9-6 In Fig. 9.17, calculate the voltage across the capacitor and the current flowing through it.

Industrial application

9-7 The nameplate on a 50 kVA transformer shows a primary voltage of 480 V and a secondary voltage of 120 V. We wish to determine the approximate number of turns on the primary and secondary windings. Toward this end, three turns of wire are wound around the external winding, and a voltmeter is connected across this 3-turn coil. A voltage of 76 V is then applied to the 120 V winding, and the voltage across the 3-turn winding is found to be 0.93 V. How many turns are there on the 480 V and 120 V windings (approximately)?

9-8 A coil with an air core has a resistance of 14.7 Ω. When it is connected to a 42 V, 60 Hz ac source, it draws a current of 1.24 A. Calculate the following:

a. The impedance of the coil
b. The reactance of the coil, and its inductance
c. The phase angle between the applied voltage (42 V) and the current (1.24 A).

9-9 Two coils are set up as shown in Fig. 9.4. Their respective resistances are small and may be neglected. The coil having terminals 1, 2 has 320 turns while the coil having terminals 3, 4 has 160 turns. It is found that when a 56 V, 60 Hz voltage is applied to terminals 1-2, the voltage across terminals 3-4 is 22 V. Calculate the peak values of ϕ, ϕ_{f1}, and ϕ_{m1}.

9-10 A 40 μF, 600 V paper capacitor is available, but we need one having a rating of about 300 μF. It is proposed to use a transformer to modify the 40 μF so that it appears as 300 μF. The following transformer ratios are available: 120 V/330 V; 60 V/450 V; 480 V/150 V. Which transformer is the most appropriate and what is the reflected value of the 40 μF capacitance? To which side of the transformer should the 40 μF capacitor be connected?

CHAPTER 10
Practical Transformers

10.0 Introduction

In Chapter 9 we studied the ideal transformer and discovered its basic properties. However, in the real world transformers are not ideal and so our simple analysis must be modified to take this into account. Thus, the windings of practical transformers have resistance and the cores are not infinitely permeable. Furthermore, the flux produced by the primary is not completely captured by the secondary. Consequently, the leakage flux must be taken into account. And finally, the iron cores produce eddy-current and hysteresis losses, which contribute to the temperature rise of the transformer.

In this chapter we discover that the properties of a practical transformer can be described by an equivalent circuit comprising an ideal transformer and resistances and reactances. The equivalent circuit is developed from fundamental concepts. This enables us to calculate such characteristics as voltage regulation and the behavior of transformers that are connected in parallel. The per-unit method is also used to illustrate its mode of application.

10.1 Ideal transformer with an imperfect core

The ideal transformer studied in the previous chapter had an infinitely permeable core. What happens if such a perfect core is replaced by an iron core having hysteresis and eddy-current losses and whose permeability is rather low? We can represent these imperfections by two circuit elements R_m and X_m in parallel with the primary terminals of the ideal transformer (Fig. 10.1a). The primary is excited by a source E_g that produces a voltage E_1.

The resistance R_m represents the iron losses and the resulting heat they produce. To furnish these losses a small current I_f is drawn from the line. This current is in phase with E_1 (Fig. 10.1b).

The magnetizing reactance X_m is a measure of the permeability of the transformer core. Thus, if the permeability is low, X_m is relatively low. The current I_m flowing through X_m represents the magnetizing current needed to create the flux Φ_m in the core. This current lags 90° behind E_1.

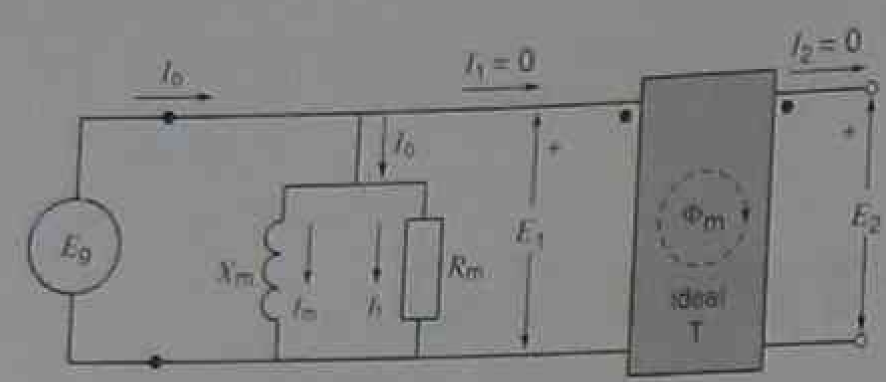

Figure 10.1a
An imperfect core represented by a reactance X_m and a resistance R_m.

The values of the impedances R_m and X_m can be found experimentally by connecting the transformer to an ac source under no-load conditions and measuring the active power and reactive power it absorbs. The following equations then apply:

$$R_m = E_1^2/P_m \quad (10.1)$$

$$X_m = E_1^2/Q_m \quad (10.2)$$

where

R_m = resistance representing the iron losses [Ω]
X_m = magnetizing reactance of the primary winding [Ω]
E_1 = primary voltage [V]
P_m = iron losses [W]
Q_m = reactive power needed to set up the mutual flux Φ_m [var]

The total current needed to produce the flux Φ_m in an imperfect core is equal to the phasor sum of I_f and I_m. It is called the *exciting current* I_o. It is usually a small percentage of the full-load current. The phasor diagram at no-load for this less-than-ideal

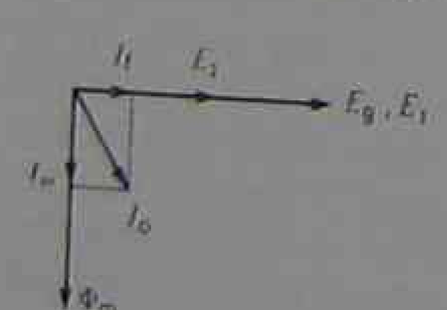

Figure 10.1b
Phasor diagram of a practical transformer at no-load.

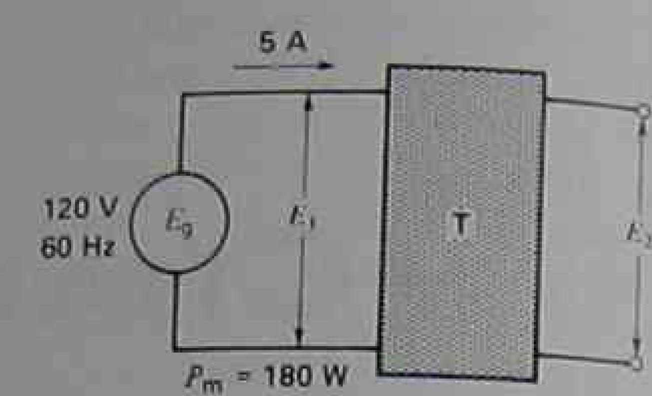

Figure 10.2a
See Example 10-1.

transformer is shown in Fig. 10.1b. The peak value of the mutual flux Φ_m is again given by Eq. 9.2:

$$\Phi_m = E_1/(4.44\, fN_1) \quad (9.2)$$

Example 10-1

A large transformer operating at no-load draws an exciting current I_o of 5 A when the primary is connected to a 120 V, 60 Hz source (Fig. 10.2a). From a wattmeter test it is known that the iron losses are equal to 180 W.*

Calculate

a. The reactive power absorbed by the core
b. The value of R_m and X_m
c. The value of I_f, I_m, and I_o

Solution

a. The apparent power supplied to the core is

$$S_m = E_1 I_o = 120 \times 5$$
$$= 600 \text{ VA}$$

The iron losses are

$$P_m = 180 \text{ W}$$

The reactive power absorbed by the core is

$$Q_m = \sqrt{S_m^2 - P_m^2} = \sqrt{600^2 - 180^2}$$
$$= 572 \text{ var}$$

* Iron losses are discussed in Sections 2.26 to 2.29.

b. The impedance corresponding to the iron losses is

$$R_m = E_1^2/P_m = 120^2/180$$
$$= 80\ \Omega$$

The magnetizing reactance is

$$X_m = E_1^2/Q_m = 120^2/572$$
$$= 25.2\ \Omega$$

c. The current needed to supply the iron losses is

$$I_f = E_1/R_m = 120/80$$
$$= 1.5 \text{ A}$$

The magnetizing current is

$$I_m = E_1/X_m = 120/25.2$$
$$= 4.8 \text{ A}$$

The exciting current I_o is

$$I_o = \sqrt{I_f^2 + I_m^2} = \sqrt{1.5^2 + 4.8^2}$$
$$= 5 \text{ A}$$

The phasor diagram is given in Fig. 10.2b.

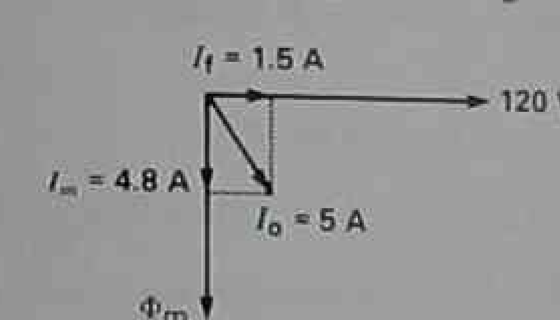

Figure 10.2b
Phasor diagram.

10.2 Ideal transformer with loose coupling

We have just seen how an ideal transformer behaves when it has an imperfect core. We now assume a transformer having a perfect core but rather loose coupling between its primary and secondary windings. We also assume that the primary and secondary windings have negligible resistance and the turns are N_1, N_2.

Consider the transformer in Fig. 10.3 connected to a source E_g and operating at no-load. The voltage

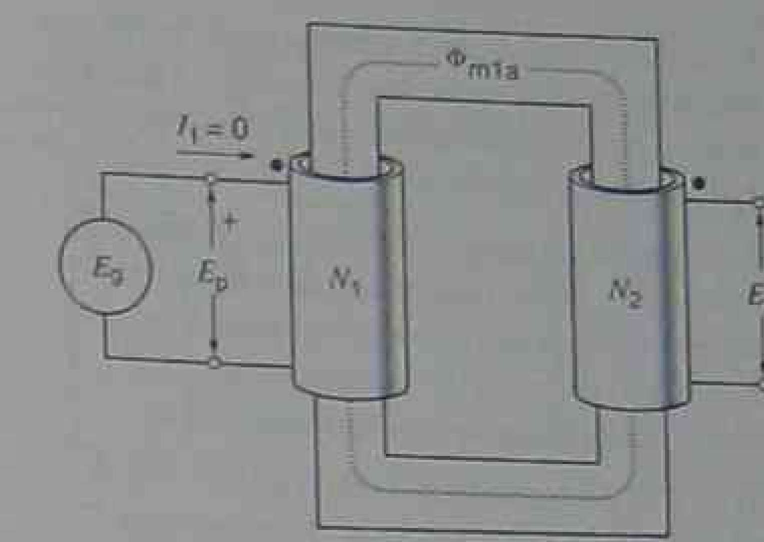

Figure 10.3
Transformer with infinitely permeable core at no-load.

across the primary is E_p and it sets up a mutual flux Φ_{m1a} in the core. This flux lags 90° behind E_p and its peak value is given by $\Phi_{m1a} = E_p/(4.44\, fN_1)$. Because the core is infinitely permeable and because it has no losses, the no-load current $I_1 = 0$. The voltage E_2 is given by $E_2 = (N_2/N_1)\, E_p$. Owing to the current being zero, no mmf is available to drive flux through the air; consequently, there is no leakage flux linking with the primary.

Let us now connect a load Z across the secondary, keeping the source voltage E_p fixed (Fig. 10.4). This simple operation sets off a train of events which we list as follows:

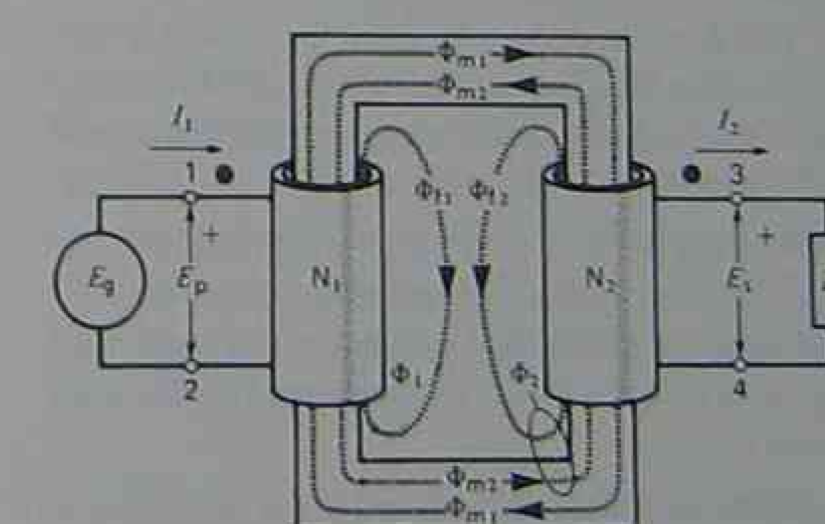

Figure 10.4
Mutual fluxes and leakage fluxes produced by a transformer under load. The leakage fluxes are due to the imperfect coupling between the coils.

1. Currents I_1 and I_2 immediately begin to flow in the primary and secondary windings. They are related by the ideal-transformer equation $I_1/I_2 = N_2/N_1$; hence $N_1I_1 = N_2I_2$.
2. I_2 produces an mmf N_2I_2 while I_1 produces an mmf N_1I_1. These magnetomotive forces are equal and in direct opposition because when I_1 flows into the polarity-marked terminal 1, I_2 flows out of polarity-marked terminal 3.
3. The mmf N_2I_2 produces a total ac flux Φ_2. A portion of Φ_2 (Φ_{m2}) links with the primary winding while another portion (Φ_{f2}) does not. Flux Φ_{f2} is called the *secondary leakage flux.*
4. Similarly, the mmf N_1I_1 produces a total ac flux Φ_1. A portion of Φ_1 (Φ_{m1}) links with the secondary winding, while another portion (Φ_{f1}) does not. Flux Φ_{f1} is called the *primary leakage flux.*

The magnetomotive forces due to I_1 and I_2 upset the magnetic field Φ_{m1a} that existed in the core before the load was connected. The question is, how can we analyze this new situation?
Referring to Fig. 10.4, we reason as follows:

First, the total flux produced by I_1 is composed of two parts: a new mutual flux Φ_{m1} and a leakage flux Φ_{f1}. (The mutual flux Φ_{m1} in Fig. 10.4 is not the same as Φ_{m1a} in Fig. 10.3.)

Second, the total flux produced by I_2 is composed of a mutual flux Φ_{m2} and a leakage flux Φ_{f2}.

Third, we combine Φ_{m1} and Φ_{m2} into a single mutual flux Φ_m (Fig. 10.5). This mutual flux is created by the joint action of the primary and secondary mmfs.

Fourth, we note that the primary leakage flux Φ_{f1} is created by N_1I_1, while the secondary leakage flux is created by N_2I_2. Consequently, leakage flux Φ_{f1} is in phase with I_1, and leakage flux Φ_{f2} is in phase with I_2.

Fifth, the voltage E_s induced in the secondary is actually composed of two parts:

1. A voltage E_{f2} induced by leakage flux Φ_{f2} and given by

$$E_{f2} = 4.44\, fN_2\Phi_{f2} \tag{10.3}$$

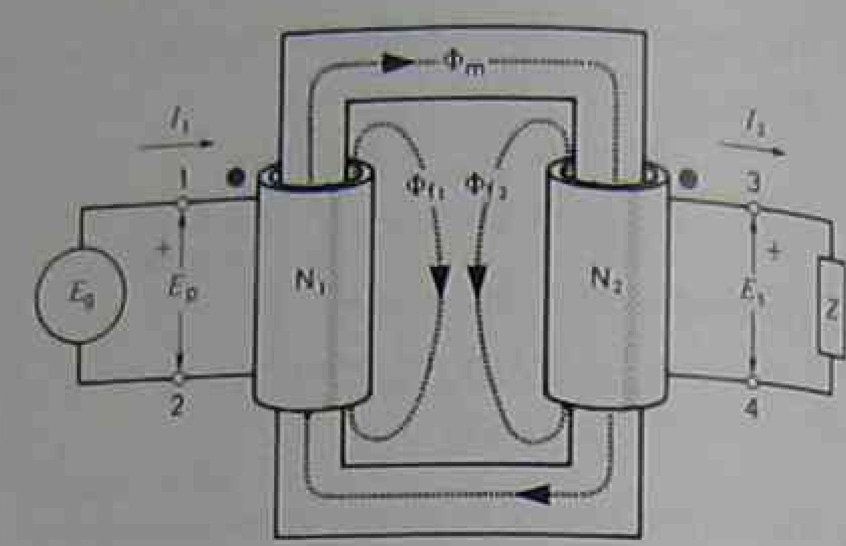

Figure 10.5
A transformer possesses two leakage fluxes and a mutual flux.

2. A voltage E_2 induced by mutual flux Φ_m and given by

$$E_2 = 4.44\, fN_2\Phi_m \tag{10.4}$$

In general, E_{f2} and E_2 are not in phase.
Similarly, the voltage E_p induced in the primary is composed of two parts:

1. A voltage E_{f1} induced by leakage flux Φ_{f1} and given by

$$E_{f1} = 4.44\, fN_1\Phi_{f1} \tag{10.5}$$

2. A voltage E_1 induced by mutual flux Φ_m and given by

$$E_1 = 4.44\, fN_1\Phi_m \tag{10.6}$$

Sixth, induced voltage E_p = applied voltage E_g.

Using these six basic facts, we now proceed to develop the equivalent circuit of the transformer.

10.3 Primary and secondary leakage reactance

We can better identify the four induced voltages E_1, E_2, E_{f1}, and E_{f2} by rearranging the transformer circuit as shown in Fig. 10.6. Thus, the secondary winding is drawn twice to show even more clearly that the N_2 turns are linked by two fluxes, Φ_{f2} and Φ_m. This rearrangement does not change the value of the induced voltages, but it does make each voltage stand out by itself. Thus, it becomes clear that

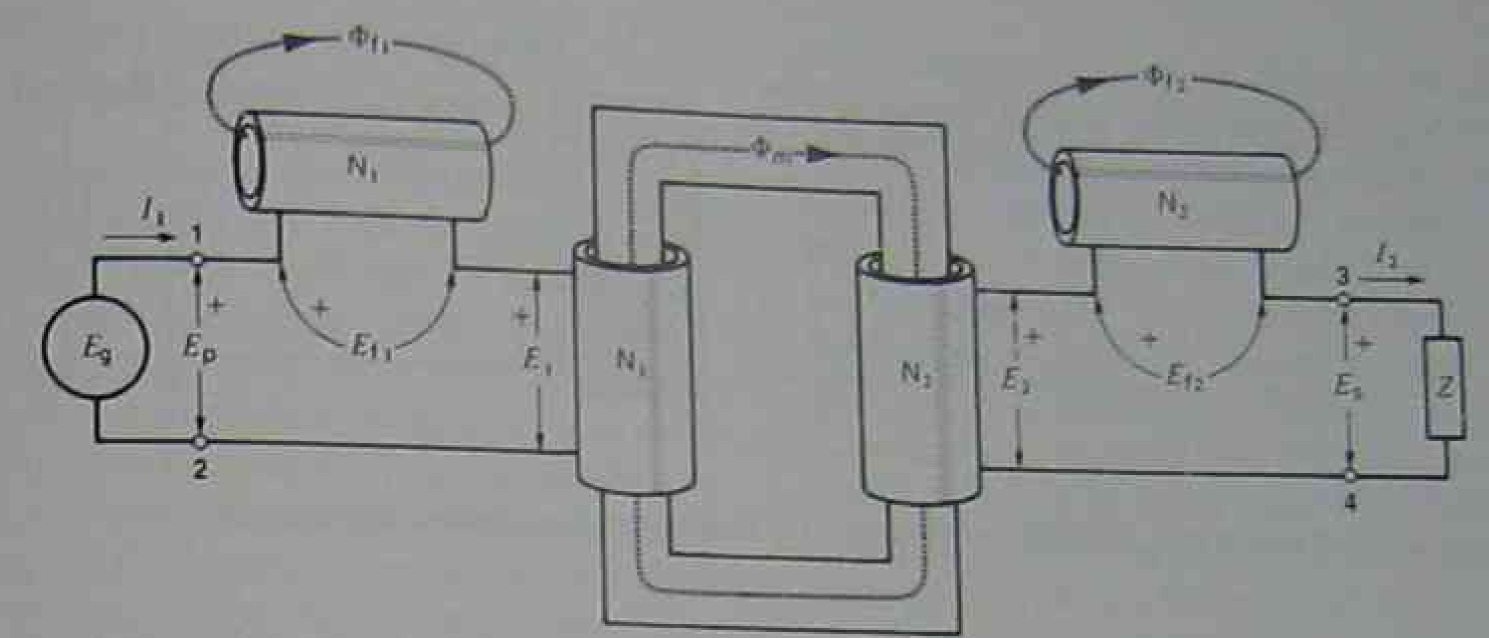

Figure 10.6
Separating the various induced voltages due to the mutual flux and the leakage fluxes.

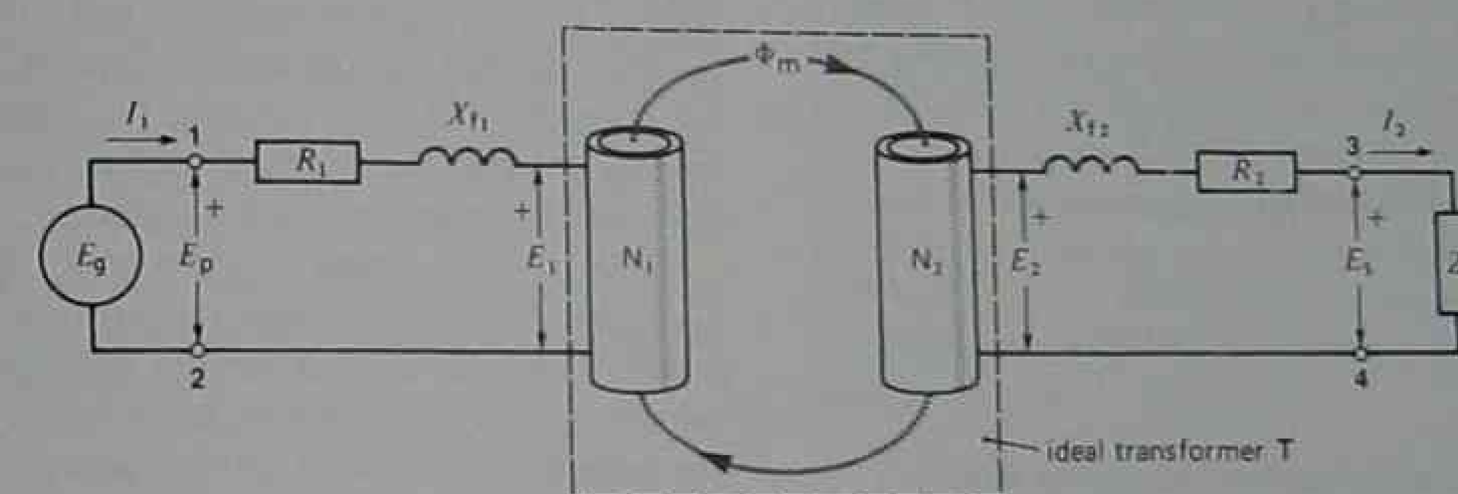

Figure 10.7
Resistance and leakage reactance of the primary and secondary windings.

E_{f2} is really a voltage drop across a reactance. This *secondary leakage reactance* X_{f2} is given by

$$X_{f2} = E_{f2}/I_2 \tag{10.7}$$

The primary winding is also shown twice, to separate E_1 from E_{f1}. Again, it is clear that E_{f1} is simply a voltage drop across a reactance. This *primary leakage reactance* X_{f1} is given by

$$X_{f1} = E_{f1}/I_1 \tag{10.8}$$

The primary and secondary leakage reactances are shown in Figure 10.7. We have also added the primary and secondary winding resistances R_1 and R_2, which, of course, act in series with the respective windings.

Example 10-2

The secondary winding of a transformer possesses 180 turns. When the transformer is under load, the secondary current has an effective value of 18 A, 60 Hz. Furthermore, the mutual Φ_m has a peak value of 20 mWb. The secondary leakage flux Φ_{f2} has a peak value of 3 mWb.

Calculate

a. The voltage induced in the secondary winding by its leakage flux
b. The value of the secondary leakage reactance
c. The value of E_2 induced by the mutual flux Φ_m

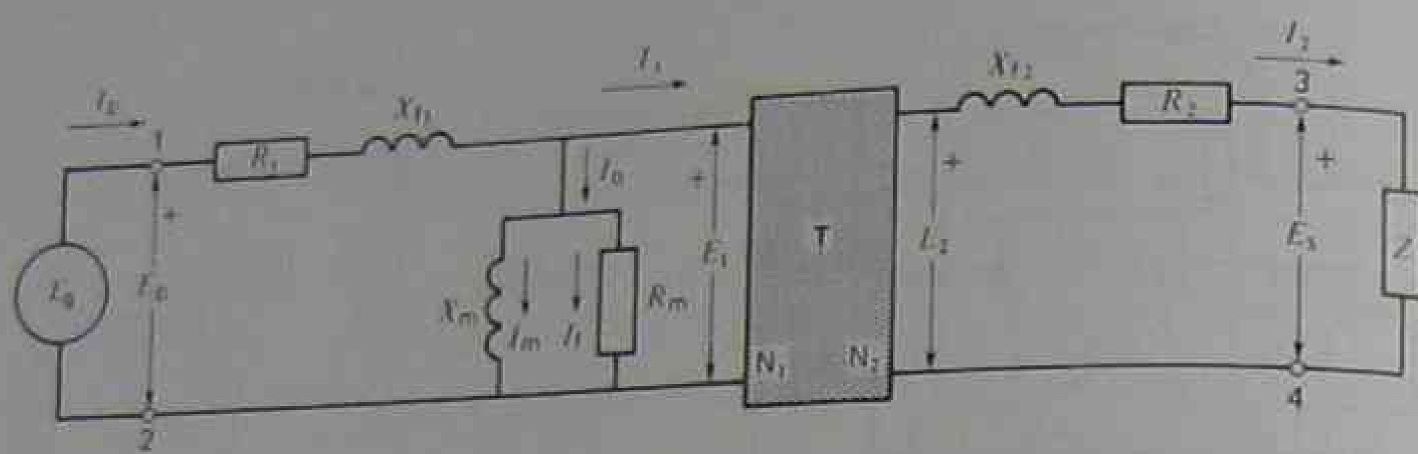

Figure 10.8
Complete equivalent circuit of a practical transformer. The shaded box T is an ideal transformer.

Solution

a. The effective voltage induced by the secondary leakage flux is

$$E_{f2} = 4.44\, fN_2\Phi_{f2} \qquad (10.3)$$
$$= 4.44 \times 60 \times 180 \times 0.003$$
$$= 143.9 \text{ V}$$

b. The secondary leakage reactance is

$$X_{f2} = E_{f2}/I_2 \qquad (10.7)$$
$$= 143.9/18$$
$$= 8\ \Omega$$

c. The voltage induced by the mutual flux is

$$E_2 = 4.44\, fN_2\Phi_m \qquad (10.4)$$
$$= 4.44 \times 60 \times 180 \times 0.02$$
$$= 959 \text{ V}$$

10.4 Equivalent circuit of a practical transformer

The circuit of Fig. 10.7 is composed of resistive and inductive elements (R_1, R_2, X_{f1}, X_{f2}, Z) coupled together by a mutual flux Φ_m, which links the primary and secondary windings. The leakage-free magnetic coupling enclosed in the dotted square is actually an ideal transformer. It possesses the same properties and obeys the same rules as the ideal transformer discussed in Chapter 9. For example, we can shift impedances to the primary side by multiplying their values by $(N_1/N_2)^2$, as we did before.

If we add circuit elements X_m and R_m to represent a practical core, we obtain the complete equivalent circuit of a practical transformer (Fig. 10.8). In this circuit T is an ideal transformer, but only the primary and secondary terminals 1-2 and 3-4 are accessible; all other components are "buried" inside the transformer itself. However, by appropriate tests we can find the values of all the circuit elements that make up a practical transformer.

Table 10A shows typical values of R_1, R_2, X_{f1}, X_{f2}, X_m and R_m for transformers ranging from 1 kVA to 400 MVA. The nominal primary and secondary voltages E_{np} and E_{ns} range from 460 V to 424 000 V. The corresponding primary and secondary currents I_{np} and I_{ns} range from 0.417 A to 29 000 A.

The exciting current I_o for the various transformers is also shown. It is always much smaller than the rated primary current I_{np}.

Note that in each case $E_{np}I_{np} = E_{ns}I_{ns} = S_n$, where S_n is the rated power of the transformer.

TABLE 10A ACTUAL TRANSFORMER VALUES

S_n	kVA	1	10	100	1000	400000
E_{np}	V	2400	2400	12470	69000	13800
E_{ns}	V	460	347	600	6900	424000
I_{np}	A	0.417	4.17	8.02	14.5	29000
I_{ns}	A	2.17	28.8	167	145	943
R_1	Ω	58.0	5.16	11.6	27.2	0.0003
R_2	Ω	1.9	0.095	0.024	0.25	0.354
X_{f1}	Ω	32	4.3	39	151	0.028
X_{f2}	Ω	1.16	0.09	0.09	1.5	27
X_m	Ω	200000	29000	150000	505000	460
R_m	Ω	400000	51000	220000	432000	317
I_o	A	0.0134	0.0952	0.101	0.210	52.9

10.5 Construction of a power transformer

Power transformers are usually designed so that their characteristics approach those of an ideal transformer. Thus, to attain high permeability, the core is made of iron (Fig. 10.9a). The resulting magnetizing current I_m is at least 5000 times smaller than it would be if an air core were used. Furthermore, to keep the iron losses down, the core is laminated, and high resistivity, high-grade silicon steel is used. Consequently, the current I_f needed to supply the iron losses is usually 2 to 4 times smaller than I_m.

Leakage reactances X_{f1} and X_{f2} are made as small as possible by winding the primary and secondary coils on top of each other, and by spacing them as closely together as insulation considerations will permit. The coils are carefully insulated from each other and from the core. Such tight coupling between the coils means that the secondary voltage at no-load is almost exactly equal to N_2/N_1 times the primary voltage. It also guarantees good voltage regulation when a load is connected to the secondary terminals.

Winding resistances R_1 and R_2 are kept low, both to reduce the I^2R loss and resulting heat and to ensure high efficiency. Figure 10.9a is a simplified version of a power transformer in which the primary and secondary are wound on one leg. In practice, the primary and secondary coils are distributed over both core legs in order to reduce the amount of copper. For the same reason, in larger transformers the cross section of the laminated iron core is not square (as shown) but is built up so as to be nearly round. (See Fig. 12.10a).

Figure 10.9b shows how the laminations of a small transformer are stacked to build up the core. Figure 10.9c shows the primary winding of a much bigger transformer.

The number of turns on the primary and secondary windings depends upon their respective voltages. A high-voltage winding has far more turns than a low-voltage winding. On the other hand, the current in a HV winding is much smaller, enabling us to use a smaller size conductor. As a result, the amount of copper in the primary and secondary windings is about the same. In practice, the outer coil (coil 2, in Fig. 10.9a) weighs more because the length per turn is greater. Aluminum or copper conductors are used.

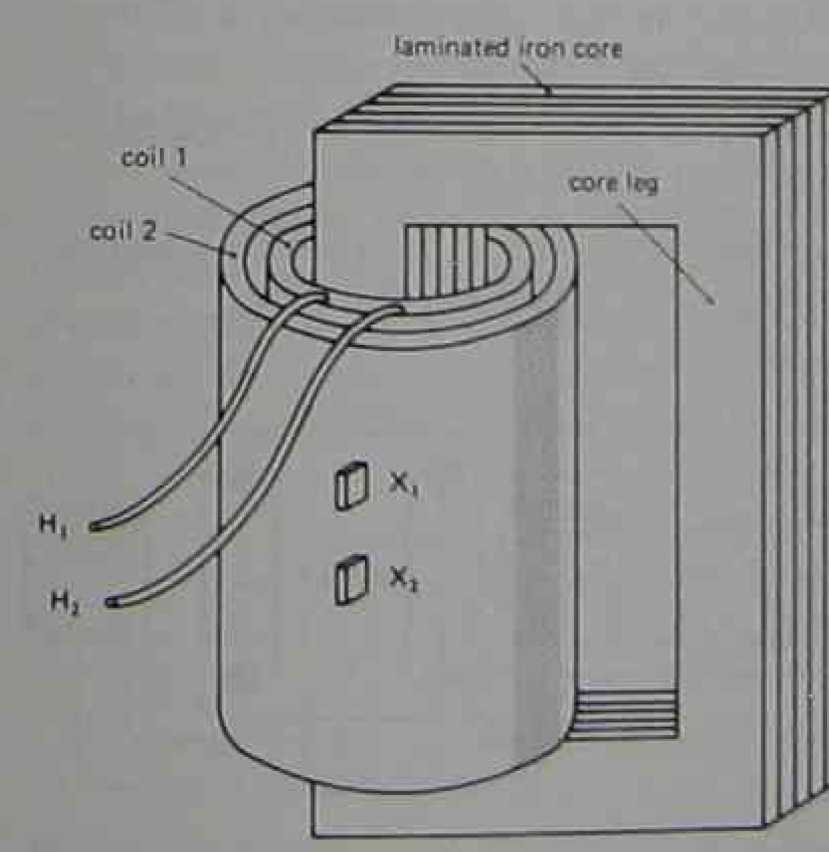

Figure 10.9a
Construction of a simple transformer.

Figure 10.9b
Stacking laminations inside a coil.

Figure 10.9c
Primary winding of a large transformer; rating 128 kV, 290 A.
(Courtesy ABB)

A transformer is reversible in the sense that either winding can be used as the primary winding, where *primary* means the winding that is connected to the source.

10.6 Standard terminal markings

We saw in Section 9.4 that the polarity of a transformer can be shown by means of dots on the primary and secondary terminals. This type of marking is used on instrument transformers. On power transformers, however, the terminals are designated by the symbols H_1 and H_2 for the high-voltage (HV) winding and by X_1 and X_2 for the low-voltage (LV) winding. By convention, H_1 and X_1 have the same polarity.

Although the polarity is known when the symbols H_1, H_2, X_1, and X_2 are given, in the case of power transformers it is common practice to mount the four terminals on the transformer tank in a standard way so that the transformer has either *additive* or *subtractive* polarity. A transformer is said to have

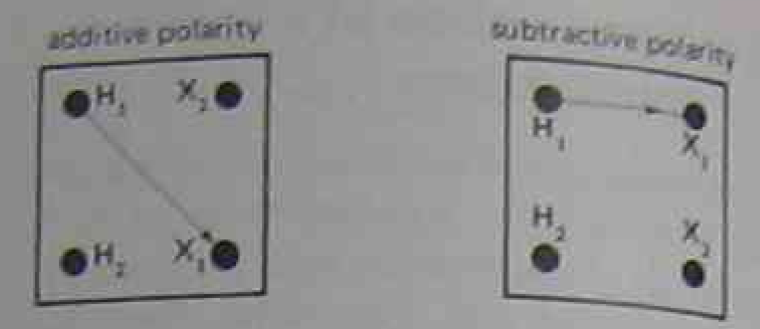

Figure 10.10
Additive and subtractive polarity depend upon the location of the H_1-X_1 terminals.

additive polarity when terminal H_1 is diagonally opposite terminal X_1. Similarly, a transformer has *subtractive* polarity when terminal H_1 is adjacent to terminal X_1 (Fig. 10.10). If we know that a power transformer has additive (or subtractive) polarity, we do not have to identify the terminals by symbols.

Subtractive polarity is standard for all single-phase transformers above 200 kVA, provided the high-voltage winding is rated above 8660 V. All other transformers have additive polarity.

10.7 Polarity tests

To determine whether a transformer possesses additive or subtractive polarity, we proceed as follows (Fig. 10.11):

1. Connect the high-voltage winding to a low-voltage (say 120V) ac source E_g.
2. Connect a jumper J between any two adjacent HV and LV terminals.
3. Connect a voltmeter E_x between the other two adjacent HV and LV terminals.

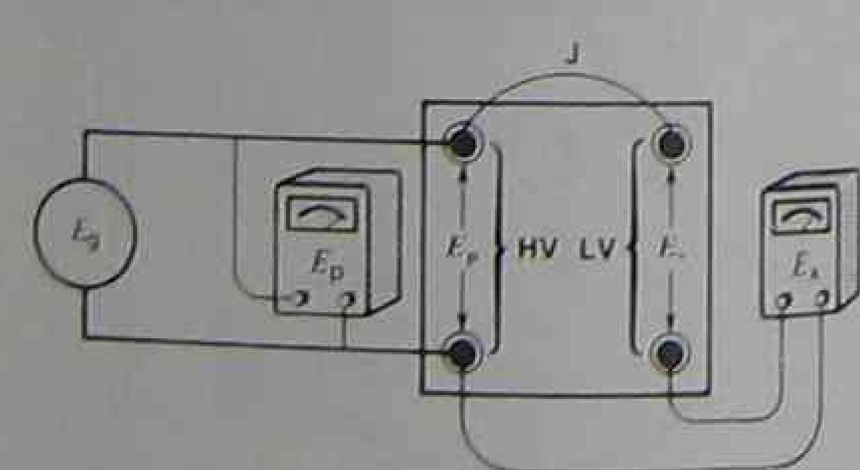

Figure 10.11
Determining the polarity of a transformer using an ac source.

