15

Switched-mode and Resonant dc-to-dc Power Supplies

A switched-mode power supply (smps) or switching regulator, efficiently converts a dc voltage level to another dc voltage level, usually at power levels below a few kilowatts.

Shunt and series linear regulator power supplies dissipate much of their energy across the regulating transistor, which operates in the linear mode. An smps achieves regulation by varying the on to off time duty cycle of the switching element. This minimises losses, irrespective of load conditions.

Figure 15.1 illustrates the basic principle of the ac-fed smps in which the ac mains input is rectified, capacitively smoothed, and supplied to a high-frequency transistor chopper. The chopped dc voltage is transformed, rectified, and smoothed to give the required dc output voltage. A high-frequency transformer is used if an isolated output is required. The output voltage is sensed by a control circuit that adjusts the duty cycle of the switching transistor in order to maintain a constant output voltage with respect to load and input voltage variation. Alternatively, the chopper can be configured and controlled such that the input current tracks a scaled version of the input ac supply voltage, therein producing unity (or controllable) power factor *I-V* input conditions.

The switching frequency can be made much higher than the 50/60Hz line frequency; then the filtering and transformer elements used can be made small, lightweight, low in cost, and efficient.

Depending on the requirements of the application, the dc-to-dc converter can be one of four basic converter types, namely

- forward
- flyback
- balanced
- resonant.

15.1 The forward converter

The basic *forward converter,* sometimes called a *buck converter,* is shown in figure 15.2a. The input voltage E_i is chopped by transistor T. When T is on, because the input voltage E_i is greater than the load voltage v_o, energy is transferred from the dc supply E_i to L, C, and the load R. When T is turned off, stored energy in L is transferred via diode D to C and the load R.

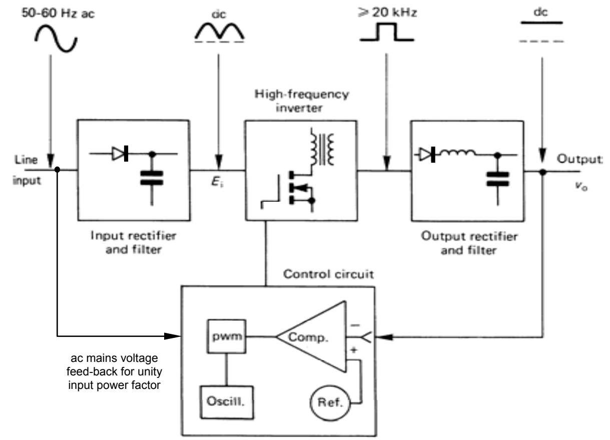

Figure 15.1. *Functional block diagram of a switched-mode power supply.*

If all the stored energy in L is transferred to C and the load before T is turned back on, operation is termed *discontinuous,* since the inductor current has reached zero. If T is turned on before the current in L reaches zero, that is, if continuous current flows in L, operation is termed *continuous*.

Parts b and c respectively of figure 15.2 illustrate forward converter circuit current and voltage waveforms for continuous and discontinuous conduction of L.

For analysis it is assumed that components are lossless and the output voltage v_o is maintained constant because of the large magnitude of the capacitor C across the output. The input voltage E_i is also assumed constant, such that $E_i \geq v_o$.

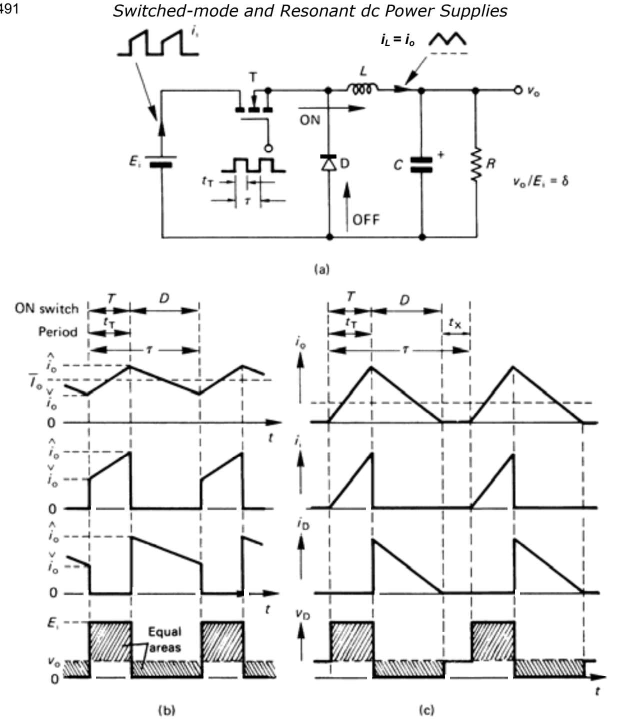

Figure 15.2. *Non-isolated forward converter (buck converter) where $v_0 \le E_1$: (a) circuit diagram; (b) waveforms for continuous output current; and (c) waveforms for discontinuous output current.*

15.1.1 Continuous inductor current

The inductor current is analysed first when the switch is on, then when the switch is off. When transistor T is turned on for period t_T, the difference between the supply voltage E_i and the output voltage v_0 is impressed across L. From $V=Ldi/dt=L\Delta i/\Delta t$, the current change through the inductor will be

$$\Delta i_L = \hat{i}_L - \check{i}_L = \frac{E_i - v_o}{L} \times t_T \tag{15.1}$$

When T is switched off for the remainder of the switching period, τ-t_T, the freewheel diode D conducts and -v_0 is impressed across L. Thus, assuming continuous conduction

$$\Delta i_L = \frac{v_o}{L} \times (\tau - t_T) \tag{15.2}$$

Equating equations (15.1) and (15.2) gives

$$(E_i - v_o)\, t_T = v_o\, (\tau - t_T) \tag{15.3}$$

This expression shows that the inductor average voltage is zero, and after rearranging:

$$\frac{v_o}{E_i} = \frac{\bar{I}_i}{\bar{I}_o} = \frac{t_T}{\tau} = \delta \qquad 0 \le \delta \le 1 \tag{15.4}$$

This equation shows that for a given input voltage, the output voltage is determined by the transistor conduction duty cycle and the output is always less than the input voltage. This confirms and validates the original analysis assumption that $E_i \ge v_o$. The voltage transfer function is independent of circuit inductance L and capacitance C.
The inductor rms ripple current (and capacitor ripple current in this case) is given by

$$i_{Lr} = \frac{\Delta i_L}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}\frac{v_o}{L}(1-\delta)\tau = \frac{1}{2\sqrt{3}}\frac{E_i}{L}(1-\delta)\delta\tau \tag{15.5}$$

while the inductor total rms current is

$$i_{L\,rms} = \sqrt{\bar{I}_L^2 + i_{Lr}^2} = \sqrt{\bar{I}_L^2 + \left(\frac{½\Delta i_L}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3}\left(\hat{i}_L^2 + \hat{i}_L \times \check{i}_L + \check{i}_L^2\right)} \tag{15.6}$$

The switch and diode average and rms currents are given by

$$\begin{aligned} \bar{I}_T &= \bar{I}_i = \delta \bar{I}_o & I_{Trms} &= \sqrt{\delta}\, i_{Lrms} \\ \bar{I}_D &= \bar{I}_o - \bar{I}_i = (1-\delta)\bar{I}_o & I_{Drms} &= \sqrt{1-\delta}\, i_{Lrms} \end{aligned} \tag{15.7}$$

If the average inductor current, hence output current, is $\bar{I}_L$, then the maximum and minimum inductor current levels are given by

$$\begin{aligned} \hat{i}_L &= \bar{I}_L + ½\Delta i_L = \bar{I}_o + ½\frac{v_o}{L}(1-\delta)\tau \\ &= v_o\left[\frac{1}{R} + \frac{1-\delta}{2fL}\right] \end{aligned} \tag{15.8}$$

and

$$\begin{aligned} \check{i}_L &= \bar{I}_L - ½\Delta i_L = \bar{I}_o - ½\frac{v_o}{L}(1-\delta)\tau \\ &= v_o\left[\frac{1}{R} - \frac{1-\delta}{2fL}\right] \end{aligned} \tag{15.9}$$

respectively, where Δi_L is given by equation (15.1) or (15.2). The average output current is $\bar{I}_L = ½\left(\hat{i}_L + \check{i}_L\right) = \bar{I}_o = v_o / R$. The output power is therefore v_o^2 / R. Circuit waveforms for continuous conduction are shown in figure 15.2b.

Switch utilisation ratio

The switch utilisation ratio, SUR, is a measure of how fully a switching device's power handling capabilities are utilised in any switching application. The ratio is defined as

$$SUR = \frac{P_{out}}{p\,\hat{V}_T\,\hat{I}_T} \tag{15.10}$$

where p is the number of power switches in the circuit; $p=1$ for the forward converter. The switch maximum instantaneous voltage and current are $\hat{V}_T$ and $\hat{I}_T$ respectively. As shown in figure 15.2b, the maximum switch voltage supported in the off-state is E_i, while the maximum current is the maximum inductor current $\hat{i}_L$ which is given by equation (15.8). If the inductance L is large such that the ripple current is small, the peak inductor current is approximated by the average inductor current $\hat{I}_T \approx \bar{I}_L = \bar{I}_o$, that is

$$SUR = \frac{v_o \bar{I}_o}{1 \times E_i \times \bar{I}_o} = \frac{v_o}{E_i} = \delta \tag{15.11}$$

which assumes continuous inductor current. This result shows that the higher the duty cycle, that is the closer the output voltage v_o is to the input voltage E_i, the better the switch *I-V* ratings are utilised.

15.1.2 Discontinuous inductor current

The onset of discontinuous inductor operation occurs when the minimum inductor current $\check{i}_L$, reaches zero. That is, with $\check{i}_L = 0$ in equation (15.9), the last equality

$$\frac{1}{R} - \frac{(1-\delta)}{2 f L} = 0 \tag{15.12}$$

relates circuit component values (R and L) and operating conditions (f and δ) at the verge of discontinuous inductor current. Also, with $\check{i}_L = 0$ in equation (15.9)

$$\bar{I}_L = \bar{I}_o = ½\Delta i_L \tag{15.13}$$

which, after substituting equation (15.1) or equation (15.2), yields

$$\bar{I}_L = \bar{I}_o = \frac{(E_i - v_o)}{2L}\tau\delta \quad \text{or} \quad \frac{E_i}{2L}\tau\delta(1-\delta) \tag{15.14}$$

If the transistor on-time t_T is reduced (or the load current is reduced), the discontinuous condition dead time t_x is introduced as indicated in figure 15.2c. From equations (15.1) and (15.2), with $\check{i}_L = 0$, the output voltage transfer function is now derived as follows

$$\hat{i}_L = \frac{(E_i - v_o)}{L} t_T = \frac{v_o}{L}(\tau - t_T - t_x) \tag{15.15}$$

that is

$$\frac{v_o}{E_i} = \frac{\delta}{1 - \dfrac{t_x}{\tau}} \qquad 0 \le \delta < 1 \tag{15.16}$$

This voltage transfer function form may not be particularly useful since the dead time t_x is not expressed in term of circuit parameters. Accordingly, from equation (15.15)

$$\hat{i}_L = \frac{(E_i - v_o)}{L} t_T \tag{15.17}$$

and from the input current waveform in figure 15.2c:

$$\bar{I}_i = ½\,\hat{i}_L \times \frac{t_T}{\tau} \tag{15.18}$$

Eliminating $\hat{i}_L$ yields

$$\frac{2\bar{I}_i}{\delta} = (1 - \frac{v_o}{E_i})\frac{\tau\delta E_i}{L} \tag{15.19}$$

that is

$$\frac{v_o}{E_i} = 1 - \frac{2L\bar{I}_i}{\delta^2 \tau E_i} \tag{15.20}$$

Assuming power-in equals power-out, that is, $E_i\bar{I}_i = v_o\bar{I}_o = v_o\bar{I}_L$, the input average current can be eliminated, and after re-arranging yields:

$$\frac{v_o}{E_i} = \frac{1}{1 + \dfrac{2L\bar{I}_o}{\delta^2 \tau E_i}} = \frac{1}{1 + \dfrac{2L\bar{I}_i}{\delta^2 \tau v_o}} \tag{15.21}$$

At a low output current or high input voltage, there is a likelihood of discontinuous inductor conduction. To avoid discontinuous conduction, larger inductance values are needed, which worsen transient response. Alternatively, with extremely low on-state duty cycles, a voltage-matching transformer can be used to increase δ. Once using a transformer, any smps technique can be used to achieve the desired output voltage. Figures 15.2b and c show that the input current is always discontinuous.

15.1.3 Load conditions for discontinuous inductor current

As the load current decreases, the inductor average current also decreases, but the inductor ripple current magnitude is unchanged. If the load resistance is increased sufficiently, the bottom of the triangular inductor current, $\check{i}_L$, eventual reduces to zero. Any further increase in resistance causes discontinuous inductor current and the linear voltage transfer function given by equation (15.4) is no longer valid and equations (15.16) and (15.20) are applicable. The critical load resistance for continuous inductor current is specified by

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{v_o}{\tfrac{1}{2}\Delta i_L} \tag{15.22}$$

Substitution for v_o from equation (15.2) and using the fact that $\overline{I}_o = \overline{I}_L$, yields

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{\Delta i_L L}{\overline{I}_L(\tau - t_T)} \tag{15.23}$$

Eliminating Δi_L by substituting the limiting condition given by equation (15.13) gives

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{\Delta i_L L}{\overline{I}_L(\tau - t_T)} = \frac{2\overline{I}_L L}{\overline{I}_L(\tau - t_T)} = \frac{2L}{(\tau - t_T)} \tag{15.24}$$

Divide throughout by τ and substituting $\delta = t_T / \tau$ yields

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{2L}{(\tau - t_T)} = \frac{2L}{\tau(1-\delta)} \tag{15.25}$$

The critical resistance can be expressed in a number of forms. By substituting the switching frequency ($f_s = 1/\tau$) or the fundamental inductor reactance ($X_L = 2\pi f_s L$) the following forms result.

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{2L}{\tau(1-\delta)} = \frac{2 f_s L}{(1-\delta)} = \frac{X_L}{\pi(1-\delta)} \qquad (\Omega) \tag{15.26}$$

If the load resistance increases beyond R_{crit}, the output voltage can no longer be maintained with duty cycle control according to the voltage transfer function in equation (15.4). Notice that equation (15.26) is in fact equation (15.12), re-arranged.

15.1.4 Control methods for discontinuous inductor current

Once the load current has reduced to the critical level as specified by equation (15.26), the input energy is in excess of the load requirement. Open loop load voltage regulation control is lost and the capacitor *C* tends to overcharge.
Hardware approaches can be used to solve this problem

- increase *L* thereby decreasing the inductor current ripple p-p magnitude
- step-down transformer impedance matching to effectively reduce the apparent load impedance

Two control approaches to maintain output voltage regulation when $R > R_{crit}$ are

- vary the switching frequency f_s, maintaining the switch on-time t_T constant so that Δi_L is fixed or
- reduce the switch on-time t_T , but maintain a constant switching frequency f_s, thereby reducing Δi_L.

If a fixed switching frequency is desired for all modes of operation, then reduced on-time control, using output voltage feedback, is preferred. If a fixed on-time mode of control is used, then the output voltage is control by varying inversely the frequency with output voltage.

15.1.4i - fixed on-time t_T, variable switching frequency f_{var}

The operating frequency f_{var} is varied while the switch-on time t_T is maintained constant such that the ripple current remains unchanged. Operation is specified by equating the input energy and the output energy, thus maintaining a constant capacitor charge, hence output voltage. That is, equating energies

$$\tfrac{1}{2}\Delta i_L E_i t_T = \frac{v_o^2}{R}\frac{1}{f_{var}} \tag{15.27}$$

Isolating the variable switching frequency f_{var} gives

$$\begin{aligned} f_{var} &= \frac{v_o^2}{\tfrac{1}{2}\Delta i_L E_i t_T}\frac{1}{R} \\ f_{var} &= f_s R_{crit} \times \frac{1}{R} \\ f_{var} &\;\alpha\; \frac{1}{R} \end{aligned} \tag{15.28}$$

That is, once discontinuous inductor current occurs, if the switching frequency is varied inversely with load resistance and the switch on-state period is maintained constant, output voltage regulation can be maintained.

Load resistance *R* is not a directly or readily measurable parameter for feedback proposes. Alternatively, since $v_o = \overline{I}_o R$ substitution for *R* in equation (15.28) gives

$$\begin{aligned} f_{var} &= f_s \frac{R_{crit}}{v_o} \times \overline{I}_o \\ f_{var} &\;\alpha\; \overline{I}_o \end{aligned} \tag{15.29}$$

That is, for $\overline{I}_o < \tfrac{1}{2}\Delta i_L$ or $\overline{I}_o < v_o / R_{crit}$, if t_T remains constant and f_{var} is varied proportionally with load current, then the required output voltage v_o will be maintained.

15.1.4ii - fixed switching frequency f_s, variable on-time $t_{T\text{var}}$

The operating frequency f_s remains fixed while the switch-on time $t_{T\text{var}}$ is reduced, resulting in the ripple current being reduced. Operation is specified by equating the input energy and the output energy as in equation (15.27), thus maintaining a constant capacitor charge, hence voltage. That is

$$\tfrac{1}{2}\Delta i_L E_i t_{T\text{var}} = \frac{v_o^2}{R}\frac{1}{f_s} \tag{15.30}$$

Isolating the variable on-time $t_{T\text{var}}$ yields

$$t_{T\text{var}} = \frac{v_o^2}{\tfrac{1}{2}\Delta i_L E_i f_s}\frac{1}{R}$$

Substituting Δi_L from equation (15.2) gives

$$t_{T\,\mathrm{var}} = t_T\sqrt{R_{crit}} \times \frac{1}{\sqrt{R}}$$
$$t_{T\,\mathrm{var}} \quad \alpha \quad \frac{1}{\sqrt{R}} \qquad (15.31)$$

That is, once discontinuous inductor current commences, if the switch on-time is varied inversely to the square root of the load resistance, maintaining the switching frequency constant, regulation of the output voltage can be maintained.

Again, load resistance R is not a directly or readily measurable parameter for feedback proposes and substitution of $v_o / \overline{I}_o$ for R in equation (15.31) gives

$$t_{T\,\mathrm{var}} = t_T\sqrt{\frac{R_{crit}}{v_o}} \times \sqrt{\overline{I}_o}$$
$$t_{T\,\mathrm{var}} \quad \alpha \quad \sqrt{\overline{I}_o} \qquad (15.32)$$

That is, if f_s is fixed and t_T is reduced proportionally to $\sqrt{\overline{I}_o}$, when $\overline{I}_o < ½\Delta i_L$ or $\overline{I}_o < v_o / R_{crit}$, then the required output voltage magnitude v_o will be maintained.

15.1.5 Output ripple voltage

Three components contribute to the output voltage ripple

- Ripple charging of the ideal capacitor
- Capacitor equivalent series resistance, ESR
- Capacitor equivalent series inductance, ESL

The capacitor inductance and resistance parasitic series component values decrease as the quality of the capacitor increases. The output ripple voltage is the vectorial summation of the three components that are shown in figure 15.3 for the forward converter.

Ideal Capacitor: The ripple voltage for a capacitor is defined as

$$\Delta v_C = \tfrac{1}{C}\int i\,dt$$

Figures 15.2 and 15.3 show that for continuous inductor current, the inductor current which is the output current, swings by Δi around the average output current, $\overline{I}_o$, thus

$$\Delta v_C = \tfrac{1}{C}\int i\,dt = ½\,\tfrac{1}{C}\,\tfrac{\Delta i}{2}\,\tfrac{\tau}{2} \qquad (15.33)$$

Substituting for Δi_L from equation (15.2)

$$\Delta v_C = \tfrac{1}{C}\int i\,dt = ½\,\tfrac{1}{C}\,\tfrac{\Delta i}{2}\,\tfrac{\tau}{2} = \tfrac{1}{8}\,\tfrac{1}{C}\,\tfrac{v_o}{L} \times (\tau - t_T)\tau \qquad (15.34)$$

If ESR and ESL are ignored, after rearranging, equation (15.34) gives the percentage voltage ripple (peak to peak) in the output voltage

$$\frac{\Delta v_C}{v_o} = \frac{\Delta v_o}{v_o} = \tfrac{1}{8}\,\frac{1}{LC} \times (1-\delta)\tau^2 = ½\pi^2(1-\delta)\left(\frac{f_c}{f_s}\right)^{½} \qquad (15.35)$$

In complying with output voltage ripple requirements, from this equation, the switching frequency $f_s = 1/\tau$ must be much higher that the cut-off frequency given by the forward converter low-pass, second-order LC output filter, $f_c = 1/2\pi\sqrt{LC}$.

ESR: The equivalent series resistor voltage follows the ripple current, that is, it swings linearly about

$$V_{ESR} = \pm ½\Delta i \times R_{ESR} \qquad (15.36)$$

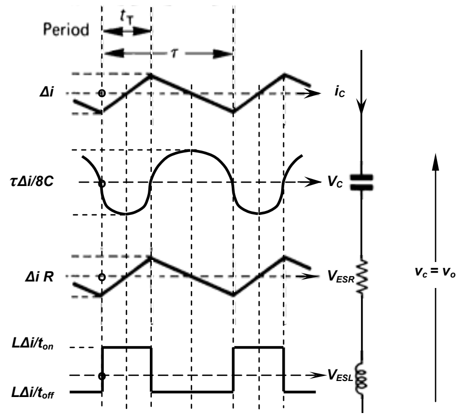

Figure 15.3. *Forward converter, three output ripple components, showing: left -voltage components; centre – waveforms; and right - capacitor model.*

ESL: The equivalent series inductor voltage is derived from $v = L\,di/dt$, that is when the switch is on

$$V^+_{ESL} = L\Delta i / t_{on} = L\Delta i / \delta\tau \qquad (15.37)$$

When the switch is off

$$V^-_{ESL} = -L\Delta i / t_{off} = -L\Delta i / (1-\delta)\tau \qquad (15.38)$$

The total ripple voltage is

$$\Delta v_o = \Delta v_C + V_{ESR} + V_{ESL} \qquad (15.39)$$

Forming a time domain solution for each component, then differentiating, gives a maximum ripple when

$$t = 2CR_{ESR}(1-\delta) \qquad (15.40)$$

This expression is independent of the equivalent series inductance, which is expected since it is constant during each state. If dominant, the inductor will affect the output voltage ripple at the switch turn-on and turn-off instants.

Example 15.1: ***Buck (step-down forward) converter***

The step-down converter in figure 15.2a operates at a switching frequency of 10 kHz. The output voltage is to be fixed at 48 V dc across a 1 Ω resistive load. If the input voltage E_i =192 V and the choke L = 200μH:

i. calculate the switch T on-time duty cycle δ and switch on-time t_T.
ii. calculate the average load current $\overline{I}_o$, hence average input current $\overline{I}_i$.
iii. draw accurate waveforms for
  - the voltage across, and the current through L; v_L and i_L
  - the capacitor current, i_c
  - the switch and diode voltage and current; v_T, v_D, i_T, i_D.

  Hence calculate the switch utilisation ratio as defined by equation (15.11).
iv. calculate the mean and rms current ratings of diode D, switch T and L.
v. calculate the capacitor average and rms current, i_{Crms} and output ripple voltage if the capacitor has an internal equivalent series resistance of 20mΩ (C = ∞).
vi. calculate the maximum load resistance R_{crit} before discontinuous inductor current. Calculate the output voltage and inductor non-conduction period , t_x, when the load resistance is triple the critical resistance R_{crit}.
vii. if the maximum load resistance is 1Ω, calculate
  - the value the inductance L can be reduced to be on the verge of discontinuous inductor current and for that L
  - the peak-to-peak ripple and rms, inductor and capacitor currents.
viii. Specify two control strategies for controlling the forward converter in a discontinuous inductor current mode.
ix. Output ripple voltage hence percentage output ripple voltage, for C=1000μF and an equivalent series inductance of ESL=0.5uH, assuming ESR = 0Ω.

Solution

i. From equation (15.4) the duty cycle δ is

$$\delta = \frac{v_o}{E_i} = \frac{48\text{V}}{192\text{V}} = ¼ = 25\%$$

Also, from equation (15.4), for a 10kHz switching frequency, the switching period τ is 100μs and the transistor on-time t_T is given by

$$\frac{v_o}{E_i} = \frac{t_T}{\tau} = \frac{48\text{V}}{192\text{V}} = \frac{t_T}{100\mu\text{s}}$$

whence the transistor on-time is 25μs and the diode conducts for 75μs.

ii. The average load current is $\overline{I}_o = \dfrac{v_o}{R} = \dfrac{48\text{V}}{1\Omega} = 48\text{A} = \overline{I}_L$

From power-in equals power-out, the average input current is

$$\overline{I}_i = v_o\overline{I}_o / E_i = 48\text{V}\times 48\text{A}/192\text{V} = 12\text{A}$$

iii. From equation (15.1) (or equation (15.2)) the inductor peak-to-peak ripple current is

$$\Delta i_L = \frac{E_i - v_o}{L}\times t_T = \frac{192\text{V-}48\text{V}}{200\mu\text{H}}\times 25\mu\text{s} = 18\text{A}$$

From part ii, the average inductor current is the average output current, 48A. The required circuit voltage and current waveforms are shown in the following figure.
The circuit waveforms show that the maximum switch voltage and current are 192V and 57A respectively. The switch utilising ratio is given by equation (15.11), that is

$$SUR = \frac{P_{out}}{E_i\times\hat{i}_o} = \frac{v_o^2/R}{E_i\times\hat{i}_o} = \frac{48\text{V}^2/1\Omega}{192\text{V}\times 57\text{A}} \equiv 21\%$$

If the ripple current were assume small, the resulting SUR value of δ = 33% gives a misleading under-estimate indication.

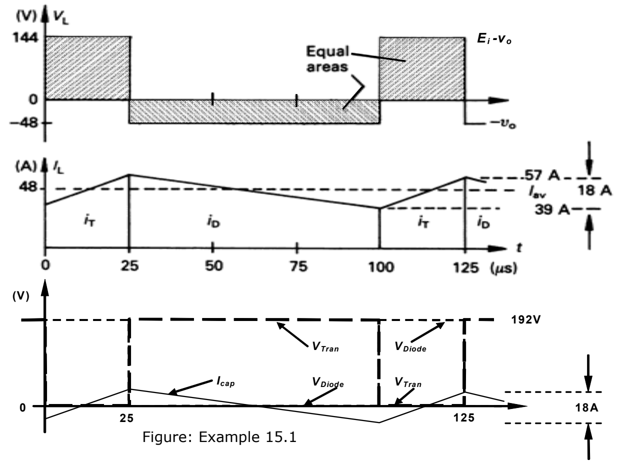

Figure: Example 15.1

iv. Current i_D through diode D is shown on the inductor current waveform. The average diode current is

$$\overline{I}_D = \frac{\tau - t_T}{\tau} \times \overline{I}_L = (1-\delta)\times \overline{I}_L = (1-¼)\times 48\text{A} = 36\text{A}$$

The rms diode current is given by

$$i_{D\text{rms}} = \sqrt{\frac{1}{\tau}\int_0^{\tau - t_T} (\hat{i}_L - \frac{\Delta i_L}{\tau - t_T} t)^2 dt} = \sqrt{\frac{1}{100\mu\text{s}}\int_0^{75\mu\text{s}} (57\text{A-} \frac{18\text{A}}{75\mu\text{s}} t)^2 dt}$$

$$= 41.8\text{A}$$

Current i_T through the switch T is shown on the inductor current waveform. The average switch current is

$$\overline{I}_T = \frac{t_T}{\tau}\overline{I}_L = \delta \overline{I}_L = ¼\times 48\text{A} = 12\text{A}$$

Alternatively, from power-in equals power-out

$$\overline{I}_T = \overline{I}_i = v_o \overline{I}_o / E_i = 48\text{V}\times 48\text{A}/192\text{V} = 12\text{A}$$

The transistor rms current is given by

$$i_{T\text{rms}} = \sqrt{\frac{1}{\tau}\int_0^{t_T} (\check{i}_L + \frac{\Delta i_L}{t_T} t)^2 dt} = \sqrt{\frac{1}{100\mu\text{s}}\int_0^{25\mu\text{s}} (39\text{A+} \frac{18\text{A}}{25\mu\text{s}} t)^2 dt}$$

$$= 24.1\text{A}$$

The mean inductor current is the mean output current, $\overline{I}_o = \overline{I}_L = 48A$.

