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Optimization methods applied for solving the short-term hydrothermal coordination problem

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ABSTRACT

Short-term hydrothermal coordination (STHTC) is a very complicated optimization problem. It is a dynamic large-scale non-linear problem and requires solving unit commitment and economic power load dispatch problems. From this perspective, many successful and powerful optimization methods and algorithms have been employed to solve this problem. These optimization methodologies and techniques are widely diverse and have been the subject of ongoing enhancements over the years. This paper presents a survey of literature on the various optimization methods applied to solve the STHTC problem. A review and a methodology-based classification of most of the publications on the topic are presented.

1. Introduction

Short-term hydrothermal coordination consists of determining the optimal usage of available hydro and thermal resources during a scheduling period of time (1 day–1 week) [1,2]. This is to determine, optimally, which of the thermal generating units should run as well as the power generated by the hydro and thermal plants so that the total cost is minimized. Minimizing the total cost in this optimization problem is subject to many control and operational constraints. In addition to reliability and security requirements, hydraulic and thermal constraints may include load balance, generation limits, water discharge, starting and ending storage volume of water and spillage discharge rate. Further, in order to solve the hydrothermal coordination problem, thermal unit commitment and economic load dispatch problems should be solved all together with the hydro schedules. Therefore, the STHTC is a large-scale non-linear and complicated constrained power system optimization problem. Mathematically, the STHTC optimization problem can be formulated, in general, as follows [1]:

$$\min \quad F_T = \sum_{t=1}^{T}\sum_{j=1}^{N} F_j(P_S(j,t))$$

where F_T is the total production cost function; $P_S(j,t)$ the power generation of thermal unit j at time interval t; $F_j(P_S(j,t))$ the production cost for $P_S(j,t)$; N the number of thermal units; T the number of time intervals.

The cost function of the thermal power production is expressed as follows:

$$F_j(P_S(j,t)) = a_j + b_j P_S(j,t) + c_j P_S^2(j,t)$$

The objective function is subject to many constraints including the following:

- Load balance:

$$\sum_{i=1}^{M} P_H(i,t) + \sum_{j=1}^{N} P_S(j,t) = P_D(t) - P_L(t) = 0$$

  where M is the number of hydro units; $P_H(i,t)$ the power generation of hydro unit i at time interval t; $P_D(t)$ the system load demand at time interval t; $P_L(t)$ the system total losses at time interval t.
- Thermal and hydro generation capacity:

$$P_S(j)^{\min} \le P_S(j,t) \le P_S(j)^{\max}$$

$$P_H(i)^{\min} \le P_H(i,t) \le P_H(i)^{\max}$$

  where $P_S(j)^{\min}$ is the minimum power generation for thermal unit j; $P_S(j)^{\max}$ the maximum power generation for thermal unit j; $P_H(i)^{\min}$ the minimum power generation for hydro unit i; $P_H(i)^{\max}$

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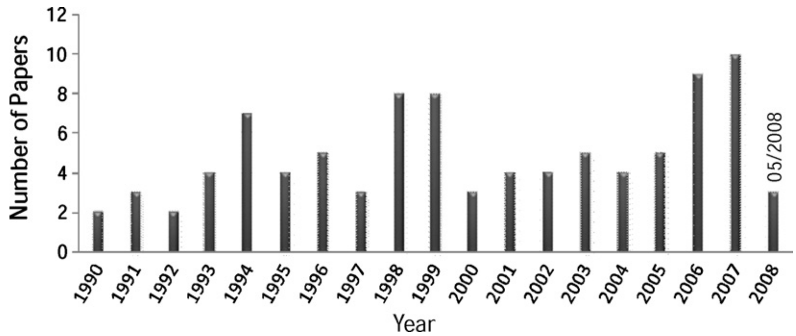

Fig. 1. Number of papers published in each year on the subject of STHTC problem.

the maximum power generation for hydro unit i.

- Total water discharge:

$$Q_{\mathrm{tot}}(i)=\sum_{t=1}^{T}q(i,t)$$

where $q(i,t)$ is the water discharge rate for the hydro unit i at time interval t.

- Hydraulic continuity equation:

$$V(i,t)=V(i,t-1)+[r(i,t)-q(i,t)-s(i,t)]n_t$$

where $V(i,t)$ is the storage volume of the reservoir i at the end of time interval t; $q(i,t)$ the water discharge rate for the hydro unit i during time interval t; $r(i,t)$ the inflow rate into reservoir i during time interval t; $s(i,t)$ the spillage discharge rate of the reservoir i during time interval t; n_t the length of time interval t.

- Water storage limits:
  - Volume limits $V^{\min}\le V(i,t)\le V^{\max}$
  - Starting volume $V(i,t)|_{t=0}=V_{\mathrm{s}}$
  - Ending volume $V(i,t)|_{t=T}=V_{\mathrm{E}}$
- Water discharge rate:
  - Flow limits $q^{\min}\le q(i,t)\le q^{\max}$
  - Fixed discharge $q(i,t)=Q(i,t)$

A variety of optimization methods and techniques have been proposed to solve the problems of power systems optimal operations and planning since the beginning of the last century [2]. Among the earliest optimization techniques applied to the problem were the so-called the base load procedure, the best point loading and the incremental method. A historical survey, which highlights the earliest works in the field, is offered in [2]. At present, several methods and algorithms have been in use to solve power system optimization problems [3,4]. These include mathematical methods, iterative approaches, artificial intelligence tools, and hybrid techniques. Over the years, different methodologies have been applied. With the development of the mathematical and computational techniques, additional details of the problem have been addressed. In the beginning, only the thermal plants were considered and before long, the hydraulic operational and topological constraints were tackled. Fig. 1 statistically illustrates the number of the published research papers on the subject of the STHTC problem during the last 15–20 years (based on IEEE/IET/Elsevier databases).

2. Optimization methods for short-term hydrothermal coordination problem

A wide range of optimization techniques has been applied to solve the STHTC problem. These techniques are principally based on the criterion of local search through the feasible region of solution [3]. Applied optimization methods can be mathematical programming algorithms such as linear and non-linear programming, dynamic programming and interior-point methods [5,6]. Among the other methods are the artificial intelligence techniques including neural networks, fuzzy systems and the evolutionary methods such as genetic algorithms and the simulated annealing. The methods considered in this survey can be classified as follows:

- Lagrangian relaxation and Benders decomposition-based methods
- Mixed-integer programming
- Dynamic programming
- Evolutionary computing methods
- Artificial intelligence methods
- Interior-point methods

These optimization methods can be generally classified into two main groups: deterministic methods and heuristic methods. Deterministic methods include Lagrangian relaxation and Benders decomposition methods, mixed-integer programming, dynamic programming and interior-point methods. Genetic algorithms, particle swarm optimization and other evolutionary methods are heuristic. Most of the methods that have been used to solve the STHTC problem are deterministic in nature. However, modern heuristic methods are getting more attention in solving large-scale optimization problems. To search for the optimal solution, classical deterministic methods, also known as derivative-based optimization methods, apply techniques such as the gradient and Hessian operators. They use single path search methods while heuristic methods use population-based search techniques to search the solution hyperspace. This difference, in fact, is an advantage for the heuristic methods as it helps searching in spaces with non-smooth characteristics. It also improves the convergence for heuristic methods and makes it less dependent on the initial solution points. Being derivative-free, modern methods are applicable to any optimization problem regardless of the linearity or non-linearity of its objective function and constraints. In contrast, different deterministic methods are required for different optimization problems. Another main difference between the two classes is that heuristic methods use stochastic techniques and include randomness in moving from one solution to the next while determinist methods follow deterministic transition rules. This, of course, gives an advantage to heuristic methods in avoiding local minima. In spite of the advantages of the heuristic techniques, classical methods have been used by the majority of research papers covered in this review. The reason is their efficiency in solving optimization problems, the solid mathematical foundation and the availability of software tools [7].

Fig. 2 shows the number of publications and the method applied to solve the STHTC problem in the specified period. In this survey we included, to the best of our knowledge (based on IEEE/IET/Elsevier

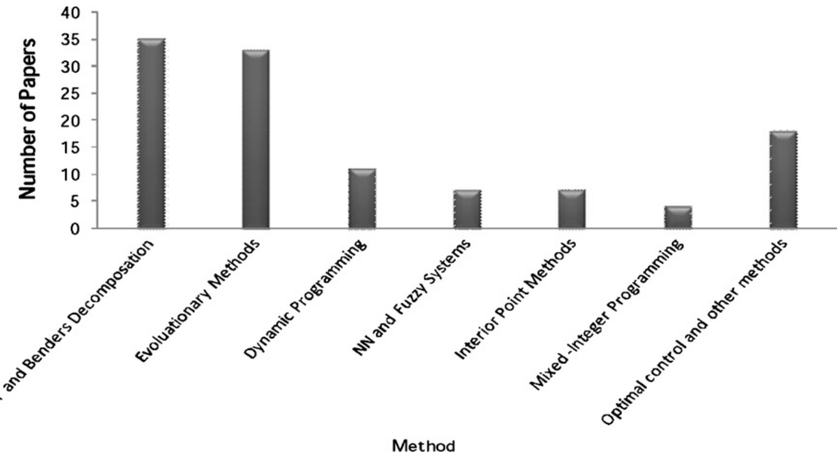

Fig. 2. Number of papers published on different optimization methods used.

databases), most of the papers that have been published during the past two decades or so.

2.1. Lagrangian relaxation and benders decomposition-based methods

Large-scale optimization problems, such as the STHTC problem, are usually divided into a set of independent sub-problems. Decomposition-based methods are used to solve this kind of problem using an iterative-based methodology. Lagrangian relaxation and augmented Lagrangian techniques are among the most popular decomposition-based methods. The original Lagrangian relaxation was improved by introducing the augmented Lagrangian techniques and by the multiplier updating methods. In these techniques, the solution is based on computing the optimal values of the Lagrangian multipliers for the sub-problems to find a solution to the dual problem. On the other hand, the solution to the primal problem is usually infeasible due to the non-convexity of the problem. Therefore, a method to find a feasible optimal or near-optimal solution has to be applied. Lagrangian multipliers are updated each iteration using several updating tools such as Bundle techniques. Benders decomposition and Dantzig–Wolfe are also well-known decomposition methods [3]. A study of the relaxed classical hydrothermal scheduling algorithm from a theoretical point of view was presented in [8,9]. The focus of these papers was the convergence issues of the Lagrangian formulation of the hydrothermal coordination problem. A relaxation coefficient was proposed to guarantee the algorithm convergence and practical procedures to compute the coefficient were presented. To illustrate the performance of the algorithm in [9], it was implemented on a real system although many details were ignored in the formulation of the problem. This was because, as mentioned, the scheduling problem itself was not the subject of this paper.

