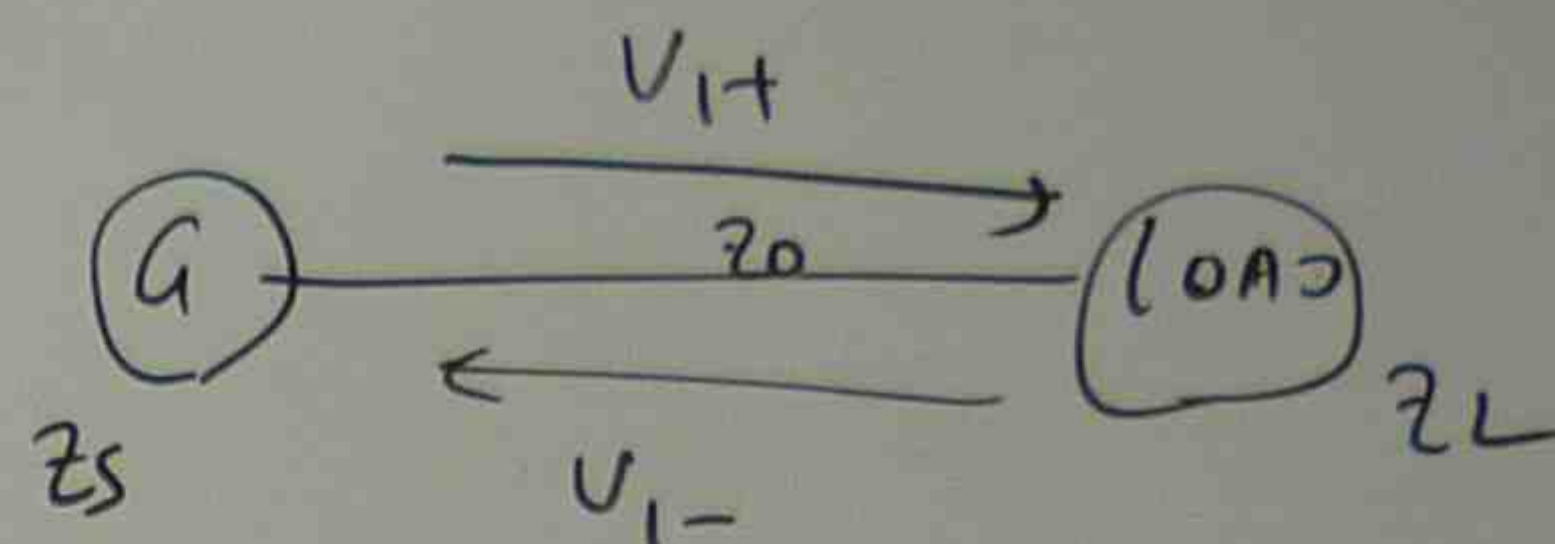
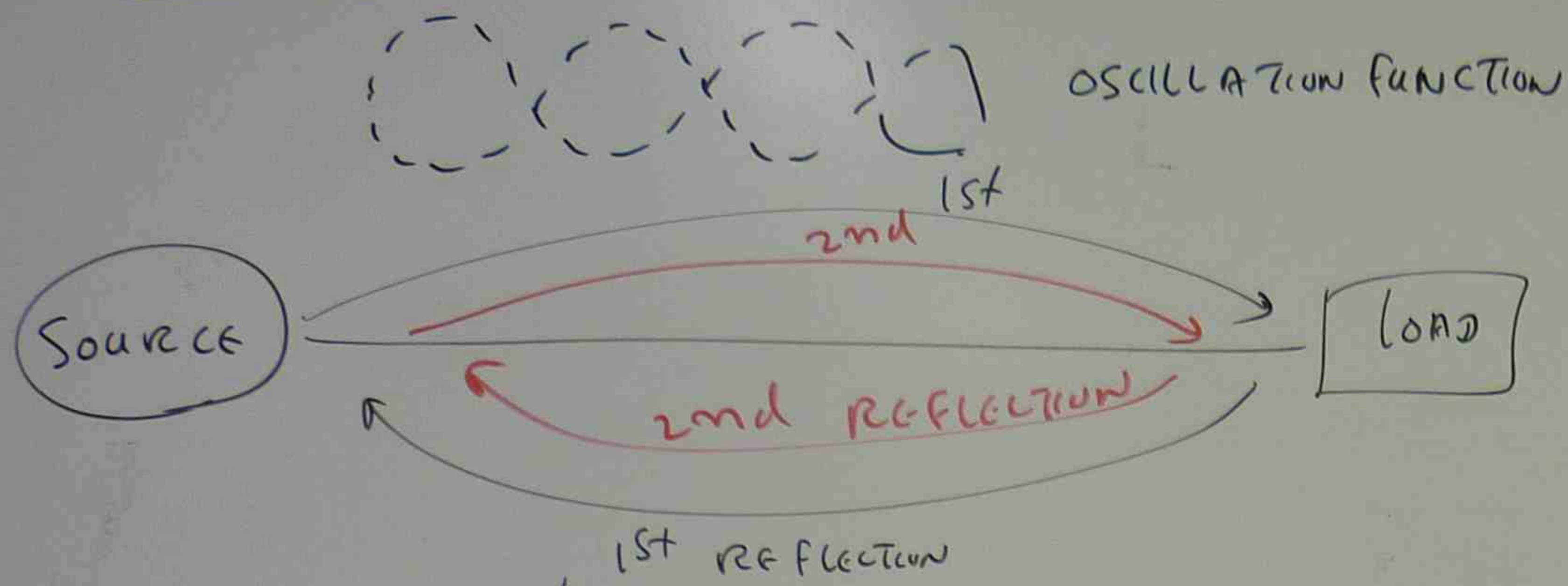


Q047



Z_L = LOAD IMPEDANCE
 Z_0 = LINE IMPEDANCE
 Z_S = SOURCE IMPEDANCE

1st REFLECTION AT LOAD

$$V_t = V_{1+} + V_{1-}$$

$$V_{1+} = V_s \frac{Z_0}{Z_0 + Z_S}$$

$$V_{1-} = V_{1+} \times \Gamma_L \leftarrow \frac{Z_L - Z_0}{Z_L + Z_0}$$

2nd REFLECTION AT SOURCE $\Rightarrow V_t = V_{1+} + V_{1-} + V_{2+}$

$$V_{2+} = V_{1-} \Gamma_S \leftarrow \frac{Z_S - Z_0}{Z_S + Z_0}$$

3rd REFLECTION AT LOAD \Rightarrow

$$V_t = V_{1+} + V_{1-} + V_{2+} + V_{2-}$$

$$V_{2-} = V_{2+} \Gamma_L \leftarrow \frac{Z_L - Z_0}{Z_L + Z_0}$$

Ex

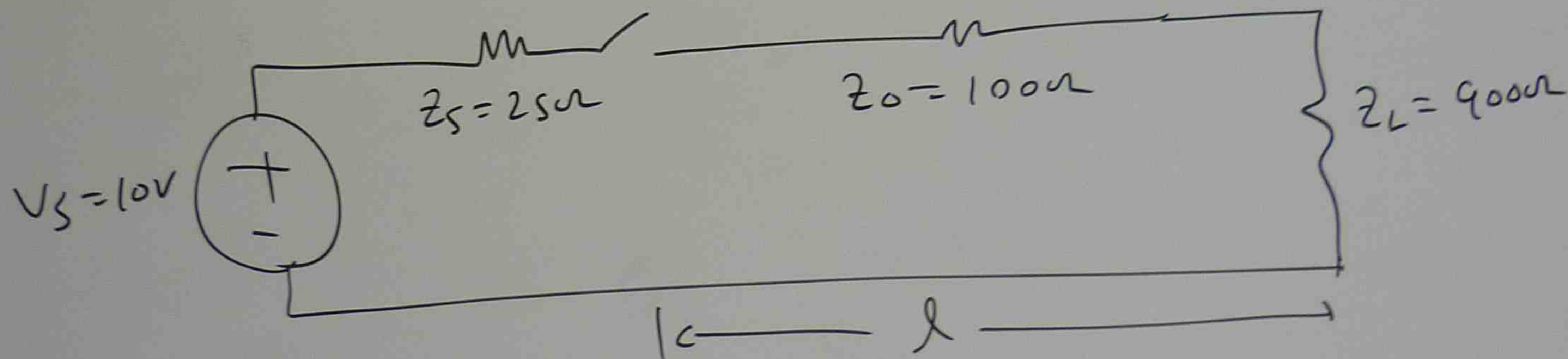
A 10V DC SOURCE WITH AN INTERNAL RESISTANCE OF 25Ω IS CONNECTED TO A TRANSMISSION LINE OF LENGTH "L" HAVING AN IMPEDANCE OF 100Ω BY SWITCH. THE TRANSMISSION LINE IS TERMINATED WITH A 900Ω RESISTOR. T = AMOUNT OF TIME REQUIRED FOR SIGNAL TO TRAVEL THE LENGTH OF THE LINE.

CALCULATE (a) THE VOLTAGE WHEN THE SWITCH IS CLOSED AT $T=0$

(b) FIRST REFLECTION AT LOAD

(c) SECOND REFLECTION AT SOURCE

(d) THIRD REFLECTION AT LOAD



(a) VOLTAGE AT $t=0$

$$V_{1+} = V_S \times \frac{Z_0}{Z_0 + Z_S}$$

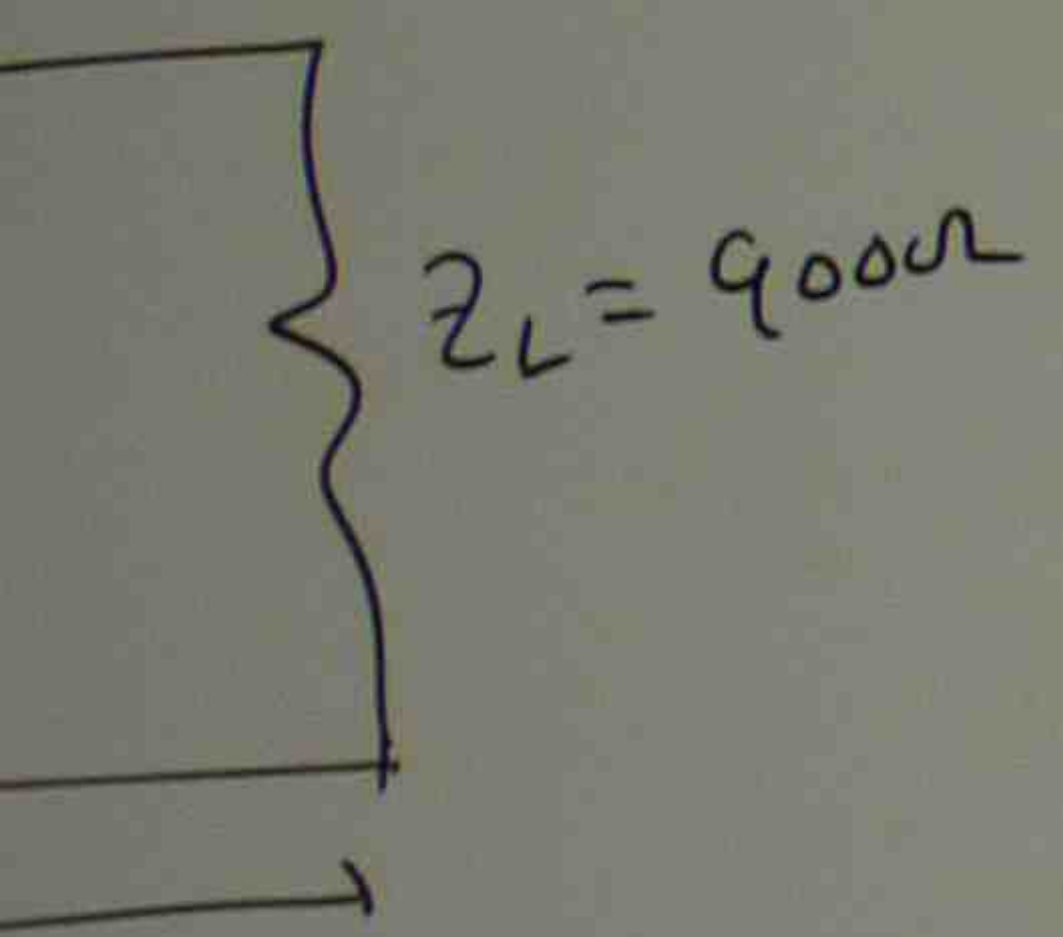
$$= 10 \times \frac{100}{100 + 25} = 8V$$

(b) $V_{1(-)}$

FIRST REFLECTION AT LOAD

(c) V_2

2 IS
 AN
 S
 TIME
 E.
 T=0



$V_S = 8V$

$$\begin{aligned}
 (b) \quad V_{1(-)} &= V_{1(+)} \Gamma_L \\
 &= 8 \times \frac{Z_L - Z_0}{Z_L + Z_0} \\
 &= 8 \times \frac{900 - 100}{900 + 100} \\
 &= 8 \times \frac{800}{1000} = 6.4V
 \end{aligned}$$

FIRST REFLECTION AT LOAD

$$\begin{aligned}
 V_L(t) &= V_{1(+)} + V_{1(-)} \\
 &= 8 + 6.4 \\
 &= 14.4V
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad V_{2+} &= V_{(-)} \times \Gamma_S \\
 &= 6.4 \times \frac{Z_S - Z_0}{Z_S + Z_0} \\
 &= 6.4 \times \frac{25 - 100}{25 + 100} \\
 &= -3.84V
 \end{aligned}$$

$$\begin{aligned}
 2^{nd} \text{ REFLECTION AT SOURCE} &= V_{1(+)} + V_{1(-)} + V_{2(+)} \\
 &= 8 + 6.4 + (-3.84) \\
 &= 10.56V
 \end{aligned}$$

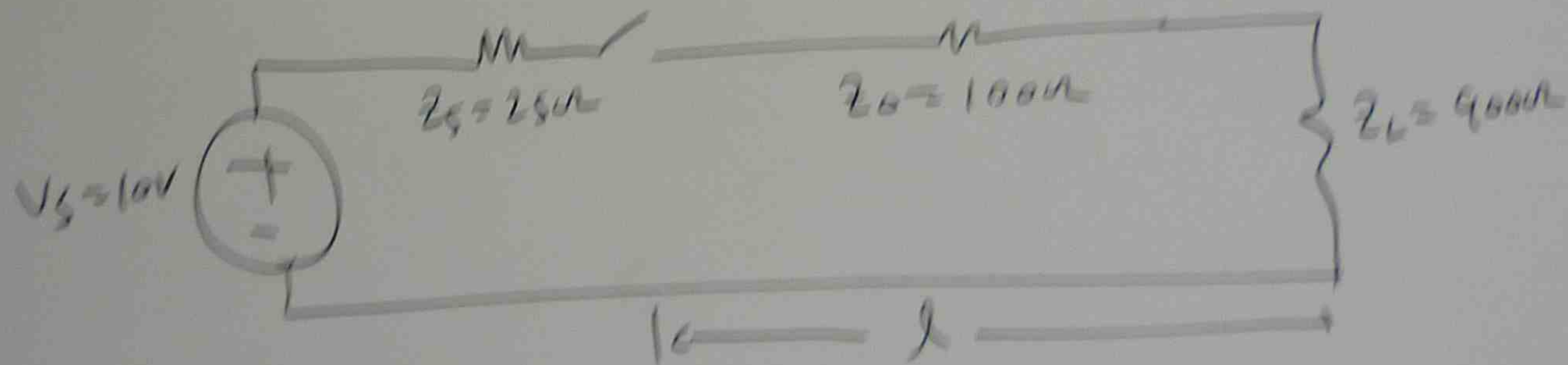
$$\begin{aligned}
 (d) \quad V_{2(-)} &= V_{2(+)} \Gamma_L \\
 &= (-3.84) \times \frac{Z_L - Z_0}{Z_L + Z_0} \\
 &= -3.84 \times \frac{900 - 100}{900 + 100} \\
 &= -3.072V
 \end{aligned}$$

$$\begin{aligned}
 3^{rd} \text{ REFLECTION AT LOAD} &= V_{1(+)} + V_{1(-)} + V_{2(+)} + V_{2(-)} \\
 &= 8 + 6.4 + (-3.84) + (-3.072) \\
 &= 7.488V
 \end{aligned}$$

mill
 R₁
 E₁
 E_{eg} =
 R_{eg} =
 E_{eg}

Ex

A 10V DC source with an internal resistance of 25Ω is connected to a transmission line of length "L" having an impedance of 100Ω by switch. The transmission line is terminated with a 900Ω resistor. T = amount of time required for signal to travel the length of the line. Calculate (a) the voltage when the switch is closed at $T=0$
 (b) first reflection at load
 (c) second reflection at source
 (d) third reflection at load



(a) voltage at $t=0$

$$V_{1+} = V_s \times \frac{Z_o}{Z_o + Z_s}$$

$$= 10 \times \frac{100}{100 + 25} = 8V$$

(b) $V_{1-} = V_{1+}$

$= 8V$

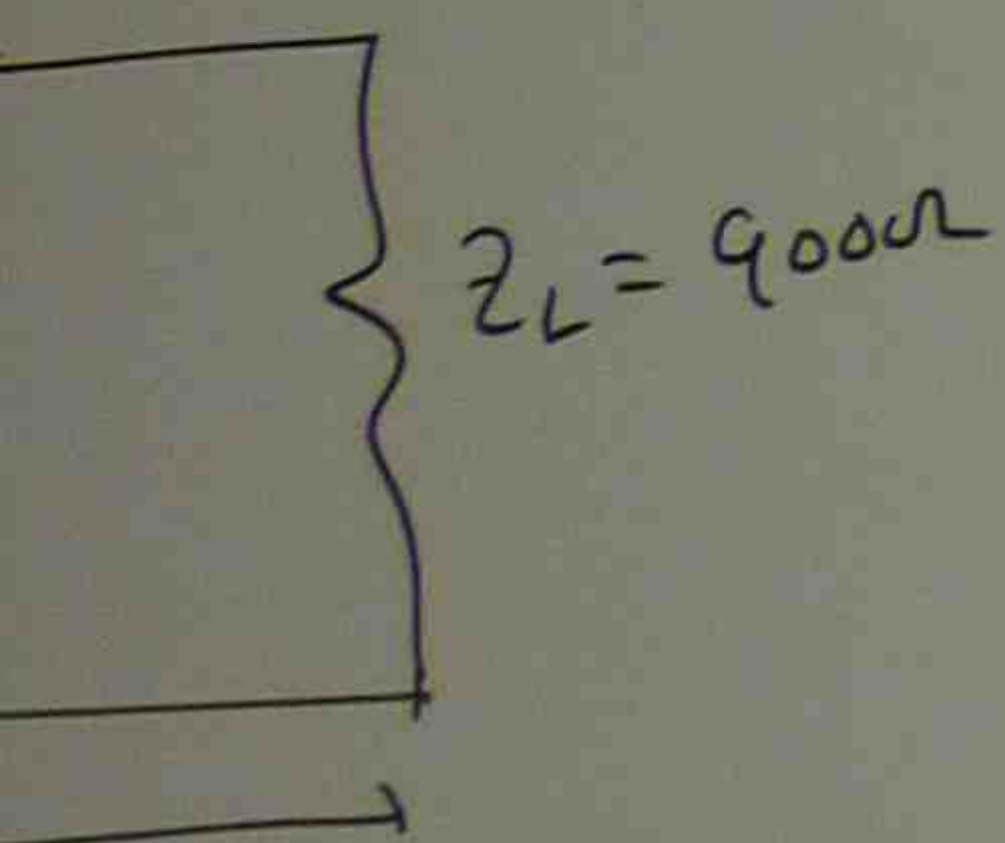
$= 8V$

$= 8V$

first reflection at load V_{1-}

(c) $V_{2+} =$

z_L is
 900
 IS
 TIME
 LINE.
 AT $T=0$



$$\frac{8}{25} = 8V$$

$$\begin{aligned}
 (b) \quad V_{1(-)} &= V_{1(+)} \Gamma_L \\
 &= 8 \times \frac{z_L - z_0}{z_L + z_0} \\
 &= 8 \times \frac{900 - 100}{900 + 100} \\
 &= 8 \times \frac{800}{1000} = 6.4V
 \end{aligned}$$

FIRST REFLECTION AT LOAD

$$\begin{aligned}
 V_L(t) &= V_{1(+)} + V_{1(-)} \\
 &= 8 + 6.4 \\
 &= 14.4V
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad V_{2+} &= V_{(-)} \times \Gamma_S \\
 &= 6.4 \times \frac{z_S - z_0}{z_S + z_0} \\
 &= 6.4 \times \frac{25 - 100}{25 + 100} \\
 &= -3.84V
 \end{aligned}$$

2nd REFLECTION AT SOURCE

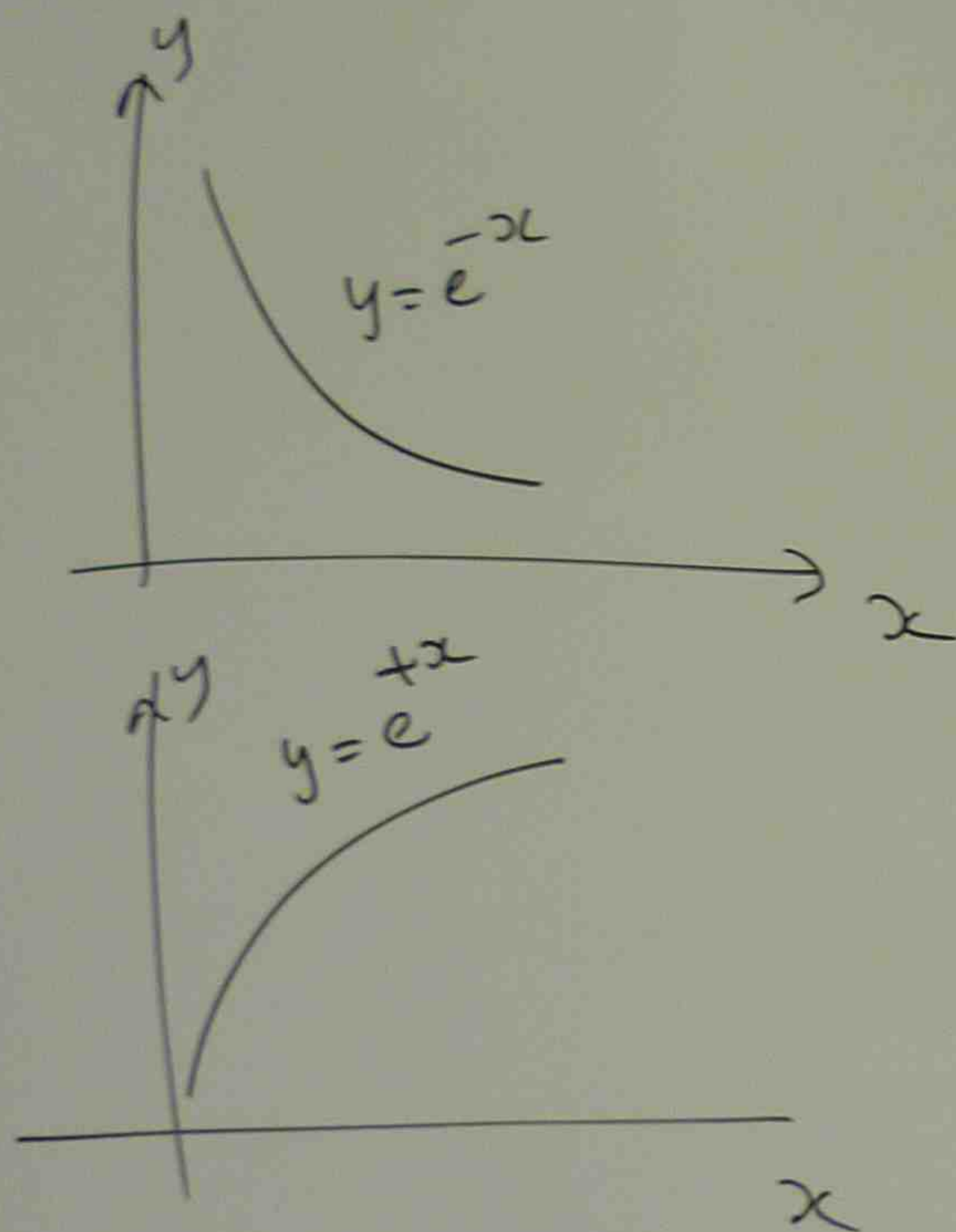
$$\begin{aligned}
 &= V_{1(+)} + V_{1(-)} + V_{2(+)} \\
 &= 8 + 6.4 + (-3.84) \\
 &= 10.56V
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad V_{2(-)} &= V_{2(+)} \Gamma_L \\
 &= (-3.84) \times \frac{z_L - z_0}{z_L + z_0} \\
 &= -3.84 \times \frac{900 - 100}{900 + 100} \\
 &= -3.072V
 \end{aligned}$$

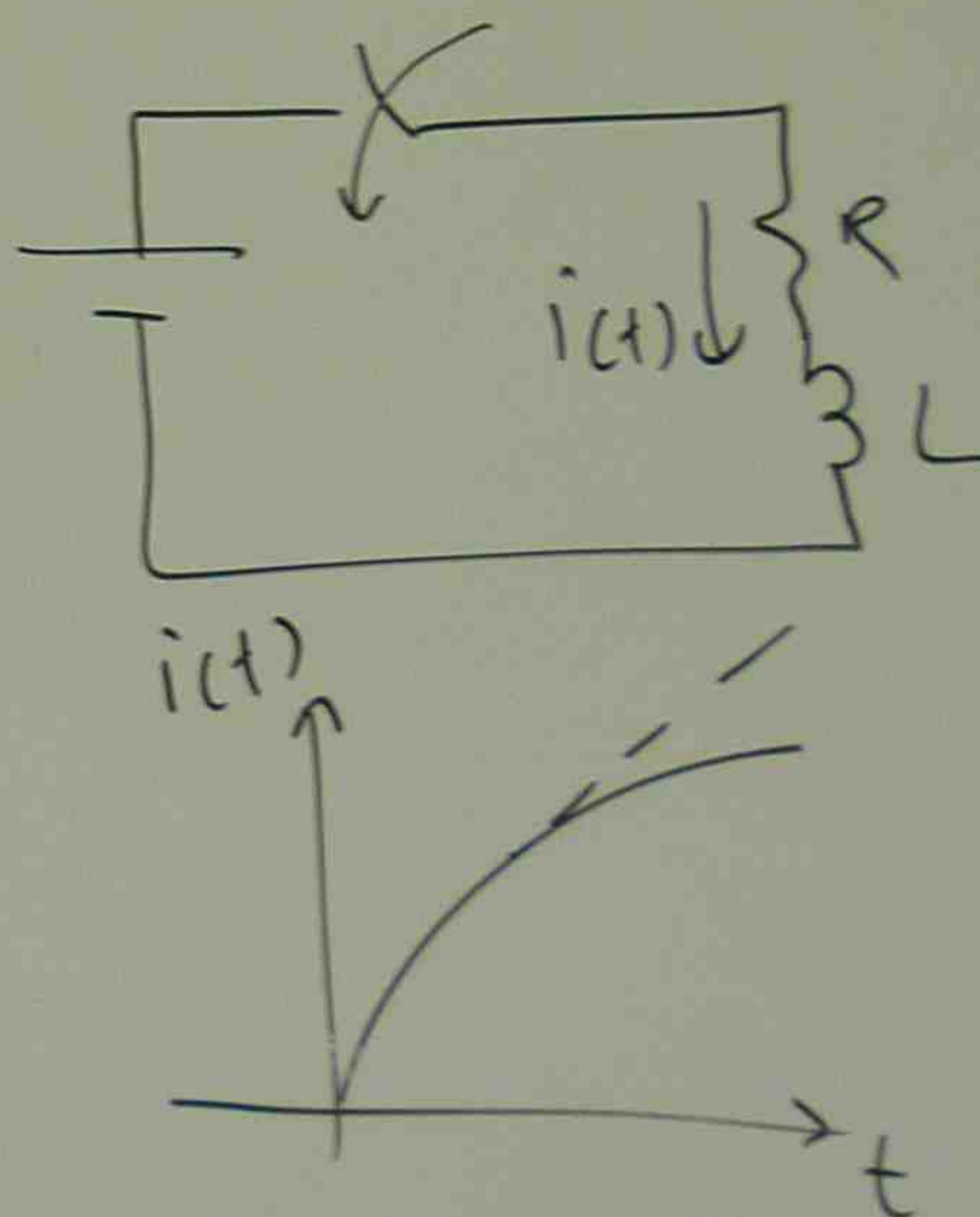
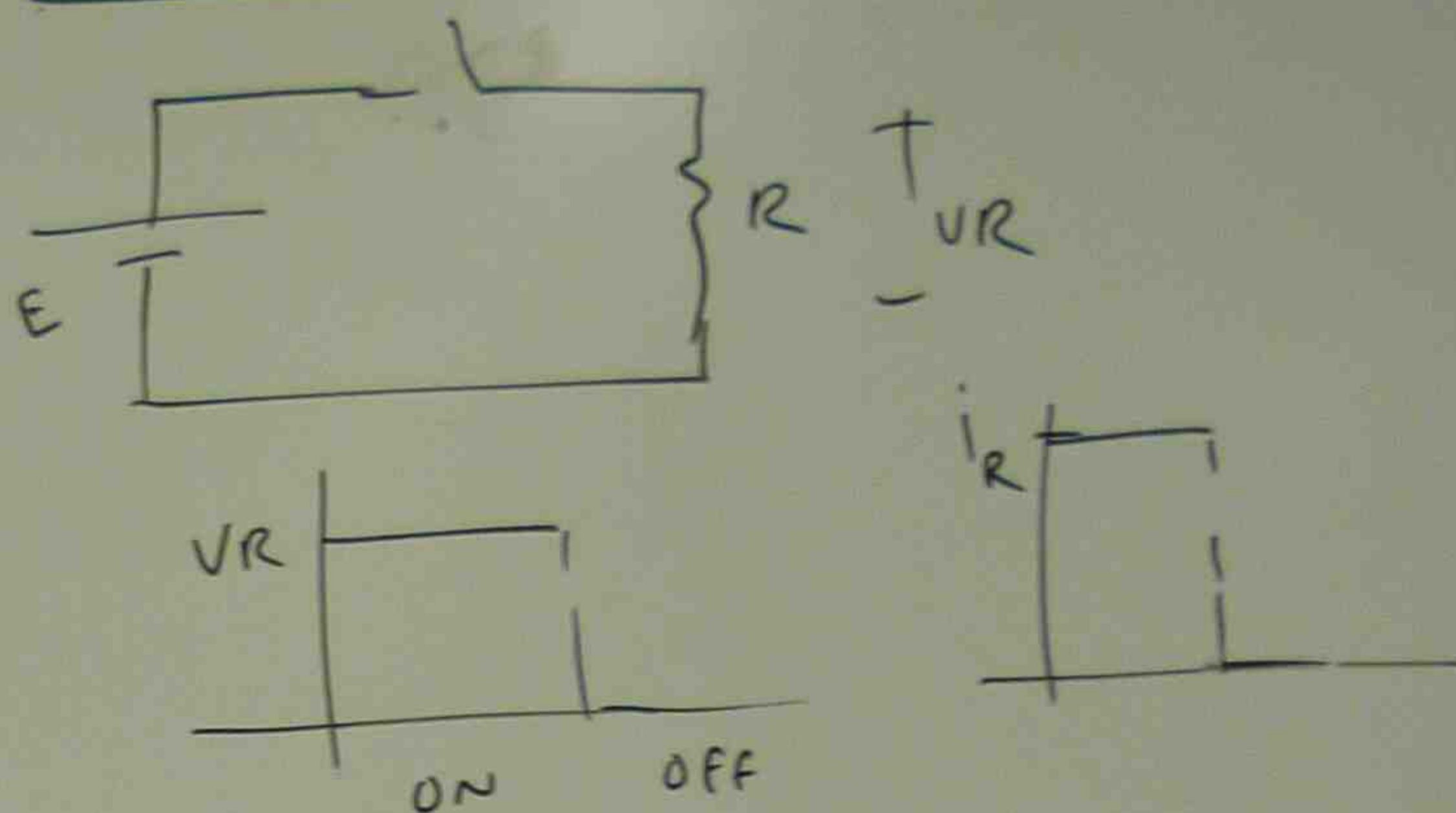
3rd REFLECTION AT LOAD

$$\begin{aligned}
 &= V_{1(+)} + V_{1(-)} + V_{2(+)} + V_{2(-)} \\
 &= 8 + 6.4 + (-3.84) + (-3.072) \\
 &= 7.488V
 \end{aligned}$$

