

Fracture Mechanics 3

Fracture Mechanics 3

Applied Quality Control

Ammar Grous

ISTE

 WILEY

First published 2013 in Great Britain and the United States by ISTE Ltd and John Wiley & Sons, Inc.

Apart from any fair dealing for the purposes of research or private study, or criticism or review, as permitted under the Copyright, Designs and Patents Act 1988, this publication may only be reproduced, stored or transmitted, in any form or by any means, with the prior permission in writing of the publishers, or in the case of reprographic reproduction in accordance with the terms and licenses issued by the CLA. Enquiries concerning reproduction outside these terms should be sent to the publishers at the undermentioned address:

ISTE Ltd
27-37 St George's Road
London SW19 4EU
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

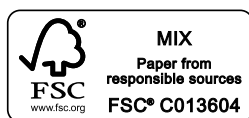
www.wiley.com

© ISTE Ltd 2013

The rights of Ammar Grous to be identified as the author of this work have been asserted by him in accordance with the Copyright, Designs and Patents Act 1988.

Library of Congress Control Number: 2012950201

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN: 978-1-84821-442-2



Printed and bound in Great Britain by CPI Group (UK) Ltd., Croydon, Surrey CR0 4YY

Table of Contents

Preface	ix
Glossary	xvii
Chapter 1. Quality Control	1
1.1. Introduction to statistical process control	1
1.2. Classical tolerancing and quality control	4
1.2.1. ISO norms in quality control	5
1.2.2. QS9000 approach	6
1.2.3. Capabilities of CNOMO E41.36.110.N control means	8
1.2.4. Some capability indicators according to ISO/TS 16949, ISO 8258 and FORD.	10
1.2.5. Metrological audits	11
1.3. The Pareto law – ABC method	14
1.3.1. Practical application	17
1.3.2. Malfunction spread over 16 so-called “neuralgic” points	18
1.4. Lot inspection by attributes	18
1.4.1. Areas on which lot inspection operates	18
1.4.2. (np) chart for controlling the number of defects	20
1.4.3. (c) card to control the number of defects	20
1.4.4. Average lot quality after control	23
1.4.5. Choosing a sample plan	32
1.4.6. Wald plan case study	34
1.4.7. Average long-term controlled quantity	37
1.4.8. Case study of a simple sampling plan	42
1.4.9. Poisson distribution of cumulative probabilities	43
1.5. Theoretical basics of control by measurement	46
1.5.1. Choosing a simple sampling plan	47

1.5.2. Choosing a simple or double sampling plan	48
1.6. Analysis of process capability	49
1.6.1. Capability indices for a normally distributed variable: $X(\mu, \sigma)$	51
1.6.2. Machine capability C_m and C_{mk}	55
1.7. Capability for a non-normal distribution	56
1.8. Control by measurement charts	57
1.8.1. Size of the samples to be taken and the frequency of control	60
1.8.2. Factor for control of X (mean) and range (R)	63
1.9. Production and reception control	64
1.9.1. Machine adaptation with respect to production tolerances	64
1.9.2. Proportion of faulty units	65
1.10. Control charts	68
1.10.1. Control by measurements charts for the mean $\bar{X}$ and the range R	69
1.10.2. Calculating control limits and case study	70
1.10.3. Study of $\bar{X}/R$ control charts – quality control lab	72
1.10.4. Graphical representation of the rejections due to non-conformity to TI	78
1.10.5. Performance case study: Capabilities	80
1.10.6. Calculating machine capability indices: C_m and C_{mk}	84
1.11. Conclusion	85
1.12. Bibliography	85
Chapter 2. Quality Control Case Studies	87
2.1. The tools of quality, as per W. Deming	87
2.2. Failure modes, effects and criticality analysis	88
2.3. Total productive maintenance method	88
2.4. The LMMEM “5M” process method	93
2.5. Estimations of times in mechanical productions (machining)	94
2.5.1. Optimizing times and costs in mechanical production	98
2.6. Stock management and supply methods	111
2.6.1. Hypothesis of a general method to improve stock management	113
2.7. Short summary of control charts	116
2.7.1. The various control charts	117
2.7.2. Measurement control charts – stability control charts (Shewhart)	118
2.7.3. Estimating μ_0 and σ_0	119
2.7.4. Efficiency – chart of the average	120
2.7.5. Control chart by attributes – determining the limits	121

2.8. CUSUM charts	123
2.8.1. EWM charts	124
2.8.2. Shewhart charts for the average $\bar{X}$, the range R and the standard deviation (s)	126
2.8.3. Control charts for the average $\bar{X}$ and the range R	127
2.9. Individual control charts	130
2.9.1. Average lengths or CUSUM-Run	132
2.10. EWMA statistics – comparison between Shewhart graph control and the EWMA chart control techniques	133
2.10.1. Poisson normal distribution approximation	138
2.10.2. Decision limits of the $\bar{X} / R$ control chart through the SPC method	141
2.11. Main statistical tests used in quality control	144
2.11.1. So-called “Henry Line” or probability diagram test	144
2.12. Partial conclusion	149
2.13. Bibliography	149
Chapter 3. Case Studies	153
3.1. Quality control case study: calculating and plotting efficiency curves in simple/double control	153
3.1.1. Simple binomial distribution	153
3.1.2. Calculation results	154
3.2. Calculating the efficiency curves of simple/double control	156
3.2.1. Calculation results	157
3.3. Calculating efficiency curves in double control: binomial distribution, double efficiency plan (Excel version)	159
3.3.1. Calculation results	159
3.4. Progressive control (Excel)	160
3.4.1. Calculating the acceptance and refusal curves in progressive control	160
3.4.2. Calculation results	161
3.4.3. Graph	162
3.4.4. Measurement acceptance test when sigma (σ^*) is known	163
3.4.5. Determining the efficiency curve and the acceptance limits	163
3.4.6. Acceptance: known standard deviation, sigma = 0.01	164
3.4.7. Measurement acceptance test when sigma* (SD) is known	165
3.4.8. Choosing a plan	166
3.4.9. Plot (Excel)	167
3.5. R&R study in quality control and dimensional metrology	168
3.5.1. R&R (EV Repeatability and AR reproducibility)	168
3.5.2. R&R study	169

3.6. X/S control chart study (average and standard deviation, σ)	173
3.7. Case study: capability of a specific control method	178
3.7.1. Measurements from a digital micrometer.	178
3.7.2. Definition drawing or clear picture	178
3.7.3. Table of the measurements taken using a digital micrometer (directly read the value from Caliper).	179
3.7.4. Table of calculation results (Excel and MathCAD).	180
3.8. Case study on type A and B uncertainties	181
3.8.1. Uncertainties guide to the expression of uncertainties in measurements (GUM).	181
3.8.2. Estimating the compound uncertainty U_C	184
3.9. Case study: uncertainties	184
3.9.1. Uncertainty study as per GUM	185
3.9.2. Graph	200
3.10. Conclusion	200
3.11. Bibliography	201
Appendix.	203
Index	257

Preface

This book is intended for technicians, engineers, designers, students and teachers working in engineering and vocational education. Our main goal is to give a succinct evaluation of quality indicators and reliability as decision-making tools. To this end, we recommend an *intuitive* and practical approach, based on mathematical rigor.

The first part of this series presents the fundamental basis of data analysis in both quality control and in studying the mechanical reliability of materials and structures. Results from laboratory and workshop are discussed, keeping the technological procedures inherent to the subject matter in mind. We also discuss and interpret the standardization of manufacturing processes as a causal link with geometrical and dimensional specifications (geometrical product specification, GPS). This is, in fact, the educational innovation offered in these books compared to the praiseworthy publications quoted here.

We examine many laboratory examples, thereby covering a new industrial organization of work. We also use mechanical components from our own real mechanisms, which we built and designed at our production labs. Finite element modification is thus relevant to real machined pieces, controlled and soldered in a dimensional metrology laboratory.

We also examine mechanical component reliability. Since statistics are common to both this field and quality control, we will simply mention reliability indices in the context of using the structure for which we are performing the calculations.

Scientists in specialized schools and corporations often take interest in measurement quality, and thus in measurement uncertainties. So-called cutting-edge

endeavors such as the aeronautics, automotive and nuclear industries, to mention but a few, put an increasing emphasis on *just measurement*. The educational content of this series stands out due to the following:

- 1) the rigor of the probabilistic methods that support statistical-mathematical treatments of experimental or simulated data;
- 2) the presentation of varied lab models that come at the end of each chapter: this should help the student to better understand how to:
 - i) define and justify a quality and reliability control target,
 - ii) identify the appropriate tools to quantify reliability with respect to capabilities,
 - iii) interpret quality (capability) and reliability (reliability indices) indicators,
 - iv) choose the adequation test for the distribution (whether justified or used *a priori*),
 - v) identify how trials can be accelerated and their limits,
 - vi) analyze the quality and reliability of materials and structures,
 - vii) size and *tolerance* (GPS) design structures and materials.

What about uncertainty calculations in applied reliability and quality control?

Structure behavior at breaking point is often characterized (in linear mechanics) as a local variation of the material's elastic properties. This inevitably leads to sizing calculations that seek to secure the structures from which the materials have come. Much work has been, and still is, put into a large range of fields that go from civil engineering to the various flavors of mechanics. Here, we do not consider continuum mechanics, but rather probabilistic laws for cracking. We are aware that some laws are systematically repeated here and there in order to better approach reliability.

Less severe adequation tests would confirm the fissure propagation hypothesis. wherever safety is a priority, such as medicine (surgery and biomechanics), aviation and nuclear power plants, where theorizing unverifiable concepts would be unacceptable. The relevant reliability calculations must therefore be as rigorous as possible.

Defining safety coefficients would be an important (or even major) element of structure sizing. This definition does not really offer any real guarantees on safety provisions (unlike security provisions). Today, the interpretation and very

philosophy behind these coefficients are reinforced by increasingly precise probabilistic calculations. Well-developed computer tools largely contribute to the calculation time and effort. Thus, we will use software commonly found in various schools (Auto Desk Inventor Pro and ANSYS in modeling and design; MathCAD, GUM and COSMOS in quality control, metrology and uncertainty calculations).

Much work has been done toward rationalizing the concept of applied reliability, but no “united method” between the mechanical and statistical interpretations of rupture has appeared yet. Some of the many factors for this non-*consensus* are unpredictable events that randomly create the fault, its propagation and the ensuing damage. Many researchers have worked on various random probabilistic and deterministic methods. This resulted in many simulation methods, the most common of which was the Monte Carlo simulation.

In these books, we will present some documented applied cases so as to help teachers present probabilistic problems (reliability and/or degradation) succinctly. The intuitive approach is a large part of our problem-solving methods, and one of the main goals of this book is to offer up this humble contribution. Many praiseworthy works and books have described reliability, quality control and uncertainty perfectly well, but as separate entities. However, our task here is to verify measurements and ensure that the measurand is well-taught. As Lord Kelvin said, “if you cannot measure it, you cannot improve it”. Indeed, measuring identified quantities is an unavoidable part of laboratory life. Theoretical confirmation of physical phenomena must go through *measurement reliability* and its effects on the function attributed to the material and/or structure, among other things.

Mechanical models (rupture criteria) of continuum mechanics discussed (Chapter 1, volume 2) make up a reference pool of work used here and there in our case studies, such as the Paris–Erdogan law, the Manson-Coffin law, SN curves (Wöhler curves), Weibull’s law (solid mechanics), etc. We could probably (and justly) wonder how this chapter is appropriate in works dedicated to reliability. The reason is that these criteria are targeted deliberately. We used mechanical modeling here to avoid the reader having to “digress” into specialized books.

Establishing *confidence* in our results is critical. Measuring a characteristic does not simply mean finding the characteristic’s value. We must also give it an *uncertainty* so as to show the measurement’s *quality*. In this book, we will show educational laboratory examples of uncertainty (GUM: Guide to the Expression of Uncertainty in Measurement).

Why then publish another book dedicated to quality control, uncertainties and reliability?

This book covers quality control including uncertainties and some case studies upon reliability. In quality control, the process is often already known or appears to be under control beforehand, hence the intervention of capability indices (statistical process control – SPC). Furthermore, the goal is sometimes the competitiveness between manufactured products. Security is shown in secondary terms. Indeed, it is in terms of maintainability and durability that quality control joins reliability as a means to guarantee the functions attributed to a mechanism, component or even an entire system.

When considering the mechanical reliability of materials and structures, the reliability index is inherently a *safety* indicator. It is often very costly computationally and has very serious consequences. The common aspect of both fields is still the probabilistic approach. Probabilities and statistical-mathematical tools are necessary to supply theoretical justifications for the computational methods. Again, this book intends to be pragmatic and leaves reasonable room for the intuitive approach of the hypotheses stated here and there.

Finally, we will give a succinct glossary in order to soften the understanding of dimensional analysis terms (VIM – International Metrology Vocabulary) and those in structural mechanical reliability. This educational method allows us to “agree” on the international language used to define the measurand, reliability index or a succinct definition of the capability indicators largely used in quality control.

Component reliability (for both materials and structures) is absolutely unavoidable in the field of safety and performance.

The field of reliability is used in many fields of engineering, from civil engineering to mechanical and electrical engineering: it is thus manifold. It often aims to estimate the functions for the various component lifespans, depending on the study. Reliability users increasingly rely on reproducible software, though they struggle to determine whether the component is active or passive, the size of the experience return and its imperative validation, the phenomena that tend to decrease the likelihood of failure or the reliability index, etc.

These three volumes use some methods used here and there to estimate operational or target reliabilities. The apparent controversy between frequentist and Bayesian probabilistic approaches will be irrelevant in our humble opinion if we know how to set the problem *a priori*. Setting boundaries for the likelihood of

rupture (failure or even degradation) is worth doing. As for us, we prefer calculating rupture through the damage indicator integral, made explicit by Madsen's work.

Just as estimating reliability can allow us to understand the history of something to better anticipate the future, we must show pragmatism in measuring the factors responsible for the likely rupture. Since the measurement is always inherently flawed and uncertain, we must include uncertainty calculations in our reliability methods. Without such calculations, our result would lead to doubt.

Reliability has its own, specific terminology (see Glossary) that, like for metrology, affects the decision's terms. Thus, we will abide by the EN 13306 standard (see Appendix, Table A1.45). In Appendix and the glossary, definitions for reliability, durability, failure and degradation can be found.

Reliability data are necessary to:

- prioritize between components → occurrence rate of each mode (system failure mode and effects analysis – SFMEA);
- identify the degradation mechanism for a clear experience feedback;
- optimize preventative and corrective maintenance;
- gather data efficient for and necessary to the correct calculation of reliability parameters and especially their uncertainties.

Analysis and *validation* are done by analyzing the experience feedback with respect to critical failure criteria, such as failure modes, mean time between failures (MTBF), probability of failure on demand (P_s) and its reliability index according to a “selected criterion”, the repair and/or material unavailability time, the confidence intervals or even the sample size.

We note that reliability is usually taken into account from the design aspect, based on the specifications. It is calculated and compared to the allocated reliability (reliability demand). It includes all phases of life (design, manufacture, development trials).

During exploitation, the expected reliability is calculated and compared to a threshold (e.g. rate of failure) such as physical calculations, with the intent of extending it beyond the lifespan anticipated during design.

Reliability is mostly measured, therefore making its *metrology* a serious business: hence, the calculation of its uncertainties including instrument and measurement equipment calibration.

Among the various difficulties that rise against the reliability function, we note the component type (repairable or not, redundant or passive) and even some controversial methods and models (frequential/Bayesian), among others.

To calculate reliability indicators for components that present active redundancy, we use:

- various degradation mechanisms with different modes of failure;
- unknown or even complex physical modeling;
- sometimes we use appropriate classical and Bayesian reliability;
- incomplete or even truncated data;
- modeling using a Gaussian distribution, an exponential law, a two or three parameter Weibull law or especially a Birnbaum–Saunders law (as is often the case in cracking rupture: wear and tear in zone II of the Ritchie curve – see Figures 1.2 and 1.17 in Chapter 1 volume 2).

The component reliability can be with active and passive redundancy using:

- few degradation mechanisms;
- degradation type: slow or progressive;
- few failures (if any);
- physical modeling of degradation: fissure initiation and propagation;
- numerical methods (energetic methods, integral calculations, finite elements);
- Bayesian approach for available failure data.

Whether a physicist, statistical mathematician or mere engineer, “controversies” sometimes appear in many schools of thought on the method or model (frequential/Bayesian). In these three books, we will try to remain pragmatic and synthesize opinions.

From a *physicist’s* perspective, the experimental conditions of data gathering are known, and their uncertainties well-bound. Its so-called frequential analysis is based only on objective data, because they were measured *correctly*. We know that measurements are costly and time-consuming. If the data from “our physicist’s” experiments are insufficient and if the process turns out to be non-repetitive or the number of parameters to estimate is high, the frequential approach falsely introduces a confirmation bias in the analysis. The paradox is that the calculations are correct, but they only answer to a logically mathematical demand. In other words, the mathematics are correct but are superficially stuck on an inappropriate case, hence a rejection of the solution and the birth of controversy...

The *engineering* approach is *attractive* due to its “applied arts and crafts” aspect (i.e. learning). Its analysis includes the knowledge that we must apply an *a priori* law, which must by definition be biased. Without rejecting the Bayesian approach, this is where we favor the *engineering* approach because it uses *decision-making tools* for which preferences are clearly expressed. At the end of this approach, the uncertainty function greatly helps make the decision...

Finally, it is important to specify and frame the problem well: its context, hypotheses, available data, etc. Simulations (using software) are a helpful educational tool, but they should not be treated as *replacements* for real experiments. Relying on *real data* from the *experiment feedback* of the collection conditions is more suitable. Indeed, experiments and “real” data are a strategic necessity in the case of preemptive validation.

In this book, we present (see Chapters 1 and 2) the qualitative analysis elements preceding quantitative, deterministic and probabilistic analysis. The laws and tests discussed in the first two chapters (volume 1 of this series) are required reading for any probabilistic study of physical phenomena, and it falls to us to be pragmatic.

Regardless of the approach used, we must analyze the sensibility of factors and always use common sense. Among many other methods of analysis, reliability is a tool for understanding the past. For example, many failures, degradations and ruptures or ruin (damage) cannot be explained by deterministic models such as aging, degradation mechanisms, models and laws (see Chapter 1, volume 2: Fracture Mechanics by Fatigue). Studying reliability allows us to find the components and subcomponents to critique, the important variables (initial faults, factor of intensity of constraint (*f.i.c.*), etc.) for which uncertainties should be reduced, and so on through a sound knowledge of physical phenomena.

Reliability anticipates and prepares for the future in order to improve performance and safety by optimizing exploitation strategies.

However, reliability alone cannot replace an experimental understanding of physical phenomena.

We present in the Appendix, not less than fifty reference tables to appreciate the acceptance quality level (AQL) and typical Dodge-Romig tables for a sampling plan AOQL and LQL (Average Outgoing Quality Limit) and (Limiting quality Level) corresponding to a consumer’s risk.

A. GROUS
November 2012

Glossary

5S

A Japanese method for application in manufacturing, office, etc. acronym for Seiri (to sort), Seiton (to set in order), Seiso (to shine), Seiketsu (to standardize), Shitsuke (to sustain).

Abbreviations

CITAC	Cooperation on International Traceability in Analytical Chemistry
CSA	Canadian Standards Association
EA	European Cooperation for Accreditation
Eurachem	Focus for Analytical Chemistry in Europe
EUROLAB	European Federation of National Associations of Measurements, Testing and Analytical Laboratories
GUM	Guide to the expression of Uncertainty in Measurement (reference document recognized by the CSA, EUROLAB, Eurachem and EA)
IEC	International Electrotechnical Commission
ILAC	International Laboratory Accreditation Co-operation
ISO	International Organization for Standardization
S (or σ)	Standard deviation
SAS	<i>Service d'accréditation suisse</i> (Swiss Accreditation Service)
U	Uncertainty
VIM	International Vocabulary of basic and general terms in Metrology

Abrasion resistance

Hard materials also show good abrasion resistance: in other words, they are not easily worn down by friction. In practice, they are harder to grind down.

Acceptable risk

Acceptable risk describes the structural and non-structural measures to be put in place to reduce probable damage to a reference level. A risk scale is often associated with dangers in order to classify them in order of seriousness.

Availability

Availability is a (dimensionless) attribute of *dependability*. It is the capacity of a system to properly deliver the service (quality) when the user has need of it. Availability is a unitless measurement; it corresponds to the ratio of uptime to total execution time of the system.

Chance

Imaginary cause of something that occurs for no apparent or explicable reason (dictionary definition).

Conditional probability

(Bayesian) probability of a consequence when the causal event will definitely occur. If we suppose that a fracture has reached the limit suggested by a pre-established hypothesis, the probability of cracking is a conditional probability.

Corrosion resistance

This denotes the ability of a material to withstand damage from the effects of the chemical reaction of oxygen with the metal. A ferrous metal that is resistant to corrosion does not rust.

Degradation

Irreversible evolution of one or more characteristics of a product related to time, to the duration of use or to an *external cause – alteration of function, constant phenomenon, physical ageing*.

Dependability

This is the property that enables users to justifiably place their faith in the service provided to them: *reliability, availability, safety, maintainability, security*.

Dilation and contraction

When a material is heated, it expands slightly: this is called dilation. Conversely, if it shrinks (cold), this is a contraction. The level of dilation and contraction of a metal affects its weldability. The more the metal is expanded or contracted, the greater the risk of *cracks* or deformations appearing.

Distribution function

This is an integral function of the probability density (or cumulative probability function), calculated in order of ascending values of the random variable. It expresses the probability of the random variable assuming a value less than or equal to a given value.

Ductility

Ductility represents the ability of a metal to be deformed without breaking. It can be stretched, elongated or subjected to torsion forces. Ductile materials are difficult to break because the cracks or defects created by a deformation do not easily propagate.

Durability

This is the ability of a product to perform its required function, in given conditions of use and maintenance, until a critical state is reached.

Elasticity

The ability of a material to return to its original form after a deformation.

Failure

Alteration or suspension of the ability of a system to perform its required function(s) to the levels of performance defined in the technical specifications.

Fault (error) resistance

Fault resistance is implemented to detect and handle errors.

Fault tree

This is a *logical* diagram using a tree structure to represent the causes of failures and their combinations leading to a *feared state* (Bayes). Fault trees enable us to calculate the *unavailability* or the *reliability* of the system model.

FMECA – risk analysis

Failure mode, effects and criticality analysis is a method for systematic risk analysis of the causes and effects of failures that might affect the components of a system. FMECA analyses the seriousness of each type of failure. It enables us to evaluate the impact of such failures on the reliability and safety of the system.

Frailty

Frailty describes the characteristic of a metal that breaks easily on impact or from a deformation.

Hardness

The ability of a body to resist penetration from another body harder than it. It is also characterized by its scratch resistance.

HAZ Heat-affected zone

The HAZ represents the heat affected region of the base metal that was not melted during welding process. Metallurgists usually define the HAZ as the area of a base material which has had its microstructure and properties altered by welding or heat.

Hazard

Describes any event, unpredictable phenomenon or human activity that would result in the loss of human lives, or damage to materials or the environment.

Maintainability

Maintainability is one of the aspects of *dependability*. The maintainability of a system expresses its capacity for repair and evolution, with maintenance supposedly completed under certain conditions with prescribed procedures and means.

Corrective maintenance or restoration to a state of proper function: The maintenance performed when a breakdown is detected, aimed at *restoring* a product to a state where it can fulfill its required function.

Preventive maintenance: *To avoid loss of function; thus, it is a probabilistic notion, one of anticipation and prediction.* Such maintenance is performed at predetermined intervals, in accordance with prescribed criteria, intended to reduce the probability of failure or degradation of the function of a product.

Malleability

This is a characteristic that allows the metal to be molded. It is the relative resistance of a metal subjected to compression forces. The malleability of a material increases with increasing temperature.

Markov chains

Used to evaluate the *dependability* of systems in a quantitative manner, this technique is based on the hypothesis that failure and repair rates are constant and that the stochastic process modeling its behavior is Markovian (a memoryless process). When the space of potential states of the system is a discrete set, the Markovian process is called a Markov chain.

Materialized measure

A measuring instrument that replicates or permanently provides different kinds of values during use, each with an assigned value.

Measurand

A value to be measured.

Measuring accuracy

Proximity between a measured value and the true value of a measurand.

NOTE.—

– Measuring accuracy is not a value and is not expressed numerically. A measurement is sometimes considered *accurate* if it offers a *smaller uncertainty*.

– Although linked to the concepts of *correctness* and *fidelity*, it is better not to use the term *measuring accuracy* for measuring *correctness* or the term *measuring fidelity* for *measuring accuracy*.

– Measuring accuracy is occasionally associated with the proximity between the measured values attributed to the measurand.

Measuring instrument, measuring apparatus

Usually a device used for making *measurements*, on its own or possibly in conjunction with other devices.

Measuring repeatability

This is the measuring fidelity according to a set of repeatability conditions.

Measuring reproducibility¹

This is the measuring fidelity according to a set of *reproducibility* conditions.

Measuring uncertainty

The non-negative parameter that characterizes the dispersion of values attributed to a measurand, arising from information used according to the method (e.g. A or B of the GUM).

N.B.—

Measuring uncertainty includes elements caused by systematic effects (associated with the corrections and the assigned benchmark values) as well as definitional uncertainty. Estimated systematic effects are not always corrected. Elements associated with uncertainty would be added. The parameter is often the standard uncertainty (from the standard deviation, σ) or the half range ($U/2$) of an interval with a determined coverage probability (e.g. $k = 2$ for 95% confidence). Certain elements are evaluated with the type-A GUM from the statistical distribution of measured values characterized by standard deviations. The evaluation using the

¹ For statistical terminology refer to ISO 5725-1:1994 and ISO 5725-2:1994.

type-B GUM is characterized by standard deviations of probability density functions based on experience, among other things.

Metrology

The science of measurements and its different applications, which encompasses all theoretical and practical aspects of measuring, regardless of the uncertainty of the measurement or the domain to which it relates.

PHA

This is a method for identifying and evaluating hazards, their causes, their consequences and the seriousness of these consequences. The aim of this analysis is to determine the appropriate methods and corrective actions to eliminate or control dangerous situations or potential accidents.

Probability

Statistical concept that can either express a degree of confidence or a measurement of uncertainty (subjective probability) or be taken as the limit of a relative frequency in an infinite series (statistical probability).

Probability density (or distribution function)

Function describing the *relative likelihood* of a random variable assuming a particular value. It assigns a probability to each value of a random variable.

Q9000 series

Standards that refer to ANSI/ISO/ASQ Q9000 series of standards.

QS-9000

It is a harmonized quality management system requirements “ISO/TS 16949.”

Quality

This is a subjective term for which each person or sector has its own definition.

- The characteristics of a product or service that bear on its ability to satisfy stated or implied needs;
- A product or service free of deficiencies. According to Joseph Juran, quality means “fitness for use”.

Quality assurance/quality control (QA/QC): ANSI/ISO/ASQ A3534-2, Statistics

“Assurance” can mean the act of *giving confidence*, the state of *being* certain or the act of *making* certain; “control” can mean an evaluation to indicate needed corrective responses.

Quality audit

This is a systematic independent examination and review to determine whether quality activities and related results comply with plans and whether these plans are implemented effectively to suits the achievement of objectives.

Quality loss function

This is a parabolic approximation of the quality loss that occurs when a quality characteristic deviates from its target value. The quality loss function was first introduced in this form by Doctor G. Taguchi.

Quality management (QM)

This is the application of a quality management system in managing a process to achieve maximum customer satisfaction at the lowest overall cost to the organization.

Quality plan

This is a set of documents that describe the standards, quality practices, resources and processes pertinent to a specific product, service and project.

Quality score chart

This is a control chart for evaluating the stability of a process.

Quality tool

This is a technique to support and improve the activities of process quality management and improvement.

Random

Process in which the result varies even if the input data set remains identical (a protocol leads to different results).

Reliability

The reliability of a system (work) is its aptitude to meet its design objectives over a specified period of time, in the environmental conditions to which it is subjected. Reliability is based on the probabilities used to evaluate it.

Reliability is one of the aspects of *dependability*. It corresponds to the continuity of service that the system must provide to its users, with the system being considered as irreparable. Any accidental failure is taken into account, regardless of its severity. Reliability measures the rate of failure, and is the inverse of the MTTF (mean time to failure).

Risk

Risk is “a more or less predictable potential danger”, or in other words a drawback that is more or less probable to which we are exposed. The scientific definition of risk involves an aspect of hazard and an aspect of loss, both expressed as probabilities.

Risk analysis

A system inevitably contains design errors, regardless of the amount of validation work done. The “zero error” criterion is not a realistic goal, in view of the development costs this would entail. Thus, it is important for so-called critical systems to evaluate the risks for users:

- FMECA: failure mode, effects and criticality analysis;
- SEEA: software error effects analysis;
- PHA: preliminary hazard analysis.

Risk assessment

Procedure to determine the probability of a hazard *occurring* and its possible consequences.

s.i.f. (ΔK) Stress intensity factor

s.i.f (ΔK) is a function of the stress, crack size and crack shape. Stress intensity factors do not have variability. They have uncertainty and modeling errors. The crack shape may be unknown and be approximated by a semicircle.

In Mode 1 (during a fatigue cycle, in mechanic of rupture) (s.i.f = ΔK) is a measure of the stress-field intensity near the tip of an ideal crack in a linear-elastic solid when the crack surfaces are displaced in the opening mode. The *stress intensity factor range* (ΔK) in fatigue crack growth rate is the variation in a cycle, that is, $K_{\max} - K_{\min}$.

Safety case

The *safety case* approach is mainly used in the oil, nuclear and rail transport sectors. In practice, this procedure facilitates the monitoring of studies.

Sample

One or more observations from a larger collection (population) – samples need to be random, independent and unbiased.

Sampling Plan

This is a scheme used for determining sample sizes and actions in order to make clear decisions.

“The decision, based on counting the number of defectives in a sample, can be to *accept the lot*, *reject the lot*, or even, for multiple (sequential) sampling schemes in order to take another sample and then repeat the decision process”.

Security

We distinguish between safety and security. Thus:

- Safety guards against catastrophic failures, for which the consequences are unacceptable in relation to the *risk*;
- Security relates to the prevention of unauthorized access to information.

SFMEA

Maintenance method, acronym for system failure mode and effects analysis.

Significant Characteristics

Product and process characteristics designated by the customer, including governmental regulatory and safety, and/or selected by the supplier through knowledge of the product and process.

Software reliability

Software inevitably contains *design errors*, no matter how strict the rules of design and validation. The ability of a software suite to provide acceptable service in spite of its residual errors defines its *reliability*.

Specifications (term which is largely used in quality control)

Specifications are engineering requirements for judging the acceptability of a component characteristic. For the production part approval process, every feature of the product as identified by engineering specifications must be measured. Actual measurement and test results are required.

Stability (known as drift)

This is the total variation in the measurements obtained with a measurement system (e.g. gage) on the same master or part when measuring a single characteristic over an extended time period.

Stable Process (in quality control)

This is a quantitative condition for the process which describes a process that is free of assignable/special causes of variation).

Statistical Control

Is the condition of a process from which all special causes of variation have been eliminated and only common causes remain. Statistical control is evidenced on a control chart by the absence of points beyond the control limits.

Subsystem

A major part of a system which itself has the characteristics of a system.

SURVIVAL for Quality Professional

S → Size up the issue on hand
U → Use process knowledge
R → Reach out to the experts
V → Vanquish defensive attitudes
I → Improve on old ideas
V → Value the customer
A → Assess the changes
L → Limit the scope to the problem

SWOT Analysis (Brainstorming, for terms used to)

This is a methodology used to aid strategic planning.

SWOT analysis' main considerations are:

Strengths:

- What are your advantages?
- What do you do well?

Weaknesses:

- What could be improved?
- What is done poorly by the company?
- What are the skills not covered?

Opportunities:

- What are the current trends?

Threats:

- What obstacles do you face?
- What is your competition doing?
- Are requirements changing?
- What are the current threats?
- Do you have resource problems?

Tenacity

Tenacity is the ability of materials to resist shock without breaking or chipping.

Trend test

Trend tests are used in *reliability* to obtain indicators of reliability, from data on failures, and determine fluctuations in reliability over time.

Undesirable event

An event that should not occur or that should be improbable in view of the objectives in terms of *dependability*.

Unit (measuring)

Real scalar value defined and adopted by convention, to which we can compare any other similar value to express the ratio between the two values as a number.

Value

Property of a phenomenon (body, length, weight) expressed quantitatively by a number and a reference.

Chapter 1

Quality Control

1.1. Introduction to statistical process control

Regardless of manufacturing processes or techniques, two components vary due to various factors, no matter how rigorously precise the components might be. The aim is to minimize this variability as much as possible and to restrict it within acceptable parameters. One of the tools to accomplish this task is statistical process control (SPC).

Many statistical sampling techniques are used to avoid individual verification of every single component of a given batch. In some cases, verifying all components is preferred. For example, while we would like a vehicle's clutch to work, controlling the gear boxes of a sample from a batch of cars does not guarantee that none of the clutches will jam. When controlling the sample of clutch mechanisms, the sample is either accepted (i.e. conforms to norms and the sample is delivered) or rejected (i.e. fails to conform to norms and the sample is sent back to manufacturer). Furthermore, the options required to accept the solution are expensive. If the manufactured component is outside parameters (rejected), however, the manufacturer has done a poor job, and SPC tends to aim to avoid producing non-conforming batches while monitoring production and intervening as soon as anomalies are found.

The production process is said to be *under control*, that is *controlled*, when the manufactured product's characteristics of the components are sufficiently similar (or identical) over time. Obviously, there is no precise cause for the product's characteristics to vary: the variability is due to the manufacturing process' tolerance intervals (TIs), that is random causes.

To determine whether the process is under control, small samples are taken at regular intervals. If the controlled characteristic is a measurement (bearing TI in

mind with respect to machine sequencing and design), the process is said to be under control when each sample's mean of this characteristic is equal to a target μ_0 value and when the standard deviation in each sample is equal to a natural standard deviation σ_0 . μ_0 is chosen to reach the target product quality, thereby meeting the specifications. σ_0 mainly depends on the production process and the technology used. If the controlled characteristic is a proportion of non-conforming items in each batch, the process is under control when this proportion (on average, in each sample) is equal to a target value p_0 . This proportion p_0 is set to satisfy quality control. Of course, p_0 (the amount of non-conforming components) is ideally near-zero. We often tolerate minor flaws such as aspect flaws (e.g. roughness).

A process said to be *out of control* is the opposite of a process said to be under control. Such a process has large variability or unexpected characteristics (the TI targeted by the design range). This can be due to many causes: these are then said to be *specific* or *assignable* to this dysfunction. A change in staff, in machine-tool technician, a machine-tool malfunction, etc., could all be the cause of an out-of-control process.

In this chapter, we will show the main statistical tools used in quality control and material and structure reliability.

To this end, we will use the following method:

- 1) Prepare the methodological data collection.
- 2) Structure the collection as a crib sheet:
 - i) Define the sampling $\rightarrow$ experimentation.
 - ii) Define the design of experiments $\rightarrow$ observation.
- 3) Analyze data:
 - i) Tables and graphs of frequencies.
 - ii) Tables and graphs of results.
 - iii) Statistical parameters of sample.
 - iv) Quality indices of machines and processes (capabilities: C_m , C_{mk} , C_p , C_{pk} , etc.).
- 4) Inferential analysis of data statistics:
 - i) Estimations of statistical parameters (see Chapter 2).
 - ii) Various statistical tests (see Chapter 2).
 - iii) Check that parameters conform to the theoretical law used during the intuitive approach to the problem.
- 5) Translation of the results (including the error risk analysis) so as to take a decision (see Chapters 2 and 3 of volume 1).

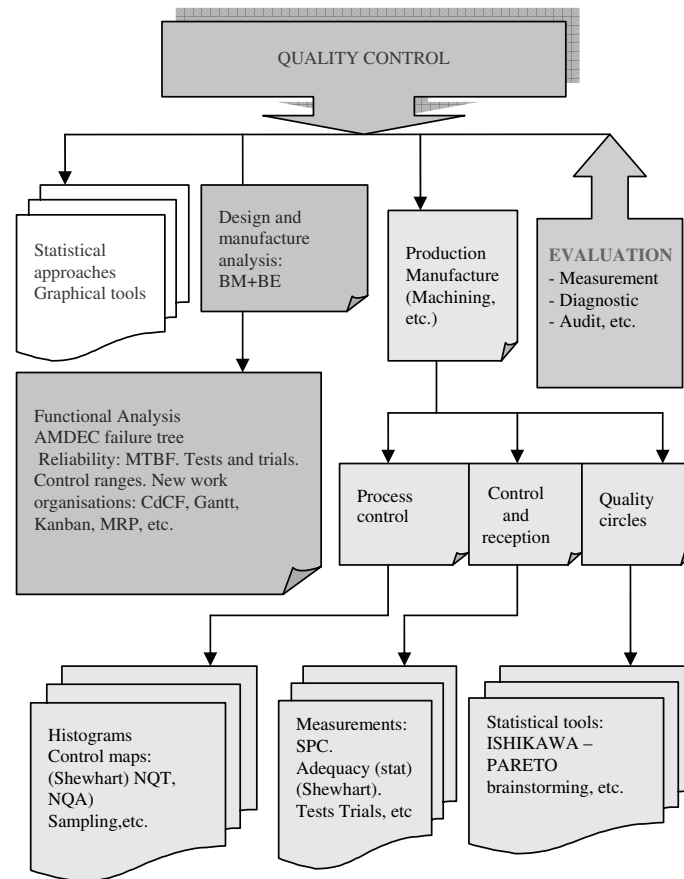


Figure 1.1. *Quality control flowchart in mechanical engineering*

Nature of the problems faced by the quality function	
Globally	Locally
Vague formulation of data	Numbered data formulation
Chronic lack of quality	Insufficient capability
Poorly managed stocks	Implementation of an SPC process
Too few quality initiatives	Rate of rejection or recovery
Disorganized delays (ignored simo charts), etc.	Identification of the true non-quality factors, etc.

Table 1.1. *The quality function*

1.2. Classical tolerancing and quality control

If the statistical approaches were used, classical tolerancing ($\bar{X} \pm TI$) would become penalizing. It adds to the confusion between two-dimensional definition systems in mechanical manufacturing, which is to say that it opposes the statistical fact of production averages (control) and tolerancing, which sets calculated objectives. The latter ignores statistical reality. The latter is nevertheless more *pragmatic*, because it includes all variability factors. Tolerance limits are usually set at more or less six standard deviations from the mean value (target value). These limits are not used to produce and control production. We must not confuse classical *tolerance limits* with *decision limits*, which are used to determine whether a batch is accepted or rejected, or even decide how to set the means of production (machines and tooling).

Problem	Analysis and quality control preparation tools						
Method YES = Y	Brainstorming	QQQCCP	Causes- effets Ishikawa	Pareto	Weighted vote	Compatibility Matrix	Logigram
State the problem		Y					
Choose a problem				Y	Y		
Set the problem appropriately in time	Y	Y		Y			Y
State all possible causes for gaps		Y	Y				
Find major causes				Y	Y		Y
Offer realistic solutions with respect to the means at hand		Y					
Choose an adequate solution					Y	Y	
Implement the solution and control the results	Y			Y			

Table 1.2. Table of analysis and quality control preparation elements

In serial production, the decision limits are completely independent from the tolerance limits (when they exist). The decision limits included in a statistical control plan correspond to operational data, whereas tolerance limits are justified by *abstract* objectives, which we (wrongfully) think can be reasonably attained. It is

high time to leave the off-putting, *classical* educational constraints where we think that verifying each product is possible. In our opinion, there is much to gain by being pragmatic and stop taking for granted that classical tolerancing is the solution.

Capability indicators (C_p , C_{pk} , C_m , C_{mk} , P_p and P_{pk}) become a pragmatic concordance between traditional, non-statistical tolerances and the results obtained by the means of production (statistical results). Respecting the target values while tolerating as little dispersion as possible is paramount. In the classical system, common practice is to use the full tolerance range around the *true value*. We think we control the fluctuations that cause the variability, but this is not the case. For now, only statistics can truly quantify (through measurements and qualitative analysis) manufacturing control effectively.

The above could cause some long-standing attitudes to *reel*. Questioning educational programs on this subject could prove very fruitful, in our opinion, following the example set by GPS (geometrical products specifications) with respect to what is still positive in classical tolerancing methods. To successfully create quality, we recommend following a clearly established plan by associating the use of statistical–mathematical decision-making tools with the causes and means implemented.

1.2.1. ISO norms in quality control

In the 1990s, ISO 9000 became a reference in terms of organizing a quality system. Companies that sought to improve quality management throughout the world rallied under the ISO banner for many reasons, the two main reasons being: the proof that their products were of quality and, of course, to satisfy their clients, thereby becoming more competitive than other manufacturers.

The quality guarantee defines the organization, materialized in a handbook, the aim of which is to guarantee quality. It is the client's responsibility to check how appropriately the reference satisfies their need. The quality guarantees are then evaluated by ACAQ auditors, from the Canadian Standards Association (CSA). After some work in a metrology or quality control laboratory, a succinct report is issued. Of course, all instruments and measurement apparatus are calibrated and verified. Knowing and abiding by manufacturer calibration norms and regulations is imperative. In Canada, the CAN3-85-Z299 norm classifies instruments into four groups to establish calibration periodicity, including:

- Reference calibration gauges 12 months
- Caliper 6 months
- Micrometers 1 month

The Canadian norm above is similar to the American MIL-STD-120. Continuous online quality control is based on quality management from the design stage. As such, it recommends a combined, continuous communication between everyone concerned with the quality. Figure 1.2 shows the above explained quality management cycle.

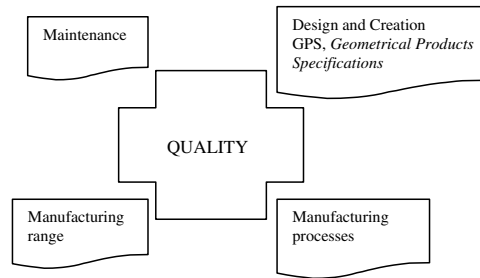


Figure 1.2. *Quality management cycle*

1.2.2. *QS9000 approach*

It was in 1994, in the United States, that the *QS9000* reference was created by the main automotive manufacturers (Ford, General Motors and Chrysler). The goal was to improve the quality of companies by decreasing waste and reinforcing ongoing improvements. The reference was designed based on ISO 9001, with the addition of a few specifications from the US automotive sector. The *QS9000* reference comes with six manuals, including:

- 1) QSA → quality system assessment;
- 2) PPAP → production part approval process;
- 3) APQP → advanced product quality planning and control plan;
- 4) FMEA → failure mode effect and analysis;
- 5) MSA → measurement system analysis;
- 6) SPC → statistical process control.

The calibration ranges are indexed by date. They change over time, and the standard could also change. Sometimes, a criterion, etc., has been changed. Since the calibration ranges are mentioned in index files for devices and their assembly, these indices (dates) can be used to find an optimal traceability with respect to verifications. The integration of blueprints for assembly, equipment, instruments, etc., is recommended. This module of the life file and verification certificates for the assembly equipment is meant for all so-called conventional devices (marking gage, V-control block, etc.) or specific assemblies.

– *Two kinds of device files exist*: The first kind relies on a defined calibration range. The range's contents must be scrupulously adhered to. The second is a file that does not call upon any existing calibration range. For such files, no criterion is defined, and the only relevant factor is the history of verifications. Generally speaking, the characteristics managed for devices and assemblies are:

- 1) device designation,
- 2) the date of the corresponding range,
- 3) the manufacturer, the device's reception date,
- 4) the original classification, and classification after verification,
- 5) the date at which the device was implemented,
- 6) the serial number, affectation, device ID, point of reference,
- 7) the device's capability, the reference norm,
- 8) the maintenance procedure attached to the file,
- 9) the calibration procedure attached to the file,
- 10) the resolution, the periodicity, etc.,
- 11) the creation visa, the date of the next verification,
- 12) the acquisition cost, the device's owner.

– *Managing calibration or verification procedures*: These procedures, added to the database, can be called upon and checked during verifications or calibrations. The verification campaigns are the equivalent of a verification planning. For example:

- campaign 1: micrometer manufactured on the [day j /month M /year A] to [day j /month M /year A];
- campaign 2: marking gage caliper manufactured on the [day j /month M /year A] to [day j /month M /year A].

– *Managing mechanical comparators*: This module deals exclusively with mechanical comparators with a dial test indicator and is based on the Canadian norm. It does apply to mechanical comparators. The characteristics managed for indicators are thus:

- 1) Managing calipers: this essentially relies on:
 - i) bias error at zero,
 - ii) the maximum bias error,
 - iii) the maximum precision error,

- iv) the maximum difference between means,
- v) the intrinsic error,
- vi) the class after verification.

2) Calibrating external micrometers is based mostly on:

- i) the bias error at the zero point,
- ii) the precision error at the zero point,
- iii) the maximum bias error,
- iv) the maximum precision error,
- v) the intrinsic error,
- vi) the class after verification.

3) Calibration is based mostly on:

- i) the flatness of the measuring surfaces,
- ii) the maximum bias error,
- iii) the maximum intrinsic error,
- iv) the class after verification.

1.2.3. Capabilities of CNOMO E41.36.110.N control means

This module is based on the *CNOMO E41.36.110.N* norm (Canada). For any technical information, the corresponding document can be looked up. The capability agreement applies to all specific control means that rely on dimensional measurements (e.g. control mounts, specific automatic control devices, etc.). Analogically, its application can be extended to control means for measurable quantities. The capability agreement allows us to:

- ensure that the material and process can perform the operation they were designed for,
- verify that the measurement uncertainty is compatible with the controlled tolerance.

ISO 8402, 1991 defines “quality” as “The set of an entity’s characteristics which confer the ability to satisfy either implicit or explicit demands”:

Within the context of federal legislation (directives), quality is part of the core requirements. Any product that fails to satisfy ISO 8402, 1991 is by definition

banned. Globally speaking, quality includes three critical factors: normalization, certification and metrology. The Canadian National Research Center (CNRC) makes recommendations in measurements and the means implemented. As for certification, measurement equipment is crucial to obtain quality products as per ISO 90004. The requirements for measurement equipment qualification are subject to the following international laws: ISO 9001 and ISO 9002 & 4.11 or ISO 10012.

1.2.3.1. QS9000 and CNOMO methods

Remark. Using means and error bars, we use trial and error to find various imprecision quantities on measurements: the intrinsic variability in metrology (measurement instruments, operators, materials, sensibility, hysteresis, etc.), the values of the instrument–operator combination as well as the influence of the real gaps between parts. The CNOMO method relies on “correctness” rather than reproducibility (operators and/or methods, laboratories, etc.) In its operational mode, the notions of control averages and metrology show that it applies perfectly well to geometrical measurements (GPS, [GRO 11])				
Sources and conditions	QS 9000 method			CNOMO method
	Range R (or W)	Means μ and Range R	ANOVA (Analysis of variance)	CNOMO method metrology and uncertainties
Estimated sources of variability	All characteristics	Instruments Operators (1 and 2) Component Others (total)	Instruments Operators Interaction: 1 and 2 Component Interaction: 2 and 4 Others	Errors Instruments Instruments and components
Typical conditions of use	Two operators each measure five components once	Three operators each measure 10 components twice	Three operators each measure five components 20 times	One operator measures a prototype five times and a component on the control mean five times, then five different components with measurements representative of the tolerance interval (TI) on the control mean (workshop) and in metrology.

Table 1.3. Summary of QS900 and CNOMO methods

1.2.4. Some capability indicators according to ISO/TS 16949, ISO 8258 and Ford

Capability and Capability Index	The normal law is in effect	Non-Gaussian expressions
$C_p \{P_p\}$	$C_{p_p} \{P_p\} = \left(\frac{LSS - LSI}{k \times \sigma} \right)$	$C_p \{P_p\} = \left(\frac{LSS - LSI}{percent > - percent <} \right)$
$C_{pk} \{P_{pk}\}$	$C_{pk} \{P_{pk}\} = \min \left(\frac{LSS - \bar{x}}{k \times \sigma}; \frac{\bar{x} - LSI}{k \times \sigma} \right)$	$C_{pk} \{P_{pk}\} = \min \left(\frac{LSS - \bar{x}}{percent > - \bar{x}}; \frac{\bar{x} - LSI}{\bar{x} - percent <} \right)$
$C_{pm} \{P_{pm}\}$	$C_{pm} \{P_{pm}\} = \frac{1}{k} \left(\frac{LSS - LSI}{\sqrt{\sigma^2 + (\bar{x} - Target)^2}} \right)$	$C_{pm} \{P_{pm}\} = \frac{1}{k} \left(\frac{LSS - LSI}{\sqrt{\sigma^2 + (\bar{x} - Target)^2}} \right)$

Notes:

- LSS and LSI are the upper and lower tolerance limits (TS and Ti).
- $\bar{x}$ (or μ) is the mean for the normal distribution. It is increased by 50% for other distributions.
- The target is the theoretically ideal value to be reached (true value, according to *Vocabulaire International de Métrologie (VIM)*).
- σ is the standard deviation.
- K , coefficient = 6 for C_p , P_p and C_{pm} , $k = 3$ for C_{pk} , P_{pk} .
- $Percentiles_{Sup}$ and $percentil_{inf}$ are limits that usually include 99.73% of production.

Table 1.4. Capability indicators according to ISO/TS 16949, ISO 8258 and Ford

1.2.4.1. Validation of the measurement process

Index	Definition	Observations
Mean control capability (CMC)	$C_{\text{mean}} = \left(\frac{\text{Tolerance}}{k \times \sigma_{(\text{instruments_operators})}} \right)$	If studying capability decreases $\rightarrow k = 6$
Percentage variation of the process	$\%VA_{\text{Process}} = \left(\frac{\sigma_{(\text{instruments_operators})}}{\sigma_{(\text{Total})}} \right)$	Independent of the tolerance interval
Instrument capability index (ICI)	$\text{Index}_{\text{instrument capability}} = \left(\frac{k \times \sigma_{(\text{instruments_operators})}}{\text{Tolerance}} \right)$	Generally, $k = 5.15$. Often used with the “6 σ ” method.
Number of degrees of freedom	$N_{\text{distincts levels}} = 1.41 \left(\frac{\sigma_{(\text{part})}}{\sigma_{(\text{instruments_operators})}} \right)$	*

Note: *The repeatability and reproducibility studies (see uncertainties) can be used to validate or disqualify a measurement process. The measurement process is accepted or rejected based on the values of the coefficients (indices). The indices of capability show the importance of deducing from trials, “instrument and operator” standard deviations (R&R guarantee) and component-to-component standard deviations, and of identifying the source of variations (e.g. the manufacturer’s mount) on the TI. With the Study Bureau approval (and thus, the client’s), we can act on the TIs to make them less severe, and thus cheaper in terms of time and precision.

Table 1.5. Validation of the measurement process

1.2.5. Metrological audits

There are also different methods to evaluate the metrological function. The audit is a definite process to analyze and verify the efficiency of metrological organizations. Here, we distinguish between the *internal* and the *external* audits. The first determines the competence of subcontractors, measurement and analysis laboratories, companies and the maintenance and calibration contractors. The second allows us to determine our own performance level. It considers what is applied through the document. This latter audit requires more technical knowledge. It can also better detect gaps and can take corrective or preventative actions as well as improve the system in place. The quality audit is defined by the *ISO 8402* norm.

The audit differs from the diagnostic in that the *diagnostic* is the situation of an organization at a time t . The audit is the comparison between what should be, and what is seen in reality, at time t . An audit is a carefully thought-out action, prepared well ahead of time. Its preparation depends on the following points:

- Establishing an audit program.
- Choosing and organizing an audit team.
- Preparing a questionnaire that will often use [WWWWHW]

This reads like the Japanese kaizen (brainstorming)

- Who? Who does it?
- What? Who does what?
- Where? Does it where?
- When? Does it when?
- What? What to do?
- How? How does it?
- Why? For what reason is it done?

Questions are asked in audits. For example, for an audit to certify that the ISO 9001 (9002) norm is being met, we can ask about 11 main questions and 11 secondary questions in addition to the analysis of the quality *manual*.

Some sample questions to ask yourself and others are:

- 1) Is there a procedure concerning the management of the measurement tools used?
- 2) Are the reference materials (prototypes) subject to a type of particular management?
- 3) Are the handling, protection and storage conditions of the measurement equipment (metrology) well defined and respected?
- 4) Are the conditions of (3) compatible with the constructor's conditions?
- 5) Is there a preventive maintenance procedure for the measurement equipment?
- 6) Does the instrument verification procedure account for acceptance criteria?
- 7) Is the equipment in use subject to any particular management?
- 8) How are everyone's responsibilities defined?
- 9) Who (competence and rigor) is responsible for managing metrology?
- 10) Are personnel trained to use and verify measurement methods?

- 11) Is there information (research) about such training?
- 12) How are the measurements to be taken defined, and what is the required exactitude?
- 13) How is the measurement equipment chosen?
- 14) Are uncertainty calculations done as per the Guide to the Expression of Uncertainty in Measurement (GUM)?
- 15) How are the updates of these standards performed?
- 16) Are there measurement devices for which calibration procedures do not exist? If not, how is calibration performed (verification)?
- 17) How are the orders for new measurement instruments defined (criteria)?
- 18) Do we proceed with a qualification of machinable medium software)?

The list is not limited. Other questions can be asked when required. In short, metrology (calibration and/or trials) is the very reason for the endeavor, and the greatest care should be taken to audit it. Nowadays, SPC-type measurement equipment verification results are highly recommended. Canadian regulations give designers the limits within which they are allowed to take their sizing hypotheses in order to ensure that the quality standard is met. These regulations are the synthesis of the constructors' knowledge when they were set. The following Canadian norms are relevant:

- CAN3-B232: keys, lodgings and key grooves.
- CAN3-B78.1: technical drawings; general principles.
- CAN3-B78.3: building drawings.
- CAN 97.1: normalized tolerances for inches-linear and metric dimensions.
- CAN 97.2: interpretation of tolerance and limits.
- CAN 97.3: tolerances and normalized adjustments for assembly components (SI).
- CAN3/CSA-Z234.1: Canadian guide to the metric system.
- CAN3/CSA-B78.5: Computer-aided design drafting (buildings).

Based on the product being produced or service being performed, the certification is given based on one of the three following models:

Model I, defined by *ISO 9001*, regards the entirety of a product's lifetime and is particularly necessary in design with product quality in mind. It encompasses model II, and therefore model III.

Model II, defined by *ISO 9002*, regards products that are both non-controllable and with very well-defined design. It covers the areas of realization, control and trials, therefore making model III an integral part of it. The product's traceability must be done from fabrication as asked by the client, right up until it is delivered.

Model III, defined by *ISO 9003*, only regards products with controllable requirements. It only covers the areas of control and trials, until delivery.

1.3. The Pareto law – ABC method

The method originated with the Marquis of Pareto *Vilfredo Samoso* (1848–1923). This Italian economist showed, using graphical representation, that 20% of the population owned 80% of the wealth (80-20 law or ABC in three distinct classes), hence the expression attributed to Dr Juran: “vital few and many trivial”, which is indicative of the Pareto effect. The Pareto tool aims to select the most representative subjects of a population with respect to quantifiable criterion. The aim of the study is the analysis of the downtimes *of a machining component manufacturing line* in an operating workshop over a duration of time (τ). The study allows us to assign the appropriate downtime (h) to each subsystem. Indeed, an easy problem-solving strategy consists of starting with cause analysis and then deciding which corrective action is appropriate, leading to a plan such as the following:

- 1) Defining the analysis elements.
- 2) Assigning a (quantifiable) value to the elements.
- 3) Classifying the elements.
- 4) Drawing the distinctive (ABC) Pareto diagram.

The elements' definition depends on the object of the study. In metrology, during the calculations for an instrument's calibration uncertainties, we assume that the elements are represented by the various causes of uncertainty.

- 1) We put the subsystems on the x -axis, in decreasing order of the downtime hours assigned to them.
- 2) We put the cumulative downtime (in hours) on the y -axis.
- 3) We then plot the resulting (ABC) curve.

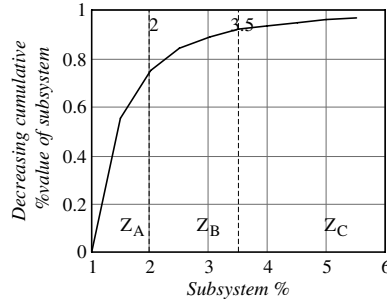


Figure 1.3. Zones A, B and C of a Pareto curve

In our calculation table, we then classify the elements in decreasing order of value. The Pareto graph shows the cumulative values as a function of the elements. The graph shows three zones, indicated in Figure 1.3. These zones are called A, B and C, hence the nickname “ABC method” instead of 80-20 or simply Pareto. In our case study, we will clearly distinguish between the three zones Z_A , Z_B and Z_C .

– Zone A, in which a small percentage of the elements represents a large percentage of the value. Thirty percent of the subsystems account for 70% of the downtime;

– Zone B, in which the percentage of the elements varies more or less with the percentage of the value. Sixty percent of the subsystems account for 91% of the downtime;

– Zone C, in which a large percentage of the elements accounts for a small percentage of the value. A mere 9% of the subsystems account accounts for 91% of the downtime.

From a mathematical perspective, the Pareto law is characterized by two strictly positive whole numbers α and β . The probability density for a Pareto law being defined for $x \in [\alpha; +\infty[$ can be written as:

$$f_{(\alpha, \beta)} = \frac{\alpha^\beta \times \beta}{x^{\beta+1}} ; x \in [\alpha; +\infty[\quad [1.1]$$

Furthermore, for $x \in [\alpha; +\infty[$ the repartition function can be written as:

$$f_{(\alpha, \beta)} = 1 - \left(\frac{\alpha}{x} \right)^\beta ; x \in [\alpha; +\infty[\quad [1.2]$$

A plot of the probability density is given for $\alpha = 1$ and $\beta = 2$:

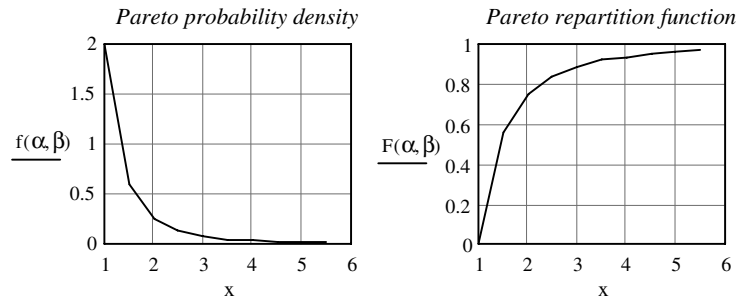
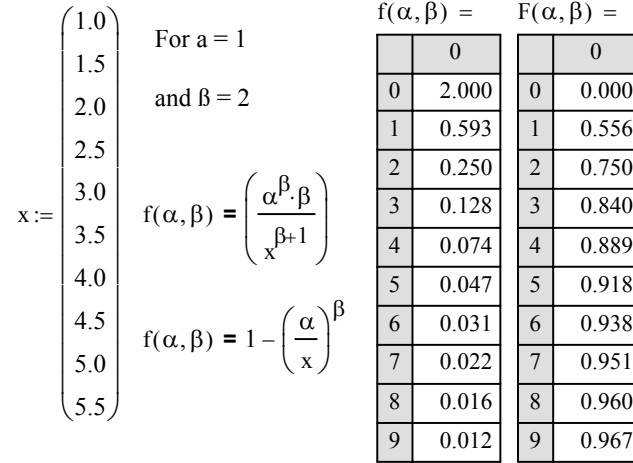


Figure 1.4. Probability density $f(\alpha, \beta)$ and repartition function $F(\alpha, \beta)$ for the Pareto law with parameters $\alpha = 1$ and $\beta = 2$

The function never reaches 1: it asymptotically tends toward $y = 1$ as x tends to infinity. Sample curves for $\beta > 1$ and $\beta > 2$ are shown below:

$$\mu = \left(\frac{\alpha \times \beta}{\beta - 1} \right) \text{ if } \beta > 1 \text{ and } \sigma = \left(\frac{\alpha \times \sqrt{\beta}}{(\beta - 1) \times \sqrt{\beta - 2}} \right) \text{ if } \beta > 2 \quad [1.3]$$

Random number generation following the Pareto law is done through the reciprocal transformation of the repartition function.

$$\frac{1}{f(\alpha, \beta)} = \frac{\alpha}{\sqrt[p]{1-p}} \text{ for } \{0 \leq p < 1\} \quad [1.4]$$

Some software can generate very large ($>10^6$) samples by MC simulation. The mean is quickly calculated with great precision. We do not tackle this subject here, but would like to point out that such (software) tools greatly facilitate research.

- 1) Field of application and lot inspection.
- 2) Types of control.
- 3) Sampling techniques.
- 4) Sampling plan.
- 5) Sampling plan's efficiency curve.
- 6) Quality acceptance level.
- 7) Outgoing quality control and its limitations.

Decisions must be taken in light of the Pareto graph. The malfunction repartition diagram is useful in this respect.

1.3.1. *Practical application*

Consider halts made on the wired Electro discharge machine (EDM) with respect to the machine's subsystems (machine parts being put out, hydraulic failure, stuck cables, etc.). Sampling results are shown in Figure 1.5.

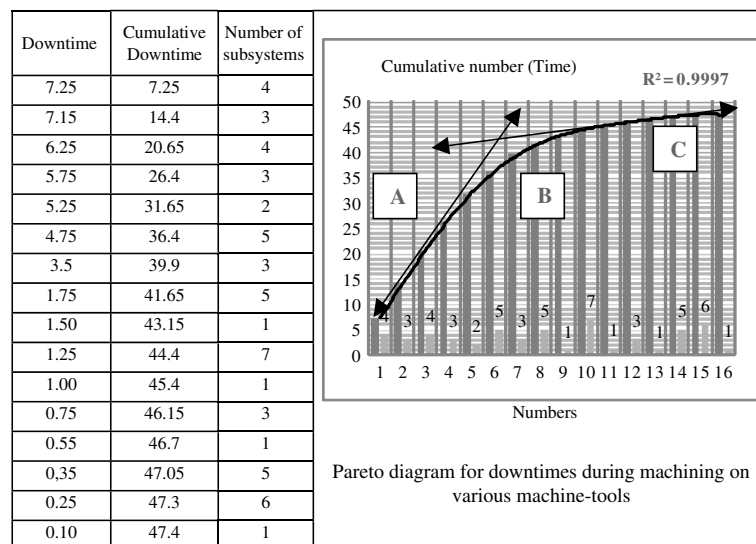


Figure 1.5. Pareto diagram for machine understaffing

1.3.2. Malfunction spread over 16 so-called “neuralgic” points

- $Z_A \rightarrow$ Here is where most failures occurred.

Decision: We have three very fragile points in the machine: 20% account for 80% of the interventions resulting in high maintenance costs because of the changes in machine elements to be performed.

- $Z_B \rightarrow$ Here is where some of the failures occurred.

Decision: We have five relatively fragile points in the machine, which are an inherent part of preventive maintenance

- $Z_C \rightarrow$ Here is where fewest failures occurred.

Decision: We have eight non-influential points in the machine, which account for 20% of the interventions resulting in “manageable” (corrective) maintenance costs.

Maintenance actions would emphasize taking action against apparent flaws. We can thus see, in light of the above, that the Pareto approach is a decision-making tool. It is however, not always a very precise method.

1.4. Lot inspection by attributes

Sometimes, quality conditions do not allow for a measurement for various (often economic) reasons. We then use a method known as characteristic quality control: charts (p , np and c). For example, instead of measuring a depth or diameter we use a range of “fits”, “doesn’t fit” to then make a decision to accept or refuse. This is a reduced judgment allocation, but is used in many fields. The characteristic principle then becomes:

- p_chart : fraction defective chart);
- np_chart : number of defectives chart);
- c_chart : number of defects chart).

1.4.1. Areas on which lot inspection operates

- productions from outside entities,
- lots made up of products likely to be individualized

Control at 100% (census) and/or by sampling occurs during a delivery, and therefore before production. The aim of lot inspection applied to a *fully homogeneous* lot allows us to:

- *decide whether to accept or reject the lot,*
- *compare the quality of lots from different providers,*
- *assess the quality of manufacturing to soften or harden control laws (series of lots).*

Work on control limits for the (p) chart is relatively straightforward. Starting from the fact that the size of all samples is same, we calculate the proportion of defects in each sample with:

$$P = d/n \quad [1.5]$$

– (d) is the number of defects (rejected) and (n) the number of items in the sample (N).

As for the $\bar{X}$ (mean) and R (range) charts (see the following section) we determine provisional limits over 20 samples. The proportion of defects $\bar{P}$ over all samples can then be written as:

$$\bar{P} = \sum_N d / \sum_N n \quad [1.6]$$

The control limits on either side of the mean as per $\pm 3\sigma$.

$$\left\{ \text{UCL} = \bar{P} + 3\sqrt{\frac{\bar{P} \times (1 - \bar{P})}{n}} \text{ and } \text{LCL} = \bar{P} - 3\sqrt{\frac{\bar{P} \times (1 - \bar{P})}{n}} \right\} \quad [1.7]$$

where UCL = upper control limit and LCL = lower control limit.

We say that the process is “under statistical control” when no point lies outside of the limits. Note that this practice is less appropriate for small samples. For this reason, samples of at least 50 items are strongly recommended. Normally, among the various probabilistic models found in the literature, Poisson (or more exactly binomial) distribution is most appropriate. The sample size can vary. The control limits for the (p) chart can then be written as:

$$\bar{P} = \sum_{i=1}^N d_i / \sum_{i=1}^N n_i \quad [1.8]$$

Similarly to the above, the LSC and LCL can then be written as:

$$\left\{ \text{UCL} = \bar{P} + 3\sqrt{\frac{\bar{P} \times (1 - \bar{P})}{n_i}} \text{ and } \text{LCL} = \bar{P} - 3\sqrt{\frac{\bar{P} \times (1 - \bar{P})}{n_i}} \right\} \quad [1.9]$$

1.4.2. ($\bar{np}$) chart for controlling the number of defects

Similarly to the above, this chart is a graph where the number of defects found in each sample is shown instantly. It is found as follows:

$$n\bar{P} = \sum_n d / \kappa \text{ with } \bar{P} = \sum_n d / n \times \kappa \quad [1.10]$$

And the LSC and LCL would then be written as:

$$\left\{ \text{LSC} = n\bar{P} + 3\sqrt{n\bar{P} \times (1 - \bar{P})} \text{ and } \text{LIC} = n\bar{P} - 3\sqrt{n\bar{P} \times (1 - \bar{P})} \right\} \quad [1.11]$$

Bear in mind that (d) represents the number of defects, which is a binomially distributed random variable.

1.4.3. ($\bar{c}$) card to control the number of defects

This is the most restrictive method. Similarly to the above, it is a graph of the number of defects found in each controlled unit (instantly). It is found as follows:

$$\bar{c} = \sum c_i / \kappa \quad [1.12]$$

and UCL and LCL are written as:

$$\left\{ \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} \text{ and } \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} \right\} \quad [1.13]$$

(c) is the number of defects and follows a random Poisson distribution with an average (expected value) of $E(C) = C^*$. When controlling subgroups from various units, we must first calculate an average number of defects per subgroup of (n) units. For (κ) subgroups, we also calculate the general number of group defects and thus:

$$\bar{c} = \frac{\sum c_i}{n} \text{ then } \bar{\bar{c}} = \frac{\sum \bar{c}}{\kappa} \quad [1.14]$$

and finally:

$$\left\{ \text{UCL} = \bar{\bar{c}} + 3\sqrt{\frac{\bar{\bar{c}}}{n}} \text{ and } \text{LCL} = \bar{\bar{c}} - 3\sqrt{\frac{\bar{\bar{c}}}{n}} \right\} \quad [1.15]$$

Value of the ratio (P_2/P_1) for $\alpha = 0.05$ and $\beta = 0.10; \beta = 0.05; \beta = 0.01$					Value of the ratio (P_2/P_1) for $\alpha = 0.01$ and $\beta = 0.10; \beta = 0.05; \beta = 0.01$				
C	$\beta = 0.10$	$\beta = 0.05$	$\beta = 0.01$	(np_1)	C	$\beta = 0.10$	$\beta = 0.05$	$\beta = 0.01$	(np_1)
0	44.890	58.404	89.781	0.052	0	229.105	298.073	458.21	0.010
1	10.946	13.349	18.681	0.355	1	26.184	31.933	44.686	0.149
2	6.5090	7.699	10.280	0.818	2	12.206	14.439	19.278	0.436
3	4.8900	5.675	7.352	1.366	3	8.115	9.418	12.202	0.823
4	4.0570	4.646	5.890	1.970	4	6.249	7.156	9.072	1.279
5	3.5490	4.023	5.017	2.613	5	5.195	5.889	7.343	1.785
6	3.206	3.604	4.435	3.286	6	4.520	5.082	6.253	2.330
7	2.957	3.303	4.019	3.981	7	4.050	4.524	5.506	2.906
8	2.768	3.074	3.707	4.695	8	3.705	4.115	4.962	3.507
9	2.618	2.895	3.462	5.426	9	3.440	3.803	4.548	4.130
10	2.497	2.750	3.265	6.169	10	3.229	3.555	4.222	4.771
11	2.397	2.630	3.104	6.924	11	3.058	3.354	3.959	5.428
12	2.312	2.528	2.968	7.690	12	2.915	3.188	3.742	6.099
13	2.240	2.442	2.852	8.464	13	2.795	3.047	3.559	6.782
14	2.177	2.367	2.752	9.246	14	2.692	2.927	3.403	7.477
15	2.122	2.302	2.665	10.035	15	2.603	2.823	3.269	8.181
16	2.073	2.244	2.588	10.831	16	2.524	2.732	3.151	8.895
17	2.029	2.192	2.520	11.633	17	2.455	2.652	3.048	9.616
18	1.990	2.145	2.458	12.442	18	2.393	2.580	2.956	10.346
19	1.954	2.103	2.403	13.254	19	2.337	2.516	2.874	11.082
20	1.922	2.065	2.352	14.072	20	2.287	2.458	2.799	11.825
21	1.892	2.030	2.307	14.894	21	2.241	2.405	2.733	12.574
22	1.865	1.999	2.265	15.719	22	2.200	2.357	2.671	13.329
23	1.840	1.969	2.226	16.548	23	2.162	2.313	2.615	14.088
24	1.817	1.942	2.191	17.382	24	2.126	2.272	2.564	14.853
25	1.795	1.917	2.158	18.218	25	2.094	2.235	2.516	15.623
26	1.775	1.893	2.127	19.058	26	2.064	2.200	2.472	16.397
27	1.757	1.871	2.098	19.900	27	2.035	2.168	2.431	17.175
28	1.739	1.850	2.071	20.746	28	2.009	2.138	2.393	17.957
29	1.723	1.831	2.046	21.594	29	1.985	2.110	2.358	18.742
30	1.707	1.813	2.023	22.444	30	1.962	2.083	2.324	19.532

Table 1.6. *J.M. Cameron tables for a simple (c and n) sample [CAM 52]*

NOTE: Our case studies use Tables given in the Appendix A.10 – A.42. The Cameron table [CAM 52] gives the (P_2/P_1) ratio as a function of (c) and (n) if the (α) and (β) risks agree with those shown in the table.

Table 1.6 gives the (n) and (c) values for which the sampling plan's efficiency curve goes through the following points: $(AQL, 1 - \alpha)$ and $(LTPD, \beta)$.

- AQL = acceptable quality lot;
- LTPD = lot tolerance percent defective;
- (α) = producer's risk;
- (β) = consumer's risk.

Concretely speaking, the above is done as follows. Consider a given control plan (see the Appendix) with the following conditions:

- AQL = 1%; with producer's risk $(\alpha) = 0.05$; and
- LTPD = 5%; with consumer's risk $(\beta) = 0.10$.

With these conditions in mind, the sampling plan should accept lots with a set quality of 2% (why not better?), 95 times $(1 - \alpha)$ of 100 on average. This plan should also only accept lots with a set quality of 2% (which could also have been worse), only 10 times of 100 on average.

- $P_a = 0.95\%$; $\rightarrow$ if $p = AQL = 2\%$ and $P_a = 0.10\%$; $\rightarrow$ if $p = LTPD = 5\%$.

Based on the *a priori* specifications, this allows us to set two points on the efficiency curve to represent this: $\rightarrow [AQL; 1 - \alpha]$ and $[LTPD; \beta]$. Note that the shape of the efficiency curve will depend on (n) and (c) as follows:

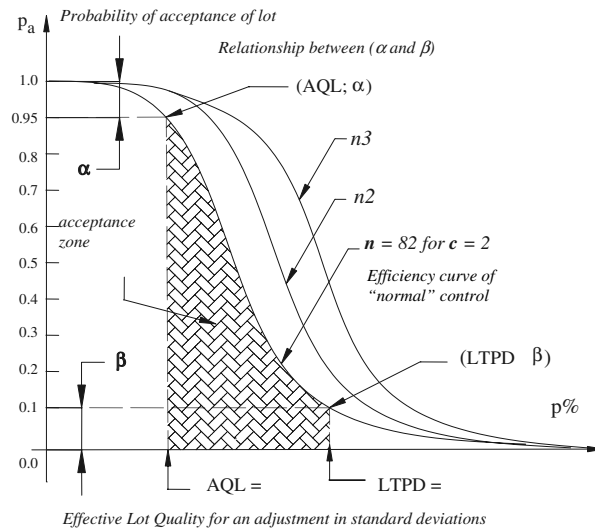


Figure 1.6. Shape of the efficiency curve as a function of (n) and (c)

Using the Cameron table, we will solve the problem as follows:

HYPOTHESIS.— AQL = 1% for $(\alpha) = 0.05$ and LTPD = 5% for $(\beta) = 0.10$

1) Using the Cameron table, we define:

$$R_{\text{ratio}} = \text{LTPD}/\text{AQL} = 5\%/1\% = 0.05/0.01 = 5.$$

2) Reading the column for $\alpha = 0.05$ and $\beta = 0.10$ allows us to find that $(P_2/P_1) = 4.890$ (the closest value) for $c = 3$ (see line 2 of the J.M. Cameron table) and $(np_1) = 1.366$.