The inductor rms current is given by equation (15.6), that is

$$I_{L\,rms} = \sqrt{\overline{I}_L^2 + \left(\frac{½\Delta i_L}{\sqrt{3}} \right)^2} = \sqrt{48\text{A}^2 + \left(\frac{½\times 18\text{A}}{\sqrt{3}} \right)^2} = 48.3\text{A}$$

v. The average capacitor current $\overline{I}_C$ is zero and the rms ripple current is given by

$$i_{C\text{rms}} = \sqrt{\frac{1}{\tau}\left[\int_0^{t_T} (-½\Delta i_L + \frac{\Delta i_L}{t_T} t)^2 dt + \int_0^{\tau - t_T} (½\Delta i_L - \frac{\Delta i_L}{\tau - t_T} t)^2 dt\right]}$$

$$= \sqrt{\frac{1}{100\mu\text{s}}\left[\int_0^{25\mu\text{s}} (\text{-9A+} \frac{18\text{A}}{25\mu\text{s}} t)^2 dt + \int_0^{75\mu\text{s}} (\text{9A-} \frac{18\text{A}}{75\mu\text{s}} t)^2 dt\right]}$$

$$= 5.2\text{A} \quad (= \Delta i_L / 2\sqrt{3})$$

The capacitor voltage ripple (hence the output voltage ripple), is determined by the capacitor ripple current which is equal to the inductor ripple current, 18A p-p, that is

$$v_{o\text{ripple}} = \Delta i_L \times R_{Cesr}$$

$$= 18\text{A}\times 20\text{m}\Omega = 360\text{mV p-p}$$

and the rms output voltage ripple is

$$v_{o\text{rms}} = i_{C\text{rms}} \times R_{Cesr}$$

$$= 5.2\text{A rms}\times 20\text{m}\Omega = 104\text{mV rms}$$

vi. Critical load resistance is given by equation (15.26), namely

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{2L}{\tau(1-\delta)}$$

$$= \frac{2\times 200\mu\text{H}}{100\mu\text{s}\times(1\text{-}¼)} = 16/3\Omega$$

$$= 5⅓\Omega \text{ when } \overline{I}_o = 9\text{A}$$

Alternatively, the critical load current is 9A (½Δi_L), thus from the equation immediately above, the load resistance must not be greater than $v_o / \overline{I}_o$ = 48V/9A=5⅓Ω, if the inductor current is to be continuous.

When the load resistance is tripled to 16Ω the output voltage is given by equation (15.20), which is shown normalised in table 15.2. That is

$$v_o = E_i\, k\left[-1 + \sqrt{1 + \frac{2}{k}} \right] \text{ where } k = \frac{\delta^2 R\tau}{4L} = \frac{¼^2 \times 16\Omega\times 100\mu\text{s}}{4\times 200\mu\text{H}} = ⅛ \text{ thus}$$

$$v_o = 192\text{V}\times ⅛ \times\left[-1 + \sqrt{1 + \frac{2}{⅛}} \right] = 75\text{V}$$

The inductor current is zero for an interval of the 100μs switching period, and the time is given by the appropriate normalised expression involving t_x for the forward converter in table 15.2 or by equation (15.16), which when re-arranged to isolate t_x becomes

$$t_x = \tau\left(1 - \frac{\delta}{v_o / E_i} \right) = 100\mu\text{s}\times\left(1 - \frac{¼}{75\text{V}/50\text{V}} \right) = 36\mu\text{s}$$

vii. The critical resistance formula given in equation (15.26) is valid for finding critical inductance when inductance is made the subject of the equation, that is, rearranging equation (15.26) gives

$$L_{crit} = ½\times R\times(1-\delta)\times\tau \qquad \text{(H)}$$

$$= ½\times 1\Omega\times(1\text{-}¼)\times 100\mu\text{s} = 37½\mu\text{H}$$

This means the inductance can be reduced from 200μH with a 48A mean and 18A p-p ripple current, to 37½μH with the same 48A mean plus a superimposed 96A p-p ripple current. The rms capacitor current is given by

$$i_{C\text{rms}} = \Delta i_L / 2\sqrt{3}$$

$$= 96\text{A}/2\sqrt{3} = 27.2\text{A rms}$$

The inductor rms current requires the following integration

$$i_{L\text{rms}} = \sqrt{\frac{1}{\tau}\left[\int_0^{t_T}(\check{i}_L + \frac{\Delta i_L}{t_T}t)^2\,dt + \int_0^{\tau - t_T}(\hat{i}_L - \frac{\Delta i_L}{\tau - t_T}t)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{100\mu\text{s}} \times \left[\int_0^{25\mu\text{s}}(0 + \frac{96\text{A}}{25\mu\text{s}}t)^2\,dt + \int_0^{75\mu\text{s}}(96\text{A} - \frac{96\text{A}}{75\mu\text{s}}t)^2\,dt\right]}$$

$$= 96/\sqrt{3} = 55.4\text{ A rms}$$

or from equation (15.6)

$$i_{L\text{rms}} = \sqrt{\overline{I}_L^2 + i_{L\text{ripple}}^2}$$

$$= \sqrt{48^2 + (96/2\sqrt{3})^2}$$

$$= 55.4\text{ A rms}$$

viii. For $R > 16/3\Omega$, or $\overline{I}_o < 9\text{A}$, equations (15.29) or (15.32) can be used to develop a suitable control strategy.

(a) From equation (15.29), using a variable switching frequency of less than 10kHz,

$$f_{\text{var}} = f_s \frac{R_{crit}}{v_o}\overline{I}_o = 10\text{kHz}\ \frac{5\tfrac{1}{3}\Omega}{48\text{V}}\ \overline{I}_o$$

$$f_{\text{var}} = \frac{10}{9} \times \overline{I}_o \quad \text{kHz}$$

(b) From equation (15.32), maintaining a fixed switching frequency of 10kHz, the on-time duty cycle is reduced for $\overline{I}_o < 9\text{A}$ according to

$$t_{T\,\text{var}} = t_T\sqrt{\frac{R_{crit}}{v_o}}\sqrt{\overline{I}_o} = 25\mu\text{s}\sqrt{\frac{5\tfrac{1}{3}\Omega}{48\text{V}}}\ \sqrt{\overline{I}_o}$$

$$t_{T\,\text{var}} = \frac{25}{3} \times \sqrt{\overline{I}_o} \quad \mu\text{s}$$

ix. From equation (15.33) the output ripple voltage due the pure capacitor is given by

$$\Delta v_C = \frac{\Delta i\ \tau}{8\ C}$$

$$= \frac{18\text{A} \times 100\mu\text{s}}{8 \times 1000\mu\text{F}} = 225\text{mV p-p}$$

The voltage produced because of the equivalent series 0.5 µH inductance is

$$V_{ESL}^{+} = L\Delta i / \delta\tau$$

$$= 0.5\mu\text{H} \times 18\text{A}/0.25 \times 100\mu\text{s} = 360\text{mV}$$

$$V_{ESL}^{-} = -L\Delta i / (1 - \delta)\tau$$

$$= -\ 0.5\mu\text{H} \times 18\text{A}/(1\ -\ 0.25) \times 100\mu\text{s} = -120\text{mV}$$

Time domain summation of the capacitor and ESL inductor voltages show that the peak to peak output voltage swing is determined by the ESL inductor, giving

$$\Delta v_o = V_{ESL}^{+} - V_{ESL}^{-}$$

$$= 360\text{mV} + 120\text{mV} = 480\text{mV}$$

The percentage ripple in the output voltage is 480mV/48V = 1%.

♣

15.1.6 *Underlying mechanisms of the forward converter*

The inductor current is pivotal to the analysis and understanding of any smps. For analysis, the smps internal and external electrical conditions are in steady-state on a cycle-by-cycle basis and the input power is equal to the output power.

The first concept to appreciate is that the net capacitor charge change is zero over each switching cycle. That is, the average capacitor current is zero:

$$\overline{I}_c = \frac{1}{\tau}\int_t^{t+\tau} i_c(t)\,dt = 0$$

In so being, the output capacitor provides any load current deficit and stores any load current surplus associated with the inductor current within each complete cycle. Thus, the capacitor is a temporary storage component where the capacitor voltage is fixed on a cycle-by-cycle basis, and because of its large capacitance does not vary significantly within a cycle.

The second concept involved is that the average inductor voltage is zero. Based on $v = L\,di/dt$ the equal area criteria in chapter 11.1.1i

$$i_{t+\tau} - i_t = \frac{1}{L}\int_t^{t+\tau} v_L(t)\,dt = 0 \quad \text{since} \quad i_{t+\tau} = i_t \ \text{in steady-state}$$

Thus the average inductor voltage is zero:

$$\overline{V}_L = \frac{1}{\tau}\int_t^{t+\tau} v_L(t)\,dt = 0$$

The most enlightening way to appreciate the operating mechanisms is to consider how the inductor current varies with load resistance *R* and inductance *L*. The figure 15.4 shows the inductor current associated with the various parts of example 15.1.

For continuous inductor current operation, the two necessary and sufficient equations are $I_o = v_o / R$ and equation (15.2). Since the duty cycle and on-time are fixed for a given output voltage requirement, equation (15.2) can be simplified to show that the ripple current is inversely proportional to inductance, as follows

$$\Delta i_L = \frac{v_o}{L} \times (\tau - t_T) \qquad (15.41)$$
$$\Delta i_L \; \alpha \; \frac{1}{L}$$

Since the average inductor current is equal to the load current, then the average inductor current is inversely proportional to the load resistance, that is

$$\overline{I}_L = \overline{I}_o = v_o / R \qquad (15.42)$$
$$\overline{I}_L \; \alpha \; \frac{1}{R}$$

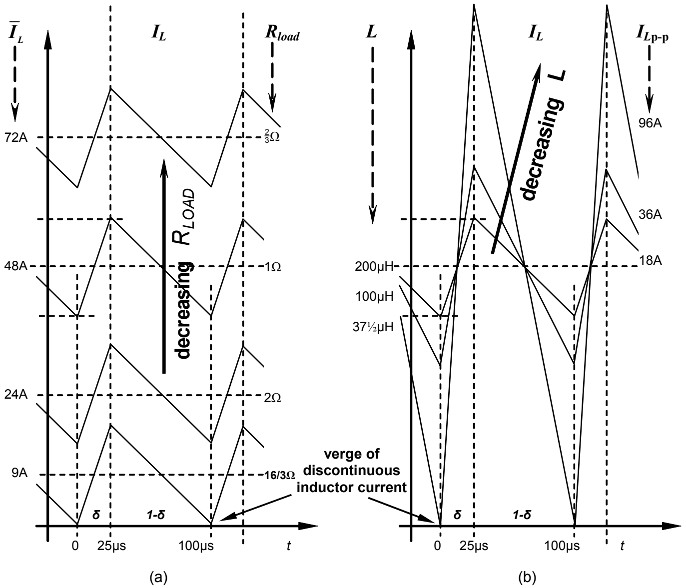

Figure 15.4. *Forward converter (buck converter) operational mechanisms showing that: (a) the average inductor current is inversely proportional to load resistance R and (b) the inductor ripple current magnitude is inversely proportional to inductance L.*

Equation (15.42) predicts that the average inductor current is inversely proportional to the load resistance, as shown in figure 15.4a. As the load is varied, the triangular inductor current moves vertically, but importantly the peak-to-peak ripple current is constant, that is the ripple current is independent of load. As the load current is progressively decreased, by increasing *R*, the peak-to-peak current is unchanged; the inductor minimum current eventually reduces to zero, and discontinuous inductor current operation occurs.

Equation (15.41) indicates that the inductor ripple current is inversely proportional to inductance, as shown in figure 15.4b. As the inductance is varied the ripple current varies inversely, but importantly the average current is constant, and specifically the average current value is not related to inductance *L* and is solely determined by the load current, v_o /R. As the inductance decreases the magnitude of the ripple current increases, the average is unchanged, and the minimum inductor current eventually reaches zero and discontinuous inductor current operation results.

15.2 Flyback converters

Flyback converters store energy in an inductor, termed ‘choke’, and transfer that energy to the load storage capacitor such that output voltage magnitudes in excess of the input voltage are attained. Flyback converters are alternatively known as *ringing choke* converters. Two versions of the flyback converter are possible

- The step-up voltage flyback converter, called the *boost converter,* where no output voltage polarity inversion occurs.
- The step-up/step-down voltage flyback converter, called the *buck-boost converter,* where output voltage polarity inversion occurs.

15.3 The boost converter

The *boost converter* transforms a dc voltage input to a dc voltage output that is greater in magnitude but has the same polarity as the input. The basic circuit configuration is shown in figure 15.5a. It will be seen that when the transistor is off, the output capacitor is charged to the input voltage E_i. Inherently, the output voltage v_o can never be less than the input voltage level.

When the transistor is turned on, the supply voltage E_i is applied across the inductor *L* and the diode D is reverse-biased by the output voltage v_o. Energy is transferred from the supply to *L* and when the transistor is turned off this energy is transferred to the load and output capacitor. While the inductor is transferring its stored energy into *C,* energy is also being provided from the input source.

The output current is always discontinuous, but the input current can be either continuous or discontinuous. For analysis, we assume $v_o > E_i$ and a constant input and output voltage. Inductor currents are then linear and vary according to $v = L\, di/dt$.

15.3.1 Continuous inductor current

The circuit voltage and current waveforms for continuous inductor conduction are shown in figure 15.5b. The inductor current excursion, which is the input current excursion, during the switch on-time t_T and switch off-time τ- t_T , is given by

$$\Delta i_L = \frac{(v_o - E_i)}{L}(\tau - t_T) = \frac{E_i}{L} t_T \tag{15.43}$$

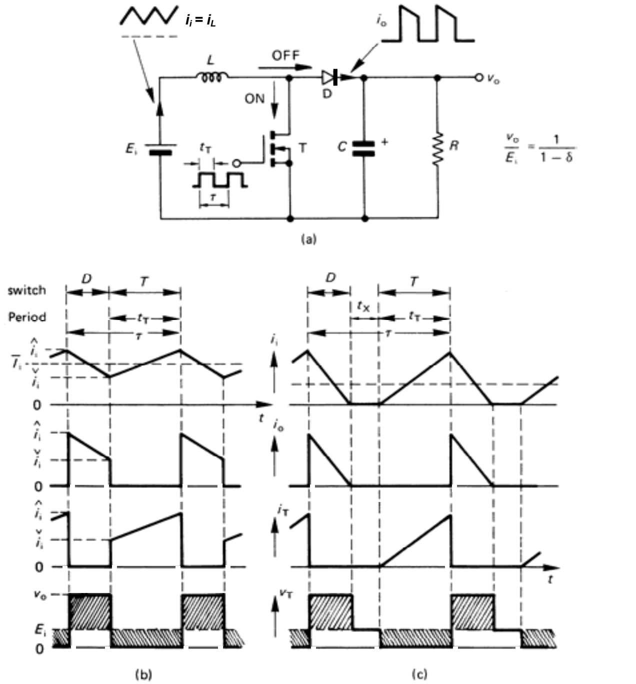

Figure 15.5. *Non-isolated, step-up, flyback converter (boost converter) where $v_0 \ge E_1$: (a) circuit diagram; (b) waveforms for continuous input current; and (c) waveforms for discontinuous input current.*

that is, after rearranging, the voltage transfer function is given by

$$\frac{v_o}{E_i} = \frac{\bar I_i}{\bar I_o} = \frac{1}{1-\delta} \tag{15.44}$$

where $\delta = t_T/\tau$ and t_T is the transistor on-time. The maximum inductor current, which is the maximum input current, $\hat i_L$, using equation (15.43), is given by

$$\begin{aligned}\hat i_L &= \bar I_L + ½\Delta i_L = \bar I_i + ½\frac{E_i t_T}{L}\\ &= \frac{\bar I_o}{1-\delta} + ½\frac{v_o}{L}(1-\delta)\delta\tau = v_o\left[\frac{1}{(1-\delta)R} + \frac{(1-\delta)\delta\tau}{2L}\right]\end{aligned} \tag{15.45}$$

while the minimum inductor current, $\check i_L$ is given by

$$\begin{aligned}\check i_L &== \bar I_L - ½\Delta i_L = \bar I_i - ½\frac{E_i t_T}{L}\\ &= \frac{\bar I_o}{1-\delta} - ½\frac{v_o}{L}(1-\delta)\delta\tau = v_o\left[\frac{1}{(1-\delta)R} - \frac{(1-\delta)\delta\tau}{2L}\right]\end{aligned} \tag{15.46}$$

For continuous conduction $\check i_L \ge 0$, that is, from equation (15.46)

$$\bar I_L \ge ½\frac{E_i t_T}{L} = ½\frac{v_o(1-\delta)t_T}{L} \tag{15.47}$$

The inductor rms ripple current (and input ripple current in this case) is given by

$$i_{Lr} = \frac{\Delta i_L}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}\frac{v_o}{L}(1-\delta)\delta\tau \tag{15.48}$$

The harmonic components in the input current are

$$I_{in} = \frac{\sqrt{2}\, E_i\tau \sin n\delta\pi}{2\pi^2 n^2 (1-\delta)L} = \frac{\sqrt{2}\, v_o\tau \sin n\delta\pi}{2\pi^2 n^2 L} \tag{15.49}$$

while the inductor total rms current is

$$i_{Lrms} = \sqrt{\bar I_L^2 + i_{Lr}^2} = \sqrt{\bar I_L^2 + \left(½\Delta i_L \big/ \sqrt{3}\right)^2} = \sqrt{\frac{1}{3}\left(\hat i_L^2 + \hat i_L \times \check i_L + \check i_L^2\right)} \tag{15.50}$$

The switch and diode average and rms currents are given by

$$\begin{aligned}&\bar I_T = \bar I_i - \bar I_o = \delta\,\bar I_i = \delta\,\bar I_L \qquad && I_{Trms} = \sqrt{\delta}\; i_{Lrms}\\ &\bar I_D = (1-\delta)\bar I_i = \bar I_o && I_{Drms} = \sqrt{1-\delta}\; i_{Lrms}\end{aligned} \tag{15.51}$$

Switch utilisation ratio

The switch utilisation ratio, SUR, is a measure of how fully a switching device’s power handling capabilities are utilised in any switching application. The ratio is defined as

$$SUR = \frac{P_{out}}{p\,\widehat{V}_T\,\hat{I}_T} \tag{15.52}$$

where p is the number of power switches in the circuit; $p=1$ for the boost converter. The switch maximum instantaneous voltage and current are $\widehat{V}_T$ and $\hat{I}_T$ respectively. As shown in figure 15.5b, the maximum switch voltage supported in the off-state is v_o, while the maximum current is the maximum inductor current $\hat{i}_L$ which is given by equation (15.45). If the inductance L is large such that the ripple current is small, the peak inductor current is approximated by the average inductor current such that $\hat{I}_T \approx \bar{I}_L = \bar{I}_o / 1-\delta$, that is

$$SUR = \frac{v_o \bar{I}_o}{v_o \times \bar{I}_o / 1-\delta} = 1-\delta \tag{15.53}$$

which assumes continuous inductor current. This result shows that the lower the duty cycle, that is the closer the step-up voltage v_o is to the input voltage E_i, the better the switch *I-V* ratings are utilised.

15.3.2 Discontinuous capacitor charging current in the switch off-state

It is possible that the input current (inductor current) falls below the output (resistor) current during a part of the cycle when the switch is off and the inductor is transferring energy to the output circuit. Under such conditions, towards the end of the off period, part of the load current requirement is provided by the capacitor even though this is the period during which its charge is replenished by inductor energy. The circuit independent transfer function in equation (15.44) remains valid. This discontinuous charging condition occurs when the minimum inductor current and the output current are equal. That is

$$\begin{aligned} &\check{I}_L - \bar{I}_o \le 0 \\ &\bar{I}_L - ½\Delta i_L - \bar{I}_o \le 0 \\ &\frac{\bar{I}_o}{1-\delta} - ½\frac{E_i \delta \tau}{L} - \bar{I}_o \le 0 \end{aligned} \tag{15.54}$$

which yields

$$\delta \le 1 - \sqrt{\frac{2L}{\tau R}} \tag{15.55}$$

15.3.3 Discontinuous inductor current

If the inequality in equation (15.47) is not satisfied, the input current, which is also the inductor current, reaches zero and discontinuous conduction occurs during the switch off period. Various circuit voltage and current waveforms for discontinuous inductor

conduction are shown in figure 15.5c.

The onset of discontinuous inductor operation occurs when the minimum inductor current $\check{i}_L$, reaches zero. That is, with $\check{i}_L = 0$ in equation (15.46), the last equality

$$\frac{1}{(1-\delta)R} - \frac{(1\text{-}\delta)\delta\tau}{2L} = 0 \tag{15.56}$$

relates circuit component values (R and L) and operating conditions (f and δ) at the verge of discontinuous inductor current.

With $\check{i}_L = 0$, the output voltage is determined as follows

$$\hat{i}_L = \frac{E_i t_T}{L} = \frac{(v_o - E_i)}{L}(\tau - t_T - t_x) \tag{15.57}$$

yielding

$$\frac{v_o}{E_i} = \frac{1-\frac{t_x}{\tau}}{1-\frac{t_x}{\tau}-\delta} \tag{15.58}$$

Alternatively, using

$$\hat{i}_L = \frac{E_i t_T}{L}$$

and

$$\bar{I}_L - \bar{I}_o = ½\delta\hat{i}_L$$

yields

$$\frac{2}{\delta}(\bar{I}_L - \bar{I}_o) = \frac{E_i t_T}{L}$$

Assuming power-in equals power-out

$$\frac{2}{\delta}\bar{I}_o(\frac{v_o}{E_i} - 1) = \frac{E_i t_T}{L}$$

that is

$$\frac{v_o}{E_i} = 1 + \frac{E_i t_T \delta^2}{2L\bar{I}_o} = 1 + \frac{v_o t_T \delta^2}{2L\bar{I}_i} \tag{15.59}$$

or

$$\frac{v_o}{E_i} = \frac{1}{1 - \frac{E_i t_T \delta^2}{2L\bar{I}_i}} \tag{15.60}$$

On the verge of discontinuous conduction, these equations can be rearranged to give

$$\bar{I}_o = \frac{E_i}{2L}\tau\delta(1-\delta) \tag{15.61}$$

At a low output current or low input voltage, there is a likelihood of discontinuous inductor conduction. To avoid discontinuous conduction, larger inductance values are needed, which worsen transient response. Alternatively, with extremely high on-state duty cycles, (because of a low input voltage E_i) a voltage-matching step-up transformer can be used to decrease δ. Figures 15.5b and c show that the output current is always discontinuous.

15.3.4 Load conditions for discontinuous inductor current

As the load current decreases, the inductor average current also decreases, but the inductor ripple current magnitude is unchanged. If the load resistance is increased sufficiently, the bottom of the triangular inductor current, $\check{i}_L$, eventually reduces to zero. Any further increase in load resistance causes discontinuous inductor current and the voltage transfer function given by equation (15.44) is no longer valid and equations (15.58) and (15.59) are applicable. The critical load resistance for continuous inductor current is specified by

$$R_{crit} \le \frac{v_o}{\overline{I}_o} \tag{15.62}$$

Eliminating the output current by using the fact that power-in equals power-out and $\overline{I}_i = \overline{I}_L$, yields

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{v_o^2}{E_i \overline{I}_L} \tag{15.63}$$

Using $\overline{I}_L = ½\Delta i_L$ then substituting with the right hand equality of equation (15.43), halved, gives

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{v_o^2}{E_i \overline{I}_L} = \frac{v_o^2 2L}{E_i^2 t_T} = \frac{2L}{\tau\delta(1-\delta)^2} \tag{15.64}$$

The critical resistance can be expressed in a number of forms. By substituting the switching frequency ($f_s = 1/\tau$) or the fundamental inductor reactance ($X_L = 2\pi f_s L$) the following forms result.

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{2L}{\tau\delta(1-\delta)^2} = \frac{2 f_s L}{\delta(1-\delta)^2} = \frac{X_L}{\pi\delta(1-\delta)^2} \qquad (\Omega) \tag{15.65}$$

If the load resistance increases beyond R_{crit}, the output voltage can no longer be maintained with duty cycle control according to the voltage transfer function in equation (15.44). Equation (15.65) is equation (15.56), re-arranged.

15.3.5 Control methods for discontinuous inductor current

Once the load current has reduced to the critical level as specified by equation (15.65), the input energy is in excess of the load requirement. Open loop load voltage regulation control is lost and the capacitor C tends to overcharge, thereby increasing v_o.

Hardware approaches can be used to solve this problem

- increase L thereby decreasing the inductor current ripple p-p magnitude
- step-down transformer impedance matching to effectively reduce the apparent load impedance

Two control approaches to maintain output voltage regulation when $R > R_{crit}$ are

- vary the switching frequency f_s, maintaining the switch on-time t_T constant so that Δi_L is fixed or
- reduce the switch on-time t_T , but maintain a constant switching frequency f_s, thereby reducing Δi_L.

If a fixed switching frequency is desired for all modes of operation, then reduced on-time control, using output voltage feedback, is preferred. If a fixed on-time mode of control is used, then the output voltage is control by inversely varying the frequency with output voltage.

15.3.5i - fixed on-time t_T, variable switching frequency f_{var}

The operating frequency f_{var} is varied while the switch-on time t_T is maintained constant such that the ripple current remains unchanged. Operation is specified by equating the input energy and the output energy, thus maintaining a constant capacitor charge, hence output voltage. That is, equating energies

$$½\Delta i_L E_i \tau = \frac{v_o^2}{R}\frac{1}{f_{var}} \tag{15.66}$$

Isolating the variable switching frequency f_{var} gives

$$\begin{aligned} f_{var} &= \frac{v_o^2}{½\Delta i_L E_i \tau}\frac{1}{R} \\ f_{var} &= f_s R_{crit} \times \frac{1}{R} \\ f_{var} &\;\alpha\; \frac{1}{R} \end{aligned} \tag{15.67}$$

Load resistance R is not a directly or readily measurable parameter for feedback proposes. Alternatively, since $v_o = \overline{I}_o R$, substitution for R in equation (15.67) gives

$$\begin{aligned} f_{var} &= f_s \frac{R_{crit}}{v_o} \times \overline{I}_o \\ f_{var} &\;\alpha\; \overline{I}_o \end{aligned} \tag{15.68}$$

That is, for discontinuous inductor current, namely $\overline{I}_i < ½\Delta i_L$ or $\overline{I}_o < v_o / R_{crit}$, if the switch on-state period t_T remains constant and f_{var} is either varied proportionally with load current or varied inversely with load resistance, then the required output voltage v_o will be maintained.

15.3.5ii - fixed switching frequency f_s, variable on-time $t_{T\text{var}}$

The operating frequency f_s remains fixed while the switch-on time $t_{T\text{var}}$ is reduced such that the ripple current can be reduced. Operation is specified by equating the input energy and the output energy as in equation (15.66), thus maintaining a constant capacitor charge, hence voltage. That is

$$\tfrac{1}{2}\Delta i_L E_i t_{T\,\text{var}} = \frac{v_o^2}{R}\frac{1}{f_s} \tag{15.69}$$

Isolating the variable on-time $t_{T\text{var}}$ gives

$$t_{T\,\text{var}} = \frac{v_o^2}{\tfrac{1}{2}\Delta i_L E_i f_s}\frac{1}{R}$$

Substituting Δi_L from equation (15.43) gives

$$t_{T\,\text{var}} = t_T\sqrt{R_{crit}} \times \frac{1}{\sqrt{R}}$$
$$t_{T\,\text{var}}\ \alpha\ \frac{1}{\sqrt{R}} \tag{15.70}$$

Again, load resistance R is not a directly or readily measurable parameter for feedback proposes and substitution of $v_o/\overline{I}_o$ for R in equation (15.70) gives

$$t_{T\,\text{var}} = t_T\sqrt{\frac{R_{crit}}{v_o}} \times \sqrt{\overline{I}_o}$$
$$t_{T\,\text{var}}\ \alpha\ \sqrt{\overline{I}_o} \tag{15.71}$$

That is, if the switching frequency f_s is fixed and switch on-time t_T is reduced proportionally to $\sqrt{\overline{I}_o}$ or inversely to $\sqrt{R}$, when discontinuous inductor current commences, namely $\overline{I}_i < \tfrac{1}{2}\Delta i_L$ or $\overline{I}_o < v_o/R_{crit}$, then the required output voltage magnitude v_o will be maintained.

15.3.6 Output ripple voltage

The output ripple voltage is the capacitor ripple voltage. The ripple voltage for a capacitor is defined as

$$\Delta v_o = \tfrac{1}{C}\int i\,dt$$

Figure 15.5 shows that for continuous inductor current, the constant output current $\overline{I}_o$ is provided solely from the capacitor during the period t_{on} when the switch is on, thus

$$\Delta v_o = \tfrac{1}{C}\int i\,dt = \tfrac{1}{C}t_{on}\overline{I}_o$$

Substituting for $\overline{I}_o = v_o/R$ gives

$$\Delta v_o = \tfrac{1}{C}\int i\,dt = \tfrac{1}{C}t_{on}\overline{I}_o = \tfrac{1}{C}t_{on}\,{}^{v_o}\!/_{R}$$

Rearranging gives the percentage voltage ripple (peak to peak) in the output voltage

$$\frac{\Delta v_o}{v_o} = \frac{\delta\,\tau}{R\,C} \tag{15.72}$$

The capacitor equivalent series resistance and inductance can be account for, as with the forward converter, 15.1.4. When the switch conducts, the output current is constant and is provided from the capacitor. No ESL voltage effects result during this constant capacitor current portion of the switching cycle.

Example 15.2: ***Boost (step-up flyback) converter***

The boost converter in figure 15.5 is to operate with a 50μs transistor fixed on-time in order to convert the 50 V input up to 75 V at the output. The inductor is 250μH and the resistive load is 2.5Ω.

i. Calculate the switching frequency, hence transistor off-time, assuming continuous inductor current.
ii. Calculate the mean input and output current.
iii. Draw the inductor current, showing the minimum and maximum values.
iv. Calculate the capacitor rms ripple current.
v. Derive general expressions relating the operating frequency to varying load resistance.
vi. At what load resistance does the instantaneous input current fall below the output current.

