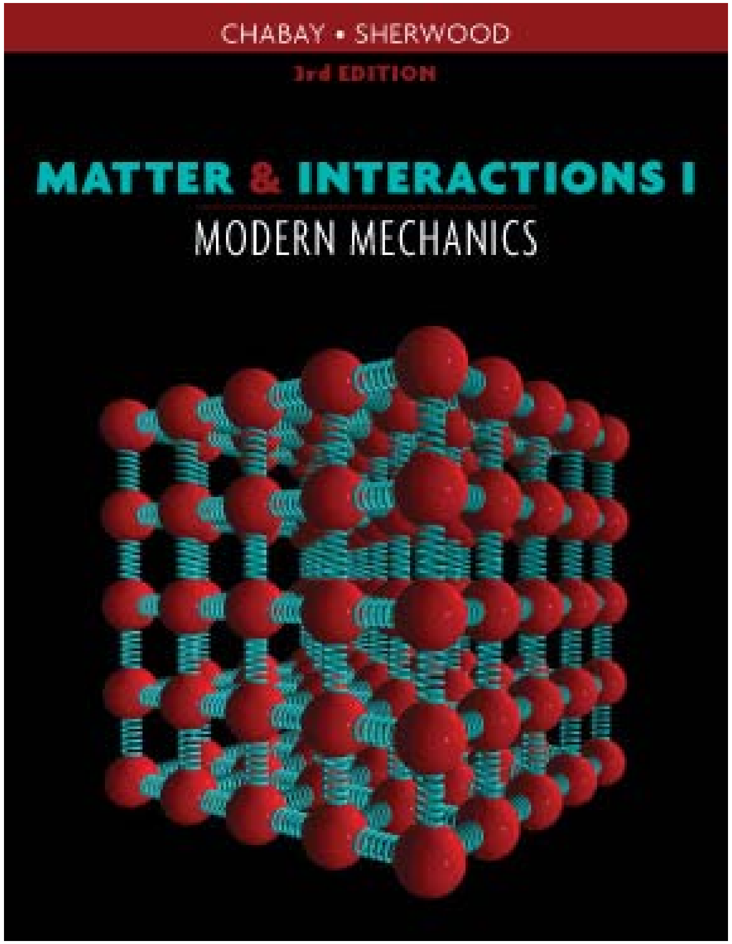


PHY1004W 2011
Modern Mechanics
Part 3

Prof Andy Buffler
Room 503 RW James
andy.buffler@uct.ac.za

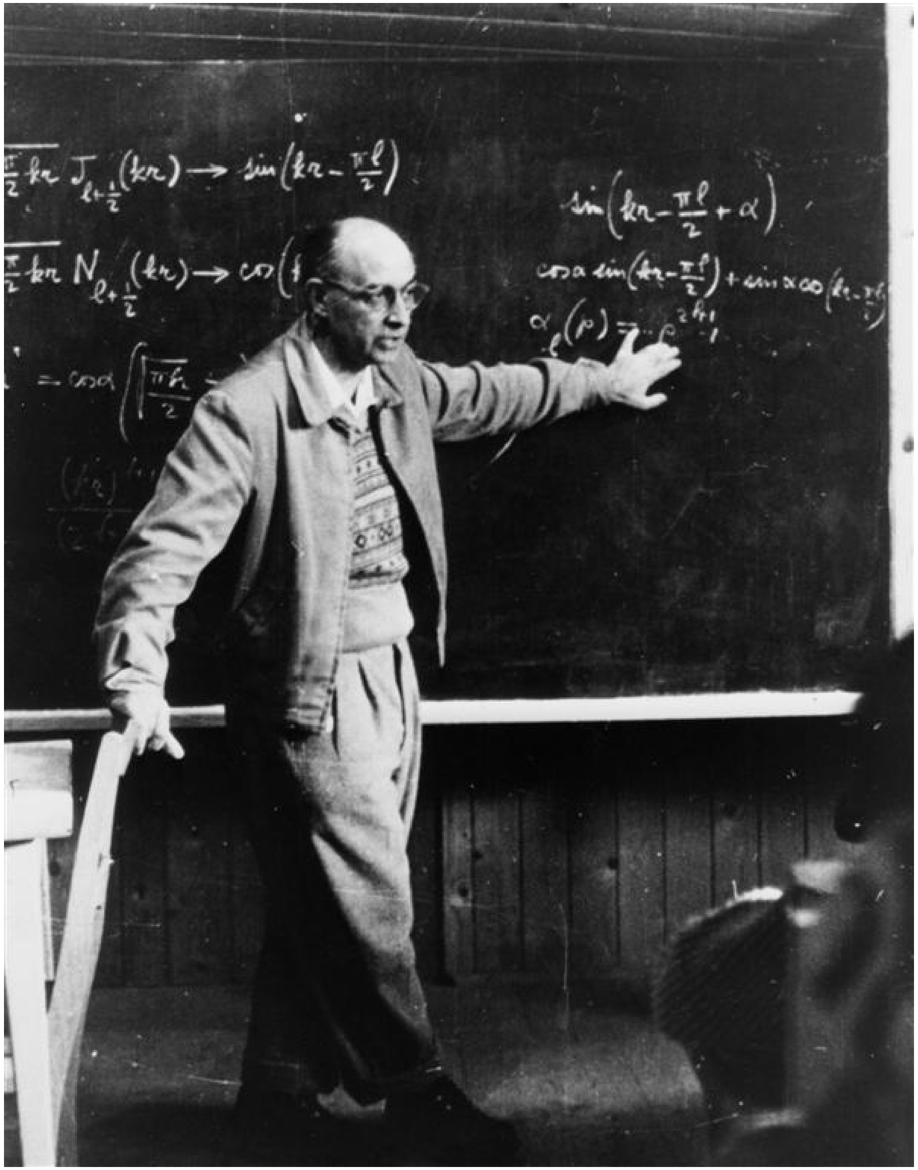
These slides have benefited from significant guidance from the notes of Roger Fearick (UCT Physics) and the resources provided by the textbook authors.

These slides are available on … UCT OpenContent

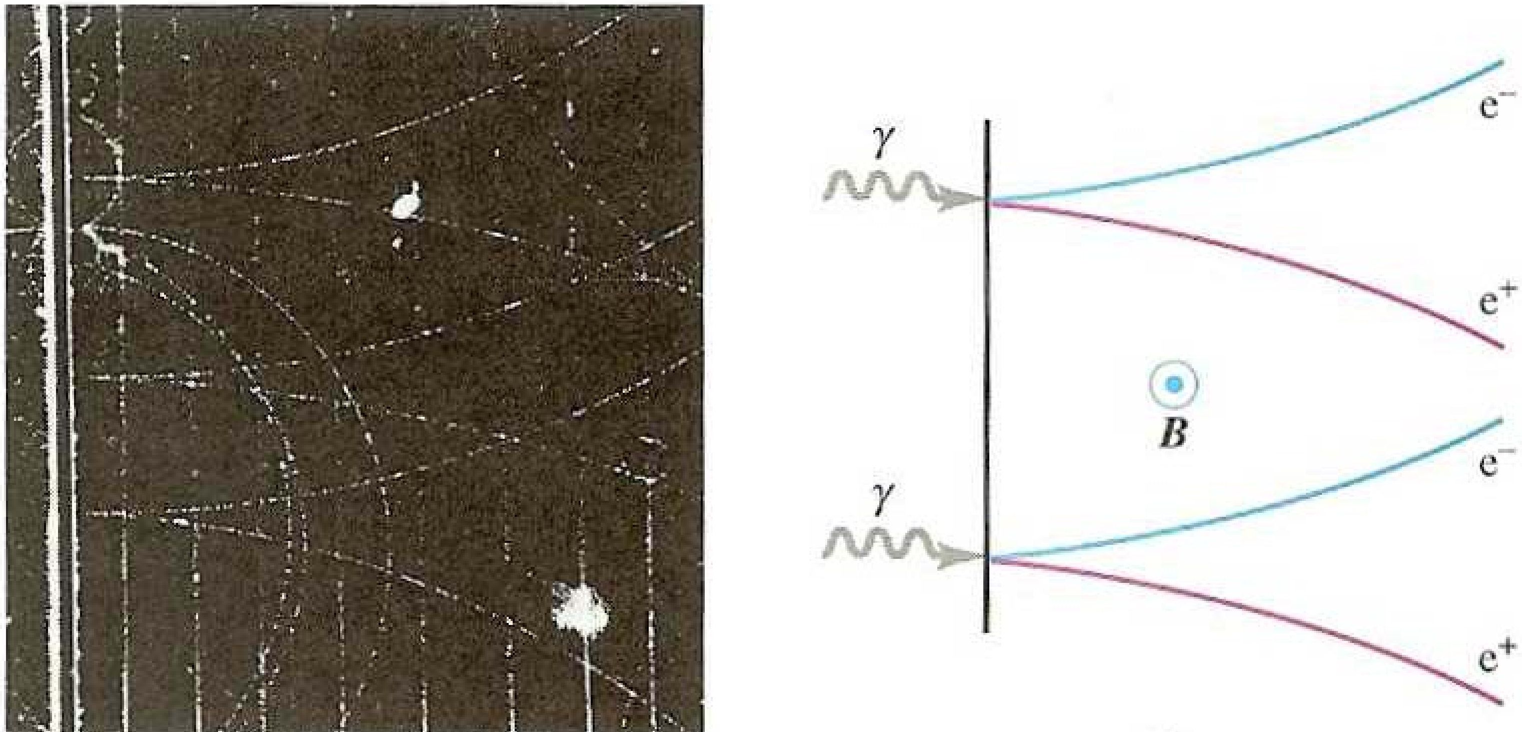
M&I

Chapter 6

The Energy Principle



Conservation of energy and charge in e^+e^- production



(a) (b)

The momentum principle

… is a fundamental principle in physics …

… states that the change in momentum of a system is equal to the net force acting on the system times the duration of the interaction…

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\text{net}}\Delta t$$

where $$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$$

and $$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \ldots$$

$$\Delta\vec{\mathbf{p}}_{\text{system}} + \Delta\vec{\mathbf{p}}_{\text{surroundings}} = 0$$

Principle of conservation of momentum

M&I
3E 6.1

The energy principle

The momentum principle (Newton's 2nd law) tells us how systems evolve, given the forces involved.

Energy is a concept that allows us to say what is possible …

… what are the constraints on behaviour.

The evolution of a system can be related to the transfer of energy

… leads to the conservation of energy as a central principle.

$$\boxed{\Delta E_{\text{system}} = W_{\text{surr}} + \text{ other energy transfers}}$$

The energy principle

The change in energy (ΔE_{system}) of a system is equal to the work done on the system by the surroundings (W_{surr}) , and to other kinds of energy transfers between system and surroundings.

Conservation of energy

Write $$\boxed{\Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0}$$

principle of conservation of energy

Thus the energy of a system is changed by transferring energy to or from the surroundings.

There are basically two modes of energy we need consider:

- Energy of particles.
- Energy arising from the interaction between particles.

All others are semantics.

Energy of a particle

What is the energy of a particle?

… what is a particle ..?

A particle is any object that does not undergo any internal changes during the process of interest.

What we regard as a particle depends on our model …
… e.g. is the Earth a particle?

Special Relativity

Newtonian mechanics works well at low speeds, but fails when applied to objects whose speed approaches *c*.

Albert Einstein: Special theory of relativity 1905
General theory of relativity 1917

The special theory is based on two postulates …

The relativity postulate: The laws of physics are the same for all observers in all inertial reference frames.
(Inertial reference frames move at constant velocity with respect to each other.)

The speed of light postulate: The speed of light in free space (vacuum) has the same value ($c = 3\times10^8$ m s^{-1}) in all directions and in all reference frames.

The energy of a single particle system

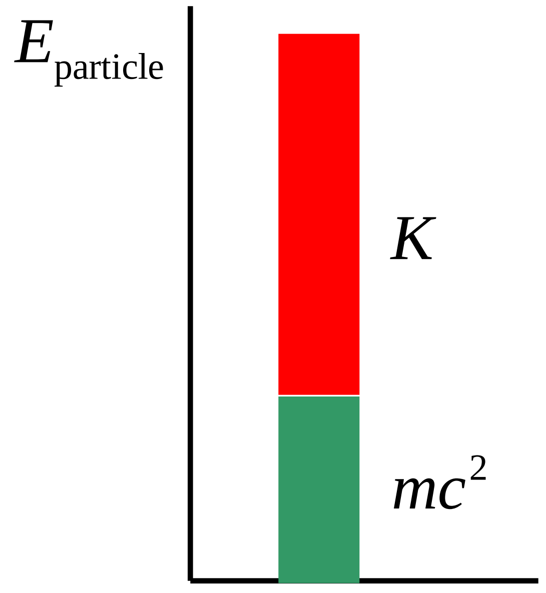
Einstein (1905): $E_{\text{particle}} \equiv \gamma mc^2$ where $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$.

"Rest energy" of a particle at rest = mc^2 .

Therefore the kinetic energy of a particle: $K = \gamma mc^2 - mc^2$

$$= (\gamma - 1) mc^2$$

or $\boxed{E_{\text{particle}} = mc^2 + K}$



As v approaches c,
K (and hence E) become very large
(and approach each other in magnitude).

Units for energy

The unit of energy is 1 N m = 1 joule.

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}.$$

Another unit in wide use is the electron volt.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}.$$

This is used mainly when dealing with (sub-)atomic particles.

Kinetic energy

At low speeds, the kinetic energy becomes a useful concept.

For $v << c$:

$$E_{\text{particle}} = \frac{mc^2}{\sqrt{1-v^2/c^2}} = mc^2\left[1+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\left(\frac{v^2}{c^2}\right)^2+\frac{5}{16}\left(\frac{v^2}{c^2}\right)^3+\ldots\right]$$

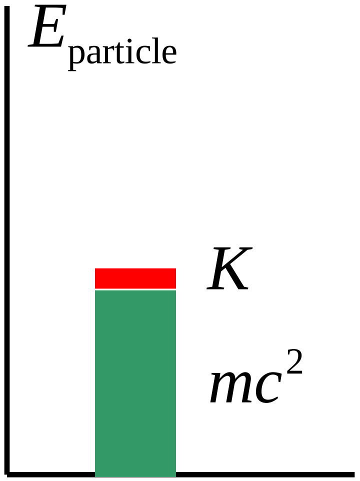
Taking the lowest order terms:

$$E_{\text{particle}} = \frac{mc^2}{\sqrt{1-v^2/c^2}} \approx mc^2 + \frac{1}{2}mv^2$$

Thus kinetic energy K at low speeds is

$$K \approx \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$K << mc^2 \text{ when } v << c$$



Kinetic energy …2

Now we can write:

$$\boxed{K + mc^2 = \gamma mc^2 = E}$$

kinetic energy

rest energy

total energy

Therefore $E = \gamma mc^2 = \gamma E_0$

$\Longrightarrow$ mass is a form of energy.

A small mass can produce an enormous amount of energy

It can be shown that $E^2 = p^2c^2 + m^2c^4$.

For an object at rest, $p = 0$, so $E = E_0 = mc^2$

… total energy = rest energy

For particles which have zero mass, e.g. photons, $E = pc$.

Work: mechanical energy transfer

How can we change the speed of a particle (and hence its energy)?

We can accelerate a particle in two ways:

1. Apply force $\vec{\mathbf{F}}$ for time Δt .
As a result the momentum changes: $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$

2. Apply force $\vec{\mathbf{F}}$ for a displacement $\Delta\vec{\mathbf{r}}$.
In 1D, the **kinetic energy** changes: we find $\Delta K = F\Delta r$

The quantity $F\Delta r$ is known as **work**
(recall that $F\Delta t$ is the impulse.)

Work and impulse

Impulse $\vec{\mathbf{F}}\Delta t$ and $\Delta\vec{\mathbf{p}}$ are in the same direction.
With work, in general, we have two vectors: $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{r}}$.

Given two vectors, we can construct ...

(a) a scalar, or (b) a vector perpendicular to both.

What makes sense here?

Let’s consider accelerating a block (a “particle”!) with a force.

Work and impulse

Consider a force applied to block at angle θ to direction of motion. Choose x-axis in direction of motion.

If the force is not variable, i.e. $\vec{\mathbf{F}}$ is constant, and $\Delta\vec{\mathbf{r}}$ is in a straight line

No vertical displacement so $F_y \Delta r_y = F_y 0 = 0$

Only component in direction of motion does work:

$$F_x \Delta r_x = F \cos\theta \Delta r$$

Dot product or "scalar product"

Given two vectors

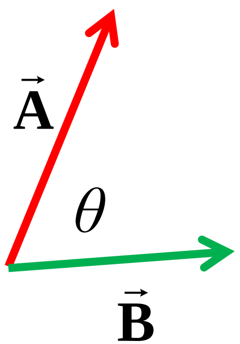
$$\vec{\mathbf{A}} = \left\langle A_x, A_y, A_z \right\rangle = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \left\langle B_x, B_y, B_z \right\rangle = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

Then $\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z = d$

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = \vec{\mathbf{B}}\bullet\vec{\mathbf{A}} \qquad \vec{\mathbf{A}}\bullet\vec{\mathbf{A}} = \mathrm{A}^2$$

$$\hat{\mathbf{i}}\bullet\hat{\mathbf{i}} = \hat{\mathbf{j}}\bullet\hat{\mathbf{j}} = \hat{\mathbf{k}}\bullet\hat{\mathbf{k}} = 1 \qquad \hat{\mathbf{i}}\bullet\hat{\mathbf{j}} = \hat{\mathbf{j}}\bullet\hat{\mathbf{k}} = \hat{\mathbf{k}}\bullet\hat{\mathbf{i}} = 0$$



In polar form in 2D: $\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos\theta$

where θ is the angle between tails of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

Work and the scalar product

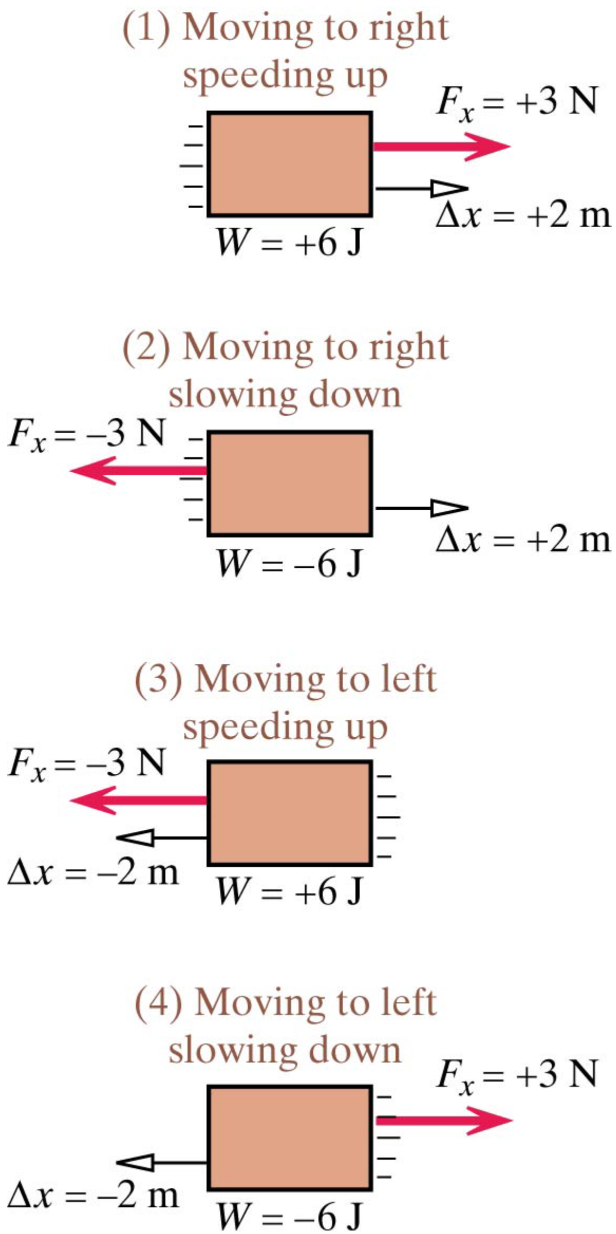
The work done by a force is defined as

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \bullet \Delta\vec{\mathbf{r}}$$

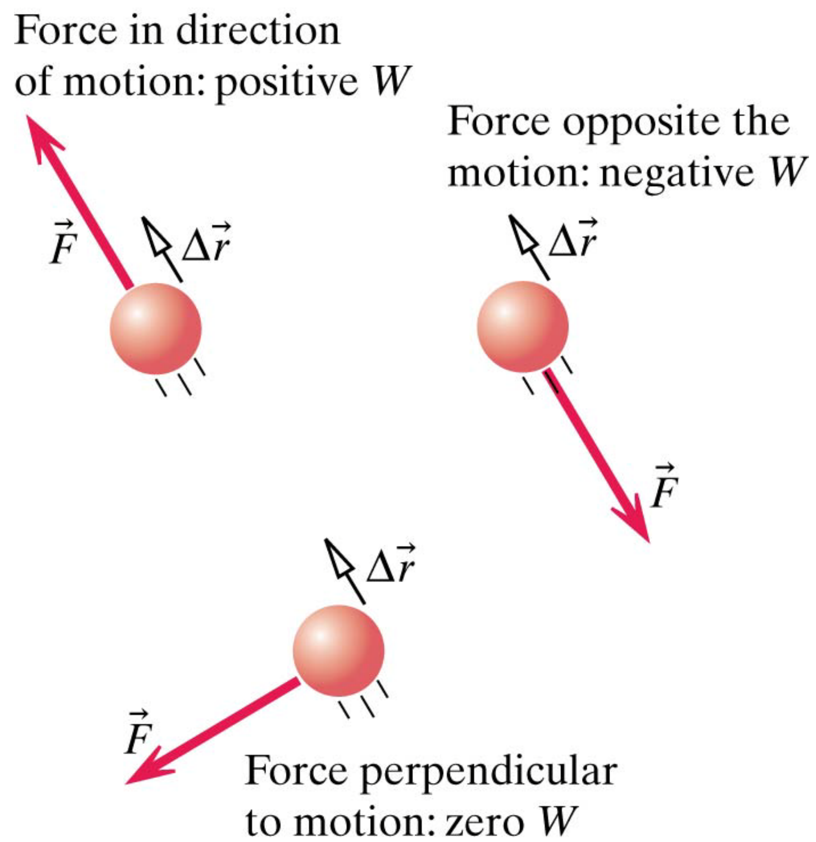
The unit of energy or work is 1 N m = 1 joule.

Note: *W* can be positive or negative.

Positive and negative work

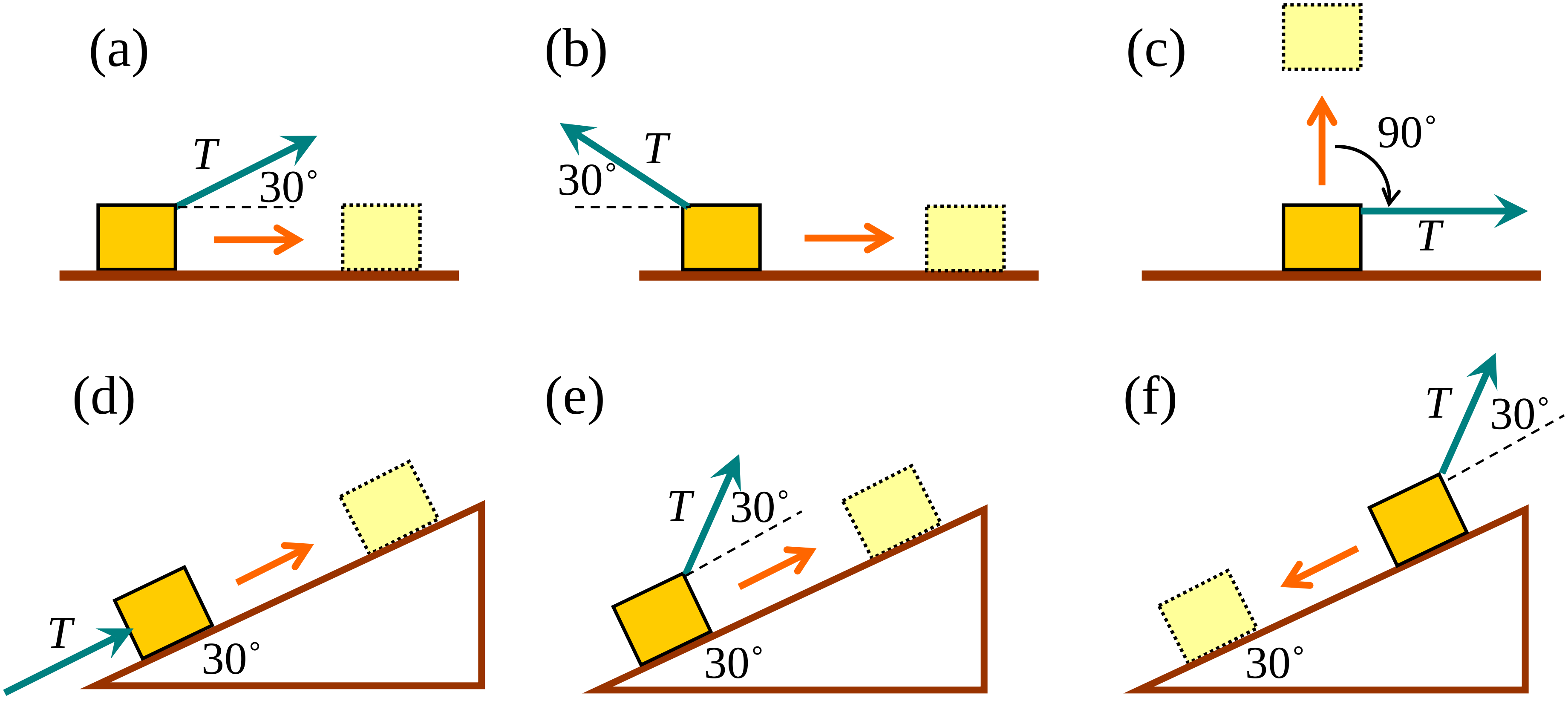


Zero work



The work done by a force.

