2a

$$\mathcal{L}^{-1}\frac{5/2\, s}{s^2+25} = 5/2\,\mathcal{L}^{-1}\frac{s}{s^2+5^2} = \frac{5}{2}\sin 5t$$

$$\mathcal{L}^{-1}\frac{25/2}{s^2+25} = \frac{25}{2}\,\mathcal{L}^{-1}\frac{1}{s^2+5^2} = \frac{25}{2}\cos 5t$$

$$\mathcal{L}^{-1}\frac{5/2}{s+5} = 5/2\,\mathcal{L}^{-1}\frac{1}{s+5} = \frac{5}{2}e^{-5t}$$

$$I(t) = \frac{5}{2}e^{-5t} - \frac{5}{2}\cos 5t + \frac{5}{2}\sin 5t$$

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week 7/8/9 1

Ordinary Differential Equations

Integrating

Pb1 Find a general solution of the differential equation

$$\frac{dy}{dx} = 3x^2$$

$$\therefore \frac{dy}{dx} = 3x^2 \longrightarrow dy = 3x^2\,dx$$

$$\int dy = \int 3x^2\,dx$$

$$y = 3 \times \frac{x^{2+1}}{2+1} + C = 3\frac{y^3}{3} + C$$

$$y = y^3 + C$$

Pb2 Solve $y'' = 3x - 2$, $y(0) = 2$, $y'(1) = -3$

$$y'' = 3x - 2$$

$$\int y''\,dy = \int 3x\,dx - \int 2\,dx + c_1 = 3\frac{x^2}{2} - 2x + c_1 = y'$$

$$y'(1) = -3 \Longrightarrow -3 = 3\frac{1^2}{2} - 2\times 1 + c_1$$

$$\therefore c_1 = -3 - \frac{3}{2} + 2 = -1 - \frac{3}{2} = -\frac{5}{2}$$

$$\therefore y' = \frac{3x^2}{2} - 2x - \frac{5}{2}$$

$$\int y'\,dy = \int \frac{3x^2}{2}\,dx - \int 2x\,dx - \int \frac{5}{2}\,dx + C_2$$

$$= 3\frac{x^{2+1}}{2\times[2+1]} - 2\frac{x^{1+1}}{1+1} - \frac{5}{2}x + C_2$$

$$y = \frac{x^3}{2} - \frac{\cancel{2}x^2}{\cancel{2}} - \frac{5}{2}x + C_2$$

$$\text{But } y(0) = 2 \longrightarrow 2 = \frac{0^3}{2} - (0)^2 - \frac{5}{2}\times 0 + C_2$$

$$\therefore C_2 = 2$$

Separation of variables

Pb3
(a) Find the general solution of $(4x + xy^2)dx + (y + x^2y) = 0$
(b) Find the particular solution of $y(1) = 2$

(a) $(4x + xy^2)dx + (y + x^2y)dy = 0$

$x(4 + y^2)dx + y(1 + x^2)dy = 0$

Divide by $(1+x^2) \times (4+y^2)$

$$\frac{x(4+y^2)dx}{(1+x^2)(4+y^2)} + \frac{y(1+x^2)dy}{(4+y^2)(1+x^2)} = 0$$

$$\frac{x\,dx}{1+x^2} + \frac{y\,dy}{4+y^2} = 0$$

Integrate $\int \frac{x\,dx}{1+x^2} + \int \frac{y\,dy}{4+y^2} = \int 0 = 0$

$d(1+x^2) = 2x\,dx$ $\quad d(4+y^2) = 2y\,dy$

$\therefore x\,dx = \frac{d(1+x^2)}{2}$, $\quad y\,dy = \frac{d(4+y^2)}{2}$

Substitute $\int \frac{\frac{d(1+x^2)}{2}}{1+x^2} + \int \frac{\frac{d(4+y^2)}{2}}{4+y^2} = 0$

$$\frac{1}{2}\int \frac{d(1+x^2)}{1+x^2} + \frac{1}{2}\int \frac{d(4+y^2)}{4+y^2} = 0$$

$$\frac{1}{2}\ln(1+x^2) + \frac{1}{2}\ln(4+y^2) = C_1$$

$$\ln(1+x^2) + \ln(4+y^2) = 2C_1$$

$$\ln(1+x^2)\cdot(4+y^2) = 2C_1$$

$$\therefore (1+x^2)(4+y^2) = e^{2C_1} = C$$

For $y(1)=2 \Rightarrow$ when $x=1$, $y=2$ $\quad (1+1)^2 + (4+2^2) = c$
$4 + 12 = c = 16$

$(1+x^2)(4+y^2)$
$= 16$

4 Solve $\frac{dy}{dx} + 3y = 8, \quad y(0) = 2$

Find equation of dy & dx

$$\frac{dy}{dx} = 8 - 3y$$

$$dy = (8-3y)dx \longrightarrow dx = \frac{dy}{(8-3y)}$$

$$d(8-3y) = d8 - d3y = -3dy \longrightarrow dy = -\frac{d(8-3y)}{3}$$

$$\therefore dx = \frac{-\frac{d(8-3y)}{3}}{(8-3y)} = -\frac{1}{3}\frac{d(8-3y)}{8-3y}$$

$$\int dx = -\frac{1}{3}\int \frac{d(8-3y)}{(8-3y)} = -\frac{1}{3}\ln(8-3y) + C$$

$$x = -\frac{1}{3}\ln(8-3y) + C$$

$$y(0) = 2 \longrightarrow x = 0 \longrightarrow y = 2$$

$$(0) = -\frac{1}{3}\ln(8-3\times 2) + c$$

$$0 = -\frac{1}{3}\ln 2 + c$$

$$\therefore c = \frac{1}{3}\ln 2$$

$$\therefore x = -\frac{1}{3}\ln(8-3y) + \frac{1}{3}\ln 2$$

Pb 5 Solve $\frac{dy}{dx} = \sec y \tan x$

$$\frac{dy}{dx} = \frac{1}{\cos y}\frac{\sin x}{\cos x}$$

$$\cos y\, dy = \frac{\sin x}{\cos x}dx$$

$d\cos x = -\sin x\, dx$

$\therefore \sin x\, dx = -d\cos x$

$\longrightarrow \cos y\, dy = -\frac{d\cos x}{\cos x}$

$\int \cos y\, dy = -\int \frac{d\cos x}{\cos x}$

$\sin y = -\ln\cos x + c$

$\therefore c = \sin y + \ln\cos x$

$\therefore$ ~~$\sin y$~~

#

Exact equation

$m(x,y)\,dx + n(x,y)\,dy = 0$

Given $\frac{\partial m}{\partial y} = \frac{\partial n}{\partial x}$

Then $\int m\,\partial x + \int \left(n - \frac{\partial}{\partial y}\int m\,\partial x\right) dy = c$

pb 6 Show that $(3x^2 + y\cos x)\,dx + (\sin x - 4y^3)\,dy = 0$ is exact differential equation and find its general solution

Solution

$(3x^2 + y\cos x)\,dx + (\sin x - 4y^3)\,dy = 0$

compare $m(x,y)\,dx + n(x,y)\,dy = 0$

$\therefore m = 3x^2 + y\cos x, \quad n = \sin x - 4y^3$

$\frac{\partial m}{\partial y} = \frac{\partial}{\partial y}(3x^2 + y\cos x) = \cancel{\frac{\partial 3x^2}{\partial y} + y\frac{\partial \cos x}{\partial y}}$

$= \cancel{3x^2}_{0} + \cos x = \cancel{6x + y\sin x}$

$\frac{\partial n}{\partial x} = \frac{\partial}{\partial x}(\sin x - 4y^3) = \cos x$

$\therefore \frac{\partial m}{\partial y} = \frac{\partial n}{\partial x}$

Then $\int m\,\partial x + \int \left[n - \frac{\partial}{\partial y}\int m\,\partial x\right)\Big]\partial y = c$

$\int (3x^2 + y^2\cos x)\,\partial x + \int\Big[(\sin x - 4y^3) - \frac{\partial}{\partial y}\left(\int (3x^2 + y\cos x)\,\partial x\right)\Big]\partial y = c$

$3\frac{x^{2+1}}{2+1} + y^2\sin x + \int\Big[(\sin x - 4y^3) - \cancel{3\frac{x^2}{}x + \frac{\partial}{\partial y}[y\sin x]}$

$- \frac{\partial}{\partial y}\left(3\frac{x^{2+1}}{2+1} + y\sin x\right)\Big]\partial y = c$

$$x^3 + y\sin x + \int \left[(\sin x - 4y^3) - \frac{\partial}{\partial y}(x^3 + y\sin x) \right] \partial y$$

$$x^3 + y\sin x + \int (\sin x - 4y^3) - 0 - \sin x \,] \,\partial y$$

$$x^3 + y\sin x + \cancel{\int \sin x \,\partial y}_{0} - \int 4y^3 \partial y - \cancel{\int \sin x \,\partial y}_{0}$$

$$x^3 + y\sin x - 4\,\frac{y^{3+1}}{3+1} = c$$

$$x^3 + y\sin x - 4\,\frac{y^4}{4} = x^3 + y\sin x - y^4 = c$$

Integrating factor

$$m(x,y)\,dx + N(x,y)\,dy = 0$$

when $\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$

Apply Integrating factor.

$$\mu\, m\,dx + \mu N(x,y)\,dy = 0$$

$$\frac{\partial \mu m}{\partial y} = \frac{\partial \mu N}{\partial x}$$

Integrating factor $\mu = e^{\int f(x)\,dx}$

Integrating factor

$$m(x,y)\,dx + N(x,y)\,dy = 0$$

where $\dfrac{\partial m}{\partial y} \neq \dfrac{\partial N}{\partial x}$

Integrating factor

(a) $\dfrac{x\,dy - y\,dx}{x^2} = d\left(\dfrac{y}{x}\right)$

(b) $\dfrac{x\,dy - y\,dx}{y^2} = -d\left(\dfrac{x}{y}\right)$

(c) $\dfrac{x\,dy - y\,dx}{x^2+y^2} = d\left(\tan^{-1}\dfrac{y}{x}\right)$

(d) $\dfrac{x\,dy - y\,dx}{x^2-y^2} = \dfrac{1}{2}\,d\left(\ln\dfrac{x-y}{x+y}\right)$

(e) $\dfrac{x\,dx + y\,dy}{x^2+y^2} = \dfrac{1}{2}\,d\left(\ln(x^2+y^2)\right)$

Pb 7 Solve (a) $(y + x^4)\,dx - x\,dy = 0$

$$y\,dx + x^4\,dx - x\,dy = 0$$

$$\underline{y\,dx - x\,dy} + x^4\,dx = 0$$

Integrating factor $= \dfrac{1}{x^2}$

$\therefore$ divide by $x^2 \Rightarrow \dfrac{y\,dx - x\,dy}{x^2} + \dfrac{x^4\,dx}{x^2} = 0$

$$\frac{y\,dx \cancel{-} x\,dy}{\cancel{x^2}\; x^2\; \cancel{x}} + x^2\,dx = 0$$

$$\frac{d}{dx}\left(\frac{y}{x}\right) - \cancel{d\left(\frac{y}{x}\right)} + d\left(\frac{x^3}{3}\right)$$

$$-\frac{y}{x} + \frac{x^3}{3} = 0$$

Pb (8) $(x^3 + xy^2 - y)\,dx + x\,dy = 0$

$$\times \frac{1}{x^2+y^2} \Rightarrow \cancel{\frac{(x^3+xy^2-y)\,dx}{x^2+y^2}} +$$

$$x^3\,dx + xy^2\,dx - y\,dx + x\,dy = 0$$

$$x^3\,dx + xy^2\,dx - (y\,dx - x\,dy) = 0$$

Divide by $\frac{1}{x^2+y^2}$ $\quad \cancel{x(x^2\,dx + y^2} d$

$$x\,dx(x^2+y^2) + x\,dy - y\,dx = 0$$

divide by $x^2+y^2 \Rightarrow$ $\frac{x\,dx(x^2+y^2)}{(x^2+y^2)} + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$

$$x\,dx + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$$

$$x\,dx + d\left(\tan^{-1}\frac{y}{x}\right) = 0$$

$$\cancel{\int x\,dx + \int \frac{d}{dx}\left(\frac{y}{x}\right) = 0}$$

$$\cancel{\frac{x^2}{2} + y\int \frac{d}{dx}\frac{1}{x}}$$

$$x\,dx + \frac{d}{dx}\tan^{-1}\frac{y}{x} = 0$$

$$\frac{d}{dx}\left(\frac{x^2}{2}\right) + \frac{d}{dx}\left(\tan^{-1}\frac{y}{x}\right) = 0$$

