Q.6.(20 marks). Determine the first four terms of the trigonometric Fourier series for the waveform shown in figure 3.

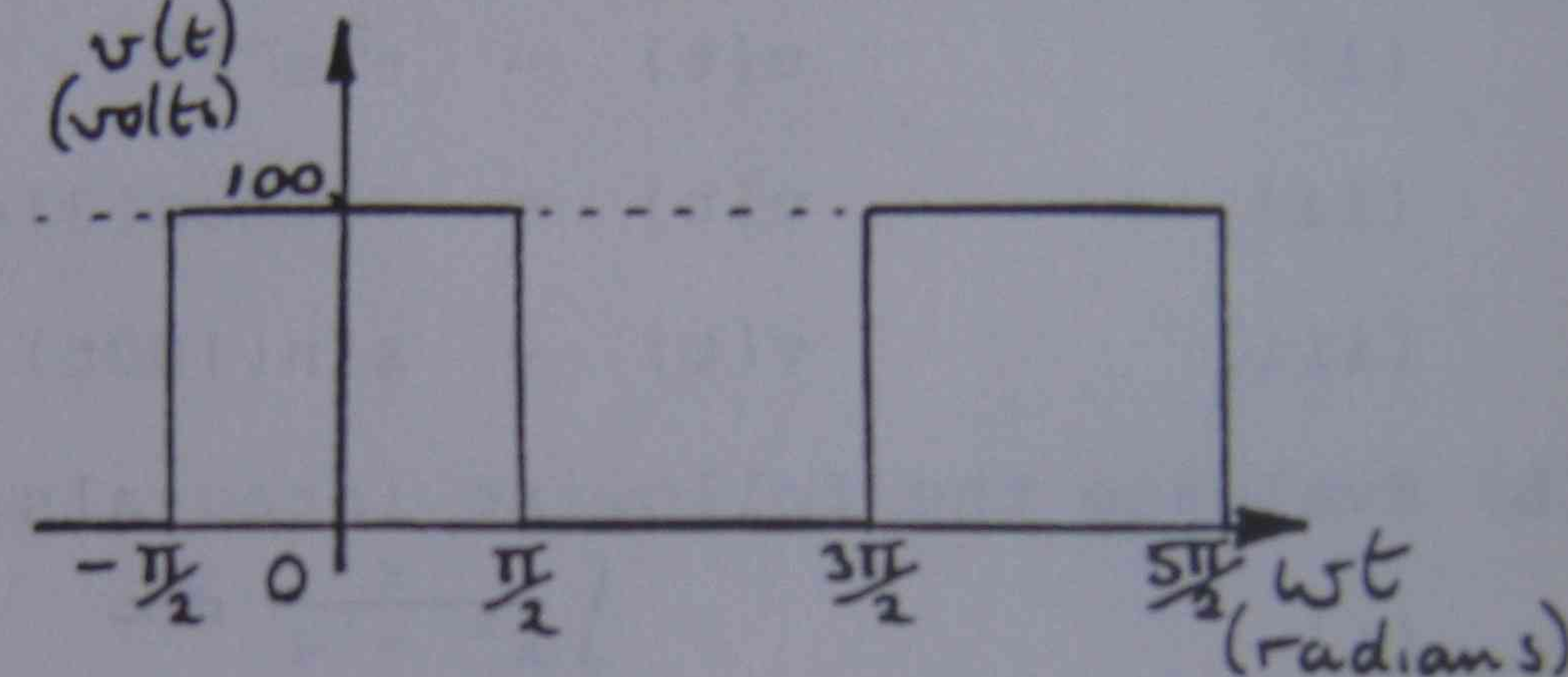
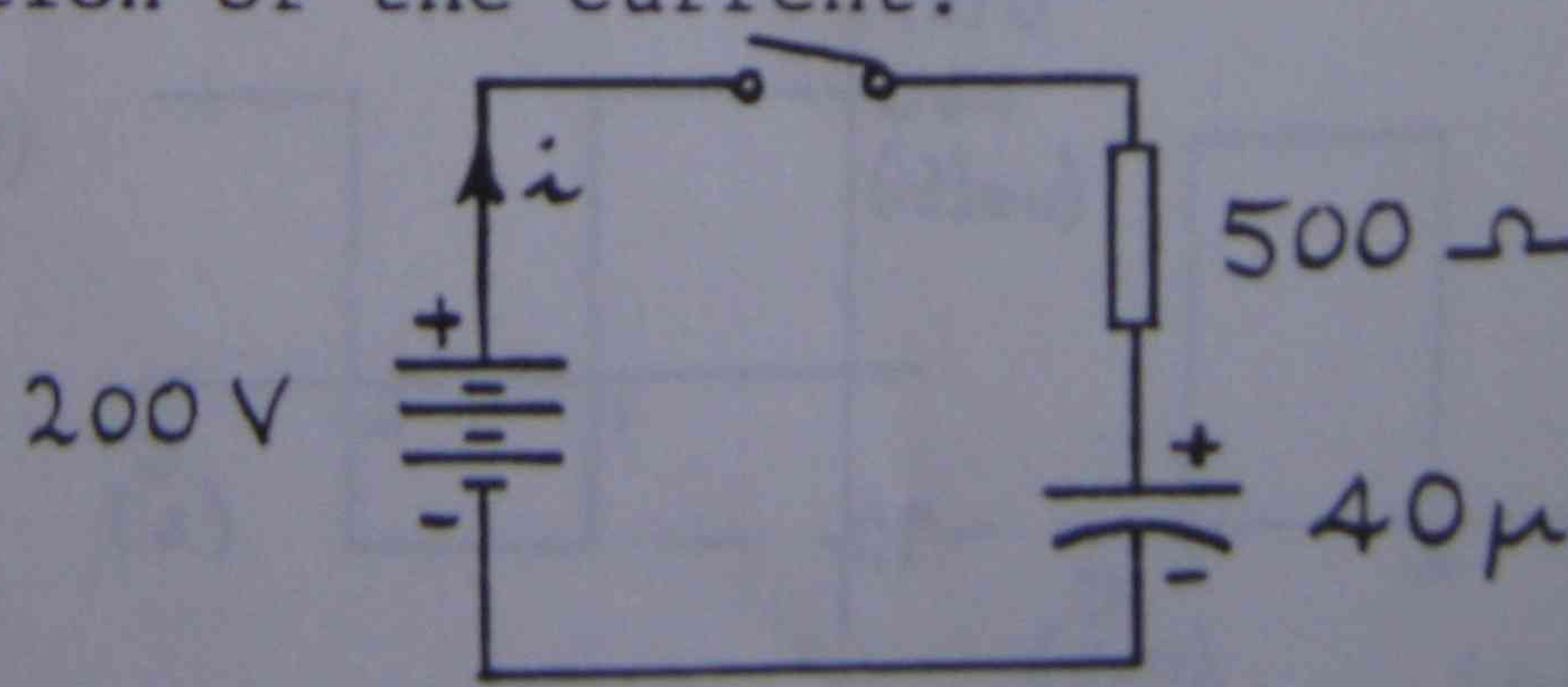


Figure 3

Q.7.(10 marks). After closing the switch in the circuit shown in figure 4 a current will flow. Determine the following:-

(a) the final value of current;
(b) the initial value of current;
(c) the time constant of the circuit;
(d) the equation of the current.



wk 1+2

Power Circuit Principles

2840BP

UNIT 3

Harmonics and Fourier analysis

OTEN

Department of School Education

TAFE NSW

Contents

Introduction

All the circuit analysis work you have experienced in studying ac principles up to this point has involved only pure sine waves. Because we have used only pure sine waves, the calculations of impedances involving inductive and capacitive reactances and complex powers have been reasonably simple.

You may not agree with the previous statement, but consider how you would analyse the effect of a square wave on a circuit containing inductance and capacitance. In this unit you will be shown a method called Fourier analysis to help you work out problems similar to that of a square wave in complex circuits.

Much electrical and electronic equipment involve waveforms which are non-sinusoidal. Some of these are deliberately created, others are the results of distortions produced by equipment. In the analysis of these non-sinusoidal waveforms you will be shown that all repetitive waveforms can be made up of combinations of many sinusoidal waves. Also any waveform can be analysed to determine the component quantities. These sine wave components may be used when investigating the response of a complex impedance circuit to a non-sinusoidal supply voltage. The Fourier method provides the means for solving this type of problem.

When you are studying this unit, the reference from the textbook, Edminister, is Chapter 12. The textbook, in covering this topic uses a complete mathematical approach using integral calculus. For the work in this unit, you are not expected to use calculus. Where calculus is required (to obtain the values of constants etc which you need to use in your analysis of waveforms), such values will be given to you. I would advise that the textbook only be used as a supplement for the material presented in this unit. The self-assessment exercises will be your guide to the depth to which each topic has to be understood.

Objectives

The following list of objectives is for your guidance when learning the work in this unit. Do not be concerned if there are terms or concepts stated here which you do not understand. These will become clear as you progress through the unit. At the completion of this unit, review these objectives—you should be able to understand how to:

- determine the trigonometric Fourier series for repetitive waveforms;
- recognise waveform symmetry and hence simplify the Fourier analysis;
- synthesise the Fourier series and show how the series represents the original waveform;
- apply the concepts of Fourier analysis to explain the harmonic content of waveforms;
- state the sources of the production of harmonics;
- explain the problems of non-sinusoidal waveforms in the supply network;
- calculate the effective value of the waveform and the power consumed in the load;
- analyse the effects of harmonics on the value of the rms current in a load.

Harmonics in waveforms

Reference: Edminister, page 190

Most of the circuit analysis in ac work you have done up to this stage has been possible because you have always considered the supply voltage and current to be sinusoidal. The formulae required the use of sinusoidal waves.

The waveshapes in Figure 1 are common in both electronic and electrical circuits and the calculation of circuit responses is possible using the theory already applied to sine waves.

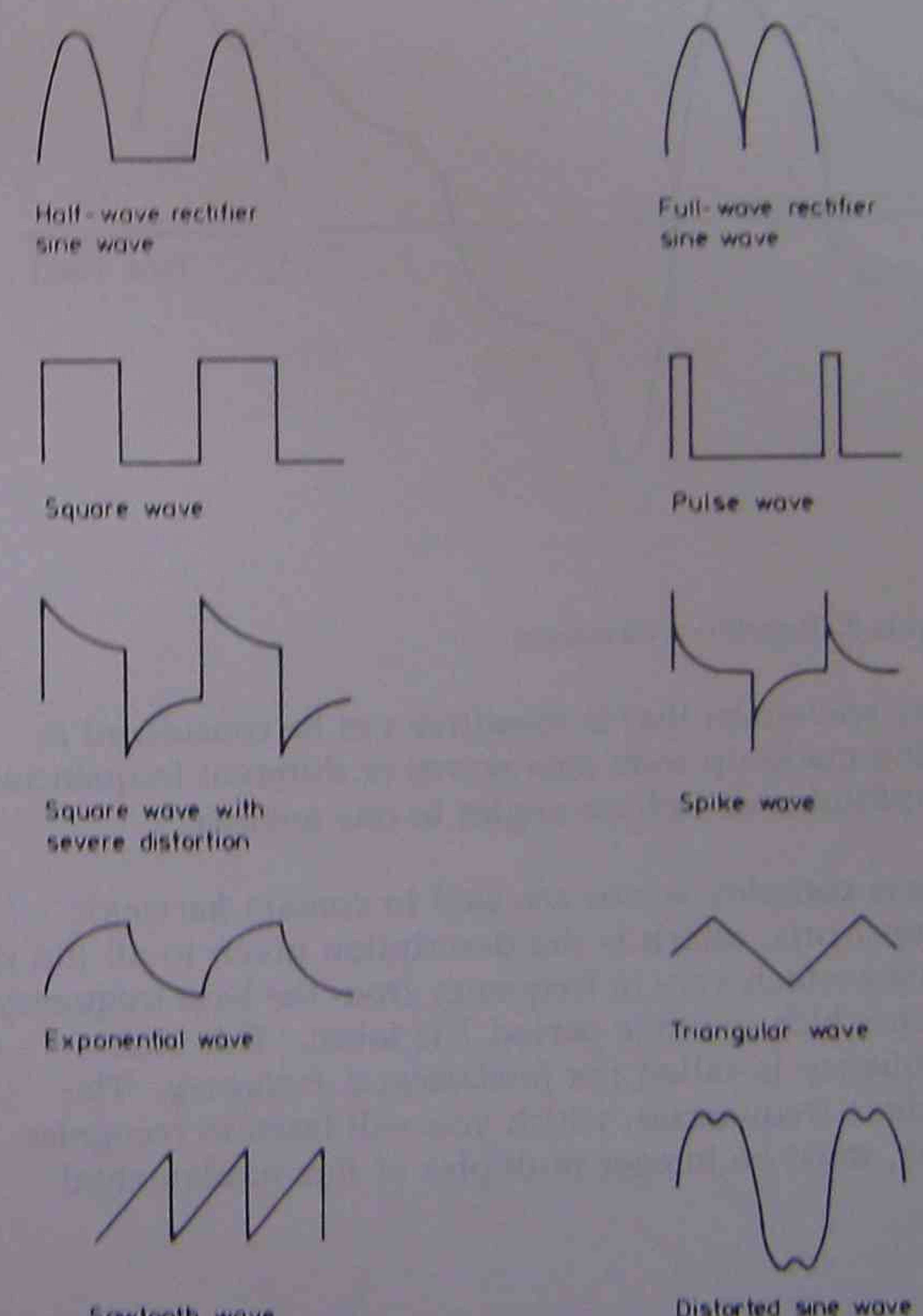


Figure 1

The first important detail to be known about any waveshape is whether it is *repetitive*. The term 'periodic' is used sometimes instead of 'repetitive'.

Consider the waveshape in Figure 2. If we take the value of the wave at any time t_1 then the wave must have exactly the same value at $t_2 = t_1 + T$, where T is the period of the wave and again at $t_3 = t_1 + 2T$. In other words the wave must reproduce itself exactly during each time period T(hence the term 'periodic'), during the time that the supply is considered stable.

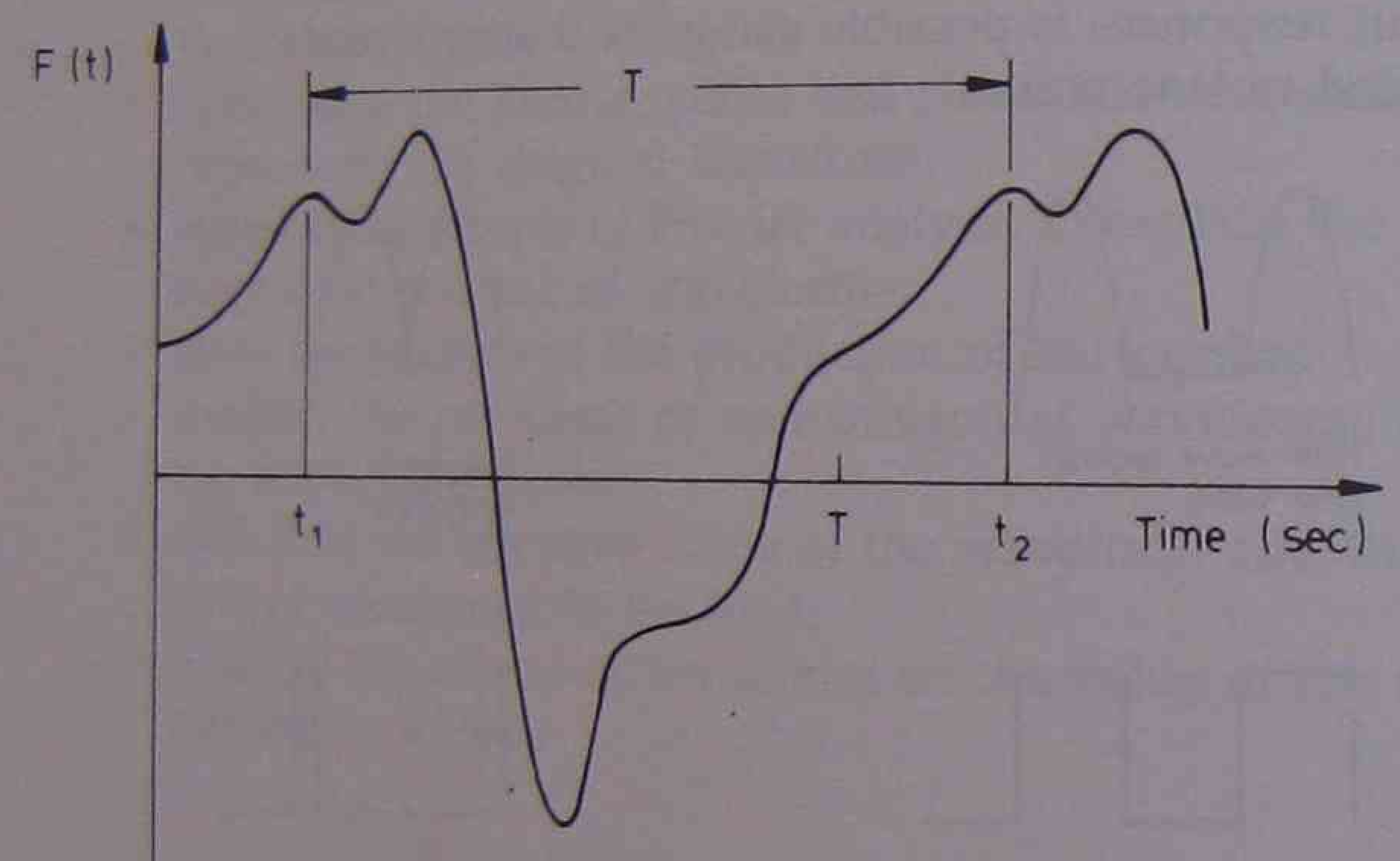


Figure 2: Repetitive waveform

Any waveform that is repetitive can be considered as being made up from sine waves of different frequencies, amplitudes and phase angles to one another.

These complex waves are said to contain *harmonic components*, which is the description given to all the sine waves which vary in frequency from the base frequency from which the time period T is taken. This base frequency is called the *fundamental frequency*. The various frequencies, which you will learn to recognise later, must be integer multiples of this fundamental.

These harmonics take the form shown below:

fundamental = base frequency (sin ωt)
2nd harmonic = 2 x base frequency (sin $2\omega t$)
3rd harmonic = 3 x base frequency (sin $3\omega t$)
to the nth harmonic = n x base frequency (sin $n\omega t$)

Waveform synthesis using Fourier series

Reference: Edminister, page 196

To develop the concept of complex waves being made up of components of sine waves, let us consider an example and develop it through two stages.

If two generators are connected in series, as in Figure 3, with generator 2 having a frequency three times the frequency of generator 1, the output of the system is the sum of all instantaneous amplitudes to give a combined wave shape.

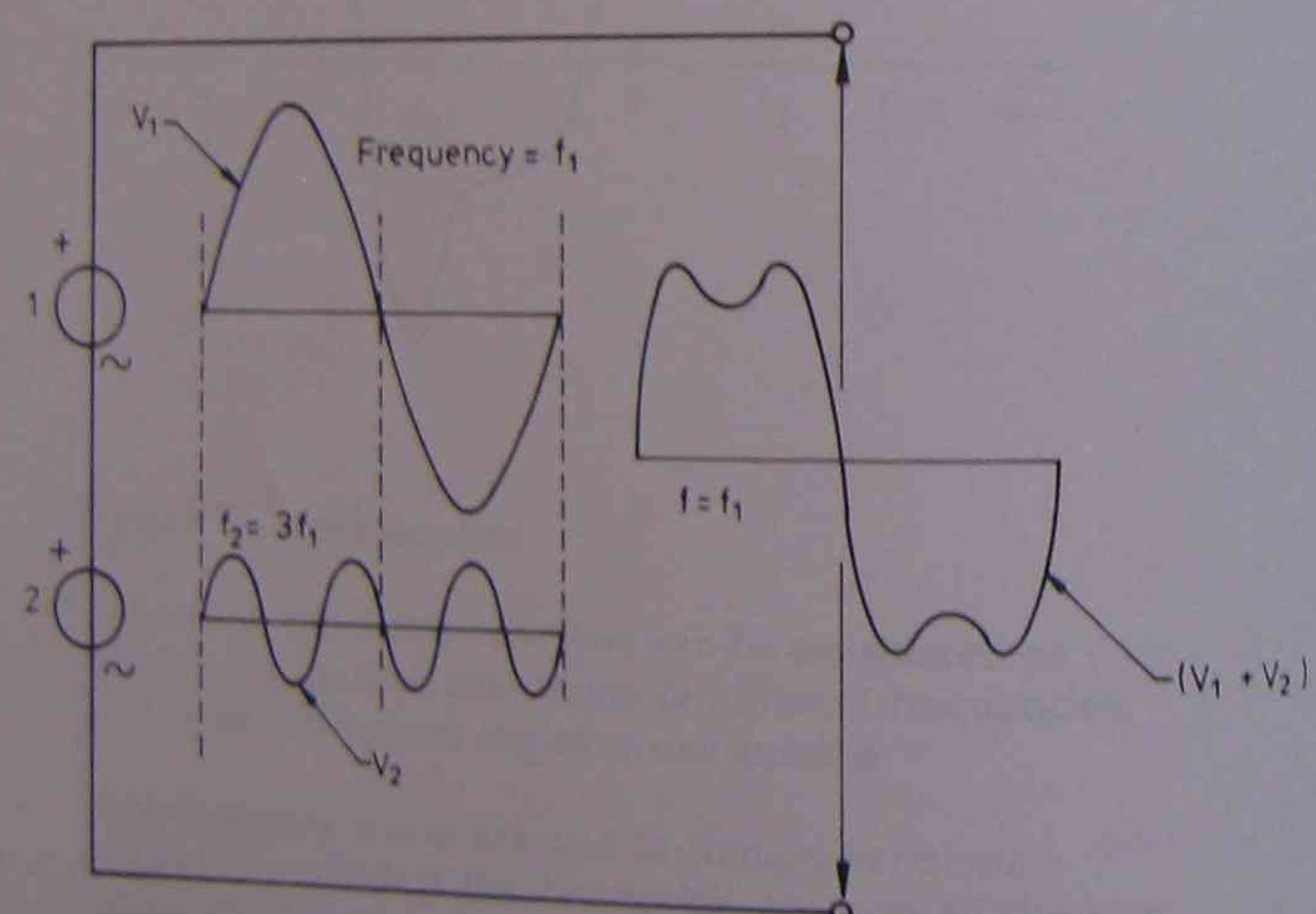


Figure 3

Have you noted that the amplitude of the wave from generator 2 is much less than that of generator 1? Both waves also start at the same point and both rise in a positive direction. If wave 2 were to start in the negative direction, the output wave would be quite different.

Let us now add a third generator to the system with a frequency of five times that of generator 1. This system is shown in Figure 4.

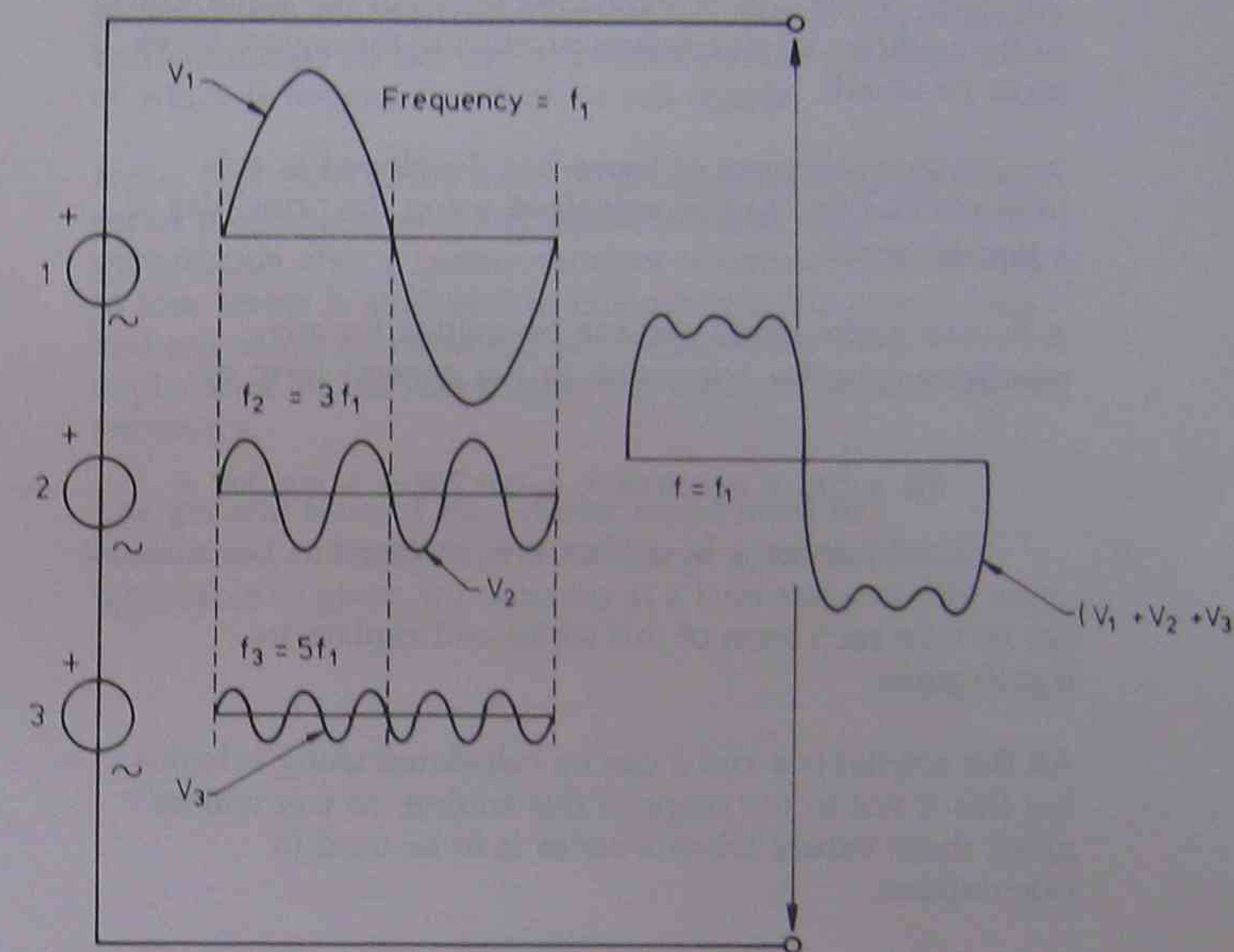


Figure 4

With the addition of the third generator the output wave is now beginning to resemble a poor quality square wave. If time and space permitted, we would add further generators to this system containing the frequencies of seven times, nine times, 11 times the fundamental frequency, and so on. Each additional wave would produce a squarer square wave with less ripple on the top, until, with sufficient generators, a practical pure square wave would be produced.

In this example a number of points must be noted:

- Each successive generator produces a frequency which is an odd number multiple of the fundamental (base) frequency.

- These frequencies are called the third, fifth, seventh (etc) harmonic frequencies.
- Each successive harmonic was at a reduced magnitude to its predecessor.
- All started at the same point and in a positive direction.

All these points will be explained later but the significance of the addition of sine waves producing the square wave must be noted.

An obvious sequence of terms has developed in this example and this can be explained using the principles of a Fourier series.

A Fourier series exists and can be written for any repetitive complex waveform in the general form of:

$$f(t) = \frac{1}{2}a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \ldots\ldots$$

$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \ldots\ldots$$

Let us take each term of this series and explain its significance.

All the constants *a* and *b* can be calculated using calculus but this is not in the scope of this subject, so you will be given these values when a series is to be used in calculations.

The first term $\frac{1}{2}a_0$ is an average value, hence it is the value of any dc component which may be present. In the previous example there were only ac generators, but there could have been a dc generator or battery in the system, which would have offset the wave up or down depending on its polarity. You will see this effect in later examples.

Because some complex waves do not start with zero amplitude at time zero, they can be expressed in the terms of cosine functions. So the Fourier series either contains both cosine and sine terms or is made up exclusively of sine, or of cosine, terms. Later in this unit the difference between the sine and cosine terms will be explained and also when either, or both, are used.

The reason we use *a* and *b* constants is to be able to know which constants go with sine terms and which go with cosine terms. In an example where all the *b* terms are zero, then there are no sine terms in the series. Where

the *a* terms are zero, then there are no cosine terms in the series.

At the end of each section of the general series, after the $a_3 \cos 3\omega t$ and the $b_3 \sin 3\omega t$ there is a row of dots. These dots represent the continuation of the terms in the series to infinity in the mathematical sense. In reality it would be determined by the circuit conditions, the consideration of which is beyond the scope of this course.

Note: Sufficient terms must be given to indicate how the series progresses, as it will repeat the arithmetic progression after a certain number of terms. Usually three to four terms is sufficient to demonstrate the progression and any more terms would not be necessary for an explanation of the waveshape, but they can be calculated if necessary.

The general form of the Fourier series must be memorised so that you will be able to construct series applicable to given waveshapes at a later stage of this unit.

Fourier series simplification using waveform symmetry

Reference: Edminister, page 193

There are two basic conditions which are used to simplify the Fourier series.

Condition 1: Half-wave symmetry

When you look at the two examples in Figure 5, you will notice that the horizontal axis is through the centre of the waves. The shape of the positive section of the wave is the same as that of the negative section of the wave.

The test for half-wave symmetry is that both positive and negative half cycles of the wave are indentical. This can be checked by superimposing the negative half cycle between the two positive half cycles and if each wave pulse is identical to the next, then the wave is half-wave symmetric. Compare Figure 5 to Figure 6.

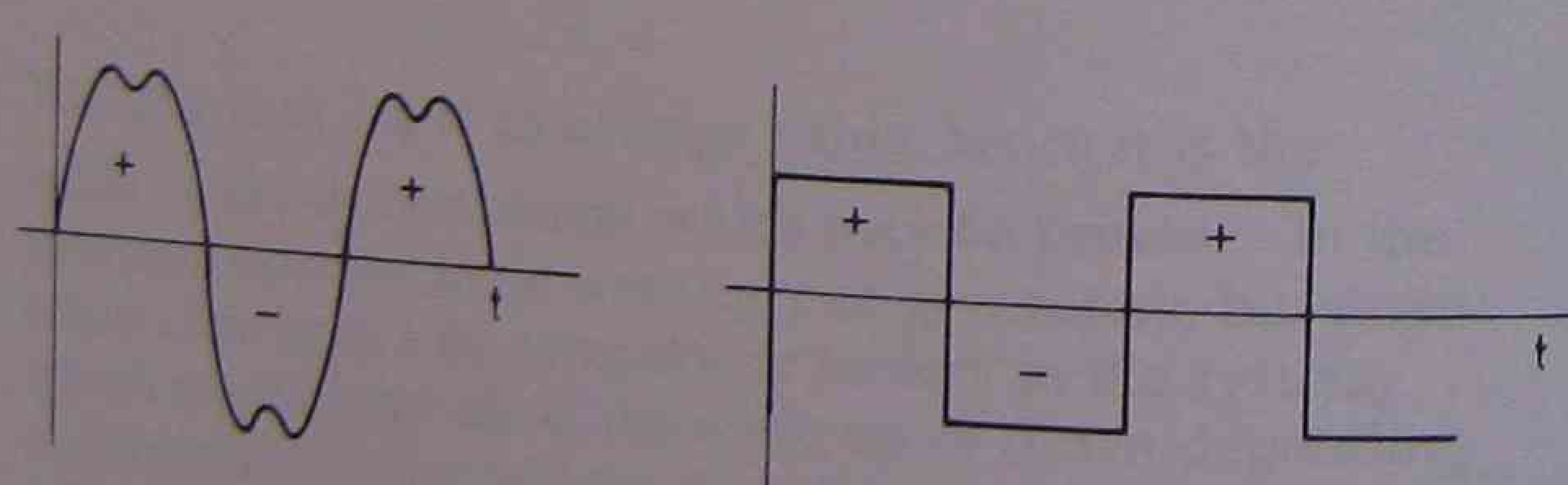


Figure 5: Waveforms with half-wave symmetry

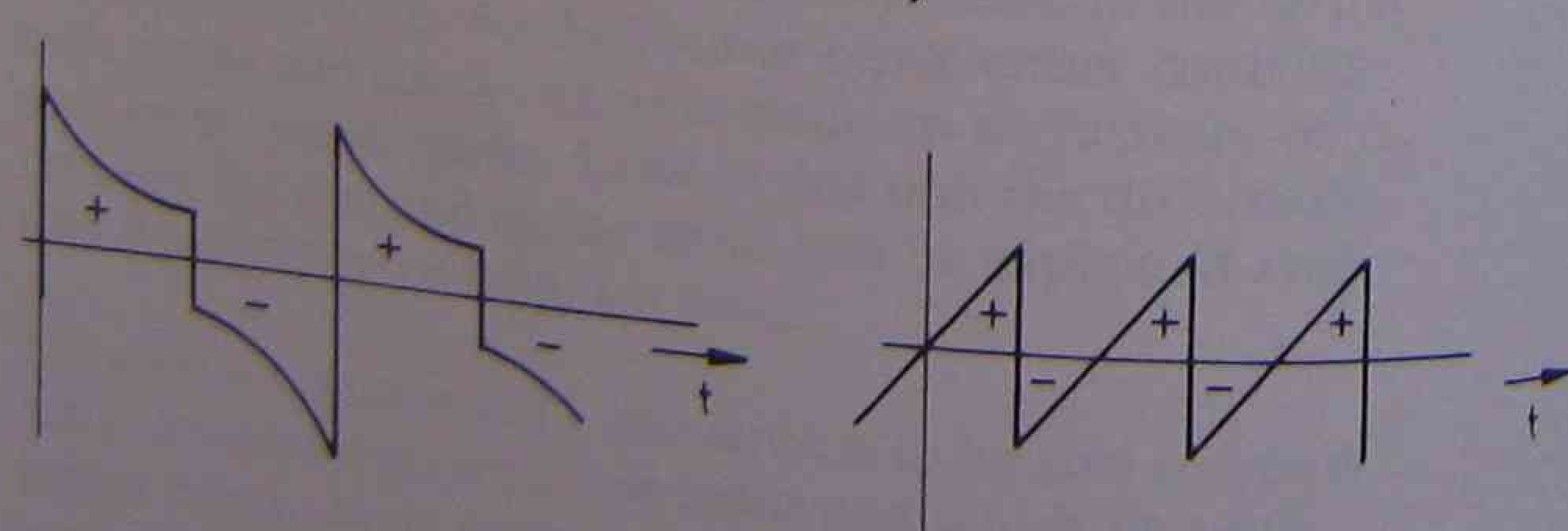


Figure 6: Waveforms not having half-wave symmetry

A function is said to be half-wave symmetric if

$$f(t) = -f(t + \frac{T}{2})$$

where T is the period of the wave.

