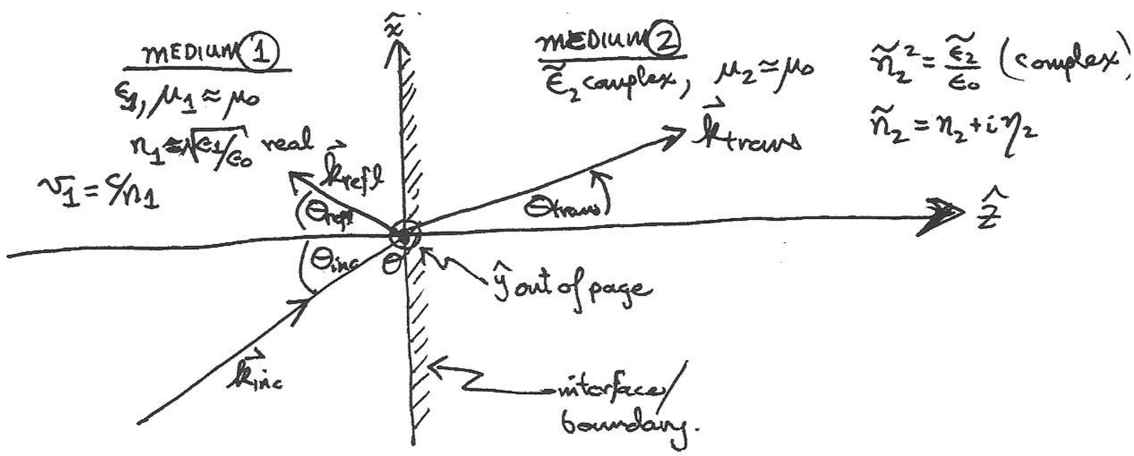
LECTURE NOTES 8.5

Reflection and Refraction of *EM* Waves at the Boundary of a Dispersive/Absorbing/Conducting Medium

Consider a situation where monochromatic plane *EM* waves are incident on a boundary between two media {located at $z = 0$ and lying in the x-y plane} as shown in the figure below. For the sake of simplicity, the 1st medium ($z < 0$) is linear/homogeneous/isotropic, non-absorbing / non-dispersive and non-magnetic. The 2nd medium is also linear/homogeneous/isotropic and non-magnetic, but is absorbing/dispersive *and* conductive.



Because of the above-stated *EM* properties of the two media, in medium (1) the incident and reflected wavevectors $\vec{k}_{inc}$ and $\vec{k}_{refl}$ are purely real, whereas in medium (2), the transmitted wavevector is complex: $\tilde{\vec{k}}_{trans}(\omega) = \vec{k}_{trans}(\omega) + i\vec{\kappa}_{trans}(\omega)$. Note that the monochromatic plane *EM* wave(s) have the same frequency ω, independent of the medium they are propagating in.

THE ELECTRIC FIELDS:

Medium (1) (non-absorbing):
$$\begin{cases} \tilde{\vec{E}}_{inc}(\vec{r},t) = \tilde{\vec{E}}_{o_{inc}}(\vec{r})\, e^{i(\vec{k}_{inc}\cdot\vec{r}-\omega t)} \\ \tilde{\vec{E}}_{refl}(\vec{r},t) = \tilde{\vec{E}}_{orefl}(\vec{r})\, e^{(\vec{k}_{refl}\cdot\vec{r}-\omega t)} \end{cases}$$
$\boxed{\vec{k}_{inc},\ \vec{k}_{refl}}$ $\Leftarrow$ real, *constant* wavevectors

Medium 2) (absorbing / conducting):
$$\tilde{\vec{E}}_{trans}(\vec{r},t) = \tilde{\vec{E}}_{o_{trans}}(\vec{r})\, e^{i(\tilde{\vec{k}}_{trans}(\omega)\cdot\vec{r}-\omega t)}$$
$\boxed{\tilde{\vec{k}}_{trans}(\omega) = \vec{k}_{trans}(\omega) + i\vec{\kappa}_{trans}(\omega)}$ $\Leftarrow$ *complex* wavevector

On the boundary/interface (lying in the x-y plane at $z = 0$) we *must* have (for *arbitrary* times, t):

$$\boxed{e^{i(\vec{k}_{inc}\cdot\vec{r}-\omega t)} = e^{i(\vec{k}_{refl}\cdot\vec{r}-\omega t)}} \quad \underline{and}: \quad \boxed{e^{i(\vec{k}_{inc}\cdot\vec{r}-\omega t)} = e^{i(\tilde{\vec{k}}_{trans}(\omega)\cdot\vec{r}-\omega t)} = e^{i(\vec{k}_{trans}(\omega)\cdot r-\omega t)}e^{-\vec{\kappa}_{trans}(\omega)\cdot\vec{r}}}$$

$$\Rightarrow \boxed{\vec{k}_{inc}\cdot\vec{r} = \vec{k}_{refl}\cdot\vec{r}} \quad \underline{and}: \boxed{\vec{k}_{inc}\cdot\vec{r} = \tilde{\vec{k}}_{trans}(\omega)\cdot\vec{r} = \left(\vec{k}_{trans}(\omega) + i\vec{\kappa}_{trans}(\omega)\right)\cdot\vec{r} = \vec{k}_{trans}(\omega)\cdot\vec{r} + i\vec{\kappa}_{trans}(\omega)\cdot\vec{r}}$$

On the interface/boundary lying in the *x-y* plane at $z = 0$:

> ***n.b.*** θ_{inc}, θ_{refl} and θ_{trans} are defined with respect to the $+\hat{z}$ unit normal of the interface/boundary.

The 1st equation: $\boxed{\vec{k}_{inc} \cdot \vec{r} = \vec{k}_{refl} \cdot \vec{r}}$ gives usual Law of Reflection:

$$\boxed{k_{inc} r \sin\theta_{inc} = k_{refl} r \sin\theta_{refl}}$$

but: $\boxed{k_{inc} = \omega/v_1 = k_{refl} = \omega/v_1}$ because both the incident and reflected waves are in the same non-dispersive/non-absorbent medium {medium (1)}.

$$\Rightarrow \quad \boxed{\sin\theta_{inc} = \sin\theta_{refl}} \quad \Rightarrow \quad \boxed{\theta_{inc} = \theta_{refl}}$$

The 2nd equation: $\boxed{\vec{k}_{inc} \cdot \vec{r} = \tilde{\vec{k}}_{trans}(\omega) \cdot \vec{r} = \left(\vec{k}_{trans}(\omega) + i\vec{\kappa}_{trans}(\omega)\right) \cdot \vec{r} = \vec{k}_{trans}(\omega) \cdot \vec{r} + i\vec{\kappa}_{trans}(\omega) \cdot \vec{r}}$, after equating real and imaginary parts, gives:

$$\boxed{\mathrm{Re}(\ ):\quad \vec{k}_{inc} \cdot \vec{r} = \vec{k}_{trans}(\omega) \cdot \vec{r}} \quad \text{and} \quad \boxed{\mathrm{Im}(\ ):\quad 0 = \vec{\kappa}_{trans}(\omega) \cdot \vec{r}}$$

$\Rightarrow$ In general, $\vec{k}_{trans}(\omega)$ and $\vec{\kappa}_{trans}(\omega)$ are ***not*** parallel to each other!!

i.e. In general, $\vec{k}_{trans}(\omega)$ and $\vec{\kappa}_{trans}(\omega)$ ***will*** point in ***different*** directions!! Why/How???

Physically, the requirement that $\vec{\kappa}_{trans}(\omega) \cdot \vec{r} = 0$ on the interface/boundary {lying in the *x-y* plane at $z = 0$} means that $\vec{\kappa}_{trans}(\omega) = \mathrm{Im}\left(\tilde{\vec{k}}_{trans}(\omega)\right)$ *must* be $\perp$ to the boundary (*i.e.* $\vec{\kappa}_{trans} \parallel +\hat{z}$), since the position vector $\vec{r}$ {pointing from the origin $\vartheta(0,0,0)$ to an arbitrary point $(x, y, z = 0)$ on the boundary} lies in the *x-y* plane.

