Stress and Strain

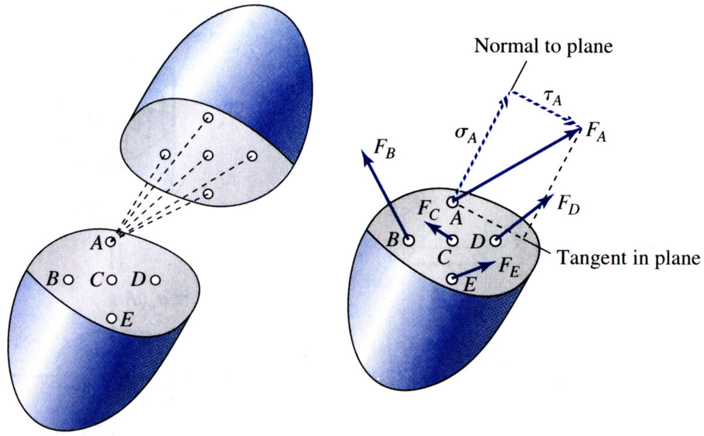
The learning objectives in this chapter are:

- Understanding the concept of stress and the use of double subscripts in determining the direction of stress components on a surface.

- Understanding the concept of strain and the use of small strain and finite difference approximation.

- Understanding the stress and strain transformation in three dimension.

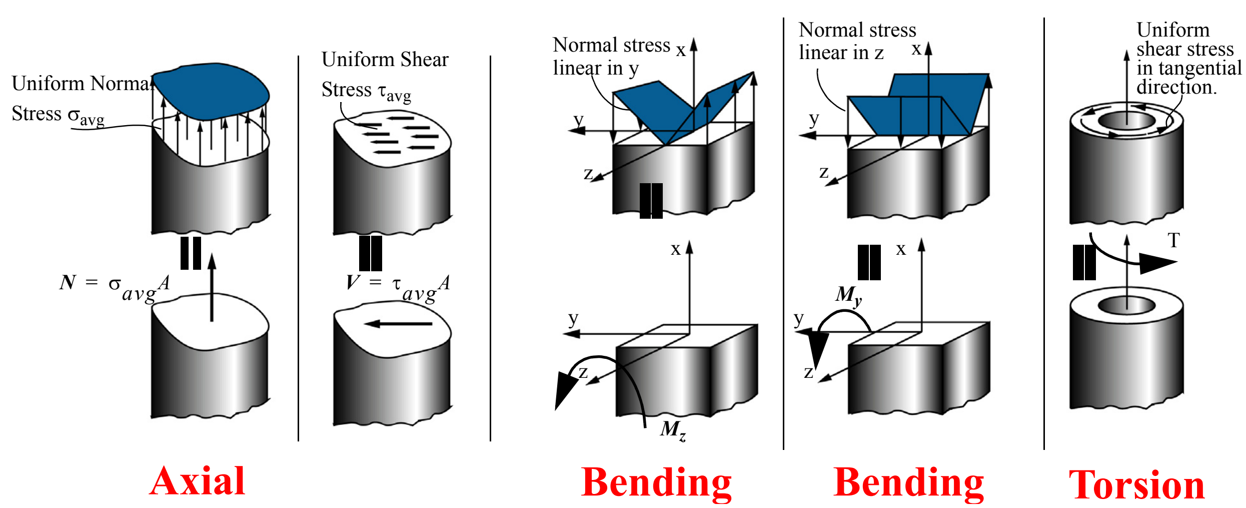
Internally Distributed Force System



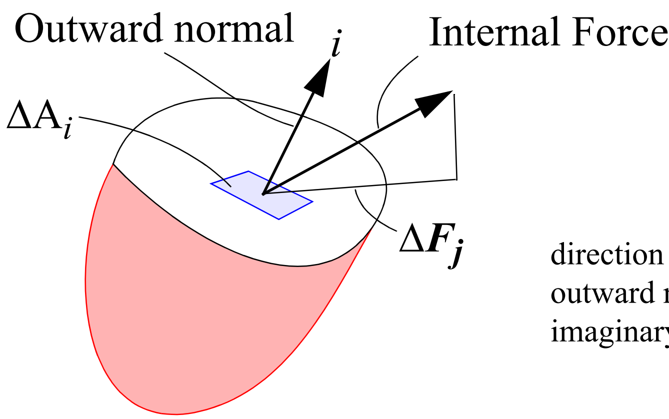
- The intensity of internal distributed forces on an imaginary cut surface of a body is called the *stress on a surface*.
- The intensity of internal distributed force that is normal to the surface of an imaginary cut is called the *normal stress* on a surface**.**
- The intensity of internal distributed force that is parallel to the surface of an imaginary cut surface is called the *shear stress* on the surface.
- Relating stresses to external forces and moments is a two step process.

Stresses ⟷ (Static Equivalency) ⟷ Internal Forces and Moments ⟷ (Equilibrium) ⟷ External Forces and Moments

Static equivalency



Stress at a Point



$$\sigma_{ij} = \lim_{\Delta A_i \to 0}\left(\frac{\Delta F_j}{\Delta A_i}\right)$$

direction of outward normal to the imaginary cut surface.

direction of the internal force component.