3) We calculate the sample size (n) using the following relation:

$$n = np_1 / \alpha = 4.89/0.01 = 48.9 \text{ or } 50 \text{ components.}$$

We deduce that the simple sampling plan is $n = 50$ for $c = 3$. We must randomly sample 50 components from a lot of $N = 1,000$ components to control them individually (1/4 of the manufacture) and accept the lot if $d > 2$ or refuse it if $d < 2$ (we have already mentioned that d represents the number of defects (rejected components)).

4) We must now check the risks if this were a sampling plan that corresponded to set risks. For a lot with quality $p = \text{AQL} = 1\%$ (or 0.01), the probability that the lot would be accepted with the 50 and $c = 3$ plan is:

$$P_a = 0.95 \Rightarrow P\{d \leq 2 \mid np = 50 \times 0.01 = 0.50\}$$

but the probability was set at 0.95 (95%). We can thus interpolate that $(1 - \alpha) = 0.95 \rightarrow (\alpha) = 0.05$.

For a lot with quality 0.05, this plan's acceptance probability is then:

$$P_a = 0.04 \Rightarrow P\{d \leq 2 \mid np = 50 \times 0.05 = 5/2\}$$

We wanted to control for $(\beta = 0.10)$. Our calculations give a better result. We will thus refuse (on average) lots with quality as poor as LPTD = 5%, 96 times 100, rather than 90 times.

1.4.4. Average lot quality after control

For various technical reasons, it is possible that a lot will be returned to the manufacturer (supplier). It is also possible to accept the lot and control all the components ourselves without omitting a single one (100%). To rectify refused

scores, we must make rectifications that confirm the designer's scores. It is common practice to redo the design project, or to at least get the authorization from the design/planning department (manufacture range's design) and O & M (machining range and simo charts).

To answer the above, we calculate the average outgoing lot quality. This is the control of the percentage of the remaining defective components. We assume that the lot's number N is much larger than (n) . This percentage is usually written as an average outgoing quality (AOQ) and is found as follows:

$$AOQ = \frac{P_a \times p(N-n)}{N} = P_a \times p \left(1 - \frac{n}{N}\right) \approx P_a \times p \quad [1.16]$$

Since the sample of the (n) rejected components is 100% controlled, we state that no component is outside the tolerance. The accepted lot (p) will still have $p(N-n)$ defective components on average. This situation only presents itself in the long term, or our reasoning would be contradictory (100% controlled and accepted?). The fact that in the long term P_a out of 100 times does not allow us to deduce that on average and after control means that we have $P_a p(N-n)$ defects. The AOQ percentage can then be written as:

$$AOQ = \frac{P_a \times p(N-n)}{N - p\{np_a + N(1-P_a)\}} \quad [1.17]$$

Clearly, there is little difference between both relations of AOQ even for a small (p) . Let us translate this into a simple example: we wish to know the average outgoing lot quality in the previous hypothesis, that is $n = 50$ and $c = 3$ for lots with $N = 1,000$. Outgoing control is done with $p = 5\%$ (0.05).

1.4.4.1. Solving this problem

Let $n = 50$, $c = 3$. Using the cumulative Poisson probability table, we find that

$$P_a = P\{d \leq 2 \mid np = 50 \times 0.05 = 2.5 = 1/4\} = 0.544$$

The mean outgoing lot quantity will then be of the order of:

$$\left\{ \begin{aligned} AOQ &= \frac{P_a \times p(N-n)}{N} = \frac{0.544 \times 0.05(1000-50)}{1000} = 0.0268 \\ \text{This is equivalent to: } P_a \times p &= (0.544 \times 0.05) = 0.0272 \end{aligned} \right\}$$

From Poisson distribution and with the following data:

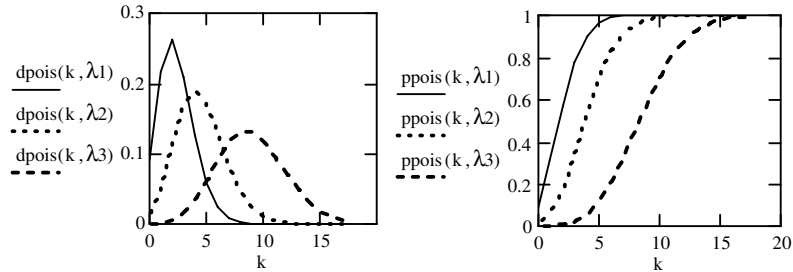
$$P(\lambda_i, k) = \frac{(n_i \times p_n)^k}{k!} \times \text{Exp}\{-n_i \times p_n\} = \text{dpois}(k, \lambda_i) \text{ (MathCAD syntax)}$$

$n_1 = 60$	$\lambda_1 = 2.4$	$p_1 = \lambda_1 / n_1 = 4\%$
$n_2 = 90$	$\lambda_2 = 4.5$	$p_1 = \lambda_2 / n_2 = 5\%$
$n_3 = 150$	$\lambda_1 = 9$	$p_1 = \lambda_3 / n_3 = 6\%$

We have calculated the values for the Poisson distribution ourselves, rather than read them from the tables. Our results are then:

Individual probability	Cumulative probability	Individual probability	Cumulative probability	Individual probability	Cumulative probability
$=\text{dpois}(k, \lambda_1)$	$=\text{ppois}(k, \lambda_1)$	$=\text{dpois}(k, \lambda_2)$	$=\text{ppois}(k, \lambda_2)$	$=\text{dpois}(k, \lambda_3)$	$=\text{ppois}(k, \lambda_3)$
7.4168×10^{-10}	1.0000	3.9739×10^{-6}	1.0000	5.7863×10^{-3}	0.9947
5.2536×10^{-9}	1.0000	1.5013×10^{-5}	1.0000	0.0109	0.9889
3.5024×10^{-8}	1.0000	5.3378×10^{-5}	1.0000	0.0194	0.9780
2.1890×10^{-7}	1.0000	1.7793×10^{-4}	0.9999	0.0324	0.9585
1.2769×10^{-6}	1.0000	5.5355×10^{-4}	0.9997	0.0504	0.9261
6.9166×10^{-6}	1.0000	1.5991×10^{-3}	0.9992	0.0728	0.8758
3.4583×10^{-5}	1.0000	4.2644×10^{-3}	0.9976	0.0970	0.8030
1.5850×10^{-4}	1.0000	0.0104	0.9933	0.1186	0.7060
6.6044×10^{-4}	0.9998	0.0232	0.9829	0.1318	0.5874
2.4766×10^{-3}	0.9991	0.0463	0.9597	0.1318	0.4557
8.2555×10^{-3}	0.9967	0.0824	0.9134	0.1171	0.3239
0.0241	0.9884	0.1281	0.8311	0.0911	0.2068
0.0602	0.9643	0.1708	0.7029	0.0607	0.1157
0.1254	0.9041	0.1898	0.5321	0.0337	0.0550
0.2090	0.7787	0.1687	0.3423	0.0150	0.0212
0.2613	0.5697	0.1125	0.1736	4.9981×10^{-3}	6.2322×10^{-3}
0.2177	0.3084	0.0500	0.0611	1.1107×10^{-3}	1.2341×10^{-3}
0.0907	0.0907	0.0111	0.0111	1.2341×10^{-4}	1.2341×10^{-4}

We have also programmed the probability density functions and repartition functions using our own data. The graphs for our own calculations are shown in Figure 1.7.



Figures 1.7. Distribution and repartition functions for Poisson distribution

for $n = 90$; $d \leq 2$; $p =$ (from 0 to 10) and $AOQ = (P_{axp})$

Recall our results for the initial efficiency plans, with the following graphs on their respective efficiency curves:

$P_a = 0.05$	$P_R = 0.165$	$Pa_1 = \text{Plan}_1 (60, 2.4, 4\%)$
		$Pa_2 = \text{Plan}_2 (90, 4.5, 5\%)$
For: $\alpha = 1 - 0.95 = 0.05$	For: $\beta = (0.180)$	$Pa_3 = \text{Plan}_3 (150, 9, 6\%)$
AQL = Acceptable Quality Level		
LTPD = Lot Tolerance per cent Defective (see the Appendix 1)		

The graph for the P_a efficiency curves as a function of (p) is:

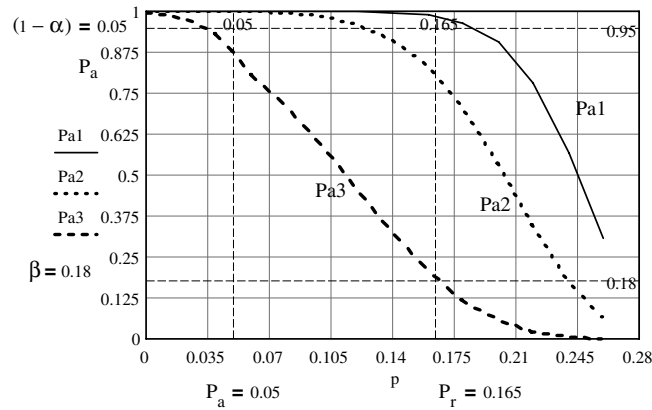


Figure 1.8. Respective efficiency curves per plan

To plot the mean outgoing batch quality curve for the $n = 90$, $c = 3$ plan (for example) for different qualities p of the lots was controlled with a correction.

p =		Pa =		n·p =		AOQ(p) =	
	0		0		0		0
0	0	0	0	0	0	0	0
1	0.01	1	0.95	1	0.9	1	$9.5 \cdot 10^{-3}$
2	0.02	2	0.773	2	1.8	2	0.016
3	0.03	3	0.554	3	2.7	3	0.017
4	0.04	4	0.364	4	3.6	4	0.015
5	0.05	5	0.224	5	4.5	5	0.011
6	0.06	6	0.132	6	5.4	6	$8 \cdot 10^{-3}$
7	0.07	7	0.075	7	6.3	7	$5 \cdot 10^{-3}$
8	0.08	8	0.041	8	7.2	8	$3 \cdot 10^{-3}$
9	0.09	9	0.023	9	8.1	9	$2 \cdot 10^{-3}$
10	0.1	10	0.012	10	9	10	$1 \cdot 10^{-3}$

We thus obtain the graph for the average outgoing lot quality.

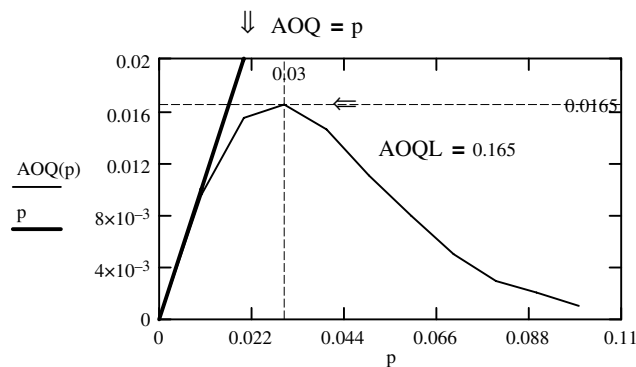


Figure 1.9. Average outgoing lot quality curve

We note from our results (numerical and graphical) that AOQL = 1.65% corresponds to 1.65 and that AOQ = p to 3%. This allows us to claim that if the “rejected” lots were not 100% controlled (rectified) then AOQ = p and that the curve must thus go through the value’s maximum. Generally, we have two main types of control.

– *Control by counting or attributes:* After controlling one or more qualitative (esthetics, colors) or quantitative (nominal scores, tolerances, geometrical constraints, roughness, etc.) aspects [GRO 11], the scores are classified as “conforming” or “non-conforming”.

– *Control by measurement*: The observed characteristic is a measurable quantity (e.g. $\varnothing 20H7/g6$) for which one or two tolerance limits ($\pm TI$) exist, with the [COTES] outside tolerance being non-conforming as per the definition and product (manufacture) drawing plan.

– *Sampling procedures*: The schematics given here apply to control by counting. For control by measurement, the principles are the same but the criteria are based on the mean rather than a number of non-conforming.

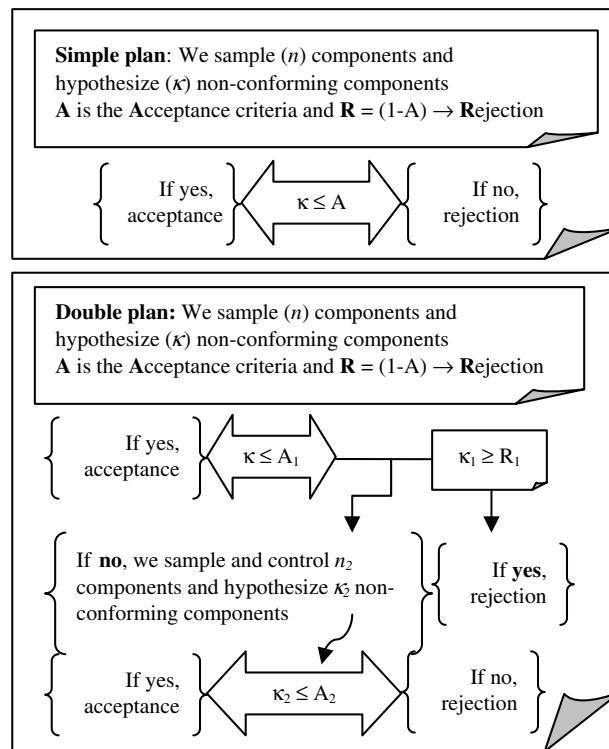


Figure 1.10. Sampling procedures (plans)

– *Multiple plan*: This is simply a generalization of the double plan. The Canadian norm (French Canada) includes at least seven representative samples.

– *Progressive plan*: This is a generalization of the multiple plan. Here, a decision is taken after controlling each score. The control duration and sample size are not known *a priori*. Although it is less efficient, the simple plan is cheaper and easier to implement. This plan is thus more expensive, harder to implement but also much more efficient than previous plans.

– *Sampling plan*: A sampling plan is based on defining a decision rule leading to either accepting or rejecting the batch. It is defined by:

- *the control type (counting or true measurement in metrology);*
- *the sampling procedure (simple, double, multiple, progressive plan) or by the sample's size;*
- *the appropriate relation between the results and the measured decision (e.g. $\varnothing 20H7/g6$ if the score is outside the imposed tolerance limits $\leftrightarrow$ we accept the score if it is within the imposed tolerance limits (drawing plan), we decide to reject the component or modify it with the agreement of the design/planning department and O&M, Research and Development and Organization and Methods).*

The considerations that affect the plan choice are varied, and include:

- *the nature of the controlled characteristic (qualitative, quantitative);*
- *the importance of accepted risks (drawing plan in design);*
- *the size of the lots (an incorrect decision would inevitably have more consequences for large lots than for small lots);*
- *the information available on quality (commonly shared);*
- *the cost of control (metrology: Coordinate Measurement Machine (CMM), profile projector, X-rays, etc.).*

– *Plan efficiency*: The decision to accept or refuse a lot is essentially test-based. The plan's efficiency then depends on reading the test's efficiency curve correctly: in other words, on the value of the acceptance probability P_a to accept the lot, knowing that the non-conformity proportion is p .

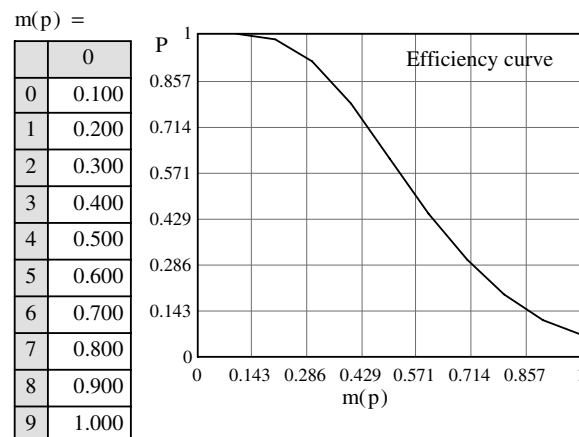


Figure 1.11. Shape of a plan's efficiency curve

– *Supplier and client risks*: Using the efficiency curve graph is not very practical. Instead, the efficiency curve is characterized by two points: (p_1, α) and (p_2, β) where:

- $\alpha = P[\text{reject the lot}/p = p_1] = \text{supplier's risk,}$
- $1 - \alpha = P[\text{accept the lot}/p = p_1] = P_a(p_1),$
- $\beta = P[\text{accept the lot}/p = p_2] = \text{client's risk} = P_a(p_2),$

with $p_2 > p_1$. The usual values for α and β are 5% and 10%, respectively, and we denote the corresponding proportions p_1 and p_2 by p_{95} and p_{10} .

– *Discrimination relation*

- The plan's quality can be measured by the discrimination relation $(DS) = p_2/p_1 > 1$. The closer (DS) comes to one, the better the plan. A plan is even better if α and β tend to 0. The ideal plan would be the one below, where $\alpha = \beta = 0$.

– *Acceptable quality level*: The AQL is a contractual parameter between client and supplier.

It is the maximum percentage of non-conforming scores for the lot to still be acceptable.

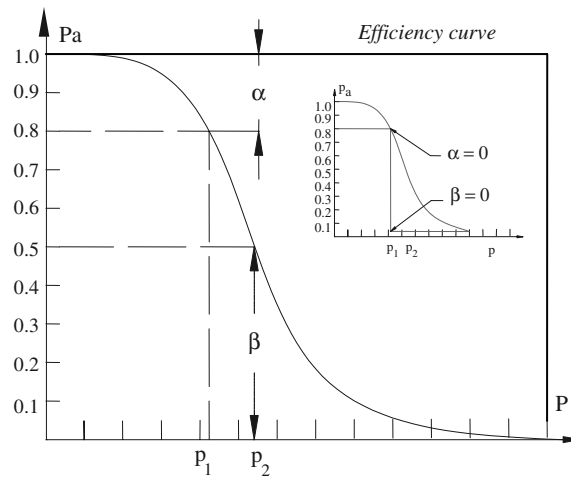


Figure 1.12. Discrimination ratio on an efficiency curve

– *Average outgoing quality (AOQ)*: The AOQ is the stock's average quality. It is found after controlling a series of lots in which all non-conforming components of the controlled samples were removed, and after 100% triage of the refused lots.

– *Average outgoing quality limit (AOQL)*: For $p = 0$ and $p = 1$, $AOQ = 0$. The AOQ function goes through a maximum AOQL, which characterizes the worst average quality that risks being found in outgoing stock.

– *Theoretical basics of control by counting*: The decision at the end of the test is taken based on the number of non-conforming components (K) or the average number of non-conformities per component (κ). In the control of the non-conforming proportion, K follows:

- a *binomial* distribution with parameters n and p if the sampling rate n/N is less than 10%;
- a *hypergeometric* distribution with parameters N , n and p if the sampling rate n/N is greater than 10%;
- for the control of the average number of non-conformities per component, K follows a Poisson distribution with parameters (n, p) ;
- knowing K 's distribution allows us to plot the efficiency curve as a function of n and A , acceptance criterion.

1.4.4.2. Sampling rules

For the tables and efficiency chart: (*see Weblink*)

The norm defines three control types (see the Appendix):

- 1) *Normal*: adapted to begin the control of a series of lots or for an isolated lot.
- 2) *Reinforced*: less economical, but offers better client protection (smaller β). The client imposes it when normal control leads to doubts on the quality.
- 3) *Reduced*: more economical. The client calls for it when normal control leads us to believe that the quality is acceptable.

The Canadian norm gives empirical rules for going between these three levels of control. Specialized tables to choose a simple sampling plan give reinforced and reduced plans from normal control.

– *Levels of control*: The notion of levels of control comes in when the plan is chosen based on the lot size. The norm defines three levels of control:

- Level I: reserved for particularly difficult-to-control criteria;
- Level II: this is the norm level, usually taken;

NOTE.— See Weblink means that we should refer to specialized textbooks or on the Internet, on which the following Weblinks are useful: <http://www.sqconline.com/about-acceptance-sampling> and <http://www.sqconline.com/control-charts>.

- Level III: reserved for particularly easy-to-control criteria.

In addition to these general-case levels, the norm has five levels for special uses, which must be reserved for special cases such as destructive control. The controlled quantities are small, but efficiency is mediocre.

- *Lot size*: When the size of the lots to be delivered must be determined, it can be defined based on the level of control (presumably Level II) and the sample size. The norm gives a code letter to help determine a bracket in which the lot size can fit.

- *Size of the samples to be taken*: In most cases, the lot size is fixed. The norm gives a code letter to determine the size of the sample to be taken as a function of the level of control (presumably Level II) and the lot's size.

- *Choice of a simple sampling plan*: In most cases, the lot size is fixed and an NQA value is chosen. For the usual Level II, the table (*see Weblink*) gives the sample size. Specialized tables (*see Weblink*) give a plan with an Acceptable Quality Level (AQL) close to what is desired, rejection and acceptance criteria, along with p_{95} and p_{10} values that correspond to the supplier and client risks.

- *Control of the percent of non-conforming individuals – Simple plan* (*see Appendix/Weblink*).

- *Control of the average number of non-conformities – Simple plan* (*see Appendix/Weblink*).

Other ways of choosing a plan can be considered based on the parameters we wish to emphasize: NQA, batch size, sample size, p_{10} , p_{95} , AOQL, level of control. Depending on the case, other tables (found in ACNOR (CSA) norms) might be necessary.

1.4.5. Choosing a sample plan

Choosing a double or multiple sample plan, based on the selected criteria, of the appropriate simple plan, we obtain a sample size. The numbers and acceptance/rejection criteria of double and multiple plans with an equivalent efficiency can be found in the tables given in the Appendix. The advantage of the double or multiple plans against simple plans is the savings in terms of the size of the controlled sample. The abaci given by the ACNOR norm (CSA) allow for these savings by using the mean effective controlled ratio in a double or multiple plan by number in the simple plan.

- *Correspondence between double and simple plans* $\leftrightarrow$ *Tables* (*see Appendix 1/Weblink*).

- *Correspondence between multiple and simple plans* $\leftrightarrow$ *Tables* (*see Appendix 1/Weblink*).

As said above, *progressive control* allows us to take a decision after controlling each component. The plan is built with two points on the efficiency curve: (p_1, α) and (p_2, β) . The decision-making rule, resulting from the Wald sequential test theory, can be written as:

Decision	Formula
Lot is accepted if	$\kappa n \leq -(h_1 - sn)$
Lot is refused if	$\kappa n \geq (h_2 + sn)$
Sampling continues if	$(sn - h_1) < \kappa n < (h_2 + sn)$

Table 1.7. *Progressive control*

where (κ, n) is the number of non-conforming components after sampling n components.

This kind of control takes place over continuous series of components from a same manufacture range. The plan's truncation is explained by the fact that if we controlled between the tolerance limits for long enough, stopping at component $n_t = (3/2) n_0$ is easy. In this case, n_0 is the number of components in the simple plan of same efficiency as the *Wald* plan. If the curve lies within the acceptance zone, control stops. If not (rejection zone), we reject the lot and perform a triage. The graphical illustration of the above (decision rule) is as follows:

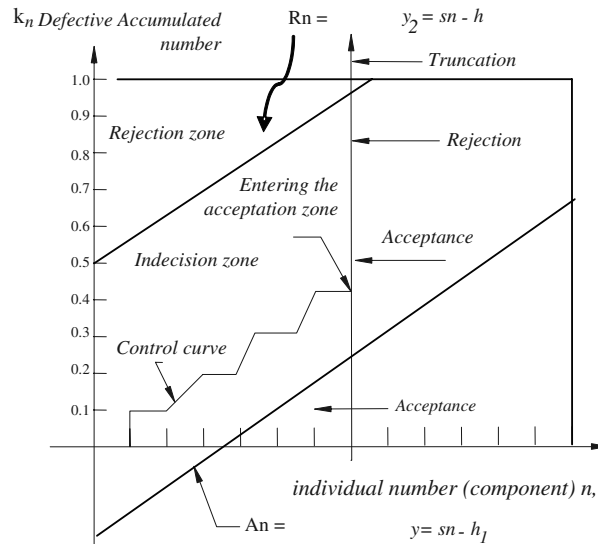


Figure 1.13. *Discrimination ratio on an efficiency curve*

To avoid the indecision zone, we stop control by truncating when two of three of the components in a simple plan are sampled. The constants s , h_1 , h_2 , function of κ are given by the literature formulas below:

$$\left\{ \begin{array}{l} \kappa = \text{Log} \left(\frac{p_2 \times (1-p_1)}{p_1 \times (1-p_2)} \right); s = \frac{\text{Log} \left(\frac{1-p_1}{1-p_2} \right)}{\kappa} \\ h_1 = \frac{\text{Log} \left(\frac{1-\alpha}{\beta} \right)}{\kappa} \quad ; \quad h_2 = \frac{\text{Log} \left(\frac{1-\beta}{\alpha} \right)}{\kappa} \end{array} \right\} \quad [1.18]$$

1.4.6. Wald plan case study

We programmed the plan in MathCAD, for which the results on a lot of 50 components are shown:

$$\left\{ \begin{array}{l} \alpha = 0.075 ; P_A = 0.075 ; \beta = 0.075 \text{ and } P_R = 0.15 \\ \text{Then : } (1-\alpha) = 0.925 \text{ and } (P_R - P_A) = 0.075 \end{array} \right\}$$

$$\left\{ \begin{array}{l} U = \ln \left(\frac{1-\alpha}{\beta} \right) = 2.512 ; V = \ln \left(\frac{1-\beta}{\alpha} \right) = 2.512 \\ X = \ln \left(\frac{P_R}{P_A} \right) = 0.693 ; Y = \ln \left(\frac{1-P_A}{P_A} \right) = 1.819 \end{array} \right\}$$

CASE 1.– Control of the non-conforming proportion of individuals

$$s = \left(\frac{Y}{X+Y} \right) = 0.724; H_A = \left(\frac{U}{X+Y} \right) = 1; H_R = \left(\frac{V}{X+Y} \right) = 1$$

CASE 2.– Control of the proportion of non-conforming individuals

$$s = \left(\frac{P_R - P_A}{X} \right) = 0.108; h_a = \left(\frac{U}{X} \right) = 3.624; h_r = \left(\frac{V}{X} \right) = 3.624$$

1.4.6.1. Plotting the limitation curves: $F(An)$ and $F(Rn)$

$$F(n) = \{s \times n - h_a\} \text{ and } f(n) = \{s \times n + h_r\} \quad [11.19]$$

DECISION.– We reject the lot if $D_n \geq F(n)$ or we accept the lot if $D_n \leq F(n)$.

We programmed our experimental data based on the relations given previously to find the data given in the control tables.

For the control of the average number of non-conforming characteristics per individual component see tables in the Appendix, under Control.

We applied the An and Rn relations and find the results given in the table below:

n	An	Rn	Individual results	Cumulative results	Decision (C = continue and A = accept)	n	An	Rn	Individual results	Cumulative results	Decision (C = continue and A = accept)
1		2	0	0	C	26	0	5	0	2	C
2		3	0	0	C	27	1	5	1	2	C
3		3	0	0	C	28	1	5	0	2	C
4		3	0	0	C	29	1	5	0	2	C
5		3	0	0	C	30	1	5	0	2	C
6		3	0	0	C	31	1	5	0	2	C
7		3	0	0	C	32	1	5	0	2	C
8		3	1	1	C	33	1	5	0	2	C
9		3	0	1	C	34	1	6	0	2	C
10		3	0	1	C	35	1	6	0	2	C
11		3	0	1	C	36	1	6	0	2	C
12		4	0	1	C	37	1	6	0	2	C
13		4	0	1	C	38	2	6	1	2	C
14		4	1	1	C	39	2	6	0	3	C
15		4	0	2	C	40	3	7	0	3	C
16	0	4	0	2	C	41	3	7	0	3	C
17	0	4	0	2	C	42	3	7	0	3	C
18	0	4	0	2	C	43	3	7	0	3	C
19	0	4	0	2	C	44	3	7	0	3	A
20	0	4	0	2	C	45	3	7	0	3	Stop
21	0	4	0	2	C	46	3	7	0	3	Stop
22	0	4	0	2	C	47	3	7	0	3	Stop
23	0	5	0	2	C	48	3	7	0	3	Stop
24	0	5	0	2	C	49	3	7	0	3	Stop

Table 1.8. Control of the average number of non-conforming characters per individual

1.4.6.2. Graph curves and discussion of the calculation results

The graphs below clearly show the limits framing the control function, as per the Wald plan.

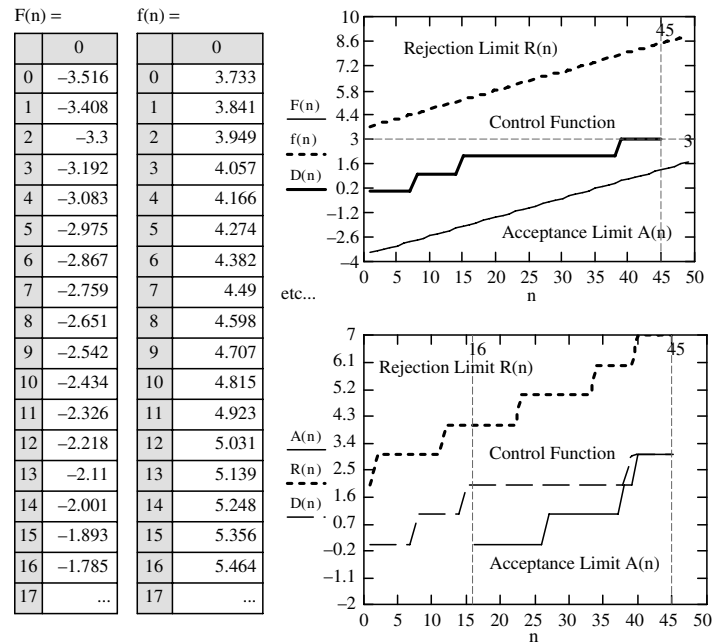


Figure 1.14. Case study of a Wald plan

The efficiency curve that shows our results is the following:

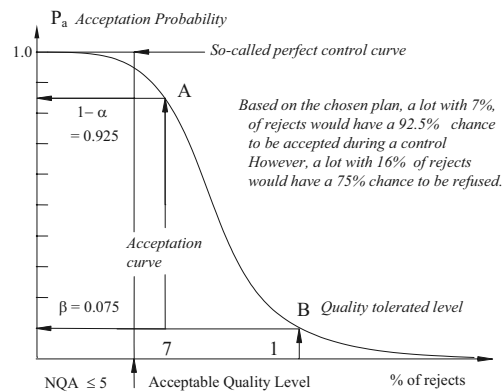


Figure 1.15. Efficiency curve linked to specialized tables (see <http://www.sqconline.com/about-acceptance-sampling>; <http://www.sqconline.com/control-charts>)

1.4.7. Average long-term controlled quantity

In order to manage the “workforce”, it is common practice (especially in the United States) to know the costs of the control. It was then suggested that quantifying the mean control per lot can be quantified as follows:

$$\ell = n + (1 - P_a) \times (N - n) \quad [1.20]$$

- P_a is the likelihood of acceptance;
- N is the sample size and n the lot's size.

1.4.7.1. Plans and inspection tables for Dodge–Romig sampling plans

The inspection tables and the main norms governing sampling plans are given in the Appendix, under Quality Control.

- ISO 2859 – ANSI/ASQ Z1.4
- Dodge–Romig (see index and Weblink)
- ISO 3951 – ANSI/ASQC Z1.9
- MIL-STD-1916

Controlling vast quantities is very demanding in time and resources. Dodge and Romig, two American engineers (*Bell Telephone Co.*), have deduced tables regarding this, which can be found on the Web. The main goal of the Dodge–Romig tables [DOD 59], given in the Appendix, is the minimization of the total inspection of long-term control of average quality, including screening. These useful tables can help find simple sampling and double sampling plans:

- Simple sampling tables based on the limiting tolerable quality level (LTPD);
- Double sampling tables based on the limiting tolerable quality level (LTPD);
- Sampling tables based on the average outgoing quality level's limit (AOQL).

The plan's first two groups are tabulated by limiting tolerance percent defective (LTPD). The latter is at 0.5%, 1.0%, 2.0%, 3.0%, 4.0%, 5%, 7.0% and 10.0% with a consumer risk of 0.10. The last two plan groups are tabulated based on their AOQL at 0.1%, 0.25%, 0.5%, 0.75%, 1.0%, 1.5%, 2.0%, 2.5%, 3.0%, 4.0%, 5.0%, 7.0% and 10.0% (see tables in the Appendix)

Of these three plan types, double AOQL sampling has the largest acceptance. The first two plans give the considerable guarantee that individual inferior lots will be rejected. These plans are applicable in cases where the lots keep their identity

after inspection (large quantity expedited during inspection). The last two plans (the AOQL plans) give the long-term limit on poor quality, but do not maintain the uniform guarantee that individual low-quality lots will be rejected. The various plans given in the Dodge–Romig table for a simple sampling plan (LTPD = 3%) give the same quality protection as that measured by tolerance.

This table is easy to use: for each lot, a sample (n) must be inspected. If it contains defective pieces $\leq c$, the lot is accepted; otherwise, it is rejected. When rejected, the entire lot must be inspected. When the average percentage of defectives of some lots is known and no process changes are expected, the plan must be chosen in agreement with the column with the same process average. If, for example, the inspected article had reached an average of 0.7% defectives in the past and no changes in the process are expected, we can predict the minimum quantity of the inspection total if a plan was chosen from the columns 0.61–0.90.

The table for a double sampling plan (AOQL) allows for the random inspection of a first sample n_1 . If it contains no more than c_1 defectives, the entire lot is accepted. If the defectives are greater than c_1 , a second random article sample n_2 is taken. If the total number of defectives found in the first and second samples is less than the acceptance number given for the second sample (c_2), the entire lot is accepted or it is rejected and the whole lot is inspected (“ALL”). Table 1.9 shows a simple sampling plan (LTP = 3%).

The so-called “*rectifying* inspection” control consists of controlling 100% of each rejected lot, followed by replacing all defective items by items that conform to specifications. In this case, the proportion is of course smaller than it was before inspection because the defective items were replaced by good items. This proportion is also a function of (p), the proportion of defective items that existed before inspection. There are some online calculation Websites that can be used for direct calculations such as <http://www.aqlcalculator.com>. (see the appendix).

There are also Dodge–Romig tables for control by sampling (BETA): simple sampling for declared values of the AOQL. This application gives the sampling plan for a “rectifying inspection by attributes”. For 100,000 points with an AOQL of 1.00% and an average process value of 0.25%, an example of inspection plan is as follows:

255 articles. If the number of non-conforming elements is as follows:

- 0 → accept the lot;
- 1–4 → rectify the non-conforming element(s) and accept the lot;
- 5 or more → inspect the entire lot and rectify all non-conforming elements.

p_t (quality limit for a single *lot*) for this process is 3.10% ($p\% = 100p$).

Process Average %			0–0.03		0.04–0.30		0.31–0.60		0.61–0.90		0.91–1.20		1.21–1.50				
Lot size	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %		
1–40	All	0	0	All	0	0.08	All	0	0	All	0	0	All	0	0		
41–55	40	0	0.18	40	0	0.18	40	0	0.18	40	0	0.18	40	0	0.18		
56–100	55	0	0.30	55	0	0.30	55	0	0.30	55	0	0.30	55	0	0.30		
101–200	65	0	0.38	65	0	0.38	65	0	0.38	65	0	0.38	65	0	0.38		
201–300	70	0	0.40	70	0	0.40	110	1	0.48	110	1	0.48	110	1	0.48		
301–400	70	0	0.43	70	0	0.43	115	1	0.52	115	1	0.52	115	2	0.54		
401–500	70	0	0.43	70	0	0.45	120	1	0.53	160	2	0.58	160	2	0.58		
501–600	75	75	0.43	75	0	0.45	120	1	0.56	160	2	0.63	200	3	0.65		
6001–800	75	75	0.47	125	1	0.57	125	1	0.57	165	2	0.66	205	4	0.74		
801–1,000	75	75	0.48	125	1	0.59	170	2	0.67	210	3	0.73	250	4	0.76		
1,001–2,000	75	75	0.63	130	1	0.60	175	2	0.72	260	4	0.85	300	5	0.90		
2,001–3,000	75	75	0.63	130	1	0.62	220	CO	0.82	300	5	0.95	385	7	1.0		
3,001–4,000	130	130	0.63	175	2	0.75	220	3	0.84	305	5	0.96	425	8	1.0		
4,001–5,000	130	130	0.64	175	2	0.76	260	4	0.91	345	6	1.0	465	9	1.1		
5,001–7,000	130	130	0.64	175	2	0.76	265	4	0.92	390	7	1.1	505	10	1.2		
7,001–10,000	130	130	0.64	175	2	0.77	265	4	0.93	390	7	1.1	550	11	1.2		
10,001–20,000	130	130	0.64	175	2	0.78	305	5	1.0	430	8	1.2	630	13	1.3		
20,001–50,000	130	130	0.65	225	3	0.86	350	6	1.1	520	10	1.2	750	16	1.4		
50,001–100,000	130	130	0.65	265	2	0.96	390	7	1.1	590	12	1.3	830	18	1.5		
n = sample size, c = acceptance number (allowable defects)												in a AOQL sample				average outgoing quality limit	

Table 1.9. Dodge–Romig table (see Table A.25 in Appendix); source: <http://what-when-how.com/metrology/dodge-romig-tables-metrology>

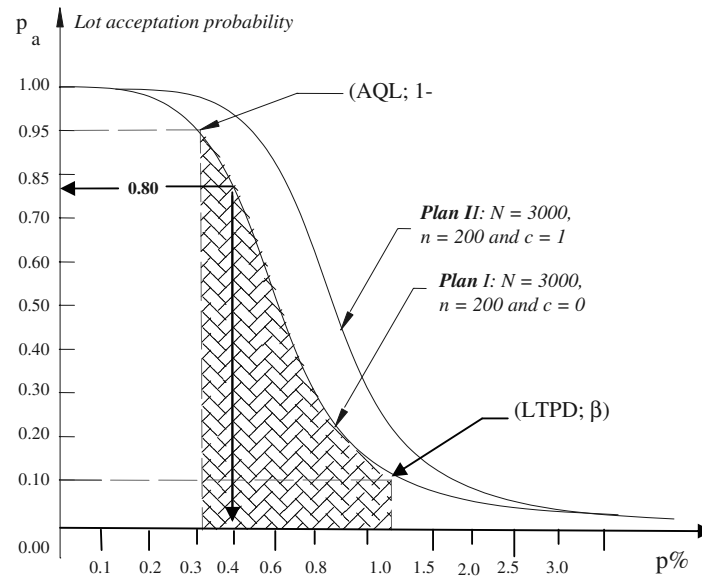


Figure 1.16. Average percentage of items outside tolerance limits in the set of deliveries

These tables show the most economical sampling plans. In metrology (control), the least possible items should be manipulated. These tables allow us to read the AOQL directly for each plan. The item's buyer is then well informed on the outgoing quality level. When a lot buyer (of machined items, for example) knows nothing of the number of items that do *not conform to the design's tolerance*, they will apply one of the plans in the Dodge–Romig table. For example, if the delivery contained 5% of defective items (items outside tolerance limits), nine boxes out of ten would be refused. Because the refused boxes would have to undergo total, 100% verification, we see why one should choose the appropriate line in the table.

In the literature, Dodge and Romig have provided eight tables relative to each of the limits for which nine out of ten boxes would be refused: 0.5%, 1.0%, 2.0%, 3.0%, 4.0%, 5.0%, 7.0% and 10.0%. The purchaser will thus choose the table that suits them. Here is how we read the Dodge–Romig table: Let us assume that they choose that nine boxes out of ten will be refused. If the delivery contained 3% defectives, they would then have to use the 3% table given in the Dodge–Romig table below (Table 1.10). A clear and concise idea of the percentage of defective items in a delivery can thus be made very quickly. All that needs to be done is to choose the corresponding column in the table. Thus the question arises, “Why these intellectual gymnastics?”. Because this is the minimum handling required, leading to the least cost in terms of time and resources.

Process average %	0–0.03			0.04–0.30			0.31–0.60			0.61–0.90			0.91–1.20			1.21–1.50		
	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %
Lot size	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
301–400	70	0	0.43	70	0	0.43	115	1	0.52	115	1	0.52	115	1	0.52	115	2	0.54
401–500	70	0	0.43	70	0	0.45	120	1	0.53	120	1	0.53	160	2	0.58	160	2	0.58
501–600	75	75	0.43	75	0	0.45	120	1	0.56	160	2	0.63	160	2	0.63	200	3	0.65
6001–800	75	75	0.47	125	1	0.57	125	1	0.57	165	2	0.66	205	CO	0.71	240	4	0.74
801–1,000	75	75	0.48	125	1	0.59	170	2	0.67	210	3	0.73	250	4	0.76	290	5	0.78
1,001–2,000	75	75	0.63	130	1	0.60	175	2	0.72	260	4	0.85	300	5	0.90	380	7	0.95
2,001–3,000	75	75	0.63	130	1	0.62	220	CO	0.82	300	5	0.95	385	7	1.0	460	9	1.2
3,001–4,000	130	130	0.63	175	2	0.75	220	3	0.84	305	5	0.96	425	8	1.0	540	11	1.2
4,001–5,000	130	130	0.64	175	2	0.76	260	4	0.91	345	6	1.0	465	9	1.1	620	13	1.2
n = sample size; c = acceptable number of defects	In a “AOQL” sample																average outgoing quality limit=AOQL	

Table 1.10. Table of values: n , c and $AOQL$ as a function of the lot to control
(source: <http://what-when-how.com/metrology/dodge-romig-tables-metrology/>)

A concrete example: consider a box with 1,000 items machined by Blaireau LeNain. We will read that the sample size must be of the order of $n = 75$ items. Within this “mysterious” box, we will find either 0, 1 or 2 items outside tolerance limits (defectives) for which p_a (or p_t for a tolerated percentage) = 8.1 (see Dodge–Romig table above). Starting from a percentage of 8.1% of the delivery (from the machinist Blaireau LeNain) we will then refuse, based on this plan, nine out of ten boxes. Furthermore, Dodge–Romig have also created 13 different tables for each of the following AOQL values: 0.1%, 0.25%, 0.5%, 0.75%, 1.0%, 1.5%, 2.0%, 2.5%, 3.0%, 4.0%, 5.0%, 7.0% and 10.0%. Evidently, starting from the “assumed” fact that operations happen on quantities of boxes (and thus machined items), which are very large in terms of production becomes very important.

In this example, we have often considered in terms of probabilities. *However, probabilities must not be confused with frequencies.* This “confusion” might be acceptable if we were considering very large numbers of items within these famous boxes. The law of large numbers would then apply. For an example on the efficiency curve (see Figure 1.8), we deliberately represented an acceptance probability $P_a = 8/10$ (0.80) for a box. We then deduce that eight of the ten boxes are accepted, hence the refusal of the remaining two boxes.

1.4.8. Case study of a simple sampling plan

We will solve a concrete problem of the descriptive diagram of the simple sampling plan. To this end, we took a sample of size n and counted the defectives (items rejected at machining). From this number, we will make a justified decision to accept or reject. The mathematical nature of the simple plan is in fact defined exhaustively by $f(p)$ where P is the lot acceptance probability (purchasers of our machined items) and p the quality of the lots controlled by the three-dimensional measurement machine from the metrology lab. On the basis of the binomial distribution, this function can be written:

$$P = \sum_{k=0}^{k=A} C_n^k \times p^k \times (1-p)^{n-k} \quad [1.21]$$

We used MathCAD to plot this binomial distribution curve:

$$P(\tau) \binom{20}{\tau} \times (0.05)^\tau \times (0.95)^{(20-\tau)} \text{ or } b(\tau, n, p) = \frac{20!}{\tau!(20-\tau)!} (0.05)^\tau (0.95)^{(20-\tau)}$$

Our simulation result is then plotted:

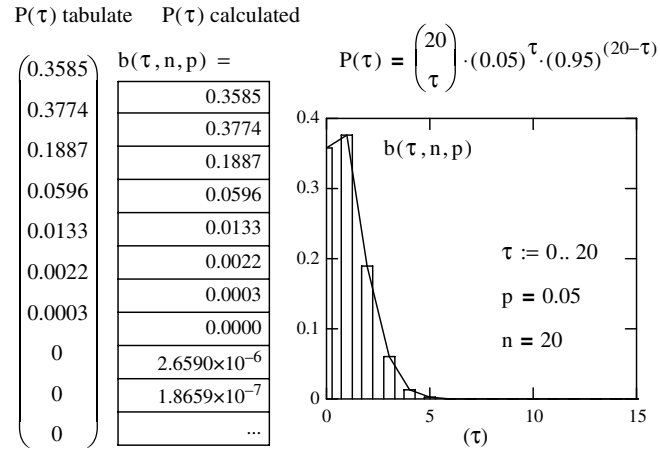


Figure 1.17. *Applied binomial distribution*

In practice, p is less than 10%. We will then use the Poisson approximation, for which the mathematical expression is given in the following section.

1.4.9. Poisson distribution of cumulative probabilities

We present the Poisson distribution in a manner similar to that used in quality control on three plans.

$$P(\lambda, k) = \sum_{k=0}^{k=A} \text{Exp}^{-np} \times \frac{(np)^k}{k!} \quad [1.22]$$

For $k = 0-18$, we used the following hypotheses:

- $n_1 = 60$; $\lambda_1 = 2.4$; $p_1 = \frac{\lambda_1}{n_1} = 4\% = 0.04$
- $n_2 = 90$; $\lambda_2 = 4.5$; $p_2 = \frac{\lambda_2}{n_2} = 4\% = 0.04$
- $n_3 = 150$; $\lambda_3 = 9$; $p_3 = \frac{\lambda_3}{n_3} = 6\% = 0.06$

The Poisson distribution was programmed, giving us the following results:

$$P(\lambda, k) = \left(\frac{(np)^k}{k!} \right) \times \text{Exp}^{-n.p} \quad [1.23]$$

Because the aim is to plot the three efficiency curves and comment on them, we used the MathCAD functions (which also coincide with our programmed functions), hence the $\text{ppois}(k, \lambda)$ syntax.

Pa1 =		Pa2 =		Pa3 =		p =	
	0		0		0		0
0	1	0	1	0	0.995	0	0
1	1	1	1	1	0.989	1	0.01
2	1	2	1	2	0.978	2	0.02
3	1	3	1	3	0.959	3	0.03
4	1	4	1	4	0.926	4	0.04
5	1	5	0.999	5	0.876	5	0.05
6	1	6	0.998	6	0.803	6	0.06
7	1	7	0.993	7	0.706	7	0.08
8	1	8	0.983	8	0.587	8	0.1
9	0.999	9	0.96	9	0.456	9	0.12
10	0.997	10	0.913	10	0.324	10	0.14
11	0.988	11	0.831	11	0.207	11	0.16
12	0.964	12	0.703	12	0.116	12	0.18
13	0.904	13	0.532	13	0.055	13	0.2
14	0.779	14	0.342	14	0.021	14	0.22
15	0.57	15	0.174	15	$6.232 \cdot 10^{-3}$	15	0.24
16	0.308	16	0.061	16	$1.234 \cdot 10^{-3}$	16	0.26
17	0.091	17	0.011	17	$1.234 \cdot 10^{-4}$	17	

We deliberately set the risks (less than 10% but of poor quality) for the three plans:

- $P_{a1} = \text{Plan}[n_1, \lambda_1, p_1] = \text{Plan}[60; 2.4; 4\%];$
- $P_{a2} = \text{Plan}[n_2, \lambda_2, p_2] = \text{Plan}[90; 4.5; 5\%];$
- $P_{a3} = \text{Plan}[n_3, \lambda_3, p_3] = \text{Plan}[150; 9.0; 6\%].$

Poisson tables A.12, A.13 (also A.16) for a confidence interval of 95%:

n	P in %	Upper limit	Lower limit
80	4% = 0.04	13.8	1.8
90	5% = 0.05	11.7	1.5
150	6% = 0.06	11.1	2.8

Reading and interpreting our results:

– *Plan I* (P_{a1}): if we see a defectiveness percentage of $p = 4\%$ of $n = 80$ machined items, we can state (with a risk of 5%) that the lot's percentage will lie between 1.8% and 13.8%.

– *Plan II* (P_{a2}): if we see a defectiveness percentage of $p = 5\%$ of $n = 90$ machined items, we can state (with a risk of 5%) that the lot's percentage will lie between 1.5% and 11.7%.

– *Plan II* (P_{a3}): if we see a defectiveness percentage of $p = 4\%$ of $n = 150$ machined items, we can state (with a risk of 5%) that the lot's percentage will lie between 2.8% and 11.1%.

We should mention that our choices conveniently gave us the exact values given in the range Poisson Table A.16 (See example presented in the Appendix). If the value (say, 5.25%) were not in the table, we would have proceeded by interpolation. Considering the data treated here, the decision to take in consequence would be:

– $P_a = 0.95$ (given), that is $\alpha = 1 - 0.95 = 0.05 = 5\%$ (acceptation);

– $P_R = 0.165$ (given), with $\beta = 0.18$ (plan rejection).

We can thus deduce that sensible risks would be as follows: $\alpha = 0.05$ and 0.09 . The client must now decide, knowing the risk sensibility $(P_a, 1 - \alpha) = (P_R, \beta) \rightarrow (0.95, 0.05) = (0.165, 0.18)$. We then plotted $P_a = f(p)$ as the expression for the efficiency curve. Our results are shown in Figure 1.8.

Efficiency curve “discrimination”: the efficiency curve of a simple sampling plan depends on n , the sample size and A the number of acceptations. For the same value of A , the higher the value of n , the faster the efficiency curve decreases. This allows us to have a better selectivity in control. This is also why the decision rules must be well thought out.

We made many simulations with the previous curve before getting there. Using software such as MathCAD, Mathematica, MathLAB, simulating many cases is an easy task. This is what we have done here. For a same-sized sample, increasing A shifts the efficiency curve farther from the origin. It also flattens, showing that the control is “relaxed” due to lower selectivity.

1.5. Theoretical basics of control by measurement

The controlled characteristic can be measured; it follows a normal or near-normal distribution. There are one or two tolerance limits allowing us to class each individual as conforming or non-conforming. The decision at the end of the test is taken on the basis of the number of non-conforming items by way of the sample average. Knowing the average's distribution allows us to build the efficiency curve.

Sampling rules: Canadian norms define three types of control:

- *Normal:* used at the beginning of the control of a series of lots, or an isolated lot.
- *Reinforced:* less economical, but offers better client protection (smaller β). It is imposed by the client when normal control leaves doubts regarding quality.
- *Reduced:* more economical. It is decided by the client when normal control seems to indicate that the quality is satisfying.

The norm gives empirical rules to pass between these three types of control. The corresponding tables for a *simple sampling plan* give the reinforced and reduced plans of normal control. The notion of control levels intervenes when the plan is chosen on the basis of the size of the lots. The norm defines three levels of control:

- *Level I:* reserved for particularly difficult-to-control criteria;
- *Level II:* the standard level, usually chosen;
- *Level III:* reserved for particularly easy-to-control criteria.

In addition to these levels, the norm has two special levels: S_3 and S_4 , which are adopted only if imperious reasons force us to take only small samples. We will not discuss this subject in this chapter because the controlled quantities are often small and the efficiency mediocre. When the size of the lots to be delivered (see the table in the Appendix) is to be determined, it can be defined on the basis of the level of sampling (presumably level II) and the sample size. The table (see corresponding Table A.12. *Distribution de Poisson. (n.p/c)* see Weblink: <http://www.itl.nist.gov/div898/handbook> see also www.aqlcalculator.com) gives a code letter that allows us to determine a range in which the lot size could fit from the corresponding table. In most cases, the lot size is fixed. The table (see the Appendix) gives a code letter as a function of the level of sampling (presumably level II) and the lot size. The table (see the Appendix) then gives the size of the sample to be taken.

Known or unknown standard deviation:

- “ σ ” method: used when the standard deviation is known;
- “ s ” method: used when the standard deviation is unknown. During control, we use the estimated σ calculated for each sample.

In both cases, we can build:

- an “average control chart” where the acceptance limits defined by the plan would appear;
- a standard deviation or range control chart where the classical control limits would appear (see production control).

The first chart will allow us to accept or refuse the lot, based on the average’s position with respect to the acceptance limits (TI). The second chart will allow us to do the following:

- if σ is known, to verify the dispersion’s stability. If it is not known, we must use the “s” method;
- if σ is unknown, we used the “ σ ” method if the dispersion has been stable for a certain amount of time.

Only one or two tolerance limits: the decision is taken on the basis of the sample’s average compared to a function criterion of the tolerances T_i , T_s and a constant K which we find in the following tables (see the Appendix).

Tolerance limits	Method	Formula
< tolerance limit	σ	$\bar{x} \geq T_i + K\sigma$
Lot accepted if	s	$\bar{x} \geq T_i + Ks$
> tolerance limit	σ	$\bar{x} \leq T_s - K\sigma$
Lot accepted if	s	$\bar{x} \leq T_s - Ks$
Two tolerance limits	σ	$T_i + K\sigma \leq \bar{x} \leq T_s - K\sigma$
Lot accepted if	s	$T_i + Ks \leq \bar{x} \leq T_s - Ks$

Table 1.11. *Decision: one or two tolerance limits*

The two tolerance limits are treated as separate. The norm considers an additional case in which both limits would be combined, that is the quality criterion (e.g. NQA) would be fixed by the set of both limits, indistinctly from one another. We note that control cannot be undertaken if the standard deviation is sufficiently less than the tolerance interval (TI). The norm gives maximum values as a function of NQA.

1.5.1. Choosing a simple sampling plan

In the most common cases, the lot size is fixed and we choose an NQA value. For the usual level II, Tables A.20, A.21, A.22 and A.23 give a code letter. See also this Weblink: www.aqlcalculator.com. For this code letter, Tables II or III give a

plan with the constant allowing us to calculate the acceptance or rejection criteria as well as the p_{95} and p_{10} values, which correspond to the supplier and client risks.

– Unknown standard deviation: “s” method – see corresponding table (Appendix/Weblink: <http://www.itl.nist.gov/div898/handbook>). See also this Weblink: www.aqlcalculator.com);

– Known standard deviation: “σ” method – see corresponding table (Appendix/Weblink: <http://www.itl.nist.gov/div898/handbook>). See also this Weblink: www.aqlcalculator.com).

Other ways of choosing a plan can be considered on the basis of the following preferred parameters: NQA, lot size, sample size, p_{10} , p_{95} , AOQL, level of control. Depending on the case, using other tables might be necessary.

1.5.2. Choosing a simple or double sampling plan

The following case arises when the standard deviation is known and when a single tolerance limit is considered (e.g. Upper limit or lower limit considered in the study, not both tolerances). We need to separately control both limits (upper and lower). The lot will be accepted only if it is accepted for both limits separately. For progressive control, a decision is taken after controlling each individual. We build the plan by giving ourselves two points on the efficiency curve: (p_1, α) and (p_2, β) . The decision rule based on the Wald sequential test theory is as follows: with x_j the measurement done on individual j , we write $y_j = (x_j - T_l)$ for a lower limit and $y_j = (T_s - x_j)$ for an upper limit.

Decision	Formula
Lot accepted if	$\sum_{j=1}^n y_j \geq (h_1 + sn)$
Lot refused if	$\sum_{j=1}^n y_j \leq (sn - h_2)$
Continue sampling if	$(sn - h_2) < \sum_{j=1}^n y_j < (sn + h_1)$

Table 1.12. Double or multiple sampling plan

This decision rule can be graphically represented, such as progressive control by counting. To avoid staying in the indecision zone indefinitely, we stop control by

truncating when we have, say, sampled two-thirds of the individuals expected in a simple plan. The decision is taken by looking at the closest line. The constants s , h_1 , h_2 , functions of k , are given by the following formulas:

$$\left\{ \begin{array}{l} \kappa = (u_1 - p_1) - (u_1 + p_2); \quad s = \frac{\sigma}{2}(u_1 - p_1) + (u_1 - p_2) \\ h_1 = \frac{\text{Ln}\left(\frac{1-\alpha}{\beta}\right)}{\kappa} \times \sigma; \quad h_2 = \frac{\text{Log}\left(\frac{1-\beta}{\alpha}\right)}{\kappa} \times \sigma \end{array} \right\} \quad [1.24]$$

$U_{(1-p)}$ is the fractile of order $(1-p)$ of the centered, reduced and normal variable

OC Curve	ARL Curve
– Sample size as a function of the risks inherent to the factors: the clients, suppliers, target shifts, etc.	– Average number of samples to detect a risk-based misalignment: customers, suppliers, shift the target, etc.
– Control curves for μ (average), R (range) and σ (standard deviation)	– Common comparison tools between curves: μ , R , Exponentially Weighted Moving Average (EWMA), CUMulative SUM (CUSUM)... – Sampling frequency estimate
Statistical Process Control (SPC) devices have efficiency curves and Operational Period Averages curves (ARL Curve). The efficiency curve relates to the sample size, the ARL curve is used to compare the performances of various control curves.	

Table 1.13. Comparison between efficiency curves and ARL curves

1.6. Analysis of process capability

Among control tools, statistical process control (SPC) is used more and more commonly. Suited to workstation operators, it allows us to tend toward zero flaws. SPC is based on three fundamental principles:

- priority is given to prevention (intervention before producing rejects);
- referring to the process as it functions (machine qualification);
- production responsibility and active operator participation.

Interpreting data is essential and should be done extremely rigorously. There are two main data categories:

- (discontinuous) measurement data such as length, mass, temperature which can all take any value in a given interval;
- countable (discrete) data such as the number of flaws, the percent of defectives.

Process: A combination of labor, materials, machines, methods, metrology = measurements with uncertainty. For example, X represents the quality characteristic.

Capability: “Aptitude of a production process or a means to respect specifications” (ISO, ACNOR, CSA, Canada).

A process is declared “fit” if it has shown (for the selected characteristics) that it was able to produce during a sufficiently long period with a theoretical rate of non-conformities (TI) under the contractual or corporation’s internal standards. This rate is often fixed at 0.27 (per the 6σ method):

– *Histogram*: $n = 50$ – 100 or more $\rightarrow$ advantage: immediate view of the process’ performance;

– *Henry line*: $\rightarrow$ advantage: no division into classes.

When implementing SPC (and after verifying, when controlling by measurements, that the distribution was normal), we must compare the machine’s possibilities with the specified tolerances. The indicators used are as follows:

– C_p : process’ *capability index*;

– C_{pk} : process’ *capability coefficient (maladjustment indicator)*.

Formula to calculate the instantaneous capability index:

$$C_p = \left\{ \frac{T_{upper} - T_{lower}}{6 \times \sigma_{inst}} \right\} \quad [1.25]$$

Generally, $C_p = 1.33$ is used as a capability limit.

– T_{upper} : upper tolerance and T_{lower} : lower tolerance;

– σ_i : instantaneous standard deviation;

– $T_{upper} - T_{lower} = TI$.

Steps taken to implement control by measurement curves:

– *Henry line*: in the Appendix, there is an example of the gaussio-arithmetic graph to plot. (The Henry line is given in Chapters 1 and 2, Volume II).

– The Henry line is a graphical method used to verify how well measured data fit a normal (Gaussian) distribution.

– The Pearson (χ^2) test, a general test to verify how well measured data fit a given distribution.

- The Shapiro–Wilk test, used specifically to verify a distribution’s normality.
- Kolmogorov–Smirnov (KS) test, used to verify how well a distribution fits a theoretical distribution.

Steps taken to implement control by measurement curves:

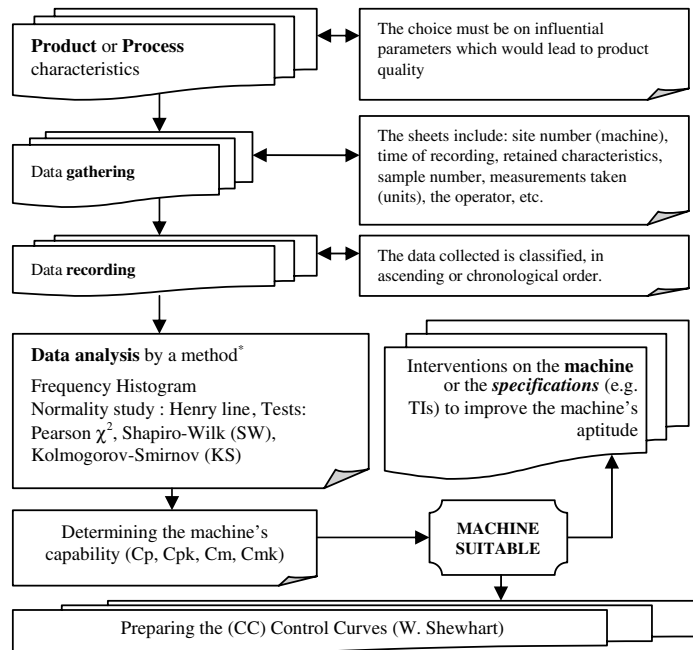


Figure 1.18. Implementation of control curves

1.6.1. Capability indices for a normally distributed variable: $X(\mu, \sigma)$

We present the calculation method of capabilities applied to a normal law (Gauss) known as $X(\mu, \sigma)$.

Statement:

- T_l lower specification limit;
- T_u upper specification limit;
- The X values outside the limits are called “non-conforming”;
- The rate of non-conformity is minimized if $\mu = (T_{\text{lower}} + T_{\text{upper}})/2$ (centered process).

$$\left\{ \begin{array}{l} C_p = \left(\frac{T_{upper} - T_{lower}}{6\sigma_{inst}} \right) \Rightarrow \text{two specifications, } \{T_u \text{ et } T_l\} \\ C_p = \left(\frac{T_{upper} - \mu}{3\sigma} \right) \Rightarrow \text{Only one specification, } T_u \\ C_p = \left(\frac{\mu - T_{lower}}{3\sigma} \right) \Rightarrow \text{Only one specification, } T_l \end{array} \right\} \quad [1.26]$$

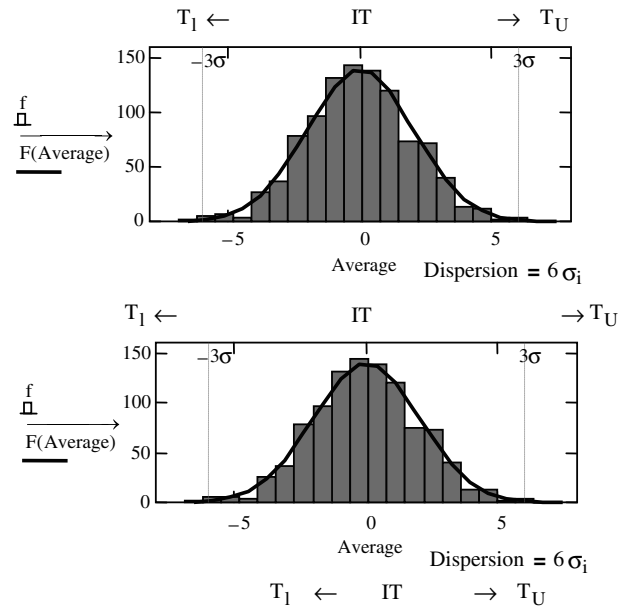


Figure 1.19. Capability index illustration (Gauss' Law)

1.6.1.1. Calculation of the capability coefficient C_p

The classical relations used in mechanical engineering, especially in the automotive industry (Ford, Chrysler) can be written as follows:

$$C_p = \min \left\{ \left(\frac{T_{Upper} - \bar{\bar{x}}}{3 \times \sigma_{inst}} \right) \text{ or } \left(\frac{\bar{\bar{x}} - T_{Lower}}{3 \times \sigma_{inst}} \right) \right\} \quad [1.27]$$

To be capable, a process must not produce defective items. The criterion to remember is C_{pk} , which includes both the intrinsic capability and maladjustment.

A process is capable if its C_{pk} is greater than **1.33**.

If that is true, $C_{pmin} = D/3 \times \sigma_{inst}$, where D represents the dispersion.

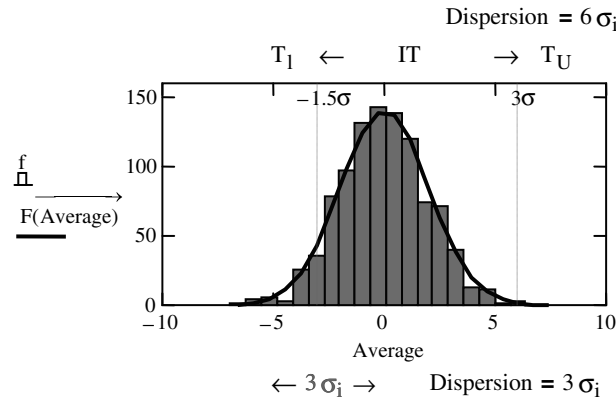


Figure 1.20. Capability index (dispersion D)

$C_p = 1$ does not guarantee that there can only be 0.27% non-conformities. The desired NC (non-conformity) proportion will never be less than 0.27% (X follows a normal distribution). The large C_p values do not guarantee acceptability without information regarding the process' centered value. Automobile manufacturers like Ford, for example, suggest $C_p = 1.67$ and $C_{pk} = 1.33$. The minimum values recommended for C_p by Montgomery [MON 00] are given in Table 1.14.

Submitted process	2 specifications	1 specification
Existing process	1.33	1.25
New process	1.50	1.45
Security or critical parameter, existing process	1.50	1.45
Security or critical parameter, new process	1.67	1.60

Table 1.14. Minimum recommended values for C_p

On the basis of the ISO 8258 norm on control curves, we set them as shown in Table 1.15.

Designation	Meaning of the capability indicator
Performance indices. Delivered quality indicators P_p and P_{pk} .	Denotes the process' capacity to produce in the long term. Includes the effects of assignable causes.
Intrinsic process capability C_p and C_{pk} .	Denotes the process' possibilities without assignable causes.
Short-term capability machine capability C_m and C_{mk} .	Denotes the capability of the means of production from a stack test.
Capability C_{pm} .	Global index including the average and standard deviation.

Table 1.15. C_p values, ISO 8258 norm for control curves

C_p	Number of controlled values										
	10	20	30	40	50	75	100	125	150	200	250
1.00	1.65	1.37	1.28	1.23	1.20	1.16	1.13	1.12	1.11	1.09	1.08
1.33	2.19	1.82	1.70	1.64	1.60	1.54	1.51	1.49	1.47	1.45	1.44
1.50	2.47	2.06	1.92	1.85	1.80	1.74	1.70	1.68	1.66	1.64	1.62
1.67	2.75	2.29	2.14	2.06	2.01	1.93	1.89	1.87	1.85	1.82	1.80
2.00	3.29	2.74	2.56	2.46	2.40	2.32	2.27	2.24	2.21	2.18	2.16

Table 1.16. C_{pk} table

C_p	Number of controlled values										
	10	20	30	40	50	75	100	125	150	200	250
1.00	1.42	1.29	1.24	1.21	1.18	1.15	1.13	1.12	1.11	1.09	1.08
1.33	1.81	1.66	1.60	1.56	1.54	1.50	1.48	1.46	1.45	1.43	1.42
1.50	2.01	1.85	1.78	1.74	1.72	1.68	1.65	1.64	1.63	1.61	1.60
1.67	2.20	2.04	1.97	1.93	1.90	1.86	1.83	1.81	1.80	1.78	1.7
2.00	2.58	2.40	2.32	2.28	2.25	2.20	2.17	2.16	2.14	2.12	2.11

Table 1.17. C_{pk} table (source: Ford SPC guide, version 95)

C_p estimation $\rightarrow$ Consider $x_1, \dots, x_n$ with $n = 50$ to 100 or more

$$\hat{\sigma} = s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad [1.28]$$

Excel can easily be used for serious calculations. Here is a simple sample calculation of the standard deviation $s = \text{STDEV}(\text{number1}, \text{number2}, \text{etc.})$

If X is normally distributed then: $\chi_{n-1}^2 \approx s^2(n-1)/\sigma^2$ and the $(1 - \alpha)$ th confidence interval for 1σ can be written as:

$$\left\{ s \left(\sqrt{\frac{(n-1)}{\chi_{(1-\alpha/2)}^2}} \right) \leq \sigma \leq s \left(\sqrt{\frac{(n-1)}{\chi_{\alpha/2}^2}} \right) \right\} \quad [1.29]$$

Using Excel is easy here, too. Chi-square.INVERSE (p, $n-1$ degrees of freedom) = $k/p(x > k) = p$.

The $(1 - \alpha)$ th confidence interval for C_p can be written as:

$$\left\{ \hat{C}_p \left(\frac{\chi_{\alpha/2}^2}{\sqrt{(n-1)}} \right) < C_p \leq \hat{C}_p \left(\frac{\chi_{1-\alpha/2}^2}{\sqrt{(n-1)}} \right) \right\} \quad [1.30]$$

C_{pk} index: C_p does not require knowledge of the average to be evaluated. C_{pk} is thus introduced to give the mean some influence.

$$\left\{ \begin{array}{l} C_{pk} = \min \left\{ C_p^{T_l}, C_p^{T_u} \right\} \Rightarrow \text{Two specifications, } \{T_U \text{ et } T_l\} \\ C_{pk}^{T_s} = \min \left\{ \frac{T_U - \mu}{3 \times \sigma} \right\} \Rightarrow \text{Only one specification, } T_{\text{Upper}} \\ C_{pk}^{T_i} = \min \left\{ \frac{\mu - T_l}{3 \times \sigma} \right\} \Rightarrow \text{Only one specification, } T_{\text{lower}} \end{array} \right\} \quad [1.31]$$

- if C_p and $C_{pk} > 1$ then the process is said to be both capable and well centered;
- if $C_p > 1$ and $C_{pk} < 1$ then the process is said to be capable and poorly centered;
- if $C_p < 1$ then the process is said to be non-capable.

1.6.2. Machine capability C_m and C_{mk}

C_m and C_{mk} calculations are done from 50 items manufactured consecutively and without changing settings. When an item goes through various machines (phases and subphases, and milling), variances are added. In addition, requirements follow for C_m and C_p . C_m must be greater than 1.33 (four times the standard deviation).

- If C_m and $C_{mk} > 1$ then the process is said to be both capable and well – centered.

- If $C_m > 1$ and $C_{mk} < 1$ then the process is said to be capable and poorly-centered.
- If $C_m < 1$ then the process is said to be non-capable.

On the basis of the values of C_m or C_{mk} , we generally class capability as follows:

- $C < 0.67 \rightarrow$ Very poor;
- $0.67 < C < 1 \rightarrow$ Poor;
- $1 < C < 1.33 \rightarrow$ Very average;
- $1.33 < C < 1.67 \rightarrow$ Average;
- $1.67 < C < 2 \rightarrow$ Good;
- $C > 2 \rightarrow$ Very good.

The process is said to be capable if C_{pk} is greater than 1: we accept that three items in 1,000 can be out of tolerance. If the process is not considered capable, then we must improve it.

1.7. Capability for a non-normal distribution

The distribution of variables can sometimes be non-Gaussian (non-normal). Verifying normality beforehand is extremely important (we will show a verification case later). Assuming that all distributions are normal is incorrect.

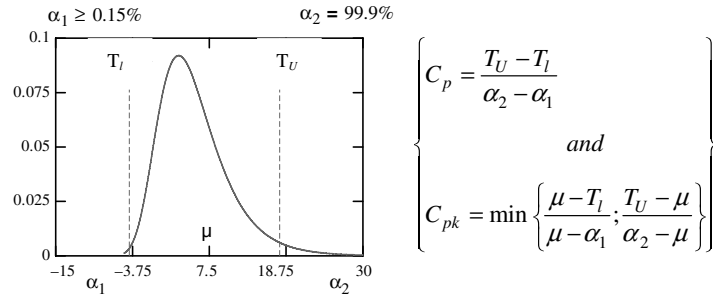


Figure 1.21. Asymmetrical law (see kurtosis)

Before calculating C_p and C_{pk} , we must question the lack of assignable causes, for example, a change of lot. If the law is strongly asymmetrical, the literature recommends using the median to calculate C_{pk} . Oftentimes, after a serious capability study, we must confront the following duality:

1) The machine (or process) is capable. It would be professional to proceed to further verifications (this is costly!); otherwise, control charts (Shewhart) should be used with a logbook.

2) The machine (or process) is not capable. The reasons for this must be found: variances or gaps. We then appeal to experience plans to proceed to testing in an easily interpretable fashion.

1.8. Control by measurement charts

Control charts are a good tool to monitor quality regularly. There are two kinds of control charts, depending on the number of characteristics of the monitored process. *Univariate* control, which characterizes quality, and *multivariate* control, the statistics of which are characterized by more than one criterion. The graph contains a central line that represents the average value of the control process. Two other horizontal lines, called upper control limit (UCL) and lower control limit (LCL) are also shown on the graph. These control limits are chosen such that almost all data points will *fall within* these limits as long as the process remains under control, as shown below.

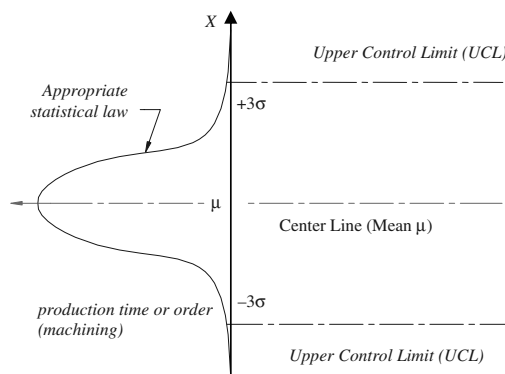


Figure 1.22. Basic control curve representation graph

In mechanical engineering (and therefore our case studies), the principles of control by measurement charts are based on studying the variability of a machined score. Among the causes that can justify the variation around the *true value* (nominal score), which is now our target, there are many factors that induce “fluctuations”, sometimes dependent on the machine. This is not always true, and can be simplistic: the fluctuations have origins as varied as they are diverse.

The so-called assignable causes are certainly easily identified, as they are physical and often fairly obvious, for example loose stop, worn tool (due to lack of

sharpening), untuned chuck wagon causing considerable dispersions. The causes are easy to identify, making derivation, gap and dispersion corrections in order. All this influences the average's tendency to go toward the true value (average nominal score, as tolerated by the research department, in design). In machining workshops, we say that the production is "controlled", referring to the central tendency and the dispersion of the machined scores staying "stable".

If we were waiting for the control from the metrology lab, that is after machining, it would be too late to control the "production line flaws". We then see why monitoring and controlling the machining is inevitable in order to detect abnormal process fluctuations "in time"

- processus: a system of activities that uses resources to transform inbound elements into outgoing elements by adding value;
- process: transformation mode of a product implementing finite means;
- procedures: reference documents that explain the planning, control and improvement of various processus.

