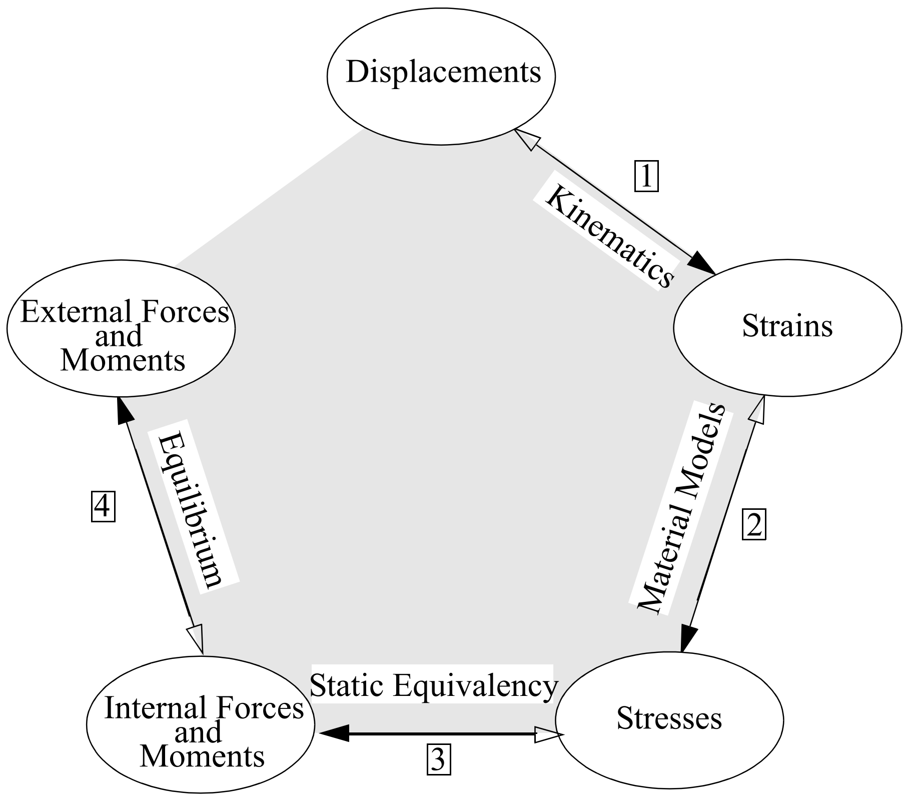
Composite Structural Members

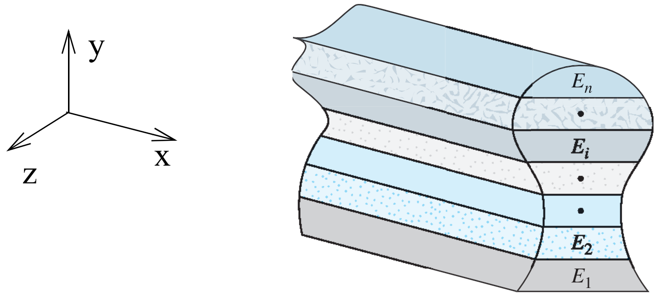
The learning objective of this chapter is:

Understand the incorporation and implications of material in-homogeneity across the cross-section in the theories for axial members, circular shafts in torsion, and symmetric bending of beams.



- Assumption 8 on material homogenity across the cross-section is not valid.The assumption is made in the link of static equivalency.

Composite Axial Members



$$\sigma_{xx} = E\frac{du}{dx}(x)$$

Internal Forces and Moments

$$N = \int_A \sigma_{xx} dA \qquad M_z = -\int_A y\sigma_{xx} dA = 0 \quad \text{or} \quad N = \frac{du}{dx}(x)\int_A E dA$$

Location of origin: $$\int_A yE dA = 0$$

Formulas for composite axial rods

$$N = \frac{du}{dx}\int_A E dA = \frac{du}{dx}\left[\int_{A_1} E_1 dA + \int_{A_2} E_2 dA + \cdot + \cdot + \int_{A_n} E_n dA\right]$$

$$\boxed{N = \frac{du}{dx}\sum_{j=1}^{n} E_j A_j}$$

$$\boxed{(\sigma_{xx})_i = \frac{NE_i}{\sum_{j=1}^{n} E_j A_j}} \qquad \boxed{u_2 - u_1 = \frac{N(x_2 - x_1)}{\sum_{j=1}^{n} E_j A_j}}$$