- ΔA_i will be considered positive if the outward normal to the surface is in the positive i direction.
- A stress component is positive if numerator and denominator have the same sign. Thus σ_{ij} is positive if: (1) ΔF_j and ΔA_i are both positive. (2) ΔF_j and ΔA_i are both negative.
- Stress Matrix in 3-D: $\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$

Table 1.1. Comparison of number of components

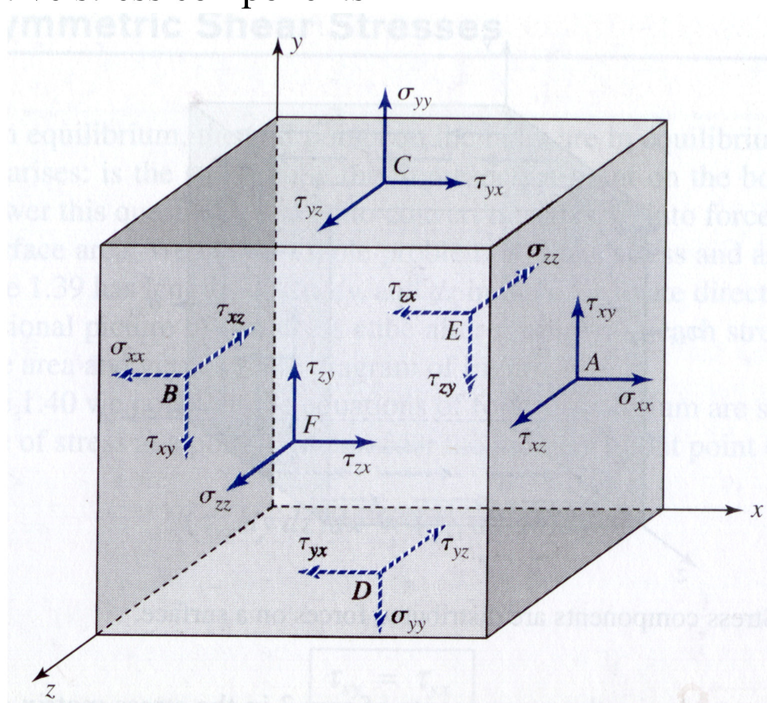
Quantity	1-D	2-D	3-D
Scaler	$1=1^0$	$1=2^0$	$1=3^0$
Vector	$1=1^1$	$2=2^1$	$3=3^1$
Stress	$1=1^2$	$4=2^2$	$9=3^2$

Stress Element

- Stress element is an imaginary object that helps us visualize stress at a point by constructing surfaces that have outward normal in the coordinate directions.

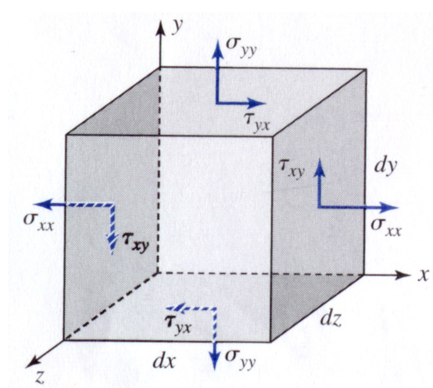
Stress cube showing all positive stress components

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$



Plane Stress: All stress components on a plane are zero.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Symmetric Shear Stresses: $\tau_{xy} = \tau_{yx}$ $\tau_{yz} = \tau_{zy}$ $\tau_{zx} = \tau_{xz}$

- A pair of symmetric shear stress points towards the corner or away from the corner.

C1.1 Show the non-zero stress components on the A,B, and C faces of the cube shown in Figure P1.3 and Figure P1.4.

$$\begin{bmatrix} \sigma_{xx} = 0 & \tau_{xy} = -15ksi & \tau_{xz} = 0 \\ \tau_{yx} = -15ksi & \sigma_{yy} = 10ksi(C) & \tau_{yz} = 25ksi \\ \tau_{zx} = 0 & \tau_{zy} = 25ksi & \sigma_{zz} = 20ksi(T) \end{bmatrix}$$

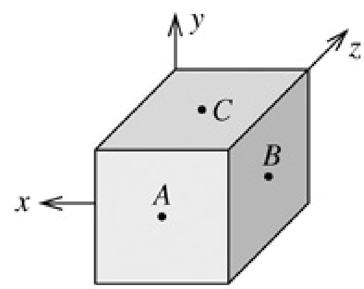


Figure P1.3

Fig. P1.1

Class Problem 1.1

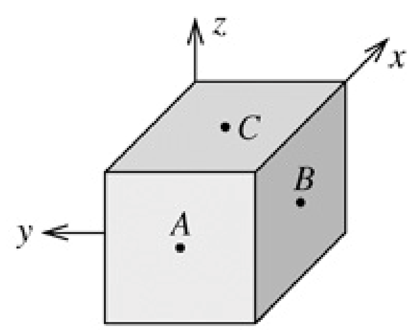
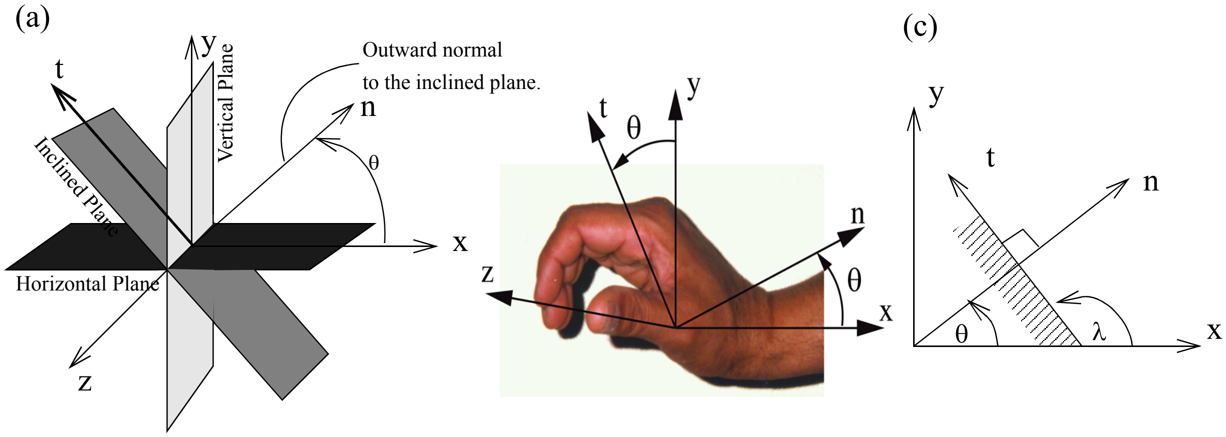


Figure P1.4

Stress transformation in two dimension



$$\sigma_{nn} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\tau_{nt} = -\sigma_{xx}\cos\theta\sin\theta + \sigma_{yy}\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\sigma_{tt} = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

