PHY123H

Mechanics
Part A

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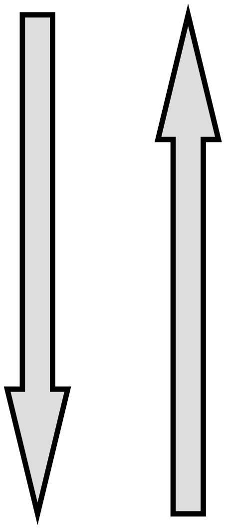
... see Chapters 2, 3 & 4 in
University Physics
by Ronald Reese

What is physics?

Physics world

(The world of physics models)



Real (everyday) **world**

(Infinitely complex)

Under what conditions (idealizations) can I use my model?

Modeling tools in the physics world

Words

Pictures

Physics diagrams

Mathematics ← Not the best place to start

Problem solving in this course will require you to use all of these representations.

In order to succeed in this course, you need to learn how to reflect upon your own learning (of physics)

When you are working on some physics task, ask yourself ...

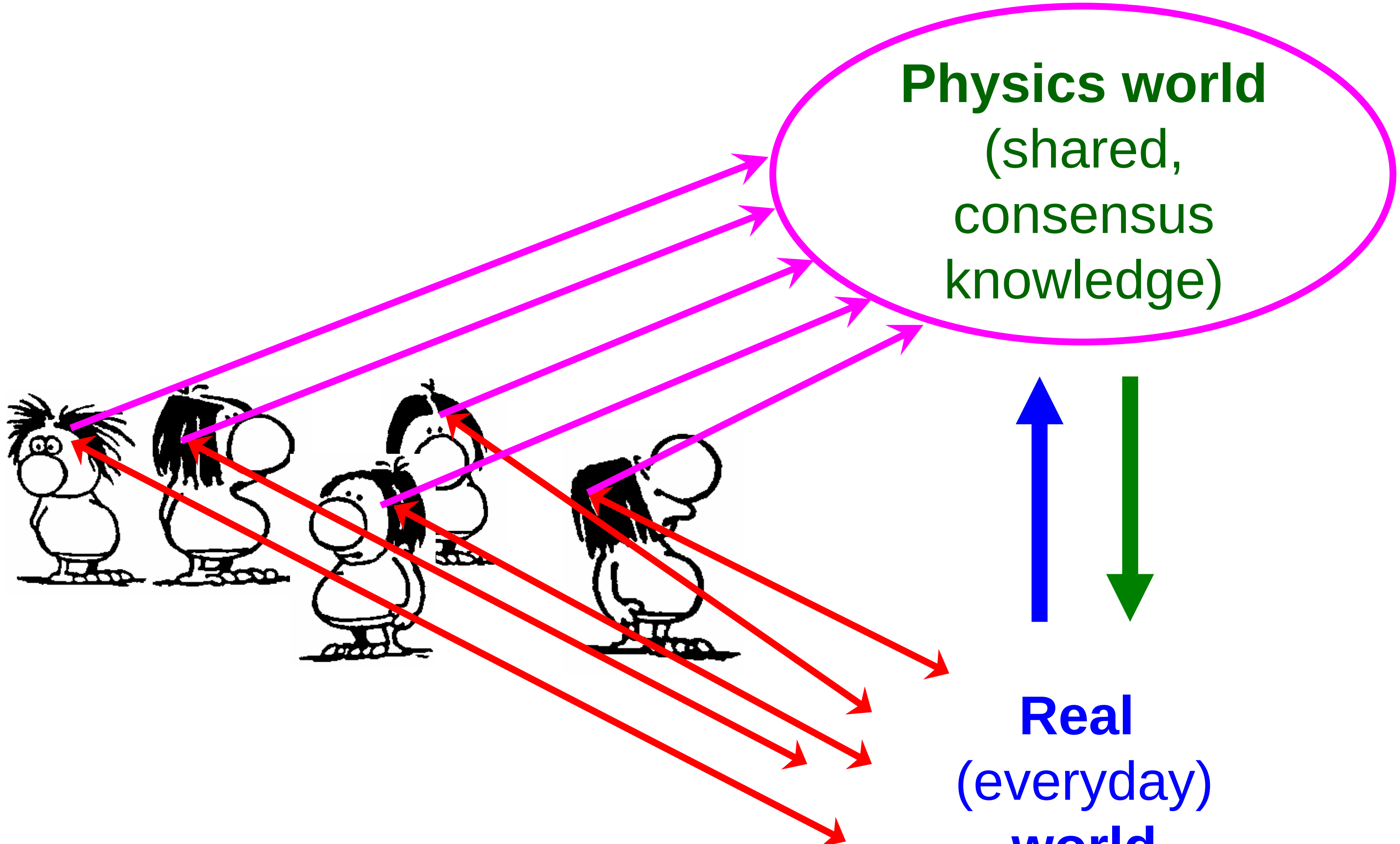
- **What (exactly) are you doing?**
- **Why are you doing it?**
- **How is it helping you?**

Remember that not everyone learns in the same way

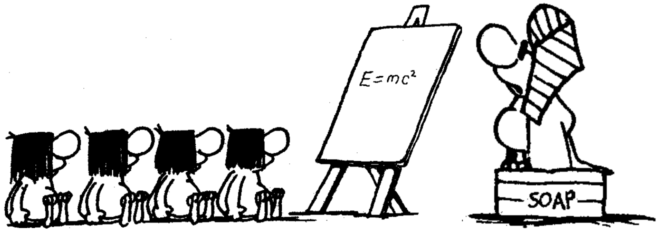
We all have **different** approaches to learning.

Everyone constructs his or her own **mental structures**.

There is no unique answer to the question:
"What is the best way to learn a particular topic?"


Physics world
(shared,
consensus
knowledge)
Real
(everyday)
world

What to expect in this course ...



Lectures ... what are they good for?

Friday morning tutorials ... use these to understand your weekly problem sets

Tuesday afternoon tutorials ... more time to work through problems

Homework ... weekly problem sets, other tasks, self study

Read your **textbook** ! ... Reese, *University Physics*

Laboratories ... Tuesday afternoons

Where to go for help ...

- The course tutors ... Mrs Celia Spargo and Mr Trevor Volkwyn
- Your friends in the course
- Your textbook
- Students who did the course previously

An approach to solving physics problems

Step 1. Think carefully about the problem situation and draw a picture of what is going on (Pictorial Representation).

- Draw one or more **pictures** which show all the important objects, their motion and any interactions.
- Now ask "**What is being asked**?" "Do I need to calculate something?"
- Think about what **physics concepts** and principles you think will be useful in solving the problem and when they will be most useful.
- Construct a **mental image** of the problem situation - do your friends have the same image?
- Specify a convenient **system** to use - circle this on your picture.
- Identify any **idealisations and constraints** present in the situation - write them down!
- Specify any **approximations** or simplifications which you think will make the problem solution easier, but will not affect the result significantly.

Step 2. Describe the physics (Physics Representation).

- Draw a **coordinate axis** (or axes) onto your picture (decide where to put the origin and on the direction of the axes).
- Translate your pictures into one or more **diagrams** (with axes) which only gives the essential information for a mathematical solution.
- If you are using kinematic concepts, draw a motion diagram specifying the object's velocity and acceleration at definite positions and times.
- If interactions or statics are important, draw idealised, free body and force diagrams.
- When using conservation principles, draw "initial" and "final" diagrams to show how the system changes.
- For optics problems draw a ray diagram.
- For circuit problems, a circuit diagram will be useful.
- Define a **symbol** for every important physics variable in your diagram and write down what information you know (e.g. T_1 = 30 N).
- Identify your **target** variable? ("What unknown must I calculate?").

Step 3. Represent the problem mathematically and plan a solution (Mathematical Representation).

- Only now think about what **mathematical expressions** relate the physics variables from your diagrams.
- Using these mathematical expressions, construct specific **algebraic equations** which describe the specific situation above.
- Think about **how** these equations can be combined to find your target variable.
- Begin with an equation that contains the target variable.
- Identify any unknowns in that equation
- Find equations which contain these unknowns
- Do not solve equations numerically at this time.
- Check your equations for **sufficiency**... You have a solution if your plan has as many independent equations are there are unknowns. If not, determine other equations or check the plan to see if it is likely that a variable will cancel from your equations.
- **Plan** the best order in which to solve the equations for the desired variable.

Step 4. Execute the plan

- **Do the algebra** in the order given by your outline.
- When you are done you should have a single equation with your target variable.
- **Substitute** the values (numbers with units) into this final equation.
- Make sure **units** are **consistent** so that they will cancel properly.
- **Calculate** the numerical **result** for the target variable.

Step 5. Evaluate your solution

- Do **vector** quantities have both magnitude and direction ?
- Does the **sign** of your answer make sense ?
- Can someone else follow your solution ? Is it **clear** ?
- Is the **result reasonable** and within your experience ?
- Do the **units** make sense ?

Have you answered the question ?

In this course you will need to be familiar with particular **modeling tools** in physics...

See the *Tools and Skills for Physics I* book.

In particular ...

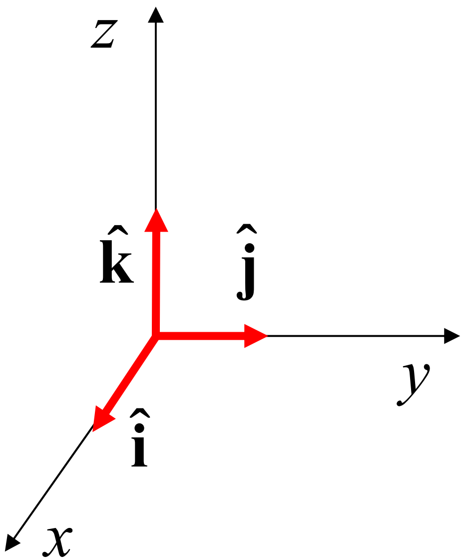
- Using coordinate systems
- Vectors and component vectors ($\hat{\mathbf{i}}\ \hat{\mathbf{j}}\ \hat{\mathbf{k}}$ notation)
- Vector addition and subtraction
- Dot and cross products
- Rates of change (differentiation)
- Simple integration

Vector algebra

Right handed coordinate system:

Unit vectors $\hat{\mathbf{i}}$ $\hat{\mathbf{j}}$ $\hat{\mathbf{k}}$

$$\left|\hat{\mathbf{i}}\right| = \left|\hat{\mathbf{j}}\right| = \left|\hat{\mathbf{k}}\right| = 1$$



$$\vec{\mathbf{A}} = (A_x, A_y, A_z) = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = (B_x, B_y, B_z) = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x, A_y + B_y, A_z + B_z)$$

$$-\vec{\mathbf{A}} = (-A_x, -A_y, -A_z)$$

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}}) = (A_x - B_x, A_y - B_y, A_z - B_z)$$

$$c\vec{\mathbf{A}} = (cA_x, cA_y, cA_z)$$

$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z = \mathrm{d}$$

$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = \vec{\mathbf{B}} \bullet \vec{\mathbf{A}} \qquad \vec{\mathbf{A}} \bullet \vec{\mathbf{A}} = \mathrm{A}^2$$

$$\hat{\mathbf{i}} \bullet \hat{\mathbf{i}} = \hat{\mathbf{j}} \bullet \hat{\mathbf{j}} = \hat{\mathbf{k}} \bullet \hat{\mathbf{k}} = 1 \qquad \hat{\mathbf{i}} \bullet \hat{\mathbf{j}} = \hat{\mathbf{j}} \bullet \hat{\mathbf{k}} = \hat{\mathbf{k}} \bullet \hat{\mathbf{i}} = 0$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} + (A_zB_x - A_xB_z)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}$$

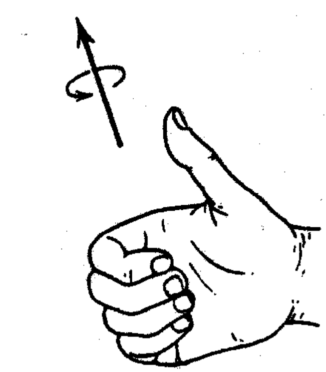
$$= \vec{\mathbf{G}} \quad \text{where} \quad \vec{\mathbf{G}} \perp \vec{\mathbf{A}} \quad \text{and} \quad \vec{\mathbf{G}} \perp \vec{\mathbf{B}}$$

easy to remember: $\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ always —

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = -(\vec{\mathbf{B}}\times\vec{\mathbf{A}}) \qquad \vec{\mathbf{A}}\times\vec{\mathbf{A}} = 0$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0 \qquad \hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}};\ \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}};\ \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

In polar form in 2D:



$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos\theta$ and $\vec{\mathbf{A}}\times\vec{\mathbf{B}} = AB\sin\theta\,\hat{\mathbf{k}}$

where θ is the angle between tails of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

Differentiation of vector functions

If $\vec{\mathbf{A}}(t) = A_x(t)\hat{\mathbf{i}} + A_y(t)\hat{\mathbf{j}} + A_z(t)\hat{\mathbf{k}}$

then $$\frac{d}{dt}\vec{\mathbf{A}}(t) = \frac{dA_x(t)}{dt}\hat{\mathbf{i}} + \frac{dA_y(t)}{dt}\hat{\mathbf{j}} + \frac{dA_z(t)}{dt}\hat{\mathbf{k}}$$

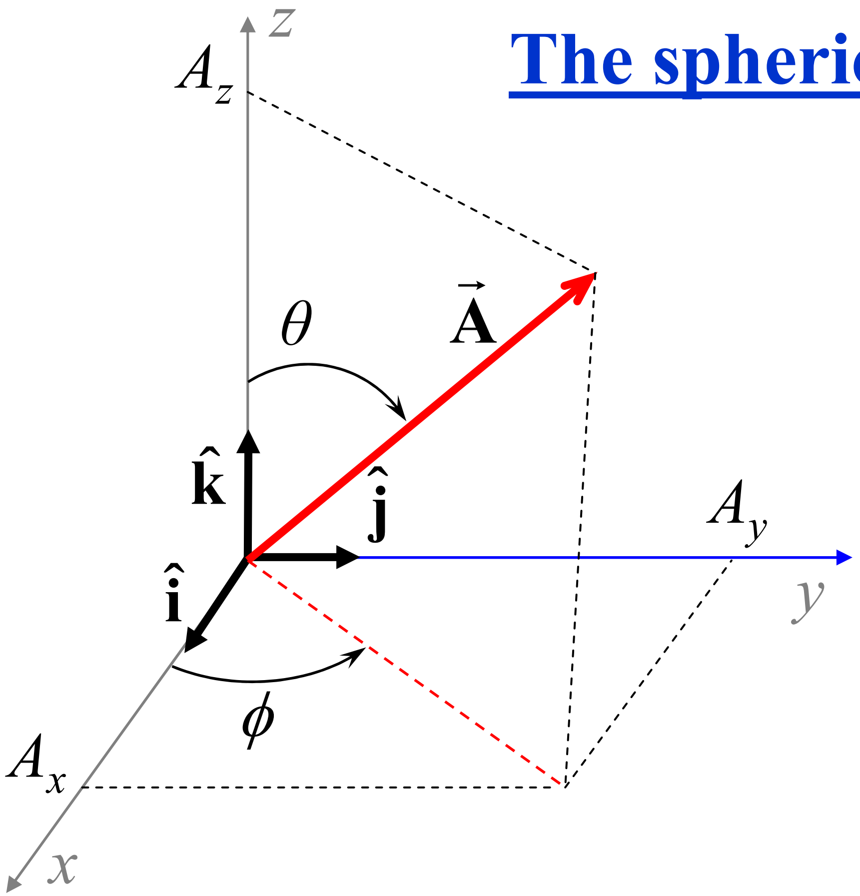
Also: $$\frac{d}{dt}\left[\vec{\mathbf{A}}(t) + \vec{\mathbf{B}}(t)\right] = \frac{d\vec{\mathbf{A}}(t)}{dt} + \frac{d\vec{\mathbf{B}}(t)}{dt}$$

$$\frac{d}{dt}\left[c(t)\vec{\mathbf{A}}(t)\right] = \frac{dc(t)}{dt}\vec{\mathbf{A}}(t) + c(t)\frac{d\vec{\mathbf{A}}(t)}{dt}$$

$$\frac{d}{dt}\left[\vec{\mathbf{A}}(t)\bullet\vec{\mathbf{B}}(t)\right] = \vec{\mathbf{A}}(t)\bullet\frac{d\vec{\mathbf{B}}(t)}{dt} + \frac{d\vec{\mathbf{A}}(t)}{dt}\bullet\vec{\mathbf{B}}(t)$$

$$\frac{d}{dt}\left[\vec{\mathbf{A}}(t)\times\vec{\mathbf{B}}(t)\right] = \vec{\mathbf{A}}(t)\times\frac{d\vec{\mathbf{B}}(t)}{dt} + \frac{d\vec{\mathbf{A}}(t)}{dt}\times\vec{\mathbf{B}}(t)$$

The spherical polar coordinate system



$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$A_x = A\ \cos\phi\ \sin\theta$$

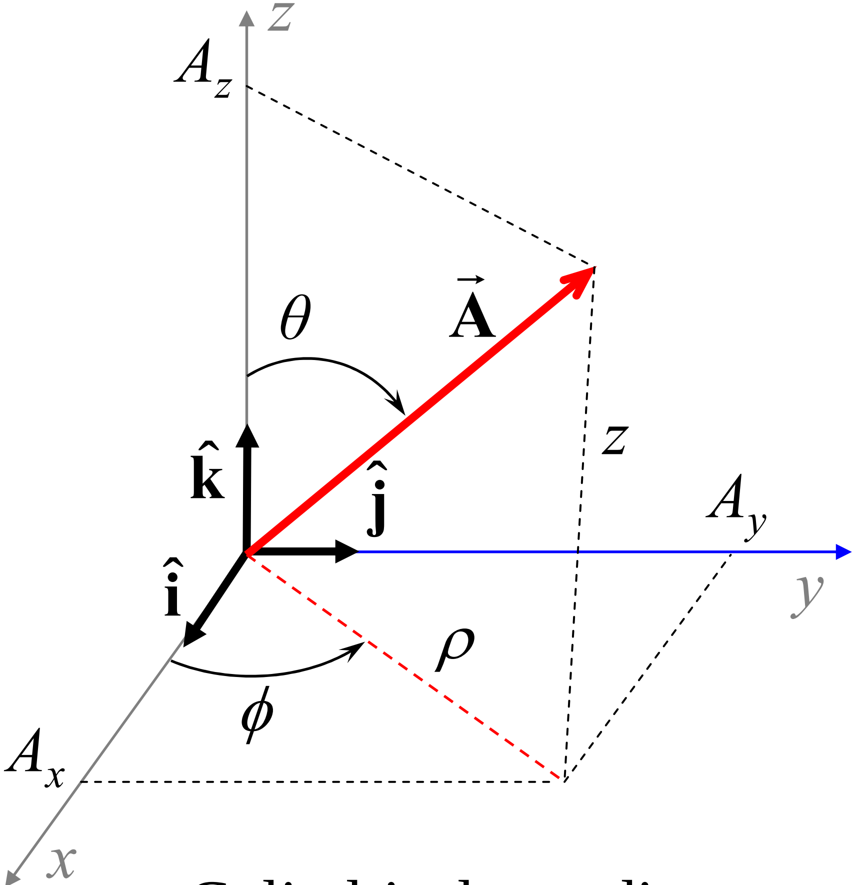
$$A_y = A\ \sin\phi\ \sin\theta$$

$$A_z = A\ \cos\theta$$

Spherical coordinates: A, θ, ϕ :

$$\left\{ \begin{aligned} A &= \sqrt{A_x^{\ 2} + A_y^{\ 2} + A_z^{\ 2}} \\ \cos\theta &= \frac{A_z}{A} \\ \tan\phi &= \frac{A_y}{A_x} \end{aligned} \right.$$

The cylindrical polar coordinate system



$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$A_x = \rho\cos\phi$$

$$A_y = \rho\sin\phi$$

$$A_z = z$$

Cylindrical coordinates: ρ, θ, z :

$$\begin{cases} \rho = \sqrt{A_x^2 + A_y^2} \\ \tan\phi = \dfrac{A_y}{A_x} \\ z = A_z \end{cases}$$

Learning how to **estimate** quantities realistically is an important skill to learn ...

- How many molecules of nitrogen are there in this room?
- How long would it take for you to walk from here to the Bremner Building?
- What is the temperature of the air in this room?
- What is the mass of the lecturer?
- How many hairs are there on your head?
- What is the volume of this building?

An estimation problem ...

One hot day you find the fossil remains of a dinosaur in the Karoo. You estimate that the beast probably weighed five tonnes. Being very hot, you drink a mouthful of Coke, and suddenly realise that if all the water in the urine excreted by the dinosaur became randomly mixed with all the water molecules in the oceans and atmosphere over the past sixty five million years, there is a chance that some of the water molecules you have just drunk may have come from the urine of that very same dinosaur. Aghast, you proceed to calculate just how many molecules of that dinosaur's urine you have just drunk! Make reasonable assumptions (i.e. guesses) about any quantities that you are not sure of.

(from DGA)

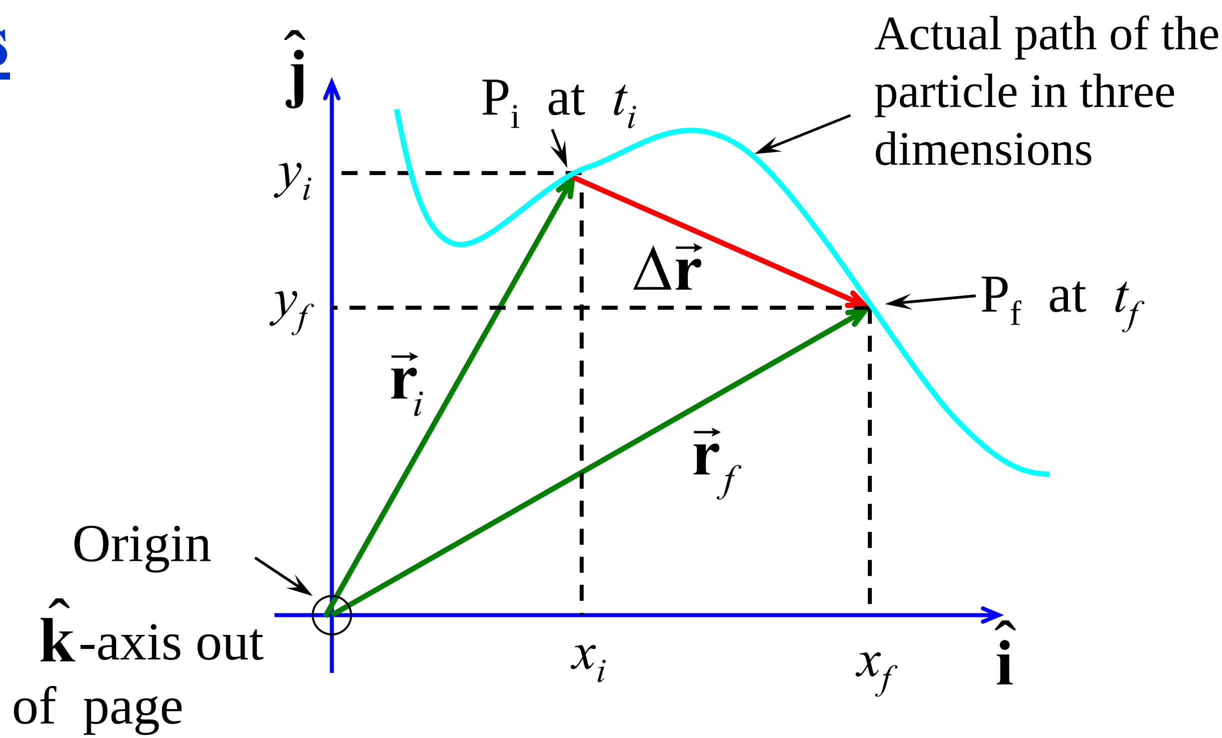
Mechanics

... has to do with the way bodies interact with each other.

Coming up in this section ...

