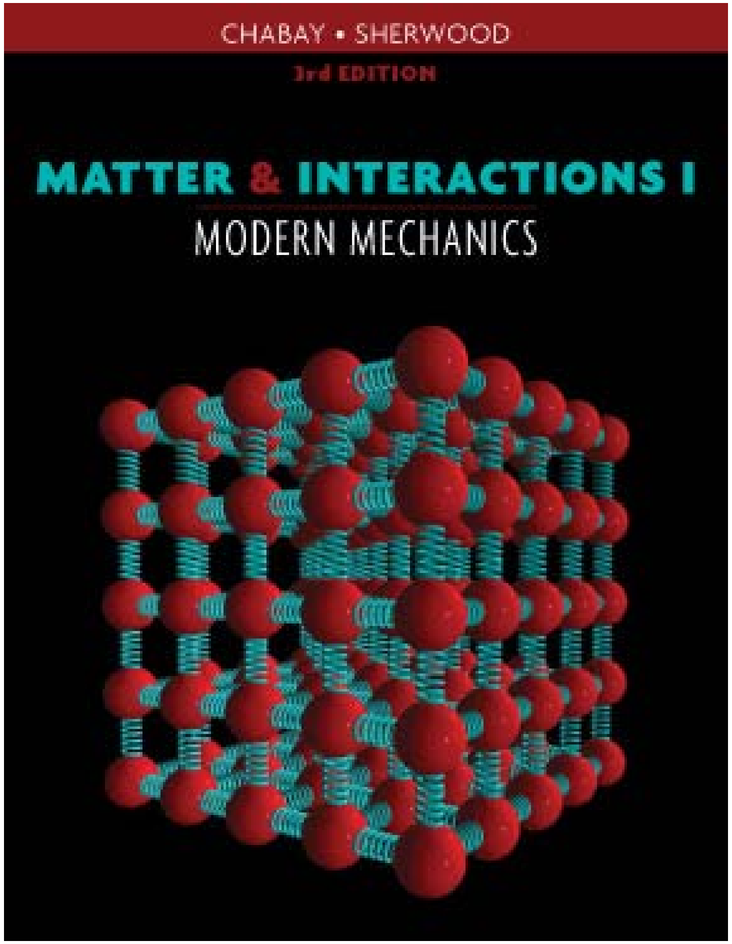

PHY1004W 2012
Modern Mechanics
Part 2

Prof Andy Buffler
Room 503 RW James
andy.buffler@uct.ac.za

These slides have benefited from significant guidance from the notes of Roger Fearick (UCT Physics) and the resources provided by the textbook authors.

These slides are available on … UCT OpenContent

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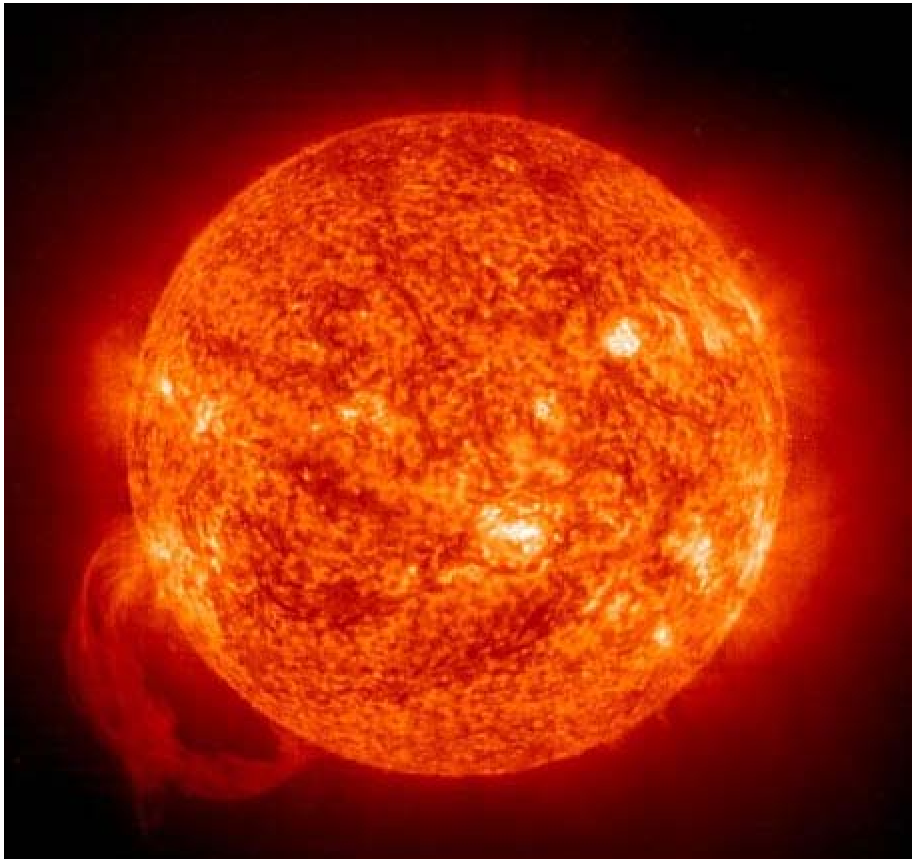
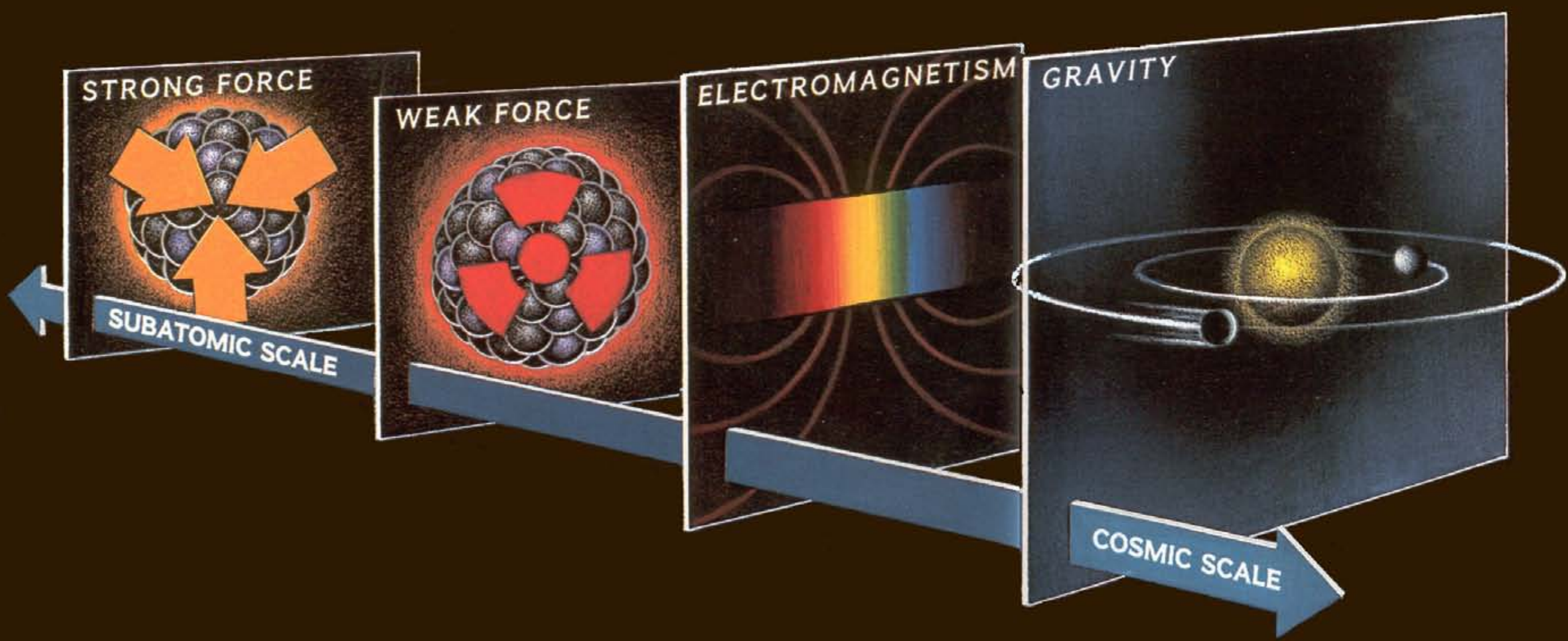
Chapter 3

The fundamental forces



PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20



Menu of forces

- Gravitational force
- Electromagnetic force
- Strong nuclear force
- Weak nuclear force

- Pushes and pulls
- Normal forces
- Tension in ropes
- Restoring forces in springs
- Friction

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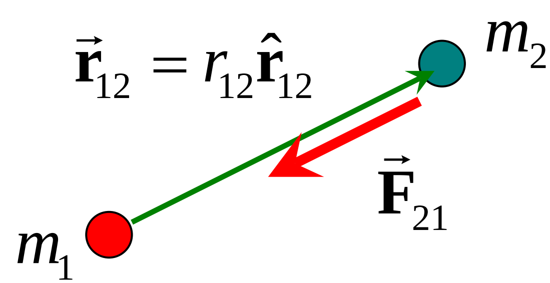
3E 3.2

The gravitational force

Every particle in the universe **attracts** every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. The force acts along the line joining the two particles.

force on 2
due to 1

$$\vec{\mathbf{F}}_{21} = G\frac{m_1 m_2}{r_{12}^2}(-\hat{\mathbf{r}}_{12})$$



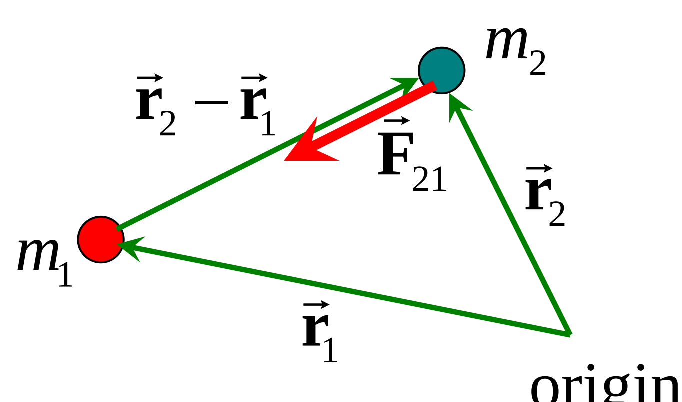
G : Universal gravitational constant.

... measured to be 6.672×10^{-11} N m^2 kg^{-2}

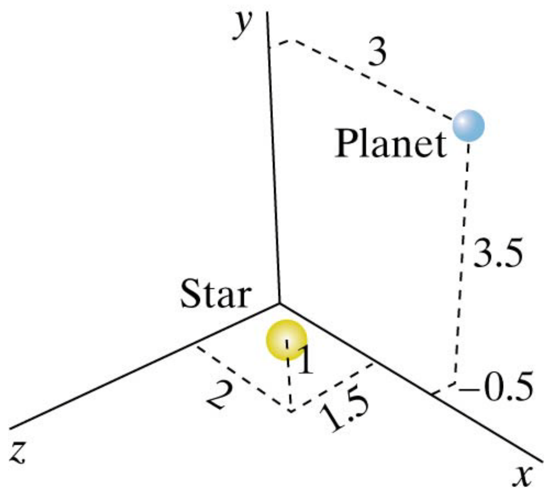
If m_1 is not at the origin ...

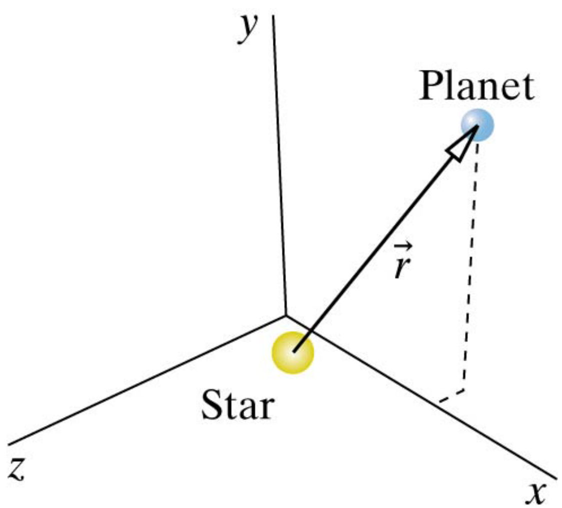
$$\vec{\mathbf{F}}_{21} = G\frac{m_1 m_2}{\left|\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1\right|^2}(-\hat{\mathbf{r}}_{12})$$

$$= G\frac{m_1 m_2}{r_{12}^2}(-\hat{\mathbf{r}}_{12}) \qquad = G\frac{m_1 m_2}{r_{12}^3}(\vec{\mathbf{r}}_{12})$$



The gravitational force
Important worked example: force on a planet by a star





A star of mass 4×10^{30} kg is located at position $\left\langle 2\times10^{11}, 1\times10^{11}, 1.5\times10^{11} \right\rangle$ m

and a planet of mass 3×10^{24} kg is located at position $\left\langle 3\times10^{11}, 3.5\times10^{11}, -0.5\times10^{11} \right\rangle$ m

What is the gravitational force exerted on the planet by the star?

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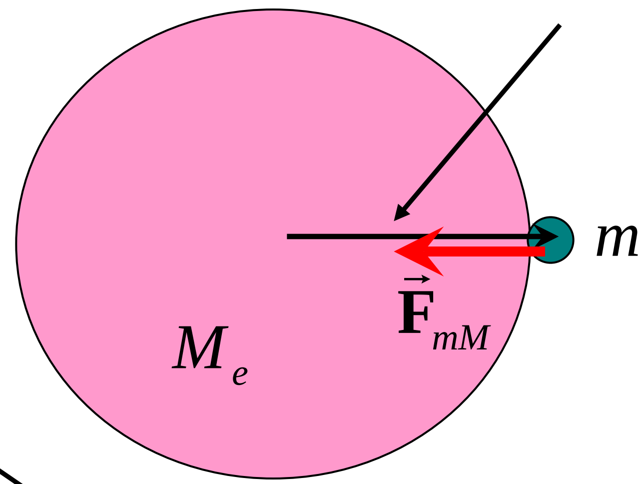
If one of the objects is the earth ...

$$F_{mM} = G\frac{mM_e}{R_e^2}$$

$r_{Mm} \simeq R_e$ (radius of the earth)



$$\therefore F_{mM} = G\frac{mM_e}{R_e^2} = mg$$

... introduce the “local gravitation strength” $\vec{\mathbf{g}}$

where $g = G\dfrac{M_e}{R_e^2}$ = 9.80 m s^{-2} in Cape Town

and write $\vec{\mathbf{F}}_{grav} = \vec{\mathbf{W}} = m\vec{\mathbf{g}}$

W is called the “weight” of the object.

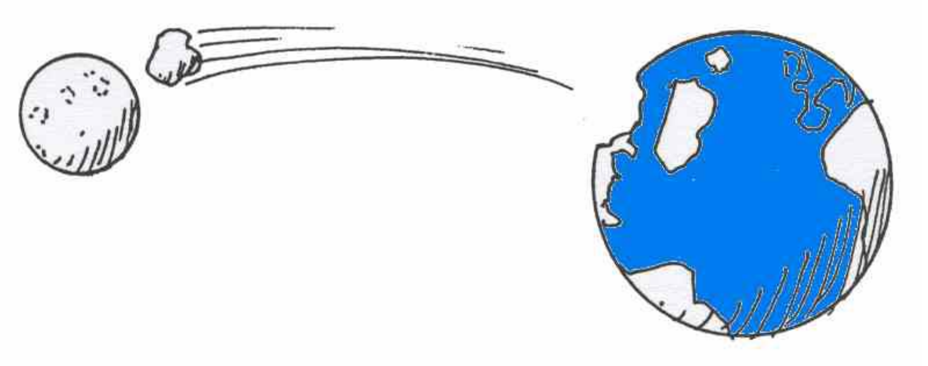
1	2	3	4	5

The gravitational force between the earth and the moon is about 4.7×10^{22} newtons.
If, somehow, part of the earth were suddenly transferred to the moon, then the gravitational force between the earth and the moon would …

(1) increase

(2) decrease

(3) remain the same



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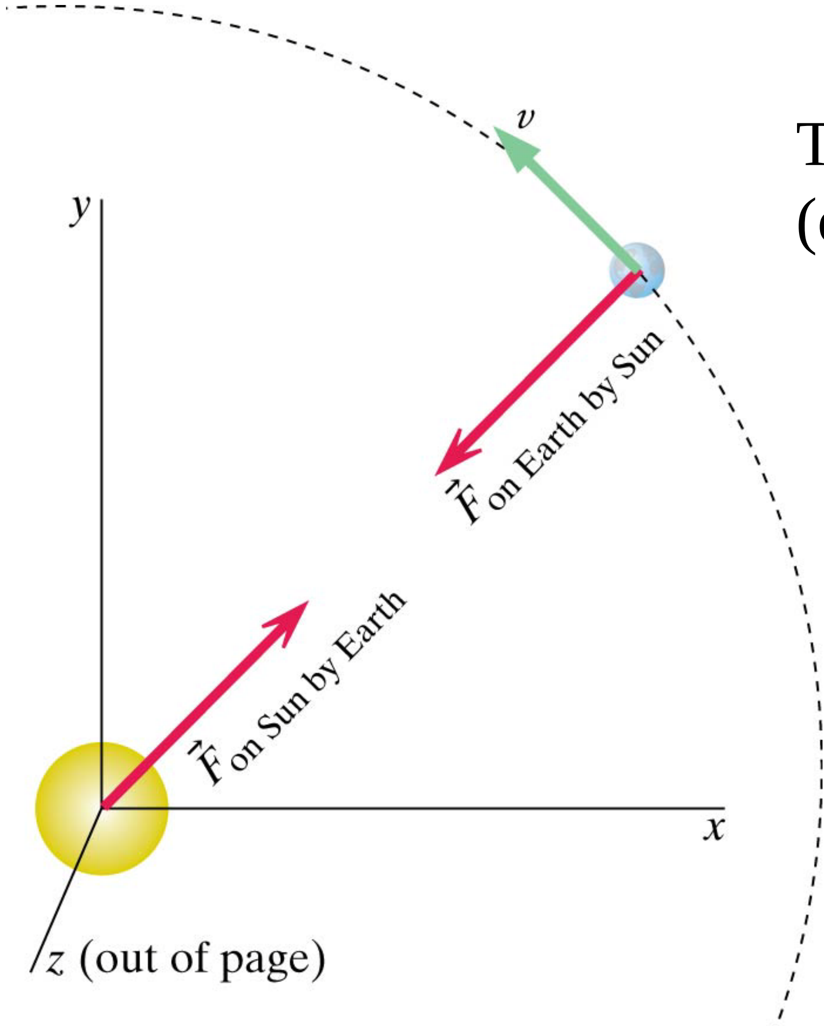
3E 3.4

Reciprocity

For both gravitational and electric forces ... $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$



This is called "reciprocity"
(or "Newton's third law of motion"):

The force exerted by 2 on 1 is equal in magnitude and opposite in direction to that exerted by 1 on 2.
(forces are **interactions**).

1	2	3	4	5

The gravitational force exerted by a planet on one of its moons is 3×10^{23} newtons when the moon is at a particular location.

If the **mass of the moon were three times as large**, then what would the force on the moon be?

(1) 1×10^{23} N
(2) 3×10^{23} N
(3) 6×10^{23} N
(4) 9×10^{23} N

1	2	3	4	5

The gravitational force exerted by a planet on one of its moons is 3×10^{23} newtons when the moon is at a particular location.

If **the distance between the moon and the planet was doubled**, what would the force on the moon be?

(1) 6×10^{23} N
(2) 3×10^{23} N
(3) 1.5×10^{23} N
(4) 0.75×10^{23} N
(5) 0.33×10^{23} N

1	2	3	4	5

Mass of Mars: 6.4×10^{23} kg; radius of Mars: 3.4×10^{6} m
Mass of Earth: 6×10^{24} kg; radius of Earth: 6.4×10^{6} m

If the Mars rover measured the value of g on Mars, it would be

(1) 9.8 N/kg

(2) less than 9.8 N/kg

(3) more than 9.8 N/kg

1	2	3	4	5

The Earth has a mass of 6×10^{24} kg.
The Sun is much more massive; its mass is 2×10^{30} kg.
Which of the following statements is correct?

(1) The gravitational force on the Sun by the Earth is smaller in magnitude than the gravitational force on the Earth by the Sun.

(2) The gravitational force on the Sun by the Earth is exactly the same in magnitude as the gravitational force on the Earth by the Sun.

(3) Neither (1) nor (2) is correct.

1 2 3 4 5

You hold a tennis ball at rest above your head, then open your hand and release the ball, which begins to fall.
At this moment, which statement about the magnitudes of the gravitational forces between the Earth and ball is correct?

(1) The force on the ball by the Earth is larger than the force on the Earth by the ball.
(2) The force on the ball by the Earth is smaller than the force on the Earth by the ball.
(3) The force on the ball by the Earth is equal to the force on the Earth by the ball.
(4) There is not enough information to determine this.

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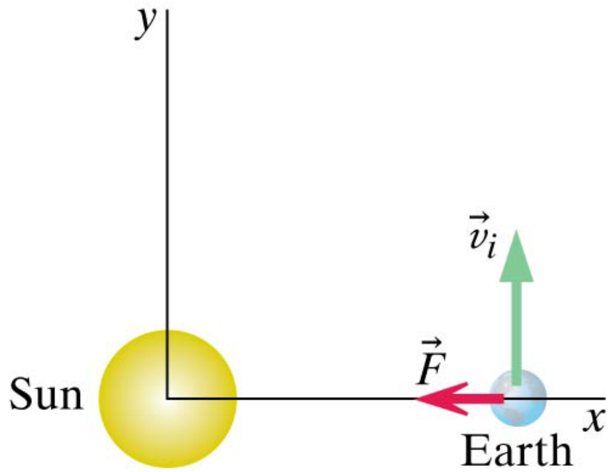
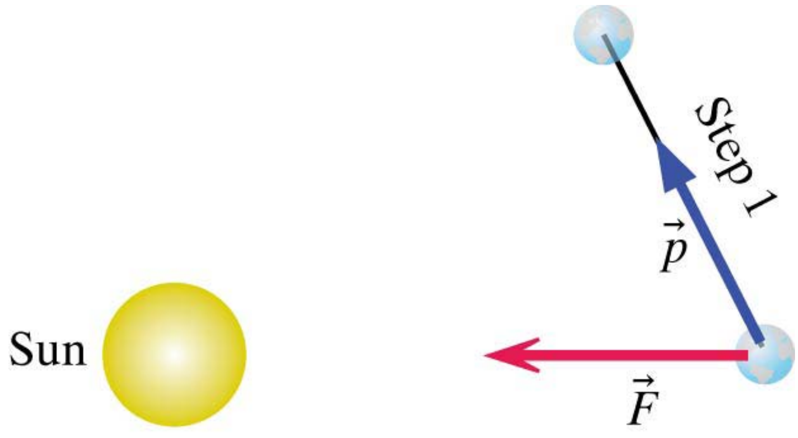
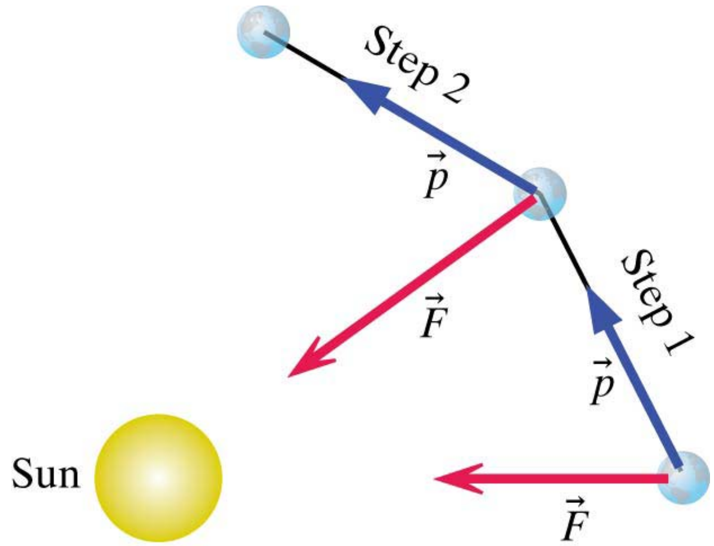
3E 3.5

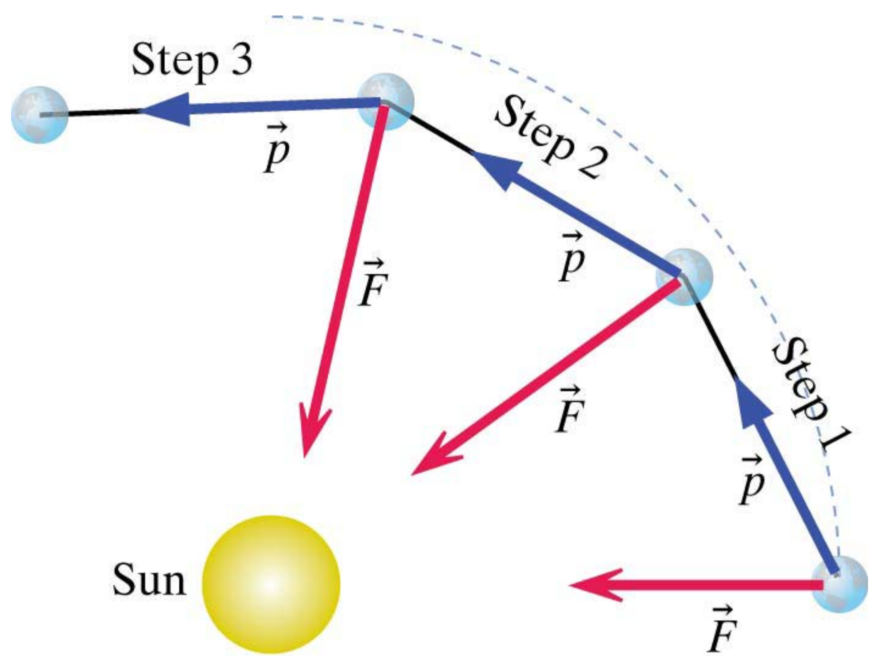
Example: planet in orbit







Step 2
$\vec{p}$
$\vec{F}$
Step 1
$\vec{p}$
Sun
$\vec{F}$

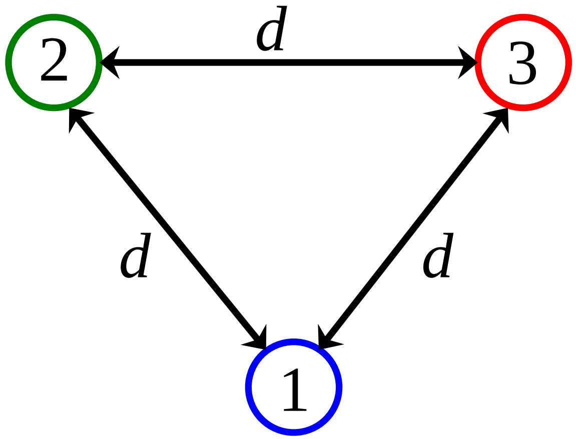


Step 3
$\vec{p}$
Step 2
$\vec{F}$
$\vec{p}$
$\vec{F}$
Step 1
$\vec{p}$
Sun
$\vec{F}$

```
from visual import *
Orbit1-2.py 04/2006 rwf
Based on Orbit1-1.py
Simple numerical integration of Earth-Sun system.
Circular orbit.
Allow sun to move as well.
#
G = 6.67e-11   # grav. const.
t=0            # initial time (in s)
dt=24*3600     # time step 1 day
# simulation objects
sun   = sphere( pos=(0,0,0), radius=1.5e10, color=color.yellow )
earth = sphere( pos=(1.5e11,0,0), radius=6.4e9, color=color.blue )
sun.M=2e30
earth.M=6e24
earth.p=earth.M*vector(0,sqrt(G*sun.M/mag(earth.pos-sun.pos)),0)
earth.trail=curve(pos=earth.pos, color=earth.color)
sun.p=sun.M*vector(0,0,0)
sun.trail=curve(pos=sun.pos, color=sun.color)
scene.autoscale=0
# scene administration
scene.autoscale=0
# integration loop
while t < 1000*dt:
    rate(50)  # limit frame rate
    R=earth.pos-sun.pos
    magR=mag(R)
    force = -G*sun.M*earth.M*R/magR**3
    earth.p=earth.p+force*dt
    earth.pos=earth.pos+(earth.p/earth.M)*dt
    earth.trail.append(pos=earth.pos)
    sun.p=sun.p-force*dt
    sun.pos=sun.pos+(sun.p/sun.M)*dt
    sun.trail.append(pos=sun.pos)
    t=t+dt
```

Example

Shown in the figure are three planets. Determine the resultant gravitational force on planet 1 due to the presence of the other two.



Mass of planet 1 $= 6.0 \times 10^{20}$ kg

Mass of planet 2 $= 2.6 \times 10^{22}$ kg

Mass of planet 3 $= 4.5 \times 10^{21}$ kg

$d = 2.0 \times 10^{12}$ km

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

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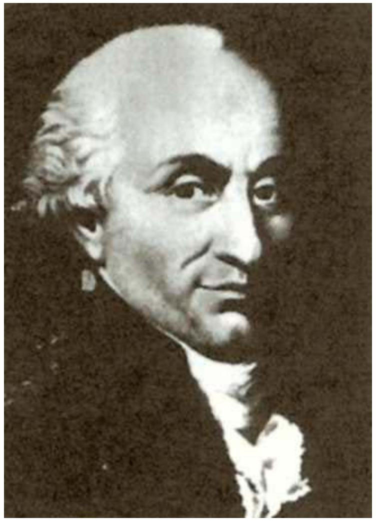
3E 3.6

The electric force



The electromagnetic force is a combination of electrical and magnetic forces.

For stationary charges, the electric force between charges is described by Coulomb's Law (1785):

$$\left|\vec{\mathbf{F}}_{21}\right| \propto \frac{q_1 q_2}{r_{12}^2} \quad \text{or} \quad \boxed{\left|\vec{\mathbf{F}}_{21}\right| = k\,\frac{q_1 q_2}{r_{12}^2}} \quad \text{Coulomb's Law}$$

where: $\vec{\mathbf{F}}_{21}$ is the force on q_2 due to q_1

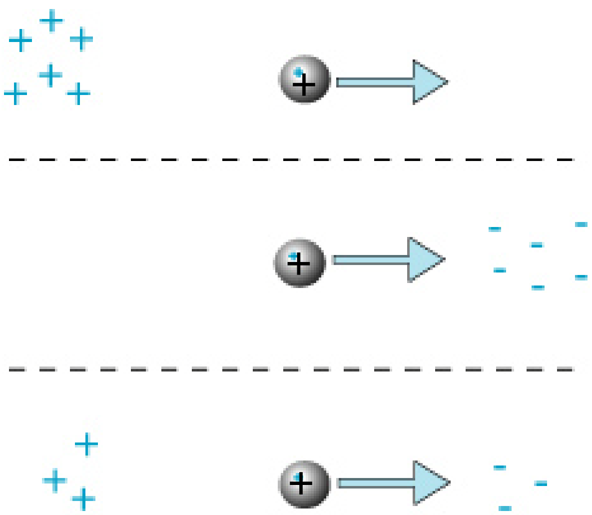
$$k = \text{constant (from experiment)} = \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ \text{N m}^2\ \text{C}^{-2}$$

$$\varepsilon_0 = \text{permittivity of free space} = 8.85\times10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

$$\vec{\mathbf{F}}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

Coulomb's Law

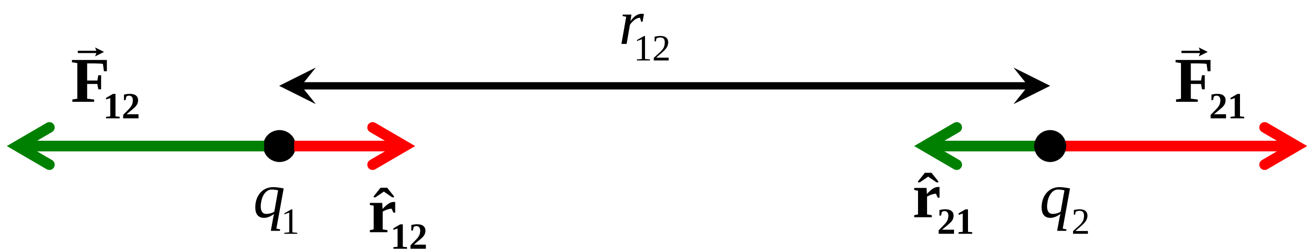
Where $\hat{\mathbf{r}}_{12}$ is the **unit vector** (magnitude = 1) which indicates the direction along which the force is acting i.e. from q_1 to q_2.