A force T = 50 N acts on a block as it moves a distance of 30 m in a straight line as shown in each case below. Calculate the work done by the force T in each case.



Work done by a force: example

A force $\vec{\mathbf{F}} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ N acts on a 10 kg mass and moves it from an initial position $\vec{\mathbf{r}}_i = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ m to a final position $\vec{\mathbf{r}}_f = 5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ m. How much work is done by this force in moving the block through this displacement?

1	2	3	4	5

A ball whose mass is 2 kg travels at a velocity of
< 0, –3, 4> m/s.

What is the kinetic energy of the ball?

(1) < 0, –6, 8 > J
(2) < 0, –3, 4 > J
(3) 2 J
(4) 10 J
(5) 25 J

1	2	3	4	5

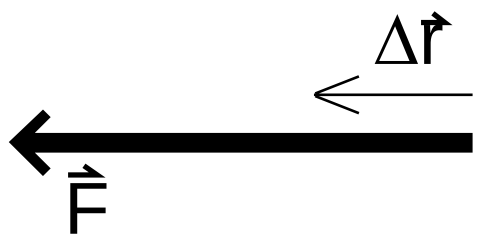
A ball whose mass is 2 kg travels at a velocity of $< 0, -3, 4>$ m/s.

What is the rest energy of the ball?

(1) 0 J
(2) 25 J
(3) 6e8 J
(4) 9e16 J
(5) 1.8e17 J

1	2	3	4	5

A fancart moves in the –*x* direction. The fan is on, and the force on the cart by the air is also in the –*x* direction. Is the work done by the air positive, negative, or zero?



(1) positive
(2) negative
(3) zero

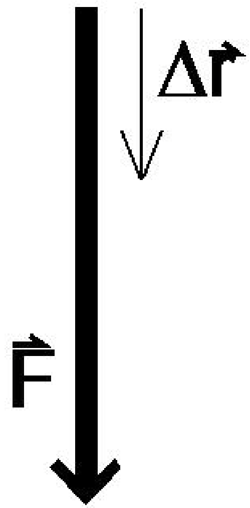
1	2	3	4	5

A skater on a skateboard coasts in the +*x* direction. He is about to run into his friend, so she pushes him in the –*x* direction, to slow him down. What is the sign of the work done by the friend?

(1) positive
(2) negative
(3) zero

1	2	3	4	5

A tennis ball is moving in the –*y* direction. You hit it downward with a tennis racket. During the time your racket is in contact with the ball, do you do positive, negative, or zero work on the ball?



(1) positive
(2) negative
(3) zero

Update form of the energy principle

Our technical definition of work is not much use unless we connect it with energy.

Work is the amount of (mechanical) energy transferred to a system from the surroundings through the agency of forces.

This implies that, if E is energy, $\Delta E_{\text{system}} = W_{\text{surr}}$

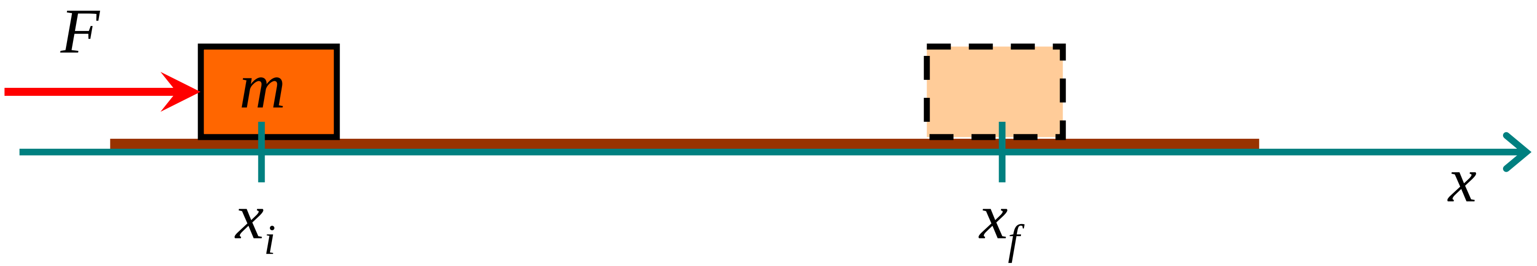
Energy principle

Energy principle in update form: $E_{\text{system}, f} = E_{\text{system}, i} + W_{\text{surr}}$

For a single particle: $E_f = E_i + W_{\text{surr}}$

The work-energy theorem

Consider a mass m moving along the x-axis acted upon by a constant force F:



Then the work done by F in moving m from x_i to x_f :

$$W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} madx$$

But $\quad madx = m\frac{dv}{dt}dx = m\frac{dx}{dt}dv = mvdv$

$$\therefore W = \int_{v_i}^{v_f} mvdv = m\int_{v_i}^{v_f} mvdv = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\therefore W = K_f - K_i = \Delta K$$

If W is positive, then the mass will speed up.

If W is negative, then the mass will slow down.

Change of rest energy

If the particle in question does not change its identity, then

$$E_{\text{particle}} = mc^2 + K_f = mc^2 + K_i + W$$

or $$K_f = K_i + W$$

In many situations, the rest mass of a particle can change,
e.g. Decay of a neutron at rest: $n \rightarrow p + e^- + \bar{\nu}$

$$E_f = E_i + W$$

$$\left(m_p c^2 + K_p\right) + \left(m_e c^2 + K_e\right) + \left(m_{\bar{\nu}} c^2 + K_{\bar{\nu}}\right) = \left(m_n c^2 + K_n\right) + W$$

$$\left(938.3\ \text{MeV} + K_p\right) + \left(0.511\ \text{MeV} + K_e\right) + \left(0 + K_{\bar{\nu}}\right) = \left(939.6\ \text{MeV} + 0\right) + 0$$

$$\therefore K_p + K_e + K_{\bar{\nu}} = 0.8\ \text{MeV}$$

... positive, as it should be.

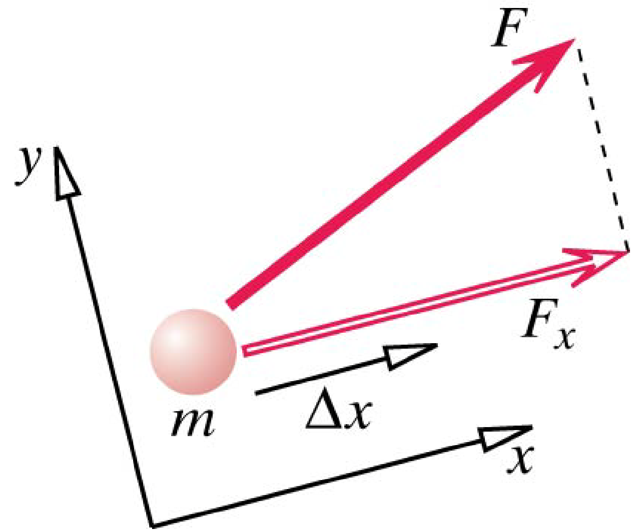
Proof of the energy principle for a particle

Take x-axis in direction of motion, i.e. along $\Delta\vec{\mathbf{r}}$.

$$\Delta E = \vec{\mathbf{F}} \bullet \Delta\vec{\mathbf{r}} = F_x \Delta x = \frac{\Delta p_x}{\Delta t}\Delta x$$

Thus $\dfrac{\Delta E}{\Delta x} = \dfrac{\Delta p_x}{\Delta t}$

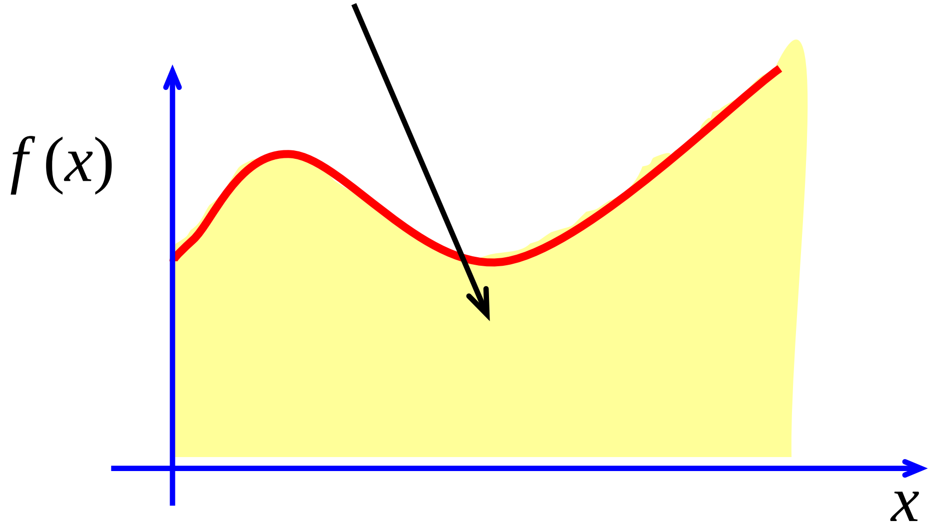
or in the limit $\Delta t \to 0$: $\dfrac{dE}{dx} = \dfrac{dp_x}{dt}$



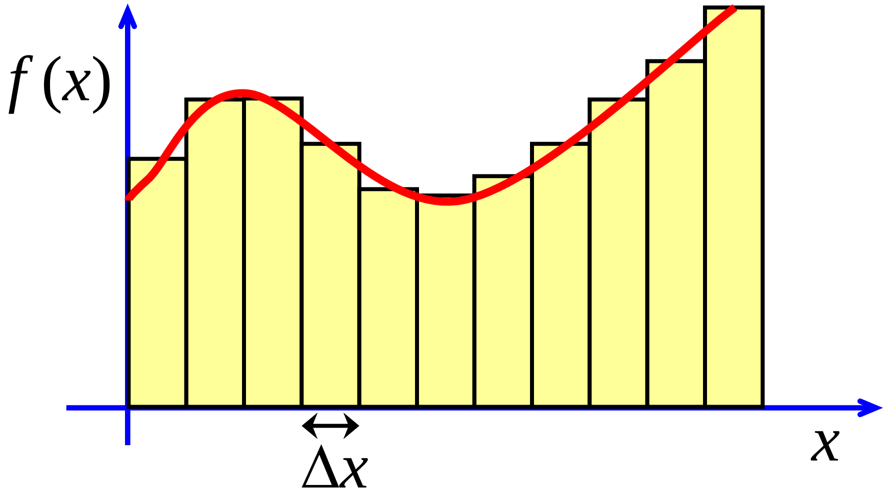
A excursion into … Integration in physics

Basically, integration is the inverse operation of differentiation, and is therefore sometimes referred to as anti-differentiation.

Consider a function $f(x)$ as shown below.
How can we find the area under such a curve?

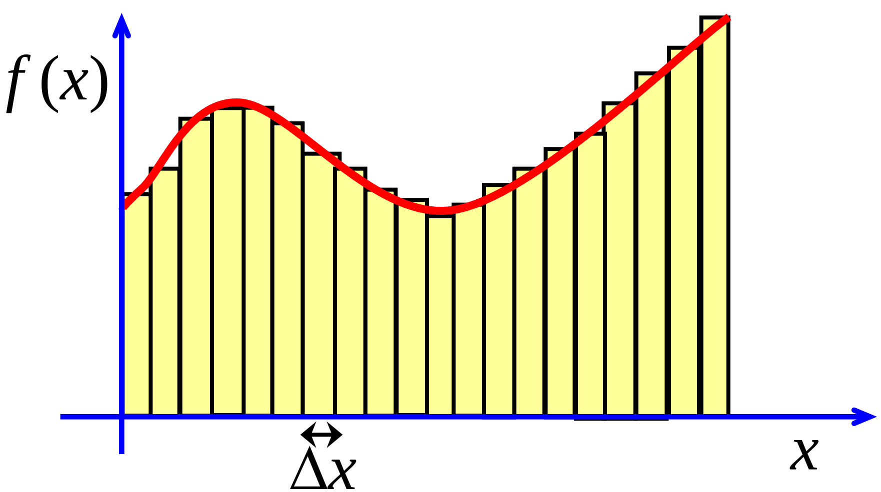


One way would be to construct thin, equally wide rectangles under the curve, as shown.



If each rectangle has width Δx, then the area under the curve $f(x)$ is approximated by $\sum f_i(x_i)\Delta x$

As we use rectangles with smaller Δx, $\sum f_i(x_i)\Delta x$ approaches the area under the curve …



Of course, we would try to make these rectangles as small as possible, infinitesimally small if we could.
The operation of integration does precisely that for you …

The "integral of $f(x)$" = $\int f(x)dx$

$= A(x)$ = area under the function $f(x)$.

This is true if $\dfrac{dA(x)}{dx} = f(x)$

… therefore $A(x)$ is called the anti-derivative of $f(x)$.

$A(x)$ is not a unique function since …

$$\frac{d}{dx}(A(x)+C) = \frac{d}{dx}A(x) = f(x)$$ for <u>any constant</u> C

$\int f(x)dx$ is called an indefinite integral since it provides the function which generally describes the "area" under the function $f(x)$. Note that the "area" can be positive or negative.

Note that there are many, many more complicated cases of integration which you will learn about in your mathematics course. For now we are only interested in dealing with the case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

Exercises:

(a) $\int 4x dx =$

(b) $\int (9x^2 + 16x^3) dx =$

(c) $\int \frac{x^4}{2} dx =$

(d) $\int x^{-3} dx =$

If we are only interested in the area between two particular x values, then we write

$$\int_{x_i}^{x_f} f(x)\,dx$$

where x_{f} and x_{i} are called the limits of the definite integral. We interpret this mathematical statement as "determine the area under the function $f(x)$ between $x = x_i$ and $x = x_f$."

A definite integral may result in a positive number, a negative number or even zero as the result.

For example… $\displaystyle\int_1^2 \left(x^3+3\right)dx = \left.\frac{x^4}{4}+3x+C\right|_1^2$

$$= \left(\frac{(2)^4}{4}+3(2)+C\right)-\left(\frac{(1)^4}{4}+3(1)+C\right)$$

$$= 4+6-0.25-3 \quad = \quad 6.75$$

Exercises:

(a) $\int_{-1}^{1} 4x\,dx =$

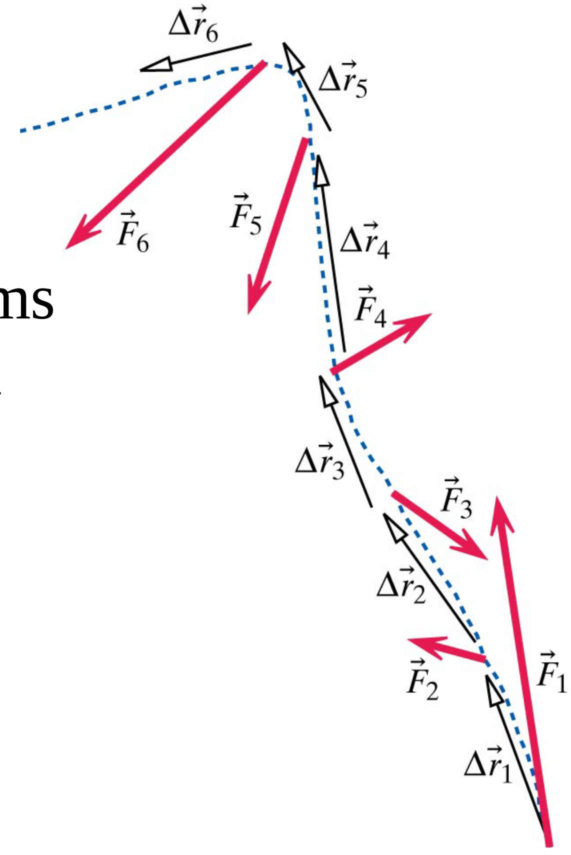
(b) $\int_{1}^{3} (3x^2 - 4x + 8)\,dx =$

Work done by a nonconstant force

If the force is not constant along the path, we have to break up the path into small segments:

$$W = \vec{\mathbf{F}}_1 \bullet \Delta\vec{\mathbf{r}}_1 + \vec{\mathbf{F}}_2 \bullet \Delta\vec{\mathbf{r}}_2 + \vec{\mathbf{F}}_3 \bullet \Delta\vec{\mathbf{r}}_3 + \ldots$$

We can write this as $W = \sum_i \vec{\mathbf{F}}_i \bullet \Delta\vec{\mathbf{r}}_i$



We can make our increments smaller and smaller, in such away that the number of terms in the sum becomes infinite as the increment becomes infinitesimal (infinitely small).

This can be done consistently, leading to the result

$$W = \int_f^i \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$$

This is a (line) integral.

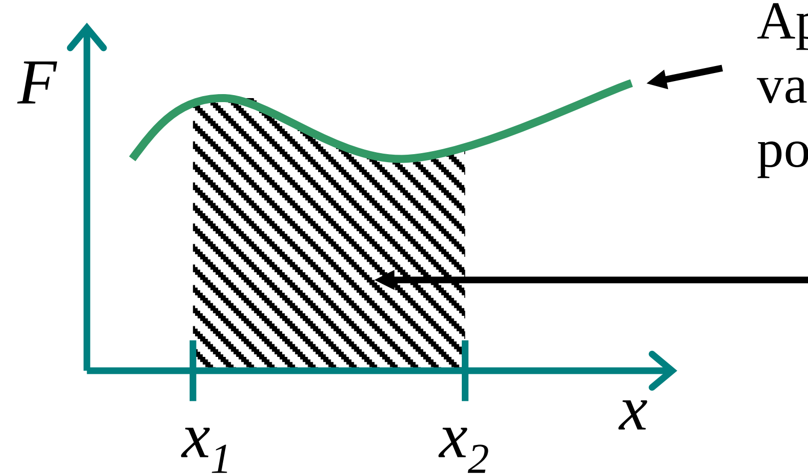
We will generally stick with sums.

If $\vec{\mathbf{F}}$ is variable, but the displacement is still in only one direction,

then $W = \int_i^f \vec{\mathbf{F}}(\vec{\mathbf{r}}) \bullet d\vec{\mathbf{r}}$ simplifies to $W = \int_{x_1}^{x_2} \vec{\mathbf{F}}(\vec{\mathbf{x}}) \bullet d\vec{\mathbf{x}}$

This integral may be interpreted as the area under the F - x curve.

For example:

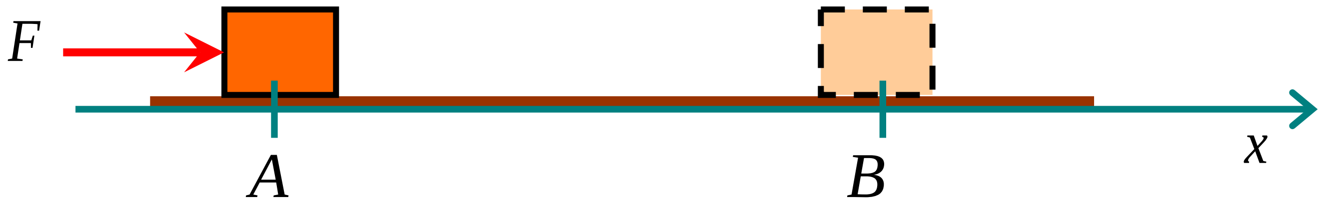


Applied force which varies as a function of position x.

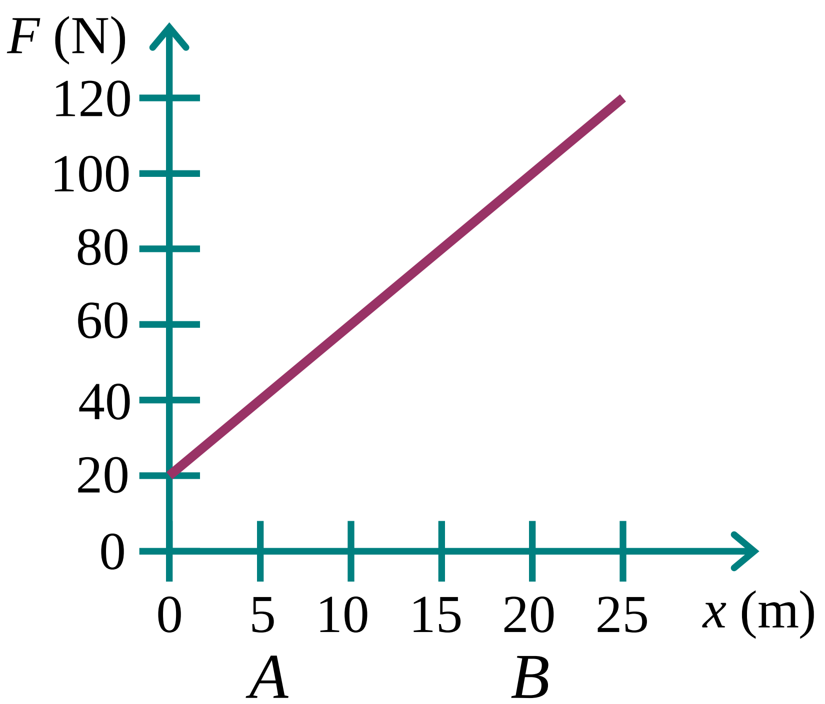
Work done by F in moving the object from x_1 to x_2.

Work done by a force: example

A force F moves a box from point A to point B along the x-axis.



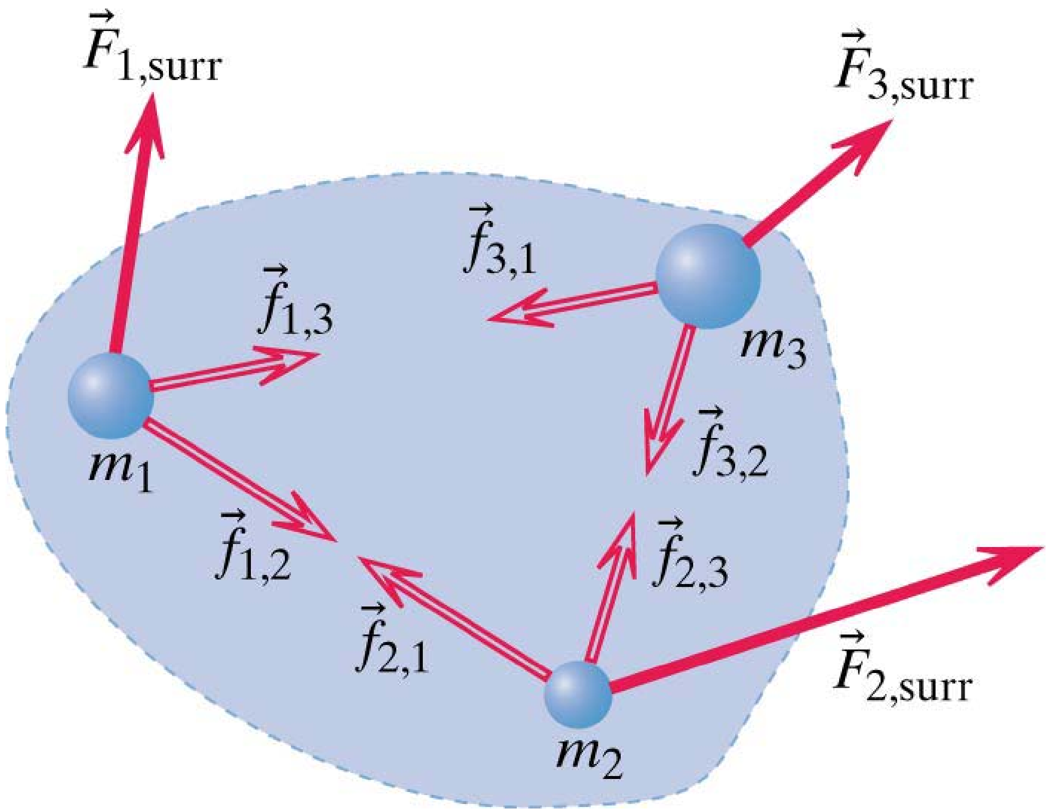
Shown alongside is the force-distance graph for this situation.