To explain this expression for the function of *t* look at Figure 7 and note the value of the wave at t_1. If you now move along the horizontal axis, half the period T, to the negative half cycle and to the position $(t_1 + \frac{T}{2})$ then the value must be the negative of the value at t_1.

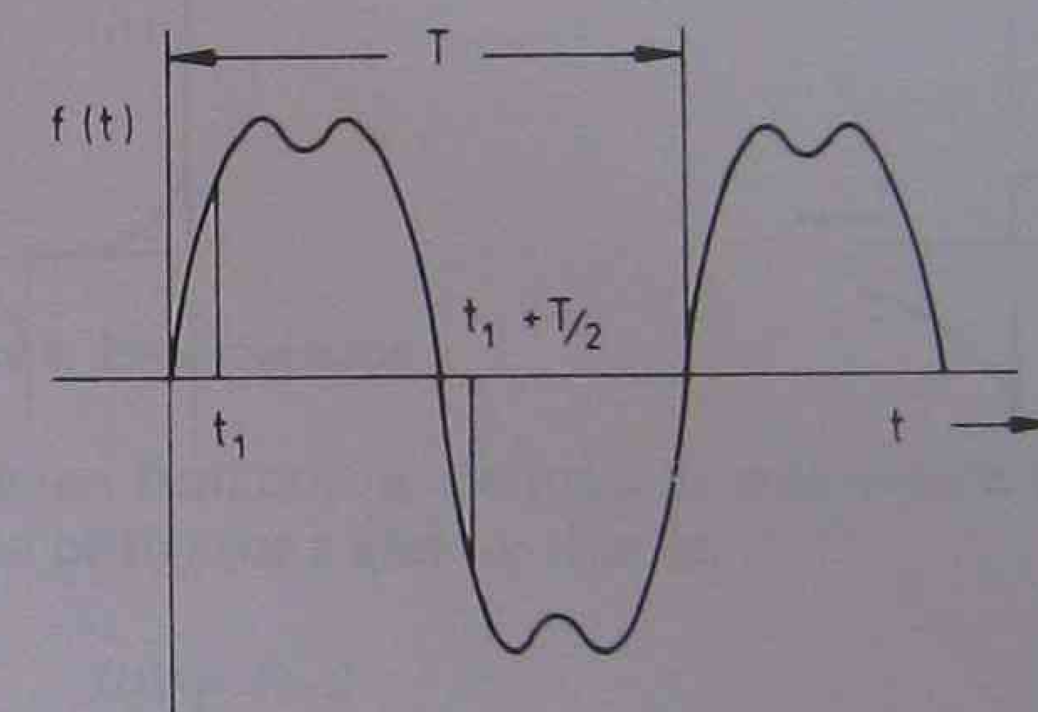


Figure 7

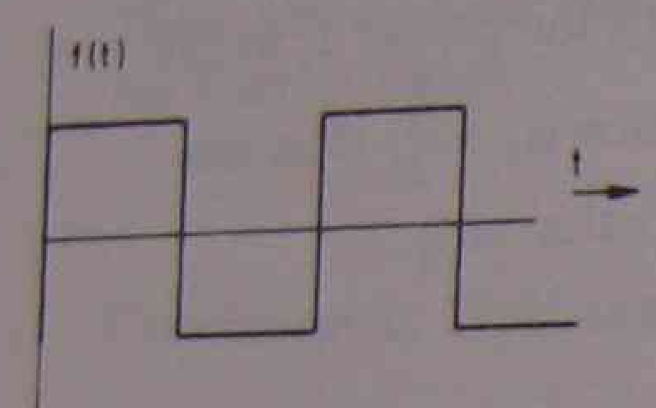
This condition of half-wave symmetry can only exist if **odd harmonics** only are present. Odd harmonics are the 3rd, 5th, 7th, 9th etc. When even harmonics are present, the positive and negative half cycles are different.

Condition 1 must be memorised so you can interpret wave shapes for the purpose of constructing Fourier series.

Now try the exercise in Self-assessment 1. This question is for your self-testing only. Do *not* send your answer to OTEN. Check your answers with those given at the end of the unit.

Self-assessment 1

Indicate whether the following waves are half-wave symmetric.



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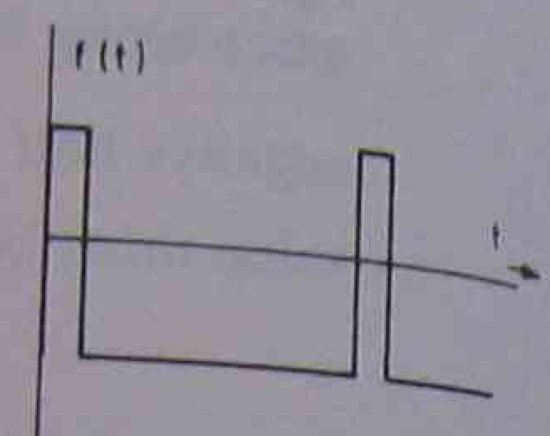
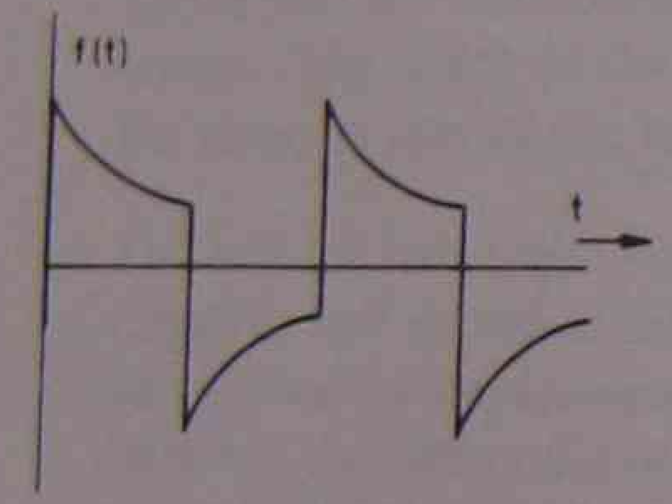


(2)



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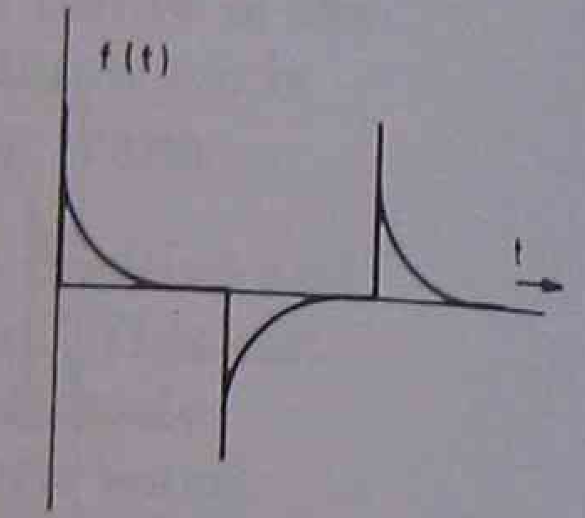
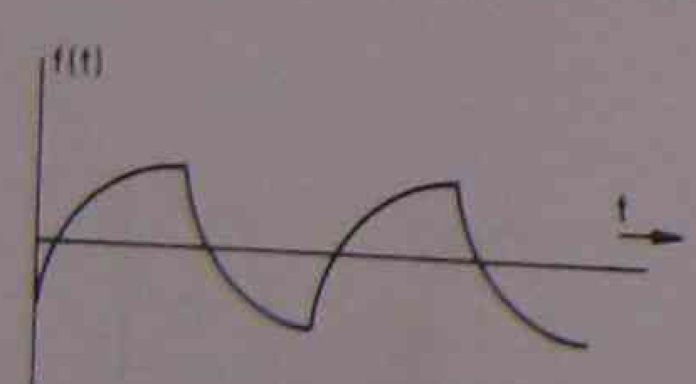
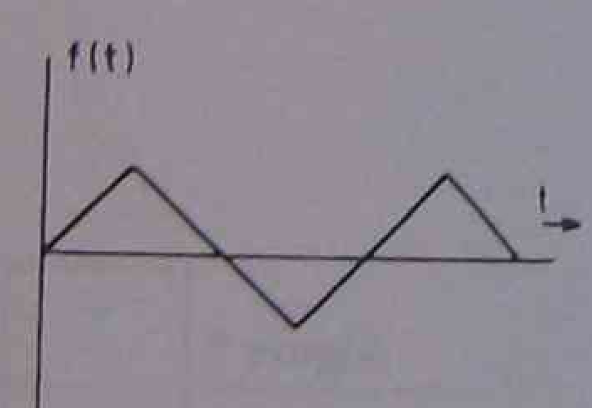


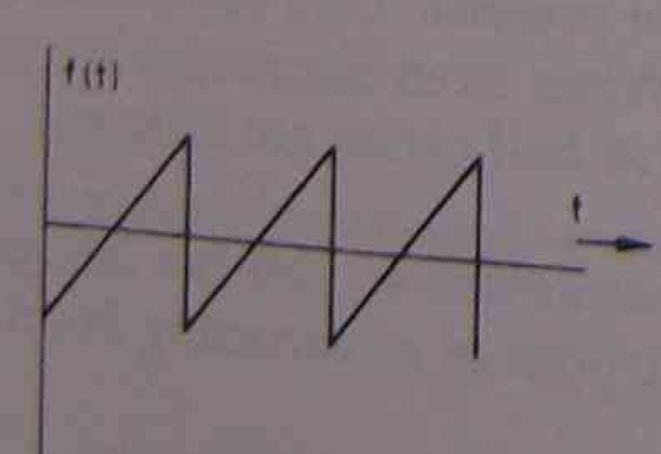
(4)



(5)



(6)



(7)

Condition 2: Odd and even functions

When you look at Figure 8, the two wave shapes have a feature in common which classifies them as **even** functions.

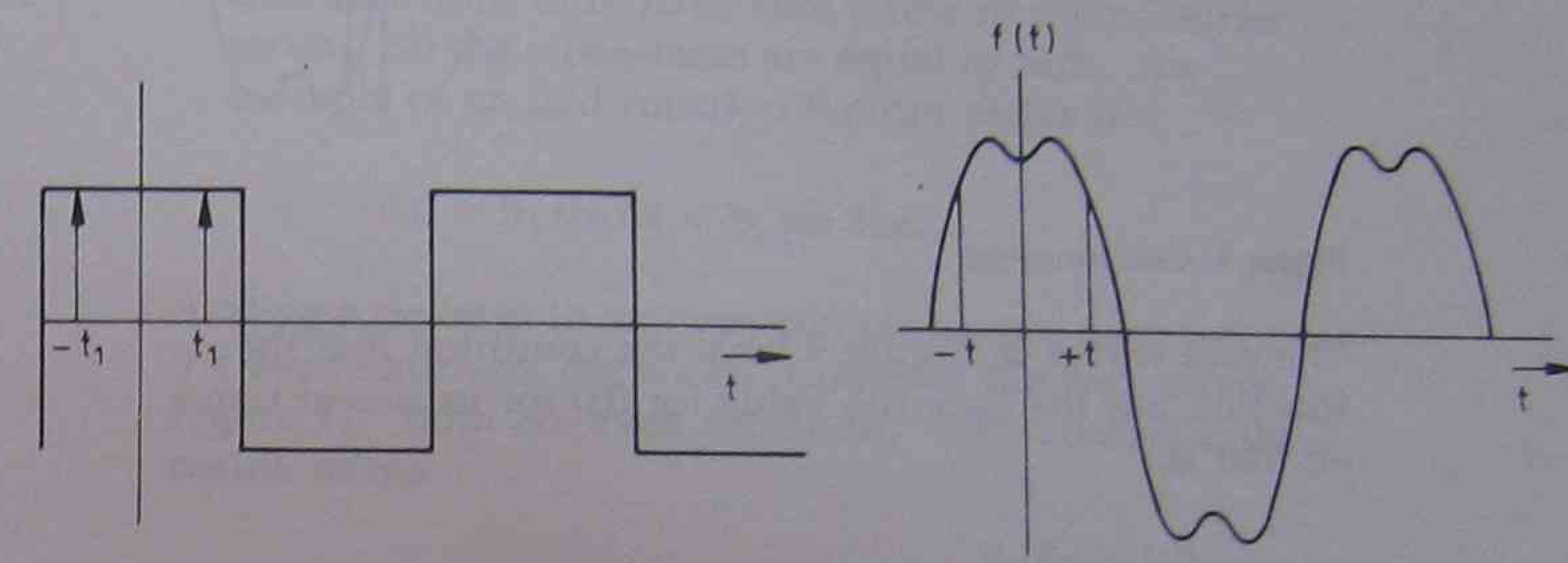


Figure 8: Even functions

An even function is defined as one which has the same value of f(t) for t and −t; that is,

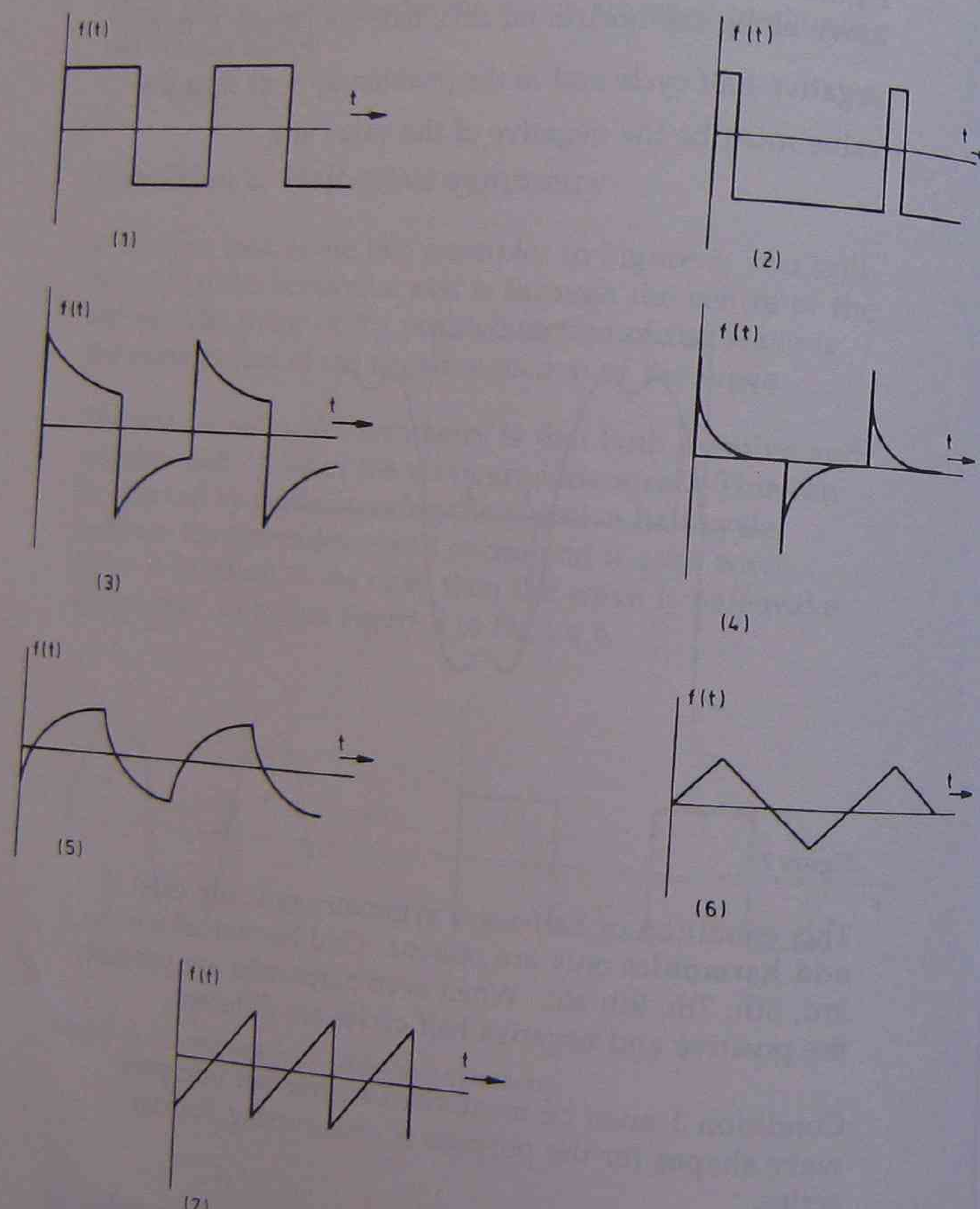
$$f(t) = f(-t)$$

This means the wave must be the same both sides of the vertical axis. One way physically to determine this is to imagine a mirror placed along the vertical axis and the reflection in the positive direction on the time axis is identical to the wave in the negative direction.

Compare the even waves in Figure 8 with the odd waves of Figure 9.

Self-assessment 1

Indicate whether the following waves are half-wave symmetric.



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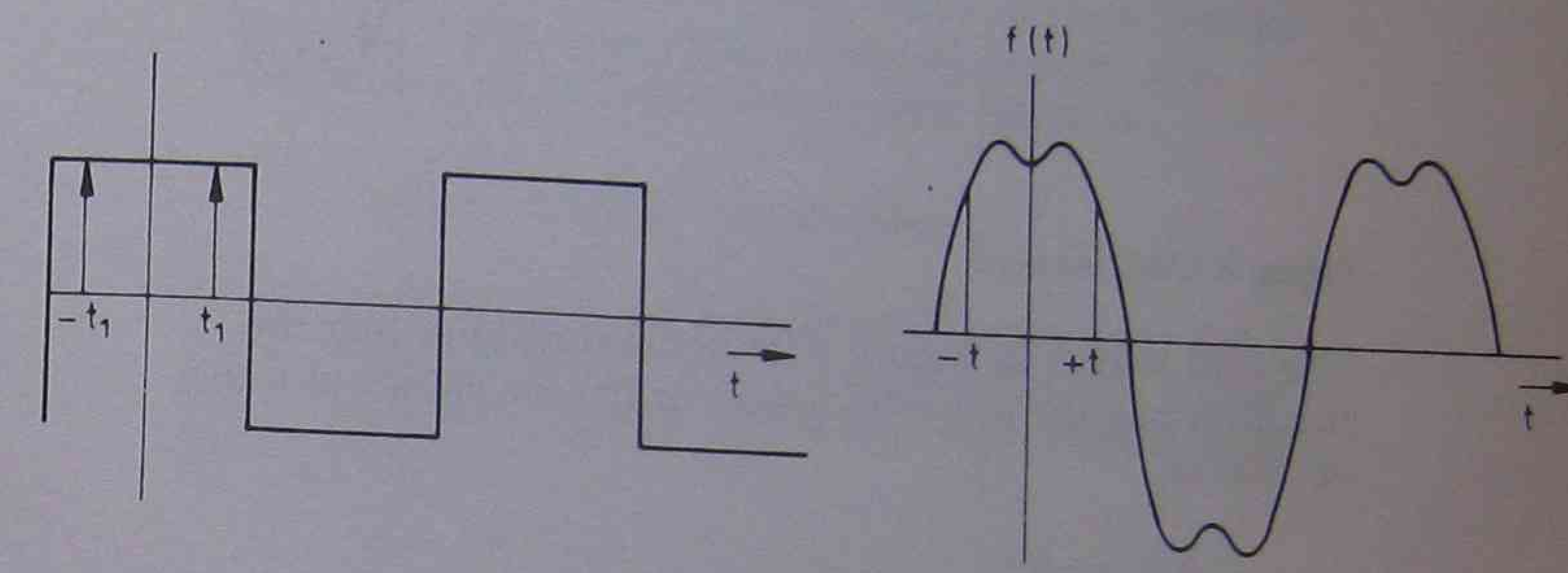

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Compare the even waves in Figure 8 with the odd waves of Figure 9.

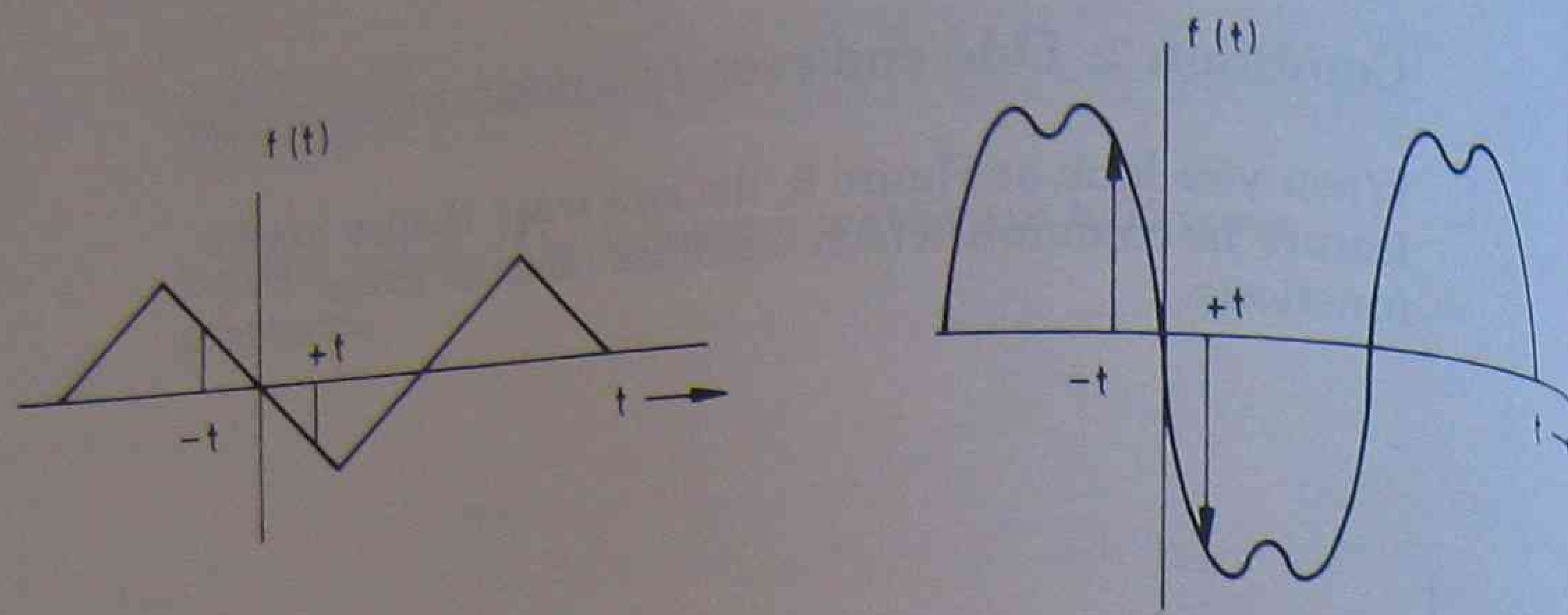
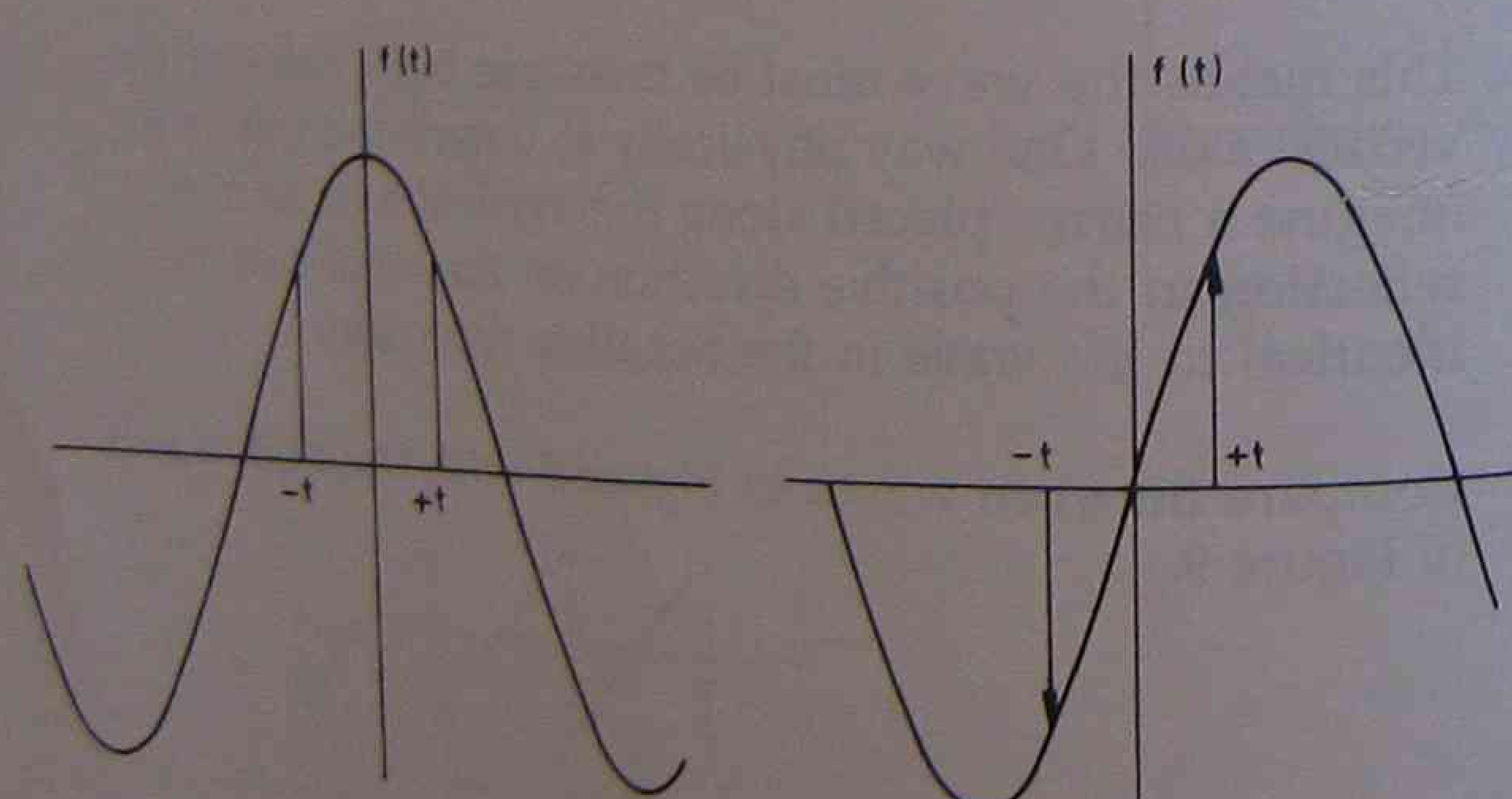


Figure 9: Odd functions

The odd waves in Figure 9 have the condition that the function has the opposite value for f(t) for values of t and –t; that is,

$$f(t) = -f(-t)$$

The most common even and odd functions are the cosine and sine waves respectively. These are shown in Figure 10.



Cosine—even function

Sine—odd function

Figure 10

Figure 10 leads us to the condition that:

- Even functions only have cosine terms in their Fourier series. All the *b* constants are equal to zero. An example of an even function Fourier series is

$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t........$$

- Odd functions only have sine terms in their Fourier series. All the a constants are equal to zero. An example of an odd function Fourier series is

$$f(t) = b_1 \sin \omega t + b_2 \sin 2\omega t........$$

Adding a dc level to an even complex wave does not alter the even nature of the wave. Consider the two waves in Figure 11. Both are even functions. Both contain only cosine terms.

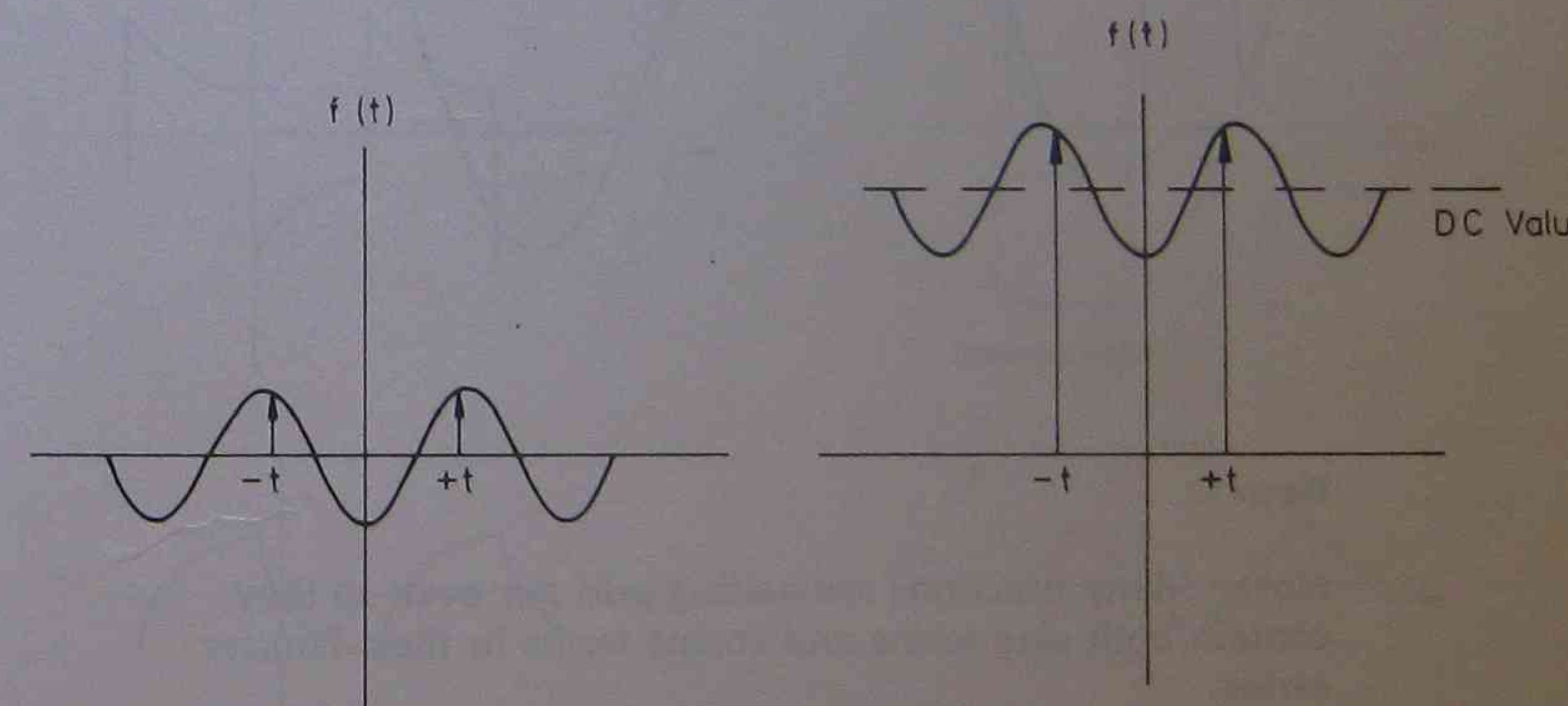


Figure 11

Note: The addition of a constant to an **odd** function removes the **odd** nature of the function and the function will no longer be odd, nor is it even. The function could then contain both sine and cosine terms in the Fourier series.

Moving the wave on the time axis

If we move the wave along the time axis you may make an odd function even or an even function odd.

Consider the even function in Figure 12. The displacement of the vertical axis one quarter wavelength to the right will, in this case, make the function odd. This has made a cosine wave into a sine wave.

