

# 11

## Naturally Commutating Converters

The converter circuits considered in this chapter have in common an ac supply input and a dc load. The function of the converter circuit is to convert the ac source into controlled dc load power, mainly for high inductive loads. Turn-off of converter semiconductor devices is brought about by the ac supply reversal, a process called *line commutation* or *natural commutation*.

Converter circuits employing only diodes are termed *uncontrolled* while the incorporation of only thyristors results in a *controlled converter*. The functional difference is that the diode conducts when forward-biased whereas the turn-on of the forward-biased thyristor can be controlled from the gate. An uncontrolled converter provides a fixed output voltage for a given ac supply.

Converter circuits employing a combination of both diodes and thyristors are generally termed *half-controlled*. Both fully controlled and half-controlled converters allow an adjustable output voltage by controlling the phase angle at which the forward biased thyristors are turned on. The polarity of the load voltage of a fully controlled converter can reverse, allowing power flow into the supply, a process called *inversion*. Thus a fully controlled converter can be described as a *bidirectional converter* as it facilitates power flow in either direction.

The half-controlled converter, as well as the uncontrolled converter, contains diodes which prevent the output voltage from going negative. Such converters only allow power flow from the supply to the load, termed *rectification*, and can therefore be described as *unidirectional converters*.

Although all these converter types provide a dc output, they differ in characteristics such as output ripple and mean voltage as well as efficiency and supply harmonics. Another converter characteristic is that of pulse number, which is defined as the repetition rate in the direct output voltage during one complete cycle of the input ac supply.

The general analysis in this chapter is concerned with single and three-phase supplies feeding inductive loads. A load back emf is used in modelling the dc machine.

### 11.1 Single-phase uncontrolled converter circuits

#### 11.1.1 Half-wave circuit with an R-L load

A simple half-wave diode rectifying circuit is shown in figure 11.1a, while various circuit electrical waveforms are shown in figure 11.1b. Load current starts to flow when the supply goes positive at  $\omega t = 0$ . It will be seen that load current flows not only during the positive part of the supply voltage,  $0 \leq \omega t \leq \pi$ , but also during a portion of the negative supply voltage,  $\pi \leq \omega t \leq \beta$ . The load inductor stored energy maintains the load current and the inductor's terminal voltage reverses so as to overcome the negative supply and keep the diode forward-biased and conducting. This current continues until all the inductor energy,  $\frac{1}{2}Li^2$ , is released ( $i = 0$ ) at the *current extinction angle*,  $\omega t = \beta$ .

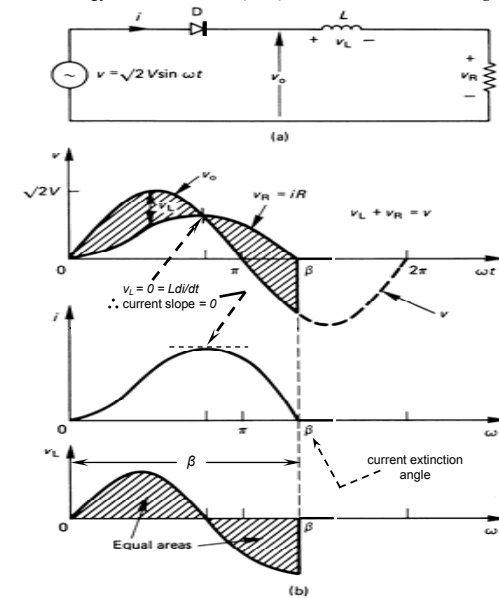


Figure 11.1. Half-wave rectifier with an R-L load: (a) circuit diagram and (b) waveforms, illustrating the equal area and zero current slope criteria.

During diode conduction the circuit is defined by the Kirchhoff voltage equation

$$L \frac{di}{dt} + Ri = \sqrt{2} V \sin \omega t \quad (\text{V}) \quad (11.1)$$

where  $V$  is the rms ac supply voltage. Solving equation (11.1) yields the load current

$$i(\omega t) = \frac{\sqrt{2}V}{Z} \left\{ \sin(\omega t - \phi) + \sin \phi e^{-\omega t / \tan \phi} \right\} \quad (\text{A}) \quad (11.2)$$

$$0 \leq \omega t \leq \beta \quad (\text{rad})$$

$$\text{where } Z = \sqrt{R^2 + \omega^2 L^2} \quad (\text{ohms})$$

$$\tan \phi = \omega L / R$$

$$i(\omega t) = 0 \quad (\text{A}) \quad (11.3)$$

$$\beta \leq \omega t \leq 2\pi \quad (\text{rad})$$

The current extinction angle  $\beta$  is determined by the load impedance  $Z$  and can be solved from equation (11.2) when  $i = 0$  with  $\omega t = \beta$ , such that  $\beta > 0$ , that is

$$\sin(\beta - \phi) + \sin \phi e^{-\beta / \tan \phi} = 0 \quad (11.4)$$

This is a transcendental equation which can be solved by iterative techniques. Figure 11.2a can be used to determine the extinction angle  $\beta$ , given any load impedance angle  $\phi = \tan^{-1} \omega L / R$ .

The mean value of the rectified current,  $\bar{I}_o$ , is given by integration of equation (11.2)

$$\bar{I}_o = \frac{1}{2\pi} \int_0^\beta i(\omega t) d\omega t \quad (\text{A}) \quad (11.5)$$

$$\bar{I}_o = \frac{\sqrt{2}V}{2\pi R} (1 - \cos \beta) \quad (\text{A})$$

while the mean output voltage  $V_o$  is given by

$$V_o = \bar{I}_o R = \frac{\sqrt{2}V}{2\pi} (1 - \cos \beta) \quad (\text{V}) \quad (11.6)$$

since the mean voltage across the load inductance is zero (see the equal area criterion below). Figure 11.2b shows the normalised output voltage  $V_o / V$  as a function of  $\omega L / R$ . The rms output voltage is given by

$$V_{rms} = \left[ \frac{1}{2\pi} \int_0^\beta (\sqrt{2}V)^2 \sin^2 \omega t d\omega t \right]^{1/2} \quad (11.7)$$

$$= \sqrt{2} V \left[ \frac{1}{2\pi} \left\{ \beta - \frac{1}{2} \sin 2\beta \right\} \right]^{1/2}$$

### 11.1.1i - Equal area criterion

The average output voltage  $V_o$ , given by equation (11.6), is based on the fact that the average voltage across the load inductance, in steady state, is zero. The inductor voltage is given by

$$v_L = L di / dt \quad (\text{V})$$

which for the circuit in figure 11.1a can be expressed as

$$\int_0^{\beta/\omega} v_L(t) dt = \int_{i_o}^{i_\beta} L di = L(i_\beta - i_o) \quad (11.8)$$

If the load current is in steady state then  $i_\beta = i_o$ , which is zero here, and in general

$$\int v_L dt = 0 \quad (\text{Vs}) \quad (11.9)$$

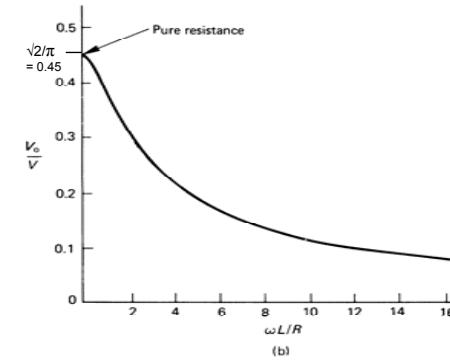
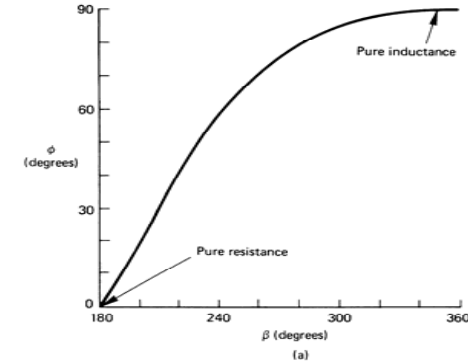


Figure 11.2. Single-phase half-wave converter characteristics: (a) load impedance angle  $\phi$  versus current extinction angle  $\beta$  and (b) variation in normalised mean output voltage  $V_o / V$  versus  $\omega L / R$ .

The inductor voltage waveform for the circuit in figure 11.1a is shown in the last plot in figure 11.1b. The equal area criterion implies that the shaded positive area must equal the shaded negative area, in order to satisfy equation (11.9). The net inductor energy is zero. This is a useful aid in predicting and drawing the load current waveform.

It is useful to superimpose the supply voltage  $v$ , the load voltage  $v_o$ , and the resistor voltage  $v_R$  waveforms on the same time axis,  $\omega t$ . The load resistor voltage,  $v_R = Ri$ , is directly related to the load current,  $i$ . The inductor voltage  $v_L$  will be the difference between the load voltage and the resistor voltage and this bounded area must be zero. The equal voltage areas associated with the load inductance are shown shaded in two plots in figure 11.1b.

### 11.1.1ii - Load current zero slope criterion

The load inductance voltage polarity changes from positive to negative as energy initially transferred into the inductor, is released. The stored energy in the inductor allows current to flow after the input ac voltage has reversed. At the instant when the inductor voltage reverses, its terminal voltage is zero, that is

$$v_L = L di/dt = 0$$

that is  $di/dt = 0$

(11.10)

The current slope changes from positive to negative, whence the voltage across the load resistance ceases to increase and starts to decrease, as shown in figure 11.1b. That is, the  $Ri$  waveform crosses the supply voltage waveform with zero slope, whence when the inductor voltage is zero, the current begins to decrease. The fact that the resistor voltage slope is zero when  $v_L=0$ , aids prediction and sketching of the various circuit waveforms in figure 11.1b, and subsequent waveforms in this chapter.

### 11.1.2 Half-wave circuit with an R-L load and freewheel diode

The circuit in figure 11.1a, which has an R-L load, is characterised by discontinuous ( $i = 0$ ) and high ripple current. Continuous load current can result when a diode is added across the load as shown in figure 11.3a. This freewheel diode prevents the voltage across the load from reversing during the negative half-cycle of the supply voltage. The stored energy in the inductor cannot reduce to zero instantaneously, so the current is forced to find an alternative path whilst decreasing towards zero. When diode  $D_1$  ceases to conduct at zero volts it blocks, and diode  $D_f$  provides an alternative load current freewheeling path, as indicated by the waveforms in figure 11.3b.

The output voltage is the positive half of the sinusoidal input voltage. The mean output voltage is

$$V_o = \frac{1}{2\pi} \int_0^\pi \sqrt{2}V \sin \omega t d\omega t$$

$$V_o = \sqrt{2}V / \pi = 0.45V \quad (V) \quad (11.11)$$

### Naturally commutating converters

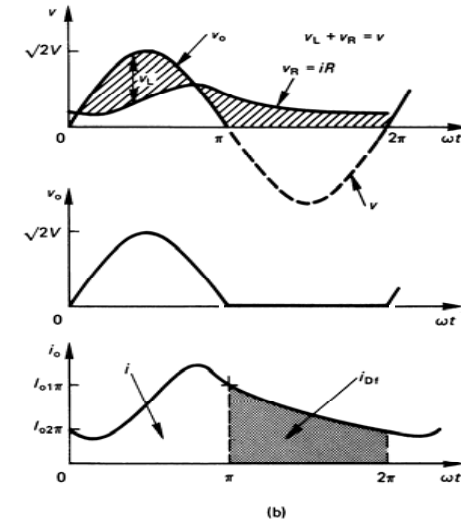
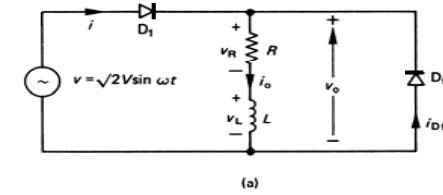


Figure 11.3. Half-wave rectifier with a freewheel diode and an R-L load: (a) circuit diagram and parameters and (b) circuit waveforms.

The rms value of the load circuit voltage  $v_o$  is given by

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\pi (\sqrt{2}V \sin \omega t)^2 d\omega t}$$

$$= \sqrt{2}V / 2 = 0.71V \quad (V) \quad (11.12)$$

The output ripple voltage is defined as

$$V_{Rl} \triangleq \sqrt{V_{rms}^2 - V_o^2} = \sqrt{\left(\frac{\sqrt{2}V}{2}\right)^2 - \left(\frac{\sqrt{2}V}{\pi}\right)^2} = 0.545V \quad (11.13)$$

hence the voltage ripple factor is defined as

$$K_v \triangleq V_{Rl} / V_o = \sqrt{\left(\frac{V_{rms}}{V_o}\right)^2 - 1} = \sqrt{1/4\pi^2 - 1} = 1.211 \quad (11.14)$$

After a large number of ac supply cycles, steady-state load current conditions are established, and the load current is defined by

$$L \frac{di}{dt} + Ri = \sqrt{2}V \sin \omega t \quad (\text{A}) \quad 0 \leq \omega t \leq \pi \quad (11.15)$$

and when the freewheel diode conducts

$$L \frac{di}{dt} + Ri = 0 \quad (\text{A}) \quad \pi \leq \omega t \leq 2\pi \quad (11.16)$$

During the period  $0 \leq \omega t \leq \pi$ , when the freewheel diode current is given by  $i_{Df} = 0$ , the supply current and load current are given by

$$i(\omega t) = i_o(\omega t) = \sqrt{2}V/Z \sin(\omega t - \phi) + (I_{o2\pi} + \sqrt{2}V/Z \sin \phi) e^{-\omega t / \tan \phi} \quad (\text{A}) \quad (11.17)$$

$$0 \leq \omega t \leq \pi$$

for

$$I_{o2\pi} = \sqrt{2}V/Z \sin \phi \frac{1 + e^{-\pi / \tan \phi}}{e^{\pi / \tan \phi} - e^{-\pi / \tan \phi}} \quad (\text{A})$$

$$\text{where } Z = \sqrt{R^2 + (\omega L)^2} \quad (\text{ohms})$$

$$\tan \phi = \omega L / R$$

During the period  $\pi \leq \omega t \leq 2\pi$ , when the supply current  $i = 0$ , the diode current and hence load current is given by

$$i_o(\omega t) = i_{Df}(\omega t) = I_{o1\pi} e^{-(\omega t - \pi) / \tan \phi} \quad (\text{A}) \quad \pi \leq \omega t \leq 2\pi \quad (11.18)$$

for

$$I_{o1\pi} = I_{o2\pi} e^{\pi / \tan \phi} \quad (\text{A})$$

In figure 11.3b it will be seen that although the load current is continuous, the supply current is discontinuous and therefore has a high harmonic content.

The output voltage Fourier series is

$$v(t) = \frac{\sqrt{2}V}{\pi} + \frac{\sqrt{2}V}{2} \sin(\omega t) - \sum_{n=2,4,6}^{\infty} \frac{2\sqrt{2}V}{(n^2 - 1)\pi} \cos(n\omega t) \quad (11.19)$$

By dividing each component by the corresponding load impedance at that frequency

gives the harmonic current, whence rms current. That is

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega L|} = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}} \quad (11.20)$$

and

$$I_{rms} = \sqrt{I_o^2 + \sum_{n=1,2,4,6,\dots} 1/2 I_n^2} \quad (11.21)$$

### Example 11.1: Half wave rectifier

In the circuit of figure 11.3 the source voltage is  $240\sqrt{2} \sin(2\pi 50t)$  V,  $R = 10$  ohms, and  $L = 50$  mH. Calculate

- the mean and rms values of the load voltage,  $V_o$  and  $V_{rms}$
- the mean value of the load current,  $\bar{I}_o$
- the current boundary conditions, namely  $I_{o1\pi}$  and  $I_{o2\pi}$
- the average freewheel diode current, hence average rectifier diode current
- the rms load current, hence load power
- the power factor

### Solution

- i. From equation (11.11), the mean output voltage is given by

$$V_o = \sqrt{2}V/\pi = \sqrt{2} \times 240/\pi = 108\text{V}$$

From equation (11.12) the load rms voltage is

$$V_{rms} = \sqrt{2}V/2 = 240/\sqrt{2} = 169.7\text{V}$$

- ii. The mean output current, equation (11.5), is

$$\bar{I}_o = V_o/R = \sqrt{2}V/\pi R = \sqrt{2} \times 240/\pi \times 10 = 10.8\text{A}$$

- iii. The load impedance is characterised by

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi \times 50 \times 0.05)^2} = 18.62\text{ohms}$$

$$\tan \phi = \omega L / R$$

$$= 2\pi \times 50 \times 0.05 / 10 = 1.57\text{rad or } \phi = 57.5^\circ$$

From section 11.1.2, equation (11.17)

$$I_{o2\pi} = \frac{\sqrt{2}V}{Z} \sin \phi \frac{1 + e^{-\pi/\tan \phi}}{e^{\pi/\tan \phi} - e^{-\pi/\tan \phi}}$$

$$I_{o2\pi} = \frac{\sqrt{2} \times 240}{18.62} \times \sin(\tan^{-1} 1.57) \times \frac{1 + e^{-\pi/1.57}}{e^{\pi/1.57} - e^{-\pi/1.57}} = 3.41 \text{ A}$$

Hence, from equation (11.18)

$$I_{o1\pi} = I_{o2\pi} e^{\pi/\tan \phi} = 3.41 \times e^{\pi/1.57} = 25.22 \text{ A}$$

iv. Integration of the diode current given in equation (11.18) yields the average diode current.

$$\begin{aligned} \bar{I}_{Df} &= \frac{1}{2\pi} \int_0^{\pi} i_{Df}(\omega t) d\omega t = \frac{1}{2\pi} \int_0^{\pi} I_{o1\pi} e^{-\omega t / \tan \phi} d\omega t \\ &= \frac{1}{2\pi} \int_0^{\pi} 25.22 \text{ A} \times e^{-\omega t / 1.57 \text{ rad}} d\omega t = \frac{25.22 \text{ A}}{2\pi} \times 1.57 \text{ rad} \times \left[ 1 - e^{-\frac{\pi}{1.57}} \right] = 5.46 \text{ A} \end{aligned}$$

