**Introduction**

Resistance, inductance and capacitance exist in all electrical circuits to a greater or lesser degree. Because inductance and capacitance affects the relationship between current and voltage in a.c. circuits, an additional set of mathematical tools are needed to analyse and interpret the operation of these circuits.

These tools include trigonometry, Pythagoras' Theorem and sinusoidal wave forms. You will also need to understand the purpose and operation of oscilloscopes which are electrical measurement instruments that can provide a visual display of the a.c. voltage and current relationships.

This topic will help you develop your understanding of trigonometry ratios, Pythagoras' Theorem, oscilloscope operation and a.c. sinusoidal wave forms.

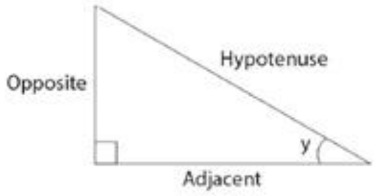
**Trigonometry Ratios**

Trigonometry can be used to analyse the relationships between voltage and current in a.c. circuits. Trigonometry ratios are based on the relationship between the length of the sides, and the angles, of right angle triangles.

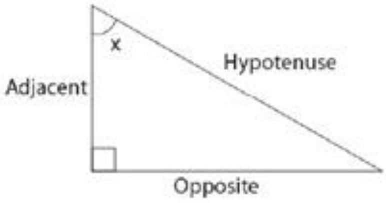
Points to note about right angle triangles:

- One angle is always 90°.
- The sum of the three angles is 180°.
- The side opposite to the right angle is called the 'hypotenuse'.
- The other two sides are called the 'opposite' or 'adjacent' side, depending on the angle of reference.

In the diagram below, angle 'y' is shown in relationship to the hypotenuse, the opposite side and the adjacent side.




In this diagram, the new reference angle is 'x'. Notice how the locations of the adjacent side and opposite side have reversed when compared to the diagram above. The hypotenuse is unchanged.



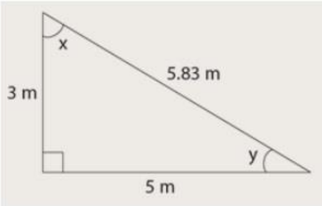

**Sine, Cosine and Tangent**

Sine, cosine and tangent represent ratios between the lengths of any two sides of a right angle triangle. These ratios can be used to find the angle (in degrees) between any two sides.

| Trigonometry Ratios | | | |
|---|---|---|---|
| **Ratio** | **Short Abbreviation** | **Equation** | **Mnemonic*** |
| Sine | Sin | Sin θ = Opposite/Hypotenuse | SOH |

**Worked Example – Calculating the sin, cos and tan ratios of an angle 1**

In relation to the right angle triangle shown below, calculate the sin, cos and tan ratios for angle y. Also calculate the angle of y in degrees, using the ratios.




sin - SOH

$$\sin y = \frac{Opp}{Hyp}$$

$$\sin y = \frac{3}{5.83}$$

$$\underline{\sin y = 0.52}$$

$\underline{y = \sin^{-1}(0.52) = 31.3°}$

*[* precise answer (no rounding at intermediate steps) y = 31.0°]*

cos – CAH

$$\cos y = \frac{Adj}{Hyp}$$

$$\cos y = \frac{5}{5.83}$$

$$\underline{\cos y = 0.86}$$

$\underline{y = \cos^{-1}(0.86) = 30.7°}$

*[* precise answer (no rounding at intermediate steps) y = 31.0°]*

-

tan – TOA

$$\tan y = \frac{Opp}{Adj}$$

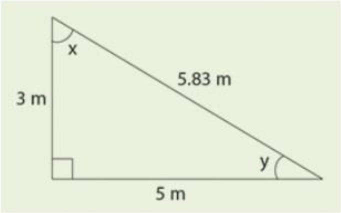
$$\tan y = \frac{3}{5}$$

$$\underline{\tan y = 0.6}$$

$\underline{y = \tan^{-1}(0.6) = 31.0°}$

**Worked Example – Calculating the sin, cos and tan ratios of an angle 2**

In relation to the right angle triangle shown below, calculate the sin, cos and tan ratios for angle x. Also calculate the angle of x in degrees, using the ratios.




sin - SOH

$$\sin x = \frac{Opp}{Hyp}$$

$$\sin x = \frac{5}{5.83}$$

$$\underline{\sin x = 0.86}$$

$\underline{x = \sin^{-1} (0.86) = 59.3°}$

*[* precise answer (no rounding at intermediate steps) y = 59.05°]*

cos – CAH

$$\cos x = \frac{Adj}{Hyp}$$

$$\cos x = \frac{3}{5.83}$$

$$\underline{\cos x = 0.52}$$

$\underline{x = \cos^{-1} (0.52) = 58.7°}$

*[* precise answer (no rounding at intermediate steps) y = 59.1°]*

-

tan – TOA

$$\tan x = \frac{Opp}{Adj}$$

$$\tan x = \frac{5}{3}$$

$$\underline{\tan x = 1.67}$$

$\underline{x = \tan^{-1} (1.67) = 59.1°}$

*[* precise answer (no rounding at intermediate steps) y = 59.0°]*

View

**Pythagoras' Theorem**

Pythagoras's Theorem is another tool that is used to analyse the relationships between voltage and current in an a.c. circuit.

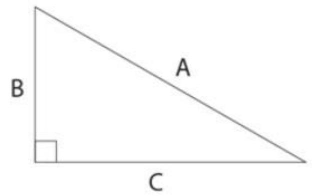
Pythagoras' Theorem states:

*'In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.'*

Pythagoras Theorem is expressed as:

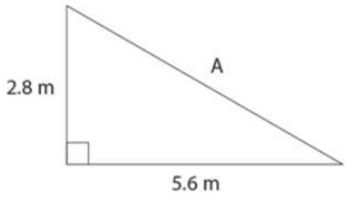
$$A^2 = B^2 + C^2$$

In this diagram, side A is the hypotenuse, and sides B and C are the other two sides.




**Worked Example – Pythagoras' Theorem 1**

In relation to the right angle triangle shown below, calculate the value of the unknown side, A.

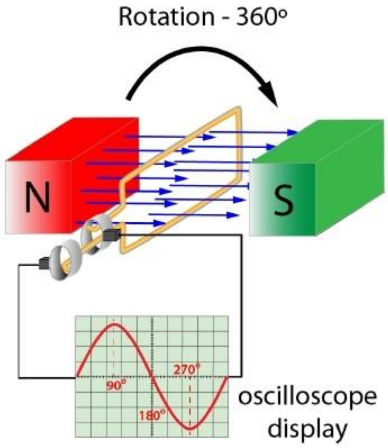



side A = $\sqrt{(2.8^2 + 5.6^2)}$

side A = $\sqrt{(7.84 + 31.4)}$

side A = $\sqrt{39.2}$

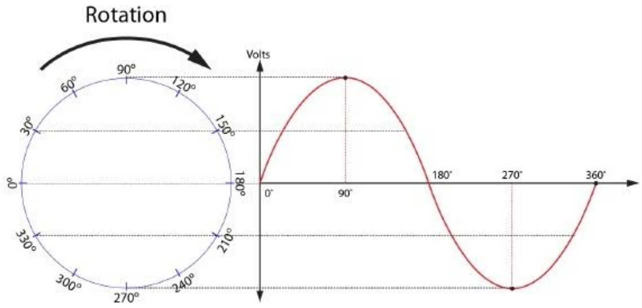
<u>side A = 6.26 m</u>




**a.c. Sinusoidal Waveform**

The shape of the waveform shown on the oscilloscope above is called an 'a.c. sinusoidal waveform', or sine wave for short. The advantage of a sine wave is that it is the only a.c. waveform that can be transformed and keep its shape.

The sine wave is the fundamental wave shape for the voltage and current in the electricity supply and distribution network.




The sine wave shape is based on the sine function, and the voltage at any point on the wave can be calculated using this equation.

$$e = V_{max} \sin \theta$$

Where:

- e = instantaneous induced e.m.f. at some angle of rotation (volts)
- $V_{max}$ = maximum induced e.m.f. (volts)
- θ = angle at which the conductor cuts the magnetic field (Greek letter theta)

e = -322 volts

*[* precise answer (no rounding at intermediate steps) e = -320 volts]*

**Sine Wave Terms**

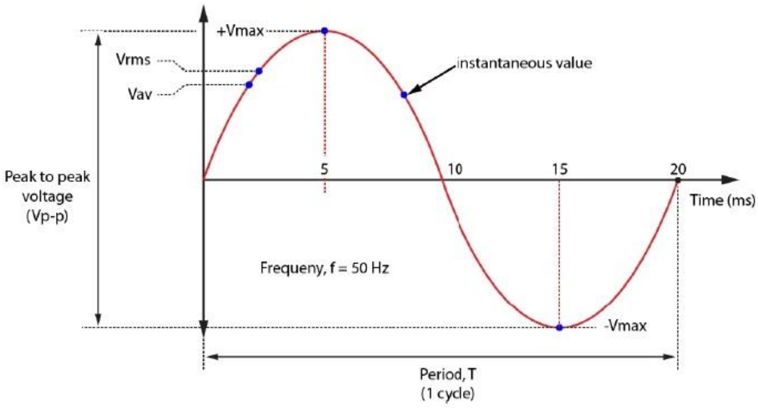
As an aid to analysing the operation of a.c. circuits and to enable the voltage and current values to be calculated, some key terms are used to identify important points on the sine wave.

The terms include:

- Period
- Frequency
- Maximum value
- Peak-to-peak value
- Instantaneous value
- Average value
- Root-mean-square (r.m.s.) value

These terms are identified on the sine wave below. Note that the x-axis is now labelled 'Time', and the units are milliseconds (ms). The maximum values are labelled +Vmax and –Vmax. If the sine wave represents current, then the maximum values will be labelled +Imax and –Imax.

This sine wave has a frequency of 50 Hz, which is the standard supply frequency through Australia.




| Sine Wave Terms | | |
|---|---|---|
| Period, T | Time to complete one cycle | T = 1/f seconds (s) |
| Frequency, f | The number of cycles per second | f = 1/T hertz (Hz) |
| Maximum (peak) value | Highest value of the waveform in one direction | +Vmax<br>-Vmax |
| Peak-to-peak value ($V_{p-p}$) | Value from +Vmax to –Vmax | 2 x Vmax |
| Instantaneous value | Value at any instantaneous point on the sine wave | e = Vmax sin θ |
| Average value ($V_{av}$) | Average value of all instantaneous values for **one** half cycle | $V_{av}$ = Vmax(0.637) |
| Root-mean-square (r.m.s.) value ($V_{rms}$) | Known as the 'effective' value of a.c.<br>The r.m.s. value of a.c. produces the same heating effect as the equivalent value of d.c. | $V_{rms}$ = Vmax(0.707) |

### Introduction

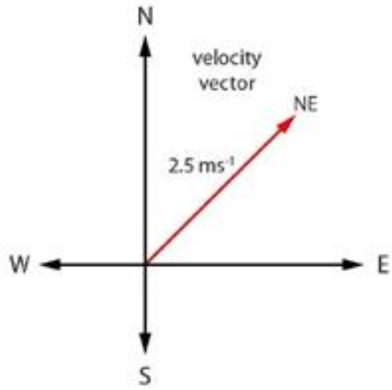
Phasor diagrams are a tool which can help you understand and measure the relationship between voltages and currents in a.c. circuits. 'Phasors' are straight lines drawn to scale to show the r.m.s. values of voltages and/or currents, and the angle of 'lead' or 'lag' between a.c. quantities.

This topic will help you develop your understanding of the terms 'vector' and 'phasor', the purpose of phasor diagrams and the conventions used to represent a.c. quantities in phasor diagrams.

### Vectors

In mathematics, a 'vector' is a physical quantity that has both a magnitude and a direction in space. Vectors are static, i.e. they do not rotate. Two vectors of the same type, e.g. two velocity vectors, can be added together using 'vector addition'.

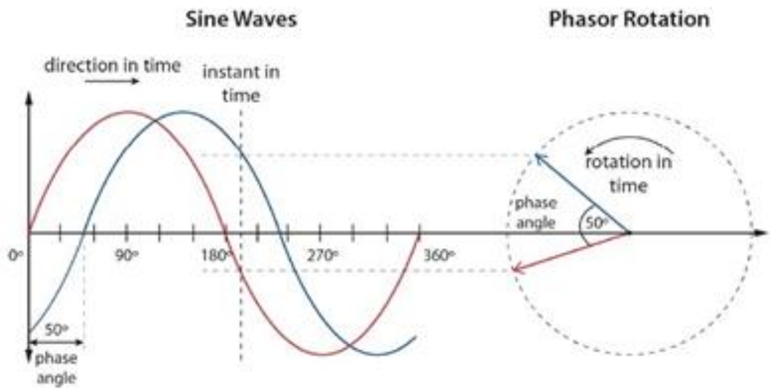
This diagram shows a velocity vector, with a magnitude of 2.5 $ms^{-1}$ and a direction of north east (NE).




### Phasors

A 'phasor' is a rotating vector that can represent the instantaneous magnitude and phase relationships of an a.c. sinusoidal quantity. Phasors are represented by straight lines drawn to scale on a 'phasor diagram'.

In this diagram two sine waves, with the same maximum value, are shown to be 'out-of-phase' by a constant 'phase angle' of 50°. On the right side of the diagram, two phasors are drawn representing the magnitude of the two sine waves at one instant in time. Notice that the phase angle between the two sine waves is exactly the same as the phase angle between the two phasors. By definition, phasors rotate in an anti-clockwise direction.



**Phasor Diagrams**

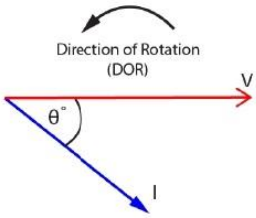
Phasor diagrams show the phase relationship between voltages and/or currents in a.c. circuits. Phasors usually represent the r.m.s. values of a sine wave and are drawn with respect to a horizontal reference phasor.

The voltage rating displayed on electrical equipment rating plates is the r.m.s. value of the equipment's rated operating voltage.

This diagram shows two phasors, a voltage phasor V, and a current phasor I, separated by angle θ°. In this diagram, V is the reference phasor and is drawn horizontally to the right. Because phasors rotate in an anti-clockwise direction, I 'lags' V by angle θ°.

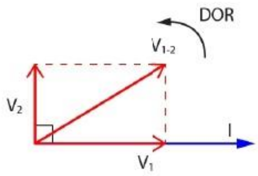
Voltage phasors are drawn with an open arrow head. Current phasors are drawn with a closed arrow head.

Phasor Diagram




Two phasors of the same type, i.e. two voltage phasors or two current phasors, can be added together using 'phasor addition'.

This diagram shows two voltage phasors $V_1$ and $V_2$ out-of-phase by 90° being added together using phasor addition.




Check your understanding of the content by clicking on the link below then undertaking the activity.

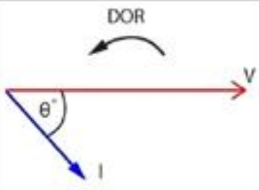
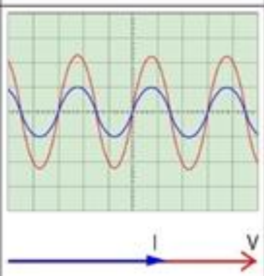
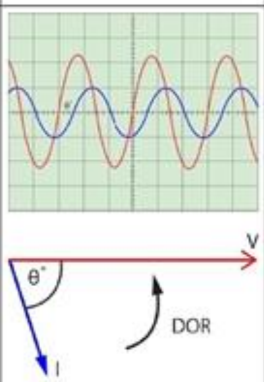
**Load the Activity**

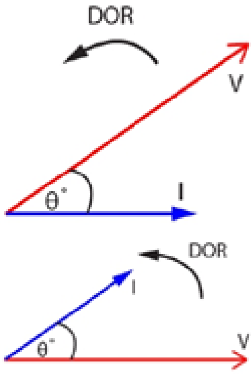
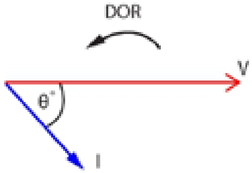
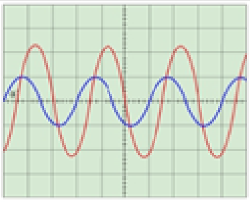
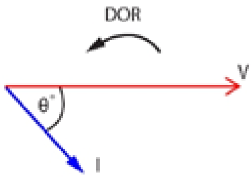
**Phasor Relationships**

As stated in Topic 1, every circuit contains some value of resistance, inductance and capacitance which can affect the phasor relationship between the voltages and currents in an a.c. circuit.