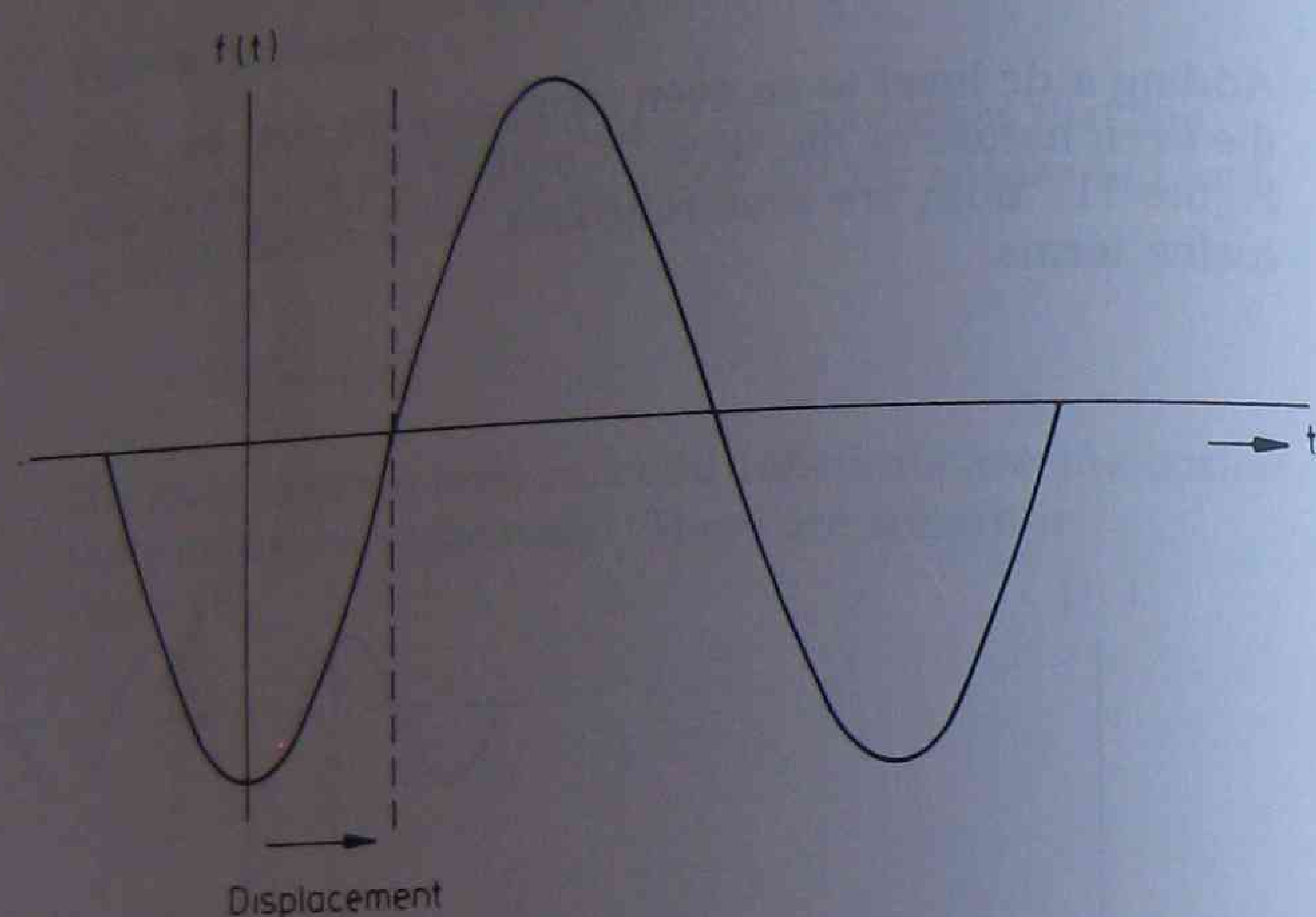


Figure 12

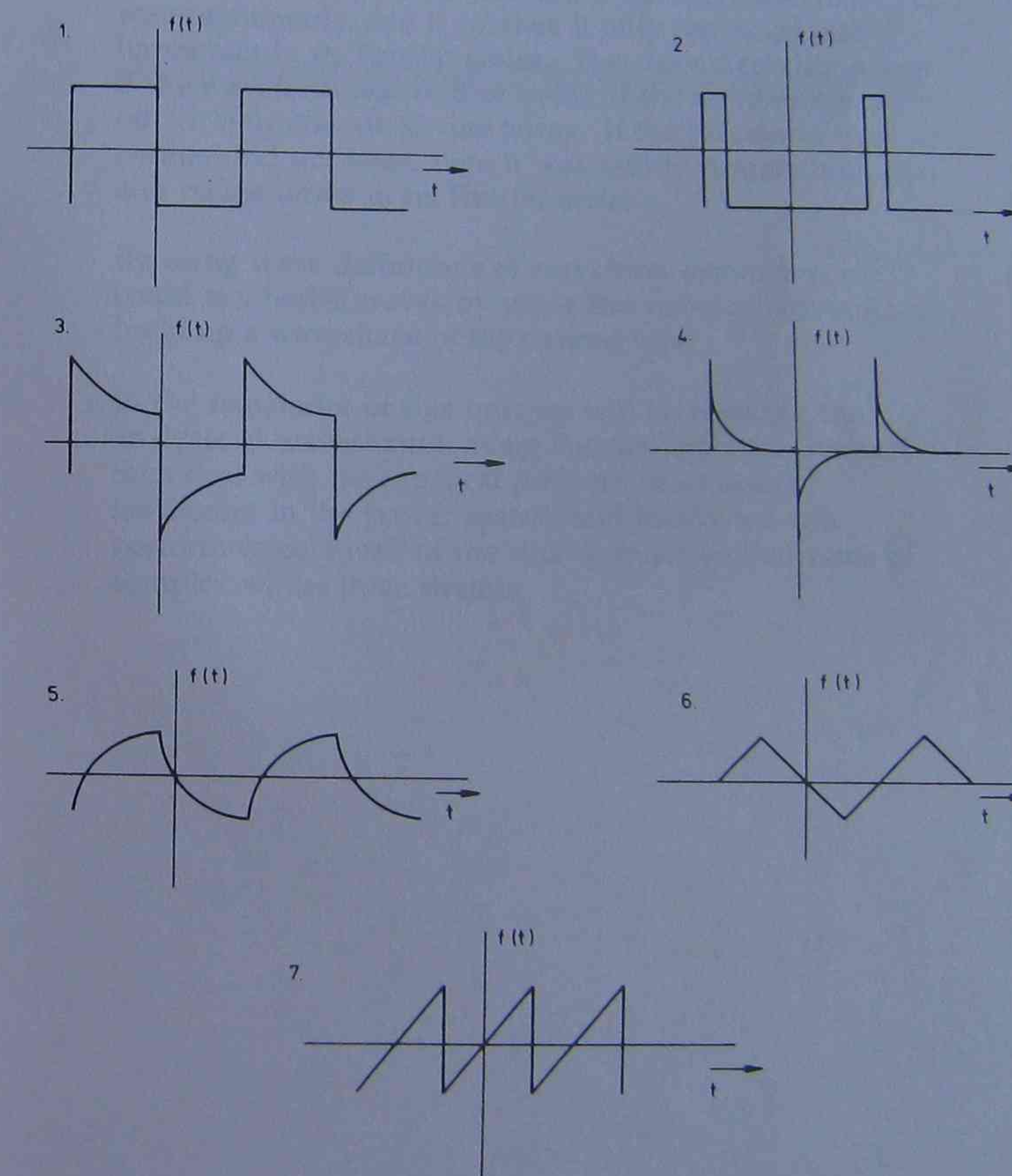
Note: Many functions are neither odd nor even so they contain both sine terms and cosine terms in their Fourier series.

Condition 2 and the variations listed in this section must be memorised so you can interpret wave shapes for the purpose of constructing Fourier series.

Now try the exercise in Self-assessment 2. These questions are for your self testing only. Do *not* send your answers to OTEN. Check your answers with those given at the end of the unit.

Self-assessment 2

1 Indicate whether the following waves are odd, even or neither odd nor even.



2 Indicate whether the following series are even or odd:

(a) $f(t) = \frac{2}{\pi}\sin\omega t + \frac{3}{2\pi}\sin 3\omega t$........

(b) $f(t) = 70\sin\omega t + 20\sin 2\omega t + 10\sin 3\omega t$......
$+ 40\cos\omega t + 10\cos 2\omega t$........

(c) $f(t) = 35 - \frac{10}{\pi}\sin\omega t - \frac{10}{2\pi}\sin 2\omega t$........

Review

So far in this unit you have covered the topics of waveform symmetry and waveform synthesis using the Fourier series.

The topic of waveform symmetry was related to two conditions. Condition 1 occurred if the wave was half-wave symmetric, and if so, then it only contained odd harmonics in its Fourier series. The second condition was if the waveform was odd or even. If the waveform was odd, it only contained sine terms. If the waveform was neither odd nor even, then it was said to contain both sine and cosine terms in its Fourier series.

By using these definitions of waveform symmetry, we could synthesise waves by using the various harmonics to build up a waveshape of the desired type.

In the remainder of this unit we will be covering the analysis of waveshapes, using Fourier analysis. We will then deal with the practical problem of sources of harmonics in the power system and finally we will perform calculations of the rms voltages and currents of complex waves in ac circuits.

Fourier analysis of repetitive waveforms

In the previous sections you were shown how to make up (synthesise) complex waves from a series of sine waves using the Fourier series principle. Using the same principle, waveforms can be analysed to determine their harmonic content.

As an example a perfect square wave as shown in Figure 13 can be shown by Fourier analysis to be represented by

$$e = \frac{4Em}{\pi}\left\{ \sin \omega t + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \frac{\sin 7\omega t}{7} + \ldots\ldots \right\}$$

where e is the instantaneous voltage at time t, E_m is the peak value of the square wave, and ω is 2π x (fundamental frequency). The $\sin \omega t$ component is the fundamental, the $\sin 3\omega t$ quantity is the third harmonic, $\sin 5\omega t$ represents the fifth harmonic etc. The symmetrical square wave can be said to be made up of a fundamental, odd-numbered harmonics, no even harmonics, and no dc component.

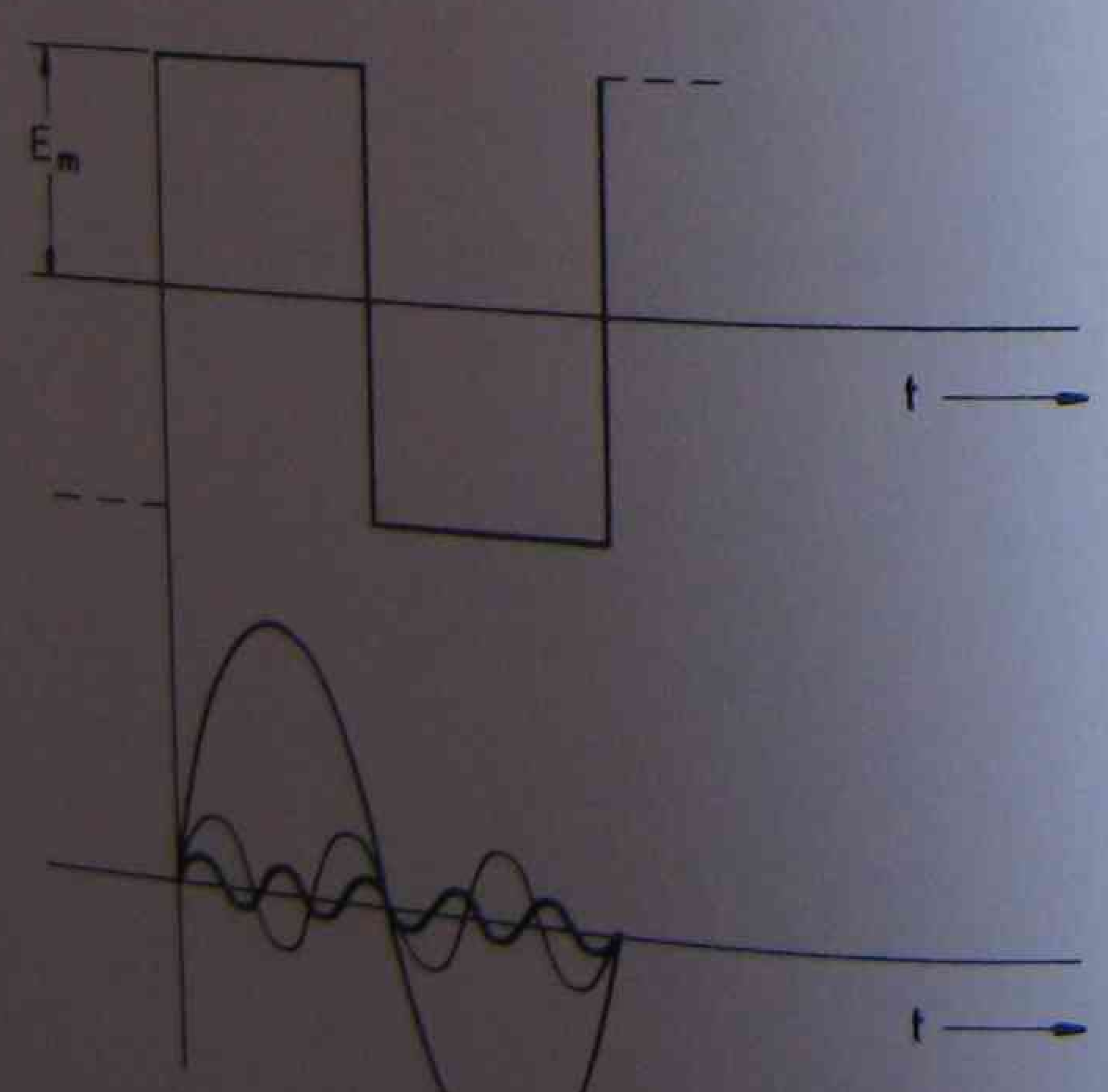


Figure 13

For the level of understanding required by this subject you should be able to analyse the wave using the two conditions in the previous section of this unit.

Let us consider the square wave of Figure 13.

Condition 1: *Is it half-wave symmetric?*
Yes. Then the waveform contains only odd harmonics.
Condition 2: *Is it an odd or even function?*
Odd. Then the waveform contains only sine functions.
Is the wave offset by a dc component?
No, because the positive and negative half cycles have the same maximum value.

We can, from these details, construct a Fourier series as follows:

$$e(t) = b_1 \sin \omega t + b_3 \sin 3\omega t + b_5 \sin 5\omega t + b_7 \sin 7\omega t\ldots\ldots$$

which has the same terms as the equation given except that the values of the constants b_1, b_3 etc are not evaluated as we do not have to perform that level of maths in this subject.

A second example is the sawtooth wave shown in Figure 14, which, by the method of Fourier analysis produces the following equation:

$$e = \frac{2\,Em}{\pi}\left(\sin \omega t - \frac{\sin 2\omega t}{2} + \frac{\sin 3\omega t}{3} - \frac{\sin 4\omega t}{4} + \frac{\sin 5\omega t}{5}\ldots\ldots\right)$$

In this case all the harmonics are present and once again there is no dc component.

Note: In general, a waveform has no dc component when it is symmetrical above and below the horizontal time axis.

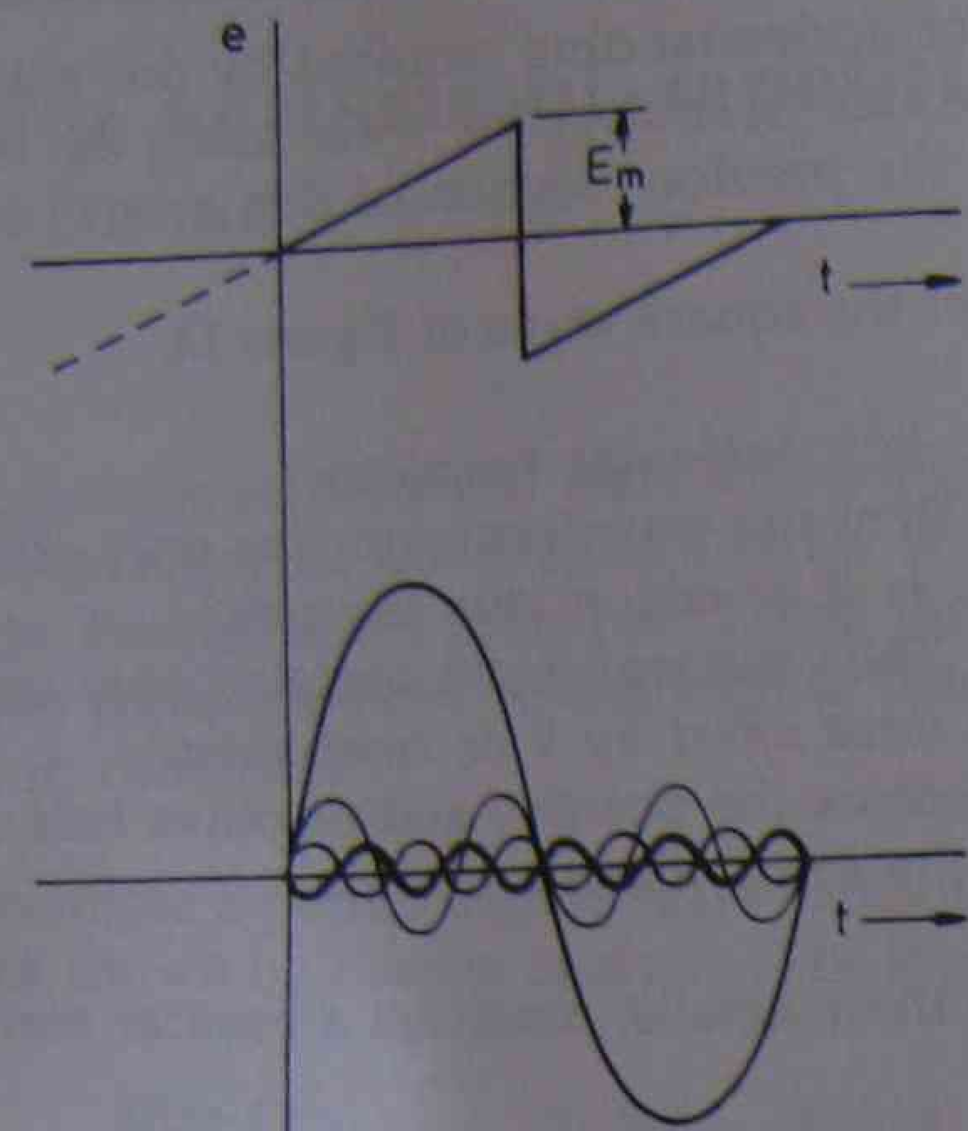


Figure 14

Let us again consider the sawtooth wave of Figure 14 within the scope of this subject.

Condition 1: *Is it half-wave symmetric?*
No. Then the waveform contains both odd and even harmonics.
That means that all harmonics are present.

Condition 2: *Is it an odd or even function?*
Odd. Then the waveform contains only sine functions.
Is the wave offset by a dc component?
No.

We can, from these details, construct a Fourier series as follows:

$$e(t) = b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + b_4 \sin 4\omega t \ldots\ldots$$

Now try the exercises in Self-assessment 3. These questions are for your self-testing only. Do *not* send your answers to OTEN. Check your answers with those given at the end of the unit.

Self-assessment 3

Construct basic Fourier series for the following waveshapes:

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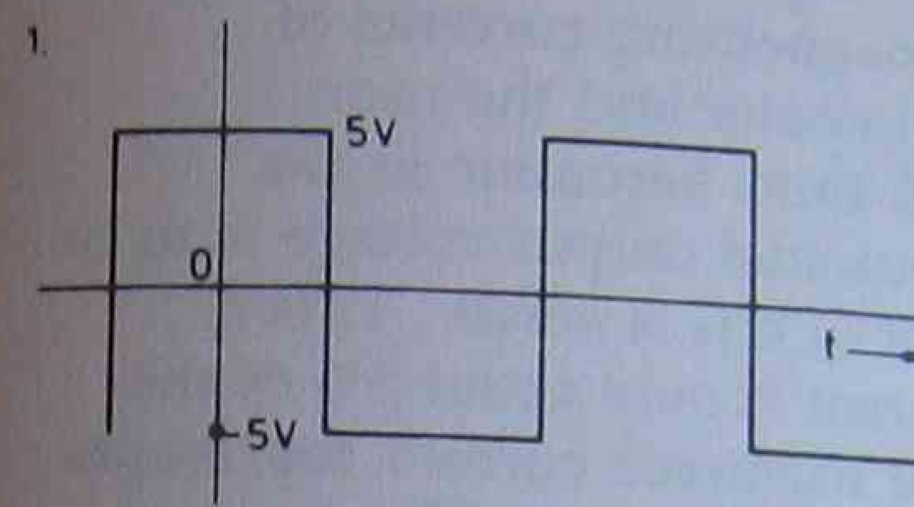

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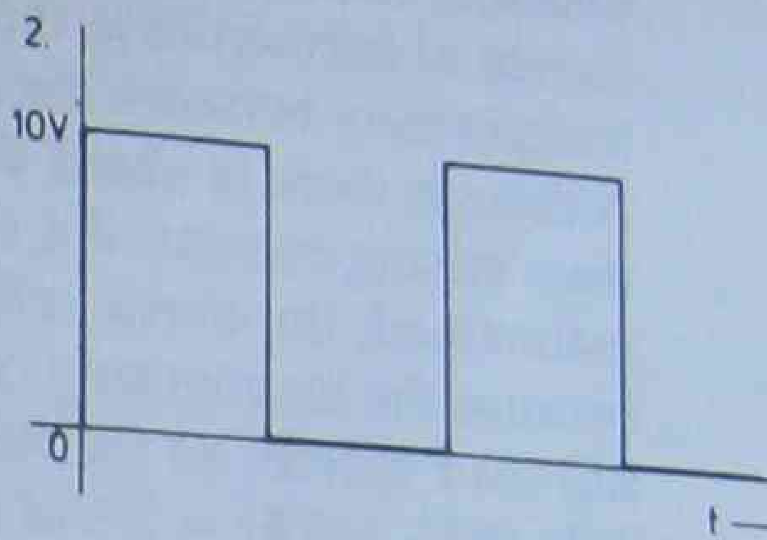

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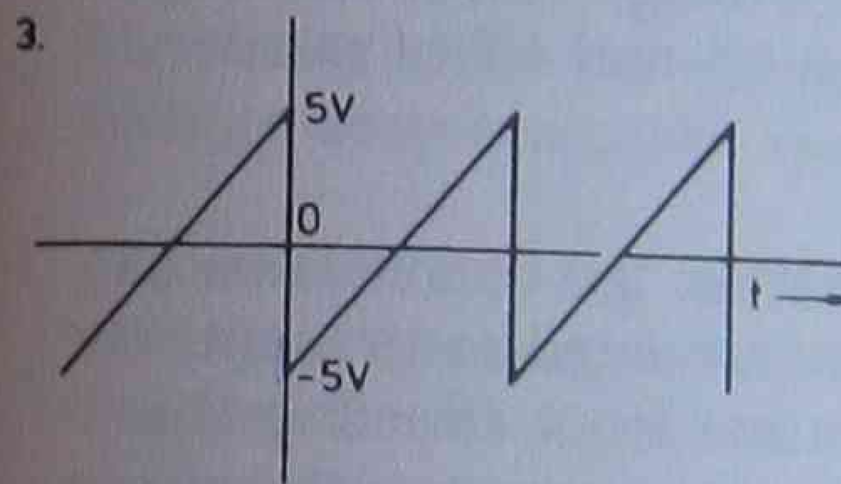

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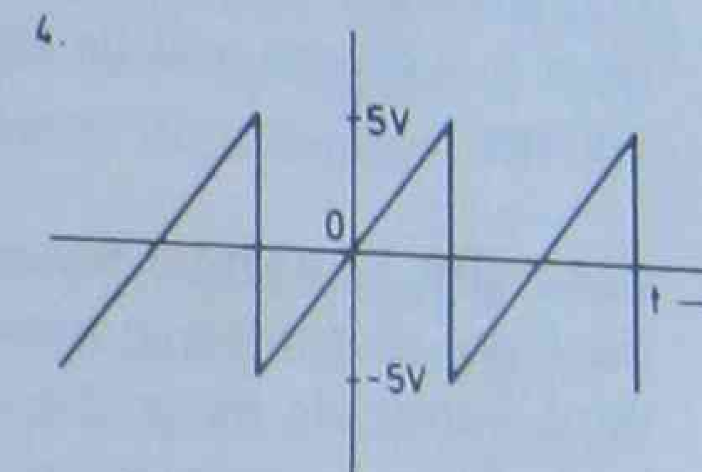

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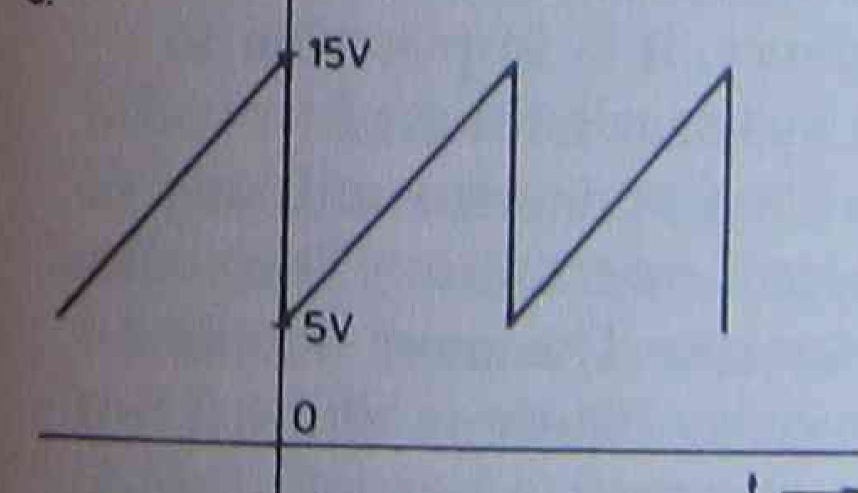

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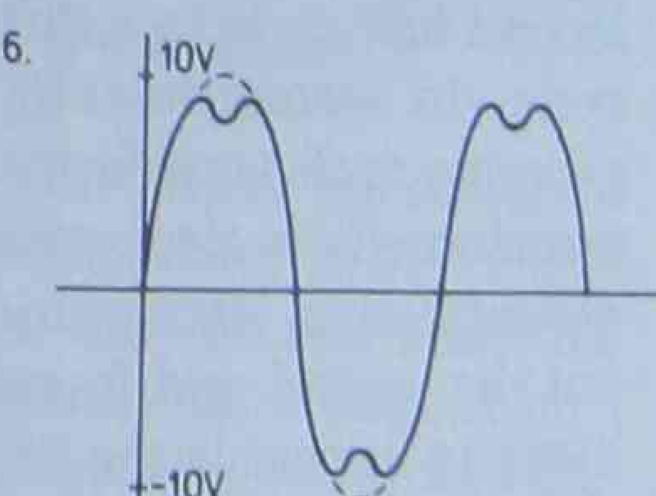

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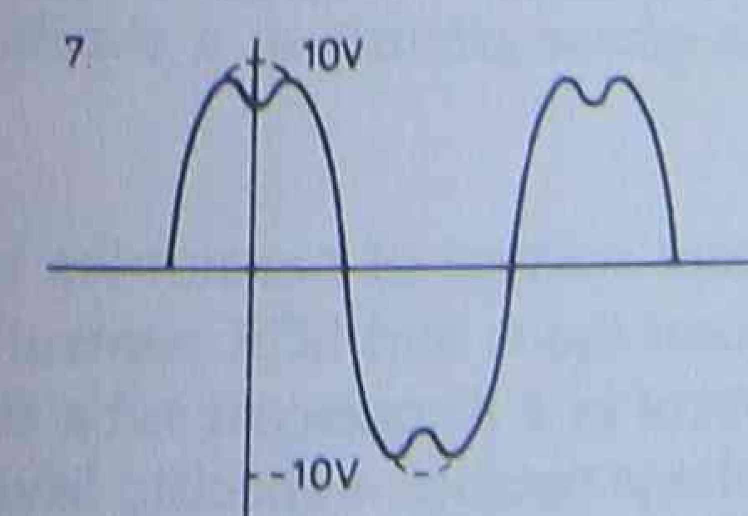


Sources of harmonics

The source of harmonics in a power system, at the most basic level, is the shape of the magnetic field from the alternator rotor and the layout of the stator coils in their slots. In practice, these two combine to produce an emf which is remarkably close to sinusoidal. The next possible source of harmonics is the magnetising currents of transformers between the alternator and the user. Although there is about 40% third harmonic in the magnetising current, if a sinusoidal output voltage is to be maintained, the overall effect of this is small. This is because the magnetising current is only about 5% of the full load current so the third harmonic current represents only (40% x 5%) = 2% of full load current. This means that the level of third harmonic voltage which can occur from this source will lie between 1% and 4% of rated voltage.

Generally, in normal power systems, the connections of the three-phase transformers are arranged to prevent third harmonic magnetising current from affecting the load. Even harmonics are very rare in power systems as they require the generation of waveforms which start the second half cycle in a different way from the first half cycle. In normal rotating machinery, it is impossible to generate such waveforms. The hysteresis loop of transformers is also symmetrical and so is also unlikely to generate even harmonics in normal operation. This rules out the second and fourth harmonics. The next that is likely to appear is the fifth harmonic. There is about 1% fifth harmonic in a transformer magnetising current and this behaves in three-phase circuits in a similar manner to the fundamental.

Increasingly important sources of harmonics in power systems are fluorescent lights and SCR control systems for dc motors. The current in a fluorescent tube does not start to flow until the voltage reaches a specific level in the cycle and it turns off before the voltage returns to zero. Thus the wave shape of current taken is as shown in Figure 15.

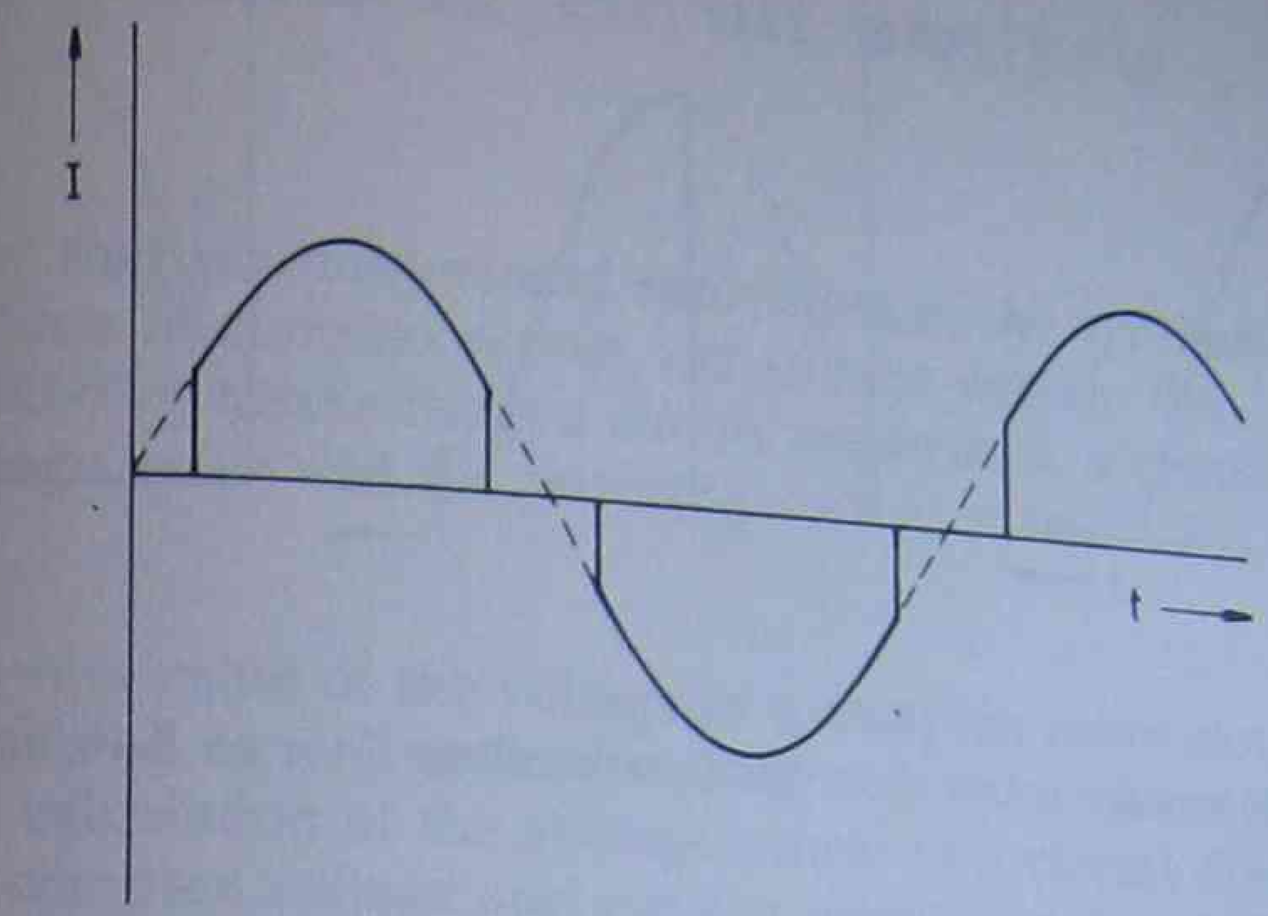
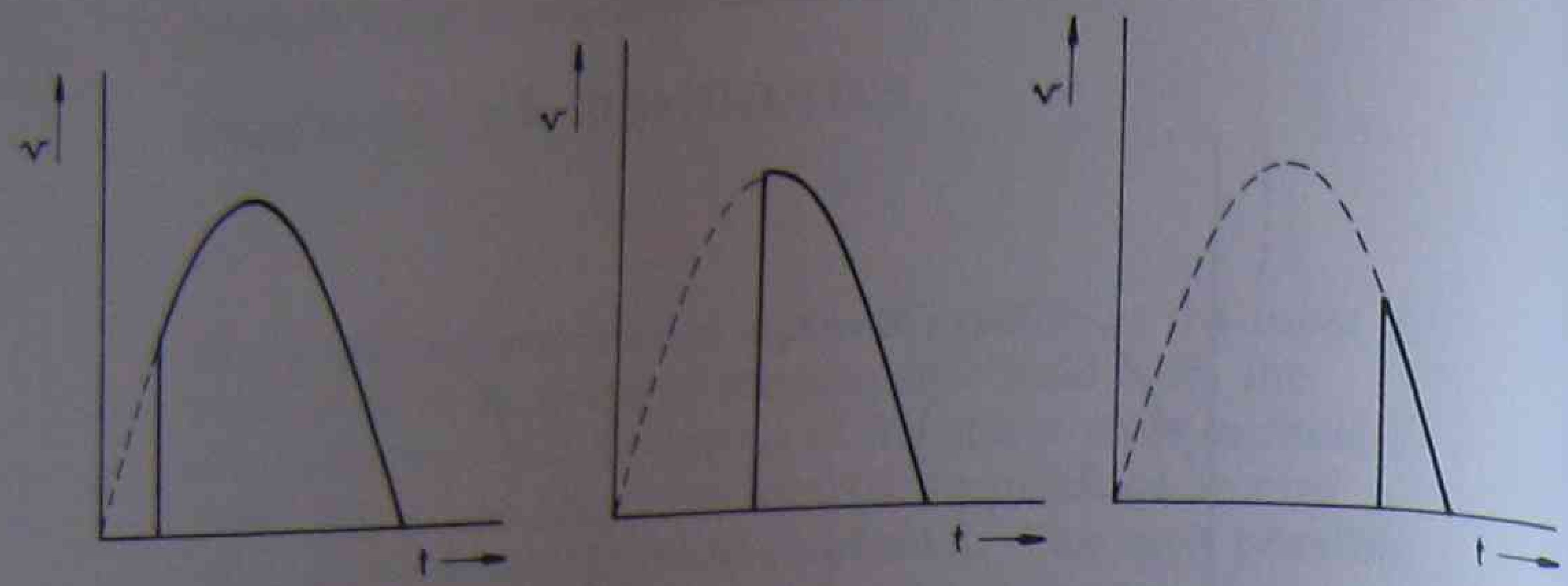


Figure 15

Such a waveform must contain harmonics. These current harmonics affect the voltage waveform by causing a voltage drop across the transformer impedance.

The current flowing in fluorescent tubes contains a variety of harmonics and the third and ninth (*triple-n harmonics*) and so on will add up in the neutral if banks of fluorescent lamps are wired phase-to-neutral and spread over the three phases. This can cause overheating of the neutral connection in large installations.

Silicon controlled rectifier (SCR) units affect the current drawn from the supply in a similar manner to fluorescent tubes, except the wave shape of the current, and hence its harmonic content, varies as the average dc voltage from the rectifier system is varied by phase angle control of the SCRs. Typical voltage waveforms are shown in Figure 16.



High dc voltage Medium dc voltage Low dc voltage

Figure 16

If the SCR unit is driving a motor and the motor must produce constant torque at all speeds, the harmonic content of the current waveform will increase at lower speeds. This is so because $T \propto I$ and the flux under low speed conditions will be constant—therefore, for constant torque, the average value of I must be constant. For I_a average to be constant when the SCR is conducting for a small part of each cycle, it follows that the peak value of the current will be high. As this high current occurs over a small part of a cycle it must contain a large harmonic content. As with fluorescent tubes these current harmonics will show up in the system voltage because of the voltage drop across the transformer impedance.

