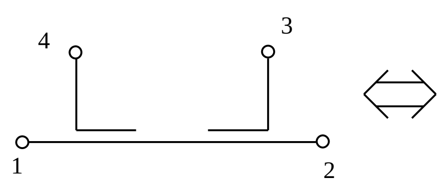
CHAPTER
7

CALIBRATION AND MEASUREMENT OF S-PARAMETERS

7.1 MEASURING S-PARAMETERS

The measurement of s-parameters normally takes one of two forms. Originally slotted lines were used and the VSWR and locations of nulls was measured and the circuit s-parameters were deduced as discussed in chapter 3. . A second technique which is the basis of modern network analyzer is to use a four port circuit known as a coupled line to sample forward and reverse propagating waves. A symbolic representation of coupler together with ideal s-parameters for the coupler are shown in figure 7.1. Using an ideal coupler the following system could be used to measure the s-parameters of a one port "device under test" by measuring the complex voltage from ports 3 and 4, i.e., v_3 and v_4 and computing the ratio referred to here as the *measured* reflection coefficient,

$$\Gamma_m = \frac{v_4}{v_3}\,.$$



$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Figure 7.0 Schematic symbol for a coupler with ideal s-parameters

These voltage terms are proportional to the counter-propagating signals at those points, i.e.,

$$v_3 = a_3 + b_3 = b_3(1+\Gamma_3)\sqrt{Z_o}$$
$$v_4 = a_4 + b_4 = b_4(1+\Gamma_4)\sqrt{Z_o}$$

If matched loads are placed at ports 3 and 4 then $\Gamma_3 = \Gamma_4 = 0$ and then $v_3 = b_3\sqrt{Z_o}$, and $v_4 = b_4\sqrt{Z_o}$. In this case the measured reflection coefficient is given by $\Gamma_m = \dfrac{b_4}{b_3}$. Also, if ports "3" and "4" are matched then the entering signals are zero, $a_3 = a_4 = 0$. Therefore,

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ 0 \\ 0 \end{pmatrix}$$

and $b_2 = a_1$, $b_3 = a_1$, $b_4 = a_2$ implying that the measured reflection coefficient equals the reflection coefficient of port "2," Γ_2., since $\Gamma_m = \dfrac{b_4}{b_3} = \dfrac{a_2}{a_1} = \dfrac{a_2}{b_2} = \Gamma_2$. If Γ_A designates the actual reflection coefficient for a one port device then

$$s_{11} = \Gamma_A = \Gamma_m \, .$$

Figure 7.1 illustrates a system to implement the ideal measurement system. In practice two effects degrade the previous system. First the terminating system at the measurement ports, 3 and 4, can *not* be expected to be matched ($\Gamma_3 \neq 0$ and $\Gamma_4 \neq 0$), particularly over a very wide band of frequencies, and second the coupler circuit can not be expected to be ideal..

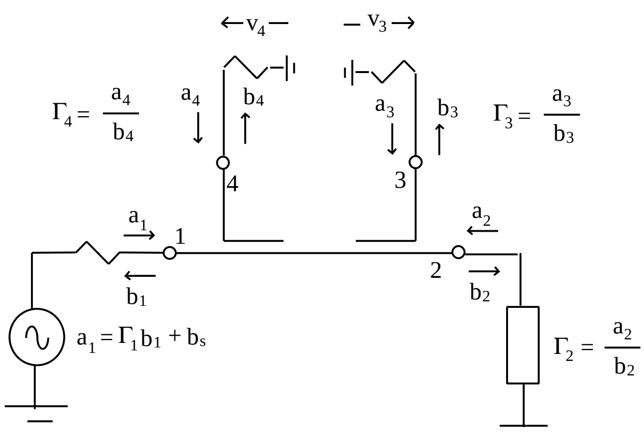


Figure 7.1 A system to measure the reflection coefficient.

The results of the first degradation is that $\Gamma_m = \dfrac{v_4}{v_3} = \dfrac{b_4}{b_3}\left(\dfrac{1+\Gamma_4}{1+\Gamma_3}\right) = k\dfrac{b_4}{b_3}$, where $k = \left(\dfrac{1+\Gamma_4}{1+\Gamma_3}\right)$. The second degradation results from the non-ideal nature of the coupler system. In general the coupler system can be represented by a 4x4 scattering matrix, where the elements of the matrix are unknown constants. Including the source term results in the following matrix equation

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix} \begin{pmatrix} \Gamma_1 b_1 + b_s \\ \Gamma_2 b_2 \\ \Gamma_3 b_3 \\ \Gamma_4 b_4 \end{pmatrix}$$

Expansion of the matrix equation gives

$$b_1 = s_{11}\Gamma_1 b_1 + s_{12}\Gamma_2 b_2 + s_{13}\Gamma_3 b_3 + s_{14}\Gamma_4 b_4 + s_{11} b_s$$

$$b_2 = s_{21}\Gamma_1 b_1 + s_{22}\Gamma_2 b_2 + s_{23}\Gamma_3 b_3 + s_{24}\Gamma_4 b_4 + s_{21} b_s$$

$$b_3 = s_{31}\Gamma_1 b_1 + s_{32}\Gamma_2 b_2 + s_{33}\Gamma_3 b_3 + s_{34}\Gamma_4 b_4 + s_{31} b_s$$

$$b_4 = s_{41}\Gamma_1 b_1 + s_{42}\Gamma_2 b_2 + s_{43}\Gamma_3 b_3 + s_{44}\Gamma_4 b_4 + s_{41} b_s$$

Manipulating these equations results in

$$(s_{11}\Gamma_1 - 1)b_1 + s_{12}\Gamma_2 b_2 + s_{13}\Gamma_3 b_3 + s_{14}\Gamma_4 b_4 = -s_{11} b_s$$

$$s_{21}\Gamma_1 b_1 + (s_{22}\Gamma_2 - 1)b_2 + s_{23}\Gamma_3 b_3 + s_{24}\Gamma_4 b_4 = -s_{21} b_s$$

$$s_{31}\Gamma_1 b_1 + s_{32}\Gamma_2 b_2 + (s_{33}\Gamma_3 - 1)b_3 + s_{34}\Gamma_4 b_4 = -s_{31} b_s$$

$$s_{41}\Gamma_1 b_1 + s_{42}\Gamma_2 b_2 + s_{43}\Gamma_3 b_3 + (s_{44}\Gamma_4 - 1)b_4 = -s_{41} b_s$$

which can be put in the following matrix form

$$\begin{pmatrix} (s_{11}\Gamma_1 - 1) & s_{12}\Gamma_2 & s_{13}\Gamma_3 & s_{14}\Gamma_4 \\ s_{21}\Gamma_1 & (s_{22}\Gamma_2 - 1) & s_{23}\Gamma_3 & s_{24\Gamma_4} \\ s_{31}\Gamma_1 & s_{32}\Gamma_2 & (s_{33}\Gamma_3 - 1) & s_{34}\Gamma_4 \\ s_{41}\Gamma_1 & s_{42}\Gamma_2 & s_{43}\Gamma_3 & (s_{44}\Gamma_4 - 1) \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = -b_s \begin{pmatrix} s_{11} \\ s_{21} \\ s_{31} \\ s_{41} \end{pmatrix}$$

