interconnected capacity. The biggest single generator has a rating of 500 MW. For this reason the performance of a single machine is unlikely to affect appreciably the voltage and frequency of the whole system. A machine connected to such a system, where the capacity of any one machine is small compared with the total interconnected capacity, is often said to be connected to *infinite busbars*. The outstanding electrical characteristics of such busbars are that they are constant-voltage constant-frequency busbars.

When the machine is connected to the infinite busbars the terminal voltage and frequency becomes fixed at the values maintained by the rest of the system. Unless the machine is grossly overloaded or under-excited, no change in the mechanical power supply, load or excitation will alter the terminal voltage or frequency. If the machine is acting as a generator and the mechanical driving power is increased the power output from the machine to the busbars must increase, assuming that the efficiency does not greatly change. In the same way, a decrease of mechanical driving power or the application of a mechanical load (motoring) will produce a decrease in output power or the absorption of power from the busbars.

12.12 Synchronizing

The method of connecting an incoming alternator to the live busbars will now be considered. This is called *synchronizing*.

A stationary alternator must not be connected to live busbars, or, since the induced e.m.f. is zero at standstill, a short-circuit will result. The alternator induced e.m.f. will prevent dangerously high switching currents only if the following conditions are almost exactly complied with:

1. The frequency of the induced voltages in the incoming machine must equal the frequency of the voltages of the live busbars.
2. The induced voltages in the incoming machine must equal the live busbar voltages in magnitude and phase.
3. The phase sequence of the busbar voltages and the incoming-machine voltages must be the same.

In modern power stations alternators are synchronized automatically. The principles may be illustrated by the three-lamp method, which, along with a voltmeter, may be used for synchronizing low-power machines.

Fig. 12.11(*a*) is the connexion diagram from which it will be noted that one lamp is connected between corresponding phases while the two others are cross-connected between the other two phases. In the complexor diagram at (*b*) the machine induced voltages are represented by E_R, E_Y and E_B, while the live supply voltages are

represented by V_R, V_Y, and V_B. The lamp symbols have been added to the complexor diagram to indicate the instantaneous lamp voltages. It will be realized that the speed of rotation of the complexors will correspond to the frequencies of the supply and the machine—if these are the same then the lamp brilliancies will be constant. The speed of the machine should be adjusted until the machine frequency is nearly that of the supply, but exact equality is inconvenient for there would then be, in all probability, a permanent phase difference between corresponding voltages. The machine excitation should now be varied until the two sets of voltages