$$\frac{x^2}{2} + \tan^{-1}\frac{y}{x} = 0$$

Linear Equation

solve $\frac{dy}{dx} + P(x)\,y = Q(x)$

$$y = e^{-\int P dx} \int Q\, e^{\int P dx}\, dx + c\, e^{-\int P dx} \quad \#$$

Pb (1) solve $x \frac{dy}{dx} - 2y = x^3 \cos 4x$

Compare with $\frac{dy}{dx} + P(x)\,y = Q(x)$

Change $\frac{dy}{dx} - \frac{1}{x} \times 2y = x^2 \cos 4x$

Then $P = -\frac{2}{x}$, $Q = x^2 \cos 4x$

$$\therefore y = e^{-\int P dx} \int Q\, e^{\int P dx}\, dx + c\, e^{-\int P dx}$$

$$= e^{-\int \frac{2}{x} dx} \int x^2 \cos 4x\, e^{\int -\frac{2}{x} dx}\, dx + c\, e^{-\int -\frac{2}{x} dx}$$

$$= e^{-2\ln x} \int x^2 \cos 4x \cdot e^{-\ln x}\, dx + c\, e^{2\ln x}$$

$\boxed{e^{\ln x} = x}$

$$= e^{\ln x^{-2}} \int x^2 \cos 4x\, e^{\ln x^{-2}}\, dx + c\, e^{\ln x^2}$$

$e^{\ln x^{-2}} = \cancel{x}^{2}$
$= \cancel{x}$
$= 1/x^2$

$$= \frac{1}{x^2} \int x^2 \cos 4x \times \frac{1}{x^2}\, dx + c \times x^2$$

$$= \frac{1}{x^2} \int \cos 4x\, dx + c\, x^2$$

$$= \frac{1}{x^2} \times \frac{1}{4} \sin 4x + c\, x^2$$

Homogeneous Equation

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

Then $\frac{y}{x} = v$ or $y = vx$

Then $v + x\frac{dv}{dx} = F(v)$ (or) $x\,dv + (F(v) - v)\,dx = 0$

$$\ln x = \int \frac{dv}{F(v) - v} + c$$

where $v = \frac{y}{x}$ $\longrightarrow$ Solution $y = cx$

Pb (10) solve $(2x^3 + y^3)\,dx - 3xy^2\,dy = 0$

Rewrite $\frac{dy}{dx} = \frac{2x^3 + y^3}{3xy^2}$

$$\frac{2x^3 + y^3}{3xy^2} = \frac{2x^3}{3xy^2} + \frac{y^3}{3xy^2} = \frac{2x^2}{3y^2} + \frac{y}{3x}$$

$$\frac{dy}{dx} = \frac{2}{3(y/x)^2} + \frac{1}{3}\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{2}{3(y/x)^2} + \frac{1}{3}\left(\frac{y}{x}\right) = F(y/x)$$

Homogeneous equation,

Then $\frac{y}{x} = v$ (or) $y = vx$

Then $\ln x = \int \frac{dv}{F(v) - v} + c$

$$\cancel{\ln x = \int \frac{dv}{\frac{2}{3v^2} +}}$$

$$\ln x = \int \frac{dv}{\frac{2}{3v^2} + \frac{1}{3}v} + c$$

$$= \int \frac{dv}{\frac{1}{3}\left[\frac{2}{v^2} - v\right]} + c$$

$$= \int \frac{1}{\frac{1}{3}\left(\frac{2 - v^3}{v^2}\right)} + c$$

$$= \int \frac{3v^2\,dv}{2 - v^3} + c$$

$$d(2 - v^3) = -3v^2\,dv$$

$$\therefore \ln x = \int \frac{-d(2 - v^3)}{2 - v^3} + c$$

$$\ln x = -\ln(2 - v^3) + c$$

$$\ln x = -\ln\left(2 - \left(\frac{y}{x}\right)^3\right) + c$$

$$\ln x + \ln\left(2 - \frac{y^3}{x^3}\right) = c$$

$$\cancel{\ln \frac{x}{2 - \frac{y^3}{x^3}}} \neq \ln x\left(2 - \frac{y^3}{x^3}\right) = c$$

$$\cancel{\ln x^4}$$

$$\ln 2x - \frac{y^3}{x^2} = c$$

$$\frac{2x^3 - y^3}{x^2} = e^c = c_1$$

$2x^3 - y^3 = c_1 x^2$

Pb (11) Solve $\dfrac{dy}{dx} = \dfrac{5x+y}{2x-y}$

$\dfrac{dy}{dx} = \dfrac{5 + y/x}{2 - y/x}$

$\therefore$ RHS is function of LHS

Homogeneous equation

Then $\dfrac{y}{x} = v \qquad y = vx$

$\ln x = \displaystyle\int \frac{dv}{F(v) - v} + c$

$\ln x = \displaystyle\int \frac{dv}{\frac{5+v}{2-v} - v} + c$

$\ln x = \displaystyle\int \frac{dv}{\frac{5+v-2+v^2}{2-v}} + c$

$= \displaystyle\int \frac{(2-v)(dv)}{3+v+v^2} + c$

$d(3+v+v^2) = (\cancel{1+2v)dv} \quad 1 + 2v\,dv \Rightarrow v\,dv = \dfrac{d(3+v+v^2)}{2}$

$\therefore v = \dfrac{d(3+v+v^2) - 1}{2}$

$2 - v = 2 - \dfrac{d(3+v+v^2) - 1}{2}$

$y = vx$ 12

$\cancel{x \frac{dv}{dx} + v}$

$$\frac{d}{dx}(vx) = \frac{5+4v}{2-v}$$

$$x\frac{dv}{dx} + v = \frac{5+4v}{2-v}$$

$$\frac{x\,dv}{dx} = \frac{5+4v}{2-v} - v$$

$$\frac{x\,dv}{dx} = \frac{5+4v-2v+v^2}{2-v}$$

$$\frac{x\,dv}{dx} = \frac{v^2+2v+5}{2-v}$$

$$\frac{dx}{x\,dv} = \cancel{\frac{v^2+2v+5}{2-v}} \quad \frac{2-v}{v^2+2v+5}$$

$$\frac{dx}{x} = \frac{2-v}{v^2+2v+5}\,dv$$

$$\frac{dx}{x} - \frac{2-v}{v^2+2v+5}\,dv = 0$$

$$\frac{dx}{x} + \frac{(v-2)}{v^2+2v+5}\,dv = 0$$

$$\cancel{\int \frac{dx}{x} + \frac{v\,dv}{v^2+2v+5} - \frac{2}{v^2+2v+5}\,dv = 0}$$

$$\cancel{\int \frac{dx}{x} + \frac{\frac{1}{2}d(v^2+2v+5)-2}{v^2+2v+5}}$$

$$\frac{dx}{x} + \int\frac{(v+1)\,dv}{v^2+2v+5} + \int\frac{-3\,dv}{v^2+2v+5}$$

$$\frac{dx}{x} + \frac{1}{2}\ln(v^2+2v+5) + \int\frac{-3\,dv}{(v+1)^2+4}$$

$$\frac{dx}{x} + \frac{1}{2}\ln(v^2+2v+5) - \frac{3}{2}\tan^{-1}\left(\frac{v+1}{2}\right)$$

$d(v^2+2v+5)$

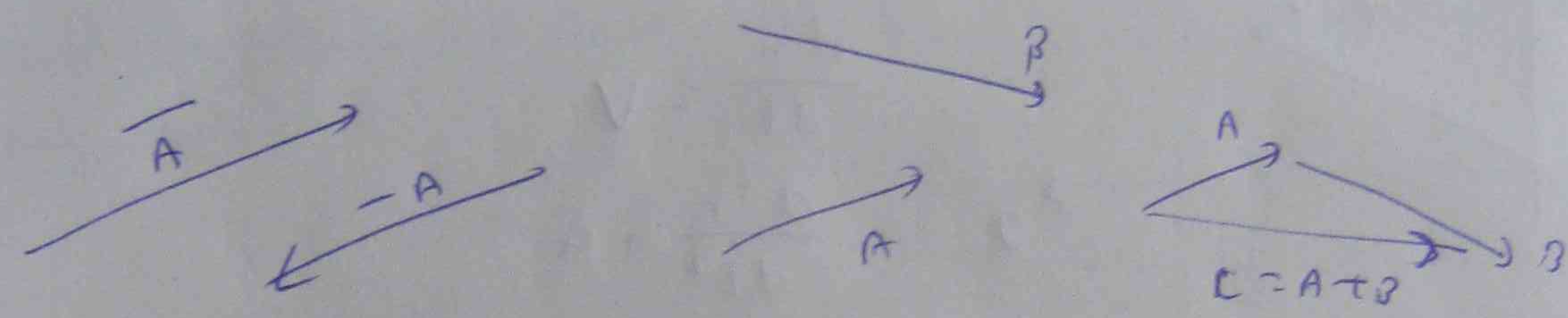
$= 2v\,dv + 2$

$\therefore d(v^2+2v+5) - 2 = 2v\,dv$

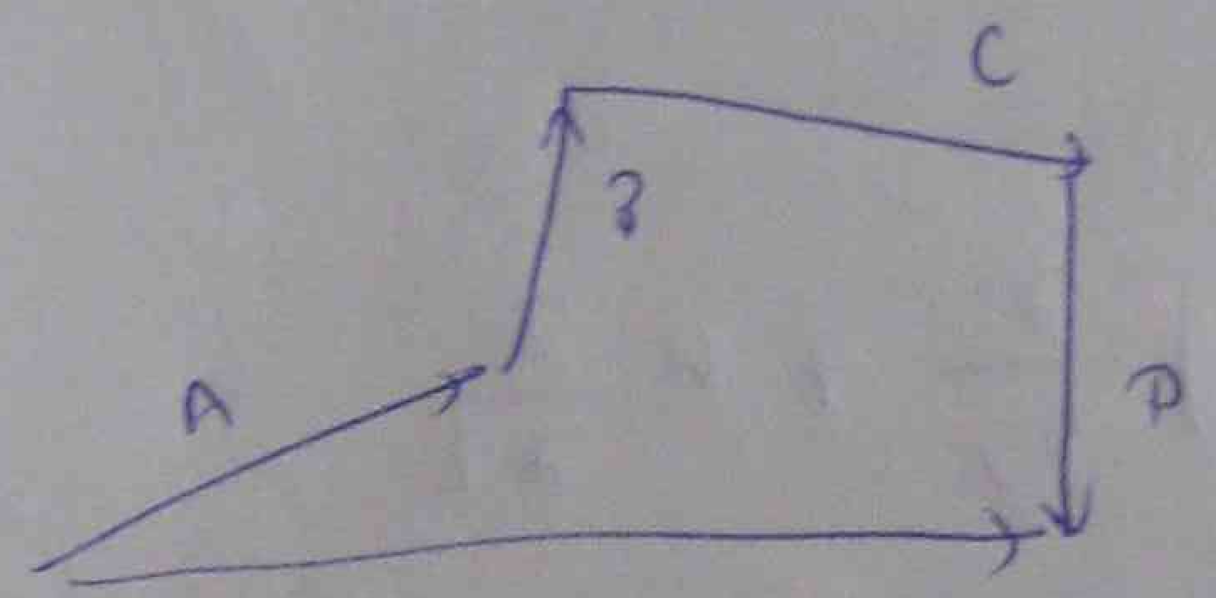
$v\,dv = \frac{1}{2}\left(d(v^2+2v+5) - 2\right.$

