CHAPTER

9

COUPLERS, COMBINERS AND DIVIDERS

9.1 DIRECTIONAL COUPLERS

Directional couplers are an important category for passive microwave circuits. They are implemented in a variety of ways resulting in a range of capabilities and constraints. Directional couplers are used to sample propagating microwave energy for the purpose of monitoring or measuring. Also directional couplers are used to divide signals from a single channel into multiple channels in both small signal and large signal applications.

The most general definition for a directional coupler describes it as a multi-port (usually 4 port) matched, lossless, reciprocal circuit that has an isolated port which depends upon the incident signal port. Four port directional couplers are usually represented schematically by two single line representations of transmission lines with a crossed line between them to represent the coupling between the lines, as shown in Figure 1. We will see that this coupling between lines can occur by providing an electrical connection between the lines (branch line or Wilkinson's coupler) or by bringing two lines close together so that their magnetic and electric fields interact (parallel line or Lange couplers).

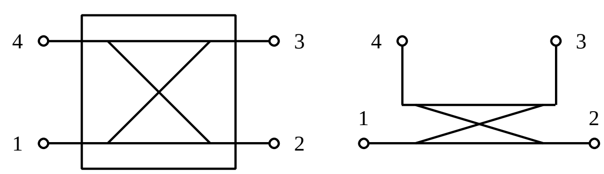


Figure 1. Schematics of four port directional coupler.

The above definition of a directional coupler requires that a number of the s-parameters be equal to zero. These are shown together with the part of the definition that requires that they vanish.

$$\text{Matched} \Rightarrow S_{11} = S_{22} = S_{33} = S_{44} = 0$$

$$\text{Isolated Output} \Rightarrow S_{41} = S_{23} = 0$$

$$\text{Reciprocal} \Rightarrow S_{ij} = S_{ji}$$

A passive lossless circuit implies that the matrix of s-parameters must be a unitary matrix. This means that the matrix when multiplied by its transposed complex conjugate results in the identity matrix (i.e. ones on the diagonal and zeros otherwise). This is a mathematical condition which results from requiring that the total power exiting the circuit exactly equals the total power entering the circuit regardless of the load or source configurations. Such a product is

$$\begin{pmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & S_{12}^* & S_{13}^* & 0 \\ S_{12}^* & 0 & 0 & S_{24}^* \\ S_{13}^* & 0 & 0 & S_{34}^* \\ 0 & S_{24}^* & S_{34}^* & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Carrying out the matrix multiplication results in a set of equations for the various s-parameters. Two such equations are shown

$$S_{12}S_{24}^* = -S_{13}S_{34}^* \text{ and } S_{12}S_{13}^* = -S_{24}S_{34}^*$$

with the first resulting from the top row multiplied with the last column and the second resulting from the second row multiplied with the third column. Division of the two equations results in

$$\frac{S_{24}^*}{S_{13}^*} = \frac{S_{13}}{S_{24}} \Rightarrow S_{24}S_{24}^* = S_{13}S_{13}^* \Rightarrow |S_{24}| = |S_{13}|$$

Similarly, another pair of equation can be obtained to show that

$$|S_{12}| = |S_{34}|$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{21}|^2 + |S_{31}|^2 = 1$$

This latter equation shows that the power into port 1 equals the power out of ports 2 and 3. Note that in this case the power out of port 4 is zero, i.e., port 4 is the isolated port when power is incident into port 1. The port designation of a four port directional coupler with port 1 as the input port is shown below.

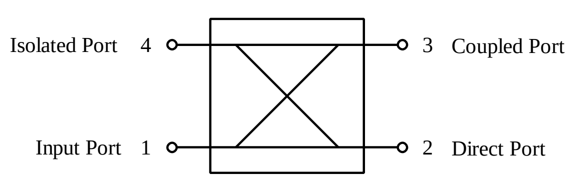


Figure 2. Port designation of a four port directional coupler.

PERFORMANCE MEASURES

Coupler performance is characterized by five parameters defined below.

$$\textit{Return Loss} = -20\log|S_{11}|$$

$$\begin{aligned} \textit{Insertion Loss} &= -20\log\left|S_{21}\right| \\ \textit{Coupling} &= -20\log\left|S_{31}\right| \\ \textit{Isolation} &= -20\log\left|S_{41}\right| \\ \textit{Directivity} &= -20\log\left|\frac{S_{41}}{S_{31}}\right| \end{aligned}$$

Return Loss is a measure of the input impedance match. The term loss is somewhat anomalous. If the input match is good then the input reflection coefficient is small and only a small amount of incident power is reflected back. Abstractly, the power which passes into the circuit can be thought of as power lost to the feeding circuit. If the reflection coefficient is low then the loss is high. If the reflection coefficient is high (i.e., $|\Gamma| \approx 1$) then the return loss is low.

Insertion Loss is a measure of the amount of power which enters the coupler but fails to make it out of the direct port. A large fraction of the power coupled away would represent a large loss of power to the direct port. When couplers are inserted into a circuit for the purpose of measuring the propagating energy, a low insertion loss is desired to insure that the original circuit is not modified by the measuring system.

The *Coupling* coefficient is a measure of the amount of power coupled away from the direct port. This coefficient is often used alone to identify a coupler. A 3 dB coupler is one in which half the power is coupled away from the direct port, i.e., the power is split between the coupled and direct port. A 20 dB coupler is one in which one hundredth of the power is coupled away.

The *Isolation* is a measure of how well the coupler prevents power from exiting the isolated port. For an ideal coupler the isolation would equal $-\infty$.

Directivity is a measure of the fraction of coupled power which exits from the coupled port when compared to that which exits from the isolated port.

COUPLER SCATTERING MATRIX

Since $|S_{12}| = |S_{34}|$ then sections of transmission lines can be added to ports 2 and 4, as shown in Figure 3, so that these s-parameters are equal in phase as well as magnitude.

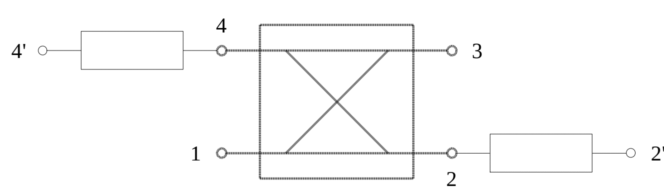


Figure 3. Transmission lines added to ports 2 and 4.

In fact, by adding the right length lines the phase can be made equal to zero, i.e., the s-parameters are real numbers. Therefore,

$$S_{12'} = S_{34'} = positive real number$$

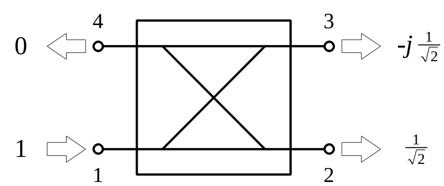
We now assume that the original coupler had been modified so that this is true and the prime notation is therefore dropped. S_{12} is represented as a postive real number C_1 and S_{13} is then represented as a complex number with magnitude C_2 and phase θ. All of the other s-parameters are then determined in terms of C_1, C_2, and θ. C_2 is called the coupling coefficient and $|C_1|^2 + |C_2|^2 = 1$. The general coupler scattering matrix is therefore

$$S = \begin{pmatrix} 0 & C_1 & C_2 e^{jq} & 0 \\ C_1 & 0 & 0 & C_2 e^{j(p-q)} \\ C_2 e^{jq} & 0 & 0 & C_1 \\ 0 & C_2 e^{j(p-q)} & C_1 & 0 \end{pmatrix}$$

TWO TYPES OF 4-PORT COUPLERS

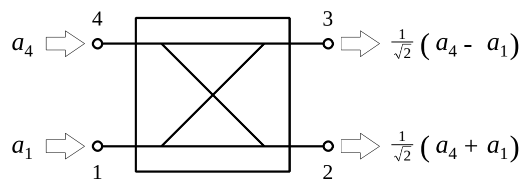
If $q = -p/2$ and $C_1 = C_2 = \frac{1}{\sqrt{2}}$, then the resulting coupler is called a 3 dB hybrid. The magnitude of the direct and coupled output are equal but the phase of the coupled output lags behind the direct port by 90 degrees. The output contains an in-phase and quadrature channel. The scattering matrix, as well as the schematic showing the incident and exiting voltages at each port, are shown below.

$$S = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -j\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -j\frac{1}{\sqrt{2}} \\ -j\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$



If $q = p$ and $C_1 = C_2 = 1/\sqrt{2}$, then a sum and difference coupler results. Incident power in port 1 splits with a ± phase. However, incident power in the isolated port splits evenly with both outputs having a + phase.

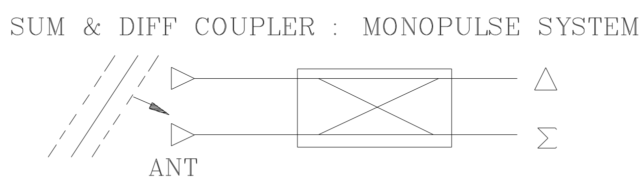
$$S = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$



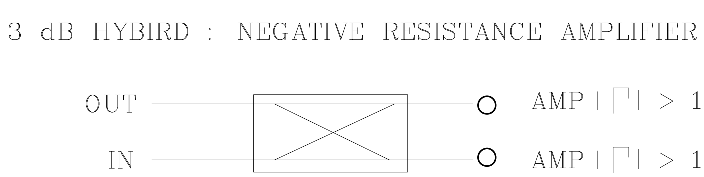
EXAMPLES OF COUPLER APPLICATIONS

Couplers are used in circuits to generate separate signal channels with desirable characteristics. Several examples serve to illustrate such applications.

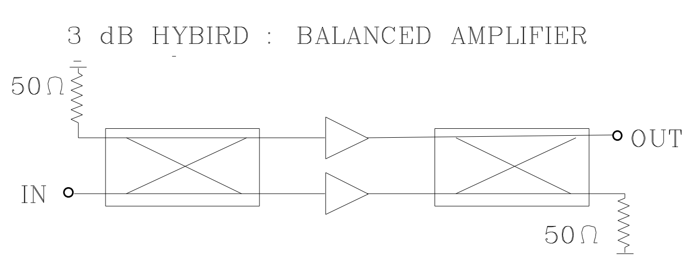
A sum and difference coupler can be used in a monopulse receiver/direction finding system. The input and isolated ports are connected to two identical antennas. A received signal then produces a sum Σ and difference Δ signal which depends upon the angle of incidence of the receive signal. If the signal is incident from a normal direction, for example, the Δ signal would be zero.



A 3dB hybrid is important in the application of negative resistance amplifiers. Negative resistance devices have a reflection coefficient which is greater than one, i.e., the reflected power exceeds the incident power. The 3dB hybrid can be used to separate the input and the output channels for such devices.

