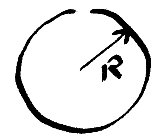
Last time: Scattering by a small dielectric sphere

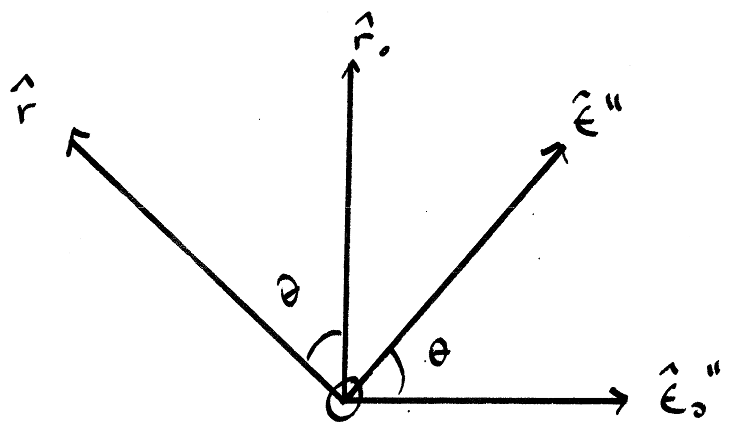
$$\frac{d\sigma}{d\Omega} = k^4 R^6 \left|\frac{\epsilon-1}{\epsilon+2}\right|^2 |\hat{\epsilon}^* \cdot \hat{\epsilon}_0|^2$$



Rayleigh's law.

Even unpolarized light becomes polarized after scattering.

Consider the scattering plane and the following basis vectors for polarization:



$\hat{\epsilon}^{\perp} = \hat{\epsilon}_0^{\perp}$ (perpendicular to plane)

For

- light polarized along $\hat{\epsilon}_0^{\parallel}$: $\frac{d\sigma_{\parallel}}{d\Omega} \propto \cos^2\theta$; $\frac{d\sigma_{\perp}}{d\Omega} \propto 0$

- light polarized along $\hat{\epsilon}_0^{\perp}$: $\frac{d\sigma_{\parallel}}{d\Omega} \propto 0$; $\frac{d\sigma_{\perp}}{d\Omega} \propto 1$

For unpolarized light, the polarization matrix is

$$\rho = \frac{1}{2}\,\hat{\epsilon}_0^{\parallel}(\hat{\epsilon}_0^{\parallel})^{+} + \frac{1}{2}\,\hat{\epsilon}_0^{\perp}(\hat{\epsilon}_0^{\perp})^{+} = \frac{1}{2}\underbrace{(\mathbb{1} - \hat{r}_0\,\hat{r}_0^{+})}_{\text{Projects onto plane } \perp \text{ to } \hat{r}_0}$$

Note that ρ is invariant under those rotations that preserve $\hat{r}_0$. For this ρ,

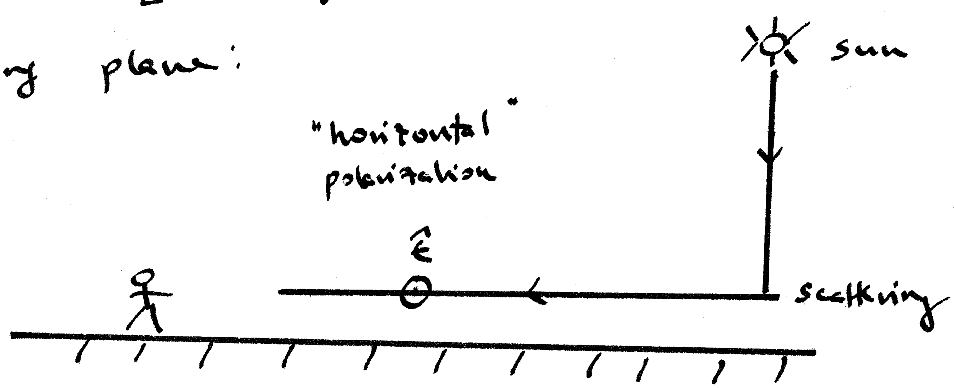
$I = 1, \quad Q = 0, \quad U = 0, \quad V = 0$

(in a basis with $\hat{\epsilon}_1 \equiv \hat{\epsilon}_0^{\parallel}$, $\hat{\epsilon}_2 = \hat{\epsilon}_0^{\perp}$).

