

# Multiobjective Power System Optimization Including Security Constraints

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**Abstract--** In this paper, an optimal power flow method is proposed to minimize the total cost of generation and minimize the transmission loss. The key concept is the inclusion of security constraints so that the obtained solutions are secure for credible contingencies. The power system operator can select the solution that meets the desired objectives from the available multiple solutions. Case studies are presented to illustrate the proposed method.

**Index Terms—**power system security, multiobjective optimization, particle swarm optimization.

## I. INTRODUCTION

Optimization is one of the challenging problems in power system operation. The goal of optimization is to minimize (or maximize) a specific objective function subject to the operational constraints of the power system [1]. In recent years there has been interest in applying multiobjective optimization for power system problems [2,3]. Multiobjective optimization can be considered as optimizing many objective functions subject to different constraints. For power system applications, these objective functions can be cost, transmission loss, voltage deviation etc. Many tools are available to solve multiobjective optimization problems. Particle Swarm Optimization and Genetic Algorithms [4,5] are some of the techniques that have been proposed recently.

For power systems applications, many of the proposed methods for multiobjective optimization focus on the constraints related to the steady state operation. Security constraints (i.e. operation of the power system under credible contingencies) are not considered in detail. The objective of this paper is to consider multiobjective optimization for power systems including security constraints. The goal is to achieve the different objectives while maintaining security constraints.

This paper is organized as follows: section II presents an overview of multiobjective optimization. Section III discusses the different tools for multiobjective optimization. A simple example using Particle Swarm Optimization is also presented. In section IV, the formulation of the problem for power system applications including security constraints is discussed. Case studies using simple power system models are presented in section V. Section VI provides some concluding remarks.

## II. MULTIOBJECTIVE OPTIMIZATION

Engineering design often deals with multiple, possibly conflicting, objective functions or design criteria [6]. As an example, one may want to maximize the performance of a system while minimizing its cost. Such design problems are the subject of multiobjective optimization and can generally be formulated as:

$$\begin{aligned} \text{min } & J(x, p) \\ \text{subject to } & g(x, p) \leq 0 \\ & h(x, p) = 0 \end{aligned} \quad (1)$$

$$J = [J_1(x) \dots J_z(x)]^T$$

$$x = [x_1 \dots x_n]^T$$

$$g = [g_1(x) \dots g_{m_1}(x)]^T$$

$$h = [h_1(x) \dots h_{m_2}(x)]^T$$

In equation (1),  $J$  is an objective function vector,  $x$  is a design vector,  $p$  is a vector of fixed parameters,  $g$  is an inequality constraint vector and  $h$  is an equality constraint vector. There are  $z$  objectives,  $m_1$  inequality constraints and  $m_2$  equality constraints. Compared to single objective problems, multiobjective problems are more difficult to solve, since there is no unique solution. There is a set of acceptable trade-off optimal solutions. This set is called Pareto front. The preferred solution, the one most desirable to the decision maker, is selected from the Pareto set. The Pareto set allows the decision maker to make an informed decision by observing a wide range of options since it contains the solutions that are optimum from an overall standpoint. A vector of decision variable is *Pareto Optimal* if there is no feasible vector of decision variables  $x$  which will decrease some criterion without causing a simultaneous increase in at least one another criterion.

One of the most widely used methods for solving multiobjective optimization problems is to transform a multiobjective problem into a series of single objective problems. The weighted sum method is a traditional method that parametrically changes the weights among the objective functions to obtain the Pareto front. The weight of an objective

can be chosen in proportion to the objective's relative importance in the problem.  $\epsilon$ -constraint method, weighted metric methods, value function methods, goal programming methods are some of the other available methods [4, 7]. For the power system studies, presented in this paper, the weighted sum method for multiobjective optimization is used [7].

### III. TOOLS FOR MULTIOBJECTIVE PROBLEMS

Different software tools have been proposed to solve multiobjective optimization problems. Some tools are commercially available. Some tools are in the research stage. This section discusses three of the software tools.

#### A. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a heuristic search technique [8,9] that simulates the movements of a flock of birds which aim to find food. The relative simplicity of PSO and the fact that it is a population-based technique have made it a natural candidate to be extended for multi-objective optimization [5,10].

