Ministry of Science and Technology

Department of Technical and Vocational Education

Sample Questions & Answers

for

ME 5019
Computer application in Mechanical Engineering (II)

B.E
(Mechanical Engineering)

Sample Questions

Chapter	Example	Problem
1	5 examples	4 problems
2	1 examples	4 problems
3	1 examples	4 problems
4	1 examples	3 problems
5	3 examples	3 problems
Sub total	**12 examples**	**18 problems**
Total 30 problems		

Chapter 1

Example 1 See in the Art.1.2.1 of **Variational Method**
(Page 5 in ME 5019(2))

Example 2 See in the Art.1.2.2 of **Collational Method**
(Page 6 in ME 5019(2))

Example 3 See in the Art.1.2.3 of **Subdomain Method**
(Page 6 in ME 5019(2))

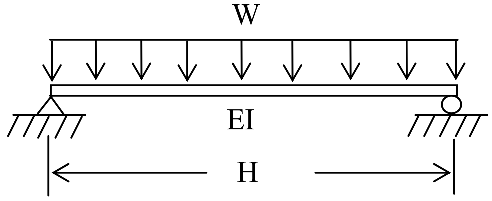
Example 4 See in the Art.1.2.4 of **Galerkin's Method**
(Page 7 in ME 5019(2))

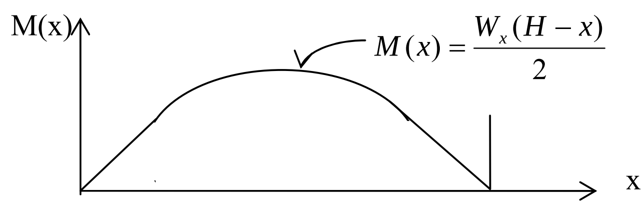
Example 5 See in the Art.1.2.1 of **Least Square Method**
(Page 8 in ME 5019(2))

Problems 1.1 , 1.2 , 1.3 and 1.4 in ME 5019(2) and its solutions are as follows:

Prob (1.1~1.4) Obtain an approximate displacement equation for the simply supported beam of length H and section property EI shown in Figure P1.1. Assume that the ial displacement equation is $y(x) = A\sin \pi\, x / H$. Compare the deflection at the center with the theoretical value $y = -5WH^4/384EI$. The governing differential equation is

$$EI\frac{d^2y}{d\,x^2} - \frac{Wx(H-x)}{2} = 0$$





Prob 1.1 Evaluate A by minimizing the integral

$$\Pi = \int_0^H \left[\frac{EI}{2}\left(\frac{dy}{dx}\right)^2 + \left(\frac{Wx(H-x)}{2}\right)y\right]dx$$

Solution

Governing Differential Equation

$$\mathrm{EI}\frac{d^2y}{dx^2} - \frac{Wx(H-x)}{2} = 0 \qquad \text{……………………(1)}$$

Trial displacement equation

$$y(x) = A\,Sin\frac{\pi x}{H} \qquad \text{……………………..(2)}$$

$$\frac{dy}{dx} = A\frac{\pi}{H}Cos\frac{\pi x}{H} \quad , \quad \frac{d^2y}{d\,x^2} = -A\frac{\pi^2}{H^2}Sin\frac{\pi x}{H}$$

The integral of variational method

$$\Pi = \int_0^H \left[\frac{\mathrm{EI}}{2}\left(\frac{dy}{dx}\right)^2 + \left(\frac{Wx(H-x)}{2}\right)y\right]dx \text{ ……………(3)}$$

Substituting $\frac{dy}{dx}$ and y in the equation (3)

$$\Pi = \int_0^H \frac{EI}{2}\left(-\frac{A\pi^2}{H^2}Sin\frac{\pi x}{H}\right)^2 dx + \int_0^H \frac{Wx(H-x)}{2}A\,Sin\frac{\pi x}{H}dx$$

$$= \frac{\text{EI}}{2}\left(\frac{A\pi}{H}\right)^2 \int_0^H \frac{1}{2}\left(Cos\frac{2\pi x}{H}+1\right)dx + \int_0^H \frac{WA}{2}\left[xH\,Sin\frac{\pi x}{H} - x^2 Sin\frac{\pi x}{H}\right]dx$$

$$= \frac{\text{EI}}{2}\left(\frac{A\pi}{H}\right)^2 \frac{1}{2}\left[\frac{H}{2\pi}Sin^2\frac{\pi x}{H}+x\right]_0^H + H\left[-\frac{H}{\pi}xCos\frac{\pi x}{H} - \frac{H^2}{\pi^2}Sin\frac{\pi x}{H}\right]_0^H$$

$$+\left[-\frac{H}{\pi}x^2 Cos\frac{\pi x}{H} + 2x\frac{H^2}{\pi_2}Sin\frac{\pi x}{H} + 2\frac{H^3}{\pi^3}Cos\frac{\pi x}{H}\right]_0^H$$

$$= \frac{\text{EI}}{4}\frac{A^2\pi^2}{H} + \frac{H^3}{H} + \left(\frac{H^3}{\pi^3} - 2\frac{H^3}{\pi^3} - 2\frac{H^3}{\pi^3}\right)$$

$$= \frac{\text{EI}}{4}\frac{A^2\pi^2}{H} + 4\frac{WAH^3}{2\pi^3}$$

Minimizing Π yield, $\frac{d\prod}{2A} = 0$

$$\frac{d\prod}{dA} = \frac{\text{EI}}{2}\frac{\pi^2}{H} \times A + \frac{4WH^3}{2\pi^3} = 0$$

$$A = -\frac{8WH^4}{\text{EI}\pi^5} \qquad \text{Ans:}$$

Prob 1.2 Evaluate A by requiring that the residual vanish at (a) $x = H/3$, and (b) $x = H/2$.

Solution

$$\text{EI}\frac{d^2y}{dx^2} - \frac{Wx(H-x)}{2} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Trial equation $y(x) = A\,Sin\frac{\pi x}{H}$

$$\frac{dy}{dx} = \frac{AH}{\pi}Cos\frac{\pi x}{H} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\frac{d^2y}{dx^2} = -\frac{AH^2}{\pi^2}Sin\frac{\pi x}{H} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

By substituting $\frac{d^2y}{dx^2}$ in equation (1)

The residual equation $R(x)$ is obtained as follows

$$R(x) = \text{EI}\left[\frac{-AH^2}{\pi^2}Sin\frac{\pi x}{H}\right] - \frac{Wx(H-x)}{2}$$

The residual $R(x)$ vanished at $x = \frac{H}{3}$

$$\therefore \quad R(x) = \text{EI}\left[\frac{-AH^2}{\pi^2}Sin\left(\frac{\pi \times \frac{H}{3}}{H}\right)\right] - \frac{W \times \frac{H}{3}(H - \frac{H}{3})}{2} = 0$$

$$A = -\frac{WH^4}{9\text{EI}\pi^2 Sin\frac{\pi}{3}}$$

$$= -0.0129\frac{WH^4}{\mathrm{EI}} \qquad \text{(Ans:)}$$

Similarly at $x = \frac{H}{2}, R(\frac{H}{2}) = 0$

$$\text{Gives} \quad A = -0.0127\frac{WH^4}{\mathrm{EI}} \qquad \text{(Ans:)}$$

Prob. 1.3 Evaluate A using the subdomain method.