- Kinematics in one dimension (including falling bodies)
- Kinematics in two dimensions (projectiles)
- Relative motion
- Rotational kinematics
- Forces and Newton's Laws (linear dynamics)
- Rotational dynamics
- Centre of mass
- Work and Energy
- Linear momentum
- Torque, moments and angular momentum
- Statics

Kinematics

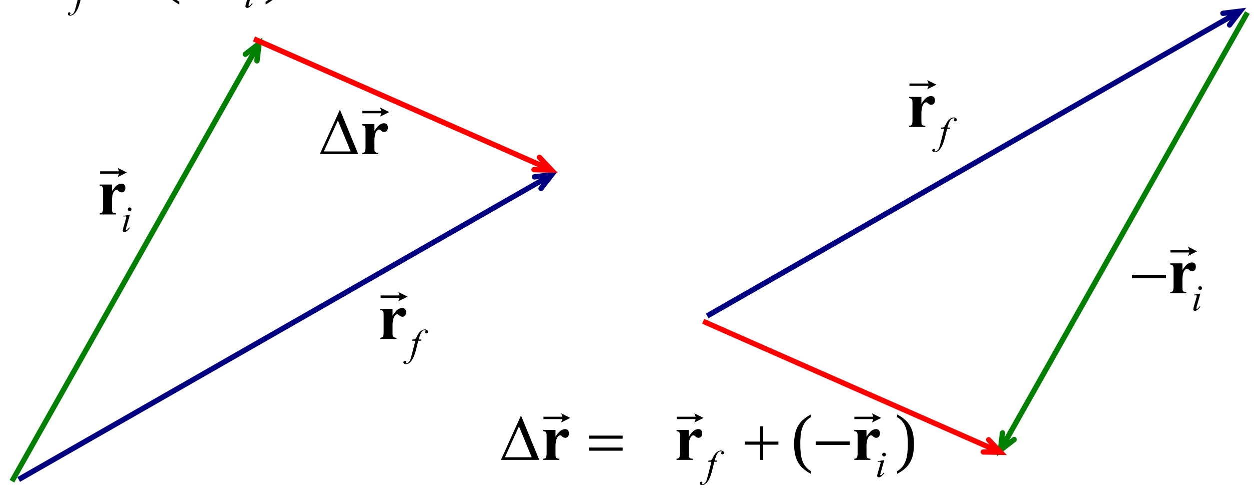


Consider a particle that follows the curved (blue) path in space.
At time t_i it is at position P_i and time t_f it is at position P_f.
To describe the motion of the particle, we use a three dimensional Cartesian coordinate system as shown.
Therefore at $t = t_i$, the particle is at position $\vec{\mathbf{r}}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$
and $t = t_f$, the particle is at position $\vec{\mathbf{r}}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} + z_f\hat{\mathbf{k}}$

The **displacement vector** is

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \left(x_f - x_i\right)\hat{\mathbf{i}} + \left(y_f - y_i\right)\hat{\mathbf{j}} + \left(z_f - z_i\right)\hat{\mathbf{k}}$$

$$= \vec{\mathbf{r}}_f + (-\vec{\mathbf{r}}_i)$$



Of course $\left|\Delta\vec{\mathbf{r}}\right|$ is <u>not</u> necessarily the distance from P_i to P_f.

In general an **instantaneous position vector** $\vec{\mathbf{r}}(t)$ describes the position of a particle at a particular instant in time relative to the origin of a set of coordinate axes:

$$\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$$

The velocity vector

The **average velocity** vector $$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i}{t_f - t_i}$$

$$= \frac{\left(x_f - x_i\right)\hat{\mathbf{i}} + \left(y_f - y_i\right)\hat{\mathbf{j}} + \left(z_f - z_i\right)\hat{\mathbf{k}}}{t_f - t_i}$$

The **instantaneous velocity** vector $$\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt}$$

$$= \frac{d}{dt}\left(x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}\right)$$

$$= \frac{dx(t)}{dt}\hat{\mathbf{i}} + \frac{dy(t)}{dt}\hat{\mathbf{j}} + \frac{dz(t)}{dt}\hat{\mathbf{k}}$$

The acceleration vector

The **average acceleration** vector $\vec{\mathbf{a}}_{av} = \dfrac{\Delta \vec{\mathbf{v}}}{\Delta t} = \dfrac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}$

The **instantaneous acceleration** vector

$$\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt}$$

$$= \frac{dv_x(t)}{dt}\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\hat{\mathbf{j}} + \frac{dv_z(t)}{dt}\hat{\mathbf{k}}$$

$$= \frac{d^2x(t)}{dt^2}\hat{\mathbf{i}} + \frac{d^2y(t)}{dt^2}\hat{\mathbf{j}} + \frac{d^2z(t)}{dt^2}\hat{\mathbf{k}}$$

Example 1

The position of a 3 kg object as a function of time is given by:

$$\vec{\mathbf{r}}(t) \;=\; (5t^2 - 2t)\hat{\mathbf{i}} + (3t + 8)\hat{\mathbf{j}} \quad \text{metres}$$

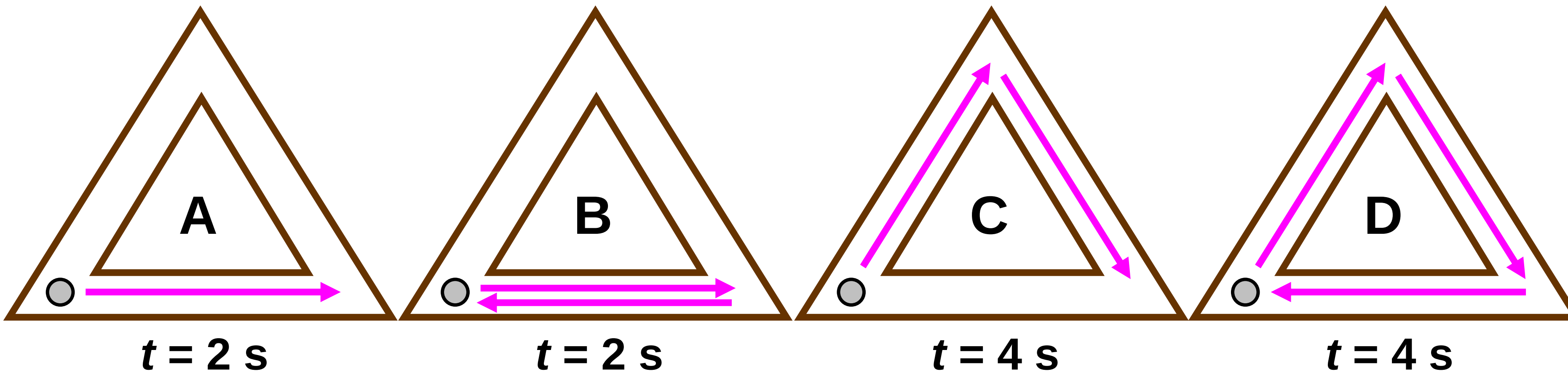
(a) What is the position of the object at $t = 2$ s?
(b) Write down an expression for the velocity of the object as a function of time.
(c) What is the velocity of the object at $t = 2$ s?
(d) What is the displacement of the object between $t = 0$ s and $t = 2$ s ?
(e) What is the average velocity of the object between $t = 0$ s and $t = 2$ s?

Example 2

Bugs walks 3 metres in the $\hat{\mathbf{i}}$-direction for 6 seconds, and then 6 metres in a direction 30 to the $\hat{\mathbf{i}}$-direction for another 6 seconds. What is his average velocity?

Example 3

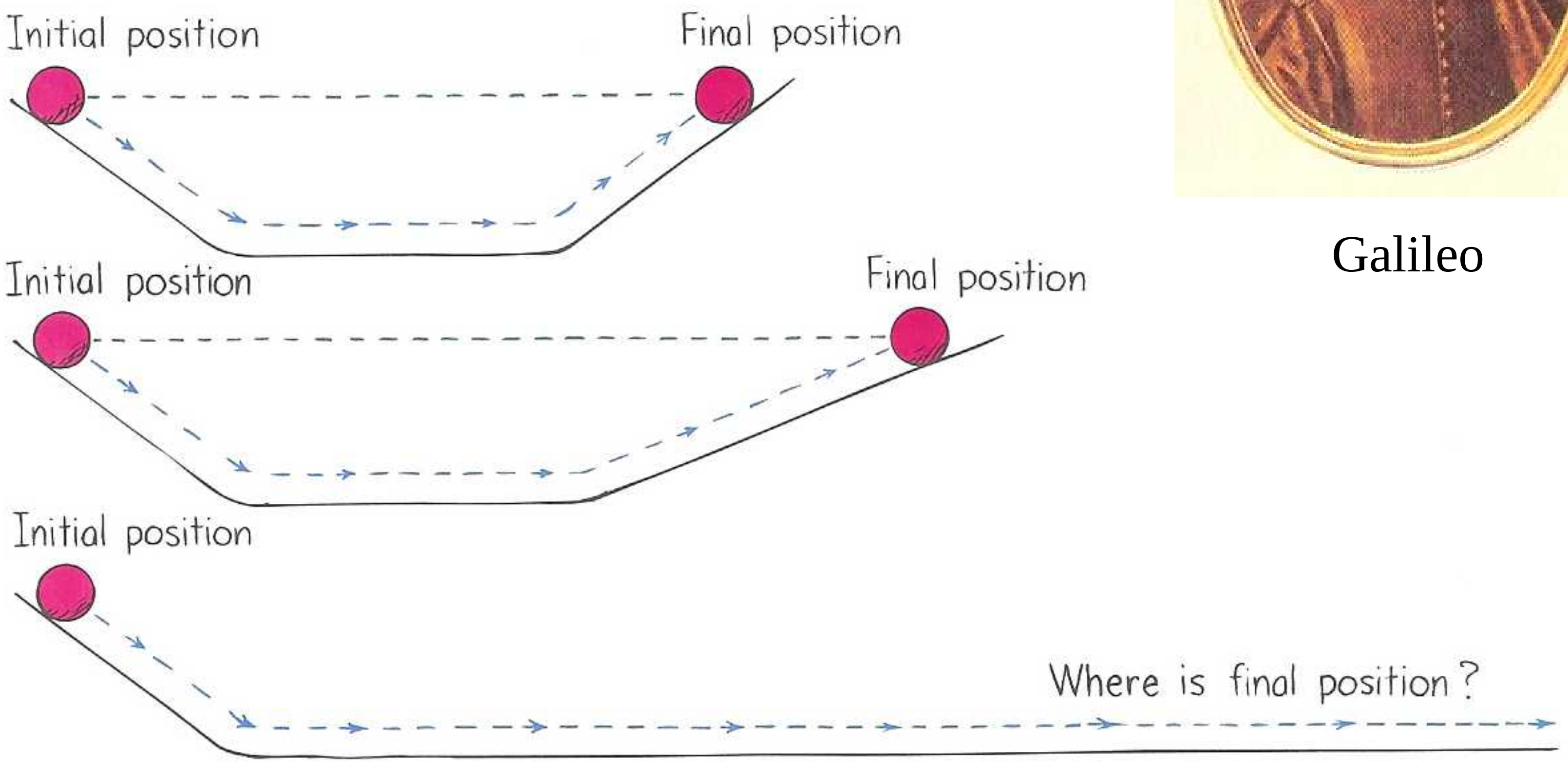
Four different mice (labeled A, B, C and D) ran the triangular maze shown below. They started in the lower left hand corner and followed the paths of the arrows. The times they took are shown below each figure.



For each item below, write down the letters of all the mice that fit the description.

(a) This mouse had the greatest average speed.
(b) This mouse had the greatest total displacement.
(c) This mouse had an average velocity that points in this direction $\rightarrow$
(d) This mouse had the greatest average velocity.

Kinematics with constant acceleration



Galileo

Kinematics with constant acceleration

We introduce of two kinematic equations of motion for the case of constant acceleration:

$$\vec{\mathbf{r}}(t) \;=\; \vec{\mathbf{r}}_o \;+\; \vec{\mathbf{u}}t \;+\; \tfrac{1}{2}\vec{\mathbf{a}}t^2$$

$$\vec{\mathbf{v}}(t) \;=\; \vec{\mathbf{u}} \;+\; \vec{\mathbf{a}}t$$

$\vec{\mathbf{r}}(t)$	:	position vector at time t
$\vec{\mathbf{r}}_o = \vec{\mathbf{r}}(t=0)$	:	position vector at time $t = 0$ (the initial position)
$\vec{\mathbf{v}}(t)$	:	velocity vector at time t
$\vec{\mathbf{u}} = \vec{\mathbf{v}}(t=0)$	:	velocity vector at time $t = 0$ (the initial velocity)
$\vec{\mathbf{a}}$	:	acceleration vector (constant)

The displacement vector is then $\Delta\vec{\mathbf{r}} \;=\; \vec{\mathbf{r}}(t) - \vec{\mathbf{r}}_o$

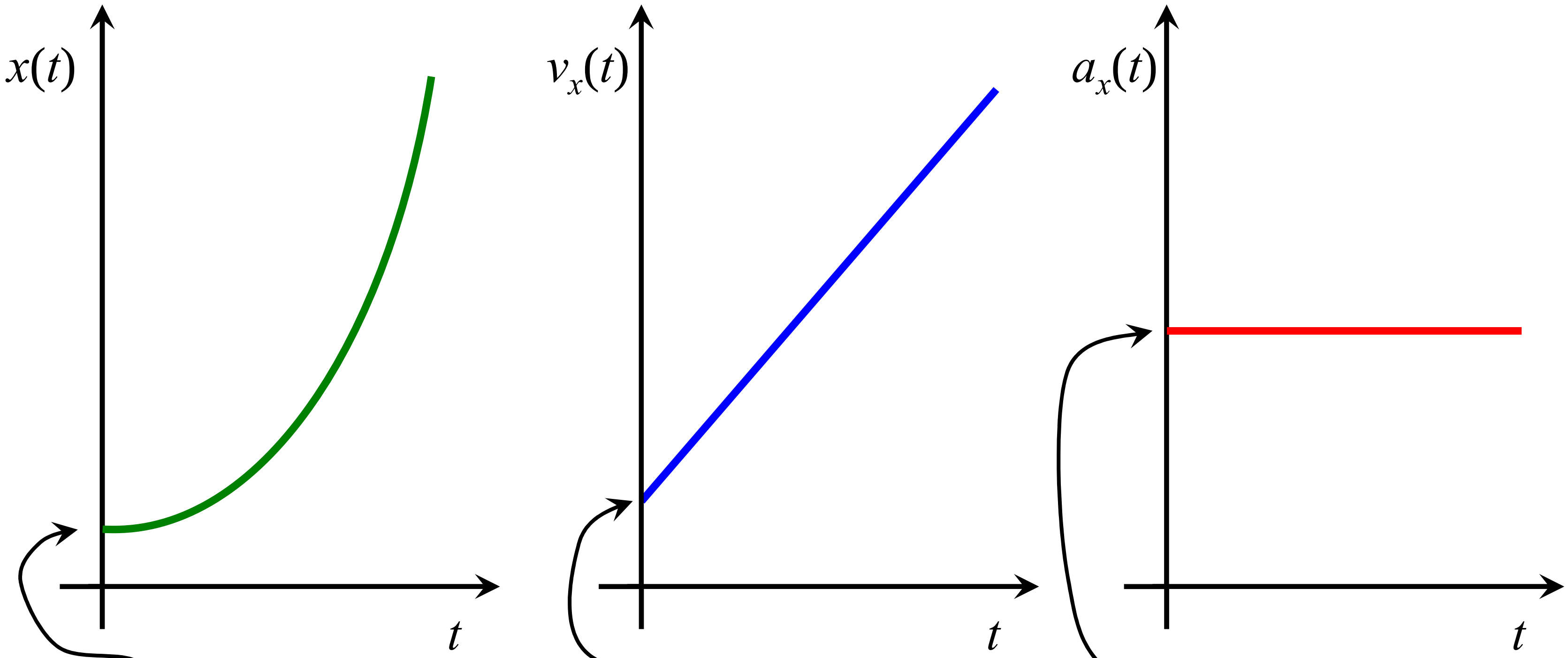
When the motion is only in one dimension, we can use slightly different notation:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2 \qquad \vec{\mathbf{v}}_x(t) = \vec{\mathbf{u}}_x + \vec{\mathbf{a}}_x t$$

$$\vec{\mathbf{y}}(t) = \vec{\mathbf{y}}_o + \vec{\mathbf{u}}_y t + \tfrac{1}{2}\vec{\mathbf{a}}_y t^2 \qquad \vec{\mathbf{v}}_y(t) = \vec{\mathbf{u}}_y + \vec{\mathbf{a}}_y t$$

... which sometimes helps to remind us what is going on.

Graphical representations of constant accelerated motion



$$x(t) = x_o + u_x t + \tfrac{1}{2} a_x t^2$$

$$\frac{d}{dt} x(t) = \frac{d}{dt}\left(x_o + u_x t + \tfrac{1}{2} a_x t^2\right) \rightarrow v_x(t) = u_x + a_x t$$

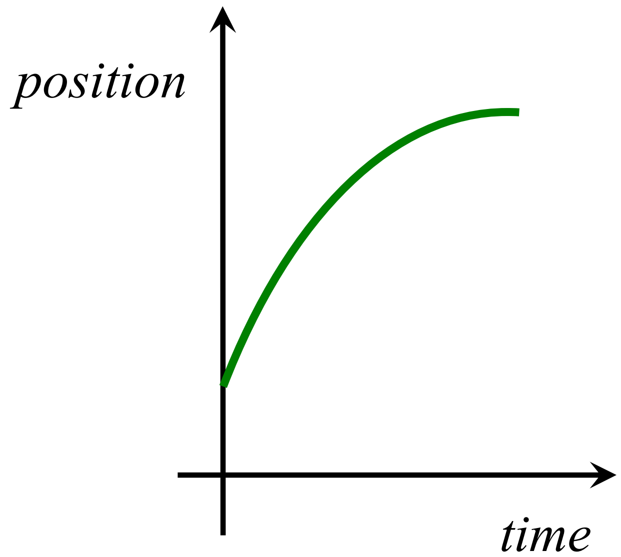
$$\frac{d}{dt} v_x(t) = \frac{d}{dt}(u_x + a_x t) \rightarrow a_x = \text{constant}$$

FIGURING PHYSICS

A car moves along a straight road. The graph below shows the position of the car as a function of time.

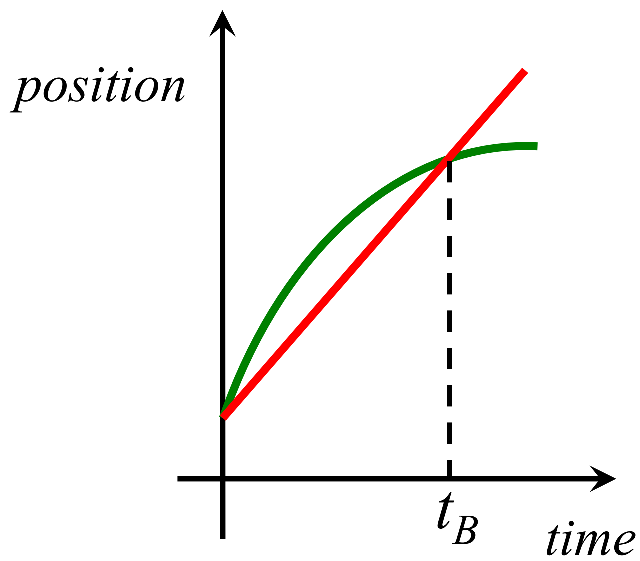
The graph shows that the car:

(A) speeds up all the time
(B) slows down all the time
(C) moves at a constant velocity



Figuring Physics

The graph shows position as a function of time for two trains running on parallel tracks. Which is true:



(A) At time t_B both trains have the same velocity
(B) Both trains speed up all the time
(C) Both trains have the same velocity at some time before t_B

Example of motion in a straight line

A car is initially at rest. It starts to move, accelerating uniformly, and reaches a speed of 15 m s^{-1} after 20 s. It travels at a constant speed for 2 minutes after which time it slows down uniformly to stop in 30 seconds.
Assume that the entire motion takes place in a straight line in the $\hat{\mathbf{i}}$ -direction.

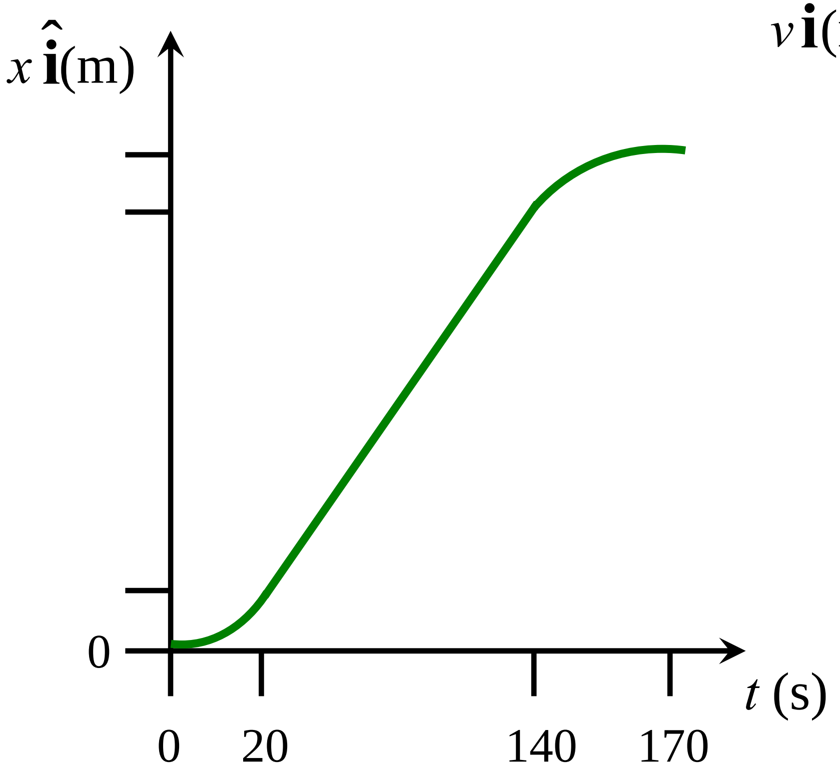
Determine:
(a) the average acceleration of the car during (i) the first 20 seconds; (ii) the last 30 seconds; and (iii) the whole trip.
(b) the total displacement of the car.

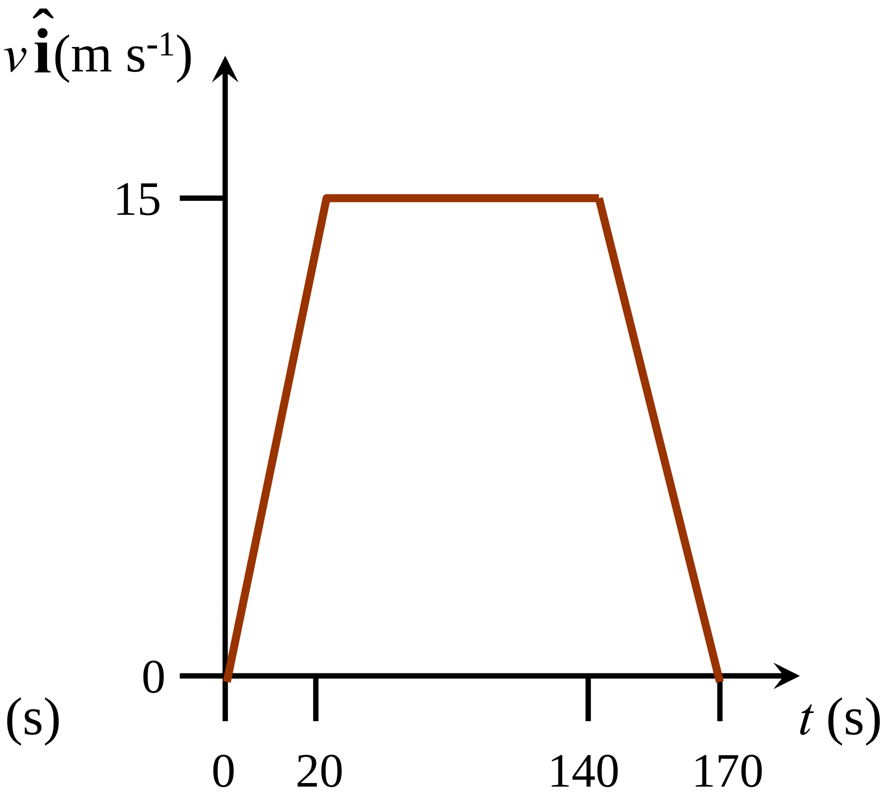
First draw a **picture** of what is happening.
Add in a coordinate system and include all known and unknown variables ...

a = ? $\qquad$ a = 0 $\qquad$ a = ?