Inside Absorbing/Conducting Medium (2) (*i.e.* $z > 0$):

Because $\tilde{\vec{k}}_{trans} = \vec{k}_{trans} + i\vec{\kappa}_{trans}$, then: $\tilde{\vec{E}}_{trans}(z,t) = \tilde{\vec{E}}_{o_{trans}}(\vec{r}) e^{i\left(\tilde{\vec{k}}_{trans} \cdot \vec{r} - \omega t\right)} = \tilde{\vec{E}}_{o_{trans}}(\vec{r}) e^{-\kappa_{trans} z} e^{i\left(\vec{k}_{trans} \cdot \vec{r} - \omega t\right)}$;

Thus, we see that:

$\vec{\kappa}_{trans} = \mathrm{Im}\left(\tilde{\vec{k}}_{trans}\right)$ defines planes ($\parallel$ to the boundary/interface) of constant electric field ***amplitude*** in medium (2).

$\hat{\kappa}_{trans} = \mathrm{Im}\left(\tilde{\hat{k}}_{trans}\right)$ is the unit normal to the planes of constant electric field ***amplitude*** in medium (2).

Furthermore:

$\vec{k}_{trans} = \mathrm{Re}\left(\tilde{\vec{k}}_{trans}\right)$ defines planes of constant ***phase*** in medium (2)

$\hat{k}_{trans} = \mathrm{Re}\left(\tilde{\hat{k}}_{trans}\right)$ is the unit normal to the planes of constant ***phase*** in medium (2)

{*n.b.* in general, planes of constant ***phase*** could be in ***any*** direction, depending on the material!}

See the following figure for a explicit diagram of exactly what is occurring in this physics problem:

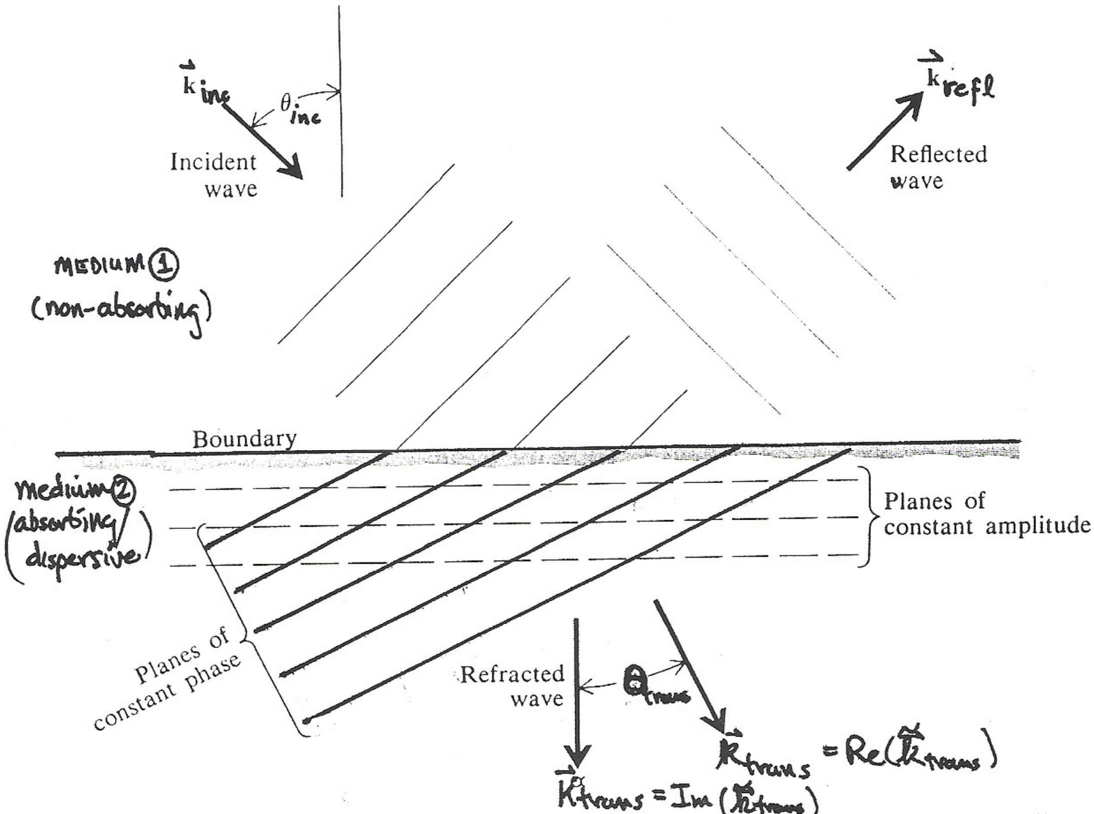


Figure 5.5. Real and imaginary parts of the wave vector in an absorbing medium for the case of oblique incidence of light at the boundary.

On the interface/boundary {lying in the *x*-*y* plane at *z* = 0}, at an arbitrary point $(x, y, z = 0)$:

$$\boxed{\mathrm{Re}(\ ):\quad \vec{k}_{inc}\bullet\vec{r} = \vec{k}_{trans}(\omega)\bullet\vec{r}} \quad \text{means:} \quad \boxed{k_{inc} r\sin\theta_{inc} = k_{trans}(\omega) r\sin\theta_{trans}}$$

$$\text{or:} \quad \boxed{k_{inc}\sin\theta_{inc} = k_{trans}(\omega)\sin\theta_{trans}}$$

n.b. θ_{inc}, θ_{refl} and θ_{trans} are defined with respect to the $+\hat{z}$ unit normal of the interface/boundary.

Because the wave vector $\vec{\tilde{k}}_{trans}(\omega)$ is complex, we ***do not*** have a simple relation between the wavenumber $k_{trans}(\omega)$ and the {angular} frequency ω in the dispersive, conducting medium (2), *i.e.* $k_{trans}(\omega) \neq \omega/v_2$ as we did for the incident and reflected wavevectors $k_{inc} = \omega/v_1 = k_{refl} = \omega/v_1$ associated with their respective *EM* waves propagating in the non-dispersive, non-conducting, non-magnetic medium (1).

In medium (1), the index of refraction n_1 is purely real and independent of frequency (*i.e.* medium (1) is *non-dispersive*), thus the {real} relation $v_1 = c/n_1$ *is* valid in medium (1), whereas in the *dispersive*, *conductive* medium (2), the {frequency-dependent!} complex wavenumber $\tilde{k}_2(\omega)$ and index of refraction $\tilde{n}_2(\omega)$ are related to each other by $\tilde{n}_2(\omega) = (c/\omega)\tilde{k}_2(\omega)$, thus the index of refraction in medium (2) is complex *and* frequency-dependent $\tilde{n}_2(\omega) = n_2(\omega) + i\eta_2(\omega)$, and thus the speed of propagation in medium (2) $\tilde{v}_2(\omega) = c/\tilde{n}_2(\omega)$ is also complex.

Note that we can also determine the relationship between complex wave vector $\tilde{\vec{k}}_2(\omega)$ and complex index of refraction, $\tilde{n}_2(\omega)$ of the absorbing/dispersive, non-magnetic, conducting medium (2) from (either of) the wave equation(s) associated with the tranmitted $\vec{E}$ and $\vec{B}$-fields in medium (2), which can be written (*e.g.* for complex $\vec{E}_{trans}$) as:

$$\boxed{\nabla^2\tilde{\vec{E}}_{trans}(\vec{r},t) = \frac{1}{\tilde{v}_2^2(\omega)}\frac{\partial^2\tilde{\vec{E}}_{trans}(\vec{r},t)}{\partial t^2} = \frac{\tilde{n}_2^2(\omega)}{c^2}\frac{\partial^2\tilde{\vec{E}}_{trans}(\vec{r},t)}{\partial t^2}}$$

For plane harmonic (*i.e.* monochromatic) *EM* waves propagating in absorbing/dispersing non-magnetic medium (2), noting that: $\nabla\tilde{\vec{E}}_{trans}$ gives: $\underline{\underline{i\tilde{\vec{k}}_{trans}(\omega)}}$ and: $\partial\tilde{\vec{E}}_{trans}/\partial t$ gives: $\underline{\underline{-i\omega}}$

Thus, the <u>characteristic equation</u> associated with the above differential equation is:

$$\boxed{i\tilde{\vec{k}}_{trans}(\omega)\bullet i\tilde{\vec{k}}_{trans}(\omega) = \frac{\tilde{n}_2^2(\omega)}{c^2}(-i\omega)(i\omega)} \;\Rightarrow\; \boxed{-\tilde{k}_{trans}^2(\omega) = -\left(\frac{\tilde{n}_2(\omega)}{c}\right)^2\omega^2} \;\; \underline{\text{or}}\text{:} \; \boxed{\tilde{k}_{trans}^2(\omega) = \left(\frac{\omega}{c}\right)^2\tilde{n}_2^2(\omega)}$$

<u>But</u>: $k_o \equiv \left(\frac{\omega}{c}\right)$ = <u>*vacuum*</u> wavenumber = $\frac{2\pi}{\lambda_o}$, where: $\lambda_o = \frac{c}{f}$.