A method based on the Lagrangian relaxation was also presented by Yan *et al.* in Ref. [10] where the STHTC problem was decomposed into two sub-problems. A method of merit order allocation was implemented to solve the hydro sub-problem while the thermal sub-problem was solved by applying a dynamic programming approach. The method was tested using limited water resources hydro units. The hydraulic coupling among these water resources and the upper and lower constraints were not considered. The hydro sub-problem was formulated as a linear programming problem without accounting for the non-linear characteristics. The non-linearity that could be caused by the start up cost function for the thermal units was not taken into consideration. Power exchange with other utilities was also not included in this paper. The authors did not consider the pumped-storage power units in this paper but in another paper [11], they presented a method to incorporate them where a solution methodology for pumped-storage units was presented. The dynamic transition regarding the operation status (generation, pumping and idle) of the pumped-storage units was considered by the Lagrangian relaxation based solution. The algorithm was integrated with a scheduling package and implemented to solve the STHTC problem where the importance of the pumped-storage units was demonstrated. It should be noted that the pumped-storage unit that was implemented was always running as either a generator or a pump with short off time. The convergence of the algorithm was reported to be very good although a few additional iterations were required for updating the Lagrangian multiplier using a subgradient approach.

A peak-shaving method was presented in [12] to study the influence of the interchange resource scheduling on the STHTC problem. The interchange was formulated as a decomposed sub-problem with a proposed scheduling strategy to provide a smoother hydro generation profile. The methodology was claimed to be suitable for practical systems although it was only applied to two test systems. Results showed that the method was beneficial especially for the systems that could be augmented by interchange purchases. In [13] a network flow programming based algorithm was presented to solve the STHTC problem of dominantly hydro systems. The hydraulic subsystem was simulated while the transmission system was modeled as an optimal DC load flow. The performance of the approach was found efficient when applied to two synthetic test systems. The tests were performed using amateur codes that made the time of convergence an issue of a concern. Network flow programming was applied to solve the hydro sub-problem in a number of papers such as [14]. This paper presented an implementation of an incremental network flow programming and integrated it with an existing unit commitment and economic dispatching software. The objective was to make a comprehensive industrial hydrothermal coordination product for applications of energy management system. The approach was implemented in a realistic system and showed a good performance with good convergence properties in spite of employing a modified golden search algorithm. It might be worth mentioning that the golden search could fail into local minima due to non-linearity. However, the authors reported that, fortunately, their algorithm gave very satisfactory results.

In [15], a network flow programming algorithm was applied in a hydro dominated power system. The problem was decomposed into hydro and electric sub-problems. Constraints were represented in detail by network models including transmission DC power flow model. The approach was tested using a practical system; however, convergence required extended time. Hydrothermal scheduling problem in a hydro dominated power system was the subject of [16] too. In this paper, a case study was presented considering that the solution of the unit commitment problem was predetermined and, hence, thermal start up and shut down operations and power rate restrictions were not considered. On the other hand, many constraints were formulated such as reservoir release targets, power flow transmission limits, reservoir storage limits and discharge variation limits.

Augmented Lagrangian relaxation is also another decomposition-based method that was presented in a number of papers such as [17]. In this paper, the augmented Lagrangian decomposition and coordination technique was applied to the STHTC problem instead of the standard Lagrangian relaxation approach. Reducing the oscillation of the solutions to the sub-problems in the standard Lagrangian relaxation technique was an objective of the augmented Lagrangian approach. By applying this method, the linearity and piecewise linearity of the cost functions of the sub-problems was avoided and hence the oscillation was reduced. Compared to the standard Lagrangian relaxation, the augmented Lagrangian approach required less computational time with better convergence characteristics although, oscillation was not eliminated. This lead to a smooth movement of the solutions to the sub-problems with a slight change of the multipliers. The approach was tested using a practical system consisting of thermal, hydro and pumped-storage units with many practical constrains were considered. It should be pointed out that the selection of the penalty coefficient was not easy, as it could be not fitting for all different units. The oscillations of the solutions to the sub-problems in the Lagrangian relaxation technique as well as the singularity of these solutions were also discussed in [18]. In this paper, a non-linear approximation method was presented. Quadratic non-linear functions were used to approximate linear cost functions. The algorithm was tested and applied to a practical system and the results demonstrated its efficiency although compared to the standard Lagrangian relaxation method, no difference in the computational time was reported.

In [19,20] a Lagrangian relaxation-based algorithm and a dynamic programming technique were integrated into an expert system to solve the STHTC problem. Steam and gas turbines were considered as well as many constraints such as the non-linear functions of thermal generation cost, transmission losses and water discharge rate. The algorithm was reported to reach a feasible solution in an acceptable time although additional iterations were required in some test cases to find the optimal Lagrangian multipliers for the nearest feasible solution. Guan *et al.* presented a Lagrangian relaxation-based technique for the STHTC problem in [21]. They concentrated on the solution methodology for the hydro sub-problem with the cascaded reservoirs and discrete hydro constraints. In the formulation of the problem, the hydro units were represented by one unit and only one existing river catchment was assumed. Ramp rates and system losses constraints were not included in the problem formulation. The algorithm was tested using a realistic system and a near-optimal solution was confirmed. The computational time was assumed good although the results were not compared to those of other approaches.

Updating the Lagrangian multipliers was addressed by several papers to overcome the problems associated with the commonly used subgradient updating method. In [22] an application of a bundle method called "the Bundle Trust Region Method" was presented and applied to update the Lagrangian multipliers. The categorization of the presented method as a trust region method was a point of debate since a method could be defined as a trust region method if a dynamic modification is applied to the feasibility region while this approach could be better classified as a dual penalty cutting plane method. The method was compared to the subgradient method and other bundle type methods and found to have faster convergence although only the line search rule was used to compute the ascent direction.

An alternative bundle method to update the multipliers was presented in [23]. This algorithm, which is a kind of a bundle method, was called "reduced complexity bundle method" and used to obtain a better convergence and to avoid the zigzagging behavior of the conventional bundle method, which could cause a slow convergence. Tests were run using practical data sets with various realistic considerations included. Results showed better feasible solutions to the dual cost function compared to the subgradient method although no guarantee of convergence was verified. It should be mentioned that some details in the methodology were not made clear such as the procedure of selecting the bundle elements and how they were projected.

Dual programming algorithms were applied to solve the thermal generation scheduling sub-problem as a part of the STHTC problem in [24]. Two methods were implemented and tested using both small and large-scale test systems. Results were considered to have good convergence characteristics. In [25], an updating technique for the Lagrangian multipliers was presented. This technique was called the optimal distance method. The method was tested and compared to the subgradient method and proved to have better convergence properties and accuracy of solution. However, convergence in some test cases was not reached easily and more iteration was required to find a feasible solution. Another updating approach for the Lagrangian relaxation multipliers was presented in [26]. This approach was referred to as "a novel, non-oscillating and efficient multiplier updating procedure". The procedure was tested and showed superiority when compared to the subgradient and bundle methods based on the number of iterations required.

Ref. [27] presented a novel relaxation algorithm to solve river catchment sub-problems taking into consideration the continuous and discontinuous dynamics and constraints as well as hydraulic coupling of cascaded reservoirs. In this approach, another set of multipliers was used to relax minimum hydro generation of each hydro unit. The algorithm considered the pumped-storage units in addition to thermal and hydro units, which were represented by one unit. A near-optimal feasible solution was obtained when the algorithm was tested. However, there were some concerns regarding convergence issues compared to a previously applied heuristic method. Bidding in energy markets and its influence on the hydrothermal problem was the subject of [28]. The paper presented a novel formulation to integrate bidding and hydrothermal scheduling based on a Lagrangian relaxation approach. A Markov chain model was employed to present the hourly market cleaning prices considering the reserve market and the self-scheduling requirements. The problem was decomposed into a number of sub-problems that were solved using a stochastic dynamic programming approach. Pumped-storage units were also included and many practical considerations were formulated although, for simplicity, some assumptions regarding the market clearing prices and the "perfect market" were applied.

In [29] an augmented Lagrangian method was used to solve the STHTC problem considering the transmission constraints and including pumped-storage units in the model. The method was tested using the IEEE 24-bus system and results showed that a feasible solution was reached with no iterations required with economic dispatch algorithms. The method was considered accurate and practical though it was not applied to realistic systems. With respect to convergence properties, it was suggested that the augmented Lagrangian approach could be improved by using suitable updating methods. In [30,31], various techniques were applied to relax the Lagrangian relaxation multipliers. These are Dantzig–Wolfe linear programming, subgradient method and Branch and Bound. Real generation data were used to perform the application of the methods considering many practical constraints such as system losses and other thermal and hydro limits but not the ramp rates. Results were shown to demonstrate the effectiveness and optimization capabilities of the applied techniques, however, convergence issues and stopping criterion were not discussed.

A Lagrangian relaxation approach was also presented in [32]. The dual problem was solved using a Lagrangian relaxation approach with a disaggregated bundle method. In order to improve the performance of the bundle method, a warm-starting procedure was also introduced. Results showed better convergence characteristics compared to the aggregated bundle methods although the implementation was only run on a simple system.