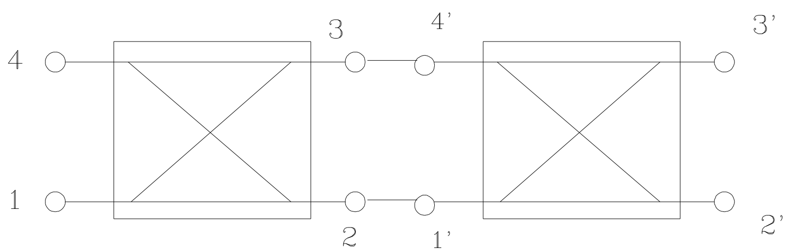


Two 3dB hybrids can be used to build a balanced amplifier. Such circuits combine two identical amplifying elements with couplers at the input and output. Mismatches at the input or output of the amplifying elements cancel after passing through the couplers. The couplers allow one to design the amplifying elements to maximize performance of gain, noise figure, or bandwidth without having to be constrained by matching requirements. Reflected signals resulting from mismatches of the amplifying elements are absorbed by the matched loads at the input and output couplers.

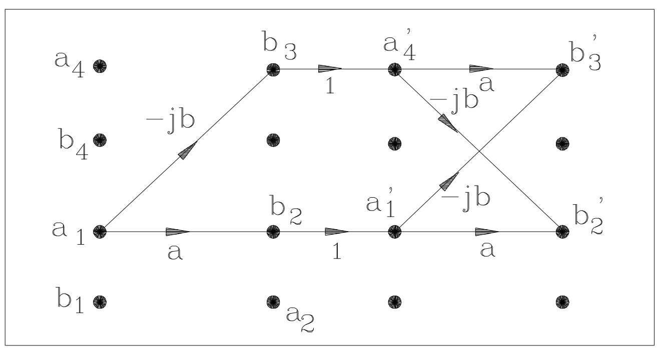


Loosely coupled direction coupler can be cascaded to increase the overall coupling coefficient as shown below. This arrangement is useful if tight coupling is difficult to achieve with a single coupler, e.g. parallel line coupler.

CASCADED DIRECTIONAL COUPLERS



$$S_{21} = a \qquad S_{31} = -jb$$



$$\tilde{S}_{21} = a^2 - b^2 = 1/\sqrt{2} \qquad\Longrightarrow\qquad a = -.68 \text{ dB}$$

$$\tilde{S}_{31} = -j2ab = -j\,1/\sqrt{2} \qquad\qquad b = -8.34 \text{ dB}$$

$$\boxed{\text{TWO CASCADED 8.34 dB COUPLERS} \equiv \text{3 dB COUPLER}}$$

9.2 EVEN & ODD MODE ANALYSIS

Another analysis technique which is often useful in the analysis of multi-port circuits is the use of even and odd modes. Since s-parameters are determined by feeding an incident signal (analytically this is usually taken as 1) into a port and look at the signals that come out of all ports (assumed to be matched), the approach consists of decomposing the input signal into a set of mode signals where the superposition of the modes equals the original incident signal at the input port and equals zero at all other ports. Usually this decomposition results in modes having the same sign (+) for the incident port and opposite signs (±) for all other ports. Thus, the set of signals for one mode all have the same sign (+) and the set of signals for the other mode have a mixture of (+) and (-). The first set is referred to as the *even mode* while the second set is called the *odd mode*.

The circuit is then analyzed by looking at each of the modes. Usually, the circuit can be immediately simplified on the basis of symmetry. That is, for the even mode (input signals have same sign) points in the circuit can be identified where the current must go to zero. These circuit points can be replaced with open circuits. Likewise for the odd mode circuit points can be identified where the current must vanish. These points can be replaced by short circuits to ground.

The output for each of the modes is then determined with the subscript "EV" and "OD" indicating that the output signal results from the *even* and *odd* mode sources, respectively. The output from the original problem is then the combined output of the even and odd modes. The combined output equals the various s-parameters if the input was specified to be equal to 1.

The analysis of the individual circuits associated with the modes often requires the determination of s-parameters which can be facilitated by the Kirchhoff's Law approach described earlier.

EVEN AND ODD MODE ANALYSIS

APPLICATION
Symmetric Circuits
Multi-port Circuits

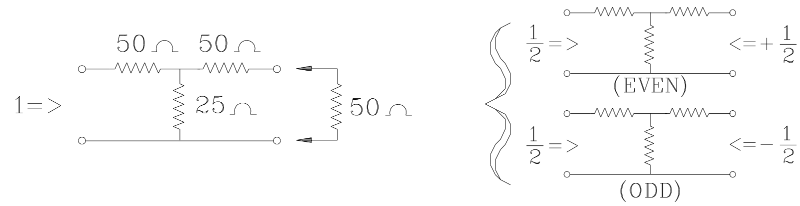
APPROACH
Decompose Voltage signal (Sum & Difference)
Solve via Superposition
Use symmetry to simplify individual circuits
Combined Output Signal = S-paraemters

NOTE
Kirchhoff Law Approach Useful to Compute S-Parameters

EVEN & ODD MODE ANALYSIS (EXAMPLE)

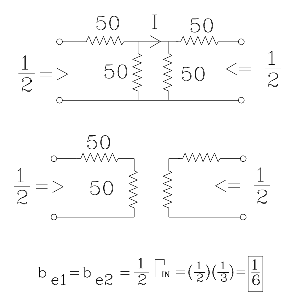
The even and odd mode analysis technique is illustrated first by solving a simple circuit consisting of a configuration of resistors. To find the s-parameters of the circuit we must assume an incident signal equal to 1 with all ports matched and determine the reflected (b_1) and transmitted signal (b_2) which will equate to the S_{11} and S_{22}, respectively.

Thus, we start with an incident signal of 1 into port 1 and an incident signal of 0 into port 2. This is equivalent to the superposition of even and odd modes consisting of +1/2 , +1/2 , and +1/2 , -1/2 signals into ports 1 and 2, respectively.



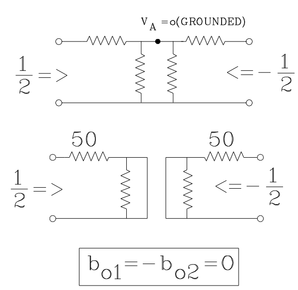
The 25Ω shunt resistor can be divided into two 50Ω parallel resistors. When we do that we see that for the even mode the current in the middle of the circuit must vanish since the even inputs would exactly oppose each other. The two ports are then separated and the output for each port is independent. However, the circuits are seen to be the same so only one circuit needs to be solved. The even output will always be seen to be equal.

EVEN MODE ANALYSIS



For the odd mode the voltage at the circuit center is seen to be zero since the odd inputs are equal magnitude but opposite sign. This point in the circuit can be replaced with a short circuit. When this is done two independent (but identical) circuits result. The reflection coefficient can be obtained by finding the input impedance and the output signals obtained. For the odd modes the output signals will always be the negative of each other.

ODD MODE ANALYSIS



The s-parameters for the original circuit are then found by combining the even and odd output for the respective ports.

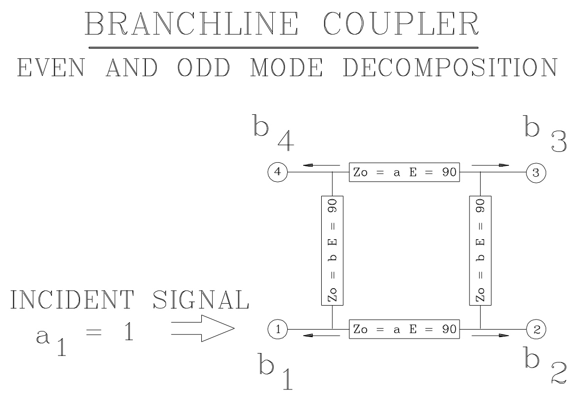
$$S_{11} = \mathrm{b}_{e1} + \mathrm{b}_{o1} = \tfrac{1}{6}$$
$$S_{21} = \mathrm{b}_{e2} + \mathrm{b}_{o2} = \tfrac{1}{6}$$

9.3 BRANCHLINE COUPLER

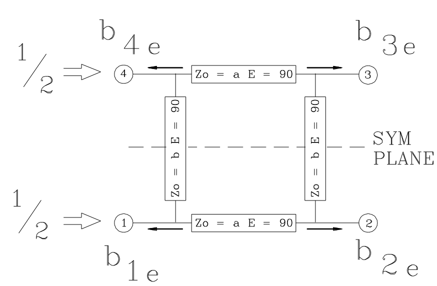
A branchline coupler is a four port circuit consisting to two quarter wave lines coupled together by means of two quarter wave branch lines connected at the ends. In the case that we will consider the branch lines have a characteristic impedance $Z_0 = \mathrm{b}$ while the other pair have a characteristic impedance $Z_0 = \mathrm{a}$. After computing the s-parameters as a function of a and b we will see that we can adjust the performance of the coupler by selection of these values.

EVEN AND ODD MODE DECOMPOSITION

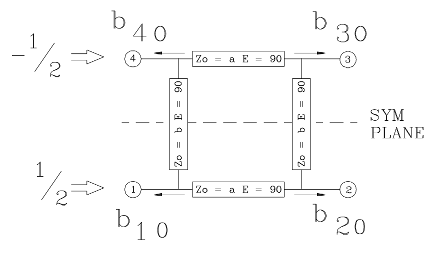
The s-parameters are found by assuming that all ports are matched and that a signal of 1 is incident upon port 1. The s-parameters are equivalent to the resulting signals exiting from the respective ports. The exiting signals are determined by decomposing the incident signal into even and odd modes similar to the previous example.



The even mode consists of signals of +1/2 and +1/2 incident upon ports 1 and 4. The odd mode consists of signals of +1/2 and -1/2 incident upon ports 1 and 4, respectively. The exiting signals are denoted by and additional subscript "e" or "o" to designate, respectively, the even or odd mode from which they were produced.



EVEN MODE

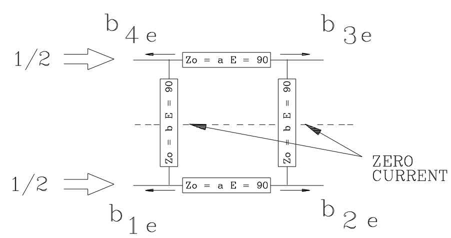


ODD MODE

This procedure determines S_{11}, S_{12}, S_{13}, and S_{14}. From the symmetry of the circuit one can see that all other s-parameters are equal to one of these. Therefore this procedure will determine all of the s-parameters associated with the branchline coupler.

EVEN MODE ANALYSIS

For the even mode analysis we see that two center points exist where the line of symmetry intersects the circuit. The current at these two points must be zero. Consequently, the circuit can be opened at these points without changing the circuit performance. The even mode circuit results in two independent and identical circuits (as seen in the earlier examples) referred to as reduced circuits. The reduced circuits consist of two shunt λ/8 open stubs connected at the ends of λ/4 lines. From symmetry we see that the exiting signals from ports 1, 4, and 2, 3 are respectively, equal.