In the electronics area of engineering non-sinusoidal waveshapes are more common than sinusoidal, so that Fourier analysis is essential to analyse and synthesise waves. The operation of most electronic musical instruments depends on these principles.

Harmonics in circuit analysis

Now that you understand something of the synthesis and analysis of complex waves, I shall now explain the method of determining a circuit response to a complex voltage wave and the resulting current wave.

The rms value of the voltage of a complex wave can be calculated as well as the resulting rms value of current. The calculation of the average power in a circuit due to the complex voltage and current waves can then be performed.

Consider the circuit shown in Figure 17:

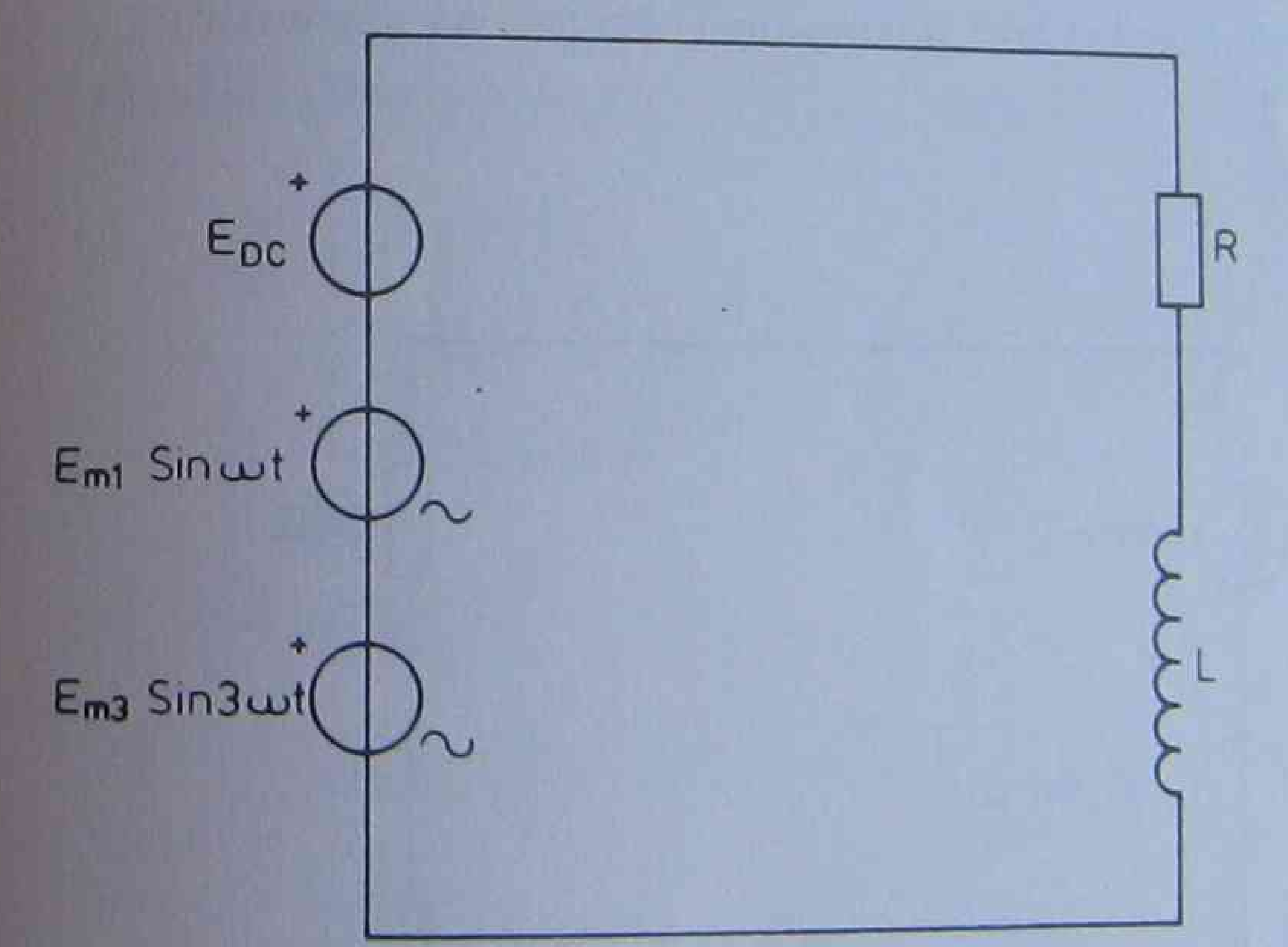


Figure 17

To calculate the total voltage, current and power in the circuit, it is necessary to consider each voltage source separately, calculate its respective current and then to combine these values to obtain the resultant overall values.

This is the *superposition method* which is similar to the method used in multi-sourced dc circuits.

To demonstrate this method you can work through the following example.

The superposition method

A Series R–L circuit has a resistance of 20 ohms and an inductance of 0.2 henry. The applied voltage is

$$V(t) = 25 + 80 \sin \omega t + 20 \sin 3\omega t$$
$$\text{where } \omega = 250 \text{ rad/sec}$$

Find:

1 the instantaneous current
2 rms voltage and current
3 average power supplied to the circuit

Solution

Draw up the circuit with separate voltage sources for the dc, fundamental and harmonic voltages, as shown in Figure 18.

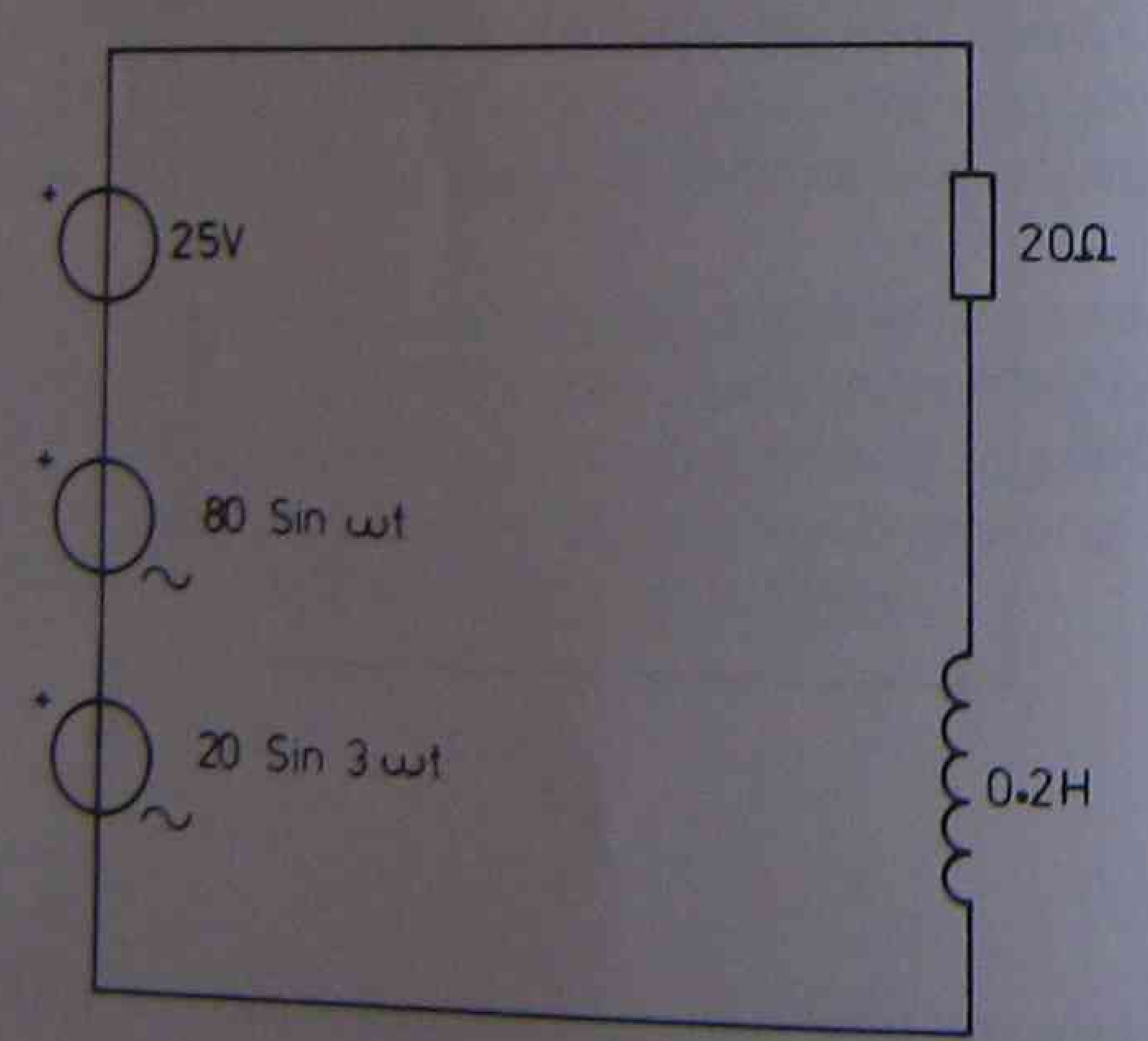


Figure 18

1 The instantaneous current

Now consider each voltage source separately acting on the circuit.

(a) dc = 25 volts

Note: The inductance has no effect on dc.

Then $I = \dfrac{E}{R}$

$= \dfrac{25}{20}$

$= 1.25$ amperes

(b) ac fundamental = 80 sin ωt

Calculate the inductive reactance at $\omega = 250$ rad/s

$$X_L = \omega L$$
$$= 250 \times 0.2$$
$$= 50 \text{ ohms}$$

Calculate impedance of circuit at $\omega = 250$ rad/s

$$Z = \sqrt{R^2 + X_L^2}$$
$$= \sqrt{(20)^2 + (50)^2}$$
$$= 53.85 \text{ ohms}$$

$$\theta = \tan^{-1} \frac{X}{R}$$
$$= \tan^{-1} \frac{50}{20}$$
$$= 68.2°$$

$$Z = 53.85 \angle 68.2° \text{ ohms}$$

Now calculate the current

$$i = \frac{e}{Z}$$
$$= \frac{80 \sin 250t}{53.85 \angle 68.2°}$$

Then $i = 1.49 \sin (250t - 68.2°)$

For the fundamental wave

$$V_{rms} = \frac{80 \angle 0^\circ}{\sqrt{2}}$$

$$= 56.6 \angle 0^\circ \text{ volts}$$

$$I_{rms} = \frac{1.49 \angle -68.2^\circ}{\sqrt{2}}$$

$$= 1.05 \angle -68.2^\circ \text{ amperes}$$

(c) *ac, third harmonic = 20 sin 3ωt*

Calculate the inductive reactance at ω = 750 rad/s

$$X_L = \omega L$$
$$= 750 \times 0.2$$
$$= 150 \text{ ohms}$$

Calculate impedance of circuit at ω = 750 rad/s

$$Z = \sqrt{R^2 + X_L^2}$$
$$= \sqrt{(20)^2 + (150)^2}$$
$$= 151.3 \text{ ohms}$$

$$\theta = \tan^{-1}\frac{X}{R}$$
$$= \tan^{-1}\frac{150}{20}$$
$$= 82.4^\circ$$

$$\bar{Z} = 151.3 \angle 82.4^\circ$$

Now calculate the current for the third harmonic

$$i = \frac{e}{Z}$$
$$= \frac{20 \sin 3wt}{151.3 \angle 82.4^\circ}$$
$$= 0.132 \sin(750t - 82.4^\circ)$$

For the third harmonic wave

$$V_{rms} = \frac{20 \angle 0^\circ}{\sqrt{2}}$$
$$= 14.14 \angle 0^\circ \text{ volts}$$

$$I_{rms} = \frac{0.132 \angle 82.4^\circ}{\sqrt{2}}$$
$$= 0.093 \angle 82.4^\circ \text{ amperes}$$

Instantaneous current $i = 1.25 + 1.49 \sin(250t - 68.2^\circ) + 0.132 \sin(750t - 82.4^\circ)$

2 Voltage and current (rms)

To calculate the total rms voltage in the circuit we *cannot* use simple arithmetic with phasor values.

$$P_{dc} = \frac{E_{dc}^2}{R}, \quad P_1 = \frac{E_1^2}{R}, \quad P_3 = \frac{E_3^2}{R}$$

$$P_{total} = P_{dc} + P_1 + P_2$$

$$\frac{E_{rms}^2}{R} = \frac{E_{dc}^2}{R} + \frac{E_1^2}{R} + \frac{E_3^2}{R}$$

Then

$$\boxed{E_{rms} = \sqrt{E_{dc}^2 + E_1^2 + E_3^2}}$$

If we consider the power as

$$P = I^2R$$

Then using the same principle as above

$$\boxed{I_{rms} = \sqrt{I_{dc}^2 + I_1^2 + I_3^2}}$$

Note: These formulae for E_{rms} and I_{rms} can be used for any number of harmonics and *must be memorised*.

For the example above,

$$E_{rms} = \sqrt{(25)^2 + (56.6)^2 + (14.14)^2}$$
$$= 63.5 \text{ volts}$$

and $$I_{rms} = \sqrt{(1.25)^2 + (1.05)^2 + (0.093)^2}$$
$$= 1.64 \text{ amperes}$$

Average power supplied to the circuit

To calculate the average power supplied to the circuit use the total rms current and the resistance of the circuit and then check by calculating each component of power.

$$\text{total power} = I^2R = (1.64)^2 \times 20 = 53.8 \text{ watts}$$

Check

$$\text{dc power} = I^2R = (1.25)^2 \times 20 = 31.25 \text{ watts}$$

$$\text{fundamental wave power} = I^2R = (1.05)^2 \times 20 = 22.05 \text{ watts}$$

$$\text{third harmonic power} = I^2R = (0.093)^2 \times 20 = 0.17 \text{ watts}$$

$$\text{total power} = 31.25 + 22.05 + 0.17 = 53.5 \text{ watts}$$

Notes on the example:

- When calculating dc values, only the resistance of the circuit is taken into consideration.
- The inductive reactance increased proportionally to the order (value) of the harmonic.
- The current lags at an increasing angle for higher value harmonics.
- The harmonic currents contribute to the power dissipation in a circuit.

Now try the exercise in Self-assessment 4. This question is for your self-testing only. Do *not* send your answer to OTEN. Check your answers with those given at the end of the unit.

Self-assessment 4

A circuit containing a coil with resistance of 25 ohms and inductance of 0.25 henrys is in series with a 45 ohm resistor. The supply voltage is given by the expression

$$e(t) = 13 + 120 \sin \omega t + 35 \sin 3\omega t$$

The fundamental frequency is 50 Hz.

Determine:

1 the expression for the instantaneous current

2 the rms value of current

3 the total power dissipated in the coil.

Review

In the second part of this unit you have covered the analysis of a simple repetitive waveform to determine its basic Fourier series components. This was achieved by using waveform symmetry. The next topic contained an explanation of the sources of harmonics in power systems and concentrated on the three main areas of magnetic circuits, transformers, fluorescent lights and SCR-controlled motors. Finally, the effect of non-sinusoidal waveforms on complex impedances in circuits was detailed.

You have to be able to calculate the total rms voltages and current in these circuits, remembering that to add individual rms values, the formula

$$I_{rms} = \sqrt{(I_{1rms})^2 + (I_{2rms})^2 + (I_{3rms})^2}$$

must be used for current and the similar one for voltage. The calculation of the individual rms currents for each harmonic was shown by calculating each harmonic current from the harmonic voltage in the complex impedance. The individual currents we then added in the formula shown above.

Answers to self-assessment questions

Self-assessment 1

1 half-wave symmetric
2 not half-wave symmetric
3 half-wave symmetric
4 half-wave symmetric
5 half-wave symmetric
6 half-wave symmetric
7 not half-wave symmetric

Self-assessment 2

1 1 odd
2 even
3 neither odd nor even
4 neither odd nor even
5 neither odd nor even
6 odd
7 odd

2 (a) odd
(b) neither odd nor even
(c) neither odd nor even

Self-assessment 3

1 $e(t) = a_1 \cos \omega t + a_3 \cos 3\omega t + a_5 \cos 5\omega t$.......

2 $e(t) = 5 + b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t$........
$+ a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t$........

3 $e(t) = b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t$......
$+ a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t$......

4 $e(t) = b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t$......

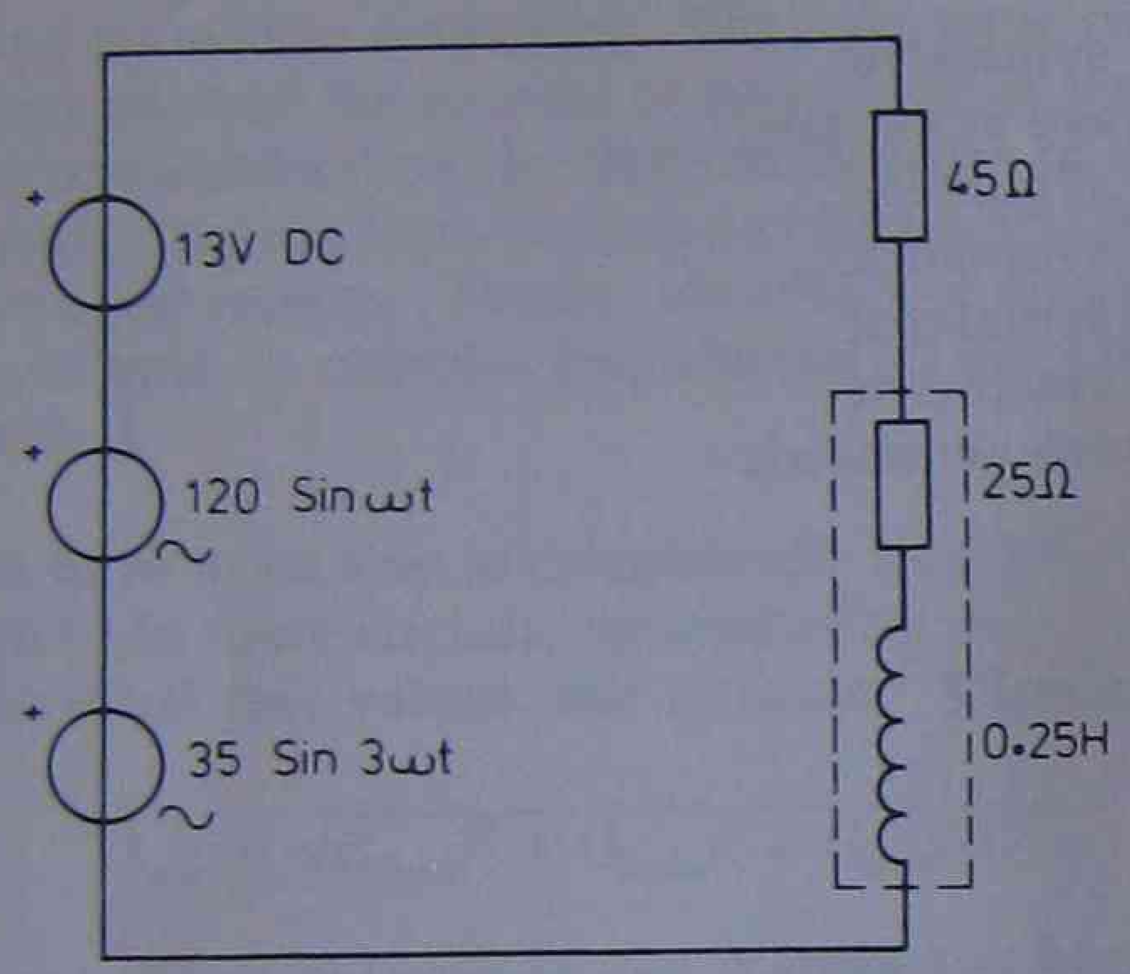
5 $e(t) = 10 + b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t$......
$+ a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t$........

6 $e(t) = b_1 \sin \omega t + b_3 \sin 3\omega t + b_5 \sin 5\omega t$........

7 $e(t) = a_1 \cos \omega t + a_3 \cos 3\omega t + a_5 \sin 5\omega t$........

Self-assessment 4

Circuit diagram



Frequency = 50 Hz = $2\pi \times 50$ rad/s = 314 rad/s

1 *Expression for instantaneous current*

dc: $i = \frac{E}{R_T}$

$= \frac{13}{70}$

$= 0.186$ amperes

$R_T = 45 + 25$
$= 70$ ohms

Fundamental:

$X_L = \omega L$
$= 314 \times 0.25$
$= 78.5$ ohms

$\bar{Z} = \sqrt{R^2 + X^2} \angle \tan^{-1} \frac{X}{R}$

$= \sqrt{(70)^2 + (78.5)^2} \angle \tan^{-1} \frac{78.5}{70}$

$= 105.2 \angle 48.3°$ ohms

$i_1 = \frac{120 \sin 314t}{105.2 \angle 48.3°}$

$= 1.14 \sin (314t - 48.3°)$ amperes

Third harmonic:

$X_L = \omega L$
$= 942 \times 0.25$
$= 235.5$ ohms

$Z = \sqrt{R^2 + X_L^2} \angle \tan^{-1} \frac{X}{R}$

$= \sqrt{(70)^2 + (235.5)^2} \angle \tan^{-1} \frac{235.5}{70}$

$= 245.7 \angle 73.4°$ ohms

$i_3 = \frac{35 \sin 942t}{245.7 \angle 73.4°}$

$= 0.14 \sin (942t - 73.4°)$

Then expression is

$i(t) = 0.186 + 1.14 \sin (314t - 48.3°) + 0.14 \sin (942t - 73.4°)$ amperes

2 *rms value of current*

$I = \sqrt{(0.186)^2 + \left(\frac{1.14^2}{\sqrt{2}}\right) + \left(\frac{0.14^2}{\sqrt{2}}\right)}$

$= \sqrt{0.035 + 0.65 + 0.0098}$

$= \sqrt{0.695}$

$= 0.83$ amperes

3 Power in coil

$R = 25$ ohms
$= I^2R$
$= (0.83)^2 \times 25$
$= 17.4$ watts

Glossary of terms

analysis	breaking down a waveshape into its component parts (harmonics).
distortion	any variation from the original waveshape.
even function	a function which has the same value of f(t) for t and –t. It contains only cosine terms in its Fourier series.
Fourier series	a mathematical series which can be used to represent a complex repetitive wave form.
fundamental frequency	the basic frequency of a complex wave. It is the reciprocal of the period of the complete complex wave.
half-wave symmetry	the shapes of the positive half cycle and negative half cycle of the waveform are identical. The Fourier series of such waves has only odd harmonics.
harmonic	a sine wave of some frequency which is a multiple of the base or fundamental frequency.
integral calculus	a mathematic method which can be used to find areas under waveshapes.
non-sinusoidal	a waveshape which has a form different to a sine wave.
odd function	a function which has the opposite value of f(t) for t and –t. It contains only sine terms in its Fourier series.

wk 3+4+5

Ref 29

Electronic Signals and Systems

7761L

Student Workbook

17902

UEENEEH014B - Troubleshoot frequency dependent circuits

National Module No. EA190
Electrical Engineering
St George TAFE
Sydney Institute

3. State which of the following electrical/electronic systems would use closed loop control.

- The speed control on a kitchen food processor.
- The laser tracking system in a compact disk player.
- A remote controller for a model aeroplane.
- An AGC system in a radio receiver.
- An oven for the crystal in a high stability oscillator.
- The pressure controller in a steel rolling mill.

4. Briefly describe the action of the feedback in a voltage amplifier.

5. Two motor speed controllers are controlled by potentiometers. One uses open loop control and the other uses closed loop control. Briefly compare the likely control characteristics.

Section 2: Signals, spectra and non-linearity

WK3

SUGGESTED DURATION	PREAMBLE
7 hrs	To introduce you to Fourier concepts and the spectra of some common signals, and to extend these concepts in explaining the effect of non-linearity.
This section covers learning outcomes 3 and 4 of the Module Descriptor.	

Objectives

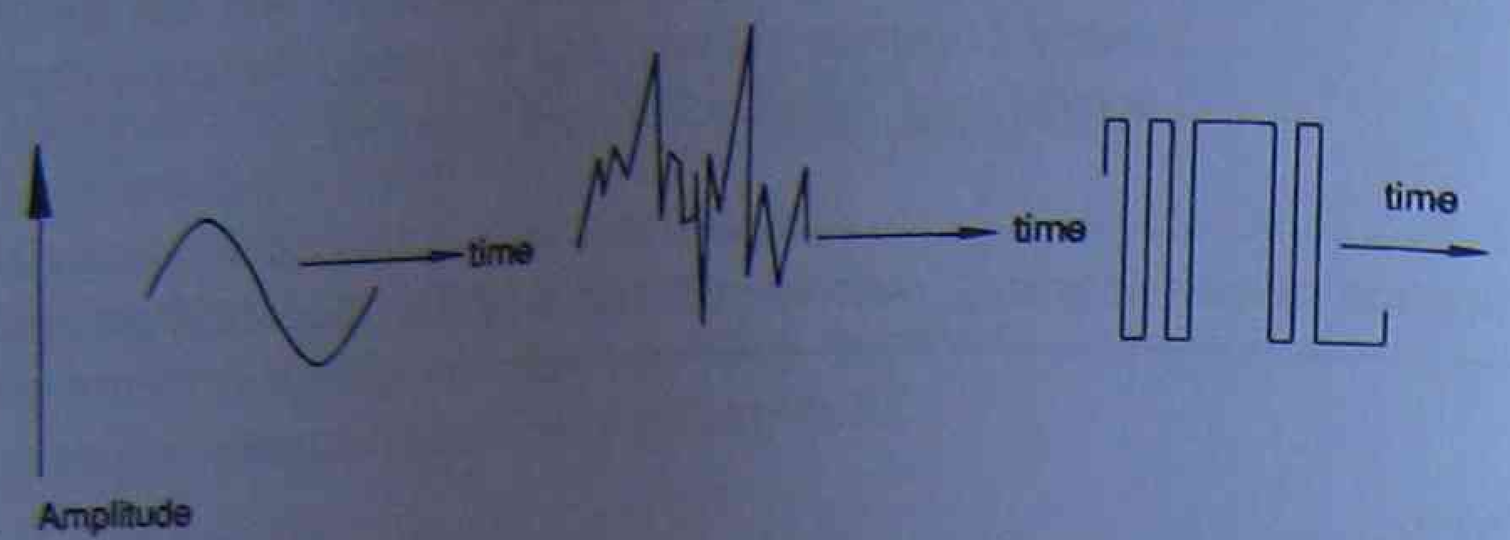
At the end of this section you should be able to:

- ☐ predict the spectral frequencies of a given periodic wave
- ☐ recognise that a non-periodic signal such as speech contains a continuous band of frequencies
- ☐ sketch typical time and frequency domain diagrams for white noise, speech, music, video and random binary data
- ☐ relate one line of video to a grey scale
- ☐ define non-linearity
- ☐ calculate harmonic and intermodulation distortion frequencies.

Time and frequency domains

Time domain

The traditional method of observing electrical signals is to view them in the time domain, using an oscilloscope.



Time domain displays

The information displayed is amplitude (voltage) versus time, which is adequate for most low frequency audio and digital waveform measurements involving timing and phase.

However, time domain measurements are not usually adequate when studying RF devices such as amplifiers, oscillators, filters, mixers, modulators and antennas. The reasons for this are given below.

1. Oscilloscopes cannot normally view waveforms above several hundred MHz. Their internal amplifiers are not capable of amplifying many of the high frequency signals found in communications equipment. For example, AUSSAT signals return to earth at 12GHz (12000 MHz!!!). An oscilloscope can not be used to view these signals.

2. Oscilloscopes are often not sensitive enough to display the tiny signals found in communications equipment. For example, most oscilloscopes have 1mV/cm as the most sensitive range, and could not display signals with uV levels. It would be even more difficult to display a 1uV signal at the same time as a 10V signal.

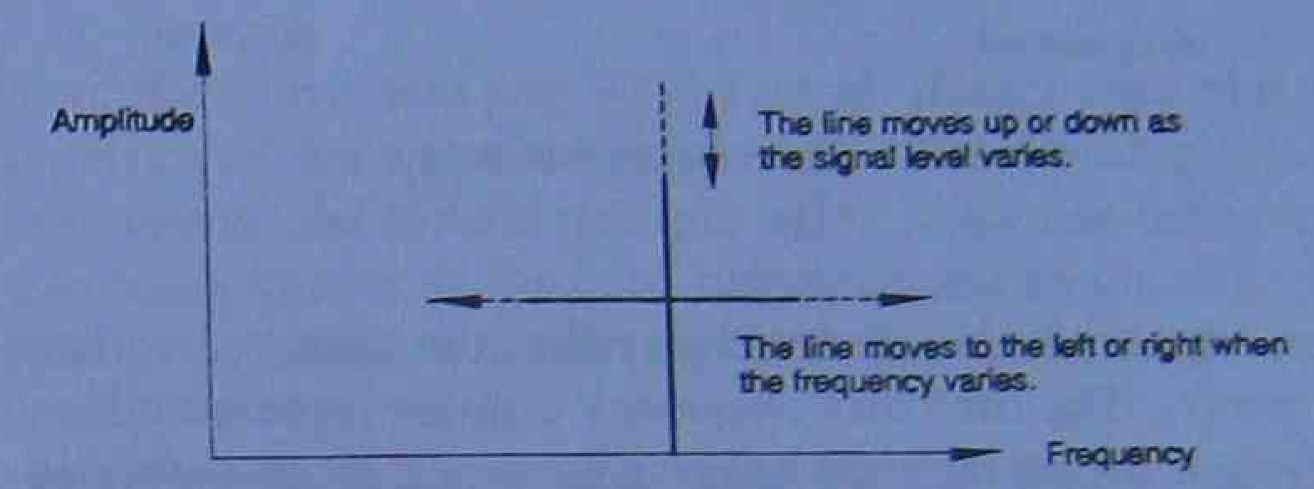
3. An oscilloscope cannot break a complex signal down into its constituent parts; it displays them all added together. Many signals are complex; that is, they are composed of more than one frequency component. It is impossible with an oscilloscope to examine individual components of a complex wave.

Are you ready to throw your oscilloscope away? Don't! Even though it can't do the things mentioned above, it is probably the most versatile general purpose laboratory instrument. Besides, the instrument that can do everything mentioned above may cost between $10,000 and $100,000. This expensive instrument is called a spectrum analyser.

Frequency domain

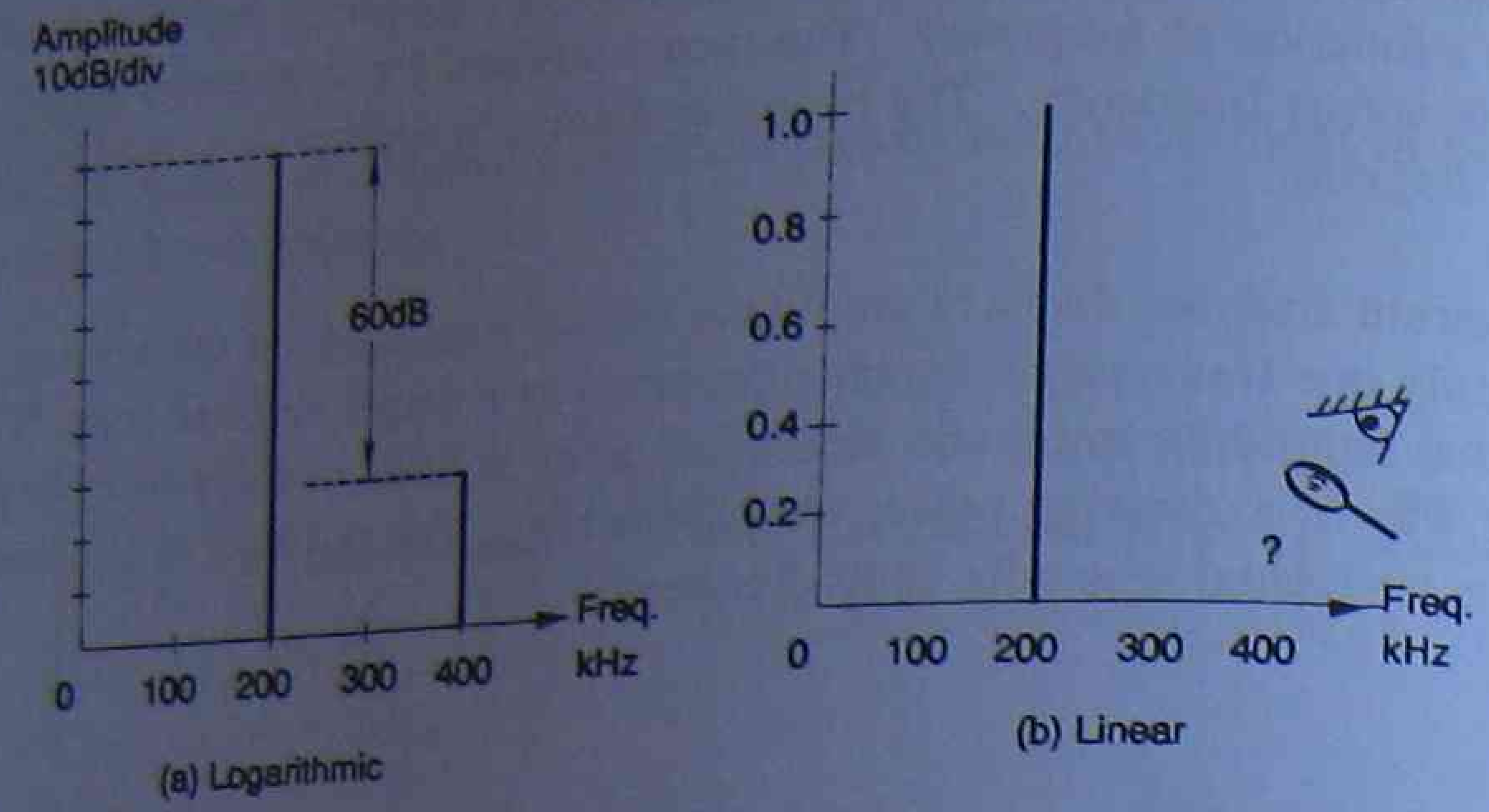
In general, the term frequency domain refers to any graph or measurement which is taken as a function of frequency. The most commonly encountered measurement is amplitude versus frequency. The resulting display is known as a spectrum or a spectral diagram.

The spectrum analyser displays amplitude versus frequency on the screen. A signal having only one frequency component appears as a single vertical line. The height of this line represents amplitude measured either in volts or milliwatts or dBm (another way of expressing power measurements). The position of this vertical line along the horizontal axis tells us its frequency.