$\therefore$ $\boxed{\tilde{k}_{trans}^2(\omega) = \left(\frac{\omega}{c}\right)^2\tilde{n}_2^2(\omega) = \tilde{n}_2^2(\omega)k_o^2}$ <u>where</u>: $\boxed{k_o \equiv \left(\frac{\omega}{c}\right)}$ = purely <u>real</u> quantity.

If we explicitly write out the <u>real</u> and <u>imaginary</u> parts of $\tilde{\vec{k}}_{trans}(\omega) = \vec{k}_{trans}(\omega) + i\vec{\kappa}_{trans}(\omega)$ associated with the above $i\tilde{\vec{k}}_{trans}(\omega)\bullet i\tilde{\vec{k}}_{trans}(\omega)$ term and the <u>real</u> and <u>imaginary</u> parts of $\tilde{n}_2(\omega) = n_2(\omega) + i\eta_2(\omega)$ associated with the above $\tilde{n}_2^2(\omega)$ term:

$$\boxed{\left(\vec{k}_{trans} + i\vec{\kappa}_{trans}\right)\bullet\left(\vec{k}_{trans} + i\vec{\kappa}_{trans}\right) = (n_2 + i\eta_2)(n_2 + i\eta_2)k_o^2}$$

$$\boxed{(\underbrace{\vec{k}_{trans}\bullet\vec{k}_{trans}}_{=k_{trans}^2} + \underbrace{2i\vec{k}_{trans}\bullet\vec{\kappa}_{trans}}_{=2ik_{trans}\kappa_{trans}\cos\theta_{trans}} - \underbrace{\vec{\kappa}_{trans}\bullet\vec{\kappa}_{trans}}_{=\kappa_{trans}^2}) = \left(n_2^2 - 2in_2\eta_2 - \eta_2^2\right)k_o^2}$$

$$\therefore \boxed{\left(k_{trans}^2 - \kappa_{trans}^2\right) + i\left(2k_{trans}\kappa_{trans}\cos\theta_{trans}\right) = \left[\left(n_2^2 - \eta_2^2\right) + i\left(2n\eta_2\right)\right]k_o^2}$$

Equating the <u>real</u> and <u>imaginary</u> parts of the LHS and RHS of the above equation, we see that:

$$\boxed{\left(k_{trans}^2(\omega) - \kappa_{trans}^2(\omega)\right) = \left(n_2^2(\omega) - \eta_2^2(\omega)\right)k_o^2} \;\text{ and: }\; \boxed{k_{trans}(\omega)\kappa_{trans}(\omega)\cos\theta_{trans} = n_2(\omega)\eta_2(\omega)k_o^2}$$

Thus, for $\tilde{k}_{trans}(\omega) = k_{trans}(\omega) + i\kappa_{trans}(\omega)$ and $\tilde{n}_2(\omega) = n_2(\omega) + i\eta_2(\omega)$ we have the complex relations:

1.) $\boxed{\left(k_{trans}^2(\omega) - \kappa_{trans}^2(\omega)\right) = \left(n_2^2(\omega) - \eta_2^2(\omega)\right)k_o^2}$ with vacuum wavenumber $k_o \equiv \dfrac{2\pi}{\lambda_o} = \dfrac{\omega}{c}$

2.) $\boxed{k_{trans}(\omega)\kappa_{trans}(\omega)\cos\theta_{trans} = n_2(\omega)\eta_2(\omega)k_o^2}$ and vacuum wavelength $\lambda_o \equiv c/f$, $\omega = 2\pi f$

We also have the relation:

3.) $\boxed{k_{inc}\sin\theta_{inc} = k_{trans}(\omega)\sin\theta_{trans}}$ where: $\boxed{k_{inc} = \left(\dfrac{\omega}{c}\right)n_1 = k_o n_1}$

Inserting relation 3.) into relations 1.) and 2.) above, after some algebra these relations yield the following relation:

$$\boxed{k_{trans}(\omega)\cos\theta_{trans} + i\kappa_{trans}(\omega) = n_1 k_o\sqrt{\frac{\left(n_2^2(\omega) - \eta_2^2(\omega) + 2in_2(\omega)\eta_2(\omega)\right)}{n_1^2} - \sin^2\theta_{inc}} = n_1 k_o\sqrt{\frac{\tilde{n}_2^2(\omega)}{n_1^2} - \sin^2\theta_{inc}}}$$

{***n.b.*** if medium (2) is L/H/I non-conductive/non-magnetic/non-dispersive medium (***i.e.*** like medium (1)), then $\kappa_{trans} = \eta_2 = 0$ and it is easy to show that this relation then reduces to: $k_{inc}\sin\theta_{inc} = k_{trans}\sin\theta_{trans} \neq fcn(\omega)$ }

<u>Let us define</u>: $\boxed{\tilde{\mathscr{N}}(\omega) \equiv \dfrac{\tilde{n}_2(\omega)}{n_1} = \sqrt{\dfrac{\tilde{n}_2^2(\omega)}{n_1^2}} = \sqrt{\dfrac{\left(n_2^2(\omega) - \eta_2^2(\omega) + 2in_2(\omega)\eta_2(\omega)\right)}{n_1^2}}}$ $\Leftarrow$ complex!

<u>Then</u>: $\boxed{k_{trans}(\omega)\cos\theta_{trans} + i\kappa_{trans}(\omega) = n_1 k_o\sqrt{\tilde{\mathscr{N}}^2(\omega) - \sin^2\theta_{inc}}}$

We define the <u>Law of Complex Refraction</u> {for this particular boundary/interface situation} as:

$$\boxed{n_1\sin\theta_{inc} = \tilde{n}_2(\omega)\sin\tilde{\theta}_{trans}(\omega)}$$

<u>where</u>: $\tilde{\theta}_{trans}(\omega) \equiv$ <u>complex</u> angle: $\boxed{\tilde{\theta}_{trans}(\omega) \equiv \theta_{trans}(\omega) + i\Theta_{trans}(\omega)}$

<u>with</u>: $\boxed{\theta_{trans}(\omega) \equiv \mathrm{Re}\left(\tilde{\theta}_{trans}(\omega)\right)}$ <u>and</u>: $\boxed{\Theta_{trans}(\omega) \equiv \mathrm{Im}\left(\tilde{\theta}_{trans}(\omega)\right)}$

Physically, $\theta_{trans}(\omega) \equiv \mathrm{Re}\left(\tilde{\theta}_{trans}(\omega)\right)$ has the usual physical meaning (except that it is now frequency-dependent), whereas $\Theta_{trans}(\omega) \equiv \mathrm{Im}\left(\tilde{\theta}_{trans}(\omega)\right)$ has no simple/easy physical meaning.

The Law of Complex Refraction can be rewritten as:

$$\boxed{\tilde{\mathscr{N}}(\omega) \equiv \frac{\tilde{n}_2(\omega)}{n_1} = \sqrt{\frac{\tilde{n}_2^2(\omega)}{n_1^2(\omega)}} = \frac{\sin\theta_{inc}}{\sin\tilde{\theta}_{trans}(\omega)}}$$

Then: $\boxed{\tilde{\mathscr{N}}^2(\omega)\sin^2\tilde{\theta}_{trans}(\omega)=\sin^2\theta_{inc}} \Rightarrow \boxed{\tilde{\mathscr{N}}^2(\omega)\left(1-\cos^2\tilde{\theta}_{trans}(\omega)\right)=\sin^2\theta_{inc}}$

$\Rightarrow \boxed{\left(1-\cos^2\tilde{\theta}_{trans}(\omega)\right)=\sin^2\theta_{inc}/\tilde{\mathscr{N}}^2(\omega)} \Rightarrow \boxed{\cos\tilde{\theta}_{trans}(\omega)=\sqrt{1-\left(\sin^2\theta_{inc}/\tilde{\mathscr{N}}^2(\omega)\right)}}$

But:

$$\boxed{k_{trans}(\omega)\cos\tilde{\theta}_{trans}+i\kappa_{trans}(\omega)=n_1k_o\sqrt{\tilde{\mathscr{N}}^2(\omega)-\sin^2\theta_{inc}}=n_1k_o\tilde{\mathscr{N}}(\omega)\sqrt{1-\left(\sin^2\theta_{inc}/\tilde{\mathscr{N}}^2(\omega)\right)}}$$