(a) From the graph, calculate the work done by this force in moving the box from A to B.

(b) Using an appropriate integral, also calculate this work.

Potential energy in multi-particle systems



Isolated particles have only the energy $E = \dfrac{mc^2}{\sqrt{1-v^2/c^2}}$.

If a particle interacts with others, there is an energy associated with the <u>interactions</u>: this is called **potential energy**.

Potential energy

Suppose a system of several (or many) particles interact with one another and external forces act as well.

Then $$\Delta\left(E_1 + E_2 + E_3 + \ldots\right) = W_{\text{internal}} + W_{\text{external}}$$

We can write this as $$\Delta\left(E_1 + E_2 + E_3 + \ldots\right) - W_{\text{internal}} = W_{\text{external}}$$

or $$\Delta\left(E_1 + E_2 + E_3 + \ldots\right) + \Delta U = W_{\text{external}}$$

Hence we define the work done by the internal forces W_{internal} to be the change in potential energy U.

$$\boxed{\Delta U \equiv -W_{\text{internal}}}$$

We can define the energy of the system to be

$$E_{\text{system}} = (E_1 + E_2 + E_3 + \ldots) + U$$

For example, for a 3 particle system:

$$E_{\text{system}} = (m_1c^2 + m_2c^2 + m_3c^2 + K_1 + K_2 + K_3) + U_{12} + U_{23} + U_{13}$$

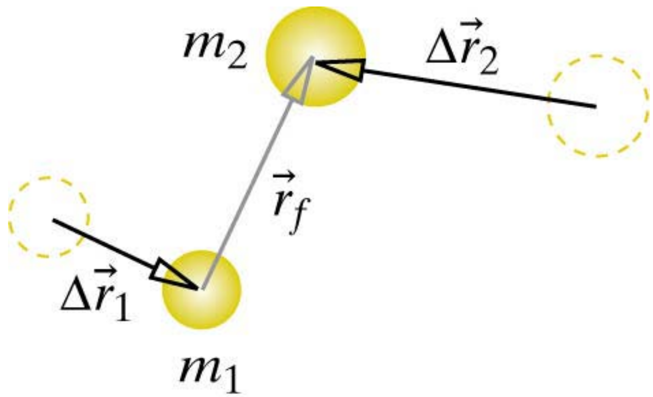
Since $\Delta E_{\text{system}} = W_{\text{surroundings}}$

and $\Delta E_{\text{surroundings}} = -W_{\text{surroundings}}$

Write $\boxed{\Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0}$ **principle of conservation of energy**

Thus the energy of a system is changed by transferring energy to or from the surroundings.

Gravitational potential energy



Gravitational force: $\vec{\mathbf{f}}_{1,2} = -\vec{\mathbf{f}}_{2,1}$

Then

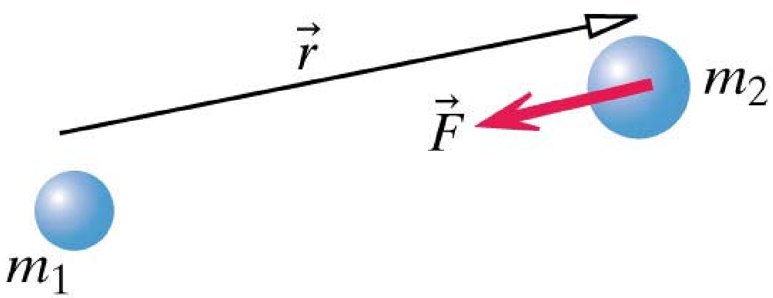
$$\Delta U_g = -W_{\text{int}} = -\left(\vec{\mathbf{f}}_{1,2} \bullet \Delta\vec{\mathbf{r}}_1 + \vec{\mathbf{f}}_{2,1} \bullet \Delta\vec{\mathbf{r}}_2\right)$$

$$\therefore \Delta U_g = -\vec{\mathbf{f}}_{2,1} \bullet \left(\Delta\vec{\mathbf{r}}_2 - \Delta\vec{\mathbf{r}}_1\right)$$

Write $\Delta U_g = -\vec{\mathbf{f}}_{2,1} \bullet \Delta\vec{\mathbf{r}}$

If $F_{g,r}$ is the component of the gravitational force on m_2 in the direction of $\vec{\mathbf{r}}$ ...

$$dU_g = -F_{g,r}\, dr$$



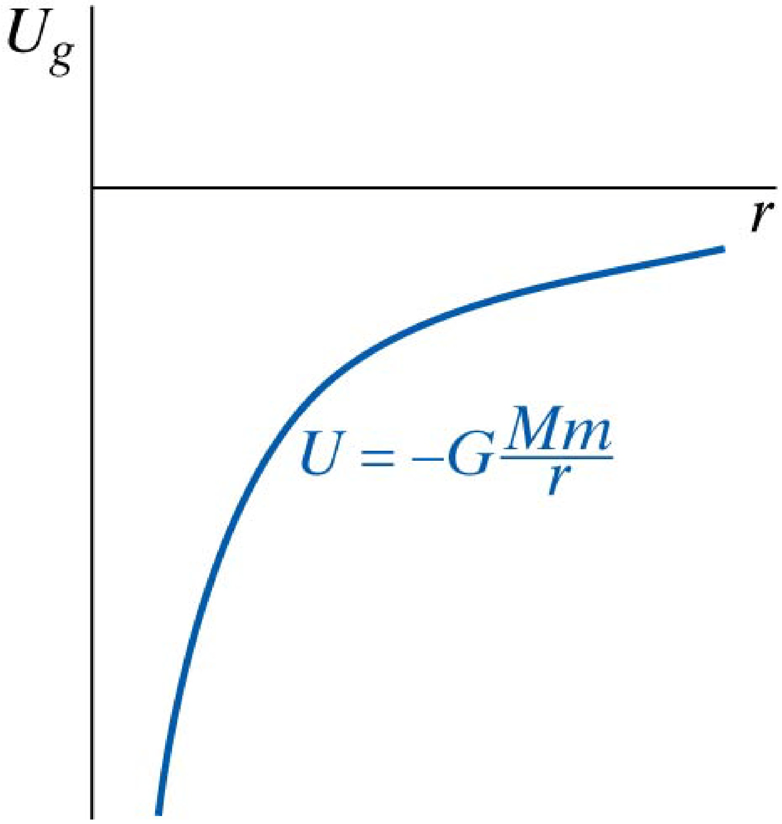
Gravitational potential energy ... continued

$$dU_g = -F_{g,r}\,dr$$

Then $$F_{g,r} = -\frac{dU_g}{dr}$$

since $$F_{g,r} = -G\frac{m_1 m_2}{r^2}$$

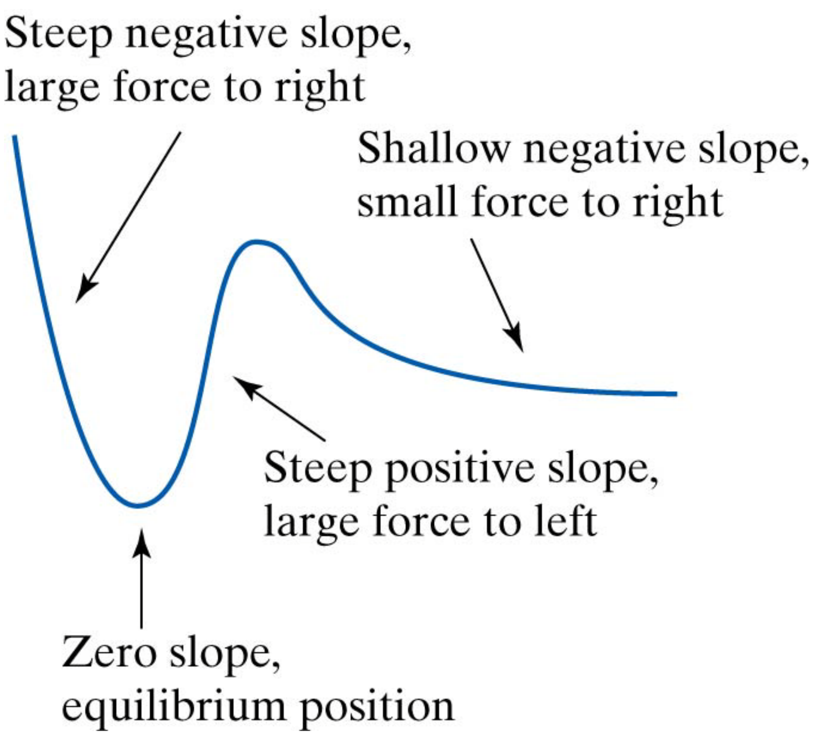
We find that $$U_g = -G\frac{m_1 m_2}{r}$$



Force and potential energy

In general, each component of the force can be related to a derivative of the potential energy with respect to the appropriate coordinate.

Thus $$F_x = -\frac{dU}{dx}$$



Such a vector object obtained from a scalar function is known as a **gradient.**

$$\vec{\mathbf{F}} = \left\langle -\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z} \right\rangle$$

or $$\vec{\mathbf{F}} = -\vec{\nabla} U$$

(wait for Physics II)

M&I
3E 6.10

General properties of potential energy

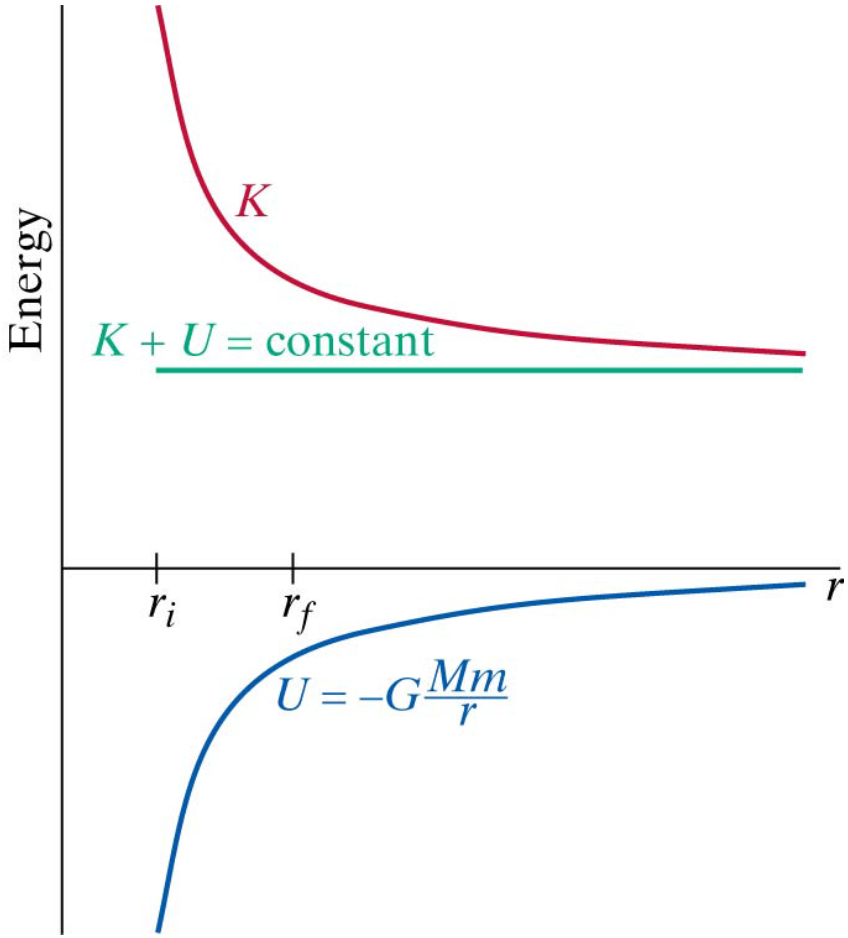
... depends on the separation between pairs of particles, not their individual positions.

... must approach zero as the separation of the particles becomes very large.

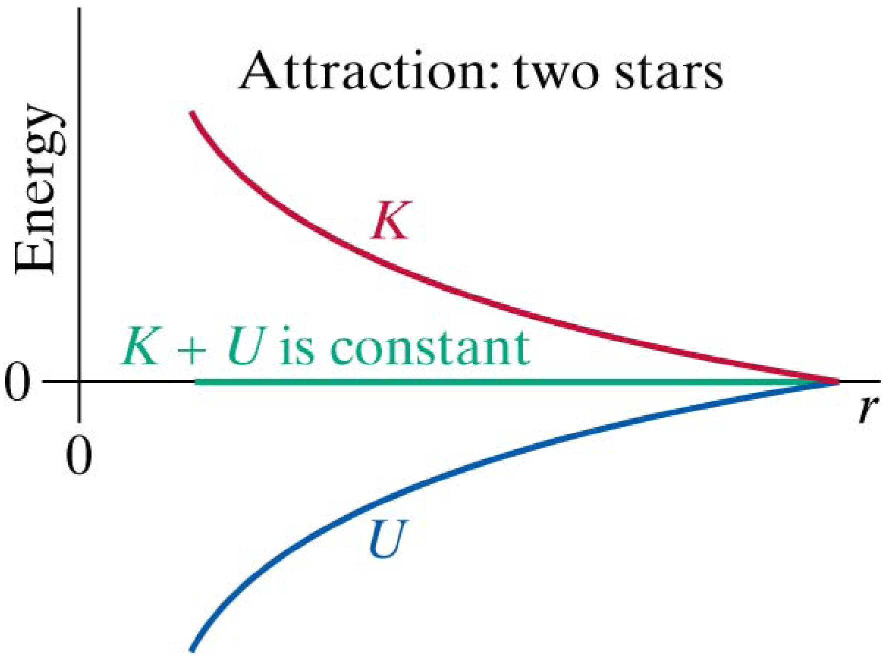
... becomes negative as the distance between particles decreases (if an interaction is **attractive**).

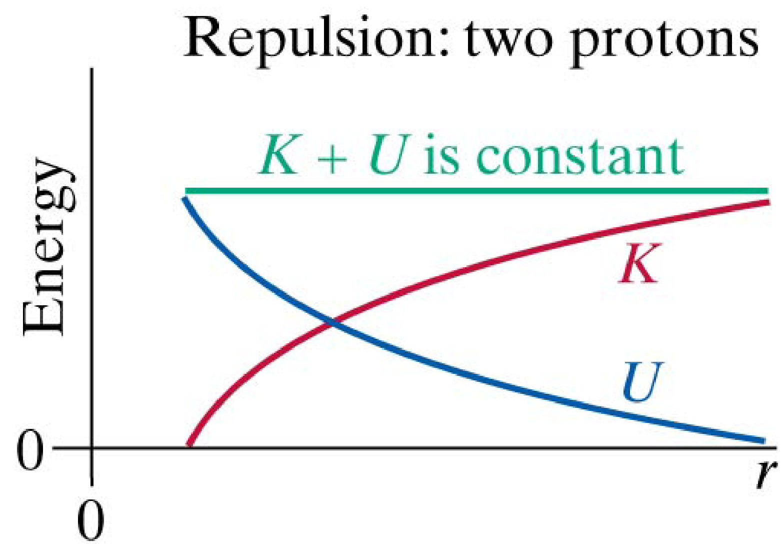
... becomes positive as the distance between particles decreases (if an interaction is **repulsive**).

Plotting energy versus separation



A spacecraft initially parked on an asteroid, blasts off and travels away from the asteroid.


Attraction: two stars
Energy
K
K + U is constant
0
r
0
U


Repulsion: two protons
K + U is constant
Energy
K
U
0
r
0

M&I 3E 6.13

Gravitational potential energy near the earth's surface

$$U_g = -G\frac{m_1 m_2}{r}$$

An important special case: $U_g(y) = -G\frac{M_E m_2}{R_E + y} = -G\frac{M_E m_2}{R_E\left(1 + y/R_E\right)}$

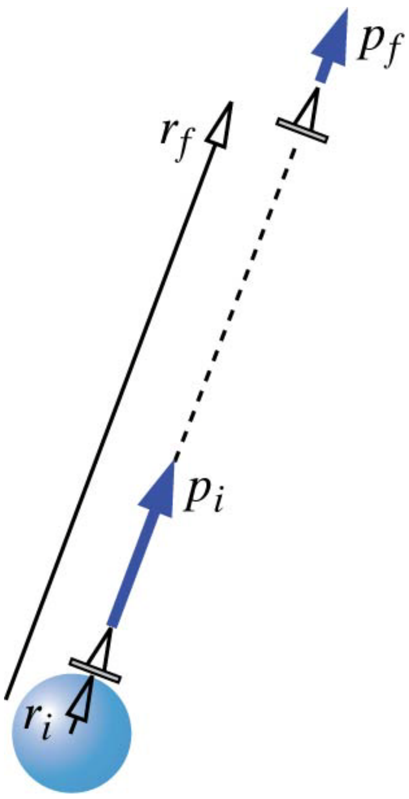
$$\approx -G\frac{M_E m_2}{R_E}\left(1 - y/R_E\right)$$

If we are interested in changes in U_g near the surface of the earth:

$$\Delta U_g = U_g(y+h) - U_g(y) \; = \; G\frac{M_E m_2}{R_E^2}h \; = \; mgh$$

M&I
3E 6.12

Applying gravitational potential energy: Escape speed



Leave the surface of the planet with p_i (mv_{esc})

Want to have $K = 0$ far away, i.e. when $U = 0$

Therefore minimal condition for escape: $K + U = 0$

$$\therefore K_i + U_{g,i} = \frac{1}{2}mv_{esc}^2 + \left(-G\frac{Mm}{R}\right) = 0$$

Thus $\quad v_{esc} = \sqrt{\frac{2GM}{R}}$

1	2	3	4	5

You drop a ball of mass *m* at a height *h* above the ground. The ball falls, speeding up, bounces off the floor, and goes upward, slowing down, until it is once again at the location where you released it (height *h*).

Initial state: Just after release
Final state: Ball back at original location

How much work was done by the Earth on the ball?

(1) mgh
(2) $-mgh$
(3) $2*mgh$
(4) $-2*mgh$
(5) 0

1	2	3	4	5

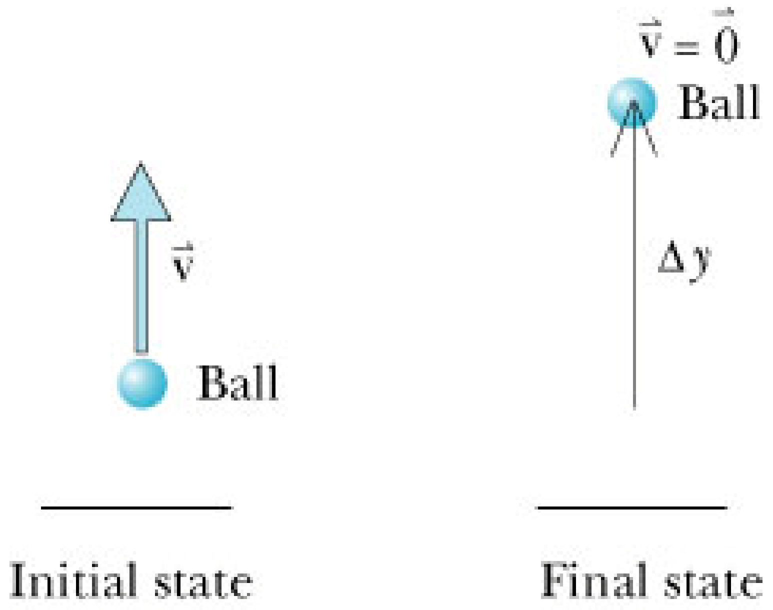
An isolated neutron decays: $n \rightarrow p^+ + e^- + \overline{\nu}$

Initial: Neutron at rest   Final: $p^+, e^-, \overline{\nu}$ far from each other

(1) The sum of the rest energies of the products equals the rest energy of the neutron.
(2) The sum of the kinetic energies of the products equals the rest energy of the neutron.
(3) The sum of the rest energies and kinetic energies of the products equals the rest energy of the neutron.
(4) The sum of the kinetic energies of the products equals the kinetic energy of the neutron.

1	2	3	4	5

A thrown ball heads straight up.
System: **Ball**

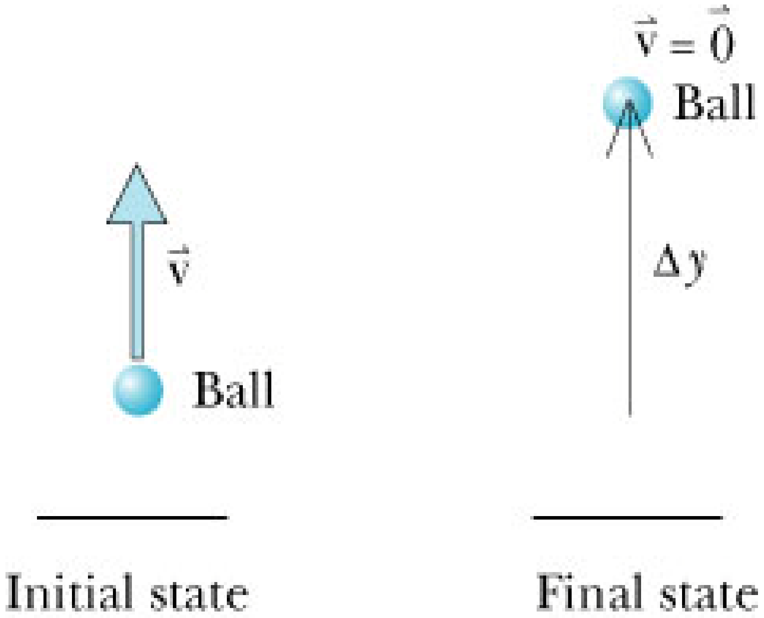


What is the work done
by the surroundings?

(1) 0
(2) $mg\Delta y$
(3) $-mg\Delta y$
(4) something else

1	2	3	4	5

A thrown ball heads straight up.
System: **Ball + Earth**

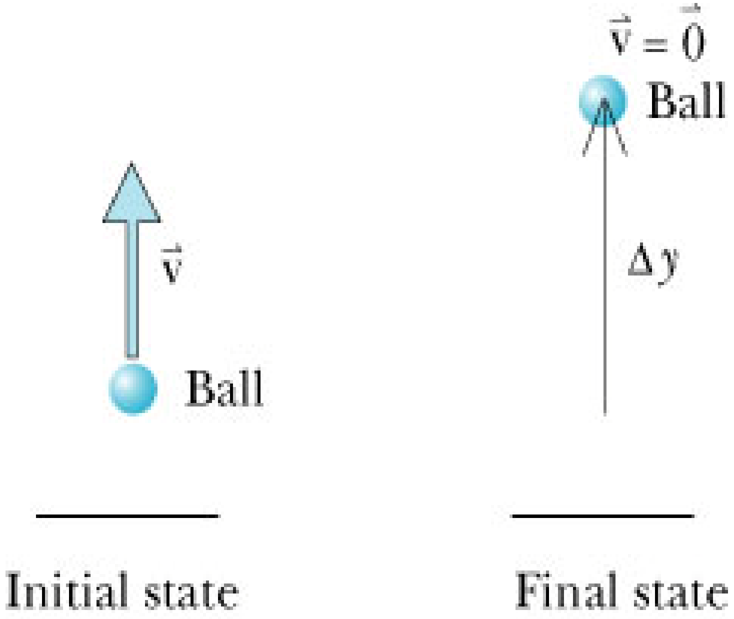


What is the work done by the surroundings?

(1) 0
(2) $mg\Delta y$
(3) $-mg\Delta y$
(4) something else

1	2	3	4	5

A thrown ball heads straight up.
System: **Ball + Earth**



How did the kinetic energy of the system change?

(1) $\Delta K > 0$
(2) $\Delta K = 0$
(3) $\Delta K < 0$

1	2	3	4	5

A ball of mass 0.1 kg is dropped from rest near the Earth. The ball travels downward 2 m, speeding up.

