

Another possible problem is related to the accumulation of sludge in the transformer tank and windings. If a transformer is taken out of service for any extended time, any sludge in the transformer oil allowed to settle in the bottom of the tank can coagulate or harden and become difficult to remove. Sludge in the oil can also become a very serious problem in the subsequent recommissioning of the transformer if the sludge has accumulated in the cooling ducts within the transformer winding. Any blockage of these cooling-oil ducts will significantly reduce the rate of oil flow through the winding and reduce the 'effective' rating below the nameplate rating. Constraints of the flow of the cooling oil will reduce the effective cooling of the windings, especially when the transformer is operating near or above full load.

Because of the very large mass of a fully operational transformer, reinforced-concrete areas are provided in switchyards on which the transformers are placed (to prevent them sinking into the ground). Most large transformer tanks are fitted with small wheels for the tank to be moved into its final position on the reinforced-concrete area and suit the local cabling and other requirements, such as clearances of other equipment.



## Check your progress 2

- 1 After the core laminations are assembled, they are fitted with strapping. Give at least two reasons for fitting the strapping.
- 2 In what basic shapes are transformer coils usually made?
- 3 Calculate the area between a square with 2 cm sides and the smallest circle into which this square will fit.
- 4 In the construction of a transformer, state the purpose of these items:
  - (a) bushings
  - (b) conservator
  - (c) tank wheels
  - (d) silica gel breather filters.(Write brief answers of about three to four lines each.)
- 5 Give brief definitions of the following terms when used in relation to transformer oil:
  - (a) acidity
  - (b) sludge
  - (c) flashpoint
  - (d) oxidation
  - (e) inhibitors.
- 6
  - (a) List the five basic requirements for transformer oil used in oil-immersed transformers.
  - (b) Explain why it is necessary to filter transformer oil.
- 7 List three factors to be considered when a power transformer is to be transported from one location to another by road trailer.
- 8 List five major parts of an oil-filled power transformer.

## Objectives

When you have worked through this section, you should be able to

- describe the basic features and operation of a transformer
- demonstrate an understanding of magnetic flux, and sketch the flux paths in core- and shell-type transformer construction
- (for a nominated coil) determine the peak value of magnetic flux using formulas, and carry out calculations relating magnetic flux, voltage, frequency and turns
- describe the construction of the cores, and the electrical and mechanical considerations of the windings.

## The principles of the transformer

One of the greatest advantages of an ac electricity supply system is the easy and efficient manner in which the voltage can be increased or decreased according to need. Electrical transformers are the equipment used to achieve this feature.

A transformer can be considered a 'matching device', since it matches a given power source with the voltage rating of a load.

The electrical transformer is a piece of static equipment with any moving parts being quite small and of minimum power consumption. This reduces the sources of possible losses and also minimises the maintenance required.

### The operation of a transformer

The operation of an electrical transformer is based on the law of electromagnetic induction which states that

if a conductor is placed in a changing magnetic field there is an emf induced into that conductor.

Regardless of their size and function, all electrical transformers rely on this principle for their operation.

In its simplest form, a transformer consists of two coils of wire mounted on a common closed magnetic circuit. The coil that receives energy from the ac supply system, and produces the magnetic field essential to the operation of the transformer, is termed the *primary* coil. The other coil, which is designed to deliver energy at a higher or lower voltage to the load but at the same frequency, is termed the *secondary* coil.

It is important for you to note that the fundamental theory of transformers is based on the premise that the voltage and current waveforms are sinusoidal, as only sinusoidal quantities are faithfully reproduced in an electrical transformation process. If you were to look at these

waveforms (on a cathode ray oscilloscope, or CRO) you would see that the waveforms are not absolutely sinusoidal, but are slightly distorted. You should be able to compensate in your calculations for this distortion, and these techniques will be shown to you in later sections.

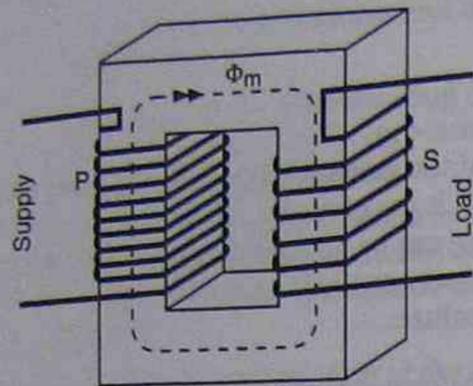


Figure 1: A simple transformer using a laminated core

Figure 1 shows an arrangement for the coils and the magnetic circuit of a simplified transformer. The primary and secondary coils are denoted by the letters 'P' and 'S'.

When the primary winding is connected to an ac supply system, an alternating flux is established in the magnetic circuit; this flux links both primary and secondary coils and acts in both directions between the primary and secondary coils. As a result, the secondary coil has an emf induced into it, and this emf produces a current in any external load which may be connected.

The primary coil also has an emf induced into it (from the action of the flux linkages of the current flowing in the secondary winding), and this (reversed) emf acts to oppose the supply voltage. The difference between the supply voltage and the (reversed) emf induced into the primary winding allows only enough current into the primary coil to provide both the energy for the load and all transformer internal losses.

### A simple experiment

Transformer action can be seen in a simple experiment. Figure 2 shows an iron ring with two pieces of wire wound on it. There are two separate circuits. The first, or primary circuit, has a battery and a switch connected into it. The second, or secondary circuit, has an ammeter.

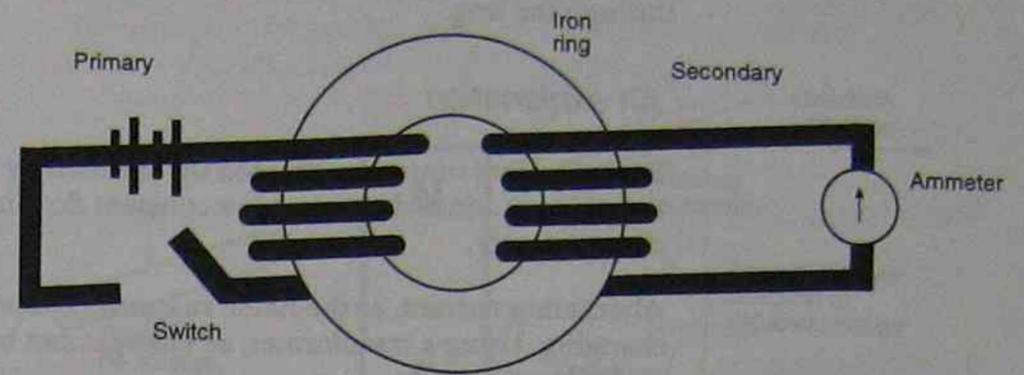


Figure 2: A simple transformer using a toroid 'doughnut' core

The ammeter can show current flow in both directions, being swapped if it is a centre-zero type, without the connections. It is better to use a galvanometer here as it is very sensitive to small currents. *Take care* when using a galvanometer because of its greater sensitivity and its likelihood of being irreparably damaged.

Current can flow in the primary winding when the switch is closed. The current is determined by the resistance of the wire and the voltage of the battery. At the time of the switch closing there is no current flow, and it takes a short period of time for the current to reach its maximum value after closing. During this short period of time, while the primary current is changing, the ammeter in the secondary circuit shows a current flowing.

While the current in the primary is constant there is no current flowing in the secondary and the ammeter shows zero amps. When the switch is opened, the ammeter in the secondary shows a current flow for a short period of time in the reverse direction, then no current when the primary current has returned to zero.

The changing primary current produces a current in the secondary. Constant direct current produces no current in the secondary.

In practical use of voltage and power transformers the secondary is normally not shorted with an ammeter. (This could have disastrous results due to the high currents produced.) You can do this with your experiment(s) because you can control the voltage and currents produced and keep them very low.

This experiment was first done by Michael Faraday in 1831, and can be done by you if you have the equipment. You will need a 'soft' iron ring and a sensitive ammeter. You

will also need plenty of patience to put many turns of wire through the ring.

### An explanation

Direct current cannot be stepped up or down by a transformer, since dc current is a constant flow in one direction only.

Alternating current, as the name suggests, is always changing. Using a transformer, ac voltages can be stepped up or down.

When a current passes through a wire a magnetic field is produced, as shown in Figure 3. Magnetic-flux lines may also be called magnetic-field lines, or magnetic lines of force.

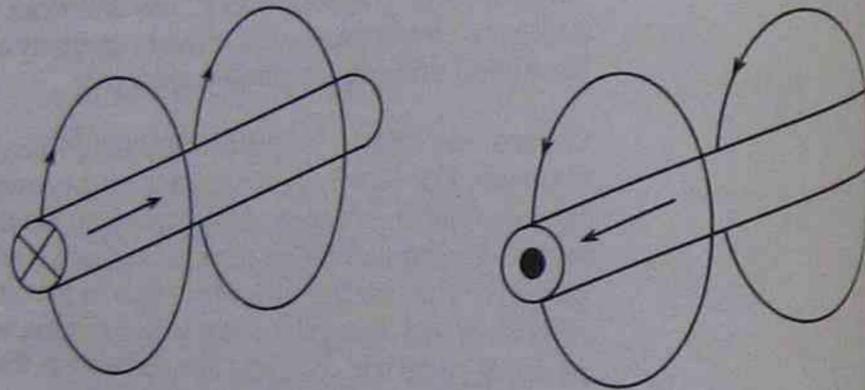


Figure 3: Magnetic-field lines circle the conductors in the directions shown

If a wire is kept stationary in a constant magnetic field a voltage is not induced; but if the wire is moved, or if the field moves or changes, a voltage is induced into the wire.

These conditions are shown in Figures 4 and 5, where the arrows on the conductors indicate the direction of induced voltage into these conductors.

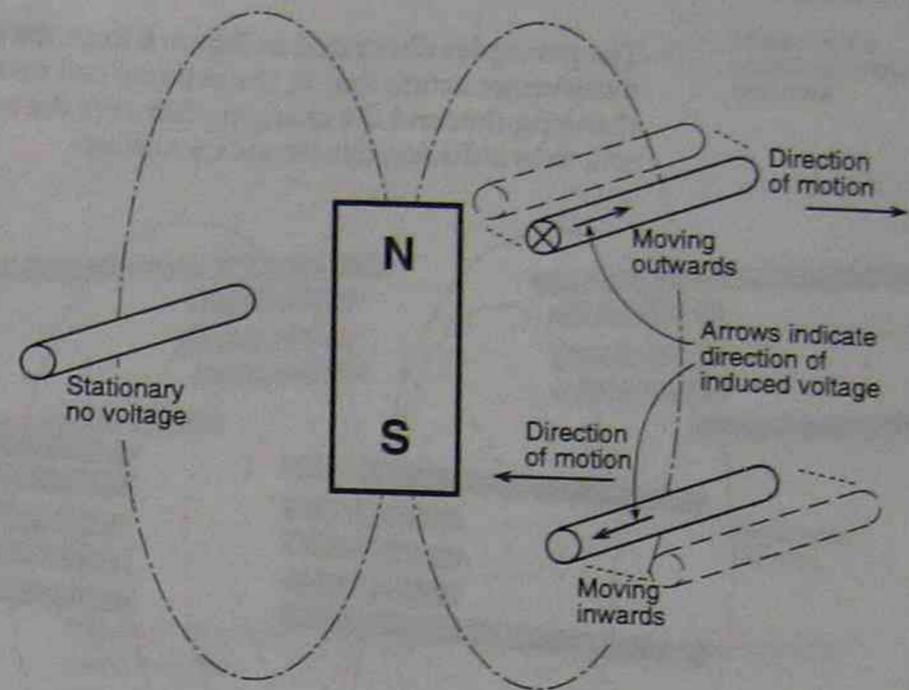


Figure 4: Inducing a voltage into a conductor that is moving in a magnetic field

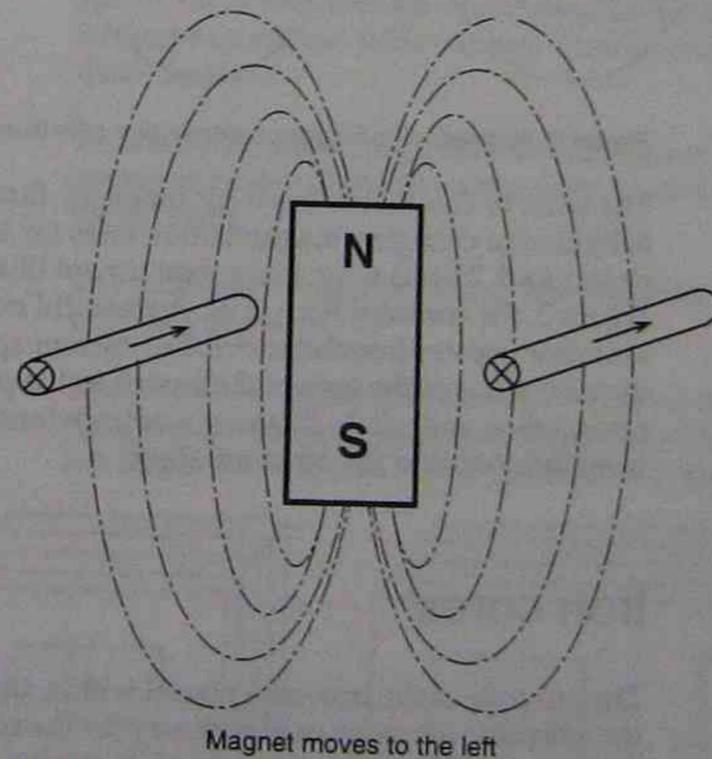


Figure 5: Inducing a voltage into a conductor when the magnetic field is moved near the stationary conductor

The principles illustrated in Figure 6 form the basis of transformer action that is, the primary coil produces a changing flux and the changing flux cuts the secondary coil, thus inducing the secondary voltage.

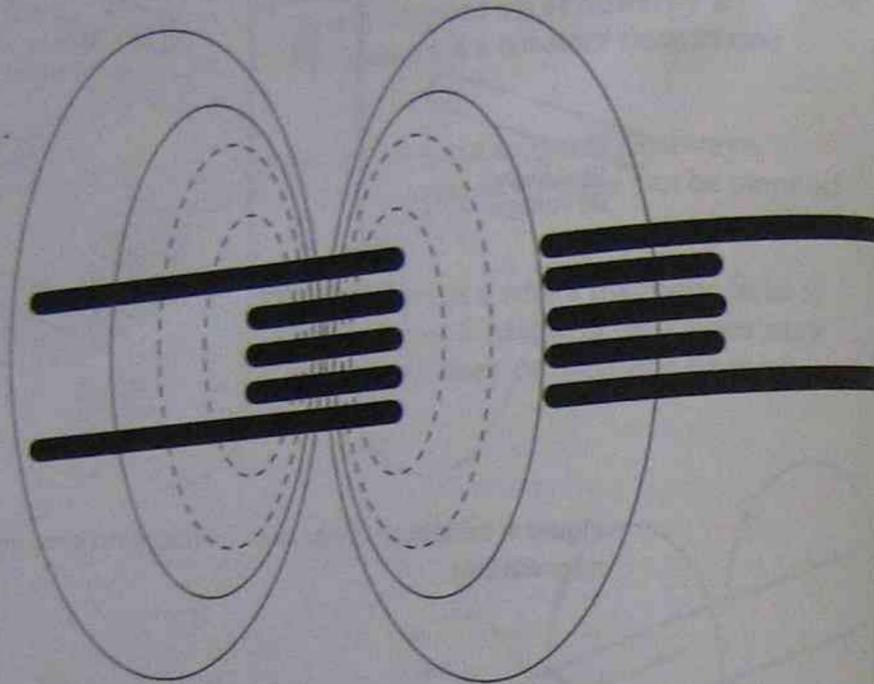


Figure 6: Magnetic-flux linkages between two coils in air

The wire, or conductor, must cut magnetic flux-lines, or be subjected to changing magnetic-flux lines for a voltage to be induced. This is why, in the experiment illustrated in Figure 3, the constant flow of dc current did not make the ammeter pointer (needle) move, but current appeared in the secondary at the instant the switch in the primary was operated on and off. In this way a voltage (and current) were induced into the other winding.

### Iron cores

The purpose of the iron core placed within and between the coils is to act as an easy pathway for the magnetic field or magnetic flux. If there is only air in the space between the two coils, the magnetic flux from the first coil will be spread uniformly in all directions around the first coil; thus there will be a very small percentage of flux lines from the primary winding linking with the secondary, as shown in Figure 7. However, when the 'soft' iron ring is used to hold the primary and secondary windings, there is a greater concentration of the linking magnetic-flux lines between the primary and secondary windings.

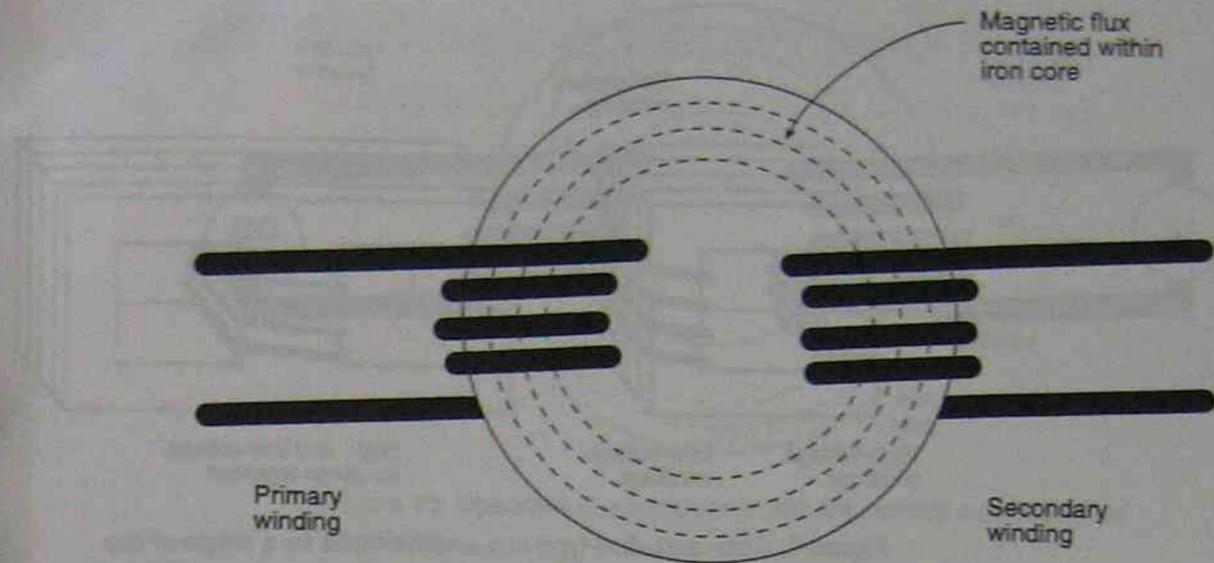


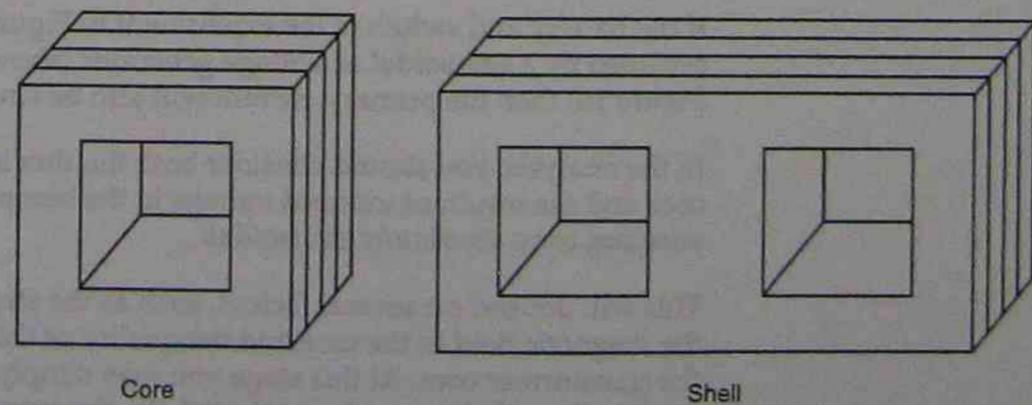
Figure 7: Magnetic-flux linkages between two coils wound onto the same iron core

The iron core is therefore a necessary part of the transformer. It is not usually made of a solid piece of iron, but sheets of transformer steel (called laminations). These laminations reduce 'eddy-current' losses, which will be discussed later.

To induce the same magnitude of voltage into the secondary without the iron ring needs a greater current in the primary.

The magnetic circuit operates more efficiently with the iron core in place.

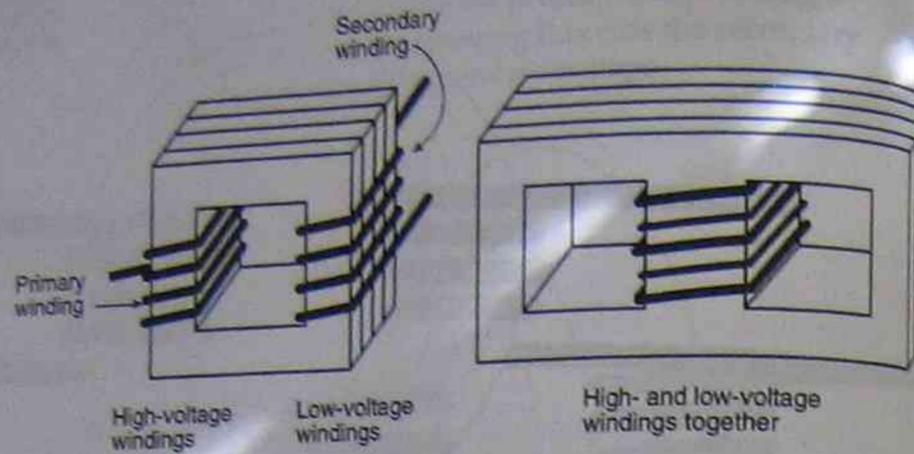
There are two shapes that are used in iron cores to which the primary and secondary windings of the transformer are fitted: *core* and *shell* types (see Figures 8 and 9).



Core

Shell

Figure 8: Core- and shell-type iron arrangements for a single-phase transformer



**Figure 9:** Core- and shell-type iron arrangements for a single-phase transformer with windings fitted

Figures 8 and 9 show the two core shapes, together with windings fitted and the resultant flux paths when the coils are energised. In the construction of larger transformers the method of placing the windings onto each of these types of cores may be different.

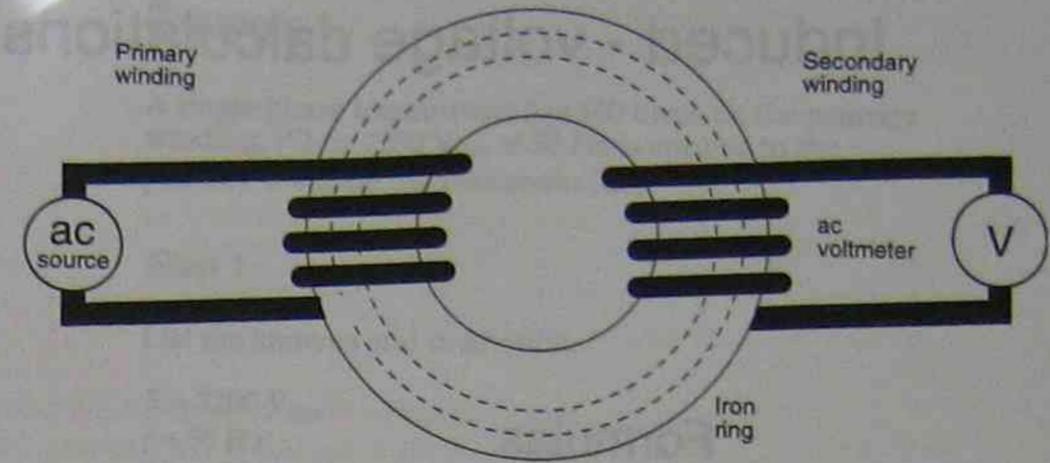
The windings are fitted to separate sides of the core form; however, for the shell form for large transformers the primary and secondary windings are formed separately and placed over each other on the same *leg* of the transformer core. For the smaller transformers (as found in rectified dc power supplies) the windings are often formed together and fitted to the core as a *single* winding.

## Sine wave primary voltage

If the battery and switch in the experiment in Figure 2 are replaced by a sinusoidal ac voltage generator (shown in Figure 10) then the primary current will also be sinusoidal.

In the analysis, you should consider both the flux in the core and the resultant induced voltage in the secondary winding to be absolutely sinusoidal.

This will depend on several factors, such as the strength of the magnetic field in the core and the quality of the steel in the transformer core. At this stage you may simply assume these factors can be neglected. So, the greater the applied voltage the greater the primary current and the strength of the magnetic flux and the greater the secondary voltage.



**Figure 10:** Operation of a transformer with the primary supplied by an ac voltage

More turns on the secondary result in a greater voltage for the same magnetic field. The frequency of the primary ac voltage determines the rate of change of the magnetic field.

You have now learnt about the principles of the transformer, iron cores and sine wave primary voltage. Next try the Check your progress questions. Working through these will confirm your readiness to proceed to the next topic.

# Induced - voltage calculations

## Formulas

The relationship between the frequency, the strength of the magnetic field, the number of turns and the applied or induced voltage can be represented by equation 1. This equation is derived in Appendix A of this section.

$$E = 2\pi f N \Phi \quad (\text{equation 1})$$

where:

$E$  is the root mean square (rms) voltage induced  
 $f$  is the frequency in hertz (or cycles per second)  
 $\Phi$  is the field strength in rms webers (Wb)  
 $N$  is the number of turns of the coil.

**Note:** In equations like these  $2\pi$  often occurs because we need to convert the value of frequency, in hertz, to radians per second to give us the result in the correct units.

Although you should refer to voltage in rms values, field strength is normally referred to as a maximum instantaneous value, or peak value. Expressed mathematically:

$$\Phi_{\text{rms}} = \frac{\Phi_{\text{max}}}{\sqrt{2}}$$

Equation 1 can then be rewritten as

$$E = \frac{2\pi}{\sqrt{2}} f N \Phi_{\text{max}}$$

$$E = 4.44 f N \Phi_{\text{max}} \quad (\text{equation 2})$$

## Example 1

A single-phase transformer has 480 turns on the primary winding. When 2200 V<sub>rms</sub> at 50 Hz is applied to the primary calculate the maximum flux.

### Step 1

List the knowns and unknowns:

$$E = 2200 \text{ V}_{\text{rms}}$$

$$f = 50 \text{ Hz}$$

$$N = 480$$

$$\Phi_{\text{max}} = ?$$

### Step 2

List the formulas and solve for the unknown:

$$E = \frac{2\pi}{\sqrt{2}} f N \Phi_{\text{max}}$$

$$\text{or } E = 4.44 f N \Phi_{\text{max}}$$

Substituting:

$$2200 = 4.44 \times 50 \times 480 \times \Phi_{\text{max}}$$

$$\Phi_{\text{max}} = \frac{2200}{4.44 \times 50 \times 480}$$

$$= 0.02063 \text{ Wb (or 20.63 mWb)}$$

If the peak value of the flux density is 1.1 teslas, calculate the cross-sectional area of the core.  
(Hint: Remember, flux = flux density  $\times$  area.)

$$\Phi_{\text{max}} = B_{\text{max}} A$$

where:

$\Phi_{\text{max}}$  is the flux in webers

$B_{\text{max}}$  is the flux density in teslas

$A$  is the core cross-sectional area in m<sup>2</sup>.

Then  $\Phi_{\text{max}} = 0.02065 \text{ Wb}$

and  $B_{\text{max}} = 1.1 \text{ T}$

Substituting gives

$$0.02063 = 1.1 \times A$$

$$\text{so } A = \frac{0.02063}{1.1} \\ = 0.01877 \text{ m}^2$$

### Guided exercise 1

A single-phase transformer has 80 turns on the primary. When  $240 \text{ V}_{\text{rms}}$  at 50 Hz is applied to the primary, the maximum flux density is 0.675 T. Calculate the maximum value of the flux, and the cross-sectional area of the core. (Express your answers in engineering notation.)

#### Step 1

List the knowns and unknowns:

$$E = 240 \text{ V}_{\text{RMS}}$$

$$f = 50 \text{ Hz}$$

$$N = 80$$

$$B = 0.675 \text{ T}$$

$$\Phi_{\text{max}} = ?$$

$$A = ?$$

#### Step 2

List the formulas and substitute:

$$E = 4.44 f N \Phi_{\text{max}}$$

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\Phi_{\text{max}} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$\Phi_{\text{max}} = B_{\text{max}} A$$

Substitute:

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\text{so } A = \underline{\hspace{2cm}}$$

(Answers:  $\Phi_{\text{max}} = 13.5 \text{ mWb}$ ;  $A = 200 \text{ cm}^2$ )

### Example 2

A single-phase transformer has 480 turns on the primary. When  $2200 \text{ V}_{\text{rms}}$  at 50 Hz is applied to the primary, we calculated the maximum flux to be 20.65 mWb. If the secondary of this transformer has 90 turns, and the primary and secondary are closely linked (ie we don't lose any flux linkages), the induced secondary voltage can be calculated.

#### Step 1

List the knowns and unknowns:

$$E_{\text{prim}} = 2200 \text{ V}_{\text{rms}}$$

$$f = 50 \text{ Hz}$$

$$\Phi_{\text{max}} = 0.02063 \text{ Wb}$$

$$N = 90$$

$$E_{\text{sec}} = ?$$

#### Step 2

List the formulas and solve for unknowns.

Let's first calculate the secondary voltage using the value of flux given:

$$E = 4.44 f N \Phi_{\text{max}}$$

Substituting:

$$E_{\text{sec}} = 4.44 \times 50 \times 90 \times 0.02063$$

so  $E_{\text{sec}} = 412.58 \text{ V}_{\text{rms}}$  measured across the secondary.

As a means of explanation, we used the formula and given value of flux to calculate the secondary voltage. There was no need to do so in this case, and the secondary voltage could have been calculated from

$$\frac{E_{\text{prim}}}{E_{\text{sec}}} = \frac{N_{\text{prim}}}{N_{\text{sec}}}$$

Substituting:

$$\frac{2200}{E_{\text{sec}}} = \frac{480}{90}$$

$$E_{\text{sec}} = 412.5 \text{ V}_{\text{rms}}$$

This means the volts-per-turn ratio is constant for every winding on the core.

The direct relationship between voltages and turns results, of course, from the fact that the values of frequency and flux are the same for each winding.

### Guided exercise 2

A single-phase 50 Hz transformer has 80 turns on the primary winding and 400 turns on the secondary winding. If the primary winding is connected to a 240 V<sub>RMS</sub> supply, determine the emf induced in the secondary winding.

#### Step 1

List the knowns and unknowns:

$$\begin{aligned} E_{\text{prim}} &= 240 \text{ V}_{\text{rms}} \\ f &= 50 \text{ Hz} \\ N &= 80 \\ E_{\text{sec}} &= ? \end{aligned}$$

#### Step 2

List the formulas and solve for unknowns:

$$\frac{E_{\text{prim}}}{E_{\text{sec}}} = \frac{N_{\text{prim}}}{N_{\text{sec}}}$$

and  $E = 4.44 f N \Phi_{\text{max}}$

From:

$$\frac{E_{\text{prim}}}{E_{\text{sec}}} = \frac{N_{\text{prim}}}{N_{\text{sec}}}$$

so  $\frac{240}{E_{\text{sec}}} = \frac{80}{400}$

$$E_{\text{sec}} = \text{---}$$

$$= \text{---}$$

(Answer:  $E_{\text{sec}} = 1200 \text{ V}_{\text{rms}}$ )

You may wish to repeat this calculation using the alternative formula, where you need to calculate  $\Phi_{\text{max}}$  as well.



### Check your progress 1

- 1 A single-phase 50 hertz transformer has 120 turns on the primary winding and 600 turns on the secondary winding. If the primary winding is connected to a 50 volt (rms) ac supply, determine the emf induced into the secondary winding.
- 2 A single-phase 50 hertz transformer with 60 turns on the primary winding is connected to a 120 rms volt supply. The resultant peak flux density is 0.45 teslas(T). Calculate the peak value of flux and the cross-sectional area of the core.

You can check your answers at the end of the section.

# Components of transformer magnetic cores

Earlier in this section you read that part of these learning materials will be calculations and part will be descriptive material to help you visualise the devices we are talking about. Let's commence descriptions by looking at the transformer magnetic circuit and windings. Both of these are related to the theory and calculations covered earlier.

After working through this material you should be able to describe the construction of the cores and the electrical and mechanical considerations of the windings.

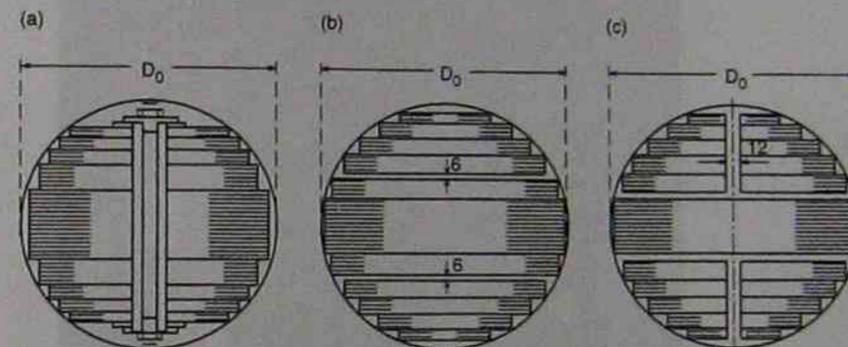
## Magnetic circuit

The magnetic circuit, or *core*, of the transformer is made up of thin laminations of selected steel. Materials in recent use include 0.335 mm-thick steel containing about 3.5 per cent silicon content, cold rolled to achieve grain orientation. One side of each lamination is coated to provide an effective high resistance between adjacent laminations. This is done to reduce *eddy-current losses* in the steel core. These and other losses appear as heat generated inside the transformer and will be considered later.

In considering the dimensions of the cross-sectional area of any limb of the magnetic circuit, it is not usual to exceed a field-strength value of 1.8 teslas. In construction, the magnetic circuit is built up, lamination on lamination, until the required cross-section of steel has been reached. The windings of the primary and secondary are usually copper but may also be aluminium. For a number of reasons, mainly to do with having a suitable shape, these windings are wound so as to have a tubular (for helical and layer windings) or discal (for flat windings) shape. Both of these have a circular centre. These shapes will be explained in some detail, although other shapes, such as 'sheet', may be encountered.

As space in a large transformer is very valuable its use is of concern. For this reason, the cross-sectional shape of each leg of the core is made to achieve the 'best fit' for the windings. Some of the core shapes possible are shown in Figure 11. Notice how the diameter ( $D_o$ ) is critical to ensure the coils will fit over the core legs in assembly and still allow free space as needed to allow for the movement of a cooling medium, such as air or oil.

Each section, or limb, of the magnetic circuit with a winding fitted to it needs to have a circular, or nearly circular, cross-section. This is difficult to achieve using thin laminations unless each is of slightly different width to its neighbour, and the cost of such a range of sheets would not be economical to manufacturers. A compromise is reached where, depending on the size of the transformer and other factors, the sides of limbs where windings are fitted are *stepped* to optimise the filling of the circular winding area of the coil (see Figure 11).



**Figure 11:** Cross-sectional diagram of stepped cores (a) without cooling ducts (b) with axial cooling ducts (c) with axial and transverse ducts

When a corner is to be constructed into the magnetic circuit it is necessary to interleave the sheets of steel. This is done to keep the airspace at the ends of the butt-joined laminations to a minimum, as this airspace inhibits the easy passage of magnetic flux in the core.

Historically, this reluctance was minimised by overlapping at each corner piece. In more recent construction these sheets have mitre joints that only overlap a short distance. This has been found to be more effective in making use of the grain orientation of the laminations.

Figures 12 and 13 show examples of mitre-lap corner joints. Although many of the construction diagrams included show three-phase transformers, the details explained are also relevant to single-phase transformers.

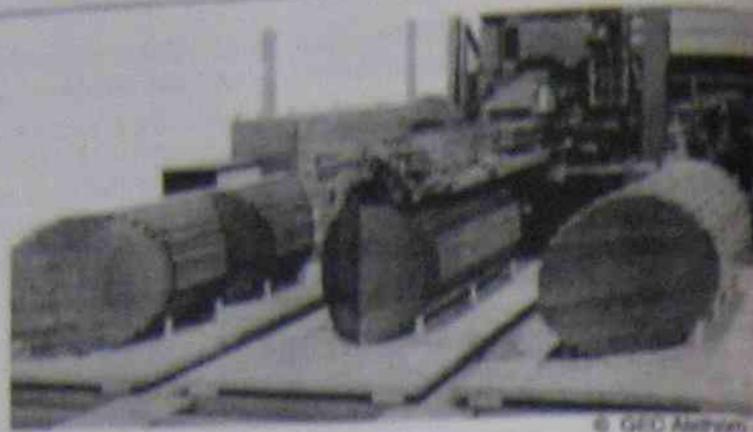


Figure 12: A laminated core with mitred joints



Figure 13: The core of a 390 MVA transformer showing mitred joints and strapping of core laminations

As you learnt earlier, for single-phase transformers the magnetic circuit is built in two different shapes called 'core' and 'shell'. The windings are placed onto these magnetic circuits in different locations, and so each of these types needs to be assembled differently to allow the later fitting of the windings, or coils.

It is necessary to construct the magnetic section in at least two parts because the coils have to be fitted onto the limbs before the rest of the magnetic path is completed. So that the assembly of laminations can be handled easily, sections need to be clamped together. In the past this was

done by fitting bolts through the sheets and bolting the laminations together, which resulted in a disturbance to the magnetic material around the holding-bolt holes. Modern methods use straps of insulating material to hold the laminations together, causing less disturbance to the magnetic characteristics of the steel.

Because of the very large forces generated inside a transformer winding and magnetic laminations under short-circuit conditions, it is necessary to have the completed assembly held together rigidly to keep the sheets fixed in place and minimise any vibration noise. As a result, there is bracing far beyond what you might expect in those sections not surrounded by coil windings. This is done by fitting large supporting frames to the outside of the laminations. Figure 14 shows an assembly with the sections clearly visible.

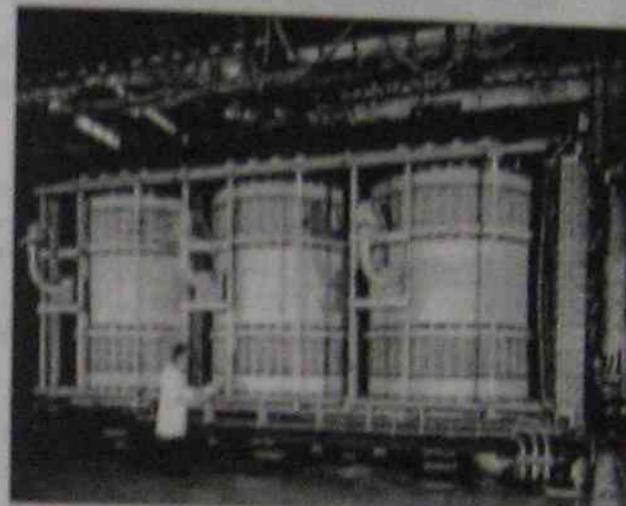
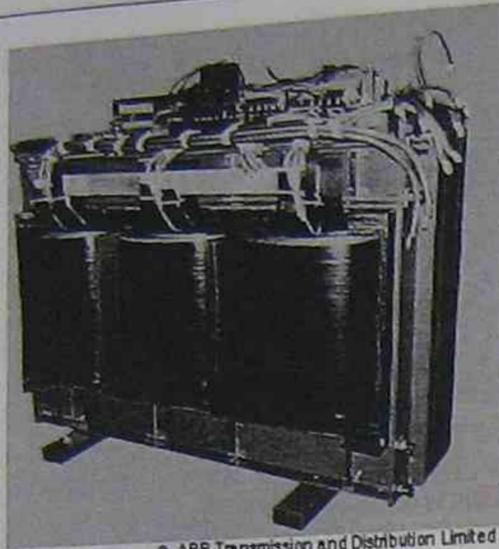


Figure 14: Core and windings of three-phase 200 kVA 50 Hz transformer

As the transformer approaches final assembly it is necessary to make the connections between the ends of the fitted coils to the large current conductors (*bus-bars*) inside the transformer. These connections are usually large copper or aluminium straps, as shown in Figure 15. (Note that this is actually a three-phase transformer, which will be covered later. It is shown here for the features nominated.)