But: $\boxed{\cos\tilde{\theta}_{trans}(\omega)=\sqrt{1-\left(\sin^2\theta_{inc}/\tilde{\mathscr{N}}^2(\omega)\right)}}$ {from above}

$$\therefore \boxed{k_{trans}(\omega)\cos\tilde{\theta}_{trans}+i\kappa_{trans}(\omega)=n_1k_o\tilde{\mathscr{N}}(\omega)\sqrt{1-\left(\sin^2\theta_{inc}/\tilde{\mathscr{N}}^2(\omega)\right)}=n_1k_o\tilde{\mathscr{N}}(\omega)\cos\tilde{\theta}_{trans}(\omega)}$$

i.e. $\boxed{k_{trans}(\omega)\cos\tilde{\theta}_{trans}+i\kappa_{trans}(\omega)=n_1k_o\tilde{\mathscr{N}}(\omega)\cos\tilde{\theta}_{trans}(\omega)}$

Solve for $\tilde{\mathscr{N}}(\omega)$:

$$\boxed{\tilde{\mathscr{N}}(\omega)=\frac{k_{trans}(\omega)\cos\tilde{\theta}_{trans}+i\kappa_{trans}(\omega)}{n_1k_o\cos\tilde{\theta}_{trans}(\omega)}=\frac{\tilde{n}_2(\omega)}{n_1}} \Rightarrow \boxed{\tilde{n}_2(\omega)=\frac{k_{trans}(\omega)\cos\tilde{\theta}_{trans}+i\kappa_{trans}(\omega)}{k_o\cos\tilde{\theta}_{trans}(\omega)}}$$

The {complex} $\vec{E}$ and $\vec{B}$ fields involved at the interface are:

Incident wave: $\boxed{\tilde{\vec{E}}_{inc}(\vec{r},t)=\tilde{\vec{E}}_{o_{inc}}(\vec{r})e^{i\left(\vec{k}_{inc}\cdot\vec{r}-\omega t\right)}}$ $\boxed{\tilde{\vec{B}}_{inc}(\vec{r},t)=\frac{1}{\omega}\vec{k}_{inc}\times\tilde{\vec{E}}_{inc}(\vec{r},t)}$

Reflected wave: $\boxed{\tilde{\vec{E}}_{refl}(\vec{r},t)=\tilde{\vec{E}}_{o_{refl}}(\vec{r})e^{i\left(\vec{k}_{refl}\cdot\vec{r}-\omega t\right)}}$ $\boxed{\tilde{\vec{B}}_{refl}(\vec{r},t)=\frac{1}{\omega}\vec{k}_{refl}\times\tilde{\vec{E}}_{inc}(\vec{r},t)}$

n.b. this form of *B* – takes care of everything!!!

Transmitted wave: $\boxed{\tilde{\vec{E}}_{trans}(\vec{r},t)=\tilde{\vec{E}}_{o_{trans}}(\vec{r})e^{i\left(\tilde{\vec{k}}_{trans}\cdot\vec{r}-\omega t\right)}}$ $\boxed{\tilde{\vec{B}}_{trans}(\vec{r},t)=\frac{1}{\omega}\tilde{\vec{k}}_{trans}\times\tilde{\vec{E}}_{trans}(\vec{r},t)}$

$$=\boxed{\frac{1}{\omega}\left(\vec{k}_{trans}\times\tilde{\vec{E}}_{trans}(\vec{r},t)+i\,\vec{\kappa}_{trans}\times\tilde{\vec{E}}_{trans}(\vec{r},t)\right)}$$

The boundary conditions at the interface {lying in the *x*-*y* plane at $z = 0$} are:

BC 1) (normal $\vec{D}$ continuous): $\boxed{\varepsilon_1\tilde{E}_1^{\perp}=\tilde{\varepsilon}_2\tilde{E}_2^{\perp}}$ ($\sigma_{free}=0$ on the interface/boundary)

BC 2) (tangential $\vec{E}$ continuous): $\boxed{\tilde{E}_1^{\parallel}=\tilde{E}_2^{\parallel}}$

BC 3) (normal $\vec{B}$ continuous): $\boxed{\tilde{B}_1^{\perp}=\tilde{B}_2^{\perp}}$

BC 4) (tangential $\vec{H}$ continuous): $\boxed{\frac{1}{\mu_1}\tilde{B}_1^{\parallel}=\frac{1}{\mu_2}\tilde{B}_2^{\parallel}}$ ($\vec{K}_{free}=0$ on the interface/boundary)

$\Rightarrow \boxed{\tilde{B}_1^{\parallel}=\tilde{B}_2^{\parallel}}$ if $\boxed{\mu_1\simeq\mu_2\simeq\mu_o}$

(medium (1) and medium (2) both non-magnetic)

On the interface/boundary at $z = 0$ (for any arbitrary space-point, *e.g.* $(x, y, z) = (0, 0, 0)$ and time t):

TE Polarization Case:

BC 2) $\boxed{\tilde{E}_{o_{inc}} + \tilde{E}_{o_{refl}} = \tilde{E}_{o_{trans}}}$ $\boxed{k_o = \omega/c = 2\pi/\lambda_o}$, $\boxed{\lambda_o = c/f}$

BC 4) $\boxed{\tilde{B}_{o_{inc}}\cos\theta_{inc} + \tilde{B}_{o_{refl}}\cos\theta_{refl} = \tilde{B}_{o_{trans}}\cos\theta_{trans}}$ $\boxed{k_{inc} = n_1 k_o}$, $\boxed{k_{refl} = n_1 k_o}$ and $\boxed{\theta_{inc} = \theta_{refl}}$

$$= \boxed{-k_{inc}\tilde{E}_{o_{inc}}\cos\theta_{inc} + k_{refl}\tilde{E}_{o_{refl}}\cos\theta_{refl} = -\left(k_{trans}\tilde{E}_{o_{trans}}\cos\theta_{trans} + i\kappa_{trans}\tilde{E}_{o_{trans}}\right)}$$

$$= \boxed{-n_1 k_o\left(\tilde{E}_{o_{inc}} - \tilde{E}_{o_{refl}}\right)\cos\theta_{inc} = -\left(k_{trans}\cos\theta_{trans} + i\kappa_{trans}\right)\tilde{E}_{o_{trans}}}$$

$$= \boxed{+n_1 k_o\cos\theta_{inc}\left(\tilde{E}_{o_{inc}} - \tilde{E}_{o_{refl}}\right) = +\left(k_{trans}\cos\theta_{trans} + i\kappa_{trans}\right)\tilde{E}_{o_{trans}}}$$

or: $$\boxed{\left(\tilde{E}_{o_{inc}} - \tilde{E}_{o_{refl}}\right) = \left(\frac{k_{trans}\cos\theta_{trans} + i\kappa_{trans}}{n_1 k_o\cos\theta_{inc}}\right)\tilde{E}_{o_{trans}}}$$

but from BC 2) $\boxed{\tilde{E}_{o_{trans}} = \tilde{E}_{o_{inc}} + \tilde{E}_{o_{refl}}}$ $\therefore$ $\boxed{\left(\tilde{E}_{o_{inc}} - \tilde{E}_{o_{refl}}\right) = \left(\frac{k_{trans}\cos\theta_{trans} + i\kappa_{trans}}{n_1 k_o\cos\theta_{inc}}\right)\left(\tilde{E}_{o_{inc}} + \tilde{E}_{o_{refl}}\right)}$

Skipping the details of the algebra, but using: $\boxed{\tilde{\mathscr{N}}(\omega) = \frac{k_{trans}(\omega)\cos\tilde{\theta}_{trans} + i\kappa_{trans}(\omega)}{n_1 k_o\cos\tilde{\theta}_{trans}(\omega)} = \frac{\tilde{n}_2(\omega)}{n_1}}$

It can be shown that:

TE Polarization: $$\boxed{\left(\frac{\tilde{E}_{o_{refl}}}{\tilde{E}_{o_{inc}}}\right)_{TE} = \frac{\cos\theta_{inc} - \tilde{\mathscr{N}}(\omega)\cos\tilde{\theta}_{trans}}{\cos\theta_{inc} + \tilde{\mathscr{N}}(\omega)\cos\tilde{\theta}_{trans}}}$$

Similarly, it can also be shown that:

n.b. these have the identical functional forms of those the lossless dielectric case!