1

Vector Analysis



$C = A + B$



$E = A + B + C + D$

Law of vector algebra

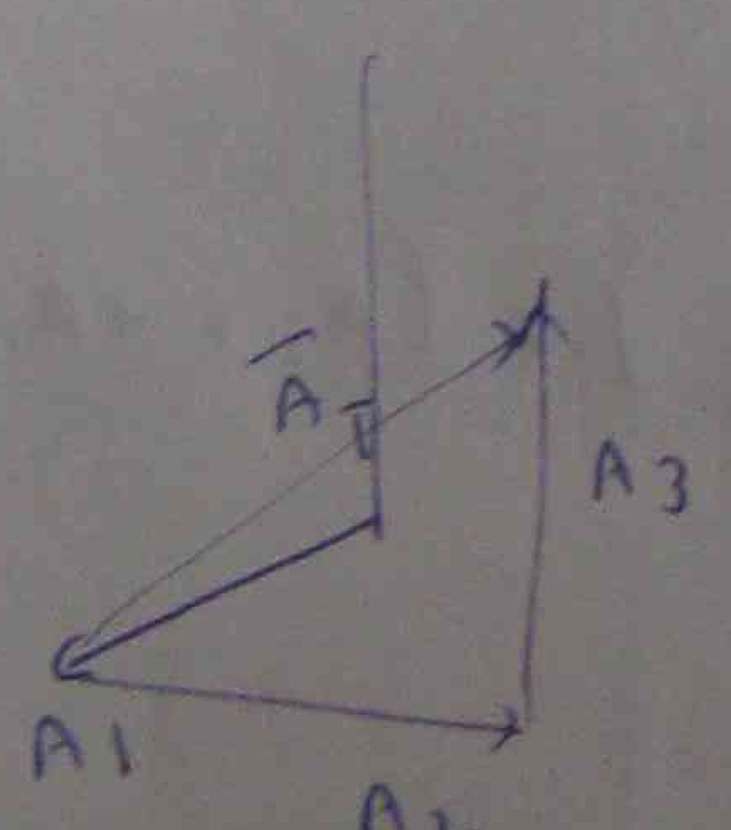
$A + B = B + A$

$A + (B + C) = (A + B) + C$

$m(nA) = (mn)A$

$(m + n)A = mA + nA$

$m(A + B) = mA + mB$



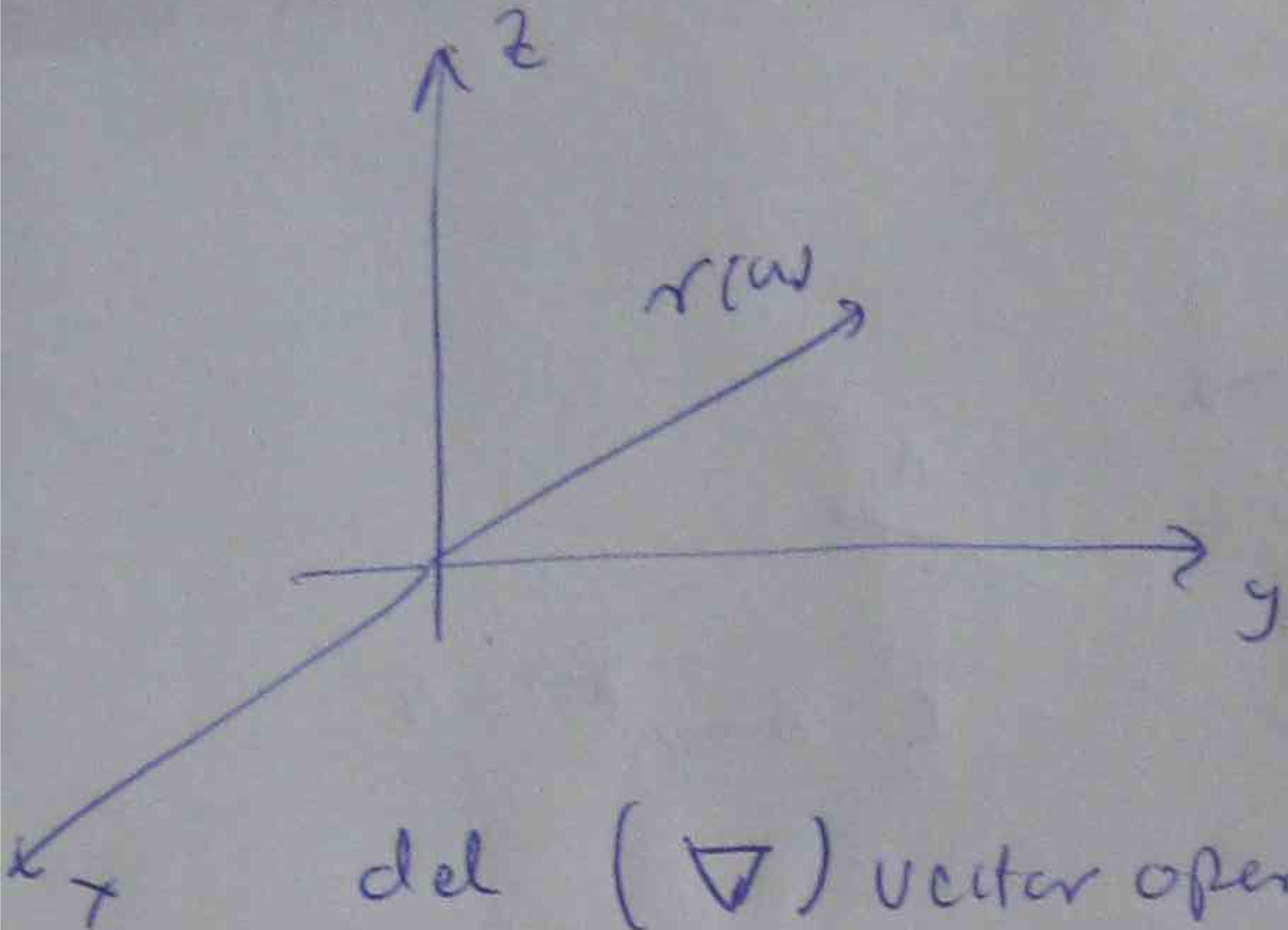
$A_T = A_1\hat{i} + A_2\hat{j} + A_3 k$

$A_T = |A| = \sqrt{A_1^2 + A_2^2 + A_3^2}$

Cross vector product

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$A \cdot (B \times C) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$



$$\frac{dr}{dt} = V$$

$$\frac{d^2r}{dt^2} = a$$

del (∇) vector operator

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Gradient ϕ $\qquad \phi(x, y, z)$

$$\text{Grad}\,\phi = \nabla\phi = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\phi$$

Divergence

$$\text{div}\,A = \nabla \cdot A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot (A_1 i + A_2 j + A_3 k)$$

$$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

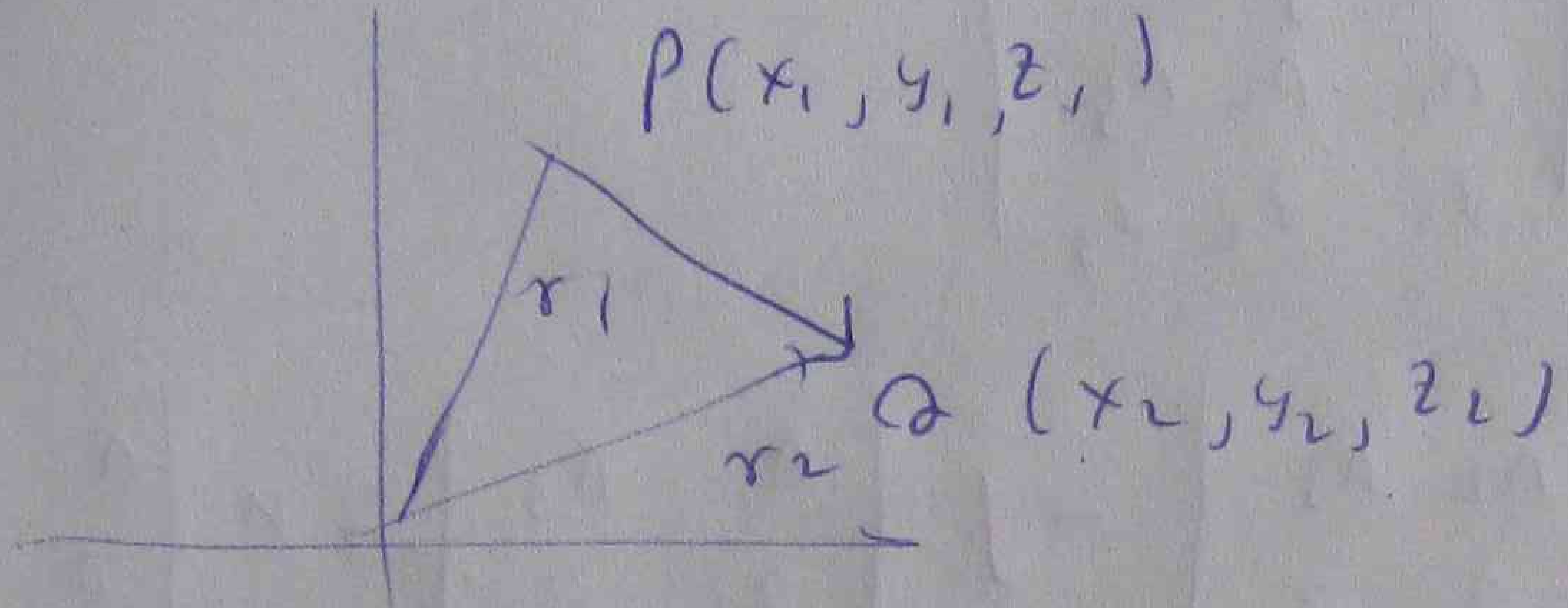
Curl

$$\text{curl}\,A = \nabla \times A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times (A_1 i + A_2 j + A_3 k)$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= \left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) i + \left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right) j + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right) k$$

Pb1 determine the vector having initial point $P(x_1, y_1, z_1)$ and the terminal point $Q(x_2, y_2, z_2)$ and find its magnitude.



$$PQ = r_2 - r_1 = (x_2 i + y_2 j + z_2 k) - (x_1 i + y_1 j + z_1 k)$$
$$= (x_2 - x_1) i + (y_1 - y_1) j + (z_2 - z_1) k$$

magnitude of PQ

$$\overline{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$\boxed{\text{dot (or) scalar product}}$

Pb2 If $A = A_1 i + A_2 j + A_3 k$ & $B = B_1 i + B_2 j + B_3 k$

Prove $A \cdot B = A_1 B_1 + A_2 B_2 + A_3 B_3$

$$A \cdot B = (A_1 i + A_2 j + A_3 k) \cdot (B_1 i + B_2 j + B_3 k)$$
$$= A_1 i \cdot (B_1 i + B_2 j + B_3 k) + A_2 j (B_1 i + B_2 j + B_3 k)$$
$$+ A_3 k (B_1 i + B_2 j + B_3 k)$$
$$= A_1 B_1 \, i \cdot i + A_1 B_2 \, ij + A_1 B_3 \, i \cdot k + B_2 B_1 \, j \cdot i + A_2 B_2 \, j \cdot j$$
$$+ A_3 B_3 \, j \cdot k + A_3 B_1 \, k \cdot j + A_3 B_3 \, k \cdot k$$

$i \cdot i = 1$, $j \cdot j = 1$, $k \cdot k = 1$ | $i \cdot j = j \cdot k = k \cdot i = 0$ | $= A_1 B_1 + A_2 B_2 + A_3 B_3$

Cross Product

$\cancel{i \times i \neq 1}$ $\quad$ $i \times i = j \times j = k \times k = 0$

$\cancel{i \times j \neq 0}$ $\quad$ $i \times j = k, \; j \times k = i, \; k \times i = j$

Pb 3

If $A = A_1 i + A_2 j + A_3 k$ and $B = B_1 i + B_2 j + B_3 k$

Prove that $A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$

$$\begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = i \begin{vmatrix} A_2 & A_3 \\ B_2 & B_3 \end{vmatrix} - j \begin{vmatrix} A_1 & A_3 \\ B_1 & B_3 \end{vmatrix} + k \begin{vmatrix} A_1 & A_2 \\ B_1 & B_2 \end{vmatrix}$$

$$= i \left(A_2 B_3 - A_3 B_2\right) - j \left(A_1 B_3 - B_1 A_3\right) + k \left(A_1 A_2 - B_1\right.$$

$$A \times B = \left(A_1 i + A_2 j + A_3 k\right) \times \left(B_1 i + B_2 j + B_3 k\right)$$

$$= A_1 i \times \left(B_1 i + B_2 j + B_3 k\right) + A_2 j \times \left(B_1 i + B_2 j + B_3 k\right)$$
$$+ A_3 k \times \left(B_1 i + B_2 j + B_3 k\right)$$

$$= A_1 B_1 i \times i + A_1 B_2 i \times j + A_1 B_3 i \times k + A_2 B_1 j \times i$$
$$+ A_2 B_2 j \times j + A_2 B_3 j \times k + A_3 B_1 k \times i + A_3 B_2 k \times j$$
$$+ A_3 B_3 k \times k$$

$$= \cancel{A_1 B_2 k +}$$

$$= A_1 B_2 i \times j + A_1 B_3 i \times k + A_2 B_1 j \times i + A_2 B_3 j \times k$$
$$+ A_3 B_1 k \times i + A_3 B_2 k \times j$$

$$= \cancel{A_1 B_2 k} + A_1 B_3 j + A_2 B_1 k + A_2 B_3 i + A_3 B_1 j$$
$$+ A_3 B_2 i$$

