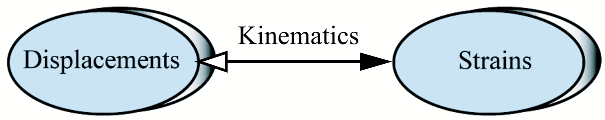
Strain

- Relating strains to displacements is a problem in geometry.



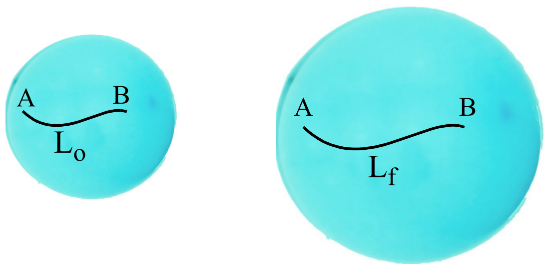
Learning objectives

- Understand the concept of strain.
- Understand the use of approximate deformed shape for calculating strains from displacements.

Preliminary Definitions

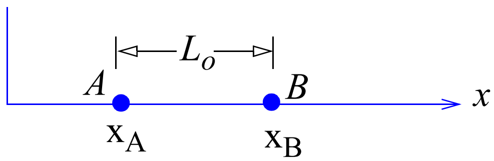
- The total movement of a point with respect to a fixed reference coordinates is called *displacement*.
- The relative movement of a point with respect to another point on the body is called *deformation*.
- *Lagrangian strain* is computed from deformation by using the original undeformed geometry as the reference geometry.
- *Eulerian strain* is computed from deformation by using the final deformed geometry as the reference geometry.

Average Normal Strain

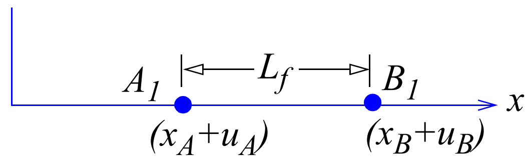


$$\varepsilon_{av} = \frac{L_f - L_o}{L_o} = \frac{\delta}{L_o}$$

- Elongations ($L_f > L_o$) result in *positive* normal strains. Contractions ($L_f < L_o$) result in *negative* normal strains.



$$L_0 = x_B - x_A$$



$$L_f = (x_B + u_B) - (x_A + u_A) = L_o + (u_B - u$$

$$\varepsilon_{av} = \frac{u_B - u_A}{x_B - x_A}$$

Units of average normal strain

- To differentiate average strain from strain at a point.
- in/in, or cm/cm, or m/m
- percentage. 0.5% is equal to a strain of 0.005
- prefix: $\mu = 10^{-6}$. 1000 μ in / in is equal to a strain 0.001 in / in

C2.1 Due to the application of the forces in Fig. C2.1, the displacement of the rigid plates in the x direction were observed as given below. Determine the axial strains in rods in sections AB, BC, and CD.

$$u_B = -1.8 \text{ mm} \qquad u_C = 0.7 \text{ mm} \qquad u_D = 3.7 \text{ mm}$$