So $\vec{\mathbf{r}}_{12} = r_{12}\,\hat{\mathbf{r}}_{12}$

Note: $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$

i.e. $\left|\vec{\mathbf{F}}_{12}\right| = \left|\vec{\mathbf{F}}_{21}\right|$ but $\hat{\mathbf{r}}_{12} = -\hat{\mathbf{r}}_{21}$

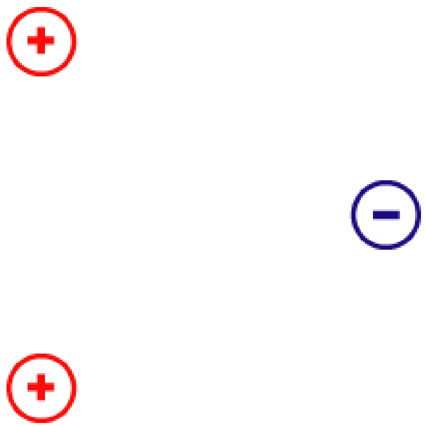


1	2	3	4	5

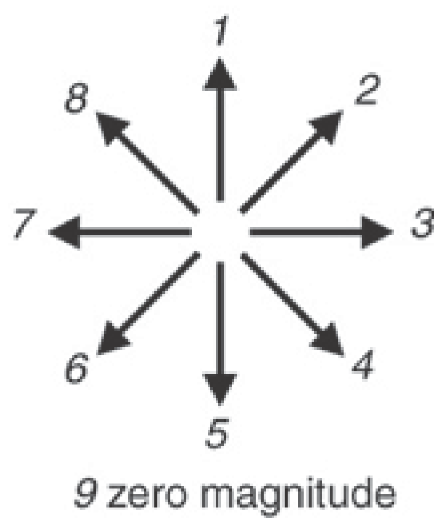
An alpha particle contains two protons and two neutrons, and has a net charge of $+2e$.
The alpha particle is 0.1 m away from a single proton, which has charge $+e$.
Which statement about the magnitudes of the electric forces between the particles is correct?

(1) The force on the proton by the alpha particle is equal to the force on the alpha particle by the proton.
(2) The force on the proton by the alpha particle is larger than the force on the alpha particle by the proton.
(3) The force on the proton by the alpha particle is smaller than the force on the alpha particle by the proton.
(4) There is not enough information to determine this.

1	2	3	4	5

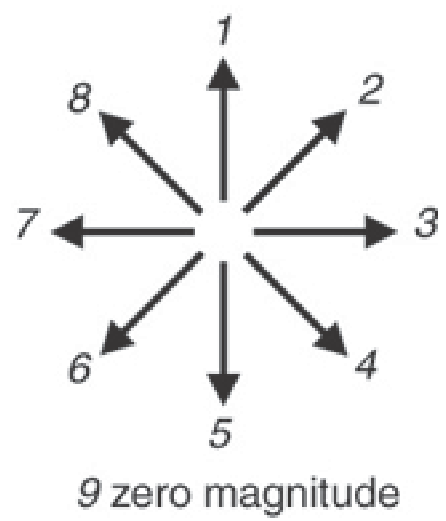


Which arrow best indicates the direction of the net electric force on the blue negatively charged object?



1	2	3	4	5

Which arrow best indicates the direction of the net electric force on the blue negatively charged object?



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3E 3.7

The strong interaction

Binds together the constituents of an atomic nucleus.

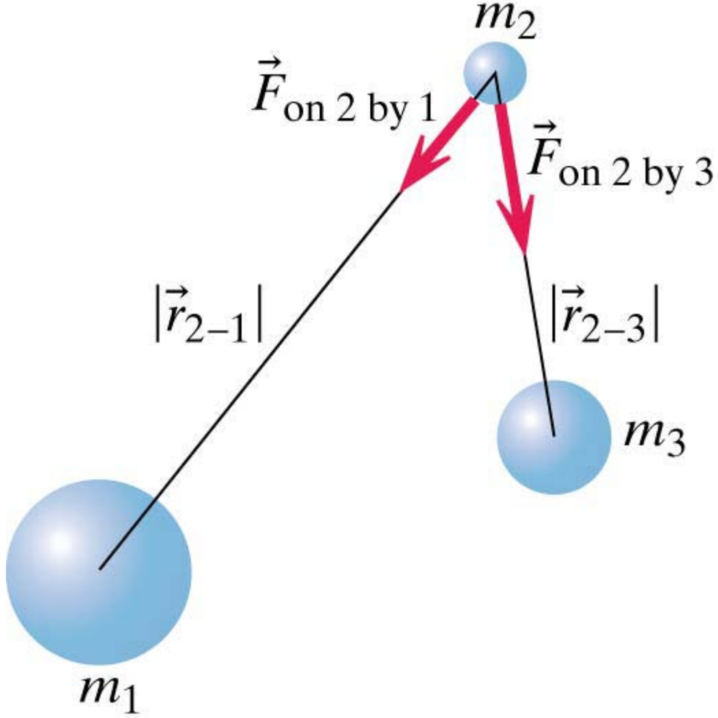
The weak interaction

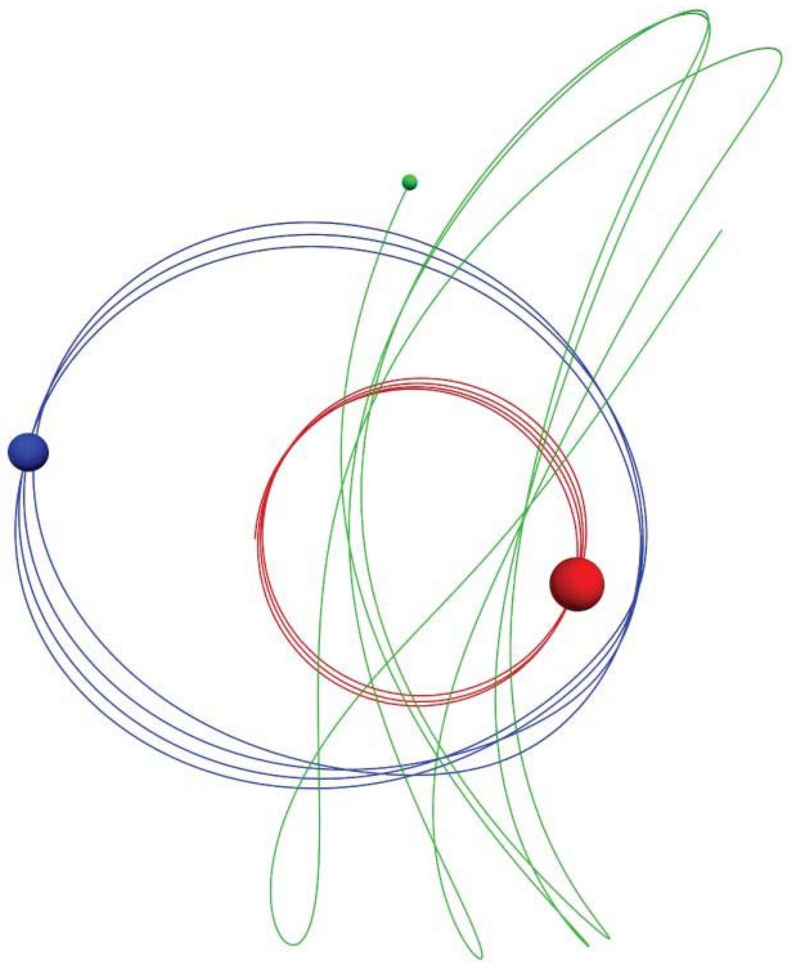
... involved in certain types of radioactive decay

Physicists are presently trying to formulate a unified theory for all the four fundamental forces.

... find out about Grand Unification Theories, Supersymmetry and String Theory ...

The three-body problem



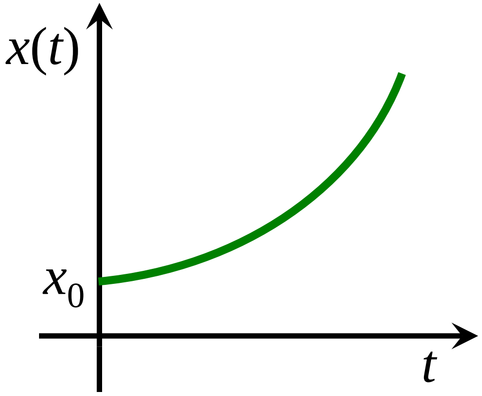


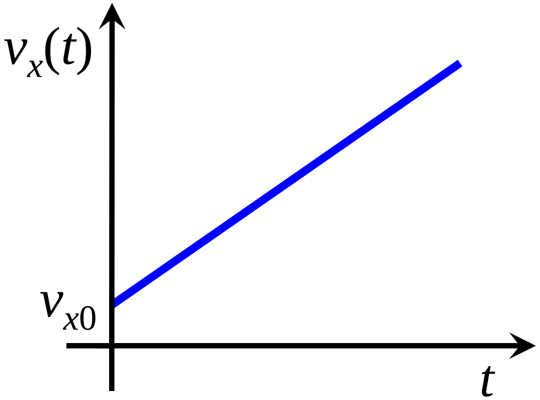
Determinism

The equations of motion …

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i \Delta t + \frac{1}{2}\frac{\vec{\mathbf{F}}_{net}}{m}\Delta t^2 \qquad \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}_{net}}{m}\Delta t$$

… allow one to calculate the position and velocity of a particle at any instant in time … they are complete descriptions of the **continuous motion** of the particle …

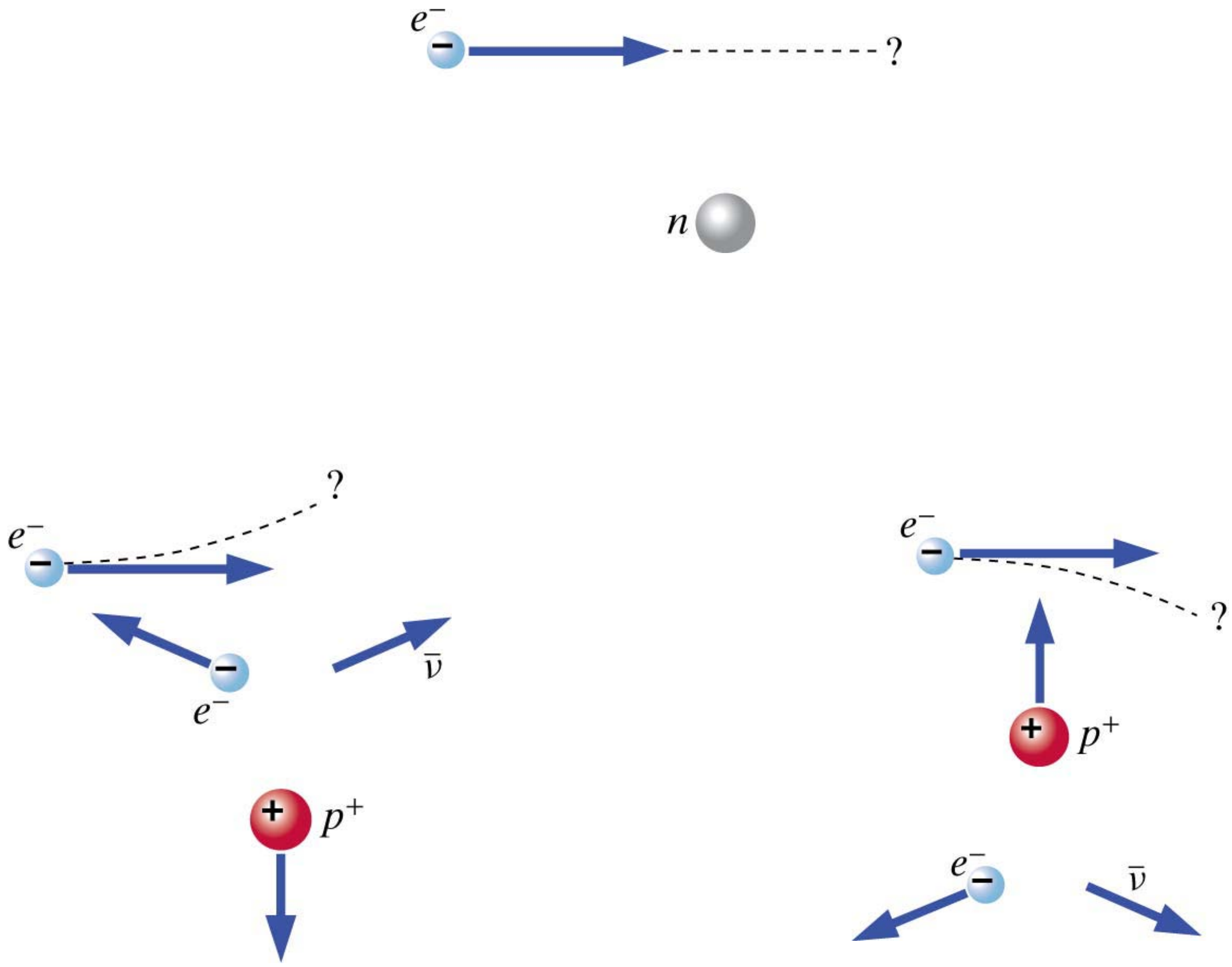




If one knows the initial conditions, then one is able to predict the motion at some time in the future … the equations are **deterministic** … later on, quantum mechanics showed that nature should be understood as being probabilistic and the determinism of Newton's mechanics applies only to the macro scale.

Neutron decay

$$n \rightarrow p^{+} + e^{-} + \overline{\nu}$$



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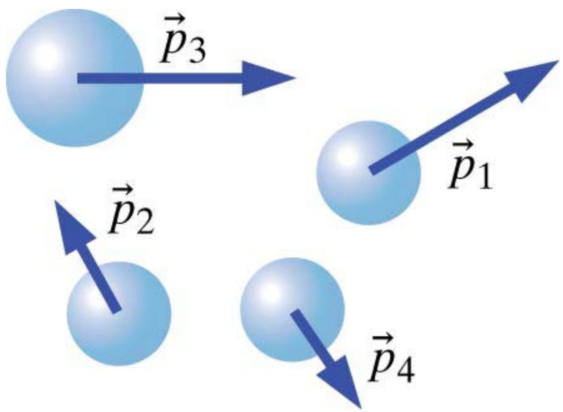
Conservation of momentum

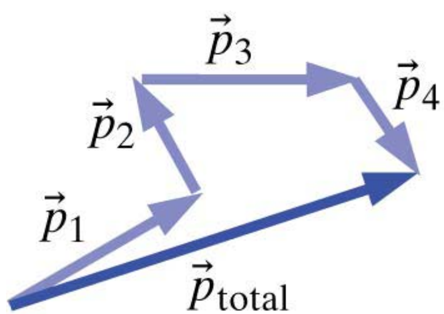
Momentum principle $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{net}\Delta t$



leads to the ...

Principle of the conservation of linear momentum …

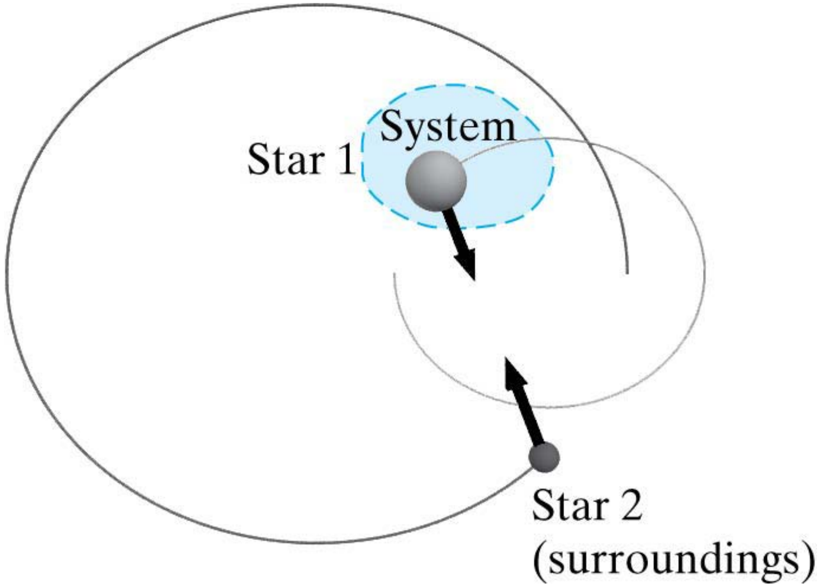
If there is no resultant external force acting on an isolated system, then the total linear momentum remains constant (is "conserved").

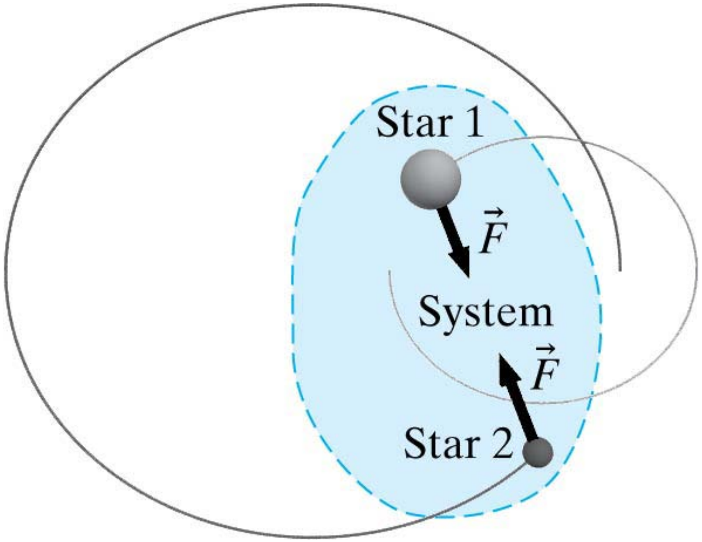


$$\Delta\vec{\mathbf{p}} = 0 \quad \text{if} \quad \vec{\mathbf{F}}_{net} = 0$$

Another way of viewing this: $\Delta\vec{\mathbf{p}}_{system} + \Delta\vec{\mathbf{p}}_{surr} = 0$

Example: Conservation of momentum in a binary star system



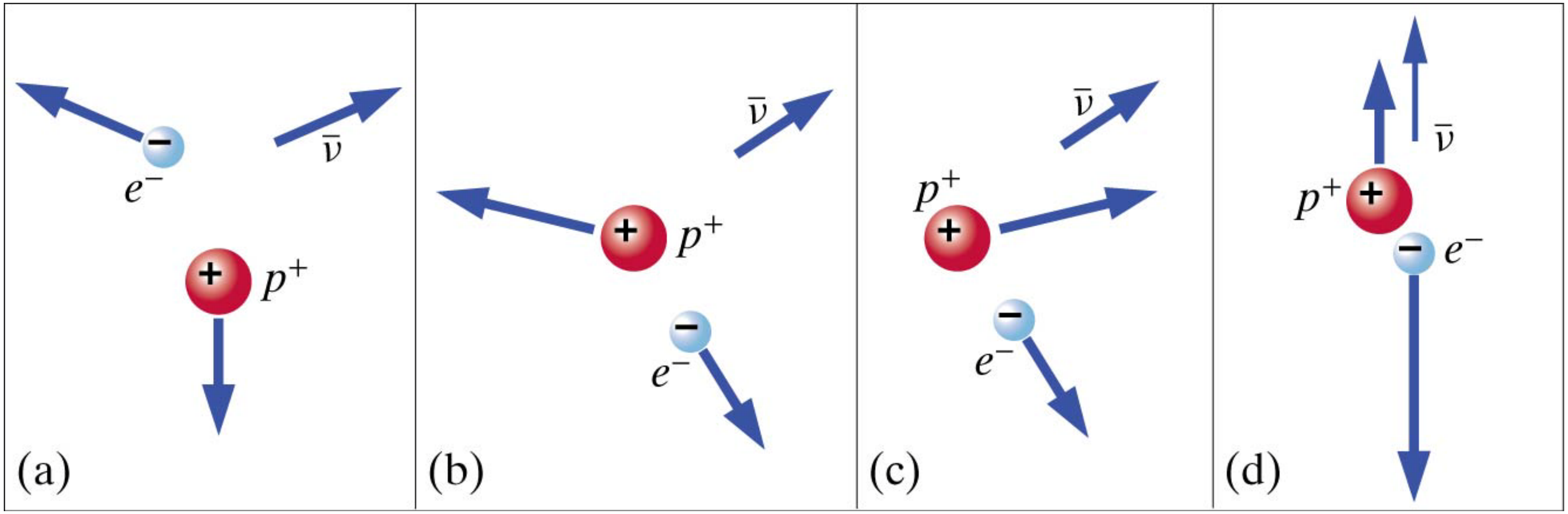


$$\Delta\vec{\mathbf{p}}_{system} + \Delta\vec{\mathbf{p}}_{surr} = 0$$

$$\Delta\vec{\mathbf{p}}_{system} = 0$$

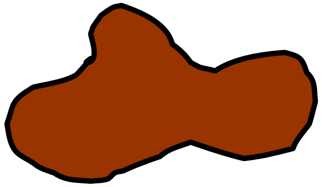
Example: Conservation of momentum in neutron decay

$$n \rightarrow p^+ + e^- + \bar{\nu}$$



Which is <u>not</u> possible?

<u>Centre of mass</u>



(can vibrate, rotate, etc.)

An extended body has size ... so far we have only considered the motion of a single particle •, but an extended body is made up of many particles.

We can show that if a body rotates, or if several particles move relative to one another, there is <u>one</u> point that moves in the same path that a single particle would move if subjected to the same net force. This is the **centre of mass**.

... this point moves as if the all mass is concentrated at that point.

We define the centre of mass for a system of particles as:

$$\vec{\mathbf{r}}_{CM} = \sum \frac{m_i \vec{\mathbf{r}}_i}{M_{total}} \qquad \text{total mass} = \sum m_i$$

$$x_{CM} = \sum \frac{m_i x_i}{M_{total}} \quad ; \quad y_{CM} = \sum \frac{m_i y_i}{M_{total}} \quad ; \quad z_{CM} = \sum \frac{m_i z_i}{M_{total}}$$

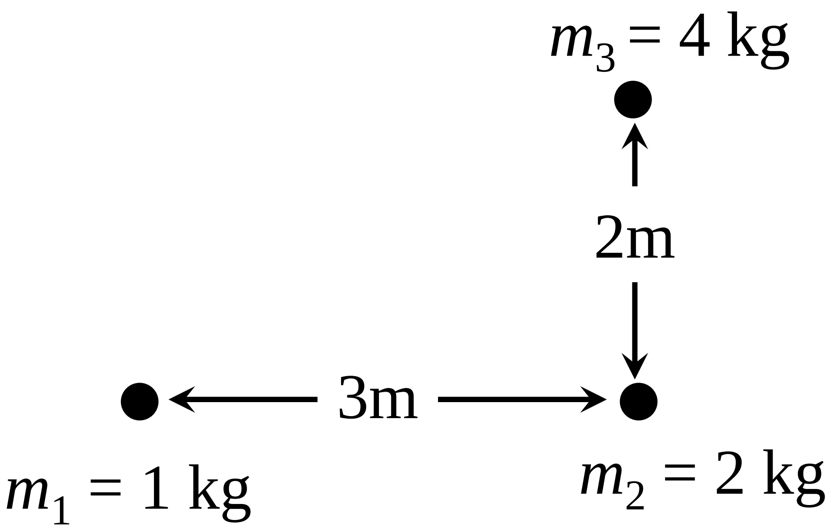
then $\vec{\mathbf{r}}_{CM} = x_{CM}\hat{\mathbf{i}} + y_{CM}\hat{\mathbf{j}} + z_{CM}\hat{\mathbf{k}}$

For an extended body (a continuous mass distribution) :

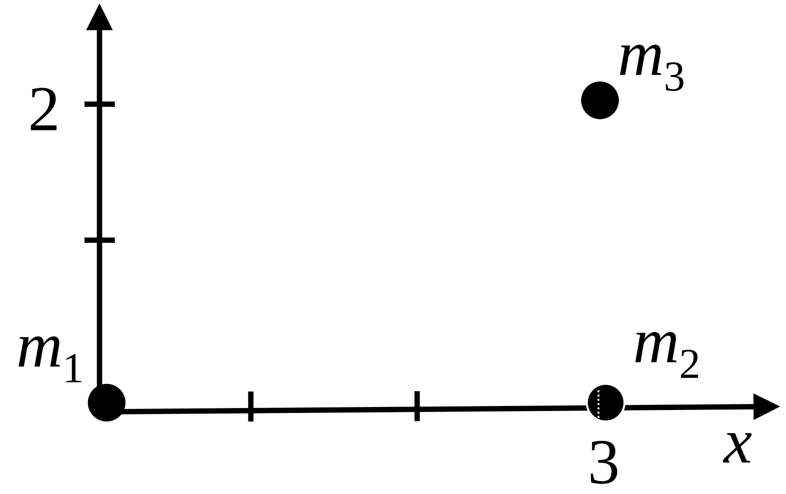
$$\vec{\mathbf{r}}_{CM} = \frac{1}{M_{total}} \int \vec{\mathbf{r}} dm = \frac{1}{M_{total}} \int x dm\, \hat{\mathbf{i}} + \frac{1}{M_{total}} \int y dm\, \hat{\mathbf{j}} + \frac{1}{M_{total}} \int z dm\, \hat{\mathbf{k}}$$

Example

Find the centre of mass of the distribution of particles shown:



Choose a coordinate axes (<u>any</u> will do)

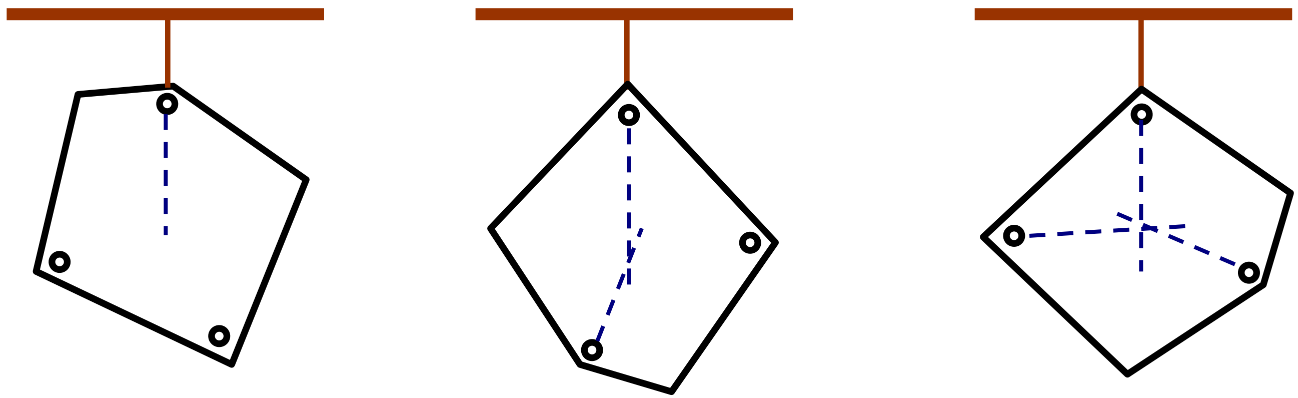


$$x_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (2)(3) + (4)(3)}{1 + 2 + 4} = \frac{18}{7} = 2.57$$

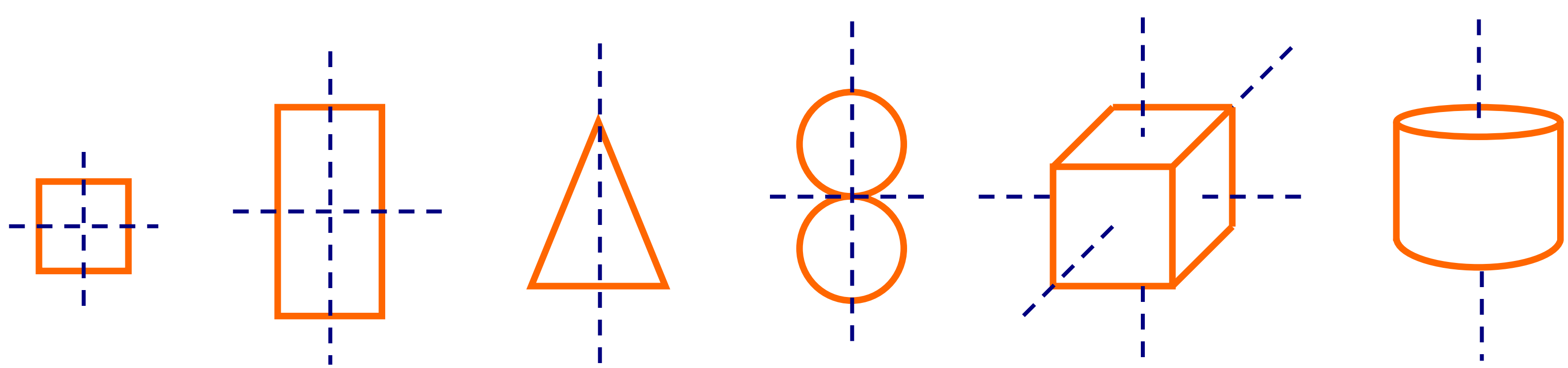
$$y_{\text{CM}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (2)(0) + (4)(2)}{1 + 2 + 4} = \frac{8}{7} = 1.14$$

$\therefore\ \vec{\mathbf{r}}_{\text{CM}} = \langle 2.57, 1.14, 0 \rangle$ m <u>in this coordinate system.</u>

A body free to rotate about a support will hang so that its centre of mass is vertically below support.



The centre of mass of a symmetrical object must lie on the line of symmetry.



Velocity of the centre of mass

The importance of centre of mass lies in the fact that the motion of the centre of mass for a system of particles (or extended body) can often be described simply since it is related to net force on the system.

Consider n particles of total mass M which remains constant.

Then: $$M_{total}\vec{\mathbf{r}}_{CM} = \sum m_i \vec{\mathbf{r}}_i$$

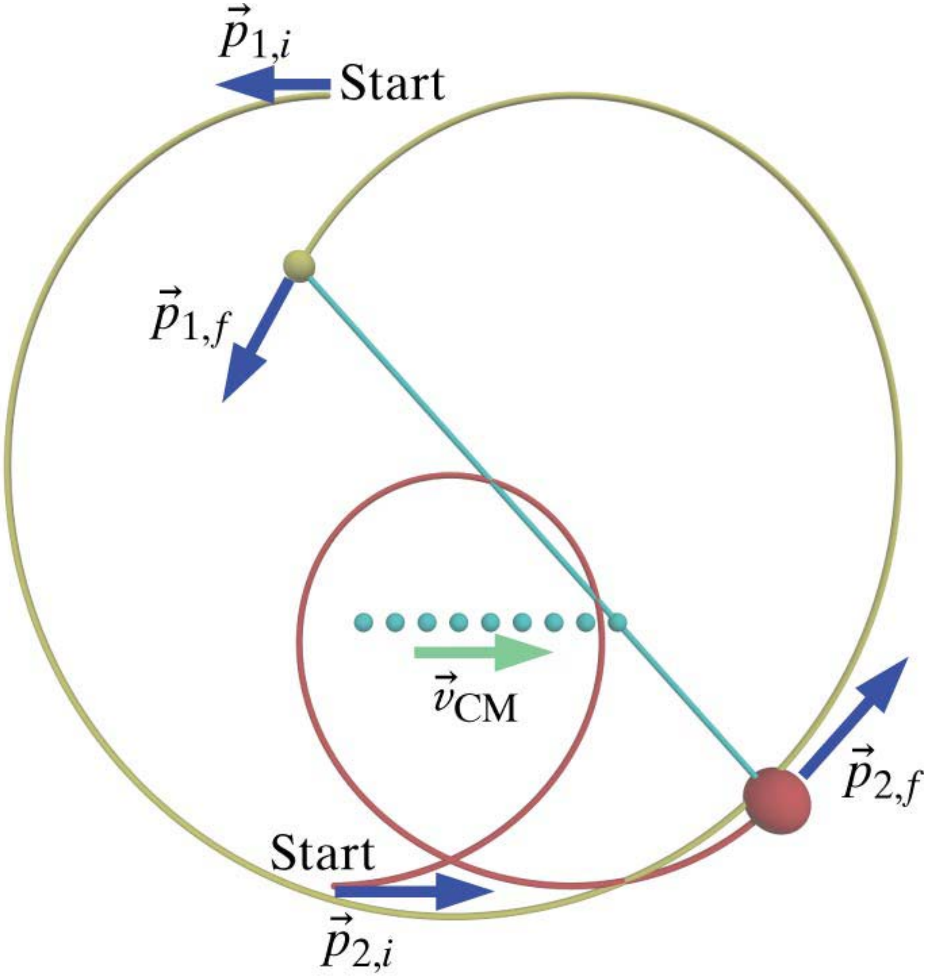
$$\therefore M_{total}\frac{d\vec{\mathbf{r}}_{CM}}{dt} = \sum m_i \frac{d\vec{\mathbf{r}}_{\mathrm{i}}}{dt}$$

$$\therefore \vec{\mathbf{p}}_{sys} = M_{total}\underbrace{\vec{\mathbf{v}}_{CM}}_{\text{Velocity of centre of mass}} = \sum m_i \underbrace{\vec{\mathbf{v}}_i}_{\text{Velocity of } i^{\text{th}} \text{ particle of mass } m}$$

Velocity of centre of mass

Velocity of i^{th} particle of mass m

Velocity of the centre of mass of a binary star system



$$\vec{\mathbf{p}}_{1,i} + \vec{\mathbf{p}}_{2,i} = (m_1 + m_2)\vec{\mathbf{v}}_{CM}$$

$$\vec{\mathbf{p}}_{1,f} + \vec{\mathbf{p}}_{2,f} = (m_1 + m_2)\vec{\mathbf{v}}_{CM}$$

$\vec{\mathbf{p}}_1$ not constant

$\vec{\mathbf{p}}_2$ not constant

$\vec{\mathbf{p}}_{sys} = (m_1 + m_2)\vec{\mathbf{v}}_{CM}$ constant

M&I
3E 3.12

The multiparticle momentum principle

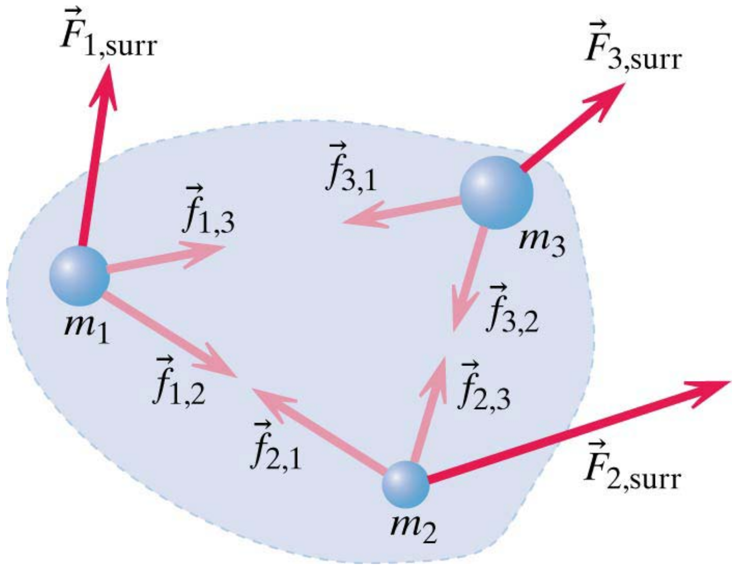
Momentum principle for multiparticle systems:

$$\Delta\vec{\mathbf{p}}_{total} = \Delta\vec{\mathbf{p}}_{total,f} - \Delta\vec{\mathbf{p}}_{total,i} = \vec{\mathbf{F}}_{net}\Delta t$$

where $\vec{\mathbf{p}}_{total} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 + \ldots$

and $\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_{1,surr} + \vec{\mathbf{F}}_{2,surr} + \vec{\mathbf{F}}_{3,surr} + \ldots$

i.e. same form as for a single particle ... because ...
internal forces cancel out in pairs by reciprocity.