The signal parameters, b_3 and b_4 can be found by solving the system of equations using Kramer's Rule. Thus, $b_3 = -b_s \dfrac{K_3}{\Delta}$ and $b_4 = -b_s \dfrac{K_4}{\Delta}$,where

$$K_3 = \det \begin{pmatrix} (s_{11}\Gamma_1 - 1) & s_{12}\Gamma_2 & s_{11} & s_{14}\Gamma_4 \\ s_{21}\Gamma_1 & (s_{22}\Gamma_2 - 1) & s_{21} & s_{24\Gamma_4} \\ s_{31}\Gamma_1 & s_{32}\Gamma_2 & s_{31} & s_{34}\Gamma_4 \\ s_{41}\Gamma_1 & s_{42}\Gamma_2 & s_{41} & (s_{44}\Gamma_4 - 1) \end{pmatrix},$$

$$K_4 = \det \begin{pmatrix} (s_{11}\Gamma_1 - 1) & s_{12}\Gamma_2 & s_{13}\Gamma_3 & s_{11} \\ s_{21}\Gamma_1 & (s_{22}\Gamma_2 - 1) & s_{23}\Gamma_3 & s_{21} \\ s_{31}\Gamma_1 & s_{32}\Gamma_2 & (s_{33}\Gamma_3 - 1) & s_{31} \\ s_{41}\Gamma_1 & s_{42}\Gamma_2 & s_{43}\Gamma_3 & s_{41} \end{pmatrix},$$

and

$$\Delta = \det\begin{pmatrix} (s_{11}\Gamma_1 - 1) & s_{12}\Gamma_2 & s_{13}\Gamma_3 & s_{14}\Gamma_4 \\ s_{21}\Gamma_1 & (s_{22}\Gamma_2 - 1) & s_{23}\Gamma_3 & s_{24\Gamma_4} \\ s_{31}\Gamma_1 & s_{32}\Gamma_2 & (s_{33}\Gamma_3 - 1) & s_{34}\Gamma_4 \\ s_{41}\Gamma_1 & s_{42}\Gamma_2 & s_{43}\Gamma_3 & (s_{44}\Gamma_4 - 1) \end{pmatrix}$$

$$K_3 = -s_{12}\Gamma_2 \det\begin{pmatrix} s_{21}\Gamma_1 & s_{21} & s_{24}\Gamma_4 \\ s_{31}\Gamma_1 & s_{31} & s_{34}\Gamma_4 \\ s_{41}\Gamma_1 & s_{41} & (s_{44}\Gamma_4 - 1) \end{pmatrix} + (s_{22}\Gamma_2 - 1)\det\begin{pmatrix} (s_{11}\Gamma_1 - 1) & s_{11} & s_{14}\Gamma_4 \\ s_{31}\Gamma_1 & s_{31} & s_{34}\Gamma_4 \\ s_{41}\Gamma_1 & s_{41} & (s_{44}\Gamma_4 - 1) \end{pmatrix} -$$

$$s_{32}\Gamma_2 \det\begin{pmatrix} (s_{11}\Gamma_1 - 1) & s_{11} & s_{14}\Gamma_4 \\ s_{21}\Gamma_1 & s_{21} & s_{24}\Gamma_4 \\ s_{41}\Gamma_1 & s_{41} & (s_{44}\Gamma_4 - 1) \end{pmatrix} + s_{42}\Gamma_2 \det\begin{pmatrix} (s_{11}\Gamma_1 - 1) & s_{11} & s_{14}\Gamma_4 \\ s_{21}\Gamma_1 & s_{21} & s_{24}\Gamma_4 \\ s_{31}\Gamma_1 & s_{31} & s_{34}\Gamma_4 \end{pmatrix} =$$

$$-s_{12}\Gamma_2 D_1 + (s_{22}\Gamma_2 - 1)D_2 - s_{32}\Gamma_2 D_3 - s_{42}\Gamma_2 D_4 =$$

$$(-s_{12}D_1 + s_{22}D_2 - s_{32}D_3 - s_{42}D_4)\Gamma_2 - D_2$$

implying that

$$K_3 = a\Gamma_2 + b$$

where α and β are constants independent of Γ_2.. Therefore,

$$b_3 = -\frac{b_s}{\Delta}(a\Gamma_2 + b)$$

In a similar manner

$$b_4 = -\frac{b_s}{\Delta}(g\Gamma_2 + d)$$

where α, β, γ, and δ are not functions of Γ_2. The signal parameter ratio is

$$\frac{b_4}{b_3} = \frac{(g\Gamma_2 + d)}{(a\Gamma_2 + b)} = \frac{\left(\frac{g}{b}\Gamma_2 + \frac{d}{b}\right)}{\left(\frac{a}{b}\Gamma_2 + 1\right)}$$

and therefore,

$$\Gamma_m = k\frac{b_4}{b_3} = \frac{\left(k\frac{g}{b}\Gamma_2 + k\frac{d}{b}\right)}{\left(\frac{a}{b}\Gamma_2 + 1\right)}$$

Letting Γ_A denote the actual reflection coefficient of the circuit being tested then the measured reflection coefficient is related to the actual reflection coefficient by the relationship below where A, B, and C are complex number independent of Γ_A.

$$\Gamma_m = \frac{A\Gamma_A + B}{C\Gamma_A + 1}$$

7.2 ERROR ADAPTER MATRICES

Suppose that a two port circuit is inserted between the device under test and an ideal measurement system. This is illustrated in figure 7.2 which also shows the SFG manipulations to obtain the transfer ration $\Gamma_M = \frac{b_M}{a_M}$.

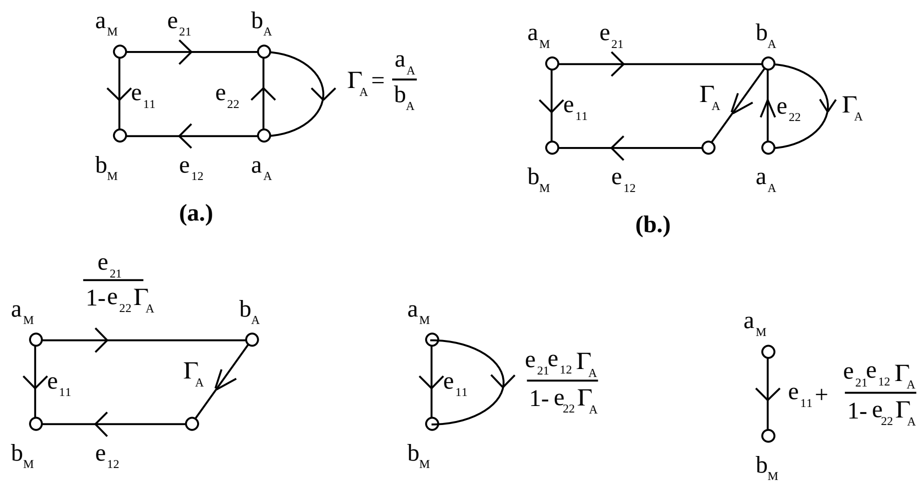


Figure 7.2 Two port circuit between measurement ports and DUT.

$$\Gamma_M = \frac{b_M}{a_M} = e_{11} + \frac{e_{21}e_{12}\Gamma_A}{1 - e_{22}\Gamma_A} = \frac{-\Delta_e\Gamma_A + e_{11}}{-e_{22}\Gamma_A + 1}$$

where $\Delta_e = e_{11}e_{22} - e_{12}e_{21}$. Recalling that

$$\Gamma_M = \frac{A\Gamma_A + B}{C\Gamma_A + 1}$$

implies

$$\begin{aligned} A &= -\Delta_e e_{11} = e_{12}e_{21} - e_{11}e_{22} \\ B &= e_{11} \\ C &= -e_{22} \end{aligned}$$

and therefore,

$$\begin{aligned} e_{11} &= B \\ e_{22} &= -C \\ e_{12}e_{21} &= A - BC \end{aligned}$$