4. Connect another voltmeter E_p across the HV winding. If E_x gives a higher reading than E_p, the polarity is additive. This tells us that H_1 and X_1 are diagonally opposite. On the other hand, if E_x gives a lower reading than E_p, the polarity is subtractive, and terminals H_1 and X_1 are adjacent.

In this polarity text, jumper J effectively connects the secondary voltage E_s in series with the primary voltage E_p. Consequently, E_s either adds to or subtracts from E_p. In other words, $E_x = E_p + E_s$ or $E_x = E_p - E_s$, depending on the polarity. We can now see how the terms additive and subtractive originated.

In making the polarity test, an ordinary 120 V, 60 Hz source can be connected to the HV winding, even though its nominal voltage may be several hundred kilovolts.

Example 10-3

During a polarity test on a 500 kVA, 69 kV/600 V transformer (Fig. 10.11), the following readings were obtained: $E_p = 118$ V, $E_x = 119$ V. Determine the polarity markings of the terminals.

Solution

The polarity is additive because E_x is greater than E_p. Consequently, the HV and LV terminals connected by the jumper must respectively be labelled H_1 and X_2 (or H_2 and X_1).

Figure 10.12 shows another circuit that may be used to determine the polarity of a transformer. A dc source, in series with an open switch, is connected to the LV winding of the transformer. The transformer terminal connected to the positive side of the source is marked X_1. A dc voltmeter is connected across the HV terminals. When the switch is closed, a voltage is momentarily induced in the HV wind-

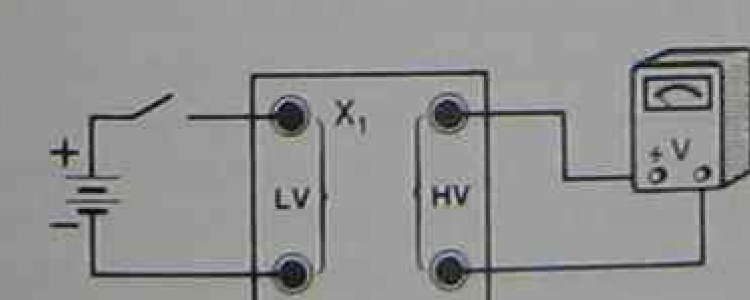

Figure 10.12
Determining the polarity of a transformer using a dc source.

ing. If, at this moment, the pointer of the voltmeter moves upscale, the transformer terminal connected to the (+) terminal of the voltmeter is marked H_1 and the other is marked H_2.

10.8 Transformer taps

Due to voltage drops in transmission lines, the voltage in a particular region of a distribution system may consistently be lower than normal. Thus, a distribution transformer having a ratio of 2400 V/120 V may be connected to a transmission line where the voltage is never higher than 2000 V. Under these conditions the voltage across the secondary is considerably less than 120 V. Incandescent lamps are dim, electric stoves take longer to cook food, and electric motors may stall under moderate overloads.

To correct this problem *taps* are provided on the primary windings of distribution transformers (Fig. 10.13). Taps enable us to change the turns ratio so as to raise the secondary voltage by $4\frac{1}{2}$, 9, or $13\frac{1}{2}$ percent. We can therefore maintain a satisfactory secondary voltage, even though the primary voltage may be $4\frac{1}{2}$, 9, or $13\frac{1}{2}$ percent below normal. Thus, referring to the transformer of Fig. 10.13, if the line voltage is only 2076 V (instead of 2400 V), we would use terminal 1 and tap 5 to obtain 120 V on the secondary side.

Some transformers are designed to change the taps automatically whenever the secondary voltage is above or below a preset level. Such *tap-changing transformers* help maintain the secondary voltage within ±2 percent of its rated value throughout the day.

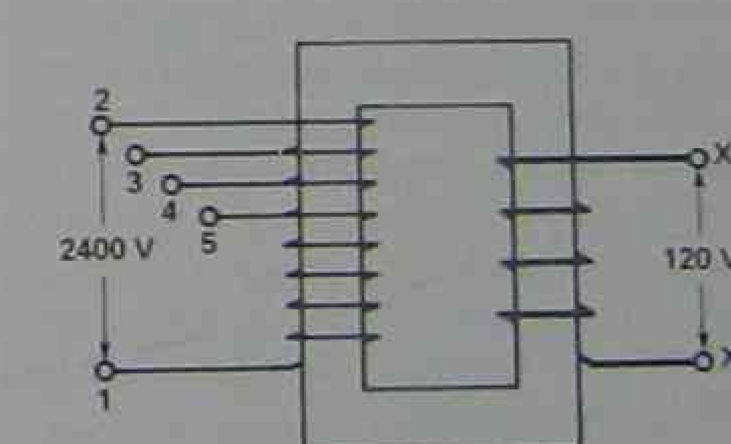

Figure 10.13
Distribution transformer with taps at 2400 V, 2292 V, 2184 V, and 2076 V.

10.9 Losses and transformer rating

Like any electrical machine, a transformer has losses. They are composed of the following:

1. I^2R losses in the windings
2. Hysteresis and eddy-current losses in the core
3. Stray losses due to currents induced in the tank and metal supports by the primary and secondary leakage fluxes

The losses appear in the form of heat and produce 1) an increase in temperature and 2) a drop in efficiency. Under normal operating conditions, the efficiency of transformers is very high; it may reach 99.5 percent for large power transformers.

The heat produced by the iron losses depends upon the peak value of the mutual flux Φ_m, which in turn depends upon the applied voltage. On the other hand, the heat dissipated in the windings depends upon the current they carry. Consequently, to keep the transformer temperature at an acceptable level, we must set limits to both the applied voltage and the current drawn by the load. These two limits determine the *nominal voltage* E_{np} and *nominal current* I_{np} of the transformer winding (primary or secondary).

The *power rating* of a transformer is equal to the product of the nominal voltage times the nominal current of the primary or secondary winding. However, the result is not expressed in watts, because the phase angle between the voltage and current may have any value at all, depending on the nature of the load. Consequently, the power-handling capacity of a transformer is expressed in voltamperes (VA), in kilovoltamperes (kVA) or in megavoltamperes (MVA), depending on the size of the transformer. The temperature rise of a transformer is directly related to the *apparent power* that flows through it. This means that a 500 kVA transformer will get just as hot feeding a 500 kvar inductive load as a 500 kW resistive load.

The rated kVA, frequency, and voltage are always shown on the nameplate. In large transformers the corresponding rated currents are also shown.

Example 10-4

The nameplate of a distribution transformer indicates 250 kVA, 60 Hz, primary 4160 V, secondary 480 V.

a. Calculate the nominal primary and secondary currents.
b. If we apply 2000 V to the 4160 V primary, can we still draw 250 kVA from the transformer?

Solution

a. Nominal current of the 4160 V winding is

$$I_{np} = \frac{\text{nominal } S}{\text{nominal } E_p} = \frac{S_n}{E_{np}} = \frac{250 \times 1000}{4160} = 60 \text{ A}$$

Nominal current of the 480 V winding is

$$I_{ns} = \frac{\text{nominal } S}{\text{nominal } E_s} = \frac{S_n}{E_{ns}} = \frac{250 \times 1000}{480} = 521 \text{ A}$$

b. If we apply 2000 V to the primary, the flux and the iron losses will be lower than normal and the core will be cooler. However, the load current should not exceed its nominal value, otherwise the windings will overheat. Consequently, the maximum power output using this far lower voltage is

$$S = 2000 \text{ V} \times 60 \text{ A} = 120 \text{ kVA}$$

10.10 No-load saturation curve

Let us gradually increase the voltage E_p on the primary of a transformer, with the secondary open-circuited. As the voltage rises, the mutual flux Φ_m increases in direct proportion, in accordance with Eq. 9.2. Exciting current I_o will therefore increase but, when the iron begins to saturate, the magnetizing current I_m has to increase very steeply to produce the required flux. If we draw a graph of E_p versus I_o, we see the dramatic increase in current as we pass the normal operating point (Fig. 10.14). Transformers are usually designed to operate at a peak flux density of about 1.5 T, which corresponds roughly to the knee of the saturation curve. Thus, when nominal voltage is applied to a transformer, the corresponding flux density is about 1.5 T. We can exceed the nominal voltage by perhaps 10 percent, but if we were to apply twice the nominal voltage, the exciting current could become even greater than the nominal full-load current.

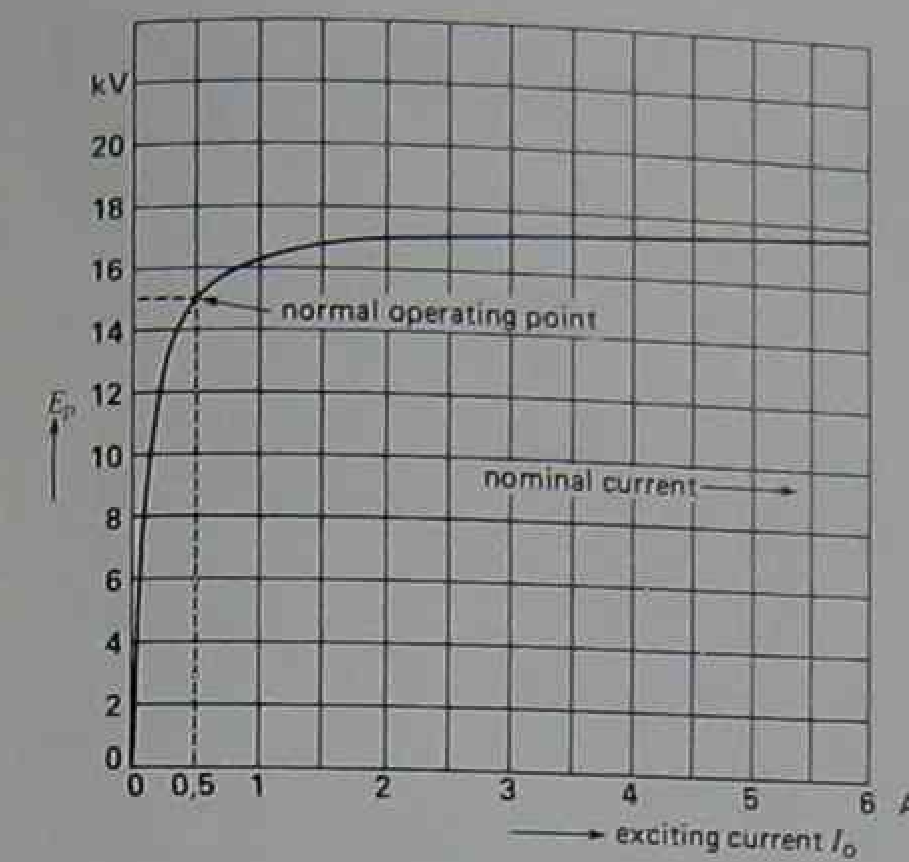

Figure 10.14
No-load saturation curve of a 167 kVA, 14.4 kV/480 V, 60 Hz transformer.

The nonlinear relationship between E_p and I_o shows that the exciting branch (composed of R_m and X_m in Fig. 10.1a) is not as constant as it appears. In effect, although R_m is reasonably constant, X_m decreases rapidly with increasing saturation. However, most transformers operate at close to rated voltage, and so R_m and X_m remain essentially constant.

10.11 Cooling methods

To prevent rapid deterioration of the insulating materials inside a transformer, adequate cooling of the windings and core must be provided.

Indoor transformers below 200 kVA can be directly cooled by the natural flow of the surrounding air. The metallic housing is fitted with ventilating louvres so that convection currents may flow over the windings and around the core (Fig. 10.15). Larger transformers can be built the same way, but forced circulation of clean air must be provided. Such *dry-type* transformers are used inside buildings, away from hostile atmospheres.

Figure 10.15
Single-phase dry-type transformer, type AA, rated at 15 kVA, 600 V/240 V, 60 Hz, insulation class 150°C for indoor use. Height: 600 mm; width: 434 mm; depth: 230 mm; weight: 79.5 kg.
(Courtesy of Hammond)

Distribution transformers below 200 kVA are usually immersed in mineral oil and enclosed in a steel tank. Oil carries the heat away to the tank, where it is dissipated by radiation and convection to the outside air (Fig. 10.16). Oil is a much better insulator than air is; consequently, it is invariably used on high-voltage transformers.

As the power rating increases, external radiators are added to increase the cooling surface of the oil-filled tank (Fig. 10.17). Oil circulates around the transformer windings and moves through the radiators, where the heat is again released to surrounding air. For still higher ratings, cooling fans blow air over the radiators (Fig. 10.18).

For transformers in the megawatt range, cooling may be effected by an oil-water heat exchanger. Hot oil drawn from the transformer tank is pumped to a heat exchanger where it flows through pipes that are

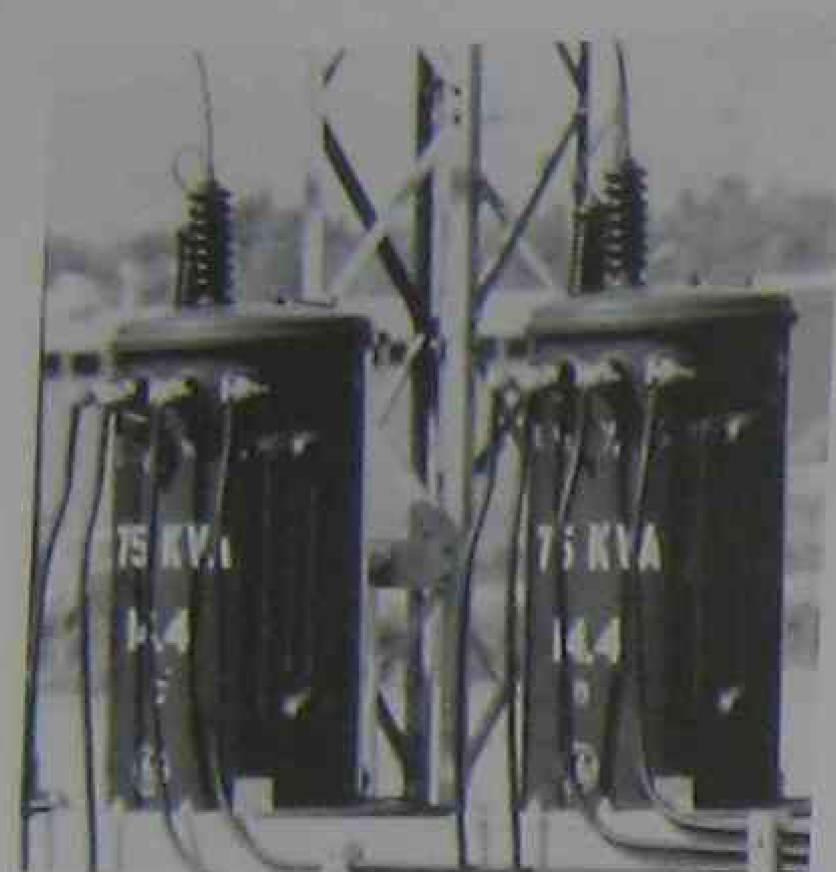

Figure 10.16
Two single-phase transformers, type OA, rated 75 kVA, 14.4 kV/240 V, 60 Hz, 55°C temperature rise, impedance 4.2%. The small radiators at the side increase the effective cooling area.

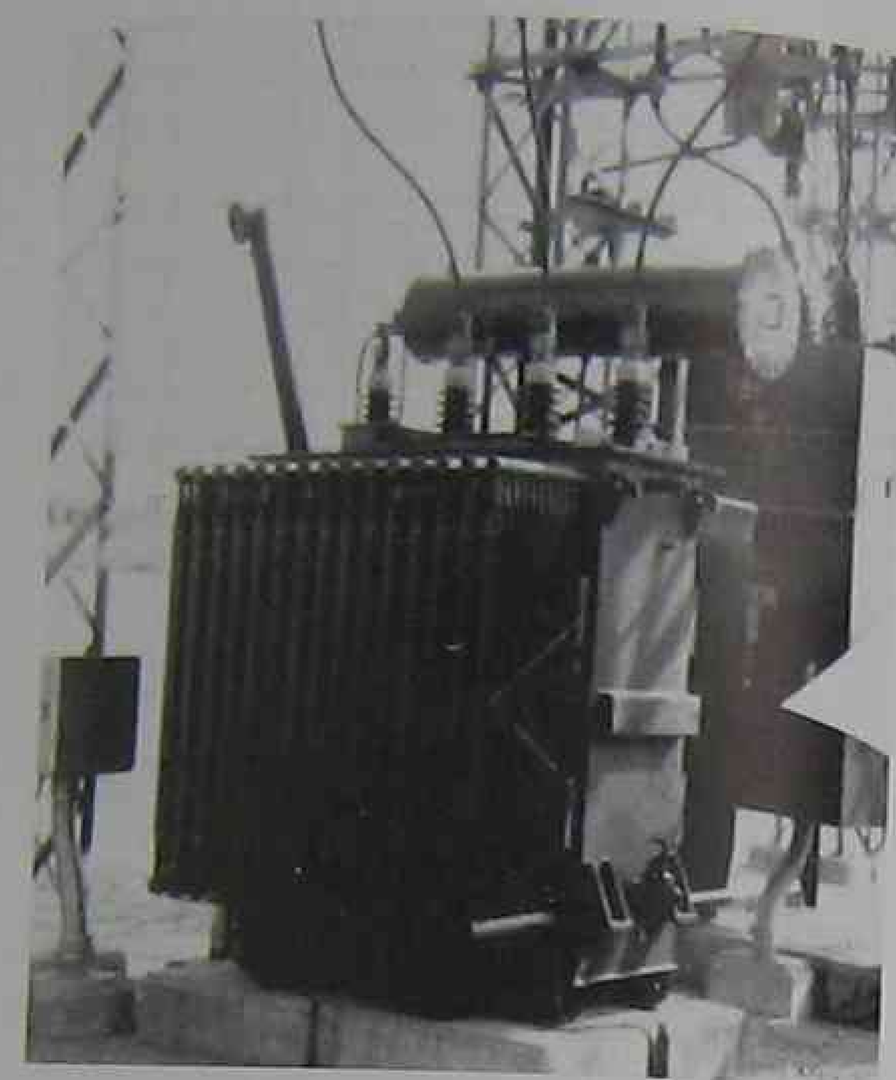

Figure 10.17
Three-phase, type OA grounding transformer, rated 1900 kVA, 26.4 kV, 60 Hz. The power of this transformer is 25 times greater than that of the transformers shown in Fig. 10.16, but it is still self-cooled. Note, however, that the radiators occupy as much room as the transformer itself.

in contact with cool water. Such a heat exchanger is very effective, but also very costly, because the water itself has to be continuously cooled and recirculated.

Some big transformers are designed to have a multiple rating, depending on the method of cooling used. Thus, a transformer may have a triple rating of 18 000/24 000/32 000 kVA depending on whether it is cooled

1. by the natural circulation of air (AO) (18 000 kVA) or
2. by forced-air cooling with fans (FA) (24 000 kVA) or
3. by the forced circulation of oil accompanied by forced-air cooling (FOA) (32 000 kVA).

These elaborate cooling systems are nevertheless economical because they enable a much bigger output from a transformer of a given size and weight (Fig. 10.19).

The type of transformer cooling is designated by the following symbols:

AA–dry-type, self-cooled
AFA–dry-type, forced-air cooled
OA–oil-immersed, self-cooled
OA/FA–oil-immersed, self-cooled/forced-air cooled
AO/FA/FOA–oil-immersed, self-cooled/forced-air cooled/forced-air, forced-oil cooled

The temperature rise by resistance of oil-immersed transformers is either 55°C or 65°C. The tempera-

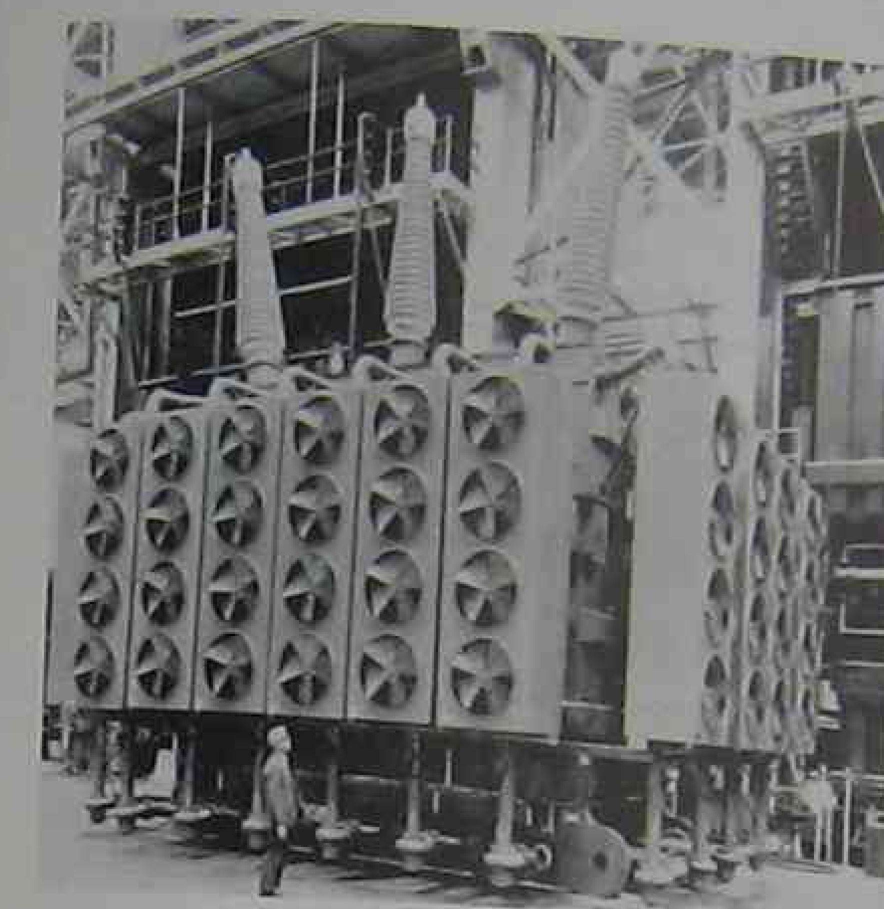

Figure 10.18
Three-phase, type FOA, transformer rated 1300 MVA, 24.5 kV/345 kV, 60 Hz 65°C temperature rise, impedance: 11.5%. This step-up transformer, installed at a nuclear power generating station, is one of the largest units ever built. The forced-oil circulating pumps can be seen just below the cooling fans.
(*Courtesy of Westinghouse*)

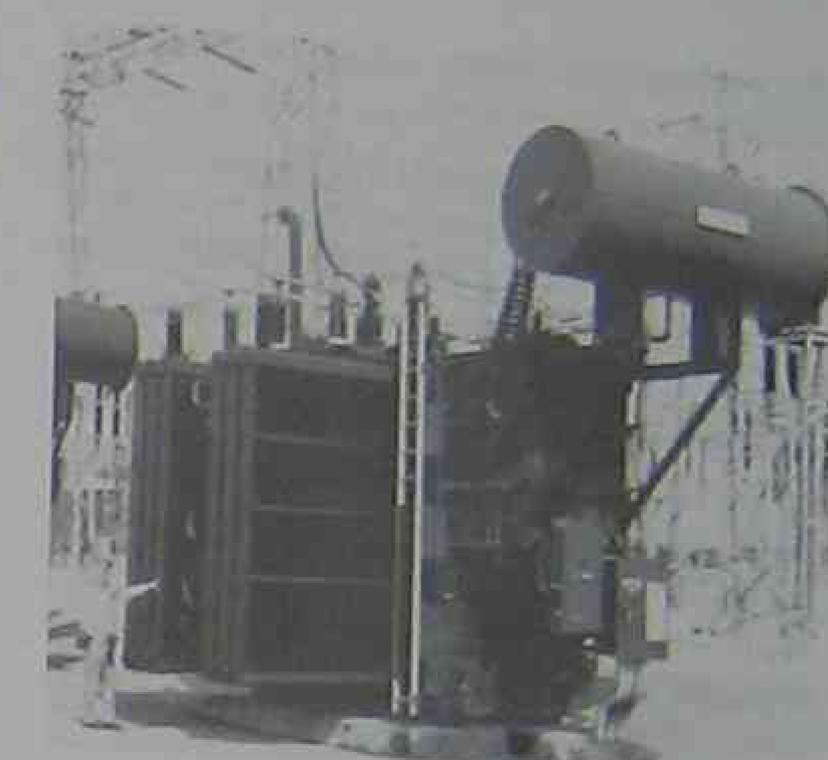

Figure 10.19
Three-phase, type OA/FA/FOA transformer rated 36/48/60 MVA, 225 kV/26.4 kV, 60 Hz, impedance 7.4%. The circular tank enables the oil to expand as the temperature rises and reduces the surface of the oil in contact with air. Other details:

weight of core and coils: 37.7 t
weight of tank and accessories: 28.6 t
weight of coil (44.8 m^3): 38.2 t
Total weight: 104.5 t

ture must be kept low to preserve the quality of the oil. By contrast, the temperature rise of a dry-type transformer may be as high as 180°C, depending on the type of insulation used.

10.12 Simplifying the equivalent circuit

The complete equivalent circuit of the transformer as shown in Fig. 10.8 gives far more detail than is needed in most practical problems. Consequently, let us try to simplify the circuit when the transformer operates 1) at no-load and 2) at full-load.

1. At *no-load* (Fig. 10.20) I_2 is zero and so is I_1 because T is an ideal transformer. Consequently, only the exciting current I_o flows in R_1 and X_{f1}. These impedances are so small that the voltage drop across them is negligible. Furthermore, the current in R_2 and X_{f2} is zero. We can, therefore, neglect these four impedances, giving us the much simpler circuit of Fig. 10.21. The turns ratio, $a = N_1/N_2$, is obviously equal to the ratio of

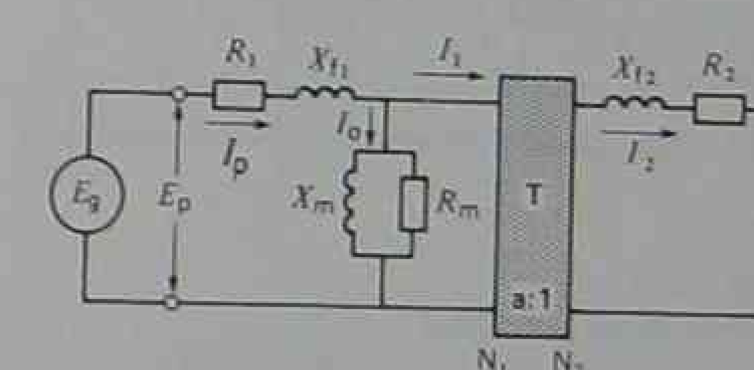

Figure 10.20
Complete equivalent circuit of a transformer at no-load.

the primary to secondary voltages E_p/E_s measured across the terminals.

2. At *full-load* I_p is at least 20 times larger than I_o. Consequently, we can neglect I_o and the corresponding magnetizing branch. The resulting circuit is shown in Fig. 10.22. This simplified circuit may be used even when the load is only 10 percent of the rated capacity of the transformer.

We can further simplify the circuit by shifting everything to the primary side, thus eliminating transformer T (Fig. 10.23). This technique was explained in Section 9.10. Then, by summing the respective resistances and reactances, we obtain the circuit of Fig. 10.24. In this circuit

$$R_p = R_1 + a^2R_2 \tag{10.9}$$

$$X_p = X_{f1} + a^2X_{f2} \tag{10.10}$$

where

R_p = total transformer resistance referred to the primary side

X_p = total transformer leakage reactance referred to the primary side

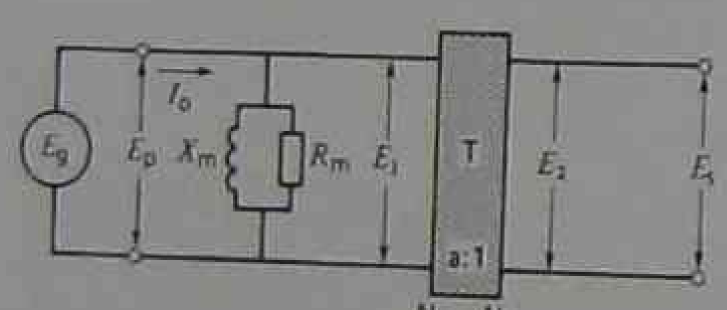

Figure 10.21
Simplified circuit at no-load.

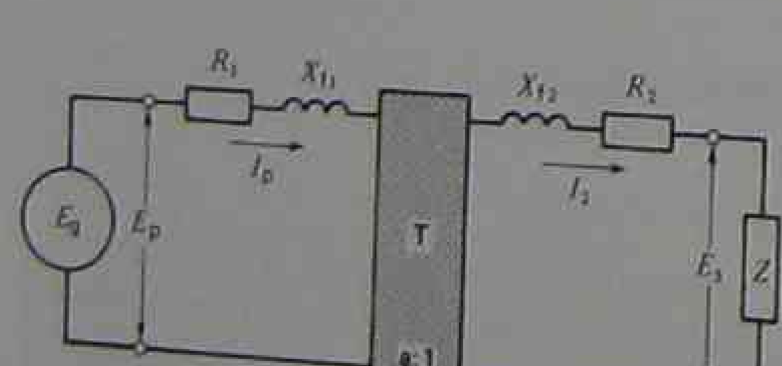

Figure 10.22
Simplified equivalent circuit of a transformer at full-load.

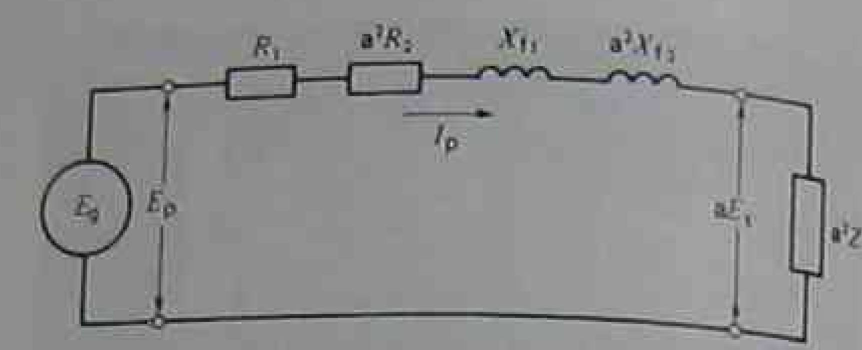

Figure 10.23
Equivalent circuit with impedances shifted to the primary side.

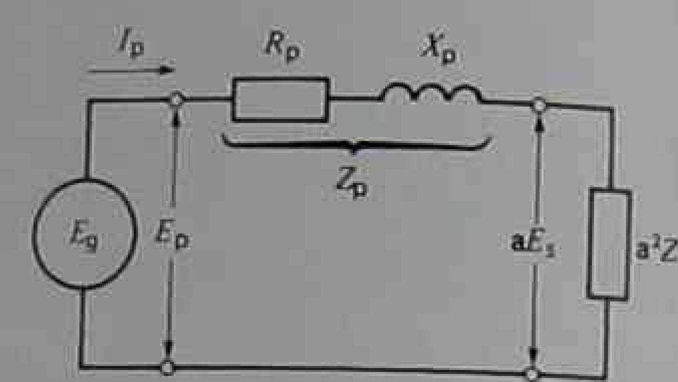

Figure 10.24
The internal impedance of a large transformer is mainly reactive.

The combination of R_p and X_p constitutes the total transformer impedance Z_p referred to the primary side. From Eq. 2.12 we have

$$Z_p = \sqrt{R_p^2 + X_p^2} \tag{10.11}$$

Impedance Z_p is one of the important parameters of the transformer. It produces an internal voltage drop when the transformer is loaded. Consequently, Z_p affects the voltage regulation of the transformer.

Transformers above 500 kVA possess a leakage reactance X_p that is at least five times greater than R_p. In such transformers we can neglect R_p, as far as voltages and currents are concerned.* The equivalent circuit is thus reduced to a simple reactance X_p between the source and the load (Fig. 10.25). It is quite remarkable that the relatively complex circuit of Fig. 10.8 can be reduced to a simple reactance in series with the load.

* From the standpoint of temperature rise and efficiency, R_p can never be neglected.

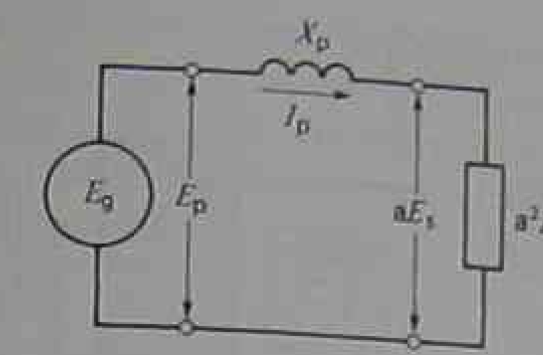

Figure 10.25
The internal impedance of a large transformer is mainly reactive.

10.13 Voltage regulation

An important attribute of a transformer is its voltage regulation. With the primary impressed voltage held constant at its rated value, the voltage regulation, in percent, is defined by the equation:

$$\text{voltage regulation} = \frac{E_{NL} - E_{FL}}{E_{FL}} \times 100 \tag{10.12}$$

where

E_{NL} = secondary voltage at no-load [V]

E_{FL} = secondary voltage at full-load [V]

The voltage regulation depends upon the power factor of the load. Consequently, the power factor must be specified. If the load is capacitive, the no-load voltage may exceed the full-load voltage, in which case the voltage regulation is negative.

Example 10-5

A single-phase transformer rated at 3000 kVA, 69 kV/4.16 kV, 60 Hz has a total internal impedance Z_p of 127 Ω, referred to the primary side.

Calculate

a. The rated primary and secondary currents
b. The voltage regulation from no-load to full-load for a 2000 kW resistive load, knowing that the primary supply voltage is fixed at 69 kV
c. The primary and secondary currents if the secondary is accidentally short-circuited.

Solution

a. Rated primary current

$$I_{np} = S_n/E_{np} = 3\,000\,000/69\,000 = 43.5 \text{ A}$$

Rated secondary current

$$I_{ns} = S_n/E_{ns} = 3\,000\,000/4160 = 721 \text{ A}$$

b. Because the transformer exceeds 500 kVA, the windings have negligible resistance compared to their leakage reactance; we can therefore write

$$Z_p = X_p = 127\ \Omega$$

Referring to Fig. 10.26a, the approximate impedance of the 2000 kW load on the secondary side is

$$Z = E_s^2/P = 4160^2/2\,000\,000$$
$$= 8.65\ \Omega$$

Load impedance referred to primary side:

$$a^2Z = (69/4.16)^2 \times 8.65 = 2380\ \Omega$$

Referring to Fig. 10.26b we have

$$I_p = 69\,000/\sqrt{127^2 + 2380^2}$$
$$= 28.95 \text{ A}$$
$$aE_s = (a^2Z)\,I_p = 2380 \times 28.95$$
$$= 68\,902 \text{ V}$$
$$E_s = 68\,902 \times (4.16/69) = 4154 \text{ V}$$

Because the primary voltage is held constant at 69 kV, it follows that the secondary voltage at no-load is 4160 V.

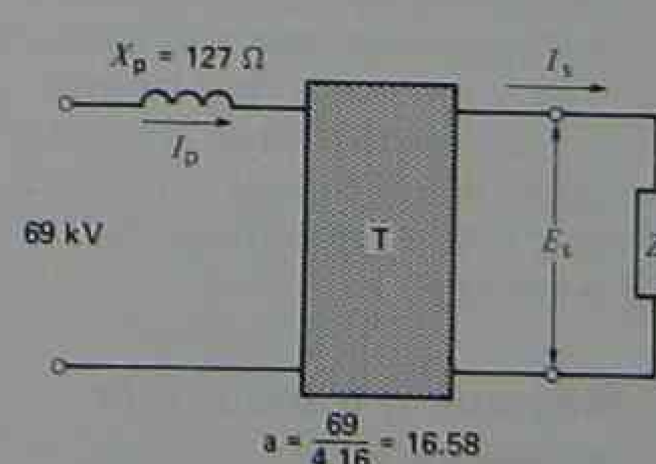

Figure 10.26a
See Example 10-7.

Voltage regulation is:

$$\text{voltage regulation} = \frac{E_{NL} - E_{FL}}{E_{FL}} \times 100 \qquad (10.12)$$

$$= \frac{4160 - 4154}{4154} \times 100$$

$$= 0.14\%$$

The voltage regulation is excellent.

c. Referring again to Fig. 10.26b, if the secondary is accidentally short-circuited, $aE_s = 0$ and so

$$I_p = E_p/X_p = 69\,000/127$$
$$= 543 \text{ A}$$

The corresponding current I_s on the secondary side:

$$I_s = aI_p = (69/4.16) \times 543$$
$$= 9006 \text{ A}$$

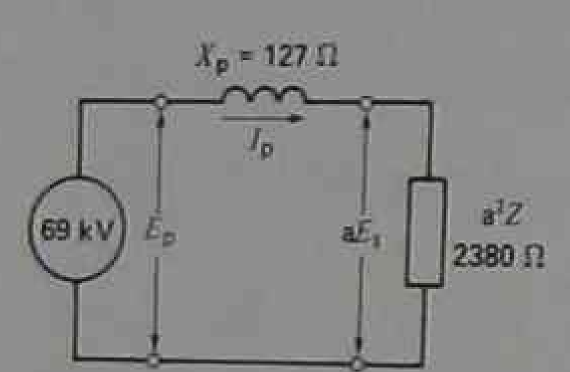

Figure 10.26b
See Example 10-7.

The short-circuit currents in both the primary and secondary windings are 12.5 times greater than the rated values. The I^2R losses are, therefore, 12.5^2 or 156 times greater than normal. The circuit-breaker or fuse protecting the transformer must open immediately to prevent overheating. Very powerful electromagnetic forces are also set up. They, too, are 156 times greater than normal and, unless the windings are firmly braced and supported, they may be damaged or torn apart.