Matrix Notation

$$n_x = \cos\theta \qquad n_y = \sin\theta \qquad t_x = \cos\lambda \qquad t_y = \sin\lambda$$

True only in 2D: $\lambda = 90 + \theta \qquad t_x = -n_y \qquad t_y = n_x$

$$\{n\} = \begin{Bmatrix} n_x \\ n_y \end{Bmatrix} \qquad \{t\} = \begin{Bmatrix} t_x \\ t_y \end{Bmatrix} \qquad [\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$

The symmetry of shear stresses $[\sigma]^T = [\sigma]$

$$\boxed{\sigma_{nn} = \{n\}^T[\sigma]\{n\}}$$

$$\boxed{\tau_{nt} = \{t\}^T[\sigma]\{n\}}$$

$$\boxed{\sigma_{tt} = \{t\}^T[\sigma]\{t\}}$$

Traction or Stress vector

Mathematically the stress vector {S} is defined as:

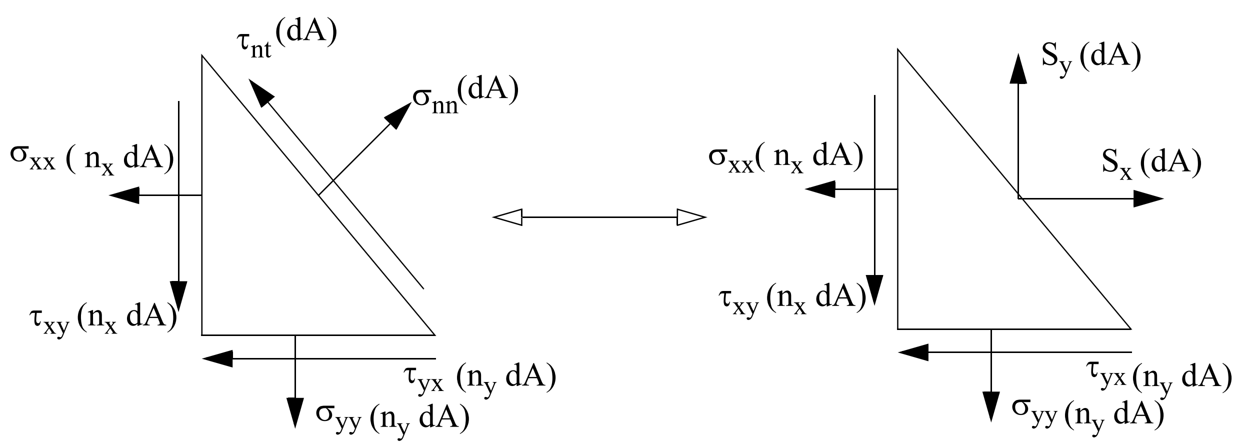
$$\{S\} = [\sigma]\{n\}$$

$$S_x = \sigma_{xx} n_x + \tau_{xy} n_y$$

$$S_y = \tau_{yx} n_x + \sigma_{yy} n_y$$

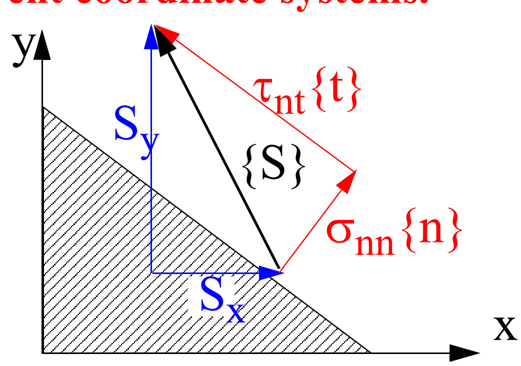
- pressure is a scaler quantity.
- traction is a vector quantity.,
- stress is a second order tensor.

Statically equivalent force wedge.



$$\{S\} = \sigma_{nn}\{n\} + \tau_{nt}\{t\}$$

Stress vector in different coordinate systems.



Principal Stresses and Directions

$$\{S\} = [\sigma]\{p\} = \sigma_p\{p\}$$

OR

$$\{S\} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \end{Bmatrix} = \begin{bmatrix} \sigma_p & 0 \\ 0 & \sigma_p \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \end{Bmatrix}$$

OR

$$\begin{bmatrix} (\sigma_{xx} - \sigma_p) & \tau_{xy} \\ \tau_{yx} & (\sigma_{yy} - \sigma_p) \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \end{Bmatrix} = 0$$

Characteristic equation

$$\sigma_p^2 - \sigma_p(\sigma_{xx} + \sigma_{yy}) + (\sigma_{xx}\sigma_{yy} - \tau_{xy}^2) = 0$$

Roots: $$\sigma_{1,2} = [(\sigma_{xx} + \sigma_{yy}) \pm \sqrt{(\sigma_{xx} + \sigma_{yy})^2 - 4(\sigma_{xx}\sigma_{yy} - \tau_{xy}^2)}]/2$$

OR

$$\sigma_{1,2} = \left[\left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}\right]$$

- The *eigenvalues* of the stress matrix are the principal stresses.
- The *eigenvectors* of the stress matrix are the principal directions.