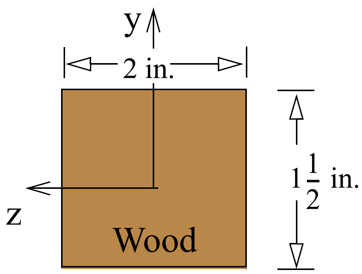
Location of axial force application (origin)

$$\eta_c = \frac{\sum_{i=1}^{n} \eta_i E_i A_i}{\sum_{i=1}^{n} E_i A_i}$$

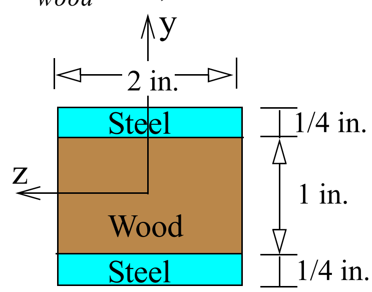
- Axial strain is uniform across entire cross section of a composite rod.
- Axial stress is uniform in each material.
- There is a discontinuity in axial stress at junction of each material.

Example 4.2

$$\varepsilon_{xx} = -200\mu \qquad E_{steel} = 30,000 \text{ ksi} \qquad E_{wood} = 8,000 \text{ ksi}$$

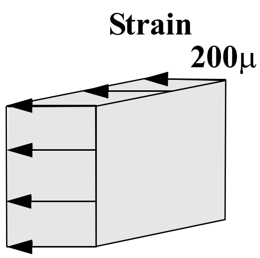


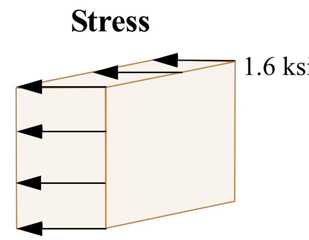
Homogenous cross section



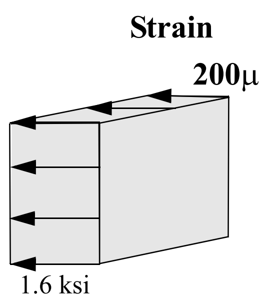
Laminated cross section

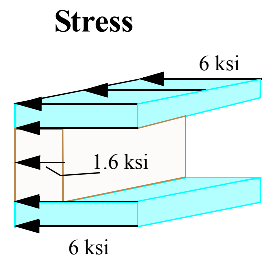
Homogenous



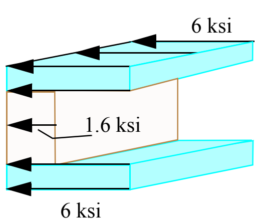


Laminated

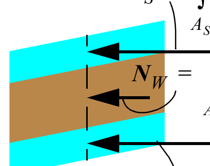




Internal Axial Force For Composite



=



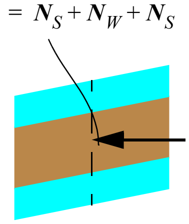
$$N_S = \int_{A_{St}} \sigma_{xx} dA = (6)(2)(1/4)$$

$$N_W = \int_{A_W} \sigma_{xx} dA = (1.6)(2)(1)$$

$$N_S = \int_{A_{Sb}} \sigma_{xx} dA = (6)(2)(1/4)$$

=

$$N = N_S + N_W + N_S$$



C4.1 A wooden rod (E_W = 2000 ksi) and steel strip (E_s = 30,000 ksi) are fastened securely to each other and to the rigid plates as shown in Fig. C1.1. Determine (a) the location h of the line along which the external forces must act to produce no bending. (b) the maximum axial stress in steel and wood.

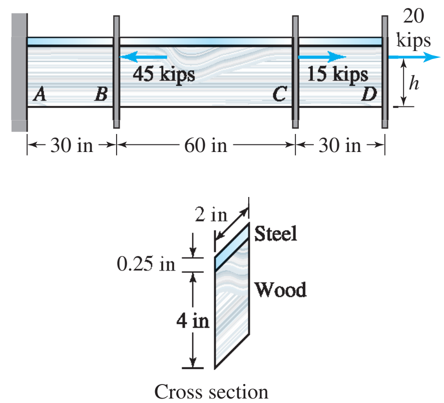
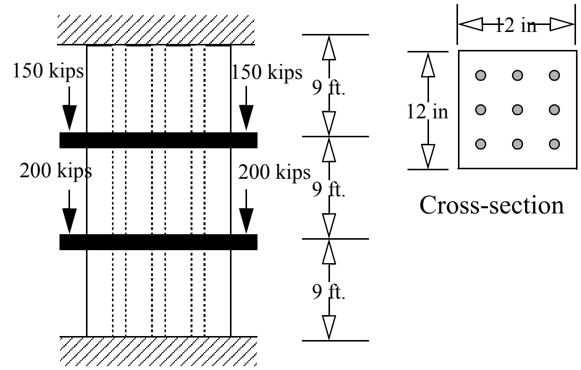