10.14 Measuring transformer impedances

For a given transformer, we can determine the actual values of X_m, R_m, R_p and X_p shown in Figs. 10.21 and 10.24 by means of an open-circuit and a short-circuit test.

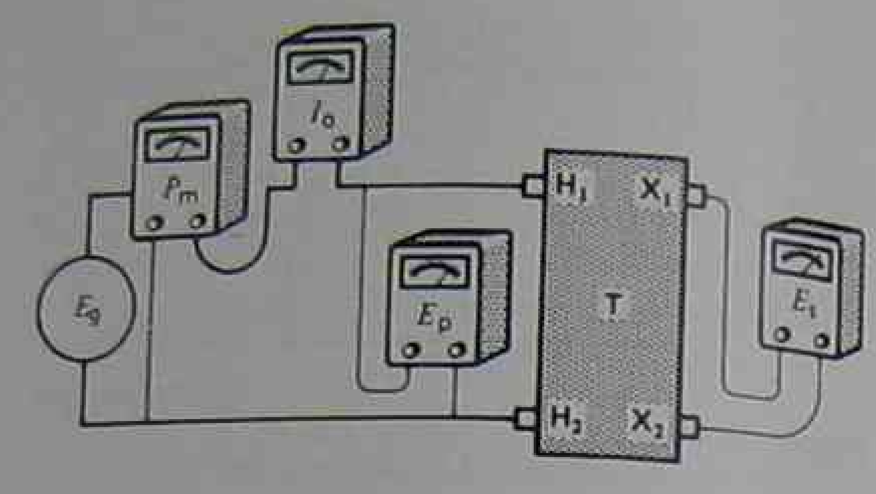

Figure 10.27
Open-circuit test and determination of R_m, X_m, and turns ratio.

During the *open-circuit test*, rated voltage is applied to the primary winding and current I_o, voltage E_p, and active power P_m are measured (Fig. 10.27). The secondary open-circuit voltage E_s is also measured. These test results give us the following information:

active power absorbed by core $= P_m$

apparent power absorbed by core $= S_m = E_pI_o$

reactive power absorbed by core $= Q_m$

$$\text{where } Q_m = \sqrt{S_m^2 - P_m^2}$$

Resistance R_m corresponding to the core loss is

$$R_m = E_p^2/P_m \qquad (10.1)$$

Magnetizing reactance X_m is

$$X_m = E_p^2/Q_m \qquad (10.2)$$

Turns ratio a is

$$a = N_1/N_2 = E_p/E_s$$

During the *short-circuit test*, the secondary winding is short-circuited and a voltage E_g much lower than normal (usually less than 5 percent of rated voltage) is applied to the primary (Fig. 10.28). The primary current I_{sc} should be less than its nominal value to prevent overheating and, particularly, to prevent a rapid change in winding resistance while the test is being made.

The voltage E_{sc}, current I_{sc}, and power P_{sc} are measured on the primary side (Fig. 10.28) and the following calculations made:

Total transformer impedance referred to the primary side is

$$Z_p = E_{sc}/I_{sc} \qquad (10.13)$$

Total transformer resistance referred to the primary side is

$$R_p = P_{sc}/I_{sc}^2 \qquad (10.14)$$

Total transformer leakage reactance referred to the primary side is

$$X_p = \sqrt{Z_p^2 - R_p^2} \qquad (10.11)$$

Example 10-6

During a short-circuit test on a transformer rated 500 kVA, 69 kV/4.16 kV, 60 Hz, the following voltage, current, and power measurements were made. Terminals X_1, X_2 were in short-circuit (see Fig. 10.28):

$$E_{sc} = 2600 \text{ V}$$
$$I_{sc} = 4 \text{ A}$$
$$P_{sc} = 2400 \text{ W}$$

Calculate the value of the reactance and resistance of the transformer, referred to the HV side.

Solution

Referring to the equivalent circuit of the transformer under short-circuit conditions (Fig. 10.29), we find the following values:

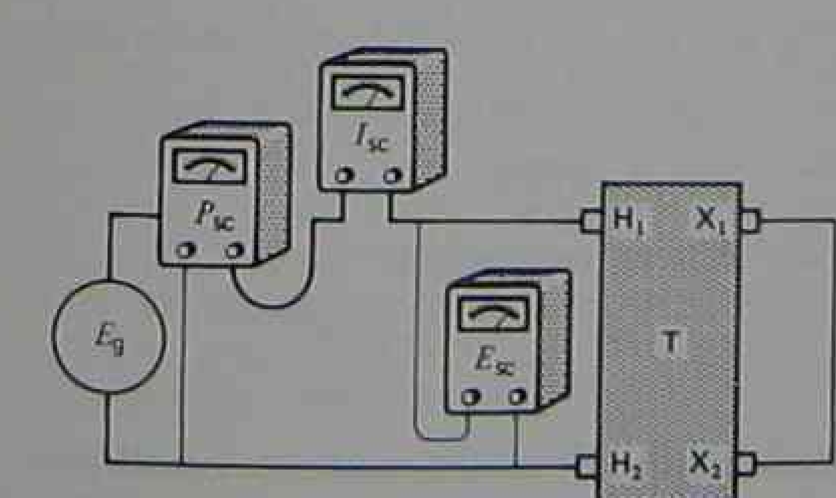

Figure 10.28
Short-circuit test to determine leakage reactance and winding resistance.

Transformer impedance referred to the primary is

$$Z_p = E_{sc}/I_{sc} = 2600/4$$
$$= 650 \ \Omega$$

Resistance referred to the primary is

$$R_p = P_{sc}/I_{sc}^2 = 2400/16$$
$$= 150 \ \Omega$$

Leakage reactance referred to the primary is

$$X_p = \sqrt{650^2 - 150^2}$$
$$= 632 \ \Omega$$

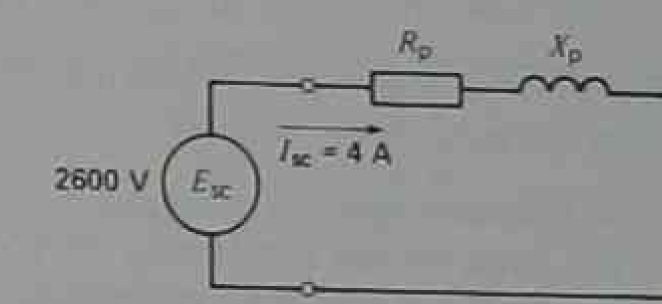

Figure 10.29
See Example 10-6.

Example 10-7

An open-circuit test was conducted on the transformer given in Example 10-6. The following results were obtained when the *low-voltage* winding was excited. (In some cases, such as in a repair shop, a 69 kV voltage may not be available and the open-circuit test has to be done by exciting the LV winding at its rated voltage.)

$$E_s = 4160 \text{ V} \qquad I_o = 2 \text{ A} \qquad P_m = 5000 \text{ W}$$

Using this information and the transformer characteristics found in Example 10-6, calculate:

a. the values of X_m and R_m on the primary side (Fig. 10.21)
b. the efficiency of the transformer when it supplies a load of 250 kVA, whose power factor is 80 % (lagging).

Solution

a. Applying Eq. 10.1 to the secondary side:

$$R_m = E_s^2/P_m$$
$$= 4160^2/5000 = 3461 \ \Omega$$

The apparent power S_m is:

$$S_m = E_s I_0 = 4160 \times 2 = 8320 \text{ VA}$$
$$Q_m = \sqrt{S_m^2 - P_m^2}$$
$$= \sqrt{8320^2 - 5000^2} = 6650 \ \Omega$$
$$X_m = E_s^2/Q_m$$
$$= 4160^2/6650 = 2602 \ \Omega$$

The values of R_m and X_m referred the primary side will be $(69\,000/4160)^2 = 275$ times greater. The values on the primary side are therefore:

$$X_m = 275 \times 2602 \ \Omega = 715 \times 10^3 \ \Omega = 715 \text{ k}\Omega$$
$$R_m = 275 \times 3461 \ \Omega = 952 \times 10^3 \ \Omega = 952 \text{ k}\Omega$$

These are the values that would have been found if the primary had been excited at 69 kV.

b. Industrial loads and voltages fluctuate all the time. Thus, when we state that a load is 250 kVA, with $\cos\theta = 0.8$, it is understood that the load is *about* 250 kVA and that the power factor is *about* 0.8. Furthermore, the primary voltage is *about* 69 kV.

Consequently, in calculating efficiency, there is no point in arriving at a precise mathematical answer, even if we were able to give it. Knowing this, we can make certain assumptions that make it much easier to arrive at a solution.

The equivalent circuit of the transformer and its load is represented by Fig. 10.30. The values of R_p and X_p are already known, and so we only have to add the magnetizing branch. To simplify the calculations, we shifted X_m and R_m from points 3, 4 to the input terminals 1, 2. This change is justified because these impedances are much greater than X_p and R_p. Let us assume that the voltage across the load is 4160 V. We now calculate the efficiency of the transformer.

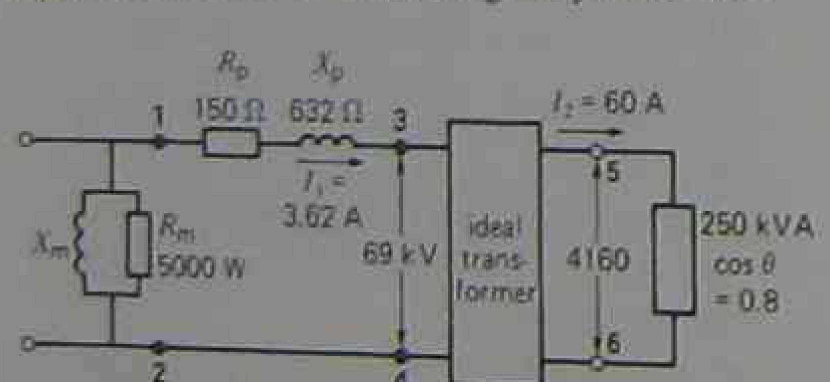

Figure 10.30
See Example 10-7.

The load current is

$$I_2 = S/E_s = 250\,000/4160$$
$$= 60 \text{ A}$$

The turns ratio is

$$a = 69 \text{ kV}/4160 \text{ V} = 16.59$$

The current on the primary side is:

$$I_1 = I_2/a = 60/16.59 = 3.62 \text{ A}$$

The total copper loss (primary and secondary) is

$$P_{\text{copper}} = I_1^2 R_p = 3.62^2 \times 150$$
$$= 1966 \text{ W}$$

The iron loss is the same as that measured at rated voltage on the LV side of the transformer.

$$P_{\text{iron}} = 5000 \text{ W}$$

Total losses are

$$P_{\text{losses}} = 5000 + 1966$$
$$= 6966 \text{ W} = 7 \text{ kW}$$

The active power delivered by the transformer to the load is

$$P_o = S\cos\theta = 250 \times 0.8$$
$$= 200 \text{ kW}$$

The active power received by the transformer is

$$P_i = P_o + P_{\text{losses}} = 200 + 7$$
$$= 207 \text{ kW}$$

The efficiency is therefore

$$\eta = P_o/P_i = 200/207$$
$$= 0.966 \text{ or } 96.6\%$$

Note that in making the calculations, we only consider the active power. The reactive power of the transformer and its load does not enter into efficiency calculations.

10.15 Introducing the per-unit method

Per-unit notation is often encountered when dealing with transformers and other electrical machines. The reason is that per-unit values give us a feel for the *relative* magnitudes of impedances, voltages, currents and powers. Thus, instead of dealing with ohms, amperes, volts and kilowatts, we simply work with numbers. Consequently, we don't have to carry along units when per-unit values are used.

The per-unit method as applied to transformers is easy to understand. However, readers who are not yet familiar with per-unit calculations will find it useful to read Sections 1.9 to 1.11 in Chapter 1 before proceeding further.

Let us begin by looking at Table 10A which is reproduced here for convenience. It displays the actual values of R_1, R_2, X_{f1}, X_{f2}, X_m and R_m of five transformers ranging from 1 kVA to 400 MVA. In scanning through the table, we see that the impedances vary from 505 000 Ω to 0.0003 Ω, a range in excess of a billion to one. Furthermore, there is no recognizable pattern to the values; they are all over the map. The reason is that the various voltages, currents and impedances are expressed in actual values using volts, amperes and ohms.

TABLE 10A ACTUAL TRANSFORMER VALUES

S_n	kVA	1	10	100	1000	400000
E_{np}	V	2400	2400	12470	69000	13800
E_{ns}	V	460	347	600	6900	424000
I_{np}	A	0.417	4.17	8.02	14.5	29000
I_{ns}	A	2.17	28.8	167	145	943
R_1	Ω	58.0	5.16	11.6	27.2	0.0003
R_2	Ω	1.9	0.095	0.024	0.25	0.354
X_{f1}	Ω	32	4.3	39	151	0.028
X_{f2}	Ω	1.16	0.09	0.09	1.5	27
X_m	Ω	200000	29000	150000	505000	460
R_m	Ω	400000	51000	220000	432000	317
I_o	A	0.0134	0.0952	0.101	0.210	52.9

Instead of expressing R_1, R_2, X_{f1}, X_{f2}, X_m and R_m in ohms, we could express them relative to another ohmic value. The question is: what value should we choose as a basis of comparison?

The best approach is to employ the *nominal* load (voltage and current) of the transformer. We can calculate its ohmic value and use it as a reference.

For example, in the case of the 10 kVA transformer listed in Table 10A, the nominal load impedance on the secondary side is

$$Z_{ns} = \frac{E_{ns}}{I_{ns}} = \frac{347 \text{ V}}{28.8 \text{ A}} = 12.0 \ \Omega$$

Using this ohmic value as a reference, the *relative* value of the secondary resistance R_2 is

$$R_2(\text{pu}) = \frac{0.095 \ \Omega}{12.0 \ \Omega} = 0.0079$$

Similarly, the nominal load impedance on the primary side is:

$$Z_{np} = \frac{E_{np}}{I_{np}} = \frac{2400 \text{ V}}{4.17 \text{ A}} = 576 \ \Omega$$

Using this load impedance as a reference, the *relative* value of the primary resistance R_1 is

$$R_1(\text{pu}) = \frac{5.16 \ \Omega}{576 \ \Omega} = 0.0090$$

The relative values R_1(pu) and R_2(pu) are pure numbers because they are the ratio of two quantities that bear the same unit.

Circuit elements on the primary side are always compared with the nominal load impedance Z_{np} on the primary side. Similarly, circuit elements on the secondary side are compared with the nominal load impedance Z_{ns} on the secondary side.

Proceeding in this way for the other impedances of the 10 kVA transformer, we obtain the relative values X_{f1}(pu), R_m(pu), etc. displayed in Table 10B.

The relative impedances of the other transformers are calculated the same way. In each case, the respective nominal load impedances Z_{np} and Z_{ns} are chosen as the reference impedances. Using the rated voltage and power of the transformer, they are given by:

$$Z_{np} = \frac{E_{np}}{I_{np}} = \frac{E_{np}}{S_n/E_{np}} = \frac{E_{np}^2}{S_n} \qquad (10.15a)$$

$$Z_{ns} = \frac{E_{ns}}{I_{ns}} = \frac{E_{ns}}{S_n/E_{ns}} = \frac{E_{ns}^2}{S_n} \qquad (10.15b)$$

In practice, the relative values of R_1, R_2, X_{f1}, etc., are called *per-unit* values and are designated by the symbols R_1(pu), R_2(pu), X_{f1}(pu), etc. The quantities used as references are called *base quantities*. Thus, Z_{np}, Z_{ns}, S_n, E_{np}, E_{ns}, I_{np}, I_{ns} listed in Table 10B are all base quantities.

In examining Table 10B, the reader will note that for a given transformer, the values of R_1(pu) and R_2(pu) are nearly the same. Similarly, the values of X_{f1}(pu) and X_{f2}(pu) are nearly the same. This pattern of similarity does not show up in Table 10A.

TABLE 10B PER UNIT TRANSFORMER VALUES

S_n	kVA	1	10	100	1000	400000
E_{np}	V	2400	2400	12470	69000	13800
E_{ns}	V	460	347	600	6900	424000
I_{np}	A	0.417	4.17	8.02	14.5	29000
I_{ns}	A	2.17	28.8	167	145	943
Z_{np}	Ω	5760	576	1555	4761	0.4761
Z_{ns}	Ω	211.6	12.0	3.60	47.61	449.4
R_1 (pu)	–	0.0101	0.0090	0.0075	0.0057	0.00071
R_2 (pu)	–	0.0090	0.0079	0.0067	0.0053	0.00079
X_{f1} (pu)	–	0.0056	0.0075	0.0251	0.0317	0.0588
X_{f2} (pu)	–	0.0055	0.0075	0.0250	0.0315	0.0601
X_m (pu)	–	34.7	50.3	96.5	106	966
R_m (pu)	–	69.4	88.5	141.5	90.7	666
I_o (pu)	–	0.032	0.023	0.013	0.015	0.0018

There is even a similarity between the per-unit values of transformers whose ratings are quite different. For example, the R_1(pu) of the 1 kVA transformer (0.0101) is of the same order of magnitude as the R_1(pu) of the 1000 kVA transformer (0.0057) despite the fact that the latter is 1000 times more powerful and the voltages are vastly different. Clearly, the per-unit method offers insights that would otherwise not be evident.

10.16 Impedance of a transformer

The total internal impedance Z_p of a transformer was defined in Section 10.12 and highlighted in Fig. 10.24. In power and distribution transformers its value is always indicated on the nameplate. However, it is expressed as a percent of the nominal load impedance. Thus, if the nameplate is marked 3.6 %, the per-unit value of Z_p is 0.036.

Example 10.8

A transformer rated 250 kVA, 4160 V/480 V, 60 Hz has an impedance of 5.1 %. Calculate:

a. the base impedance on the primary and secondary side
b. the total internal impedance Z_p of the transformer referred to the primary side

Solution

a. Base impedance on the primary side is

$$Z_{np} = E_p^2/S_n = 4160^2/250\,000 = 69\ \Omega$$

Base impedance on the secondary side is

$$Z_{ns} = E_s^2/S_n = 480^2/250\,000 = 0.92\ \Omega$$

b. The actual value of Z_p on the primary side is:

$$Z_p = 5.1\ \% \times Z_{np} = 0.051 \times 69\ \Omega = 3.52\ \Omega$$

10.17 Typical per-unit impedances

We have seen that we can get a better idea of the relative magnitude of the winding resistance, leakage reactance, etc., of a transformer by comparing these impedances with the base impedance of the transformer. In making the comparison, circuit elements located on the primary side are compared with the primary base impedance. Similarly, circuit elements on the secondary side are compared with the secondary base impedance. The comparison can be made either on a percentage or on a per-unit basis; we shall use the latter. Typical per-unit values are listed in Table 10C for transformers ranging from 3 kVA to 100 MVA. For example, the table shows that the per-unit resistance of the primary winding of a transformer ranges from 0.009 to 0.002 for all power ratings between 3 kVA and 100 MVA. Over this tremendous power range, the per-unit resistance R_1 of the primary or secondary windings varies only from 0.009 to 0.002 of the base impedance of the transformer. Knowing the base impedance of either the primary or the secondary winding, we can readily estimate the order of magnitude of the real values of the transformer impedances. Table 10C is, therefore, a useful source of information.

TABLE 10C TYPICAL PER-UNIT VALUES OF TRANSFORMERS

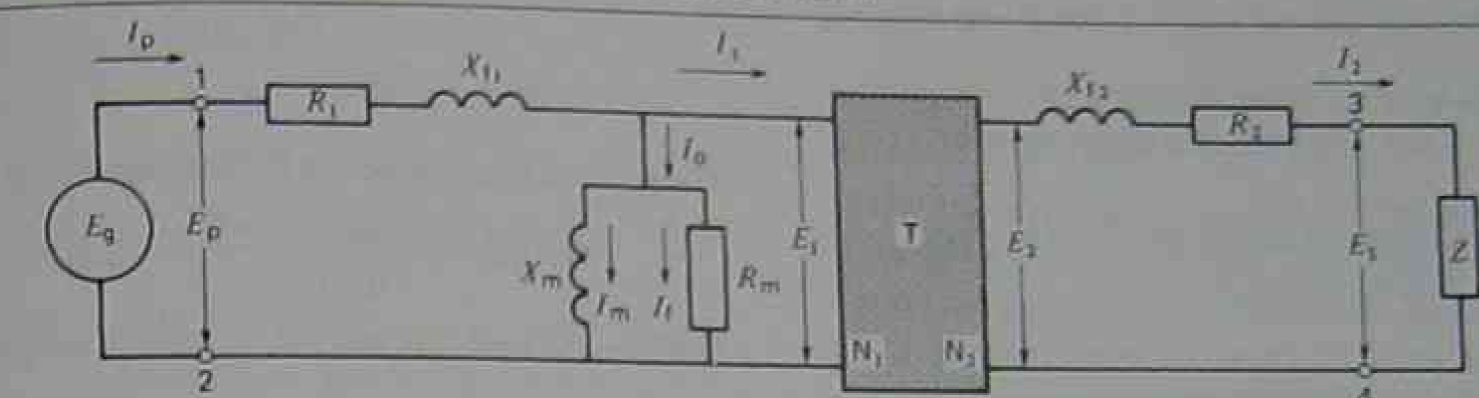

Figure 10.31
Equivalent circuit of a transformer.

Circuit element (see Fig. 10.31)	Typical per-unit values 3 kVA to 250 kVA	1 MVA to 100 MVA
R_1 or R_2	0.009–0.005	0.005–0.002
X_{f1} or X_{f2}	0.008–0.025	0.03–0.06
X_m	20–30	50–200
R_m	20–50	100–500
I_o	0.05–0.03	0.02–0.005

Example 10-9

Using the information given in Table 10C, calculate the approximate real values of the impedances of a 250 kVA, 4160 V/480 V, 60 Hz distribution transformer.

Solution

We first determine the base impedances on the primary and secondary side. From the results of Example 10-8, we have

$$Z_{np} = 69\ \Omega$$
$$Z_{ns} = 0.92\ \Omega$$

We now calculate the real impedances by multiplying Z_{np} and Z_{ns} by the per-unit values given in Table 10C. This yields the following results:

$$R_1 = 0.005 \times 69\ \Omega = 0.35\ \Omega$$
$$R_2 = 0.005 \times 0.92\ \Omega = 4.6\ \text{m}\Omega$$
$$X_{f1} = 0.025 \times 69\ \Omega = 1.7\ \Omega$$
$$X_{f2} = 0.025 \times 0.92\ \Omega = 23\ \text{m}\Omega$$
$$X_m = 30 \times 69\ \Omega = 2070\ \Omega = 2\ \text{k}\Omega$$
$$R_m = 50 \times 69\ \Omega = 3450\ \Omega = 3.5\ \text{k}\Omega$$

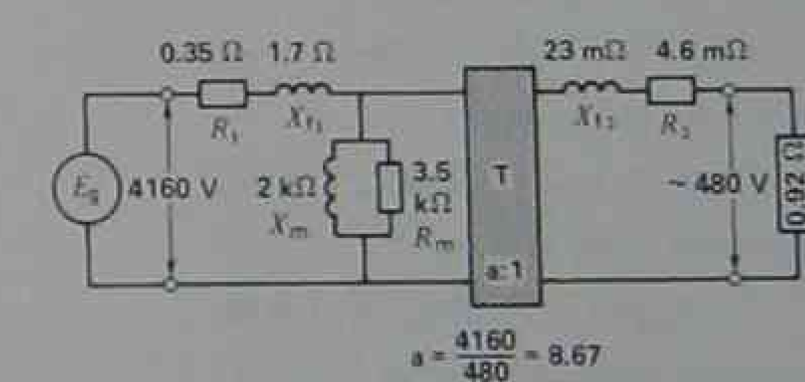

Figure 10.32
See Example 10-9.

This example shows the usefulness of the per-unit method of estimating impedances. The equivalent circuit of the 250 kVA transformer is shown in Fig. 10.32. The true values may be 20 to 50 percent higher or lower than those shown in the figure. The reason is that the per-unit values given in Table 10C are broad estimates covering a wide range of transformers.

Example 10.10

The 500 kVA, 69 kV/4160 V, 60 Hz transformer shown in Fig. 10.30 has a resistance R_p of 150 Ω and a leakage reactance X_p of 632 Ω. Using the per-unit method, calculate:

a. the voltage regulation when the load varies between zero and 250 kVA at a lagging power factor of 80%
b. the actual voltage across the 250 kVA load
c. the actual line current I_1.

Solution
In examining Fig. 10.30, it is clear that the presence of the magnetizing branch does not affect the voltage drop across R_p and X_p. Consequently, the magnetizing branch does not affect the voltage regulation.

To determine the voltage regulation, we will refer all voltages, impedances, and currents to the HV (69 kV) side. We assume the voltage between terminals 1, 2 is 69 kV, and that it remains fixed.

The base power P_B is 500 kVA
The base voltage E_B is 69 kV

Consequently, the base current is

$$I_B = P_B/E_B = 500\,000/69\,000 = 7.25 \text{ A}$$

and the base impedance is

$$Z_B = E_B/I_B = 69\,000/7.25 = 9517\ \Omega$$

The per-unit value of R_p is

$$R_p(\text{pu}) = 150/9517 = 0.0158$$

The per-unit value of X_p is

$$X_p(\text{pu}) = 632/9517 = 0.0664$$

The per-unit value of voltage E_{12} is

$$E_{12}(\text{pu}) = 69\,000/69 \text{ kV} = 1.0$$

The per-unit value of the apparent power absorbed by the load is

$$S(\text{pu}) = 250 \text{ kVA}/500 \text{ kVA} = 0.5$$

The per-unit value of the active power absorbed by the load is

$$P(\text{pu}) = S(\text{pu}) \cos\theta = 0.5 \times 0.8 = 0.4$$

The per-unit value of the reactive power absorbed by the load is

$$Q(\text{pu}) = \sqrt{S^2(\text{pu}) - P^2(\text{pu})} = \sqrt{0.5^2 - 0.4^2} = 0.3$$

The per-unit load resistance R_L corresponding to P is

$$R_L(\text{pu}) = \frac{E^2(\text{pu})}{P(\text{pu})} = \frac{1.0^2}{0.4} = 2.50$$

The per-unit load reactance X_L corresponding to Q is

$$X_L(\text{pu}) = \frac{E^2(\text{pu})}{Q(\text{pu})} = \frac{1.0^2}{0.3} = 3.333$$

We now draw the equivalent per-unit circuit shown in Figure 10.33. The magnetizing branch is not shown because it does not enter into the calculations. Note that the load appears across the primary terminals 3, 4 of the circuit shown in Figure 10.30. (These terminals are not accessible; they exist only in the equivalent circuit diagram.) The per-unit impedance between terminals 3, 4 is

$$Z_{34}(\text{pu}) = \frac{2.50 \times j\,3.33}{250 + j\,3.33} = 2\angle 36.87° = 1.6 + j\,1.2$$

The per-unit impedance between terminals 1, 2 is

$$Z_{12}(\text{pu}) = 0.0158 + 1.6 + j(1.2 + 0.0664) = 1.616 + j\,1.266 = 2.053\angle 38.07°$$

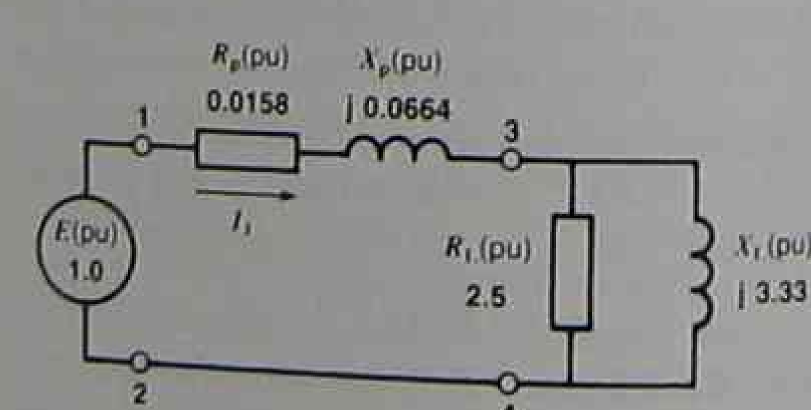

Figure 10.33
Per-unit equivalent circuit of a 500 kVA transformer feeding a 250 kVA load.

The per-unit current I_1 is

$$I_1(\text{pu}) = \frac{E_{12}(\text{pu})}{Z_{12}(\text{pu})} = \frac{1.0}{2.053\angle 38.07°} = 0.4872\angle -38.07°$$

The per-unit voltage E_{34} across the load is

$$E_{34}(\text{pu}) = I_1(\text{pu}) \times Z_{34}(\text{pu}) = (0.4872\angle -38.07°)(2\angle 36.87°) = 0.9744\angle -1.20°$$

The per-unit voltage regulation is

$$\frac{E_{34}(\text{pu}) \text{ at no-load} - E_{34}(\text{pu}) \text{ at full-load}}{E_{34}(\text{pu}) \text{ at full-load}} = \frac{1.0 - 0.9744}{0.9744} = 0.0263$$

a. The voltage regulation is therefore 2.63 %.

We can now calculate the *actual* values of the voltage and current as follows:
Voltage across terminals 3, 4 is

$$E_{34} = E_{34}(\text{pu}) \times E_B = 0.9744 \times 69\,000 = 67.23 \text{ kV}$$

b. Actual voltage across the load is

$$E_{56} = E_{34} \times (4160/69\,000) = 67.23 \times 10^3 \times 0.0603 = 4054 \text{ V}$$

c. Actual line current is

$$I_1 = I_1(\text{pu}) \times I_B = 0.4872 \times 7.246 = 3.53 \text{ A}$$

10.18 Transformers in parallel

When a growing load eventually exceeds the power rating of an installed transformer, we sometimes connect a second transformer in parallel with it. To ensure proper load-sharing between the two transformers, they must possess the following:

a. The same primary and secondary voltages
b. The same per-unit impedance

Particular attention must be paid to the polarity of each transformer, so that only terminals having the same polarity are connected together (Fig. 10.34). An error in polarity produces a severe short-circuit as soon as the transformers are excited.

In order to calculate the currents flowing in each transformer when they are connected in parallel, we must first determine the equivalent circuit of the system.

Consider first the equivalent circuit when a single transformer feeds a load Z_L (Fig. 10.35a). The primary voltage is E_p and the impedance of the transformer referred to the primary side is Z_{p1}. If the ratio

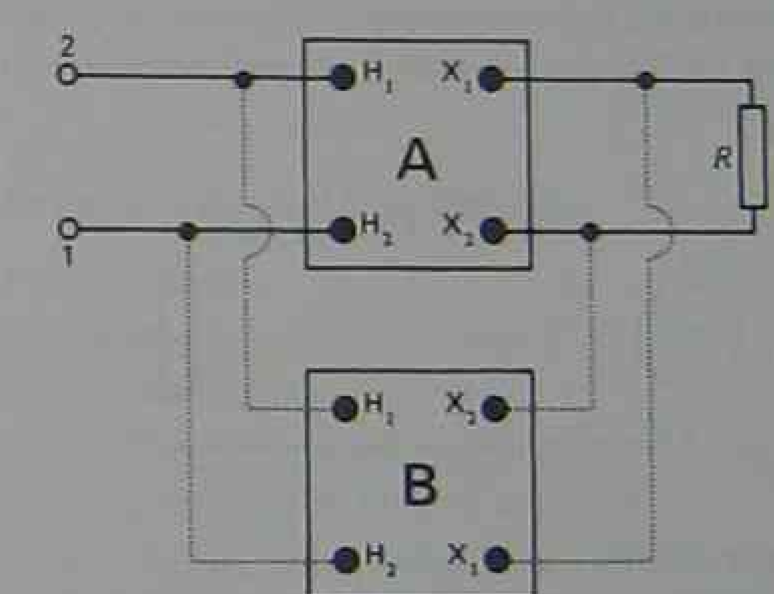

Figure 10.34
Connecting transformers in parallel to share a load.

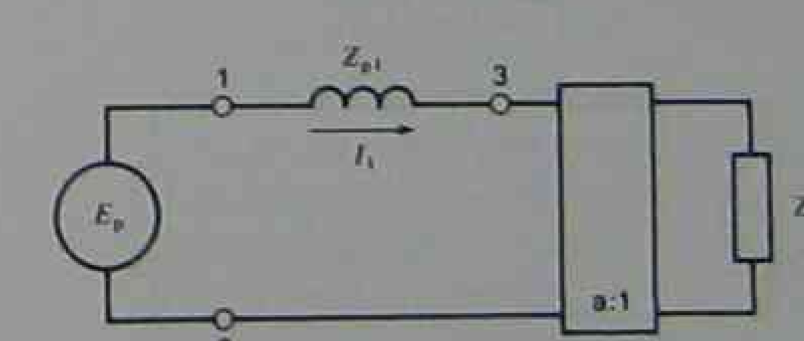

Figure 10.35a
Equivalent circuit of a transformer feeding a load Z_L.

of transformation is **a**, the circuit can be simplified to that shown in Fig. 10.35b, a procedure we are already familiar with.

If a second transformer having an impedance Z_{p2} is connected in parallel with the first, the equivalent circuit becomes that shown in Fig. 10.35c. In effect, the impedances of the transformers are in parallel. The primary currents in the transformers are respectively I_1 and I_2. Because the voltage drop E_{13} across the impedances is the same, we can write

$$I_1 Z_{p1} = I_2 Z_{p2} \tag{10.16}$$

that is,

$$\frac{I_1}{I_2} = \frac{Z_{p2}}{Z_{p1}} \tag{10.17}$$

The ratio of the primary currents is therefore determined by the magnitude of the respective primary impedances—and not by the ratings of the two transformers. But in order that the temperature rise be the same for both transformers, the currents must be proportional to the respective kVA ratings. Consequently, we want to fulfill the following condition:

$$\frac{I_1}{I_2} = \frac{S_1}{S_2} \tag{10.18}$$

From Eqs. 10.17 and 10.18 it can readily be proved that the desired condition is met if the transformers have the same per-unit impedances. The following example shows what happens when the per-unit impedances are different.

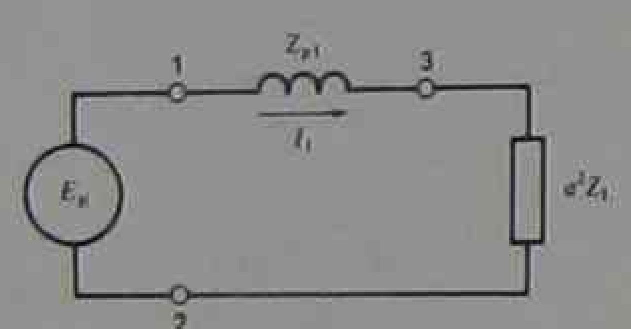

Figure 10.35b
Equivalent circuit with all impedances referred to the primary side.

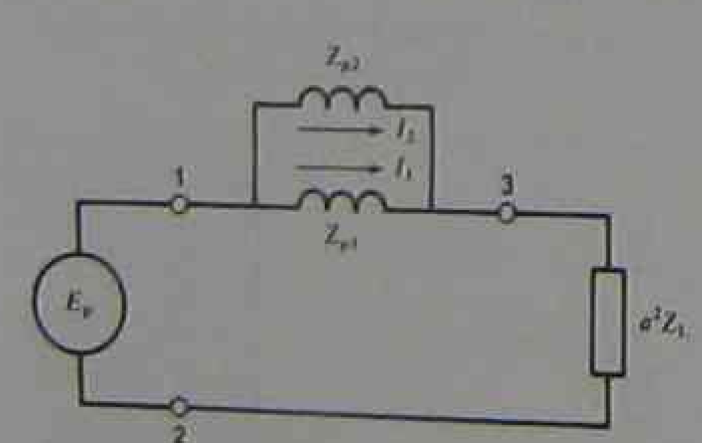

Figure 10.35c
Equivalent circuit of two transformers in parallel feeding a load Z_L. All impedances referred to the primary side.

Example 10-11

A 100 kVA transformer is connected in parallel with an existing 250 kVA transformer to supply a load of 330 kVA. The transformers are rated 7200 V/240 V, but the 100 kVA unit has an impedance of 4 percent while the 250 kVA transformer has an impedance of 6 percent (Fig. 10.36a).

Calculate

a. The nominal primary current of each transformer
b. The impedance of the load referred to the primary side
c. The impedance of each transformer referred to the primary side
d. The actual primary current in each transformer

Solution

a. Nominal primary current of the 250 kVA transformer is

$$I_{n1} = 250\,000/7200 = 34.7 \text{ A}$$

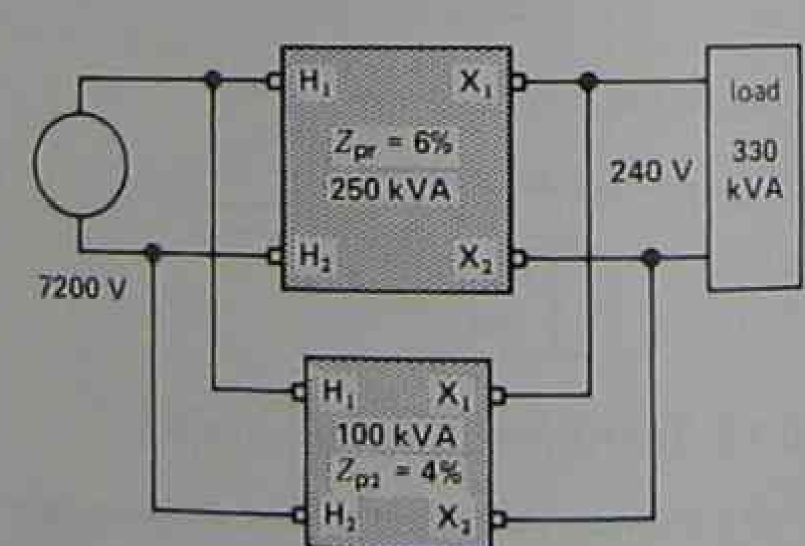

Figure 10.36a
Actual transformer connections.