In any a.c. circuit the voltages and currents can be 'in-phase' or 'out-of-phase'. If the voltage and current are out-of-phase, the voltage can either 'lead' or 'lag' the current. The angle of lead or lag is called the 'phase angle'. If the voltage and current are in-phase, then the phase angle is zero.

These and other terms used to describe phasor relationships in a.c. circuits are defined in the following table.

| Phase Relationships | | |
|---|---|---|
| Direction of Rotation (DOR) |  | • Phasor direction of rotation (DOR) is anti-clockwise. |
| Voltage Phasor |  | • Voltage phasors are drawn with an open arrow head.<br>• They usually represent r.m.s values of voltage. |
| Current Phasor |  | • Current phasors are drawn with a closed arrow head.<br>• They usually represent r.m.s. values of current. |
| 'In-phase' |  | Two waveforms are in-phase, when:<br>• The frequency, (f) of each waveform is the same.<br>• The maximum values of each waveform occur at the same time.<br>• The zero values of each waveform occur at the same time.<br>Two phasors are in-phase, when:<br>• The phasors are drawn parallel with each other, and pointing in the same direction.<br>• In this diagram I is in-phase with V. |
| 'Out-of-phase' |  | Two waveforms are out-of-phase, when:<br>• The maximum values and the zero values of each waveform do not occur at the same time.<br>Two phasors are out-of-phase, when:<br>• The phasors are not drawn parallel and are separated by some angle θ°.<br>• In this diagram I is out-of-phase with V by angle θ°. |

| | | |
|---|---|---|
| Reference Phasor |   | The reference phasor is drawn horizontally.<br>Series Circuit:<br>• The current phasor is the reference phasor in a *series* a.c. circuit, because current is the same value in all parts of a series circuit.<br>Parallel Circuit:<br>• The voltage phasor is the reference phasor in a *parallel* a.c. circuit, because the voltage across all components is the same in a parallel circuit. |
| Phase Angle |   | • The phase angle, θ°, is the phase difference between two sine waves or two phasors.<br>• In this diagram the phase angle between V and I is θ°. |
| Lead |  | • One sine wave leads another when its cycle begins first.<br>• In this diagram, the blue sine wave starts rising from zero towards its positive peak before the red sine wave.<br>• One phasor leads when it is in front of another phasor in the direction of rotation.<br>• In this diagram the current I is leading the reference voltage V by the angle θ°. |
| Lag |   | • One sine wave lags another when its cycle starts after the other.<br>• In this diagram, the blue sine wave starts rising from zero towards its positive peak after the red sine wave.<br>• One phasor lags when it is behind another phasor in the direction of rotation.<br>• In this diagram the current I is lagging the reference voltage V by the angle θ°. |

**Phasor Addition**

Sinusoidal values of voltage or current can only be added algebraically if they are in-phase.

Where voltages and current are out-of-phase, they must be added together using 'phasor addition'.

When two phasors are out-of-phase by 90° they can be added together using Pythagoras' Theorem and trigonometry.

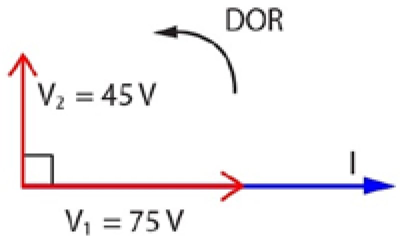
When two phasors are out-of-phase by some other angle that is not 90°, they can be added together using the parallelogram method or the 'tip to tail' method.

The following worked examples demonstrate how two phasors that are out-of-phase can be added together using phasor addition.

**Worked Example – Phasor Addition 1 – Trigonometry/Pythagoras' Theorem**

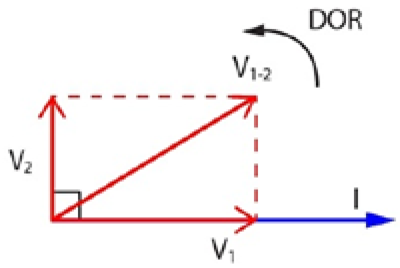
This phasor diagram shows the voltages measured across two components connected in series in an a.c. circuit. Current I is the reference phasor. $V_1$ is 75 V and in-phase with I. $V_2$ is 45 V, leading I by 90°. Use phasor addition to calculate the circuit supply voltage.

1. Draw both voltage phasors to scale, showing their phase angle with respect to the reference current, I and show their direction of rotation

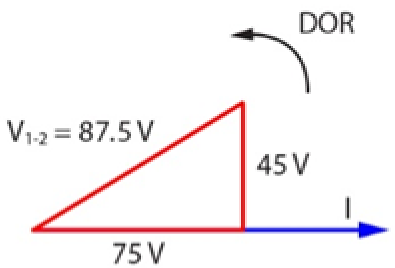



2. Construct a parallelogram from the two voltage phasors

3. Draw the resultant voltage phasor, $V_{1\text{-}2}$




4. Use Pythagoras' Theorem to determine the length of voltage phasor, $V_{1\text{-}2}$




$$V_{1\text{-}2} = \sqrt{(75^2 + 45^2)}$$

$$V_{1\text{-}2} = \sqrt{(7{,}650)}$$

$$\underline{V_{1\text{-}2} = 87.5\text{ V}}$$

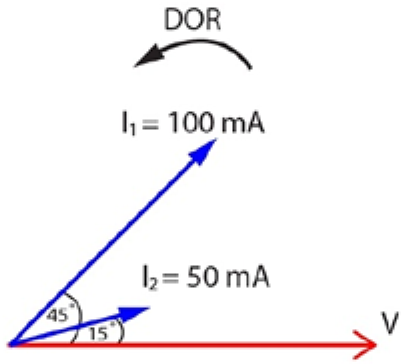
5. Use trigonometry to calculate the phase angle, θ°

**Worked Example – Phasor Addition 2 – parallelogram method**

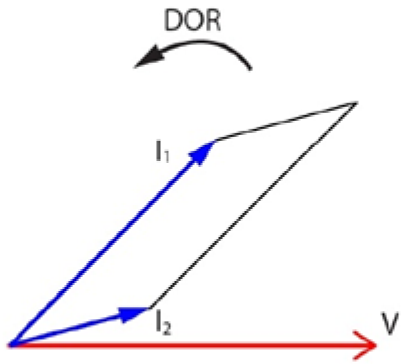
The currents in two branches of a parallel a.c. circuit are out of phase with the supply voltage V, which is the reference phasor.

$I_1$ is 100 mA leading V by 45°. $I_2$ is 50 mA leading V by 15°. Calculate the total supply current.

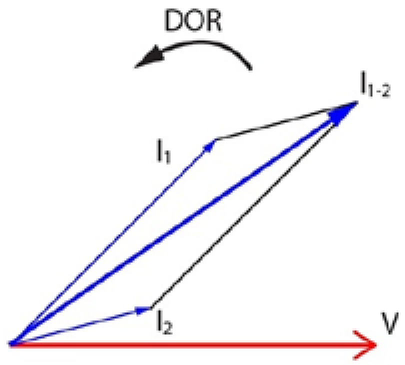
1. Draw both current phasors to scale, showing their phase angle with respect to the reference voltage, V and show their direction of rotation.



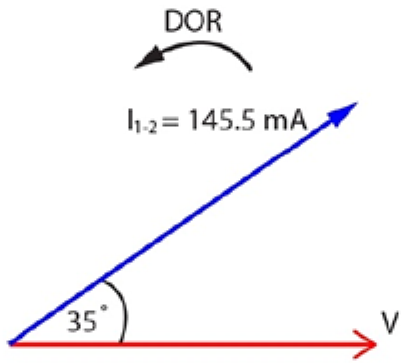

2. Construct a parallelogram from the two current phasors.




3. Draw the resultant current phasor, $I_{1\text{-}2}$.




4. Measure the length and phase angle of the resultant current phasor, $I_{1\text{-}2}$.



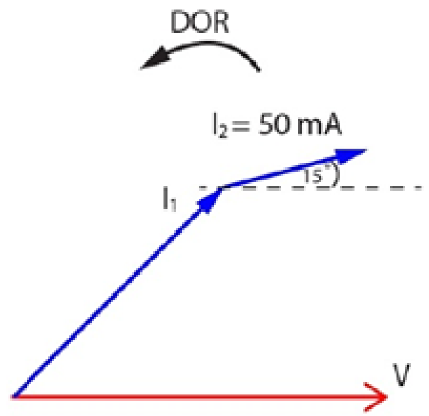
**Worked Example – Phasor addition 3 – tip to tail method**

This worked example demonstrates phasor addition using the tip to tail method. The current values and phase angles are the same as the previous worked example.

$I_1$ is 100 mA leading V by 45°. $I_2$ is 50 mA leading V by 15°. Calculate the total supply current.

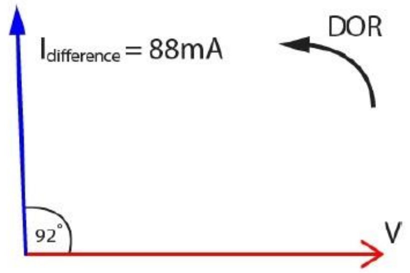
1. Draw both current phasors to scale, showing their phase angle with respect to the reference voltage, V and show their direction of rotation.

2. Move phasor $I_2$ so that its tail is joined to the tip of phasor $I_1$. It is important to maintain the length and phase angle of phasor $I_2$.




3. Draw the resultant current phasor for $I_{1\text{-}2}$ from the tail of $I_1$ to the tip of $I_2$.

4. Measure the length and phase angle of the resultant current phasor, $I_{1\text{-}2}$.



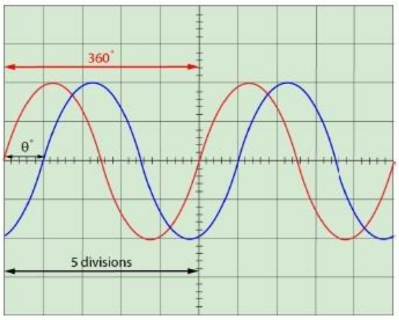

**Measuring Phase Angle**

To determine the phase angle between two sine waves displayed on an oscilloscope, measure the phase angle between the waveforms.

The following worked example demonstrates how to measure the phase angle between two sine waves displayed on an oscilloscope.

**Worked Example – Measure Phase Angle**

This diagram shows two sine waves displayed on an oscilloscope. The blue sine wave is lagging the red sine by angle θ°. Determine the phase angle between the two waveforms.




1. One cycle = 360°

2. 360° = 5 divisions on the display

3. θ = 1 division

4. θ = 360 / 5 = 72°

5. The blue sine wave is lagging the red sine wave by 72°

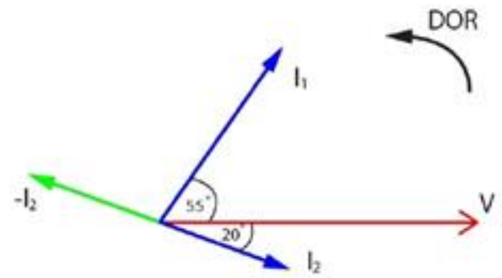
**Worked Example – Phasor subtraction 1 – parallelogram method**

The current in two branches of a parallel a.c. circuit are out of phase with the supply voltage, V. Current $I_1$, is 85 mA leading the supply voltage by 55°. Current $I_2$ is 55 mA lagging the supply voltage by 20°. Use phasor subtraction to calculate the difference between the two current phasors.

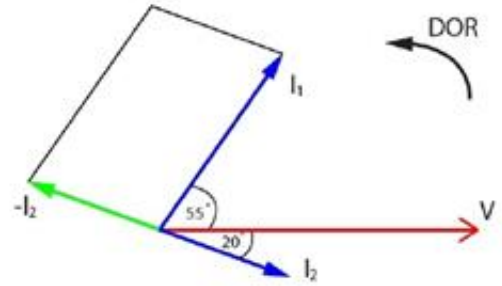
In this example $I_2$ will be subtracted from $I_1$.

1. Draw both current phasors to scale, showing their phase angle with respect to the reference voltage, V and show their direction of rotation.

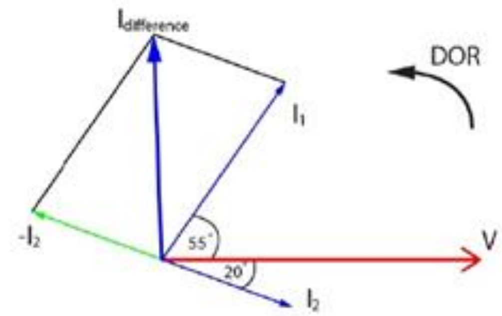
2. To subtract $I_2$ from $I_1$, draw a new phasor 180° out-of-phase with $I_2$. Label this phasor $-I_2$.




3. Construct a parallelogram from phasors $I_1$ and $-I_2$.




4. Draw the resultant current phasor, $I_{difference}$.




5. Measure the length and phase angle of the resultant current phasor, $I_{difference}$.

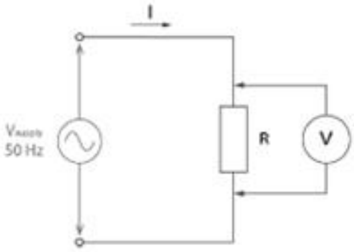
## Introduction

Many a.c. circuits contain only resistive elements, such as lamp filaments and heating elements. Most practical circuits, however, contain a combination of resistive, inductive and capacitive components. To better understand the voltage and current relationships in complex practical a.c. circuits, it is helpful to first analyse and test basic single element circuits before studying more complex circuits that combine these elements.

This topic will help you develop your understanding of single element a.c. circuits, containing resistive, inductive or capacitive components only.

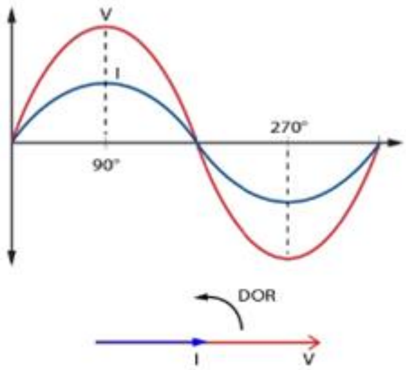
## Resistive a.c. Circuits

In a purely resistive a.c. circuit the current and voltage are *in-phase*. Resistance provides the only opposition to current and Ohm's law can be used to calculate the current, voltage and resistance in the circuit.




This diagram shows the current and voltage sine waves, and the current and voltage phasors for a purely resistive a.c. circuit.

The current and voltage are in-phase in a purely resistive circuit.




This diagram shows two resistors connected in series to an a.c. supply. Both voltages $V_1$ and $V_2$ are in-phase with the supply voltage and the current.

The supply voltage, $V_{supply} = V_1 + V_2$.

The total resistance, $R_T = R_1 + R_2$.

The current $I = V_{supply} / R_T$.

**Worked Example – Purely Resistive a.c. Series Circuit**

For the resistive a.c. series circuit pictured below, determine:

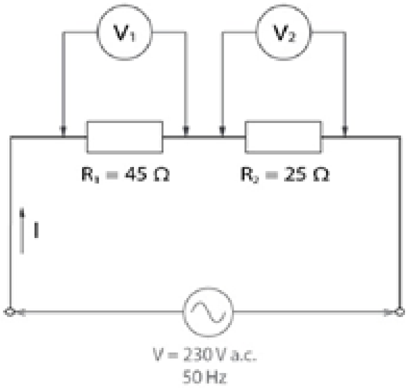
a) *The total resistance, $R_T$*

b) *The supply current, I*

c) *The voltage drop across $R_1$*

d) *The voltage drop across $R_2$*

e) *Draw the phasor diagram for the circuit*




a) Total resistance, $R_T$

$$R_T = R_1 + R_2$$

$$R_T = 45 + 25$$

$$\underline{R_T = 70\ \Omega}$$

b) Supply current, I

$$I = \frac{V}{R_T}$$

$$I = \frac{230}{70}$$

$$\underline{I = 3.29\ A}$$

c) Voltage, $V_1$

$$V_1 = I \times R_1$$

$$V_1 = 3.29 \times 45$$

$$\underline{V_1 = 148.1\ V}$$

d) Voltage, $V_2$

$$V_2 = I \times R_2$$

$$V_2 = 3.29 \times 25$$

$$\underline{V_2 = 82.3\ V}$$

e) Phasor diagram (the current is the reference phasor – all voltages and the current are in-phase).

