REVISION (STATISTICS)

① FIND THE MEAN OF THE FOLLOWING G F D

INTERVAL	FREQUENCY
$0 \rightarrow 9$	1
$10 \rightarrow 19$	2
$20 \rightarrow 29$	4
$30 \rightarrow 39$	8
$40 \rightarrow 49$	4
$50 \rightarrow 59$	1

② FIND THE VARIANCE AND STANDARD DEVIATION OF THE FOLLOWING SAMPLE OF GROUPED DATA

INTERVAL	FREQUENCY
10 — 14	4
15 — 19	7
20 — 24	5
25 — 29	3
30 — 34	1

VARIANCE

$$s^2 = \frac{\sum f(x-\bar{x})^2}{n-1} = \frac{\sum f x^2 - n\bar{x}^2}{n-1}$$

STANDARD DEVIATION

$$s = \sqrt{\frac{\sum f x^2 - n\bar{x}^2}{n-1}}$$

THE VARIANCE AND STANDARD DEVIATION OF
NG SAMPLE OF GROUPED DATA

	FREQUENCY
	4
	7
	5
	3
	1

$$\frac{f(x-\bar{x})^2}{n-1} = \frac{\sum fx^2 - n\bar{x}^2}{n-1}$$

$$\sqrt{\frac{\sum fx^2 - n\bar{x}^2}{n-1}}$$

1

INTERVAL	MEAN (x) = $\frac{\text{SMALLEST} + \text{HIGHEST}}{2}$	f	fx
$0 \to 9$	$\frac{0+9}{2} = 4.5$	1	$4.5 \times 1 = 4.5$
$10 \to 19$	$\frac{10+19}{2} = 14.5$	2	$14.5 \times 2 = 29$
$20 \to 29$	$\frac{20+29}{2} = 24.5$	4	$24.5 \times 4 = 98$
$30 \to 39$	$\frac{30+39}{2} = 34.5$	8	$34.5 \times 8 = 276$
$40 \to 49$	$\frac{40+49}{2} = 44.5$	4	$44.5 \times 4 = 178$
$50 \to 59$	$\frac{50+59}{2} = 54.5$	1	$54.5 \times 1 = 545$
		$\sum f = 20$	$\sum fx = 640$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{640}{20}$$

$$= 32$$

2

INTERVAL
10 – 14
$15 \to 19$
20 – 24
25 – 29
30 – 34

$\bar{x} = \sum$

VARIANCE = $\sum$

STANDARD DEVIATION

...AN = SMALLEST + HIGHEST / 2	f	fx
= 4.5	1	4.5×1 = 4.5
= 14.5	2	14.5×2 = 29
= 24.5	4	24.5×4 = 98
= 34.5	8	34.5×8 = 276
= 44.5	4	44.5×4 = 178
= 54.5	1	54.5×1 = 54.5

$\Sigma f = 20$ $\Sigma fx = 640$

$$\frac{\ldots x}{\ldots f} = \frac{640}{20}$$

$$= 32$$

2

INTERVAL	$\bar{x}$ = MID POINT	f	fx	fx^2
10 – 14	$\frac{10+14}{2} = 12$	4	12×4 = 48	$4\times12^2 = 576$
15 → 19	$\frac{15+19}{2} = 17$	7	17×7 = 119	$7\times17^2 = 2023$
20 – 24	$\frac{20+24}{2} = 22$	5	22×5 = 110	$5\times22^2 = 2420$
25 – 29	$\frac{25+29}{2} = 27$	3	27×3 = 81	$3\times27^2 = 2187$
30 – 34	$\frac{30+34}{2} = 32$	1	32×1 = 32	$1\times32^2 = 1024$

$\Sigma f = 20$ $\Sigma fx = 390$ $\Sigma fx^2 = 8230$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{390}{20} = 19.5 \quad , n = \Sigma f = 20$$

$$\text{VARIANCE} = s^2 = \frac{\Sigma fx^2 - n(\bar{x})^2}{n-1} = \frac{8230 - 20\times(19.5)^2}{20-1}$$

$$= \frac{8230 - 7605}{19} = 32.89$$

$$\text{STANDARD DEVIATION} = s = \sqrt{32.89} = 5.735$$

76

023

420

187

024

230

= 20

$(19.5)^2$

89 = 32.89

MATHS A+B REVISION

1. FACTORISE $8x^2 + 73x + 9$
2. $(x-7)(x+2) = 4x - 1$ FIND x
3. $\log_{10} \frac{k}{k-x} = t$, FIND x
4. PLOT GRAPH FOR $y = 2x^2 - 12x + 13$
5. THE FOLLOWING VALUES OF X AND Y ARE BELIEVED TO SATISFY THE EQUATION $Y = AX^2 + BX$. FIND A LINEAR EQUATION THAT SUITS THE INFORMATION AND EVALUATE A & B
6. CALCULATE THE AMOUNT THAT \$5000 GROWS TO WHEN IT IS INVESTED FOR 3 WEEKS AT INTEREST RATE 13% PER YEAR. INTEREST IS COMPOUNDED DAILY
7. PLOT $Y = 7\sin 2\theta$

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1. $8x^2 + 73x + 9$

$$\begin{array}{ccc} 8x & & 1 \\ & \times & \\ x & & 9 \end{array}$$

$72x + x = 73x$

$(8x+1)(x+9)$

---

2. $(x-7)(x+2) = 4x - 1$

$x^2 - 7x + 2x - 14 = 4x - 1$

$x^2 - 5x - 14 = 4x - 1$

$x^2 - 5x - 14 - 4x + 1 = 0$

$x^2 - 9x - 13 = 0$

$Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 1 \times (-13)}}{2 \times 1}$$

$x = \frac{9 \pm \sqrt{}}{2}$

$= \frac{9 \pm \sqrt{}}{}$

$= \frac{9 \pm 11.5}{2}$

$= \frac{9 + 11.53}{2}$

$= 10.265$

3. $\log_{10} \frac{k}{k-x}$

$\frac{k}{k-x}$

$\frac{k}{10^t}$

$x =$

$x =$

$x =$

$x + 9$

$\to 1$

< 9

$= 73x$

$x + 9\,]$

$x+2) = 4x - 1$

$2x - 14 = 4x - 1$

$-14 = 4x - 1$

$4 - 4x + 1 = 0$

$-13 = 0$

$+ c = 0$

$\sqrt{B^2 - 4AC}$

A

$)\pm\sqrt{(-9)^2 - 4\times1\times(-13)}$

2×1

$$x = \frac{9 \pm \sqrt{81+52}}{2}$$

$$= \frac{9 \pm \sqrt{133}}{2}$$

$$= \frac{9 \pm 11.53}{2}$$

$$= \frac{9+11.53}{2} \ (\text{or}) \ \frac{9-11.53}{2}$$

$$= 10.265 \ (\text{OR}) \ -1.265$$

③ $\log_{10} \frac{k}{k-x} = t$

$$\frac{k}{k-x} = 10^t$$

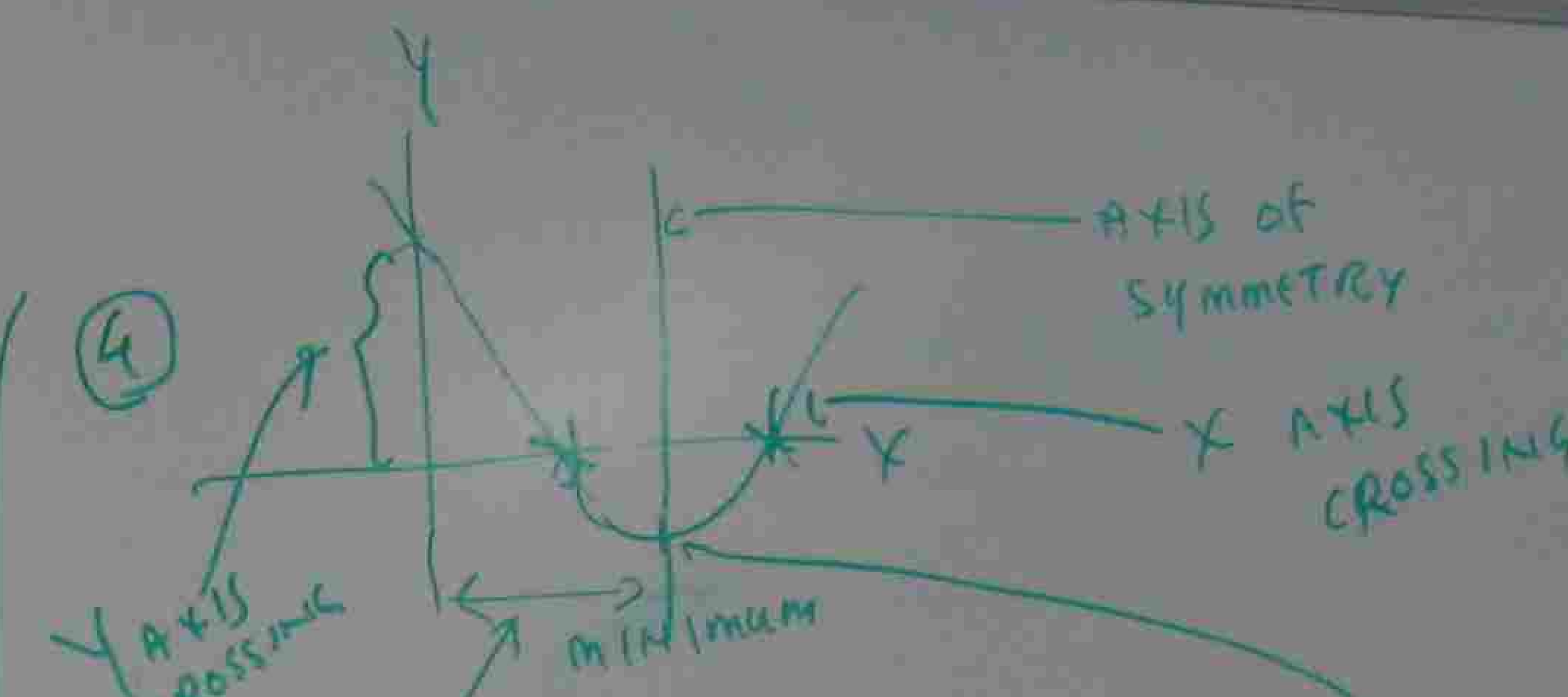
$$\frac{k}{10^t} = k - x$$

$$x = k - \frac{k}{10^t}$$

$$x = k\left(1 - \frac{1}{10^t}\right)$$

$$x = k\left(\frac{10^t - 1}{10^t}\right)$$

④



① COMPARE WITH $Y = Ax^2 + Bx + C$

THEN $\frac{-B}{2A}$ = AXIS OF SYMMETRY

② FIND Y VALUE AT AXIS OF SYMMETRY

③ FIND Y AXIS CROSSING BY PUTTING $X = 0$

④ FIND X AXIS CROSSING BY PUTTING $Y = 0$

$$Y = 2x^2 - 12x + 13$$

$$Y = Ax^2 + Bx + C$$

$A = 2$, $B = -12$, $C = 13$

① AXIS OF SY

② $Y = 2x$

Y AT AXIS

$= 2$

③ Y AX

$Y = 2$

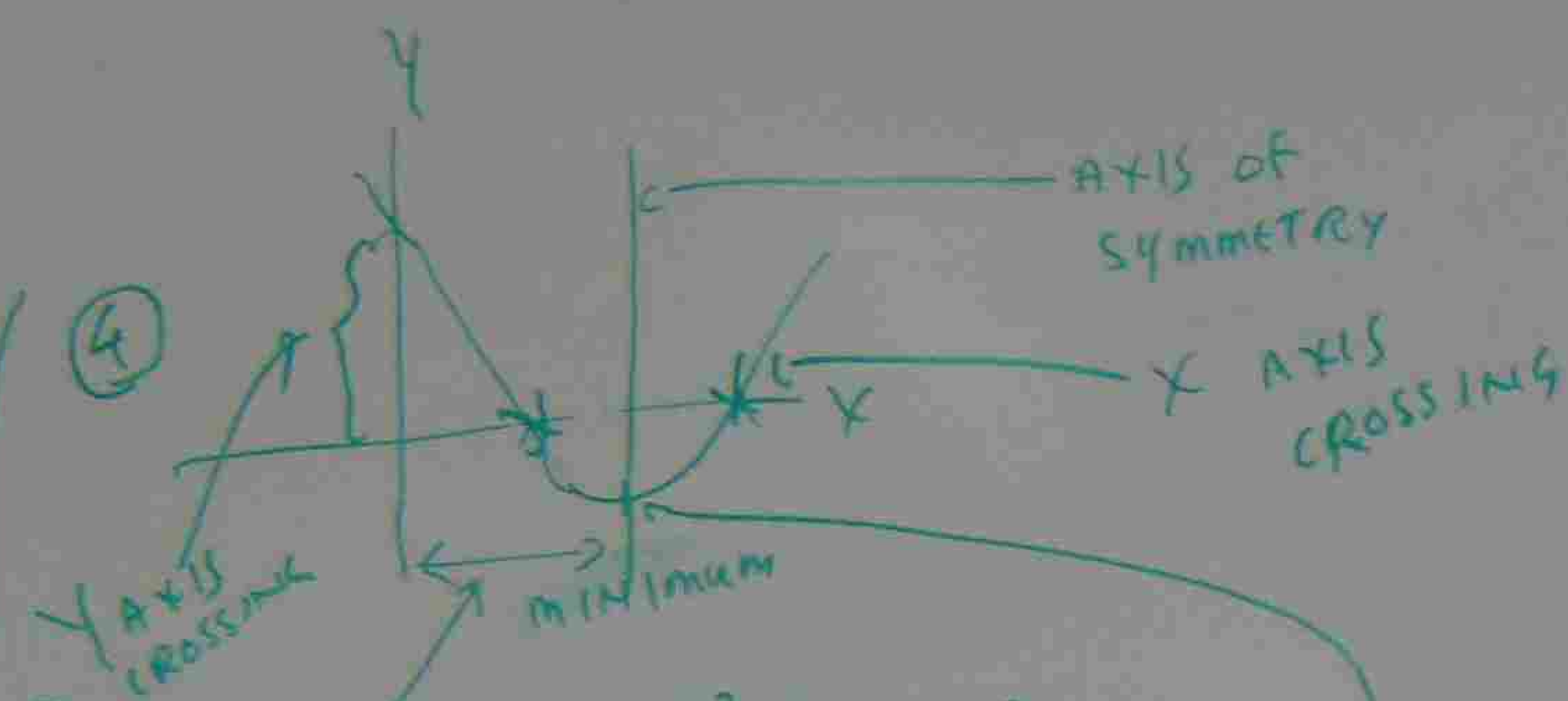
④ X A

$0 =$

$X =$

13

-5



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$Y = 2X^2 - 12X + 13$

$Y = AX^2 + BX + C$

$A = 2, \; B = -12, \; C = 13$

① AXIS OF SYMMETRY $= \frac{-B}{2A} = \frac{-(-12)}{2 \times 2} = \frac{12}{4} = 3$

