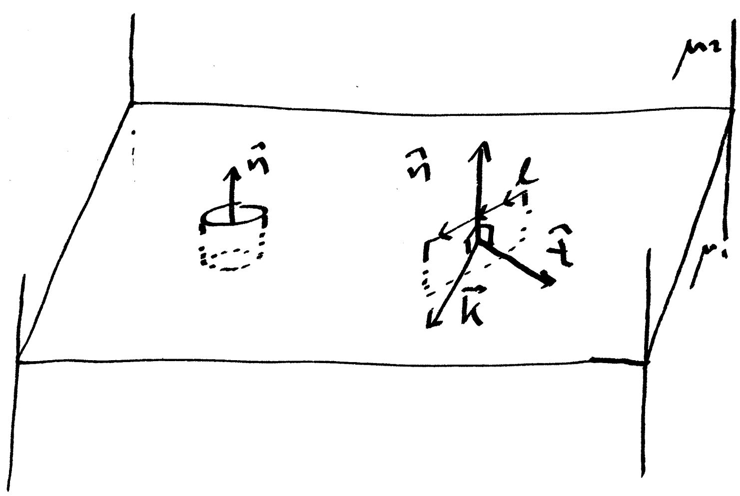
Macroscopic magnetostatics equations:

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = \frac{4\pi}{c}\vec{J}, \qquad \vec{B} = \vec{H} + 4\pi\vec{M} = \mu\vec{H}.$$

At the interface between two media, we need to impose appropriate junction conditions:



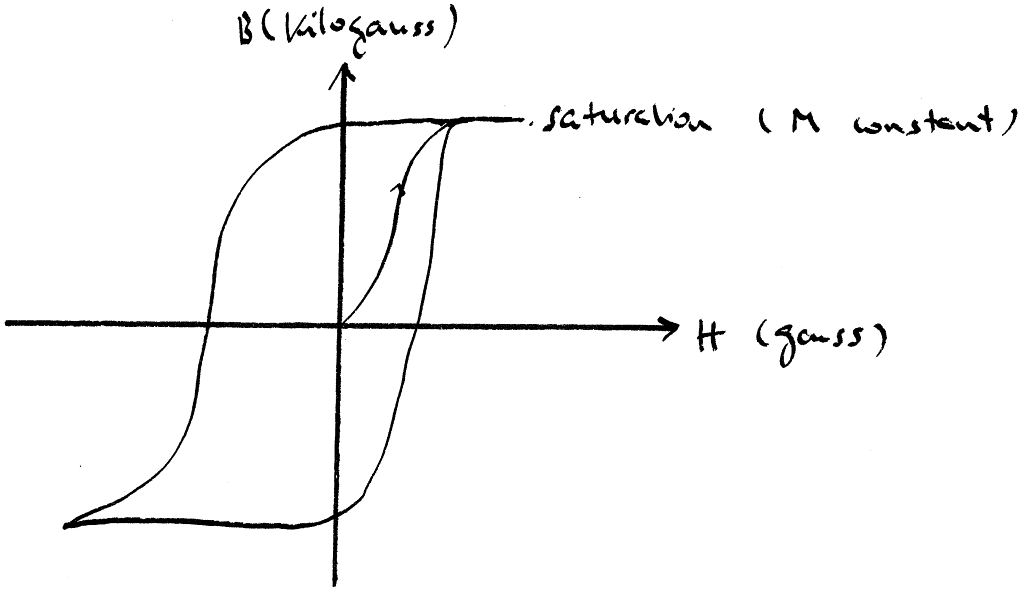
$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow (\vec{B}_2 - \vec{B}_1)\,\hat{n} = 0$$

$$\vec{\nabla}\times\vec{H} = \frac{4\pi}{c}\vec{J} \Rightarrow l\left[\vec{H}_2\cdot(\hat{t}\times\hat{n}) - \vec{H}_1\cdot(\hat{t}\times\hat{n})\right] = \overset{\frac{4\pi}{c}}{\vec{K}}\cdot\hat{t}\cdot l$$

$$\Rightarrow \hat{n}\times(\vec{H}_2 - \vec{H}_1) = \frac{4\pi}{c}\vec{K}$$

8.4. Ferromagnetism

In ferromagnetic materials (like iron), the magnetization can become much larger than $\vec{H}$. In these materials, the function $\vec{B}(\vec{H})$ shows a hysteresis curve:

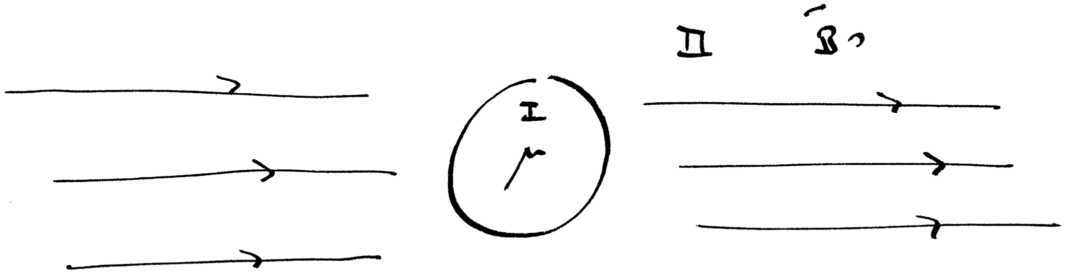


for soft ferromagnets $B = 0$ at $H = 0$

" hard " $B \neq 0$ at $H = 0$ (permanent magnetization)

We can use the similarity between the eqs. of electrostatics and magnetostatics to solve problems in ferromagnetism

Example: A hard ferromagnetic sphere in an external field $\vec{B}_0$



solve: $\vec{\nabla}\cdot\vec{B} = 0$ with $(\vec{B}_2 - \vec{B}_1)\cdot\hat{n} = 0$ $(\vec{B} = \mu\vec{H})$

$\vec{\nabla}\times\vec{H} = 0$ $\hat{n}\times(\vec{H}_2 - \vec{H}_1) = 0.$

Compare with electrostatics:

$\vec{\nabla}\cdot\vec{D} = 0$ with $(\vec{D}_2 - \vec{D}_1)\cdot\hat{n} = 0$ $(\vec{D} = \epsilon\vec{E})$

$\vec{\nabla}\times\vec{E} = 0$ $\hat{n}\times(\vec{E}_2 - \vec{E}_1) = 0.$

$\vec{D} \leftrightarrow \vec{B}$ and $\vec{E} \leftrightarrow \vec{H}$ turns one into the other.

In the case of electrostatics,

$$\vec{E}_I = \frac{3}{\epsilon+2}\vec{E}_0 \Rightarrow \vec{H}_I = \frac{3}{\mu+2}\vec{H}_0 \Rightarrow \frac{\vec{B}_I}{\mu} = \frac{3}{\mu+2}\frac{\vec{B}_0}{1}$$

or $\vec{B}_I = \dfrac{3\mu}{\mu+2}\vec{B}_0.$ In the limit $\mu\to\infty$, $\vec{B}_I = 3\vec{B}_0$

Therefore, we can identify

$\vec{B}_M = \vec{B}_I - \vec{B}_0 = 2\vec{B}_0$ as the field

due to the magnetization.

If we assume $\vec{H}$ negligible, $\vec{H} = \vec{B} - 4\pi\vec{M} \simeq 0$,

the magnetization is $\vec{B} = 4\pi\vec{M} = 3\vec{B}_0$, when

the external field is on. If we shut

the latter off, the magnetization retains its value

$\vec{M} = \frac{3\vec{B}_0}{4\pi}$. The magnetic field inside is then

$\vec{B} = 2\vec{B}_0 = \frac{8\pi}{3}\vec{M}$, and hence

$\vec{H} = \vec{B} - 4\pi\vec{M} = 2\vec{B}_0 - 4\pi\vec{M} = -\frac{4\pi}{3}\vec{M}$.

Note that $\vec{H}$ is determined by the magnetization.

The relation between $\vec{B}$ and $\vec{H}$ in a ferromagnet

depends on the geometry.

8.8. The H field for a permanent magnet

In a permanent magnet, $\vec{M}$ is not determined by $\vec{H}$ nor $\vec{B}$, and can be taken as given.

Then,

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \quad & \vec{\nabla}\cdot\vec{H} = -4\pi\,\vec{\nabla}\cdot\vec{M} \\ & \vec{\nabla}\times\vec{H} \end{aligned}\right\}$$

These eqs. work like those of electrostatics with $\vec{H} \leftrightarrow \vec{E}$ and $\rho_e \leftrightarrow \rho_m \equiv -\vec{\nabla}\cdot\vec{M}$

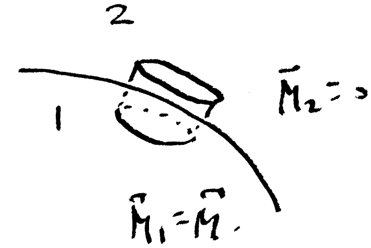
(also compare with $\rho_{pol} = -\vec{\nabla}\cdot\vec{P}$)