Solution

Using subdomain method

$$\int_0^H R(x)dx = 0$$

$$\int_0^H \left[\mathrm{EI}\left(\frac{-A\pi^2}{H^2}Sin\frac{\pi x}{H}\right) - \frac{Wx(H-x)}{2}\right]dx = 0$$

$$\mathrm{EI}\times\frac{A\pi^2}{H^2}\left[-\frac{H}{\pi}Cos\frac{\pi x}{H}\right]_0^H - \frac{W}{2}\left[H\frac{x^2}{2} - \frac{x^3}{3}\right]_0^H = 0$$

$$\mathrm{EI}\frac{2A\pi^2 H}{\pi H^2} + \frac{1}{12}WH^3 = 0$$

$$A = -\frac{WH^4}{24\mathrm{EI}\pi} \qquad \text{(Ans:)}$$

Prob. 1.4 Evaluate A using Galerkin's method.

Solution

Using Galerkin Method $\int W_i(x)R(x)dx = 0$ …………….(1)

Weighing function $W_i(x) = Sin\frac{\pi x}{H}$

Substituting $W_i(x)$ and the $R(x)$ from problem 1.2 into the equation (1)

$$\int_0^H Sin\frac{\pi x}{H}\left[\mathrm{EI}\left(\frac{-A\pi^2}{H^2}Sin\frac{\pi x}{H}\right) - \frac{Wx(H-x)}{2}\right]dx = 0$$

$$\mathrm{EI}\times\frac{2A\pi^2 H}{\pi H^2} + \frac{1}{12}WH^3 = 0$$

$$A = -\frac{WH^4}{24\mathrm{EI}\pi} \qquad \text{(Ans:)}$$

Chapter 2

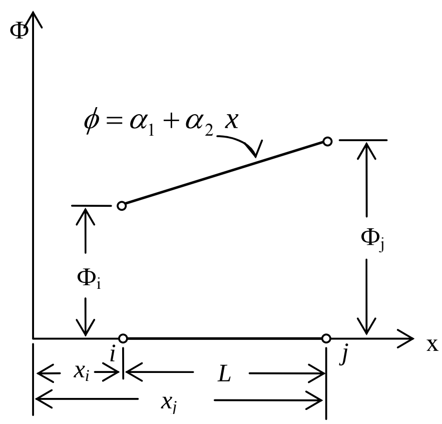
Example 1 See in Art.2.2 **Illustrative Example**
(Page 17 in ME 5019(2))

Problems 2.1 , 2.2 , 2.3 and 2.4 in ME 5019(2) and its solutions are as follows:

Prob. 2.1 The nodal coordinates X_i and X_j and the nodal values of Φ_i and Φ_j for several linear elements are given below. Evaluate ϕ at the given value of x. The x values are in centimeters, and Φ_i and Φ_j are in degrees Celsius.

	x	X_i	X_j	Φ_i	Φ_j
(a)	0.8	0.0	1.5	60	43
(b)	3.6	3.0	4.5	27	33
(c)	7.1	6.5	7.5	63	51

Solution



Given data $X_i = 0.0, X_j = 1.5, \phi_i = 60, \phi_j = 43$

Find $\phi = ?$ at $x = 0.8$

$$\phi = N_i\phi_i + N_j\phi_j$$

where $N_i = \dfrac{X_j - x}{L}$, $N_j = \dfrac{x - X_i}{L}$

$$L = X_j - X_i$$

$$\phi = \left(\frac{X_j - x}{L}\right)\phi_i + \left(\frac{x - X_i}{L}\right)\phi_j \tag{1}$$

(a) $L = X_j - X_i = 1.5 - 0.0 = 1.5$

Substituting $X_i = 0.0, X_j = 1.5, \phi_i = 60, \phi_j = 43, L = 1.5$ and $x = 0.8$ in equation (1), yields

$$\phi = \frac{(1.5-0.8)}{1.5}\times 60 + \frac{(0.8-0.0)}{1.5}\times 43$$
$$\phi = 50.93\,^{\circ}\text{C}$$

(b)Similarly $L = 4.5 - 3 = 1.5$

Substituting $X_i = 3, X_j = 4.5, \phi_i = 27, \phi_j = 3, L = 1.5$ and $x = 3.6$ in equation (1), yields

$$\phi = 29.4\,^{\circ}\text{C}$$

(c)Similarly $L = 7.5 - 6.5 = 1$

Substituting $X_i = 6.5, X_j = 7.5, \phi_i = 63, \phi_j = 51, L = 1$ in equation (1),

yields $\phi = 55.8\,^{\circ}\text{C}$ (Ans)

Prob.2.2 Evaluate $d\phi / dx$ for the corresponding element in Problem 2.1.

Solution

$$\phi = \left(\frac{X_j - x}{L}\right)\phi_i + \left(\frac{x - X_i}{L}\right)\phi_j \qquad (1)$$

$$\frac{d\phi}{dx} = -\frac{\phi_i}{L} + \frac{\phi_j}{L} = \frac{\phi_j - \phi_i}{L} \qquad (2)$$

From problem 2.1(a), $\phi_i = 60, \phi_j = 43, L = 1.5$

Substituting in equation (2), yields

$$\frac{d\phi}{dx} = \frac{43-60}{1.5} = -11.33\,^{\circ}\text{C/cm}$$

Similarly, substituting $\phi_i = 27, \phi_j = 33, L = 1.5$ in equation (2), yields

$$\frac{d\phi}{dx} = \frac{33-27}{1.5} = 4\,^{\circ}\text{C/cm}$$

Similarly, substituting $\phi_i = 63, \phi_j = 51, L = 1$ in equation (2), yields

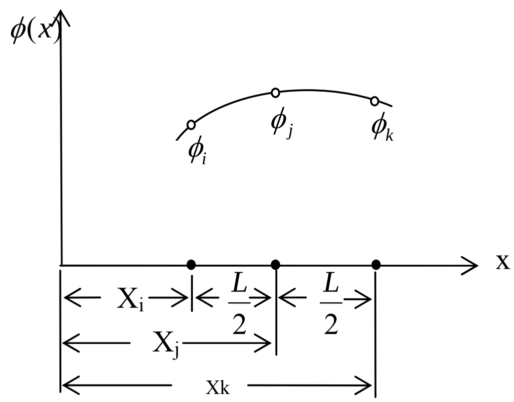
$$\frac{d\phi}{dx} = -6\ \ ^{\circ}\text{C/cm}$$

Prob. 2.3 The shape function for the quadratic element shown in Figure P2.3 are

$$N_i = \frac{2}{L^2}(x - X_j)(x - X_k)$$
$$N_j = -\frac{4}{L^2}(x - X_i)(x - X_k)$$
$$N_k = \frac{2}{L^2}(x - X_i)(x - X_j)$$

(a) Show that these shape functions equal one at their own node and are zero at the other two nodes. Also show that the shape functions sum to one .