The average input current, which is the rectifying diode mean current, is given by

$$\begin{aligned} \bar{I}_{D1} &= \bar{I}_o - \bar{I}_{Df} \\ &= 10.8 \text{ A} - 5.46 \text{ A} = 5.34 \text{ A} \end{aligned}$$

iv. The load voltage harmonics given by equation (11.20) can be used to evaluate the load current at the load impedance for that frequency harmonic.

$$v(t) = \frac{\sqrt{2}V}{\pi} + \frac{\sqrt{2}V}{2} \sin(\omega t) - \sum_{n=2,4,6}^{\infty} \frac{2\sqrt{2}V}{(n^2-1)\pi} \cos(n\omega t)$$

The following table shows the calculations for each frequency component.

harmonic n	$V_n = \frac{2\sqrt{2}V}{(n^2-1)\pi}$	$Z_n = \sqrt{R^2 + (n\omega L)^2}$	$I_n = V_n / Z_n$	$\frac{1}{2} I_n^2$
0	<b>108.04</b>	10.00	10.80	<b>116.72</b>
1	<b>169.71</b>	18.62	9.11	41.53
2	72.03	32.97	2.18	2.39
4	14.41	63.62	0.23	0.03
6	6.17	94.78	0.07	0.00
8	3.43	126.06	0.03	0.00
			$I_o^2 + \sum \frac{1}{2} I_n^2 =$	<b>160.67</b>

The rms load current is

$$I_{rms} = \sqrt{I_o^2 + \sum_{n=1}^{\infty} \frac{1}{2} I_n^2} = \sqrt{160.7} = 12.68 \text{ A}$$

The power dissipated in the load is therefore

$$P_{10\Omega} = I_{rms}^2 R = 12.68 \text{ A}^2 \times 10 \Omega = 1606.7 \text{ W}$$

v. The diode rms current is

$$\begin{aligned} I_{D1} &= \sqrt{\frac{1}{2\pi} \int_0^{\pi} (I_{o1\pi} e^{-(\omega t)/\tan \phi})^2 d\omega t} \\ &= \sqrt{\frac{1}{2\pi} \int_0^{\pi} (25.22 \text{ A} \times e^{-(\omega t)/1.57 \text{ rad}})^2 d\omega t} = 8.83 \text{ A} \end{aligned}$$

Thus the input rms current is given by

$$\begin{aligned} I_{rms} &= \sqrt{I_{o1\pi}^2 - I_{Df}^2} \\ &= \sqrt{12.68^2 - 8.83^2} = 9.09 \text{ A} \end{aligned}$$

The input power factor is

$$pf = \frac{P_{out}}{V_{rms} I_{rms}} = \frac{1606.7 \text{ W}}{240 \text{ V} \times 9.09 \text{ A}} = 0.74$$



### 11.1.3 Full-wave bridge circuit

Single-phase uncontrolled full-wave bridge circuits are shown in figures 11.4a and 11.4b. Figures 11.4a and b appear identical as far as the load is concerned. It will be seen in part b that two fewer diodes can be employed but this requires a centre-tapped secondary transformer where each secondary has only a 50% copper utilisation factor. For the same output voltage, each of the secondary windings in figure 11.4b must have the same rms voltage rating as the single secondary winding of the transformer in figure 11.4a. The rectifying diodes in figure 11.4b experience twice the reverse voltage, as that experienced by each of the four diodes in the circuit of figure 11.4a. The use of a centre tapped transformer secondary, halves the copper utilisation factor.

Figure 11.4c shows bridge circuit voltage and current waveforms. The load experiences the transformer secondary rectified voltage which has a mean voltage of

$$\begin{aligned} V_o &= \frac{1}{\pi} \int_0^{\pi} \sqrt{2} V \sin \omega t d\omega t \\ V_o = \bar{I}_o R &= \frac{2\sqrt{2}V}{\pi} = 0.90V \quad (\text{V}) \end{aligned} \quad (11.22)$$

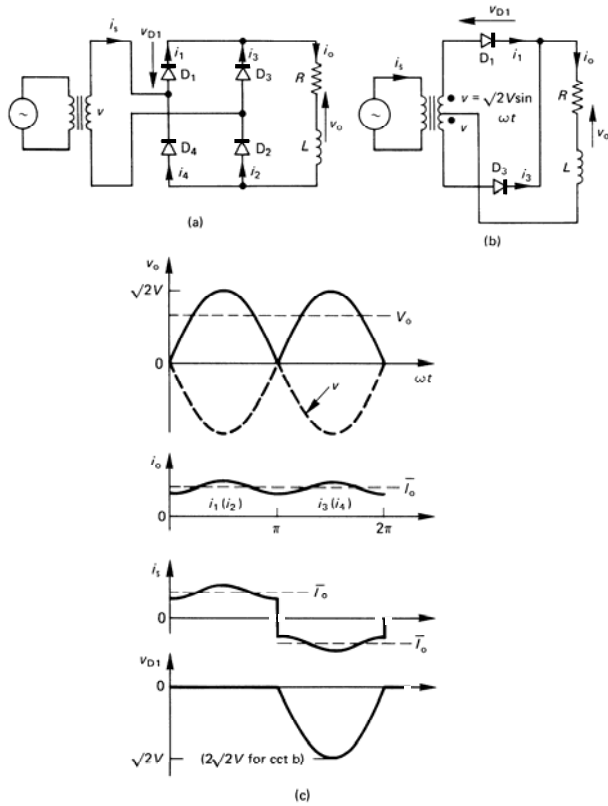


Figure 11.4. Single-phase full-wave rectifier bridge: (a) circuit with four rectifying diodes; (b) circuit with two rectifying diodes; and (c) circuit waveforms.

The rms value of the load circuit voltage  $v_o$  is

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (\sqrt{2}V \sin \omega t)^2 d\omega t} \quad (11.23)$$

$$V_{rms} = V \quad (V)$$

The ripple voltage is

$$V_{Rl} \triangleq \sqrt{V_{rms}^2 - V_o^2} = \sqrt{V^2 - (2\sqrt{2}/\pi)^2 V^2} = 0.435V \quad (V) \quad (11.24)$$

hence the voltage ripple factor is

$$K_v \triangleq V_{Rl} / V_o = \sqrt{1 - (2\sqrt{2}/\pi)^2} / (2\sqrt{2}/\pi) = 0.483 \quad (11.25)$$

which is significantly less than the half-wave rectified value of 1.211 from equation (11.14).

The output voltage in Fourier expansion form is given by

$$v_o(\omega t) = \frac{2\sqrt{2}V}{\pi} + \sum_{n=2,4,6} \frac{2\sqrt{2}V}{\pi} \times \frac{2}{n^2-1} \cos(n\omega t) \quad (11.26)$$

The output current can be derived by dividing each voltage component by the appropriate load impedance at that frequency. That is

$$I_o = \frac{V_o}{R} = \frac{2\sqrt{2}V}{\pi R} \quad (11.27)$$

$$I_n = \frac{V_n}{Z_n} = \frac{2\sqrt{2}V}{\pi} \times \frac{2}{\sqrt{R^2 + (n\omega L)^2}} \quad \text{for } n = 2, 4, 6..$$

The load rms current whence load power, are given by

$$I_{rms} = \sqrt{I_o^2 + \sum_{n=2,4,6} \frac{1}{2} I_n^2} \quad (11.28)$$

$$P_L = I_{rms}^2 R$$

With a highly inductive load, which is the usual practical case, virtually constant load current flows, as shown dashed in figure 11.4c. The bridge diode currents are then square wave blocks of current of magnitude  $\bar{I}_o$ . The diode current ratings can now be specified and depend on the pulse number  $n$ . For this full-wave single-phase application each input cycle comprises two output current pulses, hence  $n = 2$ .

The mean current in each diode is

$$\bar{I}_D = \frac{1}{2} \bar{I}_o = \frac{1}{2} \bar{I}_o \quad (A) \quad (11.29)$$

and the rms current in each diode is

$$I_D = \frac{1}{\sqrt{2}} \bar{I}_o = \bar{I}_o / \sqrt{2} \quad (A) \quad (11.30)$$

whence the diode current form factor is

$$K_{Id} = I_D / \bar{I}_D = \sqrt{n} = \sqrt{2} \quad (11.31)$$

Since the current is approximately constant, power delivered to the load is

$$P_o \approx V_o \bar{I}_o = \frac{8\sqrt{2}}{\pi^2} \times V^2 / R \quad (\text{W}) \quad (11.32)$$

## 11.2 Single-phase full-wave half-controlled converter

When a converter contains both diodes and thyristors, for example as shown in figure 11.5 parts a, b, and c, the converter is termed half-controlled. These three circuits produce identical load waveforms neglecting any differences in the number and type of semiconductor voltage drops. The power to the load is varied by controlling the angle  $\alpha$ , shown in figure 11.5d, at which the bridge thyristors are triggered. The circuit diodes prevent the load voltage from going negative, extend the conduction period, and reduce the ac ripple.

The particular application will determine which one of the three circuits should be employed. For example, circuit figure 11.5a contains five devices of which four are thyristors, whereas the other two circuits contain four devices, of which only two are thyristors. The thyristor triggering requirements of the circuit in figure 11.5b are simple since both thyristors have a common cathode connection.

Figure 11.5b may suffer from prolonged shut-down times with highly inductive loads. The diode in the freewheeling path will hold on the freewheeling thyristor, allowing conduction during that thyristors next positive cycle without any gate drive present. This does not occur in circuits 11.5a and c since freewheeling does not occur through the circuit thyristors, hence they will drop out of conduction at converter shut-down. The table in figure 11.5d shows which semiconductors are active in each circuit during the various periods of the load cycle.

Various circuit waveforms are shown in figure 11.5d.

The mean output voltage and current are

$$\begin{aligned} V_o &= \bar{I}_o R = \frac{1}{\pi} \int_{\alpha}^{\pi} \sqrt{2} V \sin(\omega t) d\omega t \\ &= \frac{\sqrt{2} V}{\pi} (1 + \cos \alpha) \quad (\text{V}) \end{aligned} \quad (11.33)$$

$$\bar{I}_o = V_o / R = \frac{\sqrt{2} V}{\pi R} (1 + \cos \alpha) \quad (\text{A})$$

where  $\alpha$  is the delay angle from the point at which the associated thyristor first becomes forward-biased and is therefore able to be turned on. The maximum mean output voltage,  $\hat{V}_o = 2\sqrt{2}V/\pi$  (also predicted by equation 11.16), occurs at  $\alpha = 0$ .

The normalised mean output voltage  $V_n$  is

$$V_n = V_o / \hat{V}_o = \frac{1}{2}(1 + \cos \alpha) \quad (11.34)$$

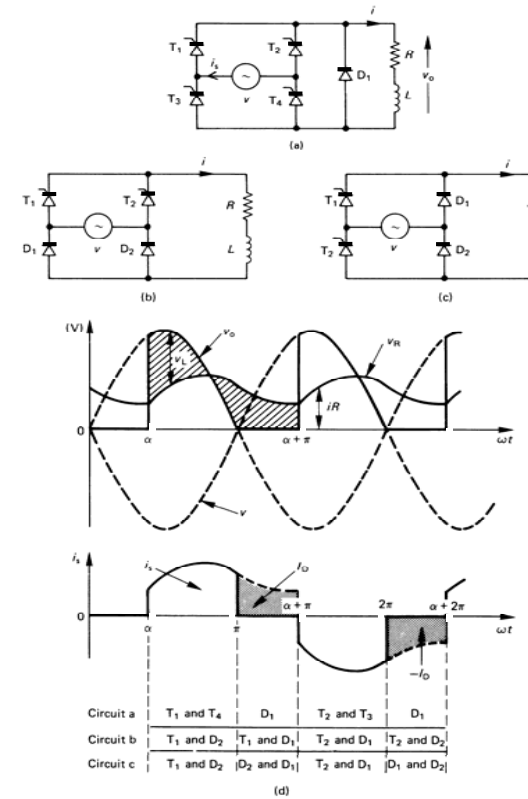


Figure 11.5. Full-wave half-controlled converter with freewheel diodes: (a), (b) and (c) different circuit configurations producing the same output; and (d) circuit voltage and current waveforms and device conduction table.

Equation (11.33) shows that the load voltage is independent of the load (because the diodes clamp the load to zero volts thereby preventing the load from going negative), and is a function only of the phase delay angle for a given supply voltage.

The rms value of the load circuit voltage  $v_o$  is

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} (\sqrt{2}V \sin \omega t)^2 d\omega t} \quad (11.35)$$

$$= V \sqrt{\frac{\pi - \alpha + \frac{1}{2} \sin 2\alpha}{\pi}} \quad (V)$$

Equations (11.33) and (11.35) can be used to evaluate the load ripple voltage, defined by equation (11.13), and load voltage ripple factor, defined by equations (11.14).

**11.2i - Discontinuous load current**, with  $\alpha < \pi$  and  $\beta - \alpha < \pi$ , the load current (and supply current) is based on equation (11.1) which gives

$$i(\omega t) = i_s(\omega t) = \sqrt{2}V/Z \left( \sin(\omega t - \phi) - \sin(\alpha - \phi) e^{-\omega t / \tan \phi} \right) \quad (A) \quad (11.36)$$

$$\alpha \leq \omega t \leq \pi$$

After  $\omega t = \pi$  the load current decreases exponentially to zero through the freewheel diode according to

$$i(\omega t) = i_{Df}(\omega t) = I_{01x} e^{-\omega t / \tan \phi} \quad (A) \quad (11.37)$$

$$0 \leq \omega t \leq \alpha$$

where for  $\omega t = \pi$  in equation (11.36)

$$I_{01x} = \sqrt{2}V/Z \sin(\phi - \alpha)(1 - e^{-\pi / \tan \phi})$$

**11.2ii - Continuous load current**, with  $\alpha < \phi$  and  $\beta - \alpha \geq \pi$ , the load current is given by equations similar to equations (11.17) and (11.18), specifically

$$i(\omega t) = i_s(\omega t) = \sqrt{2}V/Z \left( \sin(\omega t - \phi) + \left( \frac{\sin \phi e^{-\alpha / \tan \phi} - \sin(\alpha - \phi)}{1 - e^{-\pi / \tan \phi}} \right) e^{-\omega t / \tan \phi} \right) \quad (A) \quad (11.38)$$

$$\alpha \leq \omega t \leq \pi$$

While the load current when the freewheel diode conducts is

$$i(\omega t) = i_{Df}(\omega t) = I_{01x} e^{-\omega t / \tan \phi} \quad (A) \quad (11.39)$$

$$0 \leq \omega t \leq \alpha$$

where, when  $\omega t = \pi$  in equation (11.38)

$$I_{01x} = \sqrt{2}V/Z \frac{\sin \phi - \sin(\alpha - \phi) e^{-\pi / \tan \phi}}{1 - e^{-\pi / \tan \phi}} \quad (A)$$

#### Critical load inductance

The critical inductance, to prevent the current falling to zero, is given by

$$\frac{\omega L_{crit}}{R} = \theta - \alpha - \frac{1}{2}\pi + \frac{\alpha + \sin \alpha + \pi \cos \theta}{1 + \cos \alpha} \quad (11.40)$$

for  $\alpha \leq \theta$  where

$$\theta = \sin^{-1} \frac{V_o}{\sqrt{2}V} = \sin^{-1} \frac{1 + \cos \alpha}{\pi} \quad (11.41)$$

The minimum current occurs at the angle  $\theta$ , where the mean output voltage  $V_o$  equals the instantaneous load voltage,  $v_o$ . When phase delay angle  $\alpha$  is greater than the critical angle  $\theta$ ,  $\theta = \alpha$  in equation (11.41) yields

$$\frac{\omega L_{crit}}{R} = -\frac{1}{2}\pi + \frac{\alpha + \sin \alpha + \pi \cos \alpha}{1 + \cos \alpha} \quad (11.42)$$

It is important to note that converter circuits employing diodes cannot be used when inversion is required. Since the converter diodes prevent the output voltage from being negative, regeneration from the load into the supply is not achievable.

Figure 11.5a is a fully controlled converter with an  $R$ - $L$  load and freewheel diode. In single-phase circuits, this converter essentially behaves as a half-controlled converter.

### 11.3 Single-phase controlled thyristor converter circuits

#### 11.3.1 Half-wave circuit with an $R$ - $L$ load

The diode in the circuit of figure 11.1 can be replaced by a thyristor as shown in figure 11.6a to form a half-wave controlled rectifier circuit with an  $R$ - $L$  load. The output voltage is now controlled by the thyristor trigger angle,  $\alpha$ . The output voltage ripple is at the supply frequency. Circuit waveforms are shown in figure 11.6b.

The output current, hence output voltage, for the circuit is given by

$$L \frac{di}{dt} + Ri = \sqrt{2}V \sin \omega t \quad (V) \quad (11.43)$$

$$\alpha \leq \omega t \leq \beta \quad (\text{rad})$$

where phase delay angle  $\alpha$  and current extinction angle  $\beta$  are shown in the waveform in figure 11.6b and are the zero load current points.

Solving equation (11.43) yields the load and supply current

$$i(\omega t) = \frac{\sqrt{2}V}{Z} \{ \sin(\omega t - \phi) - \sin(\alpha - \phi) e^{(\alpha - \omega t) / \tan \phi} \} \quad (A) \quad (11.44)$$

$$\text{where } Z = \sqrt{R^2 + (\omega L)^2} \quad (\text{ohms}) \quad \alpha \leq \omega t \leq \beta$$

$$\tan \phi = \omega L / R$$

The current extinction angle  $\beta$  is dependent on the load impedance and trigger angle  $\alpha$ , and can be determined by solving equation (11.44) with  $\omega t = \beta$  when  $i(\beta) = 0$ , that is

$$\sin(\beta - \phi) = \sin(\alpha - \phi) e^{(\alpha - \beta) / \tan \phi} \quad (11.45)$$



This is a transcendental equation. A family of curves of current conduction angle versus delay angle, that is  $\beta - \alpha$  versus  $\alpha$ , is shown in figure 11.7. The plot for  $\phi = \frac{1}{2}\pi$  is for a purely inductive load, whereas  $\phi = 0$  is a purely resistive load.