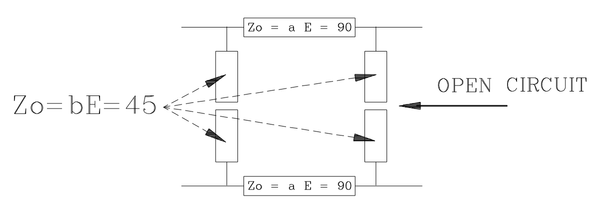
EVEN MODE

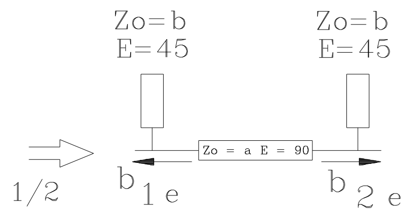
$$S_{11} = b_1 = b_{1e} + b_{1o}$$

$$S_{21} = b_2 = b_{2e} + b_{2o}$$

$$S_{31} = b_3 = b_{3e} + b_{3o}$$

$$S_{41} = b_4 = b_{4e} + b_{4o}$$

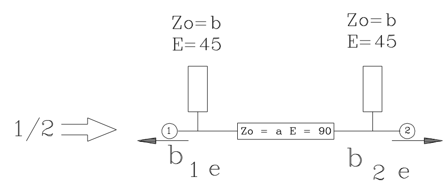




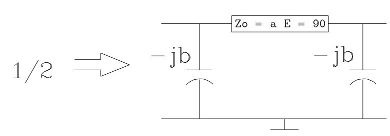
$$\mathrm{b}_{4\mathrm{e}} = \mathrm{b}_{1\mathrm{e}}$$

$$\mathrm{b}_{3\mathrm{e}} = \mathrm{b}_{2e}$$

We look first at the input side of the coupler, i.e., we will determine the s-parameters S_{11} and S_{41} first. This means we need to determine b_{1e} (since $b_{4e} = b_{1e}$) with all ports matched. We do this by determining the input reflection coefficient using the Kirchhoff's Law approach described earlier.

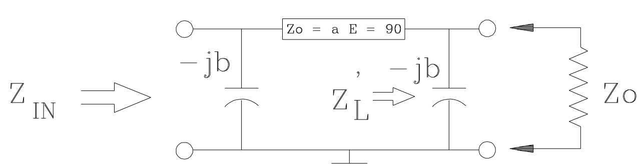


The shunt open circuit λ/8 lines appear as capacitances across the λ/4 line. This results from the fact that the input impedance of a open circuit transmission line is given by $Z_{oe} = -jZ_o \cot\theta$.



where θ = electrical length of the open line. In our case θ = 45 degrees since the open circuit line length is λ/8 and $Z_o = b$.

The combined load consisting of the parallel combination of a resistor of value Z_o and a capacitor with reactance $-j\mathrm{b}$ must be found. This is done by first connecting a load $Z_L = Z_o$ and finding Z_L' and then Z_{IN} .



$$Z_L' = \frac{-jbZ_o}{Z_o - jb}$$

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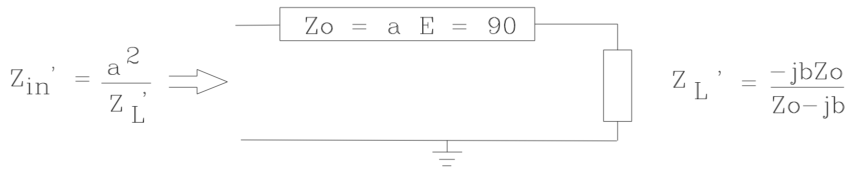
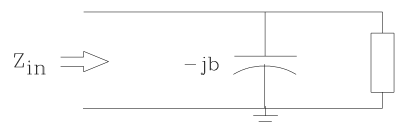
The input impedance looking into the l/4 line is found using the impedance transforming properties of quarter wave lines.



The input impedance looking into the circuit can then be found by combining that result with a shunt capacitance.



$$Z_{IN} = \frac{-ba^2(b + jZ_o)}{Z_o(a^2 - b^2) - ja^2 b}$$

The reflection coefficent is found from the input impedance Z_{IN} using the formula

$$\Gamma_{IN} = \frac{Z_{IN} - Z_o}{Z_{IN} + Z_o}$$

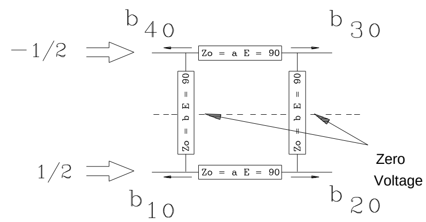
and the exiting signal b_{1e} is then found by multiplying the reflection coefficient by the incident signal which is 1/2.

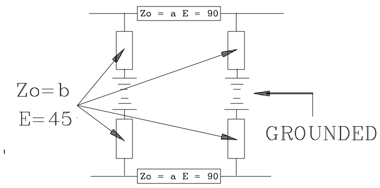
$$b_{1e} = \frac{1}{2}\Gamma_{IN}$$

$$b_{1e} = \frac{-j(a^2b^2 + a^2Z_o^2 - b^2Z_o^2)}{4Z_o a^2 b + 2j(a^2Z_o^2 - b^2Z_o^2 - a^2b^2)}$$

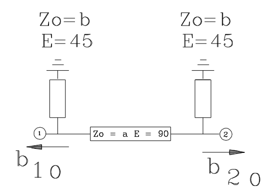
ODD MODE ANALYSIS

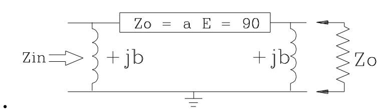
The odd mode analysis of the input side of the circuit is performed in a similar manner. Two symmetric points exist on the circuit where the voltage must be zero. A short to ground can be inserted in the circuit at these points without affecting the performance of the circuit.





This results in two independent identical circuit each consisting of a λ/4 line with shunt short circuited λ/8 stubs at each end.





From symmetry we see that $b_{4o} = -b_{1o}$ since the port 4 circuit is the same but the incident signal is negative (i.e., - 1/2). A similar relationship exists for the output side of the circuit.

The input impedance for the odd mode two port circuit can be found using the same technique described for the even mode. However, a short cut can be taken to obtain the final answer by recognizing that the input impedance for the odd mode circuit must equal the expression obtained for the even mode circuit with -*j* substituted for *j* (i.e., the shunt capacitance is substituted for a shunt inductance).

$$b_{1o} = \frac{1}{2}\Gamma_{IN}$$

$$b_{1o} = \frac{j\left(a^2b^2 + a^2Z_0^2 - b^2Z_0^2\right)}{4Z_0a^2b - 2j\left(a^2Z_0^2 - b^2Z_0^2 - a^2b^2\right)}$$

The input reflection coefficient S_{11} can now be found by adding the even and odd exiting signals from port 1 ($b_{1e} + b_{1o}$).

$$S_{11} = b_1 = b_{1e} + b_{1o}$$

$$S_{11} = \frac{AB}{4\left(Z_o a^2 b\right)^2 + B^2}$$

$$A = a^2 b^2 + a^2 Z_0^2 - b^2 Z_0^2$$

$$B = a^2 b^2 - a^2 Z_0^2 + b^2 Z_0^2$$

If we require that the input be matched i.e., $S_{11} = 0$ then either A or B must be zero. For this to be true (which we shall require) then a and b can not be independently specified but must be related as shown.

$$Case1(A = 0)a^2 = \frac{b^2 Z_0^2}{b^2 + Z_0^2}$$

$$Case2(B = 0)b^2 = \frac{a^2 Z_0^2}{a^2 - Z_0^2}$$

We will examine the Case 1 where $A = 0$. The other case results ultimately in the same coupler, but the port identifications are permuted.

We now look at the output side of the circuit, starting with the even mode circuit. The exiting signal can be found using the Kirchhoff's Law approach. Since we are assuming the case 1 condition then A = 0. This means that

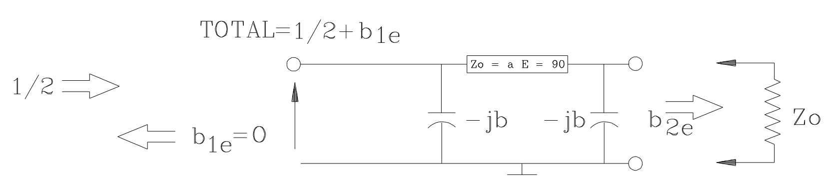
$$a^2 = \frac{b^2 Z_0^2}{b^2 + Z_0^2}$$

Substitution of this into the expression for b_{1e} results in

$$b_{1e} = 0$$

Thus, the condition for matching $S_{11} = 0$ means that the even mode circuit is also matched. The same result follows for the odd circuit.

The total input signal, therefore, equals 1/2 since there is no reflection.

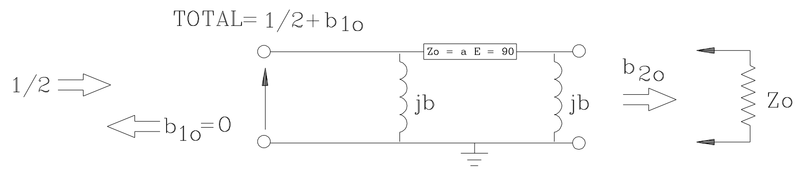


The signal at the output can then be found by using the voltage transfer relationship for a λ/4 line.

$$\frac{b_{2e}}{\frac{1}{2} + b_{1e}} = \frac{b_{2e}}{\frac{1}{2}} = -j\frac{Z_L}{a}$$

$$b_{2e} = -\frac{bZ_0}{2a}\left(\frac{1}{Z_0 - jb}\right)$$

Analysis of the odd mode circuit follows in a similar manner.



The short-cut substitution of -j for j does not apply to this case because the voltage transfer relationship includes "-j". The λ/4 voltage transfer relationship gives

$$\frac{b_{2o}}{\frac{1}{2}+b_{1o}} = \frac{b_{2o}}{\frac{1}{2}} = -j\frac{Z_L}{a}$$

$$b_{2o} = \frac{bZ_0}{2a}\left(\frac{1}{Z_0 + jb}\right)$$

The signal exiting port 2 is found by adding the exiting even and odd mode signals $b_{2e} + b_{2o}$, or

$$S_{21} = b_2 = b_{2o} + b_{2e}$$

$$S_{21} = \frac{bZ_0}{2a}\left(-\frac{1}{Z_0 - jb} + \frac{1}{Z_0 + jb}\right) = -j\frac{b^2 Z_0}{a\left(Z_0^2 + b^2\right)}$$

Substitution of the case 1 (A=0) expression

$$a = \frac{bZ_0}{\sqrt{b^2 + Z_0^2}}$$

results in the following expression

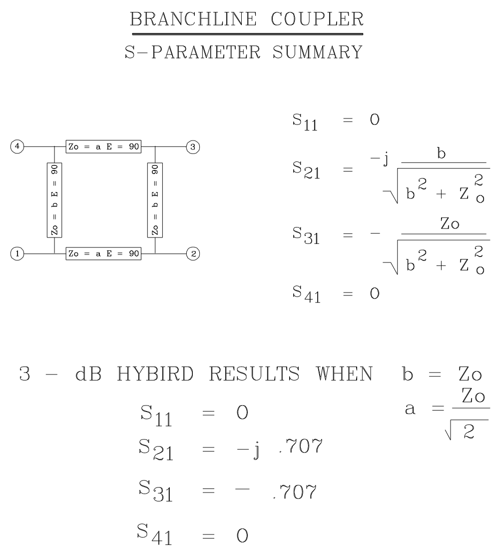
$$S_{21} = -j\frac{b}{\sqrt{b^2 + Z_0^2}} = -j\frac{a}{Z_0}$$

S-PARAMETER SUMMARY

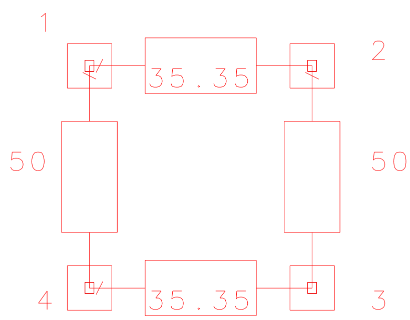
The other s-parameters can be found by adding the even and odd exiting signals from ports 3 and 4. The even mode signals are the same as those exiting ports 1 and 2 while the odd mode signals are the negative of those exiting ports 1 and 2. We see that $S_{41} = 0$. Port 4 is the isolated port.

