atmospheres. Spark gaps are incorporated to prevent current flow to earth under normal-voltage conditions. The air in the interior of the porcelain housing containing the spark-gap assembly is evacuated and then the housing is filled with nitrogen. Fig. 16.11 shows the details of the arrangement of a 33kV Metrosil diverter and spark-gap assembly.

The diverters are mounted vertically, mechanical support being also provided at the top of the assembly for larger ratings. External stress rings are provided for ratings above 110kV.

The diverters are normally set to operate on twice normal voltage, it being undesirable for them to operate on small over-voltages. Operation is extremely rapid, taking less than a microsecond. The impulse ratio is practically unity.

PROBLEMS

16.1 An overhead line of surge impedance 500Ω terminates in a transformer of surge impedance 3,500Ω. Find the amplitudes of the current and voltage surge transmitted to the transformer due to an incident voltage of 30kV. (*H.N.C.*)

Ans. 52·5kV; 0·015kA.

16.2 Derive an expression for the surge impedance of a transmission line.

A transmission line has a capacitance of 0·012 μF per km and an inductance of 1·8 mH per km. This overhead line is continued by an underground cable with a capacitance of 0·45 μF per km and an inductance of 0·3 mH per km. Calculate the maximum voltage occurring at the junction of line and cable when a 20kV surge travels along the cable towards the overhead line. (*H.N.C.*)

Ans. 37·5kV.

16.3 Obtain an expression for the surge impedance of a transmission line and for the velocity of propagation of electric waves in terms of the line inductance and capacitance.

A cable having an inductance 0·3 mH per km and a capacitance of 0·4 μF per km is connected in series with a transmission line having an inductance of 1·5 mH per km and a capacitance of 0·012 μF per km. A surge of peak value 50kV originates in the line and progresses towards the cable. Find the voltage transmitted into the cable. Use the result to explain the practice sometimes adopted of terminating a line by a short length of cable before connecting to reactive apparatus. (*H.N.C.*)

Ans. 7·2kV.

16.4 An overhead transmission line 300 km long, having a surge impedance of 500 Ω is short-circuited at one end and a steady voltage of 3 kV is suddenly applied at the other end.

Neglecting the resistance of the line explain, with the aid of diagrams, how the current and voltage change at different parts of the line, and calculate the current at the end of the line 0·0015s after the voltage is applied.

Ans. 0.

16.5 Two stations are connected together by an underground cable having a capacitance of 0·15 μF/km and an inductance of 0·35 mH/km joined to an overhead line having a capacitance of 0·01 μF/km and an inductance of 2·0 mH/km.

If a surge having a steady value of 100kV travels along the cable towards the junction with an overhead line, determine the values of the reflected and transmitted waves of voltage and current at the junction.

State briefly how the transmitted waves would be modified along the overhead line if the line were of considerable length. (*L.U.*)

Ans. 81kV; 181kV; 1·57kA; 0·404kA.

16.6 Derive an expression for the velocity with which a disturbance will be transmitted along a transmission line.

A disturbance, due to lightning, travels along an overhead line of characteristic impedance 200 Ω. After travelling 30 km along the line the disturbance reaches the end of the line where it is joined to a cable of surge impedance 50Ω and dielectric constant [relative permittivity] 6. Calculate the relative magnitude of the energy of the disturbance in the cable and the time taken between initiation and arrival at a point 15 km along the cable from the junction. (*H.N.C.*)

Ans. 0·64; 225 μs.

16.7 An overhead transmission line having a surge impedance of 500Ω is connected at one end to two underground cables, one having a surge impedance of 40Ω and the other one of 60Ω. A rectangular wave having a value of 100kV travels along the overhead line to the junction.

Deduce expressions for, and determine the magnitude of, the voltage and current waves reflected from and transmitted beyond the junction.

If the rectangular wave originated at a long distance from the junction, state how and why it would be modified in its passage along the line. (*L.U.*)

Ans. 90·8kV; 0·182kA; 9·16kV; 0·229kA; 0·153kA.

16.8 Two single transmission lines A and B with earth return are connected in series and at the junction a resistance of 2,000Ω is connected between the lines and earth. The surge impedance of line A is 400Ω and of B 600Ω. A rectangular wave having an amplitude of 100kV travels along line A to the junction.

Develop expressions for and determine the magnitude of the voltage and current waves reflected from and transmitted beyond the junction. What value of resistance at the junction would make the magnitude of the transmitted wave 100kV? (*L.U.*)

Ans. 7kV; 0·018kA; 107kV; 0·178kA; 1,200Ω.

Chapter 15

INTERCONNECTED SYSTEMS

Fig. 15.1 is a line diagram of two power stations A and B joined by an interconnector, the interconnector being connected to the busbars

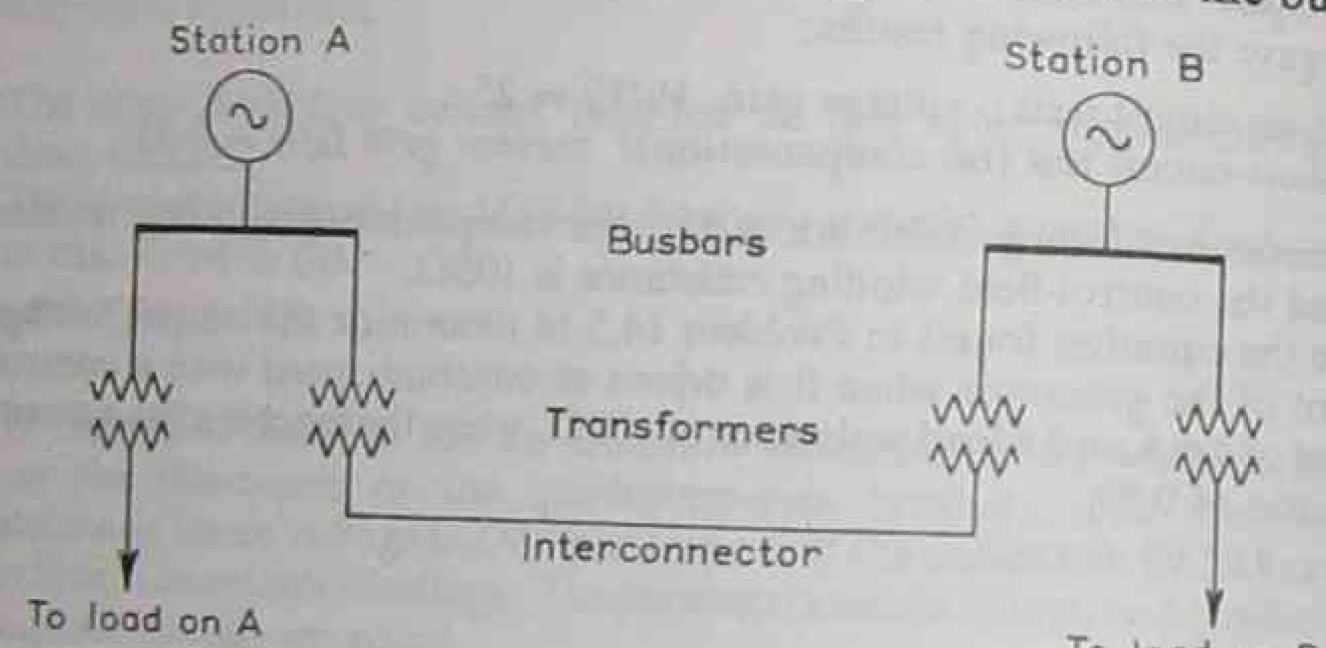

Fig. 15.1 INTERCONNEXION OF POWER STATIONS

of each station through transformers. Each station also has a feeder load connected through a transformer to its busbars.

The power sent across the interconnector will depend, ultimately, on the steam supply to the turbines of each station. For example, if the feeder loads on the busbars of A and B are each 50 MW and the output of the generators on A's busbars is 30 MW, the output of the generators on B's busbars must then be 70 MW, and 20 MW must be transmitted across the interconnector from B to A. As

was shown in Chapter 12, the output of the generators depends only on the power supply to their prime movers. Thus the power transmitted from point to point in an interconnected network depends ultimately on the steam supplies to the prime movers. Where more than one path is available between interconnected points, the proportion of power transmitted by each path may be controlled, but the total remains dependent on the load conditions.

The control of the power transmitted over the National Grid in this country is centralized in the control rooms of the Generating Divisions and in the National Control Room. These maintain communication with the generating stations coming under their control and issue instructions to station engineers to increase or reduce station loadings. The control room engineers thus control the frequency and the loading of transmission links in the network.

Apart from the question of the control of the power flow there is the question of voltage regulation. When power is transmitted across the interconnector there will be a voltage drop in the interconnector, the magnitude of which will depend on the impedance of the interconnector and on the power factor at which the power is transmitted. This voltage drop may be accommodated in a number of ways. Assuming that power is being transmitted from B to A (Fig. 15.1) these are as follows.

1. The busbar voltage at B or at A may be so adjusted that the difference in the busbar voltages is equal to the voltage drop in the interconnector and associated transformers. The disadvantage of this method is that it affects the voltages at which the loads connected to the station busbars are supplied.

2. The interconnector transformers may be equipped with on-load tap-changing gear. The voltage drop in the interconnector may then be supplied by adjusting the secondary e.m.f.s of the interconnector transformers, and the busbar voltages may be maintained constant. This method is commonly used where main transformers are, in any case, necessary.

3. A voltage boost in the appropriate direction may be injected into the interconnector either by an induction regulator or by a series boosting transformer. The latter is now of less importance due to the modern practice of incorporating on-load tap-changing gear in main transformers which, in effect, performs the same function as the series boosting transformer.

4. The secondary terminal voltages of the interconnector transformers may be held constant and the voltage drop in the interconnector may be accommodated by adjusting the relative phase of the voltages at the sending and receiving ends of the interconnector by means of a synchronous phase modifier. Synchronous phase

modifiers are used only on transmission links some hundreds of miles in length.

A further use of voltage regulating equipment is to control the division of power between two or more feeders or transmission lines operating in parallel. In the absence of voltage regulating equipment the division of the load between two lines is determined by their respective impedances. This division of the load may be modified by the introduction of a voltage boost in one line.

The control of the power division between lines in parallel by voltage boosting has the important advantage that both lines may be utilized to maximum capacity. It was shown in Chapter 9 that, when lines are operated in parallel, one may become fully loaded before the other has taken up its full load because of disproportionate impedances. A voltage boost of the appropriate magnitude and direction in such under-loaded lines may allow them to take up their full load.

15.1 Tap-changing Transformers

Fig. 15.2(*a*) shows a transformer having variable tappings in the secondary winding. As the position of the tap is varied, the effective

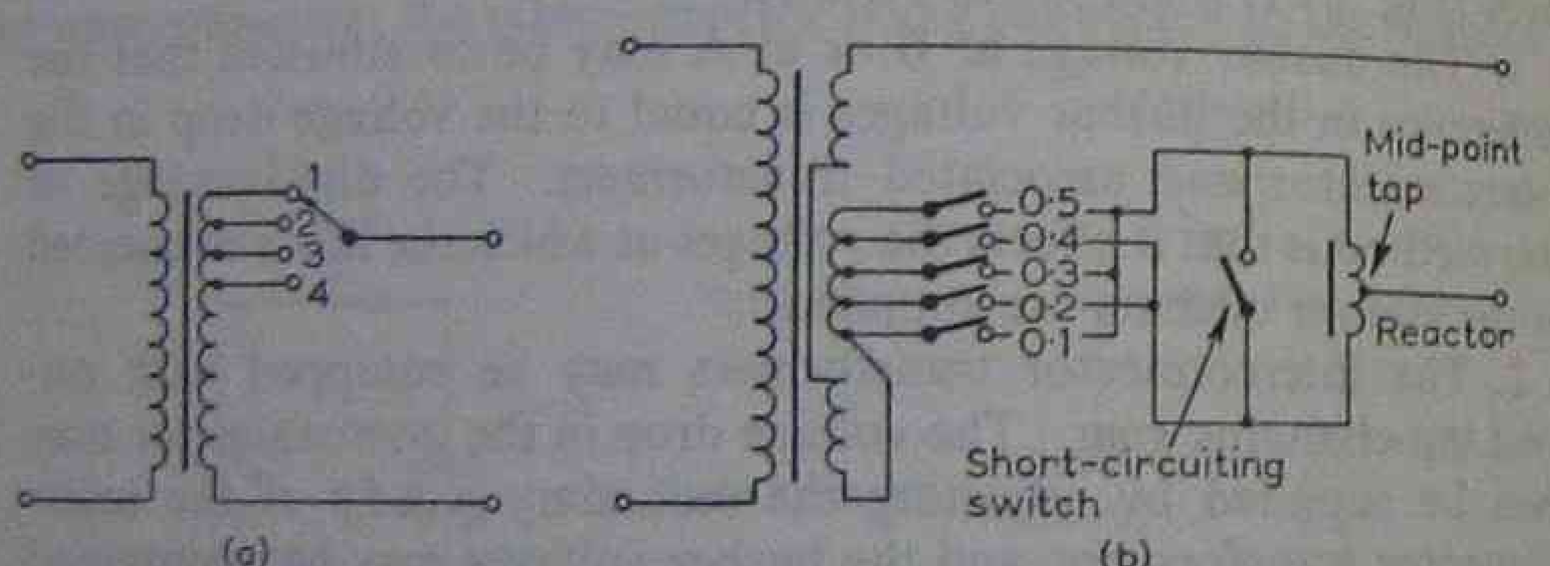

Fig. 15.2 TAP-CHANGING TRANSFORMER

number of secondary turns is varied, and hence the e.m.f. and output voltage of the secondary can be altered.

In supply networks, however, tap-changing has normally to be performed on load (that is, without causing an interruption to supply). The arrangement shown in Fig. 15.2(*a*) is unsuitable for this purpose. Suppose that the tapping is to be altered from position 1 to position 2. If contact with position 1 is broken before contact with position 2 is made, an open-circuit results. If, on the other hand contact with position 2 is made before contact with position 1 is broken, the coils connected between these two tapping points are

short-circuited, and will carry damagingly heavy currents. Moreover, in both cases, switching would be accompanied by excessive arcing.

Fig. 15.2(*b*) shows diagrammatically one type of on-load tap-changing transformer. With switch 5 closed, all the secondary turns are in circuit. If the reactor short-circuiting switch is also closed, half the total current flows through each half of the reactor—since the currents in each half of the reactor are in opposition, no resultant flux is set up in the reactor and there is no inductive voltage-drop across it.

Suppose now it is desired to alter the tapping point to position 4. The reactor short-circuiting switch is opened. The load current now flows through one-half of the reactor coil only so that there is a voltage drop across the reactor. Switch 4 is now closed, so that the coils between tapping points 4 and 5 are now connected through the whole reactor winding. A circulating current will flow through this local circuit, but its value will be limited by the reactor. Switch 5 is now opened and the reactor short-circuiting switch is closed, thus completing the operation.

The tapping coils are placed physically in the centre of the transformer limb to avoid unbalanced axial forces acting on the coils, as would arise if they were placed at either end of the limb. Electrically, the tapped coils are at one end of the winding, the practice being to connect them at the earth-potential end.

15.2 Three-phase Induction Regulator

In construction, the 3-phase induction regulator resembles a 3-phase induction motor with a wound rotor. In the induction regulator, the rotor is locked, usually by means of a worm gear, to prevent its revolving under the action of the electromagnetic force operating on it. The position of the rotor winding relative to the stator winding is varied by means of the worm gear.

If the stator winding is connected to a constant-voltage constant-frequency supply, a rotating magnetic field is set up and will induce an e.m.f. in each phase of the rotor winding. The magnitude of the induced rotor e.m.f. per phase is independent of the rotor position, since the e.m.f. depends only on the speed of the rotating field and the strength of the flux, neither of which varies with rotor position. However, variation of the position of the rotor will affect the phase of the induced rotor e.m.f. with respect to that of the applied stator voltage.

Fig. 15.3(*a*) shows the star-connected stator winding of a 3-phase induction regulator with each of the rotor phase windings in series with one line of an interconnector. In Fig. 15.3(*b*) O*a*, O*b*, O*c* represent the input values of the line-to-neutral voltages of the interconnector. The circles drawn at the extremities a, b, c of these complexors represent the loci of the rotor phase e.m.f.s as the rotor position is varied with respect to the stator. The complexors aa', bb' and cc' represent the voltage boosts introduced by the induction

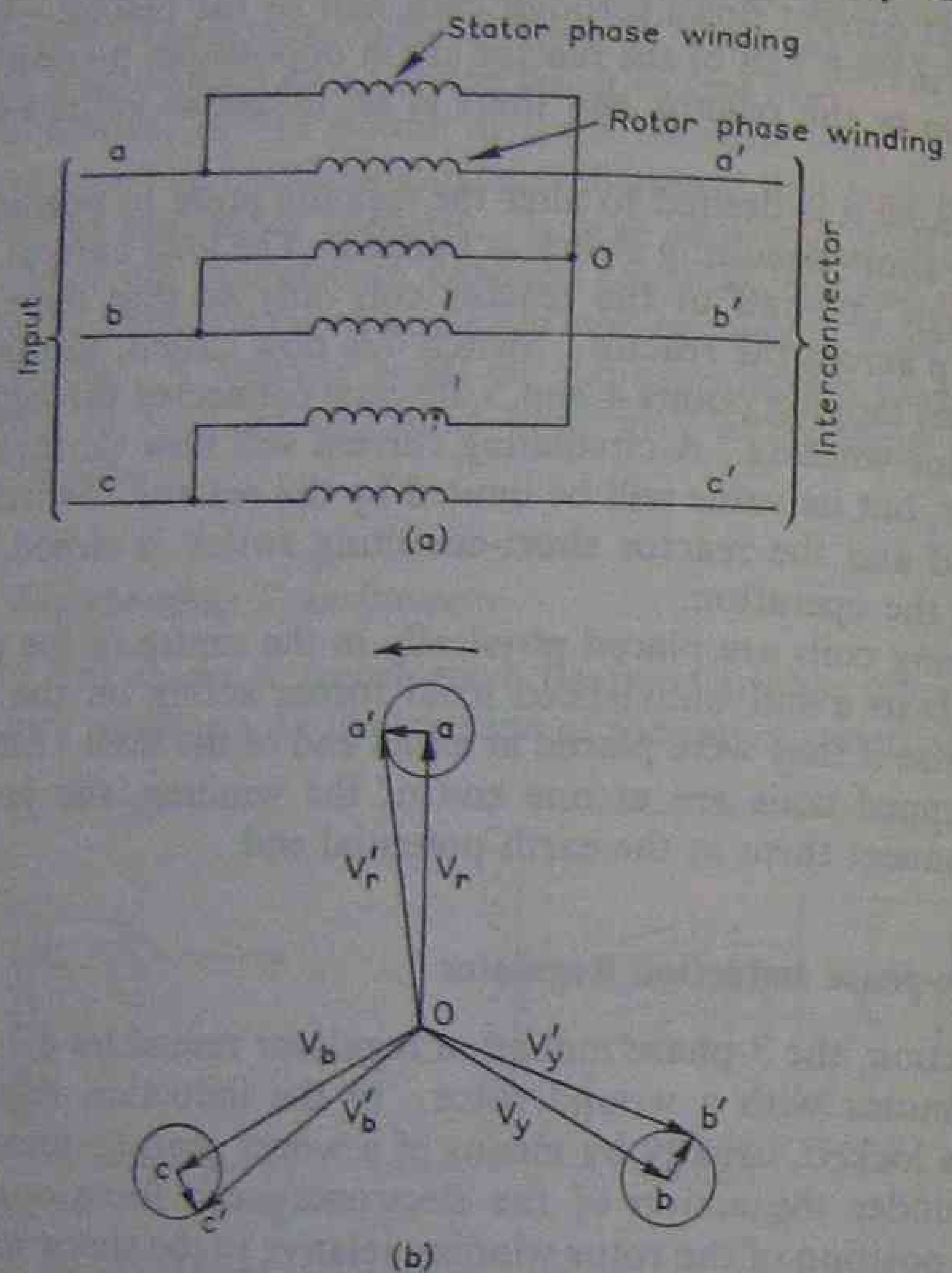

Fig. 15.3 POLYPHASE INDUCTION REGULATOR

regulator when the rotor position is such as to cause these voltage boosts to lead on their respective line-to-neutral voltages by 90°. Oa', Ob' and Oc' represent the resultant voltages V_r', V_y' and V_b'.

It will at once be seen that the induction regulator has altered the phase of the voltages as well as introducing a voltage boost.

To eliminate this phase displacement, a double polyphase induction regulator is employed, in which two rotors are assembled on a common shaft. The connexion diagram is shown in Fig. 15.4(a). The rotor windings of each regulator are connected in series with the interconnector. The stator windings are star-connected, but the

phase sequence of one regulator stator is reversed with respect to the other. This reversal has the effect of eliminating any phase displacement in the resultant voltage boost in the interconnector. Thus, when the shaft of the double regulator is displaced, both rotors move by the same angular amount, but if the e.m.f. induced in one leads its former value, then the e.m.f. induced in the other lags by the same amount since the rotating fields in the regulators rotate in

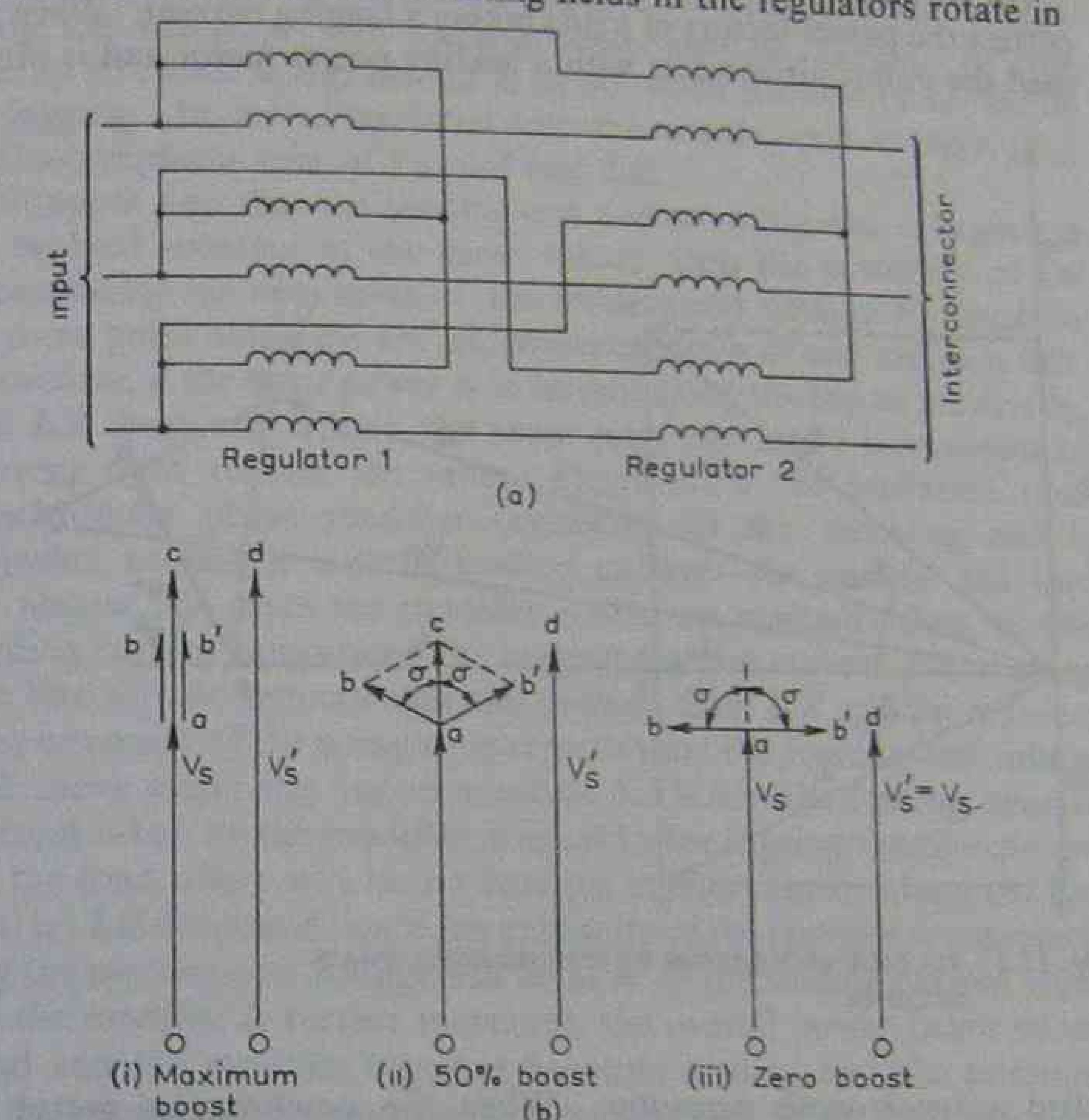

Fig. 15.4 DOUBLE POLYPHASE INDUCTION REGULATOR

opposite directions. Fig. 15.4(b) is a complexor diagram for various rotor positions. One phase only is shown for clarity. Oa represents the unboosted input-end voltage, V_s, ab and ab' represent the voltage boosts supplied by each rotor, ac represents the resultant voltage boost, and Od represents the resultant voltage V_s'.

It is often convenient to reverse the functions of the stator and the rotor windings in induction regulators used for boosting. The rotor then carries the primary winding. This has the advantage of requiring only three connexions to the rotor instead of six, and the interconnector current flows in the stator instead of the rotor.

15.3 Synchronous Phase Modifier

In Chapter 12, it was shown that variation in the excitation of a synchronous motor alters the power factor at which the machine works. As the excitation of the machine is increased, the power factor passes from a lagging, through unity, to a leading power factor.