Therefore, averaging over polarizations with $p_{\parallel} = p_{\perp} = \frac{1}{2}$

$$\begin{cases} \dfrac{d\sigma_{\parallel}}{d\Omega} = \dfrac{1}{2}\, k^4 R^6 \left|\dfrac{\epsilon-1}{\epsilon+2}\right|^2 \cos^2\theta \\[2ex] \dfrac{d\sigma_{\perp}}{d\Omega} = \dfrac{1}{2}\, k^4 R^6 \left|\dfrac{\epsilon-1}{\epsilon+2}\right|^2 \end{cases}$$

At $\theta = \frac{\pi}{2}$, light is 100 % polarized $\perp$ scattering plane:



Finally, let us consider

<u>Scattering by a point particle (an electron)</u>

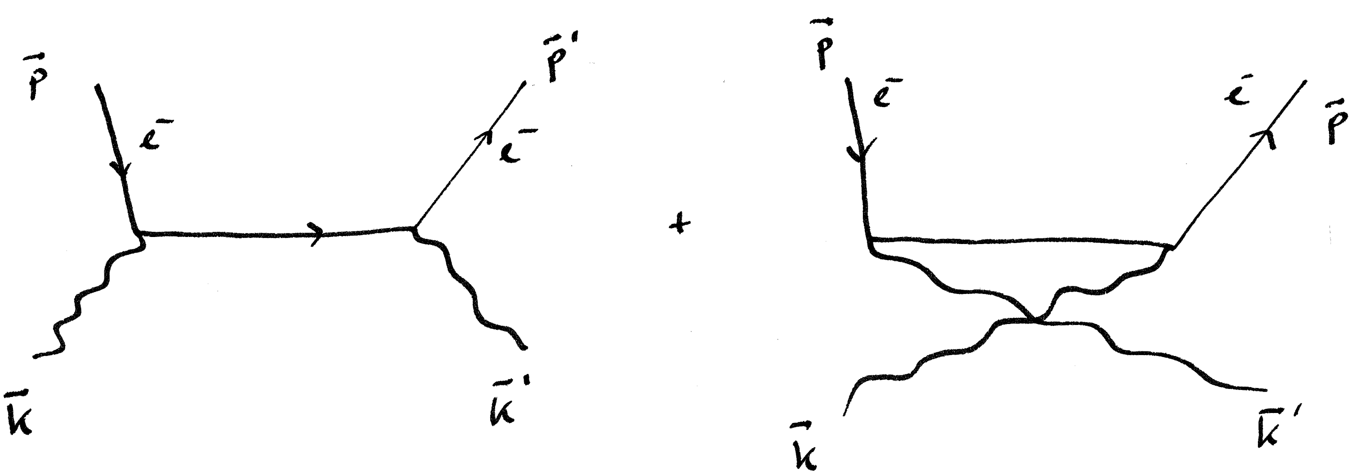
with

$$m \frac{d^2\vec{x}}{dt^2} = -e\,\vec{E}_0\, e^{-i\omega t}\,, \quad \vec{x} = \frac{e\vec{E}_0}{m\omega^2}\, e^{-i\omega t}\,, \quad \vec{p} = \frac{e^2\vec{E}_0}{m\omega^2}\, e^{-i\omega t}$$

we get (identifying $\vec{p} = \frac{\epsilon-1}{\epsilon+2} R^3 \vec{E} \leftrightarrow \vec{p} = \frac{e^2\vec{E}}{m\omega^2}$).

$$\frac{d\sigma_{tot}}{d\Omega} = \frac{d\sigma_{\parallel}}{d\Omega} + \frac{d\sigma_{\perp}}{d\Omega} = \left(\frac{e^2}{m\omega^2}\right)^2 \frac{1+\cos^2\theta}{2}\,.$$

This is the cross section for <u>Thomson scattering</u>, which agrees in the non-relativistic limit $\omega \ll m_e$ with the corresponding calculation in quantum field theory:



14. Special Relativity

We analyze now a "strange" prediction of Maxwell's eqs: <u>Light travels at a constant speed c</u>.