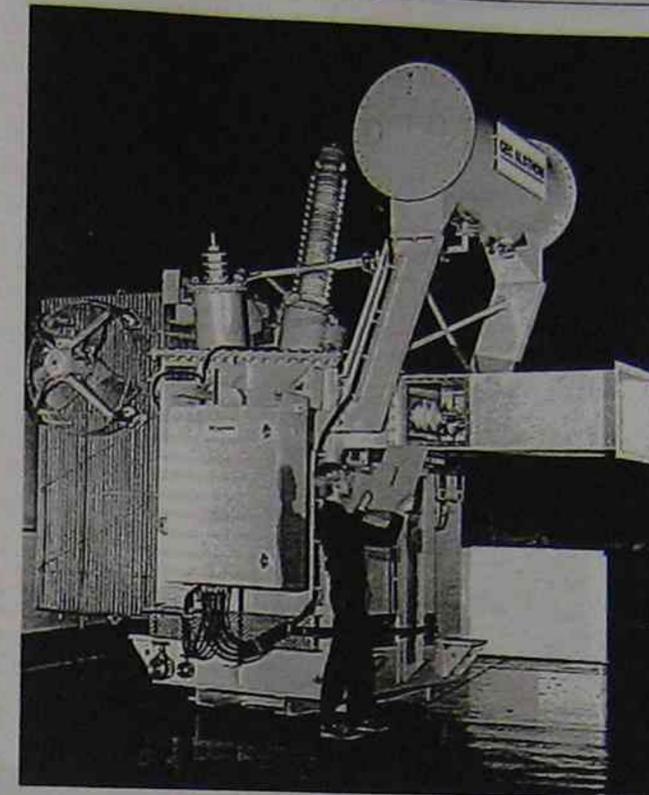
© ABB Transmission and Distribution Limited

**Figure 15:** Heavy-current bus-bars fitted to the ends of coils

On top of this assembly, the insulation wrapping and mechanical support are fitted to hold the complete assembly together.

After the transformer windings and core are fitted into the tank, the electrical connections that must pass through the tank to equipment outside it are made to the large porcelain insulator *bushings* that form part of the tank lid. The lid is then fitted and bolted to the tank to give the finished product.

Figure 16 shows an example of a complete (three-phase) transformer, including additional items that will be explained later.



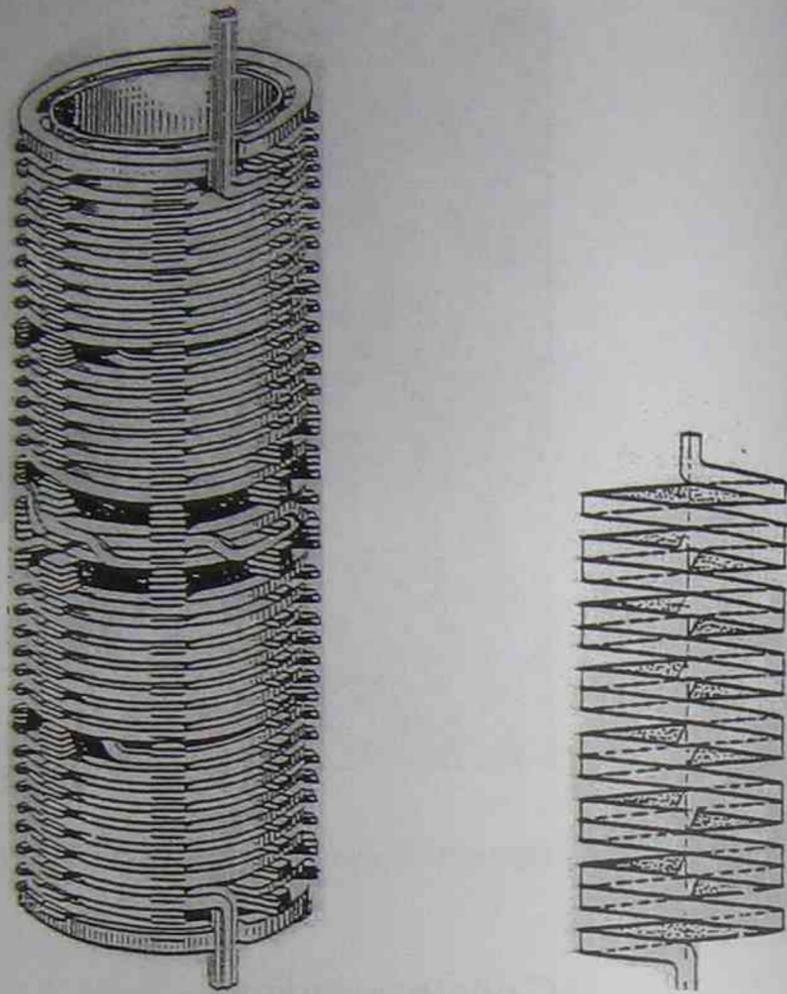
© GEC Alsthorn

**Figure 16:** Complete unit of a 22.5/45 MVA 132/33 kV transformer

## Coils or windings

The essential working parts of the transformer are the coils that carry the alternating current and create the interacting magnetic fields. Coils are generally wound from copper conductors (hence the term 'copper losses' when referring to  $I^2R$  losses) but aluminium is also widely used.

These coils are commonly made in three basic shapes: as helixes (Figure 17), layers (Figure 18), or discs (Figure 19).



(a) (b)

Figure 17: Helical windings (a) simple (b) semi-helical

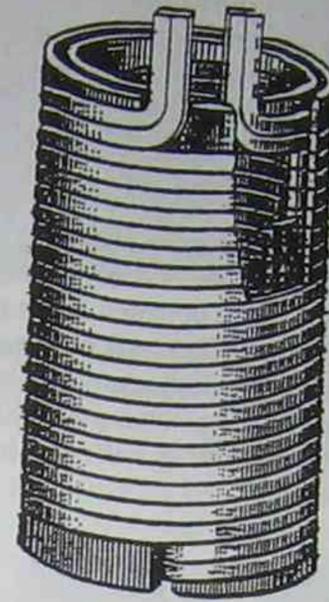


Figure 18: Layers—cylindrical double-layer winding

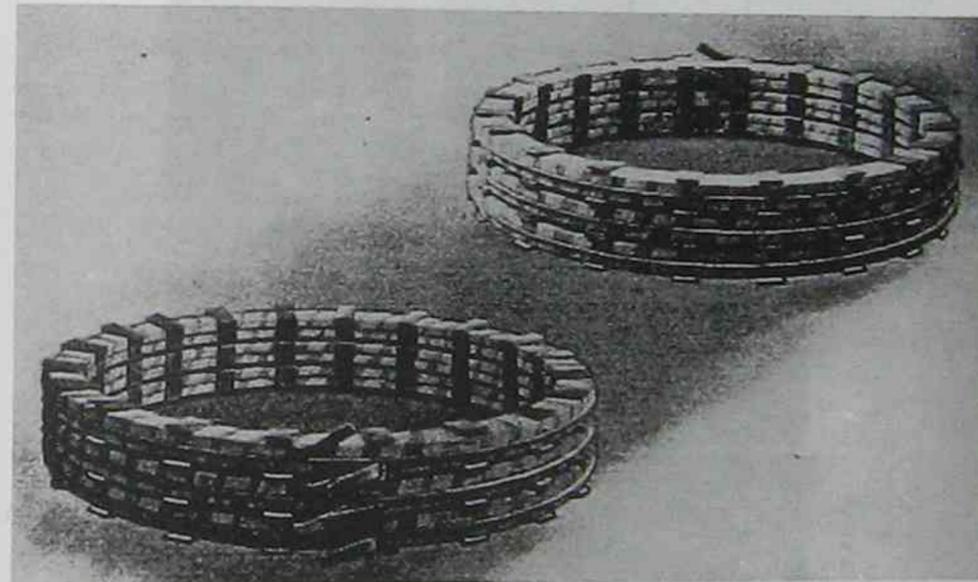


Figure 19: Discal coils

Each of these different shapes is pre-formed or wound from insulated conductor of circular or rectangular cross-section, before being fitted to the limb of the transformer core.

The insulating material used here is usually either an enamel coating on each strand (or strands), or a wrapping of paper (which becomes impregnated with oil), thus forming insulation between adjacent turns.

In addition, sheets of other insulating material are used to separate one layer of turns from those above and below it. Because of the large amount of heat generated by the current flow through the winding (due to  $I^2R$ ) it is necessary to create spaces between winding layers and coils to allow the free movement of the cooling medium that takes the heat from the coils.

In helical windings multiple conductor strands are wound in a simple helix from one end of the coil to the other, with insulating spacers fitted between turns to permit oil circulation for cooling. This type of winding is used where few turns and large conductor cross-section is required, as in a low-voltage winding (Figure 20). (Note that low voltage is the lower winding voltage, although its value can still be quite high.)

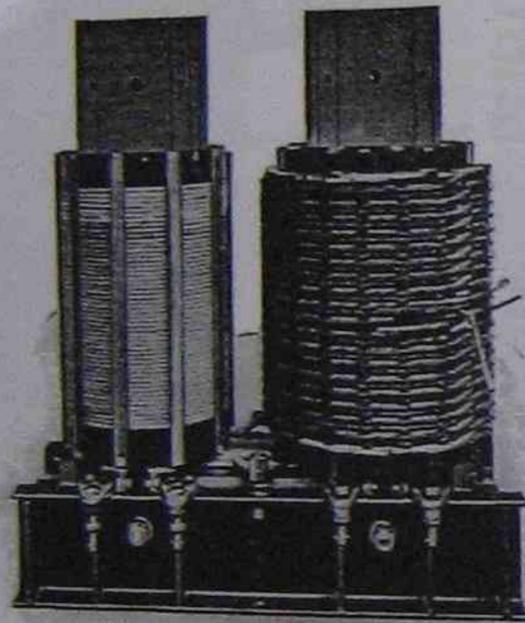


Figure 20: Helical and discal coils on single-phase 2000 kVA 50 hertz 22/10.5 kV transformer

Discal windings, as shown in Figure 21, resemble helical windings, however, unlike the helix with all conductors in one horizontal array, the discal winding has multiple turns in the horizontal array so that it appears to be a disc.

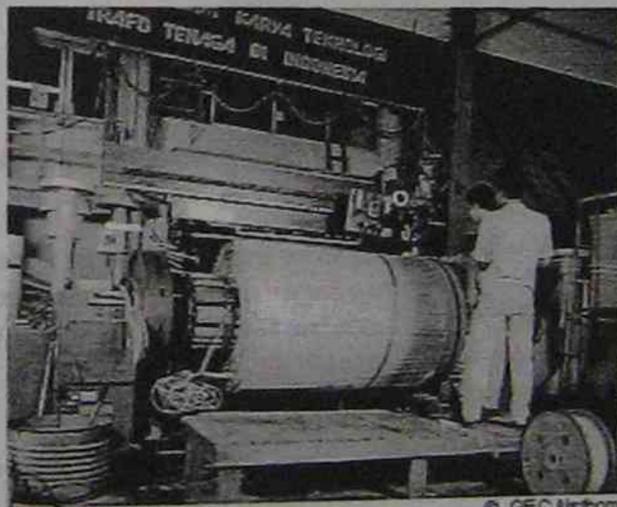
Discal windings are rarely used in single-phase transformers, but do occur more frequently in three-phase transformers. A helical winding might be regarded as the limiting case of a discal winding with one turn per discal. The usual application for a discal winding is in a high-voltage winding where the current is relatively small but many turns are needed.



© GEC Alsthorn

Figure 21: Interleaving discal coils showing internal transposition and external crossover

In a layer or cylindrical winding turns are wound like threads onto a spool from one end to the other and then a crossover is made, as shown in Figures 22 and 23. After the insertion of an insulating layer and a cooling duct, the next layer is wound back over the first in the reverse direction. The number of layers is not limited and may go up to 20 for very high-voltage windings. This process is mainly used for high-voltage windings, but may be used for low-voltage windings in which many conductor bundles may be wound in parallel for each turn.



© GEC Alsthorn

Figure 22: Continuous discal-coil winding using four wires in parallel



© GEC Alsthon

Figure 23: A three-phase distribution transformer in course of construction

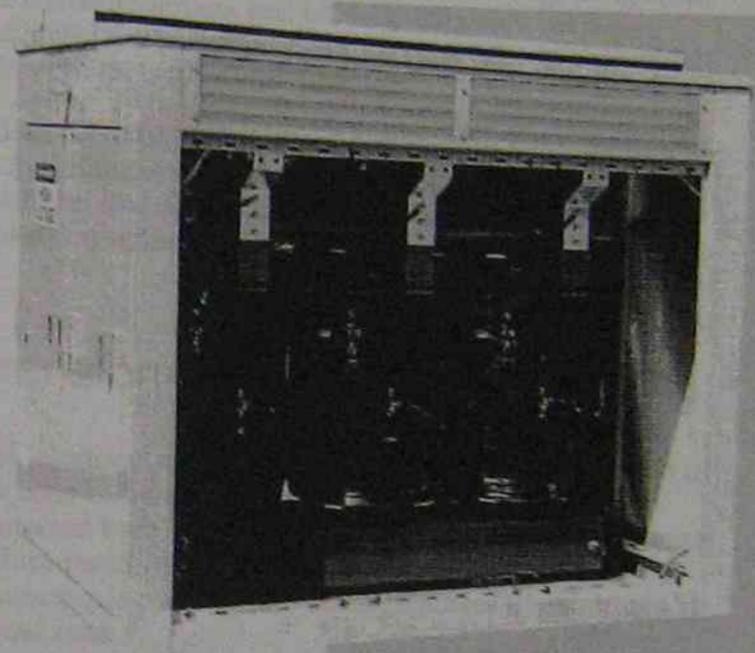
Another feature to be incorporated in windings is the means to have a range of different voltages, or *tappings*, being taken from the same coil. This is done to give a choice of voltages without having to wind a separate coil for each of the voltages sought.

For this, *tapping straps* are brazed onto the ends of the winding conductors and are then brought out under, or over, the winding. These straps are then formed to be connected to a tap changer located within the enclosure.

Because of their shape, transformer windings will appear electrically to be both an inductor and a capacitor—an inductor because of the looping shape of adjacent turns in each coil, and a capacitor because the coils consist of adjacent (insulated) conductors with a voltage difference between them. In our simplified analysis of the transformer the capacitive effect will be ignored for operation at 50 hertz, although this cannot be neglected

when considering the transformer as an element in a complete generation and transmission system.

After construction and during assembly, the windings of the transformer have to be electrically insulated (isolated) from each other and from the steel laminations, and the ends of the winding conductors need to be available for connection to other equipment outside the transformer. An example of a three-phase transformer with the high-current connections from the windings to other components is shown in Figure 24.



© ABB Transmission and Distribution Limited

Figure 24: Core and windings of 300 kVA 660/433 V three-phase distribution transformer

It should be noted that within the winding shown as (a) in Figure 17 a number of the individual conductors of coils (in the centre of the winding) are 'transposed'. This is done to assure equal load sharing of all parallel conductors of the winding.

As has been explained previously, one of the limitations in the construction of transformers is the material that is used to provide the electrical insulation of the coils.

To minimise the electrical stress that the insulating material can experience, the gradient of voltage at various parts of the winding influences the type and thickness of insulating material used.

Another significant factor is the placing of the high-voltage winding outside the low-voltage winding to maintain the voltage gradient to earthed parts of the transformer. As the magnetic core is an integral part of the transformer and is connected electrically to the tank (earthed), by placing the low-voltage winding closer to the magnetic core than the high-voltage winding we have the voltage gradient, from high to low to earth, as uniform as possible.

## Transformer construction

Continuing from our look at cores and coils, we will now examine the construction of transformers, beginning with the transformer tank and lid, following on with the bushings, the conservator, transformer oil, together with synthetic dielectrics and inhibitors.

When you have finished this next part, you should be able to describe the purpose of the tank and conservator, bushings and oil, relative to the construction and operation of the transformer.

### Transformer tank and lid

The 'electrical working parts' of the transformer consist of the coils and steel laminated core. These parts must be protected both from mechanical damage and any electrical failure resulting from insulation breakdown. This protection is provided for all large power transformers by a steel tank.

The tank must be large enough to contain the electrical windings, core and so forth with sufficient strength to allow the complete transformer to be lifted and placed in position. For mobility, wheels are usually fitted to allow final positioning. The tank is fabricated 'open', and a lid is fitted after the electrical windings and other fittings are placed inside.

Because the tank is filled with oil while the transformer is operating, a gasket is fitted under the lid to prevent any leaks. For safety, the tank is always earthed and the electrical connections are carried through the body of the tank via holes fitted with *bushings* to insulate them.

An important fitting to the tank is the *oil-drain valve*. This allows the tank to be drained of oil (to reduce its total weight when being moved between locations) and for the connection of oil-purification equipment, if the oil needs any special treatment during the life of the transformer.

## Bushings

There are generally two types of bushing used with high-voltage transformers in Australia. These are identified by their material and construction as either *ceramic* or *oil-filled*.

The ceramic type is entirely porcelain, with a hole through the centre to carry the electrical connection. Metal caps are fitted to each end to permit connections both inside and outside the tank.

An oil-filled bushing consists of a hollow porcelain cylinder with a conductor through its centre. The space between the conductor and the porcelain is filled with oil, as an insulator. Oil is fed into the bushing at the top where a glass cylinder acts as both an expansion tank for the oil on heating and an oil-level indicator. Insulating sleeves are placed around the central conductor in the oil to prevent impurities in the oil from forming radial (electrical) breakdown paths.

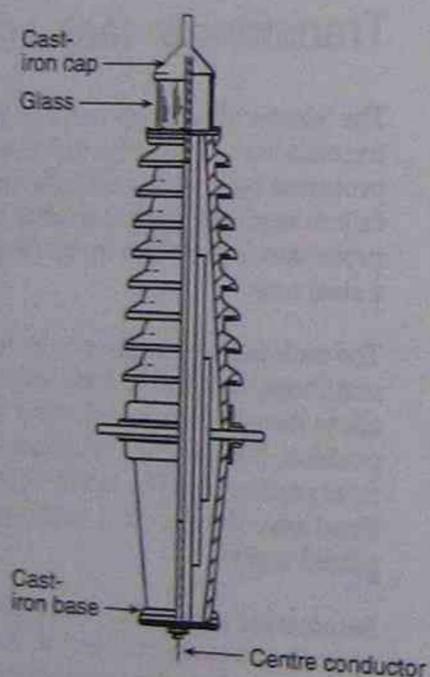


Figure 25: Oil-filled bushing

There is another kind of bushing, rarely encountered in Australia, called the *capacitive* type. The capacitive bushing involves an insulating protective case which is sealed into the hole of the transformer tank. Tubular cores of alternate conductive and insulating layers are installed around the centrally placed conductive connection to fill

the space inside the protective case, thus forming large concentric tubular capacitors inside the case. The lengths of these tubular cores are progressively reduced as the diameter increases. This maintains a uniform voltage gradient along the length of the bushing. Special care needs to be taken to weatherproof this type of bushing.

## Conservator

Oil is used in transformers as a cooling (and insulating) medium; however, the presence of moisture or dissolved oxygen in the oil can cause major problems and breakdowns. In Australia, it is general practice to have an oil-expansion tank, called a *conservator*, fitted above the transformer tank. Its purpose is to ensure that air and moisture cannot directly enter the transformer tank, by keeping the tank filled with oil (see Figure 26).

The level of oil will vary in the conservator due to the expansion of the oil in the transformer tank with temperature change. To prevent pressure failure a *breather pipe* ensures the oil pressure is always at 'atmospheric'. The displaced air is filtered through a layer of calcium chloride or silica gel, which extracts moisture from the air so avoiding contamination of the insulating oil.

It is not possible to completely prevent oxygen from entering the oil, but with the correctly sized connecting pipes the movement of any oxygenated oil into the transformer tank can be minimised.

With a conservator fitted, the oil exposed to the air in the conservator has a much lower temperature than the oil in the main tank, and the exposed surface area of oil is smaller. This ensures that sludge formation is reduced.

With the main tank completely filled, the possibility of a hydrogen-oxygen explosion due to internal arcing is eliminated since there is no longer a vacant air space in the tank.

When a conservator is fitted a pressure-relief device must also be fitted, as the use of an expansion tank complicates the effect of gas evolution in the event of internal arcing. This pressure build-up of gas may be very rapid and, without such a relief device, the tank could rupture.

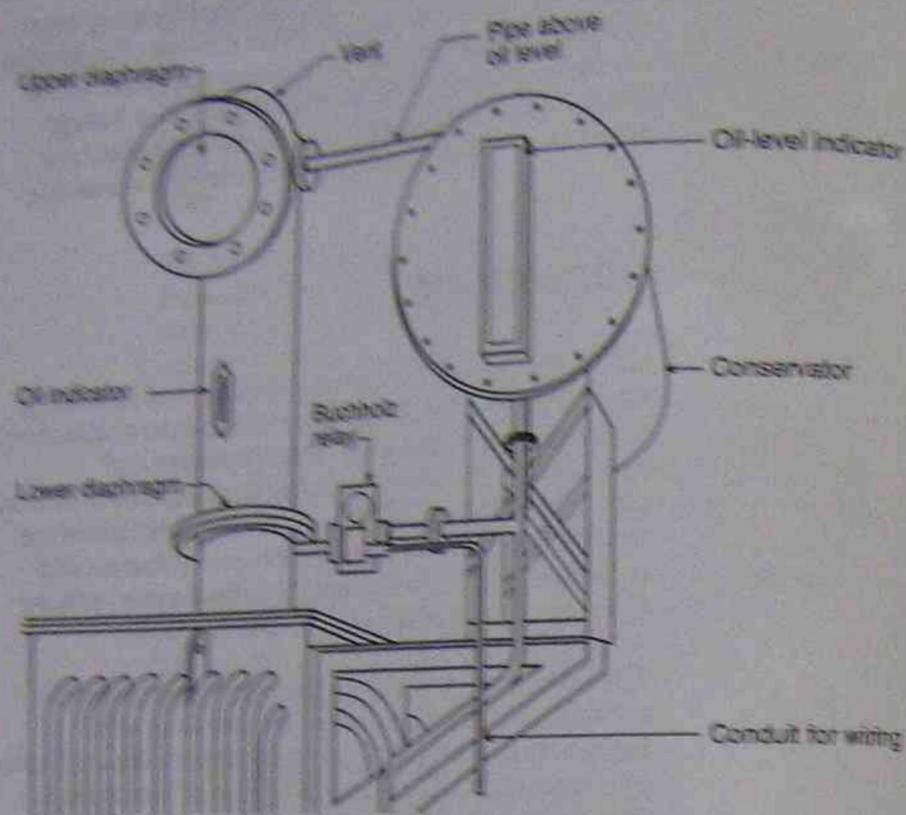


Figure 26: Position of conservator above an oil-cooled transformer

Other advantages of the conservator system are

- installation is simple and maintenance is easy
- leaks soon show up as oil stains
- as the main tank is low and under positive pressure (due to the oil in the conservator), air and moisture cannot get into the main tank if there is a leak
- a Buchholz relay (to be described in a later section) can be fitted to protect the transformer against internal electrical faults.

The main disadvantages of the conservator system are

- the conservator tank and relief pipe are tall and may interfere with incoming high-voltage electricity lines
- the high oil 'head' could feed a fire resulting from an oil leak in a broken bushing
- some degree of degradation is still possible in the conservator tank, so contaminated oil can get into the main tank.

## Transformer oil

The transformer oil used in oil-immersed transformers must satisfy the following requirements:

- It must be able to transfer heat readily from the transformer windings and core to the cooling surfaces where the heat is ultimately given to the surrounding air or to circulating water. To do this effectively it must have a low viscosity.
- The fire hazard must be reduced to a minimum. This is achieved by having a high flashpoint for the oil.
- It must be chemically inert in relation to the materials with which it comes in contact. The acidity and sludge values are kept low to achieve this.
- It must be certified as not being a risk to human health.
- It must be satisfactory as an electrical insulator. This factor is tested by the electrical strength of the oil.

## Standards for oil

There are two Australian Standards which refer to the oil used in transformers:

- AS1767 (1975, amended 1976), which sets out the requirements for transformer-insulating oil
- AS1883 (1993), which addresses the maintenance of transformer-insulating oil.

These are comprehensive documents containing important details (eg for testing) and should be used whenever questions regarding transformer oil are to be resolved.

## Tests for transformer oil

Transformer oil is a pure hydrocarbon mineral oil, clean and free from matter likely to impair its properties, and without special additives. The oil may contain oxidation inhibitors or other additives which do not preclude its compliance with AS1767.

The oil is regularly tested for the following contaminants and possible failure:

- sludge and acidity values after oxidation
- electrical strength
- corrosive sulphur
- loss tangent (and resistivity)
- water content (the Karl Fischer method using a single buffer).

## Synthetic dielectrics

Non-inflammable liquid dielectrics were used in the past in the limited range of application where fire hazard was a very important factor. These non-inflammable dielectrics were once usually chlorinated hydrocarbons known as PCBs.

Due to the serious risk to human health associated with the use of PCBs their use is now banned.

With the improvement in higher-temperature insulation, dry-type transformers are now commonly used instead of synthetic dielectrics.

## Inhibitors

Additives are now widely used in the production of refined mineral oils for insulation to give them special characteristics. One of the most important purposes of these use of additives is to provide oxidation stability of a higher order than can be achieved with mineral oil alone. Such inhibitors are chemical compounds, added to the oil in very small proportions, which have the effect of prolonging the service life of the oil.

## Materials used in dry-type transformers

As mentioned, it is not normal practice to use oil-filled transformers in certain locations where they may prove to be a hazard for example, substations in the upper floors of

high-rise buildings (where burning oil may fall on people below). Transformers manufactured without the need for the coils to be contained in an oil-filled tank are called 'dry-type' transformers.

Australian Standard AS2735 (1984) *Dry Type Power Transformer*, specifies the requirements for dry-type transformers (including auto-transformers) having a highest voltage for equipment up to and including 36 kV. This specification does not cover transformers described as:

- single-phase transformers rated at less than 1 kVA
- polyphase transformers rated at less than 5 kVA
- instrument transformers
- transformers used for semiconductor devices
- starting transformers
- testing transformers
- traction transformers mounted on rolling stock
- flameproof transformers
- welding transformers
- voltage-regulation transformers
- small power transformers where safety is a special consideration.

Cooling of dry-type transformers is usually more difficult than for oil-filled enclosures, as the heat needs to be carried away from the windings by air convection or the fitting of a 'heat sink' device in close contact with the outside of the windings. For this reason the insulation material used on the windings is normally of a higher temperature rating.

Australian Standard AS2768 (1985) *Electrical Insulating Materials—Evaluation and Classification Based on Thermal Endurance*, provides a table of classification of insulating materials and their respective assigned temperatures:

Class number	Class letter	Assigned temperature
		90°C
90	Y	105°C
105	A	120°C
120	E	130°C
130	B	155°C
155	F	180°C
180	H	200°C
200	Class C is above 180°C	220°C
220		250°C
250		Corresponding intervals of 25°C
Thereafter in intervals of 25		

The assigned temperature is determined from:

assigned value = ambient temperature + temperature rise + hot-spot allowance

Because of their lack of oil, dry type transformers do not have conservators and are usually constructed to be of minimum height so that they may be located in restricted spaces.

## Transportation of transformers

Because of their physical size and mass, the transportation of transformers from their point of manufacture to the location where they will be used in an electrical system (and in any relocation) can be a major undertaking. To carry such heavy items by either road or rail requires special 'low loader' trailers.

When transporting transformers by road the route needs to be carefully checked and such factors as road gradient, height of overhead cabling and structures, and load capacity of bridges to be crossed must be assessed beforehand.

Special railway trailer units are sometimes used, although rarely in Australia. Because of the physical size of the transformer and the limitations caused by track curvature, and clearances of tunnels and overhead electrical conductors, this option is not commonly used.

Another problem to be considered is the loading and unloading of the transformer onto and from the trailer. The loading is usually done by an overhead industrial crane at the manufacturer's works, but it is also necessary to have a crane of adequate capacity at the destination of the transformer for unloading. As these cranes invariably require a large space for safe operation this also needs to be taken into consideration at the site.

To minimise the mass (weight) of the transformer to be handled it is invariably shipped 'dry'. This means that insulating oil is delivered to the site and the tank filled at its final location. To ease handling and reduce mass all associated pumps, piping, conservator, insulator bushings and heat exchangers are removed and then reassembled at the site.

During transportation all openings to the tank must be sealed to prevent the entry of moisture or foreign matter. As the coils have all been 'dried' before leaving the manufacturer the absorption of moisture by the insulating material is to be avoided. This is sometimes achieved by filling the sealed tank with a 'dry' gas at a pressure above atmospheric for the time it is being transported.

With working lifetime of 25 or more years, a transformer may be reused at a number of different locations in this period. Their handling and transportation at times other than immediately following manufacture introduces some potential problems.

When in operation the transformer needs to be kept at a safe working temperature, however, even this temperature has an effect on the insulating material used in the windings, and can cause it to deteriorate. Usually this results in the material becoming brittle and discoloured, with any vibration causing it to disintegrate depending on the degree and duration that the windings have been operated at excessive temperatures (called 'cooking'). Insulation which has failed to this degree may allow adjacent conductors in the winding to touch, resulting in short circuiting within a coil or between high- and low-voltage coils. In addition, if the tank is held empty of oil for any considerable period this may cause the insulating material to dry excessively, with a similar result. Failure of the insulating material inside the coils during transportation has been known to render transformers unusable on delivery to their new site unless the winding assembly is to be completely rebuilt (at considerable cost).

## On-load tap changers

For electrical distribution systems where large variations in load current regularly occur and where continuity of supply is important, it becomes necessary for the supply transformers to be fitted with *on-load* tap changers.

On-load tap changers are significantly more complex than off-load tap changers because they involve the use of additional heavy duty switching contacts for making and breaking large inductive currents.

Although there are many different designs of tap change mechanisms, all possess the following features:

- a form of impedance to prevent short circuiting that portion of the winding over which a tap change operation is being performed.
- a duplicate circuit arrangement to enable one circuit to maintain the load current during switching of the other.

This involves the use of heavy duty diverter switches in addition to the normal tap position selector switches.

A typical arrangement is illustrated in Figure 18 which shows the tapped portion only of the transformer winding.

Figures 18(a) to (g) shows the six separate steps involved in a tap change from position T1 to T2.

Figure 18(a) - The transformer is operating in tap position 1 when a tap position change is initiated.

Figure 18(b) - Diverter switch D1 opens. Note that load current is maintained by diverter switch D2.

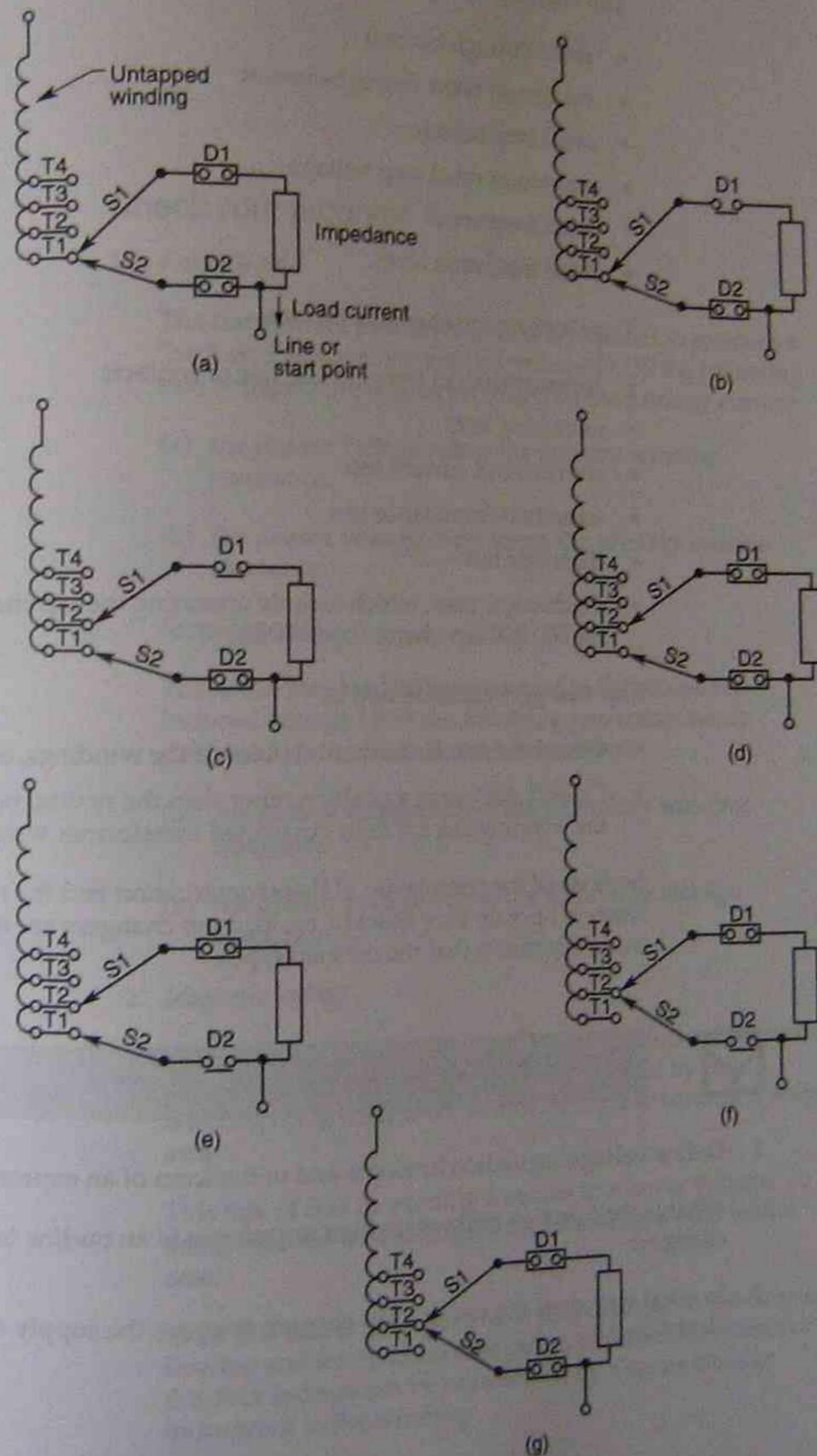
Figure 18(c) - Tap selector switch S1 moves from winding tap 1 to tap 2 without being required to break any current (D1 is open).

Figure 18(d) - D1 re-closes thereby bridging the section of winding between taps 1 and 2. The impedance prevents the value of 'short-circuit' current flowing from being excessive.

Figure 18(e) - D2 opens thereby breaking the heavy inductive current flowing while D1 maintains the load current.

Figure 18(f) - S2 move to tap position T2 without breaking any current.

Figure 18(g) - D2 re-closes, completing the tap change cycle and the circuit is now in position for any further tap change operations.



### Example 3

Using Figure 15, determine the expected % voltage regulation for a transformer with 1.0% resistance and 5.0% reactance for the following loaded conditions:

- full-load 0.6 power factor (lag)
- half-load 0.9 power factor (lag)
- half-load 0.9 power factor (lead)
- full-load 0.6 power factor (lead). If the rated output voltage of the transformer is 240 volts, what terminal voltages would be expected for each of the loads (a) to (d) above?
- What is the load-power factor at which zero regulation is achieved?

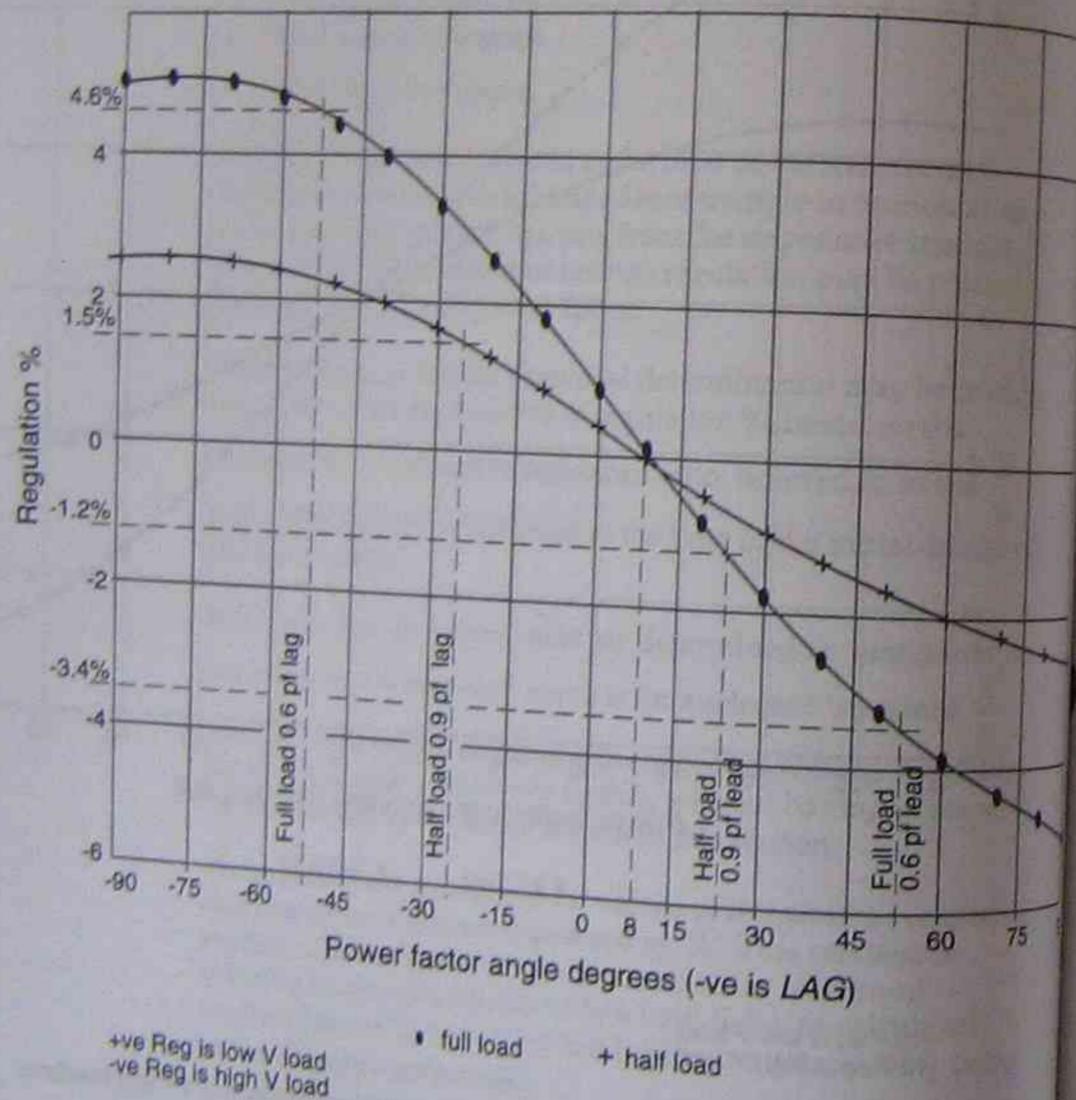


Figure 15: Marked-up copy of Figure 14

### Solution

- $4.6\%$ ;  $240 \times (1 - 0.046) = 229.0 \text{ V}$
- $1.5\%$ ;  $240 \times (1 - 0.015) = 236.4 \text{ V}$
- $1.2\%$ ;  $240 \times (1 + 0.012) = 242.9 \text{ V}$
- $3.4\%$ ;  $240 \times (1 + 0.034) = 248.2 \text{ V}$
- Power factor of 'leading 8°' or 'lead 0.99'

## Tap changers

When we discussed the concept of voltage regulation it was shown that the output voltage of the transformer will be influenced by the magnitude of the load current and the (effective) impedance of the transformer relative to the output terminals.

Generally, we see that as the load current increases, the voltage at the terminals of the secondary windings decreases. This has the effect of lowering the output-load voltage when the system is at its most vulnerable supplying its highest load.

In supplying electricity to the user, the distribution authority has a legal obligation to keep the voltage (and frequency) within prescribed limits, and if there is a variation beyond these values for an extended period of time there may be a justification for legal redress by the electricity consumer.

When a distribution authority is deciding on the installation of a new transformer, a significant decision relates to selecting the appropriate magnitude of the series circuit equivalent impedance ( $Z_{series}$ ). Transformers with high  $Z_{series}$  values act to

- limit the magnitude of fault-level current, and
- provide poor voltage regulation as the load changes.

Transformers with low  $Z_{series}$  values act to

- allow large values of fault level current, and
- give good voltage regulation as the load changes.

As system fault-level currents are very important, the solution usually is to have a high  $Z_{series}$  value and to otherwise address the problem of poor voltage regulation with load changes.

To allow the voltage to be compensated upwards when the load current increases (and downward as the load current decreases) transformers are fitted with devices called tap changers. These

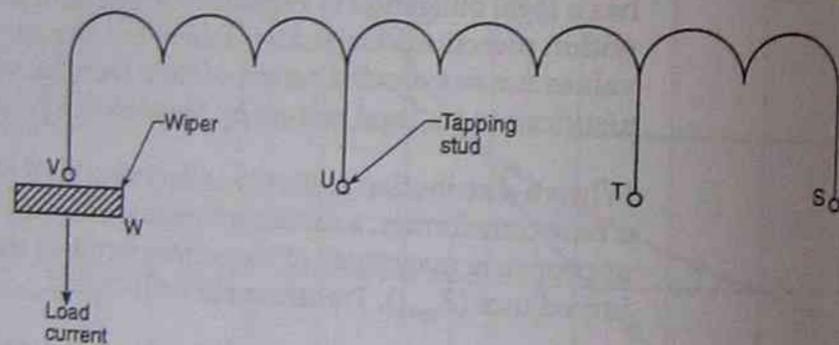
tap changers are connected to individual sets of coils at the end of the winding so that minor changes can be made to the overall output voltage as the load current varies.

