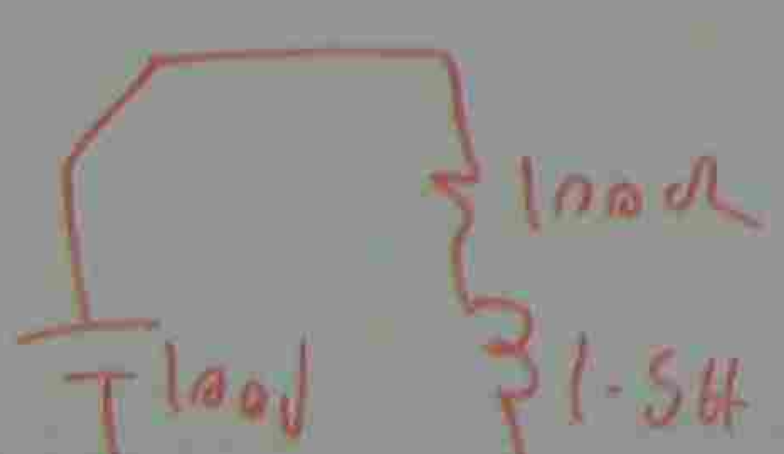



DETERMINE THE EQUATION OF THE CURRENT IN ABOVE FIGURE AFTER SWITCHING TO POSITION ②. ASSUME THAT STEADY STATE CURRENT HAD BEEN ATTAINED IN POSITION ①

INITIAL CURRENT $I(0) = \frac{200V}{100\Omega} = 2$ Amp.

(OR) $I(t)\big|_{t=0} = 2$ Amp (NOT STEADY STATE)

…N SWITCH AT ②




FINAL CURRENT $= \frac{100}{100} = 1$ Amp.

STEADY STATE CONSTANT VALUE

$$E = IR + L\frac{dI}{dt}$$

$$i(t) = I_p + A e^{-t/\tau}$$

FINAL

$$\tau = \frac{L}{R} = \frac{1.5}{100} = 0.015$$

$I_p = 1$ Amp ← FINAL

$$i(t) = 1 + A e^{-\frac{t}{0.015}}$$

WHEN TIME $(t = 0)$, THE INITIAL CURRENT IS 2 AMP.

$$i(t)\big|_{t=0} = 2$$

$$2 = 1 + A e^{-\frac{0}{0.015}}$$

$$2 = 1 + A e^{0}$$

$$2 = 1 + A$$

$$\therefore A = 2 - 1 = 1,$$

$$\therefore i(t) = 1 + 1 e^{-\frac{t}{0.015}}$$

$$= 1 + e^{-t/0.015} = 1 + e^{-6\ldots}$$