Use is made of this characteristic of the synchronous motor to correct the power factors of loads taking a lagging current. When so used the motor always acts with a leading power factor and is often

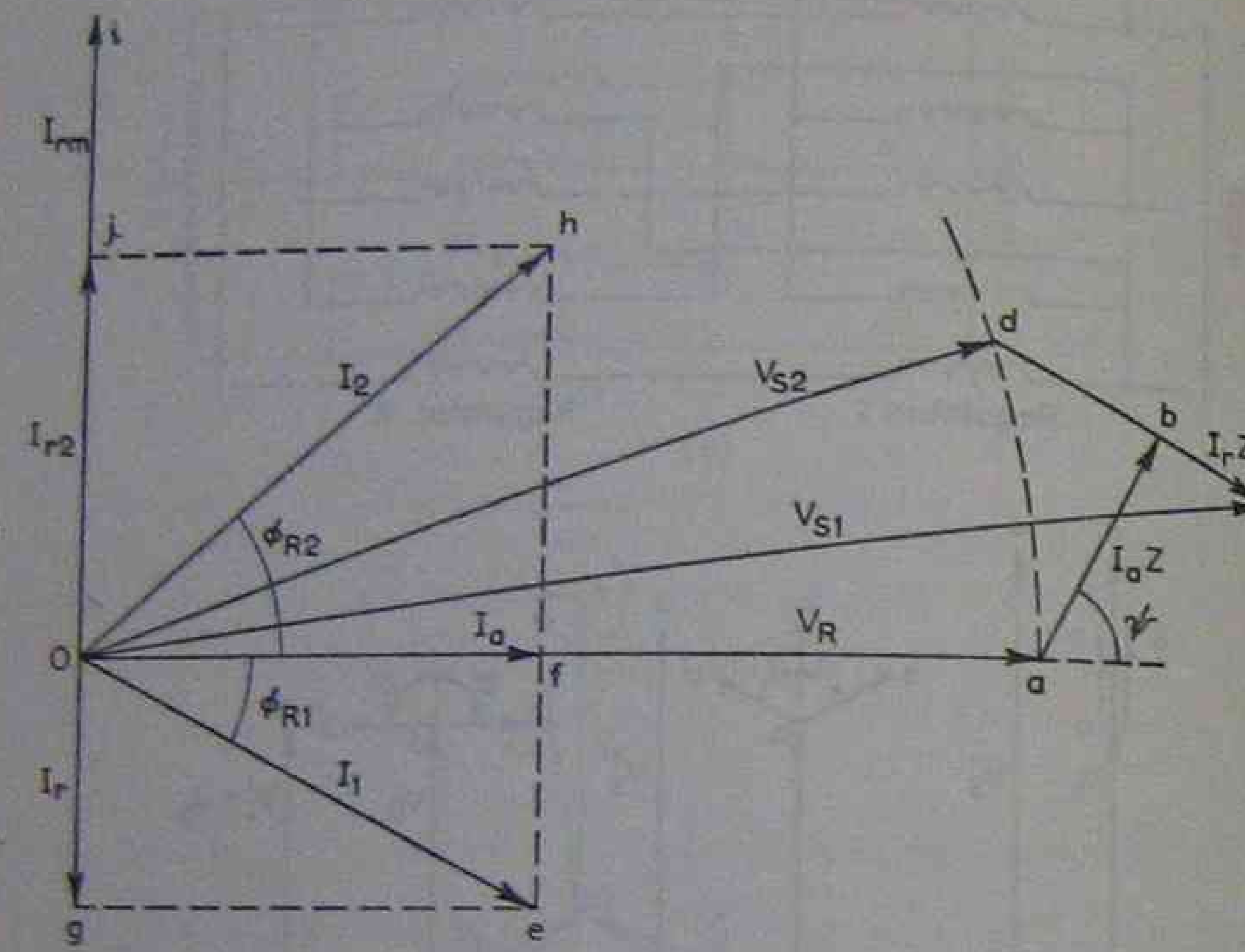

Fig. 15.5 VOLTAGE REGULATION BY SYNCHRONOUS PHASE MODIFIER

called a *synchronous capacitor*. When the synchronous motor is used as a means of controlling the voltage of a transmission line the term *synchronous phase modifier*, or *synchronous compensator* is usually preferred, since, in this application, the machine may be adjusted to take either a leading or a lagging current. The machine is connected in parallel with the load at the receiving end of the line.

The action of the synchronous phase modifier in controlling the voltage of a transmission line is best understood by reference to the complexor diagram shown in Fig. 15.5. For simplicity the diagram is that of a short line where the effects of capacitance are neglected, but it should be understood that this method of control is mostly applied to long lines where, with other methods of control, the voltage drop along the line would be excessive.

In Fig. 15.5, O*a* represents the receiving-end voltage V_R, and O*e* represents the receiving-end current, I_1, lagging behind the receiving-end voltage by a phase angle ϕ_{R1}. O*f* and O*g* represent the active and reactive components (I_a and I_r) of current, respectively, *ab* represents the voltage drop I_aZ caused by the active component of current, which leads V_R by the phase angle of the line impedance, ψ ($\tan^{-1} X_L/R$), Z being the line impedance. *bc* represents the voltage drop I_rZ caused by the reactive component of current. *bc* lags I_aZ by 90°, since I_r lags behind I_a by 90° when the load power factor is lagging. In an unregulated line the sending-end voltage, V_{s1}, is the complexor sum of V_R, I_aZ and I_rZ.

Suppose now that the sending-end and receiving-end voltages are to be held constant at the same value; then the extremity of O*d* representing the new value of the sending-end voltage V_{s2} must be at some point along the arc *ad*, whose centre is O and radius is OD. Moreover, if the same power is to be sent along the line as previously, the I_aZ drop will remain the same since the active component of current must remain the same. However, if the excitation of a synchronous phase modifier connected to the receiving end is adjusted so that it takes a leading current—the current will lead by almost 90° since the modifier works on no-load—then as this leading current is increased the lagging reactive current drawn along the line will be reduced and the voltage drop I_rZ will be reduced. The extremity of the complexor representing the sending-end voltage will move along the line *cb* towards *b*. When the leading reactive current taken by the modifier is equal to the lagging reactive current of the load, there will be no reactive current drawn along the line and no I_rZ drop, and hence the extremity of the complexor representing the sending-end voltage will be at *b*. If the leading current taken by the modifier is further increased, the overall power factor of the load and the modifier together becomes leading and the extremity of the complexor representing the sending-end voltage lies along the line *bd* between *b* and *d*. Thus if the leading current taken by the modifier is made sufficiently great the sending-end voltage complexor takes up the position O*d*.

The synchronous phase modifier may therefore be used to control the voltage drop of a transmission line. If the sending-end voltage is maintained constant, then on full-load at a lagging power factor (the usual condition) the modifier will be over-excited to take a leading current. The receiving-end voltage will thus increase compared with its value had the line been unregulated. On no-load, on the other hand, the modifier would be under-excited and would take a lagging current in order to offset the voltage rise which occurs at the receiving end of a long unregulated line when the load is removed.

The power which may be sent along a transmission line is limited by either the power loss in the line reaching its permissible maximum value or by the voltage drop along the line reaching the maximum value which can be conveniently dealt with. On long transmission lines it is the voltage drop which limits the power which can be sent. Thus, if synchronous phase modifiers are used to regulate the voltage, more power can be dealt with by the line. Moreover, since voltage drop in the line and associated plant is not the first consideration when synchronous phase modifiers are used to control the voltage, current-limiting reactors may be incorporated in the system to reduce the maximum short-circuit current should a fault occur.

The principal disadvantage, apart from cost, of using synchronous phase modifiers is the possibility of their breaking from synchronism and causing an interruption to the supply.

15.4 Sending-end Voltage

In constant-voltage transmission systems using synchronous phase modifiers, the sending-end and receiving-end voltages are held constant, but they do not necessarily have to be equal. There is, indeed, an advantage in having the sending-end voltage higher than the receiving-end voltage, particularly with short lines, since under such conditions a smaller synchronous phase modifier capacity will satisfactorily regulate the voltage. For example, referring to Fig. 15.5, if the sending-end voltage had been greater than the receiving-end voltage, the reactive voltage drop *cd*, due to the reactive current of the synchronous phase modifier, would have been smaller and a synchronous phase modifier of smaller capacity would have been sufficient.

On longer lines, the capacitive effect tends to cause a voltage rise on light loads and no load, and the synchronous phase modifier has to work with a lagging power factor in order to hold the voltage constant. Thus the longer the line the less is the advantage of having the sending-end voltage higher than the receiving-end voltage.

EXAMPLE 15.1 A 3-phase transmission line has a resistance of 8·75Ω and an inductive reactance of 15Ω per phase. The line supplies a load of 10 MW at 0·8 power factor lagging and 33 kV. Determine the kVA rating of a synchronous phase modifier operating at zero power factor such that the sending-end voltage may be maintained at 33 kV. The effect of the capacitance of the line may be neglected. Determine, also, the maximum power which may be transmitted by the line at these voltages.

The problem is most easily tackled graphically.

The line current drawn by the load is

$$I = \frac{10^7}{\sqrt{3} \times 33 \times 10^3 \times 0{\cdot}8} \angle{-\cos^{-1} 0{\cdot}8}$$

$$= 219\angle{-36{\cdot}9^\circ} = (175 - j132)\,\text{A}$$

Phase impedance $= 8{\cdot}75 + j15 = 17{\cdot}4\angle{59{\cdot}7}\,\Omega$

The voltage drop caused by the active component of the load current is

$I_aZ = 175 \times 17{\cdot}4 = 3{\cdot}04 \times 10^3\,\text{V}$

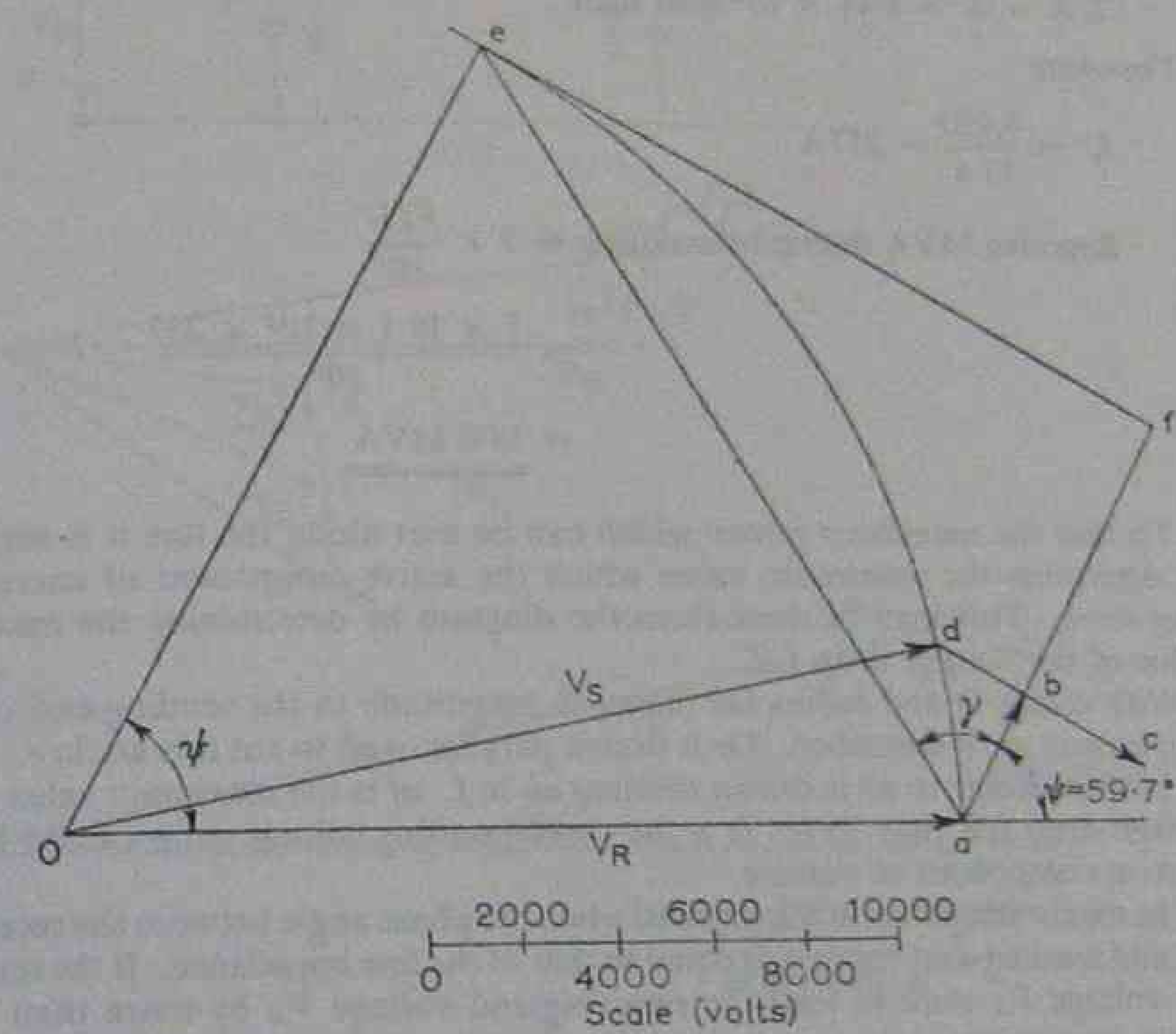

Fig. 15.6

The voltage drop caused by the reactive component of the load current is

$I_rZ = 132 \times 17{\cdot}4 = 2{\cdot}3 \times 10^3\,\text{V}$

Receiving-end voltage (phase value), $V_R = 19{\cdot}1 \times 10^3\,\text{V}$

Phase angle of line impedance, $\psi = \tan^{-1}\dfrac{X_L}{R} = \tan^{-1}\dfrac{15}{8{\cdot}75} = 59{\cdot}7^\circ$

The graphical construction is shown in Fig. 15.6.

Oa is drawn to represent V_R to a suitable scale.

ab, equivalent in length to $3{\cdot}04 \times 10^3$ V to scale, is drawn making an angle ψ ($=59{\cdot}7^\circ$) with *Oa* to represent I_aZ.

bc, equivalent in length to $2{\cdot}3 \times 10^3$ V to scale, is drawn lagging behind *ab* by 90° to represent I_rZ.

The line joining O and *c* would then represent the necessary sending-end voltage in an unregulated line. With centre O and radius *Oa* (equivalent in length to $19{\cdot}1 \times 10^3$V) an arc *ae* is drawn. The extremity of the complexor representing the sending-end voltage must lie on this arc. *cb* is now produced until it cuts arc *ae* in *d*. *Od* represents the sending-end voltage.

cd represents the voltage drop due to the reactive current of the modifier alone.

It will be observed that the resultant power factor is leading and the resultant voltage drop, *bd*, due to the reactive current leads the voltage drop, I_aZ, due to the active component of current by 90°.

If I_r' is the reactive current of the synchronous phase modifier,

$$I_r'Z = dc = 4{\cdot}46 \times 10^3\text{V to scale}$$

Therefore

$$I_r' = \frac{4{,}460}{17{\cdot}4} = 257\text{A}$$

$$\text{Reactive MVA drawn by modifier} = 3 \times \frac{V_R I_r'}{10^6}$$

$$= \frac{3 \times 19{\cdot}1 \times 10^3 \times 257}{10^6}$$

$$= \underline{\underline{14{\cdot}6\text{ MVA}}}$$

To find the maximum power which can be sent along the line it is necessary to determine the maximum value which the active component of current, I_a, may have. This may be done from the diagram by determining the maximum value of the voltage drop I_aZ.

With centre O and radius *Oa* (equal in magnitude to the sending-end voltage V_S), an arc, *ae*, is described. *Oe* is drawn parallel to *ab* to cut this arc in *e*. From *e* a perpendicular to *ab* is drawn meeting *ab* in *f*. *af* is the maximum value which voltage drop I_aZ may have; *fe* is the corresponding voltage drop caused by the reactive component of current.

The maximum power is transmitted when the phase angle between the receiving-end and sending-end voltages is equal to that of the line impedance. If the sending-end voltage V_S were to lead the receiving-end voltage V_R by more than ψ the power sent would decrease, since the projection of the sending-end voltage on *ab* would decrease. From Fig. 15.6,

$$I_aZ = af = 9{\cdot}6\text{kV} \quad \text{so that} \quad I_a = \frac{9{\cdot}6 \times 10^3}{17{\cdot}4} = 552\text{A}$$

$$\text{Power sent} = \frac{\sqrt{3}VI_a}{10^6} = \frac{\sqrt{3} \times 33{,}000 \times 552}{10^6} = \underline{\underline{31{\cdot}5\text{MW}}}$$

EXAMPLE 15.2 A 3-phase 50Hz 132 kV transmission line 100 km long has the following constants per km: resistance 0·2Ω, inductance 2mH, capacitance 0·015μF.

The sending-end voltage is 132kV and the receiving-end voltage is held constant at 132kV by means of a synchronous phase modifier. Determine the reactive kVA of the synchronous phase modifier when the load at the receiving end is 50MW at a power factor of 0·8 lagging.

One method of taking the effect of line capacitance into account is to use a nominal-π equivalent circuit. The advantage of doing this is that the graphical method developed for the short line is still applicable if, instead of the receiving-end current, I_R, the current in the mid-section of the π-circuit, I', is used.

Fig. 15.7(*a*) shows the nominal-π equivalent circuit, and Fig. 15.7(*b*) shows the complexor diagram of this circuit. The voltage drop in the line impedance is shown as the sum of $I_a'Z$ (the voltage drop caused by the active component of I' and $I_r'Z$ (the voltage drop caused by the reactive component of I'). The voltage diagram showing the relationship between V_R and V_S is now similar to that for the short line.

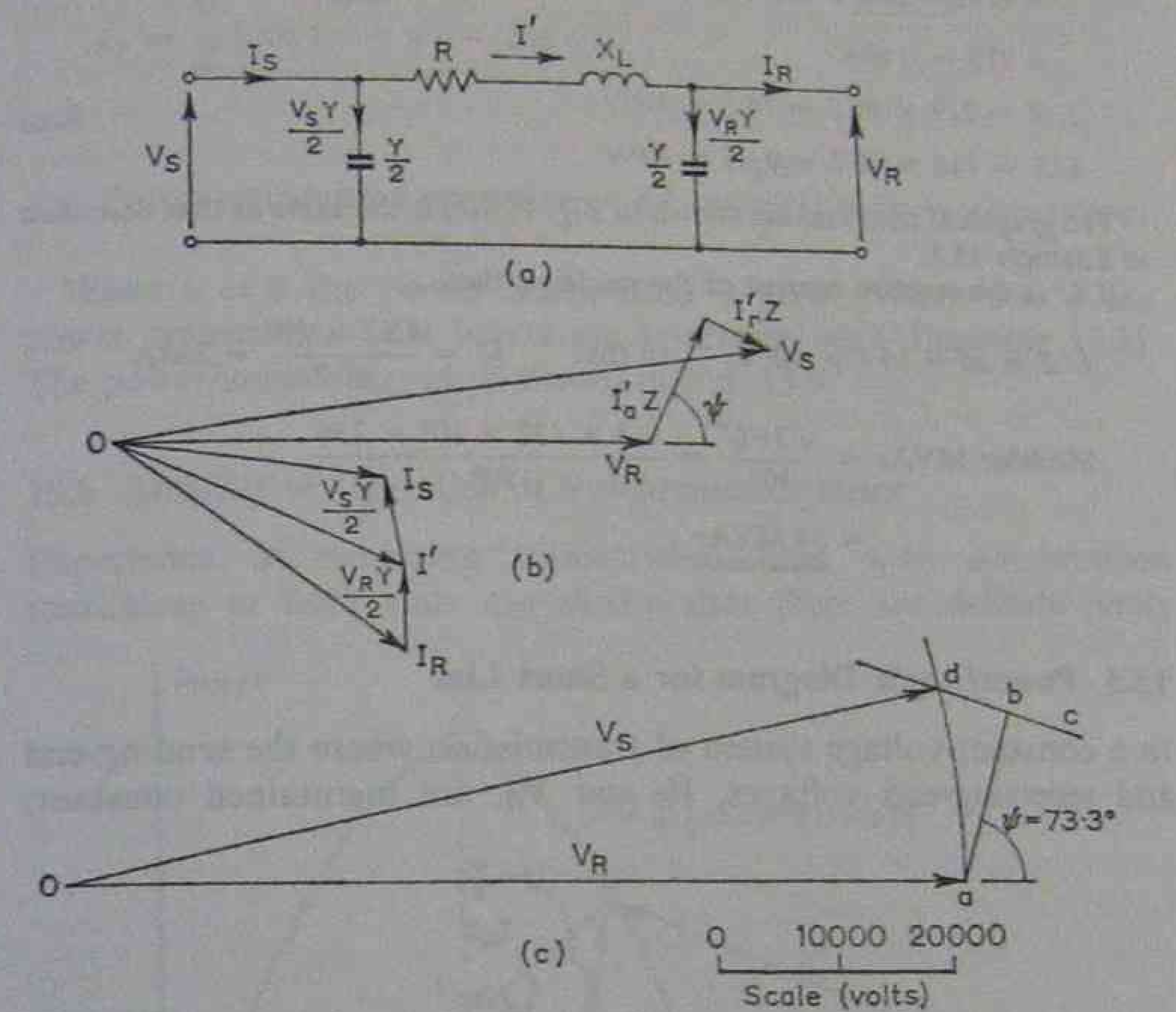

Fig. 15.7

Taking the receiving-end voltage as reference complexor and working with phase values,

$$V_R = \frac{132 \times 10^3}{\sqrt{3}} \underline{/0^\circ} = 76{\cdot}2 \times 10^3\underline{/0^\circ}\text{V}$$

$$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0{\cdot}8} \underline{/-\cos^{-1} 0{\cdot}8} = 273\underline{/-36{\cdot}9^\circ}\text{A}$$

$$Z = 0{\cdot}2 \times 100 + j2\pi \times 50 \times 2 \times 10^{-3} \times 100$$

$$= 20 + j62{\cdot}8 = 66{\cdot}2\underline{/72{\cdot}4^\circ}\Omega$$

$$\frac{Y}{2} = j\frac{2\pi \times 50 \times 0{\cdot}015 \times 10^{-6} \times 100}{2}$$

$$= j0{\cdot}236 \times 10^{-3} = 0{\cdot}236 \times 10^{-3}\underline{/90^\circ}\text{ S}$$

Before proceeding to the normal graphical construction, I' should first be calculated with its active and reactive components.

$$I' = I_R + \frac{V_R Y}{2}$$

$$= (273\angle{-36{\cdot}9^\circ}) + (76{\cdot}2 \times 10^3 \times 0{\cdot}236 \times 10^{-3}\angle{90^\circ})$$

$$= 219 - j146\,\text{A}$$

$$I_a' Z = 219 \times 66{\cdot}2 = 14{\cdot}5 \times 10^3\,\text{V}$$

$$I_r' Z = 146 \times 66{\cdot}2 = 9{\cdot}65 \times 10^3\,\text{V}$$

The graphical construction shown in Fig. 15.7(*c*) is the same as that described in Example 15.1.

If I_r'' is the reactive current of the modifier, then

$$I_r''Z = cd = 15{\cdot}7 \times 10^3\,\text{V} \quad \text{so that} \quad I_r'' = \frac{15{\cdot}7 \times 10^3}{66{\cdot}2} = 236\,\text{A}$$

$$\text{Modifier MVAr} = \frac{\sqrt{3} V I_r''}{10^6} = \frac{\sqrt{3} \times 132 \times 10^3 \times 236}{10^6}$$

$$= \underline{\underline{54\ \text{MVAr}}}$$

15.5 Power/Angle Diagram for a Short Line

In a constant-voltage system of transmission where the sending-end and receiving-end voltages, V_S and V_R, are maintained constant,

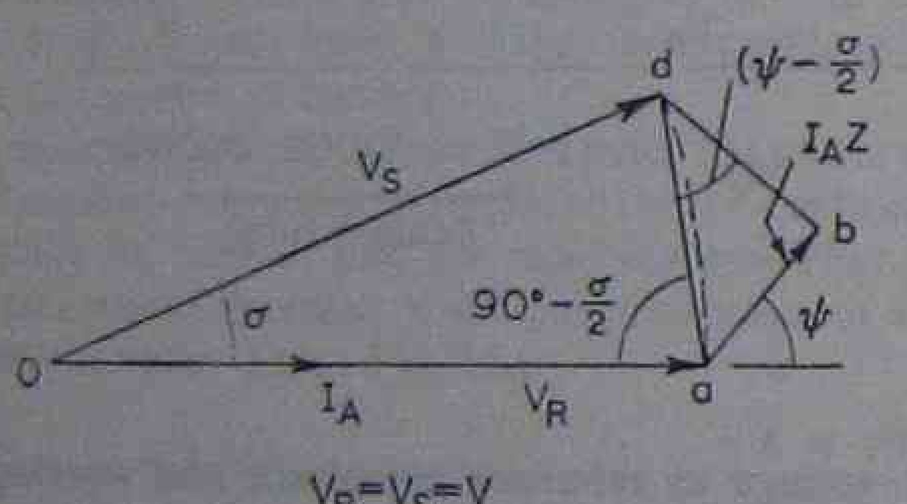

Fig. 15.8 COMPLEXOR DIAGRAM FOR A SHORT LINE

the power transmitted depends on the phase displacement between these voltages. It is possible to derive an expression for the power sent as a function of the phase displacement between V_S and V_R. To simplify the derivation it will be assumed (i) that the line is a short one, and (ii) that $V_S = V_R = V$ (phase values).