TM Polarization: $$\boxed{\left(\frac{\tilde{E}_{o_{refl}}}{\tilde{E}_{o_{inc}}}\right)_{TM} = \frac{-\tilde{\mathscr{N}}(\omega)\cos\theta_{inc} + \cos\tilde{\theta}_{trans}}{\tilde{\mathscr{N}}(\omega)\cos\theta_{inc} + \cos\tilde{\theta}_{trans}}}$$

Reflectance / Reflection Coefficient: $$\boxed{R = \left|\frac{\tilde{E}_{o_{refl}}}{\tilde{E}_{o_{inc}}}\right|^2}$$

Using the above ratios for TE and TM polarization plus realistic/detailed/full-blown $\tilde{n}_2(\omega)$ expression for metal, reflection coefficient/reflectance vs angle of incidence for TE and TM polarized EM waves (in visible light/optical region of *EM* spectrum) is shown below for a typical air-metal interface:

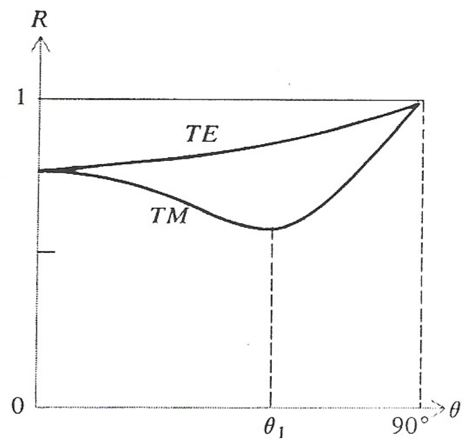


Figure 5.6. Reflectance as a function of angle of incidence for a typical metal.
θ_1 = PRINCIPAL ANGLE OF INCIDENCE FOR TM POLARIZATION

For TM polarization, a metal has no Brewster angle where $R(\theta_B) = 0$, but instead has a *dip* (*i.e.* minima) where θ_B (for a lossless dielectric) used to be. The angular location of this minima / dip for TM polarization is known as the principal angle of incidence, θ_1.

At *normal* incidence $\theta_{inc} = \theta_{refl} = \theta_{trans} = 0$, both TE and TM polarization give the same ratio:

$$\boxed{\left(\frac{\tilde{E}_{o_{refl}}}{\tilde{E}_{o_{inc}}}\right)_{\theta_{inc}=0} = \frac{1-\tilde{\mathscr{N}}(\omega)}{1+\tilde{\mathscr{N}}(\omega)}}$$

Thus the reflectance of the metal/conductor at normal incidence, $\theta_{inc} = 0$ is :

$$\boxed{R(\theta_{inc}=0) = \left|\frac{\tilde{E}_{o_{refl}}}{\tilde{E}_{o_{inc}}}\right|^2_{\theta_{inc}=0} = \left|\frac{1-\tilde{\mathscr{N}}(\omega)}{1+\tilde{\mathscr{N}}(\omega)}\right|^2} \quad \text{where:} \quad \boxed{\tilde{\mathscr{N}}(\omega) \equiv \frac{n_2(\omega)}{n_1} = \sqrt{\frac{n_2^2(\omega)}{n_1^2}}}$$

If (for simplicity) medium 1) is the vacuum, then: $n_1 = 1.0 \approx n_{air}$

Then: $$\boxed{R(\theta_{inc}=0) = \frac{(1-n_2(\omega))^2 + \eta_2^2(\omega)}{(1+n_2(\omega))^2 + \eta_2^2(\omega)}}$$

For lossless/dispersionless dielectrics $\eta_2(\omega) = 0$, then: $\boxed{R(\theta_{inc}=0) = \frac{(1-n_2(\omega))^2}{(1+n_2(\omega))^2}}$.

For metals, the extinction coefficient $\zeta_2(\omega) \equiv 2\eta_2(\omega)$ is large, *e.g.* in the visible light range.
$\Rightarrow R(\theta_{inc}=0) \to$ unity $(\approx 85-95\%)$ for many metals in visible light range.

In the low frequency region, we have shown that: $n(\omega) \approx \eta(\omega) \simeq \sqrt{\dfrac{\sigma_C}{2\varepsilon_o \omega}}$ where: $\sigma_C = \sqrt{\dfrac{n_e e^2}{m_e^* \gamma}}$

Then: $\boxed{R_{\substack{\text{low}\\ \text{frequency}}}(\theta_{inc}=0) \simeq 1 - \dfrac{2}{n} \approx 1 - \sqrt{\dfrac{8\varepsilon_o \omega}{\sigma_C}}}$ $\Leftarrow$ known as the Hagen-Rubens formula

{Works well for metals in the far-infrared portion of the *EM* spectrum – experimentally verified}

The high reflectivity of metals at optical and higher frequencies is caused by (essentially) the same physics as that for a tenuous plasma!

The complex *total* electric permittivity for an absorptive/dispersive conducting medium is:

$$\boxed{\tilde{\varepsilon}_{Tot}(\omega) = \tilde{\varepsilon}_{bound}(\omega) + \tilde{\varepsilon}_{free}(\omega) = \varepsilon_o \left(1 + \left(\frac{n_e^b e^2}{\varepsilon_o m_e^*}\right) \sum_{j=1}^{n} \frac{f_j^{\substack{bound\\ osc}}}{\left[\omega_{1j}^2 - \omega^2 - i\gamma_j \omega\right]} - \frac{\omega_P^2}{\left(\omega^2 + i\gamma_o \omega\right)}\right)}$$

Where:

$f_j^{\substack{bound\\ osc}}$ = oscillator strength of j^{th} bound resonance, with $\sum_{j=1}^{n} f_j^{\substack{bound\\ osc}} = 1$

$\omega_{1j} = \sqrt{\omega_{0j}^2 - \left(n_e^b e^2 / 3\varepsilon_o m_e^*\right)}$ = {angular} frequency of j^{th} resonance of bound valence electrons.

$\omega_{0j} \equiv \sqrt{k_{e_j} / m_e^*}$ = "natural" {angular} frequency of j^{th} resonance of bound valence electrons.

m_e^* = electron mass in medium ($\neq m_e$ for electron *e.g.* in vacuum!)

γ_j = width/damping constant of j^{th} resonance of bound valence electrons.

n_e^b = # density (#/m^3) of ***bound*** atomic electrons in the valence bands.

γ_o = width/damping constant of "free"/conduction electrons' resonance at $\omega_0 = 0$ *rad/sec*

$\omega_P \equiv \sqrt{n_e^f e^2 / \varepsilon_o m_e^*}$ = plasma frequency associated with "free"/conduction electrons

n_e^f = # density (#/m^3) of "free"/conduction electrons in the metal.

At high frequency, $\omega \gg \gamma_o$ the *total* complex permittivity of the metal/conductor takes the approximate form:

$$\boxed{\tilde{\varepsilon}_{Tot}(\omega) = \tilde{\varepsilon}_{bound}(\omega) + \tilde{\varepsilon}_{free}(\omega) \approx \tilde{\varepsilon}_{bound}(\omega) - \varepsilon_o \left(\frac{\omega_P}{\omega}\right)^2} \quad \text{for } \omega \gg \gamma_o$$

For even ***higher*** frequencies, but $\omega \ll \omega_P$, but also where $\omega \gg \omega_{1j}$ of {all of} the bound/valence band resonances in the metal, the complex electric permittivity is given approximately by:

$$\boxed{\tilde{\varepsilon}_{Tot}(\omega) \simeq \varepsilon_o \left(1 - \left(\frac{\omega_P}{\omega}\right)^2\right)} \quad \text{for } \omega \gg \gamma_o,\ \omega \gg \omega_{1j} \text{ of valence band resonances, but } \omega \ll \omega_P.$$

Visible light penetrates only a very short distance $\delta_{sc}(\omega_{vis}) = 1/\kappa(\omega_{vis}) \simeq c/\omega_P$ into the metal and is almost entirely reflected.

When the frequency of the incident *EM* wave is increased still further, into the UV and *x*-ray region then $\omega \geq \omega_P$ and the metal suddenly becomes transparent – the transmittance T increases from zero and the reflectance $R = 1 - T$ therefore decreases.

A Simplified Model of *EM* Wave Propagation in the Earth's Ionosphere and Magnetosphere

Propagation of *EM* waves in the earth's ionosphere is very similar to that in a tenuous plasma, however, the earth's weak DC magnetic dipole field:

$$\left|\vec{B}_{earth}\right| \approx 0.3 \text{ Gauss } = 0.3\times10^{-4} \text{ Tesla } = 30\mu\text{Tesla} \text{ at the earth's surface}$$

significantly changes the nature of *EM* wave propagation in the earth's ionosphere, and thus cannot be neglected in the theory formalism.