(b) Show that the derivatives of N_i, N_j and N_k with respect to x sum to zero.



Solution

For the quadratic element equation, the shape functions are

$$N_i = \frac{2}{L^2}(x - X_j)(x - X_k)$$

$$N_j = \frac{-4}{L^2}(x - X_i)(x - X_k)$$

$$N_k = \frac{2}{L^2}(x - X_i)(x - X_j)$$

$$X_j - X_i = \frac{L}{2}, X_k - X_j = \frac{L}{2}, X_k - X_i = L$$

(a) For own node i at $x = X_i$

$$N_i = \frac{2}{L^2}(X_i - X_j)(X_i - X_k)$$

$$= \frac{2}{L^2}\left(-\frac{L}{2}\right)(-L) = 1$$

Similarly, for own node j at $x = X_j$

$$N_j = \frac{-4}{L^2}(X_j - X_i)(X_j - X_k)$$

$$= \frac{-4}{L^2} \times \frac{L}{2} \times \left(-\frac{L}{2}\right) = 1$$

Similarly, for own node k at $x = X_k$

$$N_k = \frac{2}{L^2}(X_k - X_i)(X_k - X_j)$$

$$= \frac{2}{L^2} \times L \times \left(\frac{L}{2}\right) = 1$$

For other node j & k at $x = X_i$,

$$N_j = \frac{-4}{L^2}(X_i - X_i)(X_i - X_k) = 0$$

$$N_k = \frac{2}{L^2}(X_i - X_i)(X_i - X_j) = 0$$

For other node i & k at $x = X_j$,

$$N_i = \frac{2}{L^2}(X_j - X_j)(X_j - X_k) = 0$$

$$N_k = \frac{2}{L^2}(X_j - X_i)(X_j - X_j) = 0$$

For other node i & j at $x = X_k$,

$$N_i = \frac{2}{L^2}(X_k - X_j)(X_k - X_k) = 0$$

$$N_j = \frac{-4}{L^2}(X_k - X_i)(X_k - X_k) = 0$$

(b)

$$\frac{dN_i}{dx} = \frac{d}{dx}\left\{\frac{2}{L^2}(x^2 - X_k x - X_j x - X_j X_k\right\}$$

$$\frac{dN_i}{dx} = \frac{2}{L^2}(2x - X_j - X_k)$$

Similarly

$$\frac{dN_j}{dx} = \frac{-4}{L^2}(2x - X_i - X_k)$$

$$\frac{dN_k}{dx} = \frac{2}{L^2}(2x - X_i - X_j)$$

$$\frac{dN_i}{dx} + \frac{dN_j}{dx} + \frac{dN_k}{dx} = \frac{2}{L^2}(2x - X_j - X_k) - \frac{4}{L^2}(2x - X_i - X_k) + \frac{2}{L^2}(2x - X_i - X_j)$$

$$\frac{dN_i}{dx} + \frac{dN_j}{dx} + \frac{dN_k}{dx} = \frac{2}{L^2}(X_i - X_j) + \frac{2}{L^2}(X_k - X_j)$$

$$= \frac{2}{L^2}(-\frac{L}{2}) + \frac{2}{L^2}(\frac{L}{2})$$

$$= 0$$

Prob. 2.4 The implementation of the finite element method requires the evaluation of integrals that Contain the shape functions or their derivatives. Evaluate

(a) $\int_{X_i}^{X_j} N_i dx$ (b) $\int_{X_i}^{X_j} \frac{dN_i}{dx}\frac{dN_j}{dx} dx$ (c) $\int_{X_i}^{X_j} N_j^2 dx$

for the linear element.

Solution

The shape function of liner element

$$\phi = N_i \Phi_i + N_j \Phi_j$$

where $N_i = \dfrac{X_j - x}{L}$ and $N_j = \dfrac{x - X_i}{L}$

$$\int_{X_i}^{X_j} N_i dx = \int_{X_i}^{X_j} (\frac{X_j - x}{L}) dx$$

$$= \left[\frac{X_j x}{L} - \frac{x^2}{2L} \right]_{X_i}^{X_j}$$

$$= \left[\frac{X_j (X_j - X_i)}{L} - \frac{(X_j^2 - X_i^2)}{2L} \right]$$

$$= \frac{X_j \times L}{L} - \frac{(X_j - X_i)(X_j + X_i)}{2L}$$

$$= X_j - \frac{(X_i + X_j)}{2} \qquad \text{(Ans)}$$

(b)

$$\frac{dN_i}{dx} = -\frac{1}{L} \qquad , \qquad \frac{dN_j}{dx} = \frac{1}{L}$$

$$\int_{X_i}^{X_j} \frac{dN_i}{dx} \frac{dN_j}{dx} dx = \int_{X_i}^{X_j} (-\frac{1}{L}) \times (\frac{1}{L})\, dx$$

$$= \frac{1}{L^2} (X_i - X_j) \qquad \text{(Ans)}$$

(c)

$$\int_{X_i}^{X_j} N_j^2 dx = \int_{X_i}^{X_j} (\frac{x - X_i}{L})^2 dx$$

$$= \int_{X_i}^{X_j} \frac{1}{L} (x^2 - 2X_i x + X_i^2)\, dx$$

$$= \frac{1}{L} \left[\frac{x^3}{3} - \frac{2X_i x^2}{2} + X_i^2\ x \right]_{X_i}^{X_j}$$

$$= \frac{1}{3} (X_j^3 - X_i^3) - X_i X_j L \qquad \text{(Ans)}$$

Chapter 3

Example 1 See in the Art.3.4 of **Analysis of simply supported beam** (Page 27 in ME 5019(2))

Problems 3.1 , 3.2 , 3.4 and 3.5 in ME 5019(2) and its solutions are as follows:

Prob. 3.1 (a) Obtain the final system of finite element equations for the nodal deflections of the stepped beam shown in Figure P3.1.

(b) Solved the equations in (a) and calculate the deflection at x = 3H / 16.