In [33] a comparison of Lagrangian relaxation and truncated dynamic programming methods was presented. The comparison was performed based on the time required for the two methods to reach a near-optimal feasible solution. In spite of the fact that dual optimal solution obtained by the Lagrangian relaxation method was not always satisfied, it was found to be more flexible for large-scale problems with lower cost than the truncated dynamic programming approach. The approach presented in [34] was a Lagrangian relaxation technique based on variable splitting to solve the hydrothermal scheduling problem in large-scale predominantly hydro-electrical systems. The problem was decomposed into a set of sub-problems; hydro, thermal and electric. In this approach, the resulting dual problem was non-differentiable and could be solved using a bundle method. A final feasible solution was found by an augmented Lagrangian relaxation technique. However, some infeasibility was still recognized due to the representation of non-linear constraints by the piecewise linear approximation. The common problem of oscillatory effects associated with the Lagrangian relaxation technique could also be pointed out. Another decomposition method, which is widely applied to solve the hydrothermal coordination problem, is the Benders decomposition method. In this approach, the large-scale problem is usually decomposed into sets of coupling variables and the dual formulation of the sub-problems is employed to find the solution.

A multistage Benders decomposition method was presented in [35] to solve the security constrained hydrothermal scheduling problem. In this presentation, the hydro system was modeled considering many details and constraints such as cascaded reservoirs, storage and spillage and other hydraulic constraints. The DC model losses for each line was represented by a piecewise linear function. The approach was applied on a study case considering the IEEE 118-bus system. Considering not only the DC transmission losses but also the AC power flow, Benders decomposition technique was presented in [36,37]. This approach studied modeling the transmission network using AC power flow when applied to solve the STHTC problem. Congestion management and quality control of service that are typically presented in large and weakly meshed networks. The method was tested using a 9-bus system and an actual power system. It was considered to have robust convergence properties although, as expected for Benders decomposition method, there was a tailing off effect and slow convergence. To help in reducing the computational times an accelerating technique was included in the scheme and presented in [37].

2.2. Mixed-integer programming

Integer and mixed-integer programming have been widely applied to solve different optimization problems such as the hydrothermal coordination problem. The commonly shared characteristic of these problems is having continuous and discrete variables that could only take an integer value. Branch and Bound (B&B) and cutting plane are among the most widely applied mixed-integer programming methods.

The STHTC problem is one of the most complicated mixed-integer programming problems. Several mixed-integer programming approaches and algorithms have been introduced and many commercial solvers are available. To improve the performance of these solvers, a convex hydrothermal scheduling method was presented in [38]. In this paper, convexity issues of some standard commercial mixed-integer programming solvers were discussed. The non-linear mixed-integer short-term hydrothermal scheduling problem was linearized and the conditional constraints such as the on/off status of the generation units, which was a function of the minimum up/down times, were normalized. The method was tested and results were demonstrated to show good solution quality and computational speed although it was concluded that the convergence computational speed could be improved by using a better B&B procedure.

In [39], a mixed-integer model for hydroelectric systems short-term planning was presented. This model was designed to avoid the problems caused by non-linearity and non-convexity by considering only the points with good degree of efficiency. The problem was decomposed into sub-problems with relaxed coupling constraints. The model was tested practically using a power system consisting of nuclear and hydro generation units with some assumptions were applied and some constraints were not considered for the sake of simplicity. In [40], a mixed-integer programming based algorithm was used to deal with the unit commitment problem while a multi-embedded linear programming model was applied to find the optimal scheduling for hydrothermal systems. Linear programming and network flow programming were used to formulate the multi-embedded blocks. The hydro units were represented by a linear model in which water head was fixed and assumed known. To test the algorithm it was applied to a realistic large-scale case study and the results showed the total cost of the study while no information was given regarding the computer memory nor the computational time required by the algorithm, although, direct methods, in general, are expected to have good convergence properties. The problem of computational time associated with the mixed-integer programming approaches is widely recognized especially when applied to large-scale optimization problem such as the STHTC problem.

An algorithm was presented in [41] to deal with this problem by improving the Branch and Bound (B&B) search method. In order to achieve this goal an initial feasible integer solution was provided to lead the B&B to the optimal solution. An under-relaxed procedure was applied to the hydrothermal system entirely to overcome the oscillation problem. The algorithm was tested using realistic large-scale case studies showing good performance and computational features.

2.3. Dynamic programming

The dynamic programming optimization approach has been extensively used to solve the hydrothermal coordination problem because of the complexity of this problem and existence of the dynamic variables. The ability of dynamic programming to overcome the difficulty of the non-linearity and non-convexity of large-scale systems is another reason for its popularity. In dynamic programming techniques, large size problems, which consist of many state variables and dynamic elements, are decomposed into smaller problems that could be solved independently. What so-called the “curse of dimensionality” [4], which is the limited ability to solve large-sized problems with large number of variables, is the main disadvantage of dynamic programming. In spite of this drawback, it has been applied to various power system areas and studied by a considerable number of publications in the literature.

Yang and Chen presented a special form of dynamic programming techniques in [42] to solve the STHTC problem. To improve the performance of dynamic programming and overcome its disadvantages, those are the high computational time and large memory storage, a multi-pass dynamic programming technique was implemented. The algorithm was tested using real data obtained from a realistic power system containing thermal and hydro units however, some constraints were not considered. Although the cases tested in this paper did convert to reasonable solutions, but there was no indication that global optimal solutions were guaranteed. In fact, the solutions that were reached might be local, especially when we keep in mind that the used algorithm was an iterative-based process.

A multi-pass dynamic programming approach was presented again in [43] by the same authors and applied to a similar hydrothermal power system but, in this paper, the pumped-storage units were included in the system formulation. The same approach was used also in [44] considering the pumped-storage units and battery energy storage system which was simulated and integrated into the system. The energy stored in the battery storage system was presented as an additional state variable in the problem formulation. In [45], a similar approach using the multi-pass dynamic programming was presented and tested using real data although results regarding the convergence issues and computational time and memory storage required were not presented in details to be compared with other published work. A solution to the dual subproblem of thermal and hydro units was presented in [46], which was the second of two papers whereas the first, [47], was devoted to solve the primal problem. A dynamic programming method was used to solve the thermal unit sub-problem while the hydro subproblem was solved using the approximation in the state space within the multi-pass dynamic programming. The procedure was tested by running a number of simulations and results demonstrated a good performance and improved computational time.

In [48], a multi-pass dynamic programming was integrated with an evolutionary programming algorithm in order to obtain an improved solution. Two case studies were presented to implement the approach considering, in addition to thermal and hydro units, pumped-storage units which either worked in pumping mode or generation mode with no idle times. An extended differential dynamic programming and mixed coordination approach was presented in [49] to determine optimal short-term scheduling of hydrothermal systems. The problem was decomposed into thermal and hydro sub-problems. An analytical approach was used to solve the thermal sub-problem while the extended differential dynamic programming was implemented for the hydro sub-problems. Unpredictable changes in natural inflow and its impact on total cost were taken into consideration with developing a quick and practical estimate way for this change.

In [50], a priority-list-based dynamic programming was used to solve the hydro unit commitment as a part of the STHTC problem to reduce the dimension of the problem. A successive approximation method was employed to obtain better convergence properties when applied to realistic test systems.

2.4. Evolutionary computing methods

Based on evolutionary theory and inspired by the principle of “survival of the fittest” [51], evolutionary computation is one of the computational intelligence based approaches that have been applied to solve complex optimization problems [52]. Flexibility and capability to obtain good quality solutions are the advantages of evolutionary computation; however, these are highly affected by computer requirements and convergence characteristics [53]. Several evolutionary computation techniques have been introduced and applied to powers system optimization problems, these include; genetic algorithms (GA), simulated annealing (SA), evolutionary strategies, evolutionary programming and particle swarm optimization (PSO).

2.4.1. Genetic algorithms (GA)

Genetic algorithms have been widely applied to power systems since they were introduced by John Holland in his book [54] in 1975. GA is a search technique that searches for a population of solution points starting from an initial arbitrary solution within the feasible region [4]. Genetic algorithms have become one of the most popular approaches because of the many advantages that have been acknowledged such as their ability to handle any objective function with any constraints. Moreover, they are less likely to converge to local minima since their population-based search is a probabilistic transition strategy [55]. On the other hand, their main weakness is the high computational time required for convergence [53].

Various power system planning and operation problems have been solved using genetic algorithms such as economic dispatching, unit commitment and hydrothermal coordination problems. One of the earliest applications of GA to solve the STHTC problem was presented in [56]. In this work, a GA-based method was applied to the 24 h ahead generation scheduling of hydraulically coupled units. The GA was used to solve the hydro sub-problem considering the water balance as well as the effects of net head and water travel time delays. A realistic system was employed to test the method and compare its performance to a dynamic programming approach. Results showed the good performance with good solution quality and robustness of GA especially for avoiding local minima as it was theoretically stated. A good overview on GA was presented in [57] and applied to determine the optimal short-term scheduling of hydrothermal systems. In this lengthy paper, a case study system of chain cascaded hydro units and a number of thermal units was used to evaluate the algorithm where the unit commitment problem was assumed solved while the economic dispatch sub-problem was considered. Many practical constraints were included in the formulation, however, the size of the problem of the case study was small and there was no evidence that the algorithm could be successfully applied to larger size problems. The performance of the algorithm was considered good although no comparison with other approaches was carried out in order to evaluate whether the algorithm was competitive or not.

In [58], a diploid genotype GA was applied to the STHTC problem. A comparison between diploid genotype and haploid genotype GA approaches was performed and the superiority of the first was illustrated based on the results obtained from performing several simulated test examples. Robustness of the convergence and ability to satisfy constraints were also demonstrated although the algorithm was not compared to other methods. A concurrent solution of the unit commitment and the economic dispatch sub-problems in addition to the STHTC problem using GA was presented in [59]. The approach was tested using a realistic system and the results were considered good. Test results for purely thermal systems were compared to results obtained by other methods including results of previously implemented GA presented in other papers showed lower costs, however, no comparison based on computational time was presented.

