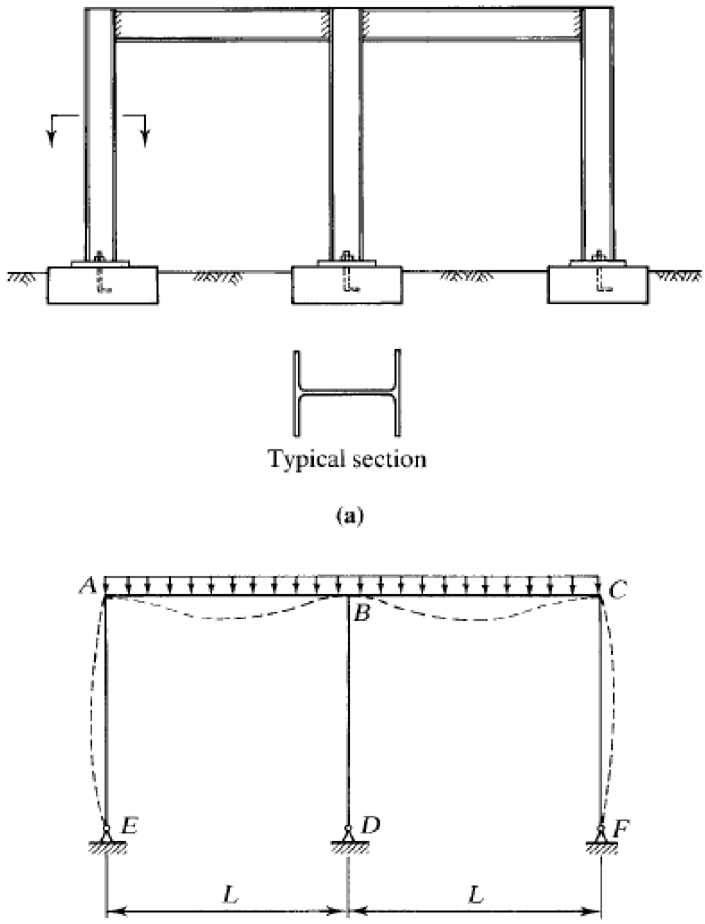
FIGURE 1.2



(b)

For the loading shown in Figure 1.2b, the frame will deform as indicated by the dashed line (drawn to a greatly exaggerated scale). The individual members of the frame can be classified according to the type of behavior represented by this deformed shape. The horizontal members *AB* and *BC* are subjected primarily to bending, or flexure, and are called *beams*. The vertical member *BD* is subjected to couples transferred from each beam, but for the symmetrical frame shown, they are equal and opposite, thereby canceling each other. Thus member *BD* is subjected only to axial compression arising from the vertical loads. In buildings, vertical compression members such as these are referred to as *columns*. The other two vertical members, *AE* and

(AISI, 2007).

1.5 STRUCTURAL STEEL

The earliest use of iron, the chief component of steel, was for small tools, in approximately 4000 B.C. (Murphy, 1957). This material was in the form of wrought iron, produced by heating ore in a charcoal fire. In the latter part of the eighteenth century and in the early nineteenth century, cast iron and wrought iron were used in various types of bridges. Steel, an alloy of primarily iron and carbon, with fewer impurities and less carbon than cast iron, was first used in heavy construction in the nineteenth century. With the advent of the Bessemer converter in 1855, steel began to displace wrought iron and cast iron in construction. In the United States, the first structural steel railroad bridge was the Eads bridge, constructed in 1874 in St. Louis, Missouri (Tall, 1964). In 1884, the first building with a steel frame was completed in Chicago.

The characteristics of steel that are of the most interest to structural engineers can be examined by plotting the results of a tensile test. If a test specimen is subjected to an axial load P, as shown in Figure 1.3a, the stress and strain can be computed as follows:

$$f = \frac{P}{A} \quad \text{and} \quad \varepsilon = \frac{\Delta L}{L}$$

where

f = axial tensile stress
A = cross-sectional area
ε = axial strain
L = length of specimen
ΔL = change in length

FIGURE 1.6

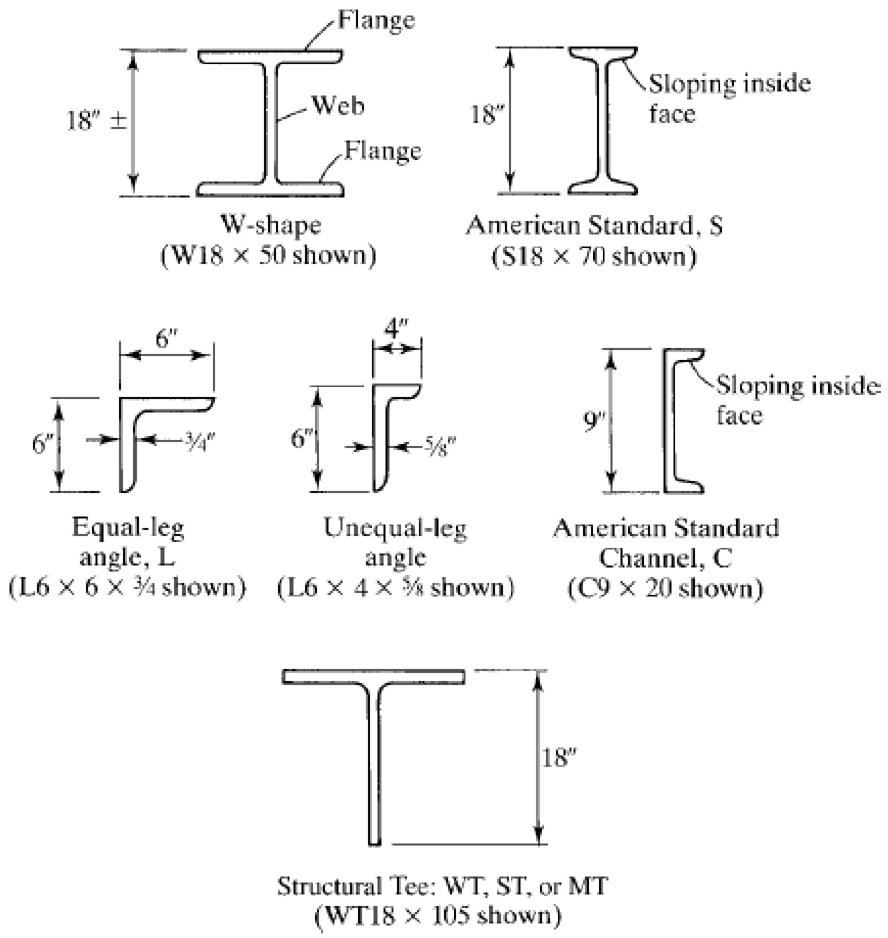


FIGURE 1.7

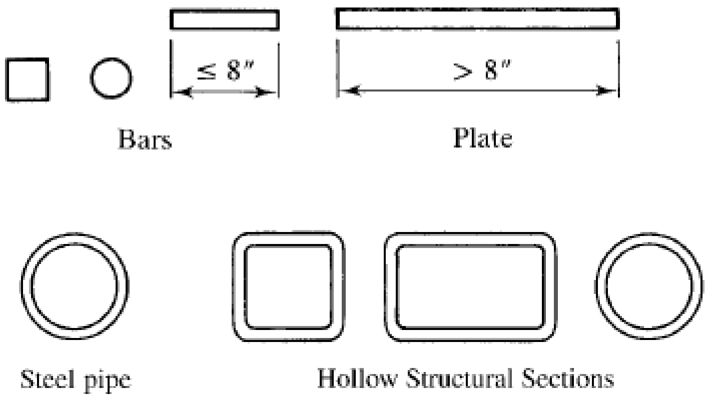
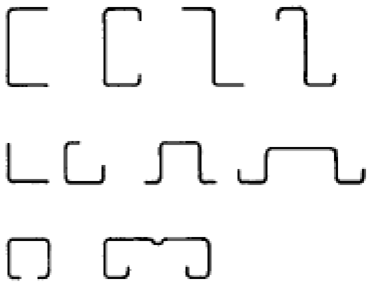


TABLE 1.2

Shape	Preferred Steel
Angles	A36
Plates	A36
S, M, C, MC	A36
HP	A572 Grade 50
W	A992
Pipe	A53 Grade B (only choice)
HSS	A500 Grade B (round) or C (rectangular)

FIGURE 1.9



2.1 DESIGN PHILOSOPHIES

As discussed earlier, the design of a structural member entails the selection of a cross section that will safely and economically resist the applied loads. Economy usually means minimum weight—that is, the minimum amount of steel. This amount corresponds to the cross section with the smallest weight per foot, which is the one with the smallest cross-sectional area. Although other considerations, such as ease of construction, may ultimately affect the choice of member size, the process begins with the selection of the lightest cross-sectional shape that will do the job. Having established this objective, the engineer must decide how to do it safely, which is where different approaches to design come into play. The fundamental requirement of structural design is that the required strength not exceed the available strength; that is,

Required strength ≤ available strength

In *allowable strength design* (ASD), a member is selected that has cross-sectional properties such as area and moment of inertia that are large enough to prevent the maximum applied axial force, shear, or bending moment from exceeding an allowable, or permissible, value. This allowable value is obtained by dividing the nominal, or theoretical, strength by a factor of safety. This can be expressed as

$$\text{Required strength} \le \text{allowable strength} \tag{2.1}$$

where

$$\text{Allowable strength} = \frac{\text{nominal strength}}{\text{safety factor}}$$

Strength can be an axial force strength (as in tension or compression members), a flexural strength (moment strength), or a shear strength.

If stresses are used instead of forces or moments, the relationship of Equation 2.1 becomes

$$\text{Maximum applied stress} \le \text{allowable stress} \tag{2.2}$$

This approach is called *allowable stress design.* The allowable stress will be in the elastic range of the material (see Figure 1.3). This approach to design is also called *elastic design* or *working stress design.* Working stresses are those resulting from the working loads, which are the applied loads. Working loads are also known as *service* loads.

Plastic design is based on a consideration of failure conditions rather than working load conditions. A member is selected by using the criterion that the structure will fail at a load substantially higher than the working load. Failure in this context means either collapse or extremely large deformations. The term *plastic* is used because, at failure, parts of the member will be subjected to very large strainsÑ large enough to put the member into the plastic range (see Figure 1.3b). When the entire cross section becomes plastic at enough locations, •plastic hingesŽwill form at those locations, creating a *collapse mechanism.* As the actual loads will be less than the failure loads by a factor of safety known as the *load factor,* members designed this way are not unsafe, despite being designed based on what happens at failure. This design procedure is roughly as follows.

1. Multiply the working loads (service loads) by the load factor to obtain the failure loads.
2. Determine the cross-sectional properties needed to resist failure under these loads. (A member with these properties is said to have sufficient strength and would be at the verge of failure when subjected to the factored loads.)
3. Select the lightest cross-sectional shape that has these properties.

Members designed by plastic theory would reach the point of failure under the factored loads but are safe under actual working loads.

Load and resistance factor design (LRFD) is similar to plastic design in that strength, or the failure condition, is considered. Load factors are applied to the service loads, and a member is selected that will have enough strength to resist the factored loads. In addition, the theoretical strength of the member is reduced by the application of a resistance factor. The criterion that must be satisfied in the selection of a member is

$$\text{Factored load} \leq \text{factored strength} \qquad (2.3)$$

In this expression, the factored load is actually the sum of all service loads to be resisted by the member, each multiplied by its own load factor. For example, dead loads will have load factors that are different from those for live loads. The factored strength is the theoretical strength multiplied by a resistance factor. Equation 2.3 can therefore be written as

$$\sum (\text{loads} \times \text{load factors}) \leq \text{resistance} \times \text{resistance factor} \qquad (2.4)$$

The factored load is a failure load greater than the total actual service load, so the load factors are usually greater than unity. However, the factored strength is a reduced, usable strength, and the resistance factor is usually less than unity. The factored loads are the loads that bring the structure or member to its limit. In terms of safety, this *limit state* can be fracture, yielding, or buckling, and the factored resistance is the useful strength of the member, reduced from the theoretical value by the resistance factor. The limit state can also be one of serviceability, such as a maximum acceptable deflection.

2.3 LOAD FACTORS, RESISTANCE FACTORS, AND LOAD COMBINATIONS FOR LRFD

Equation 2.4 can be written more precisely as

$$\Sigma \gamma_i Q_i \leq \phi R_n \tag{2.5}$$

where

Q_i = a load effect (a force or a moment)
γ_i = a load factor
R_n = the nominal resistance, or strength, of the component under consideration
ϕ = resistance factor

The factored resistance ϕR_n is called the *design strength*. The summation on the left side of Equation 2.5 is over the total number of load effects (including, but not limited to, dead load and live load), where each load effect can be associated with a different load factor. Not only can each load effect have a different load factor but also the value of the load factor for a particular load effect will depend on the combination of loads under consideration. Equation 2.5 can also be written in the form

$$R_u \leq \phi R_n \tag{2.6}$$

where

R_u = required strength = sum of factored load effects (forces or moments)

Section B2 of the AISC Specification says to use the load factors and load combinations prescribed by the governing building code. If the building code does not give them, then ASCE 7 (ASCE, 2010) should be used. The load factors and load combinations in this standard are based on extensive statistical studies and are prescribed by most building codes.

ASCE 7 presents the basic load combinations in the following form:

Combination 1: $1.4D$
Combination 2: $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
Combination 3: $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
Combination 4: $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$

Combination 5: $1.2D + 1.0E + L + 0.2S$
Combination 6: $0.9D + 1.0W$
Combination 7: $0.9D + 1.0E$

where

D = dead load
L = live load due to occupancy
L_r = roof live load
S = snow load
R = rain or ice load*
W = wind load
E = earthquake (seismic load)

In combinations 3, 4, and 5, the load factor on L can be reduced to 0.5 if L is no greater than 100 pounds per square foot, except for garages or places of public assembly. In combinations with wind or earthquake loads, you should use a direction that produces the worst effects.

The ASCE 7 basic load combinations are also given in Part 2 of the AISC *Steel Construction Manual* (AISC 2011a), which will be discussed in Section 2.6 of this chapter. They are presented in a slightly different form as follows:

Combination 1: $1.4D$
Combination 2: $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
Combination 3: $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (0.5L \text{ or } 0.5W)$
Combination 4: $1.2D + 1.0W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R)$
Combination 5: $1.2D \pm 1.0E + 0.5L + 0.2S$
Combinations 6 and 7: $0.9D \pm (1.0W \text{ or } 1.0E)$

Here, the load factor on L in combinations 3, 4, and 5 is given as 0.5, which should be increased to 1.0 if L is greater than 100 pounds per square foot or for garages or places of public assembly. ASCE 7 combinations 6 and 7 arise from the expression shown by considering combination 6 to use $1.0W$ and combination 7 to use $1.0E$. In other words,

Combination 6: $0.9D \pm 1.0W$
Combination 7: $0.9D \pm 1.0E$

Combinations 6 and 7 account for the possibility of dead load and wind or earthquake load counteracting each other; for example, the net load effect could be the difference between $0.9D$ and $1.0W$ or between $0.9D$ and $1.0E$. (Wind or earthquake load may tend to overturn a structure, but the dead load will have a stabilizing effect.)

As previously mentioned, the load factor for a particular load effect is not the same in all load combinations. For example, in combination 2 the load factor for the live load L is 1.6, whereas in combination 3, it is 0.5. The reason is that the live load

*This load does not include *ponding*, a phenomenon that we discuss in Chapter 5.

2.4 SAFETY FACTORS AND LOAD COMBINATIONS FOR ASD

For allowable strength design, the relationship between loads and strength (Equation 2.1) can be expressed as

$$R_a \le \frac{R_n}{\Omega} \tag{2.7}$$

where

R_a = required strength
R_n = nominal strength (same as for LRFD)
Ω = safety factor
R_n/Ω = allowable strength

The required strength R_a is the sum of the service loads or load effects. As with LRFD, specific combinations of loads must be considered. Load combinations for ASD are also given in ASCE 7. These combinations, as presented in the AISC *Steel Construction Manual* (AISC 2011a), are

Combination 1:	D
Combination 2:	$D + L$
Combination 3:	$D + (L_r \text{ or } S \text{ or } R)$
Combination 4:	$D + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
Combination 5:	$D \pm (0.6W \text{ or } 0.7E)$
Combination 6a:	$D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$
Combination 6b:	$D + 0.75L \pm 0.75(0.7E) + 0.75S$
Combinations 7 and 8:	$0.6D \pm (0.6W \text{ or } 0.7E)$

The factors shown in these combinations are not load factors. The 0.75 factor in some of the combinations accounts for the unlikelihood that all loads in the combination will be at their lifetime maximum values simultaneously. The 0.7 factor applied to the seismic load effect E is used because ASCE 7 uses a strength approach (i.e., LRFD) for computing seismic loads, and the factor is an attempt to equalize the effect for ASD.

Corresponding to the two most common values of resistance factors in LRFD are the following values of the safety factor Ω in ASD: For limit states involving yielding or compression buckling, $\Omega = 1.67$.* For limit states involving rupture, $\Omega = 2.00$. The relationship between resistance factors and safety factors is given by

$$\Omega = \frac{1.5}{\phi} \tag{2.8}$$

For reasons that will be discussed later, this relationship will produce similar designs for LRFD and ASD, under certain loading conditions.

If both sides of Equation 2.7 are divided by area (in the case of axial load) or section modulus (in the case of bending moment), then the relationship becomes

$$f \leq F$$

where

f = applied stress
F = allowable stress

This formulation is called *allowable stress design*.

EXAMPLE 2.1

A column (compression member) in the upper story of a building is subject to the following loads:

Dead load:	109 kips compression
Floor live load:	46 kips compression
Roof live load:	19 kips compression
Snow:	20 kips compression

a. Determine the controlling load combination for LRFD and the corresponding factored load.
b. If the resistance factor ϕ is 0.90, what is the required *nominal* strength?
c. Determine the controlling load combination for ASD and the corresponding required service load strength.
d. If the safety factor Ω is 1.67, what is the required nominal strength based on the required service load strength?

SOLUTION

Even though a load may not be acting directly on a member, it can still cause a load effect in the member. This is true of both snow and roof live load in this example. Although this building is subjected to wind, the resulting forces on the structure are resisted by members other than this particular column.

a. The controlling load combination is the one that produces the largest factored load. We evaluate each expression that involves dead load, D; live load resulting from occupancy, L; roof live load, L_r; and snow, S.

*The value of Ω is actually $1\frac{2}{3} = 5/3$ but has been rounded to 1.67 in the AISC specification.

Combination 1:	$1.4D = 1.4(109) = 152.6$ kips
Combination 2:	$1.2D + 1.6L + 0.5(L_r$ or S or $R)$. Because S is larger than L_r and $R = 0$, we need to evaluate this combination only once, using S. $1.2D + 1.6L + 0.5S = 1.2(109) + 1.6(46) + 0.5(20) = 214.4$ kips
Combination 3:	$1.2D + 1.6(L_r$ or S or $R) + (0.5L$ or $0.5W)$. In this combination, we use S instead of L_r, and both R and W are zero. $1.2D + 1.6S + 0.5L = 1.2(109) + 1.6(20) + 0.5(46) = 185.8$ kips
Combination 4:	$1.2D + 1.0W + 0.5L + 0.5(L_r$ or S or $R)$. This expression reduces to $1.2D + 0.5L + 0.5S$, and by inspection, we can see that it produces a smaller result than combination 3.
Combination 5:	$1.2D \pm 1.0E + 0.5L + 0.2S$. As $E = 0$, this expression reduces to $1.2D + 0.5L + 0.2S$, which produces a smaller result than combination 4.
Combinations 6 and 7:	$0.9D \pm (1.0W$ or $1.0E)$. These combinations do not apply in this example, because there are no wind or earthquake loads to counteract the dead load.

ANSWER Combination 2 controls, and the factored load is 214.4 kips.

b. If the factored load obtained in part (a) is substituted into the fundamental LRFD relationship, Equation 2.6, we obtain

$$R_u \leq \phi R_n$$
$$214.4 \leq 0.90R_n$$
$$R_n \geq 238 \text{ kips}$$

ANSWER The required nominal strength is 238 kips.

c. As with the combinations for LRFD, we will evaluate the expressions involving D, L, L_r, and S for ASD.

Combination 1:	$D = 109$ kips. (Obviously this case will never control when live load is present.)
Combination 2:	$D + L = 109 + 46 = 155$ kips
Combination 3:	$D + (L_r$ or S or $R)$. Since S is larger than L_r, and $R = 0$, this combination reduces to $D + S = 109 + 20 = 129$ kips
Combination 4:	$D + 0.75L + 0.75(L_r$ or S or $R)$. This expression reduces to $D + 0.75L + 0.75S = 109 + 0.75(46) + 0.75(20) = 158.5$ kips
Combination 5:	$D \pm (0.6W$ or $0.7E)$. Because W and E are zero, this expression reduces to combination 1.

Combination 6a:	$D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$. Because W and E are zero, this expression reduces to combination 4.
Combination 6b:	$D + 0.75L \pm 0.75(0.7E) + 0.75S$. This combination also gives the same result as combination 4.
Combinations 7 and 8:	$0.6D \pm (0.6W \text{ or } 0.7E)$. These combinations do not apply in this example, because there are no wind or earthquake loads to counteract the dead load.

ANSWER Combination 4 controls, and the required service load strength is 158.5 kips.

d. From the ASD relationship, Equation 2.7,

$$R_a \le \frac{R_n}{\Omega}$$

$$158.5 \le \frac{R_n}{1.67}$$

$$R_n \ge 265 \text{ kips}$$

ANSWER The required nominal strength is 265 kips.

Example 2.1 illustrates that the controlling load combination for LRFD may not control for ASD.

When LRFD was introduced into the AISC Specification in 1986, the load factors were determined in such a way as to give the same results for LRFD and ASD when the loads consisted of dead load and a live load equal to three times the dead load. The resulting relationship between the resistance factor ϕ and the safety factor Ω, as expressed in Equation 2.8, can be derived as follows. Let R_n from Equations 2.6 and 2.7 be the same when $L = 3D$. That is,

$$\frac{R_u}{\phi} = R_a\Omega$$

$$\frac{1.2D + 1.6L}{\phi} = (D + L)\Omega$$

or

$$\frac{1.2D + 1.6(3D)}{\phi} = (D + 3D)\Omega$$

$$\Omega = \frac{1.5}{\phi}$$

CHAPTER 3

Tension Members

3.1 INTRODUCTION

Tension members are structural elements that are subjected to axial tensile forces. They are used in various types of structures and include truss members, bracing for buildings and bridges, cables in suspended roof systems, and cables in suspension and cable-stayed bridges. Any cross-sectional configuration may be used, because for any given material, the only determinant of the strength of a tension member is the cross-sectional area. Circular rods and rolled angle shapes are frequently used. Built-up shapes, either from plates, rolled shapes, or a combination of plates and rolled shapes, are sometimes used when large loads must be resisted. The most common built-up configuration is probably the double-angle section, shown in Figure 3.1, along with other typical cross sections. Because the use of this section is so widespread, tables of properties of various combinations of angles are included in the AISC *Steel Construction Manual*.

The stress in an axially loaded tension member is given by

$$f = \frac{P}{A}$$

where P is the magnitude of the load and A is the cross-sectional area (the area normal to the load). The stress as given by this equation is exact, provided that the cross section under consideration is not adjacent to the point of application of the load, where the distribution of stress is not uniform.

If the cross-sectional area of a tension member varies along its length, the stress is a function of the particular section under consideration. The presence of holes in a member will influence the stress at a cross section through the hole or holes. At these locations, the cross-sectional area will be reduced by an amount equal to the area removed by the holes. Tension members are frequently connected at their ends with bolts, as illustrated in Figure 3.2. The tension member shown, a $\frac{1}{2} \times 8$ plate, is connected to a *gusset plate*, which is a connection element whose purpose is to transfer the load from the member to a support or to another member. The area of the bar at section a–a is $(\frac{1}{2})(8) = 4 \text{ in.}^2$, but the area at section b–b is only $4 - (2)(\frac{1}{2})(\frac{7}{8}) = 3.13 \text{ in.}^2$

FIGURE 3.1

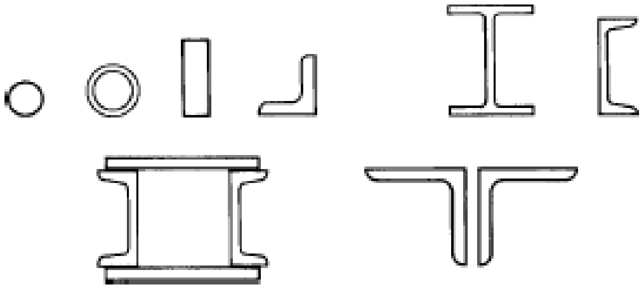
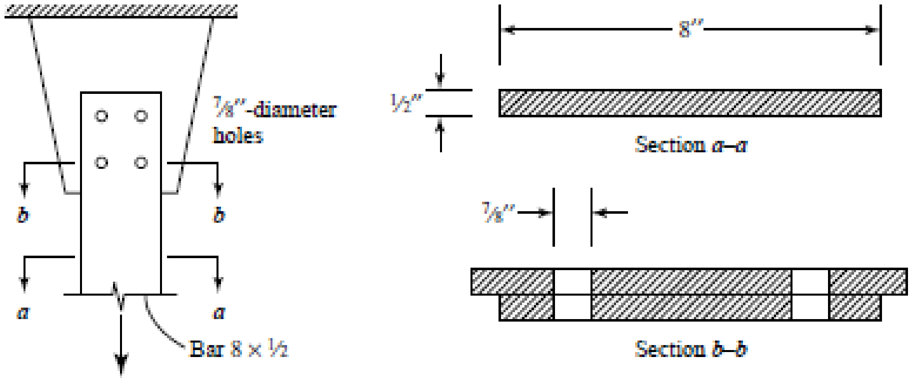


FIGURE 3.2



and will be more highly stressed. This reduced area is referred to as the *net area,* or *net section*, and the unreduced area is the *gross area*.

The typical design problem is to select a member with sufficient cross-sectional area to resist the loads. A closely related problem is that of analysis, or review, of a given member, where in the strength is computed and compared with the load. In general, analysis is a direct procedure, but design is an iterative process and may require some trial and error.

Tension members are covered in Chapter D of the Specification. Requirements that are common with other types of members are covered in Chapter B, "Design Requirements."

3.2 TENSILE STRENGTH

A tension member can fail by reaching one of two limit states: excessive deformation or fracture. To prevent excessive deformation, initiated by yielding, the load on the gross section must be small enough that the stress on the gross section is less than the yield stress F_y. To prevent fracture, the stress on the net section must be less than the tensile strength F_u. In each case, the stress P/A must be less than a limiting stress F or

$$\frac{P}{A} < F$$

Thus, the load P must be less than FA, or

$$P < FA$$

The *nominal* strength in yielding is

$$P_n = F_y A_g$$

and the nominal strength in fracture is

$$P_n = F_u A_e$$

where A_e is the *effective* net area, which may be equal to either the net area or, in some cases, a smaller area. We discuss effective net area in Section 3.3.

Although yielding will first occur on the net cross section, the deformation within the length of the connection will generally be smaller than the deformation in the remainder of the tension member. The reason is that the net section exists over a relatively small length of the member, and the total elongation is a product of the length and the strain (a function of the stress). Most of the member will have an unreduced cross section, so attainment of the yield stress on the gross area will result in larger total elongation. It is this larger deformation, not the first yield, that is the limit state.

LRFD: In load and resistance factor design, the factored tensile load is compared to the design strength. The design strength is the resistance factor times the nominal strength. Equation 2.6,

$$R_u = \phi R_n$$

can be written for tension members as

$$P_u \le \phi_t P_n$$

where P_u is the governing combination of factored loads. The resistance factor ϕ_t is smaller for fracture than for yielding, reflecting the more serious nature of fracture.

For yielding, $\phi_t = 0.90$

For fracture, $\phi_t = 0.75$

Because there are two limit states, both of the following conditions must be satisfied:

$$P_u \le 0.90 F_y A_g$$
$$P_u \le 0.75 F_u A_e$$

The smaller of these is the design strength of the member.

ASD: In allowable strength design, the total service load is compared to the allowable strength (allowable load):

$$P_a \le \frac{P_n}{\Omega_t}$$

where P_a is the required strength (applied load), and P_n/Ω_t is the allowable strength. The subscript "a" indicates that the required strength is for "allowable strength design," but you can think of it as standing for "applied" load.

For yielding of the gross section, the safety factor Ω_t is 1.67, and the allowable load is

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{1.67} = 0.6 F_y A_g$$

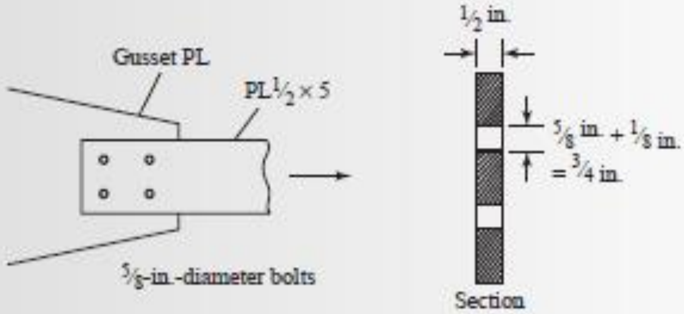
EXAMPLE 3.1

A ½ × 5 plate of A36 steel is used as a tension member. It is connected to a gusset plate with four ⅝-inch-diameter bolts as shown in Figure 3.3. Assume that the effective net area A_e equals the actual net area A_n (we cover computation of effective net area in Section 3.3).

a. What is the design strength for LRFD?

b. What is the allowable strength for ASD?

FIGURE 3.3



SOLUTION

For yielding of the gross section,

$$A_g = 5(1/2) = 2.5 \text{ in.}^2$$

and the nominal strength is

$$P_n = F_y A_g = 36(2.5) = 90.0 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 2.5 - (\tfrac{1}{2})(\tfrac{3}{4}) \times 2 \text{ holes}$$
$$= 2.5 - 0.75 = 1.75 \text{ in.}^2$$

$A_e = A_n = 1.75 \text{ in.}^2$ (This is true for this example, but A_e does not always equal A_n.)

The nominal strength is

$$P_n = F_u A_e = 58(1.75) = 101.5 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81.0 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(101.5) = 76.1 \text{ kips}$$

ANSWER The design strength for LRFD is the smaller value: $\phi_t P_n = 76.1$ kips.

b. The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{101.5}{2.00} = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Alternative Solution Using Allowable Stress: For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 21.6(2.5) = 54.0 \text{ kips}$$

(The slight difference between this value and the one based on allowable strength is because the value of Ω in the allowable strength approach has been rounded from 5/3 to 1.67; the value based on the allowable stress is the more accurate one.)
For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 29.0(1.75) = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

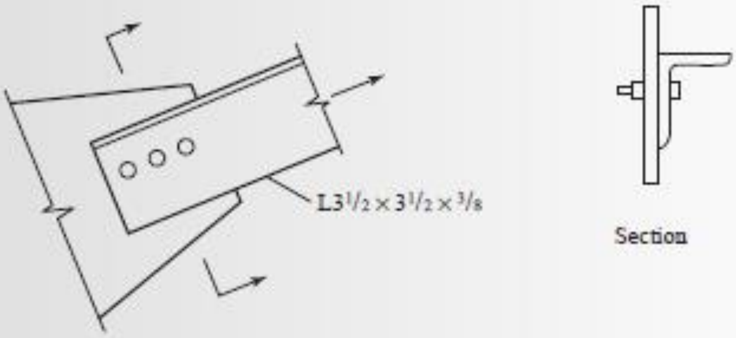
EXAMPLE 3.2

A single-angle tension member, an L3½ × 3½ × ⅜, is connected to a gusset plate with ⅞-inch-diameter bolts as shown in Figure 3.4. A36 steel is used. The service loads are 35 kips dead load and 15 kips live load. Investigate this member for compliance with the AISC Specification. Assume that the effective net area is 85% of the computed net area.

a. Use LRFD.

b. Use ASD.

FIGURE 3.4



SOLUTION

First, compute the nominal strengths.

Gross section:

$A_g = 2.50 \text{ in.}^2$ (from Part 1 of the *Manual*)

$P_n = F_y A_g = 36(2.50) = 90 \text{ kips}$

Net section:

$$A_n = 2.50 - \left(\frac{3}{8}\right)\left(\frac{7}{8}+\frac{1}{8}\right) = 2.125 \text{ in.}^2$$

$A_e = 0.85A_n = 0.85(2.125) = 1.806 \text{ in.}^2$ (in *this* example)

$P_n = F_u A_e = 58(1.806) = 104.7 \text{ kips}$

a. The design strength based on yielding is

$\phi_t P_n = 0.90(90) = 81 \text{ kips}$

The design strength based on fracture is

$\phi_t P_n = 0.75(104.7) = 78.5 \text{ kips}$

The design strength is the smaller value: $\phi_t P_n = 78.5$ kips

Factored load:

When only dead load and live load are present, the only load combinations with a chance of controlling are combinations 1 and 2.

Combination 1: $1.4D = 1.4(35) = 49$ kips

Combination 2: $1.2D + 1.6L = 1.2(35) + 1.6(15) = 66$ kips

The second combination controls; $P_u = 66$ kips.

(When only dead load and live load are present, combination 2 will always control when the dead load is less than eight times the live load. In future examples, we will not check combination 1 [1.4*D*] when it obviously does not control.)

ANSWER Since $P_u < \phi_t P_n$, (66 kips < 78.5 kips), the member is satisfactory.

b. For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

For the net section, the allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{104.7}{2.00} = 52.4 \text{ kips}$$

The smaller value controls; the allowable strength is 52.4 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$$P_a = D + L = 35 + 15 = 50 \text{ kips}$$

ANSWER Since 50 kips < 52.4 kips, the member is satisfactory.

Alternative Solution Using Allowable Stress

For the gross area, the applied stress is

$$f_t = \frac{P_a}{A_g} = \frac{50}{2.50} = 20 \text{ ksi}$$

and the allowable stress is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

For this limit state, $f_t < F_t$ (OK)

For the net section,

$$f_t = \frac{P_a}{A_e} = \frac{50}{1.806} = 27.7 \text{ ksi}$$

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi} > 27.7 \text{ ksi} \quad \text{(OK)}$$

ANSWER Since $f_t < F_t$ for both limit states, the member is satisfactory.