Frequency domain display

One feature of the spectrum analyser is that it allows either linear or logarithmic (dB) scales. The logarithmic scale permits both large and small signals to be displayed simultaneously. For example a signal which is 60dB below another is $\frac{1}{1000{,}000}$ of that signal's power. On a linear scale only the larger signal would be seen. Viewed on an oscilloscope, the effect of the smaller signal would not be noticeable.



Spectrum analyser display

In many cases a signal can be observed in either the time or frequency domains. The choice is yours. The time and frequency domain representations of a signal are complementary, and if one representation is known, then the other can be derived from it.

This field of mathematics is known as Fourier analysis, after Jean Baptiste Joseph, Baron de Fourier (1768-1830). He accompanied Napoleon on the Egyptian campaign in 1798, becoming Governor of Lower Egypt before returning to France where he produced his classic paper 'Theories Analytique de la Chaleur' (Analysis of the Flow of Heat). In it he evolved the mathematical series which bears his name today, and has found application in most branches of applied science.

Summary

- Time domain refers to signals and quantities viewed as a function of time.
- The oscilloscope displays signals in the time domain.
- Frequency domain refers to signals and quantities viewed as a function of frequency.
- The spectrum analyser displays signals in the frequency domain.

Fundamentals of Fourier Analysis

Stated in the simplest of terms, Fourier's theorem says:

> A complex periodic waveform may be analysed as a number of harmonically related sinusoidal waves.

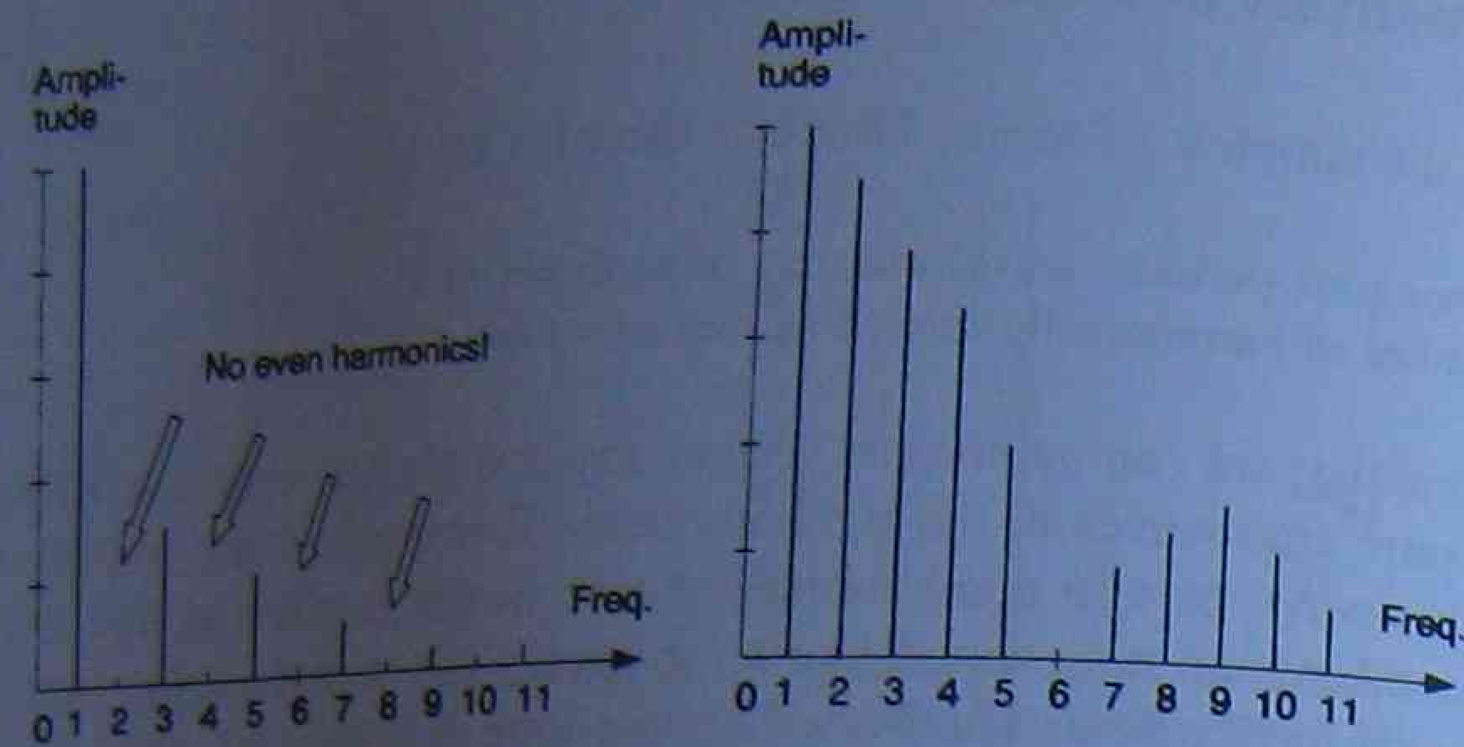
This means that we can synthesise (make) any complex periodic waveform by adding together pure sine waves in the right amounts. Electronic music can be created in exactly this way: certain combinations of sine waves may sound like a flute, while another combination may sound like a fog-horn. The term periodic simply means that the waveform repeats itself after a given time period T.

The frequencies of the constituent sine waves are all integer multiples of the fundamental frequency of the waveform concerned. These multiples are known as the harmonic frequencies, the second multiple being known as the second harmonic, the third multiple being known as the third harmonic and so on. The first harmonic is just the original frequency, and is referred to simply as the fundamental.

This applies to all complex periodic waveforms, such as square, triangle, pulsed and sawtooth signals. An ideal sine wave however, has only a fundamental component and no harmonics.

Generally speaking, the higher harmonics are weaker than the lower ones, although the individual amplitudes may vary in a complex manner. (Note that in the figure on the next page, the ninth harmonic is larger than the eighth harmonic). Also note that the fundamental or any harmonic(s) may have zero amplitude.

An example of this is the square wave which has only odd harmonics. Another property of the square wave is that the third harmonic has an amplitude $\frac{1}{3}$ that of the fundamental, the fifth harmonic has an amplitude $\frac{1}{5}$ that of the fundamental and so on.



(a) Square wave (b) Rectangular wave

Distribution of harmonics

Example

A waveform has a period T = 40mS. Calculate the frequency of the fundamental, and the second, third and fourth harmonics.

Fundamental frequenc= $\frac{1}{T}$

$= \frac{1}{40 \times 10^{-3}}$

= 25Hz

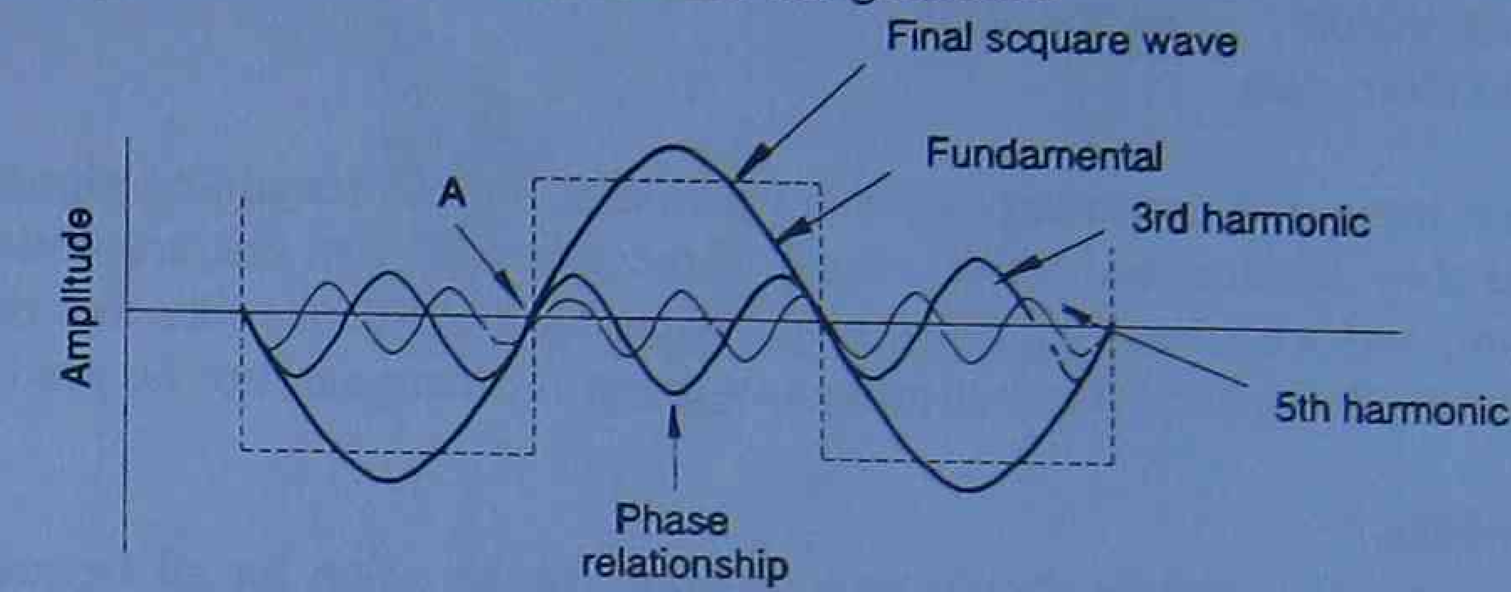
Second harmonic = 2 x 25Hz
= 50Hz

Third harmonic = 3 x 25Hz
= 75Hz

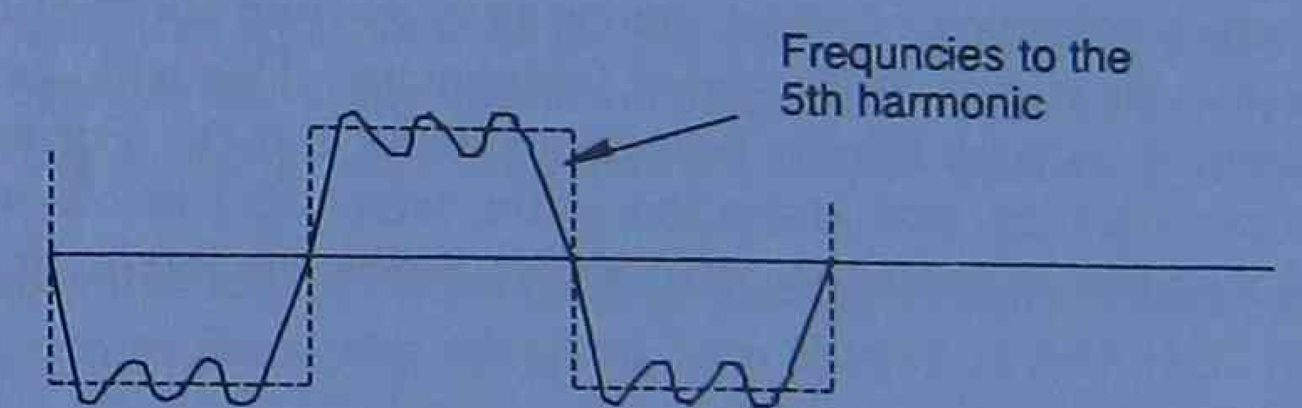
Fourth harmonic = 4 x 25Hz
= 100Hz

Let us now see how a square wave can be made by adding a fundamental frequency and a number of harmonics. You will recall that only odd harmonics are required, and that the third harmonic has $\frac{1}{3}$ the amplitude of the fundamental and so on.

Figure (a) below shows the fundamental, the third and fifth harmonics and their phase relationships, and Figure (b) shows the resultant. Note that all odd harmonics to infinity must be considered to construct a perfect square wave, although in practice the higher order harmonics become insignificant.



(a) Components of a square wave



(b) Resultant of frequencies to fifth harmonic

Waveforms and spectral diagrams for common signals

You will now be introduced to some of the waveforms commonly found in communications equipment and systems. The waveforms are:

- the sine wave
- the square wave
- white noise
- speech
- music
- television
- random data.

The sine wave and the square wave are periodic, but the remaining signals are not, because they are not predictable. Each of these has its own characteristic frequency spectrum, and we know in general terms what each one looks like, but the precise detail of the frequency distribution at any given moment can not be predicted.

Bandwidth

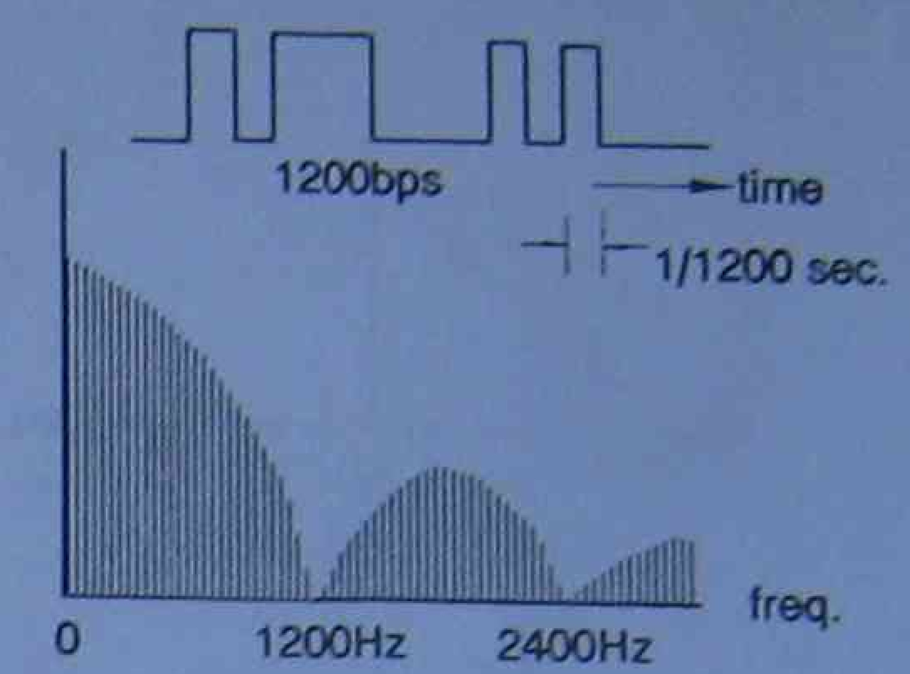
A communication system should provide good transmission for all frequencies where the signal power spectrum is significant.

Speech
For speech, the entire collection of vocal sounds extends from about 80Hz to 12kHz, with strongly decreasing energy at the higher frequencies. This wide range of frequencies is required for high fidelity broadcast quality speech. For most communications purposes (eg. taxi and police radio, telephone) such a wide range is unnecessary and it can be restricted to 300-3400Hz before the intelligibility suffers.

Music
For high fidelity music, the band from 30Hz to 15kHz is required.

Video
For video, frequencies from 0 to 5MHz are required.

Random data
For data, the required bandwidth is related to the bit rate. The higher the bit rate, the more bandwidth is required. For most applications we can say that the required bandwidth is equal to the reciprocal of duration of the narrowest pulse.



Spectrum of random data

The term bandwidth in relation to the various signals above, refers to the numerical difference between the upper and lower frequency limits of the signal. For example, the speech signal above has a minimum acceptable bandwidth of 3100Hz.

But what does this mean? Does it mean that the component frequencies in the speech that comes from your telephone stop sharply at 3400Hz?

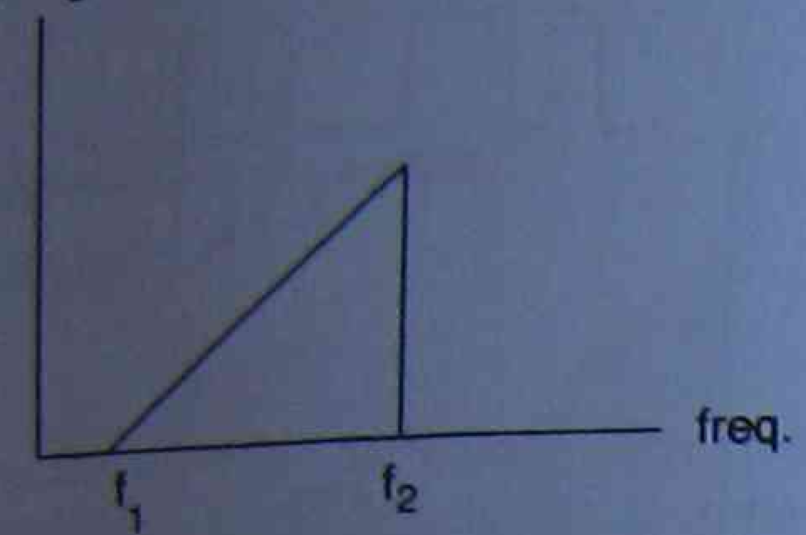
No! It means that outside the limits of 300Hz and 3400Hz, the spectral components are weaker by 3 decibels (dB) or more below the strongest spectral components within the 300 to 3400Hz band.

Note: The decibel is a logarithmic power ratio which is used throughout all fields of electronics. In particular, 3 dB means that the power has fallen by half.

In the case of speech which has been transmitted through a telephone network, the bandwidth will have been reduced so that the 3dB points are 300Hz and 3400Hz.

Symbolic representation of the baseband

Since baseband signals have varying spectral diagrams, it is necessary to adopt a standard representation for all of them, regardless of their actual spectra. The standard shape is show in the figure below, the hypotenuse of the triangle increasing in the direction of increasing frequency.



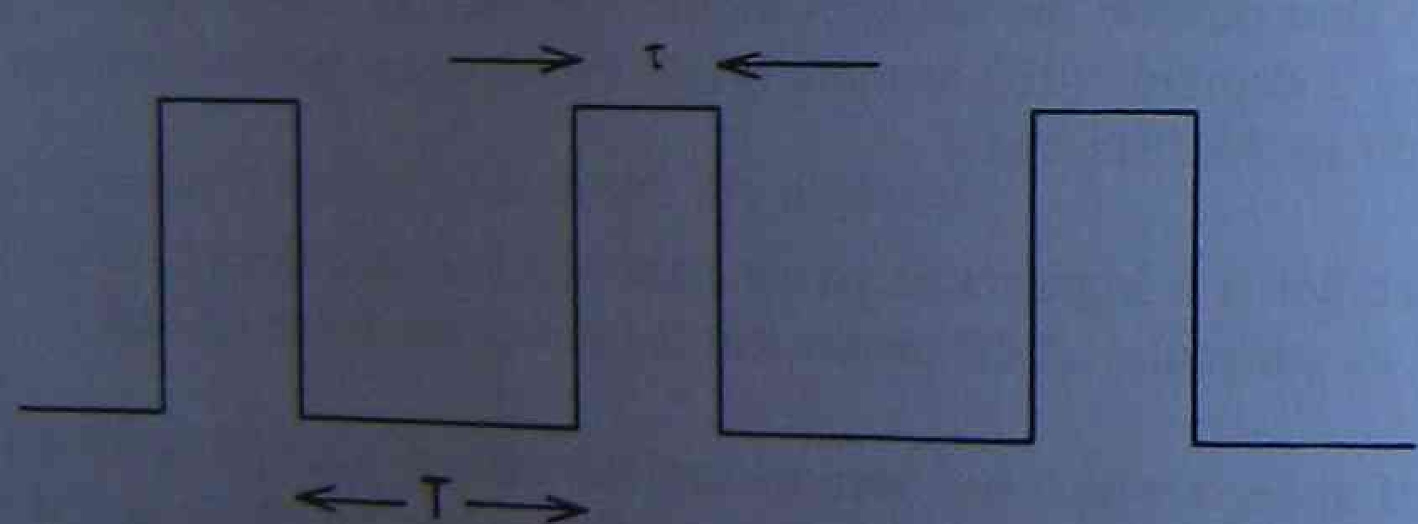
Symbolic representation of the baseband

Note that this symbolic shape usually bears no resemblance to the actual baseband signal in the system. Speech signals in fact have the opposite characteristic.

The significance of this representation will become apparent when we consider the topic Modulation in Section 4.

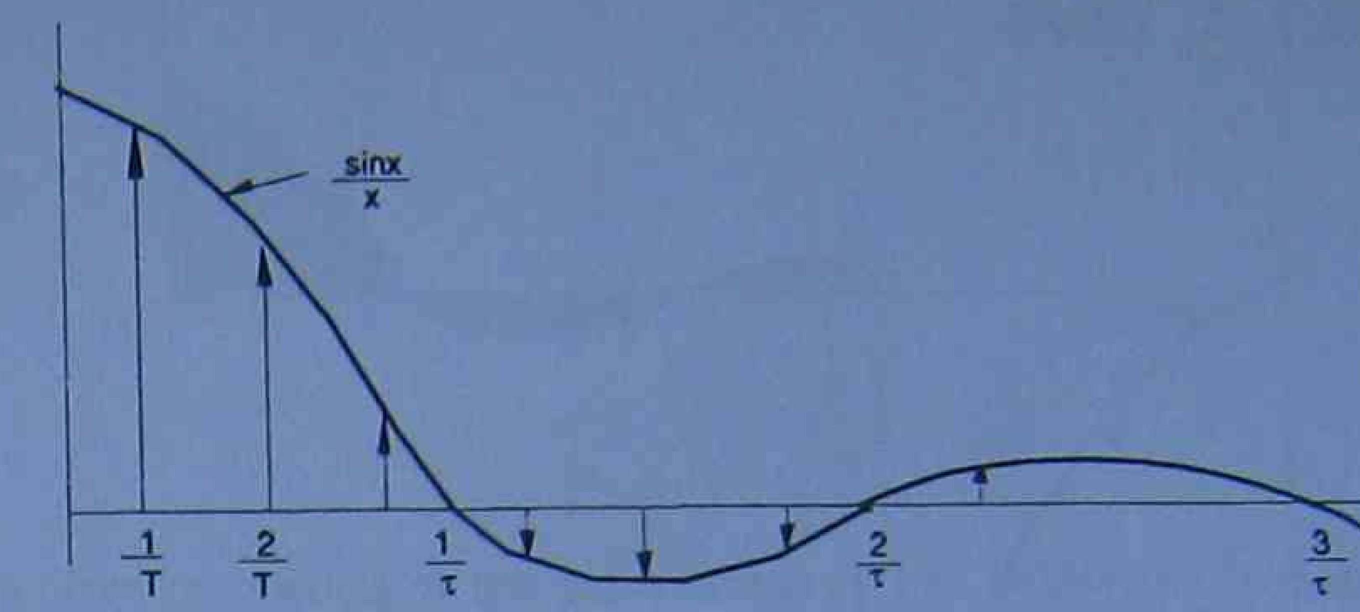
The spectrum of a pulse train

For a rectangular pulse train with period T and pulse width τ as shown below, the relative amplitudes of the spectral components can be found from the $\frac{\sin x}{x}$ curve.



The first zero point on the $\frac{\sin x}{x}$ curve corresponds to the frequency $\frac{1}{\tau}$.

The fundamental frequency is, of course, $\frac{1}{T}$.

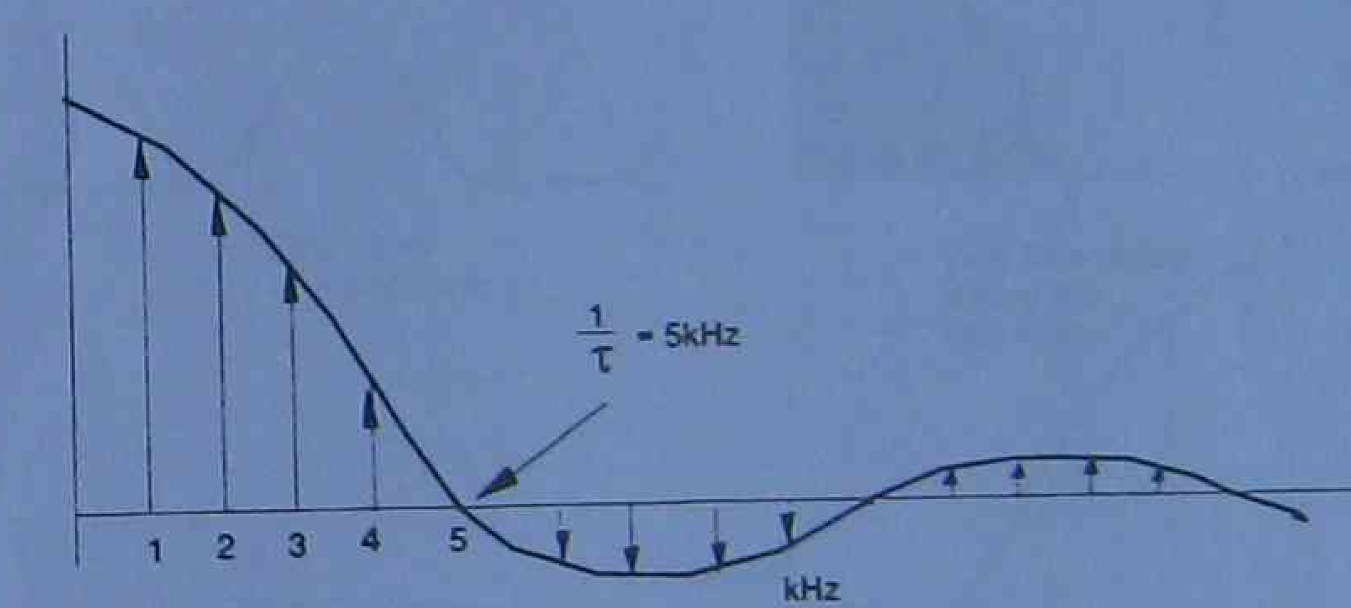


Example 1

T = 1ms, τ = 0.2ms

$\therefore \frac{1}{T} = 1kHz, \quad \frac{1}{\tau} = 5kHz$

Locate 5kHz at the first zero of the curve.



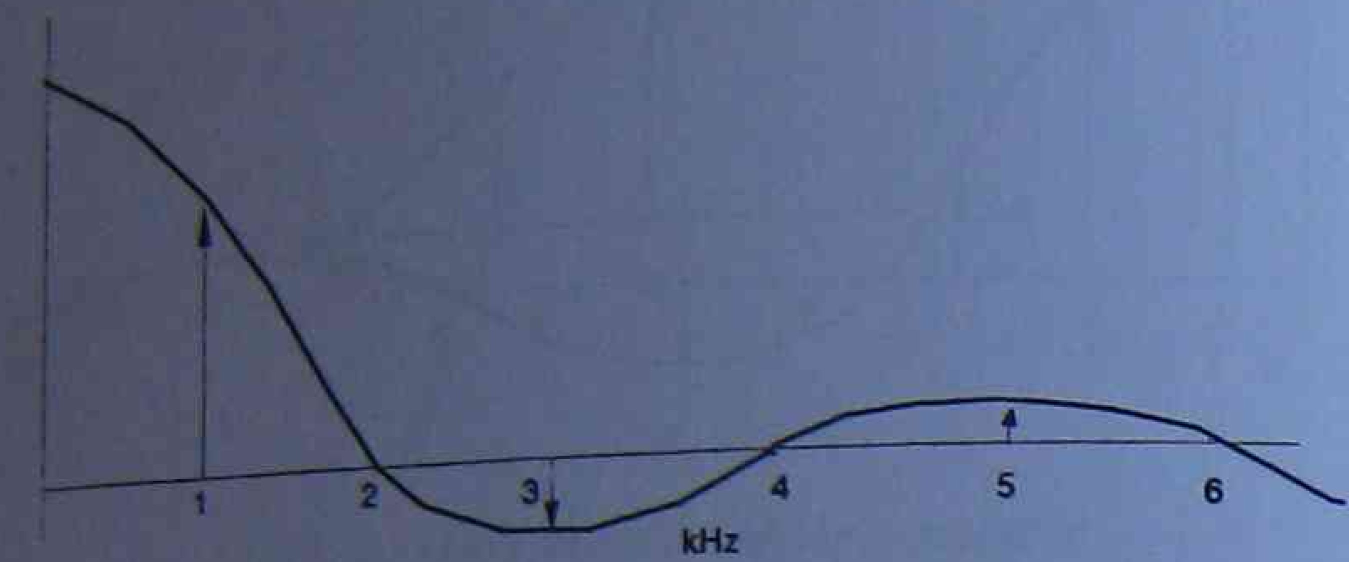
In this case the 5th harmonic (5kHz) has zero amplitude.

Note that when the above spectra are displayed on a spectrum analyzer, all the spectral lines are shown above the horizontal axis.

Example 2

T = 1ms, τ = 0.5ms (square wave)

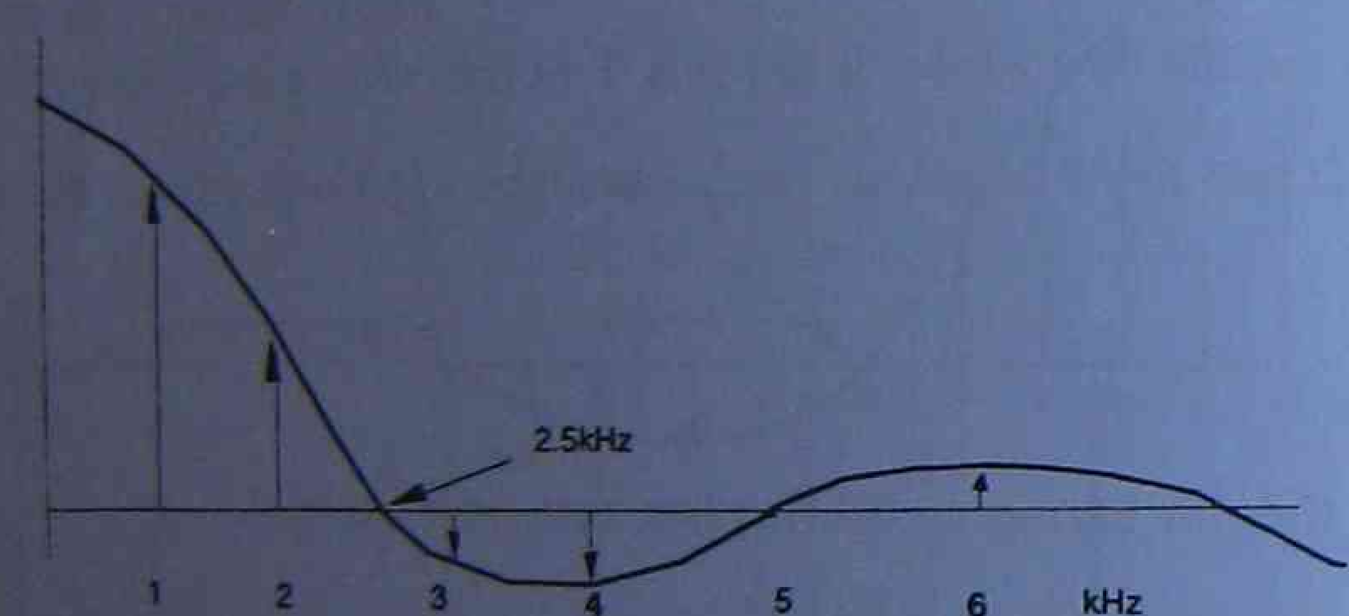
$\therefore \frac{1}{T} = 1kHz, \ \frac{1}{\tau} = 2kHz$



Example 3

T = 1ms, τ = 0.4ms

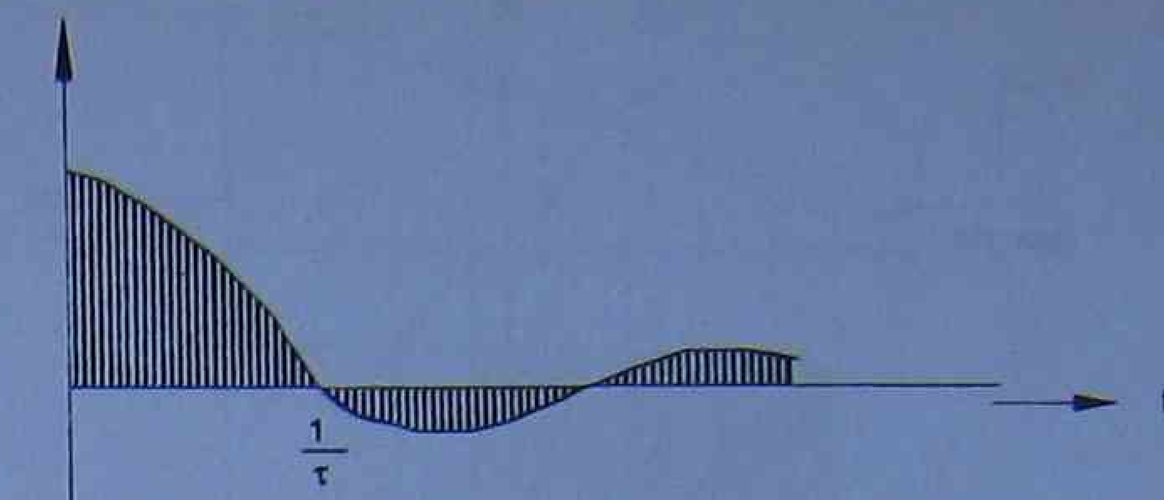
$\therefore \frac{1}{T} = 1kHz, \ \frac{1}{\tau} = 2.5kHz$



Note that in this case $\frac{1}{\tau}$ is not a harmonic, and that the first component to have zero amplitude is at 5kHz (the fifth harmonic).