1. Galilean Invariance

The laws of Newtonian mechanics (Newton's laws) apply only in a particular class of coordinate systems: <u>inertial frames</u>

- A <u>coordinate system</u> is a prescription to map an <u>event</u> to a set of coordinates (t, x, y, z).
- An <u>event</u> is anything with a well-defined location in space and time (e.g. the collision of two point particles).

- An <u>inertial frame</u> is a coordinate system in which force-free objects move at constant speed:

$$\frac{d^2\vec{x}}{dt^2} = 0 \qquad \text{(Newton's first law)}$$

(We know that an object is force-free if it moves with constant speed in an inertial frame!?)

Inertial frames move at constant speed with respect to each other. Hence, the coordinates of two inertial frames are related by a

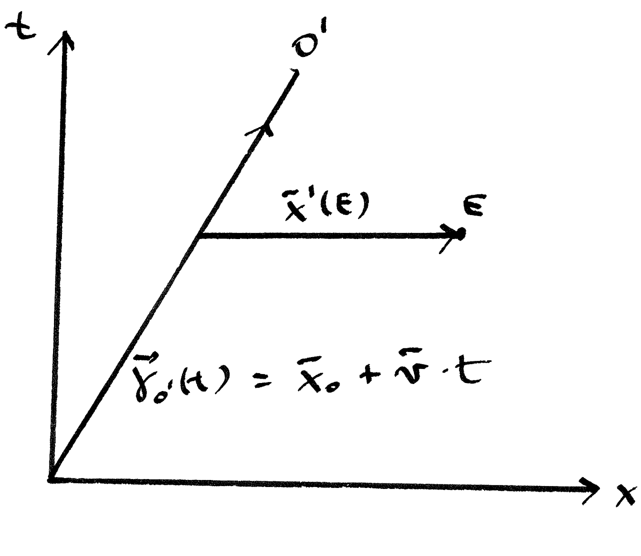
<u>galilean transformation</u>:

$$(6)\begin{cases} \vec{x}' = R(\vec{x} - \vec{x}_0 - \vec{v}\cdot t) \\ t' = t - t_0 \end{cases}$$

This is a 10-parameter group of transformations:

1 time translation, t_0; 3 spatial translations, $\vec{x}_0$; 3 rotations, R; 3 rel. velocities $\vec{v}$.

This transformation captures our intuitive notion of space and time:



$$\begin{cases} \vec{x}' = R(\vec{x} - \vec{\gamma}_0(t)) \\ t' = t - t_0 \end{cases}$$

In particular:

- The distance between two simultaneous events is universal (frame-independent)

- The time interval between two events is universal (frame-independent)

- If an object moves at speed $\vec{v}$ in a given coordinate system, it moves at speed

$$\vec{v}' = \frac{d\vec{x}'}{dt'} = \frac{d(\vec{x} - \vec{\delta}_0)}{dt} = \vec{v} - \vec{v}_0' \qquad (\text{we assume } R = \mathbb{1})$$

This is the galilean addition of velocities.

- Inertial frames are mapped to inertial frames.

The laws of Newtonian mechanics satisfy the principle of special relativity:

The laws of physics have the same form in all inertial frames

(impossible to determine if we are moving at constant speed by performing local experiments)

Because we know how inertial frames are related to each other (by Galilean transformations), we can reformulate the latter as a

<u>principle of galilean invariance</u>:

The laws of physics are (form) invariant under galilean transformations.