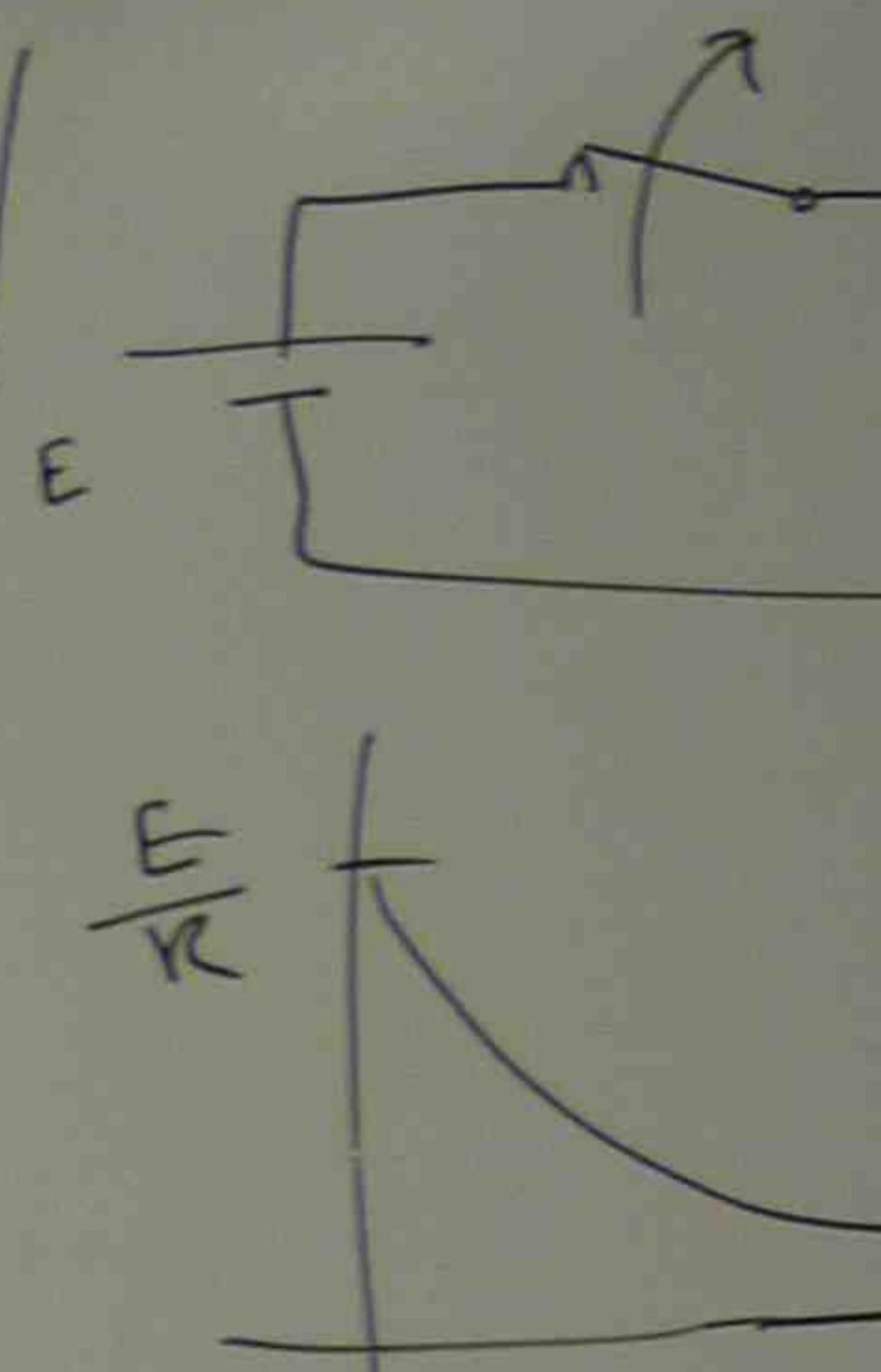
APPLICATION OF EXPONENTIAL FUNCTION IN POWER ENGINEERING CALCULATIONS



PURE RESISTANCE



$$i(t) = \frac{V}{R} \left(1 - e^{-\frac{Rt}{L}} \right)$$

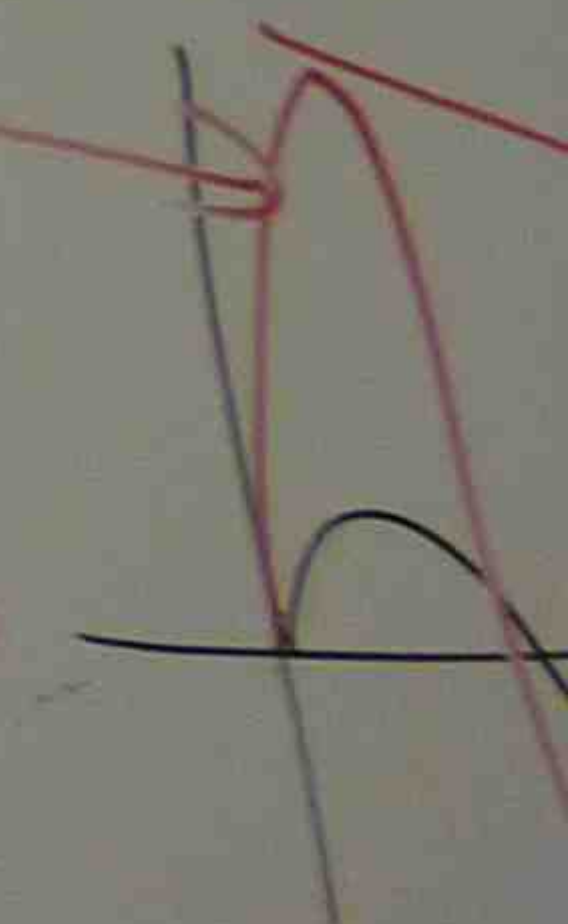


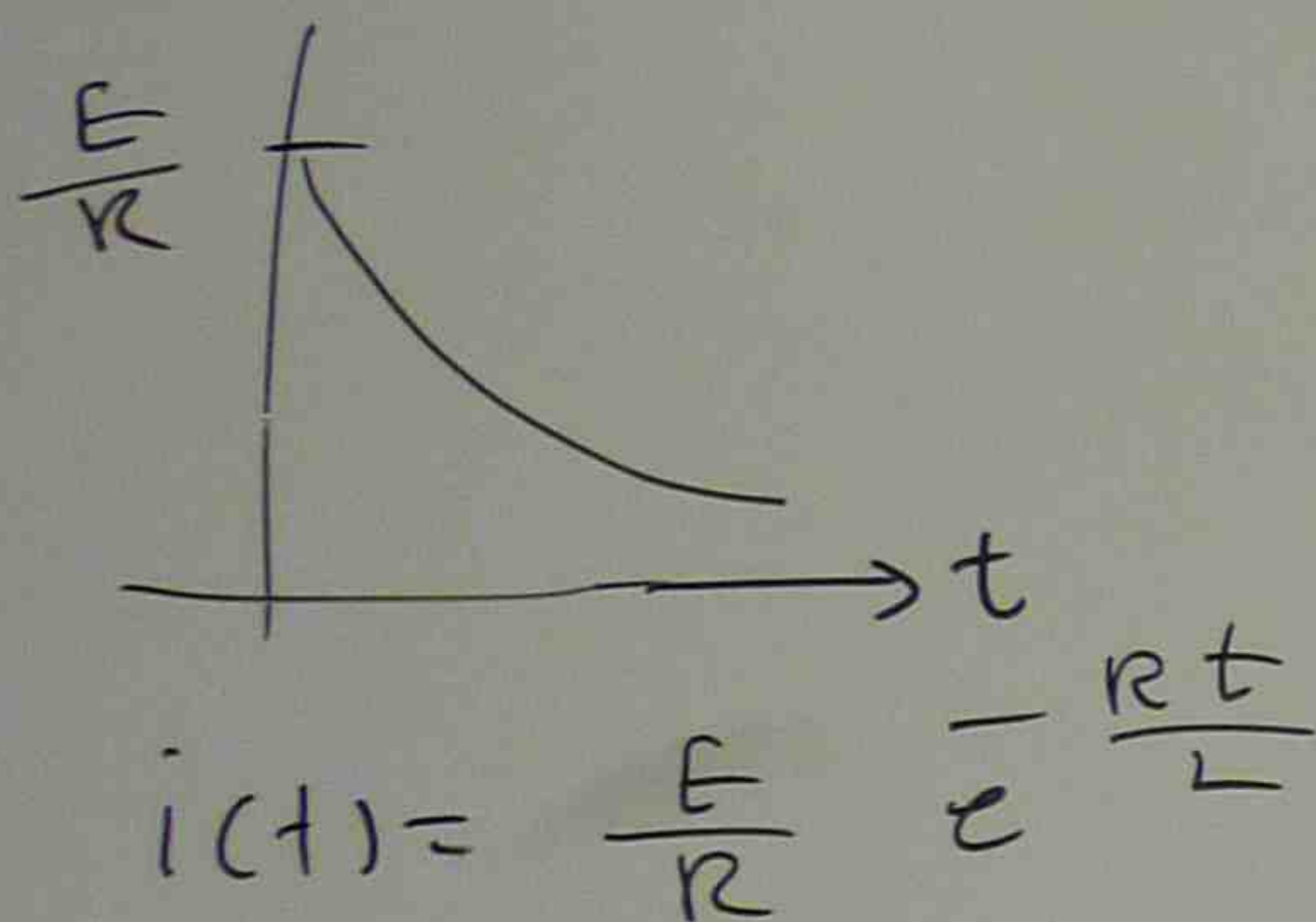
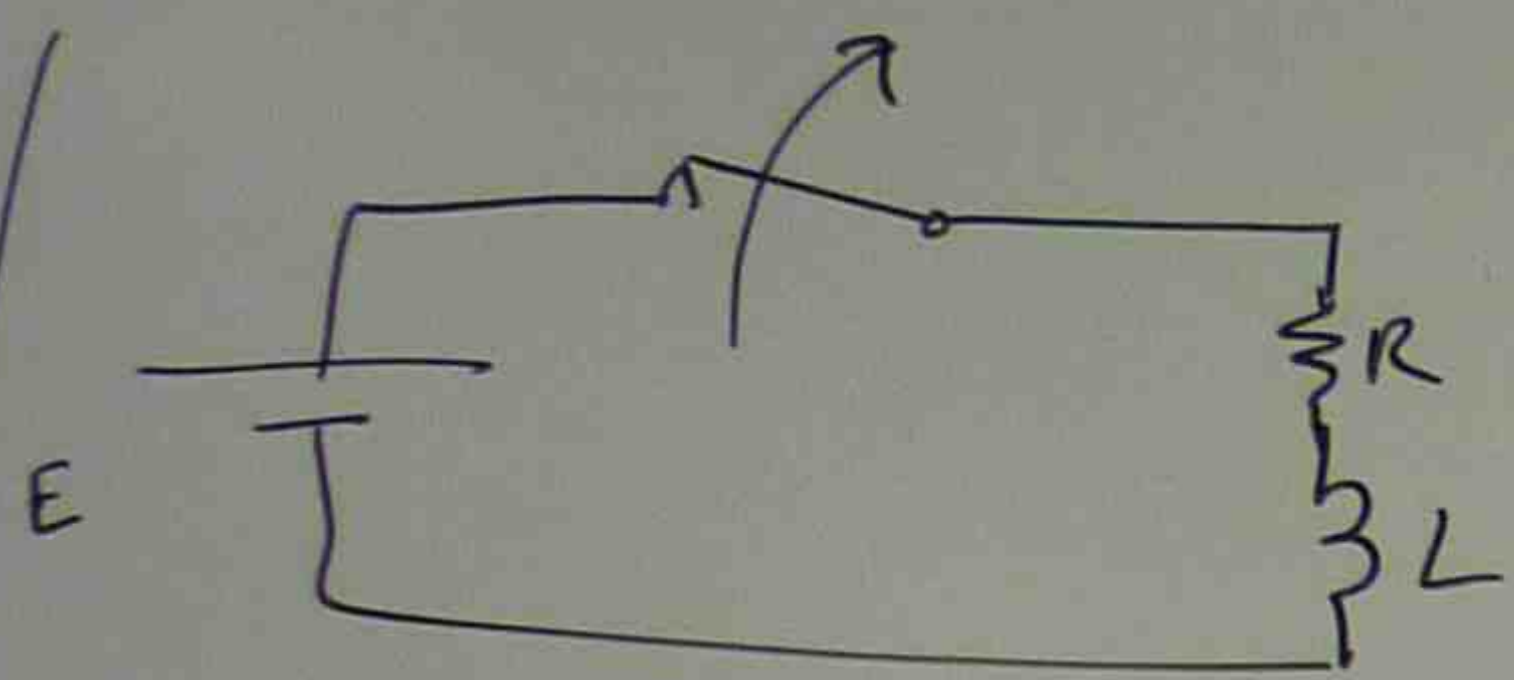
$$i(t) = \frac{E}{R}$$

TRANSIENT
USES EXP

POWER LINE

INITIAL
STATE
FAULT
CURRENT

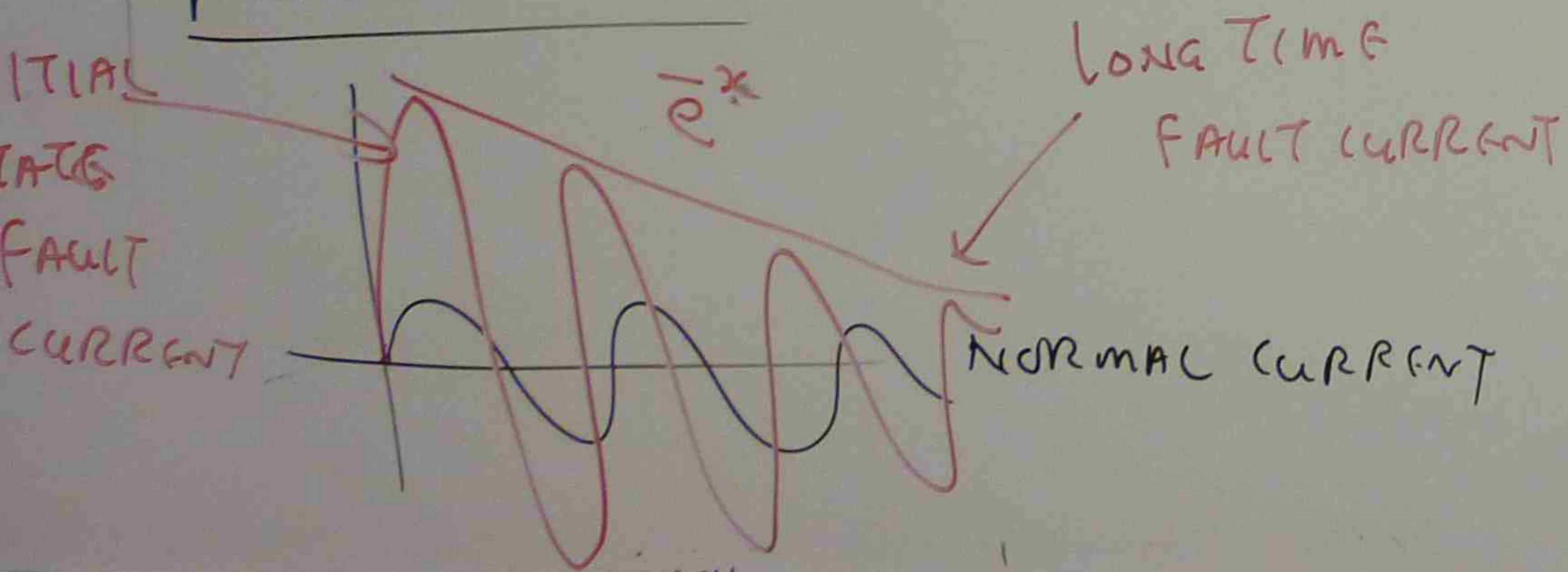




TRANSIENT CIRCUIT CALCULATION

USES EXPONENTIAL FUNCTION

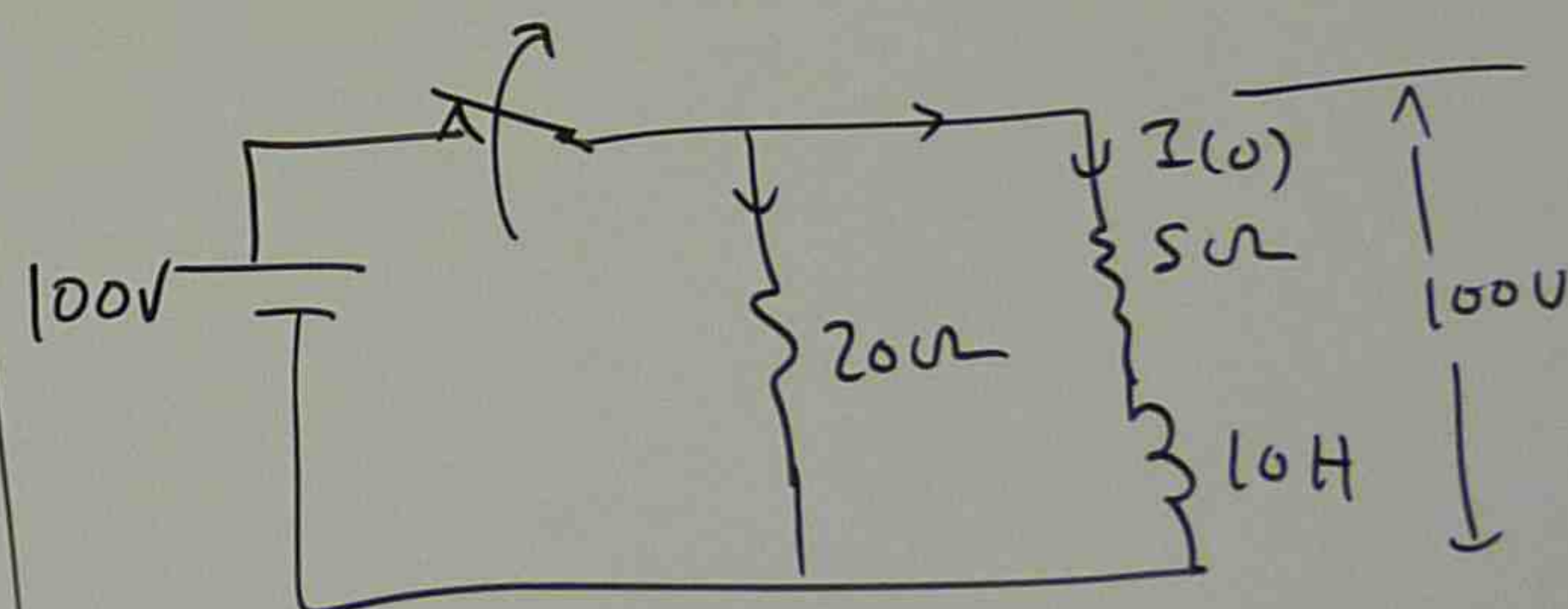
POWER LINE FAULT



EX

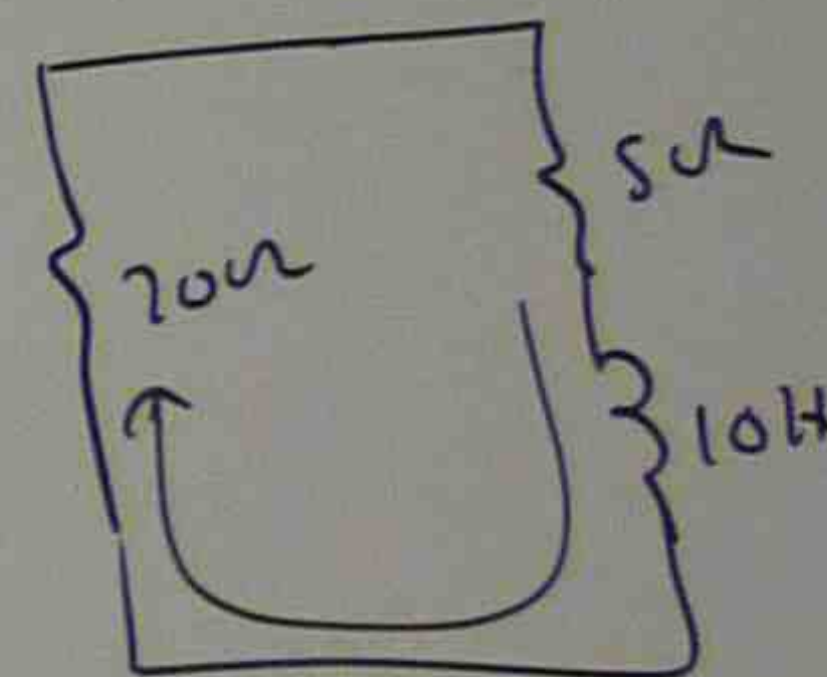
A COIL OF 10 H INDUCTANCE AND 5 OHM RESISTANCE IS CONNECTED IN PARALLEL WITH A 20 OHM RESISTOR ACROSS A 100 V DC SUPPLY WHICH IS SUDDENLY DISCONNECTED.

- FIND (a) INITIAL RATE OF CHANGE OF CURRENT AFTER SWITCHING
 (b) THE VOLTAGE ACROSS 20 OHM RESISTOR INITIALLY AFTER 0.3 S
 (c) THE VOLTAGE ACROSS THE SWITCH CONTACTS AT THE INSTANCE OF SEPARATION
 (d) THE RATE AT WHICH THE COIL IS LOSING STORED ENERGY 0.3 SEC AFTER SWITCHING.



$$I(0) = \frac{100}{5} = 20A$$

OFF THE SWITCH



$I(t)$

$R =$

$L =$

$I(t)$

20A

(a)

$\frac{dI}{dt}$

RESISTANCE IS
ACROSS A
CTED.

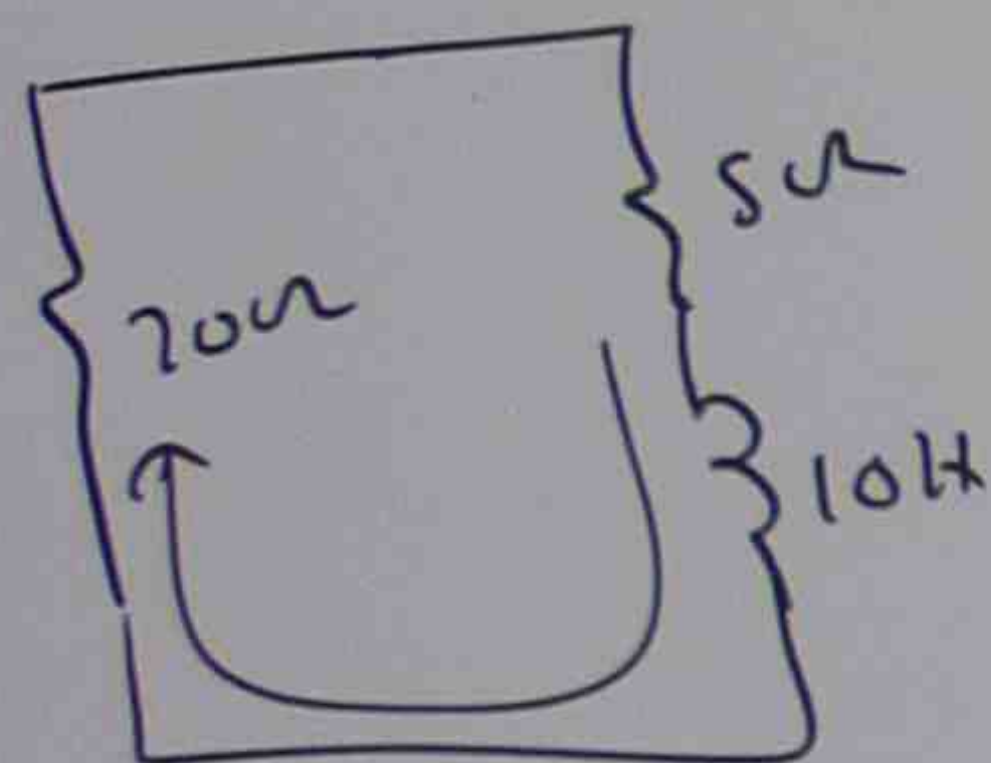
AFTER SWITCHING
INITIALLY

CTS AT THE

OSING STORED

HING.

OFF THE SWITCH



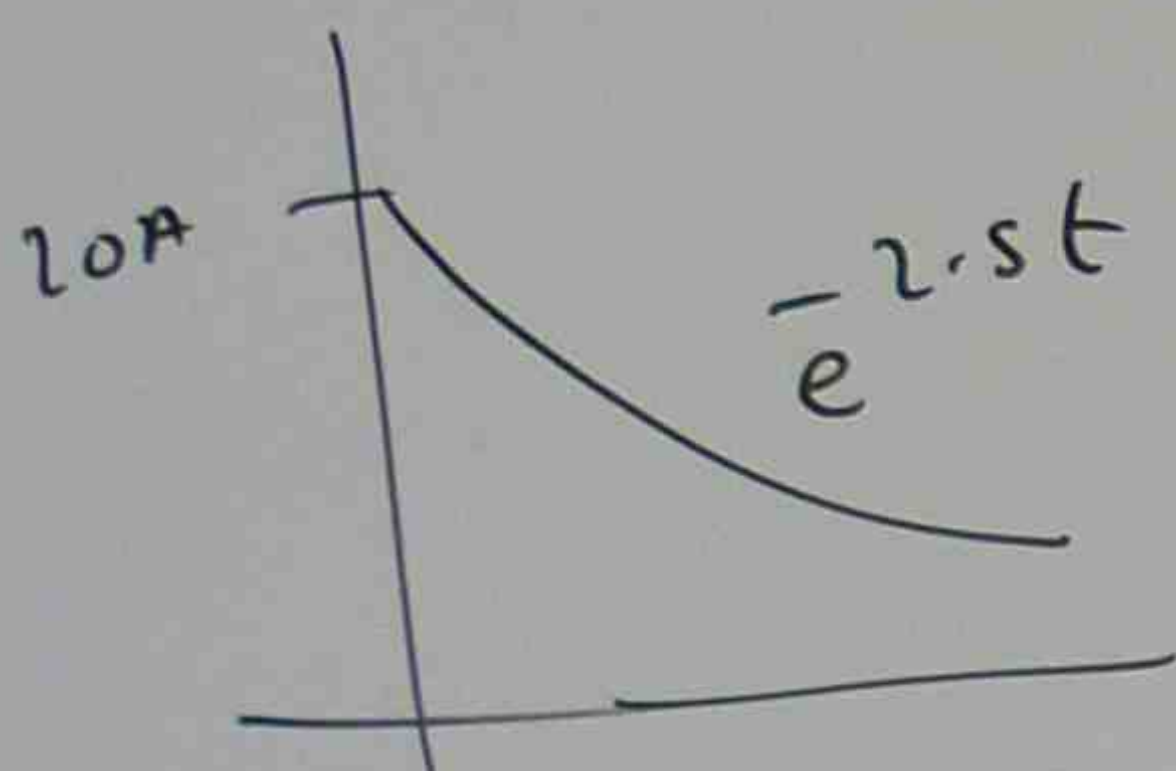
$$I(t) = I(0) e^{-\frac{Rt}{L}}$$

$$R = 20 + 5 = 25 \Omega$$

$$L = 10H$$

$$I(t) = 20 e^{-\frac{25t}{10}}$$

$$= 20 e^{-2.5t}$$



(a)

$$\frac{dI(t)}{dt} = \frac{d}{dt} (20 e^{-2.5t})$$

$$= 20 (-2.5) e^{-2.5t}$$

$$= -50 e^{-2.5t}$$

(b)

$$V_{20\Omega}(t) = I(t) \times 20 \Omega$$

$$= 20 e^{-2.5t} \times 20$$

$$= 400 e^{-2.5t}$$

$$V_{20\Omega}(0.3) = 400 \times e^{-2.5 \times 0.3}$$

$$= 400 \times e^{-0.75}$$

$$= 188V$$



$$V_{20\Omega}(0) = 20 \times 20 = 400V$$

$$(-100) + V_{sw} + (-400) = 0$$

$$V_{sw} = 500V$$

(d)