The measurement can be viewed as a combination of two circuits, an error two-port and the actual one-port, Γ_A, as shown in figure 7.3

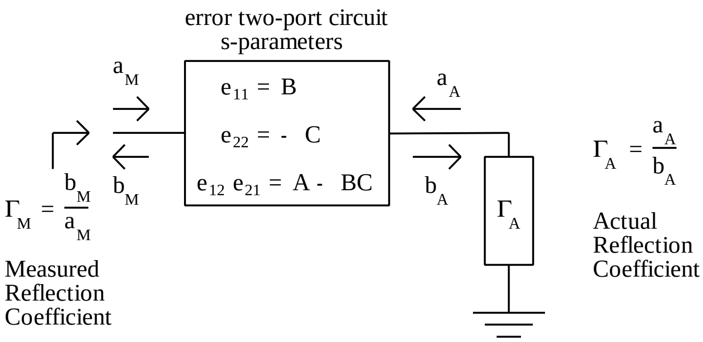


Figure 7.3 The measured reflection coefficient results from actual circuit combined with an error circuit

Representing the error circuit in terms of its transmission matrix yields

$$\begin{pmatrix} b_M \\ a_M \end{pmatrix} = \frac{1}{e_{21}} \begin{pmatrix} -\Delta_e & e_{11} \\ -e_{22} & 1 \end{pmatrix} \begin{pmatrix} a_A \\ b_A \end{pmatrix} = \frac{1}{e_{21}} \begin{pmatrix} A & B \\ C & 1 \end{pmatrix} \begin{pmatrix} a_A \\ b_A \end{pmatrix}$$

Inverting the 2x2 matrix and substituting $\Gamma_M = \frac{b_M}{a_M} \Rightarrow b_M = \Gamma_M a_M$ gives

$$\begin{pmatrix} a_A \\ b_A \end{pmatrix} = \frac{e_{21}}{A - BC} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} \begin{pmatrix} b_M \\ a_M \end{pmatrix} = \frac{e_{21} a_M}{A - BC} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} \begin{pmatrix} \Gamma_M \\ 1 \end{pmatrix}$$

Expanding the matrices results in two equations

$$a_A = \frac{e_{21} a_M}{A - BC} (\Gamma_M - B)$$
$$b_A = \frac{e_{21} a_M}{A - BC} (-C\Gamma_M + A)$$

Dividing the first equation by the second yields

$$\Gamma_A = \frac{a_A}{b_A} = \frac{\Gamma_M - B}{-C\Gamma_M + A}$$

$$-C\Gamma_A \Gamma_M + A\Gamma_A = \Gamma_M - B$$

or

$$\Gamma_A A + B - \Gamma_A \Gamma_M C = \Gamma_M$$

7.3 ONE-PORT CALIBRATION--SHORT, OPEN, LOAD (SOL)

It is desired to find the constants A, B, and C . To do this standard devices are inserted into the measurement system and the measured reflection coefficient observed. It is shown that if three calibration standards are used then the constants can be deduced. Letting subscripts S,O,L refer to short, open, and load and using primes to indicate the known reflection coefficient and unprimed for the measured reflection coefficients then substituting

$$\begin{aligned} \Gamma_A &= \Gamma'_S, \Gamma'_O, \Gamma'_L \\ \Gamma_M &= \Gamma_S, \Gamma_O, \Gamma_L \end{aligned}$$

results in

$$\begin{aligned} &\Gamma'_S A + B - \Gamma'_S \qquad \Gamma_S C = \Gamma_S \\ &\Gamma'_O A + B - \Gamma'_O \qquad \Gamma_O C = \Gamma_O \\ &\Gamma'_L A + B - \Gamma'_L \qquad \Gamma_L C = \Gamma_L \end{aligned}$$

One approach is to solve this system of equations using matrix techniques, i.e.,

$$\begin{pmatrix} \Gamma'_S & 1 & \Gamma'_S \qquad \Gamma_S \\ \Gamma'_O & 1 & \Gamma'_O \qquad \Gamma_O \\ \Gamma'_L & 1 & \Gamma'_L \qquad \Gamma_L \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} \Gamma_S \\ \Gamma_O \\ \Gamma_L \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} \Gamma'_S & 1 & \Gamma'_S \qquad \Gamma_S \\ \Gamma'_O & 1 & \Gamma'_O \qquad \Gamma_O \\ \Gamma'_L & 1 & \Gamma'_L \qquad \Gamma_L \end{pmatrix}^{-1} \begin{pmatrix} \Gamma_S \\ \Gamma_O \\ \Gamma_L \end{pmatrix}$$

A second approach is to solve the equations by taking pairs of them and subtracting. For example subtracting the second equation from the first, and the third equation from the second results in

$$\begin{aligned} &(\Gamma'_S - \Gamma'_O)A + (\Gamma'_O \qquad \Gamma_O - \Gamma'_S \qquad \Gamma_S)C = \Gamma_S - \Gamma_O \\ &(\Gamma'_O - \Gamma'_L)A + (\Gamma'_L \qquad \Gamma_L - \Gamma'_O \qquad \Gamma_O)C = \Gamma_O - \Gamma_L \end{aligned}$$

Letting

$$\boxed{\begin{array}{c} P_1 = (\Gamma'_S - \Gamma'_O) \quad Q_1 = (\Gamma'_O \qquad \Gamma_O - \Gamma'_S \qquad \Gamma_S) \quad M_1 = \Gamma_S - \Gamma_O \\ P_2 = (\Gamma'_O - \Gamma'_L) \quad Q_2 = (\Gamma'_L \qquad \Gamma_L - \Gamma'_O \qquad \Gamma_O) \quad M_2 = \Gamma_O - \Gamma_L \\ \text{(primed means known } \Gamma \text{ while unprimed means measured } \Gamma\text{)} \end{array}}$$

then

$$\begin{aligned} P_1 A + Q_1 C &= M_1 \\ P_2 A + Q_2 C &= M_2 \end{aligned}$$

These equations can be solved by multiplying the first by Q_2 and the second by Q_1

$$Q_2 P_1 A + Q_2 Q_1 C = Q_2 M_1$$
$$Q_1 P_2 A + Q_1 Q_2 C = Q_1 M_2$$

Subtraction yields

$$A = \frac{Q_2 M_1 - Q_1 M_2}{Q_2 P_1 - Q_1 P_2}$$

Repeating this process by multiplying the first equation P_2 and the second by P_1

$$P_2 P_1 A + P_2 Q_1 C = P_2 M_1$$
$$P_1 P_2 A + P_1 Q_2 C = P_1 M_2$$

Subtraction yields

$$C = \frac{P_2 M_1 - P_1 M_2}{P_2 Q_1 - P_1 Q_2}$$

With A and C determined then B is found from any one of the original three equations. Using the third yields

$$B = \Gamma_L + \Gamma_L' \qquad \Gamma_L C - \Gamma_L' A$$

With A, B, C, determined then the Device Under Test (DUT) can be measured an its actual reflection coefficient determined from the formula

$$\Gamma_A = \frac{a_A}{b_A} = \frac{\Gamma_M - B}{-C\Gamma_M + A}$$

Example 7.1. Illustration of a One Port, Short-Open-Load (SOL), Calibration Technique used for S-Band Antenna Patch Measurements o an HP 8510 (Courtesy of the J. H. U. Dorsey Center)