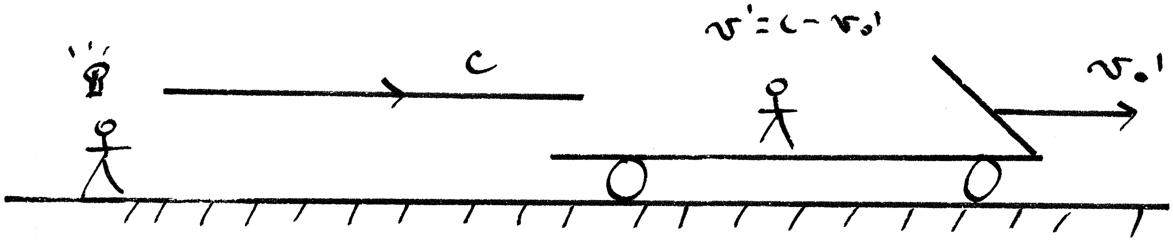
<u>2. Electromagnetic Waves</u>

Does electromagnetism satisfy the

- principle of special relativity?
- principle of galilean invariance?

Recall that Maxwell's eqs. in vacuum admit wave solutions $\vec{E} = E_0 \exp(i\vec{k}\vec{x} - i\omega t)$ with phase speed $v_p = \frac{\omega}{|\vec{k}|} = c$, where c is a constant of nature, the speed of light.

This is <u>not compatible</u> with galilean invariance: If light travels at speed c in one inertial frame, it should travel at speed $v' = c - v_0' \neq c$ in a different inertial frame



Several Alternatives:

i) The laws of electromagnetism are wrong — unlikely

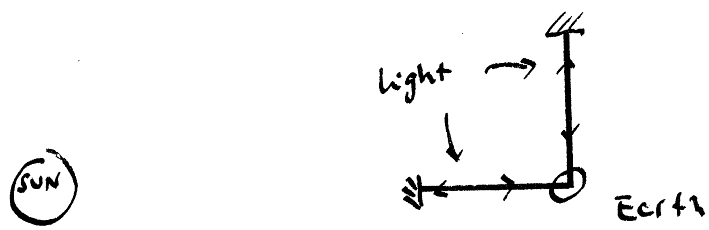
ii) The principle of relativity is wrong

iii) The principle of Galilean invariance is wrong

As to ii):

- Electromagnetic waves propagate in a medium, the "aether." Maxwell's eqs. only hold in a coordinate system in which the aether is at rest.

- Thus, we should be able to measure the earth's speed wrt the aether using light.



Using an interferometer, Michelson and Morley failed to detect our motion wrt the aether.

Einstein followed alternative iii) ...

3. Einstein's Two Postulates

Einstein developed special relativity from two simple postulates. He kept the

1. Principle of Special Relativity

- The laws of physics are the same in all inertial frames

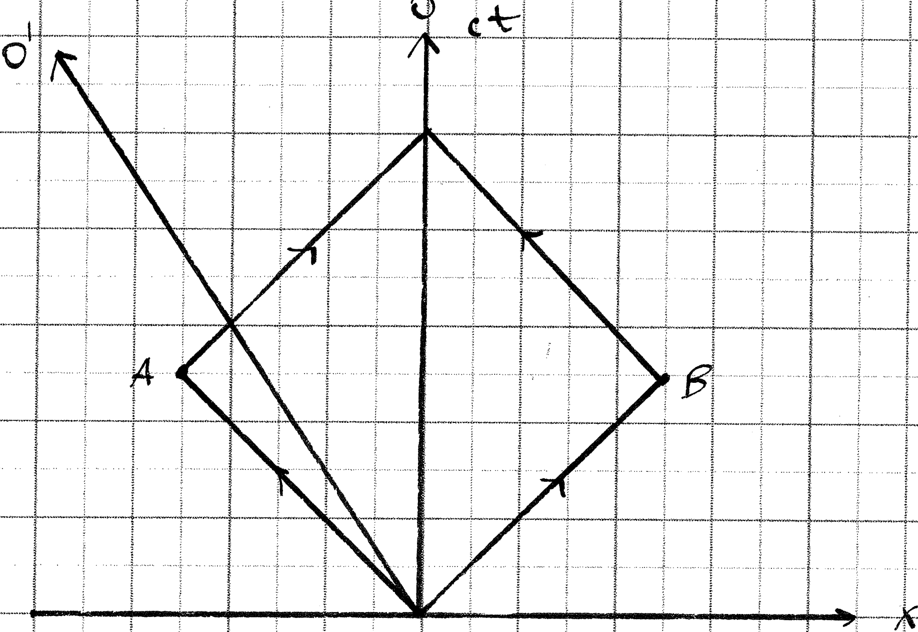
and also kept the predictions of Maxwell's eqs.:

2. Principle of the constancy of the speed of light

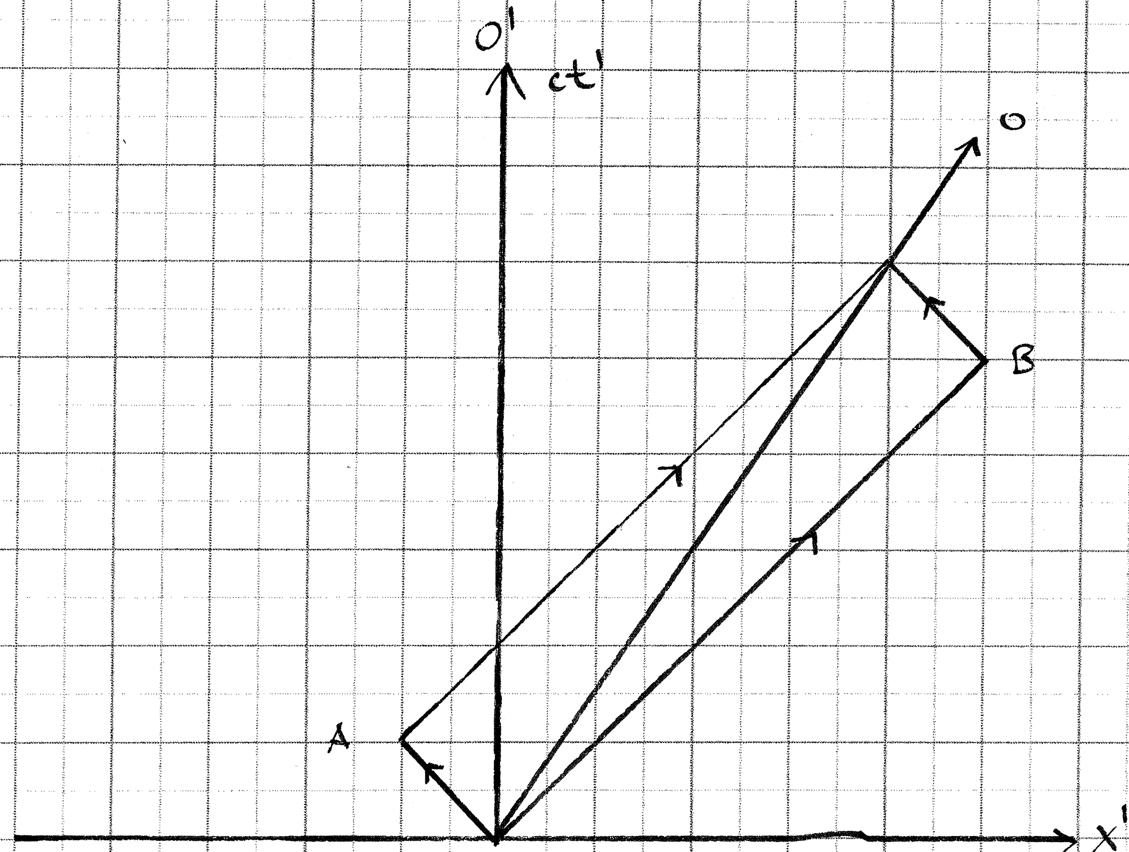
- The speed of light is a constant c, no matter what the state of motion of the source is.

Several striking consequences follow from the 2 principles:

- **Relativity of simultaneity:**



Events A & B are **simultaneous** for O



Events A & B are **not** **simultaneous** in O'

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