Control charts are graphical tools used to clearly follow the central tendency's evolution and the dispersion in the sample assumed to be representative of production. If we clearly see that the scores remain within the limits imposed by R&D tolerances, we let machining be as they are. Otherwise, we stop production to correct the "situation" due to the derivation seen on the curve. Identifying variability is useful to:

- prevent maladjustment effects and sort the production *a posteriori*;
- avoid 100% control (controlling every item, which is very costly);
- rationalize the sampling plans (control number and frequency).

Quality evaluation by controlling a sample is limited to a few units n . The chosen characteristic X is the average of n measurements taken on the n sampled units. It is thus an estimation of the average value of X for the N produced units.

A process' *total variability*: statistically, variance $s(x) = \sigma^2$ is calculated by the sum of the variances of all i elements (blueprints, machine, preparation, etc.) which make it up:

$$s(x) \sum_i s_i(x) = \text{machine variance} + \text{measurement variance} \quad [1.32]$$

We note that metrology is omnipresent in quality control. We should understand the meaning of the vocabulary (VIM) specific to this science of measurement:

– *ISO 5725 1994* accuracy (justness and fidelity) of measurement methods and results.

– *Part 1* → general principles and definitions.

– *Part 2* → basic method to determine repeatability and reproducibility.

Typical limits of more or less “3σ”: Let X represent the value of a characteristic process. If the cause system generates a variation in X , which follows a Gaussian distribution, the probability, for example 0.001 (or 0.01) is around the 3σ limits. We can move $\pm 3\sigma$ in either direction from 0.00135, or in both directions with 0.0027. In most countries (including Canada and the United States), we use control limits on some multiple of the standard deviation without considering the distribution’s normality. This multiple is usually 3σ . In the United Kingdom, for example, probability limits are used.

Interval between limits	Probability that the controlled characteristic is within the interval	Probability that the controlled characteristic is outside the interval
Between control limits	99.8%	0.2%
Between monitoring limits	95%	5%

Table 1.18. *Intervals between limits*

Generally speaking, the control limits of the central tendency and dispersion are of the order of less than 0.1% on a so-called “normal” production. In the industry, more pragmatic immediate intermediate limits are established, which would correspond to values with a chance of less than 2.5%. If these limits are overstepped, a second control is performed. If the underlying distribution is biased, the 3σ limit will be outside of the upper limit of 0.001. This means that the risk of looking for assignable causes for positive variation when none exist would be greater than 1/1,000. But the risk of finding an assignable cause for negative variation when none exist would be reduced.

The result influences the risk’s increase of a random variation above the control limits. This increase in risk will also depend on the curve’s degree of asymmetry (kurtosis). If, for example, the quality variation follows a Poisson distribution, for which $n, p = 0.8$, the risk of overstepping the limits would be increased by using the 3σ limits from 0.001 to 0.009 and the lower limit would be reduced from 0.001 to 0. For an average Poisson distribution and variance of np , the upper 3σ limit would be $\{0.8 + \sqrt{8/10}\} = 3.48$ and the lower limit = 0.

If $np = 0.8$, the probability of having more than three successes is then equal to 0.09. If a data point falls outside of the control limits, we assume that the process is

probably out of control and that an enquiry is justified to find and eliminate the cause(s). Does this mean that the process is under control if all points fall within the limits? Not necessarily.

1.8.1. Size of the samples to be taken and the frequency of control

Once the machine is under control, we must determine the size of the samples to be taken and the frequency at which the samples would be taken, based on the control efficiency. To this end, we set a risk of $\alpha = 0.002$ to reset the machine (the machine being well set). This so-called “first-species” risk α is linked to another (second-type) risk β , which is the risk of not retuning the machine when it is in fact untuned. We note that the efficiency measured by $(1 - \beta)$ directly depends on the sample size.

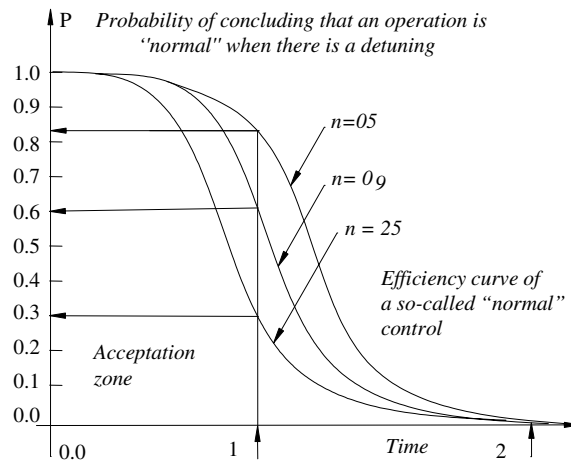


Figure 1.23. Efficiency curve for the choice of sample size

We should mention that in the precision given above, estimations vary by $1/\sqrt{n}$ for averages and $1/\sqrt{2n}$ for dispersions (and very large n). The average of samples with n individuals taken from a nominal population $N(\mu_0, \sigma_0)$ follows a Gaussian $N(\mu_0, \sigma_0/\sqrt{n})$. The range R of samples with n individuals of a dissymmetrical population with average $\delta_n \cdot \sigma_0$ is $N(\mu_0, \sigma_0)$. The limits that correspond to overstepping probabilities can be read from the tables for the repartition function of a reduced-range distribution.

The sampling frequency is a function of the sample size n , hence the control severity being directly proportional to cost. This sampling also depends on the rate of production (machining) and the tendency of the production process to misadjust

itself (age of machine tools, maintenance, conditions of use, etc.). The so-called Cavé rule [CAV 66] can be written as:

$$f_{\text{sampling}} = \left(\frac{60}{C} \right) \sqrt{n \times M} \quad [1.33]$$

C is the rate of production, number of items machined in an hour (hence 60) and M is the average (mean) number of items machined in the time interval between two detunings.

The convenient thing with curve control plans is that we always consider that the distribution is Gaussian (cumulative effects).

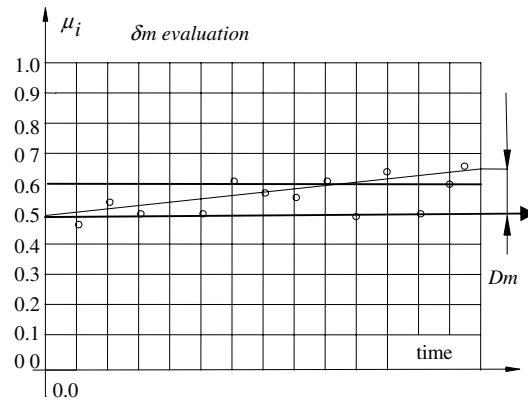


Figure 1.24. D_m tuning

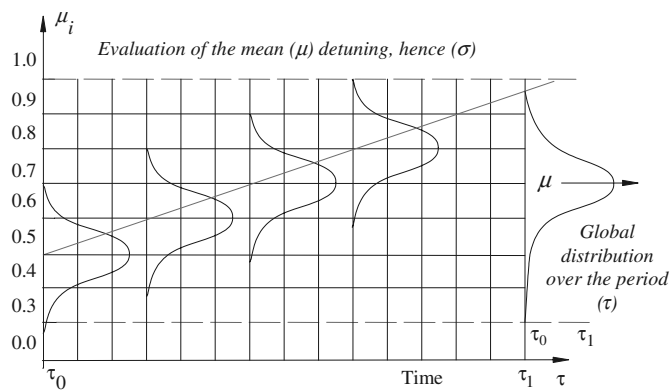


Figure 1.25. Mean (σ) tuning

Production (e.g. machining) is considered “Normal” with $N[\mu_0, \sigma_0]$ known	Characteristic submitted to SURVEILLANCE	
	Mean μ_0	R or $D_n \cdot \sigma_0$
Limits	$\pm \left(1.96 \frac{\sigma_0}{\sqrt{n}} \right)$	$D_{s1} \sigma_0$ et $D_{s2} \sigma_0$
Intervals between limits	$\left(3.92 \frac{\sigma_0}{\sqrt{n}} \right)$	$(D_{s2} - D_{s1}) \sigma_0$
Probability of being outside the interval	0.05	0.05
Probability of being outside the limit	0.025*	0.025*

Production (e.g. machining) is considered “Normal” with $N[\mu_0, \sigma_0]$ known	Characteristic submitted to CONTROL	
	Mean μ_0	R or $D_n \cdot \sigma_0$
Limits	$\pm \left(3.09 \frac{\sigma_0}{\sqrt{n}} \right)$	$D_{C1} \sigma_0$ et $D_{C2} \sigma_0$
Intervals between limits	$\left(6.18 \frac{\sigma_0}{\sqrt{n}} \right)$	$(D_{C2} - D_{C1}) \sigma_0$
Probability of being outside the interval	0.02	0.02
Probability of being outside the limit	(1%)**	(1%)**

*Limit indicator light (usually orange) on.

**Limit indicator light (usually red) on.

Table 1.19. Summary of the usual formulas used to frame the mean and range

Size of the sample, n	Average control chart						Range control chart			
	Known σ_0		Estimated σ_0				Known σ_0		σ_0 estimated by $\bar{R}$	
			By $\bar{R}$		By $\bar{s}$					
	A_c	A_s	A'_c	A'_s	A''_c	A''_s	D_{c2}	D_{s2}	D'_{c2}	D'_{s2}
2	2.185	1.386	1.937	1.229	3.874	2.457	4.65	3.17	4.12	2.81
3	1.784	1.132	1.054	0.668	2.464	1.564	5.06	3.68	2.99	2.17
4	1.545	0.980	0.750	0.476	1.936	1.228	5.31	3.98	2.58	1.93
5	1.382	0.876	0.594	0.377	1.643	1.042	5.48	4.20	2.36	1.81
6	1.262	0.800	0.498	0.316	1.452	0.921	5.62	4.36	2.22	1.72
7	1.168	0.741	0.432	0.274	1.315	0.834	5.73	4.49	2.12	1.66
8	1.092	0.693	0.384	0.244	1.209	0.767	5.82	4.61	2.04	1.62
9	1.030	0.653	0.347	0.220	1.127	0.714	8.90	4.70	1.99	1.58
10	0.977	0.620	0.317	0.202	1.059	0.672	5.97	4.79	1.94	1.56
11	0.932	0.591	0.295	0.186	1.002	0.635	6.04	4.86	1.90	1.53
12	0.892	0.566	0.274	0.174	0.953	0.605	6.09	4.92	1.87	1.51

Table 1.20. Mean and range control charts coefficients

n	d_n	b_n	n	d_n	b_n	n	d_n	b_n	n	d_n	b_n
2	1.880	0.564	6	2.534	0.869	10	3.078	0.923	14	3.407	0.945
3	1.023	0.724	7	2.704	0.888	11	3.173	0.930	15	3.472	0.949
4	0.729	0.798	8	2.847	0.903	12	3.258	0.936	20	3.735	0.962
5	0.577	0.841	9	2.970	0.914	13	3.336	0.941	25	3.931	0.970

Values of the coefficients (d_2) d_n and b_n as a function of (n).

Table 1.21. d_n (d_2) and b_n coefficients as a function of n
(sources: [BOI 01] and [PAC 69])

1.8.2. Factor for control of X (mean) and range (R)

Sample Size (n)	Factors to estimate			Factor to estimate the standard deviation	Relative efficiency
	$\bar{X}$	R		σ	
n	A_2	D_3	D_4	d_2	
2	1.880	0.000	3.267	1.128	1.000
3	1.023	0.000	2.575	1.693	0.992
4	0.729	0.000	2.282	2.059	0.975
5	0.577	0.000	2.115	2.326	0.955
6	0.483	0.000	2.004	2.534	0.930
7	0.419	0.076	1.924	2.707	
8	0.373	0.136	1.864	2.847	
9	0.337	0.184	1.816	2.970	
10	0.308	0.223	1.777	3.078	0.850
11	0.285	0.256	1.744	3.173	
12	0.266	0.284	1.716	3.258	
13	0.249	0.308	1.692	3.336	
14	0.235	0.329	1.671	3.407	
15	0.223	0.348	1.652	3.472	
16	0.212	0.364	1.636	3.532	
17	0.203	0.379	1.621	3.588	
18	0.194	0.392	1.608	3.640	
19	0.187	0.404	1.596	3.689	
20	0.180	0.414	1.586	3.735	

Table 1.22. Calculation factors for control limits of X and R charts
(source: G. Baillargeon, [BAI 80])

1.9. Production and reception control

Controlling a production process consists of stabilizing it within optimal return conditions. If a manufacturing has a high proportion of rejects, we declare that it does not conform to previously set specifications. In machining ranges, nominal TI scores are set on the basis of the demands of the drawing plan.

In make-to-order productions, quality is a completely different problem than in a serial or mass production range. The $p\%$ proportion denotes the faulty fraction, that is the items rejected for not conforming to the toleranced score. In these cases, the machine is in fact perfectly set if the production average (μ) is comparable to the nominal score (D). This happens when the causes for variations in m are not explainable by machine-dependent reasons, for example a machine tool with noticeable wear-and-tear, too loose, poorly maintained.

Considering the above, it is obvious that rejected items are inevitable, irrespective of the skills of the machinist. The problem to be solved (assuming a small $p\%$) is thus the machine's precision. Another problem is statistical tuning when we strive to minimize $p\%$ during production. Much care is taken to identify the causes that could systematically create rejects and then to take action against indiscriminate tuning.

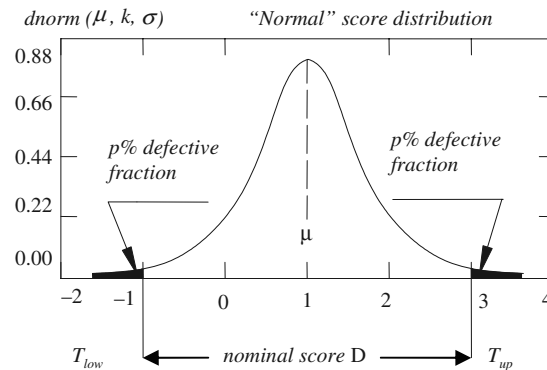


Figure 1.26. Specification graph

1.9.1. Machine adaptation with respect to production tolerances

In these cases, we consider that the machine is tuned correctly with respect to the mean (μ), that is that the mean "really" corresponds to the nominal score (see Figure 1.27 (b, c, d)). The centered interval (6σ) includes all of the normal distribution's data, or 99.8%. From there, we have three distinct cases:

Case (a): In this instance, $\left(\frac{2IT}{6\sigma}\right) > 1$ Otherwise $6\sigma < 2IT$, and we consider that there is almost no waste, even if we should notice a well-controlled (i.e. within a critical threshold) detuning around the mean. We declare the machine precise enough and we continue machining.

Of course, a choice has to be made when we know that precision is costly, and resources should be associated to it.

Case (b): In this instance, $\left(\frac{2IT}{6\sigma}\right) = 1$ Otherwise $6\sigma = 2IT$, and we consider that there is almost no waste. However, any detuning of the average will change the TI and bring about greater waste.

Case (c): In this instance, $\left(\frac{2IT}{6\sigma}\right) > 1$ Otherwise $6\sigma > 2IT$ and we consider that there is waste, represented by the $p\%$ (Figure 1.27 c). At this point, it is too late to act on the machine to improve the level of waste ($p\%$). The following duality imposes itself:

- we change machines, as the $p\%$ are rejected due to the machine's imprecision;
- we accept that we need to “manage” the waste $p\%$ while trying to find the reasons for the imprecisions in order to correct them. This is summarized in Figure 1.26.

1.9.2. Proportion of faulty units

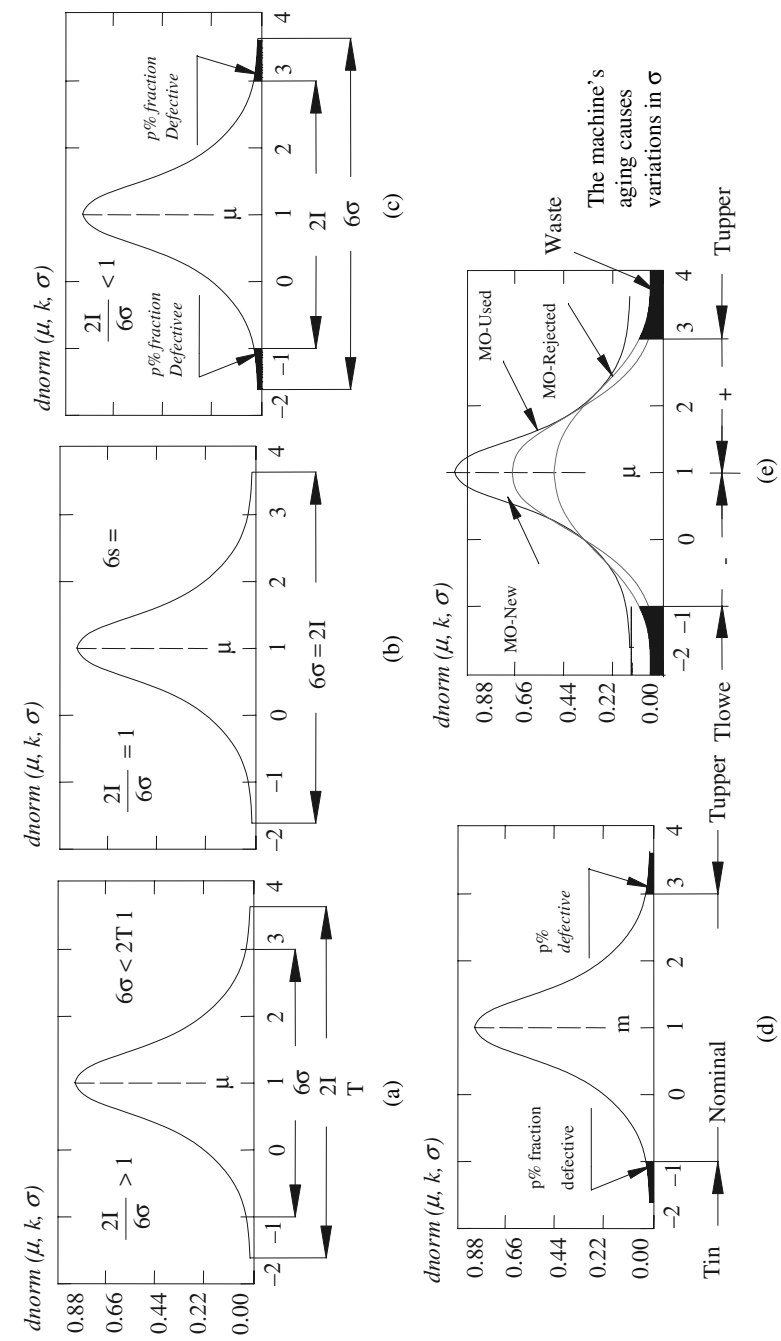
The proportion of faulty units is based on a binomial distribution. The flawed articles in a production process can be *monitored* using statistics based on the observed number of defectives in a random sample of size n taken from a continuous production process, a large population or a lot. The proportion of flawed individuals within a lot follows a binomial (or at least Poisson) distribution, where p is the probability of an individual element to be found defective. We should ask ourselves the following questions about quality control:

- Is the proportion of defectives within the prescribed limits?
- Is the proportion of defectives less than or greater than the prescribed limits?

The corresponding testable hypotheses are as follows:

- 1) $p = p_0$;
- 2) $p \leq p_0$;
- 3) $p \geq p_0$;

where p_0 is the prescribed defective proportion.



Figures 1.27. Illustrative graphs the machine wear and tear (proportions of defective)

Statistical test based on a normal approximation: Given a random population measurement sample $Y_1, \dots, Y_n$, the proportion of elements found to be defective from these n measurements is written $\hat{p}$. The statistical test is then written as:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \quad [1.34]$$

This statistic is based on a normal approximation of the binomial distribution, which is true for large n ($n > 30$). This approximation simplifies calculations by using the essential values of the normal distribution table as shown below:

p	0.001	0.005	0.010	0.025	0.050	0.100
Z_p	-3.090	-2.576	-2.326	-1.960	-1.645	-1.28
p	0.999	0.995	0.990	0.975	0.950	0.900
Z_p	3.090	2.576	2.326	1.960	1.645	1.282

Table 1.23. Reduced centered value around Z (normal distribution)

Restriction on sample size: Because the test is approximate, n must be large enough for the test to be valid. A criterion is that n must be chosen such that

$$\text{Min}\{np_0, n(1-p_0)\} \geq 5 \quad [1.35]$$

For example, if $p_0 = 0.1$, then n must be at least 50 and if $p_0 = 0.01$ then N must be at least 500.

$$\text{If } \left\{ \begin{array}{l} 1 \rightarrow |z| \geq z_{\alpha/2} \\ 2 \rightarrow z < -z_{\alpha} \\ 3 \rightarrow z \geq +z_{\alpha} \end{array} \right\} \quad [1.36]$$

The unilateral test with confidence level $(-\alpha)$ corresponds to hypotheses (1), (2) and (3). For hypothesis (1), the test statistic z is compared to the upper normal distribution's critical value (see Table 1.23), which is overstepped with a probability of $\alpha/2$, and similarly for hypotheses (2) and (3).

Conclusion: The null hypothesis is rejected.

Calculations for a unilateral defective proportion test: In a production process, 100 Stanley cups machined in 6061 were tested, and five showed a particular flaw. Thus, for $n = 100$, the proportion of flawed individuals is estimated at $\hat{p}5/100 = 0.05$.

Beforehand, the production process was able to manufacture items with a defective proportion of 0.10 at most. The question is whether the new process experienced degradation in item quality. The relevant criterion is unilateral test (3), which protects against an increase in the proportion of defective individuals. For a test with significance level $\alpha = 0.05$, the “absence of degradation” hypothesis is validated if the z -test statistic is less than the critical value of $z_{0.05} = -1.645$. The test’s statistic is thus calculated with $n = 100$.

$$\text{From [1.34], let us set: } z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.12 - 0.10}{\sqrt{\frac{0.10(0.90)}{100}}} = 0.6667$$

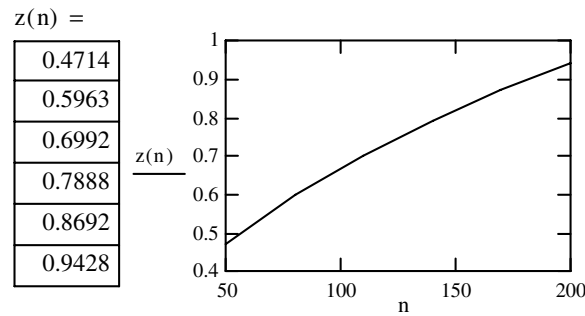


Figure 1.28. Graph of a unilateral test of the defective proportion

Interpretation: Because the test’s statistic is greater than the critical value (-1.645), we cannot reject hypothesis (3). Therefore, we also cannot conclude that the new production process presents a new deterioration in Stanley cups machined in 6061. The new process can in fact be worse, but more proof will be necessary to reach this conclusion at the 95% confidence level.

1.10. Control charts

The control maps are done on measured variables (metrology). The control charts characterized by the number of defectives are called charts of quality control by attributes. Both are suitable tools for decision-making with respect to process acceptance. Process control has two main goals:

Ensure that production is stable and limit (and if possible, eliminate) non-conforming (or outside-tolerance) production. Control charts are not the only possible method to do this. However, we wish to show a method where graphical

observation allows us to directly see a potential machine detuning. Control charts are thus an excellent tool to analyze fluctuations observed *on-line*, during the production process.

There also exist *control by measurement charts* (where measurements – metrology – are required) and *control by attribute charts* when the quantities cannot be measured.

1.10.1. Control by measurements charts for the mean $\bar{X}$ and the range R

These are the control charts most often used to measure ranges ($\bar{X}$ and R). These values characterize the central value and dispersion of each sample. The choice of these two characteristics is based on the fact that any quantity randomly distributed according to any distribution has these two characteristics if the quantities are random. This is thus a statistical fact. Traditionally, we work on the average of the set of samples within ± 3 standard deviations of the mean. This interval is the variation allowed in the mean.

The central limiting theorem largely favors using the mean. A production is called stable (or rather, “in a state of operational stability”) if the tendency ($\bar{X}$) and the dispersion (R) are statistically constant in time. To better explain our reasoning, here are two typical graphs of the above: mean $\bar{X}$ and range R .

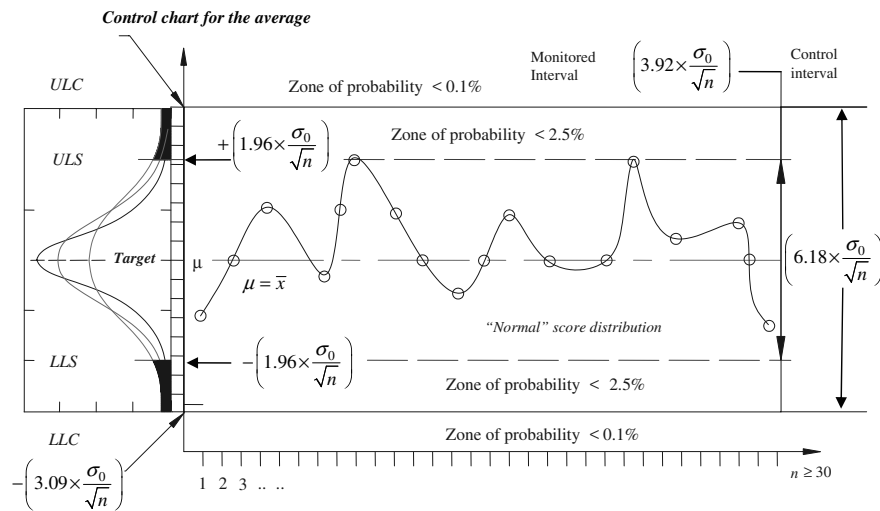


Figure 1.29. Control chart for the average

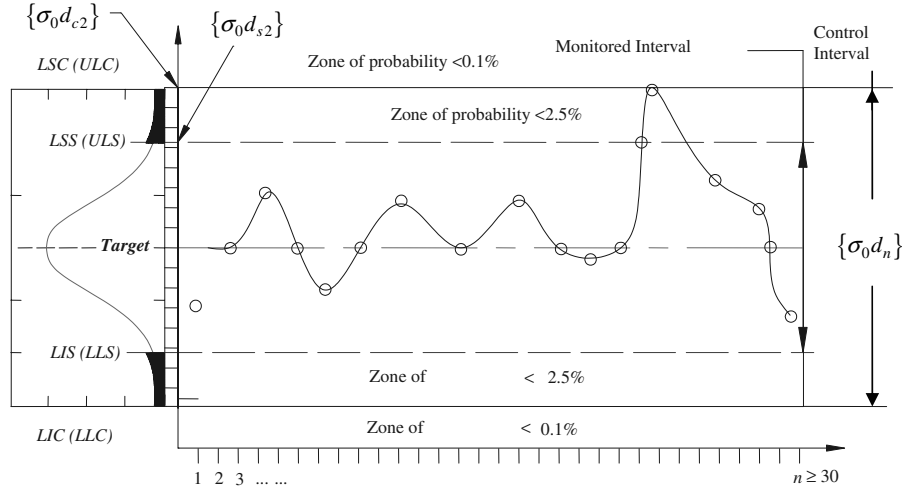


Figure 1.30. Control chart for the range

1.10.2. Calculating control limits and case study

The first step is to collect samples (20 is more than enough). On each sample, five succinct observations are necessary. The method is performed as per the following points:

- 1) Create a spreadsheet (log) used to record the observations from each sample (sample log given later).
- 2) Calculate the average ($\bar{X}$) and the range (R) for each sample, thus:

$$\bar{X} = \sum_{i=1}^n X_i \text{ and } R = \{X_{\text{larger value}} - X_{\text{smaller value}}\}.$$

- 3) Calculate the general average (average of averages $\bar{\bar{X}}$) for the observations and $\bar{R}$. For example, κ samples gives us:

$$\left\{ \bar{\bar{X}} = \sum_{j=1}^{\kappa} \bar{X}_j \text{ and } \bar{R} = \sum_{j=1}^{\kappa} R_j \right\}.$$

- 4) Calculate the control limits for the average, that is the process' tendency with the following relations:

$$\left\{ \begin{array}{l} LSC \text{ (UCL = Upper Control Limit)} = \bar{\bar{X}} + A_2 \times \bar{R} \\ LIC \text{ (UCL = Lower Control Limit)} = \bar{\bar{X}} - A_2 \times \bar{R} \end{array} \right\} \text{ When } A_2 = \left(\frac{3}{d_2 \sqrt{n}} \right).$$

The tabulated values for A_2 (and/or d_2) can be found in the tables in the Appendix. We already know that the calculation of the control limits for the average is a simple estimation of this interval:

$$\left\{ \mu - 3\sigma_{\bar{X}} \leq \bar{X} \leq \mu + 3\sigma_{\bar{X}} \right\} \text{ with } \sigma_{\bar{X}} = \left(\frac{\sigma}{\sqrt{n}} \right).$$

The estimation of $\sigma_{\bar{X}}$ is then given by $(\bar{R}/d_2\sqrt{n})$. d_2 can be read from a Table 1.22 – also see the Appendix for more details regarding this table. Its calculation is in fact not only complicated but also unnecessary in this context.

5) Calculate the control limits for the range, that is the process' dispersion with the following relations:

$$\left\{ \begin{array}{l} LSC \text{ (UCL = Upper Control Limit)} = D_4 \times \bar{R} \\ LIC \text{ (UCL = Lower Control Limit)} = D_3 \times \bar{R} \end{array} \right\}.$$

The values for D_4 and D_3 can be read directly from Table 1.22, based on the sample size (n). As before, we already know that calculating the control limits for the range R is simply estimating this interval:

$$\left\{ E(R) - 3\sigma_R \leq R \leq E(R) + 3\sigma_R \right\} \text{ with } \sigma_R = (\sigma \times d_3)$$

D_3 can also be read from the appropriate tables (see FORD or Chrysler table: A.11 in the Appendix) or, after transformations, the following can be used:

$$D_3 = \left(1 - 3 \frac{d_3}{d_2} \right), \text{ where } d_2 \text{ and } D_3 \text{ are easier to read (see tables A.11 in the}$$

Appendix)

Plot the graphs (Figures 1.29 and 1.30) for both control charts and make conclusions (tuning and monitoring deductions will follow).

Sample of laboratory presentation for control charts of $\bar{X}$ (mean) and R (Range)

Order number	Dates and time	X_1	X_2	X_3	X_4	X_5	$\bar{X}$	R
1								
2								
etc. ...	...							
Technician: lab assistant Blaireau LeNain							$\Sigma \bar{X}$	$\Sigma \bar{X}$
Formules principales * $\left\{ \begin{array}{l} LSC = D_4 \times \bar{R} = \dots\dots\dots \\ LIC = D_3 \times \bar{R} = \dots\dots\dots \end{array} \right\}$ and $\left\{ \begin{array}{l} LSC = \bar{\bar{X}} + A_2 \times \bar{R} = \dots\dots\dots \\ LIC = \bar{\bar{X}} - A_2 \times \bar{R} = \dots\dots\dots \end{array} \right\}$ (*) Cite sources		<i>Plan drawing of the scored item (norm GPS = Geometrical Product Specification) if machined</i> <i>Other specifications if necessary</i>					General Average $\bar{\bar{X}}$	General Average Range $\bar{R}$
							$\left\{ \begin{array}{l} A_2 \times \bar{R} = \dots\dots\dots \\ D_3 \times \bar{R} = \dots\dots\dots \\ D_4 \times \bar{R} = \dots\dots\dots \end{array} \right\}$	

Table 1.24. Sample log for control charts of $\bar{X}$ and R

1.10.3. Study of $\bar{X}/R$ control charts – quality control lab

Consider an item machined in the laboratory by Sam, a machinist student, during a course on machine tools. The control takes place in the dimensional metrology lab using a digital micrometer as shown in Figure 1.31. The machine tool's standard deviation is unknown.



Figure 1.31. Photograph of our measurements

The data for 100 items machined by Sam, Sarah and Nadim are grouped in the following table:

Order number	Dates hours	X_1	X_2	X_3	X_4	X_5	X_{moy}	R
1	9:00 am	5.000	5.000	4.982	5.000	5.001	4.9966	0.0180
2	10:00 am	5.000	5.000	4.876	5.045	4.989	4.9820	0.1690
3	11:00 am	4.990	4.857	4.991	5.018	4.955	4.9622	0.1610
4	1:00 pm	4.890	4.890	4.780	5.056	5.001	4.9234	0.1660
5	2:00 pm	5.020	5.020	5.012	4.980	5.055	5.0174	0.0750
6	3:00 pm	5.030	5.011	5.028	4.897	5.045	5.0022	0.1480
7	4:00 pm	5.000	5.078	5.036	4.897	4.859	4.9740	0.2190
8	9:00 am	5.070	5.001	5.000	4.893	4.891	4.9710	0.1790
9	10:00 am	4.950	4.989	4.973	5.000	4.975	4.9774	0.0500
10	11:00 am	4.990	4.955	4.991	5.000	5.000	4.9872	0.0450
11	1:00 pm	4.679	5.001	5.000	4.965	4.888	4.9066	0.1130
12	2:00 pm	5.060	5.055	5.000	4.970	4.877	4.9924	0.1830
13	3:00 pm	5.081	5.045	5.001	4.965	4.865	4.9914	0.2160
14	4:00 pm	5.011	5.000	5.002	5.000	4.878	4.9782	0.1330
15	etc.	5.004	5.001	4.892	5.000	5.000	4.9794	0.1120
16		4.999	4.983	4.968	4.859	5.052	4.9722	0.0840
17		4.897	4.889	4.984	4.891	5.000	4.9322	0.1090
18		4.957	4.957	5.009	4.975	5.078	4.9952	0.1211
19		5.000	5.005	5.000	4.800	4.999	4.9608	0.0060
20		4.857	4.875	5.035	5.089	4.988	4.9687	0.1780
Technician/Lab Assistant: Blaireau LeNain			See drawing plan	Sum (ΣX , ΣR)		99.4706	2.4851	
				Average G . $\bar{X}$		4.9735		
n	5	Table	Support table machined 6061	Average G . $\bar{R}$			0.1243	
Table	A_2	0.577		LSC (X_{general})		5.0452		
Table	d_2	2.326		LIC (X_{general})		4.9018		
Table	D4	2.115		LSC (R_{general})			0.2628	
Table	D3	0.000		LIC (R_{general})			0.0000	
Main formulas for the chart of average					Formulas for the chart of range R			
$\left\{ \begin{array}{l} \text{LSC} = \bar{X} + A_2 \times \bar{R} = \dots\dots\dots \\ \text{LIC} = \bar{X} - A_2 \times \bar{R} = \dots\dots\dots \end{array} \right\}$					$\left\{ \begin{array}{l} \text{LSC} = D_4 \times \bar{R} = \dots\dots\dots \\ \text{LIC} = D_3 \times \bar{R} = \dots\dots\dots \end{array} \right\}$			
Average G = General average $\bar{X} = \Sigma X/N$								
Average G = Average G $\bar{R} = \Sigma R/N$								
Source of coefficient data [BAI 80]					Formulas for the coefficients			
					$\left\{ \begin{array}{l} A_2 \times \bar{R} = \dots\dots\dots \\ D_3 \times \bar{R} = \dots\dots\dots \\ D_4 \times \bar{R} = \dots\dots\dots \end{array} \right\}$			

Table 1.25. Calculation results (perfect agreement between MathCAD and Excel)

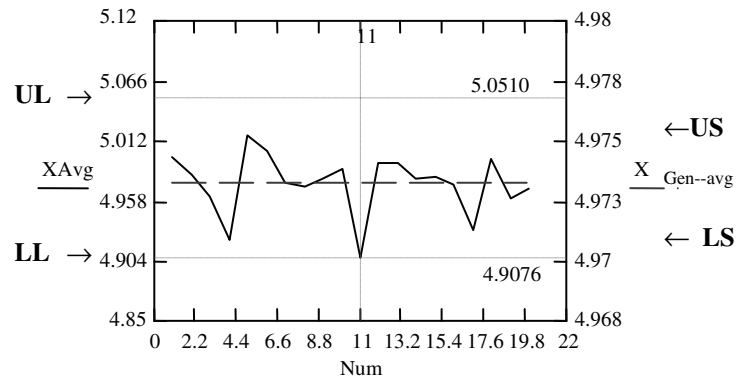


Figure 1.32. Chart of the average X_{avg} , calculated and plotted in MathCAD

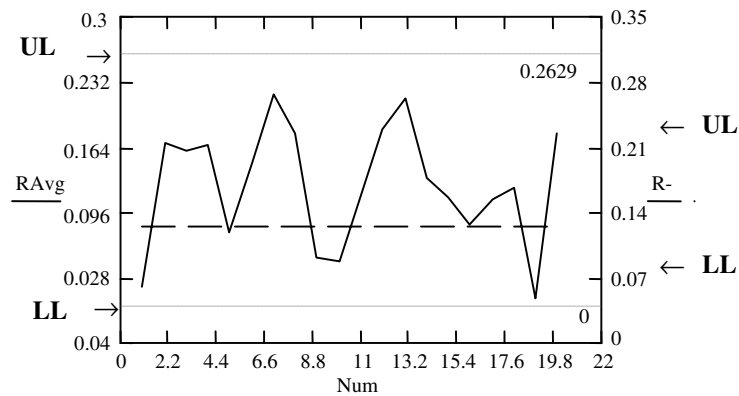


Figure 1.33. Chart of the range (R), calculated and plotted in MathCAD

DISCUSSION.— Reading the graphs of our results, we can see that there are no points outside of the control limits (after calculations: LCL, UCL). On the basis of this statistical evidence, there is no reason to operate on the machine because the machining process appears to be stable. Point 11 with a diameter of 4.679 seems a bit off, but not off enough to threaten the process' stability. We should then keep an eye out for this particular diameter.

Rigorous control if the standard deviation was known, $\sigma_{\text{Machine}} = 2.4012$.

We calculated the limits (USL and LSL) for both the average and range. We then saw that $\sigma_{\text{Machine}} = 2.4012$ exactly, with a sample size of $n = 5$.

$$\left\{ \begin{array}{l} \sigma_{\text{machine}} = \sigma_0 \rightarrow \left(\frac{\sigma_0}{\sqrt{n}} \right) = 0.025 = 2.5\% \text{ then for } \bar{\bar{X}} = 4.9735 \\ \text{we have } \rightarrow \text{LSC} = \bar{\bar{X}} + 3.09 \times \left(\frac{\sigma_0}{\sqrt{n}} \right) = 5.0510 \text{ and} \\ \rightarrow \text{LIC} = \bar{\bar{X}} - 3.09 \times \left(\frac{\sigma_0}{\sqrt{n}} \right) = 4.8960 \end{array} \right\}$$

Compared to the same calculation above, we see the perfect agreement between the limit results (UCL and LCL).

Monitoring the machine if the standard deviation was known, $\sigma_{\text{machine}} = 0.025$.

We calculated the limits (USL and LSL) for both the mean and the range. We find (with perfect precision) that $\sigma_{\text{Machine}} = 2.4012$, for a sample size $n = 5$,

$$\left\{ \begin{array}{l} \sigma_{\text{machine}} = \sigma_0 \rightarrow \left(\frac{\sigma_0}{\sqrt{n}} \right) = 0.011 \approx 1\% \text{ then for } \bar{\bar{X}} = 4.9735 \text{ we have } \rightarrow \\ \text{ULS} = \bar{\bar{X}} + 1.96 \times \left(\frac{\sigma_0}{\sqrt{n}} \right) = 4.9954 \text{ and LLS} = \bar{\bar{X}} - 1.96 \times \left(\frac{\sigma_0}{\sqrt{n}} \right) = 4.9516 \end{array} \right\}$$

The machine should now be tuned on the basis of these surveillance limits, and the production process should be continued. Beyond these monitoring limits, an orange lamp will turn on to signal that the tolerances were overstepped, and the tuning will then need to be adjusted accordingly.

COMMENTS.– In mechanical production (mid- and large-quantity of parts machining), usually we try to accomplish the following with a control chart:

First, the TIs are set by the drawing plan (design range). Various causes can affect preestablished TI, accidental or not [GRO 11]. In any case, so-called accidental variations exist in any process. We must therefore ensure that nuances in materials (e.g. 6061 in our case study), the machine tool, mounting and temperature conditions, the equipment's wear and tear (see Taylor's law used here to calculate the lifespan of cutting tools), etc. Non-accidental causes usually affect the production process abruptly, but can sometimes do so progressively. We must specify that control charts, as precise and rigorous as they may be, do not bring any apparent and explicit precision on the reasons why things might be outside of the control limits. All they do is give us a timely warning of an anomaly in the production process that threatens the process' stability.

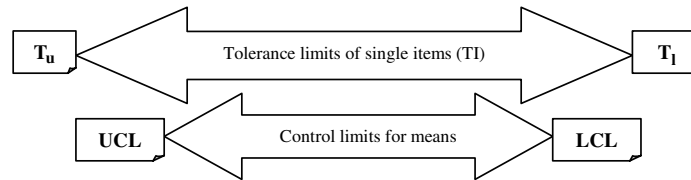
The following factors are likely to affect the process average:

- an approximate or poorly done metrology (incorrect measurements);
- non-conforming calibration of the machine and measurement instruments;
- arbitrary (and inadequate) change in measurement tool;
- calculation and/or human errors;
- an arbitrary or questionable adjustment;
- wear and tear in machine elements (poorly maintained machine tool);
- materials with respect to cutting conditions.

The following factors are likely to affect the process dispersion (range):

- the wear and tear of equipment and machine elements;
- non-tolerated aging (technician, machinist, etc.);
- abnormal looseness in adjustments, etc.

We will now use the $\bar{X}$ and R charts to evaluate the production's conformity with specifications imposed ahead of time, that is TI.



We start from the fact that machining (production) is done with stability in time. This is the indication given by the $\bar{X}_{avg}$ and R charts. Statistically, we deduce that the process takes place under control with an average (μ) and a standard deviation (σ), which support a known distribution (often a normal distribution).

The standard deviation estimation can be written as: $\hat{\sigma} = \bar{R}/d_2$. d_2 must be read from Table A.11 (see the Appendix, table A.11) and $\bar{R}$ of the control chart. The experimental tolerance standard deviation can be written as $\hat{\sigma}_{exp} = 6(\bar{R}/d_2)$, which corresponds to $\pm 3 \times \bar{R}/d_2$. If we compare the experimental tolerance standard deviation and the specification deviation (i.e. the TI from design), we get the expression for the tolerance interval of specifications (TI) as follows:

$$ETS = \{T_{Upper} - T_{Lower}\} = 6 \times \sigma$$

We declare that the machining process is accepted if ETS does not go above the specifications (TI) established by engineering consulting firm (ECF) (design) and we define:

$$6 \times \left(\frac{\bar{R}}{d_2} \right) \leq ETS, \text{ i.e. } \leq 6 \times \sigma$$

We then deduce that the general average ($\bar{\bar{X}}$) is in fact halfway within specifications and that $\left(\frac{\bar{R}}{d_2} \right)$ is less than $\left(\frac{1}{6} ETS \right)$.

Example: depending on design specifications, we define the TI (0.402) as follows, bearing in mind that we know from the previous problem that:

$$T_{up} = 5.081 \text{ in}; T_{lo} = 4.6790 \text{ in} \rightarrow \{T_{up} - T_{lo}\} = 5.081 - 4.679 = 0.4020 \text{ inches}$$

$$\left(\frac{5.081 + 4.679}{2} \right) = 4.88 \Leftrightarrow \left\{ \begin{array}{l} 4.88 + 0.201 = 5.081 \\ 4.88 - 0.201 = 4.679 \end{array} \right\} \text{ or } \left\{ \begin{array}{l} 5.0810; 4.6790 \\ 5.0403; 4.7197 \end{array} \right\}$$

$$\text{Since from (BE) data we have: } IT = 0.402 \Rightarrow 2IT = \left(\frac{0.402}{2} \right) = 0.201$$

$$\left\{ \begin{array}{l} \text{From table (XX) we read that } d_2 = 2.326 \text{ for } n = 5 \\ \text{Knowing that : } X_{AvgGeneral} = \bar{\bar{X}} = 4.9735 \text{ and } \bar{R}_{General} = 0.1243 \end{array} \right\}$$

$$\left\{ \sigma = \frac{\bar{R}}{d_2} = \frac{0.1243}{2.3260} = 0.0534 \right\} \text{ and } ETS = 6 \times \frac{\bar{R}}{d_2} = 6 \times 0.0534 = 0.3206$$

$$\left\{ 6 \times \left(\frac{\bar{R}}{d_2} \right) < 6\sigma \right\} \text{ which is correct as } \{0.3206 < 0.4020\}$$

$$\left\{ 3 \times \left(\frac{\bar{R}}{d_2} \right) = 0.1603 \right\} \text{ or } \left\{ \begin{array}{l} 4.88 + 3 \times \left(\frac{\bar{R}}{d_2} \right) = 5.0403 \\ 4.88 - 3 \times \left(\frac{\bar{R}}{d_2} \right) = 4.7197 \end{array} \right\} \rightarrow (5.0403; 4.7197)$$

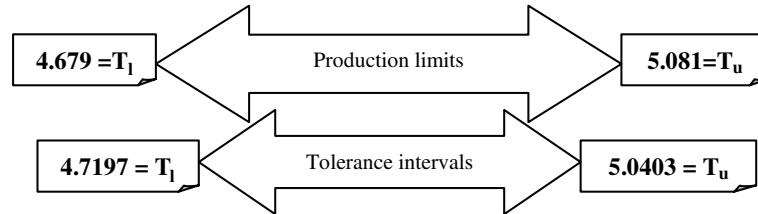
We machine at the following limits:

$$\{5.0403; 4.7197\} \text{ or } \left\{ \frac{5.0403 + 4.7197}{2} = 4.88 \right\} \rightarrow \left\{ \begin{array}{l} 4.88 + 0.201 = 5.081 \\ 4.88 - 0.201 = 4.679 \end{array} \right\} \text{ QED}$$

which corresponds exactly to ECF (design) initial conditions.

$$\{5.081; 4.679\} = \{5.0403; 4.7197\} \rightarrow \left\{ \begin{array}{l} 4.88 + 0.201 = 5.081 \\ 4.88 - 0.201 = 4.679 \end{array} \right\} = 4.88 \pm 0.2011$$

We remain within the limits imposed by ECF (*Engineering consulting firm*).



We can clearly see that the process is not centered. The machined components defined by the toleranced dimension (4.88 ± 0.2011). The table supports machined in our workshops and controlled in metrology labs remain satisfactory. Of course, we clearly see an excess of precision. This is because tuning must be performed frequently. For the process to be destabilized, we must observe a large enough shift in the average that the components would be rejected. Corrective measurements are in order when the process is not able to remain stable within the limits specified by the TI, such as:

- Widen the TI. But be careful not to change TI because we are faced with a “non-capability”. To be able to change them, we must return to design and reconsider the TI, based on the functional specifications, range specifications (BE) and the client’s demands.

- We can also reduce the production’s dispersion, that is try to get a clear machining homogeneity. This method requires know-how, time and investments, which are difficult and time-consuming to make profitable.

1.10.4. Graphical representation of the rejections due to non-conformity to TI

In machining, a component is said to be acceptable when its scores lie within the tolerance limits $\leq$ (nominal score $\pm$ IT). These strict conditions are set during design

and executed by *engineers*. *Traditional education* does not always rely on the statistical approach as it is deemed *vague*, strictly intuitive, sometimes difficult and even non-pragmatic. This is extremely wrong, especially in mass production where the cost of individual control is exponentially high. Individual control does not guarantee process stability. It could even become anachronistic, and offers no certainty regarding machine-tool settings.

For example, consider a measured characteristic that follows a normal distribution of average $(\bar{X})$ and standard deviation $\bar{R}/d_2$. We will determine the proportion of defectives (components outside tolerances). To this end, we should evaluate the probability that this value (X) of the characteristic lies outside of the tolerances specified as shown in Figure 1.34:

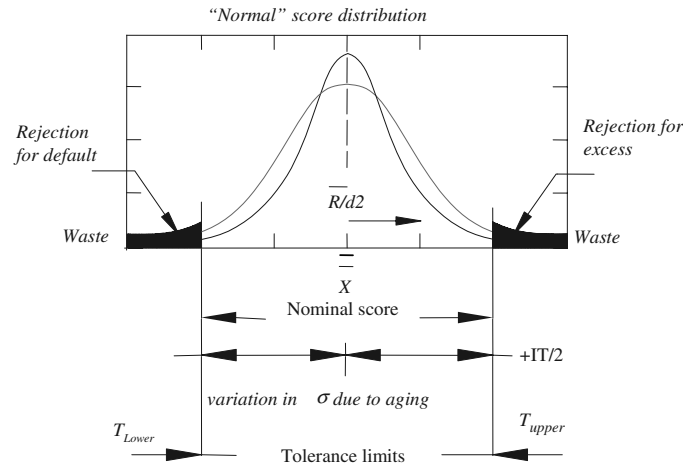


Figure 1.34. Graphical illustration of non-conformity rejections

To calculate the proportion of components either rejected or retaken (defectives with respect to specification conditions of the drawing plan), we calculate u_2 (rejection by excess) and u_1 (rejection by default) as follows:

$$\left\{ \begin{array}{l} \text{Defective by excess} \rightarrow u_2 = \left(T_{Upper} - \bar{X} \right) / \frac{\bar{R}}{d_2} \\ \text{Defective by default} \rightarrow u_1 = \left(T_{Lower} - \bar{X} \right) / \frac{\bar{R}}{d_2} \end{array} \right\} \quad [1.36]$$

On the basis of equation [1.36], we represent the proportion of components satisfying a normal law (it lies within this famous bell curve). We refer to Gauss' Law using the probabilities for a reduced centered distribution. In fact, we look for the probability of observing a value between u_1 and 0, or $P(u_1 < u < 0) = P_1$. The probability of observing a value between 0 and u_2 can then be written as $P(0 < u < u_2) = P_2$. The probability for a value to lie between u_1 and u_2 can then be written as: $P(u_1 < u < u_2) = [P_1 + P_2]$; this will then give us the probability of observing that the component conforms to the initial tolerance as shown in Figure 1.35.

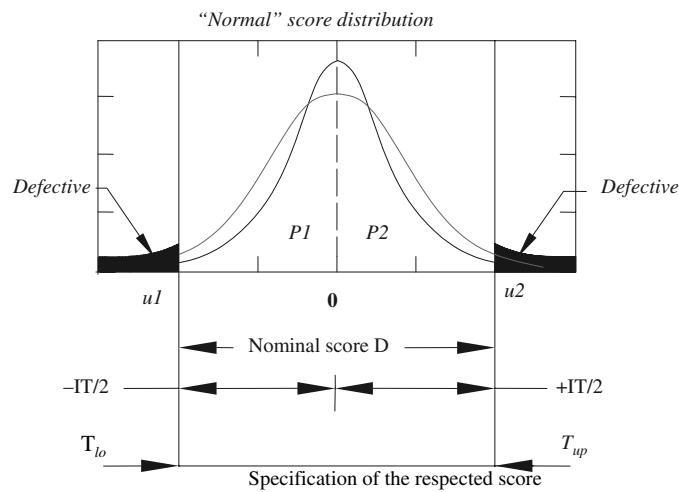


Figure 1.35. Graphical illustration of rejections due to non-conformity to TI

The total quantity (Q) of components rejected for non-conformity to tolerances can be written as: $Q = 1 - [P_1 + P_2]$. If we want to accept components by default or by excess, we must calculate [by excess ($1/2 - P_2$)] and [by default ($1/2 - P_1$)], respectively.

1.10.5. Performance case study: Capabilities

– Performance indices P_p and P_{pk} : more than 100 samples of six components, taken over 30 days of boring support (machined in our facilities, see following image, see Figure 1.36), that is 600 values.

– Study statistics

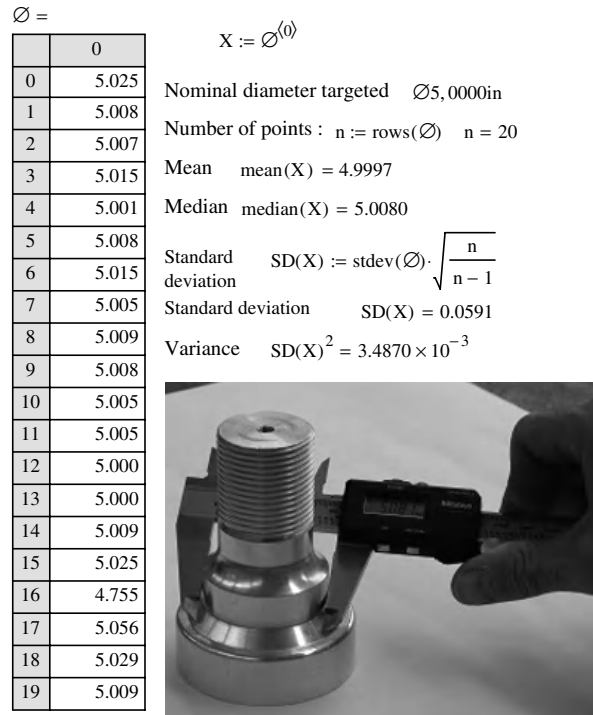


Figure 1. 36. *Quality control case study (applied statistics)*

Method used:

- 1) Fill the (n) table with X data, adding the measurement unit.
- 2) Correctly write the maximum value ($T_{\max}$) and minimum value ($T_{\min}$).
- 3) Calculate the range, $R = T_{\max} - T_{\min}$. This range is divided into multiple “classes” (number of histogram columns, using Ishikawa method).
- 4) Determine the theoretical width of the class called “class interval”, $h_t = R/k$ (class).
- 6) The practical interval, h_p , which verifies the product $h_t \times K^3$.
- 7) Do the study’s statistics.
- 8) Plot the histogram, place the specification range (i.e. T_l and T_u , lower and upper tolerances), the *mean* and the *target*.

First case: the target nominal diameter is $\varnothing 5$ inches.

T_s Score >	T_i Score <	Mean (μ)	Standard deviation (σ)	Range $R = T_{\text{sup}} - T_{\text{inf}}$	$P_p = \frac{T_{\text{sup}} - T_{\text{inf}}}{6 \times \sigma}$	$P_{pk}^{\text{sup}} = \frac{T_{\text{sup}} - \mu}{3 \times \sigma}$	$P_{pk}^{\text{inf}} = \frac{\mu - T_{\text{inf}}}{3 \times \sigma}$
5.0250	4.7500	4.9997	0.0591	0.2750	0.7755	0.1410	1.4100

Table 1.26. Calculation result table (indicative statistics)

Consider the smallest P_{pk} . As tabulated $P_{pk} = 1.33 \rightarrow$ then tabulated $P_{pk} <$ lower P_{pk} , that is $1.141 > 1.33$. We thus deduce that the process performance is variable over the considered period. We accept this *instantaneous* control and continue the still-controlled production.

Second case: for the same lab experiment, we calculated an average standard deviation of $s = 0.0591$ in.

n	10	12	16	20	24	28	30	35	40	50	75	100
C	1.64	1.55	1.43	1.37	1.32	1.30	1.28	1.26	1.24	1.21	1.16	1.13

Table 1.27. Indicative values of C and n (source: the FORD company)

On the basis of the values of C_{am} or C_{mk} , we generally classify capability as shown in Table 1.28.

Tabulated C value	Rating
$C < 0.67$	Very poor
$0.67 < C < 1$	Poor
$1 < C < 1.33$	Very average
$1.33 < C < 1.67$	Average
$1.67 < C < 2$	Good
$C > 2$	Very good

Table 1.28. Summary of the indicative values of C_{am} or C_{mk} (source: the FORD company)

The process is said to be capable if C_{pk} is greater than one: we accept that three components out of 1,000 can be out of tolerance. If the process is not considered capable, we must then improve the process.

Tabulated coefficients (C_4 and d_2) from a Ford Table 1.29 (See also A.11) according to the sampling size (n).

n	C_4	d_2	n	C_4	d_2	n	C_4	d_2
2	0.7979	1.128	7	0.9594	2.704	12	0.9776	3.258
3	0.8862	1.693	8	0.9650	2.847	13	0.9794	3.336
4	0.9213	2.059	9	0.9693	2.970	14	0.9810	3.407
5	0.9400	2.326	10	0.9727	3.078	15	0.9823	3.472
6	0.9515	2.534	11	0.9754	3.173	20	0.9869	3.735

Table 1.29. Estimated standard deviation s

The samples are defined for $n = 20$. From Table 1.29, we then find that $C_4 = 0.9869$.

The estimated standard deviation is 0.074605 in, that is

Data: $R = 0.2750$, $C_4 = 0.9869$ and $d_2 = 3.735$ (see the Ford table, Appendix A.11)

$$\bar{s}_{\text{estimate}} = \left(\frac{\bar{R}}{d_2} \right) = \left(\frac{0.2750}{3.7350} \right) = 0.0736 \text{ inch or}$$

$$\hat{\sigma}_{\text{estimate}} = \left(\frac{\hat{\sigma}_{\text{estimate}}}{C_4} \right) = \left(\frac{0.0736}{0.9869} \right) = 0.074605 \text{ inch}$$

The indices d_2 and C_4 are given in the Ford manual and the ISO 8258 norm (1991). ISO 8258 (1991) calls the C_p value the GAI (general aptitude index). The target nominal diameter is $\varnothing 5$ inches.

T_s Score >	T_i Score <	Average (μ)	Standard deviation (s)	Estimated standard deviation $\hat{\sigma}_{\text{estimate}} = \frac{\bar{s}_{\text{Avg}}}{C_4}$	$C_p = \frac{T_{\text{up}} - T_{\text{lo}}}{6 \times \sigma}$	$C_{pk}^{\text{up}} = \frac{T_{\text{up}} - \mu}{3 \times \sigma}$	$C_{pk}^{\text{lo}} = \frac{\mu - T_{\text{lo}}}{3 \times \sigma}$
5.025	4.750	4.9997	0.0591	0.07436	0.61143	0.1117	1.1170

Table 1.30. Calculation results table (indicative statistics)

On the basis of our results, we see that C_p and C_{pk} (in other words, the expressed capability) are better than the performances expressed by P_p and P_{pk} . We deduce that the assignable cause created by a large drift could be the reason for this fluctuation, and we should therefore fix the detuning.

Third case: consider the results of a capability trial taken on the sheet provided by the metrologist. Reading the log and graph, we find the following statistics:

The target nominal diameter is $\varnothing 5$ inches.

$$\text{Given, } \{8 \times s\} = 0.4000 \text{ then } s = \left(\frac{0.4000}{8} \right) = 0.0500$$

T_s Score >	T_i Score <	Average (μ)	Standard deviation (σ)	Estimated SD $s_{\text{estimated}} = \left(\frac{0.4}{8} \right)$	$C_m = \frac{T_{\text{up}} - T_{\text{lo}}}{6 \times \sigma}$	$C_{mk}^{\text{up}} = \frac{T_{\text{up}} - \mu}{3 \times \sigma}$	$C_{mk}^{\text{low}} = \frac{\mu - T_{\text{low}}}{3 \times \sigma}$
5.025	4.750	5.0000	0.0591	0.0500	0.9167	0.1667	1.6667

Table 1.31. Calculation results table (indicative statistics)

1.10.6. Calculating machine capability indices: C_m and C_{mk}

If the norm (CNOMO and ISO) demands a value of approximately 1.67 for both indices, we can consider that the machine is barely capable for C_m . It is capable for C_{mk} . We recommend thinking about an average-centering procedure to aim for the target, which is 5 inches (true metrological value: see VIM)

Fourth case: Consider the data above, with a target value of 5 inches (true metrological value: see VIM)

The target is 5.00 for $n > 100$ samples, and the statistics give us:

$$\hat{\sigma}_{\text{estimate}} = \sqrt{(\text{Target} - \mu)^2 + \frac{\sum_{i=1}^n (x_i - \mu)^2}{n-1}} \quad [1.37]$$

T_s Score >	T_i Score <	Average (μ)	SD (σ)	SD estimate $s_{\text{estimated}} = \left(\frac{0.4}{8} \right)$	$C_{mp} = \frac{T_{\text{up}} - T_{\text{lo}}}{6 \times \sigma}$	$C_{mp}^{\text{up}} = \frac{T_{\text{up}} - \mu}{3 \times \sigma}$	$C_{mp}^{\text{low}} = \frac{\mu - T_{\text{low}}}{3 \times \sigma}$
5.025	4.750	5.0000	0.0591	0.0500	0.776836	0.1412	1.4124

Table 1.32. Calculation results table (indicative statistics)

We see that the C_{mp} values are far from C_p as the average's (μ) detuning is fairly pronounced. We should stop production, and seriously check why there is a detuning.

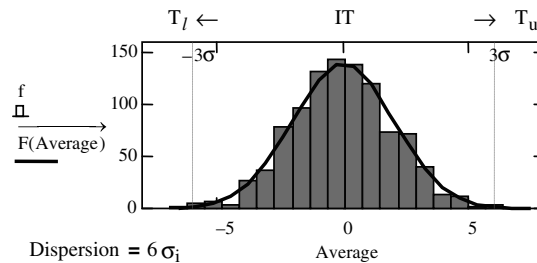


Figure 1.37. Graphical representation of the production results

1.11. Conclusion

Mastery of statistical tools (capabilities), and thus mastery of machine-tool variances (both inherent and personalized) greatly helps predict the machine tool's ability to machine a new component without recurring to new machining simulations.

Furthermore, we have seen that TIs are recurring characteristics that have stood out throughout this chapter. As these come from the drawing plan (design and machining range), simply changing the TIs is not professional, because then the entire method imposed by the specifications would be subject to doubt.

1.12. Bibliography

- [AND 84] ANDERSON T.W., *Introduction to Multivariate Statistical Analysis*, 2nd ed., Wiley, New York, NY, 1984.
- [BAI 80] BAILLARGEON G., *Introduction aux méthodes statistiques en contrôle de la qualité*, Québec, Canada, vol. 4, p. 14. Ed. SMG, 1980.
- [BIS 90] BISSEL A.F., "How reliable is your capability Index?", *Applied Statistics*, vol. 39, pp. 331–340, 1990.
- [BOI 01] BOITEUX B., *Guide du technicien qualité, Outils pour la qualité en production*, Éditions Delagrave, Paris: France, pp.165–169, 2001.
- [CAM 52] CAMERON J.M., "Tables for constructing and for computing the operating characteristics of single-sampling plans", *Industrial Quality Control*, vol. 9, pp. 37–39, July 1952.
- [CAV 66] CAVÉ R., *Le contrôle statistique des fabrication*, Eyrolles, France, 1966.
- [CER 86] CERESTA, *Aide-mémoire pratique des techniques statistiques pour ingénieurs et techniciens supérieurs*, Paris, p. 274, 1986.

- [DEM 86] DEMING. W. E., *Out of the Crisis*, MIT Center for Advanced Engineering Study, 1986.
- [DOD 59] DODGE H.F., ROMIG G.G., WALTER A., *Sampling Inspection Tables*, John Wiley and Sons, New York, NY, 1959.
- [DUN 86] DUNCAN A.J., *Quality Control and Industrial Statistics*, 5th ed., Irwin, Homewood, IL, 1986.
- [GRO 11] GROUS A., *Applied Metrology for Manufacturing Engineering*, ISTE Ltd, London and John Wiley and Sons, New York, 2011.
- [JOH 98] JOHNSON R.A., WICHERN D.W., *Applied Multivariate Statistical Analysis*, 4th ed., Prentice Hall, Upper Saddle River, NJ, 1998.
- [JUR 89] JURAN., J.M., *Planifier la qualité*, Coll. AFNOR Gestion, France, 1989.
- [LIN 65] LINDLEY D.V., *Probability and Statistics 2 Inference*, Cambridge University Press, 1965.
- [MON 00] MONTGOMERY D.C., *Introduction to Statistical Quality Control*, 4th ed., Wiley, New York, NY, 2000.
- [OTT 90] OTT E.R., SCHILLING E.G., *Process Quality Control*, 2nd ed., McGraw-Hill, New York, NY, 1990.
- [PAC 69] PACÉ P., *Contrôle des fabrications, Statistique 2: Essais-Réglages-Réception-Fiabilité*. Éditions Delagrave, Paris, France, 1969.
- [POI 92] POIRIER J., *Observation statistique*, R 240, traité Mesures et Contrôle juil, 1992.
- [QUE 93] QUESENBERRY C.P., "The effect of sample size on estimated limits for $\bar{x}$ and $\bar{X}$ control charts", *Journal of Quality Technology*, vol. 25, no. 4, pp. 237–247, 1993.
- [RYA 89] RYAN T., *Statistical Methods for Quality Improvement*, Wiley & Sons, 1989.
- [RYA 00] RYAN T.P., *Statistical Methods for Quality Improvement*, 2nd ed., Wiley, 2000.
- [SCH 82] SCHILLING E.G., *Acceptance Sampling in Quality Control*, Marcel Dekker, New York, NY, 1982.
- [SHE 39] SHEWHART W.A., *Statistical Method from the Viewpoint of Quality Control*, Dover, New York, NY, 1939.
- [SHE 80] SHEWHART WALTER A., *Economic Control of Quality of Manufactured Product*, American Society for Quality, 1980.
- [TRA 92] TRACY N.D., YOUNG J.C., MASON, R.L., "Multivariate control charts for individual observations", *Journal of Quality Technology*, vol. 24, no. 2, pp. 88–95, 1992.
- [WOO 93] WOODAL W.H., ADAM B.M.A., "The statistical design of CUSUM charts", *Quality Engineering*, vol. 5, no. 4, pp. 559–570, 1993.
- [WOO 97] WOODAL W.H., "Control charting based on attribute data: bibliography and review", *Journal of Quality Technology*, vol. 29, pp. 172–183, 1997.

Chapter 2

Quality Control Case Studies

2.1. The tools of quality, as per W. Deming

In quality control, the method given by [DEM 86, DEM 02, SHE 80, SHE 31, SHE 39, SCH 82, SCH 85] is clear, simple and pragmatic in principle. The statistical tools are less complex than those used in material and structure reliability. The basic statistical analysis elements are the same, but risk analysis is slightly different. We will give the reasons for this along the presentations below.

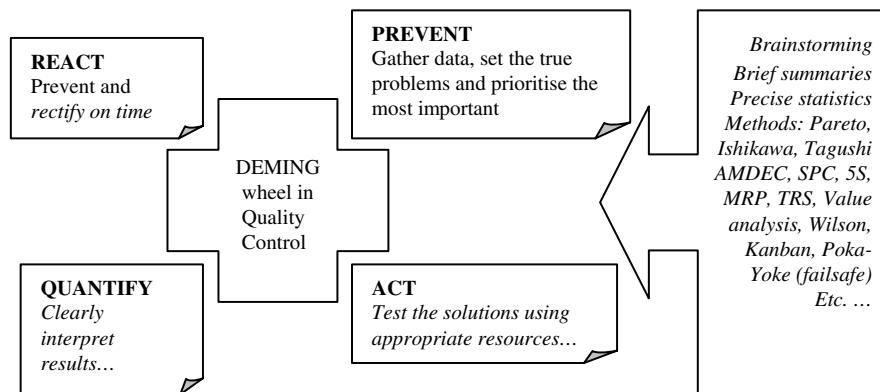


Figure 2.1. Quality tools as per W.Deming

2.2. Failure modes, effects and criticality analysis

Failure modes, effects and criticality analysis (FMECA) is a method of reliability analysis that allows us to take inventory of the failures with consequences affecting the system's operation.

1) “*Product*” FMECA → allows us to act from design while ensuring that the product performs adequately.

2) “*Process*” FMECA → concerns the product in its production phase and checks the production process' impact on the product's conformity.

3) “*Process or machine*” FMECA → concerns the resources used in manufacturing the product.

- *Approach*: The system being studied is a parallel lathe that operates for 16 h a day, with an hourly production of 50 Stanley cups with production costs of \$200 per hour.

- *Study group*: The study group consists of the maintenance manager (bear), three (03) of his/her agents and two (02) technicians.

- *Actions taken*: The greatest criticality (i.e. 18) is associated with wear and tear on the belt. The service takes a preventive action by changing the belts every four months.

2.3. Total productive maintenance method

This Japanese method puts the *responsibility of maintaining* their equipment on the people involved in production. By controlling it themselves, production would be more group oriented.

– *Maintenance*: Repair, clean, grease, and accept that some time will be required to do it.

– *Production*: Manufacture as few penalizing rejections as possible.

– *Total*: Seek a global yield for installations by getting “everyone” involved.

Method for total productive maintenance (TPM) gives first-level tasks such as clean, calibrate, lubricate. The goal is to improve the availability of materials through a detailed analysis of their productive and unproductive periods.

F = Frequency	D = Non-detection	G = Gravity (unavailability)	1) Failure analysis: The detailed study of work orders after the malfunction allowed us to record two modes of failure with their associated causes and effects (see table below) 2) Criticality calculations: With the values of F, D and G, we can calculate the criticality to give each failure.
One failure per year at most	Can be checked by operator	Production is not stopped	
One failure per trimester at most	Easily detected by the maintenance crew	Stop ≤ 1 h	
One failure per month at most	Hard to detect	$1 \text{ h} < \text{stop} \leq 1 \text{ day}$	
One failure per week at most	Undetectable	Stop $> 1 \text{ day}$	

Analysis of modes of failure, Their effects and their criticality										
System (subsystem or element)	Failures			Criticalities			Actions taken	Evolution		
	Modes	Effects	Causes	F	D	G		F	D	G
							$C = F \times D \times G$			
Kinetic movement	Stuck tool carriage	Non- conforming advances	Failure of the lead screw	1	2	3	6			
Kinetic movement	Stuck tool carriage	Non- conforming advances	Defective bearing	1	2	3	6			
Kinetic movement	Defective speed	Non- conforming advances	Unbalanced mandrel	2	3	3	18	Rectify gears every 4 months	1	3 2
										6

Table 2.1. FMECA: application on a concrete example in a design and New Work Organization (NWO) Workshop

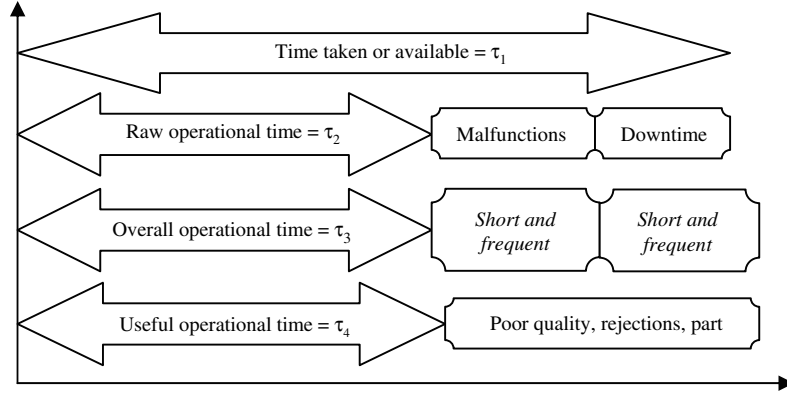


Figure 2.2. Breakdown of productive and unproductive periods

- Improving the *raw operational time* implies reducing the downtimes due to maintenance.
- Improving the *performance metric* implies reducing the small downtimes related to production.
- Improving the *quality metric* implies reducing the amount of bad components.

When differentiating between productive and unproductive times, the operational hours are the sum of the downtime periods and the raw operational time. The overall equipment effectiveness (τ_{OEE}) is a product of the other three:

$$\tau_{OEE} = (TA_F) \times (T_P) \times (T_Q) = \left(\frac{T_P}{T_{operation}} \right) \quad [2.1]$$

$$\left\{ \begin{array}{l} T_{\text{raw operational}} = \left(\frac{\text{operational time}}{\text{time needed or available}} \right) = \left(\frac{\tau_2}{\tau_1} \right) \\ T_{\text{Performance}} = \left(\frac{\text{overall operational time}}{\text{raw operational time}} \right) = \left(\frac{\tau_3}{\tau_2} \right) \\ T_{\text{Quality}} = \left(\frac{\text{useful operational time}}{\text{overall operational time}} \right) = \left(\frac{\tau_4}{\tau_3} \right) \\ T_{\text{operation}} = \left(\frac{\text{downtime}}{\text{raw operational time}} \right) = \left(\frac{\tau_5}{\tau_3} \right) \end{array} \right\} \quad [2.2]$$

The OEE is an excellent indicator of a material's availability and the quality and efficiency of its methods of exploitation and maintenance. In mechanical machining, productive time is the time spent manufacturing "good" components in terms of quality control, that is components that conform to pre-established tolerance limits.

The time spent on corrective and preventative maintenance, making machines operational again, tool calibration (spreading the cutting tools) and replacement, downtime between series (if production is serial) all makes up the so-called "unproductive" time. We must also consider time spent manufacturing rejected or replaced components, which contributes (with production time) to the overall production time. Unfortunately, time corresponding to inopportune situations inherent to a use in under-capacity must appear in a simo (*simogramme*) chart.