X VALUE

② $Y = 2X^2 - 12X + 13$

Y AT AXIS OF SYMMETRY

$= 2 \times 3^2 - 12 \times 3 + 13 = 2 \times 9 - 36 + 13 = -5$

③ Y AXIS CROSSING, $X = 0$

$Y = 2X^2 - 12X + 13 = 2 \times 0^2 - 12 \times 0 + 13 = 13$

④ X AXIS CROSSING $Y = 0$

$0 = 2X^2 - 12X + 13$

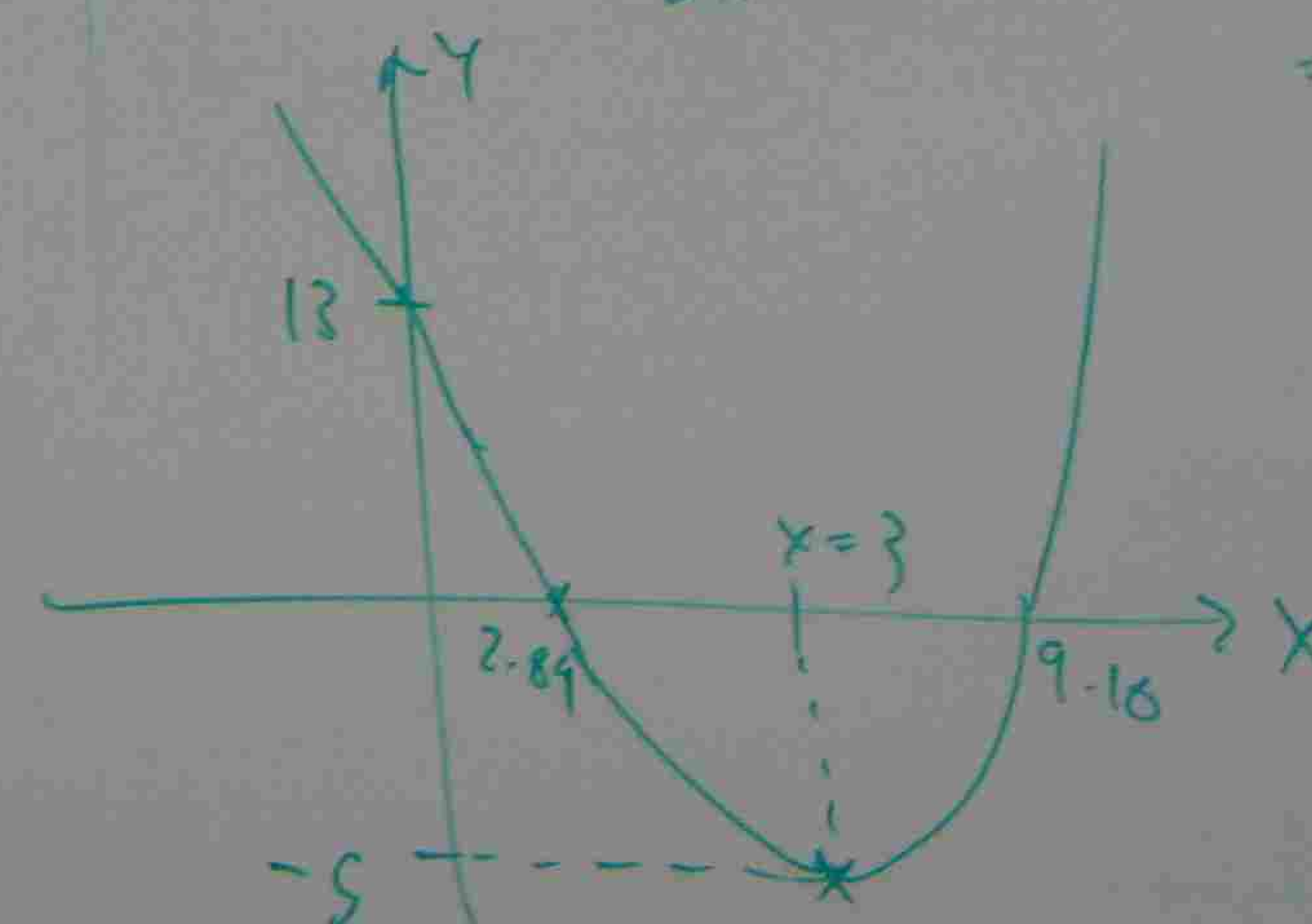
$AX^2 + BX + C = 0$

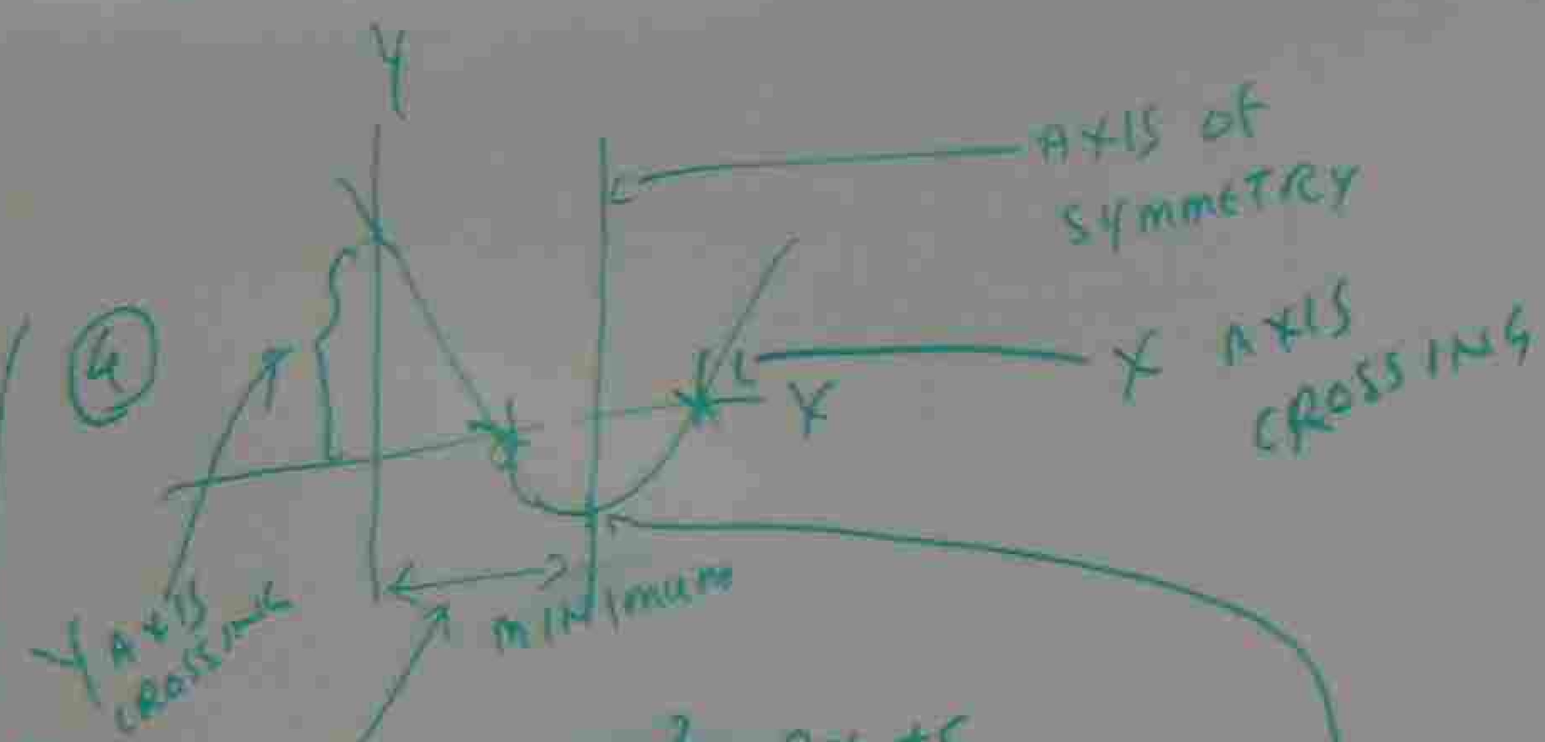
$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = -\frac{(-12) \pm \sqrt{(-12)^2 - 4 \times 2 \times 13}}{2 \times 2}$$

$$= \frac{12 \pm \sqrt{144 - 104}}{4}$$

$$= \frac{12 \pm \sqrt{40}}{4} = \frac{12 \pm 6.32}{4}$$

$= 9.16$ (OR) 2.84





① compare with $Y = AX^2 + BX + C$

Then $\frac{-B}{2A}$ = AXIS OF SYMMETRY

② FIND Y VALUE AT AXIS OF SYMMETRY

③ FIND Y AXIS CROSSING BY PUTTING $X = 0$

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$$Y = 2X^2 - 12X + 13$$

$$Y = AX^2 + BX + C$$

$A = 2$, $B = -12$, $C = 13$

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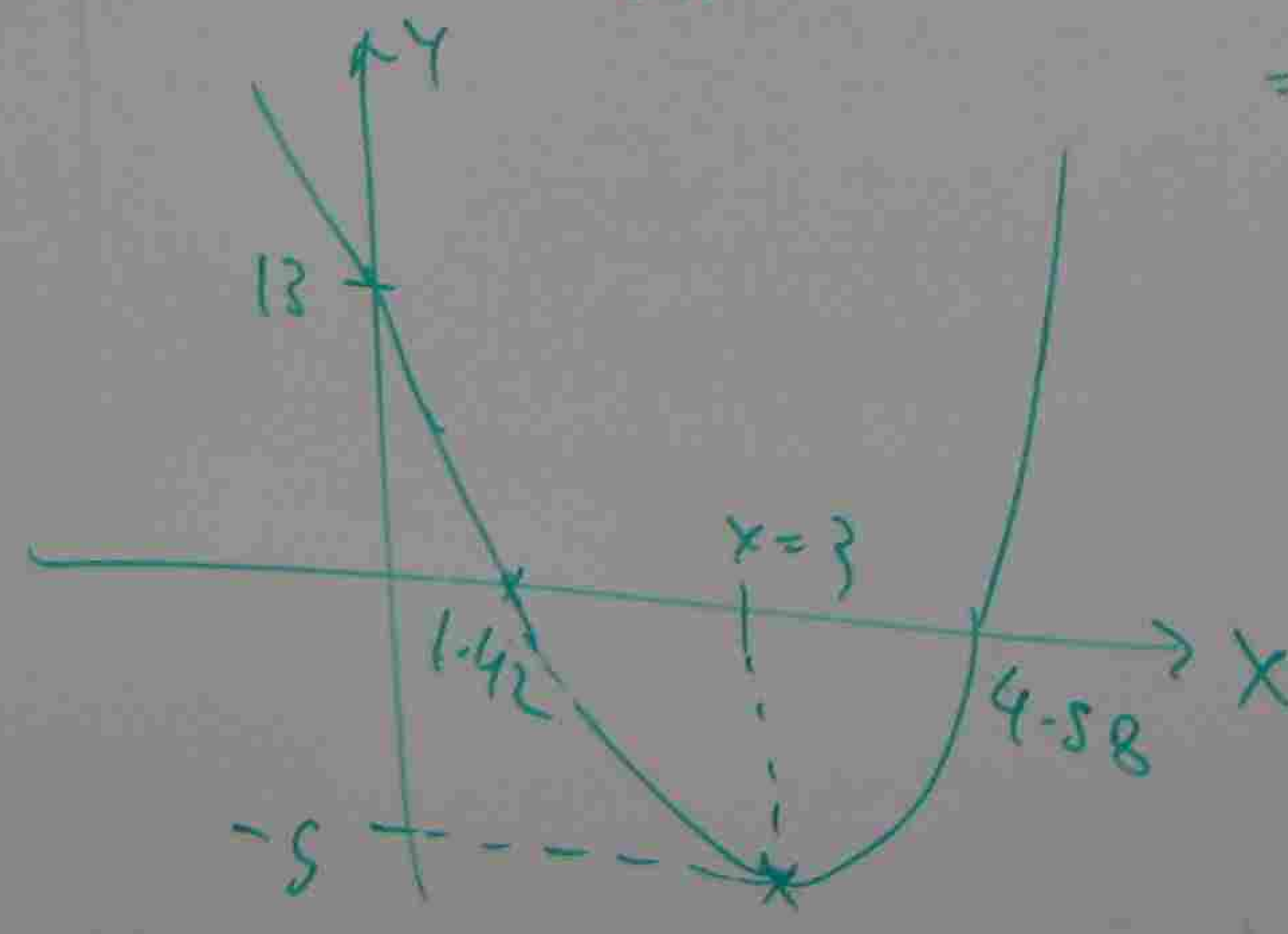
$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 2 \times 13}}{2 \times 2}$$

$$= \frac{12 \pm \sqrt{144 - 104}}{4}$$

$$= \frac{12 \pm \sqrt{40}}{4} = \frac{12 \pm 6.32}{4}$$

$= 4.58$ (OR) 1.42



MATHS A + B REVISION

① FACTORISE $8x^2 + 73x + 9$

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FIND X

③ $\log_{10} \frac{k}{k-x} = t$, FIND X

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⑤ THE FOLLOWING VALUES OF X AND Y ARE BELIEVED TO SATISFY THE EQUATION $Y = Ax^2 + Bx$

FIND A LINEAR EQUATION THAT SUITS THE INFORMATION AND EVALUATE A & B

X = 1	2	3	4	5
Y = 5	16	34	57	84

⑥ CALCULATE THE AMOUNT THAT $\$5000$ GROWS TO WHEN IT IS INVESTED FOR 3 WEEKS AT INTEREST RATE 13% PER YEAR INTEREST IS COMPOUNDED DAILY

⑦ PLOT $Y = 7 \sin 2\theta$

① $8x^2 + 73$

$8x$

x

$72x + x$

$(8x+1)($

② $(x-7)$

$x^2 - 7x$

$x^2 - 5$

$x^2 - 5x -$

$x^2 - 9x$

$Ax^2 + B$

$x = -B$

$= -($

$=$

5

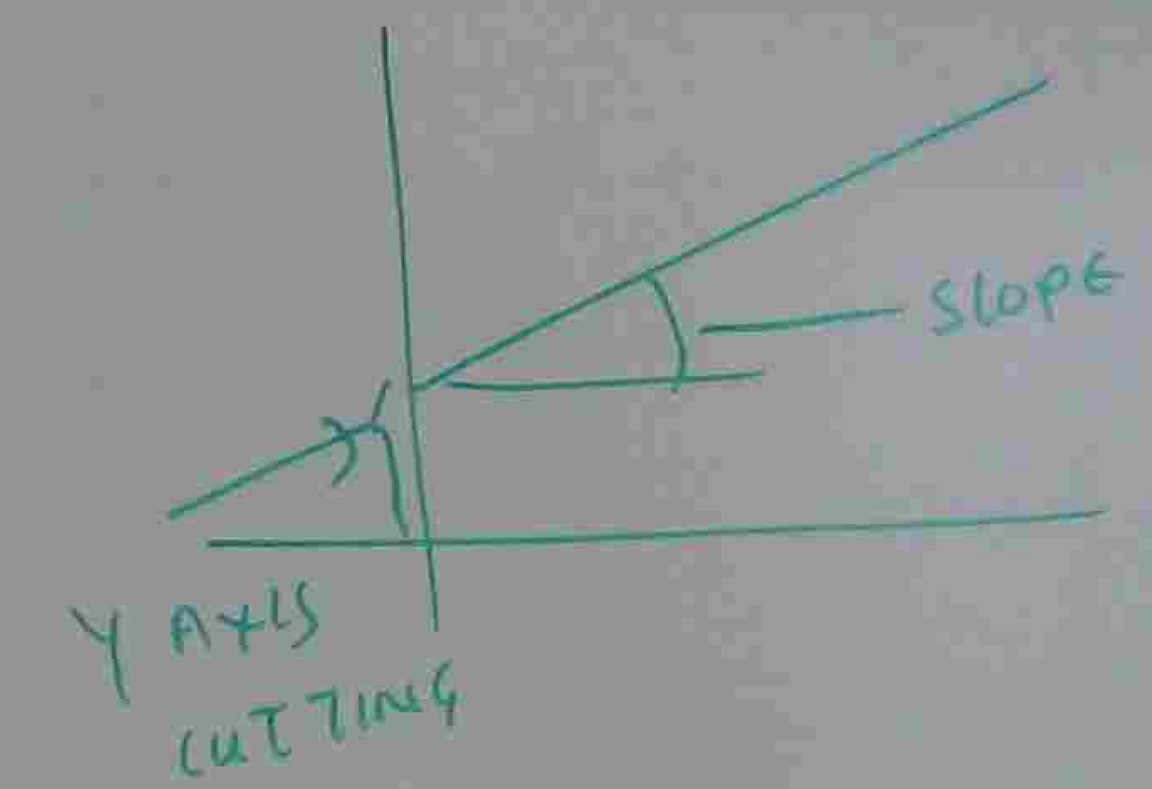
X	1	2	3	4	5
Y	5	16	34	57	84

X	1	2	3	4	5
$\frac{y}{x} = Y$	$\frac{5}{1}=5$	$\frac{16}{2}=8$	$\frac{34}{3}=11.3$	$\frac{57}{4}=14.3$	$\frac{84}{5}=16.8$

$$y = Ax^2 + Bx$$

$$\frac{y}{x} = Ax + B$$

$$Y = Ax + B$$



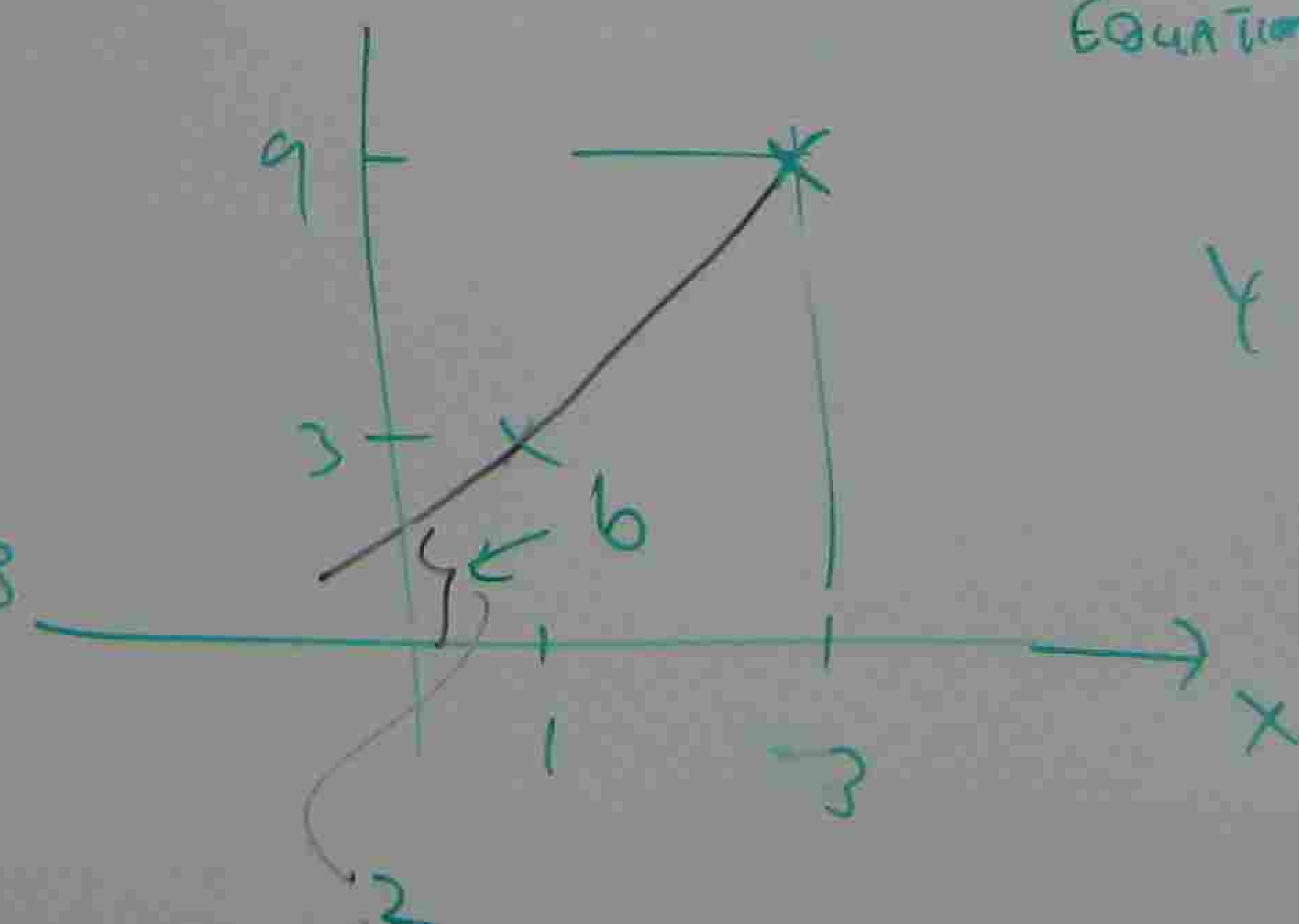
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8 - 5}{2 - 1} = 3$$

Slope = 3

$x = 1 \longrightarrow y = 3x = 3 \times 1 = 3$

$x = 3 \longrightarrow y = 3 \times 3 = 9$



EQUATION = SLOPE × X + Y CUTTING POINT

$$Y = 3x + 2$$

$$\frac{y}{x} = 3x + 2$$

$$y = 3x^2 + 2x$$

6

A

7

— SLOPE

...TION = SLOPE × X + Y CUTTING POINT

$y = 3x + 2$

$\frac{y}{x} = 3x + 2$

x

$y = 3x^2 + 2x$

6

$$A = \text{INVEST AMOUNT} \times \left(1 + \frac{\frac{\text{ANNUAL INTEREST RATE}}{365}}{100}\right)^{\text{No. of DAYS INVESTED}}$$

$$= 5000 \times \left(1 + \frac{\frac{13}{365}}{100}\right)^{3\times 7}$$

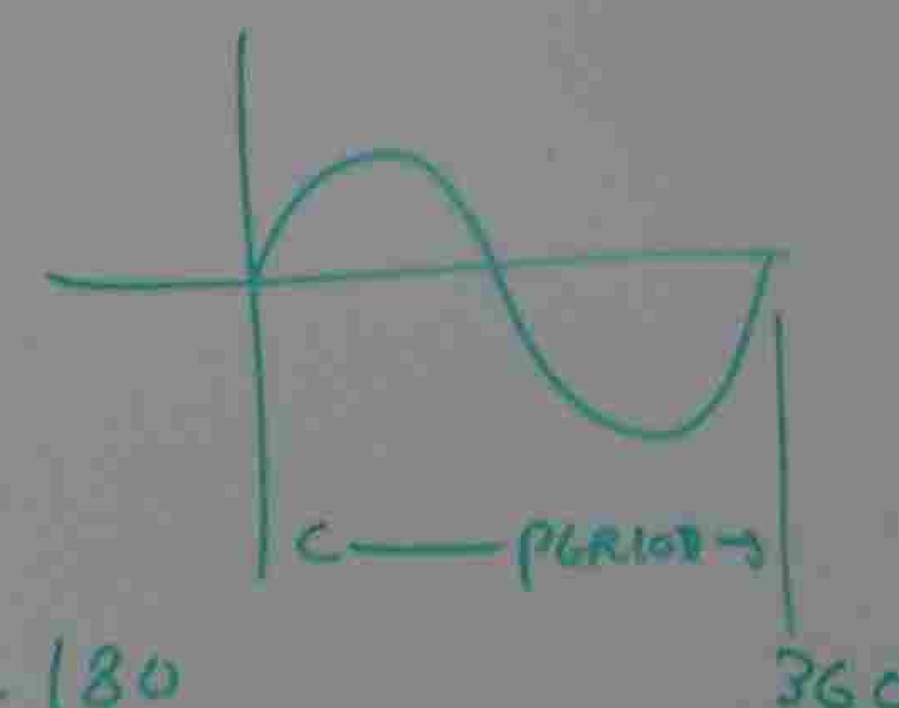
$$= 5000 \left(1 + \frac{\frac{13}{365}}{100}\right)^{21} = \$5037.53$$