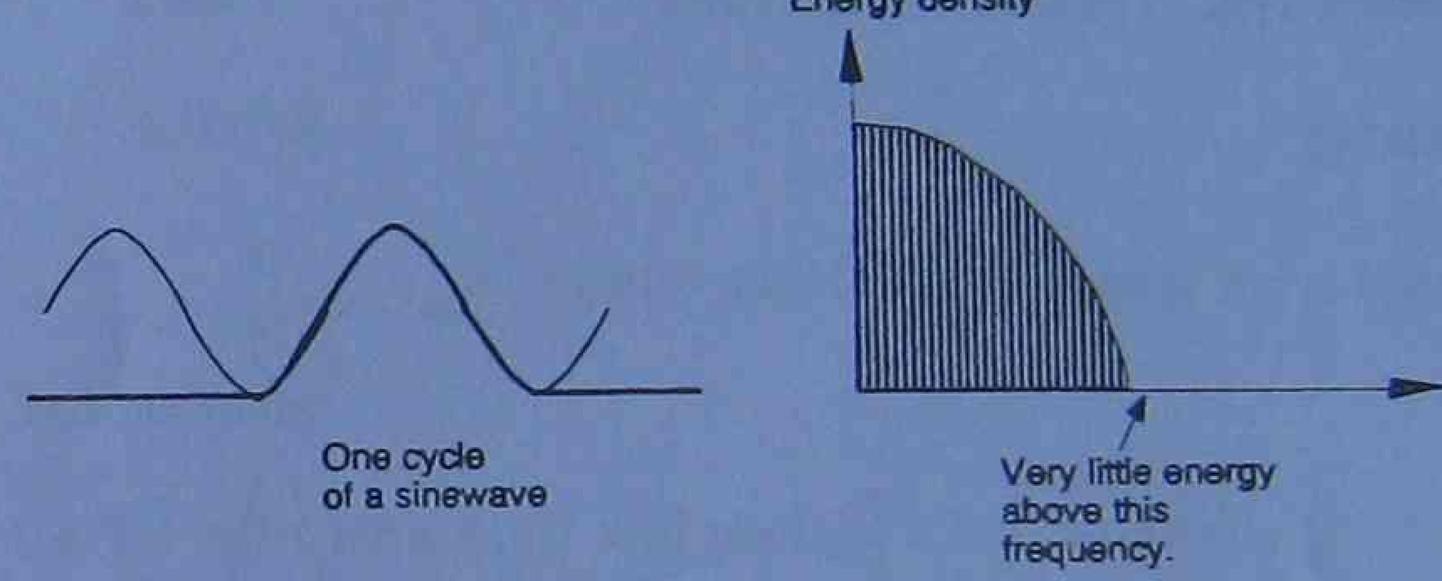
As T increases, the spectral lines get closer together.

If T $\rightarrow \infty$ (ie. we have only one pulse), the spectral lines form a continuum.

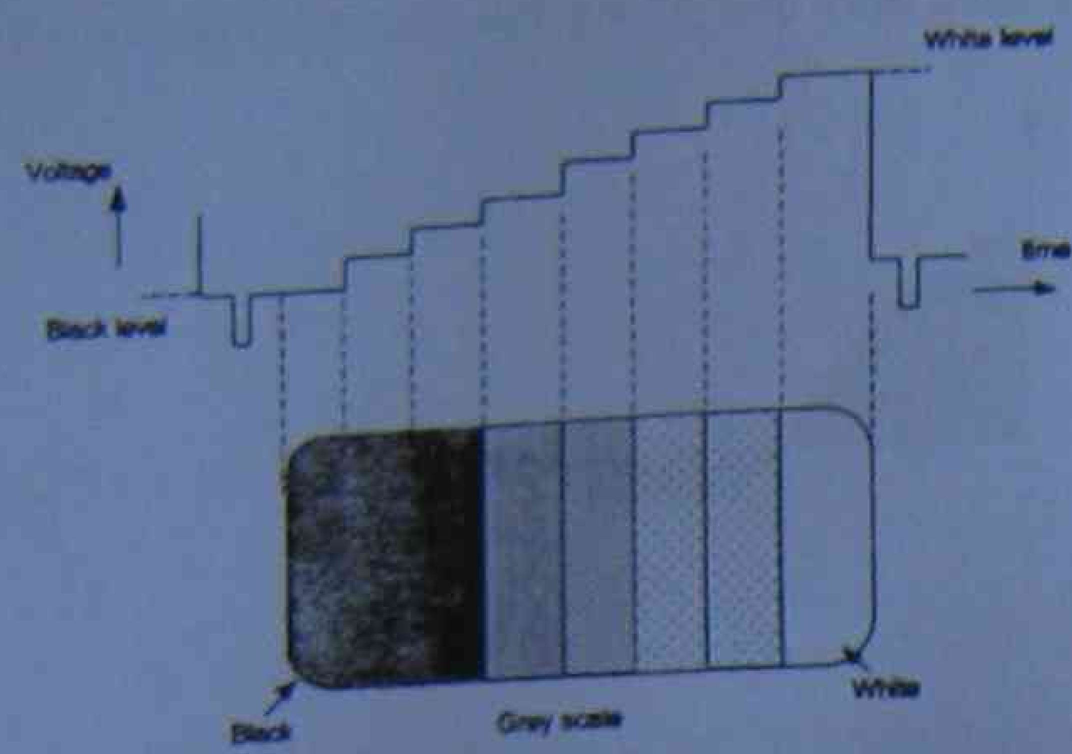


Note that a large proportion of the pulse's energy lies below the frequency $\frac{1}{\tau}$.

Even one cycle of a sinewave has a continuous spectrum. This is used as a test signal in Television.



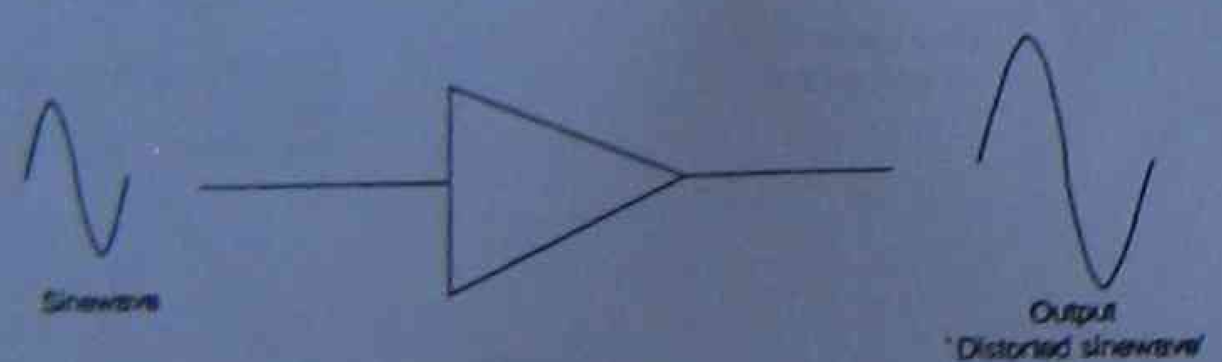
The video signal



Spectrum with varying signal:
- Luminance: 0 - 5MHz
- Chrominance: Centred on 4.43MHz
- Horizontal sync: 15625Hz + harmonics.

Non-linearity

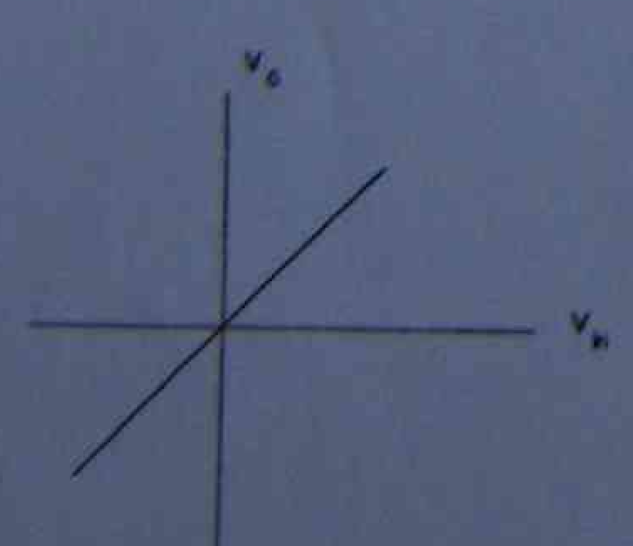
Output voltage is not proportional to input voltage.



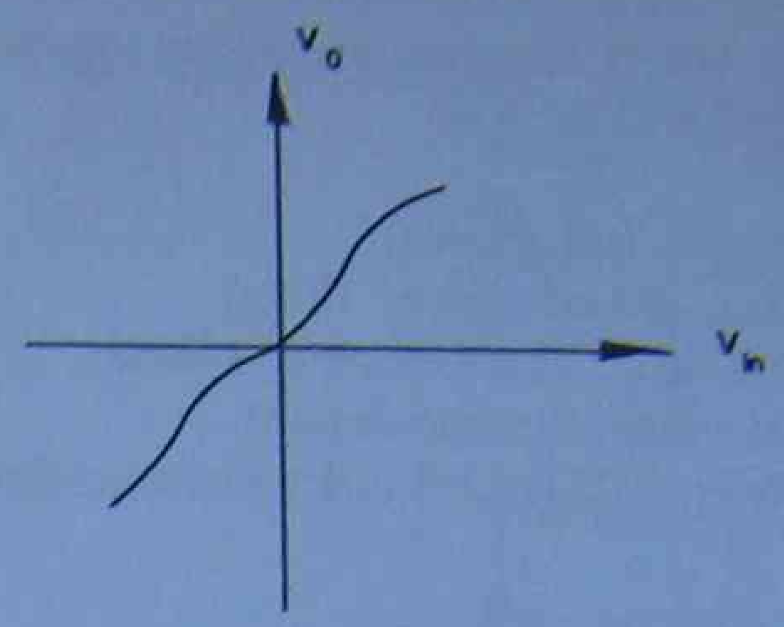
Output is not a sinewave but is still periodic. It therefore contains harmonics and has the same fundamental frequency as the input.

A linear device has $v_0 \propto v_{in}$

$$v_0 = Av_{in}$$



Non-linear device



The curve can be represented by as power series:

$v_0 = A_1 v_{in} + A_2\, v_{in}^2 + A_3\, v_{in}^3 + ...$

Harmonic distortion

If we let $v_{in} = \sin\omega t$

then $A_2\, v_{in}^2 = A_2 \sin^2\omega t$

$$= \frac{A_2}{2} - \frac{A_2}{2} \cos 2\omega t$$

$\cos 2\omega t$ = 2nd harmonic.

Likewise the 3rd order term $A_3 v_{in}^3$ will produce a 3rd harmonic.

Intermodualation distortion

Let the input consist of two sinewaves:

$v_{in} = \sin\omega_1 t + \sin\omega_2 t$

The second term in the power series above becomes
$A_2 V_{in}^2 = A_2 (\sin\omega_1 t + \sin\omega_2 t)^2$

A little trigonometry will show that this term produces not only the harmonics $2\omega_1$ and $2\omega_2$ but also sum and difference frequencies $\omega_1 \pm \omega_2$

The 3rd order term will produce $\omega_1 \pm 2\omega_2$ and $2\omega_1 \pm \omega_2$.

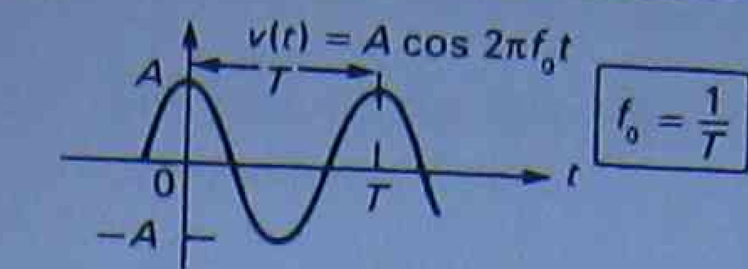
Example

A 1kHz sinewave and a 10kHz sinewave are added together and applied to a non-linear device. The output will include:

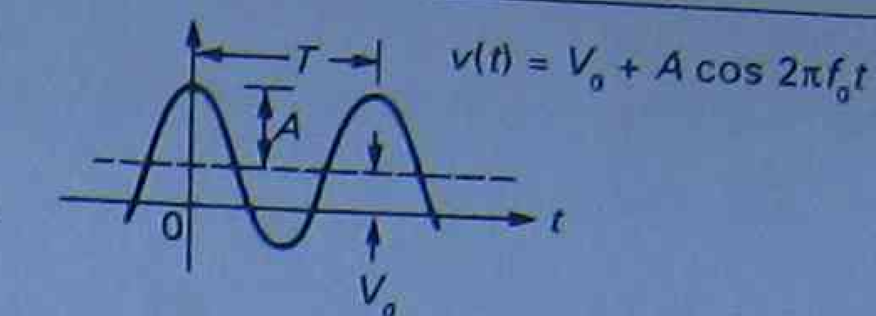
- 1kHz, 10kHz (original frequencies)
- 2kHz, 3kHz, 4kHz.....(harmonics of 1kHz)
- 20kHz, 30kHz, 40kHz ......(harmonics of 10kHz)
- 9kHz, 11kHz (2nd order intermodulation)
- 8kHz, 12kHz, 19kHz, 21kHz (3rd order intermodulation).

TABLE 3–1 Some Periodic Waveforms and Their Fourier Series Mathematical Expressions

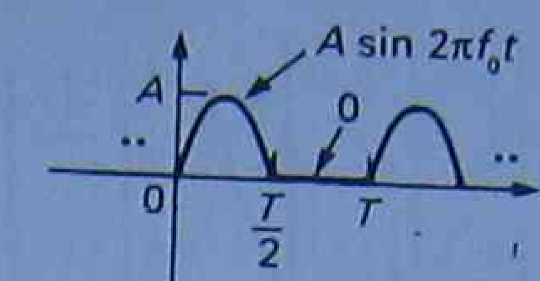
a.



b.

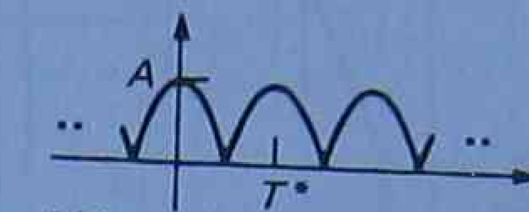


c.



$$v(t) = \frac{A}{\pi} + \frac{A}{2}\sin 2\pi f_0 t - \frac{2A}{3\pi}\cos 2\pi(2f_0)t - \frac{2A}{15\pi}\cos 2\pi(4f_0)t + \cdots$$
$$= \frac{A}{\pi} + \frac{A}{2}\sin 2\pi f_0 t + \sum_{n=2}^{\infty} \frac{A[1+(-1)^n]}{\pi(1-n^2)}\cos 2\pi(nf_0)t$$

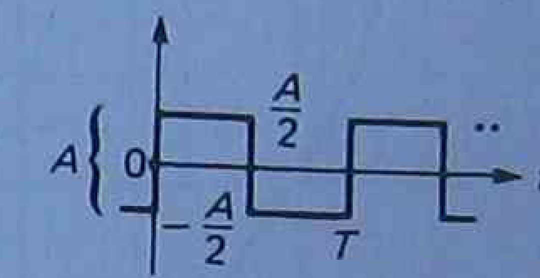
d.



* (the rectifier input signal will have a period of 2 T)

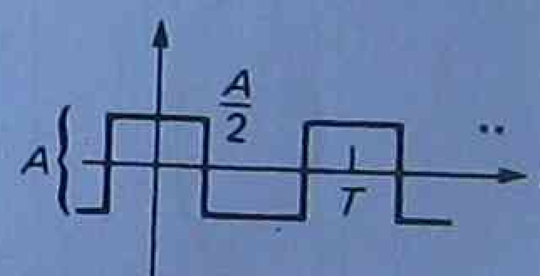
$$v(t) = \frac{2A}{\pi} + \frac{4A}{3\pi}\cos 2\pi f_0 t - \frac{4A}{15\pi}\cos 2\pi(2f_0)t + \cdots$$
$$= \frac{2A}{\pi} + \sum_{n=1}^{\infty} \frac{4A(-1)^n}{\pi[1-(2n)^2]}\cos 2\pi(nf_0)t$$

e.



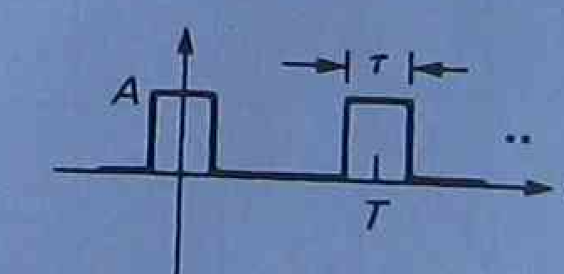
$$v(t) = \frac{2A}{\pi}\sin 2\pi f_0 t + \frac{2A}{3\pi}\sin 2\pi(3f_0)t + \cdots$$
$$= \sum_{n,\ \text{odd only}}^{\infty} \frac{2A}{n\pi}\sin 2\pi(nf_0)t$$

f.



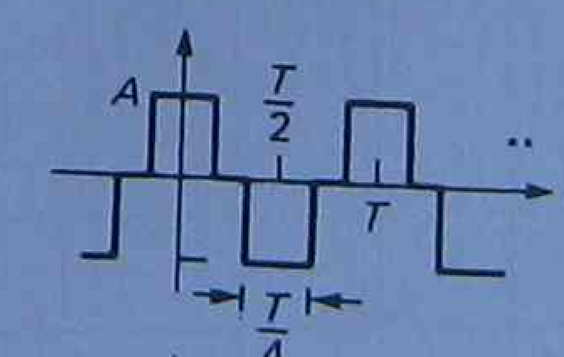
$$v(t) = \frac{2A}{\pi}\cos 2\pi f_0 t - \frac{2A}{3\pi}\cos 2\pi(3f_0)t + \frac{2A}{5\pi}\cos 2\pi(5f_0)t + \cdots$$
$$= \sum_{n=1}^{\infty}\left(A\frac{\sin n\pi/2}{n\pi/2}\right)\cos 2\pi(nf_0)t$$

g.



$$v(t) = \frac{A\tau}{T} + \sum_{n=1}^{\infty}\left(2A\frac{\tau}{T}\right)\left(\frac{\sin n\pi\tau/T}{n\pi\tau/T}\right)\cos 2\pi(nf_0)t$$

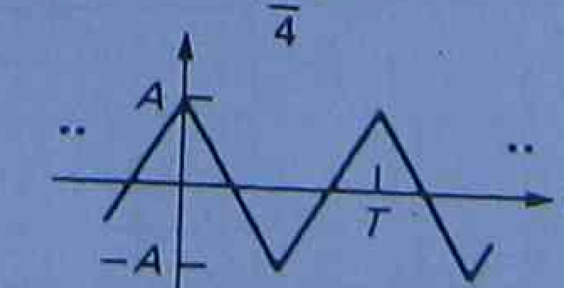
h.



$$v(t) = \sum_{n,\ \text{odd only}}^{\infty}\left(A\frac{\sin n\pi/4}{n\pi/4}\right)\cos 2\pi(nf_0)t$$

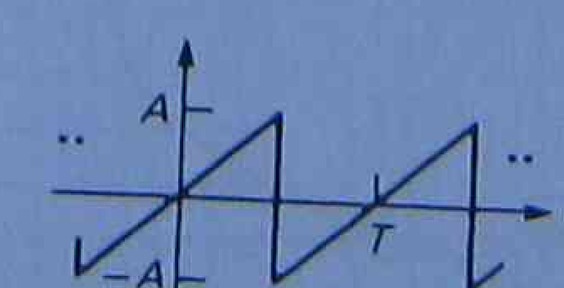
(special case of 50% "alternate inversion")

i.

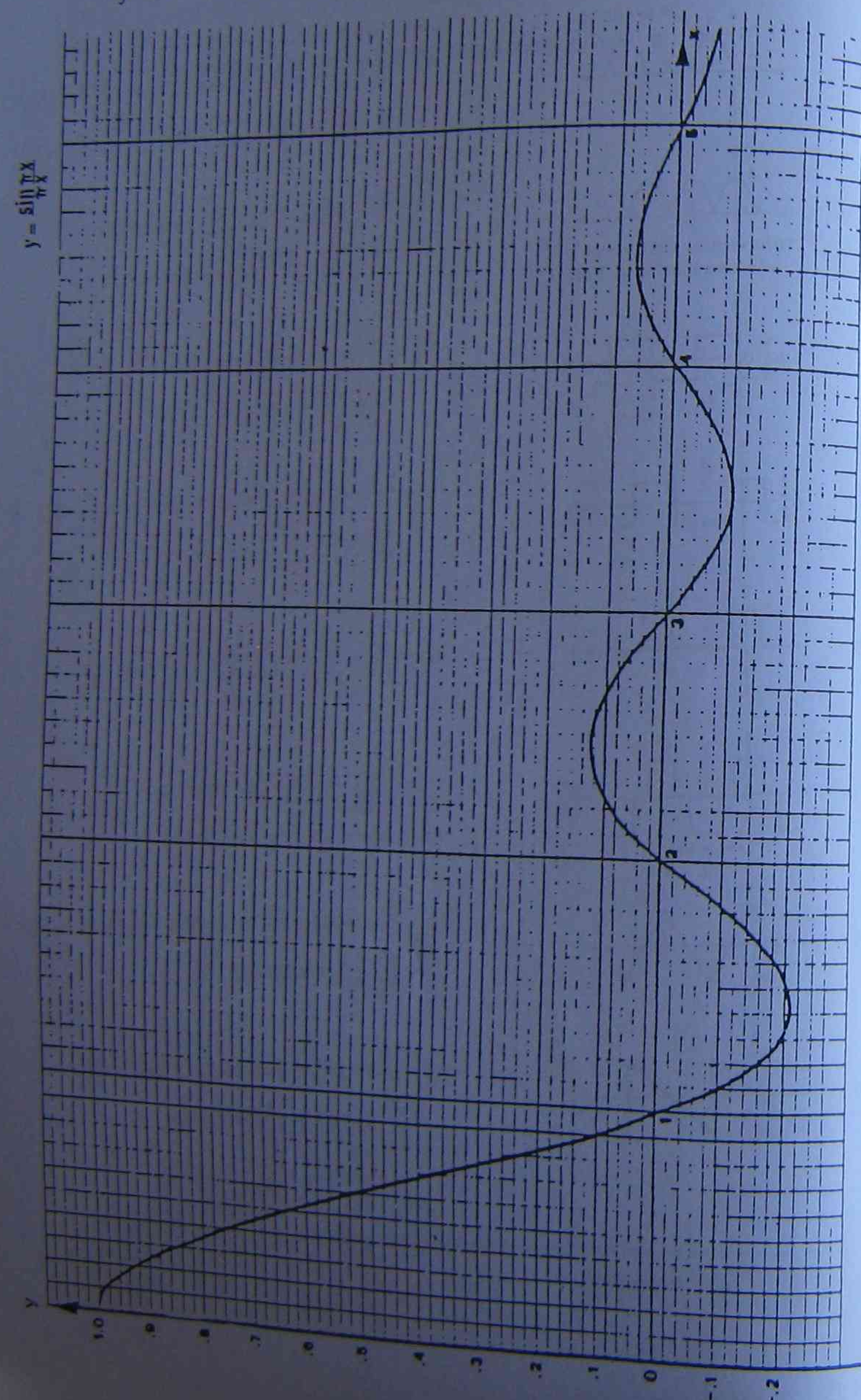


$$v(t) = \frac{8A}{\pi^2}\cos 2\pi f_0 t + \frac{8A}{9\pi^2}\cos 2\pi(3f_0)t + \frac{8A}{25\pi^2}\cos 2\pi(5f_0)t + \cdots$$
$$= \sum_{n\ \text{odd}}^{\infty}\frac{8A}{(n\pi)^2}\cos 2\pi(nf_0)t$$

j.



$$v(t) = \frac{2A}{\pi}[\sin 2\pi f_0 t - \tfrac{1}{2}\sin 2\pi(2f_0)t + \tfrac{1}{3}\sin 2\pi(3f_0)t + \cdots]$$
$$= \sum_{n=1}^{\infty}[(-1)^{n+1}]\left(\frac{2A}{n\pi}\right)\sin 2\pi(nf_0)t$$

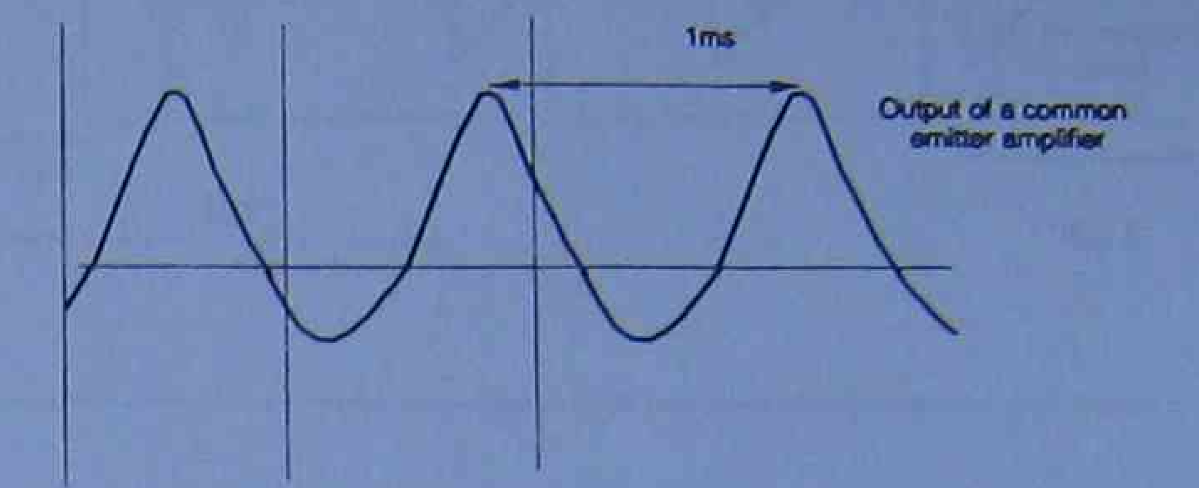


Review questions

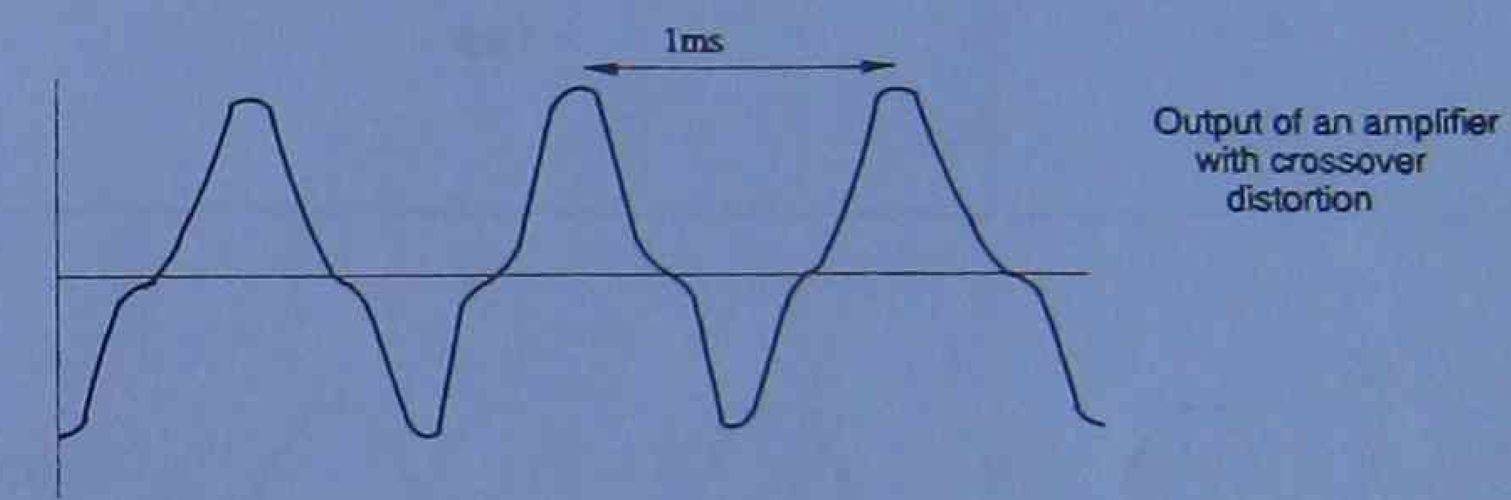
These questions will help you revise what you have learnt in Section 2.

1. For each of the following waveforms, state the frequencies of the first three components present.

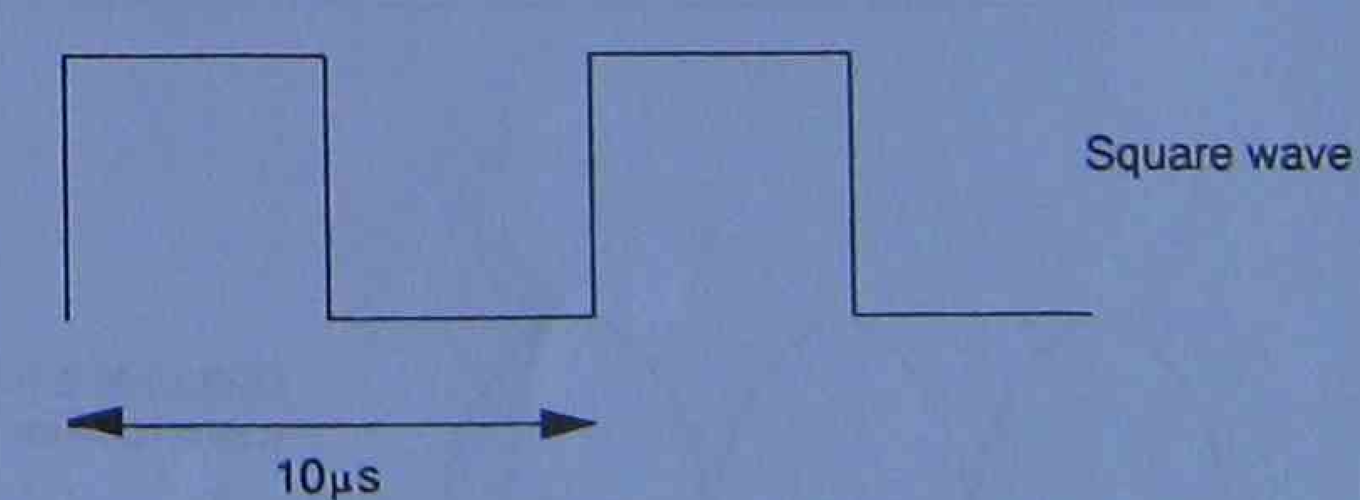
(a)



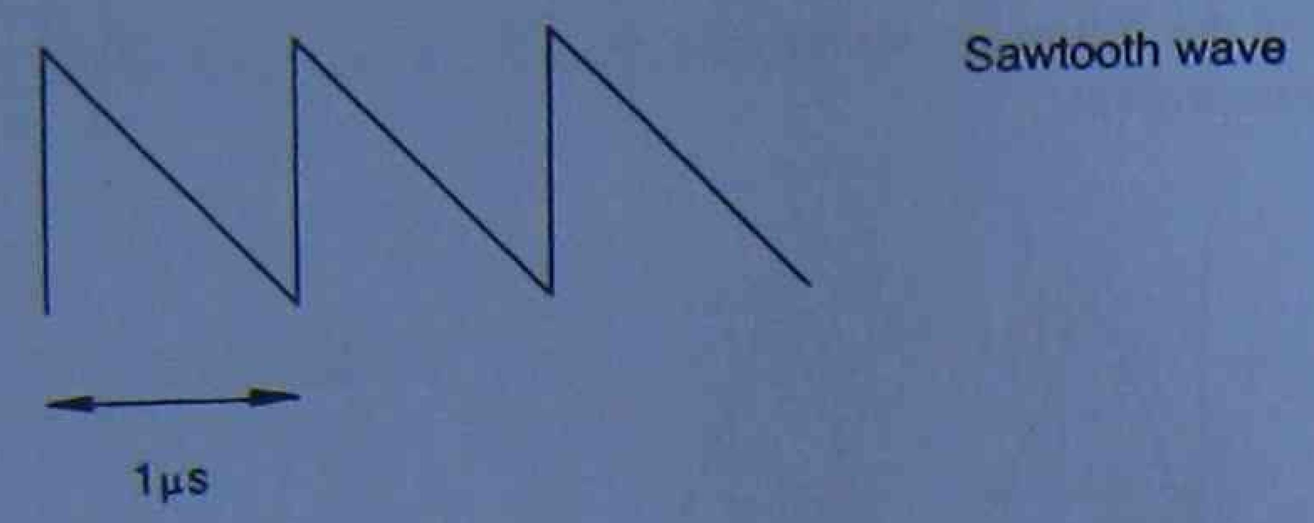
(b)



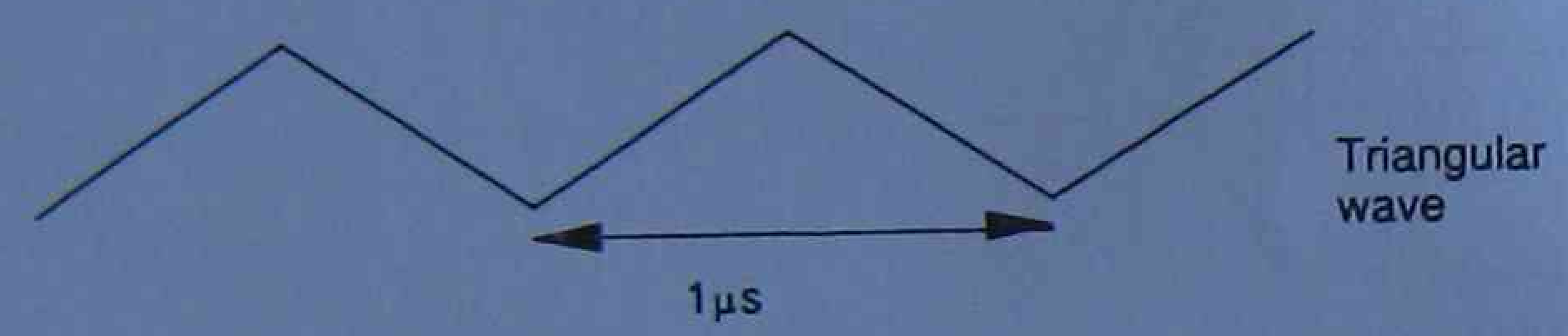
(c)



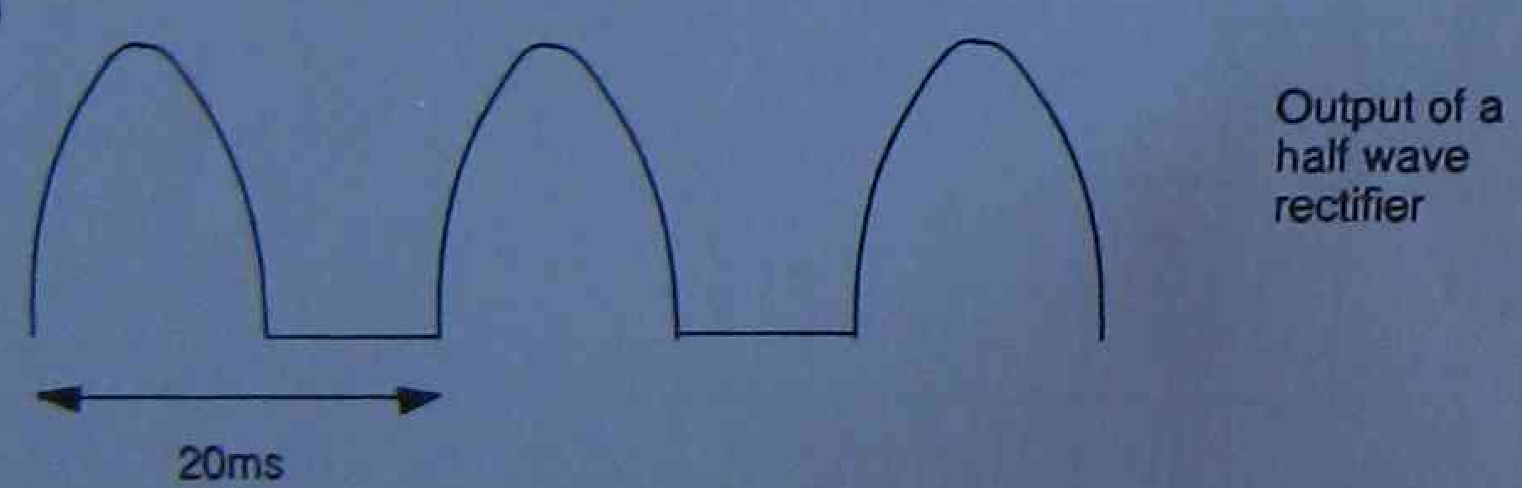
(d)



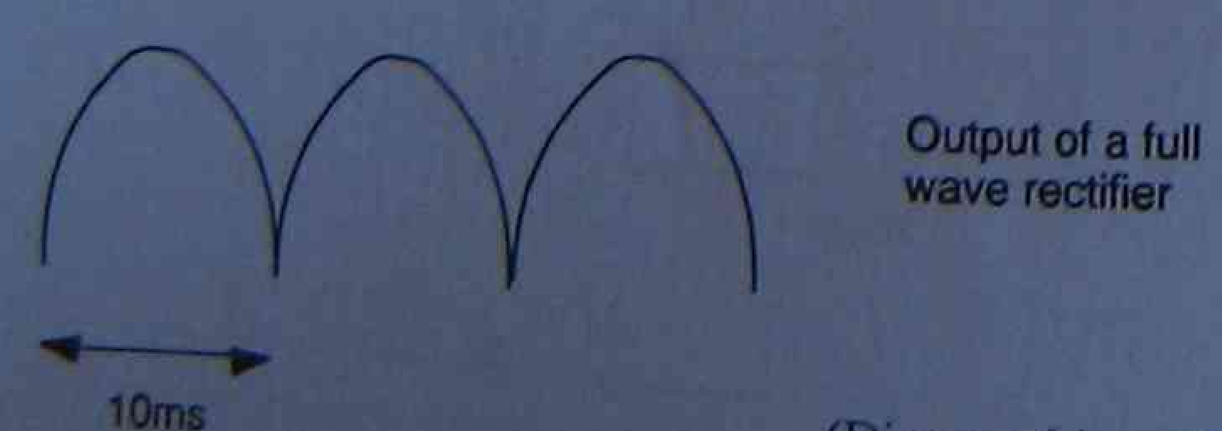
(e)



(f)

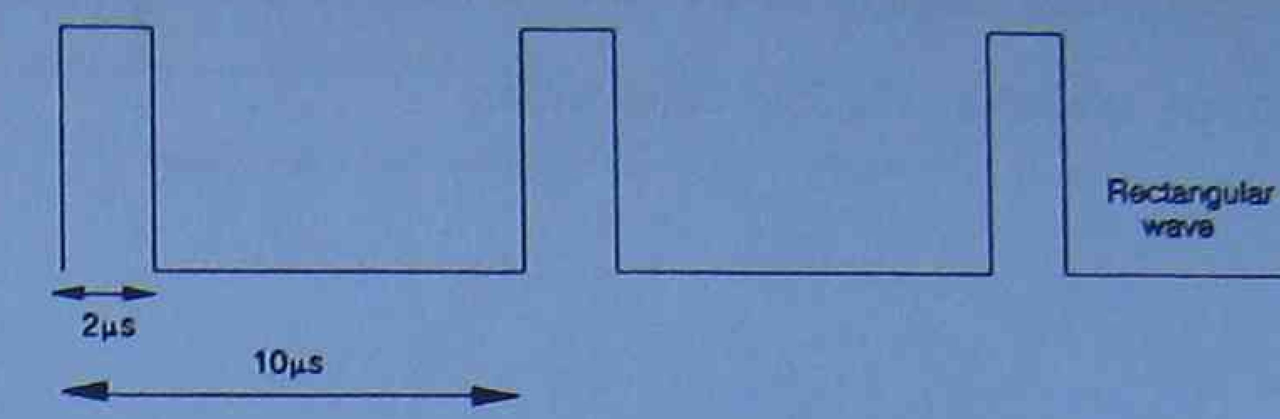


(g)



(Discuss this with your teacher!)