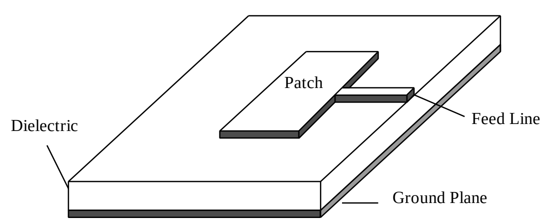


Figure 7. 4. The Device Under Test (D.U.T.) is a microstrip S-band Patch Antenna

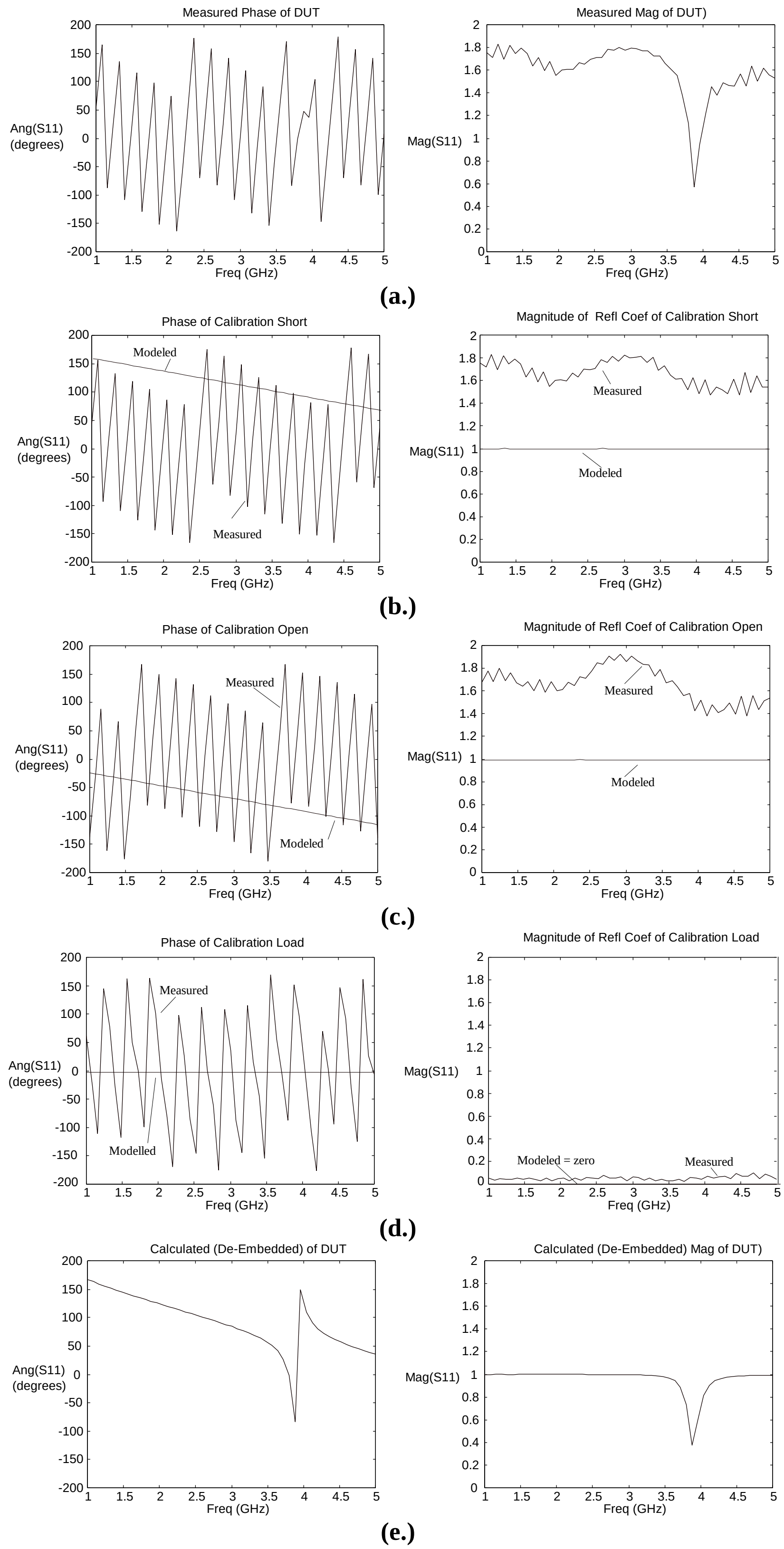


Figure 7. 5 (a.) Raw (Uncalibrated) data, (b.) Data for **Short**, (c.) Data for **Open**, (d.) Data for **Load**, and (e.) Calibrated (De-Embedded) Data for S-band Antenna Patch


```
% Filename: sol_95.m
% Date: 10/2/95

% This File will extract the A, B and C vector for an SOL
% calibration using the raw measurement data and the model
% of the cal standards. The A, B and C vector are then used
% to calculate the reflection coefficient of any 1-port
% circuit from its raw measurement data.  Uses relationship
%
% Gamma_M=(A*Gamma_A+B)/(C*Gamma_A+1)
% or equivalently
% Gamma_A=(Gamma_M-B)/(-C*Gamma_M+A)
%

load rd_shrt.s1p;     % load measured raw data for short
load rd_open.s1p;     %   "     "      "    "   "  open
load rd_load.s1p;     %   "     "      "    "   "  load


Gs=rd_shrt(:,1)+i*rd_shrt(:,2);    % convert measured data to complex numbers
Go=rd_open(:,1)+i*rd_open(:,2);
Gl=rd_load(:,1)+i*rd_load(:,2);

F=[1e9:8e7:5e9]';                  % frequency sample points

% - - - - - - - - - - model for short - - - - - - - - - - - - - -
L0=2.0765e-12;
L1=-108.54e-24;
L2=2.1705e-33;
L3=-.01e-42;

Ltau=31.785e-12;
Lrv0=2.36e9;           %atten/sec=(atten/dist)(prop. vel)
Lrv=Lrv0*sqrt(F/1e9);
Lloss=.5*Lrv/50;

Zsp=i*2*pi.*F.*(L0+L1*F+L2*F.^2+L3*F.^3);
Gsp=((Zsp-50)./(Zsp+50)).*exp(2*(-Lloss-i*2*pi*F)*Ltau);

     % Zsp=Zs-prime=impedance for short
     % Gsp=Gamma_s-prime=reflect coef for short
     %    gamma=wave number=alpha+j*beta=atten+j*prop const.
     % Gamma(d)=Gamma_L*exp(-2*gamma*d)
     % Gamma(d)=Gamma_L*exp[2*(-alpha-j*beta)*d]
     % alpha*d=alpha*vel*Tau=Lloss*Tau
     %
     %         =Gamma_L*exp[2*(-Lloss-jw)*T]
     % w=2*pi*freq,   T=time delay (tau)

% - - - - - - - - - - model for open  - - - - - - - - - - - - - -
C0=49.433e-15;
C1=-310.13e-27;
C2=23.168e-36;
C3=-.15966e-45;

Ctau=29.243e-12;
Crv0=2.2e9;            %atten/sec=(atten/dist)(prop. vel)
Crv=Lrv0*sqrt(F/1e9);
Closs=.5*Lrv/50;

Zop=-i./(2*pi.*F.*(C0+C1*F+C2*F.^2+C3*F.^3));   % impedance for open
Gop=((Zop-50)./(Zop+50)).*exp(2*(-Closs-i*2*pi*F)*Ctau);
 %  reflect coef for open

% Filename: sol_95.m  Cont'd

% - - - - - - - - - - model for load - - - - - - - - - - - - - - -
Glp=zeros(size(F))+i*zeros(size(F));

   % Gl-prime = actual load reflect coef = 0+i0
% - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
M1=Gs-Go;
M2=Go-Gl;
P1=Gsp-Gop;
P2=Gop-Glp;
Q1=Gop.*Go-Gsp.*Gs;
Q2=Glp.*Gl-Gop.*Go;

A=(Q2.*M1-Q1.*M2)./(P1.*Q2-P2.*Q1);
C=(P2.*M1-P1.*M2)./(P2.*Q1-P1.*Q2);
B=Gl+Glp.*Gl.*C-Glp.*A;

load rd_patch.s1p
load dd_patch.s1p

Grdut=rd_patch(:,1)+i*rd_patch(:,2);
Gddut=dd_patch(:,1)+i*dd_patch(:,2);

Gcdut=(Grdut-B)./(-C.*Grdut+A);     %Calibrated measurements of DUT


% ------------------------------- Plot Routines -----------------------
plot(F/1e9,angle(Grdut)*180/pi);
title('Measured Phase of DUT');
xlabel('Freq (GHz)');
ylabel('Ang(S11) (degrees)');
axis([1 5 -200 200]);


figure;
plot(F/1e9,abs(Grdut));
title('Measured Magnitude of DUT');
xlabel('Freq (GHz)');
ylabel('Mag(S11)');
axis([1 5 0 2]);


figure;
plot(F/1e9,angle(Gs)*180/pi,F/1e9,angle(Gsp)*180/pi);
title('Measured and Modeled Phase of Short');
xlabel('Freq (GHz)');
ylabel('Ang(S11) (degrees)');
axis([1 5 -200 200]);


figure;
plot(F/1e9,abs(Gs),F/1e9,abs(Gsp));
title('Measured and Modeled Magnitude of Short');
xlabel('Freq (GHz)');
ylabel('Mag(S11)');
axis([1 5 0 2]);


figure;
plot(F/1e9,angle(Go)*180/pi,F/1e9,angle(Gop)*180/pi);