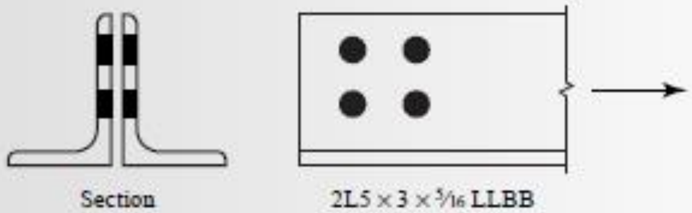
EXAMPLE 3.3

A double-angle shape is shown in Figure 3.5. The steel is A36, and the holes are for ½-inch-diameter bolts. Assume that $A_e = 0.75A_n$.

a. Determine the design tensile strength for LRFD.

b. Determine the allowable strength for ASD.

FIGURE 3.5



SOLUTION

Figure 3.5 illustrates the notation for unequal-leg double-angle shapes. The notation LLBB means "long-legs back-to-back," and SLBB indicates "short-legs back-to-back."

When a double-shape section is used, two approaches are possible: (1) consider a single shape and double everything, or (2) consider two shapes from the outset. (Properties of the double-angle shape are given in Part 1 of the *Manual.*) In this example, we consider one angle and double the result. For one angle, the nominal strength based on the gross area is

$$P_n = F_y A_g = 36(2.41) = 86.76 \text{ kips}$$

There are two holes in each angle, so the net area of one angle is

$$A_n = 2.41 - \left(\frac{5}{16}\right)\left(\frac{1}{2} + \frac{1}{8}\right) \times 2 = 2.019 \text{ in.}^2$$

The effective net area is

$$A_e = 0.75(2.019) = 1.514 \text{ in.}^2$$

The nominal strength based on the net area is

$$P_n = F_u A_e = 58(1.514) = 87.81 \text{ kips}$$

a. The design strength based on yielding of the gross area is

$$\phi_t P_n = 0.90(86.76) = 78.08 \text{ kips}$$

The design strength based on fracture of the net area is

$$\phi_t P_n = 0.75(87.81) = 65.86 \text{ kips}$$

ANSWER Because 65.86 kips < 78.08 kips, fracture of the net section controls, and the design strength for the two angles is $2 \times 65.86 = 132$ kips.

b. The allowable stress approach will be used. For the gross section,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_g = 21.6(2.41) = 52.06 \text{ kips}$$

For the net section,

$$F_t = 0.5F_u = 0.5(58) = 29 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_e = 29(1.514) = 43.91 \text{ kips}$$

ANSWER Because 43.91 kips < 52.06 kips, fracture of the net section controls, and the allowable strength for the two angles is $2 \times 43.91 = 87.8$ kips.

FIGURE 3.7

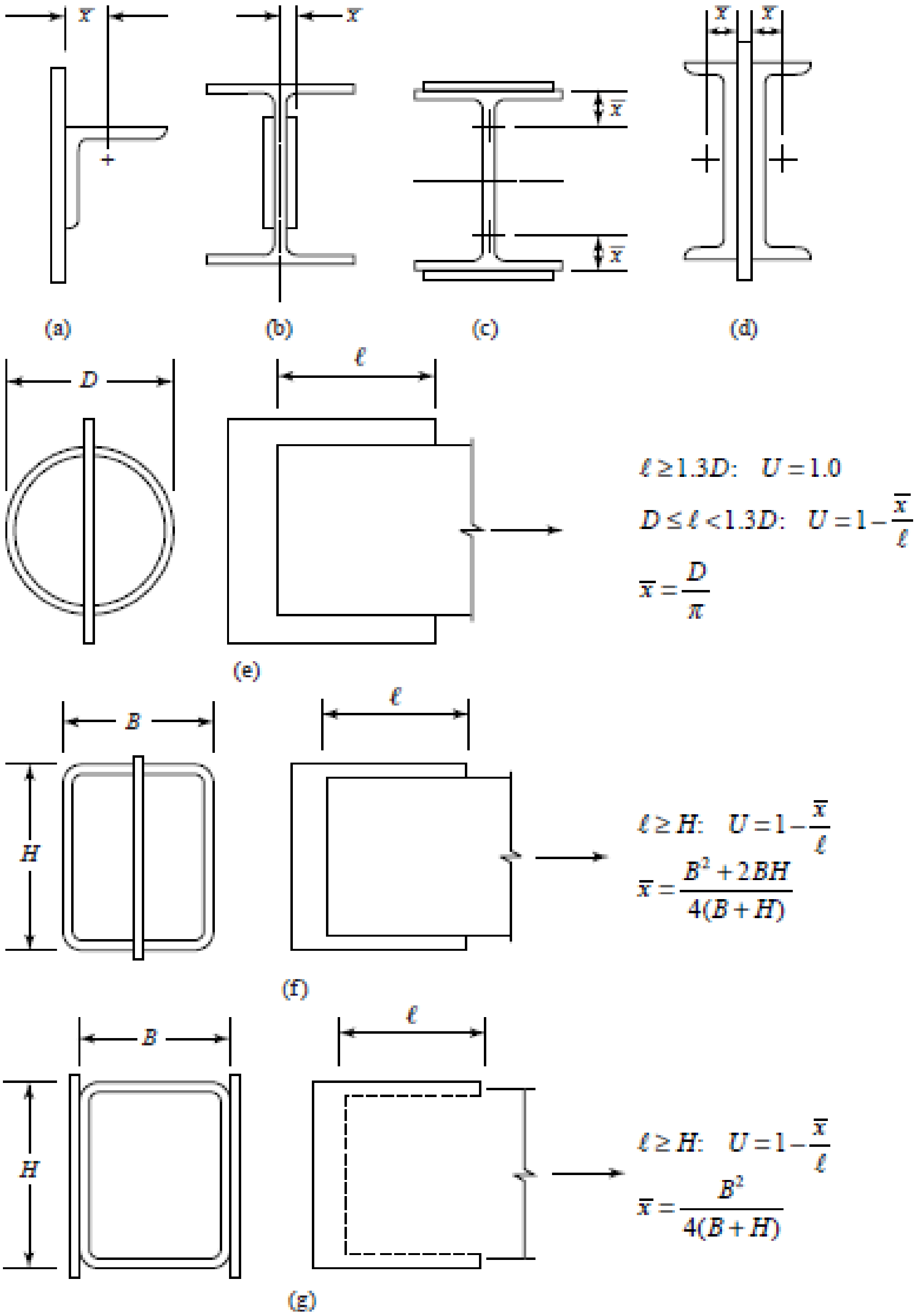
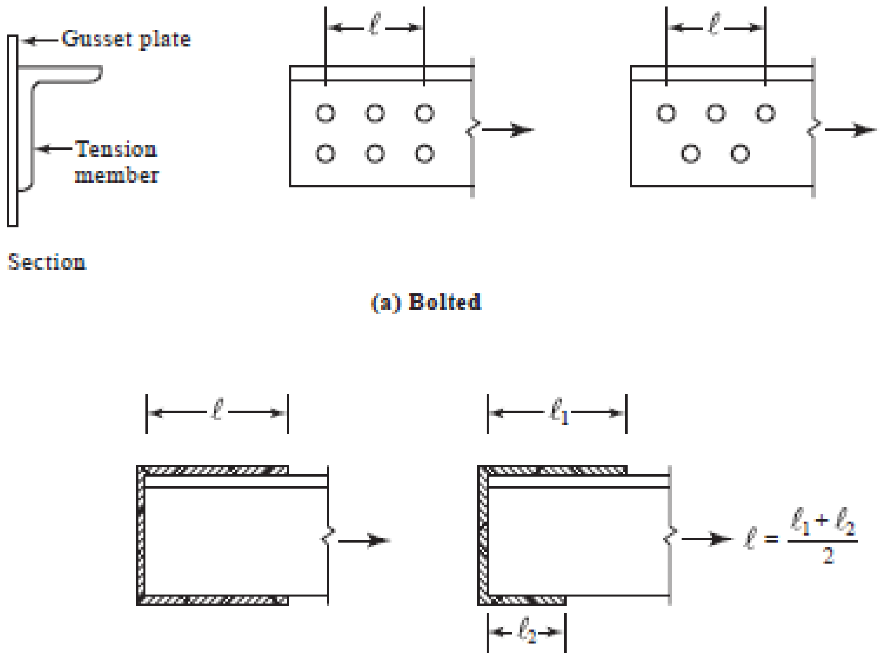

(a)
(b)
(c)
(d)
D
ℓ
$\ell \geq 1.3D$: $U = 1.0$
$D \leq \ell < 1.3D$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{D}{\pi}$
(e)
B
H
ℓ
$\ell \geq H$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{B^2 + 2BH}{4(B+H)}$
(f)
B
H
ℓ
$\ell \geq H$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{B^2}{4(B+H)}$
(g)

FIGURE 3.8



(b) Welded

The Commentary of the AISC Specification further illustrates $\bar{x}$ and ℓ. Figure C-D3.2 shows some special cases for $\bar{x}$, including channels and I-shaped members connected through their webs. To compute $\bar{x}$ for these cases, the Commentary uses the concept of the plastic neutral axis to explain the procedure. Since this concept is not covered until Chapter 5 of this book, we will use $\bar{x}$ for channels as shown in Case 2 of Specification Table D3.1 and in Figure 3.7b of this book. For I-shaped members and tees connected through the web, we can use Case 2 or Case 7 of Specification Table D3.1.

2. Plates

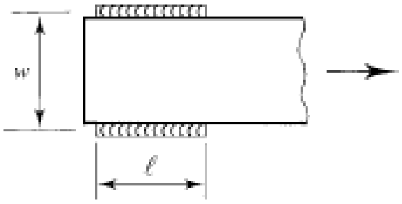
In general, $U = 1.0$ for plates, since the cross section has only one element and it is connected. There is one exception for welded plates, however. If the member is connected with longitudinal welds on each side with no transverse weld (as in Figure 3.9), the following values apply:

- For $\ell \geq 2w$ $U = 1.0$
- For $1.5w \leq \ell < 2w$, $U = 0.87$
- For $w \leq \ell < 1.5w$, $U = 0.75$

3. Round HSS with $\ell \geq 1.3D$ (see Figure 3.7e):

$U = 1.0$

FIGURE 3.9



4. Alternatives to Equation 3.1 for Single and Double Angles:

The following values may be used in lieu of Equation 3.1.

- For four or more fasteners in the direction of loading, $U = 0.80$.
- For three fasteners in the direction of loading, $U = 0.60$.

5. Alternatives to Equation 3.1 for W, M, S, HP, or Tees Cut from These Shapes:

If the following conditions are satisfied, the corresponding values may be used in lieu of Equation 3.1.

- Connected through the flange with three or more fasteners in the direction of loading, with a width at least ⅔ of the depth: $U = 0.90$.
- Connected through the flange with three or more fasteners in the direction of loading, with a width less than ⅔ of the depth: $U = 0.85$.
- Connected through the web with four or more fasteners in the direction of loading: $U = 0.70$.

Figure 3.10 illustrates the alternative values of U for various connections.

If a tension member is connected with only transverse welds, $U = 1.0$, and A_n is the area of the connected element. Figure 3.11 illustrates the difference between transverse and longtitudinal welds. Connections by transverse welds alone are not common.

There are some limiting values for the effective area:

- For bolted *splice plates*, $A_e = A_n \leq 0.85A_g$. This limit is given in a user note and is from a requirement in Chapter J of the Specification "Design of Connections."
- For open cross-sectional shapes (such as W, M, S, C, HP, WT, and ST) and (angles), the value of U need not be less than the ratio of the connected element gross area to the total gross area.

EXAMPLE 3.4

Determine the effective net area for the tension member shown in Figure 3.12.

SOLUTION

$$A_n = A_g - A_{\text{holes}}$$

$$= 5.77 - \frac{1}{2}\left(\frac{5}{8} + \frac{1}{8}\right)(2) = 5.02 \text{ in.}^2$$

Only one element (one leg) of the cross section is connected, so the net area must be reduced. From the properties tables in Part 1 of the *Manual*, the distance from the centroid to the outside face of the leg of an L6 × 6 × ½ is

$$\bar{x} = 1.67 \text{ in.}$$

FIGURE 3.10

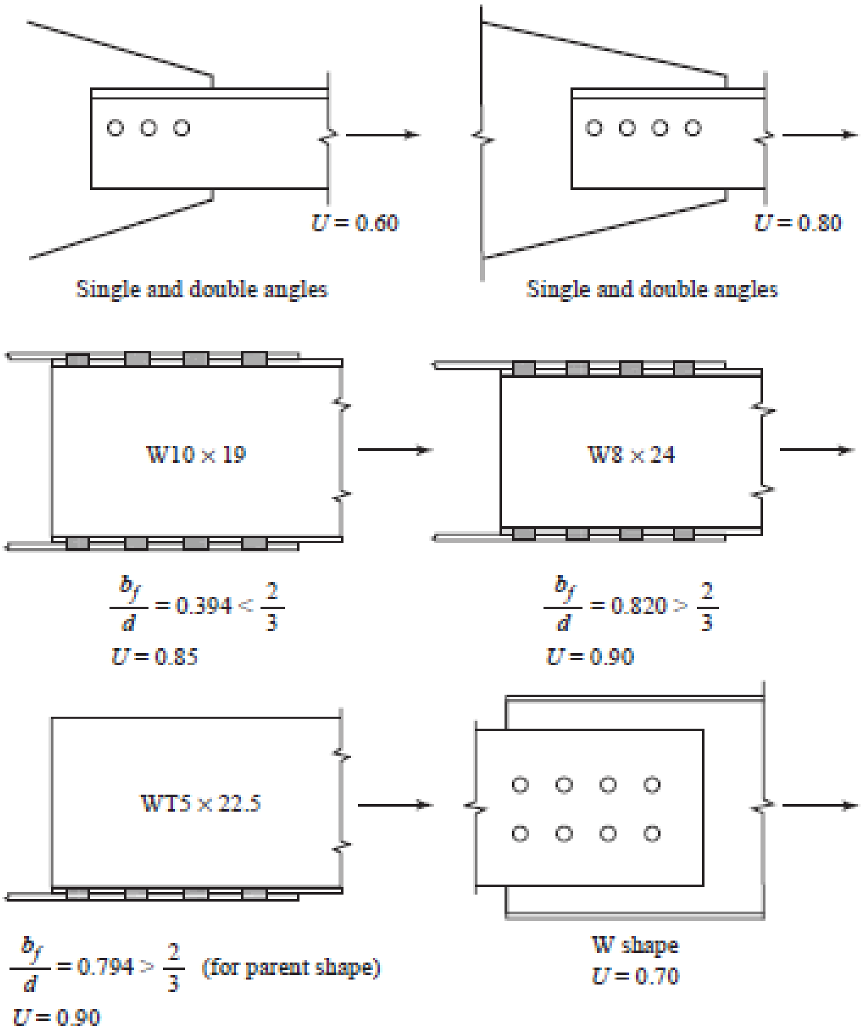


FIGURE 3.11

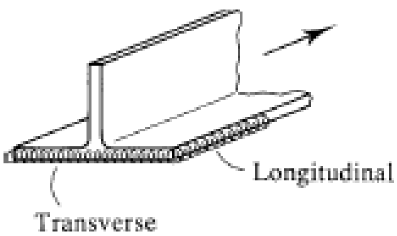
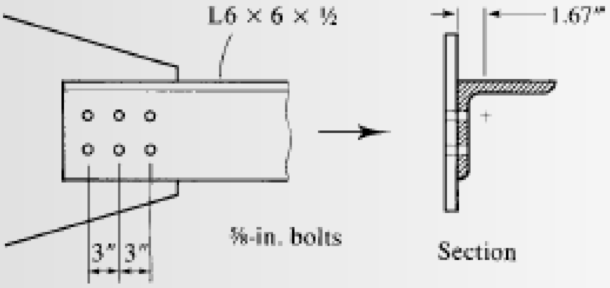


FIGURE 3.12



The length of the connection is

$$\ell = 3 + 3 = 6 \text{ in.}$$

$$\therefore U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{6}\right) = 0.7217$$

$$A_e = A_n U = 5.02(0.7217) = 3.623 \text{ in.}^2$$

The alternative value of U could also be used. Because this angle has three bolts in the direction of the load, the reduction factor U can be taken as 0.60, and

$$A_e = A_n U = 5.02(0.60) = 3.012 \text{ in.}^2$$

Either U value is acceptable, and the Specification permits the larger one to be used. However, the value obtained from Equation 3.1 is more accurate. The alternative values of U can be useful during preliminary design, when actual section properties and connection details are not known.

EXAMPLE 3.5

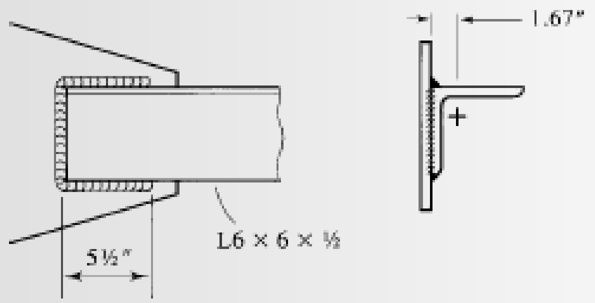
If the tension member of Example 3.4 is welded as shown in Figure 3.13, determine the effective area.

SOLUTION As in Example 3.4, only part of the cross section is connected and a reduced effective area must be used.

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{5.5}\right) = 0.6964$$

ANSWER $A_e = A_g U = 5.77(0.6964) = 4.02 \text{ in.}^2$

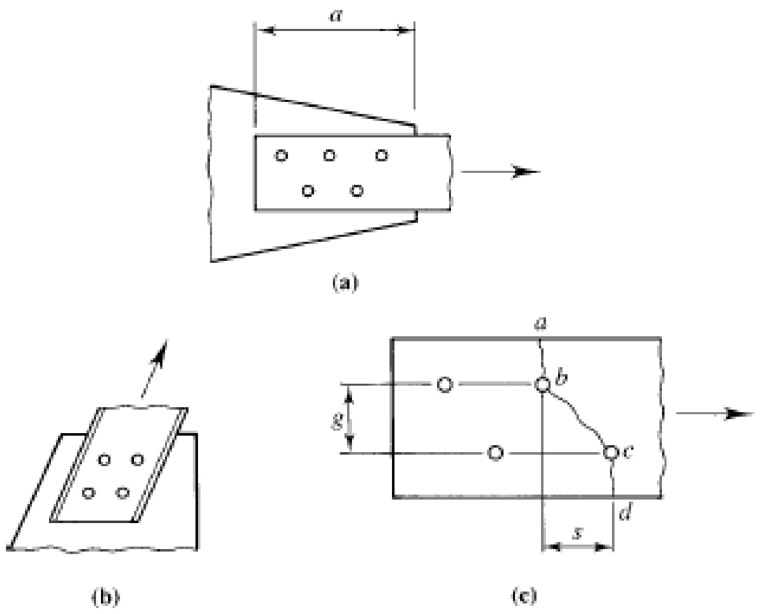
FIGURE 3.13



3.4 STAGGERED FASTENERS

If a tension member connection is made with bolts, the net area will be maximized if the fasteners are placed in a single line. Sometimes space limitations, such as a limit on dimension *a* in Figure 3.14a, necessitate using more than one line. If so, the reduction in cross-sectional area is minimized if the fasteners are arranged in a staggered pattern, as shown. Sometimes staggered fasteners are required by the geometry of a connection, such as the one shown in Figure 3.14b. In either case, any cross section passing through holes will pass through fewer holes than if the fasteners are not staggered.

FIGURE 3.14



If the amount of stagger is small enough, the influence of an offset hole may be felt by a nearby cross section, and fracture along an inclined path such as *abcd* in Figure 3.14c is possible. In such a case, the relationship $f = P/A$ does not apply, and stresses on the inclined portion *b–c* are a combination of tensile and shearing stresses. Several approximate methods have been proposed to account for the effects of staggered holes. Cochrane (1922) proposed that when deducting the area corresponding to a staggered hole, use a reduced diameter, given by

$$d' = d - \frac{s^2}{4g} \tag{3.2}$$

where d is the hole diameter, s is the stagger, or pitch, of the bolts (spacing in the direction of the load), and g is the gage (transverse spacing). This means that in a failure pattern consisting of both staggered and unstaggered holes, use d for holes at the end of a transverse line between holes ($s = 0$) and use d' for holes at the end of an inclined line between holes.

The AISC Specification, in Section B4.3b, uses this approach, but in a modified form. If the net area is treated as the product of a thickness times a net width, and the diameter from Equation 3.2 is used for all holes (since $d' = d$ when the stagger $s = 0$), the net width in a failure line consisting of both staggered and unstaggered holes is

$$\begin{aligned} w_n &= w_g - \sum d' \\ &= w_g - \sum \left(d - \frac{s^2}{4g} \right) \\ &= w_g - \sum d + \sum \frac{s^2}{4g} \end{aligned}$$

where w_n is the net width and w_g is the gross width. The second term is the sum of all hole diameters, and the third term is the sum of $s^2/4g$ for all inclined lines in the failure pattern.

When more than one failure pattern is conceivable, all possibilities should be investigated, and the one corresponding to the smallest load capacity should be used. Note that this method will not accommodate failure patterns with lines parallel to the applied load.

EXAMPLE 3.6

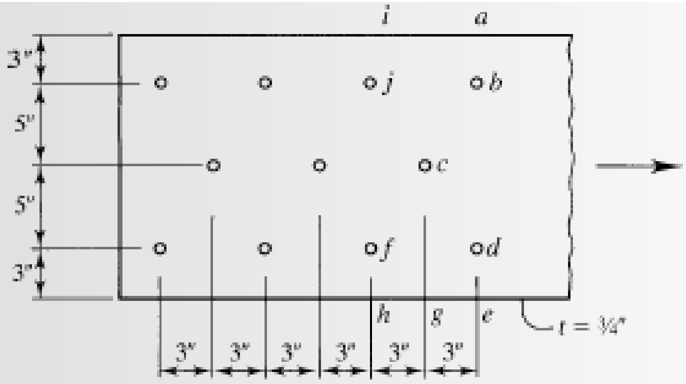
Compute the smallest net area for the plate shown in Figure 3.15. The holes are for 1-inch-diameter bolts.

SOLUTION

The effective hole diameter is $1 + \frac{1}{8} = 1\frac{1}{8}$ in. For line *abde*,

$$w_n = 16 - 2(1.125) = 13.75 \text{ in.}$$

FIGURE 3.15



For line *abcde*,

$$w_n = 16 - 3(1.125) + \frac{2(3)^2}{4(5)} = 13.52 \text{ in.}$$

The second condition will give the smallest net area:

ANSWER $A_n = tw_n = 0.75(13.52) = 10.1 \text{ in.}^2$

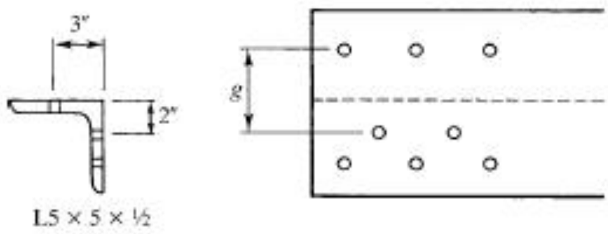
Equation 3.2 can be used directly when staggered holes are present. In the computation of the net area for line *abcde* in Example 3.6,

$$A_n = A_g - \sum t \times (d \text{ or } d')$$
$$= 0.75(16) - 0.75(1.125) - 0.75\left[1.125 - \frac{(3)^2}{4(5)}\right] \times 2 = 10.1 \text{ in.}^2$$

As each fastener resists an equal share of the load (an assumption used in the design of simple connections; see Chapter 7), different potential failure lines may be subjected to different loads. For example, line *abcde* in Figure 3.15 must resist the full load, whereas *ijfh* will be subjected to $^8/_{11}$ of the applied load. The reason is that $^3/_{11}$ of the load will have been transferred from the member before *ijfh* receives any load.

When lines of bolts are present in more than one element of the cross section of a rolled shape, and the bolts in these lines are staggered with respect to one another, the use of areas and Equation 3.2 is preferable to the net-width approach of the AISC Specification. If the shape is an angle, it can be visualized as a plate formed by "unfolding" the legs to more clearly identify the pitch and gage distances. AISC B4.3b specifies that any gage line crossing the heel of the angle be reduced by an amount that equals the angle thickness. Thus, the distance *g* in Figure 3.16, to be used in the $s^2/4g$ term, would be $3 + 2 - \frac{1}{2} = 4\frac{1}{2}$ inches.

FIGURE 3.16



EXAMPLE 3.7

An angle with staggered fasteners in each leg is shown in Figure 3.17. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

a. Determine the design strength for LRFD.
b. Determine the allowable strength for ASD.

SOLUTION From the dimensions and properties tables, the gross area is $A_g = 6.80$ in.². The effective hole diameter is $\frac{7}{8} + \frac{1}{8} = 1$ in.

For line *abdf*, the net area is

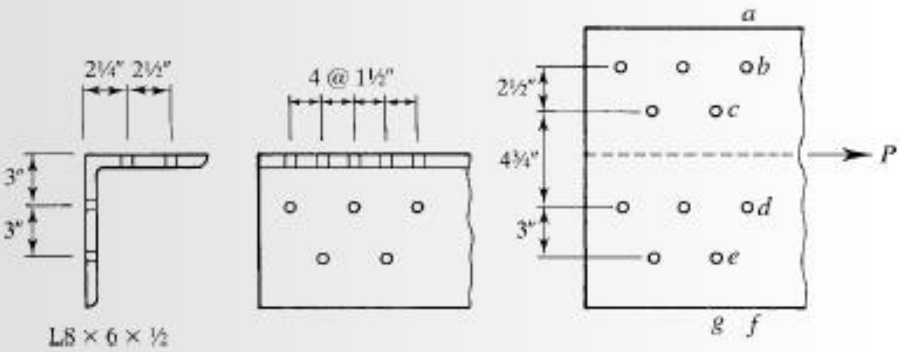
$$A_n = A_g - \sum t_w \times (d \text{ or } d')$$
$$= 6.80 - 0.5(1.0) \times 2 = 5.80 \text{ in.}^2$$

For line *abceg*,

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5(1.0) = 5.413 \text{ in.}^2$$

Because $\frac{1}{10}$ of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only $\frac{9}{10}$ of the load. Therefore, the net area

FIGURE 3.17



of 5.413 in.^{2} should be multiplied by $^{10}/_9$ to obtain a net area that can be compared with those lines that resist the full load. Use $A_n = 5.413(^{10}/_9) = 6.014$ in.^{2} For line *abcdeg*,

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(4.75)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(3)}\right]$$

$$= 5.065 \text{ in.}^2$$

The last case controls; use

$$A_n = 5.065 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.065 \text{ in.}^2$$

The nominal strength based on fracture is

$$P_n = F_u A_e = 58(5.065) = 293.8 \text{ kips}$$

The nominal strength based on yielding is

$$P_n = F_y A_g = 36(6.80) = 244.8 \text{ kips}$$

a. The design strength based on fracture is

$$\phi_t P_n = 0.75(293.8) = 220 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(244.8) = 220 \text{ kips}$$

ANSWER Design strength = 220 kips.

b. For the limit state of fracture, the allowable stress is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable strength is

$$F_t A_e = 29.0(5.065) = 147 \text{ kips}$$

For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$
$$F_t A_g = 21.6(6.80) = 147 \text{ kips}$$

ANSWER Allowable strength = 147 kips.

EXAMPLE 3.8

Determine the smallest net area for the American Standard Channel shown in Figure 3.18. The holes are for ⅝-inch-diameter bolts.

SOLUTION

$$A_n = A_g - \sum t_w \times (d \text{ or } d')$$

$$d = \text{bolt diameter} + \frac{1}{8} = \frac{5}{8} + \frac{1}{8} = \frac{3}{4} \text{ in.}$$

Line *abe:*

$$A_n = A_g - t_w d = 3.82 - 0.437\left(\frac{3}{4}\right) = 3.49 \text{ in.}^2$$

Line *abcd:*

$$A_n = A_g - t_w(d \text{ for hole at } b) - t_w(d' \text{ for hole at } c)$$

$$= 3.82 - 0.437\left(\frac{3}{4}\right) - 0.437\left[\frac{3}{4} - \frac{(2)^2}{4(3)}\right] = 3.31 \text{ in.}^2$$

ANSWER Smallest net area = 3.31 in.^{2}

FIGURE 3.18

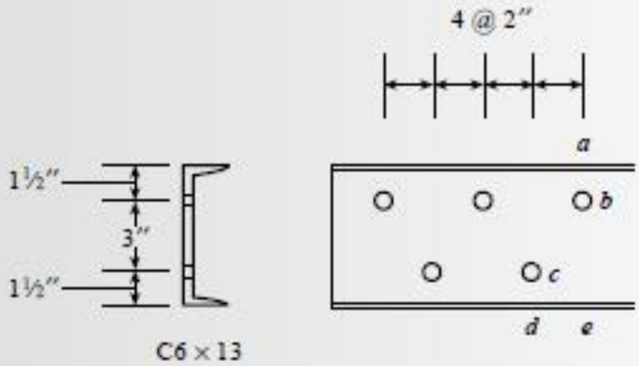
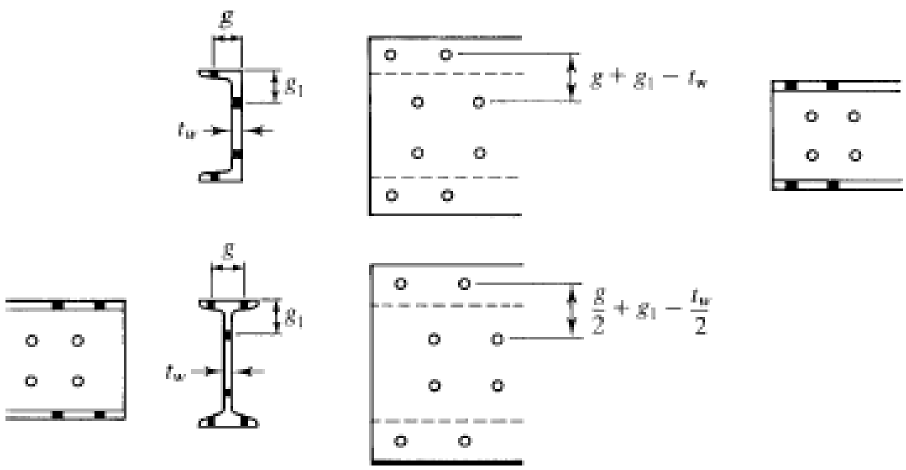


FIGURE 3.19



EXAMPLE 3.9

Find the available strength of the S-shape shown in Figure 3.20. The holes are for ¾-inch-diameter bolts. Use A36 steel.

SOLUTION Compute the net area:

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$\text{Effective hole diameter} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

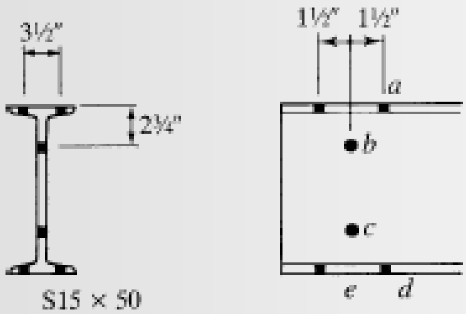
For line *ad*,

$$A_n = 14.7 - 4\left(\frac{7}{8}\right)(0.622) = 12.52 \text{ in.}^2$$

For line *abcd*, the gage distance for use in the $s^2/4g$ term is

$$\frac{g}{2} + g_1 - \frac{t_w}{2} = \frac{3.5}{2} + 2.75 - \frac{0.550}{2} = 4.225 \text{ in.}$$

FIGURE 3.20



Starting at a and treating the holes at b and d as the staggered holes gives

$$A_n = A_g - \sum t \times (d \text{ or } d')$$
$$= 14.7 - 2(0.622)\left(\frac{7}{8}\right) - (0.550)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right]$$
$$- (0.550)\left(\frac{7}{8}\right) - 2(0.622)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right] = 11.73 \text{ in.}^2$$

Line $abcd$ controls. As all elements of the cross section are connected,

$$A_e = A_n = 11.73 \text{ in.}^2$$

For the net section, the nominal strength is

$$P_n = F_u A_e = 58(11.73) = 680.3 \text{ kips}$$

For the gross section,

$$P_n = F_y A_g = 36(14.7) = 529.2 \text{ kips}$$

LRFD SOLUTION The design strength based on fracture is

$$\phi_t P_n = 0.75(680.3) = 510 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(529.2) = 476 \text{ kips}$$

Yielding of the gross section controls.

ANSWER Design strength = 476 kips.

ASD SOLUTION The allowable stress based on fracture is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_e = 29.0(11.73) = 340$ kips.

The allowable stress based on yielding is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_g = 21.6(14.7) = 318$ kips.

Yielding of the gross section controls.

ANSWER Allowable strength = 318 kips.

Read Page 64 Block Shear and Example 3.10

3.6 DESIGN OF TENSION MEMBERS

The design of a tension member involves finding a member with adequate gross and net areas. If the member has a bolted connection, the selection of a suitable cross section requires an accounting for the area lost because of holes. For a member with a rectangular cross section, the calculations are relatively straightforward. If a rolled shape is to be used, however, the area to be deducted cannot be predicted in advance because the member•s thickness at the location of the holes is not known.

A secondary consideration in the design of tension members is slenderness. If a structural member has a small cross section in relation to its length, it is said to be *slender*. A more precise measure is the slenderness ratio, L/r, where L is the member length and r is the minimum radius of gyration of the cross-sectional area. The minimum radius of gyration is the one corresponding to the minor principal axis of the cross section. This value is tabulated for all rolled shapes in the properties tables in Part 1 of the *Manual*.

Although slenderness is critical to the strength of a compression member, it is inconsequential for a tension member. In many situations, however, it is good practice to limit the slenderness of tension members. If the axial load in a slender tension member is removed and small transverse loads are applied, undesirable vibrations or deflections might occur. These conditions could occur, for example, in a slack bracing rod subjected to wind loads. For this reason, the user note in AISC D1 suggests a maximum slenderness ratio of 300. It is only a recommended value because slenderness has no structural significance for tension members, and the limit may be exceeded when special circumstances warrant it. This limit does not apply to cables, and the user note explicitly excludes rods.