Nominal primary current of the 100 kVA transformer is

$$I_{n2} = 100\,000/7200 = 13.9 \text{ A}$$

b. The equivalent circuit of the two transformers and the load, referred to the primary side, is given in Fig. 10.35c. Note that transformer impedances Z_{p1} and Z_{p2} are considered to be entirely reactive. This assumption is justified because the transformers are fairly big.
Load impedance referred to the primary side is

$$Z = E_p^2/S_{load} = 7200^2/330\,000$$
$$= 157\ \Omega$$

The approximate load current is

$$I_L = S_{load}/E_p = 330\,000/7200 = 46 \text{ A}$$

c. The base impedance of the 250 kVA unit is

$$Z_{np1} = 7200^2/250\,000 = 207\ \Omega$$

Transformer impedance referred to the primary side is

$$Z_{p1} = 0.06 \times 207 = 12.4\ \Omega$$

Base impedance of the 100 kVA unit is

$$Z_{np2} = 7200^2/100\,000 = 518\ \Omega$$

Transformer impedance referred to the primary side is

$$Z_{p2} = 0.04 \times 518 = 20.7\ \Omega$$

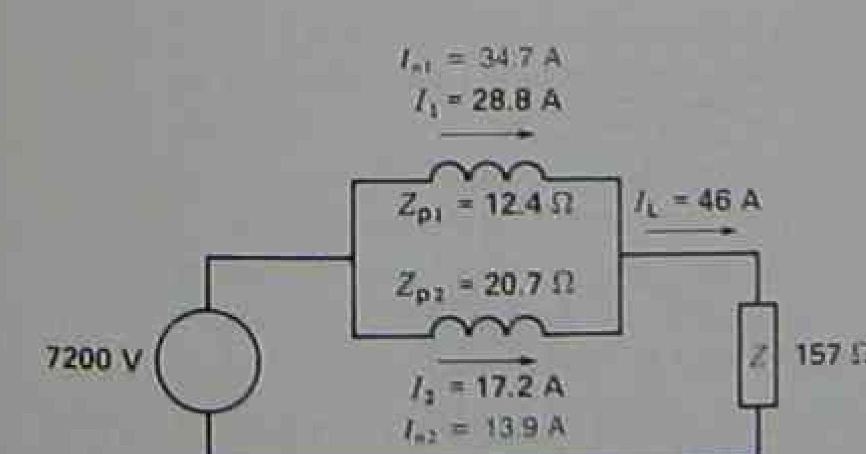

Figure 10.36b
Equivalent circuit. Calculations show that the 100 kVA transformer is seriously overloaded.

d. Referring to Fig. 10.36b, we find that the 46 A load current divides in the following way:

$$I_1 = 46 \times 20.7/(12.4 + 20.7) = 28.8 \text{ A}$$
$$I_2 = 46 - 28.8 = 17.2 \text{ A}$$

The 100 kVA transformer is seriously overloaded because it carries a primary current of 17.2 A, which is 25 percent above its rated value of 13.9 A. The 250 kVA unit is not overloaded because it only carries a current of 28.8 A versus its rated value of 34.7 A. Clearly, the two transformers are not carrying their proportionate share of the load.

The 100 kVA transformer is overloaded because of its low impedance (4 percent), compared to the impedance of the 250 kVA transformer (6 percent). A low-impedance transformer always tends to carry more than its proportionate share of the load. If the percent impedances were equal, the load would be shared between the transformers in proportion to their respective power ratings.

Questions and Problems

Practical level

10-1 Name the principal parts of a transformer.

10-2 Explain how a voltage is induced in the secondary winding of a transformer.

10-3 The secondary winding of a transformer has twice as many turns as the primary. Is the secondary voltage higher or lower than the primary voltage?

10-4 Which winding is connected to the load: the primary or secondary?

10-5 State the voltage and current relationships between the primary and secondary windings of a transformer under load. The primary and secondary windings have N_1 and N_2 turns, respectively.

10-6 Name the losses produced in a transformer.

10-7 What purpose does the no-load current of a transformer serve?

10-8 Name three conditions that must be met in order to connect two transformers in parallel.

10-9 What is the purpose of taps on a transformer?

10-10 Name three methods used to cool transformers.

10-11 The primary of a transformer is connected to a 600 V, 60 Hz source. If the primary has 1200 turns and the secondary has 240, calculate the secondary voltage.

10-12 The windings of a transformer respectively have 300 and 7500 turns. If the low-voltage winding is excited by a 2400 V source, calculate the voltage across the HV winding.

10-13 A 6.9 kV transmission line is connected to a transformer having 1500 turns on the primary and 24 turns on the secondary. If the load across the secondary has an impedance of 5 Ω, calculate the following:
a. The secondary voltage
b. The primary and secondary currents

10-14 The primary of a transformer has twice as many turns as the secondary. The primary voltage is 220 V and a 5 Ω load is connected across the secondary. Calculate the power delivered by the transformer, as well as the primary and secondary currents.

10-15 A 3000 kVA transformer has a ratio of 60 kV to 2.4 kV. Calculate the nominal current of each winding.

Intermediate level

10-16 In Problem 10-11, calculate the peak value of the flux in the core.

10-17 Explain why the peak flux in a 60 Hz transformer remains fixed as long as the ac supply voltage is fixed.

10-18 The transformer in Fig. 10.37 is excited by a 120 V, 60 Hz source and draws a no-load current I_o of 3 A. The primary and secondary windings respectively possess 200 and 600 turns. If 40 percent of the primary flux is linked by the secondary, calculate the following:
a. The voltage indicated by the voltmeter
b. The peak value of flux Φ
c. The peak value of Φ_m
d. Draw the phasor diagram showing E_1, E_2, I_o, Φ_m, and Φ_{f1}

10-19 In Fig. 10.38, when 600 V is applied to terminals H_1 and H_2, 80 V is measured across terminals X_1, X_2.
a. What is the voltage between terminals H_1 and X_2?
b. If terminals H_1, X_1 are connected together, calculate the voltage across terminals H_2, X_2.
c. Does the transformer have additive or subtractive polarity?

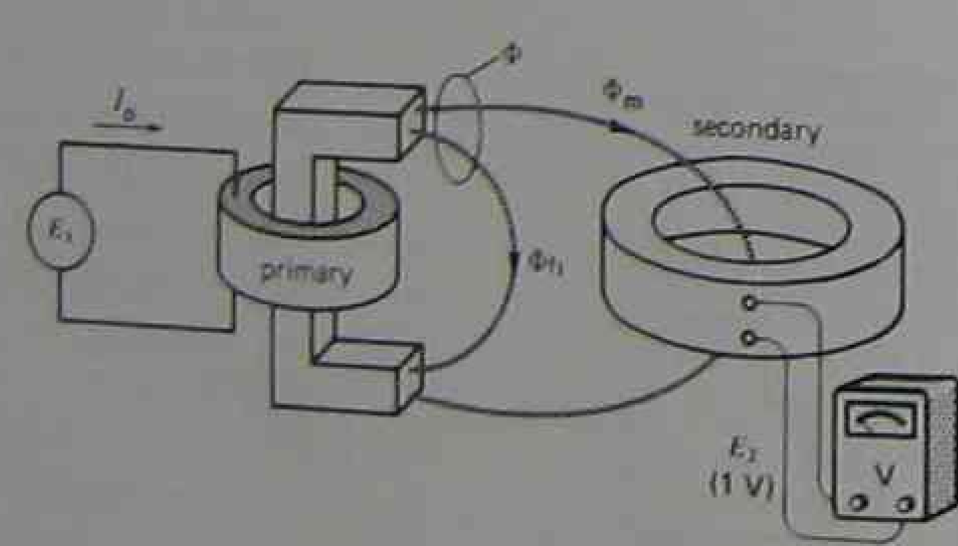

Figure 10.37
See Problem 10-18.

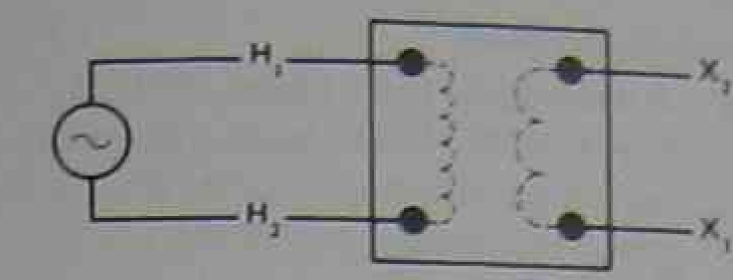

Figure 10.38
See Problem 10-19.

10-20 a. Referring to Fig. 10.34, what would happen if we reversed terminals H_1 and H_2 of transformer B?
b. Would the operation of the transformer bank be affected if terminals H_1, H_2 and X_1, X_2 of transformer B were reversed? Explain.

10-21 Explain why the secondary voltage of a practical transformer decreases with increasing resistive load.

10-22 What is meant by the following terms:
a. Transformer impedance
b. Percent impedance of a transformer

10-23 The transformer in Problem 10-15 has an impedance of 6 percent. Calculate the impedance [Ω] referred to:
a. The 60 kV primary
b. The 2.4 kV secondary

10-24 A 2300 V line is connected to terminals 1 and 4 in Fig. 10.13. Calculate the following:
a. The voltage between terminals X_1 and X_2
b. The current in each winding, if a 12 kVA load is connected across the secondary

10-25 A 66.7 MVA transformer has an efficiency of 99.3 percent when it delivers full power to a load having a power factor of 100 percent.
a. Calculate the losses in the transformer under these conditions.
b. Calculate the losses and efficiency when the transformer delivers 66.7 MVA to a load having a power factor of 80 percent.

10-26 If the transformer shown in Fig. 10.15 were placed in a tank of oil, the temperature rise would have to be reduced to 65°. Explain.

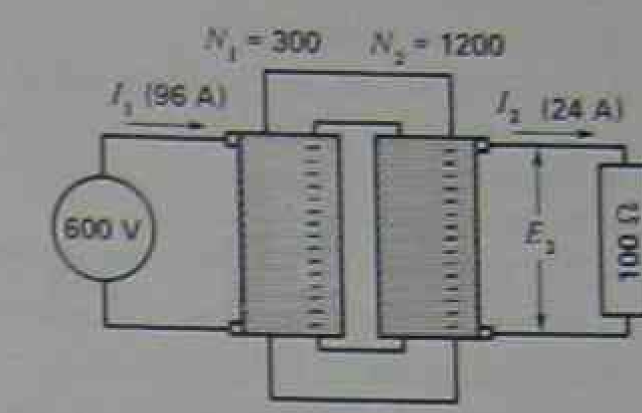

Figure 10.39
See Problem 10-33. The primary is wound on one leg and the secondary on the other.

Advanced level

10-27 Referring to Fig. 10.39, calculate the peak value of flux in the core if the transformer is supplied by a 50 Hz source.

10-28 The impedance of a transformer increases as the coupling is reduced between the primary and secondary windings. Explain.

10-29 The following information is given for the transformer circuit of Fig. 10.22.

$R_1 = 18\ \Omega$ $\quad E_p = 14.4$ kV (nominal)
$R_2 = 0.005\ \Omega$ $\quad E_s = 240$ V (nominal)
$X_{f1} = 40\ \Omega$ $\quad X_{f2} = 0.01\ \Omega$

If the transformer has a nominal rating of 75 kVA, calculate the following:
a. The transformer impedance [Ω] referred to the primary side
b. The percent impedance of the transformer
c. The impedance [Ω] referred to the secondary side
d. The percent impedance referred to the secondary side
e. The total copper losses at full load
f. The percent resistance and percent reactance of the transformer

10-30 During a short-circuit test on a 10 MVA, 66 kV/7.2 kV transformer (see Fig. 10.28), the following results were obtained:

$$E_g = 2640 \text{ V}$$
$$I_{sc} = 72 \text{ A}$$
$$P_{sc} = 9.85 \text{ kW}$$

Calculate the following:

a. The total resistance and the total leakage reactance referred to the 66 kV primary side
b. The nominal impedance of the transformer referred to the primary side
c. The percent impedance of the transformer

10-31 In Problem 10-30, if the iron losses at rated voltage are 35 kW, calculate the full-load efficiency of the transformer if the power factor of the load is 85 percent.

10-32 a. The windings of a transformer operate at a current density of 3.5 A/mm^2. If they are made of copper and operate at a temperature of 75°C, calculate the copper loss per kilogram.

b. If aluminum windings were used, calculate the loss per kilogram under the same conditions.

10-33 If a transformer were actually built according to Fig. 10.39, it would have very poor voltage regulation. Explain why and propose a method of improving it.

Industrial application

10-34 A transformer has a rating 200 kVA, 14 400 V/277 V. The high-voltage winding has a resistance of 62 Ω. What is the approximate resistance of the 277 V winding?

10-35 The primary winding of the transformer in Problem 10-34 is wound with No. 11 gauge AWG wire. Calculate the approximate cross section (in square millimeters) of the conductors in the secondary winding.

10-36 An oil-filled distribution transformer rated at 10 kVA weighs 118 kg, whereas a 100 kVA transformer of the same kind weighs 445 kg. Calculate the power output in watts per kilogram in each case.

10-37 The transformer shown in Fig. 10.13 has a rating of 40 kVA. If 80 V is applied between terminals X_1 and X_2, what voltage will appear between terminals 3 and 4? If a single load is applied between terminals 3 and 4 what is the maximum allowable current that can be drawn?

CHAPTER 11
Special Transformers

11.0 Introduction

Many transformers are designed to meet specific industrial applications. In this chapter we study some of the special transformers that are used in distribution systems, neon signs, laboratories, induction furnaces, and high-frequency applications. Although they are special, they still possess the basic properties of the standard transformers discussed in Chapter 10. As a result, the following approximations can be made when the transformers are under load:

1. The voltage induced in a winding is directly proportional to the number of turns, the frequency, and the flux in the core.
2. The ampere-turns of the primary are equal and opposite to the ampere-turns of the secondary.
3. The apparent power input to the transformer is equal to the apparent power output.
4. The exciting current in the primary winding may be neglected.

11.1 Dual-voltage distribution transformer

Transformers that supply electric power to residential areas generally have two secondary windings, each rated at 120 V. The windings are connected in series, and so the total voltage between the lines is 240 V while that between the lines and the center tap is 120 V (Fig. 11.1). The center tap, called *neutral*, is always connected to ground.

Terminal H_2 on the high-voltage winding is usually bonded to the neutral terminal of the secondary winding so that both windings are connected to ground.

The nominal rating of these *distribution transformers* ranges from 3 kVA to 500 kVA. They are mounted on poles of the electrical utility company (Fig. 11.2) to supply power to as many as 20 customers.

The load on distribution transformers varies greatly throughout the day, depending on customer demand. In residential districts a peak occurs in the morning and another peak occurs in the late afternoon. The power peaks never last for more than one or two hours, with the result that during most of the 24-hour day the transformers operate far below their normal rating. Because thousands of such transformers are connected to the public utility system, every effort is made to keep the no-load losses small. This is achieved by using special low-loss silicon-steel in the core.

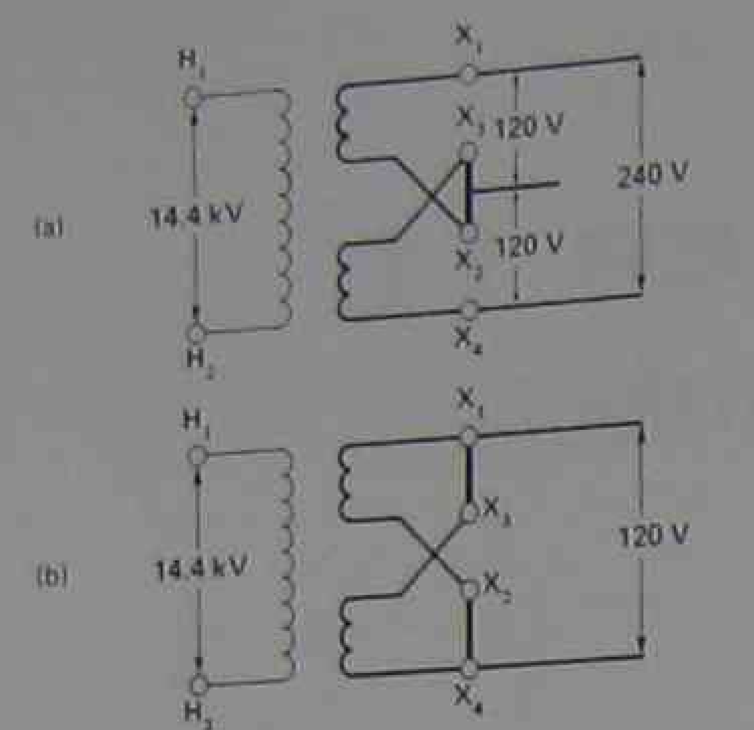

Figure 11.1
a. Distribution transformer with 120 V/240 V secondary. The central conductor is the neutral.
b. Same distribution transformers reconnected to give only 120 V.

Figure 11.2
Single-phase pole-mounted distribution transformer rated: 100 kVA, 14.4 kV/240 V/120 V, 60 Hz.

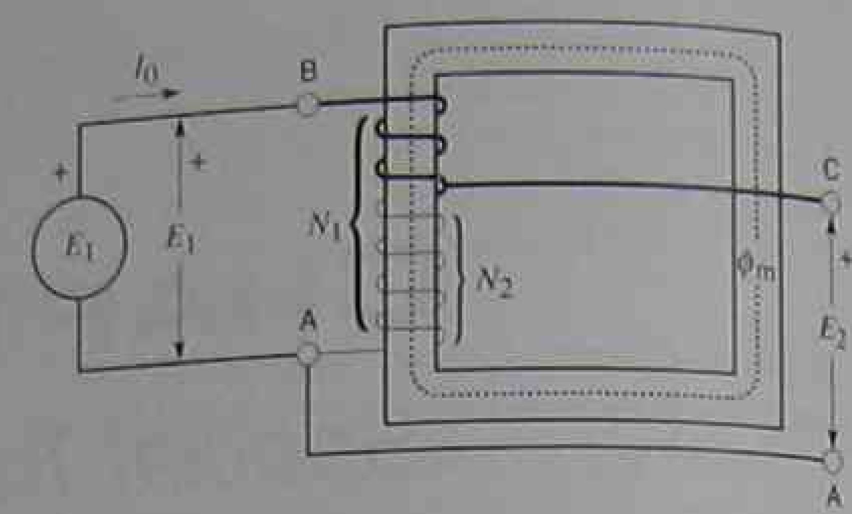

Figure 11.3
Autotransformer having N_1 turns on the primary and N_2 turns on the secondary.

11.2 Autotransformer

Consider a single transformer winding having N_1 turns, mounted on an iron core (Fig. 11.3). The winding is connected to a fixed-voltage ac source E_1, and the resulting exciting current I_o creates an ac flux Φ_m in the core. As in any transformer, the peak value of the flux is fixed so long as E_1 is fixed (Section 9.2).

Suppose a tap C is taken off the winding, so that there are N_2 turns between terminals A and C. Because the induced voltage between these terminals is proportional to the number of turns, E_2 is given by

$$E_2 = (N_2/N_1) \times E_1 \qquad (11.1)$$

Clearly, this simple coil resembles a transformer having a primary voltage E_1 and a secondary voltage E_2. However, the primary terminals B, A and the secondary terminals C, A are no longer isolated from each other, because of the common terminal A.

If we connect a load to secondary terminals CA, the resulting current I_2 immediately causes a primary current I_1 to flow (Fig. 11.4).

The BC portion of the winding obviously carries current I_1. Therefore, according to Kirchhoff's current law, the CA portion carries a current $(I_2 - I_1)$. Furthermore, the mmf due to I_1 must be equal and opposite to the mmf produced by $(I_2 - I_1)$. As a result, we have

$$I_1(N_1 - N_2) = (I_2 - I_1)\,N_2$$

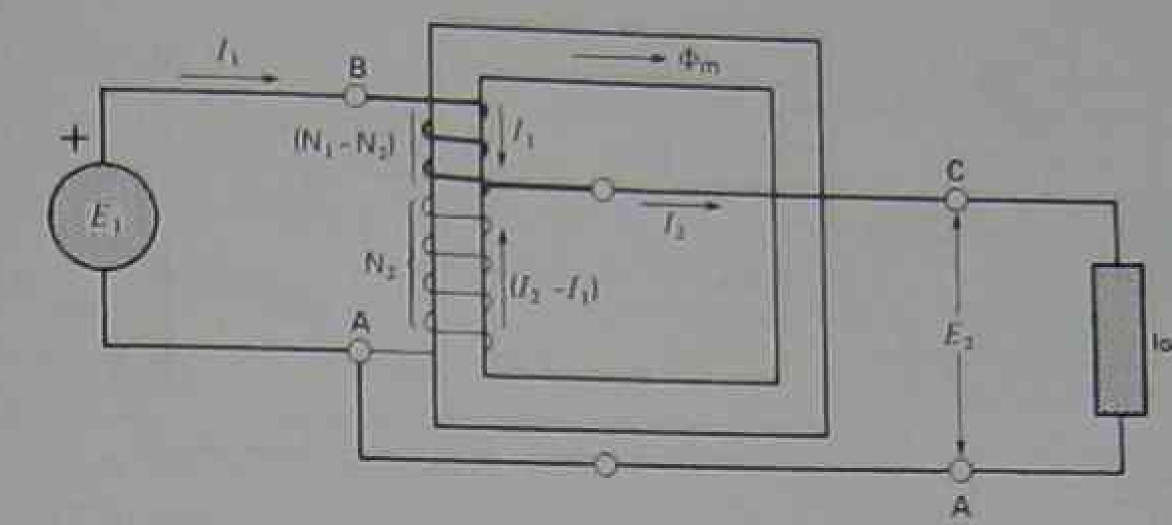

Figure 11.4
Autotransformer under load. The currents flow in opposite directions in the upper and lower windings.

which reduces to

$$I_1 N_1 = I_2 N_2 \qquad (11.2)$$

Finally, assuming that both the transformer losses and exciting current are negligible, the apparent power drawn by the load must equal the apparent power supplied by the source. Consequently,

$$E_1 I_1 = E_2 I_2 \qquad (11.3)$$

Equations 11.1, 11.2, and 11.3 are identical to those of a standard transformer having a turns ratio N_1/N_2. However, in this *autotransformer* the secondary winding is actually part of the primary winding. In effect, an autotransformer eliminates the need for a separate secondary winding. As a result, autotransformers are always smaller, lighter, and cheaper than standard transformers of equal power output. The difference in size becomes particularly important when the ratio of transformation E_1/E_2 lies between 0.5 and 2. On the other hand, the absence of electrical isolation between the primary and secondary windings is a serious drawback in some applications.

Autotransformers are used to start induction motors, to regulate the voltage of transmission lines, and, in general, to transform voltages when the primary to secondary ratio is close to 1.

Example 11-1

The autotransformer in Fig. 11.4 has an 80 percent tap and the supply voltage E_1 is 300 V. If a 3.6 kW load is connected across the secondary, calculate:

a. The secondary voltage and current
b. The currents that flow in the winding
c. The relative size of the conductors on windings BC and CA

Solution

a. The secondary voltage is

$$E_2 = 80\% \times 300 = 240 \text{ V}$$

The secondary current is

$$I_2 = P/E_2 = 3600/240 = 15 \text{ A} \quad \text{(Fig. 11.5).}$$

b. The current supplied by the source is

$$I_1 = P/E_1 = 3600/300 = 12 \text{ A}$$

the current in winding BC = 12 A

the current in winding CA = 15 − 12 = 3 A

c. The conductors in the secondary winding CA can be one-quarter the size of those in winding BC because the current is 4 times smaller (see Fig. 11.5). However, the voltage across winding BC is equal to the difference between the primary and secondary voltages, namely (300 − 240) = 60 V. Consequently, winding CA has four times as many turns as winding BC. Thus, the two windings require essentially the same amount of copper.

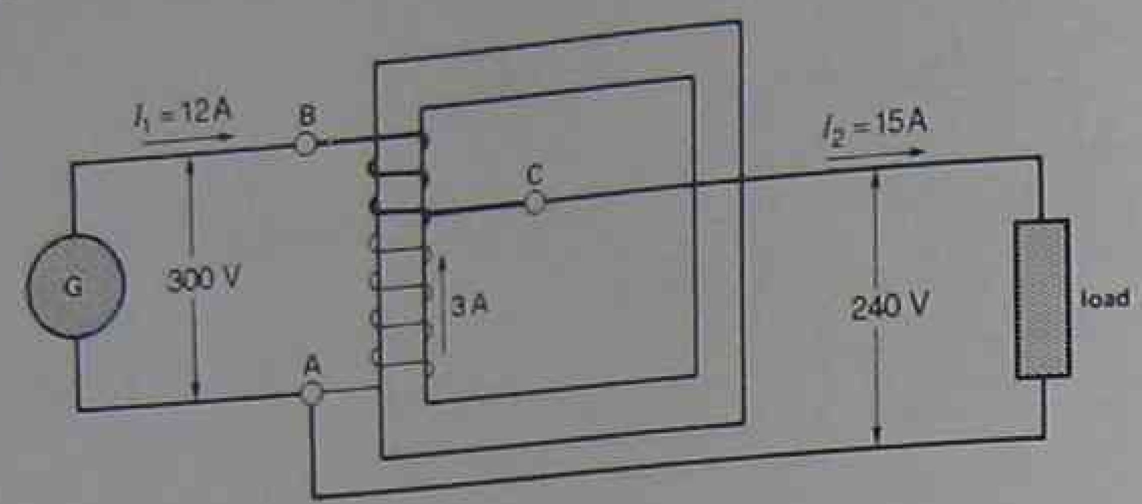

Figure 11.5
Autotransformer of Example 11-1.

11.3 Conventional transformer connected as an autotransformer

A conventional two-winding transformer can be changed into an autotransformer by connecting the primary and secondary windings in series. Depending upon how the connection is made, the secondary voltage may add to, or subtract from, the primary voltage. The basic operation and behavior of a transformer is unaffected by a mere change in external connections. Consequently, the following rules apply whenever a conventional transformer is connected as an autotransformer:

1. The current in any winding should not exceed its nominal current rating.
2. The voltage across any winding should not exceed its nominal voltage rating.
3. If rated current flows in one winding, rated current will *automatically* flow in the other winding (reason: The ampere-turns of the windings are always equal).
4. If rated voltage exists across one winding, rated voltage *automatically* exists across the other winding (reason: The same mutual flux links both windings).
5. If the current in a winding flows from H_1 to H_2, the current in the other winding must flow from X_2 to X_1 and vice versa.
6. The voltages add when terminals of opposite polarity (H_1 and X_2, or H_2 and X_1) are connected together by means of a jumper. The voltages subtract when H_1 and X_1 (or H_2 and X_2) are connected together.

Example 11-2

The standard single-phase transformer shown in Fig. 11.6 has a rating of a 15 kVA, 600 V/120 V, 60 Hz. We wish to reconnect it as an autotransformer in three different ways to obtain three different voltage ratios:

a. 600 V primary to 480 V secondary
b. 600 V primary to 720 V secondary
c. 120 V primary to 480 V secondary

Calculate the maximum load the transformer can carry in each case.

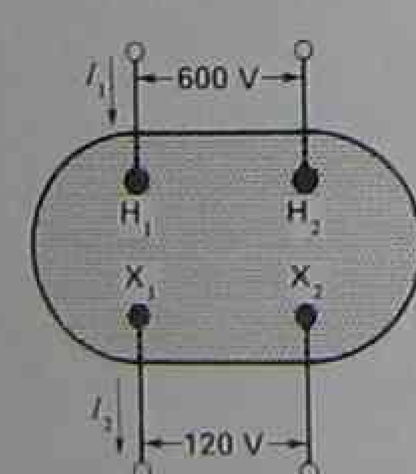

Figure 11.6
Standard 15 kVA, 600 V/120 V transformer.

Solution

Nominal current of the 600 V winding is

$$I_1 = S/E_1 = 15\,000/600 = 25 \text{ A}$$

Nominal current of the 120 V winding is

$$I_2 = S/E_2 = 15\,000/120 = 125 \text{ A}$$

a. To obtain 480 V, the secondary voltage (120 V) between terminals X_1, X_2 must subtract from the primary voltage (600 V). Consequently, we connect terminals having the same polarity together, as shown in Fig. 11.7. The corresponding schematic diagram is given in Fig. 11.8.

   Note that the current in the 120 V winding is the same as that in the load. Because this winding has a nominal current rating of 125 A, the load can draw a maximum power.

$$S_a = 125 \text{ A} \times 480 \text{ V} = 60 \text{ kVA}$$

   The currents flowing in the circuit at full-load are shown in Fig. 11.8. Note the following:

   1. If we assume that the current of 125 A flows from X_1 to X_2 in the winding, a current of 25 A must flow from H_2 to H_1 in the other winding. All other currents are then found by applying Kirchhoff's current law.
   2. The apparent power supplied by the source is equal to that absorbed by the load:

$$S = 100 \text{ A} \times 600 \text{ V} = 60 \text{ kVA}$$

b. To obtain a ratio of 600 V/720 V, the secondary voltage must *add* to the primary voltage: 600 + 120 = 720 V. Consequently, terminals of opposite polarity (H_1 and X_2) must be connected together, as shown in Fig. 11.9.

   The current in the secondary winding is again the same as that in the load, and therefore the maximum load current is again 125 A. The maximum load is now

$$S_b = 125 \text{ A} \times 720 \text{ V} = 90 \text{ kVA}$$

The previous examples show that when a conventional transformer is connected as an auto-

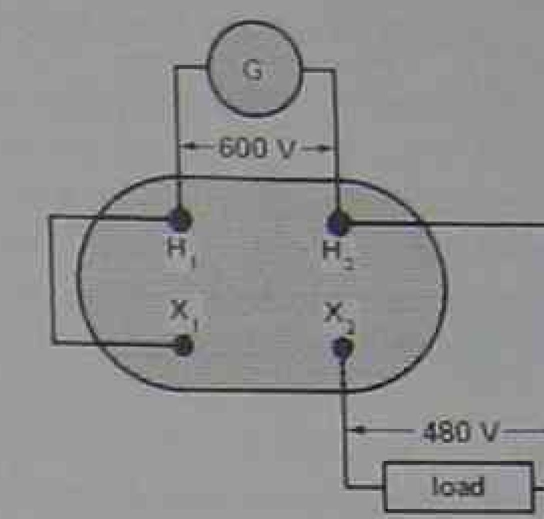

Figure 11.7
Transformer reconnected as an autotransformer to give a ratio of 600 V/480 V.

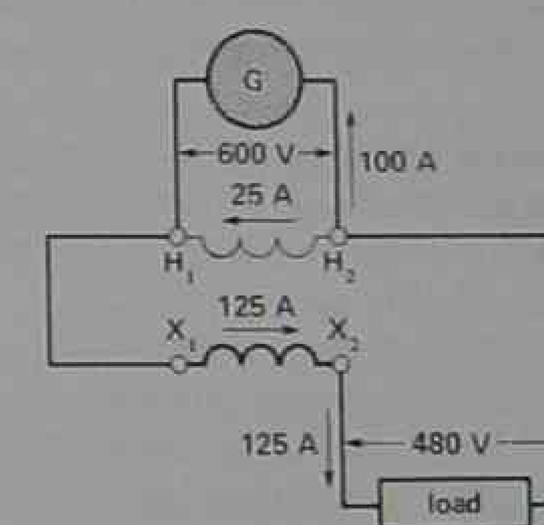

Figure 11.8
Schematic diagram of Fig. 11.7 showing voltages and current flows.

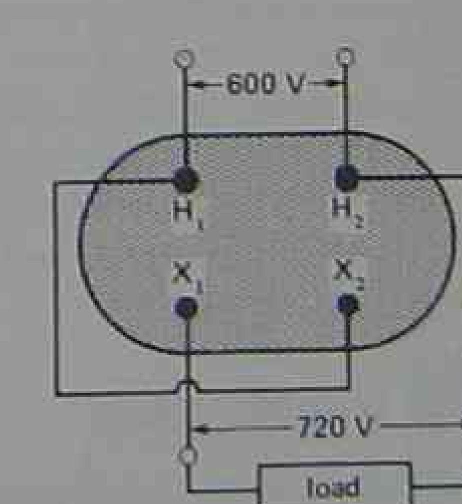

Figure 11.9
Transformer reconnected to give a ratio of 600 V/720 V.

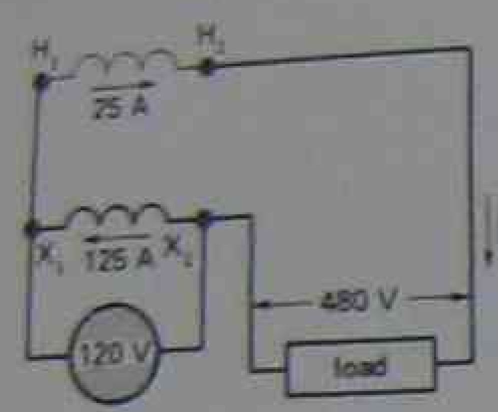

Figure 11.10
Transformer reconnected to give a ratio of 120 V/480 V.

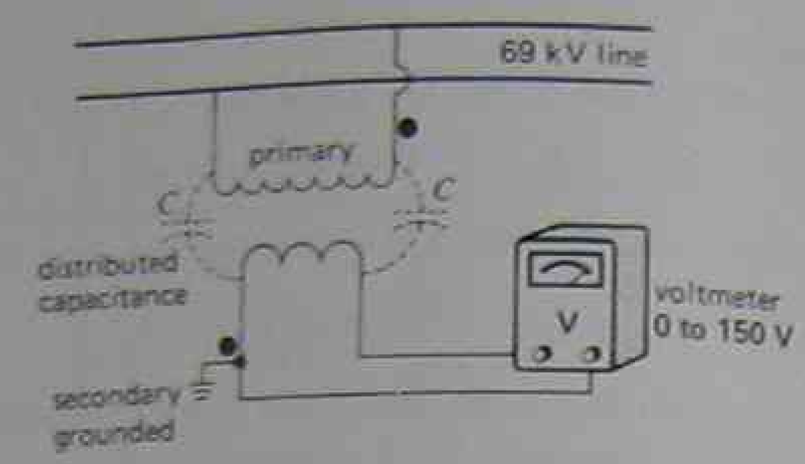

Figure 11.11
Potential transformer installed on a 69 kV line. Note the distributed capacitance between the windings.

transformer, it can supply a load far greater than the rated capacity of the transformer. As mentioned earlier, this is one of the advantages of using an autotransformer instead of a conventional transformer. However, this is not always the case, as the next part of our example shows.

c. To obtain the desired ratio of 120 V to 480 V, we again connect H_1 and X_2 (as in solution a), but the source is now connected to terminals X_1X_2 (Fig. 11.10).
This time, the current in the 600 V winding is the same as that in the load; consequently, the maximum load current cannot exceed 25 A. The corresponding maximum load is, therefore,

$$S_c = 25 \text{ A} \times 480 \text{ V} = 12 \text{ kVA}$$

This load is *less* than the nominal rating (15 kVA) of the standard transformer.

We want to make one final remark concerning these three autotransformer connections. The temperature rise of the transformer is the same in each case, even though the loads are respectively 60 kVA, 90 kVA, and 12 kVA. The reason is that the currents in the windings and the flux in the core are identical in each case and so the losses are the same.

11.4 Voltage transformers

Voltage transformers (also called *potential transformers*) are high-precision transformers in which the ratio of primary voltage to secondary voltage is a known constant, which changes very little with load.* Furthermore, the secondary voltage is almost exactly in phase with the primary voltage. The nominal secondary voltage is usually 115 V, irrespective of what the rated primary voltage may be. This permits standard instruments and relays to be used on the secondary side. Voltage transformers are used to measure or monitor the voltage on transmission lines and to isolate the metering equipment from these lines (Fig. 11.11).

The construction of voltage transformers is similar to that of conventional transformers. However, the insulation between the primary and secondary windings must be particularly great to withstand the full line voltage on the HV side.

In this regard, one terminal of the secondary winding is always connected to ground to eliminate the danger of a fatal shock when touching one of the secondary leads. Although the secondary *appears* to be isolated from the primary, the distributed capacitance between the two windings makes an invisible connection which can produce a very high voltage between the secondary winding and ground. By grounding one of the secondary terminals, the highest voltage between the secondary lines and ground is limited to 115 V.

The nominal rating of voltage transformers is usually less than 500 VA. As a result, the volume of insulation is often far greater than the volume of copper or steel.

* In the case of voltage transformers and current transformers, the load is called *burden*.

Voltage transformers installed on HV lines always measure the line-to-neutral voltage. This eliminates the need for two HV bushings because one side of the primary is connected to ground. For example, the 7000 VA, 80.5 kV transformer shown in Fig. 11.12 has one large porcelain bushing to isolate the HV line from the grounded case. The latter houses the actual transformer.

The basic impulse insulation (BIL) of 650 kV expresses the transformer's ability to withstand lightning and switching surges.