2. For the waveform shown below, will the 5th harmonic be present?



3. A rectangular pulse train has a peak amplitude of 5V, a pulse width of 2ms and a period of 5ms.

(a) Calculate the DC ('average') value of this waveform.

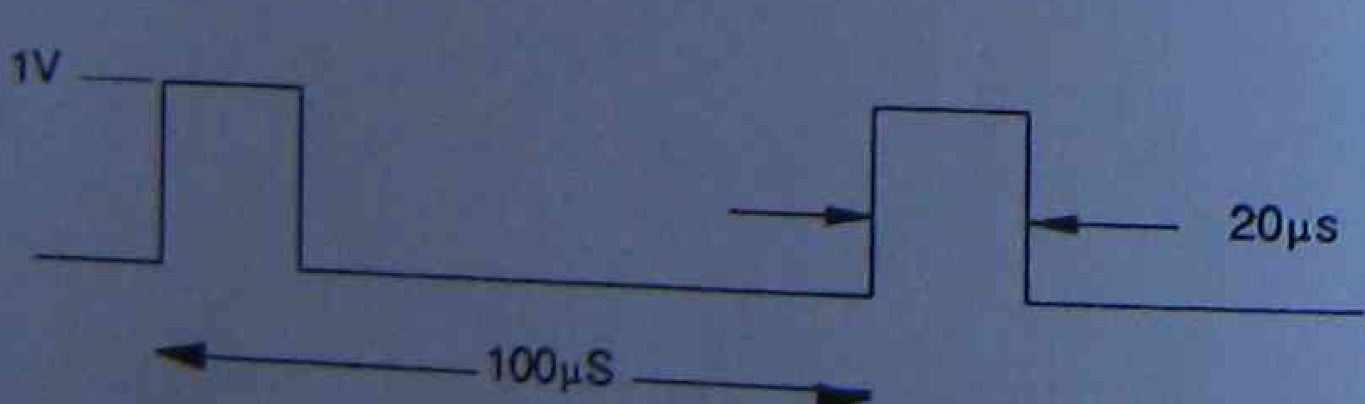
(b) Calculate the mark-space ratio.

(c) On a $\frac{1}{x}sinx$ curve, sketch the five lowest frequency components in the waveform, excluding the DC component.

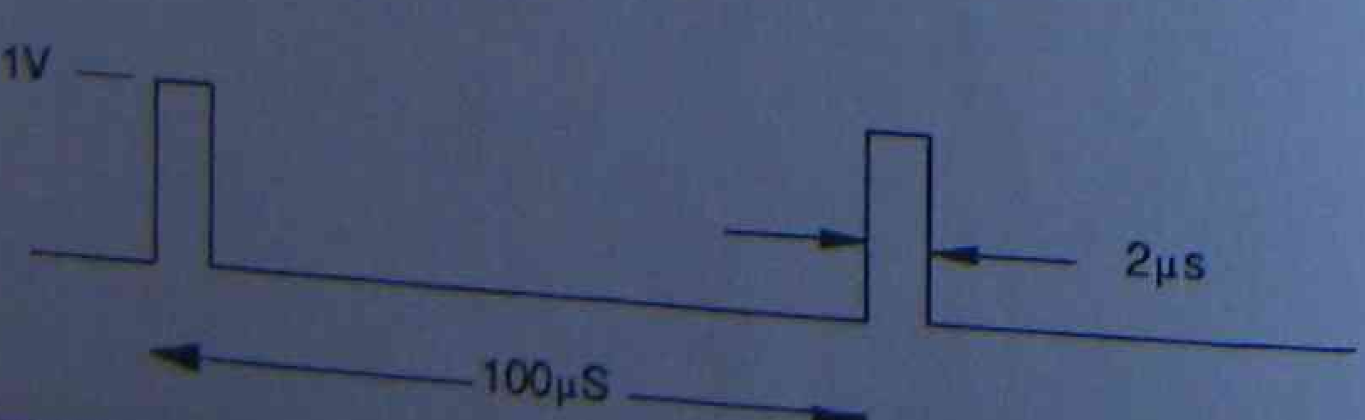
(d) Which harmonics have zero amplitude?

4. (a) Using a $\frac{1}{x}sinx$ curve, sketch the spectra of each of the pulse trains below, marking the frequency order of the harmonic at the first zero of the curve.

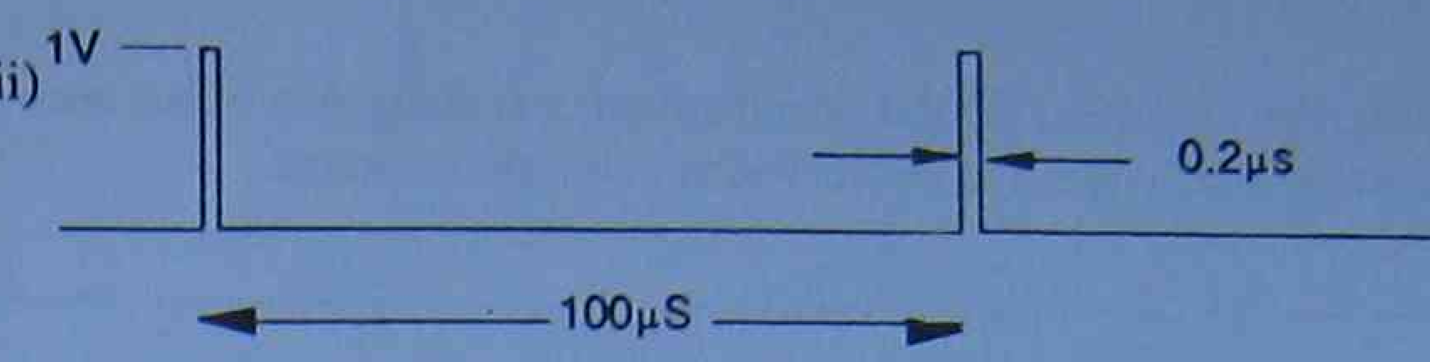
(i)



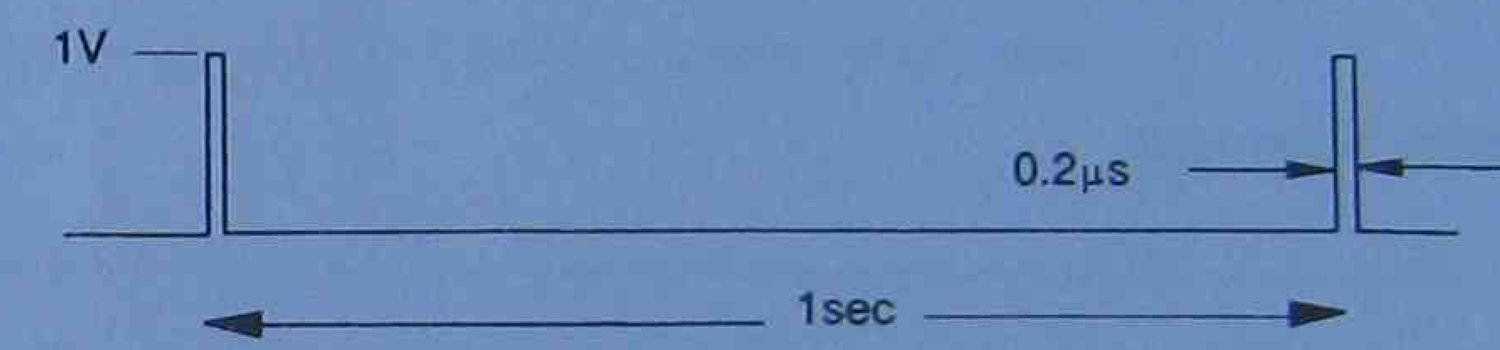
(ii)



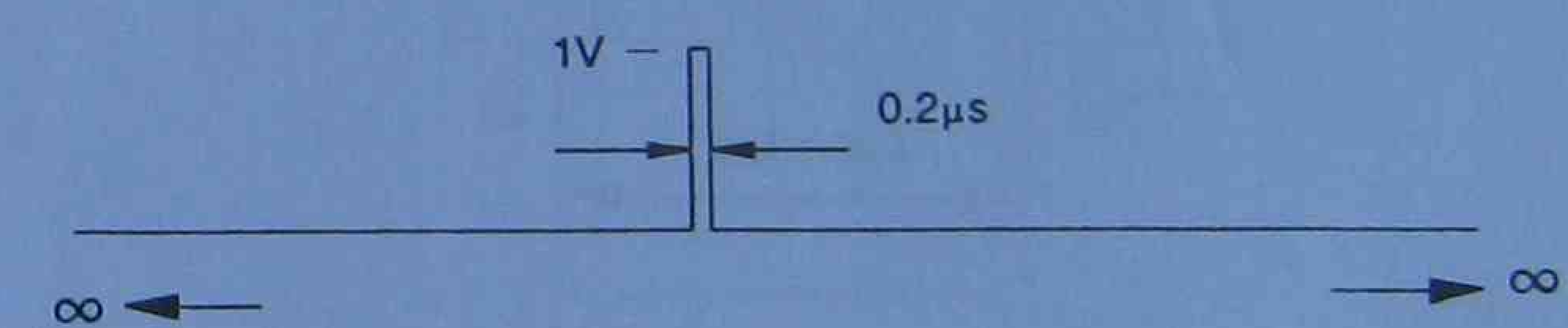
(iii)



(iv)



(v)



(b) Describe the effect on the spectrum of a rectangular pulse train if the pulse width is decreased, while the pulse period remains constant.

(c) Describe the effect on the spectrum of a rectangular pulse train if the period is increased, while the pulse width remains constant.

____

____

____

____

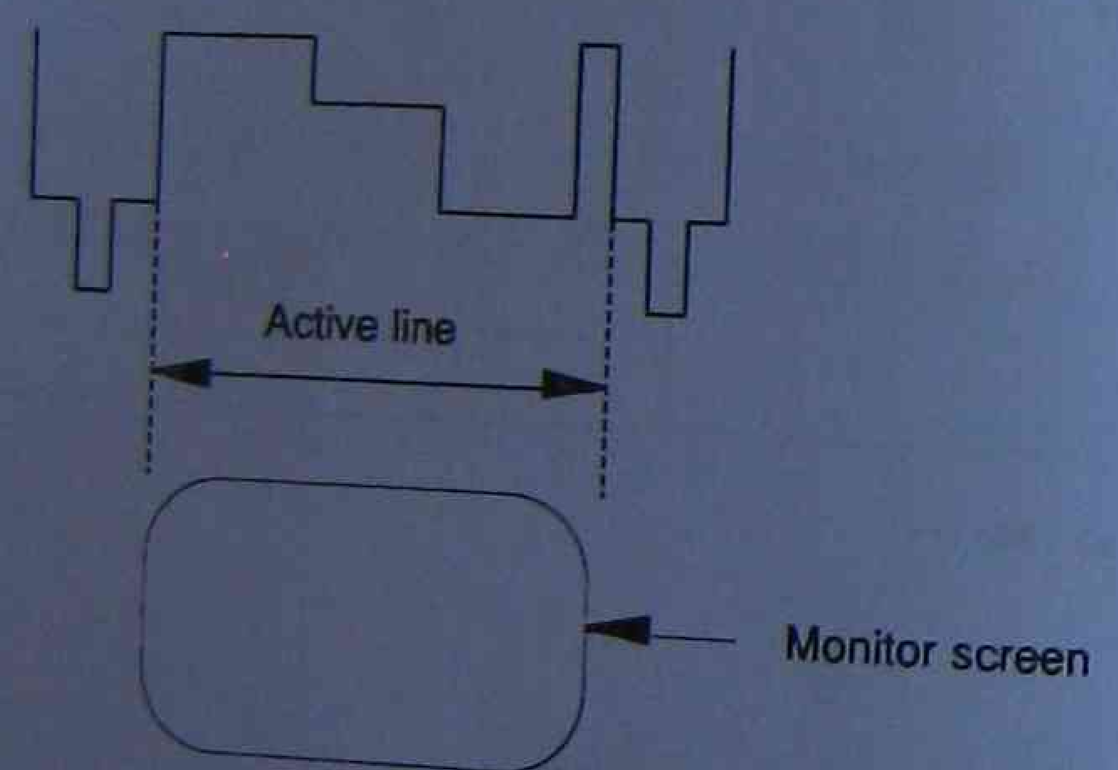
5. What range of frequencies is required for high quality transmission of music?

____

6. Define white noise.

____

7. Sketch the pattern that the following video signal will produce on a video monitor.



8. Sketch the spectrum of 2400bps random binary data.

9. A 50Hz sinewave and a 7kHz sinewave are applied to a non-linear amplifier. List the output frequencies produced by 2nd and 3rd order non-linearities.

____

____

____

____

10. Non-linearity in the 'front end' of radio receivers causes intermodulation products which may cause one station to be received at several points on the dial (and perhaps to block out your favourite station). Third order products can be particularly troublesome.

For a FM receiver receiving stations on 100HMz, 101MHz and 102MHz, calculate all the third order products which fall within the 88 - 108MHz band.

Section 3: Filters

SUGGESTED DURATION	PREAMBLE
5 hrs 20 mins	To introduce you to simple filter types, filter parameters, and the effects of filters on various signals.
This section covers learning outcome 5 of the Module Descriptor.	

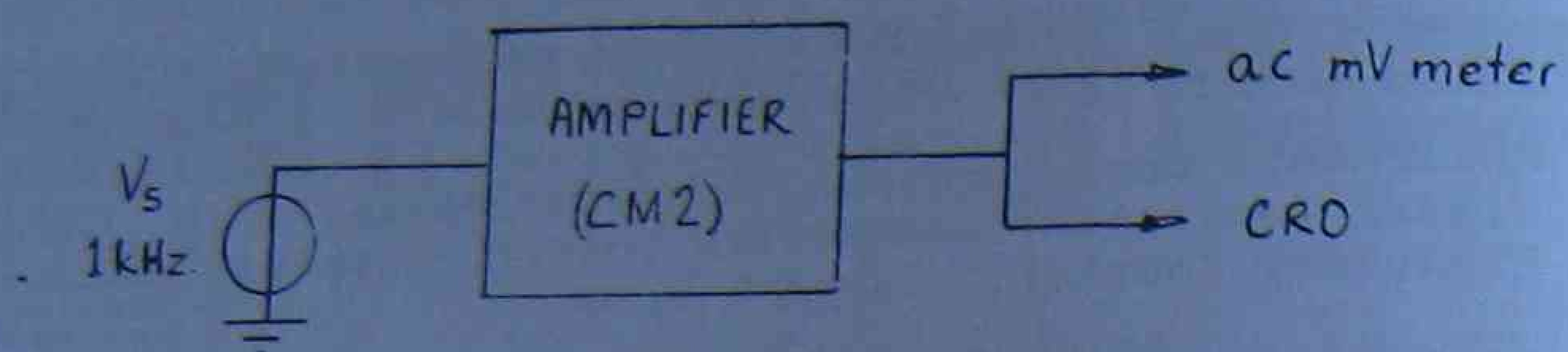
Objectives

At the end of this section you should be able to:

- ☐ use frequency domain diagrams to define both ideal and practical filters of the following types:
  - low-pass
  - high pass
  - band pass
  - band stop
- ☐ draw and identify block diagram symbols for ideal and practical filters
- ☐ for ideal and practical filters, use diagrams to explain what is meant by:
  - amplitude versus frequency response
  - phase versus frequency response
  - 3 dB bandwidth
  - insertion loss
  - roll-off slope
- ☐ describe the effects of both high and low-pass filters on audio signals
- ☐ describe the effect of low-pass filtering on video signals
- ☐ describe the effect of low-pass filtering on data signals
- ☐ give two reasons for using filters in communications equipment.

ESS Lab No 3

MEASURING AMPLIFIER SNR



AIM: To measure the Signal to Noise Ratio of an amplifier.

EQUIPMENT: Comms Trainer, CM2 Amplifier, mV meter, CRO, Decade Resistance Box, Signal Generator. 2×BNC/BNC, 2×BNC/4mm, BNC T piece (BNC/4mm adaptor)

PROCEDURE: 1) Set the amplifier volume control to maximum and leave it set in this position for the remainder of the lab. Switch the amplifier output to the internal load.

2) Set Vs to produce maximum undistorted output from the amplifier, using the CRO to monitor the output. Measure output level with the mV meter.

Max.O/P = ...V.rms = ... dBV

3) Place a decade box in series with the amplifier's input and adjust it to halve the amplifier's output. The setting on the decade box will equal the amplifier's input resistance, Ri.

Ri = ...

) Connect a resistance equal to Ri across the amplifier's input terminals and measure the amplifier's output with the mV meter. Observe the output on the CRO.

Noise O/P = ...V.rms = ... dBV

) Calculate the amplifier's SNR.

SNR = ...dB.

Repeat 2), 3) and 5) above, using the dB scale on the mV meter. Ask Teacher if not familar with this method.

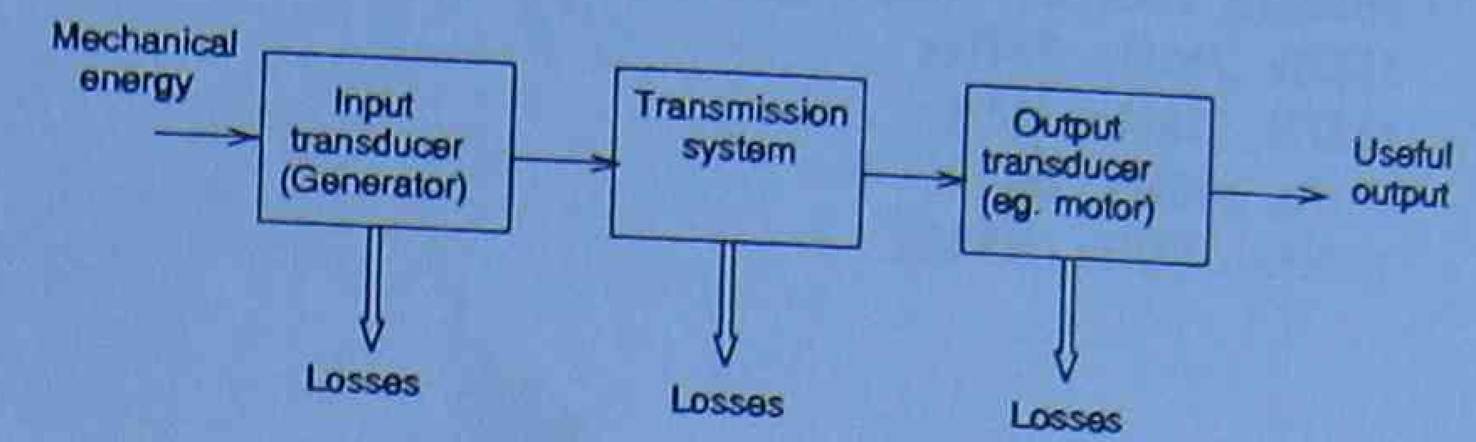
SNR = ...dB.

NCLUSIONS: What errors are involved in the SNR measurement as rried out above? Are any of these significant? Explain.

REVIEW QUESTIONS

Section 1

1.



2.
- In an electrical power transfer system, efficiency is the prime consideration. In an information transfer system, waveform integrity and signal-to-noise ratio are the prime considerations.
- In an electrical power transfer system, power is provided at the source. In an information transfer system, power is provide at various points.

3.
- The laser tracking system in a compact disk player
- an agc system in a radio receiver
- a crystal oven for a high stability oscillator
- The pressure controller in a steel rolling mill

Note that a remote controller for a model aeroplane transmits but does not receive. Therefore there can be no feedback to the controller itself, though there would be feedback in the plane.

4. The output voltage is sampled by a resistive divider. The sample is compared with the source and the difference (error signal) is amplified.

5.
- Open loop control: The motor speed will greatly depend on the motor load and supply voltage, as well as on the controller setting.
- Closed loop control: The effect of load and supply voltage will be greatly reduced by the feedback.

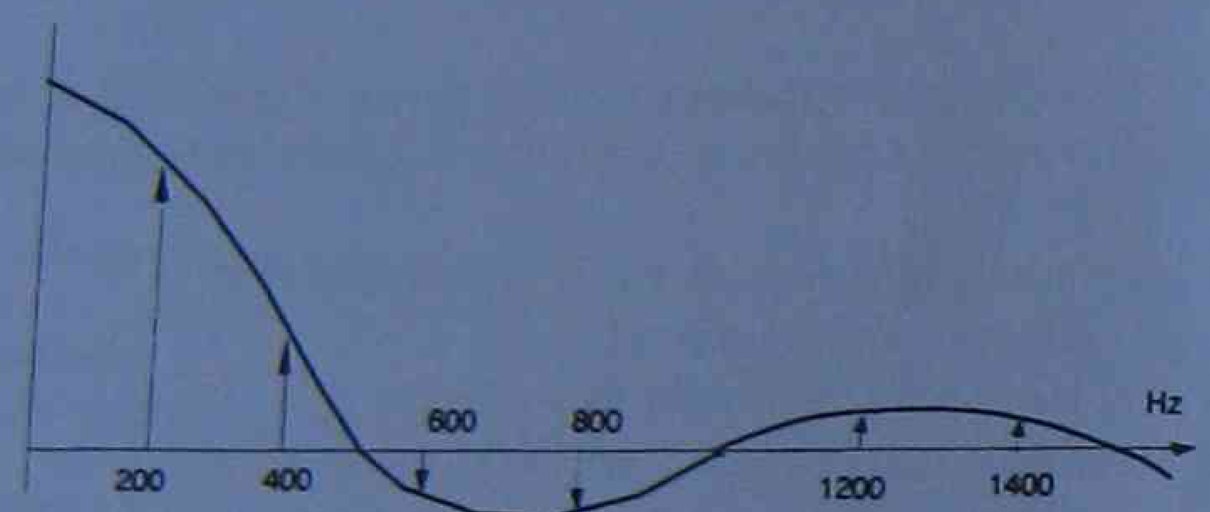
Section 2

1. (a) 1kHz, 2kHz, 3kHz
   (b) 1kHz, 3kHz, 5kHz
   (c) 100kHz, 300kHz, 500kHz
   (d) 1MHz, 2MHz, 3MHz
   (e) 1MHz, 3MHz, 5MHz
   (f) 50Hz, 100Hz, 200Hz
   (g) 100Hz, 200Hz, 300Hz.

2. No

3. (a) 2V
   (b) 2:3 or 0.667

   (c)



   (d) 5th, 10th 15th etc.

4. (a) (i) The 5th harmonic falls at the first zero of the curve.
   (ii) The 50th harmonic falls at the first zero of the curve.
   (iii) The 500th harmonic falls at the first zero of the curve.
   (iv) The 5 millionth harmonic falls at the first zero of the curve.
   (v) The spectrum is a continuum.

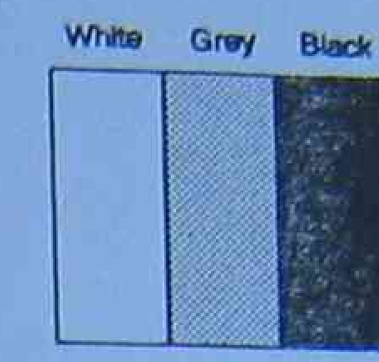
   (b) The spectral frequencies remain the same but more components fall in the first lobe of the sinx/x curve. That is, there are more harmonics with significant amplitude. Therefore, the signal requires more bandwidth.

   (c) The frequency corresponding to the first zero of the sinx/x curve remains unchanged. Therefore, the required bandwidth remains unchanged, even though there are more harmonics in the first lobe.

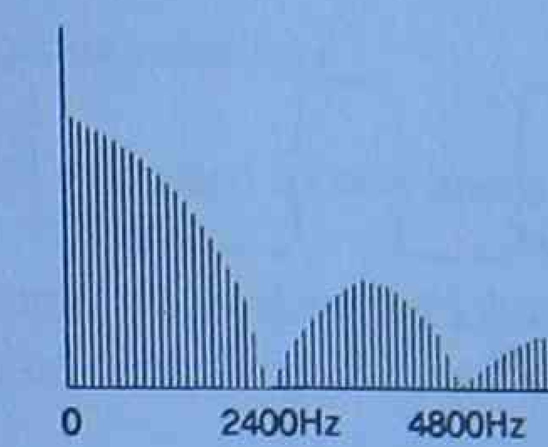
5. 30Hz - 15kHz (or perhaps 20kHz for young people with excellent hearing).

6. Noise with equal power per unit of bandwidth.

7.



8.



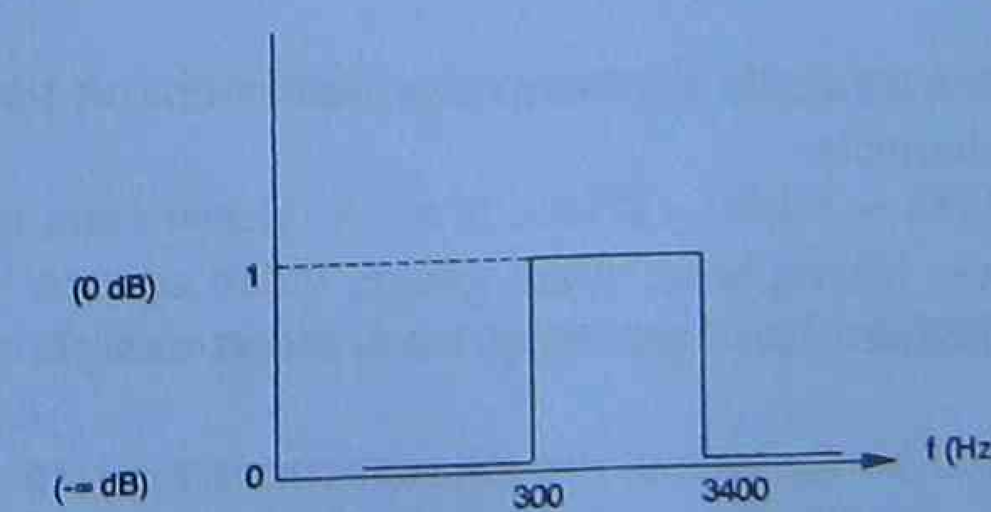
9. 2nd order:
   - 100Hz, 14kHz (Harmonics)
   - 6.95kHz, 7.05kHz (Intermod products)

   3rd order:
   - 150hz, 21kHz (Harmonics)
   - 6.9kHz, 7.1khz, 13.95kHz, 14.05kHz

10. 98, 99, 100, 102, 103 and 104MHz

Section 3

1.



2. 20°

3. Band stop filter. (Stop band = 100 - 200 kHz).

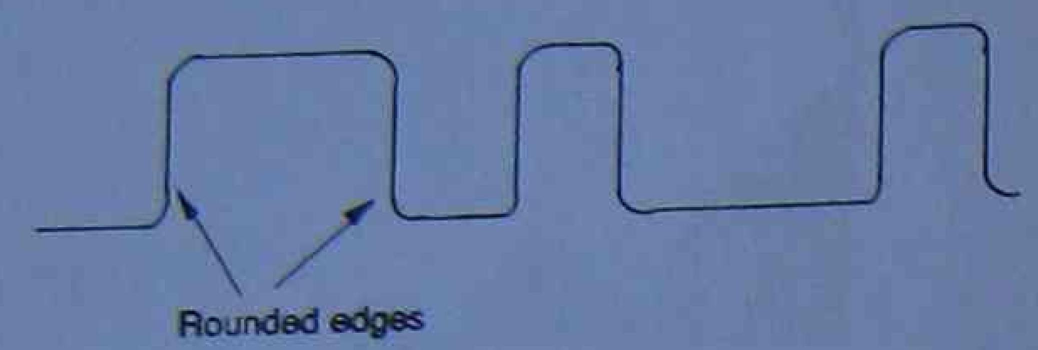
4. As the 3dB bandwidth.

5. (a) Band pass
   (b) 1dB
   (c) 4kHz (148 - 152kHz)

6. 6dB/octave.

7. ▪ loss of colour
   ▪ loss of definition

8.

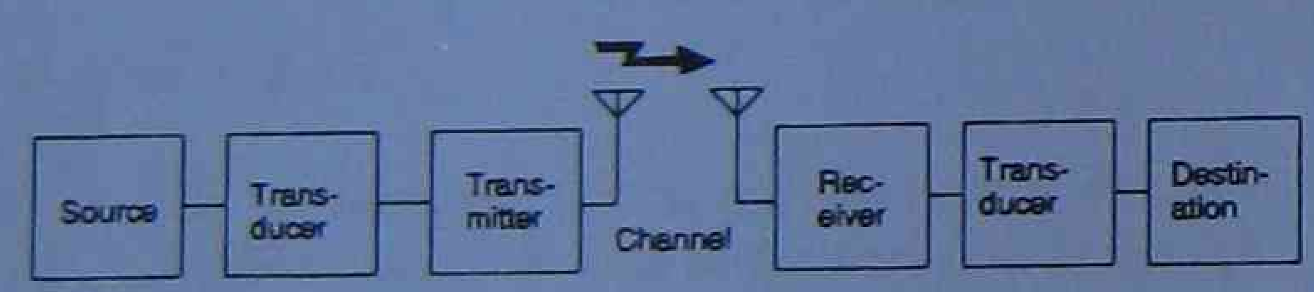


9. Both the phase/frequency response and the amplitude/frequency response affect the shape of the waveform. The shape of the waveform is very important for digital signals.

10. ▪ to select a particular signal
    ▪ to reject unwanted signals and noise

Section 4

1. (a)



   (b) A channel is a means of one-way communication. A bearer may carry a number of channels.

   (c) ▪ Source: Microphone, strain gauge, video camera
       ▪ Destination: Loudspeaker, printer, video monitor

2. No, the source would be the sound, the microphone is the transducer.

3. • To allow multiplexing
   • To allow use of a more suitable frequency range.

4. Any three of the following:
   ▪ noise
   ▪ crosstalk
   ▪ limited frequency response
   ▪ delay distortion
   ▪ non-linearities.