Internal forces cancel

$$\Delta\vec{\mathbf{p}}_1 = \left(\vec{\mathbf{F}}_{1,surr} + \vec{f}_{1,2} + \vec{f}_{1,3}\right)\Delta t$$
$$\Delta\vec{\mathbf{p}}_2 = \left(\vec{\mathbf{F}}_{2,surr} + \vec{f}_{2,1} + \vec{f}_{2,3}\right)\Delta t$$
$$\Delta\vec{\mathbf{p}}_3 = \left(\vec{\mathbf{F}}_{3,surr} + \vec{f}_{3,1} + \vec{f}_{3,2}\right)\Delta t$$

Add all 3 equations ...

$$\Delta\vec{\mathbf{p}}_1 + \Delta\vec{\mathbf{p}}_2 + \Delta\vec{\mathbf{p}}_3$$
$$= \left(\vec{\mathbf{F}}_{1,surr} + \vec{f}_{1,2} + \vec{f}_{1,3} + \vec{\mathbf{F}}_{2,surr} + \vec{f}_{2,1} + \vec{f}_{2,3} + \vec{\mathbf{F}}_{3,surr} + \vec{f}_{3,1} + \vec{f}_{3,2}\right)\Delta t$$

By reciprocity: $\vec{f}_{1,2} = -\vec{f}_{2,1};\quad \vec{f}_{1,3} = -\vec{f}_{3,1};\quad \vec{f}_{2,3} = -\vec{f}_{3,2}$

$$\therefore \Delta\vec{\mathbf{p}}_1 + \Delta\vec{\mathbf{p}}_2 + \Delta\vec{\mathbf{p}}_3 = \left(\vec{\mathbf{F}}_{1,surr} + \vec{\mathbf{F}}_{2,surr} + \vec{\mathbf{F}}_{3,surr}\right)\Delta t$$

or $\quad \Delta\vec{\mathbf{p}}_{sys} = \vec{\mathbf{F}}_{net,surr}\Delta t$

Special form of the momentum principle

Starting with $$\vec{\mathbf{p}}_{sys} = M_{total}\vec{\mathbf{v}}_{CM}$$

If the mass is constant then $$\Delta\vec{\mathbf{p}}_{sys} = M_{total}\Delta\vec{\mathbf{v}}_{CM} = \vec{\mathbf{F}}_{net}\Delta t$$

Divide by Δt $$\frac{\Delta\vec{\mathbf{p}}_{sys}}{\Delta t} = M_{total}\frac{\Delta\vec{\mathbf{v}}_{CM}}{\Delta t} = \vec{\mathbf{F}}_{net}$$

In the limit as $\Delta t \to 0$ $$\frac{d\vec{\mathbf{p}}_{sys}}{dt} = M_{total}\frac{d\vec{\mathbf{v}}_{CM}}{dt} = \vec{\mathbf{F}}_{net}$$

Thus $$M_{total}\vec{\mathbf{a}}_{CM} = \vec{\mathbf{F}}_{net}$$

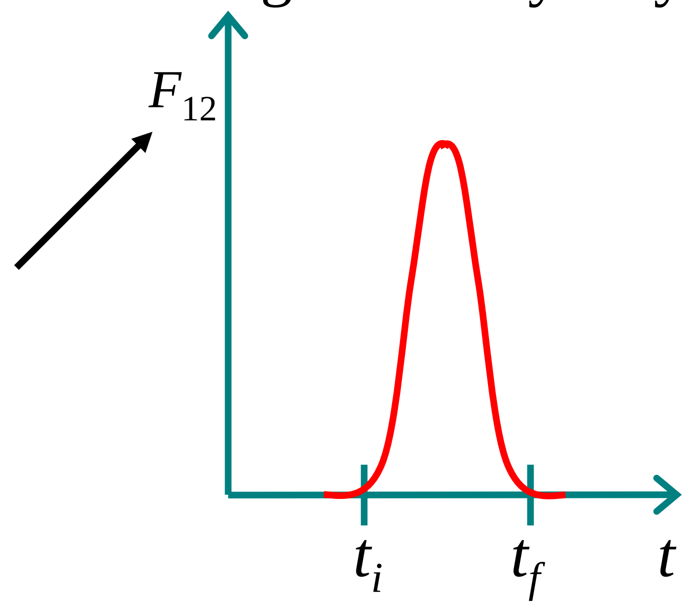
Therefore the vector sum of all the forces acting on the system equals the total mass of the system times the acceleration of the centre of mass.

The centre of mass moves as though all the mass were concentrated there, and the external forces are all applied at the centre of mass.

Collisions

A collision is when two bodies interact over a short time interval. The forces that the bodies exert on each other are usually so strong during the collision that all forces acting on a body may be ignored.

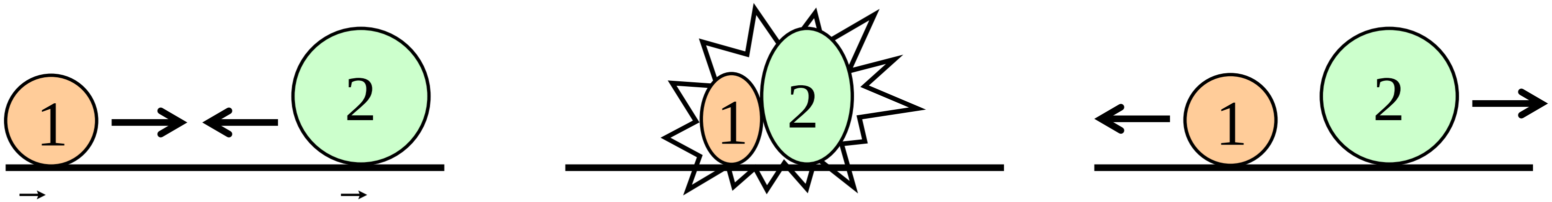
During a collision between two bodies (1 and 2), the contact force exerted by one body on the other jumps from zero to a very large value and then abruptly drops to zero again.



The time interval $\Delta t = t_f - t_i$ is usually very small.

Note that $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$ for the collision

Consider the collision of two bodies:



$\vec{\mathbf{p}}_{1,i}$ $\vec{\mathbf{p}}_{2,i}$ $\vec{\mathbf{p}}_{1,f}$ $\vec{\mathbf{p}}_{2,f}$

$$\Delta\vec{\mathbf{p}}_1 = \vec{\mathbf{p}}_{1,f} - \vec{\mathbf{p}}_{1,i}$$

and $$\Delta\vec{\mathbf{p}}_2 = \vec{\mathbf{p}}_{2,f} - \vec{\mathbf{p}}_{2,i}$$

But $$\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$$

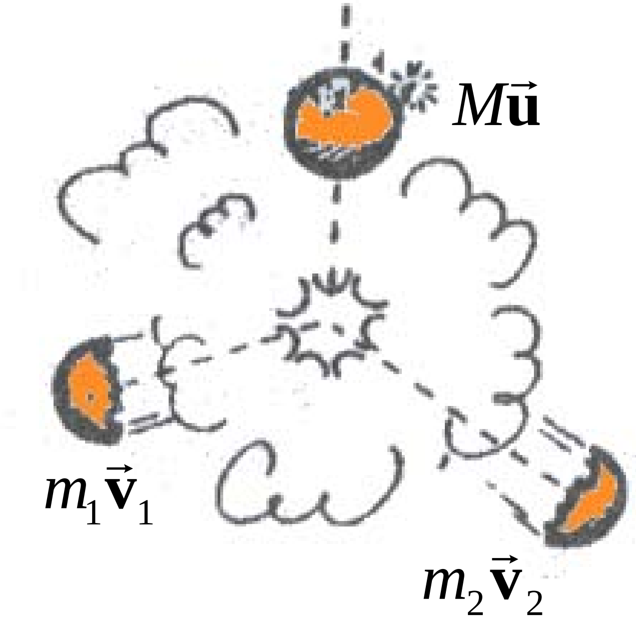
$$\therefore \Delta\vec{\mathbf{p}}_1 = -\Delta\vec{\mathbf{p}}_2$$

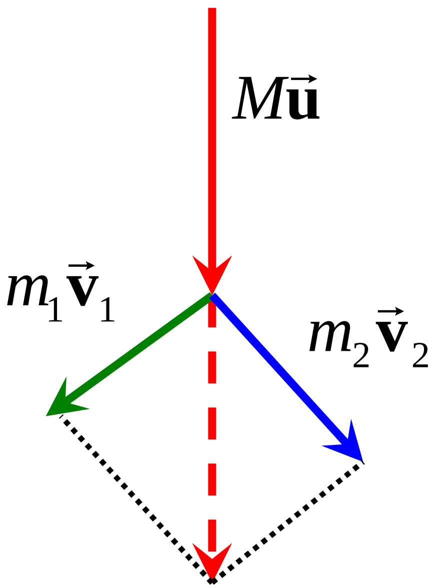
$$\therefore \vec{\mathbf{p}}_{1,f} - \vec{\mathbf{p}}_{1,i} = -(\vec{\mathbf{p}}_{2,f} - \vec{\mathbf{p}}_{2,i})$$

$$\therefore \vec{\mathbf{p}}_{1,i} + \vec{\mathbf{p}}_{2,i} = \vec{\mathbf{p}}_{1,f} + \vec{\mathbf{p}}_{2,f}$$ Momentum is conserved.

or $$m_1\vec{\mathbf{v}}_{1,i} + m_2\vec{\mathbf{v}}_{2,i} = m_1\vec{\mathbf{v}}_{1,f} + m_2\vec{\mathbf{v}}_{2,f}$$

Another case ... an exploding object



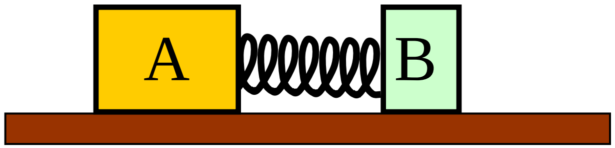


$$M\vec{\mathbf{u}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

Note that some collisions require more principles to be considered in order to solve the problem ... such as conservation of energy ... ... later ...

1	2	3	4	5

Two blocks A and B, rest on a horizontal frictionless table. The blocks are separated by a compressed spring of negligible mass. The mass of block A is twice that of block B. When the blocks are released, they move apart. Which one of the following statements is true?



I.

(1) The magnitude of the momentum of A equals that of B after release.
(2) The magnitude of the momentum of A is greater than B after release.
(3) The magnitude of the momentum of A is less than B after release.

II.

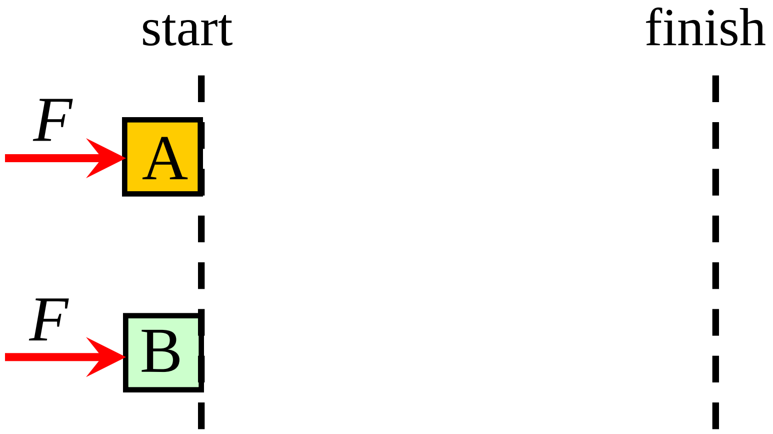
(1) The total momentum of the blocks after release is the same as before release.
(2) The total momentum of the blocks after release is greater than before release.
(3) The total momentum of the blocks after release is less than before release.

III.

(1) The speed of block A is the same as that of B at all times after release.
(2) The speed of block A is greater than that of B after release.
(3) The speed of block A is less than that of B after release.

1	2	3	4	5

Identical constant forces continuously push identical blocks A and B from the start line to the finish line.
Block A is initially at rest.
Block B is initially moving to the right.



I. Which block has the larger change in momentum?

(1) A
(2) B
(3) They have the same momenta change.

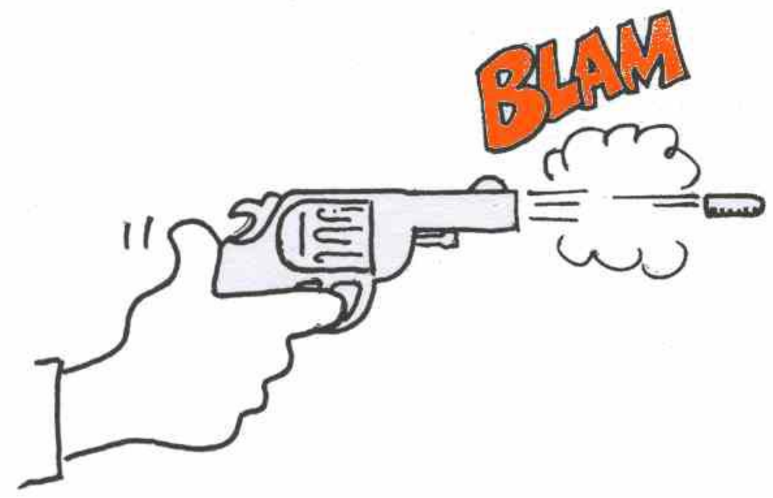
II. My reason for my answer to I is

(1) The same force acts on identical blocks for the same distance.
(2) Block B already has some momentum, so its change isn’t as great.
(3) The impulse on block B is less since the force acts for a shorter time interval.
(4) Block B is moving faster at the finish line, so its change is greater.
(5) The initial and final velocities are not given.

1	2	3	4	5

Strictly speaking, when a gun is fired, compared with the momentum of the recoiling gun, the opposite momentum of the bullet is

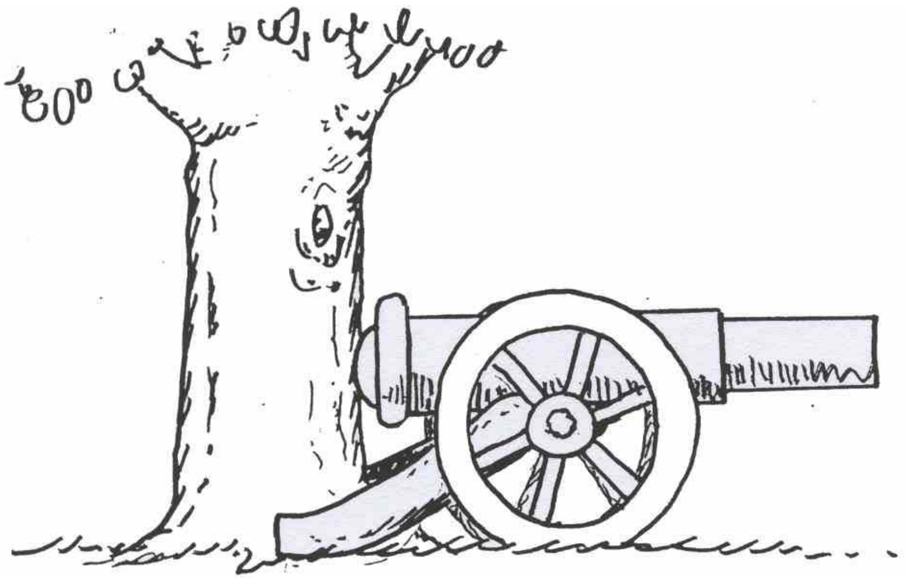
(1) less
(2) more
(3) the same



1	2	3	4	5

Suppose a cannon is propped against a massive tree to reduce recoil when it fires, Then the range of the cannonball will be

(1) increased
(2) decreased
(3) unchanged



1	2	3	4	5

A tennis ball falls for 1 second.
During this time the change in the *y* component of the ball's momentum is Δp_y = –0.6 kg m/s

What is the change in the *y* component of the Earth's momentum?

(1) –0.6 kg m/s
(2) +0.6 kg m/s
(3) zero because the ball does not exert a force on the Earth
(4) zero because the Earth's momentum can't change
(5) There is not enough information to determine this.

1	2	3	4	5

When a ping pong ball collides with a bowling ball, why is the effect on the ping pong ball more noticeable than the effect on the bowling ball?

(1) The momentum of the bowling ball does not change.

(2) The change in the bowling ball’s momentum is less than the change in the ping pong ball’s momentum.

(3) The change in the bowling ball’s velocity is less than the change in the ping pong ball’s velocity.

1	2	3	4	5

A bowling ball is initially at rest, floating in outer space. A ping pong ball moving in the $+z$ direction hits the bowling ball, and bounces off it, traveling back in the $-z$ direction. Consider a time interval Δt from slightly before to slightly after the collision.

In this time interval, what is the sign of Δp_z for the system consisting of both balls?

(1) positive
(2) negative
(3) zero – no change in p_z

1 2 3 4 5

A bullet of mass 0.04 kg traveling horizontally at a speed of 800 m/s embeds itself in a block of mass 0.5 kg that is sitting at rest on a very slippery sheet of ice.

You want to find the speed of the block just after the bullet embeds itself in the block.

What should you choose as the system?

(1) the bullet
(2) the block
(3) the bullet and the block

Example

A 60 kg person standing on a 40 kg trolley are travelling to the right at a speed of 10 m s^{-1} on a horizontal surface. The person jumps off the trolley and flies through the air at a speed of 2 m s^{-1} to the left. Determine the velocity of the trolley relative to the ground immediately after the person jumps off. Assume that the axles of the trolley have a very low coefficient of friction.

Example

Ball A of mass 10 kg is moving at 30 m s^{-1} at 180 º.
Ball B of mass 20 kg is moving at 15 m s^{-1} at 300º.
They collide elastically and thereafter ball A is moving at 25 m s^{-1} at 45°.
All angles are measured anticlockwise from the positive *x*-axis.
What is the final velocity of ball B after the collision?

[Answer: $-16.3\hat{\mathbf{i}} - 21.8\,\hat{\mathbf{j}}$ m s^{-1}]

1	2	3	4	5

A bullet of mass *m* traveling horizontally at a very high speed *v* embeds itself in a block of mass *M* that is sitting at rest on a very slippery sheet of ice.

You want to find the speed of the block just after the bullet embeds itself in the block.

What should you choose as the system?

(1) the bullet
(2) the block
(3) the bullet and the block

1	2	3	4	5

A bullet of mass m traveling horizontally at a very high speed v embeds itself in a block of mass M that is sitting at rest on a very slippery sheet of ice.

What is the speed of the block just after the bullet embeds itself in the block?

(1) v

(2) $\left(\dfrac{m}{M}\right)v$

(3) $\sqrt{\dfrac{m}{M+m}}\; v$

(4) $\left(\dfrac{M+m}{m}\right)v$

(5) $\left(\dfrac{m}{M+m}\right)v$

1	2	3	4	5

A space satellite of mass 500 kg has velocity < 12, 0, –8 > m/s just before being struck by a rock of mass 3 kg with velocity < –3000, 0, 900 > m/s.

After the collision the rock's velocity is < 700, 0, –300 > m/s. What is the velocity of the space satellite?

(1) < –5100, 0, –400 > m/s
(2) < –10.2, 0, –0.8 > m/s
(3) < 10.2, 0, 0.8 > m/s
(4) < –3688, 0, 1191 > m/s
(5) < 3688, 0, –1192 > m/s

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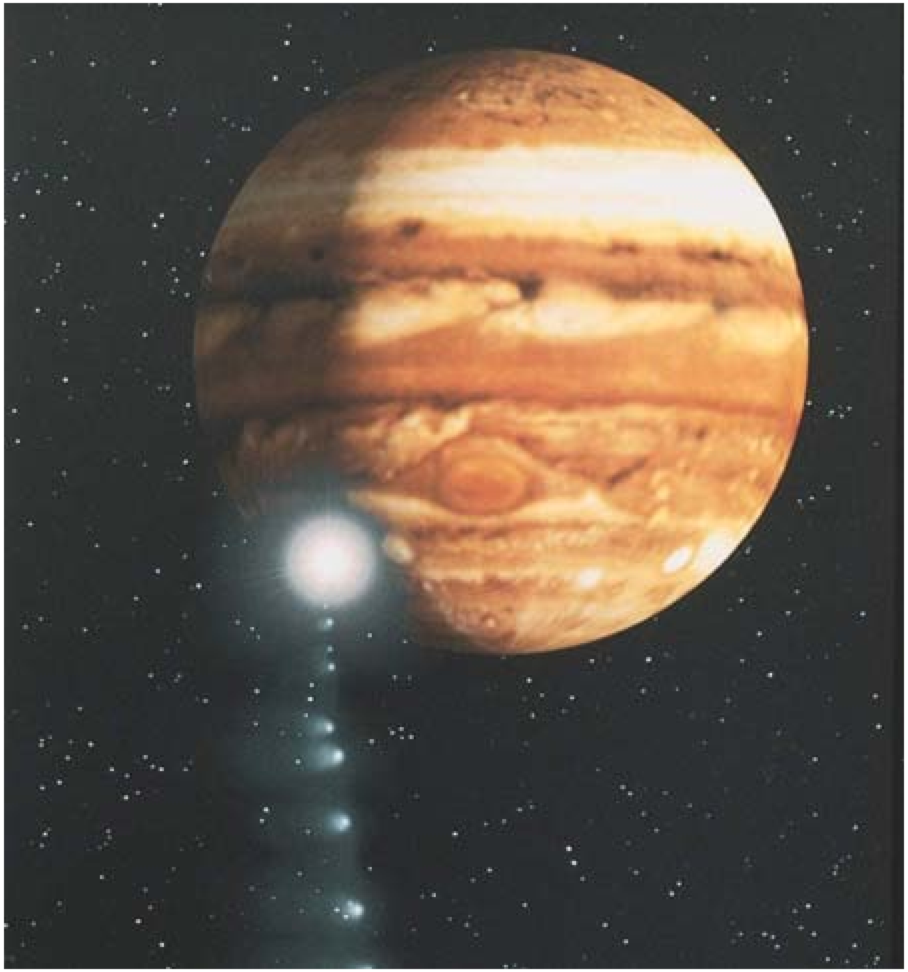
Chapter 4

Contact interactions



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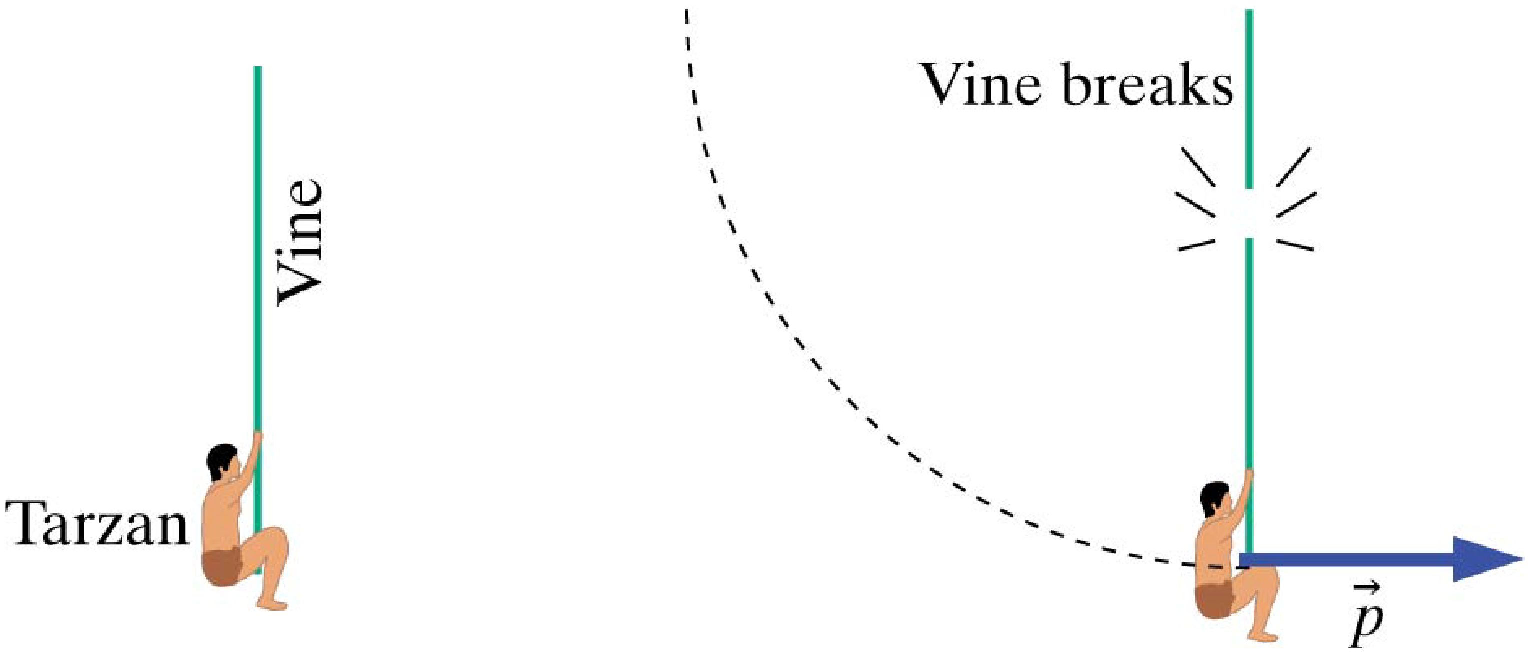
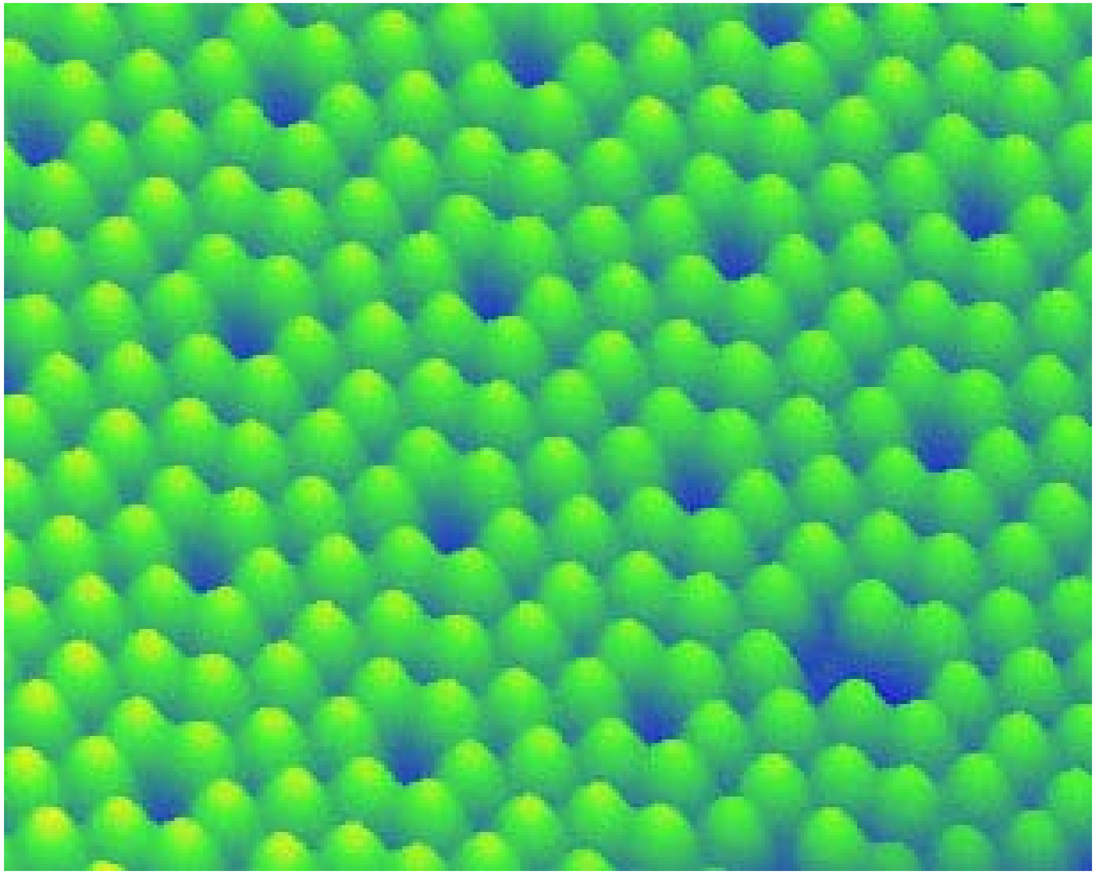
Tarzan and the vine



If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most information in the fewest words? I believe that it is the atomic hypothesis (or the atomic fact) that all things are made of atoms – little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.”

Richard Feynman
in *The Feynman Lectures on Physics*



Silicon atoms

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A model of a solid: balls connected by springs

All things are made of atoms:

atoms are about 10^{-10} m in size

atoms attract each other when close enough (0.5 nm)

atoms repel each other when too close

atoms are in perpetual motion

atoms can form more complicated structures via bonding with others

Atomic mass

Each atom is an elementary unit of a chemical element, defined by an **atomic number** Z (the number of protons in the atom).

The mass of an atom is determined basically by the number of protons and neutrons in the atom.

It is specified as the atomic mass m_{at} in atomic mass units

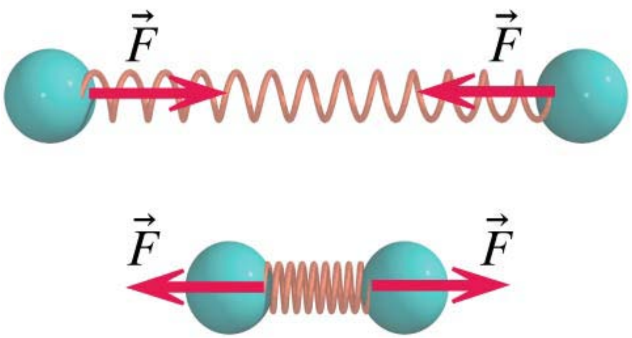
(1 u is about the mass of a proton or neutron: the atomic mass in u is roughly the number of nucleons in that atom).