$i \times \hat{j} = k$, $\hat{j} \times i = -k$

$\hat{j} \times k = i$, $k \hat{j} = -i$

$k \times i = \hat{j}$, $i k = -\hat{j}$

$= A_1 B_2 k + A_1 B_2 (-\hat{j}) + A_2 B_1 (-k) + A_2 B_3 i +$

$A_3 B_1 (\hat{j}) + A_3 B_2 (-i)$

$= A_1 B_2 k - A_1 B_2 \hat{j} - A_2 B_1 k + A_2 B_3 i + A_3 B_1 \hat{j}$

$- A_3 B_2 i$

$= A_2 B_3 i - A_3 B_2 i + A_3 B_1 \hat{j} - A_1 B_2 \hat{j} + A_1 B_2 k$

$- A_2 B_1 k$

$= A_2 B_3 i - A_3 B_2 i - A_1 B_2 \hat{j} + A_3 B_1 \hat{j} + A_1 B_2 k - A_2 B_1 k$

$= (A_2 B_3 - A_3 B_2) i - (A_1 B_2 - A_3) \hat{j} + (A_1 B_2 - A_2) k$

pb 4

If $A = 3i - \hat{j} + 2k$, $B = 2i + 3\hat{j} - k$

Find $A \times B$

$$A \times B = \begin{vmatrix} i & \hat{j} & k \\ 3 & -1 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i} \begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + k \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$$

$= \hat{i} \left| -1 \times -1 - 2 \times 3 \right| - \hat{j} \left| 3 \times (-1) - 2 \times 2 \right| + k \left[3 \times 3 - 2(-1) \right]$

$= \hat{i} [1 - 6] - \hat{j} [-3 - 4] + k [9 + 2]$

$= -5\hat{i} + 7\hat{j} + 11k$

Phs

If $A = i + j$, $B = 2i - 3j + k$, $C = 4j - 3k$

Find (a) $(A \times B) \times C$ (b) $A \times (B \times C)$

(a) $A \times B = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 2 & -3 & 1 \end{vmatrix} = i \begin{vmatrix} 1 & 0 \\ -3 & 1 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} + k \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$

$= i\,[1 \times 1 - (-3) \times 0] - j\,[1 \times 1 - 2 \times 0] + k\,[1(-3) - 2 \times 1]$

$= i\,[1] - j\,[1] + k\,(-3-2)$

$A \times B = \hat{i} - \hat{j} - 5k$

$(A \times B) \times C = (\hat{i} - \hat{j} - 5k) \times (4\hat{j} - 3k)$

$= \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -1 & -5 \\ 0 & 4 & -3 \end{vmatrix}$

$= \hat{i}\,[-1 \times (-3) - (-5) \times 4] - \hat{j}\,[1(-3) - 0 \times (-5)]$

$+ k\,[1 \times 4 - 0(-1)]$

$= \hat{i}\,[3 + 20] - j\,[-3] + k \times 4$

$= 23\hat{i} + 3\hat{j} + 4k$

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$$B\times C = \begin{array}{c} \\ B\rightarrow \\ C\rightarrow \end{array}\begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix} = i\begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$$= i\left(-3\times(-3) - 4\times 1\right) - j\left(2\times(-3) - 0\times 1\right) + k\left(2\times 4 - 0\times -3\right)$$

$$= i\left(9-4\right) - j\left(-6\right) + k\times 8$$

$$= 5i + 6j + 8k$$

$$A\times(B\times C) = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 5 & 6 & 8 \end{vmatrix}\begin{array}{l} \\ \leftarrow A \\ \leftarrow B\times C \end{array}$$

$$= i\begin{vmatrix} 1 & 0 \\ 6 & 8 \end{vmatrix} - j\begin{vmatrix} 1 & 0 \\ 5 & 8 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 5 & 6 \end{vmatrix}$$

$$= i\left(1\times 8 - 6\times 0\right) - j\left(1\times 8 - 5\times 0\right) + k\left(1\times 6 - 5\times 1\right)$$

$$= 8i - j(8) + k(1)$$

$$= 8i - 8j + k$$

Gamma, Beta & other special functions

$$\Gamma(m) = \int_0^\infty x^{m-1} e^{-x} \, dx$$

$$\Gamma(m+1) = m\,\Gamma(m)$$

$$\Gamma(m+1) = m!$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

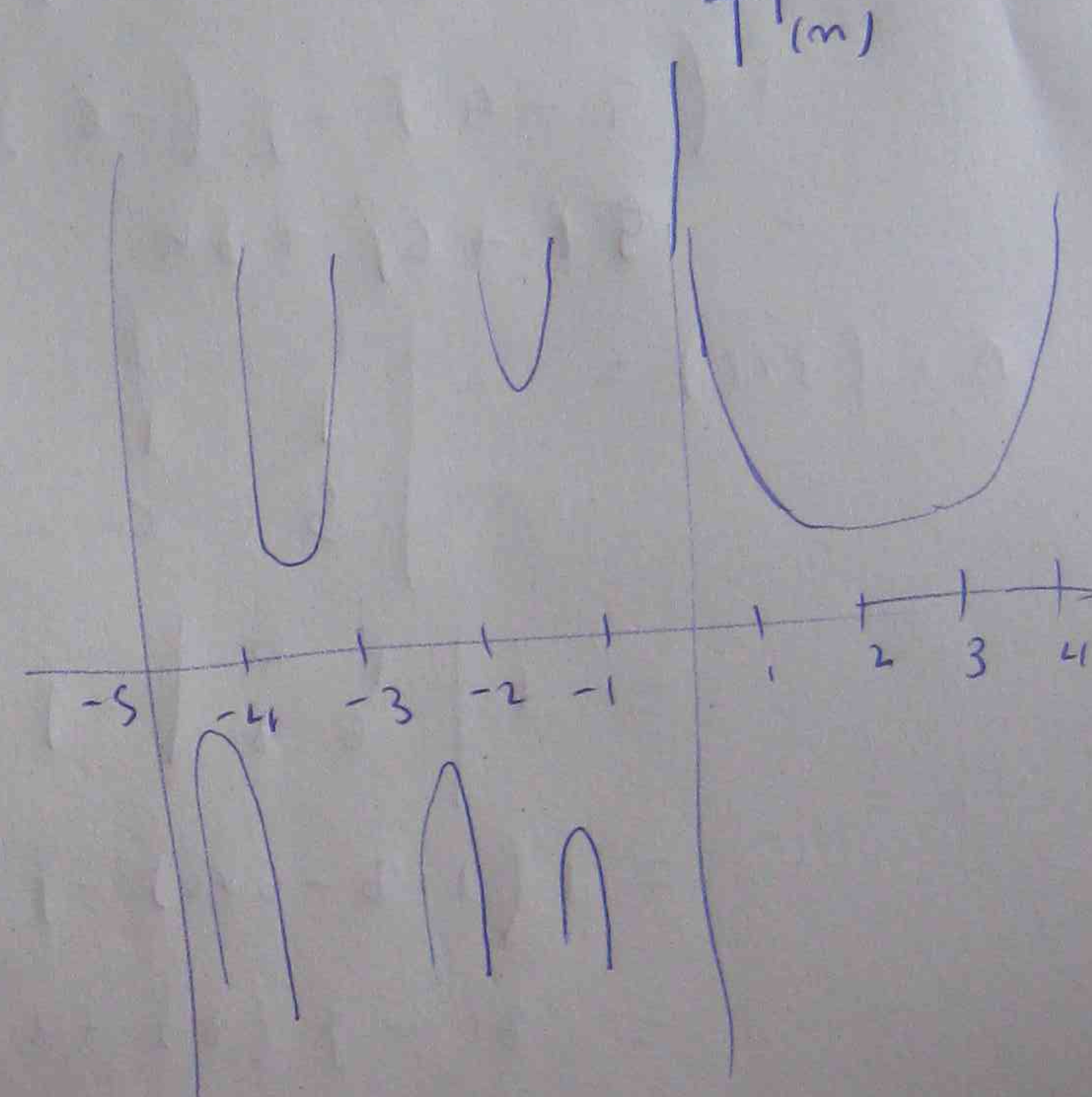

Table of Gamma Function

m	$\Gamma(m)$
1.00	1.00
1.10	0.9514
1.20	0.9182
1.30	0.8975
1.40	0.8873
1.50	0.8862
1.60	0.8935
1.70	0.9086
1.80	0.9314
1.90	0.9618
2.00	1.00

Pb (1) Evaluate each of the followings

(a) $\dfrac{\Gamma(6)}{2\Gamma(3)}$ (b) $\dfrac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})}$ (c) $\dfrac{\Gamma(3)\,\Gamma(2.5)}{\Gamma(5.5)}$

(d) $\dfrac{6\,\Gamma(\frac{8}{3})}{5\,\Gamma(\frac{2}{3})}$

(a) $\dfrac{\Gamma(6)}{2\,\Gamma(3)}$ $\Gamma(m+1) = m!$ $\dfrac{\Gamma(5+1)}{2\,\Gamma(2+1)} = \dfrac{5!}{2 \times 2!} = \dfrac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 2 \times 1}$

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$$\Gamma(m+1) = m\,\Gamma(m)$$

b) $$\frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})}$$

$$\frac{\Gamma(\cancel{1} + \frac{3}{2})}{\Gamma(\frac{1}{2})} = \frac{\frac{3}{2}\,\Gamma(\frac{3}{2})}{\Gamma(\frac{1}{2})} = \frac{\frac{3}{2}\,\Gamma(1+\frac{1}{2})}{\Gamma(\frac{1}{2})}$$

$$= \frac{\frac{3}{2} \times \frac{1}{2}\,\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2})} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

(c) $$\frac{\Gamma(3) \times \Gamma(2.5)}{\Gamma(5.5)} = \frac{\Gamma(3) \times \Gamma(\frac{5}{2})}{\Gamma(\frac{11}{2})}$$

$$\Gamma(3) = \Gamma(2+1) = 2!$$

$$= \quad \Gamma(5/2) = \Gamma(1+\tfrac{3}{2}) = \tfrac{3}{2}\,\Gamma(\tfrac{3}{2}) = \tfrac{3}{2}\,\Gamma(1+\tfrac{1}{2})$$

$$= \tfrac{3}{2} \times \tfrac{1}{2}\,\Gamma(\tfrac{1}{2})$$

$$\Gamma(5.5) = 1.5 \times 0.5\,\Gamma(0.5)$$

$$\Gamma(\tfrac{11}{2}) = \Gamma(1+\tfrac{9}{2}) = \tfrac{9}{2}\,\Gamma(\tfrac{9}{2}) = 4.5 \times \Gamma(1+\tfrac{7}{2})$$

$$= 4.5 \times \tfrac{7}{2}\,\Gamma(\tfrac{7}{2}) = 4.5 \times 3.5 \times \Gamma(1+5/2)$$

$$= 4.5 \times 3.5 \times \tfrac{5}{2}\,\Gamma(5/2)$$

$$= 4.5 \times 3.5 \times 2.5 \times \Gamma(1+3/2)$$

$$= 4.5 \times 3.5 \times 2.5 \times \tfrac{3}{2}\,\Gamma(3/2)$$

$$= 4.5 \times 3.5 \times 2.5 \times 1.5 \times \Gamma(1+1/2)$$

$$= 4.5 \times 3.5 \times 2.5 \times 1.5 \times \tfrac{1}{2}\,\Gamma(1/2)$$

$$= 4.5 \times 3.5 \times 2.5 \times 1.5 \times 0.5\,\Gamma(0.5)$$

$$\frac{2! \times 1.5 \times 0.5 \times \cancel{\Gamma(0.5)}}{4.5 \times 3.5 \times 2.5 \times 1.5 \times 0.5 \times \cancel{\Gamma(0.5)}} = \frac{2 \times 1 \times 1.5 \times 0.5}{4.5 \times 3.5 \times 2.5 \times 1.5 \times 0.5}$$

$$= \frac{16}{315}$$

(d) $$\frac{6\,\Gamma(8/3)}{5\,\Gamma(2/3)} = \frac{6\,\Gamma(1+5/3)}{5\,\Gamma(2/3)} = \frac{6 \times \frac{5}{3}\,\Gamma(5/3)}{5\,\Gamma(2/3)}$$

$$= \frac{6 \times \frac{5}{3} \times \Gamma(1+2/3)}{5 \times \Gamma(2/3)} = \frac{6 \times \frac{5}{3} \times \frac{2}{3}\,\Gamma(2/3)}{5 \times \Gamma(2/3)}$$

$$= \frac{6 \times 5 \times 2}{3 \times 3 \times 5} = \frac{4}{3}$$ #

Pb(2) find $\int_0^\infty x^3 e^{-x}\,dx = \int_0^\infty x^{4-1} e^{-x}\,dx = \Gamma(m) = \Gamma(4)$