In [60], the thermal sub-problem was addressed using a priority-list method while an enhanced GA was used for the hydro sub-problem. Several practical constraints were considered by the hydro model, which was formulated at the unit level in order to be more accurate. The method was tested using a realistic system and proofed to be more effective than standard GA approaches. The scheduling problem in [61] was decomposed into three subproblems; unit commitment, economic dispatching and STHTC sub-problems. To test the method, a test example was employed consisting of hydro and thermal units but no pumped storage ones were included. Volume and discharge water constraints were considered as well as spinning reserve and losses, however, some others such as ramp rates were not formulated. Results regarding total costs and final volume and water discharge in reservoirs but no information were revealed about computational time and memory size required.

An evolutionary algorithm called a cultural algorithm was presented in [62] to solve the daily scheduling problem of hydrothermal systems. The algorithm was compared to GA method and showed better results in terms of solution quality and convergence behavior. In Ref. [63], a real GA and a binary coded GA method were applied to solve the STHTC problem and compared from a computational efficiency point of view. The two algorithms solved the problem assuming that the unit commitment was already solved but the economic dispatch was considered in the problem formulation. Two test cases for each algorithm were run, in the first, the valve-point loading effects were considered while they were not in the second. Results supported the superiority of the real coded GA as it showed better performance than the binary coded GA; however, the two algorithms were not compared to other applied methods.

In [64], a GA was implemented to the optimal short-term scheduling where the on/off status of the thermal and hydro units was computed. In order to obtain a guaranteed feasible solution, some heuristics were applied. Several realistic test case examples were employed to evaluate the performance of the algorithm. These examples consisted of a number of thermal units and one equivalent hydro unit but no pumped-storage units were included. Practical and hydro constraints such as ramp constraints were formulated which raised the operating cost when considered. The GA was compared to an interior-point method approach and as expected, the GA gave better feasible minima while the interior-point method showed better convergence properties.

2.4.2. Simulated annealing (SA)

Although it is not always classified as one of the evolutionary methods, many researchers include simulated annealing (SA) techniques under this category. SA, which is a heuristic optimization method, was first introduced in 1983 by Kirkpatrick et al. [65]. Inspired by thermodynamics, SA technique tackles combinational optimization problems by simulating the thermal dynamics associated to the process of gradually cooling metals and forming crystals. Simplicity and ability to represent different objective functions of complicated optimization problems gave rise to the number of publications and applications of the SA techniques in different research areas of optimization. On the other hand the main disadvantage of SA is the repeated annealing since when an optimal solution is reached it cannot be detected unless another method is incorporated with the SA [53].

SA techniques have been applied to various optimization problems in power systems including unit commitment, maintenance scheduling, transmission networks and distribution systems. Wong and Wong in their paper [66] presented a sequential SA algorithm to solve the STHTC problem considering various hydro and thermal constraints although some other constraints such as ramp rates were not included. To evaluate the algorithm it was applied to a test example, however, it was a small size system consisted of equivalent thermal and hydro-plants without including any pumped-storage units. Results demonstrated the advantages of the SA techniques such as simplicity and capability to handle complex objective functions in addition to the insensitivity to the starting schedule. On the other hand, the well-known drawback of SA, which was the high computational time required, obviously came into sight. To treat this weakness and improve the speed of execution, the authors developed another SA algorithm, which was described as a coarse-grained parallel SA algorithm, and presented it in another paper [67]. The same testing system was employed to apply the developed parallel algorithm and compare it to the previous sequential one. The parallel SA algorithm showed considerable difference in computational time besides slight improvement in its performance compared to the sequential SA algorithm.

In [68], SA was implemented to solve the thermal sub-problem while the hydro sub-problem was solved using a peak-shaving method in order to find the optimal short-term scheduling for hydrothermal power systems. The proposed method was tested using a modified version of a realistic power system and was considered robust with good performance and reasonable conversion time although it was not compared to other optimization approaches.

2.4.3. Particle swarm optimization (PSO)

Particle swarm optimization (PSO) is an evolutionary method motivated by the social behavior of bird flocks and fish schools. PSO was first introduced in 1995 by Kennedy and Eberhart [69,70] as a heuristic algorithm for non-linear optimization problems. In addition to the advantages of other evolutionary methods such as simplicity and ability to handle complex objective functions, PSO algorithm does not include many parameters to adjust and does not need perfect initial points. On the other hand, PSO is not ideal from a mathematical background viewpoint and it cannot guarantee convergence to global solutions [71]. PSO algorithms have been widely applied to various complex optimization problems including power system operational planning and design problems. A comprehensive survey of PSO applications in various areas of power system optimization as well as a review of the advantages and disadvantages of this approach were presented in [71]. Umayal and Kamaraj in [72], presented a PSO application to find the short-term optimal generation schedule as a multi-objective optimization problem. In addition to minimization of operation costs, the formulation of the objective function had to consider minimizing emission in order to satisfy environmental constraints. Several practical constraints including emission control and the usual hydro and thermal constraints were considered but some of them such as ramp rates were not accounted for. In order to evaluate the proposed algorithm two testing systems were employed and good performance results were reported. The method was considered applicable for realistic large-scale systems however, the size of the testing systems was very limited and no evidence was presented to support this statement. Although it was concluded that the proposed solution was simple and required less time consumption, there was no information regarding convergence properties neither there was any comparison with previously applied methods.

In [73], PSO application for short-term hydrothermal scheduling problem was studied and compared to other meta-heuristic search algorithms. Comparison revealed that PSO approach was superior as it proved better convergence characteristics and less solution time. In [74], different PSO versions were presented, applied to solve STHTC problem and compared to each other. According to this reference, there were four versions of PSO based on the size of the neighborhood and the formulation of velocity updating. The algorithms were applied to a test system consisting of a number of hydro units and an equivalent thermal unit while no pumped-storage units were included. Compared to other evolutionary approaches, the different PSO algorithms showed better performance and in particular, the local versions of the PSO were found best as they could maintain the diversity of population. However, considering only one equivalent unit to represent all thermal units could violate the accuracy when applied to realistic systems. This weakness was treated in [75] by considering cost characteristics of individual thermal units. To solve the problem a PSO was applied considering the effect of valve point loading on the cost function as well as many, but not all, practical hydro and thermal constraints. The algorithm was implemented using a test system that consisted of a number of hydro and thermal units with no pumped-storage units were considered. A comparison to a simulated annealing (SA) as well as to an evolutionary programming method was conducted and showed the superiority of the proposed PSO algorithm.

In [76], an evolutionary PSO-based solution was presented to solve the problem of scheduling pumped-storage units in the frame of the solution to the STHTC problem. The proposed approach was a combination of binary version of PSO and a mutation operation in order to help reaching global minima and performing fast convergence. Results obtained from tests demonstrated the good quality solutions and convergence properties when compared to other existing applied methods including standard PSO algorithms.

2.4.4. Evolutionary strategies and evolutionary programming

Evolutionary strategies and evolutionary programming, that are two quite similar evolutionary computation methods, were both introduced in 1960, although they were developed separately in parallel [52]. In spite of their similarities, the two approaches still have some differences from genetic algorithms and other evolutionary based methods. Since they were proposed, the two methods have been applied to various power system problems, though not as widely and increasingly as genetic algorithms. Compared to the small number of published work on the application of evolutionary strategies to solving the STHTC problem, several research papers that applied evolutionary programming techniques have been introduced.

Werner and Verstege [77] proposed what seems to be a unique application of evolutionary strategies to the short-term hydrothermal scheduling problem. In the formulation of the problem, the researchers considered run-of-river plants and pumped-storage units without decomposing the problem into sub-problems. A hydrothermal testing system was employed to evaluate the method consisting of pumped-storage units in addition to the thermal units and hydro units that have the ability to store a limited amount of water. Results were considered good as the algorithm showed the ability to solve the problem with good solution quality, although the quality of solution was dependent on the initial solutions as it is the case in all evolutionary computation methods. It was also noted that there was no indication in the paper that convergence to global minimum could be guaranteed and hence it was concluded that more focus on convergence properties was required.

In [78], a novel evolutionary programming algorithm was presented and applied to the STHTC problem. The performance of the proposed algorithm was compared to that of gradient search, genetic algorithms and simulated annealing methods and was reported to show better results in terms of cost as well as computational time and memory size requirement. Two example systems were used to test the algorithm consisting of hydro units and thermal unit while pumped-storage unit were not considered, however, the size of the systems was relatively small. Another comparison between the evolutionary programming algorithm and the classical gradient search and simulated annealing techniques was presented in [79]. Although results agree with those of the previous reference regarding a cheaper cost and more powerful searching compared to gradient search and simulating annealing methods, but in contrast, longer execution time was required for convergence than that required by the other two methods.

Ref. [80] presented an application of several evolutionary based methods including an evolutionary programming algorithm. In this work, besides the usual constraints, environmental aspects were also taken into consideration as well as ramp rates and transmission losses. The algorithms were applied and compared to classical methods whereas results were considered good although no detailed presentation was offered about the testing system and its size, about neither the comparison results nor the convergence behavior. A constructive evolutionary programming method, which was a combination of dynamic programming and evolutionary programming, was presented in [81] to solve the scheduling optimization problem of multiple storage hydrothermal systems. In this method, the problem was decomposed into several subproblems with cost-to-go functions assuming linear variables and constraints. The method was tested using two case study systems with multi-storage plants and considered robust, efficient and suitable for a rapid optimal scheduling of practical and large-size systems although, non-linear, non-convex and non-smooth characteristics of realistic systems were not accounted for when tested. In addition to classical evolutionary programming and fast evolutionary algorithms, that are Gaussian and Cauchy mutation-based respectively, an improved faster evolutionary programming method was presented in [82]. In order to evaluate and compare the three algorithms, various aspects were studied such as the effect of the initial solutions and computational time in addition to minimum and mean cost. Effects of valve point loading and prohibited hydro-discharge zones on hydrothermal scheduling with quadratic thermal cost function were explored. Results revealed the superiority of the proposed improved fast evolutionary algorithm over the other two evolutionary programming ones. It should be pointed out, hereby, that the case study used to apply and implement the proposed algorithm was very small-sized system and there was no proof that the same good results could be obtained if applied to practical large-scale complex systems. The same results were obtained and presented by the same authors in [83].