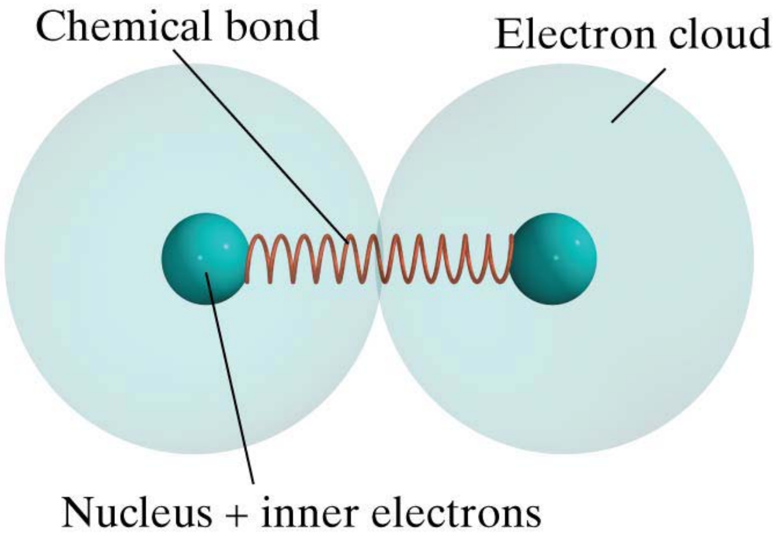
The **molar mass** m_{mol} is the mass of one mole (that is Avogadro's number $N_A = 6.023 \times 10^{23}$) of items.

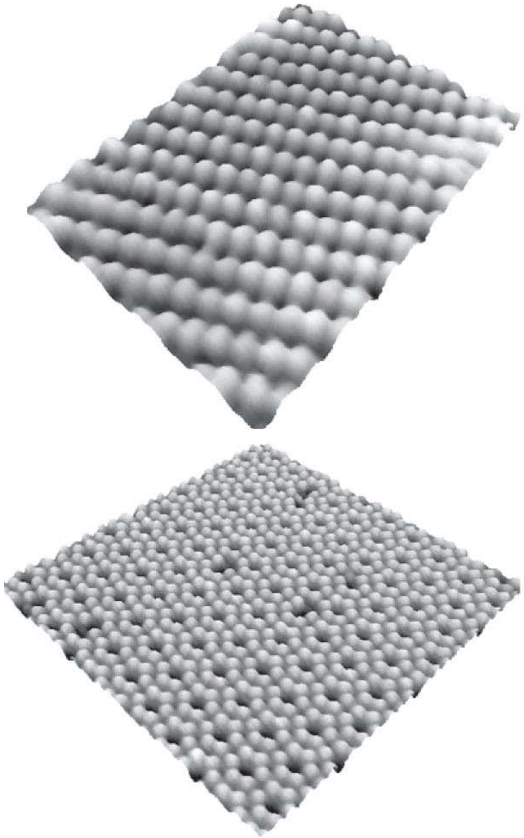
This number is defined so that m_{at} in u equals m_{mol} in g.

Substance	**Molar mass** [g]
Hydrogen molecule H_2	2.0
Carbon atom C	12.0
Oxygen atom O	16.0
Iron atom Fe	55.8
Lead atom Pb	207.2

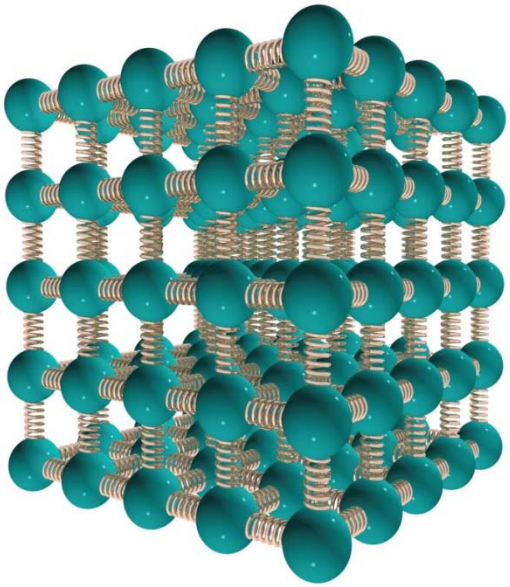
A chemical bond is like a spring







A ball-spring model for a solid object



Ball and spring model of solids.

… simple model of solid — very much a “model”

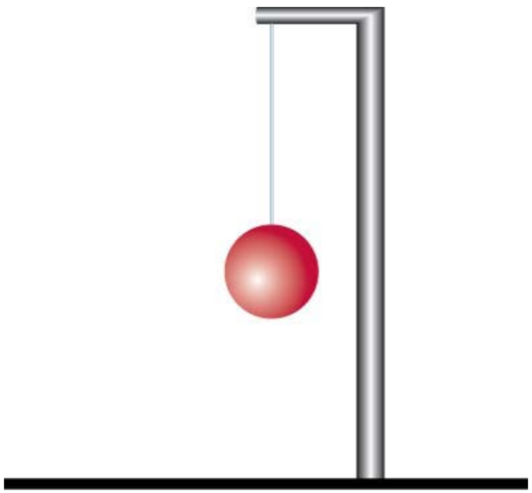
… useful for describing thermal properties

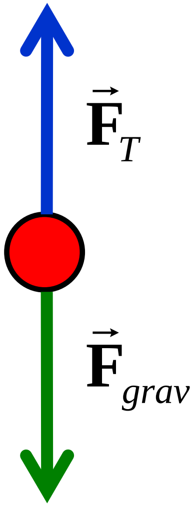
But …

… need QM to explain conductivity, etc, etc.

… emergence of properties of solids

Tension forces

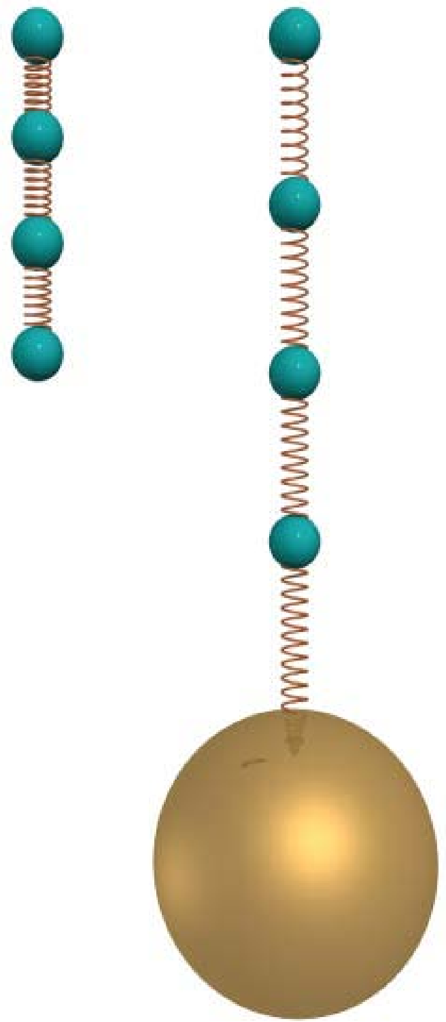




Tension in wire = Weight of ball

$$\vec{\mathbf{F}}_T = -\vec{\mathbf{F}}_{grav}$$

$$F_T - mg = 0$$



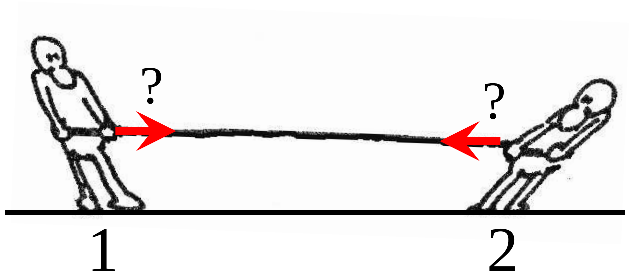
We know we can exert forces using a wire, string, rope, etc.

These forces are known as tension forces.

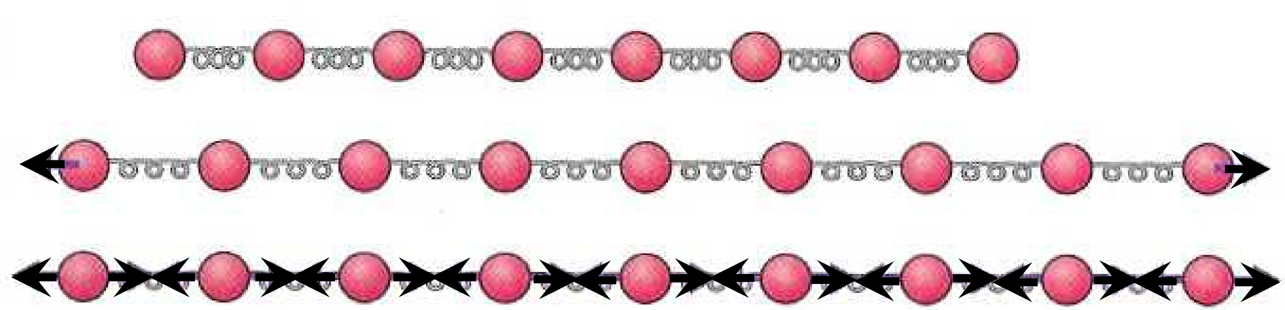
These forces arise because we can stretch the bonds a bit — they act like springs.

Tension in ropes

What is the force of Man 1 on Man 2?
What is the force of Man 2 on Man 1?

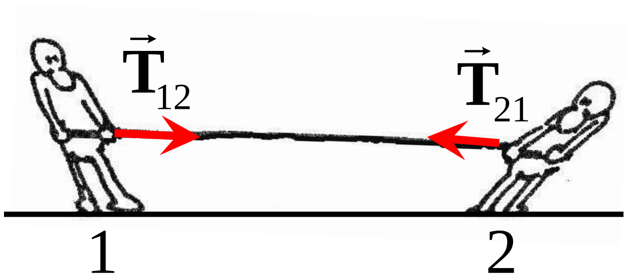


Think of the rope being made up of a long chain of single atoms, each interacting by inter-atomic (spring) forces):



Each atom exerts an equal force on its neighbour, with the resultant force on each individual atom being zero (since the rope is not accelerating).

$$\vec{\mathbf{T}}_{12} = -\vec{\mathbf{T}}_{21}$$



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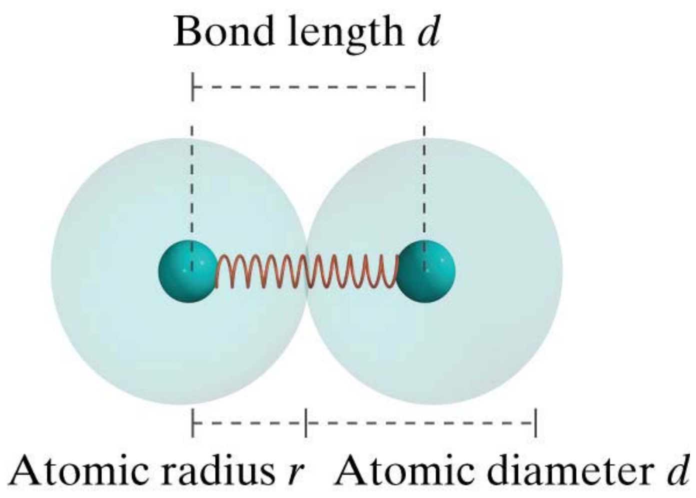
Length of an interatomic bond



Define bond length d as the centre-to-centre distance between atoms.

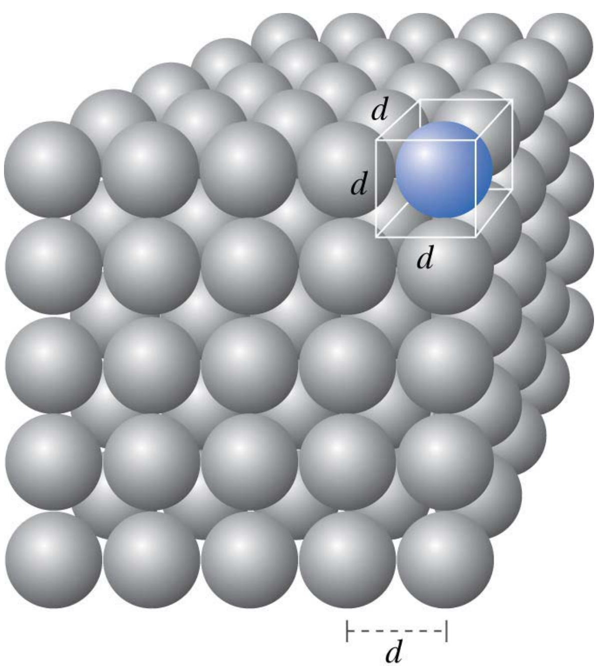
For example, one mole of copper has a mass of 64 g.
The density of copper is 8.94 g cm^{-3}.
What is the approximate diameter of a copper atom in solid copper?

Take a cube of copper: $$d^3 = \frac{V}{N} = \frac{m_{mol}}{\rho}\frac{1}{N_A}$$

Get $d = 2.28 \times 10^{-10}$ m

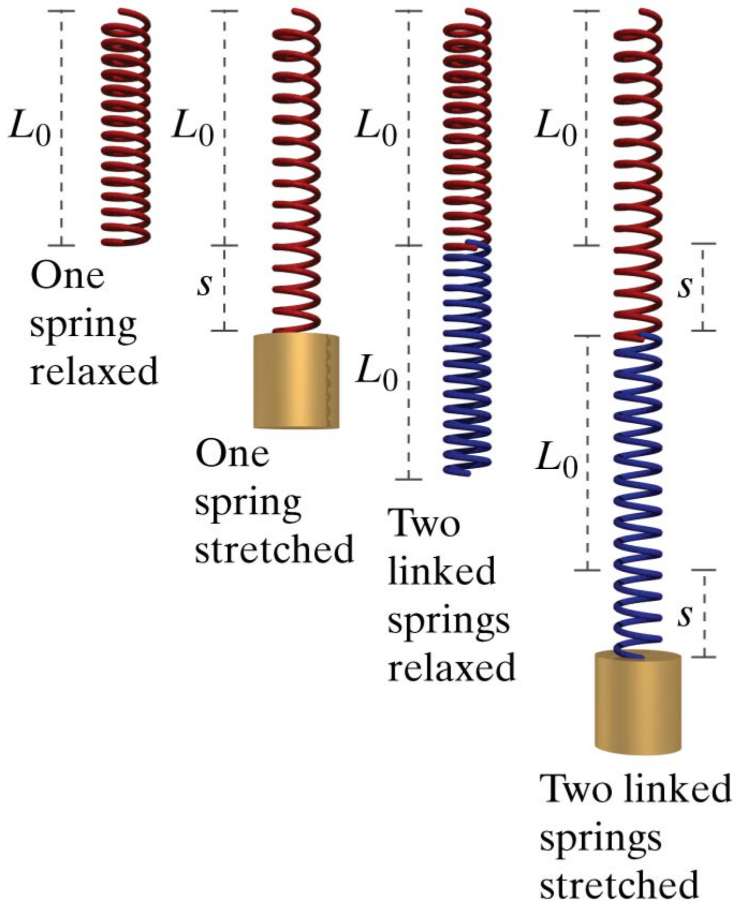
Alternative approach: use a microscopic view of density

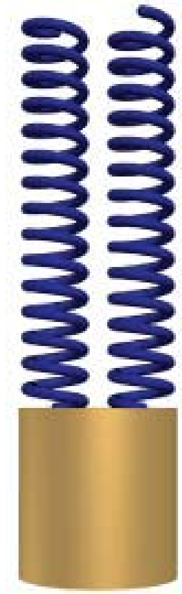
$$\text{Mass of one atom } m_a = \frac{\text{mass of one mole}}{N_A}$$



Then $\rho = \dfrac{d^3}{m_a}$

The stiffness of an atomic bond

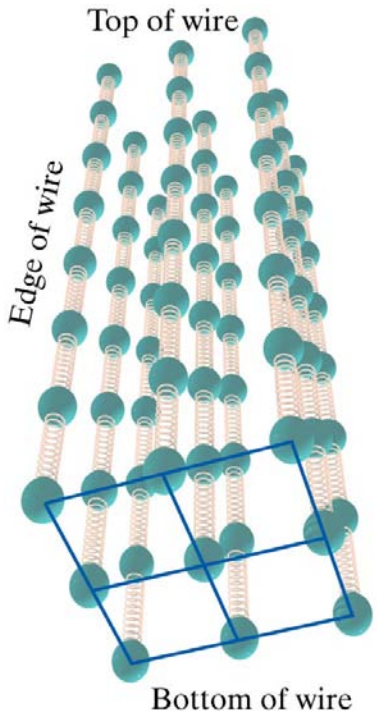




What about this situation?

The long spring (red + blue) has half the spring constant of the short spring (red only)

Interatomic bond stiffness in copper



Say that a 2 m copper wire (having square cross section) is 1 mm wide. When you hang a 10 kg mass at the end of the wire, careful measurement shows that the wire is now 1.51 mm longer.

$$k_{s,wire} = \frac{mg}{s} = \frac{(10 \text{ kg})(9.8 \text{ N/kg})}{(1.51 \times 10^{-3} \text{ m})} = 6.49 \times 10^{4} \text{ N/m}$$

Number of side-by-side atomic chains:

$$N_{chains} = \frac{A_{wire}}{A_{atom}} = \frac{(1 \times 10^{-3} \text{ m})^2}{(2.28 \times 10^{-10} \text{ m})^2} = 1.92 \times 10^{13}$$

Number of interatomic bonds in one atomic chain:

$$N_{\text{bonds in 1 chain}} = L_{wire}/d = (2 \text{ m})/(2.28 \times 10^{-10} \text{ m})^2 = 8.77 \times 10^{9}$$

Then: $$k_{s,i} = \frac{(6.49 \times 10^{4} \text{ N/m})(8.77 \times 10^{9})}{(1.92 \times 10^{13})} = 29.6 \text{ N/m}$$

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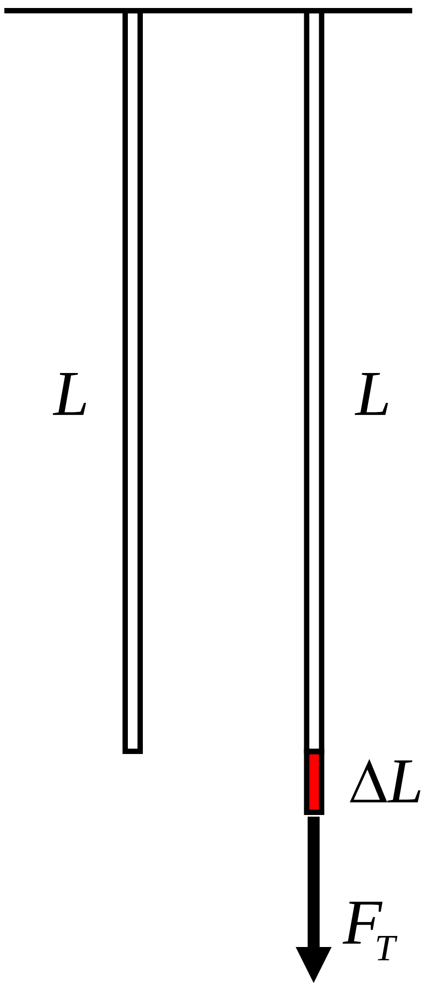
3E 4.6

Stress, strain, and Young's modulus

We take a wire of length L and stretch it by applying an external force F_T.
Say that the wire stretches by an amount ΔL.



Then the "strain" $\equiv \dfrac{\Delta L}{L}$

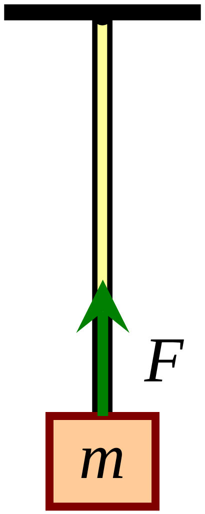
The tension force
per unit area is called the "stress" $\equiv \dfrac{F_T}{A}$

Young's modulus $Y \equiv \dfrac{\text{stress}}{\text{strain}} = \dfrac{\left(\dfrac{F_T}{A}\right)}{\left(\dfrac{\Delta L}{L}\right)}$

Young's modulus is property of the material.
Units of Y: N m^{-2} or Pa (pascal).

A bar or wire can be regarded as a non-helical macroscopic spring.

Write $Y = \dfrac{\left(\dfrac{F_T}{A}\right)}{\left(\dfrac{\Delta L}{L}\right)}$ as $F_T = \left(\dfrac{YA}{L}\right)|\Delta L|$



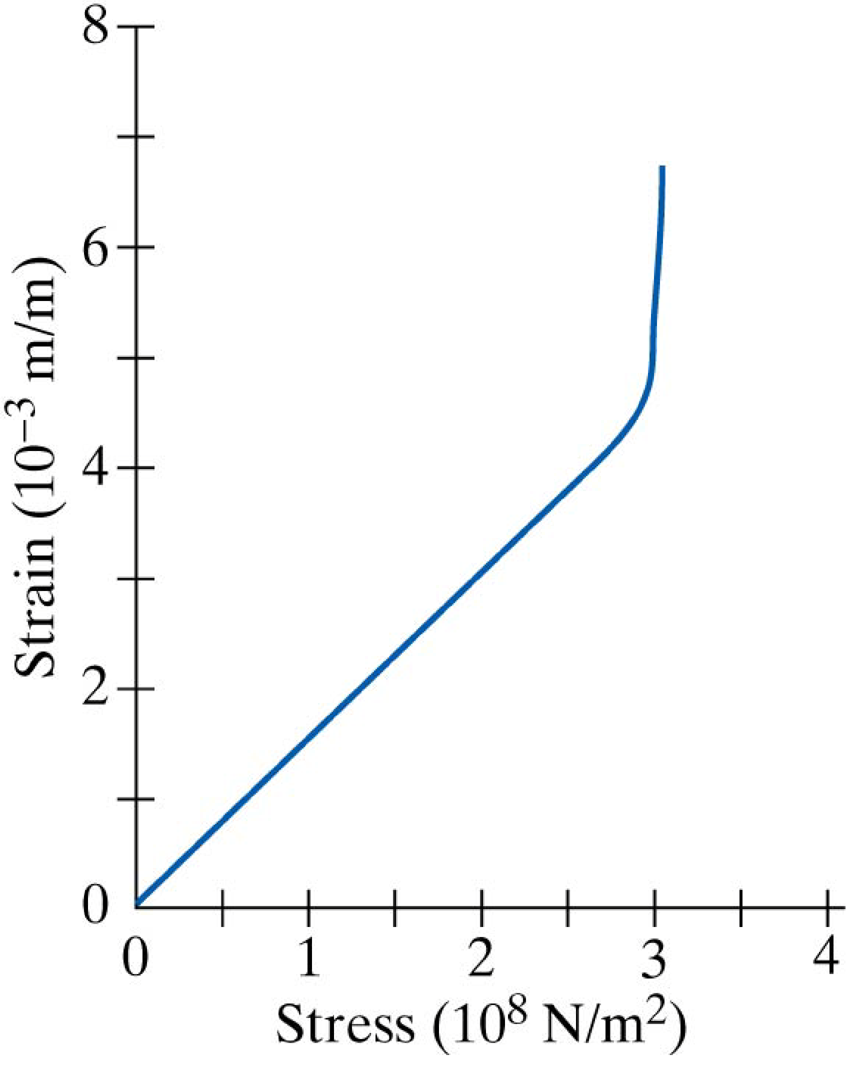
Compare with $F_{\text{restore}} = k_s|x|$ for a spring

Elastic oscillations in a wire

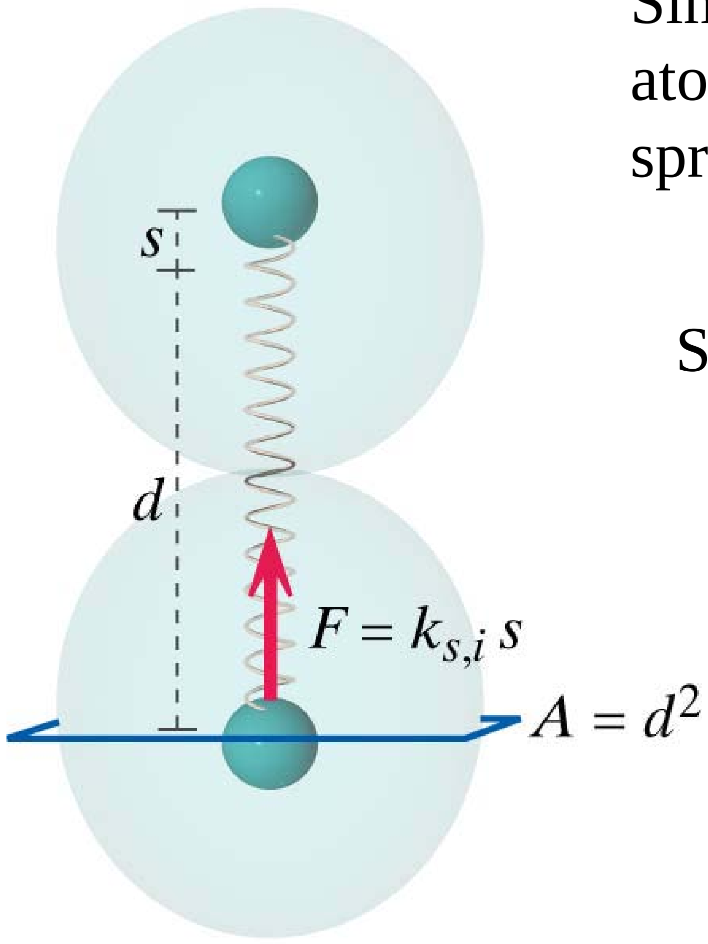
Here F is restoring force in wire

$$ma = -\frac{AYx}{L} \quad \text{and} \quad \omega_0 = \sqrt{\frac{AY}{mL}}$$

Limit of applicability of Young's Modulus



Relating Young's Modulus to interatomic spring stiffness



Simple model:
atoms d apart held together by springs of stiffness k_s.

$$\text{Strain per atom} = \frac{s}{d}$$

$$\text{Atomic stress} = \frac{k_s s}{d^2}$$

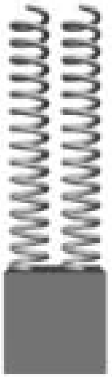
Then $$Y = \frac{k_s s}{d^2} \bigg/ \frac{s}{d} = \frac{k_s}{d}$$

1	2	3	4	5

A short spring has a stiffness of 20 N/m.
You link 4 of these springs end to end to make a longer spring. What is the stiffness of the longer spring?

(1) 0.2 N/m
(2) 5 N/m
(3) 20 N/m
(4) 80 N/m

1 2 3 4 5



You hang a 1 kg mass from a spring, which stretches 0.4 m. You place a second identical spring beside the first, so the 1 kg mass is now supported by two springs.

How much does each spring stretch?

(1) 0.2 m

(2) 0.4 m

(3) 0.5 m

(4) 0.8 m

1	2	3	4	5

Lead is much softer than aluminum, and can be more easily deformed or pulled into a wire.
What difference between the two materials best explains this?

(1) Pb and Al atoms have different sizes.

(2) Pb and Al atoms have different masses.

(3) The stiffness of the interatomic bonds is different in Pb and Al.

1	2	3	4	5

You hang a 10 kg mass from a copper wire, and the wire stretches by 8 mm.
Now you hang the same mass from two copper wires, identical to the first.

What happens?

(1) Each wire stretches 4 mm.

(2) Each wire stretches 8 mm.

(3) Each wire stretches 16 mm.

1	2	3	4	5

You hang a 10 kg mass from a copper wire, and the wire stretches by 8 mm.
Now you hang the same mass from a second copper wire, whose cross-sectional area is half as large (but whose length is the same).

What happens?

(1) The second wire stretches 4 mm.

(2) The second wire stretches 8 mm.

(3) The second wire stretches 16 mm.

1	2	3	4	5

You hang a 10 kg mass from a copper wire, and the wire stretches by 8 mm.
Now you hang the same mass from a second copper wire, which is twice as long, but has the same diameter.

What happens?

(1) The second wire stretches 4 mm.

(2) The second wire stretches 8 mm.

(3) The second wire stretches 16 mm.

1	2	3	4	5

Two wires with equal lengths are made of pure copper. The diameter of wire A is twice the diameter of wire B. When 6 kg masses are hung on the wires, wire B stretches more than wire A.

You make careful measurements and compute Young's modulus for both wires. What do you find?

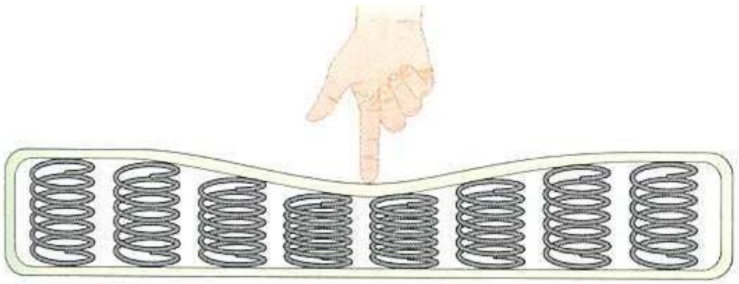
(1) $Y_A > Y_B$

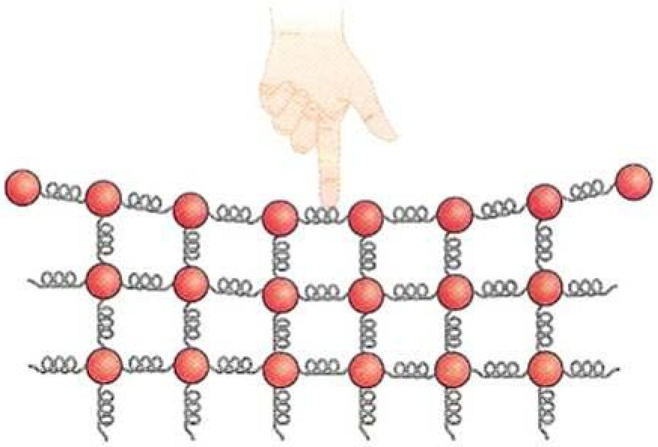
(2) $Y_A = Y_B$

(3) $Y_A < Y_B$

Compression (normal) forces

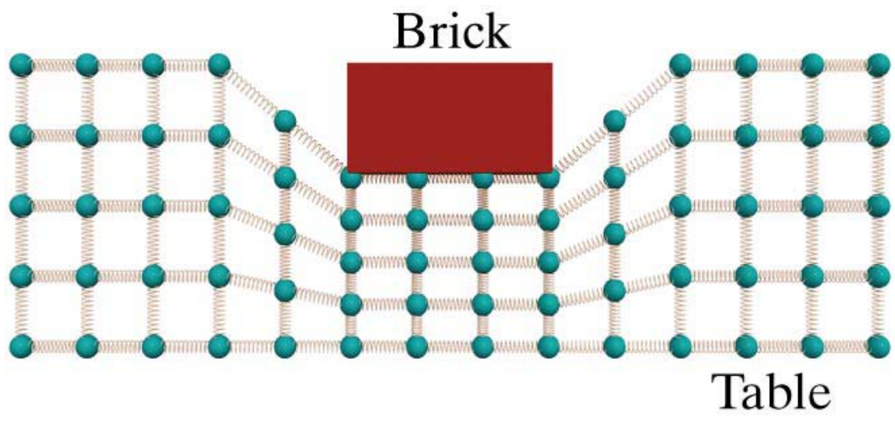
We can think of compressing a solid in the same way as compressing a spring mattress:





(very much exaggerated)

The harder you press, the more compressed the springs become.