Stress Transformation in 3-D

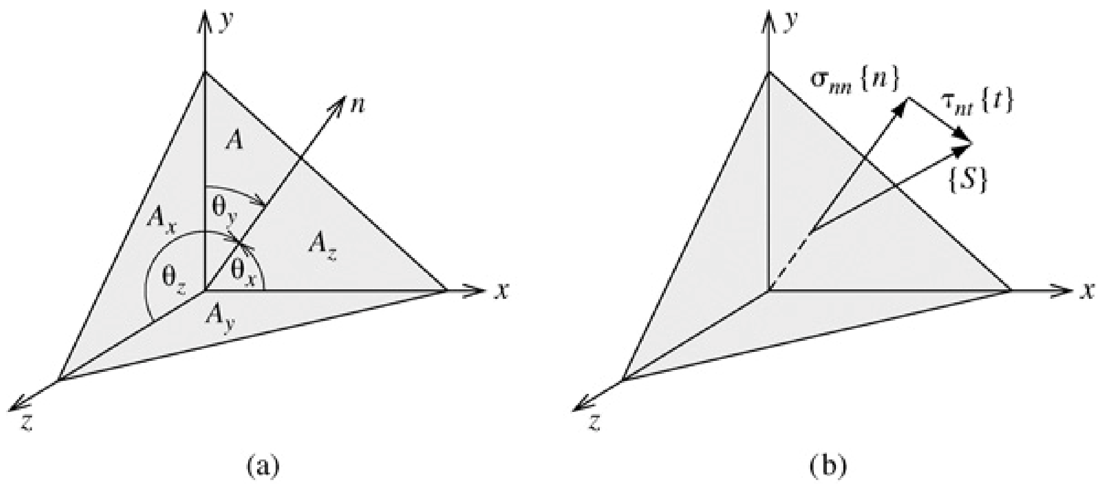


Figure 1.19 (a) Direction cosines of a unit normal. (b) Equilibrating shear stress.

$$\{n\} = \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} \qquad \{S\} = \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} \qquad [\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\boxed{\sigma_{nn} = \{n\}^T[\sigma]\{n\}}$$

$$\boxed{\tau_{nt} = \{t\}^T[\sigma]\{n\}}$$

$$\boxed{\sigma_{tt} = \{t\}^T[\sigma]\{t\}}$$

$$\{S\} = [\sigma]\{n\}$$

Equilibrium condition: $\{S\} = \sigma_{nn}\{n\} + \tau_{nt}\{t_E\}$

Principal Stresses and Directions

- Planes on which the shear stresses are zero are called the principal planes.
- The normal direction to the principal planes is referred to as the principal direction or the principal axis.
- The angles the principal axis makes with the global coordinate system are called the principal angles.

$$\{S\} = [\sigma]\{p\} = \sigma_p\{p\}$$

OR

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{bmatrix} \sigma_p & 0 & 0 \\ 0 & \sigma_p & 0 \\ 0 & 0 & \sigma_p \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix}$$

OR

$$\begin{bmatrix} (\sigma_{xx} - \sigma_p) & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & (\sigma_{yy} - \sigma_p) & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & (\sigma_{zz} - \sigma_p) \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = 0$$

- The *eigenvalues* of the stress matrix are the principal stresses.
- The *eigenvectors* of the stress matrix are the principal directions.

$$\boxed{p_x^2 + p_y^2 + p_z^2 = 1}$$

Principal stress convention

Ordered principal stresses in 3-D: $\sigma_1 > \sigma_2 > \sigma_3$

Ordered principal stresses in 2-D: $\sigma_1 > \sigma_2$

Principal Angles $0^o \le \theta_x, \theta_y, \theta_z \le 180^o$

Characteristic equation

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

Stress Invariants

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{zy} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{zx} & \sigma_{zz} \end{vmatrix}$$

$$I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

$$x^3 - I_1 x^2 + I_2 x - I_3 = 0$$

Roots:

$$x_1 = 2A\cos\alpha + I_1/3 \qquad x_{2,3} = -2A\cos(\alpha \pm 60^o) + I_1/3$$

$$A = \sqrt{(I_1/3)^2 - I_2/3}$$

$$\cos 3\alpha = [2(I_1/3)^3 - (I_1/3)I_2 + I_3]/(2A^3)$$

Principal Stress Matrix

$$[\sigma] = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix}$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

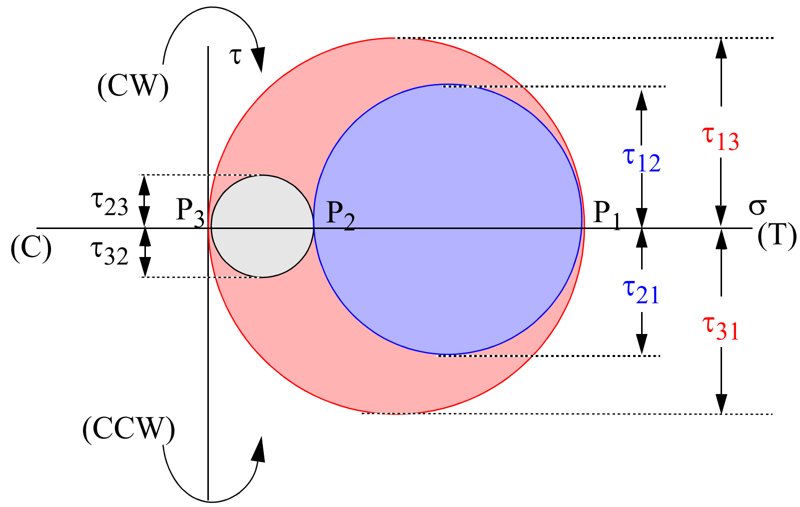
$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$

$$I_3 = \sigma_1\sigma_2\sigma_3$$

Maximum Shear Stress

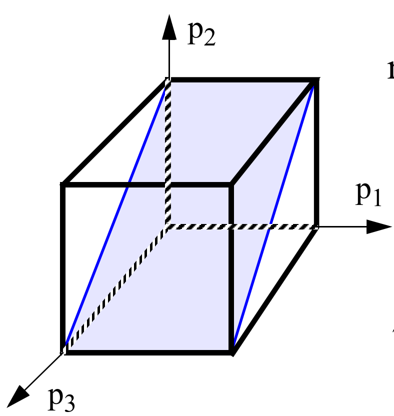
Plane Stress

$\sigma_3 = 0$



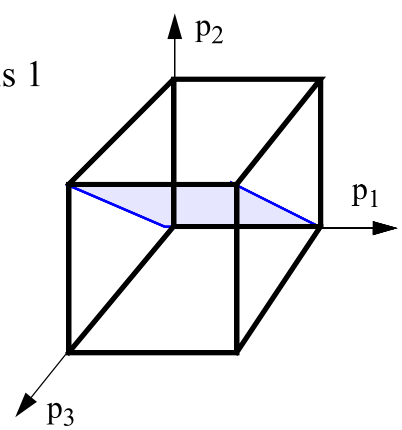
- maximum shear stress exists on two planes, each of which are 45° away from the principal planes.