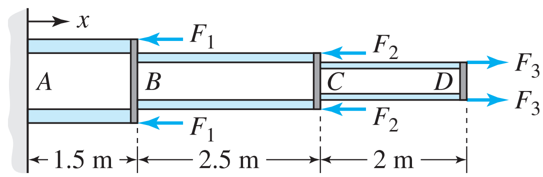


Fig. C2.1

Average shear strain

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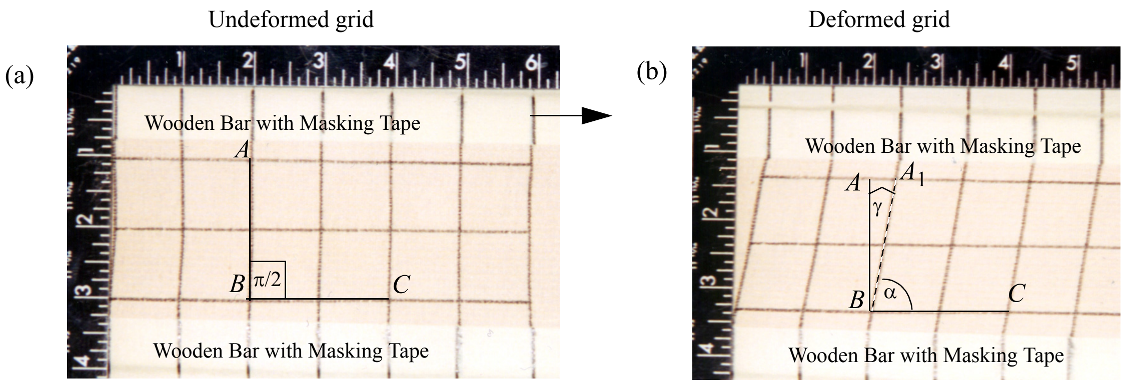


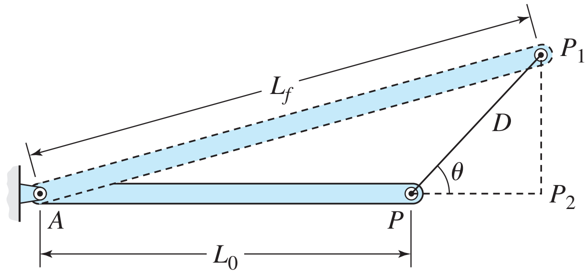
$$\gamma_{av} = \frac{\pi}{2} - \alpha$$

- Decreases in the angle $(\alpha < \pi / 2)$ result in *positive* shear strain. Increase in the angle $(\alpha > \pi / 2)$ result in *negative* shear strain

Units of average shear strain

- To differentiate average strain from strain at a point.
- rad
- prefix: $\mu = 10^{-6}$. 1000 μ rad is equal to a strain 0.001 rad

Small Strain Approximation



$$L_f = \sqrt{L_o^2 + D^2 + 2L_o D\cos\theta}$$

$$L_f = L_o\sqrt{1 + \left(\frac{D}{L_o}\right)^2 + 2\left(\frac{D}{L_o}\right)\cos\theta}$$

$$\varepsilon = \frac{L_f - L_o}{L_o} = \sqrt{1 + \left(\frac{D}{L_o}\right)^2 + 2\left(\frac{D}{L_o}\right)\cos\theta} - 1 \qquad \textbf{2.5}$$

$$\varepsilon_{small} = \frac{D\cos\theta}{L_o} \qquad \textbf{2.6}$$

ε_{small} Eq. 2.6	ε Eq. 2.5	% error
1.0	1.23607	19.1
0.5	0.58114	14.0
0.1	0.10454	4.3
0.05	0.005119	2.32
0.01	0.01005	0.49
0.005	0.00501	0.25

- Small-strain approximation may be used for strains less than 0.01
- Small normal strains are calculated by using the deformation component in the original direction of the line element regardless of the orientation of the deformed line element.
- In small shear strain (γ) calculations the following approximation may be used for the trigonometric functions: $\tan\gamma \approx \gamma$ $\sin\gamma \approx \gamma$ $\cos\gamma \approx 1$
- Small-strain calculations result in linear deformation analysis.
- Drawing approximate deformed shape is very important in analysis of small strains.

C2.2 A thin triangular plate ABC forms a right angle at point A. During deformation, point A moves vertically down by δ_A. Determine the average shear strain at point A.