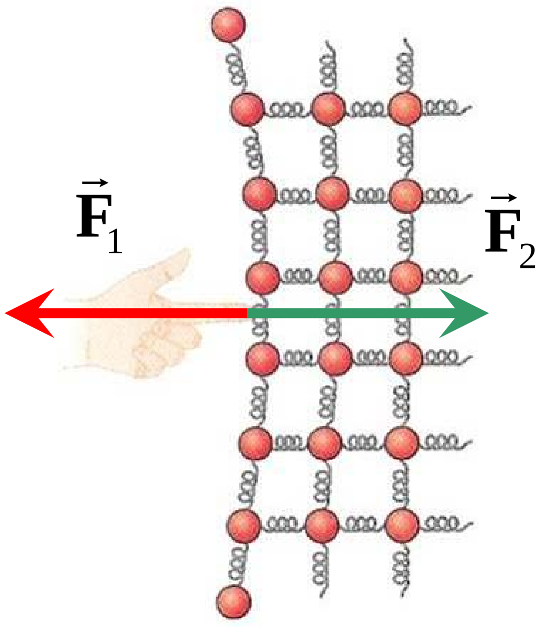


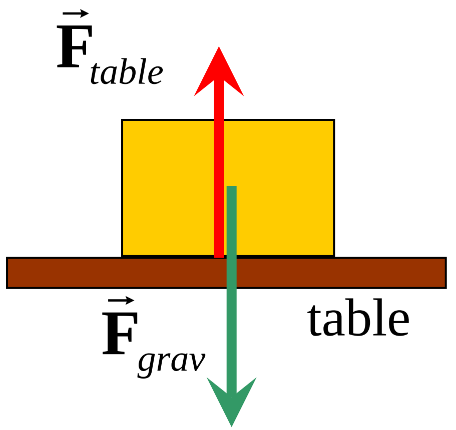
The Normal force

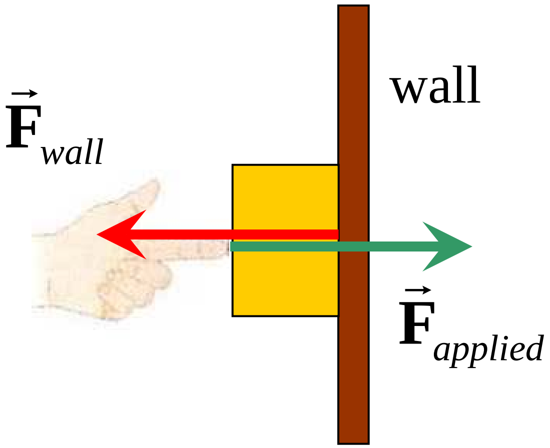
$$\vec{\mathbf{F}}_1 = -\vec{\mathbf{F}}_2$$

where:

$\vec{\mathbf{F}}_1$ Force exerted by wall on hand
$\vec{\mathbf{F}}_2$ Force exerted by hand on wall



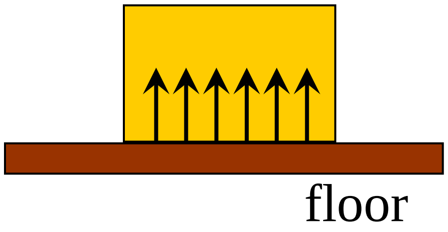


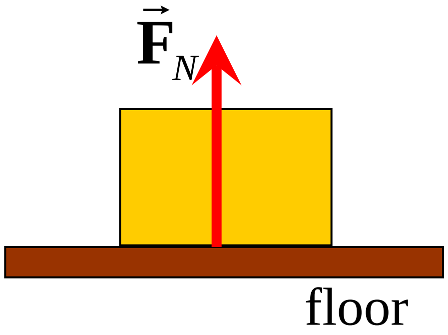


There is a normal force whenever a body is in contact with another body.

... normal force continued...

The normal force exerted by a surface on another body is actually the sum of billions of interactions between the surface atoms in the surface and the block. We use a single force vector to summarize all these forces since we are modeling the block as a point particle.





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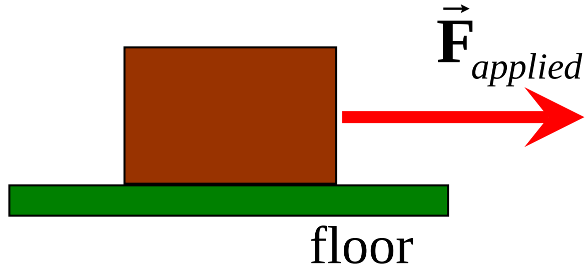
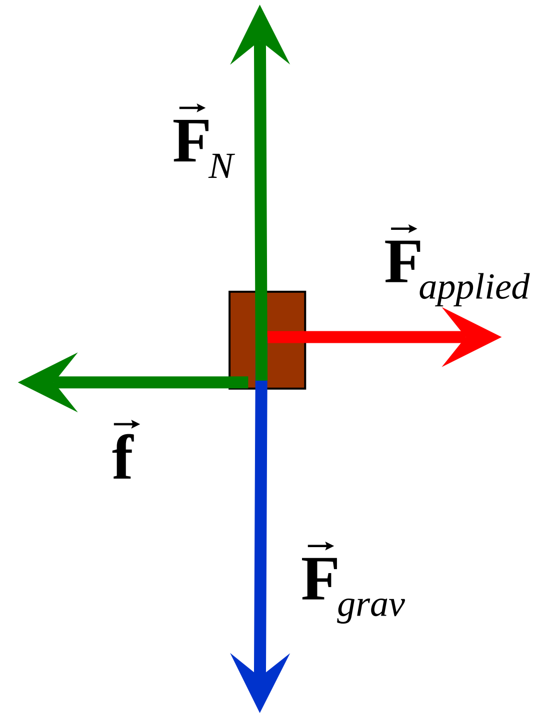
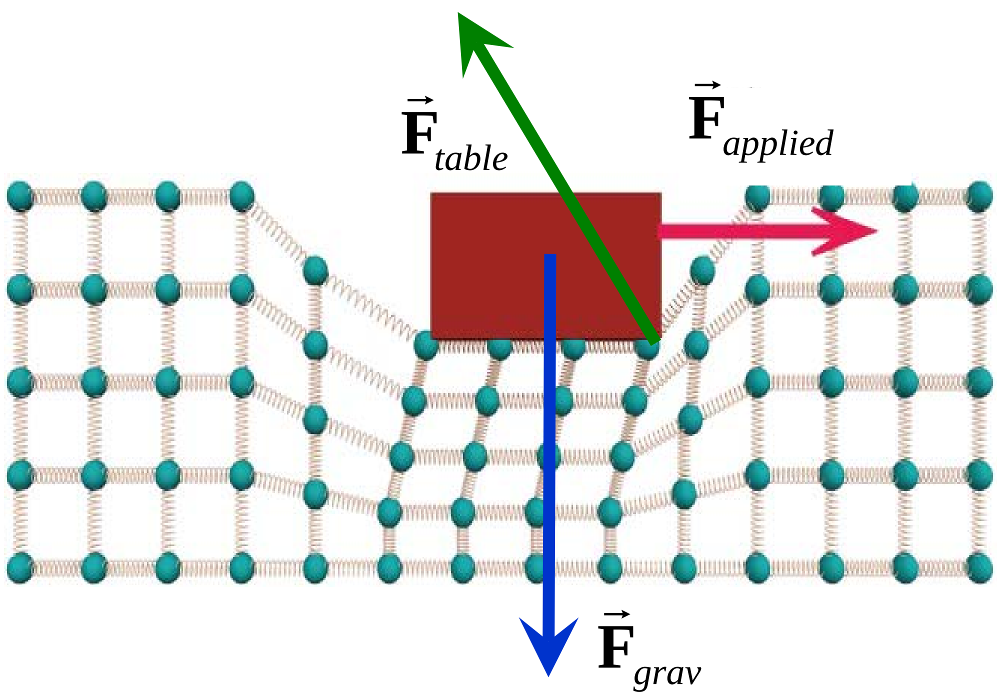
Friction



A box is pulled across a rough floor.





$$\vec{\mathbf{F}}_{table} = \vec{\mathbf{f}} + \vec{\mathbf{F}}_N$$

where $\vec{\mathbf{f}}$ is the "friction" force.

Friction

Friction acts parallel to the contact surface and opposes the motion of the object.

From experiment, we find that the frictional force:

- ... is proportional to the normal force
- ... is independent of the area of contact
- ... depends on whether the object is stationary or sliding

Two types of friction: **kinetic friction** and **static friction**

static friction: $f_s = \mu_s N$

kinetic friction: $f_k = \mu_k N$

where μ_s **coefficient of static friction**

μ_k **coefficient of kinetic friction**

$\left.\right\}$ measured in experiments

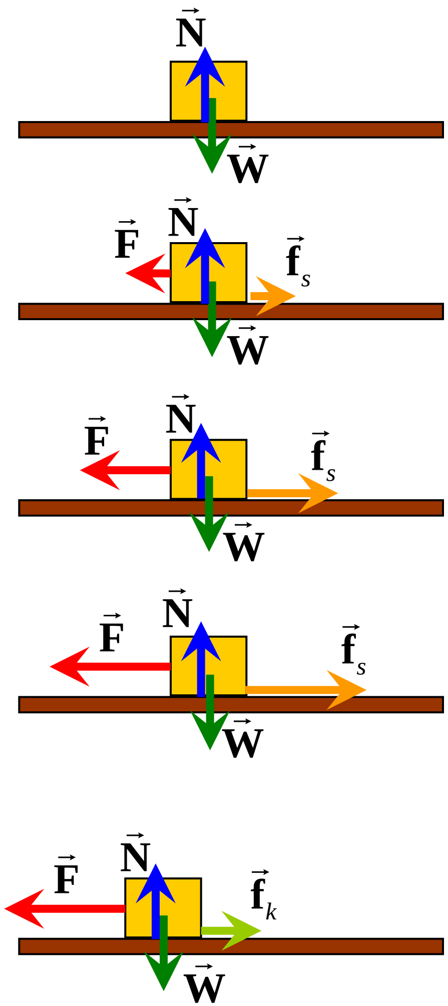
Generally $\mu_s \geq \mu_k$ for a given situation

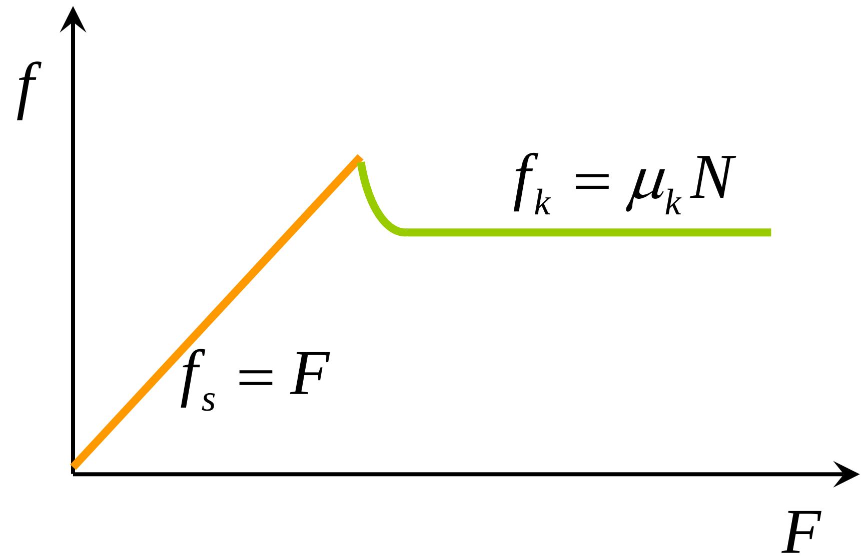
Some approximate coefficients of friction

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminium on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25 - 0.5	0.2
Glass on glass	0.94	0.4
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

Friction continued ...

Consider the motion of a block on a surface with friction:





1	2	3	4	5

She holds a book stationary against the wall as shown. Friction on the book by the wall acts

(A) upward
(B) downward
(C) can't say

1	2	3	4	5

You push a 100 kg mass on the floor with a horizontal force of 400 N.   It doesn't move. The coefficient of static friction is 0.6.

What is the magnitude of the frictional force on the block by the floor?

(1) 980 N

(2) 588 N

(3) 400 N

(4) Can't tell

1	2	3	4	5

You push an initially stationary 100 kg mass on the floor with a horizontal force. The coefficient of static friction is 0.6.

What is the minimum amount of force you need to exert on the mass in order to get it to move?

(1) 980 N

(2) 588 N

(3) 400 N

(4) Can’t tell

1	2	3	4	5

You push a 100 kg mass on the floor with a horizontal force of 400 N, and it's moving in the direction you are pushing. The coefficient of static friction is 0.3.

What happens to the speed of the block while you push it?

(1) The speed increases

(2) The speed decreases

(3) The speed does not change

(4) Can't tell

1	2	3	4	5

You push a 100 kg mass on the floor with a horizontal force, and it's moving in the direction you are pushing at a *constant* speed. The coefficient of static friction is 0.3.

How much force are you exerting on the block?

(1) 980 N

(2) 294 N

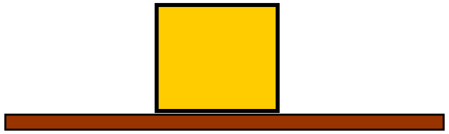
(3) 490 N

(4) Can't tell

Identifying forces

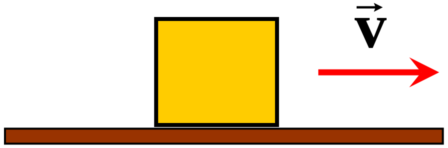
In each of the situations below, identify all the significant forces acting on the system, as specified.
Draw in each force as a vector arrow and label it.

(a)



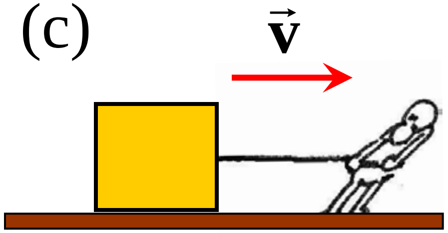
A box is at rest on the floor
System: the box

(b)

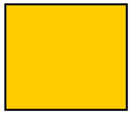


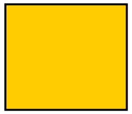
A box is moving at constant velocity on a frictionless surface.
System: the box

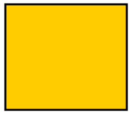
(c)



A box is pulled along a floor with friction at constant velocity.
System: the box

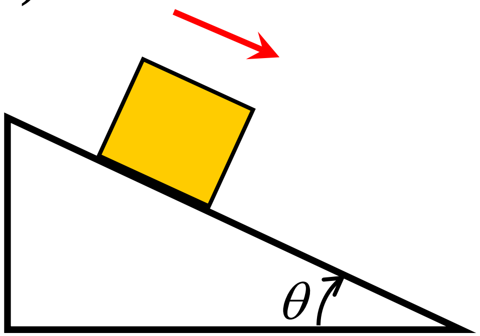




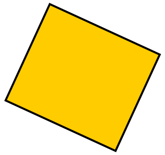


Identifying forces 2 ...

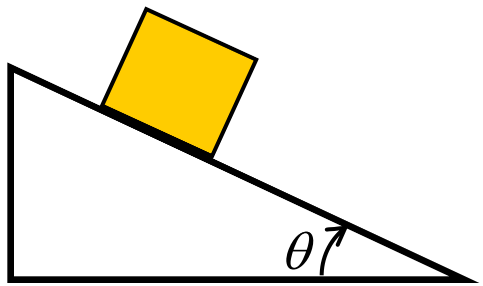
(d)



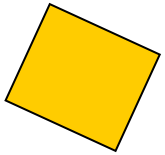
A box slides down a frictionless slope.
<u>System</u>: the box



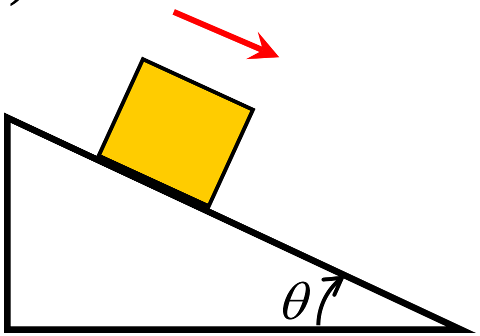
(e)



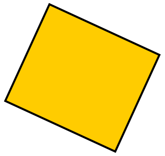
A box is at rest on a slope.
<u>System</u>: the box



(f)

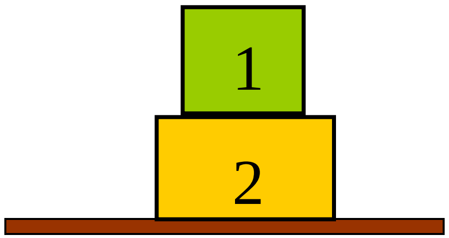


A box slides down a slope with friction.
<u>System</u>: the box



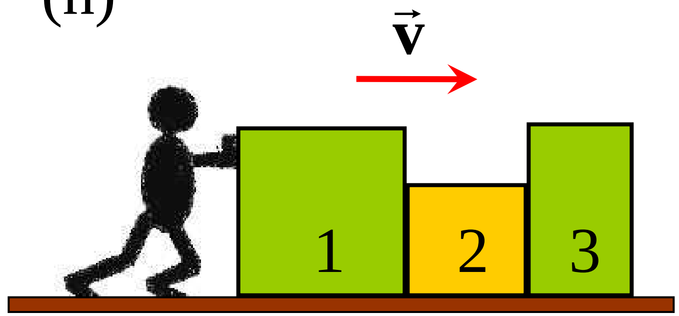
Identifying forces 3 ...

(g)



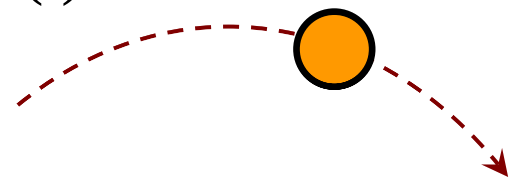
Two blocks are stacked on the floor.
System: box 2

(h)

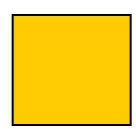


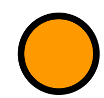
A man pushes 3 blocks across a floor with friction at a constant velocity.
System: box 2

(i)



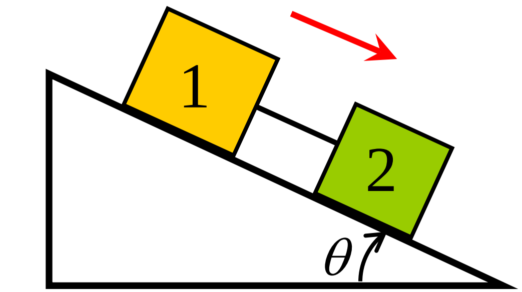
A ball moves through the air in a parabolic path.
System: the ball





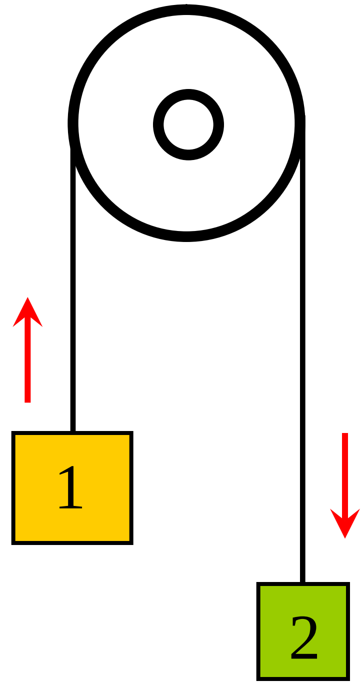
Identifying forces 4 ...

(j)



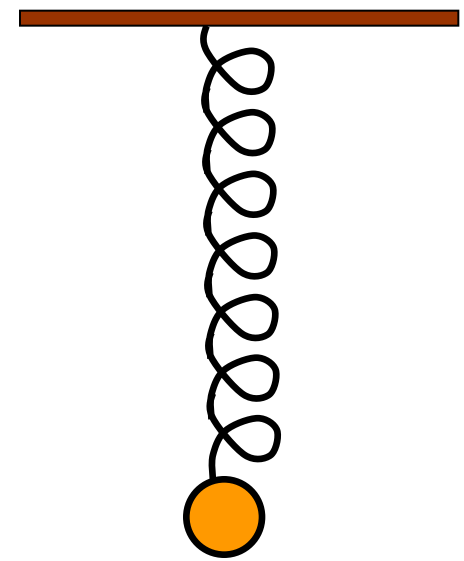
Two boxes connected by a rope slide down a slope with friction.
<u>System 1</u>: box 1
<u>System 2</u>: box 2

(k)

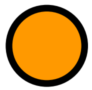


Two boxes connected by a rope slide over a frictionless pulley.
<u>System 1</u>: box 1
<u>System 2</u>: box 2

(l)

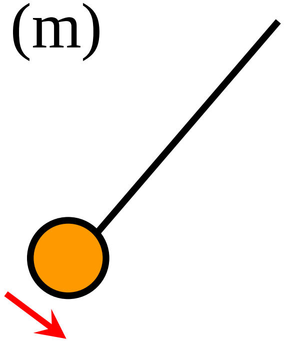


A heavy ball hangs stationary at the end of an extended spring.
<u>System</u>: the ball

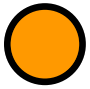


Identifying forces 5 ...

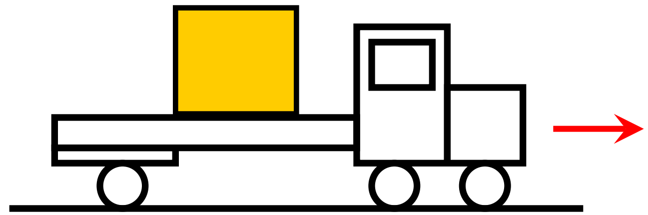
(m)



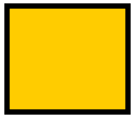
A heavy ball swings at the end of a long string.
System: the ball



(n)



A box sits at the back of a truck which is accelerating towards the right.
System: the box



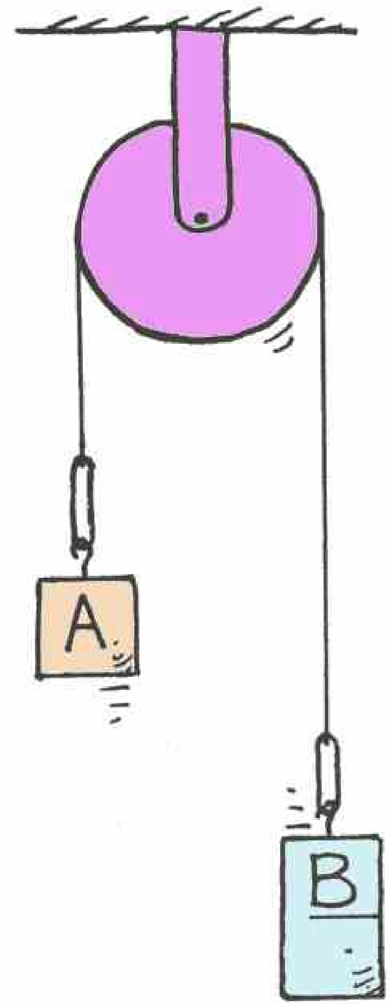
(o)

A walking man.
System: the man

1	2	3	4	5

Two identical rubber bands connect masses A and B to a string over a light, frictionless pulley. The amount of stretch is greater in the band that connects

(A) mass A
(B) mass B
(C) both the same



Speed of sound in a solid

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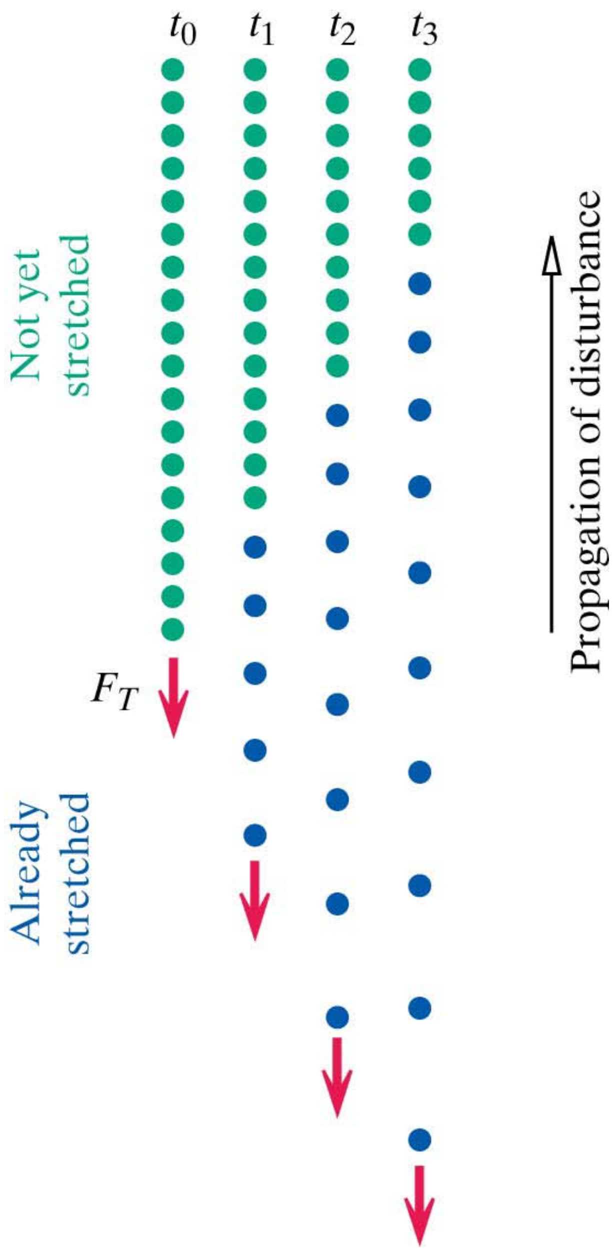
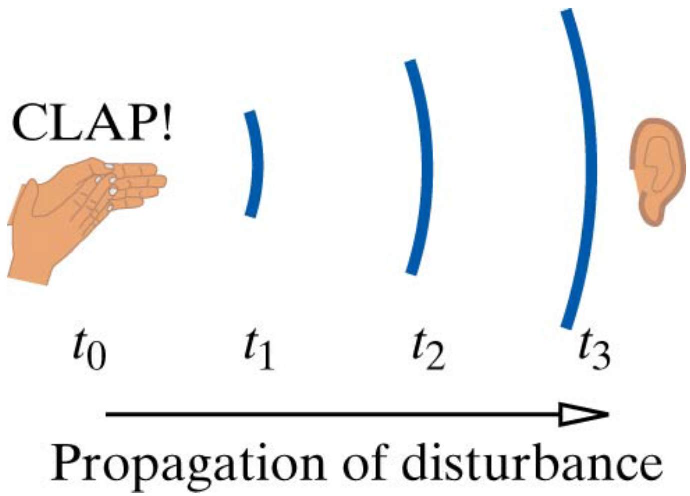
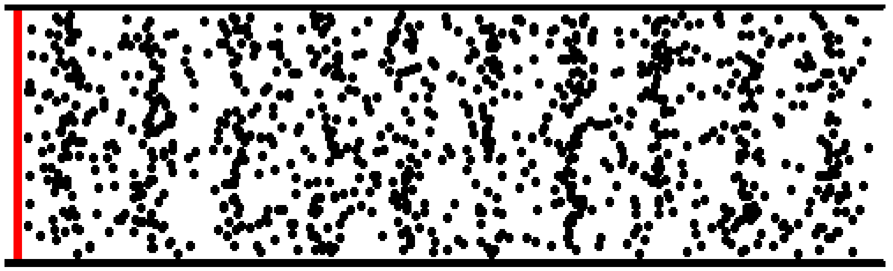
3E 4.9







longitudinal wave

M&I
3E 4.10

Derivative form of the momentum principle

Starting with $$\frac{\Delta\vec{\mathbf{p}}}{\Delta t}=\vec{\mathbf{F}}_{net}$$

then $$\lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{p}}}{\Delta t}=\frac{d\vec{\mathbf{p}}}{dt}$$

If the mass is constant then $$\frac{d\vec{\mathbf{p}}}{dt}=\frac{d\left(m\vec{\mathbf{v}}\right)}{dt}$$

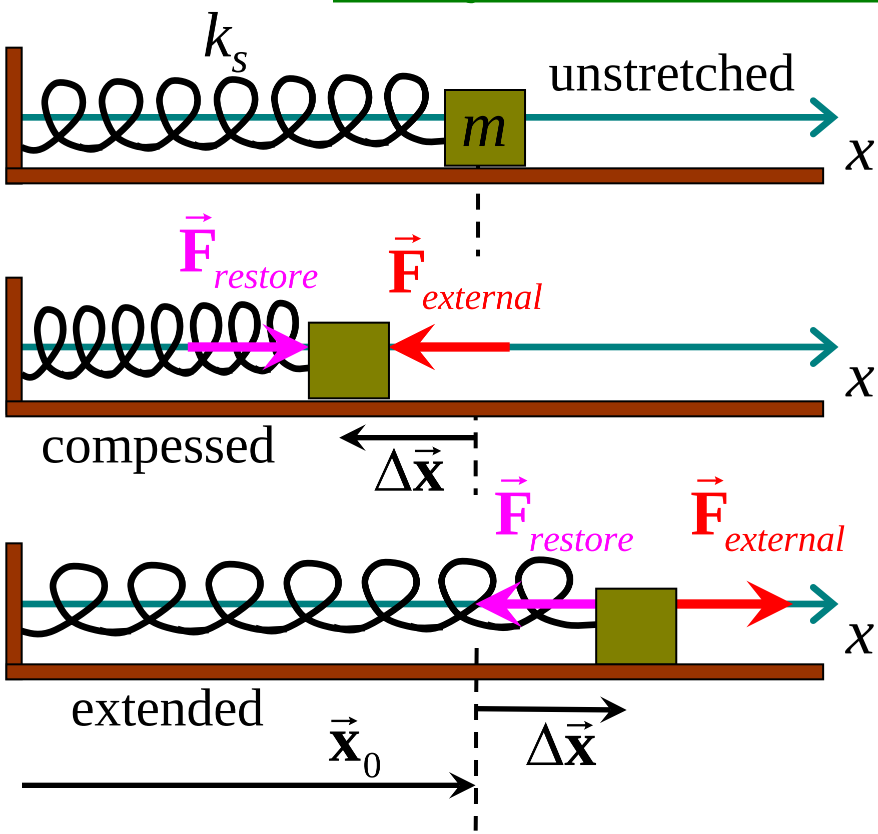
$$=m\frac{d\vec{\mathbf{v}}}{dt}+\vec{\mathbf{v}}\frac{dm}{dt}$$

$$=m\frac{d\vec{\mathbf{v}}}{dt}+0$$

Thus $$\frac{d\vec{\mathbf{p}}}{dt}=\vec{\mathbf{F}}_{net}=m\vec{\mathbf{a}}$$

Analytical solution: spring-mass system

M&I
3E 4.11



Hooke's Law:

Restoring force,

$$\vec{\mathbf{F}}_{restore} = -k_s \Delta\vec{\mathbf{x}}$$

where $\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_0$

and k_s is the "spring constant" [N m^{-1}]

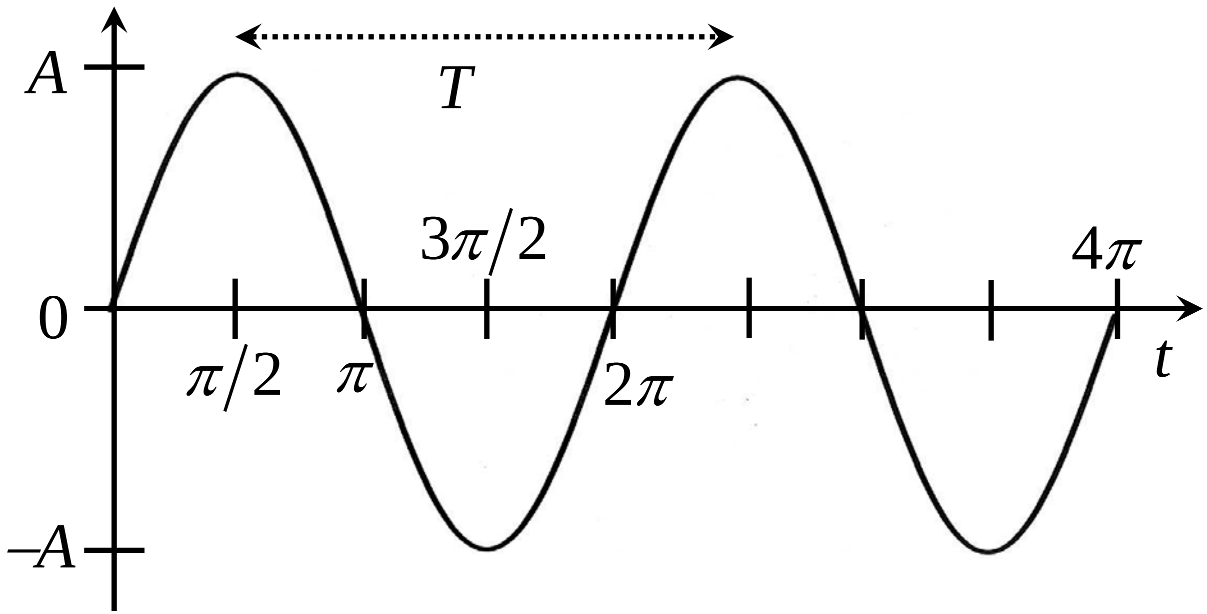
Start with the momentum principle: $\dfrac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{\text{net}}$

For horizontal forces on the mass: $\dfrac{dp_x}{dt} = -k_s x$

$$\therefore \frac{d(mv_x)}{dt} = -k_s x \quad \text{or} \quad \frac{d}{dt}\left(m\frac{dx}{dt}\right) = -k_s x$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{k_s}{m}x$$

Consider the function $x(t) = A\cos(\omega t + \phi)$



$\omega T = 2\pi$ where T : period (s)

ω : angular frequency (rad s^{-1})

some books use υ $f = \dfrac{1}{T}$ where f : frequency (Hz)

A : Amplitude

ϕ : phase angle, initial phase or phase constant

Mass-spring oscillator ...2

$$\frac{d^2x(t)}{dt^2} = \frac{-k_s}{m}x(t)$$

... a second order differential equation
... we know that if we displace a mass-spring system from its rest position and then release it, it will perform SHM ...