In [84], a hybrid evolutionary programming based algorithm was presented to solve the problem without decomposition into subproblems. The algorithm was tested using sample cases and results lead to a conclusion that the algorithm was an efficient and advantageous method to solve the problem although the test examples were very small-sized and no evidence was indicated to verify that the results and conclusion drawn would hold for complex large-size systems. Besides, many practical aspects and constraints were not taken into consideration including pumped-storage units and ramping limitations. Convergence properties as well as computational time and memory size required were not discussed in the paper although the execution time was judged acceptable. Another application of hybrid evolutionary programming was proposed in [85] using a novel evolutionary programming in the first phase and a direct search was used in the second phase. In order to validate the proposed algorithm it was tested using small size example and compared to other methods. Results showed that the proposed algorithm provided better cost results and less computational time in addition to robustness and effectiveness although, except for the cost-based comparison, no information was presented to support other results and conclusions. An evolutionary programming algorithm in [86] was applied and compared to a genetic algorithm approach and when tested showed better performance in terms of cost while no details were shown regarding computational time and memory size. A modified differential evolution-based novel approach was compared to an evolutionary programming algorithm in [87] when applied to solve the short-term hydrothermal scheduling problem. Effects of valve point loading and emission function inclusion were investigated while various thermal and hydro constraints were considered. Results showed the effectiveness of the method based on the optimal costs and emission but no presentation regarding convergence and execution time was offered in details.

2.5. Artificial intelligence techniques

In the field of power system operations and planning, very sophisticated computer programs are required and designed in such a way that they could be executed and modified frequently according to any variations. Artificial intelligence is a powerful knowledge-based approach that has the ability to deal with the high non-linearity of practical systems. Among their useful features are the ability to learn and build the experience within a period and the ability to store the knowledge they learn inspired by the human intelligence. Artificial neural networks and fuzzy logic are the most important artificial intelligence approaches that have been applied to the STHTC.

2.5.1. *Neural networks (NN)*

A neural network is a massively parallel-distributed processor that has the ability of learning and storing knowledge and making it available for use through resembling the human brain in some aspects. The learning process is performed by a learning program that changes the synaptic weights, which are the interneuron connection strengths where knowledge is stored, to attain a desired design objective. Once the network is trained, it is capable of generation. Generation refers to the capability of the neural network producing “reasonable” outputs for inputs encountered during the training process [88].

Neural network techniques have extensively been of a great interest to power system community since not very later than the publication of remarkable Hopfield’s paper in 1982 [89]. By the late 1980s and early 1990s, the application of neural network in various power system areas has become quite well established [90]. It should be noted that most of the implementations in the literature are based on the feed-forward multilayer networks [53]. Although other architectures have also been investigated but they are few in numbers. In spite of the popularity of neural networks in power systems, a quite few papers have been published that applied this technique to solve STHTC problem. Naresh and Sharma [91] proposed a two-phase neural network-based method to find the optimal short-term schedule for hydrothermal systems. In this implementation, the neural network seemed to be a feed-forward network although its structure was not indicated. The states of the analogue neurons were employed as scheduled discharge for the hydro units. Several hydro and thermal constraints taken into consideration including water transportation delay between cascaded reservoirs and transmission losses although some others were not accounted for such as ramp rates. The method was applied using a test example consisting of multi-chain cascaded hydro units and an equivalent thermal unit while no pumped-storage units were included. The performance of the method was compared to the results obtained by an augmented Lagrangian method and found very close to each other. In [92], a Hopfield neural network was applied to the problem considering a system with fixed head hydro units and a piecewise linear relationship between the water discharge and the electrical power output function. A test system with two hydro-plants and two thermal plants was employed to validate the proposed method and to compare it to Newton’s method. Results showed that the two methods did not give different cost results but no comparison regarding convergence characteristics nor computational time was offered although it was concluded that the proposed method was fast and required small computational resources in addition to its efficiency and practicality. It should be noted also that the formulation of the problem did not consider some realistic constraints and the used test example is not large enough to give an evidence that the method could be successfully applied to real word large-scale problems in addition of not considering pumped-storage units.

Pumped-storage units were included in the formulation of the problem presented in [93]. In this paper, an enhanced merit order and an augmented Lagrange Hopfield neural network was presented and applied for the STHTC problem consider various constraints. The merit order was enhanced by heuristic search algorithms while the energy function of the Hopfield neural network was an augmented Lagrangian-based function. The method was evaluated using a system with a number of hydro and thermal units as well as pumped-storage units and results were compared to those obtained by other methods such as Lagrangian relaxation and quadratic programming methods. Superiority of the proposed technique was demonstrated in terms of production cost and computational time requirement noting that the comparison was run considering only CPU chip frequency as a common base between the different methods in order to perform a reasonable and fair comparison.

2.5.2. *Fuzzy logic-based methods*

The idea of fuzzy sets was first introduced by Zadeh in 1965 as a generalization of classical set theory [94]. Fuzzy sets use membership functions to map elements of universe of discourse to the unit interval [0, 1]. The membership function always reflects the features and limitations of the studied system so that the fuzzy set accurately represents the problem and makes it easy to understand by non-experts [53]. Fuzzy sets has been applied to various power system areas such as operational planning, state estimation and load forecasting to represent uncertainties by fuzzification of ambiguous variables and employing membership functions that represent system characteristics [3]. A comprehensive practical guide to the fundamentals and application issues can be found in [94] as well as in other published works such as [95–97].

Surveys on the application of the fuzzy logic in power systems were presented in [98,99]. In the area of STHTC problem, the number of published papers is relatively small compared to the other optimization methods. Among those few papers was the one presented by Wong and Wong in 1996 [100] who proposed a combination of fuzzy set algorithm with a genetic algorithm and a simulated annealing method. A short-term generation scheduling was formulated considering take-or-pay fuel cost then the formulation was extended to include the economic dispatch problem. A test example was used to demonstrate the performance whereas the heat-rate functions of the generators included the effect of valve-point loading. Another approach that integrated a genetic algorithm and a fuzzy system based technique for scheduling hydroelectric generation was proposed in [101]. In this application, problem objective and constraints were fuzzified using genetic algorithms in order to gain better-tuned membership functions and hence more accurate representation. The method was tested using a realistic power system and found effective and practical compared to the conventional fuzzy system although some system constraints were not considered. The comparison was based on production cost and computational time between the two fuzzy-based schemes but it was not performed to include other classic methods.

Refs. [102] presented a fuzzy decision-making approach to find the optimal short-term schedule for fixed-head hydrothermal systems considering a multi-objective problem. In the formulation for the objective function, not only the cost was to be optimized but also the gaseous emission should be minimized in order to meet the environmental regulations. However, these objectives were manifold and hence a weighting technique was applied to satisfy them all in a non-inferior range. To evaluate the method, three testing case studies were employed to demonstrate the effectiveness and flexibility of the algorithm. Issues like computational time and memory size as well convergence properties were not focused on although it was pointed out that the high computational time required for the complete solution was a limitation. Another formulation of the short-term scheduling as a multi-objective problem was proposed in [103] considering the emission concern. In this paper, an interactive fuzzy satisfying method based on an evolutionary programming technique was proposed considering multi-reservoir cascaded hydro units and non-smooth characteristics of thermal units in addition to emission and other hydro and thermal constraints. The trade-off between the diverse objectives in the non-inferior domain was obtained by the fuzzy satisfying method instead of using weighting methods as in the previous reference. The method was validated using a testing system consisting of a limited number of hydro and thermal units and results were presented although no detailed information was offered regarding convergence behavior or computational time requirement.

2.6. Interior-point (IP) methods

Since the publishing of Karmarkar's revolutionary paper [104] in 1984, the field of optimization has been remarkably changed when he rediscovered the IP methods [105]. In his paper, Karmarkar proved that IP methods were capable to solve linear programs in a polynomial time manner. He also provided, for the first time, direct evidence that IP methods were faster than the simplex method especially in large-scale optimization problems. The earliest ideas for the IP methods can be traced back to 1955 when Frisch [106] proposed a log barrier function to replace the linear inequality constraints. Gill et al. [107] explained the relationship between Karmarkar's method and Fiacco and McCormick's classical logarithmic barrier method.

In general, IP methods can be categorized as; projective methods, affine scaling methods or primal-dual methods [108]. The primal-dual algorithm was first outlined by Megiddo [109]. This "primal-dual path following" approach proposed that the optimal solution should follow the center path. In 1992, Mehrotra [110] proposed the predictor–corrector technique to be integrated with the "interior-point path following" methods. IP methods have been applied to solve the problems of power system optimization since early 1990s. Clements et al. [111] was the first who applied IP methods to power system in 1991. In his research, Clements applied a non-linear programming interior-point technique to solve the state estimation problems in power systems. Afterwards, a large number of papers tackling the subject of the application of IP methods in various fields were introduced. A review of the IP methods that were applied to a variety of power system optimization problems up to 1993 is presented in [6]. IP methods have been successfully applied to power system operation and planning areas such as economic dispatch, unit commitment and hydrothermal scheduling problems. In addition to long and medium-term hydrothermal coordination, IP methods have been also employed to determine the optimal short-term schedules for hydrothermal systems.

Palacio et al. in [112] proposed a primal-dual IP method to solve the STHTC problem and studied the influence of the bilateral contracts and spot market on the optimal coordination. Transmission losses of each power transaction were calculated and the effects of the loading order on the transmission losses allocated to the pool and bilateral loads were studied. To validate the results, two test systems were used; a 6-bus system and a 27-bus system that was assumed equivalent to a specific real system. Various thermal and hydraulic constraints were considered, but some were not addressed such as the down and up ramp rates.

A genetic algorithm was combined with a primal-dual IP method in the methodology of [113]. In this reference, the determination of the on/off status of the thermal units was performed using the genetic algorithm while the IP method was employed to solve the economic dispatch problem in order to work out the STHTC problem. Results showed that the total available hydraulic energy could not be used because of the hydraulic and generation constraints. Inter-temporal constraints caused by cascaded reservoirs and maximum up and down ramps were among the considered constraints while the transmission system losses were not. Robustness and speed of convergence of primal-dual and predictor–corrector IP methods were studied in [114]. To solve the STHTC problem, the two IP methods were implemented and both reported to be robust and fast when tested. In this implementation, the thermal system related constraints were not involved because the model considered a predominant hydro system. Costs of start up and shut down of thermal generating units and the minimum up and down ramp rates were taken into consideration. In order to make it simple, the model used only pure quadratic cost functions for generation considering the same quadratic coefficients for all units. In a conclusion on the convergence issues, the results showed that the number of iterations required was increased by the active bounds, while the generation costs found to be more critical than the transmission costs and therefore led to a faster convergence.