$t = 0$	$t = 20$ s	$t = 140$ s	$t = 170$ s
$x(t = 0) = 0$	$x(t = 20) = ?$	$x(t = 140) = ?$	$x(t = 170) = ?$
$v(t = 0) = 0$	$v(t = 20) = 15$ m s^{-1}	$v(t = 140) = 15$ m s^{-1}	$v(t = 170) = 0$ m s^{-1}

We can draw position versus time and velocity versus time graphs for this situation:





Finally, use the **equations of motion**:

First calculate the accelerations:

(i) Between $t = 0$ and $t = 20$ s:

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{15\hat{\mathbf{i}} - 0}{20 - 0} = 0.75\hat{\mathbf{i}} \quad \text{m s}^{-2}$$

Between $t = 20$ and $t = 140$ s: $\vec{\mathbf{a}}_{av} = 0 \ \text{m s}^{-2}$

(ii) Between $t = 140$ and $t = 170$ s:

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{0 - 15\hat{\mathbf{i}}}{170 - 140} = -0.50\hat{\mathbf{i}} \quad \text{m s}^{-2}$$

(iii) Between $t = 0$ and $t = 170$ s:

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{0 - 0}{170 - 0} = 0$$

Now the displacements ...

We consider each of the three stages separately.

(i) Between $t = 0$ and $t = 20$ s:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$

$$\vec{\mathbf{x}}(t = 20) = (0) + (0) + \tfrac{1}{2}(0.75\hat{\mathbf{i}})(20)^2 = 150\hat{\mathbf{i}}\ \text{m}$$

(ii) Between $t = 20$ and $t = 140$ s:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$

$$\vec{\mathbf{x}}(t = 140) = (150\hat{\mathbf{i}}) + (15\hat{\mathbf{i}})(120) + \tfrac{1}{2}(0)(120)^2 = 1950\hat{\mathbf{i}}\ \text{m}$$

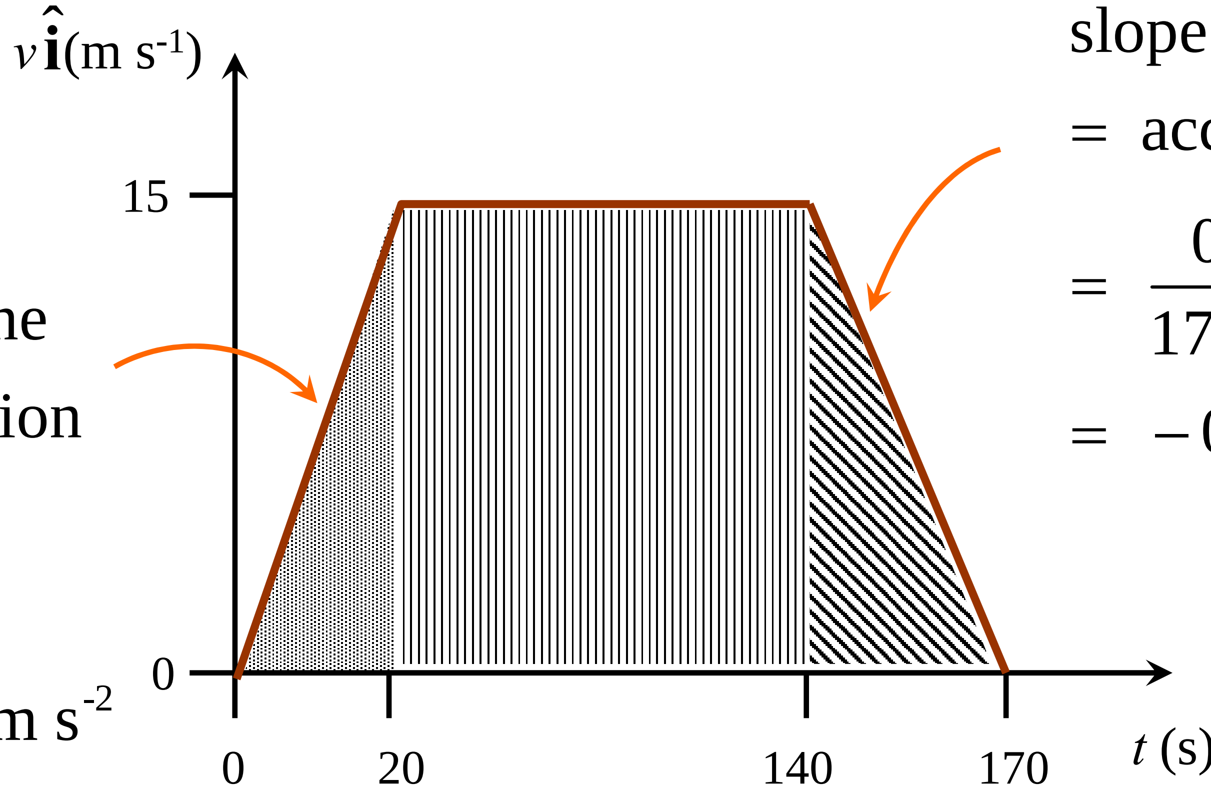
(iii) Between $t = 140$ and $t = 170$ s:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$

$$\vec{\mathbf{x}}(t = 170) = (1950\hat{\mathbf{i}}) + (15\hat{\mathbf{i}})(30) + \tfrac{1}{2}(-0.5\hat{\mathbf{i}})(30)^2 = 2175\hat{\mathbf{i}}\ \text{m}$$

Final position of the car

Back to our velocity versus time graph:



slope of line

$= $ accleration

$$= \frac{15\hat{\mathbf{i}} - 0}{20 - 0}$$

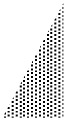
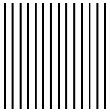
$$= 0.75\hat{\mathbf{i}} \quad \text{m s}^{-2}$$

slope of line

$= $ accleration

$$= \frac{0 - 15\hat{\mathbf{i}}}{170 - 140}$$

$$= -0.50\hat{\mathbf{i}} \quad \text{m s}^{-2}$$

displacement = area under graph =  +  +  $= 2175\hat{\mathbf{i}}$ m

An example involving two moving bodies

You are driving at a speed of 60 km h^{-1} and see a truck 20 m ahead coming directly towards you at a constant speed of 40 km h^{-1}. If you immediately hit the breaks and your car starts to slow down at 8.0 m s^{-2}, how long is it before the truck smashes into you?

The general approach when dealing with two moving bodies is to apply a **single set of coordinate axes** to the situation and then apply the equations of motion separately to each object. The equations will be linked to each other usually by one or more parameters, such as time, or the final positions (if they are the same for each object).

Another example involving two moving bodies

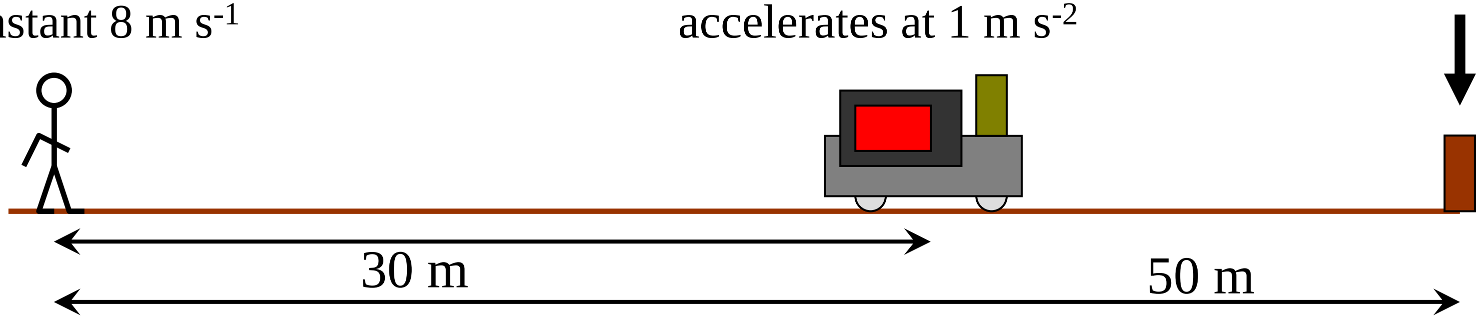
You want to visit your friend in Durban over the Winter vacation. To save money, you decide to travel there by train. But you are late finishing your physics exam, so you are late in arriving at the train station. You run as fast as you can, but just as you reach one end of the platform your train departs, 30 metres ahead of you down the platform. You can run at a maximum speed of 8 m s^{-1} and the train is accelerating at 1 m s^{-2}. You can run along the platform for 50 m before you reach a barrier. Will you catch your train?

Pictorial representation

Student runs at a constant 8 m s^{-1}

train starts from rest and accelerates at 1 m s^{-2}

end of platform (will they meet before here?)



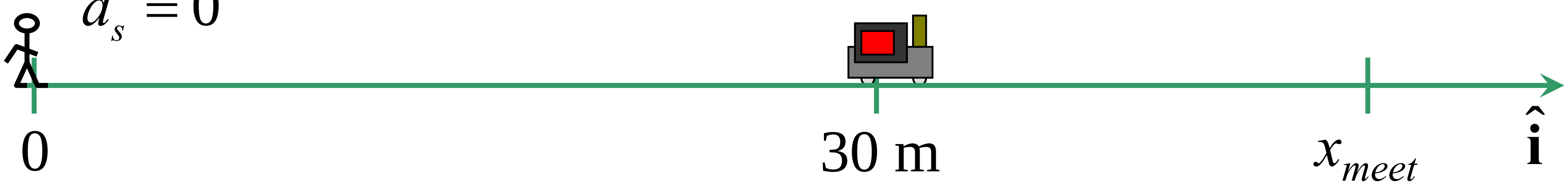
Physics representation

$\vec{u}_s = 8\,\hat{\mathbf{i}}$ m s^{-1}

$\vec{a}_s = 0$

$\vec{u}_t = 0$

$\vec{a}_t = 1\,\hat{\mathbf{i}}$ m s^{-2}



Mathematics representation

Student:

$$\vec{x}(t) = \vec{x}_o + \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$

$$x_{meet} = 0 + 8t + 0$$

Train:

$$\vec{x}(t) = \vec{x}_o + \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$

$$x_{meet} = 30 + 0 + \tfrac{1}{2}(1)t^2$$

solve for x_{meet}

Is $x_{meet} < 50$ m?

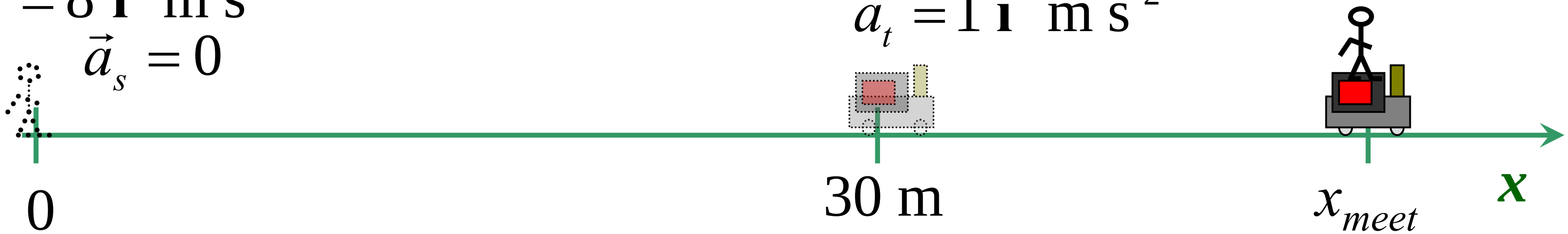
Physics representation

$\vec{u}_s = 8\,\hat{\mathbf{i}}$ m s^{-1}

$\vec{a}_s = 0$

$\vec{u}_t = 0$

$\vec{a}_t = 1\,\hat{\mathbf{i}}$ m s^{-2}

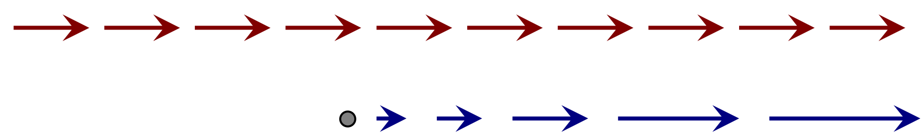


Other useful physics representations:

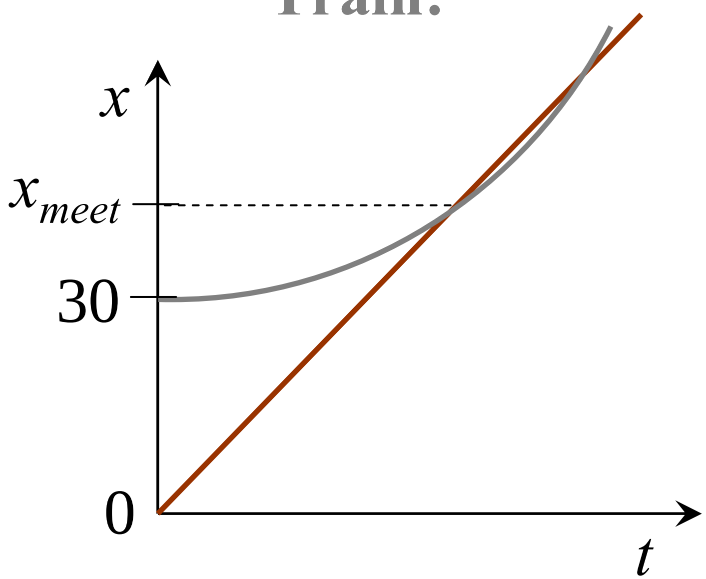
Motion diagram:

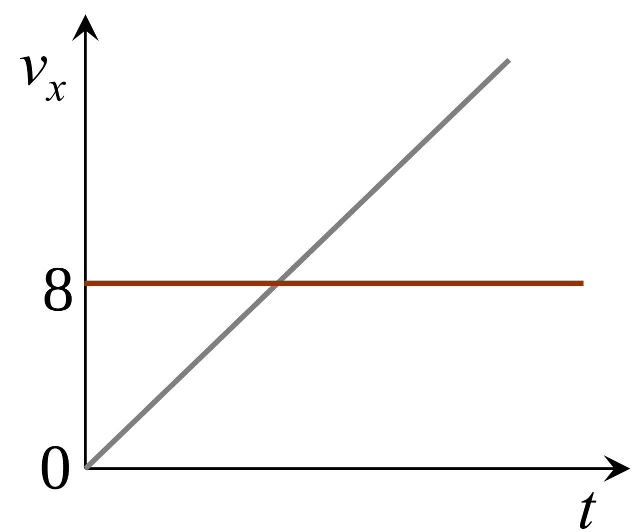
Student:

Train:



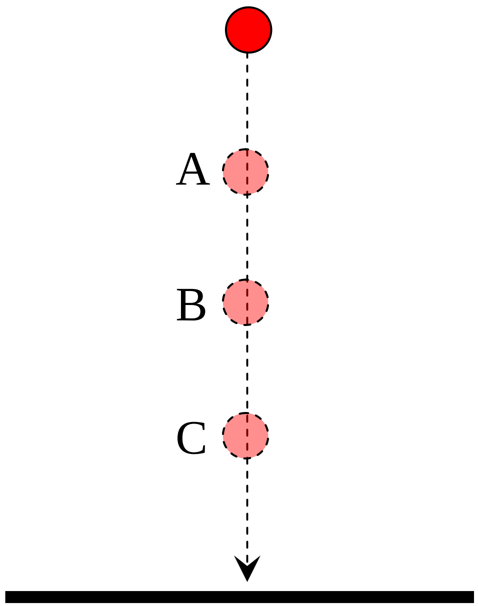
Graphical:



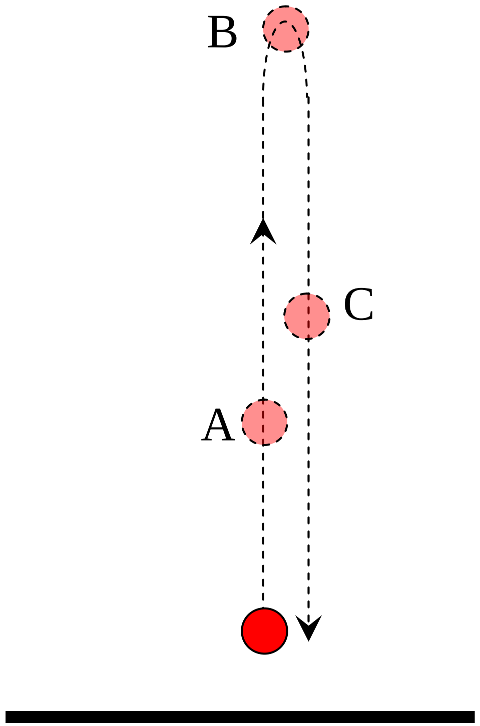


Bodies in free fall

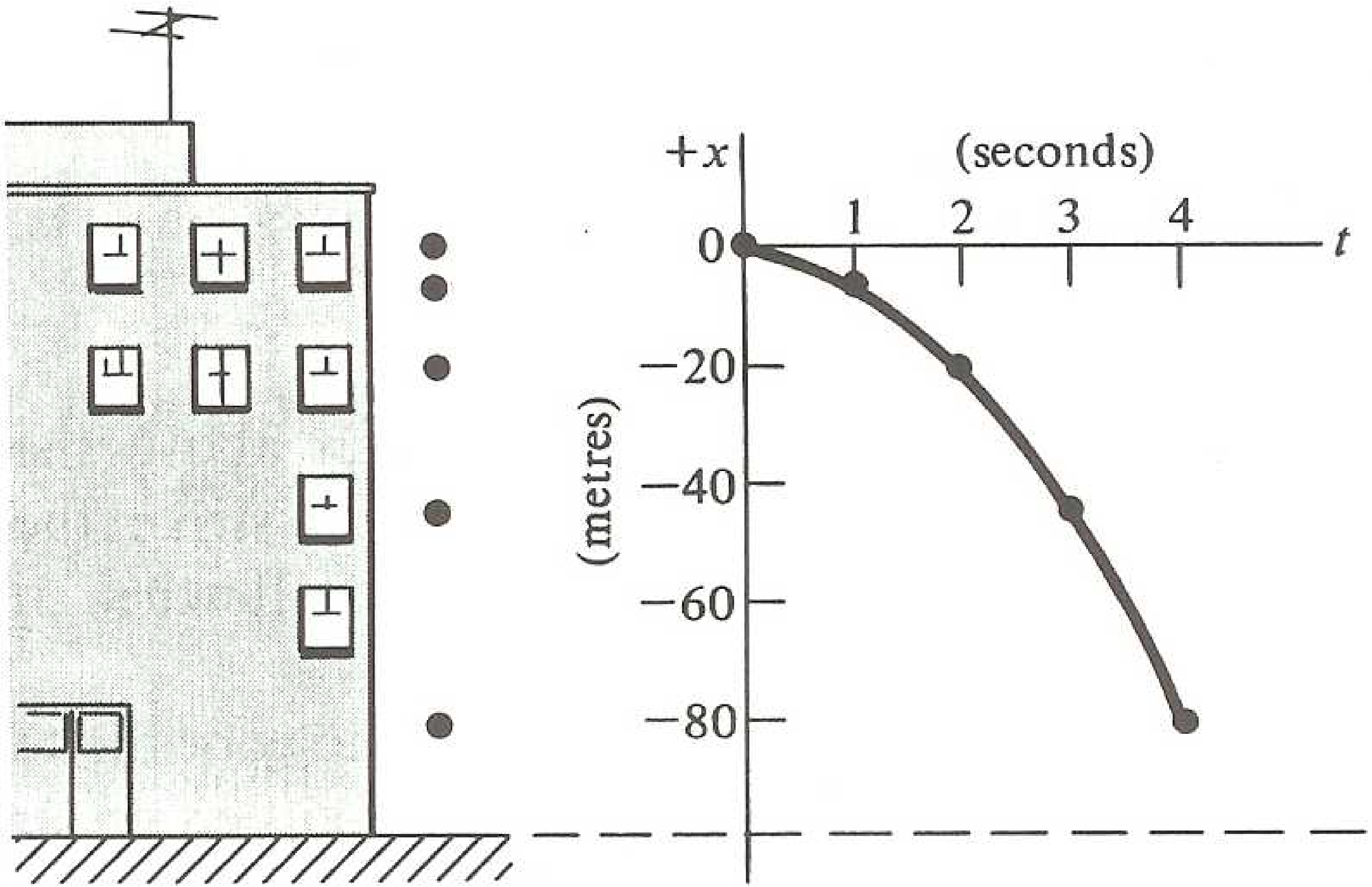
Consider the following two situations. At each position, indicate the magnitude and direction of the **resultant acceleration** of the ball.



The ball is dropped from rest from a height and allowed to fall to the floor

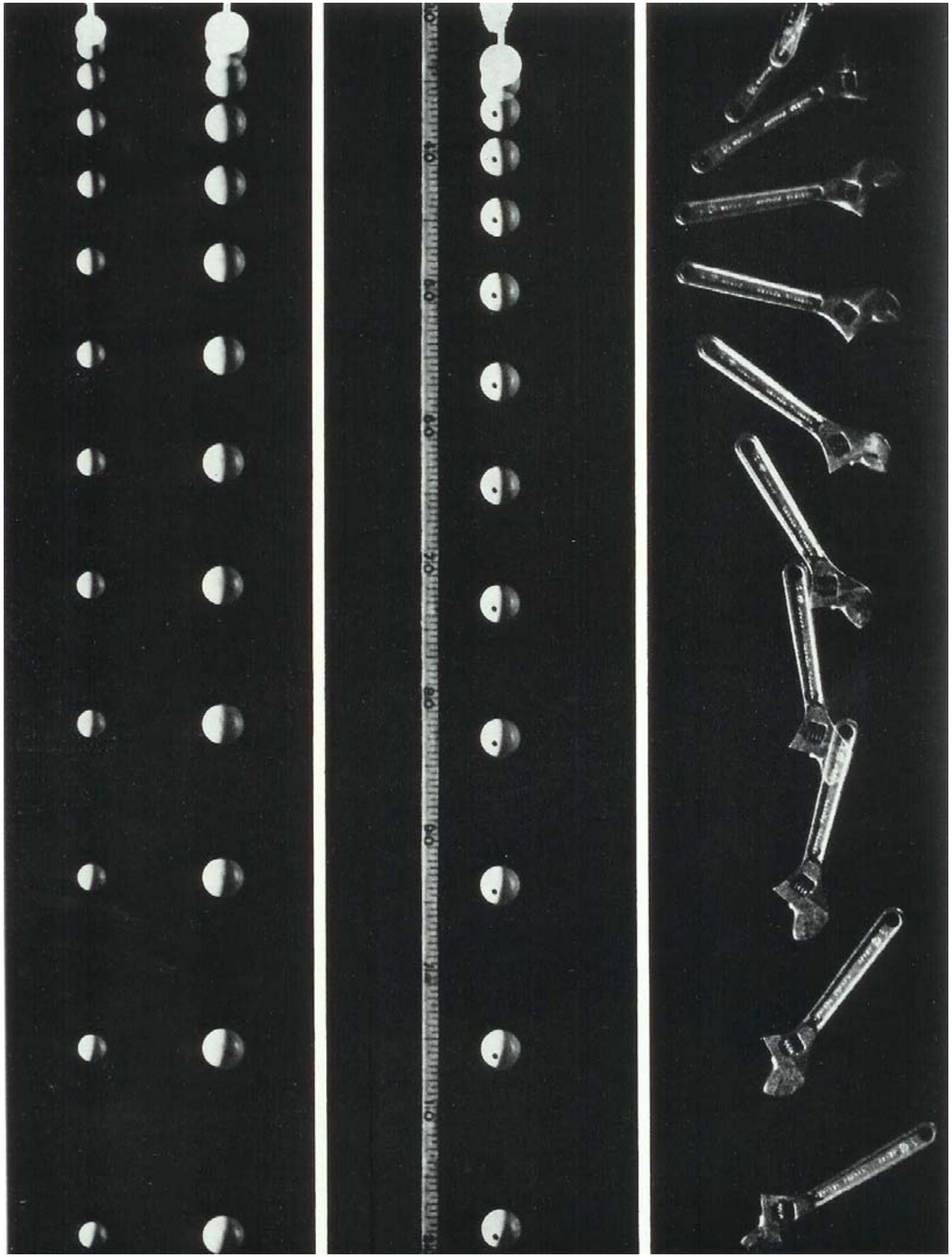


The ball is thrown upwards, reaches some height, and falls back to the floor


+x
(seconds)
1
2
3
4
t
0
−20
−40
−60
−80
(metres)

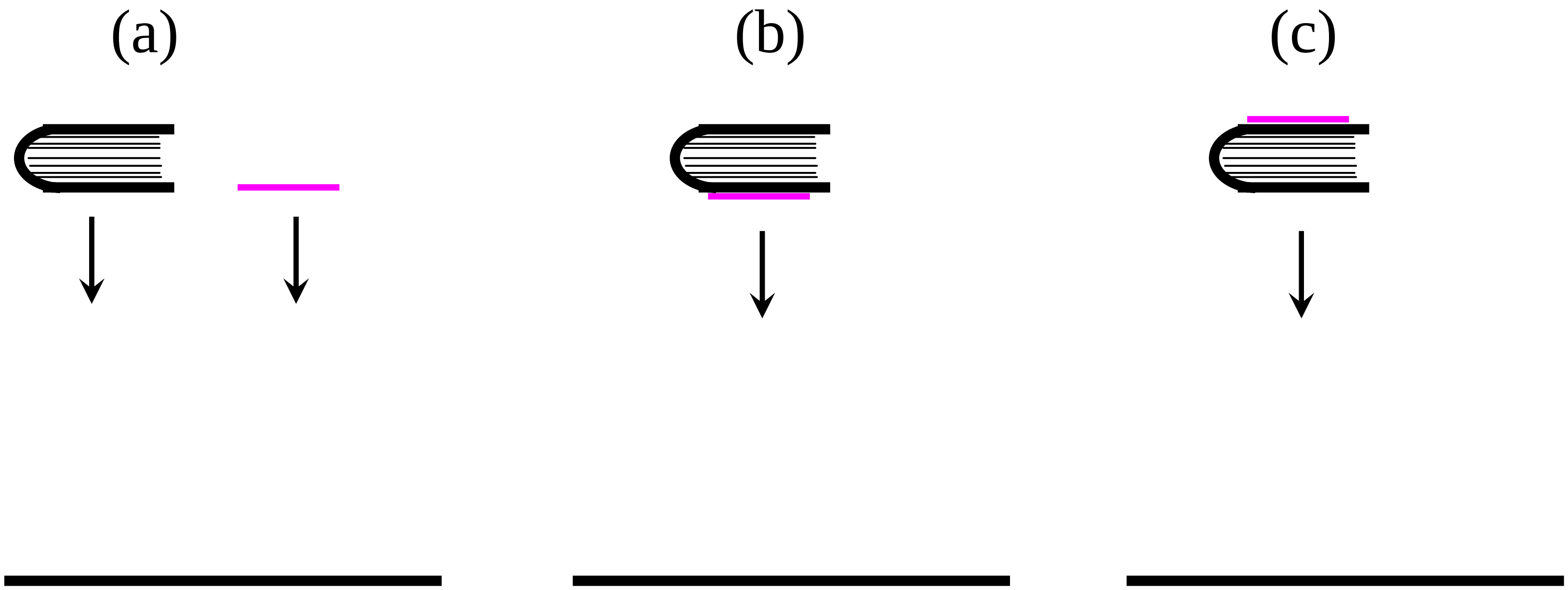
Demonstration

A large ball and a small ball are dropped from the same height in air. Which ball reaches the ground first?