Coils for transformers to be fitted with tap changers are made with a range of turns at the end of the coil, each having a lead brought out to be connected to the tap changer. These leads (called 'tails') are connected to 'studs' on the tap changer, and these studs, or wiping terminals, have a 'wiper' which is similar to the wiper inside a rheostat. As the wiper moves from one stud to the next the (different) voltage at the stud is transferred to the wiper, which is connected to the output terminal of the transformer.

The special consideration in understanding the operation of the tap changer comes when we realise the magnitude of current that needs to be interrupted, and the effects (sparking, voltage surges, etc) that come with switching such large currents.

To solve some of these problems tap changers can be either *off-load* or *on-load* devices.

## Off-load tap changers



Circuitry of OFF load tap changer

Note: that the size of the wiper is smaller than the space between adjacent tapping studs

Figure 16: Basic off-load top changing arrangement

Off-load tap changers are the simplest type commonly available for power transformers.

Connections are made from each tap of the transformer winding (which is usually immersed in the insulating oil) and are then terminated at a simple multi-position rotary switch mounted inside the transformer tank. The shaft of the rotary switch is brought out (using appropriate oil seals) to an externally mounted operating handle of the multi-position rotary switch.

An example of this arrangement is illustrated in Figure 17.

Due to simplicity of design the contacts of the tap position selector switch do not have any significant capability to make or break current and this type of tap-changer is therefore only suitable for operation when there is no load connected to the transformer.

To change the tap position of an off-load tap-changer would require the load to be temporarily disconnected during the tap change operation.

This type of tap-changer should therefore only be used in systems where temporary disconnection of the load can be tolerated.

A typical tap adjustment range is  $\pm 5\%$  in four steps of  $2\frac{1}{2}\%$  of the transformer rated voltage.

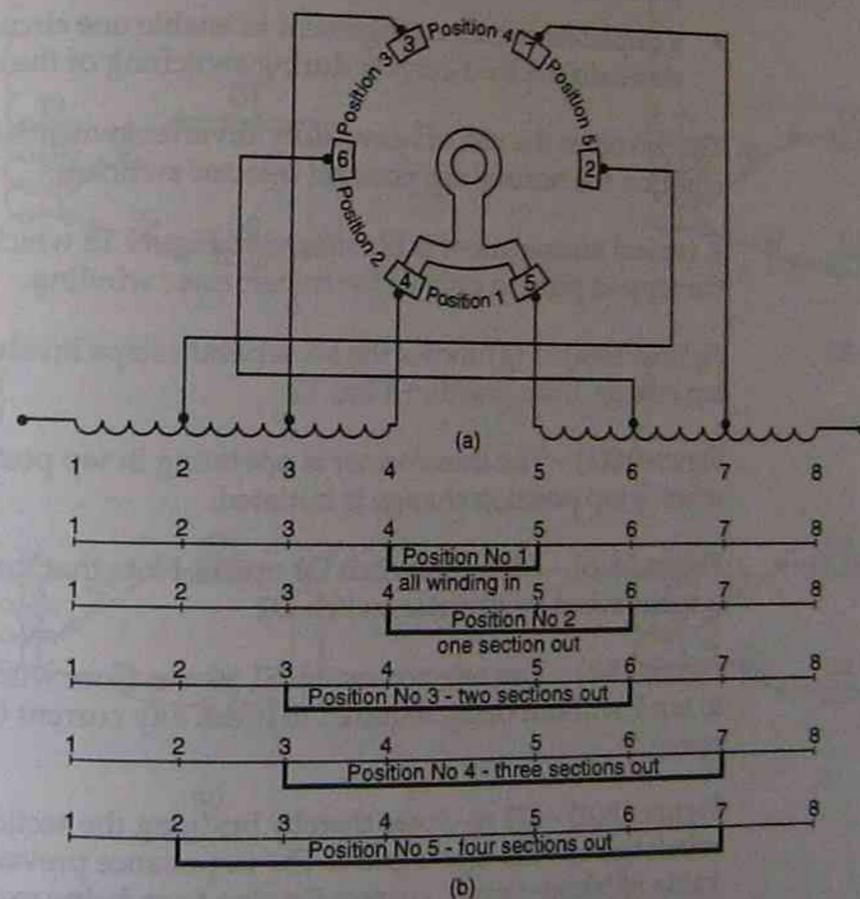


Figure 17: (a), (b) Diagram of off-load tap changer

At the rated operation of each transformer the following per-unit resistance and reactance voltage drops were determined for the transformer equivalent circuit.

Transformer A: 75 kVA transformer 0.02 pu resistive,  $j0.05$  pu reactive

Transformer B: 50 kVA transformer 0.02 pu resistive,  $j0.04$  pu reactive

Determine

- the kVA load provided by each transformer, and
- the operating power factor of each transformer.

#### Solution

The transformers are of different rating, so to use the per-unit method it is necessary to bring their per-unit values to a common base, which has been selected as 75 kVA. (You may need to briefly revise Section 4 to check the method of converting per-unit values to a different base.)

Converting the 50 kVA transformer values, 0.02 pu resistive becomes

$$\frac{0.02}{50} \times 75 = 0.03 \text{ pu}$$

and  $j0.04$  pu reactive becomes

$$\frac{0.04}{50} \times 75 = j0.06 \text{ pu}$$

to a 75 kVA base.

The load carried by transformer A ( $kVA_A$ ), then is

$$\begin{aligned} kVA_A &= 115 \angle \cos^{-1} 0.9 \times \frac{\text{transformer B}}{\text{transformer A} + \text{transformer B}} \\ &= 115 \angle -25.8^\circ \times \frac{0.03 + j0.06}{(0.02 + j0.05) + (0.03 + j0.06)} \\ &= 115 \angle -25.8^\circ \times \frac{0.067 \angle 63.4^\circ}{0.121 \angle 65.6^\circ} \\ &= 63.68 \angle -28^\circ \text{ kVA} \end{aligned}$$

$$\begin{aligned} kVA_B &= 115 \angle \cos^{-1} 0.9 \times \frac{\text{transformer A}}{\text{transformer A} + \text{transformer B}} \\ &= 115 \angle -25.8^\circ \times \frac{0.02 + j0.05}{(0.02 + j0.05) + (0.03 + j0.06)} \\ &= 115 \angle -25.8^\circ \times \frac{0.0539 \angle 68.2^\circ}{0.121 \angle 65.6^\circ} \\ &= 51.23 \angle -23.2^\circ \text{ kVA} \end{aligned}$$

Load carried by transformer A: 63.68 kVA  
Load carried by transformer B: 51.23 kVA

Note that Transformer B is marginally overloaded and will become hot if the load is sustained for too long.

Transformer A operating pf:  $\cos -28^\circ$ , which is 0.883 (lag)  
Transformer B operating pf:  $\cos -23.2^\circ$ , which is 0.919 (lag)

## Fault-level calculations

The following material will give you a better idea of the problems associated with the parallel operation of transformers. This is for understanding only and does not form part of the syllabus or objectives of this subject.

Possibly the greatest problem arising with two or more transformers connected in parallel is the increase in the value of current when there is a short circuit ('fault') on the output from the transformers. This is because each transformer acts as a separate path for supply of the fault current.

To understand what happens in this situation refer briefly to Section 2, where it was shown that when the terminals of the output of the transformer equivalent circuit are joined together a large current will pass through the output terminals.

The magnitude of this current will be limited only by the series resistance and reactance of the transformer equivalent circuit.

So, when two equal transformers (having the same transformer equivalent circuit) are connected in parallel, the fault current will be twice the magnitude had the supply been through only one transformer.

As the parallel connection of *equal* transformers is more the exception than the rule, to calculate the value of fault currents that may be expected use per-unit values.

# Introduction

## Parallel operation of transformers

So far we have looked at the operation of the transformer as if it were a single source of power to other equipment. This is the most common way that transformers are connected, but when a load is too large for a single transformer or it is necessary to always maintain the supply to a particular load even if there is a failure in the supply transformer, it is usual to connect transformers in parallel. When this happens there is a sharing of the load between the two (or more) transformers.

To connect transformers in parallel in this manner it is essential that the rated primary and secondary voltages are the same, and that the two secondary windings are connected together with the same polarity. If the transformers in parallel are of exactly the same size and equivalent circuit then the load will be equally shared between them. To carry out this calculation, you use the simplified transformer equivalent circuit referred to the primary, and ignore that part which gives the excitation and no-load losses.

## Industrial transformers

From the work on measuring instruments covered in other subjects of this course, you should be familiar with the techniques used in extending the range of dc meters by using voltage-divider circuits for dc voltmeters and shunts for dc ammeters.

When it comes to making measurements in ac, especially at high voltages, these methods are not satisfactory for a number of reasons:

- In some instances the power loss in the resistive networks may be considerable, and the dissipation of this heat would have to be allowed for in the design of switchboards and control units.

- Where the metering is associated with equipment used on systems with the voltage 11 kV or higher, the use of shunts and multipliers requires the instruments to be in direct connection with the metering points. The instruments would thus be connected at a high voltage relative to ground, which could produce some obvious dangers.

Where instrument transformers are used to extend the range of instruments both of these disadvantages are overcome. Not only are the power losses in the instrument transformers negligible, but the use of instrument transformers also gives electrical isolation between the supply and the instrument. This reduces the danger to operators and allows instruments to be changed without disconnecting the supply.

A further advantage in using instrument transformers is that whereas a shunt or multiplier can only be used for one instrument at a time, an instrument transformer can be used with several instruments concurrently, provided that the rated burden on the transformer is not exceeded.

While many transformers are seen to be large, as is the case with auto-transformers, there are also some very special transformers used for the connection of instruments and protective devices into power systems. Some of the features of these different types of transformers will be investigated.

## Generation of harmonics

With the development and use of thyristors and the subsequent use of solid-state switches and other devices, it was now possible to have discontinuities in the sine wave voltages. These discontinuities introduce harmonics into the electricity distribution system, and we will look at some of the effects of these produced by single-phase transformers.

## Objectives

After working through this section you should be able to

- state the necessary conditions to permit the successful parallel operation of transformers when the connected load exceeds the rating of either transformer, and the difficulties experienced if these conditions are not met
- state the standard terminal markings for transformers

- describe appropriate techniques to be followed when connecting transformers in parallel
- calculate the distribution of load sharing between two transformers under different load conditions, and the resulting fault levels when operating transformers in parallel
- state why instrument transformers are to be preferred to shunts and multipliers for extending the range of ammeters and voltmeters on ac circuits
- list the standard ratings of current transformers (CTs) and potential transformers (PTs) used for instrumentation and protection
- correctly describe the method of connection and construction of instrument transformers
- describe the techniques employed to minimise transformation errors
- describe the essential differences between CTs used for protection and those used for measurement purposes.

## Parallel operation

Power transformers are usually designed to ensure that maximum efficiency occurs between 75% and 100% of full load. At low loads, efficiency falls rapidly.

In the distribution of power for industrial and general domestic uses, the load often has a general pattern of being *high* between 6 am and 9 pm and *low* (below 50%) outside these hours. Under these conditions it would be uneconomical to have one large transformer alone providing this load and, as a consequence, working for a large proportion of its time on light load. In supply centres such as switchyards, where the load varies widely, it is usual to have several transformers connected in parallel, one for light-load conditions, and the others being switched in or out as required to meet load variations.

## Conditions for parallel operation

When transformers are to be operated in parallel, the primary windings are connected to the incoming supply bus-bars and the secondary windings are connected to the outgoing load. In these arrangements care must be taken to ensure that the terminals with the same polarity are connected together, otherwise the secondary emfs will act together (in the local secondary circuits) to produce a circulating current that is the equivalent of a short circuit.

There are certain requirements which must be met to ensure satisfactory parallel operation of transformers, and sharing of load in proportion to their kVA ratings:

- The primary windings must be suitable for the supply-system voltage and frequency.
- The *voltage ratio* must have the same value for each transformer.

If the voltage ratios are not equal when the secondary windings are connected in parallel, the difference between the voltages will cause circulating currents in the transformers. These will cause additional  $I^2R$  losses in the

windings, and the resultant heating can reduce the permissible output from the transformer.

- At their respective full-load currents, the transformer impedance triangles should have the same value, and the resistance: reactance ratio should also have the same value for all transformers in parallel.

This requirement could alternatively be stated in terms of the percent impedance and the percent reactance of each transformer being the same. If the impedance triangles at the rated kVAs are not identical in size and shape parallel operation is possible, but the transformers will operate at power factors different from the load-power factor and will not share the load in proportion to their kVA ratings.

## Tests for polarity

One method of assigning polarity markings to an unmarked or indistinctly marked transformer is the subtractive polarity test as shown in Figure 1.

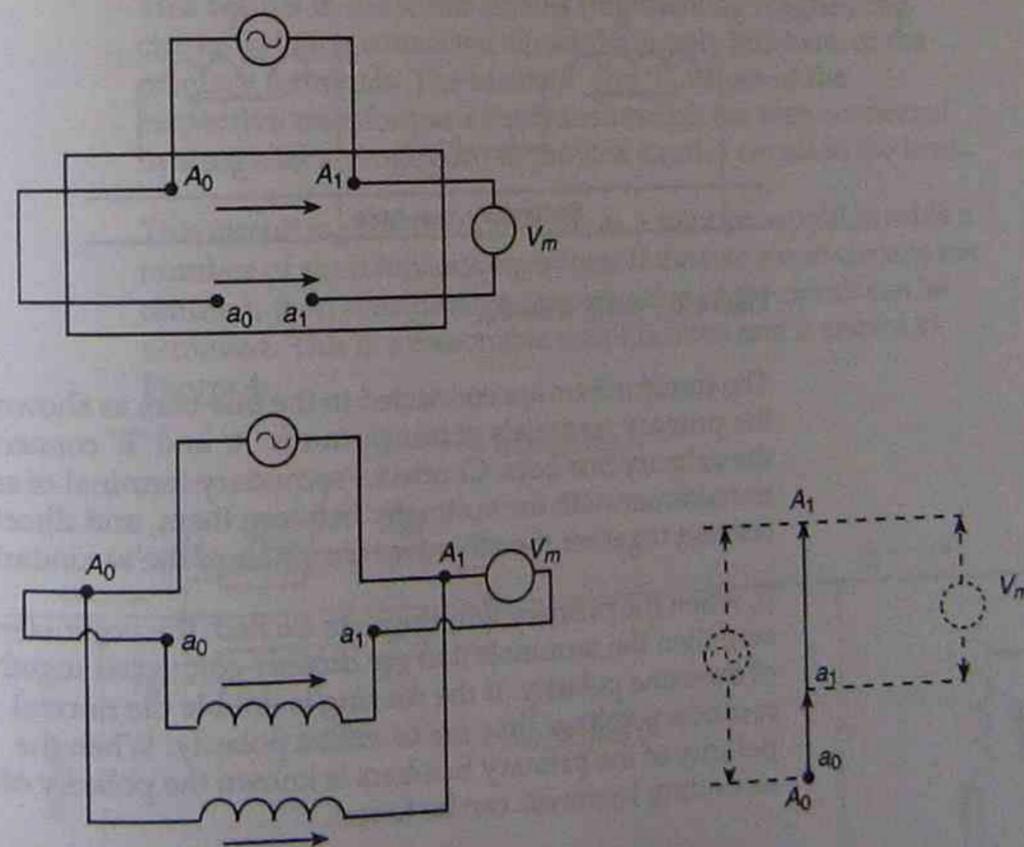


Figure 1: Subtractive polarity testing

With this method you can assign polarity markings arbitrarily to both windings, as shown in Figure 1. The windings are then interconnected so that, if the windings' terminals are correctly marked, the voltmeter reads the phasor difference of the primary and the secondary voltages.

The voltmeter must be capable of safely reading a voltage up to the sum of the primary and secondary voltages in case one winding is incorrectly marked.

Alternatively, when you have transformers that are to be connected in parallel but don't have absolutely clear terminal markings, it is necessary to check the winding polarities before putting the transformers into service. One test that can be used when the terminal markings of a transformer are indistinct is shown in the circuit diagram of Figure 2. In this case it is essential to have a voltmeter which can read twice the secondary voltage.

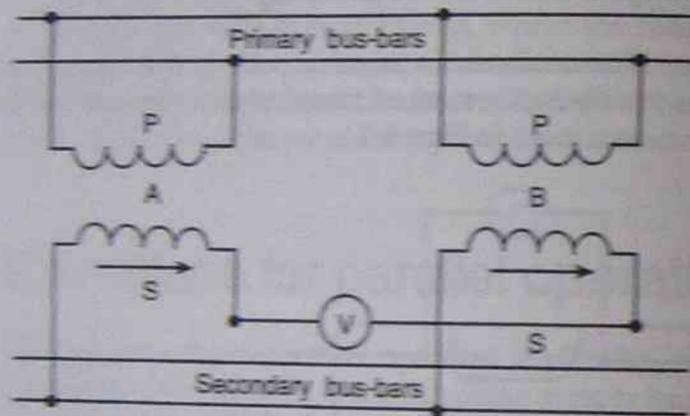


Figure 2: Polarity checking

The transformers are connected to the bus-bars as shown with the primary terminals of transformers 'A' and 'B' connected to the primary bus-bars. Connect a secondary terminal of each transformer with the voltmeter between them, and directly connect together the other two terminals of the secondaries.

If, when the primary windings are excited, the voltmeter reads zero then the terminals that are directly connected together are of the same polarity. If the reading is double the normal secondary voltage they are of unlike polarity. When the polarity of the primary bus-bars is known the polarity of the secondary terminals can be found.

## Parallel operation equivalent circuit

The equivalent circuit for two transformers operating in parallel may be drawn as in Figure 3:

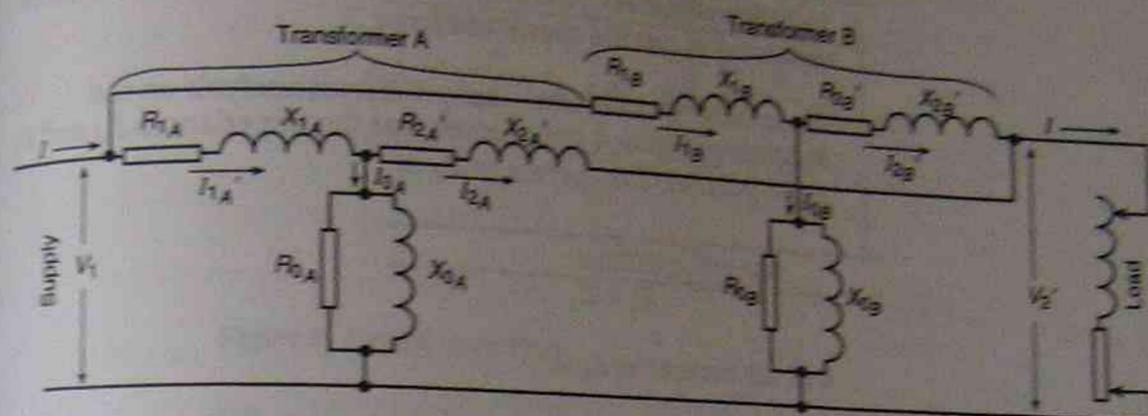


Figure 3: Exact equivalent circuit for parallel operation (primary referred values)

Basically, the circuit consists of the transformer equivalent circuits of the two transformers being connected in parallel. This results in the shunt circuits (representing magnetising currents) being connected across the supply bus-bars, or the primary terminals. The series 'R' and 'L' values of the respective transformer's equivalent circuit are then connected in a parallel arrangement to provide a series circuit to the load.

This circuit is difficult to analyse, as a solution would involve a number of simultaneous equations. If the two shunt circuits are omitted, then considerable simplification of the circuit can be achieved. This is a reasonable simplification and is shown in Figure 4:

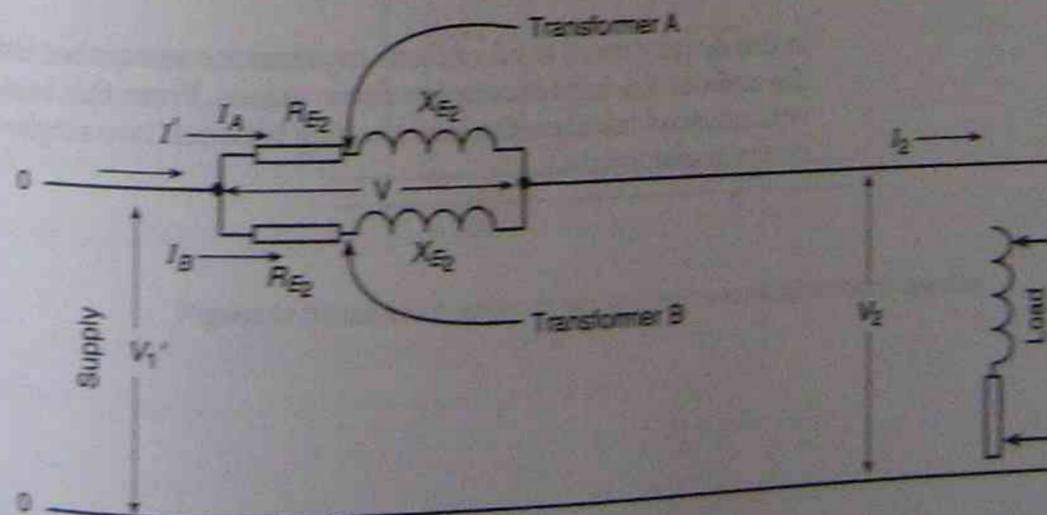


Figure 4: Simplified equivalent circuit (secondary referred values)

In the majority of cases, where the ratios of transformation are equal, the circuit of Figure 4 is accurate enough. However, when the ratios of transformation are not equal there will be circulating currents between the windings, and in order to make the appropriate calculations it is necessary to know the impedance values of the shunt circuits. In this case it would be necessary to use the circuit of Figure 3.

If you now carry out some simple circuit analysis on the diagram of Figure 4 you can see that the impedances  $Z_A$  and  $Z_B$  are in parallel. The current in  $Z_A$  is

$$I_{\text{total}} \times \frac{Z_B}{Z_A + Z_B}$$

and the current in  $Z_B$  is

$$I_{\text{total}} \times \frac{Z_A}{Z_A + Z_B}$$

Since the load voltages are the same for both transformers, similar equations can be developed for the sharing of power, or VAs, by the two transformers.

The VA power carried by transformer A (with equivalent series impedance  $Z_A$ ) is then written as

$$\text{VA}_{\text{total}} \times \frac{Z_B}{Z_A + Z_B}$$

and the VA power carried by  $Z_B$  is

$$\text{VA}_{\text{total}} \times \frac{Z_A}{Z_A + Z_B}$$

In the development of all of these expressions remember that the units of the impedances are *ohmic* values. From this basis you can now calculate the load sharing between two single-phase transformers.

### Example 1

Figure 5 is a single-line diagram of two single-phase transformers in parallel supplying a load.

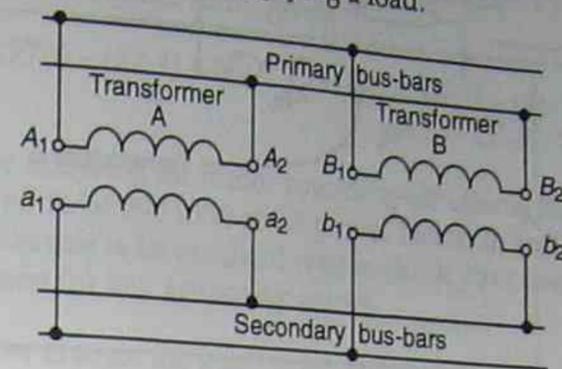


Figure 5: Parallel connection

When the transformer symbol for each is replaced by the following equivalent circuit specification referred to the primary side (ignoring the excitation and no-load losses), we have:

Transformer 1: 200 kVA 33 kV/11 kV  
series  $R_s = 5.0 \Omega$   
series  $X_{Ls} = j12.6 \Omega$

Transformer 2: 200 kVA 33 kV / 11 kV  
series  $R_s = 4.5 \Omega$   
series  $X_{Ls} = j14.0 \Omega$

Load: 200 kVA at 0.9 power factor (lag)

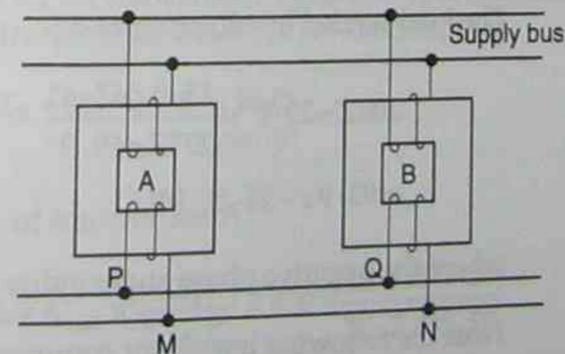


Figure 6: Schematic diagram of two single-phase transformers in parallel

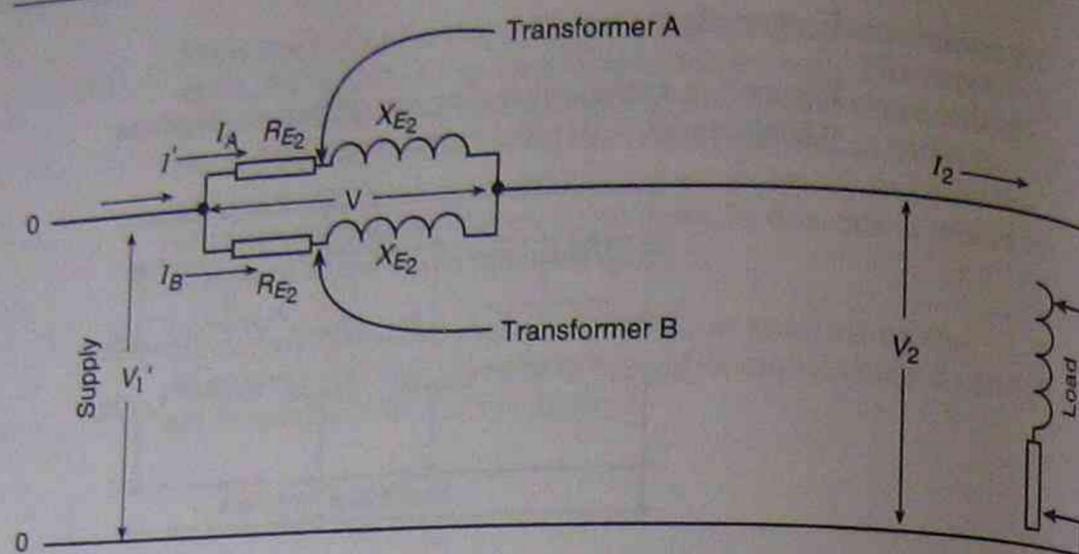


Figure 7: Series equivalent circuits of two single-phase transformers connected in parallel

If  $Z_A$ , for transformer 1 is  $5.0 + j12.0$  ohms, and if  $Z_B$ , for transformer 2 is  $4.5 + j14.0$  ohms, then

$$\begin{aligned} Z_A + Z_B &= (5.0 + j12.0) + (4.5 + j14.0) \\ &= 9.5 + j26.0 \\ &= 27.7 \angle 69.9^\circ \end{aligned}$$

The load carried by transformer 1 is

$$\begin{aligned} 200 \angle -25.8^\circ \times \frac{14.7 \angle 72.2^\circ}{27.7 \angle 69.9^\circ} \\ = 106.1 \angle -23.5^\circ \text{ VA} \end{aligned}$$

The load carried by transformer 2 is

$$\begin{aligned} 200 \angle -25.8^\circ \times \frac{13.0 \angle 67.4^\circ}{27.7 \angle 69.9^\circ} \\ = 93.9 \angle -28.3^\circ \text{ kVA} \end{aligned}$$

where the negative phase angle indicates a *lagging* power factor.

Note the following important comments regarding the above calculation:

- Both transformers are loaded within their rating.
- The sign of the angle associated with the (lagging) load is negative.

You should also note with interest that with a connected load of 200 kVA total, the total of the loads carried by the two transformers (given by  $106.1 \angle -23.5^\circ + 93.9 \angle -28.3^\circ$ ) when added, in rectangular form, gives:

$$\begin{aligned} (97.6 - j42.4) + (82.7 - j44.5) &= 180.3 - j86.9 \\ &= 200.1 \angle -25.7^\circ \end{aligned}$$

and, after allowing for minor rounding-off effects, this can be considered to be 200 kVA at 0.9 power factor. In fact, this simple exercise is an excellent way to check your own calculations for any arithmetic errors.

If you now change the load connected to the two paralleled transformers you just consider to 400 kVA at unity power factor, and repeat the calculation for load sharing, the following calculation should result.

### Example 2

Transformer A: 200 kVA 33 kV/11 kV  
series  $R_s = 5.0 \Omega$   
series  $X_{Ls} = j12 \Omega$

Transformer B: 200 kVA 33 kV/11 kV  
series  $R_s = 4.5 \Omega$   
series  $X_{Ls} = j14.0 \Omega$

Load: 400 kVA at unity power factor

Again, if  $Z_A$  for transformer 1 is  $5.0 + j12$  ohms, and if  $Z_B$  for transformer 2 is  $4.5 + j14.0$  ohms, then

$$\begin{aligned} Z_A + Z_B &= 9.5 + j26 \Omega \\ &= 27.7 \angle 69.9^\circ \end{aligned}$$

The kVA of transformer A:

$$\begin{aligned} \text{kVA}_{\text{total}} \times \frac{Z_B}{Z_A + Z_B} &= 400 \times \frac{14.7 \angle 72.2^\circ}{27.7 \angle 69.9^\circ} \\ &= 212.5 \angle 2.25^\circ \text{ kVA} \end{aligned}$$

Similarly, for transformer B:

$$\begin{aligned} \text{kVA}_{\text{total}} \times \frac{Z_A}{Z_A + Z_B} &= 400 \times \frac{13.55 \angle 67.4^\circ}{27.7 \angle 69.9^\circ} \\ &= 187.7 \angle -2.52^\circ \text{ kVA} \end{aligned}$$

Note: Transformer A is now loaded in excess of its rating. This should be avoided to prevent damage to the transformer.

Another way in which these calculations may be made is to use the values of percentage voltage regulation (explained in Section 4).

### Example 3

Two 25 kVA, 240/120 volt single-phase transformers are connected in parallel to supply a 40 kVA unity-power-factor load. Given that the regulation voltages for the equivalent circuit of each transformer are

Transformer A:  $E_R\% = 2.0\%$ ,  $E_X\% = 6.5\%$

Transformer B:  $E_R\% = 1.5\%$ ,  $E_X\% = 6.0\%$

calculate the part of the load carried by each transformer.

Now convert the regulation values to measurable values relative to the primary winding (ie the winding into which the load power will flow on its way to the load).

For transformer A:

$$E_{R\%A}, \text{ being } 2.0\% \text{ of } 240 \text{ V} = 0.02 \times 240 \\ = 4.8 \text{ V}$$

$$E_{X\%A}, \text{ being } 6.5\% \text{ of } 240 \text{ V} = 0.065 \times 240 \\ = 15.6 \text{ V}$$

For transformer B:

$$E_{R\%B}, \text{ being } 1.5\% \text{ of } 240 \text{ V} = 0.015 \times 240 \\ = 3.6 \text{ V}$$

$$E_{X\%B}, \text{ being } 6.0\% \text{ of } 240 \text{ V} = 0.06 \times 240 \\ = 14.4 \text{ V}$$

Now, the rated primary current is:

$$\frac{25\,000}{240} = 104.17 \text{ A}$$

The ohmic values of resistance and reactance in the equivalent circuits can then be calculated:

$$\text{Ohmic value for } R_A: \frac{4.8}{104.17} = 0.046 \Omega$$

$$\text{For } X_A: \frac{15.6}{104.17} = j0.15 \Omega$$

$$\text{For } R_B: \frac{3.6}{104.17} = 0.035 \Omega$$

$$\text{For } X_B: \frac{14.4}{104.17} = j0.138 \Omega$$

Then, calculating load sharing (using these ohmic values), the load carried by transformer A ( $V_{A_A}$ ) is

$$V_{A_A} = \frac{40 \angle 0^\circ \times (0.035 + j0.138)}{(0.046 + j0.15) + (0.035 + j0.138)} \\ = 40 \angle 0^\circ \times \frac{0.142 \angle 75.8^\circ}{0.299 \angle 74.3^\circ} \\ = 19 \angle 1.5^\circ \text{ kVA}$$

and the load carried by transformer B ( $V_{A_B}$ ) is

$$V_{A_B} = \frac{40 \angle 0^\circ \times (0.046 + j0.15)}{(0.046 + j0.15) + (0.035 + j0.138)} \\ = 40 \angle 0^\circ \times \frac{0.157 \angle 73.0^\circ}{0.299 \angle 74.3^\circ} \\ = 21.0 \angle -1.3^\circ \text{ kVA}$$

### Percentage values

Example 3 can also be calculated using the percentage values of regulation originally given.

This is a more useful form of calculation as it provides final answers in terms of percentage magnitudes, which is the form generally required for these problems.

### Example 4

Two 25 kVA, 240/120 volt single-phase transformers are connected in parallel to supply a 40 kVA unity power factor load.

Given that the regulation voltages for the equivalent circuit of each transformer are:

Transformer A:  $E_R\% = 2.0\%$ ,  $E_X\% = 6.5\%$

Transformer B:  $E_R\% = 1.5\%$ ,  $E_X\% = 6.0\%$

Calculate the part of the load carried by each transformer.

As an alternative to Example 3, we could use the regulation values directly to get

$$\begin{aligned} VA_A &= \frac{40\angle 0^\circ \times (1.5 + j6.0)}{(2 + j6.5) + (1.5 + j6.0)} \\ &= \frac{40\angle 0^\circ \times 6.185\angle 76.0^\circ}{12.981\angle 74.4^\circ} \\ &= 19.06\angle 1.6^\circ \text{ kVA} \end{aligned}$$

$$\begin{aligned} VA_B &= \frac{40\angle 0^\circ \times (2 + j6.5)}{(2 + j6.5) + (1.5 + j6.0)} \\ &= \frac{40\angle 0^\circ \times 6.801\angle 72.9^\circ}{12.981\angle 74.4^\circ} \\ &= 20.96\angle -1.5^\circ \text{ kVA} \end{aligned}$$

In this calculation, the loads carried by each of the transformers is less than their 25 kVA rating.

## Phasor diagram for parallel operation

In Figure 4 there are only three principal voltages:

- the supply voltage  $V_1'$
- the equivalent impedance voltage drop  $V$
- the load voltage  $V_2$ .

Since these voltages are common to both transformers, the phasor diagram for parallel operation can be drawn from these three voltages.

### Drawing phasor diagrams for parallel operation

When drawing the phasor diagram for parallel operation assume that all conditions for parallel operation are met: that is, that the primary windings are

- suitable for the supply system voltage and the frequency, and
- connected with correct polarity.

Figure 8 is drawn for the parallel operation of two transformers having the same ratio of series resistance to leakage reactance, so they can both be represented as collinear phasors on the phasor diagram:

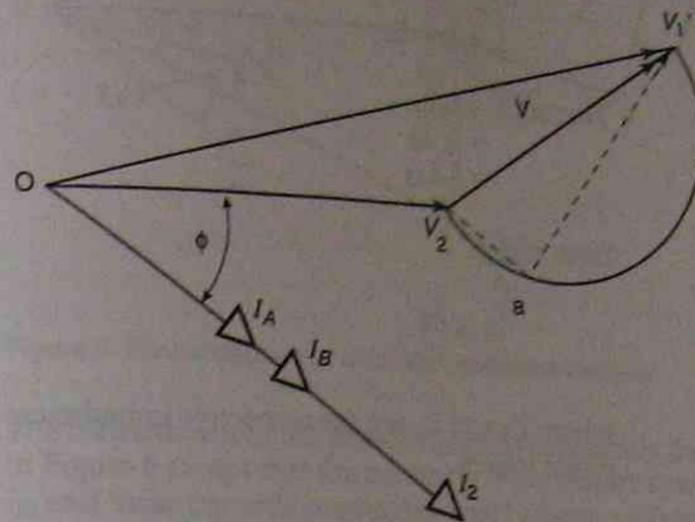


Figure 8: Phasor diagram for similar impedance triangles

The component phasors in Figure 8 may be defined as follows:

- $V_1'$  is the phasor representing the primary terminal voltage (reversed).
- $V_2$  is the phasor representing the secondary terminal voltage.
- $I_2$  is the phasor representing the load current,  $I$ .
- $I_A$  is the phasor representing the secondary current of transformer A,  $I_A$ .
- $I_B$  is the phasor representing the secondary current of transformer B,  $I_B$ .

The parallel grouping of the equivalent impedances shown in Figure 5 can be represented by an equivalent impedance,  $Z_{AB}$ .

Thus, for a given load current  $I$ , the impedance drop  $I_2 Z_{AB}$  (which equals  $V_1' - V_2$ ) will be constant in magnitude and, since  $V_2$  and  $V_1'$  are fixed, it will be fixed in direction in respect to  $V_2$  and  $V_1'$ .

Phasor  $V_2 V_1'$  as shown in Figure 8 is the phasor sum of phasors  $V_2 a$  and  $a V_1'$ , which are at right angles to one another. These represent the equivalent resistance and reactance drops respectively.

If a semi-circle is constructed with diameter  $V_2 V_1'$  then any triangle  $V_2 a V_1'$  will be a right-angled triangle.  $V_2 a$  is parallel to the load-current phasor, and  $a V_1'$  is in quadrature with it.

For any given load:

$$I_2 = I_A + I_B$$

$$\begin{aligned} \text{Hence } V &= I_A Z_A \\ &= I_B Z_B \\ &= I_2 Z_{AB} \end{aligned}$$

from which

$$\frac{I_A}{I_B} = \frac{Z_B}{Z_A}$$

where  $Z_A$  and  $Z_B$  are the respective impedances of the two transformers.

### Phasor diagram for dissimilar impedance diagrams

It is often a requirement in practice for two transformers of unequal MVA rating and internal impedance to be connected in parallel to supply a larger load than either could individually source.

It then becomes necessary to calculate each transformer's contribution to the load to ensure that the rated current of one or the other transformer is not exceeded.

Figure 9 shows the phasor diagram where the triangles have equal ratios of transformation, but dissimilar impedance triangles.

Because of the restraints shown in Figure 4, both of the transformers share the common phasor  $V_2 V_1'$  as their impedance-drop phasor.

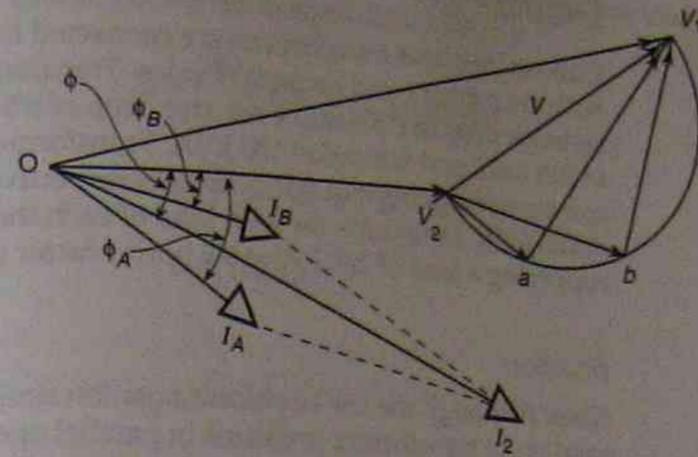


Figure 9: Phasor diagram for dissimilar impedance triangles

The construction of the phasor diagram is basically the same as in Figure 8 except that the current  $I_2$  is the phasor sum of  $I_A$  and  $I_B$ , and these currents produce two impedance voltage triangles,  $V_2 a V_1'$  and  $V_2 b V_1'$ , having a common hypotenuse,  $V_2 V_1'$ .

The resistance drops  $V_2 a$  and  $V_2 b$  are parallel to their respective secondary currents,  $I_A$  and  $I_B$ . Since  $V_2$  is common to both transformers, then the kVA of transformer A is

$$S_A = \frac{S_T \times Z_B}{Z_A + Z_B}$$

and kVA for transformer B, is

$$S_B = \frac{S_T \times Z_A}{Z_A + Z_B}$$

where;

$S_A$  = Transformer A's MVA contribution to the load

$S_B$  = Transformer B's MVA contribution to the load

$S_T$  = Total MVA load

Although there are no restrictions on the method of solution of this type of problem, it will generally be found that the easiest solution is the method that uses the polar-coordinate representation of complex numbers. This will be seen from the following example.

### Example 5

Two single-phase transformers are connected in parallel on both the primary and secondary sides. Transformer A has a resistive drop of 0.5% and a reactive drop of 8% of the voltage at full load, and is rated at 100 kVA. Transformer B has corresponding drops of 0.75% and 4% respectively, and is rated at 200 kVA. Determine the kVA load of each transformer when supplying a load of 280 kW at 0.9 power factor (lagging).