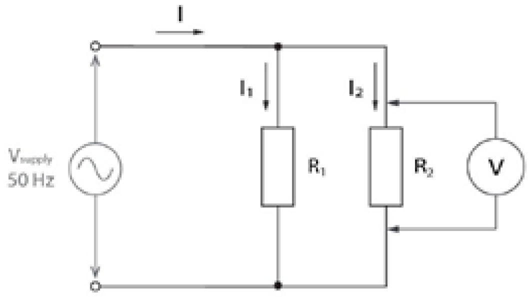

This diagram shows two resistors connected in parallel to an a.c. supply. The voltage drops across resistors $R_1$ and $R_2$ is equal to, and in-phase with the supply voltage. The current I is in-phase with supply voltage, and the currents through each resistor are also in-phase with the supply voltage.

Current $I_1 = V_{supply} / R_1$

Current $I_2 = V_{supply} / R_2$

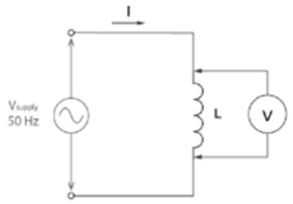
Total current, $I = I_1 + I_2$

Total resistance, $R_T = V_{supply} / I_{total}$



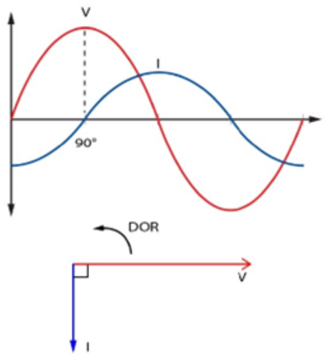
**Inductive a.c. Circuits**

In a purely inductive a.c. circuit the current *lags* the voltage by 90°. The angle of lag is created by inductance which opposes the change in current.




This diagram shows the current and voltage sine waves, and the current and voltage phasors for a purely inductive a.c. circuit.

The current lags the voltage by 90° in a purely inductive circuit.




**Inductive Reactance**

'Inductive reactance' is the property of an inductor that opposes the change in current in an a.c. circuit. The unit is ohms and Ohm's law is used to calculate the voltage and/or current in the circuit.

Inductive reactance is calculated using this equation.

$$X_L = 2\pi fL$$

Where:

- $X_L$ = inductive reactance in ohms (Ω)
- f = frequency of the supply in hertz (Hz)
- L = inductance in henrys (H)
- π = pi (3.14159....)

Points to note about inductive reactance:

- Inductive reactance is the opposition to the change in current in an a.c. circuit.
- Inductive reactance is *directly* proportional to the frequency of the supply, i.e. as the frequency increase the inductive reactance increases, and vice versa.
- Inductive reactance is *directly* proportional to inductance, i.e. as the inductance increases the inductive reactance increases, and vice versa.

The current in an inductive a.c. circuit can be calculated using Ohm's law.

$$I = \frac{V}{X_L}$$

Where:

- I = circuit current in amperes (A)
- V = supply voltage (V)
- $X_L$ = inductive reactance in ohms (Ω)

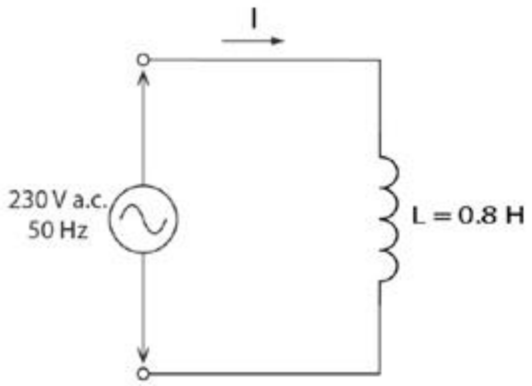
**Worked Example – Inductive Reactance**

For the inductive a.c. circuit pictured below, determine:

a) The inductive reactance, $X_L$

b) The supply current, I

c) Draw the phasor diagram for the circuit




a) Inductive reactance, $X_L$

$$X_L = 2\pi f L$$

$$X_L = 2 \times \pi \times 50 \times 0.8$$

$$\underline{X_L = 251\ \Omega}$$

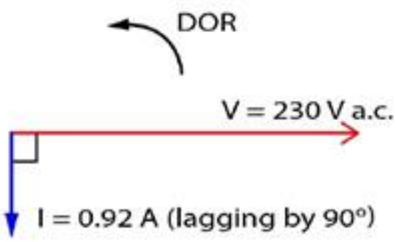
b) Supply current, I

$$I = \frac{V}{X_L}$$

$$I = \frac{230}{251}$$

$$\underline{I = 0.916\ A}$$

c) Phasor diagram (the voltage is the reference phasor – the current lags the voltage by 90°)

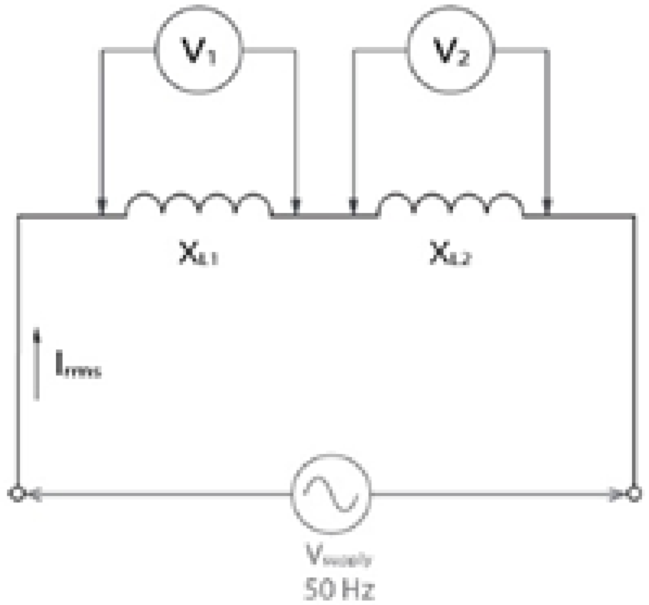



This diagram shows two inductors connected in series to an a.c. supply. Both voltages $V_1$ and $V_2$ are in-phase with the supply voltage and the current lags the supply voltage by 90°.

The supply voltage, $V_{supply} = V_1 + V_2$.

The total inductive reactance, $X_{LT} = X_{L1} + X_{L2}$.

The current $I = V_{supply} / X_{LT}$.


V1
V2
XL1
XL2
Irms
Vsupply
50 Hz

**Worked Example – Purely Inductive a.c. Series Circuit**

For the inductive a.c. series circuit pictured below, determine:

a) The inductive reactance of $L_1$

b) The inductive reactance of $L_2$

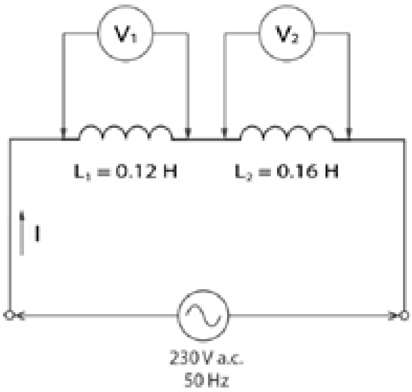
c) The total inductive reactance, $X_{LT}$

d) The supply current, I

e) The voltage drop across $X_{L1}$

f) The voltage drop across $X_{L2}$

g) Draw the phasor diagram for the circuit




a) Inductive reactance, $X_{L1}$

$$X_{L1} = 2\pi f L_1$$

$$X_{L1} = 2 \times \pi \times 50 \times 0.12$$

$$\underline{X_{L1} = 37.7\ \Omega}$$

b) Inductive reactance, $X_{L2}$

$$X_{L2} = 2\pi f L_2$$

$$X_{L2} = 2 \times \pi \times 50 \times 0.16$$

$$\underline{X_{L2} = 50.3\ \Omega}$$

c) Total inductive reactance, $X_{LT}$

$$X_{LT} = X_{L1} + X_{L2}$$

$$X_{LT} = 37.7 + 50.3$$

$$\underline{X_{LT} = 88.0\ \Omega}$$

d) Supply current, I

$$I = \frac{V}{X_{LT}}$$

$$I = \frac{230}{88}$$

$$\underline{I = 2.61\ A}$$

e) Voltage, $V_1$

$$V_1 = I \times X_{L1}$$

$$V_1 = 2.61 \times 37.7$$

$$V_1 = 98.4\ V$$

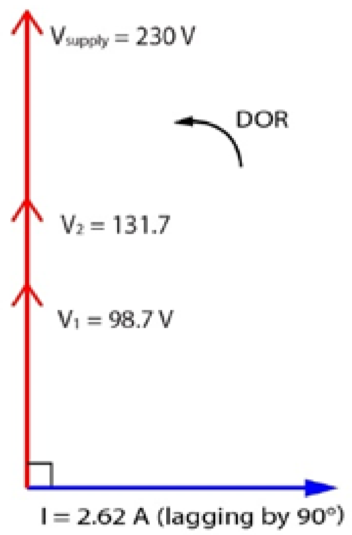
f) Voltage, $V_2$

$$V_2 = I \times X_{L2}$$

$$V_2 = 2.61 \times 50.3$$

$$V_2 = 131\ V$$

g) Phasor diagram (the circuit current is the reference phasor in a series circuit - the current 'lags' the voltages by 90°)



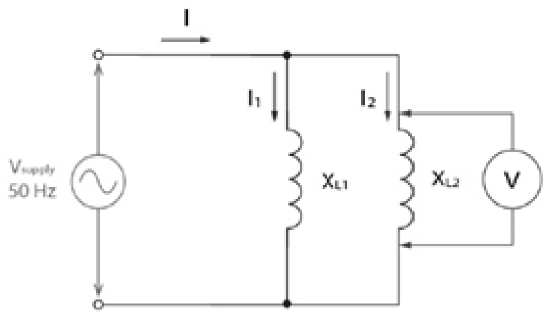

This diagram shows two inductors connected in parallel to an a.c. supply. The voltage drops across each inductor is equal to, and in-phase with the supply voltage. The current I, and the current through each inductor lags the supply voltage by 90°

Current $I_1 = V_{supply} / X_{L1}$

Current $I_2 = V_{supply} / X_{L2}$

Total current, $I = I_1 + I_2$

Total inductive reactance, $X_{LT} = V_{supply} / I$



:

**Worked Example – Purely Inductive a.c. Parallel Circuit**

For the inductive a.c. parallel circuit pictured below, determine:

a) The inductive reactance of $L_1$

b) The inductive reactance of $L_2$

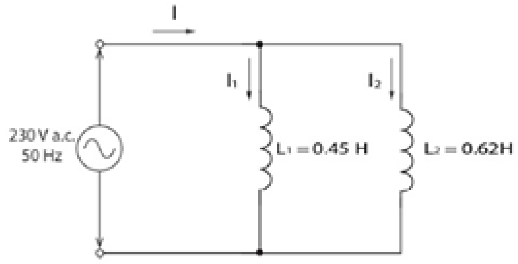
c) The current through $L_1$

d) The current through $L_2$

e) The supply current, I

f) The total inductive reactance, $X_{LT}$

g) Draw the phasor diagram for the circuit




a) Inductive reactance, $X_{L1}$

$$X_{L1} = 2\pi f L_1$$

$$X_{L1} = 2 \times \pi \times 50 \times 0.45$$

$$\underline{X_{L1} = 141\ \Omega}$$

b) Inductive reactance, $X_{L2}$

$$X_{L2} = 2\pi f L_2$$

$$X_{L2} = 2 \times \pi \times 50 \times 0.62$$

$$\underline{X_{L2} = 195\ \Omega}$$

c) Current through $L_1$

$$I_1 = \frac{V}{X_{L1}}$$

$$I_1 = \frac{230}{141}$$

$$\underline{I_1 = 1.63\ A}$$

d) Current through $L_2$

$$I_2 = \frac{V}{X_{L2}}$$

$$I_2 = \frac{230}{195}$$

$$\underline{I_2 = 1.18\ A}$$

e) Supply current, I

$$I = I_1 + I_2$$

$$I = 1.63 + 1.18$$

$$\underline{I = 2.81\ A}$$

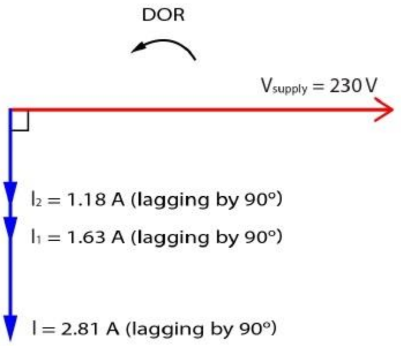
f) Total inductive reactance, $X_{LT}$

$$X_{LT} = \frac{V}{I}$$

$$X_{LT} = \frac{230}{2.81}$$

$$X_{LT} = 81.9\ \Omega$$

g) Phasor diagram (the supply voltage is the reference phasor in a parallel circuit - the currents 'lag' the voltage by 90°)




**Applications for Inductive a.c. Circuits**

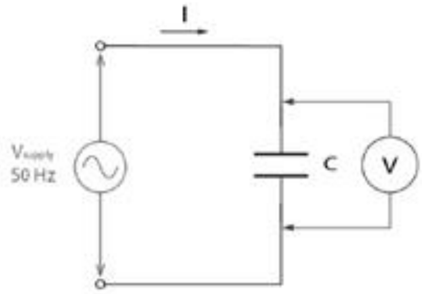
Inductive a.c. loads are common in many a.c. circuits. Some applications include:

- Fluorescent lamp ballast
- Transformer
- Electric motor
- Electromechanical relay
- Choke coil
- Line Reactor
- Inductive proximity sensor

Inductors are often used to control a.c. currents because they have a low power loss.

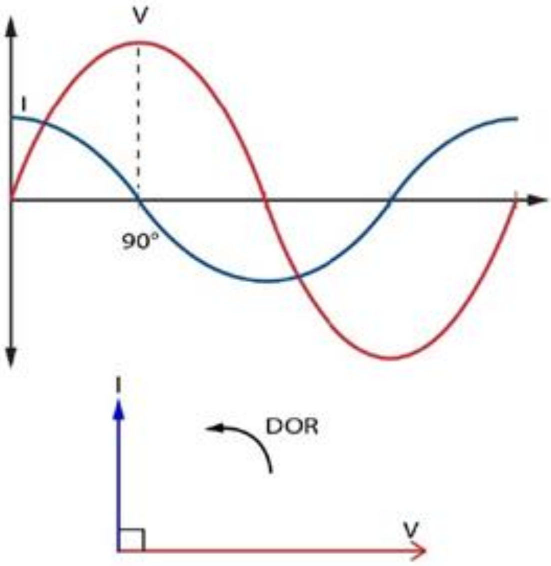
**Capacitive a.c. Circuits**

In a purely capacitive a.c. circuit the current *leads* the voltage by 90°. The angle of lead is created by the capacitance which opposes the change in voltage.




This diagram shows the current and voltage sine waves, and the current and voltage phasors for a purely capacitive a.c. circuit.

The current leads the voltage by 90° in a purely capacitive circuit.




**Capacitive Reactance**

'Capacitive reactance' is the property of an capacitor that opposes the change in voltage in an a.c. circuit. The unit is ohms and Ohm's law is used to calculate the voltage and/or current in the circuit.

Capacitive reactance is *indirectly* proportional to:

- The capacitance C, in farads (F).
- The frequency of the supply f, in hertz (Hz).

Capacitive reactance is calculated using this equation.

$$X_C = \frac{1}{2\pi fC}$$

Where:

- $X_C$ = capacitive reactance in ohms (Ω)
- f = frequency of the supply in hertz (Hz)
- C = capacitance in farads (F)
- π = pi (3.14159....)

Points to note about capacitive reactance:

- Capacitive reactance is the opposition to the change in voltage in an a.c. circuit.
- Capacitive reactance is *indirectly* proportional to the frequency of the supply, i.e. as the frequency *increases* the capacitive reactance *decreases*, and vice versa.
- Capacitive reactance is *indirectly* proportional to capacitance, i.e. as the capacitance *increases* the capacitive reactance *decreases*, and vice versa.