The central problem of all member design, including tension member design, is to find a cross section for which the required strength does not exceed the available strength. For tension members designed by LRFD, the requirement is

$$P_u \leq \phi_t P_n \quad \text{or} \quad \phi_t P_n \geq P_u$$

where P_u is the sum of the factored loads. To prevent yielding,

$$0.90 F_y A_g \geq P_u \quad \text{or} \quad A_g \geq \frac{P_u}{0.90 F_y}$$

To avoid fracture,

$$0.75 F_u A_e \geq P_u \quad \text{or} \quad A_e \geq \frac{P_u}{0.75 F_u}$$

For allowable strength design, if we use the allowable *stress* form, the requirement corresponding to yielding is

$$P_a \leq F_t A_g$$

and the required gross area is

$$A_g \geq \frac{P_a}{F_t} \quad \text{or} \quad A_g \geq \frac{P_a}{0.6 F_y}$$

For the limit state of fracture, the required effective area is

$$A_e \geq \frac{P_a}{F_t} \quad \text{or} \quad A_e \geq \frac{P_a}{0.5 F_u}$$

The slenderness ratio limitation will be satisfied if

$$r \geq \frac{L}{300}$$

where r is the minimum radius of gyration of the cross section and L is the member length.

EXAMPLE 3.11

A tension member with a length of 5 feet 9 inches must resist a service dead load of 18 kips and a service live load of 52 kips. Select a member with a rectangular cross section. Use A36 steel and assume a connection with one line of ⅞-inch-diameter bolts.

LRFD SOLUTION

$P_u = 1.2D + 1.6L = 1.2(18) + 1.6(52) = 104.8 \text{ kips}$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{P_u}{0.90F_y} = \frac{104.8}{0.90(36)} = 3.235 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{P_u}{0.75F_u} = \frac{104.8}{0.75(58)} = 2.409 \text{ in.}^2$$

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.235}{1} = 3.235 \text{ in.}$$

Try a $1 \times 3\frac{1}{2}$ cross section.

$$A_e = A_n = A_g - A_{hole}$$
$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.409 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$
$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$
$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

ANSWER Use a PL $1 \times 3\frac{1}{2}$.

ASD SOLUTION

$P_a = D + L = 18 + 52 = 70.0$ kips

For yielding, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi, and

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{70}{21.6} = 3.24 \text{ in.}^2$$

For fracture, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi, and

$$\text{Required } A_e = \frac{P_a}{F_t} = \frac{70}{29.0} = 2.414 \text{ in.}^2$$

(The rest of the design *procedure* is the same as for LRFD. The numerical results may be different)

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.241}{1} = 3.241 \text{ in.}$$

Try a $1 \times 3\frac{1}{2}$ cross section.

$$A_e = A_n = A_g - A_{\text{hole}}$$
$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.414 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

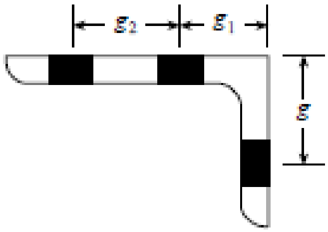
$$I_{\min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$
$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$
$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

ANSWER Use a PL $1 \times 3\frac{1}{2}$.

FIGURE 3.24



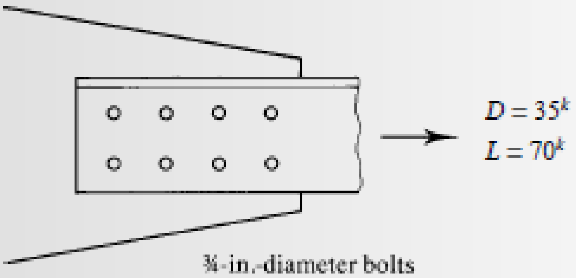
Usual Gages for Angles (inches)

Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1
g	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	⅞	⅞	¾	⅝
g_1	3	2½	2¼	2										
g_2	3	3	2½	1¾										

EXAMPLE 3.12

Select an unequal-leg angle tension member 15 feet long to resist a service dead load of 35 kips and a service live load of 70 kips. Use A36 steel. The connection is shown in Figure 3.25.

FIGURE 3.25



LRFD SOLUTION

The factored load is

$$P_u = 1.2D + 1.6L = 1.2(35) + 1.6(70) = 154 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{154}{0.90(36)} = 4.75 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{154}{0.75(58)} = 3.54 \text{ in.}^2$$

The radius of gyration should be at least

$$\frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

To find the lightest shape that satisfies these criteria, we search the dimensions and properties table for the unequal-leg angle that has the smallest acceptable gross area and then check the effective net area. The radius of gyration can be checked by inspection. There are two lines of bolts, so the connected leg must be at least 5 inches long (see the usual gages for angles in Figure 3.24). Starting at either end of the table, we find that the shape with the smallest area that is at least equal to 4.75 in.^{2} is an L6 × 4 × ½ with an area of 4.75 in.^{2} and a minimum radius of gyration of 0.864 in.

Try L6 × 4 × ½.

$$A_n = A_g - A_{\text{holes}} = 4.75 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

Because the length of the connection is not known, Equation 3.1 cannot be used to compute the shear lag factor U. Since there are four bolts in the direction of the load, we will use the alternative value of $U = 0.80$.

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)*}$$

Try the next larger shape from the dimensions and properties tables.

Try L5 × 3½ × ⅝ ($A_g = 4.93$ in.^{2} and $r_{\min} = 0.746$ in.)

$$A_n = A_g - A_{\text{holes}} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$
$$A_e = A_n U = 3.836(0.80) = 3.07 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

(Note that this shape has slightly more gross area than that produced by the previous trial shape, but because of the greater leg thickness, slightly more area is deducted for the holes.) Passing over the next few heavier shapes,

Try L8 × 4 × ½ ($A_g = 5.80$ in.^{2} and $r_{\min} = 0.863$ in.)

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$
$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.54 \text{ in.}^2 \qquad \textbf{(OK)}$$

*The notation N.G. means "No Good."

ANSWER This shape satisfies all requirements, so use an L8 × 4 × ½.

ASD SOLUTION The total service load is

$$P_a = D + L = 35 + 70 = 105 \text{ kips}$$

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{105}{0.6(36)} = 4.86 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{105}{0.5(58)} = 3.62 \text{ in.}^2$$

$$\text{Required } r_{min} = \frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

Try L8 × 4 × ½ ($A_g = 5.80$ in.² and $r_{min} = 0.863$ in.). For a shear lag factor U of 0.80,

$$A_n = A_g - A_{holes} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.62 \text{ in.}^2 \qquad \text{(OK)}$$

ANSWER This shape satisfies all requirements, so use an L8 × 4 × ½.

Tables for the Design of Tension Members

Part 5 of the *Manual* contains tables to assist in the design of tension members of various cross-sectional shapes, including Table 5-2 for angles. The use of these tables will be illustrated in the following example.

EXAMPLE 3.13

Design the tension member of Example 3.12 with the aid of the tables in Part 5 of the *Manual*.

LRFD SOLUTION From Example 3.12,

$$P_u = 154 \text{ kips}$$

$$r_{min} \geq 0.600 \text{ in.}$$

The tables for design of tension members give values of A_g and A_e for various shapes based on the assumption that $A_e = 0.75A_g$. In addition, the corresponding available strengths based on yielding and rupture (fracture) are given. All values available for angles are for A36 steel. Starting with the lighter shapes (the ones with the smaller gross area), we find that an L6 × 4 × ½, with $\phi_t P_n = 154$ kips based on the gross section and $\phi_t P_n = 155$ kips based on the net section, is a possibility. From the dimensions and properties tables in Part 1 of the *Manual*, $r_{min} = 0.864$ in. To check this selection, we must compute the actual net area. If we assume that $U = 0.80$,

$$A_n = A_g - A_{holes} = 4.75 - 2\left(\frac{3}{4}+\frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2$$

$$\phi_t P_n = \phi_t F_u A_e = 0.75(58)(3.10) = 135 \text{ kips} < 154 \text{ kips} \qquad \text{(N.G.)}$$

This shape did not work because the ratio of actual effective net area A_e to gross area A_g is not equal to 0.75. The ratio is closer to

$$\frac{3.10}{4.75} = 0.6526$$

This corresponds to a required $\phi_t P_n$ (based on rupture) of

$$\frac{0.75}{\text{actual ratio}} \times P_u = \frac{0.75}{0.6526}(154) = 177 \text{ kips}$$

Try an L8 × 4 × ½, with $\phi_t P_n = 188$ kips (based on yielding) and $\phi_t P_n = 189$ Kips (based on rupture strength, with $A_e = 0.75A_g = 4.31$ in.^{2}). From the dimensions and properties tables in Part 1 of the *Manual*, $r_{min} = 0.863$ in. The actual effective net area and rupture strength are computed as follows:

$$A_n = A_g - A_{holes} = 5.80 - 2\left(\frac{3}{4}+\frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2$$

$$\phi_t P_n = \phi_t F_u A_e = 0.75(58)(3.94) = 171 > 154 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use an L8 × 4 × ½, connected through the 8-inch leg.

ASD SOLUTION From Example 3.12,

$$P_a = 105 \text{ kips}$$

$$\text{Required } r_{min} = 0.600 \text{ in.}$$

From *Manual* Table 5-2, try an L5 × 3½ × ⅝, with $P_n/\Omega_t = 106$ kips based on yielding of the gross section and $P_n/\Omega_t = 107$ kips based on rupture of the net section. From the dimensions and properties tables in Part 1 of the *Manual*, $r_{min} = 0.746$ in.

Using a shear lag factor U of 0.80, the actual effective net area is computed as follows:

$$A_n = A_g - A_{holes} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$
$$A_e = A_n U = 3.836(0.80) = 3.069 \text{ in.}^2$$

and the allowable strength based on rupture of the net section is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{58(3.069)}{2.00} = 89.0 \text{ kips} < 105 \text{ kips} \qquad \text{(N.G.)}$$

This shape did not work because the ratio of actual effective net area A_e to gross area A_g is not equal to 0.75. The ratio is closer to

$$\frac{3.069}{4.93} = 0.6225$$

This corresponds to a required P_n/Ω_t (based on rupture), for purposes of using Table 5-2, of

$$\frac{0.75}{0.6225}(105) = 127 \text{ kips}$$

Using this as a guide, try L6 × 4 × ⅝, with $P_n/\Omega_t = 126$ kips based on yielding of the gross section and $P_n/\Omega_t = 128$ kips based on rupture of the net section. From the dimensions and properties tables in Part 1 of the *Manual*, $r_{min} = 0.859$ in.

$$A_n = A_g - A_{holes} = 5.86 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 4.766 \text{ in.}^2$$
$$A_e = A_n U = 4.766(0.80) = 3.81 \text{ in.}^2$$
$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{58(3.81)}{2.00} = 111 \text{ kips} > 105 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use an L6 × 4 × ⅝, connected through the 6-inch leg.

3.8 TENSION MEMBERS IN ROOF TRUSSES

Many of the tension members that structural engineers design are components of trusses. For this reason, some general discussion of roof trusses is in order. A more comprehensive treatment of the subject is given by Lothars (1972).

When trusses are used in buildings, they usually function as the main supporting elements of roof systems where long spans are required. They are used when the cost and weight of a beam would be prohibitive. (A truss may be thought of as a deep beam with much of the web removed.) Roof trusses are often used in industrial or mill buildings, although construction of this type has largely given way to rigid frames. Typical roof construction with trusses supported by load-bearing walls is illustrated in Figure 3.28. In this type of construction, one end of the connection of the truss to the walls usually can be considered as pinned and the other as roller-supported. Thus the truss can be analyzed as an externally statically determinate structure. The supporting walls can be reinforced concrete, concrete block, brick, or a combination of these materials.

Roof trusses normally are spaced uniformly along the length of the building and are tied together by longitudinal beams called *purlins* and by X-bracing. The primary function of the purlins is to transfer loads to the top chord of the truss, but they can also act as part of the bracing system. Bracing is usually provided in the planes of both the top and bottom chords, but it is not required in every bay because lateral forces can be transferred from one braced bay to the other through the purlins.

Ideally, purlins are located at the truss joints so that the truss can be treated as a pin-connected structure loaded only at the joints. Sometimes, however, the roof deck cannot span the distance between joints, and intermediate purlins may be needed. In such cases, top chord members will be subjected to significant bending as well as axial compression and must be designed as beam–columns (Chapter 6).

Sag rods are tension members used to provide lateral support for the purlins. Most of the loads applied to the purlins are vertical, so there will be a component parallel to a sloping roof, which will cause the purlin to bend (sag) in that direction (Figure 3.29).

Sag rods can be located at the midpoint, the third points, or at more frequent intervals along the purlins, depending on the amount of support needed. The interval is a function of the truss spacing, the slope of the top chord, the resistance of the purlin

FIGURE 3.28

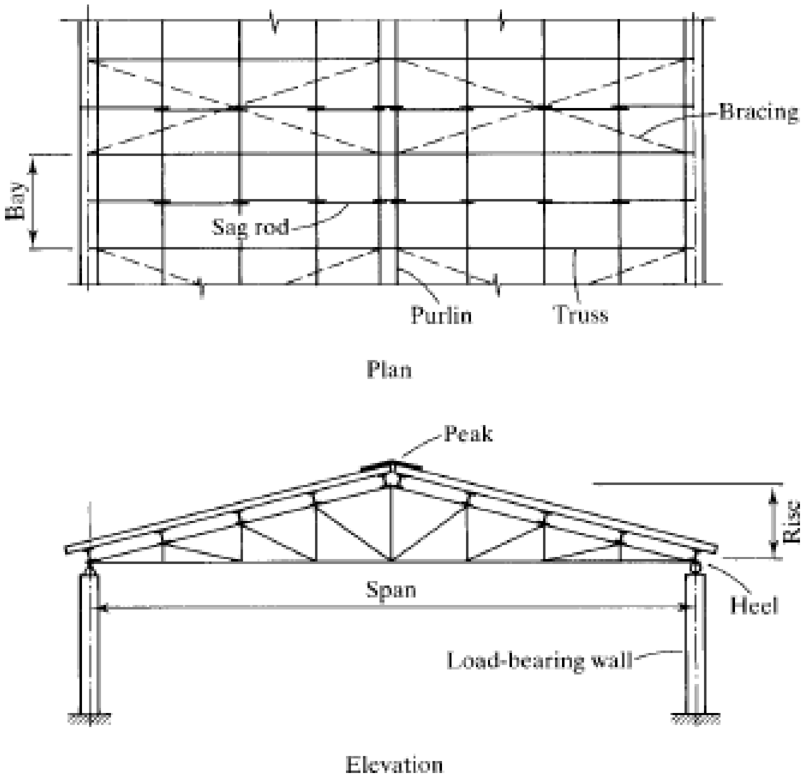
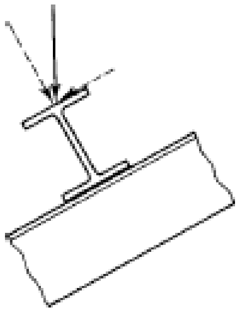


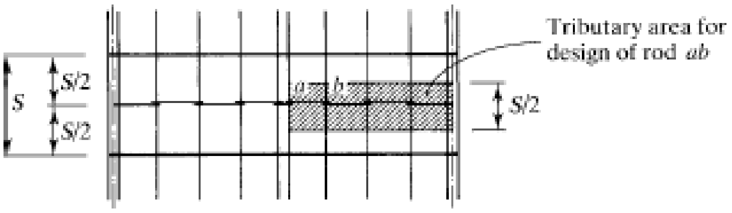
FIGURE 3.29



to this type of bending (most shapes used for purlins are very weak in this respect), and the amount of support furnished by the roofing. If a metal deck is used, it will usually be rigidly attached to the purlins, and sag rods may not be needed. Sometimes, however, the weight of the purlin itself is enough to cause problems, and sag rods may be needed to provide support during construction before the deck is in place.

If sag rods are used, they are designed to support the component of roof loads parallel to the roof. Each segment between purlins is assumed to support everything below it; thus the top rod is designed for the load on the roof area tributary to the rod, from the heel of the truss to the peak, as shown in Figure 3.30. Although the force will be different in each segment of rod, the usual practice is to use one size throughout.

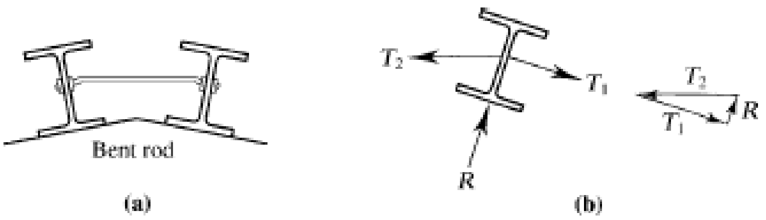
FIGURE 3.30



The extra amount of material in question is insignificant, and the use of the same size for each segment eliminates the possibility of a mix-up during construction.

A possible treatment at the peak or ridge is shown in Figure 3.31a. The tie rod between ridge purlins must resist the load from all of the sag rods on either side. The tensile force in this horizontal member has as one of its components the force in the upper sag-rod segment. A free-body diagram of one ridge purlin illustrates this effect, as shown in Figure 3.31b.

FIGURE 3.31



EXAMPLE 3.15

Fink trusses spaced at 20 feet on centers support W6 × 12 purlins, as shown in Figure 3.32a. The purlins are supported at their midpoints by sag rods. Use A36 steel and design the sag rods and the tie rod at the ridge for the following service loads.

Metal deck:	2 psf
Built-up roof:	5 psf
Snow:	18 psf of horizontal projection of the roof surface
Purlin weight:	12 pounds per foot (lb/ft) of length

SOLUTION Calculate loads.

Tributary width for each sag rod = 20/2 = 10 ft

Tributary area for deck and built-up roof = 10(46.6) = 466 ft^2

Dead load (deck and roof) = (2 + 5)(466) = 3262 lb

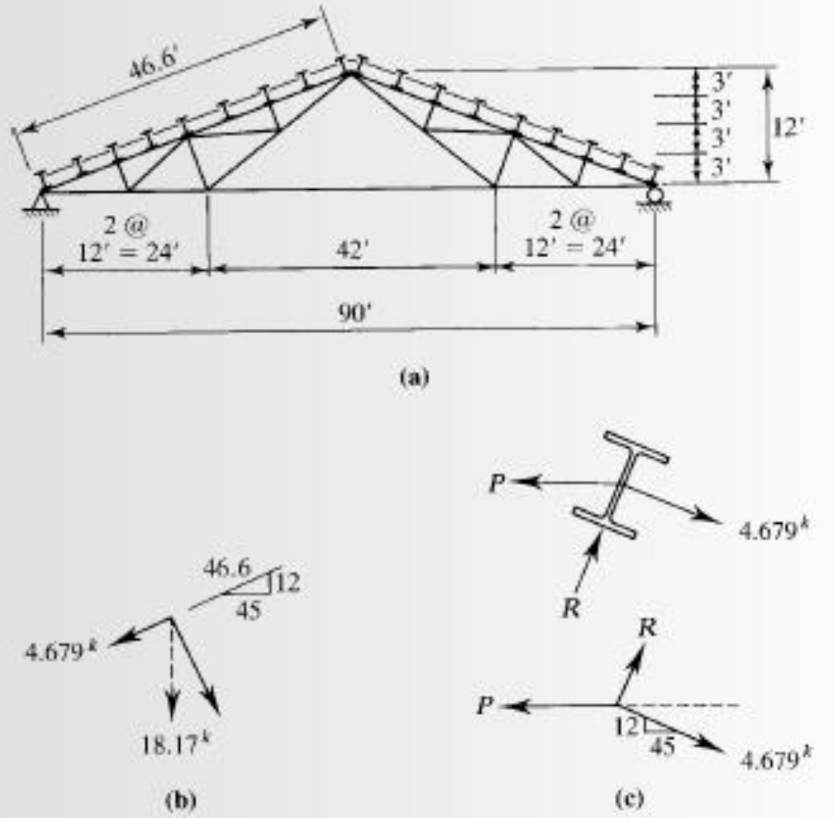
Total purlin weight = 12(10)(9) = 1080 lb

Total dead load = 3262 + 1080 = 4342 lb

Tributary area for snow load = 10(45) = 450 ft^2

Total snow load = 18(450) = 8100 lb

FIGURE 3.32



LRFD SOLUTION

Check load combinations.

Combination 2: $1.2D + 0.5S = 1.2(4342) + 0.5(8100) = 9260$ lb

Combination 3: $1.2D + 1.6S = 1.2(4342) + 1.6(8100) = 18{,}170$ lb

Combination 3 controls. (By inspection, the remaining combinations will not govern.)

For the component parallel to the roof (Figure 3.32b),

$$T = (18.17)\frac{12}{46.6} = 4.679 \text{ kips}$$

$$\text{Required } A_b = \frac{T}{\phi_t(0.75F_u)} = \frac{4.679}{0.75(0.75)(58)} = 0.1434 \text{ in.}^2$$

ANSWER

Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}).

Tie rod at the ridge (Figure 3.32c):

$$P = (4.679)\frac{46.6}{45} = 4.845 \text{ kips}$$

$$\text{Required } A_b = \frac{4.845}{0.75(0.75)(58)} = 0.1485 \text{ in.}^2$$

ANSWER Use a 5/8-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}).

ASD SOLUTION By inspection, load combination 3 will control.

$$D + S = 4342 + 8100 = 12,440 \text{ lb}$$

The component parallel to the roof is

$$T = 12.44\left(\frac{12}{46.6}\right) = 3.203 \text{ kips}$$

The allowable tensile stress is $F_t = 0.375F_u = 0.375(58) = 21.75$ ksi.

$$\text{Required } A_b = \frac{T}{F_t} = \frac{3.203}{21.75} = 0.1473 \text{ in.}^2$$

ANSWER Use a 5/8-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}) for the sag rods.

Tie rod at the ridge:

$$P = 3.203\left(\frac{46.6}{45}\right) = 3.317 \text{ kips}$$

$$\text{Required } A_b = \frac{3.317}{21.75} = 0.1525 \text{ in.}^2$$

ANSWER Use a 5/8-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}) for the tie rod at the ridge.

SOLUTION Calculate loads:

Snow = 20(40)(20) = 16,000 lb

Dead load (exclusive of purlins) =		
	Deck	2 psf
	Roof	4
	Insulation	3
	Total	9 psf

Total dead load = 9(40)(20) = 7200 lb

Total purlin weight = 6.5(20)(9) = 1170 lb

Estimate the truss weight as 10% of the other loads:

$$0.10(16{,}000 + 7200 + 1170) = 2437 \text{ lb}$$

Loads at an interior joint are

$$D = \frac{7200}{8} + \frac{2437}{8} + 6.5(20) = 1335 \text{ lb}$$

$$S = \frac{16{,}000}{8} = 2000 \text{ lb}$$

At an exterior joint, the tributary roof area is half of that at an interior joint. The corresponding loads are

$$D = \frac{7200}{2(8)} + \frac{2437}{2(8)} + 6.5(20) = 732.3 \text{ lb}$$

$$S = \frac{16{,}000}{2(8)} = 1000 \text{ lb}$$

LRFD SOLUTION Load combination 3 will control:

$$P_u = 1.2D + 1.6S$$

At an interior joint,

$$P_u = 1.2(1.335) + 1.6(2.0) = 4.802 \text{ kips}$$

At an exterior joint,

$$P_u = 1.2(0.7323) + 1.6(1.0) = 2.479 \text{ kips}$$

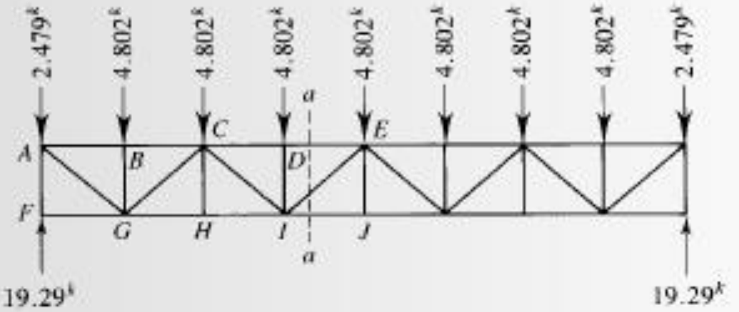
The loaded truss is shown in Figure 3.35a.

The bottom chord is designed by determining the force in each member of the bottom chord and selecting a cross section to resist the largest force. In this example, the force in member *IJ* will control. For the free body left of section *a–a* shown in Figure 3.35b,

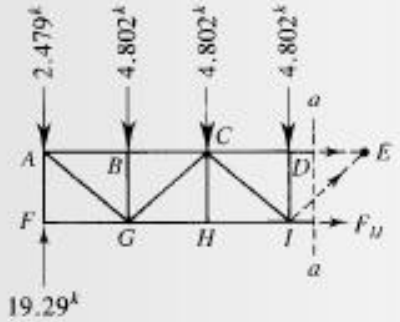
$$\sum M_E = 19.29(20) - 2.479(20) - 4.802(15 + 10 + 5) - 4F_{IJ} = 0$$

$$F_{IJ} = 48.04 \text{ kips}$$

FIGURE 3.35



(a)



(b)

For the gross section,

$$\text{Required } A_g = \frac{F_{IJ}}{0.90F_y} = \frac{48.04}{0.90(50)} = 1.07 \text{ in.}^2$$

For the net section,

$$\text{Required } A_e = \frac{F_{IJ}}{0.75F_u} = \frac{48.04}{0.75(65)} = 0.985 \text{ in.}^2$$

Try an MT5 × 3.75:

$A_g = 1.11 \text{ in.}^2 > 1.07 \text{ in.}^2$ (OK)

Compute the shear lag factor U from Equation 3.1.

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.51}{9}\right) = 0.8322$$

$$A_e = A_g U = 1.11(0.8322) = 0.924 \text{ in.}^2 < 0.985 \text{ in.}^2 \quad \text{(N.G.)}$$

Try an MT6 × 5:

$A_g = 1.48 \text{ in.}^2 > 1.07 \text{ in.}^2$ (OK)

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.86}{9}\right) = 0.7933$$

$$A_e = A_g U = 1.48(0.7933) = 1.17 \text{ in.}^2 > 0.985 \text{ in.}^2 \quad \text{(OK)}$$

If we assume that the bottom chord is braced at the panel points,

$$\frac{L}{r} = \frac{5(12)}{0.594} = 101 < 300 \qquad \text{(OK)}$$

ANSWER Use an MT6 × 5.

ASD SOLUTION Load combination 3 will control. At an interior joint,

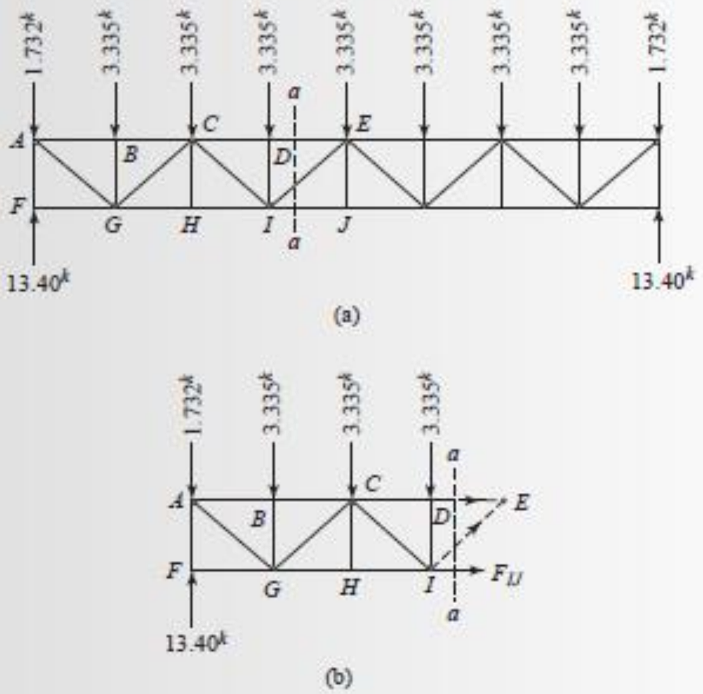
$$P_a = D + S = 1.335 + 2.0 = 3.335 \text{ kips}$$

At an exterior joint,

$$P_a = 0.7323 + 1.0 = 1.732 \text{ kips}$$

The loaded truss is shown in Figure 3.36a.

FIGURE 3.36



Member *IJ* is the bottom chord member with the largest force. For the free body shown in Figure 3.36b,

$$\sum M_E = 13.40(20) - 1.732(20) - 3.335(15 + 10 + 5) - 4F_{IJ} = 0$$

$$F_{IJ} = 33.33 \text{ kips}$$

For the gross section, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi

$$\text{Required } A_g = \frac{F_{IJ}}{F_t} = \frac{33.33}{21.6} = 1.54 \text{ in.}^2$$

For the net section, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi

$$\text{Required } A_e = \frac{F_U}{F_t} = \frac{33.33}{29.0} = 1.15 \text{ in.}^2$$

Try an MT6 × 5.4:

$$A_g = 1.59 \text{ in.}^2 > 1.54 \text{ in.}^2 \qquad \text{(OK)}$$

$$U = 1 - \frac{\overline{x}}{\ell} = 1 - \frac{1.86}{9} = 0.7933$$

$$A_e = A_g U = 1.59(0.7933) = 1.26 \text{ in.}^2 > 1.15 \text{ in.}^2 \qquad \text{(OK)}$$

Assuming that the bottom chord is braced at the panel points, we get

$$\frac{L}{r} = \frac{5(12)}{0.566} = 106 < 300 \qquad \text{(OK)}$$

ANSWER Use an MT6 × 5.4.

CHAPTER 4

Compression Members

4.1 INTRODUCTION

Compression members are structural elements that are subjected only to axial compressive forces; that is, the loads are applied along a longitudinal axis through the centroid of the member cross section, and the stress can be taken as $f = P/A$, where f is considered to be uniform over the entire cross section. This ideal state is never achieved in reality, however, because some eccentricity of the load is inevitable. Bending will result, but it usually can be regarded as secondary. As we shall see, the AISC Specification equations for compression member strength account for this accidental eccentricity.

The most common type of compression member occurring in buildings and bridges is the *column*, a vertical member whose primary function is to support vertical loads. In many instances, these members are also subjected to bending, and in these cases, the member is a *beam–column*. We cover this topic in Chapter 6. Compression members are also used in trusses and as components of bracing systems. Smaller compression members not classified as columns are sometimes referred to as *struts*.

In many small structures, column axial forces can be easily computed from the reactions of the beams that they support or computed directly from floor or roof loads. This is possible if the member connections do not transfer moment; in other words, if the column is not part of a rigid frame. For columns in rigid frames, there are calculable bending moments as well as axial forces, and a frame analysis is necessary. The AISC Specification provides for three methods of analysis to obtain the axial forces and bending moments in members of a rigid frame:

1. Direct analysis method
2. Effective length method
3. First-order analysis method

Except in very simple cases, computer software is used for the analysis. While the details of these three methods are beyond the scope of the present chapter, more will be said about them in Chapter 6 "Beam–Columns". It is important to recognize,

4.2 COLUMN THEORY

Consider the long, slender compression member shown in Figure 4.1a. If the axial load P is slowly applied, it will ultimately become large enough to cause the member to become unstable and assume the shape indicated by the dashed line. The member is said to have buckled, and the corresponding load is called the *critical buckling load*. If the member is stockier, as shown in Figure 4.1b, a larger load will be required to bring the member to the point of instability. For extremely stocky members, failure may occur by compressive yielding rather than buckling. Prior to failure, the compressive stress P/A will be uniform over the cross section at any point along the length, whether the failure is by yielding or by buckling. The load at which buckling occurs is a function of slenderness, and for very slender members this load could be quite small.

If the member is so slender (we give a precise definition of slenderness shortly) that the stress just before buckling is below the proportional limit—that is, the member is still elastic—the critical buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{4.1}$$

where E is the modulus of elasticity of the material, I is the moment of inertia of the cross-sectional area with respect to the minor principal axis, and L is the length of the member between points of support. For Equation 4.1 to be valid, the member must be elastic, and its ends must be free to rotate but not translate laterally. This end condition is satisfied by hinges or pins, as shown in Figure 4.2. This remarkable

FIGURE 4.1

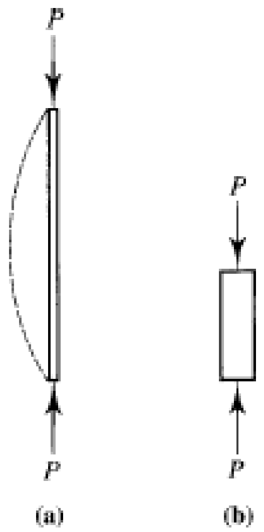
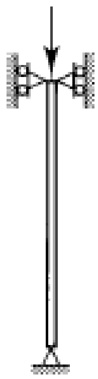


FIGURE 4.2



relationship was first formulated by Swiss mathematician Leonhard Euler and published in 1759. The critical load is sometimes referred to as the *Euler load* or the *Euler buckling load*. The validity of Equation 4.1 has been demonstrated convincingly by numerous tests. Its derivation is given here to illustrate the importance of the end conditions.

For convenience, in the following derivation, the member will be oriented with its longitudinal axis along the x-axis of the coordinate system given in Figure 4.3. The roller support is to be interpreted as restraining the member from translating either up or down. An axial compressive load is applied and gradually increased. If a temporary transverse load is applied so as to deflect the member into the shape indicated by the dashed line, the member will return to its original position when this temporary load is removed if the axial load is less than the critical buckling load. The critical buckling load, P_{cr}, is defined as the load that is just large enough to maintain the deflected shape when the temporary transverse load is removed.