#### Solution

Note: Although the use of percent notation simplifies the solution of transformer problems in parallel operation, the percent values must be referred to a common base; hence, in this example assume that the base is 200 kVA. The values for transformer A, which is rated at 100 kVA when referred to a 200 kVA base, will then be

$$R_A' \% = R_A \% \times \frac{\text{base}}{\text{rating}}$$

$$= 0.5 \times \frac{200}{100}$$

$$= 1\%$$

$$X_A' \% = X_A \% \times \frac{\text{base}}{\text{rating}}$$

$$= 8 \times \frac{200}{100}$$

$$= 16\%$$

Converting all impedances and ratings to polar coordinates, for transformer A:

$$Z_A = 1 + j16$$

$$= 16.03 \angle 86.4^\circ \Omega$$

and transformer B:

$$Z_B = 0.75 + j4$$

$$= 4.07 \angle 79.4^\circ \Omega$$

$$Z_A + Z_B = 1.75 + j20$$

$$= 20.07 \angle 85^\circ \Omega$$

Where the total load is expressed in kilowatts, the load must first be converted to equivalent kVA,  $S_T$ .

$$S_T = \frac{280}{0.9} \angle \cos^{-1} 0.9 \text{ (lagging)}$$

$$= \frac{280}{0.9} \angle -25.84^\circ \text{ kVA}$$

$$= 311 \text{ kVA}$$

$$S_A = \frac{S_T \times Z_B}{Z_A + Z_B}$$

$$= \frac{311 \angle -25.84^\circ \times 4.07 \angle 79.4^\circ}{20.07 \angle 85^\circ}$$

$$= 63.07 \angle -31.44^\circ \text{ kVA}$$

$$S_B = \frac{S_T \times Z_A}{Z_A + Z_B}$$

$$= \frac{311 \angle -25.84^\circ \times 16.03 \angle 86.4^\circ}{20.07 \angle 85^\circ}$$

$$= 248.4 \angle -24.44^\circ \text{ kVA}$$

So, in this problem, although the total load is less than the combined capability of both transformers due to the dissimilarity of the impedance triangles, one of the transformers is operating at 89% of full load and the other at only 60% of full load.

During discussion of the use of per-unit values in Section 4, it was suggested that you would appreciate the benefit of per unit values when looking at the operation of power transformers in parallel.

To demonstrate this benefit, work through this next example where per-unit values are used.

### Example 6

Two single-phase transformers 75 kVA and 50 kVA are connected in parallel to supply a load of 115 kVA at 0.9 power factor lagging.

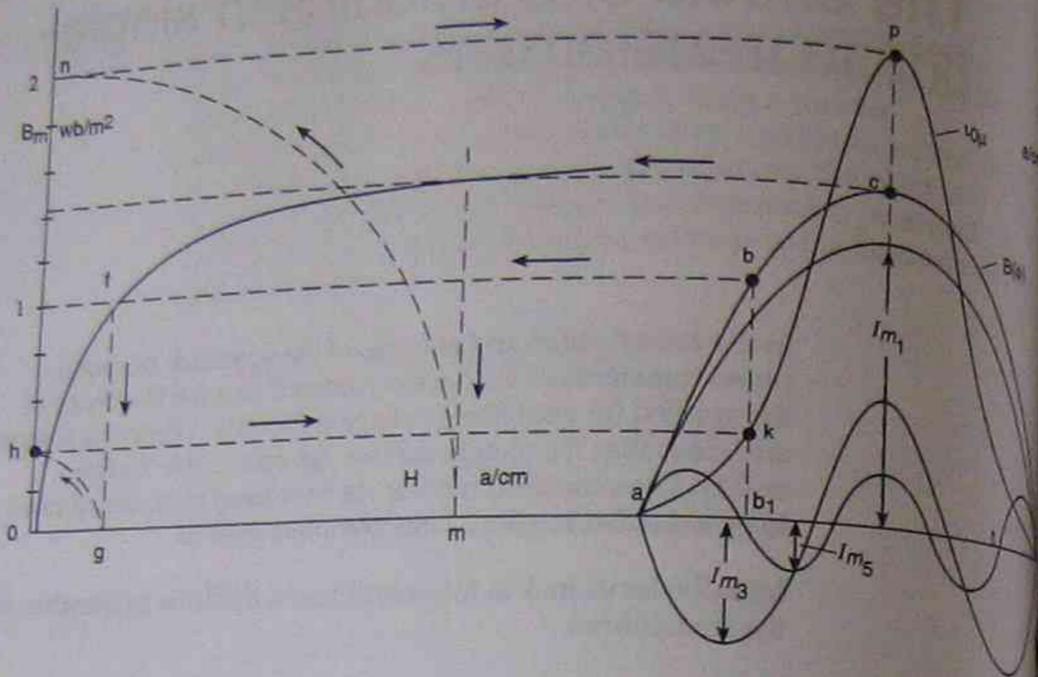


Figure 16: Plotting of curves  $i_{a1} = f(t)$  and resultant harmonic resolution

Figure 16 shows the curvature of the B-H curve (points 'g', 'f' and 'l'), and adjacent to it is shown the (assumed) sinusoidal input-current waveform and the resultant sinusoidal-flux waveform, which is based on the theory of the relationship being linear.

If you now reverse this analysis, and project (point by point) from the sinusoidal-flux curve back to find the actual shape of the excitation current waveform, this does not come to be a sine wave but a wave of another shape.

To follow this derivation, start at the flux value of point 'b' on the flux waveform. On projecting across to the B-H curve you get to 'f', which can then be projected onto 'g' on the ampere-turns scale of the B-H curve (ie 'g' is a measure of the current to produce the flux value at 'f' or 'b').

So that you can plot the current- and flux-waveshapes on the same graph you need to rotate the current waveshape through  $90^\circ$ .

This is the same as the  $90^\circ$  angle that was shown on the phasor diagram development previously. This is achieved by rotating 'g' to the point 'h'.

This value of 'h' is then projected across to find the point 'k', where the values of 'b' and 'k' occur at the same time (given by  $b_1$ ). The value of 'k' is thus the instantaneous value of current to produce the corresponding value of flux at the same instant.

When you follow this process point by point, the resultant current waveform is given by the curve 'a-k-p-d'. This shape is far from sinusoidal. On the time axis are plotted the third and fifth harmonic waveforms which, when added to the original excitation-current sinusoidal waveform, will give the resultant waveshape 'a-k-p-d'.

The derivation of this waveshape was based on the B-H curve being a line of single value; however, you also know that in any magnetic circuit that the cyclic operation through the B-H curve results in hysteresis. When the effect of hysteresis is also considered the resultant waveshapes based on a point-by-point transfer method are those shown in Figure 17:

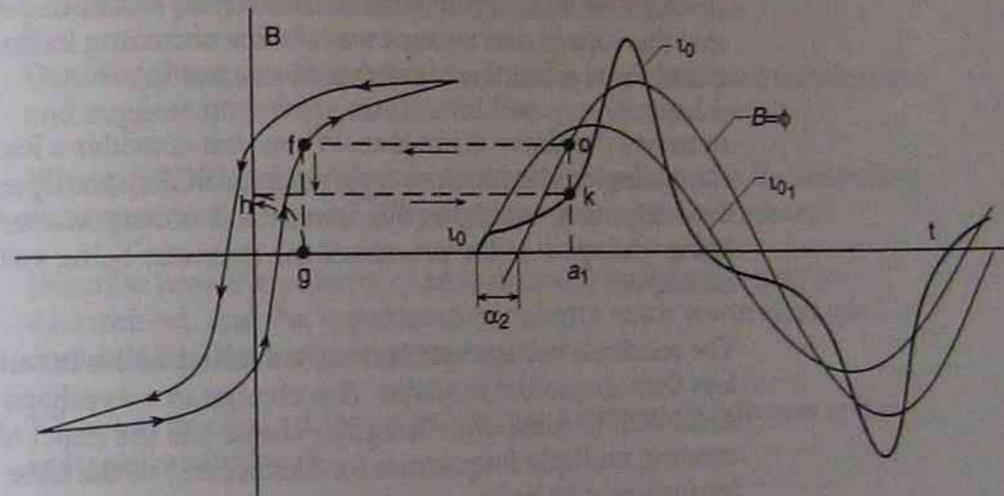


Figure 17: Effect of hysteresis on no-load current curve

It should now be obvious that the waveshape of  $I_0$  is not only markedly pointed (as previously shown), but that it has been distorted sideways and has also experienced a phase angle change, all of which will contribute to the harmonic components in the waveform of the exciting current.

Whilst all the above causes of distortion are present in all transformers, careful design and selection of materials will ensure that this distortion is minimised.

The choice of magnetic steel having the minimum area and the most vertical shape in the hysteresis loop will reduce phase displacement for the core.

# Instrument and protective transformers

Single-phase transformers are made to meet the special needs of both instrumentation applications and protective applications. A *current transformer* may be manufactured either for use in conjunction with protective equipment (protective application) or for extending the range of an ammeter (instrumentation application).

Each of these applications imposes a different set of restrictions on the design of the particular transformer. Whilst there are a number of differences, the main difference between the two applications relates to the design of the magnetic core and the value of magnetic flux that the core will allow without saturating.

While many transformers are seen to be large, as is the case with auto-transformers, there are also some very special transformers used for the connection of instruments and protective devices into power systems. Some of the features of these different types of transformers will be investigated.

With the development and use of thyristors and the subsequent use of solid-state switches and other devices, it was now possible to have discontinuities in the sine wave voltages, which introduce harmonics into the electricity distribution system. We will look at some of the effects of these in single-phase transformers.

## Applications of instrument transformers

Instrument transformers are designed to allow a selected range of standard meter movements to be used to indicate current, voltage and power values over a wide range. By using appropriate transformer ratios and suitable meter scales, you could use the same instrument movement to indicate 50 amperes or 5000 amperes in the case of current measurement or, for voltage, 11 000 or 110 000 volts for voltmeters.

between the high voltage main power circuit and the meters on the instrument which display their relevant current or voltage values.

As well as being used to extend the range of measured values, current transformers are also used in protective applications. Voltage transformers, in contrast, are primarily used in measurement applications.

The standard specification to which instrument and protective current transformers are manufactured gives standard rated secondary currents as 5, 2 and 1 amperes, with a preference for 5 and 1 amperes.

When constructed as three-phase delta-connected transformers (to be explained in later sections), these ratings of 5, 2 and 1 amperes, divided by  $\sqrt{3}$  are also permitted. With voltage transformers the standard secondary voltage is 110 volts, to allow a standard range of voltmeter instruments to be used with voltage transformers.

Current transformers are designed to give an output current which is proportional to and a 'mirror image' of the primary current waveform. The output of current transformers is expressed in VA.

Potential transformers are designed to give an output voltage which is proportional to and a 'mirror image' of the primary voltage waveform. The output of voltage transformers is expressed in millisiemens.

It is important to take into account any errors introduced by the instrument transformers in the final meter reading.

When used with wattmeters measuring at low power factor, quite definite steps must be taken to adjust the meter readings. These procedures are described later in this section.

## Construction of protective transformers

### Current transformers

When connected into a circuit, the primary winding of a current transformer is connected in series with the main supply. The secondary winding is connected to the current coil of the

measuring instrument, thus providing a current that is proportional to, and in (opposition) phase to, the current in the main circuit.

The phase relationship of the 'measurement' currents is important only with the connection of power-measuring instruments. Where transformers are used for measurement purposes, they must be consistent in output and reliable in performance. This means workmanship and insulation must be of a high standard.

The magnetic core of the transformer is very important, and a common type of construction of instrument transformers comprises a laminated 'C-core-type' split core onto which the coils are fitted. Prior to fitting, the windings are usually formed and/or wound onto a hollow bobbin to allow fitting to the C-core. In some instances the complete assembly is encapsulated.

In some instances, where heavy currents are involved, the primary winding may consist of a single straight conductor with the secondary wound about it.

Alternatively, the supply conductor may pass through the centre of a toroidal core. A diagrammatic representation of a type instrument transformer is shown in Figure 10.

The main feature of a toroidal current transformer is its extremely low leakage reactance.

Core of rectangular cross section formed by a continuous overlapping spiral of magnetic steel strip

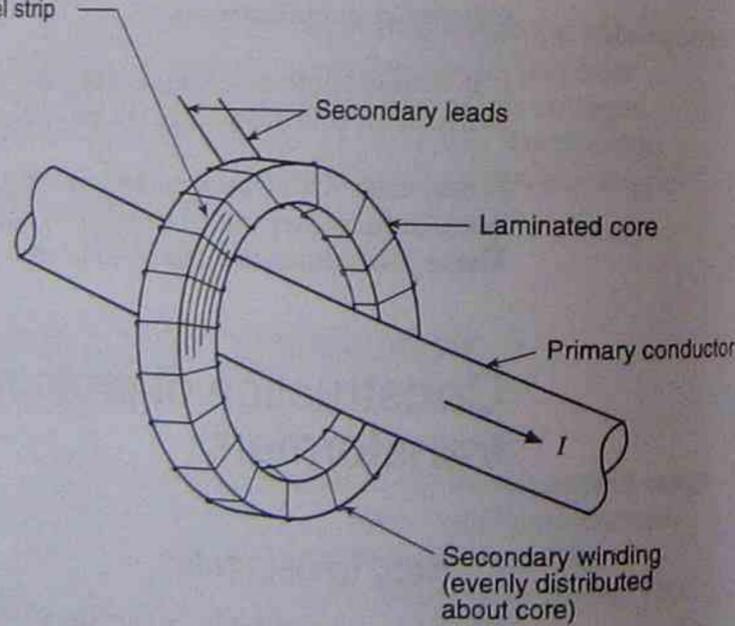


Figure 10: Toroidal-type

For a current transformer used for metering, the range over which the ratio error (to be explained later) is to be sustained need be only up to 125% of full-load current. If the transformer should saturate above this current value, the consequential drop in secondary current will tend to reduce the possibility of damage to the measuring instrument in the event of an excessive current resulting from overload conditions.

For current transformers used for protection purposes the transformers should remain accurate not only for load conditions but for short circuit conditions as well.

This usually requires them to perform satisfactorily for up to 20 times full load without saturating.

## Voltage transformers

In voltage transformers the primary winding is connected across the supply system and the secondary supplies a voltage, proportional to the supply voltage and in (opposition) phase to it, connected to the positive terminal of the meter.

Since the primary winding is across the supply voltage the greatest care must be taken when insulating this winding to withstand the high voltages encountered.

As with current transformers, the cores are made of high-quality transformer steel laminations. The laminations are made to fit together in the form of a single ring or rectangle.

Voltage instrument transformers are manufactured either as single-phase or three-phase devices. Figure 11 shows the core shape for a single-phase transformer, and the double rectangle shape, shown in Figure 12, is for a three-phase transformer (which will be explained in later sections).

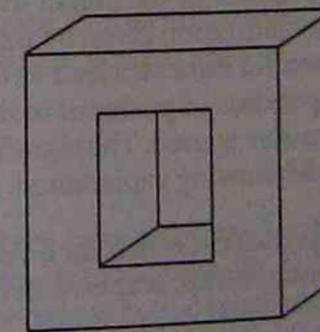


Figure 11: Single-phase instrument transformer core

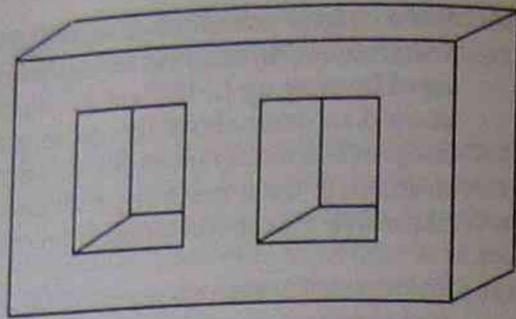


Figure 12: Three-phase instrument transformer core

The primary and secondary windings are usually placed one on top of the other on each limb of the core. For high-voltage applications the primary coils are wound in separately insulated sections connected in series.

## Error minimisation

When a high degree of accuracy is required, compensation must be made for the inherent errors that could be produced by the use of instrument transformers.

Errors arising from the use of instrument transformers may be due to the following factors:

- ratio errors, where the reflected quantity, either current or voltage, does not bear an exact nor constant relationship to the quantity in the circuit being metered
- where the phase relationship between the input and output quantities is either not exactly  $180^\circ$ , or where the phase relationship tends to vary.

The presence of the first of these errors will affect the accuracy of any indication given by the connected instruments, whereas the second error will lead to the incidence of error in the computation of power or in the indication of power, where a wattmeter is used. The significance of these errors is indicated in the following explanation.

The phasor condition for a voltage transformer, assuming a 1:1 relationship, is shown in Figure 13.

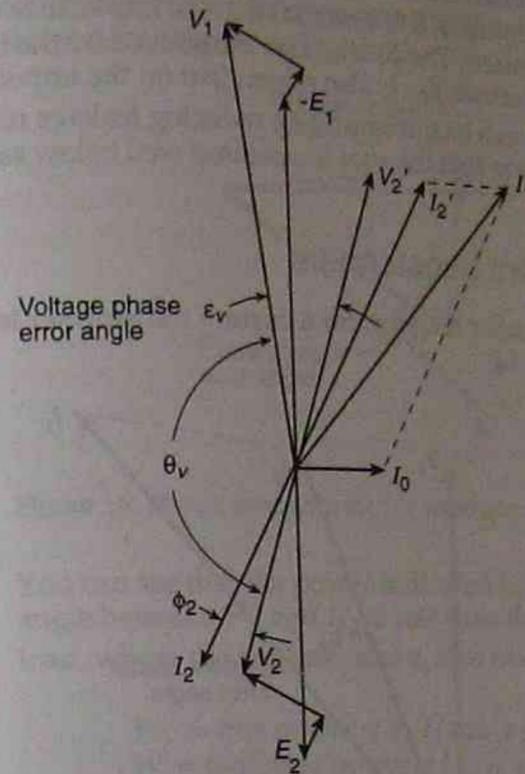


Figure 13: Phasor conditions for 1:1 voltage transformer

## Voltage transformer

For the ideal transformer, you know that the ratio between voltages and turns is given by

$$E_1 : E_2 = N_1 : N_2 \quad (\text{equation 1})$$

However, looking at the phasor conditions for the voltage transformer in Figure 13, due to the effect of leakage reactance and winding resistance you can see that

$$\frac{V_1}{V_2} \neq \frac{E_1}{E_2} \quad (\text{equation 2})$$

The magnitude of the error is primarily a function of the winding impedance drops of both windings, but it is also affected to a lesser degree by the relative magnitude of  $I_0$  and  $I_2'$ . It is also apparent from the diagram that  $\theta_v$  is not  $180^\circ$ . To achieve maximum accuracy the condition for correct operation is

$$V_1 = -V_2'$$

therefore, to minimise the transformation errors in voltage transformers, it is essential that the impedance drops be minimised. The phase error introduced by the instrument transformer,  $\epsilon_v$ , is also dependent on the impedance drops. These can be minimised by reducing leakage reactance and ensuring that the core is operated well below saturation.

### Current transformers

The phasor diagram for a current transformer is shown in Figure 14:

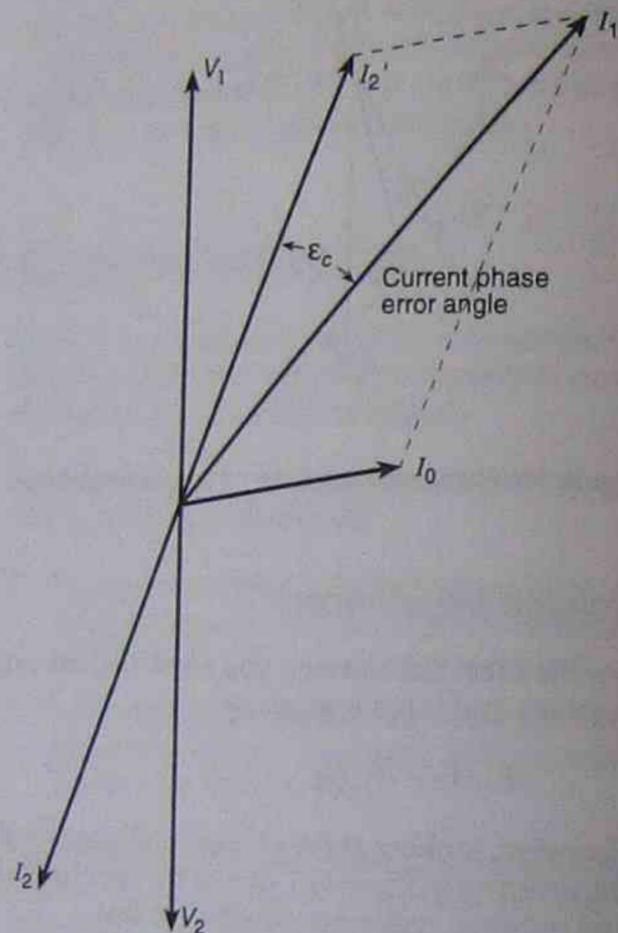


Figure 14: Phasor conditions for 1:1 current transformer

In current transformers, the major source of error both in ratio and in phase is the relative magnitude of  $I_0$  with respect to  $I_2'$ . To minimise errors in current transformers it is necessary to keep the magnetising current as small as possible. The phase error introduced by the presence of  $I_0$  is indicated by the angle  $\epsilon_c$ .

The error introduced by the phase error in the instrument transformers used in conjunction with a wattmeter is shown in the phasor diagram of Figure 15:

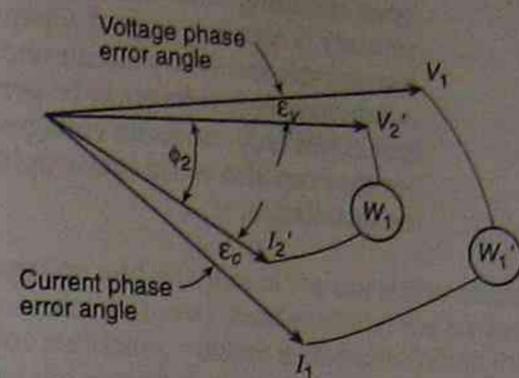


Figure 15: Phasor conditions for 1:1 wattmeter transformer (showing error)

You can see that the power indicated by the meter, based on the angle between  $V_2'$  and  $I_2'$ , is less than the actual power of the load, where the angles  $\epsilon_v$  and  $\epsilon_c$  also need to be considered:

$$W_1' = \text{true power} = V_1 I_1 \cos(\phi_2 + \epsilon_v + \epsilon_c)$$

$$W_1 = \text{indicated power} = V_2' I_2' \cos \phi_2$$

### Protective transformers

Where the transformer is intended for protective purposes the need to eliminate phase-angle error is not so important, but it is essential that the transformer faithfully reflects changes in the primary current up to 20 or more times the full-load current, as these current levels may occur on the primary side under fault conditions.

Since the design restrictions for the different purposes vary, a current transformer designed for protective purposes should not be used for metering, and a current transformer designed for metering purposes should not be used in protective circuits.

Disregarding the considerations of accuracy, a metering transformer would be unsuitable for use in protective circuits because, since the protection transformer is designed to accurately reflect fault currents into the secondary circuit without damage to the winding conductors, it is essential that transformers intended for protection are capable of a higher VA output than the equivalent measurement transformer.

### Warning

It is very important to be aware of the danger associated with open circuiting the secondary of a current transformer while its primary is carrying current. Open circuiting a current transformer secondary circuit under these conditions will result in extremely high voltages to be generated across the point of break. Not only can these voltages be dangerous to personnel but they can also cause catastrophic damage to the insulation of the installation.

## The effects of harmonics in single-phase transformers

In the investigation and analysis of the operation of single-phase transformers we have considered that the frequency of the applied (or excitation) voltage is 50 hertz. While this is ideal and simplifies the picture that we can create, it is far from reality. The simplified models we have been considering need to be extended to give a more complete picture.

Initially, let us look at the occurrence of multiple frequencies in the transformer.

### Generation of harmonics from magnetic-core saturation

In an earlier section you have been shown the relationship between the excitation flux in the magnetic core and the excitation current,  $I_e$ , from the transformer equivalent circuit.

We will now look at this relationship using graphical displays to show the effects of

- the lack of linearity in the B-H curve, and
- the area of the hysteresis loop.

In considering the effect of the lack of linearity in the B-H curve we will exaggerate the graphics to make it a little more obvious what is happening.

Examples of each of these arrangements are shown in Figure 8. If we assume that we have three windings and each winding consists of two equal sections which are connected together in series, the diagrams showing the arrangements are (a) for delta and (b) for star.

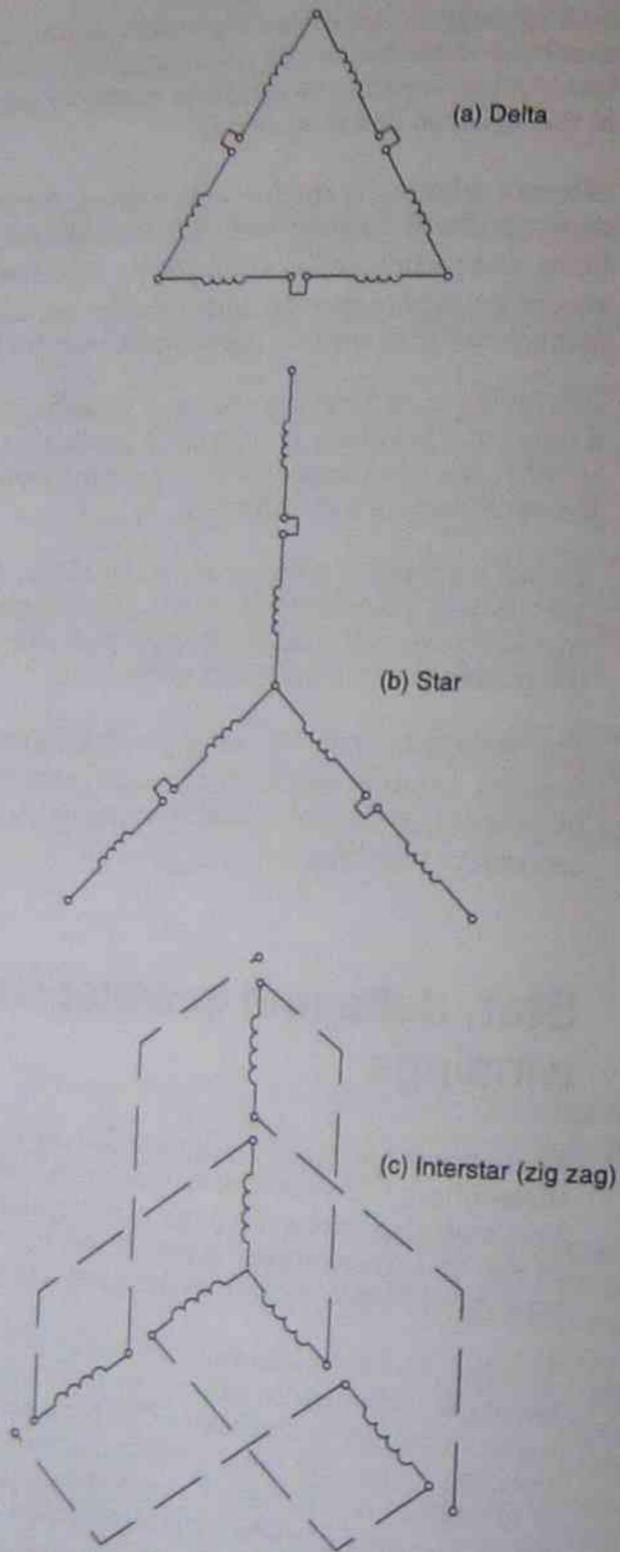


Figure 8: Winding arrangements (a) Delta (b) Star (c) Interstar

Taking the star arrangement, if we now disconnect the equal sections connected in series, and connect each with another section from another winding in such a way that the induced voltages add in a particular way, it is possible to create a third alternative connection arrangement, which is called interstar or zigzag, and is shown in (c). This same diagram is shown more simply in Figure 9.

Recall that these windings are electrically isolated from each other, and it is possible to connect part of winding 'a' in series with part of winding 'b' without causing any electrical difficulty. When this is done it is necessary to visualise that the current flowing in the combination winding of a and b will cause flux to be formed in the legs of the magnetising circuit to which both windings are fitted.

It is this feature of the creation of flux in two different legs of the magnetic core by the same current that makes the zigzag winding so useful. Figure 9 shows the single windings of a transformer interconnected in the interstar (zigzag) manner. To determine the magnitude of the output voltage (ie line-to-neutral voltage) multiply the voltage of any winding section by  $\sqrt{3}$ .

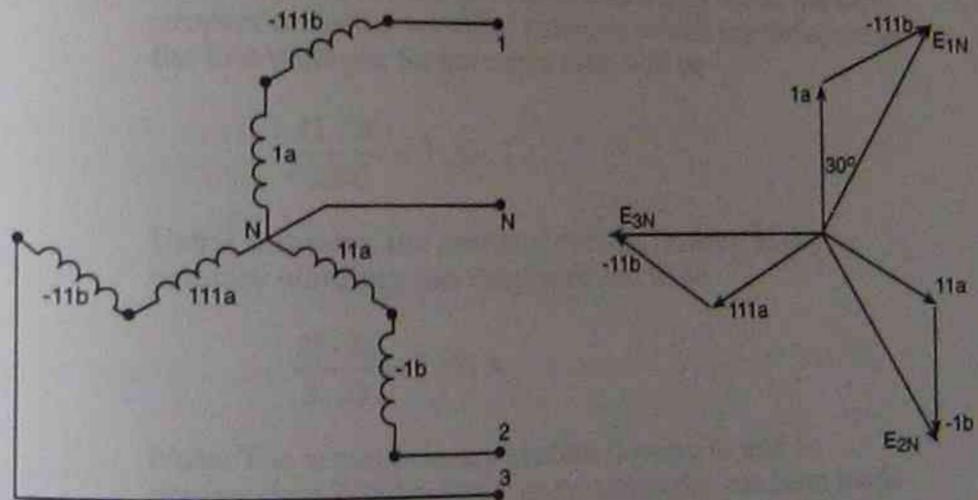


Figure 9: Zigzag-winding connections are used in transformers forming part of three-phase rectified power supplies

# Introduction

In previous electrical theory subjects, you were introduced to ac electrical circuits consisting of more than an active and neutral connection, where these circuits were described as multi-phase circuits. The most common of these is the three-phase arrangement, consisting of three active connections and a neutral connection.

You have been shown the form of the single-phase transformer equivalent circuit, and you should quickly revise this work.

As you will continue with this same form of transformer equivalent circuit, quickly revise the work of Section 2. Make sure you understand the basis on which both the series and parallel elements of this transformer equivalent circuit were developed.

In addition, you will be shown how a three-phase transformer can be considered as being formed from three single-phase transformers. This is relevant because the form of the transformer equivalent circuit for a three-phase transformer is identical to the form for a single-phase transformer.

## Objectives

After working through this section, you should be able to

- determine the transformation ratios between primary and secondary windings for three-phase transformers with various winding arrangements
- determine the values of line voltages and winding voltages for three-phase transformers with various winding arrangements
- (for three-phase transformer symbols)
  - state the vector symbol, given the winding-terminal connection diagram or the vector diagram
  - draw the vector diagram, given the vector symbol
  - describe core- and shell-type construction for three-phase transformers

- refer the transformer per-phase equivalent circuit component values to the primary or secondary side, given the voltage ratio
- determine the approximate per-phase equivalent circuit from the results of a short-circuit test and an open-circuit test
- calculate the base-line current and base impedance given the base-line voltage
- convert the per-phase equivalent component values from ohmic to per-unit values, and vice versa
- convert the per-phase equivalent circuit per-unit component values between bases
- calculate the line voltage across a specified balanced three-phase load, given the per-phase equivalent circuit component values and the supply-line voltage
- calculate the percentage voltage regulation, given the per-phase equivalent circuit component values, and the supply-line voltages, for a specified balanced three-phase load
- determine the voltage regulation using the approximate regulation equation, given the per-phase equivalent impedance in ohmic or per-unit values, for a specified balanced three-phase load
- determine the condition for maximum regulation, given the per-phase equivalent circuit component values
- determine the condition for zero regulation, given the per-phase equivalent circuit component values.

## Three-phase transformers

### Theory revision

#### Star and delta connections

Circuits are identified by describing their type of connection. One type is delta, which is based on the three active connections only, and the other is star, where all four connections are made:

The diagrams that are used to identify each of these connections are shown in Figure 1.

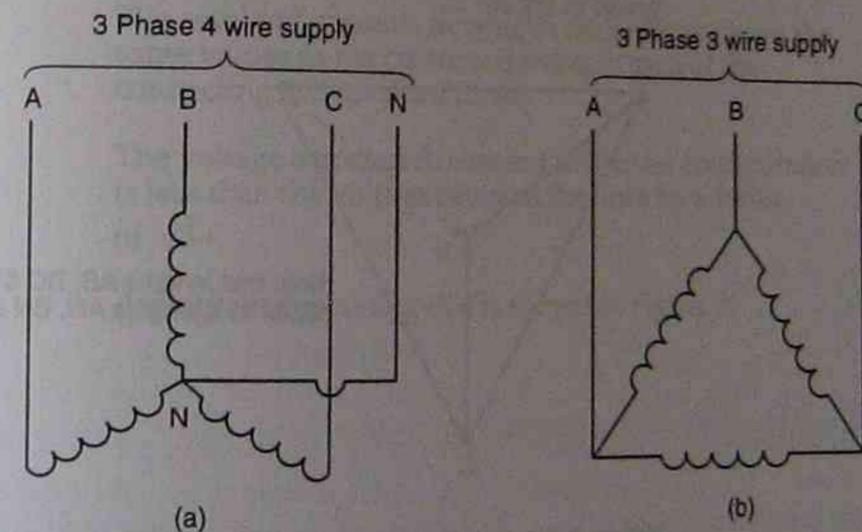


Figure 1: (a) Star connection (b) Delta connection

#### Star- and delta-voltage relationships

It is extremely important that you understand the voltage values that occur for each of these star and delta connections, as they are interdependent. The factor that exists between delta and star voltages is  $\sqrt{3}$ , where a delta voltage is  $\sqrt{3}$  times its corresponding star value.

It is from this factor that you get the 240/415 voltage value that exists in all domestic installations. With power transformers the voltages experienced are much higher than these, but the same factor prevails.

Remember that the purpose of having a star configuration is to obtain a neutral, or star point. This star point is used for a number of purposes. Possibly the most common and important purpose is to provide a connection into an electrical supply system for *earthing*. From this earth, or reference, you are then able to make measurements of voltage values, even though it is more widely recognised as a means of providing protection to the consumer.

When you extend your measuring technique to include not only the value but also the *angle* relationship of an active connection relative to the earth, then you can extend your measurements to include both the magnitude and the phase angle of any voltage in the multi-phase system relative to any other voltage in the same system.

In the past you would have done this by developing a phasor diagram from a reference value (which is usually the voltage between A-Phase and neutral). The voltage values (with both angle and magnitude) that exist in the star-delta relationship are usually shown in the form given in Figure 2.

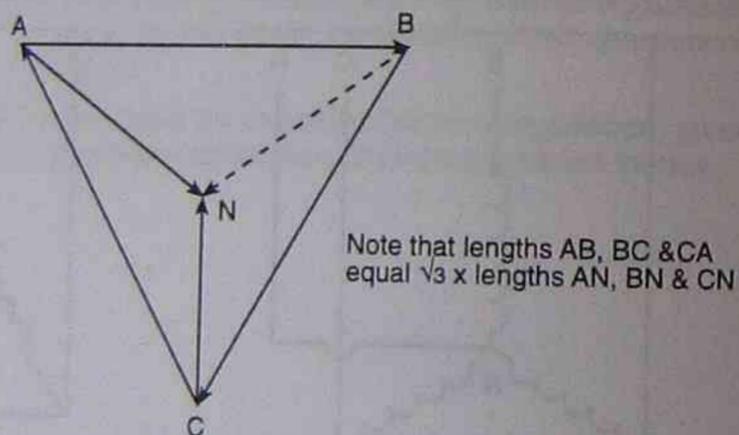


Figure 2: Line voltages  $V_{AB}$ ,  $V_{BC}$  and  $V_{CA}$  and Phase voltages  $V_{AN}$ ,  $V_{BN}$  and  $V_{CN}$

This phasor diagram can be described as a display of the instantaneous voltages in a three-wire and/or four-wire three-phase system. You are already aware of the angular displacement that exists between voltages in a star system and a delta system. In Figure 1 this can be seen when you consider the voltage at the point 'A', or the A-phase. The voltage in the star system (to neutral) is displaced  $30^\circ$  (lagging) from the voltage in the delta system (to B-phase).

Alternatively, you can see that the voltage at B-line (with respect to C) is displaced  $90^\circ$  (lagging) from the voltage at A-line (with respect to the neutral point).

This phase displacement that exists between delta and star systems is very important when you come to analyse the voltages that occur between the primary and secondary windings in three-phase transformers, and will be considered later in this section when you look at transformer-group identification.

## Star- and delta-load connections

You have, in previous subjects, been shown the conventional way in which a three-phase 'balanced' load is shown in either delta- or star-connection configuration.

For quick revision, this information is repeated below in summary form.

A star load is connected so that one end of each of the three elements of the load is connected to one of the active cables, and the remaining end of each element is connected to the remaining ends of the other two.

The resultant currents flowing in each element have the same values as the currents flowing in each of the connecting active cables (lines).

The voltage across each element in the star configuration is less than the voltage between the lines by a factor of  $\sqrt{3}$ .

A diagram explaining this is shown in Figure 3.

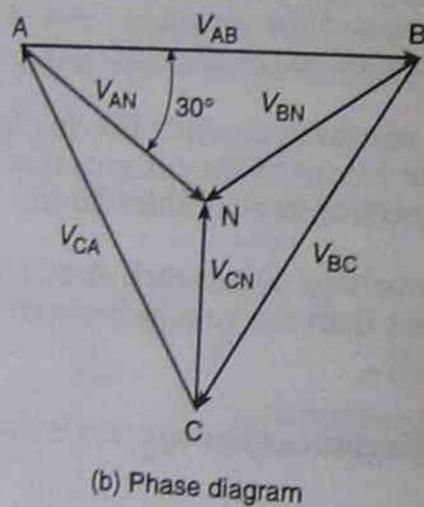
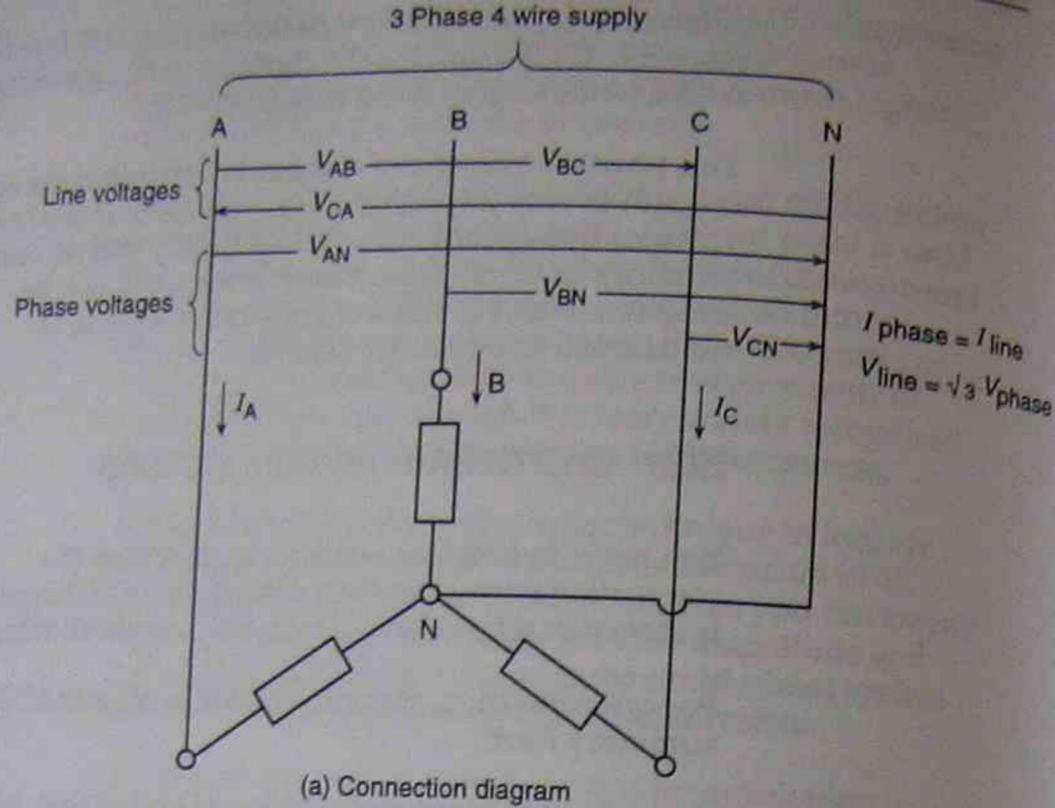


Figure 3: Star connection load

A delta load is connected so that each end of the three elements of the load is connected to one of the active cables, and no more than two ends are connected to any active cable.

The current flowing in each element in the delta configuration is less than the current flowing in each of the lines by a factor of  $\sqrt{3}$ .

The resultant voltages applied to each element have the same values as the voltages between each of the connecting active cables (lines).