As an example, consider a two-objective function to be minimized.

$$\begin{aligned} f1 &= x1^4 - 10 * x1^2 + x1 * x2 + x2^4 - x1^2 * x2^2 \\ f2 &= x2^4 - x1^2 * x2^2 + x1^4 + x1 * x2 \end{aligned} \quad (2)$$

The two functions are combined into a single objective function (unconstrained) based on the weighted sum method. The solution corresponding to different weights is determined using an unconstrained minimization method. Figure 1 shows the Pareto front obtained by PSO. The results are obtained using a source code of PSO available in the public domain [11]. Interestingly, this figure matches the Pareto front obtained by implementing Genetic Algorithm Toolbox in Matlab [12]. The greatest advantages of PSO are its simplicity, its ease of use and its high convergence rate. However, to develop PSO algorithms for multiobjective optimization, considering both equality and inequality constraints is challenging. This is typical of power system problems. The different parameters of PSO algorithm have to be selected carefully [13].

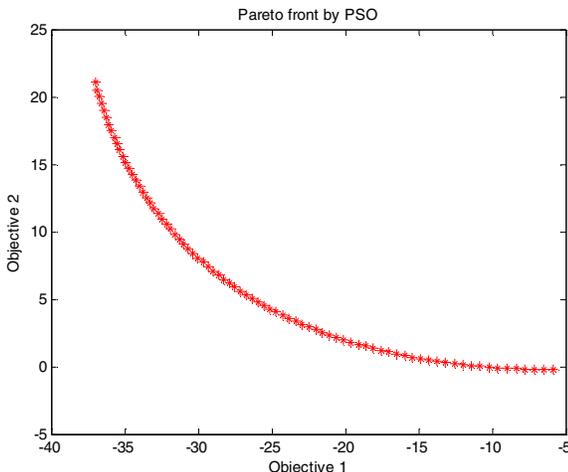


Fig. 1. Pareto front by PSO for the two objective example

#### B. Evolutionary Algorithms (EA)

There has been considerable interest recently in the investigation of evolutionary algorithms for multiobjective optimization problems [4,5]. These may be considered as non-classical, unorthodox and stochastic search and optimization algorithms. EAs use a population of solutions in each iteration, instead of a single solution. If an optimization problem has multiple optimal solutions, an EA can be used to capture multiple optimal solutions in its final population. This ability of an EA to find multiple optimal solutions in one single simulation run makes EAs unique in solving multiobjective optimization problems. Multiobjective Genetic Algorithm (MOGA), Nondominated Sorting Genetic Algorithm (NSGA), Niche-Pareto Genetic Algorithm (NPGA) are some of the proposed methods [4].

#### C. Classical Optimization Algorithms

Classical approaches consist of converting the multiobjective problem into a single objective problem, which can be solved using traditional scalar optimization techniques. These techniques have matured over the years and many powerful programs are commercially available. The focus is on satisfying Karush-Kuhn-Tucker (KKT) conditions [1]. The KKT conditions are analogous to the condition that the gradient must be zero at a minimum, modified to take constraints into account. The KKT conditions are given via an auxiliary Lagrangian function. Trust-Region-Reflective Optimization, Active-Set Optimization and Interior-Point Optimization are some of the techniques available [14]. Some methods require that the gradient of the objective function is supplied. However, single optimization techniques are computationally demanding and may converge to a local optimum.

### IV. FORMULATION FOR POWER SYSTEM STUDIES

For the power system studies presented in this research, two specific goals are considered. The first goal is to minimize the total fuel cost for specified loading conditions. All the generating units are assumed to be thermal with the fuel cost expressed as a cubic function of the output of the generating units. The second goal is to minimize the total transmission loss, again for a specified loading. In the multiobjective optimization, these two goals are combined into a single objective function using the weighted sum method.