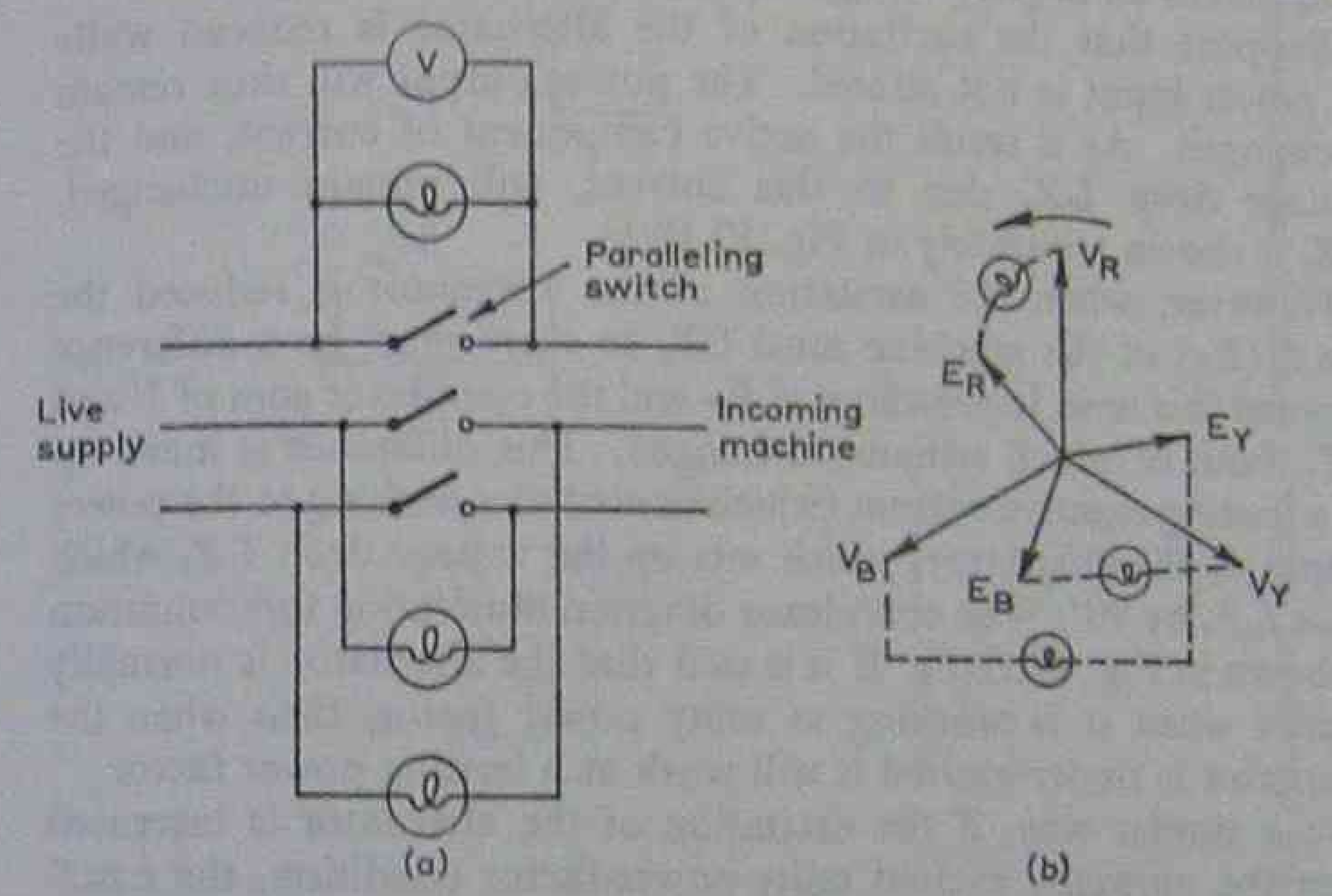

Fig. 12.11 SYNCHRONIZING BY CROSS-CONNECTED LAMP METHOD

are equal in magnitude. The correct conditions will be obtained at an instant when the straight-connected lamp is dark and the cross-connected lamps are equally bright. If the phase sequence is incorrect no such instant will occur as the cross-connected lamps will, in effect, be straight-connected and all the lamps will be dark simultaneously. In this event the direction of rotation of the incoming machine should be reversed or two lines of the machine should be interchanged. Since the dark range of a lamp extends over a considerable voltage range it is advisable to connect a voltmeter across the straight-connected lamp and to close the paralleling switch when the voltmeter reading is zero. It should be noted that the lamps and the voltmeter must be able to withstand twice the normal phase voltage.

Fig. 12.7(*b*) is the corresponding complexor diagram. The resultant e.m.f. E_R is shown for the sake of completeness but will be omitted in subsequent diagrams.