In [115], the objective of the STHTC problem was to minimize the difference between the generation costs and the consumer benefit in a predominantly hydroelectric system considering the dynamic restrictions of the consumer energy constraints. To achieve this goal, a primal-dual IP method was used and the STHTC problem was treated as a single problem. It was found that as the price-responsive load increased, the hydro generation therefore increased and as a result the thermal generation decreased which reduced the generation costs. The authors in [116] proposed a STHTC model in which the spot market trades and the bilateral transactions were discriminated. A primal-dual IP method was employed to solve the hydro and thermal sub-problems and a bundle method was implemented for the dual problem. In this method, the unit commitment status was assumed known. Although the power balance equations did not include the system transmission losses but they were assumed to be supplied by the pool. The hydropower was modeled as a function of the forebay elevation and the afterbay elevation but without considering the flow losses and the effects of the net head and the discharge rate on the joint efficiency of the turbine-generator.

Troncoso et al. in [64] used genetic algorithm to compute the optimal STHTC problem and compare its performance to that of the IP method. Results showed that the genetic algorithm reached better feasible minima while the IP method achieved better performance in CPU time. It was also shown that the IP algorithm including the constraints converged without difficulty when the problem was solved only to obtain an initialization point. On the other hand, when the same problem was solved using the former solution, the stopping criteria needed more iterations. It should be mentioned also that the constraints did not include the system power losses.

The need to decrease the computational cost of the genetic algorithm was expressed as an important issue for the real systems. Ref. [117] presented a homogenous IP method to solve the STHTC problem. Many constraints were considered in the model assuming that there existed a local large reservoir for each hydro generation unit so that the head variations were ignored. The long-term bilateral contracts and the forecasted market hourly prices for day-ahead auction were integrated in the model. In this implementation, a large-scale system with large number of constraints and variables was used to evaluate the algorithm.

2.7. Optimal control and other methods

In addition to the methods presented in the previous sections, some other approaches have been proposed to deal with the STHTC problem. It should be noted that the approaches which combined different techniques with one of the methods stated earlier are already mentioned in the preceding sections. Among the other methods is the optimal control theory that was used to solve the STHTC problem in [118,119]. Optimal control theory is an analytical optimization method that was formulated by Lev Pontryagin et al. [120]. This approach uses differential equations to minimize a cost function by identifying the paths of state and control variables.

The optimal control trajectory can be derived using Pontryagin's maximum principle as a necessary condition or by employing the Hamilton–Jacobi–Bellman equation as a sufficient condition. Refs. [2,121] present a detailed explanation of the application of the optimal control theory in hydrothermal scheduling optimization problem. In [118], it was shown that in spite of satisfying the ramp rate constraints when solving the STHTC problem, energy delivery capacity is not realized. The maximum principle was applied to formulate a set of equations to determine the energy delivery over the scheduling interval. The objective was to verify the realization of the energy delivery schedule in real-time operation. Although, the objective was different from that of the conventional STHTC problem, but the effort was to show that physical ramp rate is not equivalent to energy delivery capability as it was traditionally assumed.

Another application of optimal control theory to solve the STHTC problem was proposed in [119]. In this paper, Pontryagin's maximum principle, with the Bolza-type functional, was applied to prove a condition for the boundaries of the functional. The authors assigned a cost to the water to be included in the cost function instead of only considering the thermal cost. The Gauss–Southwell-type selection scheme was applied so as only the largest gradient value in the gradient vector was considered instead of following the direction of the negative gradient. To apply the method a Mathematica package program was used to solve hydrothermal system consisting of eight thermal plants and one hydro-plant of variable head. Two tests were carried out considering the same eight thermal units and 10 and 20 hydraulic plants respectively with the same variable-head model. Although it was reported that the method achieved good convergence and only two additional iterations were required when the number of plants was doubled, but the method was not tested for systems with a significantly larger number of hydraulic plants. In addition, it was not applied to hydrothermal systems with pumped-storage units.

A different approach to solve the STHTC problem using a non-linear network flow model was presented in [122]. This model had linear side constraints and the problem was not decomposed into hydro and thermal sub-problems. The hydrothermal coordination problem was formulated as an undecoupled problem while considering the network constraints as well as the local and spinning reserve coupling constraints. To evaluate the method it was implemented using several case examples. The results indicated that the solution was efficient and the computation requirement was reasonable. To evaluate the performance of the method, a general optimization code was used to solve the same problem. However, no comparison with a well-known standard optimization method was presented.

Ref. [123] presented an approach to improve the thermal start up and shut down costs of hydrothermal systems. This approach was applied using a program package that was not described clearly in the paper. It was indicated that the method was applied to a realistic system although no details about the model were presented. The method was reported to have a beneficial impact on the performance and overall cost; however, no numerical results were demonstrated.

3. Conclusions

This paper presents a bibliographical survey of the work published on the application of different optimization methods used to solve the short-term hydrothermal coordination problem. Various optimization techniques that tackled the problem are overviewed and classified with their advantages and limitations having been critically discussed. The paper provides a general literature survey and a list of published references on the topic aiming to offer the essential guidelines regarding this active research area.

References

[1] A.J. Wood, B.F. Wollenberg, Power Generation Operation and Control, 2nd ed., John Wiley & Sons, Inc, New York, USA, 1996.
[2] M.E. El-Hawary, G.S. Christensen, Optimal Economic Operation of Electric Power Systems, Academic Press, New York, 1979.
[3] Y.H. Song, Modern Optimization Techniques in Power Systems, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1999.
[4] J.A. Momoh, Electric Power System Applications of Optimization, Marcel Dekker, New York, 2001.
[5] J.A. Momoh, R. Adapa, M.E. El-Hawary, A review of selected optimal power flow literature to 1993. I. Nonlinear and quadratic programming approaches, IEEE Transactions on Power Systems 14 (1999) 96–104.
[6] J.A. Momoh, M.E. El-Hawary, R. Adapa, A review of selected optimal power flow literature to 1993. II. Newton, linear programming and interior point methods, IEEE Transactions on Power Systems 14 (1999) 105–111.
[7] S.S. Rao, Engineering Optimization: Theory and Practice, 3rd ed., John Wiley & Sons, New York, NY, 1996.
[8] L.A.F.M. Ferreira, A theoretical analysis of the classic hydro-thermal optimization algorithm in power system scheduling, in: Circuits and Systems 1992. ISCAS'92. Proceedings, 1992 IEEE International Symposium on, vol. 6, 1992, pp. 2757–2760.
[9] L.A.F.M. Ferreira, On the convergence of the classic hydro-thermal coordination algorithm, IEEE Transactions on Power Systems 9 (1994) 1002–1008.
[10] H. Yan, P.B. Luh, X. Guan, P.M. Rogan, Scheduling of hydrothermal power systems, IEEE Transactions on Power Systems 8 (1993) 1358–1365.
[11] X. Guan, P.B. Luh, H. Yen, P. Rogan, Optimization-based scheduling of hydrothermal power systems with pumped-storage units, IEEE Transactions on Power Systems 9 (1994) 1023–1031.
[12] R.N. Wu, T.H. Lee, E.F. Hill, Effect of interchange on short-term hydro-thermal scheduling, IEEE Transactions on Power Systems 6 (1991) 1217–1223.
[13] T. Ohishi, S. Soares, M.F. de Carvalho, A short term hydrothermal scheduling approach for dominantly hydro systems, IEEE Transactions on Power Systems 6 (1991) 637–643.
[14] C. Li, P.J. Jap, D.L. Streiffert, Implementation of network flow programming to the hydrothermal coordination in an energy management system, IEEE Transactions on Power Systems 8 (1993) 1045–1053.
[15] P.E.C. Franco, M.F. Carvalho, S. Soares, A network flow model for short-term hydro-dominated hydrothermal scheduling problems, IEEE Transactions on Power Systems 9 (1994) 1016–1022.
[16] S. Soares, T. Ohishi, Hydro-dominated short-term hydrothermal scheduling via a hybrid simulation-optimisation approach: a case study, Generation Transmission and Distribution, IEE Proceedings 142 (1995) 569–575.
[17] H. Yan, P.B. Luh, L. Zhang, Scheduling of hydrothermal power systems using the augmented Lagrangian decomposition and coordination technique, in: American Control Conference, 1994, vol. 2, 1994, pp. 1558–1562.
[18] Xi. Guan, P.B. Luh, L. Zhang, Nonlinear approximation method in Lagrangian relaxation-based algorithms for hydrothermal scheduling, IEEE Transactions on Power Systems 10 (1995) 772–778.
[19] S. Salam, K.M. Nor, A.R. Hamdan, Comprehensive algorithm for hydrothermal coordination, Generation Transmission and Distribution, IEE Proceedings 144 (1997) 482–488.
[20] M.S. Salam, K.M. Nor, A.R. Hamdam, Hydrothermal scheduling based Lagrangian relaxation approach to hydrothermal coordination, IEEE Transactions on Power Systems 13 (1998) 226–235.
[21] X. Guan, E. Ni, R. Li, P.B. Luh, An optimization-based algorithm for scheduling hydrothermal power systems with cascaded reservoirs and discrete hydro constraints, IEEE Transactions on Power Systems 12 (1997) 1775–1780.
[22] D. Zhang, P.B. Luh, Y. Zhang, A bundle method for hydrothermal scheduling, IEEE Transactions on Power Systems 14 (1999) 1355–1361.
[23] P.B. Luh, D. Zhang, R.N. Tomastik, An algorithm for solving the dual problem of hydrothermal scheduling, IEEE Transactions on Power Systems 13 (1998) 593–600.
[24] G. Demartini, T.R. De Simone, G.P. Granelli, M. Montagna, K. Robo, Dual programming methods for large-scale thermal generation scheduling, IEEE Transactions on Power Systems 13 (1998) 857–863.
[25] S. Ruzic, R. Rajakovic, Optimal distance method for Lagrangian multipliers updating in short-term hydro-thermal coordination, IEEE Transactions on Power Systems 13 (1998) 1439–1444.
[26] N.J. Redondo, A.J. Conejo, Short-term hydro-thermal coordination by Lagrangian relaxation: solution of the dual problem, IEEE Transactions on Power Systems 14 (1999) 89–95.
[27] E. Xi, X. Guan, R. Li, Scheduling hydrothermal power systems with cascaded and head-dependent reservoirs, IEEE Transactions on Power Systems 14 (1999) 1127–1132.
[28] E. Ni, P.B. Luh, Optimal integrated bidding and hydrothermal scheduling with risk management and self-scheduling requirements, in: Intelligent Control and Automation 2000. Proceedings of the 3rd World Congress on, vol. 3, 2000, pp. 2023–2028.
[29] S. Al-Agtash, Hydrothermal scheduling by augmented Lagrangian: consideration of transmission constraints and pumped-storage units, Power Engineering Review, IEEE 21 (2001) 58–59.
[30] M.M.M. Ahmed, M.R.A. Ghani, I. Hassan, Fast and optimized hydro-thermal coordination scheduling, in: Transmission and Distribution Conference and Exhibition 2002: Asia Pacific. IEEE/PES, vol. 1, 2002, pp. 261–266.