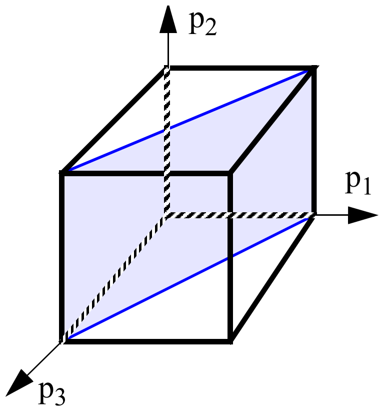
$$\tau_{max} = max\left(\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right|\right)$$



rotation about principal axis 1

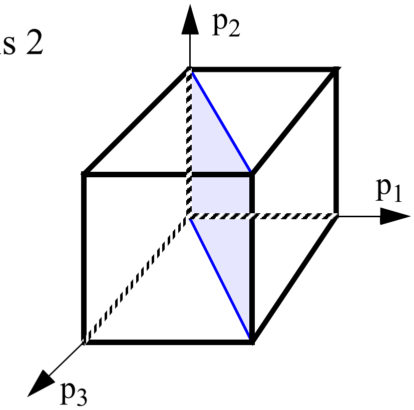
$$\tau_{23} = -\tau_{32} = \frac{\sigma_2 - \sigma_3}{2}$$

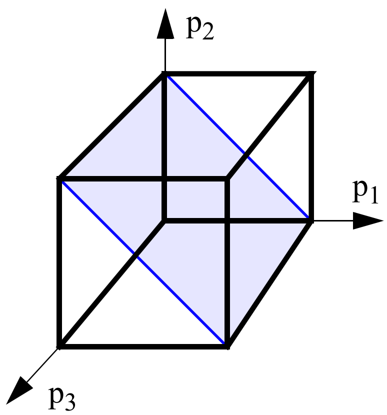




rotation about principal axis 2

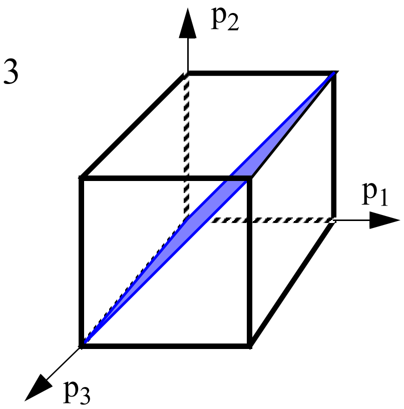
$$\tau_{31} = -\tau_{13} = \frac{\sigma_3 - \sigma_1}{2}$$





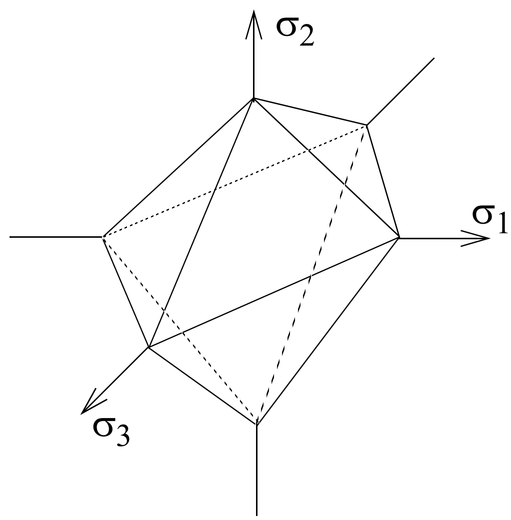
rotation about principal axis 3
(In-plane)

$$\tau_{21} = -\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$



Octahedral stresses

- A plane that makes equal angles with the principal planes is called an octahedral plane.



$$\sigma_{nn} = \sigma_1 n_1^2 + \sigma_2 n_2^2 + \sigma_3 n_3^2$$

$$\tau_{nt} = \sqrt{(\sigma_1^2 n_1^2 + \sigma_2^2 n_2^2 + \sigma_3^2 n_3^2) - \sigma_{nn}^2}$$

$$|n_1| = |n_2| = |n_3| = 1/\sqrt{3}$$

$$\sigma_{oct} = (\sigma_1 + \sigma_2 + \sigma_3)/3$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

C1.2 The stress at a point is given by the stress matrix shown. Determine: (a) the normal and shear stress on a plane that has an outward normal at 37°, 120°, and 70.43°, to x, y, and z direction respectively. (b) the principal stresses (c) the second principal direction and (d) the magnitude of the octahedral shear stress. (e) maximum shear stress

$$\begin{bmatrix} 18 & 12 & 9 \\ 12 & 12 & -6 \\ 9 & -6 & 6 \end{bmatrix} ksi$$

Strain

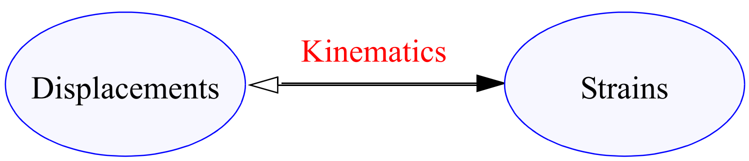
- The total movement of a point with respect to a fixed reference coordinates is called *displacement*.

- The relative movement of a point with respect to another point on the body is called *deformation*.

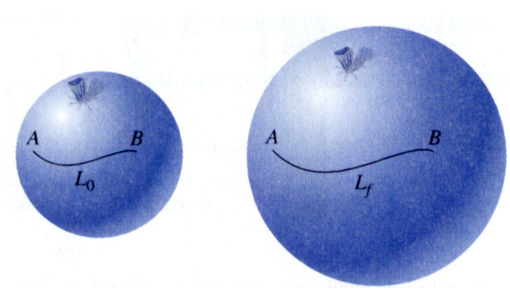
- *Lagrangian strain* is computed from deformation by using the original undeformed geometry as the reference geometry.