Example of simogram using Matssura (CNC), CNC: Computer Numerical Control

Phase	Machine – Matssura	Component		SAE	Program	
10	CNC milling machine	Precision vice base		1,070	30 components	
Series time (preparation)						
PRE.	Designate the operations and series time (preparation)					
a	Start the machine up and perform the zero machine					
b	Install the device to clamp the vice's base					1,000
c	Install and measure tools				2,000	
d	Take the zero component				750	
e	Check parameters				500	
f	Machine a first component				4,000	
g	Organize the post's metrology				600	
h	Strip and clean the post				4,000	
			Series time		13,200	

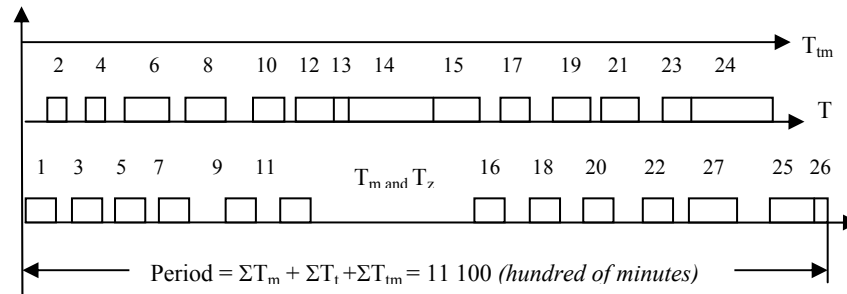
SAE– Society of Automotive Engineering, CNC– Computer Numerical Control, PRE– Time of preparation to machining, OP– Times dedicated to operations for machining

Table 2.2. *Simo chart for the production of 30 components (clamp base 1,070) – real case*

OP.	Operation	Tools	Time in min/100			
			T_t	T_{tm}	T_m	T_z
1	Set the component (clamping)	Machining clamping			200	
2	Surface side 1	Milling cutter	200			
3	Change the component's side	Machining clamping			200	
4	Surface side 2	Milling cutter	200			
5	Change the component's side	Machining clamping			200	
6	Surface side 3	Milling cutter	600			
7	Change the component's side	Machining clamping			200	
8	Surface side 4	Milling cutter	600			
9	Change the component's side	Machining clamping			200	
10	Surface side 5	Milling cutter	600			
11	Change the component's side	Machining clamping			200	
12	Surface side 6	Milling cutter	600			
13	Point and pierce (groove)	Spot drill and drill bit	300			
14	Groove boring	End-bore	1,000			
15	Groove top	End-bore	650			
16	Change the component's side	Machining clamping			200	
17	Groove side 1	Three-sized bore	550			
18	Change the component's side	Machining clamping			200	
19	Groove side 2	Three-sized bore	550			
20	Change the component's side	Machining clamping			200	
21	Groove side 3	Three-sized bore	550			
22	Change the component's side	Machining clamping			200	
23	Groove side 4	Three-sized bore	550			
24	Point and pierce (groove)	Spot drill and drill bit	1,200			
25	Dismantle clamping	Machining clamping			200	
26	Clean clamping mechanism	Control clamping			50	
27	Control the component	Metrology				700
		Total	8,150		2,250	700

Table 2.2 (continued). *Simo chart for the production of 30 components (clamp base 1,070) – real case*

Total time taken to manufacture 30 components using a CNC mill in a lab setting.



$$\tau_{total} = T_S \times \sum_{i=1}^u (T_u) = 13200(11100 \times 100) = 2\,430\,000 \text{ (Hundred of minutes)}$$

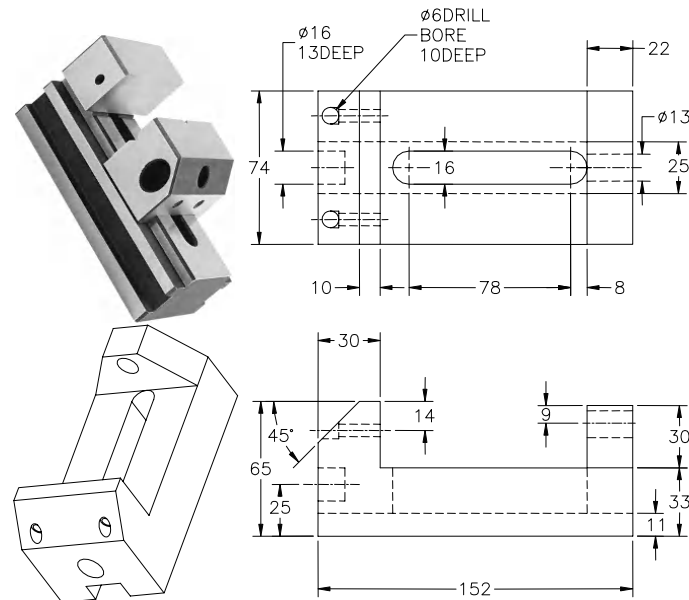


Figure 2.3. Definition drawings of the clamp base used with machine tools

2.4. The LMMEM “5M” process method

Usually, machining (and production in general) relies on the “5M, in *French* method”, because the same production processes never end up with the same product exactitude (toleranced component). This method is:

1. Labor → 2. Machine → 3. Materials → 4. Environment

We know that there are many dispersions around the target score, hence the need for the tolerance interval (TI) (e.g. $\varnothing 5 \pm 1/100$ inch). It is worth noting that variability around this tolerance is inevitable. Pragmatically, we must then be able to consider the truth around this true value. These variations are due to various factors in the production process. Production process analysis allows us to differentiate between five (5) basic elements that contribute to non-quality.

5S method: This organization technique incorporates and implicates all workers on all levels. It motivates all levels of the industry by easing the routine and improves worker comfort and safety.

5S	Meaning	Purpose	Main actions
Get rid of (<i>Seiri</i>)	Sort between useful and useless	Avoid getting lost in the workpost	Sort and identify
Clean up (<i>Seiton</i>)	Put useful items in easily accessed places	Get rid of wasted time to improve productivity	Store clearly and effectively
Keep clean (<i>Seiso</i>)	Clean your work space	Detect malfunctions	Clean and embellish the workplace (morale)
Standardize (<i>Seiketsu</i>)	Define the 5S rules	Turn the 5S into a habit	Set priorities
Implicate (<i>Shitsuke</i>)	Lead by example	Change behavior	Train workers and record results

Table 2.3. *5S method (French/Japanese)*

2.5. Estimations of times in mechanical productions (machining)

$$T_u = \frac{T_s}{N} + \sum_i \left(\left(\frac{L_c}{V_f} + T_{tm} + T_m \right) + \sum_i T_{mo} \left(\frac{T_t}{T} - \frac{1}{N} \right) \right) \quad [2.3]$$

where:

- N is the number of components in the series.
- T_u is the time per unit (component), in minutes.
- T_{mo} is the machine downtime (due to tool changes/sharpening), in minutes.
- T_m are human times, in minutes.
- T_{tm} are technical human times, in minutes.
- T_s are the preparation times, in minutes.
- T_t are the technological times, in minutes.

- T is the lifespan of a cutting tool's patch, in minutes.
- L_c is the total range of machining (e.g. tuning) in millimeters.
- V_f is the rate of advance in m/min. When turning, T_t takes the following form:

1) *Technological time in minutes*

$$T_t = \frac{L_c}{(10^3 \times V_f)} \quad [2.4]$$

The production that minimizes time, based on Taylor's law, is generally used during fast machining operations with constant speed, feed and cutting depth. Using the model of a TOWER-based turning pass, we find the following intrinsic material constant (C), which is written as:

$$C = V \times T^{-(1/K)} \quad [2.5]$$

This expression can also be written as:

$$T \times V^{-(1/K)} = C^{-(1/K)} \quad \text{or} \quad T = \frac{C^{-(1/K)}}{V^{-(1/K)}} \quad [2.6]$$

2) *Summarized problem of patch lifespan as per Taylor's law*

Consider having to calculate the lifespan of a tool with the following cutting conditions: K and C are the intrinsic material parameters (e.g. coefficients given by a manufacturer of cutting tools, such as Coroman-Sandvik); V is the cutting speed in m/min; and T is the cutting patch's (or tool's) lifespan (in minutes) according to Taylor's law, if:

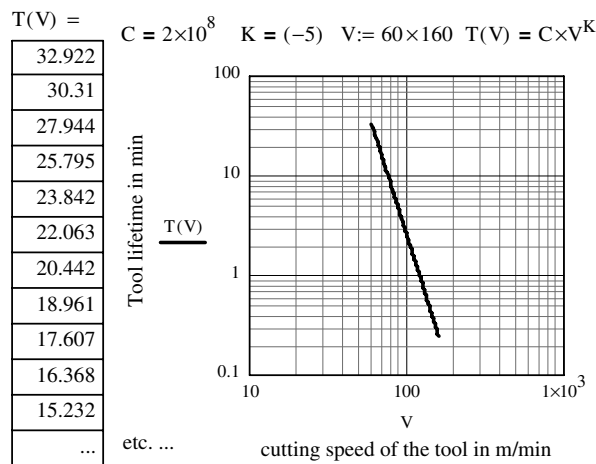


Figure 2.4. *Lifespan of the cutting patch (tool) in minutes – real case*

Tool cutting time (patch) in minutes. From [2.6], let us define

$$T_{\text{tool}} = C \times (V_{\text{cut}})^K \text{ (min)} \quad [2.7]$$

By grouping the times T_s/N , T_m and T_{im} together and replacing T and V_f with their respective expressions, the useful time becomes:

$$T_u = \left(\frac{T_s}{N} + T_{im} + T_m \right) + \left(\frac{\pi \times D \times L_c}{10^3 \times f \times V} \right) + \left(\frac{\pi \times D \times L_c \times V^{-(K+1)}}{10^3 \times f \times C^{-K}} - \frac{1}{N} \right) \times T_{mo} \quad [2.8]$$

with V the cutting speed in m/min and D the diameter (in millimeters) to be turn passed on the machine (lathe).

3) *Time assignable to preparations and to periods of non-cutting (non-machining), in minutes*

$$T_1 = \left(\frac{T_s}{N} + T_{im} + T_m \right) \quad [2.9]$$

– We clearly see that T_1 (in minutes) is independent of both the cutting speed (V) and the advance per turn (f).

– T_{im} are the technical-human times, in minutes.

– T_m are the human times, in minutes.

– As for the cutting time (T_t), it can be written in minutes:

$$T_t = \left(\frac{\pi \times D \times L_c}{10^3 \times f \times V} \right) \quad \text{or} \quad T_t = \left(\frac{L_c}{10^3 \times f \times V} \right) \quad [2.10]$$

As for the cutting time (T_t), it can be written in minutes:

$$T_3 = \left(\frac{\pi \times D \times L_c \times V^{-(K+1)}}{10^3 \times f \times C^{-K}} - \frac{1}{N} \right) \times T_{mo} = \left(\frac{\pi \times D \times L_c \times V^{-(K+1)}}{10^3 \times f \times C^{-K}} - \frac{1}{N} \right) \times T_{mo} - \left(\frac{T_{mo}}{N} \right) \quad [2.11]$$

4) *Minimum machine downtime due to tool changes, in minutes*

For cutting conditions (f , a) chosen to be the largest possible, and taking the machine–tool component set's limitations in terms of power and rigidity into

account, the optimization of time T_u from expression [2.3] is determined by finding the value for V that minimizes time. This value cancels the derivative of the expression for T_u with respect to V . We then set:

$$\frac{d^1}{dV^1} T_u = -1 \times \left(\frac{\pi \times D \times L_c}{10^3 \times f \times V^2} - \frac{1}{N} \right) - (K+1) \left(\frac{\pi \times D \times L_c \times V^{-(K+2)}}{10^3 \times f \times C^{-K}} \right) \times T_{mo} \quad [2.12]$$

For $((d^1 / dV^1) T_u) = 0$, time is minimized for (V) equal to:

$$V_{\text{time-min}} = -K \sqrt[3]{\frac{C^{-K}}{(-k-1) \times T_{mo}}} \quad [2.13]$$

– m_o is tool machine. Let then T equal to $T_{\min}$:

$$T_{\min} = (-k-1) \times T_{mo} \quad [2.14]$$

5) Numerical applications, declaring given values

$$L_c = 120(\text{mm}); \quad D = 50(\text{mm}); \quad f = 0.2 \left(\frac{\text{mm}}{\text{min}} \right); \quad V = 20 \left(\frac{\text{m}}{\text{min}} \right); \quad N = 500 \text{ parts}$$

$$T = 42(\text{min}); \quad T_s = 10(\text{min}); \quad T_{mo} = 8(\text{min}); \quad T_m = 11(\text{min}); \quad \frac{T_{mo}}{N} = \frac{8}{500} = 0.016$$

$$\text{Choice}_{\text{Manufacturer}} \rightarrow C_{\text{steel}} = 2.9 \times 10^8; \quad K_{\text{steel}} = (-5); \quad T_{lm} = 49(\text{min})$$

6) Results of our real calculations

$$\text{Linear advance speed: } V_f = \frac{V \times f \times 1,000}{\pi \times D} = \frac{20 \times 0.2 \times 1,000}{\pi \times 50} = 25.465 \left(\frac{\text{m}}{\text{min}} \right)$$

$$\text{Patch cut time: } T_1 = \left(\frac{T_s}{N} + T_{lm} + T_m \right) = \frac{10}{500} + 49 + 11 = 60.02 \text{ min}$$

where T_s are the preparation times in minutes.

7) T_t are the technological times in minutes

$$T_t = \left(\frac{\pi \times D \times L_c}{f \times V} \right) \cong \left(\frac{10^3 \times L_c}{f \times V} \right) = \left(\frac{\pi \times 50 \times 120}{0.2 \times 20} \right) \cong \left(\frac{120 \times 10^3}{0.2 \times 20} \right) = 4.712 \times 10^3 \text{ (min)}$$

The material's intrinsic constant (C) (given by the manufacturer or calculated by interpolation) can be written as:

$$C = V \times T^{-(1/K)} = 20 \times 42^{-(1/(-5))} = 42.236$$

8) Unitary time (3) for N (number of machined components) = 500 parts

$$\begin{aligned} T_3 &= \left(\frac{\pi \times D \times L_c \times V^{-(K+1)}}{10^3 \times f \times C^{-K}} - \frac{1}{N} \right) \times T_{mo} - \left(\frac{T_{mo}}{N} \right) \\ &= \left(\frac{\pi \times 50 \times 120 \times 20^{-(5+1)}}{10^3 \times 0.2 \times (2.9 \times 10^8)^{-(-5)}} - \frac{1}{500} \right) \times 8 - \left(\frac{8}{500} \right) = 0.882 \text{ (min)} \end{aligned}$$

9) T_u is the unit time (per component) in minutes

$$T_u = (T_1 + T_t + T_3) = (60.02 + 4.712 \times 10^3 + 0.882) = 4.773 \times 10^3 \text{ (min)}$$

10) True cutting speed (m/min or po/min) sampled during machining

$$V_{\text{time min}} = \sqrt[k]{\frac{C^{-K}}{(-k-1) \times T_{mo}}} = \sqrt[(-5)]{\frac{(42.236)^{-(-5)}}{(-(-5)-1) \times 8}} = 21.118 \left(\frac{\text{m}}{\text{min}} \right)$$

11) Minimum cutting time: $T_{\min} = (-k-1) \times T_{mo} = [-(-5) - 1] \times 8 = 32 \text{ (min)}$

2.5.1. Optimizing times and costs in mechanical production

The following case study takes the cutting conditions below into account.

– Starting data

$$\text{For : } T = 40 \text{ min; } T_s = 10 \text{ min; } T_m = 11 \text{ min; } T_{mo} = 8 \text{ min; } \frac{T_{mo}}{N} = 0.016$$

$$K = -5; L_c = 120 \text{ mm; } D = 50 \text{ mm; } f = 0.16 \text{ mm/tr; } V = 32 \text{ m/min; } N = 500 \text{ RPM}$$

– Simulation for a cutting speed (V) varying between 16 and 320 m/min

Designation	Formula	Result
Cutting speed based on the advance (m/min)	$V_f = \frac{V \times f \times 10^3}{\pi \times D}$	$V_f = 32.595$
T_1 in minutes	$T_1 = \left(\frac{T_s}{N} + T_{tm} + T_m \right)$	$T_1 = 60.02$
Technological cutting time in minutes	$T_t = \frac{L_c}{V \times f \times 10^3}$	$T_t = 3.682 \times 10^3$
Intrinsic material parameters	$C = V \times T^{(-1/K)}$	$C = 66.921$
Machining time (min)	$T_3 = \frac{L_c \cdot \pi \cdot D \cdot V^{(-K-1)}}{f \times C^{(-K)} \times 10^3} \times T_{mo} - \frac{T_{mo}}{N}$	$T_3 = 0.72$
Total useful time (min)	$T_u = T_1 + T_t + T_3$	$T_u = 60.744$
Minimum cutting time (m/min)	$V_{\text{time-min}} = (-K) \sqrt{\frac{C^{-K}}{(-K-1) \times T_{mo}}}$	$V_{t/m} = 33.46$
Minimum time (min)	$T_{\min} = (-K-1) \times T_{mo}$	$T_{\min} = 32$

Table 2.4. Summary of the usual cutting condition optimization formulas

– Educational example of the presentation of a quality control range

Control range during production								
Company: Badger LeNain		Control range		Quality-metrology service				
Component reference:		Range reference:		Date:				
Component sketch or product's definition drawing								
Check that the references are right angled before any measurements								
Scores to control		Allowance IT	Metrology	Score gravity/limiting values			Method of control	
Symbol	Nominal	Score	Average	Critical	Major	Minor	Calibration	Fabrication dimension
Relevant notes:								
Control chart for score x:								
Chart 1	Chart 2	...	...	...	...	...	...	CHART n
Control material available to the department, such as:						CMM	...	...
PP: Profile projector			Vernier scale		Micrometer		...	etc...
Control: during production			Sampling:		Control tuning manager:			

Table 2.5. Sample suggestive model for the range of quality control

T3(V) =		Tt(V) =		Tu(V) =		
	0		0		0	
0	0.03	0	$7.363 \cdot 10^3$	0	$7.423 \cdot 10^3$	
1	0.043	1	$6.93 \cdot 10^3$	1	$6.99 \cdot 10^3$	
2	0.058	2	$6.545 \cdot 10^3$	2	$6.605 \cdot 10^3$	
3	0.076	3	$6.201 \cdot 10^3$	3	$6.261 \cdot 10^3$	
4	0.096	4	$5.89 \cdot 10^3$	4	$5.951 \cdot 10^3$	
5	0.121	5	$5.61 \cdot 10^3$	5	$5.67 \cdot 10^3$	
6	0.148	6	$5.355 \cdot 10^3$	6	$5.415 \cdot 10^3$	
7	0.181	7	$5.122 \cdot 10^3$	7	$5.182 \cdot 10^3$	
8	0.217	8	$4.909 \cdot 10^3$	8	$4.969 \cdot 10^3$	
9	0.258	9	$4.712 \cdot 10^3$	9	$4.773 \cdot 10^3$	
10	0.305	10	$4.531 \cdot 10^3$	10	$4.591 \cdot 10^3$	
11	0.357	11	$4.363 \cdot 10^3$	11	$4.424 \cdot 10^3$	
12	0.416	12	$4.207 \cdot 10^3$	12	$4.268 \cdot 10^3$	
13	0.481	13	$4.062 \cdot 10^3$	13	$4.123 \cdot 10^3$	
14	...	14	...	14	...	etc. ...

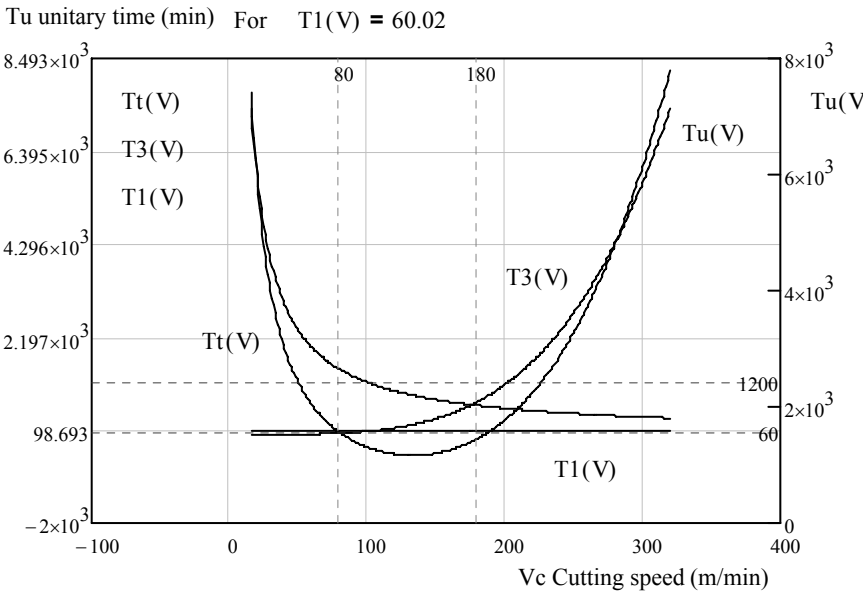


Figure 2.5. Estimated manufacture time as a function of the cutting speed – simulated case

2.5.1.1. *Optimizing the cutting parameters, in machining, as per ISO 3685*

We chose to introduce this example of an optimization of cutting parameters in quality control for reasons of industrial automation. In production ranges, the machinist is often asked to follow the production rate within the deadlines given to machining. This is achievable on paper, but in reality (in the workshop), “things” sometimes go awry due to the vagaries of metrology and quality control organization, among other things.

In mechanical production (production engineering), machining itself is no problem. Organization, on the other hand, can affect parameter choices in cut, tools, machining devices and component “movements” (outlines). A “real” machining range is also subject to these vagaries, which are often difficult to optimize depending on the resources available. To optimize means to produce within constraints toward a satisfactory goal. The various criteria that affect the optimization model can thus be grouped:

- 1) Minimum machining time versus maximum production rate (min).
- 2) Produce a (controlled) volume of shavings per edge cut (clever choices of thicknesses and various dispersions) (mm³).
- 3) Target the minimum machining cost (\$).

We show the lifespan (T) calculations for the cutting tools based on Taylor’s law (see Figure 2.4), as per ISO 3685: “the total cutting time required for a tool to reach a specific lifespan criterion”. This longevity requires regular machine downtimes to change the patch’s edge. These downtimes condition the machining time and thus the machining costs (in CND\$)

2.5.1.2. *Machining time, τ_m*

- τ_c , true cutting time – when the tool cuts through shavings;
- τ_i , unproductive time in the machining post (setup and teardown; rapid movements such as advances and speeds);
- τ_{cs} , time taken to change a cutting edge (patch, tool), that is time to change the patch on the machining post;
- T , lifespan of the cutting tool (edge) that creates the shavings, that is its longevity as per the simplified Taylor law.

A component’s machining time (in min) will thus be calculated as follows:

$$\tau_m = \tau_c + \tau_i + \left(\frac{\tau_{cs}}{P} \right) \text{in (min)} \quad [2.15]$$

where:

- P is the cost of a cutting edge (tool or patch) in CND\$;
- C_f is a set cost per component (tools, raw materials, assembly elements, etc.);
- C_m is the machine's cost in CND\$/min. It includes the machinist's salary, paying off the machine tool as well as the direct exploitation costs (electric charges, lubricants, compressed air, workshop maintenance, etc.).

The costs induced in machining a component come from the sum of the costs that make up the following model:

$$\left(\begin{array}{l} C_c = C_m \times \tau_c ; \quad C_i = C_m \times \tau_i \\ C_m = C_c \times C_i ; \quad C_s = P_o \times \frac{\tau_c}{T} \\ C_{cs} = C_m \times \left(\frac{\tau_c}{P} \right) \quad \text{et} \quad P = \frac{T}{\tau_c} \end{array} \right) \text{ in (CND\$)} \quad [2.16]$$

where:

- C_c is the machining cost per component in Canadian dollars (CND\$);
- C_i is the unproductive cost per component (CND\$);
- C_m is the machine's cost per component (CND\$);
- C_s is the machine (patch) cost per component (CND\$);
- C_{cs} is the cost of changing tools per component (CND\$);
- P is the number of components between two tool changes (edge, patch).

Based on the above, we calculate a component's machining cost by:

$$C_u = \sum_{\text{total}} \text{Costs} = \{C_c + C_i + C_{cs} + C_f\} \text{ in [CND\$]} \quad [2.17]$$

– *Application:* for $C_m = 25$ CND, $\tau_m = 1$ min and $\tau_i = 0.25$ min, we have:

$$C_c = C_m \times \tau_c = 25 \left[\frac{\text{CND\$}}{\text{part}} \right] \text{ and } C_i = C_m \times \tau_i = 6.25 \left[\frac{\text{CND\$}}{\text{part}} \right]$$

which gives us C_{machine} : $C_m = C_c + C_c = 31.25 \left[\frac{\text{CND\$}}{\text{part}} \right]$

Let the price of a single patch edge from our manufacturer Sandvik be $P_0 = 10$ CND\$ and we seek production over a lifespan (T) ranging from 1 to 10 min. We can then calculate the cost per component as a function of (T): $C_s(T)$ is then

$$C_s(T) = P_0 \times \left(\frac{\tau_c}{T} \right) \text{ in } \left[\frac{\text{CND\$}}{\text{part}} \right] \text{ with the following results:}$$

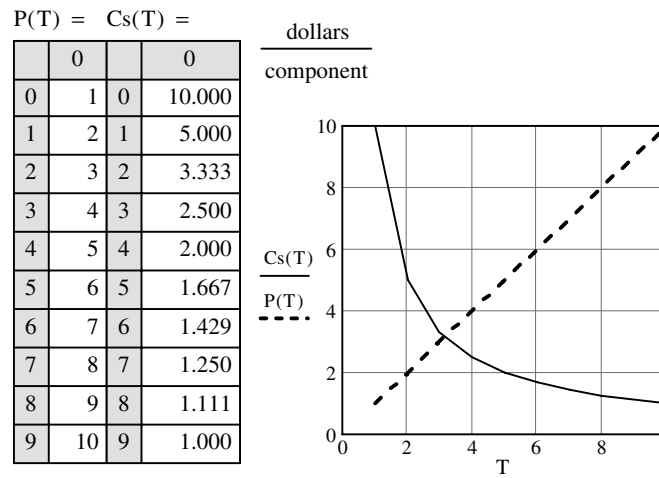


Figure 2.6. Cost evolution as a function of longevity and the number of machined components

C_{cs} is the cost to replace a tool per component. Note the evolution of the costs induced by tool changes as a function of the number of components to manufacture. $P = (1-10)$ components:

$$C_{cs}(P) = C_m \times \left(\frac{\tau_c}{P} \right) \left[\frac{\text{CND\$}}{\text{part}} \right] \text{ for } P = 1, 2, \dots, 10$$

In the same calculation conditions as above, if we consider that the tool cost per component $C_s = 8$ CND\$ and the cost of changing a tool (edge) by component $C_{cs} = 25$ CND\$, the total cost of machining a single component would then be:

$$C_u = \sum_{\text{total}} \text{Costs} = \{C_c + C_i + C_{cs} + C_f\} = 89.25 \left[\frac{\text{CND\$}}{\text{part}} \right]$$

If the tool (patch) change takes 5 min, the machining time per component $\tau_m(P)$ will thus be calculated:

$$\tau_m(P) = \tau_c + \tau_i + \left(\frac{\tau_{cs}}{P} \right) \left[\frac{\text{CND\$}}{\text{part}} \right] \text{ for } P = 1, 2, \dots, 10$$

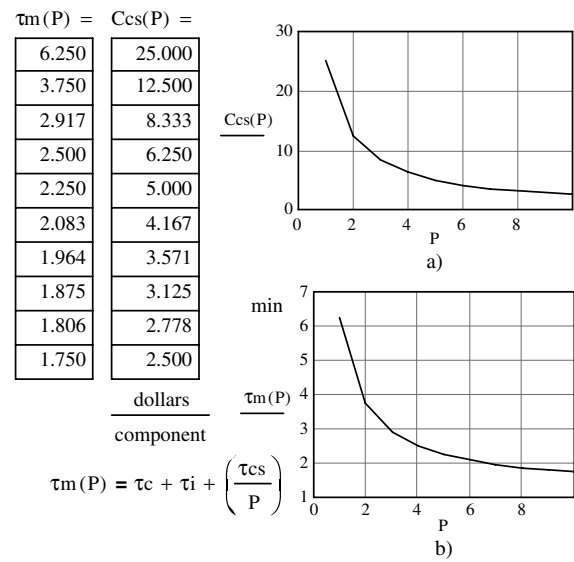


Figure 2.7. (a) Evolution of costs as a function of tool changes per component;
(b) Evolution of the machining time per component

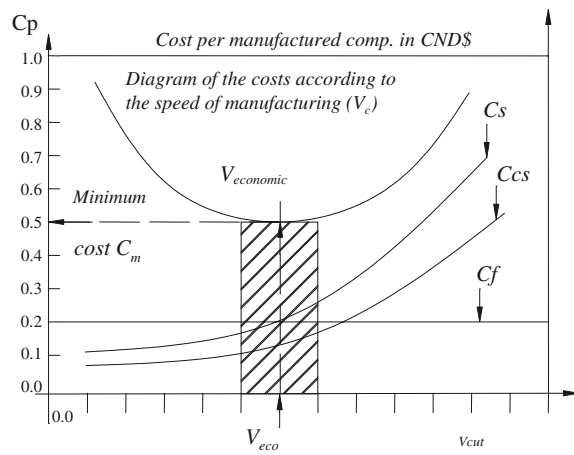


Figure 2.8. Economic costs per component as a function of the cutting speed V_c

The lifespan of cutting tools does not give us a problem to be solved. We recommend Taylor's law for simplicity, though other approaches exist. We notice that the economic lifespan (T_e) is a function of the clever choice of the economical cutting speed (V_c). As for the machining cost, we are often faced with restricting choices that force us to compromise. The machining conditions are never the same twice due to shaving and machining thicknesses, acceptable limit on the tool's aging, tool geometry, lubricants, materials, assembly stability, etc. The analysis that led us to plot the graph below is based on a machining operation (turn passing of a CM_1 tree).

We must minimize the machining cost, which has the following expression:

$$C_u = \sum_{\text{total}} \text{Costs} = \{C_c + C_i + C_{cs} + C_f\} \left[\frac{\text{CND\$}}{\text{parts}} \right] \quad [2.18]$$

We can easily see that the machining costs are minimized based on the cutting speed at the point where $V_{c(\text{economic})} = C_e$. From this, we express the minimum cost (C_u) induced by machining a component.

– Turn passing application of a tree on a machine score CM_1

Consider

$$C_u = \sum_{\text{total}} \text{Costs} = C_m \times t_c + C_m \times t_i + P_0 \frac{t_c}{T} + C_m \times \frac{t_{cs} \times t_c}{T} + C_f \left[\frac{\text{CND\$}}{\text{turn passing}} \right]$$

The turn passing is done on CM_1 of length (L) over a pass. The cutting time can be expressed as:

$$t_c = \frac{\pi \times L \times D}{1,000 \times f \times V_c} \quad [\text{min}] \quad [2.19]$$

Based on Taylor's law, the lifespan of the cutting tool's edge can be written as $T = C \times V^n$ (min) with (C and N) the intrinsic material parameters. The total machining cost as a function of the cutting speed (V_c) can then be written as:

$$C_u = C_m \frac{\pi \cdot L \cdot D}{10^3 \cdot f \cdot V_c} + C_m \cdot t_i + P_0 \frac{\pi \cdot L \cdot D}{10^3 \cdot f \cdot C \cdot V^{(n+1)}} + C_m \times t_{cs} \frac{\pi \cdot L \cdot D}{10^3 \cdot f \cdot C \cdot V^{(n+1)}} + C_f$$

After factorizing, we end up with the following:

$$C_u = \left\{ \left(\frac{\pi \cdot L \cdot D}{10^3 \cdot f \cdot V_c} \right) \times \left(\frac{C_m}{V_c} + \frac{P_0 + C_m \cdot t_{cs}}{C \cdot V^{(n+1)}} \right) + (C_m \cdot t_i) + C_f \right\} \quad [2.20]$$

Our aim is to minimize C_u . To this end, we must first find the economic speed V_e using its mathematical expression:

$$\frac{\partial C_u}{\partial V_c}(V_e) = 0 \quad [2.21]$$

After some algebra, we find that V_c is as follows:

$$\frac{\partial C_u}{\partial V_c}(V_e) = \frac{\partial C_u}{\partial V_c} \left(\frac{C_m}{V_c} + \frac{P_0 + C_m \cdot t_{cs}}{C \cdot V^{(n+1)}} \right) = 0 \quad [2.22]$$

Finally, we present the expression for the cutting speeds and economic lifespan as developed below, in 2.5.1.3.

2.5.1.3. Economic speed and cutting time in metal turning (CM_1 turn passing)

$$V_e = \sqrt[n]{\frac{-(n+1)((P_0/C_m) + t_{cs})}{C}} \quad \text{and} \quad T_e = -(n+1) \left(\frac{P_0}{C_m} + t_{cs} \right) \quad [2.23]$$

We note that these relations hold for milling, drilling and boring.

2.5.1.4. Minimum machining time and maximum production rate

In serial production (or mass production), the manufacturing cost does not affect the maximum rate of production. For this reason, it is calculated independently from the fabrication cost in minimum time. This is justified in an industrial setting by considering delays and other vagaries not mentioned here.

$$t_m = t_c + t_i + \left(\frac{t_{cs} \times t_c}{T} \right) \quad [2.24]$$

We note that the cutting speed (V_c) is the main parameter. It is controlled in the workshop. The following experimental (theoretical) diagram shows the influence of times on the total rate of production with respect to speed V_c .

The machining time is minimized for a given cutting V_c . This is also the component manufacturing speed. Optimizing thus requires writing the machine time t_m with respect to the speed V_c in order to find the production speed V_p . The volume of chips resulting from the CM_1 machining by cutting edge (patch) depends on the following parameters: m

- the cutting speed V_c relative to a certain machining volume V_d ;
- the cutting parameters, for example the advance a_p and cutting depth f ;
- the patch (edge) lifespan T_a .

If we wanted to minimize the frequency at which the cutting edge was changed during machining, the volume of metal sheared (chips) must be taken into account. Of course, the outline's thickness must be well chosen. Tuning and sharpening the tool (rapid steel, high speed steel - HSS) is costly. Again, we have the cutting speed model, with a graphical representation inspired by the machining models of equipment manufacturers (Sandvik-Coroman) shown below. (ζ) is the volume of chips between two edge changes, as a function of the cutting speed V_C .

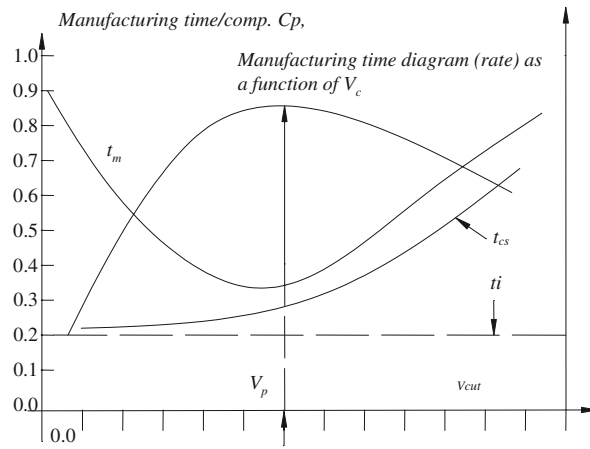


Figure 2.9. Minimum machining time and economic costs per rate of production as a function of the cutting speed V_C – simulation

The optimization technique used here is to write the volume of shavings as a function of the cutting speed V_C to find a speed V_d such that the volume of shavings from the cut be ζ_d . Let ζ be the expression for the machined volume:

$$\zeta_m = f \times a_p \times C \times V_c^{(n+1)} = f \times a_p \times C^{-1/n} \times T^{(n+1/n)} \quad [2.25]$$

For a given volume (ζ_d), the speed and cutting time V_C can be written as:

$$V_c = \left(\frac{\zeta_d}{f \times a_p \times C} \right)^{(1/(n+1))} \quad \text{and} \quad T_d = \left(\frac{\zeta_d \times C^{(1/(n+1))}}{f \times a_p} \right)^{(n/(n+1))} \quad [2.26]$$

The intrinsic parameter (C) can be calculated using various mathematical methods. It often depends on speed and time: (V_c, T_0).

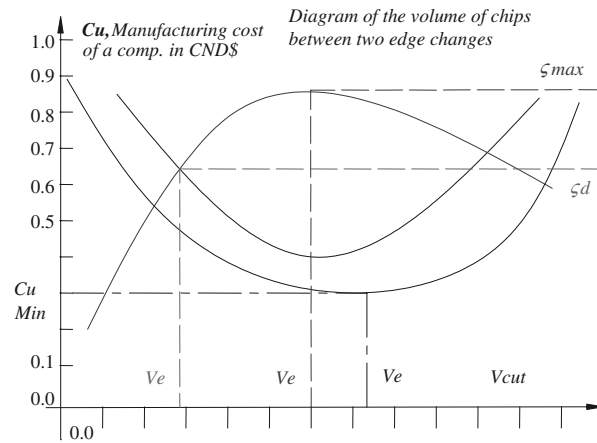


Figure 2.10. Graphical illustration of the chips volume between two edge changes, as a function of the cutting speed V_c – simulation

2.5.1.5. Cost optimization in mechanical production

In the world of, say, machining workshops, we must take control of as many materials as diverse as they are varied. They must be well stored (avoiding aging, corrosion, deformations). Often, orders of raw materials cause the managers to “tumble”. Should we order large quantities and then store them (if possible), or should they be ordered as the production goes on (as per the order book)? The question might be easy to ask, but its answer can be complex. When raw materials are ordered, it is wise to consider transport, handling, storage, etc.

Indeed, manufacturers often run into dilemmas that they always end up reducing: should we continuously consider the cost induced by commands and storage costs? Moral of the story: store as little as possible, and therefore order as little as possible! Ordering an economically profitable quantity would then be profitable. Reality is often surprising in many ways, however. The Wilson model “simplifies”, but is “telling”. We start with the assumption that demand is stable and constant: this is not the case. There are always risks of unexpected breakdowns, fluctuating prices; production may be regular but never linear, contrary to what the Wilson model suggests.

Ordering without taking the various factors that affect a rigorous control into account creates an *over stock*. This creates costs that are inherently non-profitable (stewardship costs, insurance, space). On the other hand, an *under stock* is synonymous with a shortage, and thus with a loss of production, hence client and sale losses (or migration). The relation is (wrongly) attributed to Wilson (1934).

This is actually a simple economic series that is simply exaggerated by changes in work organization models. The aim is to solve a simple capital equation: “How to keep creating capital without worrying about shortages”?

– *Calculation parameters of the economic quantity*

Factors	Designation	Terms in the calculation	Formulae
n	Number of components acquired (manufactured) per year	Number of launches per year	$\left(\frac{n}{Q}\right)$
Q	Number of components launched in machining (production)	Launch cost per year	$\left(\frac{n}{Q}\right) \times CL_F$
P_u	Cost per component (in Canadian \$)	Total cost C_{tot}	$\left(\frac{n}{Q} \times CL_F\right) + \left(\frac{Q}{2} + S_s\right) \times (t \times P_u)$
S_s	Secure stock per component	Average stock for a regularity in circulation	$\left(\frac{Q}{2}\right) + S_s$
T	% rate of possession		
CL_F	Supply or production cost (machining)	Cost of possession per year	$\left(\frac{Q}{2} + S_s\right) \times t \times P_u$
$\frac{dC_{\text{tot}}}{dQ}$	Quantity that optimizes total cost (i.e. minimal cost). This is the Q_{eco} for which the derivative of the total cost with respect to the quantity will be zero.	$\frac{dC_{\text{tot}}}{dQ}$	$\left(\frac{t \times P_u}{2}\right) - \left(\frac{n}{Q^2} \times CL_F\right)$
Q_{eco}	Economic quantity	Q_{eco}	$\sqrt{(2 \times n \times CL_F) / t \times P_u}$

Table 2.6. *Set of formulas in the Wilson model (storage optimization)*

We must avoid the “seductive” aspect of the Wilson relation, as it can create perverse effects in a world in constant change, with its economic and cultural shifts. Wilson assumes that demand is constant in overall time. This is obviously not the case, as shown by the example (2010) of Ford, GM and Chrysler in both Canada and the United States. The demand markets are almost impossible to predict. To try to optimize production criteria based on factors related to the fluctuating raw materials

is a high-risk method. In a world *set* “as the capital would have it”, the Wilson model is extremely useful. It applies to a single article, rather than a batch or a group of articles at any one time.

In other words, when considering the batch size (see section above) in a machining launch that is not optimized with respect to the client’s requirements (the purchaser being unstable in spite of itself), the approach is a tad egocentric. This is the very essence of the capital-at-all-costs, *inhuman as it is!*

The decision, based on an internal manufacture optimization, is doubtlessly excellent but still does not take external factors into account → *lean* process, without any useless operations (excessive stock which would harm performance). Rather than “reap” profits from as many resources as possible, the *Manufacturing* processes should be well managed. This is the basis of the Toyota model.

The costs and losses after the process are considerable, as per what is known as the spirit of MURI, that is unreasonable i.e. excess storage of raw materials leading to an artificial increase in the batch size, which are likely to end up unsuitable. MURI also harms the 5S approach. Furthermore, MURI also brings its own drawbacks, such as a plethora of so-called “managerial” staff employed for sociodemocratic reasons. Finally, MURI falls prey to “anachronistic” work standards.

To manufacture a component outside the tolerance limits (non-conforming, as explained in Chapter 1) is more expensive (though not necessarily twice as expensive, as claimed by the adherents of profit at all costs): we must pause the ongoing machining (production) and restart it with unexpected overtime, with the additional cost of the replacement raw materials requiring labor and energy.

Let us read what the Wilson model curves tell us:

Quantity supplied by the order	Cost per machined component in CND\$	Rate of possession of manufacture	Number of components consumed in a year	Cost of the order in CND\$
Q Pieces	$A, \$$	$\tau, \%$	n	C
100–10,000	2	5	1,000	300

Table 2.7. *Sample conforming to the Wilson model*

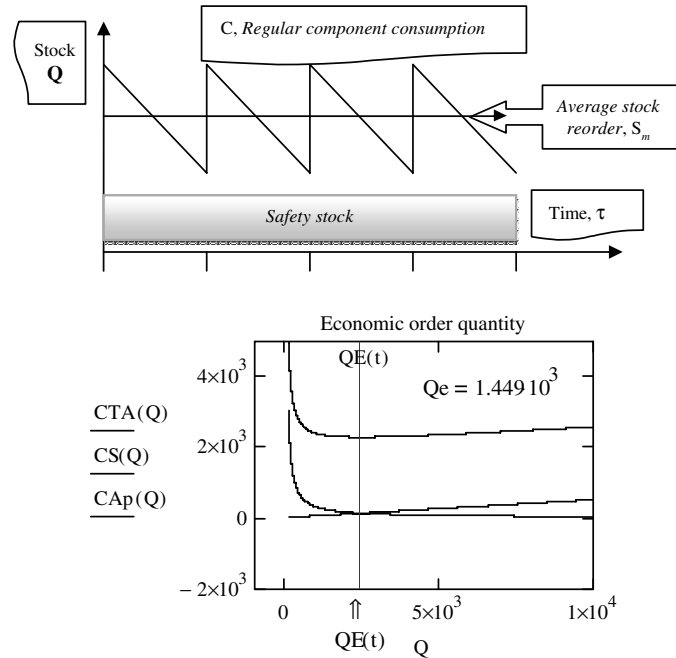


Figure 2.11. Curve of the annual cost C_a as a function of the supplied quantity Q_a

2.6. Stock management and supply methods

Wilson method hypotheses: The stocks are proportional to the number of purchased components. There is no stock shortage. The demand is regular, hence the notion of average stock being $Q/2$. The supplying cost is constant. The main parameters are:

- C is the yearly component consumption.
- Q is the supplied quantity.
- P is the price per component in Canadian dollars (CND\$).
- τ is the stock's average rate of possession.
- f is the cost to make an order in Canadian dollars (CND\$).

We try to minimize this cost to determine the economic supply quantity Q_e . To this end, the minimum of $C_r(Q)$ should be calculated.

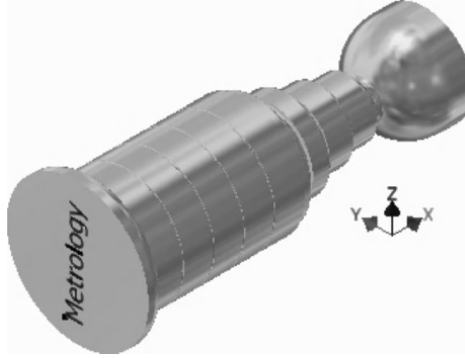


Figure 2.12. Optimizing the North American Hockey Stanley Cups stocks – real case

$P = \$8$; $C = 3,000$ components; $t = 15\%$ and $f = 18$ CND\$

$$Q_e = \sqrt{2 \times \frac{C \times f}{p \times \tau}} \quad [2.27]$$

Since $Q = Q_e = 300 \rightarrow$ then $Q_e = 300$ components (Stanley Cups)

– Cost of storage in Canadian dollars

$$C_s = \frac{1}{2} Q \times p \times \tau \text{ (AN. } \rightarrow C_s = 180 \text{ CND\$)} \quad [2.28]$$

We assume that the launch cost (in Canadian dollars) includes C_1

$$C_1 = \frac{C}{Q_e \times f} \text{ (AN. } \rightarrow C_1 = 0.556 \text{ CND\$...about 60 cents)} \quad [2.29]$$

– Cost of purchasing articles, in CND\$ (article consumption cost)

$$C_a = C \times p \quad [2.30]$$

– Global yearly cost of stock, in CND\$

$$C_{\text{total}} = C_t = \sum_n C_i = C_s + C_1 + C_a \quad [2.31]$$

$$\text{Let } C_t = \left(\frac{1}{2} Q_e \times p \times \tau \right) + \left(\frac{C}{Q_e \times f} \right) + (C \times p) = 2.418 \times 10^4 \text{ CND}$$

The economic supply number N_e

$$\text{For } C = 3,000 \text{ components, } N_e = \frac{C}{Q_e} = 10 \text{ supply runs.}$$

In conclusion, we see that the so-called Wilson method relies on hypotheses that rarely coincide with reality. The model shows that the stock is necessary. We must now minimize its cost by managing it “correctly”. Still, completing the stock as was done in the past would not be wise: feeding the flux would be more profitable. Using the Wilson model today is, in our humble opinion, a dangerous, oversimplistic approach.

2.6.1. Hypothesis of a general method to improve stock management

In this analysis, the supplier would be able to deliver “the production of their articles” within variable proportions while still ensuring a good economic value: indeed, the supply cost is proportional to the number of supply runs due to a non-negligible parameter: transport. We will show a case study that will justify the above:

- $(\beta_j \pm u)$ is the daily requirement of components with uncertainty u .
- C_a is the yearly cost of supply in Canadian dollars (CND\$).
- f is the yearly cost of supply (including transport) in CND.
- Q is the number of components per supply run (in *components*).
- N is the amount supplied per year (in components).
- F_{lp} is the frequency of component delivery.
- C is the yearly consumption in components.
- Let P be the market price in CND. Set $P = \$100$.
- (b) is the cost of purchase (excluding transport costs) and M_b is the profit margin in CND.

Starting data for controlling stocks with the aim of machining Stanley cups for the north American Hockey leagues (Canada–United States).

The Phoenix Coyotes and the Arizona Pitbulls have ordered 10,000 Stanley cups from the machinist, Blaireau LeNain. This is the hypothesis of our “patented” machinist.

$\beta_j \pm u = \{54 \pm 4\}$ Stanley cups (galvanized in non-massive Val d'Or gold).

$u = \pm 4$ is the uncertainty and (β_j) the daily need of components with uncertainty u .

$P = 100$ CND; $f = 117$ CND; $b = 35$ CND; $C = 10,000$ Stanley cups (components).

We want to calculate: Q (components), F_{ip} (frequency of delivery), M_b (in CND)

– *Yearly supply cost in CND*

$$C_a = N \times f; \text{ knowing that } N = \left(\frac{C}{Q} \right) \Rightarrow C_a = \left(\frac{C \times f}{Q} \right) \text{ in CND\$} \quad [2.32]$$

– *Profit margin per unit (component) in CND\$*

$$M_{bu} = P - \left(b + \frac{f}{Q} \right) \Rightarrow \text{qd } Q \rightarrow \infty M_{bu} = \{P - b\} \text{ CND\$} \quad [2.33]$$

If we accepted an increase of 4% in the supply cost per unit to optimize this supply, we have:

$$\left(\frac{f}{Q} \right) = \{P - b\} \times 4\% \Rightarrow Q = \left(\frac{f}{(P - b) \times 4\%} \right) \text{ pieces} \quad [2.34]$$

From which, we can deduce the delivery frequency: $F_{ip} = (Q / \beta_j)$ deliveries

This frequency depends on holidays. Bearing in mind that the maximum consumption is $(\beta_j + u)$, or $F_{ip} \times (\beta_j + u)$ components and the quantity Q of components to supply, we define:

$$\text{Secure}_{\text{stock}} = \frac{Q}{\beta_j \times (\beta_j + u) - Q} \quad [2.35]$$

This is the maximum secure stock, on which an eye should be kept.

2.6.1.1. Application 1

$$\text{From } C_a = \frac{C \times f}{Q} \text{ CND\$} \Rightarrow Q = \frac{C \times f}{C_a} \text{ and } M_{bu} = \{P - b\} = 65 \text{ CND\$}$$

Let $\left(\frac{f}{Q}\right) = \{P - b\} \times 4\%$ and find $\rightarrow Q_{\text{optimal}}$ for $f = 117$ CND\$;

$$\left(\frac{f}{\{P - b\} \times 4\%}\right) = Q_{\text{optimal}} = 45 \text{ parts and } C_{\text{yearly}} = \frac{C \times f}{Q} = 2.4 \times 10^4 \text{ parts}$$

The component delivery frequency will then be estimated at: $Q = 45$ components, which is an optimized supply quantity. The daily need $\beta_j = 45$ components with no uncertainty. The frequency of deliveries per working day with no uncertainty will then be:

$F_{lp} = (Q / \beta_j) = 1$ working day. The correction with uncertainty u : for $u = 4$.

$\beta_j = 45 + u = 54 + 4 = 49$, daily component need with uncertainty (u).

$F_{lp} = Q / \beta_j = 0.918 \approx 1$ working day .

The maximum consumption in, for example, 1 working day will be:

for a working day $\rightarrow C_{\text{max}} = \beta_j d_{\text{work}} = 49$ components per day.

We showed that the theoretical quantity to consume was 49 components. After optimizing, it is more pragmatic to ensure the stock's safety with the following relation:

$$S_{\text{secure}} = \{C_{\text{max}} - Q\} \quad [2.36]$$

We showed that the theoretical quantity to consume was 45 components. After optimizing, it is more pragmatic to ensure the stock's safety with the following relation:

$$S_{\text{secure}} = \{C_{\text{max}} - Q\} = 4 \text{ Stanley cups per working day .}$$

Let a maximum of four components be the first test of the first order. These four cups are a "just in case" stock. Badger LeNain produces 49 cups per day, in Abitibi gold (Quebec, Canada).

2.6.1.2. Application 2: economic quantity of the so-called "Wilson" model

- N , number of components consumed = 1,200 components;
- Q , quantity supplied for each order = 200–10,000 components;
- P , unit cost of component = \$15;
- $t\%$, rate of possession of company = 5%;

– L , rate of possession of company = 1,050:

- Cost of supply: $A(Q) = (N \times L) / Q$; [CND\$],

- Cost of storage: $S(Q) = (Q / 2) P_u \times T$; [CND\$],

- Cost of purchase: $C_a = N \times P_u$; [CND\$],

Total yearly cost: $C_t(Q) = (N \times P_u) + ((N \times L) / Q) \times ((Q \times P_u \times T) / 2)$; [CND\$].

– Calculation of the economic quantity (Q_e):

The minimum is reached when the derivative of the total cost with respect to Q is zero.

$$Q_e = \sqrt{\frac{2 \times N \times L}{T \times P_u}} = 3.175 \times 10^3 \text{ [CND]}$$

– Plotting the graph of our results:

Using MathCAD to plot our calculations, we are able to clearly see the influence of the quantity supplied (components) as a function of the yearly cost in Canadian dollars and thus to find the economic quantity Q_e .

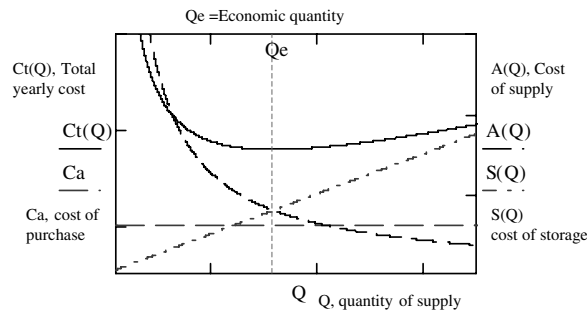


Figure 2.13. *Economic quantity as a function of the dependence parameters*

2.7. Short summary of control charts

Throughout this Chapter, we have developed what is known as statistical process control in order to control, through statistical method, the production process (machining in our case studies). The processes depend on many various 5M factors, that is labor, machine, materials, environment, methods. Because of the variability

of (5M, *French*) factors, manufactured products are never identical. There are two distinct variation categories: random and assignable causes. The goal of SPC is essentially to eliminate *assignable* causes and reduce *random* causes as much as possible. Of the various statistical–mathematical methods used, we chose the Shewhart control charts, the principle of which can be simplified as follows:

At regular intervals, samples of fixed size are taken from production. The central tendency and dispersion of these samples are logged on predefined charts (control charts) in which we examine how closely the points reach target values. If a non-random pattern emerges from these points, or if a point is found outside the predefined lines, the process is considered out of control and corrective action must be taken. The efficiency of a control chart is translated either by the efficiency curve associated with a given test, or with the average run length (ARL).

– For $\delta = 0$, $POM_0 = 1/\delta \Leftrightarrow$ the process is wrongly paused. This happens every $1/\delta$ samples on average.

– For $\delta \neq 0$, $POM_1 = 1/(1 - \beta) \Leftrightarrow$ an average of $1/(1 - \beta)$ is needed to notice the detuning.

– Good chart $\Leftrightarrow$ large POM_0 and small POM_1 .

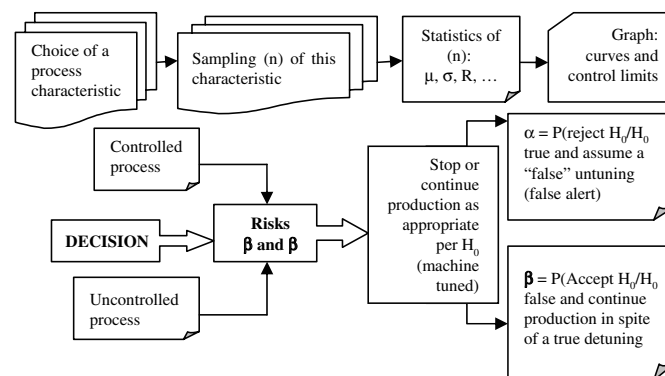


Figure 2.14. Risk flow chart

2.7.1. The various control charts

– *Shewhart control charts*: Quantitative: charts of control by average, median, range, standard deviation (see Tables below and the Appendix/Weblink)

– *Qualitative control charts*: Charts of control by attributes of the percentage of non-conforming, the number of non-conformities, the average number of non-conformities per component and value loss.

– *Control charts taking the past into account*: These are cumulative sum charts, hence the CUSUM acronym, and exponentially weighted moving average charts.

The steps of implanting these charts are as follows:

- choice of the characteristics to monitor;
- choice of the type of control (by measurements or by attributes);
- choice of the type of chart (based on the speed of detuning);
- choice of the sampling (sampling size and frequency);
- preliminary process study (parameters of the monitored characteristic);
- choice of the decision rules.

2.7.2. Measurement control charts – stability control charts (Shewhart)

The goal is to guarantee that the central tendency and dispersion of characteristic X are constant. In this context, the potential tolerances of (X) are irrelevant. The limits are thus determined:

Monitored characteristic X	Controlled statistic	Control limits CL	Monitoring limits ML
$X \rightarrow N(\mu_0, \sigma_0)$	T	$CL = \mu T \pm 3\sigma T$	$ML = \mu T \pm 2\sigma T$

Chart	$\bar{X}$ = average	m = median	S = standard deviation	R = range
Central line	$\bar{X}_0$	m_0	$c_4 s_0$	$d_2 s_0$
LCL	$\bar{X}_0 - 3 \frac{s_0}{\sqrt{n}}$	$m_0 - \tilde{A} s_0$	$B_5 s_0$	$D_1 s_0$
UCL	$\bar{X}_0 + 3 \frac{s_0}{\sqrt{n}}$	$m_0 + \tilde{A} s_0$	$B_6 s_0$	$D_2 s_0$
LML	$\bar{X}_0 - 2 \frac{s_0}{\sqrt{n}}$	$m_0 - \tilde{A}' s_0$	$B'_5 s_0$	$D'_1 s_0$
UML	$\bar{X}_0 + 2 \frac{s_0}{\sqrt{n}}$	$m_0 + \tilde{A}' s_0$	$B'_6 s_0$	$D'_2 s_0$

Table 2.8. Summary of the formulas used to calculate control and monitoring limits

The control chart coefficients come from the Shewhart technical literature, a table of which is shown below (quoted from the bibliography):

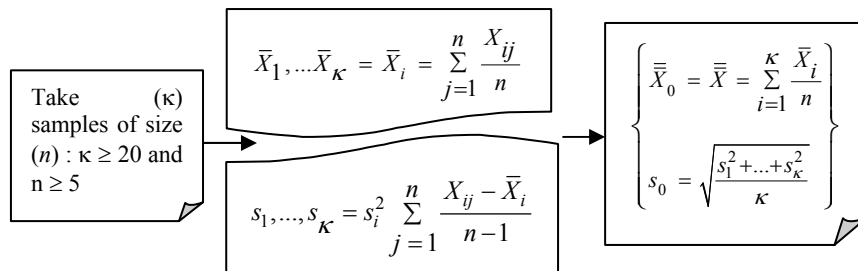
n	Control chart of the average $\bar{X}$						Control chart of $\bar{R}$			
	σ_0 known		By $\bar{R}$		By $\bar{s}$		σ_0 known		σ_0 estimated by $\bar{R}$	
	A_c	A_s	A'_c	A'_s	A''_c	A''_s	D_{c2}	D_{s2}	D'_{c2}	D'_{s2}
2	2.185	1.386	1.937	1.229	3.874	2.457	4.65	3.17	4.12	2.81
3	1.784	1.132	1.054	0.668	2.464	1.564	5.06	3.68	2.99	2.17
4	1.545	0.980	0.750	0.476	1.936	1.228	5.31	3.98	2.58	1.93
5	1.382	0.876	0.594	0.377	1.643	1.042	5.48	4.20	2.36	1.81
6	1.262	0.800	0.498	0.316	1.452	0.921	5.62	4.36	2.22	1.72
7	1.168	0.741	0.432	0.274	1.315	0.834	5.73	4.49	2.12	1.68
8	1.092	0.693	0.384	0.244	1.209	0.767	5.82	4.61	2.04	1.62
9	1.030	0.653	0.347	0.220	1.127	0.714	5.90	4.70	1.99	1.58
10	0.977	0.620	0.317	0.202	1.059	0.672	5.97	4.79	1.94	1.56
11	0.932	0.591	0.295	0.186	1.002	0.635	6.04	4.86	1.90	1.53
12	0.892	0.566	0.274	0.174	0.953	0.605	6.09	4.92	1.87	1.51

Table 2.9. Coefficients used in calculating the control and monitoring limits [BOI 01]

The Shewhart charts are determined such that only α (or POM0) be taken into account ($\alpha = 0.27\%$ for the control chart of the average, $\alpha < 1\%$ for the others). The sample size is thus arbitrary. However, the sample size can still be set based on the value of β (or POM1) for a given detuning (δ).

2.7.3. Estimating μ_0 and σ_0

Estimating (μ_0 and σ_0) must always be done in a preliminary study in which the production conditions will be especially monitored to avoid any assignable causes. Here is a classic International System Organization (ISO) and Canadian Standardization Association (CSA) illustration:



The estimation above is recommended by CSA (and ISO). Some software estimated σ using $(\bar{s}/c_4)$ for the standard deviation chart. When calculations are done by hand, using the median and range charts is preferable: in these cases, m_0 is estimated using medians and σ_0 by $\bar{R}/d_2$

2.7.4. Efficiency – chart of the average

The detuning regards the average going from m_0 to $[m_0 + d.s_0]$.

– $a = 0.27\%$ and $POM_0 = 370$;

– b depends on $d = \left\{ \frac{|m - m_0|}{s_0} \right\}$ and n .

2.7.4.1. Chart of the standard deviation and range

The detuning is the standard deviation going from s_0 to rs_0

– $a > 1\%$ (depends on n) and $POM_0 > 100$;

– *Efficiency tables for the standard deviation chart (see Weblink and the Appendix);*

– b depends on detuning $r = s/s_0$ and n ;

– *Efficiency tables for the range chart (see Weblink and the Appendix).*

2.7.4.2. Stability control charts when given a target

The monitored variable is often made to reach a target. In such conditions, the aim is for the average μ_0 of the monitored characteristic X to find itself on the target. We do not need to estimate μ_0 and the average control limits are defined by:

$$\left\{ \left(\text{target} - 3 \frac{\sigma_0}{\sqrt{n}} \right) ; \left(\text{target} + 3 \frac{\sigma_0}{\sqrt{n}} \right) \right\} \quad [2.37]$$

$$\text{Similarly, detuning is defined as: } \delta = \left\{ \frac{|m - \text{target}|}{\sigma_0} \right\} \quad [2.38]$$

NOTE.– (see Weblinks and the Appendix) Efficiency tables of the chart of the average. For coefficients (see Weblink and the Appendix): <http://www.sqconline.com/about-acceptance-sampling> and <http://www.sqconline.com/control-charts>

Control charts that aim to ensure that tolerances are conformed to: there are tolerances that need to be respected. The chart is built on a proportion of defectives that should not be exceeded. For two tolerances, this proportion is arbitrarily set to 0.27% (0.135% for a single tolerance). This results in a maximum admissible detuning:

$$\delta = \min \left\{ \frac{(T_{\text{Upper}} - 3s_0) - m_0}{s_0}; \frac{m_0 - (T_{\text{Lower}} + 3s_0)}{s_0} \right\} \quad [2.39]$$

A target is generally given and denoted by m_0 , if this is not the case, m_0 is an estimation of μ_0 . The sample size is then defined from a β value for this detuning δ . For the efficiency tables and abaci of the average chart, see Weblinks or the Appendix.

2.7.5. Control chart by attributes – determining the limits

– K = number of non-conforming elements: $K \rightarrow B(n, p_0) \rightarrow$ (Binomial distribution, see the Appendix)

– K = number of non-conformities: $K \rightarrow P(c_0) \rightarrow$ (Poisson distribution)

The laws of $F = (K/n)$, that is the proportion of non-conforming elements or number of non-conformities per unit can be deduced.

Chart	Control limits, CL	Sample size, n
np , number of non-conforming	$np_0 \pm 3\sqrt{np_0(1-p_0)}$	Constant
p , proportion of non-conforming elements	$p_0 \pm 3\sqrt{\frac{p_0(1-p_0)}{n_i}}$ and $p_0 \pm 3\sqrt{\frac{p_0(1-p_0)}{\bar{n}}}$	variable n_i $n_i \in \{0.75\bar{n}; 1.25\bar{n}\}$
c , number of non-conformities per sample	$c_0 \pm 3\sqrt{c_0}$	Constant
u , number of non-conformities by unit of production	$u_0 \pm 3\sqrt{\frac{u_0}{n_i}}$ and $u_0 \pm 3\sqrt{\frac{u_0}{n}}$	n_i variable $n_i \in \{0.75\bar{n}; 1.25\bar{n}\}$

Table 2.10. Control charts by attributes (limit determination)

The control limits of the “p” and “u” charts vary based on the sample size. To remedy this, if variations in n_i are small, that is $n_i \in \{0.75 \times \bar{n}; 1.25 \times \bar{n}\}$ then we build the control chart from a unique sample size equal to the average sample size.

– Estimating p_0 and u_0 :

- p_0 is estimated by the proportion of non-conforming elements over r samples of size n_i .

- u_0 is estimated by the average number of non-conformities per unit over r samples of size n_i .

We recommend having at least 300 observations in total in all cases.

– *Efficiency*: “p” and “np” charts. The detuning regards the proportion going from p_0 to p , with (β) a function of $\{n, LC \text{ and } p\}$.

– **Charts** “c” and “u” $\rightarrow$ the detuning regards the average number of non-conformities per unit that goes from $\{u_0 \text{ to } u\}$; with (β) depending on $\{n, LC \text{ and } u\}$.

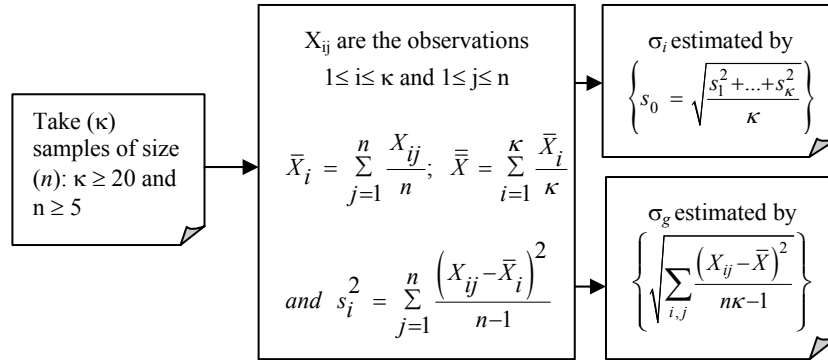
– **Capability**: we consider the case where the characteristic follows a normal distribution, with the following conditions:

- natural process dispersion: $D = 6 \cdot \sigma$ (includes 99.73% of data);

- $TI = \{T_u - T_l\}$.

$$\left\{ C = \left(\frac{IT}{D} \right) \text{ and } C_k = \min \left(\frac{\bar{X} - T_{\text{low}}}{D/2}, \frac{T_{\text{up}} - \bar{X}}{D/2} \right) \right\} \quad [12.40]$$

The index C is suitable for centered processes, whereas the C_k index is used for non-centered processes. Calculating both is useful, as comparing between the two will give an indication of how far from the centered the process might be. We distinguish between the machine’s capability C_m and C_{mk} and the process capability C_p and C_{pk} , which includes the set of variations due to 5M. The difference between calculations comes from the estimation of dispersion σ_i for the machine dispersion or instantaneous dispersion; σ_g for the process dispersion or global dispersion.



1) σ_g can also be found from 100 observations taken randomly from a batch or a week's production (acceptance test).

2) If we have a swift production, σ_t can also be estimated from about 50 consecutive measurements (if the process is stabilized during sampling).

3) The CSA standard gives the machine capability C_m in the short term and the process capability C_p in the long term. **ISO/TS 16949** speaks of *process capability* $\rightarrow C_p$ for the short term and process performance P_p for the long term.

4) We consider a process to be capable if the short-term capability is greater than 1.67 and if the long-term capability is greater than **1.33** (these arbitrary values were enforced since the 1980s. Some industries estimate that a process is capable for a value greater than 2).

2.8. CUSUM charts

As per the exponentially weighted moving average (EWMA) chart, the CUMulative SUM (CUSUM) chart is better than the Shewhart chart for detecting small variations and for samples of size 1. The CUSUM chart can be read from a *V-mask*, which is not very practical, or from horizontal control lines.

Control limits are defined by $\rightarrow$	$CL = \pm h$
We log the following points on the chart $\rightarrow$	$S_i^+ = \max\{0, S_{i-1}^+ + (z_i + \kappa)\}$
	$S_i^- = \max\{0, S_{i-1}^- + (z_i - \kappa)\}$

(κ) is a sensibility parameter, function of the detuning δ .

	Average	Proportion of non-conforming elements	Number of non-conformities
	m_0 = target value of the average	p_0 = target value of the proportion	c_0 = target value of the average number of non-conformities per sample
	m_1, m_1' = upper and lower rejectable values of the average	p_1 = upper rejectable value of the proportion	c_1 = average rejectable number of non-conformities in a sample
z_i	$\left(\frac{\bar{X}_1 - m_0}{s_0} \right) \sqrt{n}$	$\left(\frac{p_1 - p_0}{s_0} \right) \sqrt{n}$ and $s_0 = \sqrt{p_0(1-p_0)}$	$\left(\frac{c_1 - c_0}{s_0} \right) s_0 = \sqrt{c_0}$
δ	$\min \left\{ \frac{m_1 - m_0}{s_0}, \frac{m_1 - m_1'}{s_0} \right\}$	$\left(\frac{p_1 - p_0}{s_0} \right)$	$\left(\frac{c_1 - c_0}{s_0} \right)$

Table 2.11. Non-conformity proportion. For coefficients, see weblink

NOTE 2.1.— For the average chart, the date of detuning can be estimated from the last time s_0^+ or s_0^- went to zero before detuning, the detuning amplitude from $\delta\sqrt{n} \times \left(\frac{s^+}{r-1} + \kappa \right)$ or $\left(\frac{s^-}{r-1} - \kappa \right)$ with r = the number of samples between the last time 0 was reached and the detuning (counting extrema) and s^+ or s^- = the values reached at detuning.

NOTE 2.2.— For the charts of the proportion of non-conforming elements and the number of non-conformities, we can use the same coefficients as those of the average if $n.p_0 > 5$ or $m_0 > 5$.

2.8.1. EWM charts

The control limits are not horizontal, but very quickly tend to their asymptotes as defined in the tables below. We chart the points z_i . λ is a constant $0 < \lambda \leq 1$. L is a value close to three, so as to prevent too many false alerts.

	Average	Proportion of non-conforming elements	Number of non-conformities
z_0 at the start and after each tuning	$m_0 = \text{target average value}$	$p_0 = \text{target proportion value}$	$c_0 = \text{target value of the average number of non-conformities per sample}$
	$m_1, m'_1 = \text{upper and lower rejectable average values}$	$p_1 = \text{upper rejectable value of the proportion}$	$c_1 = \text{rejectable average of non-conformities per sample}$
z_i	$\lambda \bar{X}_i + (1 - \lambda) z_{i-1}$	$\lambda p_i + (1 - \lambda) z_{i-1}$	$\lambda c_i + (1 - \lambda) z_{i-1}$
CL	$m_0 \pm L \left(\frac{\sigma_0}{\sqrt{n}} \sqrt{\frac{\lambda}{2 - \lambda}} \right)$	$p_0 \pm L \left(\frac{s_0}{\sqrt{n}} \sqrt{\frac{\lambda}{2 - \lambda}} \right)$	$c_0 \pm L s_0 \left(\sqrt{\frac{\lambda}{2 - \lambda}} \right)$
		$s_0 = \sqrt{p_0(1 - p_0)}$	$s_0 = \sqrt{c_0}$
d	$\min \left\{ \frac{m_1 - m_0}{s_0}, \frac{m_0 - m'_1}{s_0} \right\}$	$\left\{ \frac{p_1 - p_0}{s_0} \right\}$	$\left\{ \frac{c_1 - c_0}{s_0} \right\}$

Table 2.12. Formulas for calculating conformities. For the coefficients, see Weblink

2.8.1.1. Coefficients for the control charts of the average $\bar{X}$ and range R

n	Control chart of the average $\bar{X}$						Control chart of $\bar{R}$			
	Known σ_0		by $\bar{R}$		by $\bar{s}$		Known σ_0		σ_0 estimated by $\bar{R}$	
	A_c	A_s	A'_c	A'_s	A''_c	A''_s	D_{c2}	D_{s2}	D'_{c2}	D'_{s2}
2	2.185	1.386	1.937	1.229	3.874	2.457	4.65	3.17	4.12	2.81
3	1.784	1.132	1.054	0.668	2.464	1.564	5.06	3.68	2.99	2.17
4	1.545	0.980	0.750	0.476	1.936	1.228	5.31	3.98	2.58	1.93
5	1.382	0.876	0.594	0.377	1.643	1.042	5.48	4.20	2.36	1.81
6	1.262	0.800	0.498	0.316	1.452	0.921	5.62	4.36	2.22	1.72
7	1.168	0.741	0.432	0.274	1.315	0.834	5.73	4.49	2.12	1.68
8	1.092	0.693	0.384	0.244	1.209	0.767	5.82	4.61	2.04	1.62
9	1.030	0.653	0.347	0.220	1.127	0.714	5.90	4.70	1.99	1.58
10	0.977	0.620	0.317	0.202	1.059	0.672	5.97	4.79	1.94	1.56
11	0.932	0.591	0.295	0.186	1.002	0.635	6.04	4.86	1.90	1.53
12	0.892	0.566	0.274	0.174	0.953	0.605	6.09	4.92	1.87	1.51

Table 2.13. Coefficients used to calculate conformities

2.8.2. Shewhart charts for the average $\bar{X}$, the range R and the standard deviation (s)

During the 1930s, Walter A. SHEWHART published the principles of control charts as an engineer at *Bell Telephone Laboratory of Western Electric*. We will show most of the charts for the average $\bar{X}$, the range R and the standard deviation (s) along with their respective graphs. In the following, we will give models based on standard tables and finally finish with the (Poisson and binomial) distribution of the characteristic of a stable process. Consider the case in which we have to estimate the standard deviation (σ) by analyzing experimental data. With m preliminary samples (from a population N) of size n , with s_i the standard deviation of the i th sample. $\bar{X}$ is the average of the m standard deviations as follows:

$$\text{If variance } S^2 = \frac{1}{n-1} \sum_{i=1}^m (x_i - \bar{x})^2 \text{ then } \rightarrow \bar{S} = \frac{1}{m} \sum_{i=1}^m S_i \quad [2.41]$$

2.8.2.1. Control limits for $\bar{X}$ of the s control charts

We will use the factor C_4 , the relation of which is shown below. The statistics $\bar{S}/C_4$ represent an *unbiased* estimate of (σ).

$$C_4 = \sqrt{\frac{2}{n-1}} \times \left[\left(\frac{n}{2} - 1 \right)! / \left(\frac{n-1}{2} - 1 \right)! \right] \quad [2.42]$$

Consequently, the limiting parameters are as follows:

$$\left\{ \begin{array}{l} \text{UCL} = \bar{S} + 3 \times \frac{\bar{S}}{C_4} \sqrt{1 - C_2^2} \\ \text{LCL} = \bar{S} - 3 \times \frac{\bar{S}}{C_4} \sqrt{1 - C_2^2} \end{array} \right\} \bar{S}, \text{ corresponds to the central line} \quad [2.43]$$

Similarly, the parameters of the average $\bar{X}$ are:

$$\left\{ \begin{array}{l} \text{UCL} = \bar{\bar{x}} + 3 \times \frac{\bar{S}}{C_4 \sqrt{n}} \\ \text{LCL} = \bar{\bar{x}} - 3 \times \frac{\bar{S}}{C_4 \sqrt{n}} \end{array} \right\} \bar{\bar{x}}, \text{ corresponds to the central line} \quad [2.44]$$

where $\bar{\bar{x}}$ is the average of all observations. It is useful for plotting the graphs of second-order average $\bar{\bar{X}}$.

2.8.3. Control charts for the average $\bar{\bar{X}}$ and the range R

If the sample is relatively small (size ≤ 10), we will use the range instead of the standard deviation to build control charts for $\bar{\bar{X}}$ and the range R . The range of a sample is simply the difference between the smallest and largest data points. There is a statistical relation called PATNAIK, 1946 [WOO 93], between the average amplitude of data from a normal distribution and the standard deviation (σ) of this distribution. This relation only depends on the sample size n . The estimate for (σ) is then R/D_2 . Based on this, we can easily develop the control charts of $\bar{\bar{X}}$ and R .