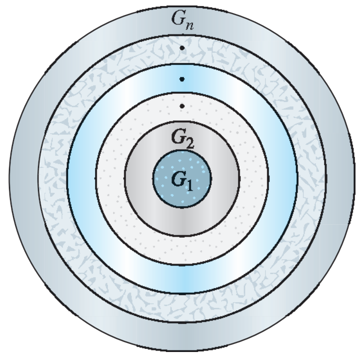
Fig. C1.1

C4.2 A for use in a building is modeled as shown in Fig. C1.2. The column is constructed by reinforcing concrete with nine steel circular bars of diameter 1 inch. The modulus of elasticity for concrete and steel are E_c= 4,500 ksi and E_s = 30,000 ksi. Determine the maximum axial stress in concrete and steel.

Fig. C1.2



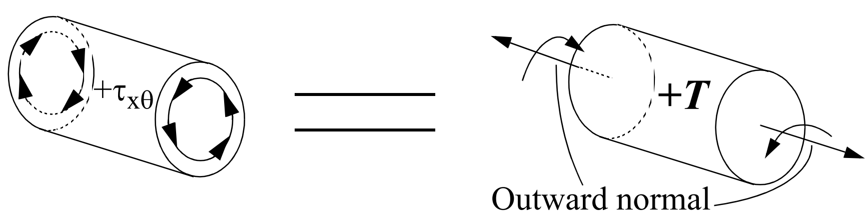
Composite Shafts



$$\tau_{x\theta} = G\rho\frac{d\phi}{dx}(x)$$

Internal Forces and Moments

$$T = \int_A \rho\tau_{x\theta}dA \quad \text{or} \quad \boldsymbol{T} = \frac{d\phi}{dx}\int_A G\rho^2 dA$$



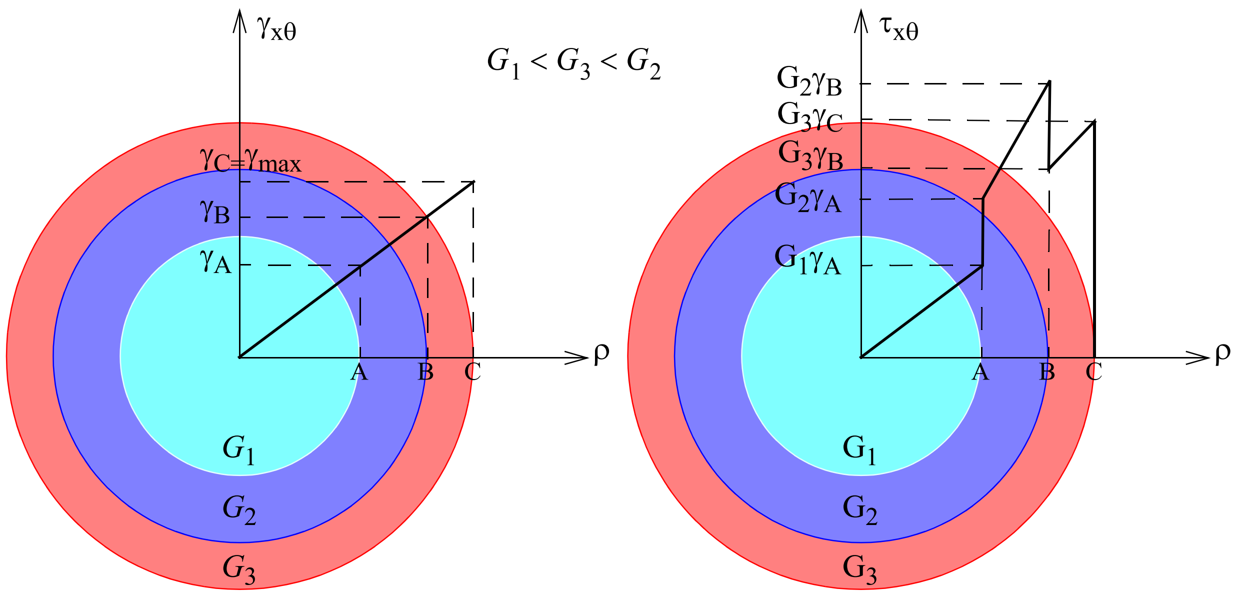
Formulas for composite shafts

$$\boldsymbol{T} = \frac{d\phi}{dx}\left[\int_{A_1} G_1\rho^2 dA + \int_{A_2} G_2\rho^2 dA + \cdot + \cdot + \int_{A_n} G_n\rho^2 dA\right]$$

$$\boxed{\boldsymbol{T} = \frac{d\phi}{dx}\left[\sum_{j=1}^{n} G_j J_j\right]}$$

$$\boxed{(\tau_{x\theta})_i = \frac{G_i\rho\boldsymbol{T}}{\left[\sum_{j=1}^{n} G_j J_j\right]}} \qquad \boxed{\phi_2 - \phi_1 = \frac{\boldsymbol{T}(x_2 - x_1)}{\left[\sum_{j=1}^{n} G_j J_j\right]}}$$