The differential equation giving the deflected shape of an elastic member subjected to bending is

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \tag{4.2}$$

where x locates a point along the longitudinal axis of the member, y is the deflection of the axis at that point, and M is the bending moment at the point. E and I were previously defined, and here the moment of inertia I is with respect to the axis of bending (buckling). This equation was derived by Jacob Bernoulli and independently by Euler, who specialized it for the column buckling problem (Timoshenko, 1953). If we begin at the point of buckling, then from Figure 4.3 the bending moment is $P_{cr}y$. Equation 4.2 can then be written as

$$y'' + \frac{P_{cr}}{EI}y = 0$$

where the prime denotes differentiation with respect to x. This is a second-order, linear, ordinary differential equation with constant coefficients and has the solution

$$y = A\cos(cx) + B\sin(cx)$$

where

$$c = \sqrt{\frac{P_{cr}}{EI}}$$

and A and B are constants. These constants are evaluated by applying the following boundary conditions:

$$\text{At } x = 0, y = 0: \quad 0 = A\cos(0) + B\sin(0) \qquad A = 0$$
$$\text{At } x = L, y = 0: \quad 0 = B\sin(cL)$$

This last condition requires that $\sin(cL)$ be zero if B is not to be zero (the trivial solution, corresponding to $P = 0$). For $\sin(cL) = 0$,

$$cL = 0, \pi, 2\pi, 3\pi, \ldots = n\pi, \qquad n = 0, 1, 2, 3, \ldots$$

From

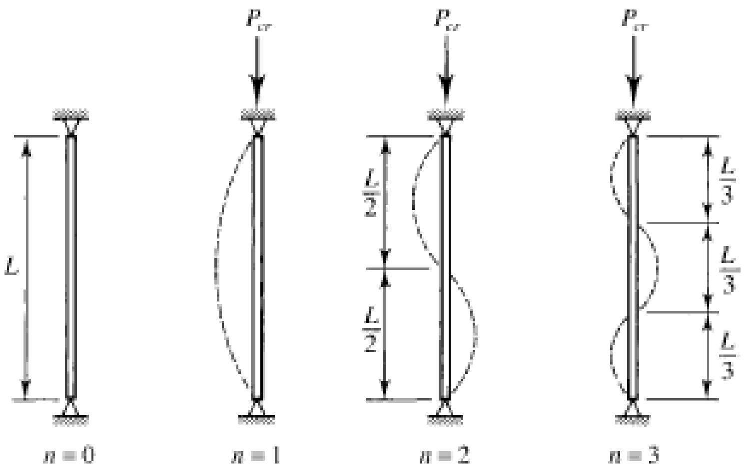
$$c = \sqrt{\frac{P_{cr}}{EI}}$$

we obtain

$$cL = \left(\sqrt{\frac{P_{cr}}{EI}}\right)L = n\pi, \quad \frac{P_{cr}}{EI}L^2 = n^2\pi^2 \quad \text{and} \quad P_{cr} = \frac{n^2\pi^2EI}{L^2}$$

The various values of n correspond to different buckling modes; $n = 1$ represents the first mode, $n = 2$ the second, and so on. A value of zero gives the trivial case of no load. These buckling modes are illustrated in Figure 4.4. Values of n larger than 1 are not possible unless the compression member is physically restrained from deflecting at the points where the reversal of curvature would occur.

FIGURE 4.4



The solution to the differential equation is therefore

$$y = B\sin\left(\frac{n\pi x}{L}\right)$$

and the coefficient B is indeterminate. This result is a consequence of approximations made in formulating the differential equation; a linear representation of a nonlinear phenomenon was used.

For the usual case of a compression member with no supports between its ends, $n = 1$ and the Euler equation is written as

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{4.3}$$

It is convenient to rewrite Equation 4.3 as

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EAr^2}{L^2} = \frac{\pi^2 EA}{(L/r)^2}$$

where A is the cross-sectional area and r is the radius of gyration with respect to the axis of buckling. The ratio L/r is the slenderness ratio and is the measure of a member's slenderness, with large values corresponding to slender members.

If the critical load is divided by the cross-sectional area, the critical buckling stress is obtained:

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} \tag{4.4}$$

At this compressive stress, buckling will occur about the axis corresponding to r. Buckling will take place as soon as the load reaches the value given by Equation 4.3, and the column will become unstable about the principal axis corresponding to the largest slenderness ratio. This axis usually is the axis with the smaller moment of inertia (we examine exceptions to this condition later). Thus the minimum moment of inertia and radius of gyration of the cross section should ordinarily be used in Equations 4.3 and 4.4.

EXAMPLE 4.1

A W12 × 50 is used as a column to support an axial compressive load of 145 kips. The length is 20 feet, and the ends are pinned. Without regard to load or resistance factors, investigate this member for stability. (The grade of steel need not be known: The critical buckling load is a function of the modulus of elasticity, not the yield stress or ultimate tensile strength.)

SOLUTION For a W12 × 50,

$$\text{Minimum } r = r_y = 1.96 \text{ in.}$$

$$\text{Maximum } \frac{L}{r} = \frac{20(12)}{1.96} = 122.4$$

$$P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = \frac{\pi^2 (29,000)(14.6)}{(122.4)^2} = 278.9 \text{ kips}$$

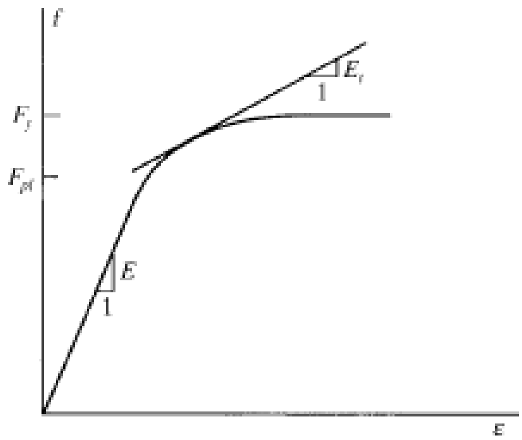
ANSWER Because the applied load of 145 kips is less than P_{cr}, the column remains stable and has an overall factor of safety against buckling of 278.9/145 = 1.92.

Early researchers soon found that Euler's equation did not give reliable results for stocky, or less slender, compression members. The reason is that the small slenderness ratio for members of this type causes a large buckling stress (from Equation 4.4). If the stress at which buckling occurs is greater than the proportional limit of the material, the relation between stress and strain is not linear, and the modulus of elasticity E can no longer be used. (In Example 4.1, the stress at buckling is $P_{cr}/A = 278.9/14.6 = 19.10$ ksi, which is well below the proportional limit for any grade of structural steel.) This difficulty was initially resolved by Friedrich Engesser, who proposed in 1889 the use of a variable tangent modulus, E_t, in Equation 4.3. For a material with a stress–strain curve like the one shown in Figure 4.5, E is not a constant for stresses greater than the proportional limit F_{pl}. The tangent modulus E_t is defined as the slope of the tangent to the stress–strain curve for values of f between F_{pl} and F_y. If the compressive stress at buckling, P_{cr}/A, is in this region, it can be shown that

$$P_{cr} = \frac{\pi^2 E_t I}{L^2} \tag{4.5}$$

Equation 4.5 is identical to the Euler equation, except that E_t is substituted for E.

FIGURE 4.5



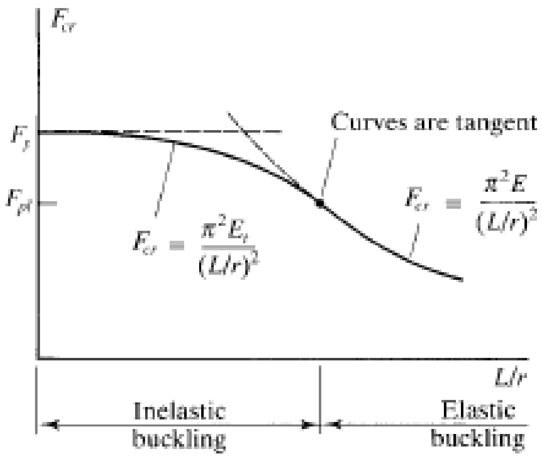
The stress–strain curve shown in Figure 4.5 is different from those shown earlier for ductile steel (in Figures 1.3 and 1.4) because it has a pronounced region of nonlinearity. This curve is typical of a compression test of a short length of W-shape called a *stub column,* rather than the result of a tensile test. The nonlinearity is primarily because of the presence of residual stresses in the W-shape. When a hot-rolled shape cools after rolling, all elements of the cross section do not cool at the same rate. The tips of the flanges, for example, cool faster than the junction of the flange and the web. This uneven cooling induces stresses that remain permanently. Other factors, such as welding and cold-bending to create curvature in a beam, can contribute to the residual stress, but the cooling process is its chief source.

Note that E_t is smaller than E and for the same L/r corresponds to a smaller critical load, P_{cr}. Because of the variability of E_t, computation of P_{cr} in the inelastic range by the use of Equation 4.5 is difficult. In general, a trial-and-error approach must be used, and a compressive stress–strain curve such as the one shown in Figure 4.5 must be used to determine E_t for trial values of P_{cr}. For this reason, most design specifications, including the AISC Specification, contain empirical formulas for inelastic columns.

Engesser's tangent modulus theory had its detractors, who pointed out several inconsistencies. Engesser was convinced by their arguments, and in 1895 he refined his theory to incorporate a reduced modulus, which has a value between E and E_t. Test results, however, always agreed more closely with the tangent modulus theory. Shanley (1947) resolved the apparent inconsistencies in the original theory, and today the tangent modulus formula, Equation 4.5, is accepted as the correct one for inelastic buckling. Although the load predicted by this equation is actually a lower bound on the true value of the critical load, the difference is slight (Bleich, 1952).

For any material, the critical buckling stress can be plotted as a function of slenderness, as shown in Figure 4.6. The tangent modulus curve is tangent to the Euler curve at the point corresponding to the proportional limit of the material. The composite curve, called a *column strength curve,* completely describes the strength of any column of a given material. Other than F_y, E, and E_t, which are properties of the material, the strength is a function only of the slenderness ratio.

FIGURE 4.6



Effective Length

Both the Euler and tangent modulus equations are based on the following assumptions:

1. The column is perfectly straight, with no initial crookedness.
2. The load is axial, with no eccentricity.
3. The column is pinned at both ends.

The first two conditions mean that there is no bending moment in the member before buckling. As mentioned previously, some accidental moment will be present, but in most cases it can be ignored. The requirement for pinned ends, however, is a serious limitation, and provisions must be made for other support conditions. The pinned-end condition requires that the member be restrained from lateral translation, but not rotation, at the ends. Constructing a frictionless pin connection is virtually impossible, so even this support condition can only be closely approximated at best. Obviously, all columns must be free to deform axially.

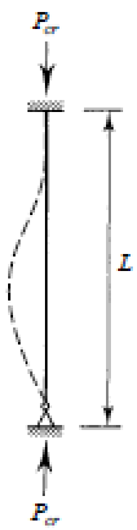
Other end conditions can be accounted for in the derivation of Equation 4.3. In general, the bending moment will be a function of x, resulting in a nonhomogeneous differential equation. The boundary conditions will be different from those in the original derivation, but the overall procedure will be the same. The form of the resulting equation for P_{cr} will also be the same. For example, consider a compression member pinned at one end and fixed against rotation and translation at the other, as shown in Figure 4.7. The Euler equation for this case, derived in the same manner as Equation 4.3, is

$$P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

or

$$P_{cr} = \frac{2.05\pi^2 EA}{(L/r)^2} = \frac{\pi^2 EA}{(0.70L/r)^2}$$

FIGURE 4.7



Thus this compression member has the same load capacity as a column that is pinned at both ends and is only 70% as long as the given column. Similar expressions can be found for columns with other end conditions.

The column buckling problem can also be formulated in terms of a fourth-order differential equation instead of Equation 4.2. This proves to be convenient when dealing with boundary conditions other than pinned ends.

For convenience, the equations for critical buckling load will be written as

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 E_t A}{(KL/r)^2} \qquad (4.6a/4.6b)$$

where KL is the *effective length*, and K is the *effective length factor*. The effective length factor for the fixed-pinned compression member is 0.70. For the most favorable condition of both ends fixed against rotation and translation, $K = 0.5$. Values of K for these and other cases can be determined with the aid of Table C-A-7.1 in the Commentary to AISC Specification Appendix 7. The three conditions mentioned thus far are included, as well as some for which end translation is possible. Two values of K are given: a theoretical value and a recommended design value to be used when the ideal end condition is approximated. Hence, unless a "fixed" end is perfectly fixed, the more conservative design values are to be used. Only under the most extraordinary circumstances would the use of the theoretical values be justified. Note, however, that the theoretical and recommended design values are the same for conditions (d) and (f) in Commentary Table C-A-7.1. The reason is that any deviation from a perfectly frictionless hinge or pin introduces rotational restraint and tends to reduce K. Therefore, use of the theoretical values in these two cases is conservative.

The use of the effective length KL in place of the actual length L in no way alters any of the relationships discussed so far. The column strength curve shown in Figure 4.6 is unchanged except for renaming the abscissa KL/r. The critical buckling stress corresponding to a given length, actual or effective, remains the same.

4.3 AISC REQUIREMENTS

The basic requirements for compression members are covered in Chapter E of the AISC Specification. The nominal compressive strength is

$$P_n = F_{cr} A_g \qquad \text{(AISC Equation E3-1)}$$

For LRFD,

$$P_u \leq \phi_c P_n$$

where

P_u = sum of the factored loads

ϕ_c = resistance factor for compression = 0.90

$\phi_c P_n$ = design compressive strength

For ASD,

$$P_a \leq \frac{P_n}{\Omega_c}$$

where

P_a = sum of the service loads
Ω_c = safety factor for compression = 1.67
P_n/Ω_c = allowable compressive strength

If an allowable stress formulation is used,

$$f_a \leq F_a$$

where

f_a = computed axial compressive stress = P_a/A_g
F_a = allowable axial compressive stress

$$= \frac{F_{cr}}{\Omega_c} = \frac{F_{cr}}{1.67} = 0.6F_{cr} \tag{4.7}$$

In order to present the AISC expressions for the critical stress F_{cr}, we first define the Euler load as

$$P_e = \frac{\pi^2 EA}{(KL/r)^2}$$

This is the critical buckling load according to the Euler equation. The Euler stress is

$$F_e = \frac{P_e}{A} = \frac{\pi^2 E}{(KL/r)^2} \qquad \text{(AISC Equation E3-4)}$$

With a slight modification, this expression will be used for the critical stress in the elastic range. To obtain the critical stress for elastic columns, the Euler stress is reduced as follows to account for the effects of initial crookedness:

$$F_{cr} = 0.877F_e \tag{4.8}$$

For inelastic columns, the tangent modulus equation, Equation 4.6b, is replaced by the exponential equation

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \tag{4.9}$$

With Equation 4.9, a direct solution for inelastic columns can be obtained, avoiding the trial-and-error approach inherent in the use of the tangent modulus equation. At the boundary between inelastic and elastic columns, Equations 4.8 and 4.9 give the same value of F_{cr}. This occurs when KL/r is approximately

$$4.71\sqrt{\frac{E}{F_y}}$$

To summarize,

$$\text{When } \frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = (0.658^{F_y/F_e})F_y \tag{4.10}$$

$$\text{When } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = 0.877F_e \tag{4.11}$$

The AISC Specification provides for separating inelastic and elastic behavior based on either the value of KL/r (as in equations 4.10 and 4.11) or the value of the ratio F_y/F_e. The limiting value of F_y/F_e can be derived as follows. From AISC Equation E3-4,

$$\frac{KL}{r} = \sqrt{\frac{\pi^2 E}{F_e}}$$

For $\quad \dfrac{KL}{r} \leq 4.71\sqrt{\dfrac{E}{F_y}},$

$$\sqrt{\frac{\pi^2 E}{F_e}} \leq 4.71\sqrt{\frac{E}{F_y}}$$

$$\frac{F_y}{F_e} \leq 2.25$$

The complete AISC Specification for compressive strength is as follows:

$$\text{When } \frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}} \quad \text{or} \quad \frac{F_y}{F_e} \leq 2.25,$$

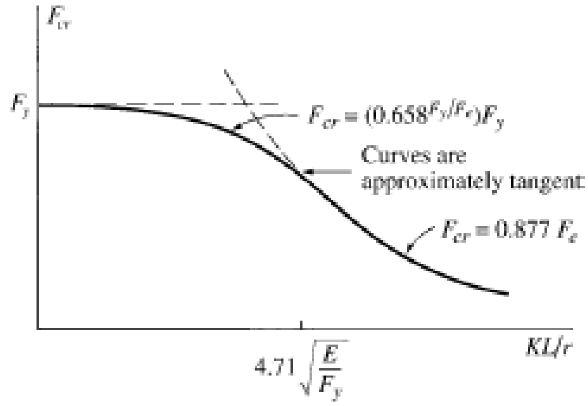
$$F_{cr} = (0.658^{F_y/F_e})F_y \qquad \text{(AISC Equation E3-2)}$$

$$\text{When } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} \quad \text{or} \quad \frac{F_y}{F_e} > 2.25,$$

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E3-3)}$$

In this book, we will usually use the limit on KL/r, as expressed in Equations 4.10 and 4.11. These requirements are represented graphically in Figure 4.8.

FIGURE 4.8



AISC Equations E3-2 and E3-3 are a condensed version of five equations that cover five ranges of KL/r (Galambos, 1988). These equations are based on experimental and theoretical studies that account for the effects of residual stresses and an initial out-of-straightness of $L/1500$, where L is the member length. A complete derivation of these equations is given by Tide (2001).

Although AISC does not require an upper limit on the slenderness ratio KL/r, an upper limit of 200 is recommended (see user note in AISC E2). This is a practical upper limit, because compression members that are any more slender will have little strength and will not be economical.

EXAMPLE 4.2

A W14 × 74 of A992 steel has a length of 20 feet and pinned ends. Compute the design compressive strength for LRFD and the allowable compressive strength for ASD.

SOLUTION Slenderness ratio:

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.48} = 96.77 < 200 \quad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since 96.77 < 113, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(96.77)^2} = 30.56 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/30.56)}(50) = 25.21 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 25.21(21.8) = 549.6 \text{ kips}$$

LRFD SOLUTION

The design strength is

$$\phi_c P_n = 0.90(549.6) = 495 \text{ kips}$$

ASD SOLUTION

From Equation 4.7, the allowable stress is

$$F_a = 0.6F_{cr} = 0.6(25.21) = 15.13 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 15.13(21.8) = 330 \text{ kips}$$

ANSWER

Design compressive strength = 495 kips. Allowable compressive strength = 330 kips.

4.6 DESIGN

The selection of an economical rolled shape to resist a given compressive load is simple with the aid of the column load tables. Enter the table with the effective length and move horizontally until you find the desired available strength (or something slightly larger). In some cases, you must continue the search to be certain that you have found the lightest shape. Usually the category of shape (W, WT, etc.) will have been decided upon in advance. Often the overall nominal dimensions will also be known because of architectural or other requirements. As pointed out earlier, all tabulated values correspond to a slenderness ratio of 200 or less. The tabulated unsymmetrical shapes—the structural tees and the single and double angles—require special consideration and are covered in Section 4.8.

EXAMPLE 4.6

A compression member is subjected to service loads of 165 kips dead load and 535 kips live load. The member is 26 feet long and pinned at each end. Use A992 steel and select a W14 shape.

LRFD SOLUTION

Calculate the factored load:

$$P_u = 1.2D + 1.6L = 1.2(165) + 1.6(535) = 1054 \text{ kips}$$

$$\therefore \text{ Required design strength } \phi_c P_n = 1054 \text{ kips.}$$

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 145 has a design strength of 1230 kips.

ANSWER

Use a W14 × 145.

ASD SOLUTION

Calculate the total applied load:

$$P_a = D + L = 165 + 535 = 700 \text{ kips}$$

$$\therefore \text{ Required allowable strength } \frac{P_n}{\Omega_c} = 700 \text{ kips}$$

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 132 has an allowable strength of 702 kips.

ANSWER Use a W14 × 132.

EXAMPLE 4.7

Select the lightest W-shape that can resist a service dead load of 62.5 kips and a service live load of 125 kips. The effective length is 24 feet. Use ASTM A992 steel.

SOLUTION The appropriate strategy here is to find the lightest shape for each nominal depth in the column load tables and then choose the lightest overall.

LRFD SOLUTION The factored load is

$$P_u = 1.2D + 1.6L = 1.2(62.5) + 1.6(125) = 275 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $\phi_c P_n \geq 275$ kips.

W10: W10 × 54, $\phi_c P_n = 282$ kips

W12: W12 × 58, $\phi_c P_n = 292$ kips

W14: W14 × 61, $\phi_c P_n = 293$ kips

Note that the strength is not proportional to the weight (which is a function of the cross-sectional area).

ANSWER Use a W10 × 54.

ASD SOLUTION The total applied load is

$$P_a = D + L = 62.5 + 125 = 188 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $P_n/\Omega_c \geq 188$ kips.

W10: W10 × 54, $\dfrac{P_n}{\Omega_c} = 188$ kips

$$\text{W12:}\quad \text{W12}\times 58,\quad \frac{P_n}{\Omega_c} = 194 \text{ kips}$$

$$\text{W14:}\quad \text{W14}\times 61,\quad \frac{P_n}{\Omega_c} = 195 \text{ kips}$$

Note that the strength is not proportional to the weight (which is a function of the cross-sectional area).

ANSWER Use a W10 × 54.

EXAMPLE 4.8

Select a W18 shape of A992 steel that can resist a service dead load of 100 kips and a service live load of 300 kips. The effective length KL is 26 feet.

LRFD SOLUTION

$P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600$ kips

Try $F_{cr} = 33$ ksi (an arbitrary choice of two-thirds F_y):

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(33)} = 20.2 \text{ in.}^2$$

Try a W18 × 71:

$$A_g = 20.9 \text{ in.}^2 > 20.2 \text{ in.}^2 \quad \text{(OK)}$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{1.70} = 183.5 < 200 \quad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(183.5)^2} = 8.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(8.5) = 7.455 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(7.455)(20.9) = 140 \text{ kips} < 600 \text{ kips} \quad \text{(N.G.)}$$

Because the initial estimate of F_{cr} was so far off, assume a value about halfway between 33 and 7.455 ksi. Try $F_{cr} = 20$ ksi.

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(20)} = 33.3 \text{ in.}^2$$

Try a W18 × 119:

$$A_g = 35.1 \text{ in.}^2 > 33.3 \text{ in.}^2 \quad \text{(OK)}$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.69} = 116.0 < 200 \quad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(116.0)^2} = 21.27 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.27) = 18.65 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.65)(35.1) = 589 \text{ kips} < 600 \text{ kips} \quad \text{(N.G.)}$$

This is very close, so try the next larger size.

Try a W18 × 130:

$$A_g = 38.3 \text{ in.}^2$$
$$\frac{KL}{r_{min}} = \frac{26 \times 12}{2.70} = 115.6 < 200 \quad \text{(OK)}$$
$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(115.6)^2} = 21.42 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.42) = 18.79 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.79)(38.3) = 648 \text{ kips} > 600 \text{ kips} \quad \text{(OK.)}$$

This shape is not slender (there is no footnote in the dimensions and properties table to indicate that it is), so local buckling does not have to be investigated.

ANSWER Use a W18 × 130.

ASD SOLUTION The ASD solution procedure is essentially the same as for LRFD, and the same trial values of F_{cr} will be used here.

$$P_a = D + L = 100 + 300 = 400 \text{ kips}$$

Try $F_{cr} = 33$ ksi (an arbitrary choice of two-thirds F_y):

$$\text{Required } A_g = \frac{P_a}{0.6F_{cr}} = \frac{400}{0.6(33)} = 20.2 \text{ in.}^2$$

Try a W18 × 71:

$$A_g = 20.9 \text{ in.}^2 > 20.2 \text{ in.}^2 \quad \text{(OK)}$$
$$\frac{KL}{r_{min}} = \frac{26 \times 12}{1.70} = 183.5 < 200 \quad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(183.5)^2} = 8.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(8.5) = 7.455 \text{ ksi}$$

$$\frac{P_n}{\Omega_c} = 0.6F_{cr}A_g = 0.6(7.455)(20.9) = 93.5 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

Because the initial estimate of F_{cr} was so far off, assume a value about halfway between 33 and 7.455 ksi. Try $F_{cr} = 20$ ksi.

$$\text{Required } A_g = \frac{P_a}{0.6F_{cr}} = \frac{400}{0.6(20)} = 33.3 \text{ in.}^2$$

Try a W18 × 119:

$$A_g = 35.1 \text{ in.}^2 > 33.3 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{KL}{r_{min}} = \frac{26 \times 12}{2.69} = 116.0 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(116.0)^2} = 21.27 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.27) = 18.65 \text{ ksi}$$

$$0.6F_{cr}A_g = 0.6(18.65)(35.1) = 393 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

This is very close, so try the next larger size.

Try a W18 × 130:

$$A_g = 38.3 \text{ in.}^2$$

$$\frac{KL}{r_{min}} = \frac{26 \times 12}{2.70} = 115.6 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(115.6^2)} = 21.42 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.42) = 18.79 \text{ ksi}$$

$$0.6F_{cr}A_g = 0.6(18.79)(38.3) = 432 \text{ kips} < 400 \text{ kips} \qquad \text{(OK)}$$

This shape is not slender (there is no footnote in the dimensions and properties table to indicate that it is), so local buckling does not have to be investigated.

ANSWER Use a W18 × 130.

Read Page 140 Ex 4.9 to 4.18

CHAPTER 5

Beams

5.1 INTRODUCTION

Beams are structural members that support transverse loads and are therefore subjected primarily to flexure, or bending. If a substantial amount of axial load is also present, the member is referred to as a *beam–column* (beam–columns are considered in Chapter 6). Although some degree of axial load will be present in any structural member, in many practical situations this effect is negligible and the member can be treated as a beam. Beams are usually thought of as being oriented horizontally and subjected to vertical loads, but that is not necessarily the case. A structural member is considered to be a beam if it is loaded so as to cause bending.

Commonly used cross-sectional shapes include the W, S, and M shapes. Channel shapes are sometimes used, as are beams built up from plates, in the form of I or box shapes. For reasons to be discussed later, doubly symmetric shapes such as the standard rolled W, M, and S shapes are the most efficient.

Coverage of beams in the AISC Specification is spread over two chapters: Chapter F, "Design of Members for Flexure," and Chapter G, "Design of Members for Shear." Several categories of beams are covered in the Specification; in this book, we cover the most common cases in the present chapter, and we cover a special case, plate girders, in Chapter 10.

Figure 5.1 shows two types of beam cross sections; a hot-rolled doubly-symmetric I shape and a welded doubly-symmetric built-up I shape. The hot-rolled I shape is the one most commonly used for beams. Welded shapes usually fall into the category classified as plate girders.

For flexure (shear will be covered later), the required and available strengths are moments. For load and resistance factor design (LRFD), Equation 2.6 can be written as

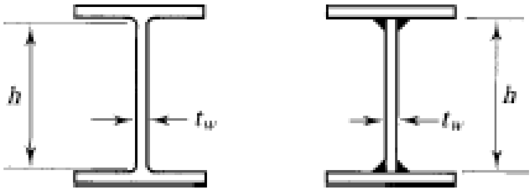
$$M_u \leq \phi_b M_n \tag{5.1}$$

where

M_u = required moment strength = maximum moment caused by the controlling load combination from ASCE 7

ϕ_b = resistance factor for bending (flexure) = 0.90

FIGURE 5.1



M_n = nominal moment strength

The right-hand side of Equation 5.1 is the design strength, sometimes called the *design moment.*

For allowable strength design (ASD), Equation 2.7 can be written as

$$M_a \le \frac{M_n}{\Omega_b} \tag{5.2}$$

where

M_a = required moment strength = maximum moment corresponding to the controlling load combination from ASCE 7

Ω_b = safety factor for bending = 1.67

Equation 5.2 can also be written as

$$M_a \le \frac{M_n}{1.67} = 0.6M_n$$

Dividing both sides by the elastic section modulus S (which will be reviewed in the next section), we get an equation for *allowable stress design:*

$$\frac{M_a}{S} \le \frac{0.6M_n}{S}$$

or

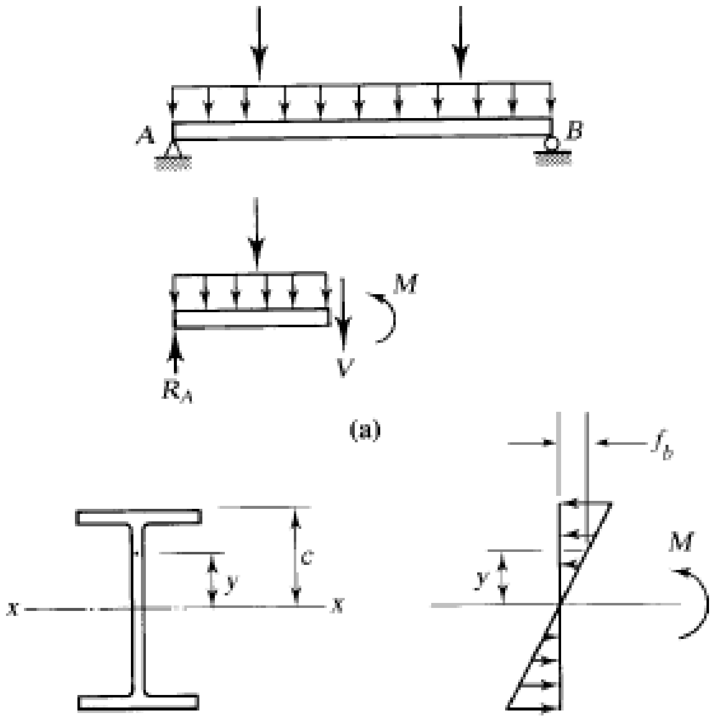
$$f_b \le F_b$$

where

f_b = maximum computed bending stress

F_b = allowable bending stress

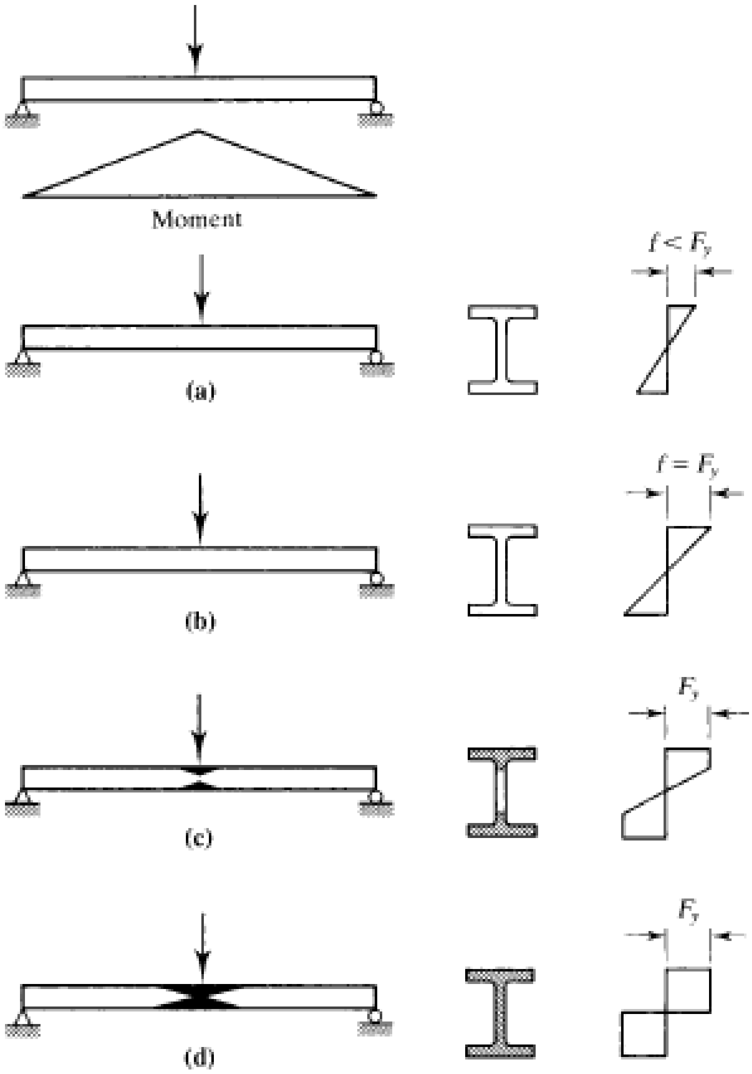
FIGURE 5.2



of the beam. (Shear is considered separately in Section 5.8.) From elementary mechanics of materials, the stress at any point can be found from the flexure formula:

$$f_b = \frac{My}{I_x} \tag{5.3}$$

FIGURE 5.3



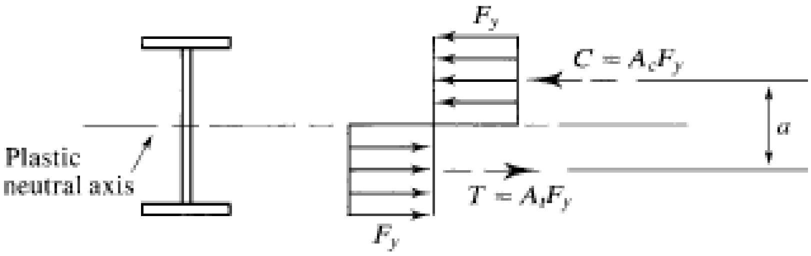
$$C = T$$
$$A_c F_y = A_t F_y$$
$$A_c = A_t$$

Thus the plastic neutral axis divides the cross section into two equal areas. For shapes that are symmetrical about the axis of bending, the elastic and plastic neutral axes are the same. The plastic moment, M_p, is the resisting couple formed by the two equal and opposite forces, or

$$M_p = F_y(A_c)a = F_y(A_t)a = F_y\left(\frac{A}{2}\right)a = F_y Z$$

FIGURE 5.4

FIGURE 5.5



where

A = total cross-sectional area

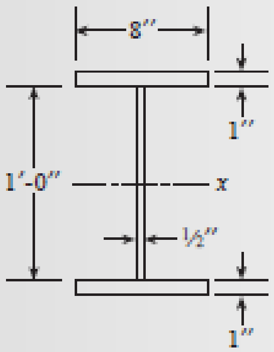
a = distance between the centroids of the two half-areas

$Z = \left(\frac{A}{2}\right)a$ = plastic section modulus

EXAMPLE 5.1

For the built-up shape shown in Figure 5.6, determine (a) the elastic section modulus S and the yield moment M_y and (b) the plastic section modulus Z and the plastic moment M_p. Bending is about the x-axis, and the steel is A572 Grade 50.

FIGURE 5.6



SOLUTION

a. Because of symmetry, the elastic neutral axis (the x-axis) is located at mid-depth of the cross section (the location of the centroid). The moment of inertia of the cross section can be found by using the parallel axis theorem, and the results of the calculations are summarized in Table 5.1.