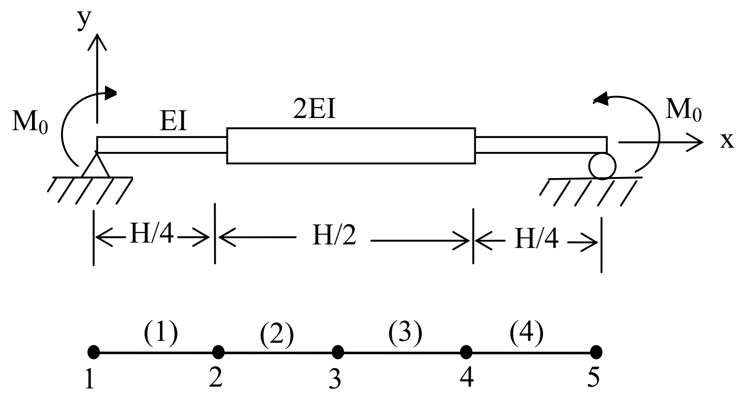


Figure P3.1

Solution

Residual equation

$$R_s = -\left(\frac{D}{L}\right)^{(S-1)} \Phi_{S-1} + \left[\left(\frac{D}{L}\right)^{(S-1)} + \left(\frac{D}{L}\right)^{(S)}\right]\Phi_S - \left(\frac{D}{L}\right)^{(S)} \Phi_{S+1}$$

$$-\left(\frac{QL}{2}\right)^{(S-1)} - \left(\frac{QL}{2}\right)^{(S)} = 0$$

Since Q and L have constant values, the R_S equation simplifies to

$$R_S = \frac{-D^{(S-1)}Y_{(S-1)} + (D^{(S-!)} + D^{(S)})Y_S - D^{(S)}\,Y_{(S+1)}}{L} - QL = 0$$

where Y = nodal deflection values

e	D	Q	L
1	EI	$-M_0$	H/4
2	2EI	$-M_0$	H/4
3	2EI	$-M_0$	H/4
4	EI	$-M_0$	H/4

Writing the residual equation for node 2, 3 and 4 gives

$$R_2 = \frac{-EI\,Y_1 + (EI + 2EI)\,Y_2 - 2EI\,Y_3}{(H/4)} - (-M_0)\,\frac{H}{4} = 0$$

$$R_2 = Y_1 - 3Y_2 + 2Y_3 - \frac{M_0 H^2}{16EI} = 0 \qquad \text{…………(1)}$$

Similarly

$$R_3 = 2Y_2 - 4Y_3 + 2Y_4 - \frac{M_0 H^2}{16EI} = 0 \qquad \text{…………..(2)}$$

$$R_4 = 2Y_3 - 3Y_4 + Y_5 - \frac{M_0 H^2}{16EI} = 0 \qquad \text{…………..(3)}$$

Substituting Y_1= 0 and Y_5 =0 in equations (1) and (3) , and solving the equations gives

$$Y_2 = \frac{-3M_0 H^2}{32EI} \quad , \quad Y_3 = \frac{-7M_0 H^2}{64EI} \quad , \quad Y_4 = \frac{-3M_0 H^2}{32EI}$$

(b) The deflection at $x = \dfrac{3H}{16}$ between node 1 and 2 of element (1)

$$Y^{(1)} = N_1 \quad Y_1 \quad + N_2 \quad Y_2$$

$$Y^{(1)} = \frac{X_2 - x}{L} \quad Y_1 \quad + \frac{x - X_1}{L} \quad Y_2$$

$$= \frac{H/4 - 3H/16}{H/4} \quad \times 0 \quad + \frac{3H/16 - 0}{H/4} \quad \times \frac{-3M_0 H^2}{32EI}$$

$$Y_2 = \frac{-9M_0 H^2}{128EI} \quad \text{(Ans)}$$

Prob.3.2 The differential equation $D^{(e)}\ d^2\Phi/dx^2 = 0$ is applicable to each section of the composite wall shown in Figure P3.2, where $D^{(e)}$ is the thermal conductivity. Calculate the temperature values within the wall and evaluate the heat flow through each material. The heat flow is given by $q = - D^{(e)}\ d\Phi/dx$. A unit of surface area is assumed.

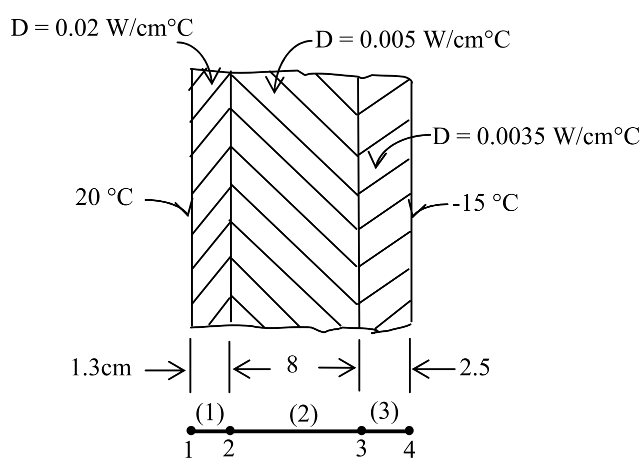


Figure P3.2

Solution

Differential equation $D^{(e)}\dfrac{d^2\Phi}{dx^2}=0$

Heat flow $q = -D^{(e)}\dfrac{d\Phi}{dx}=0$

Assume unit surface area

From the given figure,

e	D	Q	L
1	0.02	0	1.3
2	0.005	0	8
3	0.0035	0	2.5

$\Phi_1 = 20\,°\mathrm{C}$ and $\Phi_4 = -15\,°\mathrm{C}$

Residual equation

$$R_s = -\left(\frac{D}{L}\right)^{(S-1)}\Phi_{S-1} + \left[\left(\frac{D}{L}\right)^{(S-1)} + \left(\frac{D}{L}\right)^{(S)}\right]\Phi_S - \left(\frac{D}{L}\right)^{(S)}\Phi_{S+1}$$

$$-\left(\frac{QL}{2}\right)^{(S-1)} - \left(\frac{QL}{2}\right)^{(S)} = 0$$

Writing the residual equation for node 2 and 3

$$R_2 = -\left(\frac{0.02}{1.3}\right)\Phi_1 + \left[\left(\frac{0.02}{1.3}\right) + \left(\frac{0.005}{8}\right)\right]\Phi_2 - \left(\frac{0.005}{8}\right)\Phi_3 - 0 - 0 = 0$$

$R_2 = -0.0154\Phi_1 + 0.016\Phi_2 - 0.000625\Phi_3 = 0$ ………..(1)

Similarly

$R_3 = -0.000625\Phi_2 + 0.002\Phi_3 - 0.0014\Phi_4 = 0$ ………. (2)

Substituting $\Phi_1 = 20\,, \Phi_4 = -15$ in equations (1) and (2) and solving gives

$\Phi_2 = 19.07\,°\mathrm{C}$ and $\Phi_3 = -4.54\,°\mathrm{C}$ (Ans)

Heat flow through element (1)

For element 1 , $\Phi = \left(\dfrac{X_2 - x}{L_1}\right)\times\Phi_1 - \left(\dfrac{x - X_1}{L_1}\right)\times\Phi_2$

$$\frac{d\Phi}{dx} = \frac{1}{L}(-\Phi_2 + \Phi_1) = \frac{1}{1.3}(-20 + 19.07)$$
$$= -0.7154$$

Heat flow q = -0.02 x -0.7154 = 0.0143 (Ans)

Heat flow through element(2)

$$\frac{d\Phi}{dx} = \frac{1}{L}(-\Phi_2 + \Phi_3) = \frac{1}{8}(-19.07 - 4.54)$$
$$= -2.94$$

q = -0.005 x -2.94 = 0.0143 (Ans)