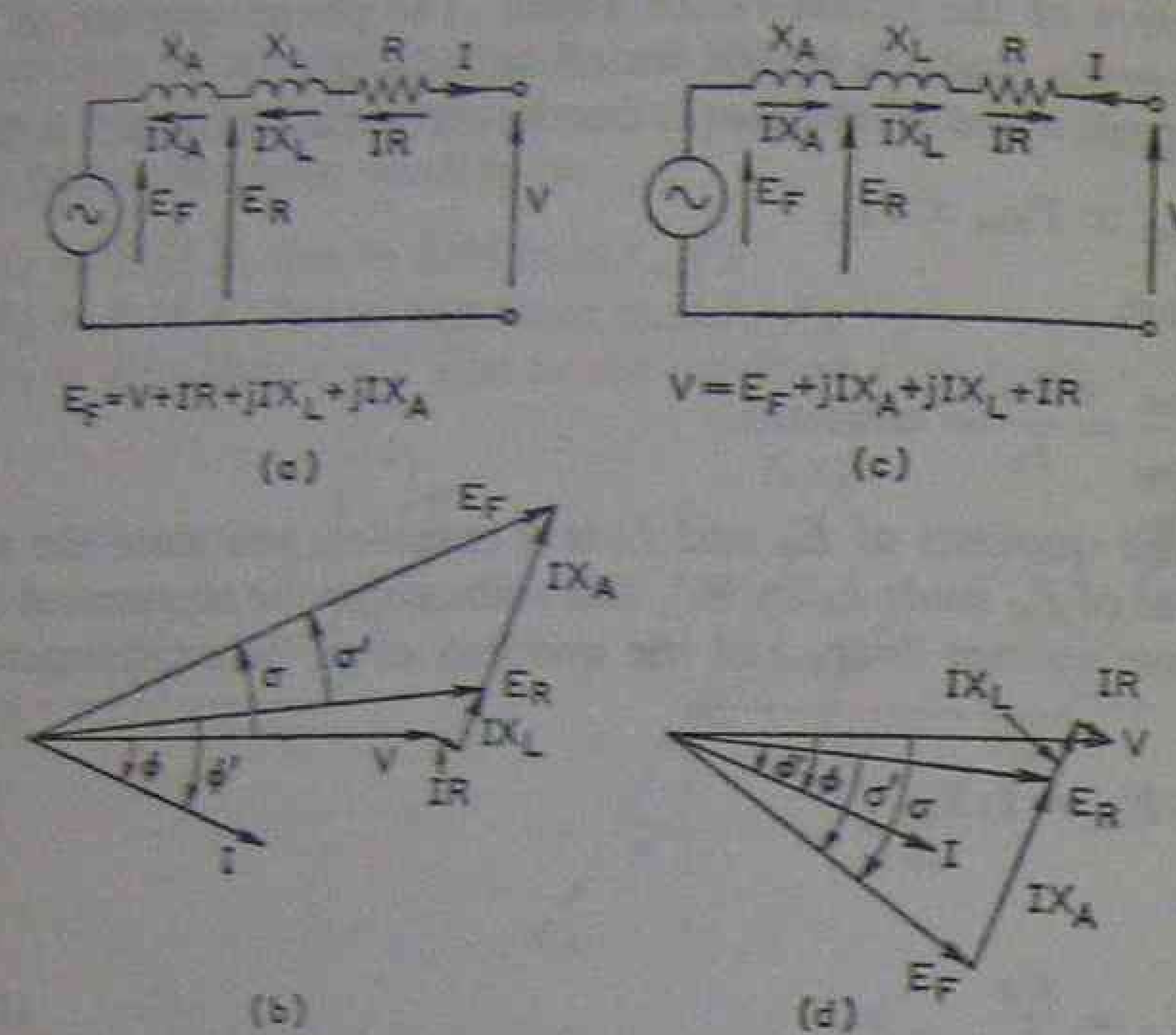

Fig. 12.7 EQUIVALENT CIRCUITS AND FULL COMPLEXOR DIAGRAMS FOR THE SYNCHRONOUS MACHINE

(*a*), (*b*) Generator (*c*), (*d*) Motor

Eqn. (12.19) may be rewritten as

$$E_F = V + IZ_s \tag{12.20}$$

where Z_s is the *synchronous impedance*.

$$Z_s = R + j(X_L + X_A) \tag{12.21}$$

or

$$Z_s = R + jX_s \tag{12.22}$$

where X_s is the *synchronous reactance*:

$$X_s = X_L + X_A \tag{12.23}$$

In polar form the synchronous impedance is

$$Z_s = Z_s\underline{/\psi} \tag{12.24}$$

where

$$\psi = \tan^{-1}\frac{X_s}{R} \tag{12.25}$$

and

$$Z_s = \sqrt{(R^2 + X_s^2)} \tag{12.26}$$

Frequently in synchronous machines $X_s \gg R$, in which case eqn. (12.26) becomes

$$Z_s = X_s\underline{/90^\circ} = jX_s \tag{12.27}$$

Fig. 12.7(*c*) shows the full equivalent circuit of the synchronous motor, in which the current flows in the conventionally positive direction for motor-mode operation (a load), i.e. entering the position terminal. Applying Kirchhoff's law to this circuit gives

$$V = E_F + jIX_A + jIX_s + IR \tag{12.28}$$

Fig. 12.7(*d*) is the corresponding complexor diagram.