System: **Ball**

What is the work done by the surroundings?

(1) 0
(2) +1.96 J
(3) –1.96 J

1	2	3	4	5

A ball of mass 0.1 kg is dropped from rest near the Earth.
The ball travels downward 2 m, speeding up.

System: **Ball + Earth**

What is the work done by the surroundings?

(1) 0
(2) +1.96 J
(3) –1.96 J

1	2	3	4	5

A ball of mass 0.1 kg is dropped from rest near the Earth.
The ball travels downward 2 m, speeding up.

System: **Ball + Earth**

Work done by surroundings: 0

However, did the kinetic energy of the **Ball + Earth** system change?

(1) K increased
(2) K decreased
(3) K did not change

1	2	3	4	5

A spacecraft travels from near the Earth toward the Moon.
System: Earth, Moon, spacecraft

How many gravitational potential energy terms U_g are there in the Energy Principle?

(1) 1
(2) 2
(3) 3
(4) 6
(5) 0

1	2	3	4	5

A comet orbits a star in a strongly elliptical orbit. The comet and star are far from other massive objects.

System: comet + star

The system has:

(1) kinetic energy
(2) kinetic energy and rest energy
(3) kinetic energy, rest energy, and potential energy

1	2	3	4	5

A comet orbits a star in a strongly elliptical orbit.
The comet and star are far from other massive objects.

System: comet + star

As the comet travels away from the star, how does the kinetic energy and potential energy of the system change?

	K	U
1)	increase	decrease
2)	increase	increase
3)	decrease	increase
4)	decrease	decrease
5)	no change	no change

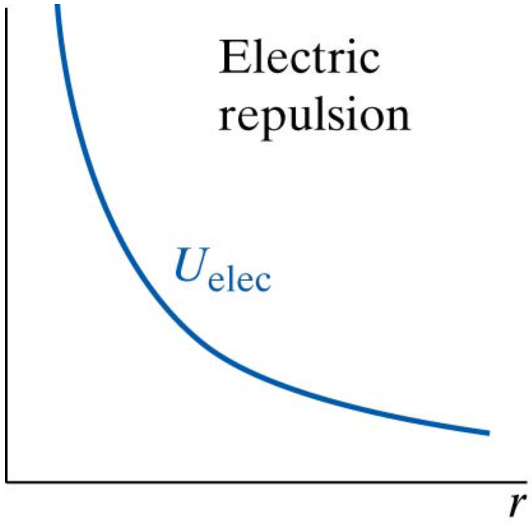
M&I
3E 6.14

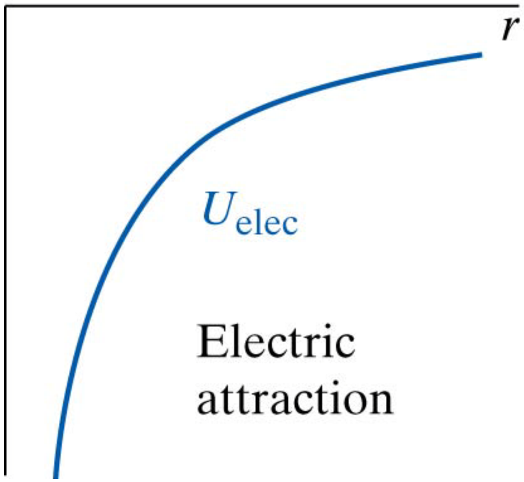
Electrical potential energy

For two charged particles: $U_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$

$$\left(U_g = -G\frac{m_1 m_2}{r}\right)$$

Charges can be positive or negative.





M&I
3E 6.15

The mass of a multi-particle system

... homework reading ...

Identifying initial and final states

For an isolated system: $\Delta K \quad + \quad \Delta U \quad = \quad W$

or $$\left(K^{f} - K^{i}\right) + \left(U_{g}^{f} - U_{g}^{i}\right) + \left(U_{other}^{f} - U_{other}^{i}\right) = W$$

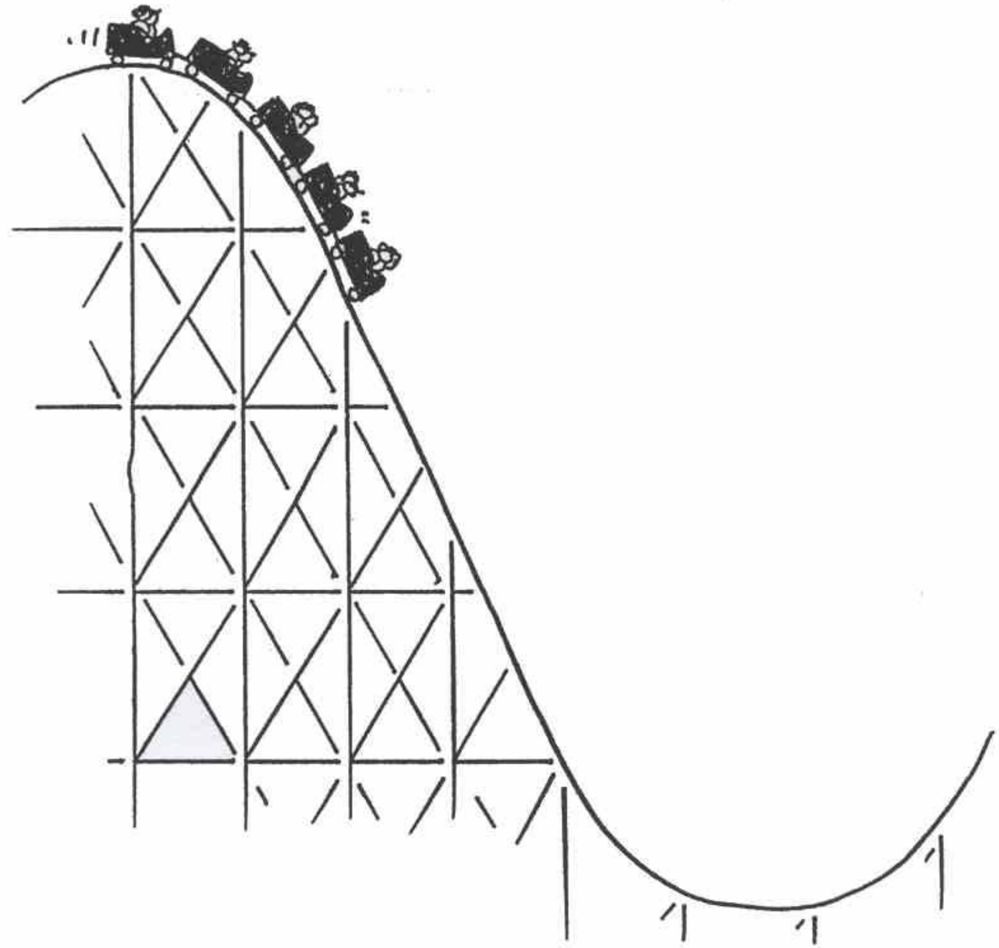
Example

A 15 kg block is released to slide from rest down a rough slope inclined at 10°. What distance down the slope is the block moving at 2 m s^{-1}? The coefficient of kinetic friction between the block and the slope is 0.1 .

1	2	3	4	5

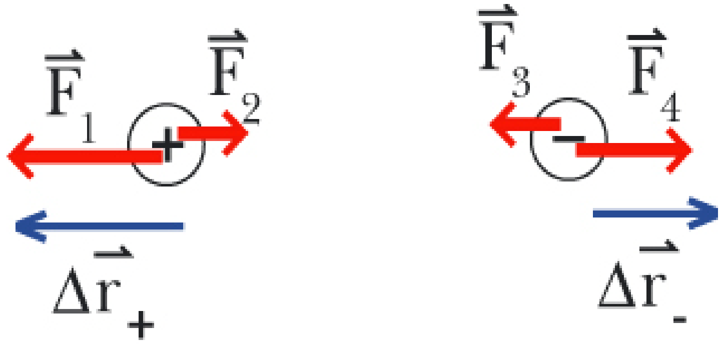
In which car will you be moving fastest at the very bottom of the incline?

(1) front car
(2) middle car
(3) rear car



1	2	3	4	5

Jack pulls to the left on a positive charge,
while Jill pulls to the right on a negative charge.
System: both charges



Which forces are external?

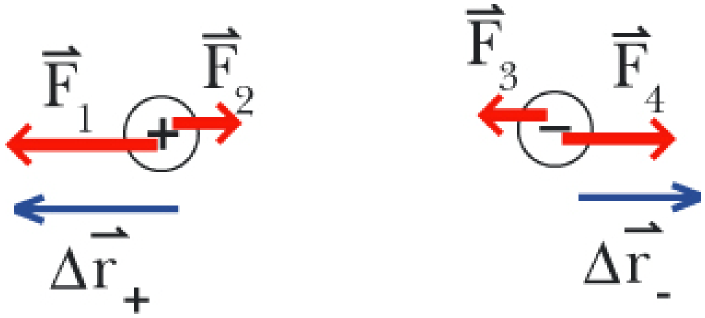
1) F_1 and F_2
2) F_2 and F_3
3) F_3 and F_4
4) F_1 and F_4

1	2	3	4	5

Jack pulls to the left on a positive charge, while Jill pulls to the right on a negative charge.

The charges are displaced, then remain at rest.

System: both charges

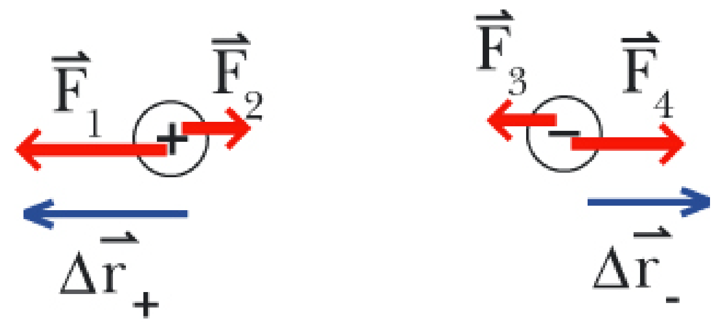


The work done by external forces was:

(1) positive
(2) negative
(3) zero
(4) need more information

1	2	3	4	5

Jack pulls to the left on a positive charge,
while Jill pulls to the right on a negative charge.



System: both charges

Initial state: charges at rest

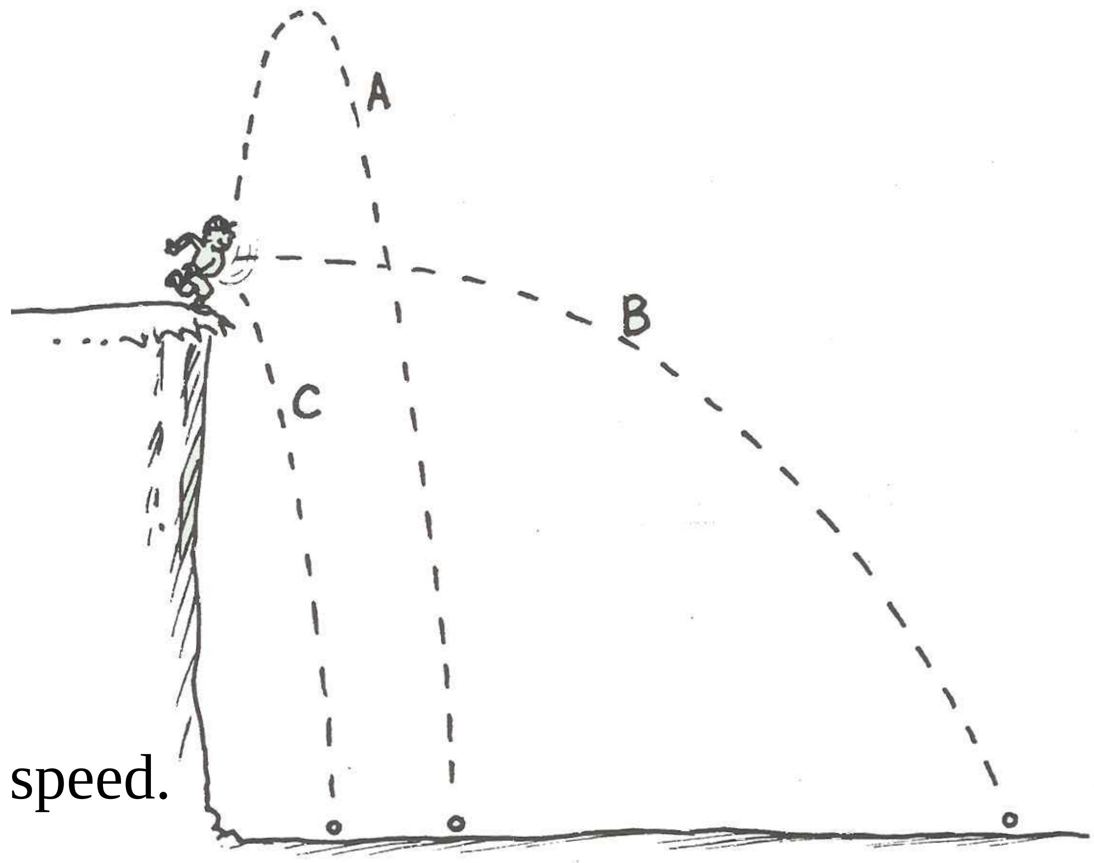
Final state: charges at rest, farther apart

The positive work done by external forces:

(1) increased *K* of the system
(2) decreased *K* of the system
(3) did not change *K* of the system

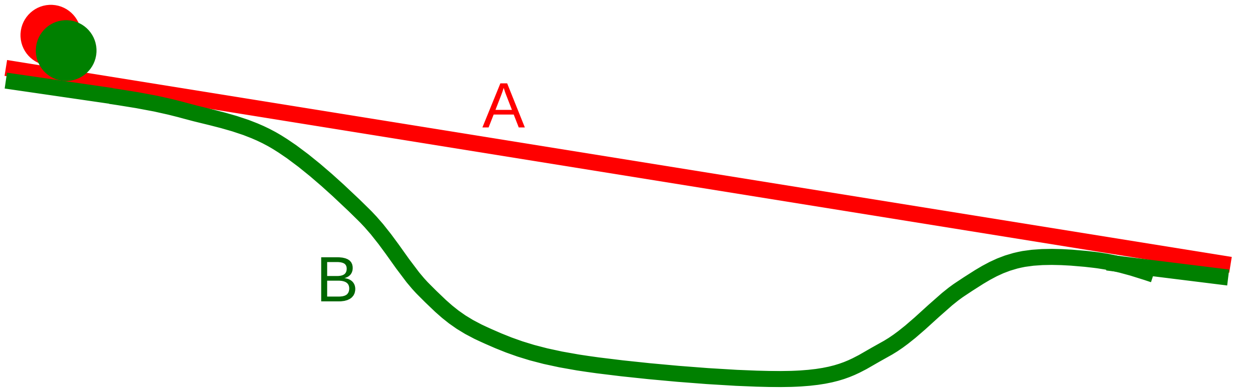
1	2	3	4	5

Three balls are thrown from the top of a cliff with the same initial speed along paths A, B and C. If there is no air resistance, which ball will strike the ground with the greatest speed?



(1) A
(2) C
(3) all strike with the same speed.

Demonstration



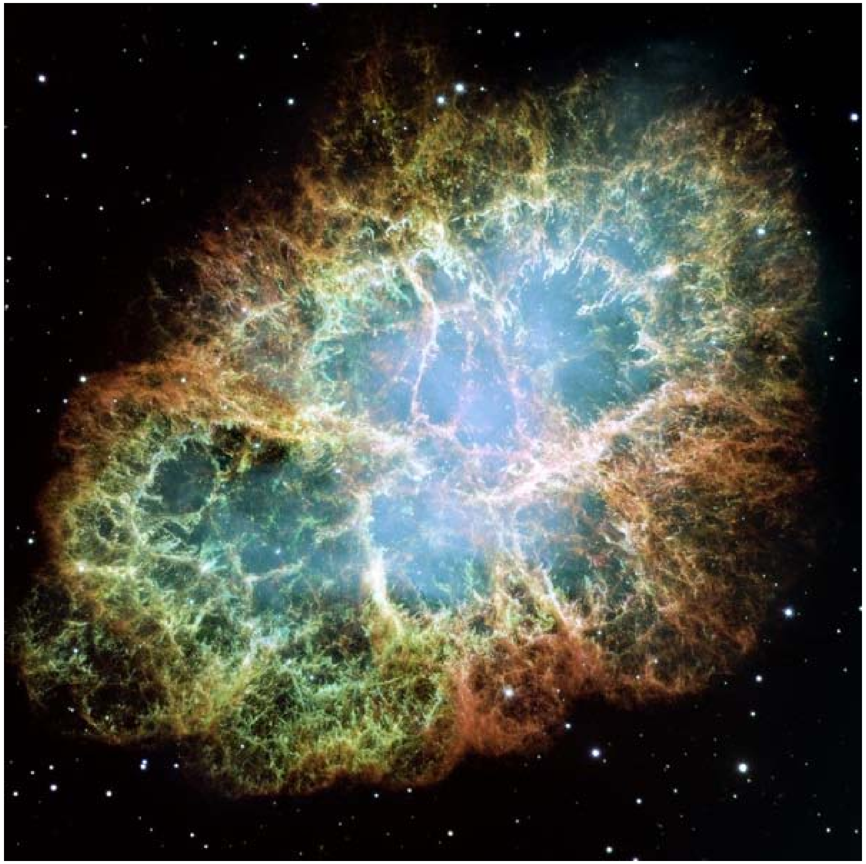
Two identical balls are able to roll long two smooth tracks. Track A is a straight downwards slope. Track B has a curved path as shown. Both tracks start and end at the same vertical position. If the two balls are released from rest at the top of the tracks, which ball will reach the bottom first?

(a) A
(b) B
(c) both A and B at the same time.

M&I

Chapter 7

Internal Energy



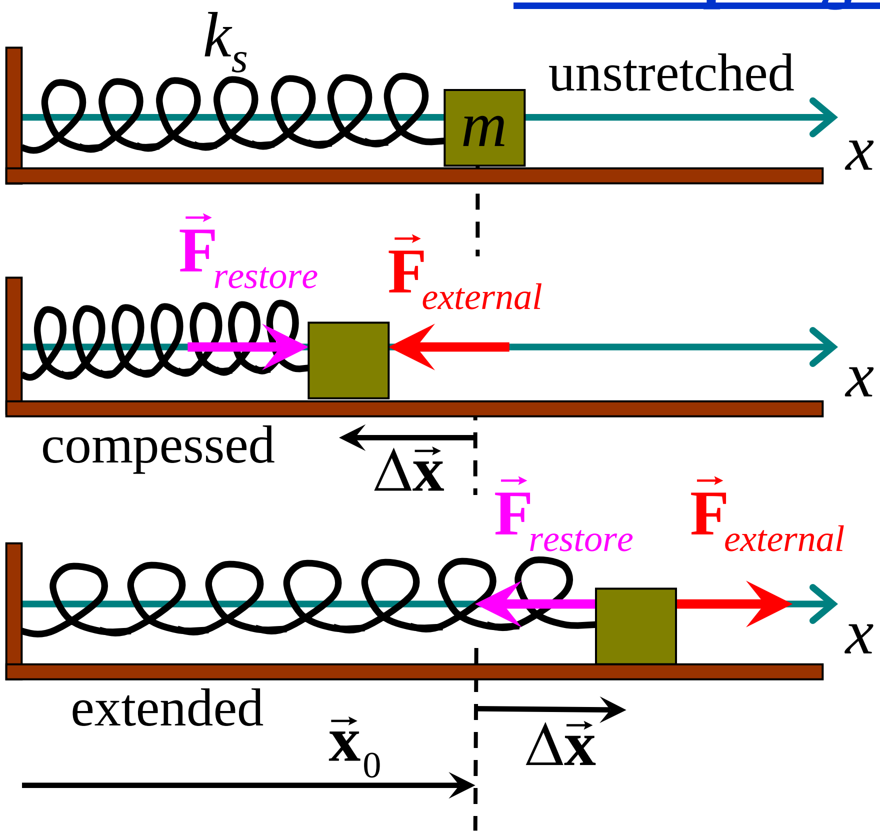
Energy in macroscopic systems

Macroscopic systems are composed of many interacting particles.

The potential energy associated with stretching or compressing interatomic bonds is similar to the potential energy of macroscopic springs.

Thermal energy is the kinetic and potential energy of the atoms and interatomic bonds within an object.

Mass-spring oscillator



Hooke's Law:

Restoring force,

$$\vec{\mathbf{F}}_{restore} = -k_s \Delta\vec{\mathbf{x}}$$

where $\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_0$

and k_s is the "spring constant" [N m^{-1}]

Start with the momentum principle: $\dfrac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{\text{net}}$

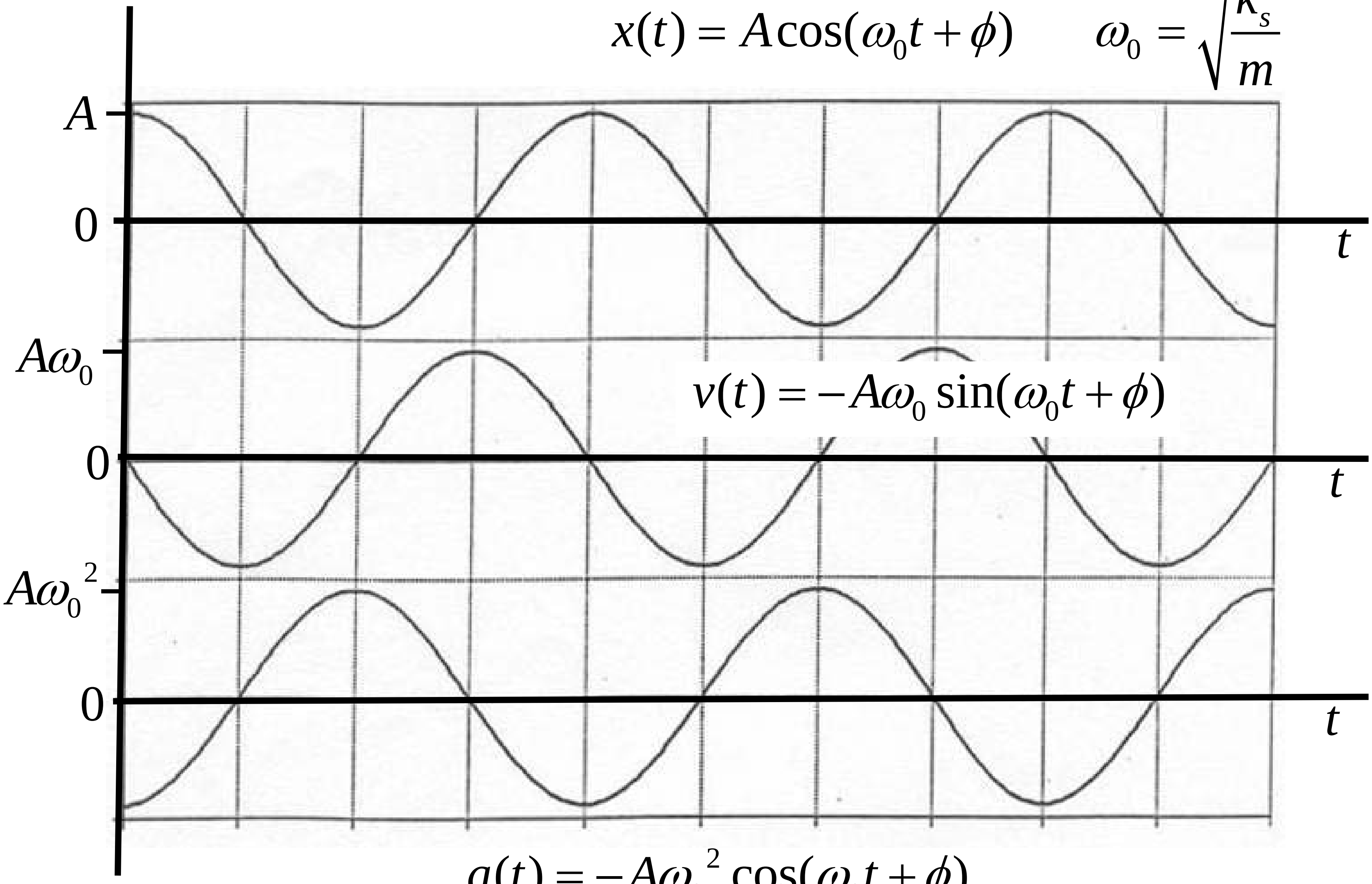
For horizontal forces on the mass: $\dfrac{dp_x}{dt} = -k_s x$

$$\therefore \frac{d(mv_x)}{dt} = -k_s x \quad \text{or} \quad \frac{d}{dt}\left(m\frac{dx}{dt}\right) = -k_s x$$

$$\therefore \frac{d^2 x}{dt^2} = -\frac{k_s}{m} x$$

Mass-spring oscillator …2

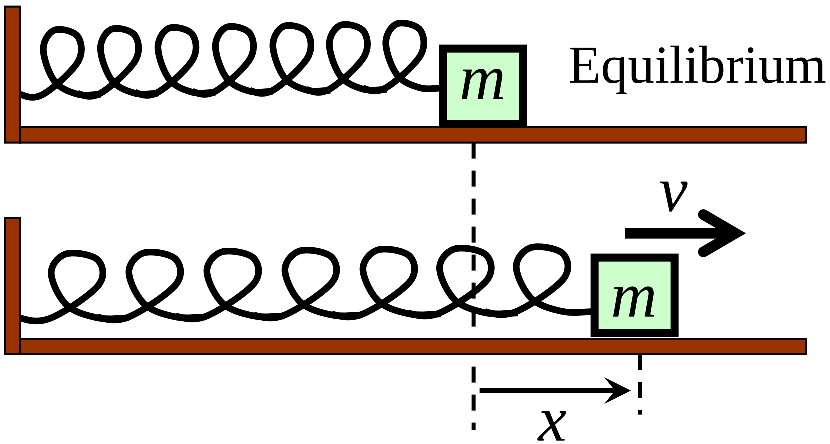
$$x(t) = A\cos(\omega_0 t + \phi) \qquad \omega_0 = \sqrt{\frac{k_s}{m}}$$



$$v(t) = -A\omega_0 \sin(\omega_0 t + \phi)$$

$$a(t) = -A\omega_0{}^2 \cos(\omega_0 t + \phi)$$

Potential energy of macroscopic springs



k_s : spring constant

Suppose that the mass has a speed v when it has displacement x

Kinetic energy of mass = $\frac{1}{2}mv^2$

Potential energy of spring = $\int_0^x F dx' = \int_0^x k_s x' dx' = \frac{1}{2}k_s x^2$