Consider a tenuous electronic (*i.e.* e^--only) plasma of uniform number density with a strong, static and uniform magnetic field $\vec{B} = \vec{B}_o$ with monochromatic plane *EM* waves propagating in the direction parallel to $\vec{B} = \vec{B}_o \parallel +\hat{z}$.

If the {complex} displacement amplitude $\tilde{\vec{r}}$ of the electronic motion is small and damping/collisions are neglected, then the approximate equation of motion is given by the following inhomogeneous 2nd order differential equation:

$$\boxed{m_e \ddot{\tilde{\vec{r}}}(\vec{r},t) - e\vec{B}_o \times \dot{\tilde{\vec{r}}}(\vec{r},t) = -e\tilde{\vec{E}}(\vec{r})e^{-i\omega t}}$$

Note that we can safely neglect the influence of the magnetic Lorentz force term $-e\tilde{\vec{v}} \times \tilde{\vec{B}}$ acting on the electrons associated with the {complex} $\vec{B}$-field of the *EM* wave, as long as $\left|\tilde{\vec{B}}_{EM}\right| \ll \left|\vec{B}_o\right|$.

We specifically/deliberately consider here circularly polarized monochromatic plane *EM* waves propagating in the $+\hat{z}$ direction $\left(\parallel \vec{B} = \vec{B}_o\right)$, which in complex notation can be succinctly written as:

$$\boxed{\tilde{\vec{E}}(\vec{r},t) = (\hat{\epsilon}_1 \overset{\text{LCP}}{\underset{\text{RCP}}{\pm}} i\,\hat{\epsilon}_2)\tilde{E}(\vec{r},t)}$$ where the polarization vectors are *e.g.* $\hat{\epsilon}_1 = \hat{x}$ and: $\hat{\epsilon}_2 = \hat{y}$

If the monochromatic plane *EM* wave's polarization vectors are: $\hat{\epsilon}_1 = \hat{x}$ and $\hat{\epsilon}_2 = \hat{y}$ and: $\vec{B} = B_o\hat{z}$, then we see that: $\vec{B} \perp \hat{\varepsilon}_1 (= \hat{x})$ and also that: $\vec{B} \perp \hat{\varepsilon}_2 (= \hat{y})$.

The magnetic Lorentz force term $-e\vec{B}_o \times \dot{\tilde{\vec{r}}}(\vec{r},t) = -eB_o\left(\hat{z} \times \dot{\tilde{\vec{r}}}(\vec{r},t)\right)$ can then *only* have components in the *x*-*y* plane - *i.e.* it can only have components along the $\hat{x} - \hat{y}$ or $\hat{\epsilon}_1 - \hat{\epsilon}_2$ axes.

A steady-state solution to the above 2nd order inhomogeneous differential equation for the electron's {complex} displacement *amplitude* $\vec{r}_e(\vec{r})$ at the space point $\vec{r}$ is:

$$\boxed{\tilde{\vec{r}}_e(\vec{r}) = \frac{e}{m_e\omega(\omega \mp \omega_B)}\tilde{\vec{E}}(\vec{r})} \quad i.e. \quad \boxed{\tilde{\vec{r}}_e(\vec{r},t) = \tilde{\vec{r}}_e(\vec{r})e^{-i\omega t} = \frac{e}{m_e\omega(\omega \mp \omega_B)}\tilde{\vec{E}}(\vec{r})e^{-i\omega t}}$$

where $\omega_B \equiv eB_o/m_e$ = electron precession frequency spiraling around the magnetic field lines and the $\mp$ sign depends on the handedness of the circular polarization {TBD, momentarily}.

We can understand this relation better in the rest frame of electrons precessing with frequency ω_B about the direction of $\vec{B} = B_o\hat{z}$ (= direction of propagation of the *EM* wave) – the static $\vec{B}$-field is eliminated – it is replaced by a rotating electric field of effective frequency $(\omega \mp \omega_B)$, where again the $\mp$ sign depends on the handedness of the circular polarization.

The {complex} harmonic oscillation of each electron's displacement $\tilde{\vec{r}}_e(\vec{r},t) = \tilde{\vec{r}}_e(\vec{r})e^{-i\omega t}$ also constitutes a {complex} oscillating electric dipole moment $\tilde{\vec{p}}(\vec{r},t) = e\tilde{\vec{r}}_e(\vec{r},t) = e\tilde{\vec{r}}_e(\vec{r})e^{-i\omega t}$, and thus results in a corresponding {complex} macroscopic electric polarization $\tilde{\vec{\mathrm{P}}}(\vec{r},t)$ (= electric dipole moment/unit volume) $\tilde{\vec{\mathrm{P}}}(\vec{r},t) = n_e\tilde{\vec{p}}(\vec{r},t)$, where n_e = electron # density and corresponding {complex} relation $\tilde{\vec{\mathrm{P}}}(\vec{r},t) = \varepsilon_o\chi_e(\omega)\tilde{\vec{E}}(\vec{r},t)$ and thus has a corresponding {real!} macroscopic electric permittivity $\varepsilon(\omega) = \varepsilon_o(1+\chi_e(\omega))$.

For circularly-polarized monochromatic plane *EM* waves propagating *parallel* to $\vec{B} = B_o\hat{z}$, the macroscopic electric permittivity is:

$$\boxed{\varepsilon^{\pm}(\omega) = \varepsilon_o\left(1 - \frac{\omega_P^2}{\omega(\omega \mp \omega_B)}\right)} \quad \text{where:} \quad \boxed{\omega_P^2 \equiv \left(\frac{n_e e^2}{\varepsilon_o m_e}\right)} \quad \text{and:} \quad \boxed{\omega_B = \frac{eB_o}{m_e}}$$

where the upper sign (–) in the *denominator* is for a LCP *EM* wave, the lower sign (+) in the *denominator* is for a RCP *EM* wave.

For circularly-polarized monochromatic plane *EM* waves propagating *anti-parallel* to $\vec{B} = B_o\hat{z}$, the macroscopic electric permittivity is:

$$\boxed{\varepsilon^{\pm}(\omega) = \varepsilon_o\left(1 - \frac{\omega_P^2}{\omega(\omega \pm \omega_B)}\right)}$$

where the upper sign (+) in the denominator is for a LCP *EM* wave, the lower sign (–) in the denominator is for a RCP *EM* wave.

$\Rightarrow$ LCP and RCP monochromatic plane *EM* waves propagate differently in a tenuous electronic plasma, depending on whether the *EM* wave propagation direction is || to (or anti-||) to $\vec{B}$.

$\Rightarrow$ The earth's ionosphere is *bi-refringent* !!!

If the direction of *EM* wave propagation not *perfectly* || to (or anti-||) to $\vec{B}$, then one simply replaces $\omega_B \to \omega_B \cos\Theta$ in the above formulae, where $\Theta \equiv$ opening angle between propagation wavevector $\vec{k}$ and $\vec{B}$, *i.e.* $\vec{k}\cdot\vec{B} = \vec{k}\cdot B_o\hat{z} = kB_o\left(\hat{k}\cdot\hat{z}\right) = kB_o\cos\Theta$

$\Rightarrow$ A tenuous electronic plasma is also *anisotropic* !!!

A typical maximum number density of free electrons in the tenuous electronic plasma of the earth's ionosphere is $n_e \sim 10^{10} - 10^{12}$ *electrons*/m^3, which corresponds to a plasma frequency of $\omega_p = \sqrt{n_e e^2/\varepsilon_o m_e} \simeq 6\times10^6 - 6\times10^7$ (*radians*/*sec*).