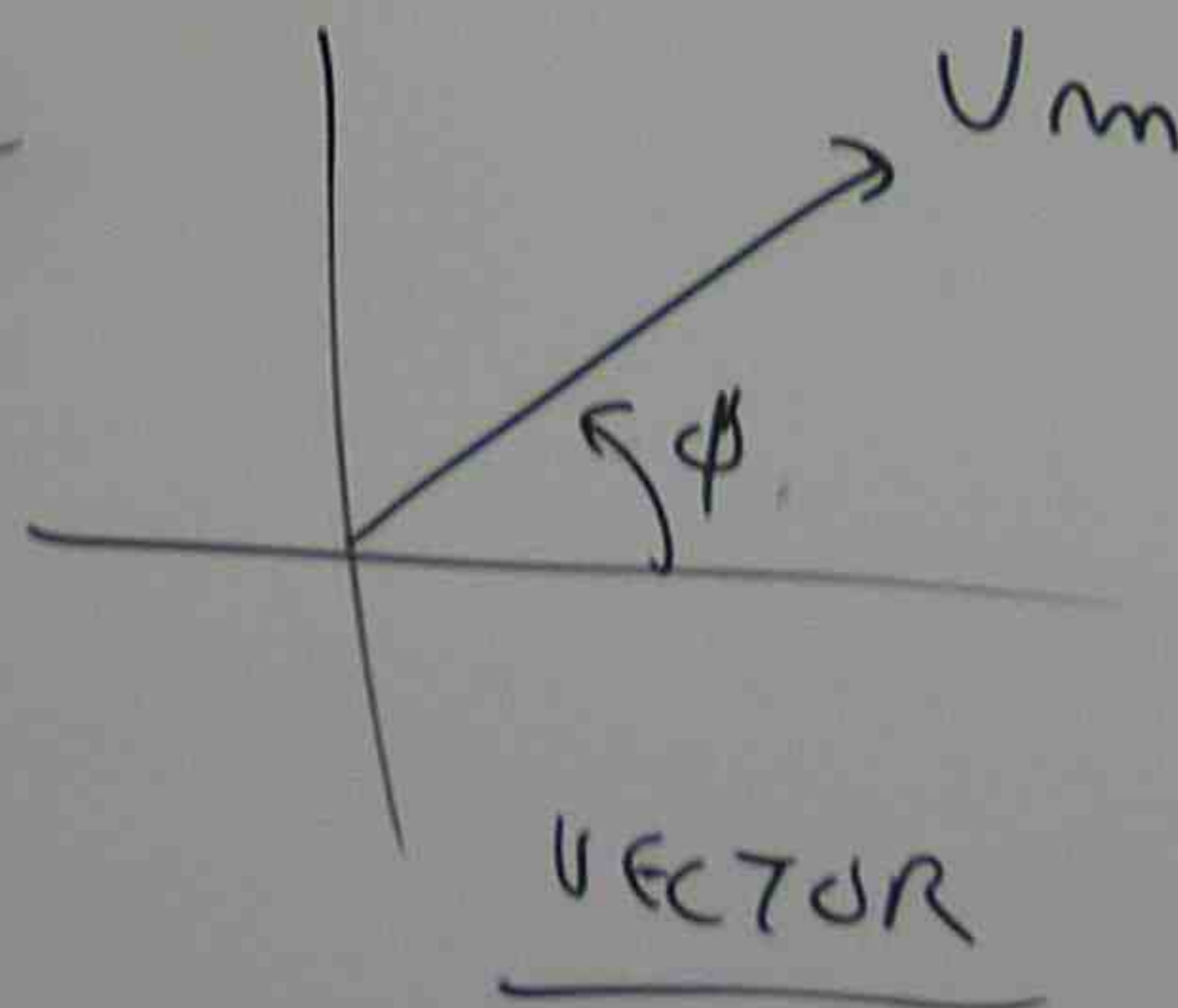
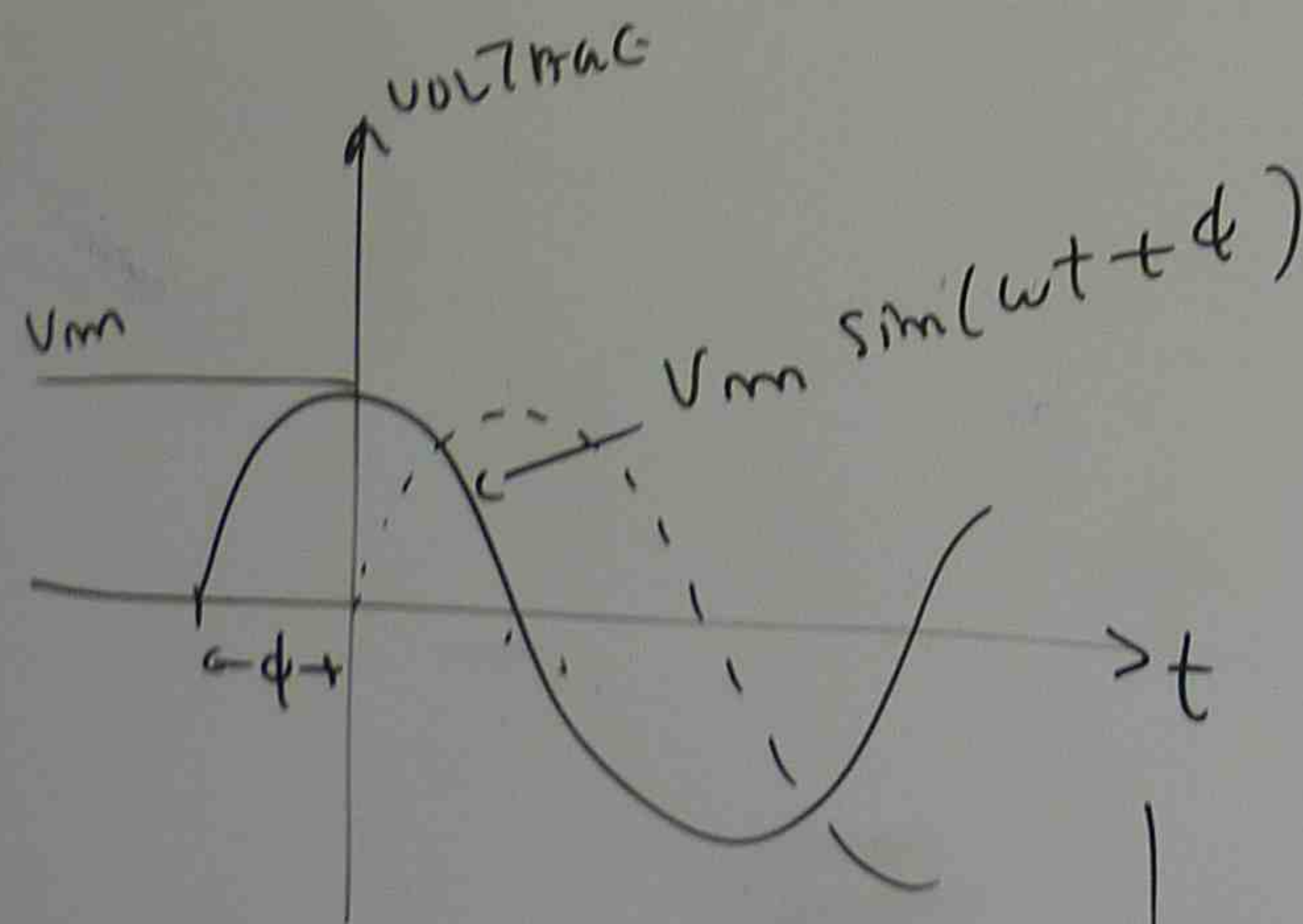
$$\text{Stored Energy} = L \frac{di(t)}{dt} \times i(t)$$

$$10 \times (-50 e^{-2.5t}) \times 20 e^{-2.5t} = -10000 e^{-5t}$$

TRANSIENT / EXPONENTIAL FUNCTION IN AC CIRCUITS

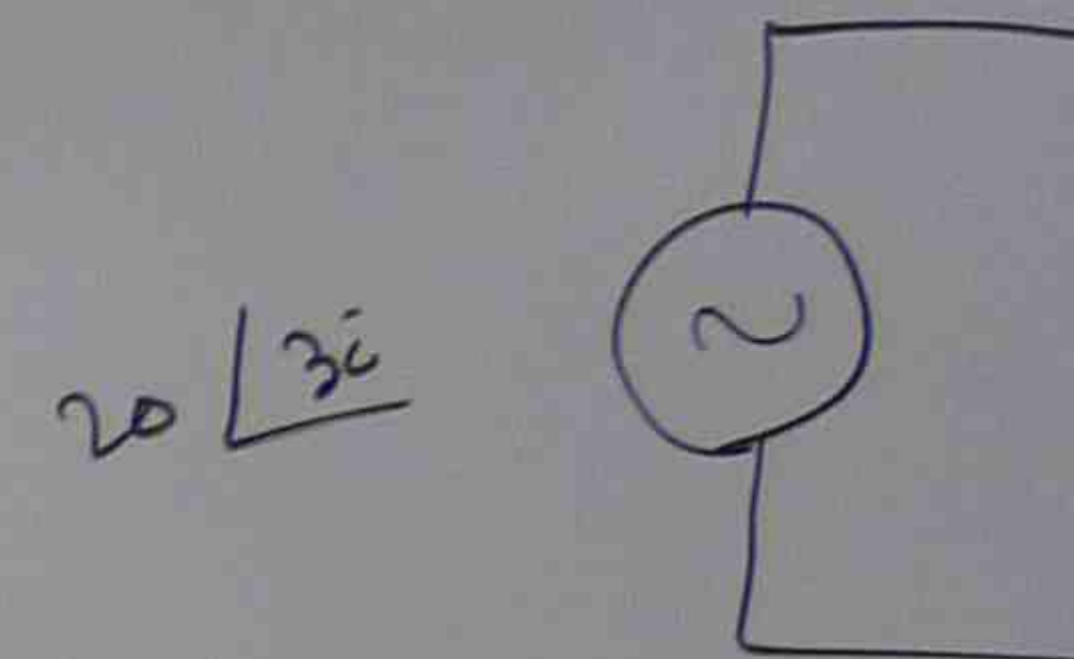
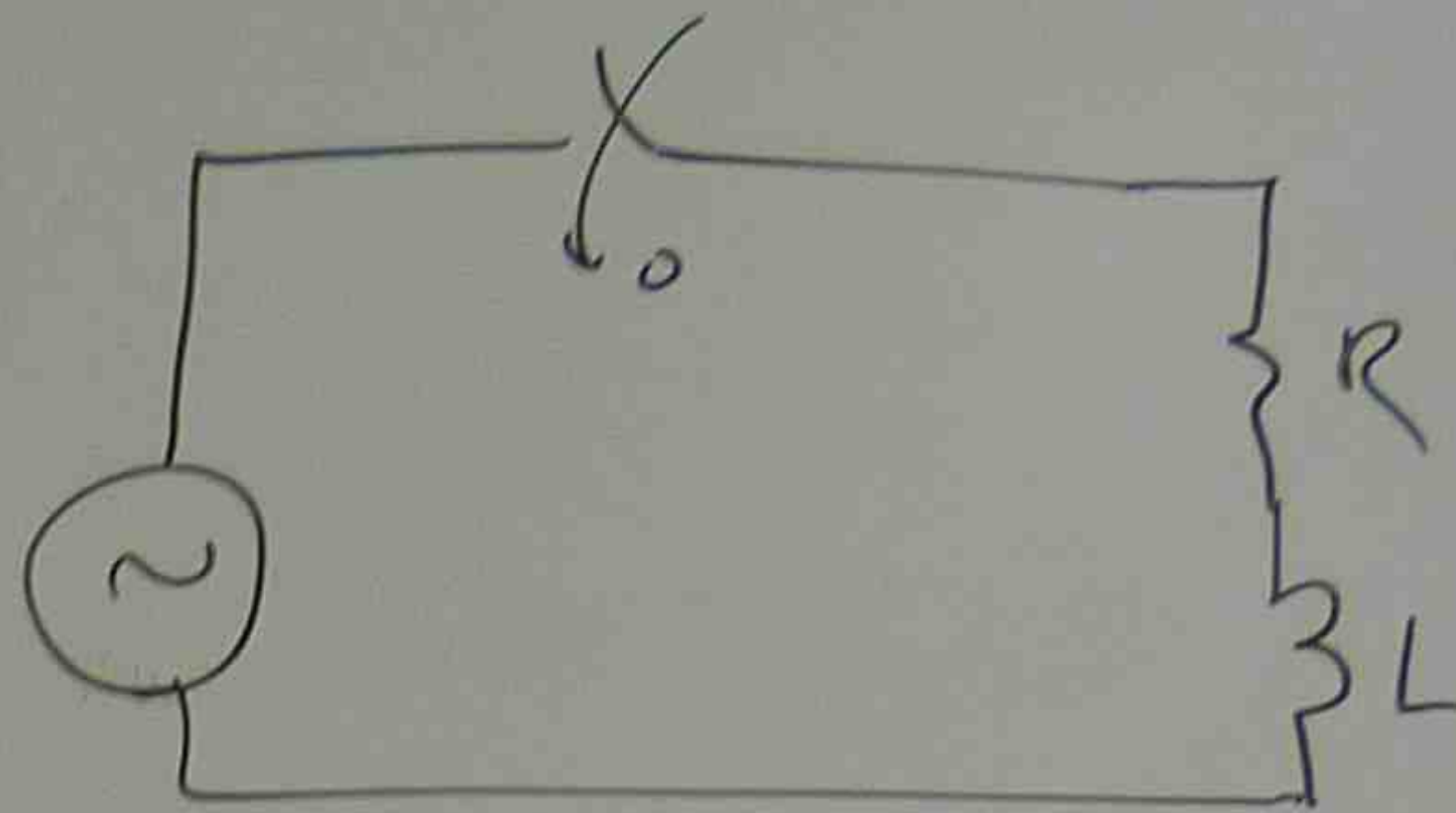
$$V = V_m \sin(\omega t + \phi)$$

TIME DOMAIN REPRESENTATION

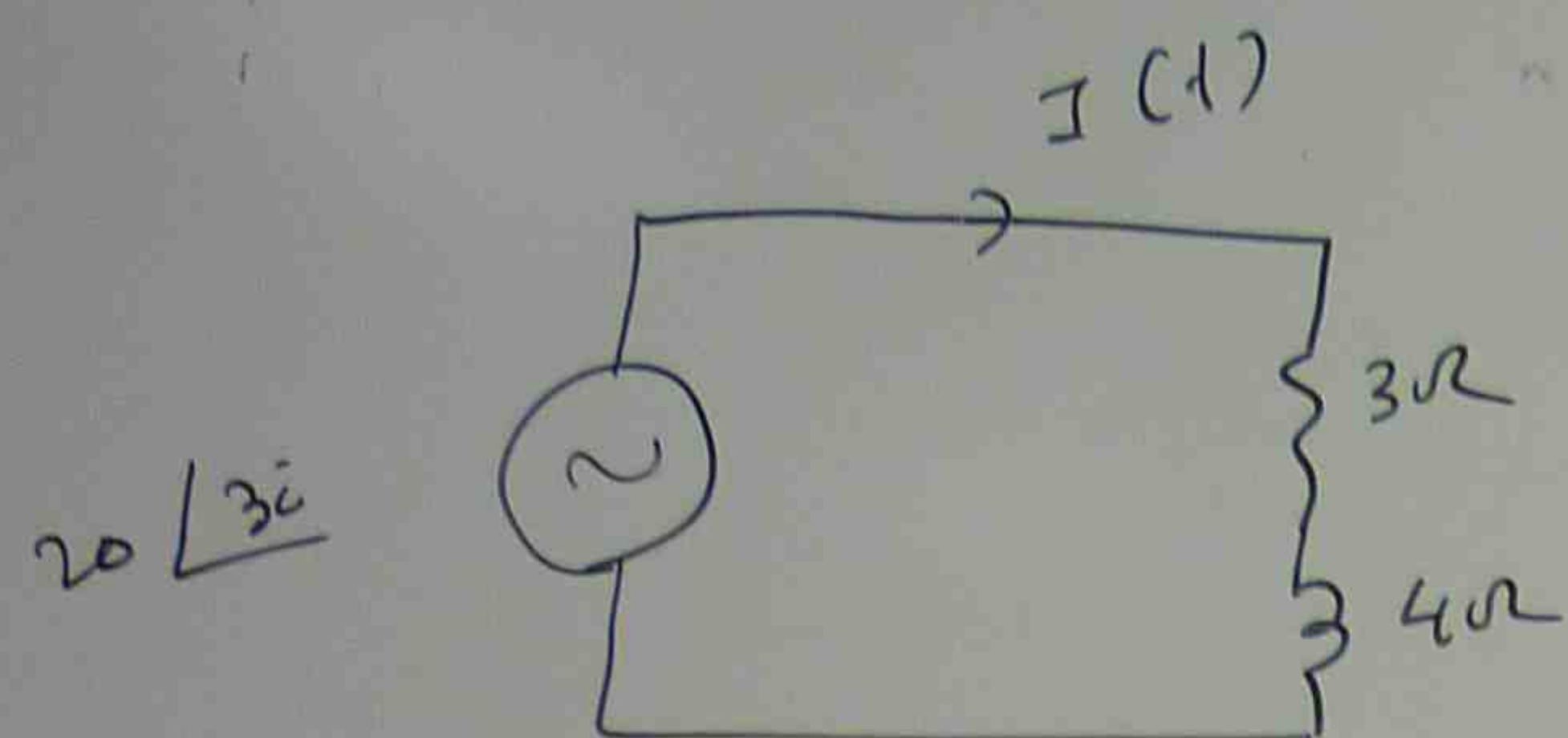


$$V = \frac{V_m}{\sqrt{2}} \angle \phi$$

FREQUENCY DOMAIN REPRESENTATION



FIND THE
(a)
(b)
F
I



FIND THE CURRENT IN

(a) FREQUENCY DOMAIN REPRESENTATION

(b) TIME DOMAIN REPRESENTATION

FREQUENCY = 50 Hz

$$I = \frac{V}{Z} = \frac{20 \angle 30}{3 + j4} = \frac{20 \angle 30}{\sqrt{3^2 + 4^2} \angle \tan^{-1} 4/3}$$

$$= \frac{20 \angle 30}{5 \angle 53.2}$$

$$= 4 \angle -23.2$$

(a) FREQUENCY
DOMAIN
CURRENT

(b) TIME DOMAIN CURRENT

$$I(t) = I_m \sin(\omega t + \phi)$$

$$= 2 \times 4 \sin(2\pi f t + (-23.2))$$

$$= 1.4142 \times 4 \sin(2 \times 3.1416 \times 50 t - 23.2)$$

$$= 5.62 \sin(314 t - 23.2)$$

$$\hat{i}(t) =$$

(b) TIME DOMAIN CURRENT

$$I(t) = I_m \sin(\omega t + \phi)$$

$$= 5.62 \sin(2\pi f t + (-23.2))$$

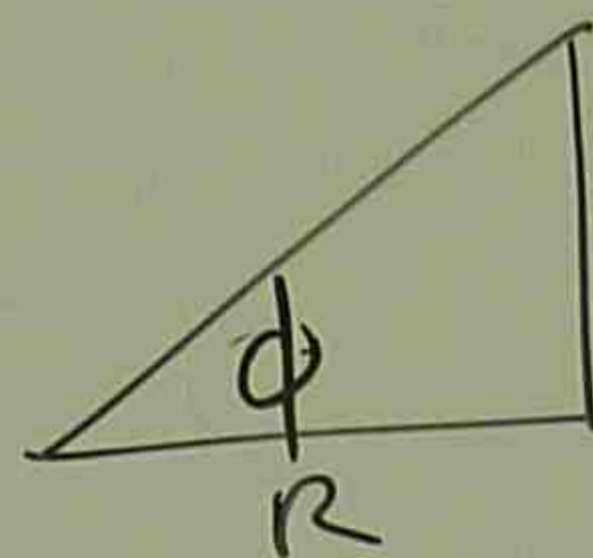
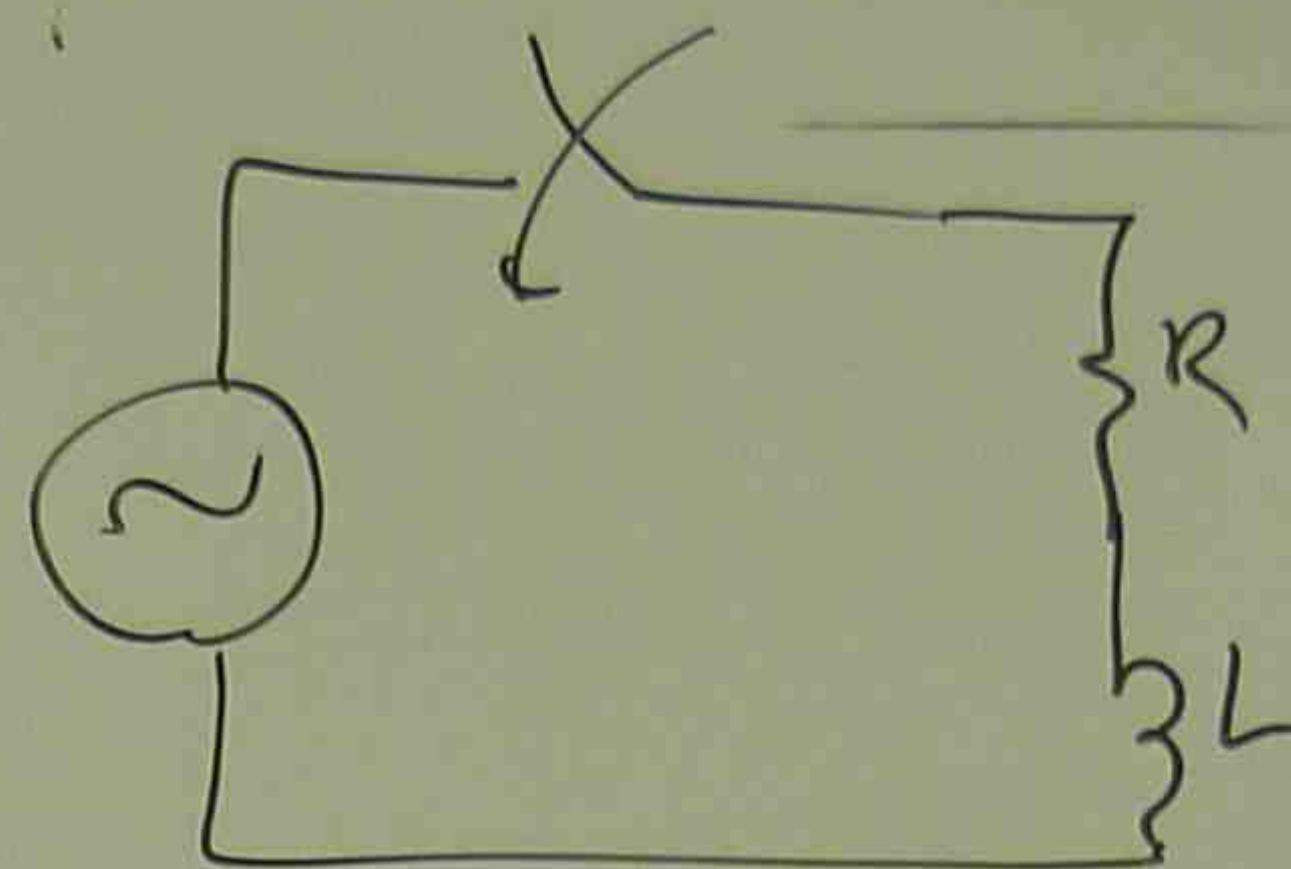
$$= 5.62 \sin(2 \times 3.1416 \times 50 t - 23.2)$$

$$= 5.62 \sin(314 t - 23.2)$$

$$\frac{20 \angle 30}{\sqrt{3^2 + 4^2}} \angle \tan^{-1} \frac{4}{3}$$

$$\frac{20 \angle 30}{5 \angle 53.2}$$

$$4 \angle -23.2$$

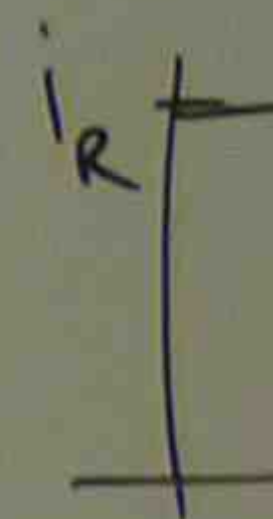
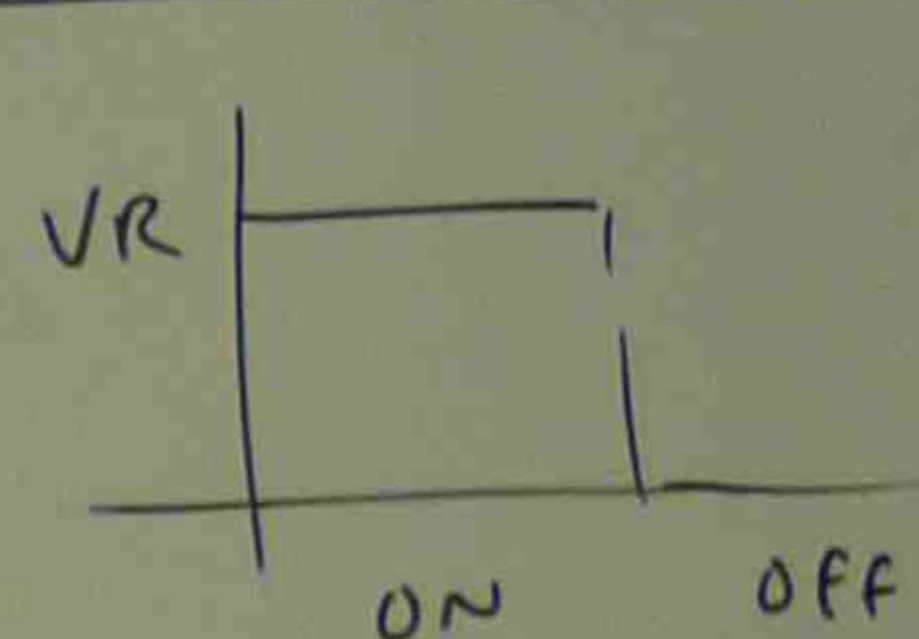
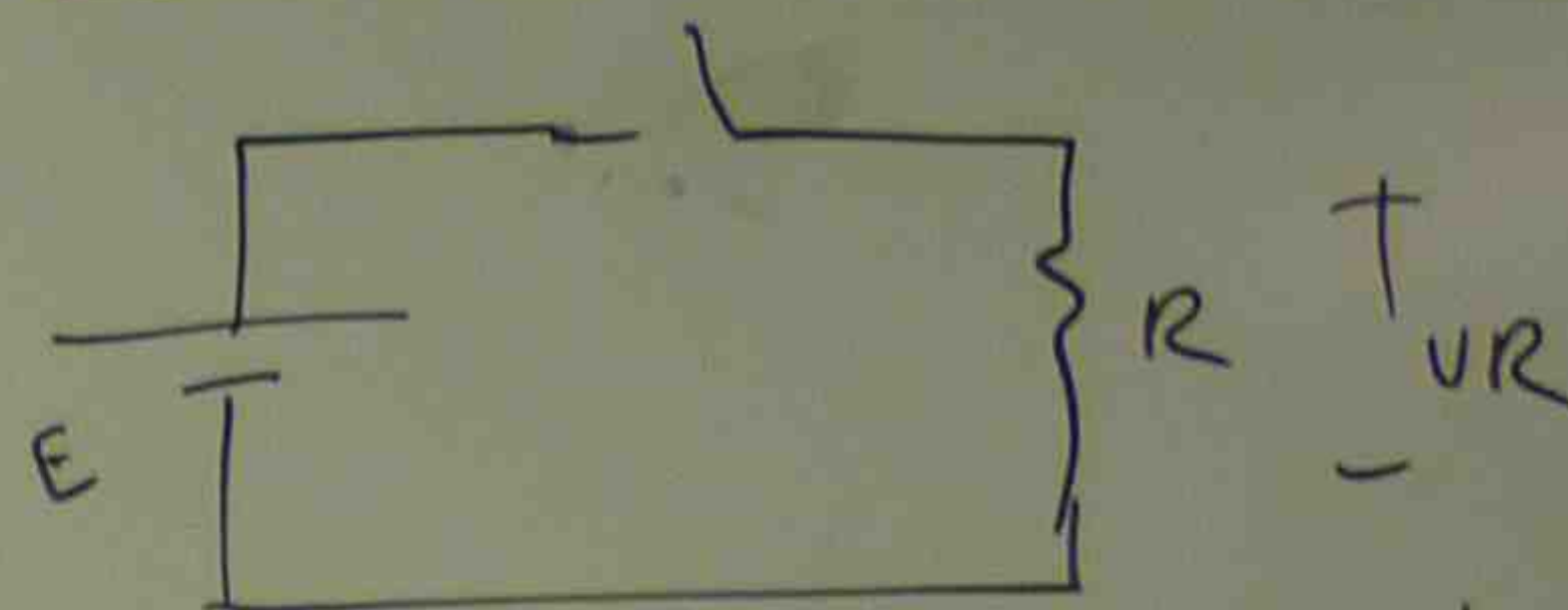


$$X_L = 2\pi f L = \omega L$$

$$\hat{I}(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \left(\sin(\omega t - \phi) + \sin \phi e^{-\frac{R}{L}t} \right)$$

$$\phi = \tan^{-1} \frac{\omega L}{R}$$

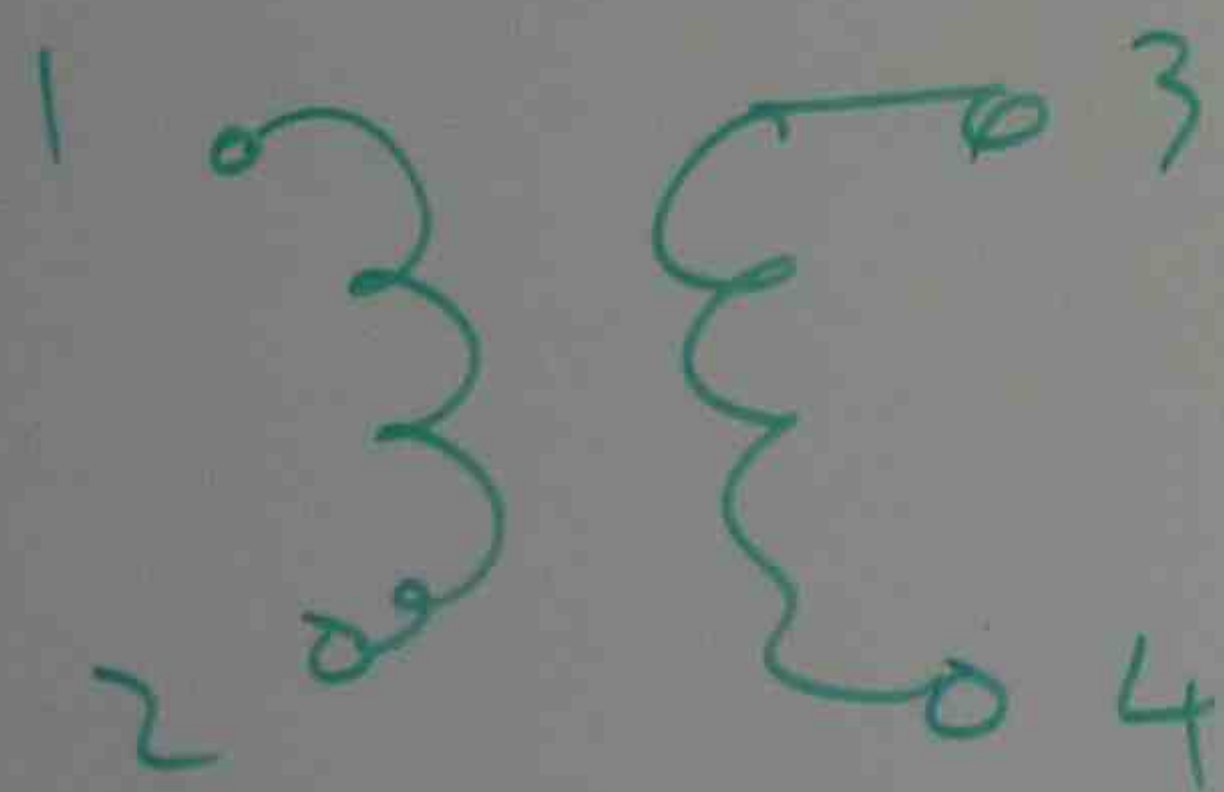
PURE RESISTANCE



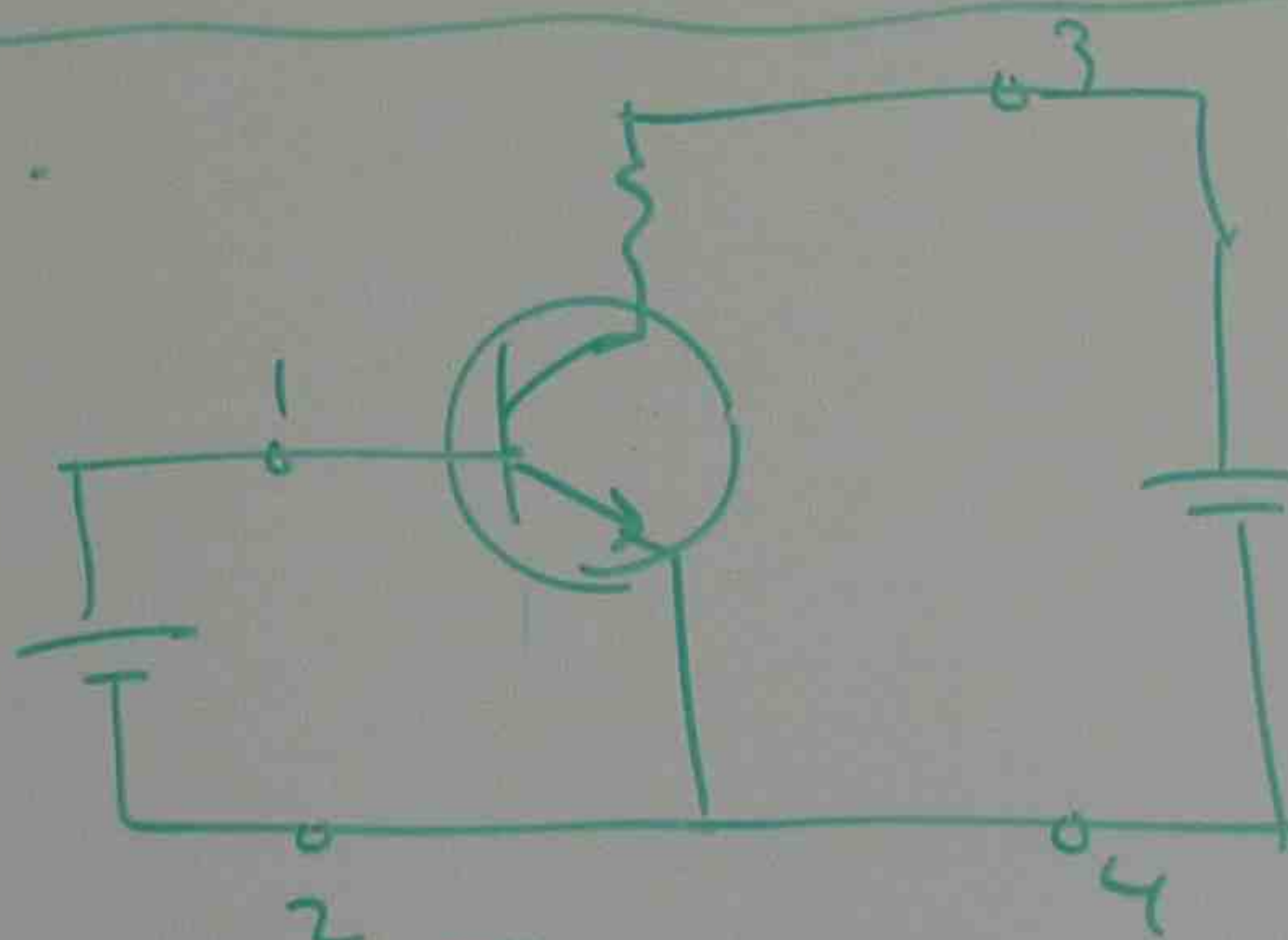
SOLVING SIMULTANEOUS EQUATIONS IN POWER ENGINEERING PROBLEMS

MATHEMATICAL MODELLING - REPRESENTING ELECTRICAL MODEL BY
MATHEMATICAL EQUATIONS.