7

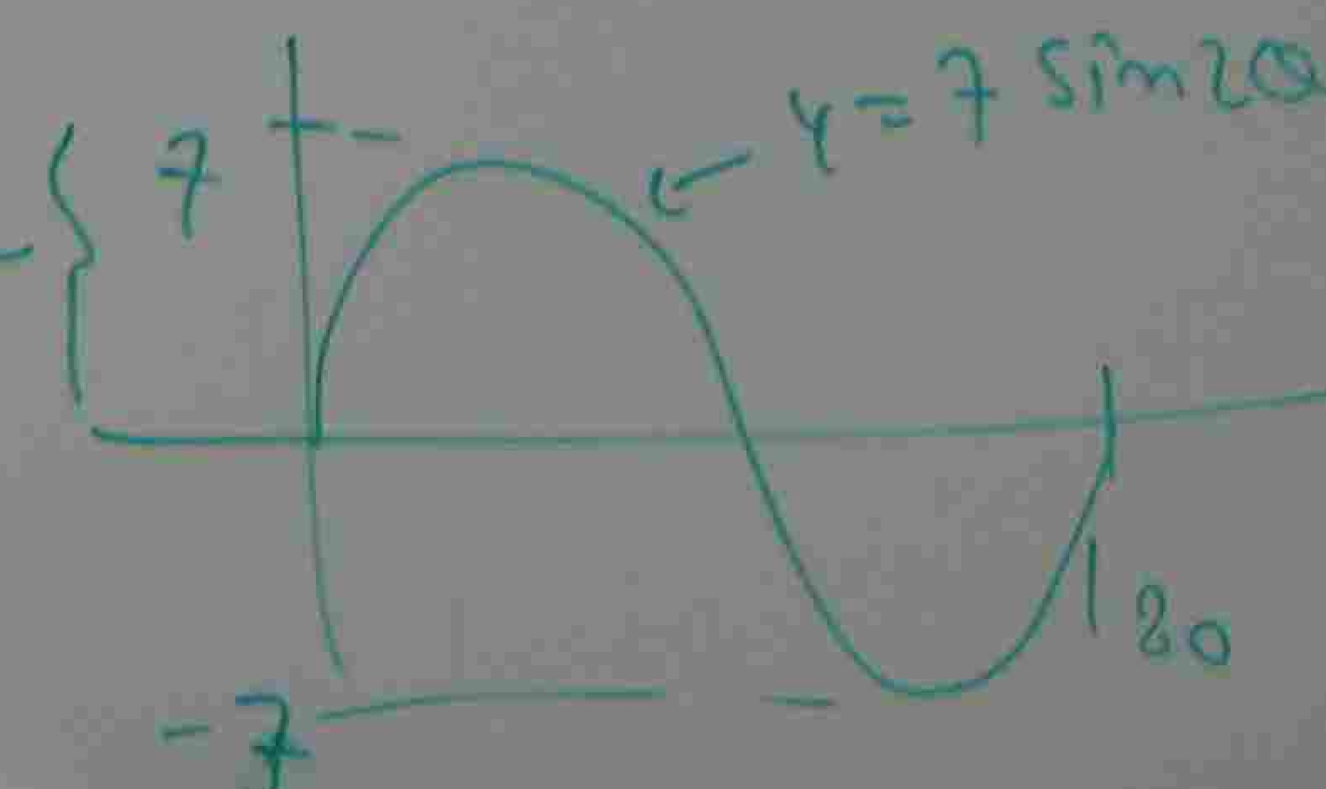
$$y = 7 \sin \underline{2\theta}$$

HALF OF AMPLITUDE

PERIOD = 360°



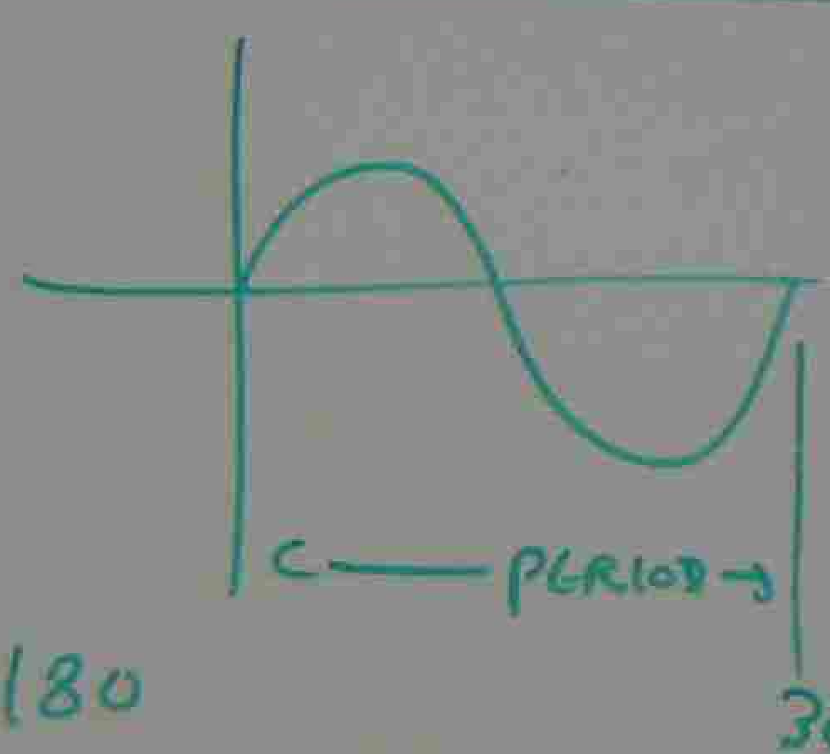
$2\theta = 360 \rightarrow \theta = 180$



$$1 + \frac{\frac{\text{ANNUAL INTEREST RATE}}{365}}{100}\Bigg)^{\text{No. of DAYS INVESTED}}$$

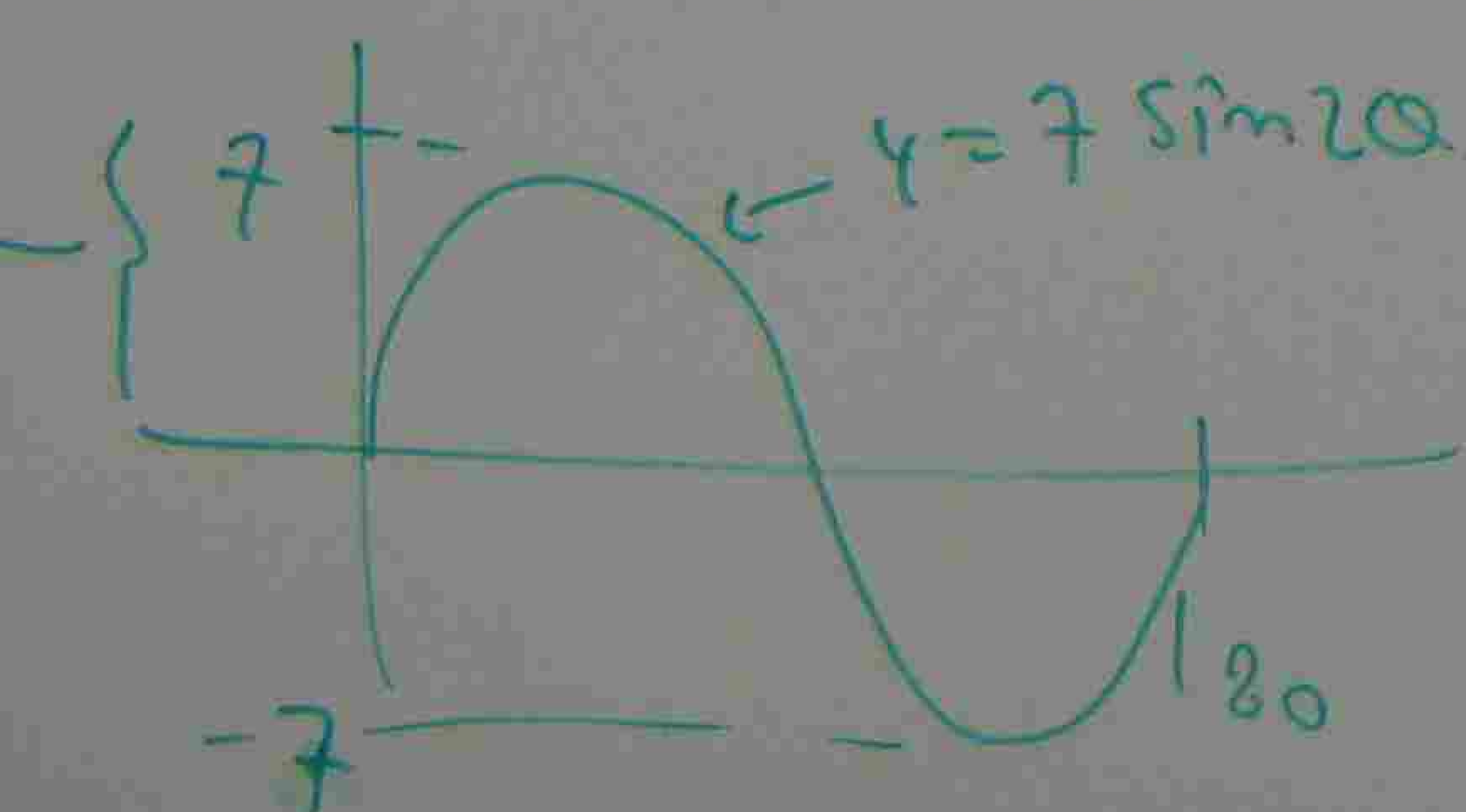
$$\frac{\frac{13}{365}}{100}\Bigg)^{3\times 7}$$

$$\Bigg)^{21} = \$\,5037.53$$



PERIOD = 360°

$= 360 \rightarrow \theta = 180$



MATHS (3)

1. SKETCH THE CURVE $5.37 \sin(136\times10^{3} t - 0.955)$ mA
   PERIOD
   PHASE SHIFT

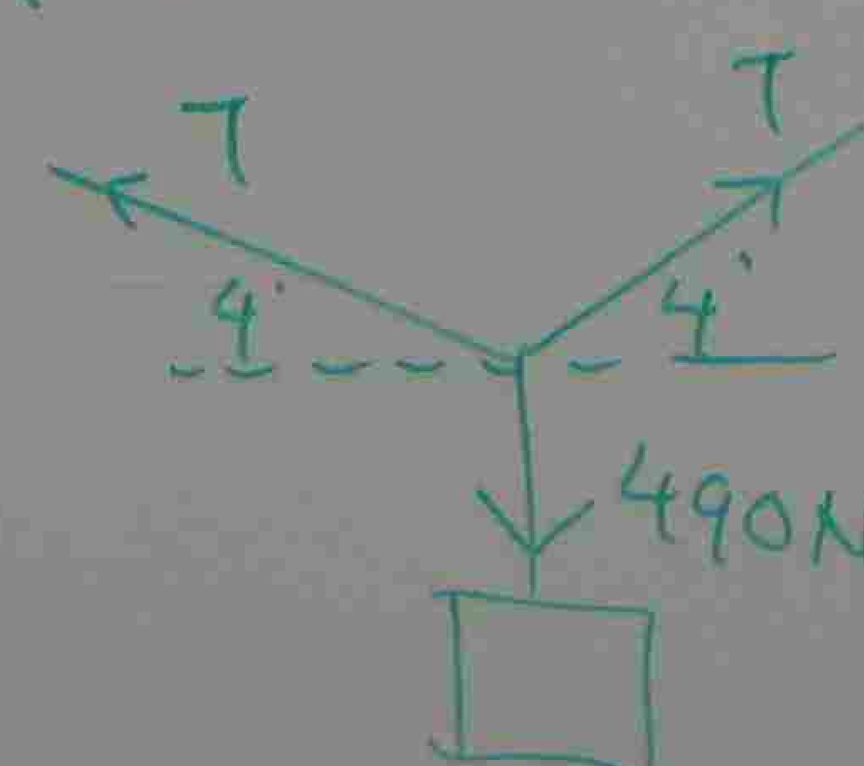
2. PROVE THAT $\dfrac{\sec^2 A - 1}{1 + \tan^2 A} = \sin^2 A$

3. FIND THE PHASE DIFFERENCE
   $\sin(\theta - 30)$ & $\sin(\theta - 100)$
   GRAPHICALLY

4. IN $\triangle ABC$, $\angle B = 65°$, $\angle C = 40°$
   AB = 5.4 m, FIND AC, BC, $\angle A$

5. SIMPLIFY $\sin 5x \cos 4x - \cos 5x \sin 4x$

6. FIND EXACT VALUE OF $\sin 15$

7. 

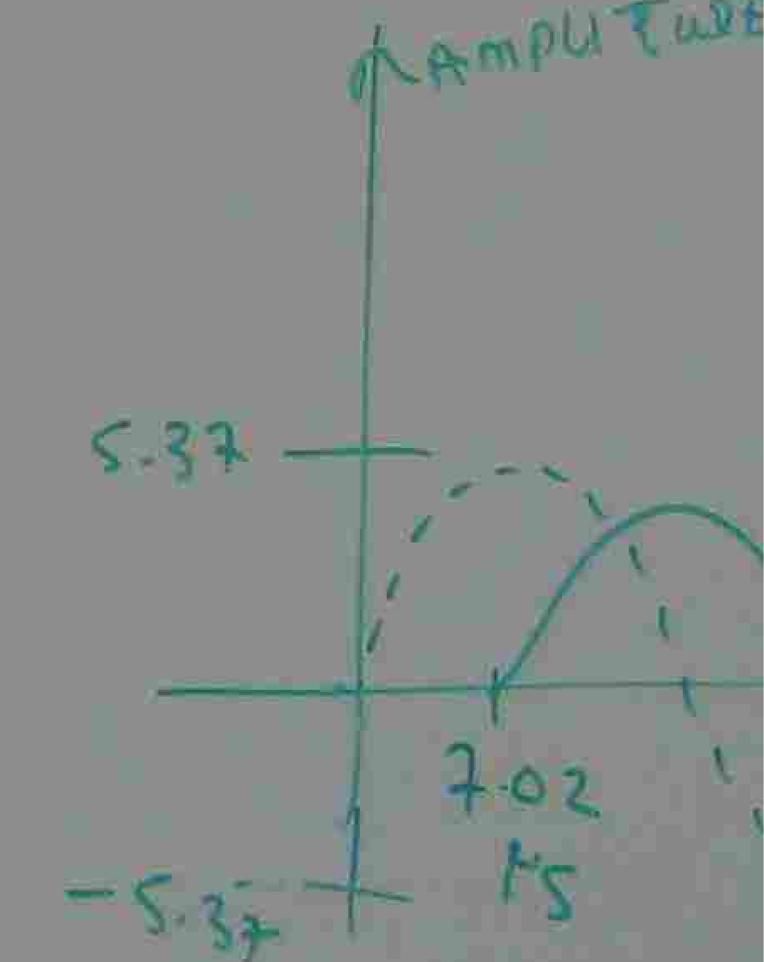
   FIND (i) TENSION IN CABLE
   (ii) RATIO OF TENSION TO LOAD SUPPORTED

1. AMPLITUDE, P

5.37 si

HALF OF AMPLITUDE

PERI



ss) mA

PHASE SHIFT

① AMPLITUDE, PERIOD, PHASE SHIFT ANGLE

$$5.37 \sin(\underline{136 \times 10^3} t - \underline{0.955})\ mA$$

5.37 ↑ HALF OF AMPLITUDE

136×10^3 ↑ ω

0.955 ↑ ϕ

$$\text{PERIOD} = T = \frac{2\pi}{\omega}$$

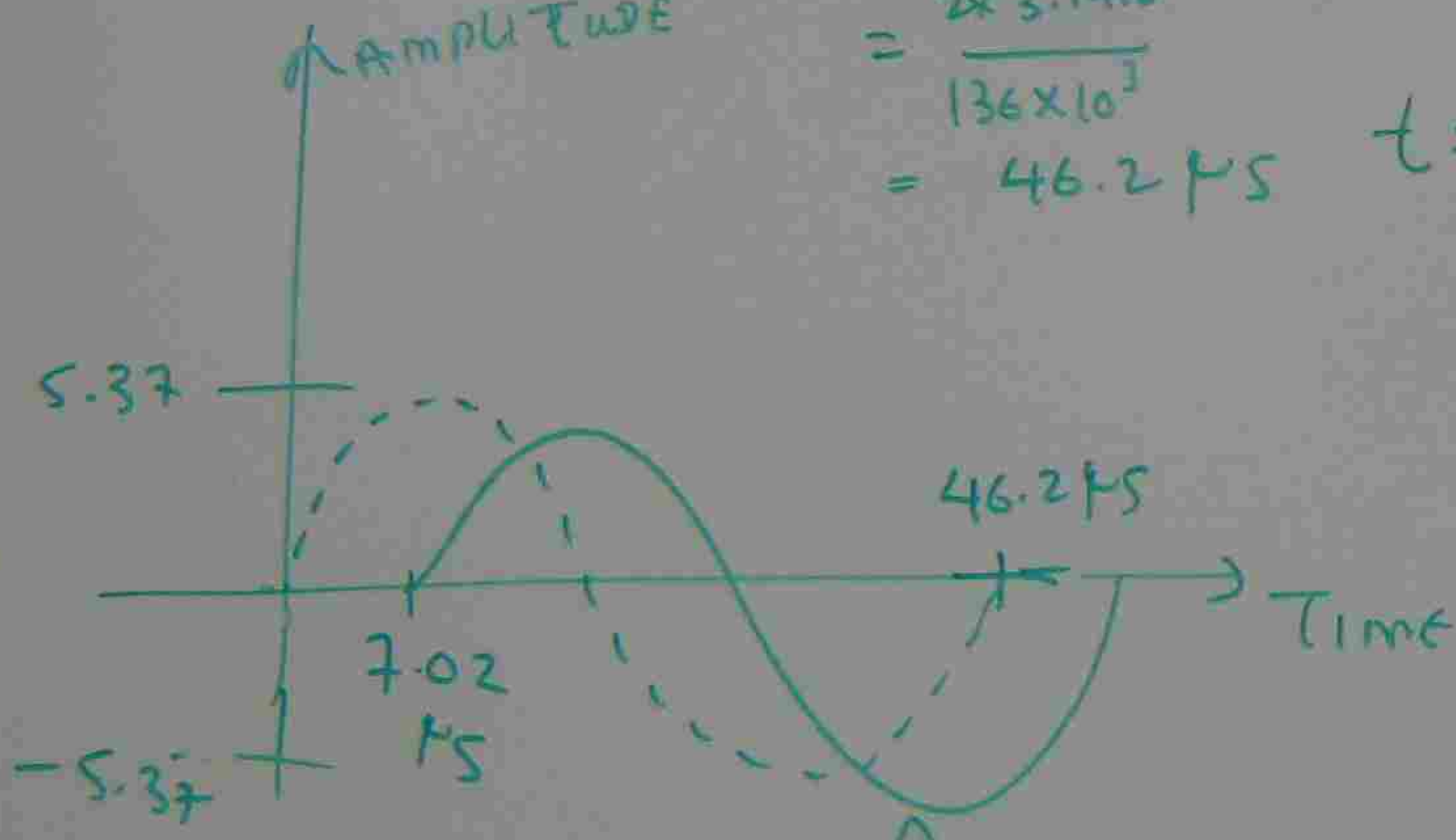
$$= \frac{2 \times 3.1416}{136 \times 10^3}$$