Solution

i. From equation (15.44), which assumes continuous inductor current

$$\frac{v_o}{E_i} = \frac{1}{1-\delta} \quad \text{where} \quad \delta = \frac{t_T}{\tau}$$

that is

$$\frac{75\text{V}}{50\text{V}} = \frac{1}{1-\delta} \quad \text{where} \quad \delta = \frac{50\mu\text{s}}{\tau} = {}^{1}\!/_{3}$$

That is, τ = 150 μs or f_s= 1/τ = 6.66 kHz, with a 100μs switch off-time.

ii. The mean output current $\overline{I}_o$ is given by

$$\overline{I}_o = v_o/R = 75\text{V}/2.5\Omega\ =\ 30\text{A}$$

From power transfer considerations

$$\overline{I}_i = \overline{I}_L = v_o\overline{I}_o/E_i = 75\text{V}\times30\text{A}/50\text{V}\ =\ 45\text{A}$$

iii. From $v = L\,di/dt$, the ripple current $\Delta i_L = E_i t_T/L$ = 50V x 50μs /250 μH = 10 A that is

$$\hat{i}_L = \bar{I}_L + ½\Delta i_L = 45\text{A} + ½\times 10\text{A} = 50\text{A}$$

$$\check{i}_L = \bar{I}_L - ½\Delta i_L = 45\text{A} - ½\times 10\text{A} = 40\text{A}$$

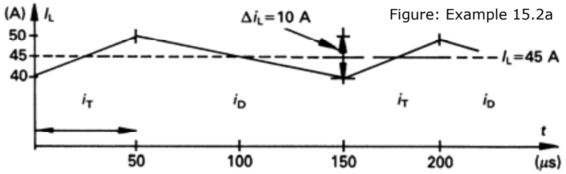

Figure: Example 15.2a

iv. The capacitor current is derived by using Kirchhoff's current law such that at any instant in time, the diode current, plus the capacitor current, plus the 30A constant load current into R, all sum to zero.

$$i_{C\text{rms}} = \sqrt{\frac{1}{\tau}\left[\int_0^{t_T} \bar{I}_o^2\, dt + \int_0^{\tau - t_T} \left(\frac{\Delta i_L}{\tau - t_T} t - \hat{i}_L + \bar{I}_o\right)^2 dt\right]}$$

$$= \sqrt{\frac{1}{150\mu\text{s}}\left[\int_0^{50\mu\text{s}} 30\text{A}^2 dt + \int_0^{100\mu\text{s}} \left(\frac{10\text{A}}{100\mu\text{s}} t - 20\text{A}\right)^2 dt\right]} \quad = 21.3\text{A}$$

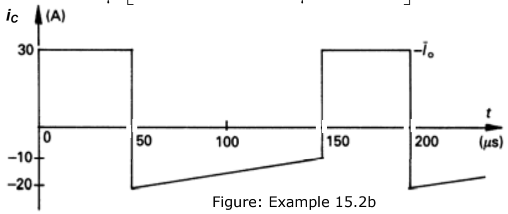

Figure: Example 15.2b

v. The critical load resistance, R_{crit}, produces an input current with Δi_L = 10 A ripple. Since the energy input equals the energy output

$$½\Delta i \times E_i \times \tau = v_o \times v_o / R_{crit} \times \tau$$

that is

$$R_{crit} = \frac{2v_o^2}{E_i \Delta i} = \frac{2\times 75\text{V}^2}{50\text{V}\times 10\text{A}} = 22½\Omega$$

Alternatively, equation (15.65) or equation (15.47) can be rearranged to give R_{crit}.

For a load resistance of less than 22½ Ω, continuous inductor current flows and the operating frequency is fixed at 6.66 kHz with δ = 1/3, that is

$$f_s = 6.66 \text{ kHz for all } R \le 22½\ \Omega$$

For load resistance greater than 22½ Ω, (< v_o / R_{crit} = 3⅓A), the energy input occurs in 150 μs burst whence from equation (15.66)

$$½\Delta i_L E_i \times 150\mu\text{s} = \frac{v_o^2}{R}\frac{1}{f_{\text{var}}}$$

that is

$$f_{\text{var}} = \frac{R_{crit}}{\tau}\frac{1}{R} = \frac{22½\Omega}{150\mu\text{s}}\frac{1}{R}$$

$$f_{\text{var}} = \frac{150}{R}\text{ kHz} \quad \text{for} \quad R \ge 22½\Omega$$

vi. The ±5A inductor ripple current is independent of the load, provided the critical resistance is not exceeded. When the average inductor current (input current) is less than 5A more than the output current, the capacitor must provide load current not only when the switch is on but also when the switch is off. The transition is given by equation (15.55), that is

$$\delta \le 1 - \sqrt{\frac{2L}{\tau R}}$$

$$\frac{1}{3} \le 1 - \sqrt{\frac{2\times 250\mu\text{H}}{150\mu\text{s}\times\text{R}}}$$

This yields $R \ge 7½\Omega$ and a load current of 10A. The average inductor current is 15A, with a minimum value of 10A, the same as the load current. That is, for $R < 7½\Omega$ all the load requirement is provided from the input inductor when the switch is off, with excess energy charging the output capacitor. For $R > 7½\Omega$ insufficient energy is available from the inductor to provide the load energy throughout the whole of the period when the switch is off. The capacitor supplements the load requirement towards the end of the off period. When $R > 22½\Omega$ (the critical resistance), discontinuous inductor current occurs, and the duty cycle dependent transfer function is no longer valid.

♣

Example 15.3: ***Alternative boost (step-up flyback) converter***

The alternative boost converters (producing a dc supply either above E_i (left) or below 0V (right)) shown in the following figure are to operate under the same conditions as the boost converter in example 15.2, namely, with a 50μs transistor fixed on-time in order to convert the 50 V input up to 75 V at the output. The energy transfer inductor is 250μH and the resistive load is 2.5Ω.

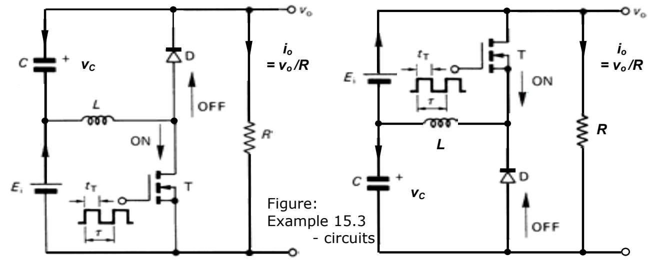

Figure: Example 15.3 - circuits

i. Derive the voltage transfer ratio and critical resistance expression for the alternative boost converter, hence showing the control performance is identical to the boost converter shown in figure 15.5.
ii. By considering circuit voltage and current waveforms, identify how the two boost converters differ from the conventional boost circuit in figure 15.5.

Solution

i. Assuming non-zero, continuous inductor current, the inductor current excursion, which for this boost converter is not the input current excursion, during the switch on-time t_T and switch off-time τ- t_T , is given by

$$L\Delta i_L = E_i t_T = v_C(\tau - t_T)$$

but $v_C = v_o - E_i$, thus substitution for v_C gives

$$E_i t_T = (v_o - E_i)(\tau - t_T)$$

and after rearranging,

$$\frac{v_o}{E_i} = \frac{\overline{I}_i}{\overline{I}_o} = \frac{1}{1-\delta} \quad \left(=1+\frac{\delta}{1-\delta}\right)$$

where $\delta = t_T/\tau$ and t_T is the transistor on-time. This is the same voltage transfer function as for the conventional boost converter, equation (15.44). This result would be expected since both converters have the same ac equivalent circuit. Similarly, the critical resistance would be expected to be the same for each boost converter variation. Examination of the switch on and off states shows that during the switch on-state, energy is transfer to the load from the input supply, independent of switching action. This mechanism is analogous to autotransformer action where the output current is due to both transformer action and the input current being directed to the load.

The critical load resistance for continuous inductor current is specified by $R_{crit} \le v_o / \overline{I}_o$. By equating the capacitor net charge flow, the inductor current is related to the output current by $\overline{I}_L = \overline{I}_o/(1-\delta)$. At minimum inductor current, $\overline{I}_L = ½\Delta i_L$ and substituting with $\Delta i_L = E_i t_T / L$, gives

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{v_o}{(1-\delta)\overline{I}_L} = \frac{v_o}{(1-\delta)½\Delta i_L} = \frac{v_o}{(1-\delta)½E_i t_T / L} = \frac{2L}{\tau\delta(1-\delta)^2}$$

Thus for a given energy throughput, some energy is provided from the supply to the load when providing the inductor energy, hence the discontinuous inductor current threshold occurs at the same load level for each boost converter.

ii. Since the boost circuits have the same ac equivalent circuit, the inductor and capacitor, currents and voltages would be expected to be the same for each circuit, as shown by the waveforms in example 15.2. Consequently, the switch and diode voltages and currents are also the same for each boost converter.
The two principal differences are the supply current and the capacitor voltage rating. The capacitor voltage rating for the alternative boost converter is v_o - E_i as opposed to v_o for the convention converter.
The supply current for the alternative converter is discontinuous, as shown in the following waveforms. This will negate the desirable continuous current feature exploited in boost converters that are controlled so as to produce sinusoidal input current.

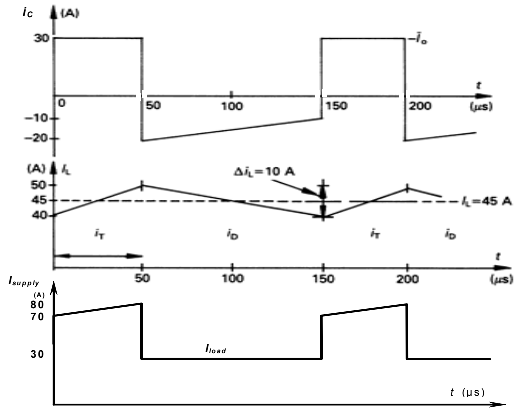

Figure: Example 15.3 - waveforms

♣

15.4 The buck-boost converter

The basic *buck-boost flyback converter* circuit is shown in figure l5.5a. When transistor T is on, energy is transferred to the inductor. When the transistor turns off, inductor current is forced through the diode. Energy stored in *L* is transferred to *C* and the load *R*. This transfer action results in an output voltage of opposite polarity to that of the input. Neither the input nor the output current is continuous, although the inductor current may be continuous or discontinuous.

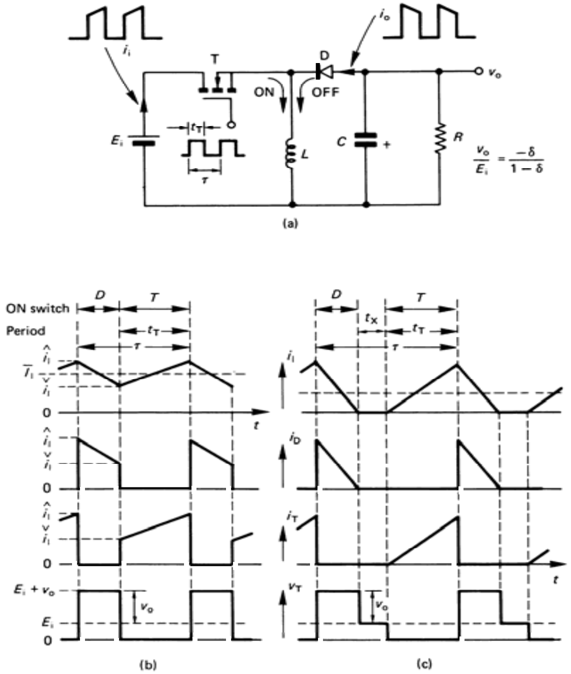

Figure 15.6. *Non-isolated, step up/down flyback converter (buck-boost converter) where $v_o \le 0$: (a) circuit diagram; (b) waveforms for continuous inductor current; and (c) discontinuous inductor current waveforms.*

15.4.1 Continuous choke current

Various circuit voltage and current waveforms for the buck-boost flyback converter operating in a continuous inductor conduction mode are shown in figure 15.6b.
Assuming a constant input and output voltage, the change in inductor current is given by

$$\Delta i_L = \frac{E_i}{L} t_T = \frac{-v_o}{L}(\tau - t_T) \tag{15.73}$$

thus

$$\frac{v_o}{E_i} = \frac{\bar{I}_i}{\bar{I}_o} = -\frac{\delta}{1-\delta} \tag{15.74}$$

where $\delta = t_T/\tau$. For $\delta<½$ the output magnitude is less than the input voltage magnitude, while for $\delta > ½$ the output is greater in magnitude than the input.
The maximum and minimum inductor current is given by

$$\hat{i}_L = \frac{\bar{I}_o}{1-\delta} + ½\frac{v_o}{L}(1\text{-}\delta)\tau = v_o\left[\frac{1}{(1-\delta)R} + \frac{(1\text{-}\delta)\tau}{2L}\right] \tag{15.75}$$

and

$$\check{i}_L = \frac{\bar{I}_o}{1-\delta} - ½\frac{v_o}{L}(1\text{-}\delta)\tau = v_o\left[\frac{1}{(1-\delta)R} - \frac{(1\text{-}\delta)\tau}{2L}\right] \tag{15.76}$$

The inductor rms ripple current (and input ripple current in this case) is given by

$$i_{Lr} = \frac{\Delta i_L}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}\frac{v_o}{L}(1\text{-}\delta)\delta\tau \tag{15.77}$$

while the inductor total rms current is

$$i_{Lrms} = \sqrt{\bar{I}_L^2 + i_{Lr}^2} = \sqrt{\bar{I}_L^2 + \left(\frac{½\Delta i_L}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3}\left(\hat{i}_L^2 + \hat{i}_L \times \check{i}_L + \check{i}_L^2\right)} \tag{15.78}$$

The switch and diode average and rms currents are given by

$$\begin{aligned} \bar{I}_T &= \bar{I}_i = \delta \bar{I}_L \qquad & I_{Trms} &= \sqrt{\delta}\; i_{Lrms} \\ \bar{I}_D &= (1-\delta)\bar{I}_L = \bar{I}_o \qquad & I_{Drms} &= \sqrt{1-\delta}\; i_{Lrms} \end{aligned} \tag{15.79}$$

Switch utilisation ratio
The switch utilisation ratio, SUR, is a measure of how fully a switching device's power handling capabilities are utilised in any switching application. The ratio is defined as

$$SUR = \frac{P_{out}}{p\,\hat{V}_T\,\hat{I}_T} \tag{15.80}$$

where *p* is the number of power switches in the circuit; *p=1* for the buck-boost converter. The switch maximum instantaneous voltage and current are $\hat{V}_T$ and

$\hat{I}_T$ respectively. As shown in figure 15.6b, the maximum switch voltage supported in the off-state is $E_i + v_o$, while the maximum current is the maximum inductor current $\hat{i}_L$ which is given by equation (15.75). If the inductance L is large such that the ripple current is small, the peak inductor current is approximated by the average inductor current which yields $\hat{I}_T \approx \bar{I}_L = \bar{I}_o / 1-\delta$, that is

$$SUR = \frac{v_o \bar{I}_o}{\left(E_i + v_o\right) \times \bar{I}_o / 1-\delta} = \delta\left(1-\delta\right) \tag{15.81}$$

which assumes continuous inductor current. This result shows that the closer the output voltage v_o is in magnitude to the input voltage E_i, that is $\delta = ½$, the better the switch *I-V* ratings are utilised.

15.4.2 Discontinuous capacitor charging current in the switch off-state

It is possible that the inductor current falls below the output (resistor) current during a part of the cycle when the switch is off and the inductor is transferring energy to the output circuit. Under such conditions, towards the end of the off period, some of the load current requirement is provided by the capacitor even though this is the period during which its charge is replenished by inductor energy. The circuit independent transfer function in equation (15.74) remains valid. This discontinuous charging condition occurs when the minimum inductor current and the output current are equal. That is

$$\begin{aligned} &\check{I}_L - \bar{I}_o \le 0 \\ &\bar{I}_L - ½\Delta i_L - \bar{I}_o \le 0 \\ &\frac{\bar{I}_o}{1-\delta} - ½\frac{\bar{I}_o R}{L}\left(1-\delta\right)\tau - \bar{I}_o \le 0 \end{aligned} \tag{15.82}$$

which yields

$$\delta \le 1 + \frac{L}{\tau R} - \sqrt{\left(1 + \frac{L}{\tau R}\right)^2 - 1} \tag{15.83}$$

15.4.3 Discontinuous choke current

The onset of discontinuous inductor operation occurs when the minimum inductor current $\check{i}_L$, reaches zero. That is, with $\check{i}_L = 0$ in equation (15.76), the last equality

$$\frac{1}{\left(1-\delta\right)R} - \frac{\left(1-\delta\right)\tau}{2L} = 0 \tag{15.84}$$

relates circuit component values (R and L) and operating conditions (f and δ) at the verge of discontinuous inductor current.

The change from continuous to discontinuous inductor current conduction occurs when

$$\bar{I}_L = ½\,\hat{i}_L = ½\Delta i_L \tag{15.85}$$

where from equation (15.73) $\quad \hat{i}_L = v_o(\tau - t_T)/L$

The circuit waveforms for discontinuous conduction are shown in figure 15.6c. The output voltage for discontinuous conduction is evaluated from

$$\hat{i}_L = \frac{E_i}{L}t = -\frac{v_o}{L}(\tau - t_T - t_x) \tag{15.86}$$

which yields

$$\frac{v_o}{E_i} = -\frac{\delta}{1-\delta-\dfrac{t_x}{\tau}} \tag{15.87}$$

Alternatively, using equation (15.86) and

$$\bar{I}_L = ½\delta\,\hat{i}_L \tag{15.88}$$

yields

$$\bar{I}_L = \frac{E_i\tau\delta}{2L} \tag{15.89}$$

The inductor current is neither the input current nor the output current, but is comprised of components of each of these currents. Examination of figure 15.6b, reveals that these currents are a proportion of the inductor current dependant on the duty cycle, and that on the verge of discontinuous conduction:

$$\bar{I}_i = ½\delta\,\hat{i}_L = \delta\bar{I}_L \quad \text{and} \quad \bar{I}_o = ½\,(1-\delta)\,\hat{i}_L = (1-\delta)\bar{I}_L \quad \text{where} \quad \hat{i}_L = \Delta i_L = ½\,\bar{I}_L$$

Thus using $\bar{I}_i = \delta\bar{I}_L$ equation (15.89) becomes

$$\bar{I}_i = \frac{E_i\tau\delta^2}{2L} \tag{15.90}$$

Assuming power-in equals power-out, that is $E_i\bar{I}_i = v_o\bar{I}_o$

$$\frac{v_o}{E_i} = \frac{E_i\tau\delta^2}{2L\bar{I}_o} = \frac{v_o\tau\delta^2}{2L\bar{I}_i} = \delta\sqrt{\frac{\tau R}{2L}} \tag{15.91}$$

On the verge of discontinuous conduction, these equations can be rearranged to give

$$\bar{I}_o = \frac{E_i}{2L}\tau\delta(1-\delta) \tag{15.92}$$

At a low output current or low input voltage there is a likelihood of discontinuous conduction. To avoid this condition, a larger inductance value is needed, which worsen transient response. Alternatively, with extremely low on-state duty cycles, a voltage-matching transformer can be used to increase δ. Once using a transformer, any smps technique can be used to achieve the desired output voltage. Figures 15.6b and c show that both the input and output current are always discontinuous.

15.4.4 Load conditions for discontinuous inductor current

As the load current decreases, the inductor average current also decreases, but the inductor ripple current magnitude is unchanged. If the load resistance is increased sufficiently, the bottom of the triangular inductor current, $\check{i}_L$, eventually reduces to zero. Any further increase in load resistance causes discontinuous inductor current and the voltage transfer function given by equation (15.74) is no longer valid and equations (15.86) and (15.91) are applicable. The critical load resistance for continuous inductor current is specified by

$$R_{crit} \le \frac{v_o}{\overline{I}_o} \qquad (15.93)$$

Substituting for, the average input current in terms of $\hat{i}_L$ and v_o in terms of Δi_L from equation (15.73) , yields

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{2L}{\tau(1-\delta)^2} \qquad (15.94)$$

By substituting the switching frequency ($f_s = 1/\tau$) or the fundamental inductor reactance ($X_L = 2\pi f_s L$) the following critical resistance forms result.

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{2L}{\tau(1-\delta)^2} = \frac{2f_s L}{(1-\delta)^2} = \frac{X_L}{\pi(1-\delta)^2} \quad (\Omega) \qquad (15.95)$$

If the load resistance increases beyond R_{crit}, the output voltage can no longer be maintained with duty cycle control according to the voltage transfer function in equation (15.74). Equation (15.95) is equation (15.84), re-arranged.

15.4.5 Control methods for discontinuous inductor current

Once the load current has reduced to the critical level as specified by equation (15.95), the input energy is in excess of the load requirement. Open loop load voltage regulation control is lost and the capacitor C tends to overcharge.
Hardware approaches can be used to solve this problem

- increase L thereby decreasing the inductor current ripple p-p magnitude
- step-down transformer impedance matching to effectively reduce the apparent load impedance

Two control approaches to maintain output voltage regulation when $R > R_{crit}$ are

- vary the switching frequency f_s, maintaining the switch on-time t_T constant so that Δi_L is fixed or
- reduce the switch on-time t_T , but maintain a constant switching frequency f_s, thereby reducing Δi_L.

If a fixed switching frequency is desired for all modes of operation, then reduced on-time control, using output voltage feedback, is preferred. If a fixed on-time mode of control is used, then the output voltage is control by inversely varying the frequency with output voltage.

15.4.5i - fixed on-time t_T, variable switching frequency f_{var}

The operating frequency f_{var} is varied while the switch-on time t_T is maintained constant such that the ripple current remains unchanged. Operation is specified by equating the input energy and the output energy, thus maintaining a constant capacitor charge, hence output voltage. That is, equating energies

$$\tfrac{1}{2}\Delta i_L E_i t_T = \frac{v_o^2}{R}\frac{1}{f_{var}} \qquad (15.96)$$

Isolating the variable switching frequency f_{var} gives

$$\begin{aligned} f_{var} &= \frac{v_o^2}{\tfrac{1}{2}\Delta i_L E_i t_T}\frac{1}{R} \\ &= f_s R_{crit} \times \frac{1}{R} \\ f_{var} &\;\alpha\; \frac{1}{R} \end{aligned} \qquad (15.97)$$

Load resistance R is not a directly or readily measurable parameter for feedback proposes. Alternatively, since $v_o = \overline{I}_o R$, substitution for R in equation (15.97) gives

$$\begin{aligned} f_{var} &= f_s \frac{R_{crit}}{v_o} \times \overline{I}_o \\ f_{var} &\;\alpha\; \overline{I}_o \end{aligned} \qquad (15.98)$$

That is, for discontinuous inductor current, namely $\overline{I}_L < \tfrac{1}{2}\Delta i_L$ or $\overline{I}_o < v_o / R_{crit}$, if the switch on-state period t_T remains constant and f_{var} is either varied proportionally with load current or varied inversely with load resistance, then the required output voltage v_o will be maintained.

15.4.5ii - fixed switching frequency f_s, variable on-time t_{Tvar}

The operating frequency f_s remains fixed while the switch-on time t_{Tvar} is reduced such that the ripple current can be reduced. Operation is specified by equating the input energy and the output energy as in equation (15.96), thus maintaining a constant capacitor charge, hence voltage. That is

$$\tfrac{1}{2}\Delta i_L E_i t_{T\,var} = \frac{v_o^2}{R}\frac{1}{f_s} \qquad (15.99)$$

Isolating the variable on-time t_{Tvar} gives

$$t_{T\,var} = \frac{v_o^2}{\tfrac{1}{2}\Delta i_L E_i f_s}\frac{1}{R}$$

Substituting Δi_L from equation (15.73) gives

$$t_{T\,\mathrm{var}} = t_T\sqrt{R_{crit}} \times \frac{1}{\sqrt{R}}$$
$$t_{T\,\mathrm{var}} \;\alpha\; \frac{1}{\sqrt{R}} \qquad (15.100)$$

Again, load resistance R is not a directly or readily measurable parameter for feedback proposes and substitution of $v_o / \overline{I}_o$ for R in equation (15.70) gives

$$t_{T\,\mathrm{var}} = t_T\sqrt{\frac{R_{crit}}{v_o}} \times \sqrt{\overline{I}_o}$$
$$t_{T\,\mathrm{var}} \;\alpha\; \sqrt{\overline{I}_o} \qquad (15.101)$$

That is, if the switching frequency f_s is fixed and switch on-time t_T is reduced proportionally to $\sqrt{\overline{I}_o}$ or inversely to $\sqrt{R}$, when discontinuous inductor current commences, namely $\overline{I}_L < ½\Delta i_L$ or $\overline{I}_o < v_o / R_{crit}$, then the required output voltage magnitude v_o will be maintained.

Alternatively the output voltage is related to the duty cycle by $v_o = \delta E_i \sqrt{R\tau / 2L}$.

15.4.6 Output ripple voltage

The output ripple voltage is the capacitor ripple voltage. Ripple voltage for a capacitor is defined as

$$\Delta v_o = \tfrac{1}{C}\int i\,dt$$

Figure 15.6 shows that the constant output current $\overline{I}_o$ is provided solely from the capacitor during the period t_{on} when the switch conducting, thus

$$\Delta v_o = \tfrac{1}{C}\int i\,dt = \tfrac{1}{C}t_{on}\overline{I}_o$$

Substituting for $\overline{I}_o = v_o / R$ gives

$$\Delta v_o = \tfrac{1}{C}\int i\,dt = \tfrac{1}{C}t_{on}\overline{I}_o = \tfrac{1}{C}t_{on}\,{}^{v_o}\!/\!_R$$

Rearranging gives the percentage peak-to-peak voltage ripple in the output voltage

$$\frac{\Delta v_o}{v_o} = \frac{1}{RC}t_{on} = \frac{\delta\tau}{RC} \qquad (15.102)$$

The capacitor equivalent series resistance and inductance can be account for, as with the forward converter, 15.1.5. When the switch conducts, the output current is constant and is provided from the capacitor. No ESL voltage effects result during this constant capacitor current portion of the switching cycle.

15.4.7 Buck-boost, flyback converter design procedure

The output voltage of the buck-boost converter can be regulated by operating at a fixed

frequency and varying the transistor on-time t_T. However, the output voltage diminishes while the transistor is on and increases when the transistor is off. This characteristic makes the converter difficult to control on a fixed frequency basis.

A simple approach to control the flyback regulator in the discontinuous mode is to fix the peak inductor current, which specifies a fixed diode conduction time, t_D. Frequency then varies directly with output current and transistor on-time varies inversely with input voltage.

With discontinuous inductor conduction, the worst-case condition exists when the input voltage is low while the output current is at a maximum. Then the frequency is a maximum and the dead time t_x is zero because the transistor turns on as soon as the diode stops conducting.

Given	Worst case
$E_{i(\min)}$	$E_i = E_{i(\min)}$
V_o	$\overline{I}_o = \overline{I}_{o(\max)}$
$\overline{I}_{o\,(\max)}$	$t_x = 0$
$f_{(\max)}$	Δe_o

Assuming a fixed value of peak inductor current $\hat{i}_i$ and output voltage v_o, the following equations are valid

$$E_{i(\min)}t_T = v_o t_D = \hat{i}_i \times L \qquad (15.103)$$

$$\tau_{(\min)} = 1/f_{(\max)} \qquad (15.104)$$

Equation (15.103) yields

$$t_D = \frac{1}{f_{(\max)}\left(\dfrac{v_o}{E_{i(\min)}} + 1\right)} \qquad (15.105)$$

Where the diode conduction time t_D is constant since in equation (15.103), v_0, $\hat{i}_i$, and L are all constants. The average output capacitor current is given by

$$\overline{I}_o = ½\,\hat{i}_i(1-\delta)$$

and substituting equation (15.105) yields

$$\overline{I}_{o\,(\max)} = ½\,\hat{i}_i \times f_{(\max)} \times \frac{1}{f_{(\max)}\left(\dfrac{v_o}{E_{i(\min)}} + 1\right)}$$

therefore

$$\hat{i}_i = 2 \times \overline{I}_{o\,(\max)} \times \left(\frac{v_o}{E_{i(\min)}} + 1\right)$$

and upon substitution into equation (15.103)

$$L = \frac{t_D v_o}{2\,\bar{I}_{o\,(\max)}\left(\dfrac{v_o}{E_{i(\min)}}+1\right)} \qquad (15.106)$$

The minimum capacitance is specified by the maximum allowable ripple voltage, that is

$$C_{(\min)} = \frac{\Delta Q}{\Delta e_o} = \frac{\hat{i}_i\, t_D}{2\Delta e_o}$$

that is

$$C_{(\min)} = \frac{\bar{I}_{o\,(\max)} t_D}{\Delta e_o\left(\dfrac{v_o}{E_{i(\min)}}+1\right)} \qquad (15.107)$$

The ripple voltage is dropped across the capacitor equivalent series resistance, which is given by

$$ESR_{(\max)} = \frac{\Delta e_o}{\hat{i}_i} \qquad (15.108)$$

The frequency varies as a function of load current. Equation (15.104) gives

$$\frac{\bar{I}_o}{f} = \frac{\hat{i}_i\, t_T}{2} = \frac{\bar{I}_{o\,(\max)}}{f_{(\max)}}$$

therefore

$$f = f_{(\max)} \times \frac{\bar{I}_o}{\bar{I}_{o\,(\max)}} \qquad (15.109)$$

and

$$f_{(\min)} = f_{(\max)} \times \frac{\bar{I}_{o\,(\min)}}{\bar{I}_{o\,(\max)}} \qquad (15.110)$$

Example 15.4: ***Buck-boost flyback converter***

The 10kHz flyback converter in figure 15.6 is to operate from a 50V input and produces an inverted non-isolated 75V output. The inductor is 300μH and the resistive load is 2.5Ω.

i. Calculate the duty cycle, hence transistor off-time, assuming continuous inductor current.
ii. Calculate the mean input and output current.
iii. Draw the inductor current, showing the minimum and maximum values.
iv. Calculate the capacitor rms ripple current and output p-p ripple voltage if C = 10,000μF.

v. Determine
   - the critical load resistance.
   - the minimum inductance for continuous inductor conduction with 2.5 Ω load
vi. At what load resistance does the instantaneous inductor current fall below the output current?
vii What is the output voltage if the load resistance is increased to four times the critical resistance?