Guess a trial solution: $x(t) = A\cos(\omega t + \phi)$

then $\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$

and substitute into our DE: $-A\omega^2\cos(\omega t + \phi) = -A\frac{k_s}{m}\cos(\omega t + \phi)$

... which is true provided $\omega^2 = \frac{k_s}{m}$

Therefore our solution is $x(t) = A\cos(\omega t + \phi)$ where $\omega = \sqrt{\frac{k_s}{m}}$

Simple harmonic oscillator

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi)$$

$$a(t) = \frac{d^2x(t)}{dt^2} = \frac{dv(t)}{dt} = -A\omega^2\cos(\omega t + \phi)$$

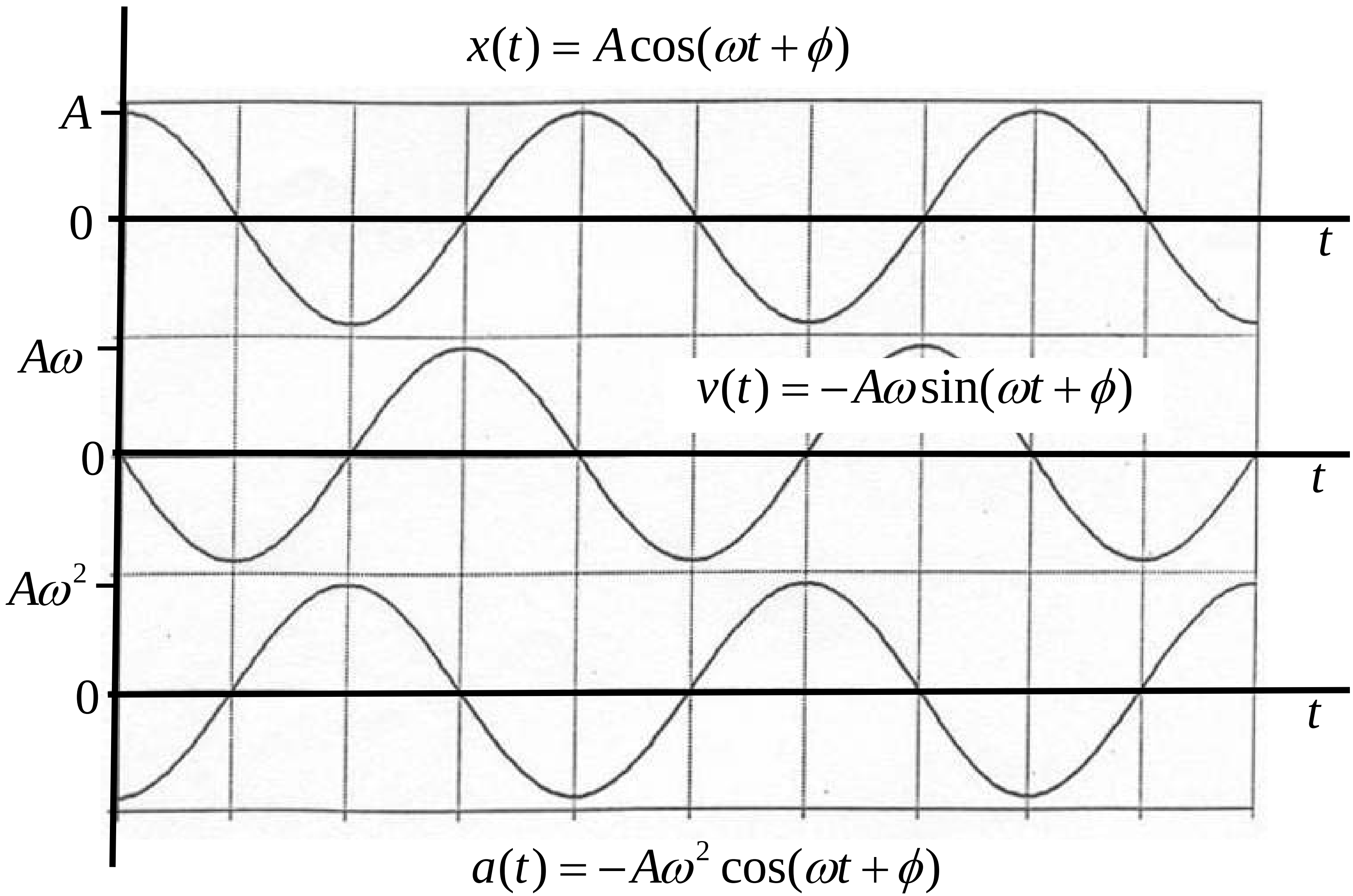
… acceleration = – (constant) . (displacement)

$$= -A\omega^2\cos(\omega t + \phi)$$

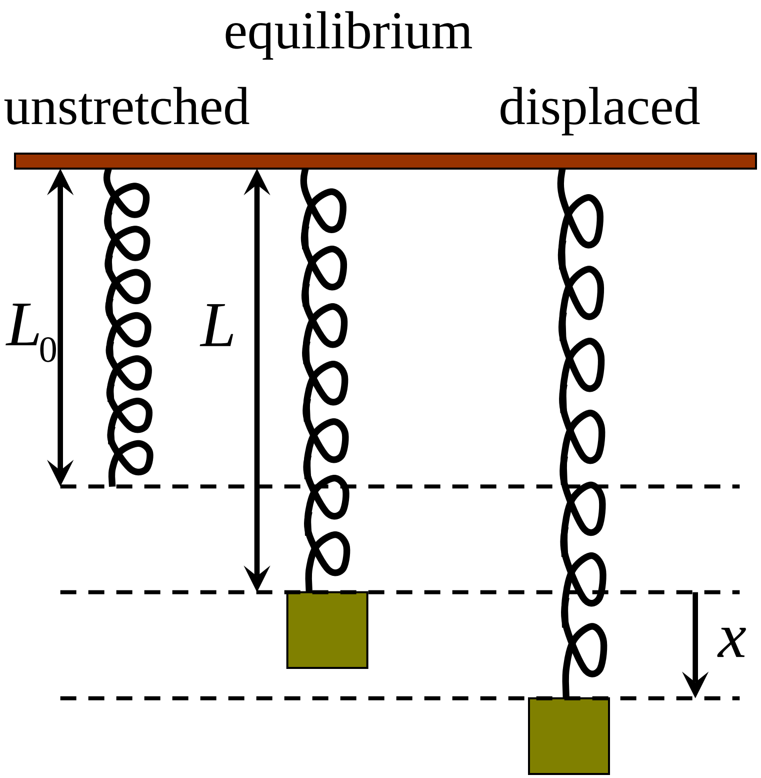
$$= A\omega^2\cos(\omega t + \phi + \pi)$$

Phase difference between acceleration and displacement is π

Phase difference between v and x (and v & a) is $\frac{\pi}{2}$


$x(t) = A\cos(\omega t + \phi)$
A
0
t
$A\omega$
$v(t) = -A\omega \sin(\omega t + \phi)$
0
t
$A\omega^2$
0
t
$a(t) = -A\omega^2 \cos(\omega t + \phi)$

Mass suspended from a light spring



Equilibrium: $k(L-L_0)=mg$

Displaced:

Force on mass due to spring:

$$=k(L-L_0)+kx \qquad =mg+kx$$

(upwards)

Net force on mass: $mg-(mg+kx)=-kx$ (downwards)

$$\therefore m\frac{d^2x}{dt^2}=-kx$$

(Same equation as for horizontal case)

1	2	3	4	5

Suppose the period of a spring-mass oscillator is 1 s.
What will be the period if we double the mass?

(1) T = 0.5 s

(2) T = 0.7 s

(3) T = 1.0 s

(4) T = 1.4 s

(5) T = 2.0 s

1	2	3	4	5

Suppose the period of a spring-mass oscillator is 1 s.
What will be the period if we double the spring stiffness?
(We could use a stiffer spring, or we could attach the mass to two springs.)

(1) $T = 0.5$ s

(2) $T = 0.7$ s

(3) $T = 1.0$ s

(4) $T = 1.4$ s

(5) $T = 2.0$ s

1	2	3	4	5

Suppose the period of a spring-mass oscillator is 1 s with an amplitude of 5 cm.
What will be the period if we increase the amplitude to 10 cm, so that the total distance traveled in one period is twice as large?

(1) $T = 0.5$ s

(2) $T = 0.7$ s

(3) $T = 1.0$ s

(4) $T = 1.4$ s

(5) $T = 2.0$ s

1	2	3	4	5

Suppose the period of a spring-mass oscillator is 1 s. What will be the period if we cut the spring in half and use just one of the pieces?

(1) T = 0.5 s

(2) T = 0.7 s

(3) T = 1.0 s

(4) T = 1.4 s

(5) T = 2.0 s

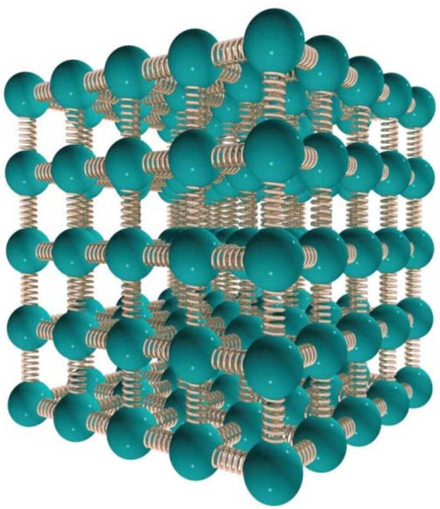
M&I
3E 4.12

Analytical expression for the speed of sound

Reasonable value for the speed of sound in air: 340 m s^{-1}

In aluminium, the speed of sound is around 5000 m s^{-1} and in lead around 1200 m s^{-1}

A disturbance in a model solid is transmitted by harmonic motion of a ball and spring system.



Dimensional analysis suggests that the magnitude of the velocity of such a disturbance is

$$v = \omega d = \sqrt{\frac{k_{s,i}}{m_a}}d$$

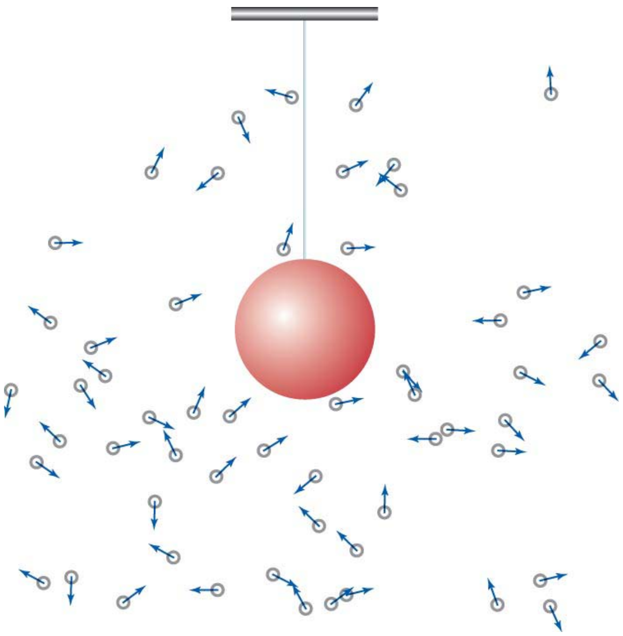
(This turns out to be the correct formula).

In solids, a macroscopic view for the speed of sound gives:

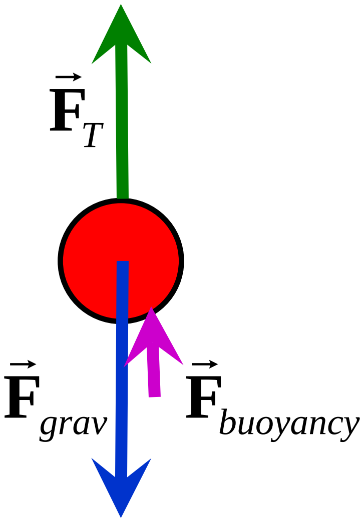
$$v^2 = \frac{k_s}{m}d^2 = \frac{k_s}{d}\frac{d^3}{m} = \frac{Y}{\rho}$$

Contact forces due to gases

Air molecules are in constant motion and exert a force on objects they collide with. Similar forces exist in liquids. We can estimate this force from macroscopic considerations. Note that it arises from atomic collisions.

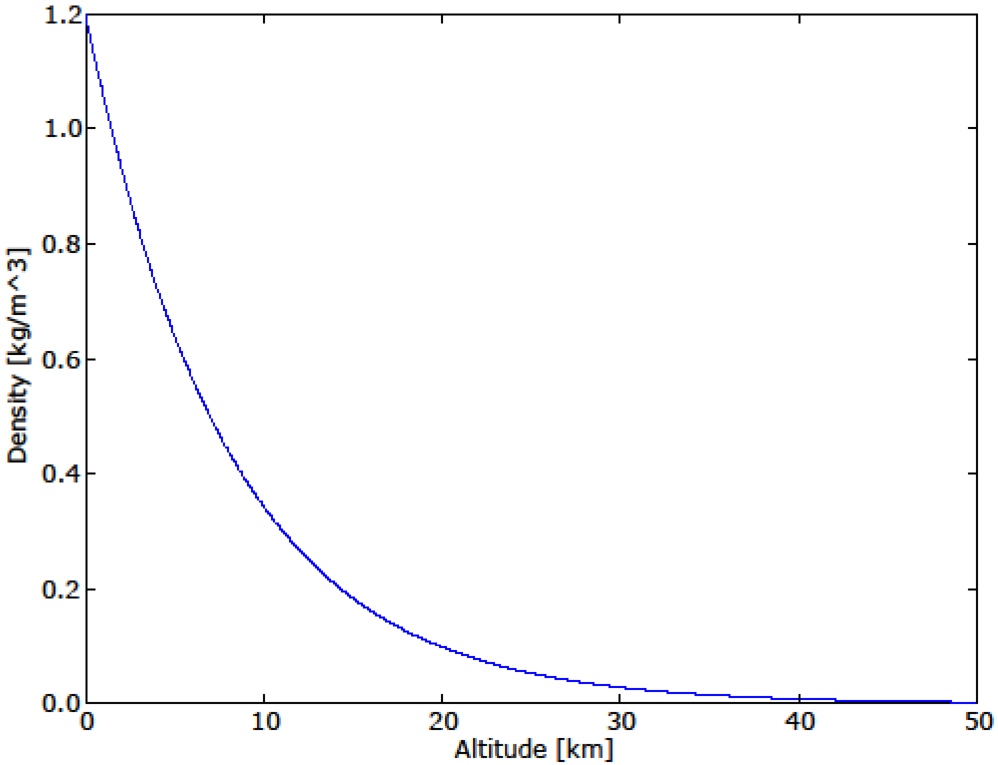


There are more collisions from air molecules at the bottom of the ball … gives rise to a **buoyancy force**.



Bouyancy

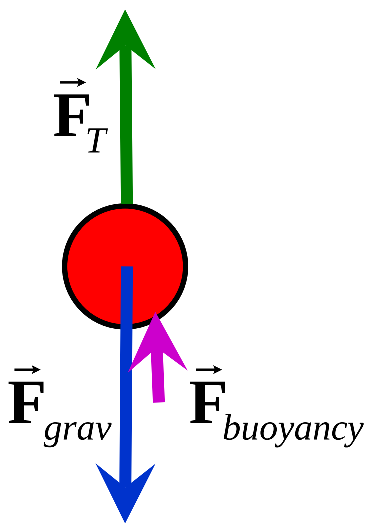
Example: Air density decreases with altitude.



Because of this variation of air density with altitude, there is an upward buoyant force on an object in the atmosphere.

Bouyancy

When can the bouyant force be neglected?



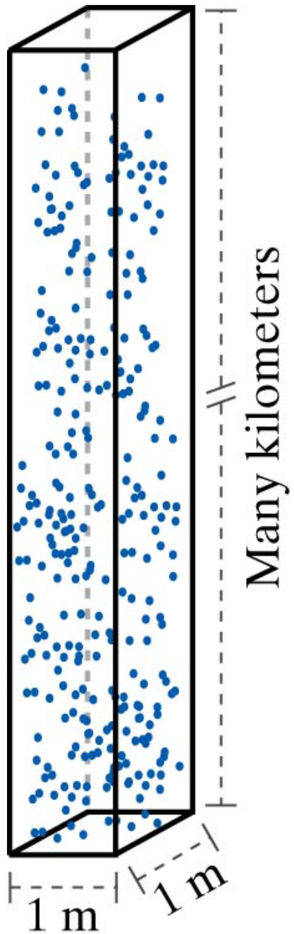
$$\frac{F_{bouyancy}}{F_{grav}} = \frac{m_{air}\,g}{M_{ball}\,g} = \frac{m_{air}}{M_{ball}} = \frac{m_{air}/V}{M_{ball}/V} = \frac{\rho_{air}}{\rho_{ball}}$$

"Reasonable air" at STP : 0 ºC and 1 atm …
.. occupies a volume of 22.4 litres.
Air about 80% N_2 (m_{mol} = 28)
and 20% O_2 (m_{mol} = 32)
Gives 29 g mol^{-1} for air

Then $$\rho_{air} \approx \frac{29\text{ g mol}^{-1}}{22.4 \times 10^{3}\text{ cm}^{-3}\text{ mol}^{-1}} \approx 1.3 \times 10^{-3}\text{ g cm}^{-3}$$

$$\left(\rho_{ball} \approx 1\text{ g cm}^{-3}\right)$$

Pressure



Pressure is defined as $P = \dfrac{F}{A}$

Units: pascals Pa (or N m^{-2})

Atmospheric pressure at sea level:
1 atmosphere $\approx 1.01 \times 10^5$ N m^{-2}

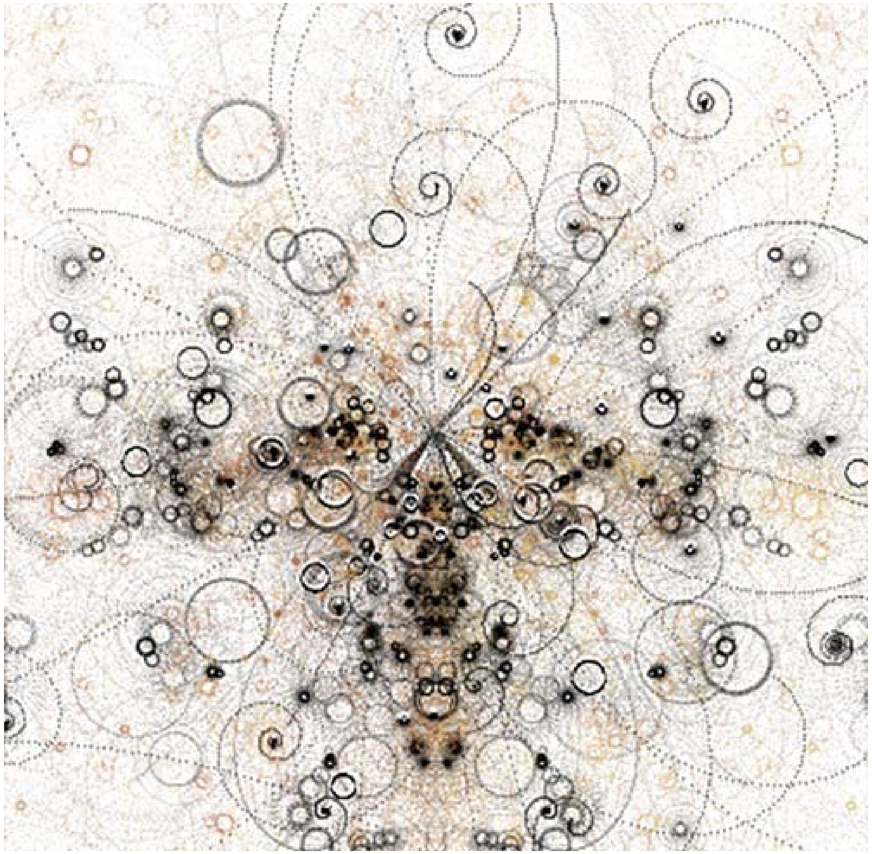
Considering a constant density model of the atmosphere …

$$P_{air} = \frac{M_{air}\,g}{A} = \frac{\rho A h g}{A} = \rho g h$$

M&I

Chapter 5

Rate of change of momentum

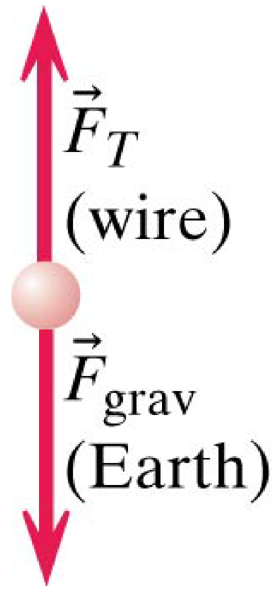


M&I
3E 5.1

Identifying forces on a system

1. Observe the rate at which a system's momentum changes $\frac{d\vec{\mathbf{p}}}{dt}$
2. List all the objects in the surroundings that exert forces on the system, and evaluate all the known forces that contribute to $\vec{\mathbf{F}}_{net}$
3. Use the momentum principle to relate $\vec{\mathbf{F}}_{net}$ to $\frac{d\vec{\mathbf{p}}}{dt}$
4. Solve for the unknowns.

Example: a hanging ball



$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_T + \vec{\mathbf{F}}_{grav}$$

$$\frac{dp_y}{dt} = F_{net,y} = F_T - mg$$

What is the system here?

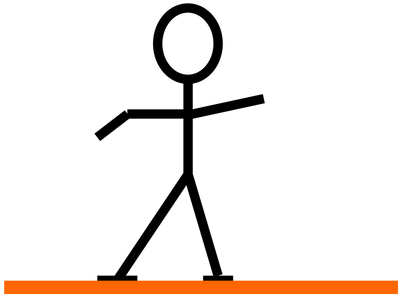
Identifying the significant forces acting on a system

What is a **system**?

... think of drawing an imaginary surface around what you want to investigate ... what is inside the imaginary surface is your **system**.

What do we mean by **significant** forces?

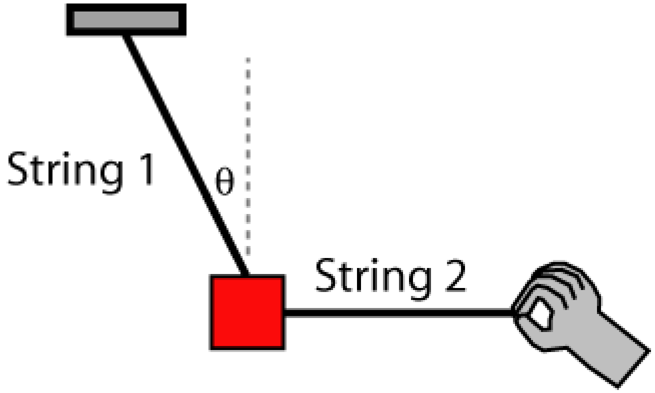
Identify all forces acting on Bob:



Menu of forces

- Gravitational force
- Electromagnetic force
- Strong nuclear force
- Weak nuclear force

- Pushes and pulls
- Normal forces
- Tension in ropes
- Restoring forces in springs
- Friction



1 2 3 4 5

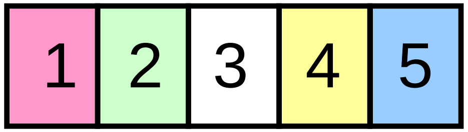
What objects exert significant forces on the red block?

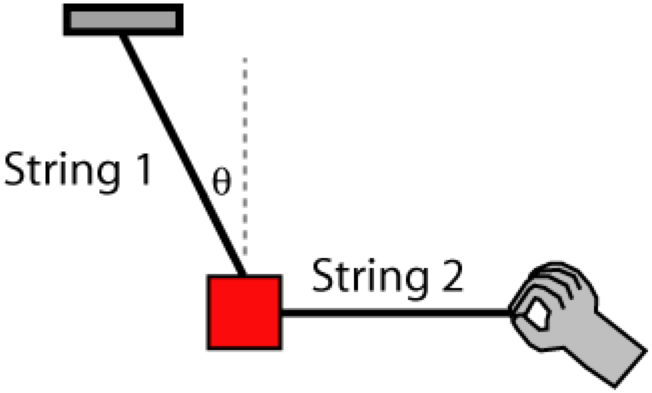
(1) Earth, String 1, String 2

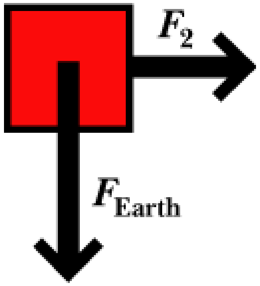
(2) Earth, String 1, String 2, Hand

(3) Earth, String 1, String 2, Hand, Ceiling

(4) Earth, Hand, Ceiling

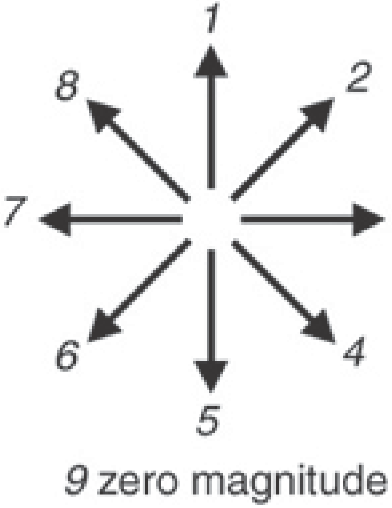


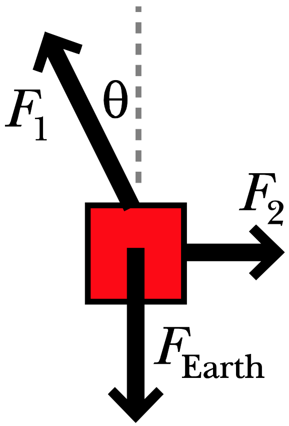


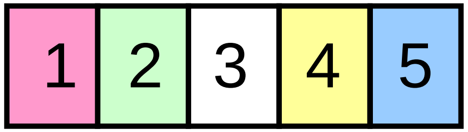


Here is an incomplete force diagram for the system of the red block.

To complete it we need to draw the force due to String 1.
Which arrow best indicates the direction of this force?







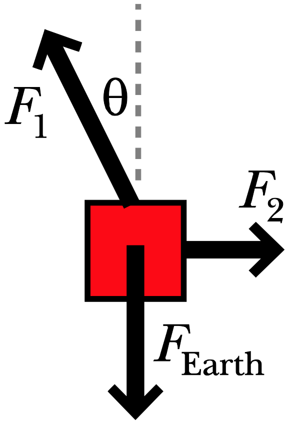
F_1 and F_2 are magnitudes of forces.
Which equation correctly states that $dp_x/dt = F_{net,x}$?

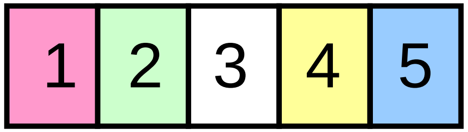
(1) $0 = -F_1 * \cos(\theta) + F_2$

(2) $0 = F_1 - F_2$

(3) $0 = F_1 + F_2 - mg$

(4) $0 = -F_1 * \sin(\theta) + F_2$





F_1 and F_2 are magnitudes of forces.
Which equation correctly states that $dp_y/dt = F_{net,y}$?

(1) $0 = F_1 - mg$

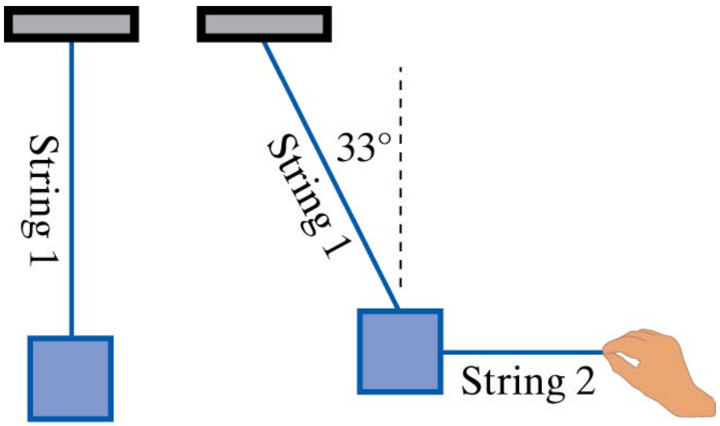
(2) $0 = F_1 + F_2 - mg$

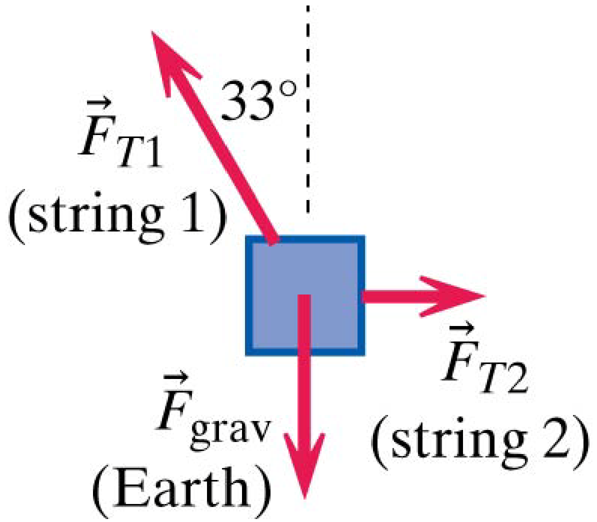
(3) $0 = -F_1 * \cos(\theta) + F_2$

(4) $0 = F_1 * \cos(\theta) - mg$

Momentum not changing

If $\dfrac{d\vec{\mathbf{p}}}{dt} = 0$ then $\vec{\mathbf{F}}_{net} = 0$

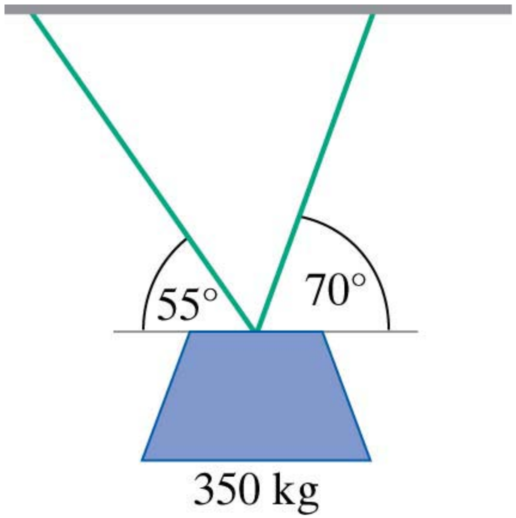


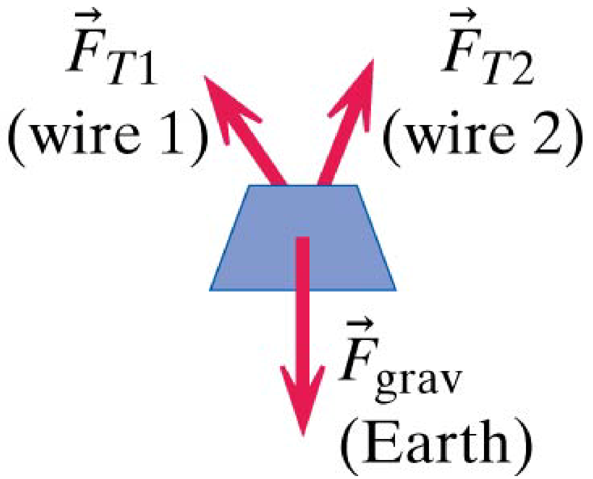


$$\vec{\mathbf{F}}_{T1} + \vec{\mathbf{F}}_{T2} + \vec{\mathbf{F}}_{grav} = 0$$

$$\langle -F_{T1}\sin 33°, F_{T1}\cos 33°, 0\rangle + \langle F_{T2}, 0, 0\rangle + \langle 0, -mg, 0\rangle = 0$$

Momentum not changing: another example





$$\vec{\mathbf{F}}_{T1} + \vec{\mathbf{F}}_{T2} + \vec{\mathbf{F}}_{grav} = 0$$

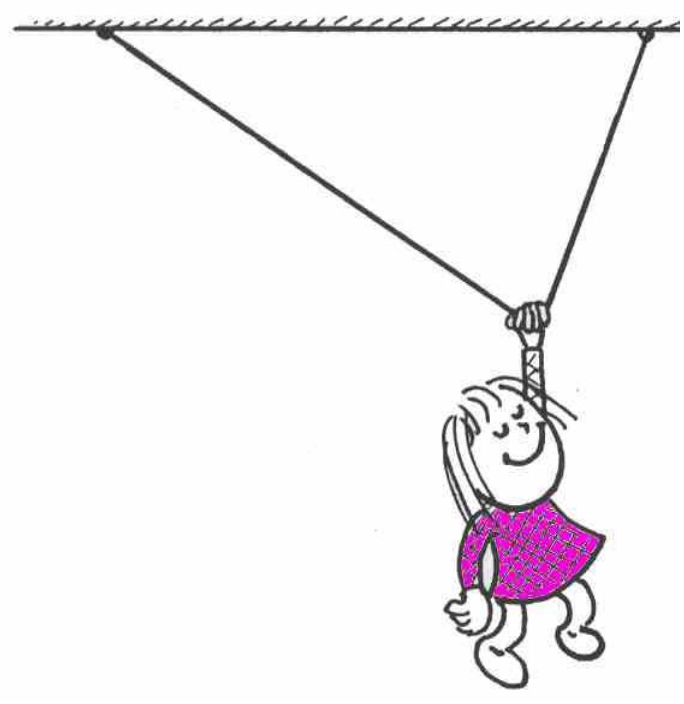
$$\left\langle -F_{T1}\cos 55°, F_{T1}\sin 55°, 0 \right\rangle + \left\langle F_{T2}\cos 70°, F_{T2}\sin 70°, 0 \right\rangle + \left\langle 0, -(350)(9.8), 0 \right\rangle = 0$$

Find $\quad F_{T1} = 1431 \text{ N}$

$\qquad\quad F_{T2} = 2402 \text{ N}$

1	2	3	4	5

Nellie Newton hangs by one hand motionless from a clothesline as shown - which is on the verge of breaking. Which side of the line is most likely to break?