#### A. Minimum Cost

The optimal power flow (OPF) can be expressed as a constrained optimization problem [15] requiring the minimization of:

$$f = f(x, u) \quad (3)$$

Subject to

$$g(x, u) = 0 \quad (4)$$

$$h(x, u) \leq 0 \quad (5)$$

$$u^{\min} \leq u \leq u^{\max}$$

$$x^{\min} \leq x \leq x^{\max}$$

In the above equations,  $f(x, u)$  is the scalar objective function,  $g(x, u)$  represents non linear equality constraints (power flow equations), and  $h(x, u)$  is the nonlinear inequality constraint of

vector arguments  $x$  and  $u$ . The vector  $x$  consists of dependent variables (for example bus voltage magnitudes and phase angles). The vector  $u$  consists of control variable (for example, real power generation). Specifically, when the objective is to minimize the total fuel cost, the objective function can be expressed as the sum of the fuel cost for all the available generating units:

$$f(x, u) = \sum_{i=1}^{N_g} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \quad (6)$$

### B. Minimum Loss

When the objective is to minimize the total transmission loss, the objective function is expressed as:

$$f(x, u) = \sum_1^{N_g} P_{gen}(x, u) - \sum_1^{Nl} P_{load} \quad (7)$$

$N_g$  = number of generators

$Nl$  = number of loads

The equality and inequality constraints are the same as that for the minimum cost problem.

### C. Multiobjective Optimization with Security Constraints

The two main differences in the multiobjective optimization problem presented in this paper are the combining of the objective functions and the inclusion of security constraints.

For multiobjective optimization, the two objectives are combined as:

$$f(x, u) = a_1 * f_1(x, u) + a_2 * f_2(x, u) \quad (8)$$

$$a_1 + a_2 = 1 \quad (9)$$

$a_1$  and  $a_2$  are the two weights which represent the importance of the objective functions.  $f_1(x, u)$  is the objective corresponding to minimizing the cost and  $f_2(x, u)$  is the objective corresponding to minimizing the total transmission loss. The operator can decide which objective is more important based on the weight factor corresponding to a specific objective.

Security assessment can be classified as static security assessment and dynamic security assessment [15]. The study presented in this paper considers only static security. The goal is to ensure that the present operating condition meets the minimum cost and loss criteria as well as remain secure considering the possibility of any credible contingency [16]. For the studies presented here, the contingencies considered are the outage of transmission lines (one at a time). For each contingency, the equality and inequality constraints (equations 4 & 5) corresponding to that operating condition must be included in the problem formulation. This will increase the number of variables ( $x$ ) in the optimization problem significantly. However, the solution will satisfy the desired operating criteria and ensure that the power system is secure.

## V. CASE STUDIES

The studies presented in this paper are based on two simple power system models. All the studies use the optimization tool box available in Matlab [17]. One of the main reasons for the use of this is the availability of the licensed software in the Faculty of Engineering at Memorial University. For some

aspects of the studies, PowerWorld Simulator is also used [18].

### A. Case Study: 7Bus Power System

Fig. 2 shows the single line diagram of the seven bus power system considered for the different studies [18]. This system has 5 generators, 11 transmission lines and 6 loads. The loads total 760 MW and 130 MVAR. The fuel costs of all the generating units are represented using cubic cost models. For the base case, the total fuel cost is 16939 \$/hr and the transmission loss is 7.9 MW. In the initial studies, security constraints are not considered. Table I summarizes the results of multiobjective optimization without considering contingency constraints. When security constraints are considered, the problem is formulated in such a way that the inequality constraints corresponding to the outage of some transmission lines are included. Table II presents the results of this study. As seen from Tables I and II, the cost is higher when security constraints are included.