EXAMPLE 12.1 A 3-phase 11·8kV 75MVA, 50Hz 2-pole star-connected synchronous generator requires a separate field m.m.f. having a maximum value of $3{\cdot}0 \times 10^4$ At/pole to give normal rated voltage on open-circuit. The flux per pole on open-circuit is approximately 5·3Wb.

Determine (*a*) the maximum armature m.m.f. per pole corresponding to rated full-load current, and (*b*) the synchronous reactance if the leakage reactance of the armature winding is 0·18Ω. Find also the p.u. value of the synchronous reactance.

Neglect the effect of space harmonics in the field and armature m.m.f.s. Assume the flux per pole to be proportional to the m.m.f. and the armature winding to be uniform and narrow spread.

The e.m.f. per phase, from eqn. (11.20), is

$$E_p = \frac{K_d K_s \omega \Phi N_p}{\sqrt{2}}$$

and the distribution factor for a uniform narrow-spread winding is

$$K_d = \frac{3}{\pi} = 0{\cdot}955 \tag{11.13}$$

$$E_p = \frac{11{\cdot}8 \times 10^3}{\sqrt{3}} = 6{,}800\,\text{V}$$

Taking $K_s = 1$, the number of turns per phase is

$$N_p = \frac{\sqrt{2}E_p}{K_s K_d \omega \Phi} = \frac{2 \times 6{,}800}{0{\cdot}955 \times 2\pi 50 \times 5{\cdot}3} = 6{\cdot}05$$

Since the number of turns per phase must be an integer, take $N_p = 6$.

The maximum armature m.m.f., from eqn. (11.28), is

$$F_{Am} = \frac{18F_{pm}}{\pi^2} = \frac{18}{\pi^2}\frac{\sqrt{2}I_p N_p}{2}$$

$$\text{Rated current per phase} = \frac{75 \times 10^6}{3 \times 6{,}800} = 3{,}680\,\text{A}$$

so that

$$F_{Am} = \frac{18}{\pi^2} \times \sqrt{2} \times \frac{3{,}680 \times 6}{2} = \underline{\underline{2{\cdot}85 \times 10^4\,\text{At/pole}}}$$

Since the flux per pole is proportional to the m.m.f.,

$$\frac{E_a}{E_f} = \frac{F_{am}}{F_{fm}}$$

$$E_a = 6{,}900 \times \frac{245 \times 10^3}{340 \times 10^3} = 6{,}450\,\text{V}$$

$$X_a = \frac{E_a}{I_a} = \frac{6{,}450}{3{,}680} = 1{\cdot}75\,\Omega$$

$$X_s = X_a + X_l = 1{\cdot}75 + 0{\cdot}18 = \underline{\underline{1{\cdot}93\,\Omega}}$$

Taking rated phase voltage and current as bases,

$$X_{spu} = \frac{3{,}680 \times 1{\cdot}93}{6{,}900} = \underline{\underline{1{\cdot}04\,\text{p.u.}}}$$

12.7 Synchronous Reactance in Terms of Main Dimensions

It is assumed that (*a*) magnetic saturation is absent; (*b*) the armature m.m.f. is sinusoidally distributed; (*c*) the air-gap is uniform; and (*d*) the reluctance of the magnetic paths in the stator and rotor is negligible.

Let D = Internal stator diameter
L = effective stator (or core) length
l_g = radial gap length

The synchronous reactance X_s is

$$X_s = X_l + X_a \tag{12.23}$$

The reactance X_a is

$$X_a = \frac{E_a}{I_a} \tag{12.17}$$

where E_a is the armature e.m.f. per phase due to the armature m.m.f. F_a, and I_a is the armature current per phase.

The value of E_a may be found by using eqn. (11.20), which gives the e.m.f. per phase of a polyphase winding:

$$E_a = K_d K_s \frac{\omega \Phi_a N_p}{\sqrt{2}} \tag{12.30}$$

where the distribution factor for a narrow-spread uniform winding is, from eqn. (11.13), $K_d = 3/\pi$; the coil span factor, $k_s = 1$; and Φ_a is the flux per pole due to the armature m.m.f. F_a.