Solution

i. From equation (15.87), which assumes continuous inductor current

$$\frac{v_o}{E_i} = -\frac{\delta}{1-\delta} \qquad \text{where} \quad \delta = t_T/\tau$$

that is

$$\frac{75\text{V}}{50\text{V}} = \frac{\delta}{1-\delta} \qquad \text{thus} \quad \delta = 3/5$$

That is, $\tau = 1/f_s$ = 100 μs with a 60μs switch on-time.

ii. The mean output current $\bar{I}_o$ is given by

$$\bar{I}_o = v_o/R = 75\text{V}/2.5\Omega \;=\; 30\text{A}$$

From power transfer considerations

$$\bar{I}_i = \bar{I}_L = v_o \bar{I}_o / E_i = 75\text{V}\times 30\text{A}/50\text{V} \;=\; 45\text{A}$$

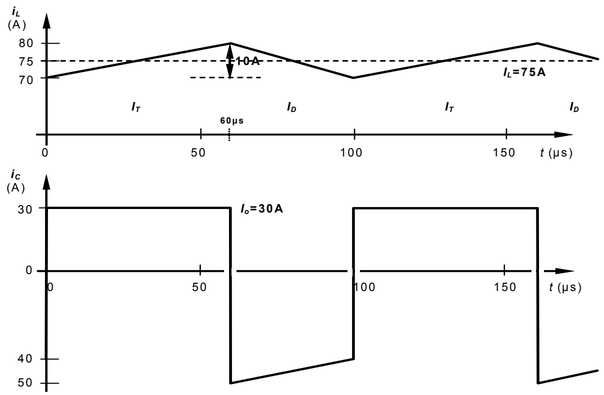

Figure: Example 15.4

iii. The average inductor current can be derived from

$$\overline{I}_i = \delta \overline{I}_L \quad \text{or} \quad \overline{I}_o = (1-\delta)\overline{I}_L$$

That is

$$\overline{I}_L = \overline{I}_i / \delta = \overline{I}_o /(1-\delta)$$
$$= 45\text{A}/{}^3\!/_5 = 30\text{A}/{}^2\!/_5 = 75\text{A}$$

From $v = L\, di/dt$, the ripple current $\Delta i_L = E_i t_T / L$ = 50V x 60μs /300 μH = 10 A, that is

$$\hat{i}_L = \overline{I}_L + \tfrac{1}{2}\Delta i_L = 75\text{A} + \tfrac{1}{2}\times 10\text{A} = 80\text{A}$$

$$\check{i}_L = \overline{I}_L - \tfrac{1}{2}\Delta i_L = 75\text{A} - \tfrac{1}{2}\times 10\text{A} = 70\text{A}$$

iv. The capacitor current is derived by using Kirchhoff's current law such that at any instant in time, the diode current, plus the capacitor current, plus the 30A constant load current into R, all sum to zero.

$$i_{\text{Crms}} = \sqrt{\frac{1}{\tau}\left[\int_0^{t_T} \overline{I}_o^{\,2} dt + \int_0^{\tau - t_T} \left(\frac{\Delta i_L}{\tau - t_T} t - \hat{i}_L + \overline{I}_o\right)^2 dt\right]}$$

$$= \sqrt{\frac{1}{100\mu\text{s}}\left[\int_0^{60\,\mu\text{s}} 30\text{A}^2 dt + \int_0^{40\,\mu\text{s}} \left(\frac{10\text{A}}{40\mu\text{s}} t - 50\text{A}\right)^2 dt\right]}$$

$$= 36.8\text{A}$$

The output ripple voltage is given by equation (15.102), that is

$$\frac{\Delta v_o}{v_o} = \frac{\delta \tau}{CR} = \frac{{}^3\!/_5 \times 100\mu\text{s}}{10{,}000\mu\text{F} \times 2\tfrac{1}{2}\Omega} \equiv 0.24\%$$

The output ripple voltage is therefore

$$\Delta v_o = 0.24 \times 10^{-2} \times 75\text{V} = 180\text{mV}$$

v. The critical load resistance, R_{crit}, produces an inductor current with Δi_L = 10 A ripple. From equation (15.95)

$$R_{crit} = \frac{2L}{\tau(1-\delta)^2} = \frac{2\times 300\mu\text{H}}{100\mu\text{s} \times (1-{}^3\!/_5)^2} = 37\tfrac{1}{2}\Omega$$

The minimum inductance for continuous inductor current operation, with a 2½Ω load, can be found by rearranging the critical resistance formula, as follows:

$$L_{crit} = \tfrac{1}{2} R\tau(1-\delta)^2 = \tfrac{1}{2}\times 2.5\Omega \times 100\mu\text{s} \times (1-{}^3\!/_5)^2 = 20\mu\text{H}$$

vi. The ±5A inductor ripple current is independent of the load, provided the critical resistance of 37½Ω is not exceeded. When the average inductor current is less than 5A more than the output current, the capacitor must provide load current not only when the switch is on but also when the switch is off. The transition is given by equation (15.83), that is

$$\delta \le 1 + \frac{L}{\tau R} - \sqrt{\left(1 + \frac{L}{\tau R}\right)^2 - 1}$$

Alternately, when

$$\overline{I}_i - \overline{I}_o = 5\text{A}$$

$$\frac{\overline{I}_o}{1-\delta} - \overline{I}_o = 5\text{A}$$

For $\delta = {}^3\!/_5$, $\overline{I}_o = 3\tfrac{1}{3}$A. whence

$$R = \frac{v_o}{\overline{I}_o} = \frac{75V}{{}^{10}\!/_3\, A} = 22\tfrac{1}{2}\Omega$$

The average inductor current is 8⅓A, with a minimum value of 3⅓A, the same as the load current. That is, for $R < 22\tfrac{1}{2}\Omega$ all the load requirement is provided from the inductor when the switch is off, with excess energy charging the output capacitor. For $R > 22\tfrac{1}{2}\Omega$ insufficient energy is available from the inductor to provide the load energy throughout the whole of the period when the switch is off. The capacitor supplements the load requirement towards the end of the off period. When $R > 37\tfrac{1}{2}\Omega$ (the critical resistance), discontinuous inductor current occurs, and the purely duty cycle dependent transfer function is no longer valid.

vii. When the load resistance is increased to 125Ω, four times the critical resistance, the output voltage is given by equation (15.91):

$$v_o = E_i\, \delta \sqrt{\frac{\tau R}{2L}} = 50\text{V} \times {}^3\!/_5 \times \sqrt{\frac{100\mu\text{s} \times 125\Omega}{2 \times 300\mu\text{H}}} = 137\text{V}$$

♣

15.5 The output reversible converter

The basic *reversible converter,* sometimes called an *asymmetrical half bridge converter* (see chapter 13.5), shown in figure 15.7a allows two-quadrant output voltage operation. Operation is characterised by both switches operating simultaneously, being either both on or both off.

The input voltage E_i is chopped by switches T_1 and T_2, and because the input voltage is greater than the load voltage v_o, energy is transferred from the dc supply E_i to L, C, and the load R. When the switches are turned off, energy stored in L is transferred via the diodes D_1 and D_2 to C and the load R but in a path involving energy being returned to the supply, E_i. This connection feature allows energy to be transferred from the load back into E_i when used with an appropriate load and the correct duty cycle.

Parts b and c respectively of figure 15.7 illustrate reversible converter circuit current and voltage waveforms for continuous and discontinuous conduction of L, in a forward converter mode, when $\delta > ½$.

For analysis it is assumed that components are lossless and the output voltage v_o is maintained constant because of the large capacitance magnitude of the capacitor C across the output. The input voltage E_i is also assumed constant, such that $E_i \ge v_o > 0$, as shown in figure 15.7a.

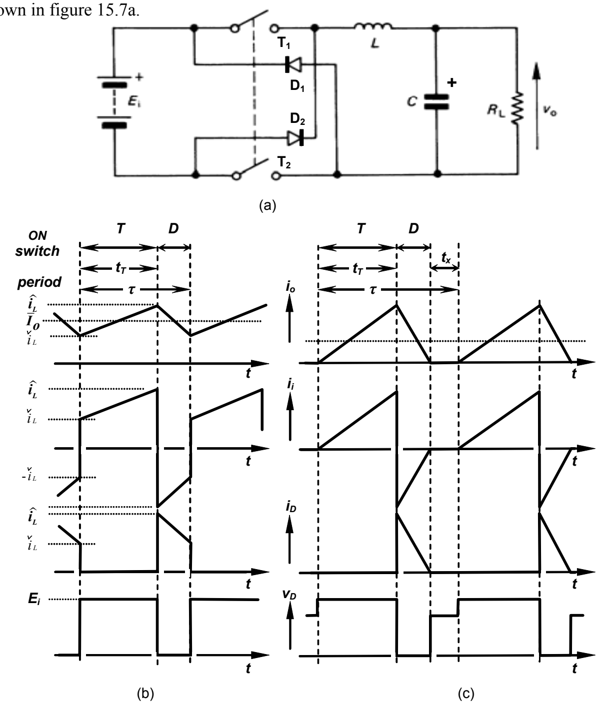

Figure 15.7. *Basic reversible converter with δ>½:*
(a) circuit diagram; (b) waveforms for continuous inductor current; and (c) discontinuous inductor current waveforms.

15.5.1 Continuous inductor current

When the switches are turned on for period t_T, the difference between the supply voltage E_i and the output voltage v_0 is impressed across L. From $V=Ldi/dt$, the rising current change through the inductor will be

$$\Delta i_L = \hat{i}_L - \check{i}_L = \frac{E_i - v_o}{L} \times t_T \qquad (15.111)$$

When the two switches are turned off for the remainder of the switching period, τ-t_T, the two freewheel diodes conduct in series and $E_i + v_o$ is impressed across L. Thus, assuming continuous inductor conduction the inductor current fall is given by

$$\Delta i_L = \frac{E_i + v_o}{L} \times (\tau - t_T) \qquad (15.112)$$

Equating equations (15.111) and (15.112) yields

$$\frac{v_o}{E_i} = \frac{\overline{I}_i}{\overline{I}_o} = \frac{2t_T - \tau}{\tau} = 2\delta - 1 \qquad 0 \le \delta \le 1 \qquad (15.113)$$

The voltage transfer function is independent of circuit inductance L and capacitance C. Equation (15.113) shows that for a given input voltage, the output voltage is determined by the transistor conduction duty cycle δ and the output voltage $|v_o|$ is always less than the input voltage. This confirms and validates the original analysis assumption that $E_i \ge |v_o|$. The linear transfer function varies between -1 and 1 for $0 \le \delta \le 1$, that is, the output can be varied between $v_o = -E_i$, and $v_o = E_i$. The significance of the change in transfer function polarity at $\delta = ½$ is that

- for $\delta > ½$ the converter acts as a forward converter, but
- for $\delta < ½$, if the output is a negative source, the converter acts as a boost converter with energy transferred to the supply E_i, from the negative output source.

Thus the transfer function can be expressed as follows

$$\frac{v_o}{E_i} = \frac{\overline{I}_i}{\overline{I}_o} = 2\delta - 1 = 2\,(\delta - ½) \qquad ½ \le \delta \le 1 \qquad (15.114)$$

and

$$\frac{E_i}{v_o} = \frac{\overline{I}_o}{\overline{I}_i} = \frac{1}{2\delta - 1} = \frac{1}{2\,(\delta - ½)} \qquad 0 \le \delta \le ½ \qquad (15.115)$$

where equation (15.115) is in the boost converter transfer function form.

15.5.2 Discontinuous inductor current

In the forward converter mode, $\delta \ge ½$, the onset of discontinuous inductor current operation occurs when the minimum inductor current $\check{i}_L$, reaches zero. That is,

$$\overline{I}_L = ½\Delta i_L = \overline{I}_o \qquad (15.116)$$

If the transistor on-time t_T is reduced or the load resistance increases, the discontinuous condition dead time t_x appears as indicated in figure 15.7c. From equations (15.111) and (15.112), with $\check{i}_L = 0$, the following output voltage transfer function can be derived

$$\Delta i_L = \hat{i}_L - 0 = \frac{E_i - v_o}{L} \times t_T = \frac{E_i + v_o}{L} \times (\tau - t_T - t_x) \qquad (15.117)$$

which after rearranging yields

$$\frac{v_o}{E_i} = \frac{2\delta - 1 - \frac{t_x}{\tau}}{1 - \frac{t_x}{\tau}} \qquad 0 \le \delta < 1 \qquad (15.118)$$

15.5.3 *Load conditions for discontinuous inductor current*

In the forward converter mode, $\delta \ge$ ½, as the load current decreases, the inductor average current also decreases, but the inductor ripple current magnitude is unchanged. If the load resistance is increased sufficiently, the bottom of the triangular inductor current, $\check{i}_L$, eventual reduces to zero. Any further increase in load resistance causes discontinuous inductor current and the linear voltage transfer function given by equation (15.113) is no longer valid. Equation (15.118) is applicable. The critical load resistance for continuous inductor current is specified by

$$R_{crit} \le \frac{v_o}{\overline{I}_o} \qquad (15.119)$$

Substituting $\overline{I}_o = \overline{I}_L$ and using equations (15.111) and (15.116), yields

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{v_o}{½\Delta i_L} = \frac{2 v_o L}{(E_i - v_o) t_T} \qquad (15.120)$$

Dividing throughout by E_i and substituting $\delta = t_T / \tau$ yields

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{(2\delta - 1) L}{(1 - \delta)\delta\tau} \qquad (15.121)$$

By substituting the switching frequency ($f_s = 1/\tau$) or the fundamental inductor reactance ($X_L = 2\pi f_s L$), critical resistance can be expressed in the following forms.

$$R_{crit} \le \frac{v_o}{\overline{I}_o} = \frac{2(\delta - ½) L}{(1 - \delta)\delta\tau} = \frac{2(\delta - ½) f_s L}{(1 - \delta)\delta} = \frac{(\delta - ½) X_L}{\pi (1 - \delta)\delta} \quad (\Omega) \qquad (15.122)$$

If the load resistance increases beyond R_{crit}, the output voltage can no longer be maintained with duty cycle control according to the voltage transfer function in equation (15.113).

15.5.4 *Control methods for discontinuous inductor current*

Once the load current has reduced to the critical level as specified by equation (15.117) the input energy is in excess of the load requirement. Open loop load voltage regulation control is lost and the capacitor *C* tends to overcharge.
As with the other converters considered, hardware and control approaches can mitigate this overcharging problem. The specific control solutions for the forward converter in section 15.3.4, are applicable to the reversible converter. The two time domain control approaches offer the following operational modes.

15.5.4i - fixed on-time t_T, variable switching frequency f_{var}

The operating frequency f_{var} is varied while the switch-on time t_T is maintained constant such that the magnitude of the ripple current remains unchanged. Operation is specified by equating the input energy and the output energy, thus maintaining a constant capacitor charge, hence output voltage. That is, equating energies

$$½\Delta i_L E_i t_T = \frac{v_o^2}{R} \frac{1}{f_{var}} \qquad (15.123)$$

Isolating the variable switching frequency f_{var} and using $v_o = \overline{I}_o R$ to eliminate R yields

$$f_{var} = f_s R_{crit} \times \frac{1}{R} = f_s \frac{R_{crit}}{v_o} \times \overline{I}_o$$
$$f_{var} \;\alpha\; \frac{1}{R} \quad \text{or} \quad f_{var} \;\alpha\; \overline{I}_o \qquad (15.124)$$

That is, once discontinuous inductor current occurs at $\overline{I}_o < ½\Delta i_L$ or $\overline{I}_o < v_o / R_{crit}$, a constant output voltage v_o can be maintained if the switch on-state period t_T remains constant and the switching frequency is varied

- proportionally with load current, $\overline{I}_o$
- inversely with the load resistance, R_{crit}
- inversely with the output voltage, v_o.

15.5.4ii - fixed switching frequency f_s, variable on-time t_{Tvar}

The operating frequency f_s remains fixed while the switch-on time t_{Tvar} is reduced, resulting in the ripple current magnitude being reduced. Equating input energy and output energy as in equation (15.27), thus maintaining a constant capacitor charge, hence voltage, gives

$$½\Delta i_L E_i t_{Tvar} = \frac{v_o^2}{R} \frac{1}{f_s} \qquad (15.125)$$

Isolating the variable on-time t_{Tvar}, substituting for Δi_L, and using $v_o = \overline{I}_o R$ to eliminate *R*, gives

$$t_{T\,\text{var}} = t_T\sqrt{R_{crit}} \times \frac{1}{\sqrt{R}} = t_T\sqrt{\frac{R_{crit}}{v_o}} \times \sqrt{\bar{I}_o}$$
$$t_{T\,\text{var}} \;\alpha\; \frac{1}{\sqrt{R}} \quad \text{or} \quad t_{T\,\text{var}} \;\alpha\; \sqrt{\bar{I}_o} \qquad (15.126)$$

That is, once discontinuous inductor current commences, if the switching frequency f_s remains constant, regulation of the output voltage v_o can be maintained if the switch on-state period t_T is varied

- proportionally with the square root of the load current, $\sqrt{\bar{I}_o}$
- inversely with the square root of the load resistance, $\sqrt{R_{crit}}$
- inversely with the square root of the output voltage, $\sqrt{v_o}$.

Example 15.5: *Reversible forward converter*

The step-down reversible converter in figure 15.7a operates at a switching frequency of 10 kHz. The output voltage is to be fixed at 48 V dc across a 1 Ω resistive load. If the input voltage E_i=192 V and the choke L = 200μH:

i. calculate the switch T on-time duty cycle δ and switch on-time t_T
ii. calculate the average load current $\bar{I}_o$, hence average input current $\bar{I}_i$
iii. draw accurate waveforms for
   - the voltage across, and the current through L; v_L and i_L
   - the capacitor current, i_c
   - the switch and diode voltage and current; v_T, v_D, i_T, i_D
iv. calculate
   - the maximum load resistance R_{crit} before discontinuous inductor current with L=200μH and
   - the value to which the inductance L can be reduced before discontinuous inductor current, if the maximum load resistance is 1Ω.

Solution

i. The switch on-state duty cycle δ can be calculate from equation (15.113), that is

$$2\delta - 1 = \frac{v_o}{E_i} = \frac{48\text{V}}{192\text{V}} = \tfrac{1}{4} \quad\Rightarrow\quad \delta = \tfrac{5}{8}$$

Also, from equation (15.113), for a 10kHz switching frequency, the switching period τ is 100μs and the transistor on-time t_T is given by

$$\delta = \frac{t_T}{\tau} = \frac{t_T}{100\mu\text{s}} = \tfrac{5}{8}$$

whence the transistor on-time is 62½μs and the diode conducts for 37½μs.

ii. The average load current is $\bar{I}_o = \dfrac{v_o}{R} = \dfrac{48\text{V}}{1\Omega} = 48\text{A} = \bar{I}_L$

From power-in equals power-out, the average input current is

$$\bar{I}_i = v_o\bar{I}_o / E_i = 48\text{V}\times 48\text{A}/192\text{V} = 12\text{A}$$

iii. The average output current is the average inductor current, 48A. The ripple current is given by equation (15.113), that is

$$\Delta i_L = \hat{i}_L - \check{i}_L = \frac{E_i - v_o}{L} \times t_T$$
$$= \frac{192\text{V - }48\text{V}}{200\mu\text{H}} \times 62.5\mu\text{s} = 45\text{A p-p}$$

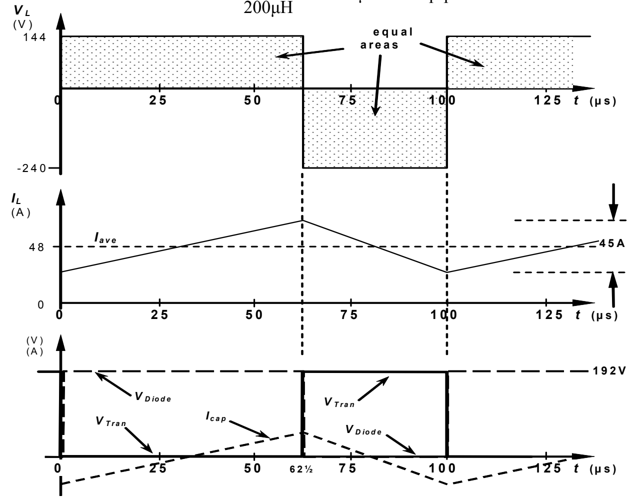

Figure: Example 15.5

iv. Critical load resistance is given by equation (15.122), namely

$$R_{crit} \le \frac{v_o}{\bar{I}_o} = \frac{(2\delta - 1)L}{\tau\delta(1-\delta)}$$

$$= \frac{(2\times \tfrac{5}{8}-1)\times 200\mu H}{100\mu s\times \tfrac{5}{8}\times (1-\tfrac{5}{8})} = 32/15\Omega$$

$$= 2\tfrac{2}{15}\Omega \text{ when } \bar{I}_o = 22\tfrac{1}{2}A$$

Alternatively, the critical load current is 22½A (½Δi_L), thus the load resistance must not be greater than $v_o/\bar{I}_o$ = 48V/22.5A=32/15Ω, if the inductor current is to be continuous.

The critical resistance formula given in equation (15.122) is valid for finding critical inductance when inductance is made the subject of the equation, that is, rearranging equation (15.122) gives

$$\begin{aligned} L_{crit} &= R\times(1-\delta)\times\delta\times\tau/(2\delta-1) \qquad \text{(H)}\\ &= 1\Omega\times(1-\tfrac{5}{8})\times\tfrac{5}{8}\times 100\mu s/(2\times\tfrac{5}{8}-1)\\ &= 93\tfrac{3}{4}\mu H \end{aligned}$$

That is, the inductance can be decreased from 200µH to 93¾µH when the load is 1Ω and continuous inductor current will flow.

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15.5.5 Comparison of the reversible converter with alternative converters

The reversible converter provides the full functional output range of the forward converter when δ>½ and provides part of the voltage function of the buck-boost converter when δ<½ but with energy transferring in the opposite direction.

Comparison of example 15.1 and 15.4 shows that although the same output voltage range can be achieved, the inductor ripple current is much larger for a given inductance L. A similar result occurs when compared with the buck-boost converter. Thus in each case, the reversible converter has a narrower output resistance range before discontinuous inductor conduction occurs. It is therefore concluded that the reversible converter should only be used if two quadrant operation is needed.

The ripple current $\bar{I}_f$ given by equation (15.2) for the forward converter and equation (15.111) for the reversible converter when $v_o > 0$, yield the following current ripple relationship.

$$\begin{aligned} &\bar{I}_f = (2-1/\delta_r)\times\bar{I}_r\\ &\text{where } 2\delta_r-1=\delta_f \text{ for } 0\le\delta_f\le 1 \text{ and } \tfrac{1}{2}\le\delta_r\le 1 \end{aligned} \qquad (15.127)$$

This equation shows that the ripple current of the forward converter $\bar{I}_f$ is never greater than the ripple current $\bar{I}_r$ for the reversible converter, for the same output voltage.

In the voltage inverting mode, from equations (15.73) and (15.111), the relationship between the two corresponding ripple currents is given by

$$\begin{aligned} &\bar{I}_{fly} = \frac{2(\delta_r-1)}{2\delta_r-1}\times\bar{I}_r\\ &\text{where } \frac{2(\delta_r-1)}{2\delta_r-1}=\delta_{fly} \text{ for } 0\le\delta_{fly}\le\tfrac{1}{2} \text{ and } 0\le\delta_r\le\tfrac{1}{2} \end{aligned} \qquad (15.128)$$

Again the reversible converter always has the higher inductor ripple current. Essentially the higher ripple current results in each mode because the inductor energy release phase involving the diode occurs back into the supply, which is effectively in cumulative series with the output capacitor voltage.

The reversible converter offers some functional flexibility, since it can operate as a conventional forward converter, when only one of the two switches is turned off. (In fact, in this mode, switch turn-off is alternated between T_1 and T_2 so as to balance switch and diode losses.

15.6 The Ćuk converter

The Ćuk converter in figure 15.8 performs an inverting boost converter function with inductance in the input and the output. As a result, both the input and output currents can be continuous. A capacitor is used in the process of transferring energy from the input to the output and ac couples the input boost converter stage (L_1, T) to the output forward converter (D, L_2). Specifically, the capacitor C_1 ac couples the switch T in the boost converter stage into the output forward converter stage.

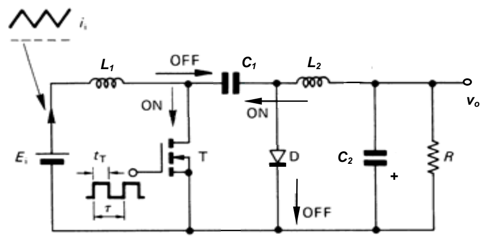

Figure 15.8. *Basic Ćuk converter.*

15.6.1 Continuous inductor current

When the switch T is on and the diode D is reversed biased

$$i_{C1(on)} = -\bar{I}_{L2} = \bar{I}_o \qquad (15.129)$$

When the switch is turned off, inductor currents i_{L1} and i_{L2} are divert through the diode and

$$i_{C1(\mathrm{off})} = \bar{I}_i \tag{15.130}$$

Over one steady-state cycle the average capacitor charge is zero, that is

$$i_{C1(\mathrm{on})}\delta\tau + i_{C1(\mathrm{off})}(1-\delta)\tau = 0 \tag{15.131}$$

which gives

$$\frac{i_{C1(\mathrm{on})}}{i_{C1(\mathrm{off})}} = \frac{\delta}{(1-\delta)} = \frac{\bar{I}_i}{\bar{I}_o} \tag{15.132}$$

From power-in equals power-out

$$\frac{v_o}{E_i} = \frac{\bar{I}_i}{\bar{I}_o} = \frac{\bar{I}_{L1}}{\bar{I}_{L2}} \tag{15.133}$$

Thus equation (15.132) becomes

$$\frac{v_o}{E_i} = \frac{\bar{I}_i}{\bar{I}_o} = \frac{\bar{I}_{L1}}{\bar{I}_{L2}} = -\frac{\delta}{(1-\delta)} \tag{15.134}$$

15.6.2 Discontinuous inductor current

The current rise in L_1 occurs when the switch is on, that is

$$\Delta i_{L1} = \frac{\delta\tau E_i}{L_1} \tag{15.135}$$

For continuous current in the input inductor L_1,

$$\bar{I}_i = \bar{I}_{L1} \geq \tfrac{1}{2}\Delta i_{L1} \tag{15.136}$$

which yields a maximum allowable load resistance, for continuous inductor current, of

$$R_{crit} \leq \frac{v_o}{\bar{I}_o} = \frac{2\delta L_1}{\tau(1-\delta)^2} = \frac{2f_s L_1\delta}{(1-\delta)^2} = \frac{\delta X_{L1}}{\pi(1-\delta)^2} \tag{15.137}$$

This is the same expression as that obtained for the boost converter, equation (15.65), which can be re-arranged to give the minimum inductance for continuous input inductor current, namely

$$\check{L}_1 = \frac{(1-\delta)^2 R\tau}{2\delta} \tag{15.138}$$

The current rise in L_2 occurs when the switch is on and the inductor voltage is E_i, that is

$$\Delta i_{L2} = \frac{\delta\tau E_i}{L_2} \tag{15.139}$$

For continuous current in the output inductor L_2,

$$\bar{I}_o = \bar{I}_{L2} \geq \tfrac{1}{2}\Delta i_{L2} \tag{15.140}$$

which yields

$$R_{crit} \leq \frac{v_o}{\bar{I}_o} = \frac{2L_2}{\tau(1-\delta)} = \frac{2f_s L_2}{(1-\delta)} = \frac{X_{L2}}{\pi(1-\delta)} \tag{15.141}$$

This is the same expression as that obtained for the forward converter, equation (15.26) which can be re-arranged to give the minimum inductance for continuous output inductor current, namely

$$\check{L}_2 = \tfrac{1}{2}(1-\delta)R\tau \tag{15.142}$$

15.6.3 Optimal inductor relationship

Optimal inductor conditions are that both inductors should both simultaneous reach the verge of discontinuous conduction. The relationship between inductance and ripple current is given by equations (15.135) and (15.139).

$$\Delta i_{L1} = \frac{\delta\tau E_i}{L_1} \text{ and } \Delta i_{L2} = \frac{\delta\tau E_i}{L_2}$$

After diving these two equations

$$\frac{L_2}{L_1} = \frac{\Delta i_{L1}}{\Delta i_{L2}} \tag{15.143}$$

Critical inductance is given by equations (15.138) and (15.142), that is

$$\check{L}_2 = \tfrac{1}{2}(1-\delta)R\tau \text{ and } \check{L}_1 = \frac{(1-\delta)^2 R\tau}{2\delta}$$

After dividing

$$\frac{\check{L}_2}{\check{L}_1} = \frac{\delta}{1-\delta} \tag{15.144}$$

At the verge of simultaneous discontinuous inductor conduction

$$\frac{\check{L}_2}{\check{L}_1} = \frac{\delta}{1-\delta} = \frac{\Delta i_{L1}}{\Delta i_{L2}} = \left|\frac{v_o}{E_i}\right| \tag{15.145}$$

That is, the voltage transfer ratio uniquely specifies the ratio of the minimum inductances and their ripple current.