Heat flow through element(3)

$$\frac{d\Phi}{dx} = \frac{1}{L}(-\Phi_3 + \Phi_4) = \frac{1}{2.5}(4.54 - 15)$$
$$= -4.184$$

q = -0.0035 x -4.184 = 0.0146 (Ans)

Prob. 3.4 Start with nodal residual equations and develop and solve the system of the finite element equation for an approximate solution to the differential equation $d^2\Phi/dx^2 + Q = 0$ using the value for Q and the boundary conditions given in the following table. Divide the interval [0, 2] into four element, each with a length 0.5 cm. The nodes and elements are numbered as shown in Figure P3.3

	Q	Φ(0)	Φ(2)
(a)	4	0	3.0
(b)	6	1	2.0
(c)	-3	2	0.5

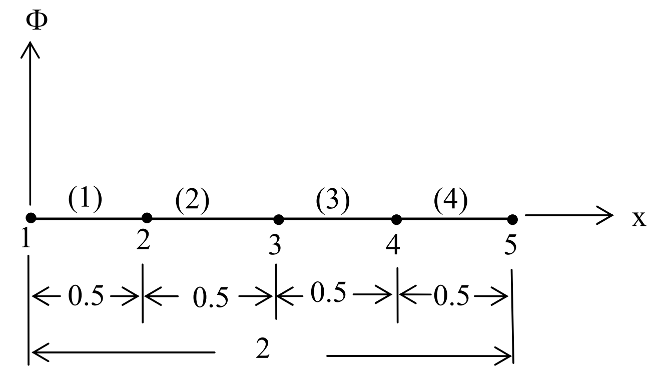


Figure P3.3

Solution

Differential equation $\frac{d^2\Phi}{d^2x} + Q = 0$

(a)

e	D	Q	L
1	1	4	0.5
2	1	4	0.5
3	1	4	0.5
4	1	4	0.5

$\Phi(0) = \Phi_1 = 0$; $\Phi(2) = \Phi_5 = 3.0$ (Given)

Writing residual equation for node 2 gives

$$R_2 = -(\frac{1}{0.5})\Phi_1 + (\frac{1}{0.5} + \frac{1}{0.5})\Phi_2 - (\frac{1}{0.5})\Phi_3 - \frac{4 \times 0.5}{2} - \frac{4 \times 0.5}{2} = 0$$

$$R_2 = -2\Phi_1 + 4\Phi_2 - 2\Phi_3 = 2 \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Similarly for node 3 and 4

$$R_3 = -2\Phi_2 + 4\Phi_3 - 2\Phi_4 = 2 \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

$$R_4 = -2\Phi_3 + 4\Phi_4 - 2\Phi_5 = 2 \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

Substituting $\Phi_1 = 0$ and $\Phi_5 = 3.0$ in the above equations and solving gives

$\Phi_2 = 2.25$, $\Phi_3 = 3.5$, $\Phi_4 = 3.75$ (Ans)

(b)

e	D	Q	L
1	1	6	0.5
2	1	6	0.5
3	1	6	0.5
4	1	6	0.5

$\Phi(0) = \Phi_1 = 1$; $\Phi(2) = \Phi_5 = 2.0$ (Given)

Similarly for node 2 , 3 and 4,

$$R_2 = -2\Phi_1 + 4\Phi_2 - 2\Phi_3 = 3 \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$
$$R_3 = -2\Phi_2 + 4\Phi_3 - 2\Phi_4 = 3 \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$
$$R_4 = -2\Phi_3 + 4\Phi_4 - 2\Phi_5 = 3 \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Substituting $\Phi_1 = 0$ and $\Phi_5 = 3.0$ in the above equations and Solving gives

$\Phi_2 = 3.5$, $\Phi_3 = 4.5$, $\Phi_4 = 4$ (Ans)

(c)

e	D	Q	L
1	1	-3	0.5
2	1	-3	0.5
3	1	-3	0.5
4	1	-3	0.5

$\Phi(0) = \Phi_1 = 2$; $\Phi(2) = \Phi_5 = 0.5$ (Given)

Writing the residual equation, applying the values of Φ_1 and Φ_5, solving the equations gives

$\Phi_2 = 0.5$, $\Phi_3 = -0.25$, $\Phi_4 = -0.25$ (Ans)

Prob. 3.5 Evaluate the residual equation for node one using the weighting function shown in Figure 3.2a. Note that the answer is the same as the equation

$R_s^{(e+1)} = D\dfrac{d\phi}{dx}\Big|_{x=X_s} + \dfrac{D}{L}(\Phi_s - \Phi_t) - \dfrac{QL}{2}$ with (e+1) = (1), s=1 and t=2.

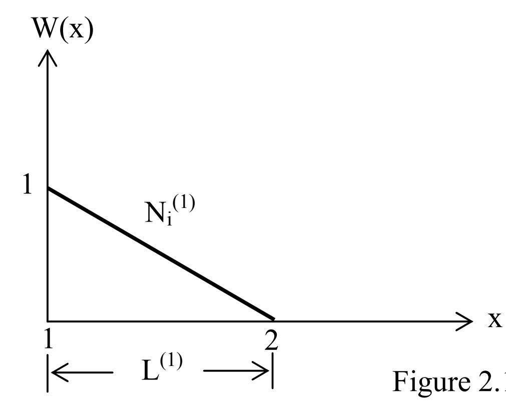


Figure 2.1 a

Solution

The shape function of liner element

$$\phi = N_i\Phi_i + N_j\Phi_j$$

where $N_i = \dfrac{X_j - x}{L}$ and $N_j = \dfrac{x - X_i}{L}$

For element 1 , $\Phi = N_1\Phi_1 + N_2\Phi_2$ ………… (1)

$$\frac{d\Phi}{dx} = \frac{1}{L}(-\Phi_1 + \Phi_2)$$

and

$$N_1 = \frac{X_2 - x}{L} \qquad \text{…………..(2)}$$

$$\frac{dN_1}{dx} = -\frac{1}{L}$$

The weighting function for node 1 of element (1)

$$W_1 = N_1 \qquad 0 \le x \le L$$

The residual equation for node 1

$$R_1 = -\int_o^L \left[N_1 (D\frac{d^2\Phi}{dx^2} + Q) \right] dx = 0$$

$$R_1 = -\int_o^L \left[\left\{ \frac{d}{dx}(N_1\frac{d\Phi}{dx}) - \frac{dN_1}{dx}\frac{d\Phi}{dx} \right\} + N_1 Q \right] dx = 0$$

$$R_1 = -(\int_o^L \frac{d}{dx}(N_1\frac{d\Phi}{dx})dx - \int_0^L \frac{dN_1}{dx}\frac{d\Phi}{dx}\, dx + \int_0^L N_1 Q dx = 0$$

$$R_1 = -[(N_1\frac{d\Phi}{dx})]_0^L + \int_0^L (-\frac{1}{L})(\frac{\Phi_2 - \Phi_1}{L})dx - \int_0^L (\frac{X_2 - x}{L})Qdx = 0$$

$$R_1 = -N_1\frac{d\Phi}{dx}\Bigg|_0^L - \left[\frac{1}{L^2}(\Phi_2 - \Phi_1)\, x \right]_0^L - Q\left[\frac{X_2}{L}\, x - \frac{x^2}{2L} \right]_0^L = 0$$

$$R_1 = -N_1\frac{d\Phi}{dx}\Bigg|_0^L - \left[\frac{1}{L}(\Phi_2 - \Phi_1) \right] - Q\left[\frac{L}{L}\, L - \frac{L^2}{2L} \right] = 0$$

$$R_1 = -N_1\frac{d\Phi}{dx}\Bigg|_0^L - \frac{1}{L}(\Phi_2 - \Phi_1) - \frac{QL}{2} = 0$$