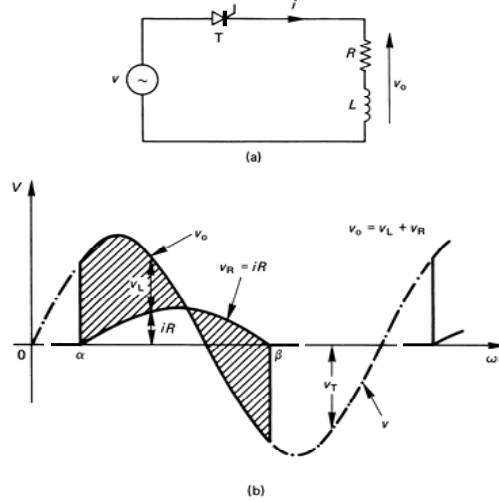


Figure 11.6. Single-phase half-wave controlled converter: (a) circuit diagram and (b) circuit waveforms for an inductive load.

The mean load voltage, whence the mean load current, is given by

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\beta} \sqrt{2} V \sin \omega t \, d\omega t \quad (11.46)$$

$$V_o = \bar{I}_o R = \frac{\sqrt{2} V}{2\pi} (\cos \alpha - \cos \beta) \quad (V)$$

where the angle  $\beta$  can be extracted from figure 11.7.

The rms load voltage is

$$V_{rms} = \left[ \frac{1}{2\pi} \int_{\alpha}^{\beta} (\sqrt{2} V)^2 \sin^2 \omega t \, d\omega t \right]^{1/2} \quad (11.47)$$

$$= \sqrt{2} V / 2 \left[ \frac{1}{\pi} \{ (\beta - \alpha) - \frac{1}{2} (\sin 2\beta - \sin 2\alpha) \} \right]^{1/2}$$

The rms current involves integration of equation (11.44), giving equation (11.67)/ $\sqrt{2}$ . Iterative solutions to equation (11.45) are shown in of figure 11.7a, where it is seen that two straight-line relationships exist between  $\alpha$  and  $\beta - \alpha$ . Exact solutions to equation (11.45) exist for these two cases. That is, exact solutions exist for the purely resistive load and the purely inductive load.

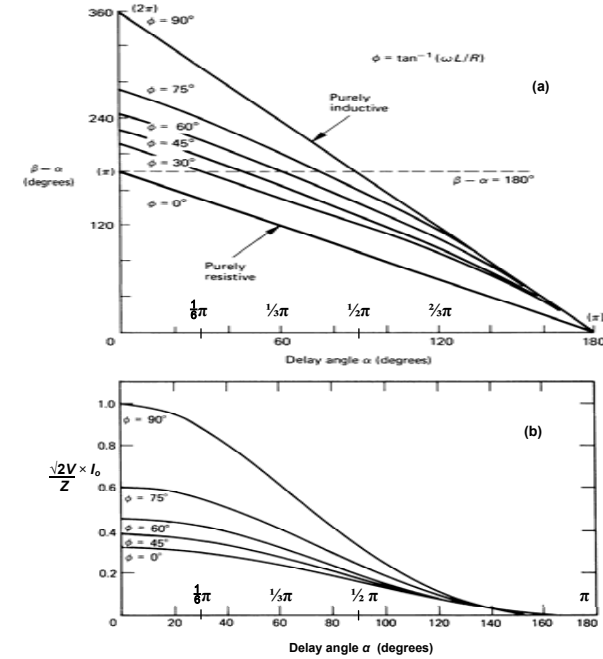


Figure 11.7. Half-wave, controlled converter thyristor trigger delay angle  $\alpha$  versus: (a) thyristor conduction angle,  $\beta - \alpha$ , and (b) normalised mean load current.

**11.3.1i - Case 1: Purely resistive load.** From equation (11.44),  $Z = R$ ,  $\phi = 0$ , and the current is given by

$$i(\omega t) = \frac{\sqrt{2} V}{R} \sin(\omega t) \quad (\text{A}) \quad (11.48)$$

$$\alpha \leq \omega t \leq \pi \text{ and } \beta = \pi \quad \forall \alpha$$

The average load voltage, hence average load current, is

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\pi} \sqrt{2} V \sin \omega t \, d\omega t \quad (11.49)$$

$$V_o = \bar{I}_o R = \frac{\sqrt{2} V}{2\pi} (1 + \cos \alpha) \quad (\text{V})$$

The rms output voltage is

$$V_{rms} = \left[ \frac{1}{2\pi} \int_{\alpha}^{\pi} (\sqrt{2} V)^2 \sin^2 \omega t \, d\omega t \right]^{1/2} \quad (11.50)$$

$$= \frac{\sqrt{2} V}{2} \left[ \frac{1}{\pi} \left\{ (\pi - \alpha) + \frac{1}{2} \sin 2\alpha \right\} \right]^{1/2}$$

Since the load is purely resistive,  $I_{rms} = V_{rms} / R$  and the power delivered to the load is  $P_o = I_{rms}^2 R$ . The supply power factor, for a resistive load, is  $P_{ou} / V_{rms} I_{rms}$ , that is

$$pf = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}} \quad (11.51)$$

**11.3.1ii - Case 2: Purely inductive load.** From equation (11.44),  $Z = \omega L$ ,  $\phi = \frac{1}{2}\pi$ , and the current is given by

$$i(\omega t) = \frac{\sqrt{2} V}{\omega L} \left( \sin(\omega t - \frac{1}{2}\pi) - \sin(\alpha - \frac{1}{2}\pi) \right) \quad (\text{A})$$

$$= \frac{\sqrt{2} V}{\omega L} (\cos \alpha - \cos \omega t) \quad (11.52)$$

$$\alpha \leq \omega t \leq \beta \text{ and } \beta = 2\pi - \alpha$$

The average load voltage, based on the equal area criterion, is zero

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{2\pi-\alpha} \sqrt{2} V \sin \omega t \, d\omega t = 0 \quad (11.53)$$

The average output current is

$$\bar{I}_o = \frac{1}{2\pi} \int_{\alpha}^{2\pi-\alpha} \frac{\sqrt{2} V}{\omega L} \{ \cos \alpha - \cos \omega t \} \, d\omega t \quad (11.54)$$

$$= \frac{\sqrt{2} V}{\pi \omega L} [(\pi - \alpha) \cos \alpha + \sin \alpha]$$

The rms output current is derived from

$$I_{rms} = \frac{\sqrt{2} V}{\omega L} \left[ \frac{1}{2\pi} \int_{\alpha}^{2\pi-\alpha} (\cos \alpha - \cos \omega t)^2 \, d\omega t \right]^{1/2} \quad (11.55)$$

The rms output voltage is

$$V_{rms} = \left[ \frac{1}{2\pi} \int_{\alpha}^{2\pi-\alpha} (\sqrt{2} V)^2 \sin^2 \omega t \, d\omega t \right]^{1/2} \quad (11.56)$$

$$= \sqrt{2} V \left[ \frac{1}{2\pi} \left\{ (\pi - \alpha) + \frac{1}{2} \sin 2\alpha \right\} \right]^{1/2}$$

Since the load is purely inductive load,  $P_o = 0$ .

By setting  $\alpha = 0$ , the equations (11.48) to (11.56) are valid for the uncontrolled rectifier considered in section 11.1.1, for a purely resistive and purely inductive load.

### Example 11.2: Half-wave controlled rectifier

The ac supply of the half-wave controlled single-phase converter in figure 11.6a is  $v = \sqrt{2} 240 \sin \omega t$ . For the following loads

Load 1:  $R = 10\Omega$ ,  $\omega L = 0\Omega$

Load 2:  $R = 0\Omega$ ,  $\omega L = 10\Omega$

Load 3:  $R = 7.1\Omega$ ,  $\omega L = 7.1\Omega$

determining in each load case, for a firing delay angle  $\alpha = \pi/6$

- the conduction angle  $\gamma$ , hence the current extinction angle  $\beta$
- the dc output voltage and the average output current
- the power dissipated in the load and power factor for the first two loads

### Solution

**Load 1:**  $R = 10\Omega$ ,  $\omega L = 0\Omega$

From equation (11.44),  $Z = 10\Omega$  and  $\phi = 0^\circ$ .

From equation (11.48),  $\beta = \pi$  for all  $\alpha$ , thus for  $\alpha = \pi/6$ ,  $\gamma = \beta - \alpha = 5\pi/6$ .

From equation (11.49)

$$V_o = \bar{I}_o R = \frac{\sqrt{2} V}{2\pi} (1 + \cos \alpha)$$

$$= \frac{\sqrt{2} V}{2\pi} (1 + \cos \pi/6) = 100.9\text{V}$$

The average load current is

$$\bar{I}_o = V_o / R = \frac{\sqrt{2} V}{2\pi R} (1 + \cos \alpha) = 100.9\text{V} / 10\Omega = 10.1\text{A}.$$

The rms load current is given by equation (11.50), that is

$$\begin{aligned} V_{rms} &= \sqrt{2} V / 2 \left[ \frac{1}{\pi} \{ (\pi - \alpha) - \frac{1}{2} \sin 2\alpha \} \right]^{\frac{1}{2}} \\ &= \sqrt{2} \times 240 / 2 \times \left[ \frac{1}{\pi} \{ (\pi - \pi/6) + \frac{1}{2} \sin \pi/3 \} \right]^{\frac{1}{2}} = 200\text{V} \end{aligned}$$

Since the load is purely resistive the power delivered to the load is

$$\begin{aligned} P_o &= I_{rms}^2 R = V_{rms}^2 / R \\ &= 100\text{V}^2 / 10\Omega = 1000\text{W} \end{aligned}$$

The power factor is

$$pf = \frac{1000\text{W}}{240\text{V} \times 10\text{A}} = 0.417$$

**Load 2:**  $R = 0\Omega$ ,  $\omega L = 10\Omega$

From equation (11.44),  $Z = 10\Omega$  and  $\phi = \frac{1}{2}\pi$ .

From equation (11.52), which is based on the equal area criterion,  $\beta = 2\pi - \alpha$ , thus for  $\alpha = \pi/6$ ,  $\beta = 11\pi/6$  whence the conduction period is  $\gamma = \beta - \alpha = 5\pi/3$ .

From equation (11.53)

$$V_o = 0\text{V}$$

The average load current is

$$\begin{aligned} \bar{I}_o &= \frac{\sqrt{2} V}{\pi \omega L} \left[ (\pi - \alpha) \cos \alpha + \sin \alpha \right] \\ &= \frac{\sqrt{2} \times 240}{\pi \times 10} \times \left[ (5\pi/6) \cos \pi/6 + \sin \pi/6 \right] = 14.9\text{A} \end{aligned}$$

Since the load is purely inductive the power delivered to the load is zero, as is the power factor.

**Load 3:**  $R = 7.1\Omega$ ,  $\omega L = 7.1\Omega$

From equation (11.44),  $Z = 10\Omega$  and  $\phi = \frac{1}{4}\pi$ .

From figure 11.7a, for  $\phi = \frac{1}{4}\pi$  and  $\alpha = \pi/6$ ,  $\gamma = \beta - \alpha = 195^\circ$  whence  $\beta = 225^\circ$ .

From equation (11.46)

$$\begin{aligned} V_o = \bar{I}_o R &= \frac{\sqrt{2} V}{2\pi} (\cos \alpha - \cos \beta) \\ &= \frac{\sqrt{2} \times 240}{2\pi} (\cos 30^\circ - \cos 225^\circ) = 85.0\text{V} \end{aligned}$$

The average load current is

$$\begin{aligned} \bar{I}_o &= V_o / R \\ &= 85.0\text{V} / 7.1\Omega = 12.0\text{A} \end{aligned}$$

Alternatively, the average current can be extract from figure 11.7b, which for  $\phi = \frac{1}{4}\pi$  and  $\alpha = \pi/6$  gives the normalised current as 0.35, thus

$$\begin{aligned} \bar{I}_o &= \sqrt{2} V / Z \times 0.35 \\ &= \sqrt{2} \times 240\text{V} / 10\Omega \times 0.35 = 11.9\text{A} \end{aligned}$$

### 11.3.2 Half-wave half-controlled

The half-wave controlled converter waveform in figure 11.6b shows that when  $\alpha < \omega t < \pi$ , during the positive half of the supply cycle, energy is delivered to the load. But when  $\pi < \omega t < 2\pi + \alpha$ , the supply reverses and some energy is returned to the supply. More energy can be retained by the load if the load voltage is prevented from reversing. A load freewheel diode can facilitate this objective.

The single-phase half-wave converter can be controlled when a load commutating diode is incorporated as shown in figure 11.8a. The diode will prevent the instantaneous load voltage  $v_o$  from going negative, as with the single-phase half-controlled converters shown in figure 11.5.

The load current is defined by equations (11.15) for  $\alpha \leq \omega t \leq \pi$  and equation (11.16) for  $\pi \leq \omega t \leq 2\pi + \alpha$ , namely:

$$\begin{aligned} L \frac{di}{dt} + Ri &= \sqrt{2} V \sin \omega t & (\text{A}) & \quad \alpha \leq \omega t \leq \pi \\ L \frac{di}{dt} + Ri &= 0 & (\text{A}) & \quad \pi \leq \omega t \leq 2\pi + \alpha \end{aligned} \quad (11.57)$$

At  $\omega t = \pi$  the thyristor is commutated and the load current, and hence diode current, is of the form of equation (11.18). As shown in figure 11.8b, depending on the delay angle  $\alpha$  and  $R$ - $L$  load time constant, the load current may fall to zero, producing discontinuous load current.

**11.3.2i - For discontinuous conduction** the load current is defined by

$$\begin{aligned} i(\omega t) = i_s(\omega t) &= \frac{\sqrt{2} V}{Z} \times \left( \sin(\omega t - \phi) - \sin(\alpha - \phi) e^{-\omega t / \tan \phi} \right) & (\text{A}) \\ & \quad \alpha \leq \omega t \leq \pi \\ i(\omega t) = i_{dy}(\omega t) &= I_{o1\pi} e^{-\omega t / \tan \phi} & (11.58) \\ &= \left\{ \frac{\sqrt{2} V}{Z} \times \sin(\phi - \alpha) (1 - e^{-\pi / \tan \phi}) \right\} e^{-\omega t + \pi / \tan \phi} & (\text{A}) \\ & \quad \pi \leq \omega t \leq 2\pi + \alpha \end{aligned}$$

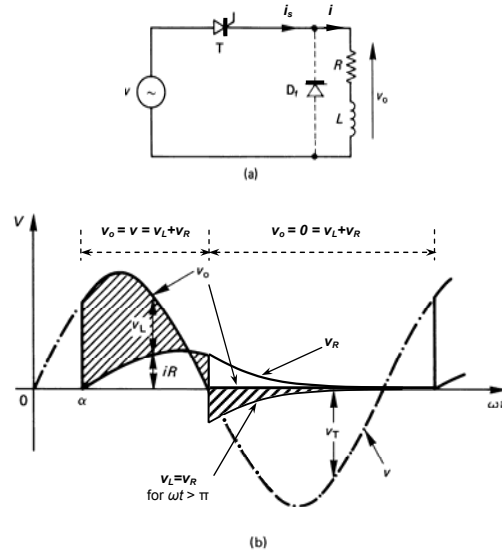


Figure 11.8. Half-wave half-controlled converter: (a) circuit diagram and (b) circuit waveforms for an inductive load.

11.3.2ii - For continuous conduction the load current is defined by

$$i(\omega t) = i_s(\omega t) = \frac{\sqrt{2}V}{Z} \times \left( \sin(\omega t - \phi) + \left( \frac{\sin \phi e^{-\alpha/\tan \phi} - \sin(\alpha - \phi)}{1 - e^{-2\pi/\tan \phi}} \right) e^{-\alpha\omega t/\tan \phi} \right) \quad \alpha \leq \omega t \leq \pi \quad (A)$$

$$i(\omega t) = i_{D1}(\omega t) = I_{012} e^{-\alpha\omega t/\tan \phi} \quad \pi \leq \omega t \leq 2\pi + \alpha \quad (A)$$

$$= \left\{ \frac{\sqrt{2}V}{Z} \times \frac{\sin \phi - \sin(\alpha - \phi) e^{-\pi/\tan \phi}}{1 - e^{-\pi/\tan \phi}} \right\} e^{-\alpha\omega t/\tan \phi} \quad \pi \leq \omega t \leq 2\pi + \alpha \quad (A)$$

The mean load voltage (hence mean output current) for all conduction cases is

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \, d\omega t \quad (11.60)$$

$$V_o = \bar{I}_o R = \frac{\sqrt{2}V}{2\pi} (1 + \cos \alpha) \quad (V)$$

which is half the mean voltage for a single-phase half-controlled converter, given by equation (11.33). The maximum mean output voltage,  $\hat{V}_o = \sqrt{2}V/\pi$  (equation (11.11)), occurs at  $\alpha = 0$ . The normalised mean output voltage  $V_n$  is

$$V_n = V_o / \hat{V}_o = \frac{1}{2} (1 + \cos \alpha) \quad (11.61)$$

The rms output voltage for both continuous and discontinuous load current is

$$V_{rms} = \left[ \frac{1}{2\pi} \int_{\alpha}^{\pi} (\sqrt{2}V)^2 \sin^2 \omega t \, d\omega t \right]^{1/2}$$

$$= \frac{\sqrt{2}V}{2} \left[ \frac{1}{\pi} \left\{ (\pi - \alpha) + \frac{1}{2} \sin 2\alpha \right\} \right]^{1/2} \quad (11.62)$$

The advantages of incorporating a load freewheel diode are

- the input power factor is improved and
- the load waveform is improved giving a better load performance

### 11.3.3 Full-wave circuit with an R-L load

Full-wave voltage control is possible with the circuits shown in figures 11.9a and b. The circuit in figure 11.9a uses a centre-tapped transformer and two thyristors which experience a reverse bias of twice the supply. At high powers where a transformer may not be applicable, a four-thyristor configuration as in figure 11.9b is suitable. The voltage ratings of the thyristors in figure 11.9b are half those of the devices in figure 11.9a, for a given input voltage.