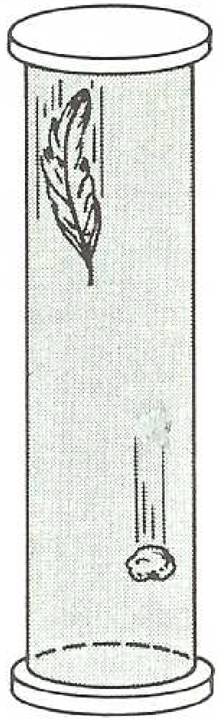
Demonstration

A book and a sheet of paper are dropped from the same height in air. (a) Which reaches the ground first?
Does it make a difference if the paper is placed
(b) under the book? ... or (c) above the book?

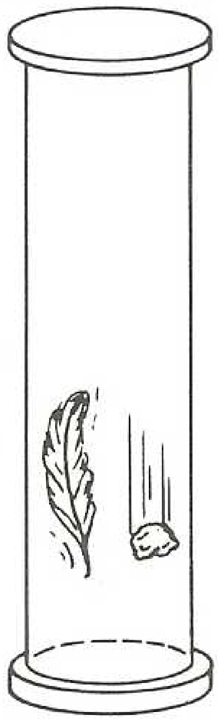


Demonstration

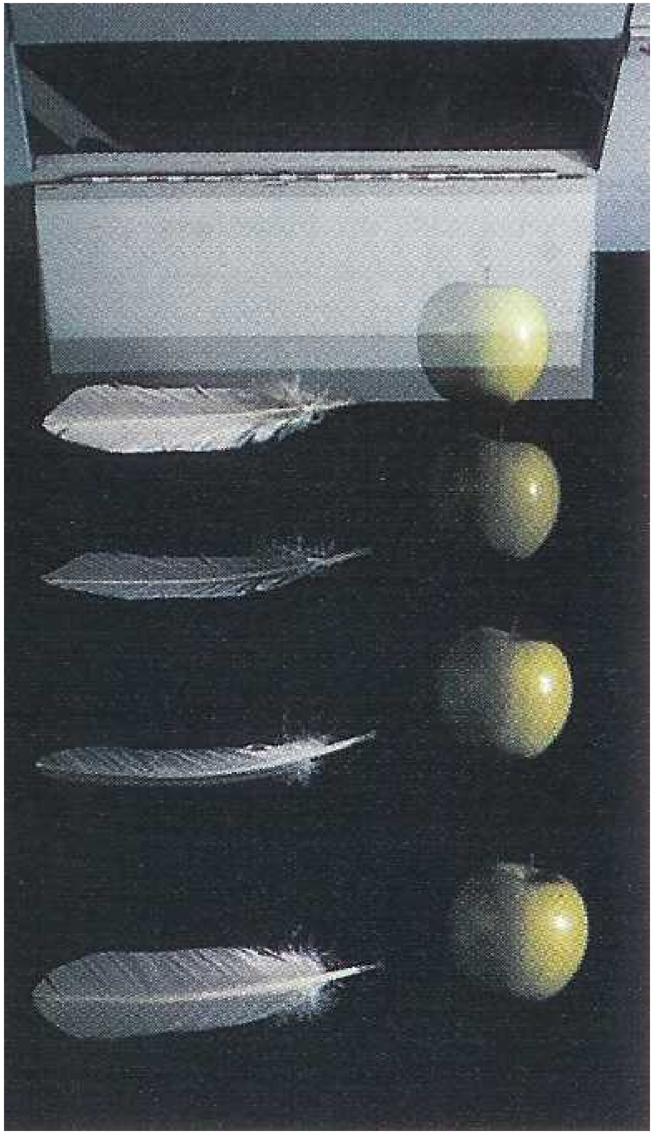
A coin (or a stone) and a disc of paper (or a feather) are dropped from a height in a cylinder filled with air. What will happen if the air is removed from the cylinder?



cylinder is filled with air



air is removed from the cylinder



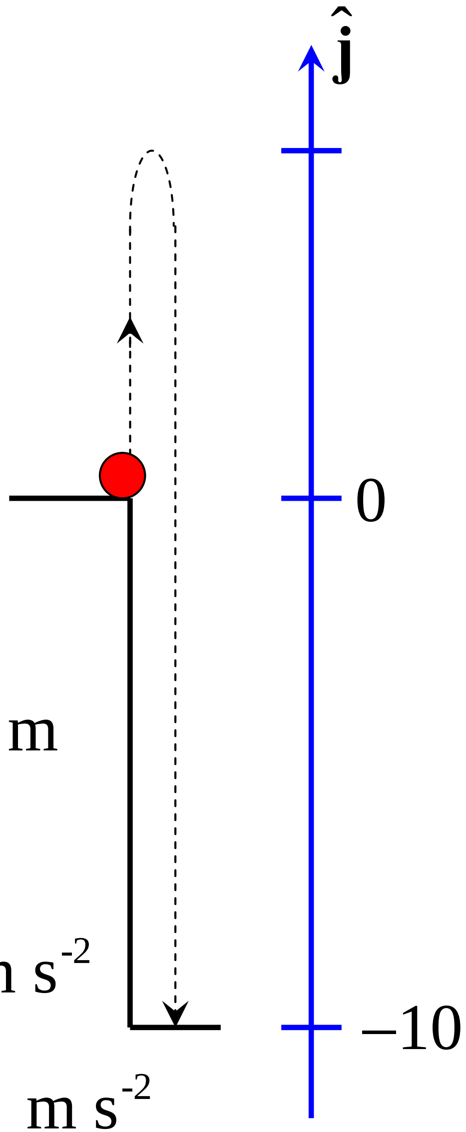
Figuring Physics

If you drop an object in the absence of air resistance, it accelerates downward at 9.8 m s^{-2}. If instead you throw it downward, then its downward acceleration after release is

(A) less than 9.8 m s^{-2}
(B) 9.8 m s^{-2}
(C) more than 9.8 m s^{-2}

Example

Bugs throws a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground. With reference to the coordinate axis given, answer the questions below:



Initial position of the ball = $0\hat{\mathbf{j}}$ m

Final position of the ball = $-10\hat{\mathbf{j}}$ m

Displacement of the ball when it reaches the ground = $-10\hat{\mathbf{j}}$ m

Initial velocity of the ball = $7\hat{\mathbf{j}}$ m s^{-1}

Acceleration of the ball while travelling upward = $-9.8\hat{\mathbf{j}}$ m s^{-2}

Acceleration of the ball while travelling downward = $-9.8\hat{\mathbf{j}}$ m s^{-2}

Acceleration of the ball at maximum height = $-9.8\hat{\mathbf{j}}$ m s^{-2}

FIGURING PHYSICS

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed.
Neglecting air resistance, the ball to hit the ground below the cliff with the greater speed is the one initially thrown

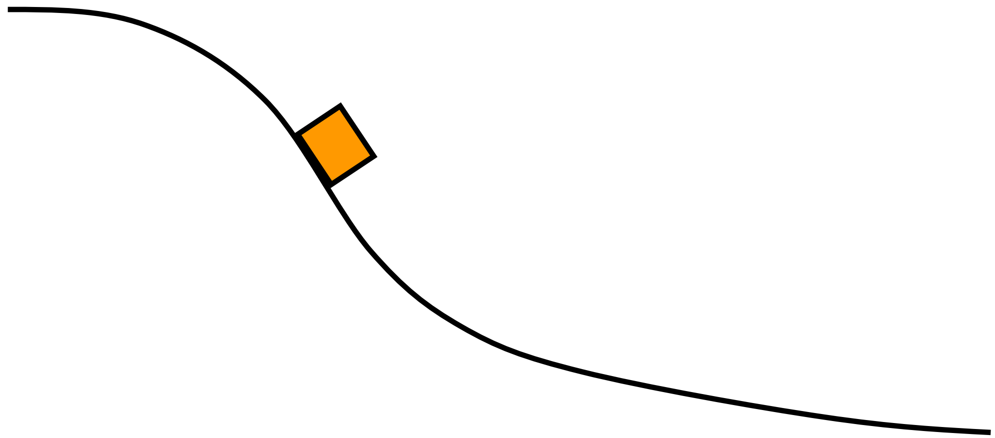
(A) upward
(B) downward
(C) neither - they both hit at the same speed.

Figuring Physics

You are throwing a ball straight up in the air.
At the highest point, the ball’s ...

(A) velocity and acceleration are zero
(B) velocity is non-zero but its acceleration is zero
(C) acceleration is non-zero but its velocity is zero

FIGURING PHYSICS



A cart on a roller-coaster slides down the track shown above. As the cart moves beyond the point shown, what happens to its speed and acceleration in the direction of motion?

(A) both decrease
(B) the speed increases, but acceleration decreases
(C) the speed decreases, but acceleration increases

Example

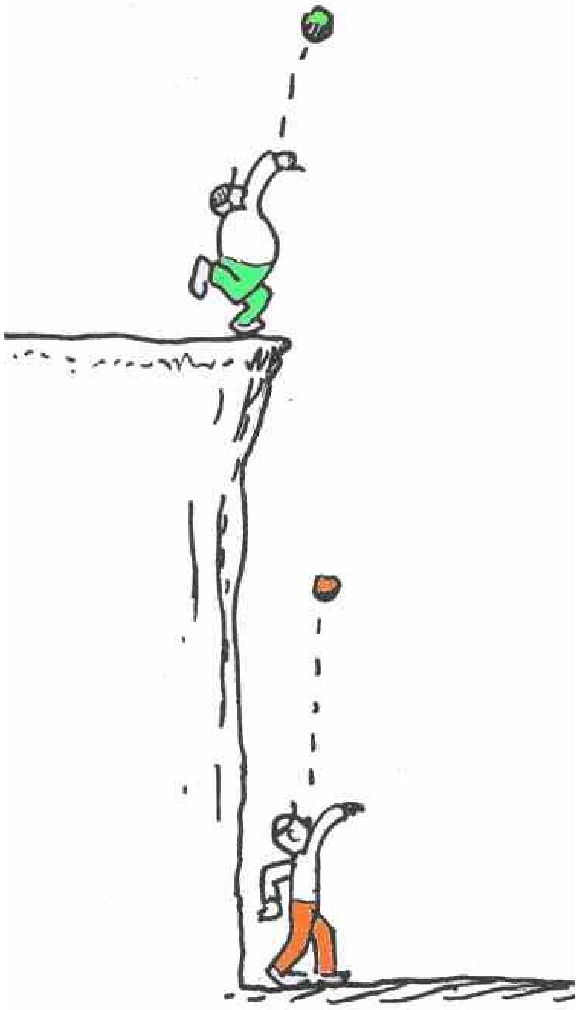
Bugs throws a ball vertically upward at 4 m s^{-1} from the edge of a cliff. It rises to its maximum height and then falls straight past him on the way down. How long does it take for the ball to reach a position 5 m below his feet?

Example

You and your brother are playing a game on the beach. He is standing on a ledge 15 m above you and throws a ball vertically upward with an initial speed of 30 m s^{-1}. The idea is for you to simultaneously throw a stone vertically upward so that it hits the ball at the apex of its flight? At what speed should you throw the stone?

FIGURING PHYSICS

Two balls are thrown straight up in the air at the same time but from different heights above the ground. They both hit the ground at the same time. Neglecting air resistance, how many times will they pass each other in flight not counting the time when they hit the ground together?

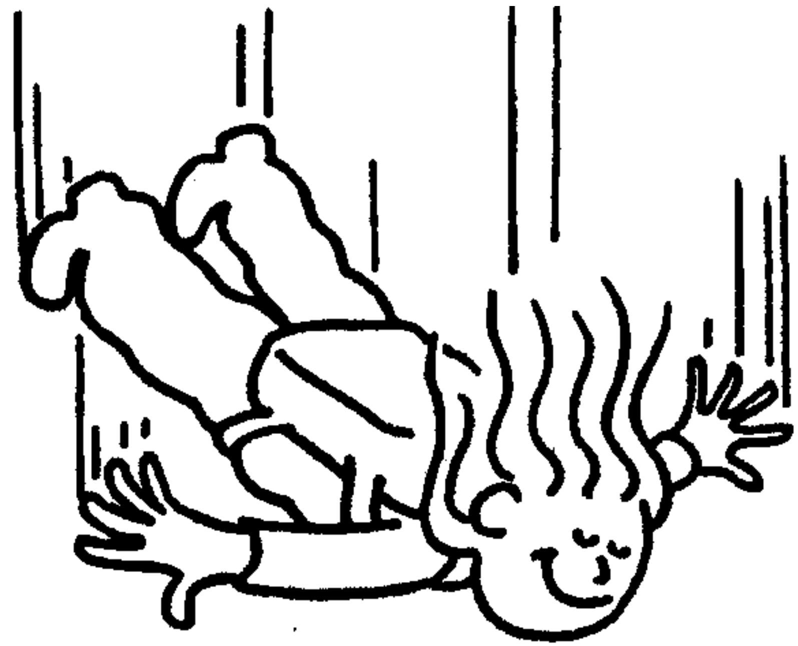


(A) Zero
(B) One
(C) Two

FIGURING PHYSICS

As she falls faster and faster <u>through air</u>, her acceleration:

(a) increases
(b) decreases
(c) remains the same

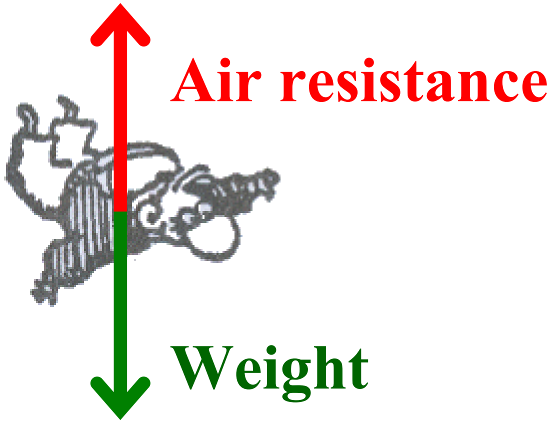


Air resistance and terminal velocity

When an object begins to fall through a fluid (such as a skydiver falling through the air), initially the only force acting on the object is gravity, and its acceleration is *g*.

As the object picks up speed, the frictional force ("air resistance" in air) increases, and the acceleration of the object decreases in magnitude.

Eventually the two forces will be balanced, and the object falls at a constant velocity, called the **terminal velocity**.

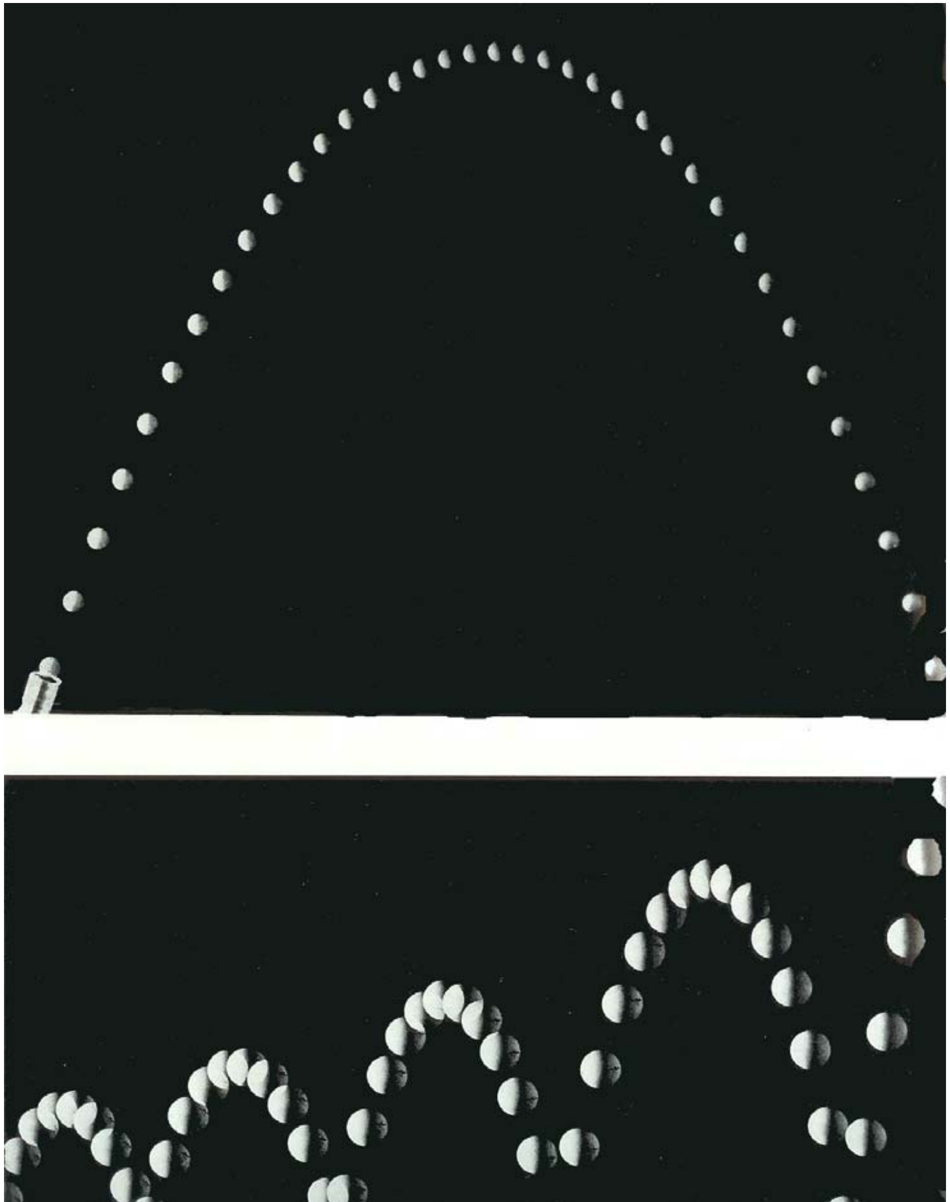


For a skydiver, terminal velocity is between 160 to 240 km h^{-1}.

Figuring Physics

When a tabletennis ball dropped from the top of a high building reaches terminal speed, its acceleration is zero. Now suppose that the same ball is projected upward with an initial speed greater than this terminal speed. At the instant its speed equals this terminal speed on the way up, the magnitude of its acceleration is

(A) zero
(B) *g*
(C) more than *g*



Projectiles

Kinematics in two dimensions: projectiles

We apply the same two equations of motion:

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_o + \vec{\mathbf{u}}t + \tfrac{1}{2}\vec{\mathbf{a}}t^2$$
$$\vec{\mathbf{v}}(t) = \vec{\mathbf{u}} + \vec{\mathbf{a}}t$$

by analyze the motion independently in the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ directions.