- *Eulerian strain* is computed from deformation by using the final deformed geometry as the reference geometry.

- Relating strains to displacements is a problem in geometry.



Average normal strain

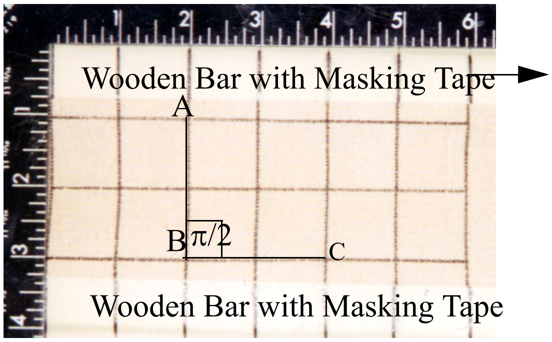


$$\varepsilon_{av} = \frac{L_f - L_o}{L_o} = \frac{\delta}{L_o}$$

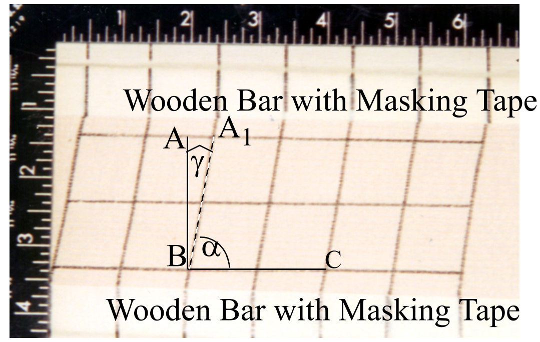
- Elongations ($L_f > L_o$) result in *positive* normal strains. Contractions ($L_f < L_o$) result in *negative* normal strains.

Average shear strain

Undeformed grid



Deformed grid



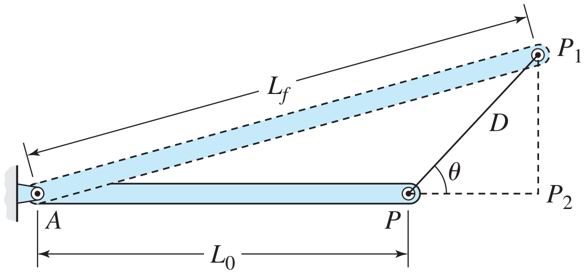
$$\gamma_{av} = \frac{\pi}{2} - \alpha$$

- Decreases in the angle ($\alpha < \pi / 2$) result in *positive* shear strain. Increase in the angle ($\alpha > \pi / 2$) result in *negative* shear strain

Units of average strain

- To differentiate average strain from strain at a point.
- in/in, or cm/cm, or m/m (for normal strains)
- rads (for shear strains)
- percentage. 0.5% is equal to a strain of 0.005
- prefix: $\mu = 10^{-6}$. 1000 μ in / in is equal to a strain 0.001 in /

Small Strain Approximation



$$L_f = \sqrt{L_o^2 + D^2 + 2L_o D\cos\theta}$$

$$L_f = L_o\sqrt{1 + \left(\frac{D}{L_o}\right)^2 + 2\left(\frac{D}{L_o}\right)\cos\theta}$$

$$\varepsilon = \frac{L_f - L_o}{L_o} = \sqrt{1 + \left(\frac{D}{L_o}\right)^2 + 2\left(\frac{D}{L_o}\right)\cos\theta} - 1 \quad \textbf{2.5}$$

$$\varepsilon_{small} = \frac{D\cos\theta}{L_o} \quad \textbf{2.6}$$

$\varepsilon_{\text{small}}$ Eq. 2.6	ε Eq. 2.5	% error
1.0	1.23607	19.1
0.5	0.58114	14.0
0.1	0.10454	4.3
0.05	0.005119	2.32
0.01	0.01005	0.49
0.005	0.00501	0.25

- Small-strain approximation may be used for strains less than 0.01
- Small normal strains are calculated by using the deformation component in the original direction of the line element regardless of the orientation of the deformed line element.
- In small shear strain (γ) calculations the following approximation may be used for the trigonometric functions: $\tan\gamma \approx \gamma$ $\sin\gamma \approx \gamma$ $\cos\gamma \approx 1$
- Small-strain calculations result in linear deformation analysis.
- Drawing approximate deformed shape is very important in analysis of small strains.

C1.3 A roller at P slides in a slot as shown. Determine the deformation in bar AP and bar BP by using small strain approximation.

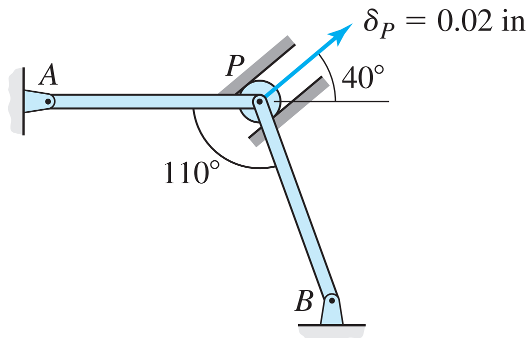
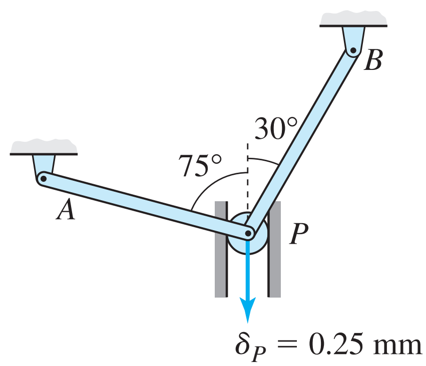


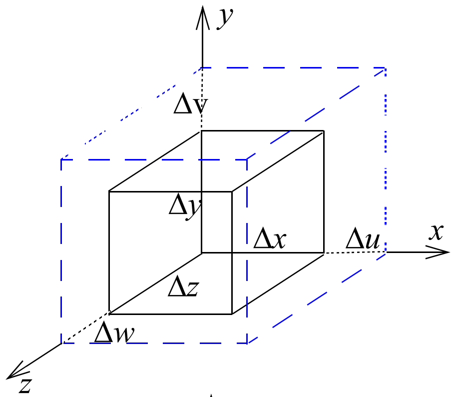
Fig. C2.3

Class Problem 1.2

Draw an approximate exaggerated deformed shape.
Using small strain approximation write equations relating δ_{AP} and δ_{BP} to δ_P.