With the incident signal into port 1 we see that port 2 is the direct port, port 3 is the coupled port, and port 4 is the isolated port.

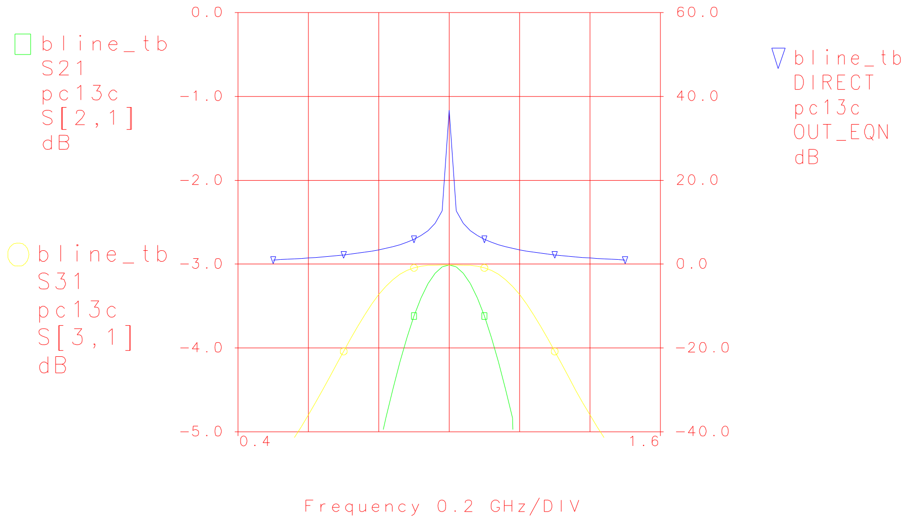
The output from port 2 has a 90 degree phase lag from the input while the output from port 3 has a phase lag of 180 degrees. Therefore port 3 is 90 degrees behind port 2. If the branch line characteristic impedance equals 50 Ω and the direct line characteristic impedance equals 35.4 Ω then the power is split equally between ports 2 and 3, i.e., the outputs are each down 3 dB. This results in a 3 dB hybrid coupler or what is called a branchline hybrid.

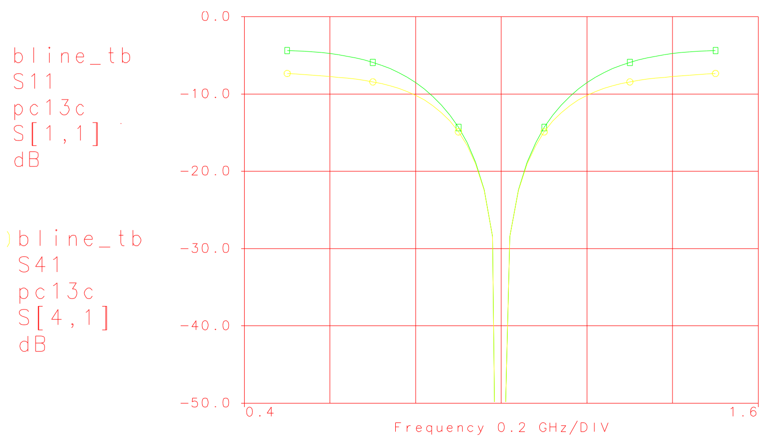


BRANCHLINE HYBRID ¾ Ideal Transmission lines

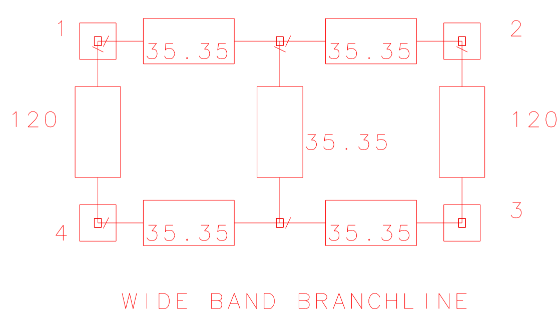


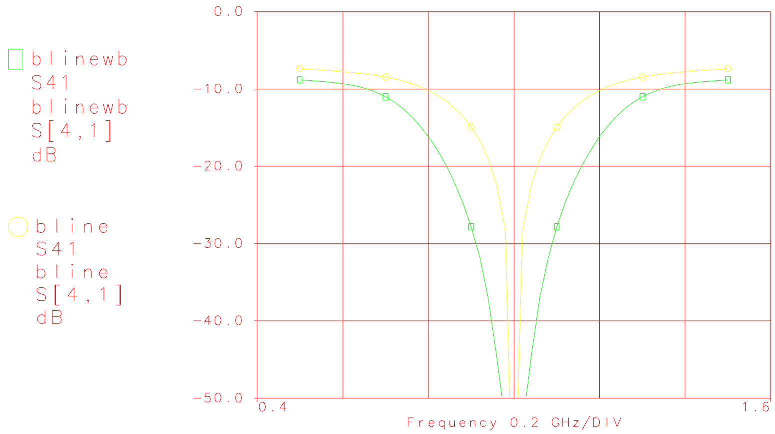
3 dB HYBRID (BRANCHLINE)

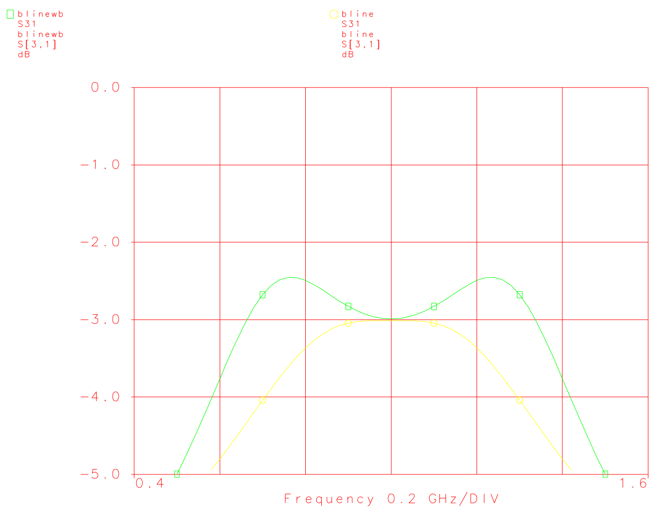




BRANCHLINE HYBRID ¾ Bandwidth Enhancement

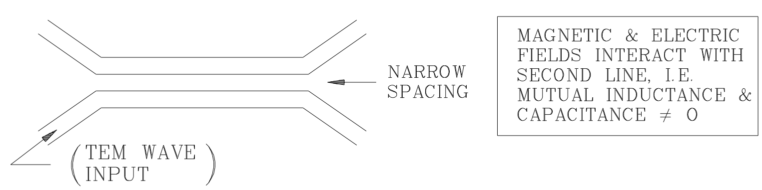






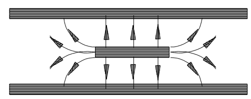
9.4 PARALLEL LINE COUPLER

A parallel line coupler is created when two transmission lines are brought close together so that their field lines interact. In looking at parallel line couplers we will consider strip line and microstrip configurations. In strip line a center conductor is surrounded by a dielectric and two ground planes. A pure transverse electromagnetic (TEM) wave can propagate in such a structure. We shall also consider microstrip structures. In microstrip a quasi-TEM wave can propagate. A pure TEM wave has a propagation velocity which is independent of the conductor geometry. A quasi-TEM on the other hand has field lines which are approximately the same as a TEM wave, but the propagation velocity depends upon the conductor geometry.

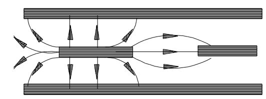


The nature of the coupling can be observed qualitatively by looking at the field lines. A cross-sectional view of a strip line structure is shown on the left with a single center conductor. Electric and magnetic field lines are shown for a wave propagating into the paper.

To the right the field lines are shown as a secondary conductor is brought near the original line. Some the electric field lines now terminate on the new conductor. Also, currents are induced in the secondary conductor. The magnetic field lines associated with the induced current circle the secondary conductor in the same direction as the original magnetic field lines circle the primary conductor.

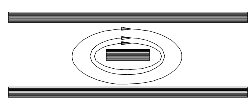


ELECTRIC FIELD LINES SINGE LINE

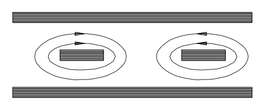


ELECTRIC FIELD LINES 1 WITH SECOND LINE ADDED

That the energy is propagating into the page for the single transmission line to the left can be seen by looking at the Poynting vector. An examination of the Poynting vector associated with the secondary fields reveals that the induced signal is propagating out of the paper. A parallel coupler results in a coupled signal which propagates opposite of the incident signal. For this reason a parallel line coupler is sometimes referred to as a backward wave coupler.



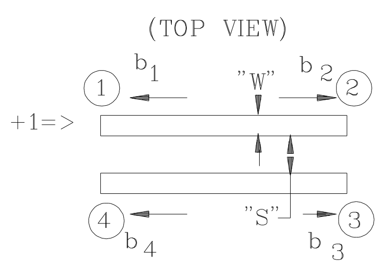
MAGNETIC FIELD LINES FOR SINGLE LINE



MAGNETIC FIELD LINES WITH SECOND LINE ADDED

S-PARAMETER DETERMINATION

A parallel line coupler is a four port circuit. To determine the s-parameters we assume all ports are matched and that a signal equal to 1 is incident upon port 1. As usual the exiting signal will be equal to the respective s-parameters. The s-parameters of S_{11}, S_{21}, S_{12}, and S_{22} are sufficient to describe the behavior of the coupler since all of the other coupler s-parameters are equal to one of them.