TABLE 5.1

Component	$\bar{I}$	A	d	$\bar{I} + Ad^2$
Flange	0.6667	8	6.5	338.7
Flange	0.6667	8	6.5	338.7
Web	72	—	—	72.0
Sum				749.4

The elastic section modulus is

$$S = \frac{I}{c} = \frac{749.4}{1 + (12/2)} = \frac{749.4}{7} = 107 \text{ in.}^3$$

and the yield moment is

$$M_y = F_y S = 50(107) = 5350 \text{ in.-kips} = 446 \text{ ft-kips}$$

ANSWER $S = 107$ in.³ and $M_y = 446$ ft-kips.

b. Because this shape is symmetrical about the x-axis, this axis divides the cross section into equal areas and is therefore the plastic neutral axis. The centroid of the top half-area can be found by the principle of moments. Taking moments about the x-axis (the neutral axis of the entire cross section) and tabulating the computations in Table 5.2, we get

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{61}{11} = 5.545 \text{ in.}$$

TABLE 5.2

Component	A	y	Ay
Flange	8	6.5	52
Web	3	3	9
Sum	11		61

FIGURE 5.7

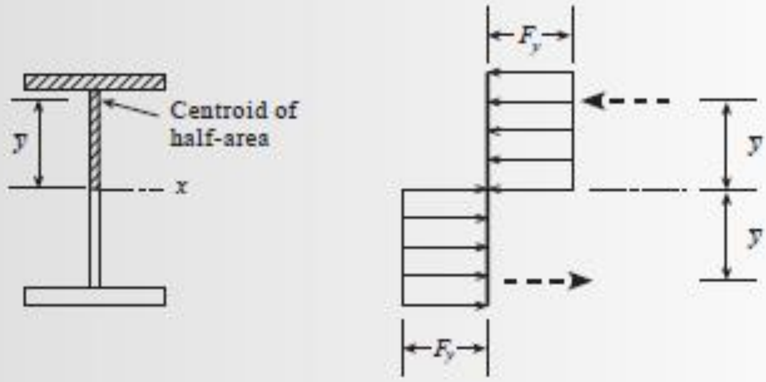


Figure 5.7 shows that the moment arm of the internal resisting couple is

$$a = 2\bar{y} = 2(5.545) = 11.09 \text{ in.}$$

and that the plastic section modulus is

$$\left(\frac{A}{2}\right)a = 11(11.09) = 122 \text{ in.}^3$$

The plastic moment is

$$M_p = F_y Z = 50(122) = 6100 \text{ in.-kips} = 508 \text{ ft-kips}$$

ANSWER $Z = 122$ in.³ and $M_p = 508$ ft-kips.

EXAMPLE 5.2

Compute the plastic moment, M_p, for a W10 × 60 of A992 steel.

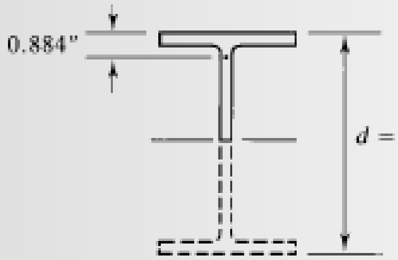
SOLUTION From the dimensions and properties tables in Part 1 of the *Manual*,

$$A = 17.7 \text{ in.}^2$$

$$\frac{A}{2} = \frac{17.7}{2} = 8.85 \text{ in.}^2$$

The centroid of the half-area can be found in the tables for WT shapes, which are cut from W shapes. The relevant shape here is the WT5 × 30, and the distance from the outside face of the flange to the centroid is 0.884 inch, as shown in Figure 5.8.

FIGURE 5.8



$$a = d - 2(0.884) = 10.2 - 2(0.884) = 8.432 \text{ in.}$$

$$Z = \left(\frac{A}{2}\right)a = 8.85(8.432) = 74.62 \text{ in.}^3$$

This result, when rounded to three significant figures, is the same as the value given in the dimensions and properties tables.

ANSWER $M_p = F_y Z = 50(74.62) = 3731$ in.-kips = 311 ft-kips.

5.3 STABILITY

If a beam can be counted on to remain stable up to the fully plastic condition, the nominal moment strength can be taken as the plastic moment capacity; that is,

$$M_n = M_p$$

Otherwise, M_n will be less than M_p.

As with a compression member, instability can be in an overall sense or it can be local. Overall buckling is illustrated in Figure 5.9a. When a beam bends, the compression region (above the neutral axis) is analogous to a column, and in a manner similar to a column, it will buckle if the member is slender enough. Unlike a column,

FIGURE 5.9

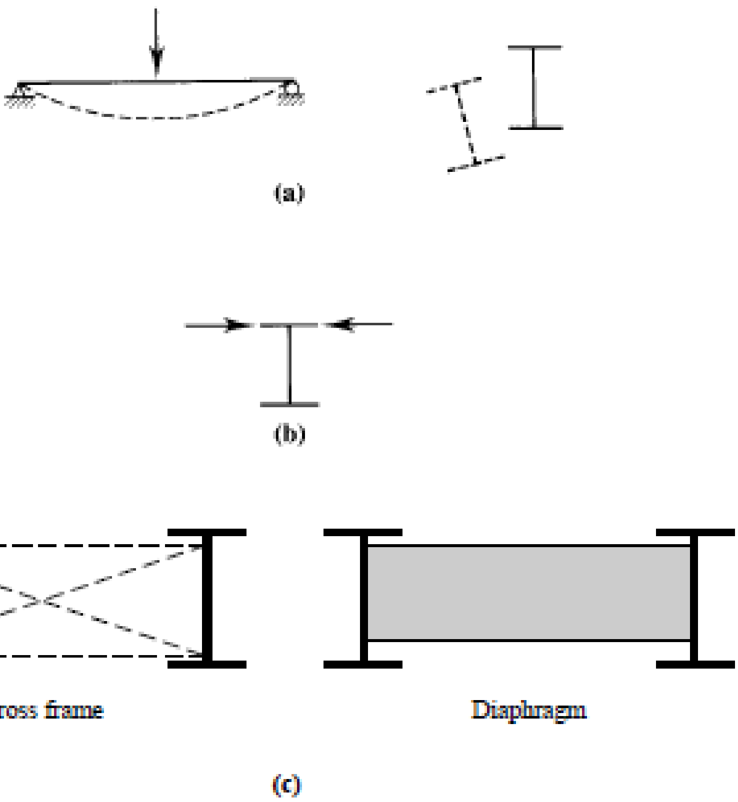
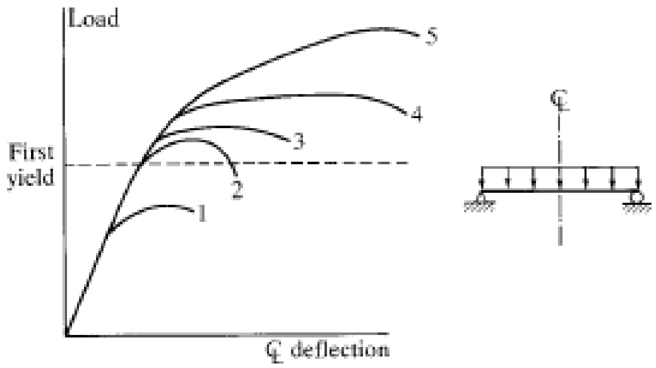


FIGURE 5.10



Curve 1 is the load-deflection curve of a beam that becomes unstable (in any way) and loses its load-carrying capacity before first yield (see Figure 5.3b) is attained. Curves 2 and 3 correspond to beams that can be loaded past first yield but not far enough for the formation of a plastic hinge and the resulting plastic collapse. If plastic collapse can be reached, the load-deflection curve will have the appearance of either curve 4 or curve 5. Curve 4 is for the case of uniform moment over the full length of the beam, and curve 5 is for a beam with a variable bending moment (moment gradient). Safe designs can be achieved with beams corresponding to any of these curves, but curves 1 and 2 represent inefficient use of material.

5.4 CLASSIFICATION OF SHAPES

AISC classifies cross-sectional shapes as compact, noncompact, or slender, depending on the values of the width-to-thickness ratios. For I shapes, the ratio for the projecting flange (an *unstiffened* element) is $b_f/2t_f$, and the ratio for the web (a *stiffened* element) is h/t_w. The classification of shapes is found in Section B4 of the Specification, "Member Properties," in Table B4.1b (Table B4.1a is for compression members). It can be summarized as follows. Let

λ = width-to-thickness ratio

λ_p = upper limit for compact category

λ_r = upper limit for noncompact category

Then

if $\lambda \le \lambda_p$ and the flange is continuously connected to the web, the shape is compact;

if $\lambda_p < \lambda \le \lambda_r$, the shape is noncompact; and

if $\lambda > \lambda_r$, the shape is slender.

The category is based on the worst width-to-thickness ratio of the cross section. For example, if the web is compact and the flange is noncompact, the shape is classified as noncompact. Table 5.3 has been extracted from AISC Table B4.1b and is specialized for hot-rolled I-shaped cross sections.

Table 5.3 also applies to channels, except that λ for the flange is b_f/t_f.

TABLE 5.3 Width-to-Thickness Parameters*

Element	λ	λ_p	λ_r
Flange	$\frac{b_f}{2t_f}$	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$
Web	$\frac{h}{t_w}$	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$

*For hot-rolled I shapes in flexure.

5.5 BENDING STRENGTH OF COMPACT SHAPES

A beam can fail by reaching M_p and becoming fully plastic, or it can fail by

1. lateral-torsional buckling (LTB), either elastically or inelastically;
2. flange local buckling (FLB), elastically or inelastically; or
3. web local buckling (WLB), elastically or inelastically.

If the maximum bending stress is less than the proportional limit when buckling occurs, the failure is said to be *elastic*. Otherwise, it is *inelastic*. (See the related discussion in Section 4.2, "Column Theory.")

For convenience, we first categorize beams as compact, noncompact, or slender, and then determine the moment resistance based on the degree of lateral support. The discussion in this section applies to two types of beams: (1) hot-rolled I shapes bent about the strong axis and loaded in the plane of the weak axis, and (2) channels bent about the strong axis and either loaded through the shear center or restrained against twisting. (The shear center is the point on the cross section through which a transverse load must pass if the beam is to bend without twisting.) Emphasis will be on I shapes. C-shapes are different only in that the width-to-thickness ratio of the flange is b_f/t_f rather than $b_f/2t_f$.

We begin with *compact shapes*, defined as those whose webs are continuously connected to the flanges and that satisfy the following width-to-thickness ratio requirements for the flange and the web:

$$\frac{b_f}{2t_f} \leq 0.38\sqrt{\frac{E}{F_y}} \quad \text{and} \quad \frac{h}{t_w} \leq 3.76\sqrt{\frac{E}{F_y}}$$

if the unbraced length is very short, the nominal moment strength, M_n, is the full plastic moment capacity of the shape, M_p. For members with inadequate lateral support, the moment resistance is limited by the lateral-torsional buckling strength, either inelastic or elastic.

The first category, laterally supported compact beams, is quite common and is the simplest case. For a doubly-symmetric, compact I- or C-shaped section bent about its major axis, AISC F2.1 gives the nominal strength as

$$M_n = M_p \qquad \text{(AISC Equation F2-1)}$$

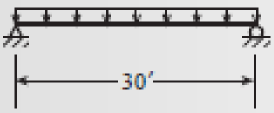
where

$$M_p = F_y Z_x$$

EXAMPLE 5.3

The beam shown in Figure 5.11 is a W16 × 31 of A992 steel. It supports a reinforced concrete floor slab that provides continuous lateral support of the compression flange. The service dead load is 450 lb/ft. This load is superimposed on the beam; it does not include the weight of the beam itself. The service live load is 550 lb/ft. Does this beam have adequate moment strength?

FIGURE 5.11



SOLUTION First, determine the nominal flexural strength. Check for compactness.

$$\frac{b_f}{2t_f} = 6.28 \qquad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15 > 6.28 \quad \therefore \text{ The flange is compact.}$$

$$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}} \qquad \therefore \text{ The web is compact.}$$

(The web is compact for all shapes in the *Manual* for $F_y \le 65$ ksi.)

This shape can also be identified as compact because there is no footnote in the dimensions and properties tables to indicate otherwise. Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(54.0) = 2700 \text{ in.-kips} = 225.0 \text{ ft kips.}$$

Compute the maximum bending moment. The total service dead load, including the weight of the beam, is

$$w_D = 450 + 31 = 481 \text{ lb/ft}$$

For a simply supported, uniformly loaded beam, the maximum bending moment occurs at midspan and is equal to

$$M_{max} = \frac{1}{8} wL^2$$

where w is the load in units of force per unit length, and L is the span length. Then

$$M_D = \frac{1}{8} w_D L^2 = \frac{1}{8}(0.481)(30)^2 = 54.11 \text{ ft-kips}$$
$$M_L = \frac{1}{8}(0.550)(30)^2 = 61.88 \text{ ft-kips}$$

LRFD SOLUTION

The dead load is less than 8 times the live load, so load combination 2 controls:

$$M_u = 1.2M_D + 1.6M_L = 1.2(54.11) + 1.6(61.88) = 164 \text{ ft-kips.}$$

Alternatively, the loads can be factored at the outset:

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.481) + 1.6(0.550) = 1.457 \text{ kips/ft}$$
$$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(1.457)(30)^2 = 164 \text{ ft-kips}$$

The design strength is

$$\phi_b M_n = 0.90(225.0) = 203 \text{ ft-kips} > 164 \text{ ft-kips} \qquad \text{(OK)}$$

ANSWER

The design moment is greater than the factored-load moment, so the W16 × 31 is satisfactory.

ASD SOLUTION

ASD load combination 2 controls.

$$M_a = M_D + M_L = 54.11 + 61.88 = 116.0 \text{ ft-kips}$$

Alternatively, the loads can be added before the moment is computed:

$$w_a = w_D + w_L = 0.481 + 0.550 = 1.031 \text{ kips/ft}$$
$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.031)(30)^2 = 116.0 \text{ ft-kips}$$

The allowable moment is

$$\frac{M_n}{\Omega_b} = \frac{M_n}{1.67} = 0.6M_n = 0.6(225.0) = 135 \text{ ft-kips} > 116 \text{ ft-kips} \qquad \text{(OK)}$$

Allowable stress solution:

The applied stress is

$$f_b = \frac{M_a}{S_x} = \frac{116.0(12)}{47.2} = 29.5 \text{ ksi}$$

The allowable stress is

$$F_b = \frac{0.6M_n}{S_x} = \frac{0.6(225.0)(12)}{47.2} = 34.3 \text{ ksi}$$

Since $f_b < F_b$, the beam has enough strength.

ANSWER The W16 × 31 is satisfactory.

5.7 SUMMARY OF MOMENT STRENGTH

The procedure for computation of nominal moment strength for I and C-shaped sections bent about the x axis will now be summarized. All terms in the following equations have been previously defined, and AISC equation numbers will not be shown. This summary is for compact and noncompact shapes (noncompact flanges) only (no slender shapes).

1. Determine whether the shape is compact.
2. If the shape is compact, check for lateral-torsional buckling as follows.

   If $L_b \le L_p$, there is no LTB, and $M_n = M_p$

   If $L_p < L_b \le L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le \boldsymbol{M_p}$$

   If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{cr}S_x \le M_p$$

where

$$F_{cr} = \frac{C_b\pi^2 E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$

3. If the shape is noncompact because of the flange, the nominal strength will be the smaller of the strengths corresponding to flange local buckling and lateral-torsional buckling.

   a. Flange local buckling:

      If $\lambda \le \lambda_p$, there is no FLB

      If $\lambda_p < \lambda \le \lambda_r$, the flange is noncompact, and

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

   b. Lateral-torsional buckling:

      If $L_b \le L_p$, there is no LTB

      If $L_p < L_b \le L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le \boldsymbol{M_p}$$

If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{cr} S_x \leq M_p$$

where

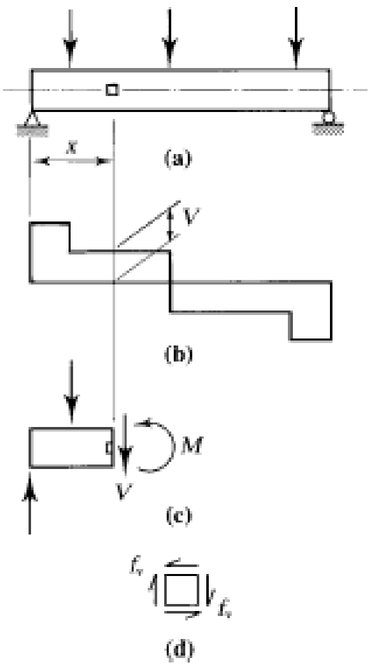
$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0} \left(\frac{L_b}{r_{ts}} \right)^2}$$

5.8 SHEAR STRENGTH

Beam shear strength is covered in Chapter G of the AISC Specification, "Design of Members for Shear." Both hot-rolled shapes and welded built-up shapes are covered. We discuss hot-rolled shapes in the present chapter of this book and built-up shapes in Chapter 10, "Plate Girders." The AISC provisions for hot-rolled shapes are covered in Section G2.1.

Before covering the AISC provisions for shear strength, we will first review some basic concepts from mechanics of materials. Consider the simple beam of Figure 5.17. At a distance x from the left end and at the neutral axis of the cross section, the state of stress is as shown in Figure 5.17d. Because this element is located at the neutral

FIGURE 5.17



axis, it is not subjected to flexural stress. From elementary mechanics of materials, the shearing stress is

$$f_v = \frac{VQ}{Ib} \tag{5.7}$$

where

f_v = vertical and horizontal shearing stress at the point of interest
V = vertical shear force at the section under consideration
Q = first moment, about the neutral axis, of the area of the cross section between the point of interest and the top or bottom of the cross section
I = moment of inertia about the neutral axis
b = width of the cross section at the point of interest

Equation 5.7 is based on the assumption that the stress is constant across the width b, and it is therefore accurate only for small values of b. For a rectangular cross section of depth d and width b, the error for $d/b = 2$ is approximately 3%. For $d/b = 1$, the error is 12% and for $d/b = \frac{1}{4}$, it is 100% (Higdon, Ohlsen, and Stiles, 1960). For this reason, Equation 5.7 cannot be applied to the flange of a W-shape in the same manner as for the web.

Figure 5.18 shows the shearing stress distribution for a W shape. Superimposed on the actual distribution is the average stress in the web, V/A_w, which does not differ much from the maximum web stress. Clearly, the web will completely yield long before the flanges begin to yield. Because of this, yielding of the web represents one of the shear limit states. Taking the shear yield stress as 60% of the tensile yield stress, we can write the equation for the stress in the web at failure as

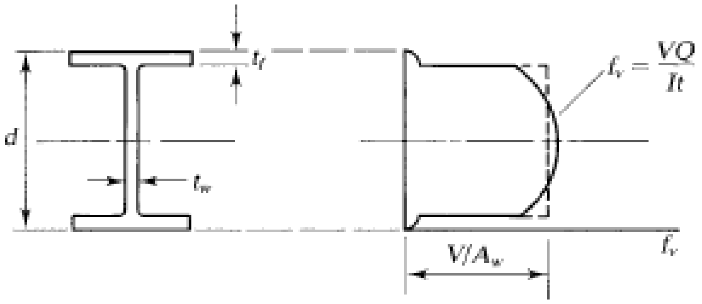
$$f_v = \frac{V_n}{A_w} = 0.6F_y$$

where A_w = area of the web. The nominal strength corresponding to this limit state is therefore

$$V_n = 0.6F_yA_w \tag{5.8}$$

and will be the nominal strength in shear provided that there is no shear buckling of the web. Whether that occurs will depend on h/t_w, the width-to-thickness ratio of the

FIGURE 5.18



AISC Specification Requirements for Shear

For LRFD, the relationship between required and available strength is

$$V_u \leq \phi_v V_n$$

where

V_u = maximum shear based on the controlling combination of factored loads
ϕ_v = resistance factor for shear

For ASD, the relationship is

$$V_a \leq \frac{V_n}{\Omega_v}$$

where

V_a = maximum shear based on the controlling combination of service loads
Ω_v = safety factor for shear

As we will see, the values of the resistance factor and safety factor will depend on the web width-to-thickness ratio.

Section G2.1 of the AISC Specification covers both beams with stiffened webs and beams with unstiffened webs. In most cases, hot-rolled beams will not have stiffeners, and we will defer treatment of stiffened webs until Chapter 10. The basic strength equation is

$$V_n = 0.6 F_y A_w C_v \qquad \text{(AISC Equation G2-1)}$$

where

A_w = area of the web ≈ dt_w
d = overall depth of the beam
C_v = ratio of critical web stress to shear yield stress

The value of C_v depends on whether the limit state is web yielding, web inelastic buckling, or web elastic buckling.

Case 1: For hot-rolled I shapes with

$$\frac{h}{t_w} \leq 2.24\sqrt{\frac{E}{F_y}}$$

The limit state is shear yielding, and

$$C_v = 1.0 \qquad \text{(AISC Equation G2-2)}$$
$$\phi_v = 1.00$$
$$\Omega_v = 1.50$$

Most W shapes with $F_y \leq 50$ ksi fall into this category (see User Note in AISC G2.1[a]).

Case 2: For all other doubly and singly symmetric shapes,

$\phi_v = 0.90$

$\Omega_v = 1.67$

and C_v is determined as follows:

For $\dfrac{h}{t_w} \le 1.10\sqrt{\dfrac{k_v E}{F_y}}$, there is no web instability, and

$$C_v = 1.0 \qquad \text{(AISC Equation G2-3)}$$

(This corresponds to Equation 5.8 for shear yielding.)

For $1.10\sqrt{\dfrac{k_v E}{F_y}} < \dfrac{h}{t_w} \le 1.37\sqrt{\dfrac{k_v E}{F_y}}$, inelastic web buckling can occur, and

$$C_v = \frac{1.10\sqrt{\dfrac{k_v E}{F_y}}}{h/t_w} \qquad \text{(AISC Equation G2-4)}$$

For $\dfrac{h}{t_w} > 1.37\sqrt{\dfrac{k_v E}{F_y}}$, the limit state is elastic web buckling, and

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \qquad \text{(AISC Equation G2-5)}$$

where

$k_v = 5$

This value of k_v is for unstiffened webs with $h/t_w < 260$. Although section G2.1 of the Specification does not give $h/t_w = 260$ as an upper limit, no value of k_v is given when $h/t_w \ge 260$. In addition, AISC F13.2, "Proportioning Limits for I-Shaped Members," states that h/t_w in unstiffened girders shall not exceed 260.

AISC Equation G2-5 is based on elastic stability theory, and AISC Equation G2-4 is an empirical equation for the inelastic region, providing a transition between the limit states of web yielding and elastic web buckling.

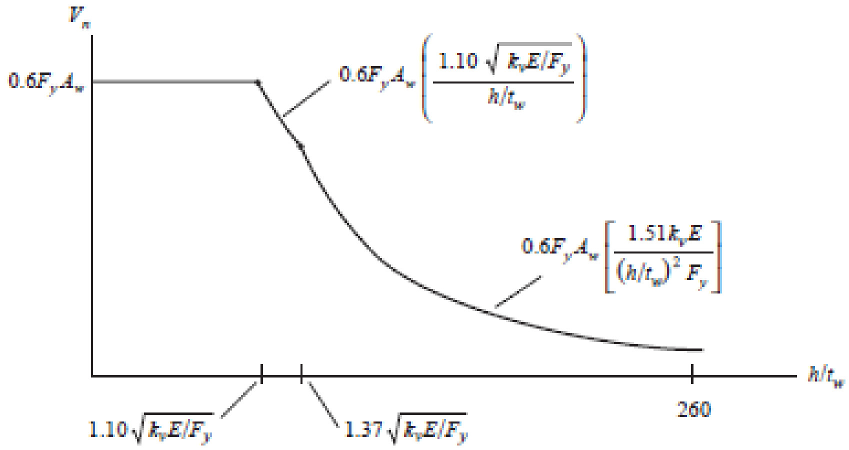
The relationship between shear strength and the web width-to-thickness ratio is analogous to that between flexural strength and the width-to-thickness ratio (for FLB) and between flexural strength and the unbraced length (for LTB). This relationship is illustrated in Figure 5.19.

Allowable Stress Formulation

The allowable strength relation

$$V_a \le \frac{V_n}{\Omega_v}$$

FIGURE 5.19



can also be written in terms of stress as

$$f_v \le F_v$$

where

$$f_v = \frac{V_a}{A_w} = \text{ applied shear stress}$$

$$F_v = \frac{V_n/\Omega_v}{A_w} = \frac{0.6F_yA_wC_v/\Omega_v}{A_w} = \text{ allowable shear stress}$$

For the most common case of hot-rolled I shapes with $h/t_w \le 2.24\sqrt{E/F_y}$,

$$F_v = \frac{0.6F_yA_w(1.0)/1.50}{A_w} = 0.4F_y$$

Shear is rarely a problem in rolled steel beams; the usual practice is to design a beam for flexure and then to check it for shear.

EXAMPLE 5.7

Check the beam in Example 5.6 for shear.

SOLUTION

From the dimensions and properties tables in Part 1 of the *Manual*, the web width-to-thickness ratio of a W14 × 90 is

$$\frac{h}{t_w} = 25.9$$

and the web area is $A_w = dt_w = 14.0(0.440) = 6.160$ in.²

$$2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29,000}{50}} = 54.0$$

Since

$$\frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}}$$

the strength is governed by shear yielding of the web and $C_v = 1.0$. (As pointed out in the Specification User Note, this will be the case for most W shapes with $F_y \leq 50$ ksi.) The nominal shear strength is

$$V_n = 0.6F_yA_wC_v = 0.6(50)(6.160)(1.0) = 184.8 \text{ kips}$$

LRFD SOLUTION

Determine the resistance factor ϕ_v.

$$\text{Since } \frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}},$$

$$\phi_v = 1.00$$

and the design shear strength is

$$\phi_v V_n = 1.00(184.8) = 185 \text{ kips}$$

From Example 5.6, $w_u = 2.080$ kips/ft and $L = 45$ ft. For a simply supported, uniformly loaded beam, the maximum shear occurs at the support and is equal to the reaction.

$$V_u = \frac{w_u L}{2} = \frac{2.080(45)}{2} = 46.8 \text{ kips} < 185 \text{ kips} \quad \text{(OK)}$$

ASD SOLUTION

Determine the safety factor Ω_v.

$$\text{Since } \frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}},$$

$$\Omega_v = 1.50$$

and the allowable shear strength is

$$\frac{V_n}{\Omega_v} = \frac{184.8}{1.50} = 123 \text{ kips}$$

From Example 5.6, the total service load is

$$w_a = w_D + w_L = 0.400 + 1.000 = 1.4 \text{ kips/ft}$$

The maximum shear is

$$V_a = \frac{w_a L}{2} = \frac{1.4(45)}{2} = 31.5 \text{ kips} < 123 \text{ kips} \quad \text{(OK)}$$

Alternately, a solution in terms of stress can be done. Since shear yielding controls ($C_v = 1.0$) and $\Omega_v = 1.50$, the allowable shear stress is

$$F_v = 0.4F_y = 0.4(50) = 20 \text{ ksi}$$

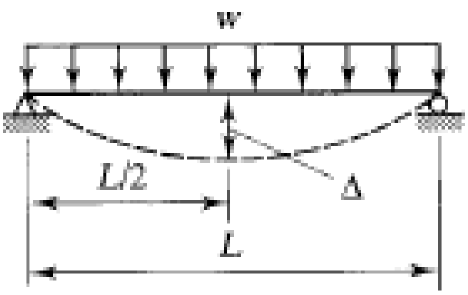
The required shear strength (stress) is

$$f_a = \frac{V_a}{A_w} = \frac{31.5}{6.160} = 5.11 \text{ ksi} < 20 \text{ ksi} \quad \text{(OK)}$$

ANSWER The required shear strength is less than the available shear strength, so the beam is satisfactory.

Read Page 222 Block Shear

FIGURE 5.22

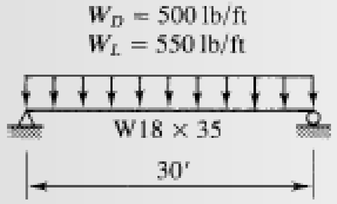


$$\Delta = \frac{5}{384}\frac{wL^4}{EI}$$

EXAMPLE 5.9

Compute the dead load and live load deflections for the beam shown in Figure 5.23. If the maximum permissible *live* load deflection is $L/360$, is the beam satisfactory?

FIGURE 5.23



SOLUTION It is more convenient to express the deflection in inches than in feet, so units of inches are used in the deflection formula. The dead load deflection is

$$\Delta_D = \frac{5}{384}\frac{w_D L^4}{EI} = \frac{5}{384}\frac{(0.500/12)(30 \times 12)^4}{29{,}000(510)} = 0.616 \text{ in.}$$

The live load deflection is

$$\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI} = \frac{5}{384}\frac{(0.550/12)(30 \times 12)^4}{29{,}000(510)} = 0.678 \text{ in.}$$

The maximum permissible live load deflection is

$$\frac{L}{360} = \frac{30(12)}{360} = 1.0 \text{ in.} > 0.678 \text{ in.} \quad \text{(OK)}$$

ANSWER The beam satisfies the deflection criterion.

Ponding is one deflection problem that does affect the safety of a structure. It is a potential hazard for flat roof systems that can trap rainwater. If drains become clogged during a storm, the weight of the water will cause the roof to deflect, thus providing a reservoir for still more water. If this process proceeds unabated, collapse can occur. The AISC specification requires that the roof system have sufficient stiffness to prevent ponding, and it prescribes limits on stiffness parameters in Appendix 2, "Design for Ponding."

5.10 DESIGN

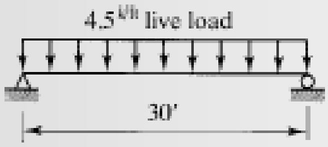
Beam design entails the selection of a cross-sectional shape that will have enough strength and that will meet serviceability requirements. As far as strength is concerned, flexure is almost always more critical than shear, so the usual practice is to design for flexure and then check shear. The design process can be outlined as follows.

1. Compute the required moment strength (i.e., the factored load moment M_u for LRFD or the unfactored moment M_a for ASD). The weight of the beam is part of the dead load but is unknown at this point. A value may be assumed and verified after a shape is selected, or the weight may be ignored initially and checked after a shape has been selected. Because the beam weight is usually a small part of the total load, if it is ignored at the beginning of a design problem, the selected shape will usually be satisfactory when the moment is recomputed.
2. Select a shape that satisfies this strength requirement. This can be done in one of two ways.
   a. Assume a shape, compute the available strength, and compare it with the required strength. Revise if necessary. The trial shape can be easily selected in only a limited number of situations (as in Example 5.10).
   b. Use the beam design charts in Part 3 of the *Manual*. This method is preferred, and we explain it following Example 5.10.
3. Check the shear strength.
4. Check the deflection.

EXAMPLE 5.10

Select a standard hot-rolled shape of A992 steel for the beam shown in Figure 5.24. The beam has continuous lateral support and must support a uniform service live load of 4.5 kips/ft. The maximum permissible live load deflection is $L/240$.

FIGURE 5.24



LRFD SOLUTION

Ignore the beam weight initially then check for its effect after a selection is made.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0) + 1.6(4.5) = 7.2 \text{ kips/ft}$$

$$\text{Required moment strength } M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(7.2)(30)^2 = 810.0 \text{ ft-kips}$$
$$= \text{required } \phi_b M_n$$

Assume that the shape will be compact. For a compact shape with full lateral support,

$$M_n = M_p = F_y Z_x$$

From $\phi_b M_n \geq M_u$,

$$\phi_b F_y Z_x \geq M_u$$

$$Z_x \geq \frac{M_u}{\phi_b F_y} = \frac{810.0(12)}{0.90(50)} = 216 \text{ in.}^3$$

The Z_x table lists hot-rolled shapes normally used as beams in order of decreasing plastic section modulus. Furthermore, they are grouped so that the shape at the top of each group (in bold type) is the lightest one that has enough section modulus to satisfy a required section modulus that falls within the group. In this example, the shape that comes closest to meeting the section modulus requirement is a W21 × 93, with $Z_x = 221$ in.^{3}, but the lightest one is a W24 × 84, with $Z_x = 224$ in.^{3}. Because section modulus is not directly proportional to area, it is possible to have more section modulus with less area, and hence less weight.

Try a W24 × 84. This shape is compact, as assumed (noncompact shapes are marked as such in the table); therefore $M_n = M_p$, as assumed.

Account for the beam weight.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.084) + 1.6(4.5) = 7.301 \text{ kips/ft}$$

$$\text{Required moment strength} = M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(7.301)(30)^2 = 821.4 \text{ ft-kips}$$

The required section modulus is

$$Z_x = \frac{M_u}{\phi_b F_y} = \frac{821.4(12)}{0.90(50)} = 219 \text{ in.}^3 < 224 \text{ in.}^3 \quad \text{(OK)}$$

In lieu of basing the search on the required section modulus, the design strength $\phi_b M_p$ could be used, because it is directly proportional to Z_x and is also tabulated. Next, check the shear:

$$V_u = \frac{w_u L}{2} = \frac{7.301(30)}{2} = 110 \text{ kips}$$

From the Z_x table,

$$\phi_v V_n = 340 \text{ kips} > 110 \text{ kips} \qquad \text{(OK)}$$

Finally, check the deflection. The maximum permissible live load deflection is $L/240 = (30 \times 12)/240 = 1.5$ inch.

$$\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI_x} = \frac{5}{384}\frac{(4.5/12)(30\times 12)^4}{29{,}000(2370)} = 1.19 \text{ in.} < 1.5 \text{ in.} \quad \text{(OK)}$$

ANSWER Use a W24 × 84.