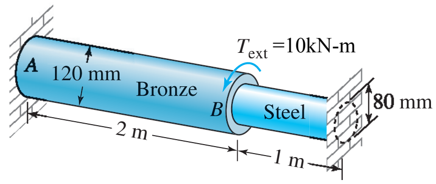
Variation of torsional shear strain and stress



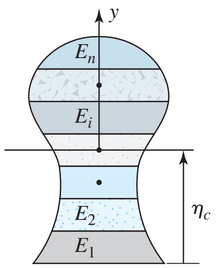
- Torsional shear strain varies linearly in the radial direction across the composite shaft.
- Maximum torsional shear strain is at the outer most radius of the shaft.
- Torsional shear stress varies linearly in the radial direction in each material with slopes that depend upon shear modulus G.
- Torsional shear stress is discontinuous at the junction of each material.
- Maximum torsional shear stress in each material is at point that is furthest from the center.
- Maximum torsional shear stress in the shaft may not be at the outer most radius of the shaft.

C4.3 A solid steel (G = 80 GPa) shaft 3 m long is securely fastened to a hollow bronze (G = 40 GPa) shaft that is 2 m long as shown Fig. C1.3. Determine (a) the magnitude of maximum shear stress in the shaft. (b) the rotation of section at 1 m from the left wall.

Fig. C1.3



Composite Beams



$$\sigma_{xx} = -Ey\frac{d^2 v}{dx^2}(x)$$

Internal Forces and Moments

$$N = \int_A \sigma_{xx} dA = 0 \qquad M_z = -\int_A y\sigma_{xx} dA \quad \text{or} \quad \boldsymbol{M}_z = \frac{d^2 v}{dx^2}\int_A Ey^2 dA$$

Location of neutral axis origin: $\int_A yEdA = 0$ $\qquad \eta_c = \dfrac{\sum_{i=1}^{n} \eta_i E_i A_i}{\sum_{i=1}^{n} E_i A_i}$

Formulas for composite beams

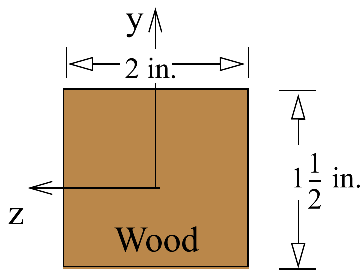
$$\boldsymbol{M_z} = \frac{d^2 v}{dx^2}\left[\int_{A_1} E_1 y^2 dA + \int_{A_2} E_2 y^2 dA + \cdot + \cdot + \int_{A_n} E_n y^2 dA\right]$$

$$\boxed{\boldsymbol{M}_z = \frac{d^2 v}{dx^2}\left[\sum_{j=1}^{n} E_j (I_{zz})_j\right]} \qquad \boxed{(\sigma_{xx})_i = -\left[\frac{E_i y \boldsymbol{M}_z}{\sum_{j=1}^{n} E_j (I_{zz})_j}\right]}$$

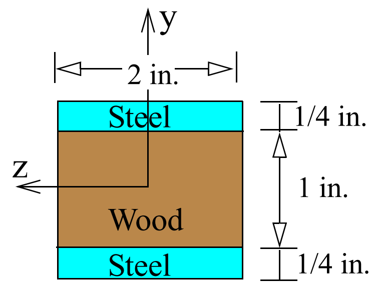
- Bending normal strain varies linearly across the cross section in the y direction.
- Bending normal strain at a cross section is maximum at a point that is furthest away from the neutral axis.
- Bending normal stress varies linearly in each material with slopes that depend upon the modulus of elasticity E.
- There is a discontinuity in bending normal stress at junction of each material.
- The maximum bending normal stress in each material is at point on the material that is furthest from the neutral axis of a composite beam.
- Maximum bending normal stress may not be at a point that is furthest away from the neutral axis.