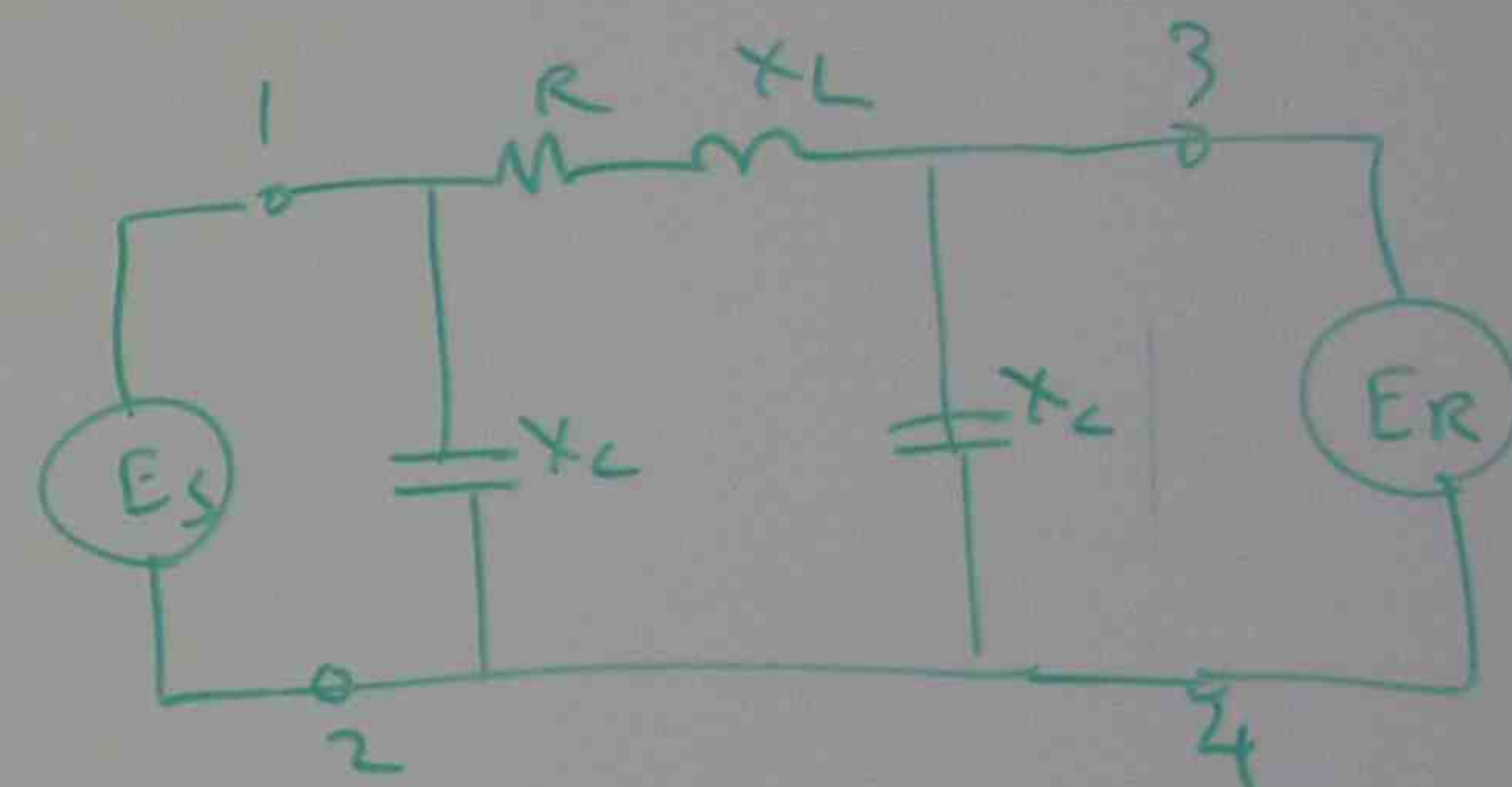
MATHEMATICAL EQUATIONS FOR FOUR TERMINAL NETWORKS



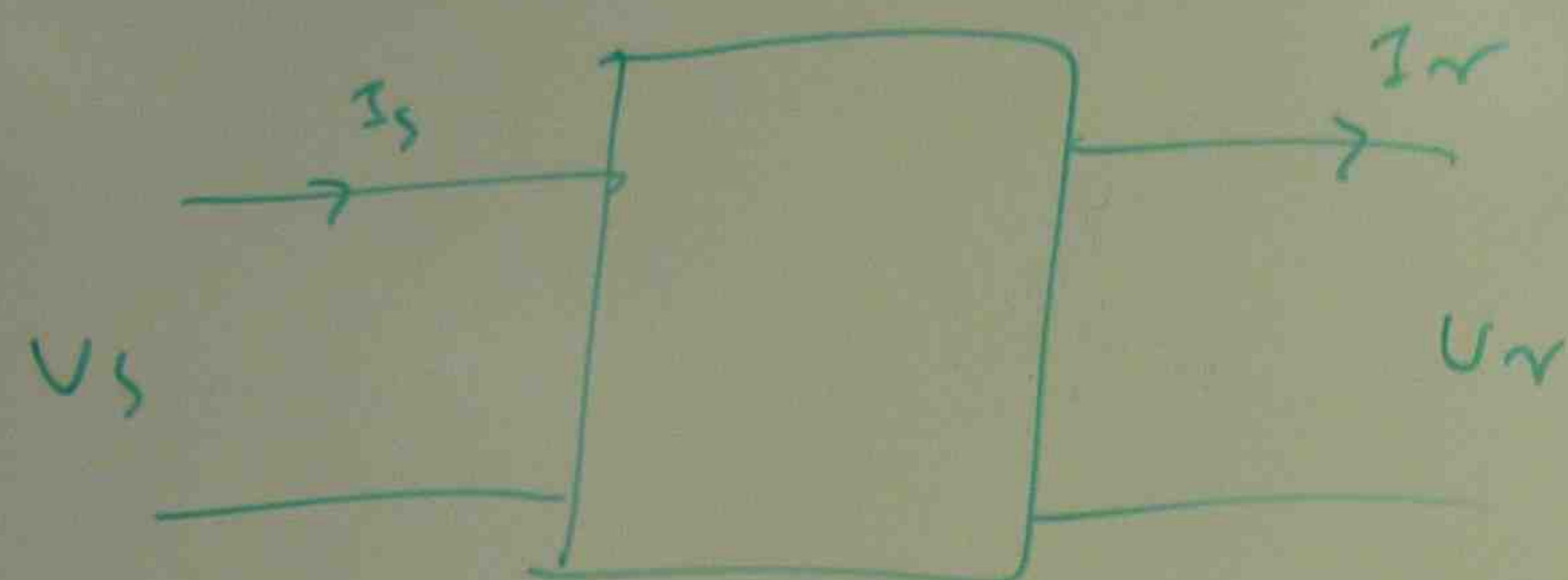
TRANSFORMER



TRANSISTOR

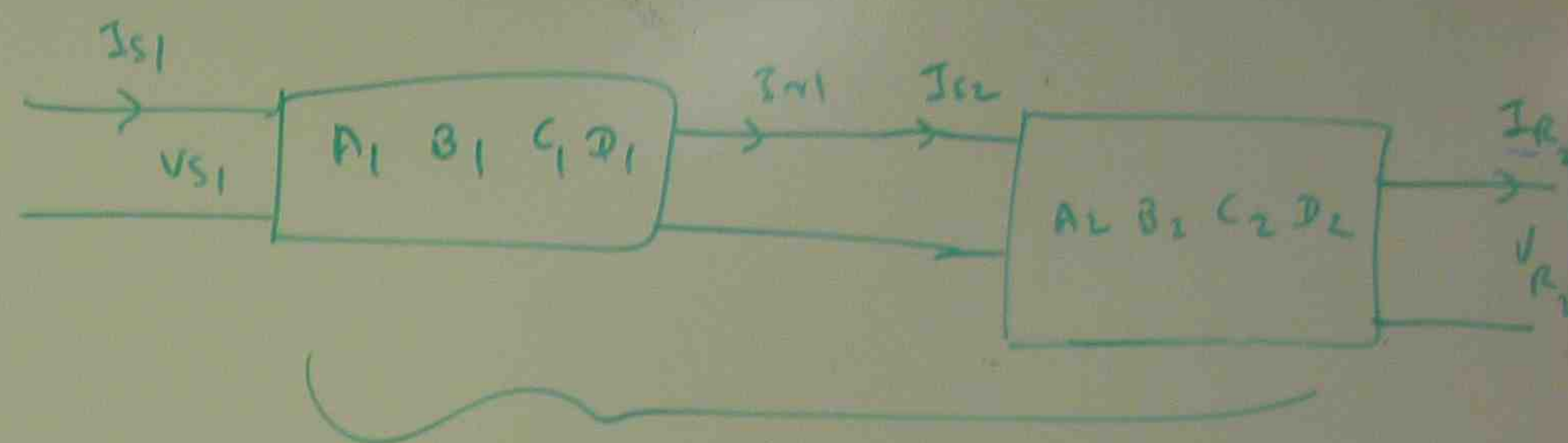
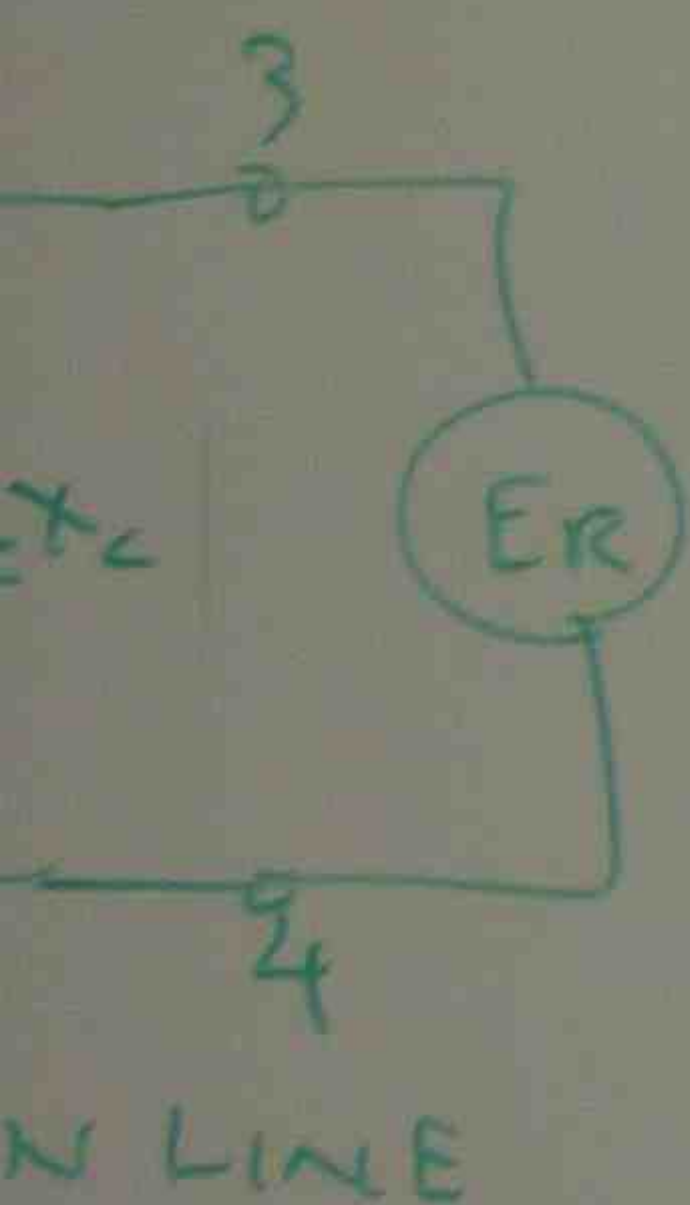


TRANSMISSION LINE



$$\bar{V}_s = A \bar{V}_r + B \bar{I}_r$$

$$\bar{I}_s = C \bar{V}_r + D \bar{I}_r$$



$$A_{eq} = A_1 A_2 + B_1 C_2$$

$$B_{eq} = A_1 B_2 + B_1 D_2$$

$$C_{eq} = C_1 A_2 + D_1 C_2$$

$$D_{eq} = C_1 B_2 + D_1 D_2$$

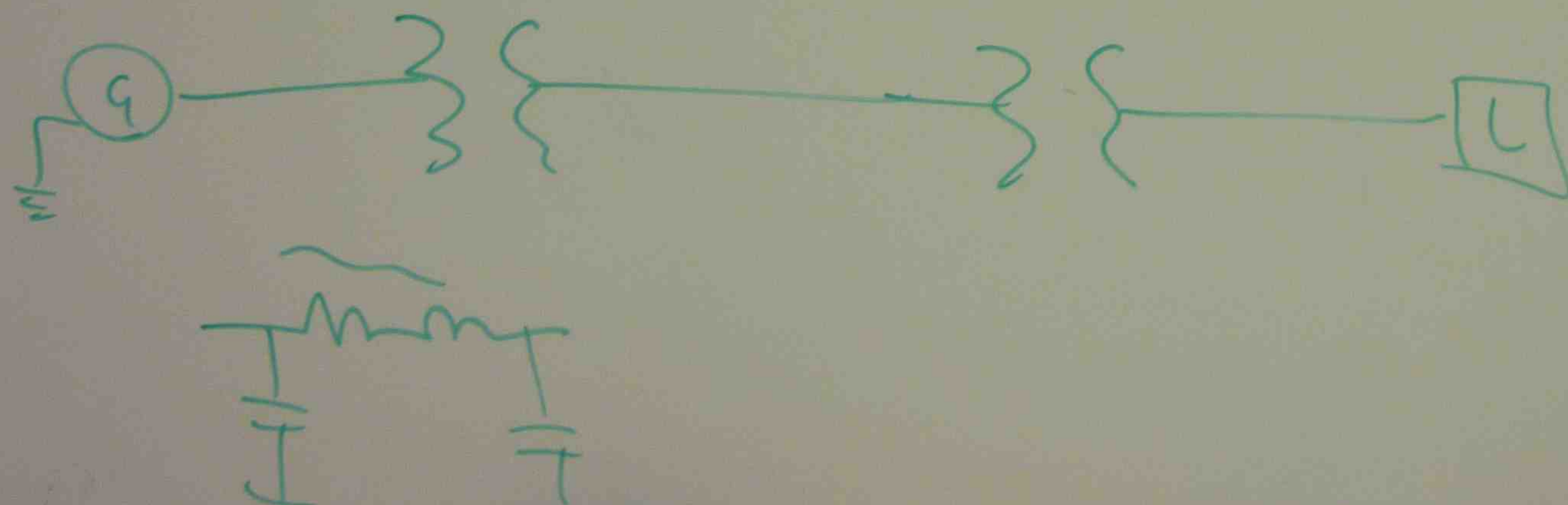
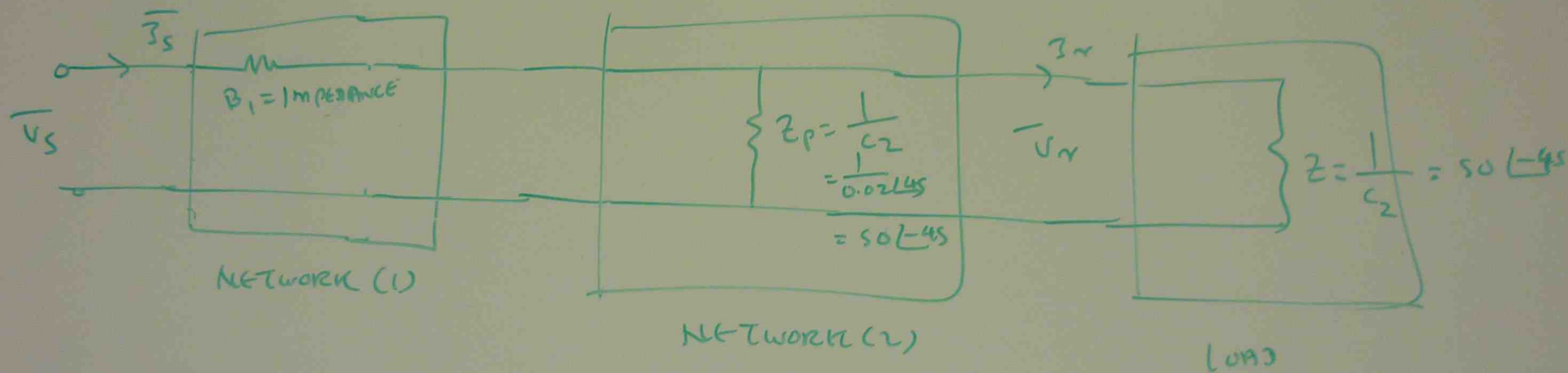
$$\bar{V}_s = A_{eq} \bar{V}_r + B_{eq} \bar{I}_r$$

$$\bar{I}_s = C_{eq} \bar{V}_r + D_{eq} \bar{I}_r$$

ph

DETERMINE THE EQUIVALENT A, B, C, D CONSTANTS OF THE GIVEN NETWORK.

$$\begin{array}{llll} A_1 = 1.0 & B_1 = 20 \angle 30^\circ \Omega & C_1 = 0 \text{ V} & D_1 = 1.0 \\ A_2 = 1.0 & B_2 = 0 & C_2 = 0.02 \angle 45^\circ \text{ V} & D_2 = 1.0 \end{array}$$



$A_{eq} =$

$B_{eq} =$

$C_{eq} =$

$$A_{eq} = A_1 A_2 + B_1 C_2 = 1.0 \times 1.0 + 20 \angle 30^\circ \times 0.02 \angle 45^\circ$$

$$= 1 + 0.4 \angle 75^\circ$$

$$= 1 + 0.4 (\cos 75^\circ + j \sin 75^\circ)$$

$$= 1 + 0.4 (0.2588 + j 0.965)$$

$$= 1 + 0.1 + j 0.4965$$

$$= 1.1 + j 0.4965$$

$$= \sqrt{1.1^2 + 0.4965^2} \angle \tan^{-1} \frac{0.4965}{1.1}$$

$$A_{eq} = 1.206 \angle 24.2^\circ$$

$$B_{eq} = A_1 B_2 + B_1 D_2 = 1.0 \times 0 + 20 \angle 30^\circ \times 1.0$$

$$= 0 + 20 \angle 30^\circ = 20 \angle 30^\circ$$

$$C_{eq} = C_1 A_2 + D_1 C_2 = 0 \times 1.0 + 1.0 \times 0.02 \angle 45^\circ$$

$$= 0.02 \angle 45^\circ$$

$$D_{eq} = C_1 B_2 + D_1 D_2$$

$$= 0 \times 0 + 1.0 \times 1.0$$

$$= 1.0$$

$$\bar{V}_S = A_{eq} \bar{V}_r + B_{eq} \bar{I}_r$$

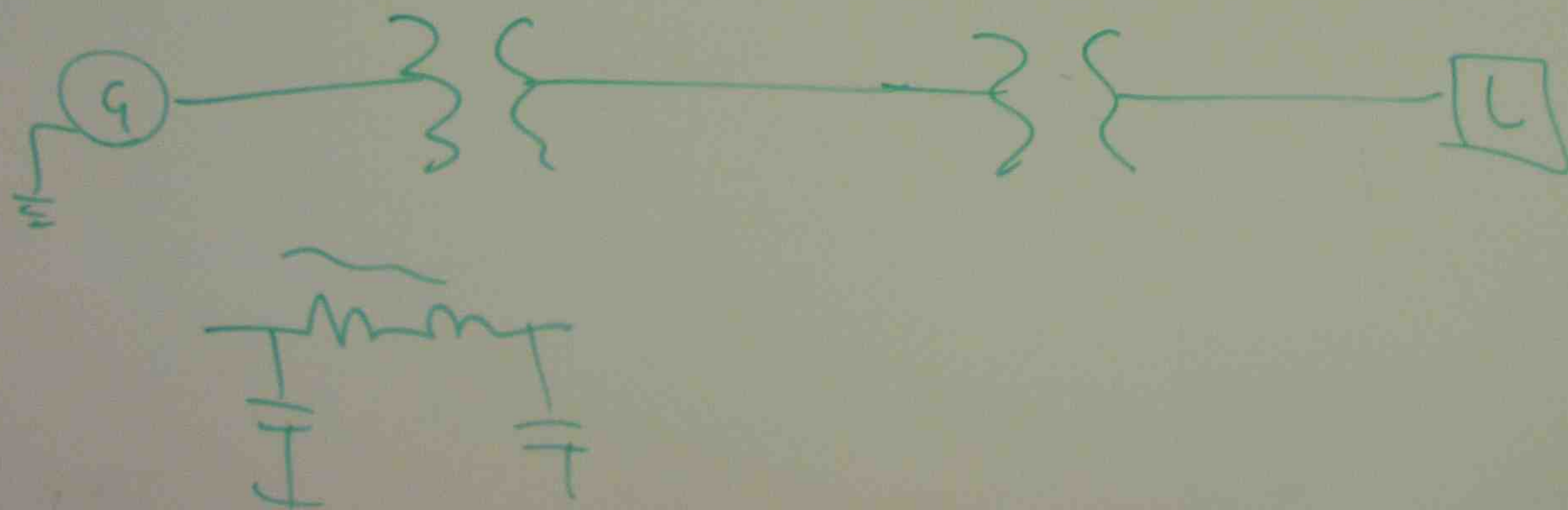
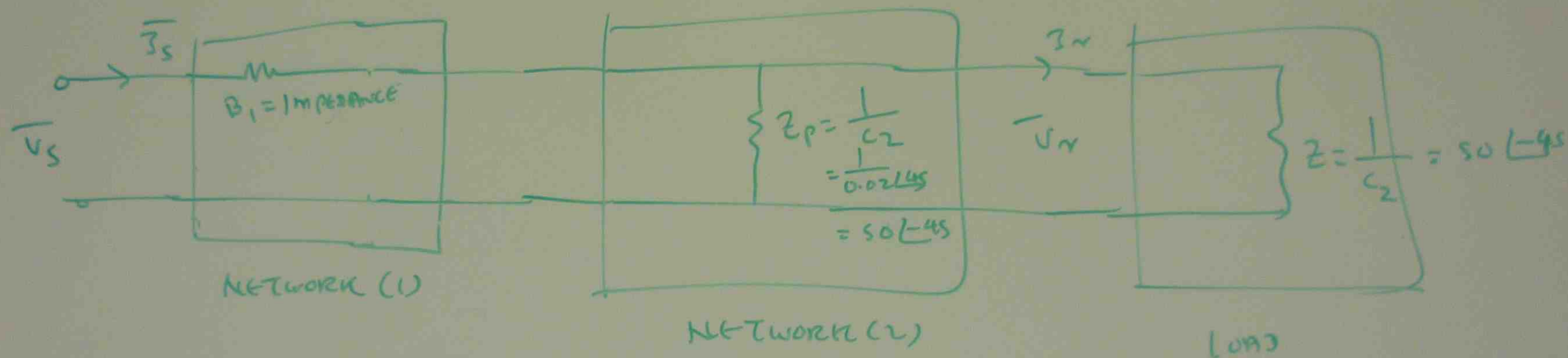
$$= 1.206 \angle 24.2^\circ \bar{V}_r + 20 \angle 30^\circ \bar{I}_r$$

$$\bar{I}_S = C_{eq} \bar{V}_r + D_{eq} \bar{I}_r$$

$$= 0.02 \angle 45^\circ \bar{V}_r + 1.0 \bar{I}_r$$

ph
DETERMINE THE EQUIVALENT A, B, C, D CONSTANTS OF
THE GIVEN NETWORK.

$$\begin{array}{llll} A_1 = 1.0 & B_1 = 20 \angle 30^\circ \Omega & C_1 = 0 \text{ V} & D_1 = 1.0 \\ A_2 = 1.0 & B_2 = 0 & C_2 = 0.02 \angle 45^\circ \text{ V} & D_2 = 1.0 \end{array}$$



$$A_{eq} =$$

$$B_{eq} =$$

$$C_{eq} =$$

$$A_{e_g} = A_1 A_2 + B_1 C_2 = 1.0 \times 1.0 + 20 \angle 30^\circ \times 0.02 \angle 45^\circ$$

$$= 1 + 0.4 \angle 75^\circ$$

$$= 1 + 0.4 (\cos 75^\circ + j \sin 75^\circ)$$

$$= 1 + 0.4 (0.2598 + j 0.9659)$$

$$= 1 + 0.1 + j 0.4965$$

$$= 1.1 + j 0.4965$$

$$= \sqrt{1.1^2 + 0.4965^2} \angle \tan^{-1} \frac{0.4965}{1.1}$$

$$A_{e_g} = 1.206 \angle 24.2^\circ$$

$$B_{e_g} = A_1 B_2 + B_1 D_2 = 1.0 \times 0 + 20 \angle 30^\circ \times 1.0$$

$$= 0 + 20 \angle 30^\circ = 20 \angle 30^\circ$$

$$C_{e_g} = C_1 A_2 + D_1 C_2 = 0 \times 1.0 + 1.0 \times 0.02 \angle 45^\circ$$

$$= 0.02 \angle 45^\circ$$

$$D_{e_g} = C_1 B_2 + D_1 D_2$$

$$= 0 \times 0 + 1.0 \times 1.0$$

$$= 1.0$$

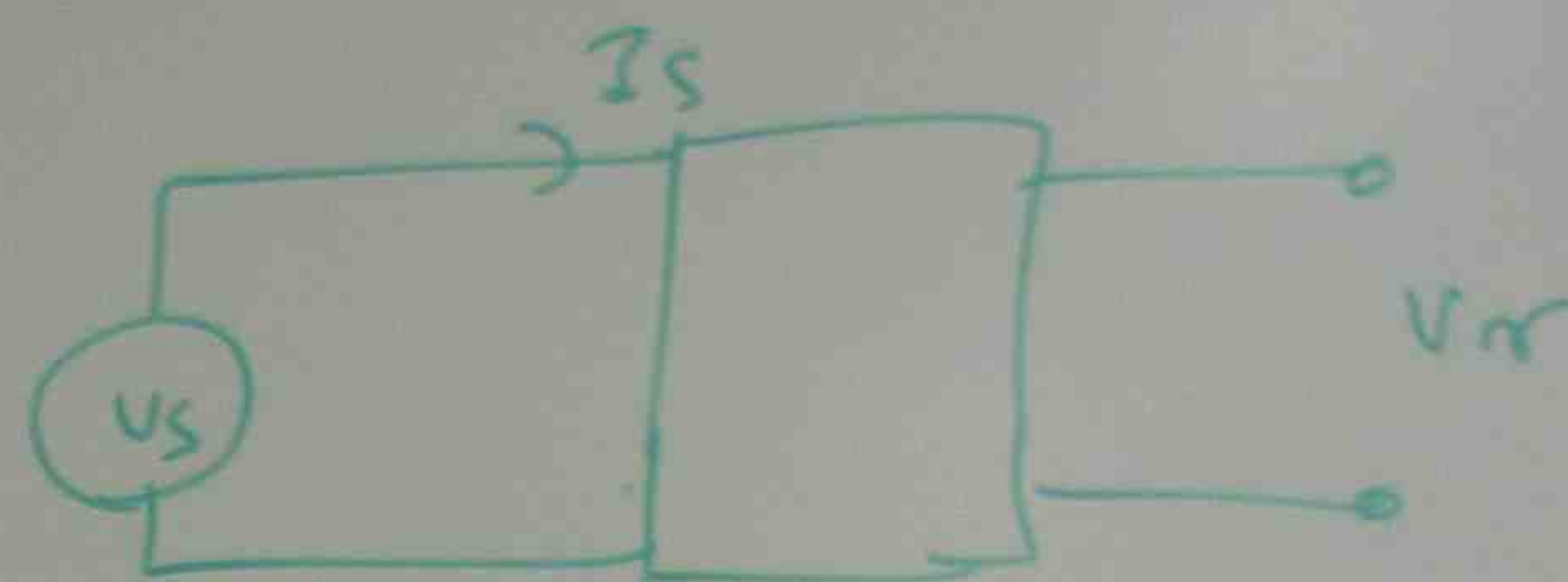
$$\bar{V}_S = A_{e_g} \bar{V}_r + B_{e_g} \bar{I}_r$$

$$= 1.206 \angle 24.2^\circ \bar{V}_r + 20 \angle 30^\circ \bar{I}_r$$

$$\bar{I}_S = C_{e_g} \bar{V}_r + D_{e_g} \bar{I}_r$$

$$= 0.02 \angle 45^\circ \bar{V}_r + 1.0 \bar{I}_r$$

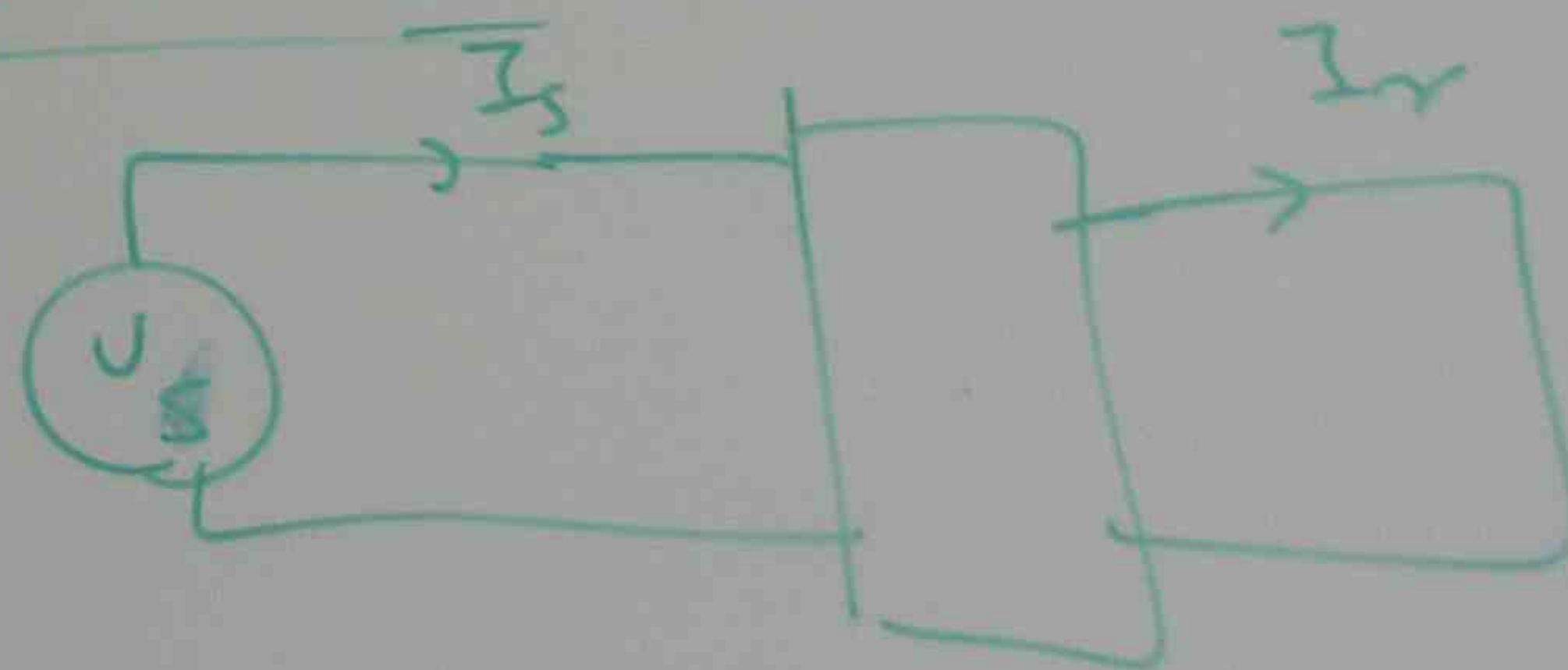
OPEN CIRCUIT



$$A_{eq} = \frac{V_s}{V_r}$$

$$C_{eq} = \frac{I_s}{V_s}$$

SHORT CIRCUIT



$$B_{eq} = \frac{V_s}{I_r}$$

$$D_{eq} = \frac{I_s}{I_r}$$

ph

DETERMINE
THE FOLLOWING

RECEIVER OPEN

$$\bar{V}_s = 100$$

$$\bar{V}_r = 70 + j$$

$$\bar{I}_s = 1.4$$

$$\bar{I}_r = 0$$

OPEN $R =$

ph

DETERMINE THE A, B, C, D CONSTANTS OF THE NETWORK IN WHICH THE FOLLOWING TEST RESULTS HAVE BEEN OBSERVED.