5. FDM and TDM

6. (a) VHF
   (b) UHF
   (c) UHF
   (d) HF

7. ▪ Super High Frequency
   ▪ 3GHz - 30GHz
   ▪ used for satellite communications

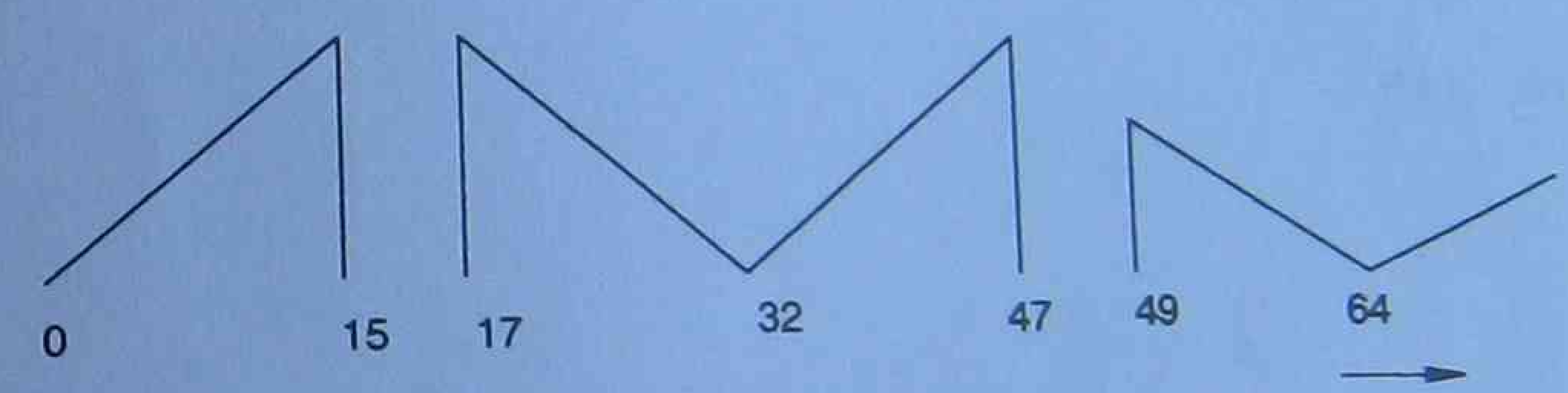
8. Its amplitude would be restricted to two levels.

9. A modem is a modulator/demodulator which converts a digital signal to a form suitable for transmission through an analog channel and reconverts to digital.

10. Codec.

Section 5

1. (a)



   (b) With a 15kHz LPF.

   (c) Yes, provided fairly sharp cut-off filters are used.

   (d) An alias would occur at 32kHz - 19kHz = 13kHz.

2. Analog to digital conversion (quantisation and encoding) and parallel to serial conversion.

3. (a) 256
   (b) 49.8dB

4. ▪ Serial to parallel conversion
   ▪ Digital to analog conversion
   ▪ Low-pass filtering

5. (a) 768 kbps
   (b) 384kHz

Section 6

1. Motor brushes, relays (virtually anything electrical!).
2. Lightning and cosmic radiation.
3. Thermal, shot and flicker.
4. To reduce shot noise. (Noise produced in the first stage of an amplifier is amplified by all the other stages.)
5. To minimise flicker noise.
6. No, it has more power at lower frequencies whereas white noise has uniform power over the spectrum.
7. White noise.
8. Impulse noise.
9. (a) 182nV.
   (b) 81dB.
10. 1uW.

wk 5+6

OPEN TRAINING & EDUCATION NETWORK TAFE

2832B Electrical Computations

Unit 4

FOURIER METHOD OF WAVEFORM ANALYSIS

CONTENTS

FOURIER METHOD OF WAVEFORM ANALYSIS

The response of linear networks to a.c. signals has been considered in other subjects within the Electrical Engineering Course. However, the analysis has been restricted to sinusoidal waveforms whereas non-sinusoidal waveforms do not have the same response characteristics.

In the consideration of the response of linear circuits to non-sinusoidal waveforms it is often convenient to express the waveform as a sum of sinusoidal functions. This method is referred to as the Fourier method of analysis.

4.1 TRIGONOMETRIC FOURIER SERIES

A periodic waveform can be expressed as a sum of pure sine waves of different frequencies and amplitudes. The component waveform with the same period as the waveform under analysis is referred to as the fundamental, the remaining components are referred to as harmonics and have frequencies which are integral multiples of the fundamental frequency.

A voltage waveform may be expressed as:

$$v(t) = V_{d.c.} + V_1 \sin(wt + \theta_1) + V_2 \sin(2wt + \theta_2) + V_3 \sin(3wt + \theta_3) + V_4 \sin(4wt + \theta_4) + \ldots \quad (1)$$

where:

$V_{d.c.}$ = d.c. component

$V_1 \sin(wt + \theta_1)$ = fundamental

$V_2 \sin(2wt + \theta_2)$ = second harmonic

$V_3 \sin(3wt + \theta_3)$ = third harmonic

$V_4 \sin(4wt + \theta_4)$ = fourth harmonic etc.

Alternatively the waveform may be expressed as:

$$v(t) = V_{d.c.} + A_1 \cos(wt) + A_2 \cos(2wt) + A_3 \cos(3wt) + \ldots + B_1 \sin(wt) + B_2 \sin(2wt) + B_3 \sin(3wt) + \ldots \quad (2)$$

where:

$A_1 = V_1 \sin\theta_1$; $B_1 = V_1 \cos\theta_1$
$A_2 = V_2 \sin\theta_2$; $B_2 = V_2 \cos\theta_2$
$A_3 = V_3 \sin\theta_3$; $B_3 = V_3 \cos\theta_3$
etc.

since

$$V_1 \sin(wt + \theta_1) = V_1 \sin(wt)\cos\theta_1 + V_1 \cos(wt)\sin\theta_1$$
$$= V_1 \sin\theta_1 \cos(wt) + V_1 \cos\theta_1 \sin(wt)$$
$$= A_1 \cos(wt) + B_1 \sin(wt)$$

To determine the component values of the harmonics we shall use the expression in equation (2) for ease of analysis.

The amplitudes of the component waveforms are given by:

$$A_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos(n\theta)\,d\theta \quad \text{for } n = 1,2,3, \ldots$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin(n\theta)\,d\theta \quad \text{for } n = 1,2,3, \ldots$$

Note that $\theta = wt$

4.2 SYMMETRICAL WAVEFORMS

Calculation of the amplitudes of component waveforms can be made easier if the waveform is symmetrical.

For an even function, all terms in the Fourier series are cosine terms with a possible constant value.

A voltage waveform v(t) is even if v(t) = v(-t).

Examples of even functions are given in figure 4.1.

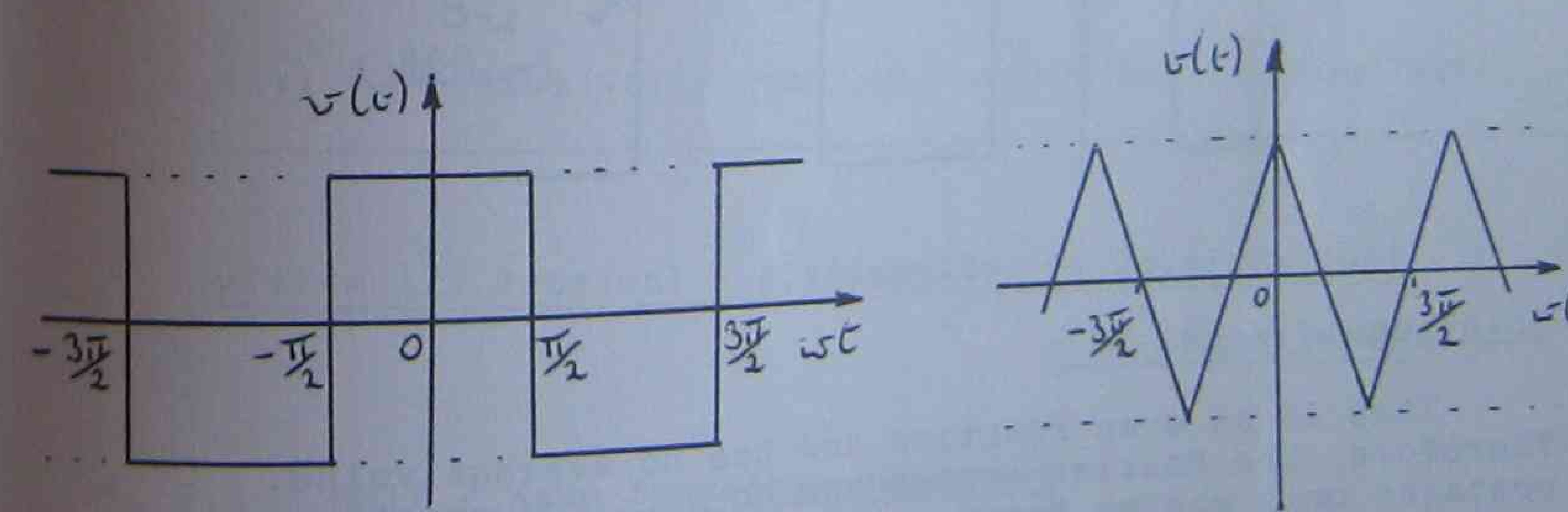


Figure 4.1

For an odd function, all terms in the Fourier series are sine terms.

A voltage waveform v(t) is odd if v(t) = -v(-t)

Examples of odd functions are given in figure 4.2.

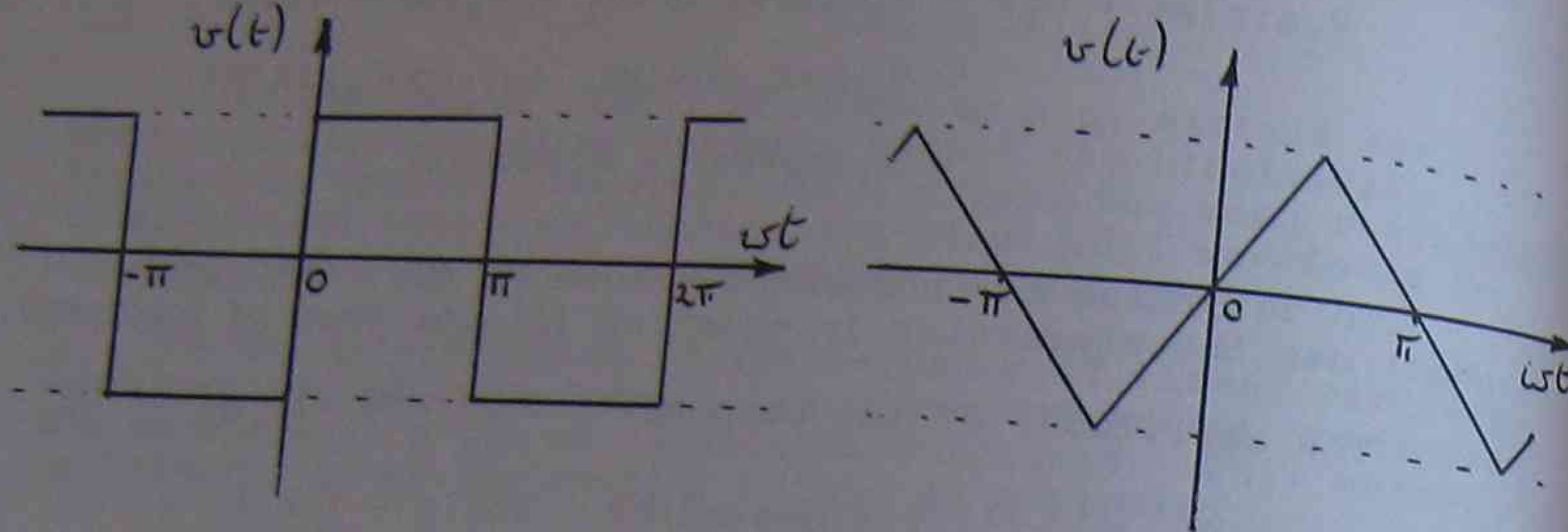


Figure 4.2

EXAMPLE 4.2.1

Find the first four terms in the trigonometric Fourier series for the square wave shown in figure 4.3.

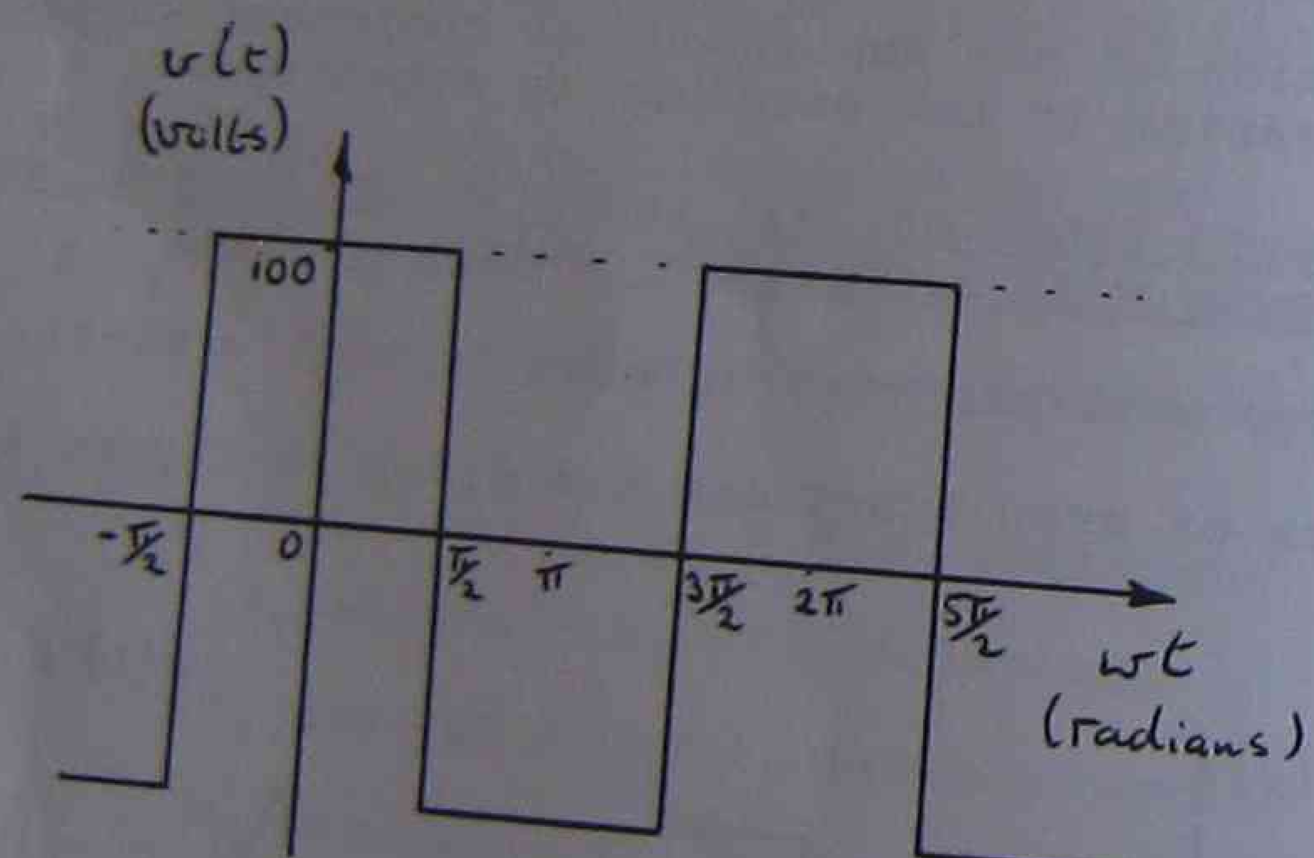


Figure 4.3

Solution:

This is an even function and has no average value. Therefore, the Fourier series has no d.c. component and contains only cosine terms.

$$v(t) = A_1\cos(wt) + A_2\cos(2wt) + A_3\cos(3wt) + \dots$$

where $A_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos(n\theta)\,d\theta$

$$= \frac{1}{\pi}\left\{\int_0^{\frac{\pi}{2}} 100\cos(n\theta)\,d\theta + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -100\cos(n\theta)\,d\theta + \int_{\frac{3\pi}{2}}^{2\pi} 100\cos(n\theta)\,d\theta\right\}$$

$$= \frac{100}{\pi}\left\{\left[\frac{\sin(n\theta)}{n}\right]_0^{\frac{\pi}{2}} + \left[\frac{-\sin(n\theta)}{n}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \left[\frac{\sin(n\theta)}{n}\right]_{\frac{3\pi}{2}}^{2\pi}\right\}$$

$$= \frac{100}{n\pi}\left\{\sin\left(\frac{n\pi}{2}\right) - \sin 0 - \sin\left(\frac{3n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right) + \sin(2n\pi) - \sin\left(\frac{3n\pi}{2}\right)\right\}$$

$$= \frac{200}{n\pi}\left\{\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{3n\pi}{2}\right)\right\}$$

n = 1 ; $A_1 = \frac{200}{\pi} \times 2 = \frac{400}{\pi}$

n = 2 ; $A_2 = 0$

n = 3 ; $A_3 = \frac{200}{\pi} \times \left(\frac{-2}{3}\right) = \frac{-400}{\pi} \times \frac{1}{3}$

n = 4 ; $A_4 = 0$

n = 5 ; $A_5 = \frac{200}{\pi} \times \frac{2}{5} = \frac{400}{\pi} \times \frac{1}{5}$

n = 6 ; $A_6 = 0$

n = 7 ; $A_7 = \frac{200}{\pi} \times \left(\frac{-2}{7}\right) = \frac{-400}{\pi} \times \frac{1}{7}$

$$v(t) = \frac{400}{\pi}\left(\cos(wt) - \frac{1}{3}\cos(3wt) + \frac{1}{5}\cos(5wt) - \frac{1}{7}\cos(7wt) + \dots\right)$$

$$v(t) = 127.3\cos(wt) - 42.44\cos(3wt) + 25.46\cos(5wt) - 18.19\cos(7wt) + \dots$$

4. 2.1 Half - Wave Symmetry: Harmonics

The nth harmonic(n is an integer) is defined as An Cosnθ, BnSin nθ.
If V(θ + π) = +V(θ) then the Fourier series for V(θ) will contain only even harmonics.
If V(θ+ π) = − V(θ) then the Fourier series for V(θ) will contain only odd harmonics.

$$B_n = \frac{50}{n\pi}\{2 - 2\cos(\frac{n\pi}{2})\}$$

$$= \frac{100}{n\pi}\{1 - \cos(\frac{n\pi}{2})\}$$

n = 1 ; $B_1 = \frac{100}{\pi}(1 - 0) = \frac{100}{\pi}$

n = 2 ; $B_2 = \frac{100}{2\pi}(1 + 1) = \frac{100}{\pi}$

n = 3 ; $B_3 = \frac{100}{3\pi}(1 - 0) = \frac{100}{\pi} \times \frac{1}{3}$

n = 4 ; $B_4 = \frac{100}{4\pi}(1 - 1) = 0$

n = 5 ; $B_5 = \frac{100}{5\pi}(1 - 0) = \frac{100}{\pi} \times \frac{1}{5}$

n = 6 ; $B_6 = \frac{100}{6\pi}(1 + 1) = \frac{100}{\pi} \times \frac{1}{3}$

$$v(t) = \frac{100}{\pi}\{\sin(wt) + \sin(2wt) + \frac{1}{3}\sin(3wt) + \frac{1}{5}\sin(5wt) + \frac{1}{3}\sin(6wt) + \ldots\}$$

$$v(t) = 31.83\sin(wt) + 31.83\sin(2wt) + 10.61\sin(3wt) + 6.37\sin(5wt) + 10.61\sin(6wt) + \ldots.$$

EXAMPLE 4.2.3

Determine the first four terms of the trigonometric Fourier series for the waveform shown in figure 4.5.

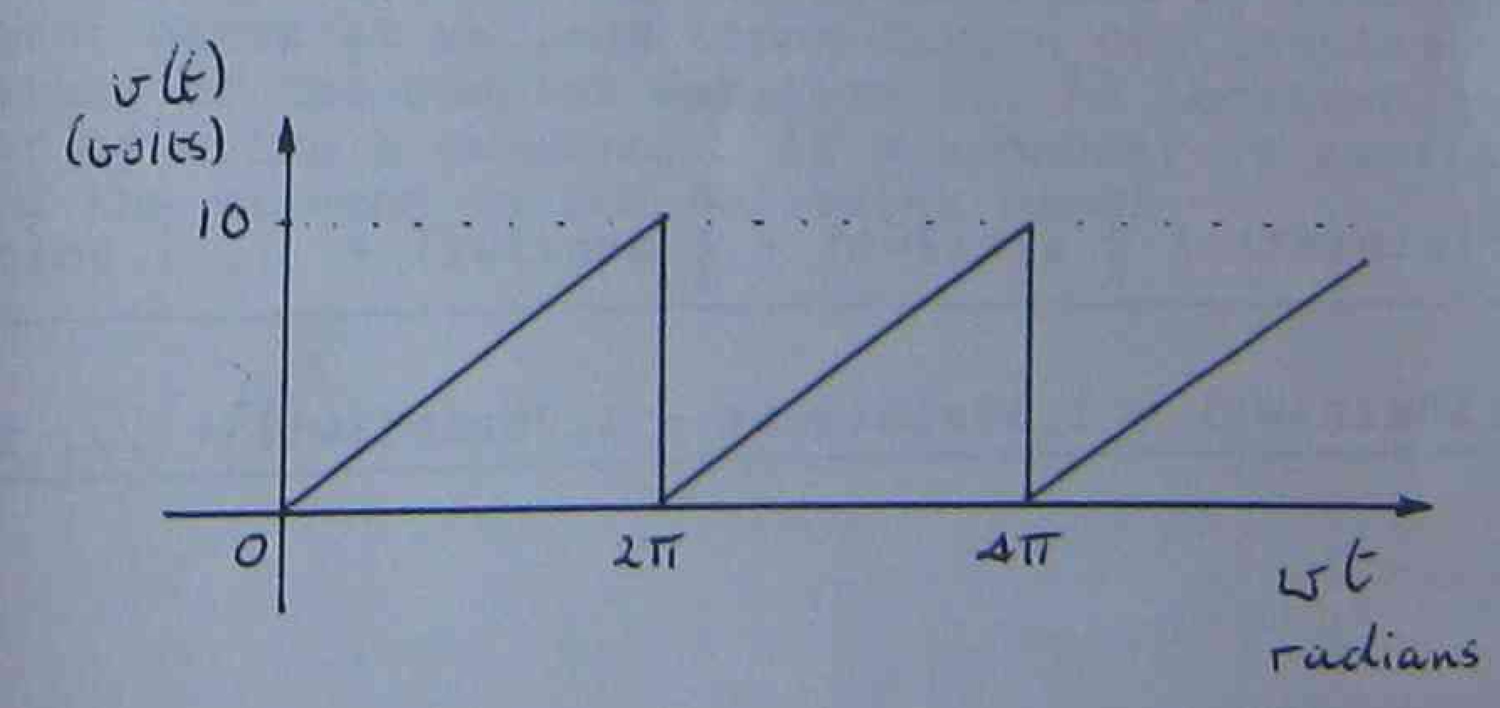


Figure 4.5

$$A_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos(n\theta)\,d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{10}{2\pi} wt \cos(n\theta)\,d\theta$$

$$= \frac{5}{\pi^2}\int_0^{2\pi}\theta\cos(n\theta)\,d\theta \quad \text{(Use integration by parts)} \quad \int u\,dv = uv - \int v\,du$$

$$= \frac{5}{\pi^2}\left\{\left[\frac{\sin(n\theta)}{n}\,\theta\right]_0^{2\pi} - \int_0^{2\pi}\frac{\sin(n\theta)}{n}\,d\theta\right\}$$

$$= \frac{5}{\pi^2}\left\{\frac{\sin(2n\pi)}{n} \times 2\pi - \frac{\sin 0}{n} \times 0 - \left[-\frac{\cos(n\theta)}{n^2}\right]_0^{2\pi}\right\}$$

$$= \frac{5}{\pi^2}\left\{\frac{0}{n} + \frac{\cos(2\pi n)}{n^2} - \frac{\cos 0}{n^2}\right\}$$

$$= 0 \text{ for all } n$$

There are no cosine terms in the series

$$V_{d.c.} = \frac{1}{2\pi}\int_0^{2\pi} v(t)\,d\theta = 5 \text{ volts} \quad \text{[i.e. average value]}$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin(n\theta)\,d\theta$$

$$= \frac{5}{\pi^2}\int_0^{2\pi}\theta\sin(n\theta)\,d\theta \quad \text{(Use integration by parts)}$$

$$= \frac{5}{\pi^2}\left\{\left[-\frac{\cos(n\theta)}{n}\,\theta\right]_0^{2\pi} - \int_0^{2\pi} -\frac{\cos(n\theta)}{n}\,d\theta\right\}$$

$$= \frac{5}{\pi^2}\left\{-\frac{\cos(2n\pi)}{n} \times 2\pi + 0 + \left[\frac{\sin(n\theta)}{n^2}\right]_0^{2\pi}\right\}$$

$$= \frac{5}{\pi^2}\left\{-\frac{2\pi}{n} + \frac{\sin(2n\pi)}{n^2} - 0\right\}$$

$$= -\frac{5}{\pi^2}\,\frac{2\pi}{n} + 0$$

$$B_n = -\frac{10}{\pi n}$$

$$v(t) = 5 - \frac{10}{\pi}\left\{\sin(wt) + \frac{1}{2}\sin(2wt) + \frac{1}{3}\sin(3wt) + \ldots\right\} \text{ volts}$$

$$v(t) = 5 - 3.18\sin(wt) - 1.59\sin(2wt) - 1.06\sin(3wt) - \ldots \text{ volts}$$

4.3 R.M.S. VALUE OF A COMPLEX WAVEFORM

The r.m.s. value of a complex waveform expressed as

$$v(t) = V_{d.c.} + V_1\sin(wt + \theta_1) + V_2\sin(2wt + \theta_2) + V_3\sin(3wt + \theta_3) + V_4\sin(4wt + \theta_4) + \ldots$$

is given by the following expression:-

$$V_{r.m.s.} = \sqrt{V_{d.c.}^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2} + \frac{V_4^2}{2} + \ldots}$$

$$= \sqrt{\text{sum of the (r.m.s. values)}^2 \text{ of the component waveforms.}}$$

If the waveform is expressed in the form:

$$v(t) = V_{d.c.} + A_1\cos(wt) + A_2\cos(2wt) + A_3\cos(3wt) + \ldots + B_1\sin(wt) + B_2\sin(2wt) + B_3\sin(3wt) + \ldots$$

Then $V_{r.m.s.} = \sqrt{V_{d.c.}^2 + \frac{A_1^2}{2} + \frac{B_1^2}{2} + \frac{A_2^2}{2} + \frac{B_2^2}{2} + \ldots}$

Also $V_1^2 = A_1^2 + B_1^2$; $V_2^2 = A_2^2 + B_2^2$; etc.

thus again $V_{r.m.s.} = \sqrt{V_{d.c.}^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2} + \ldots}$

4.4 WAVEFORM SYNTHESIS

A complex waveform may be synthesised by adding the component parts at various times during one complete cycle. The values of the complex waveform can be obtained in tabular form or by using a computer. If a computer is available try running the program on the following page:-

```
10 PRINT "THIS PROGRAMME PLOTS THE CURVE FOR A COMPLEX WAVEFORM"
20 PRINT "CONTAINING A SECOND AND THIRD HARMONIC."
30 PRINT "THE VOLTAGE WAVEFORM HAS THE EQUATION:"
40 PRINT "       E = E1*SIN(WT+A1) + E2*SIN(WT+A2) + E3*SIN(WT+A3)"
50 PRINT "ENTER THE MAGNITUDE AND PHASE ANGLE OF THE FUNDAMENTAL"
60 PRINT "E1 IN VOLTS, A1 IN RADIANS"
62 INPUT E1,A1
63 PRINT "ENTER THE MAGNITUDE AND PHASE ANGLE OF THE SECOND HARMONIC"
64 INPUT E2,A2
65 PRINT "ENTER THE MAGNITUDE AND PHASE ANGLE OF THE THIRD HARMONIC"
66 INPUT E3,A3
70 PRINT TAB(35);"O"
80 PRINT "----------------------------------------------------------------------"
90 P1=ATN(1)*4
100 FOR T=0 TO 2*P1 STEP P1/10
110 E = E1*SIN(T+A1) + E2*SIN(2*T+A2) + E3*SIN(3*T+A3)
120 M = ABS(E1)+ABS(E2)+ABS(E3)
130 X = INT(35+E*35/M)
140 PRINT TAB(X);"*"
150 NEXT T
170 END
OK
```

```
THIS PROGRAMME PLOTS THE CURVE FOR A COMPLEX WAVEFORM
CONTAINING A SECOND AND THIRD HARMONIC.
THE VOLTAGE WAVEFORM HAS THE EQUATION:
       E = E1*SIN(WT+A1) + E2*SIN(WT+A2) + E3*SIN(WT+A3)
ENTER THE MAGNITUDE AND PHASE ANGLE OF THE FUNDAMENTAL
E1 IN VOLTS, A1 IN RADIANS
? 100,0
ENTER THE MAGNITUDE AND PHASE ANGLE OF THE SECOND HARMONIC
? 20,0.35
ENTER THE MAGNITUDE AND PHASE ANGLE OF THE THIRD HARMONIC
? -40,1.41
                                   O
----------------------------------------------------------------------
                       *
                                   *
                                              *
                                                     *
                                                      *
                                                  *
                                            *
                                   *
```

WORK TO BE FORWARDED FOR COMMENT

1. A waveform has a trigonometric Fourier series of

$$v(t) = 100(\sin(wt) - \frac{1}{9}\sin(3wt) + \frac{1}{25}\sin(5wt) \ldots) \text{ volts.}$$

Write a BASIC programme that will give a sketch of the waveform between the intervals of 0 to 4π.

2. Find the trigonometric Fourier series for each waveform shown in figure 4.6.

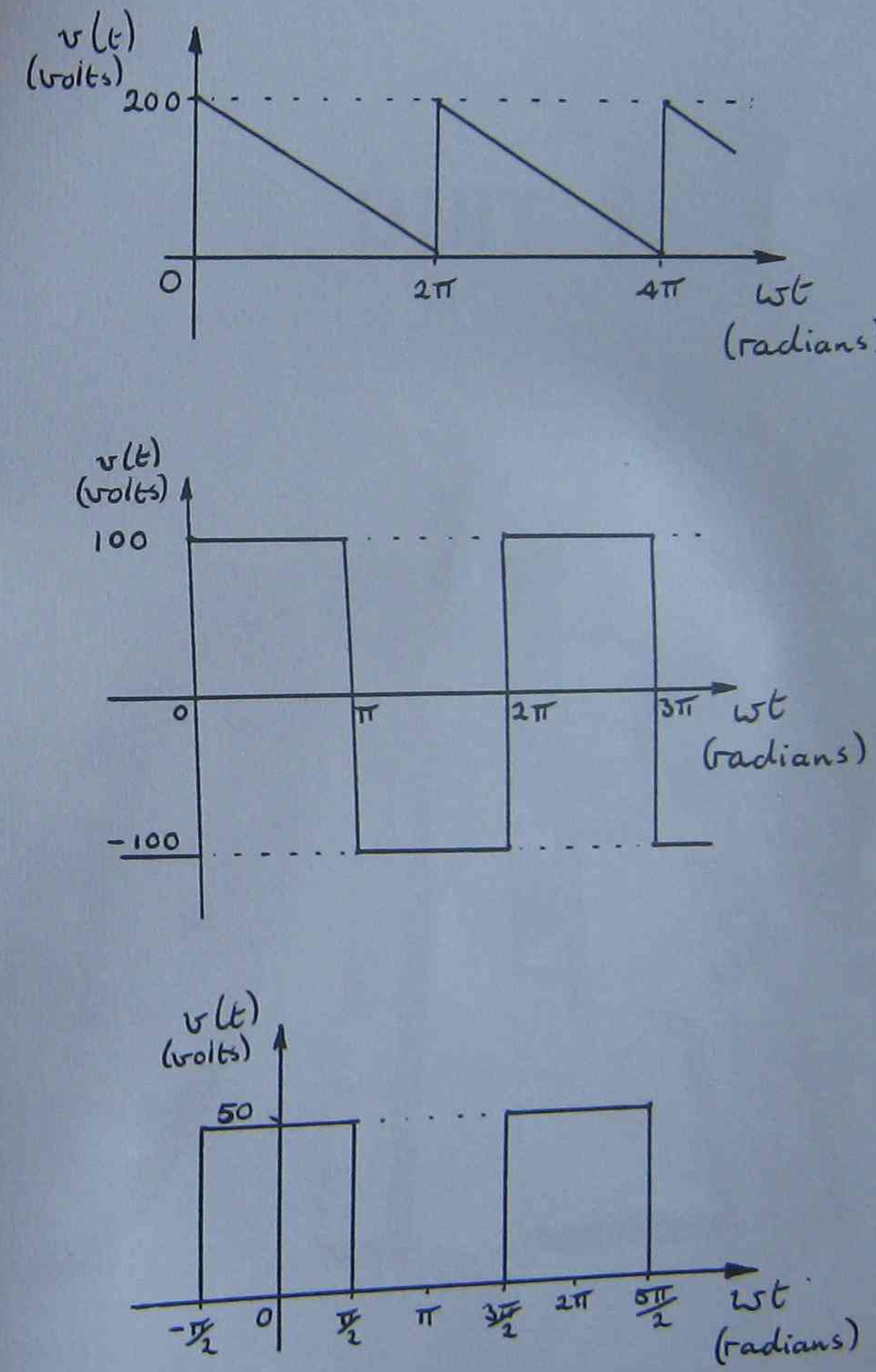


Figure 4.6

ANSWERS - UNIT 4

2 (i) $v(t) = 100 + \frac{200}{\pi}\left\{ \sin(wt) + \frac{1}{2}\sin(2wt) + \frac{1}{3}\sin(3wt) + \ldots \right\}$

(ii) $v(t) = \frac{400}{\pi}\left\{ \sin(wt) + \frac{1}{3}\sin(3wt) + \frac{1}{5}\sin(5wt) + \ldots \right\}$

(iii) $v(t) = 25 + \frac{100}{\pi}\left\{ \cos(wt) - \frac{1}{3}\cos(3wt) + \frac{1}{5}\cos(5wt) - \frac{1}{7}\cos(7wt) + \ldots \right\}$