15.6.3 Output voltage ripple

The output stage (L_2, C_2 and R) is the forward converter output stage; hence the per unit output voltage ripple on C_2 is given by equation (15.35), that is

$$\frac{\Delta v_{C2}}{v_o} = \frac{\Delta v_o}{v_o} = \tfrac{1}{8} \times \frac{(1-\delta)\tau^2}{L_2 C_2} \tag{15.146}$$

If the ripple current in L_1 is assumed constant, the per unit voltage ripple on the ac coupling capacitor C_1 is approximated by

$$\frac{\Delta v_{C1}}{v_o} = \frac{\delta\tau}{RC_1} \qquad (15.147)$$

Example 15.6: *Ćuk converter*

The Ćuk converter in figure 15.8 is to operate at 10kHz from a 50V battery input and produces an inverted non-isolated 75V output. The load power is 1.8kW.

- i Calculate the duty cycle hence switch on and off times, assuming continuous current in both inductors.
- ii Calculate the mean input and output, hence inductor, currents.
- iii At the 1.8kW load level, calculate the inductances *L1* and *L2* such that the ripple current is 1A p-p in each.
- iv Specify the capacitance for *C1* and *C2* if the ripple voltage is to be a maximum of 1% of the output voltage.
- v Determine the critical load resistance for which the purely duty cycle dependant voltage transfer function becomes invalid.
- vi At the critical load resistance value, determine the inductance value to which the non-critically operating inductor can be reduced.
- vii Determine the necessary conditions to ensure that both inductors operate simultaneously on the verge of discontinuous conduction, and the relative ripple currents for that condition.

Solution

i. The voltage transfer function is given by equation (15.134), that is

$$\frac{v_o}{E_i} = -\frac{\delta}{(1-\delta)} = -\frac{75\text{V}}{50\text{V}} = -1\tfrac{1}{2}$$

from which $\delta = \tfrac{3}{5}$. For a 10kHz switching frequency the period is 100μs, thus the switch on-time is 60μs and the off-time is 40μs.

ii. The mean output current is determined by the load and the mean input current is related to the output current by assuming 100% efficiency, that is

$$\bar{I}_o = \bar{I}_{L2} = P_o / v_o = 1800\text{W} / 75\text{V} = 24\text{A}$$

$$\bar{I}_i = \bar{I}_{L1} = P_o / E_i = 1800\text{W} / 50\text{V} = 36\text{A}$$

The load resistance is therefore $R = v_o / I_o$ = 75V/24A = 3⅛Ω.

iii The inductor ripple current for each inductor is given by the same expression, that is equations (15.135) and (15.139). Thus for the same ripple current of 1A pp

$$\Delta i_{L1} = \frac{\delta\tau E_i}{L_1} = \Delta i_{L2} = \frac{\delta\tau E_i}{L_2}$$

which gives

$$L_1 = L_2 = \frac{\delta\tau E_i}{\Delta i} = \frac{\tfrac{3}{5}\times 100\mu\text{s}\times 50\text{V}}{1\text{A}} = 3\text{mH}$$

iv The capacitor ripple voltages are given by equations (15.147) and (15.146), which after re-arranging gives

$$C_1 = \frac{v_o}{\Delta v_{C1}}\times\frac{\delta\tau}{R} = \frac{100}{1}\times\frac{\tfrac{3}{5}\times 100\mu\text{s}}{\tfrac{25}{8}\Omega} = 1.92\text{mF}$$

$$C_2 = \frac{v_o}{\Delta v_{C2}}\times\tfrac{1}{8}\times\frac{(1-\delta)\tau^2}{L_2} = \frac{100}{1}\times\tfrac{1}{8}\times\frac{(1-\tfrac{3}{5})\times 100\mu\text{s}^2}{3\text{mH}} = 16.6\mu\text{F}$$

v The critical load resistance for each inductor is given by equations (15.137) and (15.141). When both inductors are 3mH:

$$R_{crit} \le \frac{2\delta L_1}{\tau(1-\delta)^2} = \frac{2\times\tfrac{3}{5}\times 3\text{mH}}{100\mu\text{s}\times(1-\tfrac{3}{5})^2} = 225\Omega$$

$$R_{crit} \le \frac{2L_2}{\tau(1-\delta)} = \frac{2\times 3\text{mH}}{100\mu\text{s}\times(1-\tfrac{3}{5})} = 150\Omega$$

The limiting critical load resistance is 150Ω or for $I_o = v_o / R$ = 75V/150Ω = ½A, when a lower output current results in the current in L_2 becoming discontinuous although the current in L_1 is still continuous.

vi From equation (15.137), rearranged

$$L_{1\,crit} \ge \frac{\tau R(1-\delta)^2}{2\delta} = \frac{100\mu\text{s}\times 100\Omega\times(1-\tfrac{3}{5})^2}{2\times\tfrac{3}{5}} = 2\text{mH}$$

That is, if L_1 is reduced from 3mH to 2mH, then both L_1 and L_2 enter discontinuous conduction at the same load condition, 75V, ½A, and 150Ω.

vii For both converter inductors to be simultaneously on the verge of discontinuous conduction, equation (15.145) gives

$$\frac{\check{L}_2}{\check{L}_1} = \frac{\delta}{1-\delta} = \frac{\Delta i_{L1}}{\Delta i_{L2}} = \left|\frac{v_o}{E_i}\right|$$

$$\frac{3\text{mH}}{2\text{mH}} = \frac{\tfrac{3}{5}}{1-\tfrac{3}{5}} = \frac{1\text{A}}{\tfrac{2}{3}\text{A}} = \left|\frac{75\text{V}}{50\text{V}}\right| = \frac{3}{2}$$

♣

15.7 Comparison of basic converters

The converters considered employ an inductor to transfer energy from one dc voltage level to another dc voltage level. The basic converters comprise a switch, diode, inductor, and a capacitor. The reversible converter is a two-quadrant converter with two switches and two diodes, while the Ćuk converter uses two inductors and two capacitors.

Table 15.1 summarises the main electrical features and characteristics of each basic converter. Figure 15.9 shows a plot of the voltage transformation ratios and the switch utilisation ratios of the converters considered. With reference to figure 15.9, it should be noted that the flyback step-up/step-down converter and the Ćuk converter both invert the input polarity.

Every converter can operate in any one of three inductor current modes:

- discontinuous
- continuous
- both continuous and discontinuous

The main converter operational features of continuous conduction compared with discontinuous inductor conduction are

- The voltage transformation ratio (transfer function) is independent of the load.
- Larger inductance but lower core hysteresis losses and saturation less likely.
- Higher converter costs with increased volume and weight.
- Worse transient response *(L/R)*.
- Power delivered is inversely proportional to load resistance, $P = V_o^2 / R$. In the discontinuous conduction mode, power delivery is inversely dependent on inductance.

15.7.1 Critical load current

Examination of Table 15.1 show little commonality between the various converters and their performance factors and parameters. One common feature is the relationship between critical average output current $\bar{I}_o$ and the input voltage E_i at the boundary of continuous and discontinuous conduction.

Equations (15.14), (15.61), and (15.92) are identical, (for all smps), that is

$$\bar{I}_{o_{critical}} = \frac{E_i \tau}{2L}\delta(1-\delta) \qquad \text{(A)} \qquad (15.148)$$

This quadratic expression in δ shows that the critical mean output current reduces to zero as the on-state duty cycle δ tends to zero or unity. The maximum critical load current condition, for a given input voltage E_i, is when δ=½ and

$$\bar{I}_{o_c} = E_i \tau / 8L \qquad (15.149)$$

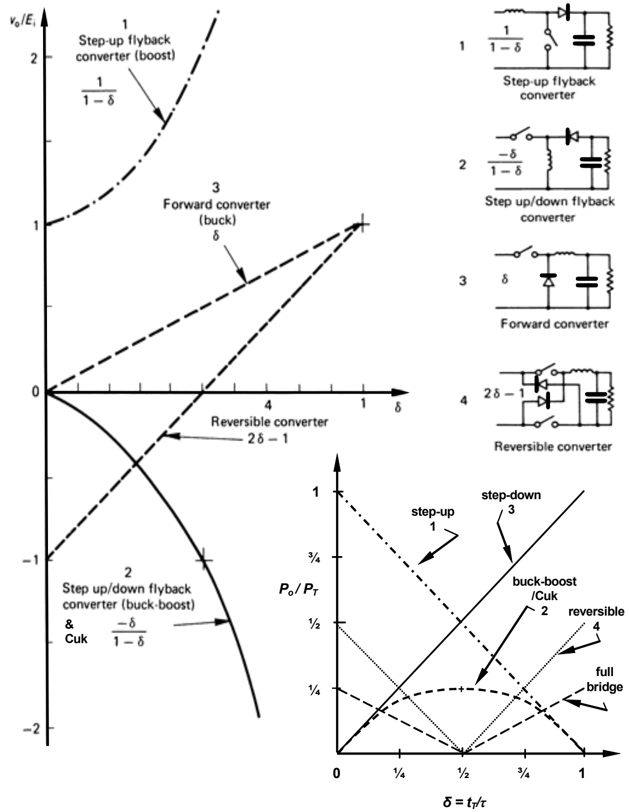

Figure 15.9. *Transformation voltage ratios and switch utilisation ratios for five converters when operated in the continuous inductor conduction mode.*

Since power in equals power out, then from equation (15.148) the input average current and output voltage at the boundary of continuous conduction for all smps are related by

$$\bar{I}_{i_{critical}} = \frac{v_o \tau}{2L}\delta(1-\delta) \quad \text{(A)} \qquad (15.150)$$

The maximum output current at the boundary, for a given output voltage, v_o, is

$$\bar{I}_{i_c} = v_o \tau / 8L \qquad (15.151)$$

Table 15.1 Converter characteristics comparison with continuous inductor current

			converter			
			Forward Step-down	*Flyback Step-up*	*Flyback Step-up/down*	*Reversible*
Output voltage continuous *I*	v_o/E_i		δ	$\frac{1}{1-\delta}$	$-\frac{\delta}{1-\delta}$	$2\delta-1$
Output voltage discontinuous *I*	v_o/E_i		$1-\frac{2L\bar{I}_i}{E_i\delta^2\tau}$	$1+\frac{E_i\delta^2 t_T}{2L\bar{I}_o}$	$\frac{E_i\delta^2\tau}{2L\bar{I}_o}$	
Output polarity with respect to input			Non-inverted	Non-inverted	inverted	any
Current sampled from the supply			discontinuous	continuous	discontinuous	bi-directional
Load current			continuous	discontinuous	discontinuous	continuous
Maximum transistor voltage	V	V	E_i	v_o	$E_i + v_o$	E_i
Maximum diode voltage	V	V	E_i	v_o	$E_i + v_o$	E_i
Ripple current	Δi	A	$E_i\delta\tau(1-\delta)/L$	$E_i\delta\tau/L$	$E_i\delta\tau/L$	$2E_i\delta\tau(1-\delta)/L$
Maximum transistor current	$\hat{i}_T$	A	$\bar{I}_o+\frac{v_o\tau(1-\delta)}{2L}$	$\bar{I}_i+\frac{E_i\tau\delta}{2L}$	$\bar{I}_L+\frac{E_i\tau\delta}{2L}$	$\bar{I}_o+\frac{(E_i-v_o)\tau\delta}{2L}$
switch utilisation ratio	*SUR*		δ	$1-\delta$	$\delta(1-\delta)$	
Transistor rms current			low	high	high	low
Critical load resistance	R_{crit}	Ω	$\frac{2L}{\tau(1-\delta)}$	$\frac{2L}{\tau\delta(1-\delta)^2}$	$\frac{2L}{\tau(1-\delta)^2}$	$\frac{2(\delta-½)L}{\tau\delta(1-\delta)}$
Critical inductance	L_{crit}	H	$½R(1-\delta)\tau$	$½R\tau\delta(1-\delta)^2$	$½R\tau(1-\delta)^2$	$\frac{½R(1-\delta)\delta\tau}{(\delta-½)}$
o/p ripple voltage **p-p**	Δv_o	V	$\frac{\tau^2(1-\delta)}{8LC}v_o$	$\frac{\tau\delta}{RC}v_o$	$\frac{\tau\delta}{RC}v_o$	$\frac{\tau\delta}{RC}v_o$

The reversible converter, using the critical resistance equation (15.122) derived in section 15.5.3, yields twice the critical average output current given by equation (15.148). This is because its duty cycle range is restricted to half that of the other converters considered. Converter normalised equations for discontinuous conduction are shown in table 15.2.

A detailed analysis summary of discontinuous inductor current operation is given in appendix 15.9.

Table 15.2 Comparison of characteristics when the inductor current is discontinuous

$k=\frac{\delta^2 R\tau}{4L}$	converter		
	Forward Step-down	*Flyback Step-up*	*Flyback Step-up/down*
$\delta_{critical}$	$\delta \le 1-\frac{2L}{R\tau}$	$\delta(1-\delta)^2 \le \frac{2L}{R\tau}$	$\delta \le 1-\sqrt{\frac{2L}{R\tau}}$
$\frac{v_o}{E_i}$	$k\left[-1+\sqrt{1+\frac{2}{k}}\right]$	$½\left[1+\sqrt{1+8k}\right]$	$\sqrt{2k}$
$\frac{t_x}{\delta\tau}$	$½\left[1+\sqrt{1+\frac{2}{k}}\right]$	$\frac{1}{4k}\left[1+4k+\sqrt{1+8k}\right]$	$1+\frac{1}{\sqrt{2k}}$
$\hat{I}_L\times\frac{\delta R}{E_i}$	$4k\left[1+k-k\sqrt{1+\frac{2}{k}}\right]$	$4k$	$4k$

15.7.2 Isolation

In each converter, the output is not electrically isolated from the input and a transformer can be used to provide isolation. Figure 15.10 shows isolated versions of the three basic converters. The transformer turns ratio provides electrical isolation as well as providing matching to obtain the required output voltage range.

Figure 15.10a illustrates an isolated version of the forward converter shown in figure 15.2. When the transistor is turned on, diode D_1 conducts and L in the transformer secondary stores energy. When the transistor turns off, the diode D_3 provides a current path for the release of the energy stored in L. However when the transistor turns off and D_1 ceases to conduct, the stored transformer magnetising energy must be released. The winding incorporating D_2 provides a path to reset the core flux. A maximum possible duty cycle exists, depending on the turns ratio of the primary winding and freewheel winding. If a 1:1 ratio (as shown) is employed, a 50 per cent duty cycle limit will ensure the required volts-second for core reset.

The step-up flyback isolated converter in part b of figure 15.10 is little used. The two transistors must be driven by complementary signals. Core leakage and reset functions are facilitated by a third winding and blocking diode D_2.

The magnetic core in the buck-boost converter of part c of figure 15.10 performs a bifilar inductor function. When the transistor is turned on, energy is stored in the core. When the transistor is turned off, the core energy is released via the secondary winding into the capacitor. A core air gap is necessary to prevent magnetic saturation and an optional clamping winding can be employed, which operates at zero load.

The converters in parts a and c of figure 15.10 provide an opportunity to compare the main features and attributes of forward and flyback isolated converters. In the comparison it is assumed that the transformer turns ratio is 1:1:1.

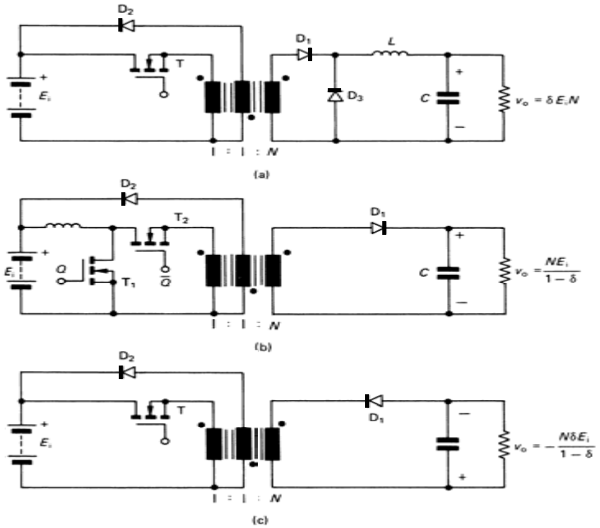

Figure 15.10. *Isolated output versions of the three basic converter configurations: (a) the forward converter; (b) step-up flyback converter; and (c) step up/down flyback converter.*

15.7.2i - *The isolated output,* ***forward converter*** *– figure 15.10a:*

- $v_o = n_T \delta E_i$ or $I_i = n_T \delta I_o$
- The magnetic element acts as a transformer, that is, because of the relative voltage polarities of the windings, energy is transferred from the input to the output, and not stored in the core, when the switch is on.
- The magnetising flux is reset by the current through the catch (feedback) winding and D_3, when the switch is off. The magnetising energy is returned to the supply E_i.
- The necessary transformer Vμs balance requirement (core energy-in equals core energy-out) means the maximum duty cycle is limited to $0 \le \delta \le 1/\left(1+n_{f/b}\right) < 1$ for $1{:}n_{f/b}{:}n_{sec}$ turns ratio. For example, the duty cycle is limited to 50%, $0 \le \delta \le ½$, with a 1:1:1 turns ratio.
- Because of the demagnetising winding, the off-state switch supporting voltage is $E_i + v_o$.
- The blocking voltage requirement of diode D_3 is E_i, v_o for D_1, and $2E_i$ for D_2.

15.7.2ii - *The isolated output,* ***flyback converter*** *– figure 15.10c:*

- $v_o = n_T E_i \delta/(1-\delta)$ or $I_i = n_T I_o \delta/(1-\delta)$
- The magnetic element acts as a storage inductor. Because of the relative voltage polarities of the windings (dot convention), when the switch is on, energy is stored in the core and no current flows in the secondary.
- The stored energy, which is due to the core magnetising flux is released (reset) as current into the load and capacitor C when the switch is off. (Unlike the forward converter, where magnetising energy is returned to E_i, not the output, v_o.) Therefore there is no flyback converter duty cycle restriction, $0 \le \delta \le 1$.
- The third winding turns ratio is configured such that energy is only returned to the supply E_i under no load conditions.
- The switch supporting off-state voltage is $E_i + v_o$.
- The diode blocking voltage requirements are $E_i + v_o$ for D_1 and $2E_i$ for D_2.

The operational characteristics of each converter change considerably when the flexibility offered by tailoring the turns ratio is exploited. A multi-winding magnetic element design procedure is outlined in section 9.1.1, where the transformer turns ratio is not necessarily 1:1.

The basic approach to any transformer (coupled circuit) problem is to transfer, or refer, all components and variables to either the transformer primary or secondary circuit, whilst maintaining power and time invariance. Thus, maintaining power-in equals power-out, and assuming a secondary to primary turns ratio of n_T is to one, gives

$$\frac{v_s}{v_p} = \frac{n_s}{n_p} = n_T \qquad \frac{i_p}{i_s} = \frac{n_s}{n_p} = n_T \qquad \frac{Z_s}{Z_p} = \left(\frac{n_s}{n_p}\right)^2 = n_T^2 \qquad (15.152)$$

Time, that is switching frequency, power, and per unit values (δ, $\Delta v_o/v_o$), are invariant. The circuit is then analysed. Subsequently, the appropriate parameters are referred back to their original side of the magnetically coupled circuit.

If the coupled circuit is used as a transformer, magnetising current (flux) builds, which must be reset to zero each cycle. Consider the transformer coupled forward converter in figure 15.10a. From Faraday's equation, $v = Nd\phi / dt$, and for maximum on-time duty cycle $\hat{\delta}$ the conduction V-µs of the primary must equal the conduction V-µs of the feedback winding which is returning the magnetising energy to the supply E_i.

$$E_i t_{on} = \frac{E_i}{n_{f/b}} t_{off} \qquad (15.153)$$

$$t_{on} + t_{off} = \tau$$

That is

$$E_i \hat{\delta} = \frac{E_i}{n_{f/b}} \left(1 - \hat{\delta}\right)$$

$$\hat{\delta} = \frac{1}{1 + n_{f/b}} \qquad (15.154)$$

$$0 \le \delta \le \frac{1}{1 + n_{f/b}}$$

From Faraday's Law, the magnetizing current starts from zero and increases linearly to

$$\hat{I}_M = E_i t_{on} / L_M \qquad (15.155)$$

where L_M is the magnetizing inductance referred to the primary. During the switch off period, this current falls linearly, as energy is returned to E_i. The current must reach zero before the switch is turned on again, whence the energy taken from E_i and stored as magnetic energy in the core, has been returned to the supply.

Two examples illustrate the features of magnetically coupled circuit converters. Example 15.7 illustrates how the coupled circuit in the flyback converter acts as an inductor, storing energy from the primary source, and subsequently releasing that energy in the secondary circuit. In example 15.8, the forward converter coupled circuit act as a transformer where energy is transferred through the core under transformer action, but in so doing, self-inductance (magnetising) energy is built up in the core, which must be periodically released if saturation is to be avoided. Relative orientation of the windings, according to the flux dot convention shown in figure 15.10, is thus important, not only the primary relative to the secondary, but also relative to the feedback winding.

Example 15.7: *Transformer coupled flyback converter*

The 10kHz flyback converter in figure 15.10c operates from a 50V input and produces a 225V dc output from a 1:1:3 (1:$n_{f/b}$:n_{sec}) step-up transformer loaded with a 22½Ω resistor. The transformer magnetising inductance is 300µH, referred to the primary:

i. Calculate the switch duty cycle, hence transistor off-time, assuming continuous inductor current.
ii. Calculate the mean input and output current.
iii. Draw the transformer currents, showing the minimum and maximum values.
iv. Calculate the capacitor rms ripple current and p-p voltage ripple if C = 1100µF.
v. Determine
   - the critical load resistance
   - the minimum inductance for continuous inductor conduction for a 22½ Ω load

Solution

The feedback winding does not conduct during normal continuous inductor current operation. This winding can therefore be ignored for analysis during normal operation.

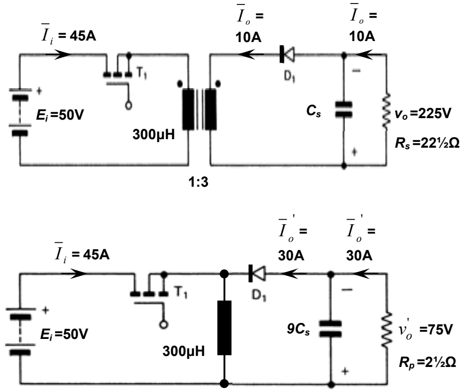

Figure 15.11. *Isolated output step up/down flyback converter and its equivalent circuit when the output is referred to the primary.*

Figure 15.11 shows secondary parameters referred to the primary, specifically

$$v_o = 225\text{V} \quad v_o' = v_o / n_T = 225\text{V}/3 = 75\text{V}$$

$$R_s = 225\Omega \quad R_p = R_s / n_T^2 = 225\Omega / 3^2 = 22½\Omega$$

Note that the output capacitance is transferred by a factor of nine, n_T^2, since capacitive reactance is inversely proportion to capacitance.

It will be noticed that the equivalent circuit parameter values to be analysed, when referred to the primary, are the same as in example 15.4. The circuit is analysed as in example 15.4 and the essential results from example 15.4 are summarised in Table 15.3 and transferred to the secondary where appropriate. The waveform answers to part iii are shown in figure 15.12.

Table 15.3 Transformer coupled flyback converter analysis

parameter		value for primary analysis	transfer factor $n_T = 3$ →	value for secondary analysis
E_i	V	50	3	150
v_o	V	75	3	225
R_L	Ω	2½	3^2	22½
C_o	μF	10,000	3^{-2}	1100
$I_{o(ave)}$	A	30	⅓	10
P_o	W	2250	invariant	2250
$I_{i(ave)}$	A	45A	⅓	15A
δ	p.u.	⅗	invariant	⅗
τ	μs	100	invariant	100
t_{on}	μs	60	invariant	60
t_{off}	μs	40	invariant	40
f_s	kHz	20	invariant	20
Δi_L	A	5	⅓	1⅔
$\hat{I}_L$	A	80	⅓	80/3
$\check{I}_L$	A	70	⅓	70/3
i_{Crms}	A rms	21.3	⅓	7.1
R_{crit}	Ω	37½	3^2	337½
L_{crit}	μH	20	3^2	180
V_{Dr}	V	125	3	375
Δv_o	mV	180	3	540
$\Delta v_o / v_o$	p.u.	0.24%	invariant	0.24%

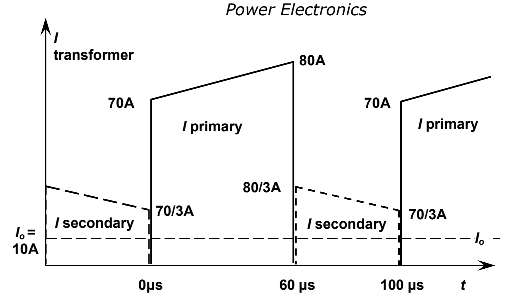

Figure 15.12. *Currents for the transformer windings in example 15.7.*

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Example 15.8: ***Transformer coupled forward converter***

The 10kHz forward converter in figure 15.10a operates from a 192V dc input and a 1:3:2 ($1{:}n_{f/b}{:}n_{sec}$) step-up transformer loaded with a 4Ω resistor. The transformer magnetising inductance is 1.2mH, referred to the primary. The secondary smps inductance is 800μH.

i. Calculate the maximum switch duty cycle, hence transistor off-time, assuming continuous inductor current.

At the maximum duty cycle:

ii. Calculate the mean input and output current.
iii. Draw the transformer currents, showing the minimum and maximum values.
iv. Determine
- the critical load resistance
- the minimum inductance for continuous inductor conduction for a 4 Ω load

Solution

The maximum duty cycle is determined solely by the transformer turns ratio between the primary and the feedback winding which resets the core flux. From equation (15.154)

$$\hat{\delta} = \frac{1}{1+n_{f/b}}$$

$$= \frac{1}{1+3} = ¼$$

The maximum conduction time is 25% of the 100μs period, namely 25μs. The secondary output voltage is therefore

$$v_{sec} = \delta n_T E_i$$
$$= ¼×2×192 = 96\text{V}$$

The load current is therefore 96V/4Ω=24A, as shown in figure 15.13a.
Figure 15.13b shows secondary parameters referred to the primary, specifically

$$R_s = 4\Omega \quad R_p = R_s / n_T^2 = 4\Omega / 2^2 = 1\Omega$$
$$v_o = 96\text{V} \quad v_o^{'} = v_o / n_T = 96\text{V}/2 = 48\text{V}$$
$$L_o = 800\mu\text{H} \quad L_o^{'} = L_o / n_T^2 = 800\mu\text{H}/2^2 = 200\mu\text{H}$$

Note that the output capacitance is transferred by a factor of four, n_T^2, since capacitive reactance is inversely proportion to capacitance.
Inspection of example 15.1 will show that the equivalent circuit in figure 15.13b is the same as the circuit in example 15.1, except that a magnetising branch has been added. The various operating condition and values in example 15.1 are valid for example 15.8.

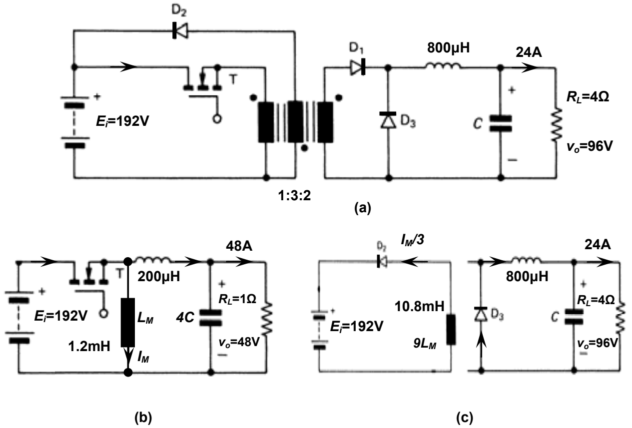

Figure 15.13. *Isolated output forward converter and its equivalent circuits when the output is referred to the primary.*

ii. The mean output current is the same for both circuits, 48A, or 24 A when referred to the secondary circuit. The mean input current from E_i remains 12A, but the switch mean current is not 12A. Magnetising current is provided from the supply E_i through the switch, but returned to the supply E_i through diode D2, which bypasses the switch. The net magnetising energy flow is zero. The magnetising current maximum value is given by equation (15.155)

$$\hat{I}_M = E_i t_{on} / L_M$$
$$= 192\text{V}×25\mu\text{s}/1.2\text{mH} = 4\text{A}$$

This current increases the switch mean current to

$$\bar{I}_T = 12\text{A} + ½×\delta×4\text{A} = 12½\text{A}$$

Figure 15.13c show the equivalent circuit when the switch is off. The output circuit functions independently of the input circuit, which is returning stored core energy to the supply E_i via the feedback winding and diode D2. Parameters have been referred to the feedback winding which has three times the turns of the primary, $n_{f/b}$=3. The 192V input voltage remains the circuit reference. Equation (15.155), Faraday's law, referred to the feedback winding, must be satisfied during the switch off period, that is

$$\frac{\hat{I}_M}{n_{f/b}} = \frac{E_i t_{off}}{n_{f/b}^2 L_M}$$
$$\frac{4}{3} = \frac{192\text{V}×75\mu\text{s}}{3^2×1.2\text{mH}}$$

The diode D2 voltage rating is $(n_{f/b}+1)×E_i$, 768V and its mean current is

$$\bar{I}_{D2} = ½(1-\delta)\frac{I_M}{n_{f/b}} = ½×(1 - 0.25)×\frac{4\text{A}}{3} = ½\text{A}$$

iii. The three winding currents for the transformer are shown in figure 15.14.