$$= 46.2\ \mu s$$

$$\text{Time} = \frac{\phi}{\omega}$$

$$t = \frac{0.955}{136 \times 10^3}$$

$$= 7.02\ \mu s$$



$$5.37 \sin(136 \times 10^3 t - 0.955)$$

mA

② LHS

$$\frac{\sec^2 A - 1}{1 + \tan^2 A}$$

$$\boxed{\sec A = \frac{1}{\cos A} \qquad \tan A = \frac{\sin A}{\cos A}}$$

$$\boxed{\sin^2 A + \cos^2 A = 1}$$

$$\frac{\frac{1}{\cos^2 A} - 1}{1 + \frac{\sin^2 A}{\cos^2 A}}$$

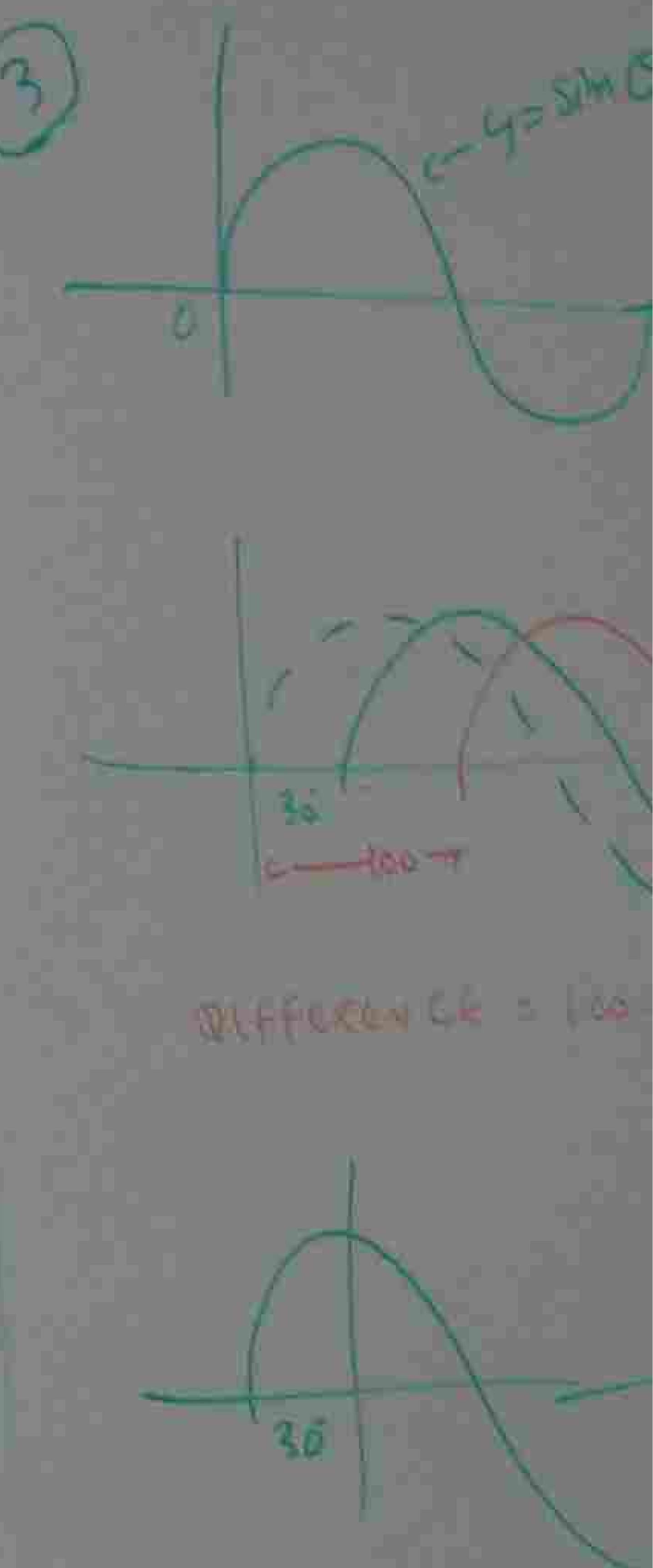
$$\frac{\frac{1 - \cos^2 A}{\cancel{\cos^2 A}}}{\frac{\cos^2 A + \sin^2 A}{\cancel{\cos^2 A}}} = \frac{1 - \cos^2 A}{1}$$

$$= 1 - \cos^2 A$$

$$= \sin^2 A$$

RHS

③



$\boxed{\sin^2 A + \cos^2 A = 1}$

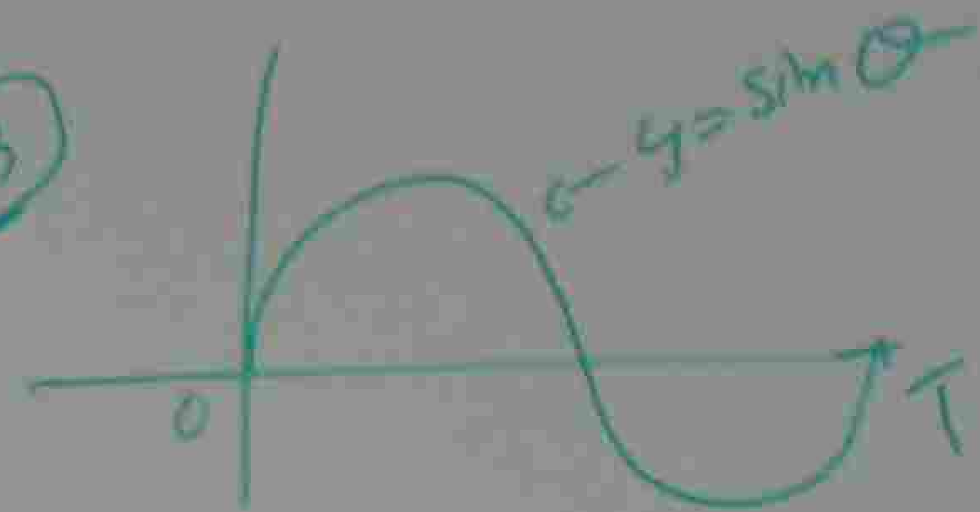
$= \dfrac{1-\cos^2 A}{1}$

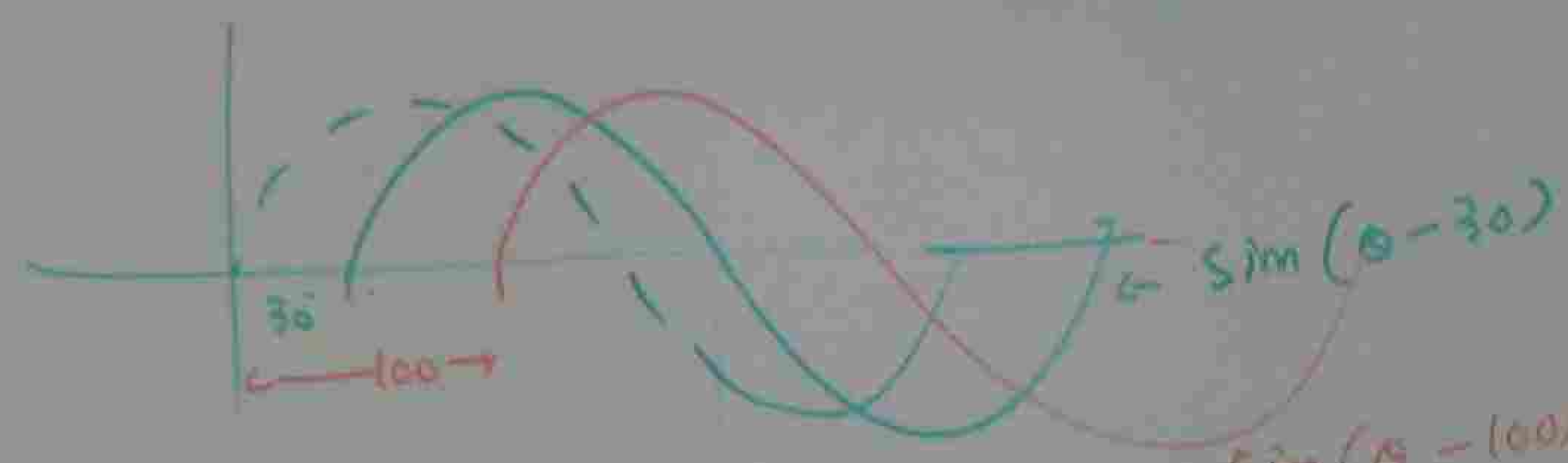
$= 1-\cos^2 A$

$= \sin^2 A$

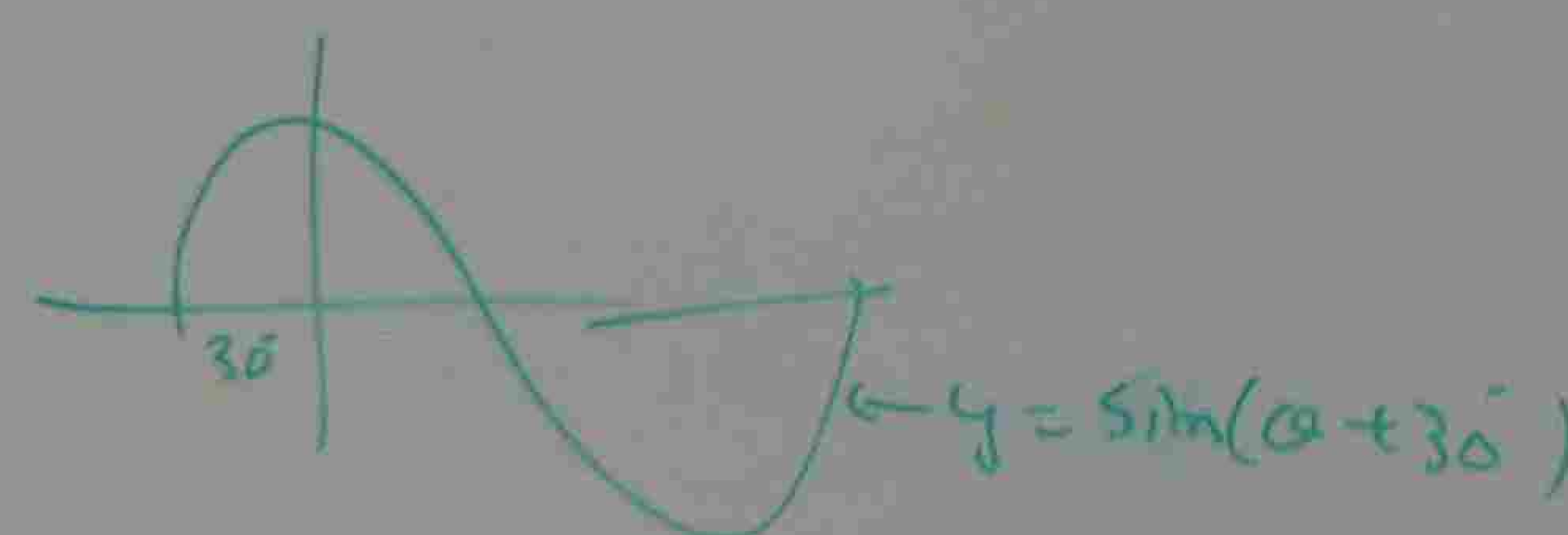
RHS

③

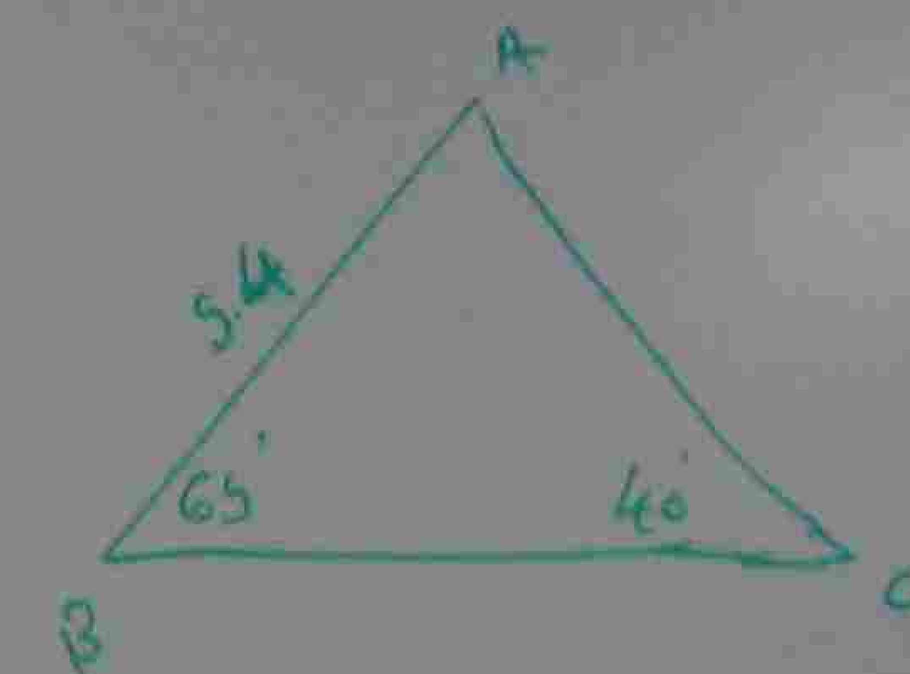




DIFFERENCE = 100 − 30 = 70°



④



$\hat{A} = 180 - (\hat{B} + \hat{C})$

$= 180 - (65 + 40)$

$= 180 - 105 = 75°$

$\dfrac{AB}{\sin C} = \dfrac{AC}{\sin B}$

$\dfrac{5.4}{\sin 40} = \dfrac{AC}{\sin 65}$

$AC = \dfrac{5.4 \sin 65}{\sin 40}$

$= \dfrac{5.4 \times 0.906}{0.642}$

$= 7.62$

$\dfrac{BC}{\sin A} = \dfrac{A}{\sin}$

$\dfrac{BC}{\sin 75} = \dfrac{S}{S}$

$BC = \dfrac{5.4}{\sin}$

$= \dfrac{5.4 \times}{0.6}$

$= 8.1$

⑤ $\sin(A+B)$

$\sin(A-B)$

$\sin 5x \cos$

$= \sin($

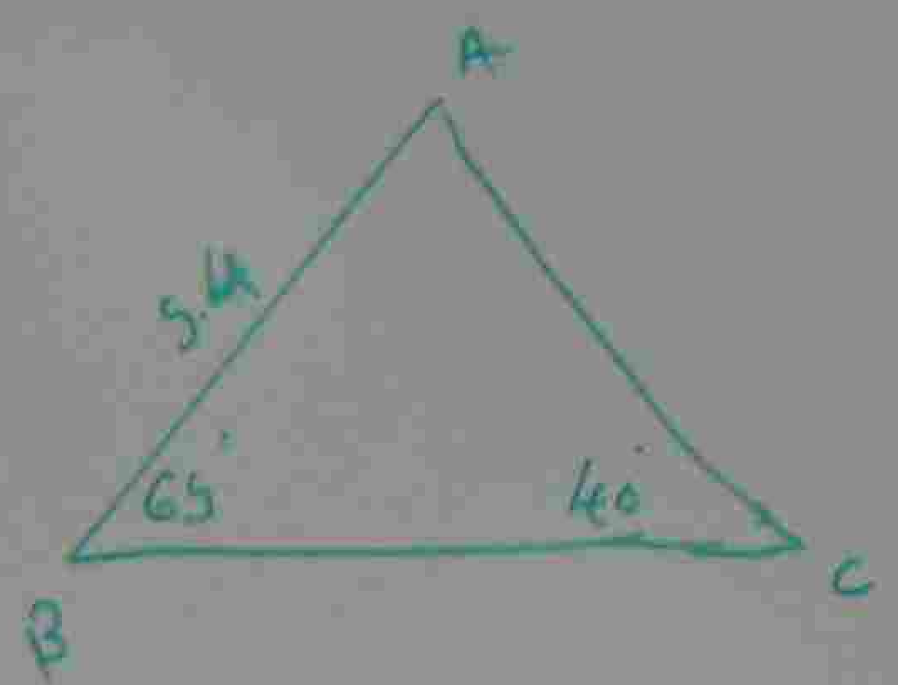
$= \sin$

$= \sin(\theta - 30)$

$\sin(\theta - 100)$

$= \sin(\theta + 30)$

④



$\hat{A} = 180 - (\hat{B} + \hat{C})$

$= 180 - (65 + 40)$

$= 180 - 105 = 75$

$\frac{AB}{\sin C} = \frac{AC}{\sin B}$

$\frac{5.4}{\sin 40} = \frac{AC}{\sin 65}$

$AC = \frac{5.4 \sin 65}{\sin 40}$

$= \frac{5.4 \times 0.906}{0.642}$

$= 7.62$

$\frac{BC}{\sin A} = \frac{AB}{\sin C}$

$\frac{BC}{\sin 75} = \frac{5.4}{\sin 40}$

$BC = \frac{5.4 \sin 75}{\sin 40}$

$= \frac{5.4 \times 0.965}{0.642}$

$= 8.11$

⑤ $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin 5x \cos 4x - \cos 5x \sin 4x$