$S_{11} = b_1$
$S_{21} = b_2$
$S_{31} = b_3$
$S_{41} = b_4$

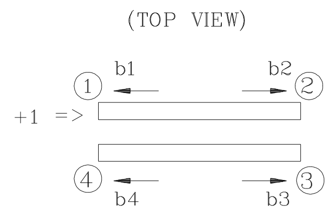
SYMMETRY =>

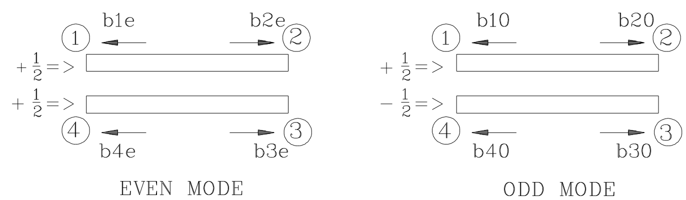
$$S_{22} = S_{33} = S_{44} = \underline{S_{11}}$$
$$S_{34} = S_{43} = S_{12} = \underline{S_{21}}$$
$$S_{42} = S_{24} = S_{13} = \underline{S_{31}}$$
$$S_{32} = S_{23} = S_{14} = \underline{S_{41}}$$

$$\boxed{b_1, b_2, b_3, b_4 \text{ DETERMINES ALL S-PARAMETERS}}$$

MODE DECOMPOSITION

The incident signal of 1 into port 1 can be decomposed into an even mode and odd mode.

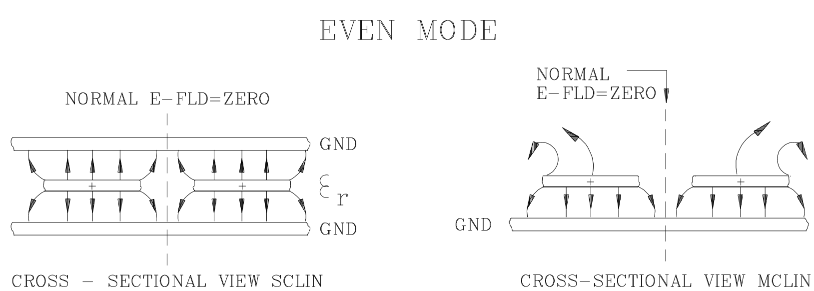




We examine each of the modes separately.

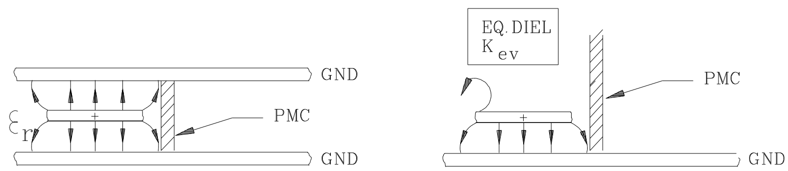
EVEN MODE

Electric field lines for the even mode are shown. Since a signal of +1/2 and +1/2 are incident upon ports 1 and 4 then the field lines associated with each center conductor are the same. Also, no electric field lines can terminate on the opposite conductor. Electric field lines do not cross the center plane between the two conductors.



A perfect magnetic conductor (PMC) is a hypothetical surface in which the tangential H field are required to vanish. The tangential H field vanishing is equivalent to requiring that the normal E field be zero. A perfect electric conductor (PEC) is a surface in which the tangential electric field is required to vanish. The normal H field must go to zero on a perfect electric conductor. A perfect magnetic conductor is the dual of a perfect electric conductor.

The second conductor can be replaced if a PMC is inserted in the center perpendicular to the ground plane.



Even mode propagation results in two independent circuits separated by a PMC. (This is similar to the open circuit separation in the earlier analyses) The characteristic impedance of the even mode transmission line is given by

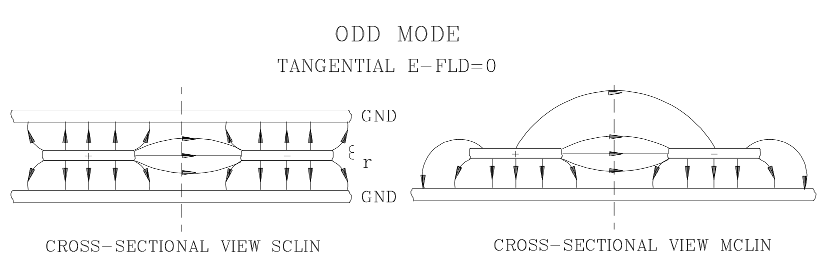
$$
\begin{aligned}
Z_{EV} &= \frac{\sqrt{e_r}}{cC_{EV}} \\
e_r &= \text{relative dielectric constant} \\
c &= \text{speed of light} \\
C_{EV} &= \text{even mode capacitance}
\end{aligned}
$$

In the case of microstrip the inhomogenous dielectric is replaced with an equivalent homogeneous dielectric with an effective dielectric constant k. This constant k depends upon the specific geometry of the conductors.

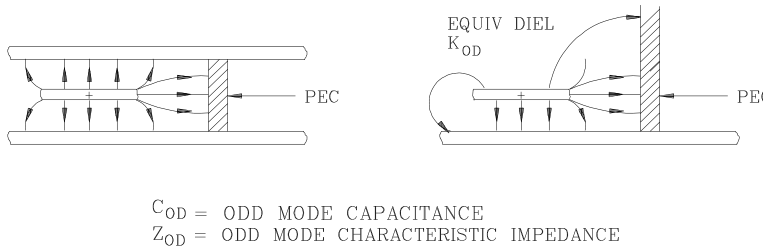
$$
\boxed{
\begin{aligned}
C_e &= \text{EVEN MODE CAPACITANCE} \\
Z_{ev} &= \text{EVEN MODE CHARACTERISTIC IMPEDANCE} = \begin{cases} \dfrac{\sqrt{\varepsilon_r}}{cC_{ev}} & \text{(SCLIN)} \\[2ex] \dfrac{\sqrt{K_{ev}}}{cC_{ev}} & \text{(MCLIN)} \end{cases}
\end{aligned}
}
$$

ODD MODE

For the odd mode and plus voltage and minus voltage exists on the separate center conductors. The electric field lines on the positive conductor originate there and terminate on either the ground plane or the negative conductor. On the other hand the field lines on the negative conductor all terminate there and originate from the positive conductor or the ground plane.



The two lines can be separated by a perfect electric conductor. The characteristic impedance of the odd transmission line can now be found from the capacitance including the PEC.



$$Z_{OD} = \frac{\sqrt{e_r}}{cC_{OD}}$$

e_r = relative dilectric constant

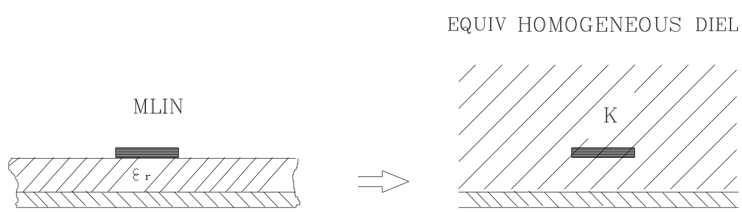
c = speed of light

C_{OD} = odd mode capacitance

In the case of microstrip e_r would be replaced with the effective dielectric constant. Z_{OD} for stripline is independent of the conductor geometry while Z_{OD} for microstrip depends upon the geometry since k does.

OBSERVATIONS

Notice that $C_{OD} > C_{EV}$ since the presence of the PEC represents a capacitor with more surfaces. Also, the even mode effective dielectric constant $k_{EV} > k_{OD}$, the odd mode effective dielectric constant since in the even mode more field lines are forced into the dielectric, e_r. Therefore, $Z_{EV} > Z_{OD}$.



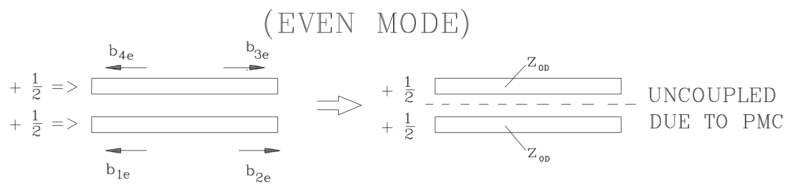
For a strip line the even mode velocity equals the odd mode velocity. However, for microstrip the even mode velocity is less than the odd mode velocity, $V_{EV} < V_{OD}$.

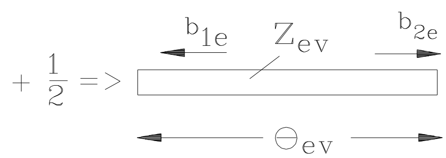
- ▷ K IS WEIGHTED AVERAGE OF $\varepsilon = \varepsilon_r$ (SUBSTRATE) AND ε (AIR) $1 \leq K \leq \varepsilon_r$
- ▷ MORE E–FLD LINES IN SUBSTRATE => K CLOSER TO ε_r
- ▷ EVEN MODE E – FLD FORCED INTO SUBSTRATE
- ▷ ODD MODE SOME E – FLD LINES NOW GO THRU AIR TO SECOND CONDUCTOR, HENCE $\boxed{K_{ev} > K_{OD}}$
- ▷ FOR MICROSTRIP COUPLER LINE

$$
\boxed{
\begin{aligned}
&C_{ev} < C_{OD} \\
&v_{ev} = \frac{c}{\sqrt{K_{ev}}} < \frac{c}{\sqrt{K_{OD}}} = v_{OD} \\
&Z_{ev} = \frac{1}{v_{ev} C_{ev}} > \frac{1}{v_{OD} C_{OD}} = Z_{OD}
\end{aligned}
}
$$

EVEN MODE

The s-parameters for the coupled line can now be found by computing the exiting signals from each of the modes. We need only compute the reflected and transmitted signal for a transmission line with characteristic impedance Z_{EV} and electrical length q_{EV}.

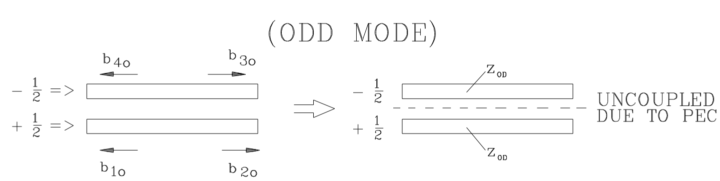


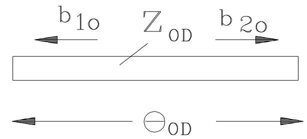


$$
\begin{aligned}
b_{1e} &= \frac{j\left(\frac{Z_{ev}}{Z_o} - \frac{Z_o}{Z_{ev}}\right)\sin\Theta_{ev}}{2\,D_{ev}} \\
b_{2e} &= \frac{1}{D_{ev}} \\
D_{ev} &= 2\cos\Theta_{ev} + j\left(\frac{Z_{ev}}{Z_o} + \frac{Z_o}{Z_{ev}}\right)\sin\Theta_{ev} \\
b_{3e} &= b_{2e} \\
b_{4e} &= b_{1e}
\end{aligned}
$$

ODD MODE

The reflected and transmitted signal for a transmission line with characteristic impedance Z_{OD} determines b_{1o} and b_{2o}. q_{OD} is the electrical length of the odd mode transmission line. For strip line $q_{EV} = q_{OD}$, however, for microstrip, $q_{EV} < q_{OD}$.



