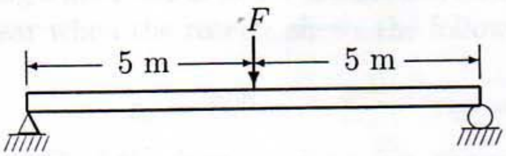
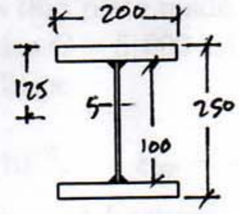
1.





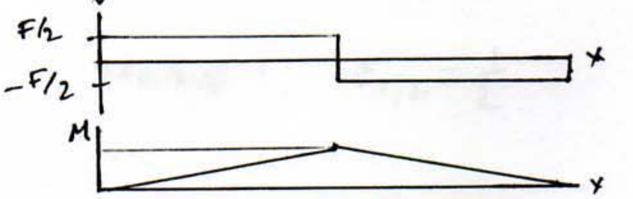
$$I = \frac{1}{12}\left(200 \times 250^3 - 195 \times 200^3\right) \text{mm}^4 = 1.304 \times 10^8 \text{ mm}^4$$

$$c = 125 \text{ mm}$$

The beam with the loading and cross-section shown is made by fillet-welding together two 25-mm by 200-mm steel flanges and a 5-mm by 200-mm steel web.

Assuming elastic behavior, find the maximum allowable value of the load F if the allowable stresses in the steel are $\sigma_{\text{all}} = 165$ MPa and $\tau_{\text{all}} = 100$ MPa, and the allowable shear flow in **each** weld is 0.2 kN/mm. (Be sure to check all criteria, and be careful with units. Show all needed diagrams.)

Formulas: $\sigma_{\max} = Mc/I$, $q = VQ/I$, $\tau = q/t$, $Q = \int_{A'} y\,dA$



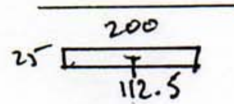
$$V_{max} = \frac{F}{2}$$

$$M_{max} = \frac{5}{2} \text{ m} \times F$$

Bending stress: $M_{all} = \dfrac{\sigma_{all} I}{c}$

$$\rightarrow F_{all} = \frac{M_{all}}{2.5 \text{ m}} = \frac{165 \,(\text{N/mm}^2) \times 1.304 \times 10^8 \text{ mm}^4}{2.5 \times 10^3 \text{ mm} \times 125 \text{ mm}} = 68.9 \text{ kN}$$

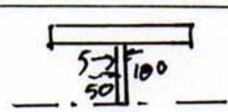
Weld shear:



$$Q = 25 \times 200 \times 112.5 \text{ mm}^3 = 5.625 \times 10^5 \text{ mm}^3$$

$$F_{all} = 2V_{all} = 2 \times \frac{2 \times 0.2 \,(\text{kN/mm}) \times 1.304 \times 10^8 \text{ mm}^4}{5.625 \times 10^5 \text{ mm}^3} = 185.5 \text{ kN}$$

Web shear:



$$Q = (5.625 \times 10^5 + 5 \times 100 \times 50) \text{ mm}^3 = 5.875 \times 10^5 \text{ mm}^3$$

$$F_{all} = 2V_{all} = 2\frac{\tau_{all}\, t\, I}{Q} = 2\,\frac{100 \,(\text{N/mm}^2) \times 5 \text{ mm} \times 1.304 \times 10^8 \text{ mm}^4}{5.875 \times 10^5 \text{ mm}^3} = 222 \text{ kN}$$

$$\Longrightarrow F_{all} = 68.9 \text{ kN}$$

2. A 0°-45°-90° strain-gage rosette is applied to a thin plate made of a brittle material with Young's modulus $E = 13,000$ ksi and shear modulus $G = 5,000$ ksi. The first fracture cracks appear when the rosette shows the following readings: $E = 2(1+\nu)G \Rightarrow \nu = 0.3$

$$\varepsilon_{0^\circ} = 600 \times 10^{-6}, \qquad \varepsilon_{45^\circ} = 750 \times 10^{-6}, \qquad \varepsilon_{90^\circ} = -300 \times 10^{-6}$$

(a) Find the fracture stress, i.e. the value of $\sigma_{\max}$ at fracture.

(b) Show the orientation of the fracture cracks with respect to the xy axes.

Hint: Use Mohr's circle as a qualitative aid.

Formulas: $\sigma_x = [E/(1-\nu^2)](\varepsilon_x + \nu\varepsilon_y)$ etc.

$\varepsilon_\theta = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta;\qquad \gamma_\theta = \gamma_{xy}\cos 2\theta + (\varepsilon_y - \varepsilon_x)\sin 2\theta$

$$\varepsilon_{45^\circ} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y + \gamma_{xy}) \Rightarrow \gamma_{xy} = 2\varepsilon_{45^\circ} - \varepsilon_{0^\circ} - \varepsilon_{90^\circ} = 1200 \times 10^{-6}$$

To find principal axes: $\gamma_\theta = 0 \Rightarrow \tan 2\theta = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{1200}{900} = \dfrac{4}{3}$

$$\theta_1 = 26.565^\circ \Rightarrow$$

$$\rightarrow \cos\theta_1 = \sqrt{\tfrac{4}{5}}, \quad \sin\theta_1 = \sqrt{\tfrac{1}{5}}$$

$$\theta_2 = -63.435 \rightarrow \cos\theta_2 = \sqrt{\tfrac{1}{5}}, \quad \sin\theta_2 = -\sqrt{\tfrac{4}{5}}$$

$$\varepsilon_1 = \left[\tfrac{4}{5} \times 600 + \tfrac{1}{5}(-300) + \tfrac{2}{5} \times 1200\right] \times 10^{-6} = 900 \times 10^{-6}$$

$$\varepsilon_2 = \left[\tfrac{1}{5} \times 600 + \tfrac{4}{5}(-300) - \tfrac{2}{5} \times 1200\right] \times 10^{-6} = -600 \times 10^{-6}$$

$$\sigma_{\max} = \sigma_1 = \frac{13{,}000 \text{ ksi}}{1 - 0.3^2}\left[900 + 0.3(-600)\right] \times 10^{-6} = \underline{10.3 \text{ ksi}}$$