There are no dissipative mechanisms in our model (no friction).
… the total energy of the mass-spring system is conserved.

$$\frac{1}{2}mv^2 + \frac{1}{2}k_s x^2 = \text{constant}$$

Mass-spring oscillator: an energy approach …2

For our mass-spring system: $\frac{1}{2}mv^2 + \frac{1}{2}k_s x^2 = \text{constant}$

$$\therefore \frac{d}{dt}\left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}k_s x^2\right) = 0$$

$$\therefore mv\frac{dv}{dt} + k_s x\frac{dx}{dt} = 0$$

$$\therefore mv\frac{dv}{dt} + k_s xv = 0$$

$$\therefore m\frac{dv}{dt} + k_s x = 0$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{k_s}{m}x$$

Which has solution $x(t) = A\cos(\omega_0 t + \phi)$ where $\omega_0 = \sqrt{\frac{k_s}{m}}$

Mass-spring oscillator: an energy approach …3

For the mass-spring system: $x(t) = A\cos(\omega_0 t + \phi)$

Potential energy $U = \tfrac{1}{2}k_s x^2 = \tfrac{1}{2}k_s A^2 \cos^2(\omega_0 t + \phi)$

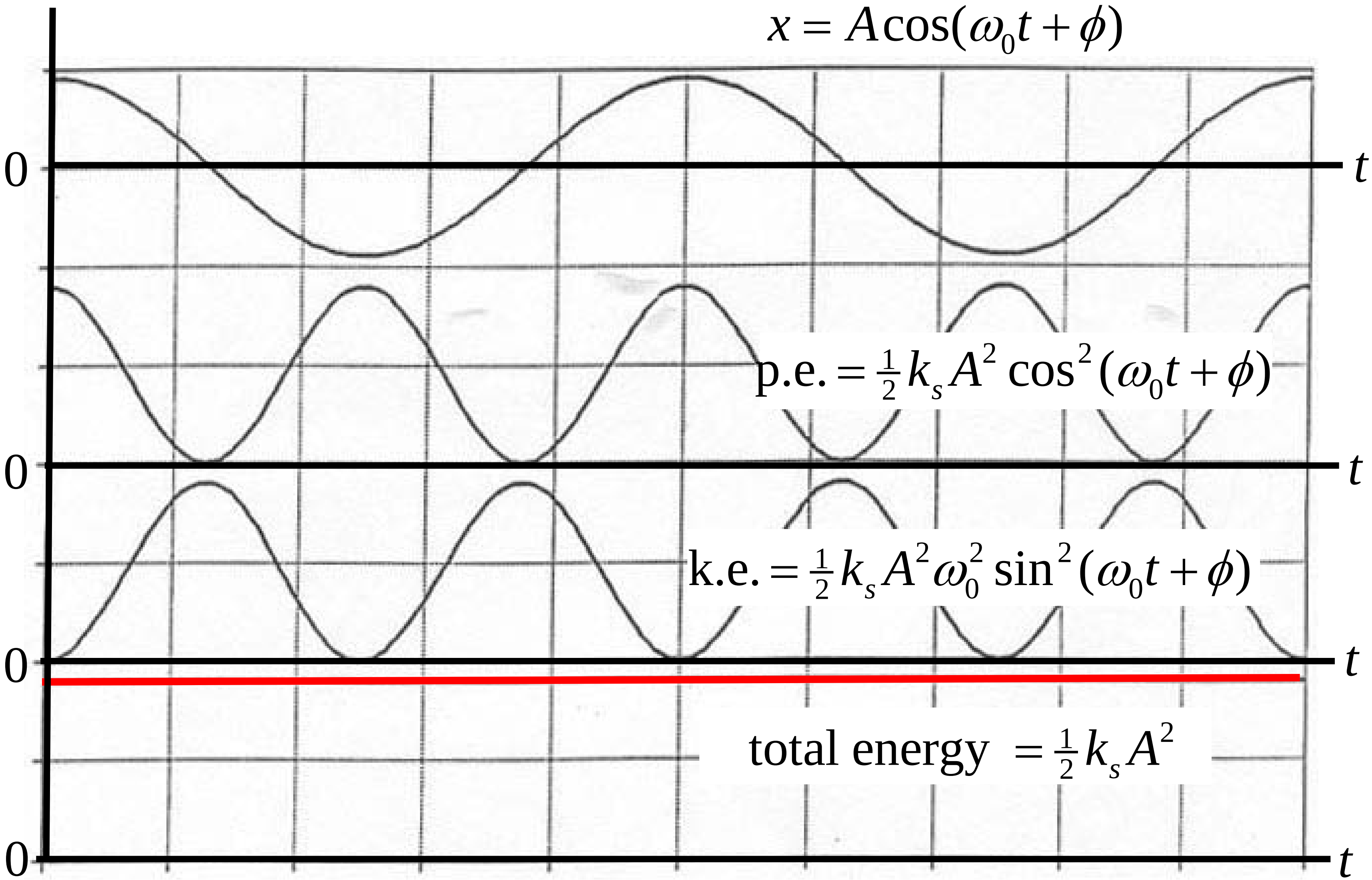
$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m[-A\omega_0 \sin(\omega_0 t + \phi)]^2 = \tfrac{1}{2}mA^2\omega_0^2 \sin^2(\omega_0 t + \phi)$$

Total energy $= K + U$

$$= \tfrac{1}{2}k_s A^2 \cos^2(\omega_0 t + \phi) + \tfrac{1}{2}mA^2\omega_0^2 \sin^2(\omega_0 t + \phi)$$

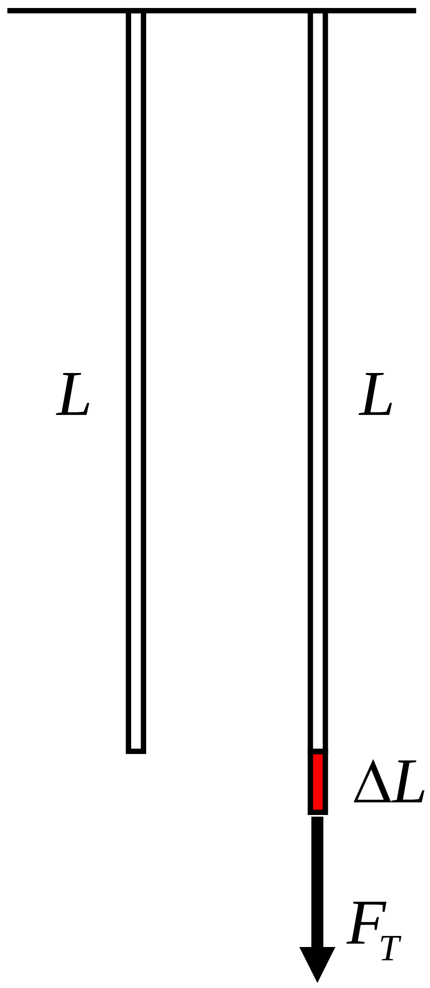
$$= \tfrac{1}{2}k_s A^2 \qquad (\therefore E \propto A^2)$$

Energy of the mass-spring simple harmonic oscillator



Young's modulus

We take a wire of length L and stretch it by applying an external force F_T.
Say that the wire stretches by an amount ΔL.



Then the "strain" $\equiv \dfrac{\Delta L}{L}$

The tension force per unit area is called the "stress" $\equiv \dfrac{F_T}{A}$

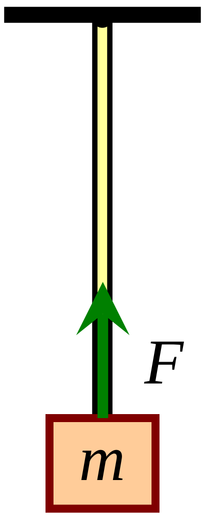
Young's modulus $Y \equiv \dfrac{\text{stress}}{\text{strain}} = \dfrac{\left(\dfrac{F_T}{A}\right)}{\left(\dfrac{\Delta L}{L}\right)}$

Young’s modulus …2

A bar or wire can be regarded as a non-helical macroscopic spring.

Write $Y = \dfrac{\left(\dfrac{F_T}{A}\right)}{\left(\dfrac{\Delta L}{L}\right)}$ as $F_T = \left(\dfrac{YA}{L}\right)\left|\Delta L\right|$

Compare with $F_{\text{restore}} = k_s \left|x\right|$ for a spring



Elastic oscillations in a wire

Here F is restoring force in wire

$$ma = -\frac{AYx}{L} \quad \text{and} \quad \omega_0 = \sqrt{\frac{AY}{mL}}$$

1	2	3	4	5

A horizontal spring has a mass attached which can move with negligible friction. You stretch the spring and release the mass from rest. For the resulting motion, which of the following statements is **true**?

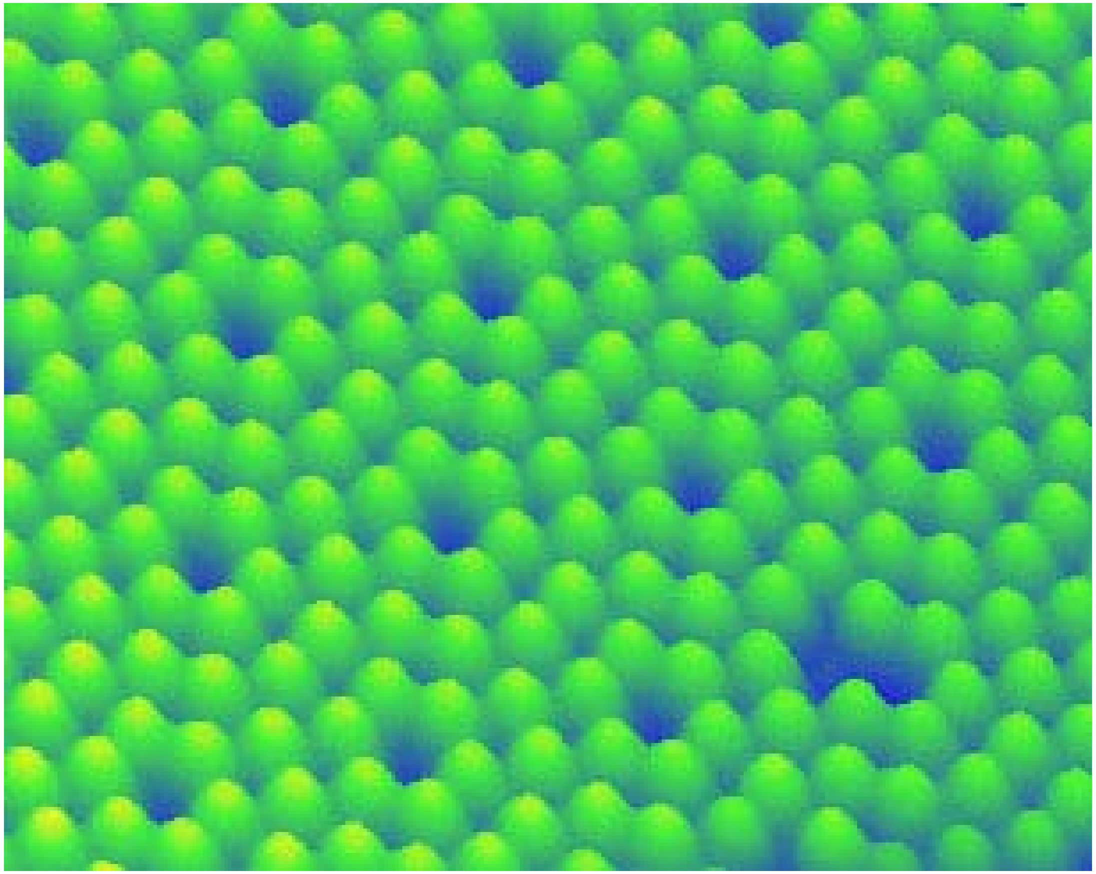
(1) When the spring is (momentarily) fully compressed, K has its largest value.
(2) When the spring (momentarily) has its relaxed length, U has its largest value.
(3) When the spring (momentarily) has its relaxed length, K has its smallest value.
(4) When K is large, U is small, and vice versa.
(5) When K is large, U is large, and vice versa.

1	2	3	4	5

An ideal mass-spring system is oscillating with angular frequency $\omega = \sqrt{k_s / m}$.

More energy is put into the mass-spring system.
What is now true?

(1) The frequency of the oscillations increases
(2) The frequency of the oscillations decreases
(3) The frequency increases and amplitude increases
(4) The frequency is unchanged and the amplitude increases
(5) The frequency decreases and amplitude decreases

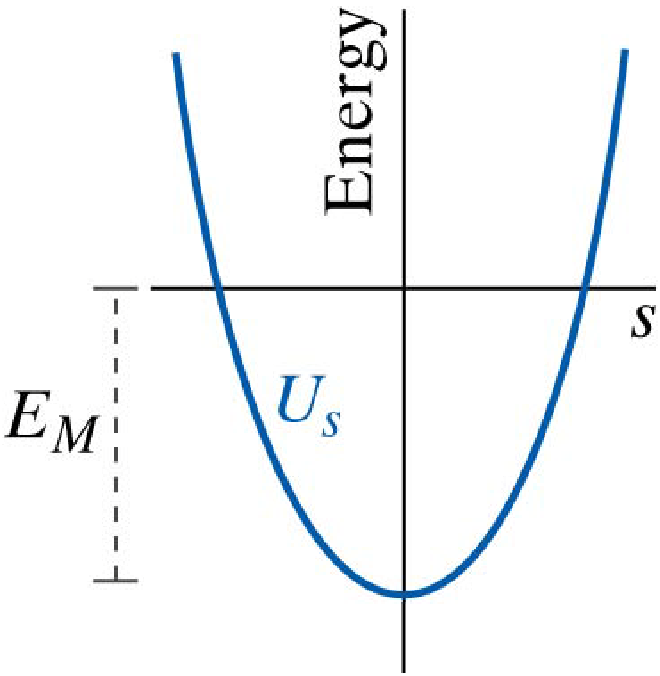


Silicon atoms

M&I
3E 7.2

Potential energy of a pair of neutral atoms

The interatomic potential energy is ultimately electrical in origin.



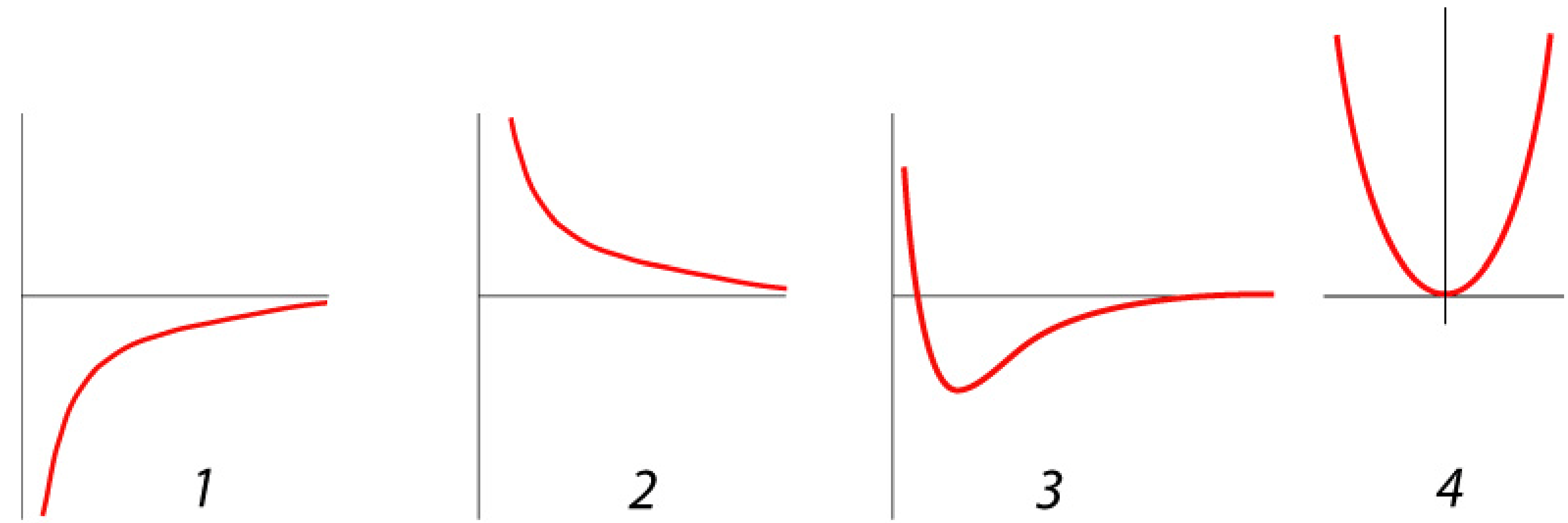
$$U = \tfrac{1}{2} k_s s^2 + E_M$$

This is a model for the interatomic potential energy close to the equilibrium separation.

1	2	3	4	5

y-axis: energy; *x*-axis: separation

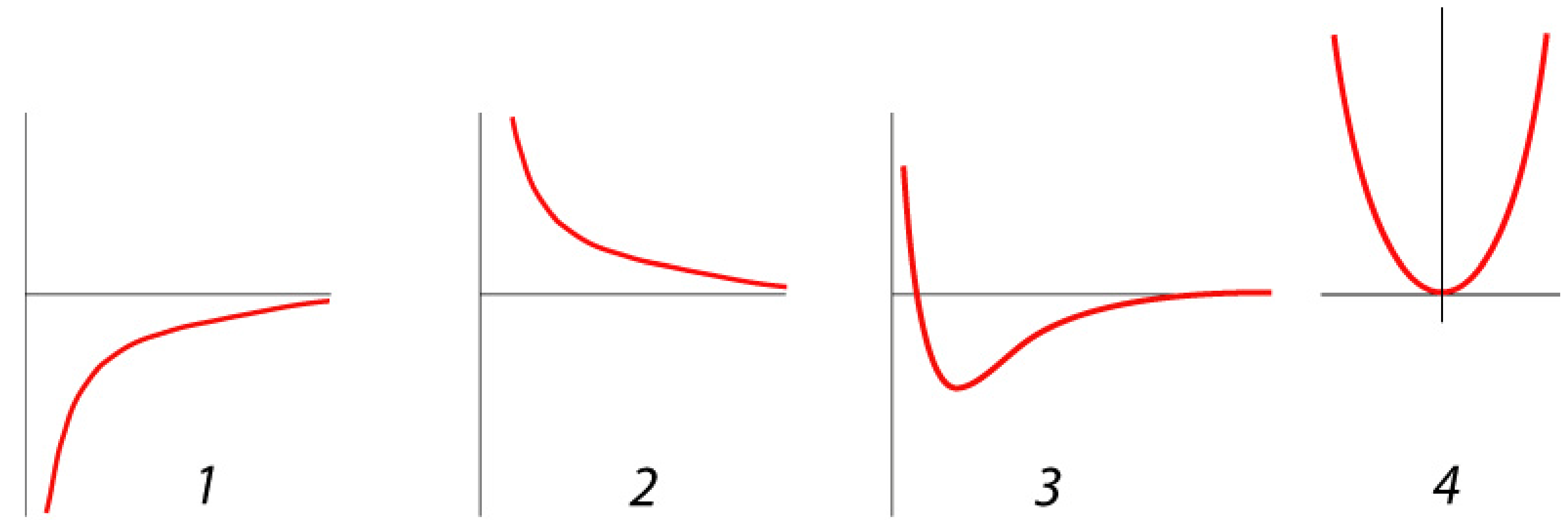
Which graph correctly shows U for two interacting electrons?



5: none of the above

1	2	3	4	5

y-axis: energy; *x*-axis: separation.
Which graph correctly shows *U* for two atoms in a diatomic molecule?

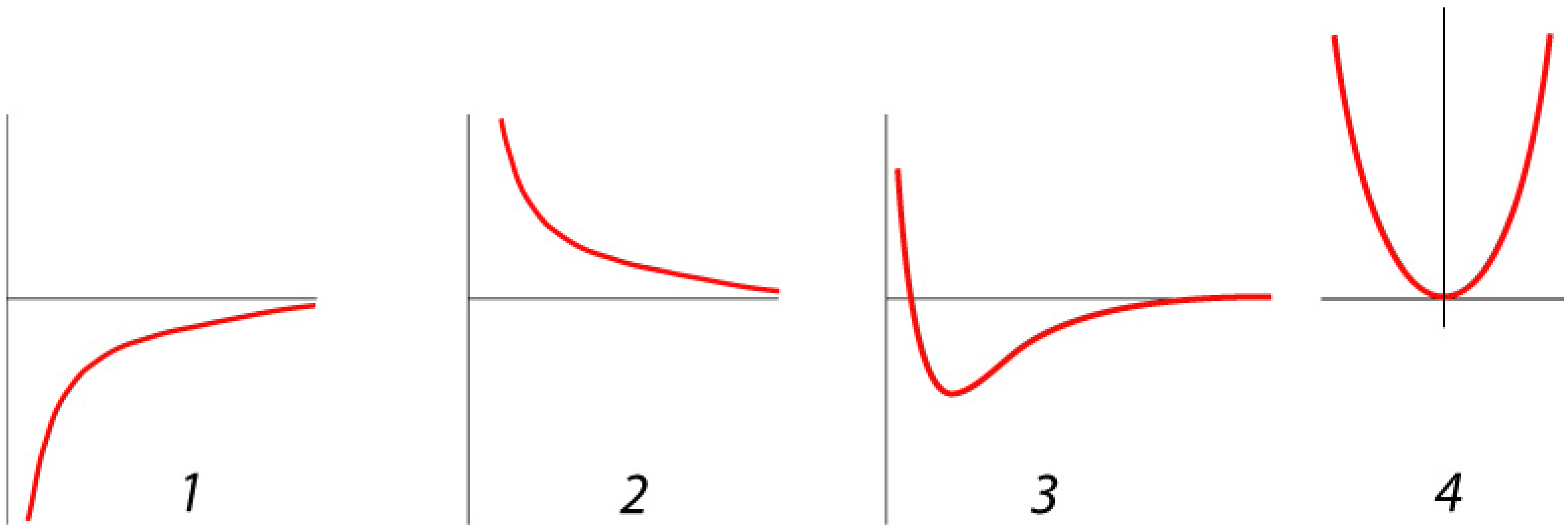


5: none of the above

1	2	3	4	5

y-axis: energy; *x*-axis: separation

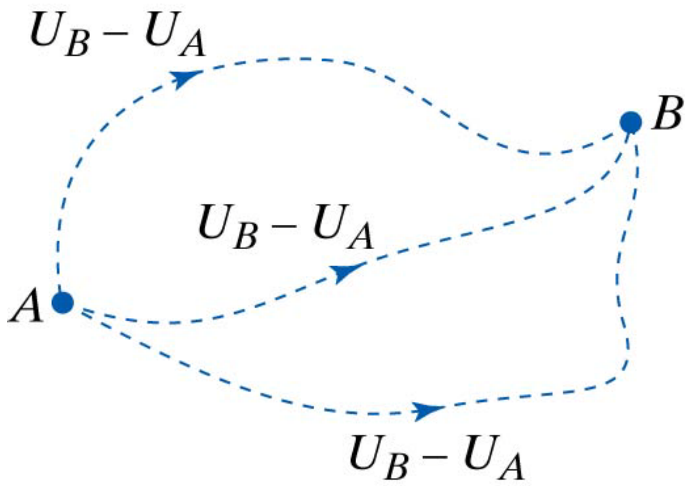
Which graph correctly shows *U* for the Moon and the Earth?