$\Rightarrow$ The precession frequency of electrons in this plasma, in the earth's magnetic field is: $\omega_B = \left(eB_o/m_e\right) \simeq 5.3\times10^6$ (*radians*/*sec*) for $B_o = B_{earth} \simeq 30$ μ*Tesla* .

$$\boxed{\vec{k} \parallel \vec{B}: \quad \varepsilon^{\pm}(\omega) = \varepsilon_o\left(1 - \frac{\omega_P^2}{\omega(\omega \mp \omega_B)}\right)} \qquad \boxed{\vec{k} \text{ anti-}\parallel \vec{B}: \quad \varepsilon^{\pm}(\omega) = \varepsilon_o\left(1 - \frac{\omega_P^2}{\omega(\omega \pm \omega_B)}\right)}$$

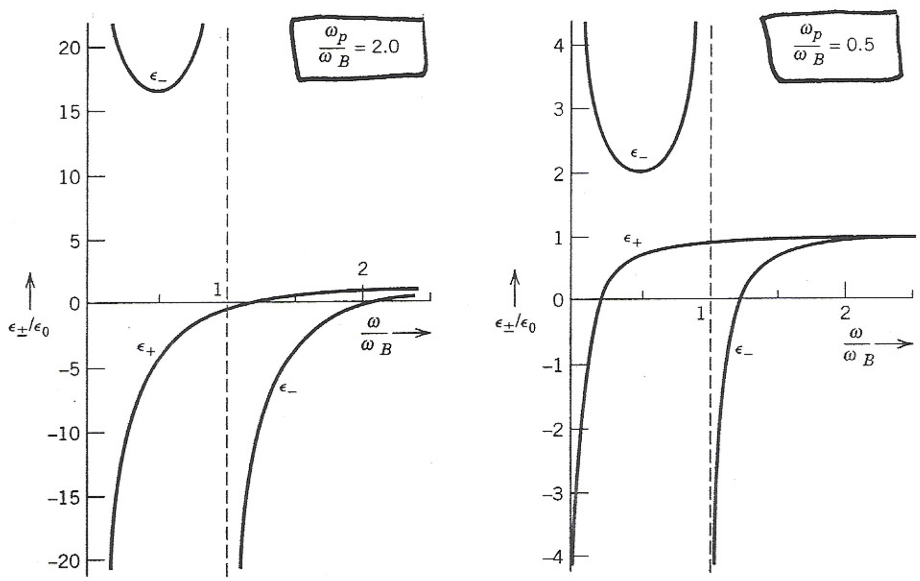


Figure 7.10 Dielectric constants as functions of frequency for model of the ionosphere (tenuous electronic plasma in a static, uniform magnetic induction). $\epsilon_\pm(\omega)$ apply to the right and left circularly polarized waves propagating parallel to the magnetic field. ω_B is the gyration frequency; ω_p is the plasma frequency. The two sets of curves correspond to $\omega_p/\omega_B = 2.0, 0.5$.

Note that circularly polarized *EM* waves with $\varepsilon^{\pm}(\omega) < 0$ *cannot* propagate in plasma because they are *exponentially* attenuated.

⇒ An incident monochromatic plane *EM* wave with circular polarization such that $\varepsilon^{\pm}(\omega) < 0$ in the tenuous electronic plasma of the earth's ionosphere will be totally reflected, the other circular polarization state (with $\varepsilon^{\pm}(\omega) > 0$) will be partially transmitted/partially reflected.

⇒ A linearly-polarized monochromatic plane *EM* wave incident on the tenuous electronic plasma of the earth's ionosphere will have a reflected wave that is elliptically polarized with its major axis rotated away from the direction of the polarization of incident wave.

The earth's ionosphere has several layers of plasma with electron densities characteristic of that/each layer, which can also vary in time and space, *e.g.* depending on the solar wind / solar storms, as well as earth's own weather (thunderstorms, etc.) as well as geological stresses in earth's crust – fault lines/earth quakes and volcanic activity….

The number density of free electrons in each ionosphere layer has a maximum at a certain height – inferred from studying reflected pulses of varying frequency, sent vertically upwards from the ground.

A short *EM* wave pulse of frequency ω_1 sent upwards from the ground actually enters the bottom of the ionospheric layer, because the number density of electrons is small there and also because the slope dn_e/dh is shallow. However, when the electron number density n_e reaches a critical value for the incident, upward-going *EM* wave, *i.e.* $\omega_1 = \omega_P = \sqrt{n_e e^2/\varepsilon_o m_e}$, the *EM* wave is reflected back, as shown in the figure below:

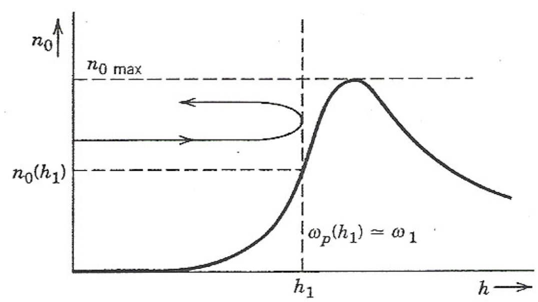


Figure 7.11 Electron density as a function of height in a layer of the ionosphere (schematic).

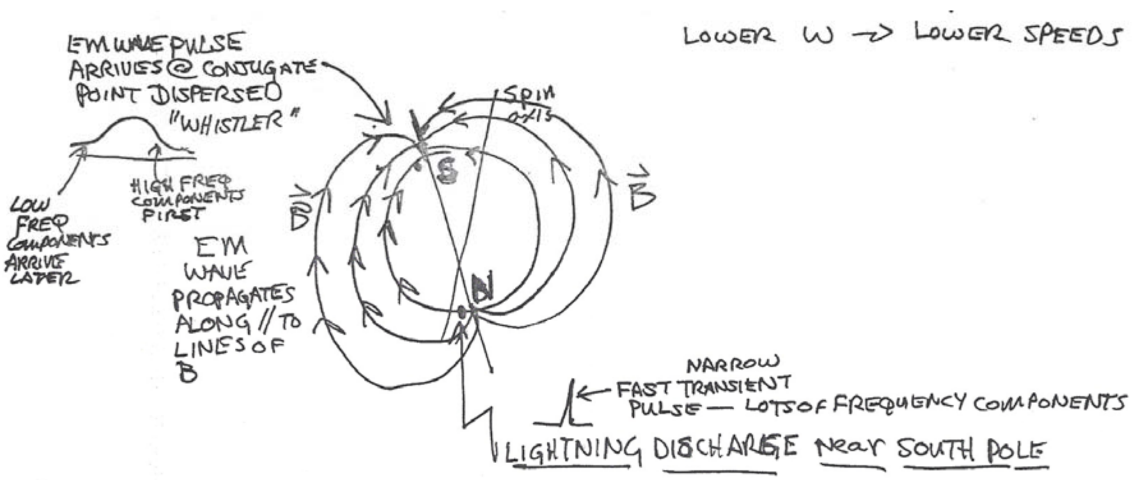
The behavior of $\varepsilon^{-}(\omega)$ at low frequencies is responsible for the magnetospheric propagation phenomenon known as "*whistlers*". As $\omega \to 0$, $\varepsilon^{-}(\omega) \to \infty$ (see graph on page 12) because:

$$\boxed{\varepsilon^{-}(\omega) \approx \varepsilon_o \left(\frac{\omega_P^2}{\omega\omega_B}\right)} \text{ for } \omega \to 0$$

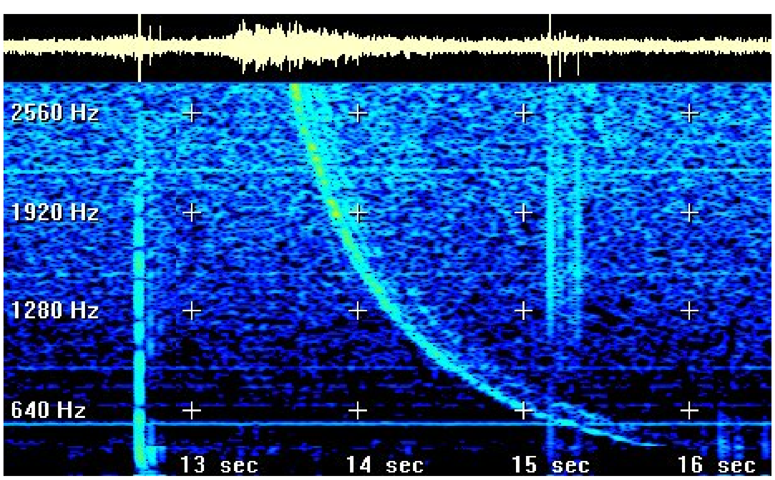
Propagation in the tenuous electronic plasma of the earth's ionosphere occurs {because $\varepsilon^{-}(\omega) > 0$ } but the wavenumber $k \simeq \left(\frac{\omega_P}{c}\right)\sqrt{\frac{\omega}{\omega_B}}$ corresponds to a highly dispersive medium!

Energy transport is governed by the group velocity, here: $\boxed{v_g(\omega) \simeq 2v_p(\omega) \approx 2c\sqrt{\frac{\omega_B\omega}{\omega_P^2}}}$

$\Rightarrow$ Pulses of *EM* waves (*e.g.* created in/during a lightning discharge) have frequency components that propagate in the earth's ionosphere at different speeds – higher $\omega \to$ higher propagation speeds, lower $\omega \to$ lower propagation speeds.