The current in a capacitive a.c. circuit can be calculated using Ohm's law.

$$I = \frac{V}{X_C}$$

Where:

- I = circuit current in amperes (A)
- V = supply voltage (V)
- $X_C$ = capacitive reactance in ohms (Ω)

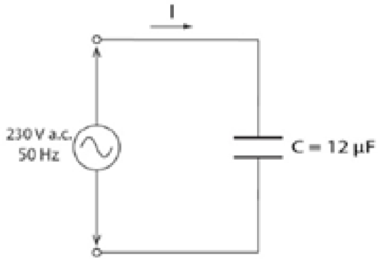
**Worked Example – Capacitive Reactance**

*For the capacitive a.c. circuit pictured below, determine:*

*a) The capacitive reactance, $X_C$*

*b) The supply current, I*

*c) Draw the phasor diagram for the circuit*




a) Capacitive reactance, $X_C$

$$X_c = \frac{1}{2\pi f C}$$

$$X_c = \frac{1}{2 \times \pi \times 50 \times 12 \times 10^{-6}}$$

$$X_c = \frac{1}{3.77 \times 10^{-3}}$$

$$\underline{X_c = 265\,\Omega}$$

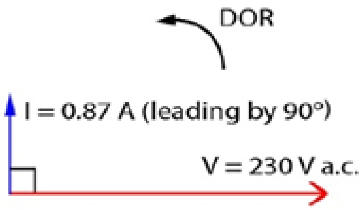
b) Supply current, I

$$I = \frac{V}{X_c}$$

$$I = \frac{230}{265}$$

$$\underline{I = 0.868\,A}$$

c) Phasor diagram (the voltage is the reference phasor – the current leads the voltage by 90°)

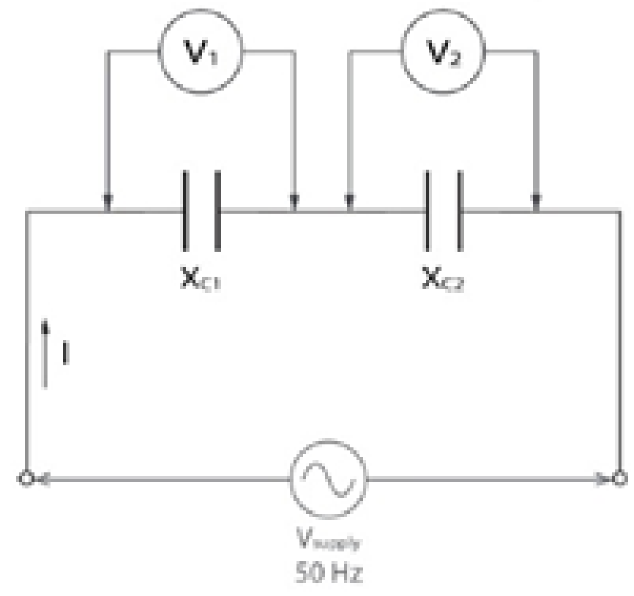



This diagram shows two capacitors connected in series to an a.c. supply. Both voltages $V_1$ and $V_2$ are in-phase with the supply voltage and the current leads the supply voltage by 90°.

The supply voltage, $V_{supply} = V_1 + V_2$.

The total capacitive reactance, $X_{CT} = X_{C1} + X_{C2}$.

The current $I = V_{supply} / X_{CT}$.


V1
V2
XC1
XC2
I
Vsupply
50 Hz

**Worked Example – Purely Capacitive a.c. Series Circuit**

For the capacitive a.c. series circuit pictured below, determine:

*a) The capacitive reactance of $C_1$*

*b) The capacitive reactance of $C_2$*

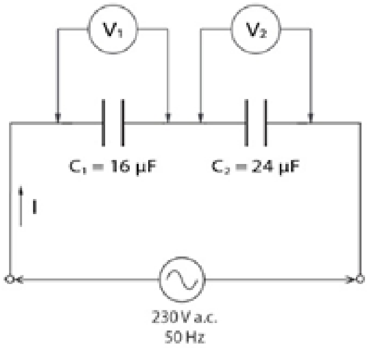
*c) The total capacitive reactance, $X_{CT}$*

*d) The supply current, I*

*e) The voltage drop across $X_{C1}$*

*f) The voltage drop across $X_{C2}$*

*g) Draw the phasor diagram for the circuit*




a) Capacitive reactance, $X_{C1}$

$$X_{C1} = \frac{1}{2\pi f C_1}$$

$$X_{C1} = \frac{1}{2 \times \pi \times 50 \times 16 \times 10^{-6}}$$

$$X_{C1} = \frac{1}{5.03 \times 10^{-3}}$$

$$\underline{X_{C1} = 199\ \Omega}$$

b) Capacitive reactance, $X_{C2}$

$$X_{C2} = \frac{1}{2\pi f C_2}$$

$$X_{C2} = \frac{1}{2 \times \pi \times 50 \times 24 \times 10^{-6}}$$

$$X_{C2} = \frac{1}{7.54 \times 10^{-3}}$$

$$\underline{X_{C2} = 133\ \Omega}$$

c) Total capacitive reactance, $X_{CT}$

$$X_{CT} = X_{C1} + X_{C2}$$

$$X_{CT} = 199 + 133$$

$$\underline{X_{CT} = 332\ \Omega}$$

d) Supply current, I

$$I = \frac{V}{X_{CT}}$$

$$I = \frac{230}{332}$$

$$\underline{I = 0.693\ A}$$

e) Voltage, $V_1$

$$V_1 = I \times X_{C1}$$

$$V_1 = 0.693 \times 199$$

$$\underline{V_1 = 138\ V}$$

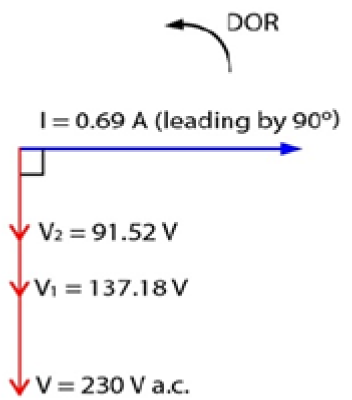
f) Voltage, $V_2$

$$V_2 = I \times X_{C2}$$

$$V_2 = 0.693 \times 133$$

$$V_2 = 92.2\ V$$

g) Phasor diagram (the circuit current is the reference phasor in a series circuit - the current 'leads' the voltages by 90°)



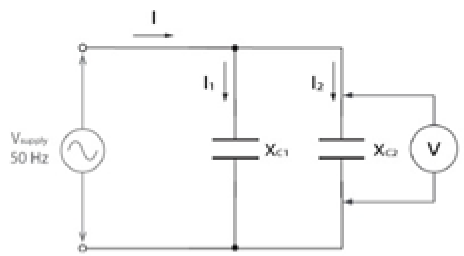

This diagram shows two capacitors connected in parallel to an a.c. supply. The voltage drop across each capacitor is equal to, and in-phase with the supply voltage. The total current I, and the current through each capacitor leads the supply voltage by 90°

Current $I_1 = V_{supply} / X_{C1}$

Current $I_2 = V_{supply} / X_{C2}$

Total current, $I = I_1 + I_2$

Total capacitive reactance, $X_{CT} = V_{supply} / I$



**Worked Example – Purely Capacitive a.c. Parallel Circuit**

For the capacitive a.c. parallel circuit pictured below, determine:

*a)* *Capacitive reactance of $C_1$*

*b)* *Capacitive reactance of $C_2$*

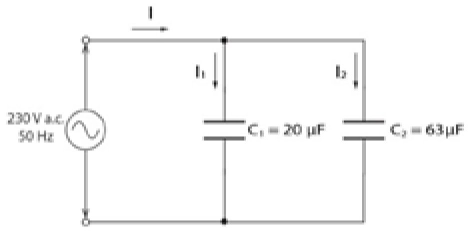
*c)* *Current $I_1$*

*d)* *Current $I_2$*

*e)* *The supply current, I*

*f)* *Total capacitive reactance, $X_{CT}$*

*g)* *Draw the phasor diagram for the circuit*




a) Capacitive reactance, $X_{C1}$

$$X_{C1} = \frac{1}{2\pi f C_1}$$

$$X_{C1} = \frac{1}{2 \times \pi \times 50 \times 20 \times 10^{-6}}$$

$$X_{C1} = \frac{1}{6.28 \times 10^{-3}}$$

$$\underline{X_{C1} = 159\,\Omega}$$

b) Capacitive reactance, $X_{C2}$

$$X_{C2} = \frac{1}{2\pi f C_2}$$

$$X_{C2} = \frac{1}{2 \times \pi \times 50 \times 63 \times 10^{-6}}$$

$$X_{C2} = \frac{1}{1.98 \times 10^{-2}}$$

$$\underline{X_{C2} = 50.5\,\Omega}$$

c) Current $I_1$

$$I_1 = \frac{V}{X_{C1}}$$

$$I_1 = \frac{230}{159}$$

$$\underline{I_1 = 1.44\,A}$$

d) Current $I_2$

$$I_2 = \frac{V}{X_{C2}}$$

$$I_2 = \frac{230}{50.5}$$

$$\underline{I_2 = 4.55\ A}$$

e) Total current, I

$$I = I_1 + I_2$$

$$I = 1.44 + 4.55$$

$$\underline{I = 5.99}$$

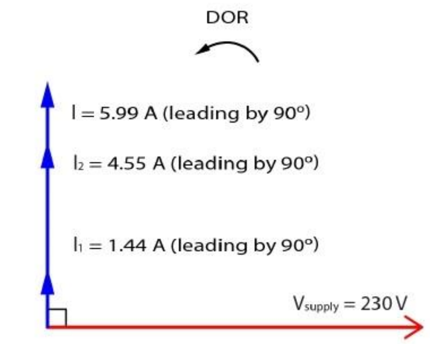
f) Total capacitive reactance, $X_{CT}$

$$X_{CT} = \frac{V}{I}$$

$$X_{CT} = \frac{230}{5.99}$$

$$\underline{X_{CT} = 38.4\ \Omega}$$

g) Phasor diagram (the supply voltage is the reference phasor in a parallel circuit - the currents 'lead' the supply voltage by 90°)




**Applications for Capacitive a.c. Circuits**

Capacitive a.c. loads are common in many a.c. circuits. Some applications include:

- Capacitive proximity sensor
- Touchscreen
- Low pass filter
- Power Factor Correction (PFC) unit
- Start/run capacitor in a single phase motor

**Introduction**

This topic will help you develop your understanding of impedance, and the effects on circuits containing a resistor and capacitor connected in series or a resistor and inductor connected in series.

**Impedance**

Impedance is the opposition to current in an a.c. circuit which results from a combination of resistance, inductive reactance and capacitive reactance in the circuit.

Impedance is defined as the ratio of the r.m.s. value of voltage to the r.m.s. value of current. The unit of impedance is ohms.

Impedance is calculated using this equation.

$$Z = \frac{V}{I}$$

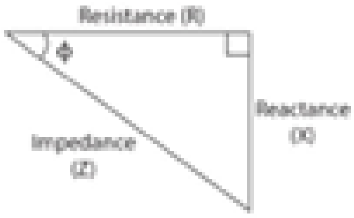
Where:

- $Z$ = impedance in ohms (Ω)
- $V$ = r.m.s. voltage in volts (V)
- $I$ = r.m.s. current in amperes (A)

**Impedance Triangle**

An 'impedance triangle' is a graphical tool which helps to analyse the effects of resistors, capacitors and inductors in an a.c. circuit.

This diagram shows an impedance triangle, which is a right angled triangle with sides labeled Resistance (R), Reactance (X) and Impedance (Z). Trigonometry and Pythagoras' Theorem are used to analyse these relationships.



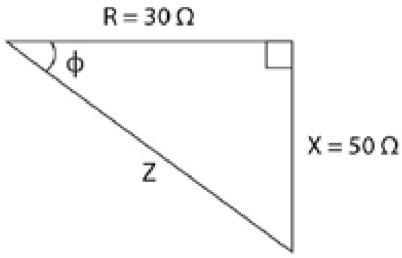

Points to note about the impedance triangle:

- Resistance is drawn horizontally.
- Reactance is drawn at right angles to the resistance (can be capacitive reactance, inductive reactance or a combination of both).
- Impedance is the hypotenuse of the triangle.
- Φ is the angle between the sides of the triangle representing resistance and impedance.

Note: Φ is the angle of lead or lag between the voltage and the current in the circuit.

**Worked Example – Impedance Triangle**

A resistance of 30 ohms is connected in series with a reactance of 50 ohms, calculate the:




*a) Impedance, Z*

*b) Phase angle, Φ*

a) Impedance, Z

$$Z = \sqrt{(R^2 + X^2)}$$

$$Z = \sqrt{(30^2 + 50^2)}$$

$$Z = \sqrt{(900 + 2500)}$$

$$Z = \sqrt{(3400)}$$

$$\underline{Z = 58.3\ \Omega}$$

b) Phase angle, Φ

$$\cos \Phi = \frac{R}{Z}$$

$$\cos \Phi = \frac{30}{58.3}$$

$$\underline{\cos \Phi = 0.52}$$

$$\Phi = \cos^{-1}(0.52)$$

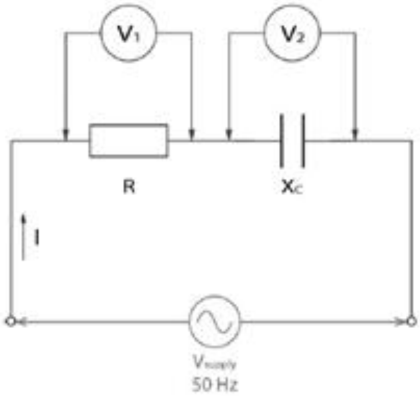
$$\underline{\Phi = 58.7°}$$

*[* precise answer (no rounding at intermediate steps) Φ = 59.0°]*

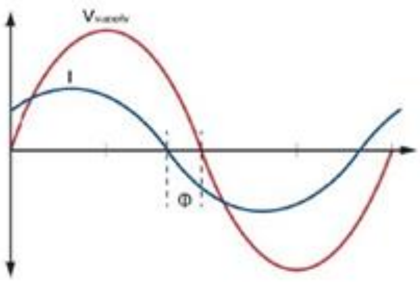
**RC Series a.c. Circuits**

In a purely capacitive a.c. circuit the current leads the voltage by 90°. When a resistor and a capacitor are connected in series the current leads the voltage, but the angle of lead is *less* than 90°.

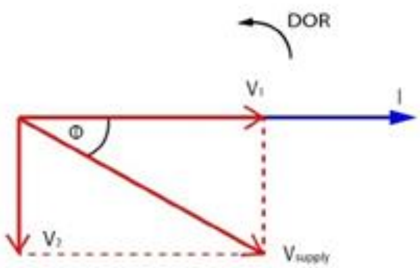
This diagram shows a capacitor and resistor connected in series.




This diagram shows the sinusoidal waveform for an RC series circuit. The current leads the supply voltage by angle Φ.




This diagram shows the phasor diagram for an RC series circuit.




Points to note about the phasor diagram of an RC series circuit:

- The current I, is the reference phasor.
- The voltage drop across the resistor, $V_1$, is drawn in-phase with the current.
- The voltage drop across the capacitor, $V_2$, is drawn lagging the current by 90°, i.e. the current in a capacitor *leads* the voltage across it by 90°.
- The supply voltage, $V_{supply}$, is the *phasor sum* of voltages V1 and V2.
- Angle Φ, is the phase difference between $V_{supply}$ and the circuit current.

**Impedance Triangle for an RC Circuit**

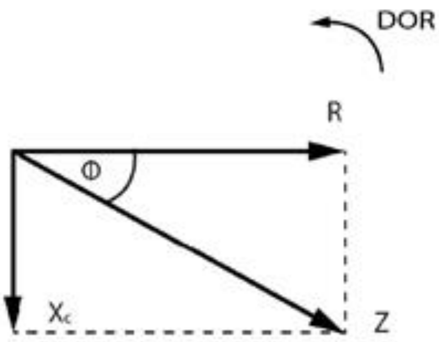
To create an impedance triangle for an RC circuit:

1. First replace the voltages shown in the phasor diagram with the resistance, capacitive reactance and the impedance of the circuit.

a) Resistance phasor drawn horizontally to the right (in the same direction as the voltage drop ($V_1$) across the resistor).

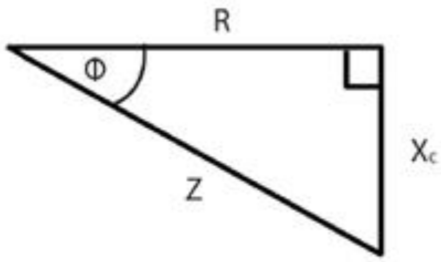
b) Capacitive reactance phasor drawn at 90° down the page (in the same direction as the voltage drop ($V_2$) across the capacitor.

c) Impedance is the phasor sum of R and $X_C$.




2. Complete the triangle by correctly labeling the sides of the triangle – resistance, capacitive reactance and the impedance.

This diagram shows the impedance triangle for an RC series circuit.