ASD SOLUTION Ignore the beam weight initially, then check for its effect after a selection is made.

$$w_a = w_D + w_L = 0 + 4.5 = 4.5 \text{ kips/ft}$$

$$\text{Required moment strength} = M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(4.5)(30)^2 = 506.3 \text{ ft-kips}$$

$$= \text{required } \frac{M_n}{\Omega_b}$$

Assume that the shape will be compact. For a compact shape with full lateral support,

$$M_n = M_p = F_y Z_x$$

From $\frac{M_p}{\Omega_b} \geq M_a$,

$$\frac{F_y Z_x}{\Omega_b} \geq M_a$$

$$Z_x \geq \frac{\Omega_b M_a}{F_y} = \frac{1.67(506.3 \times 12)}{50} = 203 \text{ in.}^3$$

The Z_x table lists hot-rolled shapes normally used as beams in order of decreasing plastic section modulus. They are arranged in groups, with the lightest shape in each group at the top of that group. For the current case, the shape with a section modulus closest to 203 in.^{3} is a W18 × 97, but the lightest shape with sufficient section modulus is a W24 × 84, with $Z_x = 224$ in.^{3}

Try a W24 × 84. This shape is compact, as assumed (if it were noncompact, there would be a footnote in the Z_x table). Therefore, $M_n = M_p$ as assumed. Account for the beam weight:

$$w_a = w_D + w_L = 0.084 + 4.5 = 4.584 \text{ kips/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(4.584)(30)^2 = 515.7 \text{ ft-kips}$$

The required plastic section modulus is

$$Z_x = \frac{\Omega_b M_a}{F_y} = \frac{1.67(515.7 \times 12)}{50} = 207 \text{ in.}^3 < 224 \text{ in.}^3 \quad \text{(OK)}$$

Instead of searching for the required section modulus, the search could be based on the required value of M_p/Ω_b, which is also tabulated. Because M_p/Ω_b is proportional to Z_x, the results will be the same.

Read Page 231 to 277

CHAPTER 6

Beam–Columns

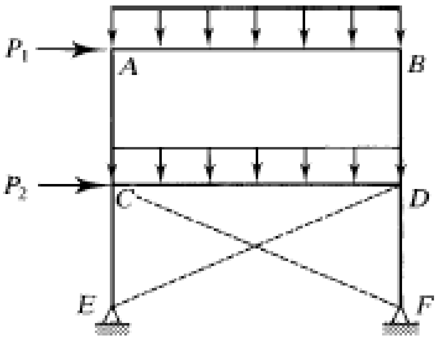
6.1 DEFINITION

While many structural members can be treated as axially loaded columns or as beams with only flexural loading, most beams and columns are subjected to some degree of both bending and axial load. This is especially true of statically indeterminate structures. Even the roller support of a simple beam can experience friction that restrains the beam longitudinally, inducing axial tension when transverse loads are applied. In this particular case, however, the secondary effects are usually small and can be neglected. Many columns can be treated as pure compression members with negligible error. If the column is a one-story member and can be treated as pinned at both ends, the only bending will result from minor accidental eccentricity of the load.

For many structural members, however, there will be a significant amount of both effects, and such members are called *beam–columns*. Consider the rigid frame in Figure 6.1. For the given loading condition, the horizontal member AB must not only support the vertical uniform load but must also assist the vertical members in resisting the concentrated lateral load P_1. Member CD is a more critical case, because it must resist the load $P_1 + P_2$ without any assistance from the vertical members. The reason is that the X-bracing, indicated by dashed lines, prevents sidesway in the lower story. For the direction of P_2 shown, member ED will be in tension and member CF will be slack, provided that the bracing elements have been designed to resist only tension. For this condition to occur, however, member CD must transmit the load $P_1 + P_2$ from C to D.

The vertical members of this frame must also be treated as beam–columns. In the upper story, members AC and BD will bend under the influence of P_1. In addition, at A and B, bending moments are transmitted from the horizontal member through the rigid joints. This transmission of moments also takes place at C and D and is true in any rigid frame, although these moments are usually smaller than those resulting from lateral loads. Most columns in rigid frames are actually beam–columns, and the effects of bending should not be ignored. However, many isolated one-story columns can be realistically treated as axially loaded compression members.

FIGURE 6.1



Another example of beam–columns can sometimes be found in roof trusses. Although the top chord is normally treated as an axially loaded compression member, if purlins are placed between the joints, their reactions will cause bending, which must be accounted for. We discuss methods for handling this problem later in this chapter.

6.2 INTERACTION FORMULAS

The relationship between required and available strengths may be expressed as

$$\frac{\text{required strength}}{\text{available strength}} \leq 1.0 \tag{6.1}$$

For compression members, the strengths are axial forces. For example, for LRFD,

$$\frac{P_u}{\phi_c P_n} \leq 1.0$$

and for ASD,

$$\frac{P_a}{P_n/\Omega_c} \leq 1.0$$

These expressions can be written in the general form

$$\frac{P_r}{P_c} \leq 1.0$$

where

P_r = required axial strength
P_c = available axial strength

If more than one type of resistance is involved, Equation 6.1 can be used to form the basis of an interaction formula. As we discussed in Chapter 5 in conjunction with biaxial bending, the sum of the load-to-resistance ratios must be limited to unity. For example, if both bending and axial compression are acting, the interaction formula would be

$$\frac{P_r}{P_c} + \frac{M_r}{M_c} \leq 1.0$$

where

M_r = required moment strength
= M_u for LRFD
= M_a for ASD

M_c = available moment strength
= $\phi_b M_n$ for LRFD

$= \dfrac{M_n}{\Omega_b}$ for ASD

For biaxial bending, there will be two moment ratios:

$$\frac{P_r}{P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \tag{6.2}$$

where the x and y subscripts refer to bending about the x and y axes.

Equation 6.2 is the basis for the AISC formulas for members subject to bending plus axial compressive load. Two formulas are given in the Specification: one for small axial load and one for large axial load. If the axial load is small, the axial load term is reduced. For large axial load, the bending term is slightly reduced. The AISC requirements are given in Chapter H, "Design of Members for Combined Forces and Torsion," and are summarized as follows:

For $\dfrac{P_r}{P_c} \geq 0.2$,

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC Equation H1-1a)}$$

For $\dfrac{P_r}{P_c} < 0.2$,

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC Equation H1-1b)}$$

These requirements may be expressed in either LRFD or ASD form.

LRFD Interaction Equations

For $\frac{P_u}{\phi_c P_n} \geq 0.2$,

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \tag{6.3}$$

For $\frac{P_u}{\phi_c P_n} < 0.2$,

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \tag{6.4}$$

ASD Interaction Equations

For $\frac{P_a}{P_n/\Omega_c} \geq 0.2$,

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) \leq 1.0 \tag{6.5}$$

For $\frac{P_a}{P_n/\Omega_c} < 0.2$,

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) \leq 1.0 \tag{6.6}$$

Example 6.1 illustrates the application of Equations 6.3–6.6.

EXAMPLE 6.1

The beam–column shown in Figure 6.2 is pinned at both ends and is subjected to the loads shown. Bending is about the strong axis. Determine whether this member satisfies the appropriate AISC Specification interaction equation.

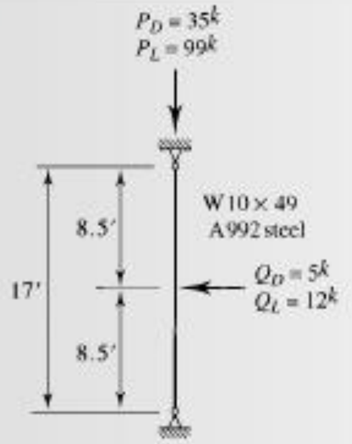
LRFD SOLUTION

From the column load tables, the axial compressive design strength of a W10 × 49 with $F_y = 50$ ksi and an effective length of $K_yL = 1.0 \times 17 = 17$ feet is

$$\phi_c P_n = 405 \text{ kips}$$

Since bending is about the strong axis, the design moment, $\phi_b M_n$, for $C_b = 1.0$ can be obtained from the beam design charts in Part 3 of the *Manual*.

FIGURE 6.2



For an unbraced length $L_b = 17$ ft,

$$\phi_b M_n = 197 \text{ ft-kips}$$

For the end conditions and loading of this problem, $C_b = 1.32$ (see Figure 5.15c).

For $C_b = 1.32$, the design strength is

$$\phi_b M_n = C_b \times 197 = 1.32(197) = 260 \text{ ft-kips}$$

This moment is larger than $\phi_b M_p = 226.5$ ft-kips (also obtained from the beam design charts), so the design moment must be limited to $\phi_b M_p$. Therefore,

$$\phi_b M_n = 226.5 \text{ ft-kips}$$

Factored loads:

$$P_u = 1.2P_D + 1.6P_L = 1.2(35) + 1.6(99) = 200.4 \text{ kips}$$
$$Q_u = 1.2Q_D + 1.6Q_L = 1.2(5) + 1.6(12) = 25.2 \text{ kips}$$

The maximum bending moment occurs at midheight, so

$$M_u = \frac{25.2(17)}{4} = 107.1 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{200.4}{405} = 0.4948 > 0.2 \quad \therefore \text{ Use Equation 6.3 (AISC Eq. H1-1a).}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{200.4}{405} + \frac{8}{9}\left(\frac{107.1}{226.5} + 0\right) = 0.915 < 1.0 \quad \text{(OK)}$$

ANSWER This member satisfies the AISC Specification.

ASD SOLUTION From the column load tables, the allowable compressive strength of a W10 × 49 with $F_y = 50$ ksi and $K_yL = 1.0 \times 17 = 17$ feet is

$$\frac{P_n}{\Omega_c} = 270 \text{ kips}$$

From the design charts in Part 3 of the Manual, for $L_b = 17$ ft and $C_b = 1.0$,

$$\frac{M_n}{\Omega_b} = 131 \text{ ft-kips}$$

From Figure 5.15c, $C_b = 1.32$. For $C_b = 1.32$,

$$\frac{M_n}{\Omega_b} = C_b \times 131 = 1.32(131) = 172.9 \text{ ft-kips}$$

This is larger than $M_p/\Omega_b = 151$ ft-kips, so the allowable moment must be limited to M_p/Ω_b. Therefore,

$$\frac{M_n}{\Omega_b} = 151 \text{ ft-kips}$$

The total axial compressive load is

$$P_a = P_D + P_L = 35 + 99 = 134 \text{ kips}$$

The total transverse load is

$$Q_a = Q_D + Q_L = 5 + 12 = 17 \text{ kips}$$

The maximum bending moment is at midheight

$$M_a = \frac{17(17)}{4} = 72.25 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{134}{270} = 0.4963 > 0.2 \qquad \therefore \text{ Use Equation 6.5 (AISC Equation H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{134}{270} + \frac{8}{9}\left(\frac{72.25}{151} + 0\right) = 0.922 < 1.0 \quad \text{(OK)}$$

ANSWER This member satisfies the AISC Specification.

In addition to the required moment strength, the required axial strength must account for second-order effects. The required axial strength is affected by the displaced geometry of the structure during loading. This is not an issue with member displacement (δ), but it is with joint displacement (Δ). The required axial compressive strength is given by

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{(AISC Equation A-8-2)}$$

where

P_{nt} = axial load corresponding to the braced condition

P_{lt} = axial load corresponding to the sidesway condition

We cover the evaluation of B_1 and B_2 in the following sections.

6.6 MEMBERS IN BRACED FRAMES

The amplification factor given by Expression 6.7 was derived for a member braced against sidesway—that is, one whose ends cannot translate with respect to each other. Figure 6.6 shows a member of this type subjected to equal end moments producing *single-curvature bending* (bending that produces tension or compression on

FIGURE 6.6

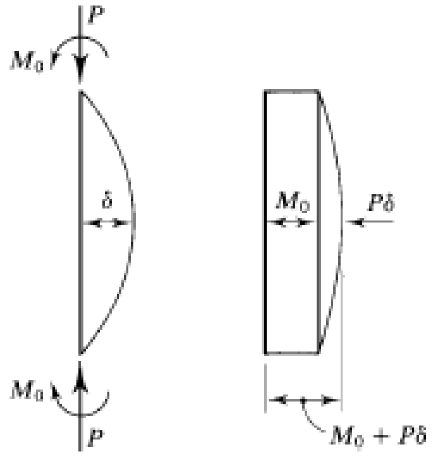
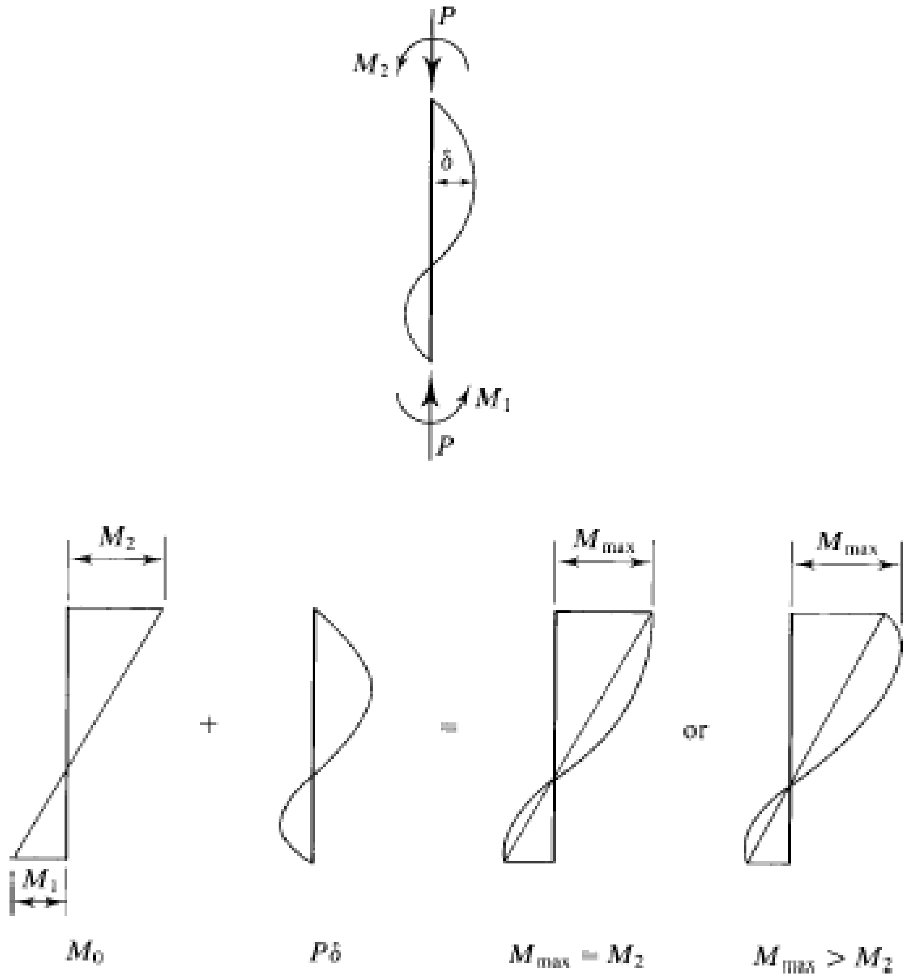


FIGURE 6.7



factor given by Expression 6.7 was derived for the worst case, so C_m will never be greater than 1.0. The final form of the amplification factor is

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} \geq 1 \qquad \text{(AISC Equation A-8-3)}$$

where

P_r = required unamplified axial compressive strength ($P_{nt} + P_{lt}$)
= P_u for LRFD
= P_a for ASD

α = 1.00 for LRFD
= 1.60 for ASD

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2} \qquad \text{(AISC Equation A-8-5)}$$

EI^* = flexural rigidity

In the direct analysis method, EI^* is a reduced stiffness obtained as

$$EI^* = 0.8\tau_b EI \qquad (6.8)$$

where

τ_b = a stiffness reduction factor

$$= 1.0 \text{ when } \frac{\alpha P_r}{P_y} \leq 0.5 \qquad \text{(AISC Equation C2-2a)}$$

$$= 4\left(\alpha\frac{P_r}{P_y}\right)\left(1-\alpha\frac{P_r}{P_y}\right) \text{ when } \frac{\alpha P_r}{P_y} > 0.5 \qquad \text{(AISC Equation C2-2b)}$$

Evaluation of C_m

The factor C_m applies only to the braced condition. There are two categories of members: those with transverse loads applied between the ends and those with no transverse loads. Figure 6.8b and c illustrate these two cases (member AB is the beam–column under consideration).

1. If there are no transverse loads acting on the member,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \qquad \text{(AISC Equation A-8-4)}$$

   M_1/M_2 is a ratio of the bending moments at the ends of the member. M_1 is the end moment that is smaller in absolute value, M_2 is the larger, and the ratio is positive for members bent in reverse curvature and negative for single-curvature bending (Figure 6.9). Reverse curvature (a positive ratio) occurs when M_1 and M_2 are both clockwise or both counterclockwise.
2. For transversely loaded members, C_m can be taken as 1.0. A more refined procedure for transversely loaded members is provided in the Commentary to Appendix 8 of the Specification. The factor C_m is given as

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) \qquad \text{(AISC Equation C-A-8-2)}$$

   The factor Ψ has been evaluated for several common situations and is given in Commentary Table C-A-8.1.

FIGURE 6.8

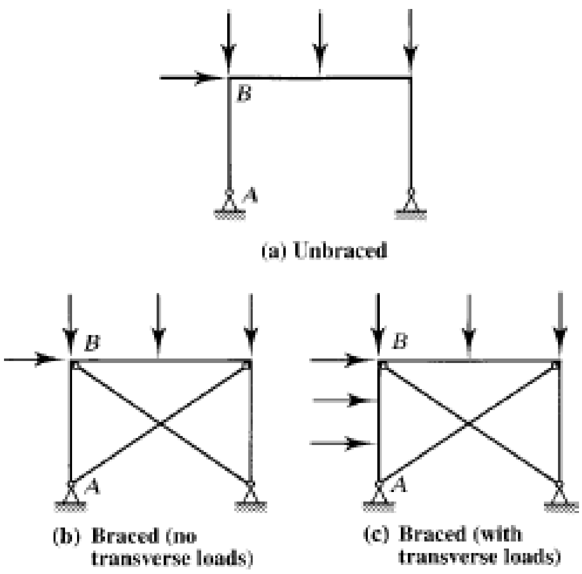
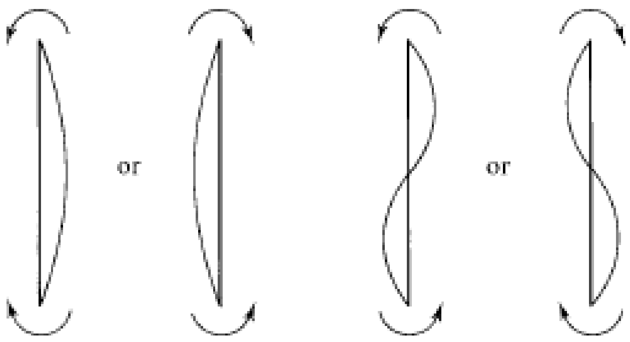


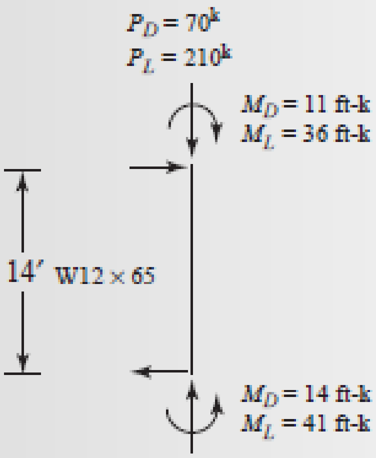
FIGURE 6.9



EXAMPLE 6.3

The member shown in Figure 6.10 is part of a braced frame. An analysis consistent with the effective length method was performed; therefore, the flexural rigidity, EI, was unreduced. If A572 Grade 50 steel is used, is this member adequate? $K_x = K_y = 1.0$.

FIGURE 6.10

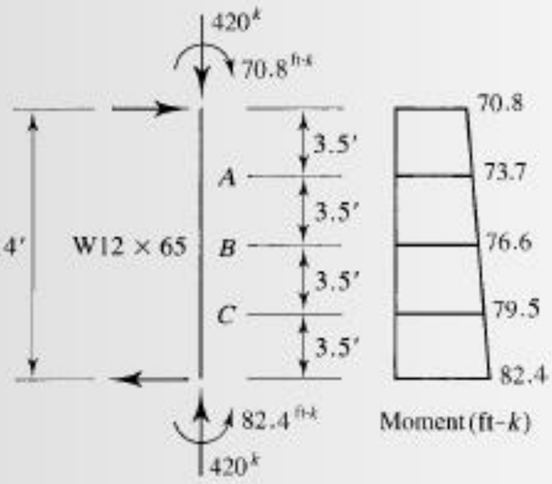


LRFD SOLUTION

The factored loads, computed from load combination 2, are shown in Figure 6.11. Determine which interaction formula to apply. The required compressive strength is

$$P_r = P_u = P_{nt} + B_2 P_{lt} = 420 + 0 = 420 \text{ kips} \qquad (B_2 = 0 \text{ for a braced frame})$$

FIGURE 6.11



From the column load tables, for $KL = 1.0 \times 14 = 14$ feet, the axial compressive strength of a W12 × 65 is

$$\phi_c P_n = 685 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{420}{685} = 0.6131 > 0.2 \qquad \therefore \text{ Use Equation 6.3 (AISC Equation H1-1a).}$$

In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(533)}{(1.0 \times 14 \times 12)^2} = 5405 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{70.8}{82.4}\right) = 0.9437$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{0.9437}{1-(420/5405)} = 1.023$$

From the Beam Design Charts in Part 3 of the *Manual* with $C_b = 1.0$ and $L_b = 14$ feet, the moment strength is

$$\phi_b M_n = 345 \text{ ft-kips}$$

For the actual value of C_b, refer to the moment diagram of Figure 6.11:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(82.4)}{2.5(82.4) + 3(73.7) + 4(76.6) + 3(79.5)} = 1.060$$

$$\therefore \quad \phi_b M_n = C_b(345) = 1.060(345) = 366 \text{ ft-kips}$$

But $\phi_b M_p = 356$ ft-kips (from the charts) < 366 ft-kips $\quad \therefore$ Use $\phi_b M_n = 356$ ft-kips.

(Since a W12 × 65 is noncompact for $F_y = 50$ ksi, 356 ft-kips is the design strength based on FLB rather than full yielding of the cross section.) The factored load moments are

$$M_{nt} = 82.4 \text{ ft-kips} \quad M_{\ell t} = 0$$

From AISC Equation A-8-1, the required moment strength is

$$M_r = M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.023(82.4) + 0 = 84.30 \text{ ft-kips} = M_{ux}$$

From Equation 6.3 (AISC Equation H1-1a),

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.6131 + \frac{8}{9}\left(\frac{84.30}{356} + 0\right) = 0.824 < 1.0 \qquad \text{(OK)}$$

ANSWER The member is satisfactory.

ASD SOLUTION The service loads, computed from load combination 2, are shown in Figure 6.12. Determine which interaction formula to apply. The required compressive strength is

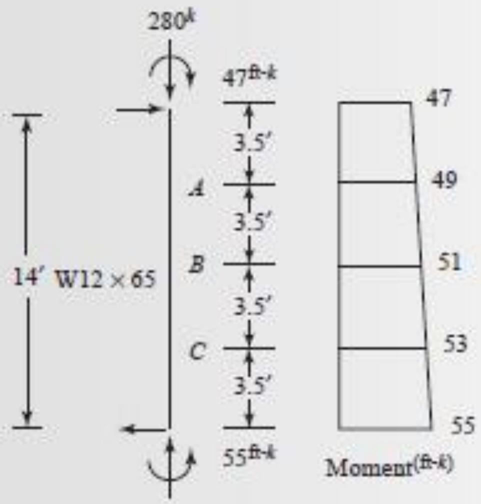
$$P_r = P_a = P_{nt} + B_2 P_{\ell t} = 280 + 0 = 280 \text{ kips} \qquad (B_2 = 0 \text{ for a braced frame})$$

From the column load tables, for $KL = 1.0 \times 14 = 14$ feet, the axial compressive strength of a W12 × 65 is

$$\frac{P_n}{\Omega_c} = 456 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{280}{456} = 0.6140 > 0.2 \qquad \therefore \text{ Use Equation 6.5 (AISC Equation H1-1a).}$$

FIGURE 6.12



In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(533)}{(1.0 \times 14 \times 12)^2} = 5405 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{47}{55}\right) = 0.9418$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{0.9418}{1-(1.60 \times 280/5405)} = 1.027$$

Next, from the Beam Design Charts with $C_b = 1.0$ and $L_b = 14$ feet, the moment strength is

$$\frac{M_n}{\Omega_b} = 230 \text{ ft-kips}$$

For the actual value of C_b, refer to the moment diagram of Figure 6.12:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(55)}{2.5(55) + 3(49) + 4(51) + 3(53)} = 1.062$$

$$\therefore \frac{M_n}{\Omega_b} = C_b(230) = 1.062(230) = 244.3 \text{ ft-kips}$$

But $\frac{M_p}{\Omega_b} = 237$ ft-kips (from the charts) < 244.3 ft-kips, so use $\frac{M_n}{\Omega_b} = 237$ ft-kips.

(Since a W12 × 65 is noncompact for $F_y = 50$ ksi, 237 ft-kips is the strength based on FLB rather than full yielding of the cross section.) The unamplified moments are

$$M_{nt} = 55 \text{ ft-kips} \qquad M_{lt} = 0$$

From AISC Equation A-8-1, the required moment strength is

$$M_r = M_a = B_1M_{nt} + B_2M_{lt} = 1.027(55) + 0 = 56.49 \text{ ft-kips} = M_{ax}$$

From Equation 6.5 (AISC Equation H1-1a),

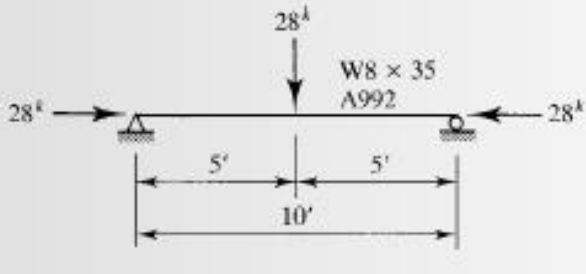
$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{280}{456} + \frac{8}{9}\left(\frac{56.49}{237} + 0\right) = 0.826 < 1.0 \qquad \text{(OK)}$$

ANSWER The member is satisfactory.

EXAMPLE 6.4

The horizontal beam–column shown in Figure 6.13 is subject to the service live loads shown. This member is laterally braced at its ends, and bending is about the x-axis. Check for compliance with the AISC Specification. $K_x = K_y = 1.0$.

FIGURE 6.13



LRFD SOLUTION

The factored axial load is

$$P_u = 1.6(28) = 44.8 \text{ kips}$$

The factored transverse loads and bending moment are

$$Q_u = 1.6(28) = 44.8 \text{ kips}$$

$$w_u = 1.2(0.035) = 0.042 \text{ kips/ft}$$

$$M_u = \frac{44.8(10)}{4} + \frac{0.042(10)^2}{8} = 112.5 \text{ ft-kips}$$

This member is braced against sidesway, so $M_{lt} = 0$.

Compute the moment amplification factor. For a member braced against sidesway and transversely loaded, C_m can be taken as 1.0. A more accurate value can be found in the Commentary to AISC Appendix 8:

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) \qquad \text{(AISC Equation C-A-8-2)}$$

From Commentary Table C-A-8.1, $\Psi = -0.2$ for the support and loading conditions of this beam–column. For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29{,}000)(127)}{(10 \times 12)^2} = 2524 \text{ kips}$$

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) = 1 - 0.2\left(\frac{1.00 P_u}{P_{e1}}\right) = 1 - 0.2\left(\frac{44.8}{2524}\right) = 0.9965$$

The amplification factor is

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{0.9965}{1-(44.8/2524)} = 1.015$$

The amplified bending moment is

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.015(112.5) + 0 = 114.2 \text{ ft-kips}$$

From the beam design charts, for $L_b = 10$ ft and $C_b = 1$,

$$\phi_b M_n = 123 \text{ ft-kips}$$

Because the beam weight is very small in relation to the concentrated live load, C_b may be taken from Figure 5.15c as 1.32. This value results in a design moment of

$$\phi_b M_n = 1.32(123) = 162.4 \text{ ft-kips}$$

This moment is greater than $\phi_b M_p = 130$ ft-kips, so the design strength must be limited to this value. Therefore,

$$\phi_b M_n = 130 \text{ ft-kips}$$

Check the interaction formula. From the column load tables, for $KL = 10$ ft,

$$\phi_c P_n = 358 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} + \frac{44.8}{358} = 0.1251 < 0.2 \qquad \therefore \text{ Use Equation 6.4 (AISC Equation H1-1b).}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{0.1251}{2} + \left(\frac{114.2}{130} + 0\right)$$
$$= 0.941 < 1.0 \text{ (OK)}$$

ANSWER A W8 × 35 is adequate.

ASD SOLUTION The applied axial load is

$$P_a = 28 \text{ kips}$$

The applied transverse loads are

$$Q_a = 28 \text{ kips and } w_a = 0.035 \text{ kips/ft}$$

and the maximum bending moment is

$$M_{nt} = \frac{28(10)}{4} + \frac{0.035(10)^2}{8} = 70.44 \text{ ft-kips}$$

The member is braced against end translation, so $M_{lt} = 0$.

Compute the moment amplification factor. For a member braced against sidesway and transversely loaded, C_m can be taken as 1.0. A more accurate value can be found in the Commentary to AISC Appendix 8:

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) \qquad \text{(AISC Equation C-A-8-2)}$$

From Commentary Table C-A-8.1, $\Psi = -0.2$ for the support and loading conditions of this beam–column. For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29{,}000)(127)}{(10 \times 12)^2} = 2524 \text{ kips}$$

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) = 1 - 0.2\left(\frac{1.60 P_a}{P_{e1}}\right) = 1 - 0.2\left(\frac{1.60 \times 28}{2524}\right) = 0.9965$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / \boldsymbol{P}_{e1})} = \frac{C_m}{1 - (1.60 P_a / P_{e1})} = \frac{0.9965}{1 - (1.60 \times 28/2524)} = 1.015$$

$$M_a = B_1 M_{nt} = 1.015(70.44) = 71.50 \text{ ft-kips}$$

From the Beam Design Charts with $C_b = 1.0$ and $L_b = 10$ feet, the moment strength is

$$\frac{M_n}{\Omega_b} = 82.0 \text{ ft-kips}$$

Because the beam weight is very small in relation to the concentrated live load, C_b may be taken from Figure 5.15c as 1.32. This results in an allowable moment of

$$\frac{M_n}{\Omega_b} = 1.32(82.0) = 108.2 \text{ ft-kips}$$

This result is larger than $\dfrac{M_p}{\Omega_b} = 86.6$; therefore, use $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 86.6$ ft-kips.

Compute the axial compressive strength. From the column load tables, for $KL = 10$ ft,

$$\frac{P_n}{\Omega_c} = 238 \text{ kips}$$

Determine which interaction formula to use:

$$\frac{P_a}{P_n/\Omega_c} = \frac{28}{238} = 0.1176 < 0.2 \qquad \therefore \text{ Use Equation 6.6 (AISC Equation H1-1b).}$$

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{0.1176}{2} + \left(\frac{71.50}{86.6} + 0\right)$$

$$= 0.884 < 1.0 \text{ (OK)}$$

ANSWER The W8 × 35 is adequate.

Read Page 322 to 362

CHAPTER 7

Simple Connections

7.1 INTRODUCTION

Connections of structural steel members are of critical importance. An inadequate connection, which can be the "weak link" in a structure, has been the cause of numerous failures. Failure of structural *members* is rare; most structural failures are the result of poorly designed or detailed connections. The problem is compounded by the confusion that sometimes exists regarding responsibility for the design of connections. In many cases, the connections are not designed by the same engineer who designs the rest of the structure, but by someone associated with the steel fabricator who furnishes the material for the project. The structural engineer responsible for the production of the design drawings, however, is responsible for the complete design, including the connections. It is therefore incumbent upon the engineer to be proficient in connection design, if only for the purpose of validating a connection designed by someone else.

Modern steel structures are connected by welding or bolting (either high-strength or "common" bolts) or by a combination of both. Until fairly recently, connections were either welded or riveted. In 1947, the Research Council of Riveted and Bolted Structural Joints was formed, and its first specification was issued in 1951. This document authorized the substitution of high-strength bolts for rivets on a one-for-one basis. Since that time, high-strength bolting has rapidly gained in popularity, and today the widespread use of high-strength bolts has rendered the rivet obsolete in civil engineering structures. There are several reasons for this change. Two relatively unskilled workers can install high-strength bolts, whereas four skilled workers were required for riveting. In addition, the riveting operation was noisy and somewhat dangerous because of the practice of tossing the heated rivet from the point of heating to the point of installation. Riveted connection design is no longer covered by the AISC Specification, but many existing structures contain riveted joints, and the analysis of these connections is required for the strength evaluation and rehabilitation of older structures. Section 5.2.6 of AISC Appendix 5, "Evaluation of Existing Structures," specifies that ASTM A502 Grade 1 rivets should be assumed unless there is evidence to the contrary. Properties of rivets can be found in the ASTM Specification (ASTM, 2010c). The analysis of riveted connections is essentially the same as for connections with common bolts; only the material properties are different.

Welding has several advantages over bolting. A welded connection is often simpler in concept and requires few, if any, holes (sometimes erection bolts may be required to hold the members in position for the welding operation). Connections that are extremely complex with fasteners can become very simple when welds are used. A case in point is the plate girder shown in Figure 7.1. Before welding became widely used, this type of built-up shape was fabricated by riveting. To attach the flange plates to the web plate, angle shapes were used to transfer load between the two elements. If cover plates were added, the finished product became even more complicated. The welded version, however, is elegant in its simplicity. On the negative side, skilled workers are required for welding, and inspection can be difficult and costly. This last disadvantage can be partially overcome by using shop welding instead of field welding whenever possible. Quality welding can be more easily ensured under the controlled conditions of a fabricating shop. When a connection is made with a combination of welds and bolts, welding can be done in the shop and bolting in the field. In the single-plate beam-to-column connection shown in Figure 7.2, the plate is shop-welded to the column flange and field-bolted to the beam web.