Therefore, we can use results and techniques from electrostatics with permanent magnets:

$$\vec{H}(\vec{r}) = -\vec{\nabla}\phi_M, \quad \text{with}$$

$$\phi_M(\vec{r}) = -\int_V \frac{\vec{\nabla}' M(\vec{r}')}{|\vec{r}-\vec{r}'|}\,d^3r' + \underbrace{\oint_{\partial V} \frac{M(\vec{r}')\,\hat{n}}{|\vec{r}-\vec{r}'|}\,dA}_{\text{contribution from surface "charge" density}}$$

As in electrostatics, at an interface



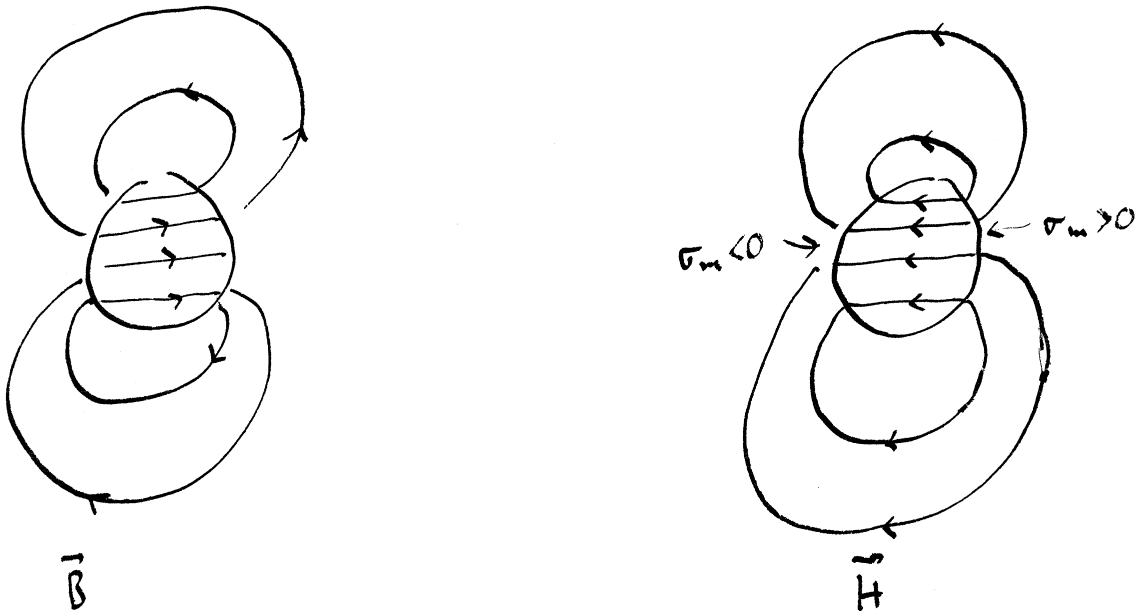
$$\sigma_m = -\left(\vec{M}_2 - \vec{M}_1\right)\hat{n} = 0 \quad \Rightarrow \quad \sigma_m = \vec{M}\cdot\hat{n}$$

if $\vec{M}_2 = 0$

(from $\vec{\nabla}\cdot\vec{M} = -\rho$)

σ_m is the magnetization surface charge.

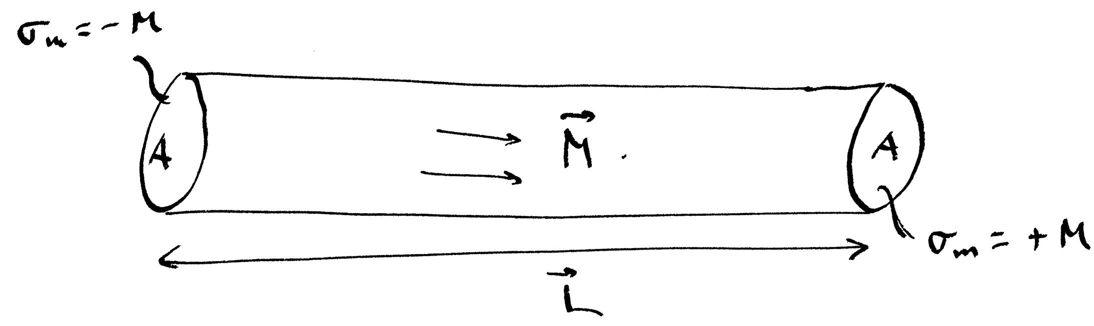
In the case of the sphere



8.9. Bar Magnet

Finally, consider a <u>bar magnet</u>:

Cylindrical hard ferromagnet of cross section area A, and length L ($L^2 >> A$), with constant magnetization $\vec{M}$ along its axis