Fig. 15.8 shows the complexor diagram in which the receiving-end voltage V_R is taken as the reference. By geometry,

$$\angle adb = (\psi - \sigma/2) \quad \text{so that} \quad ab = ad \sin(\psi - \sigma/2)$$

But $ad = 2V \sin \sigma/2$; therefore

$$ab = 2V \sin(\sigma/2) \sin(\psi - \sigma/2) = V\{\cos(\sigma - \psi) - \cos\psi\}$$

Therefore

$$I_A = \frac{V}{Z}\{\cos(\sigma - \psi) - \cos\psi\}$$

and

$$\text{Power transmitted per phase} = VI_A = \frac{V^2}{Z}\{\cos(\sigma - \psi) - \cos\psi\}$$

When $\sigma = 0$ the power transmitted is zero. When $\sigma = \psi$ the power transmitted is a maximum (compare with Example 15.1). The power/angle diagram is shown in Fig. 15.9.

15.6 Stability of Operation of Synchronous Systems

Experience in operating transmission lines with synchronous machinery at both ends has shown that there are definite limits

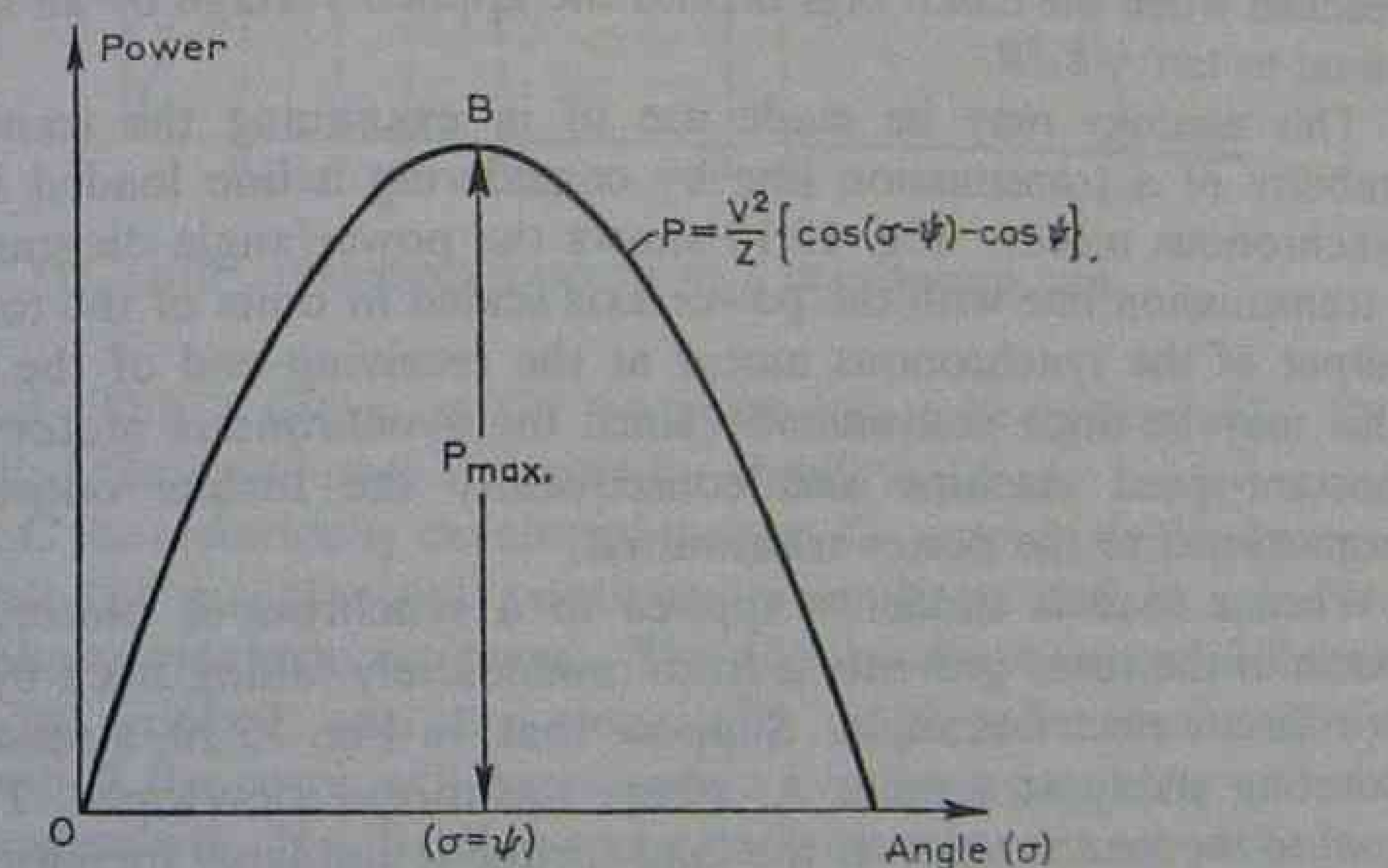

Fig. 15.9 POWER/ANGLE DIAGRAM FOR A SHORT LINE WITH EQUAL SENDING- AND RECEIVING-END VOLTAGES

beyond which operation becomes unstable, resulting in a loss of synchronism between the sending and receiving ends. It is possible to distinguish two limits of stability, a static limit and a dynamic or transient limit.

For given constant values of sending-end and receiving-end voltages, the load on a transmission line can be gradually increased until a condition is reached corresponding to B in Fig. 15.9. At this point the power transmitted is a maximum and corresponds to an angle of phase difference between the sending-end and receiving-end voltages of ψ ($\tan^{-1} X_L/R$). This point represents the static limit of stability (i.e. for a gradually applied load), and any attempt to impose further load on the line results in loss of synchronism between the ends of the line.

Short lines, where constant-voltage transmission using synchronous phase modifiers is not used, cannot be operated near the limit of stability, since the voltage drop along the line would become excessive. On long lines, however, where synchronous phase modifiers are used to control the voltage this limit may be approached and becomes of practical importance. Because of the high capital cost of transmission lines, the more power which can be transmitted over a given line the more economical becomes the operation.

The limit of stability of transmission lines is analogous to the limit of stability of a synchronous motor, where, as load is imposed on the machine, the rotor shifts backwards relative to the rotating field of the stator, and the angle of phase difference between the applied voltage and the e.m.f. increases. The limit of stability is reached when the e.m.f. lags behind the applied voltage by an angle equal to $\tan^{-1} X_s/R$.

This analogy may be made use of in examining the transient stability of a transmission line by considering a line loaded by a synchronous motor. Fig. 15.10 shows the power/angle diagram of a transmission line with the power axis scaled in units of the torque output of the synchronous motor at the receiving end of the line. This may be done conveniently since the synchronous motor is a constant-speed machine and consequently the torque output is proportional to the power transmitted.

When a load is suddenly applied to a synchronous motor, the inertia of the rotor prevents it from immediately falling back by the appropriate electrical angle. Suppose that, in Fig. 15.10, a motor is operating stably at a point A, where the torque developed, T_A, is equal to the load torque. If the load torque is suddenly increased to T_B, the angular displacement of the rotor remains momentarily at σ_A so that the electrically developed torque remains at T_A. The difference torque $(T_B - T_A)$ must therefore slow down the rotor causing its angular displacement to increase. The rotor, in slowing down, loses an amount of kinetic energy proportional to the area of the triangle DBA (energy = torque × angular displacement), this energy being transferred to the load.

At B the rotor is running below synchronous speed, so that the angular displacement will continue to increase. However, for angular displacements greater than σ_B the generated torque will exceed the

load torque and the rotor will be accelerated; it will regain synchronous speed at an angular displacement σ_C. During this time the load torque remains constant at T_B, so that the torque represented by the difference between the curve BC and the line BE must be that which accelerates the rotor. The area BCE is then proportional to the energy stored in the rotor due to its acceleration. The areas ADB and BCE must be equal if the kinetic energy taken from the rotor during deceleration is all to be returned during the accelerating

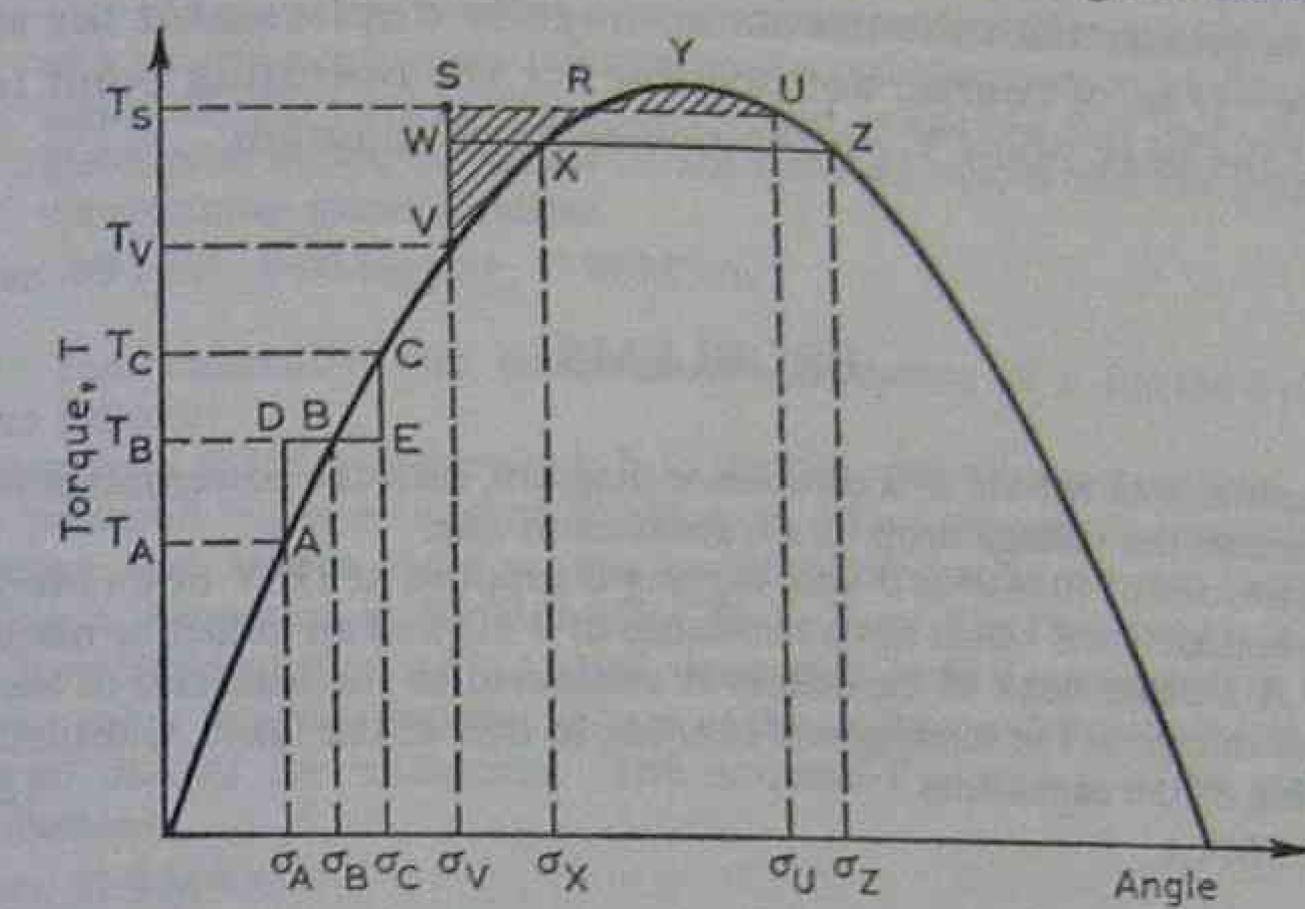

Fig. 15.10 TRANSIENT STABILITY OF A TRANSMISSION LINE

period. This is necessary for the operation to be again stable. This is called the *equal-area criterion for stability*.

At C the electrically developed torque, T_C, exceeds the load torque, so that the machine will continue to accelerate and its speed will rise above synchronous speed. The angular displacement will therefore be reduced, and the electrically developed torque will fall. Beyond B the rotor will experience a synchronizing torque tending to decelerate it. It will fall in speed until it again runs at synchronous speed at A. The whole cycle of events will then be repeated, giving rise to phase swinging or hunting between A and C.

In actual fact the damping which takes place means that the swing of the rotor will becomes less and less, and the machine will eventually run stably at B. The frequency of the oscillation may be determined from eqn. (12.48).

Consider now the motor operating at V with a load torque T_V. If the load torque suddenly increases to T_S, then the shaded area VSR represents the energy which is taken from the rotor as it slows down to accommodate the increased torque. The energy returned to the rotor while it accelerates under the influence of the synchronizing torque is the shaded area RUY. This is less than area VSR, so that the rotor cannot have as much energy returned to it as has been taken from it. It must therefore fall out of step (i.e. lose synchronism).

The maximum load which can be suddenly applied at V is represented by VW, where area VWX is just equal to area XYZ. Note that in this case it is possible for the rotor to swing through an angle which is greater than the maximum angular displacement for static stability. It is, of course, not possible for the operating point to lie beyond the peak point Y on the power/angle diagram.

PROBLEMS

15.1 Explain, with the aid of a complexor diagram, how the power factor of the load influences the voltage drop in a transmission line.

A 3-phase load of 10 MW at 0·8 p.f. lagging is supplied at 33 kV by an overhead line, each conductor of which has a resistance of 2·9 Ω and an inductive reactance of 6·5 Ω. A 3-phase bank of capacitors is connected to the load end of the line so that the voltage at the sending end is equal to that at the load. Calculate the MVA rating of the capacitors. (*L.U.*)

Ans. 12·2 MVA.

15.2 A 3-phase transmission line 25 km in length supplies a load of 10 MW at 0·8 p.f. lagging at a voltage of 33 kV. The resistance and reactance per km per conductor are 0·35 Ω and 0·6 Ω respectively. Neglecting the capacitance of the line, determine the rating of a synchronous capacitor, operating at zero power factor, connected at the load end of the line such that the sending-end voltage may be 33 kV. (*L.U.*)

Ans. 14·7 MVA.

15.3 A 3-phase transmission line is automatically regulated to zero voltage regulation by means of a synchronous phase modifier at the load end. If the full-load output is 50 MW at 0·8 p.f. lagging delivered at 200 kV and the line-to-neutral impedance is $(20 + j60)\,\Omega$, find the input to the synchronous set under these conditions. Deduce any formula employed. (*H.N.C.*)

Ans. 60·3 MVAr.

15.4 A 3-phase overhead line has resistance and reactance of 12 Ω and 40 Ω respectively per phase. The supply voltage is 132 kV and the load-end voltage is maintained constant at 132 kV for all loads by an automatically controlled synchronous phase modifier. Determine the kVAr of the modifier when the load at the receiving end is 120 MW at power factor 0·8 lagging. (*H.N.C.*)

Ans. 145 MVAr.

15.5 A 3-phase transmission line has a resistance of 6 Ω/phase and a reactance of 20 Ω/phase. The sending-end voltage is 66 kV and the voltage at the receiving end is maintained constant by a synchronous phase modifier.

Determine the MVAr of the synchronous phase modifier when the load at the receiving end is 75 MW at 0·8 p.f. lagging, and also the maximum load which can be transmitted over the line, the voltage being 66 kV at both ends. (*L.U.*)

Ans. 96·8 MVAr; 148 MW.

15.6 A 3-phase transmission line has a resistance per phase of 5 Ω and an inductive reactance per phase of 12 Ω, and the line voltage at the receiving end is 33 kV.

(*a*) Determine the voltage at the sending end when the load at the receiving end is 20 MVA at 0·8 p.f. lagging.

(*b*) The voltage at the sending end is maintained constant at 36 kV by means of a synchronous phase modifier at the receiving end, which has the same rating at zero load at the receiving end as for the full load of 16 MW. Determine the power factor of the full-load output and the rating of the synchronous phase modifier. (*L.U.*)

Ans. 40·1 kV; 0·90 lagging; 7·92 MVAr.

15.7 The "constants" per kilometre per conductor of a 150 km 3-phase line are as follows:

Resistance, 0·25 Ω; inductance, 2×10^{-3} H; capacitance to neutral, 0·015 μF.

A balanced 3-phase load of 40 MVA at 0·8 p.f. lagging is connected to the receiving end, and a synchronous capacitor operating at zero power factor leading, is connected to the mid-point of the line. The frequency is 50 Hz.

If the voltage at the load is 120 kV, determine the MVA rating of the synchronous capacitor in order that the voltage at the sending end may be equal in magnitude to that at the mid-point. The nominal-T circuit is to be used for the calculations. (*L.U.*)

Ans. 31·9 MVA.

15.8 A 3-phase 50 Hz transmission line has the following values per phase per km: $R = 0{\cdot}25\,\Omega$; $L = 2{\cdot}0$ mH; $C = 0{\cdot}014\,\mu$F. The line is 50 km long, the voltage at the receiving end is 132 kV and the power delivered is 80 MVA at 0·8 power factor lagging.

If the voltage at the sending end is maintained at 140 kV by a synchronous phase modifier at the receiving end, determine the kVAr of this machine (i) with no load, (ii) with full load at the receiving end. (*L.U.*)

Ans. 30·4 MVAr lagging; 42·6 MVAr leading.

15.9 A 3-phase transmission line has a resistance of 10 Ω per phase and a reactance of 30 Ω per phase.

Determine the maximum power which could be delivered if 132 kV were maintained at each end.

Derive a curve showing the relationship between the power delivered and the angle between the voltage at the sending and receiving ends, and explain how this curve could be used to determine the maximum additional load which could be suddenly switched on without loss of stability if the line were already carrying, say, 50 MW. (*L.U.*)

Ans. 380 MW.

15.10 Develop an expression connecting the power received with the angle between the sending-end and receiving-end voltages of a regulated transmission line in terms of these voltages and the line constants, ignoring capacitance between lines.

Sketch the curve which this equation represents and use it to describe briefly what is meant by (a) static, and (b) transient stability.

Find the maximum power which can be transmitted over the following line:

Impedance per conductor: $(24 + j45)\Omega$
Receiving-end voltage: 110 kV
Regulation: Zero

(H.N.C.)

Ans. 126 MW.

15.11 Describe the necessary conditions under which power can be transmitted between two interconnected power stations.

Two power stations are linked by a 3-phase interconnector the impedance per line of which (including transformers and reactors) is $(10 + j40)\Omega$. The busbar voltage of each station is 66 kV. Calculate the angular displacement between the two station voltages in order to transmit 8 MW from one station to the other.

(H.N.C.)

Ans. 4·8°.

Chapter 16

TRAVELLING VOLTAGE SURGES

A voltage surge in a transmission system consists of a sudden voltage rise at some point in the system, and the transmission of this voltage to other parts of the system, at a velocity which depends on the medium in which the voltage wave is travelling.

The initiation of voltage surges on overhead transmission lines is frequently caused by lightning discharges. The voltage may be induced in the line due to a lightning discharge in the vicinity without the discharge occurring directly to the line. A most severe voltage rise may be caused where the lightning discharge is direct to a line conductor; such direct strokes, however, are not common.

Voltage surges may also be initiated by switching. Such transient disturbances are due to the rapid redistribution of the energy associated with electric and magnetic fields. For example when the current in an inductive circuit is interrupted the energy stored in the magnetic field must be rapidly transferred to the associated electric field and will give rise to a sudden increase in voltage.

Fig. 16.1 shows the waveform of a typical surge voltage. This is, in effect, a graph of the build-up of voltage at a particular point to a base of time. The steepness of the wavefront is of great importance, since the steeper the wavefront the more rapid is the build-up of voltage at any point on the network, and the properties of insulators depend on the rate of rise of voltage. In most cases the build-up is comparatively rapid, being of the order of 1–5 μs.

Surge voltages are usually specified in terms of the rise time and the time to decay to half maximum value. For example, a 1/50 μs wave is one which reaches its maximum value in 1 μs and decays to

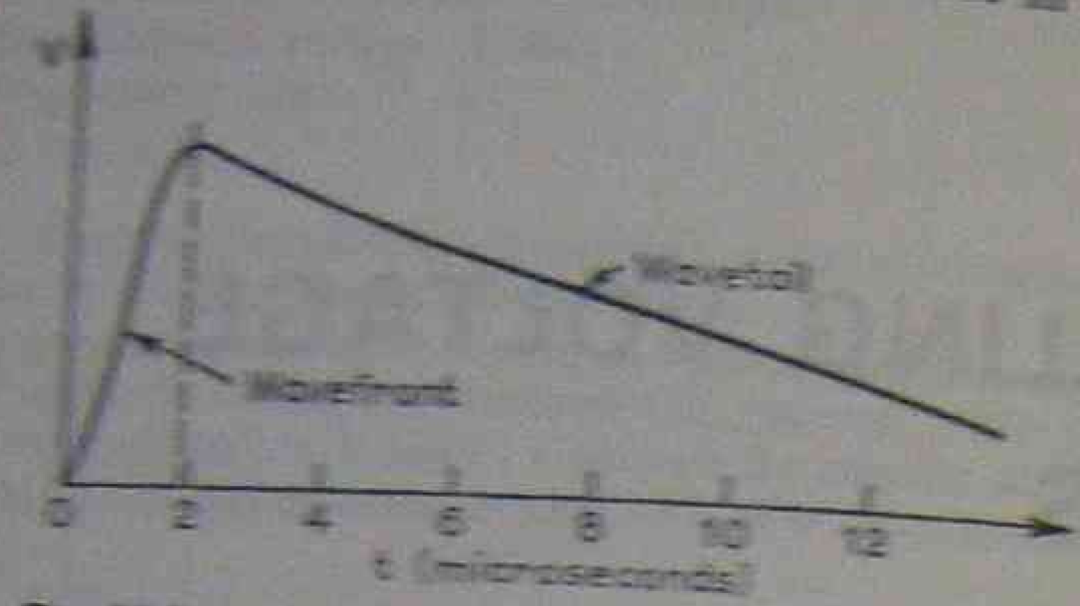

Fig. 16.1 WAVEFORM OF A VOLTAGE SURGE

half its maximum value in 50 μs. Impulse voltage tests are usually carried out with a wave of this shape.

16.1 Velocity of Propagation of a Surge

In the circuit of Fig. 16.2 the high-voltage source is assumed to give a constant high voltage E, which is applied to one end of the

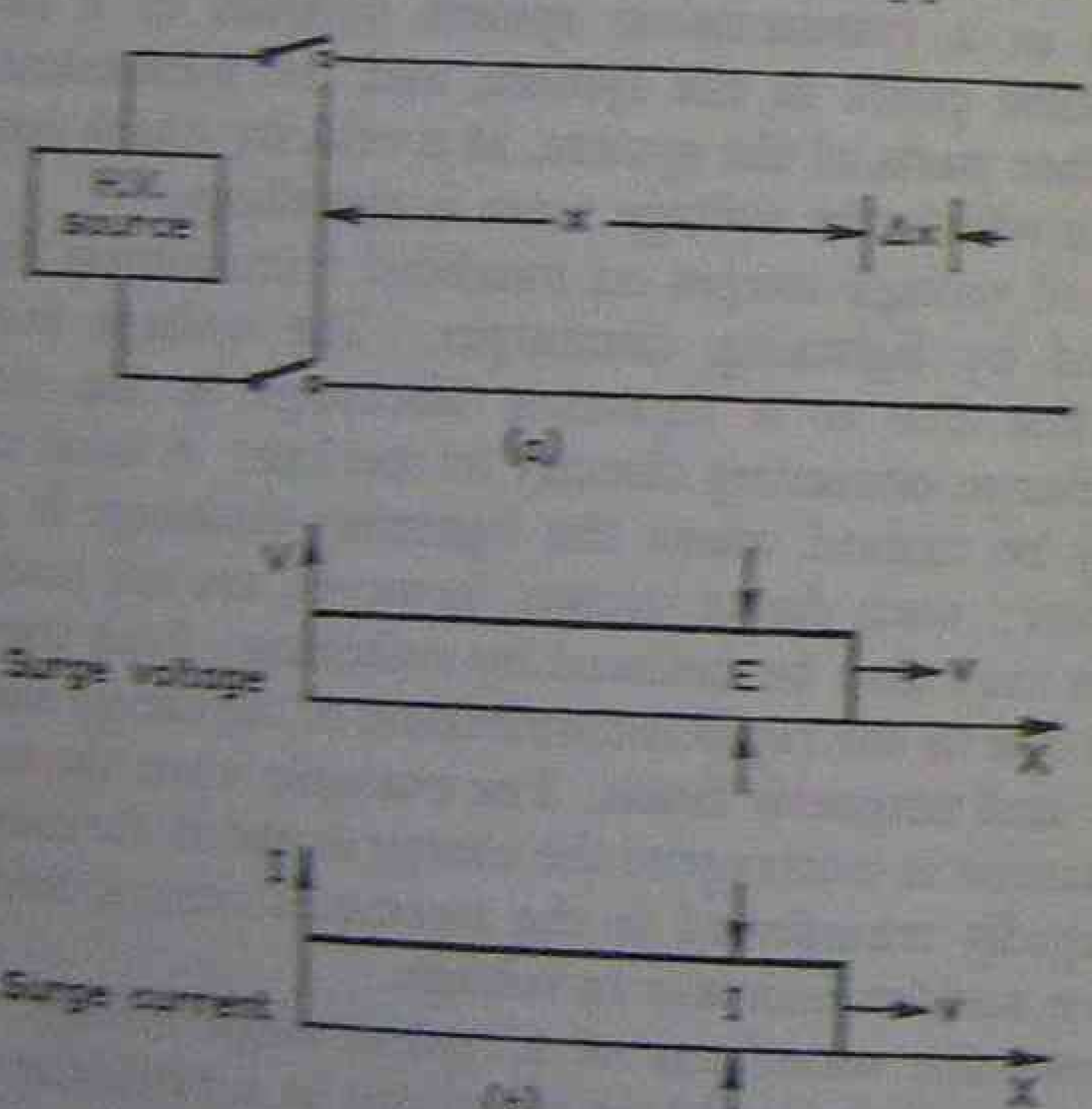

Fig. 16.2 CONSTANT VOLTAGE SURGE

line when the switch is closed. This will simulate a lightning stroke with a rapid rise of voltage and a long tail. It will also represent 50 Hz phenomena on short lines for reasons which will be discussed

later. The lines will be assumed loss-free, i.e. no conductor resistance or leakage conductance. This greatly simplifies the theory and errs on the right side, since the effect of losses is to reduce the size of a surge and the rate of rise of voltage. Real surges then will not be quite so severe as those calculated by the present theory.