From eqn. (11.28), and assuming that the armature m.m.f. is sinusoidally distributed,

Maximum armature m.m.f. per pole: $F_{am} = \dfrac{18 F_{pm}}{\pi^2} = \dfrac{18}{\pi^2}\dfrac{\sqrt{2} I_a N_p}{2p}$

Maximum air-gap field strength: $H_{gm} = \dfrac{F_{am}}{l_g} = \dfrac{1}{l_g}\dfrac{18}{\pi^2}\dfrac{\sqrt{2} I_a N_p}{2p}$

Maximum air-gap flux density: $B_{gm} = \mu_0 H_{gm} = \dfrac{\mu_0}{l_g}\dfrac{18}{\pi^2}\dfrac{\sqrt{2} I_a N_p}{2p}$

Since the air-gap flux density is sinusoidally distributed,

Average air-gap flux density: $B_{av} = \dfrac{2}{\pi} B_{gm}$

Flux per pole due to armature m.m.f.: $\Phi_a = B_{av} \times \text{Pole area}$

$$= \frac{2}{\pi}\frac{\mu_0}{l_g}\frac{18}{\pi^2}\frac{\sqrt{2}\, I_a N_p}{2p}\frac{\pi D L}{2p}$$

Substituting for K_d, K_s and Φ_a in eqn. (12.30), and then substituting the resulting expression for E_a in eqn. (12.17),

$$X_a = \frac{E_a}{I_a} = \frac{\dfrac{3}{\pi}\,\omega\,\dfrac{2}{\pi}\dfrac{\mu_0}{l_g}\dfrac{18}{\pi^2}\dfrac{\pi D L}{2p}\dfrac{\sqrt{2} I_a N_p}{2p} N_p}{\sqrt{2} I_a}$$

$$= \omega\left(\frac{18}{\pi^2}\right)^2 \frac{\mu_0}{3 l_g}\pi D L \left(\frac{N_p}{2p}\right)^2 \tag{12.31}$$

To obtain the synchronous reactance, an allowance for the leakage reactance X_l must be added to X_a. The leakage reactance is mainly due to (*a*) slot leakage flux, which links individual slots and is not, therefore, part of the main flux, and (*b*) end-turn leakage flux, which links the end turns of the stator winding following mainly air paths. The evaluation of these leakage fluxes, particularly the latter, presents some difficulties and is beyond the scope of the present volume.

The e.m.f. per phase, E_a, is due to the armature current itself and is therefore an e.m.f. of self-induction. The reactance X_a is therefore a *magnetizing reactance*.

Eqn. (12.31) shows that the value of this reactance may be reduced by increasing the gap length l_g.

Looked at in another way, for a given machine rating the rated armature current is fixed as is also, as a consequence, the maximum armature m.m.f. per pole. This fixed value of armature m.m.f. has a progressively smaller effect as the air-gap is lengthened.

EXAMPLE 12.2 A 3-phase 13·8kV 100MVA 50Hz 2-pole star-connected cylindrical-rotor synchronous generator has an internal stator diameter of 1·08m and an effective core length of 4·6m. The machine has a synchronous reactance of 2p.u. and a leakage reactance of 0·16p.u. The average flux density over the pole area is approximately 0·6Wb/m². Estimate the gap length.

Assume that the radial air-gap is constant and the armature winding uniform. Neglect the reluctance of the iron core and the space harmonics in the armature m.m.f.

With the above assumptions the reactance X_A is

$$X_A = \omega\left(\frac{18}{\pi^2}\right)^2 \frac{\mu_0}{3l_g}\pi DL\left(\frac{N_p}{2p}\right)^2 \qquad (12.31)$$

$$\text{Base voltage, } V_B = V_p = \frac{13{\cdot}8 \times 10^3}{\sqrt{3}} = 7{,}960\,\text{V}$$

$$\text{Base current, } I_B = \frac{\text{VA/phase}}{V_B} = \frac{100 \times 10^6}{3 \times 7{,}960} = 4{,}180\,\text{A}$$

$$\text{Base impedance, } Z_B = \frac{V_B}{I_B} = \frac{7{,}960}{4{,}180} = 1{\cdot}91\,\Omega$$