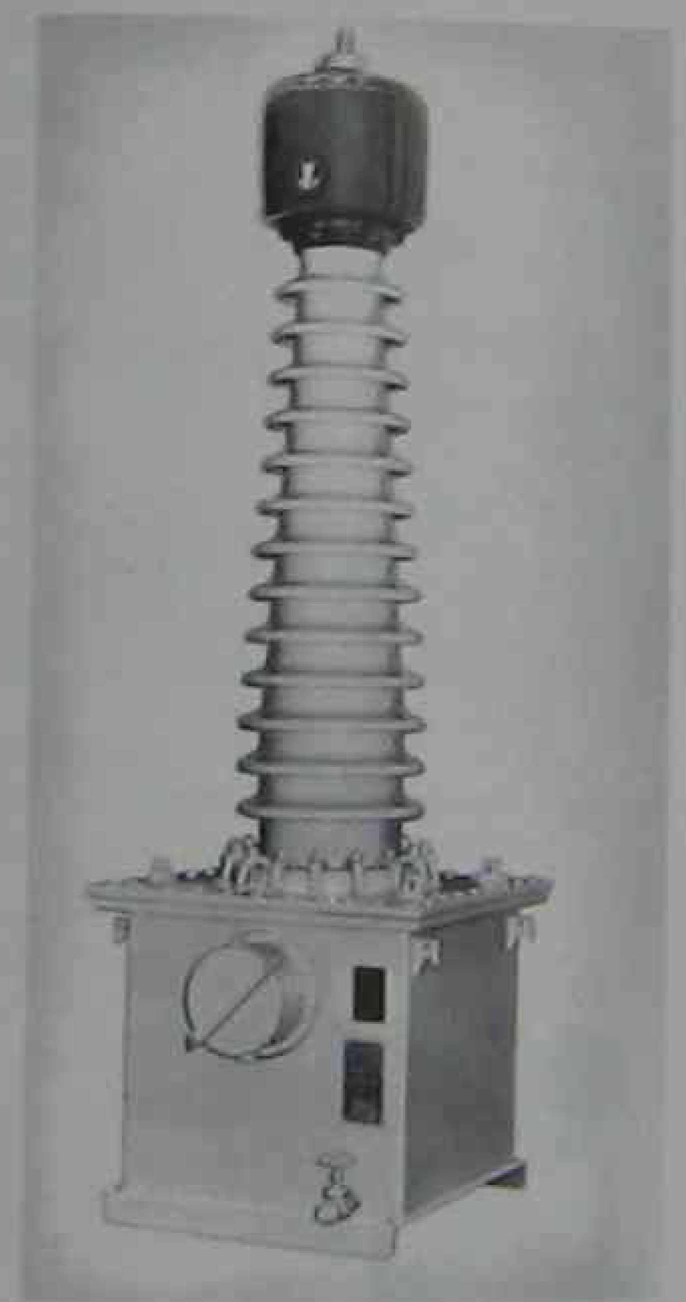

Figure 11.12
7000 VA, 80.5 kV, 50/60 Hz potential transformer having an accuracy of 0.3% and a BIL of 650 kV. The primary terminal at the top of the bushing is connected to the HV line while the other is connected to ground. The secondary is composed of two 115 V windings each tapped at 66.4 V. Other details: total height: 2565 mm; height of porcelain bushing: 1880 mm; oil: 250 L; weight: 740 kg.
(*Courtesy of Ferranti-Packard*)

11.5 Current transformers

Current transformers are high-precision transformers in which the ratio of primary to secondary current is a known constant that changes very little with the burden. The phase angle between the primary and secondary current is very small, usually much less than one degree. The highly accurate current ratio and small phase angle are achieved by keeping the exciting current small.

Current transformers are used to measure or monitor the current in a line and to isolate the metering and relay equipment connected to the secondary side. The primary is connected in series with the line, as shown in Fig. 11.13. The nominal secondary current is usually 5 A, irrespective of the primary current rating.

Because current transformers (CTs) are only used for measurement and system protection, their power rating is small—generally between 15 VA and 200 VA. As in the case of conventional transformers, the current ratio is inversely proportional to the number of turns on the primary and secondary windings. A current transformer having a ratio of 150 A/5 A has therefore 30 times more turns on the secondary than on the primary.

For safety reasons current transformers must always be used when measuring currents in HV transmission lines. The insulation between the primary and secondary windings must be great enough to withstand the full line-to-neutral voltage, including line surges. The maximum voltage the CT can withstand is always shown on the nameplate.

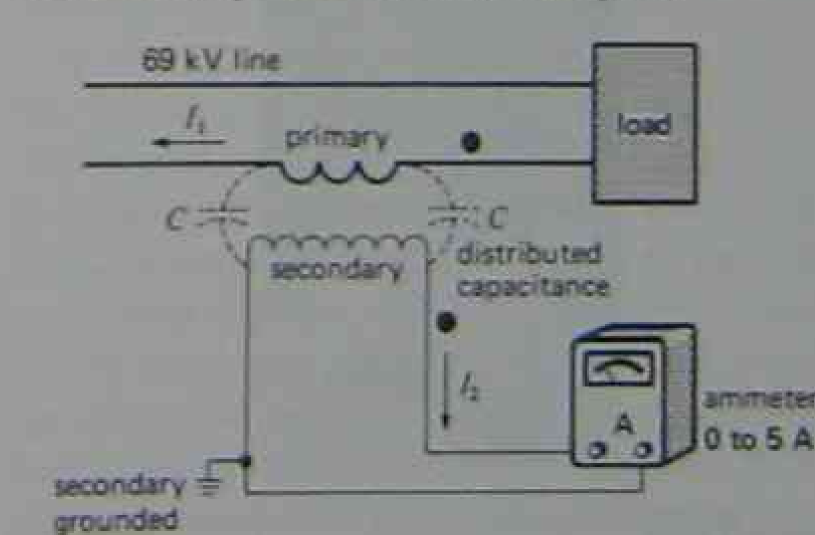

Figure 11.13
Current transformer installed on a 69 kV line.

As in the case of voltage transformers (and for the same reasons) one of the secondary terminals is always connected to ground.

Figure 11.14 shows a 500 VA, 100 A/5 A current transformer designed for a 230 kV line. The large bushing serves to isolate the HV line from the ground. The CT is housed in the grounded steel case at the lower end of the bushing. The upper end of the bushing has two terminals that are connected in series with the HV line. The line current flows into one terminal, down the bushing, through the primary of the transformer, then up the bushing and out by the other terminal. The internal construction of a CT is shown in Fig. 11.15 and a typical installation is shown in Fig. 11.16.

By way of comparison, the 50 VA current transformer shown in Fig. 11.17 is much smaller, mainly because it is insulated for only 36 kV.

Example 11-3

The current transformer in Fig. 11.17 has a rating of 50 VA, 400 A/5 A, 36 kV, 60 Hz. It is connected into an ac line, having a line-to-neutral voltage of 14.4 kV, in a manner similar to that shown in Fig. 11.13. The ammeters, relays, and connecting wires on the secondary side possess a total impedance (burden) of 1.2 Ω. If the transmission-line current is 280 A, calculate

a. The secondary current
b. The voltage across the secondary terminals
c. The voltage drop across the primary

Solution

a. The current ratio is

$$I_1/I_2 = 400/5 = 80$$

The turns ratio is

$$N_1/N_2 = 1/80$$

The secondary current is

$$I_2 = 280/80 = 3.5 \text{ A}$$

Figure 11.14
500 VA, 100 A/5 A, 60 Hz current transformer, insulated for a 230 kV line and having an accuracy of 0.6%.
(Courtesy of Westinghouse)

Figure 11.15
Current transformer in the final process of construction.
(Courtesy of Ferranti-Packard)

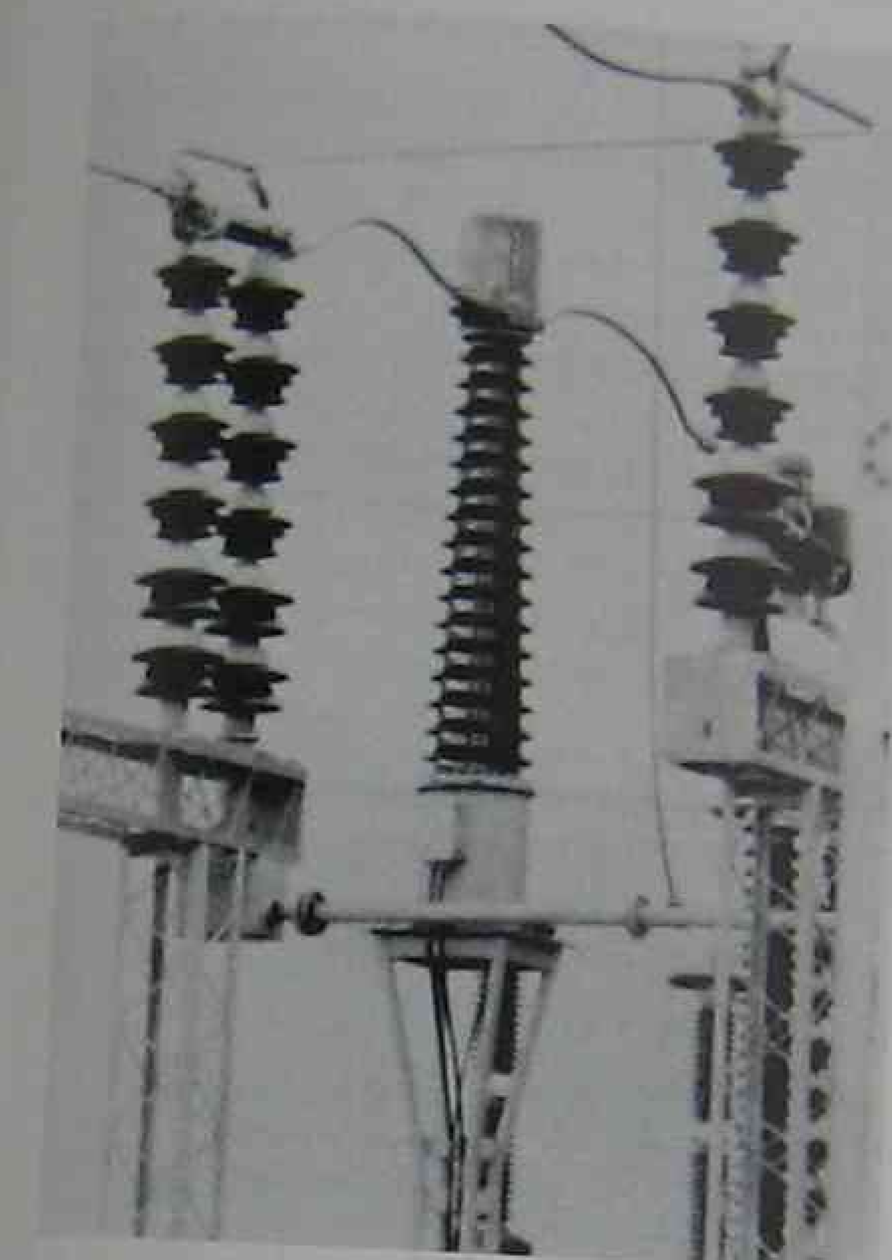

Figure 11.16
Current transformer in series with one phase of a 220 kV, 3-phase line inside a substation.

b. The voltage across the burden is

$$E_2 = IR = 3.5 \times 1.2 = 4.2 \text{ V}$$

The secondary voltage is therefore 4.2 V.

c. The primary voltage is

$$E_1 = 4.2/80 = 0.0525 = 52.5 \text{ mV}$$

This is a miniscule voltage drop, compared to the 14.4 kV line-to-neutral voltage.

11.6 Opening the secondary of a CT can be dangerous

Every precaution must be taken to *never* open the secondary circuit of a current transformer while current is flowing in the primary circuit. If the secondary is accidentally opened, the primary current

Figure 11.17
Epoxy-encapsulated current transformer rated 50 VA, 400 A/5 A, 60 Hz and insulated for 36 kV.
(Courtesy of Montel, Sprecher & Schuh)

I_1 continues to flow unchanged because the impedance of the primary is negligible compared to that of the electrical load. The line current thus becomes the *exciting* current of the transformer because there is no further bucking effect due to the secondary ampere-turns. Because the line current may be 100 to 200 times greater than the normal exciting current, the flux in the core reaches peaks much higher than normal. The flux is so large that the core is totally saturated for the greater part of every half cycle. Referring to Fig. 11.18, as the primary current I_1 rises and falls during the first half cycle, flux Φ in the core also rises and falls, but it remains at a fixed, saturation level Φ_s for most of the time. The same thing happens during the second half-cycle. During these saturated intervals, the induced voltage across the secondary winding is negligible because the flux changes very little. However, during the unsaturated intervals, the flux changes at an extremely high rate, inducing voltage peaks of several hundred volts across the open-circuited secondary. This is a dangerous situation because an unsuspecting operator could easily receive a bad shock. The voltage is particularly high in current transformers having ratings above 50 VA.

In view of the above, if a meter or relay in the secondary circuit of a CT has to be disconnected, we must first short-circuit the secondary winding and *then* remove the component. Short-circuiting a current transformer does no harm because the primary current remains unchanged and the secondary current can be no greater than that determined by the turns ratio. The short-circuit across the winding may be removed *after* the secondary circuit is again closed.

11.7 Toroidal current transformers

When the line current exceeds 100 A, we can sometimes use a *toroidal* current transformer. It consists of a laminated ring-shaped core that carries the secondary winding. The primary is composed of a single conductor that simply passes through the center of the ring (Fig. 11.19). The position of the primary conductor is unimportant as long as it is more or less centered. If the secondary possesses N turns, the ratio of transformation is N. Thus, a toroidal CT having a ratio of 1000 A/5 A has 200 turns on the secondary winding.

Toroidal CT's are simple and inexpensive and are widely used in low-voltage (LV) and medium-voltage (MV) indoor installations. They are also incorporated in circuit-breaker bushings to monitor the line current (Fig. 11.20). If the current exceeds a predetermined limit, the CT causes the circuit-breaker to trip.

Example 11-4

A potential transformer rated 14 400 V/115 V and a current transformer rated 75/5 A are used to measure the voltage and current in a transmission line. If the voltmeter indicates 111 V and the ammeter reads 3 A, calculate the voltage and current in the line.

Solution

The voltage on the line is

$$E = 111 \times (14\,400/115) = 13\,900\ \text{V}$$

The current in the line is

$$I = 3 \times (75/5) = 45\ \text{A}$$

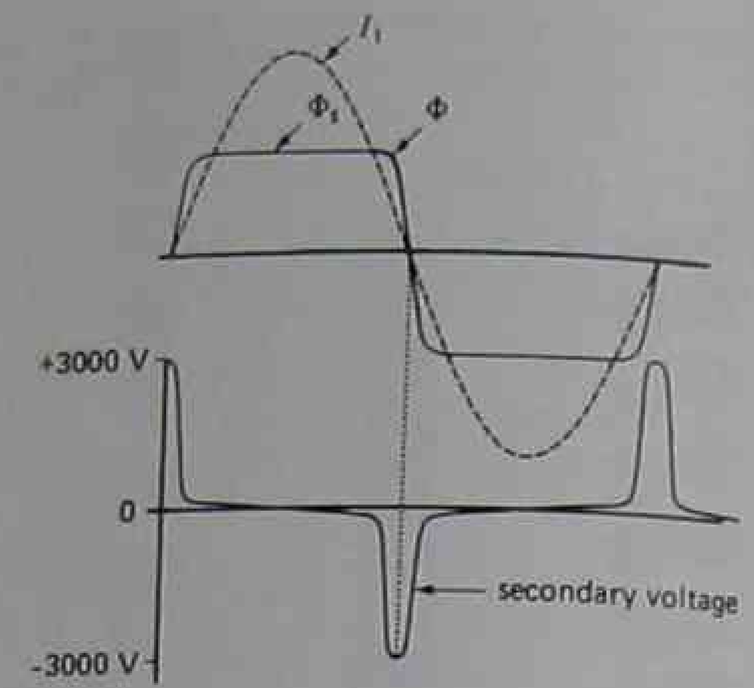

Figure 11.18
Primary current, flux, and secondary voltage when a CT is open-circuited.

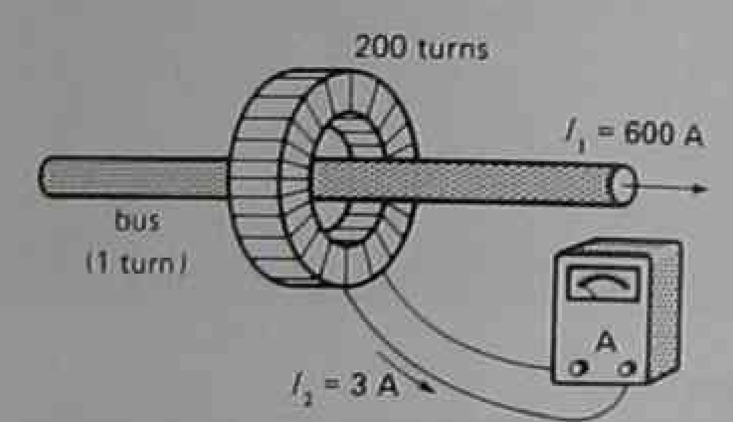

Figure 11.19
Toroidal transformer having a ratio of 1000 A/5 A, connected to measure the current in a line.

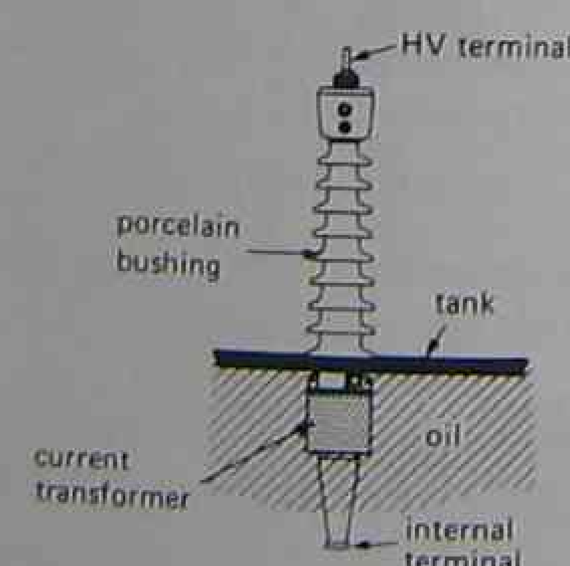

Figure 11.20
Toroidal transformer surrounding a conductor inside a bushing.

11.8 Variable autotransformer

A variable autotransformer is often used when we wish to obtain a variable ac voltage from a fixed-voltage ac source. The transformer is composed of a single-layer winding wound uniformly on a toroidal iron core. A movable carbon brush in sliding contact with the winding serves as a variable tap. The brush can be set in any position between 0 and 330°. Manual or motorized positioning may be used (Figs. 11.21 and 11.23).

As the brush slides over the bared portion of the winding, the secondary voltage E_2 increases in proportion to the number of turns swept out (Fig. 11.22). The input voltage E_1 is usually connected to a fixed 90 percent tap on the winding. This enables E_2 to vary from 0 to 110 percent of the input voltage.

Variable autotransformers are efficient and provide good voltage regulation under variable loads. The secondary line should always be protected by a fuse or circuit-breaker so that the output current I_2 never exceeds the current rating of the autotransformer.

Figure 11.21
Cutaway view of a manually operated 0-140 V, 15 A variable autotransformer showing (1) the laminated toroidal core; (2) the single-layer winding; (3) the movable brush.
(Courtesy of American Superior Electric)

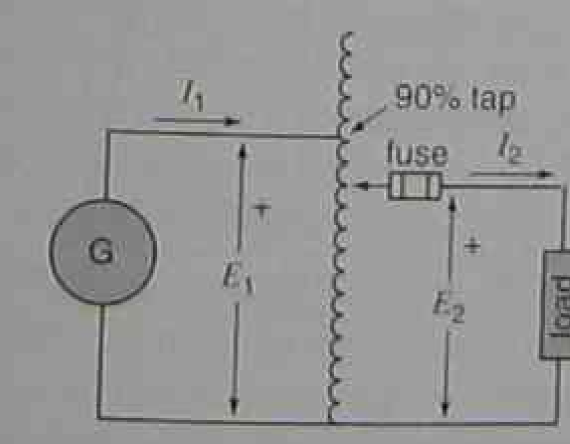

Figure 11.22
Schematic diagram of a variable autotransformer having a fixed 90% tap.

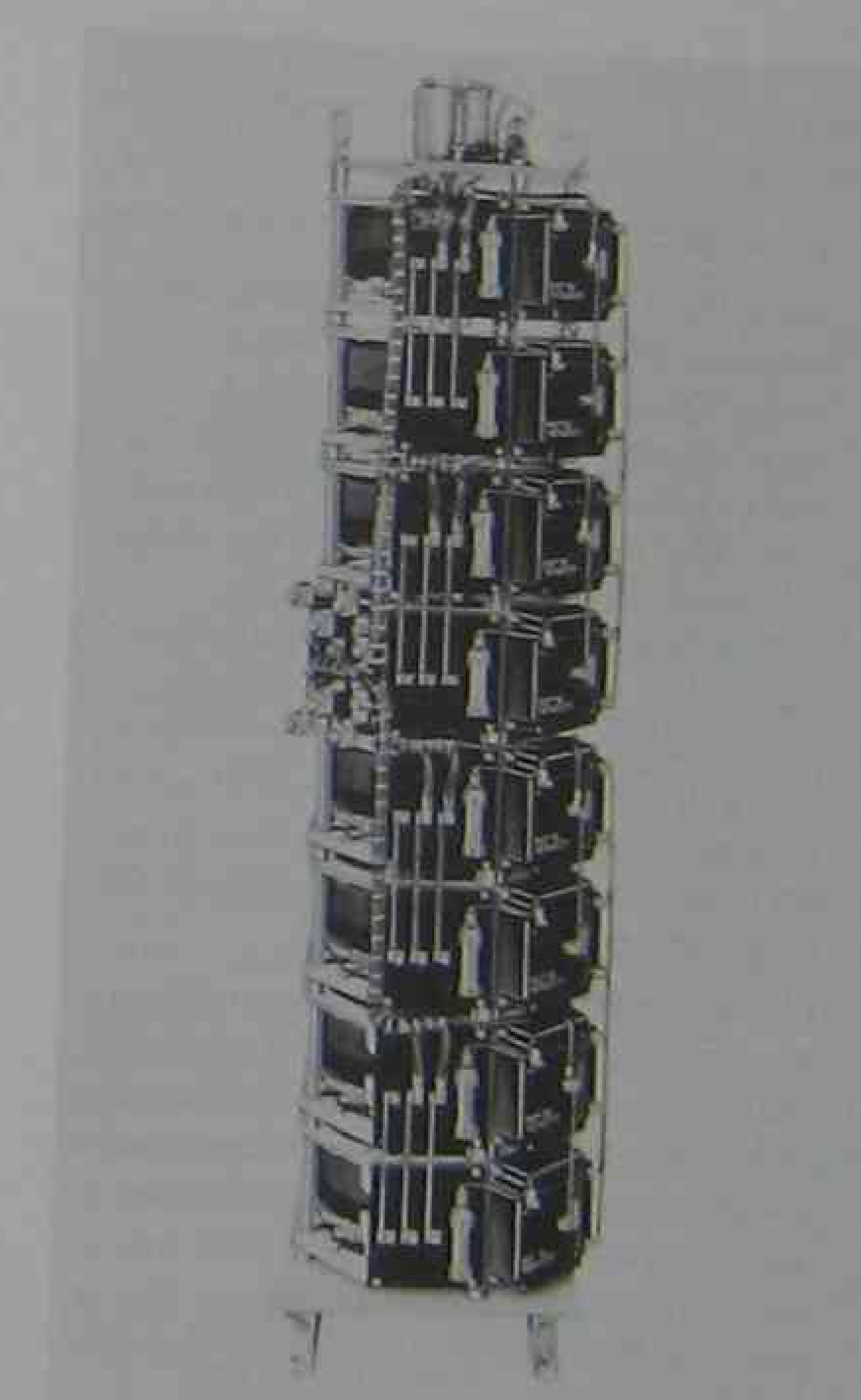

Figure 11.23
Variable autotransformer rated at 200 A, 0-240 V, 50/60 Hz. It is composed of eight 50 A, 120 V units, connected in series-parallel. This motorized unit can vary the output voltage from zero to 240 V in 5 s. Dimensions: 400 mm × 1500 mm.
(Courtesy of American Superior Electric)

11.9 High-impedance transformers

The transformers we have studied so far are all designed to have a relatively low leakage reactance, ranging perhaps from 0.03 to 0.1 per unit (Section 10.13). However, some industrial and commercial applications require much higher reactances, sometimes reaching values as high as 0.9 pu. Such high-impedance transformers are used in the following typical applications:

electric toys	arc welders
fluorescent lamps	electric arc furnaces
neon signs	reactive power regulators
oil burners	

Let us briefly examine these special applications.

1. A toy transformer is often accidentally short-circuited, but being used by children it is neither practical nor safe to protect it with a fuse. Consequently, the transformer is designed so that its leakage reactance is so high that even a permanent short-circuit across the low-voltage secondary will not cause overheating.

   The same remarks apply to some bell transformers that provide low-voltage signalling power throughout a home. If a short-circuit occurs on the secondary side, the current is automatically limited by the high reactance so as not to burn out the transformer or damage the fragile annunciator wiring.

2. Electric arc furnaces and discharges in gases possess a negative *E/I* characteristic, meaning that once the arc is struck, the current increases as the voltage falls. To maintain a steady arc, or a uniform discharge, we must add an impedance in series with the load. The series impedance may be either a resistor or reactor, but we prefer the latter because it consumes very little active power.

   However, if a transformer is used to supply the load, it is usually more economical to incorporate the reactance in the transformer itself, by designing it to have a high leakage reactance. A typical example is the neon-sign transformer shown in Fig. 11.24.

   The primary winding P is connected to a 240 V ac source, and the two secondary windings S are connected in series across the long neon tube. Owing to the large leakage fluxes Φ_a and Φ_b, the secondary voltage E_2 falls rapidly with increasing current, as seen in the regulation curve of the transformer (Fig. 11.24c). The high open-circuit voltage (20 kV) initiates the discharge, but as soon as the neon tube lights up, the secondary current is automatically limited to 15 mA. The corresponding voltage across the neon tube falls to 15 kV. The power of these transformers ranges from 50 VA to 1500 VA. The secondary voltages

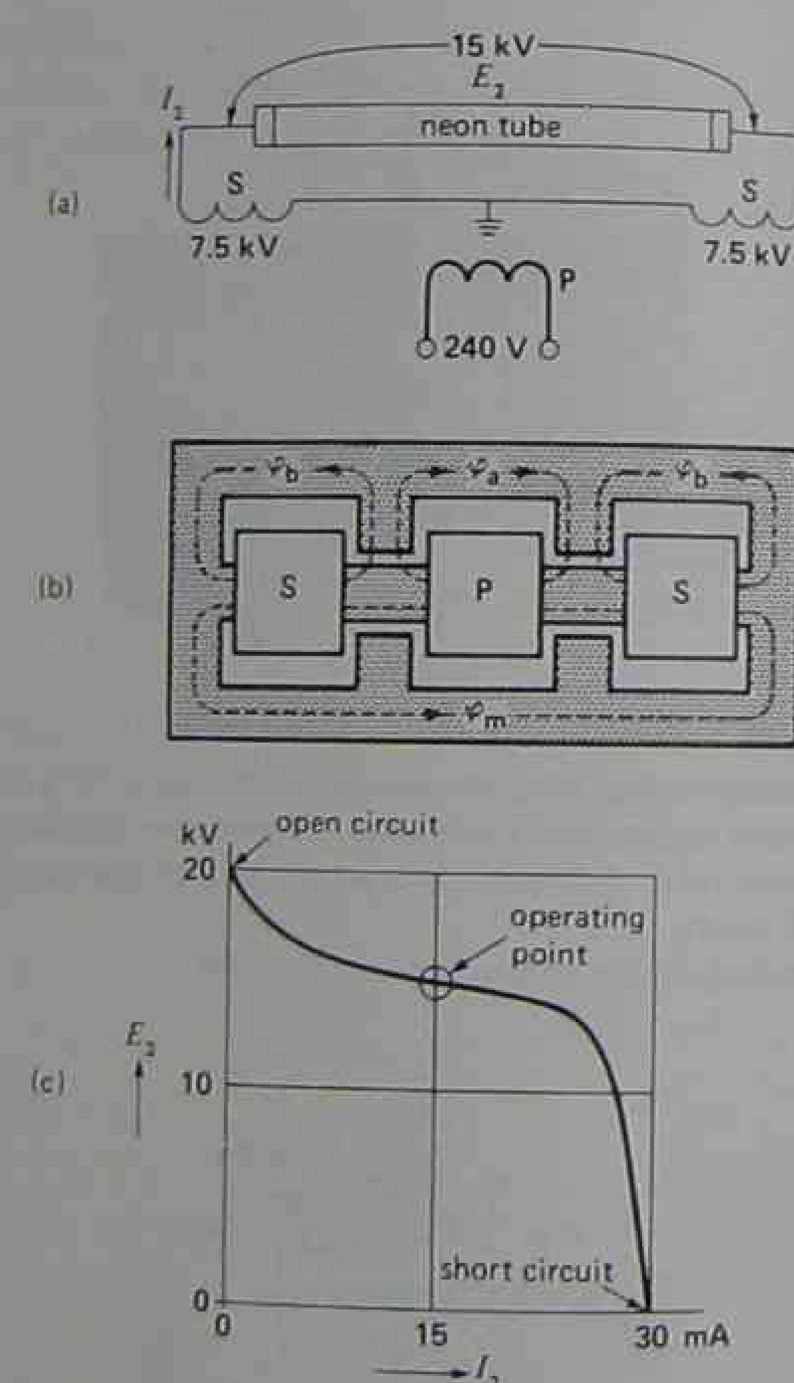

Figure 11.24
a. Schematic diagram of a neon-sign transformer.
b. Construction of the transformer.
c. Typical *E-I* characteristic of the transformer.

range from 2 kV to 20 kV, depending mainly upon the length of the tube.

Returning to Fig.11.24a, we note that the center of the secondary winding is grounded. This ensures that the secondary line-to-ground voltage is only one-half the voltage across the neon tube. As a result, less insulation is needed for the high-voltage winding.

Fluorescent lamp transformers (called ballasts) have properties similar to neon-sign transformers. Capacitors are usually added to improve the power factor of the total circuit.

Oil-burner transformers possess essentially the same characteristics as neon-sign transformers do. A secondary open-circuit voltage of about 10 kV creates an arc between two closely spaced electrodes situated immediately above the oil jet. The arc continually ignites the vaporized oil while the burner is in operation.

3. Some electric furnaces generate heat by maintaining an intense arc between two carbon electrodes. A relatively low secondary voltage is used and the large secondary current is limited by the leakage reactance of the transformer. Such transformers have ratings between 100 kVA and 500 MVA. In very big furnaces, the leakage reactance of the secondary, together with the reactance of the conductors, is usually sufficient to provide the necessary limiting impedance.

4. Arc-welding transformers are also designed to have a high leakage reactance so as to stabilize the arc during the welding process. The open-circuit voltage is about 70 V, which facilitates striking the arc when the electrode touches the work. However, as soon as the arc is established, the secondary voltage falls to about 15 V, a value that depends upon the length of the arc and the intensity of the welding current.

5. As a final example of high-impedance transformers, we mention the enormous 3-phase units that absorb reactive power from a 3-phase transmission line. These transformers are intentionally designed to produce leakage flux and, consequently, the primary and secondary windings are very loosely coupled. The three primary windings are connected to the HV line (typically between 230 kV and 765 kV) while the three secondary windings (typically 6 kV) are connected to an electronic controller (Fig. 11.25). The controller permits more or less secondary current to flow, causing the leakage flux to vary accordingly. A change in the leakage flux produces a corresponding change in the reactive power absorbed by the transformer. The transformer, incorporated in a static var compensator, is further discussed in Section 25.27.

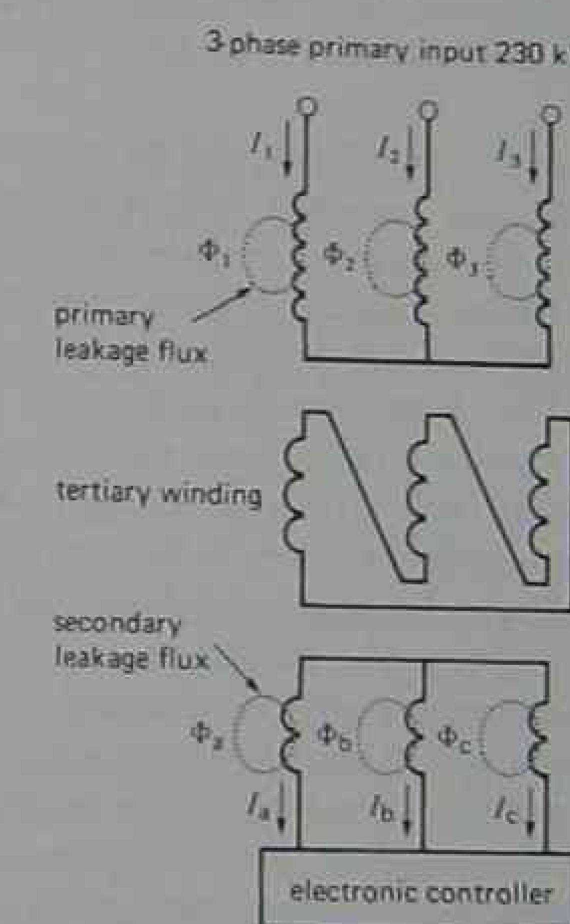

Figure 11.25
Three-phase static var compensator having high leakage reactance.

11.10 Induction heating transformers

High-power induction furnaces also use the transformer principle to produce high-quality steel and other alloys. The induction principle can be understood by referring to Fig. 11.26. A relatively high-frequency 500 Hz ac source is connected to a coil that surrounds a large crucible containing molten

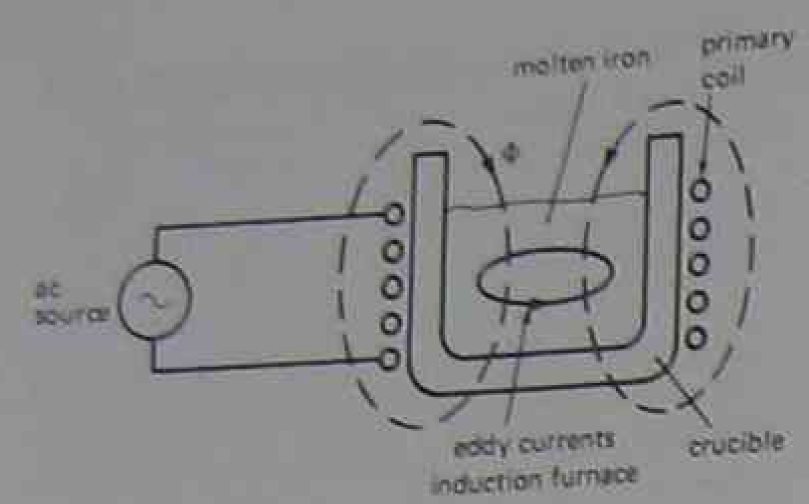

Figure 11.26
Coreless induction furnace. The flux Φ produces eddy currents in the molten metal. The capacitor furnishes the reactive power absorbed by the coil.

iron. The coil is the primary, and the molten iron acts like a single secondary turn, short-circuited upon itself. Consequently, it carries a very large secondary current. This current provides the energy that keeps the iron in a liquid state, melting other scrap metal as it is added to the pool.

Such induction furnaces have ratings between 15 kVA and 40 000 kVA. The operating frequency becomes progressively lower as the power rating increases. Thus, a frequency of 60 Hz is used when the power exceeds about 3000 kVA.

The power factor of coreless induction furnaces is very low (typically 20 percent) because a large magnetizing current is required to drive the flux through the molten iron and through the air. In this regard, we must remember that the temperature of molten iron is far above the Curie point, and so it behaves like air as far as permeability is concerned. That is why these furnaces are often called coreless induction furnaces.

Capacitors are installed close to the coil to supply the reactive power it absorbs.

In another type of furnace, known as a *channel furnace*, a transformer having a laminated iron core is made to link with a channel of molten iron, as shown in Fig. 11.27. The channel is a ceramic pipe that is fitted to the bottom of the crucible. The primary coil is excited by a 60 Hz source, and the secondary current I_2 flows in the liquid channel and through the molten iron in the crucible. In effect, the channel is equivalent to a single turn short-circuited on itself.