title('Measured and Modeled Phase of Open');
xlabel('Freq (GHz)');
ylabel('Ang(S11) (degrees)');
axis([1 5 -200 200]);
% Filename: sol_95.m  Cont'd

figure;
plot(F/1e9,abs(Go),F/1e9,abs(Gop));
title('Measured and Modeled Magnitude of Open');
xlabel('Freq (GHz)');
ylabel('Mag(S11)');
axis([1 5 0 2]);


figure;
plot(F/1e9,angle(Gl)*180/pi,F/1e9,angle(Glp)*180/pi);
title('Measured and Modeled Phase of Load');
xlabel('Freq (GHz)');
ylabel('Ang(S11) (degrees)');
axis([1 5 -200 200]);
```

```
figure;
plot(F/1e9,abs(Gl),F/1e9,abs(Glp));
title('Measured and Modeled Magnitude of Load');
xlabel('Freq (GHz)');
ylabel('Mag(S11)');
axis([1 5 0 2]);


figure;
plot(F/1e9,angle(Gcdut)*180/pi);
title('Calculated (De-Embedded) Phase of DUT');
xlabel('Freq (GHz)');
ylabel('Ang(S11) (degrees)');
axis([1 5 -200 200]);


figure;
plot(F/1e9,abs(Gcdut));
title('Calculated (De-Embedded) Magnitude of DUT');
xlabel('Freq (GHz)');
ylabel('Mag(S11)');
axis([1 5 0 2]);
```

```
% Filename: HP2ML1P.M
% Date: 9/30/94

% This file reads 1-port data files saved by HP8510 and create
% a data matrix under the same name.

% HP8510 saved files must be included in Fname matrix.

Fname=['rd_short';'rd_open ';'rd_load ';'rd_patch'];
Listsize=size(Fname,1);  %Number of entries (rows) in file list

for m=1:Listsize;      % Main loop to go through each file

   name=Fname(m,:);
   Letters=length(name);
   fid=fopen(name);
   Test='SEG';                % To skip until 'SEG' is found.
   A=zeros(size(Test));
   while ~all(A==Test);
      B=fscanf(fid,'%s',1);
      size(B);
        if size(B)==size(A);  % can only compare if size is the same
        A=B;
```

```
        end
   end
   for n=1:3                    % Read in the frequency data from the file.
      finfo(m,n)=fscanf(fid,'%f',1);  % since it follows immediately "SEG'.
   end
   Test='BEGIN';                % To skip until 'BEGIN' is found
   A=zeros(size(Test));
   while ~all(A==Test);
      B=fscanf(fid,'%s',1);
      size(B);
        if size(B)==size(A);
        A=B;
        end
   end
   for g=1:finfo(m,3)           % Start reading data
      for h=1:2                        % read both real and imaginary part
      Temp(g,h)=fscanf(fid,'%f',1);  % Put data in Temp matrix
      delimit=fscanf(fid,'%c',1);  % takes care of ',' between R and I parts
      end
   end
   fclose(fid);                        % Close file
   eval([name,'=Temp;'])       % Use original filename as name of the data matrix
end
```

7.4 TWO-PORT CALIBRATION--SHORT, OPEN, LOAD, THRU (SOLT)

The following measurement system can be used to determine two port s-parameters

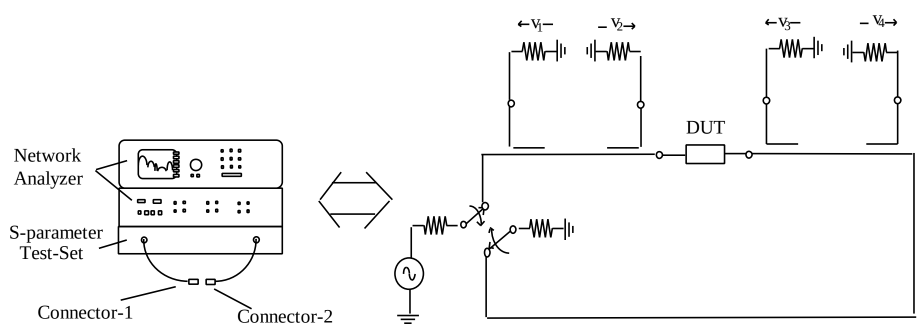


Figure 7.6 Two port S-parameter measurements

This system is equivalent to the following signal flow graph which illustrated that their are now two error circuits associated with each of the two connectors. The analyzer can be view as follows

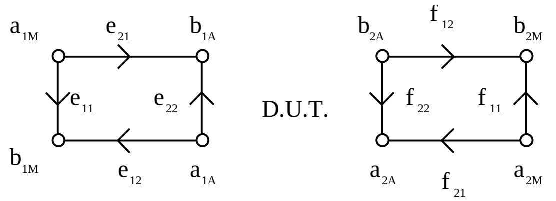


Figure 7.7 Error adapter circuits for two port S-parameter measurements

where, { e_{11} , e_{21} e_{12} , e_{22} } represent the error circuit associated with one terminal (connector 1) and { f_{11} , f_{21} f_{12} , f_{22} } are the s-parameters associated with the second terminal (connector 2). Letting $\{A_{11,}A_{21,}A_{12,}A_{22}\}$ represent the "actual" s-parameters for the D.U.T. then when it is connected to the Network Analyzer the system can be described as follows

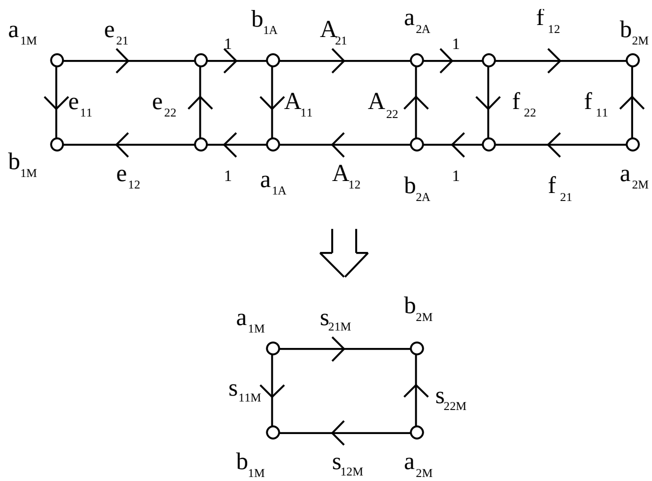


Figure 7.8 Error adapter circuits connected to actual two port combined results in the measured S-parameters.