RECEIVER OPEN CIRCUIT

$$\bar{V}_S = 100 \angle 0^\circ \text{ V}$$

$$\bar{V}_R = 70.7 \angle -45^\circ \text{ V}$$

$$\bar{I}_S = 1.41 \angle -45^\circ \text{ A}$$

$$\bar{I}_R = 0$$

RECEIVER SHORT CIRCUIT

$$\bar{V}_R = 0$$

$$\bar{V}_S = 100 \angle 0^\circ \text{ V}$$

$$\bar{I}_S = 2.0 \angle -90^\circ \text{ A}$$

$$\bar{I}_R = 2.0 \angle -90^\circ \text{ A}$$

SHORT

$$B = \frac{V_S}{I_R}$$

$$= \frac{100 \angle 0^\circ}{2.0 \angle 90^\circ}$$

$$= 50 \angle 90^\circ$$

$$D = \frac{I_S}{I_R} = \frac{2 \angle -90^\circ}{2 \angle -90^\circ} = 1$$

OPEN

$$A = \frac{V_S}{V_R} = \frac{100 \angle 0^\circ}{70.7 \angle -45^\circ} = 1.41 \angle 45^\circ$$

$$C = \frac{I_S}{V_S} = \frac{1.41 \angle -45^\circ}{100 \angle 0^\circ} = 0.0141 \angle -45^\circ$$

C, D CONSTANTS OF THE NETWORK IN WHICH
S HAVE BEEN OBSERVED.

RECEIVER SHORT CIRCUIT

$$\bar{V}_r = 0$$

$$\bar{V}_s = 100 \angle 0^\circ \text{ V}$$

$$\bar{I}_s = 2.0 \angle -90^\circ \text{ A}$$

$$\bar{I}_r = 2.0 \angle -90^\circ \text{ A}$$

$$\frac{100 \angle 0}{2 \angle -90} = 1.41 \angle 45$$

$$\frac{1.41 \angle -45}{2.0 \angle 0} = 0.0141 \angle -45$$

SHORT

$$B = \frac{V_s}{I_r}$$

$$= \frac{100 \angle 0}{2.0 \angle 90}$$

$$= 50 \angle 90$$

$$D = \frac{I_s}{I_r} = \frac{2 \angle -90}{2 \angle -90}$$

$$= 1$$

$$\bar{V}_s = A \bar{V}_r + B \bar{I}_r$$

$$\bar{V}_s = 1.41 \angle 45 \bar{V}_r + 50 \angle 90 \bar{I}_r$$

$$\bar{I}_s = C \bar{V}_r + D \bar{I}_r$$

$$\bar{I}_s = 0.0141 \angle -45 \bar{V}_r + 1 \bar{I}_r$$

ph

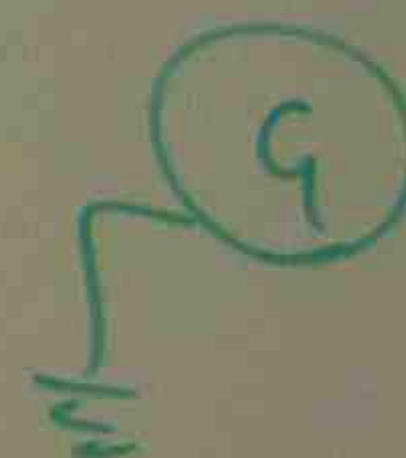
DETER
THE GL

$$A_1 =$$

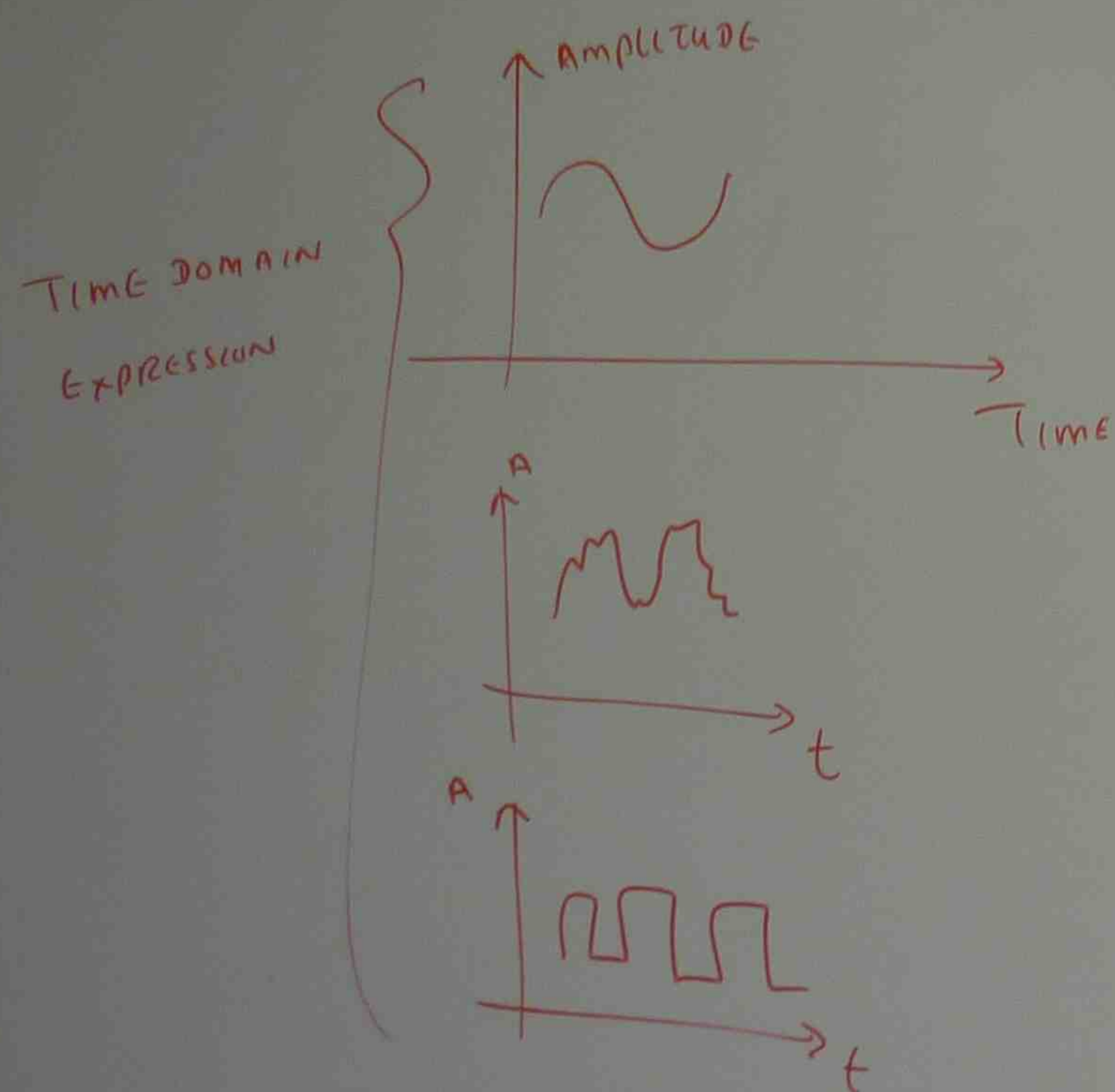
$$A_2 =$$

→

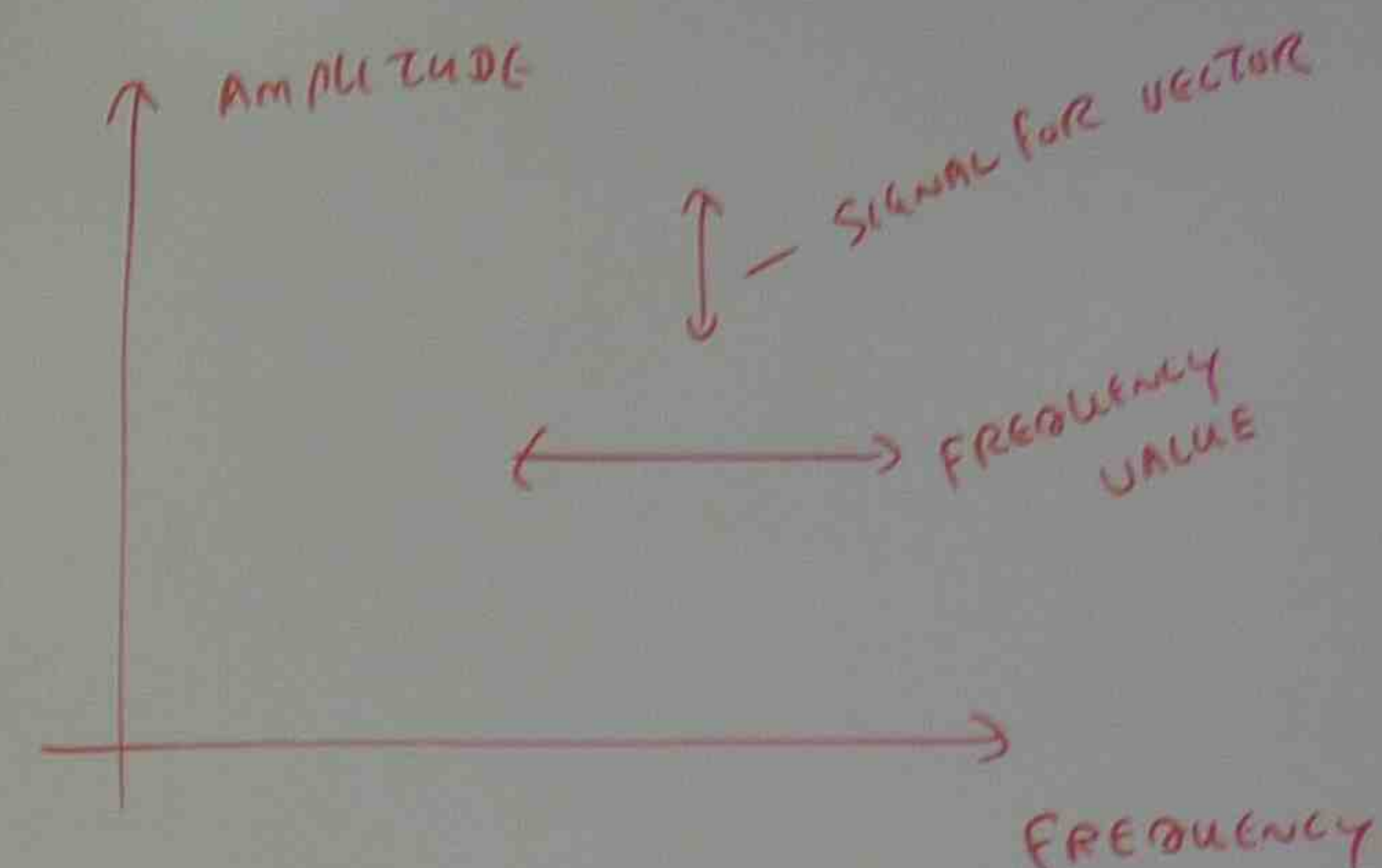
$$\bar{V}_s$$



APPLICATION OF PERIODIC WAVE FUNCTION IN POWER ENGINEERING



FREQUENCY
DOMAIN
EXPRESSION



$$F(\omega) = A \angle \pm \phi$$

↑
STEADY STATE
CALCULATION.

$$f(t) = A \sin \omega t \pm \phi \leftarrow \text{TRANSIENT CALCULATIONS}$$

→ FUNDAMENTAL FREQUENCY VECTOR

FREQUENCY VALUE

→ FREQUENCY

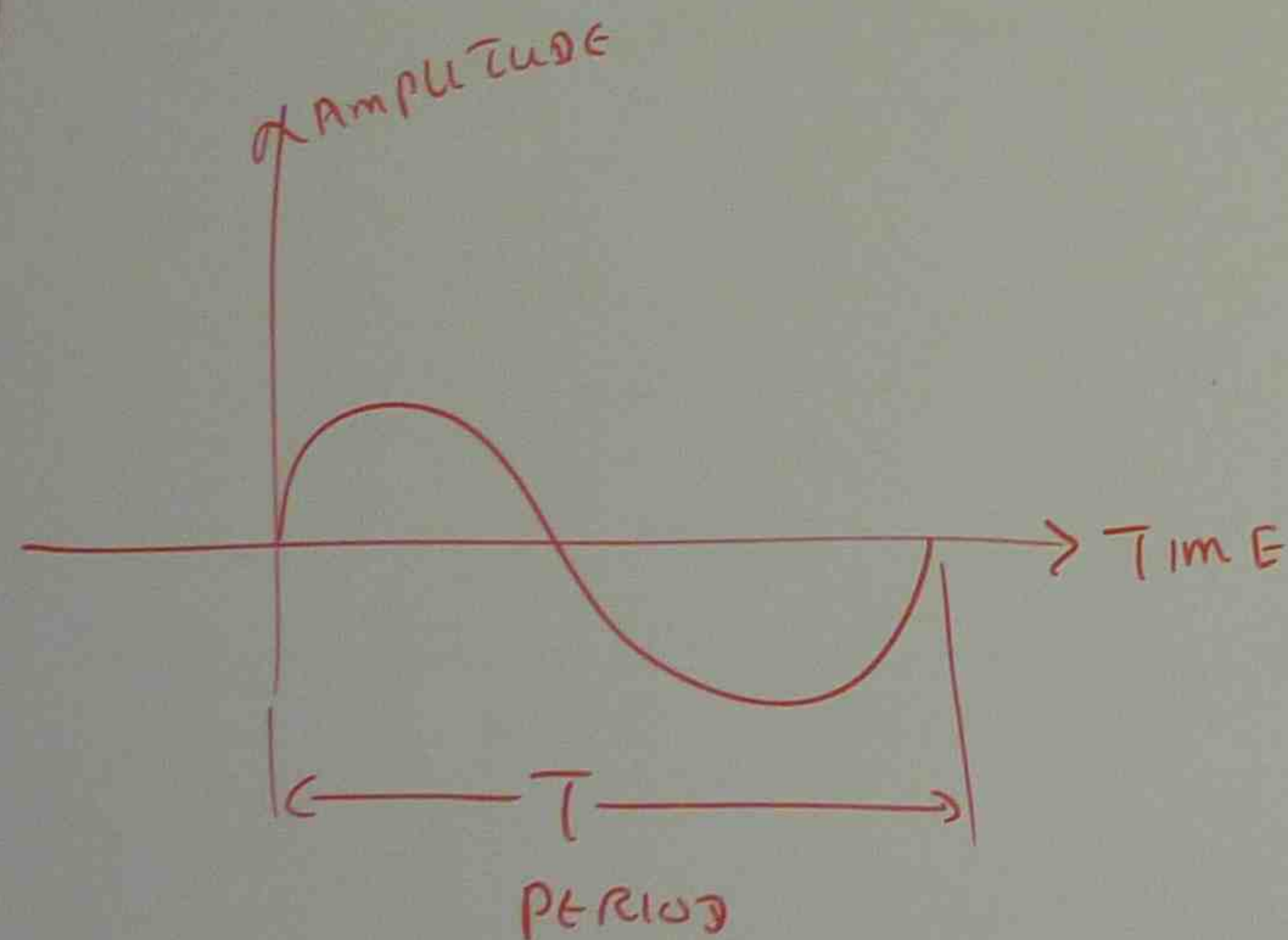
HARMONICS

MULTIPLE FACTOR OF FUNDAMENTAL FREQUENCY

FUNDAMENTAL $f_0 = 50 \text{ Hz}$

SECOND HARMONIC $f_2 = 2 \times 50 \text{ Hz} = 100 \text{ Hz}$

THIRD HARMONIC $f_3 = 3 \times 50 \text{ Hz} = 150 \text{ Hz}$



$$f = \frac{1}{T}$$

$f = \text{FREQUENCY}$

$T = \text{PERIOD}$

pb

A WAVE FORM HAS A PERIOD $T = 40 \text{ ms}$. CALCULATE THE FREQUENCY OF THE FUNDAMENTAL, THE SECOND AND THIRD HARMONICS.

$$\begin{aligned} \text{FUNDAMENTAL FREQUENCY } f &= \frac{1}{T} = \frac{1}{40 \text{ ms}} = \frac{1}{40 \times 10^{-3}} \\ &= \frac{10^3}{40} = 25 \text{ Hz} \end{aligned}$$

$$\text{SECOND HARMONICS} = 2f = 2 \times 25 \text{ Hz} = 50 \text{ Hz}$$

$$\text{THIRD HARMONICS} = 3f = 3 \times 25 \text{ Hz} = 75 \text{ Hz}$$

$$\text{FOURTH HARMONICS} = 4f = 4 \times 25 \text{ Hz} = 100 \text{ Hz}$$

AL FREQUENCY

$$12 = 100 \text{ Hz}$$

$$2 = 150 \text{ Hz}$$

IME

only

D

pb

A WAVE FORM HAS A PERIOD $T = 40 \text{ ms}$. CALCULATE THE FREQUENCY OF THE FUNDAMENTAL, THE SECOND AND THIRD HARMONICS.

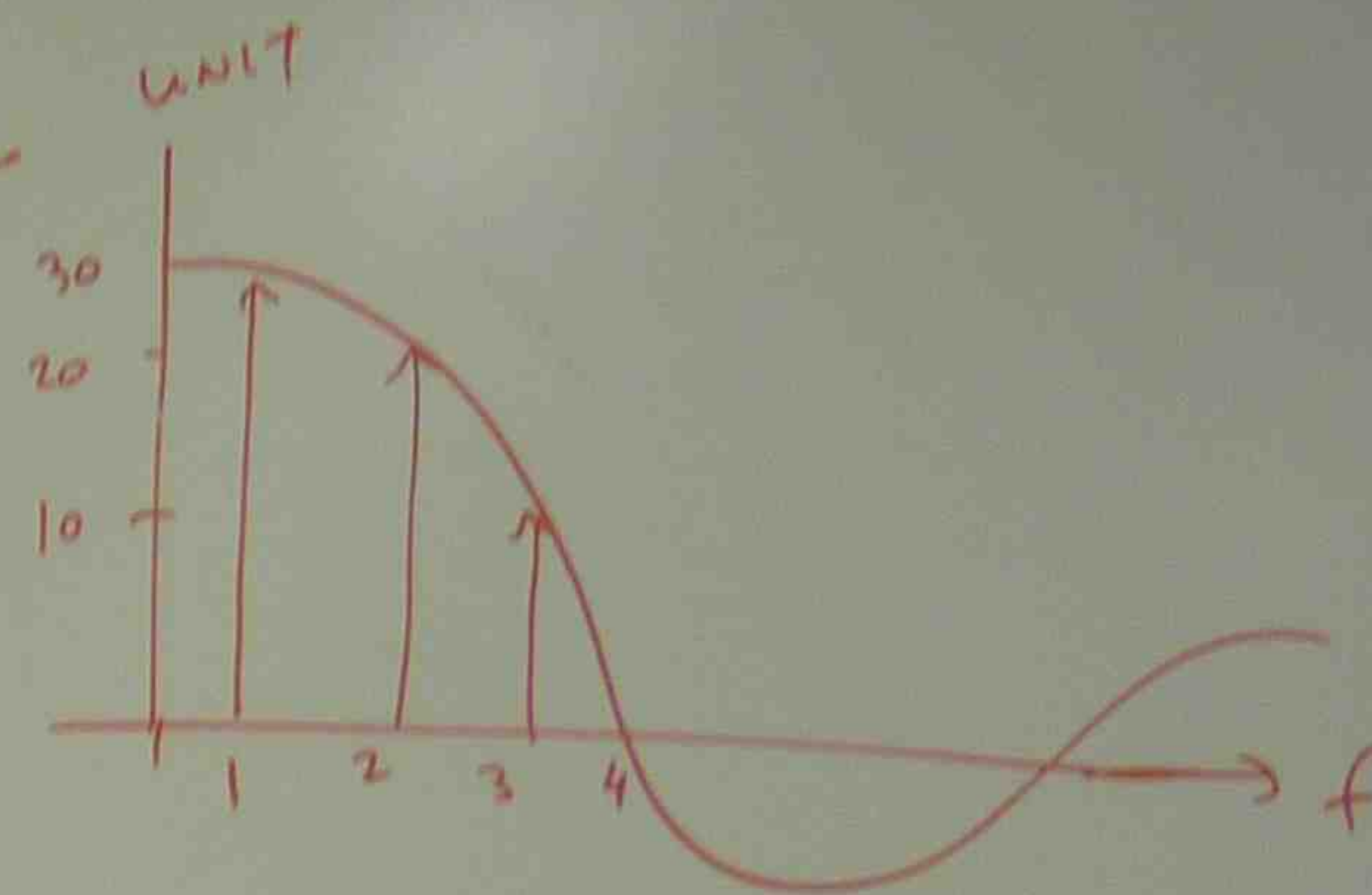
$$\begin{aligned} \text{FUNDAMENTAL FREQUENCY } f &= \frac{1}{T} = \frac{1}{40 \text{ ms}} = \frac{1}{40 \times 10^{-3}} \\ &= \frac{10^3}{40} = 25 \text{ Hz} \end{aligned}$$

$$\text{SECOND HARMONICS} = 2f = 2 \times 25 \text{ Hz} = 50 \text{ Hz}$$

$$\text{THIRD HARMONICS} = 3f = 3 \times 25 \text{ Hz} = 75 \text{ Hz}$$

$$\text{FOURTH HARMONICS} = 4f = 4 \times 25 \text{ Hz} = 100 \text{ Hz}$$

pb



FIND 1st, 2nd, 3rd AND 4th HARMONICS

AMPLITUDES OF GIVEN WAVE FORM.

$$1^{\text{st}} = \text{FUNDAMENTAL} = 30 \text{ UNIT.}$$

$$2^{\text{nd}} = \text{HARMONICS} = 20 \text{ UNIT}$$

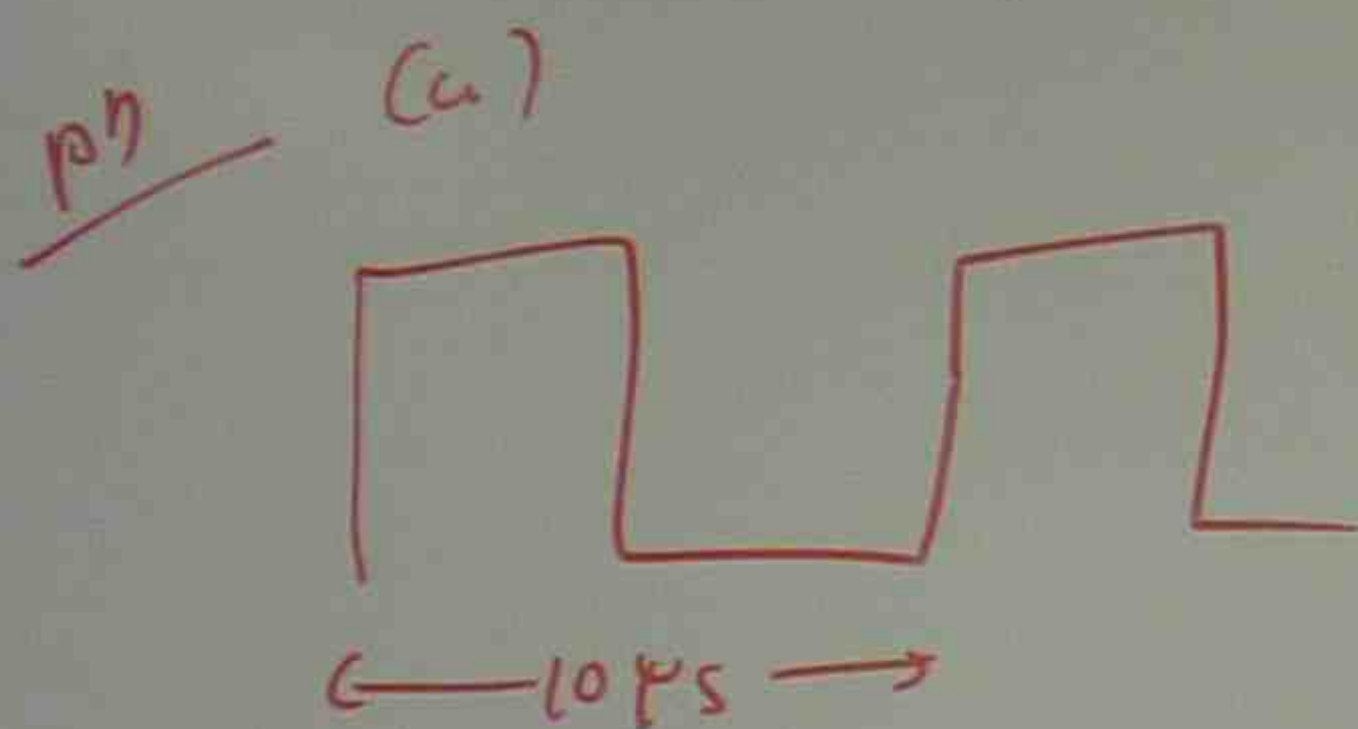
$$3^{\text{rd}} = \text{HARMONICS} = 10 \text{ UNIT}$$

pb

(a)