$\Gamma(m) = (m-1)! = (4-1)! = 3! = 3\times 2 = 6$

Beta Function

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

Pb 1 $$\int_0^1 x^4(1-x)^3\,dx = ?$$

$$\int_0^1 x^{5-1}(1-x)^{4-1}\,dx = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

$$= \frac{\Gamma(5)\,\Gamma(4)}{\Gamma(5+4)} = \frac{\Gamma(5)\,\Gamma(4)}{\Gamma(9)}$$

$$= \frac{(5-1)!\,(4-1)!}{(9-1)!} = \frac{4! \times 3!}{8!} = \frac{4\times3\times2\times1 \times 3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1} = \frac{1}{280}$$

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{2\,\Gamma(m+n)}$$

Pb (2) Solve $\int_0^2 \frac{x^2\,dx}{\sqrt{2-x}}$

when $x = 2v$ Limit is divide by 2

$0 \longrightarrow {}_0\int^2 = {}_{0/2}\int^{2/2} = \int_0^1$

let $x = 2v$

$$\int_0^1 \frac{(2v)^2\, d(2v)}{\sqrt{2-2v}} = \int_0^1 \frac{4v^2\,2\,dv}{\sqrt{2}\,(\sqrt{1-v})} = 4\sqrt{2}\int_0^1 \frac{v^2}{\sqrt{1-v}}\,dv$$

$$= 4\sqrt{2}\int_0^1 v^2 (1-v)^{-\frac{1}{2}}\,dv$$

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dv = \frac{\Gamma(m)\,\Gamma(n)}{2\,\Gamma(m+n)}$$

$m-1 = 2 \Rightarrow m = 1+2 = 3$

$n-1 = -\frac{1}{2} \quad n = 1-\frac{1}{2} = \frac{1}{2}$

$$= \frac{\Gamma(3)\,\Gamma(\frac{1}{2})}{2\,\Gamma(3+\frac{1}{2})} = \frac{\cancel{\Gamma 3}\,(3-1)!\;\Gamma(\frac{1}{2})}{2\,\Gamma(7/2}$$

$$= \frac{2!\;\Gamma(\frac{1}{2})}{2\,\Gamma(1+5/2)} = \frac{2!\;\Gamma(\frac{1}{2})}{2\times\frac{5}{2}\,\Gamma(5/2)} = \frac{2!\;\Gamma(\frac{1}{2})}{2\times\frac{5}{2}\times\Gamma(1+3/2)}$$

$$= \frac{2!\;\Gamma(\frac{1}{2})}{5\times\frac{3}{2}\,\Gamma(3/2)} = \frac{2!\;\Gamma(\frac{1}{2})}{7.5\times\Gamma(1+\frac{1}{2})} = \frac{2!\;\cancel{\Gamma(\frac{1}{2})}}{7.5\times\frac{1}{2}\,\cancel{\Gamma(\frac{1}{2})}}$$

$$= \frac{2\times 1}{7.5\times 0.5} =$$

Pb ③ Evaluate $\int_0^{\pi/2} \sin^6\theta \, d\theta$

compare $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

$2m-1=6$, $2n-1=0$ because $\cos^{2n-1}\theta = 0$

$\therefore m = 7/2 \quad n = 1/2$ $\therefore 2n-1=0$

$$\therefore \int_0^{\pi/2} \sin^6\theta \, d\theta = \frac{\Gamma(m)\,\Gamma(n)}{2\times\Gamma(m+n)} = \frac{\Gamma(7/2)\,\Gamma(\frac{1}{2})}{2\times\Gamma(\frac{7}{2}+\frac{1}{2})}$$

$$= \frac{\Gamma(7/2)\;\Gamma\frac{1}{2}}{2\times\Gamma(4)} = \frac{\Gamma(1+5/2)\;\Gamma\frac{1}{2}}{2\times(4-1)!}$$

$$= \frac{\frac{5}{2}\,\Gamma(5/2)\;\Gamma(\frac{1}{2})}{2\times 3!} = \frac{2.5\times\Gamma(1+3/2)\times\Gamma(\frac{1}{2})}{2\times 3\times 2}$$

$$= \frac{2.5\times\frac{3}{2}\,\Gamma(3/2)\times\Gamma(\frac{1}{2})}{2\times 6} = \frac{2.5\times 1.5\times\Gamma(1+\frac{1}{2})\,\Gamma(\frac{1}{2})}{2\times 6}$$

$$= \frac{2.5\times 1.5\times\frac{1}{2}\,\Gamma(\frac{1}{2})\;\Gamma(\frac{1}{2})}{2\times 6}$$

$$= \frac{2.5\times 1.5\times 0.5}{2\times 6}\times\sqrt{\pi}\times\sqrt{\pi} = \frac{2.5\times 1.5\times 0.5\times\pi}{2\times 6}$$

$$= \frac{3\pi}{6}$$

$$= \frac{5/2\times 3/2\times 1/2\;\pi}{12}$$

$$= \frac{5\times\cancel{3}\times\pi}{2\times 2\times 2\times\cancel{12}_4} = \frac{5\pi}{32}$$

(1) $\displaystyle\int_0^{\pi/2} \sin^4\theta \cos^5\theta \, d\theta = ?$

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$

$2m-1 = 4 \Rightarrow m = 5/2$

$2n-1 = 5 \Rightarrow n = 6/2 = 3$

$\therefore \frac{1}{2}(B(m,n)) = \frac{1}{2} B(5/2, 3)$

$$= \frac{\Gamma(\frac{5}{2}) \times \Gamma(3)}{2 \times \Gamma(5/2+3)}$$

$$= \frac{\Gamma(1+3/2) \times (3-1)!}{2 \times \Gamma(\frac{11}{2})}$$

$$= \frac{\frac{3}{2}\Gamma(\frac{3}{2}) \times 2!}{\Gamma(1+\frac{9}{2})} = \frac{\frac{3}{2} \times \Gamma(1+\frac{1}{2}) \times 2}{\frac{9}{2}\Gamma(9/2)}$$

$$= \frac{\frac{3}{2} \times \frac{1}{2}\Gamma(\frac{1}{2}) \times 2}{\frac{9}{2} \times \Gamma(1+7/2)} = \frac{1.5 \times 0.5 \times 2 \quad \Gamma(\frac{1}{2})}{4.5 \times \frac{7}{2}\Gamma(\frac{7}{2})}$$

$$= \frac{1.5 \times 0.5 \times 2 \quad \Gamma(\frac{1}{2})}{4.5 \times 3.5 \quad \Gamma(1+5/2)} = \frac{1.5 \times \Gamma(\frac{1}{2})}{4.5 \times 3.5 \times 5/2\,\Gamma(5/2)}$$

$$= \frac{1.5 \times \Gamma(\frac{1}{2})}{4.5 \times 3.5 \times 2.5 \quad \Gamma(1+3/2)} = \frac{1.5 \times \sqrt{\pi}}{4.5 \times 3.5 \times 2.5 \times \frac{3}{2}\Gamma(3/2)}$$

$$\frac{1.5 \times \sqrt{\pi}}{4.5 \times 3.5 \times 2.5 \times 1.5 \times \Gamma(1 + 1/2)} = \frac{1.5 \times \sqrt{\pi}}{4.5 \times 3.5 \times 2.5 \times 1.5 \times \frac{1}{2}\Gamma\frac{1}{2}}$$

$$= \frac{1.5 \times \cancel{\sqrt{\pi}}}{4.5 \times 3.5 \times 2.5 \times 1.5 \times 0.5 \times \cancel{\sqrt{\pi}}} = \frac{2}{31.5} \quad \#\#$$

<u>Other special functions</u>

Error function $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^3} du$

$$= 1 - \frac{2}{\pi} \int_x^\infty e^{-u^2} du$$

Exponential integral $Ei(x) = \int_x^\infty \frac{e^{-u}}{u} du$

Sine Integral $= Si(x) = \int_0^x \frac{\sin u}{u} du = \frac{\pi}{2} - \int_x^\infty \frac{\sin u}{u} du$

cosine integral $Ci(x) = \int_x^\infty \frac{\cos u}{u} du$

~~Fres~~ fresnel sine integral $S(x) = \sqrt{\frac{2}{\pi}} \int_0^x \sin u^3 du$

$$= 1 - \sqrt{\frac{2}{\pi}} \int_0^x \sin u^3 du$$

~~Fresnel~~

fresnal cosine integral =

$$C(x) = \sqrt{\frac{2}{\pi}} \int_0^x \cos^2 u\, du = 1 - \sqrt{\frac{2}{\pi}} \int_x^\infty \cos u^2 du$$

Laplace Transform

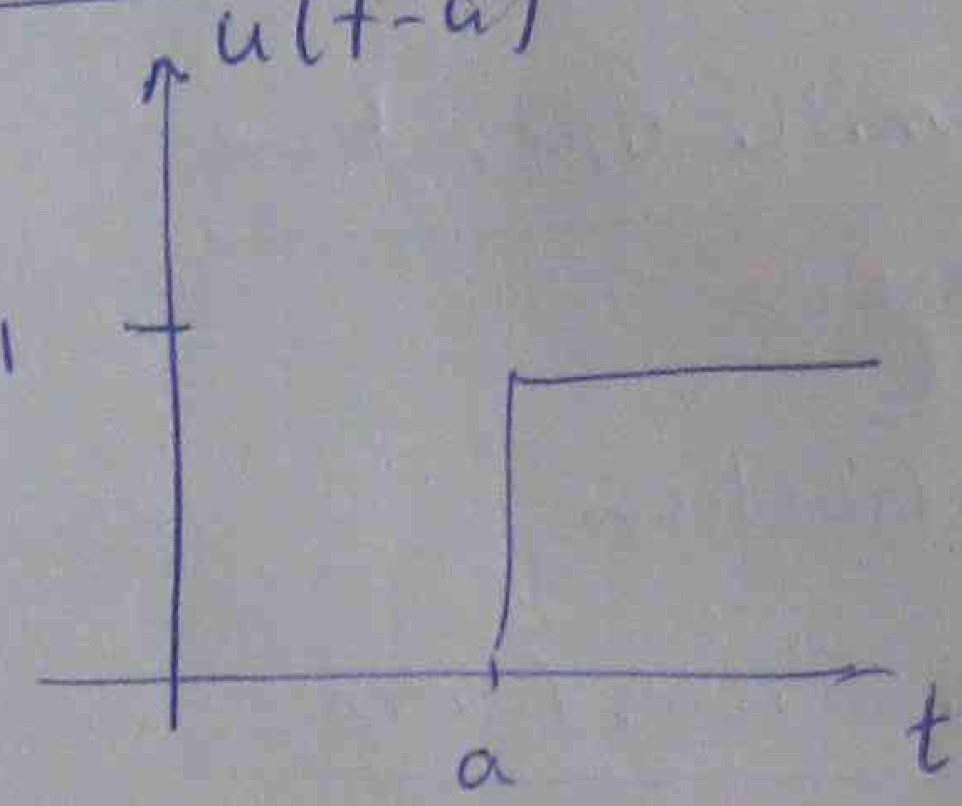
Numerical operation	Logarithmic operation
multiply	Addition
Divide	Substraction

Calculus	Laplace Transform
Integration	multiplication
differentiation	Division

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$$

$f(t)$	$\mathcal{L} f(t) = F(s)$		$\mathcal{L}^{-1} F(s)$	$f(t)$
1	$\frac{1}{s}$	$s > 0$	$\mathcal{L}^{-1} \frac{1}{s}$	1
t^n $n = 1, 2, 3, 4 \dots$	$\frac{n!}{s^{n+1}}$	$s > 0$	$\mathcal{L}^{-1} \frac{n!}{s^{n+1}}$	t^n
t^p $p > -1$	$\frac{p(p+1)}{s^{p+1}}$	$s > 0$	$\mathcal{L}^{-1} \frac{p(p+1)}{s^{p+1}}$	t^p
e^{at}	$\frac{1}{s-a}$	$s > a$	$\mathcal{L}^{-1} \frac{1}{s-a}$	e^{at}
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$s > 0$	$\mathcal{L}^{-1} \frac{s}{s^2 + \omega^2}$	$\cos \omega t$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$s > 0$	$\mathcal{L}^{-1} \frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$
$\cosh at$	$\frac{a}{s^2 - a^2}$		$\mathcal{L}^{-1} \frac{a}{s^2 - a^2}$	$\cosh at$

The unit step function



$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}^{-1}\frac{e^{-as}}{s} = u(t-a)$$

Pb (1) Prove that $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$ if $s > a$

$$\boxed{\int e^u\,du = e^u \qquad \int e^u\,du = e^u}$$

$$\mathcal{L}f(t) = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}\,e^{at} = \int_0^\infty e^{-st} \cdot e^{at}\,dt = \int_0^\infty e^{-(s-a)t}\,dt$$

$$d(-(s-a)t) = -(s-a)\,dt$$

$$\therefore dt = \frac{d(-(s-a)t)}{-(s-a)}$$

$$\int_0^\infty \frac{e^{-(s-a)t}\,d(-(s-a)t)}{-(s-a)}$$

$$\left.\frac{e^{-(s-a)t}}{-(s-a)}\right|_0^\infty = \cancel{\frac{e^{-(s-a)\infty}}{-(s-a)}}_{\to 0} - \left[\frac{e^{-(s-a)\times 0}}{-(s-a)}\right]$$

$$= +\frac{1}{s-a} \quad s > a$$

No. 2

Find the laplace transforms of each of the followings.