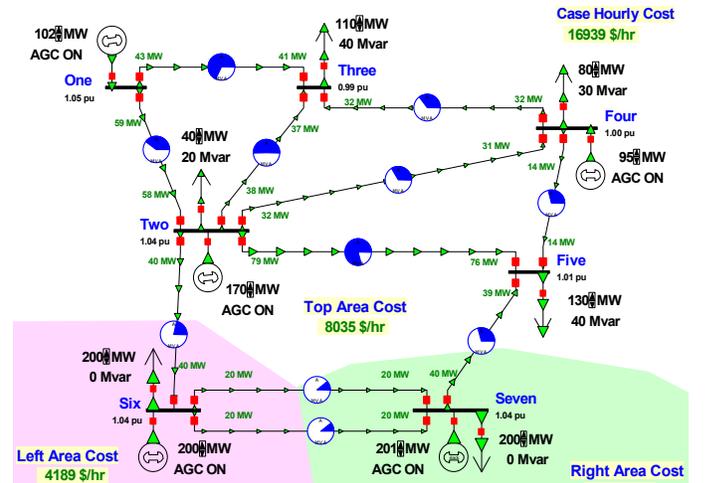


Fig. 2. Seven bus power system

TABLE I  
MULTIOBJECTIVE OPF OF SEVEN BUS POWER SYSTEM (WITHOUT SECURITY CONSTRAINTS)

Weight Factor of Loss	Transmission Loss (MW)	Weight Factor of Cost	Generation Cost (\$/hr)
0	10.050	1	16371
0.2	3.933	0.8	16804
0.4	3.408	0.6	17003
0.6	3.310	0.4	17095
0.8	3.292	0.2	17132
1	3.290	0	17150

TABLE II  
MULTIOBJECTIVE OPF OF SEVEN BUS POWER SYSTEM (SECURE)

Weight Factor of Loss	Transmission Loss (MW)	Weight Factor of Cost	Generation Cost (\$/hr)
0	4.41	1	17021
0.2	3.58	0.8	17150
0.4	3.38	0.6	17213
0.6	3.34	0.4	17243
0.8	3.34	0.2	17243
1	3.34	0	17243

Fig. 3 shows the Pareto front provided by the multiobjective optimization. The power system operator can select a particular solution from these multiple solutions. Any solution from this front is secure with respect to the outage of a single transmission line. To simplify the problem, contingencies with respect to the outage of only two transmission lines are considered.

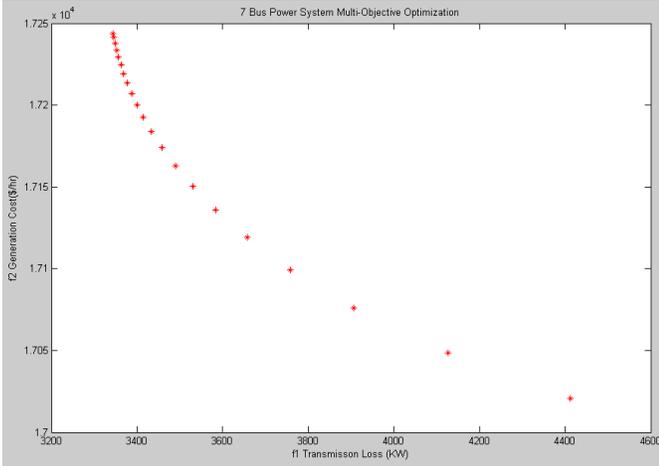


Fig. 3. Pareto front for the seven bus power system (secure)

### B. Case Study: 26Bus Power System

Fig. 4 shows the single line diagram of the 26 bus power system considered for the studies [19]. This system has 6 generators, 46 transmission lines/ transformers and 26 loads. The loads total 947.3 MW and 484.5 MVAR.