Load voltage and current waveforms are shown in figure 11.9 parts c, d, and e for three different phase control angle conditions.

The load current waveform becomes continuous when the phase control angle  $\alpha$  is given by

$$\alpha = \tan^{-1} \omega L / R = \phi \quad (\text{rad}) \quad (11.63)$$

at which angle the output current is a rectified sine wave. For  $\alpha > \phi$ , discontinuous load current flows as shown in figure 11.9c. At  $\alpha = \phi$  the load current becomes continuous as shown in figure 11.9d, whence  $\beta = \alpha + \pi$ . Further decrease in  $\alpha$ , that is  $\alpha < \phi$ , results in continuous load current that is always greater than zero, as shown in figure 11.9e.

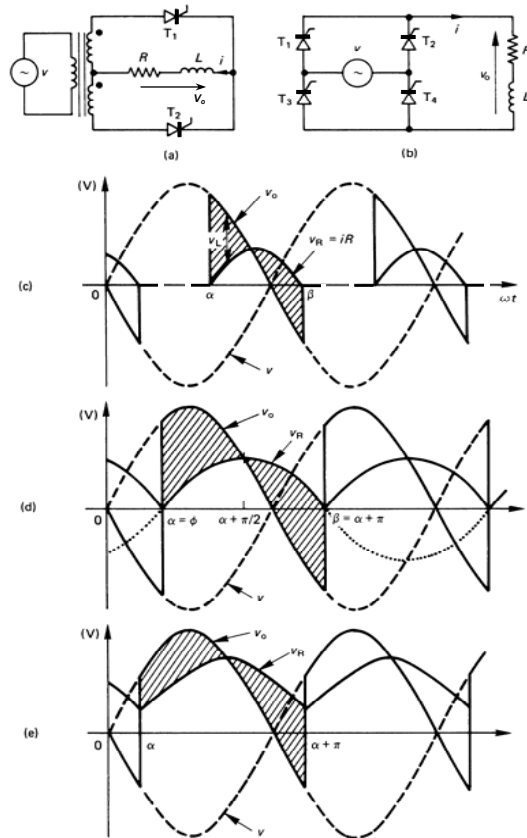


Figure 11.9. Full-wave controlled converter: (a) and (b) circuit diagrams; (c) discontinuous load current; (d) verge of continuous load current, when  $a = 0$ ; and (e) continuous load current.

### 11.3.3i - $\alpha > \phi$ , $\beta - \alpha < \pi$ , discontinuous load current

The load current waveform is the same as for the half-wave situation considered in section 11.3.1, given by equation (11.44). That is

$$i(\omega t) = \frac{\sqrt{2}V}{Z} [\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{[(\alpha - \omega t)/\tan \phi]}] \quad (\text{A}) \quad (11.64)$$

$$\alpha \leq \omega t < \beta \quad (\text{rad})$$

The mean output voltage for this full-wave circuit will be twice that of the half-wave case in section 11.3.1, given by equation (11.46). That is

$$V_o = \bar{I}_o R = \frac{1}{\pi} \int_{\alpha}^{\beta} \sqrt{2} V \sin \omega t \, d\omega t$$

$$= \frac{\sqrt{2} V}{\pi} (\cos \alpha - \cos \beta) \quad (\text{V}) \quad (11.65)$$

where  $\beta$  can be extracted from figure 11.7. The average output current is given  $\bar{I}_o = V_o / R$ .

The rms load voltage is

$$V_{rms} = \sqrt{2} V \left[ \frac{1}{\pi} \int_{\alpha}^{\beta} \sin^2 \omega t \, d\omega t \right]^{1/2}$$

$$= \sqrt{2} V \left[ \frac{1}{2\pi} \{ (\beta - \alpha) + \frac{1}{2} (\sin 2\beta - \sin 2\alpha) \} \right]^{1/2} \quad (11.66)$$

The rms load current is

$$I_{rms} = \frac{\sqrt{2} V}{2\pi R} [\cos \phi \cos(\beta - \alpha) - \sin \phi \cos(\alpha + \phi + \beta)]^{1/2} \quad (11.67)$$

The load power is therefore  $P = I_{rms}^2 R$ .

### 11.3.3ii - $\alpha = \phi$ , $\beta - \alpha = \pi$ , verge of continuous load current

When  $\alpha = \phi = \tan^{-1} \omega L / R$ , the load current given by equation (11.64) reduces to

$$i(\omega t) = \frac{\sqrt{2} V}{Z} \sin(\omega t - \phi) \quad (\text{A}) \quad (11.68)$$

$$\text{for } \phi \leq \omega t \leq \phi + \pi \quad (\text{rad})$$

and the mean output voltage, on reducing equation (11.65) using  $\beta = \alpha + \pi$ , is given by

$$V_o = \frac{2\sqrt{2} V}{\pi} \cos \alpha \quad (\text{V}) \quad (11.69)$$

which is dependent of the load such that  $\alpha = \phi = \tan^{-1} \omega L / R$ . From equation (11.66), with  $\beta - \alpha = \pi$ , the rms output voltage is  $V$ ,  $I = V/Z$ , and power =  $VI \cos \phi$ .

### 11.3.3iii - $\alpha < \phi$ , $\beta - \pi = \alpha$ , continuous load current

Under this condition, a thyristor is still conducting when another is forward-biased and

is turned on. The first device is instantaneously reverse-biased by the second device which has been turned on. The first device is commutated and load current is instantaneously transferred to the oncoming device.

The load current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{Z} \left[ \sin(\omega t - \phi) - \frac{2 \sin(\alpha - \phi)}{1 - e^{-(\alpha - \omega t)/\tan \phi}} e^{[(\alpha - \omega t)/\tan \phi]} \right] \quad (11.70)$$

This equation reduces to equation (11.68) for  $\alpha = \phi$ .

The mean output voltage, whence mean output current, are defined by equation (11.69)

$$V_o = \bar{I}_o R = \frac{2\sqrt{2}V}{\pi} \cos \alpha \quad (V) \quad (11.71)$$

which is uniquely defined by  $\alpha$ . The maximum mean output voltage,  $\hat{V}_o = 2\sqrt{2}V/\pi$  (equation (11.22)), occurs at  $\alpha=0$ . The normalised mean output voltage  $V_n$  is

$$V_n = V_o / \hat{V}_o = \cos \alpha \quad (11.71)$$

The rms output voltage is equal to the rms input supply voltage and given by

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} (\sqrt{2}V)^2 \sin^2 \omega t \, d\omega t} = V \quad (11.72)$$

The ac component harmonic magnitudes in the load are given by

$$V_n = \frac{\sqrt{2}V}{2\pi} \times \left( \frac{1}{(2n-1)^2} + \frac{1}{(2n+1)^2} - \frac{2 \cos 2\alpha}{(2n-1)(2n+1)} \right) \quad (11.73)$$

for  $n = 1, 2, 3 \dots$

The current harmonics are obtained by division of the voltage harmonic by its load impedance at that frequency, that is

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}} \quad (11.74)$$

#### Critical load inductance

The critical inductance, to prevent the current falling to zero, is given by

$$\frac{\omega L_{crit}}{R} = \frac{\pi}{2 \cos \alpha} \left( \cos \theta + \frac{2}{\pi} \sin \alpha - \frac{2}{\pi} \cos \alpha \left( \frac{1}{2} \pi + \alpha + \theta \right) \right) \quad (11.75)$$

for  $\alpha \leq \theta$  where

$$\theta = \sin^{-1} \frac{V_o}{\sqrt{2}V} = \sin^{-1} \frac{2 \cos \alpha}{\pi} \quad (11.76)$$

The minimum current occurs at the angle  $\theta$ , where the mean output voltage  $V_o$  equals the instantaneous load voltage,  $v_o$ . When phase delay angle  $\alpha$  is greater than the critical angle  $\theta$ , substituting  $\alpha=\theta$  in equation (11.75) gives

$$\frac{\omega L_{crit}}{R} = -\tan \alpha \quad (11.77)$$

#### Example 11.3: Controlled full-wave converter – continuous conduction

The fully controlled full-wave converter in figure 11.9a has a source of 240V rms, 50Hz, and a 10Ω 50mH series load. The delay angle is 45°.

Determine

- the average output voltage and current, hence thyristor mean current
- the rms load voltage and current, hence thyristor rms current
- the power absorbed by the load and the power factor

#### Solution

The load natural power factor angle is given by

$$\phi = \tan^{-1} \omega L / R = \tan^{-1} (2\pi 50 \times 50\text{mH} / 10\Omega) = 57.5^\circ$$

Since  $\alpha < \phi$  ( $45^\circ < 57.5^\circ$ ), continuous load current flows.

- The average output current and voltage are given by equation (11.69)

$$V_o = \bar{I}_o R = \frac{2\sqrt{2}V}{\pi} \cos \alpha = \frac{2\sqrt{2}V}{\pi} \cos 45^\circ = 152.8\text{V}$$

$$\bar{I}_o = V_o / R = 152.8\text{V} / 10\Omega = 15.3\text{A}$$

Each thyristor conducts for 180°, hence the thyristor mean current is ½ of 15.3A = 7.65A

- The rms load current is determined by harmonic analysis. The voltage harmonics are given by equation (11.73)

$$V_n = \frac{\sqrt{2}V}{2\pi} \times \left( \frac{1}{(2n-1)^2} + \frac{1}{(2n+1)^2} - \frac{2 \cos 2\alpha}{(2n-1)(2n+1)} \right)$$

and the corresponding current is given from equation (11.74)

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}}$$

The dc output voltage component is given by equation (11.69).

From the calculations in the following table, the rms load current is

$$I_{rms} = \sqrt{I_o^2 + \frac{1}{2} \sum I_n^2} = \sqrt{243.90} = 15.45\text{A}$$

Since each thyristor conducts for 180°, the thyristor rms current is  $1/\sqrt{2}$  of 15.3A = 10.8A

The rms load voltage is given by equation (11.72), that is 240V.

harmonic n	$V_n$	$Z_n = \sqrt{R^2 + (n\omega L)^2}$	$I_n = \frac{V_n}{Z_n}$	$\frac{1}{2}I_n^2$
0	<b>152.79</b>	10.00	15.28	<b>233.44</b>
1	60.02	18.62	3.22	5.19
2	8.16	32.97	0.25	0.03
3	3.26	48.17	0.07	0.00
4	1.77	63.62	0.03	0.00
			$I_o^2 + \sum \frac{1}{2}I_n^2 =$	243.90

iii. The power absorbed by the load is

$$P_L = I_{rms}^2 R = 15.45A^2 \times 10\Omega = 2387W$$

The power factor is

$$pf = \frac{P_L}{V_{rms} I_{rms}} = \frac{2387W}{240V \times 15.45A} = 0.64$$

♣

#### 11.3.4 Full-wave circuit with R-L and emf load

An emf source and R-L load can be encountered in dc machine modelling. The emf represents the machine speed back emf, defined by  $E = k\phi\omega$ . These machines can be controlled by a fully controlled converter configuration as shown in figure 11.10a.

If in each half sine period the thyristor firing delay angle occurs after the rectified sine supply has fallen below the emf level  $E$ , then no load current flows since the bridge thyristors will always be reverse-biased. Thus the zero current firing angle  $\alpha_o$ , for  $\alpha_o > \frac{1}{2}\pi$  is given by

$$\alpha_o = \sin^{-1}(E/\sqrt{2}V) \quad (\text{rad}) \quad (11.78)$$

where it has been assumed the emf has the polarity shown in figure 11.10a. Load current can flow with a firing angle defined by

$$0 \leq \alpha \leq \alpha_o \quad (\text{rad}) \quad (11.79)$$

whence  $\hat{\alpha} = \pi - \alpha_o$ .

The load circuit current can be evaluated by solving

$$\sqrt{2}V \sin \omega t = L \frac{di}{dt} + Ri + E \quad (V) \quad (11.80)$$

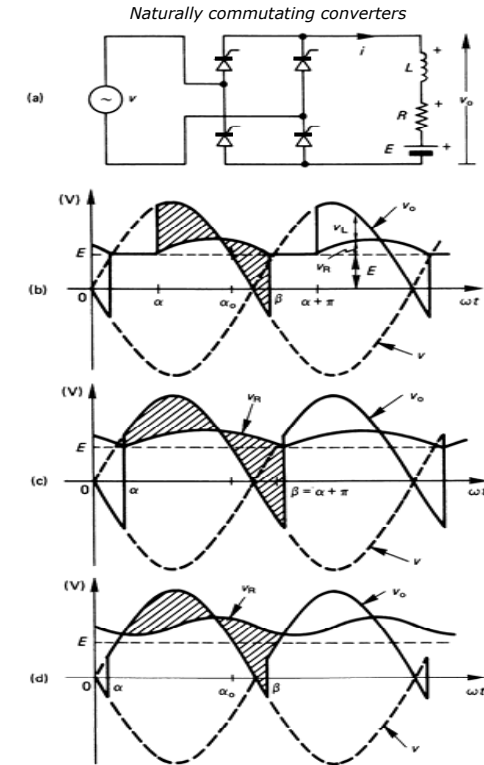


Figure 11.10. A full-wave fully controlled converter with an inductive load which includes an emf source: (a) circuit diagram; (b) voltage waveforms with discontinuous load current; (c) verge of continuous load current; and (d) continuous load current.

**11.3.4i - Discontinuous load current**

The load current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{Z} [\sin(\omega t - \phi) - \frac{E}{\sqrt{2}V} / \cos \phi + \left\{ \frac{E}{\sqrt{2}V} / \cos \phi - \sin(\alpha - \phi) \right\} e^{[(\alpha - \omega t) / \tan \phi]}] \quad (11.81)$$

$$\alpha \leq \omega t \leq \beta < \pi + \alpha \quad (\text{rad})$$

For discontinuous load current conduction, the current extinction angle  $\beta$ , shown on figure 11.10b, is solved by iterative techniques. The mean output voltage can be obtained from equation (11.31), which is valid for  $E = 0$ . For non-zero  $E$

$$V_o = \frac{1}{\pi} \int_{\alpha}^{\beta} (\sqrt{2} V \sin \omega t + E) d\omega t$$

$$V_o = \frac{\sqrt{2}V}{\pi} \left( \cos \alpha - \cos \beta + \frac{E}{\sqrt{2}V} (\beta - \alpha) \right) \quad (\text{V}) \quad (11.82)$$

$$0 < \beta - \alpha < \pi \quad (\text{rad})$$

The current extinction angle  $\beta$  is load-dependent, being a function of  $\alpha$ ,  $Z$ , and  $E$ .

Since  $V_o = E + I_o R$ , the mean load current is given by

$$\bar{I}_o = \frac{V_o - E}{R} = \frac{\sqrt{2}V}{\pi R} \left( \cos \alpha - \cos \beta - \frac{E}{\sqrt{2}V} (\beta - \alpha) \right) \quad (\text{A}) \quad (11.83)$$

$$0 < \beta - \alpha < \pi \quad (\text{rad})$$

The rms output voltage is given by

$$V_{rms} = \left( V^2 \frac{\beta - \alpha}{\pi} + E^2 \left( 1 - \frac{\beta - \alpha}{\pi} \right) - \frac{V^2}{2\pi} (\sin 2\beta - \sin 2\alpha) \right)^{1/2} \quad (\text{V}) \quad (11.84)$$

The rms voltage across the  $R$ - $L$  part of the load is given by

$$V_{RLrms} = \sqrt{V_{rms}^2 - E^2} \quad (11.85)$$

**11.3.4ii - Continuous load current**

With continuous load current conduction, the load rms voltage is  $V$ .

The load current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{Z} [\sin(\omega t - \phi) - \frac{E}{\sqrt{2}V} / \cos \phi + \frac{2 \sin(\alpha - \phi)}{e^{-\pi/\tan \phi} - 1} e^{[(\alpha - \omega t) / \tan \phi]}] \quad (11.86)$$

$$\alpha \leq \omega t \leq \pi + \alpha \quad (\text{rad})$$

The minimum current is given by

$$I = \frac{\sqrt{2}V}{Z} \sin(\alpha - \phi) \frac{e^{-\pi/\tan \phi} + 1}{e^{-\pi/\tan \phi} - 1} - \frac{E}{\sqrt{2}V} \quad (11.87)$$

For continuous load current conditions, as shown in figures 11.10c and 11.10d, the mean output voltage is given by equation (11.82) with  $\beta = \pi - \alpha$

$$V_o = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} \sqrt{2} V \sin \omega t d\omega t$$

$$= \frac{2\sqrt{2}V}{\pi} \cos \alpha \quad (\text{V}) \quad (11.88)$$

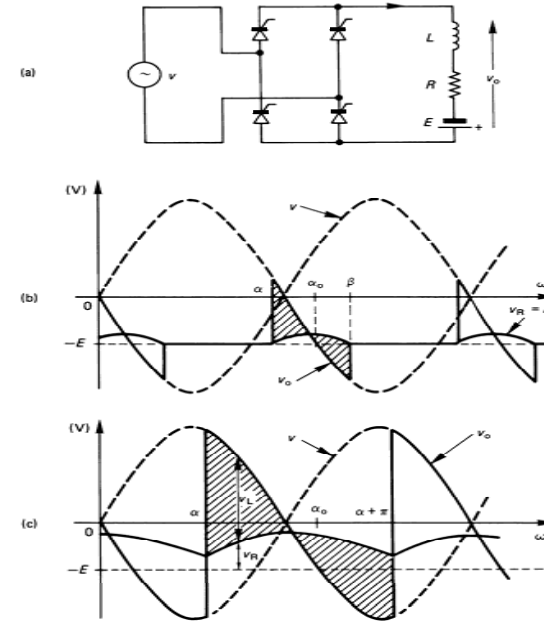


Figure 11.11. A full-wave controlled converter with an inductive load and negative emf source: (a) circuit diagram; (b) voltage waveforms for discontinuous load current; and (c) continuous load current.