(a) $3e^{-4t}$ (b) $2t^2$ (c) $4\cos 5t$ (d) $\sin \pi t$ (e) $-3/\sqrt{t}$

$$\mathcal{L}\, e^{at} = \frac{1}{s-a}$$

(a) $\mathcal{L}\, 3e^{-4t} = 3\,\mathcal{L}\, e^{-4t} = 3 \times \dfrac{1}{s-(-4)} = \dfrac{3}{s+4}$

$s > -4$

(b) $\mathcal{L}\, t^n = \dfrac{n!}{s^{n+1}}$

$\mathcal{L}\, 2t^2 = 2\,\mathcal{L}\, t^2 = \dfrac{2!}{s^{2+1}} = \dfrac{2\times 2\times 1}{s^3} = \dfrac{4}{s^3}$

(c) $\mathcal{L} \cos wt = \dfrac{s}{s^2+w^2}$ $s > 0$

$\mathcal{L}\, 4\cos 5t = 4\,\mathcal{L} \cos 5t = 4 \times \dfrac{s}{s^2+5^2} = \dfrac{4s}{s^2+25}$

$s > 0$

(d) $\mathcal{L} \sin wt = \dfrac{w}{s^2+w^2}$

$\mathcal{L} \sin \pi t = \dfrac{\pi}{s^2+\pi^2}$ $s > 0$

(e) $\cancel{\mathcal{L}\, e^{at} = \frac{1}{s-a}}$ $\mathcal{L}\, t^n = \dfrac{n!}{s^{n+1}}$

$\mathcal{L}\, \dfrac{-3}{\sqrt{t}} = \mathcal{L}\, -3\, t^{-1/2} = \cancel{-3\,\mathcal{L}\, t^{-1/2}\quad 3\times}$

$= \dfrac{-3 \times (-1/2)\,(-1/2+1)}{s^{-\frac{1}{2}+1}}$

$= \dfrac{3/2 \times 1/2}{s^{1/2}} = \dfrac{3}{4\sqrt{s}}$

4

Pb 3 Find Laplace transform of $5\sin 2t - 3\cos 2t$

$$\mathcal{L}\sin\omega t = \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}\sin 2t = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$\mathcal{L}\cos 2t = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$\mathcal{L}(5\sin 2t - 3\cos 2t) = 5\times\frac{2}{s^2+4} - 3\times\frac{s}{s^2+4} = \frac{10-3s}{s^2+4}$$

Pb 4 Find $\mathcal{L}(\sin t\cos t)$

$$\mathcal{L}\sin\omega t = \frac{\omega}{s^2+\omega^2}$$

$\sin 2t = 2\sin t\cos t$

$\therefore\ \sin t\cos t = \frac{\sin 2t}{2}$

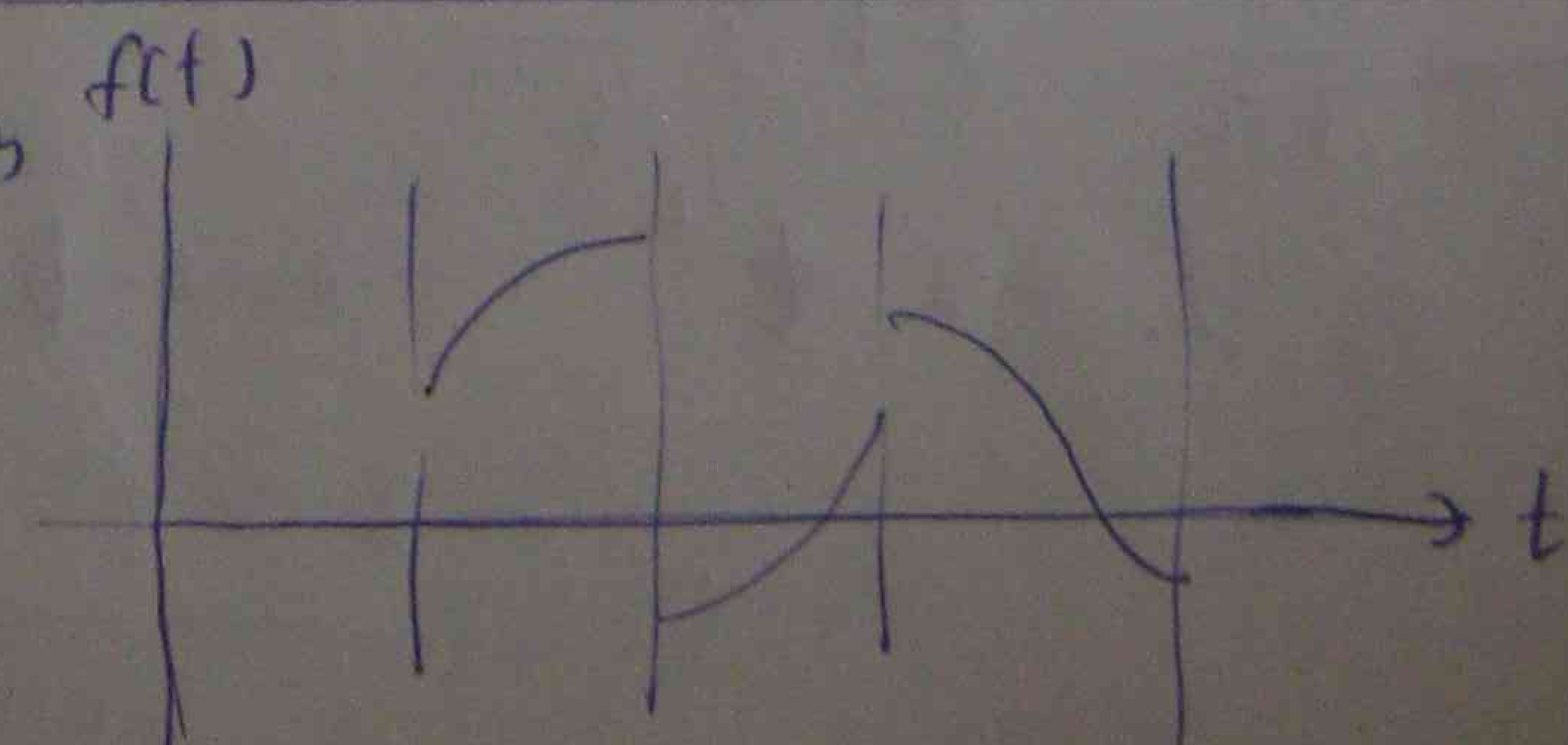
$$\mathcal{L}\sin t\cos t = \mathcal{L}\frac{\sin 2t}{2} = \frac{1}{2}\mathcal{L}\sin 2t = \frac{1}{2}\times\frac{2}{s^2+2^2}$$

$$= \frac{1}{2}\times\frac{2}{s^2+4}$$

$$= \frac{1}{s^2+4}$$

Sufficient conditions for existance of Laplace Transform

(1) Piecewise continuity



(2) $f(t) \leq M e^{\alpha t}$ for $t > T$ (exponential order)

Some special Theorems on Laplace Transform

$$\mathcal{L}\, F(t) = F(s)$$

$$\mathcal{L}\left\{ e^{at} f(t) \right\} = F(s-a)$$

$$\mathcal{L}\left\{ u(t-a)\, f(t-a) \right\} = e^{-as} F(s)$$

$$\mathcal{L}\, f(t) = F(s)$$

$$\mathcal{L}\, F(at) = \frac{1}{a} F\left(\frac{s}{a}\right)$$

$$\mathcal{L}\, t^n f(t) = (-1)^n \frac{d^n F}{ds^n} = (-1)^n F^n(s)$$

<u>periodic function</u> $p > 0$

$$\mathcal{L}\, f(t) = \int_0^p \frac{e^{-st} f(t)\,dt}{1 - e^{-sp}}$$

<u>Integration</u> $\mathcal{L} \int_0^t f(u)\,du = \frac{F(s)}{s}$

$\mathcal{L}^{-1} F(s) = f(t)$ then $\mathcal{L}^{-1} \frac{F(s)}{s} = \int_0^t f(u)\,du$

$$\mathcal{L}\, \frac{f(t)}{t} = \int_0^\infty F(u)\,du$$

If $\mathcal{L}\{f(t)\} = F(s)$, $\mathcal{L}\{g(t)\} = G(s)$

Then $\mathcal{L}\left\{\int_0^t f(u)\, g(t-u)\, du\right\} = F(s)\, G(s)$

If $\mathcal{L}^{-1}\{F(s)\} = f(t)$, $\mathcal{L}^{-1}\{G(s)\} = g(t)$

Then $\mathcal{L}^{-1} F(s)\, G(s) = \int_0^t f(u)\, g(t-u)\, du$

Pb 5 Find (a) $\mathcal{L}^{-1} \dfrac{5}{s+2}$ (b) $\mathcal{L}^{-1} \dfrac{4s-3}{s^2+4}$

(c) $\mathcal{L}^{-1} \dfrac{2s-5}{s^2}$ ~~(d) $\mathcal{L}^{-1} \dfrac{1}{s^u}$~~ $u>0$

(a) $\mathcal{L}^{-1} \dfrac{5}{s+2}$ To apply $\mathcal{L}^{-1} \dfrac{1}{s-a} = e^{at}$

modify $\mathcal{L}^{-1} \dfrac{1}{s+a} = e^{-at}$

$\therefore \mathcal{L}^{-1} \dfrac{5}{s+2} = 5\, \mathcal{L}^{-1} \dfrac{1}{s+2} = 5 \times e^{-2t}$

(b) $\mathcal{L}^{-1} \dfrac{4s-3}{s^2+4} = \mathcal{L}^{-1} \dfrac{4s}{s^2+4} - \mathcal{L}^{-1} \dfrac{3}{s^2+4}$

use $\mathcal{L}^{-1} \dfrac{s}{s^2+\omega^2} = \cos \omega t$, $\mathcal{L}^{-1} \dfrac{\omega}{s^2+\omega^2} = \sin \omega t$

$\mathcal{L}^{-1} \dfrac{4s}{s^2+4} = 4\, \mathcal{L}^{-1} \dfrac{s}{s^2+2^2} = 4 \cos 2t$

$\mathcal{L}^{-1} \dfrac{3}{s^2+4} = 3\, \mathcal{L}^{-1} \dfrac{1}{s^2+2^2} = \dfrac{3}{2}\, \mathcal{L}^{-1} \dfrac{2}{s^2+2^2} = \dfrac{3}{2} \sin 2t$

$\therefore \mathcal{L}^{-1} \dfrac{4s}{s^2+4} - \mathcal{L}^{-1} \dfrac{3}{s^2+4} = 4 \cos 2t - \dfrac{3}{2} \sin 2t$

$$\boxed{\mathcal{L}^{-1}\frac{1}{s^{p}} = \frac{t^{p-1}}{\Gamma(p)}}$$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$\Gamma(n+1) = n\Gamma(n)$