$$X_{Apu} = X_{spu} - X_{Lpu} = 2{\cdot}00 - 0{\cdot}16 = 1{\cdot}84\,\text{p.u.}$$

$$X_A = X_{Apu}Z_B = 1{\cdot}84 \times 1{\cdot}91 = 3{\cdot}52\,\Omega$$

$$\text{Flux per pole, } B_{av} \times \text{Pole area} = B_{av}\frac{\pi DL}{2} = \frac{0{\cdot}6 \times \pi \times 1{\cdot}08 \times 4{\cdot}6}{2}$$

$$= 4{\cdot}68\,\text{Wb}$$

$$E_p = K_d K_s \frac{\omega \Phi N_p}{\sqrt{2}} \qquad (11.20)$$

For a uniform winding, $K_d = 3/\pi$ and $K_s = 1$, so that

$$N_p = \frac{\sqrt{2}E_p}{K_d K_s \omega \Phi} = \frac{\sqrt{2} \times 7{,}960}{3/\pi \times 2\pi \times 50 \times 4{\cdot}68} = 8{\cdot}02$$

The number of turns per phase must be an integer, say 8. This will require a slightly higher flux per pole and average value of flux density. From eqn. (12.31),

$$l_g = 2\pi \times 50 \times \left(\frac{18}{\pi^2}\right)^2 \times \frac{4\pi \times 10^{-7}}{3 \times 3{\cdot}52} \times \pi \times 1{\cdot}08 \times 4{\cdot}6 \times \left(\frac{8}{2}\right)^2$$

$$= \underline{\underline{3{\cdot}10 \times 10^{-2}\,\text{m}}}$$

12.8 Determination of Synchronous Impedance

The ohmic value of the synchronous impedance, at a given value of excitation may be determined by open-circuit and short-circuit tests (Fig. 12.8).

On open-circuit the terminal voltage depends on the field excitation and the magnetic characteristics of the machine. Fig. 12.9 includes a

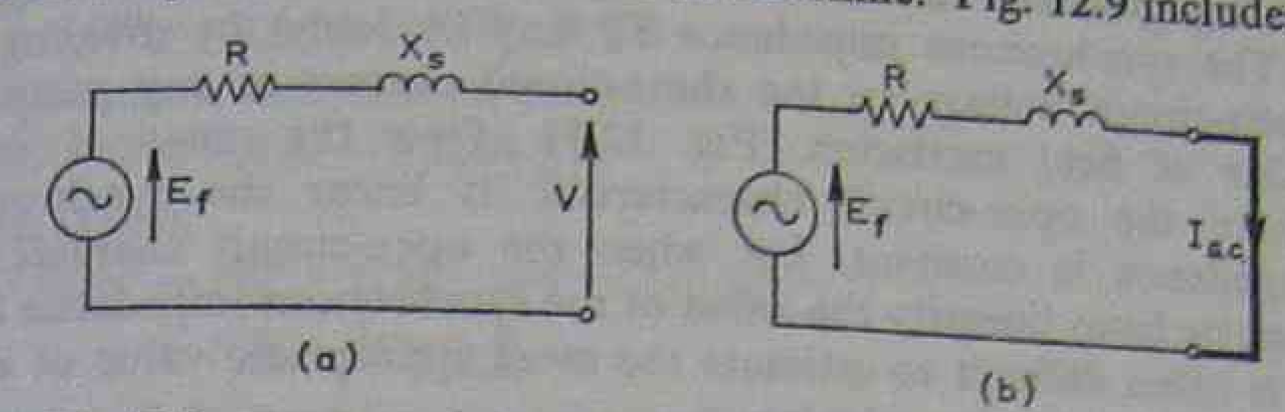

Fig. 12.8 DETERMINATION OF SYCHRONOUS IMPEDANCE
(*a*) Open-circuit test (*b*) Short-circuit test

typical open-circuit characteristic showing the usual initial linear portion and subsequent saturation portion of a magnetization curve.