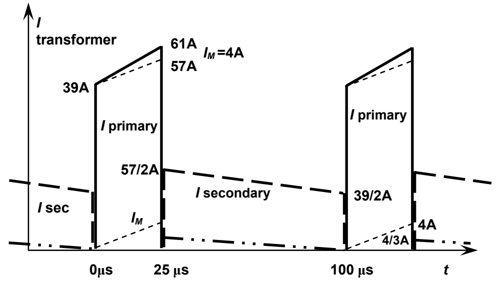

Figure 15.14. *Currents for the three transformer windings in example 15.8.*

iv. The critical resistance and inductance, referred to the primary, from example 15.1 are 5⅓Ω and 37½µH. Transforming into secondary quantities, by multiplying by 2^2 give critical values of R_L = 21⅓Ω and L = 150µH.

♣

15.8 Multiple-switch, balanced, isolated converters

The basic single-switch converters considered have the limitation of using their magnetic components only in a unipolar mode. Since only one quadrant of the *B-H* characteristic is employed, these converters are generally restricted to lower powers because of the limited flux swing, which is reduced by the core remanence flux.

The high-power forward converter circuits shown in figure 15.15 operate the magnetic transformer component in the bipolar or push-pull flux mode and require two or four switches. Because the transformers are fully utilised magnetically, they tend to be almost half the size of the equivalent single transistor isolated converter at power levels above 100 W. Also core saturation due to the magnetising current (flux) not being fully reset to zero each cycle, is not a major issue, since with balanced bidirectional fluxing, the average magnetising current is zero.

15.8.1 The push-pull converter

Figure 15.15a illustrates a push-pull forward converter circuit which employs two switches and a centre-tapped transformer. Each switch must have the same duty cycle in order to prevent unidirectional core saturation. Because of transformer coupling action, the off switch supports twice the input voltage, $2E_i$, plus any voltage associated with leakage inductance stored energy. Advantageously, no floating gate drives are required.

The voltage transfer function, for continuous inductor conduction, is based on the equivalent secondary output circuit show in figure 15.16. Because of transformer action the input voltage is $N \times E_i$ where N is the transformer turns ratio. When a primary switch is on, current flows in the loop shown in figure 15.16. That is

$$\Delta i_L = \hat{i}_L - \check{i}_L = \frac{N \times E_i - v_o}{L} \times t_T \qquad (15.156)$$

When the primary switches are off, the secondary voltage falls to zero and current continues to flow through the secondary winding due to the energy stored in L. Efficiency is increased if the diode D_f is used to bypass the transformer winding, as shown in figure 15.16. The secondary winding i^2R losses are decreased and minimal voltage is coupled from the secondary back into the primary circuit. The current in the off loop shown in figure 15.16 is given by

$$\Delta i_L = \frac{v_o}{L} \times (\tau - t_T) \qquad (15.157)$$

Equating equations (15.156) and (15.157) gives the following voltage and current transfer functions

$$\frac{v_o}{E_i} = \frac{\bar{I}_i}{\bar{I}_o} = 2N\frac{t_T}{\tau} = 2N\delta \qquad 0 \le \delta \le ½ \qquad (15.158)$$

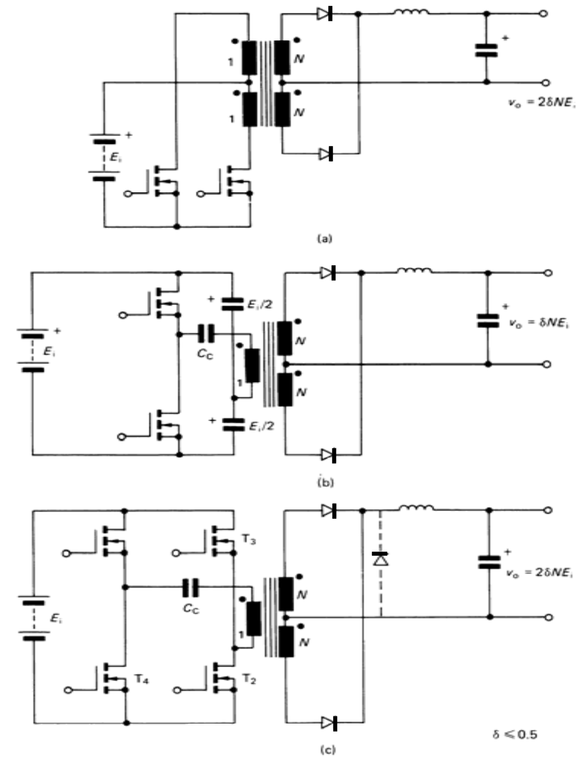

Figure 15.15. *Multiple-switch, isolated output, pulse-width modulated converters: (a) push-pull; (b) half-bridge; and (c) full-bridge.*

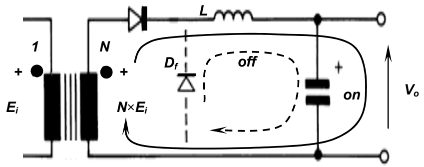

Figure 15.16. *Equivalent circuit for transformer bridge converters based on a forward converter in the secondary.*

The output voltage ripple is similar to that of the forward converter

$$\frac{\Delta v_C}{v_o} = \frac{\Delta v_o}{v_o} = \frac{(1-2\delta)\,\tau^2}{32\,LC} \tag{15.159}$$

15.8.2 Bridge converters

Figures 15.15b and c show half and full-bridge isolated forward converters respectively.

i. Half-bridge

In the half-bridge the transistors are switched alternately and must have the same conduction period. This ensures the core volts-second balance requirement to prevent saturation due to bias in one direction.

Using similar analysis as for the push-pull converter in 15.8.1, the voltage transfer function of the half bridge with a forward converter output stage, for continuous inductor conduction, is given by

$$\frac{v_o}{E_i} = \frac{\overline{I}_i}{\overline{I}_o} = N\frac{t_T}{\tau} = N\delta \qquad 0 \le \delta \le ½ \tag{15.160}$$

A floating base drive is required. Although the maximum winding voltage is ½E_i, the switches must support E_i in the off-state, when the complementary switch conducts.

The output ripple voltage is given by

$$\frac{\Delta v_C}{v_o} = \frac{\Delta v_o}{v_o} = \frac{(1-2\delta)\,\tau^2}{16\,LC} \tag{15.161}$$

ii. Full-bridge

The full bridge in figure 15.15c replaces the capacitor supplies of the half-bridge converter with switching devices. In the off-state each switch must support the rail voltage E_i and two floating gate drive circuits are required. This bridge converter is usually reserved for high-power applications.

Using similar analysis as for the push-pull converter in 15.8.1, the voltage transfer function of the full bridge with a forward converter output stage, with continuous conduction is given by

$$\frac{v_o}{E_i} = \frac{\overline{I}_i}{\overline{I}_o} = 2N\frac{t_T}{\tau} = 2N\delta \qquad 0 \le \delta \le ½ \tag{15.162}$$

Any volts-second imbalance (magnetising flux build-up) can be minimised by using dc block capacitance C_c, as shown in figures 15.15b and c.

The output ripple voltage is given by

$$\frac{\Delta v_C}{v_o} = \frac{\Delta v_o}{v_o} = \frac{(1-2\delta)\,\tau^2}{32\,LC} \tag{15.163}$$

In each forward converter in figure 15.15, a single secondary transformer winding and full-wave rectifier can be used. If the output diode shown dashed in figure 15.15c is used, the off state loop voltage is decreased from two diode voltage drops to one.

If the output inductor is not used, conventional unregulated transformer square-wave voltage ratio action occurs for each transformer based smps, where, independent of δ:

$$\frac{v_o}{E_i} = \frac{\overline{I}_i}{\overline{I}_o} = \frac{n_s}{n_o} = N \tag{15.164}$$

15.9 Resonant dc-to-dc converters

Converter switching losses may be significantly reduced if zero current or voltage switching can be utilised. This switching loss reduction allows higher operating frequencies hence smaller L and C components (in size and value). Also radiated switching noise is significantly reduced.

Two main techniques can be used to achieve near zero switching losses

- a resonant load that provides natural voltage or current zero instances for switching
- a resonant circuit across the switch which feeds energy to the load as well as introducing zero current or voltage instances for switching.

15.9.1 Series loaded resonant dc-to-dc converters

The basic converter operating concept involves a H-bridge producing an ac square-wave voltage $V_{H\text{-}B}$. When fed across a series L-C filter, a near sinusoidal oscillation current results, provided the square wave fundamental frequency is near the natural resonant frequency of the L-C filter. Because of L-C filter action, only fundamental current flows and harmonics of the square wave are attenuated due to the gain roll-off of the second order L-C filter. The sinusoidal resonant current is rectified to produce a load voltage $v_{o/p}$. The output voltage is highly dependant on the frequency relationship between the square-wave drive voltage period and the L-C filter resonant frequency.

Figure 15.17a shows the circuit diagram of a series resonant converter, which uses an output rectifier bridge to converter the resonant ac oscillation into dc. The converter is based on the series converter in figure 14.27b. The rectified ac output charges the dc output capacitor, across which is the dc load, R_{load}. The non-dc-decoupled resistance, which determines the circuit Q, is account for by resistor R_c. The dc capacitor C capacitance is assumed large enough so that the output voltage $v_{o/p}$ is maintained constant, without significant ripple voltage. Figures 15.17b and c show how the dc output circuit can be transformed into an ac square wave in series with the L-C circuit, and finally this source is transferred to the dc link as a constant dc voltage source $v_{o/p}$ which opposes the dc supply V_s. These transformation steps enable the series L-C-R resonant circuit to be analysed with square wave inverter excitation, from a dc source V_s - $v_{o/p}$. This highlights that the output voltage must be less than the dc supply, that is V_s - $v_{o/p} \geq 0$, if current oscillation is to occur. The analysis in chapter 14.3.2 is valid for this circuit, where V_s is replaced by V_s - $v_{o/p}$. The equations, modified, are repeated for completeness.

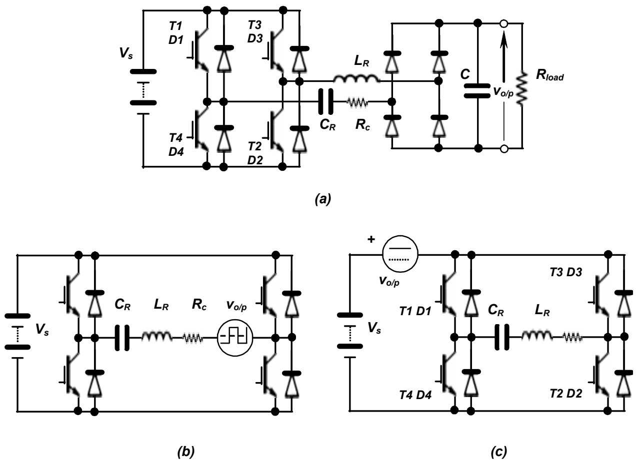

Figure 15.17. *Series resonant converter and its equivalent circuit derivation.*

The series L-C-R circuit current for a step input voltage V_s-$v_{o/p}$, with initial capacitor voltage v_o, assuming zero initial inductor current, is given by

$$i(\omega t) = \frac{(V_s - v_{o/p}) - v_o}{\omega L} \times e^{-\alpha t} \times \sin \omega t \qquad (15.165)$$

where

$$\omega^2 = \omega_o^2(1-\xi^2) \qquad \omega_o = \frac{1}{\sqrt{L_R C_R}} \qquad \alpha = \frac{R_c}{2L_R} \qquad \frac{1}{2Q_s} = \xi = \frac{R_c}{2\omega_o L_R} \qquad Z_o = \sqrt{\frac{L_R}{C_R}}$$

ξ is the damping factor. The capacitor voltage is important because it specifies the energy retained in the L-C-R circuit at the end of each half cycle.

$$v_c(\omega t) = (V_s - v_{o/p}) - \left((V_s - v_{o/p}) - v_o\right)\frac{\omega_o}{\omega} e^{-\alpha t}\cos(\omega t - \phi) \qquad (15.166)$$

where

$$\tan\phi = \frac{\alpha}{\omega} \quad \text{and} \quad \omega_o^2 = \omega^2 + \alpha^2$$

At the series circuit resonance frequency ω_o, the lowest possible circuit impedance results, $Z = R_c$. The series circuit quality factor or figure of merit, Q_s is defined by

$$Q_s = \frac{\omega_o L}{R_c} = \frac{1}{2\xi} = \frac{Z_o}{R_c} \qquad (15.167)$$

Operation is characterised by turning on switches T1 and T2 to provide energy from the source during one half of the cycle, then having turned T1 and T2 off, T3 and T4 are turned on for the second resonant half cycle. Energy is again drawn from the supply, and when the current reaches zero T3 and T4 are turned off.

Without bridge freewheel diodes, the switches support high reverse bias voltages, but the switches control the start of each oscillation half cycle. With freewheel diodes the oscillations can continue independent of the switch states. The diodes return energy to the supply, hence reducing the energy transferred to the load. Correct timing of the switches minimises currents in the freewheel diodes, hence minimises the energy needlessly being returned to the supply. Energy to the load is maximised. The switches can be used to control the effective load power factor. By advancing turn-off to occur before the switch current reaches zero, the load can be made to appear inductive, while delaying switch turn-on produces a capacitive load effect.

The series circuit steady-state current at resonance for the H-bridge with a high circuit Q can be approximated by assuming $\omega_o \approx \omega$, such that:

$$i(\omega t) = \frac{2}{1 - e^{\frac{-\alpha\pi}{\omega}}} \times \frac{(V_s - v_{o/p})}{\omega L_R} \times e^{-\alpha t} \times \sin \omega t \qquad (15.168)$$

which is valid for the ± $(V_s$ - $v_{o/p})$ voltage loops of cycle operation at resonance. For a high circuit Q this equation is approximately

$$i(\omega t) \approx \frac{4}{\pi} \times Q \times \frac{(V_s - v_{o/p})}{\omega_o L_R} \times \sin \omega_o t = \frac{4}{\pi} \times \frac{(V_s - v_{o/p})}{R_c} \times \sin \omega_o t \qquad (15.169)$$

The maximum current is

$$\hat{I} \approx \frac{4}{\pi} \times \frac{\left(V_s - v_{o/p}\right)}{R_c} \qquad (15.170)$$

while the average current with this peak value must equal to load current, that is

$$\bar{I} = \frac{2}{\pi} \times \hat{I} = \frac{8}{\pi^2} \times \frac{\left(V_s - v_{o/p}\right)}{R_c} = \frac{v_{o/p}}{R} \qquad (15.171)$$

The output voltage is obtained from equation (15.171) by isolating $v_{o/p}$

$$v_{o/p} = \frac{V_s}{1 + \frac{8}{\pi^2} \times \frac{R_c}{R}} \qquad (15.172)$$

In steady-state the capacitor voltage maxima are

$$\begin{aligned} \hat{V}_c &= \left(V_s - v_{o/p}\right)\frac{1+e^{-\alpha\pi/\omega}}{1-e^{-\alpha\pi/\omega}} = \left(V_s - v_{o/p}\right) \times \coth\left(\alpha\pi/2\omega\right) = -\check{V}_c \\ &\approx \left(V_s - v_{o/p}\right) \times 2\omega_o/\alpha\pi = \frac{4}{\pi} \times Q \times \left(V_s - v_{o/p}\right) \end{aligned} \qquad (15.173)$$

The peak-to-peak capacitor voltage, by symmetry is therefore

$$V_{c_{p-p}} \approx \frac{8}{\pi} \times Q \times \left(V_s - v_{o/p}\right) \qquad (15.174)$$

The energy lost in the coil resistance R_c, per half sine cycle (per current pulse) is

$$\begin{aligned} W &= \int_0^{\pi/\omega} i^2 R_c \, dt \approx \int_0^{\pi/\omega_o} \left(4\frac{2}{\pi} \times \frac{\left(V_s - v_{o/p}\right)}{R_c} \times \sin\omega_o t\right)^2 R_c \, dt \\ &= \frac{8}{\pi\,\omega_o R_c}\left(V_s - v_{o/p}\right)^2 \end{aligned} \qquad (15.175)$$

The relationship between the output voltage $v_{o/p}$ and H-bridge switching frequency, ω_s, is not a simple linear function. Because of the *L-C* series filter cut-off frequency $\omega_s \approx \omega_o$, only fundamental current flows as a result of the fundamental of the square-wave $V_{H\text{-}B}$, which has a magnitude of $4\!/\!_\pi V_s$. A series *R-C-L* ac circuit at frequency ω_s can be used to derive a relationship between the output voltage $v_{o/p}$ and ω_s. The effective load resistance, from equation (15.171) becomes $R_{eq} = 8\!/\!_{\pi^2} R_{load}$ such that Kirchhoff's voltage law for the series circuit, shown in figure 15.18a, is

$$\begin{aligned} \frac{4}{\pi}V_s &= i \times \left(R_{eq} + R_c + j\left(\omega_s L_R - \frac{1}{\omega_s C_R}\right)\right) \\ \frac{4}{\pi}v_{o/p} &= i \times R_{eq} \end{aligned} \qquad (15.176)$$

The equivalent output voltage is therefore given by

$$v_o = V_s \times \frac{R_{eq}}{R_{eq} + R_c + j\left(\omega_s L_R - \frac{1}{\omega_s C_R}\right)} \qquad (15.177)$$

where the H-bridge switching frequency is $\omega_s = 2\pi f_s$.

Figure 15.18 shows equation (15.177) for different circuit Q. The plot can be used to extract output voltage $v_{o/p}$ and H-bridge switching frequency. The output voltage is scaled to eliminate the coil resistance component from the total resistive voltage.

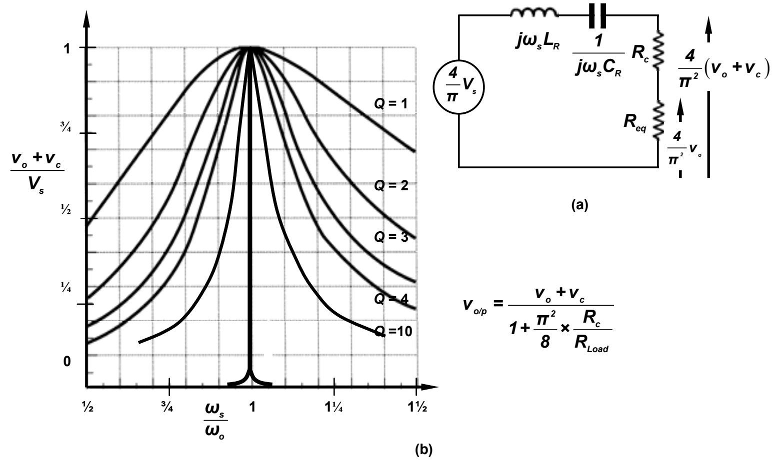

Figure 15.18. *Series resonant converter (a) equivalent circuit and (b) normalised output voltage curves.*

15.9.1i - Modes of operation - series resonant circuit

The basic series converter can be operated in any of three difference modes, depending on the switching frequency in relation to the *L-C* circuit natural resonant frequency. In all cases, the controlled output voltage is less than the input voltage, that is V_s - $v_{o/p} \geq 0$. The switching frequency involves one complete symmetrical square-wave output cycle from the inverter bridge. Waveforms for the three operational modes are shown in figure 15.19.

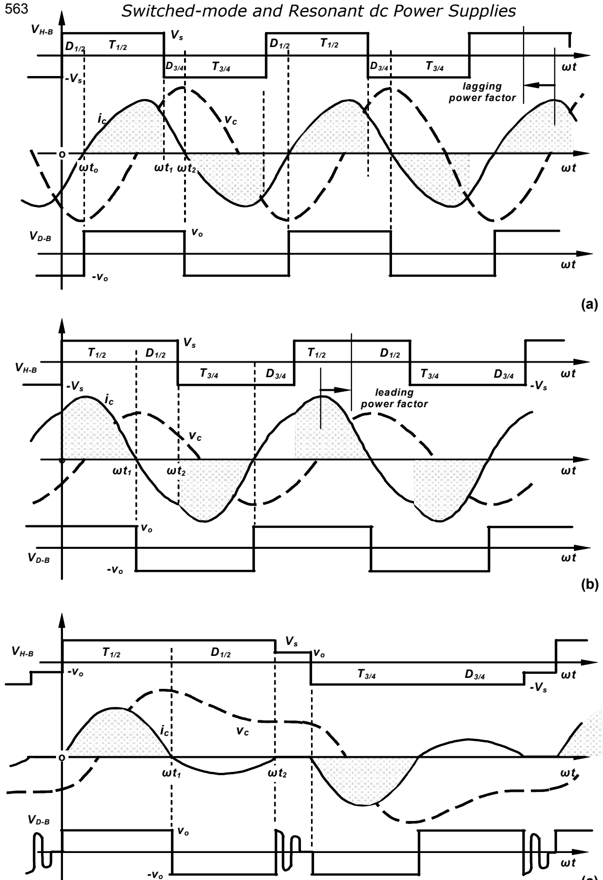

Figure 15.19. *Three modes of series load resonant converter operation (a) $f_s > f_o$; (b) $\frac{1}{2}f_o < f_s < f_o$; and (c) $f_s < \frac{1}{2}f_o$.*

i. **$f_s < \frac{1}{2}f_o$** :- discontinuous inductor current (switch conduction $1/2f \le t_T \le 1/f$)

If the switching frequency is less than half the *L-C* circuit natural resonant frequency, as shown in figure 15.19c, then discontinuous inductor current results. This because once one complete *L-C* resonant ac cycle occurs and current stops, being unable to reverse, since the switches are turned off when the diodes conduct and the capacitor voltage is less than V_s+v_o. Turn-off occurs at zero current. Subsequent turn-on occurs at zero current but the voltage is determined by the voltage retained by the capacitor. Thyristors are therefore applicable switches in this mode of operation. The freewheel diodes turn on and off with zero current. Since the capacitor current rectified provides the load current average current, the H-bridge switching frequency controls the output voltage. Therefore at low switching frequencies, relative to the resonance frequency, the peak resonant current will be relatively high.

ii. **$\frac{1}{2}f_o < f_s < f_o$** :- continuous inductor current

If the switching frequency is just less than natural resonant frequency, as shown in figure 15.19b, such that turn-on occurs after half an oscillation cycle but before a complete ac oscillation cycle is complete, continuous inductor current results. Switch turn-on occurs with finite inductor current and voltage conditions, with the diodes freewheeling. Diode reverse recovery losses occur and noise in injected into the circuit at voltage recovery snap. Fast recovery diodes are therefore necessary. Switch turn-off occurs at zero voltage and current, when the inductor current passes through zero and the freewheel diodes take up conduction. Thyristors are applicable as switching devices with this mode of control.

iii. **$f_s > f_o$** :- continuous inductor current

If turn-off occurs before the resonance of half a resonant cycle is complete, as shown in figure 15.19a, continuous inductor current flows, hard switching results, and commutable switches must be used. Switch turn-on occurs at zero voltage and current hence no diode recovery snap occurs. This zero electrical condition turn-on allows lossless turn-off snubbers to be employed (a capacitor in parallel with each switch).

For continuous inductor conduction, under light load conditions, resonant energy is continuously transferred to the output stage, which tends to progressively overcharge the output voltage towards the input voltage level, V_s. The charging progressively decreases as the V_s is backed off by the increasing output voltage. That is V_s - $v_{o/p}$ shown in figure 15.17c tends to zero such that the effective square wave input is reduced to zero, as will the input energy.

15.9.1ii - Circuit variations

The number of semiconductors can be reduced by using a split dc rail as in figure 15.20a, at the expense of halving the bridge output voltage swing $V_{H\text{-}B}$ to ± $\frac{1}{2}V_s$. Although the number of semiconductors is halved, the already poor switch utilisation associated with any resonant converter, is further decreased. The switches and diodes support V_s. In the full-bridge case the corresponding switch and diode voltages are both V_s.

Voltage and impedance matching, for example voltage step-up, can be obtained by using a transformer coupled circuit as shown in figure 15.20b. When a transformer is used as in figure 15.20c, a centre tapped secondary can reduce the number of high frequency rectifying diodes from four to two, but diode reverse voltage rating is doubled from V_s to $2V_s$. Secondary copper winding utilisation is halved.

A further modification to any series converter is to used the resonant capacitor to form a split dc rail as shown in figure 15.20c, where each capacitance is $½C_R$. In ac terms the resonant capacitors are in parallel, with one charging while the discharges, and visa versa, such that their voltage sum is V_s.

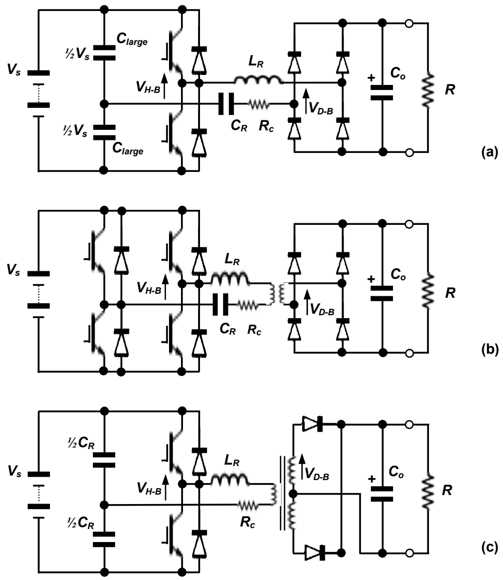

Figure 15.20. *Series load resonant converters variations: (a) half bridge, split dc supply rail; (b) transformer couple full bridge; and (c) split resonant capacitor, with a centre tapped output rectifier stage.*

15.9.2 Parallel loaded resonant dc-to-dc converters

The basic parallel load resonant dc-to-dc converter is shown in figure 15.21a and its equivalent circuit is shown in figure 15.21b. The inductor L_o in the rectified output circuit produces a near constant current. A key feature is that the output voltage $v_{o/p}$ can be greater than the input voltage V_s, that is $0 \le v_{o/p} \ge V_s$.

The capacitor voltage and inductor current equations for the equivalent circuit in figure 15.21b, for a constant current load I_o, are

$$\begin{aligned} i_L(t) &= I_o + \left(i_{Lo} - I_o\right)\cos\omega_o t + \frac{V_s - v_{Co}}{Z_o}\sin\omega_o t \\ &= I_o + \frac{V_s}{Z_o}\sin\omega_o t \quad \text{for } v_{Co} = 0 \text{ and } i_{Lo} = I_o \end{aligned} \tag{15.178}$$

$$\begin{aligned} v_c(t) &= V_s - \left(V_s - v_{Co}\right)\cos\omega_o t + Z_o\left(i_{Lo} - I_o\right)\sin\omega_o t \\ &= V_s\left(1 - \cos\omega_o t\right) \quad \text{for } v_{Co} = 0 \text{ and } i_{Lo} = I_o \end{aligned} \tag{15.179}$$

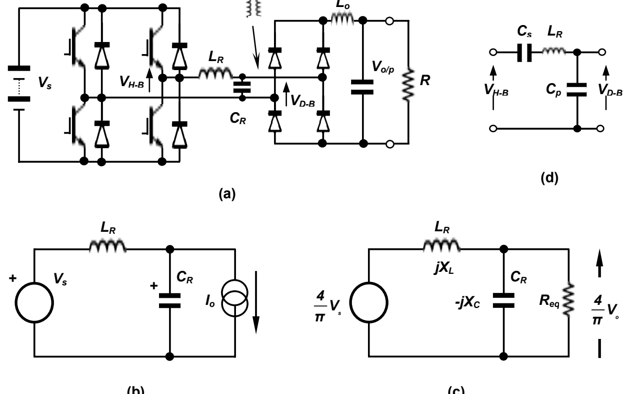

Figure 15.21. *Parallel resonant dc-to-dc converter: (a) circuit (b) equivalent ac circuit; (c) equivalent fundamental input voltage circuit; and (d) series-parallel resonant circuit stage.*

The relationship between the output voltage and the bridge switching frequency can determined from the equivalent circuit shown in figure 15.21c where the output resistance has been replaced by its equivalent resistance related to the H-bridge output fundament frequency magnitude, $4/\pi\, V_s$. The voltage across the resonant capacitor C_R is assumed to be sinusoidal.

Kirchhoff analysis of the equivalent circuit in figure 15.22b gives

$$\frac{v_o}{V_s}=\frac{8}{\pi^2}\times\left|\frac{1}{1-\dfrac{X_L}{X_C}+\dfrac{X_L}{R_{eq}}}\right|=\frac{8}{\pi^2}\times\frac{1}{\left(1-\dfrac{X_L}{X_C}\right)^2+\left(\dfrac{X_L}{R_{eq}}\right)^2} \qquad (15.180)$$

where the load resistance R is related to the equivalent resistance, at the switching frequency ω_s, by $R_{eq}=\frac{8}{\pi^2}\times R$. Series stray non-load resistance has been neglected.