**Worked Example – Impedance Triangle – RC Series Circuit**

For the series RC circuit shown below, determine:

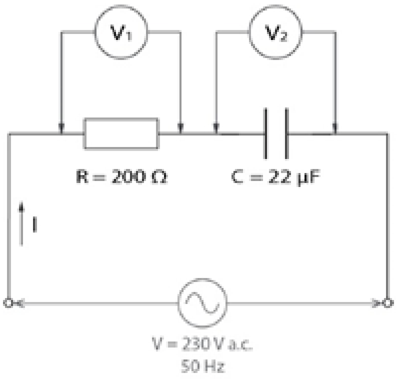
*a) Capacitive reactance, $X_C$*

*b) Impedance, Z*

*c) Supply current, I*

*d) Phase angle, Φ*

*e) Draw the impedance triangle*




a) Capacitive reactance, $X_C$

$$X_c = \frac{1}{2\pi f C}$$

$$X_c = \frac{1}{2 \times \pi \times 50 \times 22 \times 10^{-6}}$$

$$X_c = \frac{1}{6.91 \times 10^{-3}}$$

$$\underline{X_c = 145\,\Omega}$$

b) Impedance, Z

$$Z = \sqrt{(R^2 + Xc^2)}$$

$$Z = \sqrt{(200^2 + 145^2)}$$

$$Z = \sqrt{(40000 + 21025)}$$

$$Z = \sqrt{(61025)}$$

$$\underline{Z = 247\,\Omega}$$

c) Supply current, I

$$I = \frac{V}{Z}$$

$$I = \frac{230}{247}$$

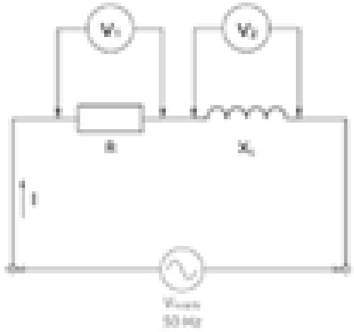
$$\underline{I = 0.93\,A}$$

d) Phase angle, Φ

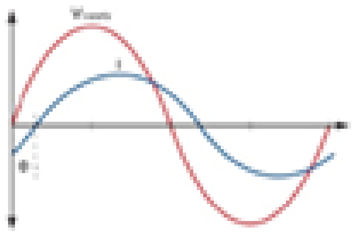
**RL Series a.c. Circuits**

In a purely inductive a.c. circuit, the current lags the voltage by 90°. When a resistor and an inductor are connected in series, the current lags the voltage, but the angle of lag is *less* than 90°.

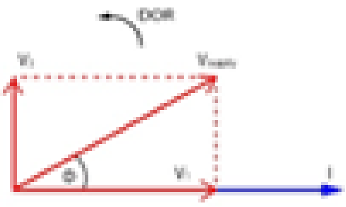
This diagram shows an inductor and resistor connected in series.




This diagram shows the sinusoidal waveform for an RL series circuit. The current lags the supply voltage by angle Φ.




This diagram shows the phasor diagram for an RL series circuit.




Points to note about the phasor diagram of an RL series circuit:

1. The current I, is the reference phasor.
2. The voltage drop across the resistor, $V_1$, is drawn in-phase with the current.
3. The voltage drop across the inductor, $V_2$, is drawn leading the current by 90°, i.e. the current in the inductor lags the voltage across it by 90°.
4. The supply voltage, $V_{supply}$, is the phasor sum of voltages $V_1$ and $V_2$.
5. Angle Φ, is the phase difference between $V_{supply}$ and the circuit current.

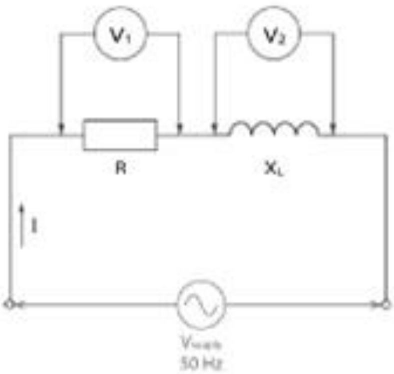
**Impedance Triangle for an RL Circuit**

To create an impedance triangle for an RL circuit:

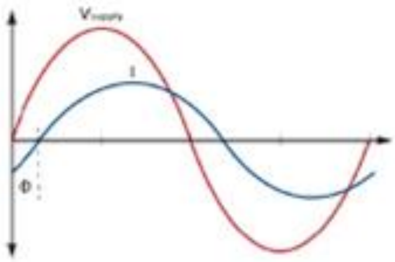
**RL Series a.c. Circuits**

In a purely inductive a.c. circuit, the current lags the voltage by 90°. When a resistor and an inductor are connected in series, the current lags the voltage, but the angle of lag is *less* than 90°.

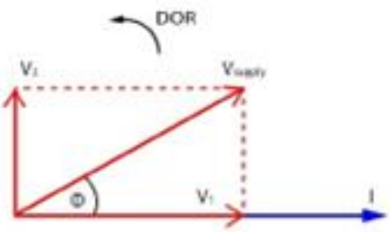
This diagram shows an inductor and resistor connected in series.




This diagram shows the sinusoidal waveform for an RL series circuit. The current lags the supply voltage by angle Φ.




This diagram shows the phasor diagram for an RL series circuit.




Points to note about the phasor diagram of an RL series circuit:

1. The current I, is the reference phasor.
2. The voltage drop across the resistor, $V_1$, is drawn in-phase with the current.
3. The voltage drop across the inductor, $V_2$, is drawn leading the current by 90°, i.e. the current in the inductor lags the voltage across it by 90°.
4. The supply voltage, $V_{supply}$, is the phasor sum of voltages $V_1$ and $V_2$.
5. Angle Φ, is the phase difference between $V_{supply}$ and the circuit current.

**Impedance Triangle for an RL Circuit**

To create an impedance triangle for an RL circuit:

**Worked Example – Impedance Triangle – RL Series Circuit**

For the series RL circuit shown below, determine:

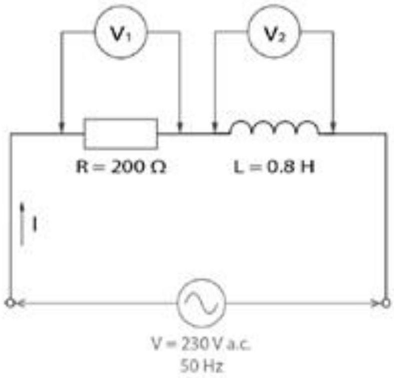
a) Inductive reactance, $X_L$

b) Impedance, Z

c) Supply current, I

d) Phase angle, Φ

e) Draw the impedance triangle




a) Inductive reactance, $X_L$

$$X_L = 2\pi f L$$

$$X_L = 2 \times \pi \times 50 \times 0.8$$

$$\underline{X_L = 251\ \Omega}$$

b) Impedance, Z

$$Z = \sqrt{(R^2 + X_L^2)}$$

$$Z = \sqrt{(200^2 + 251^2)}$$

$$Z = \sqrt{(40000 + 63001)}$$

$$Z = \sqrt{(103001)}$$

$$\underline{Z = 321\ \Omega}$$

c) Supply current, I

$$I = \frac{V}{Z}$$

$$I = \frac{230}{321}$$

$$\underline{I = 0.72\ A}$$

d) Phase angle, Φ

$$\cos\Phi = \frac{R}{Z}$$

$$\cos\Phi = \frac{200}{321}$$

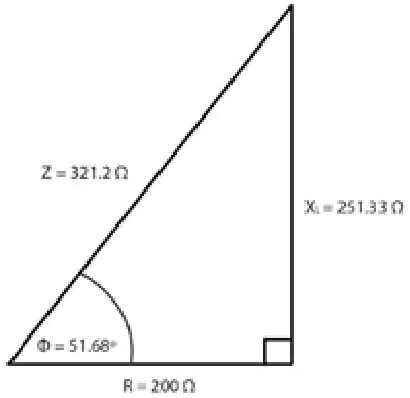
$$\underline{\cos\Phi = 0.62}$$

$$\underline{\Phi = \cos^{-1}(0.62)}$$

$$\underline{\Phi = 51.7°}$$

*[* precise answer (no rounding at intermediate steps) Φ = 51.5°]*

e) Impedance triangle

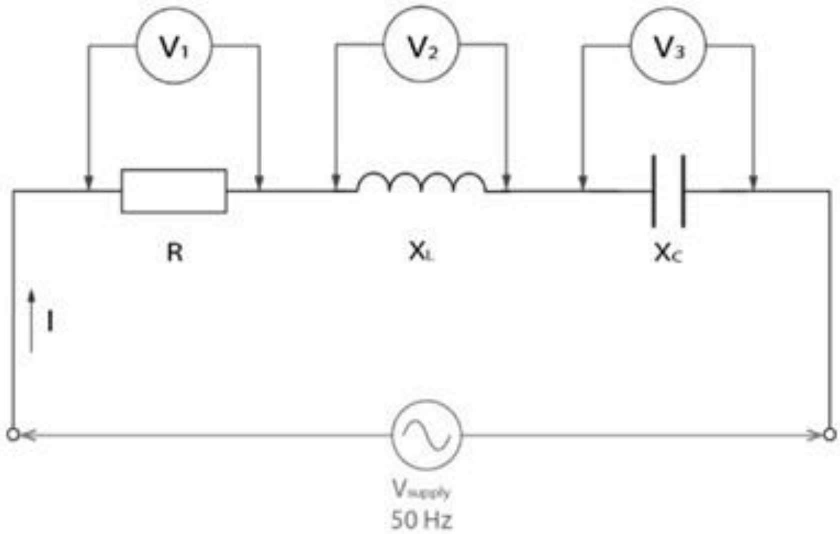


**Introduction**

This topic will help you develop your understanding of a.c. circuits that contain inductance, capacitance and resistance connected in series.

RLC series a.c. circuits contain resistance, inductance and capacitance connected in series to a voltage supply. The circuit current and the phase angle is determined by the relative values of resistance, inductive reactance and capacitive reactance in the circuit.

This diagram shows a RLC series a.c. circuit. The phasor sum of $V_1$, $V_2$ and $V_3$ equals the supply voltage.



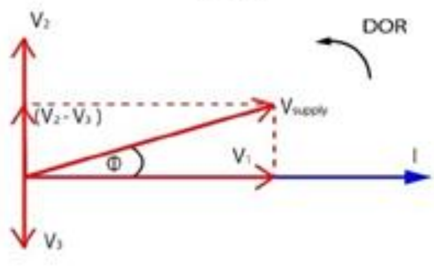

The relationships between voltage and current in a RLC series circuit can be represented using a phasor diagram.

This phasor diagram for a RLC series circuit shows the phasor relationships where the inductive reactance is *greater* than the capacitive reactance.

When the inductive reactance is greater than the capacitive reactance:

- The circuit is more 'inductive' than 'capacitive'.
- The current *lags* the supply voltage by angle Φ.
- The voltage drop, $V_2$ across the inductor is greater than the voltage drop, $V_3$ across the capacitor.
- The supply voltage, $V_{supply}$ is the phasor sum of $V_1$,$V_2$ and $V_3$.

$X_L > X_C$

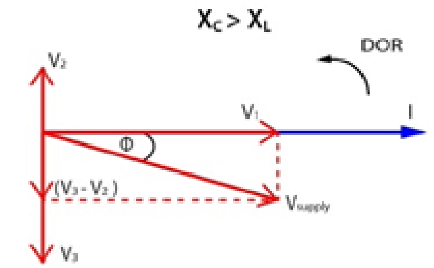



This diagram shows the phasor diagram for a RLC series circuit where the capacitive reactance is greater that the inductive reactance.

When the capacitive reactance is greater than the inductive reactance:

- The circuit is more 'capacitive' than 'inductive'.

- The current *leads* the supply voltage by angle Φ.
- The voltage drop, $V_3$ across the capacitor is greater than the voltage drop, $V_2$ across the inductor.
- The supply voltage, $V_{supply}$ is the phasor sum of $V_1$,$V_2$ and $V_3$.




**Comparison of Current Limiting Characteristics of Inductors and Resistors**

Both resistors and inductors are connected into a.c. circuits for the purpose of limiting the circuit current.

This table contrasts the advantages and disadvantages of using resistors and inductors to limit the current in an a.c. circuit.

| Comparison of Current Limiting Characteristics of Inductors and Resistors | | |
|---|---|---|
| **Component** | **Advantage** | **Disadvantage** |
| Resistor | • Resistance value does not change with frequency.<br>• Current through the resistor is in-phase with voltage drop across the resistor. | • Resistors dissipates electrical power as heat (P = I2R)<br>• Power dissipated by the resistor increases as the square of the current. |
| Inductor | • Inductor can limit the circuit current with much lower $I^2R$ losses. | • Inductor current lags the voltage across it by 90°.<br>• Inductive reactance increases and decreases with the frequency<br>• Inductor current will increase as the frequency decreases.<br>• Inductor current will decrease as the frequency increases. |

**Practical RLC Series Circuits**

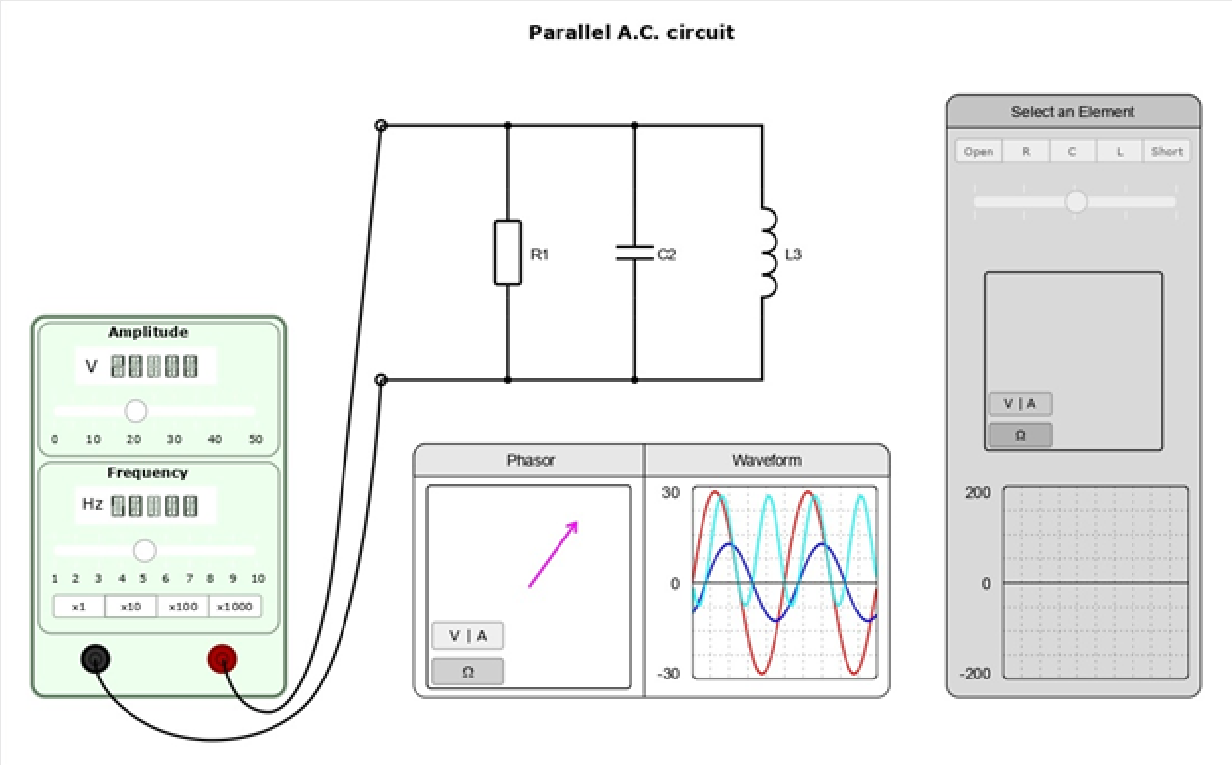
Most practical RLC series a.c. circuits are limited to electronic or motor control systems.

These systems include:

- Radio transmitter/receiver tuning circuits.
- Filtering circuits.
- Control circuits inside variable frequency drive (VFD) units.

**Check your progress**

The following interactive object demonstrates the effect of connecting a resistor in parallel with an inductor in an a.c. circuit.




For this demonstration you will connect resistor R1 in parallel with inductor L3.

1. Click on R1 and set it to 52 Ω.
2. Click on C2, and click on the 'open' button above the capacitance adjustment slider to disconnect this component.
3. Click on L3 and set it to 60 mH.
4. Set the supply amplitude to 25 V and the frequency to 50 Hz.
5. Click on R1 to select it, then click on the **V | A** button under the resistance adjustment slider to see the phasor diagram for resistor R1.
6. Notice that the resistor current and voltage are in-phase.
7. Click on L3 to select it and verify that its voltage and current are 90° out-of-phase.
8. Is the current 'leading' or 'lagging' the voltage across the inductor?
9. Move the cursor over the 'red' (voltage) and 'blue' (current) waveforms displayed immediately below the circuit diagram. Verify that the circuit obeys Ohms Law.
10. Locate the phasor diagram immediately below the circuit diagram, click on the **V | A** button to display the phasor diagram for the whole circuit.
11. What is the phase angle for the circuit? Is it 'leading' or 'lagging'?
12. Calculate the phase angle and compare it with the value given on the phasor diagram.
13. Adjust the resistance and inductance to different values and note the change in the current and phase angle.