Spectral Analysis of a Whistler - Frequency vs. Time Plot:



Hear the audio file(s) of whistlers!

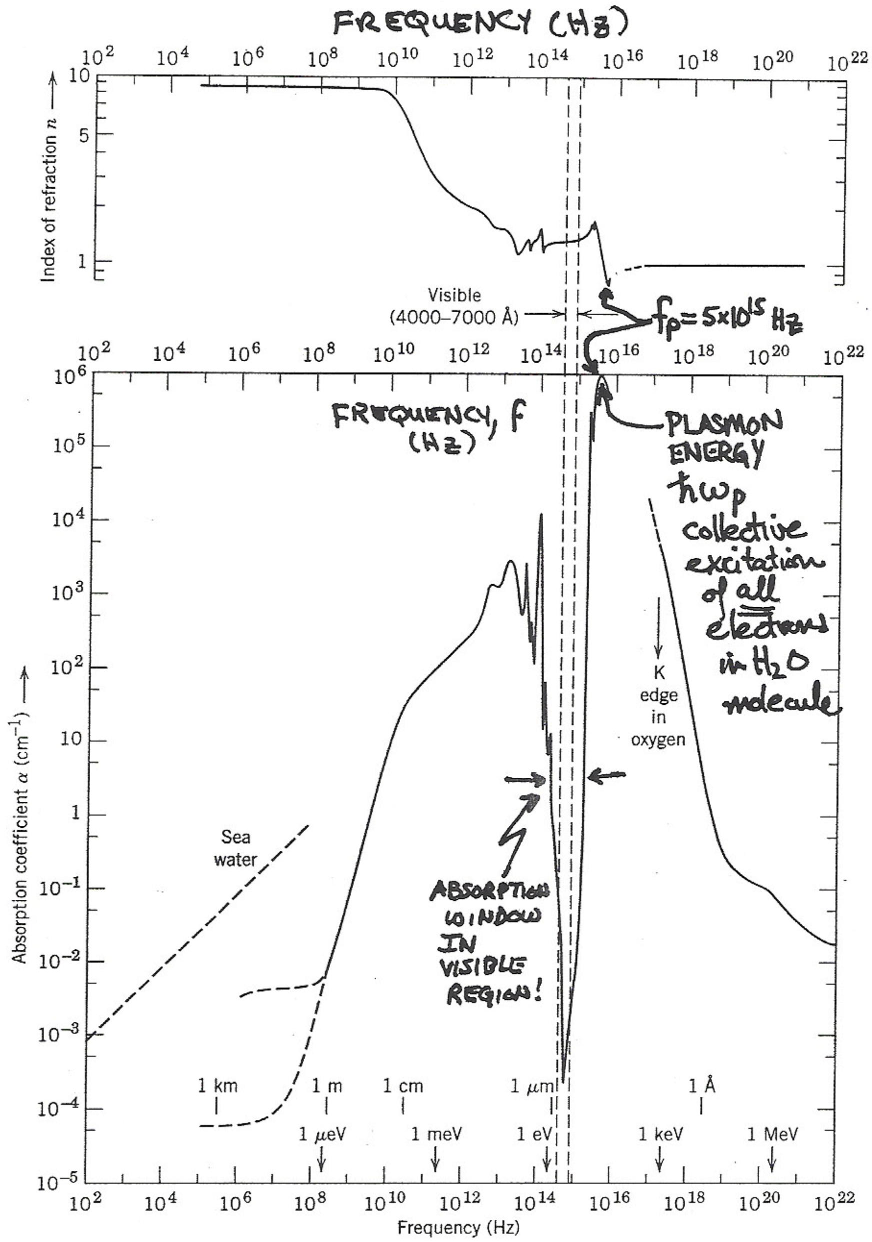
If interested in reading more about "whistlers":

See *e.g.* R. A. Helliwell, "*Whistlers & Related Ionospheric Phenomena*", Stanford University Press, Stanford, CA (1965).

Google "whistlers" & "sferics" – there are many websites where you can hear recordings of them!

Finally, we consider the complex index of refraction $\tilde{n}(\omega) = n(\omega) + i\eta(\omega)$ or equivalently, the complex wave number, $\tilde{k}(\omega) = k(\omega) + i\kappa(\omega)$ of pure water (H_2O): $\tilde{k}(\omega) = \left(\frac{\omega}{c}\right)\tilde{n}(\omega)$

The top graph in the figure below shows $n(f)\, vs.\, f$, the bottom graph shows the absorption coefficient, $\alpha \equiv 2\kappa = 2\left(\frac{\omega}{c}\right)\eta$ vs. f, λ and $E_\gamma(eV)$. Note that both plots are <u>log-log</u> plots!!!



$1\,eV = 1.6\times10^{-19}\,J$

Figure 7.9 The index of refraction (top) and absorption coefficient (bottom) for liquid water as a function of linear frequency. Also shown as abscissas are an energy scale (arrows) and a wavelength scale (vertical lines). The visible region of the frequency spectrum is indicated by the vertical dashed lines. The absorption coefficient for seawater is indicated by the dashed diagonal line at the left. Note that the scales are logarithmic in both directions.

Note the following aspects of the above plots for pure H_2O:

- At low frequencies $n(f) \simeq 9$ $\left(n^2(f) = \varepsilon(f)/\varepsilon_o = K_e(f) \simeq 81!!!\right)$ arises from partial orientation of the permanent electric dipole moment $\vec{p}$ of the H_2O molecule (Langevin equation) – the partial orientation of $\vec{p}_{H_2O}$ is due to finite-temperature thermal energy density fluctuations….

- The $n(f)$ vs. f curve falls smoothly through the infrared region – $\exists$ some "glitches" in $n(f)$ and $\eta(f)$ due to <u>molecular</u> vibrational <u>excitations</u>/<u>resonances</u> in infrared region!!

- $\exists$ more <u>resonances</u> in the UV region – due to excitations in the oxygen <u>atom</u>

- The absorption coefficient α is very small at low frequencies, but starts to rise steeply at $f \simeq 10^8\,Hz$. At $f \simeq 10^{12}\,Hz\,(\sim \text{far infrared})$, $\alpha \sim 10^4\,m^{-1} \Rightarrow \delta_{sc} \simeq 100\,\mu m$ in H_2O!!!

$\Rightarrow$ In the microwave region, $\exists$ strong absorption by H_2O $\rightarrow$ can use for microwave <u>ovens</u>!!!

$\Rightarrow$ Strong absorption by H_2O limited the trend of <u>RADAR</u> { During WWII} of going to shorter and shorter wavelengths, to achieve better spatial resolution . . .

- In the <u>infrared</u> region, the absorption coefficient for H_2O is very large, due to vibrational resonances of the H_2O molecule, $\alpha \simeq 10^4\,\text{m}^{-1}$.

- In the <u>visible</u> <u>light</u> region, there are <u>no</u> resonances of the H_2O molecule, so the absorption coefficient α drops by ~ 7-8 orders of magnitude {!!!} Thus in the visible light region H_2O/water is <u>transparent</u>/invisible.

- However, getting into the UV region, $\exists$ oxygen atom resonances (due to inner *L*, *K*-shell electrons), thus α rises again dramatically, even higher, $\alpha \simeq 10^6\,\text{m}^{-1}$ in the UV region.

$\Rightarrow$ $\exists$ an absorption *<u>window</u>* in the visible light region: $4 - 8 \times 10^{14}\,Hz$ - not very wide!!!

red light $\lambda_R = 750$ nm blue/violet light $\lambda_{BV} = 375$ nm

$\Rightarrow$ The H_2O absorption *<u>window</u>* is of fundamental importance to the evolution of life on earth Life started off in the water/ocean, aquatic critter vision/sight developed in that environment and specifically in the H_2O absorption *<u>window</u>*, where significant amounts of *EM* energy are present {thanks to the sun!} to be of use/benefit for survival…

$\Rightarrow$ The co-incidence of the H_2O absorption *<u>window</u>* and our (and other creature's) ability today to see in the visible light region of the *EM* spectrum is not a mere coincidence!

$\Rightarrow$ <u>Green</u> grass/plants at the *<u>center</u>* of visible light absorption *<u>window</u>*! Because green = <u>reflected</u> light, plants have absorption in both the red and blue/violet regions.

$\Rightarrow$ On either side of the H_2O absorption *<u>window</u>* there is not much/very little infrared or UV radiation in water after ~ few $\delta_{sc}^{IR} \sim 100\,\mu m$ $\delta_{sc}^{UV} \sim 1\,\mu m$ - because strongly attenuated !!!