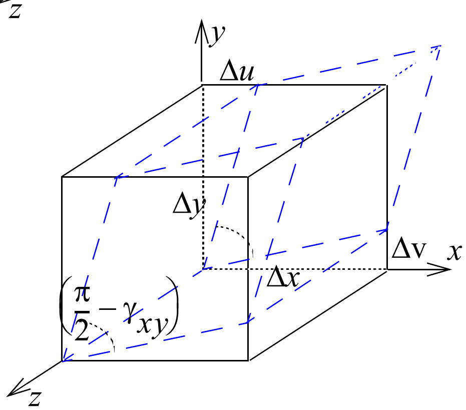
Strain Components



$$\varepsilon_{xx} = \frac{\Delta u}{\Delta x}$$

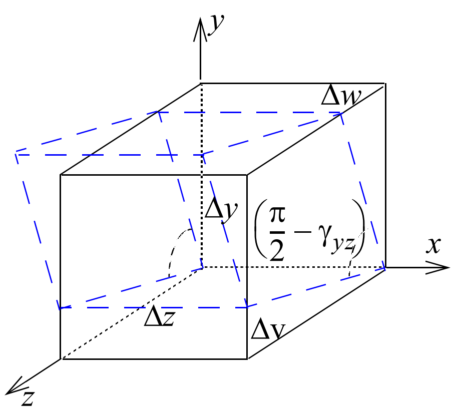
$$\varepsilon_{yy} = \frac{\Delta v}{\Delta y}$$

$$\varepsilon_{zz} = \frac{\Delta w}{\Delta z}$$



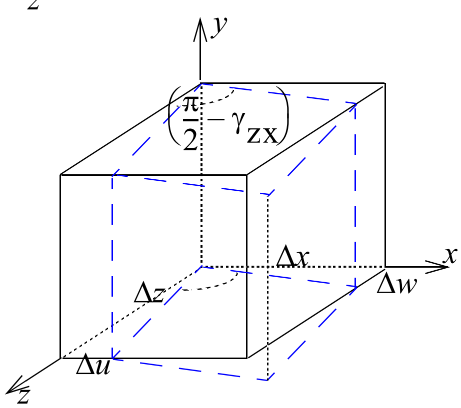
$$\gamma_{xy} = \frac{\Delta u}{\Delta y} + \frac{\Delta v}{\Delta x}$$

$$\gamma_{yx} = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y} = \gamma_{xy}$$



$$\gamma_{yz} = \frac{\Delta v}{\Delta z} + \frac{\Delta w}{\Delta y}$$

$$\gamma_{zy} = \frac{\Delta w}{\Delta y} + \frac{\Delta v}{\Delta z} = \gamma_{yz}$$



$$\gamma_{zx} = \frac{\Delta w}{\Delta x} + \frac{\Delta u}{\Delta z}$$

$$\gamma_{xz} = \frac{\Delta u}{\Delta z} + \frac{\Delta w}{\Delta x} = \gamma_{zx}$$

Strain at a point

Engineering Strain

$$\varepsilon_{xx} = \lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \gamma_{yx} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}}\left(\frac{\Delta u}{\Delta y} + \frac{\Delta \mathrm{v}}{\Delta x}\right) = \frac{\partial u}{\partial y} + \frac{\partial \mathrm{v}}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial \mathrm{v}}{\partial y} \qquad \gamma_{yz} = \gamma_{zy} = \frac{\partial \mathrm{v}}{\partial z} + \frac{\partial w}{\partial y}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \qquad \gamma_{zx} = \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

- tensor normal strains = engineering normal strains
- tensor shear strains = (engineering shear strains)/ 2

- The partial derivative with respect to a coordinate implies that during the process of differentiation the other coordinates are held constant.
- If a displacement is only a function of one coordinate, then the partial derivative with respect to that coordinate will be same as ordinary derivative.

$$\varepsilon_{xx} = \frac{du}{dx}(x)$$

Finite Difference Approximation

- *Forward difference* approximates the slope of the tangent using the point ahead of point i as:

$$(\varepsilon_{xx})_i = \frac{u_{i+1} - u_i}{x_{i+1} - x_i}$$

- *Backward difference* approximates the slope of the tangent using the point behind i as:

$$(\varepsilon_{xx})_i = \frac{u_i - u_{i-1}}{x_i - x_{i-1}}$$

- *Central difference* takes the average value using the point ahead and behind as:

$$(\varepsilon_{xx})_i = \frac{1}{2}\left[\frac{u_{i+1} - u_i}{x_{i+1} - x_i} + \frac{u_i - u_{i-1}}{x_i - x_{i-1}}\right]$$