Let C = Line capacitance per unit length (F/m)
L = Line inductance per unit length (H/m)

When the switch is closed the whole line will not immediately become charged to a voltage E, for raising the voltage of a length of line entails charging the capacitance of that length and this entails a current flow from the source. The current cannot immediately flow due to the inductance of the line and therefore the line voltage cannot immediately rise to the voltage E, though in time it will do so.

Suppose that at an instant t after the closing of the switch a length x of the line has become charged to voltage E, and that those parts of the line beyond a further distance Δx are not yet charged at all. The intermediate region of length Δx will be called the "disturbance" since it is only in this region that the voltage is changing—in front of the disturbance the voltage is zero; behind it the voltage is constant at E. The disturbance will move along the line from the switch with a uniform velocity since the line is uniform.

Let v be the velocity of the disturbance; i.e. the *surge velocity*, or *speed of propagation*.

Since the velocity is uniform equal lengths (v units) of the line will be charged up each second as the disturbance moves along the line. Therefore

$$\text{Charge required per second} = E \times \text{capacitance of length } v$$
$$= ECv$$

i.e.

$$\text{Charging current flowing along line, } I = ECv \tag{16.1}$$

I is called the *surge current*.

As the surge proceeds along the line a new length v will carry the current I each second, i.e. a length which has an inductance Lv has the current in it changed from zero to I in 1 sec.

$$\text{Potential required to increase current}$$
$$= \text{Inductance} \times \text{Rate of change of current}$$
$$= LvI \text{ volts}$$

The potential applied to increase the current is E since in front of the disturbance the potential is zero and behind it the potential is E. Therefore

$$E = Lvl \tag{16.2}$$

From eqns. (16.1) and (16.2),

$$\text{Surge velocity, } v = \frac{1}{\sqrt{(LC)}} \tag{16.3}$$

Consider, for example, the velocity of propagation in a concentric cable.

$$\text{Inductance/unit length} = \frac{\mu}{2\pi} \log_e \frac{b}{a} \text{ henrys/metre} \tag{7.29}$$

$$\text{Capacitance/unit length} = 2\pi \frac{\epsilon}{\log_e \frac{b}{a}} \text{ farads/metre} \tag{7.7}$$

where b and a are the sheath and core radii respectively, and μ and ϵ are the permeability and permittivity of the dielectric material respectively. The internal sheath linkages are not considered since a surge is a high-speed effect equivalent to a high-frequency effect and skin effect will reduce the internal linkages. If the resistance is zero, the depth of penetration is also necessarily zero.

$\mu = \mu_0$ if the dielectric is non-magnetic
$\epsilon = \epsilon_r \epsilon_0$ where ϵ_r is the relative permittivity

Therefore

$$v = \frac{1}{\sqrt{(LC)}} = \frac{1}{\sqrt{(\mu_0 \epsilon_0 \epsilon_r)}} \tag{16.4}$$

i.e. the velocity is independent of the size and spacing of the conductors. This applies to all configurations of lossless conductors, e.g. a core that is not concentric, or a twin-line system. Substituting numerical values for μ_0 and ϵ_0,

$$v = \frac{1}{\sqrt{\left(4\pi \times 10^{-7} \times \dfrac{\epsilon_r}{36\pi \times 10^9}\right)}} = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \text{ metres/second} \tag{16.5}$$

This is the velocity of electromagnetic waves in the medium concerned.

The surge velocity is seen to be extremely fast (3×10^8 m/s in air). Even in a cable with a relative permittivity of 9 (fairly high) the velocity is 10^8 m/s. If, say, 5 μs is the time of rise of the surge voltage (length of the wavefront), then the length of the disturbance, is

$5 \times 10^{-6} \times 10^8 = 500$ m, which is short for power transmission-line distances (not, of course, for high-frequency transmission-line distances). For power transmission lines, surges are often represented by a "vertical front block" as in Fig. 16.2(*b*) since the length of the disturbance is small.

In $\frac{1}{50}$th second a surge will travel $\frac{1}{50} \times 3 \times 10^8 = 6 \times 10^6$ m (3,730 miles). Thus in $\frac{1}{10}$th of a cycle, for a 50 Hz system, a surge will travel the whole length of a 600 km line (about 400 miles). In this period the voltage of the 50 Hz source will not have greatly changed, so that if a 50 Hz source is suddenly connected to a line the surge may be examined by the present theory taking the surge voltage as the instantaneous voltage when the switch is closed. Naturally the peak voltage should be considered, since this will be at the most dangerous instant at which the switch might be closed.

16.2 Surge Impedance

Dividing and simplifying eqns. (16.1) and (16.2),

$$\frac{E}{I} = \sqrt{\frac{L}{C}} \tag{16.6}$$

$\sqrt{(L/C)}$ is called the *surge impedance*, or *characteristic impedance*, Z_0, of a line. For a lossless line the surge impedance is evidently a pure resistance.

It should be noted that the surge current I, consequent on a voltage surge E, is related to the voltage surge by eqn. (16.6), i.e. by the properties of the line in which the surge travels.

The magnitude of the surge impedance for a particular conductor configuration may be determined from

$$Z_0 = \sqrt{\frac{L}{C}} \text{ ohms} \tag{16.7}$$

For an overhead power line the surge impedance is usually about 300 Ω; for a power cable it usually is about 50 Ω. The inductance per unit length of a line increases with the spacing of the conductors while the capacitance per unit length decreases as the spacing is increased. Thus, by eqn. (16.7), the surge impedance will increase with the spacing.

16.3 Power Input and Energy Storage

When the switch of Fig. 16.2(*a*) is closed, the high-voltage source maintains a potential E volts across the input to the line and supplies a current I amperes:

Power input to line $= EI$ watts

This energy input must become the energy stored in the line, for until the surge reaches the termination, there can be no output of energy from the line.

If v is the surge velocity, then in one second a length v stores electrostatic energy $\frac{1}{2}CvE^2$ and electromagnetic energy $\frac{1}{2}LvI^2$.

Energy input per second = Energy stored per second

i.e.

$$EI = \tfrac{1}{2}CvE^2 + \tfrac{1}{2}LvI^2 \qquad (16.8)$$

Now,

$$\tfrac{1}{2}CvE^2 = \tfrac{1}{2}E\frac{C}{\sqrt{(LC)}}E = \tfrac{1}{2}E\frac{E}{\sqrt{\frac{L}{C}}} = \tfrac{1}{2}EI$$

and

$$\tfrac{1}{2}LvI^2 = \tfrac{1}{2}I\frac{L}{\sqrt{(LC)}}I = \tfrac{1}{2}I\sqrt{\frac{L}{C}}\,I = \tfrac{1}{2}EI$$

Thus the electrostatic and electromagnetic stored energies are equal.

16.4 Terminations

Consider a surge, of voltage E_i, impinging on the termination of a transmission line; if the characteristic or surge impedance of the line is Z_0, then the surge current I_i in the transmission line is given by

$$I_i = \frac{E_i}{Z_0} \qquad (16.6a)$$

This surge impinging on the termination is called the *incident surge*; E_i is the *incident surge voltage* and I_i is the *incident surge current*.

Power conveyed to termination with incident surge $= E_iI_i$

Consider the particular case of a line terminated in a pure resistor equal to the surge impedance of the line. When the surge arrives the current in the resistor is E_i/Z_0.

$$\text{Power absorbed by resistor} = E_i \times \frac{E_i}{Z_0} = E_iI_i$$
$$= \text{Power transmitted by the surge}$$

In this case the surge power is exactly absorbed by the terminating impedance and there will be no further changes, i.e. the line will continue to be charged to potential E_i, and will continue to carry a current I_i.

If the terminating resistor has a resistance R, either higher or lower than the surge impedance Z_0, then changes in both voltage and current will occur. For instance if $R > Z_0$, then, on the arrival of the surge at the termination, the current through the terminating resistor will be E_i/R, which will be less than E_i/Z_0. The incident current on the line is in excess of the current which can be absorbed by the terminating resistor at the surge voltage. Since the current cannot instantaneously decrease due to the inductance of the line, the excess current will increase the charge on the capacitance at the end of the line. This increases the voltage at the termination to a value higher than the incident voltage E_i.

Let the voltage at the termination rise to E_T. Then

$$\text{Current through terminating resistor} = \frac{E_T}{R} = I_T$$

Excess voltage appearing at termination $= E_T - E_i$

Excess current at termination $= I_T - I_i$

The whole line is now charged to a potential E_i, but there has been a further rise of potential to E_T at the end of the line, i.e. an excess potential suddenly appears at the termination of the line. This is a similar condition to the initial closure of the switch which produced the incident surge, and so, in a similar manner, a surge will now travel from the termination back along the line. This is called the *reflected surge*.

Reflected surge voltage, $E_r = E_T - E_i$

Therefore

$$E_i + E_r = E_T \qquad (16.9)$$

Reflected surge current, $I_r = I_T - I_i$

Thus

$$I_i + I_r = I_T \qquad (16.10)$$

where $I_i = E_i/Z_0$, $I_r = -E_r/Z_0$, $I_T = E_T/R$.

The positive current direction is the direction of the current in the incident surge. Since the reflected current has the opposite direction the minus sign is necessary.

Substituting in eqn. (16.10),

$$\frac{E_i}{Z_0} - \frac{E_r}{Z_0} = \frac{E_T}{R}$$

Substituting for E_T from eqn. (16.9),

$$\frac{E_i}{Z_0} - \frac{E_r}{Z_0} = \frac{E_i + E_r}{R}$$

Therefore

$$RE_i - RE_r = Z_0E_i + Z_0E_r$$

and

$$E_r = \frac{R - Z_0}{R + Z_0} E_i \tag{16.11}$$

Substituting for E_r in eqn. (16.9),

$$E_i + \frac{R - Z_0}{R + Z_0} E_i = E_T$$

Thus

$$E_T = \frac{2R}{R + Z_0} E_i \tag{16.12}$$

Also

$$I_r = -\frac{E_r}{Z_0} = -\frac{1}{Z_0}\frac{R - Z_0}{R + Z_0} E_i \tag{16.13}$$

$$I_T = \frac{E_T}{R} = \frac{2}{R + Z_0} E_i \tag{16.14}$$

Graphs of the voltage and current distributions along the line are shown in Fig. 16.3 for instants before and after the incident surge reaches the termination. Since $R > Z_0$, the current at the termination is reduced and the reflected current surge is negative.

If the terminating resistance R is less than Z_0, then on the arrival of the incident surge the current, E_i/R, in the terminating resistor will be greater than the surge current $I_i = E_i/Z_0$. Due to the line inductance the line current cannot suddenly increase. The capacitance at the end of the line is then discharged, so reducing the voltage at the termination. The reduction of voltage at the termination is equivalent to the sudden application of a negative voltage surge at the termination which travels back along the line. This is the *reflected surge*.

Since the reflected voltage is negative the reflected current will be a negative current in the negative direction, i.e. equivalent to a positive current.

All the previous equations apply without change. The voltage and current distributions are shown in Fig. 16.4.

For an open-circuited line, $R \to \infty$, and hence the terminating current tends to zero. Therefore, by eqn. (16.11),

$$\text{Reflected voltage, } E_r = \frac{\infty - Z_0}{\infty + Z_0} E_i = E_i$$

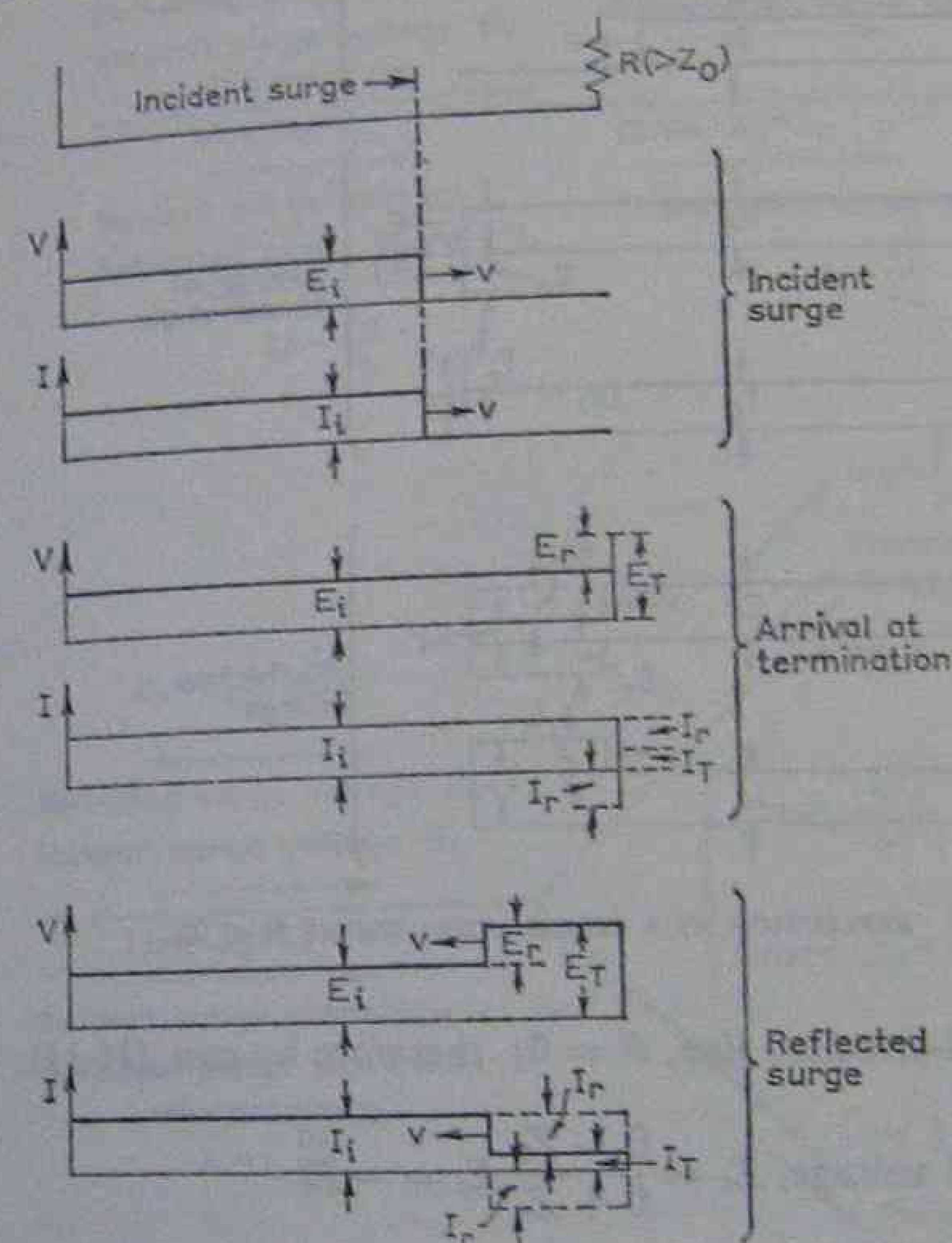

Fig. 16.3 REFLECTION AT A TERMINATION WHERE $R > Z_0$

and

$$\text{Termination voltage, } E_T = E_i + E_i = 2E_i$$

$$\text{Reflected current, } I_r = -\frac{E_r}{Z_0} = -\frac{E_i}{Z_0} = -I_i$$

Thus

$$\text{Termination current, } I_T = I_r + I_i = 0$$

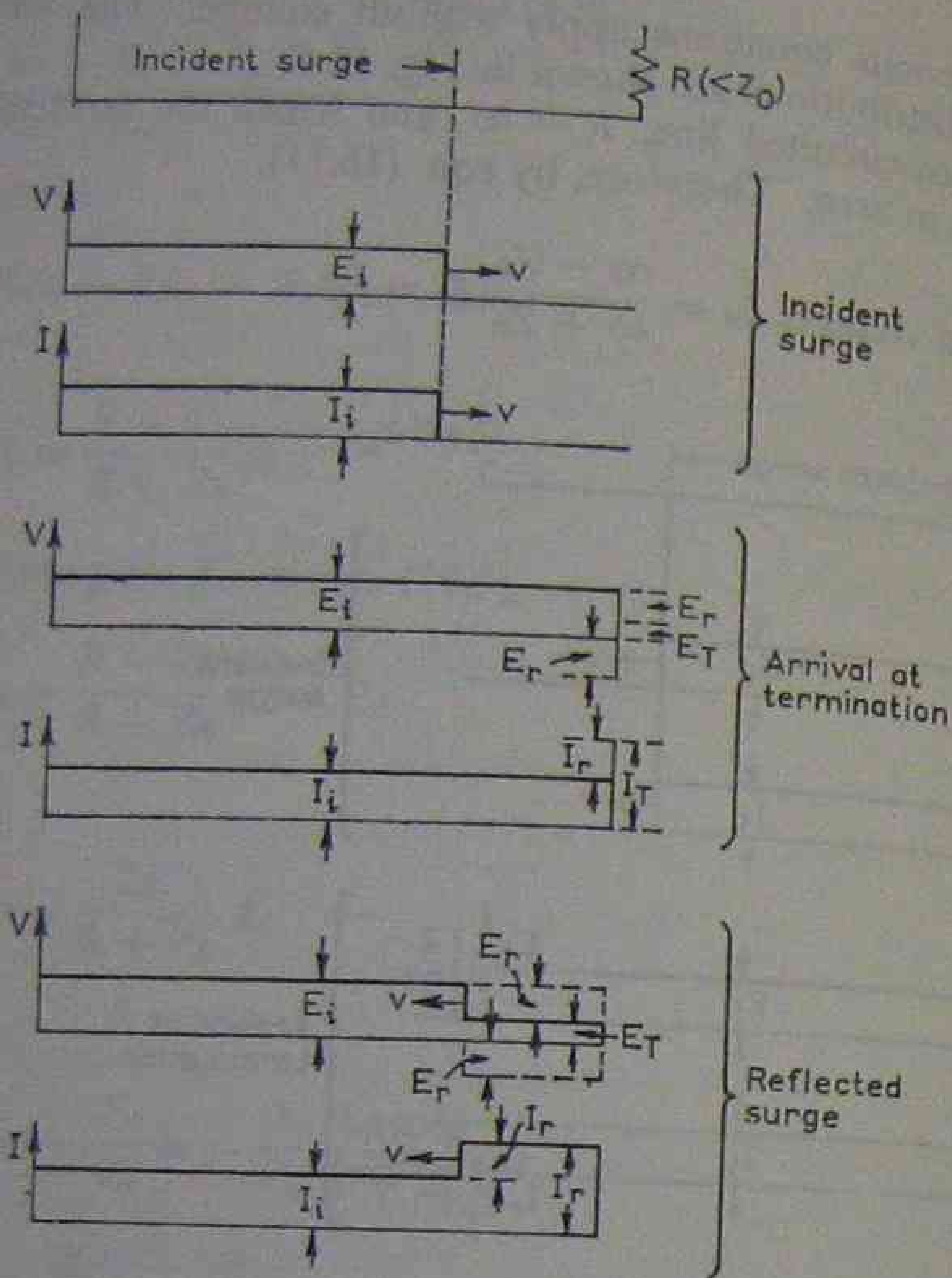

Fig. 16.4 REFLECTION AT A TERMINATION WHERE $R < Z_0$

For a short-circuited line, $R = 0$; therefore by eqn. (16.11),

$$\text{Reflected voltage, } E_r = \frac{0 - Z_0}{0 + Z_0} E_i = -E_i$$

and

$$\text{Termination voltage, } E_T = E_i - E_i = 0$$

$$\text{Reflected current, } I_r = -\frac{E_r}{Z_0} = \frac{E_i}{Z_0} = I_i$$

Thus

$$\text{Termination current, } I_T = I_i + I_i = 2I_i$$

Obviously there can be no voltage at the short-circuit. It should be noted that the terminating voltage and current are often called the *transmitted voltage* and *current*.

To summarize, if $R > Z_0$ the voltage surge is reflected unchanged in sign, while the current surge is reflected with the opposite sign. If $R < Z_0$ the sign of the reflected voltage surge is reversed, while that of the current surge is unchanged.

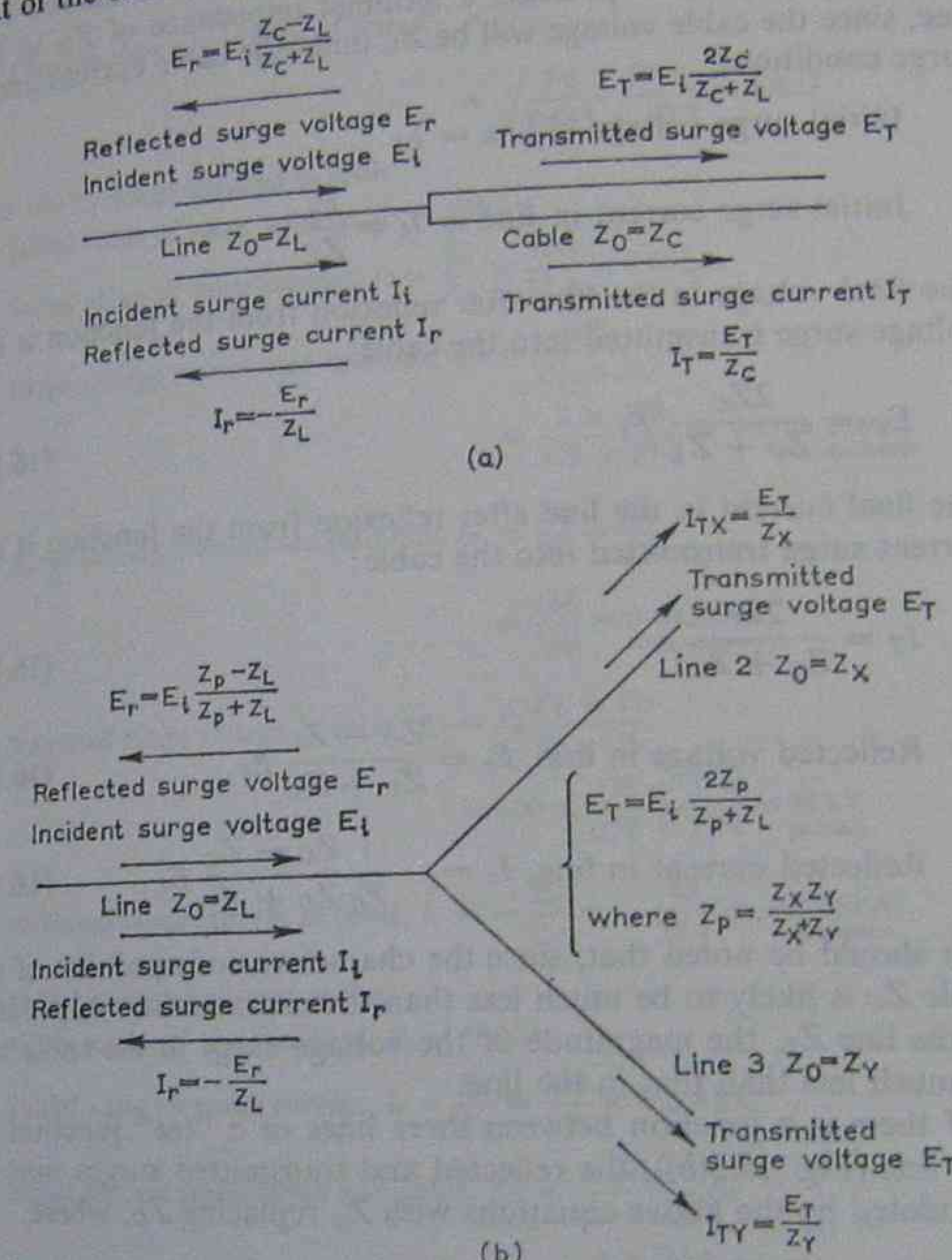

Fig. 16.5 SURGES AT LINE JUNCTIONS

16.5 Junctions of Lines having Different Characteristic Impedances

If a voltage surge travels along a line towards a junction where the characteristic impedance of the line changes, reflexion will take place at the junction and the surge transmitted into the section beyond the junction will be modified in value (Fig. 16.5(*a*)).

Consider the case of a junction between an overhead line and a cable. Let the characteristic impedance of the line be Z_L and that of the cable be Z_C. Suppose a surge voltage E_i is initiated in the line and travels down the line to the junction between the line and the cable. The cable presents a terminal impedance of Z_C to the line, since the cable voltage will be Z_C times the cable current under surge conditions.

$$\text{Initial surge voltage in line} = E_i$$

$$\text{Initial surge current in line} = I_i = \frac{E_i}{Z_L}$$

The final voltage in the line after reflexion from the junction is the voltage surge transmitted into the cable:

$$E_T = \frac{2Z_C}{Z_C + Z_L} E_i \tag{16.12}$$

The final current in the line after reflexion from the junction is the current surge transmitted into the cable:

$$I_T = \frac{2E_i}{Z_C + Z_L} \tag{16.14}$$

$$\text{Reflected voltage in line, } E_r = \frac{Z_C - Z_L}{Z_C + Z_L} E_i \tag{16.11}$$

$$\text{Reflected current in line, } I_r = -\frac{1}{Z_0}\frac{Z_C - Z_L}{Z_C + Z_L} E_i \tag{16.13}$$

It should be noted that, since the characteristic impedance of the cable Z_C is likely to be much less than the characteristic impedance of the line Z_L, the magnitude of the voltage surge in the cable will be much less than that in the line.