$$f = \frac{1}{T}$$

CALCULATE THE FREQUENCIES

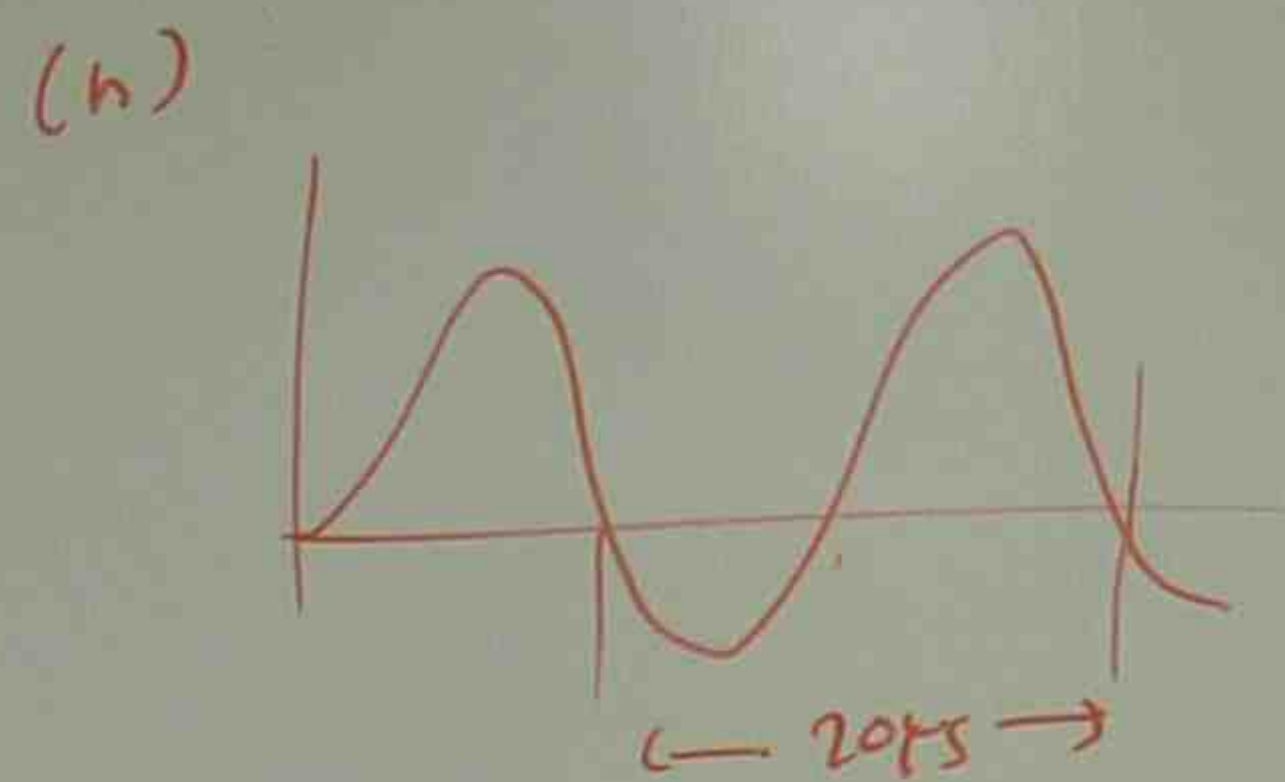


$$f = \frac{1}{T} = \frac{1}{10 \times 10^{-6}}$$

$$= \frac{10^6}{10}$$

$$= 10^5 \text{ Hz}$$

$$= 100 \text{ kHz}$$



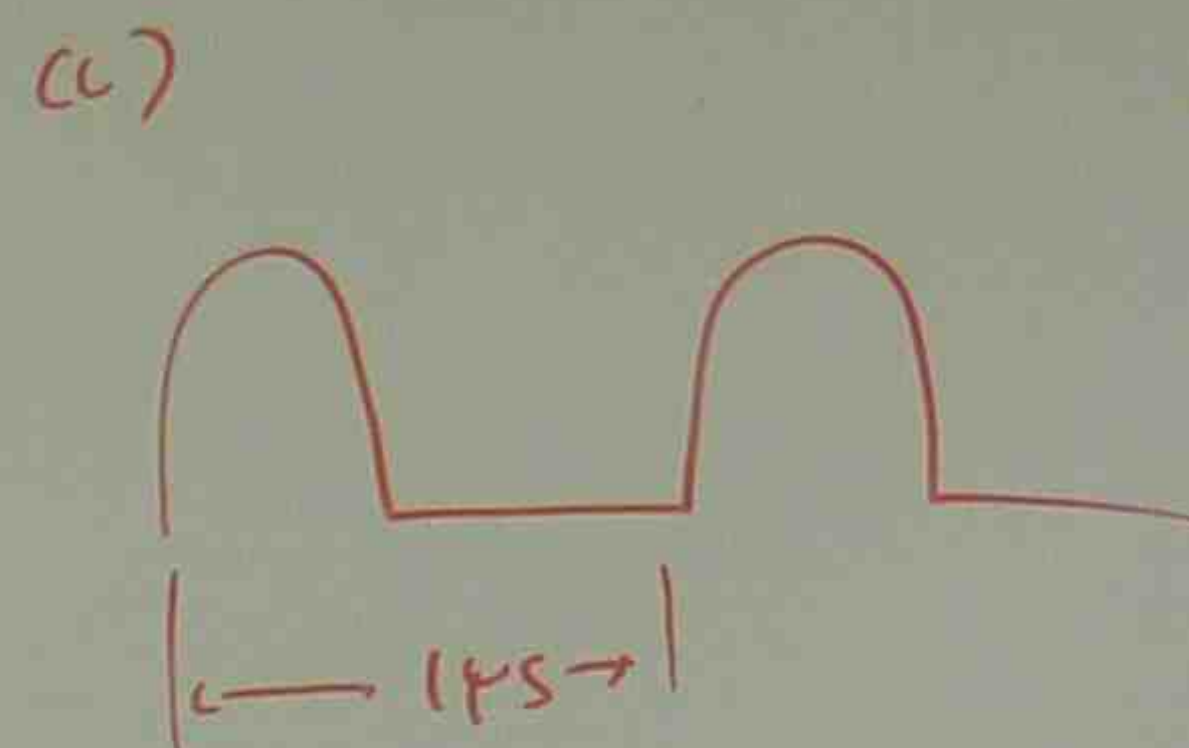
$$f = \frac{1}{T}$$

$$= \frac{1}{20 \times 10^{-6}}$$

$$= \frac{10^6}{20}$$

$$= \frac{1000 \times 10^3}{20}$$

$$= 50 \text{ kHz}$$

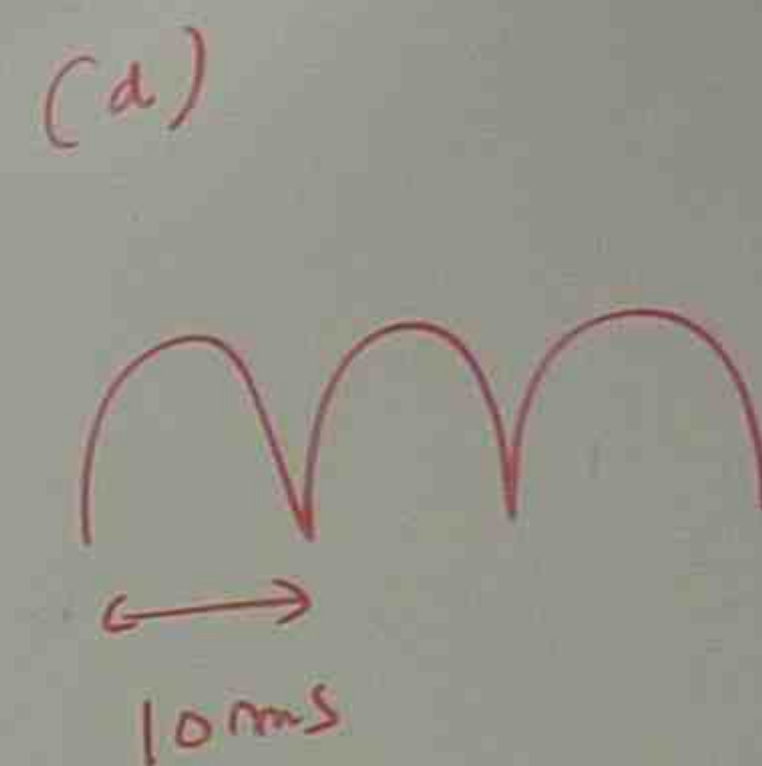


$$f = \frac{1}{T}$$

$$= \frac{1}{1 \times 10^{-6}}$$

$$= 10^6 \text{ Hz}$$

$$= 1 \text{ MHz}$$



$$f = \frac{1}{T}$$

$$= \frac{1}{10 \times 10^{-3}}$$

$$= \frac{10^3}{10}$$

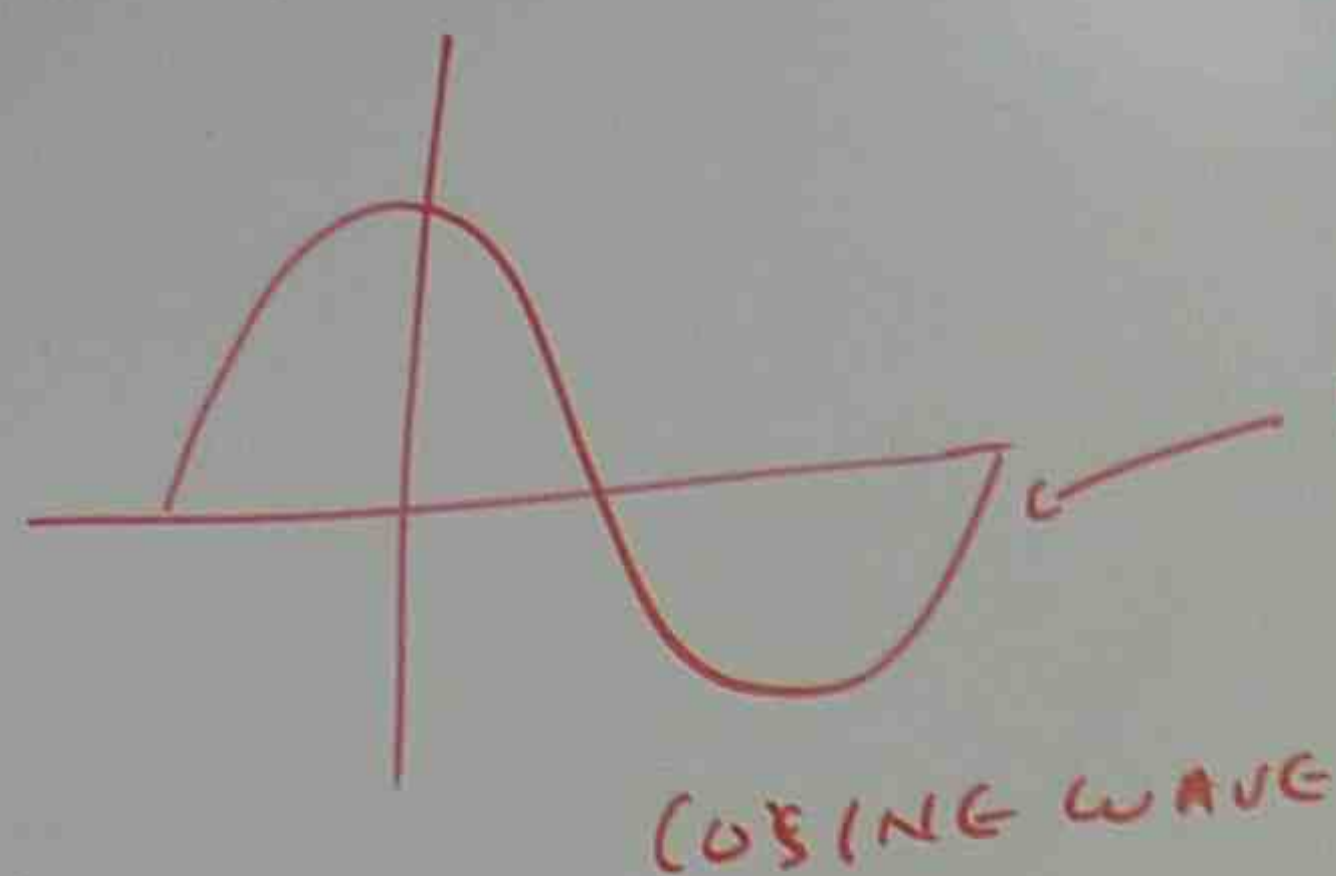
$$= 100 \text{ Hz}$$

(a)

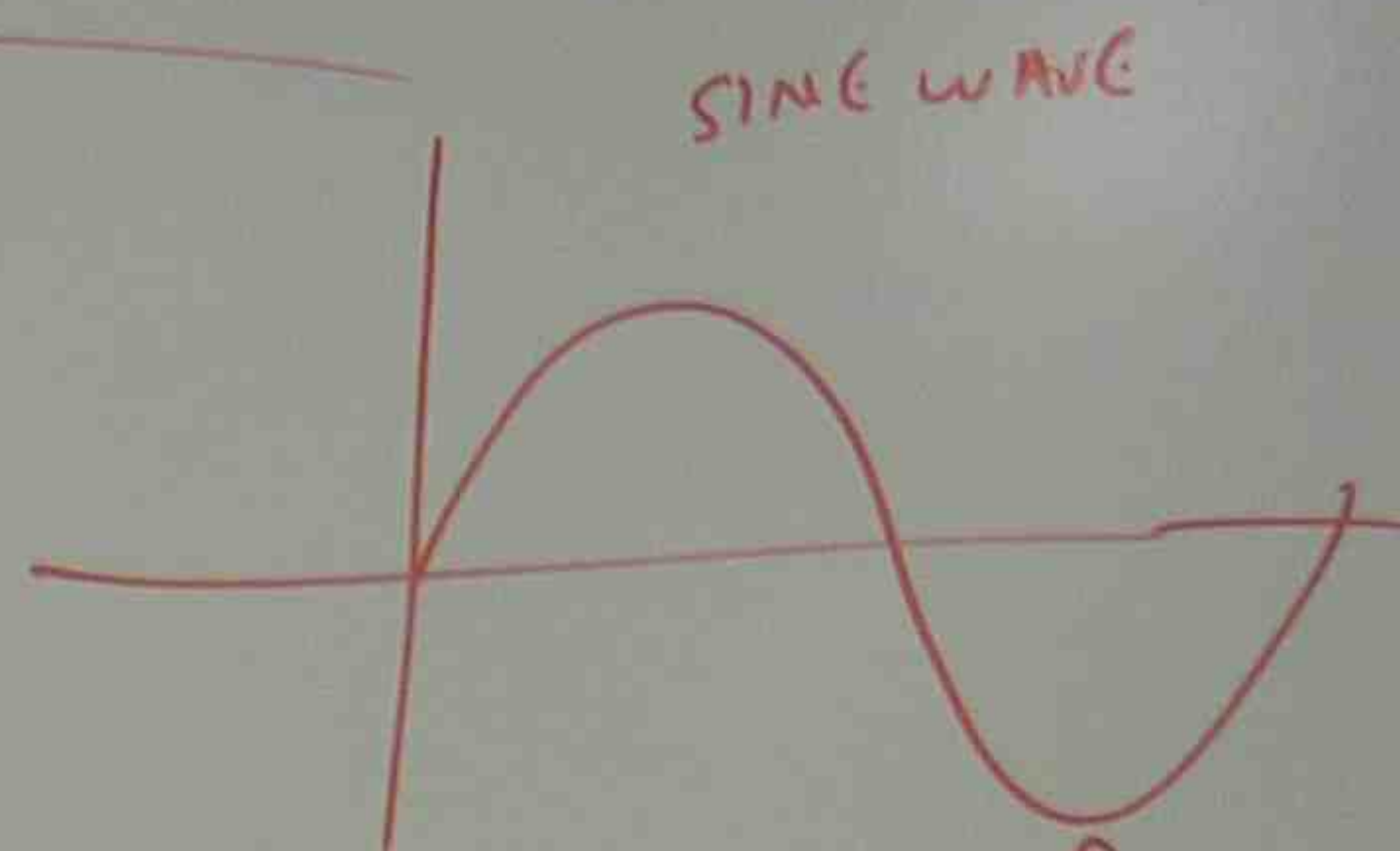


$$f = \frac{1}{T}$$
$$= \frac{1}{10 \times 10^{-3}}$$
$$= \frac{10^3}{10}$$
$$= 100 \text{ Hz}$$

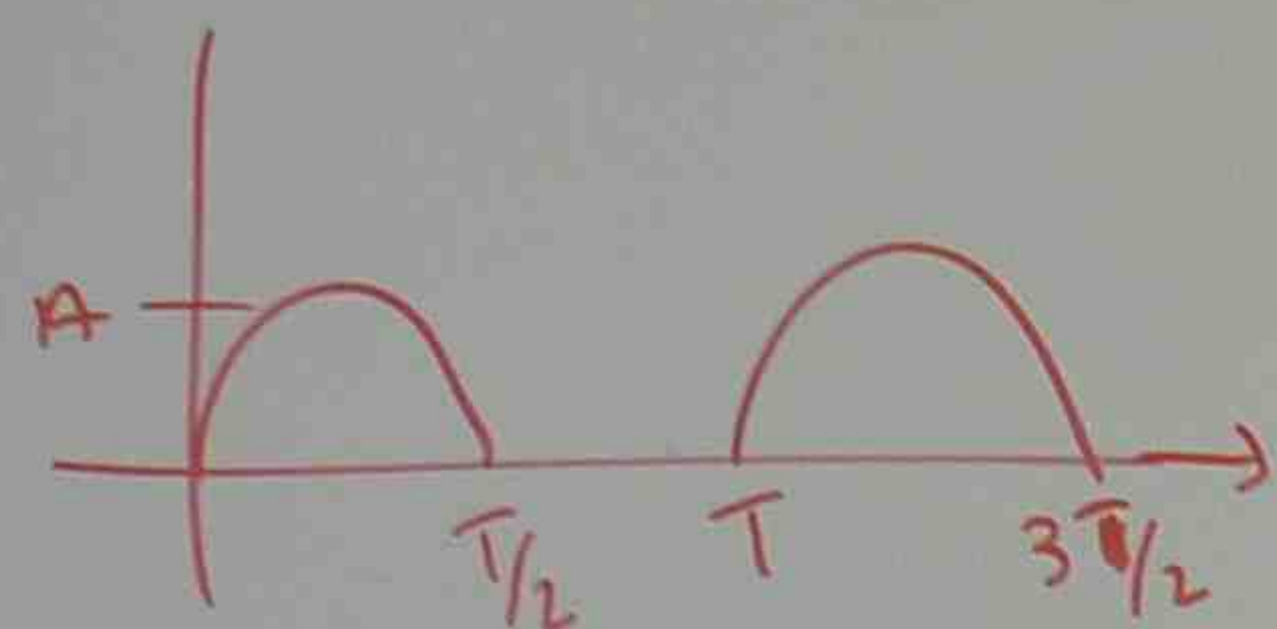
MATHEMATICAL EXPRESSION OF HARMONICS WAVE FORMS



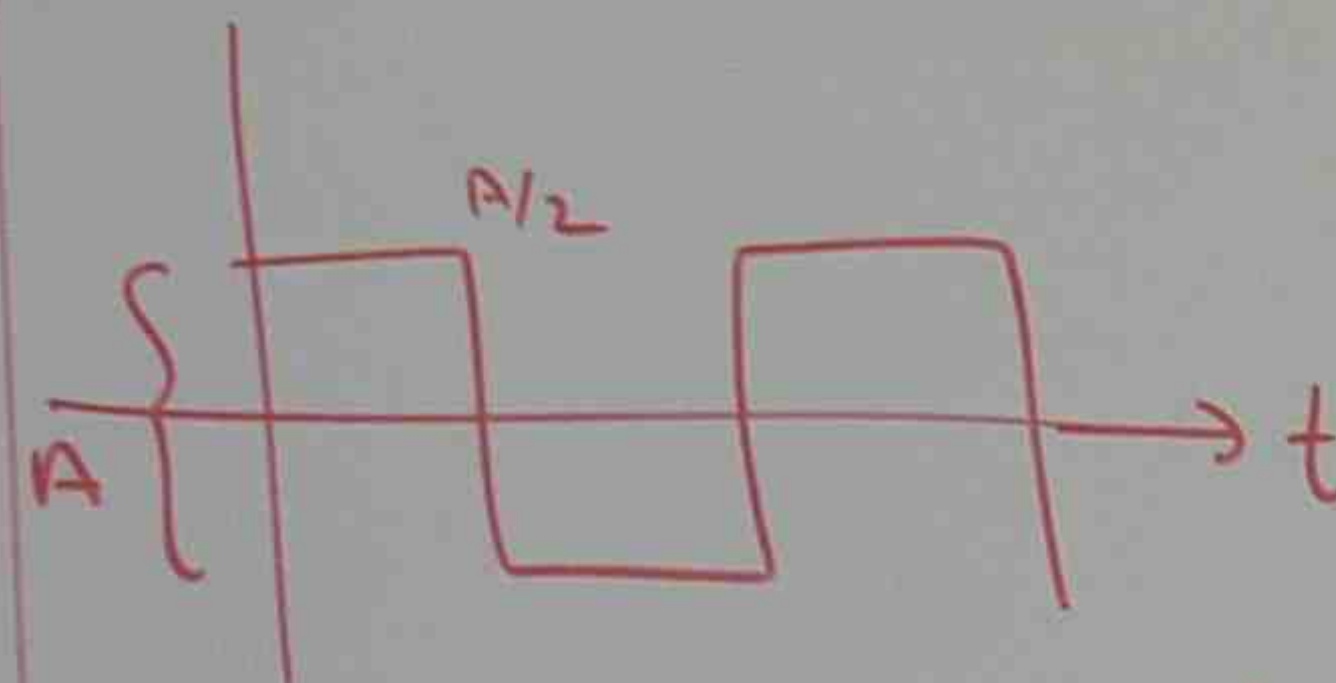
$$V(t) = A \cos 2\pi f t$$



$$V(t) = A \sin 2\pi f t$$



$$V(t) = \frac{A}{2} + \frac{A}{2} \sin 2\pi f_0 t + \sum_{n=2}^{\infty} \frac{A [1 + (-1)^n]}{n\pi (1 - n^2)} \cos 2\pi (n f_0) t$$



$$V(t) = \sum_{n=0}^{\infty} \frac{2A}{n\pi} \sin 2\pi n f_0 t$$

ncy

pb

A WAVE FORM HAS A PERIOD $T = 40 \text{ ms}$. CALCULATE THE FREQUENCY OF THE FUNDAMENTAL, THE SECOND AND THIRD HARMONICS.

$$\text{FUNDAMENTAL FREQUENCY } f = \frac{1}{T} = \frac{1}{40 \text{ ms}} = \frac{1}{40 \times 10^{-3}} \\ = \frac{10^3}{40} = 25 \text{ Hz}$$

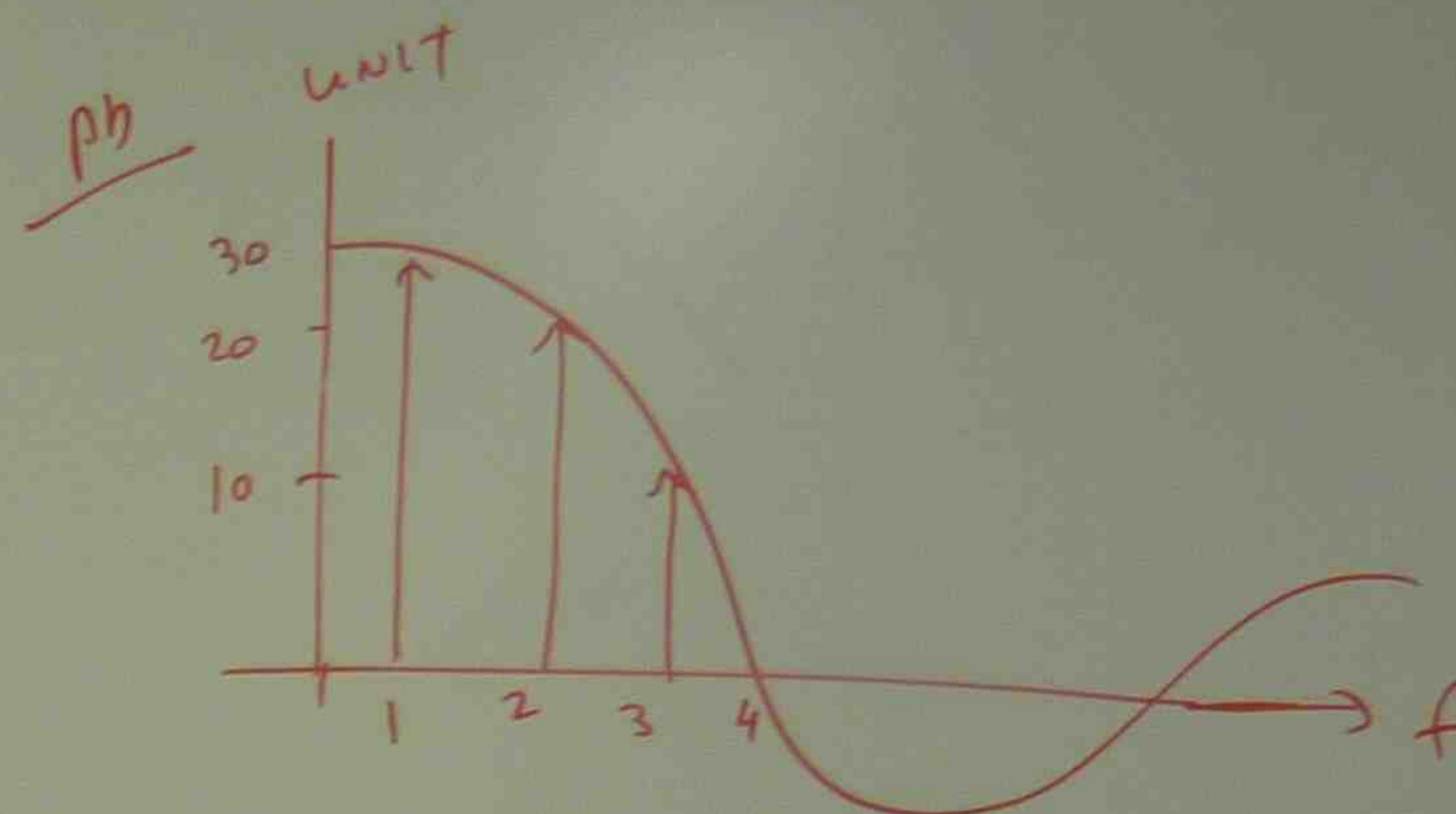
$$\text{SECOND HARMONICS} = 2f = 2 \times 25 \text{ Hz} = 50 \text{ Hz}$$

$$\text{THIRD HARMONICS} = 3f = 3 \times 25 \text{ Hz} = 75 \text{ Hz}$$

$$\text{FOURTH HARMONICS} = 4f = 4 \times 25 \text{ Hz} = 100 \text{ Hz}$$

$$\sin(2\pi f_0)t$$

$$\left(\frac{2A}{n\pi} \right) \sin 2\pi(nf_0)t$$



FIND 1st, 2nd, 3rd AND 4th HARMONICS

AMPLITUDES OF GIVEN WAVE FORM.

$$1^{\text{st}} = \text{FUNDAMENTAL} = 30 \text{ UNIT}$$

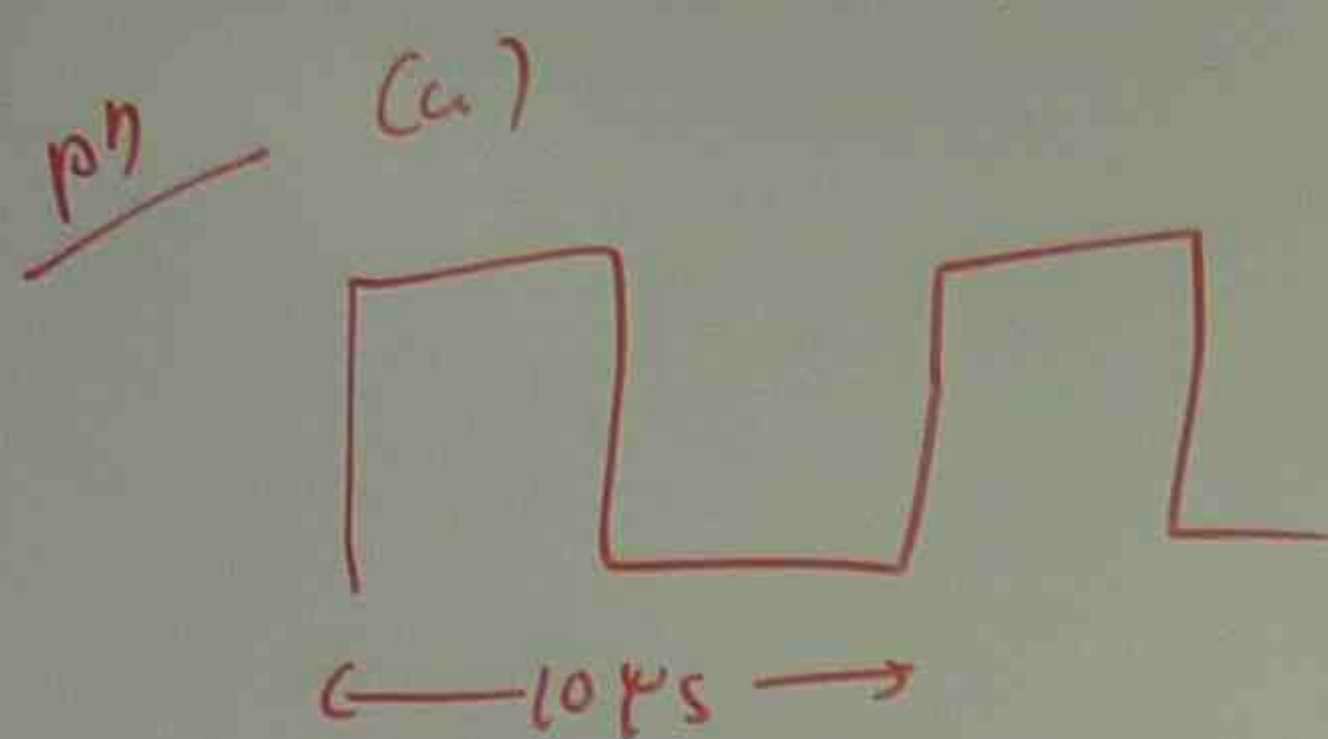
$$2^{\text{nd}} = \text{HARMONICS} = 20 \text{ UNIT}$$