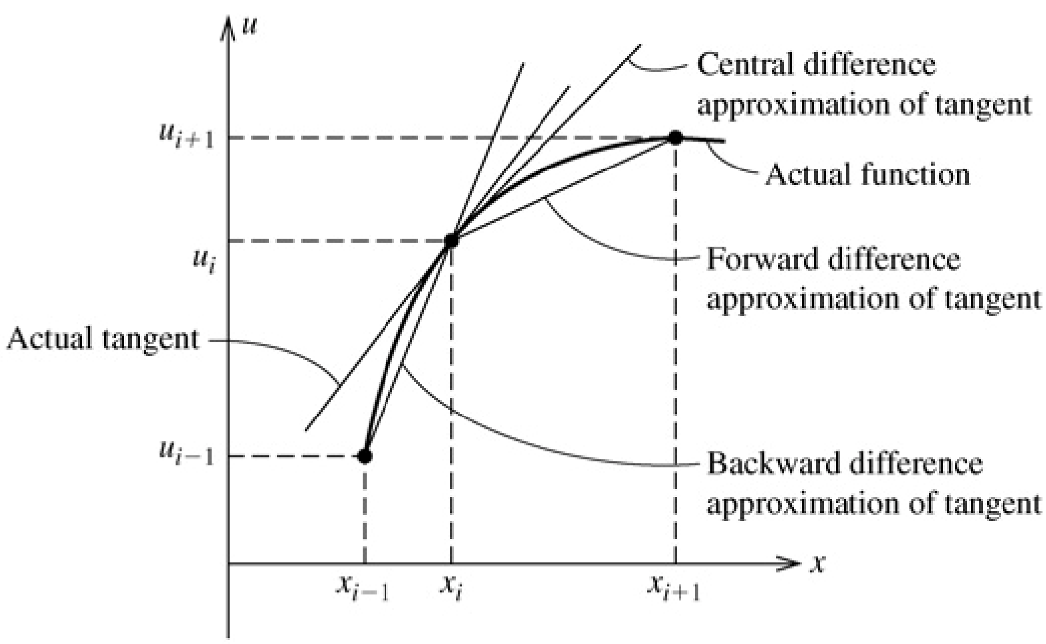
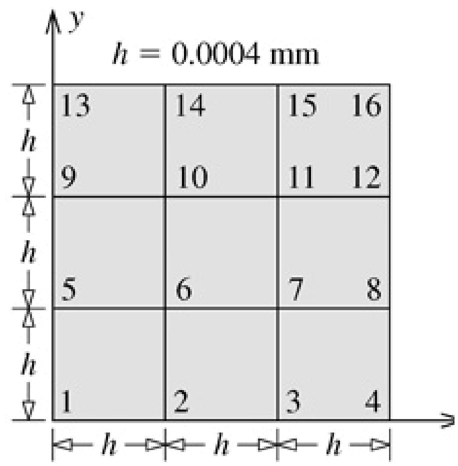


Figure 1.27 The three finite difference methods.

C1.4 The displacements u and v in the x and y directions respectively were measured by Moire' interferometry. Displacements of 16 points on the body and are as given below.

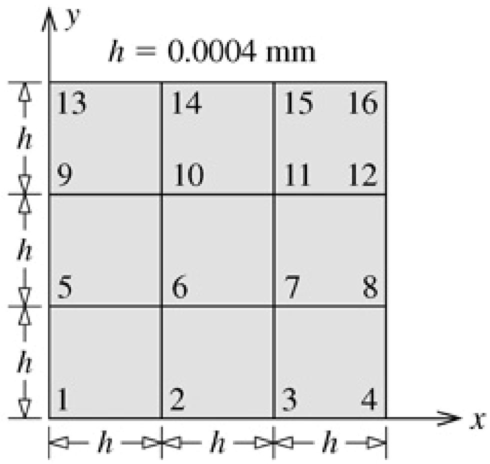
Point	u (μmm)	v (μmm)	Point	u (μmm)	v (μmm)
1	0.000	0.000	9	0.128	0.384
2	-0.112	0.144	10	-0.048	0.336
3	-0.128	0.256	11	-0.128	0.256
4	-0.048	0.336	12	-0.112	0.144
5	0.112	0.176	13	0.048	0.624
6	-0.032	0.224	14	-0.160	0.480
7	-0.080	0.240	15	-0.272	0.304
8	-0.032	0.224	16	-0.288	0.096



Determine the strains ε_{xx} , ε_{yy} , *and* γ_{xy} at points 1 and 4.

Class Problem 1.3

In terms of u, v, x, y, and node numbers, write the equations to determine the strains ε_{xx} , ε_{yy} , and γ_{xy} at point 6.



Strain Transformation

Strain transformation equations in 2-D

$$\varepsilon_{nn} = \varepsilon_{xx}\cos^2\theta + \varepsilon_{yy}\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta \qquad \varepsilon_{tt} = \varepsilon_{xx}\sin^2\theta + \varepsilon_{yy}\cos^2\theta - \gamma_{xy}\sin\theta\cos\theta$$

$$\gamma_{nt} = -2\varepsilon_{xx}\sin\theta\cos\theta + 2\varepsilon_{yy}\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

Stress transformation equations in 2-D

$$\sigma_{nn} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \qquad \sigma_{tt} = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

$$\tau_{nt} = -\sigma_{xx}\cos\theta\sin\theta + \sigma_{yy}\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

- tensor normal strains = engineering normal strains
- tensor shear strains = (engineering shear strains)/ 2

Tensor strain matrix from engineering strains

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} = \gamma_{xy}/2 & \varepsilon_{xz} = \gamma_{xz}/2 \\ \varepsilon_{yx} = \gamma_{yx}/2 & \varepsilon_{yy} & \varepsilon_{yz} = \gamma_{yz}/2 \\ \varepsilon_{zx} = \gamma_{zx}/2 & \varepsilon_{zy} = \gamma_{zy}/2 & \varepsilon_{zz} \end{bmatrix}$$

$$\varepsilon_{nn} = \{n\}^T[\varepsilon]\{n\} \qquad \varepsilon_{nt} = \{t\}^T[\varepsilon]\{n\} \qquad \varepsilon_{tt} = \{t\}^T[\varepsilon]\{t\} \qquad \gamma_{nt} = 2\varepsilon_{nt}$$

Characteristic equation

$$\varepsilon_p^3 - I_1\varepsilon_p^2 + I_2\varepsilon_p - I_3 = 0$$

Strain invariants

$$I_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$I_2 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{vmatrix} + \begin{vmatrix} \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} + \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_{zz} \end{vmatrix} = \varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_3\varepsilon_1$$

$$I_3 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} = \varepsilon_1\varepsilon_2\varepsilon_3$$

Maximum shear strain

$$\frac{\gamma_{max}}{2} = max\left(\left|\frac{\varepsilon_1 - \varepsilon_2}{2}\right|, \left|\frac{\varepsilon_2 - \varepsilon_3}{2}\right|, \left|\frac{\varepsilon_3 - \varepsilon_1}{2}\right|\right)$$