The average output voltage is dependent only on the phase delay angle  $\alpha$ . The mean load current is given by

$$\bar{I}_o = \frac{V_o - E}{R} = \frac{\sqrt{2}V}{R} \left( \frac{2}{\pi} \cos \alpha - \frac{E}{\sqrt{2}V} \right) \quad (\text{A}) \quad (11.89)$$

The power absorbed by the emf source in the load is  $P = \bar{I}_o E$

The output voltage harmonic magnitudes for continuous conduction, are given by equation (11.73), that is

$$V_n = \frac{\sqrt{2}V}{2\pi} \times \left( \frac{1}{(2n-1)^2} + \frac{1}{(2n+1)^2} - \frac{2 \cos 2\alpha}{(2n-1)(2n+1)} \right) \quad (11.90)$$

The dc component across the R-L part of the load is

$$\begin{aligned} V_{oR-L} &= V_o - E \\ &= \frac{2\sqrt{2}V}{\pi} \times \cos \alpha - E \end{aligned} \quad (11.91)$$

#### Critical load inductance

From equation (11.87) set to zero, the boundary between continuous and discontinuous inductor current must satisfy

$$\frac{R}{Z} \sin(\alpha - \phi) \frac{e^{-\pi/\tan \phi} + 1}{e^{-\pi/\tan \phi} - 1} > \frac{E}{\sqrt{2}V} \quad (11.92)$$

#### inversion

If the polarity of  $E$  is reversed as shown in figure 11.11a, waveforms as in parts b and c of figure 11.11 result. The emf supply can provide a forward bias across the bridge thyristors even after the supply polarity has gone negative. The zero current angle  $\alpha_o$  now satisfies  $\pi < \alpha_o < 3\pi/2$ , as given by equation (11.78). Thus load and supply current can flow, even for  $\alpha > \pi$ .

The relationship between the mean output voltage and current is now given by

$$V_o = -E + \bar{I}_o R \quad (11.93)$$

That is, the emf term  $E$  in equations (11.78) to (11.92) is appropriately changed to  $-E$ . The load current flows from the emf source and the average load voltage is negative. Power is being delivered to the ac supply from the emf source in the load, which is an energy transfer process called *power inversion*. In general

$$\begin{array}{ll} 0 < \alpha < 90^\circ & v_o > 0 \quad \text{rectification} \\ 90^\circ < \alpha < 180^\circ & v_o < 0 \quad \text{inversion} \end{array}$$

#### Example 11.4: Controlled converter – continuous conduction and back emf

The fully controlled full-wave converter in figure 11.9a has a source of 240V rms, 50Hz, and a 10 $\Omega$ , 50mH, 50V emf series load. The delay angle is 45°. Determine

- the average output voltage and current
- the rms load voltage and the rms voltage across the  $R$ - $L$  part of the load
- the power absorbed by the 50V load back emf
- the rms load current hence power dissipated in the resistive part of the load
- the load efficiency, that is percentage of energy into the back emf and power factor

#### Solution

From example 11.3, continuous conduction occurs since  $\alpha < \phi$  ( $45^\circ < 57.5^\circ$ ).

- The average output voltage is given by equation (11.88)

$$\begin{aligned} V_o &= \frac{2\sqrt{2}V}{\pi} \cos \alpha \\ &= \frac{2\sqrt{2} \times 240}{\pi} \times \cos 45^\circ = 152.8V \end{aligned}$$

The average current, from equation (11.89) is

$$\bar{I}_o = \frac{V_o - E}{R} = \frac{152.8V - 50V}{10\Omega} = 10.28A$$

- From equation (11.72) the rms load voltage is 240V. The rms voltage across the  $R$ - $L$  part of the load is

$$\begin{aligned} V_{RLrms} &= \sqrt{V_{rms}^2 - E^2} \\ &= \sqrt{240V^2 - 50V^2} = 234.7V \end{aligned}$$

- The power absorbed by the 50V back emf load is

$$P = \bar{I}_o E = 10.28A \times 50V = 514W$$

- The  $R$ - $L$  load voltage harmonics are given by equations (11.90) and (11.91):

$$\begin{aligned} V_{oR-L} &= \frac{2\sqrt{2}V}{\pi} \times \cos \alpha - E \\ V_n &= \frac{\sqrt{2}V}{2\pi} \times \left( \frac{1}{(2n-1)^2} + \frac{1}{(2n+1)^2} - \frac{2 \cos 2\alpha}{(2n-1)(2n+1)} \right) \end{aligned}$$

The harmonic currents and voltages are shown in the table to follow.

The rms load current is given by

$$I_{rms} = \sqrt{I_o^2 + \frac{1}{2} \sum I_n^2} = \sqrt{10.88} = 10.53A$$

The power absorbed by the 10 $\Omega$  load resistor is

$$P_L = I_{rms}^2 R = 10.53A^2 \times 10\Omega = 1108.8W$$

harmonic n	$V_n$	$Z_n = \frac{Z_s}{\sqrt{R^2 + (n\omega L)^2}}$	$I_n = \frac{V_n}{Z_n}$	$\frac{1}{2}I_n^2$
0	<b>102.79</b>	10.00	10.28	<b>105.66</b>
1	60.02	18.62	3.22	5.19
2	8.16	32.97	0.25	0.03
3	3.26	48.17	0.07	0.00
			$I_o^2 + \sum \frac{1}{2}I_n^2 =$	110.88

vi. The load efficiency, that is, percentage energy into the back emf  $E$  is

$$\eta = \frac{514\text{W}}{514\text{W} + 1108.8\text{W}} \times 100\% = 31.7\%$$

The power factor is

$$pf = \frac{P_t}{V_{rms} I_{rms}} = \frac{514\text{W} + 1108.8\text{W}}{240\text{V} \times 10.53\text{A}} = 0.49$$

#### 11.4 Three-phase uncontrolled converter circuits

Single-phase supply circuits are adequate below a few kilowatts. At higher power levels, restrictions on unbalanced loading, line harmonics, current surge voltage dips, and filtering require the use of three-phase (or higher) converter circuits. Generally it will be assumed that the output current is both continuous and smooth. This assumption is based on the dc load being highly inductive.

##### 11.4.1 Half-wave rectifier circuit with an inductive load

Figure 11.12 shows a half-wave, three-phase diode rectifier circuit along with various circuit voltage and current waveforms. A transformer having a star connected secondary is required for neutral access, N.

The diode with the highest potential with respect to the neutral conducts a rectangular current pulse. As the potential of another diode becomes the highest, load current is transferred to that device, and the previously conducting device is reverse-biased and naturally (line) commutated.

In general terms, for an  $n$ -phase system, the mean output voltage is given by

$$V_o = \frac{\sqrt{2}V}{2\pi/n} \int_{-\pi/n}^{\pi/n} \cos \omega t d\omega t \quad (\text{V})$$

$$= \sqrt{2}V \frac{\sin(\pi/n)}{\pi/n} \quad (\text{V}) \quad (11.94)$$

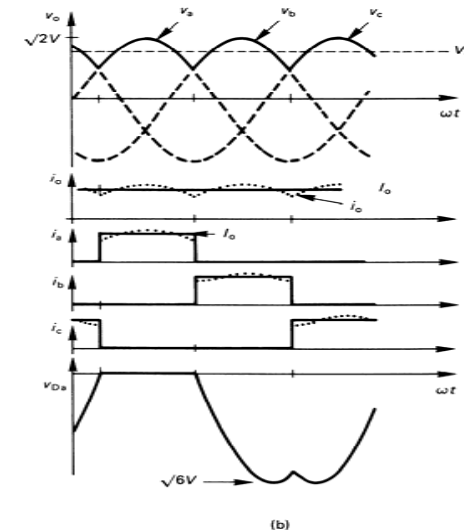
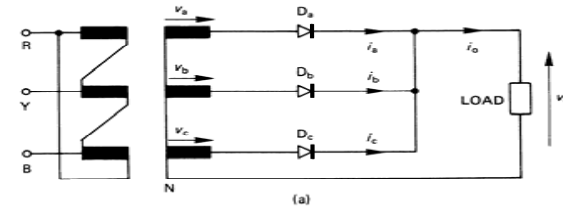


Figure 11.12. Three-phase half-wave rectifier:  
(a) circuit diagram and (b) circuit voltage and current waveforms.

For a three-phase, half-wave circuit ( $n = 3$ ) the mean output voltage is

$$V_o = \frac{1}{2\pi/3} \int_{\pi/6}^{5\pi/6} \sqrt{2} V \sin \omega t \, d\omega t \quad (11.95)$$

$$= \sqrt{2} V \frac{\sqrt{3}}{\pi/3} \quad (V)$$

The diode conduction angle is  $2\pi/n$ , namely  $2\pi/3$ . The peak diode reverse voltage is given by the maximum voltage between any two phases,  $\sqrt{3}\sqrt{2} V = \sqrt{6} V$ .

From equations (11.29), (11.30), and (11.31), for a constant output current,  $\bar{I}_o = I_{o \text{ rms}}$ , the mean diode current is

$$\bar{I}_D = \frac{1}{n} \bar{I}_o = \frac{1}{3} \bar{I}_o \quad (A) \quad (11.96)$$

and the rms diode current is

$$I_D = \frac{1}{\sqrt{n}} I_{o \text{ rms}} \approx \frac{1}{\sqrt{3}} \bar{I}_o = \frac{1}{\sqrt{3}} \bar{I}_o \quad (A) \quad (11.97)$$

The diode current form factor is

$$K_D = I_D / \bar{I}_D = \sqrt{3} \quad (11.98)$$

The rms load voltage is

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi/3} \int_{\pi/6}^{5\pi/6} (\sqrt{2} V)^2 \sin^2 \omega t \, d\omega t} \quad (11.99)$$

$$= 1.19V$$

The load form factor is

$$V_{\text{FF}} = V_{\text{rms}} / V = 1.19V / 1.17V = 1.01 \quad (11.100)$$

$$\text{The ripple factor} = \frac{\text{ac voltage at the load}}{\text{dc voltage at the load}}$$

$$= \sqrt{\frac{V_{\text{rms}}}{V} - 1} = 0.185 \quad (11.101)$$

If neutral is available, a transformer is not necessary. The full load current is returned via the neutral supply. This neutral supply current is generally not acceptable other than at low power levels. The simple delta-star connection of the supply in figure 11.12a is not appropriate since the unidirectional current in each phase is transferred from the supply to the transformer. This may result in increased magnetising current and iron losses if dc magnetisation occurs. This problem is avoided in most cases by the special interconnected star winding, called zig-zag, shown in figure 11.13a. Each transformer limb has two equal voltage secondaries which are connected such that the magnetising forces balance. The resultant phasor diagram is shown in figure 11.13b.

As the number of phases increases, the windings become less utilised per cycle since the diode conduction angle decreases, from  $\pi$  for a single-phase circuit, to  $2\pi/3$  for the three-phase case.

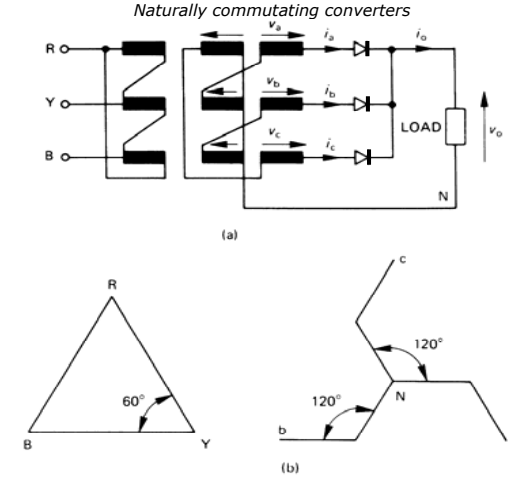


Figure 11.13. Three-phase zig-zag interconnected star winding: (a) transformer connection and (b) phasor diagram of transformer voltages.

#### 11.4.2 Full-wave rectifier circuit with an inductive load

Figure 11.14a shows a three-phase full-wave rectifier circuit where no neutral is necessary and it will be seen that two series diodes are always conducting. One diode can be considered as being in the feed circuit, while the other is in the return circuit. As such, the line-to-line voltage is impressed across the load. The rectifier circuit waveforms in figure 11.14b show that the load ripple frequency is six times the supply. Each diode conducts for  $\pi/3$  and experiences a reverse voltage of the peak line voltage. The mean load voltage is given by twice equation (11.95), that is

$$V_o = I_o R = \frac{1}{\pi/3} \int_{\pi/3}^{2\pi/3} \sqrt{2} V_L \sin \omega t \, d\omega t \quad (V) \quad (11.102)$$

$$= \sqrt{2} V_L \frac{\sqrt{3}}{\pi/3} = \frac{3}{\pi} \sqrt{2} V_L = 1.35 V_L$$

where  $V_L$  is the line-to-line rms voltage.

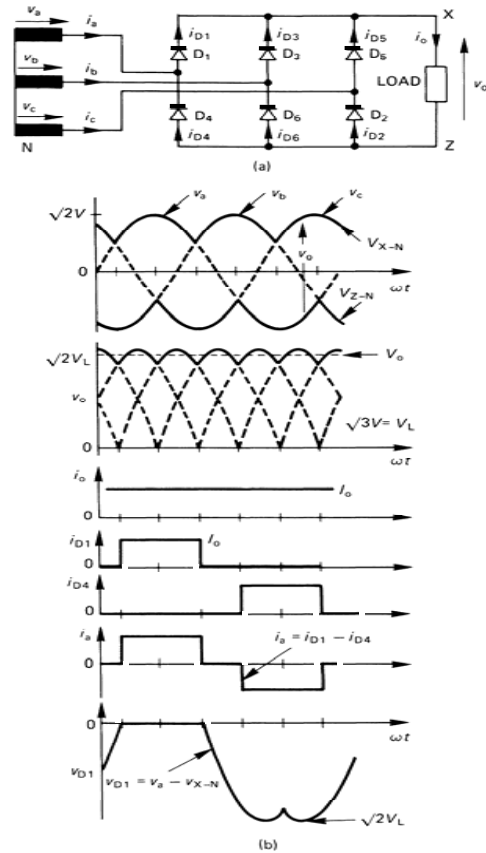


Figure 11.14. Three-phase full-wave bridge rectifier:  
(a) circuit connection and (b) voltage and current waveforms.

The output voltage harmonics are given by

$$V_{on} = \frac{6 \hat{V}_L}{\pi(n^2 - 1)} \quad (11.103)$$

for  $n=6, 12, 18, \dots$

The rms output voltage is given by

$$V_{rms} = \left( \frac{1}{2\pi/6} \int_{\pi/3}^{2\pi/3} \sqrt{2} V_L \sin^2 \omega t d\omega t \right)^{1/2} \quad (11.104)$$

$$= V_L \sqrt{1 + \frac{3\sqrt{3}}{2\pi}} = 1.352 V_L$$

The load form factor =  $1.352/1.35 = 1.001$  and the ripple factor =  $\sqrt{\text{form factor} - 1} = 0.06$ . For highly inductive load, that is a constant load current:

$$\bar{I}_D = \frac{1}{6} \bar{I}_o = \frac{1}{3} \bar{I}_o \quad (A) \quad (11.105)$$

and the rms diode current is

$$I_{D,rms} = \frac{1}{\sqrt{6}} I_{o,rms} \approx \frac{1}{\sqrt{6}} \bar{I}_o = \frac{1}{\sqrt{6}} \bar{I}_o \quad (A) \quad (11.106)$$

and the power factor for a constant load current is

$$pf = \frac{3}{\pi} = 0.955 \quad (11.107)$$

The rms input line currents are

$$I_{L,rms} = \sqrt{\frac{2}{3}} I_{o,rms} \quad (11.108)$$

The diode current form factor is

$$K_{ID} = I_{D,rms} / \bar{I}_D = \sqrt{3} \quad (11.109)$$

#### Example 11.5: Three-phase full wave rectifier

The full-wave three-phase rectifier in figure 11.14a has a three-phase 415V 50Hz source (240V phase), and a  $10\Omega$ , 50mH, series load.

Determine

- the average output voltage and current
- the rms load voltage and the ac output voltage
- the rms load current hence power dissipated and power factor
- the load power percentage error in assuming a constant load current
- the diode average and rms current requirements

**Solution**

- i. From equation (11.102) the average output voltage and current are

$$V_o = I_o R = 1.35 V_L = 1.35 \times 415 \text{ V} = 560.45 \text{ V}$$

$$I_o = \frac{V_o}{R} = \frac{560.45 \text{ V}}{10 \Omega} = 56.045 \text{ A}$$

- ii. The rms load voltage is given by equation (11.104)

$$V_{rms} = 1.352 V_L = 1.352 \times 415 \text{ V} = 560.94 \text{ V}$$

The ac component across the load is

$$V_{ac} = \sqrt{V_{rms}^2 - V_o^2} = \sqrt{560.94^2 - 560.45^2} = 23.45 \text{ V}$$

- iii. The rms load current is calculated from the harmonic currents, which are calculated from the harmonic voltage given by equation (11.103).

harmonic n	$\frac{V_n}{6 \hat{V}_L} = \frac{1}{\pi(n^2 - 1)}$	$\frac{Z_n}{\sqrt{R^2 + (n\omega L)^2}}$	$I_n = \frac{V_n}{Z_n}$	$\frac{1}{2} I_n^2$
0	<b>560.45</b>	10.00	56.04	<b>3141.01</b>
6	32.03	94.78	0.34	0.06
12	7.84	188.76	0.04	0.00
			$I_o^2 + \sum \frac{1}{2} I_n^2 =$	3141.07

The rms load current is

$$I_{rms} = \sqrt{I_o^2 + \sum \frac{1}{2} I_n^2} = \sqrt{3141.07} = 56.05 \text{ A}$$

The power absorbed by the 10Ω load resistor is

$$P_L = I_{rms}^2 R = 56.05^2 \times 10 \Omega = 31410.7 \text{ W}$$

The power factor is

$$pf = \frac{P_L}{V_{rms} I_{rms}} = \frac{31410.7 \text{ W}}{\sqrt{3} \times 415 \text{ V} \times \sqrt{\frac{2}{3}} \times 56.05 \text{ A}} = 0.955$$

This power factor value of 0.955 is as predicted by equation (11.107),  $\frac{3}{\pi}$ , for a constant current load current.