$\Gamma(1+\frac{1}{2}) = 1\ \Gamma(1/2) = 1 \times \sqrt{\pi} = \sqrt{\pi}$

Thus $\Gamma(3/2) = \sqrt{\pi}$

<u>Ph 6</u> Find $\mathcal{L}^{-1}\frac{4-5s}{s^{3/2}}$

$$\mathcal{L}^{-1}\frac{4}{s^{3/2}} - \mathcal{L}^{-1}\frac{5s}{s^{3/2}}$$

$$4\mathcal{L}^{-1}\frac{1}{s^{3/2}} - 5\mathcal{L}^{-1}\frac{1}{s^{1/2}}$$

$$4 \times \frac{t^{3/2-1}}{\Gamma(3/2)} - \cancel{\ }5\frac{t^{1/2-1}}{\Gamma(1/2)}$$

$$4\frac{t^{1/2}}{\sqrt{\pi}} - 5\frac{t^{-1/2}}{\sqrt{\pi}} = \frac{4t^{1/2} - 5t^{-1/2}}{\sqrt{\pi}}$$

<u>Ph 7</u> Find $\mathcal{L}^{-1}\frac{1}{s^{k}}$, $k>0$

<u>To cover fraction</u> $\mathcal{L}^{-1}\frac{1}{s^{k}} = \frac{t^{k-1}}{\Gamma(k)}$

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t

(c) $\mathcal{L}^{-1}\dfrac{2s-5}{s^2} = \mathcal{L}^{-1}\dfrac{2s}{s^2} - \mathcal{L}^{-1}\dfrac{5}{s^2}$

$\boxed{\mathcal{L}^{-1}\dfrac{n!}{s^{n+1}} = t^n}$ $\quad = \mathcal{L}^{-1}\dfrac{2}{s} - \mathcal{L}^{-1}\dfrac{5}{s^2}$

$0!$ $\quad s^{1+1}$

$s^1 = s^{0+1}$

$\mathcal{L}\dfrac{0!}{s^{0+1}} = t^1$ $\quad = 2\,\mathcal{L}^{-1}\dfrac{0!}{s^{0+1}} - 5\mathcal{L}\dfrac{1!}{s^{1+1}}$

$0! = 1$ $\quad = 2t^0 - 5t^1$

$= 2 - 5t$

Pb6 (d) $\mathcal{L}^{-1}\dfrac{4-5s}{s^{3/2}}$ $\quad$ (h) $\mathcal{L}^{-1}\dfrac{1}{s^2+2s}$

Gamma Function

$$\Gamma(p+1) = \int_0^\infty x^p e^{-x}\,dx \qquad p > -1$$

$$\Gamma(p+1) = p\,\Gamma(p) \quad \cdot \quad \Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi} \quad \Gamma(1) = 1$$

used For non integer power / fractional power

$$\boxed{\mathcal{L}\,t^p = \frac{\Gamma(p+1)}{s^{p+1}}} \Longleftrightarrow \boxed{\mathcal{L}^{-1}\frac{\Gamma(p+1)}{s^{p+1}} = t^p}$$

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Pb 10 Find $\mathcal{L}^{-1} \frac{1}{s^2+2s}$

$$\mathcal{L}^{-1} \frac{1}{s^2+2s} = \mathcal{L}^{-1} \frac{1}{s(s+2)}$$

$$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$\frac{1}{s(s+2)} = \frac{A(s+2) + Bs}{(s)(s+2)}$$

$$\therefore A(s+2) + Bs = 1$$

If $s=0 \Rightarrow A(2) + B(0) = 1$

$\therefore 2A + 0 = 1 \Longrightarrow 2A = 1 \longrightarrow A = 1/2$

If $s=1 \Rightarrow A(1+2) + B(1) = 1$

$3A + B = 1$

$3 \times 1/2 + B = 1 \Longrightarrow B = 1 - 3/2 = -1/2$

$$\therefore \frac{1}{s(s+2)} = \frac{1/2}{s} + \frac{(-1/2)}{s+2}$$

$$= \frac{1}{2} \times \frac{1}{s} - \frac{1}{2} \times \frac{1}{(s+2)}$$

$$\mathcal{L}^{-1} \frac{1}{s(s+2)} = \mathcal{L}^{-1} \frac{1}{2} \times \frac{1}{s} - \mathcal{L}^{-1} \frac{1}{2} \times \frac{1}{s+2}$$

$$= \frac{1}{2} \mathcal{L}^{-1} \frac{1}{s} - \frac{1}{2} \mathcal{L}^{-1} \frac{1}{s+2}$$

$$= \frac{1}{2} \times 1 - \frac{1}{2} e^{-2t} = \frac{1}{2}(1 - e^{-2t})$$

$\mathcal{L}^{-1} \frac{1}{s+a} = e^{-at}$

Laplace Tranform Derivatives

$$\mathcal{L}\, f'(t) = \int_0^{\infty} e^{-st} f'(t)\, dt$$

If $u(t-a) = \begin{cases} 0 & t < a \\ 1 & t > a \end{cases}$

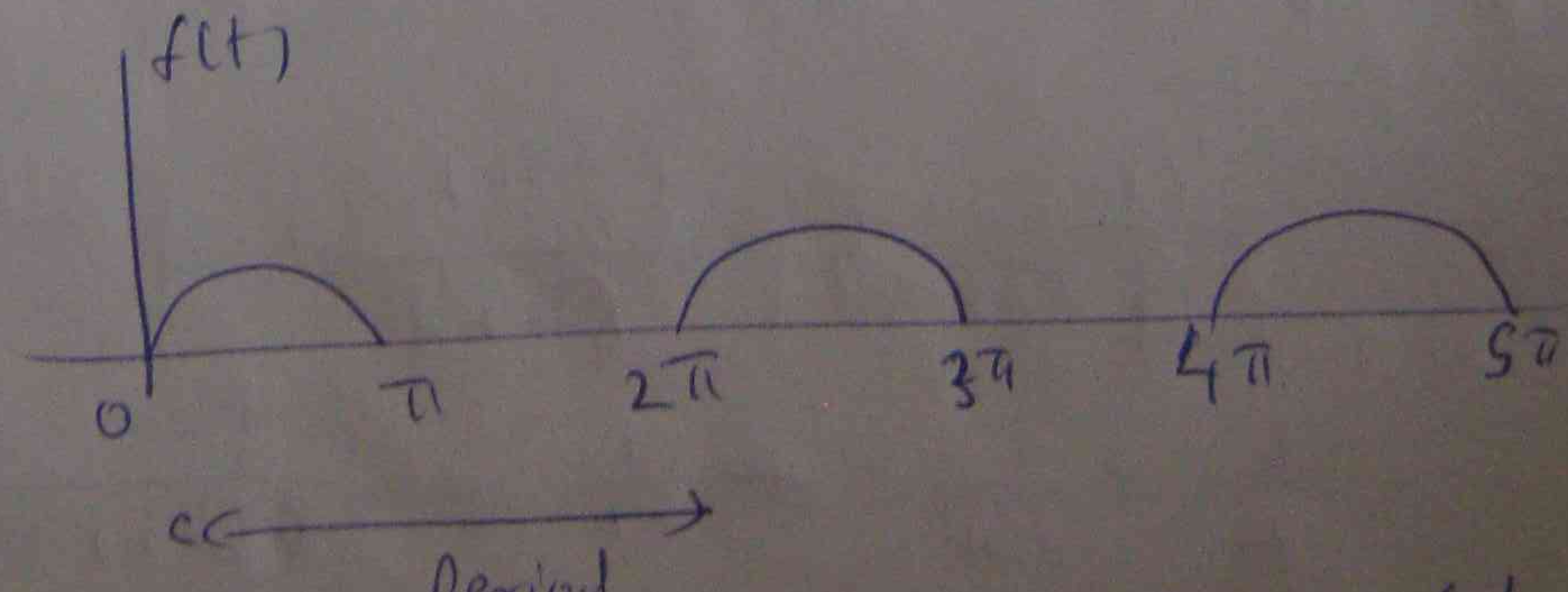
Then $\mathcal{L}\, u(t-a) = \dfrac{e^{-as}}{s}$

If ~~u(t-a)~~ $f(t) = \begin{cases} \sin t & t < \pi \\ t & t > \pi \end{cases}$

Then $\mathcal{L}\, f(t) = \mathcal{L} \sin t + e^{-\pi s}\left[\dfrac{\pi}{s} + \dfrac{1}{s^2} + \dfrac{1}{s^2+1}\right]$

$$= \frac{1}{s^2+1} + e^{-\pi s}\left[\frac{\pi}{s} + \frac{1}{s^2} + \frac{1}{s^2+1}\right]$$

Pb 4 Find the function of following graph



$0 \leq t \leq 2\pi \longrightarrow f(t) = \begin{cases} \sin t & 0 \leq t \leq \pi \\ 0 & \pi \leq t \leq 2\pi \end{cases}$

$f(t)$ has period $p > 0$ $\quad f(t+p) = f(t)$

Then $\mathcal{L}\, f(t) = \int_0^{p} e^{-st} f(t)\, dt$

$$\mathcal{L}(f(t)) = \frac{\int_0^{2\pi} e^{-st} \sin t \, dt}{1-e^{-s \times p}} \qquad \text{where } p = 2\pi$$

$$= \frac{\int_0^{\pi} e^{-st} \sin t \, dt}{e^{-s \times 2\pi}} + \cancel{\frac{\int_{\pi}^{2\pi} e^{-st} \times 0 \, dt}{e^{-s \times 2\pi}}} \to 0$$

$$= \frac{\int_0^{\pi} e^{-st} \sin t \, dt}{1-e^{-2\pi s}}$$

$$\int_0^{\pi} e^{-st} \sin t \, dt = \int_0^{\pi} u$$

$$\mathcal{L} \sin t \, \cancel{dt} = \frac{1}{s^2+1^2}$$

$$\mathcal{L} \sin 1t \, \cancel{dt} = \frac{1}{s^2+1}$$

$$\mathcal{L}\, e^{+st} \sin \omega t = \frac{(1+e^{+\pi \times s})\omega}{s^2+\omega^2}$$

$$\therefore \cancel{\mathcal{L}\, e^{-st} \sin t \, dt} = \int_0^{\pi} e^{-st} \sin t \, dt = \mathcal{L}\, e^{-st} \sin t = \frac{1+e^{\pi(-s)}}{} \times \frac{1}{s^2+1^2}$$

$$= \frac{1+e^{-\pi s}}{s^2+1}$$

$$\therefore = \frac{\frac{1+e^{-\pi s}}{s^2+1}}{1-e^{-2\pi s}} = \frac{1+e^{-\pi s}}{(1-e^{-2\pi s})(s^2+1)}$$

$$= \frac{1+e^{-\pi s}}{(1^2-(e^{-\pi s})^2)(s^2+1)} = \frac{\cancel{1+e^{-\pi s}}}{(\cancel{1+e^{-\pi s}})(1-e^{-\pi s})(s^2+1)}$$

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$$= \frac{1}{(s^2+1)(1-e^{-\pi s})}$$

Solution to differential equations

Pb 12

Solve $y''(t) + y(t) = 1$ given $y(0) = 1$, $y'(0) = 0$

$$y''(t) + y(t) = 1$$

$$\mathcal{L}\, y''(t) + \mathcal{L}\, y(t) = \mathcal{L}\, 1$$

$$\mathcal{L}\, 1 = \frac{1}{s}$$

s^2

$$\mathcal{L}\, f^{(n)}(t) = s^n \mathcal{L}\, f(t) - s^{n-1} \mathcal{L}\, f(0) - s^{n-2} \mathcal{L}\, f'(0) - \dots$$

$n = 2$

$$\mathcal{L}\, y''(t) = s^2\, \underline{\mathcal{L}\, y(t)} - s^{2-1} \mathcal{L}\, y(0) - s^{2-2} \mathcal{L}\, f'(0)$$

$$= s^2 Y - s \times y(0) - s^0 \times y'(0)$$

$$= s^2 Y - s \times 1 - 1 \times 0$$

$$= s^2 Y - s$$

$$\mathcal{L}\, y(t) = Y$$

$$\therefore \mathcal{L}(y''(t) + \mathcal{L}\, y(t) = \mathcal{L}\, 1$$

$$s^2 Y - s + Y = \frac{1}{s}$$

$$s^2 Y + Y = s + \frac{1}{s}$$

$$(s^2+1) Y = \frac{s^2+1}{s}$$

$$Y = \frac{1}{s}$$

$$y = \mathcal{L}^{-1} Y = \mathcal{L}^{-1} \frac{1}{s}$$

$$= 1$$

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$$Y = \frac{2s^2 - 5s - 5}{(s^2 - 3s + 2)(s+1)}$$