A diagram explaining this is shown in Figure 4:

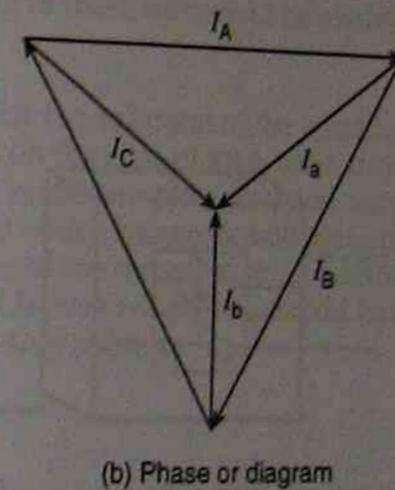
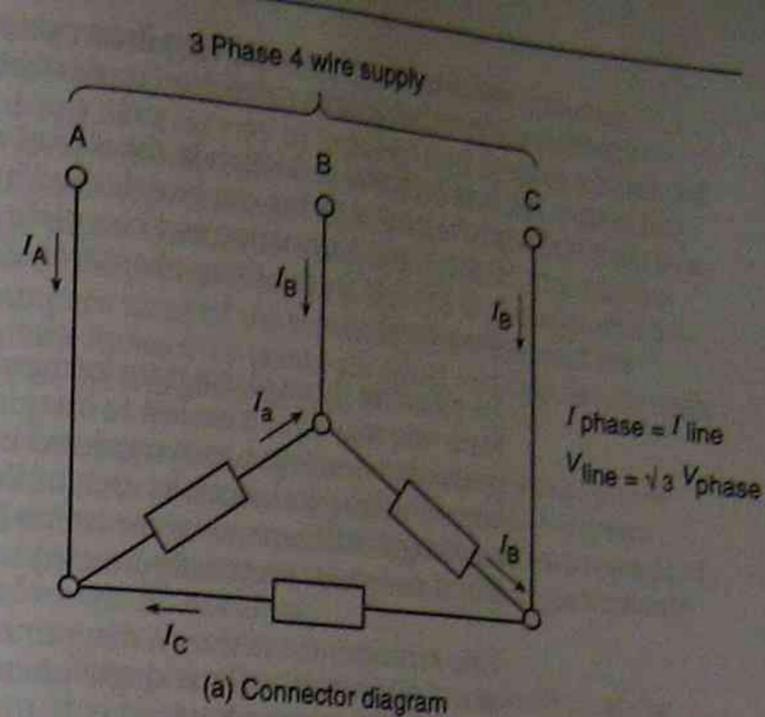


Figure 4: Delta connection load

## What is a three-phase transformer?

Up until now we have been examining the transformer as a device which operates on a single-phase supply. This is a very common application and a transformer of this type, although quite small physically, is to be found in a variety of equipment.

As the physical size of a transformer is related to its ability to distribute magnitudes of power, the larger power transformers would be enormous if they were to operate as a single-phase device. Such transformers do exist in a three-phase transmission system for very specialised applications. One example is a group of three single-phase auto-transformers (connected in delta) being the ending

connection of transmission lines operating at 330 kV (and above) at a large substation or switching yard.

A substantial reduction in the size of a transformer for a given power rating can be achieved by carrying out the voltage transformation as a complete three-phase system, and for this reason three-phase transformers were developed.

To visualise three-phase transformers in their simplest form you may find it easiest to imagine that three single-phase transformers were connected into a three-phase supply where, for example, each of the transformers is connected between one of the active lines and neutral (as in a three-phase star configuration).

This arrangement is shown diagrammatically in Figure 5 as a combination of three single-phase transformers star connected on three phases.

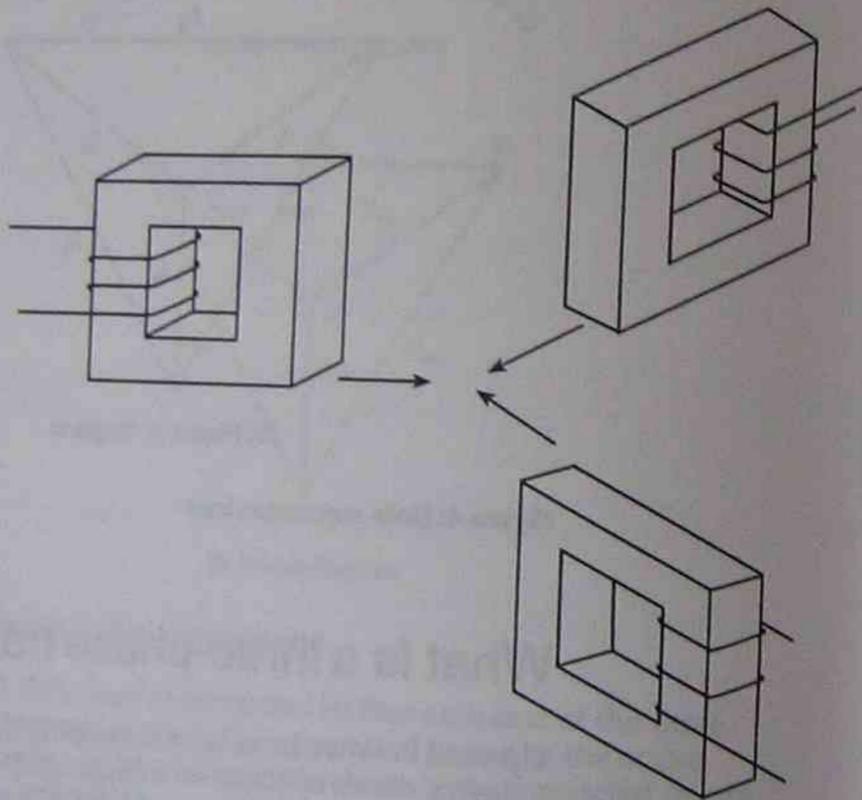


Figure 5: Three separate single-phase transformers

This arrangement of transformers is satisfactory to supply an essentially balanced load, but, there are difficulties if the loading of the transformer windings is not equal, as occurs when an unbalanced load is supplied.

To appreciate why this is so, visualise that the flux from the primary windings of each transformer is restrained within the magnetic circuit of that transformer only, and that this arrangement constitutes not one transformer but three separate transformers.

What is missing in this arrangement is the common path(s) whereby the fluxes from each of the primary windings have no way of 'mixing' and finding a balanced flux level between each other. Recall that the reason for basing this explanation on flux is due to the fact that, in a single-phase transformer supplying a load, the ampere turns of the primary winding are equal and opposite to the ampere turns of the secondary winding, and the resultant fluxes they create are equal and opposite to each other in the transformer core.

This is an important requirement of three-phase power transformers, as the *linking* of the flux paths between primary windings from different phases allows aspects of transformer operation to take place which three separate transformers do not allow.

When a three-phase transformer is being built it can be constructed either as a core or shell type, and the windings can be connected together in star, delta or interstar forms. The meaning of these terms will be explained later in this section.

The reason for the selection of the winding connections will depend on the type of load, the location of the transformer in the complete electrical supply system, and the degree to which harmonic voltages can be tolerated in the operation of the transformer. In addition, there are other critical factors which you should have been made aware of in other subjects.

## Magnetic circuit and core construction

In Section 1 the shapes of commonly used magnetic circuits, or cores, were labelled as either shell types or core types, and the same descriptions are used with three-phase transformers.

To appreciate the significance of these types being applied to three-phase transformers, visualise that the three single-phase transformers you considered previously (in star configuration) have been *pushed together* to form a single transformer which could have a symmetrical core, as shown in Figure 6.

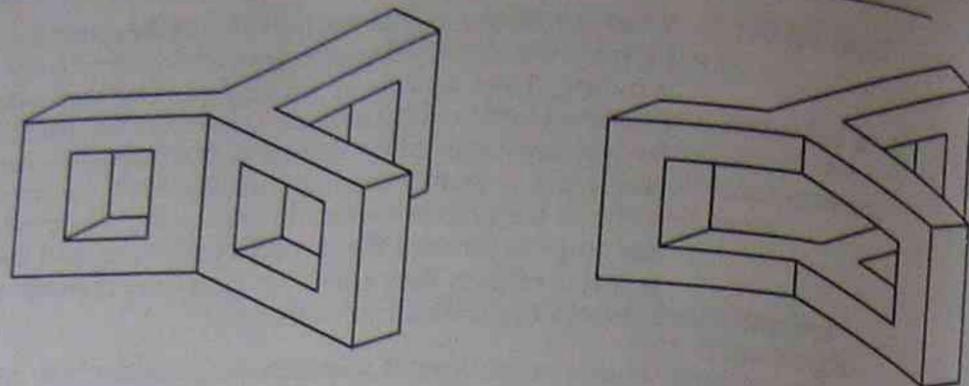


Figure 6: Three 'pushed together' single-phase transformers

For a three-phase transformer to have a core shape like that in Figure 6 would cause immense technical problems, both in the construction of the core and in the overall size and shape of the three-phase transformer when constructed. For practical reasons, the cores of three-phase transformers are constructed *flat*. Examples of these are given in Figure 7.

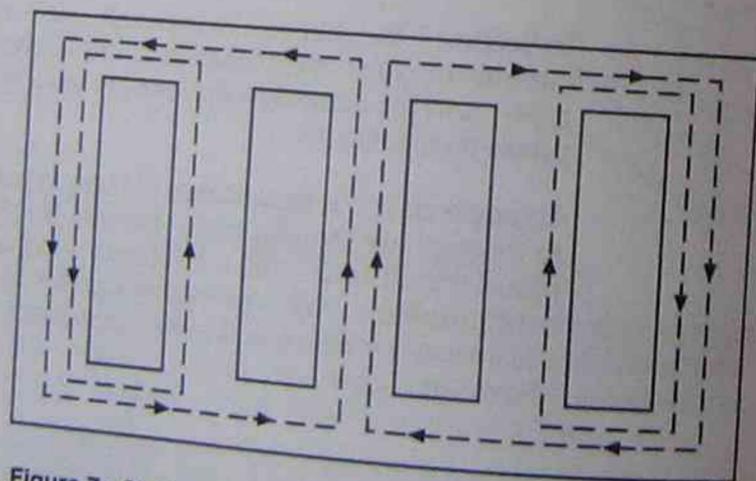
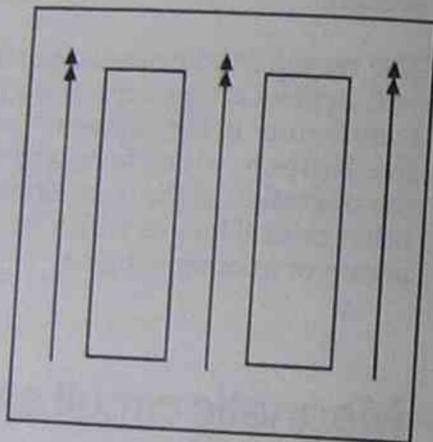


Figure 7: Shell- and core-type magnetic circuits for three-phase transformers

The magnetic circuits of three-phase transformers are generally of the core-type construction, where the primary and secondary windings of each phase are constructed as concentric windings or coils, fitted to the same leg of the core for each phase.

If a shell-type construction is used it may have four or five legs (where the diagram in Figure 7 shows five legs). In this case the central three legs are fitted with the windings, and the outer legs are used to carry any out-of-balance magnetic flux. The five-leg core affords a low-reluctance path to the fluxes from third-harmonic voltages. As a result, transformers having cores of this type may have significant third-harmonic voltages present unless action is taken to avoid these.

A five-leg core arrangement in transformers allows a reduction in the height of the core (and the resultant transformer overall) at the expense of a small increase in its length.

Because the lengths of the magnetic circuit for each of the three windings is different there will be some different values in the fluxes of each phase, which can be tolerated because of the alternative magnetic-flux path in the fourth and fifth legs. As a result, three-phase transformers with shell cores can be operated in *open delta*. This is an emergency mode of operation when a faulty winding is short-circuited and disconnected, which allows the transformer to operate up to 58% of rated full load.

It is also possible to have less steel in a shell-core construction. For this, the connections of the winding of phase two needs to be reversed, which will change the angular relationship of the voltage phasors in the primary and secondary.

To further reduce the steel in the core, a four-leg construction can be used instead of the five legs, although this causes further distortion to the flux balance in the legs of the core.

### Phasor diagrams for three-phase transformers

In Section 3 you were shown the phasor diagrams for a single-phase transformer when operating loaded and with no load.

Continuing the model of a three-phase transformer being created by bringing three single-phase transformers together, the phasor diagrams for a three-phase transformer could be created by bringing three single-

phase phasor diagrams together, ensuring that they are each displaced from each other by  $120^\circ$ .

However because of their complexity, complete (three-phase) phasor diagrams for a three-phase transformer are rarely drawn. It is more usual to assume that the transformer is supplying a 'balanced' load and to use only a single-phase diagram, it being relevant to any one of the balanced phases.

## Fitting windings to the core

In three-phase transformers, the coils of the windings are placed on top of each other on the same limb of the core. The low-voltage windings are located closest to the core and the high-voltage windings on the outer. This is done to minimise the cost of insulation, so that the high-voltage insulation need only withstand the primary-to-secondary voltage level, and the low-voltage level then needs to be insulated from the ground potential of the transformer core. In addition, the high-voltage windings are usually constructed, and connected together in sections, in order to limit the voltage between adjacent turns.

Heavy-current windings are frequently wound in single or double-layer spiral (because of the heat generated in heavy-current windings) and may be connected in a transposed manner, to equalise their impedance and ensure equal current sharing.

## Polarity of windings

The significance of the polarity of windings has already been discussed in the work on single-phase transformers and instrument transformers.

The polarity of individual windings in a three-phase transformer takes on an additional importance when you appreciate the instantaneous voltage levels that can exist between windings in a three-phase assembly. Also consider how this can influence the construction and type of insulation used. In addition, consider situations where an individual winding may be formed in a number of parts. In some transformers, sections of different windings can be interconnected to achieve specific effects. Transformers having interstar or zigzag connections are an example, and will be discussed later.

The marking of terminals connected to the ends of individual windings is carried out in accordance with

Australian Standard AS2374 (1982). The practice is to have capital letters 'A', 'B' and 'C' to identify the terminals of the high-voltage windings, and lower-case letters 'a', 'b' and 'c' to identify the corresponding terminals of like polarity of the lower-voltage windings.

A single high-voltage winding would thus have two terminals identified as  $A_1$  and  $A_2$ , and the corresponding low-voltage winding would have two terminals identified in the terminal box as  $a_1$  and  $a_2$ .

Where a winding is split or sectionalised, the individual sections would be identified from the terminals,  $a_1$  and  $a_2$  being one section and  $a_3$  and  $a_4$  being the other section, where the terminals with odd subscripts would have the same polarity as the odd subscripted primary terminals.

Increasing subscripts are also used to identify tapings on a particular winding. For example terminals  $a_1$  to  $a_5$  would identify the five progressively increasing winding taps on the secondary of a transformer.

To put it simply, the terminals of transformer windings are marked to show that the induced voltages within the windings are in the same direction when the sequence of the numbers in the suffixes are the same.

The voltage between  $A_1$  and  $A_2$  on the primary, or high-voltage, winding would therefore have the same direction, or polarity, as the voltage between  $a_1$  and  $a_2$  of the secondary winding.

## Star, delta and interstar (zigzag) windings

There are three types of winding that are common in three-phase transformers: star, delta and interstar (zigzag). You may also encounter other winding arrangements such as six- and twelve-phase types; however, in this subject you should limit yourself to the three common types.

The star and delta windings should be self-explanatory, since the individual windings are interconnected in either a star or a delta arrangement. The interstar (zigzag) winding requires a descriptive explanation. Each winding is formed in two parts: for a secondary winding the  $a$ -phase winding has terminals identified as  $a_1$ ,  $a_2$ ,  $a_3$  and  $a_4$ , where  $a_1$  and  $a_2$  belong to one half winding, and  $a_3$  and  $a_4$  belong to the other half winding. The windings of  $b$ - and  $c$ -phases are similarly formed.

equipment is to be connected and operated. Because of the very high cost of transformers and the associated electricity supply considerable expenditure is made on protective devices.

Details of the ways in which these different transformer winding connections are used and the reasons for these particular configurations are covered in other subjects. A simplified explanation is provided here:

- as delta 132/33kV three phase. There are only three active conductors on both the primary and secondary sides, and there is no neutral current.
- as delta/star 33kV/415-240V three phase. This provides an 'star point' connection for earth return currents. This transformer may also have a delta tertiary winding.
- Transformer C would be constructed star/star 415/special voltage three phase to supply large power to equipment operating at voltages other than 415 V and 240 V. This type may also have a delta tertiary winding.

A zigzag winding, as previously discussed, would be a special variation of a star/star configuration.

Within the New South Wales system large 500 kV auto-transformers are manufactured as single-phase units and are then connected in three-phase arrangements in the switchyards.

Less common winding arrangements include interstar (Zigzag) and multiphase.

Interstar zigzag are used in welding machine and arc-furnace power supplies, as the interaction of the magnetic fluxes between different legs resulting from unbalanced loading acts to stabilise the performance of the system to the unbalanced load.

Multiphase windings (ie creating a six- or twelve-phase supply from a three-phase supply) are used in high-current rectified power supplies as they can perform with controlled SCRs to provide a very 'smooth' rectified-output supply.



## Check your progress 1

- 1 Name the three most common winding configurations used in a three-phase transformer.
- 2 Explain why a tertiary (delta) winding is fitted to some three-phase transformers.
- 3 Ignoring the no-load current, calculate the current flowing in the primary and secondary windings of the following transformers supplying the nominated loads:
  - (a) a 250 kVA 11 000/415 volt delta/star three-phase transformer, with a balanced load of 240 kVA
  - (b) a 500 kVA 33 000/11 000 volt delta/delta three-phase transformer, with a balanced load of 450 kVA.

You can check your answers at the end of this section.

Because of the star/delta winding arrangement, the turns ratio (and the voltage ratio) will be

$$\frac{11\,000}{3300 \times \sqrt{3}} = 1.92 : 1$$

and when this factor is applied to the secondary current, the primary-winding current becomes

$$\frac{15.15}{1.92} = 7.89 \text{ A}$$

## Tertiary windings

From your three-phase theory, you are aware that third-harmonic currents do not flow in the lines connected to a balanced three-phase delta-connected load.

However, if the same three load elements were to be connected in star, any third-harmonic flux present in the core would cause the voltage at the star point to oscillate, or vary about a mean value, at the third-harmonic frequency.

This same voltage oscillation will occur in a transformer having star-connected windings which is connected to a three-phase 3-wire system. If, however, either the primary or secondary windings are connected in delta, any third-harmonic currents present will circulate in the delta winding. This then allows the flux waveforms to be maintained in sinusoidal form, and stabilises the potential of the neutral or star point.

If neither of the main windings are delta a third set of windings, connected in delta can be provided to accommodate this feature. Such windings are called 'tertiary' (being less significant than the other two windings), and provide a low impedance path for the flow of third-harmonic currents. For this reason they are usually constructed from a substantial size of winding wire even though they do not directly provide power to an external load.

In addition, tertiary windings can be included to provide power to an additional load, which must be kept separate from the secondary for reasons such as providing supply for instrumentation and power-factor correction equipment. In this case external connections would be made to the ends of the windings at the terminals.

## Transformer - group identification

The types of winding arrangements of three-phase transformers are identified by using group coding letters, which identify the primary and secondary winding arrangement and the phase displacement that exists between the primary and secondary voltages.

Explained simply, the system is based on the premises that

- the voltages of corresponding primary and secondary windings on the same limb of the core are 180° phase displaced, and the two induced emfs are in phase
- the emfs induced in the three phases are equal, balanced, displaced mutually by one-third of the periodic time, and have a definite sequence.

Complete details of the group identification system are given in Australian Standard AS2374 (1982), Part 4.

The system makes use of the clock-hour figure, and sets of different winding arrangements are designated in groups identifying both the phase displacement between primary and secondary reference voltages in degrees, and the corresponding clock-hour figure.

Individual winding types are identified by the letters 'Y' (star), 'D' (delta) and 'Z' (interstar), the primary winding configuration being expressed in upper case and the secondary in lower case.0

So a transformer with a primary star winding, a secondary star winding, and no angular change between the primary and secondary reference voltages would be identified by the vector group 'Yy0'.

Another, having an interstar primary, a delta secondary and an angular displacement between primary and secondary windings of 180° would be identified by the vector group 'Zd6'.

Figures 10 and 11 show this information in the form that is used in AS 2374:

Vector symbols	Line terminal markings and vector diagram of induced voltages		Winding connections
	HV winding	LV winding	
Yy6			
Dd6			
Dz6			
Zd6			

(a)

Vector symbols	Line terminal markings and vector diagram of induced voltages		Winding connections
	HV winding	LV winding	
Yy0			
Dd0			
Dz0			
Zd0			

(b)

Figure 10: Vector diagrams for standard three-phase transformer connections (a) Group 2 – 180° phase displacement (b) Group 1 – 0° phase displacement

Vector symbols	Line terminal markings and vector diagram of induced voltages		Winding connections
	HV winding	LV winding	
Dy11			
Yd11			
Yz11			
Zy11			

(a)

Vector symbols	Line terminal markings and vector diagram of induced voltages		Winding connections
	HV winding	LV winding	
Dy1			
Yd1			
Yz1			
Zy1			

(b)

Figure 11: Vector diagrams for standard three-phase transformer connections (a) Group 3 – -30° phase displacement (b) +30° phase displacement

Although transformers of the same group should be used together for compatibility when interconnected, there are combinations of transformers from different groups which can be 'mixed' and provide acceptable operation. These will be investigated later when considering the operation of three-phase transformers in parallel.

## Transformer nameplate

In an earlier section a transformer nameplate was identified as an essential item associated with transformers. At that stage of this subject it would have been confusing to provide a detailed explanation of the information to be shown on a nameplate. Having progressed to this stage, you should now be in a better position to understand this information.

Information to be provided on the nameplate of a power transformer includes

- manufacturer's name
- manufacturer's serial number
- the number of the Australian Standard to which the transformer manufacture complies
- rated kVA or MVA
- number of phases
- rated voltage at no load, higher voltage limit, lower voltage limit
- rated current(s) when operating at higher-voltage limit and at lower-voltage limit
- impedance expressed in 'voltage per cent'—winding connections and phase-displacement symbol of vector-group identification and the phase sequence to which it refers
- weight of complete transformer
- type of cooling and class of insulation, especially if a dry-type insulated transformer
- rate of flow of cooling medium (ie circulating oil, water or air)
- the total volume of the insulating liquid—if the insulating liquid is synthetic, its description is to be provided
- weight of core and winding assembly
- frequency.

The nameplate is to be permanently fixed to the transformer tank, or frame for an open transformer, and should be easily accessible and readable.

## Terminal marking plate

This plate also contains essential information: the relative physical position of the terminals and their identification markings. The manufacturer's drawing number for the diagram of connections should be stated.

## Manufacturer's connection diagrams

The manufacturer's drawing of the diagram of connections should provide the following information:

- the information given on the terminal marking plate
- the vector diagram and group number
- the marking of all terminals and tapings together with an indication of their relative electrical position in the winding
- the information on the rating plate
- the insulation level of each winding.

## Likely connections for a range of transformer applications

Most transformers used in power-supply applications use either delta or star connections for their primary and secondary arrangements. (As explained previously, tertiary windings are delta type.)

The choice of connections can be influenced by a range of factors, including the voltage level at which the transformer operates and the way the transformer is to be connected into the system.

The way in which any earth-current imbalance is to be met is a factor in the choice of having a star winding, as is the need to have both line (415 V) and phase (240 V) voltages available for users.

On the other hand, as a delta winding can act to 'trap' any third-harmonic voltages and currents which may not be wanted in the system, as well as needing only three connecting cables instead of four, this arrangement can also be justified in certain situations.

Another factor which can influence the choice of winding is the way in which system and transformer protective

losses are normally inherent in the design of a particular size and type of transformer, and various techniques are used by manufacturers to minimise them.

### Temperature rises in transformers operating with different types of cooling systems

It should now be obvious that it is possible to have different permissible temperature rises for transformers of different methods of manufacture, even though in some instances essentially the same insulating material may be used.

This is due to the manner in which the heat is dissipated from the windings (and core) and the way that heat is transferred from the cooling oil to the atmosphere.

To explain this further let's look at a particular transformer and analyse the cooling process taking place.

For a transformer with the cooling-method code of ONAN/ONAF, as previously described, when operating in the ONAN mode the dissipation of heat from the cooling system is by 'natural' methods of radiation and some air convection. This can be likened to the cooling process in a car engine that does not have a water-filled radiator, such as a Volkswagen. As long as the load supplied and consequent heat losses within the transformer stay within the range of heat that can be dissipated by natural cooling, the transformer will continue to supply the load.

As the load rises, the heat generated within the transformer rises, causing the temperature within the transformer to also rise. When the temperature within the transformer reaches the level limit for the type of insulation used, possibly one of two things should happen: either the transformer-temperature protection should function and disconnect the transformer from the load, or the temperature-sensing system should change the method of cooling to 'ONAF'. In this mode the fans on the cooling panels will start and blow air through the cooling panels. The automotive equivalent of this is to have a temperature-controlled fan blowing air through the car radiator. With this method of cooling the quantity of heat dissipated to atmosphere is much greater than for natural cooling, and the transformer can continue to function supplying a greater load.

Because of this the transformer has two ratings: a rating when operating in the ONAN mode, and a higher rating when operating in the ONAF mode.



### Check your progress 1

- 1 Explain the purpose of the thermometer well on a power transformer tank.
- 2 Describe the effects of heating on the transformer windings.
- 3 Explain the meaning of the following letter symbols when used to describe the method of cooling transformers: O, A, N, D.
- 4 Explain the meaning of the term 'hot spot'.

You can check your answers at the end of this section.

- describe the effect the temperature of the windings has on transformer impedance and efficiency
- describe the basis on which the rating of a transformer may be changed, considering the effects of different cooling methods
- briefly describe how transformers may be protected against overloads, phase-to-phase and phase-to-earth faults, and voltage surges
- determine the load at which maximum efficiency occurs
- calculate the losses at a given load
- calculate maximum efficiency and all-day efficiency as required
- calculate, for a specified load, the efficiency of a three-phase transformer
  - using the per-phase equivalent circuit component values
  - using the results of a short-circuit and an open-circuit test
- determine the loading of a three-phase transformer to achieve maximum efficiency
  - using the per-phase equivalent circuit component values
  - using the results of a short-circuit and an open-circuit test
- calculate the all-day efficiency of a transformer given the 24-hour duty cycle
  - using the per-phase equivalent circuit component values
  - using the results of a short-circuit and an open-circuit test.

# Transformer heating and cooling

## Heating and temperature rise

To assist your visualisation of the various processes that are continuing within a transformer, and as it is directly related to the losses occurring in a transformer, we will now look at the effects that the losses have on a transformer.

As with all electrical apparatus the input power is always greater than the output power. The difference between these two values, being the losses, is converted into heat. A steady temperature throughout the transformer is reached when the rate at which heat is produced by losses is equal to the rate at which this heat is dissipated by various means from the transformer.

The permissible value of the final temperature rise of the windings for constant operation at the rated full load depends upon the methods of cooling and the type of insulation used.

## Methods of cooling

For the smaller sizes of transformer, where simplicity of design is of prime importance to keep costs down, natural (convection) cooling of the transformer is used.

The larger the transformer size the more cost effective it becomes to use some type of assisted cooling for the core and windings. Transformer insulating oil is usually the cooling medium used.

In the simplest form heat from the transformer causes the oil to circulate in the transformer tank, carrying the heat from the windings to the enclosing tank. To improve on this further the oil can be pumped round the tank.

Dissipation of heat from the tank depends upon the surface area of the tank.

Fins, tubes, corrugations or radiator tanks are fitted to the transformer tank to increase its cooling surface.

In more sophisticated types, the heat is extracted from the transformer oil using air blown over the surface of finned coils containing the circulating hot transformer oil, or by using separate, free standing water oil.

## Temperature testing of transformers

### Testing of transformers

Because of the important processes they carry out in a highly expensive and valuable electrical-power supply system, it is essential that power transformers are tested to verify their reliability, and that these tests are consistent and repeatable.

The variety of tests carried out are identified in AS2374.1 (1987), *Power Transformers - General Requirements*, and include the following:

- routine tests: measurement of winding resistance, voltage ratio and voltage vector relationship, impedance voltage (on principal winding), short circuit impedance and load loss, no-load loss and current, and dielectric tests, tests on on-load tap changers, tests on insulating liquid and insulation resistance
- type tests: temperature rise and dielectric
- special tests: dielectric, measurement of zero-sequence impedance in three-phase transformers, measurement of sound-pressure level, measurement of harmonics in the no-load current, measurement of power taken by fans and pump motors, tests on insulating liquid and insulation resistance, partial discharge tests and measurement of impedance voltage and load loss at other than principal tapping.

Details of the following test procedures to be should be ascertained by reference to the appropriate Standards Association document:

- Dielectric tests relate to checking insulation against voltage impulses due to switching and lightning, and clearance (distance) between terminals. (The test practice is described in IEC publication 60-2, *High Voltage Test Techniques, Part 2: Test Procedures.*)

Insulation and dielectric tests require the value of the highest test voltage for equipment to be different in the testing of multiple winding and auto-transformers. The procedure is different for test voltages below and above 300 kV. For a transformer which has already successfully withstood a complete dielectric acceptance test, any repeat tests are carried out at 75% of the test-voltage level.

Insulation testing is based on either directly applied voltage or induced voltage.

- Lightning impulse tests are carried out in accordance with AS1931.2 (1996) *High Voltage Techniques – Application Guide for Measuring Devices*.
- The switching-impulse test is a routine test for windings having a test voltage greater than 300 kV.
- The purpose of the short-circuit test is to ascertain the thermal ability of the transformer to withstand a short circuit, as may be demonstrated by calculations.
- Measurement of sound-pressure level examines the sound generated by vibrations within the core and other structural components of the transformer and tank. Tests are carried out using A-weighted sound-pressure levels, and are carried out at no load with rated excitation voltage applied. If a reactor-type tap changer is fitted a tapping where the reactor is permanently energised is to be selected. Details of the varying measurement techniques for different types of transformers and enclosures need to be ascertained from the appropriate Standards Association document.

## Methods of temperature measurement

Among the fittings that should be an integral part of the larger power transformers is a pocket for a *mercury-in-glass* thermometer which is used to measure the oil temperature in an oil-filled transformer. The placement of this pocket is critical, so Australian Standard AS2374.2 (1982) *Power Transformers – Temperature Rise* identifies suitable locations.

The most common practice in the measurement of transformer-winding temperatures is to monitor the temperature of the transformer oil, a fairly simple procedure. Any temperature changes detected are *historic* (ie the increase in temperature of the oil will follow some time after the actual increase in

temperature at the conductor). For this reason, effective appreciation of the temperature of the windings within the transformer depends on using past records or testing to ensure that the actual temperature at critical locations within the windings does not exceed acceptable limits, (taking into account the temperature of the insulating oil).

As with all electrical machinery it is common for critical locations, or *hot spots*, to be identified. This is done either at the design stage or when standard heat tests are conducted by manufacturers as part of the construction program.

Obviously, if the temperature at these hot spots can be accurately monitored this will help ensure that the transformer continues to operate within its temperature limits.

In the process of conducting temperature testing of transformers it also normal to determine the temperature of the windings using a method based on the change of resistance of copper, or aluminium, as its temperature changes.

The exact procedure for carrying out this temperature (electrical resistance) measurement is described in Australian Standard AS2374.2.

To give you a simplified explanation of the process, the resistance of the transformer windings are measured (to possibly four significant figures of magnitude) before a controlled current is passed through the winding (ie when the winding is 'cold'). The winding temperature is then measured to the same degree of accuracy immediately after the controlled current is disconnected.

In this test, resistance measurements are made before ( $R_1$ ) and after ( $R_2$ ) the test period, the temperature of the windings being measured at the start of the test period ( $\theta_1$ ). The final temperature ( $\theta_2$ ) is then determined from the formulas.

$$\theta_2 = \frac{R_2}{R_1} (235 + \theta_1) - 235 \text{ for copper windings}$$

or

$$\theta_2 = \frac{R_2}{R_1} (225 + \theta_1) - 225 \text{ for aluminium windings}$$

where the temperatures are measured in degrees Celsius.  
These formulas are based on the notion that copper and aluminium both have a linear relationship of resistance to

temperature in the temperature range 0° – 200°C. When this linear relationship is extrapolated below the 0°C value there is a situation where the apparent resistance of copper becomes zero at the value –235°C, and the apparent resistance of aluminium becomes zero at the value –235°C. These temperature values are called the inferred temperatures, and values of resistance calculated from these formulas are called *inferred resistance values*.

Measurement of top-oil temperature is made by a thermometer placed in the oil-filled pocket of the transformer tank.

Temperature measurements are usually made with either a 'mercury-in-glass' thermometer or thermocouples. Where there is a risk of the thermocouple wires coming into contact with any other voltages thermocouples would not normally be used.

The operation of the cooling fans and oil-circulation pumps are also under temperature control. This control is usually a vapour-pressure temperature sensor located in a pocket near the top of the transformer. As the temperature rises so does the vapour pressure, and this acts to turn on the cooling fans then the oil-circulation pumps. If the temperature continues to rise the next change in temperature (pressure) acts to set off an audible alarm. At the next temperature setting the transformer-isolating switches disconnect it from the electricity supply.

For dry-type transformers (as set out in AS2735 (1984) *Dry Type Transformers*), the measurement of cooling air temperature is made in one of the following ways:

- For natural cooling, at least three thermometers are placed at different locations around the transformer, approximately halfway up the cooling surface at a distance to 2 metres from the cooling surface.
- For forced air cooling, the thermometers are placed in the inlet and exit air streams if there is a well-directed flow of air from the outside towards the intakes of the coolers without much recirculation of warm air.

If these conditions cannot be fulfilled the temperature is measured around the complete transformer, outside the recirculation stream, and preferably on a side without a cooler.

Cooling-water temperature is measured at the intake of the cooler. The temperature at the discharge of the cooler is taken as the average of at least three different readings at approximately equal time intervals (not greater than one hour apart). These readings are taken in the last quarter of the test period.

In critical situations, it is also possible for the transformer manufacturer to have thermistors placed in the windings. This is not common practice with high-voltage transformers, however, due to problems associated with maintaining the very high electrical insulation levels required, and also because of the risk of the electrical circuitry allowing high voltages to emerge from within the transformer tank (via the thermistor circuitry).

## Insulation of windings

### Classes of insulation and their applications

As you know, most power transformers use mineral oil as the cooling medium for the coils. There are also a small number of transformers manufactured as 'dry type'.

Since both these types of manufacture have different insulation materials for their windings they should be considered separately.

For oil-filled transformers the insulation material used conforms to the temperature class of insulation A. Australian Standard AS2374.2 sets the following temperature limits:

**Table 1:** Temperature limits for oil-filled transformers

Insulation loss	Maximum operating temperature
Windings—temperature class of insulation A (temperature measured by the resistance method)	65°C when oil flow is natural or forced non-directional 70°C when oil flow is forced and directed
Top oil (temperature measured by thermometer)	60°C when the transformer is fitted with a conservator or the tank is sealed 55°C when no conservator is fitted nor the tank sealed.

There is also a requirement for air-cooled transformers that the operating temperatures be reduced if the transformer is to be operated at an elevation more than 1000 metres above sea level. The percentage reductions vary between 2% and 5%, depending on the method of insulation used. Again, for specific details reference should be made to AS2374.2.

## Temperature rise for classes of insulation

Where oil-filled transformers use class A insulation, there is a range of insulating materials that are used for dry-type transformers. These are identified in Australian Standard AS2735 (1982), from which the following table has been extracted:

**Table 2:** Insulating classes for dry-type transformers

Part	Cooling method	Class of insulation	Temperature rise max.
Windings (with temperature measured by the resistance method)	Air - natural or forced	A	60°C
		E	75°C
		B	80°C
		F	100°C
		H	125°C
Cores (as for windings)			

In considering these temperature values you should note that AS2735 specifies that dry-type transformers be tested (and designed) to operate in ambient conditions within the temperature values in Table 3:

**Table 3:** Temperature limits for dry-type transformers

Outdoors	Not less than -25°C and no more than 40°C
Indoors	Not less than -5°C and no more than 40°C
Average temperature in any day	Not greater than 30°C
Average temperature in any year	Not greater than 20°C

## Temperature rise in transformers

### Methods of cooling

Consider now how a transformer may be cooled; that is, how to remove the heating losses from the core and windings of the transformer to prevent it from overheating and destroying itself. There are three basic techniques:

- natural convection flow within the windings and heat exchanger
- natural convection flow within the windings, and forced (pumped) flow in the heat exchanger
- forced (pumped) flow within the windings and the heat exchanger.

Each method will provide a different *thermal profile* of the operation of the cooling of the transformer, and will result in different values of temperature occurring throughout the transformer.

The oil moves at a very slow speed when depending on natural convection, so with this type of cooling there is usually a large temperature difference between the top and bottom, either in the windings inside the tank or the heat exchanger.

Forced, or pumped, flow invariably involves much higher flow rates, so there is a smaller temperature differential between the top and bottom temperature values.

When the type of flow is the same within the windings and the heat exchanger, the temperature profiles within the windings and the heat exchangers external to the tank are essentially the same.

If, however, the flow through the heat exchangers is pumped while the winding flow is by natural convection, the temperature profiles will be different. In this case, the pumps circulate cooled oil through the tank and only a small proportion passes through the oil ducts of the windings. Having the windings immersed in cool oil improves the natural convection flow within the winding ducts, so the heat-transfer efficiency using this method is enhanced.

For large power transformers, radiation is almost insignificant in the dissipation of heat from the tank: convection of the cooling medium is dominant. Natural-convection heat exchangers (radiators) are made as small units, each with a

given value of heat-dissipation capability. They are assembled in banks allowing different numbers to become operational as the heat to be dissipated varies. Groups of cooling fans can also be arranged to operate within these banks of radiators to help dissipate the total transformer-heating losses.

The method of cooling used for the heat dissipation from the transformer windings and tank is critical to the operation of an oil-filled transformer, and this information is stated on the transformer nameplate. The methods used are identified by the code letters in Table 4:

**Table 4:** Oil-filled transformer cooling media codes

Type of cooling medium	Letter
Mineral oil or flammable synthetic insulating liquid	O
Non-flammable synthetic insulating liquid	L
Gas	G
Water	W
Air	A

The methods of circulation of the cooling medium are identified by the code letters in Table 5:

**Table 5:** Oil-filled transformer cooling-medium circulation codes

Type of circulation	Letter
Natural	N
Forced (oil not being 'directed')	F
Forced (with the oil being 'directed')	D

To explain the way the codes are used, here are some sets of letters marked on the nameplate with their associated meanings:

**ODAF** oil immersed, with force-directed oil circulation, and forced air circulation

**ONAN/ONAF** oil immersed, with alternative methods of natural or forced cooling having non-directional oil flow

For the dry-type transformer, the method of cooling used for the heat dissipation from the transformer windings and core is critical for its continued operation. This information is also stated on the transformer nameplate. The methods used are identified by the code letters in Table 6:

**Table 6:** Dry-type transformer cooling media codes

Type of cooling	Letter
Air	A
Gas	G

The methods of circulation of the cooling medium are identified by the code letters in Table 7:

**Table 7:** Dry-type transformer cooling-medium circulation codes

Type of circulation	Letter
Natural	N
Forced	F

To explain the way the above codes are used, the following sets of letters marked on the nameplate would have the associated meanings:

**AN** no enclosure with natural cooling  
**ANAN** within an enclosure, with natural air cooling both inside and outside  
**GNAN/GNAF** contained in a sealed enclosure, with natural gas flow inside and alternatives of natural or forced air flow outside

In addition to heating from the windings, the tanks, core clamps, tie bars or plates and even laminations on the face of the core are subject to heating by the leakage-flux field.

This is sometimes referred to as *stray-flux heating* and becomes an even greater problem with increasing MVA size. These

These answers can now be checked. By adding the phasor components of the power carried by each transformer you should get the total load.

For transformer A:

$$471\angle-40.4^\circ \text{ kVA} = 359 - j305 \text{ kVA}$$

For transformer B:

$$281\angle-31.1^\circ \text{ kVA} = 241 - j145 \text{ kVA}$$

These add to give  $600 - j450 \text{ kVA}$ , and this is the same as  $750 \text{ kVA}$  at a 0.8 power factor lag.

Note that a lagging power factor in these calculations indicates the VARs to be *negative* when you express the load in polar form. This is normal for calculations of this type.

## Using neutral earthing compensators

In an electricity-distribution system safety is an object of the greatest importance. The major reason for the use of such devices as earth leakage circuit breakers (ELCBs) is to have the main system arranged to achieve maximum safety to personnel and protection of equipment against damage.

In any installation, the vast majority of electrical faults are those involving earth (as opposed to phase-to-phase faults). If the value of prospective earth fault current can be reduced, protection against damage to equipment can be significantly improved.

A common method used to achieve this is to have a reactor connected between the neutral and the earth connection made at each point of supply of electricity to a user. These connections are made inside the main switchboard and result in a safer and more stable electricity supply.

Earth-fault currents at a local level are usually not large enough to cause very much damage to an electrical installation (although the visible blackening and burning do cause concern to many people); however, when an earth fault occurs in the high-voltage system, the magnitudes of the (fault) earthing current can be very large (eg 200 000 amperes). This large current does have a number of effects, which can include damage to conductors and cables and destabilisation of voltage values in the supply system.