$+\ \frac{1}{2} \Rightarrow$ 

$$b_{1o} = \frac{j\left(\frac{Z_{OD}}{Z_o} - \frac{Z_o}{Z_{OD}}\right)\sin\Theta_{OD}}{2\,D_{OD}}$$

$$b_{2o} = \frac{1}{D_{OD}}$$

$$D_{OD} = 2\cos\Theta_{OD} + j\left(\frac{Z_{OD}}{Z_o} + \frac{Z_o}{Z_{OD}}\right)\sin\Theta_{OD}$$

$$b_{3o} = -b_{2o}$$

$$b_{4o} = -b_{1o}$$

PURE TEM CASE

The s-parameters are found by adding the respective exiting signals. For TEM propagation such as in stripline $q_{EV} = q_{OD} = q$. If

$$Z_{EV} Z_{OD} = Z_0^2$$

then the input is matched, i.e., $S_{11} = 0$.

S-PARAMTER SUMMARY

$$S_{11} = b_{1e} + b_{10}$$

$$S_{21} = b_{2e} + b_{30}$$

$$S_{31} = b_{3e} + b_{30} = b_{2e} - b_{20}$$

$$S_{41} = b_{4e} + b_{40} = b_{1e} - b_{10}$$

FOR PURE TEM CASE $\boxed{\Theta_{ev} = \Theta_{OD} = \Theta}$

IF WIDTH "W" & SPACING "S" CHOSEN SUCH THAT $Z_{ev} Z_{OD} = Z_o^2$
THEN

$$b_{1e} = -b_{1o} = \frac{j(Z_{ev} - Z_{OD})\sin\Theta}{2Z_o\cos\Theta + j(Z_{ev} + Z_{OD})\sin\Theta}$$

$$b_{2e} = b_{2o} = \frac{2Z_o}{2Z_o\cos\Theta + j(Z_{ev} + Z_{OD})\sin\Theta}$$

$$S_{11} = 0$$

$$S_{21} = \frac{2Z_o}{2Z_o\cos q + j(Z_{EV} + Z_{OD})\sin q}$$

$$S_{31} = 0$$

$$S_{41} = \frac{j(Z_{EV} - Z_{OD})\sin q}{2Z_o\cos q + j(Z_{EV} + Z_{OD})\sin q}$$

If $q = 90°$, i.e., the lines are a quarter wave length long then

$$S_{11} = 0$$
$$S_{21} = \frac{-j2Z_o}{Z_{EV} + Z_{OD}}$$
$$S_{31} = 0$$
$$S_{41} = \frac{Z_{EV} - Z_{OD}}{Z_{EV} + Z_{OD}} = CouplingCoefficient$$

$S_{31} = 0$ indicating that the coupled wave is propagating backwards and that port 3 is the isolated port. The maximum coupling occurs when $q = p/2$ or odd multiples thereof. For that case the s-parameters simplify as shown. The coupling coefficient is S_{41} and is given by

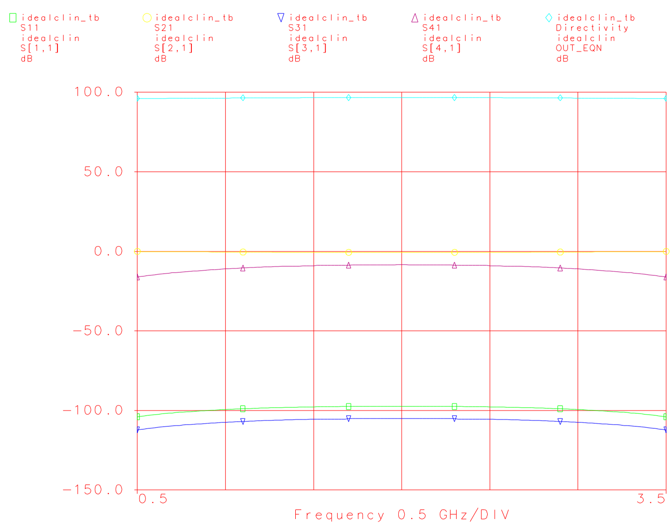
$$\frac{Z_{EV} - Z_{OD}}{Z_{EV} + Z_{OD}}$$

As the coupled lines are brought close together $Z_{EV} >> Z_{OD}$ and the coupling coefficient increases. For line separated by at least a dielectric substrate thickness $Z_{EV} \approx Z_{OD}$ and the coupling coefficient decreases.

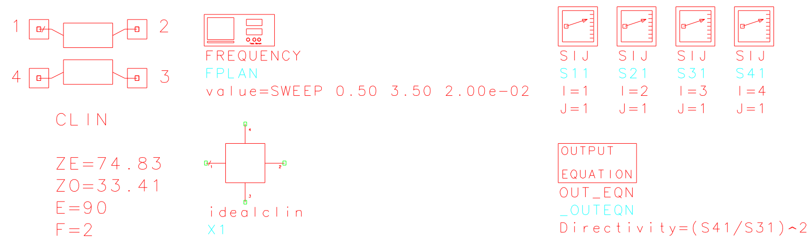
PERFORMANCE COMPARISON

Ideal Parallel Line Coupler:

<u>Performance</u>

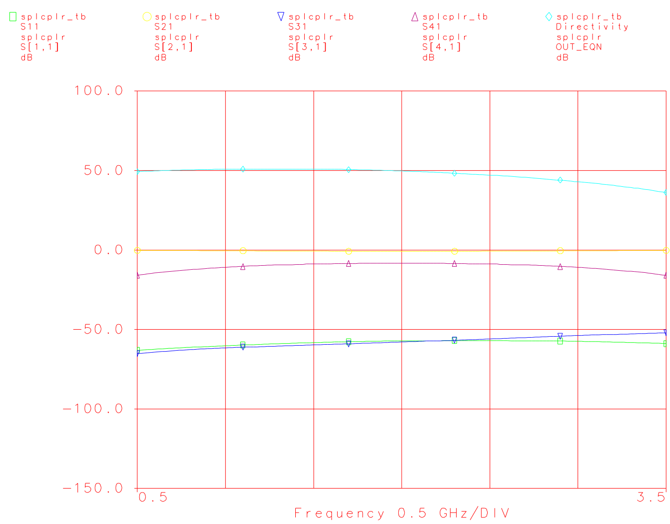


<u>Schematic</u> <u>Test Setup</u>

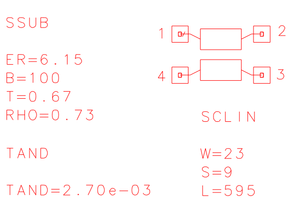


Stripline Parallel Line Coupler:

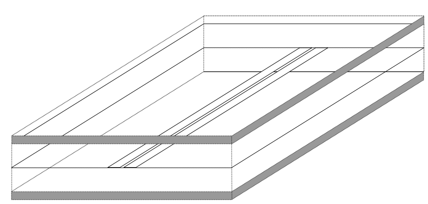
Performance



Schematic

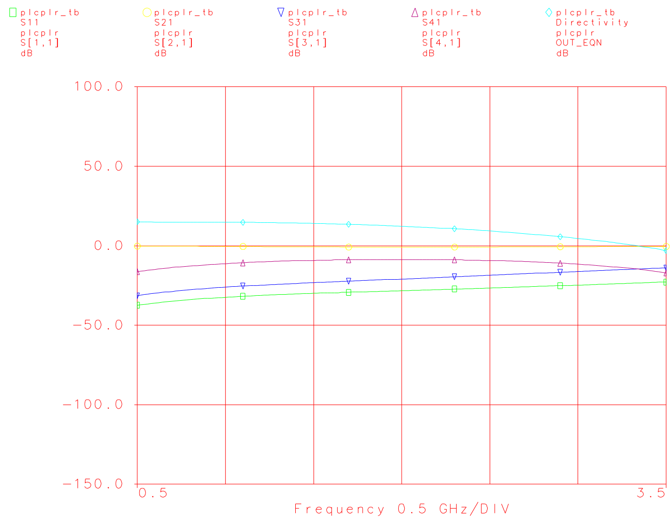


Layout



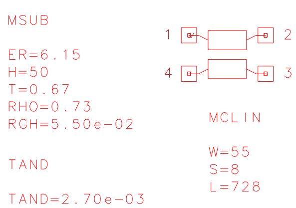
Microstrip Parallel Line Coupler:

Performance

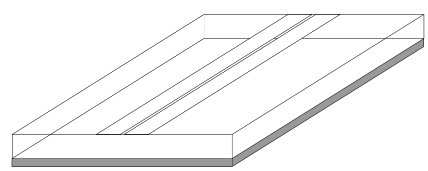


Comment: The isolation is degraded over that of stripline because of the difference in propagation velocities for the even and odd modes. This is reflected in the plot of directivity where microstrip achieves approximately 10 dB.

Schematic



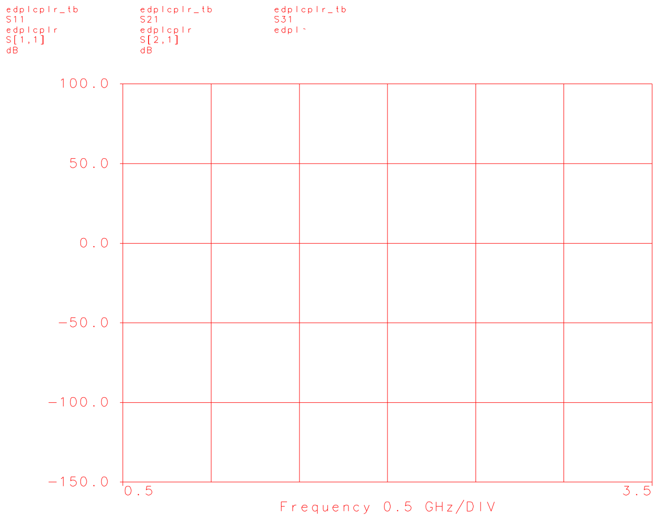
Layout



Directivity Enhancement Techniques for Microstrip Parallel Line Couplers:

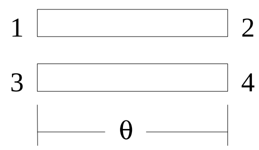
1. Adding inter-line capacitance to slow down odd mode:

Performance

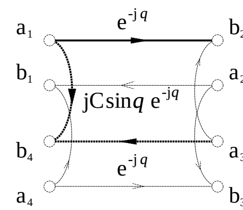

edplcplr_tb
S11
edplcplr
S[1,1]
dB
edplcplr_tb
S21
edplcplr
S[2,1]
dB
edplcplr_tb
S31
edpl-
100.0
50.0
0.0
-50.0
-100.0
-150.0
0.5
3.5
Frequency 0.5 GHz/DIV

Analysis

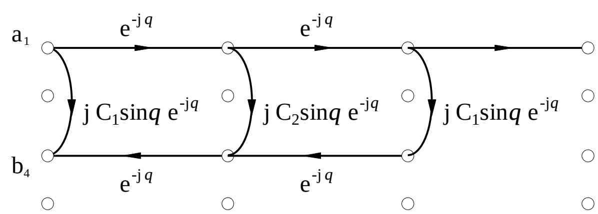
With small coupling (C) assumed, the s-parameters and signal flow graph of an ideal parallel line coupler can be approximated as below.