If there is a junction between three lines or a "tee" junction on one line (Fig. 16.5(*b*)), the reflected and transmitted surges may be calculated by the above equations with Z_p replacing Z_C, where

$$Z_p = \frac{Z_X Z_Y}{Z_X + Z_Y}$$

EXAMPLE 16.1 An underground cable having an inductance of 0·3 mH/km and a capacitance of 0·4 μF/km is connected in series with an overhead line having an inductance of 2·0 mH/km and a capacitance of 0·014 μF/km.

Calculate the values of the transmitted and reflected waves of voltage and current at the junction, due to a voltage surge of 100 kV travelling to the junction (*a*) along the cable, and (*b*) along the line. (*L.U.*)

Characteristic impedance of cable, $Z_C = \sqrt{\dfrac{L}{C}}$

$$= \sqrt{\frac{0{\cdot}3 \times 10^{-3}}{0{\cdot}4 \times 10^{-6}}} = 27{\cdot}4\,\Omega$$

Characteristic impedance of line, $Z_L = \sqrt{\dfrac{L}{C}}$

$$= \sqrt{\frac{2{\cdot}0 \times 10^{-3}}{0{\cdot}014 \times 10^{-6}}} = 378\,\Omega$$

(*a*) 100 kV surge initiated in cable

Initial value of surge voltage, $E_i = 100\,\text{kV}$

Initial value of surge current, $I_i = \dfrac{E_i}{Z_C} = \dfrac{100}{27{\cdot}4} = 3{\cdot}65\,\text{kA}$

Surge voltage transmitted into line, $E_T = \dfrac{2Z_L}{Z_L + Z_C} E_i$

$$= \frac{2 \times 378}{378 + 27{\cdot}4} \times 100 = \underline{\underline{186\,\text{kV}}}$$

Surge current transmitted into line, $I_T = \dfrac{E_T}{Z_T} = \dfrac{E_T}{Z_L}$

$$= \frac{186}{378} = \underline{\underline{0{\cdot}492\,\text{kA}}}$$

Reflected surge voltage in cable, $E_r = E_i \dfrac{Z_L - Z_C}{Z_L + Z_C}$

$$= 100 \times \frac{378 - 27{\cdot}4}{378 + 27{\cdot}4} = \underline{\underline{86\,\text{kV}}}$$

Reflected surge current in cable, $I_r = -\dfrac{E_r}{Z_C} = -\dfrac{86}{27{\cdot}4} = \underline{\underline{-3{\cdot}16\,\text{kA}}}$

(*b*) 100 kV surge initiated in line

Initial value of surge voltage, $E_i = 100\,\text{kV}$

Initial value of surge current, $I_i = \dfrac{E_i}{Z_L} = \dfrac{100}{378} = 0{\cdot}264\,\text{kA}$

Surge voltage transmitted into cable, $E_T = \dfrac{2Z_C}{Z_C + Z_L} E_i$

$$= \frac{2 \times 27{\cdot}4}{27{\cdot}4 + 378} = 100 = \underline{\underline{13{\cdot}5\,\text{kV}}}$$

Surge current transmitted into cable, $I_T = \dfrac{E_T}{Z_C} = \dfrac{13{\cdot}5}{27{\cdot}4} = \underline{\underline{0{\cdot}494\,\text{kA}}}$

Reflected surge voltage in line, $E_r = E_T - E_i = \underline{\underline{-86{\cdot}5\,\text{kV}}}$

Reflected surge current in line $= -\dfrac{E_r}{Z_L} = +\dfrac{86{\cdot}5}{378} = \underline{\underline{0{\cdot}23\,\text{kA}}}$

EXAMPLE 16.2 A single-phase overhead line is 50 km long and has a surge impedance of 300 Ω. The line has a circuit-breaker at the input end. A dead

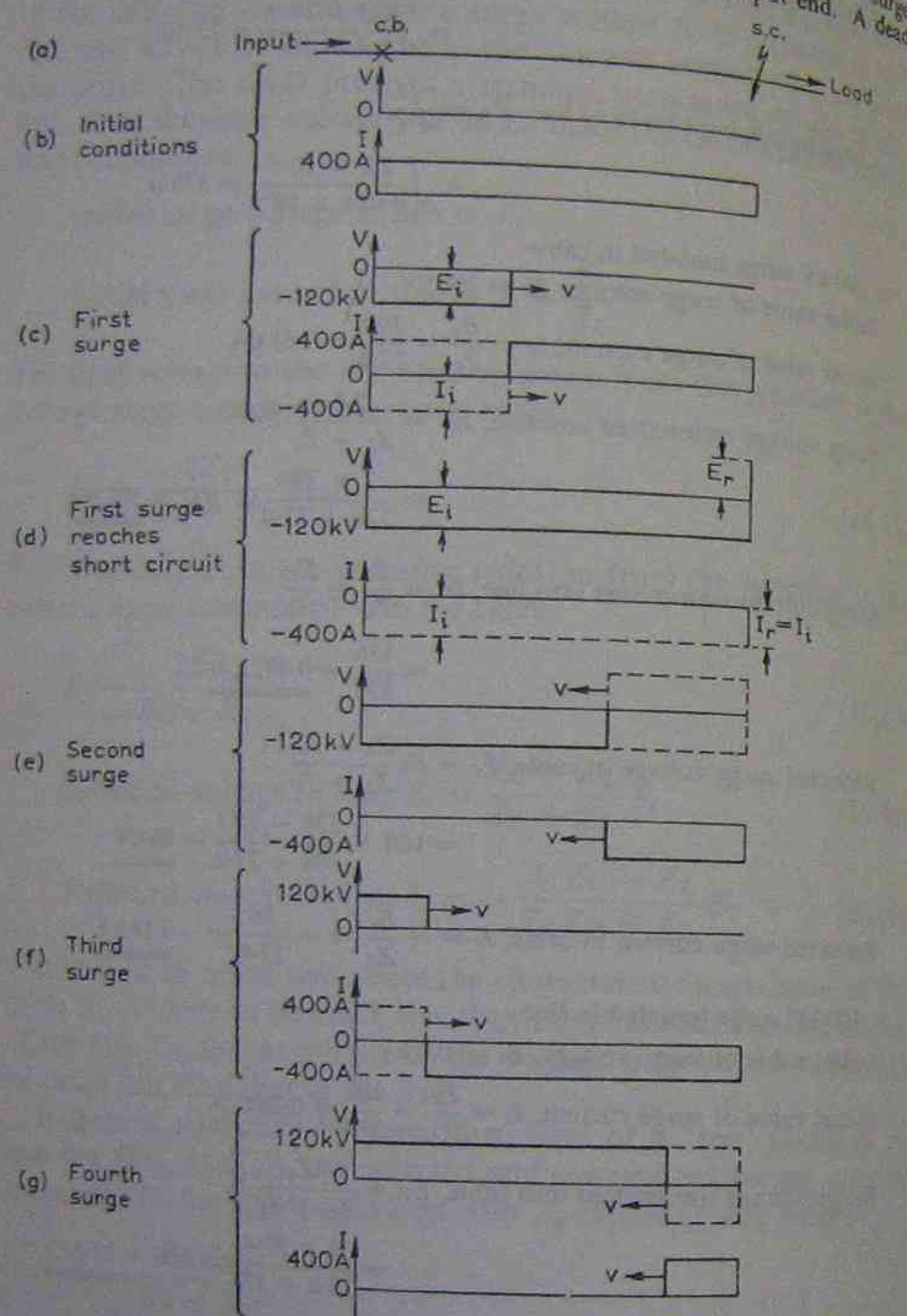

Fig. 16.6

short-circuit occurs at the terminating end and the circuit-breaker suddenly interrupts the short-circuit current when it has an instantaneous value of 400 A. Describe the surge phenomena which will occur in the line.

The line is shown in Fig. 16.6(*a*). Under the initial short-circuit conditions the voltage along the line may be taken as zero and the current as uniform at 400 A.

When the circuit-breaker opens, the current of 400 A cannot immediately cease due to the line inductance; thus the line capacitance at the circuit-breaker end must become negatively charged by the instantaneous continuation of the 400 A current. A negative voltage then arises at the circuit-breaker end while the rest of the line is uncharged; hence a negative voltage surge travels from the circuit-breaker end. The current associated with this surge will be −400 A, since there can be no resultant current at the open-circuited circuit-breaker end. Therefore

$$\text{Voltage of surge} = -400 \times Z_0 = -400 \times 300\,\text{V} = -120\,\text{kV}$$

i.e. a surge of −120 kV and −400 A travels down the line to the short-circuit termination.

At the short-circuit the termination voltage is necessarily zero; thus a reflected surge arises with a voltage of +120 kV and a current of −400 A. (At a short-circuit the surge voltage is reflected with change of sign and the surge current is reflected without change of sign, see Section 16.4.) When the reflected surge reaches the open-circuited circuit-breaker end it will in turn be reflected back down the line. The third surge voltage will be +120 kV and the third surge current will be +400 A. (At an open-circuit the surge voltage is reflected without change of sign and the surge current is reflected with change of sign; see Section 16.4.) These reflexions obey the rules: (i) the resultant voltage at a short-circuit must be zero, and (ii) the resultant current at an open-circuit must be zero.

The fourth surge stage is shown in Fig. 16.6(*g*).

It will be seen that after the fourth stage the resultant voltage and current are the same as the initial voltage and current. The fifth surge would then be the same as the first surge. In all real lines the losses will continually be reducing the magnitude of the surges so that the later surges are much smaller than the first ones.

Since this line is an overhead line, the surge velocity will approach 3×10^8 m/s. Thus the time required for a surge to travel the length of the line will be $50{,}000/(3 \times 10^8)$ s, i.e. 0·167 ms.

16.6 Surges of Short Duration

The previous theory has been developed on the assumption of a sudden rise of voltage followed by the steady application of the same voltage. In practice the sudden rise of voltage is usually followed by a slow fall of voltage back to normal. If the surge front reaches the far end of the line before the input voltage falls appreciably, then the previous theory gives a good representation of the surge conditions on the line. If the sudden rise of voltage is followed by a rapid fall so that the voltage surge becomes a voltage pulse, the previous theory does not give a clear representation of the conditions on the line. The theory is, however, still applicable and a further simple assumption gives a good representation of the actual conditions.

Fig. 16.7(*a*) shows a pulse or short-duration-surge source connected to a line. The output voltage of the source is shown in Fig. 16.7(*b*).

This output voltage may be considered as (i) a positive voltage surge of infinite duration followed at a discrete interval by (ii) a negative voltage surge of infinite duration and the same magnitude. The negative surge will then cancel the "tail" of the positive surge. Since both the positive and negative long-duration surges will obey the previous equations, the pulse voltage will also obey them.

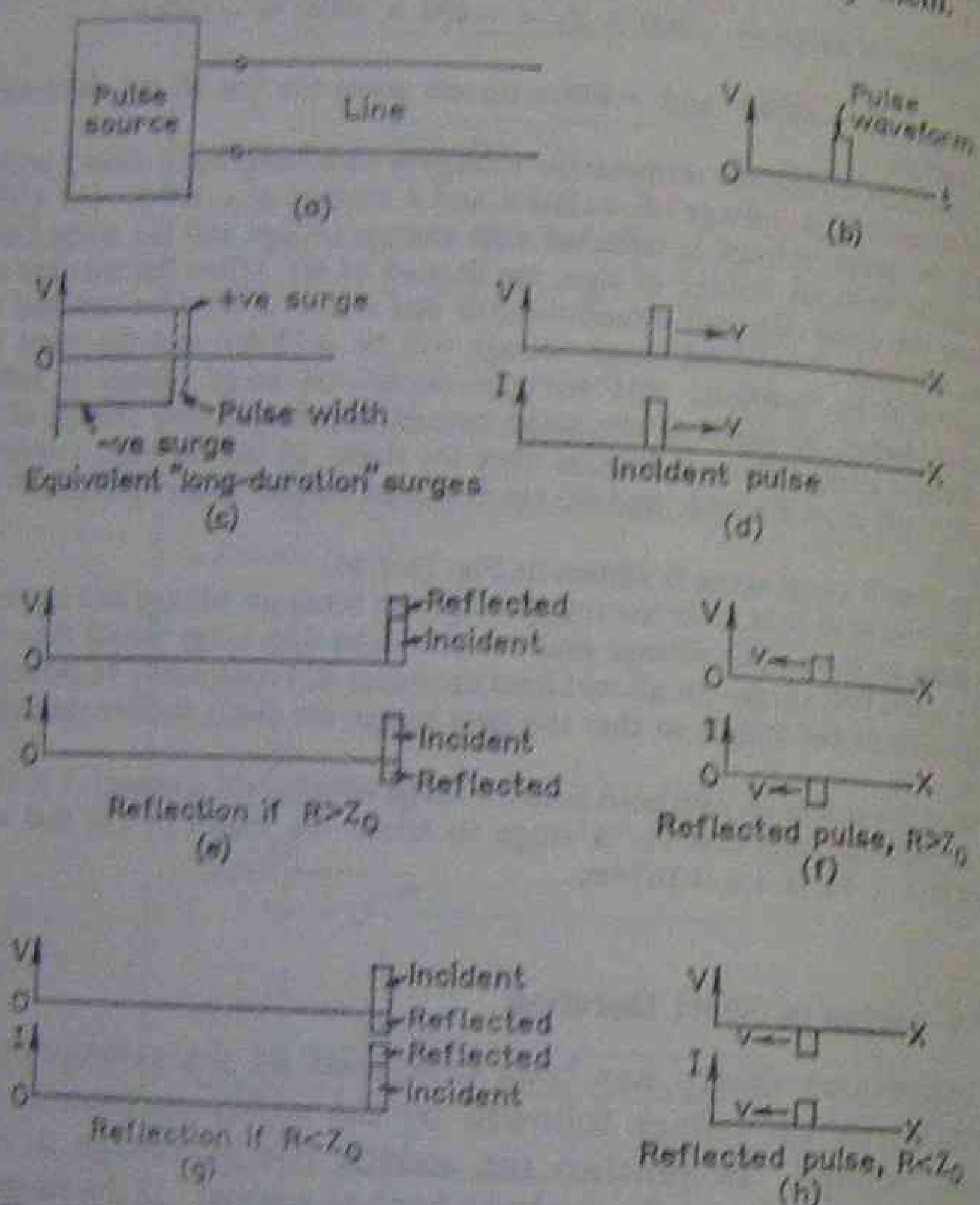

Fig. 16.7. SURGES OF SHORT DURATION

Incident and reflected pulses are shown in Figs. 16.7(*d*)-(*h*). Losses along the line will tend to decrease the pulse magnitude and to broaden the pulse front and pulse tail.

EXAMPLE 16.3 A short-duration pulse of magnitude 10 kV travels along a very long line of characteristic impedance 300 Ω. The line is joined to a similar very long line by a cable of characteristic impedance 30 Ω and with a relative permittivity of 4. If the cable is 1·5 km long, calculate the magnitude of the first and second pulses entering the second line. What is the time interval between the two pulses?

$$\text{Magnitude of pulse transmitted into cable} = 10 \times \frac{2 \times 30}{30 + 300} \qquad (16.12)$$

$$= 1{\cdot}82\,\text{kV}$$

Part of the input pulse will be reflected back along the first very long line and thus conveyed away from the cable. Assuming no losses in the cable, the pulse of 1·82 kV will be incident on the junction of the cable with the second length of line.

Magnitude of pulse transmitted into second line from cable

$$= 1{\cdot}82 \times \frac{2 \times 300}{300 + 30} = \underline{3{\cdot}31\,\text{kV}}$$

This is the first pulse to travel along the second length of line.

Magnitude of pulse reflected back into cable from junction with second length of line

$$= 1{\cdot}82 \times \frac{300 - 30}{300 + 30} = 1{\cdot}49\,\text{kV} \qquad (16.11)$$

This reflected pulse will travel back along the cable to the junction with the first length of line. Part of the reflected pulse will be transmitted into the first length of line and part of it will be reflected and will again pass down the cable toward the second length of line.

Magnitude of pulse reflected back into cable from junction with first length of line

$$= 1{\cdot}49 \times \frac{300 - 30}{300 + 30} = 1{\cdot}22\,\text{kV} \qquad (16.11)$$

This pulse will form a second pulse incident on the junction of the cable and the second length of line. Therefore

Magnitude of second pulse transmitted into second line

$$= 1{\cdot}22 \times \frac{2 \times 300}{300 + 30} = \underline{2{\cdot}22\,\text{kV}}$$

The time interval between the two pulses will be the time required for a pulse to travel first back and then forward along the length of cable, i.e. a total distance of 3 km in the cable.

$$\text{Velocity of pulse in cable} = \frac{3 \times 10^8}{\sqrt{4}} = 1{\cdot}5 \times 10^8\,\text{m/s}$$

Therefore

$$\text{Time interval} = \frac{3 \times 10^3}{1{\cdot}5 \times 10^8} = \underline{2 \times 10^{-5}\,\text{s}}$$

Note. The surge pulses in the second line were much smaller than those in the first. Equipment in stations at the ends of overhead lines is sometimes protected from overvoltage surges by bringing the overhead lines through a short length of cable before reaching the station.

16.7 Mitigation of High-voltage Surges

High-voltage surges are mainly due to either (*a*) lightning discharges, or (*b*) switching. The voltages set up by lightning discharges are reduced by stringing one or two earth wires above the main conductors. The voltages set up by switching are reduced by using *resistance switching*.

A direct lightning stroke to a line causes enormous voltages and there is little possibility of preventing these. Fortunately direct lightning strokes are rare. It is more common for high-voltage surges to be caused by the charge induced on the conductors of an overhead line when a charged cloud passes over or near to the line. The charge induced on the conductors will have the opposite polarity to that in the cloud. If the cloud passes slowly away, the charges induced on the conductors will gradually flow to earth and no disturbance will be caused. If, however, the cloud is suddenly discharged by a lightning stroke to earth or another cloud, then the induced charge on the line will be suddenly released and a surge voltage will travel along the line in either direction.

The charge induced on an overhead transmission line system by a charged cloud is mainly concentrated in the uppermost conductor, since the other conductors are to some extent electrostatically shielded by the uppermost one. If the uppermost conductor is made an earth wire and not one of the system conductors, then the charges induced on the system conductors are considerably reduced. This reduces the surges in the system conductors. It also affords some mitigation of the effects of a direct lightning stroke.

Where an earth wire is present the resistance of the tower footings must be kept low or *back flashover* from the earth wire may occur. For example, in the extreme case of the resistance to earth of the tower footing being infinite, any surge voltage wavefront reaching the tower base is doubled and reflected back to the earth wire. Eventual build-up of earth-wire potential to a value well above that of the system conductors may result in a discharge from the earth wire to the system conductors.

Transient currents and voltages naturally occur with most switching operations. Generally switching-in and disconnecting can be performed without dangerous disturbances arising. The interruption of a high short-circuit current by an efficient circuit-breaker does, however, tend to give high-voltage surges. These can be mitigated by arranging that the circuit-breaker will be opened in stages. During the stages one or more resistance sections carry the current which is being interrupted and part, at least, of the energy stored in the line inductance is dissipated in the switch resistors.

16.8 Protection of Insulation

The insulators which support an overhead line and the insulation of cables, switches or transformers will, under some surge conditions, have voltages impressed on them which are greater than the breakdown strength of the insulators or insulation. To prevent the breakdown of these costly units and to prevent the interruption of the supply which would result from their breakdown, the insulation is usually protected by an air-gap so arranged that the surge voltage will produce breakdown of the air-gap rather than of the insulation. A string of insulators for an overhead line, or the bushing of a transformer, has frequently a rod gap across it (Fig. 16.8), so that a

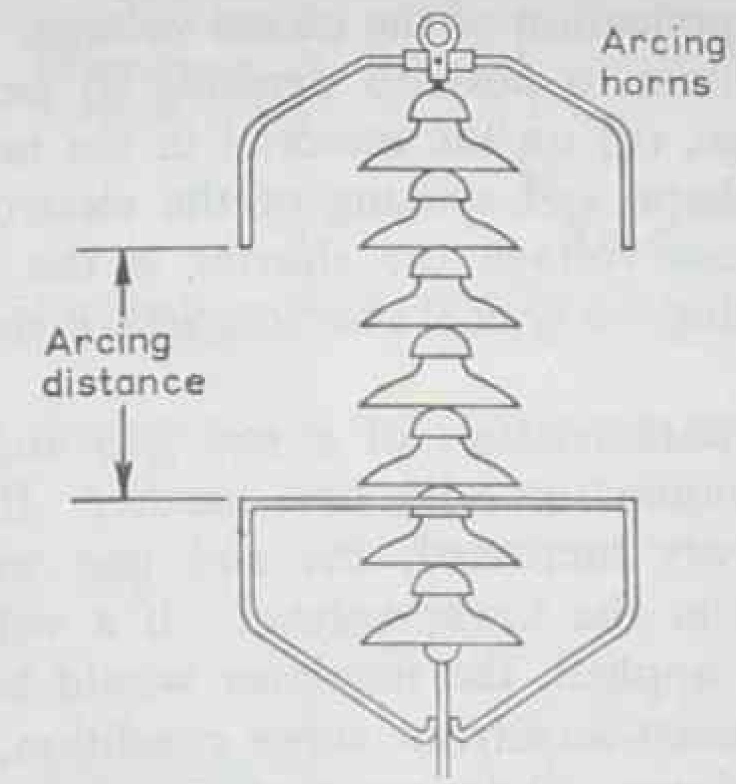

Fig. 16.8 ROD GAP PROTECTING AN INSULATOR STRING

spark or an arc will jump across the rod gap rather than down the insulator or the bushing.

Alternatively a metal ring concentric with the insulator string and about level with the third insulator shed may be used as the lower electrode in place of a rod electrode.

When setting the rod gap two factors must be taken into account: (*a*) impulse ratio and (*b*) time factor.

$$\text{Impulse ratio} = \frac{\text{Breakdown voltage under surge conditions}}{\text{Breakdown voltage under low-frequency conditions}}$$

It is found that the breakdown voltage under surge, i.e. rapidly changing or high-frequency conditions, is often higher than the breakdown voltage under steady or low-frequency conditions. The *impulse ratio* is a measure of this difference. Supposing the breakdown voltage of a string of insulators is, say, 300 kV at 50 Hz and that the string is protected by a rod gap with a breakdown voltage of, say, 200 kV at 50 Hz. If the impulse ratio for the insulators is, say, 1·3, then the surge breakdown voltage for the insulators will be 390 kV; and if the impulse ratio for the rod gap is, say, 2·1, then the surge breakdown voltage for the rod gap will be 420 kV. The rod gap does not then protect the insulators under surge conditions. Either the impulse ratio for the rod gap must be improved or the 50 Hz setting for the rod gap must be reduced. The impulse ratio is found to depend on the geometry of the air-gap. A sphere gap, with relatively close spacing, has an impulse ratio of unity—a needle gap may have an impulse ratio of between 2 and 3.

Time Factor. The breakdown of insulation or an air-gap does not occur instantaneously on the application of the excess voltage. The time for the complete breakdown to develop depends (i) on the magnitude of the excess voltage, (ii) on the material in the breakdown path, and (iii) on the shape and spacing of the electrodes. Naturally the greater the excess voltage the shorter is the time required for breakdown to develop—a typical characteristic is shown in Fig. 16.9(*a*).

Fig. 16.9(*b*) compares the characteristics of a rod gap and an insulator which are used in conjunction with one another. If the voltage across them were slowly increased, the rod gap would correctly break down first, i.e. at the lower voltage. If a voltage greater than V_c were suddenly applied, the insulator would break down first and thus, under a steep-wavefront surge condition, the rod gap does not protect the insulator. The correct relative characteristics for a rod gap to protect the insulation for all surge voltages is shown in Fig. 16.9(*c*).

The time delay is relatively short for sphere gaps and relatively long for needle gaps.

16.9 Surge Diverters

Rather than permit a surge to impinge on the terminal apparatus it is advantageous to eliminate the surge if possible. The elimination may be carried out in two ways, either (*a*) the surge may be diverted to earth, i.e. short-circuited, or (*b*) the surge energy may be absorbed. The latter method is not now used.

Modern surge diverters consist essentially of elements having a non-linear volt/ampere characteristic, and made of a ceramic material consisting of silicon carbide bonded with clay.