FIGURE 7.1

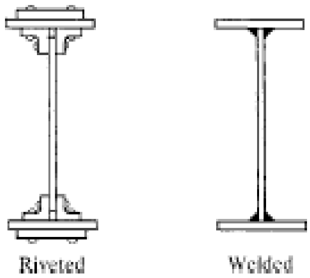


FIGURE 7.2

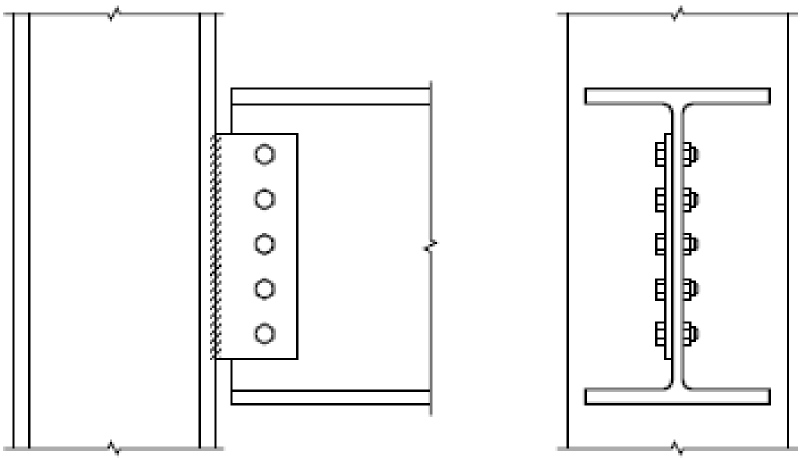
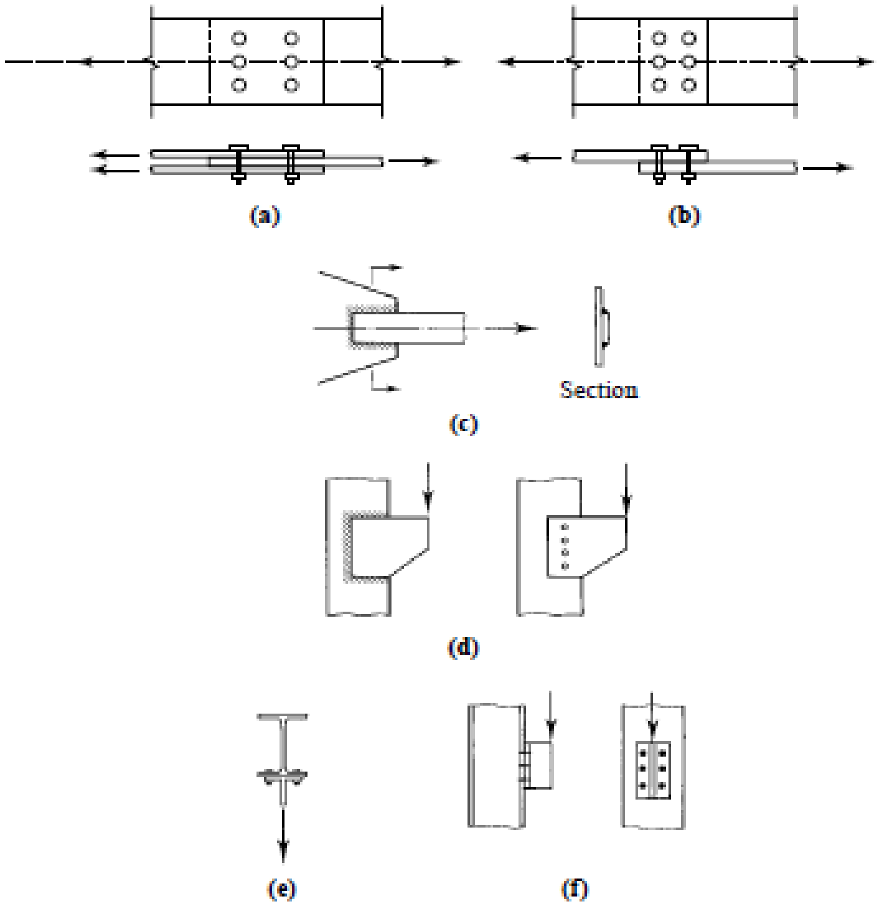


FIGURE 7.3



7.2 BOLTED SHEAR CONNECTIONS: FAILURE MODES

Before considering the strength of specific grades of bolts, we need to examine the various modes of failure that are possible in connections with fasteners subjected to shear. There are two broad categories of failure: failure of the fastener and failure of the parts being connected. Consider the lap joint shown in Figure 7.4a. Failure of the fastener can be assumed to occur as shown. The average shearing stress in this case will be

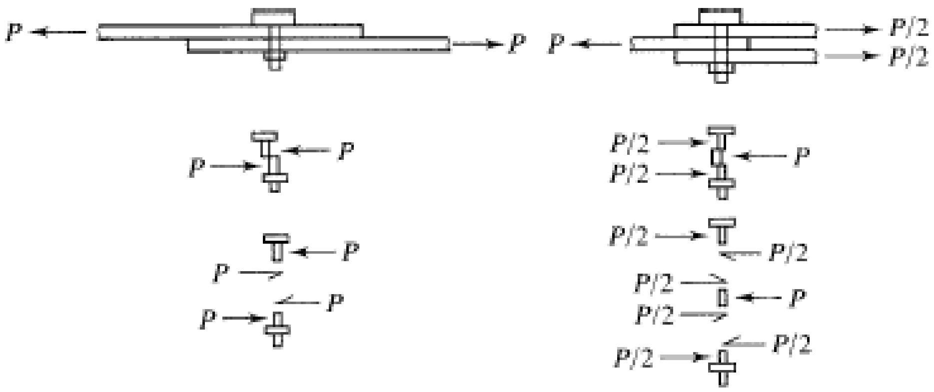
$$f_v = \frac{P}{A} = \frac{P}{\pi d^2/4}$$

where P is the load acting on an individual fastener, A is the cross-sectional area of the fastener, and d is its diameter. The load can then be written as

$$P = f_v A$$

Although the loading in this case is not perfectly concentric, the eccentricity is small and can be neglected. The connection in Figure 7.4b is similar, but an analysis

FIGURE 7.4



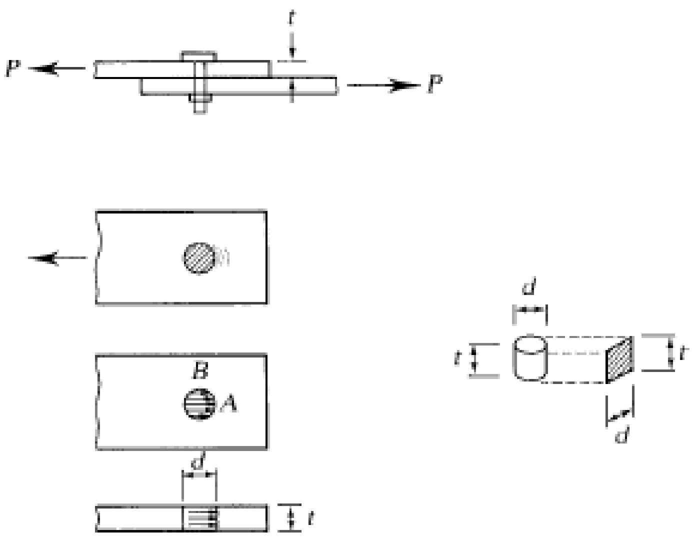
(a) Single Shear (b) Double Shear

*There is actually a small eccentricity in the connections of Figure 7.3b and c, but it is usually neglected.

connected and fall into two general categories.

1. **Failure resulting from excessive tension, shear, or bending in the parts being connected.** If a tension member is being connected, tension on both the gross area and effective net area must be investigated. Depending on the configuration of the connection, block shear might also need to be considered. Block shear must also be examined in beam-to-column connections in which the top flange of the beam is coped. (We covered block shear in Chapters 3 and 5, and it is described in AISC J4.3.) Depending on the type of connection and loading, connection fittings such as gusset plates and framing angles may require an analysis for shear, tension, bending, or block shear. The design of a tension member connection will usually be done in parallel with the design of the member itself because the two processes are interdependent.
2. **Failure of the connected part because of bearing exerted by the fasteners.** If the hole is slightly larger than the fastener and the fastener is assumed to be placed loosely in the hole, contact between the fastener and the connected part will exist over approximately half the circumference of the fastener when a load is applied. This condition is illustrated in Figure 7.5. The stress will vary from a maximum at *A* to zero at *B;* for simplicity, an average stress, computed as the applied force divided by the projected area of contact, is used.

FIGURE 7.5



7.3 BEARING STRENGTH, SPACING, AND EDGE-DISTANCE REQUIREMENTS

Bearing strength is independent of the type of fastener because the stress under consideration is on the part being connected rather than on the fastener. For this reason, bearing strength, as well as spacing and edge-distance requirements, which also are independent of the type of fastener, will be considered before bolt shear and tensile strength.

The AISC Specification provisions for bearing strength, as well as all the requirements for high-strength bolts, are based on the provisions of the specification of the Research Council on Structural Connections (RCSC, 2009). The following discussion, which is based on the commentary that accompanies the RCSC specification, explains the basis of the AISC specification equations for bearing strength.

A possible failure mode resulting from excessive bearing is shear tear-out at the end of a connected element, as shown in Figure 7.7a. If the failure surface is idealized as shown in Figure 7.7b, the failure load on one of the two surfaces is equal to the shear fracture stress times the shear area, or

$$\frac{R_n}{2} = 0.6F_u\ell_c t$$

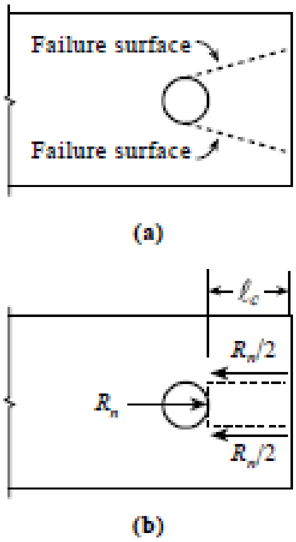
where

$0.6F_u$ = shear fracture stress of the connected part

ℓ_c = distance from edge of hole to edge of connected part

t = thickness of connected part

FIGURE 7.7



The total strength is

$$R_n = 2(0.6F_u \ell_c t) = 1.2F_u \ell_c t \tag{7.1}$$

This tear-out can take place at the edge of a connected part, as shown, or between two holes in the direction of the bearing load. To prevent excessive elongation of the hole, an upper limit is placed on the bearing load given by Equation 7.1. This upper limit is proportional to the projected bearing area times the fracture stress, or

$$R_n = C \times \text{bearing area} \times F_u = CdtF_u \tag{7.2}$$

where

C = a constant
d = bolt diameter
t = thickness of the connected part

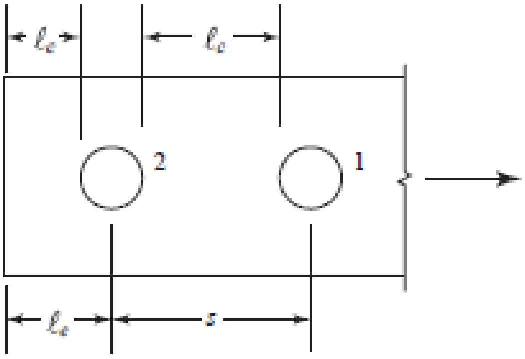
The AISC Specification uses Equation 7.1 for bearing strength, subject to an upper limit given by Equation 7.2. If excessive deformation at service load is a concern, and it usually is, C is taken as 2.4. This value corresponds to a hole elongation of about ¼ inch (RCSC, 2009). In this book, we consider deformation to be a design consideration. The nominal bearing strength of a single bolt therefore can be expressed as

$$R_n = 1.2\ell_c t F_u \leq 2.4dtF_u \qquad \text{(AISC Equation J3-6a)}$$

where

ℓ_c = clear distance, in the direction parallel to the applied load, from the edge of the bolt hole to the edge of the adjacent hole or to the edge of the material
t = thickness of the connected part
F_u = ultimate tensile stress of the connected part (*not* the bolt)

FIGURE 7.8



For load and resistance factor design, the resistance factor is $\phi = 0.75$, and the design strength is

$$\phi R_n = 0.75 R_n$$

For allowable strength design, the safety factor is $\Omega = 2.00$, and the allowable strength is

$$\frac{R_n}{\Omega} = \frac{R_n}{2.00}$$

Figure 7.8 further illustrates the distance ℓ_c. When computing the bearing strength for a bolt, use the distance from that bolt to the adjacent bolt or edge in the direction of the bearing load on the connected part. For the case shown, the bearing load would be on the left side of each hole. Thus the strength for bolt 1 is calculated with ℓ_c measured to the edge of bolt 2, and the strength for bolt 2 is calculated with ℓ_c measured to the edge of the connected part.

For the edge bolts, use $\ell_c = \ell_e - h/2$. For other bolts, use $\ell_c = s - h$,

where

ℓ_e = edge-distance to center of the hole
s = center-to-center spacing of holes
h = hole diameter

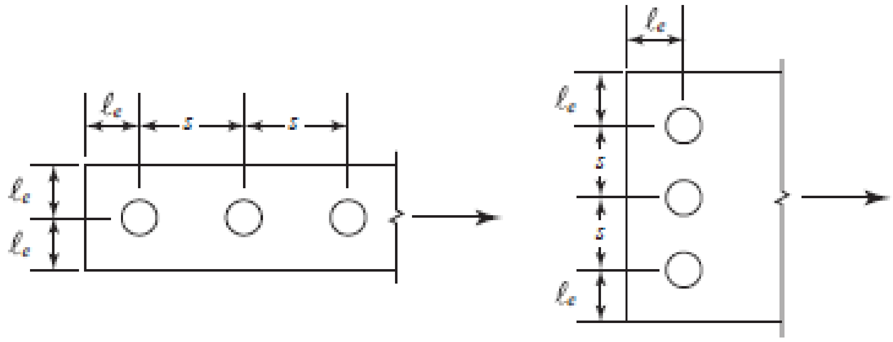
AISC Equation J3-6a is valid for standard, oversized, short-slotted and long-slotted holes with the slot parallel to the load. We use only standard holes in this book (holes 1⁄16-inch larger than the bolt diameter). For those cases where deformation is not a design consideration, and for long-slotted holes with the slot perpendicular to the direction of the load, AISC gives other strength expressions.

When computing the distance ℓ_c, use the actual hole diameter (which is 1⁄16-inch larger than the bolt diameter), and do not add the 1⁄16 inch as required in AISC B4.3b for computing the net area for tension and shear. In other words, use a hole diameter of

$$h = d + \frac{1}{16} \text{ in.}$$

not d + 1⁄8 inch (although if d + 1⁄8 were used, the slight error would be on the conservative side).

FIGURE 7.9



Spacing and Edge-Distance Requirements

To maintain clearances between bolt nuts and to provide room for wrench sockets, AISC J3.3 requires that center-to-center spacing of fasteners (in any direction) be no less than $2\frac{2}{3}d$ and preferably no less than $3d$, where d is the fastener diameter. Minimum edge distances (in any direction), measured from the center of the hole, are given in AISC Table J3.4 as a function of bolt size. The spacing and edge distance to be considered, denoted s and ℓ_e, are illustrated in Figure 7.9.

Summary of Bearing Strength, Spacing, and Edge-Distance Requirements (Standard Holes)

a. Bearing strength:

$$R_n = 1.2\ell_c t F_u \le 2.4 d t F_u \qquad \text{(AISC Equation J3-6a)}$$

b. Minimum spacing and edge distance: In any direction, both in the line of force and transverse to the line of force,

$s \ge 2\frac{2}{3}d$ (preferably $3d$)
$\ell_e \ge$ value from AISC Table J3.4

For single- and double-angle shapes, the usual gage distances given in Table 1-7A in Part 1 of the *Manual* (see Section 3.6) may be used in lieu of these minimums.

EXAMPLE 7.1

Check bolt spacing, edge distances, and bearing for the connection shown in Figure 7.10.

SOLUTION From AISC J3.3, the minimum spacing in any direction is

$$2\tfrac{2}{3}d = 2.667\left(\frac{3}{4}\right) = 2.00 \text{ in.}$$

Actual spacing = 2.50 in. > 2.00 in. (OK)

The bearing strength for the tension member is

$$R_n = 2(29.36) + 2(52.20) = 163.1 \text{ kips}$$

For the gusset plate and the holes nearest the edge of the plate,

$$\ell_c = \ell_e - \frac{h}{2} = 1.25 - \frac{13/16}{2} = 0.8438 \text{ in.}$$
$$R_n = 1.2\ell_c tF_u \leq 2.4dtF_u$$
$$1.2\ell_c tF_u = 1.2(0.8438)\left(\frac{3}{8}\right)(58) = 22.02 \text{ kips}$$
$$\text{Upper limit} = 2.4dtF_u = 2.4\left(\frac{3}{4}\right)\left(\frac{3}{8}\right)(58)$$
$$= 39.15 \text{kips} > 22.02 \text{ kips} \quad \therefore \text{ Use } R_n = 22.02 \text{ kips/bolt.}$$

For the other holes,

$$\ell_c = s - h = 2.5 - \frac{13}{16} = 1.688 \text{ in.}$$
$$R_n = 1.2\ell_c tF_u \leq 2.4dtF_u$$
$$1.2\ell_c tF_u = 1.2(1.688)\left(\frac{3}{8}\right)(58) = 44.06 \text{ kips}$$

Upper limit $= 2.4dtF_u = 39.15$ kips < 44.06 kips $\quad \therefore$ Use $R_n = 39.15$ kips/bolt.

The bearing strength for the gusset plate is

$$R_n = 2(22.02) + 2(39.15) = 122.3 \text{ kips}$$

The gusset plate controls. The nominal bearing strength for the connection is therefore

$$R_n = 122.3 \text{ kips}$$

LRFD SOLUTION The design strength is $\phi R_n = 0.75(122.3) = 91.7$ kips.

The required strength is

$$R_u = 1.2D + 1.6L = 1.2(15) + 1.6(45) = 90.0 \text{ kips} < 91.7 \text{ kips} \qquad \text{(OK)}$$

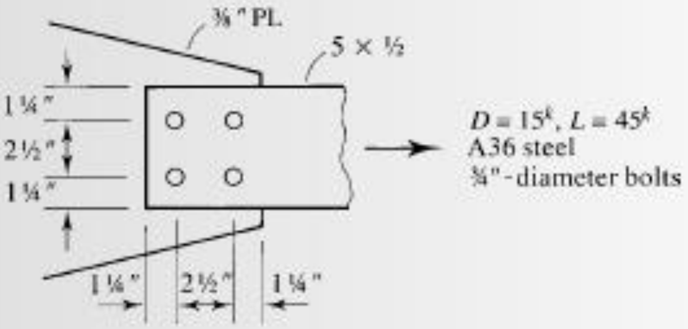
ASD SOLUTION The allowable strength is $\frac{R_n}{\Omega} = \frac{122.3}{2.00} = 61.2$ kips.

The required strength is

$$R_a = D + L = 15 + 45 = 60 \text{ kips} < 61.2 \text{ kips} \qquad \text{(OK)}$$

ANSWER Bearing strength, spacing, and edge-distance requirements are satisfied.

FIGURE 7.10



From AISC Table J3.4, the minimum edge distance in any direction is 1 inch.

$$\text{Actual edge distance} = 1\frac{1}{4}\text{ in.} > 1\text{ in.} \quad \text{(OK)}$$

For computation of the bearing strength, use a hole diameter of

$$h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}\text{ in.}$$

Check bearing on both the tension member and the gusset plate. For the tension member and the holes nearest the edge of the member,

$$\ell_c = \ell_e - \frac{h}{2} = 1.25 - \frac{13/16}{2} = 0.8438\text{ in.}$$

$$R_n = 1.2\ell_c tF_u \leq 2.4dtF_u$$

$$1.2\ell_c tF_u = 1.2(0.8438)\left(\frac{1}{2}\right)(58) = 29.36\text{ kips}$$

Check upper limit:

$$2.4dtF_u = 2.4\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)(58) = 52.20\text{ kips}$$

$$29.36\text{ kips} < 52.20\text{ kips} \quad \therefore \text{ Use } R_n = 29.36\text{ kips/bolt.}$$

(This result means that ℓ_c is small enough so that it must be accounted for.)

For the other holes,

$$\ell_c = s - h = 2.5 - \frac{13}{16} = 1.688\text{ in.}$$

$$R_n = 1.2\ell_c tF_u \leq 2.4dtF_u$$

$$1.2\ell_c tF_u = 1.2(1.688)\left(\frac{1}{2}\right)(58) = 58.74\text{ kips}$$

Upper limit (the upper limit is independent of ℓ_c and is the same for all bolts):

$$2.4dtF_u = 52.20\text{ kips} < 58.74\text{ kips} \quad \therefore \text{ Use } R_n = 52.20\text{ kips/bolt.}$$

(This result means that ℓ_c is large enough so that it does not need to be accounted for. Hole deformation controls.)

Read the rest of Chapter 7

CHAPTER 8

Eccentric Connections

8.1 EXAMPLES OF ECCENTRIC CONNECTIONS

An eccentric connection is one in which the resultant of the applied loads does not pass through the center of gravity of the fasteners or welds. If the connection has a plane of symmetry, the centroid of the shear area of the fasteners or welds may be used as the reference point, and the perpendicular distance from the line of action of the load to the centroid is called the *eccentricity*. Although a majority of connections are probably loaded eccentrically, in many cases the eccentricity is small and may be neglected.

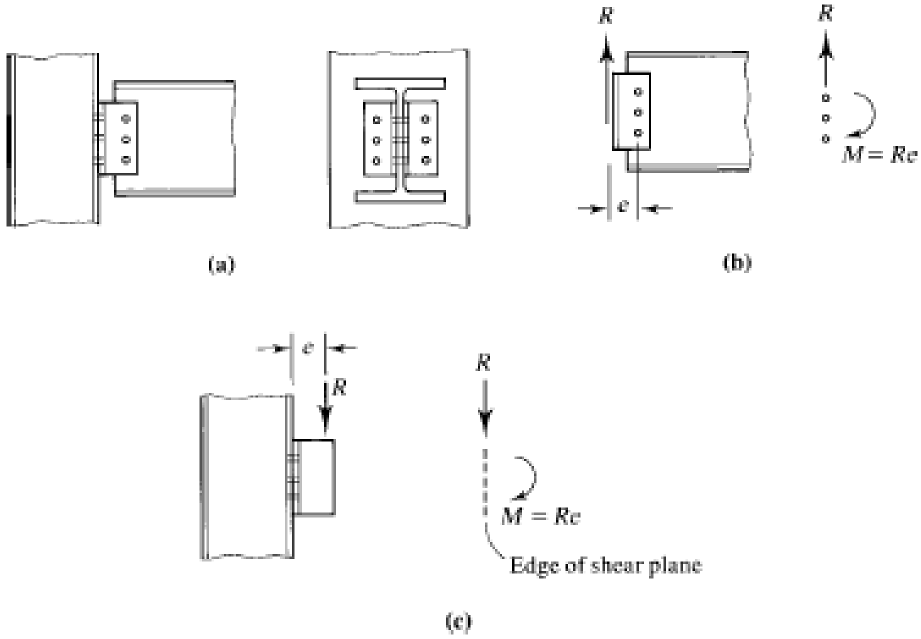
The *framed beam* connection shown in Figure 8.1a is a typical eccentric connection. This connection, in either bolted or welded form, is commonly used to connect beams to columns. Although the eccentricities in this type of connection are small and can sometimes be neglected, they do exist and are used here for illustration. There are actually two different connections involved: the attachment of the beam to the framing angles and the attachment of the angles to the column. These connections illustrate the two basic categories of eccentric connections: those causing only shear in the fasteners or welds and those causing both shear and tension.

If the beam and angles are considered separately from the column, as shown in Figure 8.1b, it is clear that the reaction R acts at an eccentricity e from the centroid of the areas of the fasteners in the beam web. These fasteners are thus subjected to both a shearing force and a couple that lies in the plane of the connection and causes torsional shearing stress.

If the column and the angles are isolated from the beam, as shown in Figure 8.1c, it is clear that the fasteners in the column flange are subjected to the reaction R acting at an eccentricity e from the plane of the fasteners, producing the same couple as before. In this case, however, the load is not in the plane of the fasteners, so the couple will tend to put the upper part of the connection in tension and compress the lower part. The fasteners at the top of the connection will therefore be subjected to both shear and tension.

Although we used a bolted connection here for illustration, welded connections can be similarly categorized as either shear only or shear plus tension.

FIGURE 8.1

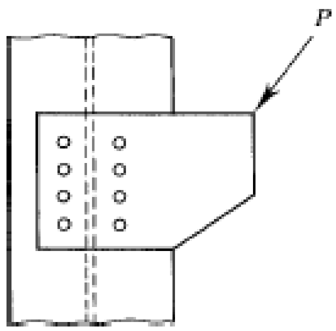


Available strengths (maximum reaction capacities) for various framed beam connections are given in Tables 10-1 through 10-12 in Part 10 of the *Manual*, "Design of Simple Shear Connections."

8.2 ECCENTRIC BOLTED CONNECTIONS: SHEAR ONLY

The column bracket connection shown in Figure 8.2 is an example of a bolted connection subjected to eccentric shear. Two approaches exist for the solution of this problem: the traditional elastic analysis and the more accurate (but more complex) ultimate strength analysis. Both will be illustrated.

FIGURE 8.2



Elastic Analysis

In Figure 8.3a, the fastener shear areas and the load are shown separate from the column and bracket plate. The eccentric load P can be replaced with the same load acting at the centroid plus the couple, $M = Pe$, where e is the eccentricity. If this replacement is made, the load will be concentric, and each fastener can be assumed to resist an equal share of the load, given by $p_c = P/n$, where n is the number of fasteners. The fastener forces resulting from the couple can be found by considering the shearing stress in the fasteners to be the result of torsion of a cross section made up of the cross-sectional areas of the fasteners. If such an assumption is made, the shearing stress in each fastener can be found from the torsion formula

$$f_v = \frac{Md}{J} \tag{8.1}$$

where

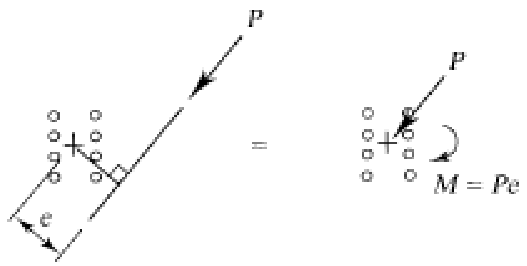
d = distance from the centroid of the area to the point where the stress is being computed
J = polar moment of inertia of the area about the centroid

and the stress f_v is perpendicular to d. Although the torsion formula is applicable only to right circular cylinders, its use here is conservative, yielding stresses that are somewhat larger than the actual stresses.

If the parallel-axis theorem is used and the polar moment of inertia of each circular area about its own centroid is neglected, J for the total area can be approximated as

$$J = \sum Ad^2 = A\sum d^2$$

FIGURE 8.3



Elastic Analysis

In Figure 8.3a, the fastener shear areas and the load are shown separate from the column and bracket plate. The eccentric load P can be replaced with the same load acting at the centroid plus the couple, $M = Pe$, where e is the eccentricity. If this replacement is made, the load will be concentric, and each fastener can be assumed to resist an equal share of the load, given by $p_c = P/n$, where n is the number of fasteners. The fastener forces resulting from the couple can be found by considering the shearing stress in the fasteners to be the result of torsion of a cross section made up of the cross-sectional areas of the fasteners. If such an assumption is made, the shearing stress in each fastener can be found from the torsion formula

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where

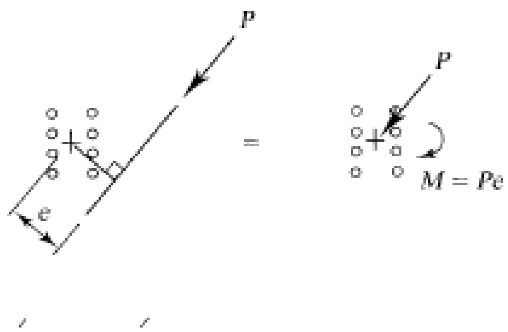
d = distance from the centroid of the area to the point where the stress is being computed
J = polar moment of inertia of the area about the centroid

and the stress f_v is perpendicular to d. Although the torsion formula is applicable only to right circular cylinders, its use here is conservative, yielding stresses that are somewhat larger than the actual stresses.

If the parallel-axis theorem is used and the polar moment of inertia of each circular area about its own centroid is neglected, J for the total area can be approximated as

$$J = \sum Ad^2 = A\sum d^2$$

FIGURE 8.3



and the total fastener force is

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2}$$

where

$$\sum p_x = p_{cx} + p_{mx}$$
$$\sum p_y = p_{cy} + p_{my}$$

If P, the load applied to the connection, is a factored load, then force p on the fastener is the factored load to be resisted in shear and bearing—that is, the required design strength. If P is a service load, then p will be the required allowable strength of the fastener.

EXAMPLE 8.1

Determine the critical fastener force in the bracket connection shown in Figure 8.5.

SOLUTION

The centroid of the fastener group can be found by using a horizontal axis through the lower row and applying the principle of moments:

$$\bar{y} = \frac{2(5) + 2(8) + 2(11)}{8} = 6 \text{ in.}$$

FIGURE 8.5

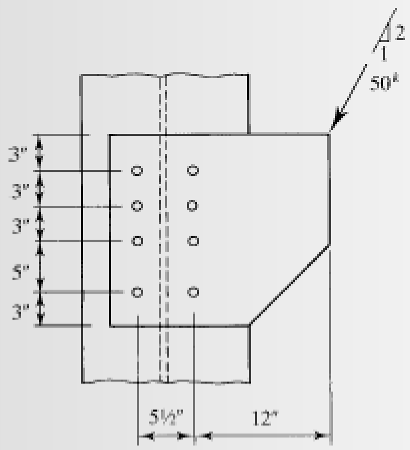
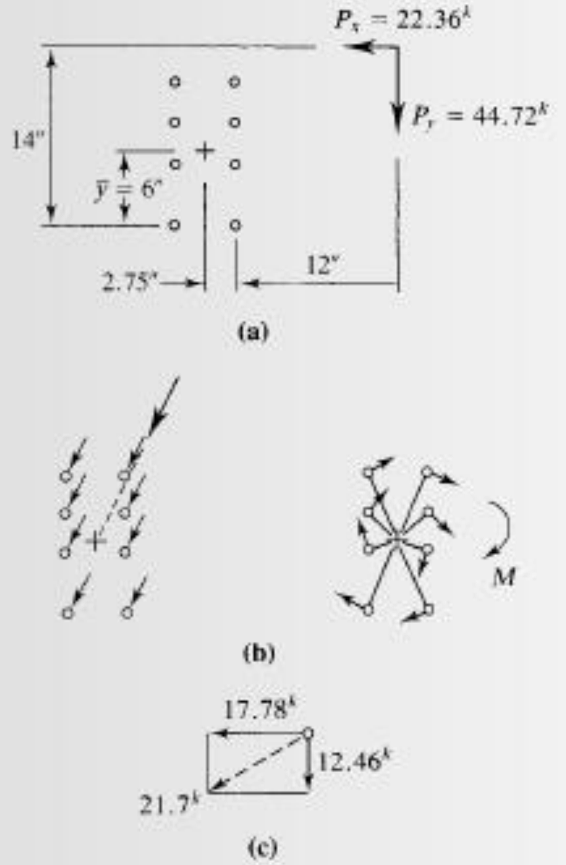


FIGURE 8.6



The horizontal and vertical components of the load are

$$P_x = \frac{1}{\sqrt{5}}(50) = 22.36 \text{ kips} \leftarrow \quad \text{and} \quad P_y = \frac{2}{\sqrt{5}}(50) = 44.72 \text{ kips} \downarrow$$

Referring to Figure 8.6a, we can compute the moment of the load about the centroid:

$$M = 44.72(12 + 2.75) - 22.36(14 - 6) = 480.7 \text{ in.-kips} \qquad \text{(clockwise)}$$

Figure 8.6b shows the directions of all component bolt forces and the relative magnitudes of the components caused by the couple. Using these directions and relative magnitudes as a guide and bearing in mind that forces add by the parallelogram law, we can conclude that the lower right-hand fastener will have the largest resultant force.

The horizontal and vertical components of force in each bolt resulting from the concentric load are

$$p_{cx} = \frac{22.36}{8} = 2.795 \text{ kips} \leftarrow \quad \text{and} \quad p_{cy} = \frac{44.72}{8} = 5.590 \text{ kips} \downarrow$$

For the couple,

$$\sum(x^2+y^2) = 8(2.75)^2 + 2[(6)^2 + (1)^2 + (2)^2 + (5)^2] = 192.5 \text{ in.}^2$$

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{480.7(6)}{192.5} = 14.98 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{480.7(2.75)}{192.5} = 6.867 \text{ kips} \downarrow$$

$$\sum p_x = 2.795 + 14.98 = 17.78 \text{ kips} \leftarrow$$

$$\sum p_y = 5.590 + 6.867 = 12.46 \text{ kips} \downarrow$$

$$p = \sqrt{(17.78)^2 + (12.46)^2} = 21.7 \text{ kips} \quad \text{(see Figure 8.6c)}$$

ANSWER The critical fastener force is 21.7 kips. Inspection of the magnitudes and directions of the horizontal and vertical components of the forces confirms the earlier conclusion that the fastener selected is indeed the critical one.

CHAPTER 9

Composite Construction

9.1 INTRODUCTION

Composite construction employs structural members that are composed of two materials: structural steel and reinforced concrete. Strictly speaking, any structural member formed with two or more materials is composite. In buildings and bridges, however, that usually means structural steel and reinforced concrete, and that usually means composite beams or columns. Composite columns are being used again in some structures after a period of disuse; we cover them later in this chapter. Our coverage of beams is restricted to those that are part of a floor or roof system. Composite construction is covered in AISC Specification Chapter I, "Design of Composite Members."

Composite beams can take several forms. The earliest versions consisted of beams encased in concrete (Figure 9.1a). This was a practical alternative when the primary means of fireproofing structural steel was to encase it in concrete; the rationale was that if the concrete was there, we might as well account for its contribution to the strength of the beam. Currently, lighter and more economical methods of fireproofing are available, and encased composite beams are rarely used. Instead, composite behavior is achieved by connecting the steel beam to the reinforced concrete slab it supports, causing the two parts to act as a unit. In a floor or roof system, a portion of the slab acts with each steel beam to form a composite beam consisting of the rolled steel shape augmented by a concrete flange at the top (Figure 9.1b).