$$\delta_A = 0.008 \quad in$$

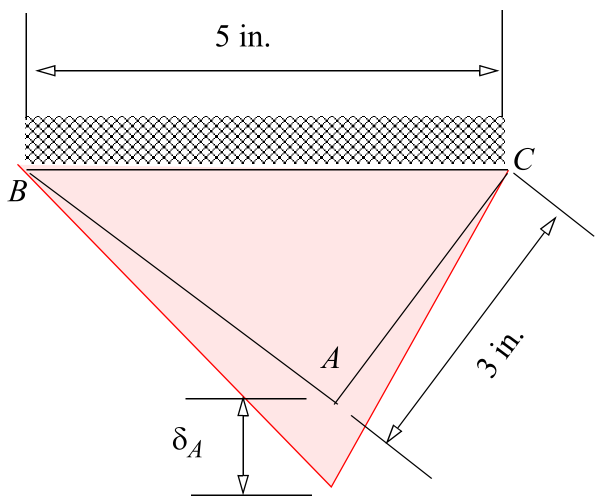


Fig. C2.2

C2.3 A roller at P slides in a slot as shown. Determine the deformation in bar AP and bar BP by using small strain approximation.

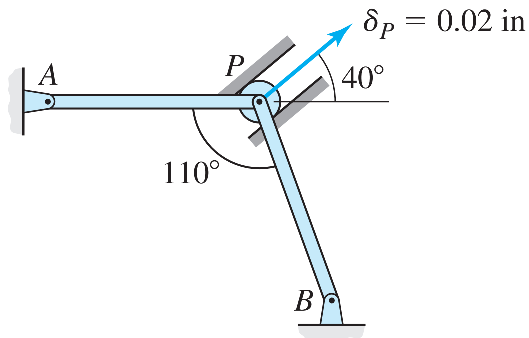
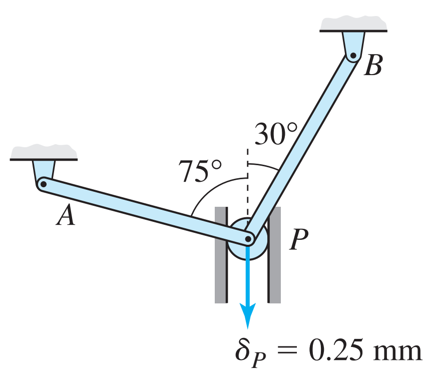


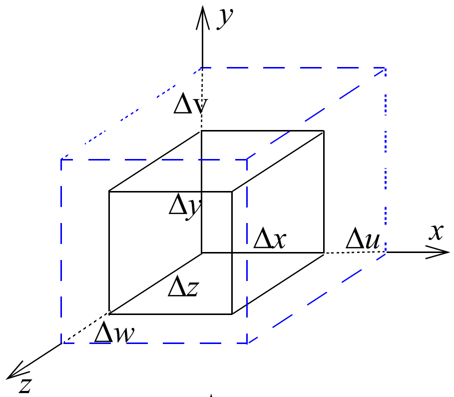
Fig. C2.3

Class Problem 1

Draw an approximate exaggerated deformed shape.
Using small strain approximation write equations relating δ_{AP} and δ_{BP} to δ_P.



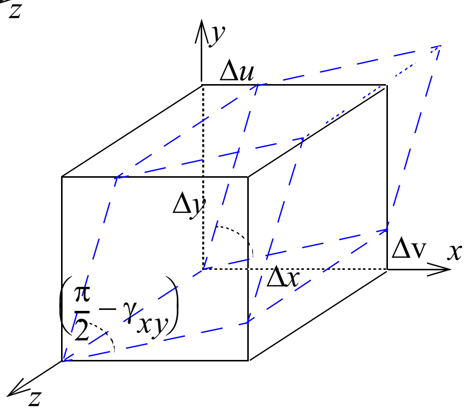
Strain Components



$$\varepsilon_{xx} = \frac{\Delta u}{\Delta x}$$

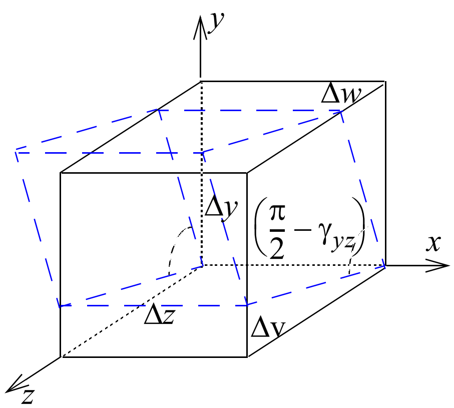
$$\varepsilon_{yy} = \frac{\Delta v}{\Delta y}$$