To protect plant successfully against high-voltage travelling waves the surge diverter must operate, as far as possible, simultaneously with the incidence of the surge. Since the surge is travelling at

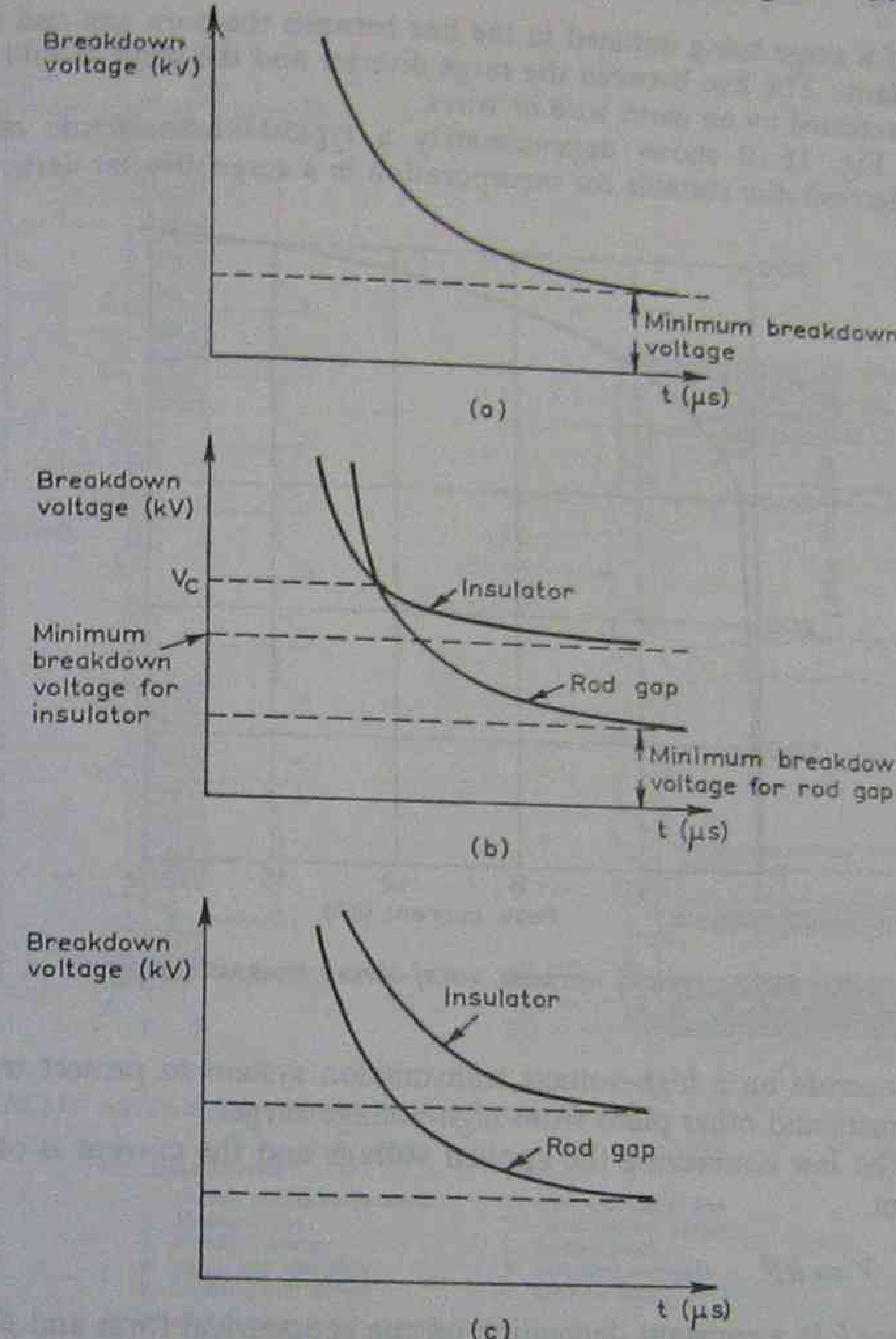

Fig. 16.9 BREAKDOWN-VOLTAGE/TIME CHARACTERISTICS

(*a*) Typical rod gap characteristic
(*b*) Unsuitable combination of insulator and rod gap
(*c*) Correct relative characteristics for insulator and rod gap protection

approximately 3×10^8 m/s a short delay will permit the surge to pass the diverter and be transmitted into the plant which the diverter is intended to protect. Moreover, the surge diverter should be placed as close as possible to the plant to be protected to obviate the risk of a surge being initiated in the line between the horn gap and the plant. The line between the surge diverter and the plant should be protected by an earth wire or wires.

Fig. 16.10 shows approximately a typical characteristic of a Metrosil disc suitable for incorporation in a surge diverter designed

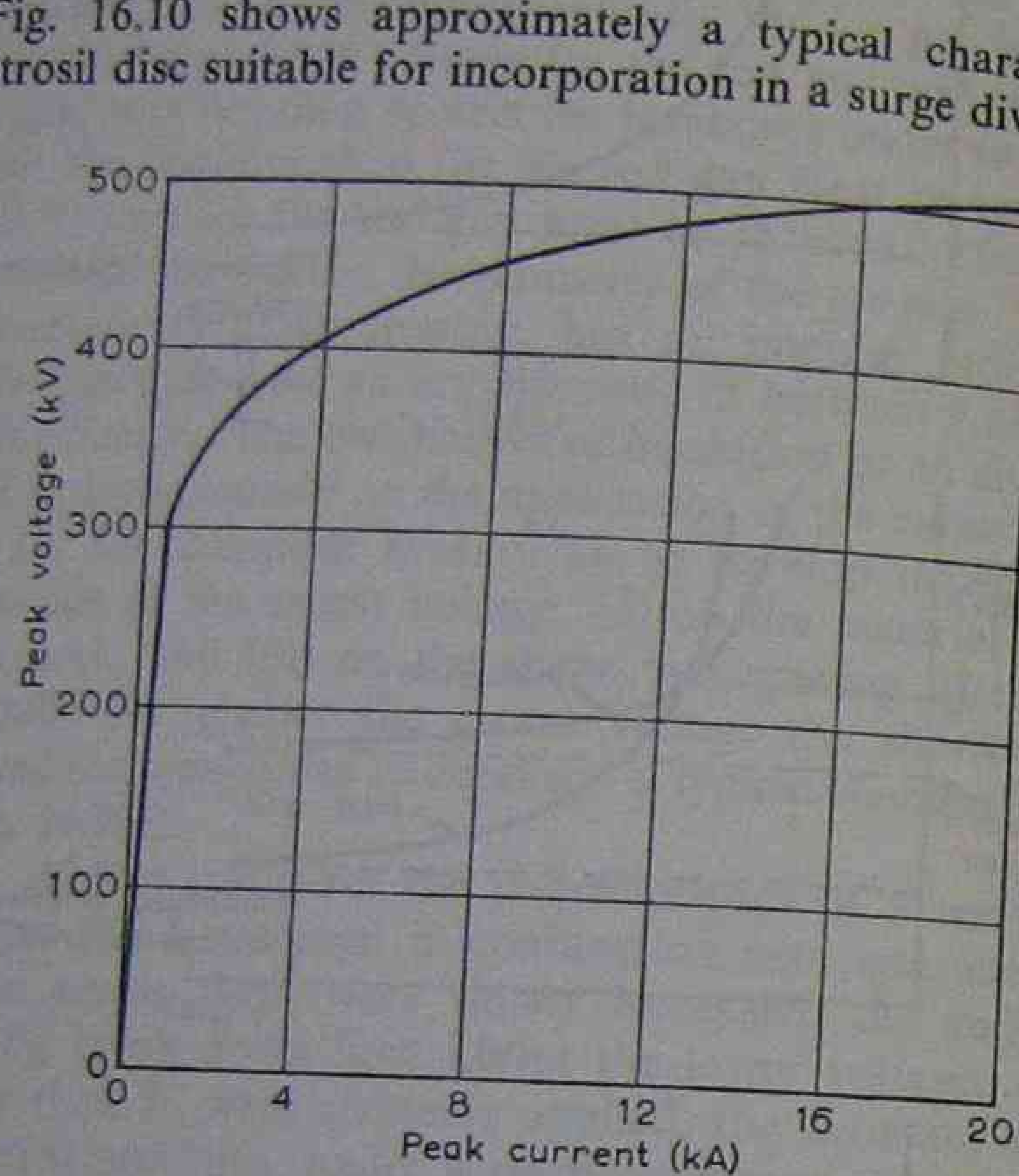

Fig. 16.10 TYPICAL METROSIL VOLT/AMPERE CHARACTERISTIC

to operate on a high-voltage transmission system to protect transformers and other plant from high-voltage surges.

The law connecting the applied voltage and the current is of the form

$$V = kI^{\beta}$$

where k is a constant depending on the geometrical form and β is a constant depending on the composition and treatment of the substance. Ideally β should be zero, so that whatever the value of the surge current the voltage would be constant. In practice values for β of the order of 0·2 are achieved.

The principle of operation is that a stack of Metrosil discs is connected between line and earth close to the transformer (or other plant) to be protected. Because of the nature of the volt/ampere characteristic, at normal voltage the diverter passes only a very small current to earth, but when a high over-voltage occurs the resistance

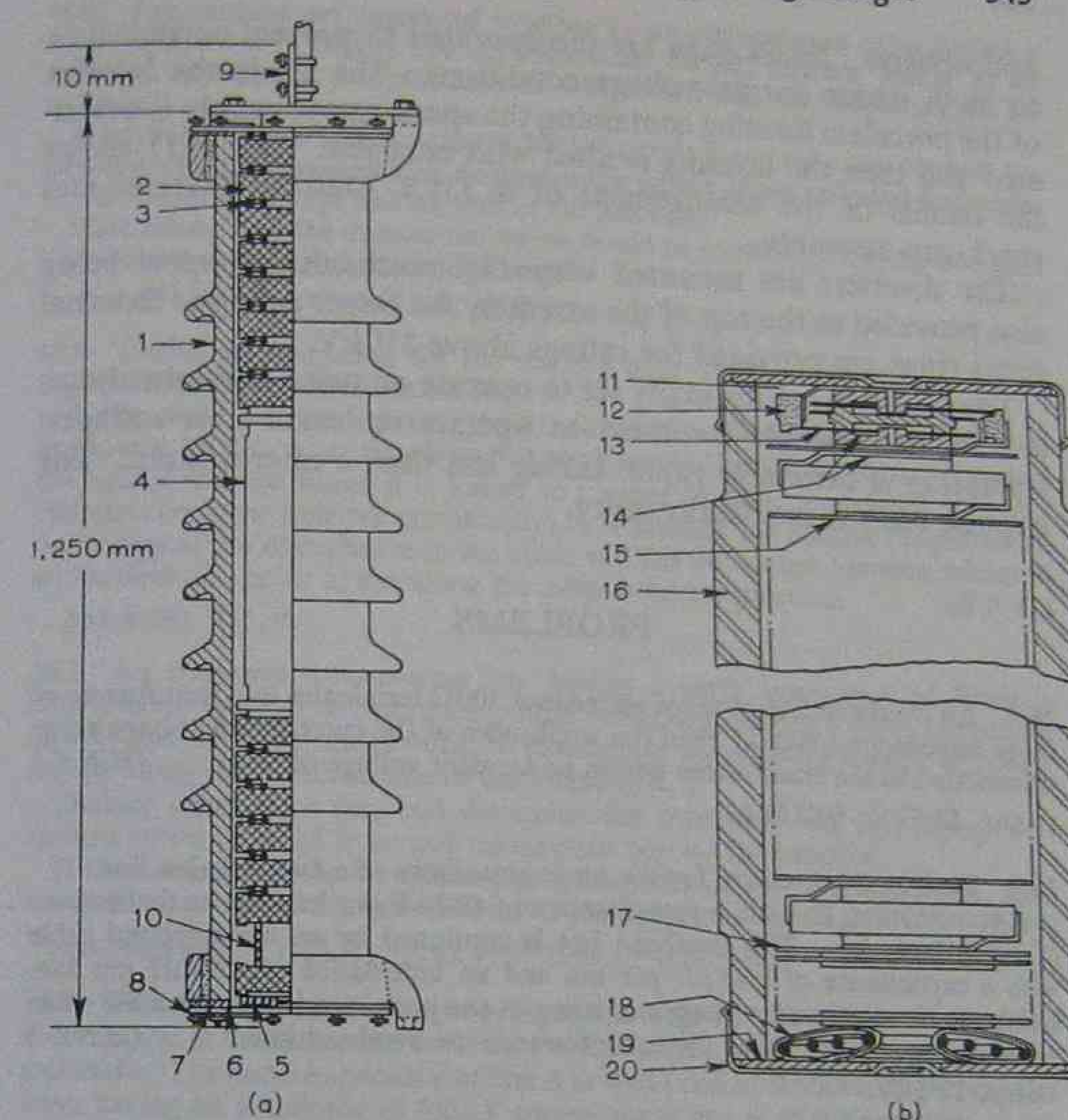

Fig. 16.11 METROSIL DIVERTER AND SPARK-GAP ASSEMBLY

(*AEI Ltd.*)

(*a*) Surge diverter

1. Glazed porcelain housing
2. Metrosil disc
3. Metallic spacers
4. Spark-gap assembly
5. Compression spring
6. Inner sealing gasket
7. Outer sealing gasket
8. Sealing plate
9. Terminal assembly
10. Spacing tube

(*b*) Section through spark-gap assembly

11. Contact clips
12. Metrosil grading ring
13. Mica disc
14. Electrode
15. Locating disc
16. Porcelain housing
17. Metal spacers
18. Compression spring
19. Contact plate
20. Sealing cap

of the diverter falls and the diverter passes a high current, diverting the surge energy to earth.

In practice, in a diverter suitable for operation on a 132 kV system, a stack of Metrosil discs 6 in. in diameter is assembled in a glazed porcelain housing which is provided on its exterior with rain sheds, which may be of a special shape for operation in dirt-laden *velodyne.* The output shaft drives a d.c. tachogenerator, and the voltage which it produces is compared with a set value derived from a reference voltage-divider, which can be calibrated in terms of output speed. With the polarities shown, the error voltage v_e is

$$v_e = v_i - K_T\omega_o \tag{18.32}$$

where K_T is the tachometer constant in volts per rad/s and ω_o $(= d\theta_o/dt)$ is the angular velocity of the output shaft.

The motor torque (assuming a constant armature current, an amplifier transconductance K_A, and a motor torque constant K_m) is $K_AK_mv_e$, so that, neglecting friction and loading, the dynamic equation for the system is

$$J\frac{d\omega_o}{dt} = K_AK_mv_e = K_AK_m(v_i - K_T\omega_o) \tag{18.33}$$

where J is the total inertia at the motor shaft.

Under conditions of steady output speed, $d\omega_o/dt = 0$, and this condition is fulfilled only when $v_i = K_T\omega_o$, so that the output speed is

$$\omega_o = \frac{v_i}{K_T} \tag{18.34}$$

If there is a constant load torque, T_L, coupled through an n:1 reduction gear, with the tachogenerator direct on the motor shaft, then the tachogenerator output will be $nK_T\omega_o$ (where ω_o is the output velocity). If the total inertia referred to the output shaft is J', the dynamic equation becomes

$$J'\frac{d\omega_o}{dt} + T_L = K_AK_m(v_i - nK_T\omega_o) \tag{18.35}$$

In this case the condition for steady output-shaft angular velocity (i.e. when $d\omega_o/dt = 0$) is

$$T_L = K_AK_m(v_i - nK_T\omega_o) = K_AK_m(nK_T\omega_i - nK_T\omega_o) \tag{18.36}$$

since the reference input voltage, v_i, can be calibrated in terms of the desired speed, ω_i, by eqn. (18.34), taking the gear ratio into account in this case. The difference between the desired speed, ω_i, and the actual speed, ω_o, is given by

$$\varepsilon = \omega_i - \omega_o = \frac{T_L}{nK_TK_AK_m} \tag{18.37}$$

This error is called the *droop*, and it should be noted that it is independent of the output speed. Thus at high shaft speeds it will represent a smaller percentage error than at low shaft speeds. It may be eliminated by the use of *integral-of-error compensation* as in the r.p.c. servo.

EXAMPLE 18.3 In the velodyne speed control shown in Fig. 18.12 the constants are as follows: amplifier transconductance, 200 mA/V; motor torque constant, 5×10^{-3} N-m/mA; tachogenerator constant 10 V per 1,000 rev/min. Determine the input voltage to give a speed of 2,000 rev/min. If the input setting is at half this value, find the droop when a load torque of 6×10^{-2} N-m is applied.

The tachogenerator constant in volts per rad/s is

$$K_T = \frac{10 \times 60}{1{,}000 \times 2\pi}$$

Hence, since there is no gearing, eqn. (18.34) gives

$$v_i = \omega_0 K_T = \underline{\underline{20\,\text{V}}}$$

When the input is set at 10 V, the no-load shaft speed will be 1,000 rev/min. If the load torque is now applied there must be an amplifier input voltage given by

$$v_e = \frac{T_L}{K_AK_m} = \frac{6 \times 10^{-2}}{200 \times 5 \times 10^{-3}} = 6 \times 10^{-2}\,\text{V}$$

Hence the tachogenerator output is $10 - (6 \times 10^{-2})$ V, and the actual shaft speed is $(10 - 6 \times 10^{-2} \times 1{,}000/10)$ rev/min, i.e. the droop is

$$6 \times 10^{-2} \times 100 = \underline{\underline{6\,\text{rev/min}}}$$

Note that this result can also be obtained by applying eqn. (18.37).

The speed regulation is $6/1{,}000 \times 100 = \underline{\underline{0{\cdot}6 \text{ per cent.}}}$

18.13 Some Limitations of the Simple Theory

In the simple theory developed in the preceding sections no account has been taken of any non-linearities in the system, such as the saturation of the amplifier, backlash in gearing, and stiction. Nor have the effects of actual servo-motor characteristics on the system damping, the mechanical limit of system acceleration (to keep acceleration stresses within reason), or the effect on dynamic response of the introduction of integral compensation been considered. Only velocity-feedback stabilization has been dealt with, and other forms of stabilization have been omitted. Such subjects, together with the important questions of system stability and harmonic response are the concern of full courses on servo-mechanism, as are considerations of large power systems using magnetic or rotating amplifiers.

Find the current fed into a symmetrical 3-phase short-circuit at the distant end of the feeder. (H.N.C.)

Ans. 513 A.

17.9 In a generating station there are four busbar sections with a 60 MVA 33 kV 3-phase generator having 15 per cent leakage reactance connected to each section. The sections are connected through a 10 per cent reactor to a common tie-bar. A 1 MVA 3-phase feeder joined to one of the busbar sections has a resistance of 60 Ω/phase and a reactance of 70 Ω/phase.

If a symmetrical 3-phase short-circuit occurs at the receiving end of the feeder, determine the MVA and also the voltage on the four busbar sections.

Ans. 11·6 MVA, 3·83 MVA; 32·3 kV, 32·7 kV. (L.U.)

17.10 Define the terms "symmetrical breaking current" and "asymmetrical breaking current" as applied to oil circuit-breakers. Show how these quantities are determined from oscillograms of short-circuit tests on a circuit-breaker. Explain why, on symmetrical 3-phase short-circuit tests, some initial asymmetry in the current always occurs.

A 50 MVA generator of 18 per cent leakage reactance and a 60 MVA generator of 20 per cent leakage reactance are connected to separate busbars which are interconnected by a 50 MVA reactor. Calculate the percentage reactance this reactor must possess in order that switches rated at 500 MVA may be employed on feeders connected to each of the busbars.

Ans. 7 per cent, or 0·07 p.u. (L.U.)

17.11 Enumerate the positions in which current-limiting reactors may be connected. What advantage does the tie-bar system have over the ring system?

A station busbar has three sections A, B and C, to each of which is connected a 20 MVA generator of reactance 8 per cent. Two similar reactors are to be connected, one between the busbar sections A and B and one between the busbar sections B and C. Calculate the percentage reactance of these reactors if the MVA fed into a symmetrical short-circuit on section A busbar is not to exceed 400. The reactors are to be rated at 10 MVA. (H.N.C.)

Ans. 4 per cent, or 0·04 p.u.

17.12 Three 11 kV 40 MVA alternators are connected to three sets of 33 kV busbars A, B and C by means of three 11/33 kV 40 MVA transformers. The busbars are joined by two similar reactor sets, one set being connected between A and B and the other between B and C. The reactance of each alternator is 20 per cent and that of each transformer is 6 per cent at 40 MVA.

Determine the percentage reactance of each set of reactors at 10 MVA in order that the symmetrical short-circuit on busbars A should be limited to 350 MVA. (L.U.)

Ans. 1·5 per cent, or 0·015 p.u.

17.13 The 33 kV busbars of a generating station are divided into three sections A, B and C, which are connected to a common tie-bar by a reactance *X* ohms. To section A is connected a 60 MVA generator having a leakage reactance of 15 per cent, to B a 40 MVA generator of leakage reactance 12 per cent, and to C a 30 MVA generator of leakage reactance 10 per cent.

If the breaking capacity of the circuit-breaker connected to section A is not to exceed 500 MVA, determine the minimum value of the reactance *X*.

Ans. 6·12 Ω.

Chapter 18

CLOSED-LOOP CONTROL SYSTEMS

In almost every sphere of human endeavour there is a need to exercise control of physical quantities. Manual control, involving a human operator, suffers from several disadvantages among which may be numbered fatigue, slow reaction time (some 0·3 s), lack of exact reproducibility, limited power, tendency to step-by-step action and variations between one operator and another. The demand for precision control of physical quantities has led to the development of *automatic control systems* or *servo systems*. It is the purpose of this chapter to examine some simple servo systems, as an introduction to a subject which is of ever-growing importance.

All precision control involves the feedback of information about the controlled quantity, in such a way that if the controlled quantity differs from the desired value an error is observed, and the control system operates to reduce this error. This type of control is called *closed-loop control* and can be either manual or automatic. In simple regulating systems there is no feedback of information, and precise control cannot be achieved. This is called *open-loop control*, since there is no feedback loop.

18.1 Open-loop Control

The operation of an open-loop regulating system may be understood by considering one or two illustrations of such systems. For example, the flow of water in a pipe may be controlled by a valve. The opening of the valve can be measured on a scale, but for any one setting the actual flow of water will depend on the available head at the inlet to the valve, and on the loading at the outlet, as well as on the valve setting. The accuracy of the setting is thus dependent on external disturbances.

Again, consider the speed control of a d.c. shunt motor by field resistance. Increasing the field resistance increases the motor speed, but at any setting of the field rheostat the actual speed will depend on the supply voltage and on the load on the machine. The rheostat cannot be calibrated accurately in terms of speed, and the system is an open-loop control. If now we connect a tachometer to the shaft, and mark on its scale the desired speed, then a human operator may adjust the field rheostat as required to keep the actual speed as near as he can to the desired speed. The operator then acts as the feedback loop, and the system has become a closed-loop system, where the control action depends on the observed error between actual and desired speed. An increased accuracy of control is thus achieved. An automatic control is achieved by replacing the operator by an error-measuring device and output controller.

18.2 Basic Closed-loop Control

The basic elements of a simple closed-loop control system are illustrated in Fig. 18.1. In this case the output of some industrial

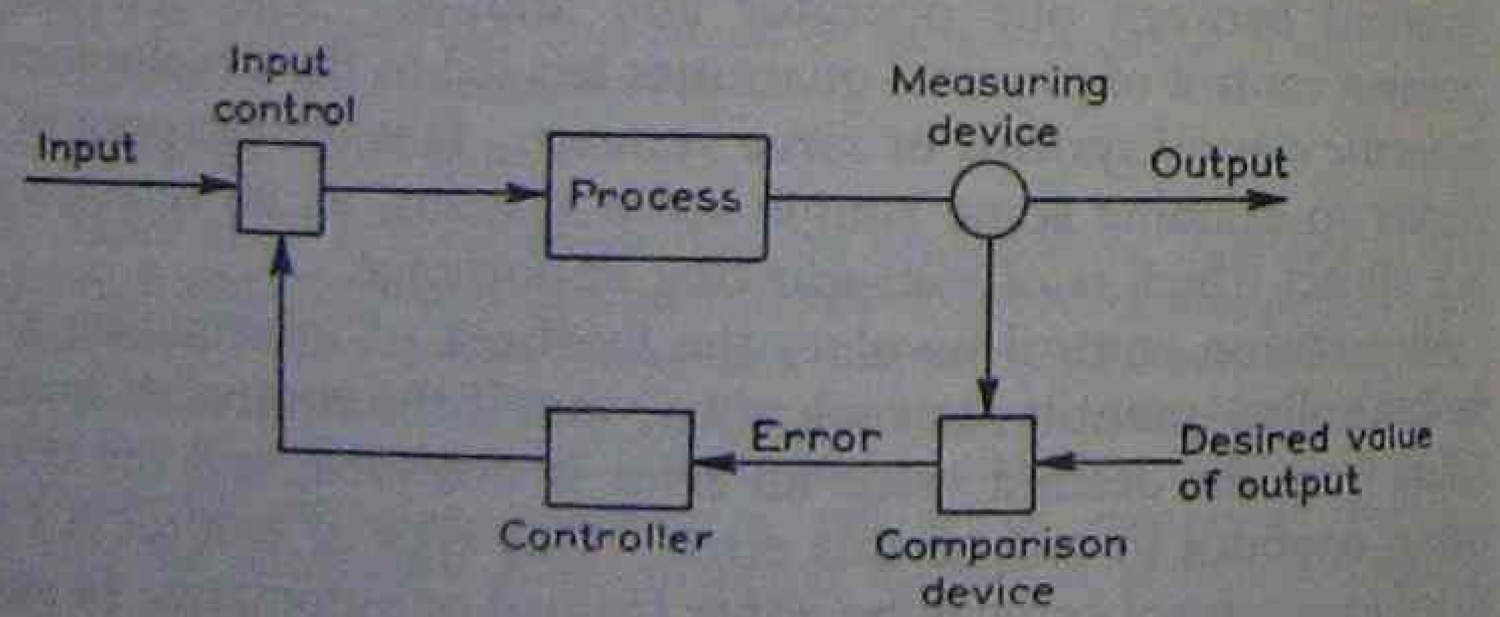

Fig. 18.1 SIMPLE CLOSED-LOOP PROCESS CONTROL

process is being controlled by a control element. The actual output from the process is measured and compared with a desired value in the comparison device. The magnitude and sense of any difference between the desired and actual values of the output is fed as an error signal to the controller which in turn actuates the correcting device in such a way as to tend to reduce the error. Note that correction

takes place no matter how the error arises, e.g. by external disturbances or changes in input conditions. The control gear forms a closed loop with the process.