$\hat{\mathbf{i}}$ direction

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$
$$\vec{\mathbf{v}}_x(t) = \vec{\mathbf{u}}_x + \vec{\mathbf{a}}_x t$$

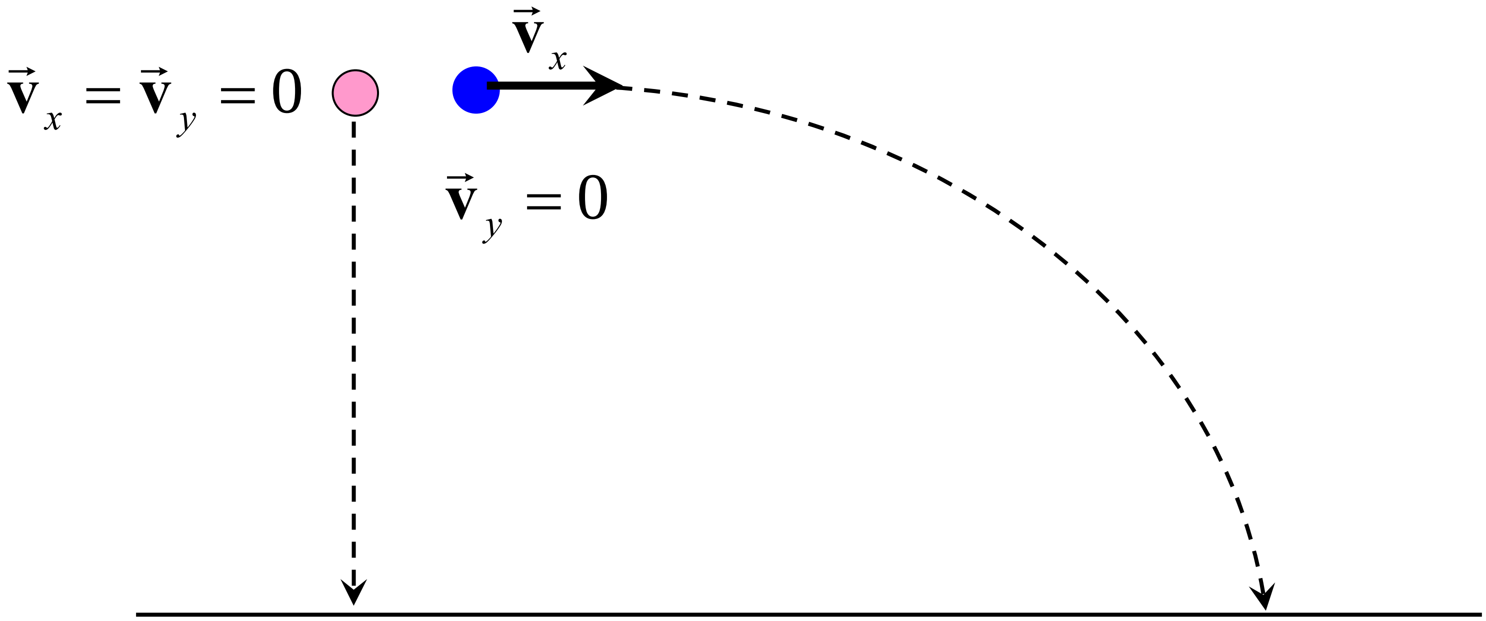
$\hat{\mathbf{j}}$ direction

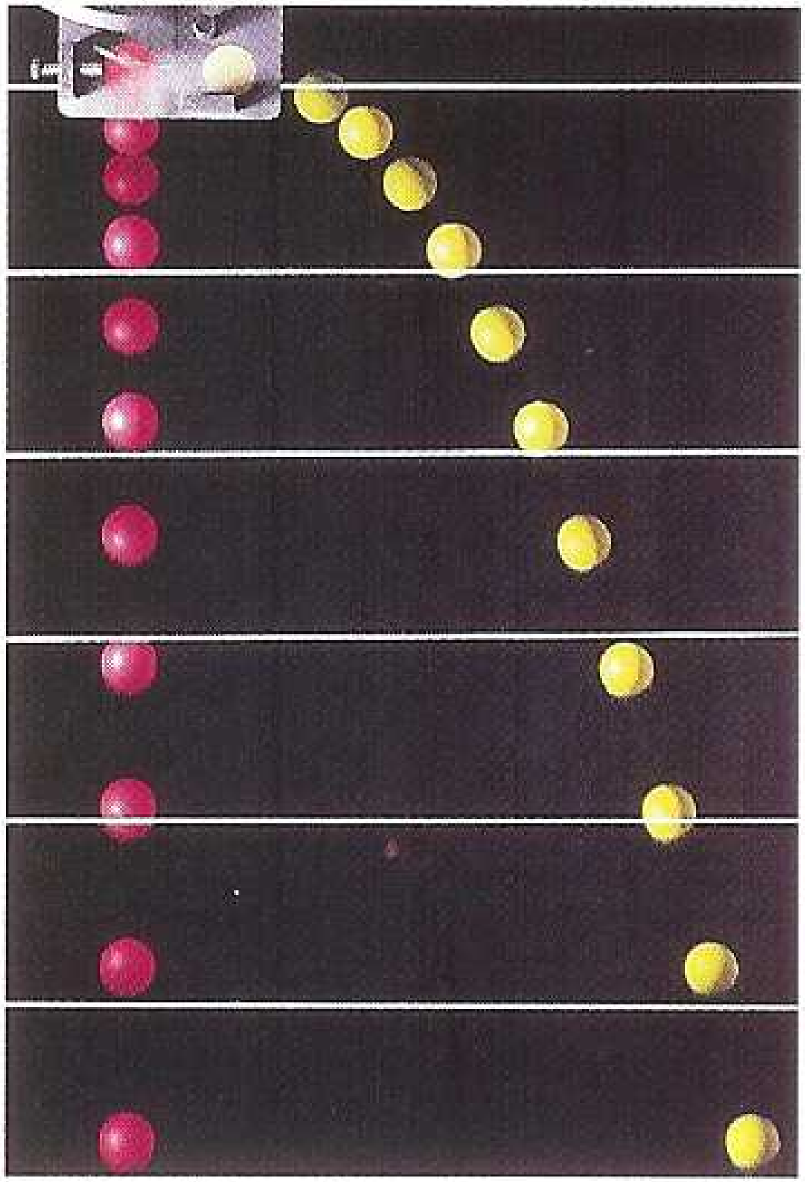
$$\vec{\mathbf{y}}(t) = \vec{\mathbf{y}}_o + \vec{\mathbf{u}}_y t + \tfrac{1}{2}\vec{\mathbf{a}}_y t^2$$
$$\vec{\mathbf{v}}_y(t) = \vec{\mathbf{u}}_y + \vec{\mathbf{a}}_y t$$

where $\vec{\mathbf{u}}_x = u\cos\theta\hat{\mathbf{i}}$ $\quad$ $\vec{\mathbf{u}}_y = u\sin\theta\hat{\mathbf{j}}$

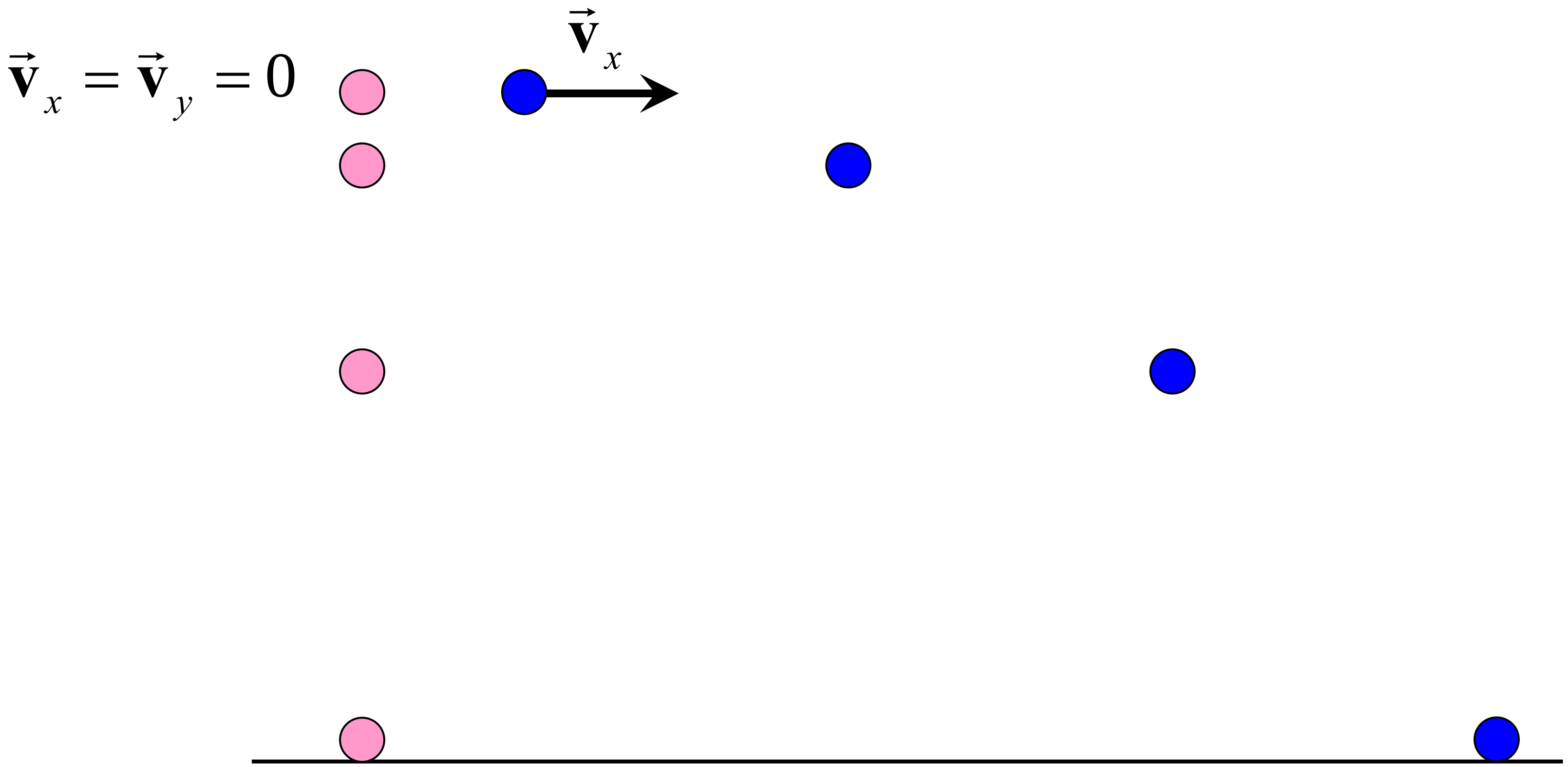
Demonstration

A ball in dropped from a height. At the same instant, a second ball is projected horizontally from the same height.
Which ball hits the ground first?

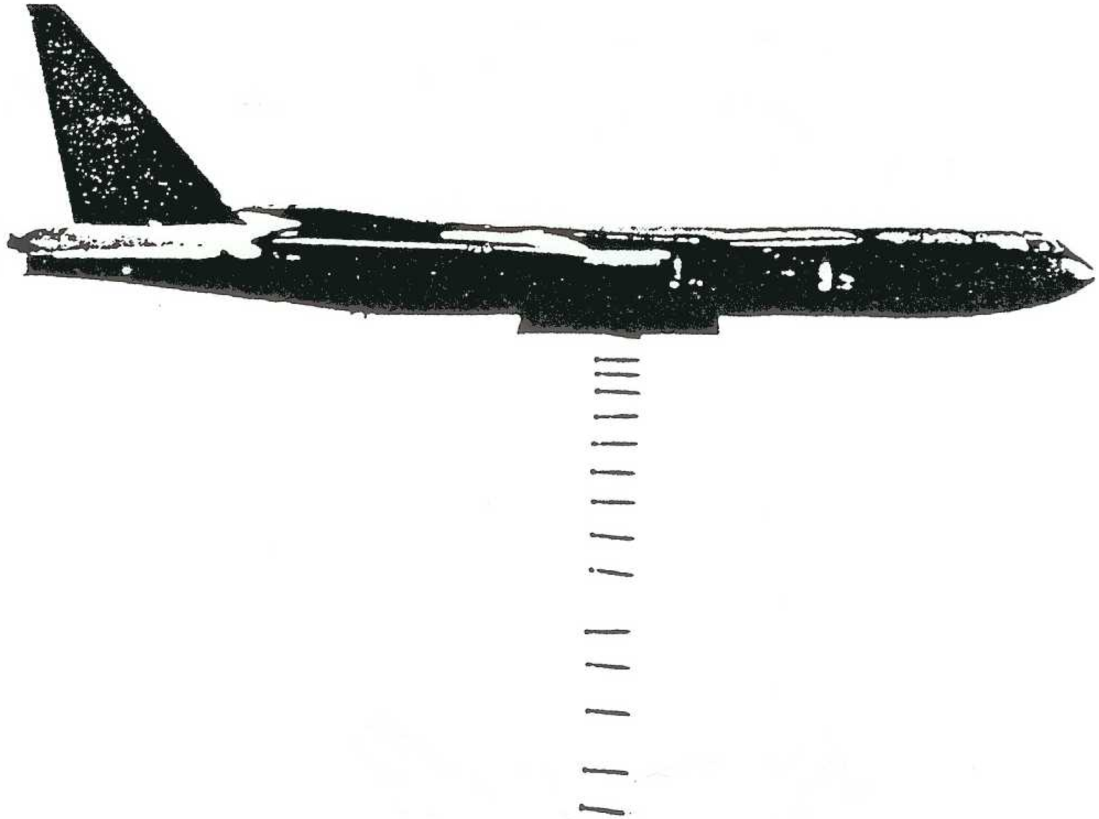




At each position shown, draw in the *x* and *y* components of the velocity vectors of both balls.



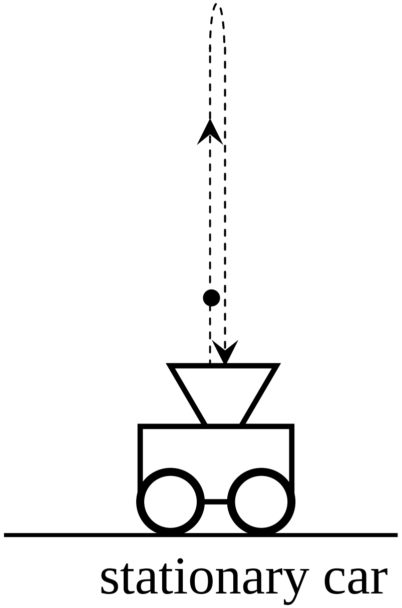
What is the magnitude and direction of the acceleration of each ball at each position shown?



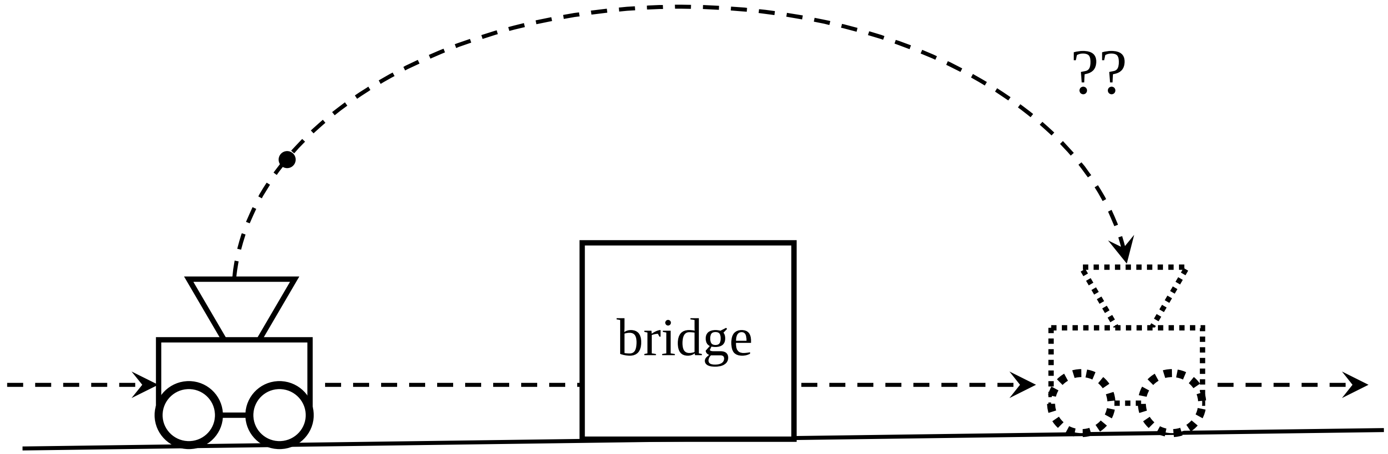
Demonstration

Rolling cart catches own ball

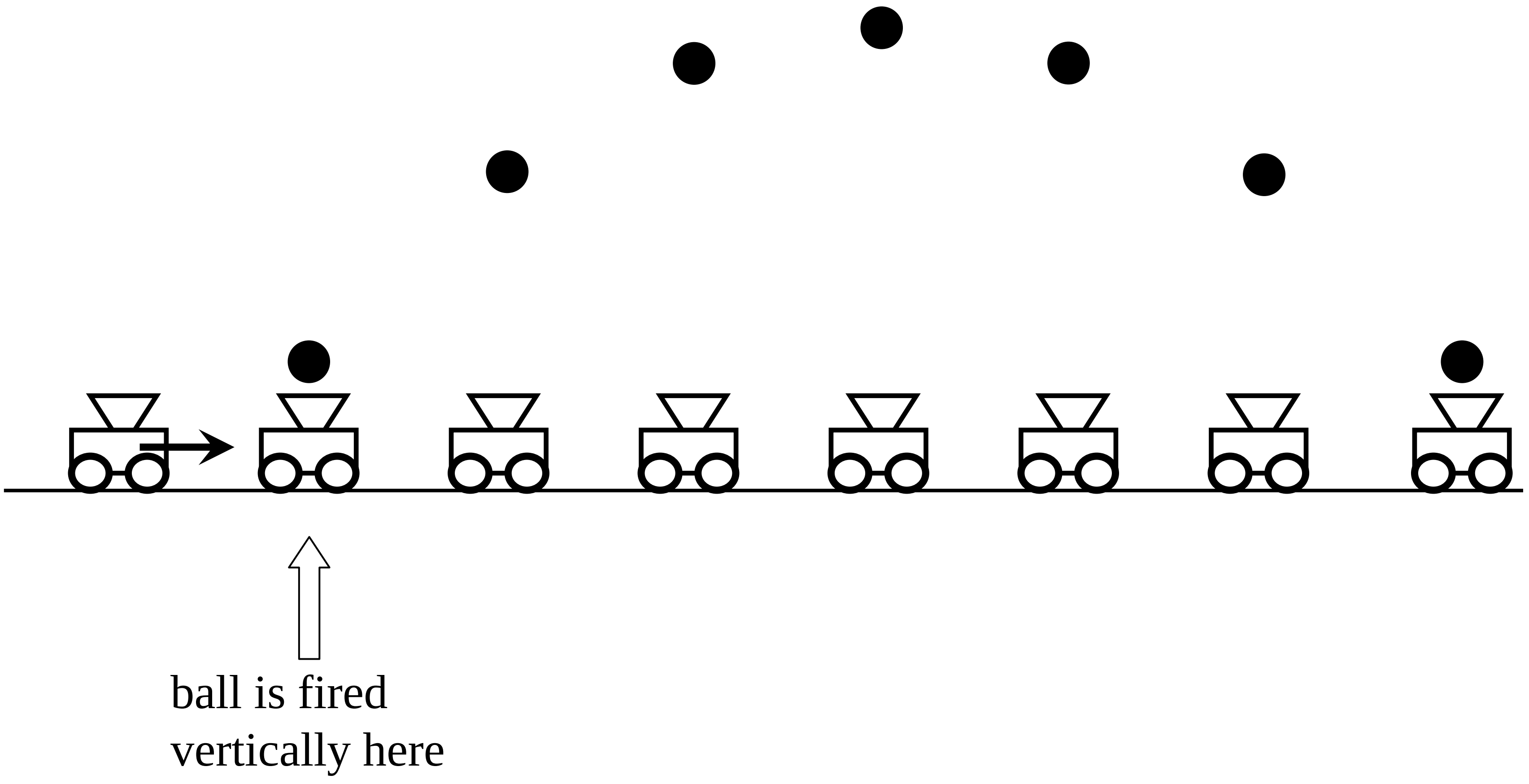
A special car is designed with low friction axles and has a spring-loaded mechanism which projects a small steel ball vertically upward.
A needle attached to a string is used to release the ball and an upward-facing funnel catches the ball.



What will happen if the ball is fired upward while the car is moving horizontally at a constant speed? Where will the ball land?

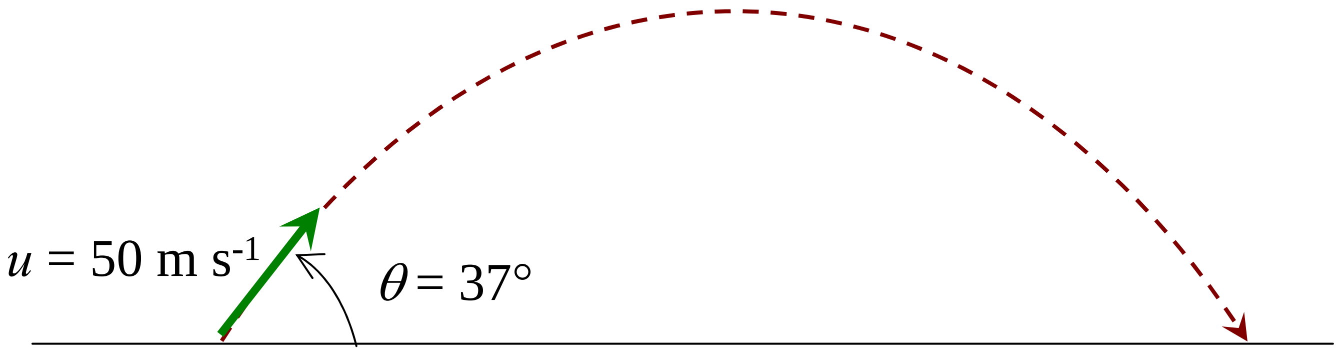


At each position shown, draw in the *x* and *y* components of the velocity vectors of the car and the ball.

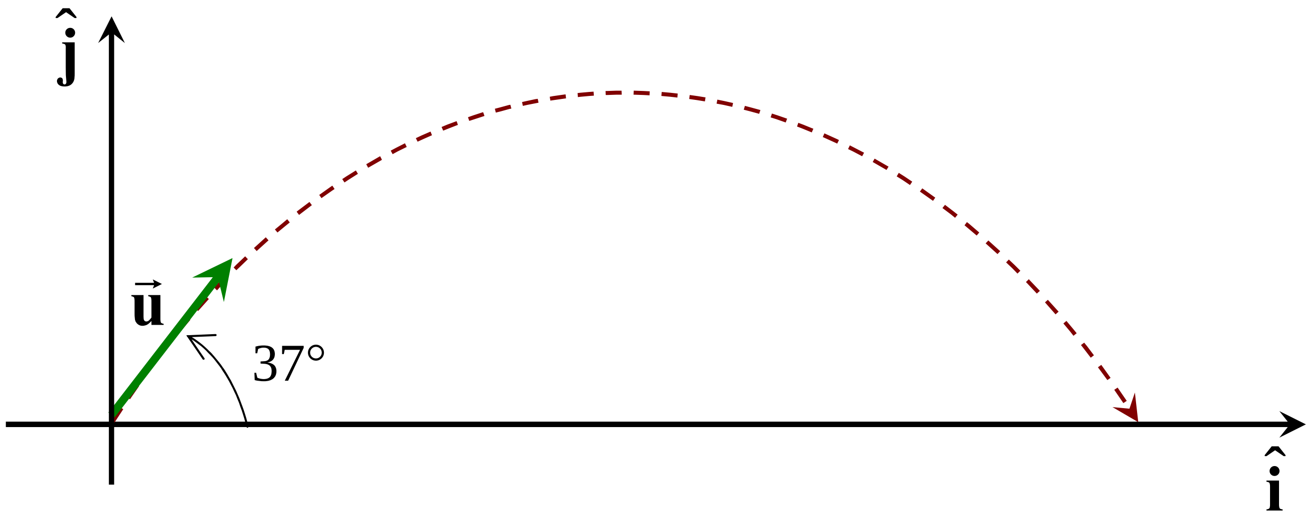


Example

A ball is projected from the origin with initial velocity having magnitude 50 m s^{-1} at an angle of 37° to the horizontal. We are interested in knowing the position and velocity of the ball as a function of time.



$$\vec{\mathbf{u}}_x = u\cos\theta\,\hat{\mathbf{i}} \qquad \vec{\mathbf{u}}_y = u\sin\theta\,\hat{\mathbf{j}}$$



We apply the same equations of motion

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_o + \vec{\mathbf{u}}t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 \qquad \vec{\mathbf{v}}(t) = \vec{\mathbf{u}} + \vec{\mathbf{a}}t$$

but analyze the motion independently in the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ directions.

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2 \qquad \vec{\mathbf{y}}(t) = \vec{\mathbf{y}}_o + \vec{\mathbf{u}}_y t + \tfrac{1}{2}\vec{\mathbf{a}}_y t^2$$

$$\vec{\mathbf{v}}_x(t) = \vec{\mathbf{u}}_x + \vec{\mathbf{a}}_x t \qquad \vec{\mathbf{v}}_y(t) = \vec{\mathbf{u}}_y + \vec{\mathbf{a}}_y t$$

where $\vec{\mathbf{u}}_x = |\vec{\mathbf{u}}|\cos\theta\hat{\mathbf{i}}$ $\qquad \vec{\mathbf{u}}_y = |\vec{\mathbf{u}}|\sin\theta\hat{\mathbf{j}}$

The $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ equations are linked by the time-of-flight t.