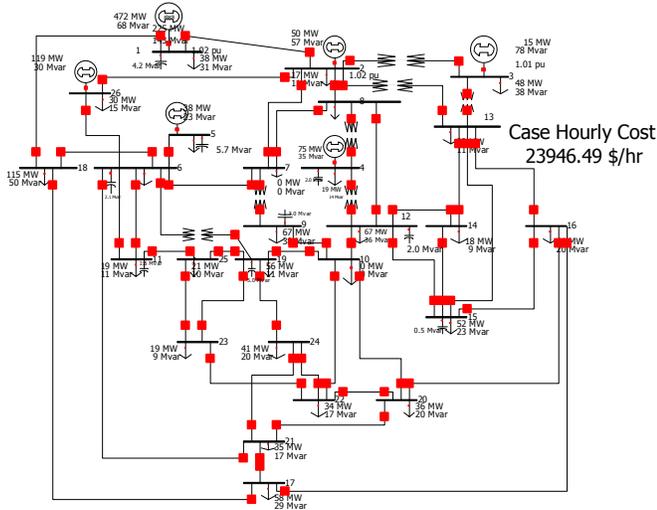


Fig. 4. Twenty six bus power system

The fuel costs of all the generating units are represented using cubic cost models. For the base case, the total fuel cost is 23946 \$/hr and the transmission loss is 9.4MW. In the initial studies, security constraints are not considered. Table III summarizes the results of multiobjective optimization without considering contingency constraints. Table IV presents the results of OPF studies when the security constraints are considered. As seen from Tables III and IV, the cost is increased in order to maintain a secure power operation system.

TABLE III  
MULTIOBJECTIVE OPF OF TWENTY SIX BUS POWER SYSTEM  
(WITHOUT SECURITY CONSTRAINTS)

Weight Factor of Loss	Transmission Loss (MW)	Weight Factor of Cost	Generation Cost (\$/hr)
0	7.25	1	22646
0.2	7.15	0.8	22648
0.4	7.01	0.6	22654
0.6	6.81	0.4	22676
0.8	6.47	0.2	22764
1	6.26	0	22974

TABLE IV  
MULTIOBJECTIVE OPF OF TWENTY SIX BUS POWER SYSTEM  
(SECURE)

Weight Factor of Loss	Transmission Loss (MW)	Weight Factor of Cost	Generation Cost (\$/hr)
0	6.83	1	22846
0.2	6.79	0.8	22846
0.4	6.75	0.6	22848
0.6	6.62	0.4	22862
0.8	6.39	0.2	22915
1	6.30	0	23076

Fig. 5 shows the optimal solutions set for the 26 bus power system provided by the multiobjective optimization. The power system operator can select a particular solution from these multiple solutions. Any solution from this front is secure with respect to the outage of a single transmission line. To simplify the problem, contingency with respect to the outage of only one transmission line is considered.

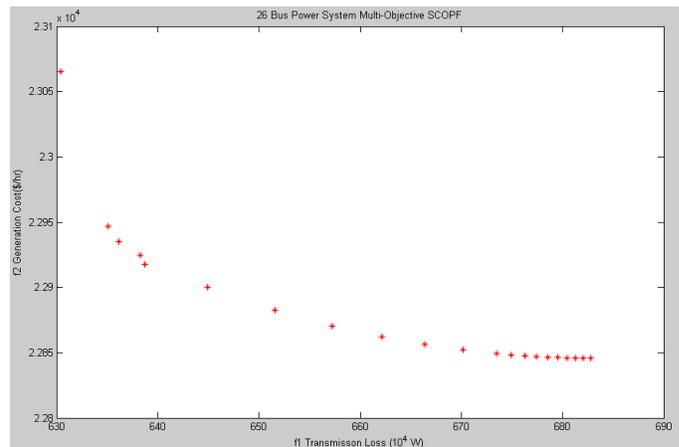


Fig. 5. Pareto front for the 26bus power system (secure)

When considering security constraints, the problem is formulated in such a way that the equality and inequality constraints corresponding to the outage of some transmission lines are included. Since the number of variables has increased, the gradient-based algorithm takes a lot of time to converge. This leads one to investigate alternate strategies for multiobjective optimization. A recently proposed method called “multiobjective particle swarm optimization” (MOPSP) [20] is a strong candidate to meet these challenges. Further research is required to explore this and investigate its

suitability for multiobjective power system optimization problems including security constraints.

## VI. CONCLUSION

This paper has presented multiobjective optimization for power system including security constraints. Case studies on simple power systems models illustrate that the objectives can be achieved while maintaining security constraints. For larger power systems, when security constraints are included, the number of variables in the optimization problem will increase significantly. This presents a challenge for the conventional gradient-based optimization methods. The potential of Particle Swarm Optimization for solving multiobjective problems should be exploited. This can lead to the possibility of ensuring that multiple objectives are satisfied while maintaining both steady state and dynamic security constraints.

## VII. ACKNOWLEDGEMENT

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## IX. BIOGRAPHIES

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