(A) left side

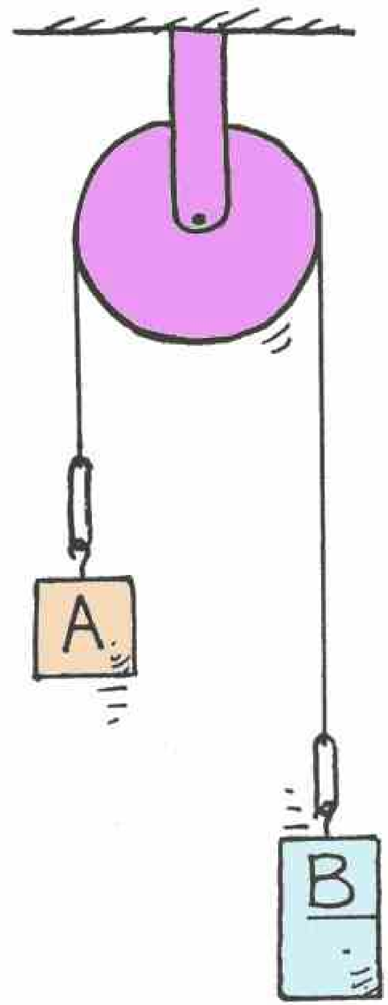
(B) right side

(C) 50/50 chance of either side breaking

1 2 3 4 5

Two identical rubber bands connect masses A and B to a string over a light, frictionless pulley. The amount of stretch is greater in the band that connects

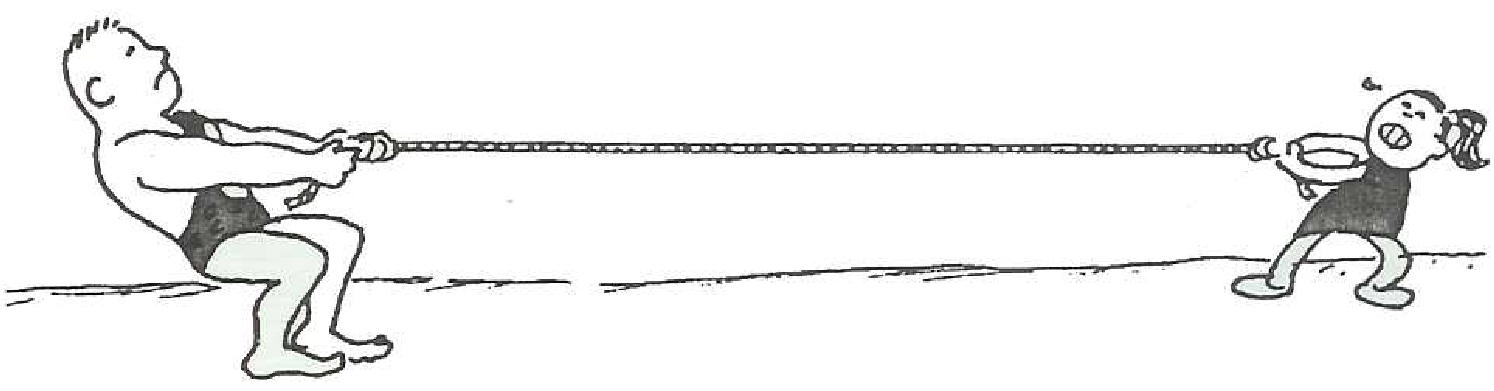
(A) mass A
(B) mass B
(C) both the same



1	2	3	4	5

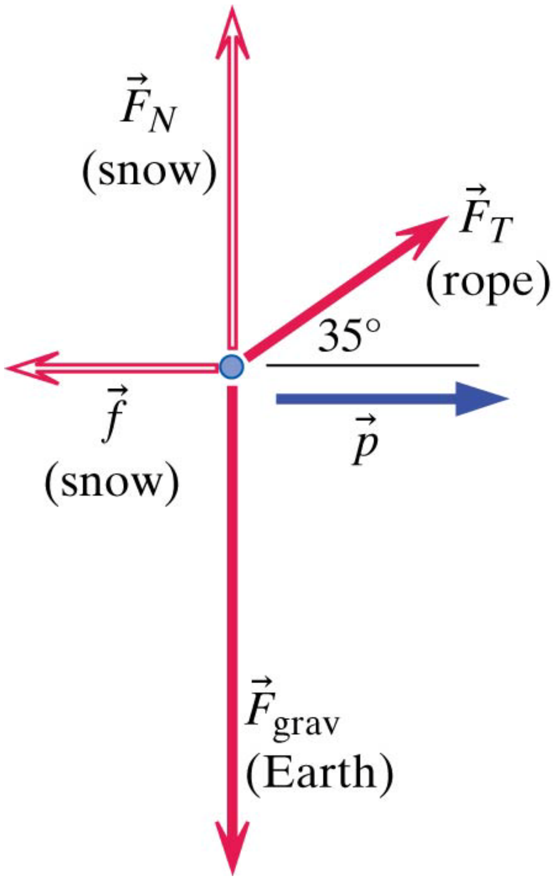
Arnold Strongman and Suzie Small pull on opposite ends of a rope in a tug of war. The greatest force exerted on the rope is by

(A) Arnold
(B) Suzie
(C) ... both the same.



Uniform motion: Momentum not changing: example

You pull a loaded sled whose mass is 40 kg at constant speed in the *x*-direction with a rope at an angle of 35°. The coefficient of kinetic friction between the snow and the sled is 0.2. What is the tension in the rope?



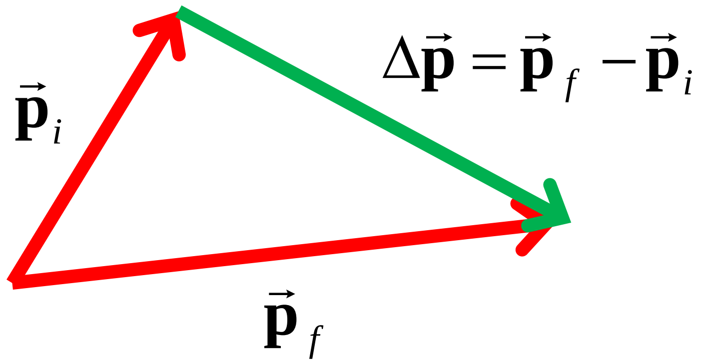
$$\vec{\mathbf{F}}_N + \vec{\mathbf{F}}_T + \vec{\mathbf{F}}_{grav} + \vec{\mathbf{f}} = \frac{d\vec{\mathbf{p}}}{dt} = 0$$

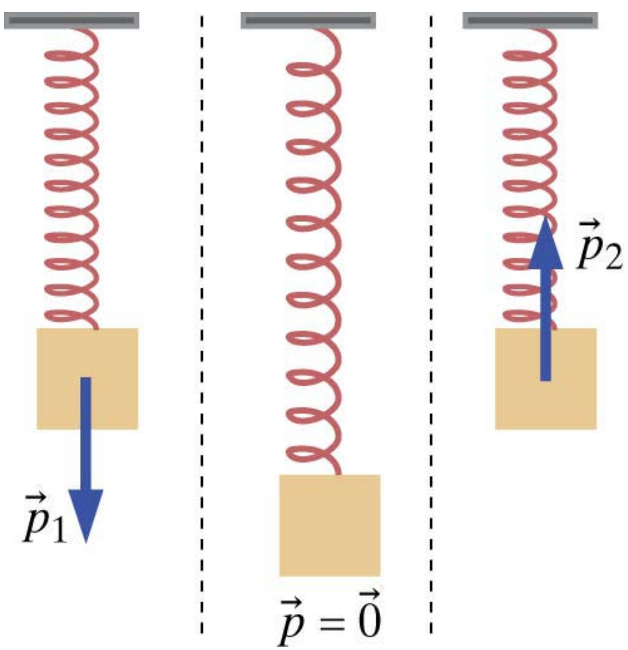
$$\langle 0, F_N, 0\rangle + \langle F_T \cos 35°, F_T \sin 35°, 0\rangle$$
$$+\langle 0, -(40)(9.8), 0\rangle + \langle -(0.2)F_N, 0, 0\rangle = 0$$

Find $F_T = 84\ \text{N}$

M&I
3E 5.3

Finding the rate of change of momentum





What is $\dfrac{d\vec{\mathbf{p}}}{dt}$ for the oscillating mass, at the bottom of its path?

... and $\vec{\mathbf{F}}_{net}$?

1	2	3	4	5

A ball hangs from the bottom of a vertical spring. You pull the ball downwards and release it, and the ball oscillates up and down.

At the bottom of each oscillation, where the ball's instantaneous momentum is zero, what is the direction of $\Delta\vec{\mathbf{p}}$?

1) $\uparrow$

2) $\downarrow$

3) **zero magnitude (no direction)**

1	2	3	4	5

A ball hangs from the bottom of a vertical spring. You pull the ball downwards and release it, and the ball oscillates up and down. At the bottom of each oscillation, where the ball's instantaneous momentum is zero, what is the direction of $\vec{F}_{\text{net}}$?

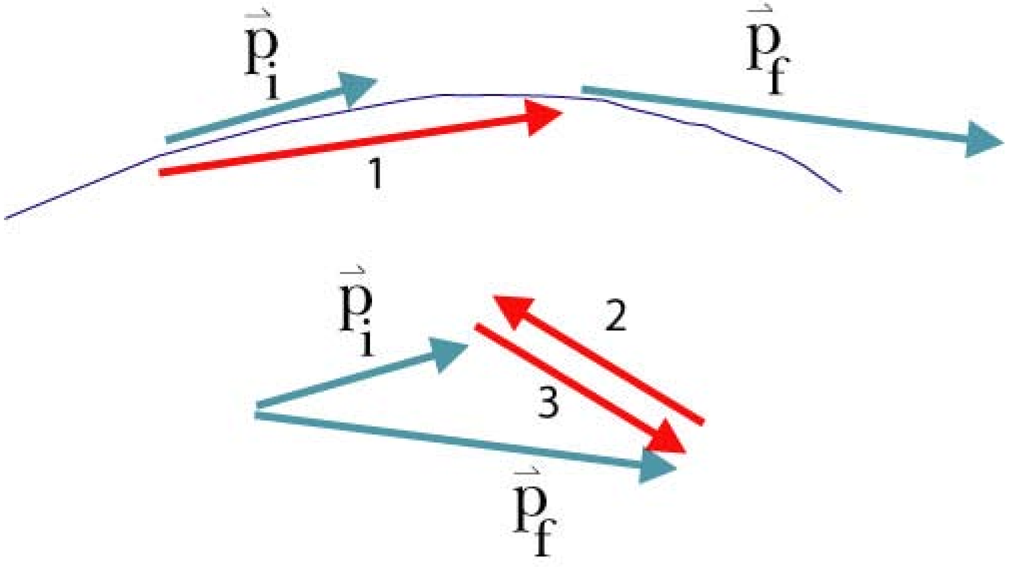
1) $\uparrow$

2) $\downarrow$

3) zero magnitude (no direction)

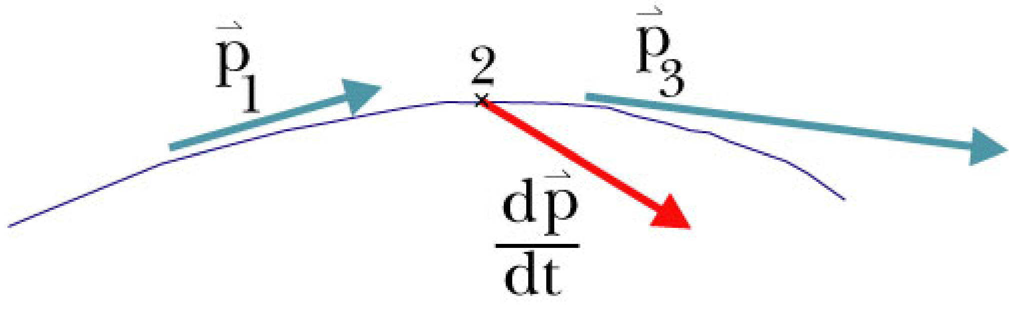
1	2	3	4	5

Which of the red arrows labeled 1, 2, or 3 represents the vector $\Delta\vec{\mathbf{p}}$, the change in the momentum of the comet?



1 2 3 4 5

The red arrow shows the rate of change of the comet's momentum when it is at location 2.

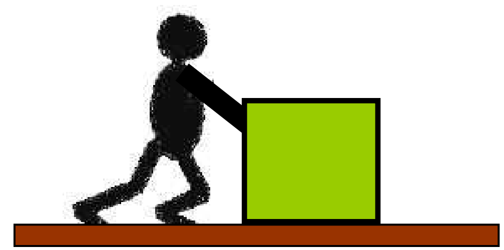


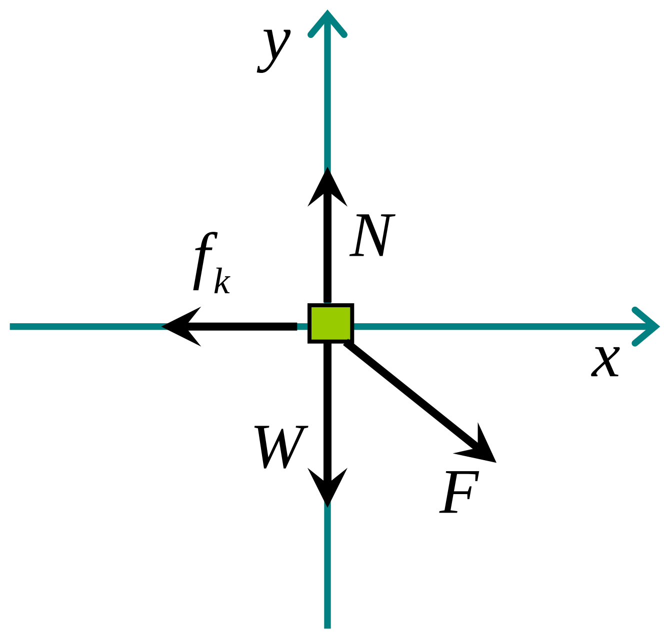
At this instant, which components of $d\vec{p}/dt$ are zero?

(1) The parallel component
(2) The perpendicular component
(3) Both the parallel and perpendicular components
(4) Neither component

1	2	3	4	5

A person pushes a crate so that it moves at constant velocity toward the right. A free body diagram for the crate is shown (where the arrows are not necessarily the correct relative lengths. Which choice below best represents the relative magnitude of the forces ?

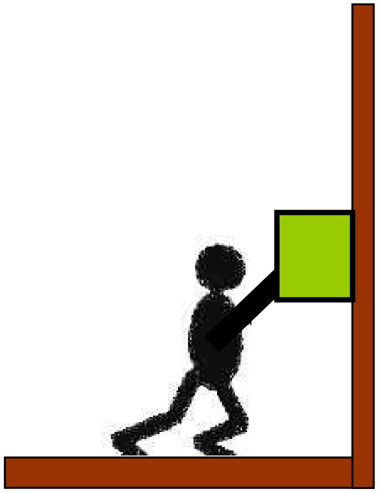




(1) $F = f_k$ and $N = W$
(2) $F = f_k$ and $N > W$
(3) $F = f_k$ and $N < W$
(4) $F > f_k$ and $N = W$
(5) $F > f_k$ and $N > W$

1	2	3	4	5

A person pushes a crate as it moves down a vertical wall at constant velocity. A free body diagram for the crate is shown (where the arrows are not necessarily the correct relative lengths. Which choice below best represents the relative magnitude of the forces ?



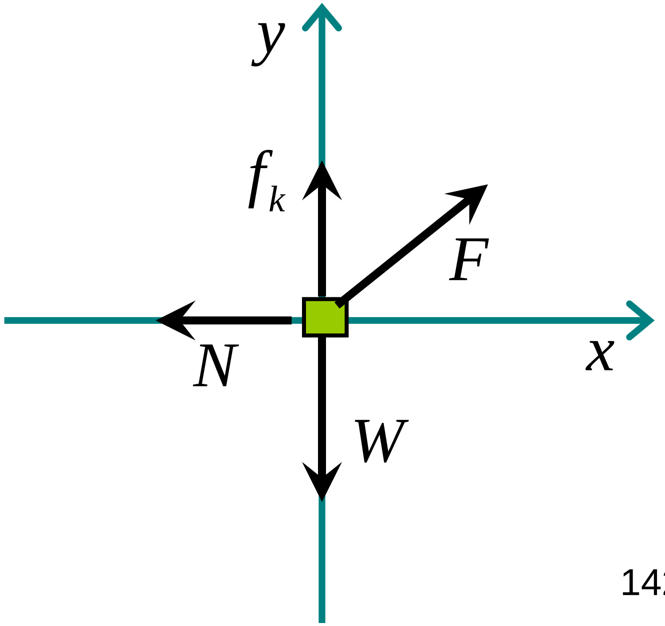
(1) $W = f_k$ and $N = F$

(2) $W = f_k$ and $N > F$

(3) $W = f_k$ and $N < F$

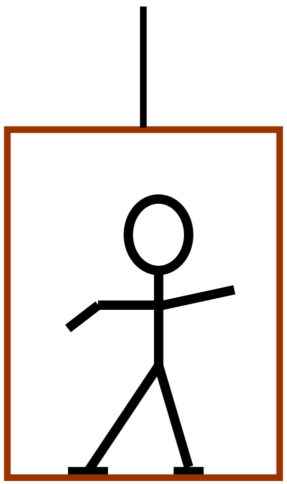
(4) $W > f_k$ and $N = F$

(5) $W > f_k$ and $N > F$



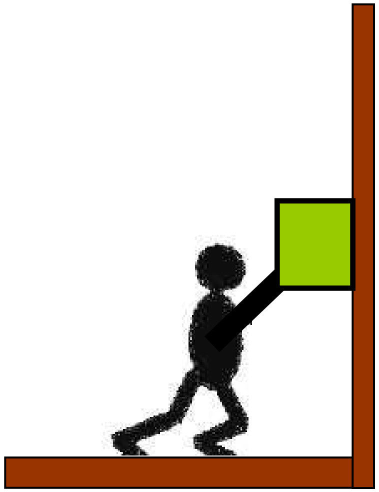
Example 1

The speed of a lift initially moving down at 10 m s^{-1} decreases to 4 m s^{-1} in a time of 2 seconds.
Determine the force of the lift’s floor on a 100 kg person standing in the lift.



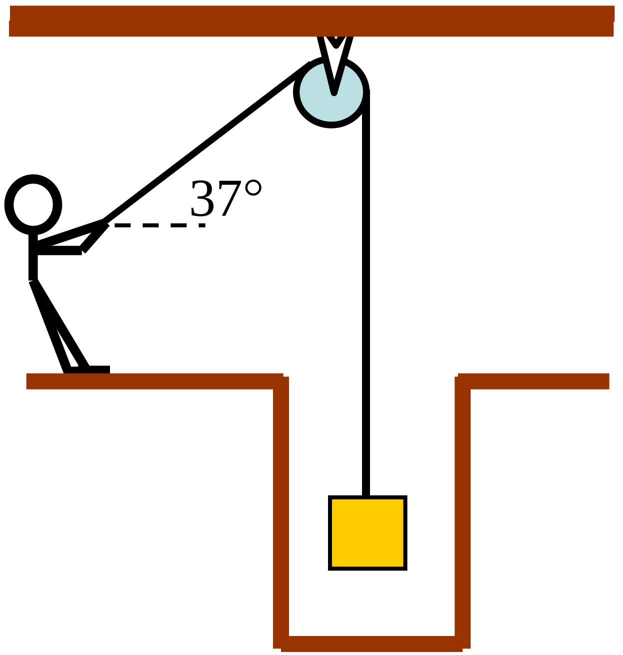
Example 2

The 20 kg crate shown is initially moving down at a speed of 2.0 m s^{-1}. Determine the magnitude of the average force exerted by the person in order to stop the crate's downward movement in a time of 5 seconds. The person pushes in a direction 53° above the horizontal and the coefficient of kinetic friction between the crate and the wall is 0.40 .



Example 3

A farmer wishes to lift a heavy box by pulling a light rope that passes over a pulley and down to the box below. What is the minimum coefficient of static friction between her shoes and the floor that will prevent her from sliding across the surface while lifting the box at a constant speed? The farmer's mass is 70 kg and the box's weight is 250 N. Assume that there is no friction in the pulley.

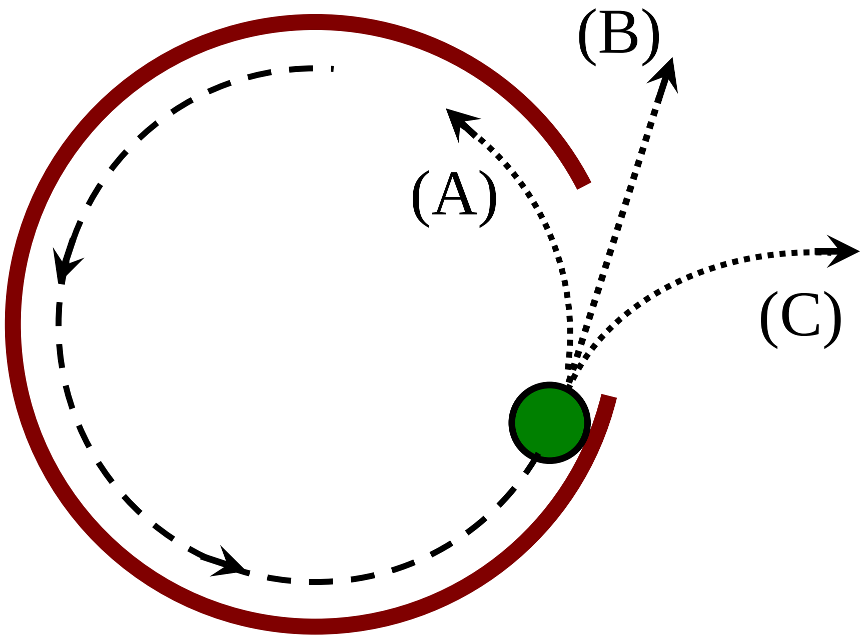


Example 4

A 40 kg box accidentally slides down a ramp and onto a level surface. On this surface, the box is initially moving at 9.0 m s^{-1} towards a large glass window 14 m away, when a man starts to push the box to prevent it from smashing the window. The force that the man exerts on the box is 100 N at an angle of 37° below the horizontal. (The man moves backward while exerting a force on the box.) A 40 N friction force opposes the box's motion. Will the box break the window?

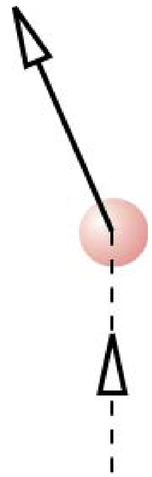
Demonstration

A marble rolls along the inside of a horizontal circular loop. When the marble reaches the gap, which path will the marble follow?

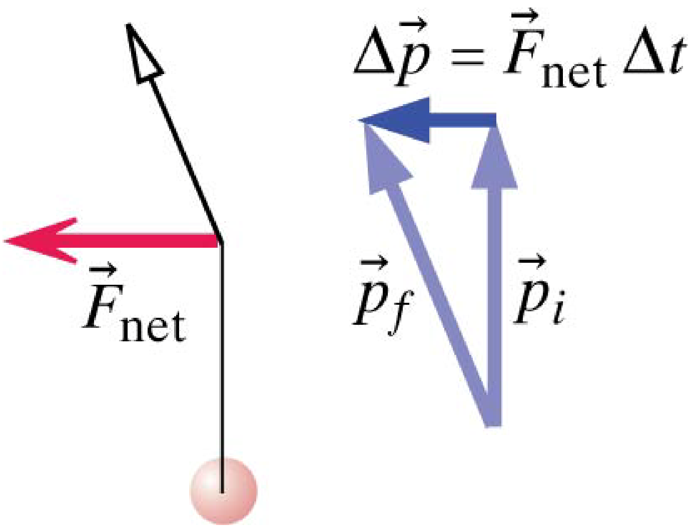


M&I
3E 5.4

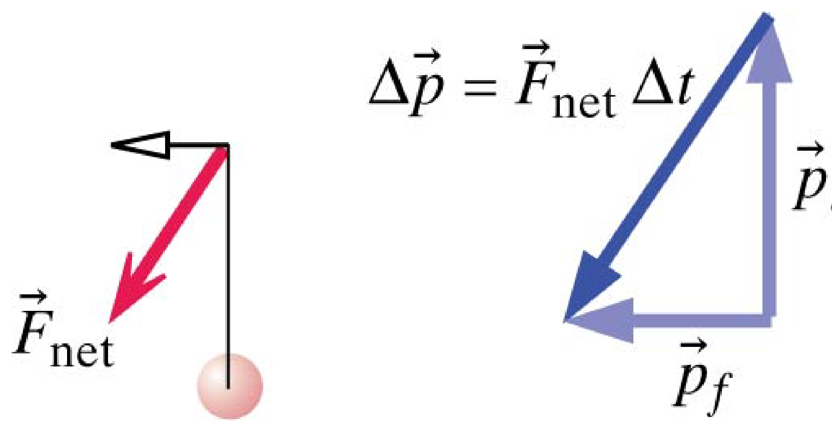
Curving motion



In order to change the direction of the ball to the left, in what direction should you give it a kick?

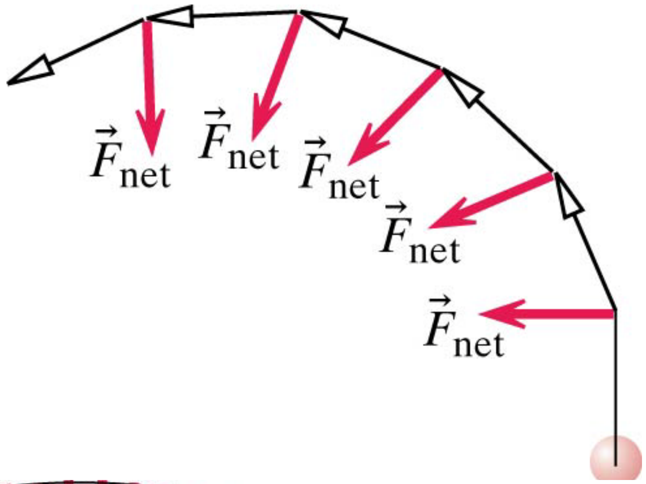


A small turn

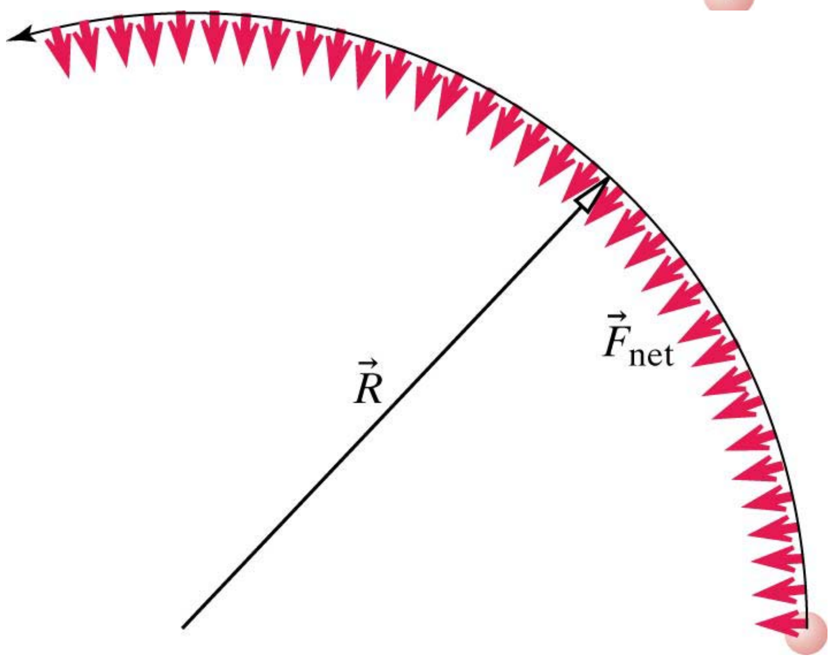


A 90° turn

Curving motion: repeated kicks



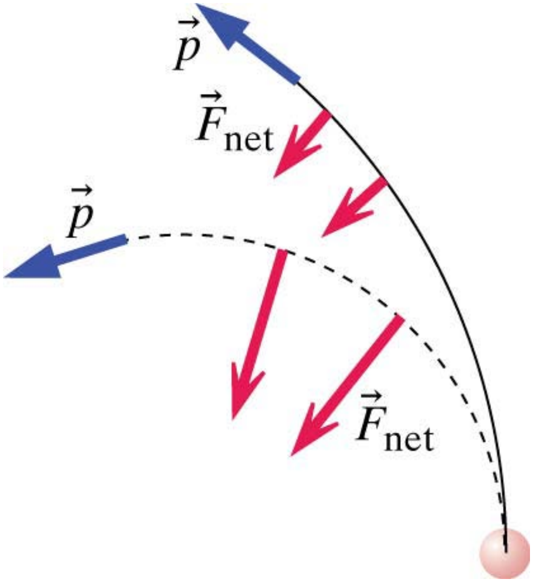
... get a roughly circular path



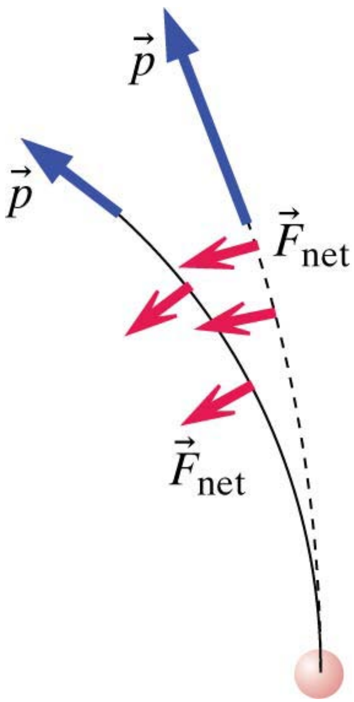
... if these kicks are equally spaced in time, and directed perpendicular to the momentum ...

... get a (very nearly) circular path.

Curving motion



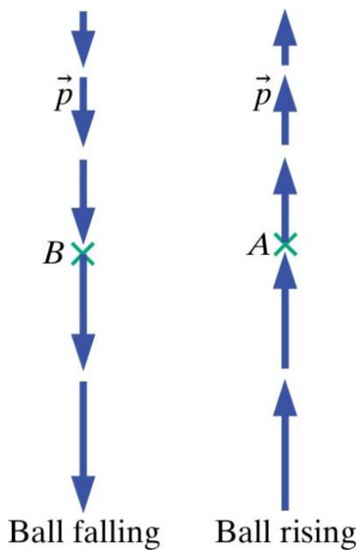
The bigger the force,
the smaller the radius of
the circular path.



The bigger the momentum,
the larger the radius of the
circular path.