Let $R_1, R_2, \dots, R_k$ be the range of (k) samples. The average range is:

$$\bar{R} = \frac{1}{k} \times \{R_1 + R_2 + \dots + R_k\} \quad [2.45]$$

$$\text{The } (\sigma) \text{ estimate is thus calculated: } \hat{\sigma} = (\bar{R}/d_2) \quad [2.46]$$

2.8.3.1. Control charts for second-order average $\rightarrow \bar{\bar{X}}$

If we use $\bar{\bar{x}}$ (or a given target) as an estimate for ($\bar{x}$) and $\bar{R}/d_2$ as an estimate for (σ), the parameters of the $\bar{\bar{X}}$ chart are delimited by the UCL and LCL as shown below:

$$\left\{ \begin{array}{l} \text{UCL} = \bar{\bar{x}} + 3 \times \frac{\bar{R}}{d_2 \sqrt{n}} \\ \text{LCL} = \bar{\bar{x}} - 3 \times \frac{\bar{R}}{d_2 \sqrt{n}} \end{array} \right\} \bar{\bar{x}}, \text{ corresponds to the central line} \quad [2.47]$$

The easiest way to describe the limits is to define the factor $A_2 = 3/(d_2 \sqrt{n})$ to then construct the averages chart $\bar{\bar{X}}$ as follows:

$$\{ \text{UCL} = \bar{\bar{x}} + A_2 \bar{R} \text{ and } \text{LCL} = \bar{\bar{x}} - A_2 \bar{R} \} \bar{\bar{x}}, \text{ corresponds to the central line} \quad [2.48]$$

The factor A_2 only depends on n and is read as follows:

Limiting factors in calculating $\bar{\bar{x}}$ and the graphs for R			
n	A_2	D_3	D_4
2	1.880	0	3.267
3	1.023	0	2.575
4	0.729	0	2.282
5	0.577	0	2.115
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777

Table 2.14. Table of the coefficients A_2 , D_3 and D_4

2.8.3.2. Range control charts $\rightarrow R$

Controlling process variability from the sample range is related to the process' standard deviation. *The line at the center of the R chart is the range of the average.* To calculate the control limits, we need an estimate of the true, unknown standard deviation $W = R/\sigma$. This comes from the Gaussian distribution (assuming the elements we measure follow a normal distribution).

The standard deviation of W is d_3 , which is a known function of the sample size n . It is given in the form of tables in many textbooks on statistical quality control. Since $R = W\sigma$, the standard deviation of R is $\hat{\sigma}_R = D_3$. The true standard deviation is unknown, and we can estimate $\hat{\sigma}_R$ through σ

$$\hat{\sigma}_R = \left\{ d_3 \times (\bar{R}/d_2) \right\} \quad [2.49]$$

$\bar{R}$ corresponds to the central line. The parameters of the graphical chart R , with the usual three-sigma control limits, can be written as:

$$\left\{ \begin{array}{l} \text{UCL} = \bar{R} + 3\hat{\sigma}_R = \bar{R} + 3 \times \left(d_3 \frac{\bar{R}}{d_2} \right) \\ \text{LCL} = \bar{R} - 3\hat{\sigma}_R = \bar{R} - 3 \times \left(d_3 \frac{\bar{R}}{d_2} \right) \end{array} \right\} \quad [2.50]$$

Similarly to the above with the parameters of the control chart for the average subgroups, we define another set of factors that will simplify calculations: specifically, the following yields:

$$D_3 = \left(1 - \frac{3D_2}{D_1}\right) \text{ and } D_4 = \left(1 + \frac{3d_2}{d_1}\right) \quad [2.51]$$

$$\left\{ \text{UCL} = \bar{R}D_4 \text{ and LCL} = \bar{R}D_3 \right\} \quad \bar{R}, \text{ corresponds to the central line} \quad [2.52]$$

The factors D_3 and D_4 only depend on n (see table above).

Generally speaking, the approach is satisfactory enough for sample sizes up to around 10. For larger samples, using the standard deviations of the subgroups is preferable. For small samples, the relative efficiency of using the large approach rather than the standard deviations is given in the following table.

n	Relative efficiency	n	Relative efficiency
2	1.000	5	0.955
3	0.992	6	0.930
4	0.975	10	0.850

Table 2.15. Efficiency of R compared to S/C_4

ARL, that is the detection time or “out-of-control” *signal waiting time* will now be discussed.

In this context, two main questions should be asked.

1) *How many false alarms will be needed before an assignable cause will be sought, when nothing in the production process will have changed?*

2) *How long will we detect some systematic changes?*

The signal lag (ARL) tells us, for a given situation, the average inherent time. This allows us to plot the control charts of the successive points before detecting a point outside the control limits. For an average $\bar{X}$ without any changes in the process, we will expect on average $(1/p)$ points before a false alarm, with p the probability of an observation being found outside the control limits.

For example, for a normal distribution with an *average* ≈ 0.25 ; Shewhart's ARL is roughly **281.14**; for $(k = 1/2) \rightarrow h\sqrt{n/\sigma} = 74.2$ and for $(k = 1/2) \rightarrow h\sqrt{n/\sigma} = 140$.

Average	$h\sqrt{n/\sigma}$	$h\sqrt{n/\sigma}$	Shewhart
For $(k = 1/2)$	4.0	5.0	$\bar{X}$
0.00	336	930	371.00
0.25	74.2	140	281.14
0.50	26.6	30.0	155.22
0.75	13.3	17.0	81.22
1.00	8.38	10.4	44.0
1.50	4.75	5.75	14.97
2.00	3.34	4.01	6.30
2.50	2.62	3.11	3.24
3.00	2.19	2.57	2.00
4.00	1.71	2.01	1.19

Table 2.16. Seeking ARL: comparison between $\bar{X}$ and CUSUM ARLs

2.9. Individual control charts

In this case, samples are individual measurements. *The range's displacement is used to calculate the upper and lower limits.* In this case, the control charts for individual measurements such as, say, the sample size = 1, we use the *displacement* range of two successive observations to measure the process variability. The field of action is defined by $MR_i = |x_i - x_{i-1}|$, which is the absolute value of the first difference (e.g. the difference between two consecutive data points) between data. Similarly to Shewhart control charts, we can plot both the data (which are the individuals) and the mobile range. For the control chart of individual measurements, the plots are as follows:

$$\left\{ \begin{array}{l} \text{UCL} = \bar{x} + 3 \times \frac{\overline{MR}}{1.128} \\ \text{LCL} = \bar{x} - 3 \times \frac{\overline{MR}}{1.128} \end{array} \right\} \bar{x}, \text{ corresponds to the central line} \quad [12.53]$$

where $\bar{X}$ is the average of all individuals and $\overline{MR}$ is the average of all mobile ranges of two observations. The averages can be replaced with a target if available (e.g. 1.128 is the value for D_2 for $n = 2$).

Application example: The following example shows the control charts for individual observations. A new process was studied to monitor the volume of machined metal. The first 10 batches have yielded the following:

Batch Number	Volume of machined metal (Chip) x	Moving Range MR
1	49.6	-
2	47.6	2.0
3	49.9	2.3
4	51.3	1.4
5	47.8	3.5
6	51.2	3.4
7	52.6	1.4
8	52.4	0.2
9	53.6	1.2
10	52.1	1.5
	$\bar{X} = 50.81$	$\overline{MR} = 1.8778$

Table 2.17. Calculation examples, control chart

The limits observed are:

$$\left\{ \begin{array}{l} \text{UCL} = \bar{x} + 3 \times \frac{\overline{MR}}{1.128} = 50.81 + 3 \times \frac{1.8778}{1.128} = 55.8041 \\ \text{LCL} = \bar{x} - 3 \times \frac{\overline{MR}}{1.128} = 50.81 - 3 \times \frac{1.8778}{1.128} = 45.8159 \end{array} \right\} \rightarrow \bar{x} = 50.81$$

We plot the representative diagram for individuals on a graph delimited by the upper limit control (above) and the lower limit control (below). “In between”, we will put the central line which corresponds to the “true” average. After plotting this, the process is declared to be under control, since none of the points plotted were

outside the UCL or LCL. Note that the table of individuals can also be built using standard deviations. We can finally have the graph for $\bar{x} \pm (3s/c_4)$.

2.9.1. Average lengths or CUSUM-Run

Finding samples to be used with a cumulative sum control chart (*Run Length cumulative average of sum control charts*) is done by taking samples of size n and plotting their cumulative sums against the number of samples r , keeping $\bar{x}_i$ as the average sample number and κ as a reference value.

$$S_\tau = \sum_{i=1}^{\tau} (\bar{x}_i - \kappa) = \sum_{i=1}^{\tau} \frac{(\bar{x}_i - \kappa)}{\sigma_{\bar{x}}} \quad (\text{Standard}) \quad [2.54]$$

In practice, κ can be equal to $(\mu_0 + 1/2)$, sometimes known as the *acceptable quality level*. A_1 is called the *rejectable quality level* ($\mu \times \hat{\mu}_0$).

If the distance between one point and previous lower points is $\geq h$, we conclude that the average process has changed. h is the *decision limit*. n is the sample size. The reference value κ and h are parameters needed to use a unilateral CUSUM. If we must control both positive and negative gaps, as is usually the case, two unilateral graphs are used with their respective values for κ_1 , κ_2 , ($\kappa_1 > \kappa_2$) and decision limits h and $(-h)$. The average displacement can be expressed in $(\mu - \kappa)$. When the measurements are normally distributed, we normalize this evolution by setting:

$$\kappa_s = \left(\frac{\mu - \kappa}{\sigma/\sqrt{n}} \right) = \left(\frac{(\mu - \kappa) \times \sqrt{n}}{\sigma} \right) \quad \text{and} \quad h_s = \left(\frac{h}{\sigma/\sqrt{n}} \right) = \left(\frac{h \times \sqrt{n}}{\sigma} \right) \quad [2.55]$$

with h_s being the decision limit.

2.9.1.1. Determining the ARL with known h and k

The ARL at a given quality level is the average number of samples (subgroups) taken before an action signal is sent. The normalized parameters κ_s and h_s and the sample size n are usually chosen to find the approximate ARL of $(L_0$ and $L_1)$ at acceptable and rejectable quality levels (μ_0 and μ_1), respectively. We would like to have a high ARL, L_0 , when the process is on-target (i.e. under control). Conversely,

we have a low ARL, L_1 , when the process shifts toward an unsatisfying level (μ). There are abaci that can be used to find the ARL when the standardized h and κ are given. Some abaci are better than the (integral) equations that make up the exact solutions, and approximate using a *System of Linear Algebraic Equations*. An example is given below for standardized h and κ , which rely on Table 2.16.

Sample search for ARL given standardized H and K: Using Table 2.16 for $\kappa = 0.5$, we see that the average (in multiples of the average's standard deviations) is found by adding 0.5 to the first column. For example, to find an average change of 1 sigma at $h = 4$, ARL = 8.38 (beginning of first column 0.5).

2.10. EWMA statistics – comparison between Shewhart graph control and the EWMA chart control techniques

Exponential weighted moving average (EWMA) is a statistic dedicated to process monitoring for data averages to give a *weight* to data when they are *spread* in time. For Shewhart graph control, the decision technique considers the process at any time, t depends only on the most recent process measurement and the degree of *correctness* of the command limit estimation is made based on historical data. For EWMA control, the decision depends on the EWMA statistic.

By choosing the weight factor (λ), EWMA control can be sensitive to a small or progressive process deviation. Shewhart control, however, can only react when the last data point is outside the control limits. To write the EWMA, the Roberts equation [STI 78, STI 86] can be used. The statistic is calculated as follows:

$$EWMA_t = \{\lambda Y_t + (1 - \lambda) EWMA_{t-1}\} \text{ for } t = 1, 2, 3, \dots, n \quad [2.56]$$

- $EWMA_0$ is the average of historical data (the target).
- Y_t is the observation at time t .
- n is the number of (monitored) observations including $EWMA_0$.
- $0 < \lambda < 1$ is a constant that determines the depth of the EWMA memory $\leq \lambda$.

Choosing the weight factor: λ determines the speed at which previous data comes in the EWMA statistic. For example, a value of $(\lambda) = 1$ indicates that only the most recent value influences the EWMA. Thus, greater values of (λ) give greater weight to recent data and less weight to older data, and a smaller value for (λ) gives more weight to older data. The value of (λ) is generally set between (0.2 and 0.3), though this choice is slightly arbitrary.

We can use the J.M. Lucas and M.S. Saccucci tables [LUC 90] to choose (λ) correctly. The same goes for (κ) . The estimated variance of EWMA is around:

$$S^2 \text{EWMA}_t = \left\{ \left(\frac{\lambda}{2 - \lambda} \right) \times S^2 \right\} \quad [2.57]$$

This expression is valid when t is not small. The standard deviation s is calculated from historical data. The control chart's central line is the target value or EWMA_0 . The control limits are as follows:

$$\left\{ \begin{array}{l} \text{Upper control limit} = \text{UCL} = \text{EWMA}_0 + ks_{\text{EWMA}} \\ \text{Lower control limit} = \text{LCL} = \text{EWMA}_0 - ks_{\text{EWMA}} \end{array} \right\} \quad [2.58]$$

We assume independent data. The tables (see Web/the Appendix) also assume a normal population. As with all control procedures, the procedure depends on EWMA, a database of measurements that are representative of the process. Once the average value and standard deviation are calculated from this database, the process can enter the control phase, under the condition that the process was not controlled during data collection. The literature [RAY 97, RAY 00] distinguishes between the *flaws*. This corresponds to a distinction between *non-conformities* and *non-conforming units*. A non-conforming element has at least one flaw. We should note that a crank-connecting rod system can have multiple flaws, but still be considered conforming. Flaws can be found on places that are not system-critical. If the number of so-called unimportant flaws becomes worrying, a production verification is in order. Approximating the Poisson law for counting or flaw numbers is explained through the following example.

Example.— Consider a mechanism such as a speed reducer. Data flaws can be fairly large. However, the probability of a flaw occurring on a single arbitrarily chosen point is likely to be very low. In such a case, anomaly incidence could be modeled by a Poisson distribution. In fact, the binomial law approximation is most appropriate in this case (see Chapter 1, Volume 1).

The sample size n must be ≥ 20 and the probability of a single success, p , must be ≤ 0.05 . If $n \geq 100$, the approximation is excellent, hence $(n \times p) \leq 10$. To illustrate the use of the Poisson distribution as an approximation of the binomial distribution, consider the following: let p be the probability of a single success with $n = 200$ trials $\rightarrow n \times p = 0.025$. From the Poisson distribution, we set:

$$p(x) = \frac{\text{Exp}\{-C\} \times C^x}{x!} = \frac{\text{Exp}\{-5\} \times 5^3}{3!} = 0.140373896 \text{ because } C = (20) \times (0.25) = 5$$

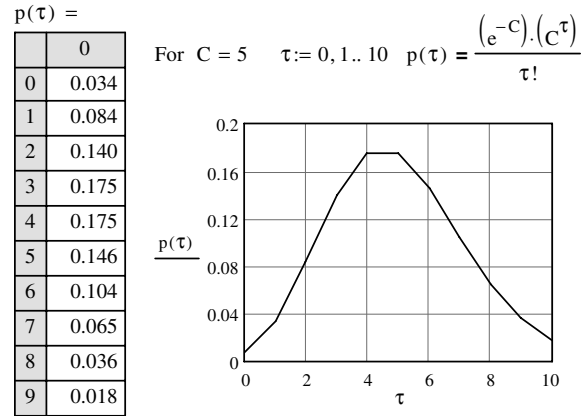


Figure 2.15. Experimental results from using the Poisson distribution

Let us now find the probability of exactly three successes. If we assume that the probability p stays constant, the solution follows a binomial distribution, that is:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{200}{3} 0.25^3 (0.975)^{197} = 0.1399995$$

We will count the number of flaws within a single given *inspection unit*. An inspection unit is a single unit or a product element (crank-connected rod system). The size of the inspection units can depend on the measurement equipment, the operators, etc. Let us assume that flaws are produced in a given inspection unit as a Poisson distribution, with parameter c (often called np or λ). The control charts using the Poisson distribution are written as $p(x)$.

$$p(x) = \frac{\text{Exp}\{-C\} \times C^x}{x!} \quad [2.59]$$

where x is the number of flaws and $c > 0$, the Poisson distribution parameter. We know that the average and variance of this distribution are c .

$$\left\{ \text{UCL} = c + \kappa\sqrt{c} \text{ and } \text{LCL} = c - \kappa\sqrt{c} \right\}, c \text{ is the central line} \quad [2.60]$$

There is no lower control limit if the LCL is negative. This control system assumes that a standard value for c is available. If not, c will be estimated as the average number of flaws in a preliminary inspection unit sample $\bar{c}$ (see the Appendix).

Usually, κ is set to 3. This is why we chose $\kappa = 3$ in our example. The latter is used to show how control limits are built. We inspected 25 pierced circles, successively injected on the plastic injection machine. Each container had 100 mechanical engineering circles. The table of our inspection palette is below.

The number of real observed flaws is given by the following table:

Circle number	Number of flaws in circles	Circle number	Number of flaws of circles
1	5	14	5
2	4	15	5
3	10	16	10
4	12	17	14
5	12	18	10
6	8	19	11
7	10	20	10
8	2	21	11
9	1	22	19
10	10	23	16
11	9	24	15
12	17	25	10
13	14	Total flaws = 250	

Table 2.18. *Number of flaws on mechanical engineering circles of production processes*

From these tables whose values come from our lab experiments, we have the following:

$$UCL = \text{flawed} + k \times \sqrt{\text{flawed}} = 19.487$$

$$\bar{c} = \frac{\text{total number of defectives}}{\text{total number of samples}} = \frac{250}{25} = 10$$

$$\left\{ \begin{array}{l} UCL (LSC) = \bar{c} + \kappa\sqrt{\bar{c}} = 10 + 3\sqrt{10} = 19.487 \\ LCL (LIC) = \bar{c} - \kappa\sqrt{\bar{c}} = \end{array} \right\}$$

– Example of a counting control chart

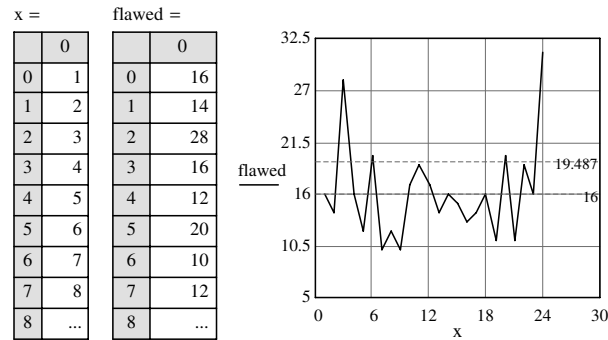


Figure 2.16. Counting control chart

– Example of a control chart inspired by a model by Ford manufacturer

Inspired from the Ford models, this (slightly modified) table is given as an example of an effective laboratory model: we used it in our workshops and mechanical engineering labs.

Process control charts $\bar{X}$ and R			Chart number	Sample chart source: FORD manufacturer	
Component	Characteristic	Specification	Sampling frequency	Operation	Machine
$\bar{X}$	Graph plot			Results summary	
				$\bar{\bar{X}} =$	
				$\bar{\bar{R}} =$	
				Process capability indices	
				$\hat{\sigma}$	
				$C_p =$	
		$C_{pk_s} =$			
Component	Characteristic	Specification	Sampling frequency	Operation	Machine
Specific comments					
R	Graph plot			Results summary	
				$C_{pk_{inf}} =$	
				$C_{pk} = \min (C_{pk_{sup}}, C_{pk_{nf}}) =$	
				<i>Control limits</i>	
				Average UCL =	
				Average LCL =	
		Range UCL =			

Table 2.19. Sample Ford manufacturer chart

Process control charts $\bar{X}$ and R			Chart number	Sample chart source: Ford manufacturer			
Operators	Nadim	Sarah	Sam	Range LCL =			
Hour				<i>Observations</i>			
Date				See Tables in Appendix for coefficient values			
X_i	1						
	2			n	<i>Tabulated constants</i>		
	3				A_2	D_3	D_4
	4			2	1.88	0.0	3.628
	5			3	1.023	0.0	2.574
$\bar{\bar{X}}$				4	0.729	0.0	2.282
$\bar{\bar{R}}$				5	0.577	0.0	2.114
Observations:							

Table 2.19 (continued). Sample Ford manufacturer chart

2.10.1. Poisson normal distribution approximation

This approximation is appropriate when the Poisson distribution's average is at least equal to five. We saw that the 3σ limits, for a table of c where c is the number of non-conformities (see Tables in the Appendix), are given by $\bar{c} \pm 3\sqrt{\bar{c}}$. The normal approximation by Poisson distribution shows the symmetry of the control limits. This requirement is often encountered in practice when the average is less than (9), where there are no LCL. To avoid such problems, we can use a transformation that gives us normally distributed data equivalents. Such a transformation is described by Ryan [RAY 00], who writes it as $Y=2\sqrt{c}$: for a large sample size, this is more or less normally distributed with an average of $2\sqrt{\lambda}$ and unit variance, where λ is the average of the Poisson distribution. Similar transformations were offered in classical literature, with the following expressions:

$$\left\{ y_1 = 2\sqrt{c + \frac{3}{8}} \text{ and } y_1 = \sqrt{c} + \sqrt{c+1} \right\} \quad [2.61]$$

The control limits are as follows, respectively:

$$\{\bar{y} \pm 3; \bar{y}_1 \pm 3 \text{ and } \bar{y}_2 \pm 3\} \quad [2.62]$$

The transformations can lead to significant control limits. We must keep in mind that the user manipulates data on a different scale than the original data. There is another way to fix the problem of symmetrical limits applied to a non-symmetrical case. We must use the limiting *probability*. We do not recommend using the 3σ limits for strongly biased distributions [RYA 97]. The proportion of control charts is as follows:

Let p be the defective fraction in a batch (population). The proportion or a non-conforming (defective) fraction in a population is defined as the ratio between the number of defectives in the population and the total number of elements in that population. The goal is to have one or more quality characteristics inspected simultaneously. If at least one characteristic does not conform with the norm, the article is declared non-conforming. The fraction or proportion can be expressed as a decimal or percent. The underlying statistical principles of a control chart of the average of non-conforming elements are based on the binomial distribution.

Let us assume that the production process operates in a stable fashion, such that the probability of a given unit not conforming to specifications is p . We assume that units produced successively are independent. In these conditions, each unit produced is the realization of a random Bernoulli variable of parameter p . If a random sample of n product units is chosen and if D is the number of non-conforming elements, D follows a binomial distribution of parameters n and p . The number of flawed units in a sample causes the binomial distribution model to act as follows:

$$p(D = x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, 2, \dots, \text{ where } \binom{n}{x} = \frac{n!}{x!(n-x)!} \quad [2.63]$$

The average of D is (n,p) and the variance is $n,p(1-p)$. The non-conforming *samples* correspond to the proportion of the ratio between the number of non-conforming individuals in the sample, D , and the sample size n .

$$\left\{ \hat{p} = (D/n); \mu = p \text{ and } \sigma_{\hat{p}}^2 = p(1-p)/n \right\} \quad [2.64]$$

These expressions are enough to develop the control chart for the proportion or a fraction of non-conforming individuals. This is the *p-chart* table.

2.10.1.1. Control charts for the proportion of defective lots p

If the fraction of p conforming elements is known (or a given standard value), then the median line and the control limits of the control chart of the fraction (proportion) of non-conforming elements can be written as:

$$\left\{ \text{UCL} = p + 3\sqrt{\frac{p(1-p)}{n}} \text{ and } \text{LCL} = p - 3\sqrt{\frac{p(1-p)}{n}} \right\} \quad p = \text{central line} \quad [2.65]$$

When the process fraction is known, it must be estimated from the available data. This is done by choosing m preliminary samples of size n . If there are D_i defectives in sample i , the fraction of non-conforming elements in sample i can be written as:

$$\hat{p}_i = (D_i/n); \quad i = 1, 2, 3, \dots, m \quad [2.66]$$

The average of these fractions (individuals) in the sample can be written as:

$$\bar{p} = \frac{\sum_{i=1}^m \hat{p}_i}{m} = \frac{\sum_{i=1}^m D_i}{m \times n}; \quad i = 1, 2, 3, \dots, m \quad [2.67]$$

$\bar{p}$ is used instead of p in graphical control configuration.

1) *Application example:* Consider a divider plate with 50 holes. On each section (i.e. $\frac{1}{4}$ of the plate), diameters are measured. Each time a poor superposition is logged, the corresponding diameter is controlled and measured. For example, for $P = 0.25$ (quarter plate), the observed results are:

No.	Defective fraction	No.	Defective fraction	No.	Defective fraction	No.	Defective fraction	No.	Defective fraction
1	0.25	11	0.15	21	0.41	31	0.47	41	0.28
2	0.29	12	0.12	22	0.33	32	0.36	42	0.30
3	0.15	13	0.25	23	0.48	33	0.40	43	0.16
4	0.20	14	0.24	24	0.32	34	0.30	44	0.29
5	0.12	15	0.35	25	0.21	35	0.18	45	0.25
6	0.14	16	0.16	26	0.24	36	0.24	46	0.14
7	0.18	17	0.20	27	0.19	37	0.16	47	0.32
8	0.18	18	0.18	28	0.21	38	0.26	48	0.19
9	0.25	19	0.25	29	0.18	39	0.18	49	0.28
10	0.20	20	0.28	30	0.12	40	0.17	50	0.24

Table 2.20. *Application example: defective fraction (also see the Appendix)*

2) *Sample proportions*: The corresponding control chart is given below, for $n = 50$ and $p = .25$; the average is $\text{mean}(\text{Flaw}) = 0.24$.

$$\left\{ \text{UCL} = p + 3\sqrt{\frac{p \times (1-p)}{n}} = 0.434 \text{ and } \text{LCL} = p - 3\sqrt{\frac{p \times (1-p)}{n}} = 0.066 \right\}$$

– *Control chart graph*:

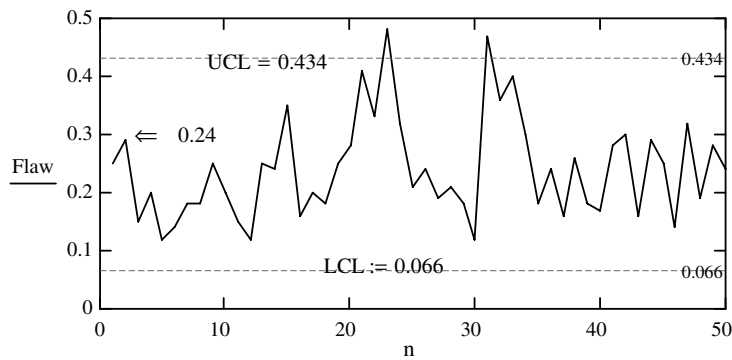


Figure 2.17. Proportion of the sample of a control chart

2.10.2. Decision limits of the $\bar{X}/R$ control chart through the SPC method

- n is the size of each sample.
- $\bar{X}$ is the (first-order) average of each sample.
- $\bar{\bar{X}}$ is the average of averages (second-order average) of samples.
- R is the range of a sample.
- $\bar{R}$ is the (first-order) average of the ranges of a sample.
- $\hat{\sigma} = (\bar{R}/d_n)$ is the estimate of the standard deviation of the population.

1) *Table of constant values*: This table is inspired by technical literature in the formulas used in quality control.

Graph of $\bar{X}$ or R	Decision limit notation	Observations notations	International ISO formulas ISO 8258	Formulas of the manufacturer Ford (USA)
$\bar{X}$ See constants	UCL of $\bar{X}$	Upper control limit	$\bar{\bar{X}} + A'_c \times \bar{R}$	$\bar{\bar{X}} + 2A_2 \times \bar{R}$
	LCL of $\bar{X}$	Lower control limit	$\bar{\bar{X}} - A'_c \times \bar{R}$	$\bar{\bar{X}} - 2A'_2 \times \bar{R}$
	UML of $\bar{X}$	Upper monitoring limit	$\bar{\bar{X}} + A'_s \times \bar{R}$	
	LML of $\bar{X}$	Lower monitoring limit	$\bar{\bar{X}} - A'_s \times \bar{R}$	
R See constants	UCL of R	Upper control limit	$D'_{c_2} \times \bar{R}$	$D_4 \times \bar{R}$
	LCL of R	Lower control limit	$D'_{c_1} \times \bar{R}$	$D_3 \times \bar{R}$
	UML of R	Upper monitoring limit	$D'_{s_1} \times \bar{R}$	
	LML of R	Lower monitoring limit	$D'_{s_1} \times \bar{R}$	

Table 2.21. Tables for the averages $\bar{X}$ and the range R (see [POI 92] and [BOI 01])

This constants table is inspired from the relevant technical literature.

Size (n) of each sample	Averages		Ranges					Average s	Ranges	
	A'_c	A'_s	D'_{c_1}	D'_{c_2}	D'_{s_1}	D'_{s_2}		A_2	D_3	D_4
2	1.937	1.229	0.00	4.12	0.04	2.81	1.128	1.880	-	3.267
3	1.054	0.668	0.04	2.99	0.18	2.17	1.693	1.023	-	2.574
4	0.750	0.476	0.10	2.58	0.29	1.93	2.059	0.729	-	2.282
5	0.594	0.377	0.16	2.36	0.37	1.81	2.326	0.577	-	2.114
6	0.498	0.316	0.21	2.22	0.42	1.72	2.534	0.483	-	2.004
7	0.432	0.274	0.26	2.12	0.46	1.68	2.704	0.419	0.076	1.924
8	0.384	0.244	0.29	2.04	0.50	1.62	2.847	0.373	0.136	1.864
9	0.347	0.220	0.32	1.99	0.52	1.58	2.970	0.337	0.184	1.816
10	0.317	0.202	0.35	1.94	0.54	1.56	3.078	0.308	0.223	1.777
11	0.295	0.186		1.90		1.53	3.173			
12	0.274	0.174		1.87		1.51	3.258			

Table 2.22. Table of constant values (see [POI 92] and [BOI 01])

2) *Risk*: For a controlled production, that is a perfectly stable production process, the risk of going above the upper control limit is limited to 1%. The same holds for the lower limit. As for the monitoring limits, we usually choose 2.5%. This generally corresponds to probabilities (99.8% and 95%) within which the production's variability is likely to be found.

Interpretation of the production processes based on the graph of the exploited control chart ($\bar{X}/R$):

– $\bar{X}$ chart

$\bar{X}$ graph	Interpretations, discussion and comments. We say that:
<i>The process is under statistical control (e.g. the graph is normal)</i>	<i>If $(-2/3)$ of the points are within the central third and $(-1/3)$ of the points are in the outside $(2/3)$.</i>
<i>The process is not under statistical control</i>	If there is (at least) one point outside of the control limits, we must understand that there is a proof of <i>assignable</i> causes relative to this/these point(s). We then receive an alarm signal that can be interpreted as: the control limit or the assigned point is false, and this is an isolated incident or we see a change of the system measurement logged on the machine's log.
<i>The process is not under statistical control</i>	We see tendencies, the nature of which can be the proof of a change in capability when:
	– Seven points are found on the same side of the average. – Regular increase or decrease of seven consecutive intervals.
	One or the other are indications of a derivation or a started tendency.
<i>The process is not under control: long series above and below $\bar{X}$</i>	The average $\bar{X}$ would be modified or (see log). This is the point that triggers a decision to pause or continue production.
<i>The process is not under control: accentuated convergence between points and control limits</i>	Unusual point spread $\rightarrow (-2/3)$ is within the center $1/3$.
	Also, over $2/3$ are within the central $1/3$, hence a sharp convergence of the mean: $\bar{X}$.
	Check the sampling methods and relevant calculations.

Table 2.23. Table of the first-order averages of $\bar{X}$

– Range (R) tables

(R) Graph	Interpretations, discussions and commentary. We say that
<i>The process is under statistical control</i>	The graph is normal.
<i>The process is not under statistical control</i>	If there is (at least) one point outside the control limits. We should know that there is proof of lack of control relative to this/these points. The assigned points will start a sound/visual signal that will inevitably lead to corrective action. For example: a point above the UCL shows an erroneous plot, a component-to-component variability that affects the range R . A point above the LCL shows the same phenomenon, with a decrease of the population's range (R).
<i>The process is not under statistical control (Long series)</i>	<i>Consecutive points around the same side of $\bar{X}$. Furthermore, we see a regular increase (or decrease) of consecutive intervals. The tendency is about to be seriously affected. We say that there is a shift in tendency.</i>
<i>The process is not under control: long series on either side of R, that is above and below R</i>	We observe a series greater than R -average. We should verify the machine's state (e.g. machine-tool parts and geometry of the cutting tools). We also notice an obvious difference between the nuance batches (material), a change in metrology (conventional measurements, CMM or others).
<i>The process is not under control: Sharp convergence of the control limits</i>	Same as for the analysis of the average $\bar{X}$.

Table 2.24. First-order range (R) table**2.11. Main statistical tests used in quality control**

Limiting ourselves to a mere two or three tests, no matter how precise and adequate, would be wrong: there exist many. In the following, we will apply three tests, namely: Pearson χ^2 , Kolmogorov–Smirnov, Student (t) and Henry. In this section, we tackle the most important statistical tests by applying them to concrete situations in our own metrology, quality control, design and production range labs.

2.11.1. So-called “Henry Line” or probability diagram test

We begin with a clarification: this is not a line but rather a series of points that “align” on a straight line, which would in turn be more like a regression line with a

tendency function and a correlation coefficient that confirms or denies the correlation between the points and this “line”. A useful visual tool in the less mathematically inclined schools and workshops, the Henry line allows us to clearly see where gaps are. This is, in fact, a graph comparing quantiles, or even a quantile–quantile graph. It is useful to see whether the data distribution is Gaussian straight away. This is when the transformation of the “Gaussian bell curve” into a regular line happens.

The observed values (e.g. our metrological data) are centered and reduced. If their distribution was normal, their respective frequencies would be compared to the repartition function of the centered reduced normal distribution (which would need a change of variables). There is a *gausso-arithmetic* paper (see Figure 2.21) to linearize the repartition function (cumulative probability function). The result is quick, simple and allows us to estimate the average and standard deviation of the observation distribution. We will show a case study of the normality test based on the following data:

Data	Values	Frequencies	Probabilities	Cum- frequencies	Φu
	0	1	2	3	4
0	3.000000000	3.000000000	$3.003000000 \times 10^{-3}$	$3.003000000 \times 10^{-3}$	-2.747453307
1	3.500000000	9.000000000	$9.000000000 \times 10^{-3}$	$1.200030000 \times 10^{-3}$	-2.257033109
2	4.000000000	23.000000000	0.023000000	0.035003000	-1.811871811
3	4.500000000	45.000000000	0.045000000	8.003000000	-1.405051361
4	5.000000000	77.000000000	0.077000000	0.157000000	-1.006851782
5	5.500000000	90.000000000	0.090000000	0.247000000	-0.683951161
6	6.000000000	152.000000000	0.152000000	0.399000000	-0.255928554
7	6.500000000	201.000000000	0.201000000	0.600000000	0.253354876
8	7.000000000	156.000000000	0.156000000	0.756000000	0.693502920
9	7.500000000	120.000000000	0.120000000	0.876000000	1.155235517
10	8.000000000	58.000000000	0.058000000	0.934000000	1.506285129
11	8.500000000	38.000000000	0.038000000	0.972000000	1.911082389
12	9.000000000	18.000000000	0.018000000	0.990000000	2.326460563
13	9.500000000	8.000000000	$8.000000000 \times 10^{-3}$	0.998000000	2.878635708
14	10.000000000	2.000000000	$2.002000000 \times 10^{-3}$	0.999900000	3.090232306

Table 2.25. Normality test case study

We programmed our calculations. Here are the results:

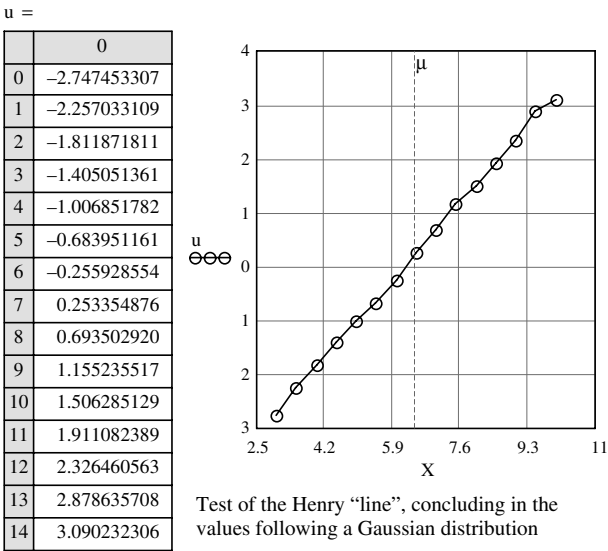


Figure 2.18. Plot of the points with the Henry line

2.11.1.1. Calculation results using MathCAD

Number of points n	Average μ	Standard deviation σ	Variance
$n : \text{rows}(\text{data})$	$\mu := \text{mean}(X)$	$\text{SD}(x) := \text{stdev}(x) \cdot \sqrt{\frac{n}{n-1}}$	$\text{SD}(X)^2 = 5$
15	6.5	2.236	5

Let Z be the centered, reduced value of the observations. We note that it can be written as a linear function, as shown below:

$$Z = \left(\frac{X - \mu}{\sigma} \right) = \left(\frac{1}{\sigma} X - \frac{\mu}{\sigma} \right) \tag{2.68}$$

This transformation proves that for any Z , there is a single corresponding X (see repartition function). This relation is indeed linear. The point is then to compare this line with the repartition function of the normal distribution, when the graphical alignment of its cumulative frequencies becomes linear. There is no quantity measuring the gap with a normal distribution as is the case for tests: we must judge the gap between both distributions visually.

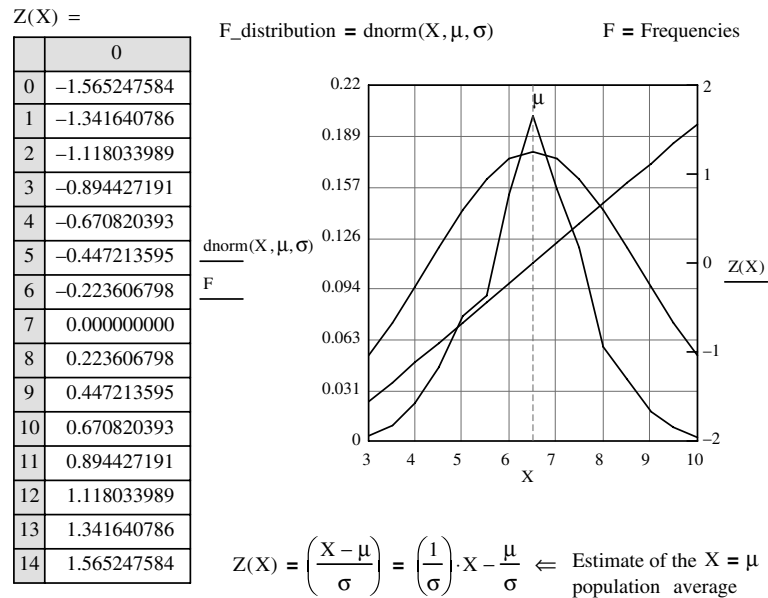


Figure 2.19. Theoretical and observed probability density functions and Z-curve

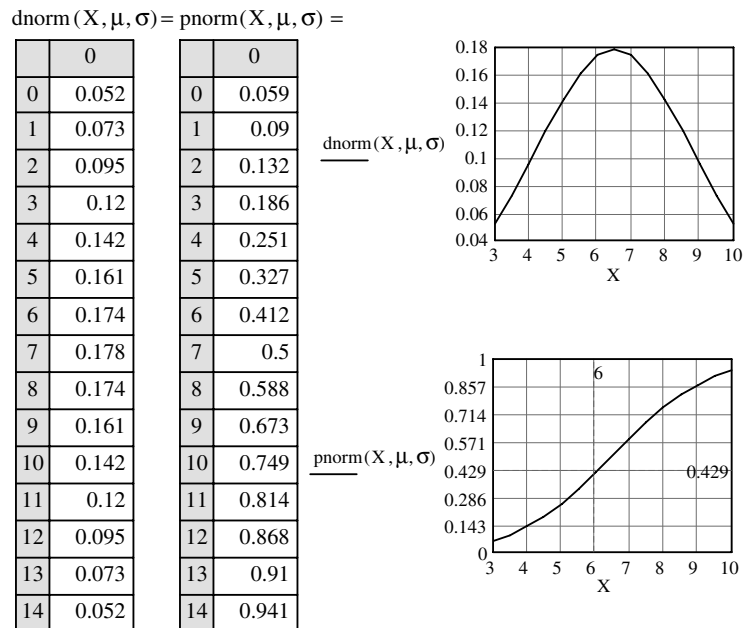


Figure 2.20. Plot of the probability density and repartition functions

Here is an illustration from the `gausso-arithmetic` paper

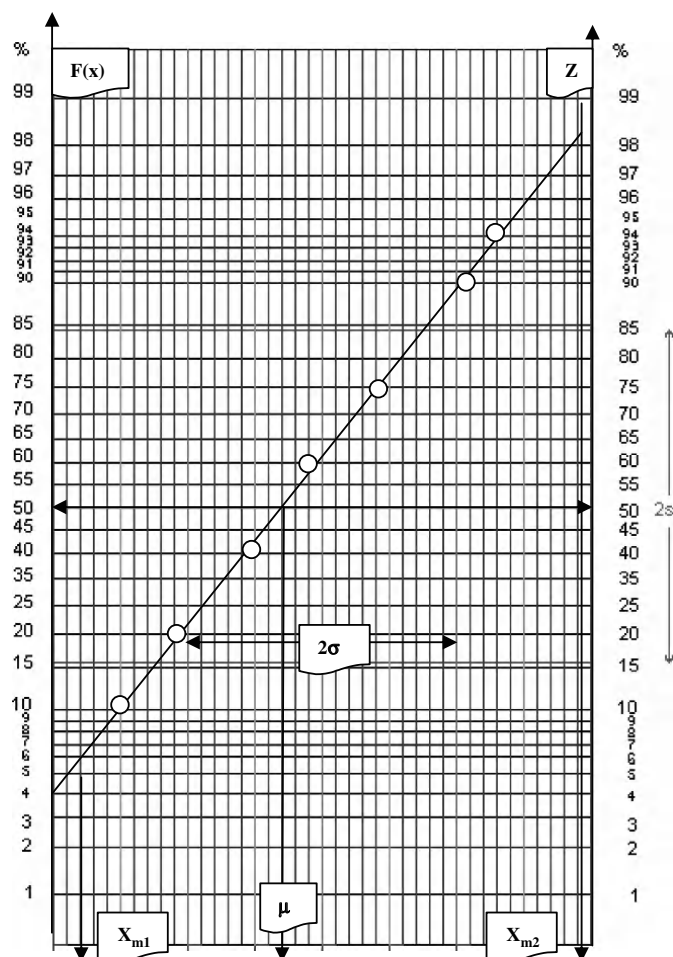


Figure 2.21. *Gausso-arithmetic graph to plot the Henry line*

We can use Excel for calculations and graphs. In fact, we calculated the last column u using the *NORMSINV* (for the standard normal distribution) function. We can also use normalized tables. The advantage of Excel or some other mathematical tool is that they allow us to draw a tendency line. Be careful, however, as we are not plotting the Henry line, but rather a mere linear regression line.

For a normal distribution, this graph (Figure 2.21) naturally gives us the average corresponding to the median, that is the x -coordinate of the point at $y = 50$. It also

gives us the standard deviation fairly easily using confidence intervals. Finally, we can realistically deduce that this approach is more pragmatic than the Wilson method. The latter is valuable, but too “*unrealistic*” for everyday use.

2.12. Partial conclusion

In this chapter, we showed the main tools used to best conduct a quality control process over mechanical components. We made deliberate choices to show representative examples of laboratories and other workshops of our own courses.

In Chapter 1 (Volume 1), we also developed themes relevant to calculating uncertainties. Quality control calculations should be framed by appropriated measurement uncertainty calculations. Technical literature [STI 78, STI 86] insists on this point, and we agree with [GRO 11] as to how to do it.

2.13. Bibliography

- [AND 84] ANDERSON T.W., *Introduction to Multivariate Statistical Analysis*, Wiley, New York, NY, 1984.
- [BIS 90] BISSEL A.F., “How reliable is your capability index ?” *Applied Statistics*, vol. 39, pp. 331–340, 1990.
- [BOI 01] BOITEUX B., *Guide du technicien qualité Outils pour la qualité en production*, Éditions Delagrave Paris, France, pp. 165–169, 2001
- [CAM 52] CAMERON J.M., “Tables for constructing and for computing the operating characteristics of single-sampling plans”, *Industrial Quality Control*, vol. IX, pp. 37–39, 1952.
- [CAV 66] CAVÉ R., *Le contrôle statistique des fabrications*, Eyrolles, France, 1966.
- [CER 86] CERESTA, (Centre d’enseignement et de recherche de la statistique appliquée), *Aide-mémoire pratique des techniques statistiques pour ingénieurs et techniciens supérieurs*, Paris, p. 274, 1986.
- [DEM 86] DEMING W.E., *Out of the Crisis*, Massachusetts Institute of Technology Center for Advanced Engineering Study, MIT Press, Cambridge, MA, 1986.
- [DEM 02] DEMING W.E., “Deming’s first major book giving the basis for his approach to management and quality”, *Out of the Crisis*, Cambridge, MIT Press, Cambridge, MA, 1982.
- [DOD 59] DODGE H.F., ROMIG H.G., *Sampling Inspection Tables*, John Wiley and Sons, NY, 1959.
- [GRO 94] GROUS A., *Étude probabiliste du comportement des Matériaux et structure d’un joint en croix soudé*, PhD in Engineering Sciences, UHA, France, 1994.

- [GRO 10] GROUS A., *Métrologie appliquée aux Sciences et Technologies. Tome 1 et 2 Éditions Hermes-Lavoisier Collection Capteurs et instrumentation dirigée par Dominique PLACKO*, Volume 1 and Volume 2. 2010.
- [GRO 11] GROUS A., *Applied Metrology for Manufacturing Engineering*, ISTE Ltd, London and John Wiley and Sons, New York, 2011.
- [JUR 89] JURAN J.M., *Planifier la qualité*, Coll. AFNOR Gestion, 1989.
- [KOT 92] KOTZ S., JOHNSON N.L., *Process Capability Indices*, Chapman & Hall, London, 1992.
- [LIN 65] LINDLEY D.V., *Probability and Statistics 2 Inference*, Cambridge University Press, 1965.
- [LUC 85] LUCAS J.M. "Counted data CUSUM", *Technometrics*, vol. 27, pp. 129–144, 1985.
- [LUC 90] LUCAS J.M., SACCUCCI M.S., "Exponentially weighted moving average control schemes: properties and enhancements", *Technometrics*, vol. 32, no. 1, pp. 1–30, 1990.
- [MON 00] MONTGOMERY D.C., *Introduction to Statistical Quality Control*, Wiley, 2000.
- [OTT 90] OTT E.R., SCHILLING E.G., *Process Quality Control*, 2nd ed., McGraw-Hill, New York, NY, 1990.
- [PAC 69] PACÉ P., *Contrôle des fabrications – Statistique 2: Essais-Réglages-Réception-Fiabilité*, Éditions Delagrave Paris, France, 1969.
- [POI 92] POIRIER J., *Observation statistique Tables statistiques*. R 240R 270, traité Mesures et Contrôle July, 1992.
- [QUE 93] QUESENBERRY C.P., "The effect of sample size on estimated limits for $\bar{X}$ and X control charts", *Journal of Quality Technology*, vol. 25, no. 4, pp. 237–247, 1993.
- [RYA 89] RYAN T.P., *Statistical Methods for Quality Improvement*, Wiley, 1989.
- [RYA 00] RYAN T.P., *Statistical Methods for Quality Improvement*, 2nd ed., Wiley, 2000.
- [RYA 97] RYAN T.P., SCHWERTMAN N.C., "Optimal limits for attributes control charts", *Journal of Quality Technology*, vol. 29, no. 1, pp. 86–98, 1997.
- [SCH 82] SCHILLING E.G., "Acceptance sampling in quality control", *Statistics: Text Book and Monographs*, Marcel Dekker, Inc., New York, vol. 42. 1982.
- [SCH 85] SCHILLING E.G., "The role of acceptance sampling in modern quality control", *Communications in Statistics Theory and Methods*, vol. 14, no. 11., pp. 2769–2777, 1985.
- [SHE 31] SHEWHART W.A., *Economic Control of Quality of Manufactured Products*, Re-edited in 1980 by the American Society for Quality, Van Nostrand and London: MacMillan, New York, p. 501, 1931.
- [SHE 39] SHEWHART W.A., *Statistical Method from the Viewpoint of Quality Control*, Re-edited by Dover Publications, NY, 1986, The Graduate School, U.S. Department of Agriculture, Washington, p. 155, 1939.

- [STI 78] STIGLER S.M., “Mathematical statistics in the early states”, *The Annals of Statistics*, vol. 6, pp. 239–265, 1978.
- [STI 86] STIGLER S.M., *The History of Statistics: The Measurement of Uncertainty Before 1900*, The Belknap Press of Harvard University Press, Cambridge, MA, 1986.
- [WOO 93] WOODAL W.L., ADAMS B.M., “The statistical design of CUSUM charts”, *Quality Engineering*, vol. 5, no. 4, pp. 559–570, 1993.
- [WOO 97] WOODALL W.H., “Control charting based on attribute data: bibliography and review”, *Journal of Quality Technology*, vol. 29, pp. 172–183, 1997.

Chapter 3

Case Studies

3.1. Quality control case study: calculating and plotting efficiency curves in simple/double control

In this chapter, we will consider applied cases from our own mechanical engineering workshops and labs. We will comment on the solutions and graphs of our results. We apply the statistical distributions shown throughout Chapters 1 and 2 of Volume 1 in these case studies. We also give examples solved in our workshops and labs on components and mechanisms designed as an introduction to design and automation.

3.1.1. Simple binomial distribution

EXAMPLE 3.1.– Consider a sample of size $n = 72$. We give ourselves an acceptance threshold (column two) and refusal threshold (column three), and then find and plot the efficiency curve for 20 data points representative of the experiment. We use binomial distribution for this.

Sample size (n)	Acceptance threshold (A)	Refusal threshold (R)	Plot minimum (%)	Plot maximum (%)
72	2	3	0	12

Table 3.1. Data from a sample (n). Simple binomial distribution

We can write the binomial distribution's probability density as:

$$P\{\kappa, n, q\} = \frac{n!}{\kappa!(n-\kappa)!} q^\kappa (1-q)^{n-\kappa} \quad [3.1]$$

The density probability function, which is already programmed into MathCAD, is written as: $dbinom(\kappa; n, q)$. The Excel formula¹ is also fairly simple and easily accessible to most users. For example, we should choose appropriate cells in the spreadsheet and keep its links to the formula in mind.

It can be written: `=BINOMDIST ($E$5;$E$4;D11;1)`

1) In cell E5, we write the acceptance level $(A2) = 2$ (in our case)

2) In cell E4, we write the sample size $(n) = 72$ (in our case)

3) In cell D11, we write the % value = 4% (in our case)

4) We finish writing with (1). We choose a location and start the Excel calculations, which gives us the following results:

$n = 10$; Number of flawed elements k						Values of p (k at most)				
n	$p = 1 \%$	$p = 2 \%$	$p = 3 \%$	$p = 4 \%$	$p = 5 \%$	$p = 6 \%$	$p = 7 \%$	$p = 8 \%$	$p = 9 \%$	$p = 10 \%$
0	0.9044	0.8171	0.7374	0.6648	0.5987	0.5386	0.4840	0.4344	0.3894	0.3487
1	0.9957	0.9838	0.9655	0.9418	0.9139	0.8824	0.8483	0.8121	0.7746	0.7361
2	0.999	0.9991	0.9972	0.9938	0.9885	0.9812	0.9717	0.9599	0.9460	0.9298
3	1	1	0.9999	0.9996	0.999	0.998	0.9964	0.9942	0.9912	0.9872
4			1	1	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984
5					1	1	1	1	0.9999	0.9999
6									1	1

Table 3.2. *BINOMDIST* (k, n, q)

3.1.2. Calculation results

Number	K (%)	From Excel spreadsheet = LOI.BINOMIALE (k, n, q)
0	0.00 %	1
1	0.80 %	0.979722390
2	1.60 %	0.891171555
3	2.40 %	0.750710937
4	3.20 %	0.593678536
5	4.00 %	0.446418891

Table 3.3. *Results of the BINOMDIST* (k, n, q) *function from Excel*

¹ In our example, processed using Excel table, we have retained the cell numbers corresponding to our balance sheets. It is helpful to respect the address of the cell inherent to the classifier of each user.

6	4.80 %	0.322315534
7	5.60 %	0.225053764
8	6.40 %	0.152778145
9	7.20 %	0.101235045
10	8.00 %	0.065676142
11	8.80 %	0.041812465
12	9.60 %	0.026170998
13	10.40 %	0.016128050
14	11.20 %	0.009797068
15	12.00 %	0.005871806
16	12.80 %	0.003474891
17	13.60 %	0.002031778
18	14.40 %	0.001174357
19	15.20 %	0.000671267
20	16.00 %	0.000379587

Table 3.3 (continued). Results of the BINOMDIST (k , n , q) function from Excel

Using the results from the third column, we plot the efficiency curve from Excel.

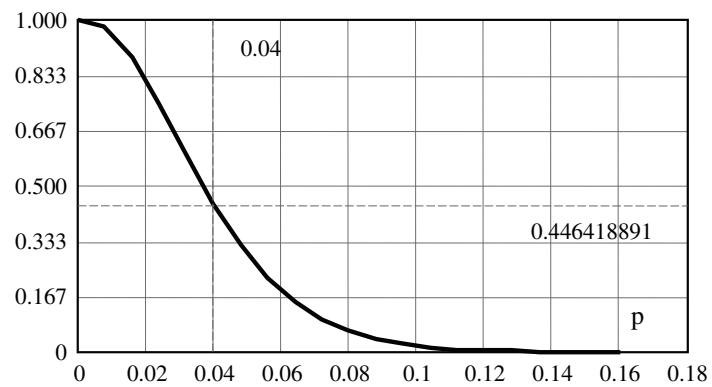


Figure 3.1. Efficiency curve from Excel calculations

Deliberately, and with different data, we performed the calculations using MathCAD. The results were as follows:

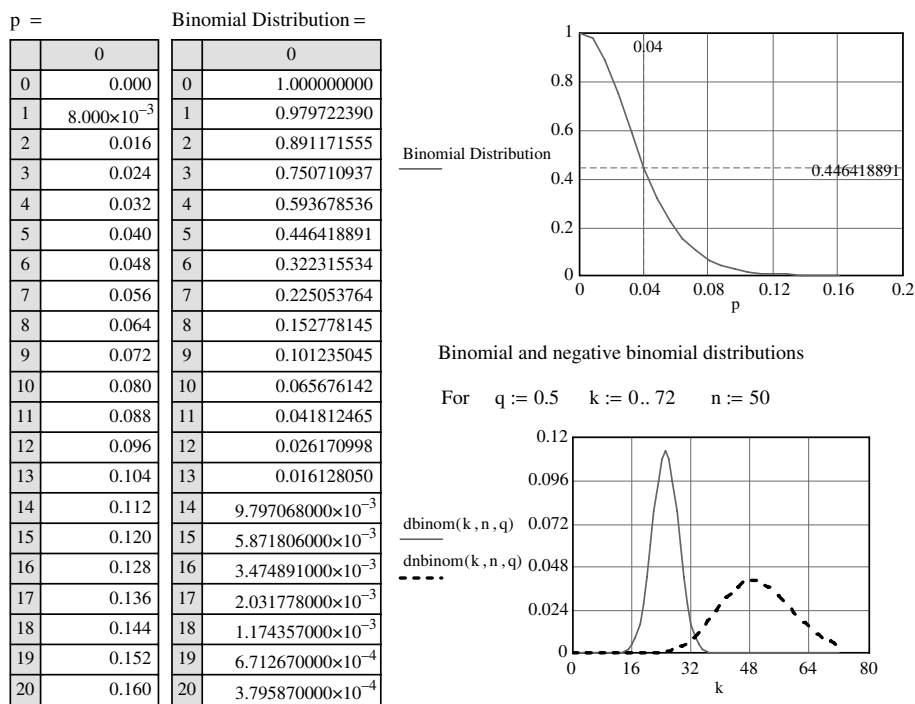


Figure 3.2. Efficiency curve using MathCAD

3.2. Calculating the efficiency curves of simple/double control

– *Hypergeometric law*: We proceed similarly to solve the problem of the hypergeometric distribution (see Chapters 1 and 2, Volume 1). Consider the data shown below. We are asked to plot the efficiency curve for this plan. This time, we used Excel only to solve the problem.

Batch size (n)	Sample size (n)	Number of flaws in batch	Acceptance level (A)	Refusal level (R)	Plot min (%)	Plot max (%)
1001	121	2 (at 100%)	3	1	0	15

Table 3.4. Data of a sample (n). Hypergeometric (Pascal) distribution

3.2.1. Calculation results

N_0	p	Efficiency curve		0	1	2	3
0	0 %	1	0	1	0	0	0
1	1 %	0.976118776	10.01	0.274020556	0.380671495	0.235736935	0.085689791
2	2 %	0.78508103	20.02	0.074041884	0.208108432	0.275224608	0.227706107
3	3 %	0.499354857	30.03	0.019721931	0.084125275	0.17180514	0.223702511
4	4 %	0.265812215	40.04	0.005176796	0.02979274	0.082796927	0.148045751
5	5 %	0.124240704	50.05	0.001338659	0.009745949	0.034438812	0.078717284
6	6 %	0.052629449	60.06	0.000340901	0.003014544	0.012982341	0.036291664
7	7 %	0.020627388	70.07	8.54640×10^{-5}	0.000892578	0.004550827	0.015098520
8	8 %	0.007584588	80.08	2.10852×10^{-5}	0.000254813	0.001505999	0.005802691
9	9 %	0.002641578	90.09	5.11738×10^{-6}	7.04530×10^{-5}	0.000475024	0.002090984
10	10 %	0.000877427	100.1	1.22131×10^{-6}	1.89216×10^{-5}	0.000143727	0.000713557
11	11 %	0.000279346	110.11	2.86505×10^{-7}	4.94602×10^{-6}	4.19002×10^{-5}	0.000232213
12	12 %	8.55588×10^{-5}	120.12	6.60373×10^{-8}	1.26000×10^{-6}	1.18063×10^{-6}	7.24265×10^{-5}
13	13 %	2.52807×10^{-5}	130.13	1.49488×10^{-8}	3.13109×10^{-7}	3.22269×10^{-6}	2.17300×10^{-5}
14	14 %	7.22157×10^{-5}	140.14	3.32192×10^{-9}	7.59424×10^{-8}	8.53584×10^{-7}	6.28872×10^{-6}
15	15 %	1.99750×10^{-5}	150.15	7.24325×10^{-10}	1.79843×10^{-8}	2.19644×10^{-7}	1.75915×10^{-6}
		1	2	0.772747253	0.212747253	0.014505495	0.000000000

Table 3.5. Results of our statistical analysis. Hypergeometric distribution

For example, to find cell E43 we should write:

=SI(\$F\$7>=E\$42;SI(\$D43<E\$42;0; HYPGEOM.DIST (E\$42;\$F\$6;\$D43;\$B\$6));""

Then, we should calculate HYPGEOM.DIST to find $\rightarrow E43 = 1$, etc.

3.2.1.1. Plot and calculation results using MathCAD

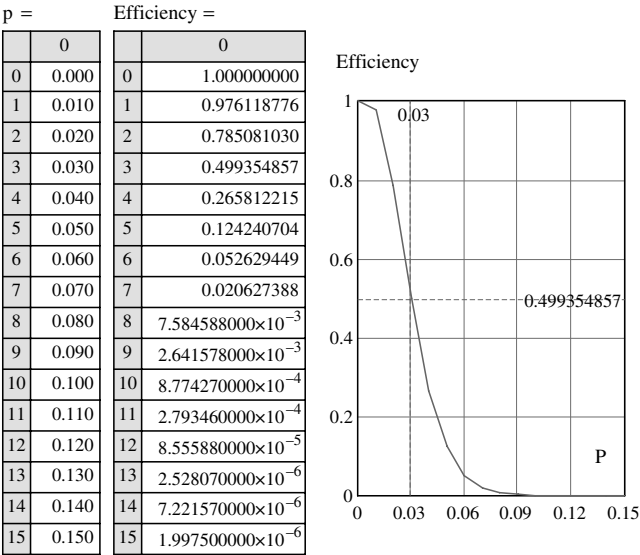


Figure 3.3. Efficiency curve as plotted with MathCAD

3.2.1.2. Plot and calculations results using Excel

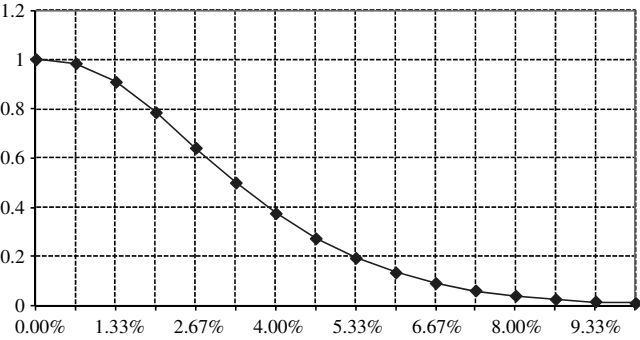


Figure 3.4. Efficiency curve (Excel) – simple control

We note that the results are exactly the same. We are then free to proceed using whatever mathematical tool is available at the time.

3.3. Calculating efficiency curves in double control: binomial distribution, double efficiency plan (Excel version)

The procedure above can be applied to the following problem, which is dedicated to a double plan.

Sample size (n)	Acceptance level (A_1)	Refusal threshold (R_1)	Plot min (%)	Plot max (%)	Percentage of defective elements	Acceptance probability
50	0	3	0	10	0.03	0.46687979

Table 3.6. Binomial law dedicated to a double plan

We plug the data in and write the Excel expression for the binomial distribution:

= BINOMDIST (\$D\$4;\$D\$3;B33;1), which gives us = 1, (accept 1st) etc. ...

3.3.1. Calculation results

No.	%	Efficiency curve	Accept 1 st	Go to 2	Refusal with 2	Accept 2 nd
0	0.00 %	1	1	5×10^{-11}	-2.2204×10^{-15}	5×10^{-11}
1	0.67 %	0.98709600	0.71573205	0.27967099	0.02970291	0.27136394
2	1.33 %	0.90205237	0.51111993	0.45969059	0.14957485	0.39093244
3	2.00 %	0.74135599	0.36416968	0.55740257	0.32331438	0.37718631
4	2.67 %	0.55495328	0.25886888	0.59264408	0.50040097	0.29608440
5	3.33 %	0.38696722	0.18358499	0.58393560	0.65170433	0.20338224
6	4.00 %	0.25682793	0.12988579	0.54682821	0.76785738	0.12694214
7	4.67 %	0.16530782	0.09167247	0.49346407	0.85077870	0.07363535
8	5.33 %	0.10482869	0.06454376	0.43276713	0.90691315	0.04028493
9	6.00 %	0.06632930	0.04533073	0.37091575	0.94338722	0.02099857
10	6.67 %	0.04226232	0.03175695	0.31189865	0.96631802	0.01050536
11	7.33 %	0.02726299	0.02219101	0.25804902	0.98034490	0.00507198
12	8.00 %	0.01783949	0.01546648	0.21050780	0.98872720	0.00237301
13	8.67 %	0.01183083	0.01075141	0.16960006	0.99363547	0.00107942
14	9.33 %	0.00793248	0.00745388	0.13512578	0.99645810	0.00047860
15	10.0 %	0.00536105	0.00515378	0.10657498	0.99805512	0.00020728
			0,21806538	0,59273270	0.58022492	0.24881442

Table 3.7. Results table

3.3.1.1. Plot of the efficiency curve (MathCAD)

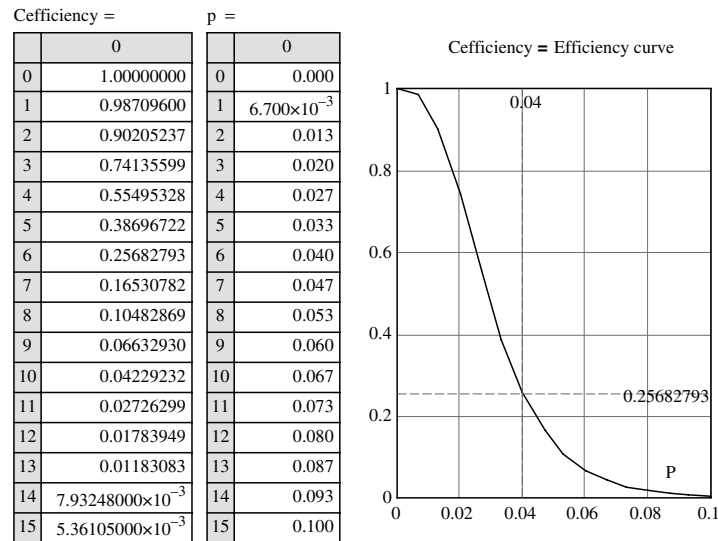


Figure 3.5. Efficiency curve – simple control

3.4. Progressive control (Excel)

3.4.1. Calculating the acceptance and refusal curves in progressive control

For the data shown in Table 3.8, we will plot the acceptance and refusal graphs.

Percentage p_1	Alpha risk	Percentage p_1	Beta risk	Acceptance h_1	Refusal h_2
1%	3%	5%	10%	1.37647799	2.06048149
Slope S	Minimum number to accept the batch	Denominator			
0.02498542	50	0.71688159			

Table 3.8. Tables for the refusal and acceptance curves

We programmed the following formulas in MathCAD and Excel (see cells) to find the results shown:

$$- \text{Denominator} = \text{Log} \left(\frac{\text{percentage } p_2}{\text{percentage } p_1} \right) + \text{Log} \left(\frac{1 - \text{percentage } p_1}{1 - \text{percentage } p_2} \right) = 0.71688159$$

- Acceptance $h_1 = \frac{\text{Log}(1 - \text{Risk beta/Risk alpha})}{\text{Denominator}} = 1.51166072$
- Rejection $h_2 = \frac{\text{Log}(1 - \text{Risk beta / Risk alpha})}{\text{Denominator}} = 2.07379655$
- Slope $Tg = \frac{\text{Log}(1 - \text{percentage } p_1 / 1 - \text{percentage } p_2)}{\text{Denominator}} = 0.02498542$

3.4.2. Calculation results

We write the following into the third column:

=IF(E5="" ;"" ;SI(AND(AK5>=AJ5 ;AK5<=AL5) ;"continuous" ;IF(AK5<AJ5 ;
"accept" ;"refusal")))

The calculation result will allow us to answer with → Continue

List of values corresponds to → (0 = good; 1 = bad)								
1	0	continue	51	0	continue	101	0	continue
2	0	continue	52	0	continue	102	0	continue
3	0	continue	53	0	continue	103	0	continue
4	0	continue	54	0	continue	104	0	continue
5	0	continue	55	0	continue	105	0	continue
6	0	continue	56	0	continue	106	0	continue
7	0	continue	57	0	continue	107	0	continue
8	0	continue	58	0	continue	108	0	continue
9	0	continue	59	0	continue	109	0	continue
10	0	continue	60	0	continue	110	0	continue
11	0	continue	61	0	continue	111	0	continue
12	0	continue	62	0	continue	112	0	continue
13	0	continue	63	0	continue	113	0	continue
14	0	continue	64	0	continue	114	0	continue
15	0	continue	65	0	continue	115	0	continue
16	0	continue	66	0	continue	116	0	continue
17	0	continue	67	0	continue	117	1	continue
18	0	continue	68	0	continue	118	0	continue

Table 3.9. Results for the acceptance and refusal curves in progressive control

List of values corresponds to → (0 = good; 1 = bad)								
19	0	continue	69	0	continue	119	0	continue
20	0	continue	70	0	continue	120	0	continue
21	0	continue	71	0	continue	121	0	continue
22	0	continue	72	0	continue	122	0	continue
23	0	continue	73	0	continue	123	0	continue
24	0	continue	74	0	continue	124	0	continue
25	1	continue	75	0	continue	125	0	continue
26	0	continue	76	0	continue	126	0	continue
27	0	continue	77	0	continue	127	0	continue
28	0	continue	78	0	continue	128	0	continue
29	0	continue	79	0	continue	129	0	continue
30	0	continue	80	0	continue	130	0	continue
31	0	continue	81	0	continue	131	0	continue
32	0	continue	82	0	continue	132	0	continue
33	0	continue	83	0	continue	133	0	continue
34	0	continue	84	0	continue	134	0	continue
35	0	continue	85	0	continue	135	0	continue
36	0	continue	86	0	continue	136	0	continue
37	0	continue	87	0	continue	137	0	continue
38	0	continue	88	0	continue	138	0	continue
39	0	continue	89	0	continue	139	0	continue
40	0	continue	90	0	continue	140	0	continue
41	0	continue	91	0	continue	141	0	continue
42	0	continue	92	0	continue	142	0	continue
43	0	continue	93	0	continue	143	0	continue
44	0	continue	94	0	continue	144	1	continue
45	0	continue	95	0	continue	145	0	continue
46	0	continue	96	0	continue	146	0	continue
47	0	continue	97	0	continue	147	0	continue
48	0	continue	98	0	continue	148	0	continue
49	0	continue	99	0	continue	149	0	continue
50	1	continue	100	1	continue	150	0	continue

Table 3.9 (continued). Results for the acceptance and refusal curves in progressive control

3.4.3. Graph

Based on the results found above, we plot the acceptance and/or refusal curves that frame the series of experiments. This method is a good graphical illustration.

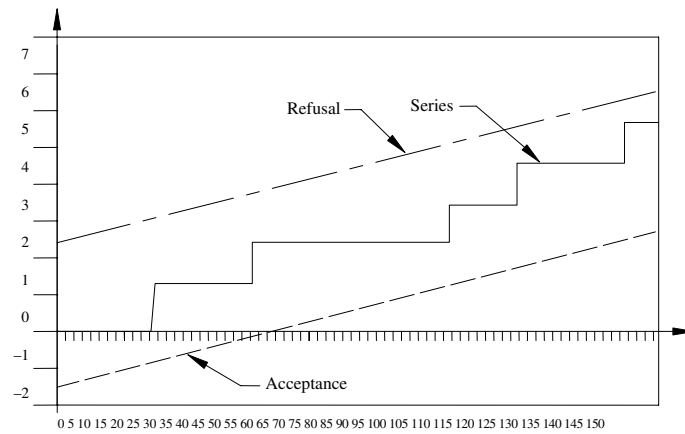


Figure 3.6. Acceptance and refusal curves (progressive control)

3.4.4. Measurement acceptance test when sigma (σ^*) is known

3.4.4.1. Choosing a plan

Flaws (%)	p	u		p	u	Sample size (n)	Discrimination (DS)	Gap at the limit (k)
Flaw p_1 (%)	0.5	2.576	Risk alpha	3	1.881	20.3	8.0	2.38
Flaw p_2 (%)	3	1.881	Risk beta	7%	1.476	Notes		

Table 3.10. Choosing a plan (see the tables in the Appendix)

3.4.5. Determining the efficiency curve and the acceptance limits

Known standard deviation		0,01	Acceptance		
Upper tolerance		10,05	10,0281		
Lower tolerance		9,95	9,9719		
Sample size (n)	Gap at the limit (k)	Plot min (%)	Plot max (%)	% value of p	% value of u
25	2,19	0	0.08	0,5	97,31%

Table 3.11. Efficiency and acceptance limits table (see the tables in the Appendix)

We chose to keep the calculation formulas used in Excel, as users might benefit from our educational drive.

$$\text{Discrimination (DS)} = \text{Flaw } P_2(\%) / \text{Flaw } P_1(\%) = 2/0.25 = 8.0.$$

$$\text{Value } (\%) = 1 - \text{NORMSDIST}(F53) \text{ and } F53 = (\$B\$15 - E53) * \text{RSQRT}(\$B\$14).$$

$$B15 = \text{standard deviation}; B14 = \text{sample size}(n) \text{ and } E53 = -\text{NORMINV}(B53).$$

$$u = -\text{NORMINV}(E6) \rightarrow E6 = \text{Risk alpha } (p) = 3.00\% = 1.881.$$

$$u = -\text{NORMINV}(E7) \rightarrow E7 = \text{Risk beta } (p) = 7.00\% = 1.476.$$

$$\text{Gap at the limit } (k) = (C6 * F7 + F6 * C7) / (F6 + F7) = 2.38.$$

$$\text{Gap at the limit } (k) = \frac{u(\text{Flaw } P_1) \times u(\text{risk beta}) + u(\text{risk alpha}) \times u(\text{Flaw } P_2)}{u(\text{risk alpha}) + u(\text{risk beta})}.$$

3.4.6. Acceptance: known standard deviation, sigma = 0.01

$$\text{Tolerance}_{\text{up}} = 10.05 = E15 - (B15 \times E14) = \text{Tol}_{\text{up}} - \{\text{Gap at the limit}(k) \times \text{known standard deviation}\}$$

$$\text{Tolerance}_{\text{low}} = 9.95 = E16 + B15 \times E14 = \text{Tol}_{\text{low}} + \{\text{Gap at the limit}(k) \times \text{known standard deviation}\}$$

3.4.6.1. Calculation results

	No. (%)	Percentage (%)		
0	0.00 %	100.00 %	7.03448383	-24.22
1	0.01 %	100.00 %	3.87490417	-8.42
2	0.01 %	100.00 %	3.70268067	-7.56
3	0.02 %	100.00 %	3.59854702	-7.04
4	0.02 %	100.00 %	3.52301534	-6.67
5	0.03 %	100.00 %	3.46343101	-6.37
6	0.03 %	100.00 %	3.41407055	-6.12
7	0.04 %	100.00 %	3.37184423	-5.91
8	0.04 %	100.00 %	3.33488939	-5.72
9	0.05 %	100.00 %	3.30199436	-5.56
10	0.05 %	100.00 %	3.27232557	-5.41
11	0.06 %	100.00 %	3.24528450	-5.28
12	0.06 %	100.00 %	3.22042652	-5.15
13	0.07 %	100.00 %	3.19741210	-5.04
14	0.07 %	100.00 %	3.17597595	-4.93
15	0.08 %	100.00 %	3.15590676	-4.830
	0.50 %	97.31 %	2.57583451	-1.930

Table 3.12. Efficiency and the acceptance limits results

3.4.6.2. Plot (Excel)

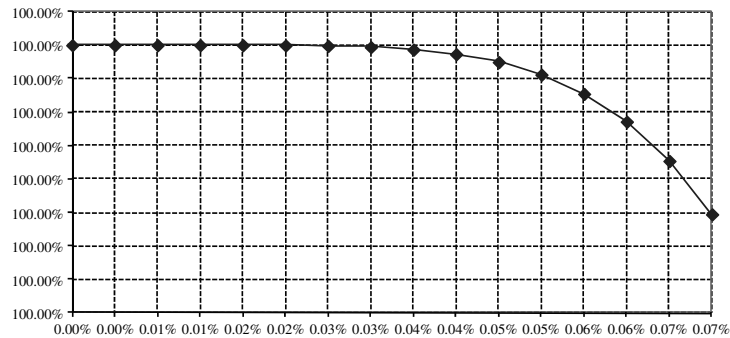


Figure 3.7. Plot of a measurement acceptance test with known σ^*

3.4.7. Measurement acceptance test when σ^* (SD) is known

3.4.7.1. Calculations

Discrimination (DS) = 5; risk alpha (p) = 3% and risk beta (p) = 8%.