$= \sin(5x - 4x)$

$= \sin x$

⑥ $\sin 15 = ?$

$\sin 15 = \sin(45 - 30)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

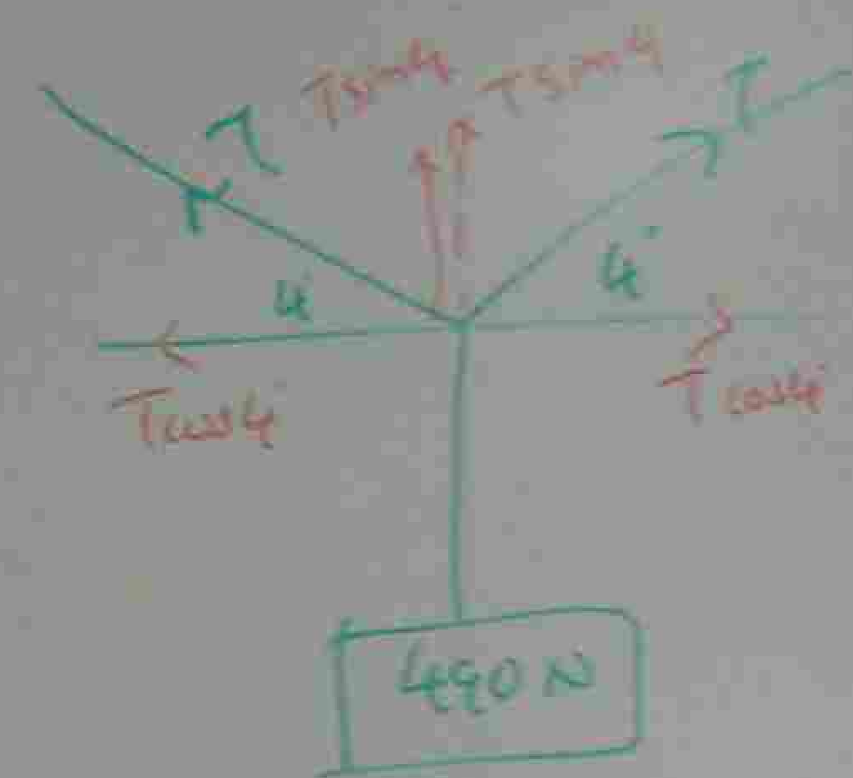
$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6} - \sqrt{2}}{2 \times 2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

7



$$2T\sin 4 = 490$$

$$T = \frac{490}{2\sin 4}$$

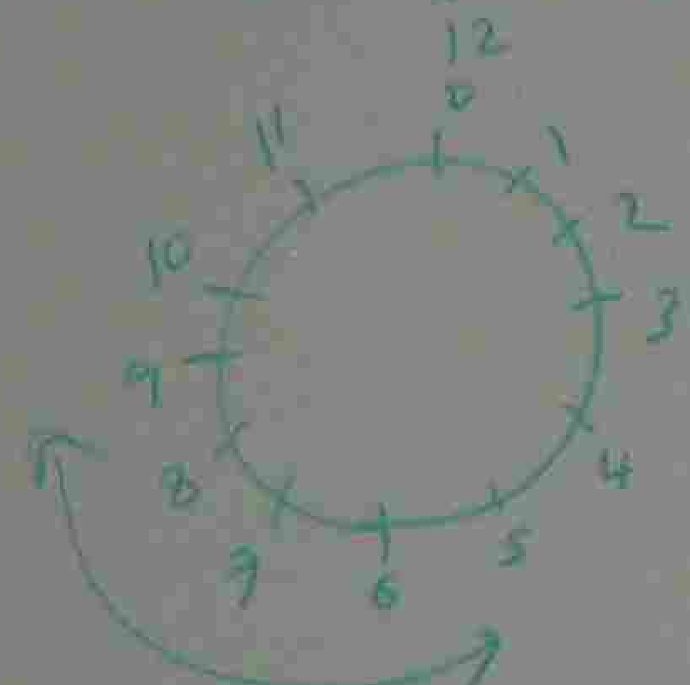
$$= \frac{490}{2\times 0.069}$$

$$= 3550$$

$$\frac{T}{W} = \frac{3550}{490} = 7.24 : 1$$

NORMAL PROBABILITY DISTRIBUTION

PROBABILITY – CHANCE (OR) LIKELIHOOD



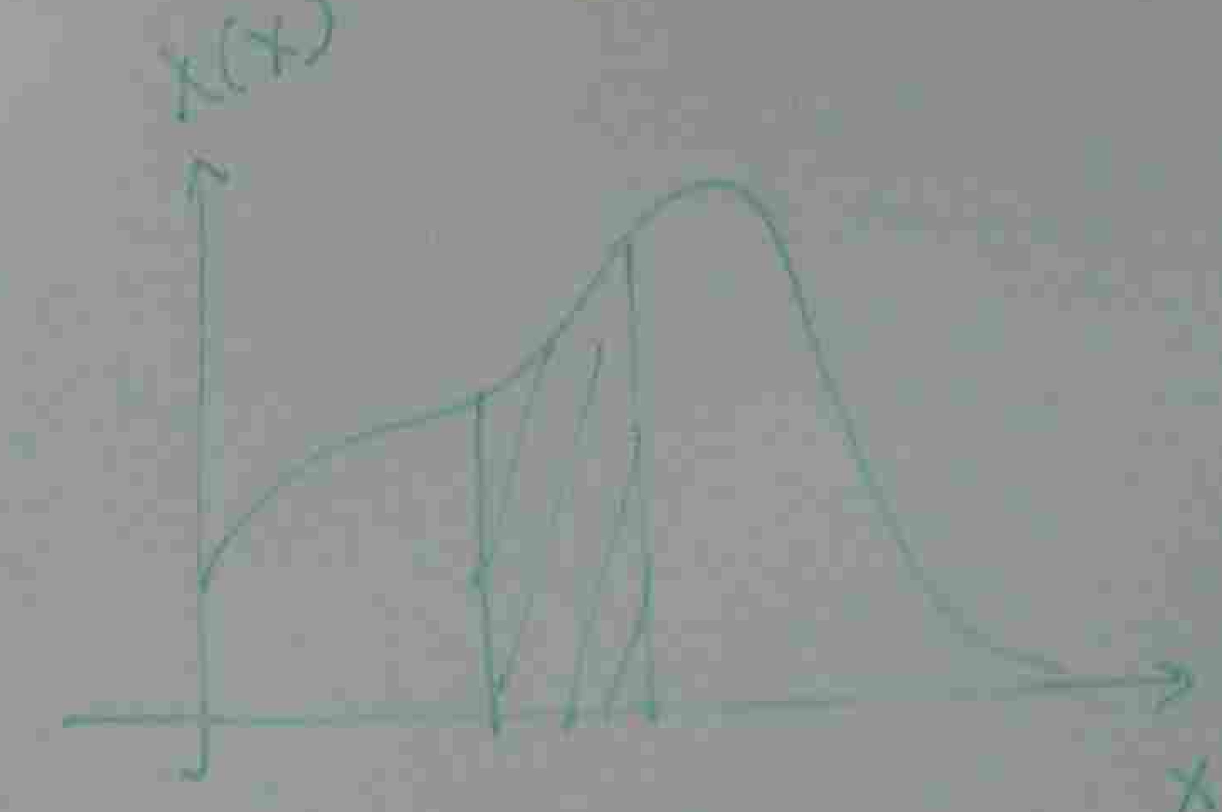
CHANCE TO STOP THE WATCH BETWEEN

5 → 9

TOTAL = 12 DIVISIONS

5 → 9 = 4 DIVISIONS.

$$P(5 < x < 9) = \frac{4}{12} = \frac{1}{3}$$



PRACTICAL PROBABILITY GRAPH

NORMAL DISTRIBUTION

STANDARD DEVIATION = $\delta = \sqrt{\dfrac{\Sigma f x^2 - n(\bar{x})^2}{n-1}}$

HOW MUCH THE NUMBER DIFFERS FROM AVERAGE VALUE

X	f
EVENT 1 →	5 TIMES
2 →	3 TIMES
3 →	4 TIMES.

$\bar{X}$ = MEAN = AVERAGE

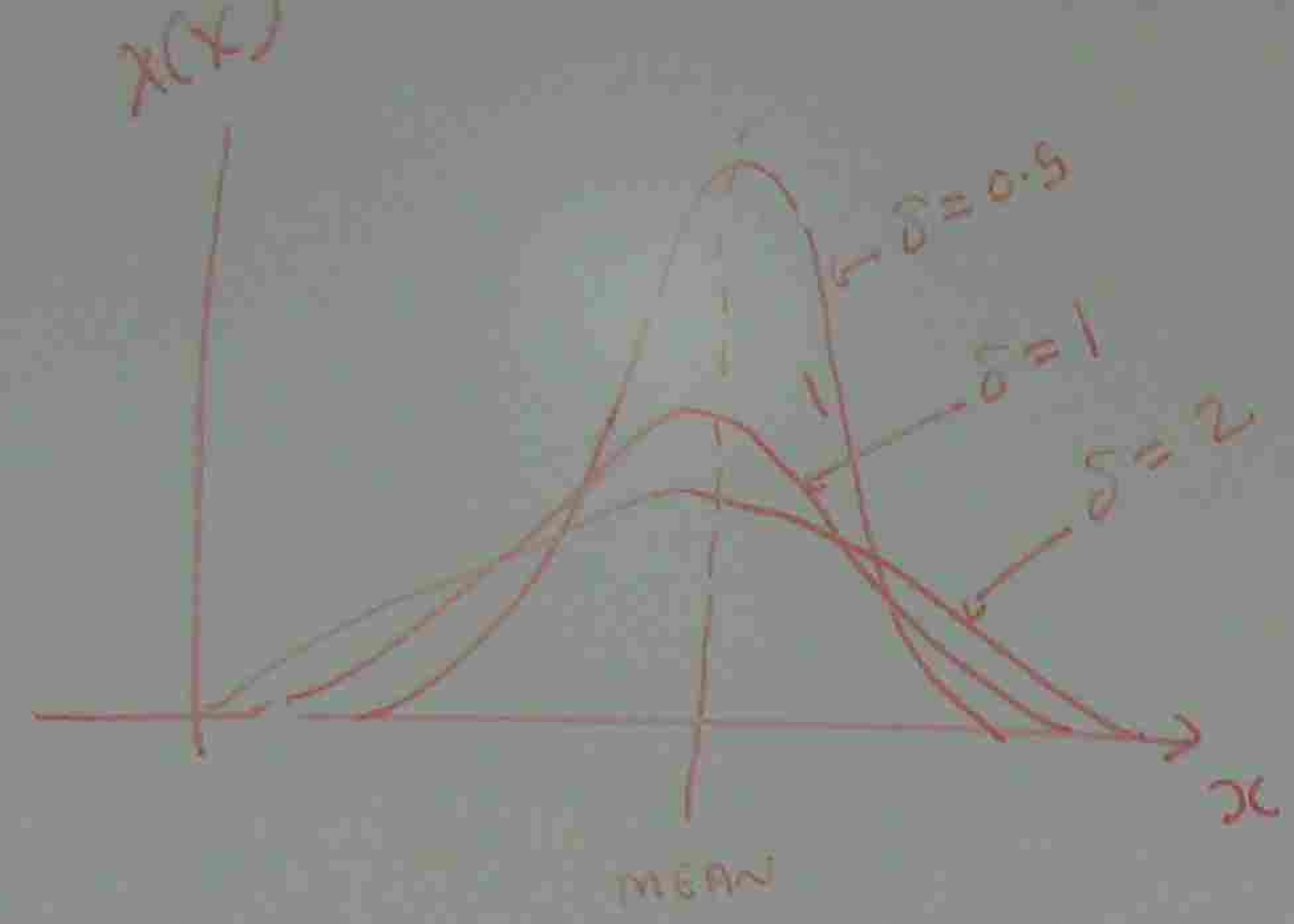
n = TOTAL NUMBER OF TIMES.

$(\overline{x})^2$

THE

DIFFERS

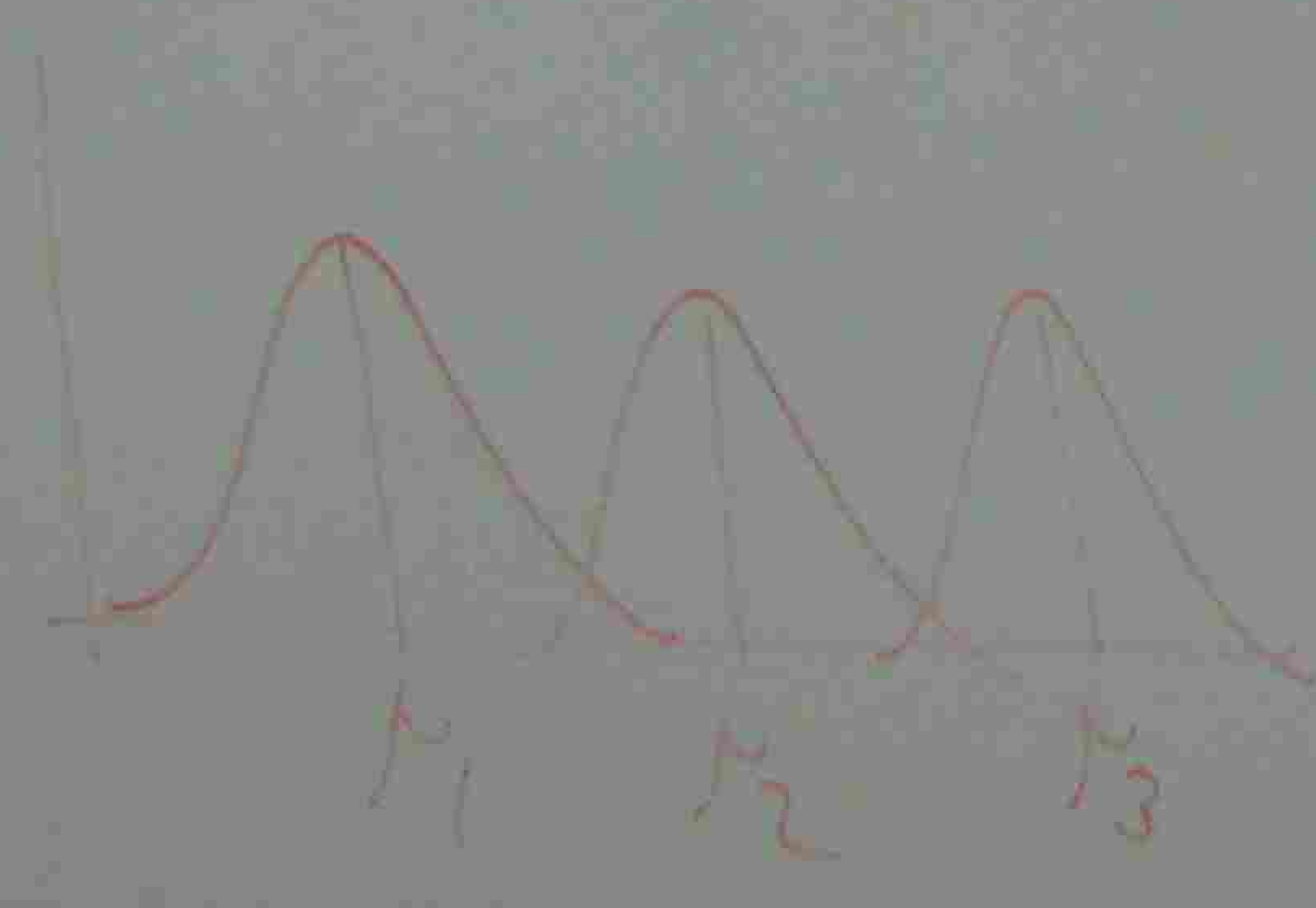
ERAGE



SAME MEN = $\mu = \frac{\sum x_1}{n_1} = \frac{\sum x_2}{n_2} = \frac{\sum x_3}{n_3}$