$$S_{11} = 0$$
$$S_{21} \approx e^{-j\theta}$$
$$S_{31} = 0$$
$$S_{41} \approx jC\sin\theta\, e^{-j\theta}$$



The coupling coefficient C_0 (at center band) of the cascaded three section coupler as shown above can be found by using the signal flow graph analysis.



$$\begin{aligned}\tilde{S}_{41} &= jC_1\sin\theta e^{-j\theta} + jC_2\sin\theta e^{-j3\theta} + jC_1\sin\theta e^{-j5\theta} \\ &= j\sin\theta e^{-j3\theta}\left(C_1e^{+j2\theta} + C_2 + C_1e^{-j2\theta}\right) \\ &= j\sin\theta e^{-j3\theta}\left(2C_1\cos 2\theta + C_2\right)\end{aligned}$$

$$\left|\tilde{S}_{41}\right| = \left|\sin\theta\left(2C_1\cos 2\theta + C_2\right)\right|$$

$$\theta = \frac{\pi}{2} \Rightarrow \left|\tilde{S}_{41}(\pi/2)\right| = C_0 = C_2 - 2C_1$$

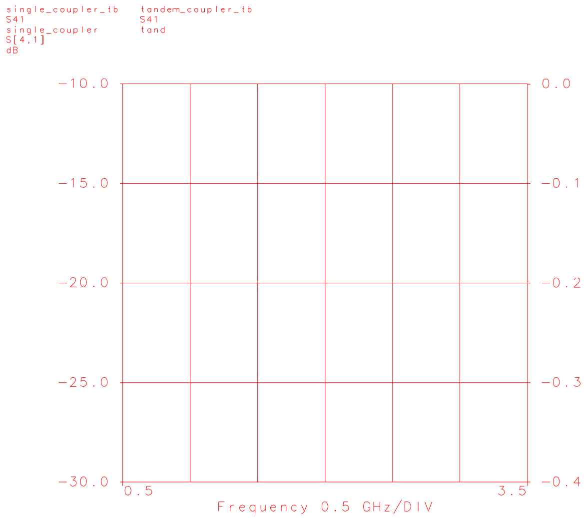
Design Procedure

1. Deterministic: Equate $\left|\tilde{S}_{41}\right|$ to the appropriate function for the desired behavior, e.g. Butterworth for maximally flat coupling or Chebyshev for even larger bandwidth with some coupling ripples.

2. Iterative: Alternatively choose values for C_1 then calculate C_2 and repeat the process until the desired coupler performance is achieved.

Performance

The following graph compares the performance of a single parallel line coupler and a three section tandem parallel line coupler both with -20 dB coupling at the center frequency.

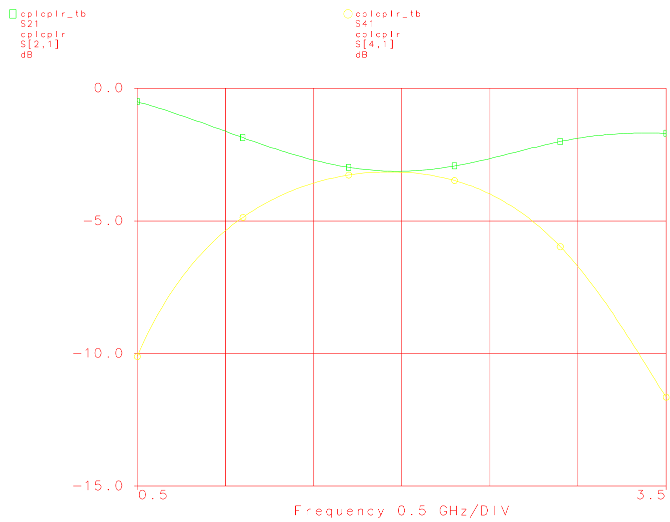

single_coupler_tb
S41
single_coupler
S[4,1]
dB
tandem_coupler_tb
S41
tand
-10.0
-15.0
-20.0
-25.0
-30.0
0.0
-0.1
-0.2
-0.3
-0.4
0.5
3.5
Frequency 0.5 GHz/DIV

$$\tilde{S}_{41} = -j2C\sqrt{1-C^2}$$

$$\left|\tilde{S}_{21}\right|^2 = \left|\tilde{S}_{41}\right|^2 \Rightarrow 8C^4 - 8C^2 + 1 = 0$$

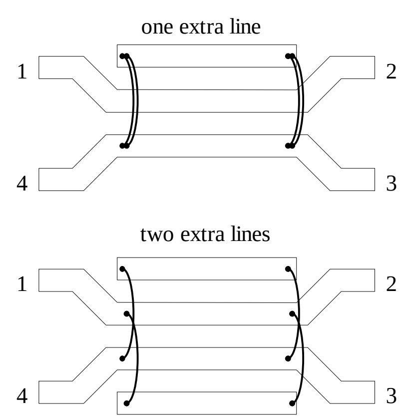
$$C = 0.383 = -8.34dB$$

Performance

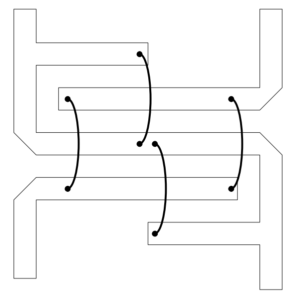


Lang Couplers

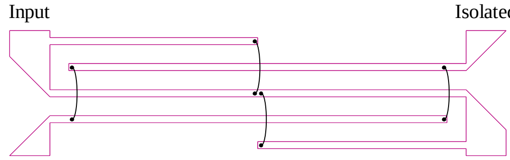
For a single parallel line coupler, the spacing between the coupled lines reduces as the coupling coefficient goes up. At a certain point, the spacing becomes narrow enough to cause difficulties in the fabrication process. By adding extra parallel lines to the coupler structure as shown below, tighter coupling can be achieved with increased spacing between the lines.



In the two extra lines case above, there exist some difficulties in wire bonding the lines. By cleverly folding the lines as shown below, the difficulties can be resolved. Note that the ports have also been rearranged.



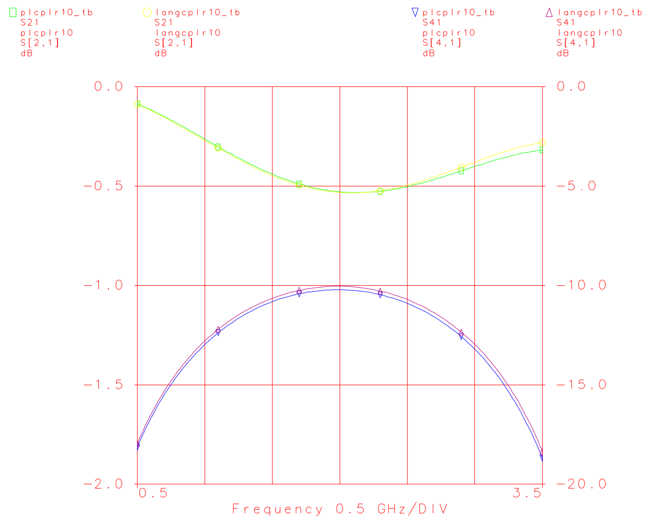
A more typical layout of the Lange coupler is shown below. This one is a 2 GHz, -10 dB coupler on the 50 mil Duroid 6006 substrate (See the MSUB statement in page 11). Note that the width of the finger is much smaller than the line width at the ports due to the impedance matching. A table that compares the physical dimensions of this Lang coupler and a -10 dB parallel line coupler on the same substrate follows the layout below.

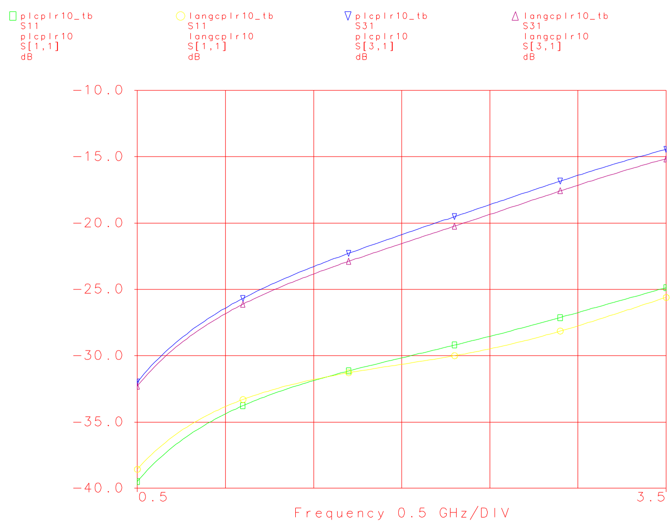


2 GHz, -10 dB Coupler on 50 mil, Duroid 6006	Width (mils)	Spacing (mils)	Length (mils)
Lang Coupler	13	35	734
Parallel Line Coupler	60	13	721

<u>Performance</u>

There is a common misconception that the bandwidth of the Lange Coupler comes about because of the folding structure. In fact, the coupling mechanism of the Lang Coupler is really the same as that of the Parallel Line Coupler. Therefore, the Lang Coupler exhibits the same bandwidth as a Parallel Line Coupler with comparable performance . As a demonstration, the following plots compare the s-parameters of the -10 dB couplers listed in the table above.

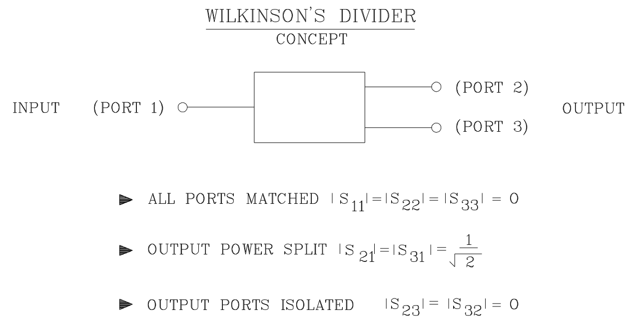




9.5 WILKINSON DIVIDER

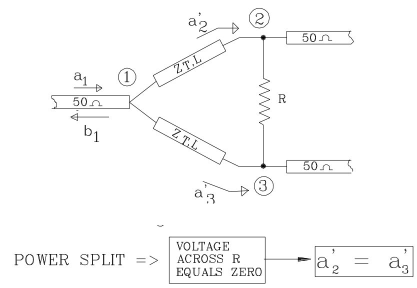
Another type of coupler is the Wilkinson's divider. This three port circuit has the property that it is matched, and the power in the input port splits between the two output ports, and the two output ports are isolated. We note that such a circuit could be made from a 3 dB branchline hybrid with port 4 terminated in a matched load. However, we will look at an alternative configuration.

Such circuits are useful where it is desired to split an input signal into two channels for further processing, e.g., amplification, etc. The fact that the output ports are isolated means that any mismatch in one output channel will not affect the other output port.