In this case: $\vec{\mathbf{u}}_x = |\vec{\mathbf{u}}|\cos\theta\hat{\mathbf{i}} = 50\cos 37°\hat{\mathbf{i}} = 40\hat{\mathbf{i}}$ m s^{-1}

$$\vec{\mathbf{u}}_y = |\vec{\mathbf{u}}|\sin\theta\hat{\mathbf{i}} = 50\sin 37°\hat{\mathbf{j}} = 30\hat{\mathbf{j}} \text{ m s}^{-1}$$

In addition: $\vec{\mathbf{a}}_x = 0$ $\quad\vec{\mathbf{a}}_y = \vec{\mathbf{g}} = -10\hat{\mathbf{j}}$ ms^{-2}

Note that we use $\vec{\mathbf{g}} = -10\hat{\mathbf{j}}$ m s^{-2} here,
but usually we will use $\vec{\mathbf{g}} = -9.8\hat{\mathbf{j}}$ ms^{-2}

Our equations of motion therefore become:

Position:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2 \qquad \vec{\mathbf{y}}(t) = \vec{\mathbf{y}}_o + \vec{\mathbf{u}}_y t + \tfrac{1}{2}\vec{\mathbf{a}}_y t^2$$

$$\vec{\mathbf{x}}(t) = (0) + (40\hat{\mathbf{i}})t + (0) \text{ m} \qquad \vec{\mathbf{y}}(t) = (0) + (30\hat{\mathbf{j}})t + \tfrac{1}{2}(-10\hat{\mathbf{j}})t^2 \text{ m}$$

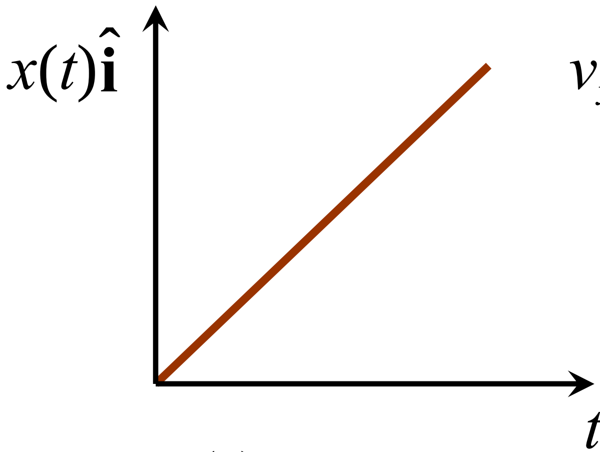
Velocity:

$$\vec{\mathbf{v}}_x(t) = \vec{\mathbf{u}}_x + \vec{\mathbf{a}}_x t \qquad \vec{\mathbf{v}}_y(t) = \vec{\mathbf{u}}_y + \vec{\mathbf{a}}_y t$$

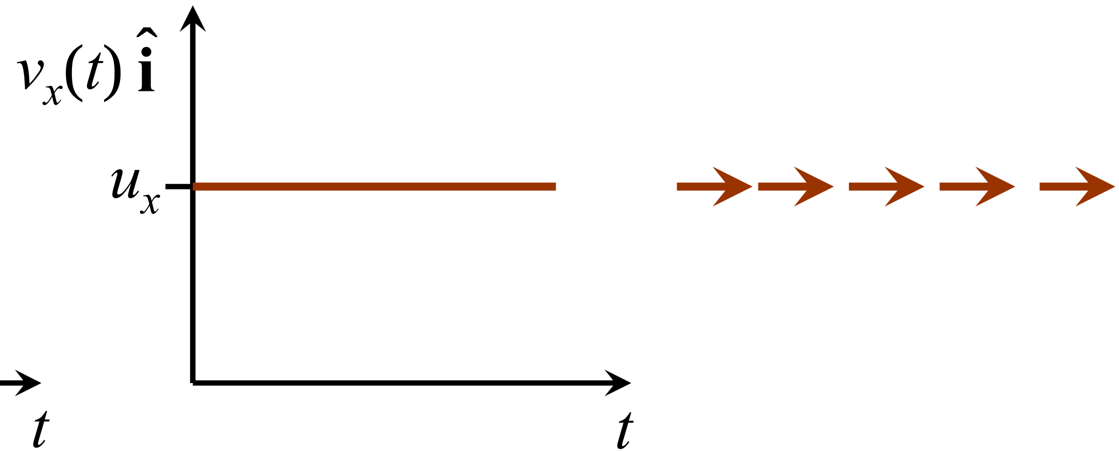
$$\vec{\mathbf{v}}_x(t) = 40\hat{\mathbf{i}} \text{ m s}^{-1} \qquad \vec{\mathbf{v}}_y(t) = (30\hat{\mathbf{j}}) + (-10\hat{\mathbf{j}})t \text{ m s}^{-1}$$

Complete the table below by calculating the components of the position and velocity vectors for this projectile.

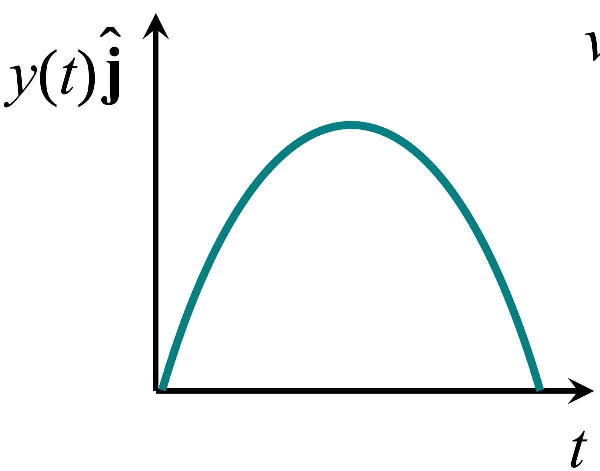
time (s)	x (m)	y (m)	v_x (m s^{-1})	v_y (m s^{-1})
0	0	0	40	30
1	40	25	40	20
2	80	40	40	10
3	120	45	40	0
4	160	40	40	–10
5	200	25	40	– 20
6	240	0	40	– 30



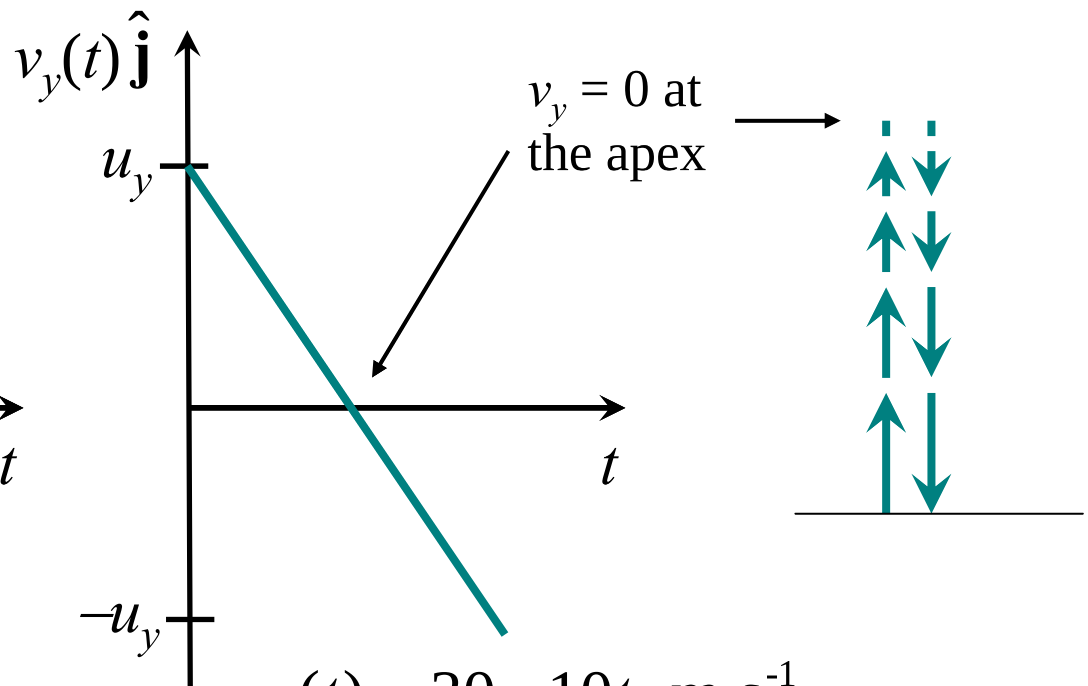
$$x(t) = 40t \text{ m}$$



$$v_x = 40 \text{ m s}^{-1}$$

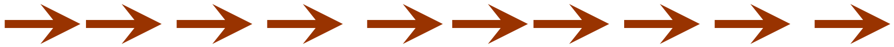


$$y(t) = 30t - 5t^2 \text{ m}$$

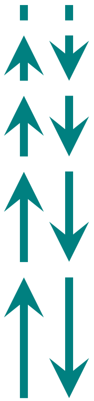


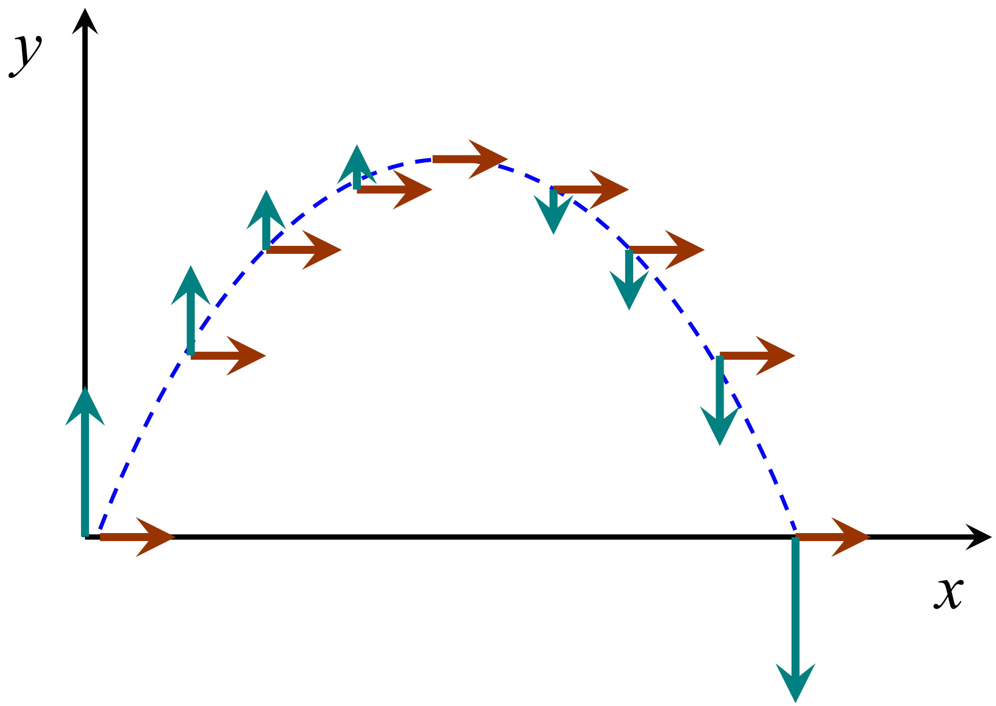
$$v_y(t) = 30 - 10t \text{ m s}^{-1}$$

We can think of the motion of a projectile as the combination of a vertical projectile and an object traveling horizontally at a constant velocity.



with

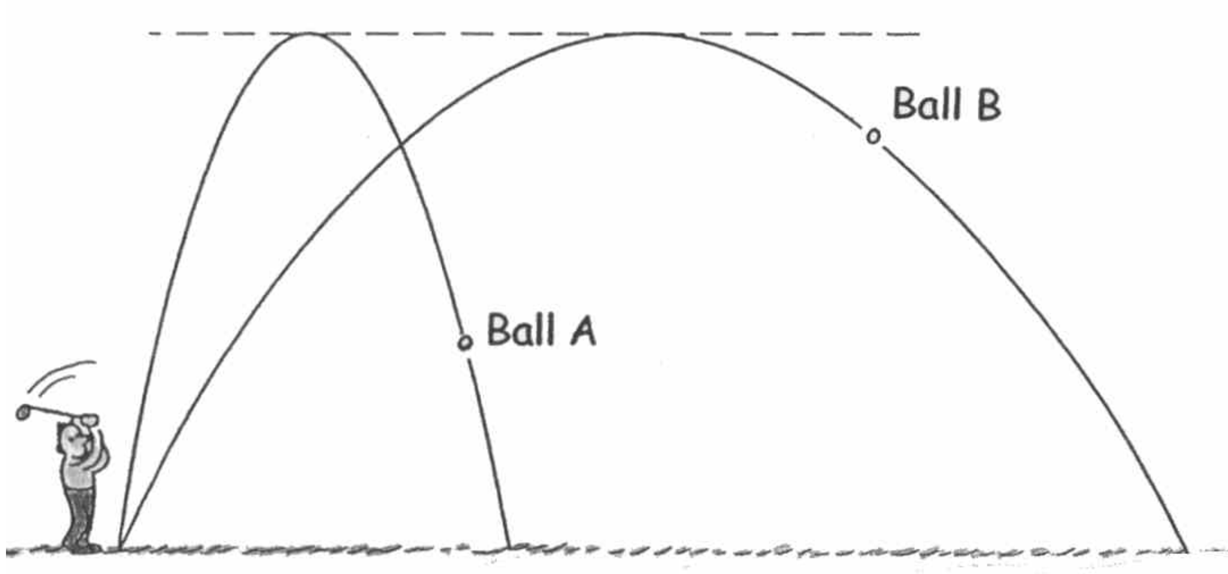




Figuring Physics

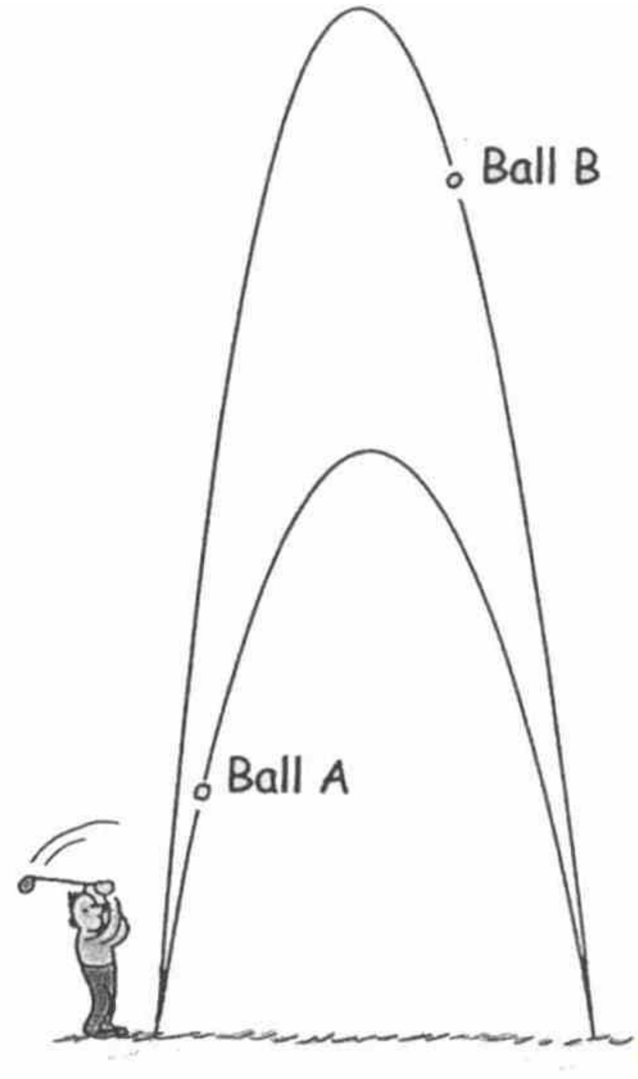
The figure shows the paths followed by two golf balls, A and B. Does Ball A spend more, the same or less time in the air than Ball B?

(A) more (B) the same (C) less



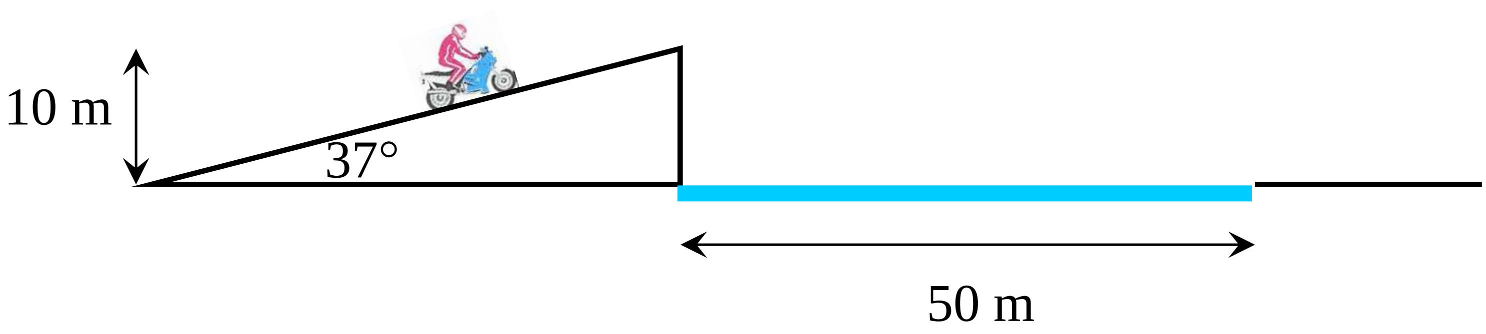
The figure shows the paths followed by two golf balls, A and B.
Does Ball A have a greater, the same or smaller launch speed than Ball B?

(A) greater
(B) the same
(C) smaller

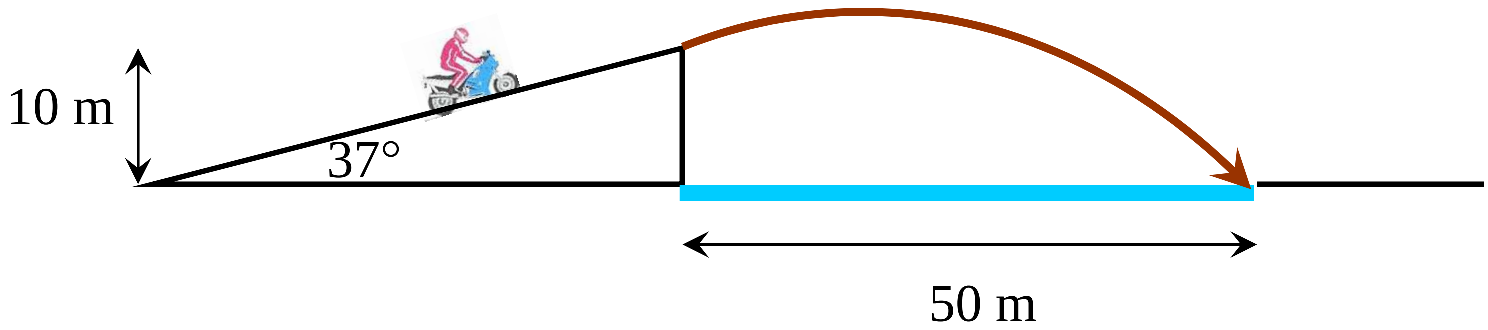


Example 1

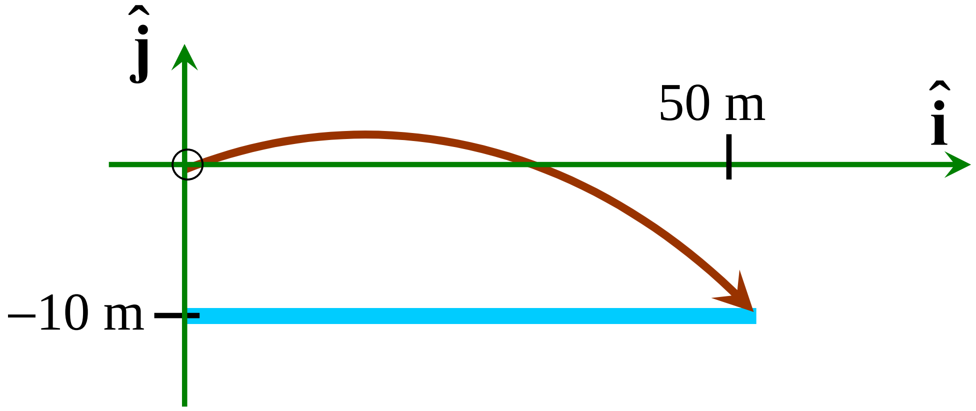
Determine the minimum speed that Bugs must have as he leaves the incline on his motorbike in order to just make it across the 50 metre wide swamp.



First draw the path of the projectile (in this case the motorbike) on your diagram.



Now draw a set of Cartesian coordinates. Usually it makes sense to set the origin at the initial position of the projectile. Mark the final position on the axes, using symbols for unknowns, if necessary.



Only now use your equations of motion

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_o + \vec{\mathbf{u}}_x t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$

$$(50\hat{\mathbf{i}}) = (0) + (u\cos 37°\hat{\mathbf{i}})t + (0)$$

or $$50 = 0.8\,ut \qquad \text{... 1}$$

$$\vec{\mathbf{y}}(t) = \vec{\mathbf{y}}_o + \vec{\mathbf{u}}_y t + \tfrac{1}{2}\vec{\mathbf{a}}_y t^2$$

$$(-10\hat{\mathbf{j}}) = (0) + (u\sin 37°\hat{\mathbf{j}})t + \tfrac{1}{2}(-10\hat{\mathbf{j}})t^2$$

or $$-10 = 0.6\,ut - 5t^2 \qquad \text{... 2}$$

Solving equations 1 and 2 for u (and t) gives

$$u = 20.2 \text{ m s}^{-1}$$

Does this answer makes sense?

FIGURING PHYSICS

A projectile is fired from a horizontal spring-loaded gun aimed directly (along the line of sight) at a distant target. The pull of gravity causes the projectile to fall in flight and hit a distance *y* beneath the bull's eye. To hit the bull's eye, the gun should be aimed along a line of sight above the bull's eye a vertical distance

(A) of *y*, exactly
(B) slightly lower than *y*
(C) slightly higher than *y*

Example 2

You are on the target range preparing to shoot a new rifle when it occurs to you that you would like to know how fast the bullet leaves the gun (the muzzle velocity). You bring the rifle up to shoulder level and aim it *horizontally* at the target centre. Carefully you squeeze off the shot at the target which is 100 m away. When you collect the target you find that your bullet hit 22 cm below where you aimed.

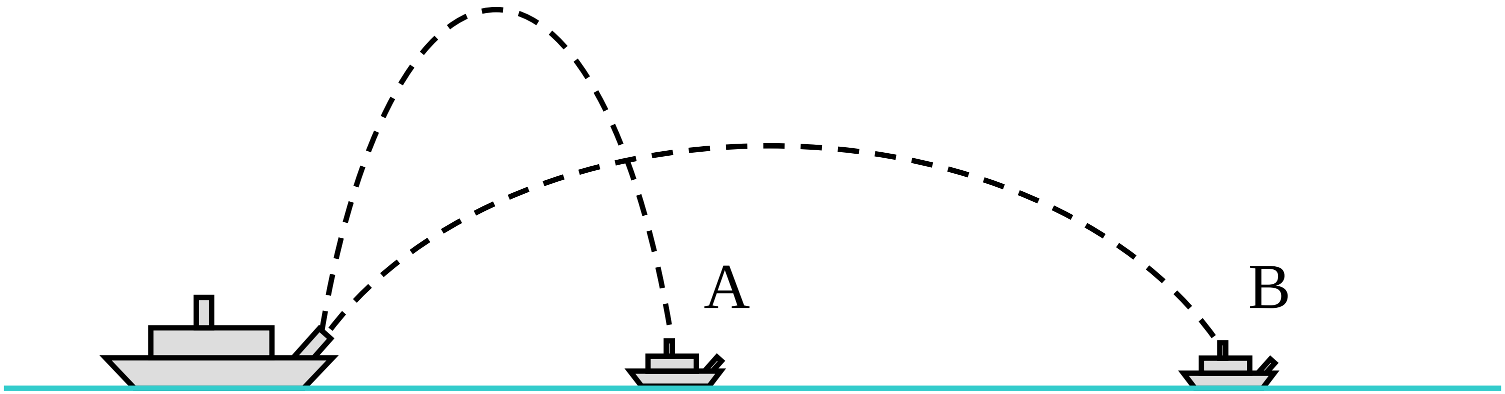
Example 3

You did so well at university you get a job in the airforce as a helicopter pilot. One of the maneuvers that you have to practice is to drop a package from a moving helicopter onto a moving truck on the ground. The difficulty is to know what speed to fly relative to the ground. You are flying horizontally at an altitude of 100 m and you know that when you drop the package, the truck will be 125 m ahead of you (measured along the road) and it will be traveling along the flat road at 60 km h^{-1}.

You estimate the height of the truck above the road to be 3 m.