Chapter 4

Example 1 See in the Art.4.3 of **Analysis of a simply supported beam** (Page 37 in ME 5019(2))

Problems 4.1 , 4.2 and 4.3 in ME 5019(2) and its solutions are as follows:

Prob. 4.1 Develop the system of equations for the problem 3.1 using the element matrix and direct stiffness concepts discussed in this chapter. Modify the system of equations to incorporate the boundary conditions and solve for the unknown nodal values.

Solution Using element matrix and direct stiffness matrix concepts

$$\{R\}=[K]\{\Phi\}-\{F\}=\{0\}$$

Where $\left[K^{(e)}\right]=\dfrac{D}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, $\left\{f^{(e)}\right\}=\dfrac{QL}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$

Given $Q = M_0$, $D^{(1)} = D^{(4)} = EI$, $D^{(2)} = D^{(3)} = 2EI$, $L = H/4$

Tabular form

e	i	j	$\frac{D}{L}$	$\frac{QL}{2}$
1	1	2	$\dfrac{4EI}{H}$	$-\dfrac{M_0H}{8}$
2	2	3	$\dfrac{8EI}{H}$	$-\dfrac{M_0H}{8}$
3	3	4	$\dfrac{8EI}{H}$	$-\dfrac{M_0H}{8}$
4	4	5	$\dfrac{4EI}{H}$	$-\dfrac{M_0H}{8}$

Element (1) matrixes

$$\left[K^{(1)}\right]=\frac{4EI}{H}\begin{array}{c}\begin{matrix}1 & \quad 2\end{matrix}\\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\end{array}\begin{matrix}1\\2\end{matrix} \quad ; \quad \left\{f^{(1)}\right\}=\frac{-M_0H}{8}\begin{Bmatrix}1\\1\end{Bmatrix}\begin{matrix}1\\2\end{matrix}$$

$$\left[K^{(2)}\right]=\frac{4EI}{H}\begin{array}{c}\begin{matrix}2 & \quad 3\end{matrix}\\ \begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}\end{array}\begin{matrix}2\\3\end{matrix} \quad ; \quad \left\{f^{(2)}\right\}=\frac{-M_0H}{8}\begin{Bmatrix}1\\1\end{Bmatrix}\begin{matrix}2\\3\end{matrix}$$

$$\left[K^{(3)}\right]=\frac{4EI}{H}\begin{array}{c}\begin{matrix}3 & \quad 4\end{matrix}\\ \begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}\end{array}\begin{matrix}3\\4\end{matrix} \quad ; \quad \left\{f^{(3)}\right\}=\frac{-M_0H}{8}\begin{Bmatrix}1\\1\end{Bmatrix}\begin{matrix}3\\4\end{matrix}$$

$$\left[K^{(4)}\right]=\frac{4EI}{H}\begin{array}{c}\begin{matrix}4 & \quad 5\end{matrix}\\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\end{array}\begin{matrix}4\\5\end{matrix} \quad ; \quad \left\{f^{(4)}\right\}=\frac{-M_0H}{8}\begin{Bmatrix}1\\1\end{Bmatrix}\begin{matrix}4\\5\end{matrix}$$

Global stiffness matrix

$$[K]=\frac{4EI}{H}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1+2 & -2 & 0 & 0 \\ 0 & -2 & 2+2 & -2 & 0 \\ 0 & 0 & -2 & 2+1 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \quad ; \quad \{F\}=-\frac{M_0H}{8}\begin{Bmatrix} 1 \\ 1+1 \\ 1+1 \\ 1+1 \\ 1 \end{Bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}$$

$$\{R\}=\frac{4EI}{H}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1+2 & -2 & 0 & 0 \\ 0 & -2 & 2+2 & -2 & 0 \\ 0 & 0 & -2 & 2+1 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{Bmatrix} - (-\frac{M_0H}{8})\begin{Bmatrix} 1 \\ 1+1 \\ 1+1 \\ 1+1 \\ 1 \end{Bmatrix}=\{0\}$$

$$R_2=-4\frac{EI}{H}Y_1+12\frac{EI}{H}Y_2-8\frac{EI}{H}Y_3+2\frac{M_0H}{8}=0$$

$$R_2=-4Y_1+12Y_2-8Y_3=-\frac{M_0H^2}{4EI} \qquad \text{............(1)}$$

$$R_3=-8Y_2+16Y_3-8Y_4=-\frac{M_0H^2}{4EI} \qquad \text{.............(2)}$$

$$R_4=-8Y_3+12Y_4-4Y_5=-\frac{M_0H^2}{4EI} \qquad \text{..............(3)}$$

Substituting $Y_1 = Y_5 = 0$ in the above equations and solving

$$Y_2=-0.094\frac{M_0H^2}{EI}$$

$$Y_3=-0.109\frac{M_0H^2}{EI}$$

$$Y_4=-0.094\frac{M_0H^2}{EI} \qquad \text{(Ans)}$$

Prob. 4.2 Develop the system of equations for the problem 3.2 using the element matrix and direct stiffness concepts discussed in this chapter. Modify the system of equations to incorporate the boundary conditions and solve for the unknown nodal values.

e	i	j	$\frac{D}{L}$	$-\frac{QL}{2}$
1	1	2	0.15385	0
2	2	3	0.00625	0
3	3	4	0.0014	0

Global stiffness matrix

$$[K]=\begin{bmatrix} 0.015385 & -0.015355 & 0 & 0 \\ -0.015385 & 0.01601 & -0.000625 & 0 \\ 0 & -0.000625 & 0.002025 & -0.0014 \\ 0 & 0 & -0.0014 & 0.0014 \end{bmatrix}$$

$$\{R\}=[K]\{\Phi\}=\{0\}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix}=\begin{bmatrix} 0.015385 & -0.015355 & 0 & 0 \\ -0.015385 & 0.01601 & -0.000625 & 0 \\ 0 & -0.000625 & 0.002025 & -0.0014 \\ 0 & 0 & -0.0014 & 0.0014 \end{bmatrix}\begin{Bmatrix} 20 \\ \Phi_2 \\ \Phi_3 \\ -15 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$\Phi_2 = 19.04$, $\Phi_3 = -4.49$

Prob. 4.3 Develop the system of equations for the problem 3.4 using the element matrix and direct stiffness concepts discussed in this chapter. Modify the system of equations to incorporate the boundary conditions and solve for the unknown nodal values.