**Is the phase angle affected by changing the resistor and inductor values?**

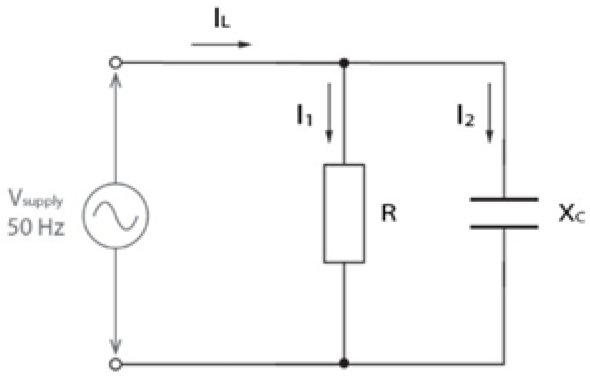
**Is the phase angle affected by changing the frequency?**

**Summary**

- A RL parallel a.c. circuit contains only resistance and inductance.
- The supply voltage is the reference phasor.
- The line current is the phasor sum of the resistive current $I_1$ and the inductive current $I_2$.

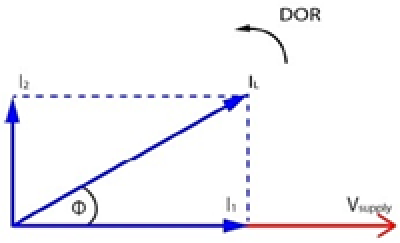
**RC Parallel a.c. Circuits**

RC parallel a.c. circuits are common in electronic circuits such as *wave shaping* circuits and current *divider* circuits. A RC parallel a.c. circuit contains only resistance and capacitance connected in parallel as shown in this circuit diagram.




This phasor diagram shows the phase relationship between the branch currents $I_1$, $I_2$ and the line current $I_L$. The supply voltage is the reference phasor. The resistive current $I_1$ is in-phase with the supply voltage. The capacitive current $I_2$ leads the supply voltage by 90°.

The line current $I_L$ is the phasor sum of $I_1$ and $I_2$ and *leads* the supply voltage by angle Φ. Pythagoras' Theorem can be used to calculate the phase angle Φ.



**Worked Example – RC Parallel Circuit**

For the RC parallel a.c circuit shown below, determine:

a) *The resistor current, $I_1$*

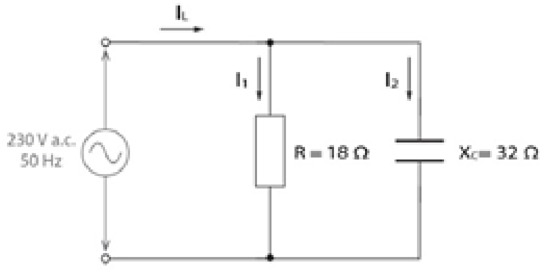
b) *The capacitor current, $I_2$*

c) *The line current, $I_L$*

d) *The impedance, Z*

e) *Phase angle, Φ*

f) *Draw the phasor diagram*




a) Resistor current $I_1$

$$I_1 = \frac{V}{R}$$

$$I_1 = \frac{230}{18}$$

$$\underline{I_1 = 12.8\ A}$$

b) Capacitor current $I_2$

$$I_2 = \frac{V}{R}$$

$$I_2 = \frac{230}{32}$$

$$\underline{I_2 = 7.19\ A}$$

c) Line current $I_L$

$$I_L = \sqrt{(I_1^2 + I_2^2)}$$

$$I_L = \sqrt{(12.8^2 + 7.19^2)}$$

$$I_L = \sqrt{(215)}$$

$$\underline{I_L = 14.7\ A}$$

$$I_L = \sqrt{(I_1^2 + I_2^2)}$$

$$I_L = \sqrt{(12.78^2 + 7.19^2)}$$

$$I_L = \sqrt{(215.03)}$$

$$\underline{I_L = 14.66\ A}$$

d) Impedance Z

$$Z = \frac{V}{I_L}$$

$$Z = \frac{230}{14.7}$$

$$\underline{Z = 15.7\ \Omega}$$

e) Phase angle Φ

$$\cos\Phi = \frac{I_1}{I_L}$$

$$\cos\Phi = \frac{12.8}{14.7}$$

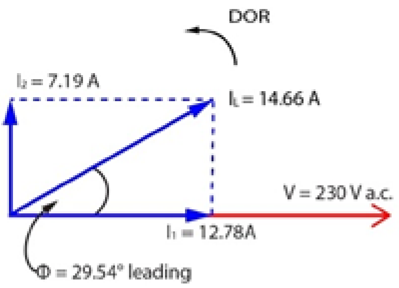
$$\underline{\cos\Phi = 0.87}$$

$$\underline{\Phi = \cos^{-1}(0.87)}$$

$$\underline{\Phi = 29.5°}$$

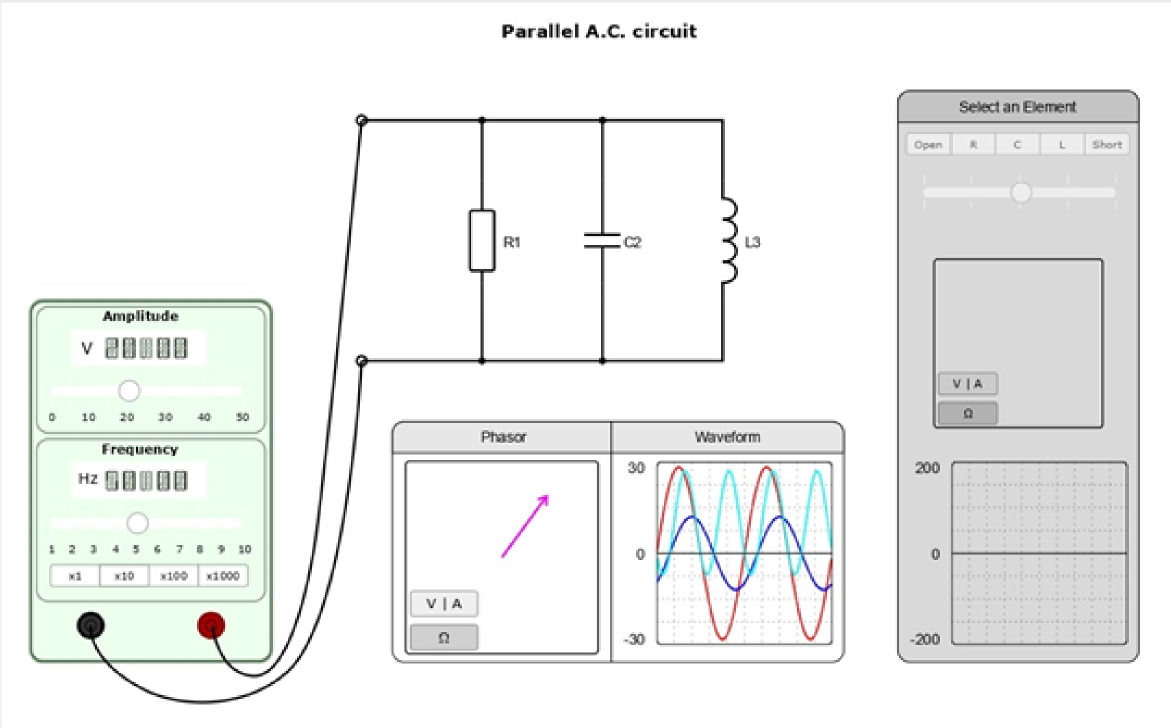
*[* precise answer (no rounding at intermediate steps) Φ = 29.3°]*

f) Phasor diagram



**Check your progress**

The following interactive object demonstrates the effect of connecting a resistor in parallel with a capacitor in an a.c. circuit.




For this demonstration you will connect resistor R1 in parallel with capacitor C2.

1. Click on R1 and set it to 60 Ω.
2. Click on C2 and set it to 100 μF.
3. Click on L3, and click on the 'open' button above the inductance adjustment slider to disconnect this component.
4. Set the supply amplitude to 40 V and the frequency to 50 Hz.
5. Click on R1 to select it, then click on the **V | A** button under the resistance adjustment slider to see the phasor diagram for resistor R1.
6. Notice that the resistor current and voltage are in-phase.
7. Click on C2 to select it and verify that its voltage and current are 90° out-of-phase.
8. Is the current 'leading' or 'lagging' the voltage across the capacitor?
9. Move the cursor over the 'red' (voltage) and 'blue' (current) waveforms displayed immediately below the circuit diagram. Verify that the circuit obeys Ohms Law.
10. Locate the phasor diagram immediately below the circuit diagram, click on the **V | A** button to display the phasor diagram for the whole circuit.
11. What is the phase angle for the circuit? Is it 'leading' or 'lagging'?
12. Calculate the phase angle and compare it with the value given on the phasor diagram.
13. Adjust the resistance and capacitance to different values and note the change in the current and phase angle.

**Is the phase angle affected by changing the resistor and capacitance values?**

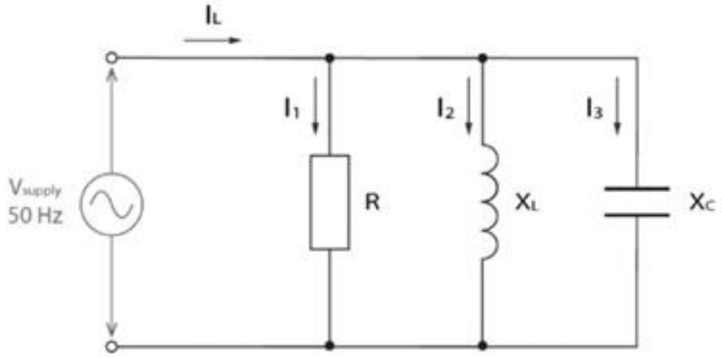
**Is the phase angle affected by changing the frequency?**

**Summary**

**RLC Parallel a.c. Circuits**

RLC parallel a.c. circuits are common in electronic circuits such as *signal filters*, *oscillators* and *voltage multipliers*. A RLC parallel a.c. circuit contains resistance, inductance and capacitance connected in parallel.

This circuit shows a RLC parallel a.c. circuit.

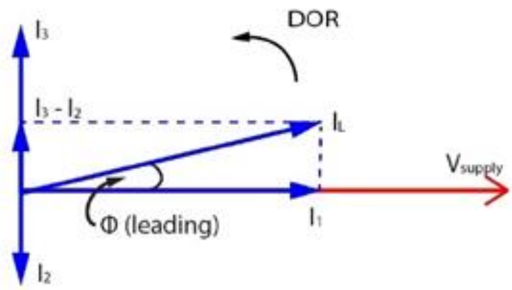



This phasor diagram shows the phase relationship between the branch currents $I_1$, $I_2$, $I_3$ and the line current $I_L$ for the parallel circuit above.

The supply voltage is the reference phasor. The resistor current $I_1$ is in-phase with the supply voltage, the inductor current $I_2$ lags the supply voltage by 90° and the capacitor current $I_3$ leads the supply voltage by 90°.

The line current $I_L$ is the phasor sum of $I_1$, $I_2$ and $I_3$ and may *lead* or *lag* the supply voltage depending on the relative values of inductive reactance and capacitive reactance in the circuit. Pythagoras' Theorem can be used to calculate the phase angle Φ.

This phasor diagram shows the line current is *leading* by angle Φ, when the inductive reactance is greater than the capacitive reactance, i.e. $I_3 > I_2$.




This phasor diagram shows the line current is *lagging* by angle Φ, when the capacitive reactance is greater than the inductive reactance, i.e. $I_2 > I_3$.

**Worked Example – RLC Parallel Circuit 1 – Inductive Reactance > Capacitive Reactance**

For the LC parallel a.c circuit shown below, determine:

a) *The Resistor current, $I_1$*

b) *The Inductor current, $I_2$*

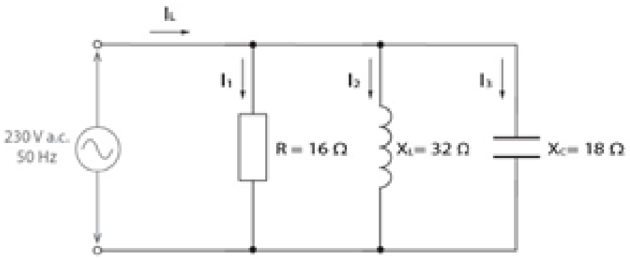
c) *The capacitor current, $I_3$*

d) *The line current, $I_L$*

e) *The impedance, Z*

f) *Phase angle, Φ*

g) *Draw the phasor diagram*




a) Resistor current $I_1$

$$I_1 = \frac{V}{R}$$

$$I_1 = \frac{230}{16}$$

$$\underline{I_1 = 14.4\ A}$$

b) Inductor current $I_2$

$$I_2 = \frac{V}{X_L}$$

$$I_2 = \frac{230}{32}$$

$$\underline{I_2 = 7.19\ A}$$

c) Capacitor current $I_3$

$$I_3 = \frac{V}{X_C}$$

$$I_3 = \frac{230}{18}$$

$$\underline{I_3 = 12.8\ A}$$

d) Line current $I_L$

$$I_L = \sqrt{(I_1^2 + (I_3 - I_2)^2)}$$

$$I_L = \sqrt{(14.4^2 + (12.8 - 7.19)^2)}$$

$$I_L = \sqrt{(14.4^2 + 5.59^2)}$$

$$I_L = \sqrt{(238)}$$

$$\underline{I_L = 15.4\ A}$$

e) Impedance Z

$$Z = \frac{V}{I_L}$$

$$Z = \frac{230}{15.3}$$

$$\underline{Z = 14.9\ \Omega}$$

f) Phase angle Φ

$$\cos \Phi = \frac{I_1}{I_L}$$

$$\cos \Phi = \frac{14.4}{15.4}$$

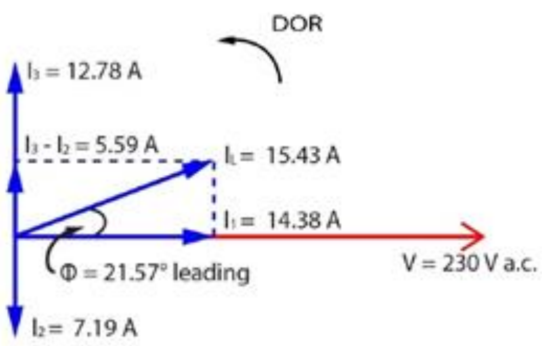
$$\underline{\cos \Phi = 0.93}$$

$$\underline{\Phi = \cos^{-1}(0.93)}$$

$$\underline{\Phi = 21.6^\circ}$$

*[* precise answer (no rounding at intermediate steps) Φ = 21.2°]*

g) Phasor diagram



**Worked Example – RLC Parallel Circuit 2 – Capacitive Reactance > Inductive Reactance**

For the LC parallel a.c circuit shown below, determine:

*a) The Resistor current, $I_1$*

*b) The Inductor current, $I_2$*

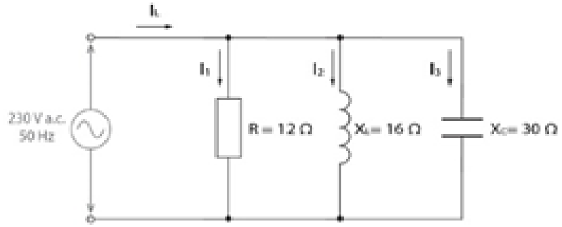
*c) The capacitor current, $I_3$*

*d) The line current, $I_L$*

*e) The impedance, Z*

*f) Phase angle, Φ*

*g) Draw the phasor diagram*




a) Resistor current $I_1$

$$I_1 = \frac{V}{R}$$

$$I_1 = \frac{230}{12}$$

$$\underline{I_1 = 19.2\ A}$$

b) Inductor current $I_2$

$$I_2 = \frac{V}{X_L}$$

$$I_2 = \frac{230}{16}$$

$$\underline{I_2 = 14.4\ A}$$

c) Capacitor current $I_3$

$$I_3 = \frac{V}{X_C}$$

$$I_3 = \frac{230}{30}$$

$$\underline{I_3 = 7.67\ A}$$

d) Line current $I_L$

$$I_L = \sqrt{(I_1^2 + (I_2 - I_3)^2)}$$

$$I_L = \sqrt{(19.2^2 + (14.4 - 7.67)^2)}$$

$$I_L = \sqrt{(19.2^2 + 6.71^2)}$$

$$I_L = \sqrt{(413)}$$

$$\underline{I_L = 20.3\ A}$$

e) Impedance Z

$$Z = \frac{V}{I_L}$$

$$Z = \frac{230}{20.3}$$

$$\underline{Z = 11.3\ \Omega}$$

f) Phase angle Φ

$$\cos\Phi = \frac{I_1}{I_L}$$

$$\cos\Phi = \frac{19.2}{20.3}$$

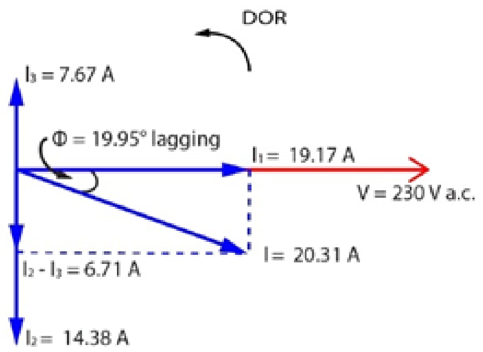
$$\underline{\cos\Phi = 0.94}$$

$$\underline{\Phi = \cos^{-1}(0.94)}$$

$$\underline{\Phi = 20.0°}$$

*[* precise answer (no rounding at intermediate steps) Φ = 19.3°]*

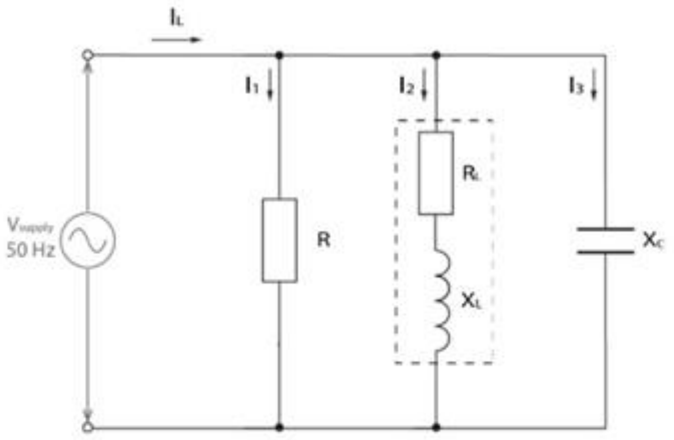
g) Phasor diagram




**Practical RLC Series a.c. Circuit**

Most inductors consist of many turns of wire wound onto an iron core. This means that practical inductors in a.c. circuits have some value of resistance as well as inductive reactance to limit the current. This is true for inductive components such as discharge lighting ballasts, a.c. motor windings, a.c. relay/contactor coils and conductor cables.

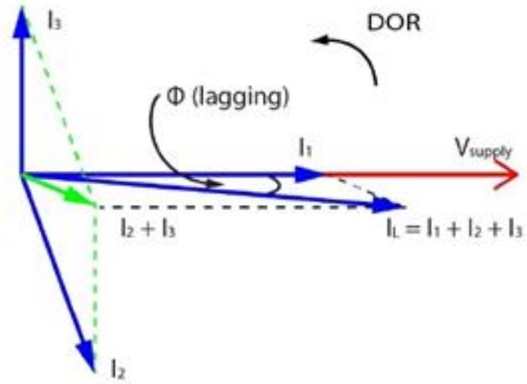
This diagram shows a RLC parallel a.c. circuit where the inductive branch is replaced with a resistor connected in series with an inductor to represent a practical inductive circuit component.

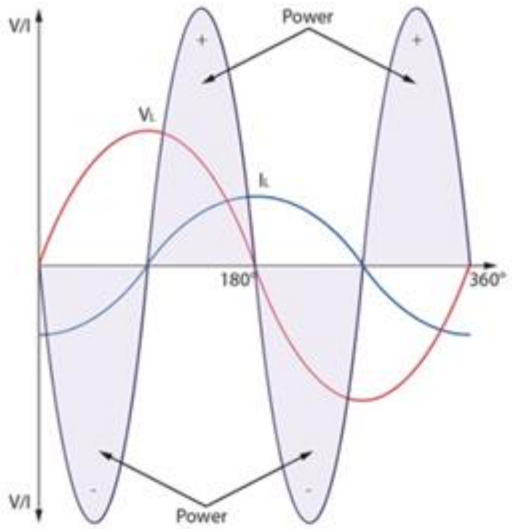



This phasor diagram shows the phase relationships in the RLC parallel a.c. circuit above. The current in the inductive branch lags the supply voltage by some angle Φ that is less than 90°. The angle of lag is dependent on the relative values of resistance and inductive reactance in that branch.

To determine the line current:

1. Calculate the phasor sum of $I_2$ and $I_3$ (the resultant phasor is shown in green below)
2. Calculate the phasor sum of $I_1$ and the resultant (green) phasor.



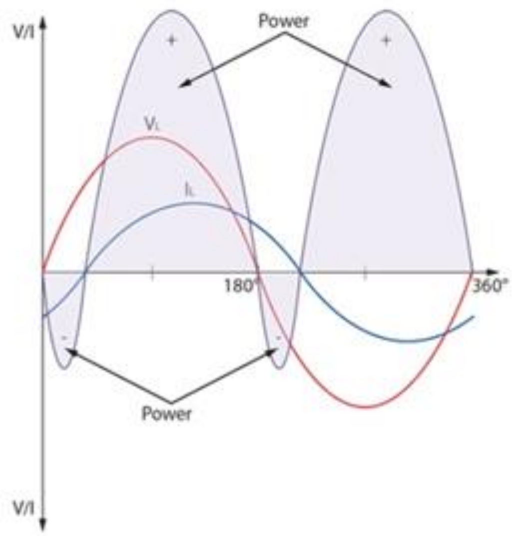



**Power in a Practical a.c. Load**

Most practical a.c. loads contain both resistive and inductive components. In a practical a.c. load the line current and line voltage are out-of-phase by some angle Φ between 0° and 90°, depending on the relative values of resistance and inductive reactance in the circuit.

This diagram shows the voltage, current and power waveforms for a practical a.c. load. The current lags the voltage by angle Φ°. The power at any point on the waveform can be calculated by multiplying the instantaneous current and voltage.

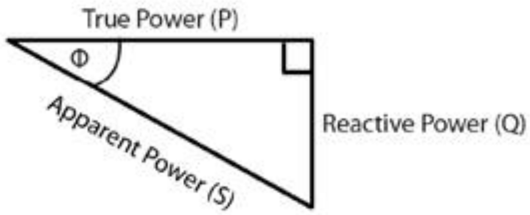
The diagram shows that the power waveform has two large positive peaks and two smaller negative peaks for each cycle of voltage and current. This shows that most of the power supplied to the load is utilised by the load and only a small proportion of the power is returned to the power supply.




**Power Triangle**

A 'power triangle' is a graphical tool, similar to an impedance triangle which helps to analyse the power in an a.c. circuit.

This diagram shows a power triangle, which is a right-angle triangle with sides labelled True Power (P), Reactive Power (Q) and Apparent Power (S).




Points to note about the power triangle:

- True power is drawn horizontally.

- Reactive power is drawn at right angles to the true power (reactive power can be due to capacitive reactance, inductive reactance or a combination of both).
- Apparent power is the hypotenuse of the triangle.
- Φ is the angle between the sides of the triangle representing true power and apparent power.
- An *increase* in reactive power (Q) will *increase* the apparent power (S) and the phase angle Φ.
- An *increase* in true power (P), with the reactive power (Q) unchanged, will *increase* the apparent power (S) and *decrease* the phase angle Φ.

**Apparent Power**

- Total power drawn from the supply.
- Can be measured using a voltmeter and ammeter and multiplying the two readings.
- Symbol S, units are volt-amperes (VA).

$$S = V_L \times I_L$$

Where:

- S = apparent power in volt-amperes (VA)
- $V_L$ = r.m.s. line voltage in volts (V)
- $I_L$ = r.m.s. line current in amperes (A)

**True Power**

- r.m.s. power being used by a circuit.
- Measured with a wattmeter.
- True power is utilised by resistive components in a circuit.
- Symbol P, units are watts (W).

$$P = V_L \times I_L \times \cos \Phi$$

Where:

- P = true power in watts (W)
- $V_L$ = r.m.s. line voltage in volts (V)
- $I_L$ = r.m.s. line current in amperes (A)
- Φ = phase angle

**Reactive Power**

- Power associated with the reactive parts of the circuit, i.e. inductive and/or capacitive components.
- The portion of power not being utilised by the load.
- Symbol Q, units are volt-amperes reactive (VAr).

$$Q = V_L \times I_L \times \sin \Phi$$

Where:

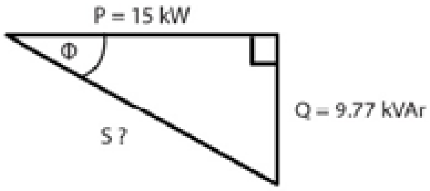
- Q = reactive power in volt-amperes reactive (VAr)
- $V_L$ = r.m.s. line voltage in volts (V)
- $I_L$ = r.m.s. line current in amperes (A)
- Φ = phase angle

**Worked Example – Power Triangle 1**

An a.c. circuit has a true power of 15 kW and a reactive power of 9.77 kVAr, as represented by the power triangle below. Determine for the circuit:

*a) The apparent power, S*

*b) Phase angle, Φ*




a) Apparent power, S

$$S = \sqrt{(P^2 + Q^2)}$$

$$S = \sqrt{(15000^2 + 9770^2)}$$

$$S = \sqrt{(320453)}$$

$$\underline{S = 17.9\ kVA}$$

b) Phase angle Φ

$$\cos\Phi = \frac{P}{S}$$

$$\cos\Phi = \frac{15000}{17900}$$

$$\cos\Phi = 0.84$$

$$\Phi = \cos^{-1}(0.84)$$

$$\underline{\Phi = 32.9°}$$

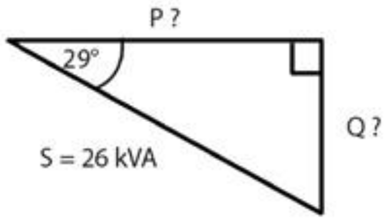
*[* precise answer (no rounding at intermediate steps) Φ = 33.1°]*

**Worked Example – Power Triangle 2**

An a.c. circuit has an apparent power of 26 kVA and a phase angle of 29° as represented by the power triangle below. Determine for the circuit:

a) The reactive power, Q

b) The true power, P




Reactive power, Q

$$Q = S \times \sin \Phi$$

$$Q = 26000 \times \sin 29$$

$$Q = 26000 \times 0.48$$

$$Q = 12.48\ kVAr$$

$$Q = S \times \sin \Phi$$

$$Q = 26000 \times \sin 29$$

$$Q = 26000 \times 0.48$$

$$Q = 12.5\ kVAr$$

*[* precise answer (no rounding at intermediate steps) Q = 12.6 kVAr]*

True power, P

$$P = S \times \cos \Phi$$

$$P = 26000 \times \cos 29$$

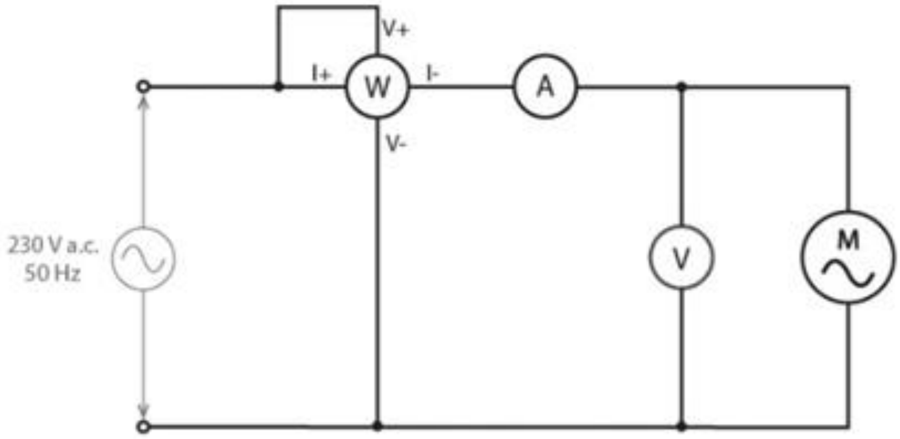
$$P = 26000 \times 0.87$$

$$P = 22.6\ kW$$

*[* precise answer (no rounding at intermediate steps) P = 22.7 kW]*

**Measuring a.c. Power**

This diagram shows the circuit arrangement for measuring the line voltage, line current and power input to a single phase a.c. motor.




The r.m.s. value of true power (P) is measured by the wattmeter. The apparent power (S) is determined by multiplying the voltmeter and ammeter readings.

The power factor is determined by this equation.

$$\lambda = \cos \Phi = \frac{P}{S}$$

Where:

- $\lambda$ = power factor ($\cos \Phi$) of the circuit (the Greek letter lambda)
- P = true power in watts (W)
- S = apparent power in volt-amperes (VA)

### Minimum Power Factor Levels

Local network providers set minimum and maximum limits for power factor in installations connected to their network. The minimum power factor for low voltage customers in Australia is 0.75 lag (Victoria), however in some jurisdictions the minimum is set at 0.8 – 0.9 lag. In some jurisdictions there is also a restriction on operating with a leading power factor, e.g. the NSW Service and Installation Rules state that 'The power factor of the installation must not become leading at any time'.

This allows the network provider to maintain the quality of the network supply and to avoid excessive loading and losses in network equipment.

Where the power factor is below the minimum specified level, the customer must install equipment to improve the installation's power factor. If this is not done, the customer can incur increased costs for their electricity or be disconnected.

### Power Factor Improvement

As stated in Topic 8.1, most installations have a lagging power factor, as inductive loads such as motors are more common place than capacitive loads. In order to improve a lagging power factor, equipment that introduces capacitive reactance into the circuit can be installed to offset the excessive inductive reactance. This equipment includes:

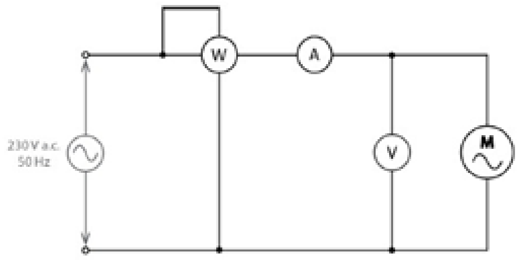
- Capacitor banks
- Synchronous motors
- Static VAR compensators

### AS/NZS 3000 Requirements for Capacitors

As previously stated in Topic 4.2, AS/NZS 3000:2018 *Clause 4.15 Capacitors* outlines specific requirements for capacitors connected into a.c. circuits. These requirements apply equally to capacitors used for power factor correction.

### Single Phase Power Factor Measurement

This diagram show the meter arrangement for measuring the power factor of a 230 V a.c. single phase motor circuit, which can be used to measure the power factor of any single phase load.




The r.m.s. value of true power (P) is measured by the wattmeter. The apparent power (S) is determined by multiplying the voltmeter (V) and ammeter readings (A).

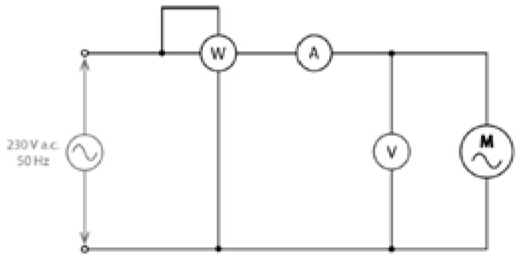
The power factor can be determined using this equation.

$$\lambda = \frac{P}{V \times I}$$

Where:

- λ = power factor (cos Φ) of the circuit
- P = true power in watts (W)
- V = load voltage in volts (V)
- I = load current in amperes (A)

**Worked Example – Power Factor Calculations 1**




Determine the power factor for the single phase motor circuit pictured above, given the following meter readings

*a) Wattmeter reading: 15.8 kW*

*b) Voltmeter reading: 230 V*

*c) Ammeter reading: 84 A*

a) Power factor, λ

$$\lambda = \frac{P}{S}$$

$$\lambda = \frac{P}{V \times I}$$

$$\lambda = \frac{15800}{230 \times 84}$$

$$\lambda = \frac{15800}{19320}$$

$$\underline{\lambda = 0.82}$$

**Worked Example – Power Factor Calculations 2**

A 230 V, 50 Hz, single phase lighting installation consists of high intensity discharge (HID) luminaires operating at a power factor of 0.52 lag.