NORMAL DISTRIBUTION { SAME MEAN, DIFFERENT STANDARD DEVIATION

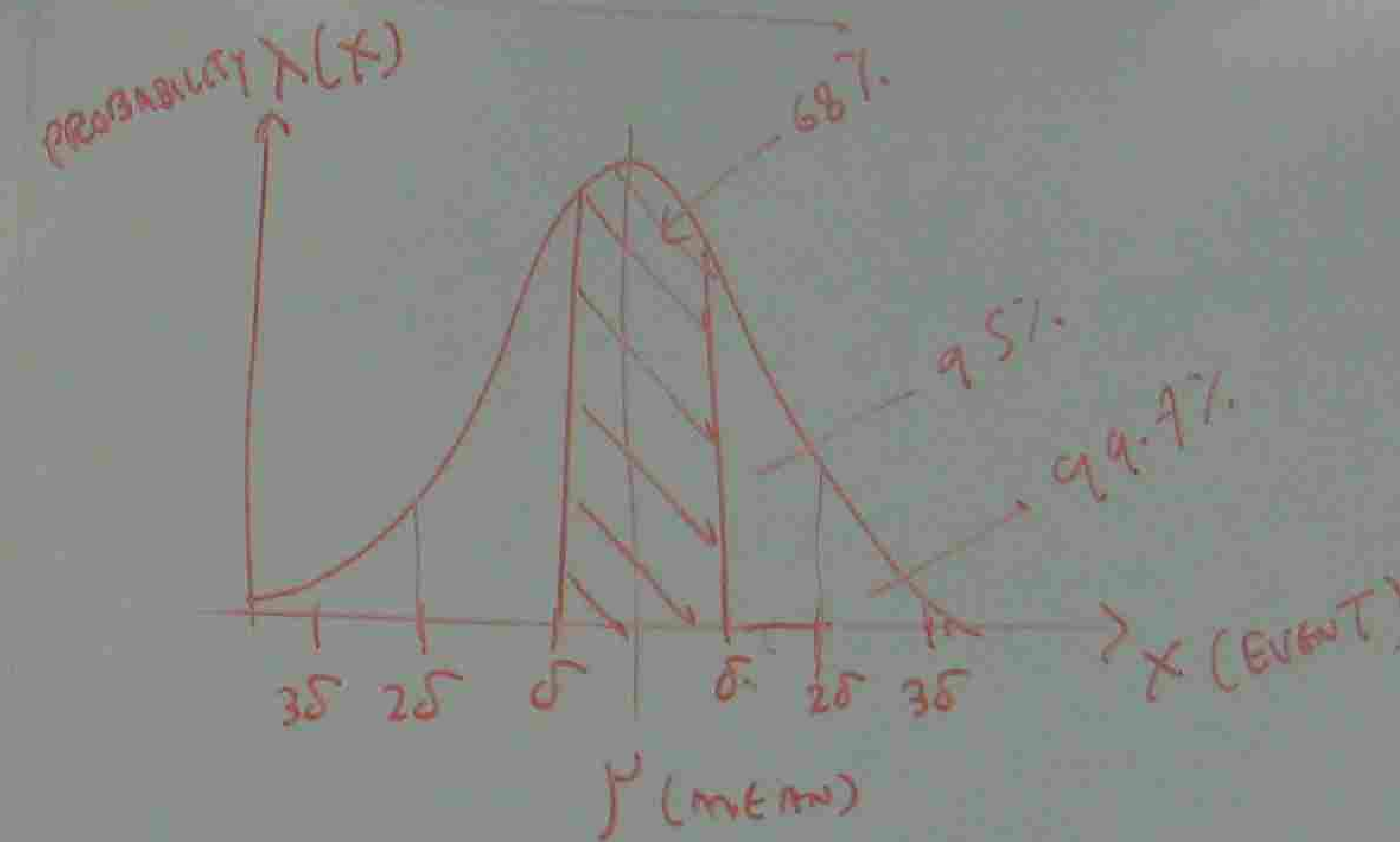
DIFFERENT MEAN, SAME STANDARD DEVIATION



s, δ

μ

NORMAL DISTRIBUTION



$P(\mu - \delta < x < \mu + \delta) = 68\% = 0.68$

$P(\mu - 2\delta < x < \mu + 2\delta) \rightarrow 0.95$

$P(\mu - 3\delta < x < \mu + 3\delta) \longrightarrow 0.997$

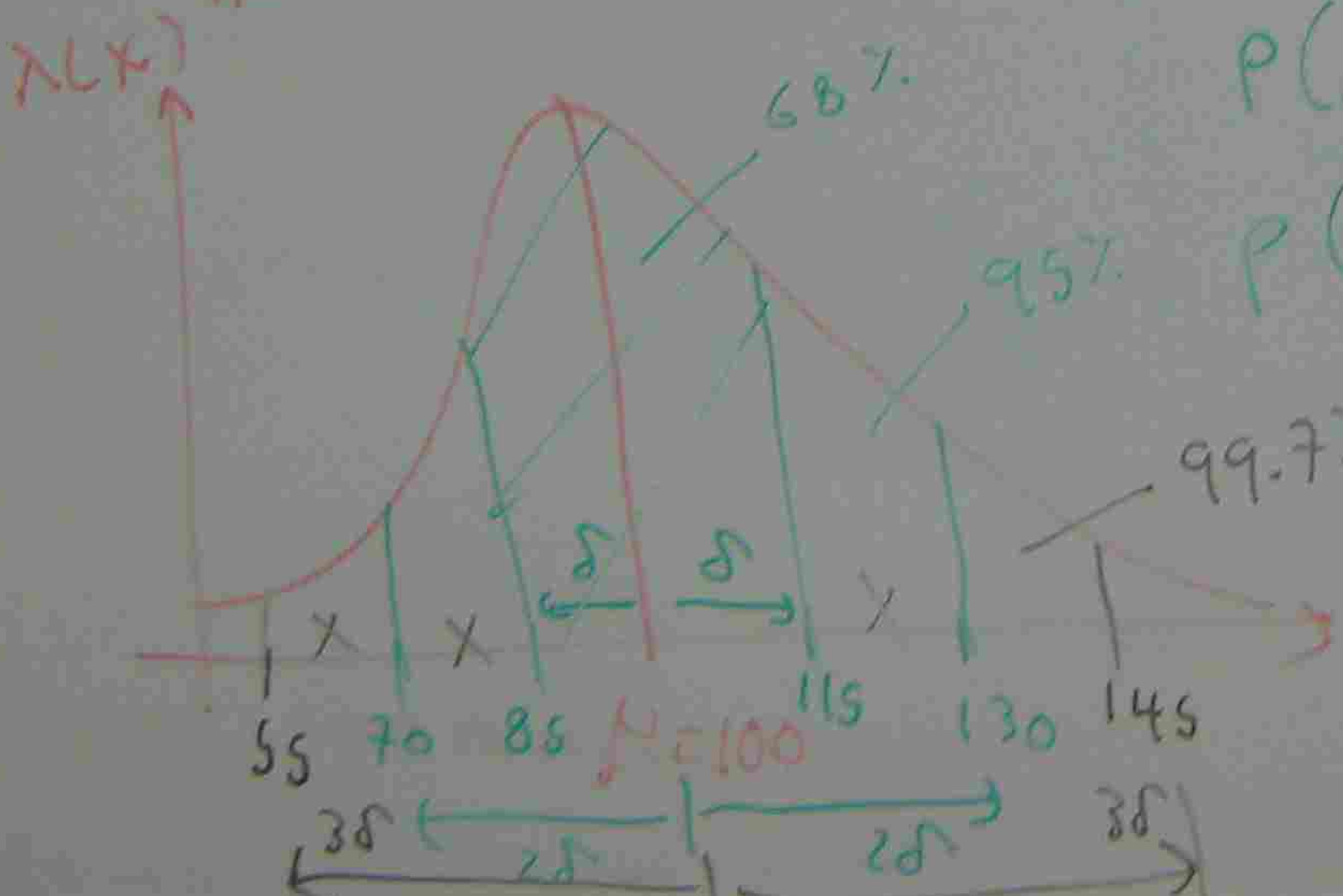
Ex (20) Suppose that IQ scores are normally distributed with a mean $\mu = 100$ and a standard deviation $\delta = 15$.

Indicate the followings on the graph.

(a) Approximately 68% of people have IQ between 85 and 115

(b) Approximately 95% of people have IQ between 70 and 130

(c) Almost every one (approximately 99.7%) have IQ between 55 and 145.



$P(\mu-\delta < x < \mu+\delta) = 0.68$

$P(\mu-2\delta < x < \mu+2\delta) = 0.95$

$P(\mu-3\delta < x < \mu+3\delta) = 0.997$

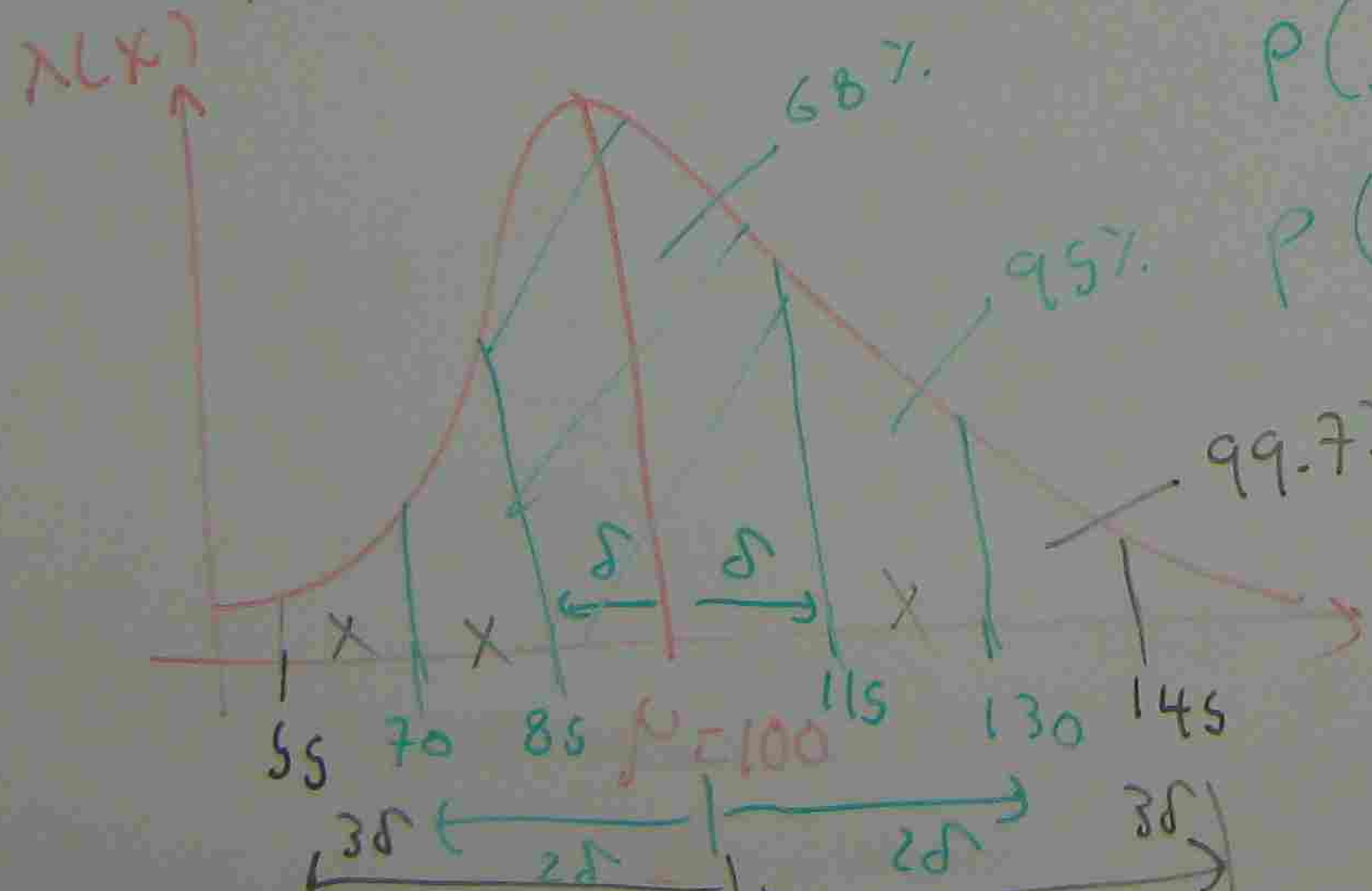
EX 20 SUPPOSE THAT IQ SCORES ARE NORMALLY DISTRIBUTED WITH A MEAN $\mu = 100$ AND A STANDARD DEVIATION $\delta = 15$.

INDICATE THE FOLLOWINGS ON THE GRAPH.

(a) APPROXIMATELY 68% OF PEOPLE HAVE IQ BETWEEN 85 AND 115

(b) APPROXIMATELY 95% OF PEOPLE HAVE IQ BETWEEN 70 AND 130

(c) ALMOST EVERY ONE (APPROXIMATELY 99.7%) HAVE IQ BETWEEN 55 AND 145.



$P(\mu - \delta < x < \mu + \delta) = 0.68$

$P(\mu - 2\delta < x < \mu + 2\delta) = 0.95$

$P(\mu - 3\delta < x < \mu + 3\delta) = 0.997$

STANDARD SCORE (

STANDARD SCORE

(Z-SCORE)

$z =$

IF $x > \mu$ $z =$

IF $x < \mu$ $z =$

IF $x = \mu$ z

EX 21 THE

HEIGHT HAS

STANDARD

FIND THE

HEIGHT TO

FEMALE HE

STANDARD SCORE (Z SCORE)

$$\text{STANDARD SCORE (Z-SCORE)} = \frac{\text{RAW DATA} - \text{MEAN}}{\text{STANDARD DEVIATION}}$$

$$Z = \frac{x - \mu}{\delta}$$

IF $x > \mu$ $\quad Z = +$

IF $x < \mu$ $\quad Z = -$

IF $x = \mu$ $\quad Z = 0$ NO ERROR

EX 21 THE DISTRIBUTION OF ADULT FEMALE HEIGHT HAS A MEAN $\mu = 165.5$ cm, STANDARD DEVIATION $\delta = 8$ cm.

FIND THE STANDARD SCORE (RELATIVE HEIGHT TO MEAN) CORRESPONDING TO A FEMALE HEIGHT OF (a) 164 cm (b) 178 cm

(a) $Z = \frac{x - \mu}{\delta} = \frac{164 - 165.5}{8} = -0.3$

(b) $Z = \frac{x - \mu}{\delta} = \frac{178 - 165.5}{8} = 1.44$

EX 22 THE WEIGHT OF A CERTAIN SPECIES OF ELEPHANT ARE DISTRIBUTED WITH A MEAN OF 20 TONNES AND A STANDARD DEVIATION OF 2 TONNES. SUPPOSE ALSO THAT WEIGHTS OF GARDEN SNAILS ARE DISTRIBUTED WITH A MEAN OF 30 GRAM AND STANDARD DEVIATION 3 GRAM.

WHICH IS HEAVIER, AN ELEPHANT WEIGHING 23 TON (OR) A SNAIL WEIGHING 34.5 GRAMS.

ELEPHANT

$\mu_1 =$

$X_1 =$

$Z = \frac{X_1}{}$

SNAIL

$\mu_2 = 3$

$X_2 =$

$Z = \frac{X}{}$

TWO AN

SAME R

IS

-0.997

MEAN

EVIATION

ADULT FEMALE

65.5 cm

m.

(RELATIVE

DING TO A

m (b) 178 cm

(a) $z = \frac{x-\mu}{\delta} = \frac{164 - 165.5}{8} = -0.3$

(b) $z = \frac{x-\mu}{\delta} = \frac{178 - 165.5}{8} = 1.44$

Ex 22 THE WEIGHT OF A CERTAIN SPECIES OF ELEPHANT ARE DISTRIBUTED WITH A MEAN OF 20 TONNES AND A STANDARD DEVIATION OF 2 TONNES. SUPPOSE ALSO THAT WEIGHTS OF GARDEN SNAILS ARE DISTRIBUTED WITH A MEAN OF 30 GRAM AND STANDARD DEVIATION 3 GRAM. WHICH IS HEAVIER, AN ELEPHANT WEIGHING 23 TON (OR) A SNAIL WEIGHING 34.5 GRAMS.

ELEPHANT

$\mu_1 = 20 \quad \delta_1 = 2$

$X_1 = 23$

$z = \frac{X_1 - \mu_1}{\delta_1} = \frac{23-20}{2} = 1.5$

SNAIL

$\mu_2 = 30, \quad \delta_2 = 3$

$X_2 = 34.5$

$z = \frac{X_2 - \mu_2}{\delta_2} = \frac{34.5-30}{3}$

$z = 1.5$

TWO ANIMALS HAVE SAME RELATIVE WEIGHT.

Ex 23

IF A STUDENT SHOWS THE FOLLOWING RESULTS IN A SERIES OF TESTS. COMPARE THE OVER ALL PERFORMANCE.

SUBJECT	STUDENT SCORE X	MEAN μ	STANDARD DEVIATION δ
MATHS	170	150	15
ENGLISH	120	120	10
HISTORY	188	200	12
SCIENCE	120	100	8

MATHS $z = \frac{X-\mu}{\delta} = \frac{170-150}{15} = 1.33$

ENGLISH $z = \frac{X-\mu}{\delta} = \frac{120-120}{10} = 0$

HISTORY $z = \frac{X-\mu}{\delta} = \frac{188-200}{12} = -1.0$

SCIENCE $z = \frac{X-\mu}{\delta} = \frac{120-100}{8} = 2.5$

PERFORMANCE

SCIENCE → BEST (2.5)

MATHS ⇒ 2nd BEST (1.33)

ENGLISH - NORMAL (0)

HISTORY : MOST POOR (−1.0)

READING Z T

Z TABLE

z_0	0.00
0.0	
0.1	
0.2	
1.1	
1.?	
3.0	

1.12 = 1.1

0.3686

0.3690

...S THE FOLLOWING RESULTS IN A
...ARE THE OVERALL PERFORMANCE.

...E	MEAN μ	STANDARD DEVIATION σ
	150	15
	120	10
	200	12
	100	8

$\frac{\ldots - 150}{\ldots} = 1.33$

$\frac{\ldots 120}{\ldots} = 0$

$\frac{\ldots - 200}{12} = -1.0$

$\frac{\ldots 0 - 100}{8} = 2.5$

PERFORMANCE

SCIENCE ⇒ BEST (2.5)

MATHS ⇒ 2nd BEST (1.33)

ENGLISH - NORMAL (0)

HISTORY = MOST POOR (−1.0)

READING Z TABLE

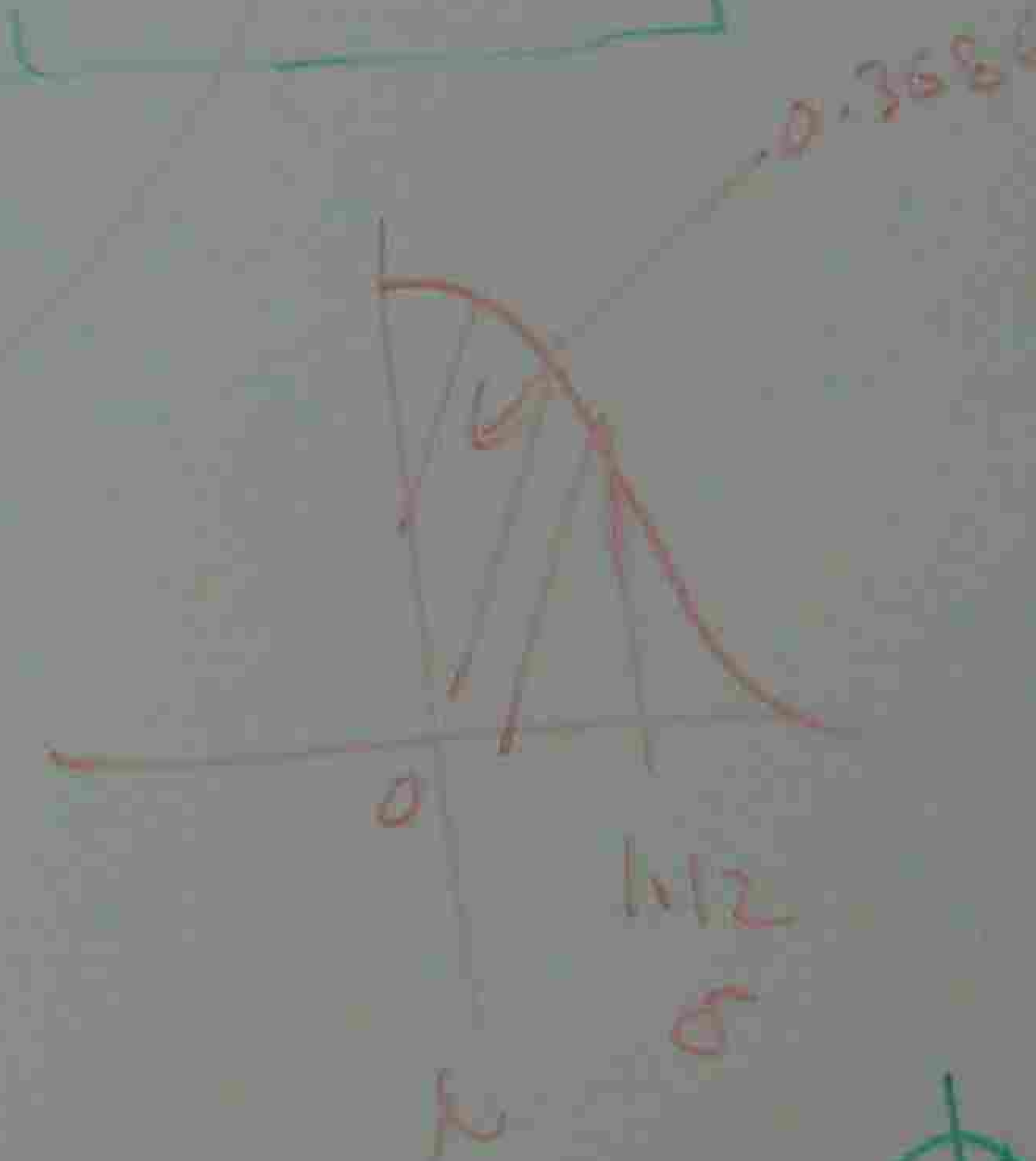
Z TABLE

z_0	0.00	0.01	0.02	0.03 - - - 0.04
0.0				
0.1				
0.2				
1.1			0.3686	
3.0				

$1.12 = 1.1 + .02$

$0.3686 \rightarrow 1.12$

$0.3690 \rightarrow 1.12$



Ex (24) USE ...
NORMA...

(a) BET...
(b) BET...
(c) BET...
(d) TO...
(e) TO...
(f) BE...

(a) P...

(b) P...

(c) P...

(e)

0.03 - - - 0.04

Ex 24 USE Z TABLE, CALCULATE THE AREA UNDER THE STANDARD NORMAL CURVE FOR THE FOLLOWING INTERVALS.