M&I
3E 5.5

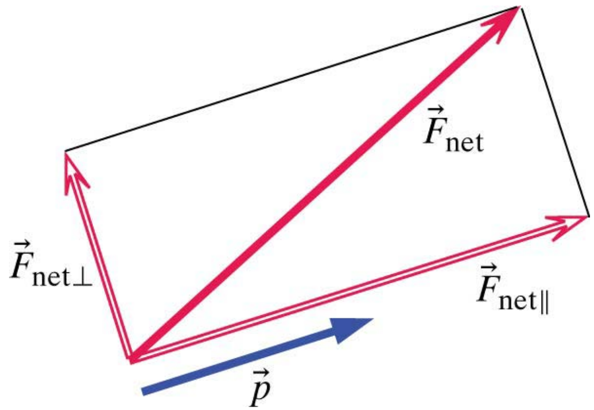
Rate of change of direction



Write $\vec{\mathbf{p}} = p\hat{\mathbf{p}}$

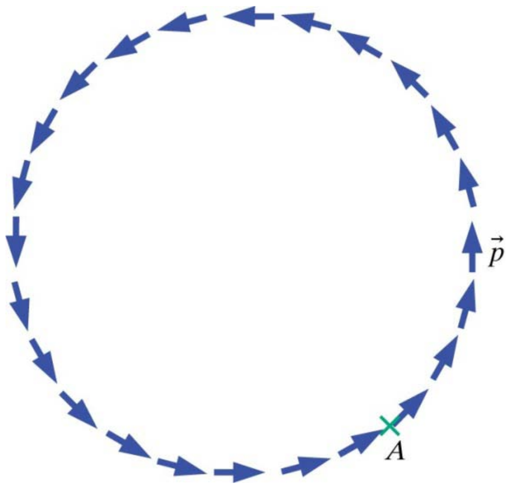
then $$\frac{d\vec{\mathbf{p}}}{dt} = \frac{dp}{dt}\hat{\mathbf{p}} + \frac{d\hat{\mathbf{p}}}{dt}p$$

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_{net||} + \vec{\mathbf{F}}_{net\perp}$$



$\dfrac{dp}{dt}\hat{\mathbf{p}}$ ... is non-zero if the speed is changing

and equal to $\vec{\mathbf{F}}_{net||}$



$\dfrac{d\hat{\mathbf{p}}}{dt}p$ ... is non-zero if the direction is changing

and equal to $\vec{\mathbf{F}}_{net\perp}$

Curving motion: a geometrical derivation

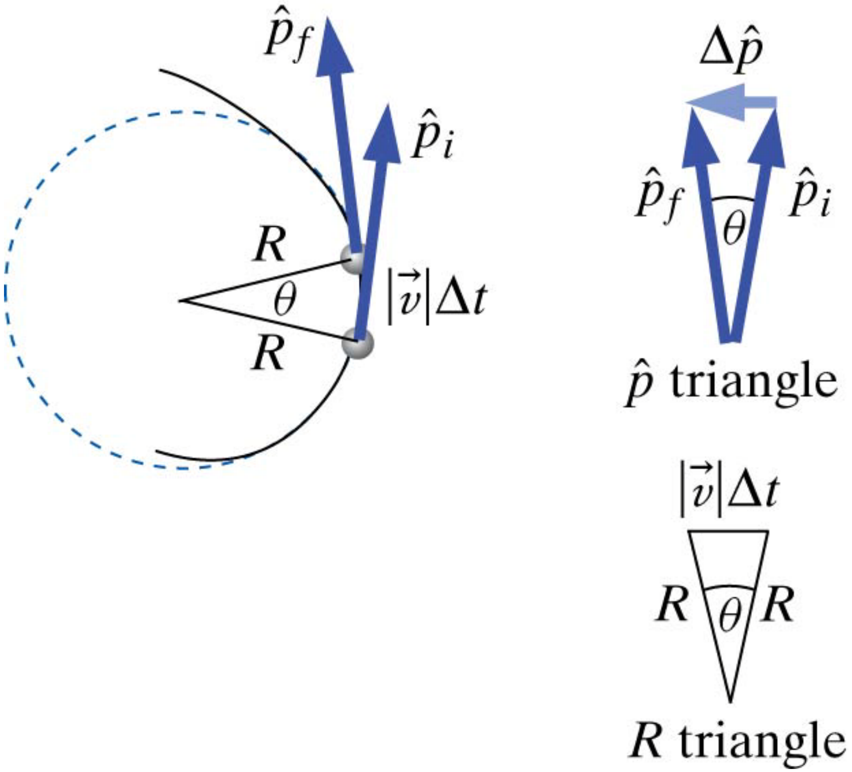
Take two positions very close to each other.

The $\hat{\mathbf{p}}$ triangle and R triangle are similar ... hence ...

$$\frac{\left|\Delta\hat{\mathbf{p}}\right|}{\left|\hat{\mathbf{p}}\right|} = \frac{\left|\vec{\mathbf{v}}\right|\Delta t}{R}$$

$$\frac{\left|\Delta\hat{\mathbf{p}}\right|}{\Delta t} = \frac{\left|\vec{\mathbf{v}}\right|}{R} \qquad \text{since} \quad \left|\hat{\mathbf{p}}\right| = 1$$

thus $$\lim_{\Delta t\to 0}\left|\frac{\Delta\hat{\mathbf{p}}}{\Delta t}\right| = \frac{d\hat{\mathbf{p}}}{dt} = \frac{\left|\vec{\mathbf{v}}\right|}{R} \qquad \text{on a curved path}$$



Then $$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_{\perp}\right| = p\frac{d\hat{\mathbf{p}}}{dt} = p\frac{v}{R} = \frac{mv^2}{R} = \vec{\mathbf{F}}_{net\perp}$$

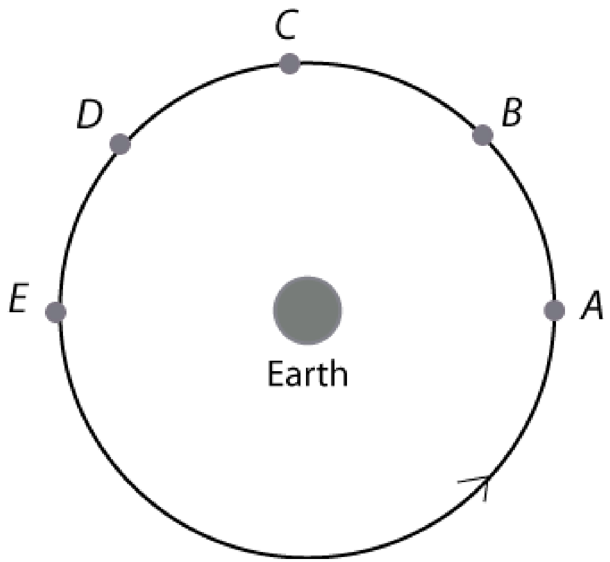
Curving motion

$$\vec{\mathbf{F}}_{net\perp} = \frac{mv^2}{R} \quad \text{on a curved path}$$

Sometimes $\vec{\mathbf{F}}_{net\perp}$ is called the **centripetal force** since it acts towards the centre of the circle.

Then $\quad \vec{\mathbf{a}}_{net\perp} = \dfrac{v^2}{R}$ is the **centripetal acceleration**.

In each situation, some force or forces must have a component towards the centre of the circle and hence "provide" the centripetal force. In other words, the centripetal force is <u>not</u> an "extra force" ... it is the name given to the resultant force in the direction of the centre of the circle.



1	2	3	4	5

The Moon travels in a nearly circular orbit around the Earth, at nearly constant speed.

When the Moon is at location *A*, which components of $d\vec{p}/dt$ are zero?

1) The parallel component
2) The perpendicular component
3) Both the parallel and perpendicular components
4) Neither component

Example: child on a merry-go-round

A 30 kg child sits on a merry-go-round at a distance of 3 m from the centre. The merry-go-round makes one revolution every 8 seconds. What is the magnitude and direction of the net force acting on the child?

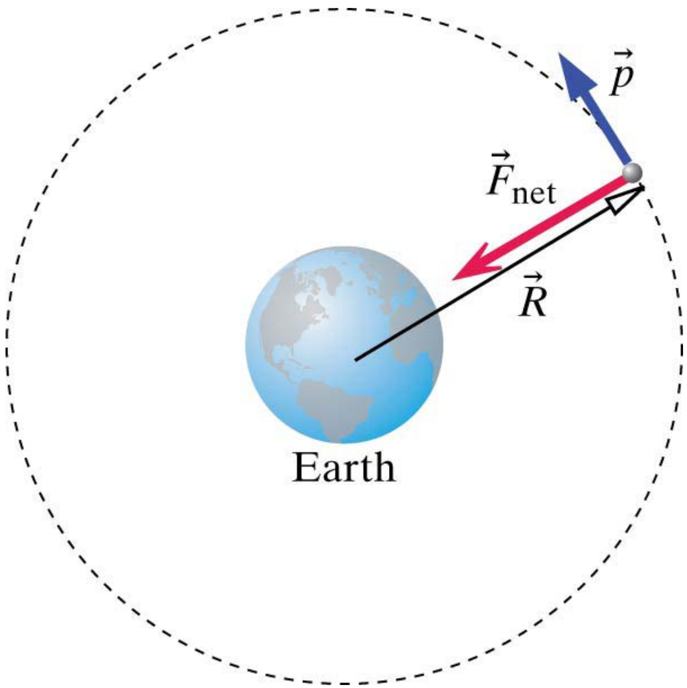
$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

$$\left| \left(\frac{d\vec{\mathbf{p}}}{dt} \right)_{||} \right| = 0 \quad \text{since speed is not changing} \qquad \therefore \vec{\mathbf{F}}_{net||} = 0$$

$$\left| \left(\frac{d\vec{\mathbf{p}}}{dt} \right)_{\perp} \right| = \frac{mv^2}{R} = \left| \vec{\mathbf{F}}_{net\perp} \right|$$

Find $F_{net\perp} = 55.5 \text{ N}$

Example: Earth satellite



$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_{||}\right| = 0$$ since speed is not changing

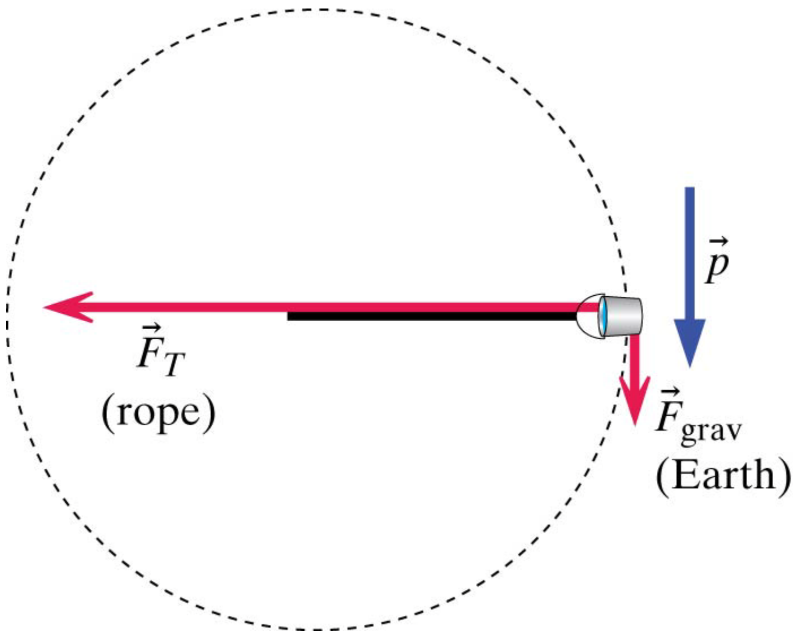
$$\therefore \vec{\mathbf{F}}_{net||} = 0$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_{\perp}\right| = \frac{mv^2}{R} = \left|\vec{\mathbf{F}}_{net\perp}\right| = \frac{GmM}{R^2}$$

$$\therefore v = \sqrt{\frac{GM}{R}}$$

or $\dfrac{2\pi R}{T} = \sqrt{\dfrac{GM}{R}}$ giving $T = \dfrac{2\pi R^{3/2}}{\sqrt{GM}}$

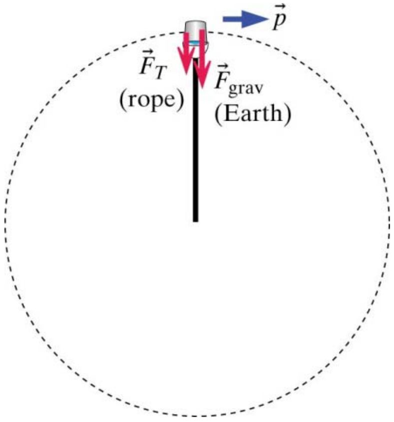
Example: swinging a bucket in a vertical plane



$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_{\|}\right| = m\left(\frac{dv}{dt}\right)_{\|} = ma_{\|} = mg$$

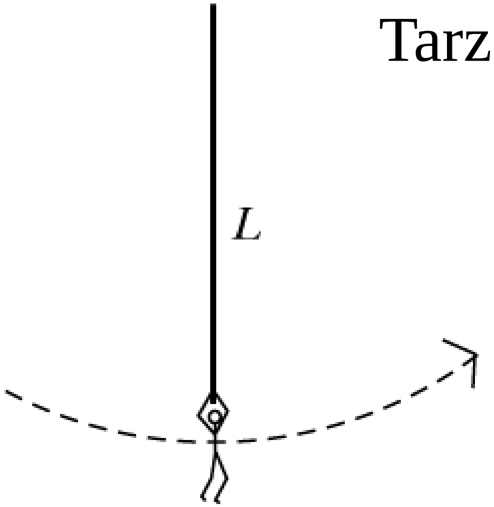
$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_{\perp}\right| = \frac{mv^2}{R} = F_T$$



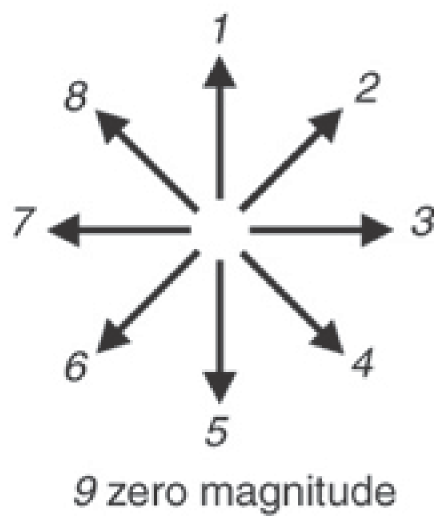
What about when the bucket is at the top of the circle?

1	2	3	4	5

Tarzan swings from a vine.

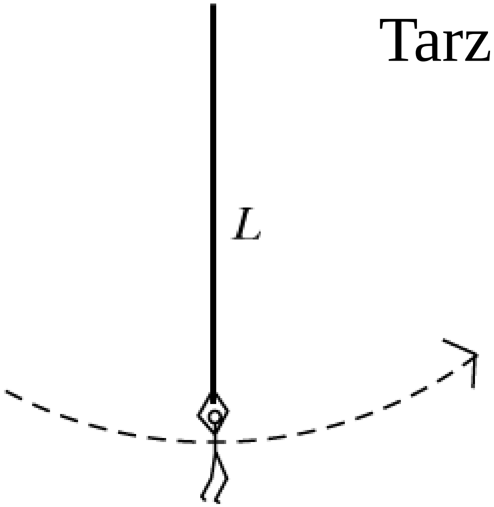


At the bottom of the swing, what is the direction of his $d\vec{p}/dt$?



1	2	3	4	5

Tarzan swings from a vine.

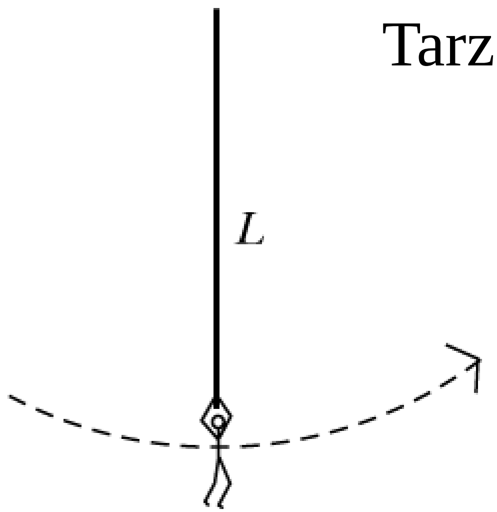


At the bottom of the swing, which components of $d\vec{p}/dt$ are zero?

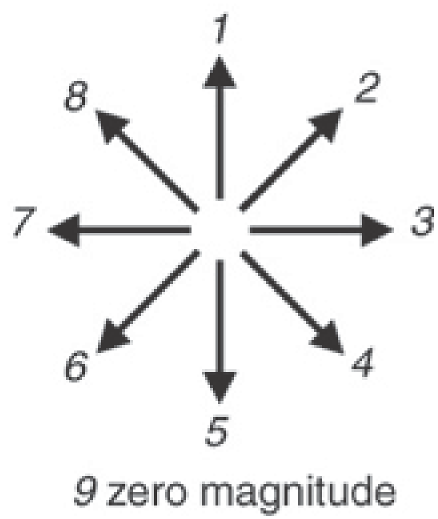
(1) The parallel component
(2) The perpendicular component
(3) Both the parallel and perpendicular components
(4) Neither component

1	2	3	4	5

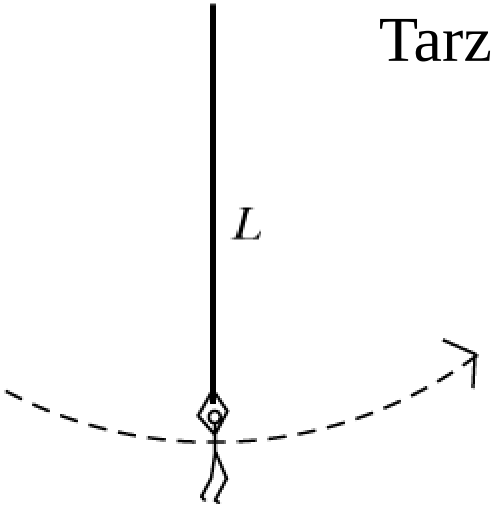
Tarzan swings from a vine.



At the bottom of the swing, what is the direction of the net force acting on Tarzan?



1	2	3	4	5

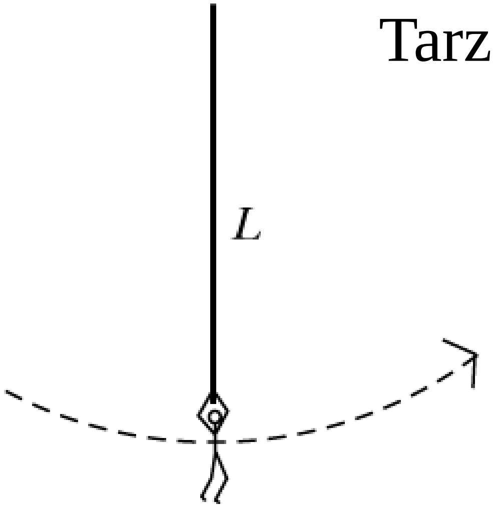


Tarzan swings from a vine.

At the bottom of the swing, what objects exert forces on Tarzan (neglecting air resistance)?

(1) Earth, vine
(2) centrifugal force only
(3) Earth and centrifugal force
(4) Earth, vine, centrifugal force

1	2	3	4	5



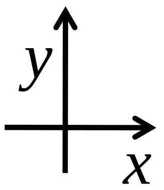
Tarzan swings from a vine.

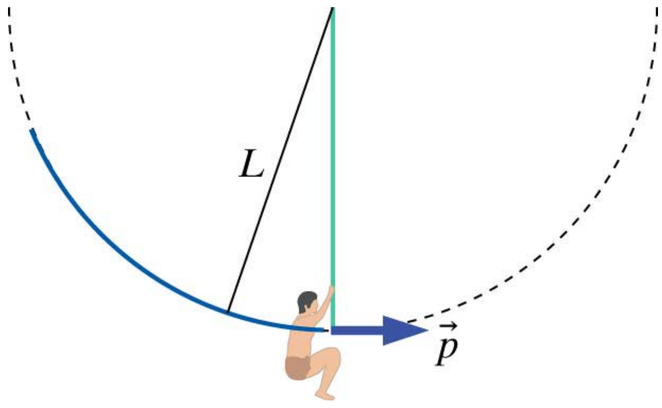
At the bottom of the swing, how does the magnitude of the force on Tarzan by the vine compare to the magnitude of the force on Tarzan by the Earth?

(1) $F_{\text{vine}} > F_{\text{Earth}}$
(2) $F_{\text{vine}} = F_{\text{Earth}}$
(3) $F_{\text{vine}} < F_{\text{Earth}}$
(4) Not enough info.

Example: Tarzan and the vine

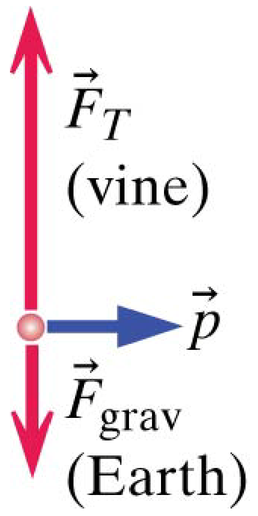
$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$





Stationary: $$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_y\right| = 0 = F_T - mg$$

Swinging: $$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_y\right| = \frac{mv^2}{L} = F_T - mg$$

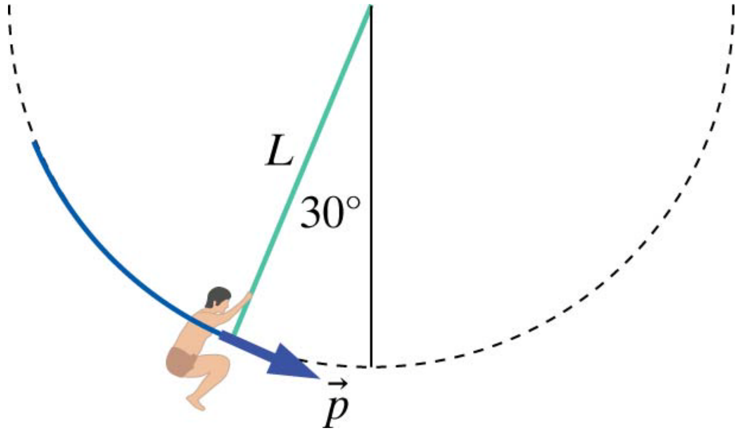


If $m = 90$ kg , $L = 8$ m ,
$v = 12$ m s^{-1} (when swinging) …

F_T (stationary) = 882 N

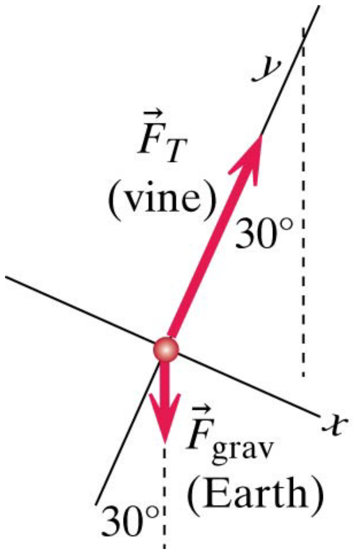
F_T (swinging) = 2502 N

Example: Tarzan and the vine



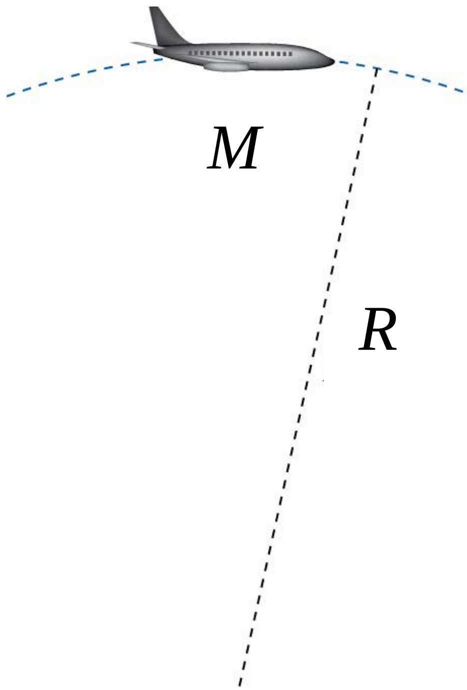
$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

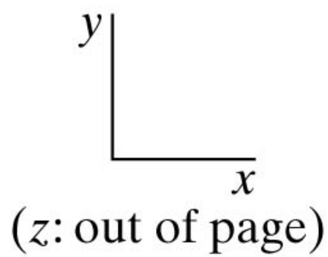
$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_y\right| = F_{net,y} = \frac{mv^2}{L} = F_T - mg\cos 30°$$

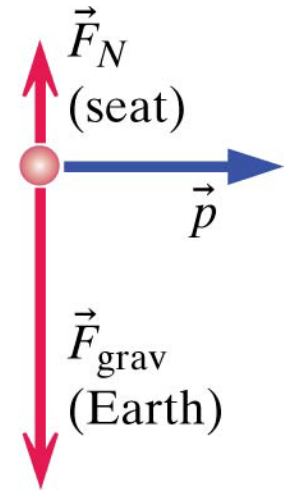


$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_x\right| = F_{net,x} = mg\sin 30°$$

Example: Compression force





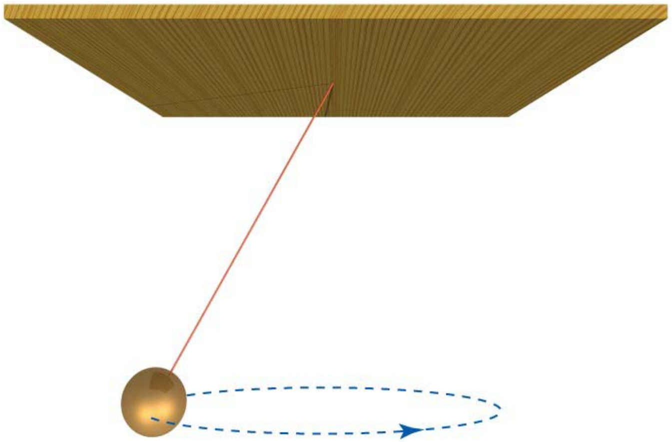


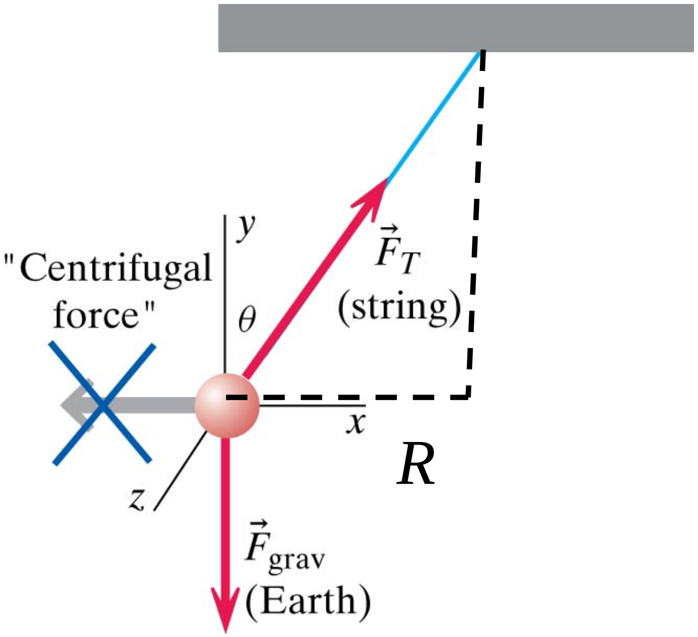
$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

$$\left| \left(\frac{d\vec{\mathbf{p}}}{dt} \right)_y \right| = F_{net,y} = -\frac{mv^2}{R} = F_N - Mg$$

$$\left| \left(\frac{d\vec{\mathbf{p}}}{dt} \right)_x \right| = F_{net,x} = 0$$

Example: Circular pendulum



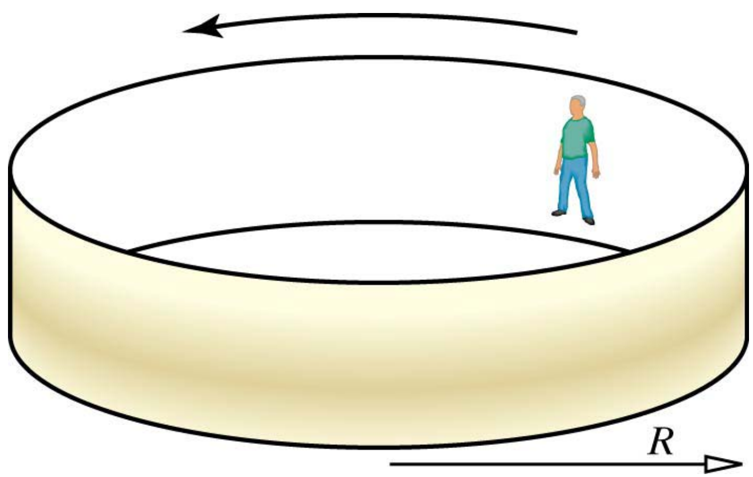


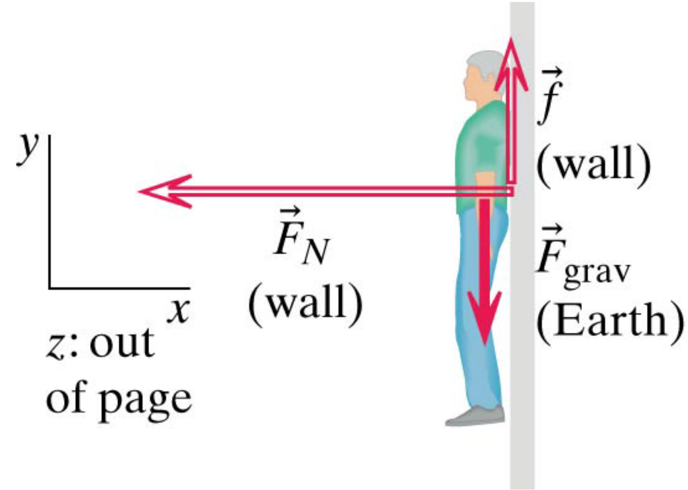
$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_y\right| = F_{net,y} = 0 = F_T\cos\theta - mg$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_x\right| = F_{net,x} = \frac{mv^2}{R} = F_T\sin\theta$$

Example: Amusement park ride





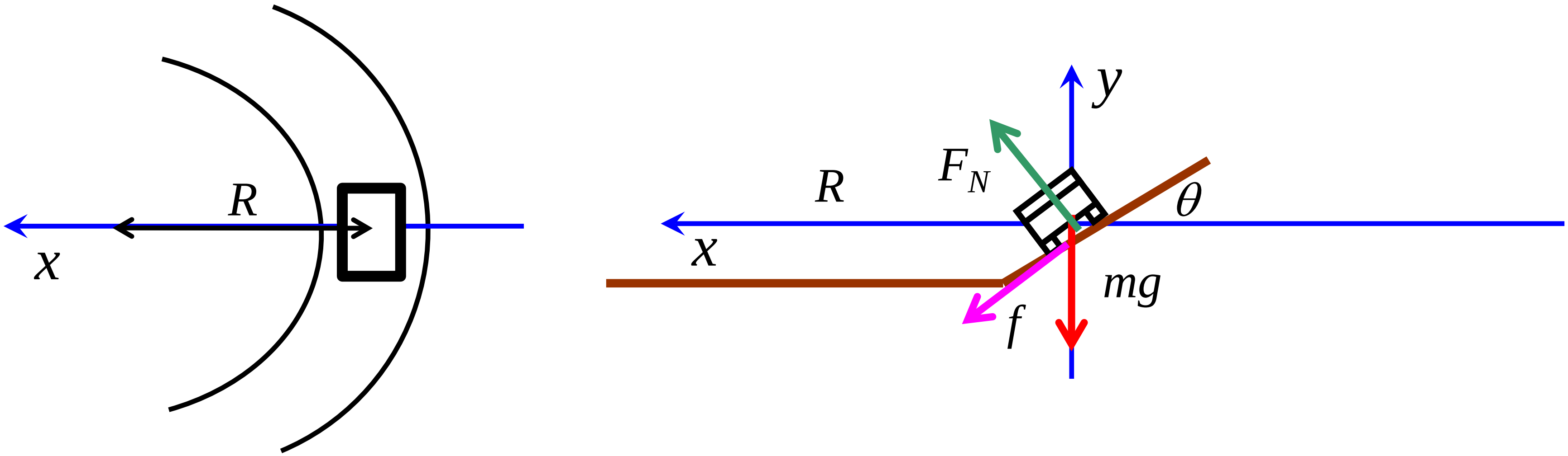
$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_y\right| = F_{net,y} = f - mg = 0$$

$$\left(f = \mu_s F_N \right)$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_x\right| = F_{net,x} = -F_N = -\frac{mv^2}{R}$$

Example: Car on a banked curve



$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net}$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_y\right| = F_{net,y} = F_N \cos\theta - f \sin\theta - mg = 0$$

$$\left|\left(\frac{d\vec{\mathbf{p}}}{dt}\right)_x\right| = F_{net,x} = \frac{mv^2}{R} = F_N \sin\theta + f \cos\theta$$