5: none of the above

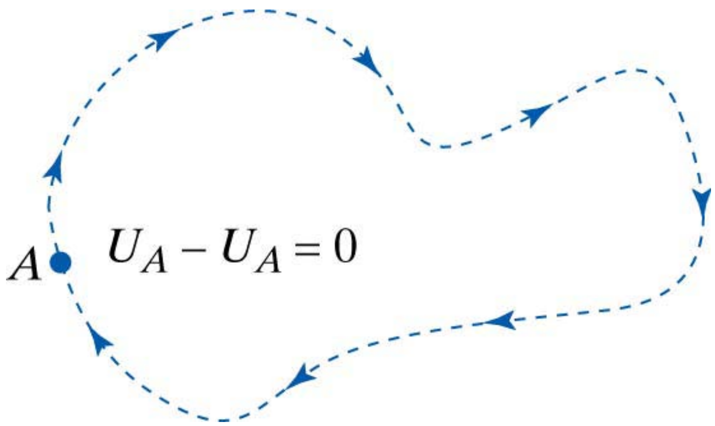
Path independence of potential energy

We have defined $\Delta U = U_B - U_A = -W_{\text{internal}} = -\sum \vec{\mathbf{F}} \bullet \Delta\vec{\mathbf{r}}$

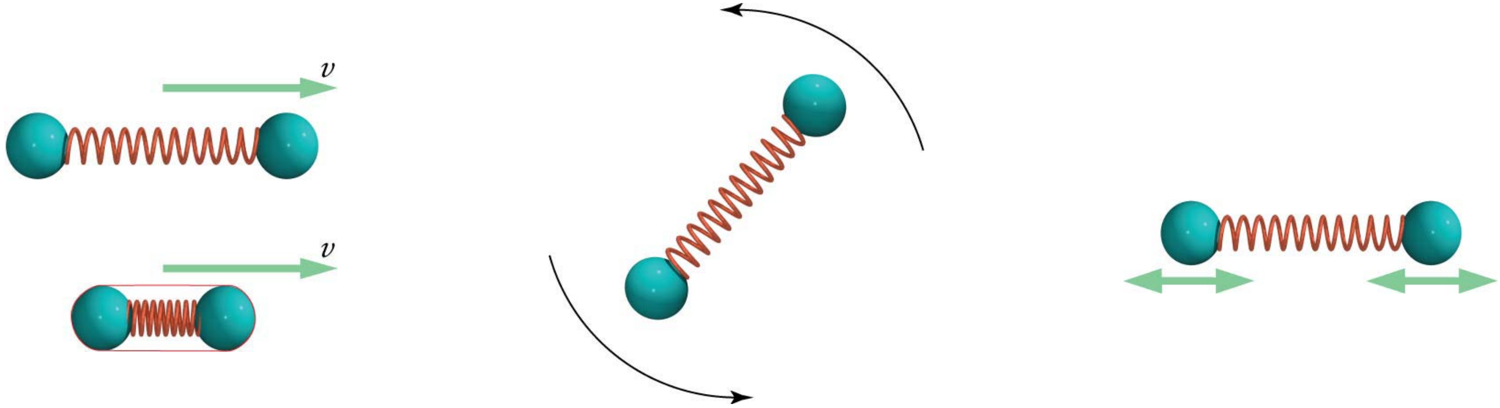


Change in potential energy (and work done by internal forces) is independent of path.

Work done by internal forces in a closed path is zero.



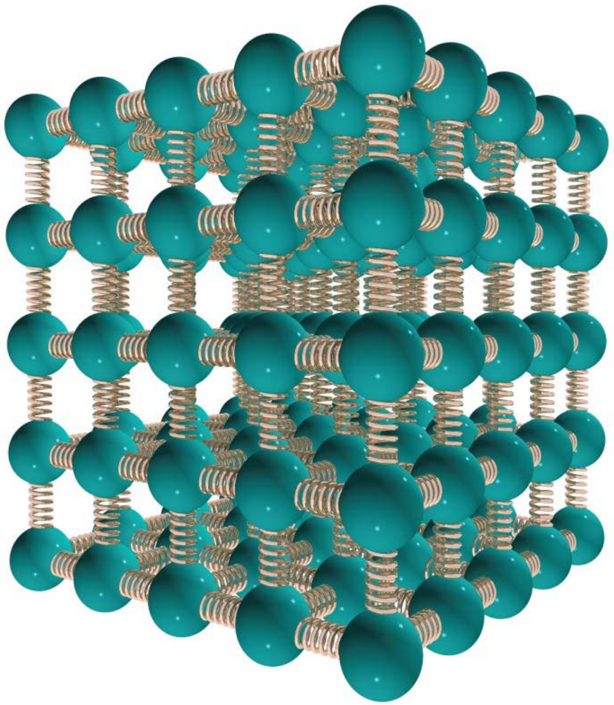
Internal energy and thermal energy



$$\text{Internal energy} = E_{thermal} + E_{rotational} + E_{vibrational} + E_{chemical} + \ldots$$

Thermal energy in solid materials

Now considering the transfer of energy to the atoms making up a solid material. We model a solid as a large number of tiny masses (the atoms) connected to their neighbours by springs (the inter-atomic bonds). The "internal energy" of a solid can be increased by increasing the kinetic energy of the atoms or the potential energy of the atom-spring system.



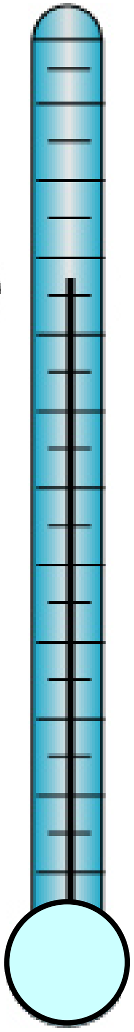
Since there are so many atoms we can only consider such things in an averaged way.
But how do we measure this energy?

Temperature

Temperature and average energy of molecules are related.
(Joule 1842: "the mechanical equivalent of heat").

A thermometer measures "temperature" T on some scale which might be "degrees" or "joules".

An ideal thermometer is the constant volume gas thermometer, in which the gas pressure is measured as a function of temperature.
By extrapolation, there is a temperature at which the pressure becomes zero.
This is defined as the zero of the Kelvin scale.
The increments are the same as the Celsius scale.
0 K is also known as "absolute zero" in temperature.

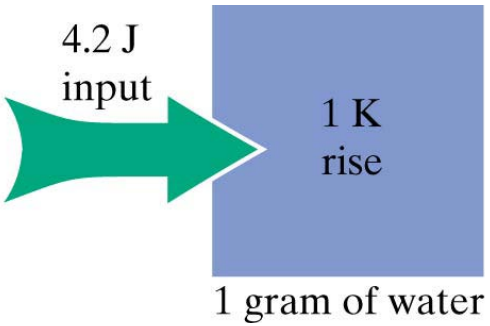


$$0\ \mathrm{K} = -273.15\ {}^{\circ}\mathrm{C}$$

Change of energy and temperature

How does change in temperature relate to change in internal energy?

Joule found that 4.2 J was required to change the temperature of 1 g of water by 1 K.



We define a **(specific) heat capacity** C by

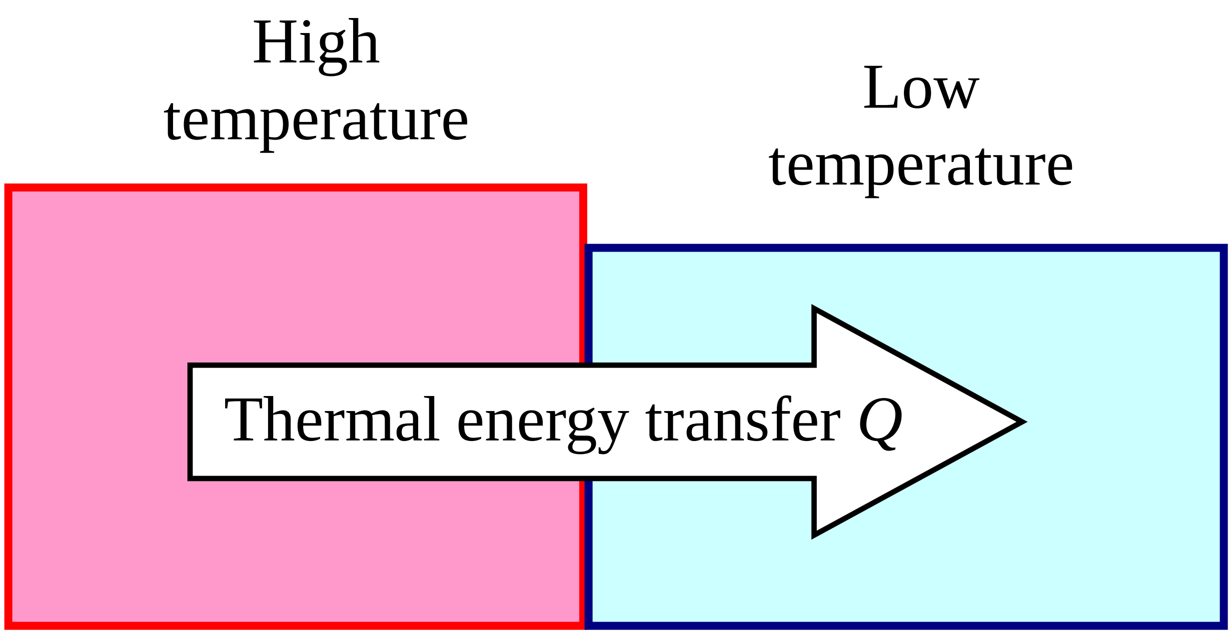
$$\Delta E_{\text{thermal}} = mC\Delta T$$

or $$C = \frac{\Delta E_{\text{thermal}}}{m\Delta T}$$ with m in grams

Specific heat capacity of ethanol = 2.4 J g^{-1} K^{-1}
Specific heat capacity of copper = 0.4 J g^{-1} K^{-1}

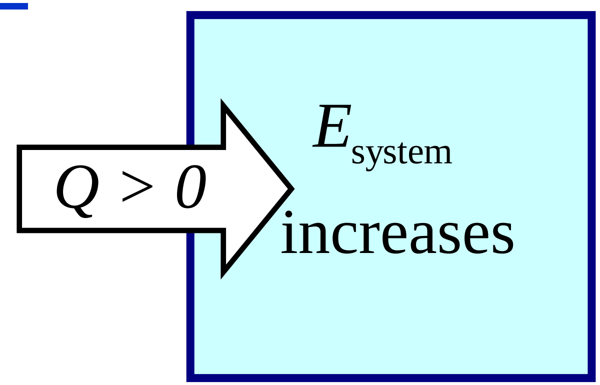
Energy transfer due to a temperature difference

If a hotter body is placed in contact with a colder body, then a certain amount of energy passes from the hotter to the colder until they are in **thermal equilibrium**, i.e. they are at the same temperature.
This happens by work done at the microscopic level.
This thermal energy transfer is usually denoted by the symbol Q.
[It is sometimes (confusingly!) known as "heat".]

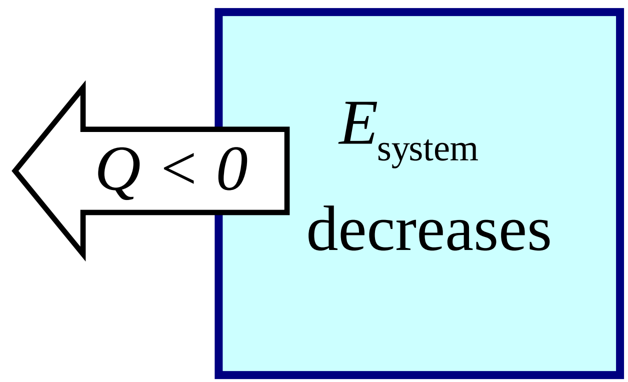


Thermal energy transfers

We normally separate the work done in changing the energy of a system into "macroscopic" work (W) and "microscopic" work (Q).



$$\Delta E_{\text{system}} = W + Q$$



This is an expression of the "First law of thermodynamics"

… a version of conservation of energy.

General form of the energy principle:

$$\Delta E_{\text{system}} = W + Q + \text{other energy transfers}$$

W: mechanical work : $\vec{\mathbf{F}} \bullet \Delta\vec{\mathbf{r}}$

Q: energy transfer due to a temperature difference

Other energy transfers:

... Matter transfer

... Mechanical waves

... Electricity

... Electromagnetic radiation

...

Reflection: forms of energy

Rest energy

Kinetic energy

Potential energy

1	2	3	4	5

Two lead bricks moving in the $+x$ and $-x$ directions, each with kinetic energy K, smash into each other and come to a stop. What happened to the energy?

(1) The kinetic energy changed into rest energy.
(2) The kinetic energy changed into thermal energy.
(3) The total energy of the system decreased by an amount $2K$.
(4) Since the blocks were moving in opposite directions, the initial kinetic energy of the system was zero, so there was no change in energy.

1	2	3	4	5

Which of the following statements is **correct**?

(1) Q and $\Delta E_{\text{thermal}}$ are the same thing.

(2) Q and $\Delta E_{\text{thermal}}$ are not the same thing, but they are always equal.

(3) $\Delta E_{\text{thermal}}$ can be nonzero even if Q is zero.

(4) Q and $\Delta E_{\text{thermal}}$ are both always positive.

Power

Power P is the rate at which work W is done.

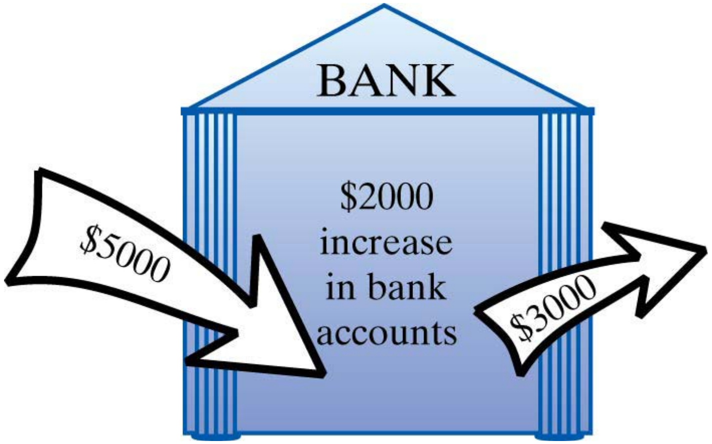
Unit of power: the watt, W (1 watt = 1 joule second^{-1})

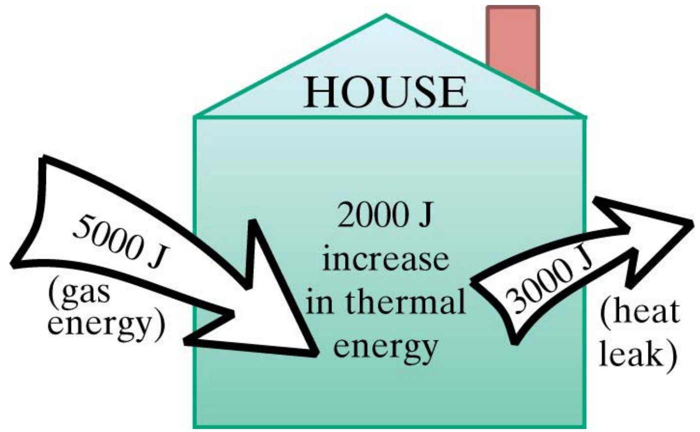
Average power: $$P_{av} = \frac{W}{\Delta t} = \frac{\Delta U}{\Delta t}$$

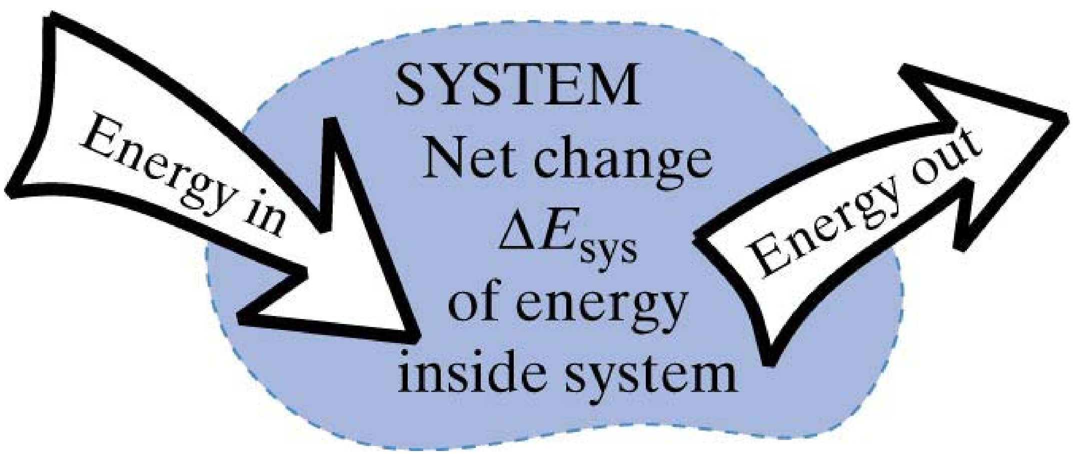
Instantaneous power: $$P = \frac{dW}{dt}$$

... can also write: $$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{\mathbf{F}} \bullet \vec{\mathbf{r}}\right) = \vec{\mathbf{F}} \bullet \frac{d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \bullet \vec{\mathbf{v}}$$

Open and Closed Systems







M&I
3E 7.9

The choice of system affects energy accounting

Read through the examples in this section very carefully.

M&I
3E 7.10

Energy dissipation

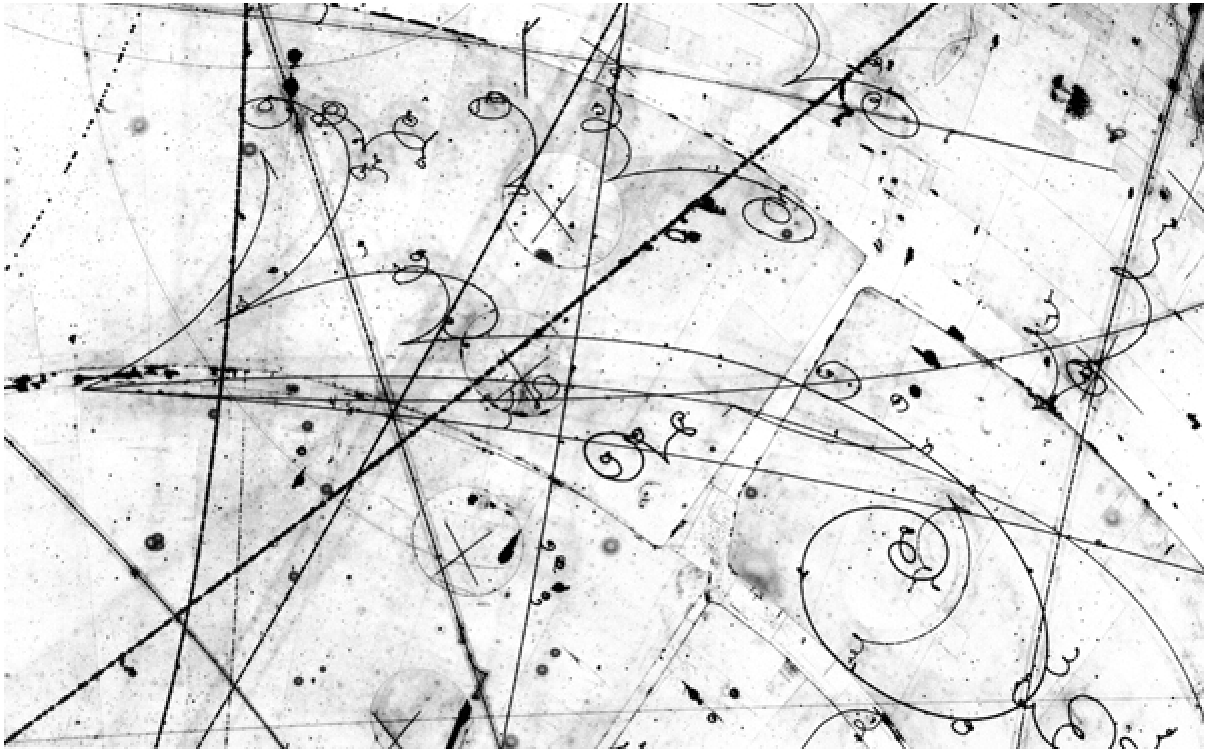
Air drag: $F_{air} = \frac{1}{2}C\rho Av^2$

Friction: $f = \mu N$

M&I

Chapter 8

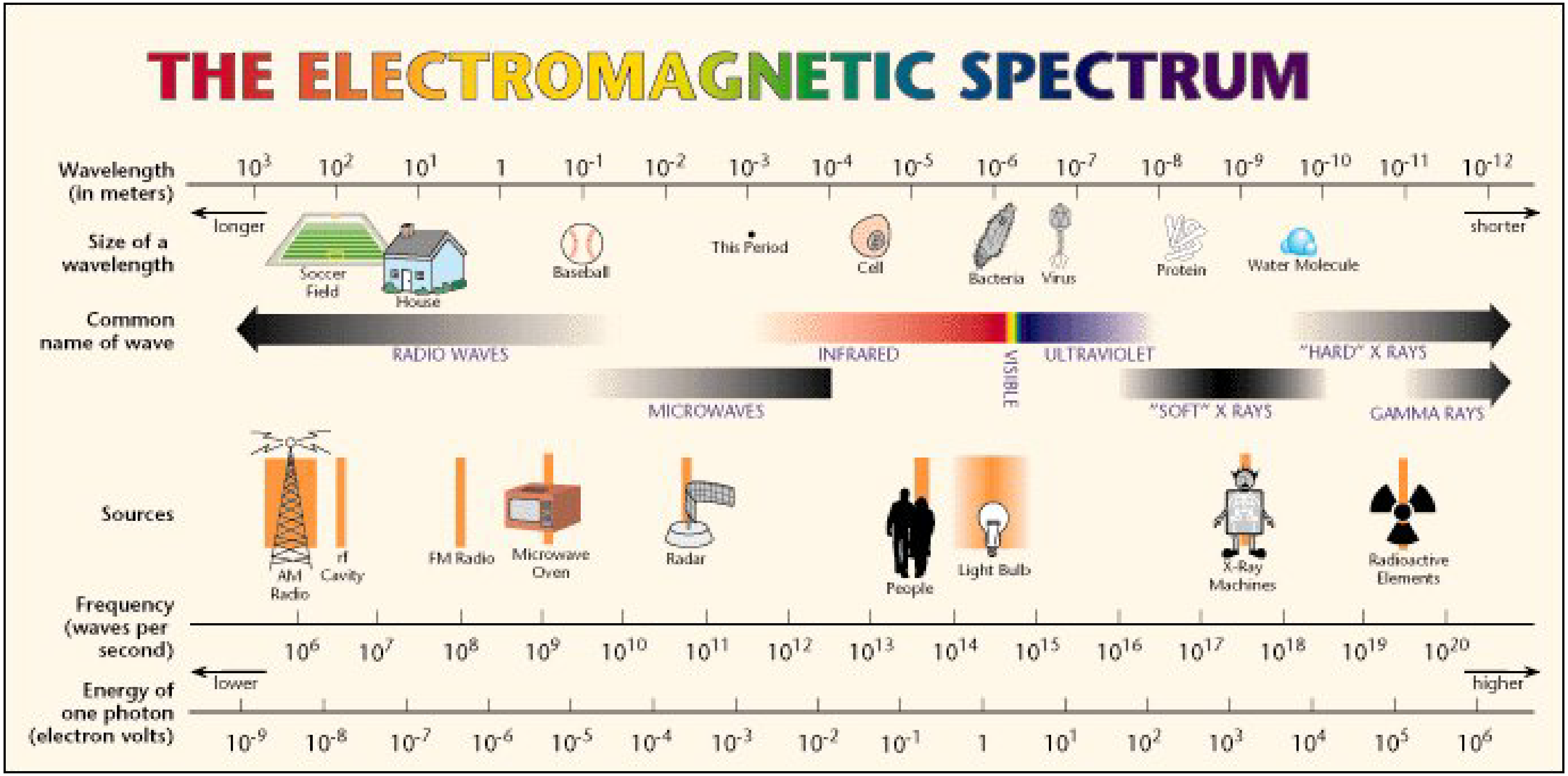
Energy Quantization



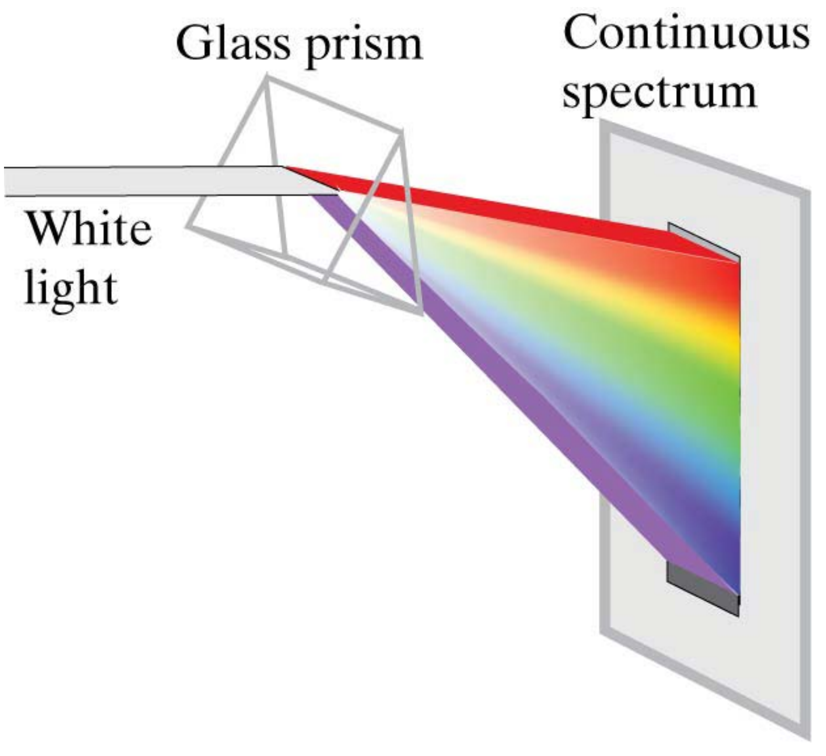
Photons

Light is an electromagnetic wave.

$$c = f\lambda$$



Visible light



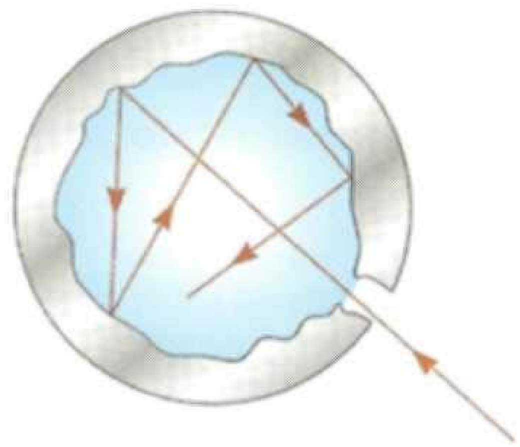
Visible light has wavelengths in the range 400-700 nm.