(a) BETWEEN $Z = 0$ AND $Z = 1.50$

(b) BETWEEN $Z = 0$ AND $Z = -2.10$

(c) BETWEEN $Z = -0.30$ AND $Z = +2.25$

(d) TO THE RIGHT of $Z = 1.95$

(e) TO THE LEFT of $Z = 1.64$

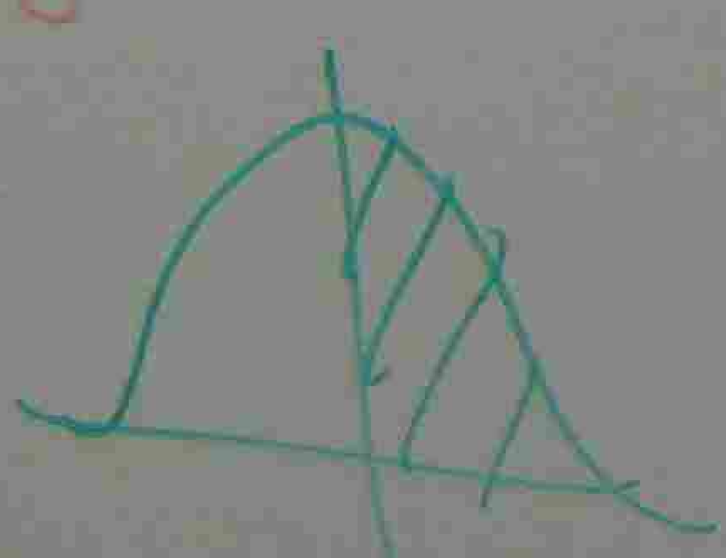
(f) BETWEEN 0.60 AND 1.80

(a) $P(0 < Z < 1.5) = 0.4332 - 0.0000$
$= 0.4332$

(b) $P(-2.1 < Z < 0) = 0 - (-0.4821)$
$= 0.4821$

(c) $P(-0.30 < Z < 2.25) = 0.4878 - (-0.1179)$
$= 0.4878 + 0.1179$
$= 0.6057$

(e) $P(Z > 1.95) =$ THE WHOLE AREA $- P(0 < Z < 1.95)$
$= 0.5 - [0.4744 - 0]$
$= 0.5 - 0.4744 = 0.0256$



(e) $P(Z < 1.64)$

$=$ THE WHOLE AREA $+ (P(0 < Z < 1.64))$

$= 0.5 + (0.4495 - 0)$

$= 0.5 + 0.4495$

$= 0.9495$

(f) $P(0.6 < Z < 1.8)$

$= P(1.8) - P(0.6)$

$= 0.4641 - 0.2257$

$= 0.2384$

EX 25

(a) If 37.7% of population has a standard score between the mean ($z=1$) and some positive z value find that z value

(b) Find the z value cutting off top 5% of population

(c) Find the z score cutting off bottom 10% of population

(d) Find the value of z so that 95% of population has a score between $-z$ and $+z$

(a) AREA = 37.7% ⇒ 0.377 ⟶ $z = ?$

	0.06
1.1	0.3770

$z = 1.16$

(b) Cutting off top 5% = the whole are − 5%

= 0.5 − 0.05 = 0.45

	0.04	0.05
1.6	0.4495	0.4505

0.4495 = 1.64

0.4505 = 1.65

$0.4500 = \frac{1.64 + 1.65}{2} = 1.645$

(c) Cutting off bottom 10% = the whole area − 10%

= 0.5 − 0.1 = 0.4

	0.08
1.2	0.3997

0.3997 ≃ 0.4 ⇒ 1.28

(d)

2 AREA = 0.95

$1\ \text{AREA} = \frac{0.95}{2} = 0.475$

0.475 ⟶ $z = ?$

	0.06
1.9	0.475

⇒ 1.96

EX 26 The height of adult males in Australia is approximately normally distributed with mean $\mu = 170$cm and standard deviation $\sigma = 7$cm.

(a) What is the probability that a male randomly selected has height (?) between 160cm & 175 cm

(ii) GREATER THAN 188CM

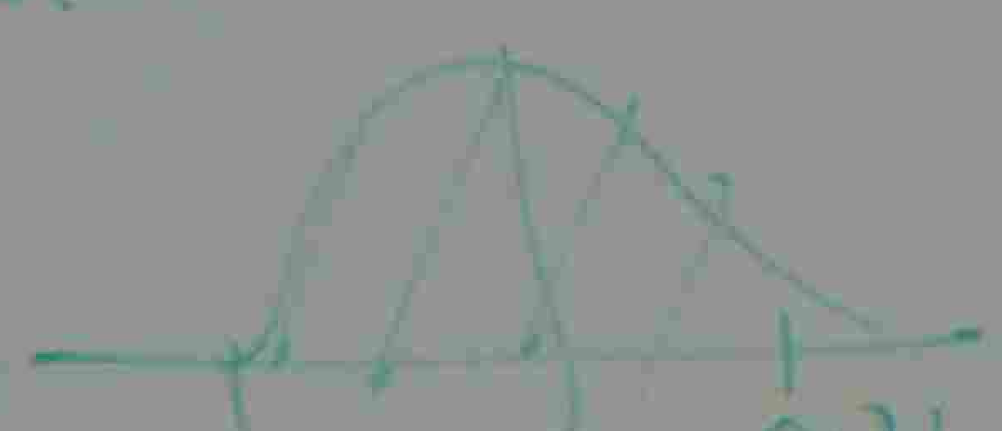
(b) WHAT IS THE GREATEST HEIGHT EXCEEDED BY 22% OF POPULATION.

(a) $P(\underset{x_1}{160} < z < \underset{x_2}{175})$ = THE AREA BETWEEN

$z_1 = \frac{x_1 - \mu}{\delta}$ AND $z_2 = \frac{x_2 - \mu}{\delta}$

$z_1 = \frac{160 - 170}{7}$ AND $z_2 = \frac{175 - 170}{7}$

$z_1 = -1.43$ AND $z_2 = 0.71$



TOTAL = AREA (1.43) + AREA (0.71)

0.03
|
1.4 — 0.4236

0.01
|
0.7 — 0.2611

= 0.4236 + 0.2611

= 0.6847

MALES

ATELY

ARD

THAT

HAS

& 175

(ii) $P(X > 188)$

= TOTAL AREA — THE AREA $z = \frac{x - \mu}{\delta}$

$= 0.5 - \left(z = \frac{188 - 170}{7} = 2.57\right)$

0.07
|
2.5 — 0.4949

= 0.5 − 0.4949 = 0.0051

(b) GREATEST HEIGHT EXCEED BY 22% OF POPULATION

= THE WHOLE AREA — 0.22

= 0.5 − 0.22 = 0.28

↳ FIND (Z)

0.07
|
0.7 — 0.2794 ≃ 0.28

∴ Z = 0.77

CATEGORY	OBSERVED FREQUENCY (O)	EXPECTED FREQUENCY (E) CUSTOMER × % SHARE	O−E	$\frac{(O-E)^2}{E}$
A	48	$200 \times \frac{30}{100} = 60$	$O-E = 48-60 = -12$	$\frac{(-12)^2}{60} = 2.4$
B	98	$200 \times \frac{50}{100} = 100$	$O-E = 98-100 = -2$	$\frac{(-2)^2}{100} = 0.04$
C	54	$200 \times \frac{20}{100} = 40$	$54-40 = 14$	$\frac{(14)^2}{40} = 4.9$

$\Sigma O-E = -12+(-2)+14 = 0$

$\Sigma \frac{(O-E)^2}{E} = 7.34$

$$X^2_{\alpha, V} = X^2_{0.05, (\text{NO. OF PRODUCTS} - 1)}$$

$$= X^2_{0.05, (3-1)} = X^2_{0.05, 2}$$

V_2	$V_1 = 2$
4	6.94

REJECTIONS POINT

FIND FIRST NUMBER LESS THAN 7.34 IN X^2 TABLE

pb 32

A CER

THERE IS A

PER MONTH AN

TO EXAMINE

EXPENDITURES

SALE VOLUME

MONTH
1
2
3
4
5
6
7
8
9
10

SAMPLING DISTRIBUTION | CORRELATION & REGRESSION ANALYSIS

$(\bar{x})$ MEAN = DATA RANGE $\left(\frac{\text{MAXIMUM NUMBER} + \text{MINIMUM NUMBER}}{2}\right)$

STANDARD DEVIATION = $S = \sqrt{\dfrac{\sum x^2 - n(\bar{x})^2}{n-1}}$

SAMPLE ERROR = $x - \mu$

= MAXIMUM NUMBER AT DATA RANGE − MEAN

MEAN OF DISTRIBUTION = $\dfrac{\sum \text{MEAN}}{\text{TOTAL SAMPLE SIZE}}$

SAMPLE MEAN = $\dfrac{\sum \text{TOTAL}}{\text{NO. OF EVENTS}}$

pb 23

A P…

TO BOT…

ONE …

IS SHO…

(a) LIST A…

(b) FOR EA…

SAMPLE …

(c) CALCUL…

(d) GRAPH …

...ION ANALYSIS

...NIMUM NUMBER)

pb (23)

A POPULATION OF 5 SALES PERSONS SELLING CAR TELEPHONES TO BOTH PRIVATE AND COMMERCIAL CUSTOMERS OVER A PERIOD OF ONE MONTH. THE NUMBER OF TELEPHONES SOLD FOR EACH PERSON IS SHOWN IN FOLLOWING TABLE

SALES PERSON	PHONES SOLD
ADAM (A)	14 ✓
BAKER (B)	20 ✓
COLLING (C)	12
DAVID (D)	8
EDWARDS (E)	16

(a) LIST ALL POSSIBLE SAMPLES OF SIZE 2

(b) FOR EACH SAMPLE, CALCULATE DATA RANGE, SAMPLE MEAN SAMPLE STANDARD DEVIATION

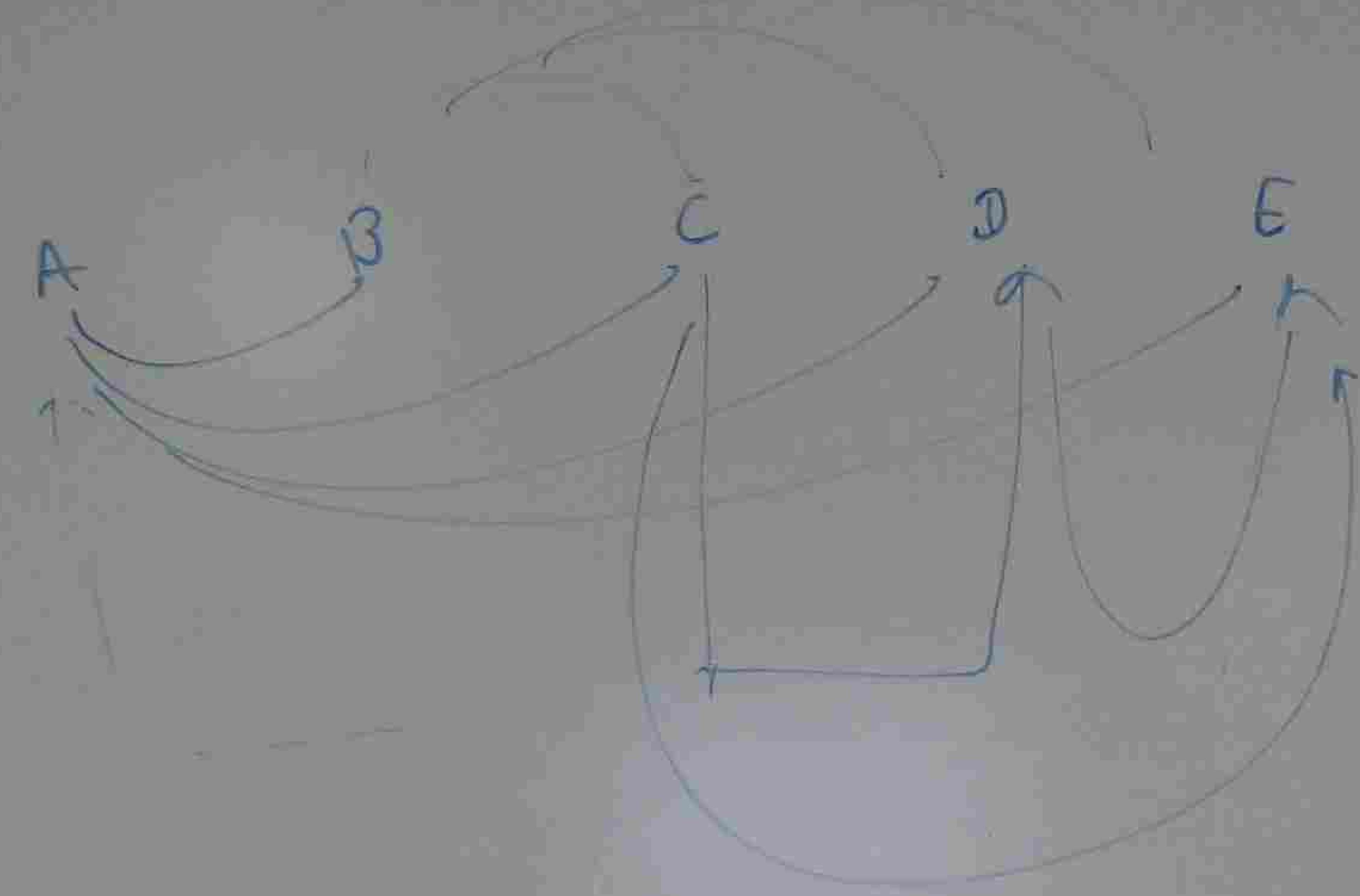
(c) CALCULATE MEAN, FREQUENCY, RELATIVE FREQUENCY

(d) GRAPH.

A B

SAMPLE COMBINATION
A B
A C
A D
A E
B C
B D
B E
C D
C E
D E

CAR TELEPHONES
OVER A PERIOD OF
SOLD FOR EACH PERSON



→ AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

LAST − MEAN

SAMPLE COMBINATION	DATA RANGE	MEAN $= \frac{MAX + MIN}{2}$, $\bar{x}$	STANDARD DEVIATION	SAMPLE ERROR $\bar{x} - \mu$
AB	14 − 20	$\bar{x} = \frac{14+20}{2} = 17$	$\sqrt{\frac{14^2+20^2-2\times17^2}{2-1}} = 4.24$	20 − 17 = 3
AC	14 − 12	$\bar{x} = \frac{14+12}{2} = 13$	$\sqrt{\frac{14^2+12^2-2\times13^2}{2-1}} = 1.414$	12 − 13 = −1
AD	14 − 8	11	4.243	−3
AE	14 − 16	15	1.414	+1
BC	20 − 12	16	5.657	+2
BD	20 − 8	14 ←	8.485	0
BE	20 − 16	18	2.828	4
CD	12 − 8	10	2.828	−4
CE	12 − 16	14 ←	2.828	0
DE	8 − 16	12	5.657	−2

LE MEAN

ENCY

BCC

B+

AE, BC, BD, BE

DE

LAST — MEAN

SAMPLE ERROR $\bar{x} - \mu$

17^2 =4.24   20 − 17 = 3

13^2 =1.414   12 − 13 = −1

−3

+1

+2

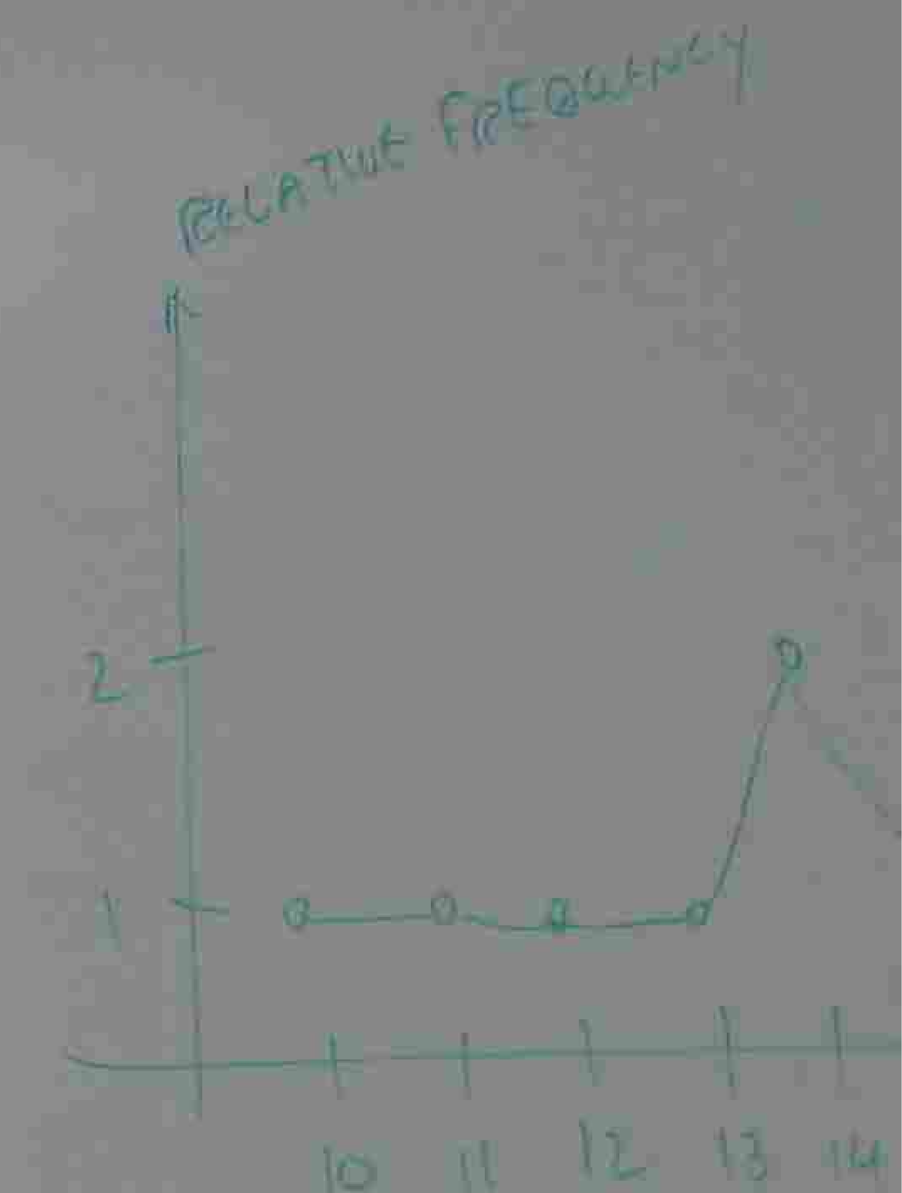
85   0

828   4

MEAN	FREQUENCY	RELATIVE FREQUENCY
10	1	$\frac{1}{\Sigma f} = \frac{1}{10} = 0.1$
11	1	$\frac{1}{10} = 0.1$
12	1	$\frac{1}{10} = 0.1$
13	1	$\frac{1}{10} = 0.1$
14	2	$\frac{2}{10} = 0.2$
15	1	$\frac{1}{10} = 0.1$
16	1	$\frac{1}{10} = 0.1$
17	1	$\frac{1}{10} = 0.1$
18	1	$\frac{1}{10} = 0.1$

$\Sigma f = 10$

ACCORDING TO FREQUENCY DISTRIBUTION, B+D, C+E CAN SELL MORE PRODUCTS.