It is possible to use a large reactor to reduce the effect of these large earth-fault currents and thus reduce their adverse impact on the electrical system. These reactors are usually connected into the electrical system between the star point of the three-phase transformer and its associated earthing connection, and act to reduce the neutral-to-earth current.

Under normal operation there will be little or no neutral-to-earth current flowing, and so there is no power loss in such a reactor. When a large neutral-to-earth fault current flows, however, the voltage drop in the reactor tends to stabilise the neutral voltage from changing too far, from its normal value. This minimises any disturbances to the stable value of the voltages in the electricity system.

Earthing transformers are three-phase transformers (or reactors) used to provide an artificial loading neutral for the earthing of a system at a point where it is otherwise unearthed, either by direct earthing or by connection of earthing reactors, resistors or arc-suppression devices with the star point earthed.

Earthing transformers are usually connected zigzag or star/delta with the star point earthed. Alternatively, the delta winding may be broken at one point to allow an adjustable resistor (or reactor) to be inserted to increase the zero-sequence impedance.

- describe those winding connections which allow the third-harmonic current component to flow
- describe the differences in the occurrence of harmonics in shell-type and core-type transformers connected Yy
- describe the types of electrical loads which create harmonics within the electrical system
- describe the hysteresis loss in a transformer
- describe the result on the phase-to-phase and/or the phase-to-neutral voltages when there is no third-harmonic component present in the excitation current
- describe those winding connections which allow the third-harmonic current component to flow
- describe the purpose of the zigzag winding connection
- describe the differences in the occurrence of harmonics in shell-type and core-type transformers connected Yy
- describe the types of electrical loads which create harmonics within the electrical system.

## Connection of three-phase transformers

Section 5 showed the operation of single-phase transformers in parallel to supply the same load. There are several circumstances where three-phase transformers may be required to operate in a similar manner. The most common of these are

- to provide more power to an existing connection
- to increase the reliability of the electricity supply.

If the load of a consumer increases beyond the capacity of the existing supply transformer, it will be necessary to either replace that transformer with a larger one or to place a second transformer in parallel to supply the (larger) existing load. It is usually less expensive to provide a second (parallel) transformer, and this is a very common practice.

There are some conditions where the reliability of the electricity supply can be very important. Examples are hospitals, communication centres, and high-rise buildings, where the failure of the electricity supply would have serious repercussions the health and safety.

In these situations it is normal to have at least two, and sometimes more, transformers connected in parallel to supply these facilities. To improve the reliability, the primary transformer connections will possibly come from different bus-bar supplies. Should one supply (or its associated transformer) fail there will be a continuing, albeit reduced, supply available to ensure the continuing operation of essential equipment.

### Conditions for parallel operation

When looking at single-phase transformers operating in parallel, you saw certain requirements that must be met to ensure satisfactory parallel operation of transformers and sharing of load in proportion to kVA rating:

- The primary winding(s) must be suitable for the supply-system voltage and frequency.

- The voltage ratio must have the same value for each transformer. (For three-phase transformers this does not necessarily mean the turns ratio must be identical.)

In addition to these requirements, when three-phase transformers are to be connected in parallel, to achieve load sharing you also need to consider the effect that internal phase displacement between primary and secondary windings will have on the phase angle(s) of the respective secondary voltages. This phase displacement was discussed in Transformer Group Identification of Section 6.

When transformers of different vector groups are connected in parallel, the primary voltages will both have the same phase angle but the secondary voltages will be displaced from each other (ie there will be a time delay between maximum instantaneous voltages of the same phase, due to the internal transformer phase displacement). The interconnecting cabling (bus-bar) between the transformers on the secondary side will then act as a short circuit to this instantaneous difference in voltage, allowing large circulating currents to flow between the secondary windings of each transformer.

These circulating currents flow within the transformers, but do not flow into the load, and so reduce the effectiveness of the transformers in parallel to meet the load requirements.

The above explanation is intended to identify the difficulties created by having transformers of different vector groups connected in parallel, but there are ways that these difficulties can be resolved. These are explained later in this section.

When connecting three-phase transformers in parallel for load sharing the following conditions need to be satisfied.

- The polarity of the secondary-winding connections must be the same so that the maximum instantaneous voltages from the secondary windings synchronise.
- The transformers must have the same angular phase shift within the transformer windings between primary and secondary voltages. (This is usually called having the same vector group, although under certain circumstances it is possible to use different vector groups, will it?)
- The secondary windings of the transformers must be connected together having the same phase sequence.
- The voltage ratios between the input-line voltage and the output-line voltage must be the same. While some variations may occur (possibly due to voltage regulation with

variations in load magnitude), only minor differences are tolerable in order to avoid large currents that would otherwise circulate between the secondary windings of each transformer.

It is also preferable to have each transformer share the load in the ratio of their respective ratings as this optimises the total capacity of the transformers connected in parallel to supply the connected load.

In addition, it is desirable to select three-phase transformers having the same impedance angle (from their transformer equivalent circuit) for operation in parallel. When the impedance angles of the two transformers are different this reduces the maximum capacity of the two transformers operating in parallel to less than the arithmetic sum of the ratings of the two transformers.

## Parallel operation of different vector groups

As previously mentioned, transformers of different vector groups (see Figures 10 and 11 of Section 6) should not be operated in parallel as they create large circulating currents between the secondary windings of the transformers. To understand this, consider the following situation.

If one of a pair of transformers to be operated in parallel is from Group I (having a phase displacement of  $0^\circ$ ) and the other is from Group III (having a phase displacement of  $-30^\circ$ ), there is a  $30^\circ$  instantaneous phase displacement between voltages of the same 'phase' on the secondary side of the transformer. This voltage sets up a circulating current between the secondary windings of the two transformers which can be several times the rated secondary current.

Three-phase transformers of these types therefore can't be operated in parallel in this manner.

Thus, by reversing the polarity of each secondary phase winding, three phase transformers of different (but compatible) phase groups can be placed together in parallel for load sharing duty.

Generally, connection of three-phase transformers in parallel should be limited to transformers having the same vector groups.

It is possible to operate a Group I Dy and a Group II Dy transformer in parallel provided the star-point connection of the secondary windings of one transformer is changed to the other end of the windings.

By reversing the polarity of the connections of the secondary winding of one of the transformers, you are creating a  $180^\circ$  phase-angle reversal from the conventional way in which this transformer would be connected. This, together with the  $180^\circ$  phase-angle displacement coming from the different group types, adds to give an effective zero phase angle displacement at the secondary terminals.

Thus, by reversing the polarity of each secondary phase winding, three-phase transformers of different (but compatible) phase groups can be placed together in parallel for load sharing duty.

**Note:** This is not a common technique, but it can be used in an emergency situation.

## Calculations of load sharing when operating in parallel

As explained earlier, when two transformers are connected in parallel to supply the same load the load is shared between them in an inverse ratio to their respective impedances. The information regarding their respective impedances can be determined in several ways, as has previously been shown. The following examples are intended to demonstrate a variety of the techniques used to calculate what proportion each of two paralleled transformers contributes to the common load.

When analysing the operation of three-phase transformers connected in parallel to supply the same load, it is extremely useful to be able to calculate the required primary voltage to maintain a given secondary voltage to a connected load. The following example demonstrates this type of calculation.

### Example 1

A load of 1000 kVA, 415 V, three-phase at 0.9 lagging power factor is to be supplied by two identical 33 000/415 V 500 kVA three-phase transformers. During construction of these transformers the tensioning of the winding for the primary coils of one transformer was excessive, and the wire was stretched when the coils were formed; therefore, there is a difference

between the transformer equivalent circuit of each transformer. Transformer A has an equivalent series impedance (referred to the primary side) of  $6.1 + j59.6$  ohms, and the corresponding value for transformer B is  $6.8 + j60.0$  ohms.

### Solution

Expressing the transformer impedances in both rectangular and polar form:

$$\begin{aligned} Z_{SA} &= 6.1 + j59.6 \\ &= 59.91 \angle 84.2^\circ \end{aligned}$$

$$\begin{aligned} Z_{SB} &= 6.8 + j60.0 \\ &= 60.38 \angle 83.5^\circ \end{aligned}$$

$$\begin{aligned} Z_{SA} + Z_{SB} &= 12.9 + j119.6 \\ &= 120.3 \angle 83.8^\circ \end{aligned}$$

The load carried by transformer A is

$$\begin{aligned} &1000 \angle -\cos^{-1} 0.9 \times \frac{60.38 \angle 83.5^\circ}{120.3 \angle 83.8^\circ} \\ &= 501.9 \angle -26.1^\circ \text{ kVA} \\ &= 450.6 - j221.1 \text{ kVA} \end{aligned}$$

The load carried by transformer B is

$$\begin{aligned} &1000 \angle -\cos^{-1} 0.9 \times \frac{59.91 \angle 84.2^\circ}{120.3 \angle 83.8^\circ} \\ &= 498.0 \angle -25.4^\circ \text{ kVA} \\ &= 449.7 - j213.9 \text{ kVA} \end{aligned}$$

Checking the calculation, the sum of the two loads carried by the transformers in parallel is  $900.3 - j435$  kVA, and this equals the connected load of 1000 kVA at 0.9 lag power factor when it is expressed in rectangular form.

Note that transformer A is carrying a load in excess of its rating. However, since the overload is so minimal that it should not cause any problems in a short period.

### Example 2

Two three-phase transformers are connected in parallel to supply a total load of 1 MVA at 0.85 power factor lag. The ratings and impedances of the transformers are:

- Transformer A: 750 kVA rating, resistance 2%, reactance 4.5%  
 Transformer B: 750 kVA rating, resistance 1.8%, reactance 4.1%

Calculate the load carried by each transformer.

Consider the percentage impedance values in the same way you would consider 'real' values of impedance. Then:

- For transformer A: % impedance is  $(2.0 + j6.5)$   
 For transformer B: % impedance is  $(1.8 + j6.1)$   
 For transformers A and B: % impedance is  $(3.8 + j12.6)$

Then, the load carried by transformer A is

$$\begin{aligned} 1000 \angle -36.8^\circ \times \frac{(1.8 + j6.1)(3.8 + j12.6)}{13.16 \angle 73.2^\circ} \\ = 1000 \angle -36.8^\circ \times \frac{6.36 \angle 73.4^\circ}{13.16 \angle 73.2^\circ} \\ = 483.3 \angle -31.4^\circ \text{ kVA} \\ = 412.5 - j251.8 \text{ kVA} \end{aligned}$$

and the load carried by transformer B is

$$\begin{aligned} 1000 \angle -36.8^\circ \times \frac{6.80 \angle 72.9^\circ}{13.16 \angle 73.2^\circ} \\ = 516.7 \angle -32.4^\circ \text{ kVA} \\ = 436.3 - j276.9 \text{ kVA} \end{aligned}$$

Checking the calculation by adding the above answers you get  $948.8 - j528.7$  kVA, which equals 1000 kVA at 0.85 power factor lag.

As stated in Section 5, the operation of single-phase transformers when connected in parallel to supply a load can be modelled by considering each transformer as its series impedance taken from the transformer equivalent circuit.

When the transformers are in parallel, the series impedance of each transformer will be in parallel. This was shown schematically as Figure 5 in Section 5. From this basic form we can derive several proportionality relationships. Use these relationships when calculating the operation and load sharing of three-phase transformers connected in parallel to supply a common load.

### Example 3

A 500 kVA three-phase transformer (transformer A) with 1% resistance and 3% reactance is connected in parallel with a 250 kVA transformer (transformer B) having 1.5% resistance and 4% reactance. The secondary voltage of each transformer is 400 V with no load. Neglecting the no-load losses and any effects from voltage regulation, find how each transformer shares a load of 750 kVA at a power factor of 0.8 lag.

Note: It is not necessary to find the ohmic values of resistance and reactance, as only the impedance ratios are required to determine the load sharing. The percentages given, however, refer to different ratings (of transformers) and so must be adjusted to the same base kVA. Since 2% reactance in a 250 kVA transformer is equivalent (in ohms) to 2.4% in a 500 kVA transformer, then the adjusted values for transformer B, to operate in parallel with transformer A, are:

$$\begin{aligned} Z_{B_{adj}} &= 2(1.5 + j4.0)\% \\ &= (3 + j8.0)\% \\ &= 8.54 \angle 69.4^\circ\% \end{aligned}$$

$$\begin{aligned} Z_A &= (1 + j3.0)\% \\ &= 3.16 \angle 71.7^\circ\% \end{aligned}$$

$$\begin{aligned} Z_A + Z_{B_{adj}} &= (4 + j11)\% \\ &= 11.6 \angle 72.9^\circ\% \end{aligned}$$

The total kVA load, vectorially, is  $750 \angle -36.9^\circ$  kVA. The angle  $-36.9^\circ$  comes from the lagging power factor of 0.8.

Now, to calculate the load carried by transformer A:

$$\begin{aligned} \text{Load}_A &= 750 \angle -36.9^\circ \times \frac{8.54 \angle 69.4^\circ}{11.6 \angle 72.9^\circ} \\ &= 571 \angle -40.4^\circ \text{ kVA} \\ &= 571 \text{ kVA at a 0.762 power factor lag} \end{aligned}$$

The load carried by transformer B is

$$\begin{aligned} \text{Load}_B &= 750 \angle -36.9^\circ \times \frac{3.16 \angle 71.7^\circ}{11.6 \angle 72.9^\circ} \\ &= 202 \angle -31.1^\circ \text{ kVA} \\ &= 202 \text{ kVA at a 0.85 power factor lag} \end{aligned}$$

Note: Transformer B is overloaded as its rating is 250 kVA.

$$Z_A + Z_B = 2.85 + j8 = 8.5 \angle 70.4^\circ\%$$

Load carried by transformer A is

$$\begin{aligned} \text{kVA}_A &= \frac{\text{kVA}_{\text{total}} \times Z_B}{Z_A + Z_B} \\ &= \frac{2000 \angle -36.87^\circ \times 3.12 \angle 74.2^\circ}{8.5 \angle 70.4^\circ} \\ &= 734 \text{ kVA at } 0.84 \text{ pf lag} \end{aligned}$$

$$\begin{aligned} \text{kVA}_B &= \frac{\text{kVA}_{\text{total}} \times Z_A}{Z_A + Z_B} \\ &= \frac{2000 \angle -36.87^\circ \times 5.385 \angle 68.2^\circ}{8.5 \angle 70.4^\circ} \\ &= 1267 \text{ kVA at } 0.78 \text{ pf lag} \end{aligned}$$

Note that the transformer B is overloaded

- 2 Transformer A values referred to 200 kVA base:

$$(4 + j10)\% = 10.77 \angle 68.2^\circ\%$$

Transformer B values:

$$(1 + j4)\% = 4.12 \angle 76^\circ\%$$

$$\begin{aligned} Z_A + Z_B &= (4 + j10) + (1 + j4) \\ &= (5 + j14)\% \\ &= 14.87 \angle 70.35\% \end{aligned}$$

Load carried by transformer A:

$$\begin{aligned} \text{kVA}_A &= \frac{\text{kVA}_{\text{total}} \times Z_B}{Z_A + Z_B} \\ &= \frac{240 \text{ k} \angle -36.87^\circ \times 4.12 \angle 76^\circ}{14.87 \angle 70.35^\circ} \\ &= 66.5 \angle -31.22^\circ \text{ kVA} \end{aligned}$$

So, kW from transformer A:

$$83.12 \times \cos -31.22^\circ = 56.87 \text{ kW}$$

$$\begin{aligned} \text{kVA}_B &= \frac{\text{kVA}_{\text{total}} \times Z_A}{Z_A + Z_B} \\ &= \frac{240 \text{ k} \angle -36.87^\circ \times 10.77 \angle 68.2^\circ}{14.87 \angle 70.35^\circ} \\ &= 173.8 \angle -39.0^\circ \text{ kVA} \end{aligned}$$

So kW from transformer B:

$$217.28 \times \cos 39.0^\circ = 135.1 \text{ kW}$$

Note: Transformer B is overloaded.

- 3 Winding transformation ratio is given by:

$$\frac{11000}{415/\sqrt{3}} = 45.91:1$$

The  $R_E$  on the primary side:

$$\begin{aligned} 7 + [0.003 \times (45.91)^2] &= 7 + 6.32 \\ &= 13.32 \Omega \end{aligned}$$

Rated primary-phase current:

$$\frac{500 \text{ kVA}}{3 \times 11 \text{ kV}} = 15.15 \text{ A}$$

Then total (three-phase) full-load copper loss:

$$3 \times 15.15^2 \times 13.32 = 9172 \text{ W}$$

- (a) Efficiency:

$$\begin{aligned} \text{Efficiency} &= \frac{\text{Output power} \times 100}{\text{Output power} + \text{losses}} = \frac{400 \text{ kW} \times 100}{400 \text{ kW} + 5 \text{ kW} + 9.172 \text{ kW}} \\ &= 96.58\% \end{aligned}$$

- (b) With a 200 kW 0.7 pf load, the transformer is loaded to 57.14%, so the copper losses at this load are

$$(57.14\%)^2 \times 9172 = 2995 \text{ kW}$$

$$\begin{aligned} \text{Efficiency} &= \frac{200 \text{ kW} \times 100}{200 \text{ k} + 5 \text{ kW} + 2.995 \text{ kW}} \\ &= 9.16\% \end{aligned}$$

- 4 Full-load copper loss = 1.2% of 1000 kW  
= 12 kW

As you would have read earlier, the currents flowing in the secondary delta are in the ratio 2:1, as indicated by the arrows adjacent to the respective secondary windings. When these are reflected into the primary delta windings again they flow in the ratio 2:1 in the primary windings, as indicated by the arrows.

Alternatively, the same analysis could be carried out using the currents identified by the subscript notation. In this instance, the current flowing into the load is made up of two parts,  $I_{ba}$  and  $I_{ca}$ , and you should note that the current  $I_{ca}$  is equal to  $I_{bc}$ , where the order of the subscripts identifies the positive value of current flow.

When these two secondary delta currents are then reflected into the primary delta, the resulting currents are  $I_{AB}$  and  $I_{AC}$ , and these *add* at nodes A and B to result in currents flowing in lines A and B of the supply.

By considering all the various single-phase loading conditions explained, you can identify a systematic way of determining the flow in the primary supply resulting from a single-phase load on the secondary of the transformer.

However, it may be easier to consider each situation in the way that it has been explained in this section and develop the currents flowing from each of the diagrams provided.



### Check your progress 2

- 1 When a three-phase transformer has a star winding fitted, explain why it is preferable to have the star point connected to the neutral.
- 2 For a three-phase transformer with a delta primary and star secondary, provide a sketch to show the primary and secondary windings in which current is expected to flow when a single load is connected to the secondary between a star winding terminal and the star point.

You can check your answers at the end of the section.

## Harmonics in three-phase transformers

In Section 5 the source of waveform distortion due to the non-linearity of the B-H curve and the hysteresis loop (from the magnetic circuit) were explained. You should re-read that particular explanation before proceeding any further to learn about the sources of harmonics in three-phase transformers.

Any distortion from the fundamental shape of the sinusoidal waveform can be mathematically expressed as a 'series' of sine and cosine terms. This is called the *Fourier series* and its terms are derived from frequencies that are multiples of the original or fundamental frequency, where the distorted wave is written mathematically as an equation consisting of three separate sections. These are

- a constant value, which is, in effect the dc component present in the distorted wave
- two series of terms, being sine and cosine expressions, with the first term of each series having the frequency of the fundamental and each successive term having a frequency value of an integral multiple of the fundamental frequency.

The following equation is a simple example of a Fourier series:

$$f(t) = 5 + 3 \sin 314t + 2 \sin 628t + 4 \sin 942t + \dots$$

$$\dots + 6 \cos 314t + 3 \cos 628t + 2 \cos 942t + \dots$$

(equation 1)

where this could represent either voltage or current in an electrical circuit.

As you read earlier, the value '5' would be the magnitude of any dc component present in the waveform. The terms containing the '314t' term are the first, or fundamental (frequency).

The terms with '628 t' (being  $314 t \times 2$ ) are called the *second harmonic*; those with '942 t' ( $314 t \times 3$ ) are called the *third harmonic*, and so the series continues until the magnitude of any harmonic term is so small that the term may be neglected.

As the frequency of the harmonics increases, so the effective inductive in the circuit at that particular frequency increases (because  $X_L = \omega L$ ). This results in the current values of the higher frequencies being quite small, and so any frequencies above the ninth harmonic can generally be ignored unless that particular frequency is being used for a special purpose.

An example of a higher frequency being used for a special purpose is the use of *frequency-injected* voltages to control the operation of off-peak appliances, or to control street lighting. In these instances, the higher frequency is usually injected into the electrical system on the residential (load) side of the transformer so that the attenuating effect of the transformer does not affect the signal.

As well as being created for a special purpose, harmonics also will occur due to certain types of load being connected to the electrical system. Any type of electrical load that does not have a linear characteristic (ie does not comply with Ohm's law) will produce harmonics.

The most noteworthy examples are

- thyristor control circuits, such as are used for motor-speed control or light-dimming applications, where the sinusoidal voltage waveform is 'chopped'
- fluorescent-light fittings, where ballasts, by saturating, are used to limit the current passing through the tube—this flattens the shape of the sine wave of the current and creates harmonics.

In this way the transformer is not only affected by the magnitude of the load connected, but also by the types and functions of the devices which constitute that load. Note that the heating effect from each separate frequency is additive.

As you read previously, the harmonics are described by the multiplying factor by which the particular frequency of the harmonic is greater than the fundamental frequency, and are generally grouped as being *odd* or *even*, as identified by the multiplying factor by which their frequency varies from the fundamental.

When only odd harmonics are present, these frequencies will add together to form a resultant waveshape which is symmetrical about the vertical axis (the waveform for positive values of time is a mirror image of that for negative values of time). As an example, waveshapes containing fundamental and third-harmonic frequencies are shown in Figure 9.

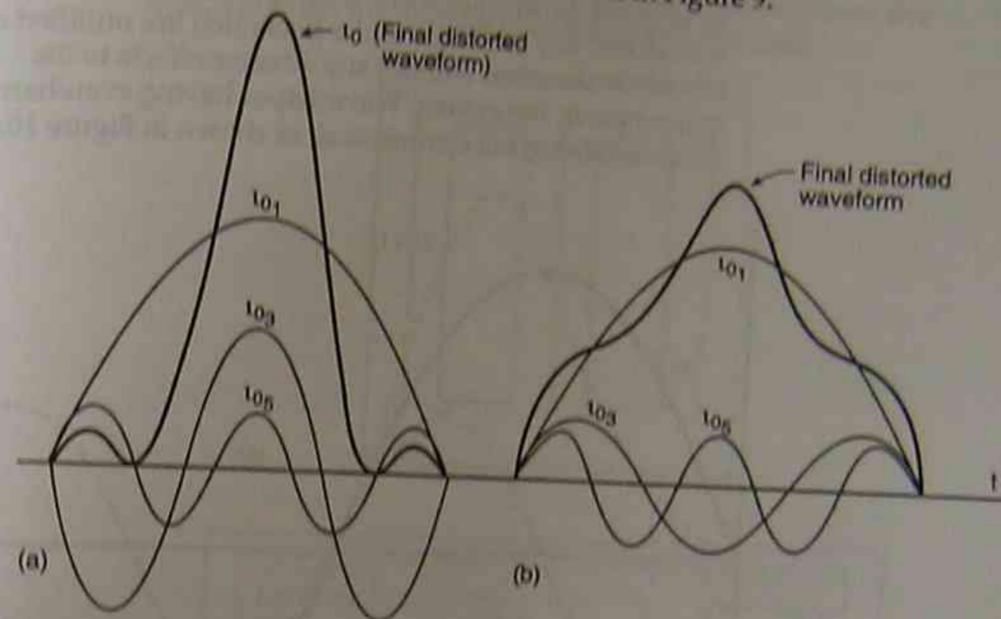


Figure 9: Waveshapes from first and third harmonic

In Figure 9, diagrams (a) and (b) show the third harmonic at a different time displacement relative to the fundamental, and the resultant change to the waveshape is obvious.

Now return to the situation regarding the effective inductive reactance of the circuit being different for each different harmonic frequency (ie  $X_L = \omega L$ ). It then follows that the phase-angle relationship between the voltage and current is different for each harmonic.

Having a different phase angle between the voltage and current for each harmonic is part of the cause of a time displacement between individual harmonics, as shown in Figure 9.

Another factor contributing towards a time displacement between the voltages of the various frequencies comes from the action of chopping the current waveform, as occurs within SCR gated circuits.

Here, the time at which the SCR is 'gated' will establish a starting time for current flow. The action of the current flowing through an impedance which has different impedance angles

for each frequency will result in the various frequency components of voltage having different phase angles.

If any even harmonics are present, the resultant waveshape is not symmetrical about the vertical axis. Having non-symmetrical waveshapes present in an electrical system should be avoided. Most equipment is designed and manufactured to ensure that any even harmonics generated are nullified or otherwise absorbed without any adverse effects to the equipment or the system. Waveshapes having even-harmonic components are not symmetrical, as shown in Figure 10.

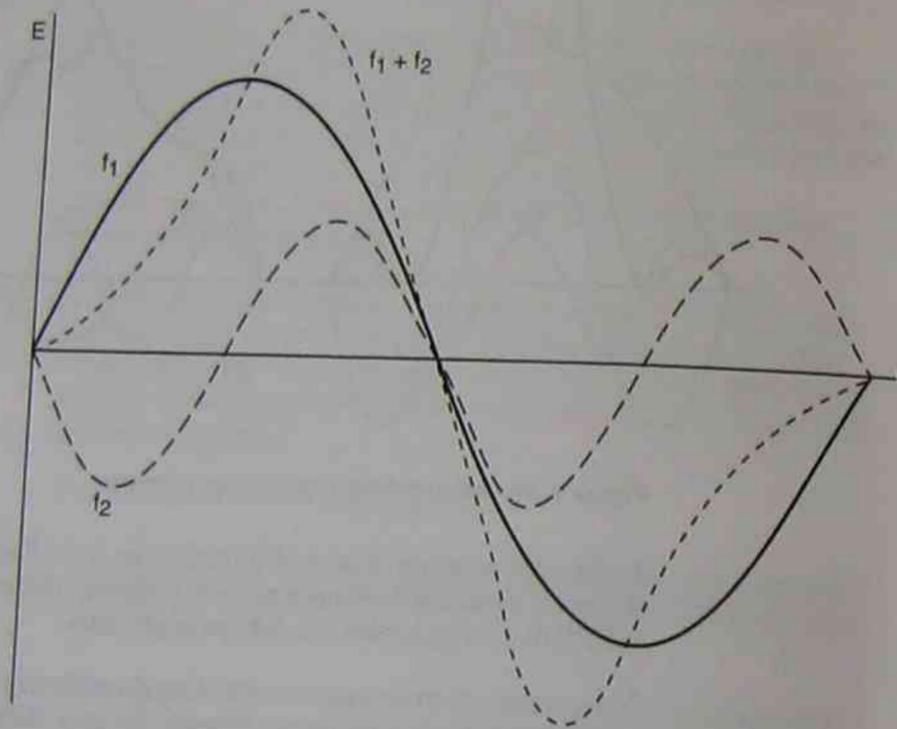


Figure 10: Even-harmonic component waveshape

## Control of harmonics in three-phase transformers

The presence of harmonic frequencies and their control in electrical systems is a very complex subject. Here, you should limit yourself to the presence of the third-harmonic frequency in three-phase transformers. There are several features associated with the construction of a three-phase transformer which have an effect on the presence of harmonics in the transformer windings. As far as the third harmonic is concerned the two main features are

- the type of magnetic core of the transformer, and
- the winding configurations of the transformer.

Three-phase transformers have two types of magnetic core (shown in Figure 11), which have been described as core and shell types. They are cores which have either three, four or five 'legs', although the four-leg core is not normally used.

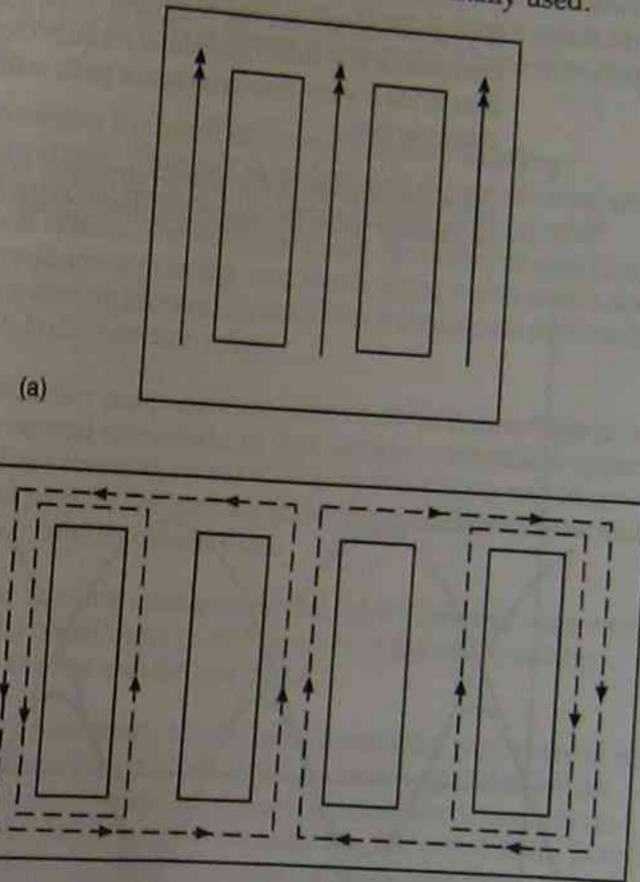


Figure 11: Three-phase magnetic core configurations (a) core type (b) shell type

The core type has only three legs, with the respective primary and secondary windings being fitted over the top of each other on each leg.

The shell type can have either four or five legs, depending on space available and other factors associated with the operation of the transformer in the system. The leg at one or both end(s) does not have a winding placed upon it. This lack of winding has a profound effect upon the flux in the magnetic circuit from the various frequencies present because the outer leg(s) allow a low-reluctance path for any magnetic flux seeking such a circulation path.

It should be obvious that there is no necessary relation between distance and time. It can be shown that a double of the distance does not double the time. Consider a system of a mass  $m$  and a spring with constant  $k$ . The period of oscillation is  $T = 2\pi\sqrt{m/k}$ . If the distance is doubled, the period remains the same. This is shown in Figure 14.

If you have a double the distance, you can double the period of the oscillation. This is shown in Figure 15. The double distance is shown in Figure 15.

In a consequence, the double distance always gives us the double period. This is shown in Figure 15.

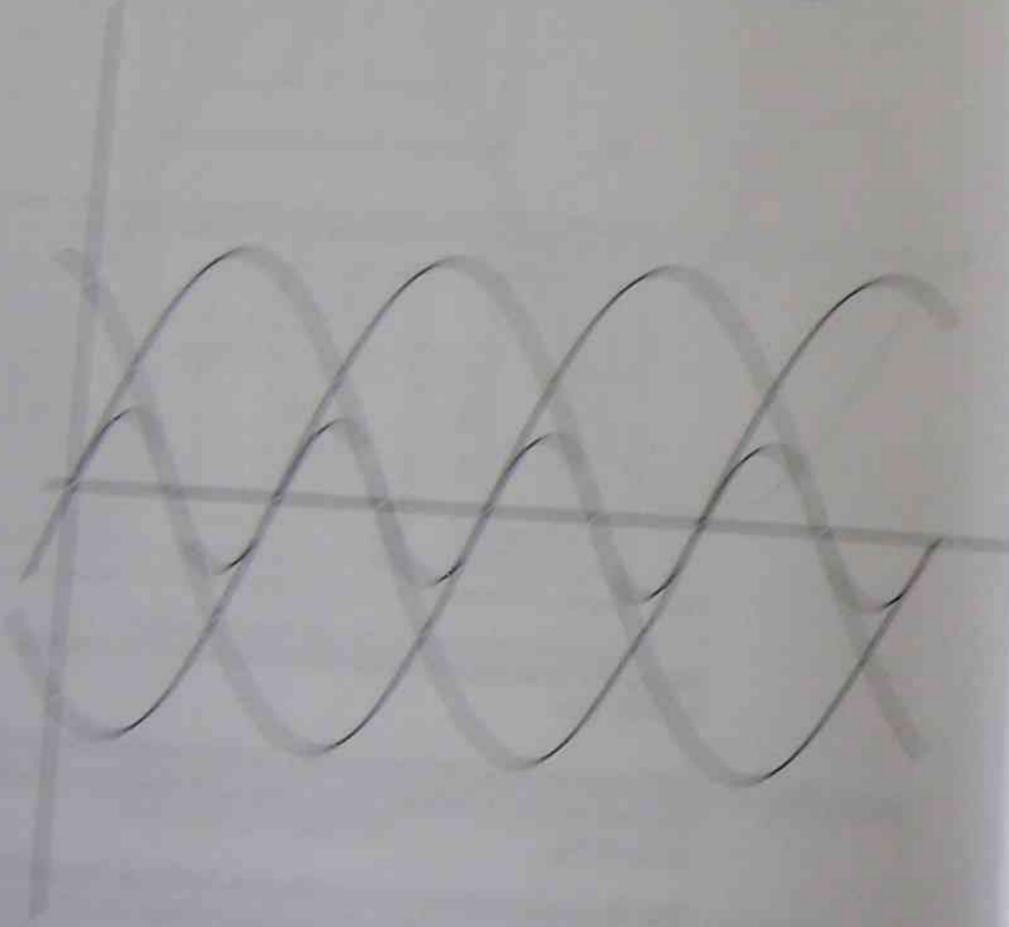


Figure 14. Double distance does not double the period.

It should be obvious that there is no necessary relation between distance and time. It can be shown that a double of the distance does not double the time. Consider a system of a mass  $m$  and a spring with constant  $k$ . The period of oscillation is  $T = 2\pi\sqrt{m/k}$ . If the distance is doubled, the period remains the same. This is shown in Figure 14.

If you have a double the distance, you can double the period of the oscillation. This is shown in Figure 15. The double distance is shown in Figure 15.

In a consequence, the double distance always gives us the double period. This is shown in Figure 15.

It should be obvious that there is no necessary relation between distance and time. It can be shown that a double of the distance does not double the time. Consider a system of a mass  $m$  and a spring with constant  $k$ . The period of oscillation is  $T = 2\pi\sqrt{m/k}$ . If the distance is doubled, the period remains the same. This is shown in Figure 14.

If you have a double the distance, you can double the period of the oscillation. This is shown in Figure 15. The double distance is shown in Figure 15.

In a consequence, the double distance always gives us the double period. This is shown in Figure 15.

It should be obvious that there is no necessary relation between distance and time. It can be shown that a double of the distance does not double the time. Consider a system of a mass  $m$  and a spring with constant  $k$ . The period of oscillation is  $T = 2\pi\sqrt{m/k}$ . If the distance is doubled, the period remains the same. This is shown in Figure 14.

for any resulting third-harmonic currents to flow through. This feature is actually used in several ways.

By having either a primary or secondary delta, or by having a tertiary delta winding manufactured into a star/star transformer, the delta winding will become a *trap* for any third-harmonic frequencies. The tertiary delta winding allows any third-harmonic currents present to circulate freely within the tertiary winding, which is effectively a short circuit for the third-harmonic frequencies. Obviously this will create heating in the tertiary delta winding and the transformer, but such a winding is designed to dissipate this heat, and the heating from the third harmonic would otherwise exist in the windings of the transformer anyway, so the benefit of using the tertiary delta to control the third-harmonic frequency is achieved.

The tertiary delta winding can also be used where it may be necessary to measure the magnitude of any present third-harmonic frequency. This measurement is achieved by making voltage measurements on the tertiary delta.

For a zigzag (interstate) arrangement, using voltages from the same winding (which is fitted to the same leg of the magnetic core) to oppose each other results in the third-harmonic voltage in the two halves of each winding being in opposition. This causes cancellation of third-harmonic frequencies in both the phase and line voltages in the electrical system connected to this type of transformer.



### Check your progress 3

- 1 List at least three types of electrical equipment that cause distortion to the current waveform.
- 2 Explain what happens to the impedance of an inductive transformer winding when harmonic components of the fundamental current flow in the winding.

## Check your progress answers

### Check your progress 1

- 1 1000 kVA secondary full-load current:

Transformer B:

$$I_{fL} = \frac{VA}{\sqrt{3}V}$$

$$= \frac{1000 \text{ kVA}}{\sqrt{3} \times 11 \text{ kV}}$$

$$= 52.49 \text{ A (line current)}$$

$$R\% = \frac{I_f R_E \times 100}{\sqrt{3} \times V_1}$$

$$= \frac{52.49 \times 2.06 \times 100}{\sqrt{3} \times 11 \text{ k}}$$

$$= 0.567\%$$

$$X\% = \frac{7.27 \times 0.567\%}{2.06}$$

$$= 2\%$$

Transformer A:

Then  $Z_A$  is

$$Z_A\% = 2 + j5$$

$$= 5.385 \angle 68.2^\circ \text{ on 1500 kVA base}$$

and  $Z_B$  is

$$(1.7 + j6) \times 1.5 = 9.35 \angle 74.2^\circ \text{ on 1500 kVA base}$$

when converted to a 1500 kVA base.

6. Explain briefly the advantages to be gained by the insertion of reactances in the bus-bars of a large generating station. A generating station contains four identical three-phase alternators  $A$ ,  $B$ ,  $C$ , and  $D$  each of 20 000 kVA., 11 kV. rating and having 20% reactance. They are connected to a bus-bar system which has a bus-bar reactor rated at 20 000 kVA. and having 25% reactance inserted between  $B$  and  $C$ . A 66 kV. feeder is taken off from the bus-bar through a 10 000 kVA. transformer having 5% reactance. If a short circuit occurs across all phases at the high voltage terminals of the transformer, calculate the current fed into the fault. (Nat. Cert., 1935.)

7. The bus-bars of a power station are split into two sections  $A$  and  $B$ , separated by a 5% reactance (based on 10 000 kVA.). A 30 000 kVA. generator with 10% reactance is connected to section  $A$  and a 50 000 kVA. generator with 12% reactance is connected to section  $B$ . Each section supplies a transmission line through a 40 000 kVA. transformer with 6% reactance which steps the voltage up to 132 kV. If a three-phase short-circuit occurs on the high-tension terminals of the transformer connected to section  $A$ , calculate the maximum initial value of current which may occur at a short circuit.

Describe how you would estimate the current which a circuit-breaker operating after 0.3 sec. would have to interrupt, and explain why this value would be different from the maximum initial value as calculated.

8. A three-phase system of voltages is given by

$$V_A = 1\,000 \angle 35^\circ \quad V_B = 3\,000 \angle 100^\circ \quad V_C = 2\,000 \angle 270^\circ$$

Resolve these voltages into their symmetrical components, namely, a balanced positive sequence component, a balanced negative sequence component and a zero sequence component.

Explain how the method of symmetrical components can be used for the calculation of short-circuit currents under unbalanced fault conditions.

(Lond. Univ., 1934.)

9. Two 3-phase, 6.6 kV. generators  $G_1$  and  $G_2$  feed into common bus-bars, which are linked by reactors  $R$  to a second set of bus-bars: the latter are also linked to a grid system of large capacity by an interconnector  $I$ . From the bus-bars, feeders  $F_1$  and  $F_2$  supply a network  $N$ , as in Fig. 191. The percentage reactances are as follows—

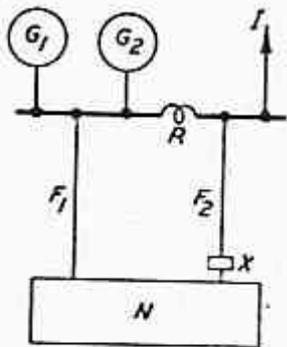


FIG. 191

$G_1$  and  $G_2$ , each 15% at 50 MVA.;  $R$ , 5% at 10 MVA.; grid system through  $I$ , 10% at 40 MVA.;  $N$ , between  $F_1$  and  $F_2$ , 10% at 100 MVA. The impedance of each feeder is  $0.01 + j0.02$  ohm per core.

Determine the maximum symmetrical fault current that a circuit-breaker at  $X$  may have to clear, in the event of either one or the other of the feeders being disconnected.

(Lond. Univ., 1949.)

## CHAPTER VIII

### SHORT CIRCUITS : SYMMETRICAL COMPONENTS

**Introduction.** When a fault occurs on a network such that a large current flows in one or more of the phases, it is said that a short circuit has occurred. The fault may be a short between one phase and earth, between two or more phases and earth, between two phases only, or across all three phases; a short between one phase and earth will cause a short circuit only if the neutral point is earthed. It is necessary to know the maximum short-circuit currents that can occur at the various points of a system in order that circuit-breakers may be selected that are adequate to withstand the currents and operate successfully to cut out the faulty section, and also in order that the protective relays may be selected for correct operation. Moreover, it is necessary to be able to calculate, approximately at least, the size of the protective reactors which must be inserted in the system to limit the short-circuit currents to a value which is not beyond that capable of being withstood by the circuit-breakers.

The short-circuit currents in an alternating current system are determined mainly by the reactance of the alternators, transformers, and lines up to the point of the fault in the case of phase-to-phase faults. When the fault is between a phase and earth, the resistance of the earth path may play an appreciable part in limiting the currents.