The essential elements of the automatic control system are thus (*a*) a measuring device, which can often be combined with (*b*) a comparison device to produce an error signal, (*c*) a controller, which normally incorporates power amplification, and (*d*) a correction device.

One very important type of control system is that in which the angular position of a shaft has to be controlled from some remote position with great accuracy. Such a system is called a *remote position control* (or r.p.c.) servo, and has applications including the automatic control of gun positions, servo-assisted steering of vehicles and ships, positioning of control rods in nuclear reactors, and automatic control of machine tools. In the following sections an electrical r.p.c. servo will be considered in more detail. In such a system, the shaft position is measured electrically, an electrical error signal is generated, amplified, and used to control an electric positioning motor.

18.3 The Summing Junction

In electrical servos it is often required to apply the sum of or difference between two or more signals to an amplifier. This can conveniently be done by means of a summing junction, as illustrated

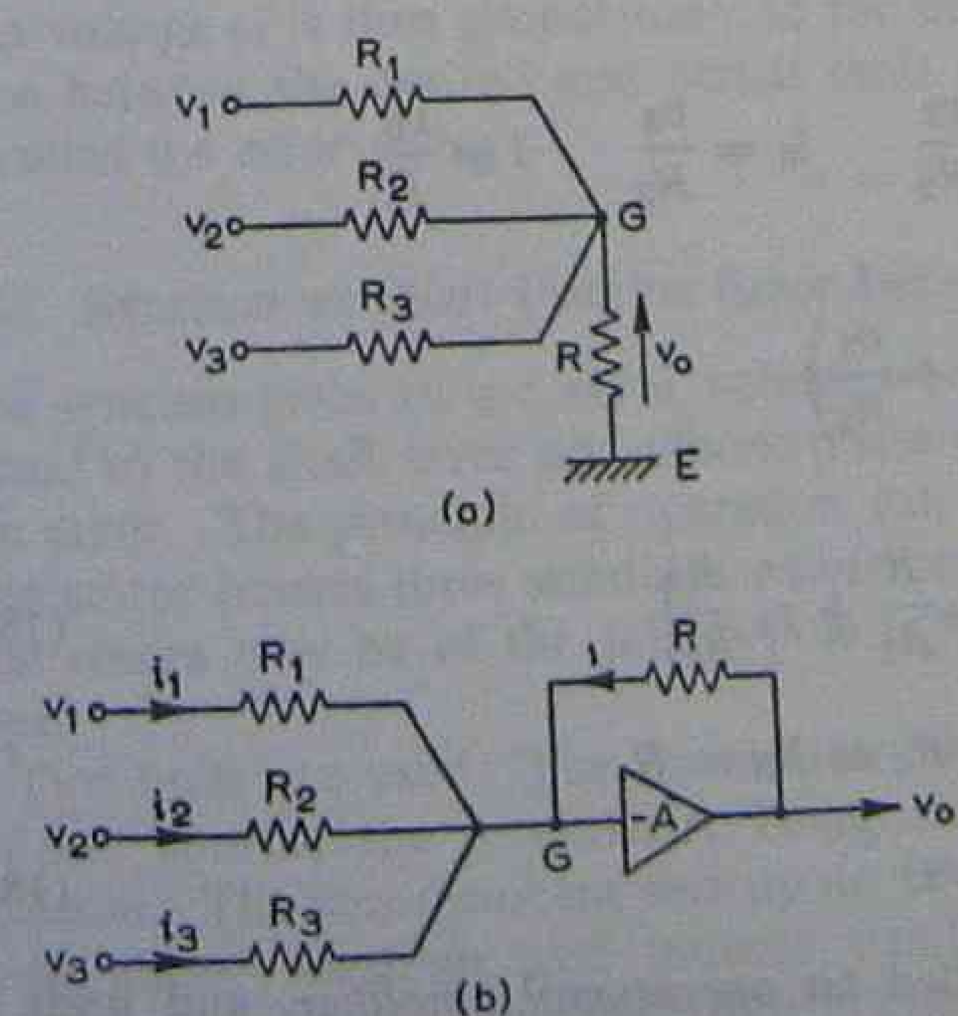

Fig. 18.2 THE SUMMING JUNCTION

in Fig. 18.2(a). Thus, if the free ends of the input resistors R_1, R_2, R_3 have voltages v_1, v_2, v_3 to earth, then by Millman's theorem,

$$v_0 = v_{GE} = \frac{\Sigma v_{ne} Y_n}{\Sigma Y_n} = \frac{\dfrac{v_1}{R_1} + \dfrac{v_2}{R_2} + \dfrac{v_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R}} \qquad (18.1)$$

The output voltage is thus dependent on the sum of the input voltages taken in proportions which depend on the ratios of the resistors. Note that, if (as is not uncommon) $R_1 = R_2 = R_3$ and $R \gg R_1$, then

$$v_0 = \tfrac{1}{3}(v_1 + v_2 + v_3) \qquad (18.2)$$

For two inputs (i.e. if R_3 were disconnected) this would reduce to

$$v_0 = \tfrac{1}{2}(v_1 + v_2)$$

A more sophisticated version of the summing junction is obtained by using a high-gain d.c. amplifier as shown in Fig. 18.2(b). If the gain of the amplifier is $-A$ and its input impedance is very high, then

$$i = -(i_1 + i_2 + i_3)$$

and the potential of point G is $-v_0/A$ which will be very small if A is large (typically A can be of the order of 10^7). Thus G can be considered to be almost at earth potential, and is called a *virtual earth*, so that

$$i_1 = \frac{v_1}{R_1} \qquad i_2 = \frac{v_2}{R_2} \qquad i_3 = \frac{v_3}{R_3} \qquad i = \frac{v_0}{R}$$

Hence

$$\frac{v_0}{R} = -\left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right)$$

or

$$v_0 = -\left\{v_1\frac{R}{R_1} + v_2\frac{R}{R_2} + v_3\frac{R}{R_3}\right\} \qquad (18.3)$$

In the case where $R_1 = R_2 = R_3 = R$,

$$v_0 = -(v_1 + v_2 + v_3) \qquad (18.4)$$

This arrangement is called an *operational amplifier*, and finds an application in analogue computers as well as in servo systems.

18.4 Measurement of Shaft Position Error by Voltage Dividers

Two methods of obtaining an electrical signal which will give a measure of the size and sense of the difference between the actual angular position of a shaft and the desired angular position will be considered. The first of these methods involves voltage dividers whose sliders are fixed to a reference and an actual output shaft respectively.

Consider the two linearly wound voltage dividers shown in Fig. 18.3, connected to a summing junction, G, though resistors which are of such high values that they give negligible loading on the voltage dividers. The slider of the reference voltage divider is set at the desired angular shaft position θ_i, so that, assuming the divider to be wound over 300°,

$$v_1 = \frac{\theta_i}{300} \times (-V)$$

The slider of the output voltage divider is connected to the output shaft, and for an output shaft angular position of θ_o,

$$v_2 = \frac{\theta_o}{300} \times (V)$$

It follows from eqn. (18.4) that

$$v_e = -\left(\frac{-\theta_i V}{300} + \frac{\theta_o V}{300}\right) = \frac{V}{300}(\theta_i - \theta_o) \qquad (18.5)$$

The voltage v_e is thus proportional to the shaft error (i.e. the difference between the desired and actual shaft positions $(\theta_i - \theta_o)$ and is called the *error voltage*.

18.5 Synchros as Shaft Position Error Detectors

The synchro gives an a.c. error voltage whose amplitude is propor-
tional to the shaft error and whose phase depends on the sense of
the error. The principle of operation can be seen from Fig. 18.4.
The stator houses three windings whose centre-lines are 120° apart.
The rotors may be of the salient-pole type or of the wound-rotor
type.

Two units are used. The transmitter has its rotor supplied from an a.c. source, the rotor shaft being set to the desired angular position. The rotor current sets up an air-gap flux which links the three stator windings and induces e.m.f.s in them according to the relative position of the rotor. The stator windings of the transmitter are linked to those of the receiver as shown in Fig. 18.4,

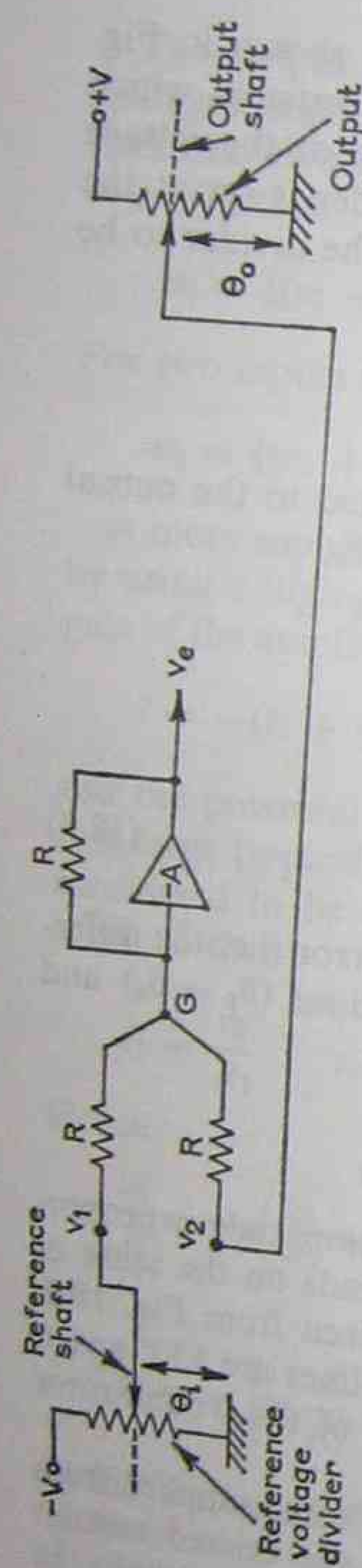

Fig. 18.3 USE OF VOLTAGE DIVIDERS FOR SHAFT ERROR MEASUREMENT

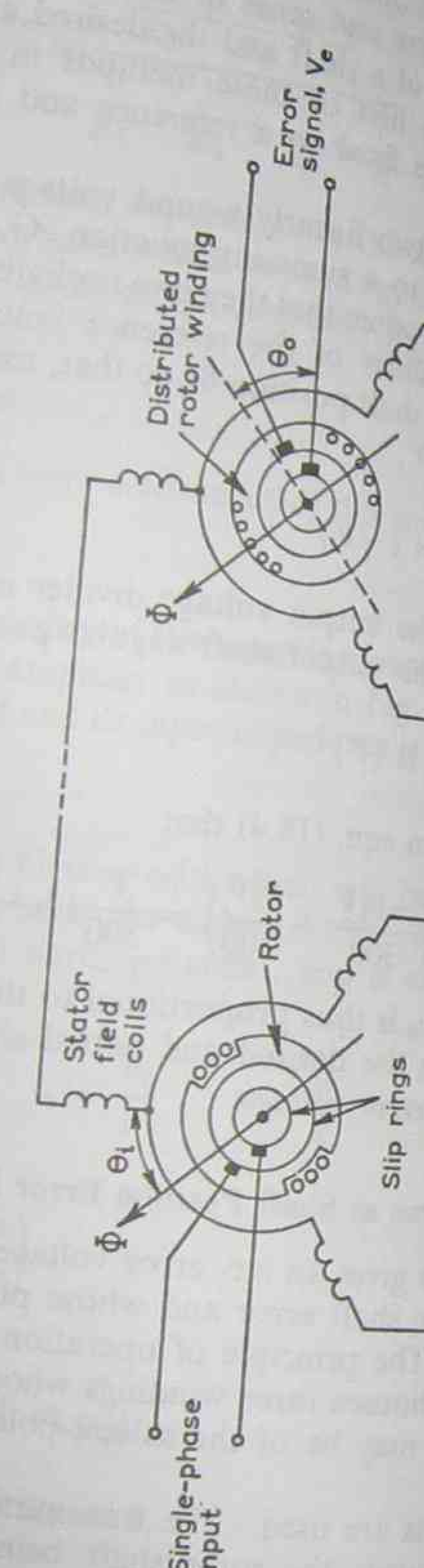

Fig. 18.4 USE OF SYNCHROS FOR SHAFT ERROR DETECTION

so that the induced e.m.f.s will set up circulating currents, which in turn cause a flux in the air-gap of the receiver. This receiver flux will have the same direction relative to the receiver stator as the transmitter air-gap flux has relative to the transmitter stator.

When the centre-line of the receiver rotor is at 90° to this flux, the e.m.f. induced in the rotor winding will be zero. For any deviation θ from the 90° position, the alternating e.m.f. induced in the receiver rotor will be

$$V_e = V \sin \theta = V \sin (\theta_i - \theta_o) \tag{18.6}$$

where V is the r.m.s. voltage induced in the receiver rotor when it links all the stator flux. For small deviations from exact quadrature between receiver flux and rotor centre-line, $\sin(\theta_i - \theta_o) \approx \theta_i - \theta_o$ and

$$V_e \approx V(\theta_i - \theta_o) \tag{18.7}$$

i.e. the error voltage is linearly related to the difference between input and output shaft positions (after allowing for the initial 90° displacement). The phase of this error voltage will be 180° different for the case when $\theta_o < \theta_i$ than it is when $\theta_o > \theta_i$, thus giving a measure of the sense of the error.

Shaft error detection is only one of several applications of these devices. As error detectors they have the advantage over voltage dividers of negligible wear, greater accuracy, and error detection over a full 360° rotation.

18.6 Small Servo Motors and Motor Drives

Electrical servos may be (*a*) entirely d.c. operated, using voltage dividers, d.c. amplifiers, and d.c. driving motors, (*b*) entirely a.c. operated using synchros, a.c. amplifiers and 2-phase a.c. driving motors, or (*c*) a.c./d.c. operated using a.c. error detection, phase-sensitive rectification and d.c. driving motors.

SPLIT-FIELD MOTOR

Small d.c. servo motors are generally of the split-field type illustrated in Fig. 18.5. Neglecting saturation and assuming a constant armature current, the output torque will be proportional to the net field current, and will reverse when this net field current reverses. The field may be fed from a push-pull amplifier stage. If the armature is not fed from a constant-current source, the build-up of armature e.m.f. with speed causes a falling torque/speed characteristic, which is equivalent to viscous-friction damping in the servo system. Approximately constant armature current can be achieved by feeding the armature through a high resistance from a high-voltage d.c. supply.

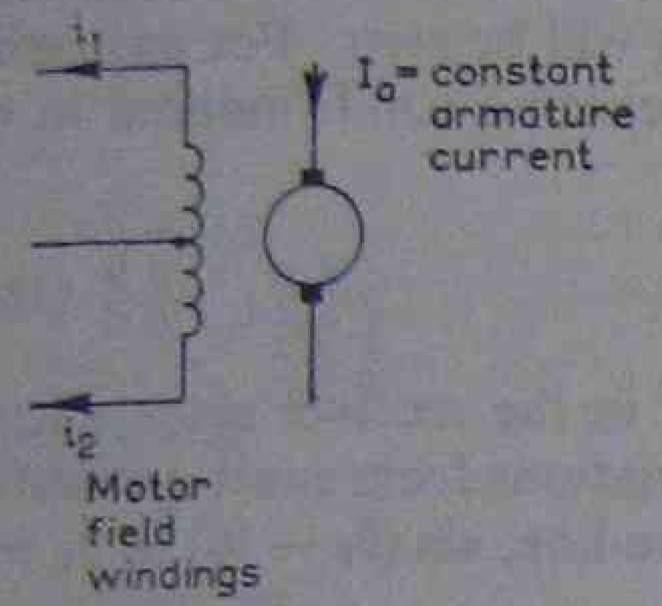

Fig. 18.5 THE D.C. SERVO MOTOR

PHASE-SENSITIVE RECTIFIER

An a.c. error signal may be used with a *phase-sensitive rectifier* (p.s.r.) to produce a d.c. error voltage. The basic operation of a p.s.r. is shown in Fig. 18.6. When there is no error voltage, each diode conducts during positive half-cycles of the reference voltage and there is no net voltage between A and B. The peak error signal,

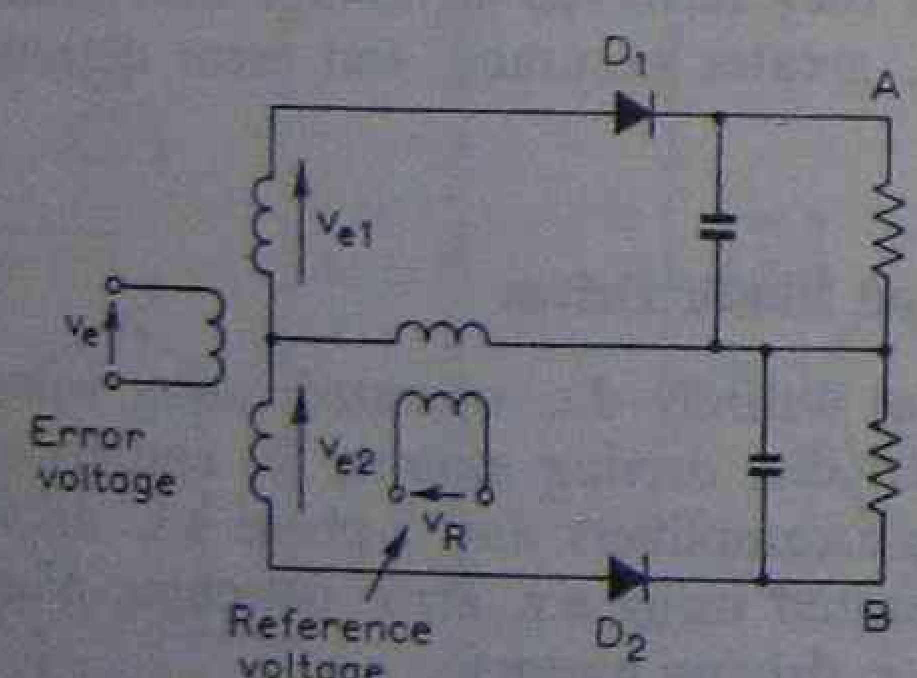

Fig. 18.6 PHASE-SENSITIVE RECTIFICATION

v_e, is arranged to be smaller than the reference voltage, v_R. When the error signal is in phase with the reference voltage, the voltage applied to D_1 during the positive half-cycles of v_R, i.e. $(v_R + v_{e1})$, is greater than that applied to D_2 $(v_R - v_{e2})$, and hence A is positive with respect to B. During the negative half-cycles of v_R both diodes remain non-conducting (since $v_R > v_e$). If the error voltage is now changed in phase by 180°, D_2 has a larger voltage applied to it during

the positive half-cycles than D_1 and B is positive with respect to A. Hence the voltage between A and B gives the magnitude and sense of the error, and may be applied direct to the bases of a long-tailed pair (Section 22.11). The capacitors provide smoothing of the output signal.

TWO-PHASE SERVO MOTOR

In an a.c. servo the error signal from a synchro is fed to an a.c. amplifier, whose output feeds one phase of 2-phase motor. The phase of the voltage applied to this winding is arranged to be in quadrature with that applied from a constant reference source to the second winding of the motor (the reference winding), as shown in Fig. 18.7(*a*).

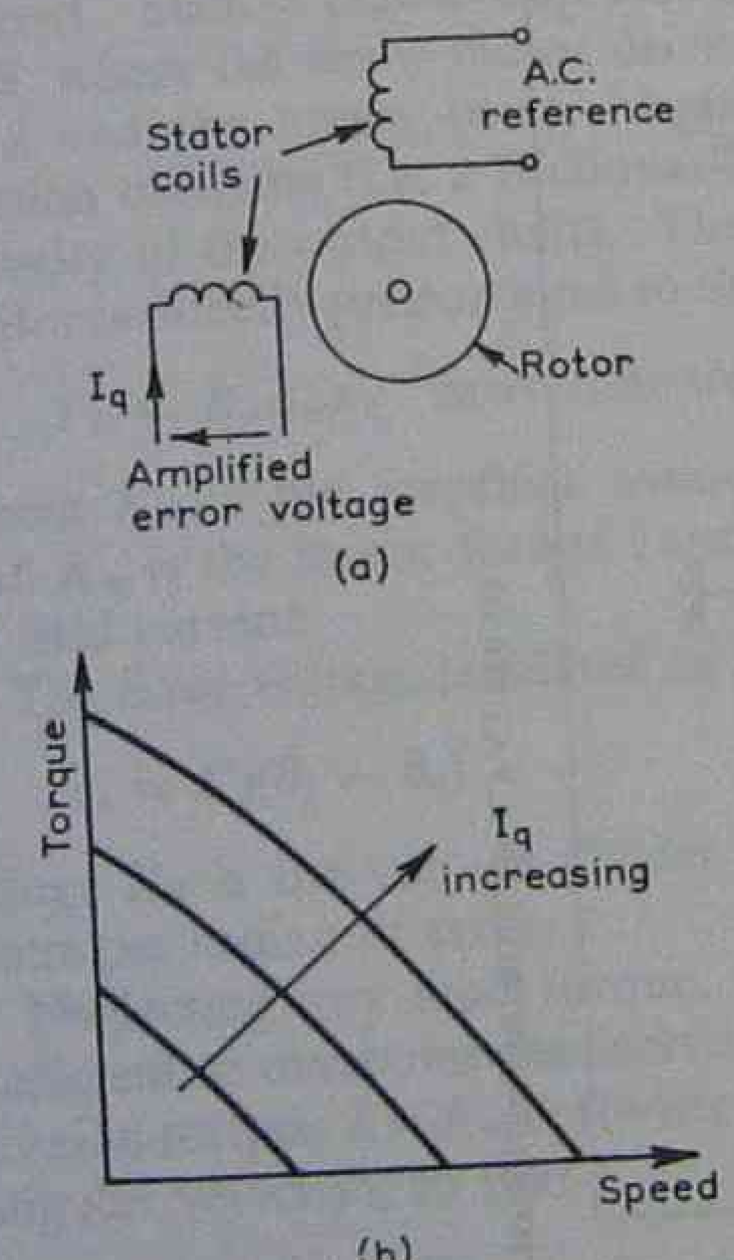

Fig. 18.7 THE 2-PHASE SERVO MOTOR

When there is no error voltage, only the reference winding is energized and the rotor is locked in position. The motor is designed to give maximum torque at standstill. The size of the output torque will depend on the magnitude of the error signal, and the direction of the torque will depend upon whether the error signal lags or leads the reference voltage by 90°. Typical characteristics are shown in Fig. 18.7(*b*).

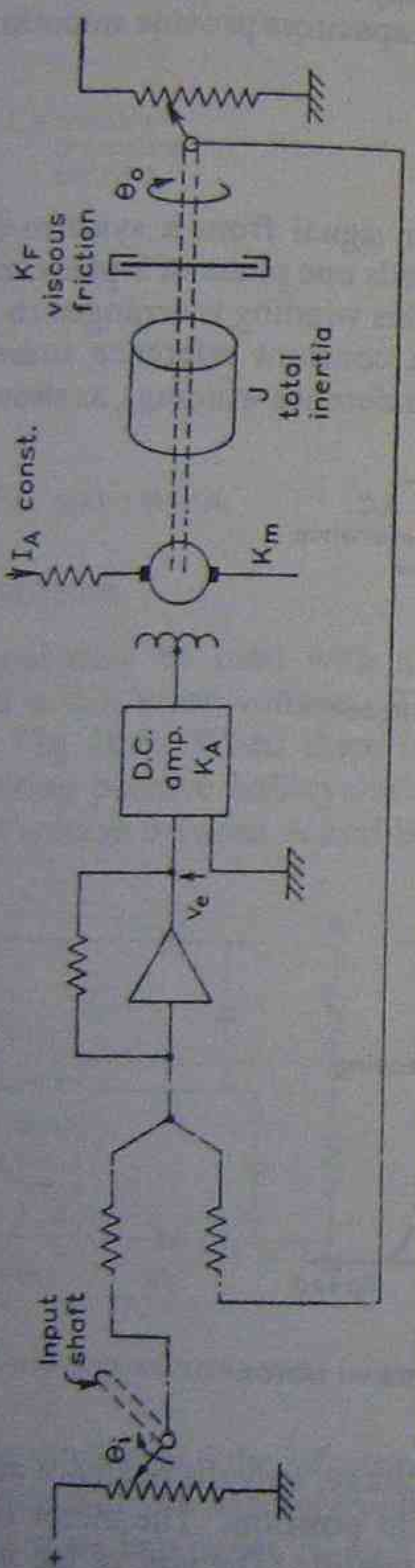

Fig. 18.8 SIMPLE TORQUE-CONTROLLED R.P.C. SERVO

The a.c. servo is not used where large power is required, because the electronic amplifier has a limited power output. In such cases rotating d.c. amplifiers (amplidyne or metadyne type) or magnetic amplifiers are used with d.c. driving motors. High-power servos will not be dealt with in this text.

Note that in both d.c. and a.c. servo motors the armatures are usually long and of small diameter in order to give a high torque/inertia ratio.

18.7 Simple Torque-controlled R.P.C. Servo

The main components of an electrical r.p.c. servo system having been briefly discussed, a simple control system can now be considered. Such a closed-loop system is shown schematically in Fig. 18.8, where the servo motor drives a load shaft (the inertia of the load and the motor being J kg-m^2), and where there is viscous-friction damping (i.e. a frictional force proportional to the angular velocity of the output shaft). The driving torque produced by the motor is directly proportional to the error voltage, v_e, and is given by

$$T_D = K_A K_m v_e \quad \text{newton-metres}$$

where K_A is the amplifier *transconductance* in amperes per volt, and K_m is the motor torque constant in newton-metres per ampere of field current.

The error voltage is related to the shaft error by the equation

$$v_e = K_S(\theta_i - \theta_o) \tag{18.12}$$

where K_S is the voltage divider and summing junction constant in volts per radian of error.