$$3^{\text{rd}} = \text{HARMONICS} = 10 \text{ UNIT}$$

pb

f

CALCULATE THE FREQUENCIES

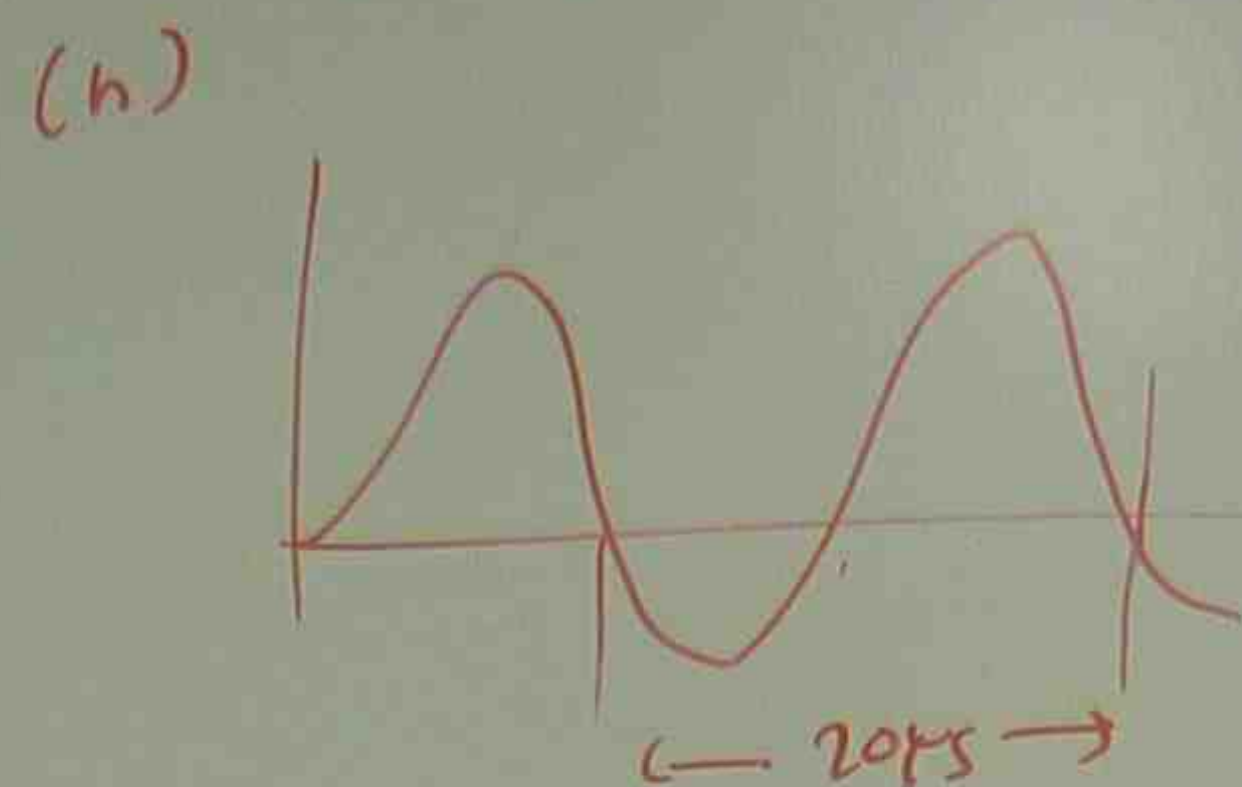


$$f = \frac{1}{T} = \frac{1}{10 \times 10^{-6}}$$

$$= \frac{10^6}{10}$$

$$= 10^5 \text{ Hz}$$

$$= 100 \text{ kHz}$$



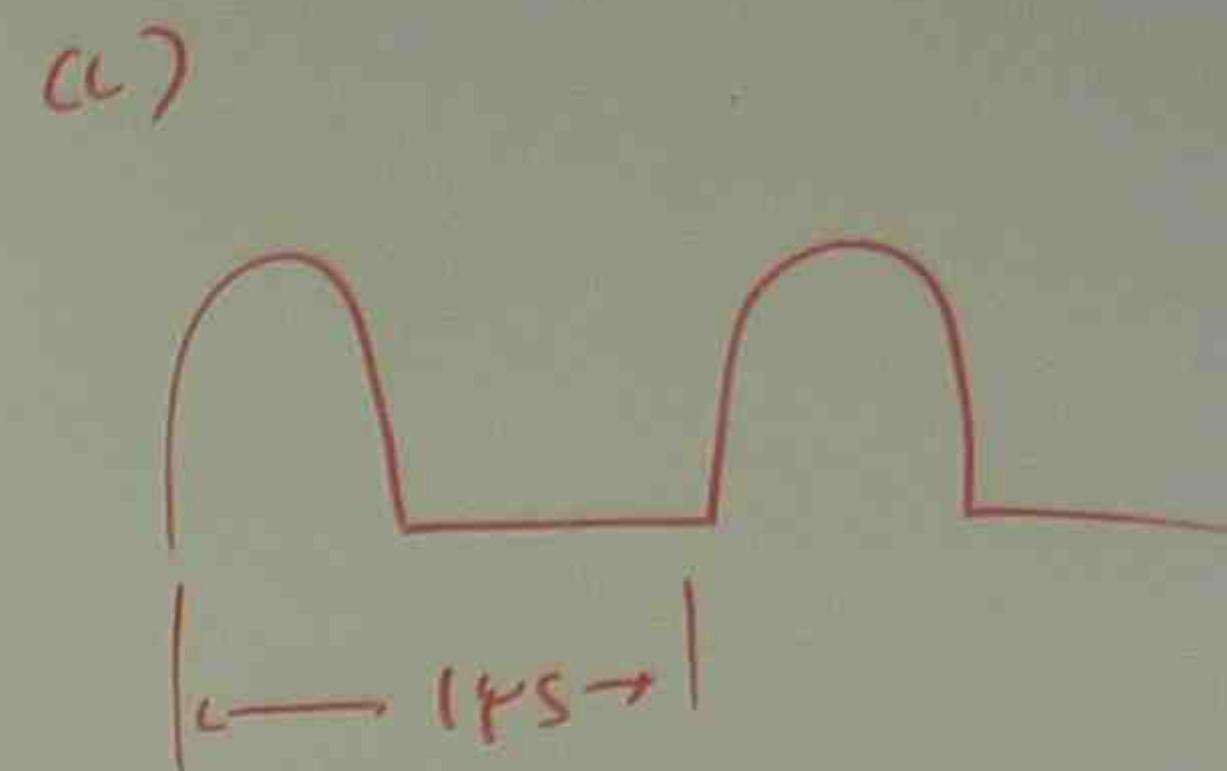
$$f = \frac{1}{T}$$

$$= \frac{1}{20 \times 10^{-6}}$$

$$= \frac{10^6}{20}$$

$$= \frac{1000 \times 10^3}{20}$$

$$= 50 \text{ kHz}$$

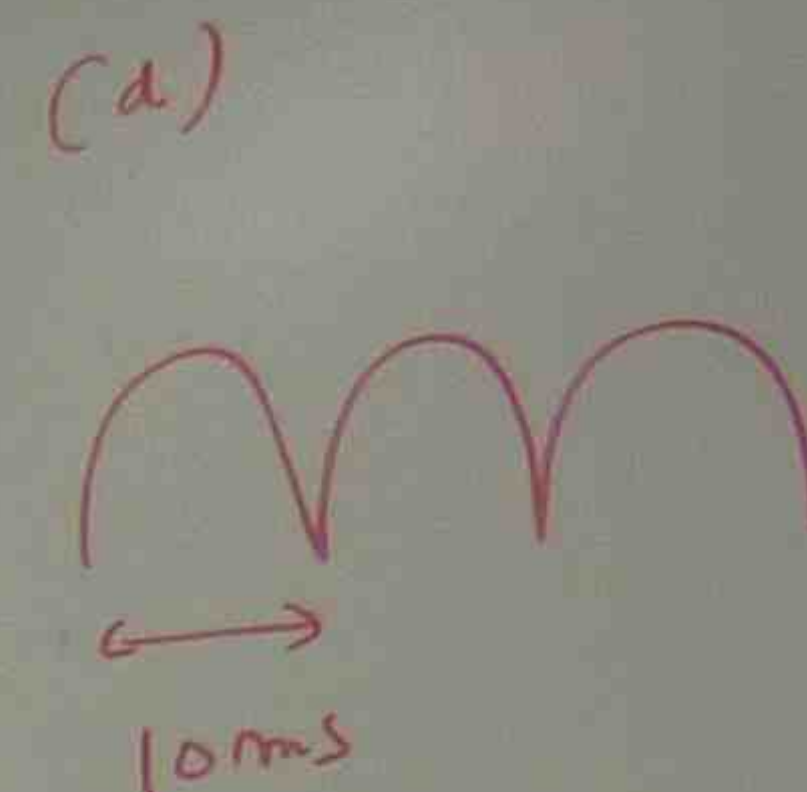


$$f = \frac{1}{T}$$

$$= \frac{1}{1 \times 10^{-6}}$$

$$= 10^6 \text{ Hz}$$

$$= 1 \text{ MHz}$$



$$f = \frac{1}{T}$$

$$= \frac{1}{10 \times 10^{-3}}$$

$$= \frac{10^3}{10}$$

$$= 100 \text{ Hz}$$

4th HARMONICS

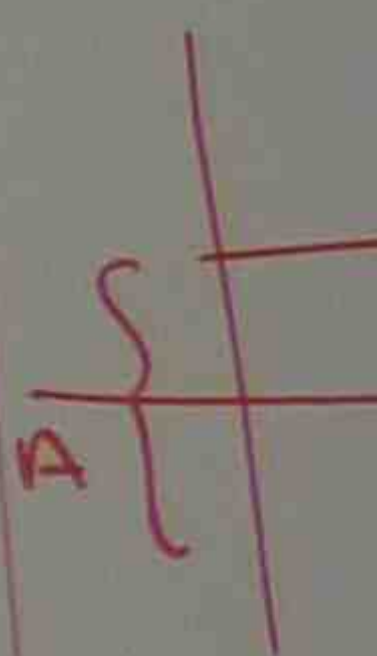
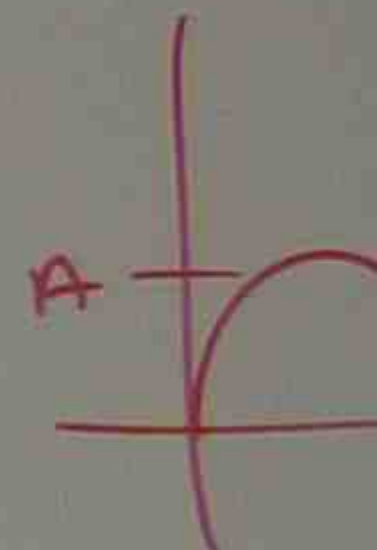
AVERAGE FORM.

UNIT.

UNIT

UNIT

MATHS



(d)



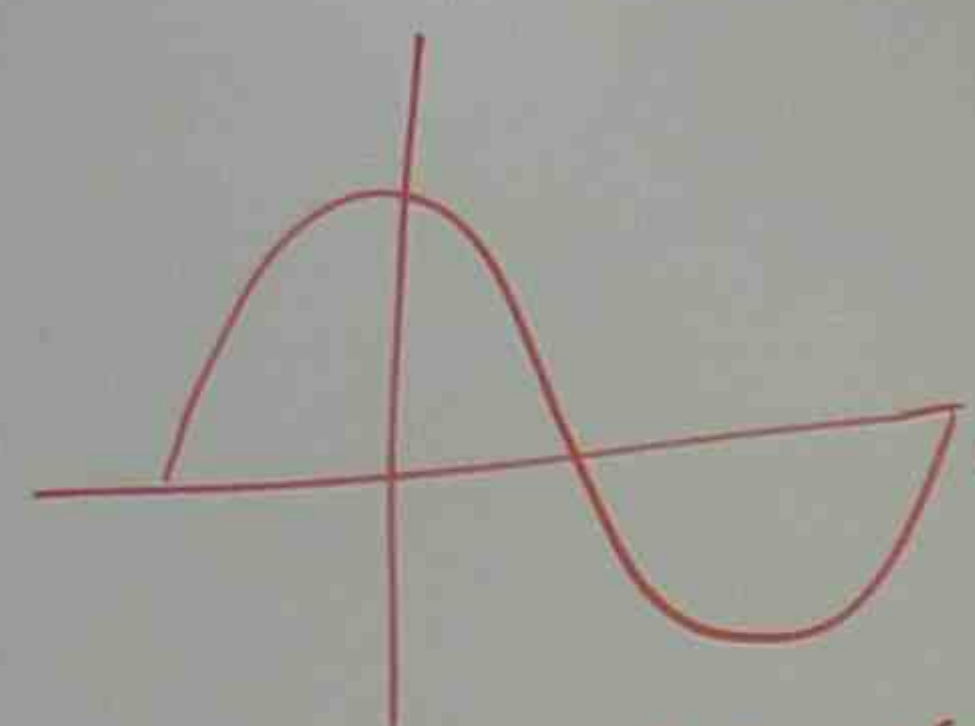
$$f = \frac{1}{T}$$

$$= \frac{1}{10 \times 10^{-3}}$$

$$= \frac{10^3}{10}$$

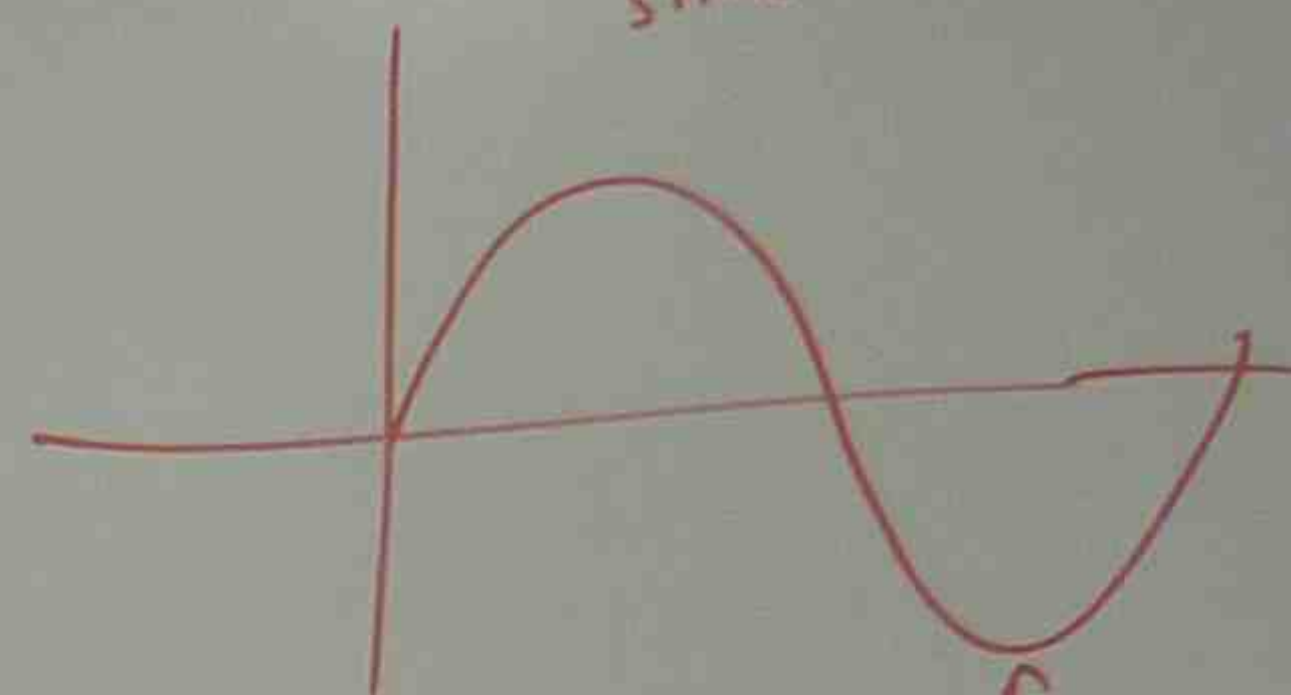
$$= 100 \text{ Hz}$$

MATHEMATICAL EXPRESSION OF HARMONICS WAVE FORMS



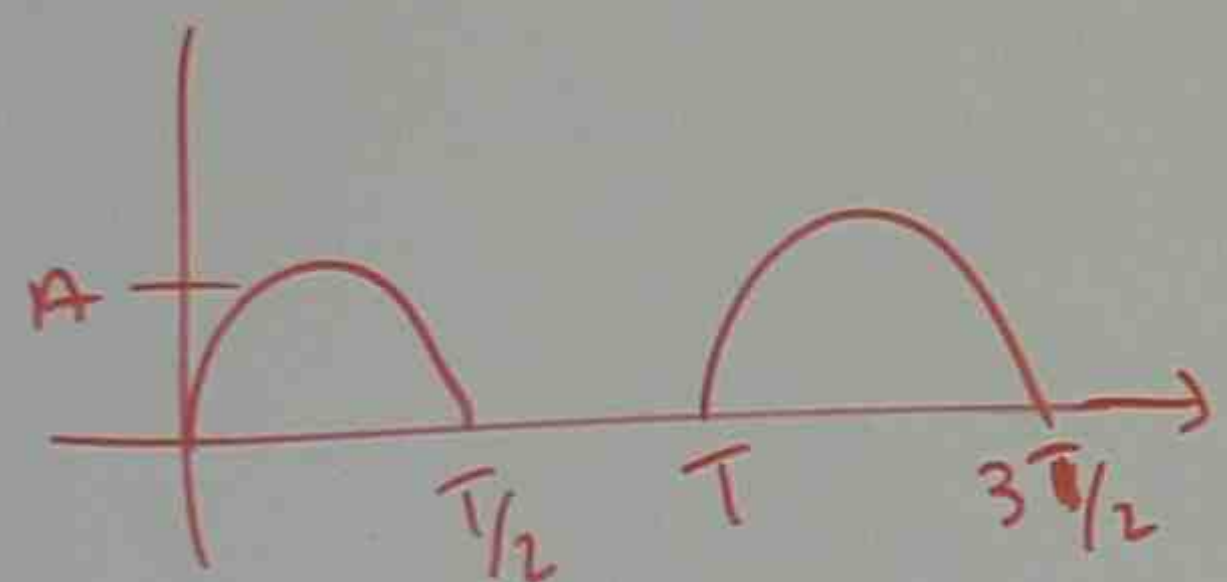
COSINE WAVE

$$V(t) = A \cos 2\pi f t$$



SINE WAVE

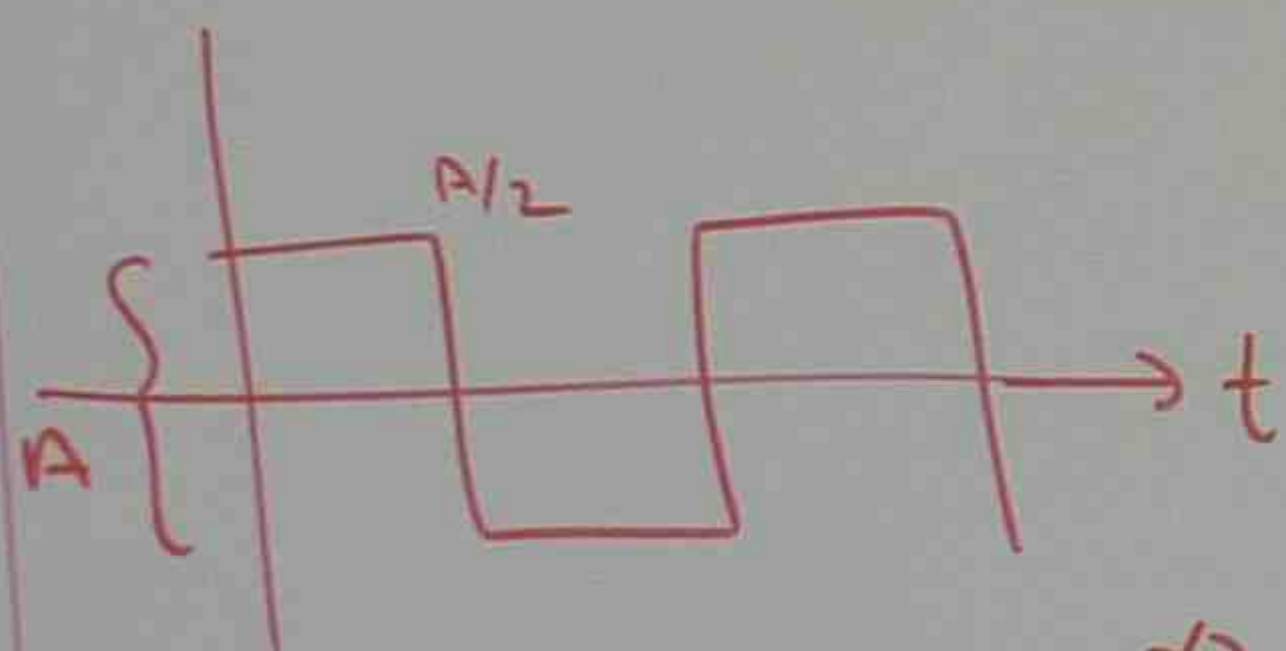
$$V(t) = A \sin 2\pi f t$$



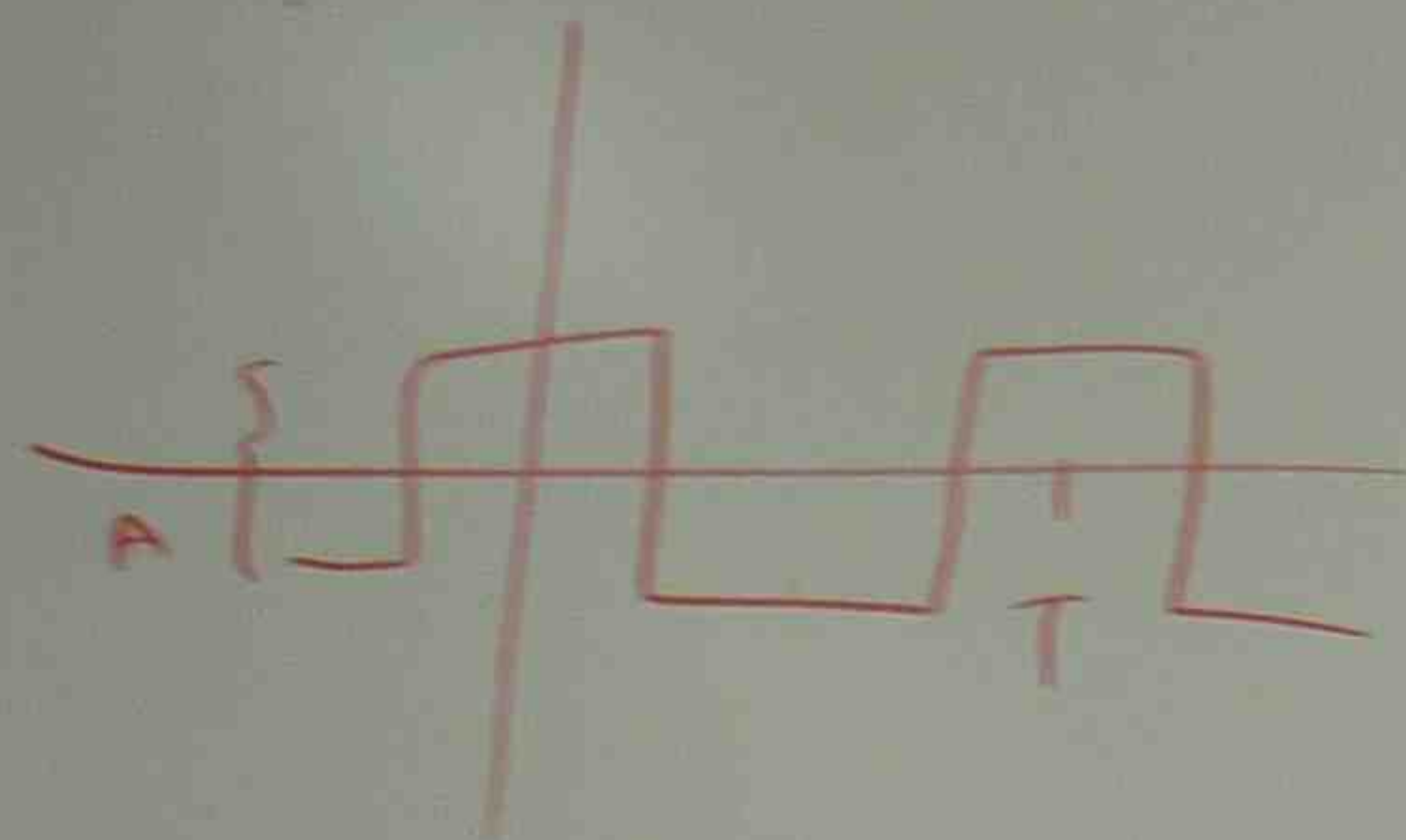
$$V(t) = \frac{A}{2} + \frac{A}{2}$$

$$\sin 2\pi f_0 t +$$

$$\sum_{n=2}^{\infty} \frac{A [1 + (-1)^n]}{\pi (1 - n^2)} \cos 2\pi (n f_0) t$$



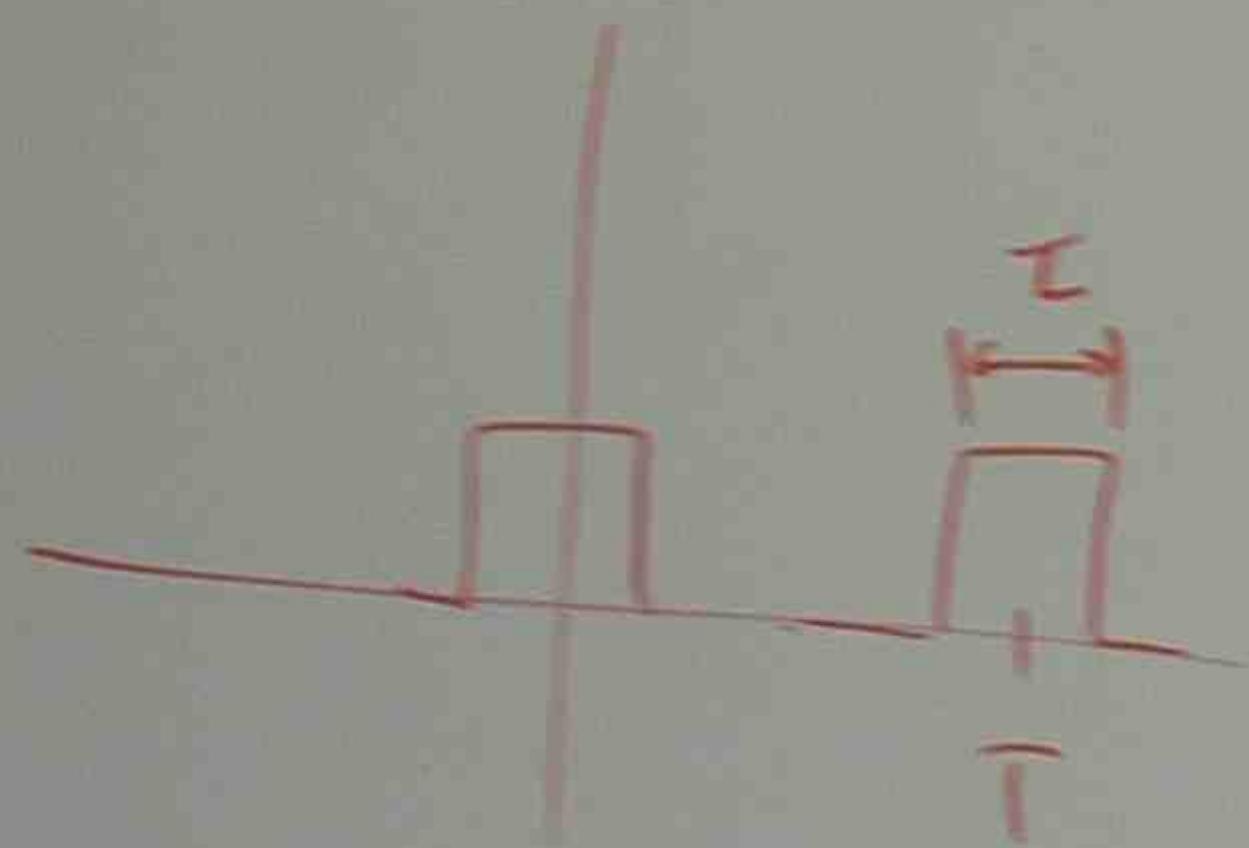
$$V(t) = \sum_{n=0}^{\infty} \frac{2A}{n\pi} \sin 2\pi n f_0 t$$



$$V(t) = \sum_{m=\text{odd}}^{\infty} \left(A \frac{\sin \frac{m\pi}{2}}{m \frac{\pi}{2}} \right) \cos 2\pi (m f_0) t$$

ODD = 1, 3, 5, 7

EVEN = 2, 4, 6, 8



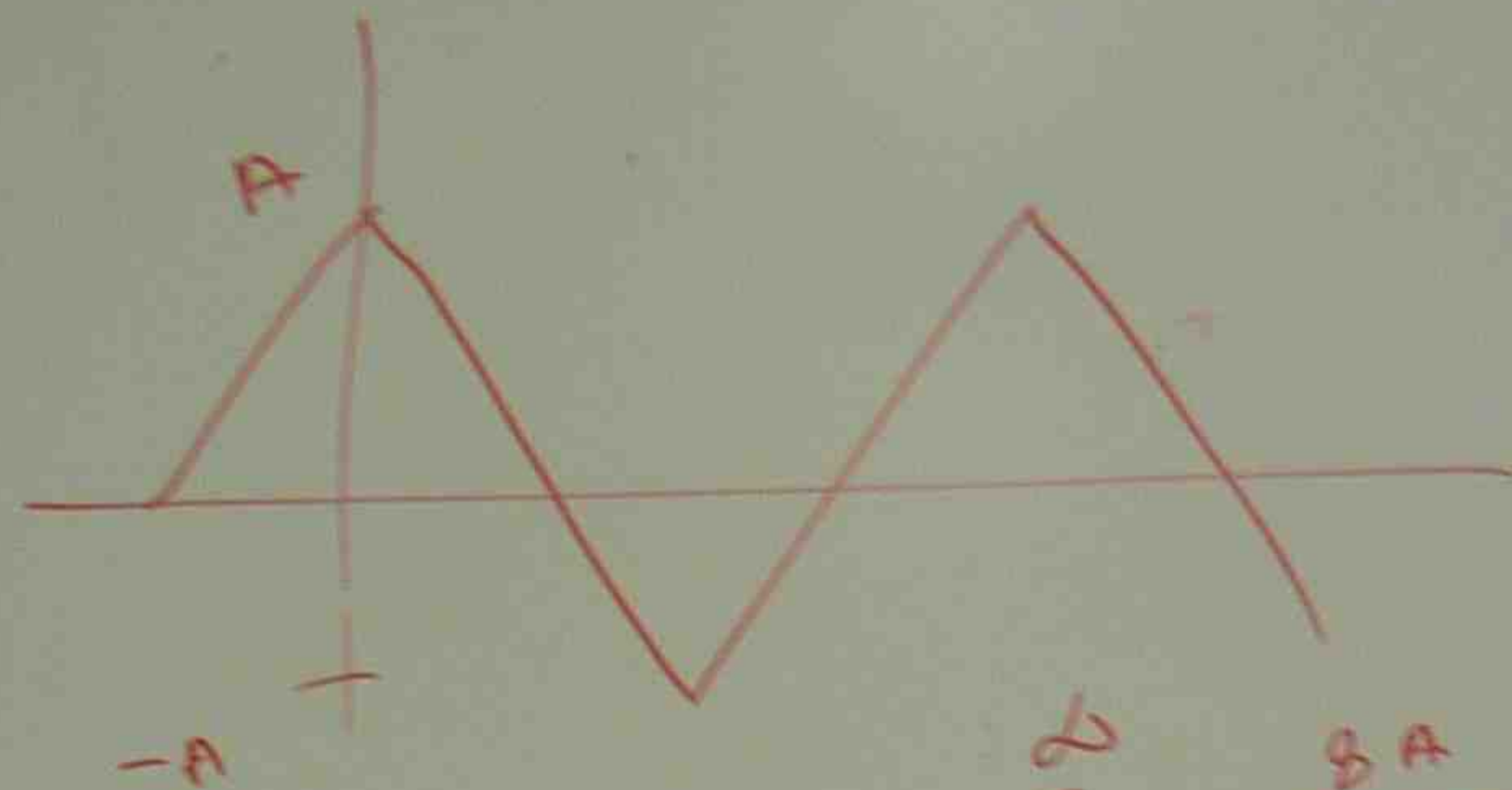
$$V(t) = \frac{A \tau}{T} + \sum_{m=3}^{\infty} \left(2A \frac{\tau}{T} \right) \left(\frac{\sin \frac{m\pi \tau/T}{2}}{2\pi \tau/T} \right) \cos 2\pi (m f_0) t$$

ODD = 1, 3, 5, 7

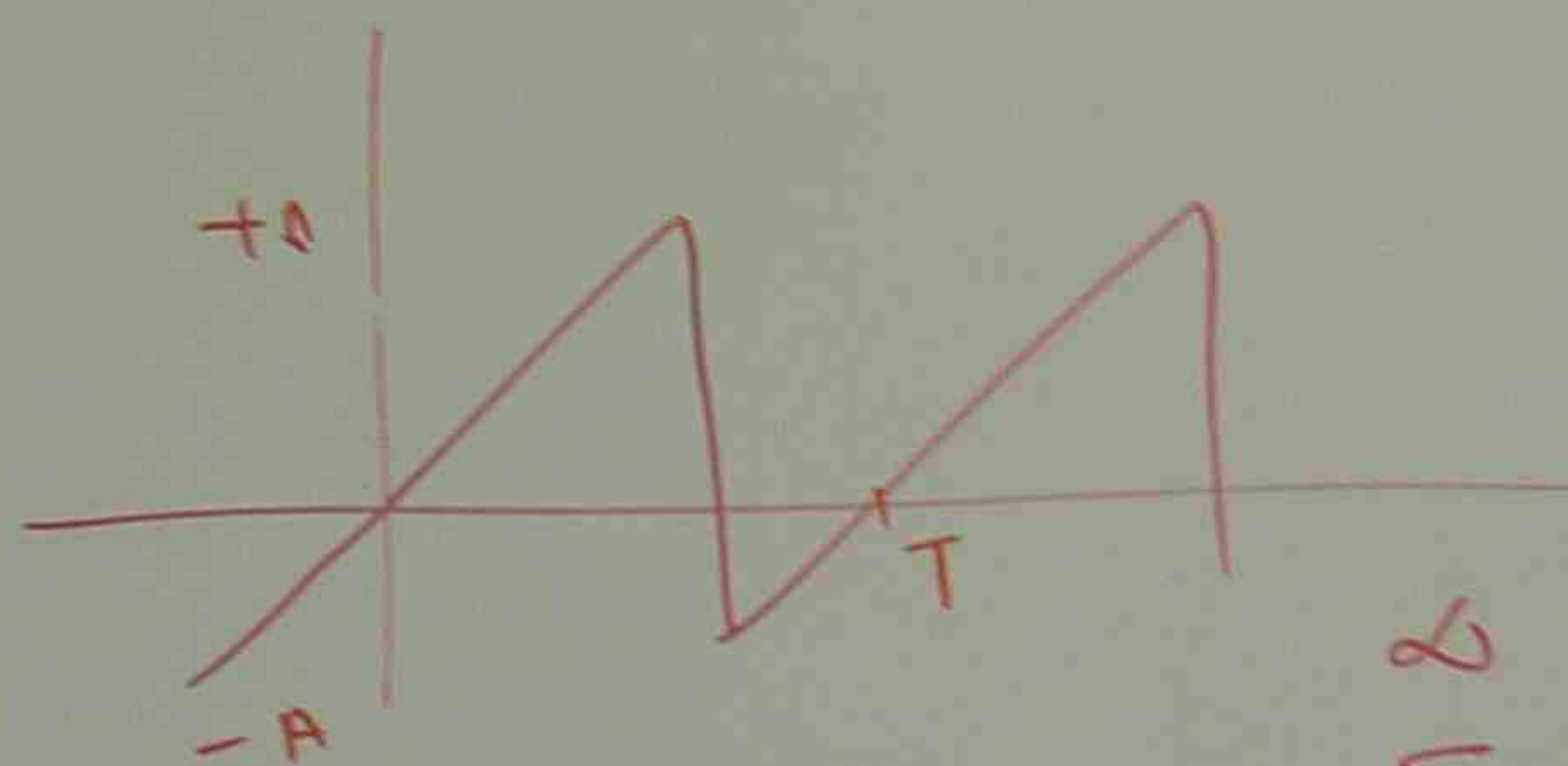
EVEN = 2, 4, 6, 8

$f_0)t$

$$\left(\frac{\sin m\pi \tau / T}{2\pi \tau / T} \right) \cos 2\pi (mf_0)t$$



$$V(t) = \sum_{m=\text{odd}}^{\infty} \frac{8A}{(m\pi)^2} \cos 2\pi (mf_0)t$$



$$V(t) = \sum_{m=1}^{\infty} (-1)^{m+1} \left(\frac{2A}{m\pi} \right) \sin 2\pi (mf_0)t$$

pb

A WAVE FOR
THE FREQUENCY

THIRD HARMONIC

FUNDAMENTAL FREQUENCY

SECOND HARMONIC

THIRD HARMONIC

FOURTH HARMONIC

REVIEW QUESTIONS FOR POWER ENGINEERING MATHS (TEST)

(1) 3 TOWNS (A, B, C) ARE LOCATED AS FOLLOWS.