The error circuits sometimes called error adapters can be represented as a Signal flow graph of s-parameters as illustrated above where the measured s-parameters would be

$$s_{11M} = \frac{b_{1M}}{a_{1M}}, \quad s_{21M} = \frac{b_{2M}}{a_{1M}}, \quad s_{12M} = \frac{b_{1M}}{a_{2M}}, \quad s_{22M} = \frac{b_{2M}}{a_{2M}}$$

Also the measurement could equivalently be represented as Transmission matrices and since the circuits are cascaded the Transmission matrices are multiplied to get the transmission matrix of the combination which represents the measured s-parameters, i.e.,

$$R_e \cdot R_A \cdot R_f = R_M$$

where

$$R_e = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \frac{1}{e_{21}} \begin{pmatrix} -\Delta_e & e_{11} \\ -e_{22} & 1 \end{pmatrix} = \frac{1}{e_{21}} \begin{pmatrix} A & B \\ C & 1 \end{pmatrix}$$

$$R_A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \frac{1}{A_{21}} \begin{pmatrix} -\Delta_A & A_{11} \\ -A_{22} & 1 \end{pmatrix}$$

$$R_f^{-1} = \begin{pmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{pmatrix} = \frac{1}{f_{21}} \begin{pmatrix} 1 & -f_{22} \\ f_{11} & -\Delta_f \end{pmatrix} = \frac{1}{f_{21}} \begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

The inverse of the matrix R_f is easily found by performing the SOL calibration using connector 2 as previous applied to a single connector for a one port. In this case the signal flow graph and the transmission matrix are naturally defined viewing the circuit from right to left as opposed to the earlier development of left to right. This is why the inverse is defined in terms of the A', B' ,C' constants. The actual transmission matrix can be found by simple manipulation to be

$$R_A = R_e^{-1} R_M R_f^{-1}$$

Since

$$R_e^{-1} = \frac{e_{21}}{A - BC} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix}$$

then

$$R_A = \left(\frac{e_{21}}{f_{21}} \right) \frac{1}{A - BC} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} R_M \begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

In this case the actual Transmission matrix is not uniquely determined by the SOL calibration even though it was applied to both test ports , i.e., both connectors 1 and 2. A forth measurement of a calibration standard is required. This can be achieved by connecting the test ports together and observing the measured data. In this case the cal standard is just a series short between the two connectors. This calibration standard is called a "Thru." The S-parameters for a thru are

$$S_T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and the transmission matrix for a thru is

$$R_T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I = identymatrix$$

If R_{MT} represents the transmission matrix for the measurements of the thru standard then

$$I = \frac{e_{21}}{f_{21}} \frac{1}{A - BC} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} R_{MT} \begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

and

$$\frac{f_{21}}{e_{21}} I = \frac{1}{A - BC} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} R_{MT} \begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}.$$

The term $\frac{f_{21}}{e_{21}}$ can be computed by taking the determinant of the previous equation which results in

$$\frac{f_{21}}{e_{21}} = \frac{1}{A-BC} \cdot \det\begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} \cdot \det(R_{MT}) \cdot \det\begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

$$\left(\frac{f_{21}}{e_{21}}\right)^2 = \frac{\det(R_{MT})}{(A-BC)} \cdot \det\begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

$$\left(\frac{f_{21}}{e_{21}}\right)^2 = \det(R_{MT}) \cdot \frac{(A'-B'C')}{(A-BC)}$$

Substitution of this allows the actual transmission matrix for the D.U.T. to be determined, i.e.,

$$R_A = \frac{e_{21}}{f_{21}} \frac{1}{A-BC} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} R \begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

$$R_A = \frac{1}{\sqrt{(A-BC)(A'-B'C')\det(R_{MT})}} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} R_M \begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

$$R_A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \frac{1}{\sqrt{(A-BC)(A'-B'C')\det(R_{MT})}} \begin{pmatrix} 1 & -B \\ -C & A \end{pmatrix} R_M \begin{pmatrix} 1 & C' \\ B' & A' \end{pmatrix}$$

$$S_M = \begin{pmatrix} s_{11M} & s_{12M} \\ s_{21M} & s_{22M} \end{pmatrix} = \frac{1}{a_{22}} \begin{pmatrix} a_{12} & \Delta_a \\ 1 & -a_{21} \end{pmatrix}$$

7.5 TWO-PORT CALIBRATION--THRU, REFLECTION, LINE (TRL)

If microstrip circuits are measured usually the board is mounted in a fixture which permit connectors to interface with the board. The SOLT calibration procedure can be used but the de-embedded data now represents the connector/transition hardware plus the microstrip board. To deduce the uncontaminated board data requires knowledge of s-parameters of the connectors/transition system. This is illustrated in figure 7.9

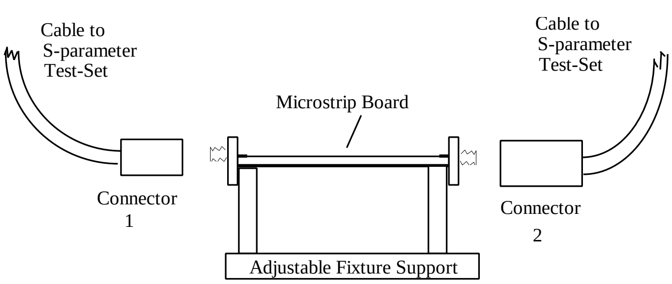


Figure 7.9 S-parameter measurement of a microstrip circuit.

It is desired now to create a calibration procedure which will permit correction for system variations from the ideal but will also permit the fixture effects to be de-embedded from the data. This is done by creating **calibration microstrip circuits on the same substrate** used for the circuit of interest. First a **Thru** is created which consists of a short 50 microstrip line. The center of this line sets the reference location for calibration and the resulting de-embedded data. This is shown in figure 7.10a. The **reflect** standard can be either an open or short circuited line 50 ohm line. For microstrip it is usually easier to use an open circuited line. The length of the line should position of the open (or short) at the reference plane of the measurement as set by the thru line. This is illustrated in figure 7.10b. The **Line** standard is a 50 line of unknown length. Amazing enough the calibration technique will determine the length of this line. The line must be longer than the thru. This is illustrated in figure 7.10c. .