Solution

e	i	j	D	Q	L	D/L	QL/2
1	1	2	1	4	0.5	2	1
2	2	3	1	4	0.5	2	1
3	3	4	1	4	0.5	2	1
4	4	5	1	4	0.5	2	1

$\Phi(0) = \Phi_1 = 0$; $\Phi(2) = \Phi_5 = 3.0$ (Given)

Global stiffness matrices

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{Bmatrix}=\begin{bmatrix} 2 & -2 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 0 & -2 & 4 & -2 & 0 \\ 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix}\begin{Bmatrix} 0 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \\ 3 \end{Bmatrix} - \begin{Bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 1 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$R_2 = 4\Phi_2 - 2\Phi_3 - 2 = 0$$
$$R_3 = -2\Phi_2 + 4\Phi_3 - 2\Phi_4 - 2 = 0$$

$$R_4 = -2\Phi_3 + 4\Phi_4 - 6 - 2 = 0$$

From the above equations,

$\Phi_2 = 2.25$, $\Phi_3 = 3.5$, $\Phi_4 = 7.75$

(b)

e	i	j	D	Q	L	D/L	QL/2
1	1	2	1	6	0.5	2	1.5
2	2	3	1	6	0.5	2	1.5
3	3	4	1	6	0.5	2	1.5
4	4	5	1	6	0.5	2	1.5

$\Phi(0) = \Phi_1 = 1$; $\Phi(2) = \Phi_5 = 2.0$ (Given)

Global stiffness matrices

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{Bmatrix} = \begin{bmatrix} 2 & -2 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 0 & -2 & 4 & -2 & 0 \\ 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \\ 2 \end{Bmatrix} - \begin{Bmatrix} 1.5 \\ 3 \\ 3 \\ 3 \\ 1.5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$R_2 = 4\Phi_2 - 2\Phi_3 - 3 = 0$$

$$R_3 = -2\Phi_2 + 4\Phi_3 - 2\Phi_4 - 3 = 0$$

$$R_4 = -2\Phi_3 + 4\Phi_4 - 4 - 3 = 0$$

From the above equations,

$\Phi_2 = 3.5$, $\Phi_3 = 4.5$, $\Phi_4 = 4$ (Ans)

(c)

e	i	j	D	Q	L	D/L	QL/2
1	1	2	1	-3	0.5	2	-0.75
2	2	3	1	-3	0.5	2	-0.75
3	3	4	1	-3	0.5	2	-0.75
4	4	5	1	-3	0.5	2	-0.75

$\Phi(0) = \Phi_1 = 2$; $\Phi(2) = \Phi_5 = 0.5$ (Given)

Global stiffness matrices

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{Bmatrix} = \begin{bmatrix} 2 & -2 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 0 & -2 & 4 & -2 & 0 \\ 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix} \begin{Bmatrix} 2 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \\ 0.5 \end{Bmatrix} - \begin{Bmatrix} -0.75 \\ -1.5 \\ -1.5 \\ -1.5 \\ -0.75 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

From the above matrices,

$\Phi_2 = 0.5$, $\Phi_3 = -0.25$, $\Phi_4 = -0.25$ (Ans)

CHAPTER 5

Example 1 See in the Art.5.2 of **Illustrative Example** (Page 45 in ME 5019(2))

Example 2 See in the Art.5.2 of **Illustrative Example** (Page 47 in ME 5019(2))

Example 3 See in the Art.5.3 of **Illustrative Example** (Page 50 in ME 5019(2))

Problems 5.1 , 5.2 , 5.3 and 5.4 in ME 5019(2) and its solutions are as follows:

Prob.5.1 Verify that N_i for the triangular element is equal to one at node *i* and equal to zero at nodes *j* and *k*.

Solutions

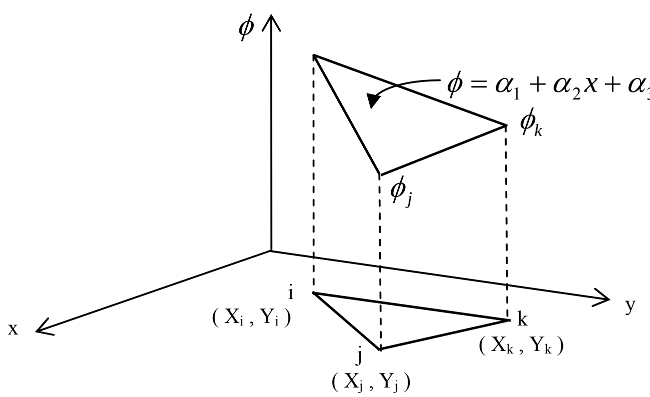
Shape function of triangular element, $N_i = \frac{1}{2A}\left[\, a_i + b_i\; x + c_i\; y\right]$ ....(1)

Verify that

$N_i = 1$ at $x = X_i$, $y = Y_i$ (node i)

$N_i = 0$ at $x = X_j$, $y = Y_j$ (node j)

$N_i = 0$ at $x = X_k$, $y = Y_k$ (node k)



At node i

Substituting $x = X_i$, $y = Y_i$,

$$a_i = X_j\, Y_k - X_k\, Y_j \qquad b_i = Y_j - Y_k \qquad and \qquad c_i = X_k - X_j$$

$$\begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix} = 2\,A\; = (X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) + (X_iY_j - X_jY_i)$$

in the above N_i equation (1)

$$N_i = \frac{(X_jY_k - X_kY_j) + (Y_j - Y_k)\times X_i + (X_k - X_j)\times Y_i}{(X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) + (X_iY_j - X_jY_i)} = 1$$

Similarly at $x = Xj, y = Yj$ (Node j)

$$N_i = \frac{(X_jY_k - X_kY_j) + (Y_j - Y_k)\times X_j + (X_k - X_j)\times Y_j}{(X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) + (X_iY_j - X_jY_i)} = 0$$

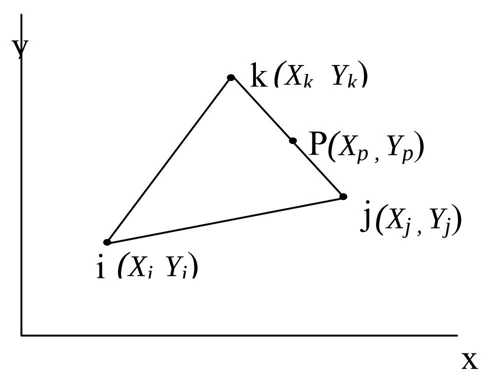
Similarly at $x = X_k, y = Y_k$ (Node k)

$$N_i = \frac{(X_jY_k - X_kY_j) + (Y_j - Y_k)\times X_k + (X_k - X_j)\times Y_k}{(X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) + (X_iY_j - X_jY_i)} = 0$$

Problem5.2 Verify that N_i for the triangular element in Figure 5.1a is zero everywhere along side *jk*. Hint ; Write an equation of the form y = d + mx for side *jk* and substitute for y in the shape function equation of N_i.