[31] M.M. Ahmed, M.R.A. Ghani, I. Hassan, Optimizing hydrothermal coordination scheduling for large scale power, in: Proceedings. PowerCon 2002. International Conference on Power System Technology, vol. 1, 2002, pp. 195–200.
[32] A. Borghetti, A. Frangioni, F. Lacalandra, C.A. Nucci, Lagrangian heuristics based on disaggregated Bundle methods for hydrothermal unit commitment, IEEE Transactions on Power Systems 18 (2003) 313–323.
[33] M.S. Salam, Comparison of Lagrangian relaxation and truncated dynamic programming methods for solving hydrothermal coordination problems, in: Proceedings of International Conference on Intelligent Sensing and Information Processing, 2004, pp. 265–270.
[34] A.L. Diniz, C. Sagastizabal, M.E.P. Maceira, Assessment of Lagrangian relaxation with variable splitting for hydrothermal scheduling, in: Power Engineering Society General Meeting 2007, IEEE, 2007, pp. 1–8.
[35] A.L. Diniz, T.N. Santos, M.E.P. Maceira, Short term security constrained hydrothermal scheduling considering transmission losses, in: Transmission & Distribution Conference and Exposition: Latin America 2006: TDC06. IEEE/PES, 2006, pp. 1–6.
[36] W. Sifuentes, A. Vargas, Short-term hydrothermal coordination considering an AC network modeling., International Journal of Electrical Power & Energy Systems 29 (2007) 488–496.
[37] W.S. Sifuentes, A. Vargas, Hydrothermal scheduling using benders decomposition: accelerating techniques, IEEE Transactions on Power Systems 22 (2007) 1351–1359.
[38] Z. Yu, F.T. Sparrow, B. Bowen, F.J. Smardo, On convexity issues of short-term hydrothermal scheduling, International Journal of Electrical Power & Energy Systems 22 (2000) 451–457.
[39] O. Nilsson, D. Sjelvgren, Mixed-integer programming applied to short-term planning of a hydro-thermal system, IEEE Transactions on Power Systems 11 (1996) 281–286.
[40] Z. Wang, G.Y. He, X. Chen, S. Zhou, A novel model of large-scale hydrothermal power system optimal scheduling, Power Systems Conference and Exposition 2004. IEEE/PES, vol. 1 (2004) 518–523.
[41] E. Parrilla, J. García-González, Improving the B&B search for large-scale hydrothermal weekly scheduling problems, International Journal of Electrical Power & Energy Systems 28 (2006) 339–348.
[42] J.-S. Yang, N. Chen, Short term hydrothermal coordination using multi-pass dynamic programming, IEEE Transactions on Power Systems 4 (1989) 1050–1056.
[43] J.-S. Yang, N. Chen, Unit commitment and hydrothermal generation scheduling by multi-pass dynamic programming, in: Decision and Control 1990, Proceedings of the 29th IEEE Conference on, vol. 6, 1990, pp. 3075–3076.
[44] T. Lee, N. Chen, The effect of pumped storage and battery energy storage systems on hydrothermal generation coordination, IEEE Transaction on Energy Conversion 7 (1992) 631–637.
[45] I. Erkmen, B. Karatas, Short-term hydrothermal coordination by using multi-pass dynamic programming with successive approximation, in: Electrotechnical Conference 1994. Proceedings, 7th Mediterranean, vol. 3, 1994, pp. 925–928.
[46] S. Ruzic, A. Vuckovic, N. Rajakovic, A flexible approach to short-term hydrothermal coordination. II. Dual problem solution procedure, IEEE Transactions on Power Systems 11 (1996) 1572–1578.
[47] S. Ruzic, N. Rajakovic, A. Vuckovic, A flexible approach to short-term hydrothermal coordination. I. Problem formulation and general solution procedure, IEEE Transactions on Power Systems 11 (1996) 1564–1571.
[48] S.N. Yu, Using hybrid EP and multi-pass dynamic programming for hydrothermal coordination considering reasonable spinning reserve, in: Transmission and Distribution Conference and Exhibition, 2005/2006 IEEE/PES, 2006, pp. 903–908.
[49] J. Tang, P.B. Luh, Hydrothermal scheduling via extended differential dynamic programming and mixed coordination, IEEE Transactions on Power Systems 10 (1995) 2021–2028.
[50] C.A. Li, A.J. Svoboda, C.L. Tseng, R.B. Johnson, E. Hsu, Hydro unit commitment in hydro-thermal optimization, IEEE Transactions on Power Systems 12 (1997) 764–769.
[51] D.B. Fogel, What is evolutionary computation? IEEE Spectrum 37 (2000) 26–32.
[52] V. Miranda, D. Srinivasan, L. Proença, Evolutionary computation in power systems, International Journal of Electrical Power & Energy Systems 20 (1998) 89–98.
[53] R.C. Bansal, Optimization methods for electric power systems: an overview, International Journal of Emerging Electric Power Systems 2 (2005), 1021.
[54] J.H. Holland, Adaptation in Natural and Artificial Systems, University of Michigan Press, 1975.
[55] J.A. Momoh, L.G. Dias, S.X. Guo, R. Adapa, Economic operation and planning of multi-area interconnected power systems, IEEE Transactions on Power Systems 10 (1995) 1044–1053.
[56] P. Chen, H. Chang, Genetic aided scheduling of hydraulically coupled plants in hydro-thermal coordination, IEEE Transactions on Power Systems 11 (1996) 975–981.
[57] S.O. Orero, M.R. Irving, A genetic algorithm modelling framework and solution technique for short term optimal hydrothermal scheduling, IEEE Transactions on Power Systems 13 (1998) 501–518.
[58] Y. Wu, C. Ho, D. Wang, A diploid genetic approach to short-term scheduling of hydro-thermal system, IEEE Transactions on Power Systems 15 (2000) 1268–1274.
[59] E. Gil, J. Bustos, H. Rudnick, Short-term hydrothermal generation scheduling model using a genetic algorithm, IEEE Transactions on Power Systems 18 (2003) 1256–1264.
[60] C.E. Zoumas, A.G. Bakirtzis, J.B. Theocharis, V. Petridis, A genetic algorithm solution approach to the hydrothermal coordination problem, IEEE Transactions on Power Systems 19 (2004) 1356–1364.
[61] P.E. Onate, J.M. Ramirez, Optimal operation of hydrothermal systems in the short term, in: Proceedings of the 37th Annual North American Power Symposium, 2005, pp. 113–119.
[62] X. Yuan, Y. Yuan, Application of cultural algorithm to generation scheduling of hydrothermal systems, Energy Conversion and Management 47 (2006) 2192–2201.
[63] S. Kumar, R. Naresh, Efficient real coded genetic algorithm to solve the non-convex hydrothermal scheduling problem, International Journal of Electrical Power & Energy Systems 29 (2007) 738–747.
[64] A. Troncoso, J.C. Riquelme, J.S. Aguilar-Ruiz, J.M. Riquelme Santos, Evolutionary techniques applied to the optimal short-term scheduling of the electrical energy production, European Journal of Operational Research 185 (2008) 1114–1127.
[65] S. Kirkpatrick, C.D. Gelatt Jr., M.P. Vecchi, Optimization by simulated annealing, Science 220 (1983) 671–680.
[66] K.P. Wong, Y.W. Wong, Short-term hydrothermal scheduling part. I. Simulated annealing approach, Generation, Transmission and Distribution, IEE Proceedings 141 (1994) 497–501.
[67] K.P. Wong, Y.W. Wong, Short-term hydrothermal scheduling. II. Parallel simulated annealing approach, Generation, Transmission and Distribution, IEE Proceedings 141 (1994) 502–506.
[68] D.N. Simopoulos, S.D. Kavatza, C.D. Vournas, An enhanced peak shaving method for short term hydrothermal scheduling, Energy Conversion and Management 48 (2007) 3018–3024.
[69] R. Eberhart, J. Kennedy, A new optimizer using particle swarm theory, in: Micro Machine and Human Science 1995. MHS95. Proceedings of the Sixth International Symposium on, 1995, pp. 39–43.
[70] J. Kennedy, R. Eberhart, Particle swarm optimization, in: Neural Networks 1995. Proceedings, IEEE International Conference on, vol. 4, 1995, pp. 1942–1948.
[71] M.R. AlRashidi, M.E. El-Hawary, A survey of particle swarm optimization applications in electric power systems, IEEE Transactions on Evolutionary Computation, in press.
[72] S.P. Umayal, N. Kamaraj, Stochastic Multi Objective Short Term Hydrothermal Scheduling Using Particle Swarm Optimization, INDICON, 2005 Annual IEEE, 2005, pp. 497–501.
[73] N. Sinha, L. Lai, Meta Heuristic Search Algorithms for Short-Term Hydrothermal Scheduling, Machine Learning and Cybernetics, 2006 International Conference on, 2006, pp. 4050–4056.
[74] B. Yu, X. Yuan, J. Wang, Short-term hydro-thermal scheduling using particle swarm optimization method, Energy Conversion and Management 48 (2007) 1902–1908.
[75] K.K. Mandal, M. Basu, N. Chakraborty, Particle swarm optimization technique based short-term hydrothermal scheduling, Applied Soft Computing 8 (2008) 1392–1399.
[76] P.H. Chen, Pumped-storage scheduling using evolutionary particle swarm optimization, IEEE Transaction on Energy Conversion 23 (2008) 294–301.
[77] T.G. Werner, J.F. Verstege, An evolution strategy for short-term operation planning of hydrothermal power systems, IEEE Transactions on Power Systems 14 (1999) 1362–1368.
[78] P. Yang, H. Yang, C. Huang, Scheduling short-term hydrothermal generation using evolutionary programming techniques, Generation Transmission and Distribution, IEE Proceedings 143 (1996) 371–376.
[79] P.K. Hota, R. Chakrabarti, P.K. Chattopadhyay, Short-term hydrothermal scheduling through evolutionary programming technique, Electric Power Systems Research 52 (1999) 189–196.
[80] F. Manzanedo, J.L. Castro, M. Perez-Donsion, Application of evolutionary techniques to short-term optimization of hydrothermal systems, in: Power System Technology 2000. Proceedings. PowerCon 2000. International Conference on, vol. 3, 2000, pp. 1539–1544.
[81] T.D.H. Cau, R.J. Kaye, Evolutionary optimisation method for multi-storage hydrothermal scheduling, Generation Transmission and Distribution, IEE Proceedings 149 (2002) 152–156.
[82] N. Sinha, R. Chakrabarti, P.K. Chattopadhyay, Fast evolutionary programming techniques for short-term hydrothermal scheduling, IEEE Transactions on Power Systems 18 (2003) 214–220.
[83] N. Sinha, R. Chakrabarti, P.K. Chattopadhyay, Fast evolutionary programming techniques for short-term hydrothermal scheduling, Electric Power Systems Research 66 (2003) 97–103.
[84] M.R. Babu, T.D. Sudhakar, K. Mohanadasse, An hybrid technique to hydrothermal scheduling, in: Power Engineering Conference, 2005. IPEC 2005. The 7th International, vol. 2, 2005, pp. 883–887.
[85] C. Nallasivan, D.S. Suman, J. Henry, S. Ravichandran, A novel approach for short-term hydrothermal scheduling using hybrid technique, in: IEEE Power India Conference, vol. 5, 2006.
[86] B.N.S. Rahimullah, T.K.A. Rahman, Short-term Hydrothermal Generation Scheduling Using Evolutionary Computing Technique, Research and Development 2006. SCOReD 2006. 4th Student Conference on, 2006, pp. 220–223.
[87] L. Lakshminarasimman, S. Subramanian, Short-term scheduling of hydrothermal power system with cascaded reservoirs by using modified differential