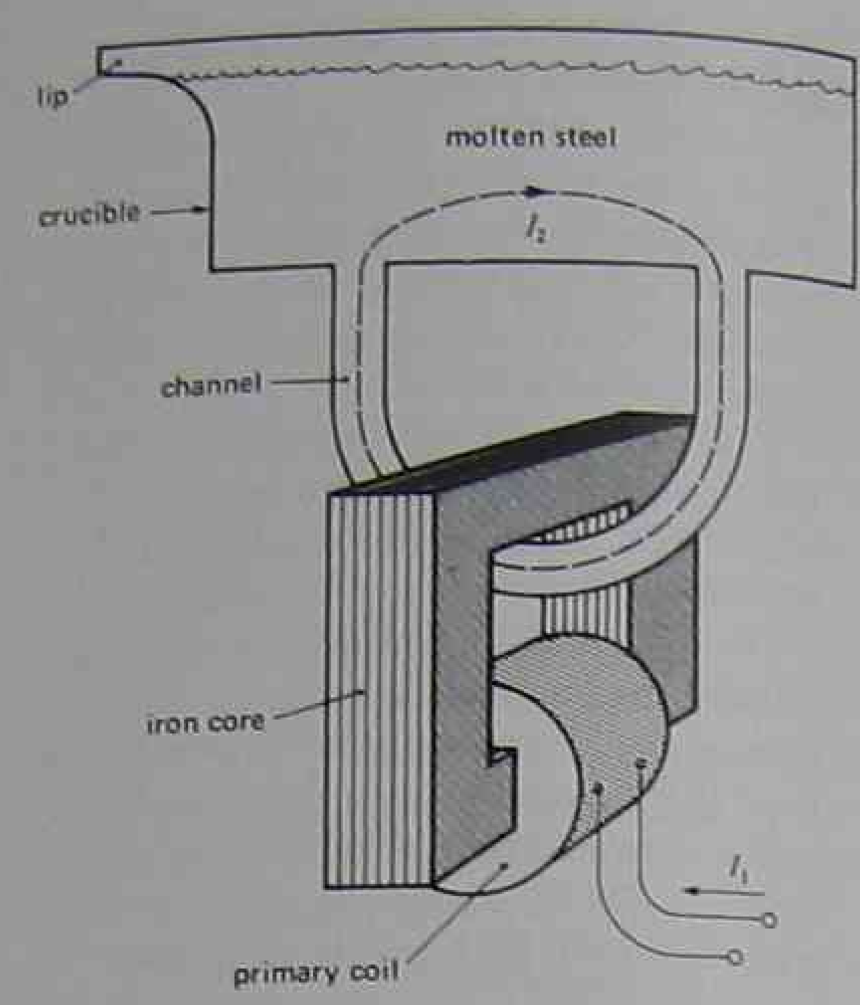

Figure 11.27
Channel induction furnace and its water-cooled transformer.

The magnetizing current is low because the flux is confined to a highly permeable iron core. On the other hand, the leakage flux is large because the secondary turn is obviously not tightly coupled to the primary coil. Nevertheless, the power factor is higher than that in Fig. 11.26, being typically between 60 and 80 percent. As a result, a smaller capacitor bank is required to furnish the reactive power.

Owing to the very high ambient temperature, the primary windings of induction furnace transformers are always made of hollow, water-cooled copper conductors. Induction furnaces are used for melting aluminum, copper, and other metals, as well as iron.

Figure 11.28 shows a very special application of the induction heating principle.

11.11 High-frequency transformers

In electronic power supplies there is often a need to isolate the output from the input and to reduce the weight and cost of the unit. In other applications, such as in aircraft, there is a strong incentive to

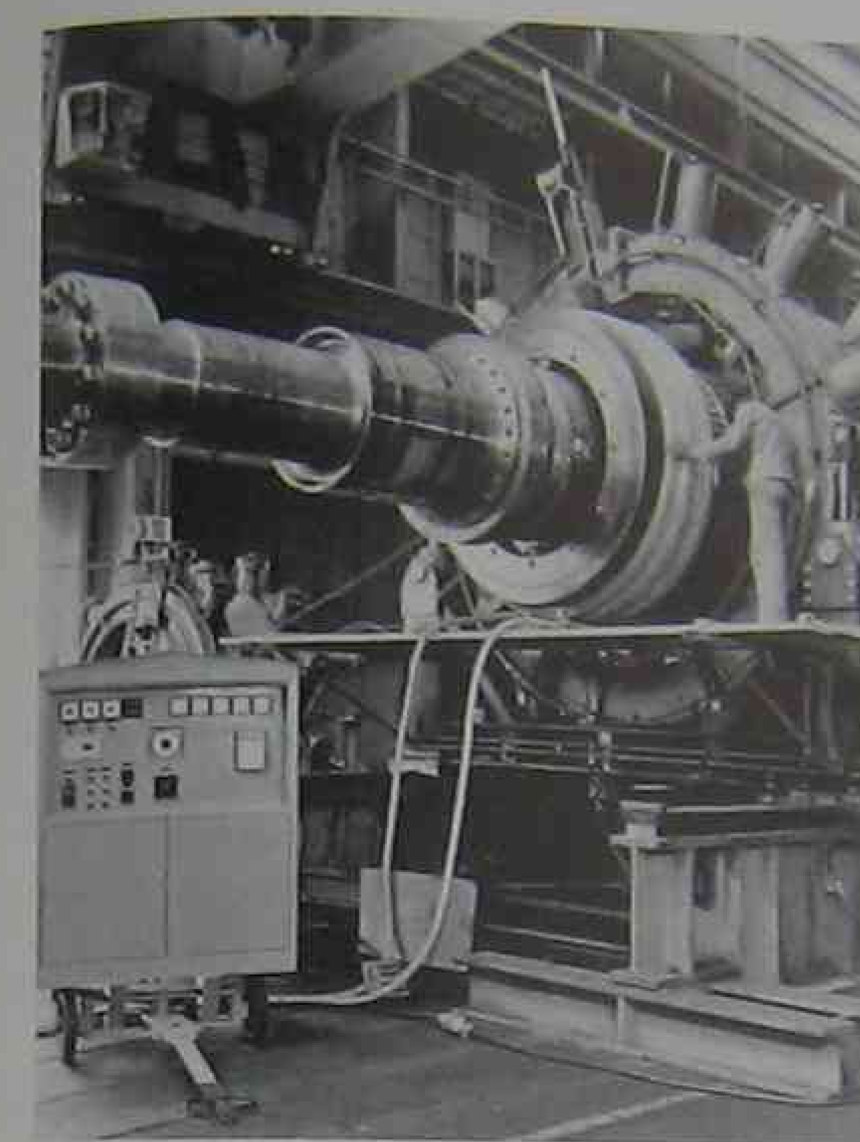

Figure 11.28
Special application of the transformer effect. This picture shows one stage in the construction of the rotor of a steam-turbine generator. It consists of expanding the diameter of a 5 t coil-retaining ring. A coil of asbestos-insulated wire is wound around the ring and connected to a 35 kW, 2000 Hz source (left foreground). The coil creates a 2000 Hz magnetic field, which induces large eddy currents in the ring, bringing its temperature up to 280°C in about 3 h. The resulting expansion enables the ring to be slipped over the coil-ends, where it cools and contracts. This method of induction heating is clean and produces a very uniform temperature rise of the large mass.
(*Courtesy of ABB*)

minimize weight. These objectives are best achieved by using a relatively high frequency compared to, say, 60 Hz. Thus, in aircraft the frequency is typically 400 Hz, while in electronic power supplies the frequency may range from 5 kHz to 50 kHz.

An increase in frequency reduces the size of such devices as transformers, inductors, and capacitors. In order to illustrate the reason for this phenomenon, we limit our discussion to transformers. Furthermore, to avoid a cumbersome theoretical analysis, we will take a practical transformer and observe how it behaves when the frequency is raised.

Consider Fig. 11.29, which shows a conventional 120 V/24 V, 60 Hz transformer having a rating of 36 VA. This small transformer weighs 0.5 kg and operates at a peak flux density of 1.5 T. The flux in the core attains a peak of 750 μWb. The laminated core is made of ordinary silicon steel having a thickness of 0.3 mm (12 mils) and the total core loss is about 1 W. The current rating is 300 mA for the primary and 1.5 A for the secondary.

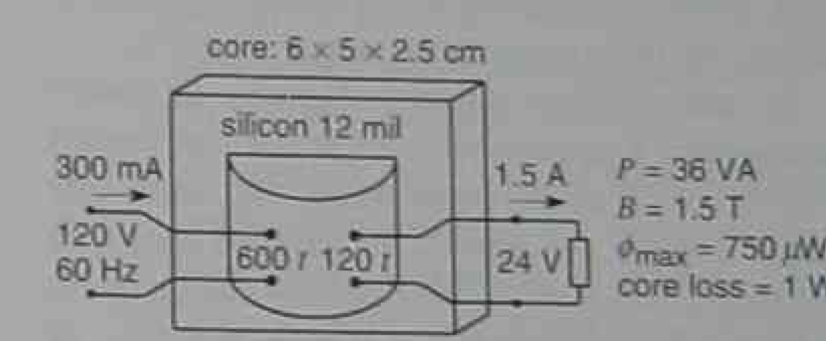

Figure 11.29

Without making any changes to the transformer, let us consider the effect of operating it at a frequency of 6000 Hz, which is 100 times higher than what it was designed for. Assuming the same peak flux density, it follows that the flux Φ_{max} will remain at 750 μWb. However, according to Eq. 9.3, this means that the corresponding primary voltage can be increased to

$$E = 4.44\, f N_1 \Phi_{max} \qquad (9.3)$$
$$= 4.44 \times 6000 \times 600 \times 750 \times 10^{-6}$$
$$= 12\,000\ \text{V}$$

which is 100 times greater than before! The secondary voltage will likewise be 100 times greater, becoming 2400 V. The operating conditions are shown in Fig. 11.30. The primary and secondary currents remain unchanged and so the power of the transformer is now 3600 VA, 100 times greater than in Fig. 11.29. Clearly, raising the frequency has had a very beneficial effect.

However, the advantage is not as great as it seems because at 6000 Hz the core loss is enormous (about 700 W), due to the increase in eddy current and hysteresis losses. Thus, the transformer in Fig. 11.30 is not feasible because it will quickly overheat.

To get around this problem, we can reduce the flux density so that the core losses are the same as they were in Fig. 11.29. Based upon the properties of 12 mil silicon steel, this requires a reduction in the flux density from 1.5 T to 0.04 T. As a result, according to Eq. 9.3, the primary and secondary voltages will have to be reduced to 320 V and 64 V, respectively. The new power of the transformer will be $P = 320 \times 0.3 = 96$ VA (Fig. 11.31). This is almost 3 times the original power of 36 VA, while retaining the same temperature rise.

By using thinner laminations made of special nickel-steel, it is possible to raise the flux density above 0.04 T while maintaining the same core losses. Thus, if we replace the original core with this special material, the flux density can be raised to 0.2 T. This corresponds to a peak flux Φ_{max} of 750 μWb × (0.2 T/1.5 T) = 100 μWb, which means that the primary voltage can be raised to

$$E = 4.44\, f N_1 \Phi_{max}$$
$$= 4.44 \times 6000 \times 600 \times 100 \times 10^{-6}$$
$$= 1600 \text{ V}$$

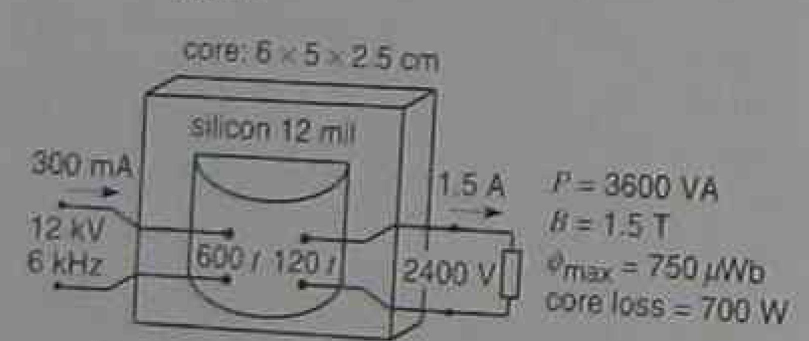

Figure 11.30

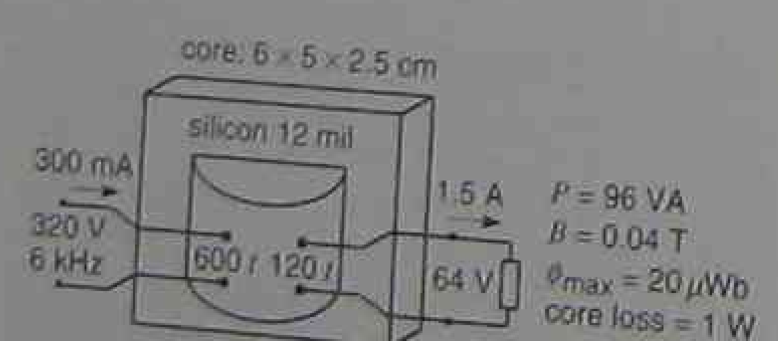

Figure 11.31

The corresponding secondary voltage is 320 V, and so the enhanced capacity of the transformer is 320 V × 1.5 A = 480 VA (Fig. 11.32).

We are interested, of course, in maintaining the original voltage ratio of 120 V to 24 V. This is readily achieved by rewinding the transformer. Thus, the number of turns on the primary will be reduced from 600 to 600 t × (120 V/1600 V) = 45 turns, while the secondary will have only 9 turns. Such a drastic reduction in the number of turns means that the wire size can be increased significantly. Bearing in mind that the capacity of the transformer is still 480 VA, it follows that the rated primary current can be raised to 4 A while that in the secondary becomes 20 A. This rewound transformer with its special core (Fig. 11.33) has the same size and weight as the one in Fig. 11.29. Furthermore, because the iron and copper losses are the same in both cases, the efficiency of the high frequency transformer is better.

It is now obvious that the increase in frequency has permitted a very large increase in the power capacity of the transformer. It follows that for a given power output a high frequency transformer is much smaller, cheaper, more efficient, and lighter than a 60 Hz transformer.

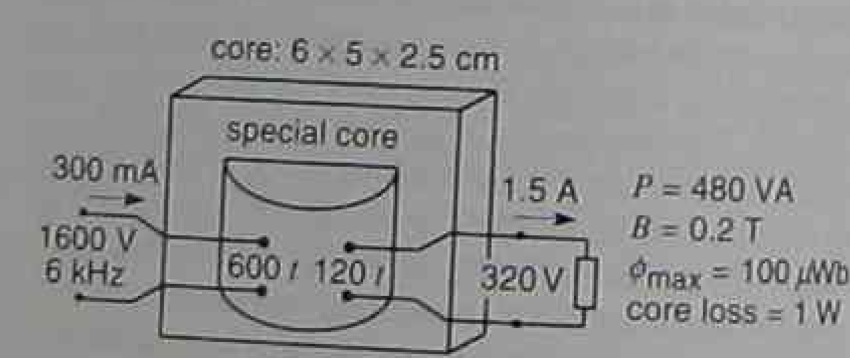

Figure 11.32

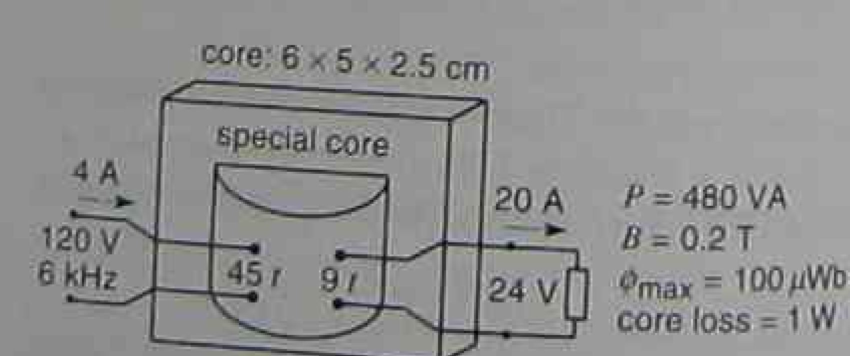

Figure 11.33

Questions and Problems

Practical level

11-1 What is the difference between an autotransformer and a conventional transformer?

11-2 What is the purpose of a voltage transformer? Of a current transformer?

11-3 Why must we never open the secondary of a current transformer?

11-4 Explain why the secondary winding of a CT or PT must be grounded.

11-5 A toroidal current transformer has a ratio of 1500 A/5 A. How many turns does it have?

11-6 A current transformer has a rating of 10 VA, 50 A/5 A, 60 Hz, 2.4 kV. Calculate the nominal voltage across the primary winding.

Intermediate level

11-7 A single-phase transformer has a rating of 100 kVA, 7200 V/600 V, 60 Hz. If it is reconnected as an autotransformer having a ratio of 7800 V/7200 V, calculate the load it can carry.

11-8 In Problem 11-7, how should the transformer terminals (H_1, H_2, X_1, X_2) be connected?

11-9 The transformer in Problem 11-7 is reconnected again as an autotransformer having a ratio of 6.6 kV/600 V. What load can it carry and how should the connections be made?

Advanced level

11-10 A current transformer has a rating of 100 VA, 2000 A/5 A, 60 Hz, 138 kV. It has a primary to secondary capacitance of 250 pF. If it is installed on a transmission line where the line-to-neutral voltage is 138 kV, calculate the capacitive leakage current that flows to ground (see Fig. 11.13).

11-11 The toroidal current transformer of Fig. 11.19 has a ratio of 1000 A/5 A. The line conductor carries a current of 600 A.

a. Calculate the voltage across the secondary winding if the ammeter has an impedance of 0.15 Ω.

b. Calculate the voltage drop the transformer produces on the line conductor.

c. If the primary conductor is looped four times through the toroidal opening, calculate the new current ratio.

Industrial application

11-12 The nameplate of a small transformer indicates 50 VA, 120 V, 12.8 V. When 118.8 V is applied to the primary, the voltage across the secondary at no-load is 13.74 V. If 120 V were available, what would the secondary voltage be? Why is this voltage higher than the indicated nameplate voltage?

11-13 Referring to Problem 11-12, the windings are encapsulated in epoxy and cannot be seen. However, the resistance of the primary is 15.2 Ω and that of the secondary is 0.306 Ω. Is the 120 V winding wound upon the 12.8 V winding, or vice versa?

11-14 Many airports use series lighting systems in which the primary windings of a large number of current transformers are connected in series across a constant current, 60 Hz source. In one installation, the primary current is kept constant at 20 A. The secondary windings are individually connected to a 100 W, 6.6 A incandescent lamp.

a. Calculate the voltage across each lamp.

b. The resistance of the secondary winding is 0.07 Ω while that of the primary is 0.008 Ω. Knowing that the magnetizing current and the leakage reactance are both negligible, calculate the voltage across the primary winding of each transformer.

c. If 140 lamps, spaced at every 50 m intervals, are connected in series using No 14 wire, calculate the minimum voltage of the power source. Assume the wire operates at a temperature of 105°C.

11-15 A no-load test on a 15 kVA, 480 V/120 V, 60 Hz transformer yields the following saturation curve data when the 120 V winding is excited by a sinusoidal source. The primary is known to have 260 turns.

a. Draw the saturation curve (voltage versus current in mA).

b. If the experiment were repeated using a 50 Hz source, redraw the resulting saturation curve.

c. Draw the saturation curve at 60 Hz (peak flux in mWb versus current in mA). At what point on the saturation curve does saturation become important? Is the flux distorted under these conditions?

E	14.8	31	49.3	66.7	90.5	110	120	130	136	142	V
I_0	59	99	144	210	430	700	1060	1740	2300	3200	mA

CHAPTER 12
Three-Phase Transformers

12.0 Introduction

Power is distributed throughout North America by means of 3-phase transmission lines. In order to transmit this power efficiently and economically, the voltages must be at appropriate levels. These levels (13.8 kV to 765 kV) depend upon the amount of power that has to be transmitted and the distance it has to be carried. Another aspect is the appropriate voltage levels used in factories and homes. These are fairly uniform, ranging from 120/240 V single-phase systems to 600 V 3-phase systems. Clearly, this requires the use of 3-phase transformers to transform the voltages from one level to another.

The transformers may be inherently 3-phase, having three primary windings and three secondary windings mounted on a 3-legged core. However, the same result can be achieved by using three single-phase transformers connected together to form a 3-phase transformer bank.

12.1 Basic properties of 3-phase transformer banks

When three single-phase transformers are used to transform a 3-phase voltage, the windings can be connected in several ways. Thus, the primaries may be connected in delta and the secondaries in wye, or vice versa. As a result, the ratio of the 3-phase input voltage to the 3-phase output voltage depends not only upon the turns ratio of the transformers, but also upon how they are connected.

A 3-phase transformer bank can also produce a *phase shift* between the 3-phase input voltage and the 3-phase output voltage. The amount of phase shift depends again upon the turns ratio of the transformers, and on how the primaries and secondaries are interconnected. Furthermore, the phase-shift feature enables us to change the *number* of phases. Thus, a 3-phase system can be converted into a 2-phase, a 6-phase, or a 12-phase system. Indeed, if there were a practical application for it, we could even convert a 3-phase system into a 5-phase system by an appropriate choice of single-phase transformers and interconnections.

In making the various connections, it is important to observe transformer polarities. An error in polarity may produce a short-circuit or unbalance the line voltages and currents.

The basic behavior of balanced 3-phase transformer banks can be understood by making the following simplifying assumptions:

1. The exciting currents are negligible.
2. The transformer impedances, due to the resistance and leakage reactance of the windings, are negligible.
3. The total apparent input power to the transformer bank is equal to the total apparent output power.

Furthermore, when single-phase transformers are connected into a 3-phase system, they retain all their basic single-phase properties, such as current ratio, voltage ratio, and flux in the core. Given the polarity marks X_1, X_2 and H_1, H_2, the phase shift between primary and secondary is zero, in the sense that $E_{X_1X_2}$ is in phase with $E_{H_1H_2}$.

12.2 Delta-delta connection

The three single-phase transformers P, Q, and R of Fig. 12.1 transform the voltage of the incoming transmission line A, B, C to a level appropriate for the outgoing transmission line 1, 2, 3. The incoming line is connected to the source, and the outgoing line is connected to the load. The transformers are connected in *delta-delta*. Terminal H_1 of each transformer is connected to terminal H_2 of the next transformer. Similarly, terminals X_1 and X_2 of successive transformers are connected together. The actual physical layout of the transformers is shown in Fig. 12.1. The corresponding schematic diagram is given in Fig. 12.2.

The schematic diagram is drawn in such a way to show not only the connections, but also the phasor relationship between the primary and secondary voltages. Thus, each secondary winding is drawn parallel to the corresponding primary winding to which it is coupled. Furthermore, if source G produces voltages E_{AB}, E_{BC}, E_{CA} according to the indicated phasor diagram, the primary windings are oriented the same way, phase by phase. For example, the primary of transformer P between lines A and B is oriented horizontally, in the same direction as phasor E_{AB}.

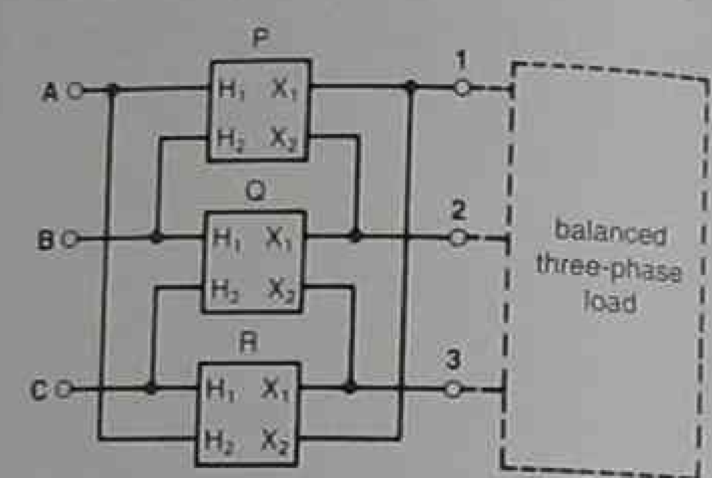

Figure 12.1
Delta-delta connection of three single-phase transformers. The incoming lines (source) are A, B, C and the outgoing lines (load) are 1, 2, 3.

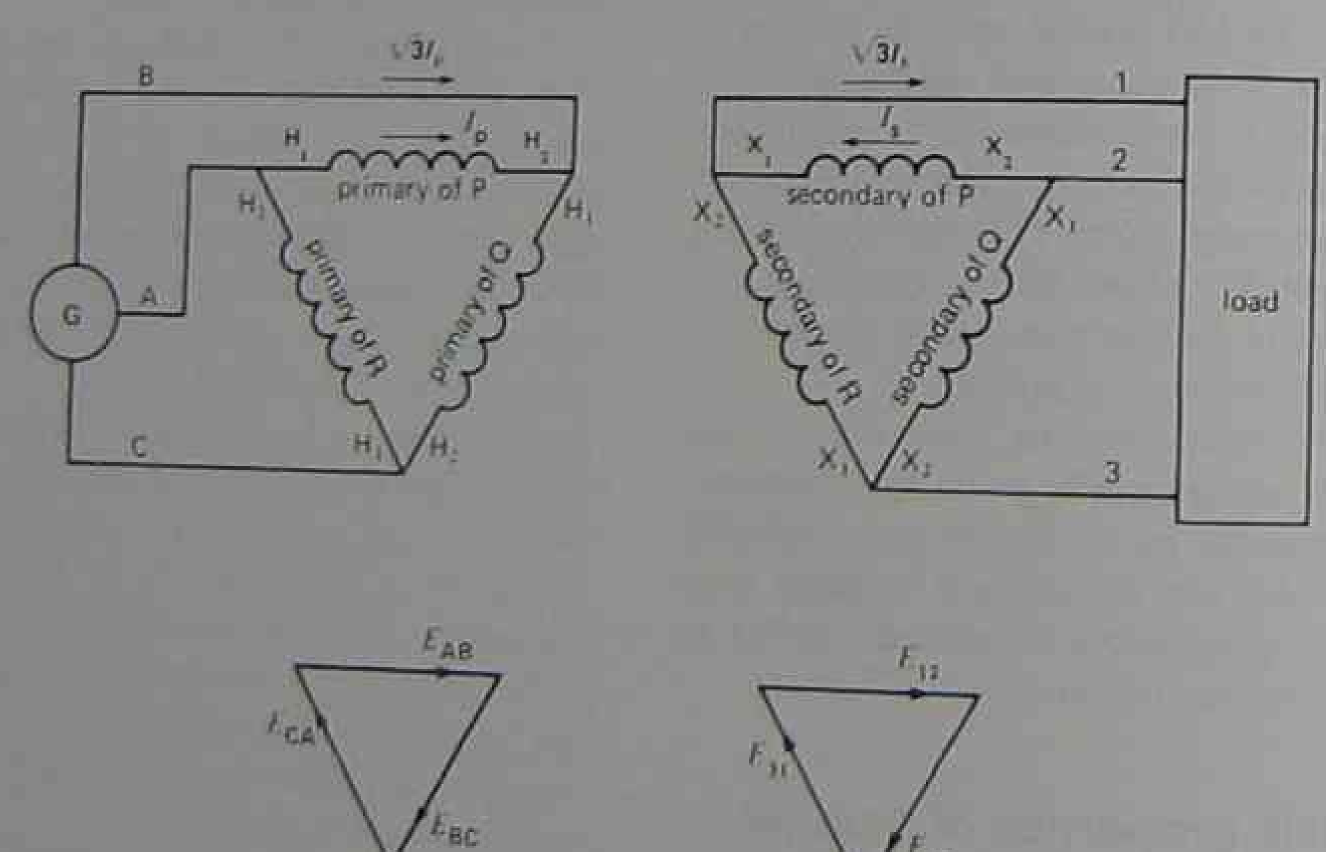

Figure 12.2
Schematic diagram of a delta-delta connection and associated phasor diagram.

Because the primary and secondary voltages $E_{H_1H_2}$ and $E_{X_1X_2}$ of a given transformer must be in phase, it follows that E_{12} (secondary voltage of transformer P) must be in phase with E_{AB} (primary of the same transformer). Similarly, E_{23} is in phase with E_{BC}, and E_{31} with E_{CA}.

In such a *delta-delta* connection, the voltages between the respective incoming and outgoing transmission lines are in phase.

If a balanced load is connected to lines 1-2-3, the resulting line currents are equal in magnitude. This produces balanced line currents in the incoming lines A-B-C. As in any delta connection, the line currents are $\sqrt{3}$ times greater than the respective currents I_p and I_s flowing in the primary and secondary windings (Fig. 12.2). The power rating of the transformer bank is three times the rating of a single transformer.

Note that although the transformer bank constitutes a 3-phase arrangement, each transformer, considered alone, acts as if it were placed in a single-phase circuit. Thus, a current I_p flowing from H_1 to H_2 in the primary winding is associated with a current I_s flowing from X_2 to X_1 in the secondary.

Example 12-1

Three single-phase transformers are connected in delta-delta to step down a line voltage of 138 kV to 4160 V to supply power to a manufacturing plant. The plant draws 21 MW at a lagging power factor of 86 percent.

Calculate

a. The apparent power drawn by the plant
b. The apparent power furnished by the HV line
c. The current in the HV lines
d. The current in the LV lines
e. The currents in the primary and secondary windings of each transformer
f. The load carried by each transformer

Solution

a. The apparent power drawn by the plant is

$$S = P/\cos\theta \qquad (7.7)$$
$$= 21/0.86$$
$$= 24.4 \text{ MVA}$$

b. The transformer bank itself absorbs a negligible amount of active and reactive power because the I^2R losses and the reactive power associated with the mutual flux and the leakage fluxes are small. It follows that the apparent power furnished by the HV line is also 24.4 MVA.

c. The current in each HV line is

$$I_1 = S/(\sqrt{3}\,E) \qquad (8.9)$$
$$= (24.4 \times 10^6)/(\sqrt{3} \times 138\,000)$$
$$= 102 \text{ A}$$

d. The current in the LV lines is

$$I_2 = S/(\sqrt{3}\,E)$$
$$= (24.4 \times 10^6)/(\sqrt{3} \times 4160)$$
$$= 3386 \text{ A}$$

e. Referring to Fig. 12.2, the current in each primary winding is

$$I_p = 102/\sqrt{3} = 58.9 \text{ A}$$

The current in each secondary winding is

$$I_s = 3386/\sqrt{3} = 1955 \text{ A}$$

f. Because the plant load is balanced, each transformer carries one-third of the total load, or 24.4/3 = 8.13 MVA.

The individual transformer load can also be obtained by multiplying the primary voltage times the primary current:

$$S = E_pI_p = 138\,000 \times 58.9$$
$$= 8.13 \text{ MVA}$$

Note that we can calculate the line currents and the currents in the transformer windings even though we do not know how the 3-phase load is connected. In effect, the plant load (shown as a box in Fig. 12.2) is composed of hundreds of individual loads, some of which are connected in delta, others in wye. Furthermore, some are single-phase loads operating at much lower voltages than 4160 V, powered by smaller transformers located inside the plant. The sum total of these loads usually results in a reasonably well-balanced 3-phase load, represented by the box.

12.3 Delta-wye connection

When the transformers are connected in *delta-wye*, the three primary windings are connected the same way as in Fig. 12.1. However, the secondary windings are connected so that all the X_2 terminals are joined together, creating a common neutral N (Fig. 12.3). In such a delta-wye connection, the voltage across each primary winding is equal to the incoming line voltage. However, the outgoing line voltage is $\sqrt{3}$ times the secondary voltage across each transformer.

The relative values of the currents in the transformer windings and transmission lines are given in Fig. 12.4. Thus, the line currents in phases A, B, and C are $\sqrt{3}$ times the currents in the primary windings. The line currents in phases 1, 2, 3 are the same as the currents in the secondary windings.

A delta-wye connection produces a 30° phase shift between the line voltages of the incoming and outgoing transmission lines. Thus, outgoing line

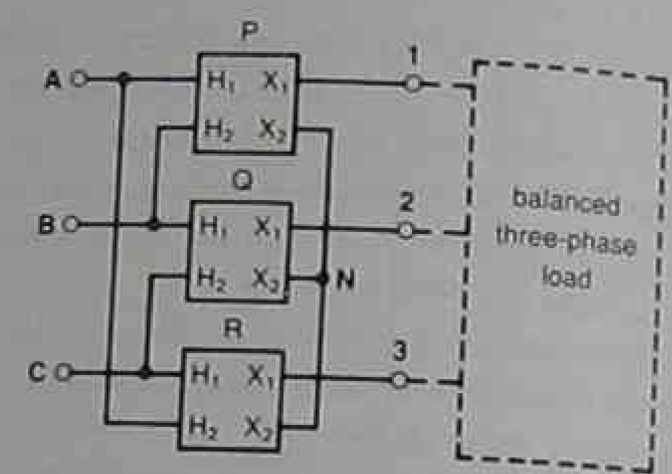

Figure 12.3
Delta-wye connection of three single-phase transformers.

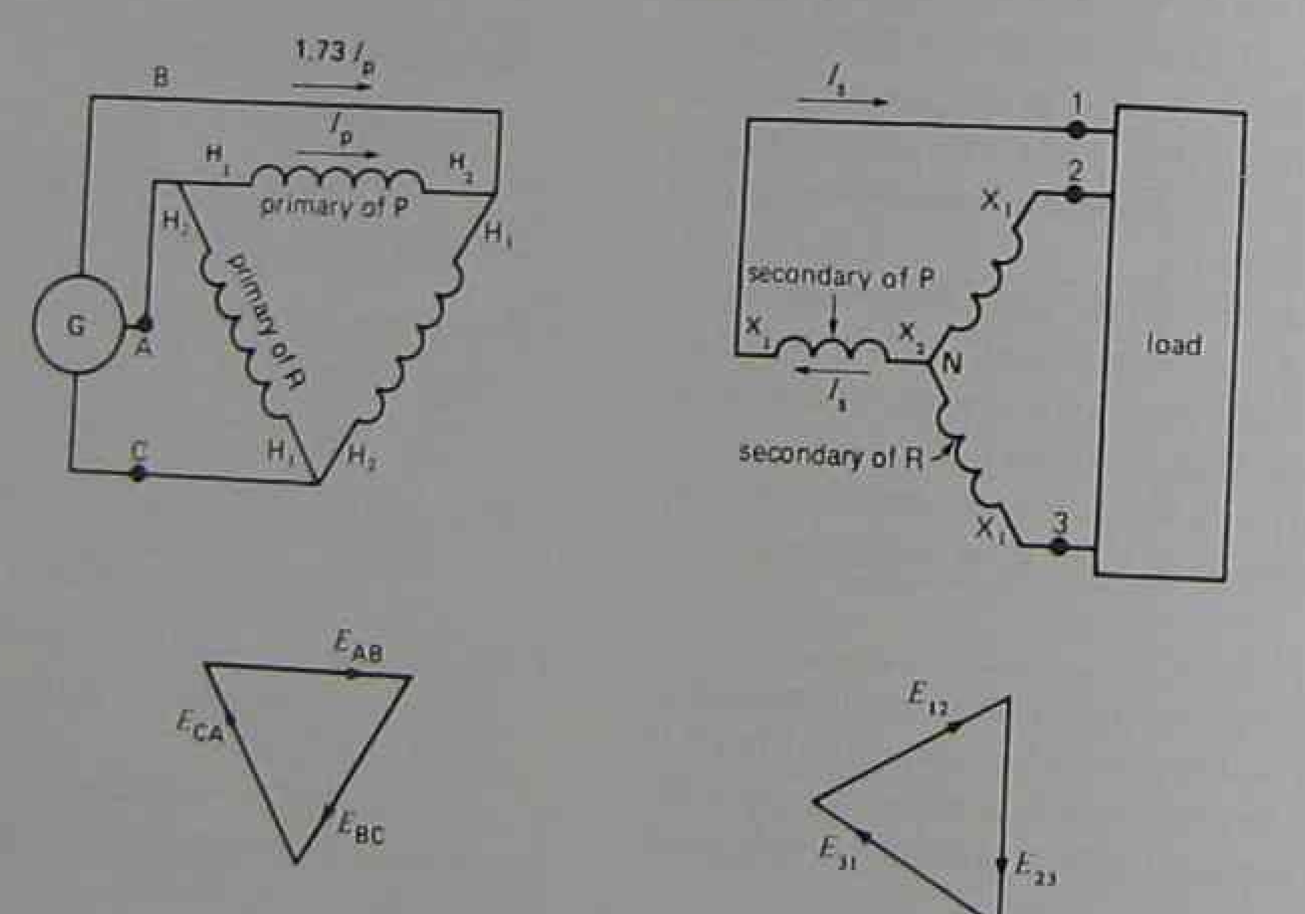

Figure 12.4
Schematic diagram of a delta-wye connection and associated phasor diagram. (The phasor diagrams on the primary and secondary sides are not drawn to the same scale.)

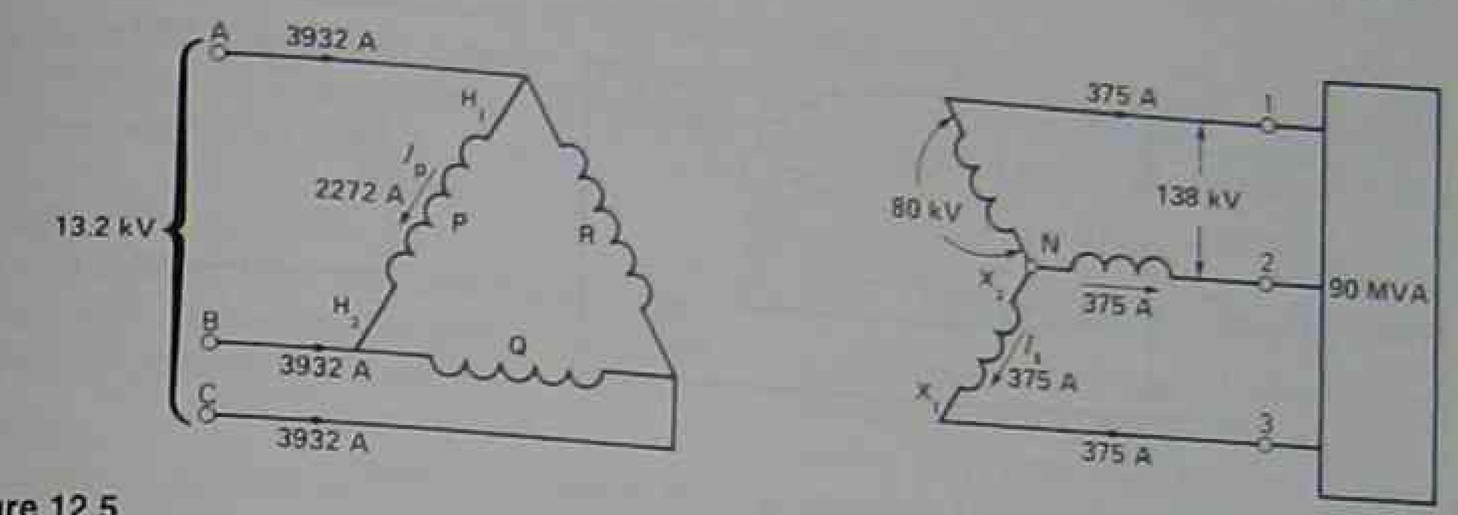

Figure 12.5
See Example 12-2.

voltage E_{12} is 30° ahead of incoming line voltage E_{AB}, as can be seen from the phasor diagram. If the outgoing line feeds an isolated group of loads, the phase shift creates no problem. But, if the outgoing line has to be connected in parallel with a line coming from another source, the 30° shift may make such a parallel connection impossible, even if the line voltages are otherwise identical.