15.9.2i - Modes of operation - parallel resonant circuit

Three modes of operation are applicable to the parallel-resonant circuit, dc-to-dc converter, and waveforms are shown in figure 15.22x.

i. $f_s < ½f_o$:- discontinuous inductor current (switch conduction $1/2f_o \le t_T \le 1/f_o$)

Initially all switches are off and the load current energy stored in L_o freewheels through the bridge diodes.

Both the inductor current and capacitor voltage are zero at the beginning of the cycle and at the end of the cycle. Thus switch turn-on and turn-off occur with zero current losses. At H-bridge turn-on the resonant inductor current increases linearly according to $i=V_s t/L_R$ until the output current level I_o is reached at time $t_I = L_R I_o/V_s$, when the capacitor is free to resonant. The capacitor voltage is given by

$$v_c(t)=V_s(1-\cos\omega_o t) \qquad (15.181)$$

while the inductor current is given by

$$i_L(t)=I_o+\frac{V_s}{Z_o}\sin(\omega_o t) \qquad (15.182)$$

The resonant circuit inductor current reverses as on-switch antiparallel freewheel diodes conduct, at which time the switches may be turned off at zero current and voltage conditions. A further inductor current reversal is therefore not possible. At the attempted reversal instant, any retained capacitor charge is discharged at a constant rate I_o in the inductor L_o. The capacitor voltage falls linearly to zero at which time the current in L_o freewheels in the output rectifier diodes.

ii. $½f_o < f_s < f_o$:- continuous inductor current

When switching below resonance, the switches commutate naturally at turn-off, as shown in figure 15.22b, making thyristors a possibility.

Hard turn-on results, necessitating the use of fast recovery diodes.

iii. $f_s > f_o$:- continuous inductor current

When switching at frequencies above the natural resonance frequency, no turn-on losses occur since turn-on occurs when a switches antiparallel diode is conducting. Turn-off occurs with inductor current flowing, hence hard turn-off occurs, with switch current commutated to a freewheel diode. In mitigation, lossless capacitive turn-off snubber can be used (a capacitor in parallel with each switch).

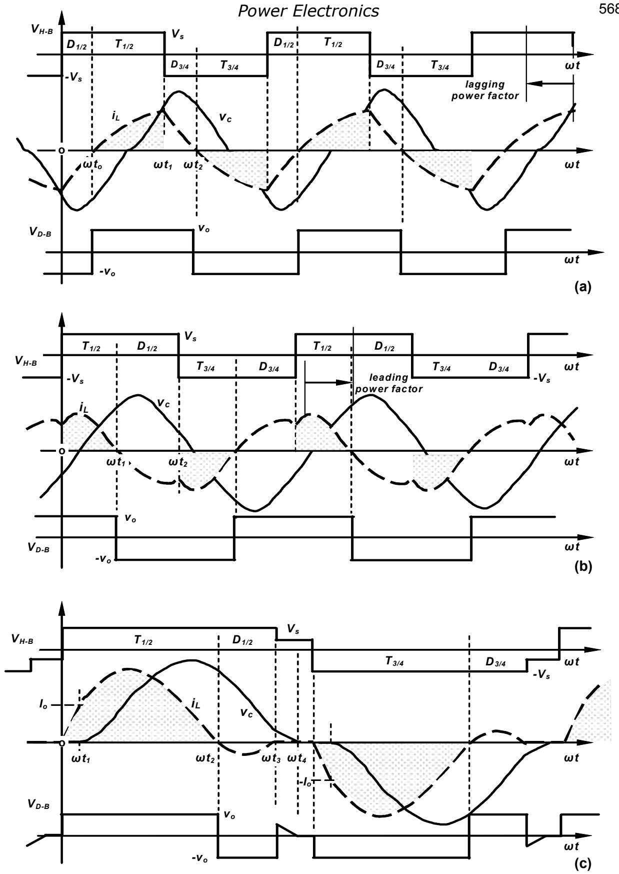

Figure 15.22. *Three modes of parallel load resonant converter operation: (a) $f_s > f_o$; (b) $½f_o < f_s < f_o$; and (c) $f_s < ½f_o$.*

15.9.2ii - Circuit variations

A parallel resonant circuit approach, with or without transformer coupling, can also be use as indicated in figure 15.21a. The centre tapped secondary approach shown in figure 15.20c is also applicable. A dc capacitor split dc rail can also be used, as shown for the series load resonate circuit in figure 15.20a. A further resonant circuit variation is a combined series-parallel resonant stage as shown in figure 15.21d.

Table 15.4 Switch and diode turn-on/turn-off conditions for resonant switch converters

	series load resonance		parallel load resonance		
	switch on / diode off	switch off / diode on	switch on / diode off	switch off / diode on	power factor
$f_s<½f_o$	zcs	zcs zvs	zcs	zcs zvs	N/A
$f_s < f_o$	hard	zcs zvs	hard	zcs zvs	leading
$f_s > f_o$	zcs zvs	hard	zcs zvs	hard	lagging

Example 15.9: ***Transformer-coupled, series-resonant, dc-to-dc converter***

The series resonant dc step down voltage converter in figure 15.17a is operated at just above the resonant frequency of the load circuit and is used with a step up transformer, 1:2 (n_T = ½), as shown in figure 15.19a. It produces an output voltage for the armature of a high voltage dc motor that has a voltage requirement that is greater than the 50Hz ac mains rectified, 340V dc, with an L-C dc link filter. The resonant circuit parameters are L=100µH, C=0.47µF, and the coil resistance is R_c = 1Ω.
For a 10Ω armature resistance, R_{load}, calculate

i. the circuit Q and ω_o
ii. the output voltage, hence dc armature current and power delivered
iii. the secondary circuit dc filter capacitor voltage and rms current rating
iv. the resonant circuit rms ac current and capacitor rms ac voltage
v. the converter average input current and efficiency
vi. the ac current in the input L-C dc rectifier filter decoupling capacitor
vii. the H-bridge square-wave switching frequency ω_s, greater than ω_o.

Solution

i. The resonant circuit Q is

$$Q = \sqrt{\frac{L_R}{C_R}} / R_c = \sqrt{\frac{100\mu\text{H}}{0.47\mu\text{F}}} / 1\Omega = 14.6$$

For this high Q, the circuit resonant frequency and damped frequency will be almost the same, that is

$$\begin{aligned}\omega \approx \omega_o &= 1/\sqrt{L_R C_R}\\ &= 1/\sqrt{100\mu\text{H}\times 0.47\mu\text{F}} = 146\text{ krad/s}\\ &= 2\pi f\\ f &= 146\text{ krad/s}/2\pi = 23.25\text{ kHz}\end{aligned}$$

ii. From equation (15.171), which will be accurate because of a high circuit Q of 14.6,

$$\begin{aligned}\bar{I} &= \frac{2}{\pi}\hat{I} = \frac{8}{\pi^2}\frac{\left(V_s - n_T\times v_{o/p}\right)}{R_c} = \frac{8}{\pi^2}\times\frac{\left(340\text{ V} - ½v_{o/p}\right)}{1\Omega}\\ &= 0.81\times\left(340\text{ V} - ½v_{o/p}\right)\end{aligned}$$

Note that the output voltage $v_{o/p}$ across the dc decoupling capacitor has been referred to the primary by n_T, hence halved, due to the turns ratio of 1:2.
The rectified resonant current provides the load current, that is

$$\begin{aligned}\bar{I} &= \frac{1}{n_T}\times\frac{v_{o/p}}{R_{load}} = 2\times\frac{v_{o/p}}{10\Omega}\\ &= \frac{v_{o/p}}{5}\end{aligned}$$

Again the secondary current has been referred to the primary. Solving the two average primary current equations gives

$$\bar{I} = 0.81\times\left(340 - ½v_o\right) = \frac{v_{o/p}}{5}$$

$$v_{o/p} = 456\text{V and } \bar{I} = 91.2\text{A}$$

That is, the load voltage is 456V dc and the load current is 456V/10Ω = 91.2A/2 = 45.6A dc. The power delivered to the load is 456^2/10Ω = 20.8kW.

iii. From part ii, the capacitor dc voltage requirement is at least 456V dc. The secondary rms current is

$$\begin{aligned}I_{Srms} &= n_T\times I_{Prms} = n_T\times\frac{1}{\sqrt{2}}\times\hat{I}_P = ½\times\frac{1}{\sqrt{2}}\times\frac{\pi}{2}\times\bar{I}_P\\ &= 0.555\times\bar{I}_P = 0.555\times 91.2\text{A}\\ &= 50.65\text{A rms}\end{aligned}$$

The primary rms current is double the secondary rms current, 101.3A rms.

By Kirchhoff's current law, the secondary current (50.65A rms) splits between the load (45.6A dc) and the decoupling capacitor. That is the rms current in the capacitor is

$$I_{Crms} = \sqrt{I_{Srms}^2 - \bar{I}_S^2} = \sqrt{50.65^2 - 45.6^2} = 22\text{A rms}$$

That is, the secondary dc filter capacitor has a dc voltage requirement of 456V dc and a current requirement of 22A rms at 46.5kHz, which is double the resonant frequency because of the rectification process.

iv. The primary rms current is double the secondary rms current, namely from part iii, I_{Prms}=101.3A rms. The 0.47µF resonant capacitor voltage is given by

$$\begin{aligned} v_{cap} &= I_{\text{Prms}} X_c = \frac{I_{\text{Prms}}}{\omega_o C_R} \\ &= \frac{101.3\text{A}}{146\text{krad/s} \times 0.47\mu\text{F}} = 1476\text{V rms} \end{aligned}$$

The resonant circuit capacitor has an rms current rating requirement of 101.3A rms and an rms voltage rating of 1476 V rms.

v. From part ii, the average input current is 91.2 A. The supply input power is therefore 340Vdc×91.2A ave = 31kW. The power dissipated in the resonant circuit resistance R_c =1Ω is given by $I_{\text{Prms}}^2 \times R_c = 101.3^2 \times 1\Omega = 10.26\text{kW}$. Note that the coil power plus the load power (from part ii) equals the input power (20.8kW+10.26kW = 31kW). The efficiency is

$$\begin{aligned} \eta &= \frac{\text{output power}}{\text{input power}} \times 100\% \\ &= \frac{20.8\text{kW}}{31\text{kW}} \times 100 = 67.1\% \end{aligned}$$

vi. The average input dc current is 91.2A dc while the resonant bridge rms current is 101.3A rms. By Kirchhoff's current law, the 340V dc rail decoupling capacitor ac current is given by

$$\begin{aligned} I_{ac} &= \sqrt{I_{Prms}^2 - I_{Pave}^2} \\ &= \sqrt{101.3^2 - 91.2^2} = 44.1\text{A ac} \end{aligned}$$

This is the same ac current magnitude as the current in the dc capacitor across the load in the secondary circuit, 22A, when the transformer turns ratio, 2, is taken into account.
vii. The voltage across the load resistance is given by equation (15.177)

$$\frac{v_o}{V_s} = \frac{R_{eq}}{R_{eq} + R_c + j\left(\omega_s L_R - \dfrac{1}{\omega_s C_R}\right)} = \frac{\dfrac{8}{\pi^2} \times n_T^2 \times R_{Load}}{\dfrac{8}{\pi^2} \times n_T^2 \times R_{Load} + R_c + j\left(\omega_s L_R - \dfrac{1}{\omega_s C_R}\right)}$$

$$\frac{226\text{V}}{340\text{V}} = 0.66 = \left| \frac{\dfrac{8}{\pi^2} \times ¼ \times 10\Omega}{\dfrac{8}{\pi^2} \times ¼ \times 10\Omega + 1\Omega + j\left(\omega_s 100\mu\text{H} - \dfrac{1}{\omega_s 0.47\mu\text{F}}\right)} \right|$$

$$0.66 = \left| \frac{2}{3 + j\left(\omega_s 100\mu\text{H} - \dfrac{1}{\omega_s 0.47\mu\text{F}}\right)} \right|$$

$$1 = \left| 1 + j \tfrac{1}{3}\left(\omega_s 100\mu\text{H} - \frac{1}{\omega_s 0.47\mu\text{F}}\right) \right|$$

Because of the high circuit Q = 14.6 and relatively high voltage transfer ratio $v_o / V_s = 0.66$, ω_s is very close to ω_o, as can be deduced from the plots in figure 15.18b. The output voltage control will be very sensitive to changes in the H-bridge switching frequency.

♣

15.9.3 Resonant-switch, dc-to-dc converters

There are two forms of resonant switch circuit configurations for dc-to-dc converters, namely resonant voltage and resonant current switch commutation. Each type reduces the switching losses to near zero.

- In *resonant current commutation* the switching current is reduced to zero by an *L-C* resonant circuit current greater in magnitude than the load current, such that the switch is turned on and off with zero current.
- In *resonant voltage commutation* the switch voltage is reduced to zero by the capacitor of an *L-C* resonant circuit with a voltage magnitude greater than the output voltage, such that the switch can turn on and off with zero voltage.

Figure 15.23a shows the basic single switch, forward, step-down voltage switch mode dc-to-dc converter. Resonant switch converters are an extension of the standard switch mode forward converter, but the switch is supplement with passive components L_R-C_R to provide resonant operation through the switch, hence facilitating zero current or voltage switching. A common feature is that the resonant inductor L_R is in series with the switch to be commutated. Parasitic series inductance is therefore not an issue with resonant switch converters.

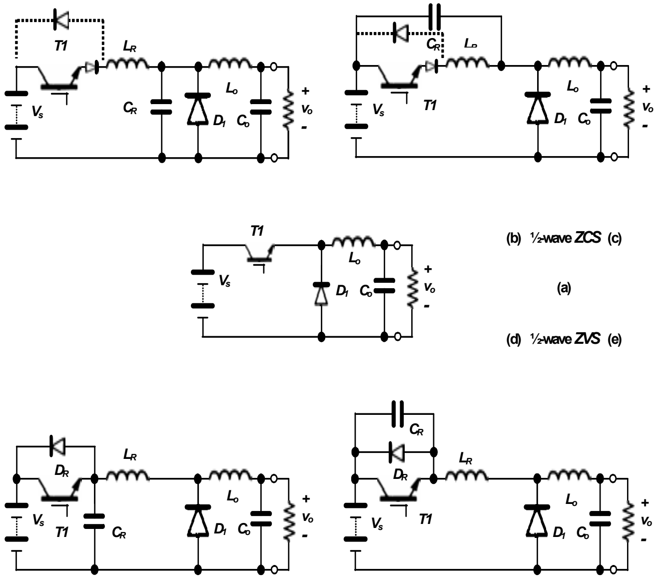

Figure 15.23. *Dc to dc resonant switch converters:*
(a) conventional switch mode forward step-down converter;
*(b) and (c) half-wave zero current switching **ZCS** resonant switch converters; and*
*(d) and (e) half-wave zero voltage switching **ZVS** resonant switch converters.*

The resonant capacitor C_R, can be either in a parallel or series arrangement as shown in figure 15.23, since small-signal ac-wise the connections are the same. A well-decoupled supply is essential when the capacitor C_R is used in the parallel switch arrangement, as shown in figure 15.23 part b and d. A further restriction is that a diode must be used in series or in antiparallel with T1 if a switch without reverse blocking capability is used. The use of an antiparallel connected diode changes the switching arrangement from half-wave resonant operation with reverse impressed voltage switch commutation to full-wave resonant operation with current displacement commutation, independent of the switch reverse blocking capabilities. Reconnecting the capacitor C_R terminal not associated with V_s, to the other end of inductor L_R in figure 15.23b-e, will

create four full-wave resonant switch circuits (the commutation type, namely voltage or current, is also interchanged). An important operational requirement is that the average load current never falls to zero, otherwise the resonant capacitor C_R can never fully discharge when performing its zero switch current turn-off function.

15.9.3i Zero-current, resonant-switch, dc-to-dc converter

The zero current switching of T1 in figure 15.24 (15.23b) can be analysed in five distinctive stages, as shown in the capacitor voltage and inductor current waveforms. The switch is turned on at t_o and turned off after t_4 but before t_5.

The circuit has attained steady state load conditions from one cycle to the next. The cycle commences, before t_o, with both the capacitor voltage and inductor current being zero, and the load current is freewheeling through D1. The current in the output inductor L_o, is large enough such that its current, I_o can be assumed constant. The switch T1 is off.

Time interval I

At t_o the switch is turned on and the series inductor L_R acts as a turn-on snubber for the switch. In the interval t_o to t_1, the supply voltage is impressed across L_R since the switch T1 is on and the diode D1 conducts the output current, thereby clamping the inductor to zero volts. Because of the fixed voltage V_s, the current in L_R increases from zero, linearly to I_o in time

$$t_I = I_o L_R / V_S \tag{15.183}$$

according to

$$i_{L_R}(t) = \frac{V_s\, t}{L_R} \tag{15.184}$$

Time interval IIA

When the current in L_R reaches I_o at time t_1, the capacitor C_R and L_R are free to resonant. The diode D1 blocks as the voltage across C_R sinusoidally increases. The constant load current component in L_R does not influence its ac performance since a constant inductor current does not produce any inductor voltage. Its voltage is specified by the resonant cycle, provided $I_o > V_s / Z_o$. The capacitor resonantly charges to twice the supply V_s when the inductor current falls back to the load current level I_o, at time t_3.

Time interval IIB

Between times t_3 and t_4 the load current is displaced from L_R by charge in C_R, in a quasi resonance process. The resonant cycle cannot reverse through the switch once the inductor current reaches zero at time t_4, because of the series blocking diode (the switch must have uni-directional conduction characteristics).

The time for period II is approximately

$$t_{II} = \left(\pi - \sin^{-1}\left(\frac{I_o Z_o}{V_s}\right)\right) / \omega_o \tag{15.185}$$

where $Z_o = \sqrt{L_R / C_R}$, while the capacitor voltage is given by

$$v_{C_R}(t) = V_s\left(1 - \cos\omega_o t\right) \tag{15.186}$$

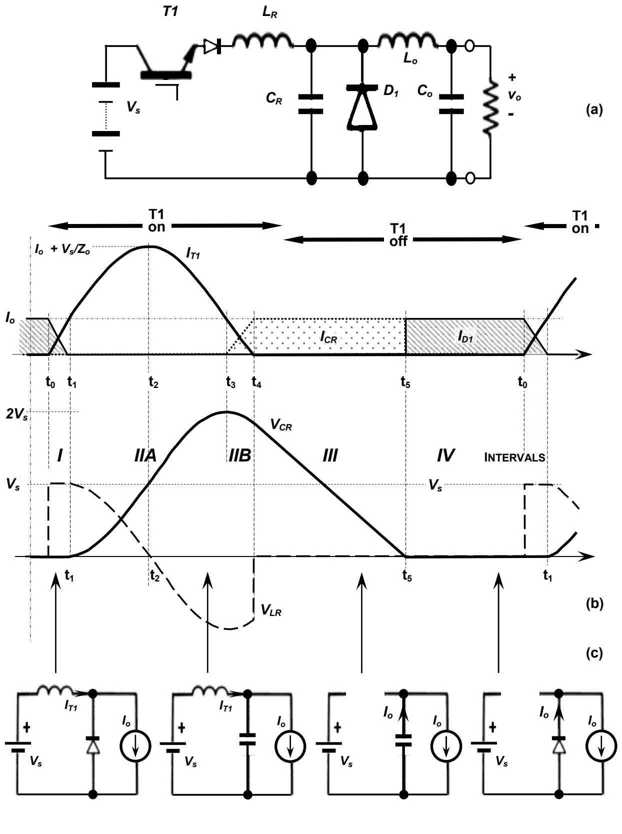

Figure 15.24. *Zero current switching, ZCS, resonant switch dc to dc converter: (a) circuit; (b) waveforms; and (c) equivalents circuits.*

Time interval III

At time t_4 the input current is zero and the switch T1 can be turned off with zero current, ZCS. The constant load current requirement I_o is provided by the capacitor, which discharges linearly to zero volts at time t_5. according to

$$v_{C_R}(t) = V_{C_R} - \frac{I_o}{(1-\cos\omega_o t_{II})C_R} \times t \tag{15.187}$$

The time for interval *III* is therefore load current dependant and is given by

$$t_{III} = \frac{V_{C_R} C_R}{I_o} \times (1-\cos\omega t_{II}) \tag{15.188}$$

Time interval IV

After t_5, the switch is off, the current freewheels through D1, the capacitor voltage is zero and the input inductor current is zero. At time t_1 the cycle recommences. The interval *IV*, t_5 to t_0, is used to control the rate at which energy is transferred to the load.

The output voltage can be specified by either evaluating the energy from the supply, through the input resonant inductor L_R, or by evaluating the average voltage across the resonant capacitor C_R which is filtered by the output filter L_o - C_o.
By considering the input inductor energy for each shown period, from the waveforms in figure 15.24b, the output voltage is given by

$$v_o = \frac{V_s}{\tau}\left(\tfrac{1}{2}t_I + t_{II} + t_{III}\right) \tag{15.189}$$

where the switching frequency $f_s = 1/\tau$.
The output voltage based on the average capacitor voltage is

$$\begin{aligned} v_o &= \frac{1}{\tau}\left[\int_{t_1}^{t_4} V_s(1-\cos\omega t)\,dt + \int_{t_4}^{t_5} V_s \frac{t}{t_5 - t_4}\,dt\right] \\ &= \frac{1}{\tau}\left[\frac{V_s}{2\pi} \times \left(\frac{3\pi}{2} + 1\right) \times \tfrac{4}{3} \times (t_4 - t_1) \; + \; \tfrac{1}{2} \times V_s \times (t_5 - t_4)\right] \end{aligned} \tag{15.190}$$

The circuit has a number of features:

i. Turn-on and turn-off occur at zero current, hence switching losses are minimal.
ii. At light load currents the switching frequency may become extreme low.
ii. The basic half resonant period is given by $t_{II} = \pi\sqrt{L_R C_R}$
iii. The capacitor discharge time is $t_{III} \le 2 \times V_s \times C_R / I_o$, thus the output voltage is load current dependant.
iv. L_R and C_R are dimensioned such that the capacitor voltage is greater than V_s at time t_4, at maximum load current I_o.
v. Supply inductance is inconsequential, decreasing the inductance L_R requirement.
vi. Being based on the forward converter, the output voltage is less than the input voltage. The output increases with increased switching frequency.

vii. If a diode in antiparallel to the switch is added as shown dashed in figure 15.23b, reverse inductor current can flow and the output voltage is $v_o \approx V_s \times f_s / f_o$.

Operation of the ZCS circuit in figure 15.23c, where the capacitor C_R is connected in parallel with the switch, is essentially the same as the circuit in figure 15.24. The capacitor connection produces the result that the capacitor voltage has a dc offset of V_s, meaning its voltage swings between ± V_s rather than zero and twice V_s, as in the circuit just considered. Any dc supply inductance must be decoupled when using the ZVS circuit in figure 15.23c.

15.9.3ii Zero-voltage, resonant-switch, dc-to-dc converter

The zero voltage switching of T1 in figure 15.25 (15.23e) can be analysed in five distinctive stages, as shown in the capacitor voltage and inductor current waveforms. The switch is turned off at t_o and turned on after t_4 but before t_5.

The circuit has attained steady state load conditions from one cycle to the next. The cycle commences, before t_o, with the capacitor C_R voltage being V_s and the load current I_o being conducted by the switch and the resonant inductor, L_R. The output inductor L_o is large enough such that its current, I_o can be assumed constant. The switch T1 is on.

Time interval I

At time t_o the switch is turned off and the parallel capacitor C_R acts as a turn-off snubber for the switch. In the interval t_o to t_1, the supply current is provided from V_s through C_R and L_R. Because the load current is constant, I_o, due to large L_o, the capacitor charges linearly from 0V until its voltage reaches V_s in time

$$t_I = \frac{V_s C_R}{I_o} \tag{15.191}$$

according to

$$v_c(t) = V_s - \frac{I_o}{C_R} \times t \tag{15.192}$$

Time interval II

When the voltage across C_R reaches V_s at time t_1, the load freewheel diode conducts, clamping the load voltage to zero volts. The capacitor C_R and L_R are free to resonant, where the initial inductor current is I_o and the initial capacitor voltage is V_s. The energy in the inductor transfers to the capacitor, which increases its voltage from V_s to a maximum at time t_2 of

$$\hat{v}_C = V_s + I_o Z_o \tag{15.193}$$

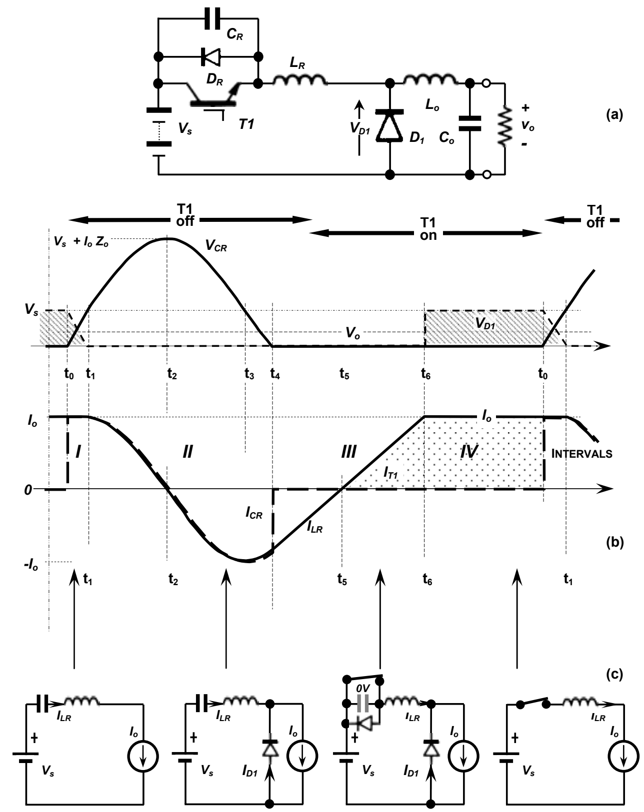

Figure 15.25. *Zero voltage switching, ZVS, resonant switch dc to dc converter: (a) circuit; (b) waveforms; and (c) equivalents circuits.*

The capacitor energy transfers back to the inductor which has resonated from + I_o to – I_o between times t_1 to time t_3. For the capacitor voltage to resonantly return to V_s, $I_o > V_s / Z_o$. Between t_3 and t_4 the voltage V_s on C_R is resonated through L_R, which conducts – I_o at t_3, as part of the resonance process. The capacitor voltage and inductor current during period II are given by

$$\begin{aligned} v_c(t) &= V_s + I_o Z_o \sin\omega_o t \\ i_L(t) &= I_o \cos\omega_o t \end{aligned} \qquad (15.194)$$

and the duration of interval II is

$$t_{II} = \left(\pi + \sin^{-1}\frac{V_s}{I_o Z_o}\right) / \omega_o \qquad (15.195)$$

At the end of interval II the capacitor voltage is zero and the inductor current is

$$i_L(t_{II}) = I_o \cos\omega_o t_{II} \qquad (15.196)$$

Time interval III

At time t_4 the voltage on C_R attempts to reverse, but is clamped to zero by diode D_R. The inductor energy is returned to the supply V_s via diode D_R and the freewheel diode D_1. The inductor current decreases linearly to zero during the period t_4 to t_5. During this period the switch T1 is turned on. No turn-on losses occur because the diode D_R in parallel with T1 is conducting during the period the switch is turned on, that is, the switch voltage is zero and the switch T1 can be turned on with zero voltage, ZVS. With the switch on at time t_5 the current in the inductor L_R reverses and builds up, linearly to I_o at time t_6. The current slope is supply V_s dependant, according to $V_s = L_R$ *di/dt*, that is

$$i_L(t) = \frac{V_s}{L_R} t + I_o \cos\omega_o t_I \qquad (15.197)$$

and the time of period *III* is load current dependant:

$$t_{III} = \frac{I_o L_R}{V_s} \times (1 - \cos\omega_o t_1) \qquad (15.198)$$

Time interval IV

At t_6, the supply V_s provides all the load current and the diode D1 recovers with a controlled *di/dt* given by V_s / L_R. The switch conduction interval *IV*, t_6 to t_o, is used to control the rate at which energy is transferred to the load.

The output voltage can be derived from the diode voltage (shown hatched in figure 15.25b) since this voltage is averaged by the output LC filter.

$$\begin{aligned} v_o &= \frac{1}{\tau}\left(\text{Volt} \times \text{second area of interval } I + \text{Volt} \times \text{second area of interval } IV\right) \\ &= \frac{1}{\tau}\left(\tfrac{1}{2}t_1 + \tau - t_5\right) = V_s\left(1 - f_s\left(t_6 - \tfrac{1}{2}t_1\right)\right) \end{aligned} \qquad (15.199)$$

The circuit has a number of features:

i. Switch turn-on and turn-off both occur at zero voltage, hence switching losses are minimal.
ii. At light load currents the switching frequency may become extreme high.
iii. The basic half-resonant period is approximately given by $t_{1-4} = \pi\sqrt{L_R C_R}$
iv. The inductor discharge time is $t_{III} \le 2 \times I_o \times L_R / V_s$, hence the output voltage is load current dependant.
v. L_R and C_R are dimensioned such that the inductor current is less than zero (being returned to the supply V_s) at time t_5, at maximum load current I_o.
vi. Being based on the forward converter, the output voltage is less than the input voltage. Increasing the switching frequency decreases the output voltage since τ-t_5 is decreased in equation (15.199).