= – NORMINV (B6/100); B6 = Flaw (P_1 %) = 0.5.

= – NORMINV (B7/100); B7 = Flaws (P_2 %) = 1.881.

u = – NORMINV (F6) = 1.881; F6 = Risk alpha (P) = 3%.

u = – NORMINV (F7) = 1.405; F7 = Risk beta (P) = 8%.

Acceptance Tol_{up} = 10.0285; Acceptance Tol_{low} = 9.9715;

Standard deviation of the sample = 0.01

Tol_{up} = 10.05; Tol_{low} = 9.95

Acceptance Tol_{up} = F16 – (B16 * F15) = Tol_{up} – (Gap at the limit (k) = 2.15)

× Standard deviation of the sample (n)

Acceptance Tol_{low} = F17 + (B16 * F15) = Tol_{low} + (Gap at the limit (k) = 2.15)

× Standard deviation of the sample (n)

Sample average: Rejection if: = If (And (F19 < G16; F19 > G17); “YES”; “rejection”)

F19 = Sample average = 10.03; G16 = Tol_{up} = 10.05; G17 = Tol_{low} = 9.95

= – NORMINV(B54); B54 = 0.50%

3.4.8. Choosing a plan

– Determining the efficiency curve (Excel and MathCAD)

Flaws	P	u	Risk	<i>p</i>	<i>u</i>	Sample size (<i>n</i>)	75.4
Flaw P_1 (%)	0.5	2.576	Risk alpha	3.00%	1.881	Discrimination (DS)	5
Flaw P_2 (%)	3	1.881	Risk beta	8.00%	1.405	Gap at the limit k	2.18

Sample size (<i>n</i>)	100	Sample's standard deviation	0.01	Acceptance	% value of P	% value of u
Gap at the limit (<i>k</i>)	2.15	Upper tolerance	10.05	10.0285	0.5	99.04 %
Plot min (%)	0	Lower tolerance	9.950	9.9715		
Plot max (%)	6	Sample average	10.03	Refusal		

Table 3.13. Efficiency curve

– Calculation results (Excel)

No.	Flaws (%)	P_{accep} (%)		
0	0.00%	100.00%	7.034483825	−26.84
1	0.40%	99.71%	2.652069808	−2.760
2	0.80%	92.26%	2.408915546	−1.420
3	1.20%	72.20%	2.257129244	−0.590
4	1.60%	48.77%	2.144410621	0.03
5	2.00%	29.84%	2.053748911	0.53
6	2.40%	17.14%	1.977368428	0.95
7	2.80%	9.46%	1.911035648	1.31
8	3.20%	5.09%	1.852179859	1.64
9	3.60%	2.69%	1.799118107	1.93
10	4.00%	1.41%	1.750686071	2.19
11	4.40%	0.73%	1.706043397	2.44
12	4.80%	0.38%	1.664562861	2.67
13	5.20%	0.20%	1.625763386	2.88
14	5.60%	0.10%	1.589267557	3.08
15	6.00%	0.05%	1.554773595	3.27
	0.50%	99.04%	2.575834515	−2.34

Table 3.14. Results of the efficiency curve

3.4.9. Plot (Excel)

3.4.9.1. Measurement acceptance test with known sigma

This plot is equivalent to the plot prepared using MathCAD. We show the plot made using Excel for the sake of simplicity.

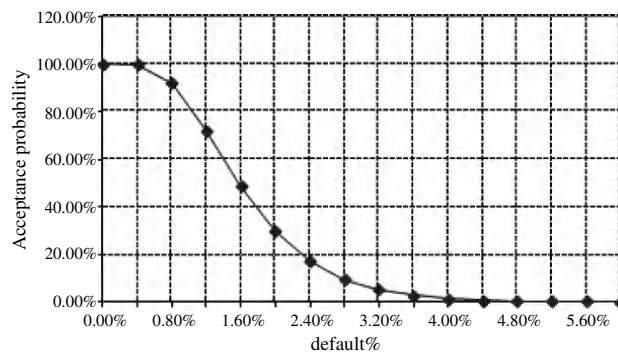


Figure 3.8. Flaw acceptance probabilities (Excel)

3.4.9.2. Results and plot from MathCAD calculations

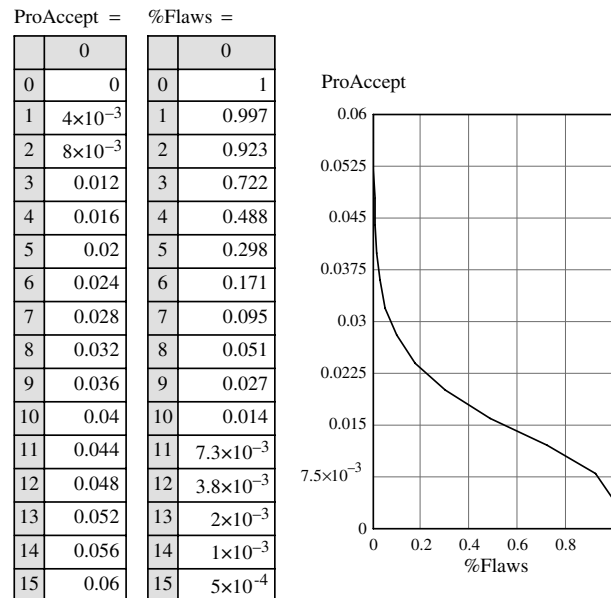


Figure 3.9. Flaw acceptance probabilities (MathCAD)

DISCUSSION.— We know that the theory developed in the respective chapters (Chapters 1 and 2, volume 1) finds concrete applications as shown here. More importantly, we see the graphs from calculations in our own dimensional metrology labs, new organization methods in work, quality control, production ranges and process design.

3.5. R&R study in quality control and dimensional metrology

The case studies we will show are relevant to many metrological applications, such as electronic dimensional metrology (coordinate measurement machine, CMM), optical metrology (profile projector) and conventional metrology (micrometer).

3.5.1. R&R (EV Repeatability and AR reproducibility)

Consider a gear shaft with threaded extremities M, 0.55×1.5 , 6g (length: 0.65 inches), designed and manufactured in workshop and triple quantity (30×3). Three operators (machinist-lab assistants: Sarah, Nadim and Sam) machined and controlled on the TMM. The results of their respective measurements have yielded the following:

Because of the QS 9000 norm, we will use the formula ($5.15 \times \sigma$). This corresponds to a probability of 99%.

$$\rightarrow R_{pv} = 0.0082 = [\text{Max}(\text{Avg}(p)) - \text{Min}(\text{Avg}(p))]$$

$$\rightarrow \text{Difference} = \text{Max } \bar{x} - \text{Min } \bar{x} = 0.0008$$

D_4 is automatically calculated (25,800) as a function of the number of operators (three in this case study). The Excel spreadsheet calculates D_4 as follows:

$$\rightarrow = \text{IF}(\text{NB}(\text{C9:E9}) > 2; 2, 58; 3, 27)$$

We should specify that cells C9 to E9 correspond to series 1, 2 and 3 of the component (series 1 is controlled by operator A, i.e. Sarah).

The upper control limit is thus calculated:

$$\bar{\bar{R}} \times D_4 = 0.0052 \times 2.580 = 0.0134$$

$$d_2 = \text{GAMMAINV} \rightarrow = \text{IF}(G38 > 2; 1.693; 1.128) = 1.6930$$

In cell G38 ($G38 = 3$), there are three distinct operators (Sarah, Nadim and Sam). Table 3.15 holds all of the experimental data used here in the R&R study. There is a lot of software related to R&R studies (MeasurLink). For the sake of education, we would rather treat our values using Excel by insisting on formulas used here, which the user will be able to use with their own values.

3.5.2. R&R study

Sample	Operator A – Sarah				Operator B – Nadim				Operator C – Samy				Average P
	Series 1	Series 2	Series 3	Range	Series 1	Series 2	Series 3	Range	Series 1	Series 2	Series 3	Range	
Component 1	0.5500	0.5600	0.5600	0.0100	0.5500	0.5600	0.5600	0.0100	0.5500	0.5600	0.5600	0.0100	
Component 2	0.5541	0.5525	0.5535	0.0016	0.5541	0.5525	0.5535	0.0016	0.5541	0.5590	0.5535	0.0055	0.5567
Component 3	0.5600	0.5605	0.5600	0.0005	0.5600	0.5605	0.5600	0.0005	0.5590	0.5605	0.5600	0.0015	0.5541
Component 4	0.5600	0.5561	0.5561	0.0039	0.5600	0.5561	0.5561	0.0039	0.5600	0.5561	0.5561	0.0039	0.5601
Component 5	0.5561	0.5566	0.5566	0.0005	0.5561	0.5566	0.5566	0.0005	0.5561	0.5600	0.5566	0.0039	0.5574
Component 6	0.5562	0.5700	0.5584	0.0138	0.5562	0.5601	0.5584	0.0039	0.5600	0.5700	0.5584	0.0116	0.5564
Component 7	0.5500	0.5600	0.5568	0.0100	0.5500	0.5568	0.5568	0.0068	0.5500	0.5568	0.5568	0.0068	0.5615
Component 8	0.5559	0.5600	0.5665	0.0106	0.5559	0.5665	0.5665	0.0106	0.5559	0.5665	0.5665	0.0106	0.5545
Component 9	0.5575	0.5580	0.5584	0.0009	0.5575	0.5580	0.5584	0.0009	0.5575	0.5580	0.5584	0.0009	0.5630
Component 10	0.5568	0.5595	0.5600	0.0032	0.5568	0.5595	0.5600	0.0032	0.5568	0.5595	0.5600	0.0032	0.5580
	Average $\bar{x} = 0.5579$				Average $\bar{x} = 0.5557$				Average $\bar{x} = 0.5584$				$R_p = 0.0082$
	Average $\bar{R} = 0.0055$				Average $\bar{R} = 0.0042$				Average $\bar{R} = 0.0058$				

Note: We should have at least 15 components controlled by operators (significant experiment)

$\bar{\bar{x}}$ Average of average $\bar{x} =$	$\bar{\bar{R}}$ Average of average $\bar{R} =$	Average maximum $\bar{\bar{x}} = 0.5584$	Average minimum $\bar{\bar{x}} = 0.5577$	Difference between averages D = 0.0008	Upper control limit LSC = 0.0103	Value of coefficient D4 = 2.58
0.5580	0.0052					

Table 3.15. R&R study calculation table

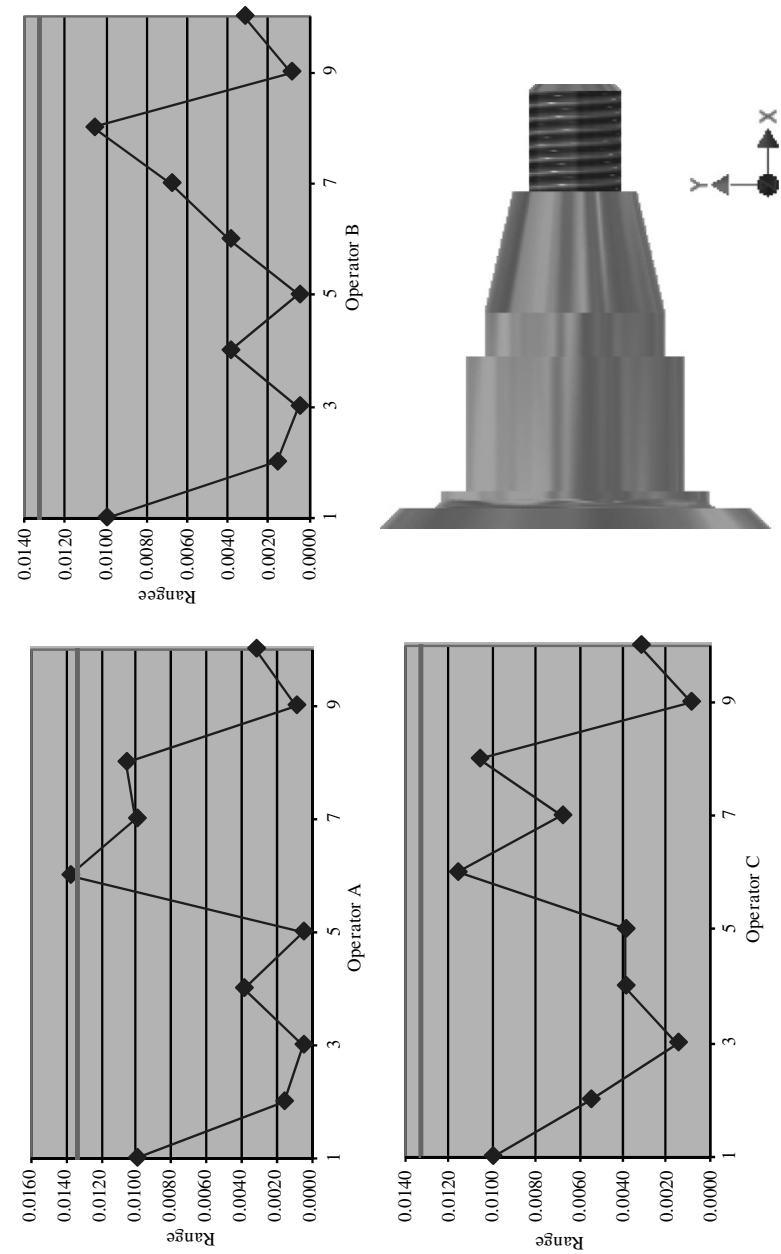


Figure 3.10. Gear shaft definition drawing. Threading $\varnothing 0.55 \times 1.5 - 6g$

Repeatability, EV (equipment variation)				Measurement series	d_2	EV	S_{EV}	Tolerance %	
$\sigma_{EV} = (\bar{R}/d_2)$				Three operators	1.693	0.0157	0.003	$\%EV = \left(\frac{100EV}{Tolerance} \right)$	
EV = $5.15 \times 0.0030 = 0.0157$								$\%EV = 74.482\%$	
Reproducibility, AV (appraiser variation)				Measurement series	d_2	AV	S_{AV}	Tolerance %	
$\sigma_{OpAvg} = \sqrt{\frac{\sigma_{EV}^2}{n \times r} + \sigma_{AV}^2} = \left(\frac{R_{\bar{x}}}{d'_2} \right)$				Three Operators	1.912	0.0000	0.000	$\%AV = \left(\frac{100AV}{Tolerance} \right)$	
AV = $5.15 \times 0.0000 = 0.0000$								$\%AV = 0.0000\%$	
Process % $\rightarrow$				$\%AV = \left(\frac{100AV}{TV} \right) \rightarrow \%AV \rightarrow$				0.0000%	
R&R = Repeatability & Reproducibility					R&R	$S_{R\&R}$	Tolerance %		
$R\&R = \sqrt{(EV)^2 + (AV)^2}$					0.0157	0.0030	$\%R\&R = \left(\frac{100R\&R}{Tolerance} \right)$		
R&R = 5.15×0.0030						$\%R\&R =$		74.4820%	
						Cmc =		1.2742	
Part variation				Total variation, TV			Process %		
sPV	R_p/d_2^*	PV	$\%PV = \frac{PV}{TV}$	$TV^2 = (R \& R)^2 + PV^2$			$\%R\&R =$		
				$TV = \sqrt{(R\&R)^2 + PV^2}$			$= \left(\frac{100R\&R}{TV} \right)$		
	0.0026	0.0132	64.3956	TV		0.0205	$\%R\&R$		
Commentary, notes and/or observations (written message)				English: Change or improve			76.5062%		
Values of tabulated coefficients: d_2 ; d_2^* and d_3									
Coef. d_2^*	1,4142	1,9118	2,2392	2,4813	2,6721	2,8294	2,9627	3,0779	3,1795
Coef. d_2	1,1280	1,6930	2,0590	2,3260	2,5340	2,7040	2,8470	2,9700	3,0780
Coef. d_3	0,8530	0,8880	0,8800	0,8640	0,8480	0,8330	0,8200	0,8080	0,7970

Table 3.16. Results of a full R&R study of the gear shaft mentioned above

$$EV = \left(\frac{\bar{\bar{R}}}{d_2} \right); \text{ from the table } d_2 = 1.6930; \bar{\bar{R}} = 0.0052 \rightarrow EV = \left(\frac{0.0052}{1.6930} \right) = 0.0157.$$

To calculate AV, we need the standard deviation, which comes from:

$$= \text{IF}(I49 > 0; \text{SQRT}(I49); 0) = (\text{in our case} = 0)$$

$$AV = \left(\frac{5.15 \times \sigma}{d_2^*} \right) \text{ from the table } d_2^* = 2.8294 \rightarrow AV = \left(\frac{0.0000}{2.8294} \right) = 0.$$

Because of the small number of components to control, we consider the corrected coefficient d_2 from the tabulated coefficient values: d_2 ; d_2^* and $d_3 \rightarrow d_2^* = 2.8294$.

In our case, to find $S_{R\&R} = \text{SQRT}(G40*G40+G49*G49)$ from Excel, we have: $\text{INVGamma} \rightarrow S_{R\&R} = 0.0030$.

$$R\&R = \text{SQRT}(G42*G42+G51*G51) = 0.0157 \text{ (with our particular cells in Excel).}$$

$$R\&R = \sqrt{(EV)^2 + (AV)^2} = 0.0157.$$

To find PV, we should apply the following formula:

$$R_{PV} = \bar{R}_{\text{Max}} - \bar{R}_{\text{Min}} = 0.0082; \text{ from the table } d_2^* = 2.8294;$$

$$PV = \left(\frac{5.15 \times \sigma}{s_{PV}} \right) = \left(\frac{5.15 \times \sigma}{R_{PV}/d_2^*} \right) = \left(\frac{5.15 \times \sigma}{0.026} \right) = 0.0132.$$

To find TV, we should apply the following formula:

$$TV = \sqrt{(R\&R)^2 + PV^2} \rightarrow \text{SQRT}(G57*G57+B60*B60) \rightarrow TV = 0.0205.$$

$$\textbf{Then: } \%PV = \left(\frac{PV}{TV} \right) = \left(\frac{0.0132}{0.0205} \right) = 64.3956\%$$

To calculate the R&R%, we should apply the following formula:

$$\%R\&R = \left(\frac{100R\&R}{TV} \right) = 76.5062\%.$$

The *final* judgment cast on the instrument used in dimensional metrology is thus written in the Excel spreadsheet using a conditional function. Keep in mind which of the user's cells correspond to the ones used here:

= IF(AND(N55<=0,1;N59<=0,1); "Instrument is good"; IF(OR(N55>0,3; N59>0,3); "change or improve instrument"; "instrument unacceptable"))

The three operators have changed the measurement instrument (micrometer) and reperformed all measurements using the following micrometer:

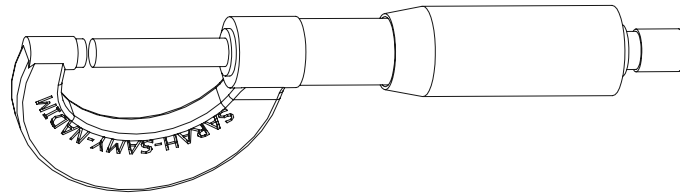


Figure 3.11. Measurements using a conventional micrometer

This study was carried out using Excel and MathCAD with our own formulas. Many programs can calculate R&R and perform complete studies with simple point and click. For educational purposes, we made the conscious decision not to use them. The user will thus be able to understand the analysis process and the interaction between the formulas used in our analyses. Once understood, the method will be able to call upon appropriate software.

3.6. X/S control chart study (average and standard deviation, σ)

Set the problem: monitored characteristic: threading of a platinum point

Max tolerated score	Min tolerated score	Tolerance (TI)	Target value	Sample size (n)	Average range (R)
1.440	1.310	0.130	1.438	6	0.0180

Table 3.17. Experimental data (measurement scores)

We designated two distinct operators (Bear and Coyote) to control the threading of a platinum point with the following simplified definition drawing:

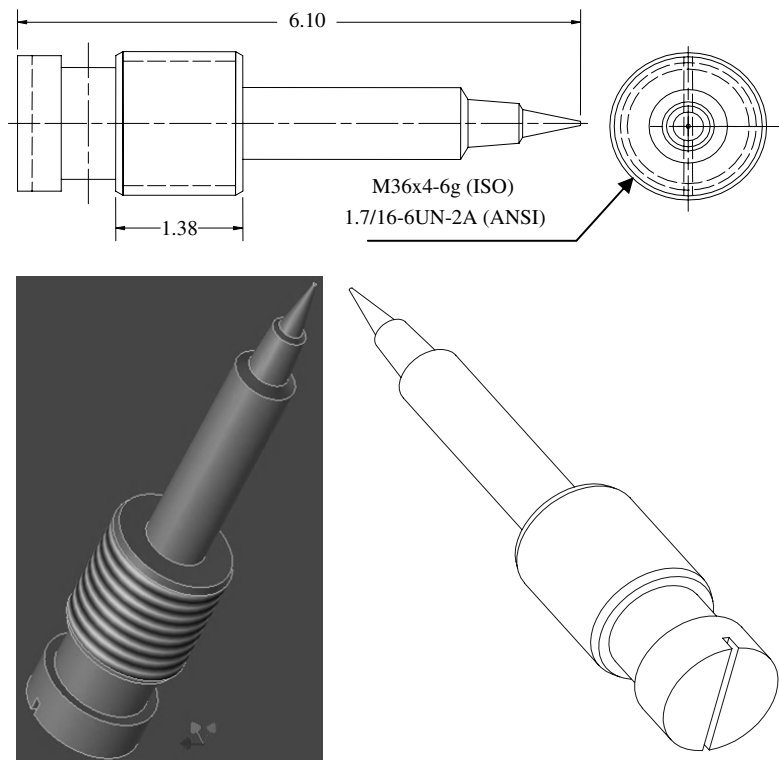


Figure 3.12. *Platinum point from an old car*

Each operator (Bear and Coyote) measured the threading from the point's machining (CNC) more than 16 days by taking six samples from a large batch every day. We are asked to analyze the averages control chart based on the theory explained in Chapters 8, volume 2 and 1, volume 3. The means used to control the threading size is a Mitutoyo profile projector [GRO 11]. The measurement results along with the statistical treatments are shown in the following tables.

Date	12 02	12 03	12 04	12 05	12 06	12 08	12 07	12 08	12 09	12 10	12 11	12 12	12 13	12 14	12 15	12 16
X1	1,430	1,422	1,421	1,422	1,428	1,425	1,427	1,431	1,400	1,422	1,400	1,428	1,430	1,425	1,427	1,431
X2	1,400	1,410	1,425	1,437	1,428	1,435	1,405	1,412	1,431	1,424	1,425	1,440	1,430	1,425	1,436	1,435
X3	1,425	1,430	1,427	1,425	1,425	1,429	1,435	1,425	1,438	1,437	1,430	1,436	1,425	1,429	1,401	1,425
X4	1,430	1,420	1,420	1,408	1,428	1,405	1,402	1,405	1,435	1,431	1,408	1,408	1,430	1,405	1,402	1,405
X5	1,425	1,421	1,408	1,407	1,432	1,411	1,428	1,415	1,425	1,412	1,408	1,330	1,436	1,400	1,428	1,401
X6	1,435	1,430	1,393	1,395	1,435	1,430	1,429	1,398	1,401	1,436	1,415	1,425	1,401	1,437	1,429	1,419
Number	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
Df	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
Average	1,4242	1,422	1,416	1,416	1,429	1,423	1,421	1,414	1,422	1,427	1,414	1,411	1,425	1,42	1,421	1,419
Coefficient A2	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483
Date	12 02	12 03	12 04	12 05	12 06	12 08	12 07	12 08	12 09	12 10	12 11	12 12	12 13	12 14	12 15	12 16
Average	1,4242	1,422	1,416	1,416	1,429	1,423	1,421	1,414	1,422	1,427	1,414	1,411	1,425	1,42	1,421	1,419
Upper limit	1,4467	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447	1,447
Lower limit	1,4293	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429	1,429
Target	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438	1,438

Table 3.18. Measurement and statistical treatment results of X/S control chart

Standard deviation	0,0124	0,007	0,013	0,015	0,004	0,012	0,014	0,012	0,017	0,01	0,011	0,041	0,012	0,014	0,015	0,014
Range	0,035	0,02	0,034	0,042	0,01	0,03	0,033	0,033	0,038	0,025	0,03	0,11	0,035	0,037	0,035	0,034
Date	12 02	12 03	12 04	12 05	12 06	12 08	12 08	12 07	12 08	12 09	12 10	12 11	12 12	12 13	12 15	12 16
d _i	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534	2,534
D ₄	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004	2,004
Range	0,035	0,02	0,034	0,042	0,01	0,03	0,033	0,033	0,038	0,025	0,03	0,11	0,035	0,037	0,035	0,034
Upper limit	0,0361	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036
Average range	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018
B ₆	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483	0,483
Weighted variance	0,0008	3E-04	8E-04	0,001	6E-05	7E-04	1E-03	8E-04	0,001	5E-04	6E-04	0,009	8E-04	0,001	0,001	1E-03

Average of averages	1,4203	Capability machine		$C_m = 1,355$			
Df number	80	Capability of the process		$C_p = 1,401$		Maximum Encountered 1,44	
Average range	0,0363	Coefficient process capability		$C_{pk} = 0,425$		Minimum Encountered 1,330	
Global standard deviation	0,0155						
Target standard deviation	0,0071	Max % except tolerance		10,1%			
Instant standard deviation	0,0160	Min % except tolerance		0,0%			
See Book Qual_Cont.	2	3	4	5	6	7	8
A ₄ Coefficient	1,88	1,023	0,729	0,577	0,483	0,419	0,373
D ₄ Coefficient	3,267	2,574	2,282	2,114	2,004	01-01	01-01
d ₂ Coefficient	1,128	1,693	2,059	2,326	2,534	2,704	2,847

Table 3.18 (continued). Measurement and statistical treatment results of X/S control chart

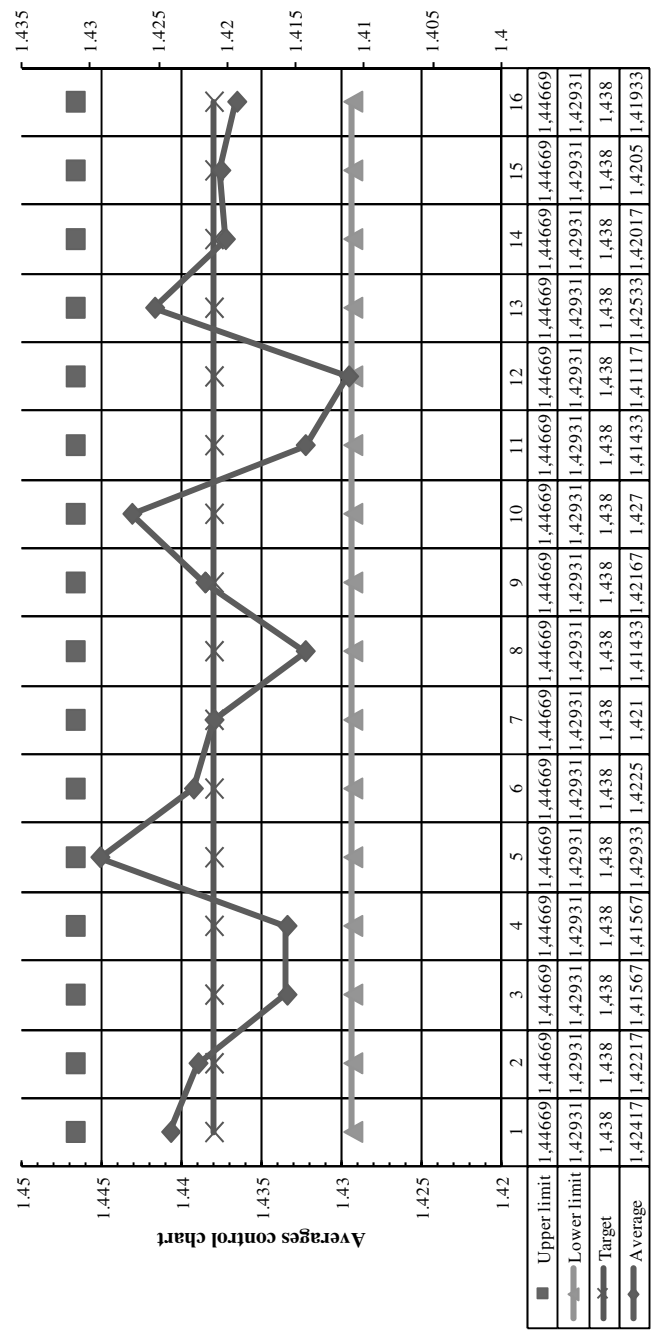


Figure 3.13. Excel plot of the averages control chart

OBSERVATION.– We can clearly see that the capability $C_p = 1.355 > 1.33$ and $C_m = 1.401 > 1.33$, *accepts* both process and machine.

3.7. Case study: capability of a specific control method

3.7.1. Measurements from a digital micrometer

Product	Center-Lab-Workshop	Date of control
6061 Rim	Dimensional metrology	2012-02-27
Title of means	Mitutoyo digital caliper	Units: mm (and inches)

3.7.2. Definition drawing or clear picture

We machined and controlled 10 rims in our industrial automation workshops (research project).

After our measurements (control) using a CMM and micrometer, we established Table 3.19 to perform a statistical analysis to support our production process decision.

For educational purposes, we used Excel and MathCAD. Our results are as follows:

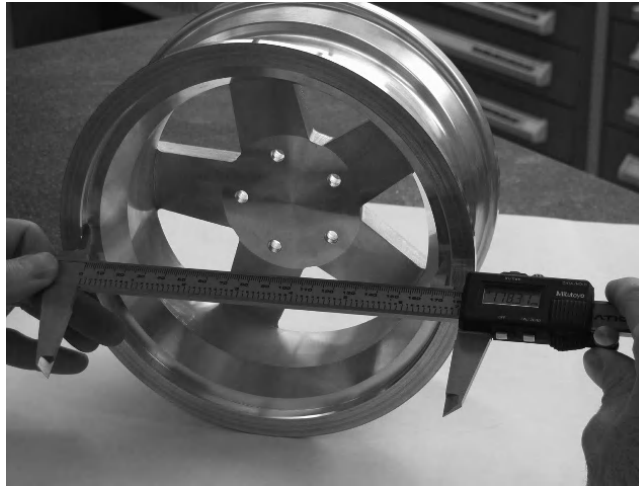


Figure 3.14. Measuring the diameter of a 6061 shade rim machined in laboratory

3.7.3. Table of the measurements taken using a digital micrometer (directly read the value from Caliper)

Parts	Operator 1: SAM				Operator 2: NADIM				Operator 3: SARAH			
	Measure- ment 1	Measure- ment 2	Measure- ment 3	Range	Measure- ment 1	Measure- ment 2	Measure- ment 3	Range	Measure- ment 1	Measure- ment 2	Measure- ment 3	Range
RIMS												
Rim 1	178.31	178.21	178.31	0.1000	178.31	178.30	178.28	0.03	178.31	178.31	178.33	0.02
Rim 2	178.25	178.28	178.25	0.0300	178.25	178.20	178.24	0.05	178.25	178.20	178.28	0.08
Rim 3	178.20	178.00	178.20	0.2000	178.31	178.20	178.25	0.11	178.18	178.20	178.24	0.06
Rim 4	178.35	178.23	178.35	0.1200	178.35	178.18	178.21	0.17	178.31	178.32	178.31	0.01
Rim 5	178.15	178.05	178.15	0.1000	178.15	178.18	178.22	0.07	178.25	178.19	178.25	0.06
Rim 6	178.22	178.30	178.22	0.0800	178.22	178.21	178.24	0.03	178.22	178.20	178.25	0.05
Rim 7	178.23	178.24	178.23	0.0100	178.19	178.23	178.17	0.06	178.32	178.23	178.28	0.09
Rim 8	178.30	178.28	178.30	0.0200	178.30	178.31	178.31	0.01	178.31	178.30	178.33	0.03
Rim 9	178.32	178.30	178.32	0.0200	178.28	178.32	178.31	0.04	178.15	178.12	178.20	0.08
Rim 10	178.35	178.32	178.35	0.0300	178.32	178.21	178.35	0.14	178.31	178.35	178.31	0.04
TOTAL	1782.68	1782.21	1782.68	0.4700	1782.68	1782.34	1782.58	0.34	1782.61	1782.42	1782.78	0.36
Statistics	Average of the ranges R				Average of the ranges R				Average of the ranges R			
	Sum of series				Sum of series				Sum of series			
	Average of measurements				Average of measurements				Average of measurements			

Table 3.19. Table of the metrological data taken using a digital micrometer

3.7.4. Table of calculation results (Excel and MathCAD)

Acronyms	Denotes		Acceptation criteria
Op	Number of operators	Three operators	– Conforms if ≥ 10 – Acceptable but to double-check >3 and <10 – Non-conforming if ≤ 3
Trials	Number of trials	Three trials	
TI	Tolerance intervals	2	Calculation formulas
$\bar{\bar{R}}$	Average of the average ranges	0.064667	$\bar{\bar{R}} = \sum_{i=1}^n \bar{R}_i = \frac{(\bar{R}_1 + \bar{R}_2 + \bar{R}_3)}{3}$
LSC	Elimination of aberrant values	0.166517	$LSC = \bar{\bar{R}} \times 2.575$
X_{diff}	Range of the averages of all three operators	0.008000	MAX (ABS(F25-K25);ABS(F25-P25);ABS(K25-P25)) (Excel spreadsheet where $\rightarrow$ F25 = 178.252; K25 = 178.253; P25 = 178.260)
EV	Repeatability depending on the coefficient	0.197233	$= LSC \times 3.05$
%EV		9.861700	$= (EV \times 100) / IT$
AV	Reproducibility depending on the coefficient	0.021600	$= X_{diff} \times 2.7$
%AV		1.08000	$= (AV \times 100) / IT$
R&R	Global uncertainty	0.198413	$\sqrt{EV^2 + AV^2}$
%R&R	% Repeatability and Reproducibility	9.92060	$(R\&R \times 100) / IT$
Cpc	Aptitude coefficient of means	10.080007	$IT / U_{global} (R\&R)$
Decision	The means is: acceptable, to revise(*)		
	(*) The series of measurement for which the range is $>UCL$ will need to be repeated, and the grid recalculated.		

Table 3.20. Calculation results and other formulas

– *Partial conclusion:* the means is just capable ($C_{pc} > 10$) = 10.080007.

3.8. Case study on type A and B uncertainties

3.8.1. *Uncertainties guide to the expression of uncertainties in measurements (GUM)*

3.8.1.1. *Cam Snowmobile SKI-DOO cam*

We are asked to control and estimate the mean's measurement uncertainty without a reference. We already know the device's measurement uncertainty (Mitutoyo, CMM). We will estimate the measurement uncertainty based on its calibration certificate (Mitutoyo, Montreal, Canada and NRC).

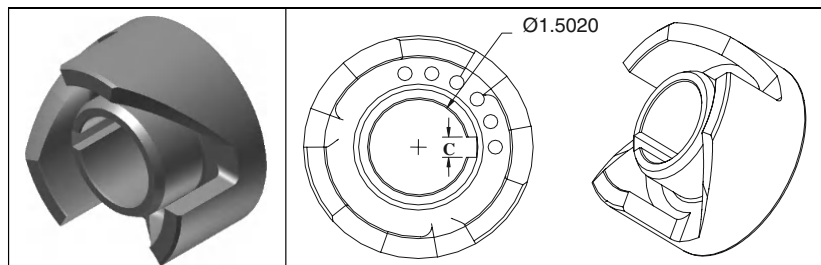


Figure 3.15. *Drawing of a Snowmobile cam (Ski-Doo)*

The method used is based on that used by the qualified technician of Mitutoyo corporation (labs accredited by ISO and NRC, Toronto, Canada), and accredited by NRC (National Research Center of Canada, section: dimensional metrology).

The estimation is made based on a calibration certificate. The mean's calibration uncertainty and the reference's calibration uncertainty are replaced by the measurement device's uncertainty (CMM), which is used as a reference.

Type A uncertainty estimation.

- We must perform three series of 10 measurements.
- Each series will be done with respect to a different standard value.

3.8.1.2. *Type A uncertainty estimation*

Fill the columns below in an Excel spreadsheet, as a function of the results in the calibration certificate. The number of measurements must be 10 measurements at most and two measurements at least. (If the repeatability cannot be estimated from

the certificate, i.e. if the same measurement is not repeated multiple times, we must perform the measurements ourselves in the trial center or arbitrarily take a repeatability of 0.)

Standard	Operator 1	Standard	Operator 2	Standard	Operator 2
True value	Sam	True value	Nadim	True value	Sarah
1.5000	1.5020	2.0000	1.4985	2.5000	1.5020
1.5000	1.5001	2.0000	1.4997	2.5000	1.5000
1.5000	1.4995	2.0000	1.4995	2.5000	1.4995
1.5000	1.5004	2.0000	1.5004	2.5000	1.5000
1.5000	1.5010	2.0000	1.5000	2.5000	1.5010
1.5000	1.5014	2.0000	1.5014	2.5000	1.5014
1.5000	1.5000	2.0000	1.4985	2.5000	1.5000
1.5000	1.4997	2.0000	1.4997	2.5000	1.4997
1.5000	1.4998	2.0000	1.4987	2.5000	1.4998
1.5000	1.5000	2.0000	1.4999	2.5000	1.5050

Table 3.21. *Dimensional metrology measurements (type A uncertainty)*

Standard value	Read value	Gap
1.5000	1.5020	0.0020
1.5000	1.5000	0.0000
1.5000	1.4995	0.0005
1.5000	1.5005	0.0005
1.5000	1.5010	0.0010
1.5000	1.5014	0.0014
1.5000	1.5000	0.0000
1.5000	1.4997	0.0003
1.5000	1.4998	0.0002
1.5000	1.5000	0.0000

Table 3.22. *Dimensional metrology measurements (type B uncertainty)*

The estimation of the repeatability parameter then gives us a component of value $R = 0$. We then have $U = 0.000749578$. (SD = standard deviation):

$$U = \left\{ \text{Max} \left\{ \left(\text{standard deviation}(B_{23} : B_{32}); \text{SD}(D_{23} : D_{32}); \text{SD}(F_{23} : F_{32}) \right) \right\} / \sqrt{A_1} \right\}.$$

In our specific case, we used a simple Excel spreadsheet to calculate U , hence $\rightarrow [U = 0,000749578]$.

3.8.1.3. Type B uncertainty estimate

Type B uncertainty essentially depends on the metrology indicators (depending on the GUM) such as *accuracy* and *resolution*.

Definitions, designations	B₁	U₁
Uncertainty due to accuracy errors: B ₁	0.0005	= B₁/2
Value to consider when calculating the combined uncertainty: U ₁	→	0.00024615
	B₂	U₂
Laboratory uncertainty on the device calibration B ₂	0.0005	(*) = B₂/2
Value to consider when calculating the combined uncertainty: U ₂ <i>*It is sometimes called absolute uncertainty in certificates</i>	→	0.00025
	B₃	U₃
Laboratory uncertainty on the device calibration B ₃	0.000025	(*) = B₃
Value to consider when calculating the combined uncertainty: U ₃ <i>*It is sometimes called measurement uncertainty in Certificates</i>	→	0,000025
	B₄	U₄
Device resolution: B ₄	0.005	= $B_4 / \sqrt{12}$
Value to consider when calculating the combined uncertainty: U ₄	→	0.00144338

Table 3.23. Calculation results for type A and type B uncertainties

3.8.2. Estimating the compound uncertainty U_c

To find the combined uncertainty, we must calculate the root mean square of the type A and type components as follows:

$$U_c = \sqrt{U + U_1^2 + U_2^2 + U_3^2 + U_4^2} = 0.001664006$$

3.8.2.1. Estimating the expanded uncertainty: U

We recall from the theory developed in Chapter 3, Volume 1 that, for example:

- $K = 1$ for a confidence interval of 68.26%;
- $K = 2$ for a confidence interval of 95.26% $\rightarrow U = 3.328012 \times 10^{-3}$;
- $K = 3$ for a confidence interval of 99.23%;
- $K = 4$ for a confidence interval of 99.99%.

In our case study, we assume that $K = 2$, with extended uncertainty, is the confidence granted to 95.26%.

U_c , for $k = 1, 2, 3, 4$	$U(K) = k \times U_c$	$U(K) = k \times U_c$
	1.664006E-3	4.992018E-3
$U_c = 0.001664006$	3.328012E-3	6.656024E-3

Table 3.24. Results of the uncertainty calculation

3.9. Case study: uncertainties

Capability of a control method $\rightarrow$ Micrometer

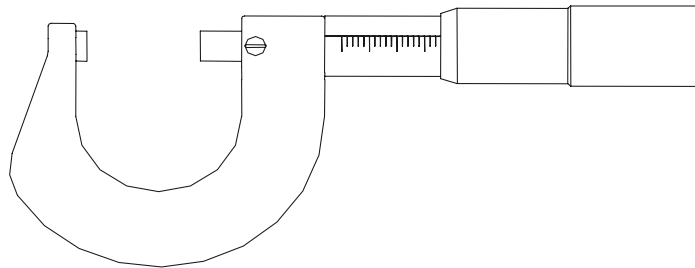


Figure 3.16. Capability of a means: micrometer

3.9.1. Uncertainty study as per GUM

Linear thermal expansion	°C	Nominal length		10 mm	Standard uncertainty	Sensitivity coefficient	Formula used: Welch-Satterthwaite			
	Value	Measure- ment unit	Evaluation type	Factor			U/c, Combined/ micrometer uncertainty	Degree of freedom Dof, v	$c^4 \times \frac{u}{n}$	
Standard GUM Procedure										
Instrument reading U (R)	1,041	µm	STATISTICAL	1	0.577	1	1.041	6	0,2934027	
Repeatability	1	µm	Normal, A		0.500	1		4		
Resolution	1	µm	Rectangle, B	3.464	0.289			1000		
u(δθ) Calibration wedge	5	µm	Normal, B	2	0.05	-1	-2.5	20	1,953125	
u(α) Linear thermal expansion coefficient		°C	Rectangle, B	1.732	1.2E-06	15000	0,017	10000	9E-11	
Temperature difference between components u(δδ)	2E-06	°C	Rectangle, B	1.732	0.577	0.11	0,095	10000	8,235E-08	
Difference between the linear expansion coefficients : u(δα)	1,1E-06	°C	Rectangle, B	1.732	6.35E-07	50000	0,032	10000	1,016E-09	
Standard temperature difference 20°C u(θ)	5	°C	Rectangle, B	1.732	4.619	0	0			
Lu(δα) u(θ) contribution						Order	0,018			24
NOTE.– We built this table in Excel. However, using GUMic would have given similar results, and would also have been wise.					Combined Standard Uncertainty U = k × u _c (y) an expanded uncertainty			µm	v _{eff}	
					Coverage factor k = 2			µm		

Table 3.25. Results of the uncertainty calculations of a method (conventional micrometer) as per GUM

Standard GUM procedure	Value	Measurement unit	Type of evaluation or distribution	Factor	Standard uncertainty	Sensitivity coefficient	Combined uncertainty/micrometer	
PUMA – one iteration only								
Standard wedge (ISO Gr. 2)	5	μm	Rectangle, B	1.732	5	μm	2.887	
Reading instrument resolution	1	μm	Rectangle, B	3.464	-		0.289	
Instrument repeatability	1	μm	Normal, A		-		1.000	Repeatability
Temperature difference between components $u(\delta\theta)$	1.5	C	"U", of Type B	1.41	0.165	μm	0.117	ISO
Difference in linear thermal expansion coefficients: $u(\alpha)$	10^{-6}	C		"U", of Type B	0.055	μm	0.039	
Standard temperature difference $20^\circ\text{C } u(\theta)$	5							
Note.— As above (GUMic)								
						Combined standard uncertainty:	3.071	μm
						Expanded uncertainty $k = 2$	6.142	μm

Table 3.25 (continued). Results of the uncertainty calculations of a method (conventional micrometer) as per GUM

DISCUSSION.— This problem was approached using both Excel and MathCAD. The uncertainty calculated on a micrometer using Excel and MathCAD gave us the following formulas:

Formulas used to fill the table above: (Excel and MathCAD)

$$L_{\text{Instrument}} = \sqrt{(I10)^2 + (I9)^2} = \sqrt{U_{\text{resolution standard}}^2 + U_{\text{repeatability standard}}^2}$$

$$= \sqrt{1.041^2 + 1^2} = 1.041 \mu\text{m}$$

$$U(\alpha) = 0.1 \times \text{linear thermal expansion}_{\text{instrument}} = \left(\frac{1}{10} \times 10^{-5} \right) = 1.1 \times 10^{-6} \text{ } ^\circ\text{C}$$

$$U(\alpha) = 0.1 \times \text{linear thermal expansion}_{\text{instrument}} = \left(\frac{1}{10} \times 10^{-5} \right) = 1.1 \times 10^{-6} \text{ } ^\circ\text{C}$$

$$\text{Resolution} = 2\sqrt{3}; \text{Expansion} = \sqrt{3}; U_{\delta\theta} = \sqrt{3}; U_{\theta} = \sqrt{3}; U_{\delta\alpha} = \sqrt{3}$$

$$U_{\text{rectangular, type B}}^{\text{ISO Gr.2}} = \sqrt{3}; U_{\text{rectangular, typeB}} = 2\sqrt{3}; U_{\text{typeB}} = \sqrt{2}; U_{\text{typeB}} = \sqrt{2}$$

$$U_{\text{standard}} = \frac{E8}{H8} = \frac{\text{value read on tool}}{\text{Factor 1}} = 1.041;$$

$$U_{\text{standard}} = \frac{E10}{H10} = \frac{\text{Tool resolution}}{\text{Factor of (3.461) } \leftrightarrow \text{ Rectangle dist.}} = 0.289$$

$$U_{\text{standard}} = \frac{E11}{H11} = \frac{\text{Tool calibration gage}}{\text{Factor of (2) } \leftrightarrow \text{ Normal dist. B}} = 2.500$$

$$U_{\text{standard}} = \frac{E13}{H13} = \frac{\text{Tool's difference in temperature } U_{\alpha}}{\text{Factor of } (\sqrt{3}) \leftrightarrow \text{ Rectangle dist. B}} = 1.2 \times 10^{-6}$$

$$U_{\text{standard}} = \frac{E15}{H15} = \frac{\text{Tool's difference in expansion coeff. } U_{\delta\theta}}{\text{Factor of } (\sqrt{3}) \leftrightarrow \text{ Rectangle dist. B}} = 0.866$$

$$U_{\text{standard}} = \frac{E17}{H17} = \frac{\text{Tool's difference in expansion } U_{\delta\alpha}}{\text{Factor of } (\sqrt{3}) \leftrightarrow \text{ Rectangle dist. B}} = 6.35 \times 10^{-7}$$

$$U_{\text{standard}} = \frac{E19}{H19} = \frac{5}{\text{Factor of } (\sqrt{3}) \leftrightarrow \text{Rectangle dist. B}} = 2.887$$

$$U_{\text{standard}} = \frac{(D6 \times 0.001) \times D5 \times E33}{0.000001} = \frac{(10 \text{ mm} \times 0.001) \times 1.1 \times 10^{-1} \times 1.5}{0.000001} = 1.165$$

$$U_{\text{standard}} = \frac{(E36 \times E37) \times D6 \times 0.001}{0.000001} = \frac{0.000001 \times 5 \times 10 \text{ mm} \times 0.001}{0.000001} = 0.05$$

$$\text{Coef}_{\text{sensitivity}} = \frac{E15 \times (D6 \times 0.001)}{0.000001} = \frac{1.5(\text{temp. diff.}) \times (20 \text{ mm} \times 0.001)}{0.000001} = 15,000$$

$$\text{Coef}_{\text{sensitivity}} = \frac{D5 \times (D6 \times 0.001)}{0.000001} = \frac{1.1 \times 10^{-5}(\text{thermal expansion}) \times (10 \text{ mm} \times 0.001)}{0.000001} = 0.110$$

$$\text{Coef}_{\text{sensitivity}} = \frac{E19 \times (D6 \times 0.001)}{0.000001} = \frac{5(\text{temperature deviation}) \times (20 \text{ mm} \times 0.001)}{0.000001} = 50,000$$

$$\text{Coef}_{\text{sensitivity}} = \frac{E19 \times (D6 \times 0.001)}{0.000001} = \frac{5(\text{temperature deviation}) \times (10 \text{ mm} \times 0.001)}{0.000001} = 0$$

$$\text{Combined}_{\text{Micrometer}}^{\text{Contribution}} = I11 \times J11 = 2.5 \times (-1) = -2.5$$

$$\text{Combined}_{\text{Micrometer}}^{\text{Contribution}} = I13 \times J13 = 1.2 \times 10^{-6} \times 15,000 = 0.017$$

$$\text{Combined}_{\text{Micrometer}}^{\text{Contribution}} = I15 \times J15 = 0.866 \times 0.11 = 0.095$$

$$\text{Combined}_{\text{Micrometer}}^{\text{Contribution}} = I17 \times J17 = 6.35 \times 10^{-7} \times 50,000 = 0.032$$

$$\text{Combined}_{\text{Micrometer}}^{\text{Contribution}} = I19 \times J19 = 2.887 \times 0 = 0.000$$

$$\begin{aligned} \text{Combined}_{\text{Micrometer}}^{\text{Contribution}} &= \frac{(D6 \times 0.001) \times (I17) \times (I19)}{0.000001} \\ &= \frac{(25 \times 0.001) \times 6.35 \times 10^{-7} U_{\text{stand}} \times 4.619 \times U_{\text{stand}}}{0.000001} = 0.073 \end{aligned}$$

$$\begin{aligned}
& \text{Combined}_{\text{Micrometer}}^{\text{Contribution}} \\
&= \sqrt{(K8^2 + K11^2 + K13^2 + K15^2 + K17^2 + K19^2 + K21^2)} = 2.710 \mu\text{m} \\
&= \sqrt{(CC_{\text{Microm.}}^2 + CC_{\text{gage}}^2 + CC_{\text{Expan.}}^2 + CC_{\text{temp.}}^2 + CC_{\text{therm. exp.}}^2 + CC_{\text{Stand.dev.}}^2 + CC_{\text{higher order}}^2)} \\
&= \sqrt{(1.041^2 + (-2.5)^2 + 0.017^2 + 0.095^2 + 0.032^2 + 0^2 + 0.018^2)} = 2.710 \mu\text{m}
\end{aligned}$$

$$\text{Combined}_{\text{Micrometer}}^{\text{Contribution}} = K \times U_c = 2 \times K23 = 5.420 \mu\text{m}$$

$$U_{\text{Standard}} = I29 \times H29 = 5(U_{\text{gage}}) \times \sqrt{3}(\text{Fact}) = 2.887$$

$$U_{\text{Standard}} = E30 \times H30 = 1(\text{Resolution}_{\text{instrument}}) \times 2\sqrt{3}(\text{Fact}) = 0.289$$

$$U_{\text{Standard}} = E9 = 1 = \text{Repeatability}$$

$$U_{\text{Standard}} = I33 \times H33 = 0.165(\text{Temperature difference}_{\text{tool and component}}) \times \sqrt{2}(\text{Fact}) = 0.117$$

$$U_{\text{Standard}} = I36 \times H36 = 0.055(\text{Expansion and temperature / deviation}) \times \sqrt{2}(\text{Fact}) = 0.039$$

$$\begin{aligned}
\text{Combined}_{\text{Uncertainty}}^{\text{Standard}} &= \sqrt{(K29^2 + K30^2 + K33^2 + K36^2 + K31^2)} = \\
&= \sqrt{2.887^2 + 0.289^2 + 0.117^2 + 0.039^2 + 1^2} = 3.071 \mu\text{m}
\end{aligned}$$

$$\text{Uncertainty}_{\text{Micrometer}}^{\text{Expanded}} = K \times U_c = 2 \times K38 = 6.142 \mu\text{m} \text{ (at 95\%)}$$

$$\text{Combined}_{\text{Micrometer}}^{\text{Contribution}} = K \times U_c = 2 \times K38 = 6.142 \mu\text{m}$$

$$D_f = \text{TRUNC} \left((K8^4) \left/ \left(\left(\frac{E9^4}{L9} \right) + \left(\frac{E10^4}{L10} \right) \right) \right. \right) = 4$$

$$\left(\frac{C^4 \times U^4}{\nu} \right) = \frac{(K8)^4}{L8} = \frac{(CC_{\text{micrometer}} = 1.041)^4}{L8 = 4 = \text{degrees of freedom } (D_f)} = 0.29340278$$

$$\left(\frac{C^4 \times U^4}{\nu} \right) = \frac{(K11)^4}{L11} = \frac{(CC_{\text{gage}} = -2.5)^4}{L11 = 20 = D_f} = 1.953125$$

$$\left(\frac{C^4 \times U^4}{\nu} \right) = \frac{(K13)^4}{L13} = \frac{(CC_{\text{gage}} = (0.017))^4}{L13 = 1,000 = D_f} = 9 \times 10^{-11}$$

$$\left(\frac{C^4 \times U^4}{\nu} \right) = \frac{(K15)^4}{L15} = \frac{(CC_{\text{temperature difference}} = 0.095)^4}{L11 = 1,000 = \text{ddl}} = 8.2356 \times 10^{-8}$$

$$\left(\frac{C^4 \times U^4}{\nu} \right) = \frac{(K17)^4}{L17} = \frac{(CC_{\text{thermal expansion}} = 0.032)^4}{L11 = 1,000 = \text{ddl}} = 1.0167 \times 10^{-9}$$

$$\left(\frac{C^4 \times U^4}{\nu} \right) = \text{TRUNC} \frac{(K23^4)}{(\text{SUM}(N8; N11; N13; N15; N17))}$$

$$= \text{TRUNC} \frac{(\text{Combined stand. uncertainty}^4)}{(\text{SUM}(N8; N11; N13; N15; N17))} = 24$$

Generated values										
0.24	-1.52	-0.06	-1.08	0.93	0.08	0.29	-0.37	-0.37	0.07	-0.11
1.89	0.97	-0.46	1.64	-1.06	0.32	-0.80	-0.77	-0.77	0.97	1.23
-0.15	-0.60	-0.61	0.42	0.80	1.96	-0.53	0.05	0.05	0.27	0.83
0.80	0.80	2.45	1.26	0.21	-1.01	-0.17	-1.03	-1.03	-0.44	-1.90
0.12	0.34	-1.37	0.14	0.20	1.18	0.09	-0.48	-0.48	0.64	1.00
-0.12	1.42	0.16	-0.41	0.50	1.11	0.77	-0.20	-0.20	0.19	1.47
0.47	1.50	-1.02	0.01	0.22	-0.21	-2.11	-1.49	-1.49	-1.30	0.95
-0.53	-1.24	0.09	1.26	-2.38	-0.59	1.48	0.40	0.40	-0.22	-1.33
0.01	-0.53	-1.98	0.35	-0.78	0.53	-0.09	-0.76	-0.76	0.45	1.00
1.00	-0.08	-0.19	0.41	-0.78	-1.43	-0.13	-0.76	-0.76	-0.17	-1.33

Table 3.26. Generating values using ordinary simulations on Excel

We will present some examples of ordinary logical simulations, followed by conventional simulations of the usual distributions (Poisson, normal and exponential processes).

– Ordinary logical simulations;

– *Starting data:* Consider a number of draws of 100 values with an average of about $\mu = 0.00000$ and standard deviation $\sigma = 1.03061$. The number of generated loops is $N_{\text{loops}} = 238$.

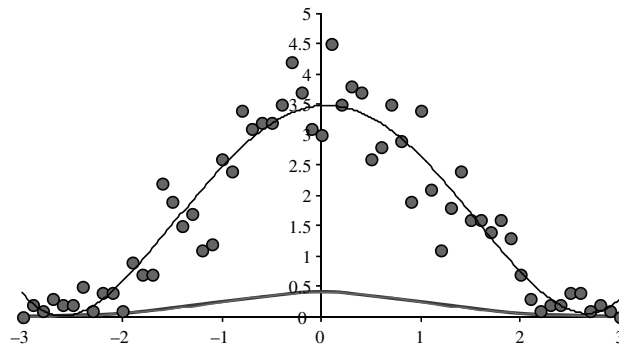


Figure 3.17. Graphical illustration for a generation of 100 values (238 loops)

In the same conditions, that is an average of about $\mu = 0.00000$ and standard deviation $\sigma = 1.03061$, but this time a number of generated loops of $N_{\text{loops}} = 2,454$.

Generated values									
1.09	0.38	0.87	1.49	0.70	-0.21	0.43	0.06	-0.10	0.03
1.78	1.60	-0.50	-1.88	-0.14	0.57	-1.34	1.73	-0.14	-0.60
0.56	0.93	-0.86	-1.01	0.29	0.93	1.42	0.02	0.00	-1.85
-1.83	-2.27	0.08	1.18	0.92	0.54	-1.56	-0.18	-0.37	0.14
1.19	-0.48	-0.54	0.96	-1.03	-0.15	1.73	0.04	-0.92	-1.22
0.93	-0.20	-0.60	0.94	-0.42	0.65	-0.49	0.27	-0.26	-0.24
2.76	0.84	1.05	-0.93	2.01	1.24	1.37	1.41	0.78	-1.41
0.58	1.33	-0.86	0.14	-0.59	-2.22	1.07	0.51	1.45	-0.20
0.07	0.87	-0.42	1.73	0.64	-1.60	-0.09	-0.22	1.28	-0.67
-0.67	0.13	0.03	-0.13	0.17	-0.20	1.15	0.17	0.85	-0.80
1.45	-1.29	-1.58	-0.50	0.08	-0.64	-1.62	0.79	0.70	-0.64
-0.23	1.06	0.02	-1.20	1.28	0.23	-0.57	-0.46	-0.35	-0.06
-0.57	0.37	-0.76	1.11	0.02	-0.60	0.74	-1.14	-0.54	-1.18
-0.20	0.82	0.61	0.18	-0.67	0.64	-0.96	-1.43	-0.36	0.09
-0.64	-1.05	0.32	-0.93	0.71	-1.87	-0.58	0.41	1.17	-1.13
-0.36	0.20	1.68	-0.66	0.83	1.05	0.21	0.13	0.26	0.59
-0.27	0.33	1.05	-0.61	-0.17	-0.23	1.29	-0.64	-0.47	0.64
-1.15	-0.85	0.22	-0.43	-0.82	0.06	-0.57	-1.13	-0.09	1.65

Table 3.27. One thousand values generated using ordinary simulation

Generated values									
1.16	0.62	-0.02	1.08	0.22	1.09	-0.50	-1.53	0.19	0.85
0.05	0.40	0.23	-0.77	-0.59	-0.05	-1.04	-0.86	0.24	-1.05
0.70	0.57	-0.30	-0.01	1.39	0.80	-2.81	0.06	-0.85	1.64
1.02	-0.40	-0.58	0.32	0.00	-0.13	-1.61	0.84	-0.20	-0.41
0.43	0.03	-0.68	-0.94	1.09	1.38	-0.80	1.55	-0.31	-1.65
0.27	-0.12	1.40	-0.35	-0.26	-0.07	0.48	1.00	-1.77	1.30
-0.92	-1.27	-0.70	0.47	0.06	-0.18	-1.67	0.38	0.08	-0.60
2.89	1.10	-1.56	1.27	1.25	0.24	1.46	-0.30	-0.80	0.78
0.19	-1.97	0.11	0.80	-0.67	1.03	-1.58	1.25	-0.64	0.86
-1.21	-0.96	-0.87	0.28	0.15	-0.23	0.45	-1.10	-0.70	0.22
-2.06	-0.72	2.61	0.65	0.08	-0.16	-1.56	-0.94	-0.71	0.07
0.40	2.15	0.80	-0.62	-2.10	0.49	0.42	0.45	-1.40	0.01
0.18	-0.52	0.94	0.90	0.64	1.07	0.17	1.14	-0.31	0.88
0.33	-0.16	-1.74	0.01	1.51	0.00	-0.76	2.00	1.42	-0.03
-2.06	-2.61	-0.74	1.77	0.70	-0.70	-1.04	1.64	-0.66	-0.09
-0.03	-1.33	-1.54	1.21	0.61	-0.72	1.11	0.03	1.96	-0.58
0.58	0.09	-0.06	1.08	1.71	0.29	-0.47	-0.29	2.69	-0.35
-0.60	-0.88	1.84	-0.21	-1.50	-1.18	-0.14	-1.22	0.31	-1.33
-0.66	-1.24	-0.43	-0.78	-0.34	-0.60	-2.55	-2.42	-0.63	-1.67
0.40	2.53	0.37	2.09	1.46	1.76	-1.49	-0.83	-1.15	0.12
-0.18	-2.59	-0.35	-2.12	-1.98	0.70	-0.26	0.83	-1.89	-0.75
0.59	-1.88	-0.40	0.45	0.87	-0.13	0.31	1.09	0.77	-1.38
-0.53	-0.46	0.00	1.78	-1.22	0.98	0.61	0.79	0.92	-0.54
-0.16	2.30	0.62	-0.45	1.68	1.00	0.32	-0.26	-0.44	0.32
-1.75	-0.96	0.57	0.60	0.22	-1.33	0.41	0.24	1.38	2.75
0.67	-0.68	0.76	-1.17	-0.31	0.46	-0.27	1.20	0.25	-0.44
1.37	-0.64	-0.49	-0.51	0.07	0.23	-1.76	-0.99	-0.42	1.01
-1.49	1.83	-0.98	1.22	0.03	-0.89	0.03	0.09	0.36	-1.64
0.72	-1.38	1.53	0.77	0.17	-1.30	-0.57	0.33	-0.03	0.26
1.42	0.78	1.59	-0.06	-0.46	0.66	-0.82	-0.02	0.44	2.67
0.82	-0.14	1.12	1.06	0.18	0.20	-0.35	0.99	0.14	0.41
0.71	-0.01	-0.96	-0.42	-1.07	0.41	0.96	-1.10	0.92	1.23
-0.22	0.20	-0.06	0.07	0.10	-0.19	1.58	-2.15	0.36	0.45
-1.34	-0.13	2.70	-2.49	-1.78	1.00	0.21	-0.50	-1.36	1.74
-0.57	1.04	0.27	1.64	-0.29	0.31	0.26	0.17	-1.09	0.64
-0.21	0.20	-2.40	-0.72	0.41	0.92	-1.24	1.60	0.49	0.22
0.25	0.02	-1.07	1.97	0.88	-1.66	1.90	-1.46	2.47	-0.10
0.38	-0.03	0.49	0.98	0.36	0.53	0.45	0.46	0.13	-0.27
0.05	1.35	-0.23	-1.44	1.20	-0.13	-0.28	0.77	1.50	-0.13
-0.62	-0.23	0.01	0.55	-1.50	-0.47	0.61	-0.49	0.15	-0.65
0.43	1.11	0.06	-0.07	0.50	2.18	1.94	-1.64	-0.21	-0.65
-2.50	1.46	-0.51	-0.46	1.89	-0.01	-0.69	-0.83	-0.03	-1.52

Table 3.27 (continued). One thousand values generated using ordinary simulation

Generated values									
1.18	-1.07	-1.34	-2.36	-0.62	0.72	-1.34	1.02	0.51	1.42
1.55	1.42	0.15	-0.02	1.22	-1.22	-0.87	0.63	-0.48	0.11
0.33	-1.39	-0.49	-0.66	1.16	0.45	-1.07	-0.87	0.39	-0.88
-0.33	1.23	0.26	-1.05	-0.15	-0.36	0.54	0.61	-2.14	-0.70
0.20	0.05	-1.57	-0.80	1.89	-0.49	2.15	0.99	-1.31	-0.41
0.16	0.39	-0.97	1.41	-0.16	0.34	-0.21	-0.45	-0.25	0.37
-1.15	0.56	-0.89	0.76	-0.45	0.47	-0.39	1.29	0.35	0.29
-0.26	0.74	-0.22	-1.68	0.18	-0.58	-1.21	0.10	-0.87	-1.55
0.17	0.45	-1.52	1.46	0.34	-0.83	-0.46	-1.25	1.17	0.06
-1.11	-1.18	-0.13	-0.29	0.25	-0.88	0.25	0.82	1.57	-0.63
0.78	-2.50	-0.65	-1.39	1.53	0.46	1.84	-0.86	1.64	-0.13
0.71	0.07	0.83	0.34	0.62	-1.71	0.08	-0.44	-0.65	-0.22
-2.14	-0.02	0.73	-0.48	0.45	0.84	0.11	-0.11	-1.02	0.66
1.54	-0.04	-0.53	0.23	0.58	0.94	-0.25	0.55	0.84	-1.29
-1.55	1.37	0.53	-2.20	0.00	-1.48	1.19	0.97	-0.18	-1.94
1.29	1.04	0.61	1.81	1.85	1.38	-0.57	-1.52	0.67	0.06
-0.62	-0.68	-0.37	-0.34	0.39	-1.02	-1.55	-1.88	-0.40	-1.08
2.08	-1.24	-0.91	2.11	1.33	-0.16	0.01	-1.56	-1.30	0.92
-2.05	0.00	-0.13	2.26	-1.25	0.45	-1.43	-0.10	-0.06	-0.19
0.31	1.21	-0.84	-1.24	-0.10	0.14	0.59	-0.37	1.98	0.12
0.15	-0.74	-0.77	-0.10	0.06	-0.63	1.29	-0.86	0.98	0.54
-1.67	0.18	-0.82	-0.01	-0.06	-0.50	-0.15	0.68	0.54	0.07
0.24	0.81	0.49	0.61	0.98	-1.61	2.73	-0.44	1.64	2.11
-1.02	0.45	-0.89	0.65	1.55	-0.28	0.81	-0.24	1.10	-0.22
-1.19	-0.14	-0.54	0.61	1.05	-0.08	-0.30	-1.21	-1.02	-0.52
-0.50	-0.73	-1.06	-0.35	-0.27	-0.45	1.47	0.61	-0.31	-0.40
1.93	2.13	-0.15	0.13	0.26	0.84	1.26	0.18	0.49	-0.86
0.80	-1.00	-0.71	0.85	-0.15	-2.93	-0.05	-0.08	-0.72	1.30
0.78	1.46	1.16	1.48	-0.07	0.36	0.10	-0.17	-0.96	-0.64
0.52	-0.03	0.76	-2.30	1.49	-1.14	-1.44	1.54	0.10	-0.68
-0.81	-2.12	-0.69	0.64	-0.46	2.40	1.37	-0.64	-1.03	0.65
0.02	-1.83	1.50	-2.31	0.22	0.10	0.57	0.42	-0.83	1.17
-0.16	-0.58	-0.03	-0.97	-0.54	-1.92	0.31	0.43	1.20	-1.94
-1.03	-1.45	-0.98	-0.59	-0.61	1.48	-1.22	-0.34	-0.40	-0.16
0.70	-0.28	-0.42	-1.37	0.08	-1.02	0.70	1.04	1.06	0.85
-0.07	0.29	-0.13	0.33	-0.81	-0.18	0.37	-0.15	1.40	1.38
0.71	1.27	0.49	1.41	-0.07	-1.69	-0.62	0.29	-0.32	-1.40
0.40	0.73	0.50	-0.71	-2.20	0.48	-0.38	-0.59	2.62	1.89
1.17	0.36	-0.27	0.44	-0.55	1.26	-1.58	1.59	-2.00	-1.33
0.42	1.80	-0.04	0.10	0.55	-1.23	0.38	0.89	1.00	-0.21

Table 3.27 (continued). One thousand values generated using ordinary simulation

Statistical distribution					
1,075370193	0,221830487	1,092462659	-0,498080134	-1,533088803	0,189301372
-0,769805789	-0,590941787	-0,046149373	-1,038268447	-0,858241439	0,237632632
-0,007562757	1,389216542	0,799604058	-2,807690144	0,058870912	-0,846284509
0,317662597	0,004243255	-0,134833217	-1,60771215	0,838438153	-0,199463725
-0,942599416	1,092308879	1,382797122	-0,801824212	1,551744103	-0,311276793
-0,354383826	-0,261267543	-0,068393111	0,476625323	0,997577548	-1,768525958
0,470273137	0,057388186	-0,179482341	-1,665270209	0,378693223	0,082226157
1,273346543	1,252250791	0,242673755	1,458999038	-0,298338532	-0,798878789
0,796901822	-0,671619773	1,025965333	-1,582771182	1,245721936	-0,642929435
0,282084346	0,146444678	-0,233163714	0,450964808	-1,100714564	-0,70251596
0,648238778	0,083280444	-0,161731839	-1,558399558	-0,935135007	-0,711699128
-0,622648358	-2,096074581	0,491783738	0,419911265	0,445217729	-1,399024129
0,902128816	0,644711852	1,072154403	0,173527122	1,138835549	-0,313768744
0,010602593	1,506612897	0,00217402	-0,76234138	2,003042698	1,421523929
1,774727225	0,698628545	-0,703473687	-1,042120099	1,636921763	-0,657351851
1,206048131	0,6124295	-0,718853831	1,111258149	0,033181787	1,957206845
1,075729251	1,70940578	0,294237256	-0,465857148	-0,293162227	2,685767174
-0,207003951	-1,500008941	-1,183302999	-0,14207375	-1,221167922	0,309408545
-0,784921288	-0,342260957	-0,604201198	-2,550384045	-2,415019989	-0,626351237
2,087180138	1,459795117	1,756550431	-1,485579371	-0,828335881	-1,149355531
-2,124364376	-1,977716804	0,702041745	-0,260081649	0,826331735	-1,889993548
0,44526279	0,866932511	-0,126343846	0,311369777	1,088945746	0,767526269
1,780184627	-1,220307469	0,980197549	0,607120156	0,787921071	0,922977805
-0,446983695	1,684632897	0,997175574	0,318166137	-0,255628467	-0,437018752
0,600793719	0,223812461	-1,327072978	0,410687327	0,242905498	1,378908992
-1,165017486	-0,306872249	0,46123302	-0,268553138	1,197972894	0,250780463
-0,514837861	0,071025968	0,227620482	-1,764941096	-0,992065072	-0,418645978
1,219102979	0,030451655	-0,88783443	0,033493638	0,0928334	0,360063672
0,765488505	0,172160268	-1,298510671	-0,568081498	0,327792764	-0,028365254
-0,06019485	-0,459730268	0,66116488	-0,818683505	-0,016855359	0,444036126
1,059119582	0,17524159	0,200100303	-0,353719354	0,987078309	0,142215371
-0,422220826	-1,072876096	0,413638473	0,960618854	-1,104521155	0,921370625
0,704483628	-0,214905381	0,433360219	0,057783723	-0,10335052	0,030507445
-0,137730718	0,571529031	-1,341878772	1,734750867	-0,138546109	-0,599351764
0,286111951	0,928533912	1,42440784	0,015694499	-0,002669692	-1,846739173
0,917492509	0,539527059	-1,559906602	-0,177405953	-0,372913241	0,140681148
-1,02615416	-0,145271659	1,732107997	0,043099523	-0,919281363	-1,220539927

Table 3.28. Distribution of the 1,000 values generated using ordinary simulation

Statistical distribution					
-0,424707055	0,650546193	-0,485094666	0,265441775	-0,262989879	-0,244032025
2,006291389	1,242971063	1,36960423	1,406658769	0,777479768	-1,409074187
-0,585392118	-2,22400713	1,065312266	0,50575912	1,450225711	-0,198722005
0,641971707	-1,601319909	-0,090235591	-0,221500754	1,281340957	-0,667299628
0,167055488	-0,204401135	1,151363254	0,166265845	0,849556088	-0,798480392
0,084658742	-0,643512368	-1,619671941	0,794483542	0,702033162	-0,63953054
1,280768037	0,230954289	-0,571775079	-0,464634061	-0,354936719	-0,056298137
0,015230298	-0,598552823	0,73686254	-1,137905002	-0,538378358	-1,183614135
-0,67496717	0,638519883	-0,960060239	-1,428052068	-0,361364722	0,089848638
0,706147313	-1,874057651	-0,575455785	0,409181714	1,169208884	-1,134787917
0,832279801	1,050521493	0,212804675	0,132447124	0,263118625	0,589259505
-0,165498376	-0,233549237	1,29242456	-0,639902472	-0,465812087	0,644260526
-0,82238853	0,059632659	-0,565412879	-1,126693368	-0,089291453	1,648104072
0,851595998	0,170784116	0,451340318	-1,519488215	1,455780387	0,344774365
-1,047933698	-1,110787511	-1,175130486	-0,133785367	-0,290214658	0,248493791
1,640565991	0,779120564	-2,500932217	-0,652332187	-1,391124129	1,530982375
-0,41009438	0,713849902	0,071305633	0,832728267	0,337405801	0,620578408
-1,647895932	-2,139699459	-0,021230578	0,72889173	-0,475450158	0,453692079
1,302237868	1,535497785	-0,037879586	-0,530690074	0,23179543	0,576985717
-0,602996707	-1,550090432	1,371337295	0,527116656	-2,196451187	7,33137E-05
0,784560084	1,289986253	1,037863612	0,606377006	1,808214784	1,848420739
0,860936522	-0,616127372	-0,682560325	-0,373138547	-0,340884805	0,385403037
0,218259931	2,075563908	-1,236131787	-0,911163926	2,110852242	1,334942937
0,067308068	-2,048696518	-0,002034545	-0,132987857	2,260348797	-1,250899673
0,011638999	0,313943267	1,209821105	-0,842363477	-1,237811923	-0,102040172
0,877316594	0,14688313	-0,736456275	-0,77061975	-0,101074576	0,055083632
-0,031292081	-1,665740132	0,17865479	-0,819055438	-0,005415559	-0,06175983
-0,090976596	0,238912225	0,808510423	0,492375255	0,61420548	0,976423144
-0,583821416	-1,01962173	0,453944564	-0,894389749	0,650084853	1,551770568
-0,351658702	-1,189674497	-0,143417001	-0,538266778	0,605751157	1,051396251
-1,330350995	-0,497070193	-0,729124904	-1,059232593	-0,35059011	-0,267797828
-1,672233224	1,925398707	2,127860546	-0,145781636	0,129461646	0,264989018
0,121932864	0,798795819	-0,997895122	-0,70646131	0,847876668	-0,151615262
-0,747879624	0,784801841	1,455083728	1,155121922	1,48223269	-0,073417068
-1,382179141	0,52328217	-0,026918292	0,758018374	-2,300914288	1,490299344
-0,540475488	-0,807921767	-2,122456551	-0,693703294	0,642460942	-0,461712956
0,317746997	0,015429854	-1,828588128	1,496032834	-2,306370258	0,222902656

Table 3.28 (continued). Distribution of the 1,000 values generated using ordinary simulation

Statistical distribution					
2,745229721	-0,160831332	-0,583391547	-0,027070642	-0,9736408	-0,537716746
-0,436671853	-1,025014043	-1,445999265	-0,976801515	-0,589278817	-0,614248395
1,014393926	0,69685328	-0,279701114	-0,419192433	-1,366881251	0,080711246
-1,63597548	-0,071264148	0,291006446	-0,130248427	0,328482985	-0,805580735
0,25636518	0,706398368	1,270529866	0,489489913	1,410414577	-0,0737468
2,669695854	0,403588414	0,726954103	0,497309089	-0,712892175	-2,202679157
0,408716083	1,1660918	0,364468932	-0,271183848	0,438567281	-0,554963708
1,231283784	0,418758988	1,796926618	-0,043577313	0,09811914	0,547423482
-0,216150641	0,203994155	-0,057536244	0,072358489	0,102903485	-0,188296437
-1,344176888	-0,127815127	2,696949959	-2,485765934	-1,776564002	0,99684298
-0,567213893	1,044486165	0,267284274	1,638991714	-0,286091924	0,314953208
-0,211753964	0,195890307	-2,400352955	-0,719628453	0,409652352	0,922424912
0,252522111	0,016110778	-1,073529124	1,972372413	0,877747893	-1,661450028
0,3757478	-0,025796771	0,490777373	0,983426929	0,361068606	0,533021808
0,050387979	1,350453258	-0,230359912	-1,443422914	1,197776914	-0,13433969
-0,621110559	-0,234306693	0,013779759	0,546879172	-1,502511621	-0,473112702
0,434282899	1,105699897	0,063007951	-0,071146846	0,503867269	2,176476479
-2,49520731	1,458792329	-0,513390183	-0,45808804	1,892820239	-0,00926435
1,176334262	-1,073592067	-1,343812108	-2,362922192	-0,622716308	0,721825719
1,54925859	1,418342471	0,147983193	-0,022106051	1,223623395	-1,219772458
0,326751351	-1,389109969	-0,486435771	-0,66438067	1,157734752	0,448989272
-0,330732465	1,230048537	0,256487489	-1,046697736	-0,154501319	-0,359826207
0,195951104	0,052310586	-1,572855592	-0,804940581	1,889977098	-0,48883903
0,161613822	0,389083028	-0,967119098	1,413354278	-0,162273288	0,343073487
-1,148956418	0,561720014	-0,887072682	0,760332227	-0,451195121	0,472043395
-0,260424972	0,737789512	-0,223422647	-1,675948262	0,179681182	-0,578670144
-0,826090693	-0,456863523	-1,249238849	1,169739604	0,063179612	
-0,877582669	0,249661088	0,824906945	1,570729852	-0,632733464	
0,463237166	1,835699916	-0,857471108	1,642875552	-0,129758477	
-1,708690524	0,078970313	-0,43738997	-0,650036216	-0,221612334	
0,84112823	0,112204671	-0,11120975	-1,018678308	0,664093852	
0,940538764	-0,251994252	0,546580195	0,835301757	-1,292581916	
-1,477033496	1,187062383	0,966201425	-0,178058267	-1,938308358	
1,376228213	-0,571704984	-1,515379071	0,670041203	0,055011392	
-1,018138289	-1,545376897	-1,881690145	-0,396770597	-1,079651713	
-0,159593225	0,008623481	-1,556555629	-1,295440078	0,921015859	
0,449507833	-1,43497932	-0,102908492	-0,059734941	-0,18877995	

Table 3.28 (continued). Distribution of the 1,000 values generated using ordinary simulation

Statistical distribution				
0,137960315	0,591638446	-0,367298484	1,975185513	0,119444489
-0,627521396	1,294908643	-0,86466372	0,976045489	0,541772962
-0,496541619	-0,150512338	0,683500171	0,536805511	0,06873858
-1,607984662	2,725074768	-0,444096923	1,637018323	2,110451221
-0,277212024	0,80897963	-0,237490296	1,104851604	-0,220681787
-0,081545234	-0,301193118	-1,206034541	-1,01818049	-0,522164226
-0,445425868	1,468173623	0,609172225	-0,305121303	-0,403963208
0,836102128	1,263960242	0,175666451	0,49211061	-0,856267333
-2,930031776	-0,051010966	-0,079541802	-0,722267032	1,295231223
0,356380105	0,101827025	-0,174749494	-0,964547753	-0,642392278
-1,13753593	-1,435625911	1,541161895	0,095542073	-0,676275373
2,401441574	1,370975375	-0,64311111	-1,032106519	0,653860688
0,102239728	0,568248868	0,417671084	-0,826632142	1,173235059
-1,924078345	0,314849496	0,425666213	1,20201695	-1,93908155
1,481267095	-1,217519403	-0,335891604	-0,404623389	-0,163055062
-1,017612576	0,701609731	1,042220235	1,061130881	0,85283339
-0,179892898	0,367814898	-0,152628064	1,402612567	1,375961423
-1,691685319	-0,621054769	0,294935346	-0,323647857	-1,402879357
0,479906917	-0,379096627	-0,591082692	2,620042801	1,887330651
1,256766915	-1,584616542	1,590516686	-2,001051903	-1,326907754
-1,232262969	0,384468198	0,890610337	0,997375846	-0,212966323
1,584718823	-2,149509907	0,357769132	0,45401752	
0,212281108	-0,498315454	-1,359009147	1,74446547	
0,257682681	0,170209765	-1,094747186	0,642317176	
-1,241298079	1,596838117	0,494059682	0,217702746	
1,904105544	-1,457988381	2,473494053	-0,100634694	
0,446714759	0,460140109	0,130962968	-0,271945596	
-0,277310729	0,76527679	1,498207211	-0,128224969	
0,607725263	-0,487404227	0,14852035	-0,64933598	
1,942776561	-1,640927911	-0,207295775	-0,647971272	
-0,69313395	-0,834524274	-0,031947255	-1,523043036	
-1,340639949	1,019135356	0,51048696	1,419018388	
-0,867900968	0,629609227	-0,477077365	0,11257875	
-1,074727178	-0,86730516	0,394396663	-0,880039573	
0,537500739	0,607238889	-2,137825489	-0,69808352	
2,153040886	0,989180446	-1,307218194	-0,41338098	
-0,211518645	-0,453614116	-0,246304393	0,367284894	
-0,391240954	1,291112781	0,347754121	0,294438243	
-1,214239955	0,102541566	-0,868643403	-1,55310452	

Table 3.28 (continued). Distribution of the 1,000 values generated using ordinary simulation

The resulting graphs are as follows:

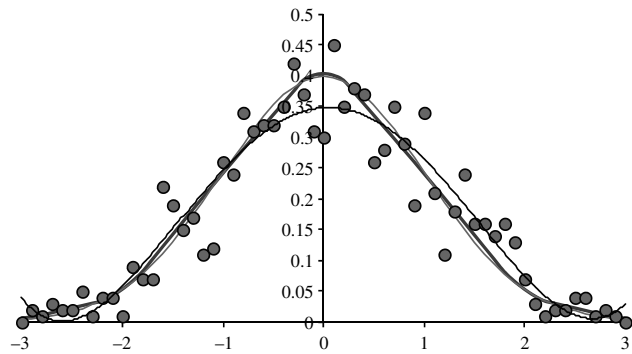


Figure 3.18. Graphical illustration for 1,000 generated values (2,454 loops)

Refusal method: example of the normal distribution: average = 2.5 and standard deviation = 1.5.

x1	2,00	x11	2,09
x2	2,05	x12	2,56
x3	2,75	x13	2,50
x4	2,00	x14	2,45
x5	2,35	x15	2,48
x6	2,50	x16	2,51
x7	2,50	x17	2,38
x8	3,00	x18	2,37
x9	2,15	x19	2,50
x10	2,55	x20	2,50
3,45422339439392			

Table 3.29. Example of simulated data → normal distribution?