DETERMINE THE MOST SUITABLE PLACE TO LOCATE
THE ELECTRICAL POWER STATION TO SUPPLY THOSE
TOWNS

A \rightarrow 100 MW (3, 5) km, B \rightarrow 300 MW (7, 10) km
C \rightarrow 400 MW (4, 17) km

(2) A TRANSMISSION LINE HAS 200m SPAN BETWEEN
SUPPORTS. THE CONDUCTOR WEIGHT IS 10 N/m AND
TENSION IN CONDUCTOR IS 11.5 kN . CALCULATE SAG.

(3) 3 PHASE 66 kV TRANSMISSION LINE IS DELIVERING
2 MW LOAD AT 0.95 PF LAGGING. LINE CONDUCTORS
HAVE 12.5 mm^2 . THE DISTANCE BETWEEN THEM ARE
 1.5 m , 2 m AND 3 m .

ING MATHS (TEST)

ATED AS FOLLOWS.

PLACE TO LOCATE
TO SUPPLY THOSE

300 MW (7, 10) km

200m SPAN BETWEEN

WEIGHT IS 10 N/m AND

IS 11.5 km . CALCULATE SAG.

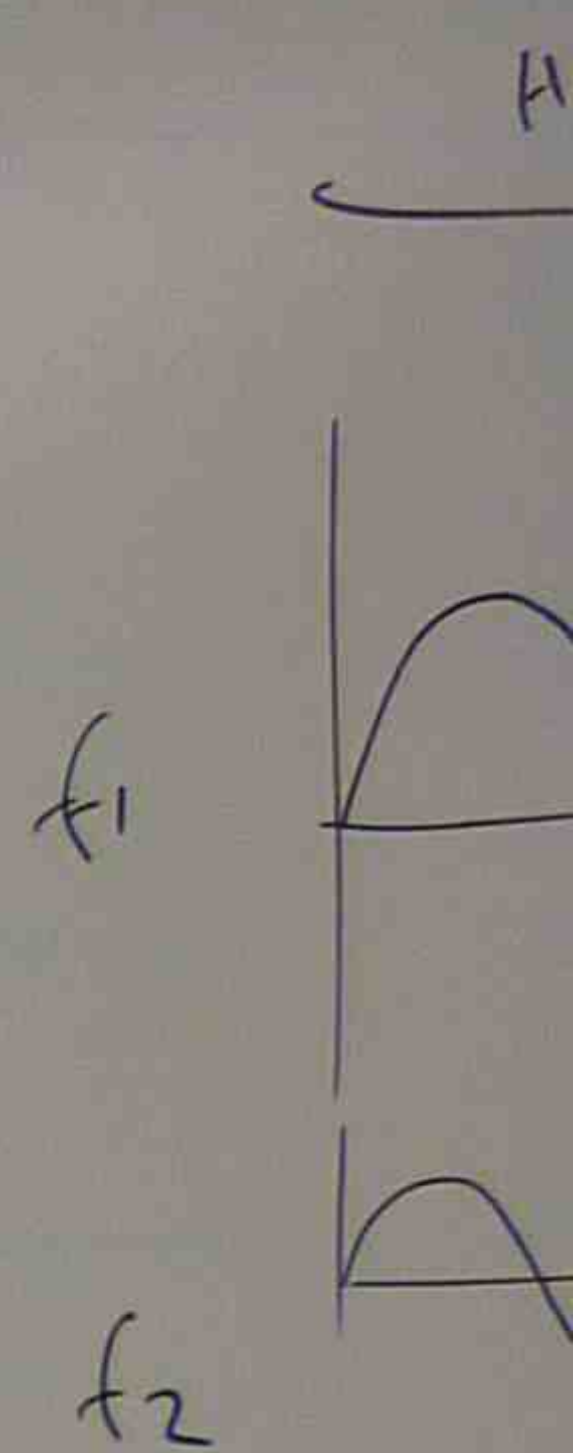
MISSION LINE IS DELIVERING

OF LAGGING. LINE CONDUCTORS

DISTANCE BETWEEN THEM ARE

THE LINE IS 300 km LONG. LINE RESISTANCE
IS $0.06 \Omega / \text{km}$. CALCULATE TOTAL LINE
INDUCTIVE REACTANCE AND CAPACTIVE REACTANCE
AT LINE FREQUENCY 50 Hz.

(4) A SINGLE-CORE CONCENTRIC CABLE IS
TO BE MANUFACTURED FOR 200KV, 60 Hz
LINE. THE PAPER USED HAS MAXIMUM
PERMISSIBLE SAFE STRESS 10^7 V/m (rms)
AND DIELECTRIC CONSTANT 4.5.
CALCULATE THE DIMENSION FOR THE MOST
ECONOMICAL CABLE AND CHARGING CURRENT
PER km.



$$f(t) =$$

Pt

NG. LINE RESISTANCE

ATE TOTAL LINE

ACTIVE REACTANCE

RIC CABLE IS

R 200KV, 60HZ

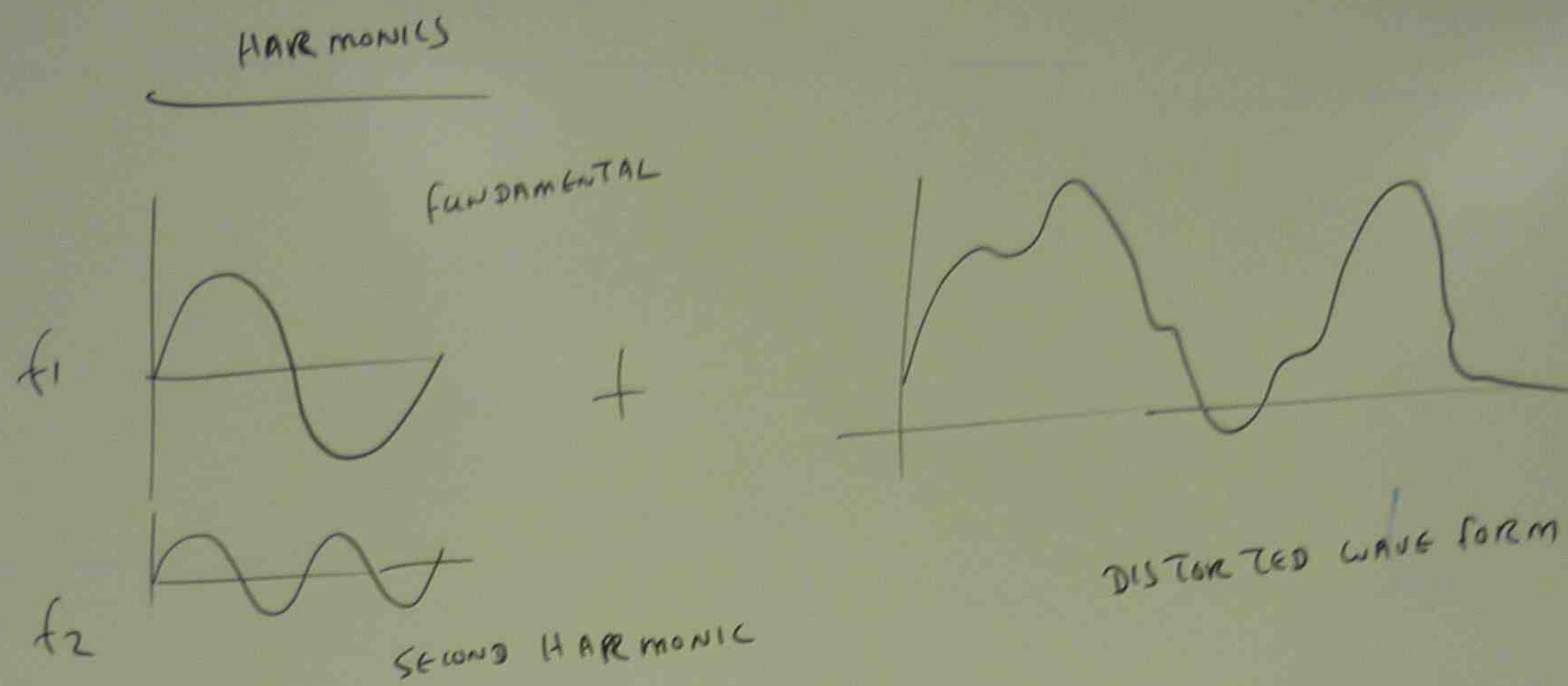
S MAXIMUM

10^7 V/m (rms)

4.5.

FOR THE MOST

CHARGING CURRENT



$$f(t) = A_1 \sin \omega t + A_2 \sin(2\omega t \pm \phi_1) + A_3 \sin(3\omega t \pm \phi_2) + \dots$$

$$P_t = \text{TOTAL POWER} = P_{1m} + P_{3m} + P_{5m} + \dots$$

$$V = \sqrt{\frac{V_{1m}^2 + V_{3m}^2 + V_{5m}^2}{2}}$$

$$I_m = \sqrt{\frac{I_{1m}^2 + I_{3m}^2 + I_{5m}^2}{2}}$$

ph

A VOLTAGE

$e = 30$

VOLT IS

IS GIVEN

$i = 0.5 \text{ S}$

Ampl.

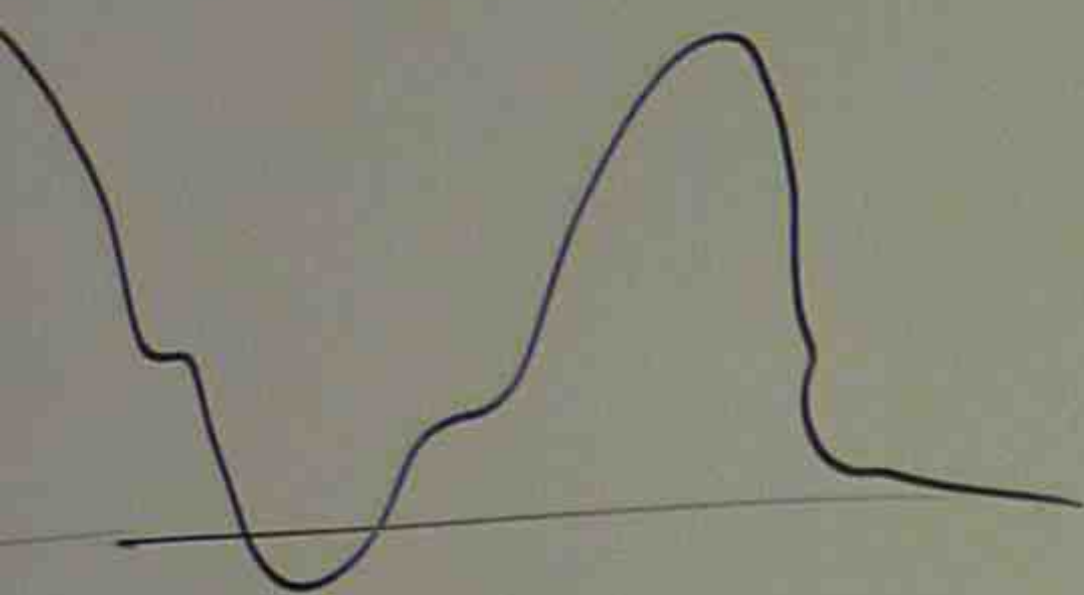
FIND TOT

$P =$
 m

$P_{1m} =$

$P_{3m} =$

$P_{5m} =$



DISTORTED WAVE FORM

$$A_1 \sin(\omega t + \phi_1) + A_3 \sin(3\omega t + \phi_2) + \dots$$

$$+ P_{sm} + \dots$$

Pb

A VOLTAGE IS GIVEN BY

$$e = 30 \sin \omega t + 20 \sin(3\omega t + 30^\circ) + 10 \sin(5\omega t - 90^\circ)$$

VOLT IS APPLIED TO A CIRCUIT AND THE RESULTING CURRENT IS GIVEN BY

$$i = 0.5 \sin(\omega t - 17^\circ) + 0.1 \sin(3\omega t - 15^\circ) + 0.09 \sin(5\omega t - 150^\circ)$$

Amp.

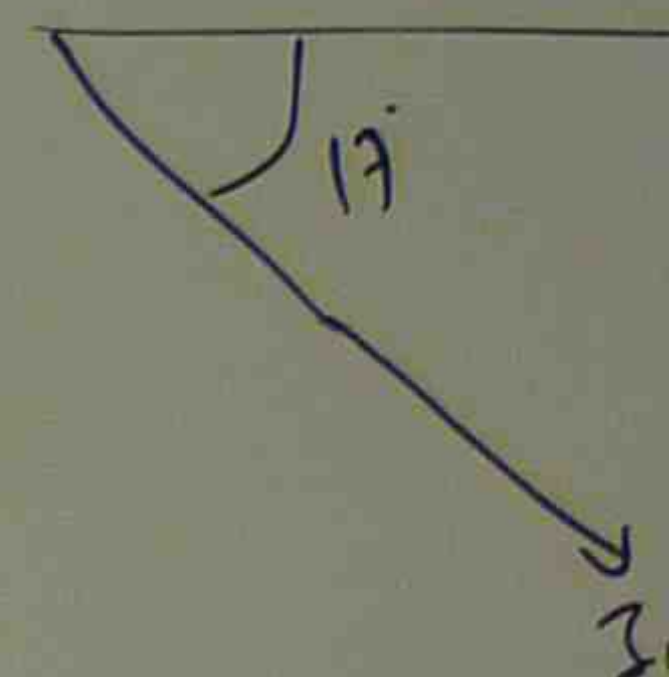
FIND TOTAL POWER APPLIED AND OVER ALL POWER FACTOR

$$P_m = \frac{V_m I_m}{2} \cos \theta$$

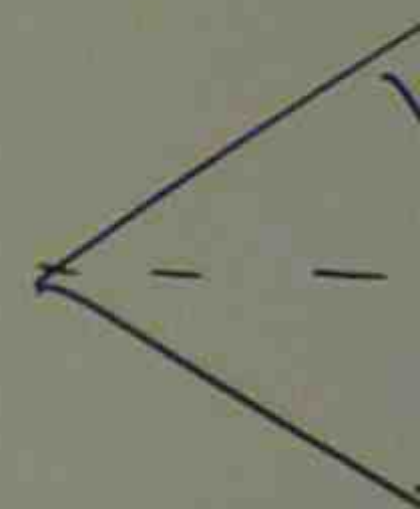
$$P_{1m} = \frac{V_{1m} I_{1m}}{2} \cos \theta$$

$$P_{3m} = \frac{V_{3m} I_{3m}}{2} \cos \theta$$

$$P_{5m} = \frac{V_{5m} I_{5m}}{2} \cos \theta$$



$$P_{1m} = \frac{30 \times 0.5}{2}$$



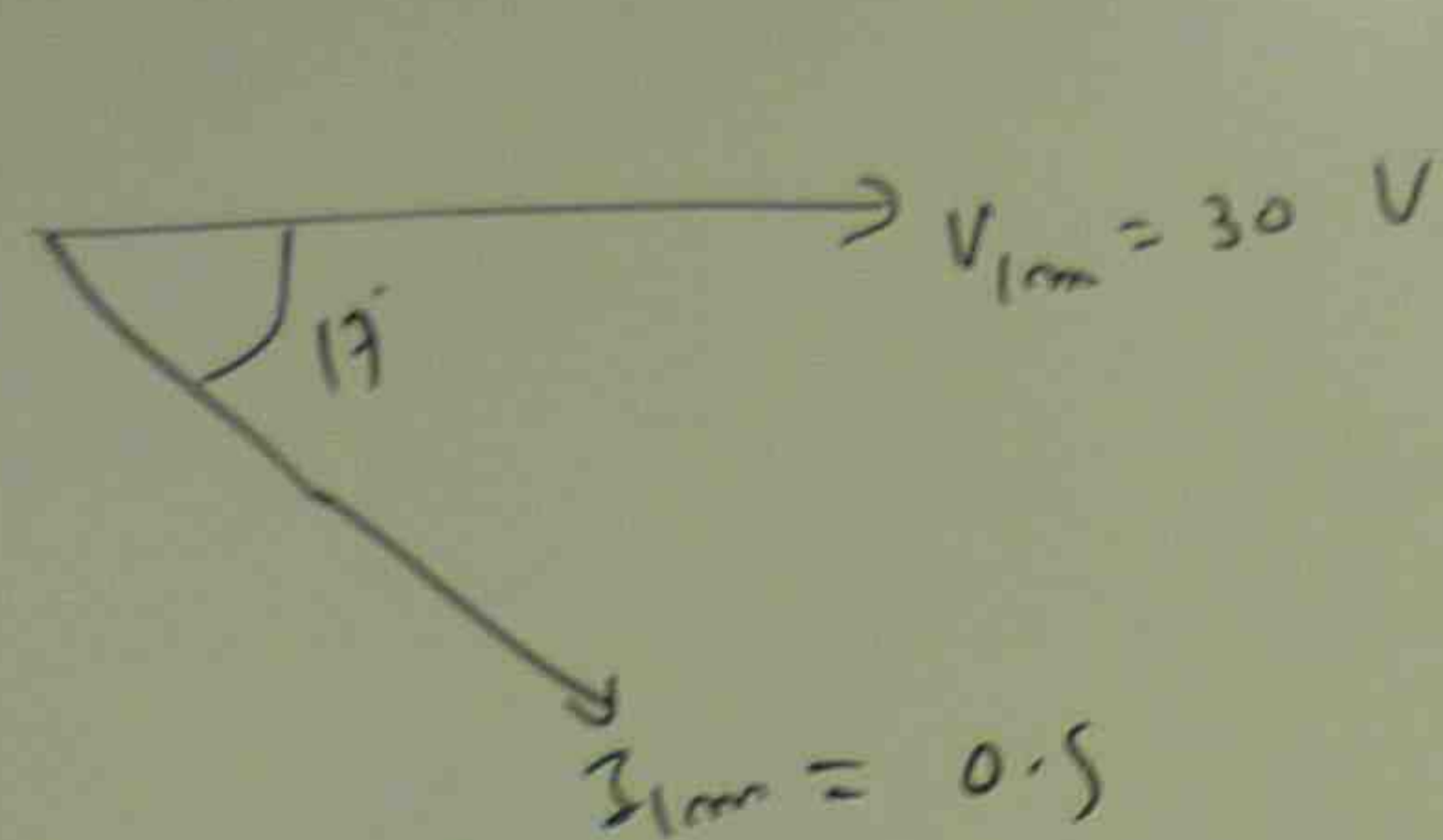
$$P_{3m} = \frac{20 \times 0.1}{2}$$

($\phi_{wt} - \phi_0$)

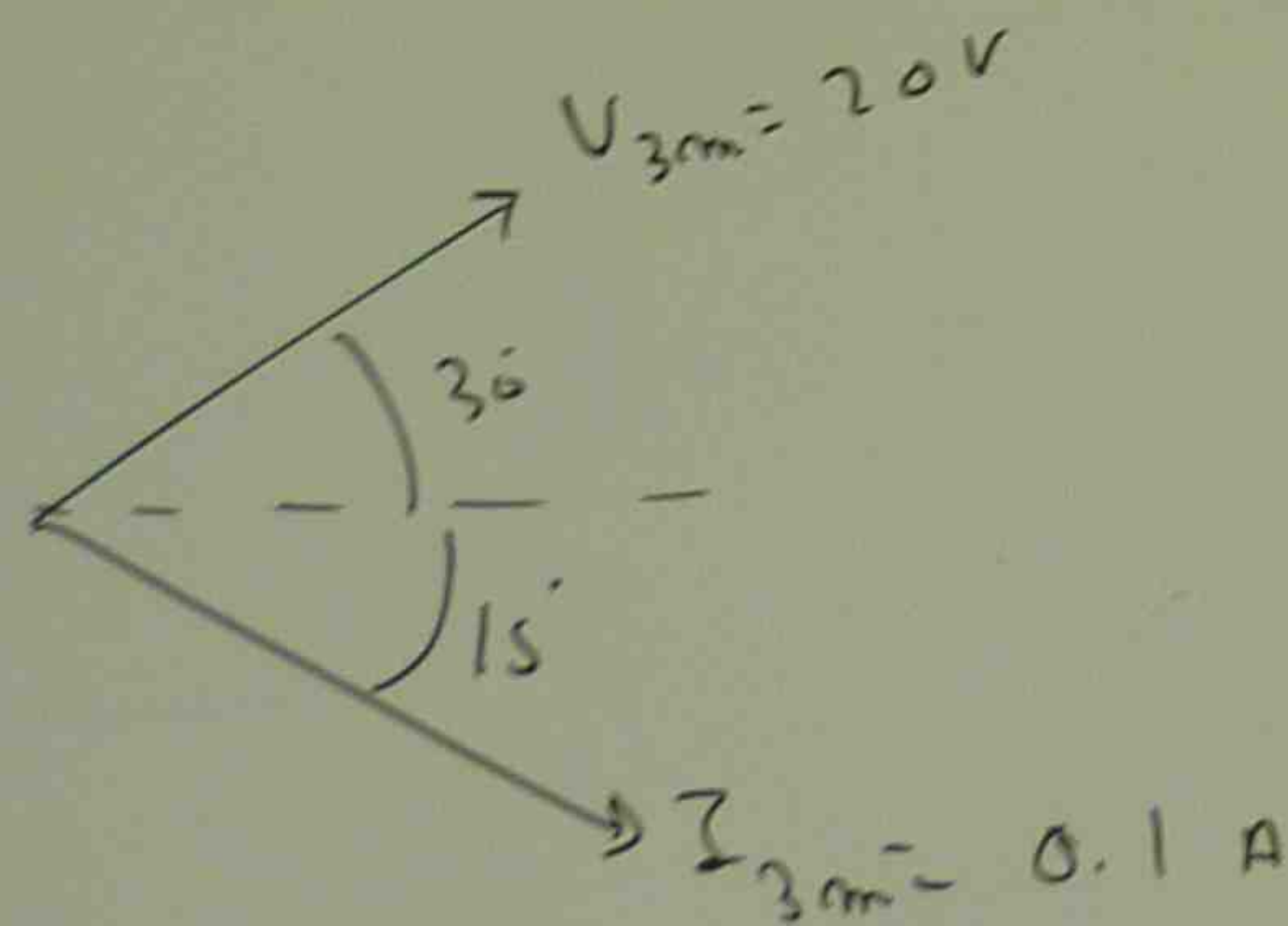
TUNG CURRENT

0.09 Sin ($\phi_{wt} - 150$)

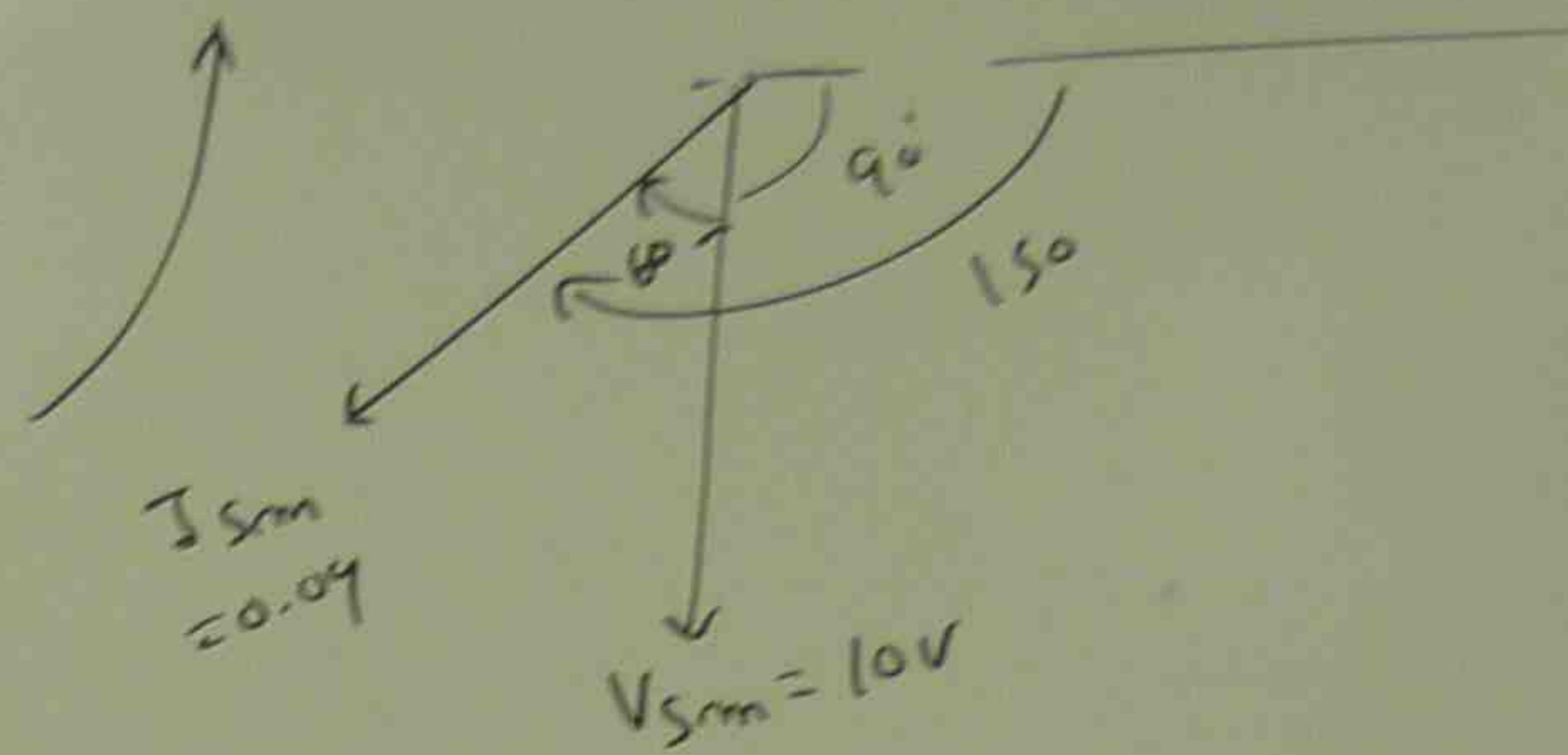
LL POWER FACTOR



$$P_{1m} = \frac{30 \times 0.5}{2} \cos 17 = 7.1 \text{ W}$$



$$P_{3m} = \frac{20 \times 0.1}{2} \cos 45 = 0.7 \text{ W}$$



$$P_{5m} = \frac{10 \times 0.09}{2} \cos 60 = 0.23 \text{ W}$$

$$\begin{aligned} P_T &= P_{1m} + P_{3m} + P_{5m} \\ &= 7.1 + 0.7 + 0.23 \\ &= 8.03 \text{ W} \end{aligned}$$

$$V_{rms} = \sqrt{\frac{V^2}{2}}$$

$$I_{rms} = \sqrt{\frac{I^2}{2}}$$

$$PF = \cos \phi$$

$$V_{rms} = \sqrt{\frac{V_{1m}^2 + V_{3m}^2 + V_{5m}^2}{2}} = \sqrt{\frac{30^2 + 20^2 + 10^2}{2}} = 18.7 \text{ V}$$

$$I_{rms} = \sqrt{\frac{I_{1m}^2 + I_{3m}^2 + I_{5m}^2}{2}} = \sqrt{\frac{0.9^2 + 0.1^2 + 0.09^2}{2}} = 1.5 \text{ Amp}$$

$$\text{TOTAL V.A} = V_{rms} I_{rms}$$

$$= 18.7 \times 1.5 = 28.06 \text{ V.A}$$

$$PF = \frac{P_T}{\text{TOTAL V.A}} = \frac{8.03}{28.06} = 0.281$$