$\cancel{14}$ $s^2 - 3s + 2 =$ $s \quad -2$, $s \quad -1$

$= (s-2)(s-1)$

$$= \frac{2s^2 - 5s - 5}{(s-2)(s-1)(s+1)}$$

$$\frac{2s^2 - 5s - 5}{(s-2)(s-1)(s+1)} = \frac{A}{(s-2)} + \frac{B}{(s-1)} + \frac{C}{(s+1)}$$

$$\frac{2s^2 - 5s - 5}{(s-2)(s-1)(s+1)} = \frac{A(s-1)(s+1) + B(s-2)(s+1) + C(s-2)(s-1)}{(s-2)(s-1)(s+1)}$$

$$2s^2 - 5s - 5 = A(s-1)(s+1) + B(s-2)(s+1) + C(s-2)(s-1)$$

$\underline{\underline{s = 1}}$

$$2(1)^2 - 5\times 1 - 5 = \cancel{A(1-1)(1+1)} + B(1-2)(1+1) + \cancel{C(1-2)(1-1)}$$

$$2 - 5 - 5 = B(-1)(2)$$

$$-8 = -2B \qquad \therefore B = 4$$

$\underline{\underline{s = -1}}$

$$2(-1)^2 - 5(-1) - 5 = \cancel{A(-1-1)(-1+1)} + \cancel{B(-1-2)(-1+1)} + C(-1-2)(-1-1)$$

$$2 + 5 - 5 = C(-3)(-2)$$

$$2 = C(6) \qquad \therefore C = 1/3$$

Pb 13 Solve $y'' - 3y' + 2y = 2e^{-t}$

where $y(0) = 2$, $y'(0) = -1$

$$\mathcal{L}(y'' - 3y' + 2y) = \mathcal{L}\,2e^{-t}$$

$$\mathcal{L}\,2e^{-t} = 2\mathcal{L}\,e^{-t} = \frac{2}{s+1}$$

$$\boxed{\begin{aligned} \mathcal{L}y'' &= s^2Y - sy(0) - y'(0) \\ \mathcal{L}y' &= sY - y(0) \\ \mathcal{L}y &= Y \end{aligned}}$$

$\mathcal{L}e^{at} = \frac{1}{s-a}$

$\mathcal{L}e^{-at} = \frac{1}{s+a}$ | $\mathcal{L}e^{-t} = \frac{1}{s+1}$

$$\therefore \left[s^2Y - sy(0) - y'(0)\right] - 3\left[sY - y(0)\right] + 2Y = \frac{2}{s+1}$$

$$\left[s^2Y - s \times 2 - (-1)\right] - 3\left[sY - 2\right] + 2Y = \frac{2}{s+1}$$

$$s^2Y - 2s + 1 - 3sY + 6 + 2Y = \frac{2}{s+1}$$

$$s^2Y - 2s - 3sY + 2Y + 7 = \frac{2}{s+1}$$

$$s^2Y - 3sY + 2Y - 2s + 7 = \frac{2}{s+1}$$

$$Y(s^2 - 3s + 2) = \frac{2}{s+1} + 2s - 7$$

$$Y(s^2 - 3s + 2) = \frac{2 + (2s-7)(s+1)}{s+1}$$

$$Y(s^2 - 3s + 2) = \frac{2 + 2s^2 - 7s + 2s - 7}{(s+1)}$$

$S = 2$

$$2(2)^2 - 5(2) - 5 = A(2-1)(2+1) + B\cancel{(2-2)}(2+1) + C\cancel{(2-2)}(2-1)$$

$$2 \times 4 - 10 - 5 = A \times 3$$

$$8 - 15 = 3A$$

$$3A = -7 \longrightarrow A = -7/3$$

$$\therefore \mathcal{L}^{-1} \frac{2S^2 - 5S - 5}{(S-2)(S-1)(S+1)} = \mathcal{L}^{-1} \frac{-7/3}{S-2} + \mathcal{L}^{-1} \frac{4}{S-1} + \mathcal{L}^{-1} \frac{1/3}{S+1}$$

$$= -7/3\, e^{2t} + 4 e^{t} + \frac{1}{3} e^{-t}$$

$$= \frac{1}{3} e^{-t} + 4 e^{t} - 7/3\, e^{2t}$$

Solving Electrical problems

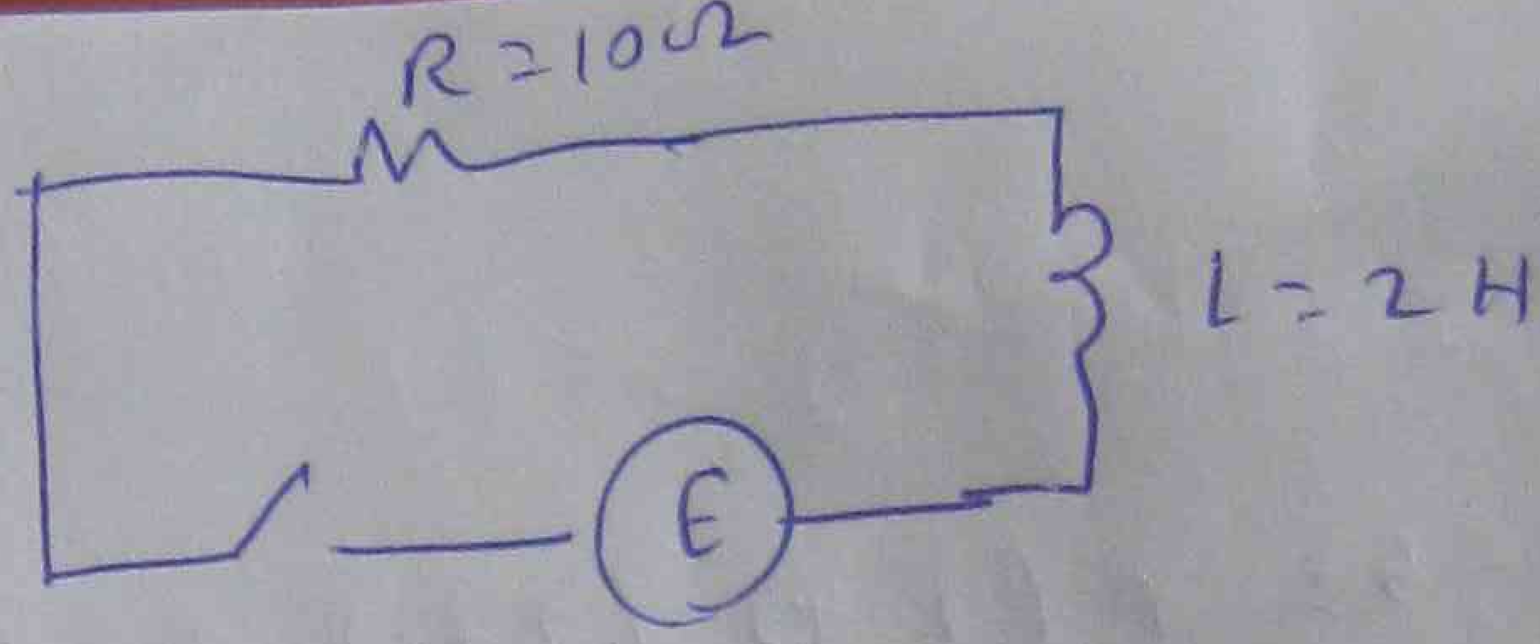
Pb 14 A resistor of $R = 10\Omega$, an inductor of $L = 2\,H$ and a battery of E volts are connected in series with a switch "S". At $t = 0$ the switch is closed and the current $I = 0$

Find I for $t > 0$ if (a) $E = 40V$

(b) $E = 20 e^{-3t}$

(c) $E = 50 \sin 5t$

$I(0) = 0$



$I(0) = 0$

$\frac{dI}{dt} + 5I = \frac{E}{2}$

$E = 40\,V$

$I' + 5I = \frac{E}{2}$

$\mathcal{L} I' + \mathcal{L} 5I = \mathcal{L} \frac{E}{2}$

$\mathcal{L} I' + \mathcal{L} 5I = \mathcal{L} \frac{40}{2} = \mathcal{L}\, 20$

$s I(s) - I(0) + 5I(s) = \frac{20}{s}$

$I(0) = 0$

$s\, I(s) + 5I(s) = \frac{20}{s}$

$I(s)\,(s + 5) = \frac{20}{s}$

$I(s) = \frac{20}{(s+5)\,s}$

$\frac{20}{(s+5)\,s} = \frac{A}{s+5} + \frac{B}{s}$

$\frac{20}{(s+5)\,s} = \frac{A s + B(s+5)}{(s+5)\,s}$

$\therefore A s + B(s+5) = 20$

$s = 0 \qquad B \times 5 = 20 \longrightarrow B = 4$

$s = -5 \longrightarrow -5A + B(-5+5) = 20 \quad \therefore A = -4$

$$\frac{20}{(s+5)(s)} = \frac{-4}{s+5} + \frac{4}{s}$$

$$\mathcal{L}^{-1}\frac{20}{(s+5)(s)} = \mathcal{L}^{-1}\frac{-4}{s+5} + \mathcal{L}^{-1}\frac{4}{s}$$

$$= -4e^{-5t} + 4$$

$$I = 4(1-e^{-5t})$$

(b) If $E = 20e^{-3t}$

$$I(s)\,(s+5) = \mathcal{L}\,\frac{20e^{-3t}}{2} = 10 \times \frac{1}{s+3}$$

$$I(s) = \frac{10}{(s+5)(s+3)}$$

$$\frac{10}{(s+5)(s+3)} = \frac{A}{s+5} + \frac{B}{s+3}$$

$$10 = A(s+3) + B(s+5)$$

$\underline{\underline{s = -3}}$ $\quad 10 = A(-3+3) + B(-3+5)$

$$2B = 10 \longrightarrow B = 5$$

$\underline{s = -5}$ $\quad 10 = A(-5+3) + B(-5+5)$

$$-2A = 10 \longrightarrow A = -5$$

$$= \frac{-5}{s+5} + \frac{5}{s+3} \quad \mathcal{L}^{-1} \Rightarrow -10(e^{-5t} + e^{-3t})$$

Ans

$(s))\,(s+5) = \mathcal{L}\,\frac{50}{2}$

$= \frac{50}{2}\,\mathcal{L}\sin 5t$

$= \frac{50}{2} \times \frac{5}{s^2+5^2}$

$I(s)\,(s+5) = \frac{125}{s^2+25}$

$I(s) = \frac{\cancel{250}\ 125}{(s^2+25)(s+5)}$

$\frac{125}{(s^2+25)(s+5)} = \frac{As+B}{s^2+25} + \frac{C}{s+5}$

$125 = (As+B)(s+5) + C(s^2+25)$

$s = 0$

$125 = (A \times 0 + B)(0+5) + C(0+25)$

$125 = 5B + 25C \longrightarrow (1)$

$s = -5$

$125 = (A(-5)+B)(-5+5) + C((-5)^2+25)$

$125 = 50C \longrightarrow C = \frac{125}{50} = \frac{5}{2}$

$$\therefore 125 = 5B + 25 \times \frac{5}{2}$$

$$125 = 5B + \frac{125}{2}$$

$$5B = \cancel{155}\ 125 - \frac{125}{2} = \frac{125}{2}$$

$$B = \frac{125}{10} = \frac{25}{2}$$

$$s = 1$$

$$125 = (A + B)(1+5) + C(1+25)$$

$$125 = \left(A + \frac{25}{2}\right) \cancel{(1+1)} \times 6 + \frac{5}{2} \times 26$$

$$125 = \left(A + \frac{25}{2}\right) \times 6 + 65$$

$$\left(A + \frac{25}{2}\right) \times 6 = 60$$

$$A + \frac{25}{2} = 10$$

$$A = 10 - \frac{25}{2} = -5/2$$

$$\frac{\cancel{20}}{\cancel{(s+5)(s)}} = \cancel{-5/2\, s} +$$

$$\frac{125}{(s^2+25)(s+5)} = \frac{-5/2\, s + \frac{25}{2}}{s^2+25} + \frac{5/2}{s+5}$$

$$= \frac{-5/2\, s}{s^2+25} + \frac{25/2}{s^2+25} + \frac{5/2}{s+5}$$