Dispersive elements can separate the different wavelengths into a spectrum: continuous or discrete.

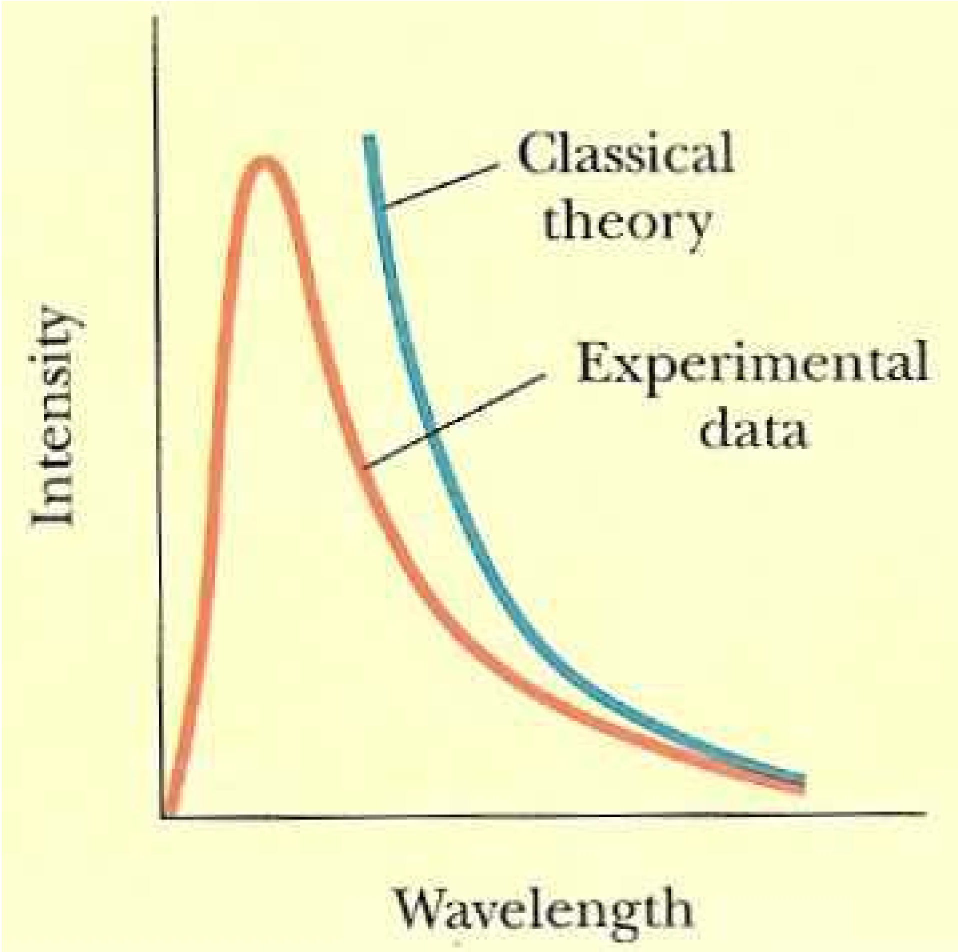
Blackbody radiation

Kirchoff showed that the most efficient radiator is also the most efficient absorber. A perfect absorber absorbs all the incident radiation and no light is reflected
$\rightarrow$ called a blackbody.

An ideal blackbody can be approximated by a cavity with a very small opening.



A blackbody will emit radiation according to the temperature of its walls.



Major disagreement between theory and experimental data, especially at short wavelengths.

$\rightarrow$ "Ultraviolet Catastrophe"

Planck's Theory (1900)

Max Planck found an empirical formula which agreed with experiment at all wavelengths:

$$I(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)}$$

h = Planck's constant = 6.63×10^{-34} J s

In deriving this formula, Planck made two radical assumptions:

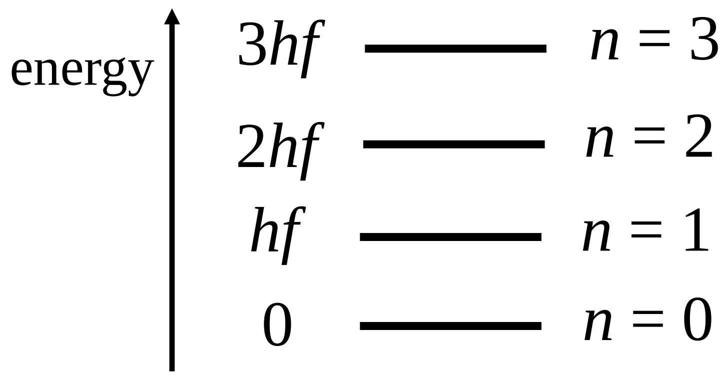
… the energy absorbed and emitted by the blackbody was "quantised" or manifested in discrete amounts.

… atoms absorb or emit energy in discrete units ("quanta") of light energy ("photons") by jumping from one quantum state to the other.

Planck's Theory

The energy and frequency of the radiation are related by:

$$\boxed{E_f - E_i = E = nhf}$$



E_f : final energy state of atom

E_i : initial energy state of atom

E : energy of absorbed or emitted photon in J (or eV)

n : quantum number

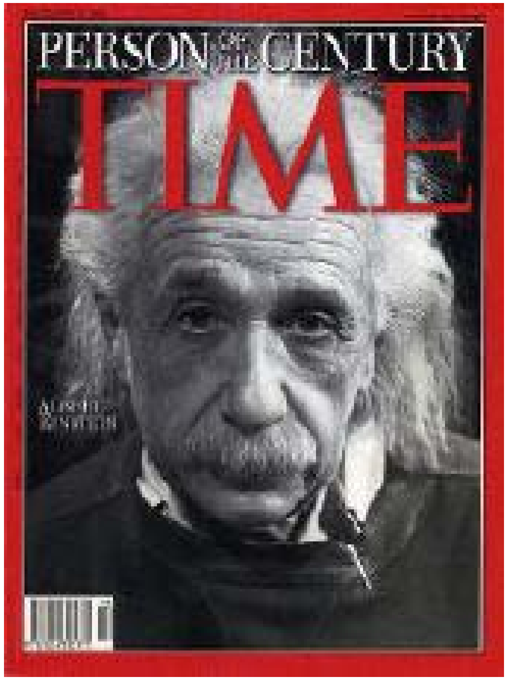
h : Planck's constant in J s (or eV s)

f : frequency of atom's oscillation = frequency of photon

hf : one "quantum" or packet of energy

The photoelectric effect

Although Planck's hypothesis worked, most physicists were sure that a classical theory was still possible and that the quantum hypothesis could be done away with. However, more and more evidence of the quanta aspects of nature emerged.



In 1905 Einstein published a paper showing that by treating the light in the interior of a blackbody as a gas consisting of particles of energy $E = hf$, one could obtain Planck's result. Furthermore, he was able to explain a phenomenon known as the photoelectric effect by using a particle description of light.

Einstein’s theory of the photoelectric effect

Einstein applied Planck’s quantum hypothesis. He assumed:

… that light propagates in discrete particle-like quanta called photons.

… each photon has energy $E = hf$

… each photon gives all its energy to a single electron.

... in the photoelectric effect, an electron in the metal completely absorbs a photon thus gaining energy hf.

Some of this energy goes into freeing the electron from the rest of the metal ϕ while the remainder appears as the kinetic energy of the electron K_{max}.

Write: $$hf = \phi + K_{\text{max}}$$

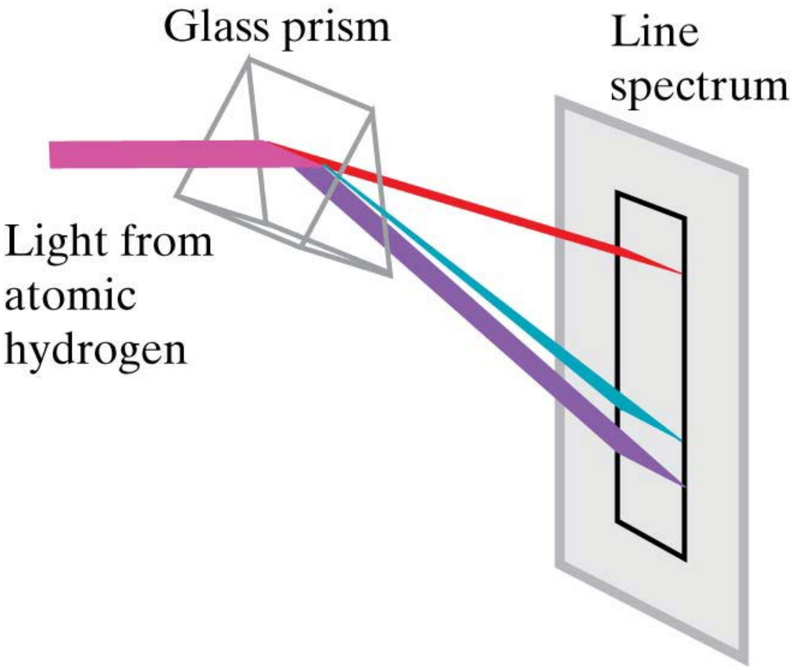
Therefore the quantum hypothesis explained both the blackbody spectrum and the photoelectric effect (and a number of other phenomena) …

… formed the foundations for the development of **quantum mechanics** …

At the same time, the classical understanding the structure of the atom was also being challenged …

Electronic energy levels

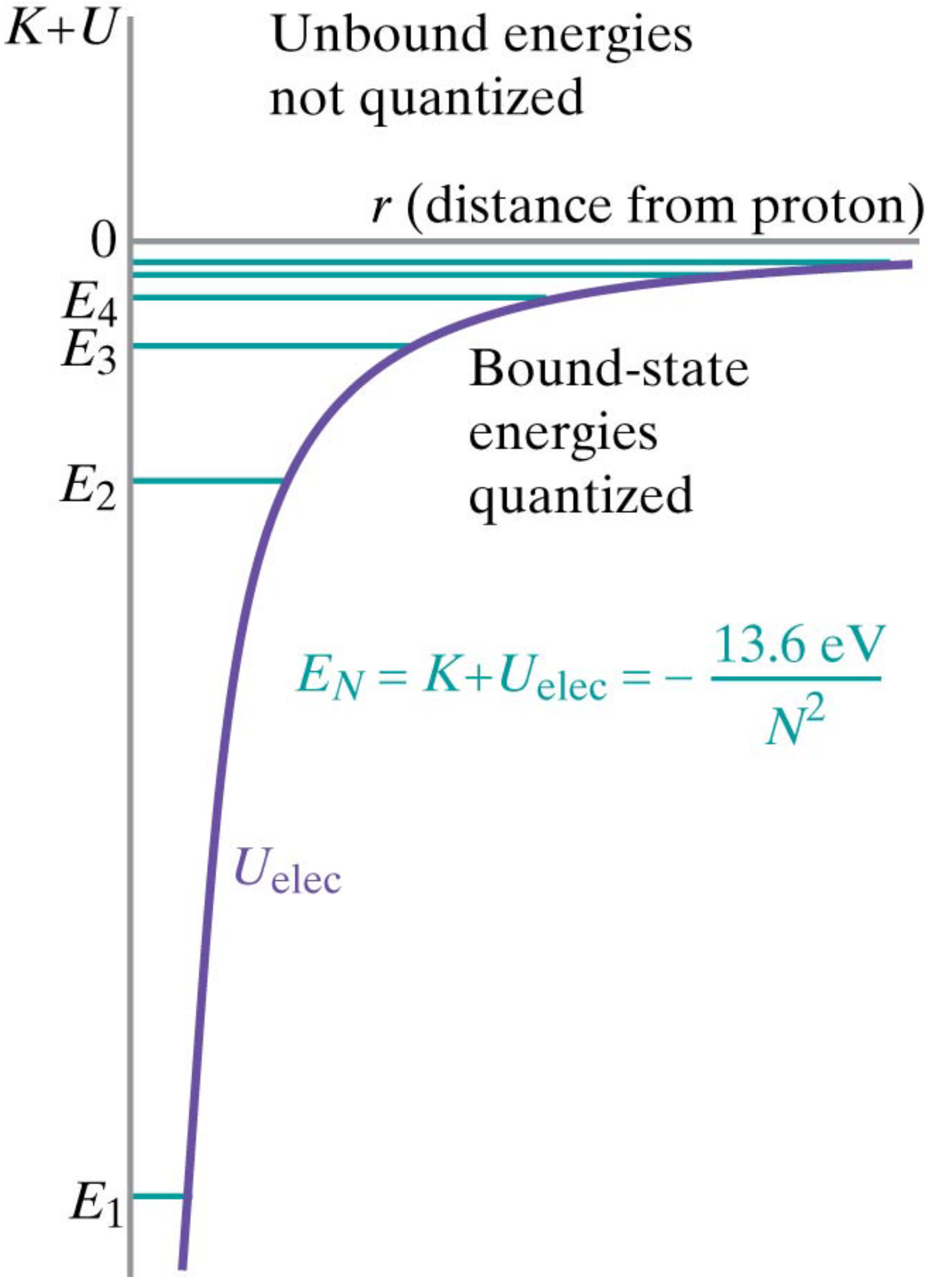
Around 1900:
The continuous spectrum understood, more or less, from thermodynamics.
Discrete spectrum known to be characteristic of atoms.
Planck (1900): black-body spectrum;
Einstein (1905) photoelectric effect: light as particle - **quanta**.



The discrete spectrum arises from transitions between energy levels:

$$\Delta E = E_n - E_m = hf$$

where $h = 6.6 \times 10^{-34}$ J s is the **Planck constant**.



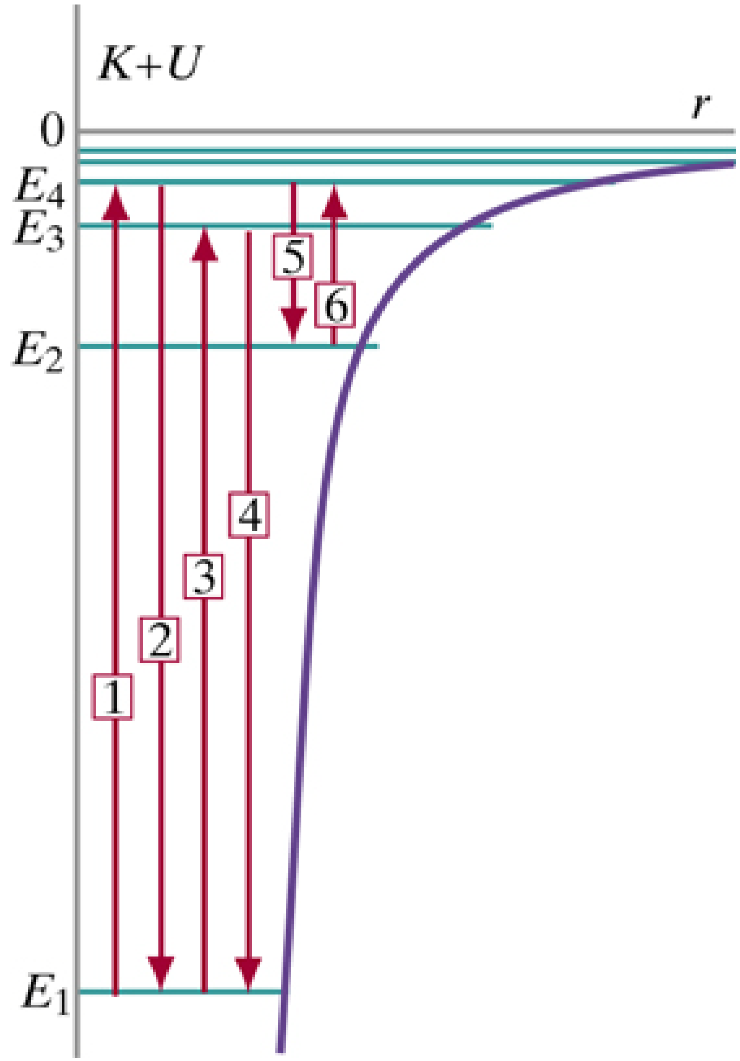
Quantised energy levels

The discrete spectrum arises from discrete energy levels in (bound) systems:
the energy $U + K$ is quantised.

Read *M&I* ...
- Emission spectra
- Absorption spectra
- Electron excitation

1	2	3	4	5

A hydrogen atom is initially in the excited state (N = 4).
It emits a photon and ends up in the state (N = 2).



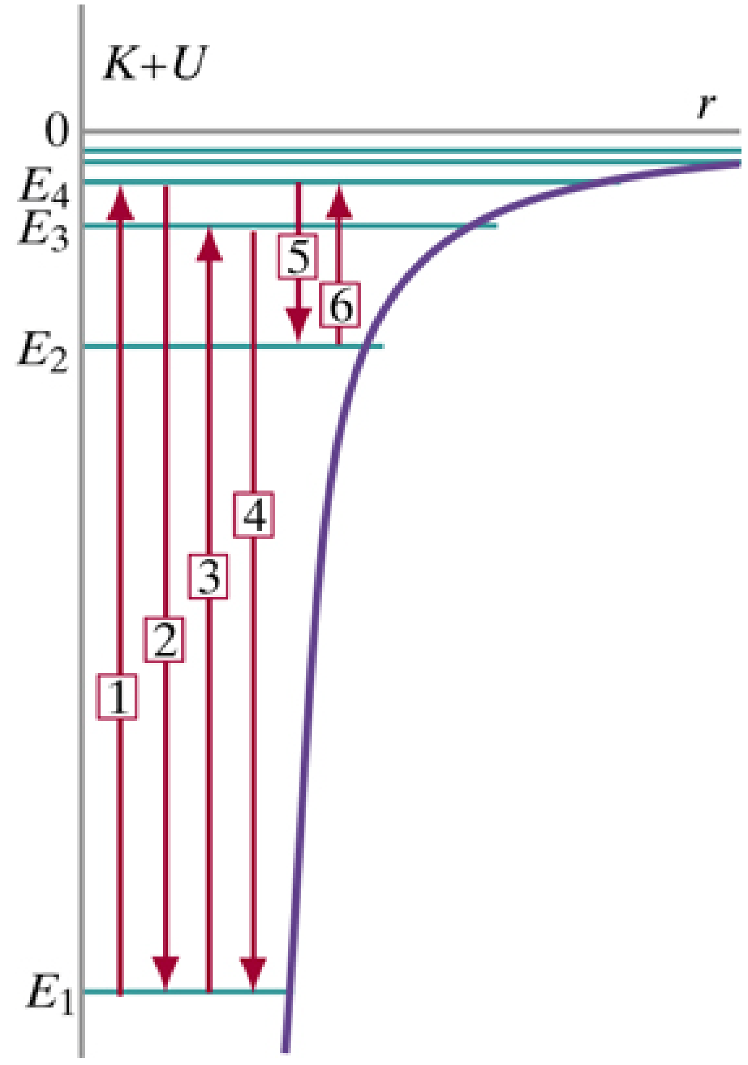
Which of the arrows in the diagram represents this process?

1	2	3	4	5

A hydrogen atom is initially in the excited state ($N = 4$).
It emits a photon and ends up in the state ($N = 2$).

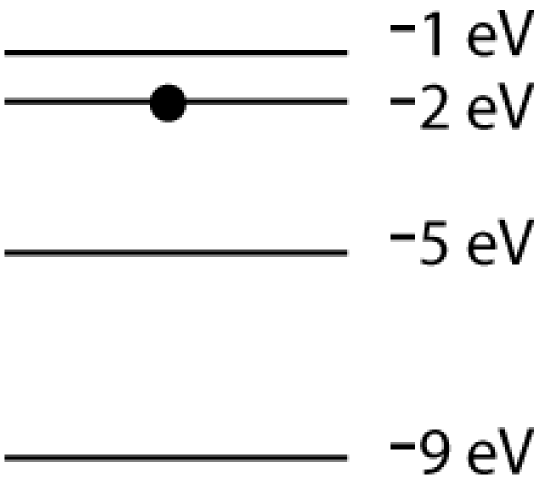
What is the energy of the emitted photon?

(1) $E_4 - E_2$
(2) $E_2 - E_4$
(3) E_4
(4) E_2
(5) $|E_4|$
(6) $|E_2|$



1	2	3	4	5

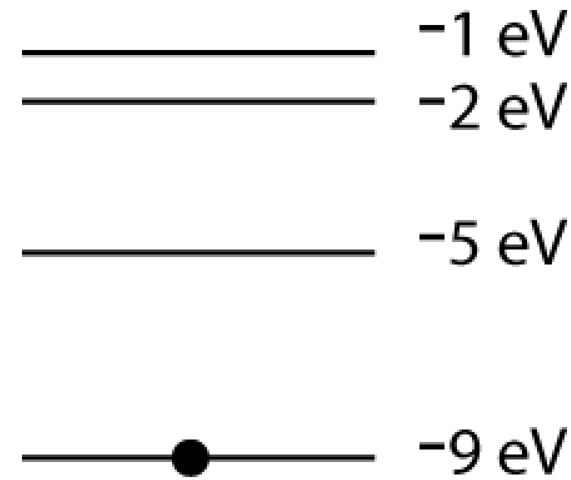
Suppose that these are the quantized electronic energy levels ($K+U$) for an atom. If the atom is excited to the second excited state (marked by a dot), what are the possible energies of photons it might emit?