One of the important advantages of the wye connection is that it reduces the amount of insulation needed inside the transformer. The HV winding has to be insulated for only $1/\sqrt{3}$, or 58 percent of the line voltage.

Example 12-2

Three single-phase step-up transformers rated at 40 MVA, 13.2 kV/80 kV are connected in delta-wye on a 13.2 kV transmission line (Fig. 12.5). If they feed a 90 MVA load, calculate the following:

a. The secondary line voltage
b. The currents in the transformer windings
c. The incoming and outgoing transmission line currents

Solution

The easiest way to solve this problem is to consider the windings of only one transformer, say, transformer P.

a. The voltage across the primary winding is obviously 13.2 kV.

The voltage across the secondary is, therefore, 80 kV.

The voltage between the outgoing lines 1, 2, and 3 is

$$E_s = 80\sqrt{3} = 139 \text{ kV}$$

b. The load carried by each transformer is

$$S = 90/3 = 30 \text{ MVA}$$

The current in the primary winding is

$$I_p = 30 \text{ MVA}/13.2 \text{ kV} = 2273 \text{ A}$$

The current in the secondary winding is

$$I_s = 30 \text{ MVA}/80 \text{ kV} = 375 \text{ A}$$

c. The current in each incoming line A, B, C is

$$I = 2273\sqrt{3} = 3937 \text{ A}$$

The current in each outgoing line 1, 2, 3 is

$$I = 375 \text{ A}$$

12.4 Wye-delta connection

The currents and voltages in a *wye-delta* connection are identical to those in the delta-wye connection of Section 12.3. The primary and secondary connections are simply interchanged. In other words, the H_2 terminals are connected together to create a neutral, and the X_1, X_2 terminals are connected in delta. Again, there results a 30° phase shift between the voltages of the incoming and outgoing lines.

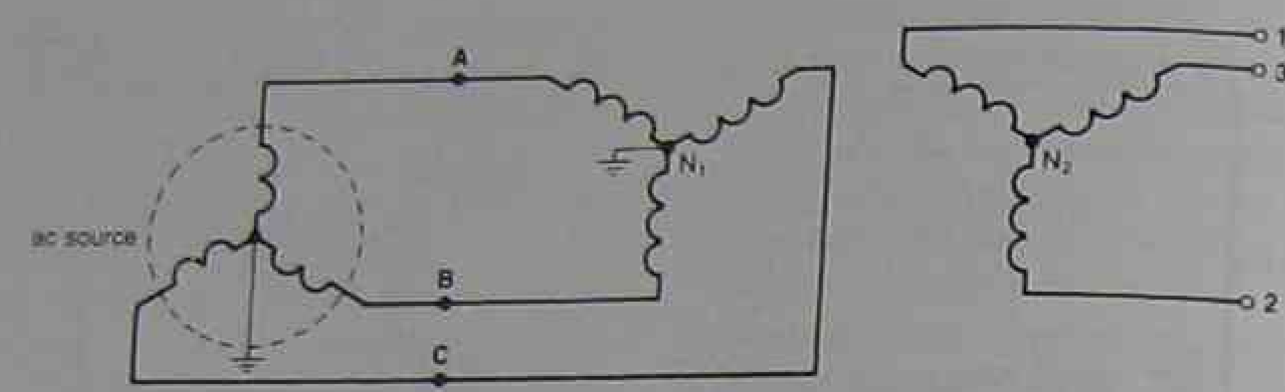

Figure 12.6
Wye-wye connection with neutral of the primary connected to the neutral of the source.

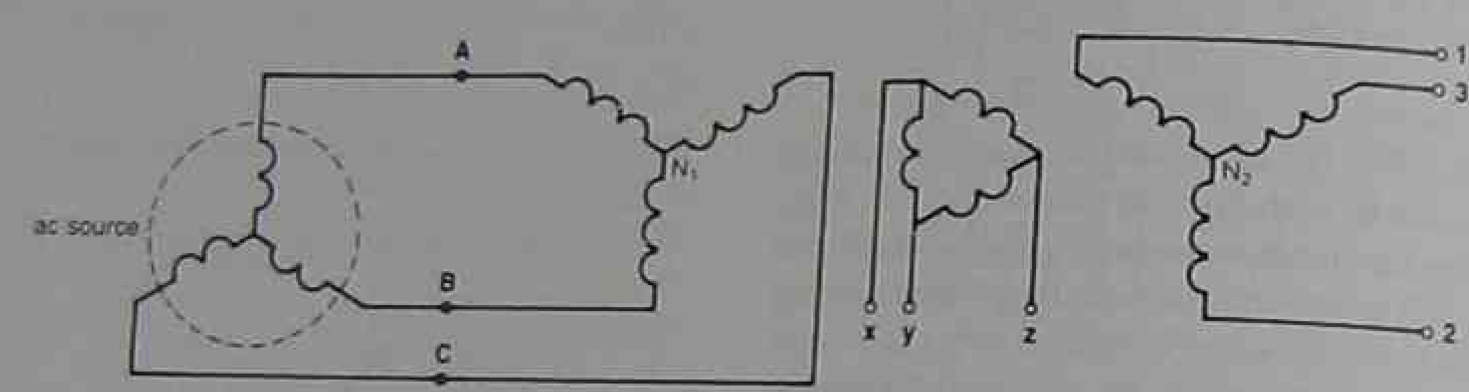

Figure 12.7
Wye-wye connection using a tertiary winding.

12.5 Wye-wye connection

When transformers are connected in wye-wye, special precautions have to be taken to prevent severe distortion of the line-to-neutral voltages. One way to prevent the distortion is to connect the neutral of the primary to the neutral of the source, usually by way of the ground (Fig. 12.6). Another way is to provide each transformer with a third winding, called *tertiary* winding. The tertiary windings of the three transformers are connected in delta (Fig. 12.7). They often provide the substation service voltage where the transformers are installed.

Note that there is no phase shift between the incoming and outgoing transmission line voltages of a wye-wye connected transformer.

12.6 Open-delta connection

It is possible to transform the voltage of a 3-phase system by using only 2 transformers, connected in open-delta. The *open-delta* arrangement is identical to a delta-delta connection, except that one transformer is absent (Fig. 12.8). However, the open-delta connection is seldom used because the load capacity of the transformer bank is only 86.6 percent of the installed transformer capacity. For example, if two 50 kVA transformers are connected in open-delta, the installed capacity of the transformer bank is obviously 2 × 50 = 100 kVA. But, strange as it may seem, it can only deliver 86.6 kVA before the transformers begin to overheat.

The open-delta connection is mainly used in emergency situations. Thus, if three transformers are connected in delta-delta and one of them becomes defective and has to be removed, it is possible to feed the load on a temporary basis with the two remaining transformers.

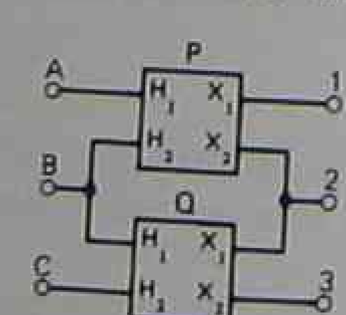

Figure 12.8a
Open-delta connection.

Example 12-3

Two single-phase 150 kVA, 7200 V/600 V transformers are connected in open-delta. Calculate the maximum 3-phase load they can carry.

Solution

Although each transformer has a rating of 150 kVA, the two together *cannot* carry a load of 300 kVA. The following calculations show why:

The nominal secondary current of each transformer is

$$I_s = 150\text{ kVA}/600\text{ V} = 250\text{ A}$$

The current I_s in lines 1, 2, 3 cannot, therefore, exceed 250 A (Fig. 12.8b). Consequently, the maximum load that the transformers can carry is

$$\begin{aligned} S &= \sqrt{3}\,EI \qquad (8.9)\\ &= \sqrt{3} \times 600 \times 250 = 259\,800\text{ VA}\\ &= 260\text{ kVA} \end{aligned}$$

Thus, the ratio

$$\frac{\text{maximum load}}{\text{installed transformer rating}} = \frac{260\text{ kVA}}{300\text{ kVA}} = 0.867, \text{ or } 86.7\%$$

12.7 Three-phase transformers

A transformer bank composed of three single-phase transformers may be replaced by one 3-phase transformer (Fig. 12.9). The magnetic core of such a transformer has three legs that carry the primary and secondary windings of each phase. The windings are connected internally, either in wye or in delta, with the result that only six terminals have to be brought outside the tank. For a given total capacity, a 3-phase transformer is always smaller and cheaper than three single-phase transformers. Nevertheless, single-phase transformers are sometimes preferred, particularly when a replacement unit is essential. For example, suppose a manufacturing plant absorbs 5000 kVA. To guarantee continued service we can install one 3-phase 5000 kVA transformer and keep a second one as a spare. Alternatively, we can install three single-phase transformers each rated at 1667 kVA, plus one spare. The 3-phase transformer option is

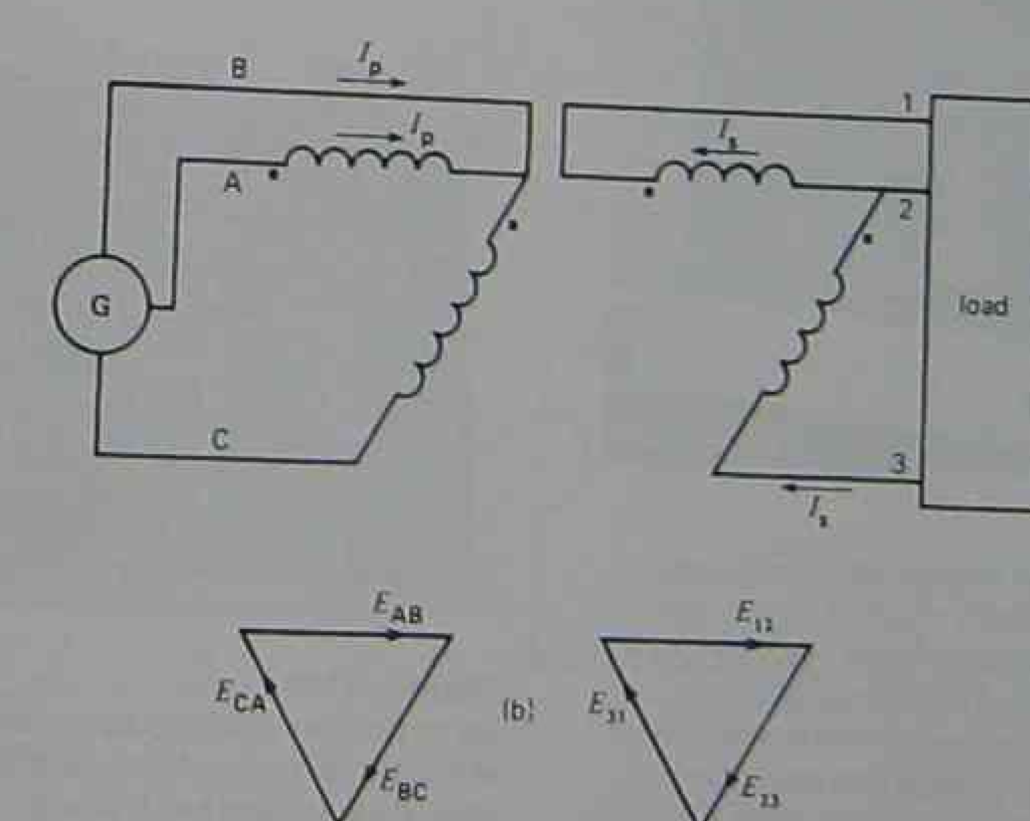

Figure 12.8b
Associated schematic and phasor diagram.

more expensive (total capacity: 2 × 5000 = 10 000 kVA) than the single-phase option (total capacity: 4 × 1667 = 6667 kVA).

Fig. 12.10 shows successive stages of construction of a 3-phase 110 MVA, 222.5 kV/34.5 kV tap-changing transformer.* Note that in addition to the three main legs, the magnetic core has two additional lateral legs. They enable the designer to reduce the overall height of the transformer, which simplifies the problem of shipping. In effect, whenever large equipment has to be shipped, the designer is faced with the problem of overhead clearances on highways and rail lines.

* A tap-changing transformer regulates the secondary voltage by automatically switching from one tap to another on the primary winding. The tap-changer is a motorized device under the control of a sensor that continually monitors the voltage that has to be held constant.

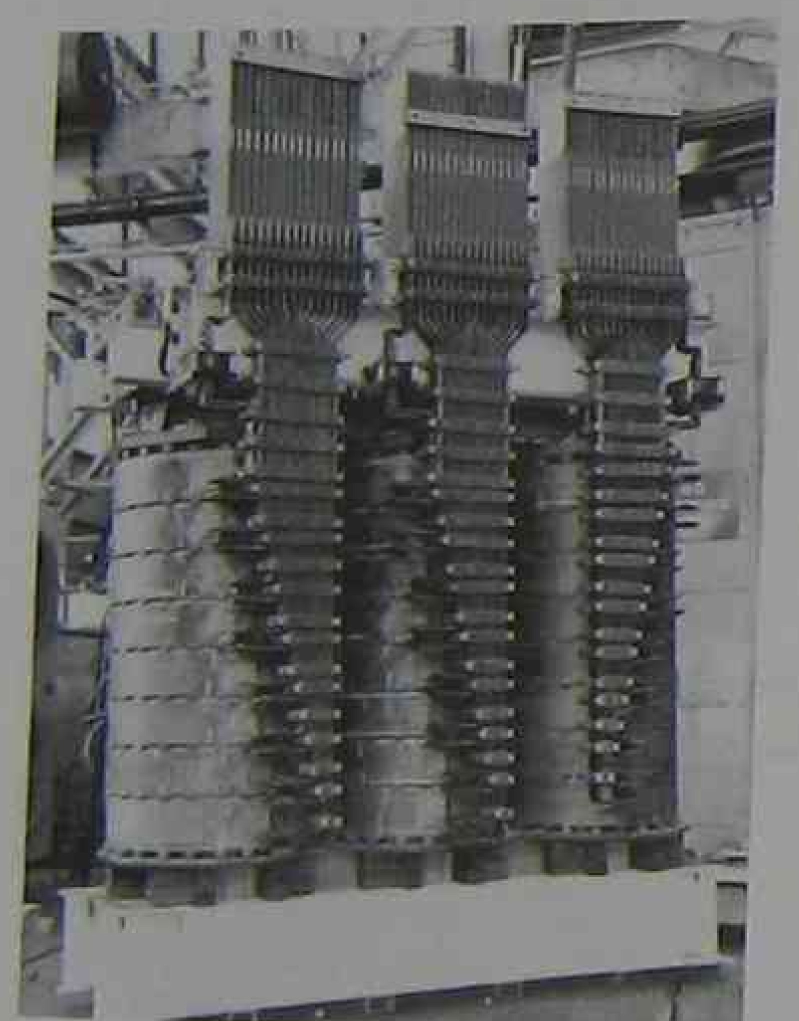

Figure 12.9
Three-phase transformer for an electric arc furnace, rated 36 MVA, 13.8 kV/160 V to 320 V, 60 Hz. The secondary voltage is adjustable from 160 V to 320 V by means of 32 taps on the primary winding (not shown). The three large busbars in the foreground deliver a current of 65 000 A. Other characteristics: impedance: 3.14%; diameter of each leg of the core: 711 mm; overall height of core: 3500 mm; center line distance between adjacent core legs: 1220 mm.
(Courtesy of Ferranti-Packard)

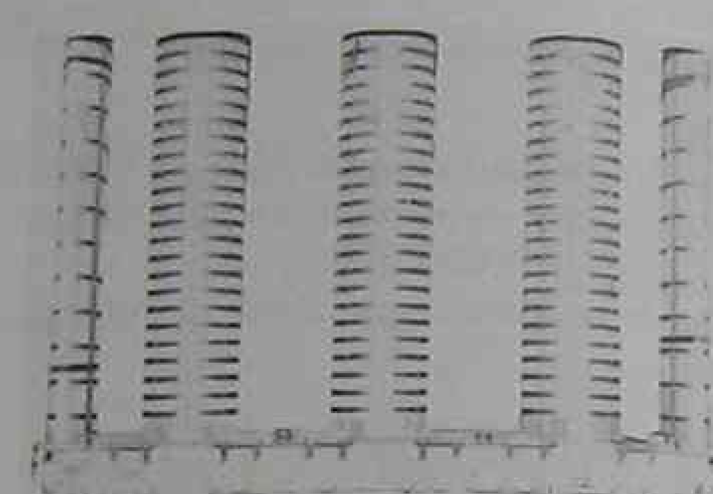

Figure 12.10a
Core of a 110 MVA, 222.5 kV/34.5 kV, 60 Hz, 3-phase transformer. By staggering laminations of different widths, the core legs can be made almost circular. This reduces the coil diameter to a minimum, resulting in less copper and lower I^2R losses. The legs are tightly bound to reduce vibration. Mass of core: 53 560 kg.

Figure 12.10b
Same transformer with coils in place. The primary windings are connected in wye and the secondaries in delta. Each primary has 8 taps to change the voltage in steps of ±2.5%. The motorized tap-changer can be seen in the right upper corner of the transformer. Mass of copper: 15 230 kg.

The 34.5 kV windings (connected in delta) are mounted next to the core. The 222.5 kV windings (connected in wye) are mounted on top of the 34.5 kV windings. A space of several centimeters separates the two windings to ensure good isolation and to allow cool oil to flow freely between them. The HV bushings that protrude from the oil-filled tank are connected to a 220 kV line. The medium voltage (MV) bushings are much smaller and cannot be seen in the photograph (Fig. 12.10c).

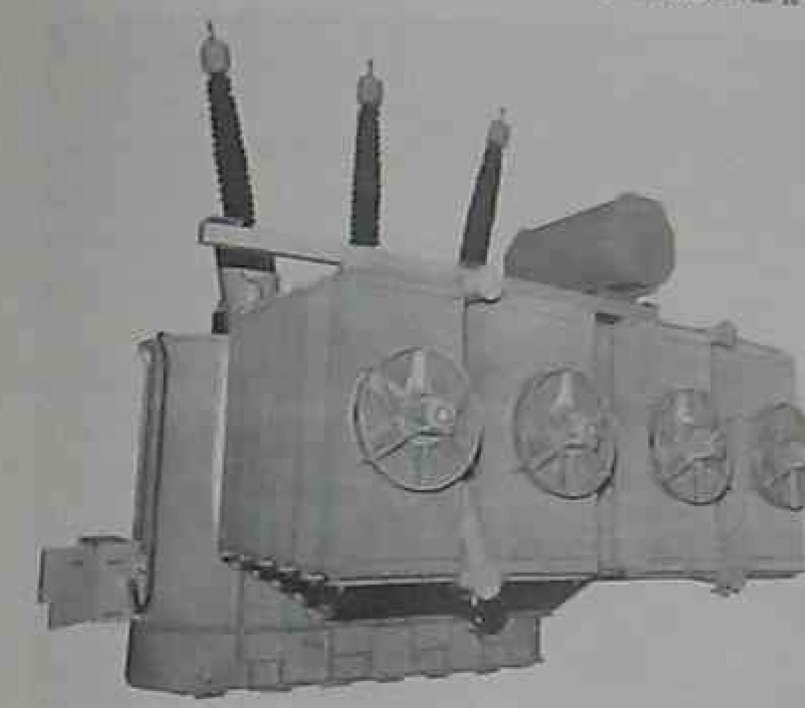

Figure 12.10c
Same transformer ready for shipping. It has been subjected to a 1050 kV impulse test on the HV side and a similar 250 kV test on the LV side. Other details: power rating: 110 MVA/146.7 MVA (OA/FA); total mass including oil: 158.7 t; overall height: 9 m; width: 8.2 m, length: 9.2 m.
(Courtesy of ABB)

12.8 Step-up and step-down autotransformer

When the voltage of a 3-phase line has to be stepped up or stepped down by a moderate amount, it is economically advantageous to use three single-phase transformers to create a wye-connected *autotransformer.* The actual physical connections are shown in Fig. 12.11a, and the corresponding schematic diagram is given in Fig. 12.11b. The respective line-to-neutral voltages of the primary and secondary are obviously in phase. Consequently, the incoming and outgoing transmission line voltages are in phase. The neutral is connected to the system neutral, otherwise a tertiary winding must be added to prevent the line-to-neutral voltage distortion mentioned previously (Section 12.5).

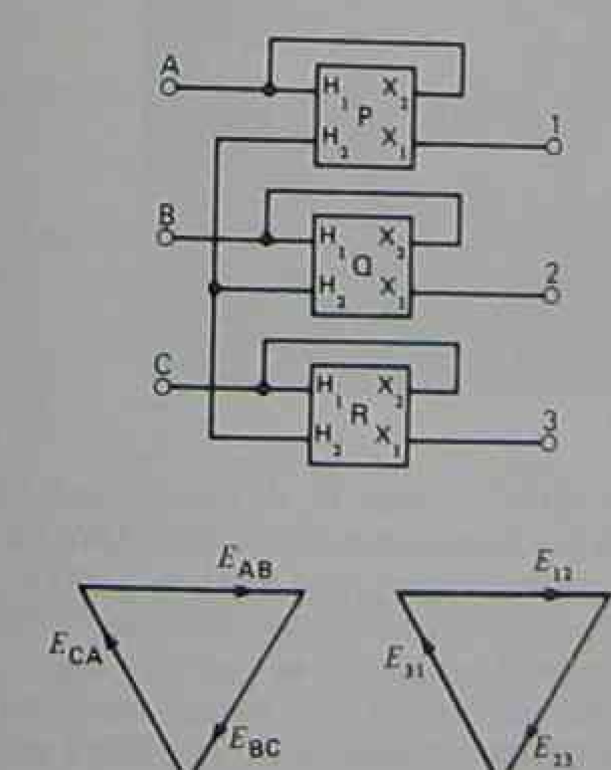

Figure 12.11a
Wye-connected autotransformer.

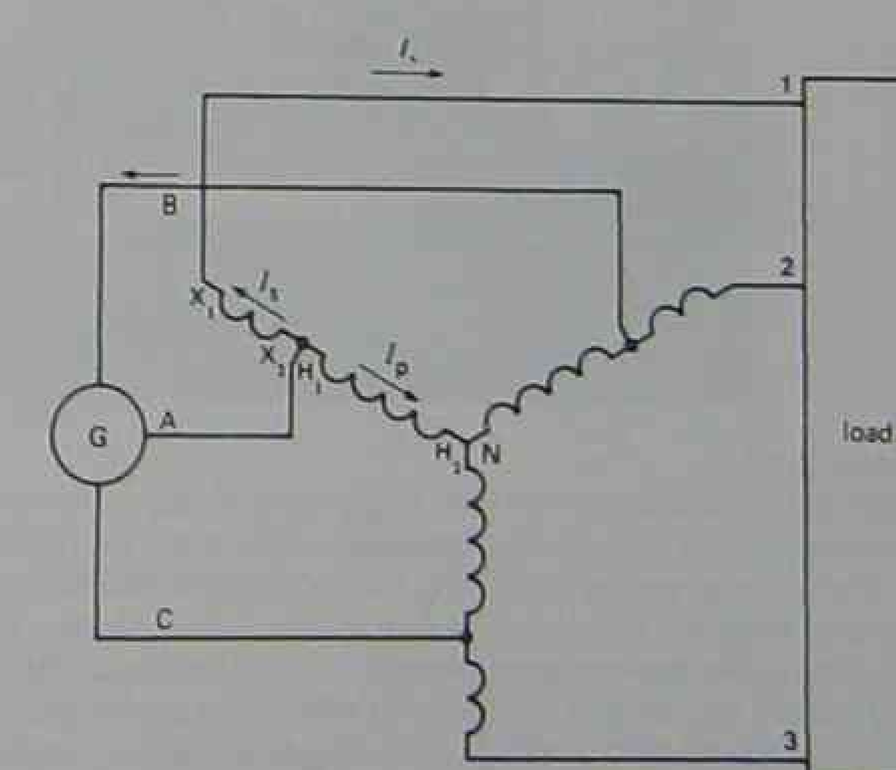

Figure 12.11b
Associated schematic diagram.

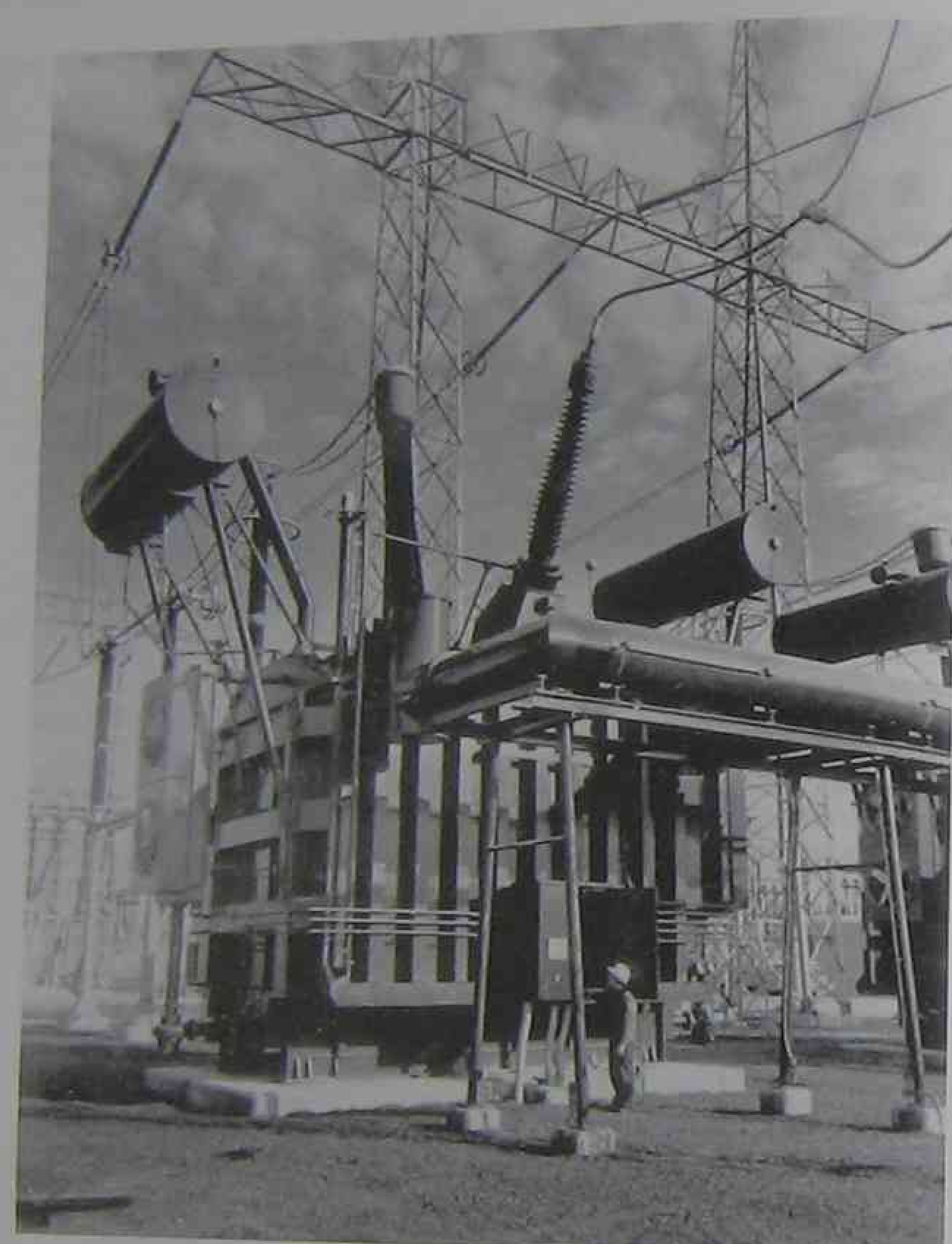

Figure 12.11c
Single-phase autotransformer (one of a group of three) connecting a 700 kV, 3-phase, 60 Hz transmission line to an existing 300 kV system. The transformer ratio is 404 kV/173 kV, to give an output of 200/267/333 MVA per transformer, at a temperature rise of 55°C. Cooling is OA/FA/FOA. A tertiary winding rated 35 MVA, 11.9 kV maintains balanced and distortion-free line-to-neutral voltages, while providing power for the substation. Other properties of this transformer: weight of core and windings: 132 t; tank and accessories: 46 t; oil: 87 t; total weight: 265 t. BIL rating is 1950 kV and 1050 kV on the HV and LV side, respectively. Note the individual 700 kV (right) and 300 kV (left) bushings protruding from the tank. The basic impulse insulation (BIL) of 1950 kV and 1050 kV expresses the transformer's ability to withstand lightning and switching surges.
(*Courtesy of Hydro-Québec*)

For a given power output, an autotransformer is smaller and cheaper than a conventional transformer (see Section 11.2). This is particularly true if the ratio of the incoming line voltage to outgoing line voltage lies between 0.5 and 2.

Figure 12.11c shows a large single-phase autotransformer rated 404 kV/173 kV with a tertiary winding rated 11.9 kV. It is part of a 3-phase transformer bank used to connect a 700 kV transmission line to an existing 300 kV system.

Example 12-4

The voltage of a 3-phase, 230 kV line has to be stepped up to 345 kV to supply a load of 200 MVA. Three single-phase transformers connected as autotransformers are to be used. Calculate the basic power and voltage rating of each transformer, assuming they are connected as shown in Fig. 12.11b.

Solution

To simplify the calculations, let us consider only one phase (phase A, say).

The line-to-neutral voltage between X_1 and H_2 is

$$E_{1N} = 345/\sqrt{3} = 199 \text{ kV}$$

The line-to-neutral voltage between H_1 and H_2 is

$$E_{AN} = 230/\sqrt{3} = 133 \text{ kV}$$

The voltage of winding X_1X_2 between lines 1 and A is

$$E_{1A} = 199 - 133 = 66 \text{ kV}$$

This means that each transformer has an effective primary to secondary voltage rating of 133 kV to 66 kV.

The current in each phase of the outgoing line is

$$\begin{aligned} I_s &= S/\sqrt{3}\,E && (8.9) \\ &= (200 \times 10^6)/(\sqrt{3} \times 345\,000) \\ &= 335 \text{ A} \end{aligned}$$

The power associated with winding X_1X_2 is

$$S_a = 66\,000 \times 335 = 22.1 \text{ MVA}$$

Winding H_1H_2 has the same power rating. The basic rating of each single-phase transformer is therefore 22.1 MVA.

The basic rating of the 3-phase transformer bank is 22.1 × 3 = 66.3 MVA.

The basic transformer rating (as far as size is concerned) is considerably less than its load-carrying capacity of 200 MVA. This is in keeping with the fact that the ratio of transformation (345/230 = 1.5) lies between 0.5 and 2.0.

12.9 Phase-shift principle

A 3-phase system enables us to shift the phase angle of a voltage very simply. Such phase shifting enables us to create 2-phase, 6-phase, and 12-phase systems from an ordinary 3-phase line. Such multiphase systems are used in large electronic converter stations and in special electric controls. Phase shifting is also used to control power flow over transmission lines that form part of a power grid.

To understand the phase shifting principle, consider a rheostat connected between phases B and C of a 3-phase line (Fig. 12.12). As we slide contact P

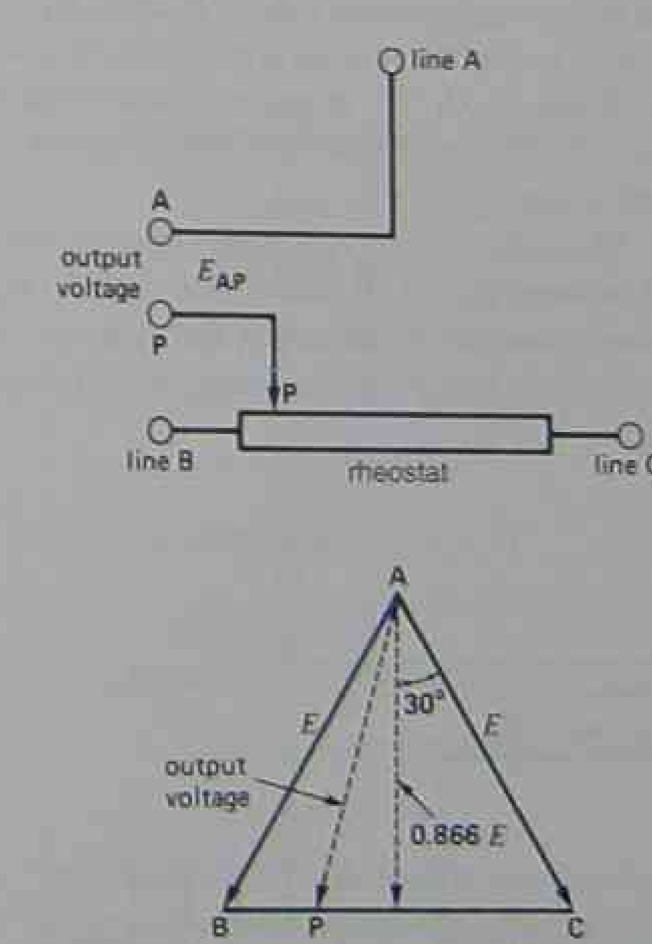

Figure 12.12
Voltage E_{AP} can be phase-shifted with respect to E_{AC} by means of a potentiometer.

from phase B toward phase C, voltage E_{AP} changes both in amplitude and phase. We obtain a 60° phase shift in moving from one end of the potentiometer to the other. Thus, as we move from B to C, voltage E_{AP} gradually advances in phase with respect to E_{AB}. At the same time, the magnitude of E_{AP} varies slightly, from E (voltage between the lines) to 0.866 E when the contact is in the middle of the rheostat.

Such a simple phase-shifter can only be used in circuits where the load between terminals A and P draws a few milliamperes. If a heavier load is applied, the resulting IR drop in the rheostat completely changes the voltage and phase angle from what they were on open-circuit.

To get around this problem, we connect a multitap autotransformer between phases B and C (Fig. 12.13). By moving contact P, we obtain the same open-circuit voltages and phase shifts as before, but this time they remain essentially unchanged when a load is connected between terminals A and P. Why is this so? The reason is that the flux in the autotransformer is fixed because E_{BC} is fixed. As a result, the voltage across each turn remains fixed (both in magnitude and phase) whether the autotransformer delivers a current to the load or not.

Fig. 12.14 shows 3 tapped autotransformers connected between lines A, B, and C. Contacts P_1, P_2, P_3 move in tandem as we switch from one set of taps to the next. This arrangement enables us to create a 3-phase source P_1, P_2, P_3 whose phase angle changes stepwise with respect to source ABC. We obtain a maximum phase shift of 60° as we move from one extremity of the autotransformers to the other. We now discuss some practical applications of the phase-shift principle.

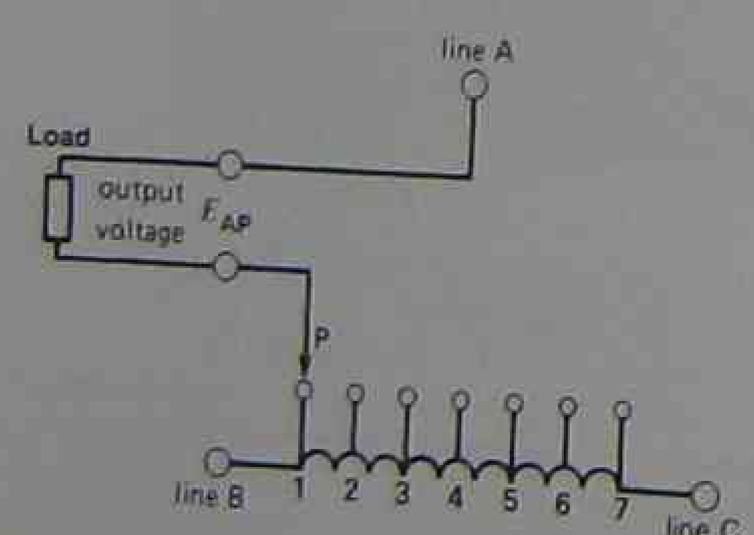

Figure 12.13
Autotransformer used as a phase-shifter.

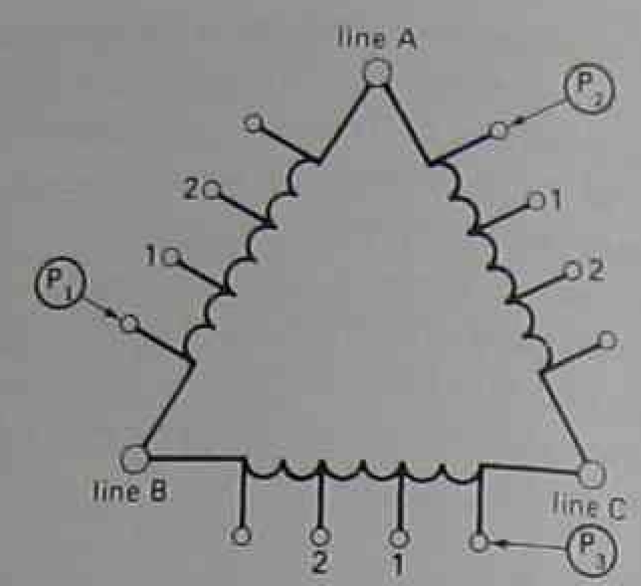

Figure 12.14
Three-phase phase shifter.