evolution, Generation Transmission and Distribution, IEE Proceedings 153 (2006) 693–700.
[88] S. Haykin, Neural Networks: A Comprehensive Foundation, 2nd ed., Prentice Hall, Inc., Upper Saddle River, NJ, USA, 1999.
[89] J.J. Hopfield, Neural networks and physical systems with emergent collective computational abilities, Proceedings of the National Academy of Sciences of the United States of America 79 (1982) 2554–2558.
[90] V.S.S. Vankayala, N.D. Rao, Artificial neural networks and their applications to power systems—a bibliographical survey, Electric Power Systems Research 28 (1993) 67–79.
[91] R. Naresh, J. Sharma, Two-phase neural network based solution technique for short term hydrothermal scheduling, Generation Transmission and Distribution, IEE Proceedings 146 (1999) 657–663.
[92] M. Basu, Hopfield neural networks for optimal scheduling of fixed head hydrothermal power systems, Electric Power Systems Research 64 (2003) 11–15.
[93] V.N. Dieu, W. Ongsakul, Enhanced merit order and augmented Lagrange Hopfield network for hydrothermal scheduling, International Journal of Electrical Power & Energy Systems 30 (2008) 93–101.
[94] M.E. El-Hawary, Electric Power Applications of Fuzzy Systems, Wiley-IEEE Press, USA, 1998.
[95] Y. Song, A.T. Johns, Applications of fuzzy logic in power systems. I. General introduction to fuzzy logic, Power Engineering Journal 11 (1997) 219–222 (see also Power Engineer).
[96] Y. Song, A.T. Johns, Application of fuzzy logic in power systems. II. Comparison and integration with expert systems, neural networks and genetic algorithms, Power Engineering Journal 12 (1998) 185–190 (see also Power Engineer).
[97] Y. Song, A.T. Johns, Applications of fuzzy logic in power systems. III. Example applications, Power Engineering Journal 13 (1999) 97–103 (see also Power Engineer).
[98] J.A. Momoh, X.W. Ma, K. Tomsovic, Overview and literature survey of fuzzy set theory in power systems, IEEE Transactions on Power Systems 10 (1995) 1676–1690.
[99] R.C. Bansal, Bibliography on the fuzzy set theory applications in power systems (1994–2001), IEEE Transactions on Power Systems 18 (2003) 1291–1299.
[100] K.P. Wong, Y.W. Wong, Combined genetic algorithm/simulated annealing/fuzzy set approach, IEEE Transactions on Power Systems 11 (1996) 128–136.
[101] S. Huang, Application of genetic based fuzzy systems to hydroelectric generation scheduling, IEEE Transaction on Energy Conversion 14 (1999) 724–730.
[102] J.S. Dhillon, S.C. Parti, D.P. Kothari, Fuzzy decision-making in stochastic multiobjective short-term hydrothermal scheduling, Generation Transmission and Distribution, IEE Proceedings 149 (2002) 191–200.
[103] M. Basu, An interactive fuzzy satisfying method based on evolutionary programming technique for multiobjective short-term hydrothermal scheduling, Electric Power Systems Research 69 (2004) 277–285.
[104] N. Karmarkar, A new polynomial-time algorithm for linear programming, Combinatorica 4 (1984) 373.
[105] A. Forsgren, P.E. Gill, M.H. Wright, Interior methods for nonlinear optimization, SIAM Review 44 (2002) 525–597.
[106] K.R. Frisch, The Logarithmic Potential Method of Convex Programming, Institute of Economics, Oslo, Norway, 1955.
[107] P.E. Gill, W. Murray, M.A. Saunders, J. Tomlin, M.H. Wright, On projected Newton barrier methods for linear programming and an equivalence to Karmarkar's projective method, Mathematical Programming 36 (1986) 183–209.
[108] V.H. Quintana, G.L. Torres, J. Medina-Palomo, Interior-point methods and their applications to power systems: a classification of publications and software codes, IEEE Transactions on Power Systems 15 (2000) 170–176.
[109] N. Megiddo, Pathways to the optimal set in linear programming on Progress in Mathematical Programming: Interior-Point and Related Methods, Pacific Grove, California, US, 1989, pp. 131–158.
[110] S. Mehrotra, On the implementation of the primal-dual interior point method, SIAM Journal on Optimization 2 (1992) 575–601.
[111] K.A. Clements, P.W. Davis, K.D. Frey, An interior point algorithm for weighted least absolute value power system state estimation, in: IEEEPES Winter Meeting, 1991.
[112] N.J.O. Palacio, K.C. Almeida, H.H. Zurn, Short term hydrothermal scheduling under bilateral contracts, in: Power Tech Proceedings, 2001 IEEE Porto, vol. 1, 2001, p. 6.
[113] J.L.M. Ramos, A.T. Lora, J.R. Santos, A.G. Exposito, Short-term hydro-thermal coordination based on interior point nonlinear programming and genetic algorithms, in: Power Tech Proceedings, 2001 IEEE Porto, vol. 3, 2001, p. 6.
[114] A.R.L. Oliveira, S. Soares, L. Nepomuceno, Short term hydroelectric scheduling combining network flow and interior point approaches, International Journal of Electrical Power & Energy Systems 27 (2005) 91–99.
[115] W. Uturbey, A.S. Costa, Dynamic optimal power flow approach to account for consumer response in short term hydrothermal coordination studies, Generation, Transmission & Distribution, IET 1 (2007) 414–421.
[116] A.J. Mezger, K.C. de Almeida, Short term hydrothermal scheduling with bilateral transactions via bundle method, International Journal of Electrical Power & Energy Systems 29 (2007) 387–396.
[117] S. Bisanovic, M. Hajro, M. Dlakic, Hydrothermal self-scheduling problem in a day-ahead electricity market, Electric Power Systems Research 78 (2008) 1579–1596.
[118] X. Guan, F. Gao, A.J. Svoboda, Energy delivery capacity and generation scheduling in the deregulated electric power market, IEEE Transactions on Power Systems 15 (2000) 1275–1280.
[119] L. Bayón, J.M. Grau, M.M. Ruiz, P.M. Suárez, A Bolza problem in hydrothermal optimization, Applied Mathematics and Computation 184 (2007) 12–22.
[120] L.S. Pontryagin, V.G. Boltayanskii, R.V. Gamkrelidze, E.F. Mishchenko, The Mathematical Theory of Optimal Processes, Wiley/Interscience, 1962.
[121] G.S. Christensen, M.E. El-Hawary, S.A. Soliman, Optimal Control Application in Electric Power Systems, Springer, 1987.
[122] F.J. Heredia, N. Nabona, Optimum short-term hydrothermal scheduling with spinning reserve through network flows, IEEE Transactions on Power Systems 10 (1995) 1642–1651.
[123] S. Ruzic, N. Rajakovic, A. Vuckovic, Operational costs modeling improvements in short term hydrothermal coordination, in: Electrotechnical Conference, 1998. MELECON 98, 9th Mediterranean, vol. 2, 1998, pp. 916–920.