Operation of the ZVS circuit in figure 15.23d, where the capacitor C_R is connected in parallel with the load circuit (the freewheel diode D1), is essentially the same as the circuit in figure 15.25. The capacitor connection produces the result that the capacitor voltage has a dc offset of V_s, meaning its voltage swings between + V_s and -$I_o Z_o$, rather than zero and V_s - $I_o Z_o$, as in the circuit just considered. Any dc supply inductance must be decoupled when using the ZVS circuit in figure 15.23d.

It will be noticed that a ZCS converter has a constant on-time, while a ZVS converter has a constant off-time.

Example 15.10: ***Zero-current, resonant-switch, dc-to-dc converter***

The ZCS resonant dc step-down voltage converter in figure 15.24a produces an output voltage for the armature of a high voltage dc motor and operates from the voltage produced from the 50Hz ac mains rectified, 340V dc, with an *L-C* dc link filter. The resonant circuit parameters are L_R = 100μH, C_R = 0.47μF, and the high frequency ac resistance of the circuit is R_c = 1Ω.

Calculate

i. the circuit Z_o, Q, and ω_o
ii. the minimum output current to ensure ZCS
iii. the maximum operating frequency, represented by the time between switch turn on and the freewheel diode recommencing conduction, at minimum load current
iv. the average diode voltage, hence load voltage at the maximum frequency.

Solution

i. The characteristic impedance is given by

$$Z_o = \sqrt{\frac{L_R}{C_R}} = \sqrt{\frac{100\mu\text{H}}{0.47\mu\text{F}}} = 14.6\Omega$$

The resonant circuit Q is

$$Q = \frac{Z_o}{R_c} = \sqrt{\frac{100\mu H}{0.47\mu F}} / 1\Omega = 14.6$$

For this high Q, the circuit resonant frequency and damped frequency will be almost the same, that is

$$\begin{aligned}\omega \approx \omega_o &= 1/\sqrt{L_R C_R}\\ &= 1/\sqrt{100\mu H \times 0.47\mu F} = 146 \text{ krad/s}\\ &= 2\pi f\\ f &= 146 \text{ krad/s} / 2\pi = 23.25 \text{ kHz}\\ \text{or} \quad T &= 43\mu s\end{aligned}$$

ii. For zero current switching, the load current must be greater than the resonant current, that is

$$I_o > V_s / Z_o = 340\text{V}/14.6\Omega = 23.3\text{A}$$

iii. The commutation period comprises the four intervals, I to IV, shown in figure 15.24b.

Interval I

The switch turns on and the inductor current rises from 0A to 23.3A in a time given by

$$\begin{aligned}t_I &= L_R \Delta I / V_s\\ &= 100\mu H \times 23.3A/340V = 6.85\mu s\end{aligned}$$

Intervals *II* and *III*

These two interval take just over half a resonant cycle, which takes 43μs/2 = 21.5μs to complete. Assuming action is purely sinusoidal resonance with a 23.3A offset, from 0A to a maximum of 23.3A and down to -23.3A then from

$$I_o = V_s / Z_o \sin\omega t \quad \text{for} \quad \omega t > \pi$$

$$23.3A = -23.3A \times \sin\omega t \text{ gives}$$

$$\begin{aligned}t &= \tfrac{3}{4} \times 43\mu s\\ &= 32.25\mu s\end{aligned}$$

The capacitor voltage at the end of this period is given by

$$\begin{aligned}V_{cIV} &= V_s (1 - \cos\omega t)\\ &= 340V \times (1 - \cos \tfrac{3}{2}\pi)\\ &= 340V\end{aligned}$$

Interval *IV*

The capacitor voltage must discharge from 340V dc to zero volts, providing the 23.3A load current. That is

$$\begin{aligned}t_{IV} &= V_{cIV} \times C_R / I_o\\ &= 340V \times 0.47\mu F/23.3A = 6.86\mu s\end{aligned}$$

The minimum commutation cycle time is therefore 6.85+32.25+6.86 = 46μs. Thus the maximum operating frequency is 21.7kHz.

iv. The output voltage v_o is the average reverse voltage of freewheel diode D_1, which is in parallel with the resonant capacitor C_R. Integration of the capacitor voltage shown in figure 15.24b gives equation (15.190)

$$\begin{aligned}v_o &= \frac{1}{t_5}\left[\int_{t_1}^{t_4} V_s (1-\cos\omega t)\,dt + \int_{t_4}^{t_5} V_s \frac{t}{t_5 - t_4}\,dt\right]\\ &= \frac{1}{46\mu s} \times \left[\int_o^{32.25\mu s} 340V \times (1-\cos\omega t)\,d\omega t + \int_0^{6.86\mu s} 340V \times \frac{t}{6.86\mu s}\,dt\right]\\ &= \frac{1}{46\mu s} \times \left[340V \times \left(\frac{3\pi}{2} + 1\right) \times \frac{43\mu s}{2\pi} \;+\; \tfrac{1}{2} \times 340V \times 6.86\mu s\right]\\ &= \frac{1}{46\mu s} \times \left[13292V\mu s \;+\; 1166V\mu s\right] = 314.3\text{Vdc}\end{aligned}$$

The maximum output voltage is 314V dc. Alternatively, using the input inductor energy based equation (15.189):

$$\begin{aligned}v_o &= \frac{V_s}{\tau}\left(\tfrac{1}{2}t_I + t_{II+III} + t_{IV}\right)\\ &= \frac{340V}{46\mu s} \times \left(\tfrac{1}{2} \times 6.85\mu s + 32.25\mu s + 6.85\mu s\right) = 314.8V\end{aligned}$$

♣

Example 15.11: *Zero-voltage, resonant-switch, dc-to-dc converter*

The zero voltage resonant switch converter in figure 15.25 operates under the following conditions**:**

V_s = 192V $\qquad$ I_o = 25A

L_R = 10μH $\qquad$ C_R = 0.1μF

Determine

i. the switching frequency f_s for v_o = 48V

ii. switch average current and

iii. the peak switch/diode/capacitor voltage.

iv. the minimum output current

Solution

i.

$$\omega_o = \frac{1}{\sqrt{L_R C_R}} = \frac{1}{\sqrt{10\mu H \times 0.1\mu F}} = 1 \times 10^6 \text{ rad/s} \quad \text{that is } f_o = 159.2\text{kHz}$$

$$Z_o = \sqrt{\frac{L_R}{C_R}} = \sqrt{\frac{10\mu H}{0.1\mu F}} = 10\Omega$$

The period of interval I is given by equation (15.191), that is

$$t_I = \frac{V_s\, C_R}{I_o} = \frac{192V \times 0.1\mu F}{25A} = 0.768\mu s$$

The period of interval II is given by equation (15.195), that is

$$t_{II} = t_3 - t_1 = \left(\pi + \sin^{-1}\frac{V_s}{I_o Z_o}\right) / \omega_o = \left(\pi + \sin^{-1}\frac{192V}{25A \times 10\Omega}\right) / 10^6 \text{rad/s} = 4.017\mu s$$

The period for the constant current period III is given by equation (15.198)

$$t_{III} = \frac{I_o\, L_R}{V_s} \times \left(1 - \cos\omega_o t_1\right) = \frac{25A \times 10\mu H}{192V} \times \left(1 - \cos\left(10^6 \times 0.768\mu s\right)\right) = 0.365\mu s$$

After re-arranging equation (15.199), the switching frequency is given by

$$f_s = \frac{\left(1 - \frac{v_o}{V_s}\right)}{t_5 - \tfrac{1}{2}t_1} = \frac{\left(1 - \frac{48V}{192V}\right)}{\left(0.768\mu s + 4.017\mu s + 0.365\mu s - \tfrac{1}{2} \times 0.768\mu s\right)} = 157.4kHz$$

ii. The switch current is shown by hatched dots in figure 15.25. The average value is dominated by interval IV, with a small contribution in interval II between t_5 and t_6.

$$\begin{aligned}\bar{I}_T &= \frac{I_o}{\tau} \times \left(\frac{\tfrac{1}{2} \times t_{III}}{1 + \left|\cos\omega_o t_{II}\right|} + \left(\tau - t_6\right)\right) \\ &= 25A \times 157.4kHz \left(\frac{\tfrac{1}{2} \times 0.365\mu s}{1 + \left|\cos\left(10^6 \times 4.017\mu s\right)\right|} + \left(\frac{1}{157.4kHz} - \left(0.768\mu s + 4.017\mu s + 0.365\mu s\right)\right)\right) \\ &= 5.17A\end{aligned}$$

iii. The peak switch/diode/capacitor voltage is given by equation (15.193), namely

$$\begin{aligned}\hat{v}_C &= V_s + I_o Z_o \\ &= 192V + 25A \times 10\Omega = 442V\end{aligned}$$

iv. For proper resonant action the minimum average output current must satisfy, $I_o > V_s / Z_o$, that is

$$\check{I}_o = \frac{V_s}{Z_o} = \frac{192V}{10\Omega} = 19.2A$$

♣

15.10 Appendix: Analysis of non-continuous inductor current operation

Operation with constant input voltage, E_i

In applications were the input voltage E_i is fixed, as with rectifier ac voltage input circuits, the output voltage v_o can be controlled by varying the duty cycle.

In the continuous inductor conduction region, the transfer function for the three basic converters is determined solely in terms of the on-state duty cycle, δ. Operation in the discontinuous current region, for a constant input voltage, can be characterised for each converter in terms of duty cycle and the normalised output or input current, as shown in figure 15.26. Key region and boundary equations, for a constant input voltage E_i, are summarised in tables 15.5 and 15.6.

Operation with constant output voltage, v_o

In applications were the output voltage v_o is fixed, as with regulated dc power supplies, the effects of varying input voltage E_i can be controlled and compensated by varying the duty cycle.

In the inductor continuous current conduction region, the transfer function is determined solely in terms of the on-state duty cycle, δ. Operation in the discontinuous region, for a constant output voltage, can be characterised in terms of duty cycle and the normalised output or input current, as shown in figure 15.27.

Key region and boundary equations, for a constant output voltage v_o, are summarised in table 15.7.

Because of the invariance of power, the output current $\bar{I}_o$ characteristics for each converter with a constant input voltage E_i , shown in figure 15.26, are the same as those for the input current $\bar{I}_i$ when the output voltage v_o is maintained constant, as shown in figure 15.27. [That is the right hand side of each plot is the same.]

Table 15.5. Transfer functions with constant input voltage, E_i, with respect to $\bar{I}_o$

E_i constant	converter step-down	converter step-up	converter step-up/down
reference equation	(15.4)	(15.44)	(15.74)
current conduction	$\frac{v_o}{E_i}=\delta$	$\frac{v_o}{E_i}=\frac{1}{1-\delta}$	$\frac{v_o}{E_i}=\frac{-\delta}{1-\delta}$
reference equation	(15.21)	(15.59)	(15.91)
discontinuous current	$\frac{v_o}{E_i}=\frac{1}{1+\frac{2L\bar{I}_o}{\delta^2\tau E_i}}$	$\frac{v_o}{E_i}=1+\frac{\delta^2 E_i t_T}{2L\bar{I}_o}$	$\frac{v_o}{E_i}=-\frac{\delta^2 E_i\tau}{2L\bar{I}_o}$
normalised $\hat{I}_o=\frac{E_i\tau}{8L}$ @ $\delta = ½$	$\frac{v_o}{E_i}=\frac{1}{1+\frac{1}{4\delta^2}\times\frac{\bar{I}_o}{\hat{I}_o}}$	$\frac{v_o}{E_i}=1+4\delta^2/\frac{\bar{I}_o}{\hat{I}_o}$	$\frac{v_o}{E_i}=-4\delta^2/\frac{\bar{I}_o}{\hat{I}_o}$
change of variable	$\frac{\bar{I}_o}{\hat{I}_o}=\frac{4\delta^2\left(1-\frac{v_o}{E_i}\right)}{\frac{v_o}{E_i}}$	$\frac{\bar{I}_o}{\hat{I}_o}=\frac{4\delta^2}{\frac{v_o}{E_i}-1}$	$\frac{\bar{I}_o}{\hat{I}_o}=\frac{-4\delta^2}{\frac{v_o}{E_i}}$
boundary	$\frac{\bar{I}_o}{\hat{I}_o}=4\frac{v_o}{E_i}\left(1-\frac{v_o}{E_i}\right)$	$\frac{\bar{I}_o}{\hat{I}_o}=\frac{4\left(\frac{v_o}{E_i}-1\right)}{\left(\frac{v_o}{E_i}\right)^2}$	$\frac{\bar{I}_o}{\hat{I}_o}=\frac{-4\frac{v_o}{E_i}}{\left(1+\frac{v_o}{E_i}\right)^2}$
duty cycle all with boundary $\frac{\bar{I}_o}{\hat{I}_o}=4\delta(1-\delta)$	$\delta=½\sqrt{\frac{\frac{\bar{I}_o}{\hat{I}_o}\times\frac{v_o}{E_i}}{1-\frac{v_o}{E_i}}}$	$\delta=½\sqrt{\frac{\bar{I}_o}{\hat{I}_o}\times\left(\frac{v_o}{E_i}-1\right)}$	$\delta=½\sqrt{\frac{\bar{I}_o}{\hat{I}_o}\times\frac{v_o}{E_i}}$

Table 15.6. Transfer functions with constant input voltage, E_i, with respect to $\bar{I}_i$

v_o constant	converter step-down	converter step-up	converter step-up/down
reference equation	(15.4)	(15.44)	(15.74)
current conduction	$\frac{v_o}{E_i}=\delta$	$\frac{v_o}{E_i}=\frac{1}{1-\delta}$	$\frac{v_o}{E_i}=\frac{-\delta}{1-\delta}$
reference equation	(15.20)	(15.60)	(15.91)
equation	$\frac{v_o}{E_i}=1-\frac{2L\bar{I}_i}{\delta^2\tau E_i}$	$\frac{v_o}{E_i}=\frac{1}{1-\frac{E_i t_T\delta^2}{2L\bar{I}_i}}$	$\frac{v_o}{E_i}=\frac{v_o\tau\delta^2}{2L\bar{I}_i}$
normalised	$\frac{v_o}{E_i}=1-\frac{4}{27\delta^2}\times\frac{\bar{I}_i}{\hat{I}_i}$	$\frac{v_o}{E_i}=\frac{1}{1-\delta^2/\left(\frac{\bar{I}_i}{\hat{I}_i}\right)}$	$1=\delta^2/\frac{\bar{I}_i}{\hat{I}_i}$
$\hat{I}_i$ max @ δ =	$\delta = ⅔$	$\delta = 1$	$\delta = 1$
$\hat{I}_i$	$\hat{I}_i=\frac{4}{27}\times\frac{E_i\tau}{2L}$	$\hat{I}_i=\frac{E_i\tau}{2L}$	$\hat{I}_i=\frac{E_i\tau}{2L}$
change of variable	$\frac{\bar{I}_i}{\hat{I}_i}=27/4\,\delta^2\left(1-\frac{v_o}{E_i}\right)$	$\frac{\bar{I}_i}{\hat{I}_i}=\frac{\delta^2\frac{v_o}{E_i}}{\left(\frac{v_o}{E_i}-1\right)}$	$\frac{\bar{I}_i}{\hat{I}_i}=-\delta^2$
boundary	$\frac{\bar{I}_i}{\hat{I}_i}=27/4\left(1-\frac{v_o}{E_i}\right)\left(\frac{v_o}{E_i}\right)^2$	$\frac{\bar{I}_i}{\hat{I}_i}=\frac{\left(\frac{v_o}{E_i}-1\right)}{\frac{v_o}{E_i}}$	$\frac{\bar{I}_i}{\hat{I}_i}=-\left(\frac{\frac{v_o}{E_i}}{\frac{v_o}{E_i}+1}\right)^2$
duty cycle	$\delta=\sqrt{4/27\,\frac{\frac{\bar{I}_i}{\hat{I}_i}}{1-\frac{v_o}{E_i}}}$	$\delta=\sqrt{\frac{\bar{I}_i}{\hat{I}_i}\times\frac{\frac{v_o}{E_i}-1}{\frac{v_o}{E_i}}}$	$\delta=\sqrt{\frac{\bar{I}_i}{\hat{I}_i}}$
boundary	$\frac{\bar{I}_i}{\hat{I}_i}=27/4\,\delta^2(1-\delta)$	$\frac{\bar{I}_i}{\hat{I}_i}=\frac{\delta}{(1-\delta)^2}$	$\delta=\sqrt{\frac{\bar{I}_i}{\hat{I}_i}}$

Table 15.7. Transfer functions with constant output voltage, v_o, with respect to $\bar{I}_o$

v_o constant	converter		
	step-down	step-up	step-up/down
reference equation	(15.4)	(15.44)	(15.74)
current conduction	$\dfrac{v_o}{E_i} = \delta$	$\dfrac{v_o}{E_i} = \dfrac{1}{1-\delta}$	$\dfrac{v_o}{E_i} = \dfrac{-\delta}{1-\delta}$
reference equation	(15.20)	(15.60)	(15.91)
equation	$\dfrac{v_o}{E_i} = 1 - \dfrac{2L\bar{I}_i}{\delta^2 \tau E_i}$	$\dfrac{v_o}{E_i} = \dfrac{1}{1 - \dfrac{E_i t_T \delta^2}{2L\bar{I}_i}}$	$\dfrac{v_o}{E_i} = \dfrac{v_o \tau \delta^2}{2L\bar{I}_i}$
normalised	$\dfrac{v_o}{E_i} = 1 - \dfrac{1}{4\delta^2} \times \dfrac{\bar{I}_o}{\hat{I}_o} \times \left(\dfrac{v_o}{E_i}\right)^2$	$\dfrac{v_o}{E_i} = \dfrac{1}{1 - 4\delta^2 / \left(\dfrac{\bar{I}_o}{\hat{I}_o} \times \left(\dfrac{v_o}{E_i}\right)^2\right)}$	$\dfrac{v_o}{E_i} = \delta^2 / \left(\dfrac{\bar{I}_o}{\hat{I}_o} \times \dfrac{v_o}{E_i}\right)$
$\hat{I}_o$ max @ δ =	$\delta = 0$	$\delta = ⅓$	$\delta = 0$
$\hat{I}_o$	$\hat{I}_o = \dfrac{v_o \tau}{2L}$	$\hat{I}_o = \dfrac{4}{27} \times \dfrac{v_o \tau}{2L}$	$\hat{I}_o = \dfrac{v_o \tau}{2L}$
change of variable	$\dfrac{\bar{I}_o}{\hat{I}_o} = \dfrac{\delta^2 \left(1 - \dfrac{v_o}{E_i}\right)}{\left(\dfrac{v_o}{E_i}\right)^2}$	$\dfrac{\bar{I}_o}{\hat{I}_o} = \dfrac{{}^{27}\!/_4\, \delta^2}{\left(\dfrac{v_o}{E_i} - 1\right)\dfrac{v_o}{E_i}}$	$\dfrac{\bar{I}_o}{\hat{I}_o} = \dfrac{-\delta^2}{\left(\dfrac{v_o}{E_i}\right)^2}$
boundary	$\dfrac{\bar{I}_o}{\hat{I}_o} = 1 - \dfrac{v_o}{E_i}$	$\dfrac{\bar{I}_o}{\hat{I}_o} = \dfrac{{}^{27}\!/_4 \left(\dfrac{v_o}{E_i} - 1\right)}{\left(\dfrac{v_o}{E_i}\right)^3}$	$\dfrac{\bar{I}_o}{\hat{I}_o} = \dfrac{-1}{\left(1 + \dfrac{v_o}{E_i}\right)^2}$
duty cycle	$\delta = \dfrac{v_o}{E_i} \sqrt{\dfrac{\dfrac{\bar{I}_o}{\hat{I}_o}}{1 - \dfrac{v_o}{E_i}}}$	$\delta = \sqrt{{}^{4}\!/_{27} \times \dfrac{\bar{I}_o}{\hat{I}_o} \left(\dfrac{v_o}{E_i} - 1\right) \dfrac{v_o}{E_i}}$	$\delta = \dfrac{v_o}{E_i} \sqrt{\dfrac{\bar{I}_o}{\hat{I}_o}}$
boundary	$\delta = 1 - \dfrac{\bar{I}_o}{\hat{I}_o}$	$\dfrac{\bar{I}_o}{\hat{I}_o} = {}^{27}\!/_4\, \delta (1 - \delta)^2$	$\delta = 1 - \sqrt{\dfrac{\bar{I}_o}{\hat{I}_o}}$

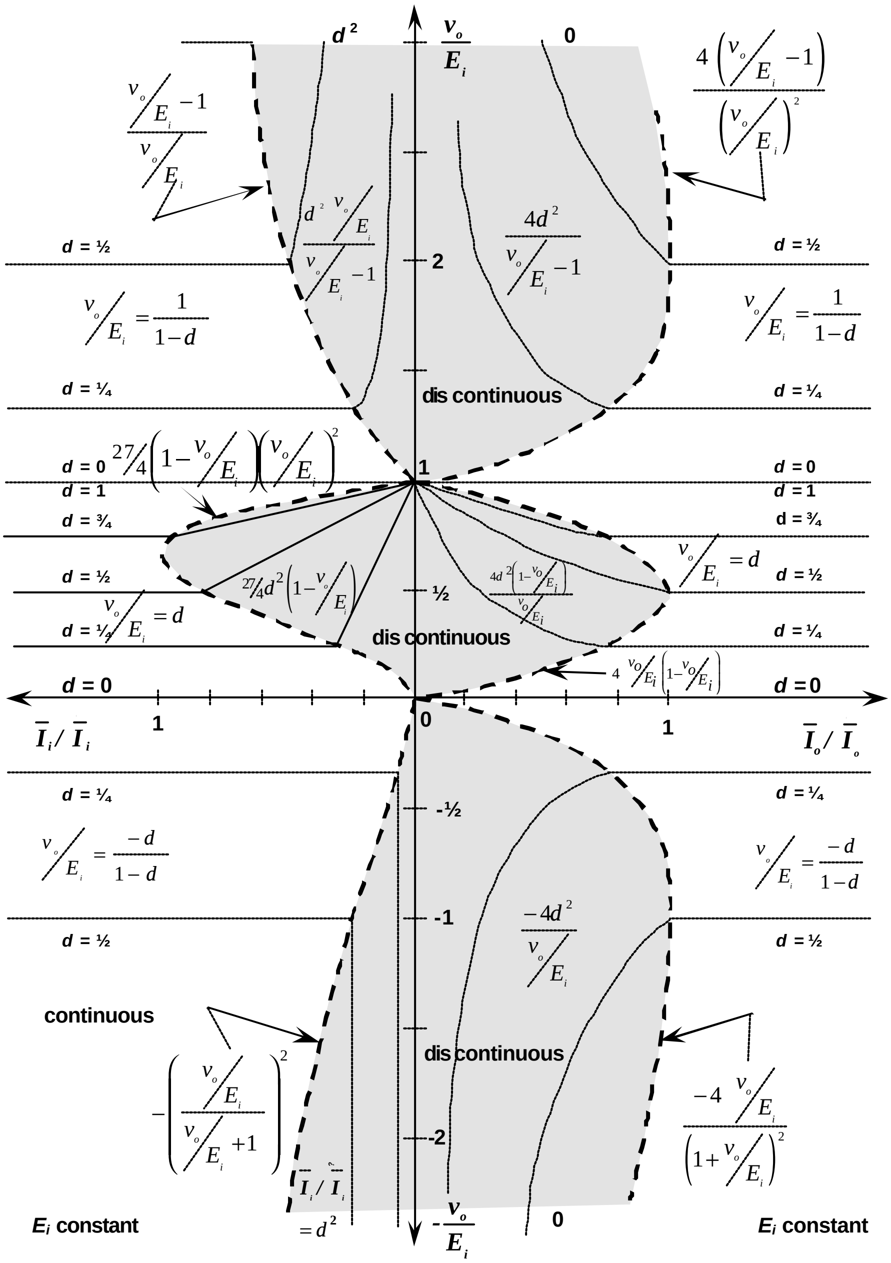

Figure 15.26. *Characteristics for three dc-dc converters, when the input voltage E_i is held constant.*

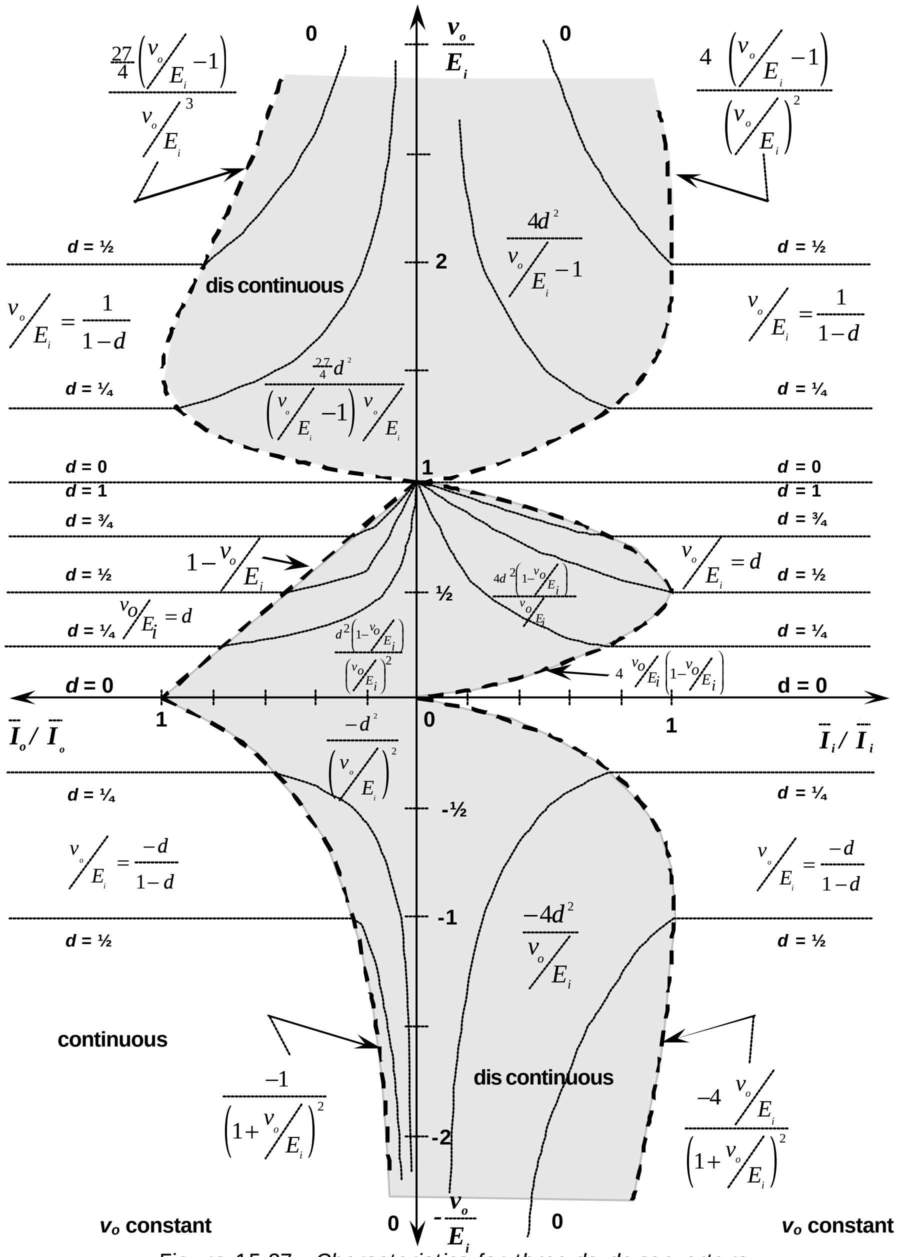

Figure 15.27. *Characteristics for three dc-dc converters, when the output voltage v_o is held constant.*

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Problems

15.1. An smps is used to provide a 5V rail at 2.5A. If 100 mV p-p output ripple is allowed and the input voltage is 12V with 25 per cent tolerance, design a flyback buck-boost converter which has a maximum switching fre quency of 50 kHz.

15.2. Derive the following design equations for a flyback boost converter, which operates in the discontinuous mode.

$$\hat{i}_i = 2\times\overline{I}_{o\,(\max)}\times\frac{v_o}{E_{i\,(\min)}} = \text{ constant} \qquad t_D = \frac{1}{f_{(\max)}\dfrac{v_o}{E_{i\,(\min)}}}$$

$$L = t_{T\,(\min)}\frac{v_o - E_{i(\min)}}{\hat{i}_i} \qquad f = \frac{1}{t} = f_{(\max)}\frac{\overline{I}_o}{\overline{I}_{o\,(\max)}}\times\frac{v_o - E_i}{v_o - E_{i(\min)}}$$

$$C_{(\min)} = \frac{\Delta Q}{\Delta e_o} = \frac{\hat{i}_i\, t_{T\,(\min)}}{2\Delta e_o} \qquad ESR_{(\max)} = \frac{\Delta e_o}{\hat{i}_i}$$

15.3. Derive design equations for the forward non-isolated converter, operating in the continuous conduction mode.

15.4. Prove that the output rms ripple current for the forward converter in figure 15.2 is given by $\Delta i_o / 2\sqrt{3}$.