On short-circuit the current in an alternator winding will normally lag behind the induced voltage by approximately 90° since the leakage

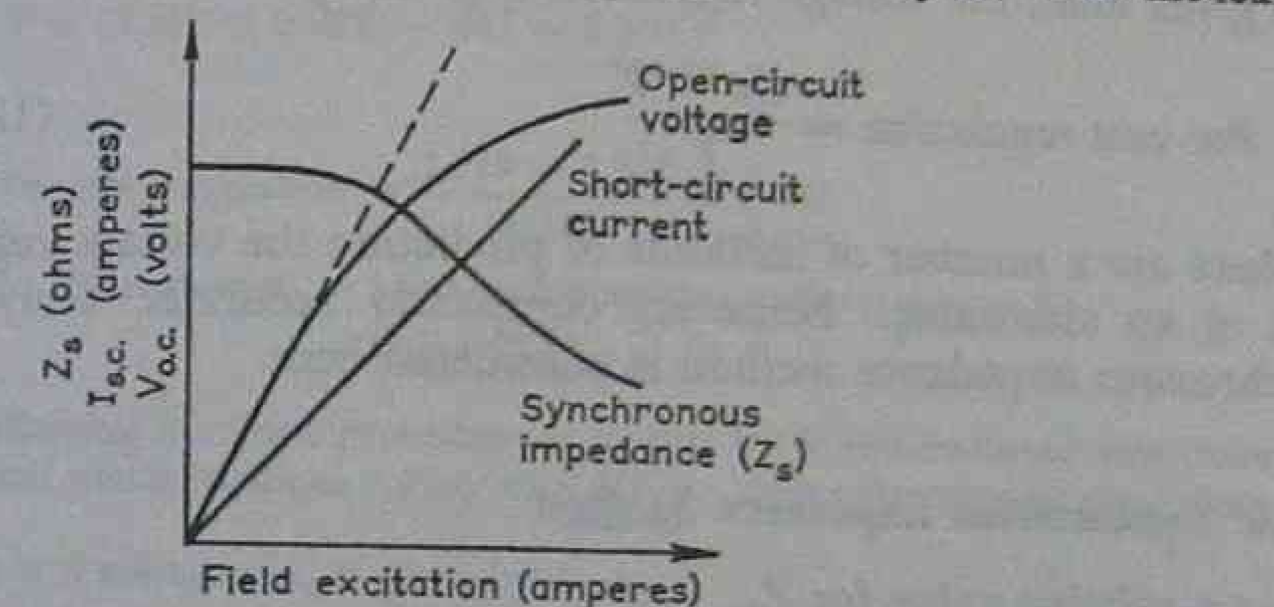

Fig. 12.9 VARIATION OF SYNCHRONOUS IMPEDANCE WITH EXCITATION

reactance of the winding is normally much greater than the winding resistance. The complexor diagram for short-circuit conditions is shown in Fig. 12.10. It is found that the armature and field m.m.f.s

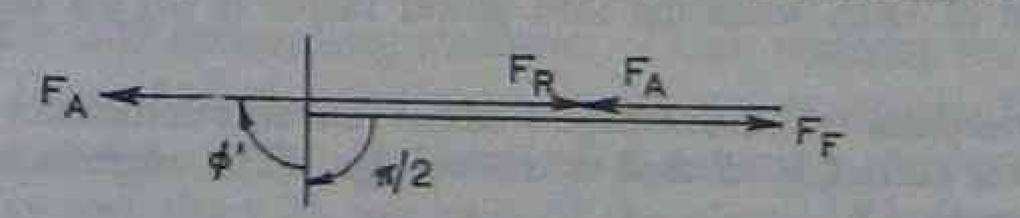

Fig. 12.10 COMPLEXOR DIAGRAM FOR SHORT-CIRCUIT CONDITIONS

are directly in opposition, so that a surprisingly large excitation is required to give full-load short-circuit current in the windings. The resultant m.m.f. and flux are small since the induced voltage is only required to overcome resistance and leakage reactance voltage drops in the windings. Since the flux is small, saturation effects will be negligible and the short-circuit characteristic is almost straight.

The synchronous impedance Z_s may be found by dividing the open-circuit voltage by the short-circuit current at any particular value of field excitation (Fig. 12.9). Over the range of values where the open-circuit characteristic is linear the synchronous impedance is constant, but when the open-circuit characteristic departs from linearity the value of the synchronous impedance falls. It is often difficult to estimate the most appropriate value of Z_s to use for a particular calculation.