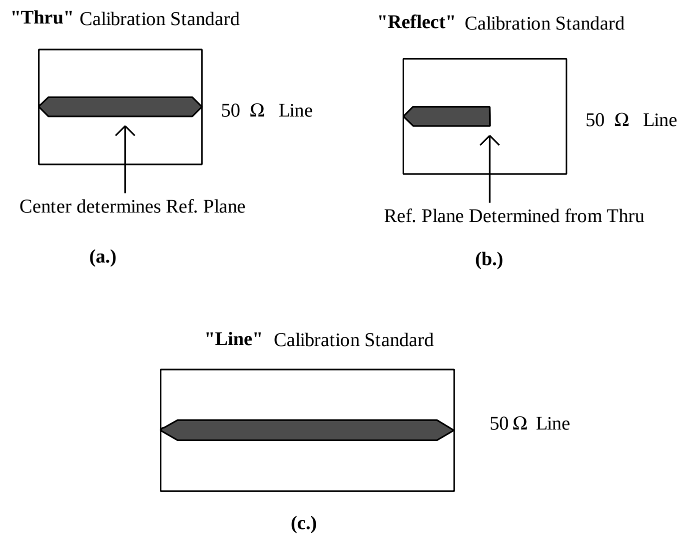


Figure 7.10. Top view of TRL microstrip calibration standards

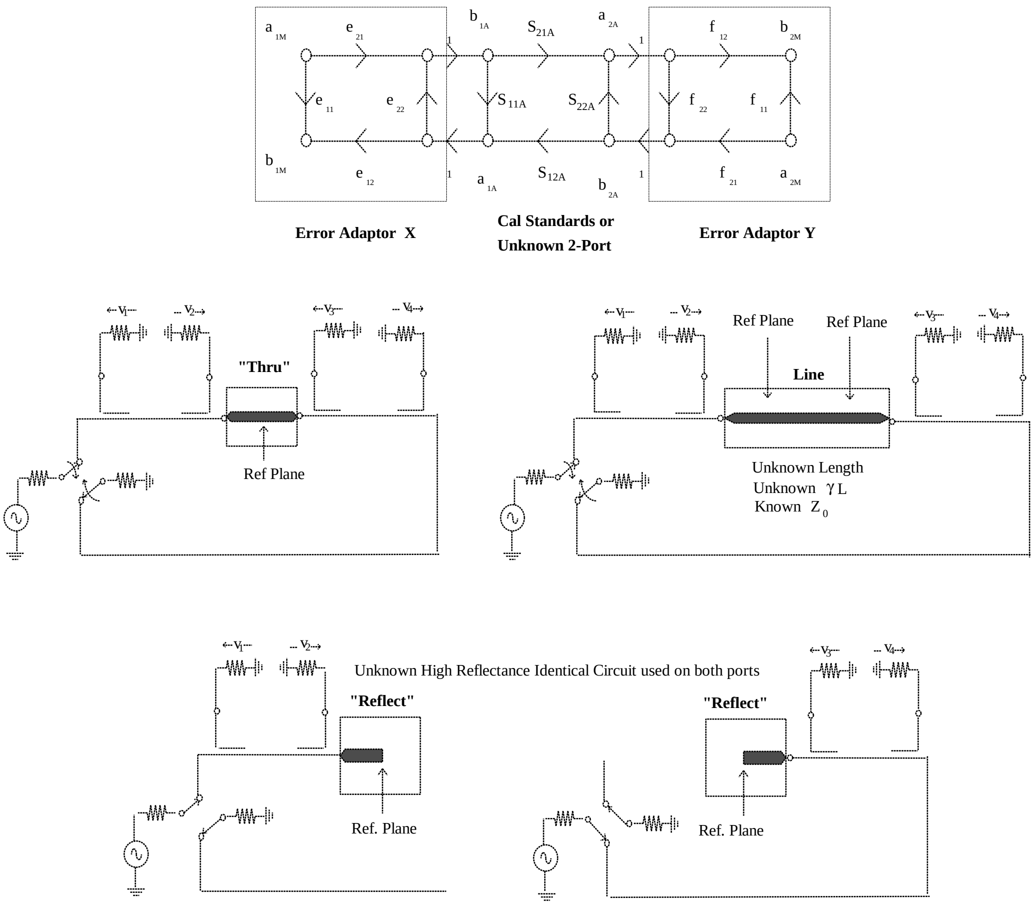


Figure 7.11 Steps in a TRL calibration

The transmission (R) matrices for the x error adapter and y error adapter are given by

$$R_X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \frac{1}{e_{21}} \begin{pmatrix} -\Delta_e & e_{11} \\ -e_{22} & 1 \end{pmatrix} \text{ and } R_Y = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \frac{1}{f_{12}} \begin{pmatrix} -\Delta_f & f_{22} \\ -f_{11} & 1 \end{pmatrix}$$

The transmission (R) matrices associated with the measured and actual S-parameters are given by

$$R_M = \begin{pmatrix} r_{11M} & r_{12M} \\ r_{21M} & r_{22M} \end{pmatrix} = \frac{1}{S_{21M}} \begin{pmatrix} -\Delta_{SM} & S_{11M} \\ -S_{22M} & 1 \end{pmatrix} \text{ and } R_A = \begin{pmatrix} r_{11A} & r_{12A} \\ r_{21A} & r_{22A} \end{pmatrix} = \frac{1}{S_{21A}} \begin{pmatrix} -\Delta_{SA} & S_{11} \\ -S_{22} & 1 \end{pmatrix}$$

The R matrices for an ideal thru (short circuit) and a line of length, "L" and characteristic impedance Z_0 are

$$R_{AT} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } R_{AL} = \begin{pmatrix} e^{-\gamma L} & 0 \\ 0 & e^{+\gamma L} \end{pmatrix}$$

In general the measured R matrix is related to the actual R matrix as shown.

$$R_M = R_X R_A R_Y$$

The measured R matrix when the thru calibration circuit is inserted is given by

$$R_{MT} = R_X R_{AT} R_Y = R_X R_Y$$

The measured R matrix when the line calibration circuit is inserted is given by

$$R_{ML} = R_X R_{AL} R_Y$$

Solving for the Y adapter R matrix gives

$$R_Y = R_X^{-1} R_{MT}$$

Substitution of R_Y into the previous matrix equation gives

$$M R_X = R_X R_{AL}$$

where

$$M = R_{ML} R_{MT}^{-1} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} \begin{pmatrix} e^{-\gamma L} & 0 \\ 0 & e^{+\gamma L} \end{pmatrix}$$

which implies

$$m_{11} x_{11} + m_{12} x_{21} = x_{11} e^{-\gamma L}$$

$$m_{21} x_{11} + m_{22} x_{21} = x_{21} e^{-\gamma L}$$

$$m_{11} x_{12} + m_{12} x_{22} = x_{12} e^{+\gamma L}$$

$$m_{21} x_{12} + m_{22} x_{22} = x_{22} e^{+\gamma L}$$

Dividing the first two equations and dividing the third and forth equation results in two quadratic equations with the same constant terms

$$m_{21} \left(\frac{x_{11}}{x_{21}} \right)^2 + (m_{22} - m_{11}) \frac{x_{11}}{x_{21}} - m_{12} = 0$$

$$m_{21} \left(\frac{x_{12}}{x_{22}} \right)^2 + (m_{22} - m_{11}) \frac{x_{12}}{x_{22}} - m_{12} = 0$$

Let the roots be a and b with $|a| > |b|$ then

$$a = \frac{x_{11}}{x_{21}} = e_{11} - \frac{e_{21} e_{12}}{e_{22}}$$

$$b = \frac{x_{12}}{x_{22}} = e_{11}$$

$$e_{11} = b$$

$$\frac{e_{21}e_{12}}{e_{22}} = b - a$$

In a similar manner one can solve for the x adapter matrix, R_X, and substitute it into the matrix equation for the line to get

$$R_Y N = R_{AL} R_Y$$

$$N = R_{MT}^{-1} R_{ML} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}$$

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}\begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} = \begin{pmatrix} e^{-\gamma \ell} & 0 \\ 0 & e^{+\gamma \ell} \end{pmatrix}\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$$

$$y_{11}n_{11} + y_{12}n_{21} = y_{11}e^{-\gamma \ell}$$

$$y_{11}n_{12} + y_{12}n_{22} = y_{12}e^{-\gamma \ell}$$

$$y_{21}n_{11} + y_{22}n_{21} = y_{21}e^{+\gamma \ell}$$

$$y_{21}n_{12} + y_{22}n_{22} = y_{22}e^{+\gamma \ell}$$

$$n_{12}\left(\frac{y_{11}}{y_{12}}\right)^2 + (n_{22} - n_{11})\frac{y_{11}}{y_{12}} - n_{21} = 0$$

$$n_{12}\left(\frac{y_{21}}{y_{22}}\right)^2 + (n_{22} - n_{11})\frac{y_{21}}{y_{22}} - n_{21} = 0$$