Solution

Verify that $N_i = 0$ along side jk



Straight line equation of line jk

$$y = d + m\,x$$

At node j $\quad Y_j = d + m\,X_j \quad \ldots\ldots\ldots \quad (1)$

At node k $\quad Y_k = d + m\,X_k \quad \ldots\ldots \quad (2)$

From equations (1) and (2)

$$d = Y_j - \frac{(Y_j - Y_k)\times X_j}{(X_j - X_K)} \quad \text{and} \quad m = \frac{(Y_j - Y_k)}{(X_j - X_K)}$$

Equation for line jk

$$y = (Y_j - \frac{(Y_j - Y_k)\times X_j}{(X_j - X_K)}) + \frac{(Y_j - Y_k)}{(X_j - X_K)} \times x$$

At any point P on the side jk

$$Y_p = (Y_j - \frac{(Y_j - Y_k)\times X_j}{(X_j - X_K)}) + \frac{(Y_j - Y_k)}{(X_j - X_K)} \times X_p$$

Shape function for triangular element

$$N_i = \frac{1}{2A}\left[\, a_i + b_i \ x + c_i \ y\right] \quad \text{where}$$

$$a_i = X_j\, Y_k - X_k\, Y_j \qquad b_i = Y_j - Y_k \qquad \textit{and} \qquad c_i = X_k - X_j$$

$$\begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix} = 2\,A \ = (X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) + (X_iY_j - X_jY_i)$$

Substituting $x = X_p$, $y = Y_p$, a_i, b_j, c_i and $2A$ in equation N_i

$$N_i = \frac{1}{2A}[(X_jY_k - X_kY_j) + (Y_j - Y_k)\times X_p + (X_k - X_j)\times$$

$$\left\{(Y_j - \frac{(Y_j - Y_k)\times X_j}{X_j - X_k}) + \frac{(Y_j - Y_k}{X_j - X_k}\times X_p\right\}]$$

$= 0$

Prob.5.3 Verify that the shape functions for the triangular element sum to one, that is, $N_i + N_j + N_k = 1$. Comment on the behavior of the following summations:

(i) $a_i + a_j + a_k$
(ii) $b_i + b_j + b_k$
(iii) $c_i + c_j + c_k$

Solution

$$N_i = \frac{1}{2A}\left(a_i + b_i x + c_i y\right)$$

$$N_j = \frac{1}{2A}\left(a_j + b_j x + c_j y\right)$$

$$N_k = \frac{1}{2A}\left(a_k + b_k x + c_k y\right)$$

Where $\begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix} = 2\,A = (X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) + (X_iY_j - X_jY_i)$

$$a_i = X_j\, Y_k - X_k\, Y_j\,, \quad b_i = Y_j - Y_k \qquad \textit{and} \qquad c_i = X_k - X_j$$

$$a_j = X_k\, Y_i - X_l\, Y_k\,, \quad b_j = Y_k - Y_i \qquad \textit{and} \qquad c_j = X_i - X_k$$

$$a_k = X_i\, Y_j - X_j\, Y_i\,, \quad b_k = Y_i - Y_j \qquad \textit{and} \qquad c_k = X_j - X_i$$

$$a_i + a_j + a_k = (X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) + (X_iY_j - X_jY_i)$$

$$b_i + b_j + b_k = 0$$

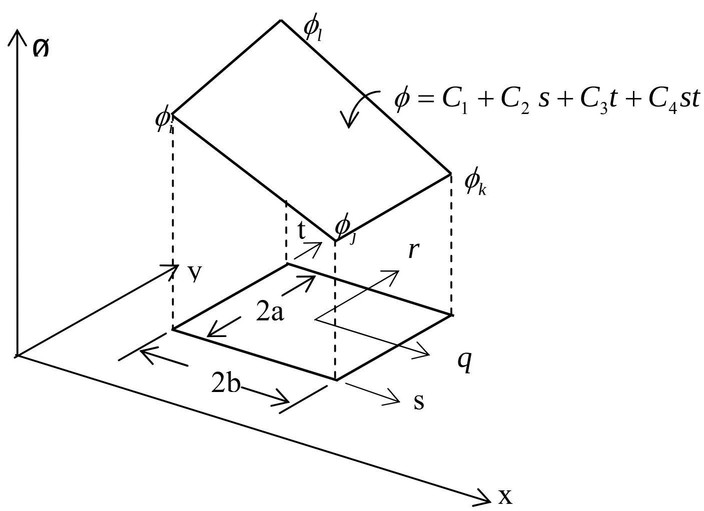
$$c_i + c_j + c_k = 0$$

$$N_i + N_j + N_k = \frac{1}{2A}\left[(a_i + a_j + a_k) + (b_i + b_j + b_k)\times x + (c_i + c_j + c_k)\times y\right]$$

$$= \frac{[(X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) - (X_iY_j - X_jY_j) - 0 - 0]}{(X_jY_k - X_kY_j) - (X_iY_k - X_kY_i) - (X_iY_j - X_jY_j)}$$

$= 1$ (Ans)

Prob. 5.4 Verify that the shape functions for the rectangular element sum to one. Also check those given by the shape function equations in term of q and r.



Shape function for rectangular element written in term of s and t

$$N_i = \left(1 - \frac{s}{2b}\right)\left(1 - \frac{t}{2a}\right)$$

$$N_j = \frac{s}{2b}\left(1 - \frac{t}{2a}\right)$$

$$N_k = \frac{st}{4ab}$$

$$N_m = \frac{t}{2a}\left(1 - \frac{s}{2b}\right)$$

$$N_i + N_j + N_k + N_m = \left(1 - \frac{s}{2b}\right)\left(1 - \frac{t}{2a}\right) + \frac{s}{2b}\left(1 - \frac{t}{2a}\right) + \frac{st}{4ab} + \frac{t}{2a}\left(1 - \frac{s}{2b}\right)$$

$= 1$ (Ans)

Shape function for rectangular element written in term of q and r.

$$N_i = \frac{1}{4}\left(1 - \frac{q}{b}\right)\left(1 - \frac{r}{a}\right)$$

$$N_j = \frac{1}{4}\left(1 + \frac{q}{b}\right)\left(1 - \frac{r}{a}\right)$$

$$N_k = \frac{1}{4}\left(1 + \frac{q}{b}\right)\left(1 + \frac{r}{a}\right)$$

$$N_m = \frac{1}{4}\left(1 - \frac{q}{b}\right)\left(1 + \frac{r}{a}\right)$$

$$N_i + N_j + N_k + N_m = \frac{1}{4}\left[(1 - \frac{q}{b})(1 - \frac{r}{a}) + (1 + \frac{q}{b})(1 - \frac{r}{a}) + (1 + \frac{q}{b})(1 + \frac{r}{a}) + (1 - \frac{q}{b})(1 + \frac{r}{a})\right]$$

$$= 1 \quad (\text{ Ans})$$