**Percentage Reactance and Short-circuit Currents.** The method of specifying the impedance or reactance of a piece of electrical plant, as described in Appendix III, pages 471-2, is very convenient for the calculation of short-circuit currents. The percentage reactance is given by

$$(\% X) = (IX/E) \times 100, \quad . \quad . \quad . \quad (99)$$

where  $X$  is the reactance,  $E$  the rated voltage, and  $I$  the full-load current. If the piece of apparatus is the only impedance in the circuit, the short-circuit current is given by  $E/X$ , which by equation (99) is

$$I_{sh} = E/X = I \times (100/\% X). \quad . \quad . \quad (100)$$

Thus the short-circuit current for a single piece of apparatus is the full-load current multiplied by (100 divided by the percentage reactance). Thus if the percentage reactance is 10 per cent, the short-circuit current is 10 times the full-load current; if it is 40 per cent, the short-circuit current is 2.5 times the full-load current.

If there are several reactances in series of magnitudes  $X_1$ ,  $X_2$ , and  $X_3$ , the short-circuit current is

$$\frac{E}{X_1 + X_2 + X_3} = \frac{E}{(E/I)(\% X_1) + (E/I)(\% X_2) + (E/I)(\% X_3)} \times 100$$

$$= I \times \frac{100}{(\% X_1) + (\% X_2) + (\% X_3)}$$

by application of equation (99) to the various reactances.

It often happens that the system contains plant of different ratings, and the percentage reactances are given for the respective values. It is then necessary to allow for the different ratings in the following way. Suppose the generator has a rating of 10 000 kVA, and a percentage reactance of 7, and a transformer has a rating of 8 000 kVA, and a percentage reactance of 5. The full-load current of the generator is  $(10\,000\,000/E) = I_1$ , say, so that its reactance is

$$X_1 = (E \times 7)/(I_1 \times 100).$$

Similarly the reactance of the transformer is

$$X_2 = (E \times 5)/(I_2 \times 100)$$

where  $I_2 = (8\,000\,000/E)$ .

The total reactance is

$$X_1 + X_2 = \frac{E \times 7}{I_1 \times 100} + \frac{E \times 5}{I_2 \times 100}$$

$$= \frac{E}{I_1 \times 100} \left[ 7 + 5 \times \frac{I_1}{I_2} \right] = \frac{E}{I_1 \times 100} \left[ 7 + 5 \frac{EI_1}{EI_2} \right]$$

$$= \frac{E}{I_1 \times 100} \left[ 7 + 5 \times \frac{10\,000}{8\,000} \right],$$

and the total percentage reactance referred to the rating of 10 000 kVA, is

$$(\% X) = \frac{I_1(X_1 + X_2)}{E} \times 100$$

$$= 7 + 5 \times \frac{10\,000}{8\,000}$$

$$= 7 + 6.25 = 13.25.$$

Thus the percentage reactance of the transformer is multiplied by the ratio of the generator rating to the transformer rating in order that it may be expressed with respect to the generator rating.

If there are several pieces of apparatus in the circuit with different ratings, we choose a basic rating and refer all percentage reactances to this rating by appropriate multipliers. We can then add the

percentages, if the pieces of apparatus are in series, and the short-circuit current is then found by equation (100). The following example illustrates the method.

**EXAMPLE.** Find the short-circuit current in the single-phase system of Fig. 165, if the fault is a short circuit between lines at the point  $F$  which is 10 miles from the transformer  $T$ . The reactance per mile is  $0.2 \Omega$ . The voltage is 6.6 kV.

We take 2 000 kVA, as the basic rating, so that full-load current is

$$I = 2\,000/6.6 = 300 \text{ A.}$$

The percentage reactance of the generator is 8, and of the transformer  $T$ ,  $7 \times (2\,000 \div 1\,200) = 11.7$ . The line reactance is  $2 \Omega$ , so that its percentage reactance is

$$\frac{300 \times 2}{6\,600} \times 100 = 9.1.$$

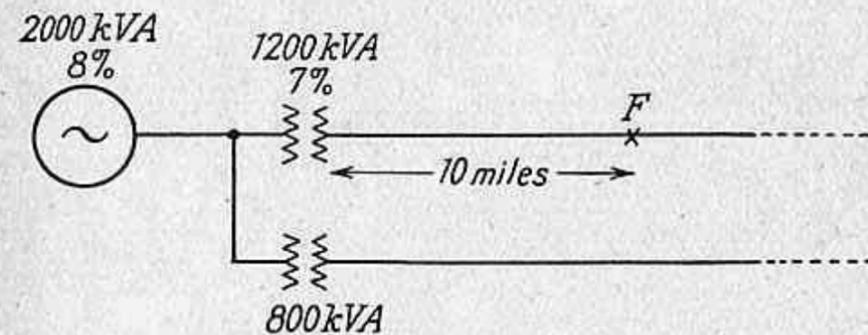


FIG. 165

The total percentage reactance is

$$8 + 11.7 + 9.1 = 28.8,$$

so that the short-circuit current is

$$I_{sh} = I \times (100/\% X) = 300 \times (100/28.8)$$

$$= \underline{1\,040 \text{ A.}}$$

We could use the *direct method*, which consists in the reduction of percentage reactances to actual reactances. Thus the reactance of the generator is

$$\frac{E \times (\% X)}{I \times 100} = \frac{6\,600 \times 8}{300 \times 100} = 1.76 \Omega.$$

The reactance of the transformer is similarly

$$\frac{6\,600 \times 7}{300 \times \left( \frac{1\,200}{2\,000} \right) \times 100} = 2.57 \Omega.$$

The total reactance is  $1.76 + 2.57 + 2.00 = 6.33$  ohms, and the short-circuit current is

$$I_{sh} = 6\,600/6.33 = 1\,040 \text{ A.}$$

If some of the pieces of apparatus are in parallel, their reactances, and hence their percentage reactances, must be compounded by the method used for parallel impedances. Thus a percentage reactance of 10 in parallel with another of 15 gives a resultant value of

$$\frac{1}{\frac{1}{10} + \frac{1}{15}} = \frac{10 \times 15}{10 + 15} = 6.$$

This method can be used for generators in parallel, as shown in the following example. In the calculation of such problems it is

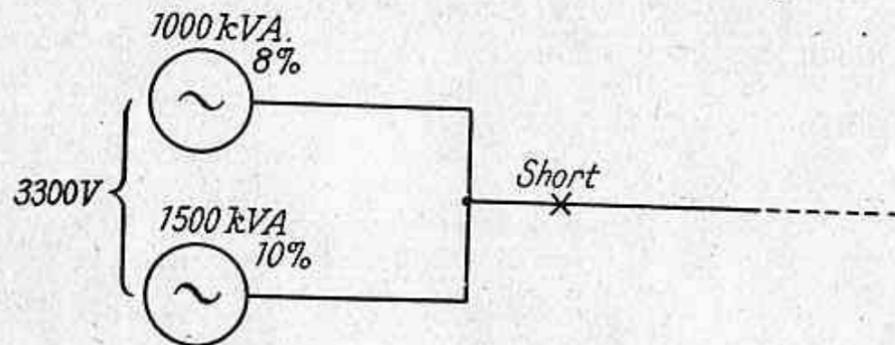


FIG. 166

assumed that the generators have equal voltages which are in phase; the error caused by this assumption should not be serious.

**EXAMPLE.** Two three-phase generators of ratings 1 000 and 1 500 kVA. and of voltage 3.3 kV. have percentage reactances of 8 and 10, with respect to their ratings. A short across all phases occurs near the common bus-bars. Find the short-circuit current.

The system is shown in Fig. 166.

Let us assume a basic kVA. of 2 500, which is the sum of both ratings. The percentage reactances with respect to this rating are

$$8 \times (2\,500/1\,000) = 20$$

and  $10 \times (2\,500/1\,500) = 16.7.$

The resultant percentage reactance is

$$\frac{1}{\frac{1}{20} + \frac{1}{16.7}} = \frac{1}{0.05 + 0.06} = \frac{1}{0.11} = 9.1.$$

The short-circuit kVA. is therefore

$$2\,500 \times (100/9.1) = 27\,500.$$

If  $I_{sh}$  is the short-circuit current per conductor, the kVA. per phase is

$$I_{sh} \times (3\,300/\sqrt{3}) = \frac{1}{3} \times 27\,500\,000,$$

so that 
$$I_{sh} = \frac{27\,500\,000 \times \sqrt{3}}{3 \times 3\,300} = 4\,810 \text{ A.}$$

The full load current is one-eleventh of this, i.e. 437 A.

**Symmetrical Short-circuit Currents.** The short-circuit currents are symmetrical, i.e. equal in the different conductors, if the short circuit occurs across both lines in a single-phase system or across the three wires of a three-phase system. The methods developed in the last section are adequate to calculate such short-circuit currents, and the two examples illustrate the methods. If there are several generators

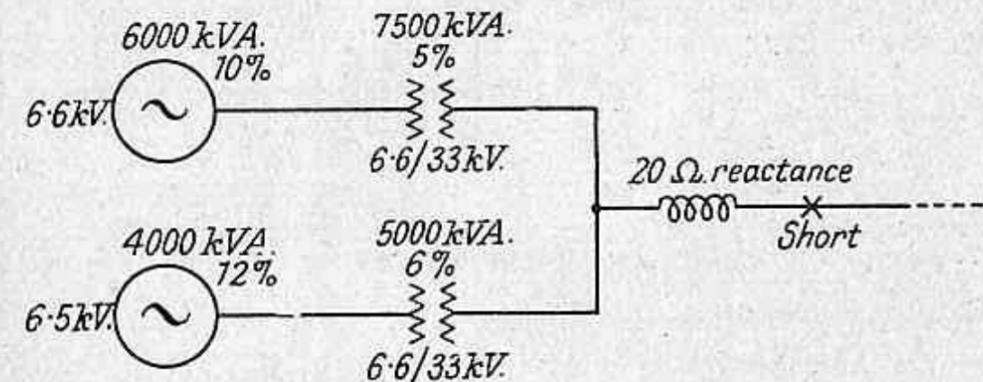


FIG. 167

in the system, it is assumed, as has already been stated, that the e.m.f.'s are equal and in phase with each other. When the system is complicated, the star-delta equivalence of Appendix III is often of great help in effecting a simplification of the network. Sometimes Thévenin's theorem is useful in obtaining the result quickly, especially when generators have unequal e.m.f.'s.

**EXAMPLE.** Find the short-circuit current in the system of Fig. 167, in which one generator is generating at 6.6 kV. and the other at 6.5 kV. The system is three-phase.

The voltages per phase are  $6.6 \text{ kV}/\sqrt{3} = 3\,820 \text{ V.}$  and  $6.5 \text{ kV.}/\sqrt{3} = 3\,750 \text{ V.}$  The full-load current of the larger generator is

$$\frac{6\,000\,000}{3 \times (6\,600/\sqrt{3})} = \frac{6\,000\,000}{\sqrt{3} \times 6\,600} = 525 \text{ A.,}$$

so that its reactance per phase is

$$\frac{(\% X) \times E}{I \times 100} = \frac{10 \times 3\,820}{525 \times 100} = 0.727 \Omega.$$

In the same way it is found that the reactance of the other generator is 1.31 Ω. It is assumed that the rated voltage in this case also is 6.6 kV., although the generated voltage is 6.5 kV.

Similarly the transformer reactances referred to the low voltage sides are 0.292 and 0.525 Ω. The line reactance transferred to the low voltage side is  $20 \div (33/6.6)^2 = 0.8 \Omega$ . The system is then as shown in Fig. 168. We then apply Thévenin's theorem to the system to the left of A. The voltage when the line is disconnected is

$$3\ 820 - \frac{(3\ 820 - 3\ 750)(0.727 + 0.292)}{0.727 + 0.292 + 1.31 + 0.525} = 3\ 795\ \text{V.}$$

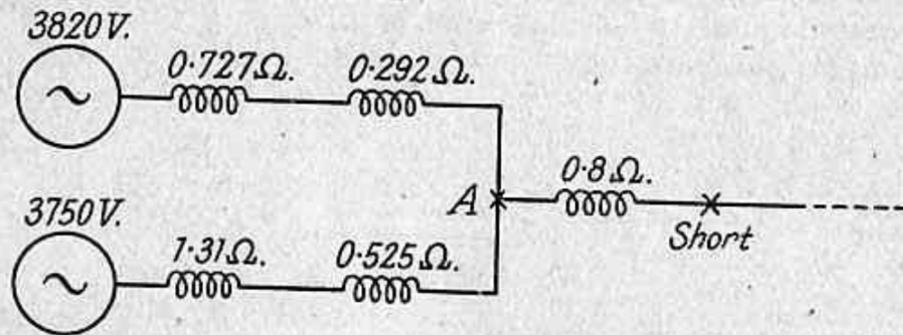


FIG. 168

The impedance is

$$\frac{(0.727 + 0.292)(1.31 + 0.525)}{0.727 + 0.292 + 1.31 + 0.525} = 0.654\ \Omega$$

The short-circuit current is thus

$$\frac{3\ 795}{0.654 + 0.8} = 2\ 600\ \text{A.}$$

The actual short-circuit current is

$$2\ 600 \times (6.6/33) = 520\ \text{A.}$$

520 A. is the r.m.s. of the steady short-circuit current, whilst the peak value is  $520 \times \sqrt{2} = 735\ \text{A}$ . There may be an increased peak value due to a "doubling effect," which, in circuits of normal value of reactance to resistance, is 1.8 times the steady peak. Thus a maximum peak value of  $1.8 \times 735 = 1\ 320\ \text{A}$ . may occur.

**Short-circuit Current of Alternators.** When an alternator is shorted, across all three phases, say, the current rises rapidly to a high value, about 18 times full-load current in turbo-alternators which have cylindrical rotors, and about 12 times in generators with salient poles. The value of the peak current is limited only by the transient or leakage reactance of the armature. Moreover if the short circuit occurs at an instant at which the voltage is zero there is a doubling effect, and the current wave is offset from the zero. Fig. 169 shows the kind of current wave obtained. If the short circuit persists, the wave becomes symmetrical; then armature reaction

*Armature Reaction*

reduces the excitation and the current falls to a steady value, which is 4 to 6 times the full-load value. Another way of considering the effect of armature reaction is to consider it as increasing the transient impedance to the synchronous impedance.

The doubling effect may be demonstrated as follows. Let the generator be considered as an e.m.f.  $E \sin(\omega t + \theta)$  in series with an impedance  $(R, L)$  which is the transient or true impedance. If a short occurs at  $t = 0$ , the equation for the short-circuit current is

$$L(di/dt) + Ri = E \sin(\omega t + \theta).$$

The complementary function is given by

$$L(di/dt) + Ri = 0,$$

i.e.

$$i = A e^{-(R/L)t} \quad (101)$$

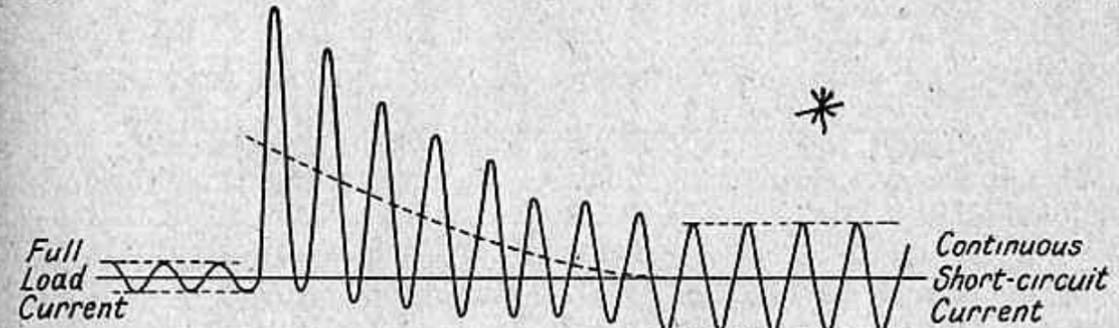


FIG. 169. DOUBLING EFFECT IN SHORT-CIRCUIT CURRENT

The particular integral is

$$i = \frac{E}{\sqrt{[R^2 + (\omega L)^2]}} \sin\left(\omega t + \theta - \tan^{-1} \frac{\omega L}{R}\right) \quad (102)$$

which is the steady current under these conditions. The actual current is the sum of the currents given in equations (101) and (102). At  $t = 0$  the current is zero. This gives

$$0 = A + \frac{E}{\sqrt{[R^2 + (\omega L)^2]}} \sin\left(\theta - \tan^{-1} \frac{\omega L}{R}\right).$$

The current is thus

$$i = -\frac{E}{\sqrt{[R^2 + (\omega L)^2]}} \sin\left(\theta - \tan^{-1} \frac{\omega L}{R}\right) e^{-(R/L)t} + \frac{E}{\sqrt{[R^2 + (\omega L)^2]}} \sin\left(\omega t + \theta - \tan^{-1} \frac{\omega L}{R}\right) \quad (103)$$

We may consider  $\omega L$  as much greater than  $R$ . Then the first term, which is considered as a d.c. component which decays exponentially, has magnitude

$$-\frac{E}{\omega L} \sin\left(\theta - \frac{\pi}{2}\right) e^{-(R/L)t} = \frac{E}{\omega L} \cos \theta \cdot e^{-(R/L)t}.$$

If  $\theta = 0$ , i.e. the voltage is zero and the current is a maximum at  $t = 0$ , the d.c. component has the initial value of  $E/\omega L$ . As the alternating part of the current has a magnitude of nearly  $E/\omega L$ , the d.c. component doubles the current at the instant when the former has its peak value, and reduces it to zero when the former reaches its negative maximum. The current is thus on one side of the zero to begin with. If, however,  $\theta = \pi/2$ , i.e. the voltage is a

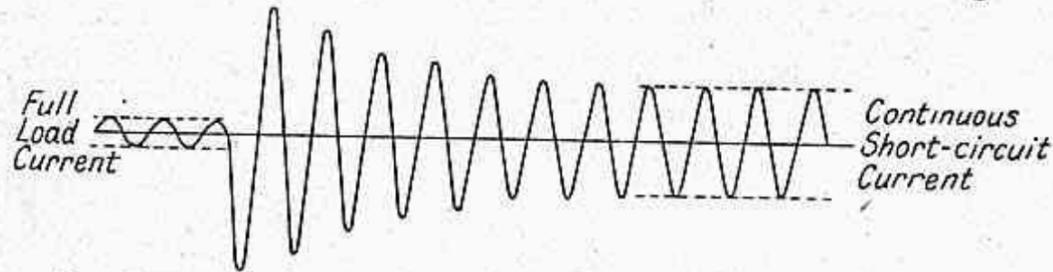


FIG. 170. SHORT-CIRCUIT CURRENT WITHOUT DOUBLING EFFECT

maximum and the current is zero at the instant of short circuit, the d.c. term is zero. The short-circuit current has then the form shown in Fig. 170.

The change from the large current at the instant of short circuit to the comparatively small current after armature reaction has asserted itself is of importance in the design of switchgear operation. The behaviour of an alternator is most easily expressed in terms of *decrement factors*, which are found by extensive tests in the

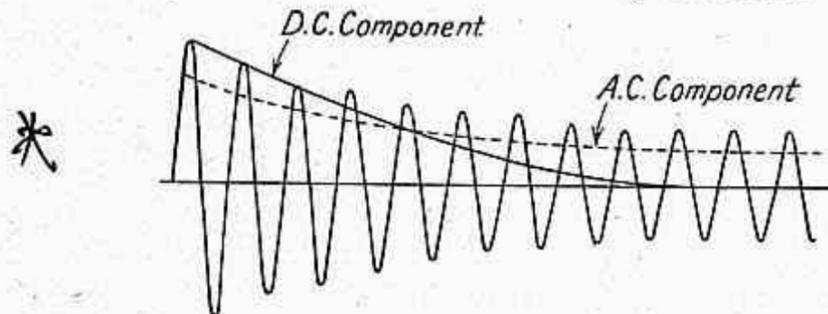


FIG. 171. D.C. AND A.C. COMPONENTS OF SHORT-CIRCUIT CURRENT

following way. The generator excitation is adjusted to the value for full load at 0.8 power factor lagging, and external reactance is put in series with the alternator to bring the value up to some definite amount, 5, 10, 20 . . . per cent. This value includes the transient reactance. A short circuit is applied and an oscillogram of the current taken. In order that the test results should be as severe as possible, it is arranged that the short circuit should take place at an instant when the full doubling effect is incurred, viz. at an instant of zero voltage. The oscillogram is analysed so that the r.m.s. current is found as a function of time. To do this the current wave of Fig. 169 is resolved into the d.c. and a.c. components shown in Fig. 171. The r.m.s. of the a.c. component is shown by the

dotted line, and has a value  $I_{a.o.}$  at time  $t$ ; the d.c. component, which decays exponentially, has a value  $I_{d.o.}$  at the same instant, and the total current has an r.m.s. value of  $\sqrt{[I_{d.o.}^2 + I_{a.o.}^2]}$ .

Curves are then drawn giving the r.m.s. of the current (as a multiple of full-load current) against time for different values of the total percentage reactance. Fig. 172 shows a set of such curves for a short circuit across all three phases.

When looking up the decrement factor, the transient reactance of the alternator is added to the external reactance to give the appropriate percentage reactance.

EXAMPLE. A 20 000 kVA. generator, whose decrement curves are shown in Fig. 172, has 15% reactance and feeds a line through a step-up transformer of 6% reactance. Find the breaking capacity of the circuit-breakers, which operate in 0.25 sec. and are on the high voltage side of the transformer.

The total reactance is 21%, and from Fig. 172 it is seen that the decrement

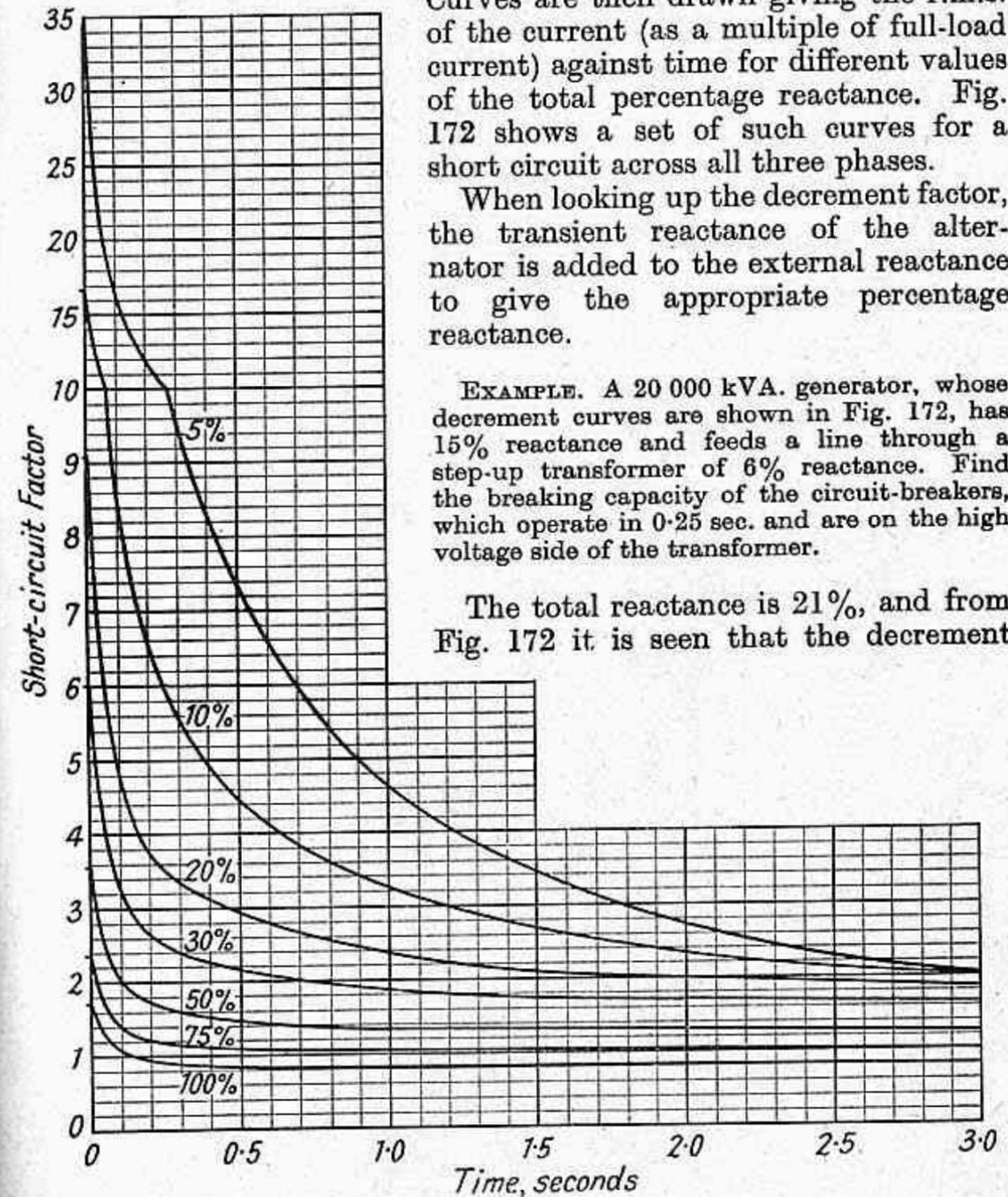


FIG. 172. DECREMENT CURVES FOR ALTERNATOR

factor at 0.25 sec. is 3.4. The current to be interrupted is thus 3.4 times the full-load current. If we assume that the recovery voltage in the breaker is equal to the normal voltage (the matter will be investigated in detail in Chapter IX), the kVA. to be broken is

$$3.4 \times 20\,000 = 68\,000 \text{ kVA.}$$

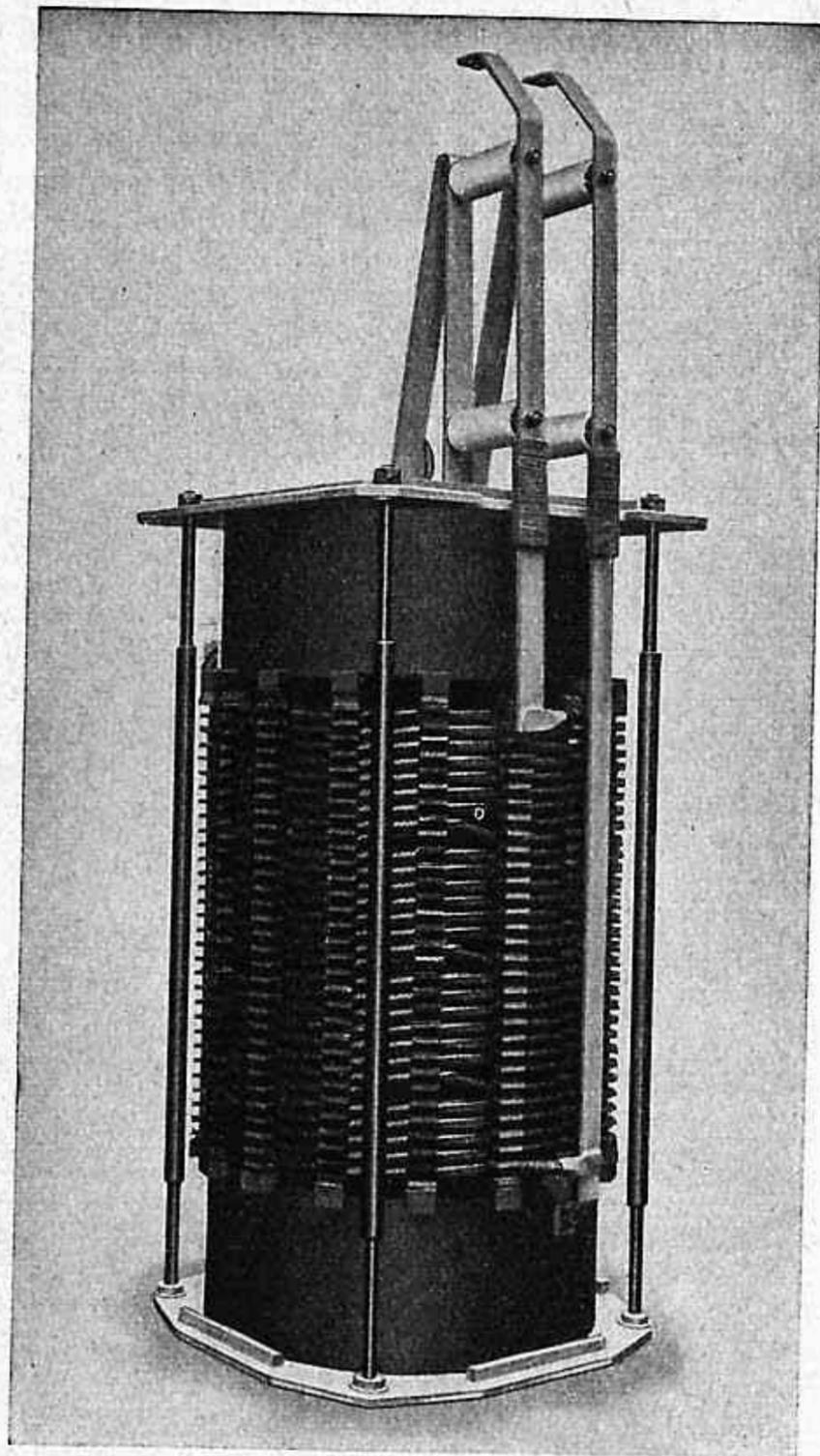


FIG. 173. CURRENT LIMITING REACTOR  
(English Electric Co.)

**Current-limiting Reactors: Sectionalization of Networks.** It is clear that the short-circuit currents are decreased by an increase of the percentage reactance in the system. In large interconnected systems the total rating of the generators is very high, and unless precautions are taken, the current fed into a fault will be enormous. The short-circuit current at a fault can be considerably reduced by the judicious placing of protective reactors in the system. It is possible to arrange the reactors so that they do not cause a large voltage drop during normal operation, but prevent a large short-circuit current being fed by most of the generators into the fault. The methods of placing reactors in a system will be considered later.

Reactors are moreover of considerable importance in limiting the currents so that the various circuit-breakers are not called upon to

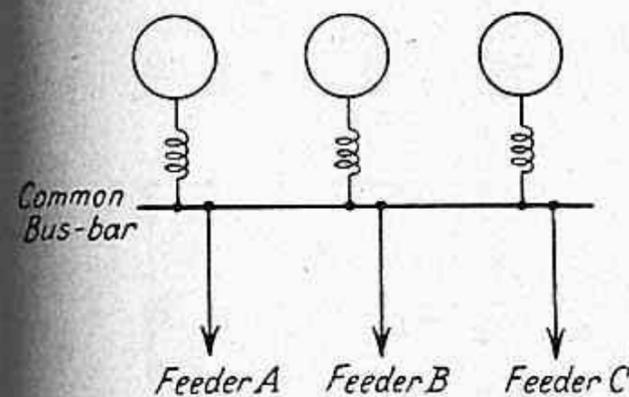


FIG. 174. GENERATOR REACTORS

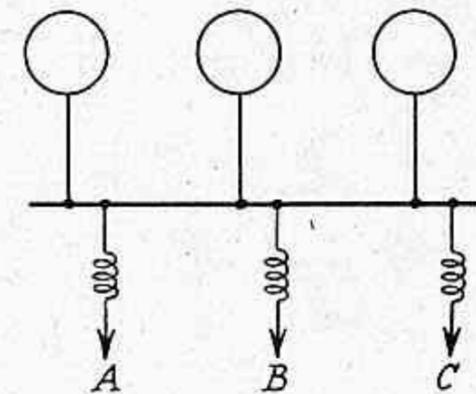


FIG. 175. FEEDER REACTORS

break currents above their rated value. If extensions are made in a system, it is essential that the additional kVA. be virtually segregated from the existing circuit-breakers when a short-circuit occurs. This is done by means of current-limiting reactors.

Fig. 173 shows a reactor. The turns, which are of copper bar or strip, experience large attractive forces under the influence of the short-circuit currents, and they are placed in concrete separators to prevent their being buckled.

**Methods of Locating Reactors.** Reactors may be inserted in series with each generator, as shown in Fig. 174. The main disadvantage of this method is that if a short occurs on one feeder, the voltage at the common bus-bar drops to a low value and the synchronous machines attached to the other feeders may fall out of step. The whole system is interrupted, and the synchronous machines must be re-synchronized when the faulty feeder is cut out. Moreover in modern alternators the transient reactance is sufficiently large to protect the machine itself against short-circuit currents, and separate reactors are used only with old alternators.

The main disadvantage of the last method is avoided by putting reactances in series with each feeder, as shown in Fig. 175. When

a short-circuit occurs on feeder *A*, the main voltage drop is in its reactor and the bus-bar voltage does not drop unduly. The remaining load and plant are therefore able to continue running. It is true that when a short circuit occurs across the bus-bars, the reactors do not protect the generators. This is, however, of no importance, as bus-bar short circuits seldom occur and the generators are protected by their internal reactances.

A disadvantage from which both the previous methods suffer is that the reactors take the full-load currents under normal operation, so that there is a constant loss and a voltage drop. The voltage drop is eliminated in a new type of reactor in which part of the windings are shunted by a carbon tetrachloride fuse. Under normal conditions the windings are such that they neutralize each other's

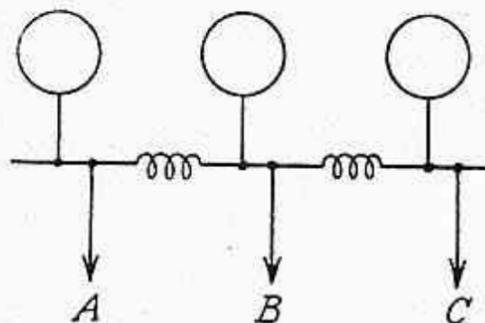


FIG. 176. BUS-BAR REACTORS, RING SYSTEM

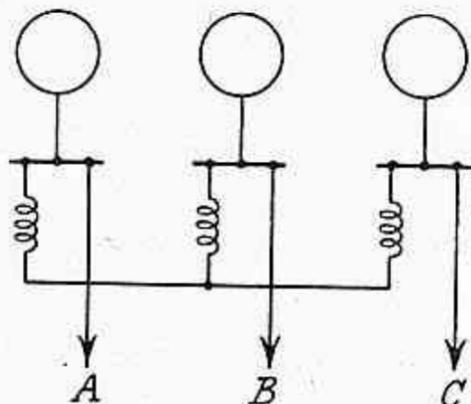


FIG. 177. BUS-BAR REACTORS, TIE-BAR SYSTEM

magnetic field and the reactor has a very small reactance; but when a short circuit occurs and the fuse blows, a large reactance is inserted into the circuit. The constant loss, however, is not eliminated.

The constant loss in reactors can be avoided by inserting the reactors in the bus-bars in the ways shown in Figs. 176 and 177. The former is the *ring* system, and the latter is the *tie-bar* system. In the ring system each feeder is normally fed by one generator, only a small amount of power flowing across the reactances. The reactors can therefore be made with a fairly high ohmic resistance and there is not much voltage drop across it. When a short circuit occurs in one feeder, the current is fed mainly by one generator, the other generators having to feed through the reactances. The tie-bar system acts in the same way, but has the following advantage. If the number of sections in the tie-bar system is increased, the current that flows into the fault will not exceed a certain value which is fixed by the size of the individual reactors. If the switch-gear is designed to operate successfully on this limiting value of

current, the system can be extended to any number of sections without modification of the switchgear.

**EXAMPLE.** Find the ratio of the percentage reactance of the reactors to that of the generators in a tie-bar system, if the short-circuit current is not to exceed three times the current of a single section.

Let the percentage reactance of a generator be  $G$  and of a reactor  $X$ , and suppose there are  $n$  sections. When there is a short circuit on a feeder, the remaining reactors and generators are in parallel, so that their percentage reactance is  $(G + X)/(n - 1)$ . This reactance is in series with the reactor of the faulty feeder, giving a reactance

$$X + (G + X)/(n - 1) = (G + nX)/(n - 1).$$

This reactance is in parallel with the reactance of the generator which is connected to the faulty feeder, so that the total reactance is

$$\frac{G \times \frac{G + nX}{(n - 1)}}{G + \frac{G + nX}{(n - 1)}} = G \frac{G + nX}{nG + nX}.$$

The short-circuit current is thus

$$I \times \frac{100}{G} \times \frac{nG + nX}{G + nX}$$

where  $I$  is the normal full-load current.

When  $n = 1$ , the current is

$$I \times (100/G).$$

The last factor gives the effect of the remaining sections, and increases from unity when  $n = 1$  to  $(G + X)/X$  when  $n$  is infinitely large. Thus if the current is not to exceed three times the short-circuit current due to a single section

$$(G + X)/X = 3$$

i.e.  $X = \frac{1}{2}G.$

If it is certain that the number of sections will not exceed a known number  $n$  we have

$$(nG + nX)/(G + nX) = 3$$

i.e.  $X = [(n - 3)/2n]G.$

Thus if  $n$  will not exceed 6,  $X$  need not be greater than  $\frac{1}{4}G$ .

**Choice of Interconnection to Limit Currents.** The cost of reactors is large and their installation is avoided if possible. It is sometimes practicable to make use of the reactance of feeders and transformers so that reactors are unnecessary.

For instance, suppose that two parallel feeders are fed by four transformers, as shown in Fig. 178, and suppose that a short circuit occurs at *B*. If the parallel feeders are not connected at their ends, the reactance from *A* to *B* is

$$\frac{X(3X + 2F)}{X + 3X + 2F} = \frac{X(3X + 2F)}{4X + 2F}$$

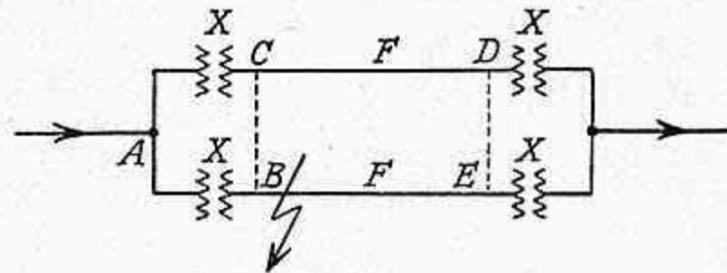


FIG. 178

If the feeders are connected at their ends (between *B* and *C*, *D* and *E*), the reactance from *A* to *B* is  $\frac{1}{2}X$ . The latter reactance is considerably less than the former; thus if  $F = X$ , the former is  $\frac{5}{8}X$  and the latter only  $\frac{1}{2}X$ .

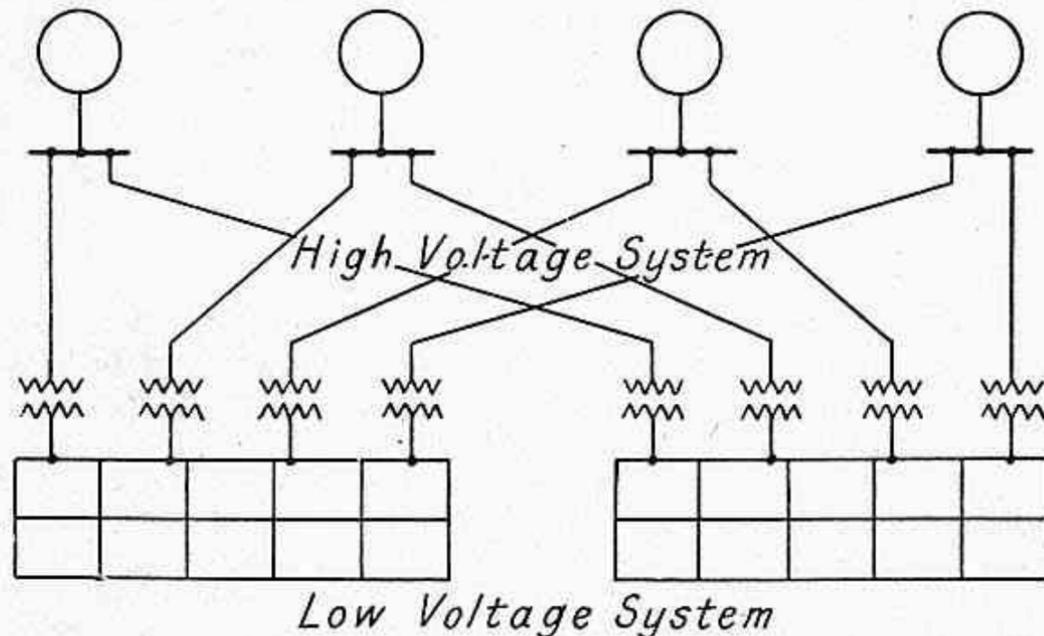


FIG. 179. SYNCHRONIZATION AT THE LOAD

As a general rule it is advisable to keep the parallel connections as few as possible.

An interesting and important application of this rule is shown by the method of interconnection of Fig. 179. The generators are unconnected in the high-tension system, but connected only at the low voltage system. This system is said to be *synchronized at the load*.

**Protection of Switchgear by Reactors.** A generating station may be extended by the addition of alternators or by a supply from the Grid. It is uneconomical to scrap the existing switchgear, which was adequate for the former output but is not of sufficient rating to meet the extensions. In such a case a protective reactor may be placed between the old system and the extensions to limit the short-circuit currents to a permissible value. An example will show how the requisite reactance is calculated.