Let roots be c and d where $|c| > |d|$

$$c = \frac{y_{11}}{y_{12}} = -f_{11} + \frac{f_{12}f_{21}}{f_{22}}$$

$$d = \frac{y_{21}}{y_{22}} = -f_{11}$$

$$f_{11} = -d$$

$$\frac{f_{12}f_{21}}{f_{22}} = c - d$$

When the high Reflectance calibration standard is measured with port 1, with the x adapter then the measured reflection coefficient is G_{MX} with actual reflectance given by G_R. In that case

$$G_{MX} = e_{11} + \frac{e_{21}e_{12}G_R}{1 - e_{22}G_R}$$

Solving for the actual reflection coefficient gives

$$G_R = \frac{1}{e_{22}} \frac{b - G_{MX}}{a - G_{MX}}$$

An identical G_R connected to port 2 and measured through the y error adapter circuit to give

$$G_{MY} = f_{11} + \frac{f_{21} f_{12} G_R}{1 - f_{22} G_R}$$

Again solving for the actual reflection coefficient gives

$$G_R = \frac{1}{f_{22}} \frac{d + G_{MY}}{c + G_{MY}}$$

Equating the two expressions for the actual high reflectance gives

$$\frac{1}{f_{22}} = \frac{1}{e_{22}} \left(\frac{b - G_{MX}}{a - G_{MX}} \right) \left(\frac{c + G_{MY}}{d + G_{MY}} \right)$$

A second equation is needed to solve for e_{22} and f_{22}. The measured input reflection coefficient obtained when the thru calibration standard is connected is called G_{M1}, and is related to the error parameters by

$$G_{M1} = e_{11} + \frac{e_{21}e_{12} f_{22}}{1 - e_{22} f_{22}}$$

Solving for e_{22} gives

$$e_{22} = \frac{1}{f_{22}} \frac{b - G_{M1}}{a - G_{M1}}$$

Substitution of $\frac{1}{f_{22}}$ from previous equation gives

$$e_{22}^2 = \left(\frac{b - G_{MX}}{a - G_{MX}} \right) \left(\frac{c + G_{MY}}{d + G_{MY}} \right) \left(\frac{b - G_{M1}}{a - G_{M1}} \right)$$

When square root is taken there is a ± ambiguity. One can use the implication for the reflectance cal standard to resolve the ambinguity, i.e., $G_R \approx +1$ for

$$G_R = \frac{1}{e_{22}} \frac{b - G_{MX}}{a - G_{MX}}$$

After determining e_{22} then

$$f_{22} = \frac{1}{e_{22}} \frac{b - G_{M1}}{a - G_{M1}}$$

The following pair of products are now also known

$$e_{21}e_{12} = (b-a)e_{22}$$

$$f_{21}f_{12} = (c-d)f_{22}$$

Additional products can be found by using the transfer measurements obtained for the thru calibration standard, i.e.,

$$S_{21MT} = \frac{e_{21}f_{12}}{1-e_{22}f_{22}} \text{ and } S_{12MT} = \frac{f_{21}e_{12}}{1-e_{22}f_{22}}$$

These result in a determination of the following products

$$e_{21}f_{12} = S_{21MT}(1-e_{22}f_{22})$$

$$f_{21}e_{12} = S_{12MT}(1-e_{22}f_{22})$$

$$R_M = \frac{1}{e_{21}}\begin{pmatrix} -\Delta_e & e_{11} \\ -e_{22} & 1 \end{pmatrix} R_A \frac{1}{f_{12}}\begin{pmatrix} -\Delta_f & f_{22} \\ -f_{11} & 1 \end{pmatrix}$$

where

$$\Delta_e = e_{11}e_{22} - e_{12}e_{21} = be_{22} - (b-a)e_{22} = ae_{22}$$

$$\Delta_f = f_{11}f_{22} - f_{12}f_{21} = -df_{22} - (c-d)f_{22} = -cf_{22}$$

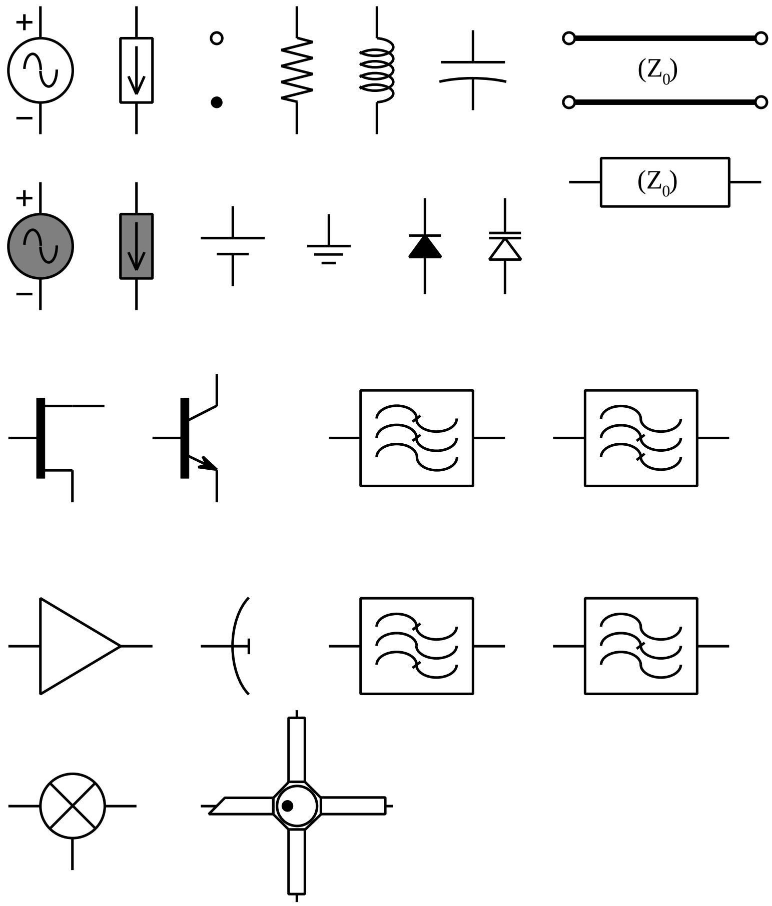
Therefore

$$R_M = \frac{1}{e_{21}f_{12}}\begin{pmatrix} -ae_{22} & e_{11} \\ -e_{22} & 1 \end{pmatrix} R_A \begin{pmatrix} cf_{22} & f_{22} \\ -f_{11} & 1 \end{pmatrix}$$

$$\boxed{R_A = (e_{21}f_{12})\begin{pmatrix} -ae_{22} & b \\ -e_{22} & 1 \end{pmatrix}^{-1} R_M \begin{pmatrix} cf_{22} & f_{22} \\ d & 1 \end{pmatrix}^{-1}}$$

The wave number, γ can now be found since

$$e^{-\gamma \ell} = m_{11} + \frac{m_{12}}{a}$$


(Z0)
(Z0)