– Conditions of the draw: $\lambda = 5.0$, period = 0.1 → Y =

X	N	Y	X	f(x)	X	N	Y	X	f(x)
–3,62	0,00	0,19	–1,00	–1,00	1,54	0,22	0,19	–1,00	–1,00
–3,50	0,00	0,19	–1,00	–1,00	1,66	0,23	0,19	–1,00	–1,00
–3,38	0,00	0,19	–1,00	–1,00	1,78	0,24	0,19	–1,00	–1,00
–3,26	0,00	0,19	–1,00	–1,00	1,90	0,25	0,19	–1,00	–1,00
–3,14	0,00	0,19	–1,00	–1,00	2,02	0,25	0,19	–1,00	–1,00

Table 3.30. Calculation results

X	N	Y	X	$f(x)$	X	N	Y	X	$f(x)$
-3,02	0,00	0,19	-1,00	-1,00	2,14	0,26	0,19	-1,00	-1,00
-2,90	0,00	0,19	-1,00	-1,00	2,26	0,26	0,19	-1,00	-1,00
-2,78	0,00	0,19	-1,00	-1,00	2,38	0,27	0,19	-1,00	-1,00
-2,66	0,00	0,19	-1,00	-1,00	2,50	0,27	0,19	-1,00	-1,00
-2,54	0,00	0,19	-1,00	-1,00	2,62	0,27	0,19	-1,00	-1,00
-2,42	0,00	0,19	-1,00	-1,00	2,74	0,26	0,19	-1,00	-1,00
-2,30	0,00	0,19	-1,00	-1,00	2,86	0,26	0,19	-1,00	-1,00
-2,18	0,00	0,19	-1,00	-1,00	2,98	0,25	0,19	-1,00	-1,00
-2,06	0,00	0,19	-1,00	-1,00	3,10	0,25	0,19	-1,00	-1,00
-1,94	0,00	0,19	-1,00	-1,00	3,22	0,24	0,19	-1,00	-1,00
-1,82	0,00	0,19	-1,00	-1,00	3,34	0,23	0,19	-1,00	-1,00
-1,70	0,01	0,19	-1,00	-1,00	3,46	0,22	0,19	-1,00	-1,00
-1,58	0,01	0,19	-1,00	-1,00	3,58	0,21	0,19	-1,00	-1,00
-1,46	0,01	0,19	-1,00	-1,00	3,70	0,19	0,19	-1,00	-1,00
-1,34	0,01	0,19	-1,00	-1,00	3,82	0,18	0,19	-1,00	-1,00
-1,22	0,01	0,19	-1,00	-1,00	3,94	0,17	0,19	-1,00	-1,00
-1,10	0,01	0,19	-1,00	-1,00	4,06	0,15	0,19	-1,00	-1,00
-0,98	0,02	0,19	-1,00	-1,00	4,18	0,14	0,19	-1,00	-1,00
-0,86	0,02	0,19	-1,00	-1,00	4,30	0,13	0,19	-1,00	-1,00
-0,74	0,03	0,19	-1,00	-1,00	4,42	0,12	0,19	-1,00	-1,00
-0,62	0,03	0,19	-1,00	-1,00	4,54	0,11	0,19	-1,00	-1,00
-0,50	0,04	0,19	-1,00	-1,00	4,66	0,09	0,19	-1,00	-1,00
-0,38	0,04	0,19	-1,00	-1,00	4,78	0,08	0,19	-1,00	-1,00
-0,26	0,05	0,19	-1,00	-1,00	4,90	0,07	0,19	-1,00	-1,00
-0,14	0,06	0,19	-1,00	-1,00	5,02	0,06	0,19	-1,00	-1,00
-0,02	0,06	0,19	-1,00	-1,00	5,14	0,06	0,19	-1,00	-1,00
0,10	0,07	0,19	-1,00	-1,00	5,26	0,05	0,19	-1,00	-1,00
0,22	0,08	0,19	-1,00	-1,00	5,38	0,04	0,19	-1,00	-1,00
0,34	0,09	0,19	-1,00	-1,00	5,50	0,04	0,19	-1,00	-1,00
0,46	0,11	0,19	-1,00	-1,00	5,62	0,03	0,19	-1,00	-1,00
0,58	0,12	0,19	-1,00	-1,00	5,74	0,03	0,19	-1,00	-1,00
0,70	0,13	0,19	-1,00	-1,00	5,86	0,02	0,19	-1,00	-1,00
0,82	0,14	0,19	-1,00	-1,00	5,98	0,02	0,19	-1,00	-1,00
0,94	0,15	0,19	-1,00	-1,00	6,10	0,01	0,19	-1,00	-1,00
1,06	0,17	0,19	-1,00	-1,00	6,22	0,01	0,19	0,19	0,01
1,18	0,18	0,19	-1,00	-1,00	6,34	0,01	-1,00	-1,00	-1,00
1,30	0,19	0,19	-1,00	-1,00	6,46	0,01	-1,00	-1,00	-1,00
1,42	0,21	0,19	-1,00	-1,00	6,58	0,01	-1,00	-1,00	-1,00
1,54	0,22	0,19	-1,00	-1,00	6,70	0,01	-1,00	-1,00	-1,00
1,66	0,23	0,19	-1,00	-1,00	6,82	0,00	-1,00	-1,00	-1,00
1,78	0,24	0,19	-1,00	-1,00	6,94	0,00	-1,00	-1,00	-1,00
1,90	0,25	0,19	-1,00	-1,00	7,06	0,00	-1,00	-1,00	-1,00

Table 3.30 (continued). Calculation results

X	N	Y	X	f(x)	X	N	Y	X	f(x)
2,02	0,25	0,19	-1,00	-1,00	7,18	0,00	-1,00	-1,00	-1,00
2,14	0,26	0,19	-1,00	-1,00	7,30	0,00	-1,00	-1,00	-1,00
2,26	0,26	0,19	-1,00	-1,00	7,42	0,00	-1,00	-1,00	-1,00
2,38	0,27	0,19	-1,00	-1,00	7,54	0,00	-1,00	-1,00	-1,00
2,50	0,27	0,19	-1,00	-1,00	7,66	0,00	-1,00	-1,00	-1,00
2,62	0,27	0,19	-1,00	-1,00	7,78	0,00	-1,00	-1,00	-1,00
2,74	0,26	0,19	-1,00	-1,00	7,90	0,00	-1,00	-1,00	-1,00
2,86	0,26	0,19	-1,00	-1,00	8,02	0,00	-1,00	-1,00	-1,00
2,98	0,25	0,19	-1,00	-1,00	8,14	0,00	-1,00	-1,00	-1,00
3,10	0,25	0,19	-1,00	-1,00	8,26	0,00	-1,00	-1,00	-1,00
3,22	0,24	0,19	-1,00	-1,00	8,38	0,00	-1,00	-1,00	-1,00

Table 3.30 (continued). Calculation results

X = 6.27388024330139 (black point).
F(X) = 0.011228160227142 (Experimental bell curve).
Y = 0.414032697677612 (Red horizontal line).

3.9.2. Graph

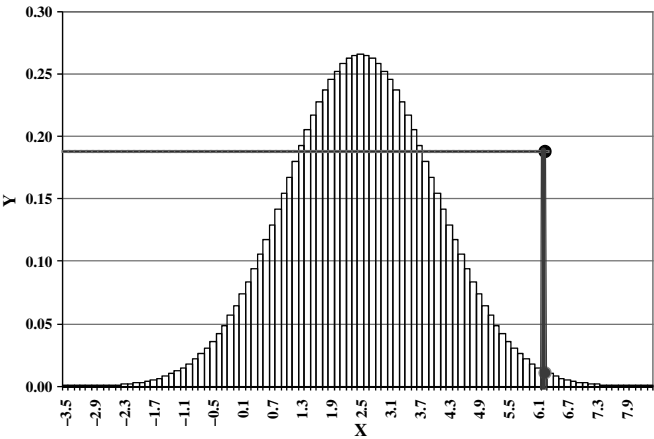


Figure 3.19. Graphical illustration of the normal distribution for 1,000 generated values

3.10. Conclusion

Collecting and analyzing reliability data must never rely on *simulations* alone when it comes to making a decision regarding safety. The decision to proceed using

any given method must be based on the exactitude of the event, such as the precise date and reason for the rupture, the implicated materials, etc. The physical characteristic (e.g. rupture) must not be an obstacle to its *true* measurement (microscopy, sampling gauge, ultrasounds, etc.). Models of reliability are not the problem. The main problem is how well the model is suited to the physical situation when it comes to calculating reliability indices. In this book, we have shown many methods. We should be careful about the adaptability of the method to the experimental cases, which are subject to analysis.

3.11. Bibliography

- [BAR 65] BARLOW C., PROSCHAN F., *Mathematical Theory of Reliability*, John Wiley, 1965.
- [BIR 68] BIRNBAUM Z.W., SAUNDERS S.C., “A probabilistic interpretation of miner’s rule”, *SIAM Journal of Applied Mathematics*, vol. 16, pp. 637–652, 1968.
- [BIR 69] BIRNBAUM Z.W., SAUNDERS S.C., “A new family of life distributions”, *Journal of Applied Probability*, vol. 6, pp. 319–327, 1969.
- [CER 86] CERESTA, (Centre d’enseignement et de recherche de la statistique appliquée), Aide-mémoire pratique des techniques statistiques pour ingénieurs et techniciens supérieurs, Paris, p. 274, 1986.
- [CHA 67] CHAKRAVARTI I.M., LAHA R.G., ROY L., *Handbook of Methods of Applied Statistics*, vol. I, John Wiley and Sons, pp. 392–394, 1967.
- [CHA 92] CHAPOUILLE P., *La fiabilité*, P.U.F., 1992.
- [COX 72] COX D.R., “Regression models and life tables”, *Journal of the Royal Statistical Society*, vol. B34, pp. 187–220, 1972.
- [COX 84] COX D.R., OAKES D., *Analysis of Survival Data*, Chapman & Hall, NY, 1984.
- [EBE 97] EBELING Ch.E., *An Introduction to Reliability and Maintainability Engineering*, McGraw-Hill, 1997.
- [ENG 82] ENGESVIK M.K., Analysis of uncertainties in fatigue capacity of welded joints, Report UR-, Norwegian Institut of Technology, University of Trondheim, Norway, 1982.
- [GRA 66] GRANT IRESON W., *Reliability Handbook*, McGraw Hill, 1966.
- [GRO 94] GROUS A., “Étude probabiliste du comportement des Matériaux et structure d’un joint en croix soudé”, PhD in Engineering Sciences, UHA, France, 1994.
- [GRO 98] GROUS A., RECHO N., LASSEN T., LIEURADE H.P., “Caractéristiques mécaniques de fissuration et défaut initial dans les soudures d’angles en fonction du procédé de soudage”, *Revue Mécanique Industrielle et Matériaux*, vol. 51, no. 1, Paris, France, 1998.
- [GRO 11] GROUS A., *Applied Metrology for Manufacturing Engineering*, ISTE Ltd, London, and John Wiley & Sons, New York, 2011.
- [HAN 72] HANDBOOK, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, New York, NY, 1972.

- [HAV 66] HAVILAND R.P., *Techniques de fiabilité et durée de vie des équipements*, Eyrolles, 1966.
- [JOH 64] JOHNSON L.G., *The Statistical Treatment of Fatigue Experiments*, Elsevier, 1964.
- [JUR 89] JURAN J.M., *Planifier la qualité*, Coll. AFNOR Gestion, 1989.
- [KOV 97] KOVALENKO I.N., KUZNETSOV N.Y., PEGGY P.A., *Mathematical Theory of Reliability of Time Dependent Systems with Practical Applications*, John Wiley & Sons, 1997.
- [LAS 92] LASSEN T., Experimental investigation and probabilistic modelling of the fatigue crack growth in welded joints, Summary Report, A. Coll. of Eng., Grimstad, Norway, 1992.
- [LLO 62] LLOYD D.K., LIPOW M., *Reliability: Management, Methods and Mathematics*, Prentice Hall, 1962.
- [MAR 74] MARCOVICI C., LIGERON J.C., *Utilisation des techniques de fiabilité en mécanique*, Tec & Doc, 1974.
- [MEE 75] MEEKER W.Q., NELSON W., “Optimum accelerated life-tests for the Weibull and extreme value distributions”, *IEEE Transactions on Reliability*, vol. R-24, no. 5, pp. 321–322, 1975.
- [MEE 85] MEEKER W.Q., HAHN G.J., *How to Plan an Accelerated Life Test – Some Practical Guidelines*, ASC Basic References in Quality Control: Statistical Techniques – Volume 10, ASQC, Milwaukee, WI, 1985.
- [MIL 78] MIL-STD-1635 (EC), Reliability Growth Testing, U.S. Government, 1978.
- [MIL 86] MIL-HDBK-217F, Reliability Prediction of Electronic Equipment, U.S. Government Printing Office, 1986.
- [MON 00] MONTGOMERY D.C., *Introduction to Statistical Quality Control*, 4th ed., Wiley, New York, NY, 2000.
- [MOO 74] MOOD A., *Introduction to the Theory of Statistics*, 3rd ed., McGraw-Hill, pp. 246–249, 1974.
- [NEL 90] NELSON W., *Accelerated Testing*, John Wiley & Sons, Inc., New York, 1990.
- [NIS 06] NIST, *Engineering Statistics Handbook*, F-Distribution, 2006.
- [OCO 91] O’CONNOR P.D.T., *Practical Reliability Engineering*, 3rd ed., John Wiley & Sons, Inc., New York, NY, 1991.
- [TOB 95] TOBIAS P.A., TRINDAD D.C., *Applied Reliability*, 2nd ed., Chapman and Hall, London, New York, NY, 1995.
- [TOV 01] TOVO R., “On the fatigue reliability evaluation of structural components under service loading”, *International Journal of Fatigue*, vol. 23, pp. 587–598, 2001.
- [WEI 51] WEIBULL W., “A statistical distribution function of wide applicability of Applied Mechanics”, *ASME Transactions*, vol. 18, no. 3, pp. 293–297, 1951.

Appendix

NOTE.–

Freely available software (including spreadsheet packages, e.g. Excel) offer access to common statistical tables and distribution laws. Hence, it would be pointless to give these tables on paper here.

There are numerous tables exploited here and in the existing body of technical literature. Here, we give the basics, that is the tables to which we refer in our chapters.

Euler's Gamma function as a (β) Weibull function

Law $\Gamma(x)$ is a second-order Euler Gamma function:

$$\left\{ a = \Gamma\left(1 + \frac{1}{\beta}\right) \text{ and } b = \sqrt{\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)} \right\}$$

Table of values (a) and (b) of the Eulerian function as a Weibull function $f(\beta)$.

β	a	b	β	a	b
0.20	120.000	190.1	1.20	0.941	0.780
0.25	24.000	199.0	1.40	0.911	0.660
0.30	9.2600	56.60	1.60	0.897	0.574
0.40	3.3233	10.50	1.80	0.889	0.511
0.50	2.0000	4.470	2.00	0.886	0.463
0.60	1.5000	2.650	2.50	0.887	0.380
0.70	1.2660	1.850	3.00	0.893	0.324
0.80	1.1330	1.430	0.35	0.900	0.285
0.90	1.0520	1.180	4.00	0.906	0.255
1.00	1.0000	1.000	5.00	0.918	0.210

Table A.1. Values of the coefficients of the Eulerian Gamma function (a and b)

Kolmogorov-Smirnov (K-S) test

Size of the sample (N)	Significance level α for $D = \text{maximum } [f_0(x) - s_n(x)]$				
	.20	.15	.10	.05	.01
1	.900	.925	.950	.975	.995
2	.684	.726	.776	.842	.929
3	.565	.597	.642	.708	.828
4	.494	.525	.564	.624	.733
5	.446	.474	.510	.565	.669
6	.410	.436	.470	.521	.618
7	.381	.405	.438	.486	.577
8	.358	.381	.411	.457	.543
9	.339	.360	.388	.432	.514
10	.322	.342	.368	.410	.490
11	.307	.326	.352	.391	.468
12	.295	.313	.338	.375	.450
13	.284	.302	.325	.361	.433
14	.274	.292	.314	.349	.418
15	.266	.283	.304	.338	.404
16	.258	.274	.295	.328	.392
17	.250	.266	.286	.318	.381
18	.244	.259	.278	.309	.371
19	.237	.252	.272	.301	.363
20	.231	.246	.264	.294	.356
25	.210	.220	.240	.270	.320
30	.190	.200	.220	.240	.290
35	.180	.190	.210	.230	.270
> 35	1.07	1.22	1.22	1.36	1.63

Table A.2. Values of (D_n, α) in Kolmogorov-Smirnov test**Student's distribution inspection table (t-Student)**

Uppermost critical values of the distribution t with v degrees of freedom ($v = \text{Dof}$) at the reliability threshold of $\alpha/2 = 5\%$							
v	$\tau_{\alpha/2}(v)$	v	$\tau_{\alpha/2}(v)$	v	$\tau_{\alpha/2}(v)$	v	$\tau_{\alpha/2}(v)$
1	12.706	26	2.056	51	2.008	76	1.992
2	4.303	27	2.052	52	2.007	77	1.991
3	3.182	28	2.048	53	2.006	78	1.991
4	2.776	29	2.045	54	2.005	79	1.990
5	2.571	30	2.042	55	2.004	80	1.990
6	2.447	31	2.040	56	2.003	81	1.990
7	2.365	32	2.037	57	2.002	82	1.989
8	2.306	33	2.035	58	2.002	83	1.989
9	2.262	34	2.032	59	2.001	84	1.989
10	2.228	35	2.030	60	2.000	85	1.988
11	2.201	36	2.028	61	2.000	86	1.988
12	2.179	37	2.026	62	1.999	87	1.988
13	2.160	38	2.024	63	1.998	88	1.987
14	2.145	39	2.023	64	1.998	89	1.987
15	2.131	40	2.021	65	1.997	90	1.987

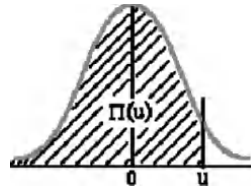
Table A.3. Student's t -distribution table

ν	$\tau_{\alpha/2}(\nu)$	ν	$\tau_{\alpha/2}(\nu)$	ν	$\tau_{\alpha/2}(\nu)$	ν	$\tau_{\alpha/2}(\nu)$
16	2.120	41	2.020	66	1.997	91	1.986
17	2.110	42	2.018	67	1.996	92	1.986
18	2.101	43	2.017	68	1.995	93	1.986
19	2.093	44	2.015	69	1.995	94	1.986
20	2.086	45	2.014	70	1.994	95	1.985
21	2.080	46	2.013	71	1.994	96	1.985
22	2.074	47	2.012	72	1.993	97	1.985
23	2.069	48	2.011	73	1.993	98	1.984
24	2.064	49	2.010	74	1.993	99	1.984
25	2.060	50	2.009	75	1.992	100	1.984

Table A.3 (continued). Student's *t*-distribution table**Binomial distribution**

Significance Level For One-Direction Test						
<i>Dof</i>	.10	.05	.025	.01	.005	.000
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
X	1.282	1.645	1.960	2.326	2.576	3.291

Table A.4. Binomial distribution table

Normal distribution table

Distribution function P of the reduced centered normal law (u). Probability of finding a value $< (u) \rightarrow P(-u) = 1 - P(u)$. Example: For $\Phi(1.73) = 0.95818$, we choose row 1.7 and column 0.03 and read $\rightarrow (1.7 + 0.03 = 1.73)$.

u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997

Table A.5. Gaussian normal law table

Values of probabilities associated with Z in a normal law

The probability in a direction rejects the null hypothesis H_0 if the value is less than that given by the value of the RCRV Z (reduced centered random variable) in the second column $Z \downarrow$ for the first decimal. The row $Z \rightarrow$ provides the second decimal, to be added to the first										
$Z \rightarrow$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0038	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007

Table A.6. Probability values associated with Z in a normal law

Karl Pearson Chi-square or χ^2 law

According to the number of degrees of freedom (first column) and the risk of error (α) (first line), we find the value of the distance (gap) χ^2 which has the probability to be exceeded.

n/α	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995
1	0.0000	0.0002	0.0010	0.0039	0.0158	0.102	0.455	1.32	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	0.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	0.0717	0.115	0.216	0.352	0.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	0.207	0.297	0.484	0.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	0.412	0.554	0.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	0.676	0.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	0.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	32.0	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.56	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.1	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7
40	20.7	22.2	24.4	26.5	29.1	33.7	39.3	45.6	51.8	55.8	59.3	63.7	66.8
50	28.0	29.7	32.4	34.8	37.7	42.9	49.3	56.3	63.2	67.5	71.4	76.2	79.5
60	35.5	37.5	40.5	43.2	46.5	52.3	59.3	67.0	74.4	79.1	83.3	88.4	92.0
70	43.3	45.4	48.8	51.7	55.3	61.7	69.3	77.6	85.5	90.5	95.0	100.4	104.2
80	51.2	53.5	57.2	60.4	64.3	71.1	79.3	88.1	96.6	101.9	106.6	112.4	116.3
90	59.2	61.8	65.6	69.1	73.3	80.6	89.3	98.6	107.6	113.1	118.1	124.1	128.3
100	67.3	70.1	74.2	77.9	82.4	90.1	99.3	109.1	118.5	124.3	129.6	135.8	140.2

Table A.7. χ^2 , Chi-square law

For large values of n , the law of probability of χ^2 tends toward a normal law of average n and variance $2n$. If n is sufficiently large, the random variable $\{\sqrt{2\chi^2} - \sqrt{2n-1}\}$ more-or-less obeys a reduced centered normal law. If the calculated value of χ^2 is greater than the value shown at the predetermined confidence threshold, we reject the null hypothesis H_0 .

<i>Dof</i>	.99	.98	.95	.90	.80	.70	.50	.30	.20	.10	.05	.02	.01	.001
1	.00016	.00063	.0039	.016	.064	.15	.46	1.07	1.64	2.71	3.84	5.41	6.64	10.83
2	.02	.04	.10	.21	.45	.71	1.39	2.41	3.22	4.60	5.99	7.82	9.21	13.82
3	.12	.18	.35	.58	1.00	1.42	2.37	3.66	4.64	6.25	7.82	9.84	11.34	16.27
4	.30	.43	.71	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	11.67	13.28	18.46
5	.55	.75	1.14	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.07	13.39	15.09	20.52
6	.87	1.13	1.64	2.20	3.07	3.83	5.35	7.23	8.56	10.64	12.59	15.03	16.81	22.46
7	1.24	1.56	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.02	14.07	16.62	18.48	24.32
8	1.65	2.03	2.73	3.49	4.59	5.53	7.34	9.52	11.03	13.36	15.51	18.17	20.09	26.12
9	2.09	2.53	3.32	4.17	5.38	6.39	8.34	10.66	12.24	14.68	16.92	19.68	21.67	27.88
10	2.56	3.06	3.94	4.86	6.18	7.27	9.34	11.78	13.44	15.99	18.31	21.16	23.21	29.59
11	3.05	3.61	4.58	5.58	6.99	8.15	10.34	12.90	14.63	17.28	19.68	22.62	24.72	31.26
12	3.57	4.18	5.23	6.30	7.81	9.03	11.34	14.01	15.81	18.55	21.03	24.05	26.22	32.91
13	4.11	4.76	5.89	7.04	8.63	9.93	12.34	15.12	16.98	19.81	22.36	25.47	27.69	34.53
14	4.66	5.37	6.57	7.79	9.47	10.82	13.34	16.22	18.15	21.06	23.68	26.87	29.14	36.12
15	5.23	5.98	7.26	8.55	10.31	11.72	14.34	17.32	19.31	22.31	25.00	28.26	30.58	37.70
16	5.81	6.61	7.96	9.31	11.15	12.62	15.34	18.42	20.46	23.54	26.30	29.63	32.00	39.29
17	6.41	7.26	8.67	10.08	12.00	13.53	16.34	19.51	21.62	24.77	27.59	31.00	33.41	40.75
18	7.02	7.91	9.39	10.86	12.86	14.44	17.34	20.60	22.76	25.99	28.87	32.35	34.80	42.31
19	7.63	8.57	10.12	11.65	13.72	15.35	18.34	21.69	23.90	27.20	30.14	33.69	36.19	43.82
20	8.26	9.24	10.85	12.44	14.58	16.27	19.34	22.78	25.04	28.41	31.41	35.02	37.57	45.32
21	8.90	9.92	11.59	13.24	15.44	17.18	20.34	23.86	26.17	29.62	32.67	36.34	38.93	46.80
22	9.54	10.60	12.34	14.04	16.31	18.10	21.24	24.94	27.30	30.81	33.92	37.66	40.29	48.27
23	10.20	11.29	13.09	14.85	17.19	19.02	22.34	26.02	28.43	32.01	35.17	38.97	41.64	49.73
24	10.86	11.99	13.85	15.66	18.06	19.94	23.34	27.10	29.55	33.20	36.42	40.27	42.98	51.18
25	11.52	12.70	14.61	16.47	18.94	20.87	24.34	28.17	30.68	34.38	37.65	41.57	44.31	52.62
26	12.20	13.41	15.38	17.29	19.82	21.79	25.34	29.25	31.80	35.56	38.88	42.86	45.64	54.05
27	12.88	14.12	16.15	18.11	20.70	22.72	26.34	30.32	32.91	36.74	40.11	44.14	46.96	55.48
28	13.56	14.85	16.93	18.94	21.59	23.65	27.34	31.39	34.03	37.92	41.34	45.42	48.28	56.89
29	14.26	15.57	17.71	19.77	22.48	24.58	28.34	32.46	35.14	39.09	42.56	46.69	49.59	58.30
30	14.95	16.31	18.49	20.60	23.36	25.51	29.34	33.53	36.25	40.26	43.77	47.96	50.89	59.70

Table A.8. *Pearson's χ^2 table: probability below H_0 such that $X \geq \chi^2$*

D/Aire	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	3.07382	3.57057	4.40379	5.27603	6.30380	8.43842	11.34032	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847
18	6.26480	7.01491	8.23075	9.39046	10.86494	13.67529	17.33790	21.60489	25.98942	28.86930	31.52638	34.80531	37.15645
19	6.84397	7.63273	8.90652	10.1701	11.65091	14.56200	18.33765	22.71781	27.20357	30.14353	32.85233	36.19087	38.58226
20	7.43384	8.26040	9.59078	10.85081	12.44261	15.45177	19.33743	23.82769	28.41198	31.41043	34.16961	37.56623	39.99685
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
26	11.16024	12.19815	13.84390	15.37916	17.29188	20.84343	25.33646	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74940	26.33634	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196

Table A.9. Table of critical values for the χ^2 distribution from the χ^2 test

The critical value for the χ^2 distribution is found at the intersection of a column, corresponding to a given probability, and a row, corresponding to the *dof*. For instance, the critical value of χ^2 with four *dof* for the probability 0.25 is equal to 5.38527. This means that the surface under the density curve of χ^2 with four *df* on the left of the value 5.38527 is equal to 0.25 (to 25% of the surface).

Various coefficients of the control chart

<i>n</i>	Chart of control of the mean						Chart of control of the extent			
	σ_0 known		σ_0 estimated				σ_0 known		σ_0 estimated	
			By $\bar{R}$		By $\bar{s}$				By $\bar{R}$	
	A_c	A_s	A'_c	A'_s	A''_c	A''_s	δ_{c2}	δ_{s2}	δ'_{c2} (D'e)	δ'_{s2} (D's)
2	2.185	1.386	1.937	0.229	3.874	2.457	4.65	3.17	4.12	2.81
3	1.784	1.132	1.054	0.668	2.464	1.564	5.06	3.68	2.99	2.17
4	1.545	0.980	0.750	0.476	1.936	1.228	5.31	3.98	2.58	1.93
5	1.382	0.876	0.594	0.377	1.643	1.042	5.48	4.20	2.36	1.81
6	1.262	0.800	0.498	0.316	1.452	0.921	5.62	4.36	2.22	1.72
7	1.168	0.741	0.432	0.274	1.315	0.834	5.73	4.49	2.12	1.66
8	1.092	0.693	0.384	0.244	1.209	0.767	5.82	4.61	2.04	1.62
9	1.030	0.653	0.347	0.220	1.127	0.714	8.90	4.70	1.99	1.58
10	0.977	0.620	0.317	0.202	1.059	0.672	5.97	4.79	1.94	1.56
11	0.932	0.591	0.295	0.186	1.002	0.635	6.04	4.86	1.90	1.53
12	0.892	0.566	0.274	0.174	0.953	0.605	6.09	4.92	1.87	1.51

Table A.10. *Coefficients for the charts of control of the mean and the extent $\bar{R}$*

Coefficients of Shewhart charts based on the sample size

The coefficients of the Shewhart charts, based on the size *n* samples, are used to calculate the limits of control according to the samples size and type of used chart.

Coefficients of Shewhart charts based on the size of the samples

<i>n</i>	<i>d</i> ₂	<i>d</i> ₃	<i>C</i> ₄	<i>A</i>	<i>A</i> ₂	<i>A</i> ₃	<i>B</i> ₃	<i>B</i> ₄	<i>B</i> ₅	<i>B</i> ₆	<i>D</i> ₃	<i>D</i> ₄	<i>D</i> ₅	<i>D</i> ₆
2	1.128	0.853	0.7979	2.121	1.880	2.659	0	3.267	0	2.606	0	3.267	0	3.686
3	1.693	0.883	0.8862	1.732	1.023	1.954	0	2.568	0	2.276	0	2.574	0	4.358
4	2.059	0.880	0.9213	1.500	0.729	1.628	0	2.266	0	2.088	0	2.282	0	4.698
5	2.326	0.864	0.9400	1.342	0.577	1.427	0	2.089	0	1.964	0	2.114	0	4.918
6	2.534	0.848	0.9515	1.225	0.483	1.287	0.030	1.970	0.029	1.874	0.076	2.004	0	5.078
7	2.704	0.833	0.9594	1.134	0.419	1.282	0.118	1.882	0.113	1.804	0.136	1.924	0.205	5.203
8	2.847	0.820	0.9650	1.061	0.373	1.099	0.185	1.815	0.178	1.752	0.184	1.864	0.387	5.307
9	2.970	0.808	0.9693	1.000	0.337	1.032	0.239	1.761	0.232	1.707	0.223	1.816	0.546	5.394
10	3.078	0.797	0.9727	0.949	0.308	0.975	0.284	1.716	0.277	1.669	0.256	1.777	0.687	5.469
11	3.173	0.787	0.9754	0.905	0.285	0.927	0.321	1.679	0.314	1.637	0.283	1.744	0.812	5.534
12	3.258	0.778	0.9776	0.866	0.266	0.886	0.354	1.646	0.346	1.609	0.307	1.717	0.924	5.592
13	3.336	0.770	0.9794	0.932	0.249	0.850	0.382	1.618	0.374	1.585	0.328	1.693	1.026	5.646
14	3.472	0.762	0.9810	0.802	0.235	0.817	0.406	1.594	0.399	1.563	0.347	1.672	1.121	5.693
15	3.735	0.755	0.9823	0.775	0.223	0.789	0.428	1.572	0.420	1.544	0.347	1.653	1.207	5.937
20	3.735	0.729	0.9869	0.671	0.180	0.680	0.510	1.490	0.503	1.471	0.415	1.585	1.548	5.922
21					0.173						0.425	1.575		
22					0.167	0.647	0.534	1.466			0.434	1.566		
23					0.162						0.443	1.557		
24						0.619	0.555	1.445						
25						0.606	0.565	1.43						
30			0.9914											
40			0.9936											

Table A.11. *Quality control in the automobile industry. Standard ISO/TS 16949*

Sampling control and efficiency curve based on Poisson's law

The purpose of the control by statistical sampling is to help to decide on the lot acceptance or its the refusal. Same as the probability of Chebyshev's approach, this type of control concerns a lot of which we don't want to fully control, for many reasons such as efficiency, technical or economic reasons.

- From a lot (N) we pick a sample (n);
- Based on (N) we target a acceptable criteria (c) and then we compute the defect of items (d);
- (c) is a acceptable criteria (c) and (r) the refusal one.

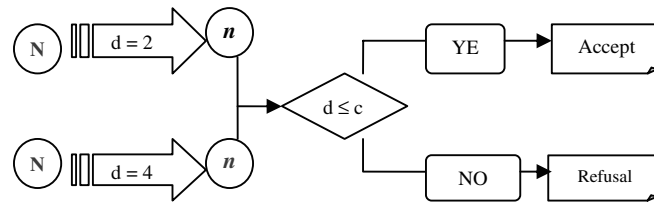


Figure A.1. Sampling control diagram

Sampling plans can be *single*, *double* or *multiple*. A single sampling plan for attributes consists of a sample of size n and an acceptance number c . The procedure operates as follows: select n items at random from the lot. If the number of defectives in the sample set is less than c , the lot is accepted. Otherwise, the lot is rejected.

We consider a lot that represents the entire items of the lot (N). We pick whatever item that has a probability (p). We also consider a accepted quality of level (AQL) while the client consider's his tolerated quality level (TQL). In order to plot the efficiency curve using a Poisson law, we need to consider a sampling plan following this method:

- For different values of p $\{p_1, p_2, p_3, \dots\}$ which is the proportion of the defect items within a lot (N) and then we calculate the values of: $n \times p$ $\{n.p_1, n.p_2, n.p_3, \dots\}$;
- for each $n.p$ value we establish the value of p_a based on the Poisson law chart below.

n	$n.p/c$	0	1	2	3	4	5	6	7	8
1	0.02	980	1000							
2	0.04	691	999	1000						
3	0.06	942	998	1000						
4	0.08	923	997	1000						
5	0.10	905	995	1000						
6	0.15	861	990	999	1000					
7	0.20	819	999	999	1000					
8	0.25	779	974	998	1000					
9	0.30	741	963	996	1000					
10	0.35	705	951	994	1000					
11	0.40	670	938	992	999	1000				
12	0.45	638	925	989	999	1000				
13	0.50	607	910	986	998	1000				
14	0.55	557	894	982	998	1000				
15	0.60	549	878	977	997	1000				
16	0.65	522	861	972	996	999	1000			
17	0.70	844	966	966	994	999	1000			
18	0.75	827	959	959	993	999	1000			
19	0.80	809	953	953	991	999	1000			
20	0.85	791	945	945	989	998	1000			
21	0.90	772	937	937	987	998	1000			
22	0.95	754	929	929	984	997	1000			
23	1.00	736	920	920	981	996	999	1000		
24	1.10	699	900	900	974	995	999	1000		
25	1.20	663	879	879	966	992	998	1000		
26	1.30	627	857	857	957	989	998	1000		
27	1.40	592	833	833	946	986	997	999	1000	
28	1.50	558	809	809	934	981	996	999	100	
29	1.60	525	783	783	921	976	994	999	1000	
30	1.70	493	757	857	907	970	992	998	1000	
31	1.80	463	731	731	891	964	990	997	999	1000
32	1.90	434	704	704	875	956	987	997	999	1000
33	2.00	406	677	677	857	947	983	995	999	1000

Table A.12. Poisson's distribution ($n.p/c$)

Defects number	Probabilities from Poisson's table. Values of P (at less k) we have $m = np$													
K	m=0.1	m=0.2	m=0.3	m=0.4	m=0.5	m=0.6	m=0.7	m=0.8	m=0.9	m=1.0	m=1.5	m=2.0	m=2.5	m=3.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0.2231	0.1353	0.0821	0.0498
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358	0.5578	0.4060	0.2873	0.1991
2	0.9998	0.9988	0.9964	0.9920	0.9856	0.9769	0.9659	0.9526	0.9372	0.9197	0.8088	0.6767	0.5438	0.4232
3	1	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9866	0.9810	0.9344	0.8571	0.7576	0.6472
4		1	1	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9814	0.9473	0.8912	0.8153
5				1	1	1	0.9999	0.9998	0.9997	0.9994	0.9955	0.9834	0.9579	0.9161
6							1	1	1	0.9999	0.9991	0.9955	0.9858	0.9665
7										1	0.9998	0.9989	0.9958	0.9881
8											1	0.9998	0.9989	0.9962
9												1	0.9997	0.9989
10													0.9999	0.9997
11													1	0.9999
K	m=3.5	m=4.0	m=4.5	m=5.0	m=5.5	m=6.0	m=6.5	m=7.0	m=7.5	m=8.0	m=8.5	m=9.0	m=9.5	m=10
0	0.0302	0.0183	0.0111	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	
1	0.1359	0.0916	0.0611	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005
2	0.3208	0.2381	0.1736	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.5366	0.4335	0.3423	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0104
4	0.7254	0.6288	0.5321	0.4405	0.3575	0.2851	0.2237	0.1713	0.1321	0.0996	0.0746	0.0550	0.0403	0.0293
5	0.8576	0.7851	0.7029	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671

Table A.13. Poisson's table

Probabilities from Poisson's table. Values of P (at less k) we have m = n.p														
6	0.9347	0.8893	0.8311	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1302
7	0.9733	0.9489	0.9134	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2203
8	0.9901	0.9786	0.9319	0.9319	0.9044	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3329
9	0.9967	0.9919	0.9682	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4580
10	0.999	0.9972	0.9933	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5831
11	0.9997	0.9991	0.9976	0.9945	0.9890	0.9799	0.9661	0.9466	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968
12	0.9999	0.9997	0.9992	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916
13	1	0.9999	0.9997	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8646
14		1	0.9999	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9166
15			1	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513
16				1	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730
17					1	1	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857
18							1	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928
19								1	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965
20									1	0.9999	0.9998	0.9996	0.9991	0.9984
21										1	0.9999	0.9998	0.9996	0.9993
22											1	0.9999	0.9998	0.9997
23												1	0.9999	0.9999

Table A.13 (continued). Poisson's table

Statistical sampling for acceptance (NQA)

This method is used in order to decide whether we accept or refuse a lot of manufactured items. we have five alternatives to the acceptance sampling:

- to deliver the whole lot without any control;
- to exempt the whole lot without any control;
- to inspect 100% of the lot;
- to accept the sample and to refuse the remaining items from the lot;
- to accept the sample and to inspect what remains from the lot at 100%.

We use a ISO 2859 norm (NQA) to measure the accepted levels of quality. The used tables are mainly American. Their equivalents at the international level of norms are (ANSI/ASQC Z1.4, ISO 2859, NF06-022, BS 6001, DIN 40080).

For example, to inspect a lot of 4000 units at a sampling level II, the table (Table A.14.) shows that the AQL (accepted quality level) of 2.5. So the lot is accepted until 10 defects are found. Beyond 10 defects the lot is refused.

SAMPLE SIZE CODE LETTERS									
Lot size	General Inspection Level				Special Inspection Level				
	I	II	III		S ₁	S ₂	S ₃	S ₄	
	2 to 8	A	A		B	A	A	A	A
	9 →15	A	B		C	A	A	A	A
	16 →25	B	B		D	A	A	B	B
	26 →50	C	D		E	A	B	B	C
51 →90	C	E	F	B	B	C	C		
91 →150	D	F	G	B	B	C	D		
151 →280	E	G	H	B	C	D	E		
281 →500	F	H	J	B	C	D	E		
501 →1200	G	J	K	C	C	E	F		
1201 →3200	H	K	L	C	D	E	G		
3201 →10000	J	L	M	C	D	F	G		
10001 →35000	K	M	N	C	D	F	H		
35001 →150000	L	N	P	D	E	G	J		
150001 →500000	M	P	Q	D	E	G	J		
500001 → and over	N	Q	R	D	E	H	K		

Table A.14. Table of code letters: sample sizes n

Sample Size Code Letter		Sample Size	Single Sampling Plans for Normal Inspection																		
			Acceptable Quality Levels for Normal Inspection																		
			.065	.10	.15	.25	.4	.65	1.0	1.5	2.5	4.0	6.5								
Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re		
A	2																				
B	3																				
C	5																				
D	8																				
E	13																				
F	20																				
G	32																				
H	50																				
J	80																				
K	125																				
L	200																				
M	315																				
N	500																				
P	800																				
Q	1250																				
R	2000																				
				Use first sampling plan below arrow. If sample size equals or exceed lot or batch size do 100% inspection																	
				Use first sampling plan above arrow																	
				Ac = Acceptance number ; Re = Rejection number																	

Table A.15. Table for acceptable quality level (AQL; normal inspection)

EXAMPLE.– We received a lot $n = 1500$ connecting rods machined by Sam. We pick a sample of 100 rods and we decide to accept the lot (N) if and only if the lot contains *five or less* defected rods. We need to plot the efficiency curve of this sampling plan with $n = 100$ rods and $c = 5$ defects. If the sample is quiet wide for the values of $p < 10\%$ we get the following results.

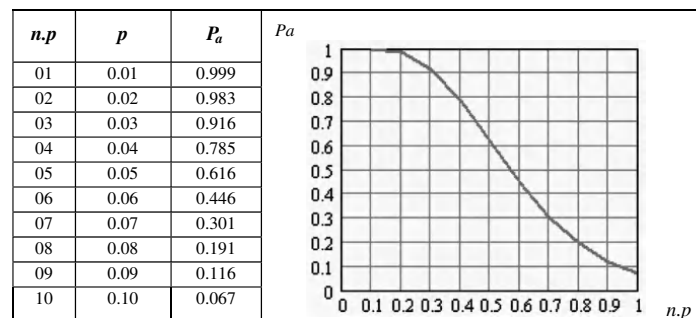


Figure A.2. Efficiency curve

According to the efficiency curve for a NQA = 2.5%, the risk (α) for the producer is 5% because the probability to accept the quality, at this level is 95%. The refusal probability is then complement it $(100\%-95\%) = 5\%$. If, for example Nadim the client wants to limit the risk (β) at 10%; his NQT could be at 9.2%. If the client can tolerate only an NQT at 7.5%, it is obvious that the present plan ($n=100$ and $c = 5$) doesn't satisfy the clients (Sam and Nadim). We use the **a** calculation table to read n and c , knowing (a) and (b) for the ration of p_2/p_1 probabilities)

c	Values of p_2/p_1 for			$n.p_1$	c	Values of p_2/p_1 for			$n.p_1$
	$\alpha = 0.05$ $\beta = 0.10$	$\alpha = 0.05$ $\beta = 0.05$	$\alpha = 0.05$ $\beta = 0.01$			$\alpha = 0.01$ $\beta = 0.10$	$\alpha = 0.01$ $\beta = 0.05$	$\alpha = 0.01$ $\beta = 0.01$	
0	44.890	58.404	89.781	0.052	0	229.105	298.073	458.21	0.010
1	10.946	13.349	18.681	0.355	1	26.184	31.9330	44.686	0.149
2	6.509	7.689	10.280	0.818	2	12.206	14.4390	19.278	0.436
3	4.890	5.675	7.352	1.366	3	8.1150	9.41800	12.202	0.823
4	4.057	4.646	5.890	1.970	4	6.2490	7.15500	9.0720	1.279
5	3.549	4.023	5.017	2.613	5	5.1950	5.88900	7.3430	1.785
6	3.206	3.604	4.435	3.286	6	4.5200	5.08200	6.2530	2.330
7	2.957	3.303	4.019	3.931	7	4.0500	4.52400	5.5060	2.906
8	2.768	3.074	3.707	4.695	8	3.7050	4.11500	4.9620	3.507
9	2.618	2.895	3.462	5.426	9	3.4400	3.80300	4.5480	4.130
10	2.497	2.750	3.265	6.169	10	3.2290	3.55500	4.2220	4.771
11	2.397	2.630	3.104	6.924	11	3.0580	3.85400	3.9590	5.428
12	2.312	2.528	2.968	7.690	12	2.9150	3.18800	3.7420	6.099
13	2.240	2.442	2.852	8.464	13	2.7950	3.04700	3.5590	6.782

Table A.16. Calculation table of n and c

c	Values of p_2/p_1 for			$n.p_1$	c	Values of p_2/p_1 for			$n.p_1$
	$\alpha = 0.05$ $\beta = 0.10$	$\alpha = 0.05$ $\beta = 0.05$	$\alpha = 0.05$ $\beta = 0.01$			$\alpha = 0.05$ $\beta = 0.10$	$\alpha = 0.05$ $\beta = 0.05$	$\alpha = 0.05$ $\beta = 0.01$	
14	2.177	2.367	2.752	9.246	14	2.6920	2.92700	3.4030	7.477
15	2.122	2.302	2.665	10.035	15	2.6030	2.82300	3.2690	8.181
16	2.073	2.244	2.588	10.831	16	2.5240	2.73200	3.1510	8.895
17	2.029	2.192	2.520	11.633	17	2.4550	2.65200	3.0480	9.616
18	1.990	2.145	2.458	12.442	18	2.3930	2.58000	2.9560	10.346
19	1.954	2.103	2.403	13.254	19	2.3370	2.51500	2.8740	11.082
20	1.922	2.065	2.352	14.072	20	2.2870	2.45500	2.7990	11.825
21	1.892	2.030	2.307	14.894	21	2.2410	2.40500	2.7330	12.574
22	1.865	1.999	2.265	15.719	22	2.2000	2.35700	2.6710	13.329
23	1.840	1.969	2.226	16.548	23	2.1620	2.31300	2.6150	14.088
24	1.817	1.942	2.191	17.382	24	2.1260	2.27200	2.5640	14.853
25	1.795	1.917	2.158	18.218	25	2.0940	2.23500	2.5160	15.623

Table A.16 (continued). Calculation table of n and c

- P_1 : is a acceptable quality level et P_2 : is a tolerate quality level (TQL).
- The ratio is: $NQT/NQA = P_2/P_1$.

Considering our example case, $NQT/NQA = p_2/p_1 = 7.5\%/2.5\% = 3$. According to a table A.16 $NQT/NQA = p_2/p_1 = 7.5\%/2.5\% = 3$ is $c = 7$. To get the sample size (n) we divide (n) by np_1 . So we get $n = 3.981/0.025 = 159.24 \approx 160$. The plan that satisfy Sam and Nadim is $n = 160$ and $c = 7$. We can also use some sampling tables such as the military norm (USA, UK and Canada) MIL-STD 105 D. The coefficients from the table A.17 are used to calculate the limits of control in relationship with the sampling size and the type of the used chart.

n	Average chart of control						Range chart of control			
	σ_0 known		σ_0 estimated by				σ_0 known		σ_0 estimated	
			$\bar{R}$		$\bar{s}$				by $\bar{R}$	
	A_c	A_s	A'_c	A'_s	A''_c	A''_s	δ_{c2}	δ_{s2}	δ'_{c2} (D'e)	δ'_{s2} (D's)
2	2.185	1.386	1.937	0.229	3.874	2.457	4.65	3.17	4.12	2.81
3	1.784	1.132	1.054	0.668	2.464	1.564	5.06	3.68	2.99	2.17
4	1.545	0.980	0.750	0.476	1.936	1.228	5.31	3.98	2.58	1.93
5	1.382	0.876	0.594	0.377	1.643	1.042	5.48	4.20	2.36	1.81
6	1.262	0.800	0.498	0.316	1.452	0.921	5.62	4.36	2.22	1.72
7	1.168	0.741	0.432	0.274	1.315	0.834	5.73	4.49	2.12	1.66
8	1.092	0.693	0.384	0.244	1.209	0.767	5.82	4.61	2.04	1.62
9	1.030	0.653	0.347	0.220	1.127	0.714	5.90	4.70	1.99	1.58
10	0.977	0.620	0.317	0.202	1.059	0.672	5.97	4.79	1.94	1.56
11	0.932	0.591	0.295	0.186	1.002	0.635	6.04	4.86	1.90	1.53
12	0.892	0.566	0.274	0.174	0.953	0.605	6.09	4.92	1.87	1.51

Table A.17. Coefficients for chart control for average ($\bar{x}$) and range $\bar{R}$, using the norm ISO/TS 16949, in the automobile field

In statistical quality control, acceptance sampling (AS) largely has been used since World War II to determine which batches of ammunition to accept and which ones to reject. We could not test every bullet to determine if it will do *its job* or not. Acceptance sampling was one of the answers. We could test a few representative bullets from the lot (N) so we could know how the rest of the bullets will perform. From this point of view, (AS) is a compromise between not *doing any inspection* at all and a *100% inspection*. The representative samples are tested to determine whether the lot is acceptable or not (*acceptance plan*). We then have two major classifications of acceptance plans based on *attributes* (“go, not-go”) and *variables*.

The *lot tolerance percent defective* (LTPD) of a sampling plan is the level of quality routinely rejected by the *sampling plan*. It is defined as the percent defective (number of defectives per hundred units X.) that the sampling plan will reject 90% of the time.

We could make a confidence statement associated with the LTPD. For example, if the lot fails the sampling plan, we can state with 90% confidence that the quality level of the lot is worse than the LTPD (defective rate of lot > LTPD). If a lot passes the sampling plan, then we can state with 90% confidence that its quality level is $\geq$ (equal to or better) than the LTPD.

Herein is a table based on the MIL-S-19500 and MIL-M-38510 norms.

Max % → Defective	20%	15%	10%	7%	5%	3%	2%	1.5%	1%	0.7%	0.5%
Acceptance Number (c) rejects = c+1	Minimum Sample Size Needed										
0	11	15	22	32	45	76	116	153	231	328	461
1	18	25	38	55	77	129	195	258	390	555	778
2	25	34	52	75	105	176	266	354	533	759	1056
3	32	43	65	94	132	221	333	444	668	953	1337
4	38	52	78	113	158	265	398	531	798	1140	1599
5	45	60	91	131	184	308	462	617	927	1323	1855

Table A.18. LTPD sampling table based on the MIL-S-19500 and MIL-M-38510

In AQL (*acceptable quality level*), the confidence statement we could make is associated with the AQL. If the lot passes the sampling plan, we can state with 95% confidence that the quality level of the lot is *equal to or better* than the AQL (defective rate of the lot $<$ AQL). If the lot fails the sampling plan, we can state with 95% confidence that the quality level of the lot is worse than the AQL.

Herein is a table based on the *MIL-STD-105D* norm.

Acceptable Quality Levels for Normal Inspection (% defective)														
LS = Lot Size and SS = Sample Size	Lot Acceptance A_c number, Maximum number of rejects to accept the lot													
	.01	.015	.025	.04	.065	.10	.15	.25	.40	.65	1.0	1.5	2.5	4.0
LS = 2 $\rightarrow$ 8 SS = 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LS = 9 $\rightarrow$ 15 SS = 3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LS = 16 to 25 SS = 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LS = 26 $\rightarrow$ 50 SS = 8	0	0	0	0	0	0	0	0	0	0	0	0	1	1
LS = 51 to 90 SS = 13	0	0	0	0	0	0	0	0	0	0	0	0	1	1
91 $\rightarrow$ 150 SS = 20	0	0	0	0	0	0	0	0	0	0	0	0	1	2
151 $\rightarrow$ 280 SS = 32	0	0	0	0	0	0	0	0	0	0	0	1	2	3
281 $\rightarrow$ 500 SS = 50	0	0	0	0	0	0	0	0	0	0	1	2	3	4
501 $\rightarrow$ 1200 SS = 80	0	0	0	0	0	0	0	0	0	1	2	3	5	7
1201 $\rightarrow$ 3.2K SS = 125	0	0	0	0	0	0	0	0	1	2	3	5	7	10
3201 $\rightarrow$ 10K SS = 200	0	0	0	0	0	0	1	1	2	3	5	7	10	14
10001 $\rightarrow$ 35K SS = 315	0	0	0	0	0	0	1	2	3	5	7	10	14	21
35001 $\rightarrow$ 150K SS = 500	0	0	0	1	1	1	2	3	5	7	10	14	21	-
150001 $\rightarrow$ 500K SS = 800	0	0	0	1	1	2	3	5	7	10	14	21	21	-

Table A.19. AQL Sampling Table based on the *MIL-STD-105D*

The AQL tables used by this calculator from Website <http://www.aqlcalculator.com> is accurate. We do not always have to search coefficients with the possibility of a mistake.

General inspection level (I)

Lot Size (LS)	Code Letter	Sample Size	0.65	1.0	1.5	2.5	4.0	6.5	10	15
2→8	A	2	0	0	0	0	0	0	1	1
9→15	A	2	0	0	0	0	0	0	1	1
16→25	B	3	0	0	0	0	0	0	1	1
26→50	C	5	0	0	0	0	0	1	1	2
51→90	C	5	0	0	0	0	0	1	1	2
91→150	D	8	0	0	0	0	1	1	2	3
151→280	E	13	0	0	0	0	1	2	3	6
281→500	F	20	0	0	1	1	2	3	5	7
501→1200	G	32	1	1	1	2	3	5	7	10
1201→3200	H	50	1	1	2	3	5	7	10	14
3201→10000	J	80	1	2	3	5	7	10	14	21
10001→35000	K	125	2	3	5	7	10	14	21	21
35001→150000	L	200	3	5	7	10	14	21	21	21
150001→500000	M	315	5	7	10	14	21	21	21	21
500001 →	N	500	7	10	14	21	21	21	21	21

Table A.20. Inspection table (Level I) (ISO 2859)**General inspection level (II)**

Lot Size	Code Letter	Sample Size	0.65	1.0	1.5	2.5	4.0	6.5	10	15
2→8	A	20	0	0	0	0	0	0	1	1
9→15	B	20	0	0	0	0	0	0	1	1
16→25	C	20	0	0	0	0	0	1	1	2
26→50	D	20	0	0	0	0	1	1	2	3
51→90	E	20	0	0	0	0	1	2	3	6
91→150	F	20	0	0	1	1	2	3	5	7
151→280	G	20	1	1	1	2	3	5	7	10
281→500	H	20	1	1	2	3	5	7	10	14
501→1200	J	20	1	2	3	5	7	10	14	21
1201→3200	K	20	2	3	5	7	10	14	21	21
3201→10000	L	20	3	5	7	10	14	21	21	21
10001→35000	M	20	5	7	10	14	21	21	21	21
35001→150000	N	20	7	10	14	21	21	21	21	21
150001→500000	P	20	10	14	21	21	21	21	21	21
500001 →	Q	20	14	21	21	21	21	21	21	21

Table A.21. Inspection table (Level II) (ISO 2859)

General inspection level (III)

Lot Size	Code Letter	Sample Size	0.65	1.0	1.5	2.5	4.0	6.5	10	15
2→8	B	20	0	0	0	0	0	0	1	1
9→15	C	20	0	0	0	0	0	1	1	2
16→25	D	20	0	0	0	0	1	1	2	3
26→50	E	20	0	0	0	0	1	2	3	6
51→90	F	20	0	0	1	1	2	3	5	7
91→150	G	20	1	1	1	2	3	5	7	10
151→280	H	20	1	1	2	3	5	7	10	14
281→500	J	20	1	2	3	5	7	10	14	21
501→1200	K	20	2	3	5	7	10	14	21	21
1201→3200	L	20	3	5	7	10	14	21	21	21
3201→10000	M	20	5	7	10	14	21	21	21	21
10001→35000	N	20	7	10	14	21	21	21	21	21
35001→150000	P	20	10	14	21	21	21	21	21	21
150001→500000	Q	20	14	21	21	21	21	21	21	21
500001 →	R	20	21	21	21	21	21	21	21	21

Table A.22. *Inspection table (Level III) (ISO 2859)*

For product inspections, the AQL tables are statistical tools from buyers. They are used for for an objective measurement of quality. They present two key question such as:

1. How many samples should be inspected?
2. Where is the limit between acceptability and refusal, when it comes to defective products?

Before using the AQL tables, we should decide on three main parameters:

1. The “lot size, n ”, inspections should be carried out for each lot. If we ordered only one product (e.g. connecting rods), the lot size is the total order quantity.
2. The inspection level levels command different number of samples to be inspected. In our example, herein, we used the so-called “level II”, under “normal severity”.
3. If customers accept *very few defects* we might want to set a lower AQL (for both major and minor defects).

We basically use two main tables. The first one shows the “code letter” to use. Then, the code letter will give us the sample size (n) and the maximum numbers of defects that can be accepted (See for example table A.23)

Sampling size (n) and letters by code

Lot size (Number of ordered products)			I	II	III
			General inspection levels		
2	to	8	A	A	B
9	→	15	A	B	C
16	→	25	B	C	D
26	→	50	C	D	E
51	→	90	C	E	F
91	→	150	D	F	G
151	→	280	E	G	H
281	→	500	F	H	J
501	→	1.200	G	J	K
1.201	→	3.200	H	K	L
3.201	→	10.000	J	L	M
10.001	→	35.000	K	N	N
35.001	→	150.000	L	P	P
150.001	→	500.000	M	Q	Q
500.001		and over	N	Q	R

Table A.23. Sampling size (n) and code by letters

How to read the table: If n is in between 91 and 151 components and the level of inspection is “Level II” then we choose a letter “G”.

Sampling plan size for the level II (normal severity)

Code Letter	Sample size	1.0	1.5	2.5	4.0	6.5
A	2	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0
B	3	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0
C	5	≤ 0	≤ 0	≤ 0	≤ 0	≤ 1
D	8	≤ 0	≤ 0	≤ 0	≤ 1	≤ 1
E	13	≤ 0	≤ 0	≤ 1	≤ 1	≤ 2
F	20	≤ 0	≤ 1	≤ 1	≤ 2	≤ 3
G	32	≤ 1	≤ 1	≤ 2	≤ 3	≤ 5
H	50	≤ 1	≤ 2	≤ 3	≤ 5	≤ 7
J	80	≤ 2	≤ 3	≤ 5	≤ 7	≤ 10
K	125	≤ 3	≤ 5	≤ 7	≤ 10	≤ 14
L	200	≤ 5	≤ 7	≤ 10	≤ 14	≤ 21
M	315	≤ 7	≤ 10	≤ 14	≤ 21	≤ 21
N	500	≤ 10	≤ 14	≤ 21	≤ 21	≤ 21
P	800	≤ 14	≤ 21	≤ 21	≤ 21	≤ 21
Q	1.250	≤ 21	≤ 21	≤ 21	≤ 21	≤ 21
R	2.000	≤ 21	≤ 21	≤ 21	≤ 21	≤ 21

Table A.24. Sampling size and code by letter (Level II, normal severity)

How to read this table? For the code letter “G”, so we will have to draw 200 components randomly from the total lot size. Besides, we assume that we have set an AQL at 2.5% for major defects and 4.0% for minor defects.

The components are accepted if NO MORE than two major defects AND NO MORE than two minor defects are found. For example, if we find three major defects and two minor defects, the products are refused. If we find one major defect and two minor defects, they are accepted.

EXAMPLE.– AQL website (www.aqlcalculator.com)

Inspection Level	Level II	Different inspection levels will command different number of samples to inspect. Usually, we will stick to the so-called “level II” under “normal severity”
AQL Level	4.0	Number of ordered products
Lot Size	1000	Number of ordered products
Defective quantity	3	The quantity of products you found to be defective
	Result	Passed or Adapted
Message	Based on a Lot Size of 1000, inspection LEVEL II and AQL 4.0 you sampled 80pcs of your lot. A defective QTY of 3 means your lot has an AQL status of PASSED!	

Table A.25a. Calculation example 2 (Passed case)

EXAMPLE 2.– AQL website (www.aqlcalculator.com)

Inspection Level	Level II	Different inspection levels will command different number of samples to inspect. Usually, we will stick to the so-called “level II” under “normal severity”
AQL Level	4.0	Number of ordered products
Lot Size	1000	Number of ordered products
Defective quantity	3	The quantity of products you found to be defective
	Result	Refused or Failed
Message	Based on a Lot Size of 1000 , inspection LEVEL II and AQL 4.0 you sampled 80pcs of your lot. A defective QTY of 8 means your lot has an AQL status of Failed!	

Table A.25b. Calculation example 2 (Failed case)

Example of presentation of quality control chart (inspired by Ford Cie charts)

Machine: TAKUMI Checked by: Sarah G		Part: Stanley cup Date of check: 2012		Operation: Turning Laboratory: Quality control		Checked parameter: Ø _i Laboratory: Metrology	
Average μ							
Range R							
μ							
R ou σ							

Determination table of sampling size of a given population

P = Population size						S = Sampling size					
P	S	P	S	P	S	P	S	P	S	P	S
10	10	85	70	220	140	440	205	1200	291	4000	351
15	14	90	73	230	144	460	210	1300	297	4500	354
20	19	95	76	240	148	480	214	1400	302	5000	357
25	24	100	80	250	152	500	217	1500	306	6000	361
30	28	110	86	260	155	550	226	1600	310	7000	364
35	32	120	92	270	159	600	234	1700	313	8000	367
40	36	130	97	280	162	650	242	1800	317	9000	368
45	40	140	103	290	165	700	248	1900	320	10000	370
50	44	150	108	300	169	750	254	2000	322	15000	375
55	48	160	113	320	175	800	260	2200	327	20000	377
60	52	170	118	340	181	850	265	2400	331	30000	379
65	56	180	123	360	186	900	269	2600	335	40000	380
70	59	190	127	380	191	950	274	2800	338	50000	381
75	63	200	132	400	196	1000	278	3000	341	75000	382
80	66	210	136	420	201	1100	285	3500	346	100000	384

Table A.27. Tests of sampling size (Runs test)**Acceptation or rejection of the hypothesis**

If the value calculated by the trial (Runs) is $\leq$ to the tabulated value according to the size of n_1 and n_2 , we reject the null hypothesis																				
n_2/n_1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
2											2	2	2	2	2	2	2	2	2	
3					2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	
4				2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	
5			2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	
6		2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6	
7		2	2	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6	6	
8		2	3	3	3	4	4	5	5	6	6	6	6	6	7	7	7	7	7	
9		2	3	3	4	4	5	5	6	6	6	7	7	7	7	7	8	8	8	
10		2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	8	
11		2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9	
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10	
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10	
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11	
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12	
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12	
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13	
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13	
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13	
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14	

Table A.28. Acceptation or the refusal (rejection) of the null hypothesis

Conversion table of AQL values

AQL Limit	0.109	0.164	0.279	0.439	0.699	1.09	1.64	2.79	4.39	6.99	10.9
AQL Table	0.10	0.15	0.25	0.40	0.65	1.00	1.5	2.5	4.0	6.5	10

Table A.29. Conversion values of AQL**Table of code by letters for sampling size (n)**

Lot size	Attribute Plan MIL STD 105 E								Measure plan Z 1.9							
n	S ₁	S ₂	S ₃	S ₄	I	II	n	III	S ₃	S ₄	I	II	n	III		
2 to 8	A	A	A	A	A	A	2	B	B	B	B	B	3	C		
9 to 15	A	A	A	A	A	B	3	C	B	B	B	B	3	D		
16 to 25	A	A	B	B	B	C	5	D	B	B	B	C	4	E		
26 to 50	A	B	B	C	C	D	8	E	B	B	C	D	5	F		
51 to 90	B	B	C	D	D	E	13	F	B	B	D	E	7	G		
91 to 150	B	C	C	E	E	F	20	G	B	C	E	F	10	H		
151 to 280	B	C	D	E	E	G	32	H	B	D	F	G	15	I		
181 to 400	B	C	D	E	E	G	32	H	C	E	G	H	20	J		
401 to 500	B	C	D	E	F	H	50	J	C	E	G	I	25	J		
501 to 1200	C	C	E	F	G	J	80	K	D	F	H	J	35	K		
1201 to 3200	C	D	E	G	H	K	125	L	E	G	I	K	50	L		
3201 to 10 000	C	D	F	G	J	L	200	M	F	H	J	L	75	M		
10 001 to 3500	C	D	F	H	K	M	315	N	G	I	K	M	100	N		
3501 to 150 000	D	E	G	J	L	N	500	P	H	J	L	N	150	P		
150 001 to 500 000	D	E	G	J	M	P	800	Q	H	K	M	P	200	P		
500 001 and more	D	E	H	K	N	Q	1250	R	H	K	N	P	200	P		
								In normal inspection								

Table A.30. Letters-codes table: Attribute plan MIL STD 105E and measure plan Z 1.9

↓	Use first sampling plan below arrow ↓ If sample size equals or exceeds, lot or batch size, do 100% inspection		
↑	Use first sampling plan above arrow ↑		
A _c →	Acceptance number	R _e →	Rejection number

More information is available at: <http://www.itl.nist.gov/div898/handbook/too-laid/pff/6-pmc.pdf>.

Normal inspection: AQL = accepted quality level

TABLE B		Single Sampling Plans for Normal Inspection											
Sample Size Code Letter	Sample Size	Acceptable Quality Levels for Normal Inspection											
		.065 Ac Re	.1 Ac Re	.15 Ac Re	.25 Ac Re	.4 Ac Re	.65 Ac Re	1.0 Ac Re	1.5 Ac Re	2.5 Ac Re	4.0 Ac Re	6.5 Ac Re	
A	2												
B	3												
C	5												
D	8												
E	13												
F	20												
G	32												
H	50												
J	80												
K	125												
L	200												
M	315												
N	500												
P	800												
Q	1250												
R	2000												
↕													

Table A.31. Main table normal inspection - simple sampling (Source : MIL STD 105E, Table II-A)

Dodge and Romig Typical table for a simple sampling plan (LTPD =1%).