12.10 Three-phase to 2-phase transformation

The voltages in a 2-phase system are equal but displaced from each other by 90°. There are several ways to create a 2-phase system from a 3-phase source. One of the simplest and cheapest is to use a single-phase autotransformer having taps at 50 percent and 86.6 percent. We connect it between any two phases of a 3-phase line, as shown in Fig. 12.15. If the voltage between lines A, B, C is 100 V, voltages E_{AT} and E_{NC} are both equal to 86.6 V. Furthermore, they are displaced from each other by 90°. This relationship can be seen by referring to the phasor diagram (Fig. 12.15c) and reasoning as follows:

1. Phasors E_{AB}, E_{BC}, and E_{CA} are fixed by the source.
2. Phasor E_{AN} is in phase with phasor E_{AB} because the same ac flux links the turns of the autotransformer.
3. Phasor E_{AT} is in phase with phasor E_{AB} for the same reason.
4. From Kirchhoff's voltage law, $E_{AN} + E_{NC} + E_{CA} = 0$. Consequently, phasor E_{NC} must have the value and direction shown in the figure.

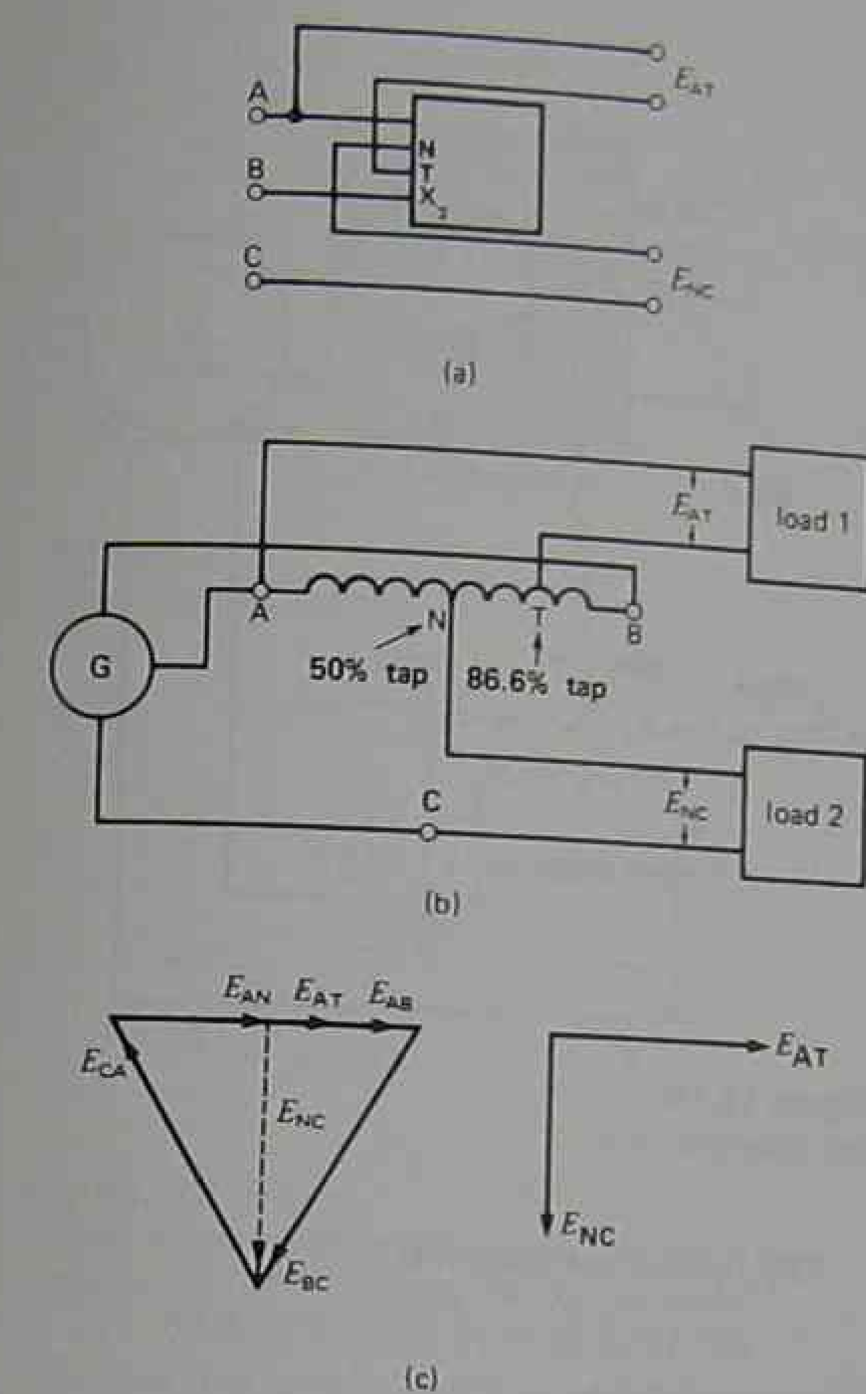

Figure 12.15
a. Simple method to obtain a 2-phase system from a 3-phase line, using a single transformer winding.
b. Schematic diagram of the connections.
c. Phasor diagram of the voltages.

Loads 1 and 2 must be isolated from each other, such as the two windings of a 2-phase induction motor. The ratio of transformation (3-phase voltage to 2-phase voltage) is fixed and given by E_{AB}/E_{AT} = 100/86.6 = 1.15.

Another way to produce a 2-phase system is to use the *Scott connection*. It consists of two identical single-phase transformers, the one having a 50 percent

tap and the other an 86.6 percent tap on the primary winding. The transformers are connected as shown in Fig. 12.16. The 3-phase source is connected to terminals A, B, C and the 2-phase load is connected to the secondary windings. The ratio of transformation (3-phase line voltage to 2-phase line voltage) is given by E_{AB}/E_{12}. The Scott connection has the advantage of isolating the 3-phase and 2-phase systems and providing any desired voltage ratio between them.

Except for servomotor applications, 2-phase systems are seldom encountered today.

Example 12-5

A 2-phase, 7.5 kW (10 hp), 240 V, 60 Hz motor has an efficiency of 0.83 and a power factor of 0.80. It is to be fed from a 600 V, 3-phase line using a Scott-connected transformer bank (Fig. 12.16c).

Calculate

a. The apparent power drawn by the motor
b. The current in each 2-phase line
c. The current in each 3-phase line

Solution

a. The active power drawn by the motor is

$$P = P_o/\eta = 7500/0.83$$
$$= 9036 \text{ W}$$

The apparent power drawn by the motor is

$$S = P/\cos\phi = 9036/0.8$$
$$= 11\,295 \text{ VA}$$

The apparent power per phase is

$$S = 11\,295/2 = 5648 \text{ VA}$$

b. The current in each 2-phase line is

$$I = S/E = 5648/240$$
$$= 23.5 \text{ A}$$

c. The transformer bank itself consumes very little active and reactive power; consequently, the 3-phase line supplies only the active and reactive power absorbed by the motor. The total apparent power furnished by the 3-phase line is, therefore, 11 295 VA.

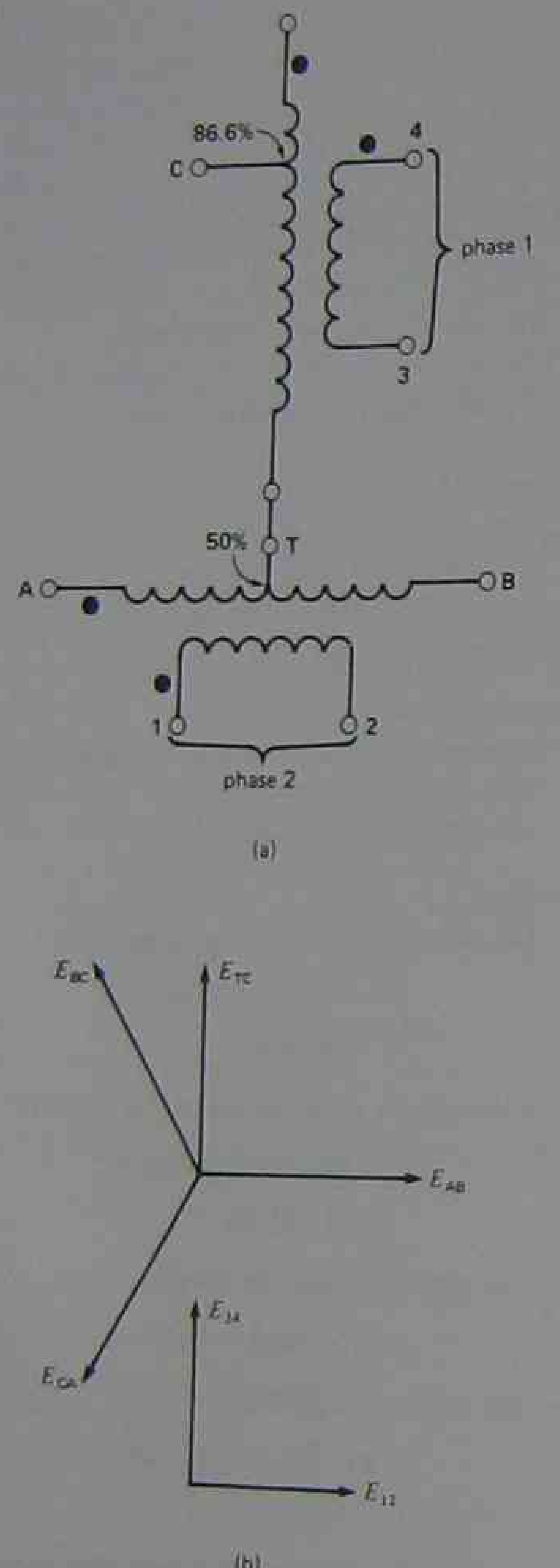

Figure 12.16
a. Scott connection.
b. Phasor diagram of the Scott connection.

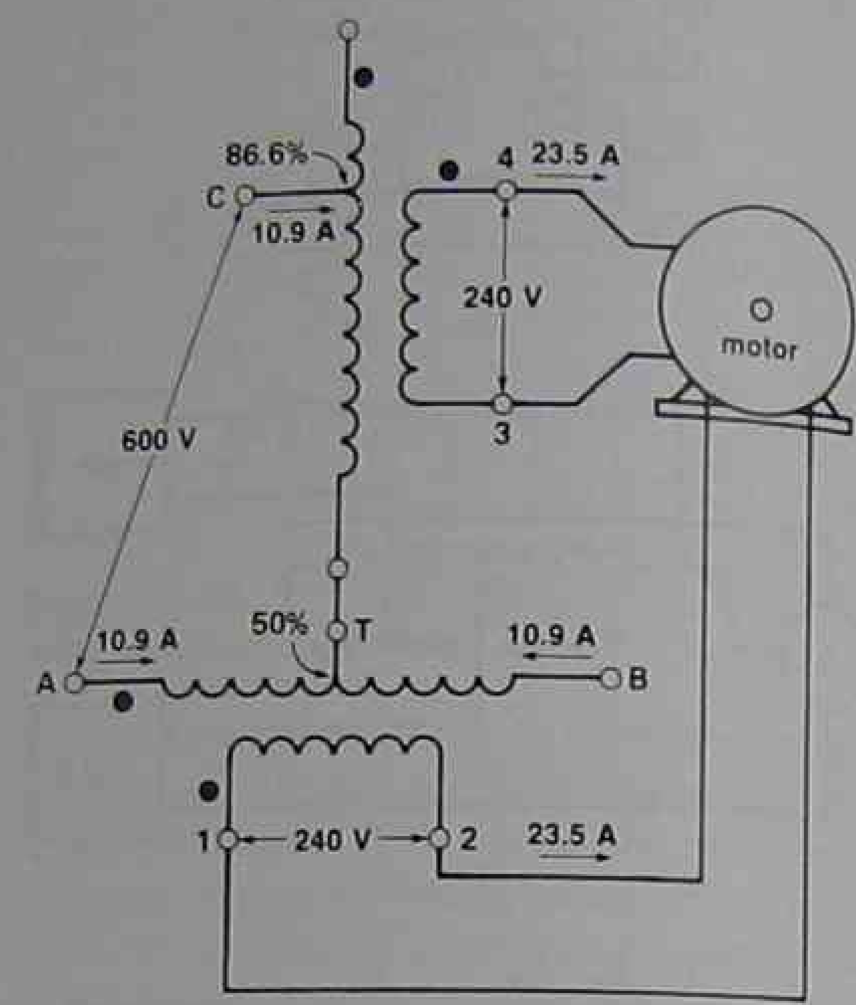

Figure 12.16c
See Example 12-5.

The 3-phase line current is

$$I = S/(\sqrt{3}\,E) = 11\,295/(\sqrt{3} \times 600)$$
$$= 10.9\ \text{A}$$

Figure 12.16c shows the power circuit and the line voltages and currents.

12.11 Phase-shift transformer

A phase-shift transformer is a special type of 3-phase autotransformer that shifts the phase angle between the incoming and outgoing lines without changing the voltage ratio.

Consider a 3-phase transmission line connected to the terminals A, B, C of such a phase-shift transformer (Fig. 12.17). The transformer twists all the incoming line voltages through an angle α without, however, changing their magnitude. The result is that all the voltages of the outgoing transmission line 1, 2, 3 are shifted with respect to the voltages of the incoming line A, B, C. The angle may be leading or lagging, and is usually variable between zero and $\pm 20°$.

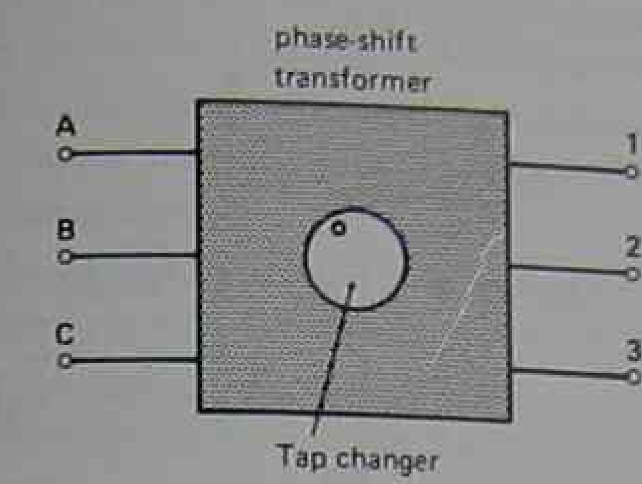

Figure 12.17a
Phase-shift transformer.

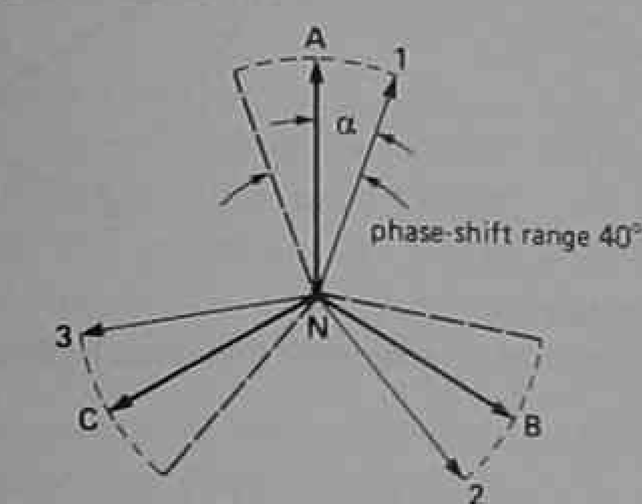

Figure 12.17b
Phasor diagram showing the range over which the phase angle of the outgoing line can be varied.

The phase angle is sometimes varied in discrete steps by means of a motorized tap-changer.

The basic power rating of the transformer (which determines its size) depends upon the apparent power carried by the transmission line, and upon the phase shift. For angles less than 20°, it is given by the approximate formula

$$S_T = 0.025\, S_L\, \alpha_{max} \qquad (12.1)$$

where

S_T = basic power rating of the 3-phase transformer bank [VA]
S_L = apparent power carried by the transmission line [VA]
α_{max} = maximum transformer phase shift [°]
0.025 = an approximate coefficient

Example 12-6

A phase-shift transformer is designed to control 150 MVA on a 230 kV, 3-phase line. The phase angle is variable between zero and $\pm 15°$.

a. Calculate the approximate basic power rating of the transformer.
b. Calculate the line currents in the incoming and outgoing transmission lines.

Solution

a. The basic power rating is

$$S_T = 0.025\, S_L\, \alpha_{max} \qquad (12.1)$$
$$= 0.025 \times 150 \times 15$$
$$= 56\ \text{MVA}$$

Note that the power rating is much less than the power that the transformer carries. This is a feature of all autotransformers.

b. The line currents are the same in both lines, because the voltages are the same. The line current is

$$I = S_L/\sqrt{3}\,E \qquad (8.9)$$
$$= (150 \times 10^6)/(\sqrt{3} \times 230\,000)$$
$$= 377\ \text{A}$$

Fig. 12.18a is an example of a 3-phase transformer that could be used to obtain a phase shift of, say, 20 degrees. The transformer has two windings on each leg. Thus, the leg associated with phase A has one winding PN with a tap brought out at terminal A and a second winding having terminals a, 3. The windings of the three phases are interconnected as shown. The incoming line is connected to terminals A, B, C and the outgoing line to terminals 1, 2, 3.

The result is that E_{1N} lags 20° behind E_{AN}. Similarly, E_{2N} lags 20° behind E_{BN}, and E_{3N} lags 20° behind E_{CN} (Fig. 12.18c).

The basic principle of obtaining a phase shift is to connect two voltages in series that are generated by two different phases. Thus, voltage E_{1b} generated by phase B is connected in series with E_{PN} generated by phase A. The values of E_{PN} and E_{1b} are selected so that the output voltage is equal to the input voltage while obtaining the desired phase angle

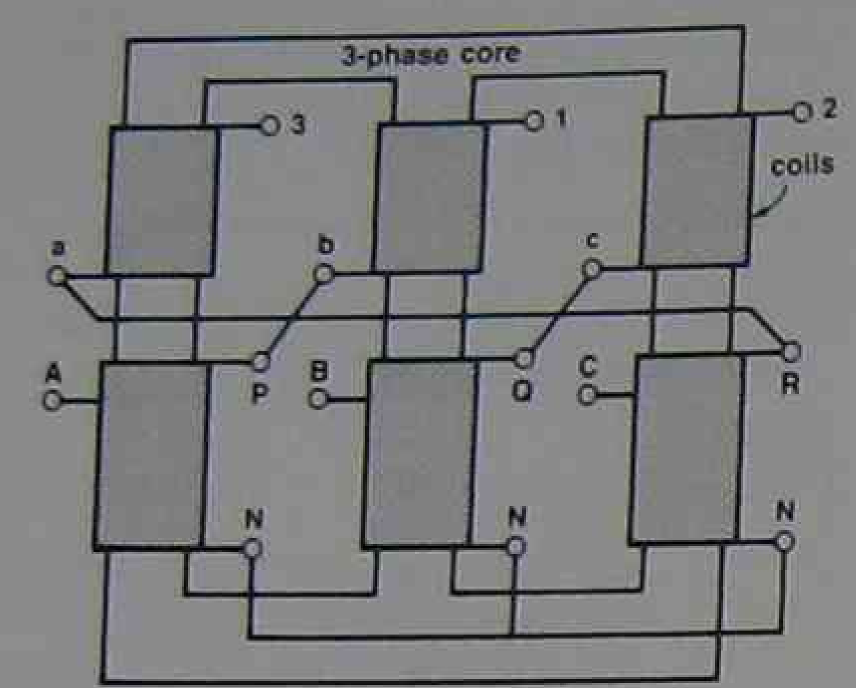

Figure 12.18a
Construction of a 3-phase-shift transformer. The incoming terminals are A, B, C; the outgoing terminals are 1, 2, 3.

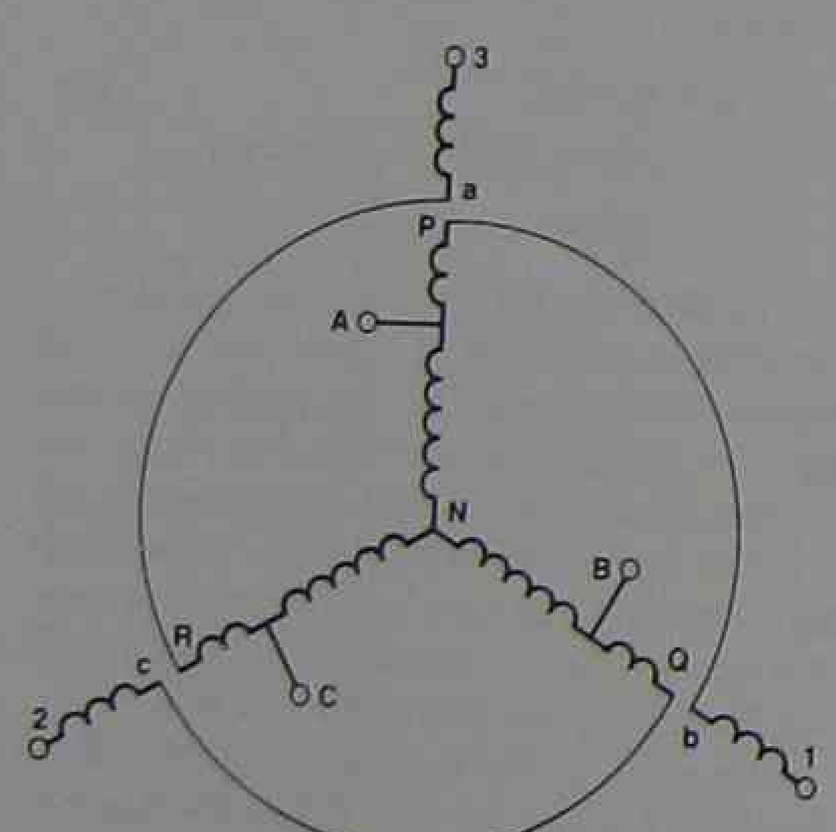

Figure 12.18b
Schematic diagram of the transformer in Fig. 12.18a.

between them. In our particular example, if E is the line-to-neutral voltage of the incoming line, the respective voltages across the windings of phase A are

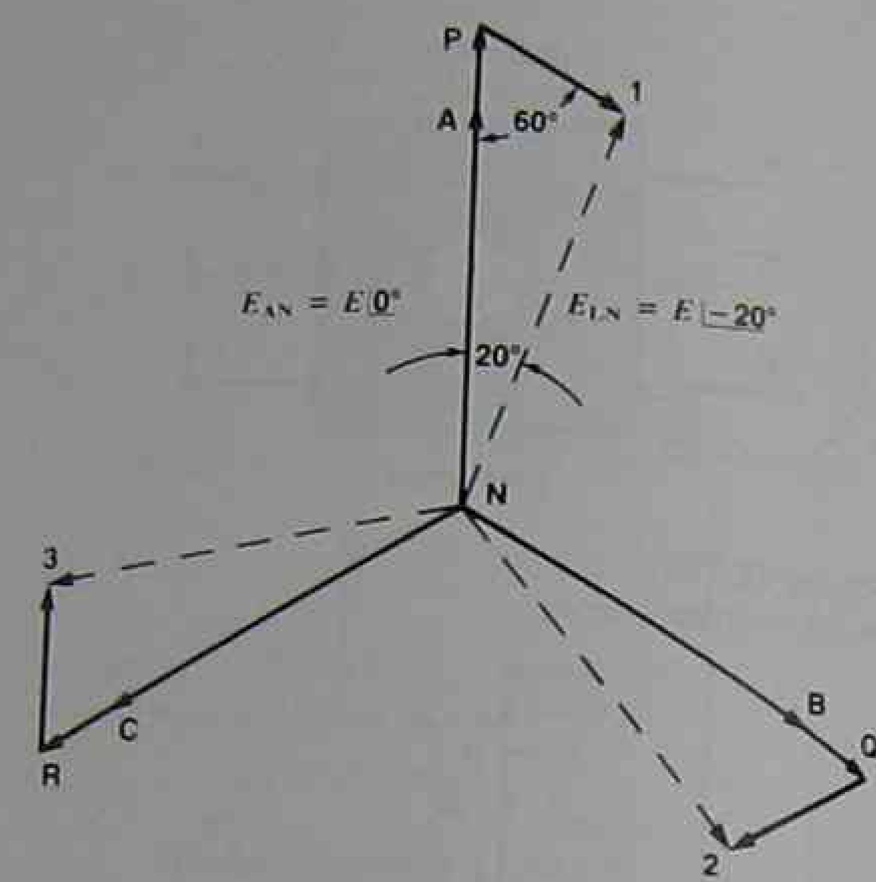

Figure 12.18c
Phasor diagram of a transformer that gives a phase-shift of 20°.

$$E_{AN} = E$$
$$E_{PN} = 1.14E$$
$$E_{3a} = 0.40E$$
$$E_{1N} = E_{AN}\angle -20°$$

In practice, the internal circuit of a tap-changing, phase-shift transformer is much more complex. However, it rests upon the basic principles we have just discussed. The purpose of such transformers will be covered in Chapter 25.

12.12 Calculations involving 3-phase transformers

The behavior of a 3-phase transformer bank is calculated the same way as for a single-phase transformer. In making the calculations, we proceed as follows:

1. We assume that the primary and secondary windings are both connected in wye, *even if they are not* (see Section 8.14). This eliminates the problem of having to deal with delta-wye and delta-delta voltages and currents.

2. We consider only one transformer (single phase) of this assumed wye-wye transformer bank.
3. The primary voltage of this hypothetical transformer is the line-to-neutral voltage of the incoming line.
4. The secondary voltage of this transformer is the line-to-neutral voltage of the outgoing line.
5. The nominal power rating of this transformer is one-third the rating of the 3-phase transformer bank.
6. The load on this transformer is one-third the load on the transformer bank.

Example 12-7

The 3-phase step-up transformer shown in Fig. 10.18 (Chapter 10) is rated 1300 MVA, 24.5 kV/345 kV, 60 Hz, impedance 11.5 percent. It steps up the voltage of a generating station to power a 345 kV line.

a. Determine the equivalent circuit of this transformer, per phase.
b. Calculate the voltage across the generator terminals when the HV side of the transformer delivers 810 MVA at 370 kV with a lagging power factor of 0.90.

Solution

a. First, we note that the primary and secondary winding connections are not specified. We don't need this information. However, we assume that both windings are connected in wye.

We shall use the per-unit method to solve this problem. We select the nominal voltage of the secondary winding as our base voltage, E_B.

The base voltage is

$$E_B = 345/\sqrt{3} = 199.2 \text{ kV}$$

Ratio of transformation is

$$a = 345/24.5 = 14.08$$

The nominal power rating of the transformer will be used as the base power S_B. Thus,

$$S_B = 1300/3 = 433.3 \text{ MVA}$$

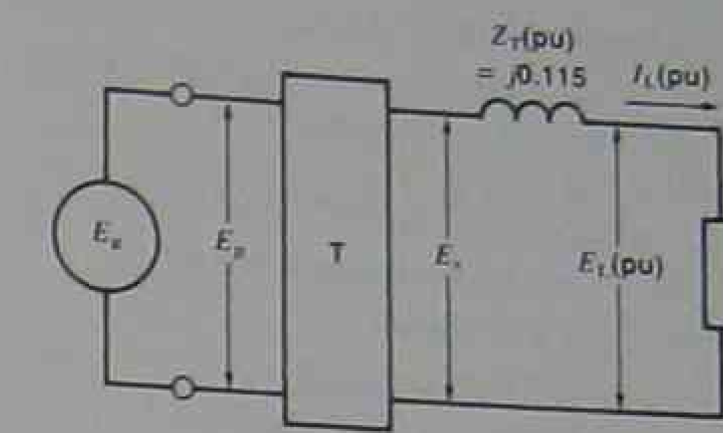

Figure 12.19
See Example 12-7.

This is a very large transformer; consequently, the transformer impedance is almost entirely reactive. The per-unit impedance is, therefore,

$$Z_T(\text{pu}) = j\,0.115$$

The equivalent circuit is shown in Fig. 12.19.

b. The power of the load per phase is

$$S_L = 810/3 = 270 \text{ MVA}$$

The voltage E_L across the load is

$$E_L = 370 \text{ kV}/\sqrt{3} = 213.6 \text{ kV}$$

The per-unit power of the load is

$$S_L(\text{pu}) = 270 \text{ MVA}/433.3 \text{ MVA} = 0.6231$$

By selecting E_L as the reference phasor, the per-unit voltage across the load is

$$E_L(\text{pu}) = 213.6 \text{ kV}/199.2 \text{ kV} = 1.0723\angle 0°$$

The per-unit current in the load is

$$I_L(\text{pu}) = \frac{S_L(\text{pu})}{E_L(\text{pu})} = \frac{0.6231}{1.0723} = 0.5811$$

The power factor of the load is 0.9. Consequently, I_L lags behind E_L by an angle of arcos 0.90 = 25.84°.

Consequently, the amplitude and phase of the per-unit load current is given by

$$I_L(\text{pu}) = 0.5811\angle -25.84°$$

The per-unit voltage E_x (Fig. 12.19) is

$$
\begin{aligned}
E_s(\text{pu}) &= E_L(\text{pu}) + I_L(\text{pu}) \times Z_T(\text{pu}) \\
&= 1.0723\angle 0^\circ + (0.5811\angle -25.84^\circ) \\
&\quad \times (0.115\angle 90^\circ) \\
&= 1.0723 + 0.0668\angle 64.16^\circ \\
&= 1.0723 + 0.0668(\cos 64.16^\circ + \\
&\quad j \sin 64.16^\circ) \\
&= 1.1014 + j\,0.0601 \\
&= 1.103\angle 3.12^\circ
\end{aligned}
$$

Therefore,

$$E_s = 1.103 \times 345\ \text{kV} = 381\ \text{kV}\angle 3.12^\circ$$

The per-unit voltage on the primary side is also

$$E_p = 1.103\angle 3.12^\circ$$

The effective voltage across the terminals of the generator is, therefore,

$$
\begin{aligned}
E_g &= E_p(\text{pu}) \times E_B(\text{primary}) \\
&= 1.103 \times 24.5\ \text{kV} \\
&= 27.02\ \text{kV}
\end{aligned}
$$

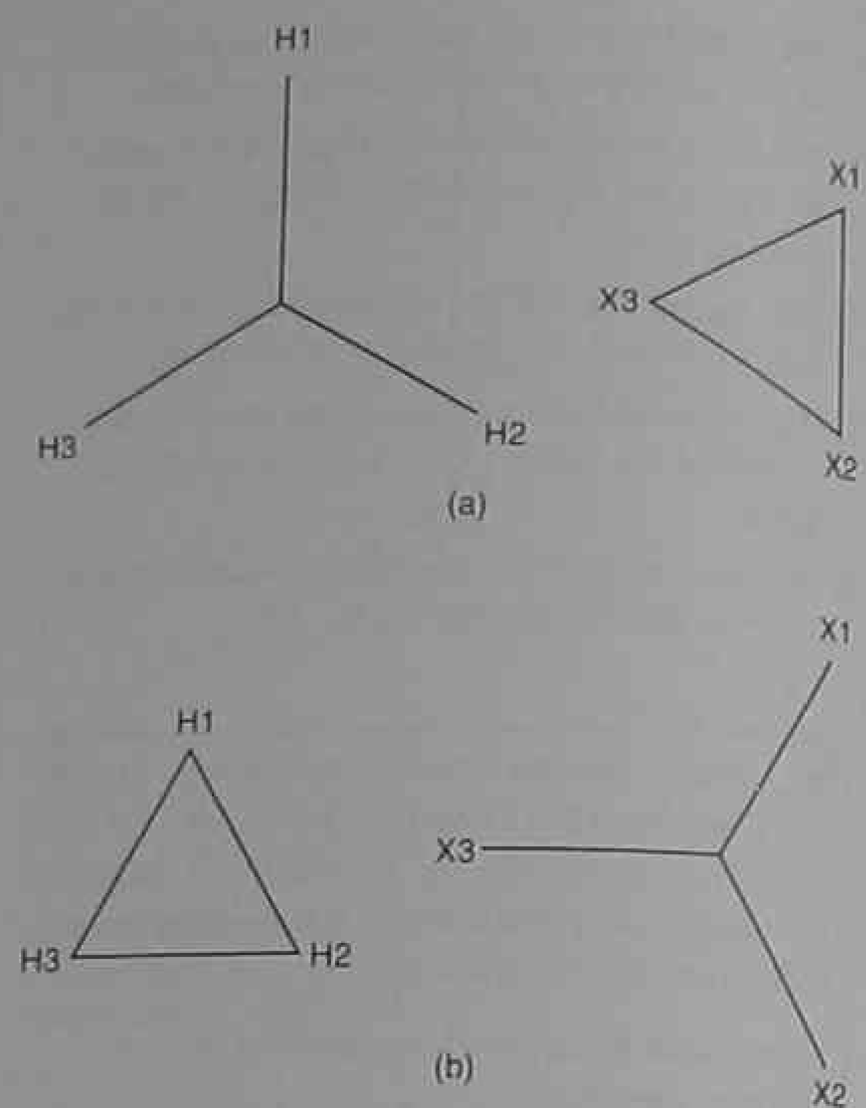

Figure 12.20
Polarity marking of 3-phase transformers.

12.13 Polarity marking of 3-phase transformers

The HV terminals of a 3-phase transformer are marked H_1, H_2, H_3 and the LV terminals are marked X_1, X_2, X_3. The following rules have been standardized:

1. If the primary windings and secondary windings are connected wye-wye or delta-delta, the voltages between similarly-marked terminals are in phase. Thus,

   $E_{H_1H_2}$ is in phase with $E_{X_1X_2}$

   $E_{H_2H_1}$ is in phase with $E_{X_2X_1}$

   $E_{H_1H_3}$ is in phase with $E_{X_1X_3}$

and so on.

2. If the primary and secondary windings are connected in wye-delta or delta-wye, there results a 30° phase shift between the primary and secondary line voltages. The internal connections are made so that the voltages on the HV side *always* lead the voltages of similarly-marked terminals on the LV side. Thus,

   $E_{H_1H_2}$ leads $E_{X_1X_2}$ by 30°

   $E_{H_2H_1}$ leads $E_{X_2X_1}$ by 30°

   $E_{H_1H_3}$ leads $E_{X_1X_3}$ by 30°

and so on.

Fig. 12.20 shows two ways of representing the delta-wye terminal markings.

3. These rules are not affected by the phase sequence of the line voltage applied to the primary side.

Questions and Problems

Practical level

12-1 Assuming that the transformer terminals have polarity marks H_1, H_2, X_1, X_2, make schematic drawings of the following connections:
  a. Delta-wye
  b. Open-delta

12-2 Three single-phase transformers rated at 250 kVA, 7200 V/600 V, 60 Hz, are connected in wye-delta on a 12 470 V, 3-phase line. If the load is 450 kVA, calculate the following currents:
  a. In the incoming and outgoing transmission lines
  b. In the primary and secondary windings

12-3 The transformer in Fig. 12.9 has a rating of 36 MVA, 13.8 kV/320 V. Calculate the nominal currents in the primary and secondary lines.

12-4 Calculate the nominal currents in the primary and secondary windings of the transformer shown in Fig. 10.18, knowing that the windings are connected in delta-wye.

Intermediate level

12-5 The transformer shown in Fig. 10.19 operates in the forced-air mode during the morning peaks.
  a. Calculate the currents in the secondary lines if the primary line voltage is 225 kV and the primary line current is 150 A.
  b. Is the transformer overloaded?

12-6 The transformers in Problem 12-2 are used to raise the voltage of a 3-phase 600 V line to 7.2 kV.
  a. How must they be connected?
  b. Calculate the line currents for a 600 kVA load.
  c. Calculate the corresponding primary and secondary currents.

12-7 In order to meet an emergency, three single-phase transformers rated 100 kVA, 13.2 kV/2.4 kV are connected in wye-delta on a 3-phase 18 kV line.
  a. What is the maximum load that can be connected to the transformer bank?
  b. What is the outgoing line voltage?

12-8 Two transformers rated at 250 kVA, 2.4 kV/600 V are connected in open-delta to supply a load of 400 kVA.
  a. Are the transformers overloaded?
  b. What is the maximum load the bank can carry on a continuous basis?

12-9 Referring to Figs. 12.3 and 12.4, the line voltage between phases A-B-C is 6.9 kV and the voltage between lines 1, 2, and 3 is balanced and equal to 600 V. Then, in a similar installation the secondary windings of transformer P are by mistake connected in reverse.
  a. Determine the voltages measured between lines 1-2, 2-3, and 3-1.
  b. Draw the new phasor diagram.

Industrial application

12-10 Three 150 kVA, 480 V/4000 V, 60 Hz single-phase transformers are to be installed on a 4000 V, 3-phase line. The exciting current has a value of 0.02 pu. Calculate the line current when the transformers are operating at no-load.

12-11 The core loss in a 300 kVA 3-phase distribution transformer is estimated to be 0.003 pu. The copper losses are 0.0015 pu. If the transformer operates effectively at no-load 50 percent of the time, and the cost of electricity is 4.5 cents per kWh, calculate the cost of the no-load operation in the course of one year.

12-12 The bulletin of a transformer manufacturer indicates that a 150 kVA, 230 V/208 V, 60 Hz, 3-phase autotransformer weighs