This unified behavior is possible only if horizontal slippage between the two components is prevented. That can be accomplished if the horizontal shear at the interface is resisted by connecting devices known as anchors (sometimes called shear connectors). These devicesÑ which can be steel headed studs or short lengths of small steel channel shapesÑ are welded to the top flange of the steel beam at prescribed intervals and provide the connection mechanically through anchorage in the hardened concrete (Figure 9.1c). Studs are the most commonly used type of anchors, and more than one can be used at each location if the flange is wide enough to accommodate them (which depends on the allowable spacing, which we consider in Section 9.4). One reason for the popularity of steel headed stud anchors is their ease

FIGURE 9.1

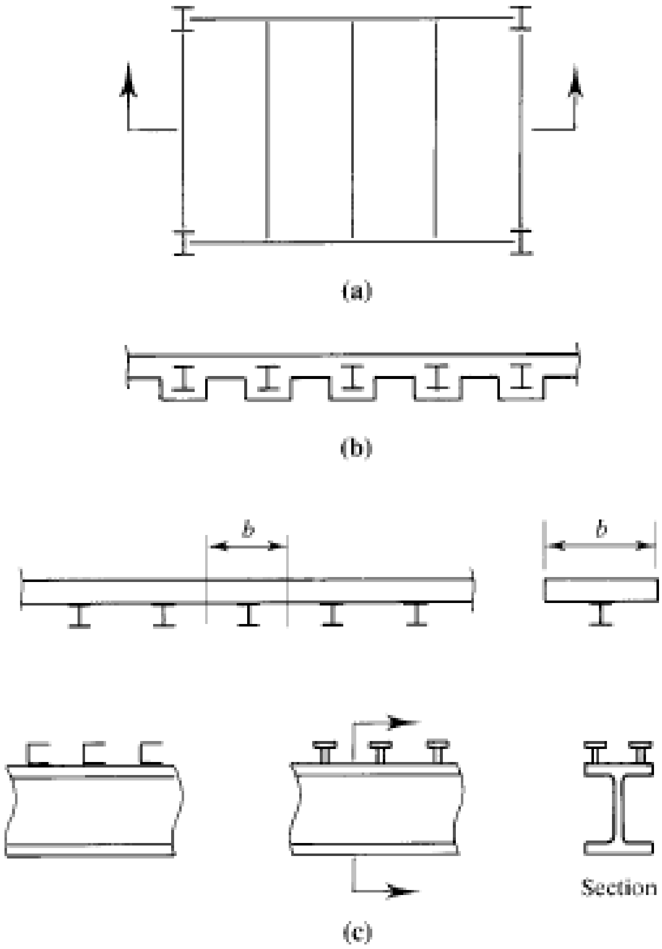
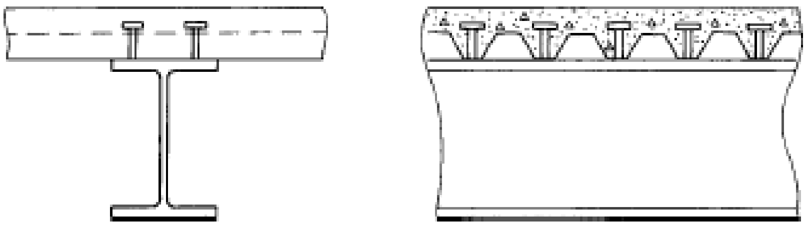


FIGURE 9.2



Almost all highway bridges that use steel beams are of composite construction, and composite beams are frequently the most economical alternative in buildings. Although smaller, lighter rolled steel beams can be used with composite construction, this advantage will sometimes be offset by the additional cost of the studs. Even so, other advantages may make composite construction attractive. Shallower beams can be used, and deflections will be smaller than with conventional noncomposite construction.

Elastic Stresses in Composite Beams

Although the available strength of composite beams is usually based on conditions at failure, an understanding of the behavior at service loads is important for several reasons. Deflections are always investigated at service loads, and in some cases, the available strength is based on the limit state of first yield.

Flexural and shearing stresses in beams of homogeneous materials can be computed from the formulas

$$f_b = \frac{Mc}{I} \qquad \text{and} \qquad f_v = \frac{VQ}{It}$$

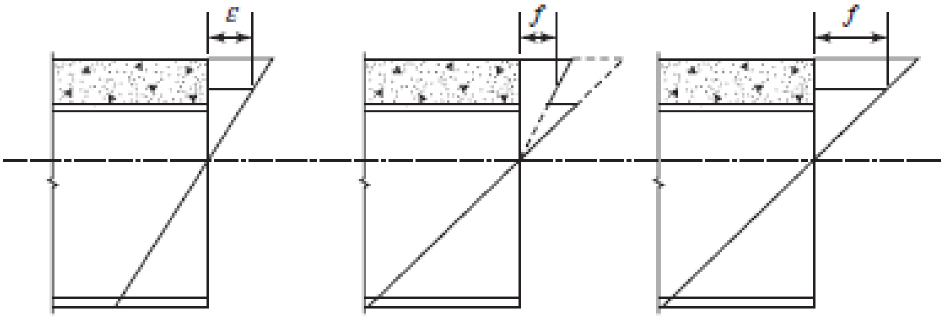
A composite beam is not homogeneous, however, and these formulas are not valid. To be able to use them, an artifice known as the *transformed section* is employed to "convert" the concrete into an amount of steel that has the same effect as the concrete. This procedure requires the strains in the fictitious steel to be the same as those in the concrete it replaces. Figure 9.3 shows a segment of a composite beam with stress and strain diagrams superimposed. If the slab is properly attached to the rolled steel shape, the strains will be as shown, with cross sections that are plane before bending remaining plane after bending. However, a continuous linear stress distribution as shown in Part c of the figure is valid only if the beam is assumed to be homogeneous. We first require that the strain in the concrete at any point be equal to the strain in any replacement steel at that point:

$$\varepsilon_c = \varepsilon_s \text{ or } \frac{f_c}{E_c} = \frac{f_s}{E_s}$$

and

$$f_s = \frac{E_s}{E_c} f_c = n f_c \tag{9.1}$$

FIGURE 9.3



(a)
Strain
(ε)

(b)
Actual Stress
($f = E\varepsilon$)

(c)
Stress in Transformed
Section

where

E_c = modulus of elasticity of concrete

$$n = \frac{E_s}{E_c} = \text{modular ratio}$$

AISC I2.lb gives the modulus of elasticity of concrete as*

$$E_c = w_c^{1.5}\sqrt{f_c'}\ \text{ksi}$$

where

w_c = unit weight of concrete in lb/ft^3. (normal-weight concrete weighs approximately 145 lb/ft^3)

f_c' = 28-day compressive strength of concrete (kips/in.^{2})

The AISC Specification also gives a metric version of the equation for E_c.

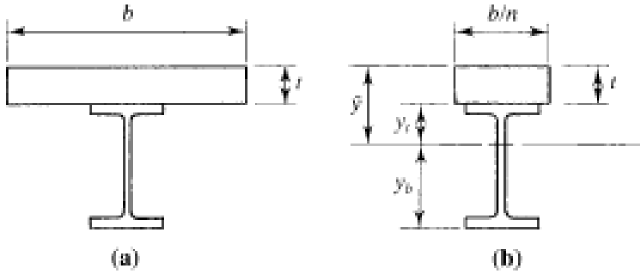
Equation 9.1 can be interpreted as follows: n square inches of concrete are required to resist the same force as one square inch of steel. To determine the area of steel that will resist the same force as the concrete, divide the concrete area by n. That is, replace A_c by A_c/n. The result is the *transformed area*.

Consider the composite section shown in Figure 9.4a (determination of the effective flange width b when the beam is part of a floor system is discussed presently). To transform the concrete area, A_c, we must divide by n. The most convenient way to do this is to divide the width by n and leave the thickness unchanged. Doing so results in the homogeneous steel section of Figure 9.4b. To compute stresses, we locate the neutral axis of this composite shape and compute the corresponding moment of inertia. We can then compute bending stresses with the flexure formula. At the top of the steel,

$$f_{st} = \frac{My_t}{I_{tr}}$$

*The ACI Building Code (ACI, 2008) gives the value of E_c as $w_c^{1.5}(33)\sqrt{f_c'}$, where f_c' is in pounds per square inch.

FIGURE 9.4



At the bottom of the steel,

$$f_{sb} = \frac{My_b}{I_{tr}}$$

where

M = applied bending moment

I_{tr} = moment of inertia about the neutral axis (same as the centroidal axis for this homogeneous section)

y_t = distance from the neutral axis to the top of the steel

y_b = distance from the neutral axis to the bottom of the steel

The stress in the concrete may be computed in the same way, but because the material under consideration is steel, the result must be divided by n (see Equation 9.1) so that

$$\text{Maximum } f_c = \frac{M\bar{y}}{nI_{tr}}$$

where $\bar{y}$ is the distance from the neutral axis to the top of the concrete.

This procedure is valid only for a positive bending moment, with compression at the top, because concrete has negligible tensile strength.

EXAMPLE 9.1

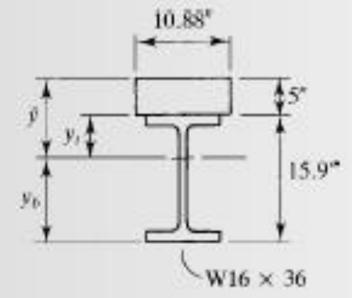
A composite beam consists of a W16 × 36 of A992 steel with a 5-inch-thick × 87-inch-wide reinforced concrete slab at the top. The strength of the concrete is $f_c' = 4$ ksi. Determine the maximum stresses in the steel and concrete resulting from a positive bending moment of 160 ft-kips.

SOLUTION

$$E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$n = \frac{E_s}{E_c} = \frac{29{,}000}{3492} = 8.3 \quad \therefore \text{ Use } n = 8.$$

FIGURE 9.5



Since the modulus of elasticity of concrete can only be approximated, the usual practice of rounding n to the nearest whole number is sufficiently accurate. Thus,

$$\frac{b}{n} = \frac{87}{8} = 10.88 \text{ in.}$$

The transformed section is shown in Figure 9.5. Although the neutral axis is shown below the top of the steel, it is not known yet whether it lies in the steel or the concrete.

The location of the neutral axis can be found by applying the principle of moments with the axis of moments at the top of the slab. The computations are summarized in Table 9.1, and the distance from the top of the slab to the centroid is

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{273.3}{65.00} = 4.205 \text{ in.}$$

Since this is less than 5 inches (the thickness of the slab) the neutral axis lies within the slab. Applying the parallel axis theorem and tabulating the computations in Table 9.2, we obtain the moment of inertia of the transformed section as

$$I_{tr} = 1530 \text{ in.}^4$$

TABLE 9.1

Component	A	y	Ay
Concrete	54.40	2.50	136.0
W16 × 36	10.6	12.95	137.3
	65.00		273.3

TABLE 9.2

Component	A	y	$\bar{I}$	d	$\bar{I} + Ad^2$
Concrete	54.40	2.50	113.3	1.705	271.4
W16 × 36	10.6	12.95	448	8.745	1259
					1530.4

The distance from the neutral axis to the top of the steel is

$$y_t = \bar{y} - t = 4.205 - 5.000 = -0.795 \text{ in.}$$

where t is the thickness of the slab. The negative sign means that the top of the steel is below the neutral axis and is therefore in tension. The stress at the top of the steel is

$$f_{st} = \frac{My_t}{I_{tr}} = \frac{(160 \times 12)(0.795)}{1530} = 0.998 \text{ ksi} \qquad \textbf{(tension)}$$

Stress at the bottom of the steel:

$$y_b = t + d - \bar{y} = 5 + 15.9 - 4.205 = 16.70 \text{ in.}$$

$$f_{sb} = \frac{My_b}{I_{tr}} = \frac{(160 \times 12)(16.70)}{1530} = 21.0 \text{ ksi} \quad \text{(tension)}$$

The stress at the top of the concrete is

$$f_c = \frac{M\bar{y}}{nI_{tr}} = \frac{(160 \times 12)(4.205)}{8(1530)} = 0.660 \text{ ksi}$$

If the concrete is assumed to have no tensile strength, the concrete below the neutral axis should be discounted. The geometry of the transformed section will then be different from what was originally assumed; to obtain an accurate result, the location of the neutral axis should be recomputed on the basis of this new geometry. Referring to Figure 9.6 and Table 9.3, we can compute the new location of the neutral axis as follows:

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{5.44\bar{y}^2 + 137.3}{10.88\bar{y} + 10.6}$$

$$\bar{y}(10.88\bar{y} + 10.6) = 5.44\bar{y}^2 + 137.3$$

$$5.44\bar{y}^2 + 10.6\bar{y} - 137.3 = 0$$

$$\bar{y} = 4.143 \text{ in.}$$

The moment of inertia of this revised composite area is

$$I_{tr} = \frac{1}{3}(10.88)(4.143)^3 + 448 + 10.6(12.95 - 4.143)^2 = 1528 \text{ in.}^4$$

FIGURE 9.6

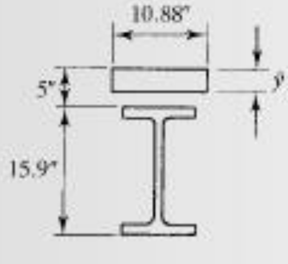


TABLE 9.3

Component	A	y	Ay
Concrete	$10.88\bar{y}$	$\bar{y}/2$	$5.44\bar{y}^2$
W16 × 36	10.6	12.95	137.3

and the stresses are

$$f_{st} = \frac{(160 \times 12)(5 - 4.143)}{1528} = 1.08 \text{ ksi} \quad \text{(tension)}$$

$$f_{sb} = \frac{(160 \times 12)(5 + 15.9 - 4.143)}{1528} = 21.1 \text{ ksi} \quad \text{(tension)}$$

$$f_c = \frac{(160 \times 12)(4.143)}{8(1528)} = 0.651 \text{ ksi}$$

The difference between the two analyses is negligible, so the refinement in locating the neutral axis is not necessary.

ANSWER The maximum stress in the steel is 21.1 ksi tension, and the maximum stress in the concrete is 0.651 ksi compression.

CHAPTER 10

Plate Girders

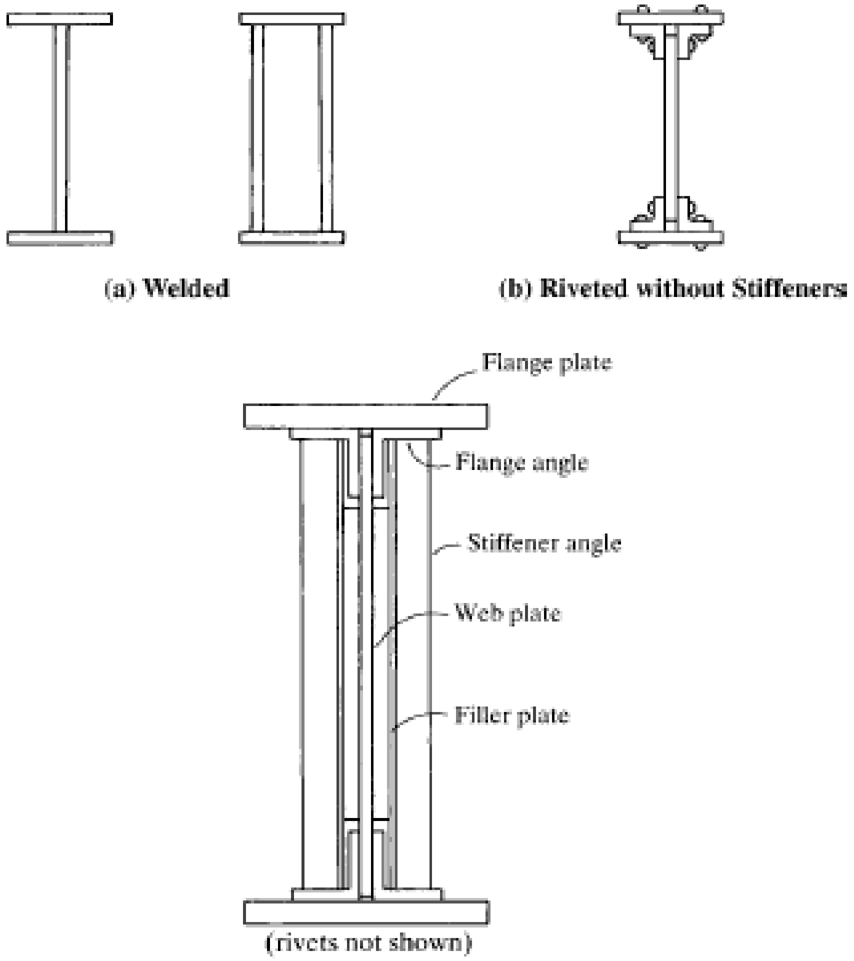
10.1 INTRODUCTION

In this chapter, we consider large flexural members (girders) that are composed of plate elements—in particular, those with noncompact or slender webs. In Chapter 5, "Beams," we covered hot-rolled shapes, and for all the standard sections in the *Manual*, the webs are compact. Some have noncompact flanges, but none have slender flanges. With shapes built up from plates, however, both flanges and webs can be compact, noncompact, or slender. These built-up shapes usually are used when the bending moments are larger than standard hot-rolled shapes can resist, usually because of a large span. These girders are invariably very deep, resulting in noncompact or slender webs.

The AISC Specification covers flexural members with slender webs in Section F5, "Doubly Symmetric and Singly Symmetric I-Shaped Members with Slender Webs Bent About Their Major Axis." This is the category usually thought of as *plate girders*. Flexural members with noncompact webs are covered in Section F4, "Other I-shaped Members with Compact or Noncompact Webs Bent About Their Major Axis." This section deals with both doubly and singly symmetric sections. Interestingly, noncompact webs are more difficult to deal with than slender webs. In a User Note in Section F4, the Specification permits members covered by Section F4 to be designed by the provisions of Section F5. In this book, we do this and use Section F5 for girders with either noncompact or slender webs. We refer to both types as plate girders. Shear provisions for all flexural members are covered in AISC Chapter G, "Design of Members for Shear." Other requirements are given in AISC F13, "Proportions of Beams and Girders."

A plate girder cross section can take several forms. Figure 10.1 shows some of the possibilities. The usual configuration is a single web with two equal flanges, with all parts connected by welding. The box section, which has two webs as well as two flanges, is a torsionally superior shape and can be used when large unbraced lengths are necessary. Hybrid girders, in which the steel in the flanges is of a higher strength than that in the web or webs, are sometimes used.

FIGURE 10.1

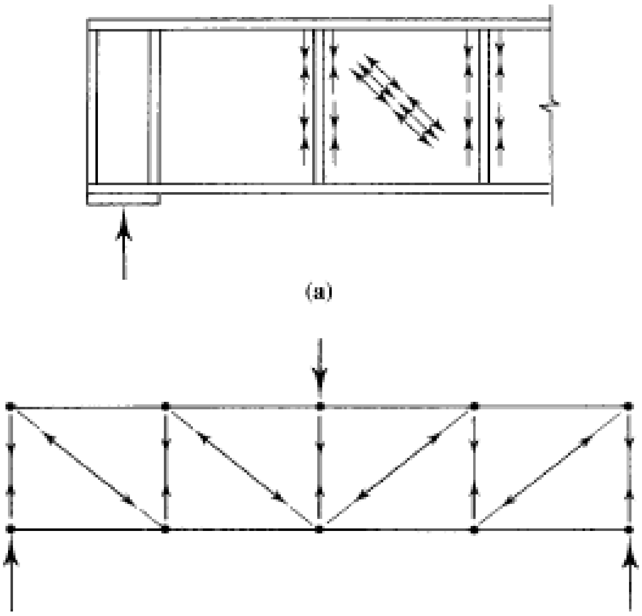


(a) Welded

(b) Riveted without Stiffeners

(c) Riveted with Stiffeners

FIGURE 10.2



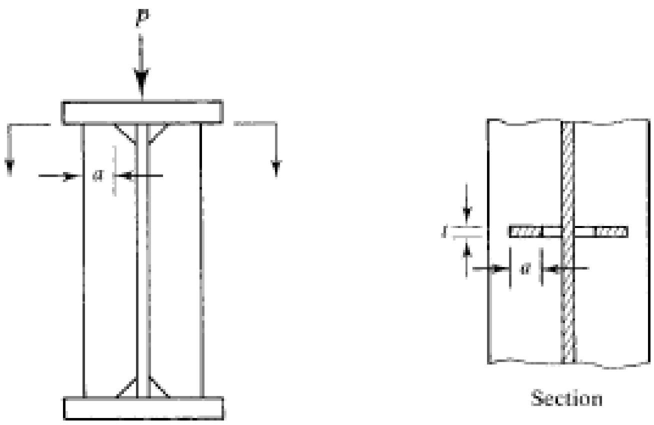
Note: The arrows indicate forces acting on members, rather than on joints

(b)

If an unstiffened web is incapable of resisting the applied shear, appropriately spaced stiffeners are used to develop tension-field action. Cross-sectional requirements for these stiffeners, called *intermediate stiffeners*, are minimal because their primary purpose is to provide stiffness rather than resist directly applied loads.

Additional stiffeners may be required at points of concentrated loads for the purpose of protecting the web from the direct compressive load. These members are called *bearing stiffeners,* and they must be proportioned to resist the applied loads. They can also simultaneously serve as intermediate stiffeners. Figure 10.3 shows a bearing stiffener consisting of two rectangular plates, one on each side of the girder web. The plates are notched, or clipped, at the inside top and bottom corners so as to

FIGURE 10.3



avoid the flange-to-web welds. If the stiffeners are conservatively assumed to resist the total applied load P (this assumption neglects any contribution by the web), the bearing stress on the contact surfaces may be written as

$$f_p = \frac{P}{A_{pb}}$$

where

A_{pb} = projected bearing area
$= 2at$ (see Figure 10.3)

or, expressing the bearing load in terms of the stress,

$$P = f_p A_{pb} \tag{10.1}$$

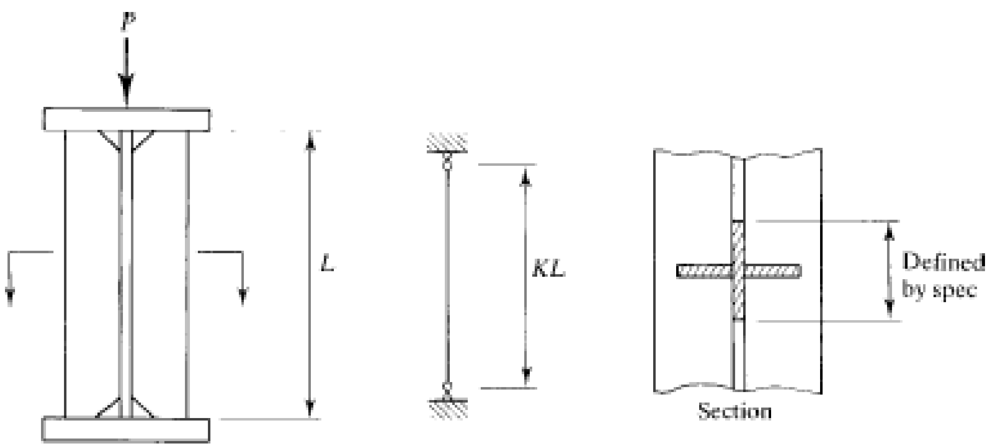
In addition, the pair of stiffeners, together with a short length of web, is treated as a column with an effective length less than the web depth and is investigated for compliance with the same Specification provisions as any other compression member. This cross section is illustrated in Figure 10.4. The compressive strength should always be based on the radius of gyration about an axis in the plane of the web, as instability about the other principal axis is prevented by the web itself.

Other limit states resulting from the application of concentrated loads to the top flange are web yielding, web crippling (buckling), and sidesway web buckling. Sidesway web buckling occurs when the compression in the web causes the *tension* flange to buckle laterally. This phenomenon can occur if the flanges are not adequately restrained against movement relative to one another by stiffeners or lateral bracing.

The welds for connecting the components of a plate girder are designed in much the same way as for other welded connections. The flange-to-web welds must resist the horizontal shear at the interface between the two components. This applied shear, called the *shear flow,* is usually expressed as a force per unit length of girder to be resisted by the weld. From Chapter 5, the shear flow, based on elastic behavior, is given by

$$f = \frac{VQ}{I_x}$$

FIGURE 10.4



10.3 AISC REQUIREMENTS FOR PROPORTIONS OF PLATE GIRDERS

Whether a girder web is noncompact or slender depends on h/t_w, the width-to-thickness ratio of the web, where h is the depth of the web from inside face of flange to inside face of flange and t_w is the web thickness. From AISC B4, Table B4.1b, the web of a doubly symmetric I-shaped section is noncompact if

$$3.76\sqrt{\frac{E}{F_y}} < \frac{h}{t_w} \leq 5.70\sqrt{\frac{E}{F_y}}$$

and the web is slender if

$$\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}}$$

For singly symmetric I-shaped sections, the web is noncompact if

$$\frac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y} - 0.09\right)^2} < \frac{h_c}{t_w} \leq 5.70\sqrt{\frac{E}{F_y}}$$

and it is slender if

$$\frac{h_c}{t_w} > 5.70\sqrt{\frac{E}{F_y}}$$

where

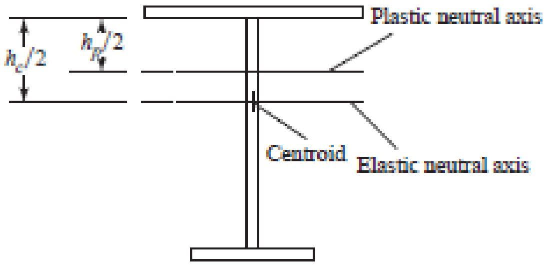
h_c = twice the distance from the elastic neutral axis (the centroidal axis) to the inside face of the compression flange. ($h_c/2$ defines the part of the web that is in compression for elastic bending. $h_c = h$ for girders with equal flanges). See Figure 10.5.

h_p = twice the distance from the plastic neutral axis to the inside face of the compression flange. ($h_p/2$ defines the part of the web in compression for the plastic moment. $h_p = h$ for girders with equal flanges). See Figure 10.5.

M_p = plastic moment = $F_y Z_x$

M_y = yield moment = $F_y S_x$

FIGURE 10.5



To prevent vertical buckling of the compression flange into the web, AISC F13.2 imposes an upper limit on the web slenderness. The limiting value of h/t_w is a function of the aspect ratio, a/h, of the girder panels, which is the ratio of intermediate stiffener spacing to web depth (see Figure 10.6).

For $\frac{a}{h} \leq 1.5$,

$$\left(\frac{h}{t_w}\right)_{max} = 12.0\sqrt{\frac{E}{F_y}} \qquad \text{(AISC Equation F13-3)}$$

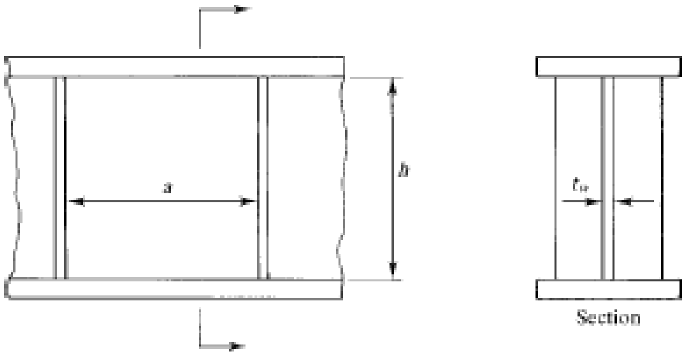
For $\frac{a}{h} > 1.5$,

$$\left(\frac{h}{t_w}\right)_{max} = \frac{0.40E}{F_y} \qquad \text{(AISC Equation F13-4)}$$

where a is the clear distance between stiffeners.

In all girders without web stiffeners, AISC F13.2 requires that h/t_w be no greater than 260 and that the ratio of the web area to the compression flange area be no greater than 10.

FIGURE 10.6



For singly symmetric sections, the proportions of the cross section must be such that

$$0.1 \le \frac{I_{yc}}{I_y} \le 0.9 \qquad \text{(AISC Equation F13-2)}$$

where
I_{yc} = moment of inertia of the compression flange about the y axis
I_y = moment of inertia of the entire cross section about the y axis

10.4 FLEXURAL STRENGTH

The nominal flexural strength M_n of a plate girder is based on one of the limit states of tension flange yielding, compression flange yielding or local buckling (FLB), or lateral-torsional buckling (LTB).

Tension Flange Yielding

From Chapter 5, the maximum bending stress in a flexural member bent about its strong axis is

$$f_b = \frac{M}{S_x}$$

where S_x is the elastic section modulus about the strong axis. Expressing the bending moment as a function of the section modulus and stress gives

$$M = f_b S_x$$

AISC F5 gives the nominal flexural strength based on tension flange yielding as

$$M_n = F_y S_{xt} \qquad \text{(AISC Equation F5-10)}$$

where S_{xt} = elastic section modulus referred to the tension side.

Compression Flange Strength

The compression flange nominal strength is given by

$$M_n = R_{pg} F_{cr} S_{xc} \qquad \text{(AISC Equation F5-7)}$$

where
R_{pg} = bending strength reduction factor
F_{cr} = critical compressive flange stress, based on either yielding or local buckling
S_{xc} = elastic section modulus referred to the compression side

The bending strength reduction factor is given by

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0 \qquad \text{(AISC Equation F5-6)}$$

where

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} \leq 10 \qquad \text{(AISC Equation F4-12)}$$

b_{fc} = width of the compression flange
t_{fc} = thickness of the compression flange

(The upper limit of 10 in Equation F4-12 is not actually part of the AISC Equation, but AISC F5.2 stipulates that limit.)

The critical compression flange stress F_{cr} depends on whether the flange is compact, noncompact, or slender. The AISC Specification uses the generic notation λ, λ_p, and λ_r to define the flange width-to-thickness ratio and its limits. From AISC Table B4.1b,

$$\lambda = \frac{b_f}{2t_f}$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}}$$

$$k_c = \frac{4}{\sqrt{h/t_w}} \text{ but } (0.35 \leq k_c \leq 0.76)$$

$F_L = 0.7F_y$ for girders with slender webs. (See AISC Table B4.1b for compact and noncompact webs.)

If $\lambda \leq \lambda_p$, the flange is compact. The limit state of yielding will control, and $F_{cr} = F_y$, resulting in

$$M_n = R_{pg} F_y S_{xc} \qquad \text{(AISC Equation F5-1)}$$

If $\lambda_p < \lambda \leq \lambda_r$, the flange is noncompact. Inelastic FLB will control, and

$$F_{cr} = F_y - 0.3F_y\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{(AISC Equation F5-8)}$$

If $\lambda > \lambda_r$, the flange is slender, elastic FLB will control, and

$$F_{cr} = \frac{0.9Ek_c}{\left(\frac{b_f}{2t_f}\right)^2} \qquad \text{(AISC Equation F5-9)}$$

Lateral-Torsional Buckling

The nominal lateral-torsional bucking strength is given by

$$M_n = R_{pg} F_{cr} S_{xc} \qquad \text{(AISC Equation F5-2)}$$

Whether lateral-torsional buckling will occur depends on the amount of lateral support—that is, the unbraced length L_b. If the unbraced length is small enough, yielding or flange local buckling will occur before lateral-torsional buckling. The length parameters are L_p and L_r, where

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}} \qquad \text{(AISC Equation F4-7)}$$

$$L_r = \pi r_t \sqrt{\frac{E}{0.7F_y}} \qquad \text{(AISC Equation F5-5)}$$

r_t = radius of gyration about the weak axis for a portion of the cross section consisting of the compression flange and one-third of the compressed part of the web. For a doubly symmetric girder, this dimension will be one-sixth of the web depth. (See Figure 10.7.) This definition is a conservative approximation of r_t (see the user note in AISC F4.2). The exact definition is given by AISC Equation F4-11.

If $L_b \le L_p$, there is no lateral torsional buckling.

If $L_p < L_b \le L_r$, Failure will be by inelastic LTB, and

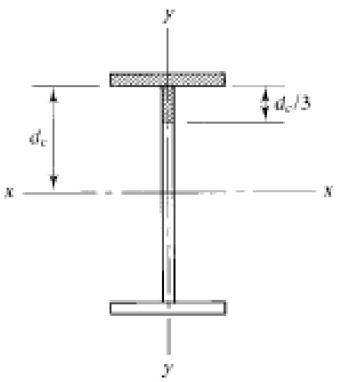
$$F_{cr} = C_b \quad F_y - 0.3F_y \left(\frac{L_b - L_p}{L_r - L_p} \right) \le F_y \qquad \text{(AISC Equation F5-3)}$$

If $L_b > L_r$, failure will be by elastic LTB, and

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_t} \right)^2} \le F_y \qquad \text{(AISC Equation F5-4)}$$

C_b is defined by AISC Equation F1-1 and is covered in Chapter 5 of this book.

FIGURE 10.7



10.5 SHEAR STRENGTH

The shear strength of a plate girder is a function of the depth-to-thickness ratio of the web and the spacing of any intermediate stiffeners that may be present. The shear capacity has two components: the strength before buckling and the postbuckling strength. The postbuckling strength relies on tension-field action, which is made possible by the presence of intermediate stiffeners. If stiffeners are not present or are spaced too far apart, tension-field action will not be possible, and the shear capacity will consist only of the strength before buckling. The AISC Specification covers shear strength in Chapter G, "Design of Members for Shear." In that coverage, the constants k_v and C_v are used. AISC defines k_v, which is a plate-buckling coefficient, in Section G2 as follows:

$$k_v = 5 + \frac{5}{(a/h)^2} \qquad \text{(AISC Equation G2-6)}$$

$$= 5 \text{ if } \frac{a}{h} > 3$$

$$= 5 \text{ if } \frac{a}{h} > {\frac{260}{(h/t_w)}}^{2}$$

$$= 5 \text{ in unstiffened webs with } \frac{h}{t_w} < 260$$

For C_v, which can be defined as the ratio of the critical web shear stress to the web shear yield stress,

If $\frac{h}{t_w} \le 1.10\sqrt{\frac{k_v E}{F_y}}$,

$$C_v = 1.0 \qquad \text{(AISC Equation G2-3)}$$

If $1.10\sqrt{\frac{k_v E}{F_y}} < \frac{h}{t_w} \le 1.37\sqrt{\frac{k_v E}{F_y}}$,

$$C_v = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad \text{(AISC Equation G2-4)}$$

If $\frac{h}{t_w} > 1.37\sqrt{\frac{k_v E}{F_y}}$,

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \qquad \text{(AISC Equation G2-5)}$$

Read the rest in Chapter 10