12.9 Voltage Regulation

The voltage regulation of an alternator is normally defined as the rise in terminal voltage when a given load is thrown off. Thus, if E_F is the induced voltage on open-circuit and V is the terminal voltage at a given load, the voltage regulation is given by

$$\text{Per-unit regulation} = \frac{E_F - V}{V} \qquad (12.32)$$

There are a number of methods of predicting the voltage regulation of an alternator. None are completely accurate. Only the synchronous impedance method is considered here.

12.10 Synchronous Impedance Method

Using a suitable value for Z_s,

$$E_F = V + IZ_s \qquad (12.20)$$

EXAMPLE 12.3 A 3-phase star-connected alternator has a resistance of 0·5Ω and a synchronous reactance of 5Ω per phase. It is excited to give 6,600 V (line) on open circuit. Determine the terminal voltage and per-unit voltage regulation on full-load current of 130A when the load power factor is (*a*) 0·8 lagging, (*b*) 0·6 leading.

It is best to take the phase terminal voltage V as the reference complexor since the phase angle of the current is referred to this voltage. (The magnitude of V is, however, not known): i.e.

Phase terminal voltage, $V = V\underline{/0°}$

The magnitude of the e.m.f E_F is known but not its phase with respect to V: i.e.

$$E_F = E_F\underline{/\sigma°} = \frac{6{,}600}{\sqrt{3}}\underline{/\sigma°} = 3{,}810\underline{/\sigma°}$$

where $\sigma°$ is the phase of E_F with respect to V as reference.

(*a*) The phase current I lags behind V by a phase angle corresponding to a power factor of 0·8 lagging, i.e.

$$I = 130\underline{/-\cos^{-1} 0{\cdot}8} = 130\underline{/-36{\cdot}9°}\text{ A}$$

The synchronous impedance per phase is

$$Z_s = (0{\cdot}5 + j5)\,\Omega = 5{\cdot}02\underline{/84{\cdot}3°}\,\Omega$$

In eqn. (12.20),

$$3{,}810\underline{/\sigma°} = V\underline{/0°} + (130\underline{/-36{\cdot}9°} \times 5{\cdot}02\underline{/84{\cdot}3°})$$
$$= V\underline{/0°} + 653\underline{/47{\cdot}4°}$$

Expressing all the terms in rectangular form,

$$3{,}810 \cos\sigma + j\,3{,}810 \sin\sigma = V + j0 + 442 + j482$$

Equating quadrate parts,

$$3{,}810 \sin\sigma = 482$$

whence $\sin\sigma = 0{\cdot}127$ and $\cos\sigma = 0{\cdot}992$

Equating reference parts,

$$3{,}810 \cos\sigma = V + 442$$
$$V = (3{,}810 \times 0{\cdot}992) - 442 = \underline{\underline{3{,}340\text{ V}}}$$

and

$$\text{Per-unit regulation} = \frac{3{,}810 - 3{,}340}{3{,}340} = \underline{\underline{0{\cdot}141}}$$

(*b*) Phase current = 130 A at 0·6 leading with respect to V

$$= 130\underline{/+53{\cdot}1°}$$

Following the same procedure as in part (*a*) it will be found that there is an on-load phase terminal voltage of $\underline{\underline{4{,}260\text{ V}}}$. Hence the per-unit regulation, since there is a voltage rise, is given by

$$\frac{3810 - 4260}{4{,}260} = \underline{\underline{-0{\cdot}106\text{ p.u.}}}$$

12.11 Synchronous Machines connected to Large Supply Systems

In Britain, electrical energy is supplied to consumers from approximately 200 generating stations. These stations vary considerably in size, the installed capacity of the largest exceeding 2,000 MW. About one-quarter of the stations have a rating of less than 50 MW and supply less than 2½ per cent of the electrical energy demanded from the public supply.

The generating stations do not operate as isolated units but are interconnected by the national *grid*, which consists of almost 10,000 miles of main transmission line, for the most part overhead lines operating at 132, 275 and 400 kV. The total generating capacity interconnected by the grid system is over 40,000 MW. The output of any single machine is therefore small compared with the total