- iv. The percentage output power error in assuming the load current is constant is given by

$$\frac{\tilde{P}_L}{P_L} = \frac{I_o^2 R}{I_{rms}^2 R} = \frac{56.045^2 \times 10 \Omega}{56.05^2 \times 10 \Omega} = \frac{31410.1 \text{ W}}{31410.7 \text{ W}} \approx 0\%$$

- v. The diode average and rms current are given by equations (11.105) and (11.106)

$$\bar{I}_D = \frac{1}{\sqrt{3}} \bar{I}_o = \frac{1}{\sqrt{3}} \times 56.045 = 18.7 \text{ A}$$

$$I_{D,rms} = \frac{1}{\sqrt{6}} I_{o,rms} = \frac{1}{\sqrt{6}} \times 56.05 = 23.4 \text{ A}$$

**11.5 Three-phase half-controlled converter**

Figure 11.15a illustrates a half-controlled converter where half the devices are thyristors, the remainder being diodes. As in the single-phase case, a freewheeling diode can be added across the load so as to allow the bridge thyristors to commutate. The output voltage expression consists of  $\sqrt{2} V \frac{3\sqrt{3}}{2\pi}$  due to the uncontrolled half of the bridge and  $\sqrt{2} V \frac{3\sqrt{3}}{2\pi} \cos \alpha$  due to the controlled half which is phase-controlled. The half-controlled bridge mean output is given by the sum, that is

$$\begin{aligned} V_o &= \sqrt{2} V \frac{3\sqrt{3}}{2\pi} (1 + \cos \alpha) = \sqrt{2} V_L \frac{3}{2\pi} (1 + \cos \alpha) \\ &= 2.34 V (1 + \cos \alpha) \quad (\text{V}) \quad (11.110) \\ &\quad 0 \leq \alpha \leq \pi \quad (\text{rad}) \end{aligned}$$

At  $\alpha = 0$ ,  $\hat{V}_o = \sqrt{2} V \frac{3\sqrt{3}}{2\pi} = 1.35 V_L$ , as in equation (11.46). The normalised mean output voltage  $V_o$  is

$$V_o = V_o / \hat{V}_o = \frac{1}{2} (1 + \cos \alpha) \quad (11.111)$$

The diodes prevent any negative output, hence inversion cannot occur. Typical output voltage and current waveforms for an inductive load are shown in figure 11.15b.

**11.5i - For  $\alpha \leq \frac{1}{2}\pi$** 

When the delay angle is less than  $\frac{1}{2}\pi$  the output waveform contains six pulses per cycle, of alternating controlled and uncontrolled phases, as shown in figure 11.15b. The output current is always continuous since no voltage zeros occur.

The rms output voltage is given by

$$\begin{aligned} V_{rms} &= \sqrt{\frac{3}{2\pi} \int_{\alpha+\pi/6}^{\alpha+5\pi/6} 3 \left( \sqrt{2} V \right)^2 \sin^2(\omega t - \pi/6) d\omega t} \\ &= \sqrt{3} \sqrt{2} V \left( \frac{3}{4\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha) \right)^{1/2} \quad (11.112) \\ &\quad \text{for } \alpha \leq \pi/3 \end{aligned}$$

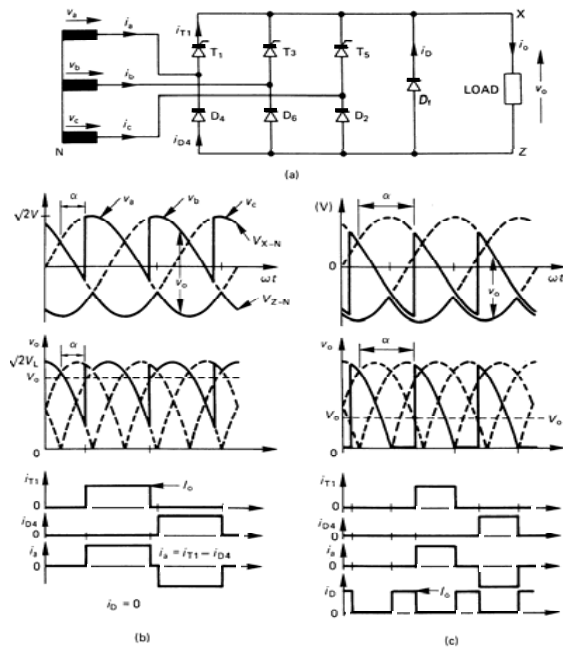


Figure 11.15. Three-phase half-controlled bridge converter: (a) circuit connection; (b) voltage and current waveforms for a small firing delay angle  $\alpha$ ; and (c) waveforms for  $\alpha$  large.

#### 11.5ii - For $\alpha \geq \pi/6$

For delay angles greater than  $\pi/6$  the output voltage waveform is made up of three controlled pulses per cycle, as shown in figure 11.15c. Although output voltage zeros result, continuous load current can flow through a diode and the conducting thyristor, or through the commutating diode if employed. The rms output voltage is given by

$$V_{rms} = \sqrt{\frac{3}{2\pi} \int_{\alpha+\pi/6}^{7\pi/6} 3(\sqrt{2}V)^2 \sin^2(\omega t - \pi/6) d\omega t} \\ = \sqrt{3}\sqrt{2}V \left( \frac{3}{4\pi} (\pi - \alpha + \frac{1}{2}\sin 2\alpha) \right)^{1/2} \quad (11.113) \\ \text{for } \alpha \geq \pi/3$$

### 11.6 Three-phase controlled thyristor converter circuits

#### 11.6.1 Half-wave circuit with an inductive load

When the diodes in the circuit of figure 11.12 are replaced by thyristors, as in figure 11.16a, a fully controlled half-wave converter results. The output voltage is controlled by the delay angle  $\alpha$ . This angle is specified from the thyristor commutation angle, which is the earliest point the associated thyristor becomes forward-biased, as shown in parts b, c, and d of figure 11.16. (The reference is not the phase zero voltage cross-over point). The thyristor with the highest instantaneous anode potential will conduct when fired and in turning on will reverse bias and turn off any previously conducting thyristor. The output voltage ripple is three times the supply frequency and the supply currents contain dc components. Each phase progressively conducts for periods of  $\pi$ . The mean output for an  $n$ -phase half-wave controlled converter is given by (see example 11.2)

$$V_o = \frac{\sqrt{2}V}{2\pi/n} \int_{\alpha-\pi/n}^{\alpha+\pi/n} \cos \omega t d\omega t \\ = \sqrt{2}V \frac{\sin(\pi/n)}{\pi/n} \cos \alpha \quad (V) \quad (11.114)$$

which for the three-phase circuit considered with **continuous load current** gives

$$V_o = \bar{I}_o R = \frac{3\sqrt{3}}{2\pi} \sqrt{2}V \cos \alpha = 1.17V \cos \alpha \quad 0 \leq \alpha \leq \pi/6 \quad (11.115)$$

For **discontinuous conduction**, the mean output voltage is

$$V_o = \bar{I}_o R = \frac{3\sqrt{3}}{2\pi} \sqrt{2}V (1 + \cos(\alpha + \pi/6)) \quad \pi/6 \leq \alpha \leq 5\pi/6 \quad (11.116)$$

The mean output voltage is zero for  $\alpha = \pi/2$ . For  $0 < \alpha < \pi/6$ , the instantaneous output voltage is always greater than zero. Negative average output voltage occurs when  $\alpha > \pi/2$  as shown in figure 11.16d. Since the load current direction is unchanged, for  $\alpha > \pi/2$ , power reversal occurs, with energy feeding from the load into the ac supply. Power inversion assumes a load with an emf to assist the current flow, as in figure 11.11. If  $\alpha > \pi$  no reverse bias exists for natural commutation and continuous load current will freewheel.

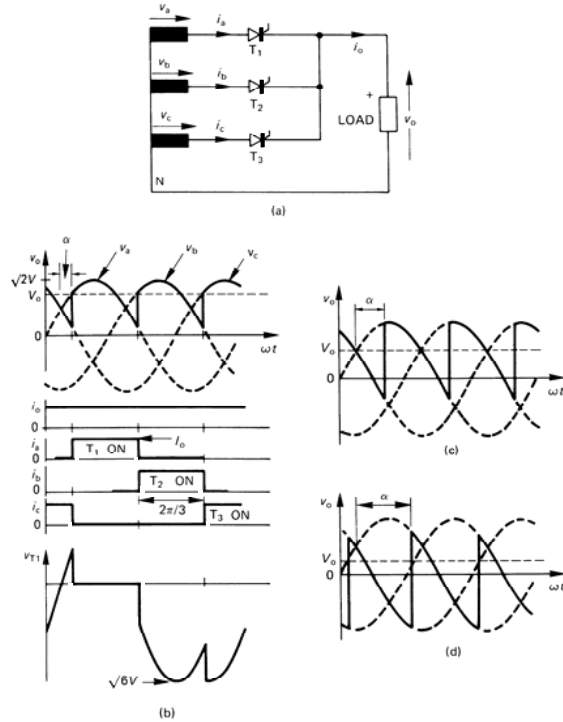


Figure 11.16. Three-phase half-wave controlled converter: (a) circuit connection; (b) voltage and current waveforms for a small firing delay angle  $\alpha$ ; (c) and (d) load voltage waveforms for progressively larger delay angles.

The maximum mean output voltage  $\hat{V}_o = \sqrt{2}V \cdot 3\sqrt{3}/2\pi$  occurs at  $\alpha = 0$ . The normalised mean output voltage  $V_n$  is

$$V_n = V_o / \hat{V}_o = \cos \alpha \quad (11.117)$$

With an  $R$ - $L$  load, at  $V_o = 0$ , the load current falls to zero. Thus for  $\alpha > \frac{1}{2}\pi$ , continuous load current does not flow for an  $R$ - $L$  load.

The rms output voltage is given by

$$V_{rms} = \sqrt{\frac{3}{2\pi} \int_{\alpha-\pi/3}^{\alpha+\pi/3} (\sqrt{2}V)^2 \sin^2(\omega t) d\omega t} \quad (11.118)$$

$$= \sqrt{3}\sqrt{2}V \left( \frac{1}{6} + \frac{\sqrt{3}}{8\pi} \sin 2\alpha \right)^{1/2}$$

### 11.6.2 Half-wave converter with freewheel diode

Figure 11.17 shows a three-phase, half-wave controlled rectifier converter circuit with a load freewheel diode. This diode prevents the load voltage from going negative, thus inversion is not possible.

**11.6.2i - For  $\alpha < \pi/6$**  the output is as in figure 11.16b, with no voltage zeros occurring. The mean output is given by equation (11.115), that is

$$V_o = \bar{I}_o R = \frac{3\sqrt{3}}{2\pi} \sqrt{2}V \cos \alpha = 1.17V \cos \alpha \quad (V) \quad (11.119)$$

$$0 \leq \alpha \leq \pi/6 \quad (\text{rad})$$

The maximum mean output  $V_o = \sqrt{2}V \cdot 3\sqrt{3}/2\pi$  occurs at  $\alpha = 0$ . The normalised mean output voltage,  $V_n$  is given by

$$V_n = V_o / \hat{V}_o = \cos \alpha \quad (11.120)$$

**11.6.2ii - For  $\alpha > \pi/6$** , voltage zeros occur and the negative portions in the waveforms in parts c and d of figure 11.16 do not occur. The mean output voltage is given by

$$V_o = \bar{I}_o R = \frac{\sqrt{2}V}{2\pi/3} \int_{\alpha-\pi/6}^{\pi} \sin \omega t d\omega t$$

$$= \frac{\sqrt{2}V}{2\pi/3} (1 + \cos(\alpha + \pi/6)) \quad (V) \quad (11.121)$$

$$\pi/6 \leq \alpha \leq 5\pi/6$$

The normalised mean output voltage  $V_n$  is

$$V_n = V_o / \hat{V}_o = [1 + \cos(\alpha + \pi/6)] / \sqrt{3} \quad (11.122)$$

The average load current is given by

$$\bar{I}_o = \frac{V_o - E}{R} \quad (11.123)$$

These equations assume continuous load current.

**11.6.2i - For  $\alpha > 5\pi/6$ .** A delay angle of greater than  $5\pi/6$  would imply a negative output voltage, clearly not possible with a freewheel load diode.

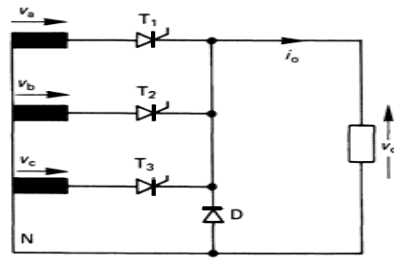


Figure 11.17. A half-wave fully controlled three-phase converter with a load freewheel diode.

### 11.6.3 Full-wave circuit with an inductive load

A three-phase bridge is fully controlled when all six bridge devices are thyristors, as shown in figure 11.18a. The frequency of the output ripple voltage is six times the supply frequency and each thyristor always conducts for  $\frac{2}{3}\pi$ . Circuit waveforms are shown in figure 11.18b. The mean output voltage is given by

$$V_o = \frac{3}{\pi} \int_{\alpha+\pi/6}^{\alpha+5\pi/6} \sqrt{2}\sqrt{3}V \sin(\omega t + \pi/6) d\omega t$$

$$= \frac{3\sqrt{3}}{\pi} \sqrt{2}V \cos \alpha = 2.34V \cos \alpha \quad (\text{V}) \quad (11.124)$$

$$0 \leq \alpha \leq \pi/6$$

which is twice the voltage given by equation (11.115) for the half-wave circuit, and

$$V_o = \frac{3\sqrt{3}}{\pi} \sqrt{2}V [1 + \cos(\alpha + \pi/6)] = 2.34V \cos \alpha \quad (\text{V}) \quad (11.125)$$

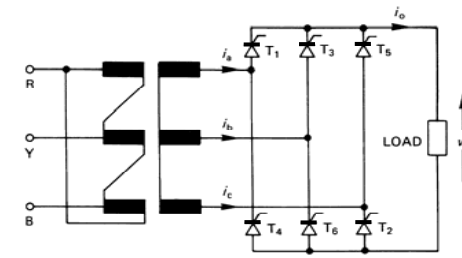
$$\pi/6 \leq \alpha \leq 5\pi/6$$

The average output current is given by  $\bar{I}_o = V_o / R$  in each case. If a load back emf exists the average current becomes

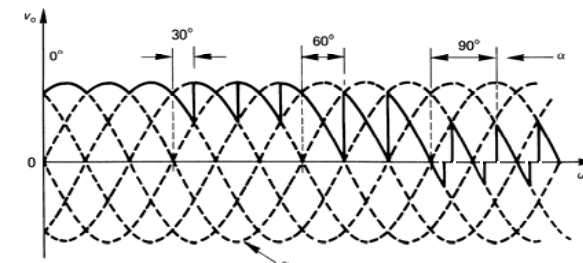
$$\bar{I}_o = \frac{V_o - E}{R} \quad (11.126)$$

The maximum mean output voltage  $\hat{V}_o = \sqrt{2}V \cdot 3\sqrt{3}/\pi$  occurs at  $\alpha = 0$ . The normalised mean output  $V_n$  is

$$V_n = V_o / \hat{V}_o = \cos \alpha \quad (11.127)$$



(a)



(b)

Figure 11.18. A three-phase fully controlled converter: (a) circuit connection and (b) load voltage waveform for four delay angles.

For delay angles up to  $\frac{1}{3}\pi$ , the output voltage is at all instances non-zero, hence the load current is continuous for any passive load. Beyond  $\frac{1}{3}\pi$  the load current may be discontinuous. For  $\alpha > \frac{1}{3}\pi$  the current is always discontinuous for passive loads and the average output voltage is less than zero. With a load back emf the critical inductance for continuous load current must satisfy

$$\frac{R}{Z} \times \left[ \sin(\alpha - \phi + \frac{1}{3}\pi) + \frac{\sin(\alpha - \phi)}{e^{-\pi/3 \tan \phi} - 1} \right] \geq \frac{E}{\sqrt{3}\sqrt{2}V} \quad (11.128)$$

where  $\tan \phi = \omega L / R$ .

The rms value of the output voltage is given by



$$V_{rms} = \left( \frac{3}{\pi} \int_{\alpha-\pi/6}^{\alpha+\pi/2} 3(\sqrt{2}V)^2 \sin^2(\omega t) d\omega t \right)^{1/2} \quad (11.129)$$

$$= \sqrt{3}\sqrt{2}V \left( 1 + \frac{3\sqrt{3}}{2\pi} \sin 2\alpha \right)^{1/2}$$

The normalise voltage harmonic peaks magnitudes in the output voltage, with continuous load current, are

$$V_{L_n} = \sqrt{2}V \frac{3\sqrt{3}}{\pi} \left( \frac{1}{(6n-1)^2} + \frac{1}{(6n+1)^2} - \frac{2\cos 2\alpha}{(6n-1)(6n+1)} \right)^{1/2} \quad (11.130)$$

for  $n = 1, 2, 3, \dots$

The actual harmonics occur at  $6 \times n$ .