Example 4.8

$$\varepsilon_{xx} = -200y\ \mu \qquad E_{steel} = 30,000 \text{ ksi} \qquad E_{wood} = 8,000 \text{ ksi}$$



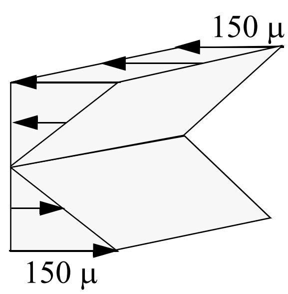
Homogenous cross section



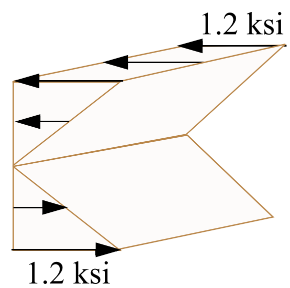
Laminated cross section

Homogenous

Strain Distribution

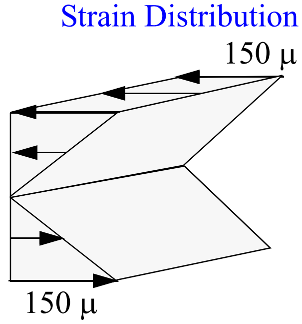


Stress Distribution

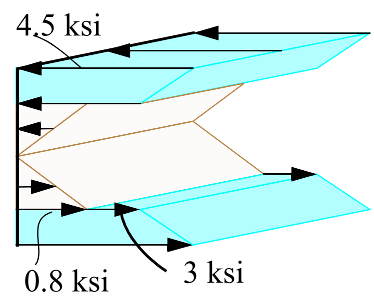


Laminated

Strain Distribution



Stress Distribution



Bending shear stress in composite beams

Equilibrium equation

$$\tau_{sx} t = -\frac{dN_s}{dx} = -\frac{d}{dx}\int_{A_s} \sigma_{xx} dA$$

$$\tau_{sx} t = -\frac{d}{dx}\int_{A_s} \frac{-Ey\boldsymbol{M}_z}{\sum_{j=1}^{n} E_j (I_{zz})_j} dA = \frac{d}{dx}\left| \frac{\boldsymbol{M}_z}{\sum_{j=1}^{n} E_j (I_{zz})_j} \int_{A_s} Ey dA \right| = \frac{d}{dx}\left| \frac{\boldsymbol{M}_z Q_{comp}}{\sum_{j=1}^{n} E_j (I_{zz})_j} \right|$$

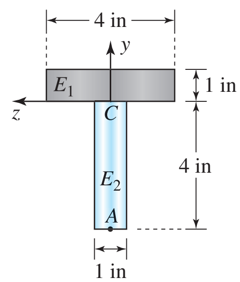
$$Q_{comp} = \int_{A_s} Ey dA$$

$$\boxed{\tau_{sx} = \tau_{xs} = -\frac{Q_{comp}\boldsymbol{V}_y}{\left[\sum_{j=1}^{n} E_j (I_{zz})_j\right] t}}$$

$$\boxed{Q_{comp} = \sum_{j=1}^{n_s} E_j (Q_z)_j}$$

- Q_{comp} is maximum at the neutral axis.
- Q_{comp} is continuous at all points on a cross section including the junctions of each material.
- The maximum bending shear stress in each material is at point closest to the neutral axis.
- The maximum bending shear stress at a cross section is at the neutral axis.
- The bending shear stress is continuous at all points on a cross section including the junctions of each material.

C4.4 The cross-section of a composite beam with a coordinate system that has an origin at C is shown. The normal strain at point A due to bending about the z-axis is $\varepsilon_{xx} = -200\ \mu$, and the modulus of elasticity of the materials are $E_1 = 8000\ ksi$ and $E_2 = 2000\ ksi$ (a) Plot the stress distribution across the cross-section. (b) Determine the maximum bending normal stress in each material. (c) Determine the equivalent internal bending moment by using $M_z = -\int_A y\sigma_{xx}dA$. (d) Determine the equivalent internal bending moment $\boldsymbol{M}_z$ by using $(\sigma_{xx})_i = \dfrac{-E_i y \boldsymbol{M}_z}{\left(\sum_{j=1}^{n} E_j (I_{zz})_j\right)}$.



C4.5 To improve the load carrying capacity of a wooden beam ($E_W = 2000$ ksi) a steel strip ($E_s = 30{,}000$ ksi) is securely fastened to it as shown. Determine (a) the maximum intensity of the load w, if the allowable bending normal stresses in steel and wood are 20 ksi, and 4 ksi.respectively. (b) the magnitude of the maximum shear stress steel and wood corresponding to the load in part (a).