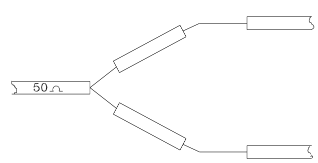


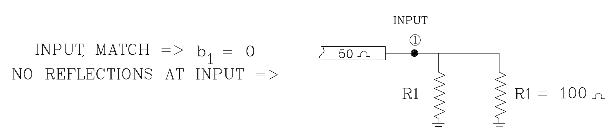
INPUT CONSIDERATIONS

The configuration which we will consider consists of an input which splits into two transmission lines each with a characteristic impedance of Z_T and length L. The output ports are connected by a resistor R. We will attempt to determine the circuit parameters which will result in the desired s-parameters

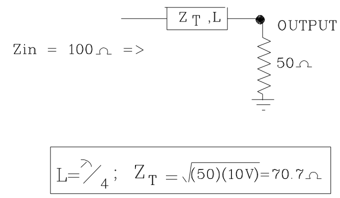


We look first at the input reflection coefficient S_{11}. If we assume all ports are matched then an incident signal into port a_1 would be split with $a_2' = a_3'$. This means that no current can flow in the resistor since the voltage at each end is the same i.e., there is no voltage drop across the resistor and consequently it can be eliminated in consideration of S_{11} effects.





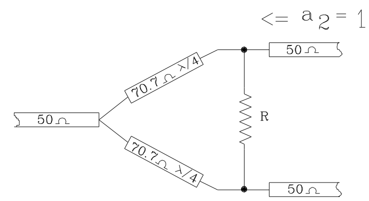
If the input is matched then the input impedance to each of the two lines must equal 100 Ω. Thus, the two input impedances when combined in parallel will result in a Wilkinson's input impedance of 50 Ω. Thus a 50 Ω load must transform to a 100 Ω input. This will occur if the line is λ/4 in length and has a characteristic impedance equal to the geometrical mean, i.e., $Z_T = 70.7\ \Omega$.



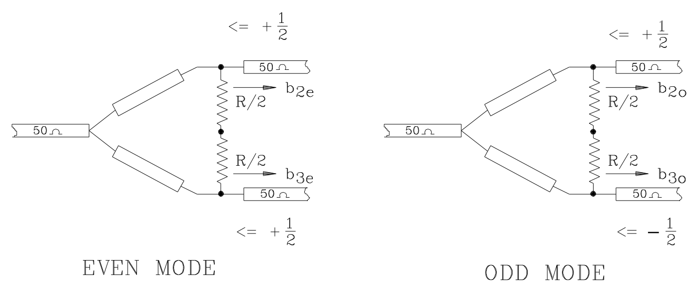
The requirement that the input be matched has determined two of the circuit parameters: Z_T and L. Note also that the lines could be any integer multiple of λ/4.

MODE DECOMPOSITION

In order to look at the output implication we assume an incident signal of 1 into port 2.



The s-parameters of S_{32} and S_{22} will equate to the exiting signals. We can then represent this situation in terms of even and odd modes. The even mode consists of $+1/2$, $+1/2$ signals into ports 2 and 3 while the odd mode consists of $+1/2$, $-1/2$ signals into ports 2 and 3, respectively. Additionally, it is helpful to replace the resistance R with two series resistors each having a value of R/2. From symmetry, the other s-parameters are known, i.e., $S_{33} = S_{22}$ and $S_{23} = S_{32}$.



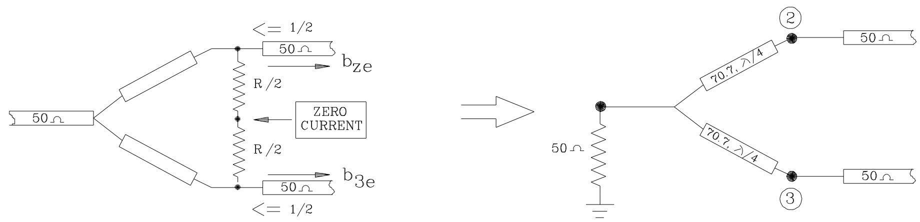
$$S_{22} = b_{2e} + b_{2o}$$

$$S_{32} = b_{3e} + b_{3o}$$

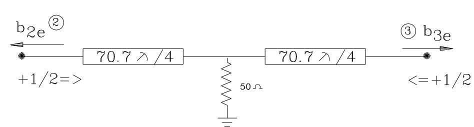
$$\text{SYMMETRY} \quad \Rightarrow \quad S_{33} = S_{22} \;\; \& \;\; S_{23} = S_{32}$$

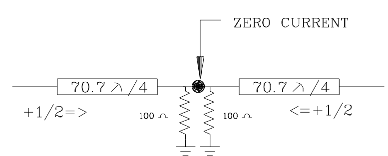
Even Mode

In analyzing the even mode the circuit point between the two $R/2$ resistors has zero current and the circuit can be opened at that point and not affect the performance. This leaves two resistors each connected at only one end and therefore they can be eliminated. The input port is terminated in a 50 Ω resistance.

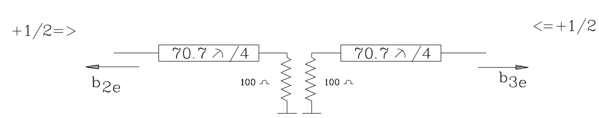


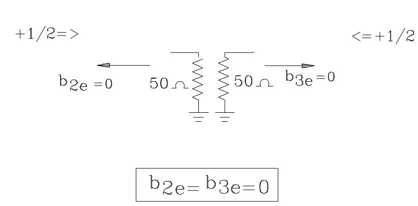
The resulting circuit consists of two $l/4$ lines with a shunt resistance in the middle. The 50Ω shunt resistance can be replaced with two 100Ω parallel resistors. From symmetry the current between the resistors equals zero and the circuit can be open at that point.





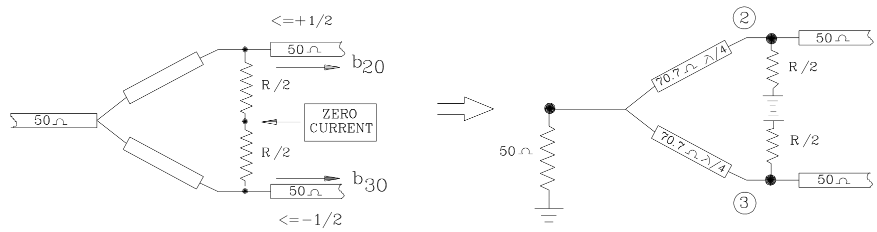
The two resulting circuits consist of a 70.7Ω $l/4$ line terminated in 100Ω. The input impedance therefore equals 50Ω and the reflection coefficient equals zero. As a result, $b_{2e} = b_{3e}$.



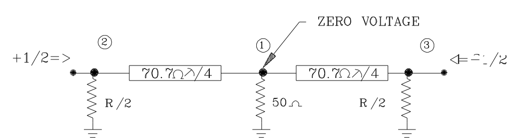


Odd Mode

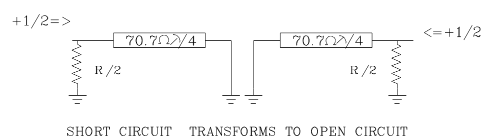
In the odd mode analysis a zero voltage exists between the two $R/2$ resistors. The resulting circuit consists of two $l/4$ lines with a 50Ω shunt resistor in the middle and two shunt $R/2$ resistors at each end.



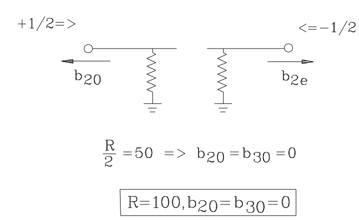
The voltage across the center resistor is zero and that point in the circuit can be replaced with a short to ground resulting in two independent circuits.



The individual circuits consists of a short circuited $l/4$ line with a shunt resitor $R/2$ at the input.

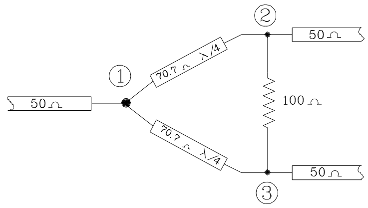


The short transforms into an open leaving the input equal to only the $R/2$ resistance. Since the even mode was matched the total exiting signal can equal zero if and only if the odd mode exiting signals equal zero. This will occur only if $R/2 = 50\Omega$ or $R = 100\Omega$ and the last parameter is determined.

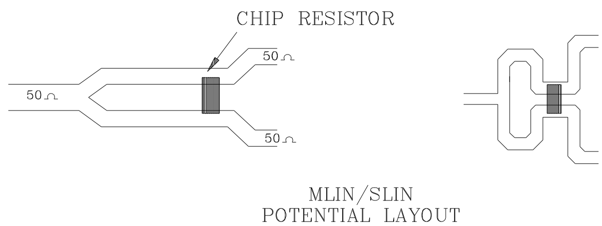


SUMMARY AND LAYOUT

The S matrix is shown. Two microstrip layouts are shown. Topologies have to be chosen such that it is easy to connect the 100 Ω resistor.

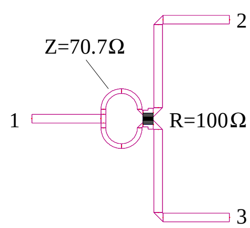


$$S=\begin{pmatrix} 0 & -j\frac{1}{\sqrt{2}} & -j\frac{1}{\sqrt{2}} \\ -j\frac{1}{\sqrt{2}} & 0 & 0 \\ -j\frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

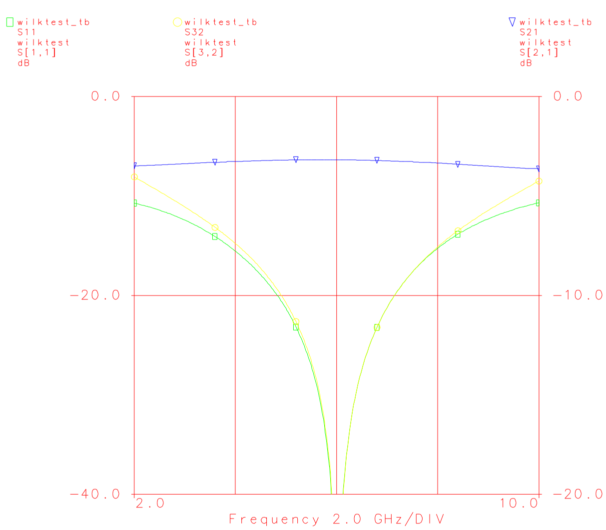


DESIGN EXAMPLES

Wilkinson's Divider

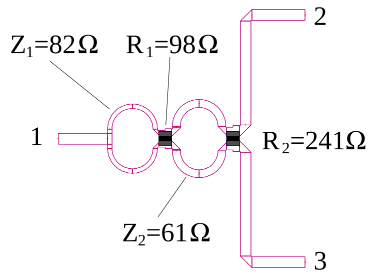


Frequency Domain Simulation



A Wilkinson's divider is simulated in the circuit file and the performance is shown as a function of frequency. The 3 dB split of power is fairly broad band. The isolation between the output ports is considerably more narrow band.

Broad Band Wilkinson's Divider



Multiple sections can be used to increase the bandwidth of a Wilkinson's divider.

Frequency Domain Simulation