#### 11.6.4 Full-wave converter with freewheel diode

Both half-controlled and fully controlled converters can employ a load freewheel diode. These circuits have the voltage output characteristic that the output voltage can never go negative, hence power inversion is not possible. Figure 11.19 shows a fully controlled three-phase converter with a freewheel diode D.

- The freewheel diode is active for  $\alpha > \pi/2$ . The output is as in figure 11.18b for  $\alpha < \pi/2$ . The mean output voltage is

$$V_o = \bar{I}_o R = \frac{3\sqrt{3}}{\pi} \sqrt{2}V \cos \alpha = 2.34V \cos \alpha \quad (V) \quad (11.131)$$

$$0 \leq \alpha \leq \pi/3 \quad (\text{rad})$$

The maximum mean output voltage  $\hat{V}_o = \sqrt{2}V 3\sqrt{3}/\pi$  occurs at  $\alpha = 0$ .

The normalised mean output voltage  $V_n$  is given by

$$V_n = V_o / \hat{V}_o = \cos \alpha \quad (11.132)$$

- while

$$V_o = \bar{I}_o R = \frac{3\sqrt{3}}{\pi} \sqrt{2}V (1 + \cos(\alpha + \pi/3)) \quad (V) \quad (11.133)$$

$$\pi/3 \leq \alpha \leq 2\pi/3 \quad (\text{rad})$$

The normalised mean output,  $V_n$  is

$$V_n = V_o / \hat{V}_o = 1 + \cos(\alpha + \pi/3) \quad (11.134)$$

- while

$$V_o = 0 \quad (V) \quad (11.135)$$

$$2\pi/3 \leq \alpha \quad (\text{rad})$$

In each case the average output current is given by  $\bar{I}_o = V_o / R$ , which can be modified to include any load back emf, that is,  $I_o = (V_o - E) / R$ .

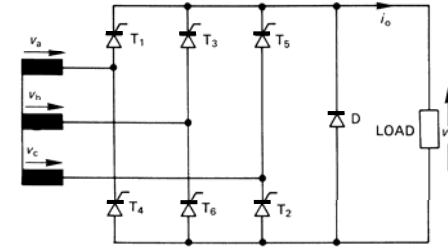


Figure 11.19. A full-wave three-phase controlled converter with a load free-wheeling diode (half-controlled).

#### Example 11.6: Converter average load voltage

Derive a general expression for the average load voltage of an  $n$ -pulse controlled converter.

#### Solution

Figure 11.20 defines the general output voltage waveform where  $n$  is the output pulse number. From the output voltage waveform

$$V_o = \frac{1}{2\pi/n} \int_{-\pi/n+\alpha}^{\pi/n+\alpha} \sqrt{2}V \cos \omega t d\omega t$$

$$= \frac{\sqrt{2}V}{2\pi/n} (\sin(\alpha + \pi/n) - \sin(\alpha - \pi/n))$$

$$= \frac{\sqrt{2}V}{2\pi/n} 2\sin(\pi/n) \cos \alpha$$

$$V_o = \frac{\sqrt{2}V}{\pi/n} \sin(\pi/n) \cos \alpha$$

$$= \hat{V}_o \cos \alpha \quad (V)$$

where

for  $n = 2$  for the single-phase full-wave controlled converter in figure 11.9.  
for  $n = 3$  for the three-phase half-wave controlled converter in figure 11.16.  
for  $n = 6$  for the three-phase full-wave controlled converter in figure 11.18.

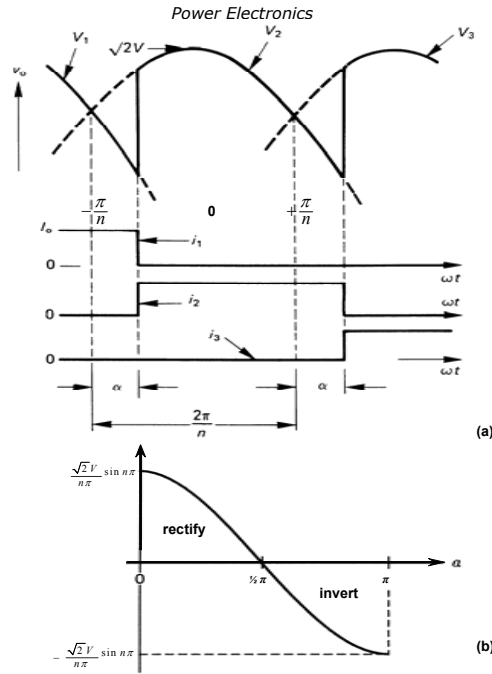


Figure 11.20. A half-wave  $n$ -phase controlled converter: (a) output voltage and current waveform and (b) transfer function of voltage versus delay angle  $\alpha$ .

♣

## 11.7 Overlap

In the previous sections, impedance of the ac source has been neglected, such that current transfers or *commutates* instantly from one switch to the other with higher anode potential. However, in practice the source has inductive reactance  $X_c$  and current takes a finite time to fall in the device turning off and rise in the device turning on.

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## Naturally commutating converters

Consider the three-phase half-wave controlled rectifying converter in figure 11.16a, where it is assumed that a continuous dc load current,  $I_o$ , flows. When thyristor  $T_1$  is conducting and  $T_2$  is turned on after delay  $\alpha$ , the equivalent circuit is shown in figure 11.21a. The source reactances  $X_1$  and  $X_2$  limit the rate of change of current in  $T_1$  as  $i_1$  decreases from  $I_o$  to 0 and in  $T_2$  as  $i_2$  increases from 0 to  $I_o$ . These current transitions in  $T_1$  and  $T_2$  are shown in the waveforms of figure 11.21d. A circulating current,  $i$ , flows between the two thyristors. If the line reactances are identical, the output voltage during commutation,  $v_o$ , is mid-way between the conducting phase voltages  $v_1$  and  $v_2$ , as shown in figure 11.21b. That is  $v_o = \frac{1}{2}(v_1 + v_2)$ , creating a series of notches in the output voltage waveform as shown in figure 11.21c. This interval during which both  $T_1$  and  $T_2$  conduct ( $i \neq 0$ ) is termed the *overlap period* and is defined by the *overlap angle*  $\gamma$ . Ignoring thyristor voltage drops, the overlap angle is calculated as follows

$$v_2 - v_1 = 2L \, di / dt$$

With reference  $t = 0$  when  $T_2$  is triggered

$$v_2 - v_1 = v_L = \sqrt{3} v_{\text{phase}} = \sqrt{3} \sqrt{2} V \sin(\omega t + \alpha)$$

where  $V$  is the line to neutral rms voltage.

Equating these two equations

$$2L \, di / dt = \sqrt{3} \sqrt{2} V \sin(\omega t + \alpha)$$

Rearranging and integrating gives

$$i(\omega t) = \frac{\sqrt{3} \sqrt{2} V}{2\omega L} (\cos \alpha - \cos(\omega t + \alpha))$$

Commutation from  $T_1$  to  $T_2$  is complete when  $i = I_o$ , at  $\omega t = \gamma$ , that is

$$I_o = \frac{\sqrt{3} \sqrt{2} V}{2\omega L} (\cos \alpha - \cos(\gamma + \alpha)) \quad (\text{A}) \quad (11.136)$$

Figure 11.21b shows that the load voltage comprises the phase voltage  $v_2$  when no source inductance exists minus the voltage due to circulating current  $v_\gamma (= \frac{1}{2}(v_1 + v_2))$  during commutation.

The mean output voltage  $V_o'$  is therefore

$$V_o' = V_o - \bar{v}_\gamma$$

$$= \frac{1}{2\pi/3} \left[ \int_{\alpha+\pi/6}^{\alpha+5\pi/6} v_2 \, d\omega t - \int_{\alpha+\pi/6}^{\gamma+\alpha+\pi/6} v_\gamma \, d\omega t \right]$$

where  $v_\gamma = \frac{1}{2}(v_1 + v_2)$

$$V_o' = \frac{3}{2\pi} \left[ \int_{\alpha+\pi/6}^{\alpha+5\pi/6} \sqrt{2} V \sin(\omega t + \alpha) \, d\omega t - \int_{\alpha+\pi/6}^{\gamma+\alpha+\pi/6} \sqrt{2} V \left\{ \sin\left(\omega t + \frac{2\pi}{3}\right) + \sin \omega t \right\} d\omega t \right]$$

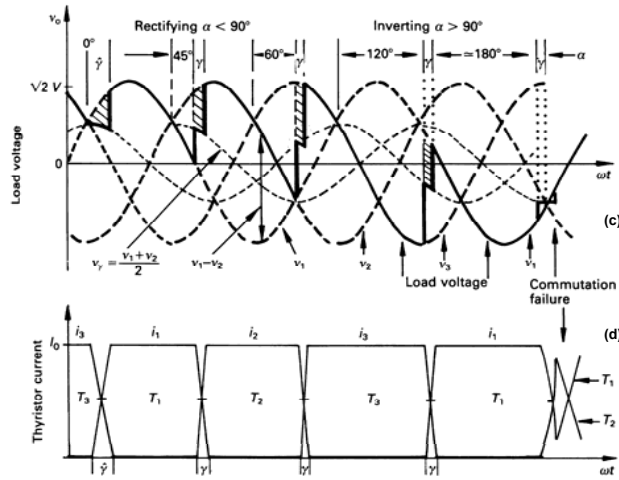
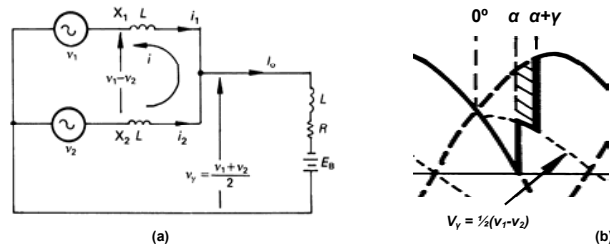


Figure 11.21. Overlap: (a) equivalent circuit during overlap; (b) angle relationships; (c) load voltage for different delay angles  $\alpha$  (hatched areas equal to  $I_o L$ ; last overlap shows commutation failure); and (d) thyristor currents showing eventual failure.

$$V_o' = \frac{3}{2\pi} \sqrt{3} \sqrt{2} V \cos \alpha - \frac{3}{2\pi} \frac{\sqrt{3}}{2} \sqrt{2} V (\cos \alpha - \cos(\alpha - \gamma)) \quad (11.137)$$

$$V_o' = \frac{3\sqrt{3}}{4\pi} \sqrt{2} V [\cos \alpha + \cos(\alpha + \gamma)] \quad (11.138)$$

which reduces to equation (11.115) when  $\gamma = 0$ . Substituting  $\cos \alpha - \cos(\alpha + \gamma)$  from equation (11.136) into equation (11.137) yields

$$V_o' = \frac{3\sqrt{3}}{2\pi} \sqrt{2} V \cos \alpha - \frac{3}{2\pi} \omega L I_o \quad (11.139)$$

$$\text{that is } V_o' = V_o - \frac{3}{2\pi} \omega L I_o \quad (11.140)$$

The mean output voltage  $V_o$  is reduced or regulated by the commutation reactance  $X_c = \omega L$  and varies with load current magnitude  $I_o$ . Converter semiconductor voltage drops also regulate the output voltage. The component  $3\omega L/2\pi$  is called the *equivalent internal resistance*. Being an inductive phenomenon, it does not represent a power loss component.

The overlap occurs immediately after the delay  $\alpha$ . The commutation voltage,  $v_2 - v_1$ , is  $\sqrt{3} \sqrt{2} V \sin \alpha$ . The commutation time is inversely proportional to the commutation voltage  $v_2 - v_1$ . As  $\alpha$  increases to  $\pi$ , the commutation voltage increases to a maximum and the overlap angle  $\gamma$  decreases to a minimum at  $\pi/2$ . From equation (11.136), with  $\alpha = \pi$

$$\gamma = \arcsin(2\omega L I_o / \sqrt{2} \sqrt{3} V)$$

The general expressions for the mean load voltage  $V_o'$  of a  $n$ -pulse, fully-controlled rectifier, with underlap, are given by

$$V_o' = \frac{\sqrt{2} V}{2\pi/n} \sin \pi/n [\cos \alpha + \cos(\alpha + \gamma)] \quad (11.141)$$

and

$$V_o' = \frac{\sqrt{2} V}{\pi/n} \sin \pi/n \cos \alpha - n X_c I_o / 2\pi \quad (11.142)$$

where  $V$  is the line voltage for a full-wave converter and the phase voltage for a half-wave converter. Effectively, as shown in figure 11.22, overlap reduces the mean output voltage by  $n I_o L$  or as if  $\alpha$  were increased. The supply voltage is effectively distorted and the harmonic content of the output is increased. Equating equations (11.141) and (11.142) gives the mean output current

$$I_o = \frac{\sqrt{2} V}{X_c} \sin \pi/n (\cos \alpha - \cos(\gamma + \alpha)) \quad (A) \quad (11.143)$$

which reduces to equation (11.136) when  $n = 3$ .

Harmonic input current magnitudes are decreased by a factor  $\sin(\pi/n\gamma)/\pi/n\gamma$ .

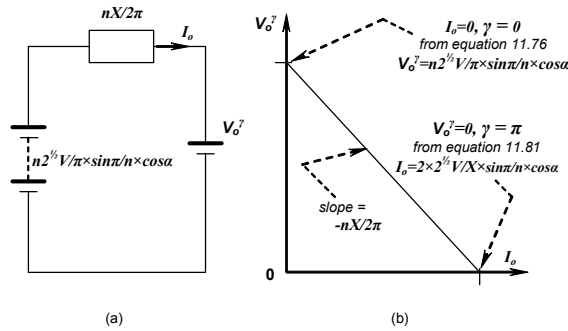


Figure 11.22. Overlap regulation model:  
(a) equivalent circuit and (b) load plot of overlap model.

### 11.8 Overlap - inversion

A fully controlled converter operates in the inversion mode when  $\alpha > 90^\circ$  and the mean output voltage is negative and less than the load back emf shown in figure 11.21a. Since the direction of the load current  $I_o$  is from the supply and the output voltage is negative, energy is being returned, *regenerated* into the supply from the load. Figure 11.23 shows the power flow differences between rectification and inversion. As  $\alpha$  decreases, the return energy magnitude increases. If  $\alpha$  plus the overlap  $\gamma$  exceeds  $\omega t = \pi$ , commutation failures occur. The output goes positive and the load current builds up uncontrolled. The last commutation with  $\alpha \approx \pi$  in figures 11.21b and c results in a commutation failure of thyristor  $T_1$ . Before the circulating inductor current  $i$  has reduced to zero, the incoming thyristor  $T_2$  experiences an anode potential which is less positive than that of the thyristor to be commutated  $T_1$ ,  $v_1 - v_2 < 0$ . The incoming device  $T_2$  fails to stay on and conduction continues through  $T_1$ , impressing positive supply cycles across the load. This positive converter voltage aids the load back emf and the load current builds up uncontrolled.

Equations (11.141) and (11.142) are valid provided a commutation failure does not occur. The controllable delay angle range is curtailed to

$$0 \leq \alpha \leq \pi - \gamma$$

The maximum allowable delay angle  $\hat{\alpha}$  occurs when  $\hat{\alpha} + \gamma = \pi$  and from equations (11.141) and (11.142) with  $\alpha + \gamma = \pi$  gives

$$\hat{\alpha} = \cos^{-1} \left\{ \frac{XI_o}{\sqrt{2}V \sin \pi / n} - 1 \right\} < \pi \quad (\text{rad}) \quad (11.144)$$

In practice commutation must be complete  $\delta$  rad before  $\omega t = \pi$ , in order to allow the outgoing thyristor to regain a forward blocking state. That is  $\alpha + \gamma + \delta < \pi$ .  $\delta$  is known as the *recovery* or *extinction* angle.

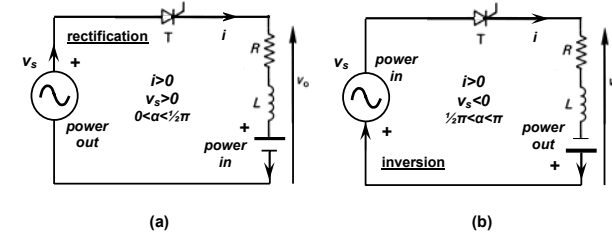


Figure 11.23. Controlled converter model showing: (a) rectification and (b) inversion.

#### Example 11.7: Converter overlap

A three-phase full-wave converter is supplied from the 415 V ac, 50 Hz mains with phase source inductance of 0.1 mH. If the average load current is 100 A continuous, determine the supply reactance voltage drop, the overlap angle, and mean output voltage for phase delay angles of (i)  $0^\circ$  and (ii)  $60^\circ$

Ignoring thyristor forward blocking time requirements, determine the maximum allowable delay angle.