We can thus think of the magnet as a cylinder with magnetic charges

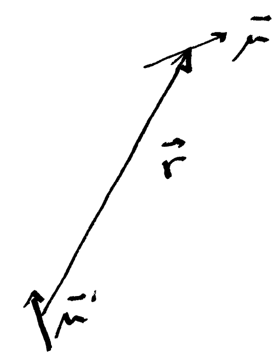
$$g = \sigma_m A = AM \quad \text{at right end}$$

$$g = \sigma_m \cdot A = -AM \quad \text{" left end.}$$

$|g| = AM$ is the magnet pole strength.

i) Therefore, at large distances, $r \gg L$, the magnet behaves like a magnetic dipole with moment $\vec{\mu} = g\vec{L}$. From Lecture 12, the magnetic field is $\vec{B} = \dfrac{3(\vec{\mu}\cdot\hat{r})\hat{r} - \vec{\mu}}{r^3}$. Therefore, the force between two magnets at $r \gg L$ is that derived from $U = -\vec{B}\cdot\vec{\mu}$, or

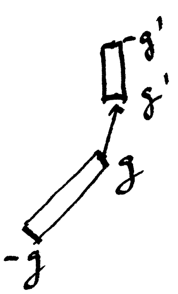
$$U = \frac{\vec{\mu}\cdot\vec{\mu}' - 3(\vec{\mu}\cdot\hat{r})(\vec{\mu}'\cdot\hat{r})}{r^3}$$



ii) If $r \ll L$ and $r^2 \gg A$, we can neglect the contribution from the "far" end of the dipole, and the charge in the "close" end can be approximated by a point charge.

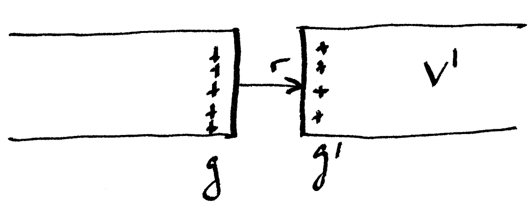
The force is then the coulomb force between two "point charges":

$$\vec{F} = \frac{g \cdot g'}{r^2}$$



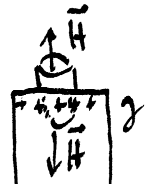
Note that there is no dependence on orientation.

iii) The force between two magnets end-to-end
($r^2 \ll A$)



Is that of the charge g' in the homogeneous field

field



$2\vec{H}\cdot\hat{n} \doteq 4\pi\sigma_m \Rightarrow H = 2\pi\sigma_m$.

$\Rightarrow B = 2\pi\sigma_m$ (outside the magnet)

To calculate this force, recall that

$$\vec{F} = \vec{\nabla}(\vec{B}\cdot\vec{\mu}) \Rightarrow d^3\vec{F} = \vec{\nabla}(\vec{B}\cdot\vec{M}')\, d^3r \Rightarrow$$

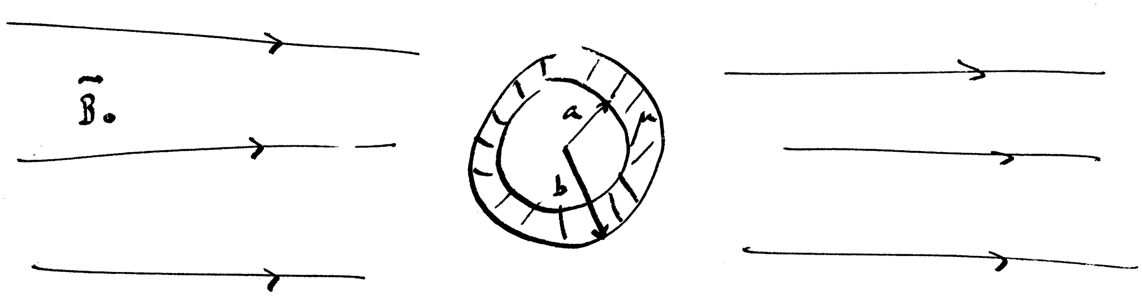
$$\vec{F} = \int_{V'} d^3r\ \vec{\nabla}(\vec{B}\cdot\vec{M}') = \oint_{\partial V'} (\vec{B}\cdot\vec{M}')\, \hat{n}\, dA \quad , \text{or}$$

$$F = B\cdot M' A = 2\pi \sigma_m M' A = 2\pi \frac{g g'}{A} .$$

As between two charged parallel plates.

Exercise 24

A cylinder shell is placed in a transverse homogeneous magnetic field, as in the figure.



Calculate the fields $\vec{H}$, $\vec{B}$ everywhere in space.

What happens in the limit $\mu \to \infty$?