Neglecting any load torque, the motor driving torque must be sufficient to overcome the inertia torque, $J(d^2\theta_o/dt^2)$, and the viscous friction torque, $K_F(d\theta_o/dt)$ (where K_F is the friction torque per unit of angular velocity), so that

$$J\frac{d^2\theta_o}{dt^2} + K_F\frac{d\theta_o}{dt} = K_A K_m v_e = K_A K_m K_S(\theta_i - \theta_o) = K(\theta_i - \theta_o) \tag{18.13}$$

where $K = K_A K_m K_S$ newton-metres per radian. Rearranging,

$$\frac{d^2\theta_o}{dt^2} + \frac{K_F}{J}\frac{d\theta_o}{dt} + \frac{K}{J}\theta_o = \frac{K}{J}\theta_i \tag{18.14}$$

Obviously, at standstill, when $d^2\theta_o/dt^2 = d\theta_o/dt = 0$, then $\theta_o = \theta_i$ and there is no error.

The response of the system (which is known as a *second-order system*) to a step change in input, θ_i, may be computed in exactly the same way as was used for the double-energy transient in Chapter 7. Rewriting eqn. (18.14) in standard form,

$$\frac{d^2\theta_o}{dt^2} + 2\zeta\omega_n\frac{d\theta_o}{dt} + \omega_n{}^2\theta_o = \omega_n{}^2\theta_i \qquad (18.14a)$$

where

$$\omega_n = \sqrt{(K/J)} = \text{undamped natural angular frequency} \qquad (18.15)$$

and ζ (zeta) is the *damping ratio given by*

$$\zeta = \frac{\text{Actual damping constant}}{\text{Damping constant for critical damping}} = \frac{K_F}{2\sqrt{(JK)}} \qquad (18.16)$$

As shown in Fig. 18.9, there are four solutions to eqn. (18.14a).

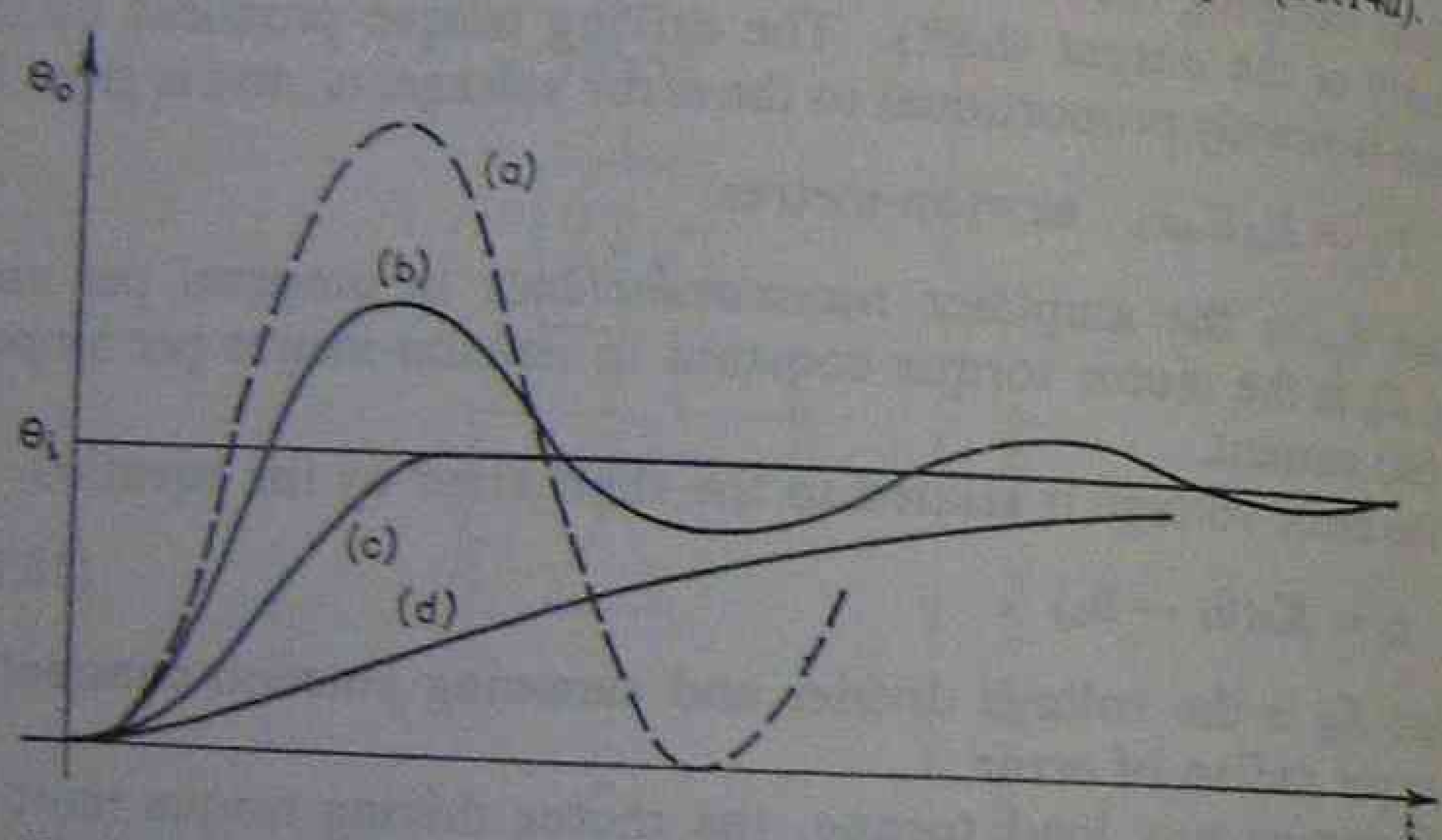

Fig. 18.9 RESPONSE OF A SECOND-ORDER SERVO TO A STEP CHANGE OF INPUT

(*a*) *Zero damping* ($\zeta = 0$; i.e. $K_F = 0$)
The response to a step input change is given by

$$\theta_o = \theta_i(1 - \cos\omega_n t) \qquad (18.17)$$

The output oscillates continuously, the amplitude of the oscillations corresponding to the input step change. This condition should obviously be avoided.

(*b*) *Underdamping* ($\zeta < 1$; i.e. $K_F < 2\sqrt{(JK)}$)
The output oscillates but finally settles down to a steady value of $\theta_o = \theta_i$. The response is characterized by an overshoot (or several

overshoots) but gives a fast rise to around the value of θ_i. The actual equation of the response is

$$\theta_o = \theta_i\left\{1 - e^{-\zeta\omega_n t}(\cos\omega t + \frac{\zeta}{\sqrt{(1-\zeta^2)}}\sin\omega t)\right\} \qquad (18.18)$$

where

$$\omega = \omega_n\sqrt{(1-\zeta^2)} \qquad (18.19)$$

The time constant of the decaying oscillation is $\tau = 1/\zeta\omega_n$. The slightly underdamped response is the type usually employed for fast-acting servos, damping ratios of the order of 0·6 being common.

(*c*) *Critical damping* ($\zeta = 1$; i.e. $K_F = 2\sqrt{(JK)}$)
This condition marks the transition between the oscillatory and the overdamped solution, and the response to a step input change is

$$\theta_o = \theta_i(1 - \zeta\omega_n t\, e^{-\zeta\omega_n t} - e^{-\zeta\omega_n t}) \qquad (18.20)$$

(*d*) *Overdamping* ($\zeta > 1$; i.e. $K_F > 2\sqrt{(JK)}$)
This represents a condition of slow response and is normally avoided in practice. The mathematical expression for the response is

$$\theta_o = \theta_i\left\{1 - e^{-\zeta\omega_n t}\left(\cosh\beta t + \frac{\zeta}{\sqrt{(1-\zeta^2)}}\sinh\beta t\right)\right\} \qquad (18.21)$$

where

$$\beta = \omega_n\sqrt{(\zeta^2 - 1)} \qquad (18.22)$$

It should be noted that exactly the same results are obtained with an a.c. servo, or a d.c. servo with synchro error detection and a phase-sensitive rectifier.

In all servos there is a lower limit to the size of error signal which will just cause a correcting action to take place. If the system is made too sensitive, spurious operation may result from random noise inputs to the amplifier. The *dead zone* of an r.p.c. servo is the range of shaft errors over which no correcting action will take place, since the motor torque will not be sufficient to overcome stiction (static friction). This dead zone will give the limit of accuracy of the system.

18.8 Gearing

It is usually economic to design servo motors which run at a much higher speed than that required for the output shaft. A reduction gear is then used to connect the motor to the load shaft (Fig. 18.10).

If the gear ratio is $n:1$, the shaft output angular rotation, velocity and acceleration will each be $1/n$ of the corresponding input quantities. Also, assuming that there is no power loss in the gearing,

Input power = Output power i.e. $T_1\omega_1 = T_2\omega_2$

or

$$T_2 = nT_1 \tag{18.23}$$

where T represents torque, ω is the angular velocity, and the subscripts 1 and 2 refer to the input and output sides of the gearbox.

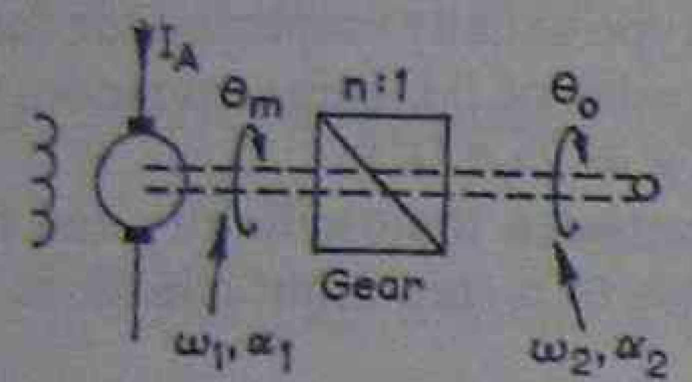

Fig. 18.10 SERVO MOTOR WITH GEAR TRAIN

The torque at the motor shaft required to overcome the motor inertia (J_m) is $J_m\alpha_1$ where α_1 is the angular acceleration at the motor. This torque at the motor shaft gives a torque of $nJ_m\alpha_1$ at the output shaft.

Hence

Output shaft torque $= nJ_m\alpha_1 = n^2J_m\alpha_2$

where α_2 is the output shaft acceleration; i.e. the motor inertia is equivalent to an inertia *at the output shaft* of

$$J_m' = n^2J_m \tag{18.24}$$

If the load inertia is J_L, the load acceleration, α_2, due to a motor torque T_m will be

$$\alpha_2 = \frac{nT_m}{J_L + n^2J_m}$$

where the total inertia referred to the load shaft is

$$J = J_L + n^2J_m \tag{18.25}$$

α_2 is a maximum as n varies when $n = \sqrt{(J_L/J_m)}$, and this gear ratio is said to match the inertias.

18.9 Velocity-feedback Damping

Normal viscous friction of the mechanical system is usually insufficient to provide enough damping for the satisfactory operation

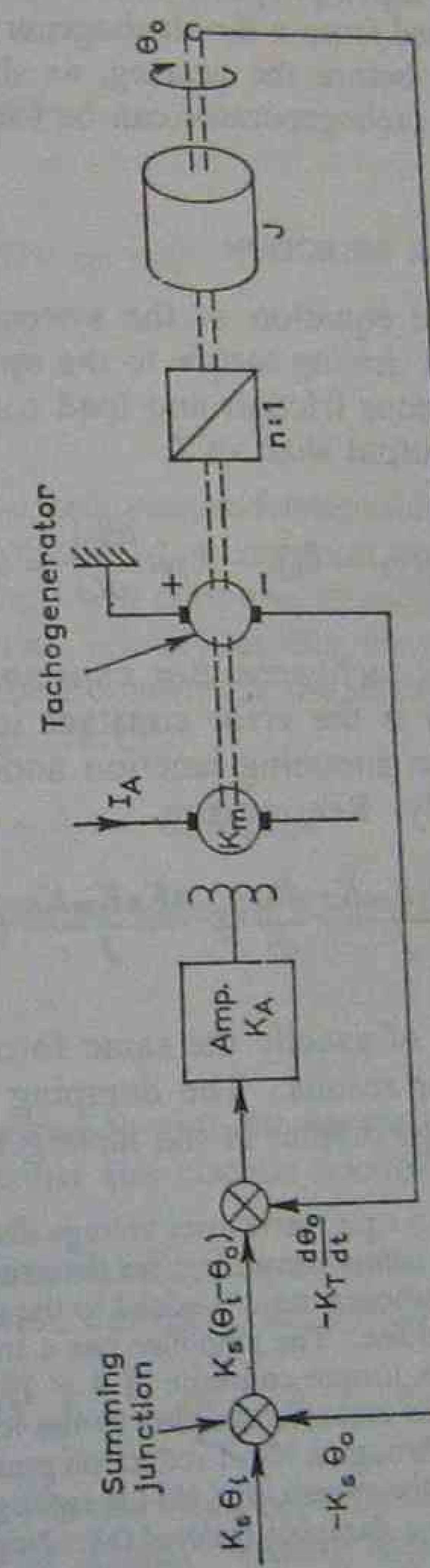

Fig. 18.11 D.C. SERVO WITH VELOCITY-FEEDBACK DAMPING

of an r.p.c. servo, and any increase in the mechanical viscous friction would involve additional power loss. The damping term in eqn. (18.14) can be readily increased, however, by *velocity feedback*, in which a second feedback loop is provided to give a negative signal at the amplifier input proportional to the shaft velocity. This signal can be obtained from a d.c. tachogenerator connected to the motor shaft, usually before the gearing, as shown in Fig. 18.11. In a.c. servos an a.c. tachogenerator can be used in the same way.

SYNCHRO ERROR DETECTION

The dynamic equation of the system is obtained as before by equating motor driving torque to the opposing torques. Neglecting mechanical viscous friction and load torque, and assuming a total inertia at the output shaft of J,

$$nK_AK_m\left\{K_S(\theta_i-\theta_o)-K_Tn\frac{d\theta_o}{dt}\right\}=J\frac{d^2\theta_o}{dt^2} \tag{18.26}$$

where K_T is the tachogenerator constant in volts per rad/s *at the motor shaft*, K_S is the error constant in volt/rad error, and it is assumed that the summing junction adds the input voltages (algebraically) directly. Rearranging,

$$\frac{d^2\theta_o}{dt^2}+\frac{n^2K_AK_mK_T}{J}\frac{d\theta_o}{dt}+\frac{nK_AK_mK_S}{J}\theta_o=\frac{nK_AK_mK_S}{J}\theta_i \tag{18.27}$$

This equation is of exactly the same form as eqn. (18.14) and will thus yield similar results. The damping can be readily varied by including a voltage divider in the tachogenerator feedback path.

EXAMPLE 18.1 An r.p.c. servo uses voltage dividers with a 300° travel and a total voltage of 30 V across them for error detection. Damping is provided by a d.c. tachogenerator, whose output is added to the shaft error voltage in an operational summing amplifier. The amplifier has a transconductance of 250 mA/V, and the motor has a torque constant of 4×10^{-4} N-m/mA and an inertia of 50×10^{-6} kg-m². The motor is coupled to the load, whose moment of inertia is 40×10^{-2} kg-m², through a 100:1 reduction gear. Calculate (*a*) the undamped natural frequency of the system, and (*b*) the tachogenerator constant in volts per 1,000 rev/min to give a damping ratio of 0·8. Neglect viscous friction.

From eqn. (18.25),

$$\text{Total inertia referred to load shaft}=(40\times10^{-2})+(10^4\times50\times10^{-6})=90\times10^{-2}\,\text{kg-m}^2$$

$$\text{Voltage divider constant}=\frac{30}{300}\times\frac{360}{2\pi}=5{\cdot}73\,\text{V/rad}$$

Hence eqn. (18.27) can be written

$$\frac{d^2\theta_o}{dt^2}+1{,}110K_T\frac{d\theta_o}{dt}+72\theta_o=72\theta_i$$

Thus from eqn. (18.15), $\omega_n=\sqrt{72}=8{\cdot}5$, so that

$$f_n=\frac{\omega_n}{2\pi}=\underline{\underline{1{\cdot}35\,\text{Hz}}}$$

Comparison with eqn. (18.14*a*) yields

$$2\zeta\omega_n=1{,}110K_T$$

Hence

$$K_T=\frac{2\times0{\cdot}8\times8{\cdot}5}{1{,}110}=0{\cdot}0123\,\text{V per rad/s}$$
$$=\underline{\underline{1{\cdot}28\,\text{V per }1{,}000\,\text{rev/min}}}$$

18.10 Velocity Lag

In a second-order r.p.c. servo with viscous friction damping, suppose that the input shaft is rotated at a constant angular velocity $\omega_i=d\theta_i/dt$. The output shaft will continue to accelerate until it is rotating at the same angular velocity as the input. Then, since $d^2\theta_o/dt^2$ will be zero under these conditions, the steady-state equation of motion will be, from eqn. (18.13),

$$K_F\frac{d\theta_o}{dt}=K_AK_mK_S(\theta_i-\theta_o)=K_AK_mK_S\varepsilon$$

where ε is the angular difference between input and output shafts. Hence

$$\varepsilon=\frac{K_F\omega_i}{K_AK_mK_S} \tag{18.28}$$

This constant error is required to give the motor torque needed to drive the output shaft against the viscous friction loading, and is called the *velocity lag*.

For an r.p.c. servo which is stabilized by negative velocity feedback in addition to viscous friction damping, the dynamic equation is

$$J\frac{d^2\theta_o}{dt^2}+K_F\frac{d\theta_o}{dt}=nK_AK_m\left\{K_S(\theta_i-\theta_o)-K_Tn\frac{d\theta_o}{dt}\right\}$$

(from eqn. (18.26)), where J is the total inertia referred to the output shaft. Under conditions of steady velocity input ($d^2\theta_o/dt^2=0$, $d\theta_o/dt=\omega_i$), the velocity lag is

$$\varepsilon=\theta_i-\theta_o=\frac{(K_F+n^2K_TK_AK_m)\omega_i}{nK_AK_mK_S}\approx\frac{nK_T\omega_i}{K_S} \tag{18.29}$$

if (as is usually the case) $K_F\ll n^2K_TK_AK_m$.

It is possible to eliminate the velocity lag in an r.p.c. servo with negative velocity feedback by arranging that the velocity feedback is removed when steady-state conditions are achieved. This is done by connecting a large capacitor in series in the velocity feedback loop. Essentially the capacitor passes any changing voltage conditions, but acts as a d.c. block to steady voltages. Thus the velocity feedback is effective only when the velocity of the output shaft is changing. This is termed *transient velocity feedback*.

EXAMPLE 18.2. An r.p.c. servo with velocity-feedback damping uses synchros as error detectors. The output of the phase-sensitive rectifier is 1·5 V/deg error and is fed to the summing junction of an operational amplifier through a 1 MΩ resistor, the feedback resistor being also 1 MΩ. The amplifier transductance is 400 mA/V, and the motor torque at standstill is 5×10^{-2} N-m when the full field current of 81 mA flows in one half of the split field and zero in the other half. The tachogenerator output is 0·3 V per rev/s and is fed through a 2 MΩ resistor to the summing amplifier, the tacho generator being on the motor shaft. The output shaft is coupled to the motor through an 80:1 reduction gear, the total inertia at this shaft being 50×10^{-2} kg-m². Determine the system damping ratio, the magnitude of the first overshoot when the input shaft is given a sudden displacement of 10°, and the velocity lag when the input shaft is rotated at 5 rev/min.

The motor torque constant is $(5 \times 10^{-2})/81 = 6{\cdot}24 \times 10^{-4}$ N-m/mA; K_T is $0{\cdot}3/2\pi$ V per rad/s; and K_S is $1{\cdot}5 \times 360/2\pi$ V/rad.

The dynamic equation of the system is

$$J\frac{d^2\theta_0}{dt^2} = nK_AK_m\left\{K_S(\theta_i - \theta_0) - \tfrac{1}{2}nK_T\frac{d\theta_0}{dt}\right\}$$

The factor of one-half is present since the tachogenerator output is fed to the summing junction through a 2 MΩ resistor. This gives

$$\frac{d^2\theta_0}{dt^2} + 76{\cdot}2\frac{d\theta_0}{dt} + 3{,}440\theta_0 = 3{,}440\theta_i$$

Comparing with eqn. (18.14*a*),

$$\omega_n = \sqrt{3{,}440} = 58{\cdot}5 \qquad \text{and} \qquad 2\zeta\omega_n = 76{\cdot}2$$

Hence the damping ratio, ζ, is $\underline{\underline{0{\cdot}65}}$.

The system is thus underdamped and the response to a step input of 10° is given by eqn. (18.18) as

$$\theta_0 = 10\{1 - e^{-38{\cdot}1t}(\cos \omega t + 0{\cdot}86 \sin \omega t)\}$$

$$= 10\left\{1 - \frac{e^{-38{\cdot}1t}}{\sqrt{(1 + 0{\cdot}86^2)}}\cos(\omega t - \tan^{-1} 0{\cdot}86)\right\}$$

where $\omega = \omega_n\sqrt{(1 - \zeta^2)} = 58{\cdot}5 \times 0{\cdot}76 = 44{\cdot}5$.

Thus θ_0 has its first maximum when $\cos(\omega t - 40{\cdot}5°) = -1$, i.e. when

$$\omega t - \frac{40{\cdot}5 \times 2\pi}{360} = \pi \qquad \text{or} \qquad t = \frac{1{\cdot}23\pi}{44{\cdot}5} = \underline{\underline{0{\cdot}087\,\text{s}}}$$

Hence

$$\theta_{0\max} = 10\{1 + e^{-3{\cdot}31{\cdot}32}\} = \underline{\underline{10{\cdot}5°}}$$

so that the magnitude of the first overshoot is $\underline{\underline{0{\cdot}5°}}$.

The velocity lag is given by eqn. (18.29) as

$$\varepsilon = \frac{nK_T\omega_i}{K_S}\ \text{rad} = \frac{80 \times 0{\cdot}3 \times 5}{1{\cdot}5 \times 60} = \underline{\underline{1{\cdot}3°}}$$

18.11 Effect of Load Torque on a Simple R.P.C. Servo

So far, position control systems where the load torque is negligible have been considered. If there is a constant load torque, the motor must supply this even at standstill, and must therefore have an input. This in turn presumes that there is an error between input and output shafts. The dynamic equation for the system of Fig. 18.11 will be

$$J\frac{d^2\theta_o}{dt^2} + T_L = nK_AK_m\left\{K_S(\theta_i - \theta_o) - nK_T\frac{d\theta_o}{dt}\right\} \tag{18.30}$$

where T_L is the constant load torque.

Under steady-state conditions when the input is set at some fixed value (θ_i = costant), $d^2\theta_o/dt^2 = d\theta_o/dt = 0$, and the expression for the error is

$$\varepsilon = \theta_i - \theta_o = \frac{T_L}{nK_AK_mK_S}\ \text{radians} \tag{18.31}$$

This error is often called the *offset*. Offset may be eliminated by altering the input signal to the servo amplifier so that it corresponds, not only to the error, but also to the error plus the time integral of the error.

18.12 Simple Speed Control—the Velodyne

The speed of the output shaft of a small servo motor can be controlled by a closed loop system similar to that used for position control. Such a system is shown in Fig. 18.12 and is commonly known as a

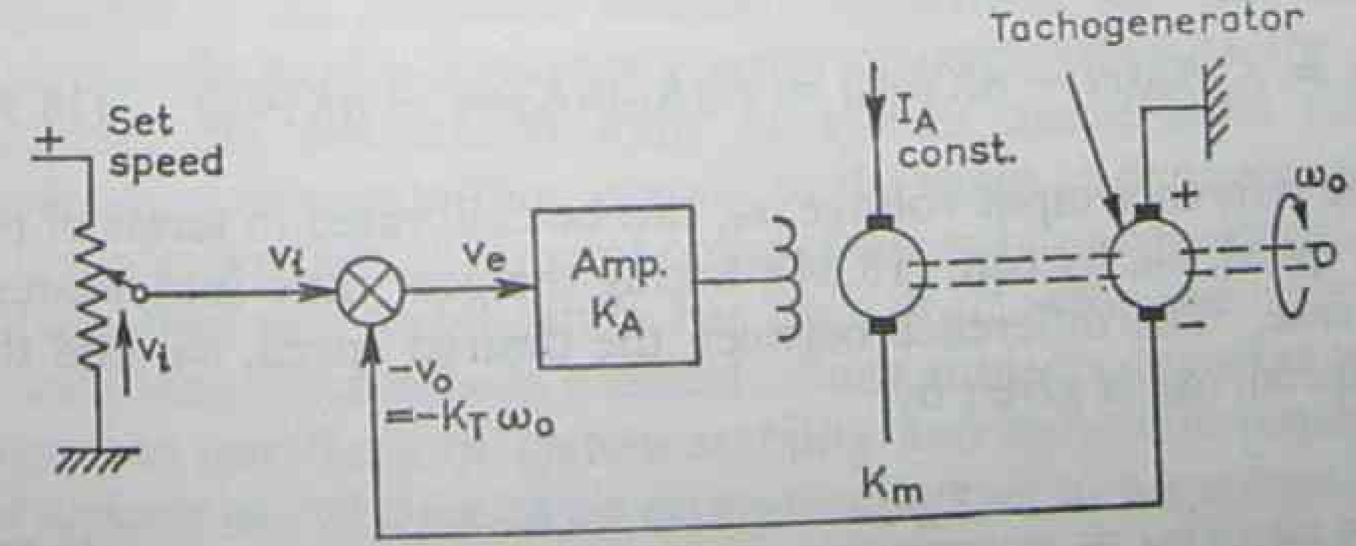

Fig. 18.12 VELODYNE SPEED CONTROL