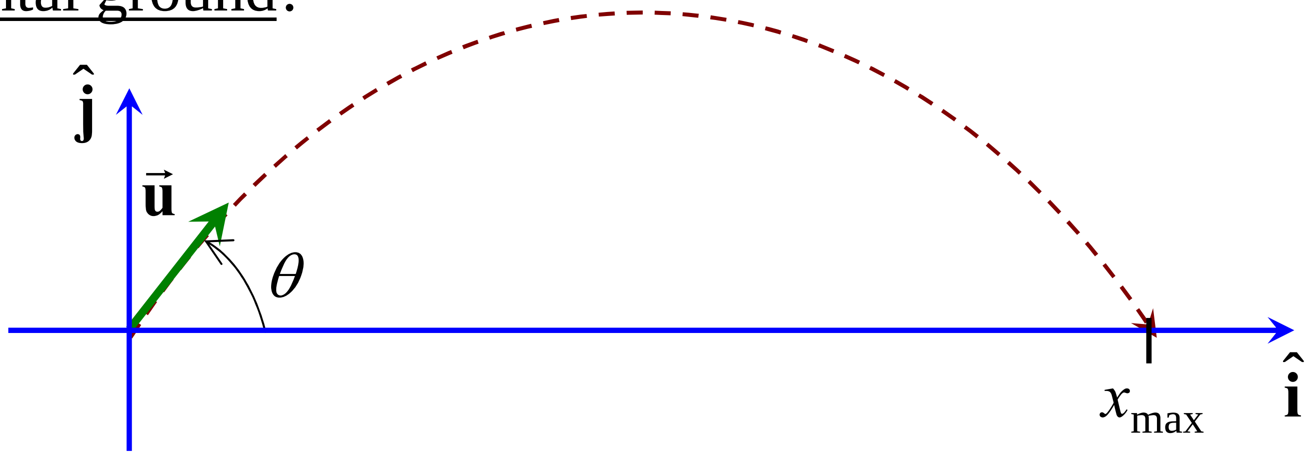
Figuring Physics

A battleship simultaneously fires two shells at the same initial speed at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



(A) A
(B) B
(C) both at the same time

Something to think about: For a given (fixed) launch velocity $\vec{\mathbf{u}}$, what launch angle θ of the projectile will given the largest range <u>on horizontal ground</u>?



For this simple case, our equations for the position are:

$$x_{\max} = \left(u\cos\theta\right)t \quad \text{and} \quad 0 = \left(u\sin\theta\right)t - \tfrac{1}{2}gt^2$$

Substituting the one into the other: $$x_{\max} = \frac{2u^2\cos\theta\sin\theta}{g}$$

Giving:

$$x_{\max} = \text{Range, } R = \frac{u^2\sin 2\theta}{g}$$ since $\sin 2\theta = 2\cos\theta\sin\theta$

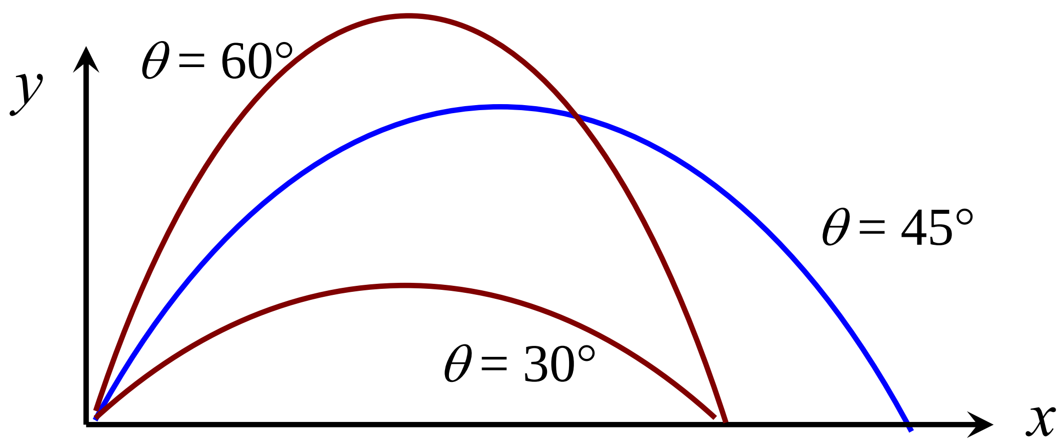
For a projectile on horizontal ground: Range, $R = \dfrac{u^2 \sin 2\theta}{g}$

It is clear that for a given u, and since g is constant, R is maximum when $\sin 2\theta$ is maximum (= 1) ,
i.e. when $2\theta = 90°$ or $\boldsymbol{\theta = 45°}$.

We can also see that (except for $\theta = 45°$), there are always two angles that give the same R.

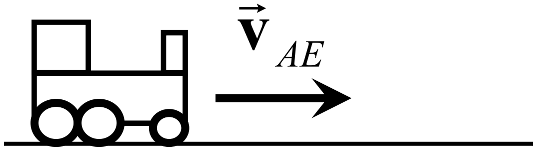
For example, $\theta = 30°$ and $\theta = 60°$ will give the same R, since
$\sin(2 \times 30°) = \sin(60°) = \sin(120°) = \sin(2 \times 60°)$

For the same launch speed u:



Relative velocities

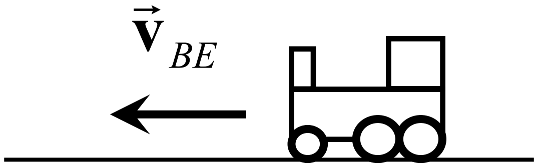
Say that you are travelling in train A moving at 20 m s^{-1} $\hat{\mathbf{i}}$



Then we can write $\vec{\mathbf{v}}_{AE} = 20\hat{\mathbf{i}}\ \text{m s}^{-1}$

(velocity of train A relative to the earth E)

A second train (B) moving at 20 m s^{-1} $(-\hat{\mathbf{i}})$ (towards you).



Then $\vec{\mathbf{v}}_{BE} = 20(-\hat{\mathbf{i}})\ \text{m s}^{-1}$

(velocity of train B relative to the earth E)

At what velocity does train B appear to be approaching you (in train A)?

In other words, what is $\vec{\mathbf{v}}_{BA}$?

(the velocity of B relative to A, or as "seen" by A, or in the reference frame of A)

Vector equation: $\vec{\mathbf{v}}_{BA} = \vec{\mathbf{v}}_{BE} + \vec{\mathbf{v}}_{EA}$

$$= \vec{\mathbf{v}}_{BE} - \vec{\mathbf{v}}_{AE}$$

$$= 20(-\hat{\mathbf{i}}) - 20(\hat{\mathbf{i}}) \text{ m s}^{-1}$$

$$= -40\hat{\mathbf{i}} \text{ m s}^{-1}$$

The **velocity of train B in the reference frame of train A** is $-40\hat{\mathbf{i}}$ m s^{-1}.

It is as if we (in train A) are at rest and we attribute all the velocity to train B.

In general:

$$\vec{\mathbf{v}}_{AX} = \vec{\mathbf{v}}_{AB} + \vec{\mathbf{v}}_{BC} + \vec{\mathbf{v}}_{CD} + \vec{\mathbf{v}}_{DE} + \ldots + \vec{\mathbf{v}}_{VW} + \vec{\mathbf{v}}_{WX}$$

note

Another example

Bugs walks with a velocity of $-12\hat{\mathbf{i}}$ m s^{-1}. At the same time Magobbi is walking nearby with a velocity of $-8\vec{\mathbf{j}}$ m s^{-1} . What is the velocity of Bugs in the reference frame of Magobbi?

$$\vec{\mathbf{v}}_{BM} = \vec{\mathbf{v}}_{BE} + \vec{\mathbf{v}}_{EM}$$

$$= \vec{\mathbf{v}}_{BE} - \vec{\mathbf{v}}_{ME}$$

$$= \quad -12\hat{\mathbf{i}} - (-8\hat{\mathbf{j}}) \text{ m s}^{-1}$$

$$\therefore \vec{\mathbf{v}}_{BM} = \quad -12\hat{\mathbf{i}} + 8\hat{\mathbf{j}} \text{ m s}^{-1}$$

... and another example

Bugs walks with a velocity of $2\vec{\mathbf{j}}$ m s^{-1} . The velocity of Bugs relative to Magobbi is 3 m s^{-1} at 30 ° to the $\hat{\mathbf{i}}$ -axis.
What is the velocity of Magobbi relative to the Earth?

$$\vec{\mathbf{v}}_{BM} = \vec{\mathbf{v}}_{BE} + \vec{\mathbf{v}}_{EM}$$

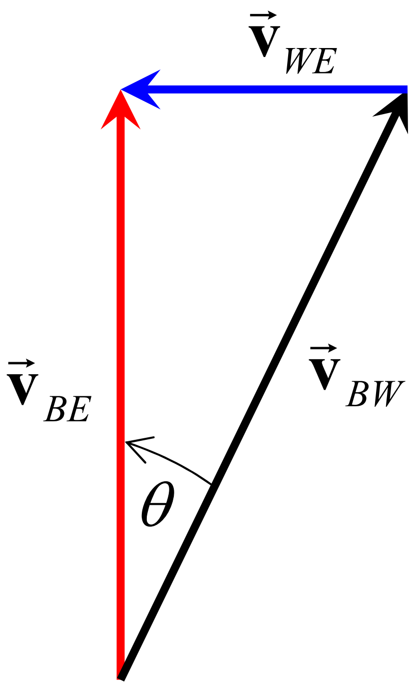
$$\vec{\mathbf{v}}_{BM} = \vec{\mathbf{v}}_{BE} - \vec{\mathbf{v}}_{ME}$$

$$\therefore \vec{\mathbf{v}}_{ME} = \vec{\mathbf{v}}_{BE} - \vec{\mathbf{v}}_{BM}$$

Relative velocities continued ...

Suppose we have a boat that can travel at a maximum speed of 40 km h^{-1} in still water. However, the river is flowing at 10 km h^{-1} parallel to the river bank.
In what direction should we head in order to travel directly across the river?
At what velocity will we be travelling relative to someone standing on the shore?

Use a velocity vector diagram:



$\vec{\mathbf{v}}_{WE}$ velocity of the water relative to the earth (the river current)

$\left|\vec{\mathbf{v}}_{WE}\right| = 10 \text{ km h}^{-1}$

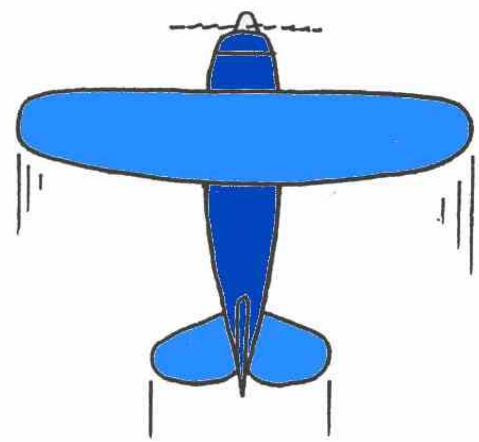
$\vec{\mathbf{v}}_{BW}$ velocity of the boat relative to the water

$\left|\vec{\mathbf{v}}_{BW}\right| = 40 \text{ km h}^{-1}$

$\vec{\mathbf{v}}_{BE}$ velocity of the boat relative to the shore (the resultant)

$$\vec{\mathbf{v}}_{BE} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WE}$$

The resultant is what an observer on the river bank sees the boat doing.



The speed of an airplane relative to the ground is affected by wind. When an airplane flies in the direction of a wind (tailwind), then it has a greater groundspeed. When an airplane flies directly into the wind (headwind), then it has a smaller groundspeed. Suppose an airplane flies with a 90-degree crosswind (the nose pointing in a direction perpendicular to the wind direction). Will its groundspeed be more, less, or the same as in still air?

(A) more

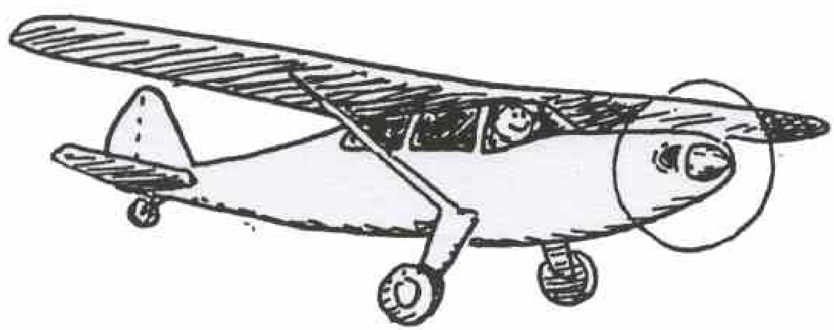
(B) less

(c) the same

Figuring Physics

An airplane makes a straight back-and-forth round trip, always at the same airspeed, between two cities. If it encounters a mild steady tailwind going, and the same steady headwind returning, will the round trip take more, less, or the same time as with no wind?

(A) more time
(B) less time
(C) the same time



Relative motion ... a harder problem

An aeroplane A can travel at a maximum speed of 500 km h^{-1} in still air. It is required to intercept plane B which is initially 60 km NE of A and flying west at 250 km h^{-1}. There is no wind. In what direction must A fly in order to intercept B?
How long will it take before they meet?

$\vec{\mathbf{v}}_{AE}$ velocity of A relative to the earth $\quad |\vec{\mathbf{v}}_{AE}| = 500 \text{ km h}^{-1}$

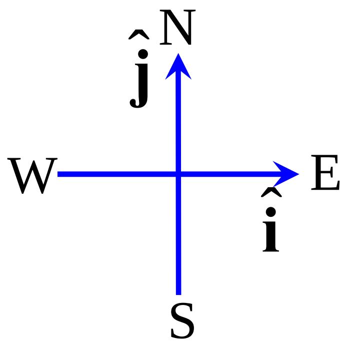
$\vec{\mathbf{v}}_{BE}$ velocity of B relative to the earth $\quad \vec{\mathbf{v}}_{BE} = 250(-\hat{\mathbf{i}}) \text{ km h}^{-1}$

$\vec{\mathbf{v}}_{AB}$ velocity of A relative to B

The direction of $\vec{\mathbf{v}}_{AB}$ is NE

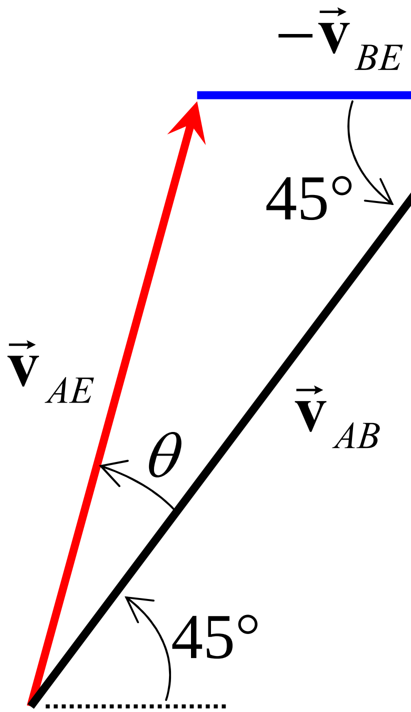
$$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} + \vec{\mathbf{v}}_{EB}$$

or $$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} - \vec{\mathbf{v}}_{BE}$$



This effectively brings B to rest.

$$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} + \left(-\vec{\mathbf{v}}_{BE}\right)$$



Get angle θ using the sine rule:

$$\frac{\sin\theta}{250} = \frac{\sin 45°}{500}$$

Giving $\theta = 20.7°$

Therefore plane A must fly in direction
45° + 20.7° = 65.7 °

Get $\left|\vec{\mathbf{v}}_{AB}\right|$ using the cosine rule:

$$\left|\vec{\mathbf{v}}_{AB}\right|^2 = \left|\vec{\mathbf{v}}_{AE}\right|^2 + \left|\vec{\mathbf{v}}_{BE}\right|^2 - \left|\vec{\mathbf{v}}_{AE}\right|\left|\vec{\mathbf{v}}_{BE}\right|\cos(180° - 65.7°)$$

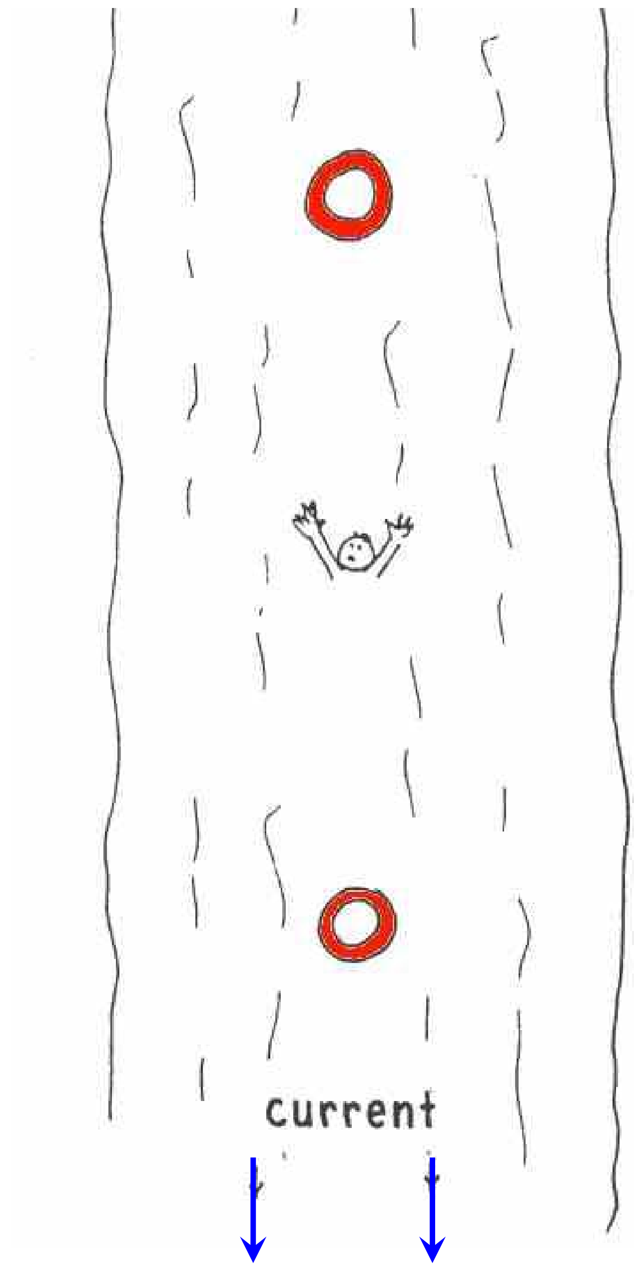
$$\therefore \left|\vec{\mathbf{v}}_{AB}\right| = \underline{644.5 \text{ km h}^{-1}}$$

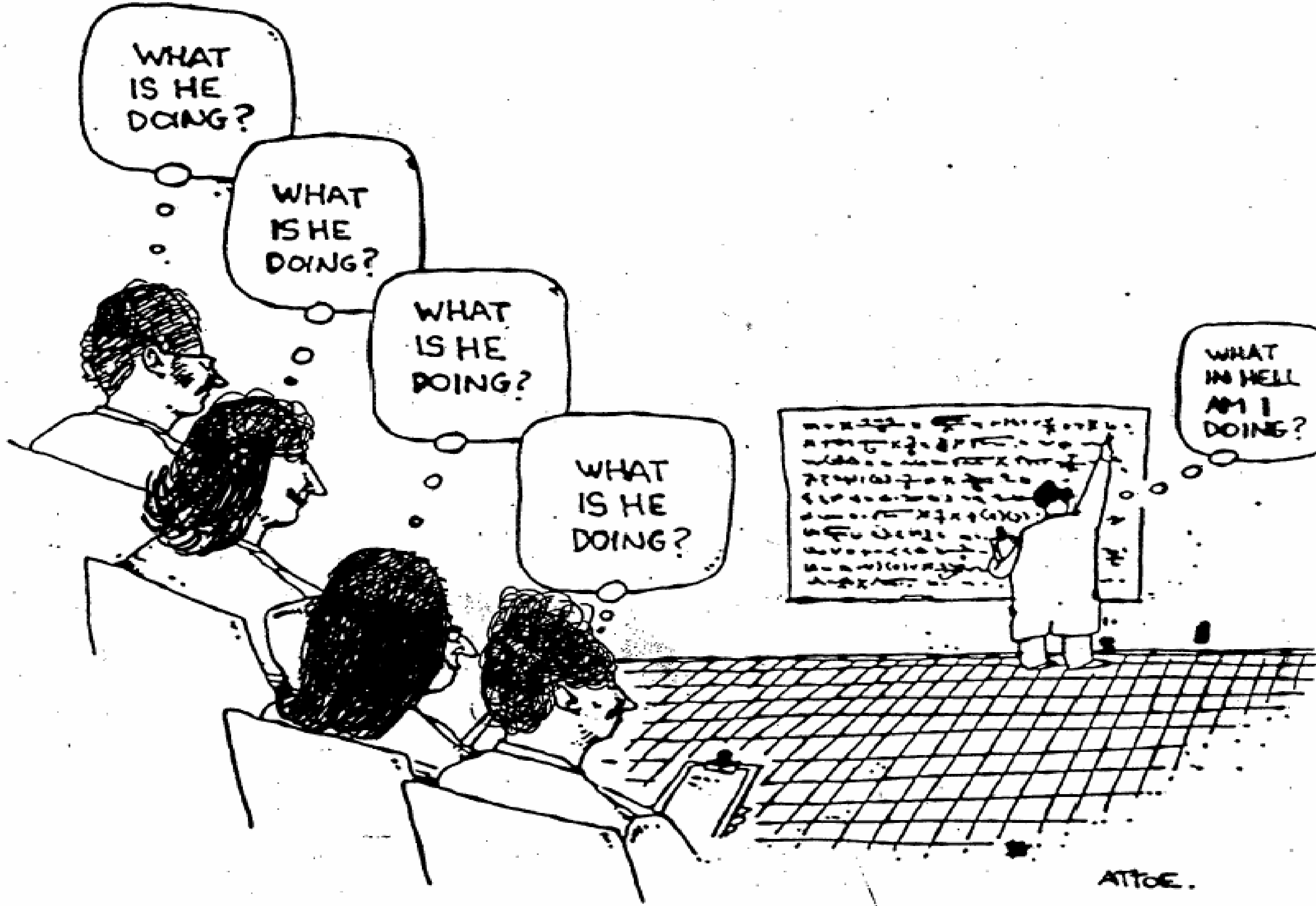
Time taken for plane A to get to plane B:

$$t = \frac{d}{\left|\vec{\mathbf{v}}_{AB}\right|} = \frac{60 \text{ km}}{644.5 \text{ km h}^{-1}} = 0.093 \text{ hours} = \underline{5.6 \text{ minutes}}$$

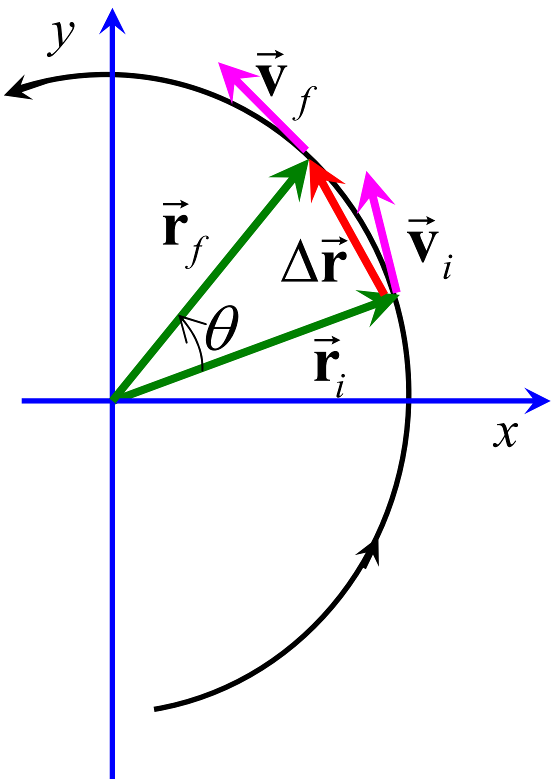
Suppose that you and a pair of life preservers are floating down a fast flowing river, as shown.
You wish to get to either of the life preservers for safety. One is 3 metres downstream from you and the other is 3 metres upstream from you.
Which can you swim to in the shortest time?

(A) the preserver upstream
(B) the preserver downstream
(C) both require the same time.




WHAT IS HE DOING?
WHAT IS HE DOING?
WHAT IS HE DOING?
WHAT IS HE DOING?
WHAT IN HELL AM I DOING?
ATTOE.

Uniform circular motion



Consider a particle travelling along a circular path at a constant speed v, where $\vec{\mathbf{v}}$ is the linear (or tangential) velocity.
We consider two positions as shown ("initial" and "final").