(1) 2, 5, and 9 eV
(2) 3, 4, and 7 eV
(3) 3 or 7 eV
(4) 5 or 9 eV
(5) 2 eV

1	2	3	4	5

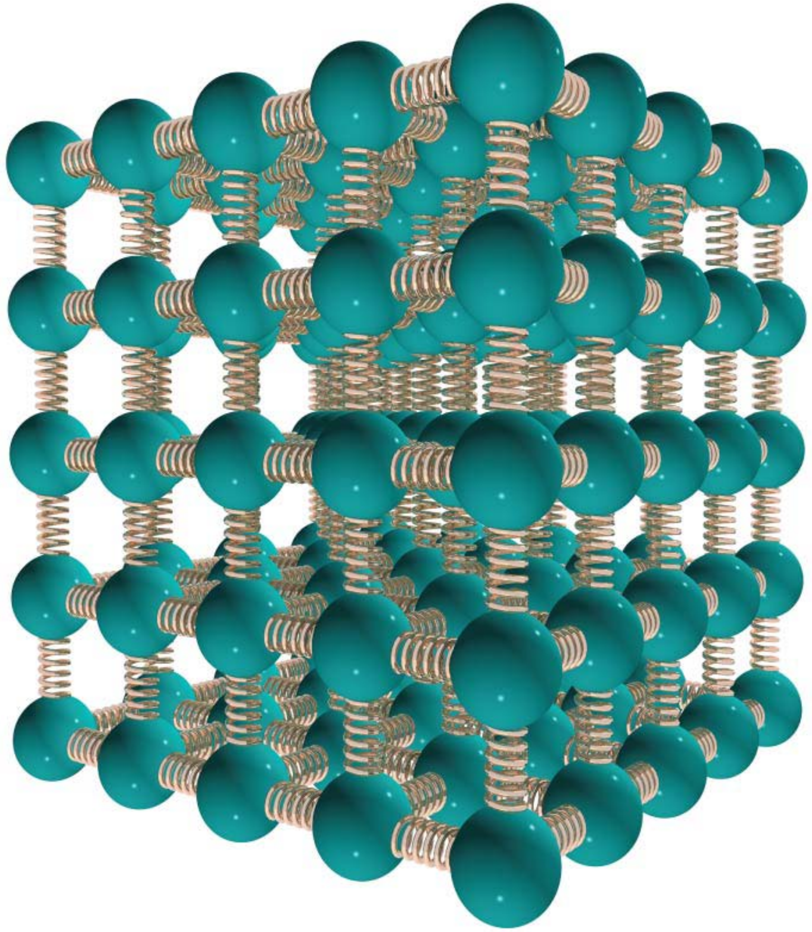
Suppose that these are the quantized energy levels ($K+U$) for an atom. Initially the atom is in its ground state (symbolized by a dot). An electron with kinetic energy 6 eV collides with the atom and excites it. What is the remaining kinetic energy of the electron?



(1) 9 eV
(2) 6 eV
(3) 5 eV
(4) 3 eV
(5) 2 eV

Vibrational energy levels

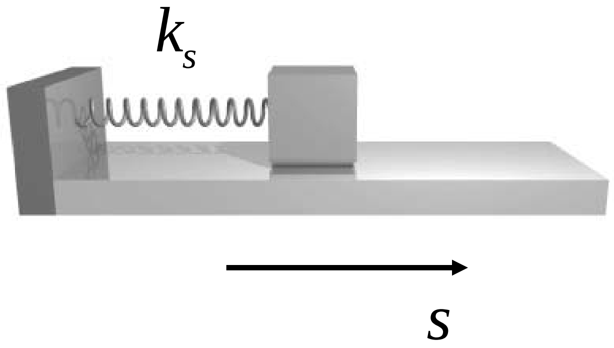
Consider a model of solid matter as a network of balls and springs.



This “classical” model explains many aspects of the behaviour of solids … but not all … for example the behaviour of solids at low temperatures and some aspects of the interaction of light with matter.

… therefore need a quantum treatment …

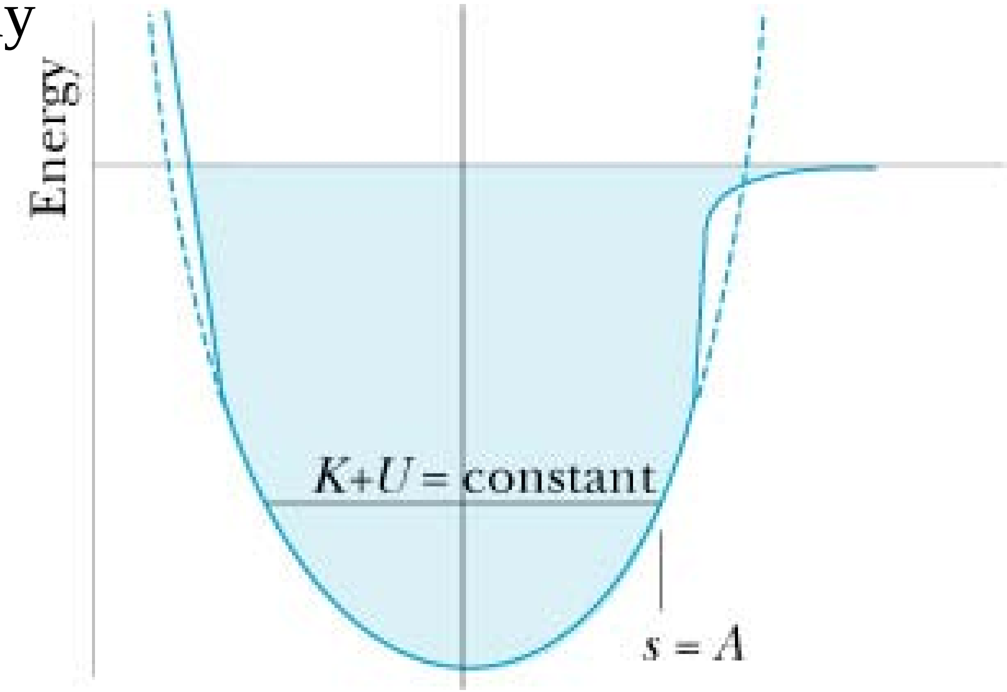
A **classical harmonic oscillator** (mass-spring system) can vibrate with any amplitude, and hence can have any energy.



$K+U$ of the system can have any value in the coloured region.

$$E_{\text{total}} = K + U = \tfrac{1}{2} k_s A^2$$

The energy is "continuous."

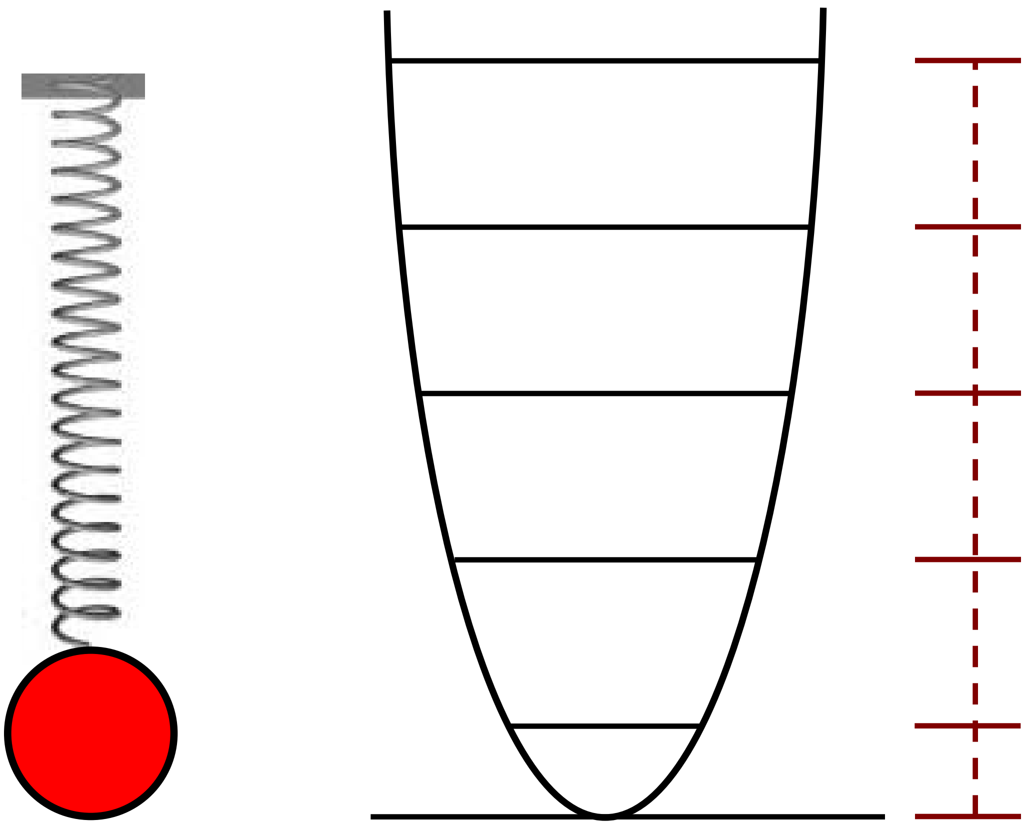


Quantised energy levels of a 1D oscillator

Quantum harmonic oscillator:

Energy can only be added in multiples of $\hbar\omega_0 = \hbar\sqrt{\dfrac{k_s}{m}}$

…where $\hbar = \dfrac{h}{2\pi} = \dfrac{6.67\times10^{-34}\ \text{Js}}{2\pi} = 1.05\times10^{-34}\ \text{Js}$



$E_4 = 4\hbar\omega_0 + E_0$

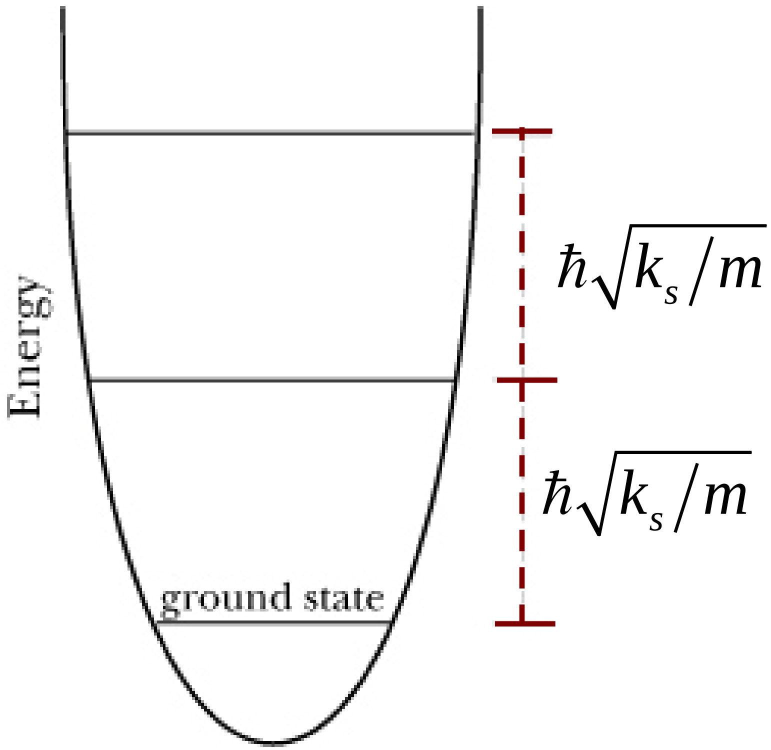
$E_3 = 3\hbar\omega_0 + E_0$

$E_2 = 2\hbar\omega_0 + E_0$

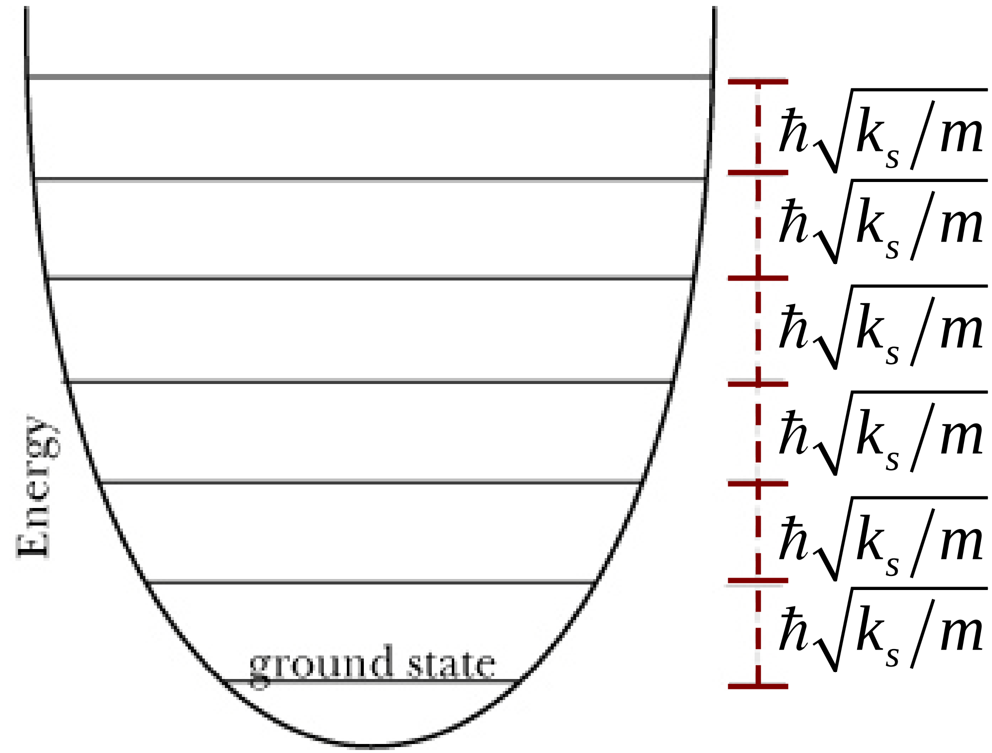
$E_1 = 1\hbar\omega_0 + E_0$

E_0 “ground state”

Effect of the "spring stiffness" k_s



Quantum oscillator
with large k_s

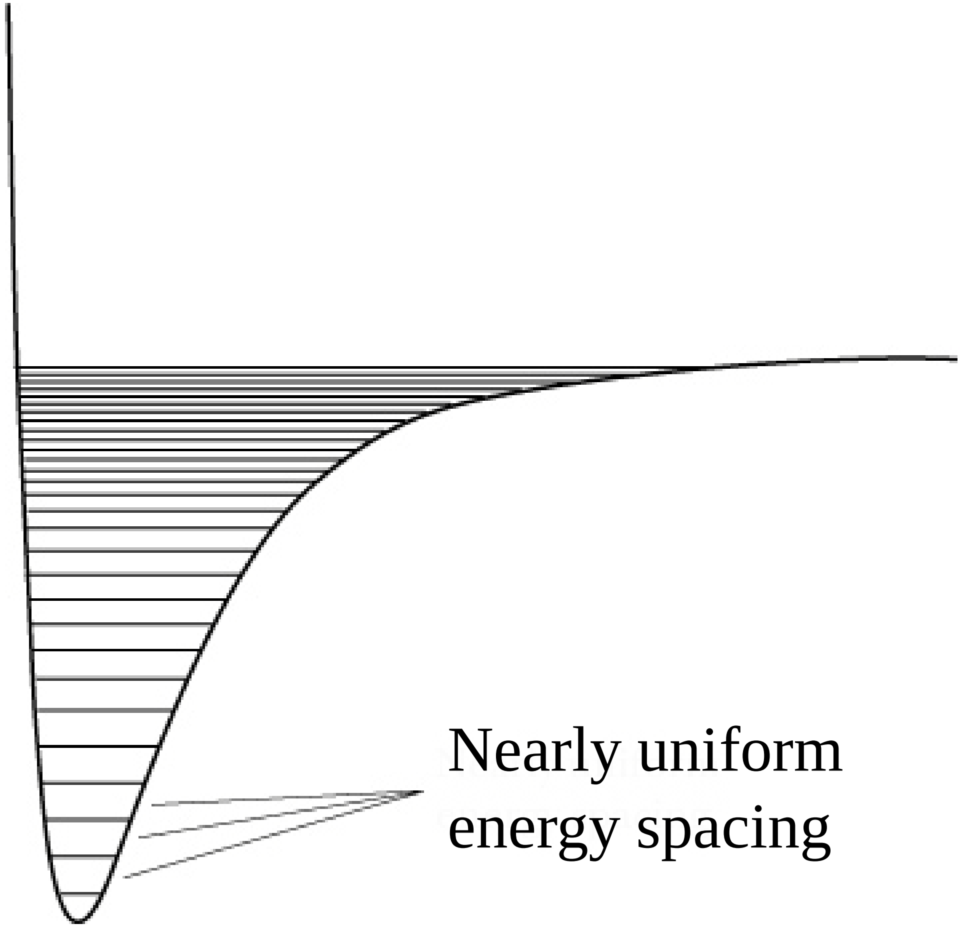


Quantum oscillator
with small k_s

Note that the "**ground state**" E_0 of the oscillator is not zero …
… from Heisenberg … usually $E_0 = \frac{1}{2}\hbar\sqrt{k_s/m}$

Energy levels for the inter-atomic potential energy

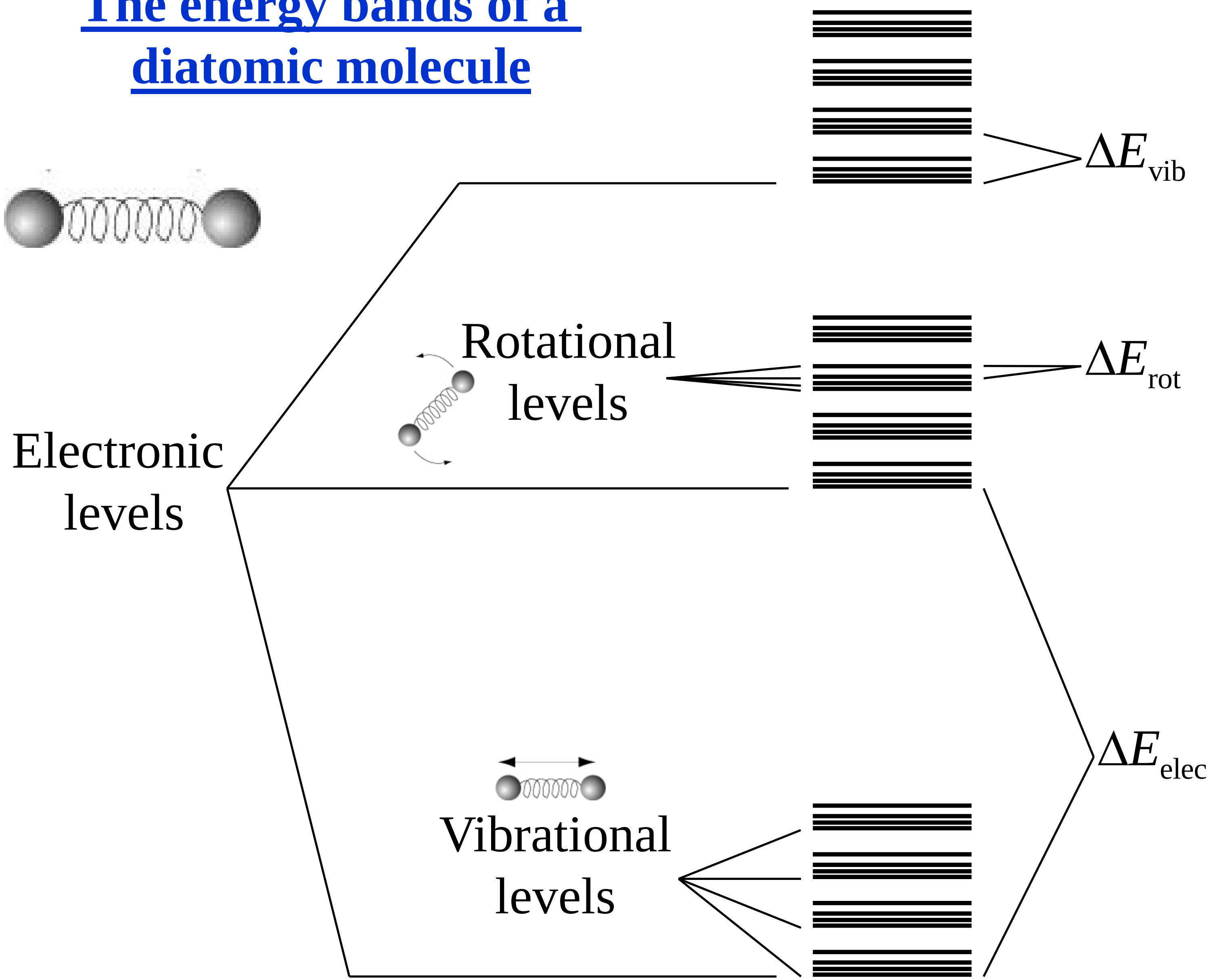
… which describes the interaction of two neighbouring atoms



Nearly uniform
energy spacing

M&I
3E 8.5

The energy bands of a diatomic molecule



1	2	3	4	5

Two atoms joined by a chemical bond can be modeled as two masses connected by a spring. In one such molecule, it takes 0.05 eV to raise the molecule from its vibrational ground state to the first excited vibrational energy state.

How much energy is required to raise the molecule from its first excited state to the second excited vibrational state?

(1) 0.0125 eV
(2) 0.025 eV
(3) 0.05 eV
(4) 0.10 eV
(5) 0.20 eV

1	2	3	4	5

Molecule A: 2 atoms of mass M_A
Molecule B: 2 atoms of mass $4*M_A$

Stiffness of interatomic bond is approximately the same for both.

Which molecule has vibrational energy levels spaced closer together?

(1) A
(2) B
(3) the spacing is the same

1	2	3	4	5

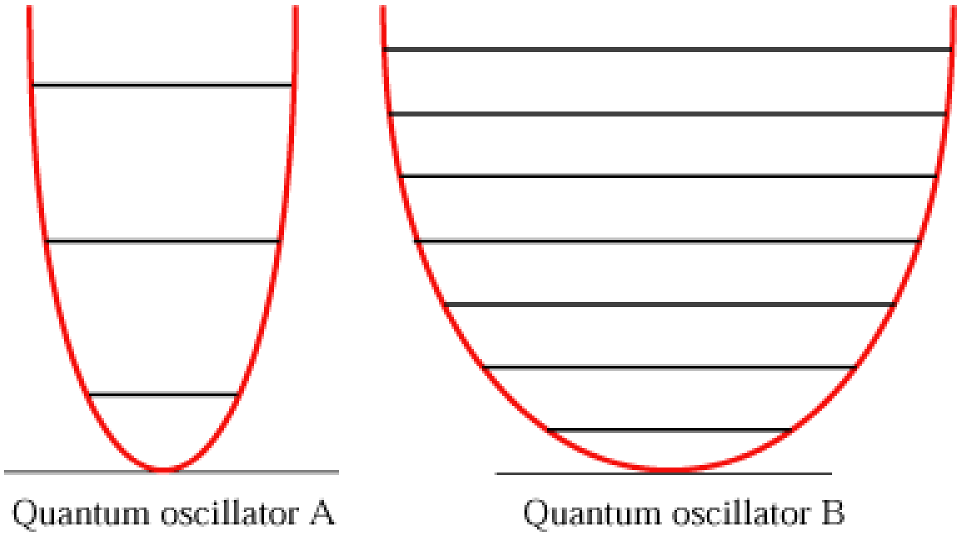
Suppose the atoms in diatomic molecules C and D had approximately the same masses, but the stiffness of the bond in C is 3 times as large as stiffness of the bond in D.

Which molecule has vibrational energy levels spaced closer together?

(1) C
(2) D
(3) the spacing is the same

1	2	3	4	5

Spacing of vibrational energy levels



Pb: $k_s \sim 5$ N/m
Al: $k_s \sim 16$ N/m

Which vibrational energy level diagram represents Pb, and which is Al?

(1) A is Pb and B is Al
(2) A is Al and B is Pb
(3) A is both Pb and Al
(4) B is both Pb and Al

M&I
3E 8.6

Type of state	Typical energy level spacing
hadronic	10^8 eV
nuclear	10^6 eV
electronic (atoms, molecules)	1 eV
vibrational (molecules)	10^{-2} eV
rotational (molecules)	10^{-4} eV