% Process Average → Lot size ↴	0-0.010			0.011-0.10			0.11-0.20			0.2-0.30			0.31-0.40			0.41-0.50		
	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %
1-120	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00
121-150	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06
151-200	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08
201-300	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10
301-400	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12
401-500	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13
501-600	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	305	1	0.13
601-800	200	0	0.13	200	0	0.14	200	0	0.14	330	1	0.15	330	1	0.15	330	1	0.14
801-1000	205	0	0.14	205	0	0.14	205	0	0.14	335	1	0.17	335	1	0.17	335	1	0.15
1001-2000	220	0	0.14	220	0	0.14	360	1	0.19	490	2	0.21	490	2	0.21	610	3	0.17
2001-3000	220	0	0.15	275	1	0.15	505	2	0.23	630	3	0.24	745	4	0.26	870	5	0.22
3001-4000	225	0	0.15	380	1	0.20	510	2	0.24	645	3	0.25	880	5	0.28	1000	6	0.26
4001-5000	225	0	0.15	380	1	0.20	520	2	0.24	770	4	0.28	895	5	0.29	1120	7	0.29
5001-7000	230	0	0.16	385	1	0.21	655	3	0.27	780	4	0.29	1020	6	0.32	1260	8	0.31
7001-10000	230	0	0.16	520	2	0.25	660	3	0.28	910	5	0.32	1150	7	0.34	1500	10	0.34
10001-20000	390	1	0.16	525	2	0.26	785	4	0.31	1040	6	0.35	1400	9	0.39	1980	14	0.43
20001-50000	390	1	0.21	530	2	0.26	920	5	0.34	1300	8	0.39	1890	3	0.44	2570	19	0.48
50001-100000	390	1	0.21	670	3	0.29	1040	6	0.36	1420	9	0.41	2120	15	0.47	3150	23	0.50

Table A.32. Dodge-Romig table based on stated value of lot tolerance per cent defective ($LTPD = 1.0\%$) and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaids/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (LTPD = 2%).

% Process Average → Lot size ↓	0-0.02			0.003-0.20			0.21-0.40			0.41-0.60			0.61-0.80			0.81-1.00		
	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %
1-75	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00
76-100	70	0	0.16	70	0	0.16	70	0	0.16	70	0	0.16	70	0	0.16	70	0	0.16
101-200	85	0	0.25	85	0	0.16	85	0	0.25	85	0	0.25	85	0	0.25	85	0	0.25
201-300	95	0	0.26	95	0	0.25	95	0	0.26	95	0	0.26	95	0	0.26	95	0	0.26
301-400	100	0	0.28	100	0	0.26	100	0	0.32	160	1	0.32	160	1	0.32	160	1	0.32
401-500	105	0	0.28	105	0	0.28	105	0	0.34	165	1	0.34	165	1	0.34	165	1	0.34
501-600	105	0	0.29	105	0	0.29	175	1	0.34	175	1	0.34	175	1	0.34	235	2	0.36
601-800	110	0	0.29	110	0	0.29	180	1	0.40	240	2	0.40	240	2	0.40	300	3	0.41
801-1000	115	0	0.28	115	0	0.28	185	1	0.42	245	3	0.42	305	3	0.44	305	3	0.44
1001-2000	115	0	0.30	190	1	0.40	255	2	0.50	325	4	0.50	380	4	0.54	440	5	0.56
2001-3000	115	0	0.31	190	1	0.41	260	2	0.58	385	5	0.58	450	5	0.60	565	7	0.64
3001-4000	115	0	0.31	195	1	0.41	330	3	0.63	450	6	0.63	510	6	0.65	690	9	0.70
4001-5000	195	1	0.41	260	2	0.50	335	3	0.63	455	7	0.63	575	7	0.69	750	10	0.74
5001-7000	195	1	0.42	265	2	0.50	335	3	0.69	515	8	0.69	640	8	0.73	870	12	0.80
7001-10000	195	1	0.42	265	2	0.50	395	4	0.69	520	10	0.69	760	10	0.79	1050	15	0.86
10001-20000	200	1	0.42	265	2	0.51	460	5	0.77	650	12	0.77	885	12	0.86	1230	18	0.94
20001-50000	200	1	0.42	335	3	0.58	520	6	0.81	710	15	0.81	1060	15	0.93	1520	23	1.00
50001-100000	200	1	0.42	335	3	0.58	585	7	0.76	770	10	0.84	1180	17	0.97	1690	26	1.10

Table A.33. Dodge-Romig table based on stated value of lot tolerance per cent defective (LTPD) = 2.0 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaims/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (LTPD = 3%).

% Process Average → Lot size ↓	0-0.03			0.04-0.30			0.31-0.60			0.61-0.90			0.91-1.20			1.21-1.50		
	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %	<i>n</i>	<i>c</i>	AOQL %
1-40	All	0	0	All	0	0.08	All	0	0	All	0	0	All	0	0	All	0	0
41-55	40	0	0.18	40	0	0.18	40	0	0.18	40	0	0.18	40	0	0.18	40	0	0.18
56-100	55	0	0.30	55	0	0.30	55	0	0.30	55	0	0.30	55	0	0.30	55	0	0.30
101-200	65	0	0.38	65	0	0.38	65	0	0.38	65	0	0.38	65	0	0.38	65	0	0.38
201-300	70	0	0.40	70	0	0.40	70	0	0.40	110	1	0.48	110	1	0.48	110	1	0.48
301-400	70	0	0.43	70	0	0.43	115	1	0.52	115	1	0.52	115	1	0.52	115	2	0.54
401-500	70	0	0.43	70	0	0.45	120	1	0.53	120	1	0.53	160	2	0.58	160	2	0.58
501-600	75	75	0.43	75	0	0.45	120	1	0.56	160	2	0.63	160	2	0.63	200	3	0.65
601-800	75	75	0.47	125	1	0.57	125	1	0.57	165	2	0.66	205	CO	0.71	240	4	0.74
801-1	75	75	0.48	125	1	0.59	170	2	0.67	210	3	0.73	250	4	0.76	290	5	0.78
1,001-2	75	75	0.63	130	1	0.60	175	2	0.72	260	4	0.85	300	5	0.90	380	7	0.95
2,001-3	75	75	0.63	130	1	0.62	220	CO	0.82	300	5	0.95	385	7	1.0	460	9	1.10
3,001-4	130	130	0.63	175	2	0.75	220	3	0.84	305	5	0.96	425	8	1.0	540	11	1.20
4,001-5	130	130	0.64	175	2	0.76	260	4	0.91	345	6	1.0	465	9	1.1	620	13	1.20
5,001-7	130	130	0.64	175	2	0.76	265	4	0.92	390	7	1.1	505	10	1.2	700	15	1.30
7,001-10	130	130	0.64	175	2	0.77	265	4	0.93	390	7	1.1	550	11	1.2	775	17	1.40
10,001-20	130	130	0.64	175	2	0.78	305	5	1.0	430	8	1.2	630	13	1.3	900	20	1.50
20,001-50	130	130	0.65	225	3	0.86	350	6	1.1	520	10	1.2	750	16	1.4	1,090	25	1.60
50,001-100	130	130	0.65	265	2	0.96	390	7	1.1	590	12	1.3	830	18	1.5	1,215	28	1.60

Table A.34. Dodge-Romig table based on stated value of lot tolerance per cent defective (*LTPD*) = 1.0 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaims/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (LTPD = 5%).

Process → Average % Lot size ↓	0-0.05			0.06-0.50			0.51-1.00			1.01-1.50			1.51-2.00			2.01-29.50		
	n	C	AOQL %	n	C	AOQL %	n	C	AOQL %	n	C	AOQL %	n	C	AOQL %	n	C	AOQL %
1-30	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00	All	0	0.00
31-50	30	0	0.49	30	0	0.49	30	0	0.49	30	0	0.49	30	0	0.49	30	0	0.49
51-100	37	0	0.63	37	0	0.63	37	0	0.63	37	0	0.63	37	0	0.63	37	0	0.63
101-200	40	0	0.74	40	0	0.74	40	0	0.74	40	0	0.74	40	0	0.74	40	0	0.74
201-300	43	0	0.74	43	0	0.74	70	1	0.92	70	2	0.92	95	2	0.99	95	2	0.99
301-400	44	0	0.74	44	0	0.74	70	1	0.99	100	3	1.00	120	3	1.00	145	4	1.10
401-500	45	0	0.75	75	1	0.95	100	2	1.10	100	3	1.10	125	3	1.20	150	4	1.20
501-600	45	0	0.76	75	1	0.98	100	2	1.10	125	4	1.20	150	4	1.30	175	5	1.30
601-800	45	0	0.77	75	1	1.00	100	2	1.20	130	5	1.20	175	5	1.40	200	6	1.40
801-1000	45	0	0.78	75	1	1.00	105	2	1.20	155	5	1.40	180	5	1.40	225	7	1.50
1001-2000	45	0	0.80	75	1	1.00	130	3	1.40	180	7	1.40	230	7	1.70	280	9	1.80
2001-3000	75	1	1.10	105	2	1.30	135	3	1.40	210	9	1.60	280	9	1.90	370	13	2.10
3001-4000	75	1	1.10	105	2	1.30	160	4	1.50	210	10	1.70	305	10	2.00	420	15	2.20
4001-5000	75	1	1.10	105	2	1.30	160	4	1.50	235	11	1.80	330	11	2.00	440	16	2.20
5001-7000	75	1	1.10	105	2	1.30	185	5	1.70	260	12	1.90	350	12	2.20	490	18	2.40
7001-10000	75	1	1.10	105	2	1.30	185	5	1.70	260	13	1.90	380	13	2.20	535	20	2.50
10001-20000	75	1	1.10	135	3	1.40	210	6	1.80	285	15	2.00	425	15	2.30	610	23	2.60
20001-50000	75	1	1.10	135	3	1.40	235	7	1.90	305	17	2.10	470	17	2.40	700	27	2.70
50001-100000	75	1	1.10	160	4	1.60	235	7	1.90	355	12	2.20	515	19	2.50	770	30	2.80

Table A.35. Dodge-Romig table based on stated value of lot tolerance per cent defective (LTPD) = 5.0 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaims/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (LTPD = 1%).

% Process Average → Lot size ↓	0-0.010					0.011-0.11					0.11-0.20				
	Trial 1		Trial 2			Trial 1		Trial 2			Trial 1		Trial 2		
	n_1	C_1	n_2	n_1+n_2	AOQL %	n_1	C_1	n_2	n_1+n_2	AOQL %	n_1	C_1	n_2	n_1+n_2	AOQL %
1-120	All	0	-	-	0.00	All	0	-	-	0.00	All	0	-	-	0.00
121-150	120	0	-	-	0.06	120	0	-	-	0.06	120	0	-	-	0.06
151-200	140	0	-	-	0.08	140	0	-	-	0.08	140	0	-	-	0.08
201-260	165	0	-	-	0.10	165	0	-	-	0.10	165	0	-	-	0.10
261-300	180	0	75	255	1	180	0	75	255	1	180	0	75	255	1
301-400	200	0	90	290	1	200	0	90	290	1	200	0	90	290	1
401-500	215	0	100	315	1	215	0	100	315	1	215	0	100	315	1
501-600	225	0	115	340	1	225	0	115	340	1	225	0	115	340	1
601-800	235	0	125	360	1	235	0	125	360	1	235	0	125	360	1
801-1000	245	0	135	380	1	245	0	135	380	1	245	0	135	380	1
1001-2000	265	0	155	420	1	265	0	155	420	2	265	0	285	550	2
2001-3000	270	0	160	430	1	270	0	300	570	2	270	0	420	690	3
3001-4000	275	0	160	435	1	275	0	305	580	2	275	0	435	710	3
4001-5000	275	0	165	440	1	275	0	310	585	2	275	0	565	840	4
5001-7000	275	0	170	445	1	275	0	315	590	2	275	0	580	855	4
7001-10000	280	0	320	600	1	280	0	460	740	3	280	0	590	870	4
10001-20000	280	0	325	605	1	280	0	465	745	3	280	0	595	875	4
20001-50000	280	0	325	605	1	280	0	605	885	4	280	0	830	1280	7
50001-100000	280	0	325	605	1	280	0	605	885	4	280	0	830	1280	7

Table A.36. Dodge-Romig table based on stated value of lot tolerance per cent defective (LTPD) = 1.0 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaims/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (LTPD = 1%).

% Process Average → Lot size ↓	0-21.30					0.31-0.40					0.41-0.50							
	Trial 1		Trial 2			AOQL %	Trial 1		Trial 2			AOQL %	Trial 1		Trial 2			
	n ₁	C ₁	n ₂	n ₁ +n ₂	C ₂		n ₁	C ₁	n ₂	n ₁ +n ₂	C ₂		n ₁	C ₁	n ₂	n ₁ +n ₂	C ₂	
1-120	All	0	-	-	-	0.00	All	0	-	-	-	0.00	All	0	-	-	-	0.00
121-150		120	0	-	-	0.06	120	0	-	-	-	0.06	120	0	-	-	-	0.06
151-200		140	0	-	-	0.08	140	0	-	-	-	0.08	140	0	-	-	-	0.08
201-260		165	0	-	-	0.10	165	0	-	-	-	0.10	165	0	-	-	-	0.10
261-300		180	0	75	255	1	0.10	180	0	75	255	1	0.10	180	0	75	255	1
301-400		200	0	90	290	1	0.12	200	0	90	290	1	0.12	200	0	90	290	1
401-500		215	0	100	315	1	0.14	215	0	100	315	1	0.14	215	0	100	315	1
501-600		225	0	115	340	1	0.15	225	0	115	340	1	0.15	225	0	205	430	2
601-800		235	0	230	465	2	0.18	235	0	230	465	2	0.18	235	0	230	465	2
801-1000		245	0	250	495	2	0.19	245	0	250	495	2	0.19	245	0	250	495	2
1001-2000		265	0	405	670	3	0.23	265	0	515	780	4	0.24	265	0	515	780	4
2001-3000		270	0	545	815	4	0.26	430	1	620	1050	6	0.28	430	1	830	1260	8
3001-4000		435	1	645	1080	6	0.29	435	1	865	1300	8	0.30	580	2	940	1520	10
4001-5000		440	1	660	1100	6	0.30	440	1	1000	1440	9	0.33	585	2	1075	1660	11
5001-7000		445	1	785	1230	7	0.33	590	2	990	1580	10	0.36	730	3	1190	1920	13
7001-10000		450	1	920	1370	8	0.35	600	2	1240	1840	12	0.39	870	4	1540	2110	17
10001-20000		605	2	1035	1640	10	0.39	745	3	1485	2230	15	0.43	1150	6	1990	3140	23
20001-50000		605	2	1295	1900	12	0.42	885	4	1845	2730	19	0.47	1280	7	2600	3880	29
50001-100000		605	2	1545	2150	14	0.44	885	4	2085	2970	21	0.49	1410	8	3280	4690	36
																		0.55

Table A.37. Dodge-Romig table based on stated value of lot tolerance per cent defective (LTPD) = 1.0 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaims/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (LTPD = 2%).

% Process Average → Lot size ↴	0.00–0.04			0.05–0.40			0.41–0.80			0.81–1.20			1.21–1.60			1.61–2.00		
	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %
1–15	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-
16–50	14	0	13.6	14	0	13.6	14	0	13.6	14	0	13.6	14	0	13.6	14	0	13.6
51–100	16	0	12.4	16	0	12.4	16	0	12.4	16	0	12.4	16	0	12.4	16	0	12.4
101–200	17	0	12.2	17	0	12.2	17	0	12.2	17	0	12.2	16	1	10.5	35	1	10.5
201–300	17	0	12.3	17	0	12.3	17	0	12.3	37	1	10.2	35	1	10.2	37	1	10.2
301–400	18	0	11.8	18	0	11.8	38	1	10.0	38	1	10.0	37	1	10.0	60	2	8.5
401–500	18	0	11.9	18	0	11.9	39	1	9.8	39	1	9.8	38	2	8.6	60	2	8.6
501–600	18	0	11.9	18	0	11.9	39	1	9.8	39	1	9.8	60	2	8.6	60	2	8.6
6001–800	18	0	11.9	40	1	9.6	40	1	9.6	65	2	8.0	65	2	8.0	85	3	7.5
801–1000	18	0	12.0	40	1	9.6	40	1	9.6	65	2	8.1	65	2	8.1	90	3	7.4
1001–2000	18	0	12.0	41	1	9.4	65	2	8.2	65	2	8.2	95	3	7.0	120	4	6.5
2001–3000	18	0	12.0	41	1	9.4	65	2	8.2	95	3	7.0	120	4	6.5	180	6	5.8
3001–4000	18	0	12.0	42	1	9.3	65	2	8.2	95	3	7.0	155	4	6.0	210	7	5.5
4001–5000	18	0	12.0	42	1	9.3	70	2	7.5	125	4	6.4	155	5	6.0	245	8	5.3
5001–7000	18	0	12.0	42	1	9.3	95	3	7.0	125	4	6.4	185	6	5.6	280	9	5.1
7001–10000	42	1	9.3	70	2	7.5	95	3	7.0	155	5	6.0	220	7	5.4	350	11	4.8
10001–20000	42	1	9.3	70	2	7.6	95	3	7.0	190	6	5.6	290	9	4.9	460	14	4.4
20001–50000	42	1	9.3	70	2	7.6	125	4	6.4	220	7	5.4	395	12	4.5	720	21	3.9
50001–100000	42	1	9.3	95	3	7.0	160	5	5.9	290	9	4.9	505	15	4.2	955	27	3.7

Table A.38. Dodge-Romig table based on stated value of lot tolerance per cent defective (LTPD) = 2.0 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaims/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (LTPD = 2.5%).

% Process Average → Lot size $\downarrow$	0.00–0.05			0.06–0.50			0.51–1.0			1.01–1.50			1.51–2.00			2.01–2.50		
	n	c	LQL %	n	c	LQL %	n	c	LQL %	n	c	LQL %	n	c	LQL %	n	c	LQL %
1–10	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-
11–50	11	0	17.6	11	0	17.6	11	0	17.6	11	0	17.6	11	0	17.6	11	0	17.6
51–100	13	0	15.3	13	0	15.3	13	0	15.3	13	0	15.3	13	0	15.3	13	0	15.3
101–200	14	0	14.7	14	0	14.7	29	0	12.9	29	1	12.9	29	1	12.9	29	1	12.9
201–300	14	0	14.9	14	0	14.9	30	1	12.7	30	1	12.7	30	1	12.7	30	1	12.7
301–400	14	0	15.0	14	0	15.0	31	1	12.3	31	1	12.3	31	1	12.3	48	2	10.7
401–500	14	0	15.0	14	0	15.0	32	1	12.0	32	1	12.0	49	2	10.6	49	2	10.6
501–600	14	0	15.1	32	1	12.0	32	1	12.0	50	2	10.4	50	2	10.4	70	3	9.3
6001–800	14	0	15.1	32	1	12.0	32	1	12.0	50	2	10.5	50	2	10.5	70	3	9.4
801–1000	15	0	14.2	33	1	11.7	33	1	11.7	50	2	10.6	70	3	9.4	90	4	8.5
1001–2000	15	0	14.2	33	1	11.7	55	2	9.3	75	3	8.8	95	4	8.0	120	5	7.6
2001–3000	15	0	14.2	33	1	11.8	55	2	9.4	75	3	8.8	120	5	7.6	145	6	7.2
3001–4000	15	0	14.3	33	1	11.8	55	2	9.5	100	4	7.9	125	5	7.4	195	8	6.6
4001–5000	15	0	14.3	33	1	11.8	75	3	8.9	100	4	7.9	150	6	7.0	225	9	6.3
5001–7000	33	1	11.8	55	2	9.7	75	3	8.9	125	5	7.4	175	7	6.7	250	10	6.1
7001–10000	34	1	11.4	55	2	9.7	75	3	8.9	125	5	7.4	200	8	6.4	310	12	5.8
10001–20000	34	1	11.4	55	2	9.7	100	4	8.0	150	6	7.0	260	10	6.0	425	16	5.3
20001–50000	34	1	11.4	55	2	9.7	100	4	8.0	180	7	6.7	345	13	5.55	640	23	4.8
50001–100000	31	1	11.4	80	3	8.4	125	5	7.4	235	9	6.1	435	16	5.2	800	28	4.5

Table A.39. Dodge-romig table based on stated value of lot tolerance per cent defective (LTPD) = 2.5 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. See Weblink: <http://www.itl.nist.gov/div898/handbook/toolaims/pff/6-pmc.pdf>.

Dodge and Romig typical table for a simple sampling plan (AOQL = 3%).

% Process Average → Lot size ↓	0.00–0.06			0.07–0.60			0.61–1.20			1.21–1.80			1.81–2.40			2.41–3.00		
	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %	<i>n</i>	<i>c</i>	LQL %
1–10	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-
11–50	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.4
51–100	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	22	0	16.4
101–200	12	0	17.0	12	0	17.0	12	0	17.0	25	1	15.1	25	1	15.1	25	1	15.1
201–300	12	0	17.0	12	0	17.0	26	1	14.6	26	1	14.6	26	1	14.6	40	1	12.8
301–400	12	0	17.1	12	0	17.1	26	1	14.7	26	1	14.7	41	2	12.7	41	2	12.7
401–500	12	0	17.2	27	1	14.1	27	1	14.1	42	2	12.4	42	2	12.4	42	2	12.4
501–600	12	0	17.3	27	1	14.2	27	1	14.2	42	2	12.4	42	2	12.4	60	3	10.8
6001–800	12	0	17.3	27	1	14.2	27	1	14.2	43	2	12.1	60	3	10.9	60	3	10.9
801–1000	12	0	17.4	27	1	14.2	44	2	11.8	44	2	11.8	60	3	11.0	80	4	9.8
1001–2000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	80	4	9.8	100	5	9.1
2001–3000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	100	5	9.1	140	7	8.2
3001–4000	12	0	17.5	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	165	8	7.8
4001–5000	28	0	13.8	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	210	10	7.4
5001–7000	28	1	13.8	45	2	11.8	65	3	10.3	105	5	8.8	145	7	8.1	235	11	7.1
7001–10000	28	1	13.9	46	2	11.6	65	3	10.3	105	5	8.8	170	8	7.6	280	13	6.8
10001–20000	28	1	13.9	46	2	11.7	85	4	9.5	125	6	8.4	215	10	7.2	380	17	6.2
20001–50000	28	1	13.9	65	3	10.3	105	5	8.8	170	8	7.6	310	14	6.5	560	24	5.7
50001–100000	28	1	13.9	65	3	10.3	125	6	8.4	215	10	7.2	385	17	6.2	690	29	5.4

Table A.40. Dodge-Romig table based on stated value of AOQL of 3% and consumer's risk = 0.10

AOQL: average outgoing quality limit. – LQL: Limiting quality Level corresponding to a consumer's risk. $\beta = 0.10$.

Dodge and Romig typical table for a simple sampling plan (AOQL) = 1.0%.

% Process Average → Lot size ↓	0.00–0.2						0.03–0.20						0.21–0.40					
	Trial 1			Trial 2			Trial 1			Trial 2			Trial 1			Trial 2		
	n_1	C_1	n_2	n_1+n_2	C_2	AOQL %	n_1	C_1	n_2	n_1+n_2	C_2	AOQL %	n_1	C_1	n_2	n_1+n_2	C_2	AOQL %
1–25	All	0	–	–	–	–	All	0	–	–	–	–	All	0	–	–	–	–
26–50	22	0	–	–	–	7.7	22	0	–	–	–	–	22	0	–	–	–	7.7
51–100	33	0	17	50	1	6.9	33	0	17	50	1	7.7	33	0	17	50	1	6.9
101–200	43	0	22	65	1	5.8	43	0	22	65	1	6.9	43	0	22	65	1	5.8
201–300	47	0	28	75	1	5.5	47	0	28	75	1	5.8	47	0	28	75	1	5.5
301–400	50	0	31	80	1	5.4	49	0	31	80	1	5.5	55	0	60	115	2	4.7
401–500	50	0	30	80	1	5.4	50	0	30	80	1	5.4	55	0	65	120	2	4.6
501–600	50	0	30	80	1	5.4	50	0	30	80	1	5.4	60	0	65	125	2	4.5
601–800	55	0	35	85	1	5.3	60	0	70	130	2	5.4	60	0	70	130	2	4.6
801–1000	55	0	30	85	1	5.2	60	0	75	135	2	4.5	60	0	75	135	2	4.4
1001–2000	65	0	35	90	1	5.1	65	0	75	140	2	4.4	75	0	120	195	3	3.8
2001–3000	65	0	80	145	2	4.2	65	0	80	145	2	4.3	75	0	125	200	3	3.7
3001–4000	70	0	80	150	2	4.1	70	0	80	150	2	4.2	80	0	175	255	4	3.5
4001–5000	70	0	80	150	2	4.1	70	0	80	150	2	4.1	80	0	180	260	4	3.4
5001–7000	70	0	80	150	2	4.1	75	0	125	200	3	4.1	80	0	180	260	4	3.4
7001–10000	70	0	80	150	2	4.1	80	0	125	205	3	3.7	85	0	180	265	4	3.3
10001–20000	70	0	80	150	2	4.1	80	0	130	210	3	3.6	90	0	230	320	5	3.2
20001–50000	75	0	80	155	2	4.0	80	0	135	2145	3	3.6	95	0	300	395	6	2.9
50001–100000	75	0	80	155	2	4.0	85	0	180	265	4	3.3	170	1	380	550	8	2.6

Table A.41. Dodge-Romig table based on stated value of $AOQL = 1.0\%$ and consumer's risk = 0.10

AOQL: average outgoing quality limit. – LQL: Limiting quality Level corresponding to a consumer's risk. $\beta = 0.10$.

Dodge and Romig typical table for a simple sampling plan (AOQL) = 1.0%, suite.

% Process Average → Lot size ↓	0.41–0.60						0.61–0.80						0.81–1.00							
	Trial 1			Trial 2			AOQL %	Trial 1			Trial 2			AOQL %	Trial 1			Trial 2		
	n_1	C_1	n_2	n_1+n_2	C_2	n_1		C_1	n_2	n_1+n_2	C_2	n_1	C_1		n_2	n_1+n_2	C_2			
1–25	All	0	-	-	-	-	-	All	0	-	-	-	-	-	-	All	0	-	-	-
26–50	22	0	-	-	-	7.7	7.7	22	0	-	-	-	7.7	22	0	-	-	-	-	7.7
51–100	33	0	17	50	1	6.9	6.9	33	0	17	50	1	6.9	33	0	17	50	1	6.9	6.9
101–200	43	0	22	65	1	5.8	5.8	43	0	22	65	1	5.8	47	0	43	90	2	5.4	5.4
201–300	55	0	50	105	2	4.9	4.9	55	0	50	105	2	4.9	55	0	50	105	2	4.9	4.9
301–400	55	0	60	115	2	4.8	4.8	55	0	60	115	2	4.8	60	0	80	140	3	4.5	4.5
401–500	55	0	65	120	2	4.7	4.7	60	0	95	155	3	4.3	60	0	95	155	3	4.3	4.3
501–600	60	0	65	125	2	4.6	4.6	65	0	100	165	3	4.2	65	0	100	165	3	4.2	4.2
601–800	65	0	105	170	3	4.1	4.1	65	0	105	170	3	4.1	70	0	140	210	4	3.9	3.9
801–1000	65	0	110	175	3	4.0	4.0	70	0	150	220	4	3.8	125	1	180	305	6	3.5	3.5
1001–2000	80	0	165	245	4	3.7	3.7	135	1	200	335	6	3.3	140	1	245	385	7	3.2	3.2
2001–3000	80	0	170	250	4	3.6	3.6	150	1	265	415	7	3.0	215	2	355	570	10	2.8	2.8
3001–4000	85	0	220	305	5	3.3	3.3	160	1	330	490	8	2.8	225	2	455	680	12	2.7	2.7
4001–5000	145	1	225	370	6	3.1	3.1	225	2	375	600	10	2.7	240	2	595	835	14	2.5	2.5
5001–7000	155	1	285	440	7	2.9	2.9	235	2	440	675	11	2.6	310	3	665	975	16	2.4	2.4
7001–10000	165	1	355	520	8	2.7	2.7	250	2	585	835	13	2.4	385	4	785	1170	19	2.3	2.3
10001–20000	175	1	415	590	9	2.6	2.6	325	3	655	980	15	2.3	520	6	980	1500	24	2.2	2.2
20001–50000	250	2	490	740	11	2.4	2.4	340	3	910	1250	19	2.2	610	7	1410	2020	32	2.1	2.1
50001–100000	275	2	700	975	14	2.2	2.2	420	4	1050	1470	29	2.1	770	9	1850	2620	41	2.0	2.0

Table A.42. Dodge-Romig table based on stated value of AOQL = 1.0 % and consumer's risk = 0.10

AOQL: average outgoing quality limit. – LQL: Limiting quality Level corresponding to a consumer's risk. $\beta = 0.10$.

Dodge and Romig typical table for a simple sampling plan (AOQL = 3.0%).

% Process Average → Lot size ↓	0.00–0.06						0.07–0.60						0.61–1.20					
	Trial 1			Trial 2			Trial 1			Trial 2			Trial 1			Trial 2		
	n_1	C_1	n_2	n_1+n_2	C_2	AOQL %	n_1	C_1	n_2	n_1+n_2	C_2	AOQL %	n_1	C_1	n_2	n_1+n_2	C_2	AOQL %
1–10	All	0	-	-	-	-	All	0	-	-	-	-	All	0	-	-	-	-
11–50	10	0	-	-	-	19.0	10	0	-	-	-	19.0	10	0	-	-	-	19.0
51–100	16	0	9	25	1	16.4	16	0	9	25	1	16.4	16	0	9	25	1	16.4
101–200	17	0	9	26	1	15.5	17	0	9	26	1	16.0	17	0	9	26	1	16.0
201–300	18	0	10	28	1	15.2	18	0	10	28	1	15.5	21	0	23	44	2	13.3
301–400	18	0	11	29	1	15.2	21	0	24	45	2	13.2	23	0	37	60	3	12.0
401–500	18	0	11	29	1	15.0	21	0	25	46	2	13.0	24	0	36	60	3	11.7
501–600	18	0	12	30	1	13.0	21	0	25	46	2	13.0	24	0	41	65	3	11.5
601–800	21	0	25	46	2	12.8	21	0	25	46	2	12.8	24	0	41	65	3	11.5
801–1000	21	0	26	47	2	12.6	21	0	26	47	2	12.6	25	0	40	65	3	11.4
1001–2000	22	0	26	48	2	12.6	22	0	26	48	2	11.4	27	0	58	65	4	10.3
2001–3000	22	0	26	48	2	12.4	25	0	40	65	3	11.0	28	0	62	85	4	10.0
3001–4000	23	0	26	49	2	12.2	25	0	45	70	3	11.0	29	0	76	90	5	9.6
4001–5000	23	0	26	49	2	12.2	26	0	44	70	3	11.0	30	0	75	105	5	9.5
5001–7000	23	0	27	50	2	12.2	26	0	44	70	3	11.0	30	0	80	105	5	9.4
7001–10000	23	0	27	50	2	12.2	27	0	43	70	3	11.0	30	0	80	110	5	9.4
10001–20000	23	0	27	50	2	12.2	27	0	43	70	3	11.0	31	0	94	110	6	9.2
20001–50000	23	0	27	50	2	12.2	28	0	67	95	4	9.7	50	1	120	175	8	8.0
50001–100000	23	0	27	50	2	12.2	31	0	84	115	5	9.0	60	1	140	200	9	7.6

Table A.43. Dodge-Romig table based on stated value of AOQL = 3% corresponding to a consumer's risk $\beta = 0.10$

AOQL: average outgoing quality limit. – LQL: Limiting quality Level corresponding to a consumer's risk. $\beta = 0.10$.

Dodge and Romig typical table for a simple sampling plan (AOQL = 3.0%) suite.

% Process Average → Lot size ↓	1.21–1.80						1.81–2.40						2.41–3.00					
	Trial 1			Trial 2			Trial 1			Trial 2			Trial 1			Trial 2		
	n_i	C_1	n_2	n_1+n_2	C_2	AOQL %	n_i	C_1	n_2	n_1+n_2	C_2	AOQL %	n_i	C_1	n_2	n_1+n_2	C_2	AOQL %
1–10	All	0	-	-	-	-	All	0	-	-	-	-	All	0	-	-	-	-
11–50	10	0	-	-	-	19.0	10	0	-	-	-	19.0	10	0	-	-	-	19.0
51–100	17	0	17	34	2	15.8	17	0	17	34	2	15.8	17	0	17	34	2	15.8
101–200	20	0	21	41	2	13.7	22	0	33	55	3	12.4	22	0	33	55	3	12.4
201–300	23	0	37	60	3	12.0	23	0	37	60	3	12.0	24	0	51	75	4	11.1
301–400	23	0	37	60	3	12.0	25	0	55	80	4	10.8	42	1	63	105	6	10.4
401–500	24	0	36	60	3	11.7	25	0	55	80	4	10.8	46	1	79	125	7	9.7
501–600	26	0	54	80	4	10.7	46	1	69	115	6	9.7	48	1	97	145	8	9.2
601–800	26	0	54	80	4	10.7	49	1	81	130	7	9.4	50	1	115	165	9	8.9
801–1000	27	0	58	85	4	10.3	49	1	86	135	7	9.2	70	2	120	190	10	8.4
1001–2000	49	1	76	125	6	9.1	50	1	150	200	10	8.0	100	3	180	280	14	7.5
2001–3000	50	1	95	145	7	8.7	80	2	165	245	12	7.6	130	4	260	390	19	6.9
3001–4000	55	1	110	165	8	8.5	105	3	200	305	14	7.0	155	5	330	485	23	6.5
4001–5000	60	1	135	195	9	7.8	110	3	225	335	15	6.7	215	7	390	605	27	6.0
5001–7000	60	1	165	225	10	7.3	110	3	250	360	16	6.6	270	9	505	775	34	5.7
7001–10000	85	2	160	245	11	7.2	115	3	290	405	18	6.5	285	9	680	965	41	5.4
10001–20000	85	2	180	265	12	7.2	140	43	315	455	20	6.3	315	10	805	1120	47	5.3
20001–50000	85	2	205	290	13	7.0	170	5	420	590	26	6.0	390	13	940	1330	56	5.2
50001–100000	90	2	245	335	15	6.8	200	6	505	705	30	5.7	445	15	1105	1550	65	5.1

Table A.44. Dodge-Romig table based on stated value of AOQL = 3% corresponding to a consumer's risk $\beta = 0.10$

AOQL: average outgoing quality limit. – LQL: Limiting quality Level corresponding to a consumer's risk. $\beta = 0.10$.

	AQL = Acceptable Quality Level %												
Sample Size Code Letter	Sample Size	T	0.10	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.0
		k	k	k	k	k	k	k	k	k	k	k	k
B C	3 4	↓	↓	↓	↓	↓	↓	↓	↓	1.12 1.17	0.958 1.01	0.765 0.814	0.566 0.617
D E F	5 7 10	↓	↓	↓	2.00 2.11	1.88 1.98	1.75 1.84	1.62 1.72	1.50 1.58	1.33 1.41	1.15 1.23	0.955 1.03	0.755 0.828
G H I	15 20 25	2.53 2.58 2.61	2.42 2.47 2.50	2.32 2.36 2.40	2.20 2.24 2.26	2.06 2.11 2.14	1.91 1.96 1.98	1.79 1.82 1.85	1.65 1.69 1.72	1.47 1.51 1.53	1.30 1.33 1.35	1.09 1.12 1.14	0.886 0.917 0.936
J	15	2.65	2.54	2.45	2.31	2.18	2.03	1.89	1.76	1.57	1.39	1.18	0.969
K L M	20 25 30	2.71 2.77 2.80	2.60 2.66 2.69	2.50 2.55 2.58	2.35 2.41 2.43	2.22 2.27 2.29	2.08 2.12 2.14	1.93 1.98 2.00	1.80 1.84 1.86	1.61 1.65 1.67	1.42 1.46 1.48	1.21 1.24 1.26	1.00 1.03 1.05
N P	50 75	2.84 2.83	2.73 2.73	2.61 2.62	2.47 2.47	2.33 2.33	2.18 2.18	2.03 2.04	1.89 1.89	1.70 1.70	1.51 1.51	1.29 1.29	1.07 1.07
		0.10	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.0	
Acceptable Quality Levels → Tightened Inspection													

Table A.45. Main table for reduced inspection of a plan based on unknown variability and use of the unknown standard deviation method → use of the sample standard deviation.
A single specification limit (k method) → Normal and tightened inspection

Reduced inspection

Sample Size Code Letter	Sample Size	AQL = Acceptable Quality Level %										
		0.10	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.0
		k	k	k	k	k	k	k	k	k	k	k
B	3	↓	↓	↓	↓	↓	↓	1.12	0.958	0.765	0.566	0.341
C	3	↓	↓	↓	↓	↓	↓	1.12	0.958	0.765	0.566	0.341
D	3	↓	↓	↓	↓	↓	↓	1.12	0.958	0.765	0.566	0.341
E	3	↓	↓	↓	↓	↓	↓	1.12	0.958	0.765	0.566	0.341
F	4	↓	↓	↓	↓	1.45	1.34	1.17	1.01	0.814	0.617	0.393
G	5	↓	↓	↓	↓	1.65	1.53	1.40	1.24	1.07	0.874	0.675
H	7	↓	2.00	1.881	1.75	1.62	1.50	1.33	1.15	0.955	0.755	0.536
I	10	2.24	2.11	.98	1.84	1.72	1.58	1.41	1.23	1.03	0.828	0.611
J	15	2.32	2.20	2.06	1.91	1.79	1.65	1.47	1.30	1.09	0.886	0.664
K	20	2.36	2.24	2.11	1.96	1.82	1.69	1.51	1.33	1.12	0.917	0.695
L	25	2.40	2.26	2.14	1.98	1.85	1.72	1.53	1.35	1.14	0.936	0.712
M	30	2.41	2.28	2.15	2.00	1.86	1.73	1.53	1.15	1.15	0.946	0.723
N	50	2.50	2.35	2.22	2.08	1.93	1.80	1.61	1.42	1.21	1.00	0.774
P	75	2.55	2.41	2.27	2.12	1.98	1.84	1.65	1.46	1.24	1.03	0.804

Table A.46. Main table for reduced inspection of a plan based on unknown variability and use of the standard deviation method

Normal and tightened inspection

AQL = Acceptable Quality Level								NORMAL INSPECTION						
Sample Size Code Letter	Sample Size	T	0.10	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.0	
		M	M	M	M	M	M	M	M	M	M	M	M	
B	3	↓	↓							7.59	18.86	26.94	33.69	
C	4	↓	↓					1.53	5.50	10.92	16.45	22.86	29.45	
D	5	↓	↓				1.33	3.32	5.83	9.80	14.39	20.19	29.56	
E	7	↓	↓		0.422	1.06	2.14	3.55	5.35	8.40	12.20	17.35	23.29	
F	10	↓	↓	0.349	0.716	1.30	2.17	3.26	4.77	7.29	10.54	15.17	20.74	
G	15	0.186	0.312	0.503	0.818	1.31	2.11	3.05	4.31	6.56	9.46	13.71	18.94	
H	20	0.228	0.365	0.544	0.846	1.29	2.05	2.95	4.09	6.17	8.92	12.99	18.03	
I	25	0.250	0.380	0.551	0.877	1.29	2.00	2.86	3.97	5.97	8.63	12.57	17.51	
J	35	0.264	0.388	0.535	0.847	1.23	1.87	2.68	3.70	5.57	8.10	11.87	16.65	
K	50	0.250	0.363	0.503	0.789	1.17	1.71	2.49	3.45	5.20	7.61	11.23	15.87	
L	75	0.228	0.330	0.467	0.720	1.07	1.60	2.29	3.20	4.87	7.15	10.63	15.13	
M	100	0.220	0.317	0.447	0.689	1.02	1.53	2.20	3.07	4.69	6.91	10.32	14.75	
N	150	0.203	0.293	0.413	0.638	0.949	1.43	2.05	2.89	4.43	6.57	9.88	14.20	
P	200	0.204	0.294	0.414	0.637	0.945	1.42	2.04	2.87	4.40	6.53	9.81	14.20	
		0.1	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.00		
Acceptable Quality Levels (tightened Inspection)														

Table A.47. Main table for normal tightened inspection of a plan based on unknown variability and use of the standard deviation method. Unknown standard deviation: use of sample standard deviation. Double specification limit → M method

AQL = Acceptable Quality Level												
Sample Size Code Letter	Sample Size	0.10	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.0
		M	M	M	M	M	M	M	M	M	M	M
B	3	↓	↓	↓	↓	↓	↓	7.59	18.86	26.94	33.69	40.47
C	3	↓	↓	↓	↓	↓	↓	7.59	18.86	26.94	33.69	40.47
D	3	↓	↓	↓	↓	↓	↓	7.59	18.86	26.94	33.69	40.47
E	3	↓	↓	↓	↓	↓	↓	7.59	18.86	26.94	33.69	40.47
F	4	↓	↓	↓	↓	1.53	5.50	10.92	16.45	22.86	29.45	36.90
G	5	↓	↓	↓	1.33	3.32	5.83	9.80	14.39	20.19	26.56	33.99
H	7	↓	0.422	1.06	2.14	3.55	5.35	8.40	12.20	17.35	23.29	30.50
I	10	0.349	0.716	1.30	2.17	3.26	4.77	7.29	10.54	15.17	20.74	27.57
J	15	0.503	0.818	1.31	2.11	3.05	4.31	6.56	9.46	13.71	18.94	25.61
K	20	0.544	0.546	1.29	2.05	2.95	4.09	6.17	8.92	12.99	18.03	24.53
L	25	0.551	0.877	1.29	2.00	2.86	3.97	5.97	8.63	12.57	17.51	23.97
M	30	0.581	0.879	1.29	1.98	2.83	3.91	5.86	4.47	12.36	17.24	23.58
N	50	0.503	0.789	1.17	1.71	2.49	3.45	5.20	7.61	11.23	15.87	22.00
P	75	0.467	0.720	1.07	1.60	2.29	3.20	4.87	7.15	10.63	15.13	21.11

Table A.48. Main table for reduced inspection of a plan based on unknown variability and use of the standard deviation method

Table p = p(Q,n): Percentage → for example: 4724 represent 47.24%.

Q	3	4	5	7	10	15	20	25	30	35	50	75	100	150	200
0.00	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000
0.10	4724	4667	4644	4626	4616	4610	4608	4606	4605	4605	4604	4603	4603	4602	4602
0.20	4446	4333	4290	4235	4224	4219	4216	4215	4215	4213	4210	4209	4208	4208	4208
0.30	4163	4000	3937	3937	3860	3844	3837	3833	3831	3829	3827	3825	3824	3822	3822
0.31	4135	3967	3902	3902	3823	3806	3799	3795	3791	3789	3789	3787	3786	3784	3784
0.32	4106	3933	3867	3867	3786	3769	3762	3758	3755	3754	3751	3749	3748	3746	3746
0.33	4077	3900	3832	3832	3749	3731	3724	3720	3718	3716	3713	3711	3710	3709	3708
0.34	4049	3867	3797	3797	3712	3694	3687	3683	3680	3678	3675	3673	3672	3671	3671
0.35	4020	3833	3762	3762	3675	3657	3649	3645	3643	3641	3638	3636	3635	3633	3633
0.36	3991	3800	3728	3728	3638	3620	3612	3608	3605	3604	3601	3598	3597	3596	3596
0.37	3962	3767	3693	3693	3602	3583	3575	3571	3568	3566	3563	3561	3560	3539	3558
0.38	3933	3733	3658	3658	3565	3546	3538	3534	3531	3529	3526	3524	3523	3522	3521
0.39	3903	3700	3623	3623	3529	3510	3501	3497	3493	3493	3489	3487	3486	3485	3484
0.40	3874	3667	3588	3588	3493	3472	3465	3460	3458	3456	3453	3450	3449	3448	3447
0.41	3845	3633	3554	3554	3457	3437	3428	3424	3421	3419	3416	3413	3412	3411	3410
0.42	3815	3600	3519	3519	3421	3400	3392	3387	3385	3383	3379	3377	3376	3374	3374
0.43	3785	3567	3485	3485	3385	3364	3356	3351	3348	3346	3343	3340	3339	3338	3357
0.44	3756	3533	3450	3450	3349	3328	3320	3315	3312	3310	3307	3304	3303	3302	3301
0.45	3726	3500	3416	3416	3313	3292	3284	3279	3276	3274	3271	3268	3267	3266	3265
0.46	3696	3467	3381	3381	3278	3257	3248	3243	3240	3238	3235	3232	3231	3230	3229
0.47	3666	3433	3347	3347	3242	3221	3212	3207	3204	3202	3199	3196	3195	3194	3193
0.48	3635	3400	3312	3312	3207	3185	3177	3172	3169	3167	3163	3161	3160	3158	3158
0.49	3605	3367	3278	3278	3172	3150	3141	3136	3133	3131	3128	3125	3124	3123	3122
0.50	3575	3333	3244	3244	3139	3115	3106	3101	3098	3096	3093	3090	3089	3087	3087
0.51	3544	3300	3210	3210	3102	3080	3071	3066	3063	3061	3057	3055	3054	3052	3052
0.52	3513	3267	3176	3176	3067	3045	3036	3031	3028	3026	3023	3020	3019	3017	3017
0.53	3482	3233	3142	3142	3032	3010	3001	2996	2993	2991	2988	2985	2984	2983	2982
0.54	3451	3200	3108	3108	2998	2976	2967	2962	2959	2957	2953	2951	2949	2948	2948
0.55	3420	3167	3074	3074	2964	2941	2932	2927	2924	2922	2919	2916	2915	2914	2943
0.56	3388	3133	3040	3040	2929	2907	2898	2893	2890	2888	2885	2882	2881	2879	2879
0.57	3357	3100	3006	3006	2895	2873	2864	2859	2856	2854	2851	2848	2847	2845	2845
0.58	3325	3067	2973	2973	2861	2839	2830	2825	2822	2820	2817	2814	2813	2812	2811
0.59	3293	3033	2939	2939	2828	2805	2796	2792	2789	2787	2783	2781	2779	2778	2777
0.60	3261	3000	2905	2905	2794	2772	2763	2758	2755	2753	2750	2747	2746	2745	2744

Table A.49. Table p(Q, n) → for example: percentage 32.28% represents 3.228

Table p = p(Q,n): Percentage → for example: 32.28% represent 3228.

Q	3	4	5	7	10	15	20	25	30	35	50	75	100	150	200
0.61	3228	2967	2872	2872	2760	2739	2730	2725	2722	2720	2716	2714	2713	2711	2711
0.62	3196	2933	2839	2839	2727	2705	2696	2692	2689	2687	2683	2681	2680	2678	2678
0.63	3163	2900	2805	2805	2694	2672	2663	2659	2656	2654	2650	2648	2647	2645	2645
0.64	3130	2867	2772	2772	2661	2639	2631	2626	2623	2621	2618	2615	2614	2613	2612
0.65	3097	2833	2739	2739	2628	2607	2598	2593	2590	2588	2585	2583	2582	2580	2580
0.66	3063	2800	2706	2706	2596	2574	2566	2561	2558	2556	2553	2551	2549	2548	2548
0.67	3030	2767	2673	2673	2563	2542	2533	2529	2526	2524	2521	2519	2517	2516	2516
0.68	2996	2733	2640	2640	2531	2510	2501	2497	2494	2492	2489	2487	2486	2484	2484
0.69	2961	2700	2607	2607	2499	2478	2470	2465	2462	2460	2457	2455	2454	2453	2452
0.70	2927	2667	2574	2574	2467	2446	2438	2433	2431	2429	2426	2424	2423	2421	2421
0.71	2892	2633	2541	2541	2435	2415	2406	2402	2399	2396	2393	2392	2391	2390	2390
0.72	2857	2600	2509	2509	2403	2383	2375	2371	2368	2367	2364	2361	2360	2359	2359
0.73	2822	2567	2476	2476	2372	2352	2344	2340	2337	2336	2333	2331	2330	2329	2328
0.74	2786	2533	2444	2444	2341	2321	2313	2309	2307	2305	2302	2300	2299	2298	2298
0.75	2750	2500	2411	2411	2310	2290	2283	2279	2276	2275	2272	2270	2269	2268	2267
0.76	2713	2467	2379	2379	2279	2260	2252	2248	2246	2244	2242	2240	2239	2238	2237
0.77	2677	2433	2347	2347	2248	2230	2222	2218	2216	2214	2212	2210	2209	2208	2208
0.78	2639	2400	2315	2315	2218	2199	2192	2189	2186	2185	2182	2180	2179	2178	2178
0.79	2602	2367	2283	2283	2219	2187	2170	2163	2159	2157	2155	2153	2151	2150	2149
0.80	2564	2333	2251	2251	2188	2157	2140	2133	2129	2127	2126	2123	2121	2120	2120
0.81	2525	2300	2219	2219	2158	2127	2110	2104	2100	2098	2097	2094	2093	2092	2091
0.82	2486	2267	2187	2187	2127	2098	2081	2075	2071	2069	2068	2065	2064	2063	2062
0.83	2447	2233	2156	2156	2097	2068	2052	2046	2042	2040	2039	2037	2035	2034	2034
0.84	2407	2200	2124	2124	2067	2039	2023	2017	2014	2012	2011	2009	2007	2006	2005
0.85	2367	2167	2093	2093	2037	2010	1994	1989	1986	1984	1982	1980	1979	1978	1977
0.86	2326	2133	2062	2062	2007	1981	1966	1960	1957	1956	1954	1953	1951	1950	1950
0.87	2284	2100	2031	2031	1978	1952	1938	1932	1930	1928	1927	1925	1924	1923	1922
0.88	2242	2067	2000	2000	1948	1923	1910	1904	1902	1900	1899	1898	1896	1895	1895
0.89	2199	2033	1969	1969	1919	1895	1882	1877	1874	1873	1872	1870	1869	1868	1868
0.90	2155	2000	1938	1938	1890	1867	1854	1850	1847	1846	1845	1843	1842	1841	1841
0.91	2111	1967	1907	1907	1861	1839	1827	1822	1820	1819	1818	1817	1816	1815	1815
0.92	2066	1933	1877	1877	1833	1811	1800	1796	1794	1792	1790	1789	1789	1788	1788
0.93	2020	1900	1846	1846	1804	1784	1773	1769	1767	1766	1765	1764	1763	1762	1762
0.94	1974	1867	1816	1816	1776	1757	1746	1743	1741	1740	1739	1738	1737	1736	1736

Table A.49 (continued). Table p(Q, n) → for example: percentage 32.28% represents 3,228

0.95	1925	1833	1786	1748	1729	1720	1717	1715	1714	1713	1712	1712	1711	1711	1711
0.96	1876	1800	1756	1720	1703	1694	1691	1689	1688	1688	1687	1686	1686	1686	1685
0.97	1825	1767	1725	1692	1676	1668	1665	1663	1663	1662	1661	1661	1661	1660	1660
0.98	1774	1733	1696	1665	1649	1642	1639	1638	1637	1637	1636	1636	1636	1636	1636
0.99	1721	1700	1666	1637	1623	1616	1614	1613	1612	1612	1612	1611	1611	1611	1611
1.00	1667	1667	1636	1610	1557	1591	1589	1588	1588	1587	1587	1587	1587	1587	1587
1.01	1611	1633	1607	1583	1572	1566	1564	1563	1563	1563	1563	1562	1562	1562	1562
1.02	1553	1600	1578	1556	1546	1541	1540	1539	1539	1538	1538	1538	1538	1538	1538
1.03	1493	1567	1548	1530	1521	1517	1515	1515	1515	1515	1515	1515	1515	1515	1515
1.04	1431	1533	1519	1503	1496	1492	1491	1491	1491	1491	1491	1491	1491	1491	1491
1.05	1366	1500	1491	1477	1471	1468	1467	1467	1467	1467	1468	1468	1468	1468	1468
1.06	1298	1467	1462	1451	1446	1444	1444	1444	1444	1444	1445	1445	1445	1445	1445
1.07	1227	1433	1433	1426	1422	1420	1420	1421	1421	1421	1422	1422	1422	1422	1423
1.08	1151	1400	1405	1400	1397	1397	1397	1398	1398	1398	1399	1399	1400	1400	1400
1.09	1071	1367	1376	1375	1373	1374	1374	1375	1375	1376	1377	1377	1378	1378	1378
1.10	984	1333	1348	1349	1350	1351	1352	1352	1353	1354	1354	1355	1355	1356	1356
1.11	889	1300	1320	1325	1326	1328	1329	1330	1331	1331	1332	1333	1334	1334	1334
1.12	782	1267	1293	1300	1303	1305	1307	1308	1309	1310	1311	1312	1312	1312	1313
1.13	660	1233	1265	1275	1280	1283	1285	1286	1287	1288	1289	1290	1291	1291	1292
1.14	508	1200	1237	1251	1257	1261	1263	1265	1266	1267	1268	1269	1270	1270	1270
1.15	29	1167	1210	1227	1234	1239	1242	1244	1245	1246	1247	1248	1249	1249	1250
1.16	0	1133	1183	1203	1212	1218	1221	1222	1224	1225	1226	1228	1229	1229	1229
1.17	0	1100	1156	1179	1190	1196	1200	1202	1203	1204	1206	1207	1208	1208	1209
1.18	0	1067	1129	1156	1168	1175	1179	1181	1182	1184	1185	1187	1188	1188	1189
1.19	0	1033	1102	1133	1146	1154	1158	1161	1162	1163	1165	1167	1168	1169	1169
1.20	0	1000	1076	1110	1124	1134	1138	1141	1142	1143	1146	1147	1148	1149	1149
1.21	0	967	1050	1087	1103	1113	1118	1121	1122	1124	1126	1128	1129	1130	1130
1.22	0	933	1023	1065	1082	1093	1093	1101	1103	1104	1107	1109	1109	1110	1111
1.23	0	900	997	1042	1061	1073	1078	1081	1084	1085	1088	1090	1091	1091	1092
1.24	0	867	972	1020	1041	1053	1059	1062	1064	1066	1069	1071	1072	1073	1073
1.25	0	833	946	998	1021	1034	1040	1043	1046	1047	1050	1052	1053	1054	1055
1.26	0	800	921	977	1000	1015	1021	1025	1027	1029	1032	1034	1035	1036	1037
1.27	0	767	896	955	981	996	1002	1006	1009	1010	1013	1016	1017	1018	1019
1.28	0	733	871	934	961	977	984	988	990	992	995	998	999	1000	1001
1.29	0	700	846	913	942	958	965	970	972	974	978	980	982	983	983
1.30	0	667	821	893	922	940	948	952	955	957	960	963	964	965	966
1.31	0	633	797	872	903	922	930	934	937	939	943	946	947	948	949
1.32	0	600	773	852	885	904	912	917	920	922	926	929	930	931	932

Table A.49 (continued). Table $p(Q, n) \rightarrow$ for example: percentage 32.28% represents 3,228

1.33	0	567	749	832	866	886	895	900	903	905	909	912	913	915	915
1.34	0	533	725	812	848	869	878	883	886	888	892	895	897	898	899
1.35	0	500	702	792	830	852	861	866	869	872	876	879	881	882	883
1.36	0	467	679	773	812	835	844	850	853	855	860	863	865	866	867
1.37	0	433	656	754	795	818	828	833	837	839	844	847	849	850	851
1.38	0	400	633	735	777	801	812	817	821	824	825	831	833	835	835
1.39	0	367	610	717	760	785	796	801	805	808	812	816	818	819	820
1.40	0	333	588	698	744	769	780	786	790	792	797	801	802	804	805
1.41	0	300	566	680	727	753	764	770	774	777	782	786	787	789	790
1.42	0	267	544	662	710	737	749	755	759	762	767	771	773	774	775
1.43	0	233	523	645	694	722	734	740	744	747	752	756	758	760	761
1.44	0	200	501	627	678	707	719	726	730	733	738	742	744	746	747
1.45	0	167	481	610	663	692	704	711	715	718	724	728	730	731	733
1.46	0	133	460	593	647	677	690	697	701	704	710	714	716	718	719
1.47	0	100	439	577	632	663	675	683	687	690	696	700	702	704	705
1.48	0	67	419	560	617	648	661	669	673	677	682	686	688	690	691
1.49	0	33	399	544	602	634	648	655	660	663	669	673	675	677	678
1.50	0	0	380	528	587	620	634	641	646	650	655	660	662	664	665
1.51	0	0	361	513	573	606	620	628	633	636	642	647	649	651	652
1.52	0	0	342	497	559	593	607	615	620	623	629	634	636	638	639
1.53	0	0	323	482	545	580	594	606	607	611	617	621	624	626	627
1.54	0	0	305	467	531	567	581	589	595	598	604	609	611	613	615
1.55	0	0	287	452	518	554	569	577	582	586	592	597	599	601	602
1.56	0	0	269	438	505	541	556	565	570	574	580	585	587	589	590
1.57	0	0	252	424	492	529	544	553	558	562	568	573	575	578	579
1.58	0	0	235	410	479	516	532	541	546	550	556	561	564	566	567
1.59	0	0	219	396	466	504	520	529	534	538	545	550	552	554	556
1.60	0	0	203	383	454	492	509	517	523	527	533	538	541	543	544
1.61	0	0	187	369	441	481	497	506	512	516	522	527	530	532	533
1.62	0	0	172	357	430	469	486	495	501	504	511	516	519	521	523
1.63	0	0	157	344	418	458	475	484	490	494	501	506	508	511	512
1.64	0	0	142	331	406	447	464	473	479	483	490	495	498	500	501
1.65	0	0	128	319	395	436	453	462	468	472	479	485	487	490	491
1.66	0	0	115	307	384	425	443	452	458	462	469	474	477	480	481
1.67	0	0	102	295	373	415	432	442	448	452	459	464	467	470	471
1.68	0	0	89	284	362	405	422	432	438	442	449	455	457	460	461
1.69	0	0	77	273	352	394	412	422	428	432	439	445	447	450	451
1.70	0	0	66	262	341	384	402	412	418	422	430	435	438	441	442

Table A.49 (continued). Table $p(Q, n) \rightarrow$ for example: percentage 32.28% represents 3,228

1.71	0	0	0	55	251	331	375	393	402	407	413	420	426	429	432	431
1.72	0	0	45	241	321	365	383	393	393	399	404	411	417	419	422	423
1.73	0	0	36	230	311	356	374	384	384	390	394	402	408	410	413	414
1.74	0	0	27	220	302	346	365	375	375	381	385	393	399	401	404	405
1.75	0	0	19	211	293	337	356	366	372	377	384	390	393	395	397	397
1.76	0	0	12	201	283	328	347	357	363	368	376	381	384	387	388	388
1.77	0	0	6	192	274	320	338	348	355	359	367	373	376	378	380	380
1.78	0	0	2	183	266	311	330	340	347	351	359	364	367	370	371	371
1.79	0	0	0	174	257	303	321	332	338	343	351	356	359	363	363	363
1.80	0	0	0	165	249	294	313	324	330	335	343	348	351	354	355	355
1.81	0	0	0	157	240	286	305	316	322	327	335	340	343	346	347	347
1.82	0	0	0	149	232	279	298	308	315	319	327	333	336	338	340	340
1.83	0	0	0	141	225	271	290	300	307	311	319	325	328	331	332	332
1.84	0	0	0	134	217	263	282	293	304	312	318	321	323	325	325	325
1.85	0	0	0	126	209	256	275	285	292	297	305	310	313	319	317	317
1.86	0	0	0	119	202	248	268	278	285	289	297	303	306	309	310	310
1.87	0	0	0	112	195	241	261	271	278	282	290	296	299	302	303	303
1.88	0	0	0	106	188	234	254	264	271	275	283	289	292	295	296	296
1.89	0	0	0	99	181	228	247	257	264	269	277	283	285	288	290	290
1.90	0	0	0	93	175	221	240	251	257	262	270	276	279	282	283	283
1.91	0	0	0	87	168	214	234	244	251	256	263	269	272	275	277	277
1.92	0	0	0	81	162	208	227	238	245	249	257	263	266	269	270	270
1.93	0	0	0	76	156	202	221	232	238	243	251	257	260	262	264	264
1.94	0	0	0	70	150	196	215	225	232	237	245	251	254	256	258	258
1.95	0	0	0	65	144	190	209	219	226	231	239	245	248	250	252	252
1.96	0	0	0	60	138	184	203	214	220	225	233	239	242	244	246	246
1.97	0	0	0	56	133	178	197	208	214	219	227	233	236	239	240	240
1.98	0	0	0	51	127	173	192	202	209	213	221	227	230	233	234	234
1.99	0	0	0	47	122	167	186	197	203	208	216	222	225	227	229	229
2.00	0	0	0	43	117	162	181	191	198	203	210	216	219	222	223	223
2.01	0	0	0	39	112	157	176	186	193	197	205	211	214	217	218	218
2.02	0	0	0	36	107	152	171	181	187	192	200	206	209	211	213	213
2.03	0	0	0	32	103	147	166	176	182	187	195	201	204	206	208	208
2.04	0	0	0	29	98	142	161	171	177	182	190	196	199	201	203	203
2.05	0	0	0	26	94	137	156	166	173	177	185	191	194	196	198	198
2.06	0	0	0	23	90	133	151	161	168	172	180	186	189	192	193	193
2.07	0	0	0	21	86	128	147	157	163	168	176	181	184	187	188	188
2.08	0	0	0	18	82	124	142	152	159	163	171	177	179	182	184	184

Table A.49 (continued). Table $p(Q, n) \rightarrow$ for example: percentage 32.28% represents 3,228

2.09	0	0	0	0	0	16	78	120	138	148	154	159	166	172	175	178	179
2.10	0	0	0	0	0	14	74	116	134	144	150	154	162	168	171	173	175
2.11	0	0	0	0	0	12	71	112	130	139	146	150	158	163	166	169	170
2.12	0	0	0	0	0	10	67	108	126	135	142	146	154	159	162	165	166
2.13	0	0	0	0	0	8	64	104	122	131	138	142	150	155	158	161	162
2.14	0	0	0	0	0	7	61	100	118	128	134	138	146	151	154	157	158
2.15	0	0	0	0	0	6	58	97	114	124	130	134	142	147	150	153	154
2.16	0	0	0	0	0	5	55	93	110	120	126	130	138	143	146	149	150
2.17	0	0	0	0	0	4	52	90	107	116	122	127	134	140	142	145	146
2.18	0	0	0	0	0	3	49	87	103	113	119	123	130	136	139	141	142
2.19	0	0	0	0	0	2	46	83	100	109	115	120	127	132	135	138	139
2.20	0	0	0	0	0	1.5	44	80	97	106	112	116	123	129	131	134	135
2.21	0	0	0	0	0	1	41	77	94	103	109	113	120	125	128	130	132
2.22	0	0	0	0	0	0.6	39	74	90	100	105	109	117	122	124	127	128
2.23	0	0	0	0	0	0.3	37	71	87	96	102	106	113	119	121	124	125
2.24	0	0	0	0	0	0.2	34	69	84	93	99	103	110	115	118	120	122
2.25	0	0	0	0	0	0.1	32	66	82	90	96	100	107	112	115	117	119
2.26	0	0	0	0	0	0	30	63	79	88	93	97	104	109	112	114	115
2.27	0	0	0	0	0	0	28	61	76	85	90	94	101	106	109	111	112
2.28	0	0	0	0	0	0	27	58	73	82	88	91	98	103	106	108	109
2.29	0	0	0	0	0	0	25	56	71	79	85	89	95	100	103	105	106

Table A.49 (continued). Table $p(Q, n) \rightarrow$ for example: percentage 32.28% represents 3,228

Graphs

There are many ways to graphically represent the distribution laws – in our case, the exponential law and the log-normal law. The graph for the Weibull distribution with two- and three-parameters is shown:

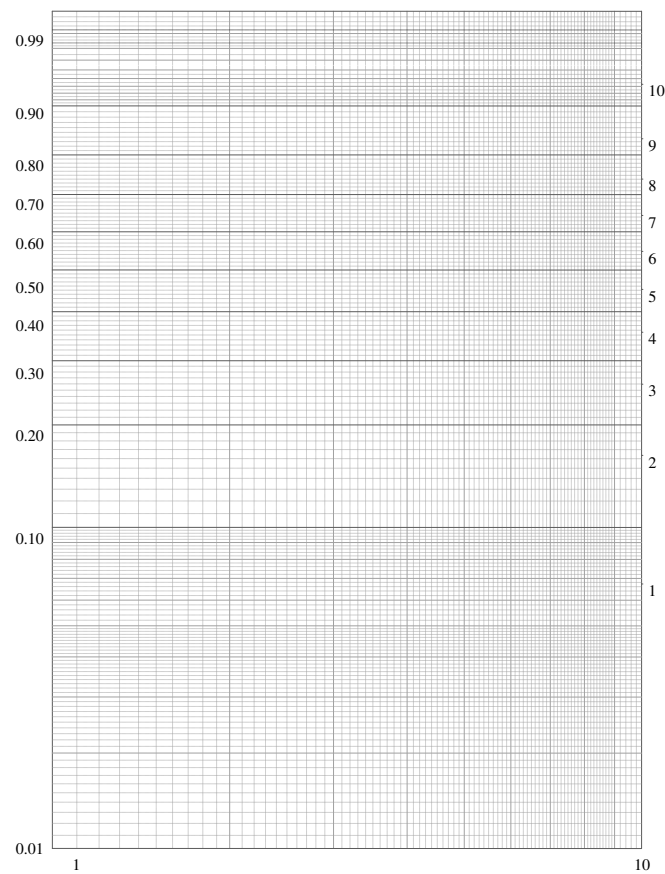


Figure A.3. *Graph of three-parameter Weibull distribution*

In the following, we present an example of this plot.

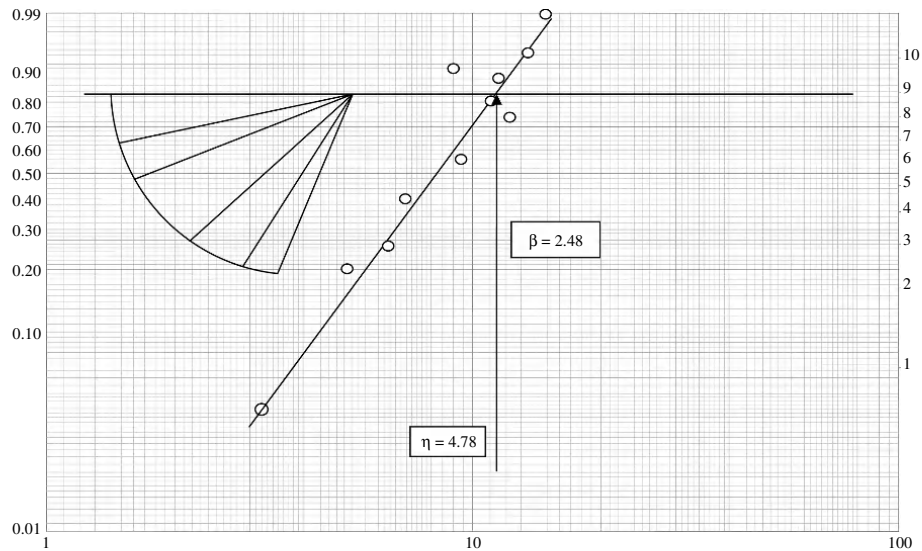


Figure A.4. Allan plait graph of 2-parameter Weibull distribution

Main regulation, codes and technical norms

The Rabaska deepwater terminal is designed in accordance with the regulations, codes and norms of Quebec and Canada, and with the American and international norms and recommendations listed here.

Reference /Acronym	TITLE	ORIGIN
CSA	Canadian Standard Association	Canada (E)
ACNOR	<i>Association Canadienne de Normalisation</i>	Canada (F)
CNRC	<i>Conseil national de recherches Canada</i>	Canada
BNQ	<i>Bureau de normalisation du Québec</i>	Canada-Québec
CISC	<i>Institut canadien de la construction en acier</i>	Canada
ACG	Association of Canadian Gas	Canada
ACI	American Concrete Institute	USA
AGA	American Gas Association	USA
ANSI	American National Standards Institute	USA
API	American Petroleum Institute	USA
ASCE	American Society of Civil Engineers	USA
ASME	American Society of Mechanical Engineers	USA
ASTM	American Society for Testing Materials	USA
BS	British Standards	UK
AFNOR	French Standards Association	France

Table A.50. References/acronyms

Earthquake resistance

The terminal's anti-earthquake design also conforms to the prescriptions of Canada's national building code: based on one earthquake that does not disrupt *normal service* (operating basis earthquake) and another that starts a *safe shutdown of the terminal*. Table A.51 presents the various norms regarding how these two reference earthquakes are to be quantified.

Norm-Ref	Operating basis earthquake	Safe shutdown earthquake
CSA Z276-01	SSN (<i>Séisme de service normal</i>) = Operating basis earthquake Earthquake with a 10% probability of being exceeded in 50 years, equating to a recurrence period of 475 years.	SAS (<i>Séisme d'arrêt sécuritaire</i>) = Safe shutdown earthquake Earthquake with a probability of being exceeded not greater than 0.1% per year, equating to a recurrence period of 1,000 years.
NFPA 59A-2001	OBE (operating basis earthquake) Earthquake with a 10% probability of being exceeded in 50 years, equating to a recurrence period of 475 years. OBE (Operating Basis Earthquake)	SSE (safe shutdown earthquake) Earthquake with a probability of being exceeded not greater than 1% per 50 years, equating to a recurrence period of 4,975 years. SSE (Safe Shutdown Earthquake)
EN 1473:1997	Earthquakes with a recurrence period equal to 475 years.	Earthquakes with a recurrence period equal to 10,000 years.
Rabaska	Application of the three norms. Recurrence period of 475 years.	Application of the European norm. Recurrence period of 10,000 years.
Emergency shutdown system		
CSA Z276	The Canadian standard prescribes: – an emergency shutdown system (ESS) to isolate or close off sources of flammable liquids and shut down machinery that would prolong or aggravate the emergency situation. The control may be manual or automatic. – that the installation be put in safety mode in case of a failure in the electricity or air supply.	
NFPA 59A	The rules are the same as the CSA.	
EN 1473 Europe	The norm describes the control systems and defines the role of emergency shutdown (ESD). The presence of an emergency electrogenic group is required.	
Rabaska	Application of the prescriptions of all three norms.	

Table A.51. Relevant international norms (Canada and elsewhere)

Regulations, codes and norms

TITLE	Edition	Reference
Handbook of Steel Construction (8th Ed)	2004	CISC
Commentary on CAN/CSA-S6-00 Canadian Highway Bridge - Design Code	2000	CSA-S6.1-00
Metric Building Code Requirement for Structural Concrete	2002	ACI 318M
Guide for the design and construction of fixed offshore concrete structures.	1984	ACI 357 R-84
State-of-the-art Report on concrete structures for the Arctic.	1997	ACI 375.1R91
Planning and Designing and Constructing Fixed Offshore Structures in Ice Environments.	1982	API Bulletin 2N
Welding of Pipe Lines and Related Facilities (19th ed.) and Errata	2001	API 1104
Flanged Safety Relief Valves (5th ed.)	2002	API 526
Steel Gate Valves – Flanged and Butt Welding Ends (11th ed.)	2001	API 600
Corrosion Resistant, Bolted Bonnet Gate Valves – Flanged and Butt-Welding Ends (6th ed.)	2001	API 603
Minimum Design Loads for Buildings & Other Structures	2002	ASCE 7
Unified Inch Screw threads (UN and UNR Thread Form)	R2001	ASME B1.1
Pipe Threads, General Purpose (inch) 1983	R2001	ASME B1.20.1
Safety Standard for Mechanical Power Transmission Apparatus	2000	ASME B15.1
Forged Fittings, socket-welding and Threaded	2001	ASME B16.11
Butt welding Ends	1997	ASME B16.25
Surface Texture, Surface Roughness, Waviness, and Lay	2002	ASME B46.1
NDT Procedures	Latest	ASME Section 5
Quality Control Standard for Control Valve Seat Leakage	2003	ASME/FCI-70-2
Appropriate Materials Standards	Latest	ASTM
Rules for the design, construction and inspection of offshore structures. Det norske Veritas, Hovik, Norway	1977	Det norske Veritas
<i>Systèmes de management de la qualité – Exigences</i>	2000	ISO 9001
<i>Systèmes de management de la qualité – Principes essentiels et vocabulaire</i>		ISO 9000
<i>Unités SI et recommandations pour l'emploi de leurs multiples et de certaines autres unités</i>	2000	ISO 1000

Table A.52. *Regulations, codes and norms (Canada, USA and Europe)*

Index

A

API, 256
Approximation, 43, 67, 138-141

B

Binomial distribution, 153-154
Birnbbaum-Saunders, xiv

C

Capability, 10-11, 49-57, 84-85,
178-180
Classical tolerancing, 4-5
CNOMO E41.36.110N, 8-9
Combination, 5
Conditional probability, xxiii
Control charts, 68-85
Convergence, 143
Corrosion, xxiv, 108
CSA, 5
CUSUM charts, 123-127

D

Deming, 88
Distribution function, xix, xxi, 208

E

EWM charts, 124-127
Exponential, xiv, 79, 118, 123, 133,
191, 254

F

Failure, 88
FORD, 10

G, H

Gauss, 51
GUM, xi, 181-184
Henry line, 144-149

I, K

Interpolation, 45, 98
Ishikawa, 81, 87
ISO 8258, 10, 53, 54, 83
ISO/TS 16949, 10, 123, 212, 220
Kolmogorov-Smimov, 144

L

Likelihood, xii, xix, 37
LMMEM, 93-94

Log-normal, 254
Longevity, 101, 103

M, N

Matrix, 4
Median, 56, 117, 120, 139, 148
Metrology, xxiii, 168-173
Nevralgic, 18

O, P

Offshore, 257
Parallel, 88
Pareto law, 14-18
Pearson, 50, 144
Poisson distribution, 43-45
Probability, 144-146
Process capability, 49-56

Q, R

QS9000, 6-11
Quality control, 1-85
R&R, 168-173
Reliability indices, ix, x, 210

S

Shewhart, 118-119, 126-127
Slope, 160, 161
Standard deviation, 126
Statistical process control, 1-3
Stock management, 111-116

V, W

VIM, xii, 10, 58
Wald plan, 33-36