If the total power of the installation is measured at 8.6 kW, determine the rating in kVAr, of a capacitor bank to be connected in parallel with the installation in order to correct the power factor to 0.9 lag.

a) Apparent power ($S_{0.52}$) at λ = 0.52 lag

$$S_{0.52} = \frac{P}{\lambda}$$

$$S_{0.52} = \frac{8600}{0.52}$$

$$\underline{S_{0.52} = 16538\ VA}$$

b) Reactive power ($Q_{0.52}$) at λ = 0.52 lag

$$Q_{0.52} = \sqrt{(16538^2 - 8600^2)}$$

$$\underline{Q_{0.52} = 14126\ \text{VAr}}$$

c) Apparent power ($S_{0.9}$) at λ = 0.9 lag

$$S_{0.9} = \frac{P}{\lambda}$$

$$S_{0.9} = \frac{8600}{0.9}$$

$$\underline{S_{0.9} = 9556\ VA}$$

d) Reactive power ($Q_{0.9}$) at λ = 0.9 lag

$$Q_{0.9} = \sqrt{(9556^2 - 8600^2)}$$

$$\underline{Q_{0.9} = 4166\ \text{Var}}$$

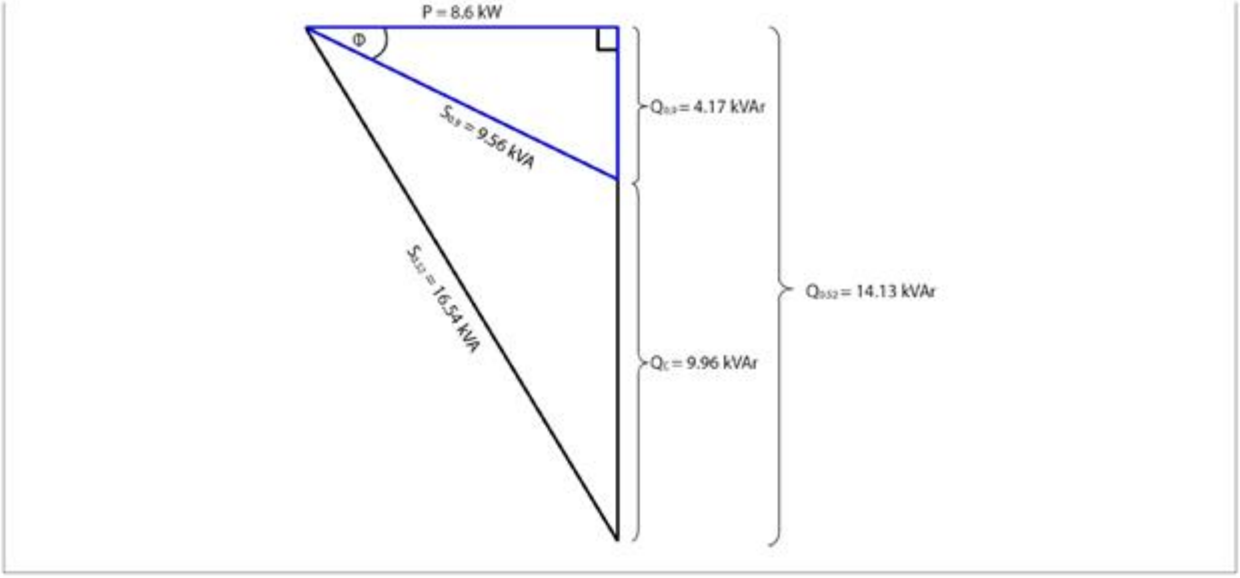
e) kVAr rating of capacitor bank, $Q_C$

$$Q_C = Q_{0.52} - Q_{0.9}$$

$$Q_C = 14126 - 4165$$

$$\underline{Q_C = 9961\ \text{VAr}}$$

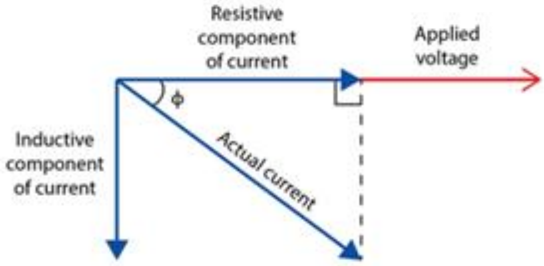
f) Power triangle




**Power Factor Correction Capacitance**

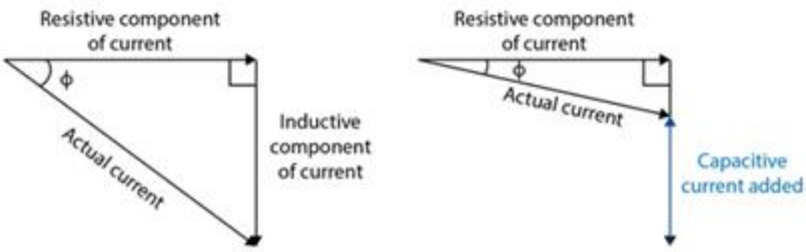
The windings of a single phase motor are both resistive and inductive. When energised, the current in the windings will lag the applied voltage because of the inductive element of the circuit. To counteract the effect of the inductance, we can connect a capacitor in parallel with the winding.

In order to determine the value of capacitance required, we need to think of the current flowing through the winding in terms of its resistive component, and its inductive component, where the resistive component of current is in phase with the voltage, and the inductive component of current lags the voltage by 90°. This can be determined using trigonometry:




(Note: in reality, the current flowing through the resistive and inductive elements of the circuit is one and the same, just as the winding itself is one and the same. We just need to think of these components of current separately, in order to determine the required value of capacitance).

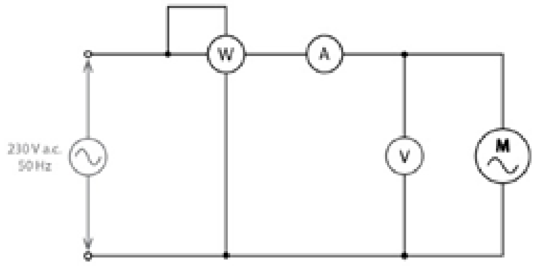
What we need to calculate, is the value of capacitance that will add a sufficient capacitive component of current into the circuit, to improve the power factor as required.




To do this:

- First we determine the desired phase angle (ϕ) for the circuit.
- Then we determine the value of capacitive current (IXC) needed to achieve this phase angle.
- Next we determine the value of capacitive reactance (XC) that would cause this capacitive current to flow.
- Finally, we determine the value of capacitance (C) that would produce this value of capacitive reactance.

Worked Example – Power Factor Calculations 3




At full load, the single phase motor pictured above draws a line current $I_L$ of 16 A at 0.71 lag. Determine the value of shunt capacitance required to improve the power factor to 0.95 lag.

a) $I_R$ at full load

$$I_R = I_L \times \lambda$$

$$I_R = 16 \times 0.71$$

$$\underline{I_R = 11.34\ A}$$

b) $I_X$ at 0.71 lag

$$I_X = \sqrt{(I_L^2 - I_R^2)}$$

$$I_X = \sqrt{(16^2 - 11.36^2)}$$

$$I_X = \sqrt{(256 - 129.05)}$$

$$\underline{I_X = 11.3\ A}$$

c) $I_L$ at 0.95 lag

$$\lambda = \frac{I_R}{I_L}$$

$$I_L = \frac{I_R}{\lambda}$$

$$I_L = \frac{11.4}{0.95}$$

$$\underline{I_L = 12.0\ A}$$

d) $I_X$ at 0.95 lag

$$I_X = \sqrt{(I_L^2 - I_R^2)}$$

$$I_X = \sqrt{(12^2 - 11.4^2)}$$

$$I_X = \sqrt{(144 - 123)}$$

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$\underline{I_X = 3.74\ A}$

e) Capacitor current, $I_{XC}$

$$I_{XC} = I_{X(0.71)} - I_{X(0.95)}$$

$$I_{XC} = 11.3 - 3.74$$

$$\underline{I_{XC} = 7.53\ A}$$

f) Capacitive reactance, $X_C$

$$X_C = \frac{V}{I_{XC}}$$

$$X_C = \frac{230}{7.53}$$

$$\underline{X_C = 30.5\ \Omega}$$

g) Capacitance, C

$$X_C = \frac{1}{2 \times \pi \times f \times C}$$

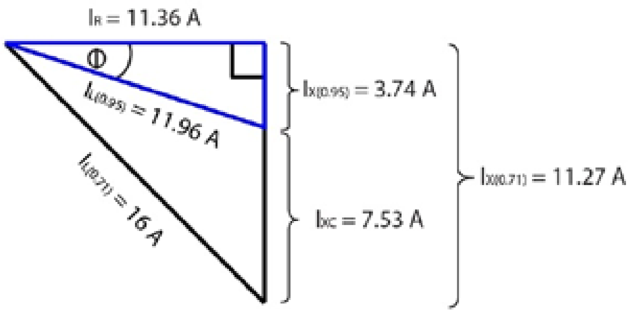
$$C = \frac{1}{2 \times \pi \times f \times X_C}$$

$$C = \frac{1}{2 \times \pi \times 50 \times 30.5}$$

$$C = \frac{1}{9596}$$

$$\underline{C = 104\ \mu F}$$

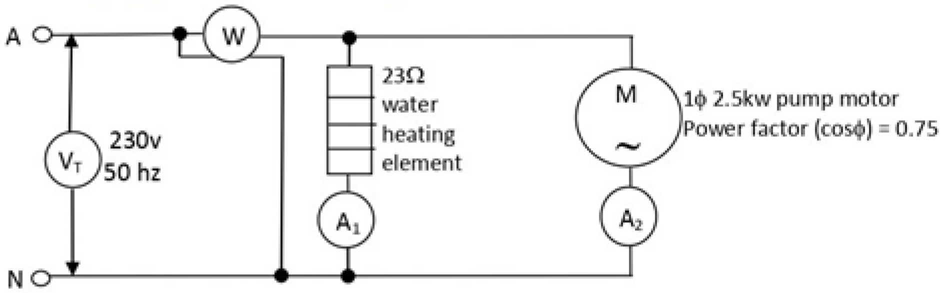
h) Phasor diagram




**Summary**

- Local network providers set the minimum level of power factor for an installation in their jurisdiction.
- When an installations power factor falls below the minimum level, power factor correction equipment must be installed.
- Power factor correction equipment adds a capacitive reactance to an installation to offset the lagging power factor created by inductive loads.
- The power factor of an installation can be calculated by using a wattmeter, voltmeter and ammeter to measure the load power, voltage and current.

## Single phase ac circuit parameters




**For the single phase ac circuit above, and using the electrical equipment details shown:**

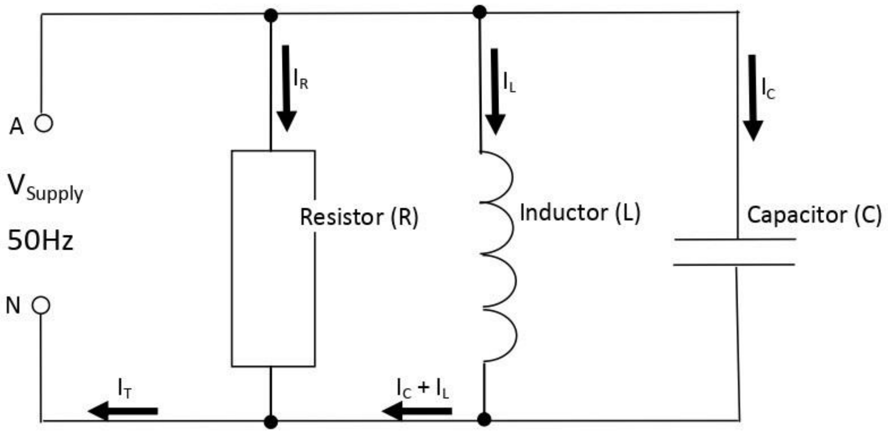
a. The reading on the ammeter $A_1$ will be = ______________

b. The reading on the ammeter $A_2$ will be = ______________

c. The reading on wattmeter W will be = ______________

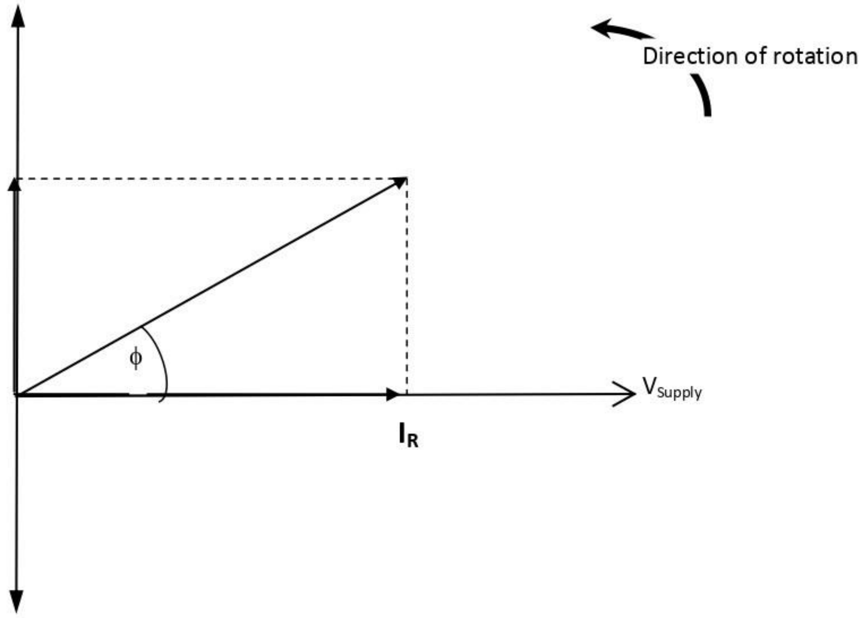
**Show your calculations here...**

## Single phase ac circuit phasor analysis



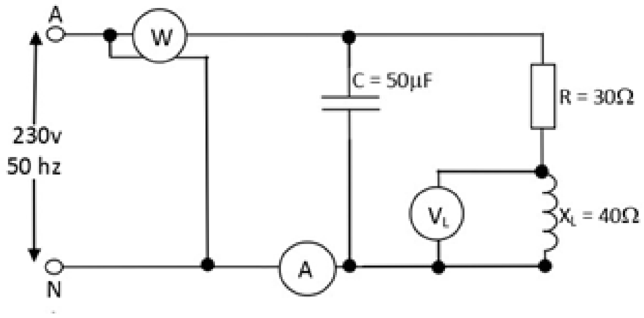

In the single phase ac circuit drawn above, arrows indicate the current in that part of the circuit. These currents are represented on the phasor diagram below. The phasor diagram shows the relationship between each branch current and the supply voltage ($V_{Supply}$).

**On this diagram, label each phasor with the corresponding circuit currents identified above.**




All phasors correctly labelled ☐

## Single phase ac circuit – effect of altering circuit components




For the single phase ac circuit above, what will be the **effect on each meter reading if the capacitor (C) was removed from the circuit?**

Put a cross ☒ in one box for each meter reading to indicate the best answer.

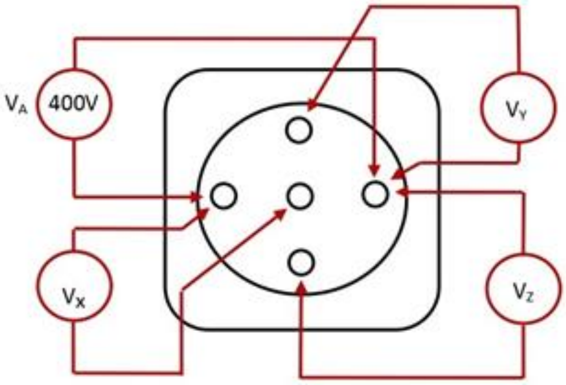
| | The reading would possibly decrease | The reading would possibly increase | The reading would definitely decrease | The reading would definitely increase | The reading would not change |
|---|---|---|---|---|---|
| Meter $V_L$ | | | | | |
| Meter A | | | | | |
| Meter W | | | | | |

**Show your reasoning or calculations here...**

## Three phase ac supply – polarity check / voltage measurement

Typical three phase ac socket outlet →


When testing a typical three phase ac socket outlet for correct polarity, the voltmeter was connected across each terminal as shown on this diagram.




**If $V_A$ is measured at 400 Volts then:**

a. the reading on voltmeter $V_X$ will be = ______________ Volts

b. the reading on voltmeter $V_Y$ will be = ______________ Volts

c. the reading on voltmeter $V_Z$ will be = ______________ Volts