Then $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$

and $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$

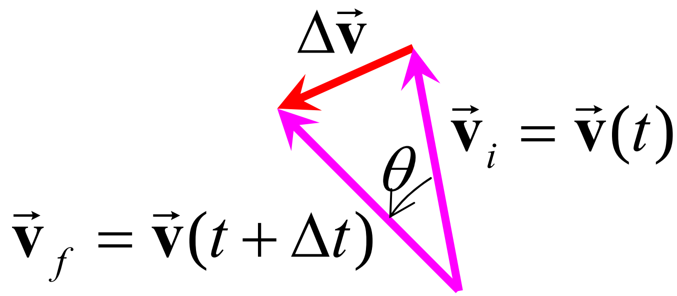
Since both $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ are both time dependent, we can write:

$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}(t+\Delta t) - \vec{\mathbf{v}}(t)$$

and $$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}(t+\Delta t) - \vec{\mathbf{r}}(t)$$

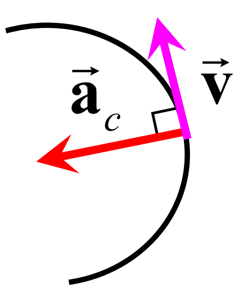
Even though $|\vec{\mathbf{v}}|$ is constant, there is an acceleration acting since the direction of $\vec{\mathbf{v}}$ is different at each $\vec{\mathbf{r}}$.
What is the magnitude and direction of this acceleration?

What is the direction of the acceleration of the particle?



As $\Delta t \to 0$, $\theta \to 0$ and the direction of $\Delta\vec{\mathbf{v}}$ points towards the centre of the circle (at right angles to $\vec{\mathbf{v}}$ itself).

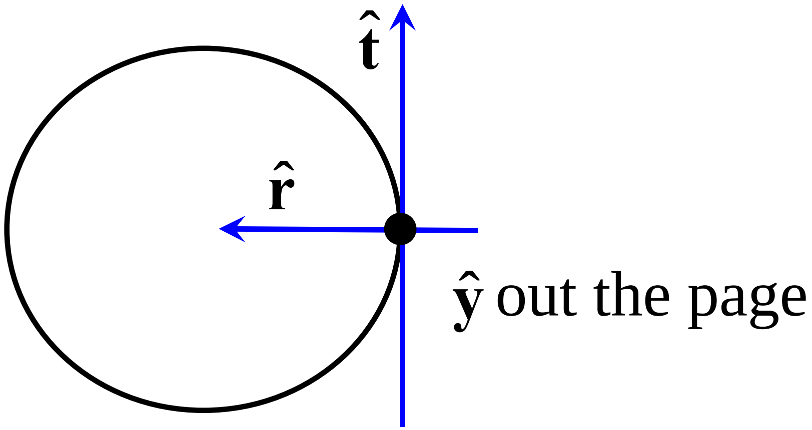
We call this acceleration the **centripetal acceleration** $\vec{\mathbf{a}}_c$.

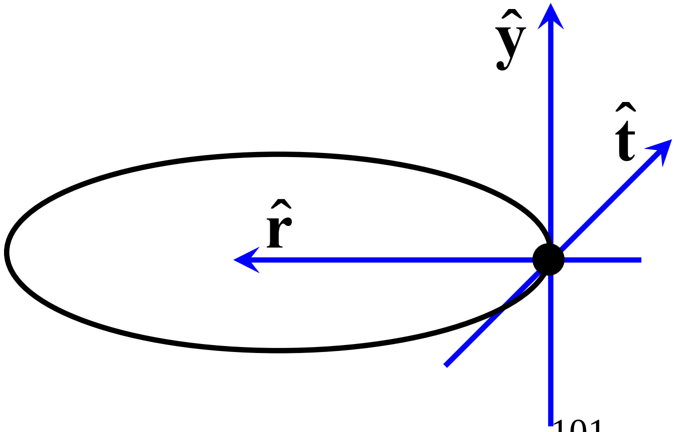


Whenever a body is moving in a circle, there must be a centripetal acceleration present (and hence a centripetal force - see later.)

It is useful to define an alternative coordinate system when describing rotation. Instead of defining a set of Cartesian coordinates $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ with the origin at the axis of the rotation, it is sometimes useful to define a set of Cartesian axes with the origin at the position of the rotating object, where:

$\hat{\mathbf{r}}$ radial direction
$\hat{\mathbf{t}}$ tangential direction
$\hat{\mathbf{y}}$ vertical direction



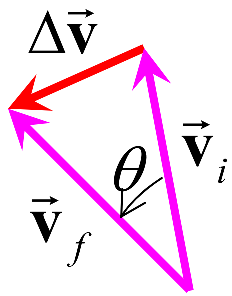


... centripetal acceleration continued ...

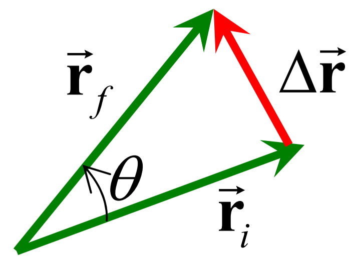
The centripetal acceleration is then $\vec{\mathbf{a}}_c = a_c \hat{\mathbf{r}}$

What about the magnitude of $\vec{\mathbf{a}}_c$?

We have two similar triangles:



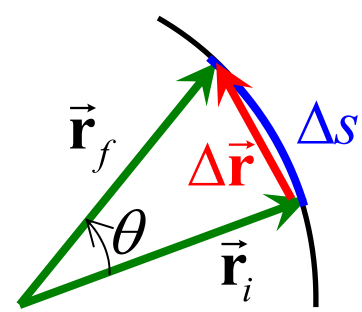
and



$\Delta\vec{\mathbf{r}}_f\vec{\mathbf{r}}_i\Delta\vec{\mathbf{r}} \parallel \Delta\vec{\mathbf{v}}_f\vec{\mathbf{v}}_i\Delta\vec{\mathbf{v}}$

$$\therefore \frac{\Delta r}{r} = \frac{\Delta v}{v}$$ since $\left|\vec{\mathbf{v}}_f\right| = \left|\vec{\mathbf{v}}_i\right| = v$ and $\left|\vec{\mathbf{r}}_f\right| = \left|\vec{\mathbf{r}}_i\right| = r$

For small θ: $\Delta r = \Delta s$ the path length



therefore $$\frac{\Delta v}{v} \approx \frac{\Delta s}{r}$$ or $$\Delta v \approx \frac{v}{r}\Delta s$$

... centripetal acceleration continued ...

$$\Delta v \approx \frac{v}{r}\Delta s$$

As $\quad \Delta t \to 0 \quad : \quad \Delta v \approx \dfrac{v}{r}\Delta s$

So $\quad \displaystyle \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \left(\frac{v}{r}\right)\frac{\Delta s}{\Delta t}$

$$a_c = \frac{v}{r} \underbrace{\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}}_{v}$$

$$\therefore a_c = \frac{v^2}{r} \qquad \text{or} \qquad \vec{\mathbf{a}}_c = \frac{v^2}{r}\hat{\mathbf{r}}$$

Whenever you observe a body moving in a circle or other curved path, there must be a centripetal force acting (which is provided by some **centripetal force** ... see later).

Non-uniform circular motion

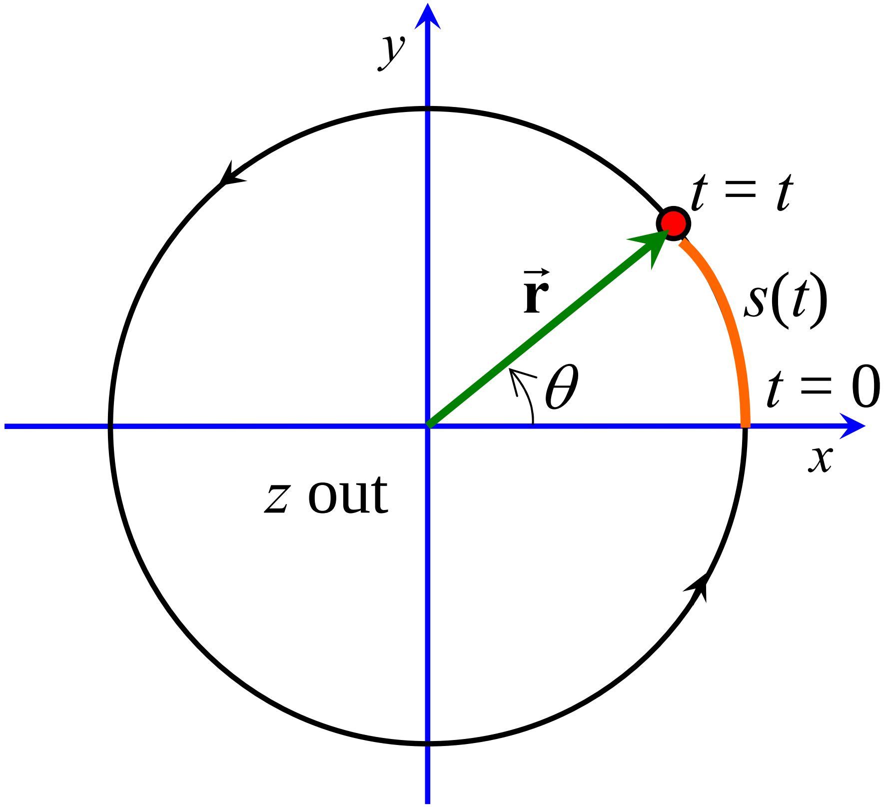
Apart from the centripetal acceleration $\vec{\mathbf{a}}_c$ which is always acting when a body is moving in a circle, there may also be a tangential acceleration $\vec{\mathbf{a}}_t$ acting, if v is no constant (i.e. the body is speeding up or slowing down as it travels on its circular path.)

where $\vec{\mathbf{a}}_t(t) = \dfrac{d\vec{\mathbf{v}}(t)}{dt}$

The total acceleration acting on the body is $\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_c$

Note that at any position on the circle, $\vec{\mathbf{a}}_c$ is always perpendicular to $\vec{\mathbf{a}}_t$.

Circular motion in terms of angular variables



Consider a particle of mass m rotating in the x-y plane about the z-axis.

We can, of course, use (x, y) to specify the position of the particle, but sometimes it's more convenient to use (r,θ).

Remember ? $\theta = \dfrac{s}{r}$ radians

Then similarly, as for translational kinematics for 1D linear motion, we can define:

average angular speed

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \qquad \text{rad s}^{-1}$$

instantaneous angular speed

$$\omega(t) = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta(t)}{dt} \qquad \text{rad s}^{-1}$$

average angular acceleration (magnitude)

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \qquad \text{rad s}^{-2}$$

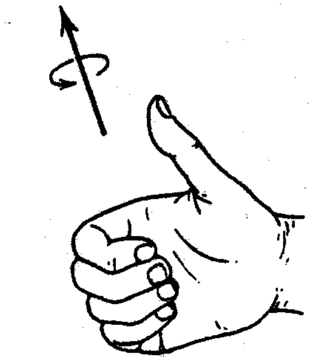
instantaneous angular acceleration (magnitude)

$$\alpha(t) = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega(t)}{dt} \qquad \text{rad s}^{-2}$$

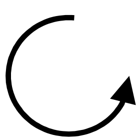
What about the direction of these angular variables?

It is possible to use a vector to describe a rotational variable, if we follow the following convention:

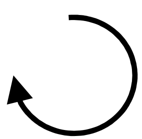
Using your **right hand** with your fingers curling in the sense of the rotation:



Directions of $\vec{\boldsymbol{\omega}}$:



anticlockwise rotation



clockwise rotation

$\vec{\boldsymbol{\omega}}$ out of page

$\vec{\boldsymbol{\omega}}$ into page

At any instant the object has a linear (tangential) velocity $\vec{\mathbf{v}}_t(t)$ and a linear (tangential) acceleration $\vec{\mathbf{a}}_t(t)$

where $\vec{\mathbf{v}}_\text{t}(t) = \dfrac{d\vec{\mathbf{r}}(t)}{dt}$ and $\vec{\mathbf{a}}_\text{t}(t) = \dfrac{d\vec{\mathbf{v}}_\text{t}(t)}{dt}$

However $\vec{\mathbf{v}}_t = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}} = \omega r \sin\theta \ \hat{\mathbf{t}}$

(For circular motion $v_t = wr$)

and $\vec{\mathbf{a}}_t = \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}} = \alpha r \sin\theta \ \hat{\mathbf{t}}$

(For circular motion $a_t = \alpha r$)

Then $\vec{\mathbf{a}}_c = \dfrac{{v_t}^2}{r} = \dfrac{\omega^2 r^2}{r} = \omega^2 r$ in $\hat{\mathbf{r}}$ direction

So we can write for circular motion :

$$\vec{\mathbf{v}}_t(t) \;=\; \omega(t)\, r(t) \quad \text{m s}^{-1} \quad (\hat{\mathbf{t}})$$

$$\vec{\mathbf{a}}_t(t) \;=\; \alpha(t)\, r(t) \quad \text{m s}^{-2} \quad (\hat{\mathbf{t}})$$

$$\vec{\mathbf{a}}_c(t) \;=\; \omega(t)^2\, r(t) \quad \text{m s}^{-2} \quad (\hat{\mathbf{r}})$$

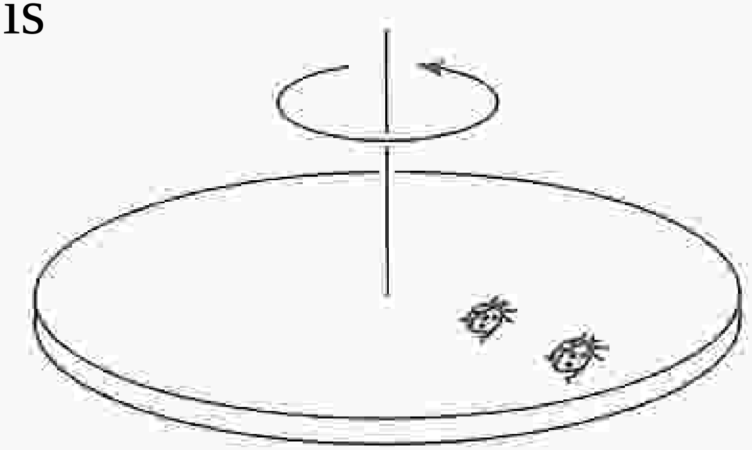
We can use the same kinematic equations what we used for the linear motion in the case of circular motion

linear motion	circular motion
$r(t) = r_o + ut + \frac{1}{2}at^2$	$\theta(t) = \theta_o + \omega_0 t + \frac{1}{2}\alpha t^2$
$v(t) = u + at$	$\omega(t) = \omega_0 + \alpha t$

Figuring Physics

A ladybug sits at the outer edge of a merry-go-round, and a gentleman bug sits halfway between her and the axis of rotation. The merry-go-round makes a complete revolution once every second.

The gentleman bug's angular speed is



(A) half the ladybug's
(B) the same as the ladybug's
(C) twice the ladybug's

Figuring Physics

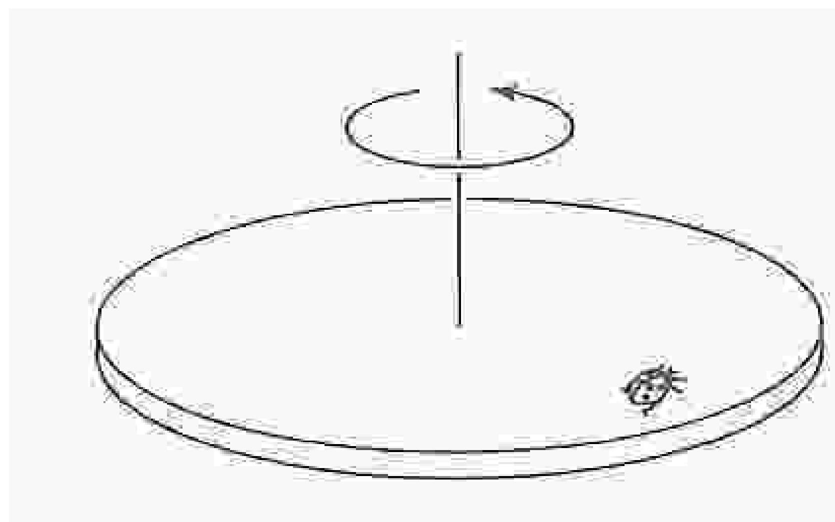
A ladybug sits at the outer edge of a merry-go-round that is turning and slowing down.

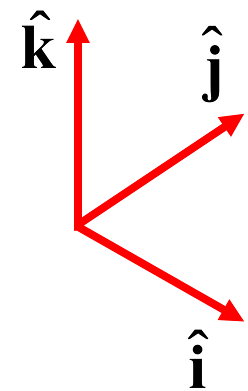
The vector expressing her **angular velocity** is in the

(A) $\hat{\mathbf{k}}$ direction

(B) $-\hat{\mathbf{k}}$ direction

(C) $-\hat{\mathbf{j}}$ direction





FIGURING PHYSICS

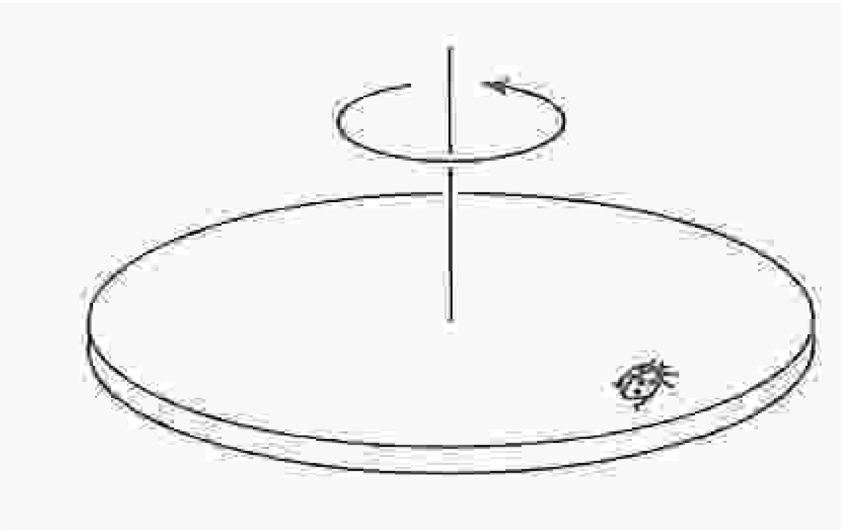
A ladybug sits at the outer edge of a merry-go-round that is turning and slowing down.

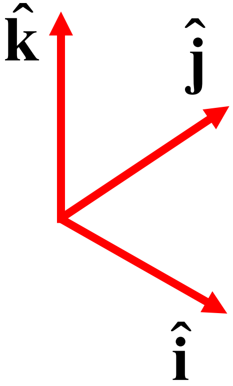
The vector expressing her **angular acceleration** is in the

(A) $\hat{\mathbf{k}}$ direction

(B) $-\hat{\mathbf{k}}$ direction

(C) $-\hat{\mathbf{j}}$ direction





Example 1

A 5 kg mass is moving in a circle of radius 2 m.
At t = 0 it is at $\theta = 45^o$ and at t = 6 s, it is at $\theta = 105^o$.

(a) If the tangential acceleration of the mass is 12 m s^{-2}, what was its initial angular velocity (i.e. at t = 0) ?

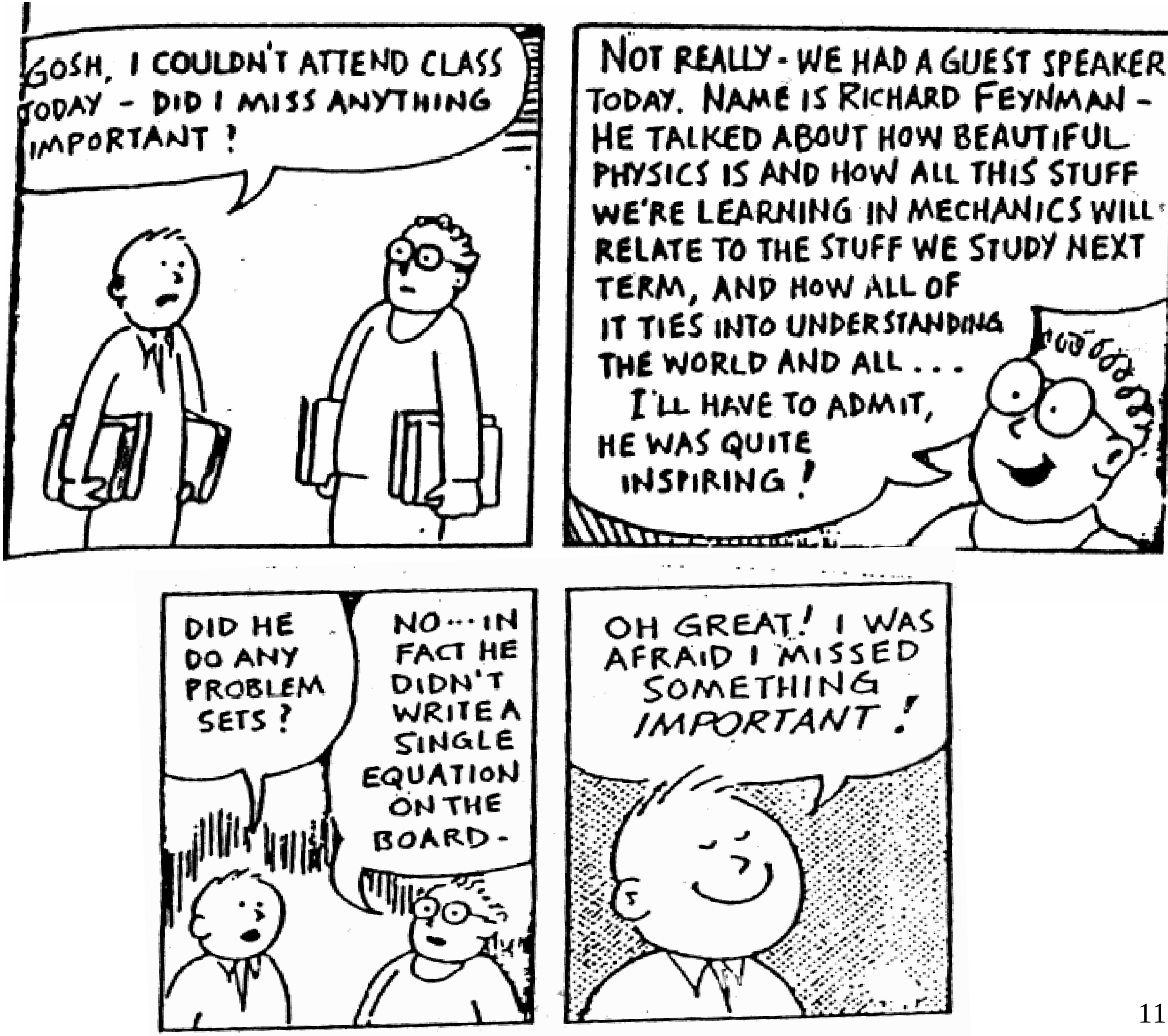
(b) What is the tangential velocity of the mass at $\theta = 105^o$?

Example 2

If the angular position of a body is given by

$$\theta(t) \;=\; (3\,t^2 \;+\; 5\,t)\,\hat{\mathbf{i}} \;+\; 5\,t^3\,\hat{\mathbf{j}} \;-\; 5\,t\,\hat{\mathbf{k}} \quad \text{radians,}$$

what is the angular acceleration of the body at $t = 2$ seconds?


GOSH, I COULDN'T ATTEND CLASS TODAY - DID I MISS ANYTHING IMPORTANT ?
NOT REALLY - WE HAD A GUEST SPEAKER TODAY. NAME IS RICHARD FEYNMAN - HE TALKED ABOUT HOW BEAUTIFUL PHYSICS IS AND HOW ALL THIS STUFF WE'RE LEARNING IN MECHANICS WILL RELATE TO THE STUFF WE STUDY NEXT TERM, AND HOW ALL OF IT TIES INTO UNDERSTANDING THE WORLD AND ALL . . .
I'LL HAVE TO ADMIT, HE WAS QUITE INSPIRING !
DID HE DO ANY PROBLEM SETS ?
NO ... IN FACT HE DIDN'T WRITE A SINGLE EQUATION ON THE BOARD -
OH GREAT! I WAS AFRAID I MISSED SOMETHING IMPORTANT !