#### Solution

Using equations (11.141) and (11.142) with  $n = 6$  and  $V = 415$  V ac, the mean supply reactance voltage

$$\bar{v}_r = \frac{n}{2\pi} 2\pi f L I_o = \frac{6}{2\pi} \times 2\pi \times 50 \times 10^{-4} \times 10^2 = 3 \text{ V}$$

- i.  $\alpha = 0^\circ$  - as for uncontrolled rectifiers. From equation (11.142), the maximum output voltage is

$$\begin{aligned} V_o^r &= \frac{\sqrt{2}V}{2\pi/n} \sin \pi / n \cos \alpha - nX_c I_o / 2\pi \\ &= \frac{\sqrt{2} \times 415}{2\pi/6} \sin \pi / 6 \cos 0 - 3 \text{ V} = 557.44 \text{ V} \end{aligned}$$

From equation (11.141)

$$V_o' = \frac{\sqrt{2}V}{2\pi/n} \sin \pi/n [\cos \alpha + \cos(\alpha + \gamma)]$$

$$557.44 = \frac{\sqrt{2} \times 415}{2\pi/6} \times \sin \pi/6 \times [1 + \cos \gamma]$$

that is  $\gamma = 8.4^\circ$

ii.  $\alpha = 60^\circ$

$$V_o' = \frac{\sqrt{2}V}{2\pi/n} \sin \pi/n \cos \alpha + nX_c I_o / 2\pi$$

$$= \frac{\sqrt{2} \times 415}{2\pi/6} \sin \pi/6 \times \cos 60^\circ - 3V = 277.22V$$

$$V_o' = \frac{\sqrt{2}V}{2\pi/n} \sin \pi/n [\cos \alpha + \cos(\alpha + \gamma)]$$

$$277.22 = \frac{\sqrt{2} \times 415}{2\pi/6} \times \frac{1}{2} \times [\cos 60^\circ + \cos(60^\circ + \gamma)]$$

that is  $\gamma = 0.71^\circ$

Equation (11.144) gives the maximum allowable delay angle as

$$\hat{\alpha} = \cos^{-1} \left\{ \frac{X I_o}{\sqrt{2} V \sin \pi/n} - 1 \right\}$$

$$= \cos^{-1} \left\{ \frac{2\pi 50 \times 10^{-4} \times 10^2}{\sqrt{2} \times 415 \times \frac{1}{2}} - 1 \right\}$$

$$= 171.56^\circ \text{ and } V_o' = -557.41V$$

♣

## 11.9 Summary

General expressions for n-phase converter mean output voltage,  $V_o$

(i) Half-wave and full-wave, fully-controlled converter

$$V_o = \sqrt{2} V \frac{\sin(\pi/n)}{\pi/n} \cos \alpha$$

where  $V$  is

the rms line voltage for a full-wave converter or  
the rms phase voltage for a half-wave converter.  
 $\cos \alpha = \cos \psi$ , the supply displacement factor

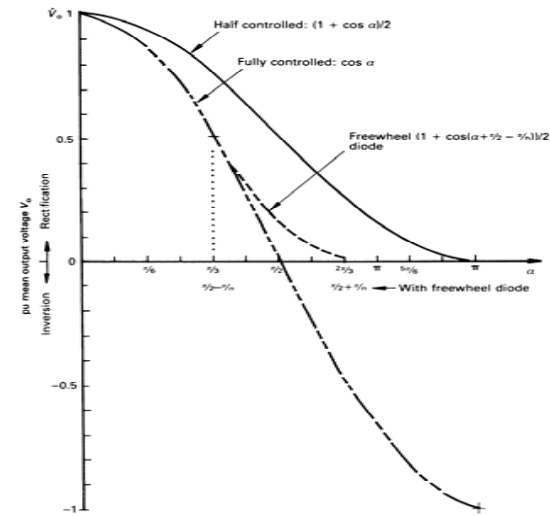


Figure 11.24. Converter normalised output voltage characteristics as a function of firing delay angle  $\alpha$ .

(ii) Full-wave, half-controlled converter

$$V_o = \sqrt{2} V \frac{\sin(\pi/n)}{\pi/n} (1 + \cos \alpha)$$

where  $V$  is

the rms line voltage.

(iii) Half-wave and full-wave controlled converter with load freewheel diode

$$V_o = \sqrt{2} V \frac{\sin(\pi/n)}{\pi/n} \cos \alpha \quad 0 < \alpha < \frac{1}{2}\pi - \pi/n$$

$$V_o = \sqrt{2} V \frac{1 + \cos(\alpha + \frac{1}{2}\pi - \pi/n)}{2\pi/n} \quad \frac{1}{2}\pi - \pi/n < \alpha < \frac{1}{2}\pi + \pi/n$$

the output rms voltage is given by

$$V_{rms} = \sqrt{2} V \sqrt{\frac{1}{2} + \frac{\cos 2\alpha \sin 2\pi/n}{4\pi/n}} \quad \alpha + \pi/n \leq \frac{1}{2}\pi$$

$$V_{rms} = \sqrt{2} V \sqrt{\frac{1}{4} + \frac{n}{8} - \frac{\alpha}{4\pi/n} - \frac{\cos(2\alpha - 2\pi/n)}{8\pi/n}} \quad \alpha + \pi/n > \frac{1}{2}\pi$$

where  $V$  is

the rms line voltage for a full-wave converter or

the rms phase voltage for a half-wave converter.

$n = 0$  for single-phase and three-phase half-controlled converters

$= \frac{1}{6}\pi$  for three-phase half-wave converters

$= \frac{1}{2}\pi$  for three-phase fully controlled converters

These voltage output characteristics are shown in figure 11.24 and the main converter circuit characteristics are shown in table 11.1.

### 11.10 Definitions

$V_o$  average output voltage  $\bar{I}_o$  average output current

$V_{rms}$  rms output voltage  $I_{rms}$  rms output current

$\hat{V}$  peak output voltage  $\hat{I}$  peak output current

Load voltage form factor  $= FF_v = \frac{V_{rms}}{V_o}$  Load voltage crest factor  $= CF_v = \frac{\hat{V}}{V_{rms}}$

Load current form factor  $= FF_i = \frac{I_{rms}}{I_o}$  Load current crest factor  $= CF_i = \frac{\hat{I}}{I_{rms}}$

$$\text{Rectification efficiency} = \eta = \frac{\text{dc load power}}{\text{ac load power} + \text{rectifier losses}}$$

$$= \frac{V_o I_o}{V_{rms} I_{rms} + \text{Loss}_{\text{rectifier}}}$$

$$\text{Waveform smoothness} = \text{Ripple factor} = K_v = \frac{\text{effective values of ac } V \text{ (or } I\text{)}}{\text{average value of } V \text{ (or } I\text{)}} = \frac{V_{Rv}}{V_o}$$

$$= \sqrt{\frac{V_{rms}^2 - V_o^2}{V_o^2}} = \sqrt{FF_v^2 - 1}$$

$$\text{where } V_{Rv} = \left[ \sum_{n=1}^{\infty} \frac{1}{2} (V_{an}^2 + V_{bn}^2) \right]^{1/2}$$

$$\text{similarly the current ripple factor is } K_i = \frac{I_{Ri}}{I_o} = \sqrt{FF_i^2 - 1}$$

$$K_i = K_v \text{ for a resistive load}$$

### Reading list

Dewan, S. B. and Straughen, A., *Power Semiconductor Circuits*, John Wiley and Sons, New York, 1975.

Sen, P.C., *Power Electronics*, McGraw-Hill, 5<sup>th</sup> reprint, 1992.

Shepherd, W *et al. Power Electronics and motor control*, Cambridge University Press, Second Edition 1995.

<http://www.ipes.ethz.ch/>

Table 11.1 Main characteristics of converter circuits

Output phase number $n$ and ripple frequency ( $\times f_s$ ):		1		2		3		6	
Type of controlled circuit:		Single-phase bridge		Two-phase half-wave		Single-phase bridge		Three-phase bridge	
Text figure number:		11.6		11.9a		11.9b		11.18	
Mean output voltage	Maximum output voltage $V_o$ $\alpha = 0$ or diode bridge $V$ is rms phase voltage	$\frac{\sqrt{2} V}{\pi}$ (0.45 $V$ )		$2\sqrt{2} V/\pi$		(0.9 $V$ )		$3\sqrt{3}/2 V/\pi$ (1.17 $V$ )	
	Normalised controlled mean output voltage $V_o/V_o$	$\frac{1 + \cos \alpha}{2}$		$\frac{1 + \cos \alpha}{2}$		$\frac{1 + \cos \alpha}{2}$		$\frac{1 + \cos \alpha}{2}$	
	Pure resistive load or with freewheel diode $D_f$ Inductive load without $D_f$	$\frac{1 + \cos \alpha}{2}$		$\frac{1 + \cos \alpha}{2}$		$\frac{1 + \cos \alpha}{2}$		$\frac{1 + \cos \alpha}{2}$	

Equivalent internal resistance $i nX/2\pi \quad X = \omega L$		0.318X	0.637X	0.477X	0.955X
Output voltage ripple ratio (per cent) ( $\alpha = 0, \gamma = 0$ )		121	48	19	4.2
Rectifying device	Average current $I_o/n$	$I_o$	$I_o/2$	$I_o/3$	$I_o/3$
	Peak voltage, $\times V$	$\sqrt{2}$	$2\sqrt{2}$	$\sqrt{2}$	$\sqrt{3}\sqrt{2}$
Supply rms currents	Fundamental $I_i$		$2\sqrt{2} I_o/\pi$	$2\sqrt{2} I_o/\pi \cos \alpha/2$	$\sqrt{6} I_o/\pi$
	Total $I_i$		$I_o$	$I_o \sqrt{1 - \alpha/\pi}$	$\sqrt{\frac{2}{3}} I_o$
Supply factors	Harmonic factor $\rho$		$\sqrt{\frac{\pi^2}{8} - 1}$	$\sqrt{\frac{\pi(\pi - \alpha)}{4(1 + \cos \alpha)}} - 1$	$\sqrt{\left(\frac{\pi}{3}\right)^2 - 1}$
	Displacement factor, $\cos \psi$		$\cos -\alpha$	$\cos -\alpha/2$	$\cos -\alpha$
	Power factor $\lambda$		$\frac{2\sqrt{2}}{\pi} \cos \alpha$	$\frac{\sqrt{2(1 + \cos \alpha)}}{\sqrt{\pi(\pi - \alpha)}}$	$\frac{3}{\pi} \cos \alpha$

## Problems

- 11.1. For the circuit shown in figure 11.25, if the thyristor is fired at  $\alpha = \frac{1}{2}\pi$
- derive an expression for the load current,  $i$
  - determine the current extinction angle,  $\beta$
  - determine the peak value and the time at which it occurs
  - sketch to scale on the same  $\omega t$  axis the supply voltage, load voltage, thyristor voltage, and load current

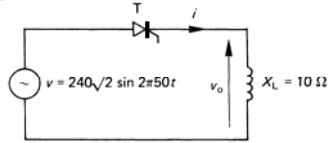


Figure 11.25. Problem 11.1.

- 11.2. For the circuit shown in figure 11.26, if the thyristor is fired at  $\alpha = \frac{1}{4}\pi$  determine
- the current extinction angle,  $\beta$
  - the mean and rms values of the output current
  - the power delivered to the source  $E$ .

Sketch the load current and load voltage  $v_o$ .

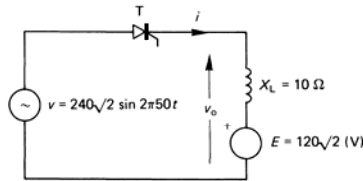


Figure 11.26. Problem 11.2.

- 11.3. Derive equations (11.17) and (11.18) for the circuit in figure 11.3.
- 11.4. Assuming a constant load current derive an expression for the mean and rms device current and the device form factor, for the circuits in figures 11.4 and 11.5.

- 11.5. Plot load ripple voltage  $K_{RI}$  and load voltage ripple factor  $K_v$ , against the thyristor phase delay angle  $\alpha$  for the circuit in figure 11.5.

- 11.6. Show that the average output voltage of a  $n$ -phase half-wave controlled converter with a freewheel diode is characterised by

$$V_o = \sqrt{2} V \frac{\sin(\pi/n)}{\pi/n} \cos \alpha \quad (V)$$

$$0 < \alpha < \frac{1}{2}\pi/n$$

$$V_o = \sqrt{2} V \frac{1 + \cos \alpha + \frac{1}{2}\pi - \frac{1}{n}\pi}{2\pi/n} \quad (V)$$

$$\frac{1}{2}\pi - \frac{1}{n}\pi < \alpha < \frac{1}{2}\pi + \frac{1}{n}\pi$$

- 11.7. Show that the average output voltage of a single-phase fully controlled converter is given by

$$V_o = \frac{2\sqrt{2} V}{\pi} \cos \alpha$$

Assume that the output current  $I_o$  is constant.

Prove that the supply current Fourier coefficients are given by

$$a_n = -\frac{4I_o}{n\pi} \sin n\alpha$$

$$b_n = \frac{4I_o}{n\pi} \cos n\alpha$$

for  $n$  odd.

Hence or otherwise determine (see section 12.6)

- the displacement factor,  $\cos \psi$
- the distortion factor,  $\mu$
- the total supply power factor  $\lambda$ .

Determine the supply harmonic factor,  $\rho$ , if

$$\rho = I_h / I_1$$

where  $I_h$  is the total harmonic current and  $I_1$  is the fundamental current.

- 11.8. Show that the average output voltage of a single-phase half-controlled converter is given by

$$v_o = \frac{\sqrt{2} V}{\pi} (1 + \cos \alpha)$$

Assume that the output current  $I_o$  is constant.

- Determine
- the displacement factor,  $\cos \psi$
  - the distortion factor,  $\mu$
  - the total supply power factor,  $\lambda$ .

Show that the supply harmonic factor,  $\rho$  (see problem 11.7), is given by



$$\rho = \sqrt{\left[ \frac{\pi(\pi - \alpha)}{4(1 + \cos \alpha)} - 1 \right]}$$

11.9. Draw the load voltage and current waveforms for the circuit in figure 11.8a when a freewheel diode is connected across the load. Specify the load rms voltage.

11.10. A centre tapped transformer, single-phase, full-wave converter (figure 11.9a) with a load freewheel diode is supplied from the 240 V ac, 50 Hz supply with source inductance of 0.25 mH. The continuous load current is 5 A. Find the overlap angles for

- the transfer of current from a conducting thyristor to the load freewheel diode and
- from the freewheel diode to a thyristor when the delay angle  $\alpha$  is  $30^\circ$ .

$$\gamma_{r-d} = \cos^{-1} \left\{ 1 - \frac{\omega L_a}{\sqrt{2} V} \right\} = 2.76^\circ;$$

$$\gamma_{r-d} = \cos^{-1} \left\{ \cos \alpha - \frac{\omega L_a}{\sqrt{2} V} \right\} - \alpha = 0.13^\circ$$

11.11. The circuit in figure 11.6a, with  $v = \sqrt{2} V \sin(\omega t + \alpha)$ , has a steady-state time response of

$$i(\omega t) = \frac{\sqrt{2} V}{Z} \left\{ \sin(\omega t + \alpha - \phi) - \sin(\alpha - \phi) e^{-Rt/L} \right\}$$

where  $\alpha$  is the trigger phase delay angle after voltage crossover, and

$$\phi = \tan^{-1}(\omega L / R)$$

Sketch the current waveform for  $\alpha = \pi/4$  and  $Z$  with

- $R \gg \omega L$
- $R = \omega L$
- $R \ll \omega L$ .

$$[(\sqrt{2} V/R) \sin(\omega t + \pi/4); (V/R) \sin \omega t; (V/\omega L) (\sin \omega t - \cos \omega t + 1)]$$

11.12. A three-phase, fully-controlled converter is connected to the 415 V supply, which has a reactance of  $0.25 \Omega/\text{phase}$  and resistance of  $0.05 \Omega/\text{phase}$ . The converter is operating in the inverter mode with  $\alpha = 150^\circ$  and a continuous 50 A load current. Assuming a thyristor voltage drop of 1.5 V at 50 A, determine the mean output voltage, overlap angle, and available recovery angle.

$$[-485.36 \text{ V } -3 \text{ V } -5 \text{ V } -11.94 \text{ V } -505.3 \text{ V}; 6.7^\circ; 23.3^\circ]$$

11.13. For the converter system in problem 11.12, what is the maximum dc current that can be accommodated at a phase delay of  $165^\circ$ , allowing for a recovery angle of  $5^\circ$ ?

$$[35.53 \text{ A}]$$

11.14. The single-phase half-wave controlled converter in figure 11.6 is operated from the 240 V, 50 Hz supply and a  $10 \Omega$  resistive load. If the mean load voltage is 50 per cent of the maximum mean voltage, determine the (a) delay angle,  $\alpha$ , (b) mean and rms load current, and (c) the input power factor.

11.15. The converter in figure 11.8a, with a freewheel diode, is operated from the 240 V, 50 Hz supply. The load consists of, series connected, a  $10 \Omega$  resistor, a 5 mH inductor and a 40 V battery. Derive the load voltage expression in the form of a Fourier series. Determine the rms value of the fundamental of the load current.

11.16. The converter in figure 11.5a is operated from the 240 V, 50 Hz supply with a load consisting of the series connection of a  $10 \Omega$  resistor, a 5 mH inductor, and a 40 V battery. Derive the load voltage expression in the form of a Fourier series. Determine the rms value of the fundamental of the load current.

11.17. The converter in figure 11.17 is operated from a Y-connected, 415 V, 50 Hz supply. If the load is 100 A continuous with a phase delay angle of  $\pi/6$ , calculate the (a) harmonic factor of the supply current, (b) displacement factor  $\cos \psi$ , and (c) supply power factor,  $\lambda$ .

11.18. The converter in figure 11.17 is operated from the 415 V line-to-line voltage, 50 Hz supply, with a series load of  $10 \Omega + 5 \text{ mH} + 40 \text{ V}$  battery. Derive the load voltage expression in terms of a Fourier series. Determine the rms value of the fundamental of the load current.

11.19. Repeat problem 11.18 for the three-phase, half-controlled converter in figure 11.15.

11.20. Repeat problem 11.18 for the three-phase, fully-controlled converter in figure 11.18.

11.21. The three-phase, half-controlled converter in figure 11.15 is operated from the 415 V, 50 Hz supply, with a 100 A continuous load current. If the line inductance is  $0.5 \text{ mH}/\text{phase}$ , determine the overlap angle  $\gamma$  if (a)  $\alpha = \pi/6$ , and (b)  $\alpha = \pi/2$ .

11.22. Repeat *example 11.1* using a 100Vac 60Hz supply.