$$\varepsilon_{zz} = \frac{\Delta w}{\Delta z}$$



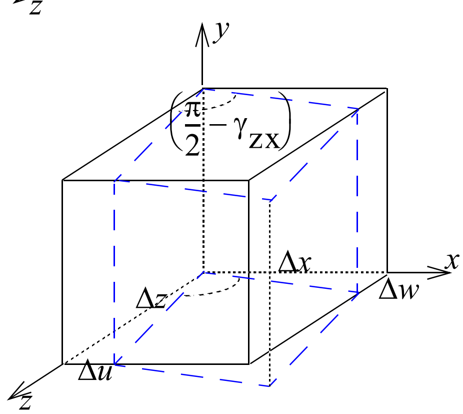
$$\gamma_{xy} = \frac{\Delta u}{\Delta y} + \frac{\Delta v}{\Delta x}$$

$$\gamma_{yx} = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y} = \gamma_{xy}$$



$$\gamma_{yz} = \frac{\Delta v}{\Delta z} + \frac{\Delta w}{\Delta y}$$

$$\gamma_{zy} = \frac{\Delta w}{\Delta y} + \frac{\Delta v}{\Delta z} = \gamma_{yz}$$



$$\gamma_{zx} = \frac{\Delta w}{\Delta x} + \frac{\Delta u}{\Delta z}$$

$$\gamma_{xz} = \frac{\Delta u}{\Delta z} + \frac{\Delta w}{\Delta x} = \gamma_{zx}$$

Engineering Strain

Engineering strain matrix

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_{zz} \end{bmatrix}$$

Plane strain matrix

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Strain at a point

$$\varepsilon_{xx} = \lim_{\Delta x \to 0} \left(\frac{\Delta u}{\Delta x}\right) = \frac{\partial u}{\partial x}$$

$$\gamma_{xy} = \gamma_{yx} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left(\frac{\Delta u}{\Delta y} + \frac{\Delta v}{\Delta x}\right) = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \gamma_{yz} = \gamma_{zy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

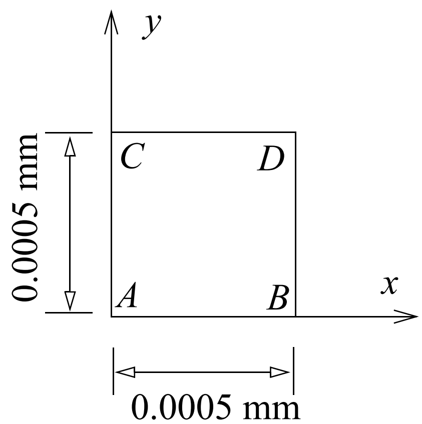
$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \qquad \gamma_{zx} = \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

- tensor normal strains = engineering normal strains
- tensor shear strains = (engineering shear strains)/ 2

Strain at a Point on a Line

$$\varepsilon_{xx} = \frac{du(x)}{dx}$$

C2.4 Displacements u and v in the x and y directions respectively were measured by Moire Interferometry method at many points on a body. Displacements of four points on a body are given below. Determine the average values of strain components ε_{xx}, ε_{yy}, *and* γ_{xy} at point A shown in Fig. C2.4.



$$u_A = 0 \qquad v_A = 0$$
$$u_B = 0.625\,\mu mm \qquad v_B = -0.3125\,\mu mm$$
$$u_C = -0.500\,\mu mm \qquad v_C = -0.5625\,\mu mm$$
$$u_D = 0.250\,\mu mm \qquad v_D = -1.125\,\mu mm$$

Fig. C2.4

C2.5 The axial displacement in a quadratic one-dimensional finite element is as given below.

$$u(x) = \frac{u_1}{2a^2}(x-a)(x-2a) - \frac{u_2}{a^2}(x)(x-2a) + \frac{u_3}{2a^2}(x)(x-a)$$

Determine the strain at Node 2.