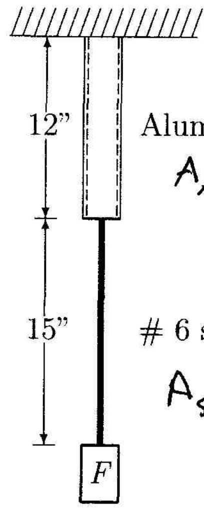
1.




Aluminum tube, $\frac{1}{2}$ in. outside diam., $\frac{1}{16}$ in. thick

$$A_{Al} = \frac{\pi}{4}\left[\left(\tfrac{1}{2}\text{ in}\right)^2 - \left(\tfrac{1}{2}-\tfrac{1}{8}\right)^2\text{in}^2\right] = 0.0859\text{ in}^2$$

# 6 steel wire (diam. = 0.192 in.)

$$A_{st} = \frac{\pi}{4}(0.192\text{ in})^2 = 0.0290\text{ in}^2$$

$E_{\text{steel}} = 30 \times 10^6$ psi, $\quad E_{\text{Al}} = 10 \times 10^6$ psi

(a) Find the magnitude of the force $F$ that will produce a total elongation of 0.020 in., and find the resulting stresses in the steel and aluminum.

(b) If the tensile yield stresses are 15 ksi in aluminum and 36 ksi in steel, find the ultimate tensile load $F_u$ for the system, and compute the safety factor under which it is working as in (a).

(a) $$\Delta = \left(\frac{L_{Al}}{E_{Al}A_{Al}} + \frac{L_{st}}{E_{st}A_{st}}\right)F = \left(\frac{12\text{ in}}{10\times 0.0859\text{ lb}} + \frac{15}{30\times 0.0290}\right) \div 10^6 \times F$$

$$= (31.2\times 10^{-6}\text{ in/lb})F$$

$$F = \frac{10^6}{31.2}\frac{\text{lb}}{\text{in}} \times 0.020\text{ in} = \underline{\underline{641\text{ lb}}}$$

$$\sigma_{Al} = \frac{F}{A_{Al}} = \frac{641\text{ lb}}{0.0859\text{ in}^2} = \underline{\underline{7.46\text{ ksi}}}$$

$$\sigma_{st} = \frac{F}{A_{st}} = \frac{641\text{ lb}}{0.0290\text{ in}^2} = \underline{\underline{22.14\text{ ksi}}}$$

(b) $$F_u = \min\left(\sigma_{yp(Al)}A_{Al},\ \sigma_{yp(st)}A_{st}\right)$$

$$= \min(15\times 0.0859,\ 36\times 0.0290)\times 10^3\text{ lb}$$

$$= \min(1288,\ 1044)\text{ lb} = 1044\text{ lb}$$

$$SF = \frac{F_u}{F} = \frac{1044}{641} = \underline{\underline{1.63}}$$

2. A shaft made of a linearly elastic material has a hollow circular cross-section, with inner radius $a$ and outer radius $\sqrt{2}a$, and is to be replaced by a shaft of solid circular cross-section of radius $c$ and made of the same material. Find the minimum value of $c$ (in terms of $a$) so that, for a given applied torque $T$, neither the maximum shear stress nor the twist exceeds the corresponding quantities in the original shaft.

Formulas
$T = (J/c)\tau_{\max} = GJ\phi'$; $\quad J = \pi(c^4 - b^4)/2$

Original: $J = \frac{\pi}{2}\left[(\sqrt{2}a)^4 - a^4\right] = \frac{3\pi}{2}a^4$

$c = \sqrt{2}a$

$$\tau_{max} = \frac{\sqrt{2}aT}{(3\pi/2)a^4} = \frac{2\sqrt{2}}{3\pi}\frac{T}{a^3}$$

$$\phi' = \frac{T}{(3\pi/2)a^4 G} = \frac{2}{3\pi}\frac{T}{a^4 G}$$

Replacement: $J = \frac{\pi}{2}c^4$

$$\tau_{max} = \frac{2}{\pi}\frac{T}{c^3}, \quad \phi' = \frac{2}{\pi}\frac{T}{c^4 G}$$

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$$\tau_{max}^{(repl)} \leq \tau_{max}^{(orig)} \Rightarrow \frac{2T}{\pi c^3} \leq \frac{2\sqrt{2}T}{3\pi a^3} \Rightarrow c^3 \geq \frac{3}{\sqrt{2}}a^3$$

$$\Rightarrow c \geq 1.285a$$

$$\phi'^{(repl)} \leq \phi'^{(orig)} \Rightarrow \frac{2T}{\pi c^4 G} \leq \frac{2T}{3\pi a^4 G} \Rightarrow c^4 \geq 3a^4$$

$$\Rightarrow c \geq 1.316a$$

$$\boxed{c \geq 1.316a}$$

3. Explain, as concisely and precisely as possible, the following pairs of concepts:
   (a) force method and displacement method
   (b) plane stress and plane strain

(a) The force method begins with the equilibrium equations to get the internal forces and stresses, uses material properties to get strains, and finally uses the compatibility conditions to get displacements.

The displacement method uses the compatibility conditions to relate strains to displacements, material properties to get strains, and finally equilibrium to get internal forces and loads.

(b) In plane stress (for example in the xy plane) the components $\sigma_z (= \tau_{zz})$, $\tau_{xz}$ and $\tau_{yz}$ are zero.

In plane strain, the components $\varepsilon_z$, $\gamma_{xz}$ and $\gamma_{yz}$ are zero.