pb 29 IN ABOVE P

DISTRIBUTIO

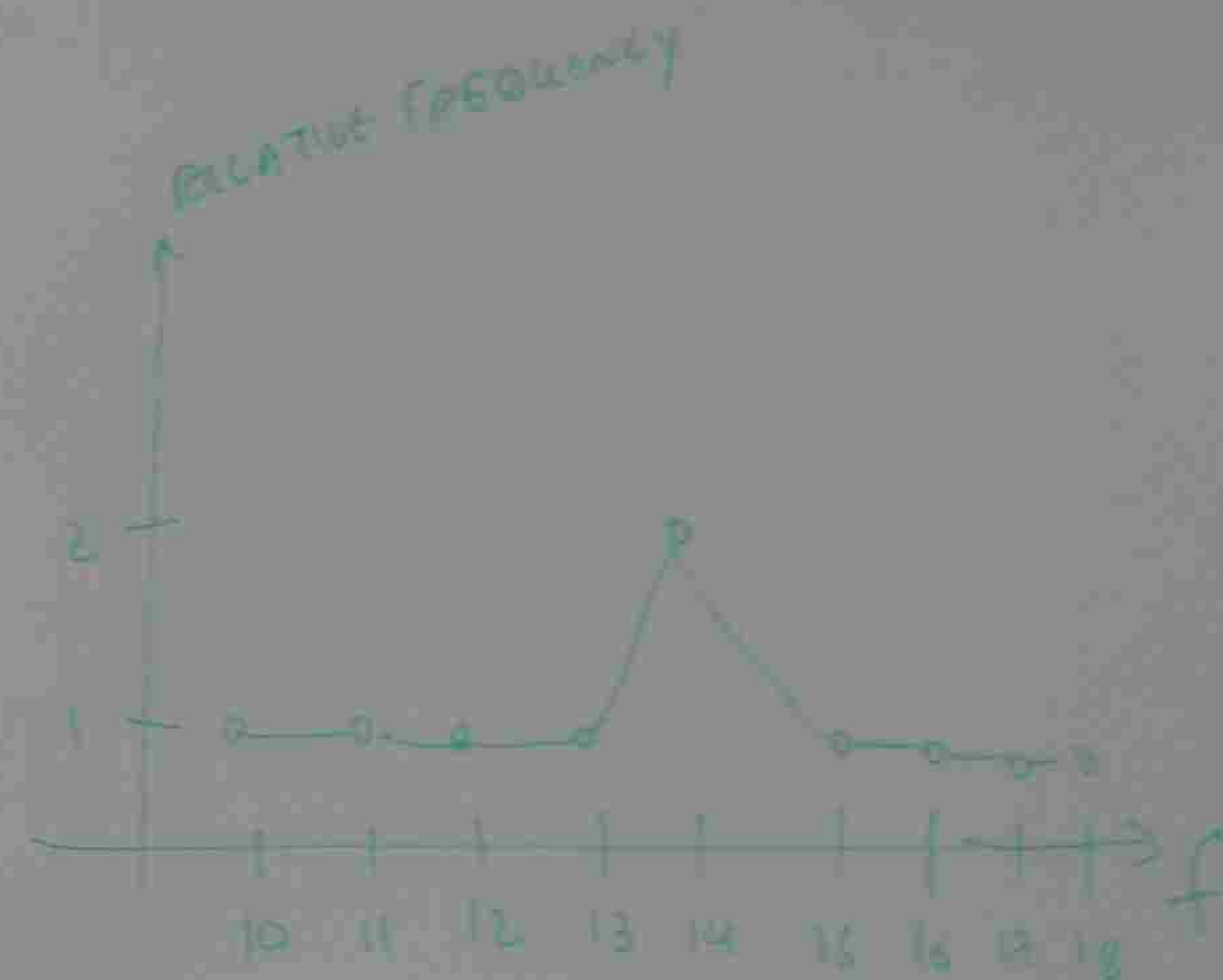
(i) MEAN OF DISTRIBUTION =

(ii) SAMPLE MEAN = Σ

FREQUENCY	RELATIVE FREQUENCY
1	$\frac{1}{\Sigma f} = \frac{1}{10} = 0.1$
1	$\frac{1}{10} = 0.1$
1	$\frac{1}{10} = 0.1$
1	$\frac{1}{10} = 0.1$
2	$\frac{2}{10} = 0.2$
1	$\frac{1}{10} = 0.1$
1	$\frac{1}{10} = 0.1$
1	$\frac{1}{10} = 0.$
1	$\frac{1}{10} = 0.1$

$\Sigma f = 10$

... TO FREQUENCY DISTRIBUTION, ...E CAN SELL MORE PRODUCTS.



Pb 29 IN ABOVE PROBLEM, CALCULATE (i) MEAN OF DISTRIBUTION μ_x, (ii) SAMPLE MEAN

(i) MEAN OF DISTRIBUTION $= \frac{\Sigma \text{MEAN}}{\text{TOTAL SAMPLE SIZE}} = \frac{17+13+11+15+16+14+18+10+14+12}{10}$

$= 14$

(ii) SAMPLE MEAN $= \frac{\Sigma \text{TOTAL EVENTS}}{\text{TOTAL PERSON}}$ ← SALES $= \frac{14+20+12+8+16}{5}$

$= 14$

STA...

STAND...

Pb 30

WHAT IS ...

MEAN = (μ)

STA...

(a) SET...

P(2...

(b...

STANDARD DISTRIBUTION OF SELECTED SAMPLE

STANDARD DISTRIBUTION OF SELECTED SAMPLE $\delta x = \dfrac{\text{PARENT STANDARD DISTRIBUTION}}{\sqrt{\text{Number of sample}}}$

Pb (30) A CERTAIN BRAND OF ROPE IS KNOWN TO HAVE A MEAN BREAKING STRENGTH 25 kg WITH A STANDARD DEVIATION OF 0.5 kg. A RANDOM SAMPLE OF ROPE IS TESTED FOR BREAKING STRENGTH. WHAT IS THE POSSIBILITY THAT SAMPLE MEAN BREAKING STRENGTH OF 50 PIECES OF ROPE WILL BE
(i) BETWEEN 24.9 AND 25.1 kg (ii) LESS THAN 24.8 kg.

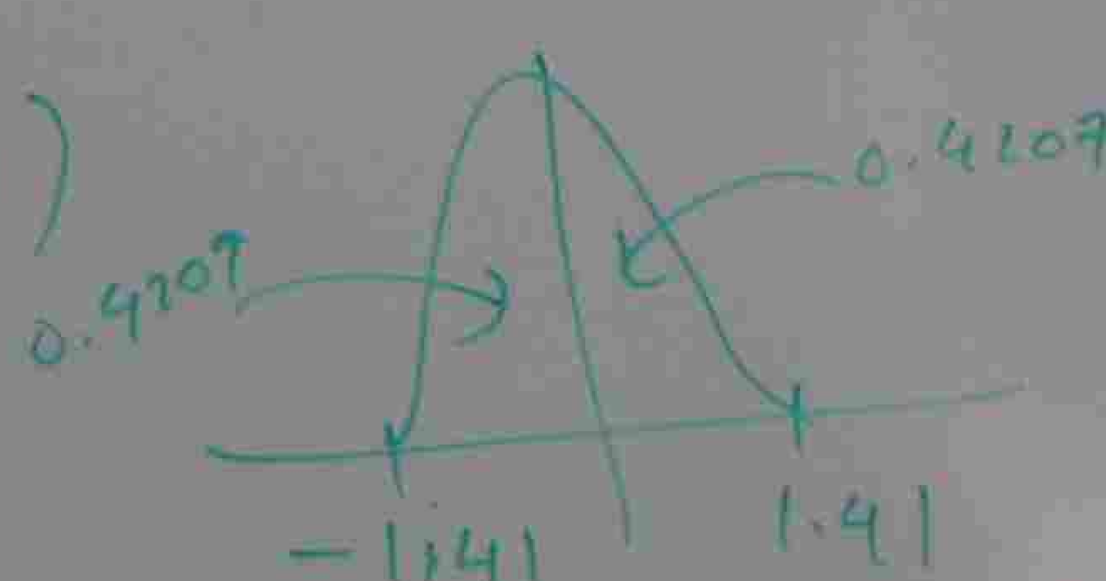
MEAN = 25 kg (μ) δ = 0.5 kg ← PARENT DISTRIBUTION

STANDARD DEVIATION OF SELECTED SAMPLE $(\delta_x) = \dfrac{\text{PARENT STANDARD DEVIATION}}{\sqrt{\text{No. of sample}}} = \dfrac{0.5}{\sqrt{50}} = 0.071$

(a) BETWEEN 24.9 AND 25.1 kg

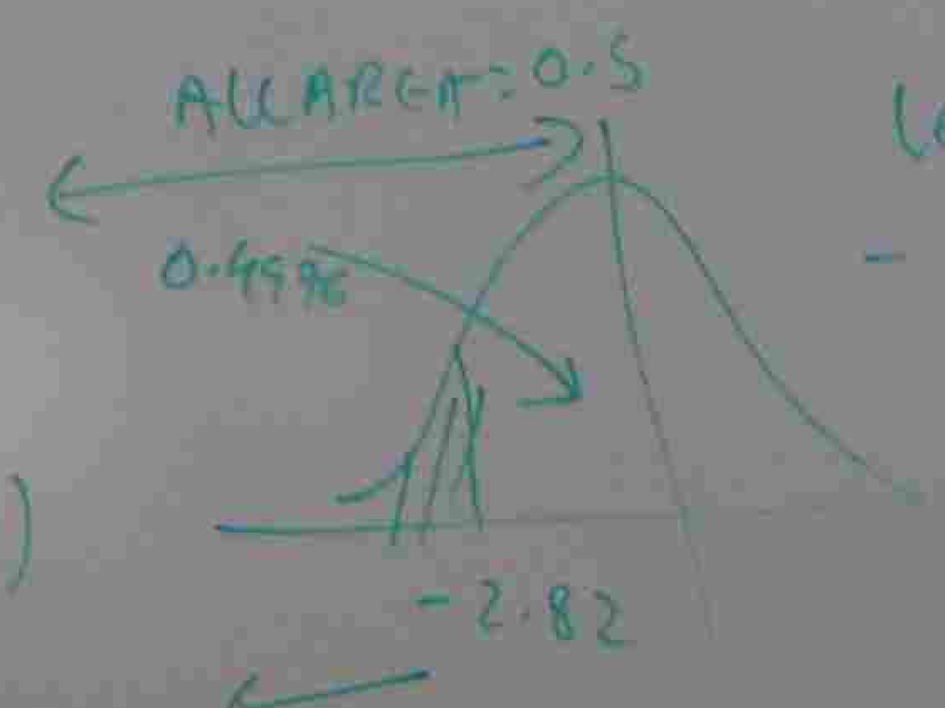
$$P(24.9 < x < 25.1) \longrightarrow P\left(\frac{x_1-\mu}{\delta x} < x < \frac{x_2-\mu}{\delta x}\right) = P\left(\frac{24.9-25}{0.071} < x < \frac{25.1-25}{0.071}\right)$$

$$P(-1.41 < x < 1.41)$$



TOTAL AREA = 0.4207 + 0.4207 = 0.8414

$$(b)\ P\left(\frac{x-\mu}{\delta x}\right) = P\left(\frac{24.8-25}{0.071}\right) = P(-2.82)$$



LESS THAN -2.82 = ALL − 2.82 AREA
= 0.5 − 0.4976
= 0.0024

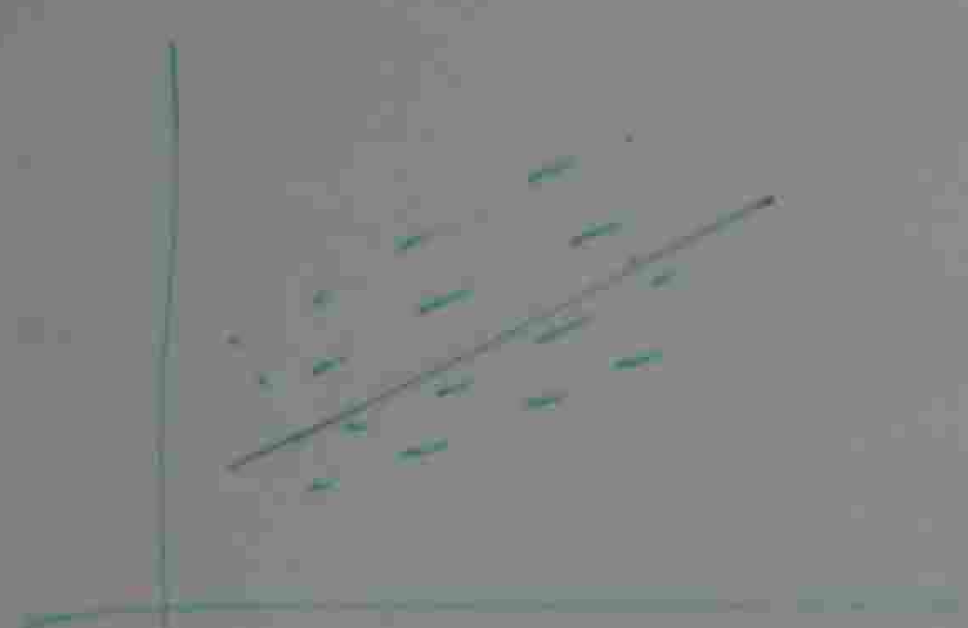
MEAN OF

10+14
+12

8+16

CORRELATION AND REGRESSION ANALYSIS

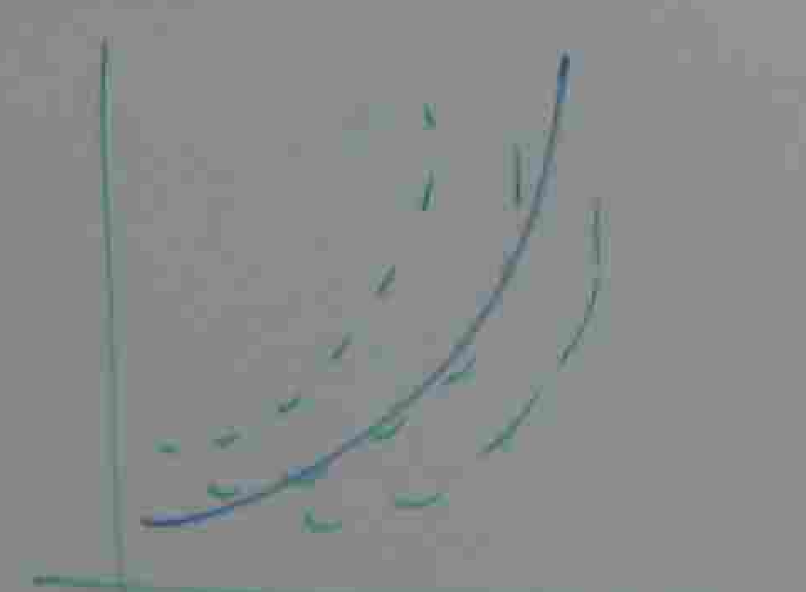
POSSIBLE RELATIONSHIPS BETWEEN X AND Y SCATTER PLOT



DIRECT LINEAR

INVERSE LINEAR

INVERSE CURVILINEAR



DIRECT CURVILINEAR

NON RELATIONSHIP

CORRELATION ANALYSIS → ACTIVITY & EFFECTIVENESS

INPUT

$$\bar{X} = \frac{X_1 + X_2 + \cdots}{n}$$

AVERAGE

STANDARD DEVIATION

$$S_X = \sqrt{\frac{X_1^2 + X_2^2 + \cdots - n(\bar{X})^2}{n-1}}$$

EFFECT

OUTPUT

$$\bar{Y}_{AVERAGE} = \frac{Y_1 + Y_2 + Y_3 + \cdots}{n}$$

$$S_y = \sqrt{\frac{y_1^2 + y_2^2 + \cdots - n(\bar{y})^2}{n-1}}$$

Σxy

$SS_{xy} = \Sigma xy - n\bar{x}\bar{y}$

$SS_x = (n-1)s_x^2$

$SS_y = (n-1)s_y^2$

CORRELATION $r = \dfrac{SS_{xy}}{\sqrt{SS_x \times SS_y}}$

IF r IS POSITIVE

CLOSE TO +1

THEN INPUT IS STRONGLY

RELATED TO OUT PUT

IF r IS NEGATIVE, INPUT

DOES NOT EFFECTED ON OUT PUT

pb 32

A CERTAIN LARGE COMPANY IS INTERESTED IN WHETHER OR NOT THERE IS A RELATIONSHIP BETWEEN THE AMOUNT SPENT ON ADVERTISING PER MONTH AND THE GROSS MONTHLY SALE VOLUME.

TO EXAMINE THIS PROBLEM, THE FOLLOWING SAMPLE OF ADVERTISING EXPENDITURES AND ASSOCIATED SALE VOLUMES.

VOLUMES ARE RANDAMLY SELECTED FOR MONTHLY DATA

MONTH	ADVERTISING EXPENDITURE (x) \$ 10000	SALE (Y) (x \$ 10000
1	1.2	101
2	0.8	92
3	1.0	110
4	1.3	120
5	0.7	90
6	0.8	82
7	1.0	93
8	0.6	75
9	0.9	91
10	1.1	108

NOT

SING

NG

ADVERTISING

MEAN $\bar{x} = \dfrac{1.2+0.8+1.0+1.3+0.7+0.8+1.0+0.6+0.9+1.1}{10} = 0.94$

$S_x = \sqrt{\dfrac{1.2^2+0.8^2+1.0^2+1.3^2+0.7^2+0.8^2+1.0^2+0.6^2+0.9^2+1.1^2-10\,(0.94)^2}{10-1}} = 0.22211$

SALE

MEAN $= \bar{y} = \dfrac{101+92+110+120+90+82+93+75+91+108}{10} = 95.9$

$S_y = \sqrt{\dfrac{101^2+92^2+110^2+120^2+90^2+82^2+93^2+75^2+91^2+108^2-10\times 95.9^2}{10-1}} = 13.37$

$\sum xy = 1.2\times101+0.8\times92+1.0\times110+1.3\times120+0.7\times90+.8\times82+1.0\times93+0.6\times75+0.9\times91+1.1\times108$
$= 924.8$

$SS_{xy} = \sum xy - n\bar{x}\bar{y} = 924.8 - 10\times0.94\times95.9 = 23.34$

$SS_x = (n-1)S_x^2 = (10-1)\times(0.22211)^2 = 0.444$

$SS_y = (n-1)S_y^2 = (10-1)\times(13.37)^2 = 1600.9$

$r = \dfrac{SS_{xy}}{\sqrt{SS_x \times SS_y}}$

$= \dfrac{23.34}{\sqrt{0.444\times1600.9}} = 0.875$

POSITIVE
CLOSE TO +1
THUS ADVERTISEMENT
AFFECTS THE SALE

$\alpha = 0.05$

V_2 \ V_1	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								

CATEGORY	OBSERVED FREQUENCY (O)	E...
		CUST...
A	48	200...
B	98	20...
C	54	20...

$\chi^2_{\alpha, V} =$