**EXAMPLE.** A small generating station has two alternators of 3 000 and 4 500 kVA., and percentage reactances 7 and 8. The circuit-breakers are rated at 150 000 kVA. It is intended to extend the system by a supply from the Grid via a transformer of 7 500 kVA. rating and 7.5% reactance. Find the reactance necessary to protect the switchgear.

Let us take as the basic rating 7 500 kVA. The reactances of *A* and *B* are then

$$(7\,500/3\,000) \times 7 = 17.5\%$$

and  $(7\,500/4\,500) \times 8 = 13.3\%$ ,

so that their combined reactance is

$$\frac{17.5 \times 13.3}{17.5 + 13.3} = 7.55\%$$

The short-circuit kVA. with respect to these alone is

$$7\,500 \times (100/7.55) = 99\,300 \text{ kVA.}$$

If no protective reactor is present the short-circuit rating due to the Grid supply is

$$(7\,500/7.5) \times 100 = 100\,000 \text{ kVA.}$$

so that the total is 199 300 kVA. In order to keep the kVA. down the rated value of 150 000 kVA. suppose that a reactor of percentage reactance  $X$  is interposed as shown in Fig. 180. The short-circuit kVA. of the Grid supply is then  $7\,500/(7.5 + X)$ , and this must not exceed the difference between the ratings of the circuit-breakers and the generators *A* and *B*

Therefore

$$\frac{7\,500}{7.5 + X} \times 100 = 150\,000 - 99\,300 \\ = 50\,700,$$

giving

$$7.5 + X = \frac{7\,500 \times 100}{50\,700}$$

$$= 14.8,$$

so that

$$X = 7.3.$$

If the voltage is 3 300 V., the full-load current per phase corresponding to 7 500 kVA. is

$$\frac{7\,500\,000}{3\,300 \times \sqrt{3}} = 1\,310 \text{ A.},$$

so that the actual reactance of the reactor per phase is

$$\frac{7.3 \times (3\,300/\sqrt{3})}{1\,310 \times 100} = \underline{0.106 \Omega}.$$

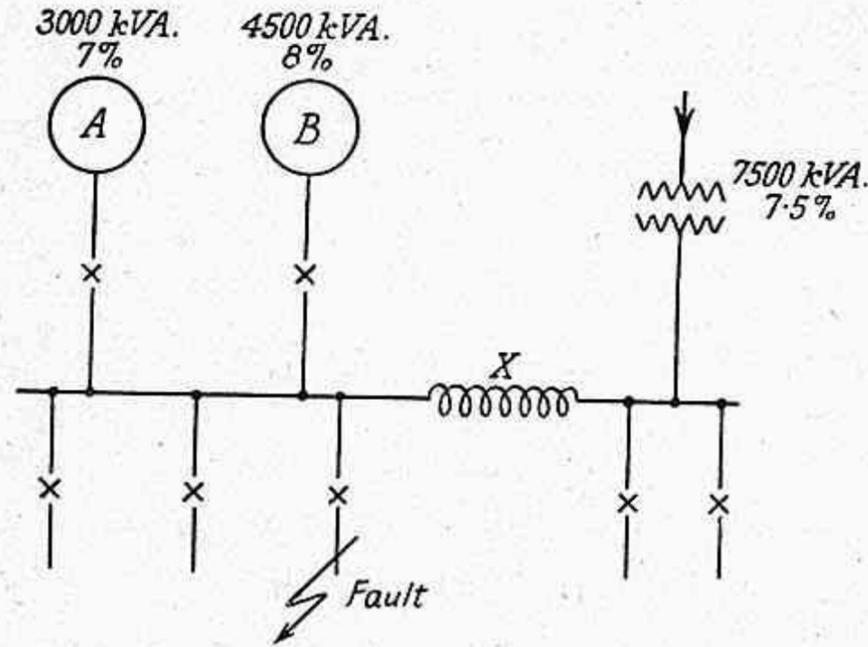


FIG. 180. REACTOR PROTECTING SWITCHGEAR

corresponding to an inductance of

$$\begin{aligned} \frac{0.106}{2\pi \times 50} &= 0.000337 \text{ H.} \\ &= 0.337 \text{ mH.} \end{aligned}$$

**Unsymmetrical Short Circuits: Symmetrical Components.** The methods of calculating short-circuit currents developed in the previous sections apply only to the cases when the fault occurs across all phases and the currents in the phases are equal. When the fault occurs across two of the three phases or between one or two phases and earth, the currents are unequal and the methods are inadequate. The currents can be found by Kirchhoff's laws, but this method is usually laborious. A method has been devised which uses *symmetrical components* of the currents and voltages.

It will first be shown how three unbalanced vectors can be expressed as the sum of three systems of balanced vectors. Three

*balanced vectors* are vectors which are equal in magnitude and differ in phase by 120°, so that they can be represented by

$$E_1, E_1' = E_1 \varepsilon^{j(4\pi/3)}, E_1'' = E_1 \varepsilon^{j(2\pi/3)},$$

if  $E_1$  is taken as a reference vector. Fig. 181 shows the vectors. It should be remembered that multiplication of a vector by the factor  $\varepsilon^{j\theta}$  results in a vector of unchanged magnitude but with a phase angle increased by  $\theta$ ; for if the vector has magnitude  $r$  and angle  $\alpha$  it is represented by

$$r \angle \alpha = r \varepsilon^{j\alpha},$$

and multiplication by  $\varepsilon^{j\theta}$  results in a vector

$$r \varepsilon^{j\alpha} \times \varepsilon^{j\theta} = r \varepsilon^{j(\alpha + \theta)} = r \angle \alpha + \theta.$$

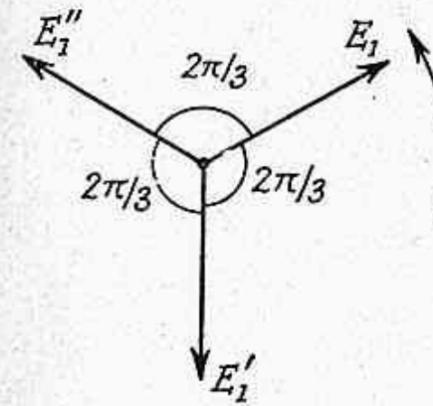


FIG. 181  
POSITIVE PHASE SEQUENCE  
 $E_1$  leads  $E_1'$  leads  $E_1''$

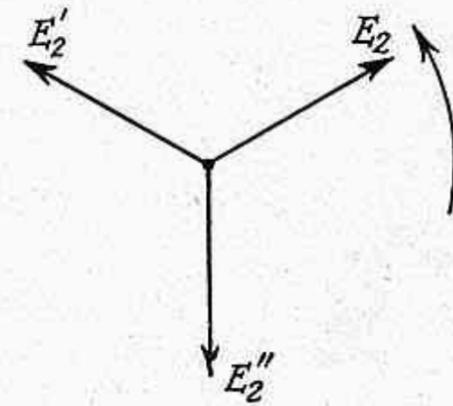


FIG. 182  
NEGATIVE PHASE SEQUENCE  
 $E_2$  lags  $E_2'$  lags  $E_2''$

As is pointed out in Appendix III, the values of the alternating quantities are the horizontal components of these vectors, which are assumed to rotate in the positive (anti-clockwise) direction with an angular velocity  $\omega = 2\pi \times \text{frequency}$ . Thus  $E_1, E_1',$  and  $E_1''$  correspond to alternating quantities which reach their maxima in the order given. The system just given is called a system of *positive phase sequence*. It is usual to replace  $\varepsilon^{j(2\pi/3)}$  by  $\lambda$ , so that

$$\left. \begin{aligned} \lambda &= \varepsilon^{j(2\pi/3)} = \cos(2\pi/3) + j \sin(2\pi/3) = -0.5 + j0.866 \\ \text{and } \lambda^2 &= \varepsilon^{j(4\pi/3)} = \cos(4\pi/3) + j \sin(4\pi/3) = -0.5 - j0.866. \end{aligned} \right\} (104)$$

It follows that  $1 + \lambda + \lambda^2 = 0.$

A positive phase-sequence system of vectors is thus represented by

$$E_1, \lambda^2 E_1, \lambda E_1. \dots \dots \dots (105)$$

It would be symmetrical but unconventional to write these as

$$\lambda^3 E_1, \lambda^2 E_1, \lambda E_1,$$

A balanced system with *negative phase sequence* is shown in Fig. 182. The order of maxima is  $E_2''$ ,  $E_2'$ ,  $E_2$ , so that the vectors are

$$E_2, E_2' = \lambda E_2, E_2'' = \lambda^2 E_2. \quad (106)$$

Suppose there is a system of unbalanced vectors  $E_A$ ,  $E_B$ , and  $E_C$ . By virtue of the relations  $(1 + \lambda + \lambda^2) = 0$  and  $\lambda^3 = 1$  we may write

$$E_A = \frac{1}{3}(E_A + E_B + E_C) + \frac{1}{3}(E_A + \lambda E_B + \lambda^2 E_C) + \frac{1}{3}(E_A + \lambda^2 E_B + \lambda E_C),$$

$$E_B = \frac{1}{3}(E_A + E_B + E_C) + (\lambda^2/3)(E_A + \lambda E_B + \lambda^2 E_C) + (\lambda/3)(E_A + \lambda^2 E_B + \lambda E_C),$$

$$\text{and } E_C = \frac{1}{3}(E_A + E_B + E_C) + (\lambda/3)(E_A + \lambda E_B + \lambda^2 E_C) + (\lambda^2/3)(E_A + \lambda^2 E_B + \lambda E_C).$$

Putting

$$\left. \begin{aligned} \frac{1}{3}(E_A + E_B + E_C) &= E_0, \\ \frac{1}{3}(E_A + \lambda E_B + \lambda^2 E_C) &= E_1, \\ \frac{1}{3}(E_A + \lambda^2 E_B + \lambda E_C) &= E_2, \end{aligned} \right\} \quad (107)$$

and

we have

$$\left. \begin{aligned} E_A &= E_0 + E_1 + E_2, \\ E_B &= E_0 + \lambda^2 E_1 + \lambda E_2, \\ E_C &= E_0 + \lambda E_1 + \lambda^2 E_2. \end{aligned} \right\} \quad (108)$$

It is seen from equations (105) and (106) that the terms in  $E_1$  represent a positive phase-sequence system of balanced vectors, whilst the terms in  $E_2$  represent a negative phase-sequence system of balanced vectors. The equal terms  $E_0$  are said to represent a *zero phase-sequence* system of vectors, which are equal. Thus any system of three unbalanced vectors can be resolved into three systems of balanced vectors, a zero phase-sequence system  $E_0$  (1, 1, 1), a positive phase-sequence system  $E_1$  (1,  $\lambda^2$ ,  $\lambda$ ), and a negative phase-sequence system  $E_2$  (1,  $\lambda$ ,  $\lambda^2$ ). The values of  $E_0$ ,  $E_1$ , and  $E_2$  are found from the unbalanced vectors  $E_A$ ,  $E_B$ ,  $E_C$  by means of equations (107) by multiplication of complex numbers or by a graphical method. The former method is described in Appendix II. The graphical method follows from the facts that, since  $\lambda = \epsilon^{j(2\pi/3)}$ , multiplication by  $\lambda$  turns a vector in the positive direction through an angle of  $2\pi/3$  radians and multiplication by  $\lambda^2$  turns a vector through  $4\pi/3$  radians.

Fig. 183 (a) shows an unbalanced system of vectors. Fig. 183 (b) shows how these are added to give  $E_A + E_B + E_C$ , one-third of which is  $E_0$ . Fig. 183 (c) shows how we obtain  $E_A + \lambda E_B + \lambda^2 E_C$ , one-third of which is  $E_1$ ; and Fig. 183 (d) shows how we obtain  $E_A + \lambda^2 E_B + \lambda E_C$ , one-third of which is  $E_2$ .

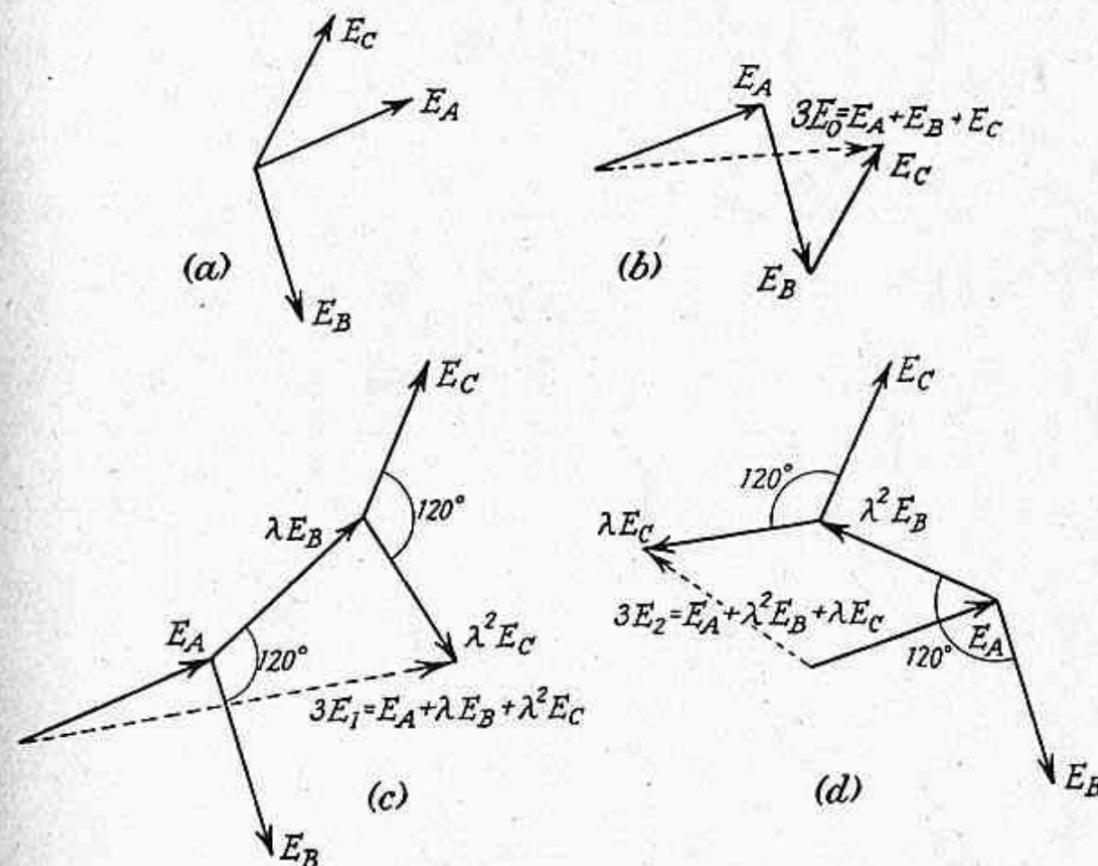


FIG. 183. GRAPHICAL METHOD OF FINDING SYMMETRICAL COMPONENTS

**EXAMPLE.** Resolve the system  $E_A = 1\,500 | 30^\circ$ ,  $E_B = 1\,800 | -70^\circ$ ,  $E_C = 2\,000 | 170^\circ$  into their symmetrical components.

$$\begin{aligned} E_0 &= \frac{1}{3}(E_A + E_B + E_C) \\ &= \frac{1}{3}(1\,500 | 30^\circ + 1\,800 | -70^\circ + 2\,000 | 170^\circ) \\ &= \frac{1}{3}(1\,500 \cos 30^\circ + 1\,800 \cos 70^\circ + 2\,000 \cos 170^\circ \\ &\quad + j1\,500 \sin 30^\circ - j1\,800 \sin 70^\circ + j2\,000 \sin 170^\circ) \\ &= \frac{1}{3}(1\,300 + 615 - 1\,970 + j750 - j1\,690 + j347) \\ &= -18 - j198 = 198 | 264^\circ 48'. \end{aligned}$$

$$\begin{aligned} E_1 &= \frac{1}{3}(E_A + \lambda E_B + \lambda^2 E_C) \\ &= \frac{1}{3}(1\,500 | 30^\circ + 1\,800 | -70^\circ + 120^\circ + 2\,000 | 170^\circ + 240^\circ) \\ &= 1\,250 + j1\,220 = 1\,740 | 44^\circ 14'. \end{aligned}$$

$$\begin{aligned} E_2 &= \frac{1}{3}(E_A + \lambda^2 E_B + \lambda E_C) \\ &= \frac{1}{3}(1\,500 | 30^\circ + 1\,800 | -70^\circ + 240^\circ + 2\,000 | 170^\circ + 120^\circ) \\ &= -70 - j270 = 280 | 252^\circ 30'. \end{aligned}$$

It is clear that unbalanced currents can be resolved into symmetrical components in the same way as voltages.

**Application of Symmetrical Components.** Let us consider the voltage drops in a symmetrically spaced transmission line due to unbalanced currents  $I_A, I_B, I_C$ . The line has self-impedances  $Z_s$  and mutual impedances  $Z_M$ , as shown in Fig. 184, where  $Z_s$  per unit length of line is the resistance plus  $j\omega L_s$ , and

$$L_s = [\frac{1}{2} + 2 \log (R/r)] \times 10^{-9} \text{ H. per cm. length}$$

according to equation (24). The mutual inductance between lines is

$$L_M = 2 \log (R/D) \times 10^{-9} \text{ H. per cm. length}$$

$R$  is a large distance beyond which the total flux is insignificant. The voltage drops in the lines are

$$\begin{aligned} V_A &= I_A Z_s + I_B Z_M + I_C Z_M \\ V_B &= I_A Z_M + I_B Z_s + I_C Z_M \\ V_C &= I_A Z_M + I_B Z_M + I_C Z_s \end{aligned}$$

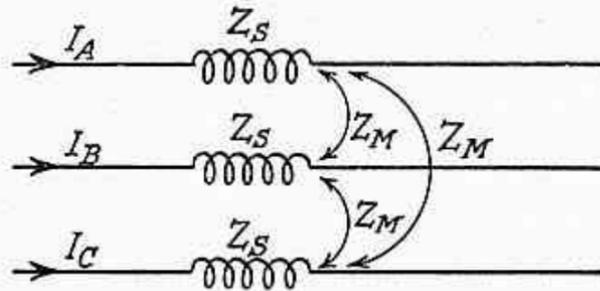


FIG. 184. GENERAL THREE-PHASE LINE

The symmetrical components of the voltages are found in the following way—

$$\begin{aligned} V_0 &= \frac{1}{3}(V_A + V_B + V_C) = \frac{1}{3}(Z_s + 2Z_M)(I_A + I_B + I_C) \\ &= I_0(Z_s + 2Z_M), \end{aligned}$$

$$\begin{aligned} V_1 &= \frac{1}{3}(V_A + \lambda V_B + \lambda^2 V_C) = \frac{1}{3}Z_s(I_A + \lambda I_B + \lambda^2 I_C) \\ &\quad + \frac{1}{3}Z_M(\lambda I_A + \lambda^2 I_A + I_B + \lambda^2 I_B + I_C + \lambda I_C) \\ &= I_1 Z_s + \frac{1}{3}Z_M(-I_A - \lambda I_B - \lambda^2 I_C), \end{aligned}$$

using the relation  $1 + \lambda + \lambda^2 = 0$ . Therefore

$$V_1 = I_1(Z_s - Z_M).$$

Similarly  $V_2 = I_2(Z_s - Z_M).$

We have therefore

$$\left. \begin{aligned} V_0 &= Z_0 I_0, \\ V_1 &= Z_1 I_1, \\ V_2 &= Z_2 I_2, \end{aligned} \right\} \dots \dots \dots (109a)$$

and

where

$$\left. \begin{aligned} Z_0 &= (Z_s + 2Z_M) \\ Z_1 &= (Z_s - Z_M) \\ Z_2 &= (Z_s - Z_M). \end{aligned} \right\} \dots \dots \dots (109b)$$

and

We have thus the important result that *the phase-sequence voltage drops are due to the separate phase-sequence currents.*

**Impedances to the Various Phase-sequence Currents.** It is seen from equations (109) that the impedances of a transmission line to the zero, positive, and negative phase-sequence currents are  $Z_0, Z_1,$  and  $Z_2$ . The last two are equal to  $(Z_s - Z_M)$ , which is due to an inductance of

$$L_s - L_M = [\frac{1}{2} + 2 \log (D/r)] \times 10^{-9} \text{ H. per cm. length,}$$

which is the inductance to neutral in a balanced system.

It is thus independent of the distance  $R$ , but depends only on the radius of the wires  $r$  and their spacing  $D$ . The zero phase-sequence impedance depends on  $R$ , or more particularly it depends upon the return path for the current  $I_0$  which flows along the three wires in parallel, since the distance  $R$  is to be chosen to include the return path. It is difficult to calculate  $Z_0$  and it is best found by experiment. If information is not available we may take  $Z_0$  as twice or three times  $Z_1$ . It will be shown later that if the neutral is earthed through an impedance, three times this impedance must be added to  $Z_0$ .

In order to calculate unsymmetrical short-circuit currents it is necessary to know the various phase-sequence properties of the generators and transformers in the system.

An alternator generates only a positive phase-sequence system of e.m.f.'s. We have already discussed in detail the impedance (or more simply the reactance) of the alternator to positive phase-sequence currents; there is the initial or transient reactance which increases to the *synchronous reactance* by reason of armature reaction. The initial reactance to negative phase-sequence currents is about 70 per cent of the previous transient reactance, and to zero phase-sequence currents 10 to 25 per cent. Under steady short-circuit conditions, the values are less. The following table gives approximate values in terms of the reactances to positive phase-sequence currents.

	REACTANCE TO NEGATIVE PHASE- SEQUENCE CURRENTS (%)	REACTANCE TO ZERO PHASE- SEQUENCE CURRENTS (%)
<i>Transient</i> . . . . .	70	10 to 25
<i>Steady Short Circuit</i>		
Salient pole . . . . .	30	5 to 15
Turbo-alternator . . . . .	15	0 to 5

Decrement factors can be used in the way described above to find the currents at intermediate times.

The impedance of a transformer to negative phase-sequence currents is the same as to the positive system. The impedance to the zero phase-sequence currents is the same as for the other sequences provided there is a through circuit for the earth currents and the compensating currents can flow, otherwise the impedance is infinite. Thus in an unearthed star/unearthed star connection the zero phase currents cannot flow and the zero phase impedance is infinite. In the unearthed star/earthed star connection, if an earth fault occurs on the primary side no zero sequence current can flow and the impedance is infinite; if the earth fault occurs on the secondary side there is a complete path for the zero sequence current, but there is no path for the compensating currents in the primary windings, and the zero phase impedance is again infinite. In the earthed star/delta connection, an earth fault in the

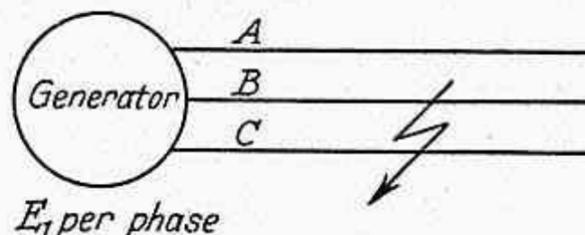


FIG. 185. APPLICATION OF SYMMETRICAL COMPONENTS

primary circuit has a complete path and the compensating currents can flow in the delta, so that the zero sequence impedance is finite; if the fault is in the secondary circuit, the zero phase impedance is infinite.

**General Method Using Symmetrical Components.** From the conditions of the fault we get three relations between the voltages  $V_A, V_B, V_C$  and the currents  $I_A, I_B, I_C$ . For instance, if there is an earth fault on line  $C$  only, we have  $I_A = 0, I_B = 0$ , and  $V_C = 0$ . If there is an earth fault on lines  $B$  and  $C$ , we have  $I_A = 0, V_B = 0$ , and  $V_C = 0$ . If the earth fault is across all three lines, we have  $V_A = 0, V_B = 0$ , and  $V_C = 0$ . If there is a short between two lines  $B$  and  $C$ , we have  $I_A = 0, I_B + I_C = 0$ , and  $V_B = V_C$ . We can then express the currents and voltages in terms of their symmetrical components. We know that the separate phase-sequence voltage drops are due to the corresponding phase-sequence currents. We thus have two three-phase balanced systems and one single-phase system (the earth return system), and we can apply the simplified method used in a balanced system, in which we replace the system by a single line and impedances to neutral.

As an example let us solve the case when there is a short between two lines, as shown in Fig. 185. The generator produces a positive phase-sequence e.m.f. only of  $E_1$  per phase; it has impedances  $Z_{g0}, Z_{g1}$ , and  $Z_{g2}$  to the sequence currents. The line has impedances

$Z_{L0}, Z_{L1}$ , and  $Z_{L2}$ . Let  $V_A, V_B, V_C$  be the voltages and  $I_A, I_B, I_C$  the currents near the fault. We have

$$I_A = 0, I_B + I_C = 0, \text{ and } V_B = V_C.$$

The phase-sequence currents are given by equations (106) as

$$\left. \begin{aligned} I_0 &= \frac{1}{3}(I_A + I_B + I_C) = 0, \\ I_1 &= \frac{1}{3}(I_A + \lambda I_B + \lambda^2 I_C) = \frac{1}{3}(\lambda - \lambda^2)I_B, \\ I_2 &= \frac{1}{3}(I_A + \lambda^2 I_B + \lambda I_C) = \frac{1}{3}(\lambda^2 - \lambda)I_B. \end{aligned} \right\} \quad (110)$$

The phase-sequence voltages are similarly given by

$$\left. \begin{aligned} V_0 &= \frac{1}{3}(V_A + V_B + V_C), \\ V_1 &= \frac{1}{3}(V_A + \lambda V_B + \lambda^2 V_C) = \frac{1}{3}(V_A + \lambda V_B + \lambda^2 V_B), \\ V_2 &= \frac{1}{3}(V_A + \lambda^2 V_B + \lambda V_C) = \frac{1}{3}(V_A + \lambda^2 V_B + \lambda V_B). \end{aligned} \right\} \quad (111)$$

The phase-sequence e.m.f.'s are  $E_1, 0, 0$ , so that applying equations (108) we get

$$\begin{aligned} V_0 &= 0 - (Z_{g0} + Z_{L0})I_0 = 0, \text{ since } I_0 = 0. \\ V_1 &= E_1 - (Z_{g1} + Z_{L1})I_1, \\ V_2 &= 0 - (Z_{g2} + Z_{L2})I_2. \end{aligned}$$

From equations (111) we see that  $V_1 = V_2$  so that

$$E_1 - (Z_{g1} + Z_{L1})I_1 = - (Z_{g2} + Z_{L2})I_2.$$

Substituting for  $I_1$  and  $I_2$  from equations (110) we get

$$\begin{aligned} E_1 &= (Z_{g1} + Z_{L1})I_1 - (Z_{g2} + Z_{L2})I_2 \\ &= (Z_{g1} + Z_{L1} + Z_{g2} + Z_{L2})\frac{1}{3}(\lambda - \lambda^2)I_B, \\ &= (Z_{g1} + Z_{L1} + Z_{g2} + Z_{L2})(j/\sqrt{3})I_B, \end{aligned}$$

so that the magnitude of  $I_B$  is

$$\frac{(\sqrt{3})E_1}{Z_{g1} + Z_{L1} + Z_{g2} + Z_{L2}}.$$

**Interference with Communication Circuits.** When a communication circuit runs parallel with a high voltage overhead line, high voltages may be induced in the former resulting in acoustic shock and noise. The induced voltages are due to electrostatic and electromagnetic induction, and are reduced considerably by *transposing* the power lines as shown in Fig. 186. See example 9 on page 236. The effect of transposition is to balance the capacitances of the lines, so that the electrostatically induced voltages balance out in the length of a complete set of transpositions; such a length is called a *barrel*. Transposition results also in a diminution of the electromagnetically induced e.m.f. on the wires, since the fluxes due to the positive and negative phase-sequence currents will add up to zero along the barrel. The flux due to the zero phase-sequence or longitudinal current is unaffected by transpositions of the power system, since it flows along the three wires in parallel. In order to

reduce the e.m.f. in the telephone circuit due to the zero phase-sequence current, which is called the *longitudinal current*, the telephone line is transposed, as shown in Fig. 186. By a proper co-ordination of transpositions of the power and communication lines, the induced voltages can be reduced to very small proportions under normal working conditions.

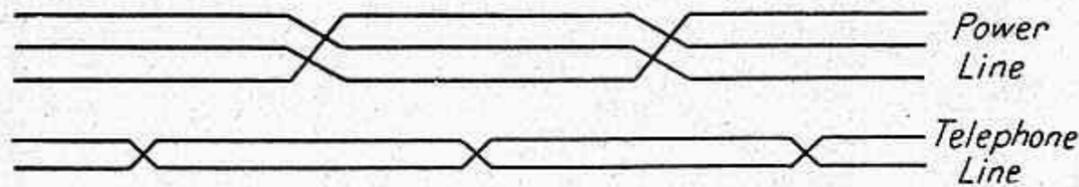


FIG. 186. CO-ORDINATED TRANSPOSITION OF POWER AND COMMUNICATION CIRCUITS

At the ends of a barrel the induced voltage is small and we have *silent points*. At points inside the barrel there may be a high voltage on the telephone line. If it is desired to tap the communication circuit at such a point, it is advisable to insert an isolating trans-

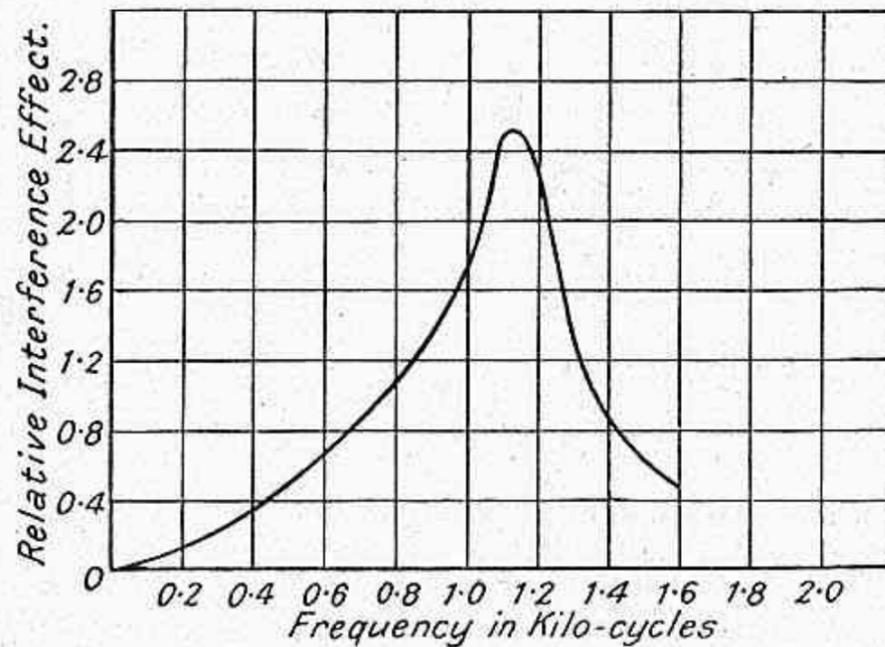


FIG. 187. TELEPHONE INTERFERENCE FACTOR CURVE (GII)

former, the insulation between the line and telephone windings being adequate to withstand the voltage; the telephone winding is earthed at the mid-point or at one end, so that a high voltage cannot reach the telephone.

The interference effect of an induced voltage or current depends greatly upon the frequency. The relative interference effect of different frequencies is shown in Fig. 187, which shows the *T.I.F.* curve, i.e. the telephone interference factor curve.

It is usual to express the interference currents in terms of a current at 800 cycles per sec. which produces the same degree of disturbance according to the curve of Fig. 187. Thus suppose that we have induced currents of  $20 \mu\text{A.}$  at 250 cycles and  $10 \mu\text{A.}$  at 350 cycles; the disturbances caused by them are the same as caused by 800 cycle currents of  $20 \times 0.25 = 5 \mu\text{A.}$  and  $10 \times 0.3 = 3 \mu\text{A.}$ , respectively.

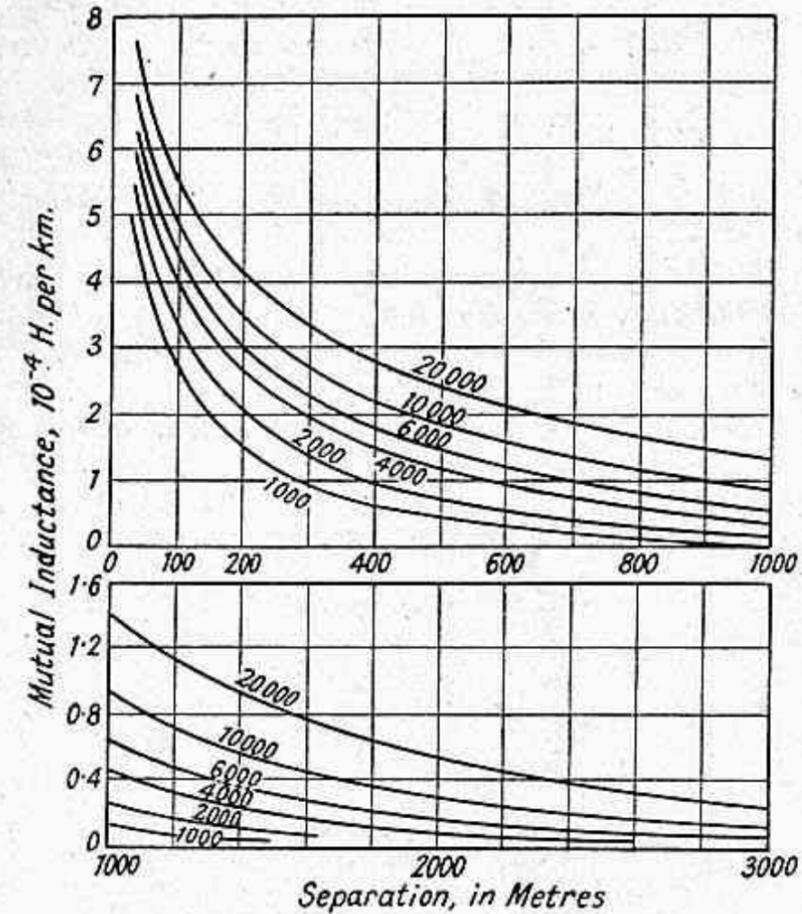


FIG. 188. MUTUAL INDUCTANCE BETWEEN A LINE AND EARTH RETURN AND ANOTHER LINE (CARSON-POLLACZEK FORMULA). FREQUENCY: 50 CYCLES  
Curves are for different resistivities of soil (in  $\Omega$ . per cm. cube).

The total disturbance is considered as due to an 800 cycle current of value  $\sqrt{(5^2 + 3^2)} = 5.8 \mu\text{A.}$

It is clear that the harmonics in the power system should be kept as low as possible, as they have a high interference factor.

When a short circuit occurs to earth, a large zero phase-sequence or longitudinal current flows along the wires in parallel and through the earth return. In this case the electromagnetic induction is large in magnitude, and depends upon the spacing between the power and telephone lines, the resistivity of the earth, and the frequency of the current. The e.m.f. induced in the telephone line is

$$E = -j\omega M I,$$

where  $I$  is the zero phase-sequence current,  $\omega = 2\pi \times$  frequency,  $l$  is the length of the parallel, and  $M$  the mutual inductance between the power line circuit (with earth return) and the telephone line. Fig. 188 gives the value of  $M$  for 1 km. parallel as calculated by

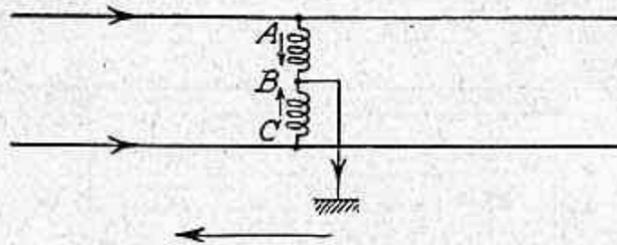


FIG. 189. DRAINAGE COIL

the Carson-Pollaczek theory for different spacings and earth resistivities, the frequency being 50 cycles per sec. It is noticed that  $M$  increases as the resistivity increases, because the current spreads out further in a soil of higher resistivity.

The e.m.f.  $E$  is induced in each of the telephone wires, so that if

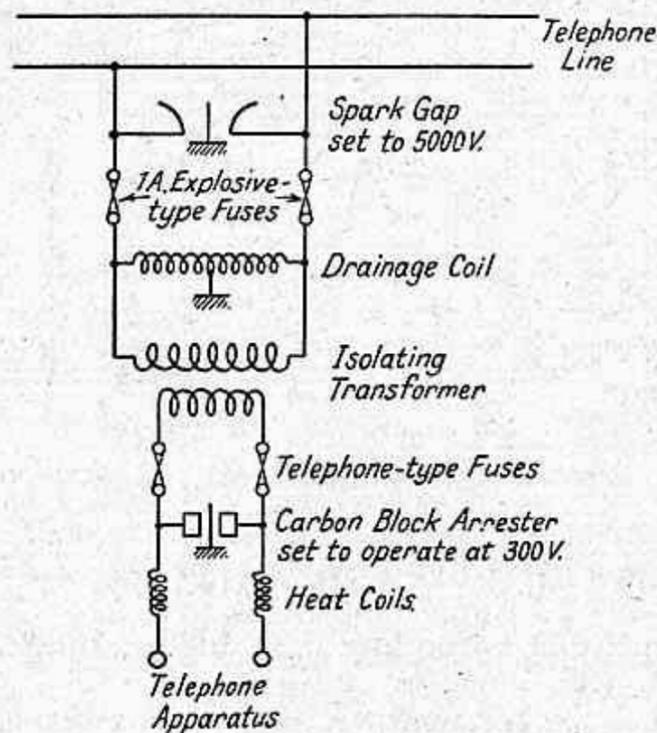


FIG. 190. TELEPHONE CIRCUIT PROTECTIVE SYSTEM

one telephone circuit were perfectly transposed and balanced there would be no voltage between the two wires of a circuit. There would, however, be the e.m.f.  $E$  between each wire and earth, and the telephone insulators must be capable of withstanding this voltage; for if one wire flashes over to earth, the full e.m.f. is applied to the telephone circuit with a resulting acoustic and electric shock. The

potential between the wires of the telephone circuit is kept low by the use of drainage coils, shown in Fig. 189. The windings  $AB$  and  $BC$  are series aiding, so that the coil has a high impedance between the terminals  $A$  and  $C$  (of the order  $6\,000 \sqrt{70^\circ}$  at 1 000 cycles), and gives an attenuation of about 2 db. to voice-frequency currents. The mid-point  $B$  is earthed, so that the impedance of the coil is very low to currents flowing along the lines in parallel and back through earth. In this way the longitudinal potential of the telephone lines is reduced from the high value  $E$  to a very low value, which depends upon the degree of balance of the windings and their resistance.

Acoustic and electric shock are minimized by the use of isolating transformers, drainage coils, spark gaps, and carbon block arresters. Fig. 190 shows an arrangement for the protection of a telephone circuit, as used in a successful installation which runs parallel with a 132 kV. power line in the Punjab.

## EXAMPLES VIII

1. Discuss the phenomena of electrostatic and electro-magnetic induction from power transmission lines to adjacent telephone lines.

State the factors upon which the magnitude of the induction depends in each case and the precautions taken in both the power and communication circuits to reduce it. (Lond. Univ., 1949.)

2. Why is automatic voltage regulation required for modern alternators?

Explain, with a diagram of connections, the operation of an automatic voltage regulator suitable for use with a large turbo-alternator. (Lond. Univ., 1950.)

3. A 3-phase overhead line has the following constants: series impedance per conductor,  $20 + j 50 \Omega$ ; shunt admittance of each conductor to neutral,  $0 + j 800$  micromhos. The line supplies, through a star/star transformer with a turn ratio of 4 to 1, a load of 40 MW. at 33 kV., unity power factor. The transformer, at 45 MVA. and 33 kV., has an impedance of  $0.5 + j 6$  per cent.

Neglecting iron loss and magnetizing current in the transformer, determine the voltage and current at the supply end of the line, and the phase angle between them. (Lond. Univ., 1954.)

4. Describe and compare the methods of interconnecting the bus-bars of a generating station.

Three generators  $A$ ,  $B$ , and  $C$ , each of 12% leakage reactance and of MVA. ratings 25, 50, and 25 respectively, are interconnected electrically by a tie-bar through reactors, each of 10% reactance based upon the MVA. rating of the machine to which it is connected. A 3-phase feeder is supplied from the bus-bars of generator  $A$  at a line voltage of 11 kV. The feeder has a resistance of  $0.1 \Omega$ /phase and an inductive reactance of  $0.3 \Omega$  phase. Estimate the maximum MVA. which can be fed into a symmetrical short-circuit at the far end of the feeder. (Lond. Univ., 1953.)

5. Explain what is meant by the symmetrical components of a 3-phase 4-wire system.

The p.d.'s to neutral of such a system are  $-36 + j 0$ ,  $0 + j 48$  and  $64 + j 0$  V. respectively, and the currents in the corresponding line wires  $R$ ,  $Y$ , and  $B$  are  $-4 + j 2$ ,  $-1 + j 5$ , and  $5 - j 3$  A. Determine the negative-sequence power and reactive voltamperes. The sequence is  $RYB$ .

(Lond. Univ., 1954.)